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Q1)

① Find $\frac{dy}{dx}$ if $x^y + y^x = a^b$

→ Let $u = x^y$; $v = y^x$

$$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx} = 0$$

$$\frac{du}{dx} = \frac{d(x^y)}{dx}$$

$$= u\left(\frac{y}{x} + \frac{dy}{dx}\log x\right)$$

$$= x^{y-1}y + x^y \log x \frac{dy}{dx} \quad \text{— ①}$$

$$\frac{dy}{dx} = \frac{d(y^x)}{dx}$$

$$= u\left(\log y + \frac{x}{y}\frac{dy}{dx}\right)$$

$$= y^x \log y + y^{x-1} x \frac{dy}{dx} \quad \text{— ②}$$

$$\frac{dy}{dx} + \frac{dy}{dx} = x^{y-1}y + x^y \log x \frac{dy}{dx} + y^x \log y + y^{x-1} x \frac{dy}{dx}$$

$$0 = x^{y-1}y - (x^y \log x + y^{x-1}x)\frac{dy}{dx} + y^x \log y$$

$$\frac{dy}{dx} = -\left(\frac{x^{y-1}y + y^x \log y}{x^y \log x + y^{x-1} x}\right)$$

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(2) Find all stationary points of the function $x^3+3xy^2-15x^2-15y^2+72x$.

Ans → $f(x,y) = x^3+3xy^2-15x^2-15y+72x$

$f_x = 3x^2+3y^2-30x+72=0 \Rightarrow x^2+y^2-10x+24=0$ — (1)
$f_y = 6xy-30y=0 \Rightarrow y(x-5)=0$ — (2)

from eqn (2) we get
$y=0$ & $x=5$
correspond to $y=0$ value of x
put $y=0$ in eqn (1)
$x^2-10x+24=0$
$x^2-6x-4x+24=0$
$x(x-6)-4(x-6)=0$
$(x-6)(x-4)=0$
$x=4, 6$

correspond to $x=5$ value of y
put $x=5$ in eqn (1)
$25+y^2-50+24=0$
$y^2-1=0$
$y^2=1$
$y=\pm 1$

Hence, critical pts on stationary points are $(4,0)$, $(6,0)$; $(5,1)$; $(5,-1)$.

③ The focal length of the mirror is found from $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$ find % error in f if u & v are both have p% error

Ans- $\frac{du}{u} \times 100 = p$ $\frac{dv}{v} \times 100 = p$ $\frac{df}{f} \times 100 = P$

$$\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$$

differentiate this we get

$$-\frac{1}{v^2} \cdot dv + \frac{1}{u^2} \cdot du = \frac{-2}{f^2} df$$

Multiply it by 100, we get

$$-\frac{1}{v}\left(\frac{dv}{v} \times 100\right) + \frac{1}{u}\left(\frac{du}{u} \times 100\right) = \frac{-2}{f}\left(\frac{df}{f} \times 100\right)$$

$$-\frac{1}{v} p + \frac{1}{u} p = \frac{-2}{f}\left(\frac{df}{f} \times 100\right)$$

$$p\left(\frac{1}{u} - \frac{1}{v}\right) = \frac{-2}{f}\left(\frac{df}{f} \times 100\right)$$

$$p\left(-\frac{\cancel{2}}{\cancel{f}}\right) = -\frac{\cancel{2}}{\cancel{f}}\left(\frac{df}{f} \times 100\right)$$

$$\frac{df}{f} \times 100 = p$$

error in f is p%

Q2)

① If $u = x+y+z$ $v = x^2+y^2+z^2$ & $w = x^3+y^3+z^3-3xyz$ are fonctionally dependent then the relation between them is

Ans -> $u = x+y+z$

$v = x^2+y^2+z^2$

$w = x^3+y^3+z^3-3xyz$

These are fonctionally dependent

$$x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

$$= \frac{1}{2}(x+y+z)\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]$$

$$= \frac{1}{2}(x+y+z)\left[2(x^2+y^2+z^2)+(x+y+z)^2 - y(xy+yz+xz)\right]$$

$$= \frac{1}{2}(x+y+z)\left[2(x^2+y^2+z^2)+(x+y+z)^2 - 2\{(x+y+z)^2-x^2-y^2-z^2\}\right]$$

$$= \frac{1}{2}u\left[2v+u^2-2(u^2-v)\right]$$

$$w = \frac{1}{2}u\left[2v+u^2-2u^2+2v\right]$$

$$w = \frac{1}{2}u\left[4v-u^2\right]$$

Relation btw u, v, w

$$2w = u(4v-u^2)$$

(3) If $x = u^2 - v^2$ $y = uv$ Find $\frac{\partial(u,v)}{\partial(x,y)} = \ldots$

Ans→ $x = u^2 - v^2$
$y = uv$

$$\frac{\partial(u,v)}{\partial(x,y)} = J$$

$$J \cdot J' = 1$$

$$J' = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix}$$

$$= 2u^2 + 2v^2$$

$$J \cdot J' = 1$$

$$J = \frac{1}{2(v^2+u^2)}$$

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{2(v^2+u^2)}$$

(4) The statement "using Lagrange's method we can find only extreme values" is true or false?

→ True

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(3) If $f(x,y) = (50 - x^2 - y^2)^{1/2}$ then find approximate value of $f(2.9, 4.1)$

Ans → $f(x,y) = (50 - x^2 - y^2)^{1/2} = z$

$x = 2.9$; $y = 4.1$

$x = x + dx$ $\quad y = y + dy$

$= 2.0 + 0.9$ $\quad = 4.0 + 0.1$

$$dz = dx \cdot \frac{df}{dx} + dy \cdot \frac{df}{dy}$$

$$dz = (0.9)\left(\frac{1}{2}\frac{(-2x)}{(50 - x^2 - y^2)^{1/2}}\right) + (0.1)\left(\frac{1}{2}\frac{(-2y)}{(50 - x^2 - y^2)^{1/2}}\right)$$

Put $x = 2$ & $y = 4$

$$dz = (0.9)\left(\frac{1}{2}\frac{(-4)}{(50 - 4 - 16)^{1/2}}\right) + (0.1)\left(\frac{-8}{2(50 - 4 - 16)^{1/2}}\right)$$

$$= 0.9\left(\frac{-2}{5}\right) + (0.1)\left(\frac{-4}{5}\right)$$

$$= \frac{-1.8 - 0.4}{5}$$

$$= \frac{-2.2}{5} = -0.44$$

$$f(2,4) = (50 - 4 - 16)^{1/2} = 5$$

$$f(2.9, 4.1) = 5 - 0.44$$

$$= 4.56$$