

Subject - LAB C

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Unit 1. Theory of Matrices

Q 1) Find the rank of the matrices using Echelon form.

$$\textcircled{1} \quad \begin{bmatrix} 4 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

$$\Rightarrow R_2 \xrightarrow{\text{R}_2 - R_1} R_2$$

$$\begin{bmatrix} 4 & 2 & -1 & 2 \\ 0 & -3/2 & 3/2 & 1/2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

Now,

$$R_3 - \frac{R_1}{R_2} \rightarrow R_3 \quad \begin{bmatrix} 4 & 2 & -1 & 2 \\ 0 & -3/2 & 3/2 & 1/2 \\ 0 & 1 & -3/2 & 1 \end{bmatrix}$$

Now,

$$R_3 - \left(-\frac{2}{3}\right)R_2 \rightarrow R_3 \quad \begin{bmatrix} 4 & 2 & -1 & 2 \\ 0 & -3/2 & 3/2 & 1/2 \\ 0 & 0 & 0 & -2/3 \end{bmatrix}$$

Rank = 3

$$\textcircled{2} \quad \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

$$R_1 \leftrightarrow R_4$$

$$R_2 \leftrightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -2 & 0 \\ 3 & 1 & 0 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$



$$\begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$R_4 - R_2$$

$$R_3 - R_2$$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$\therefore \text{Rank}(A) = 2$

$$(3) \left[\begin{array}{cccc} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{array} \right]$$

$$R_2 - 2R_1 \rightarrow R_2$$

$$\left[\begin{array}{cccc} 2 & -2 & 0 & 6 \\ 0 & 6 & 0 & -10 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{array} \right]$$

Now,

$$R_3 - R_1/2 \rightarrow R_3$$

$$R_4 - R_1/2 \rightarrow R_4$$

$$\left[\begin{array}{cccc} 2 & -2 & 0 & 6 \\ 0 & 6 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{array} \right]$$

$$R_4 - (-\frac{1}{6})R_2 \rightarrow R_4$$

$$R_4 \leftrightarrow R_3$$

$$\left[\begin{array}{cccc} 2 & -2 & 0 & 6 \\ 0 & 6 & 0 & -10 \\ 0 & 0 & 1 & -8/3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$\therefore \text{So, Rank} = 3$

Q2) Find the rank of Matrices using normal form

$$\textcircled{1} \quad \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$R_2 - 4R_1 \rightarrow$$

$$R_3 - 3/6 R_2 \rightarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 5 & -8 & -10 \\ 0 & 0 & 29/5 & 10 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$R_4 - R_2/5 \rightarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 5 & -8 & -10 \\ 0 & 0 & 29/5 & 10 \\ 0 & 0 & 8/5 & 4 \end{bmatrix}$$

$$R_4 - 8/29 R_3 \rightarrow$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 5 & -8 & -10 \\ 0 & 0 & 29/5 & 10 \\ 0 & 0 & 0 & 36/29 \end{bmatrix}$$

$\therefore \text{Rank} = 4$

$$\textcircled{2} \quad \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$

$$R_2 - (-2)R_1 \rightarrow$$

$$R_3 - R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 8 & 5 & 0 \\ 0 & -2 & 1 & 8 \end{bmatrix}$$

$$R_3 - \left(-\frac{1}{4}\right) R_2 \rightarrow R_3$$

Rank = 3

$$\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 1 \\ 0 & 8 & 5 & 0 & 0 \\ 0 & 0 & 9 & -8 & \end{array} \right]$$

$$\textcircled{3} \quad \left[\begin{array}{ccccc} 6 & 1 & 3 & 8 & \\ 4 & 2 & 6 & -1 & \\ 10 & 3 & 9 & 7 & \\ 16 & 4 & 12 & 15 & \end{array} \right]$$

$$R_2 - \frac{2}{3}R_1$$

$$\left[\begin{array}{ccccc} 6 & 1 & 3 & 8 & \\ 0 & 4/3 & 4 & -19/3 & \\ 0 & 4/3 & 4 & -19/3 & \\ 16 & 4 & 12 & 15 & \end{array} \right]$$

$$R_4 - 8/3 R_1$$

$$\left[\begin{array}{ccccc} 6 & 1 & 3 & 8 & \\ 0 & 4/3 & 4 & -19/3 & \\ 0 & 4/3 & 4 & -19/3 & \\ 0 & 4/3 & 4 & -19/3 & \end{array} \right]$$

$$R_3 - R_2$$

$$R_4 - R_2$$

$$\left[\begin{array}{ccccc} 6 & 1 & 3 & 8 & \\ 0 & 4/3 & 4 & -19/3 & \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \end{array} \right]$$

