Tutorial 8 &9

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Q1) Find Laplace Transform of the following functions.

a) $f(t) = t\sin^2 t$

by the property of multiplication by t;

$$L[t f(t)] = (-1)\frac{d}{ds}F(s)$$

$$f(t) = \sin^2 t$$

$$L[f(t)] = L[\sin^2 t] = L\left[\frac{1-\cos 2t}{2}\right] = L\left[\frac{1}{2}\right] - L\left[\frac{\cos 2t}{2}\right]$$

$$= \frac{1}{2s} - \frac{1}{2}\left[\frac{s}{s^2+4}\right] = \frac{1}{2s} - \frac{1s}{2(s^2+4)}$$

$\therefore$

$$L[t\sin^2 t] = (-1)^1 \frac{d\left[L\left[\frac{1-\cos 2t}{2}\right]\right]}{ds}$$

$$= (-1)\frac{d\left[1/2s - 1/2\left(\frac{s}{s^2+4}\right)\right]}{ds}$$

$$= \frac{(-1)}{2}\frac{d\left(1/s + s/s^2+4\right)}{ds}$$

Applying u/v differentiation Rule;

$$\Rightarrow -1/2\left[\frac{-1}{s^2} - \left[\frac{s^2+4-2s^2}{(s^2+4)^2}\right]\right]$$

$$= \frac{1}{2}\left[\frac{1}{s^2} + \frac{4-s^2}{(s^2+4)^2}\right]$$

$$F(t\sin^2 t) = \frac{6s^2+8}{s^2(s^2+4)^2}$$

b) $f(t) = e^{-t}\cos(3t+2)$

comparing the above equation with 1st shifting property, $L[e^{at}f(t)] = f(s-a)$

$a=1,\ f(t) = \cos(3t+2)$

$$L[\cos(3t+2)] = L[\cos A\cos B - \sin A\sin B]$$
$$= L[\cos 3t\cos 2 - \sin 3t\sin 2]$$
$$= \cos 2\, L[\cos 3t] - \sin 2\, L[\sin 3t]$$
$$= \cos 2\left\{\frac{s}{s^2+9}\right\} - \sin 2\left\{\frac{3}{s^2+9}\right\}$$
$$= \cos 2\left\{\frac{s}{s^2+9}\right\} - \frac{3\sin 2}{s^2+9}$$

$\therefore L[e^{-t}\cos(3t+2)] = f[s+1] \quad \because (a=-1)$

Hence,

$$L[e^{-t}\cos(3t+2)] = \left\{\frac{(s+1)}{(s+1)^2+9}\right\}\cos 2 - \frac{3\sin 2}{(s+1)^2+9}$$

$$\Rightarrow \frac{s+1}{s^2+2s+10}(\cos 2) - \frac{3\sin 2}{(s+1)^2+9}$$

c) $f(t) = \dfrac{e^{-3t}\sin 2t}{t}$

By 1st shifting property, $L[e^{at}f(t)] = f(s-a)$

$\Rightarrow f(t) = \dfrac{\sin 2t}{t} \quad a=-3$

d) $f(t) = \int_a^t e^{-2t} \sin 3t \, dt$

$$f(s) = L[e^{-2t} \sin 3t]$$

$$L[e^{-at} f(t)] = f(s+a)$$

$$f(s) = L[\sin 3t]$$

$$f(s) = \frac{3}{s^2+9}$$

$$L[e^{-2t} \sin 3t] = \frac{3}{(s+2)^2+9}$$

Now using $L\left[\int_0^t f(t)\,dt\right] = \frac{f(s)}{s}$

$$\therefore L\left[\int_0^1 e^{-2t} \sin 3t\right] = \frac{3s}{(s+2)^2+9}$$

Q2) Find inverse laplace transform of the foll-

a) $F(s) = \frac{1}{(s+4)^{3/2}}$

$$L^{-1}\left[\frac{1}{(s+4)^{3/2}}\right] = e^{-4t} L^{-1}\left[\frac{1}{s^{3/2}}\right]$$

$$= e^{-4t} t^{3/2-1}$$

$$= e^{-4t} \frac{t^{1/2}}{\Gamma 3/2} = e^{-4t} \frac{t^{1/2}}{\Gamma 3/2}$$

$$= e^{-4t} \frac{t^{1/2}}{\Gamma 1/2}$$

$$= 2e^{-4t} \frac{t^{1/2}}{\sqrt{\pi}}$$

since $3/2 = n$ we will use $L^{-1}\left[\frac{1}{s^n}\right] = \frac{\Gamma(n-)}{\Gamma n}$

