Classification of Inverters :

Classification Based on the Nature of Source :

- The inverters can be classified on the basis of a number of factors, the first one being the type of source it uses at its input.
- Depending on the nature of input power source the inverters are classified into two categories :
  (a) Voltage source inverters (VSI)
  (b) Current source inverters (CSI).
- In case of VSI the input to the inverter is provided by a ripple free dc voltage source, where as in CSI the voltage source is first converted into a current source and then used to supply the power to the inverter.

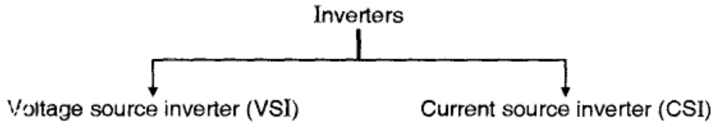


Fig. 3.1 : Classification of inverters based on the nature of source

Classification Based on the Configuration of the Inverter :

- The voltage source inverters (VSI) are classified based on the arrangement of thyristors in the power circuit. The configurations are :
  1. Series inverter
  2. Parallel inverter or push pull inverter
  3. Bridge inverter. (half bridge or full bridge)
- Any one of these configurations is selected depending on the applications. The bridge configuration however is the most widely used one.

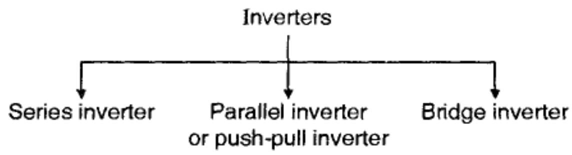


Fig. 3.2 : Classification of inverters based on the configuration

Classification Based on the Nature of Output Waveform :

- The inverter output ac waveform need not always be a sine wave. It can be a square wave, a quasi square wave or a pulse width modulated waveform.
- Filters may be used to derive a sine wave from the square or quasi square wave output.
- The percentage of harmonic frequency components largely depends on the shape of the output waveform.

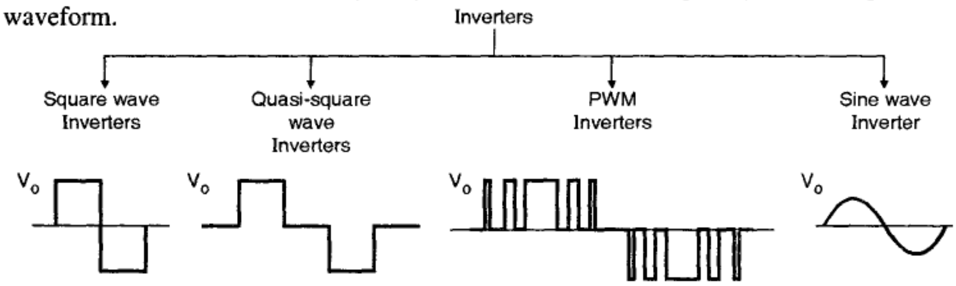


Fig. 3.3 : Inverter classification based on the type of output waveform

Classification Based on the Power Semiconductor Device Used :

- We can use any power semiconductor device as a switch to construct an inverter.
- The devices normally used are : SCR, power IGBT, power MOSFET, IGBT. The classification is as given in Fig. 6.2.4.

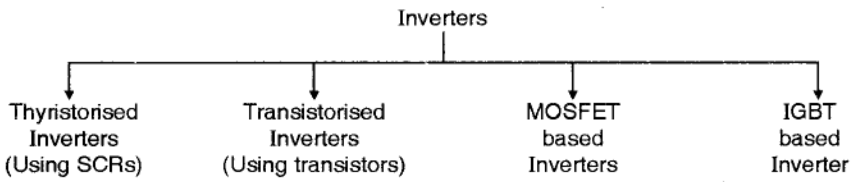


Fig. 3.4: Inverter classification based on the type of device used

Transistorised Inverters :

- The transistorised inverters use power IGBTs as power switches.
- The configurations which will be discussed in this section can be used by replacing the power IGBTs by power MOSFETs or IGBTs.
- For all these devices, the commutation circuits are not required. They can be turned off very easily just by removing their base or gate driving waveform.
- The output voltage waveforms of ideal inverters should be sinusoidal. However the waveforms of practical inverters are non-sinusoidal and contain certain harmonics.
- For low and medium power applications square wave or quasi square voltages may be acceptable and for high power applications low distortion sinusoidal waveforms are required.
- With the availability of high speed power semiconductor devices the harmonic contents of the output voltage can be minimized or reduced significantly by special switching techniques.

