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Tutorial 12 & 13

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Q1) $f(x) = 2e^{-5x} + 5e^{-2x}$

→ Using $F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\,du$

$$= \int_0^\infty (2e^{-5u} + 5e^{-u})\cos\lambda u\,du$$

$$= \int_0^\infty 2e^{-5u}\cos\lambda u\,du + \int_0^\infty 5e^{-2u}\cos\lambda u\,du$$

$$= \left[-\left(\frac{-10}{25+\lambda^2} - \frac{10}{4+\lambda^2}\right)\right]$$

$$F_c(\lambda) = \frac{10}{25+\lambda^2} + \frac{10}{4+\lambda^2}$$

Q2) $f(x) = x^2 \quad 0 \leq x \leq 1$
$= 0 \quad x > 1$

→ Using $F_s(\lambda) = \int_0^\infty f(u)\sin\lambda u\,du$

$$= \int_0^1 u^2 \sin\lambda u\,du + \int_1^\infty 0$$

$$= \left[u^2\left(\frac{-\cos\lambda u}{\lambda}\right) - 2u\left(\frac{-\sin\lambda u}{\lambda^2}\right) + 2\left(\frac{\cos\lambda u}{\lambda^3}\right)\right]_0^1$$

$$= \left[\frac{-\cos\lambda}{\lambda} + \frac{2\sin\lambda}{\lambda^2} + \frac{2\cos\lambda}{\lambda^3} - \frac{2}{\lambda^3}\right]$$

$$F_c(\lambda) = \frac{2\cos\lambda - 2}{\lambda^3} + \frac{2\sin\lambda}{\lambda^2} - \frac{\cos\lambda}{\lambda}$$

Q5) If $f(x) = 1-x^2 \quad |x|<1$
$= 0 \quad |x|>1$

S.T $F_c(\lambda) = 2\left(\frac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\right)$

ea. Evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}\,dx = \frac{-3\pi}{16}$

$\rightarrow$ $F_c(\lambda) = \int_0^\infty f(u)\cos\lambda u\,du$

$= \int_0^1 (1-u^2)\cos\lambda u\,du + \int_1^\infty 0$

$= \int_0^1 \cos\lambda u\,du - \int_0^1 u^2\cos\lambda u\,du$

$= \left[\frac{\sin\lambda u}{\lambda}\right]_0^1 - \left[\frac{u^2\sin\lambda u}{\lambda} + \frac{2u\cos\lambda u}{\lambda^2} - \frac{2\sin\lambda u}{\lambda^3}\right]_0^1$

$= \frac{\sin\lambda}{\lambda} - \left[\frac{\sin\lambda}{\lambda} + \frac{2\cos\lambda}{\lambda^2} - \frac{2\sin\lambda}{\lambda^3}\right]$

$F_c(\lambda) = 2\left(\frac{\sin\lambda - \lambda\cos\lambda}{2}\right)$

Using inverse F cosine trans

$f(x) = \frac{2}{\pi}\int_0^\infty F_c(\lambda)\cos\lambda x\,d\lambda$

$\begin{matrix} 1-x^2 & |x|<1 \\ 0 & |x|>1 \end{matrix} = \frac{2\times 2}{\pi}\int_0^\infty \frac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\cos\lambda x\,d\lambda$

$\therefore$ Put $x = \frac{1}{2}$

$$1-\left(\frac{1}{2}\right)^2 = \frac{4}{\pi}\int_0^\infty \frac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\cos\frac{\lambda}{2}\,d\lambda$$

$$\frac{3}{4} = \frac{4}{\pi}\int_0^\infty \frac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\cos\frac{\lambda}{2}\,d\lambda$$

$$\frac{3\pi}{16} = \int_0^\infty \frac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\cos\frac{\lambda}{2}\,d\lambda$$

$$\therefore \frac{-3\pi}{16} = \int_0^\infty \frac{\lambda\cos\lambda - \sin\lambda}{\lambda^3}\cos\frac{\lambda}{2}\,d\lambda$$