

Subject Name - LADC

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Q1.)

A) Determine the value of  $\alpha$  for which the equations are consistent

$$x+2y+2z=3, \quad x+y+2z=\alpha, \quad 3x+y+3z=\alpha^2$$

Find the solution for one of the value of  $\alpha$

Ans  $\Rightarrow [A][x] = [B]$  form,

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \alpha \\ \alpha^2 \end{bmatrix}$$

$$[A/B] = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & 1 & 1 & \alpha \\ 3 & 1 & 3 & \alpha^2 \end{bmatrix}$$

Now we can apply row transformation.

$$R_1 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \alpha-3 \\ 0 & -5 & 0 & \alpha^2-9 \end{bmatrix} \quad \rho(A) = 3 = \rho(A/B)$$

Hence there is unique solution to system and it is consistent. From  $\sim(A/B)$

$$x + 2y + 2z = 3$$

$$-y = \alpha - 3$$

$$-5y = \alpha^2 - 9$$

$$5 = \alpha + 3$$

$$\alpha = 2$$

This is the unique solution for  $\alpha$ .



B) Examine for linear dependence or independence of  $\alpha = (3, 1, -4)$   $\gamma = (2, 2, -3)$   $z = (0, -4, 1)$ . If dependent then find relation between them.

Ans  $\Rightarrow \alpha_1 = (3, 1, -4)$

$\alpha_2 = (2, 2, -3)$

$\alpha_3 = (0, -4, 1)$

Let  $c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 = (0, 0, 0)$

$c_1(3, 1, -4) + c_2(2, 2, -3) + c_3(0, -4, 1) = (0, 0, 0)$

$$\begin{bmatrix} 3c_1 + 2c_2 + 0c_3 \\ c_1 + 2c_2 - 4c_3 \\ -4c_1 - 3c_2 + c_3 \end{bmatrix} \begin{matrix} + 3 \text{ equation} \\ 3 \text{ variables.} \end{matrix}$$

In matrix form.

$$\begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now,  $[A|0]$

$$= \left[ \begin{array}{ccc|c} 3 & 2 & 0 & 0 \\ 1 & 2 & -4 & 0 \\ -4 & -3 & 1 & 0 \end{array} \right]$$

$$R_2 \leftrightarrow R_2 \sim \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 3 & 2 & 0 & 0 \\ -4 & -3 & 1 & 0 \end{array} \right]$$

$$\begin{matrix} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 + 4R_1 \end{matrix} \sim \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -4 & 12 & 0 \\ 0 & 5 & -15 & 0 \end{array} \right]$$



$$R_3 \rightarrow R_3 + \frac{5}{4} R_2 \sim \begin{bmatrix} 1 & 2 & -4 & 0 \\ 0 & -4 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \rho(A) = \rho(A/0) = 2 < 3$$

$\therefore$  System has infinite solutions

Now,

$$C_1 - 2C_2 - 4C_3 = 0 \quad \text{--- (1)}$$

$$-4C_2 + 12C_3 = 0$$

$$\Rightarrow C_2 - 3C_3 = 0$$

$$\therefore \text{Free variable} = 3 - 2 = 1$$

$$\text{Put } C_3 = t, \quad C_2 = 3t \quad \& \quad C_1 = -2t$$

$$\therefore C_1 x_1 + C_2 x_2 + C_3 x_3 = 0$$

$$\Rightarrow -2tx_1 + 3tx_2 + tx_3 = 0$$

$$\Rightarrow t(-2x_1 + 3x_2 + x_3) = 0$$

$$\therefore t = 1 \text{ given}$$

$$-2x_1 = 3x_2 - x_3$$

$$\therefore 2x_1 = 3x_2 + x_3$$

$$\therefore 2x_1 = 3x_2 + x_3 \text{ is the answer}$$

$\rightarrow$



c) Determine for what values of  $a, b, c$  the system have non trivial solution.

$$ax + by + cz = 0, \quad bx + cz + ay = 0, \quad cx + ay + bz = 0$$

Ans  $\rightarrow$  
$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For a non-trivial soln  $|A| = 0$

$$|A| = a \begin{vmatrix} c & a \\ a & b \end{vmatrix} - b \begin{vmatrix} b & a \\ c & b \end{vmatrix} + c \begin{vmatrix} b & c \\ c & a \end{vmatrix}$$

$$0 = a(cb - a^2) - b(b^2 - ac) + c(ab - c^2)$$

$$0 = abc - a^3 - b^3 + abc + abc - c^3$$

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$\therefore a^3 + b^3 + c^3 = 0$$

$$\approx a + b + c = 0$$

if  $a + b + c = 0$  a non-trivial soln must exist



Q2)

A) For an orthogonal matrix A

$$= \begin{pmatrix} \cos\phi \cos\theta & \sin\phi & \cos\phi \sin\theta \\ -\sin\phi \cos\theta & \cos\phi & -\sin\phi \sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} A^{-1}$$

$$A^{-1} \text{ is } = \begin{bmatrix} \cos\phi \cos\theta & -\sin\phi \cos\theta & -\sin\theta \\ \sin\phi & \cos\phi & 0 \\ \cos\phi \sin\theta & -\sin\phi \sin\theta & \cos\theta \end{bmatrix}$$

B) Is the transformation  $y_1 = x + y + z$ ,  $y_2 = 2x + 3y + 4z$ ,  $y_3 = x - y + z$  regular?  
If yes explain.

→ Yes since every component of  $f(x, y, z)$  repeats linearly.

C) For what value of  $k$  the matrix  $\begin{pmatrix} 1/2 & k \\ -k & 1/2 \end{pmatrix}$  is orthogonal.

→ For what value of  $k$  the matrix  $\begin{bmatrix} 1/2 & k \\ -k & 1/2 \end{bmatrix}$  is reg  $k=0$

D) An  $n \times n$  non-homogeneous system of linear equations  $AX=B$  with  $A$  is non-singular matrix have  $n$  solutions