Single Phase Half Bridge Inverter :

- The principle of single phase transistorised inverters can be explained with the help of Fig.3.5. The configuration is known as the **half bridge configuration.**
- The IGBT Q_1 is turned on for a time $T_o/2$, which makes the instantaneous voltage across the load $V_o = V/2$.
- If IGBT Q_2 is turned on at instant $T_o/2$, by turning Q_1 off then $- V/2$ appears across the load.
- A precaution must be taken while designing the control circuit so that both the IGBTs are not turned on simultaneously, as it will short circuit the source and may damage the IGBTs.

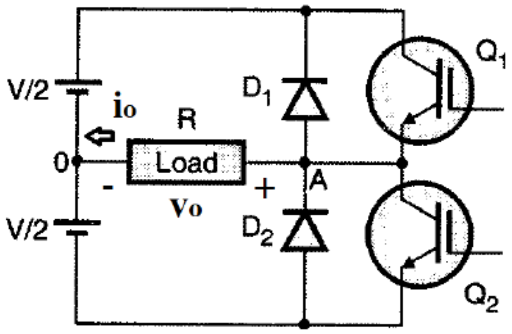


Fig. 3.5 : Half bridge inverter

- Fig. 3.6 shows the waveforms of output voltage and IGBT currents with a resistive load. The voltage across the IGBT when it is off is V and not V/2.

