Unit 2.

Network Theorems.

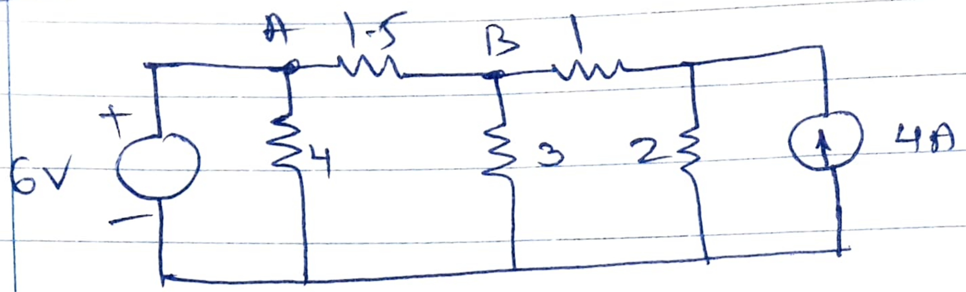
Superposition theorem.

KCL, KVL, mesh, nodal analysis give the general methods of n/w analysis. But many of the n/w problems involve only restricted analysis like - finding current in an element for diff. values of resistances, establishing condition ~~for~~ max. power, etc. Although general methods may be used here, but ~~the~~ ~~results~~ ~~w~~ it may take time or involve lot of calculations. But using n/w theorems makes the job easy.

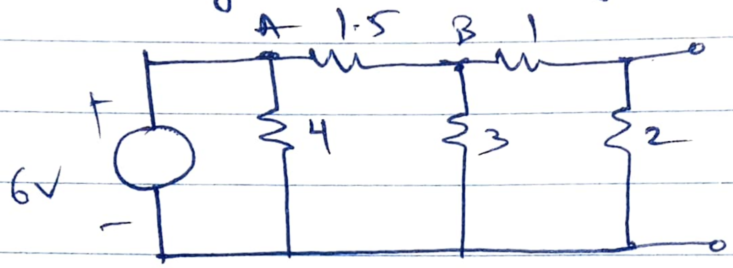
Statement - In any multisource linear bilateral n/w voltage across or current through any given element is algebraic sum of individual voltage or current considering one source at a time and replacing all remaining sources (only independent) by their internal resistances if any.

Examples.

1.



Only 6V acting

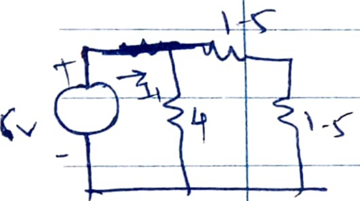


$3 \| 3 = 1.5$

$1.5 + 1.5 = 3$

$3 \| 4 = \frac{12}{7} = 1.714$

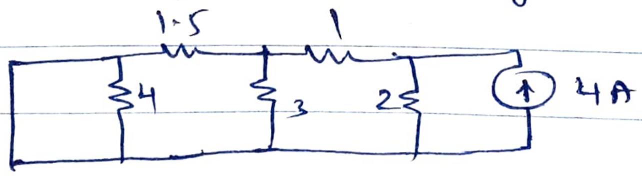
$\therefore I_1 = 6/1.714 = 3.5A$



$I_1' = 3.5 \times \frac{4}{4+1.5+1.5}$

$= 2A \rightarrow$

Only 4A acting



4A becomes redundant

$1.5 \| 3 = 1$

$I_2 = 4A$

$I_2' = 4 \times \frac{2}{2+1+1}$

$= 2A$

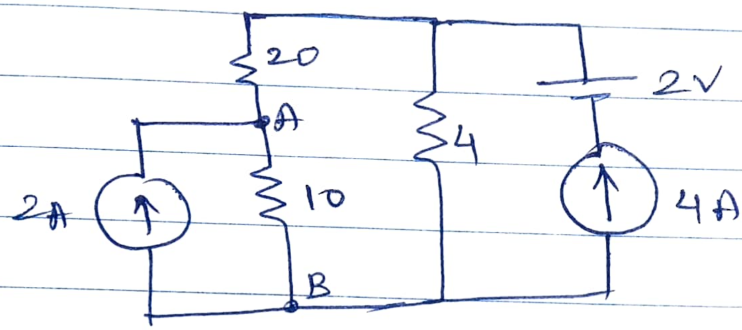
$I_2'' = 2 \times \frac{3}{3+1.5} = 1.33 A \leftarrow$