Rank = 2

(3) Examine for consistency and if consistent then solve it.

$$\text{Given} - 4x - 2y + 6z = 8$$

$$x + y - 3z = -1$$

$$15x - 3y + 9z = 21$$

The system can be written in the matrix form as:

$$\left[\begin{array}{ccc|c} 4 & -2 & 6 & x \\ 1 & 1 & -3 & y \\ 15 & -3 & 9 & z \end{array} \right] = \left[\begin{array}{c} 8 \\ -1 \\ 21 \end{array} \right] \quad S(A) = 2$$

$$S(A, B) = 1 \Rightarrow \left[\begin{array}{ccc|c} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{array} \right] \therefore$$

$$\therefore \left[\begin{array}{ccc|c} 4 & -2 & 6 & 2 \\ 0 & 3/2 & -9/2 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

System is consistent

$S(A) = S(A, B) = 2 < 3$ possesses infinite soln.

$$4x - 2y + 6z = 8 \quad \text{then } x = 1$$

$$\frac{3y - 9z}{2} = -3 \quad z = t, y = 3t$$

Then, $x = 1$

$$y = 3t - 2$$

$$z = t$$

$$\textcircled{2} \text{ Given } - 2x + 2 = 4$$

$$x - 2y + 2z = 7$$

$$3x + 2y = 1$$

The system can be written in matrix form as

$$\left[\begin{array}{ccc|c} 2 & 0 & 1 & x \\ 1 & -2 & 2 & y \\ 3 & 2 & 0 & z \end{array} \right] = \left[\begin{array}{c} 4 \\ 7 \\ 1 \end{array} \right] \quad S(A) = 2$$

$$(A, B) = \left[\begin{array}{ccc|c} 2 & 0 & 1 & 4 \\ 1 & -2 & 2 & 7 \\ 3 & 2 & 0 & 1 \end{array} \right]$$

$$\therefore \left[\begin{array}{ccc|c} 2 & 0 & 1 & 4 \\ 0 & -2 & 3/2 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$S(A, B) = 2$ System is consistent

$$S(A) = S(A, B) = 2 \neq 3$$

So not,

$$\text{By } R_1 \rightarrow 2x + 2 = 4 \quad \text{Let } z = t \text{ then}$$

$$R_2 \rightarrow 2y + 3/2z = 5 \quad x = 4 - t/2$$

so, solve are -

$$x = t$$

$$y = -\frac{5}{2} + \frac{3t}{4}$$

$$x = \frac{4-t}{2}, y = -\frac{5}{2} + \frac{3t}{4}$$

③ Given:- $2x_1 + x_2 - 5x_3 + x_4 = 8$
 $x_1 + 3x_2 - 6x_3 = 15$
 $0x_1 + 2x_2 - x_3 + 2x_4 = -5$
 $x_1 + 4x_2 - 7x_3 + 6x_4 = 0$

The system can be written in matrix form is

$$\left[\begin{array}{cccc|c} 2 & 1 & -5 & 1 & 8 \\ 1 & 3 & 0 & -6 & -15 \\ 0 & 2 & -1 & 2 & -5 \\ 1 & 4 & -7 & 6 & 0 \end{array} \right]$$

$$f(A) = 3+1=4$$

$$(A, B) = \left[\begin{array}{cccc|c} 2 & 1 & -5 & 1 & 8 \\ 1 & 3 & 0 & -6 & -15 \\ 0 & 2 & -1 & 2 & -5 \\ 1 & 4 & -7 & 6 & 0 \end{array} \right]$$

$$= \left[\begin{array}{cccc|c} 2 & 1 & -5 & 1 & 8 \\ 0 & 5/2 & 5/2 & -13/2 & -19 \\ 0 & 0 & -3 & 36/5 & 51/5 \\ 0 & 0 & 0 & -23/5 & -23/5 \end{array} \right]$$