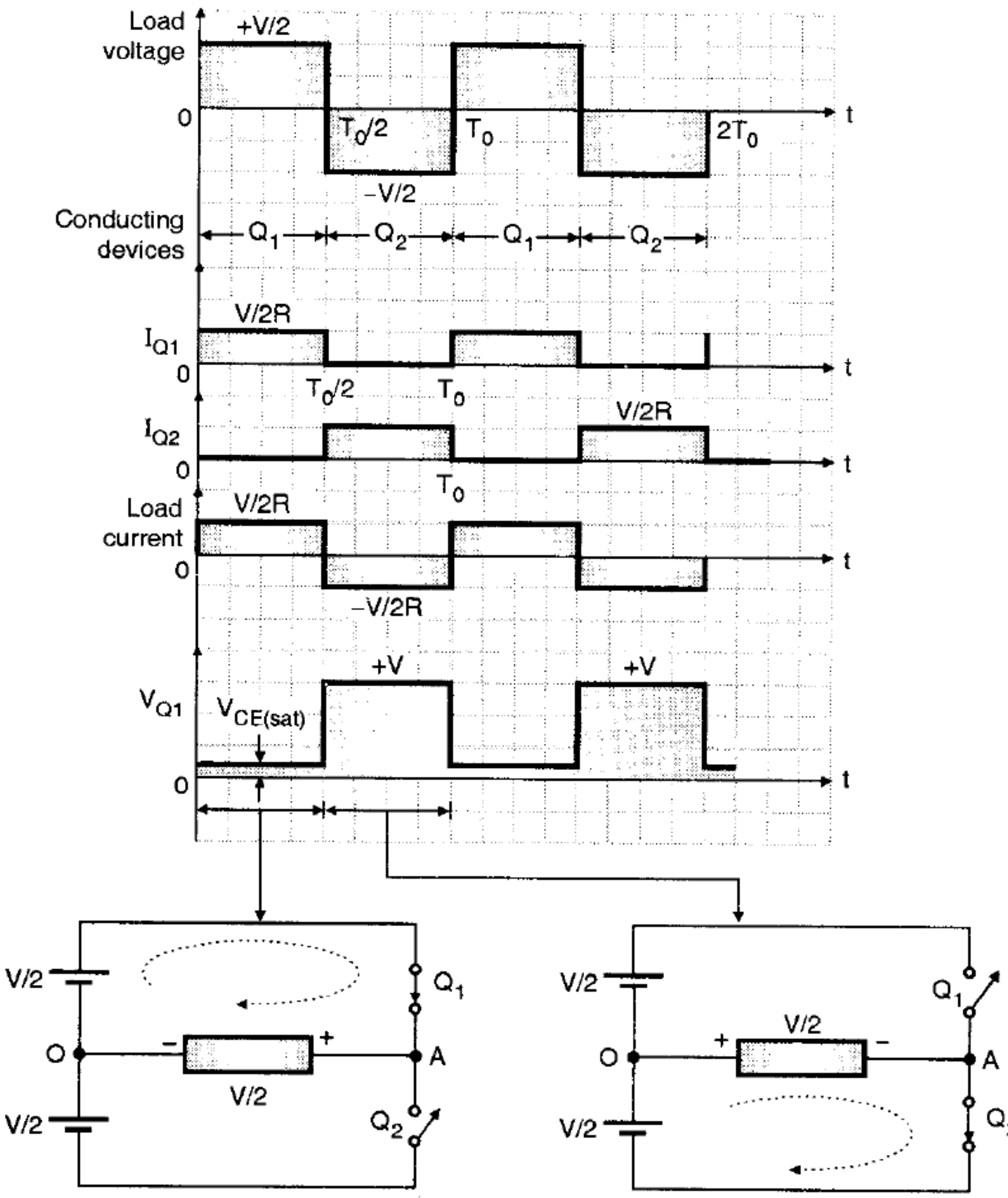


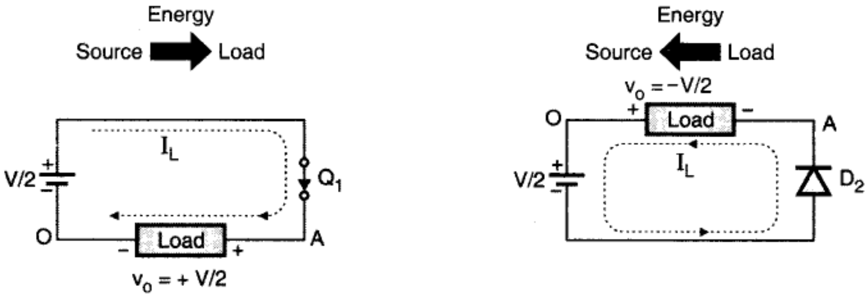
Fig. 3.6 : Load voltage and current waveforms with resistive load for a half bridge inverter

Half Bridge Inverter with Inductive Load :

- Let us divide the operation into four intervals. We start explanation from the second time interval t_1 to t_2 because at the beginning of this interval IGBT Q_1 will start conducting. Refer Fig. 3.9 for better understanding.

Interval II ($t_1 - t_2$) :

- Q_1 is turned on at instant t_1, the load voltage is equal to + V/2 and the positive load current increases gradually.
- At instant t_2 the load current reaches the peak value. The IGBT Q_1 is turned off at this instant.
- Due to the same polarity of load voltage and load current, the energy is stored by the load. Refer Fig. 3.7 (a).



(a) Equivalent circuit in interval II ($t_1 - t_2$) **(b) Equivalent circuit in interval III ($t_2 - t_3$)**

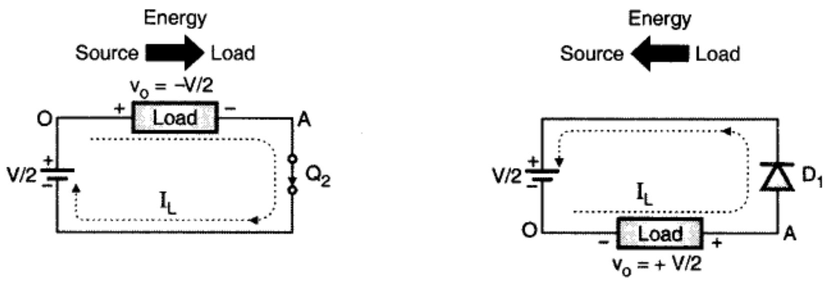
Fig. 3.7

Interval III ($t_2 - t_3$) :

- Due to inductive load, the load current direction will be maintained same even after Q_1 is turned off.
- The self induced voltage across the load will be negative. The load current flows through lower half of the supply and D_2 as shown in Fig. 3.7(b).
- In this interval the stored energy in load is fed back to the lower half of the source and the load voltage is clamped to – V/2.