www.mitwpu.edu

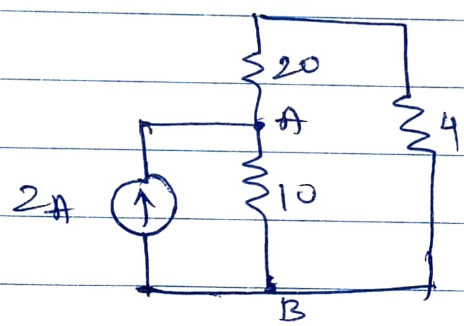
$\therefore I_{AB} = I_1' - I_2''$

$= 2 - 1.33 \quad = 0.667 A \rightarrow$

2. Find V_{AB} using superposition.



I Only 2A acting



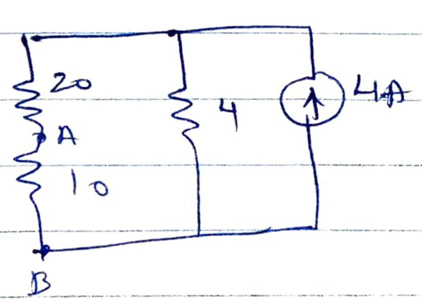
$I_{AB}' = 2 \times \frac{24}{24+10}$

$= 1.41 A \downarrow$

II Only 2 V acting;
2A & 4A open ckt

$\therefore I_{AB}'' = 0$

III Only 4A acting



$I_{AB}''' = 4 \times \frac{4}{4+20+10}$

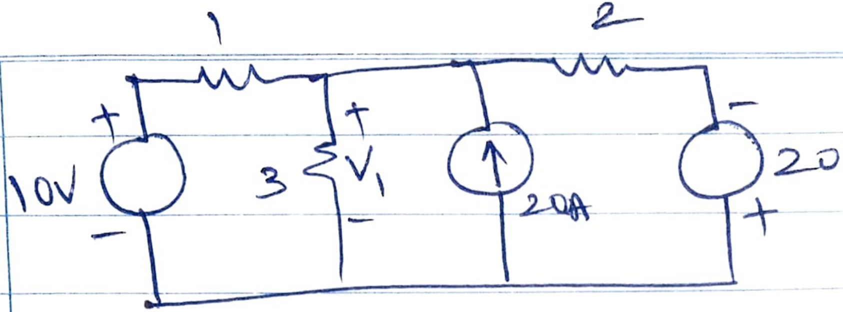
$= 0.47 \downarrow$

$\therefore I_{AB} = 1.41 + 0 + 0.47$

$= 1.88$

$\therefore V_{AB} = I_{AB} \times 10 = 18.8 V.$

3.



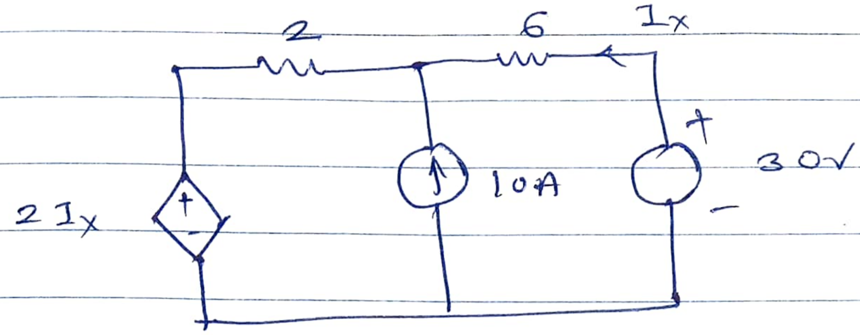
$I_1' = 1.81$ A ↓

$I_2' = 3.63$ A ↓

$I_3' = 1.81$ A ↑

$\therefore V_1 = 10.9$ V Ans

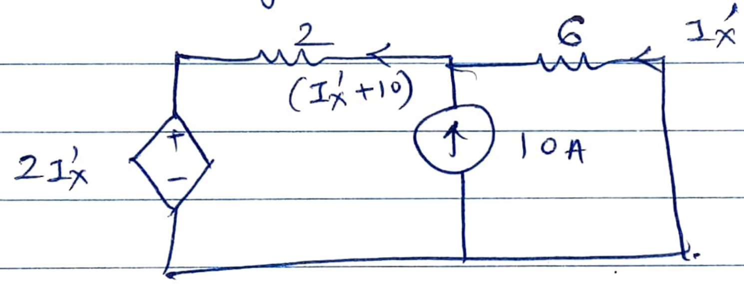
With dependent sources.