By matrix

$$\left[\begin{array}{cccc|c} 2 & 1 & -5 & 1 & 8 \\ 1 & 5/2 & 5/2 & -13/2 & -19 \\ 0 & 0 & -3 & 36/5 & 51/5 \\ 0 & 0 & 0 & -23/5 & -23/5 \end{array} \right]$$

$$R_1 \rightarrow 2x_1 + x_2 - 5x_3 + x_4 = 8$$

$$-3x_2 + \frac{36}{5}x_4 = \frac{51}{5}$$

$$-\frac{23}{5}x_4 = \frac{23}{5}$$

$$\underline{x_4 = 1} \quad \underline{x_3 = -1} \quad \underline{x_2 = -4} \quad \underline{x_1 = 3}$$

Q Given: $x_1 + x_2 + 2x_3 + x_4 = 5$
 $2x_1 + 3x_2 - x_3 - 2x_4 = 2$
 $4x_1 + 5x_2 + 3x_3 = 7$

The system can be written in matrix form as

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & x_1 \\ 2 & 3 & -1 & -2 & x_2 \\ 4 & 5 & 3 & 0 & x_3 \\ \hline & & & & x_4 \end{array} \right] = \left[\begin{array}{c} 5 \\ 2 \\ 7 \\ \hline \end{array} \right]$$

Hence the system is inconsistent as $P(A) \neq P(A, B)$

Q 4) Investigate for what values of a do the system of simultaneous

$$2x - y + 3z = 2$$

$$x + y + 2z = 2$$

$$5x - y + az = b$$

→

① No Solution

$$a=8, b=6$$

$$P(A) \neq P(A, B)$$

In matrix form

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 1 & 1 & 2 & 2 \\ 5 & -1 & a & b \end{array} \right]$$

by solving

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 2 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & a-8 & b-6 \end{array} \right]$$

$$\text{so } P(A) = P(A, B)$$

$$a=8 \rightarrow 3$$

$$a=8, b=6 \rightarrow 2$$

③ Infinite Solution

$$a=8, b=6$$

(5)

Investigate for what values of k the equations have infinite number of solutions. Hence, find solutions.

Given $x + y + z = 1$

$$2x + y + kz = k$$

$$4x + y + 10z = k^2$$

for infinite soln: $P(A) = P(A, B) \Rightarrow \leq 3$
i.e.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 2 & 1 & 4 & y \\ 4 & 1 & 10 & z \end{array} \right] \left[\begin{array}{c} x \\ y \\ z \end{array} \right] = \left[\begin{array}{c} 1 \\ k \\ k^2 \end{array} \right] Ax = B$$

so, By solving we get.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & k \\ 4 & 1 & 10 & k^2 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & k^2-3k+2 \end{array} \right]$$

$$\therefore k^2 - 3k + 2 \neq 0 \rightarrow \text{no soln}$$

$$\& k^2 - 3k + 2 = 0 \rightarrow P(2)$$

$$\text{so, } \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\therefore \boxed{k=1, 2}$$

Q6) Examine for linear dependence or independence the following system of vectors, If dependent, find relation b/w them.

① Given $x_1 = (1, -1, 1)$, $x_2 = (2, 1, 1)$, $x_3 = (3, 0, 2)$

$$c_1x_1 + c_2x_2 + c_3x_3 = 0$$

$$(c_1(1, -1, 1) + c_2(2, 1, 1) + c_3(3, 0, 2)) = 0$$

$$\Leftrightarrow c_1 + 2c_2 + 3c_3 = 0$$

$$-c_1 + c_2 + 0 = 0$$

$$c_1 + c_2 + 2c_3 = 0$$

In matrix form $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$P(A) = P(A, 2) = 2 = 2 \times 3 \therefore \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

System posses a non-finite soln.

$$c_1 + 2c_2 + 3c_3 = 0$$

$$3c_2 + 3c_3 = 0$$

$$c_2 - c_1 = 0$$

$$c_2 = c_1 = t$$

$$c_3 = -t$$

c_1, c_2, c_3 are non-zero so mostly dependent.