Interval IV ($t_3 - t_4$) :

- At the instant t_3, the load current goes to zero, indicating that all the stored energy has been returned back to the lower half of supply.
- At instant t_3, Q_2 is turned on. This will produce a negative load voltage $v_o = -V/2$ and a negative load current. Load current reaches a negative peak at the end of this interval. (See Fig. 3.8 (a)).



(a) Interval IV ($t_3 - t_4$) **(b) Interval I ($t_0 - t_1$)**

Fig. 3.8

Interval I (t_4 to t_5) or (0 to t_1) :

- IGBT Q_2 is turned off at instant t_4. The self induced voltage in the inductive load will maintain the load current.

- The load voltage changes its polarity to become positive V/2, load current remains negative and the energy stored in the load is returned back to the upper half of the dc source. (see Fig. 3.8(b)).
- At t_5, the load current goes to 0 and Q_1 can be turned on again. The cycle of operation repeats.

Note : The output voltage waveform remains square wave with purely resistive or RL type of load.

Conduction period of IGBTs :

Conduction period of the IGBTs depends upon the load power factor. For purely inductive load, an IGBT conducts only for $T_o/2$ or 90°. Depending on the load power factor, the conduction period of the IGBT will vary between 90° to 180° (180° for purely resistive load).

The voltage and current waveforms for RL load are as shown in Fig.3.9.

Interval	$(t_0 - t_1)$	$(t_1 - t_2)$	$(t_2 - t_3)$	$(t_3 - t_4)$
Mode	I	II	III	IV
Conducting Devices	D_1	Q_1	D_2	Q_2
Output Voltage	+ V/2	+ V/2	– V/2	– V/2
Output Current	Negative	Positive	Positive	Negative
Power Flow	Load to source	Source to Load	Load to source	Source to Load