Find I_x

Dependent sources to be kept as it is

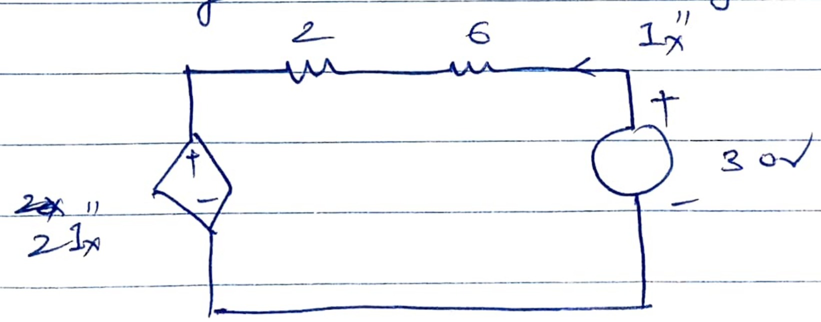
Only 10A acting



$$-6I_x' - 2(I_x'+10) - 2I_x' = 0$$

$$\therefore I_x' = -2$$

Only 30 V acting

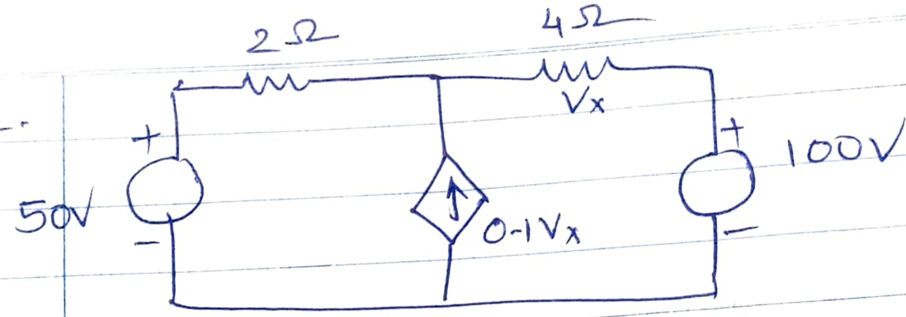


$$-6I_x'' - 2I_x'' - 2I_x'' + 30 = 0$$

$$\therefore I_x'' = \cancel{30}\ 3A$$

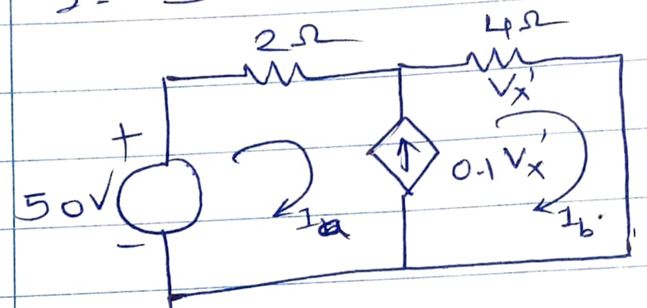
$$\therefore I_x = I_x' + I_x'' = -2 + 3 = 1A$$

2.



Use superposition th^m to evaluate Vx.

I. 50 V alone



$$I_b - I_a = 0.1V_x' \quad \text{——} \ (1)$$

Applying KVL to outer loop.

$$-2I_a - \cancel{4}4I_b + 50 = 0$$

$$I_a + 2I_b = 25 \quad \text{——} \ (2)$$

and $V_x' = -4I_b$ -- frm ckt diag ——(3)

$$\therefore \ I_b - I_a = 0.1V_x'$$

$$= 0.1(-4I_b)$$

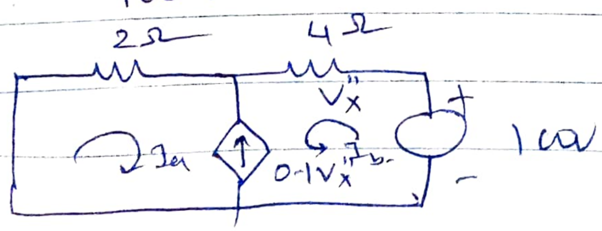
$$-I_a + 1.4I_b = 0 \quad \text{——} \ (4)$$

Add 2 & 4

$$I_b = 7.3529\,A$$

$$\therefore V_x' = -4I_b = -29.4117\,V$$ -- due to 50 V only

II 100 V alone



$$I_a + I_b = -0.1 V_x'' \quad \text{——— (5)}$$

Apply KVL to outer loop.