$\Gamma n+1 = n\Gamma n$

$\Gamma 1/2 = \sqrt{\pi}$

b) $F(s) = \log\left(\frac{s+2}{s+1}\right) - \log(s+2) - \log(s+1)$

$$L^{-1}\left[\log(s+2) - \log(s+1)\right]$$

Let $\frac{d}{ds} f(s) = f'(s)$

Hence, $f'(s) = \left(\frac{1}{s+2} - \frac{1}{s+1}\right)$

Using $L^{-1}[f(s)] = -tf(t)$

$$f(t) = \frac{1}{t} L^{-1}[f'(s)] = \frac{1}{t} L^{-1}[F'(s)]$$

$L^{-1}\left[\frac{1}{s+2}\right] = e^{-2t}$ & $L^{-1}\left[\frac{1}{s+1}\right] = e^{-1t}$

Hence, $L[f'(s)] = e^{-2t} - e^{-t} = -tf(t)$

Hence, $f(t) = -\left(\frac{e^{-2t} - e^{-t}}{t}\right)$

c) $F(s) = \dfrac{3s-8}{4s^2+25}$

$$L^{-1}\left[\frac{3s-8}{4s^2+25}\right] = L^{-1}\left[\frac{3s-8}{4\left(s^2+\frac{25}{4}\right)}\right] = \frac{1}{4}L^{-1}\left[\frac{3s-8}{s^2+\frac{25}{4}}\right]$$

$$L^{-1}\left[\frac{3s-8}{4s^2+25}\right] = \frac{1}{4}L^{-1}\left[\frac{3s-8}{s^2+(5/2)^2}\right]$$

separating,
we get

$$\frac{1}{4}L^{-1}\left\{\frac{3s}{s^2+(5/2)^2}\right\} + \frac{1}{4}L^{-1}\left\{\frac{8}{s^2+(5/2)^2}\right\}$$

$$3/4\, L^{-1}\left[\frac{s}{s^2+(5/2)^2}\right] - \frac{8}{4}L^{-1}\left\{\frac{1}{s^2+(5/2)^2}\right\}$$

$$\left\{ \text{we know, } L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at \right.$$
$$L^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{\sin at}{a}$$

Hence,

$$\frac{3}{4}\left[\cos\left(\frac{5}{2}\right)t\right] - 2\left[\frac{\sin(5/2)t}{5/2}\right]$$

which implies;

$$L^{-1}\left[\frac{3s-8}{4s^2+25}\right] = \frac{3}{4}\cos\left(\frac{5}{2}\right)t - \frac{4}{5}\sin\left(\frac{5}{2}\right)t$$

d) $f(s) = \dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$

when $s=1$ and we substitute it in denominator we get D^r as 0

Hence we can say that $(s-1)$ is one root for find rest 2 roots.

$$
\begin{array}{r|l}
 & s^2 - 5s + 6 \\
s-1 & s^3 - 6s^2 + 11s - 6 \\
 & \underline{\overset{+}{-}\, s^3 \overset{-}{+}\, s^2} \\
 & -5s^2 + 11s - 6 \\
 & \underline{\overset{-}{+}\, 5s^2 \overset{+}{-}\, 5s} \\
 & 6s - 6 \\
 & \underline{\overset{+}{-}\, 6s \overset{-}{+}\, 6} \\
 & 0 \quad 0
\end{array}
$$

$s^2 - 5s + 6$

$s^2 - 2s - 3s + 6$

$s(s-2) - 3(s-2)$

$(s-3)(s-2)$

$(s-3)$ & $(s-2)$ are other two roots of D^r

by partial fraction;

$$f(s) = \frac{2s^2 - 6s + 5}{(s-1)(s-3)(s-2)} = \frac{A}{(s-1)} + \frac{B}{(s-3)} + \frac{C}{(s-2)}$$

$$\frac{2s^2 - 6s + 5}{(s-1)(s-3)(s-2)} = \frac{A(s-3)(s-2) + B(s-1)(s-2) + C(s-1)(s-3)}{(s-1)(s-3)(s-2)}$$

$$2s^2 - 6s + 5 = A(s-3)(s-2) + B(s-1)(s-2) + C(s-1)(s-3)$$

Putting $s=3$, $2s^2 - 6s + 5 = B(2)(1)$

$$2(9) - 18 + 5 = 2B$$

$$5 = 2B$$

$$\boxed{B = 5/2}$$

similarly $\Rightarrow A = 1/2$, $C = -1$

$$L^{-1}\left[\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right] = L^{-1}\left[\frac{1/2}{(s-1)}\right] + 1\left[\frac{5/2}{(s-3}\right]$$

$$+ L^{-1}\left[\frac{-1}{(s-2)}\right]$$

$$\Rightarrow \frac{1}{2}e^{t} + \frac{5}{2}e^{3t} - e^{-2t}$$