$$c_1 + tc_2 + tc_3 = 0$$

$$\therefore \boxed{c_1 + c_2 = c_3}$$

② Given $x_1 = (1, 1, 1, 3)$, $x_2 = (1, 2, 3, 4)$, $x_3 = (2, 3, 4, 7)$
 $c_1x_1 + c_2x_2 + c_3x_3 = 0$

$$c_1(1, 1, 1, 3) + c_2(1, 2, 3, 4) + c_3(2, 3, 4, 7)$$

$$c_1 + c_2 + 2c_3 = 0$$

$$c_1 + 2c_2 + 3c_3 = 0$$

$$c_1 + 3c_2 + 4c_3 = 0$$

$$3c_1 + 4c_2 + 7c_3 = 0$$

In matrix form

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \\ 3 & 4 & 7 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Here

$$P(A) = P(A, 2) = 2 < 3$$

System possesses non-trivial soln.

$$\text{so, } c_1 + c_2 + 2c_3 = 0$$

$$c_2 + c_3 = 0$$

$$c_1 - c_3 = 0$$

$$c_1 = (3 = t)$$

$$7x_1 + 4x_2 - 4x_3 = 0$$

$$\boxed{x_1 + x_2 = x_3}$$

③ $x_1 = (3, 1, -4)$, $x_2 = (2, 2, -3)$, $x_3 = (0, -4, 1)$
 consider the matrix form

$$c_1x_1 + c_2x_2 + c_3x_3 = 0$$

$$c_1(3, 1, -4) + c_2(2, 2, -3) + c_3(0, -4, 1)$$

$$3c_1 + 2c_2 + 0c_3 = 0$$

$$c_1 + 2c_2 - 4c_3 = 0$$

$$-4c_1 - 3c_2 + c_3 = 0$$

In matrix form

$$\left[\begin{array}{ccc|c} 3 & 2 & 0 & c_1 \\ 1 & 2 & -4 & c_2 \\ -4 & -3 & 1 & c_3 \end{array} \right] \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right]$$

$$\text{So, } P(A) = P(A, 2) = 2 \left[\begin{array}{ccc|c} 3 & 2 & 0 & 0 \\ 0 & 4 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

System possesses a non trivial soln.

$$\text{So, } 3c_1 + 2c_2 + 0c_3 = 0$$

$$c_1 + 2c_2 - 4c_3 = 0$$

$$\text{So, } 2t_1 - 3t_2 - t_3 = 0$$

$$2x_1 = 3x_2 + x_3$$

\therefore Ans is $2x_1 \neq 3x_2 + x_3$

$$(4) \quad x_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, x_2 = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}, x_3 = \begin{pmatrix} 1 \\ -6 \\ -6 \end{pmatrix}$$

consider the matrix eqn $c_1x_1 + c_2x_2 + c_3x_3 = 0$

$$c_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ -6 \\ 5 \end{pmatrix} = 0$$

$$c_1 + 3c_2 + c_3 = 0$$

$$2c_1 - 2c_2 - 6c_3 = 0$$

$$3c_1 + c_2 - 5c_3 = 0$$

In matrix form

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & c_1 \\ 2 & -2 & -6 & c_2 \\ 3 & 1 & -5 & c_3 \end{array} \right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right]$$

$$\text{So, } P(A) = P(A, 2) = 2 \times 3$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Here

System posses non-trivial soln.

$$c_1 + 3c_2 + c_3 = 0$$

$$-8c_2 - 8c_3 = 0$$

$$c_2 + c_3 = 0$$

$$\text{So, } -2x_1 + x_2 - x_3 = 0$$

$$2x_1 + x_3 = x_3$$

(Q7) Find the coordinates (x_1, x_2, x_3) corresponding to $(2, 3, 0)$ in \mathcal{Y} . For

$$\rightarrow \text{Transformation} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -3 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Y = Ax \text{ i.e. } Ax = Y$$

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & -3 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