$$-2I_a + 4I_b - 100 = 0$$

$$-I_a + 2I_b = 50 \quad \text{——— (6)}$$

also $V_x'' = 4I_b$ ——— (7)

$$\therefore I_a + I_b = -0.1(4I_b)$$

$$I_a + 1.4I_b = 0 \quad \text{——— (8)}$$

Add (6) & (8),

$$I_b = 14.7058 \text{ A}.$$

$$V_x'' = 4I_b$$

$$= 58.8235 \text{ V}.$$

$$\therefore V_x = V_x' + V_x''$$

$$= -29.4117 + 58.8235$$

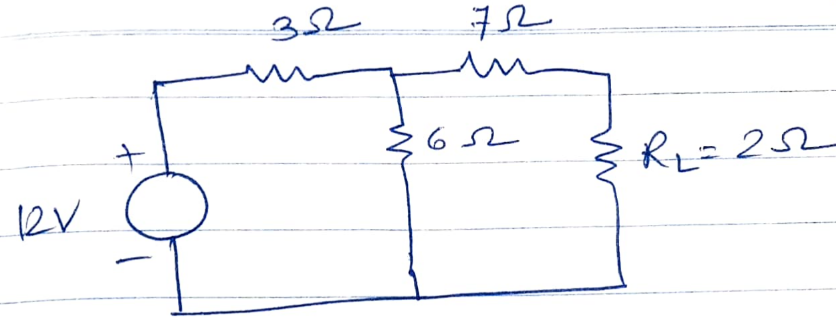
$$= 29.4117 \text{ V}.$$

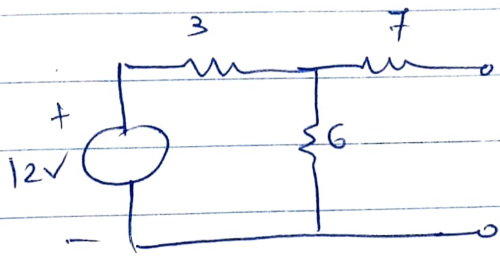
Thevenin's Theorem

Steps.

1. Remove load resistance and calculate voltage at open circuited terminals. This vltg. is V_{th}.
2. Remove sources also. Replace vltg source by S.C. & current source by open circuit. Now calculate Equivalent resistance (Req) of the circuit as if you are looking into the ckt. from open circuited terminals
3. Draw ~~E~~ Thevenin's equi. ckt.
4. Connect R_L to equi. ckt.
5. Calculate load current by using,

