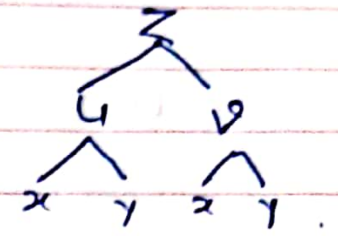
e.g. ① If $z = f(u, v)$ where $u = x^2 - 2xy - y^2$ & $v = y$ show that

$$(x+y)\frac{\partial z}{\partial x} + (x-y)\frac{\partial z}{\partial y} = (x-y)\frac{\partial z}{\partial v}$$

$\rightarrow$ $z \rightsquigarrow u, v \rightsquigarrow x, y$



$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x}.$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}(2x - 2y)$$

$$(x+y)\frac{\partial z}{\partial x} = 2(x^2 - y^2)\frac{\partial z}{\partial u} \quad \text{— ①}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}(-2x - 2y) + \frac{\partial z}{\partial v}.$$

$$(x-y)\frac{\partial z}{\partial y} = -2(x^2 - y^2) + (x-y)\frac{\partial z}{\partial v} \quad \text{— ⑪}$$

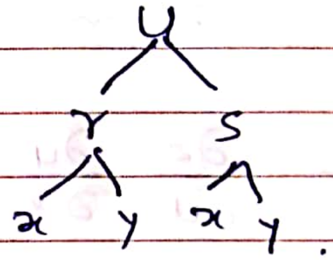
① + ⑪ gives.

$$(x+y)\frac{\partial z}{\partial x} + (x-y)\frac{\partial z}{\partial y} = (x-y)\frac{\partial z}{\partial v}.$$

2) IF $u = f(r,s)$ where $r = x^2 + y^2$, $s = x^2 - y^2$ then show that

$$y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 4xy \frac{\partial u}{\partial r}.$$

$\rightarrow$ $u \rightsquigarrow r, s \rightsquigarrow x, y$



$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial x}$$

$$= \frac{\partial u}{\partial r}(2x) + \frac{\partial u}{\partial s}(2x)$$

$$y\frac{\partial u}{\partial x} = 2xy\frac{\partial u}{\partial r} + 2xy\frac{\partial u}{\partial s} \quad \text{--- (I)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial u}{\partial s}\frac{\partial s}{\partial y}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}(2y) + \frac{\partial u}{\partial s}(-2y)$$

$$x\frac{\partial u}{\partial y} = 2xy\frac{\partial u}{\partial r} - 2xy\frac{\partial u}{\partial s} \quad \text{--- (II)}$$

(I) + (II) gives

$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 4xy\frac{\partial u}{\partial r}.$$

H.W ① If $u = x^2 - y^2$, $v = 2xy$

& $z = f(u, v)$ then show that

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2 \sqrt{u^2 + v^2} \frac{\partial z}{\partial u}.$$

② If $u = f(x-y, y-z, z-x)$ then

Find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$