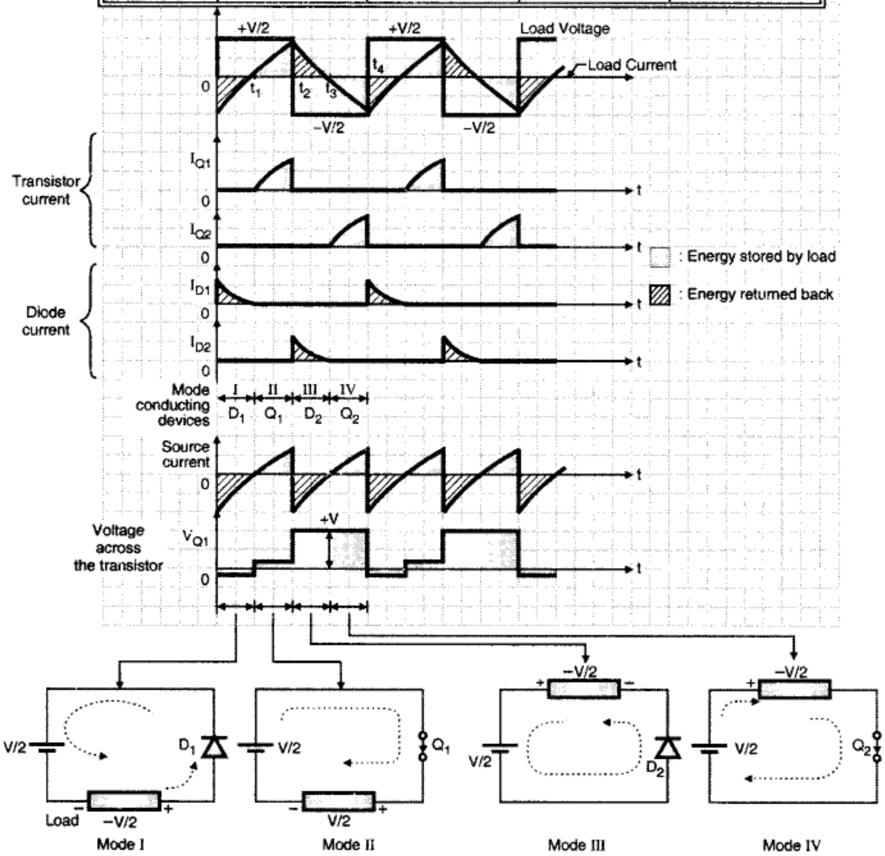


Fig. 3.9: Voltage and current waveforms for a half bridge inverter with RL load

Fourier Analysis of the Load Voltage Waveform of a Half Bridge Inverter :

Assumptions :

- The load voltage waveform is a perfect square wave with a zero average value.
- The load voltage waveform does not depend on the type of load.
- a_n, b_n and c_n are the Fourier coefficients.
- θ_n is the displacement angle for the n^{th} harmonic component of output voltage.
- Total dc input voltage to the inverter is V volts.

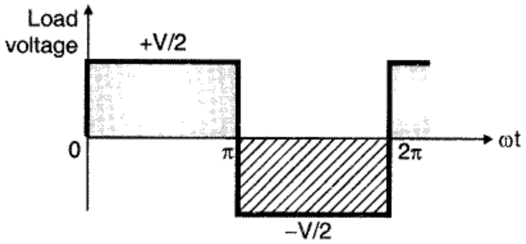


Fig. 3.10 : Load voltage of half bridge inverter

- Refer to Fig. 3.10 . The instantaneous load voltage V_o (ωt) can be expressed in the fourier series form as follows :

$$V_o(\omega t) = V_{o\,(av)} + \sum_{n=1}^{\infty} c_n \sin(n\omega t - \theta_n)$$

where $c_n = \left(a_n^2 + b_n^2\right)^{1/2}$ and $\theta_n = \tan^{-1}[a_n/b_n]$

The values of a_n and b_n can be found as follows :

1. Expression for a_n :

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\cos n\omega t\, d\omega t \qquad \text{...(3.1)}$$

but $v_o(t) = +V/2$, for $0 \le \omega t \le \pi$

and $v_o(t) = -V/2$, for $\pi \le \omega(t) \le 2\pi$

$$\therefore \quad a_n = \frac{1}{\pi}\left\{\int_0^{\pi} (V/2)\cos n\omega t\, d\omega t - \int_{\pi}^{2\pi} (V/2)\cos n\omega t\, d\omega t\right\}$$

$$= \frac{V}{2\pi n}[\sin n\pi - \sin 0] - \frac{V}{2\pi n}[\sin 2\pi n - \sin \pi n]$$

$\therefore \quad a_n = 0$ for all value of n. **...(3.2)**

2. Expression for b_n :

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_o(t)\sin n\omega t\, d\omega t \qquad \text{...(3.3)}$$

$$b_n = \frac{1}{\pi}\left\{\int_0^{\pi} (V/2)\sin n\omega t\, d\omega t - \int_{\pi}^{2\pi} (V/2)\sin n\omega t\, d\omega t\right\}$$

$$= \frac{-V}{2\pi n}[\cos n\pi - \cos 0] + \frac{V}{2\pi n}[\cos 2\pi n - \cos \pi n]$$

$$= \frac{-V}{2\pi n}[\cos n\pi - 1] + \frac{V}{2\pi n}[1 - \cos n\pi]$$

$$= \frac{V}{2\pi n}[1 - \cos n\pi + 1 - \cos \pi n]$$

$$\therefore \quad b_n = \frac{V}{\pi n}[1 - \cos n\pi] \qquad \text{...(3.4)}$$

But $\cos n\pi = +1$ for $n = 2, 4, 6,$...i.e. for n even.

$$\therefore \quad b_n = 0 \quad \text{for even values of n} \qquad \text{...(3.5)}$$

and $\cos n\pi = -1$ for $n = 1,3,5$..... i.e. for n odd

$$\therefore \quad b_n = \frac{2V}{n\pi} \quad \text{for odd values of n} \qquad \text{...(3.6)}$$

