DETT

Tutorial-1

Page No.
Date 30/8/22

Name: Shreerang Mhatre
Roll no: 33
Batch: A2
PRN: 10322111745

Q1) $\frac{d^3y}{dx^3} + \frac{6d^2y}{dx^2} + \frac{11dy}{dx} + 6y = 0$

⇒ A. $D^3 + 6D^2 + 11D + 6 = 0$

So one root is -2

$$
\begin{array}{r|l}
 & D^2+4D+3 \\
\hline
D+2 & D^3+6D^2+11D+6 \\
 & D^3+2D^2 \\
\hline
 & 4D^2+11D \\
 & 4D^2+8D \\
\hline
 & 3D+6 \\
 & 3D+6 \\
\hline
 & 0\ 0
\end{array}
$$

$\therefore (D+2)(D^2+4D+3) = D^3+6D^2+11D+6$

$D^2+4D+3 = D^2+3D+D+3$

$= D(D+3)+1(D+3)$

$(D+2)(D+1)(D+3) = D^3+6D^2+11D+6$

So the roots are $-1, -2, -3$

$$y = C_1e^{-x} + C_2e^{-2x} + C_3e^{-3x}$$

(Q2) $\frac{d^3y}{dx^3} - \frac{4dy}{dx} = 0$

→ A.E - $D^3 - 4D = 0$

$D(D^2 - 4) = 0$

$\therefore D = 0$, $D^2 - 4 = 0$

$D^2 = 4$

$D = +2, -2$

$\therefore$ Soln: $y = C_1 + C_2 e^{2x} + C_3 e^{-2x}$

(Q3) $\frac{d^2y}{dx^2} - 5y = 0$

→ A.E → $D^2 - 5 = 0$

$D^2 = 5$

$D = \pm\sqrt{5}$

Soln: $C_1 e^{\sqrt{5}x} + C_2 e^{-\sqrt{5}x}$

(Q4) $(D^4 + 25)\, y = 0$

A.E → $D^4 + 25 = 0$

$(D^2)^2 + (5)^2 = 0$

$(D^2)^2 + 10D^2 + (5)^2 = 10D^2$

$(D^2 + 5)^2 = 10D^2$

$\therefore (D^2+5)^2 - 10D^2 = 0$

$\therefore (D^2+5+\sqrt{10}D)(D^2+5-\sqrt{10}D) = 0$

$\frac{-\sqrt{10} \pm \sqrt{10-20}}{2}$ & $\frac{\sqrt{10} \pm \sqrt{10-20}}{2}$

$\therefore \frac{-\sqrt{10} \pm \sqrt{10}i}{2}$ & $\frac{\sqrt{10} \pm \sqrt{10}i}{2}$

are the four roots.

$\frac{-\sqrt{10}}{2}(1 \pm i)$ $\quad$ $\frac{\sqrt{10}}{2}(1 \pm i)$

$-\frac{\sqrt{5}}{2}(1 \pm i)$ & $\frac{\sqrt{5}}{2}(1 \pm i)$

$$y = e^{-\sqrt{5}/2 x}\left(C_1 \cos\sqrt{\frac{5}{2}}x + C_2 \sin\sqrt{\frac{5}{2}}x\right) + e^{\frac{\sqrt{5}}{2}x}\left(C_3 \cos\sqrt{\frac{5}{2}}x + C_4 \sin\sqrt{\frac{5}{2}}x\right)$$

(Q5) $y^4 - 9y'' + 20y$

$$y - \frac{9d^2y}{dx^2} + 20y = 0$$

→ A.E → $D^4 - 9D^2 + 20 = 0$

$D-2$ is a factor by trial & error

$$\therefore D-2 \;\big|\; D^4 - 9D^2 + 20 \;\big|\; D^3+2D^2-5D-10$$

$D^4 - 2D^3$

$2D^3 - 9D^2$

$2D^3 - 4D^2$

$-5D^2 + 20$

$-5D^2 + 10D + 20$

$-10D + 20$

$-10D + 20$

$0\;0$

$$\therefore (D-2)(D^3+2D^2-5D-10)$$

$$(D+2) \;\big|\; D^3+2D^2-5D-10 \;\big|\; D^2-5$$

$D^3 + 2D^2$

$-5D - 10$

$-5D - 10$

$0\;0$

$$= (D-2)(D+2)(D^2-5)$$

$$= (D-2)(D+2)(D+\sqrt{5})(D-\sqrt{5})$$

$\therefore$ S/n — $y = C_1e^{\sqrt{5}x} + C_2e^{\sqrt{5}x} + C_3e^{2x} + C_4e^{-2x}$