This is non-homogeneous system of eqn

$$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 0 & -3 \\ -1 & 1 & 1 \end{vmatrix} |A| = 19$$

$$|A| = 19 \neq 0$$

System possesses a unique soln $x = A^{-1}y$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$\text{So, } A^{-1} = \begin{vmatrix} 3/19 & 5/19 & 6/19 \\ -5/19 & -2/19 & 9/19 \\ 2/19 & -3/19 & 4/19 \end{vmatrix}$$

$$x = \begin{vmatrix} 3/19 & 5/19 & 6/19 & 2 \\ -5/19 & -2/19 & 9/19 & 9 \\ 2/19 & -3/19 & 4/19 & 0 \end{vmatrix} = \begin{vmatrix} 21/19 \\ -16/19 \\ -9/19 \end{vmatrix}$$

$$\text{So, } x_1 = \frac{21}{19}, x_2 = \frac{-16}{19}, x_3 = \frac{-9}{19}$$

- (B) Express each of the transformation $x_1 = 3y_1 + 5y_2$
 $x_2 = -y_1 + 2y_2$
 $y_1 = z_1 + 3z_2$
 $y_2 = 4z_2$

In the matrix form and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2 .

- The transformation $x_1 = 5y_1 + 5y_2$ } in the
matrix form can $x_2 = -y_1 + 2y_2$ }
be written as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$x = Ay$$

Also the transformation $y_1 = z_1 + 3z_2$
 $y_2 = 4z_2$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad y = Bz$$

∴ Required composite transformation.

$$x = Ay = \begin{bmatrix} 3 & 6 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

so

$$= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 23 & 9 \\ 27 & -3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$x_1 = 23z_1 + 9z_2$$

$x_2 = 27z_1 - 3z_2$ is the required
transformation.

Ques a) Verify whether the following matrices are orthogonal or not, if so write A^{-1} :

$$\textcircled{1} \quad A = \begin{bmatrix} \sqrt{3} & 0 & 2\sqrt{3} \\ \sqrt{3} & \sqrt{2} & -\sqrt{3} \\ -\sqrt{3} & \sqrt{2} & \sqrt{3} \end{bmatrix}$$

If A is orthogonal then $AA^{-1} = I$

$$AA^{-1} = \begin{bmatrix} \sqrt{3} & 0 & 2\sqrt{3} \\ \sqrt{3} & \sqrt{2} & -\sqrt{3} \\ -\sqrt{3} & \sqrt{2} & \sqrt{3} \end{bmatrix} \begin{bmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{2} & \sqrt{2} \\ 2\sqrt{3} & -\sqrt{3} & \sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \therefore A \text{ is Orthogonal}$$

$$\textcircled{2} \quad A = \frac{1}{3} \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix} \quad \text{If } A \text{ is orthogonal}$$

$$\text{then } AA^{-1} = I$$

$$= \frac{1}{3} \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -1 & 2 \end{bmatrix} \times \frac{1}{3} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2/5 \\ 0 & 1 & 1/5 \\ 2/5 & 1/5 & 2/3 \end{bmatrix} \quad \text{So, } AA^{-1} = I$$

Then A is not orthogonal.

$$A^{-1} = \frac{1}{7} \begin{bmatrix} -4 & -5 & 3 \\ 6 & 3 & -6 \\ 1 & 4 & 6 \end{bmatrix}$$

Q10) If $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal, find a, b, c

→ Here A is orthogonal
 $\therefore AA^{-1} = I$

$$AA^{-1} = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ a & b & c \end{bmatrix} \cdot \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & a+2b+2c/3 \\ 0 & 1 & 2a+b-2c/3 \\ a+2b+2c & 2a+b-2c & a^2+b^2+c^2 \end{bmatrix}$$

out $AA^{-1} = I$

$$\frac{a+2b+2c}{3} = 1 \Rightarrow \frac{a+2b}{3} = \frac{-2c}{3}$$

$$\frac{2a+b-2c}{3} = 1 \quad a+2b+2c = 3$$

$$4a^2 + 4b^2 - 8ab + c^2 + 4ac - 4b = 1$$

$$\therefore \boxed{a = \pm \frac{2}{3}, b = \pm \frac{2}{3}, c = \pm \frac{1}{3}}$$

