Subject Name - LADC (TUT)
Subject code -
Name - Shreerang Mhatre
Division - 11
Roll no - 111056
Batch - K3

Q1) Solve the following -

1) Find the rank of $\begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 2 & 3 \\ 0 & -1 & -3 & -2 \end{bmatrix}$

Ans ⇒ $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 2 & 3 \\ 0 & -1 & -3 & -2 \end{bmatrix}$

$R_1 \leftrightarrow R_2 \begin{bmatrix} 1 & 2 & 2 & 3 \\ 2 & 3 & 1 & 4 \\ 0 & -1 & -3 & -2 \end{bmatrix}$

$R_2 \to R_2 - 2R \begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\boxed{\rho(A) = 2}$

→

2) Find the rank of $\begin{bmatrix} 1 & 3 & 8 & 6 \\ 2 & 6 & -1 & 4 \\ 3 & 9 & 7 & 10 \end{bmatrix}$

Ans ⇒ $A = \begin{bmatrix} 1 & 3 & 8 & 6 \\ 2 & 6 & -1 & 4 \\ 3 & 9 & 7 & 10 \end{bmatrix}$.

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$ $\begin{bmatrix} 1 & 3 & 8 & 6 \\ 0 & 0 & -17 & -8 \\ 0 & 0 & -17 & -8 \end{bmatrix}$

$R_3 \to R_3 - R_2$ $\begin{bmatrix} 1 & 3 & 8 & 6 \\ 0 & 0 & -17 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\boxed{\rho(A) = 2}$

Q3) Find normal form of $\begin{bmatrix} 3 & 1 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$

Ans $\Rightarrow$

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 3 & 2 \\ 3 & 1 & 2 & 1 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 3R_1$
$R_5 \to R_5 - 2R_1$
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -3 & -6 \\ 0 & -5 & -7 & -11 \\ 0 & -5 & -5 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$
$C_3 \to C_3 - 3C_1$
$C_4 \to C_4 - 4C_1$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & -3 & -6 \\ 0 & -5 & -7 & -11 \\ 0 & -5 & -5 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to (-1) R_2$
$R_3 \to (-1) R_3$
$R_4 \to (-1) R_4$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 4 & 3 & 6 \\ 0 & 5 & 7 & 11 \\ 0 & 5 & 5 & 11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 3 & 6 \\ 0 & 5 & 7 & 11 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 5 \\ 0 & 4 & 3 & 6 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 4 & 3 & 6 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rightarrow$

$R_3 \to R_3 - R_4$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 4 & 1 & 6 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - C_2$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 4 & 1 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 - 4C_2$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & -14 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to \left(\frac{-1}{14}\right) C_4$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 4C_4$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_4 \to R_4 \times \frac{1}{2}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to R_3 - R_4$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \leftrightarrow R_4$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} I_4 \\ 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 4}$$

Q2) Fill in the blanks -

1) If the matrix A have at least one minor of order 3 is non-zero & every minor of order 4 zero then rank of A is 3

2) By performing elementary operations if any non-zero matrix A of order 4x4 is reduced to the normal form $\begin{bmatrix} I_4 & 0 \\ 0 & 0 \end{bmatrix}$ then rank of A is 4

3) Normal form of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is

$\rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} I_4 & 0 \\ 0 & 0 \end{bmatrix}$

4) When a system of equations have infinitely many solutions

$\rightarrow$ If $\rho(A) = \rho(A/B) <$ no. of varieties and the matrix is consistent.