$$I_L = \frac{V_{th}}{R_{eq} + R_L}$$

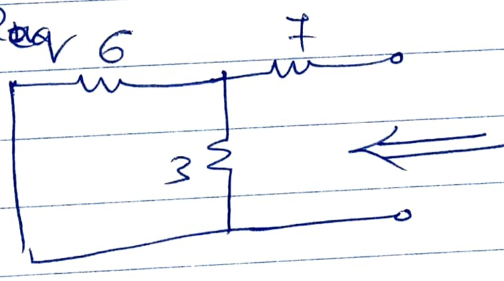




$$I = \frac{12}{9} = 1.33\ A$$

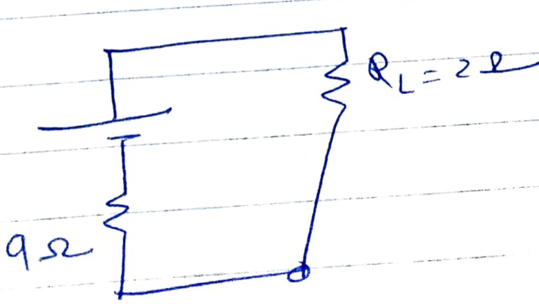
$$\therefore V_{th} = 6 \times 1.33$$

For Req

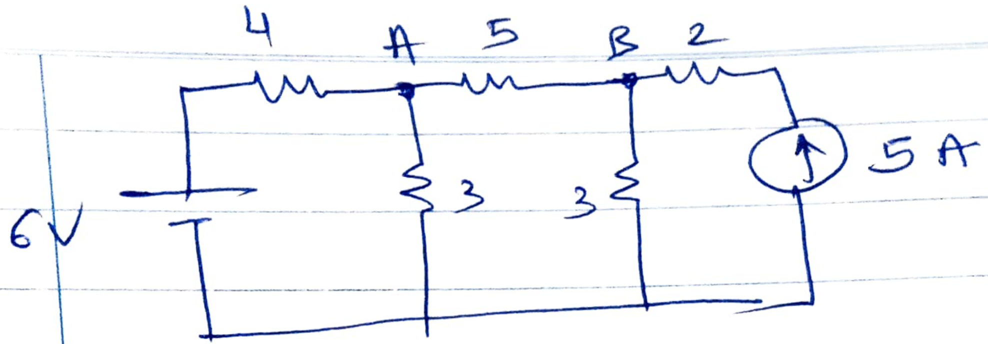


$$(6 \| 3) + 7$$

$$R_{eq.}\ \frac{18}{9} + 7 = 2 + 7 = 9\Omega$$

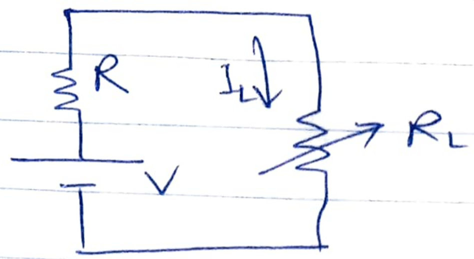


2.



$I_{4} - I_{AB} = ?$

Maximum Power Transfer Theorem



Consider a ckt. with DC vltg same V and resistance R Ω connected to variable load resistance R_L.

Load current ~~c~~ can be calculated here as,

$$I_L = \frac{V}{R+R_L}$$

The power consumed by the load resistance R_L is,

$$P = I_L^2 R_L$$

$$= \left(\frac{V}{R+R_L}\right)^2 \cdot R_L$$

If R_L is changed, I_L will also change and at a particular value of R_L, the power transferred to the load will be ma

Hence the power depends on the value of R_L. To get the ~~max~~ value of R_L at which power will be max., let's differentiate above eqn of power w.r.t. R_L & equate to zero.

$$\therefore \frac{dP}{dR_L} = 0$$

$$\therefore \frac{d}{dR_L}\left[\frac{V}{R+R_L}\right]^2 \cdot R_L = 0$$

$$V^2 \cdot \frac{d}{dR_L}\left[\frac{R_L}{(R+R_L)^2}\right] = 0$$

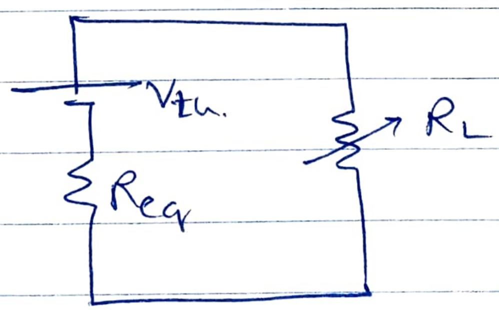
after solving this, we get;

$$R + R_L - 2R_L = 0$$

$$\therefore \boxed{R_L = R}$$

Thus when load resistance is equal to ~~internal~~ resis. of the ckt; max. power transfer takes place.

We represent the complex n/w with Thevenin's equi. ckt



If we compare ~~to~~ this ckt with above drawn ckt., ~~a~~ we get;

$$\boxed{R_L = R_{eq}}$$ for max. power transfer.

And max. power will be calculated as,

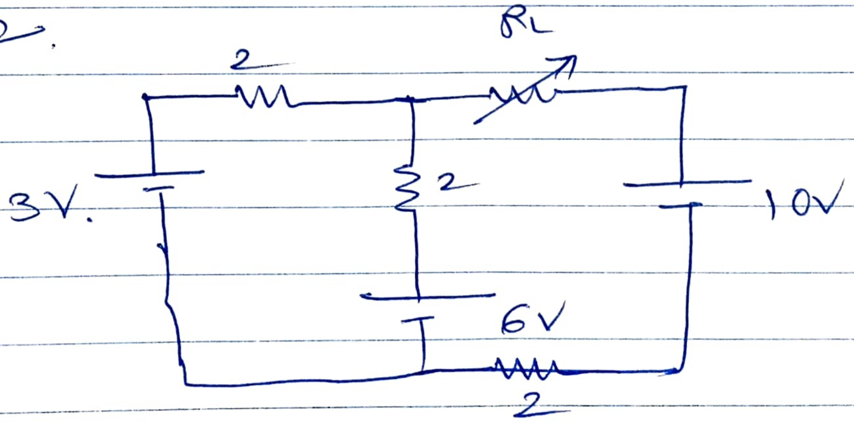
$$P_m = \left[\frac{V_{th}}{R_{eq} + R_L}\right]^2 \cdot R_L$$

with $R_L = R_{th}$;

$$P_m = \left[\frac{V_{th}}{2R_{eq}}\right]^2 \cdot R_{eq}$$

$$\boxed{P_m = \frac{V_{th}^2}{4R_{eq}} \text{ watts.}}$$

Examples.



$V_{th} = 5.5V$

$R_{eq} = 3\Omega$.

$P_m = 2.5208W$.