(Q 12) Verify Cayley-Hamilton theorem for the following matrix and use it find inverse:-

$$\textcircled{1} \quad A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \quad |A - \lambda I| = 0 \quad \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)((2-\lambda)^2-1) + 1(-2+\lambda+1) + 1(-2+3) = 0$$

$$-x^3 + 6x^2 - 9x + 4 = 0$$

$$\text{So, } A^3 - 6A^2 + 9A - 4I_3 = 0$$

$$A^2 = AXA = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{pmatrix} 22 & -21 & 22 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} \quad A^3 - 6A^2 + 9A - 4I_3$$

$$\begin{bmatrix} 22 & -21 & 22 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} \quad \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/6 & 5/6 & 1/2 \\ -1/6 & 1/6 & 1/2 \end{bmatrix}$$

$$\textcircled{2} \quad A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \quad |A - \lambda I| = 0$$

$$\begin{bmatrix} 1-\lambda & 0 & 2 \\ 2 & 2-\lambda & 4 \\ 0 & 0 & 2-\lambda \end{bmatrix} = 0 \quad -\lambda^3 + 5\lambda^2 - 3\lambda + 4 = 0$$

From this $\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 6 \\ 6 & 4 & 20 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 14 \\ 14 & 8 & 68 \\ 0 & 0 & 8 \end{bmatrix} \quad \text{so, } A^3 - 5A^2 + 8A - 4 = 0$$

$$\begin{bmatrix} 1 & 0 & 14 \\ 14 & 8 & 68 \\ 0 & 0 & 8 \end{bmatrix} \xrightarrow{-5} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 0 & 4 \end{bmatrix} \xrightarrow{+8} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{-9} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{So, } A^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}^{-1} = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$$



(Q 13) Find A^4 with the help of Cayley Hamilton theorem.

→ The characteristic eqn of A is $|A - I\lambda| = 0$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} \text{ so } |A - I\lambda| = \begin{bmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{bmatrix}$$

$$\lambda^3 - 5\lambda^2 + 5\lambda - |A| = 0$$

$$S_1 = 1 + 2 + 3 = 6$$

$$S_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$

$$S_2 = 4 + 5 + 2 = 11$$

$$|A| = 6$$

Hence, the characteristic of $\text{eqn of } A = \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$
 $-3 = 0$, By using Cayley Hamilton Thm.

$$A^3 - 6A^2 + 11A - 6 = 0$$

$$\text{so, } A^3 - 6A^2 + 11A - 6 = 0.$$

$$\text{so, } A^2 = \begin{bmatrix} -1 & -2 & -4 \\ 5 & 6 & 4 \\ 10 & 10 & 9 \end{bmatrix} \quad A^3 = \begin{bmatrix} -11 & -12 & -13 \\ 19 & 20 & 13 \\ 28 & 38 & 27 \end{bmatrix}$$

$$A^4 = 6A^3 - 11A^2 + 6A$$

$$A^4 = \begin{bmatrix} -49 & -50 & -40 \\ 65 & 66 & 40 \\ 130 & 130 & 81 \end{bmatrix}$$

→

(Q14) If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ then express

$$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$$
 in terms of A .

$$\rightarrow A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$\text{then } A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I \text{ in terms of } A$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix}$$

$$\text{then } A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I = 0$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix} |A - \lambda I| = \lambda^2 - 4\lambda - 5 = 0$$

Now, we substitute the matrix (A) in the characteristics of λ in accordance to cayley hamilton theorem.

$$A^2 - 4A - 5I = 0$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 13 & 17 \end{bmatrix}$$

$$4 \times A(4A) = 4 \times \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix}$$

$$5I = 5 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$



$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

\therefore Cayley Hamilton theorem has been verified for the given matrix A .

Now the given polynomial is

$$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$$

Let us divide this by the term obtained from Cayley Hamilton theorem.

$$\text{Remainder} = (A+5I)$$

$$\text{Quotient} = A^2 - 2A + 3I$$

$$\text{Thus, polynomial} = (A^2 - 2A + 3I) \times (A^2 - 4A - 5I) + (A+5I)$$

$$\therefore \text{we know that } (A^2 - 4A - 5I) = 0$$

So, now we can express the poly as

$$(A^2 - 2A + 3I) \times 0 + (A+5I) = A+5I.$$

\therefore The reactive solution is

$$\boxed{A+5I}$$

$$\boxed{\frac{A^2}{A+5I} = \frac{1}{5} \times \frac{A^2 - 10I}{A+5I}}$$