Expression for c_n :

- Therefore the peak value of n^{th} harmonic component of output voltage is given by,

$$c_n = \left[a_n^2 + b_n^2\right]^{1/2}$$

$$\therefore \quad c_n = b_n = \frac{2V}{n\pi} \qquad \text{...(3.7)}$$

- This is the peak amplitude of n^{th} harmonic component of the output voltage.

and $\theta_n = \tan^{-1} 0 = 0$ **...(3.8)**

and $V_{o\,(av)} = 0$

- Therefore the instantaneous output voltage of a half bridge inverter can be expressed in Fourier series form as,

$$V_o(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V}{n\pi}\sin n\omega t \qquad \text{...(3.9)}$$

$$= 0 \quad \text{for even values of n.}$$

- Equation (3.9) indicates that the frequency spectrum of the output voltage waveform consists of only odd order harmonic components. i.e. 1,3,5,7....etc.
- The even order harmonics are automatically cancelled out.
- The amplitudes of different harmonic components in output voltage can be found with the help of Equation (3.7).

RMS Output Voltage :

From the output voltage waveform shown in Fig. 3.5 the rms value of output voltage is given by,

$$V_{o\,rms} = \left\{\frac{1}{\pi}\int_0^{\pi} (V/2)^2\, d\omega t\right\}^{1/2} \qquad \text{...(3.10)}$$

$$= \left\{\frac{V^2}{4\pi} \times \pi\right\}^{1/2}$$

$$\therefore \qquad V_{o\,rms} = \frac{V}{2} \text{ volts} \qquad ...(3.11)$$

RMS Value of Fundamental Component of Output Voltage :

- In order to find the value of fundamental component of output voltage substitute n = 1 in the Equation (3.7).

$$\text{we get} \qquad V_{o1\,(peak)} = \frac{2V}{\pi}$$

- As the fundamental component is a sine wave, its rms value is given by,

$$V_{o1\,rms} = \frac{2V}{\sqrt{2}\,\pi} = \frac{\sqrt{2}V}{\pi} = 0.45\text{ V} \qquad ...(3.12)$$

Expression for Instantaneous Output Current :

1. With R load :

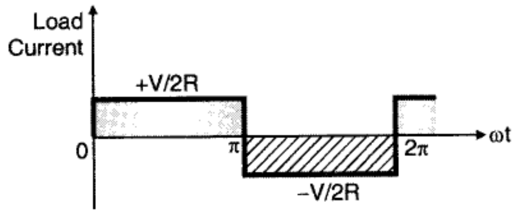


Fig. 3.7 : Load current of a half bridge inverter with resistive load

Refer to Fig. 3.7. The instantaneous output current is in phase with the output voltage waveform and has peak value of V/2R. The Fourier analysis for this waveform yields identical results to those obtained for the output voltage waveform.

$$\therefore \qquad a_n = 0, \qquad b_n = \frac{2V}{n\pi R}$$

$$\therefore \qquad c_n = \frac{2V}{n\pi R} \quad \text{and} \quad \theta_n = 0$$

$$\therefore \qquad i_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V}{n\pi R} \sin n\omega t \qquad ...(3.13)$$

2. With RL load :

For RL load the equation for the instantaneous current i_o can be found using Equation (6.3.9).

$$i_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V}{n\pi\sqrt{R^2 + (n\,\omega\,L)^2}} \sin(n\omega t - \theta_n) \qquad ...(3.14)$$

In this equation, $Z_n = \sqrt{R^2 + (n\,\omega\,L)^2}$ is the impedance offered by the load to the n^{th} harmonic component, $\frac{2v}{n\pi}$ is the peak amplitude of n^{th} harmonic voltage.

$$\text{and} \qquad \theta_n = \tan^{-1}\left\{\frac{n\omega L}{R}\right\} \qquad ...(3.15)$$

Fundamental Output Power :

The output power at fundamental frequency (n = 1) is given by

$$P_{01\,rms} = V_{o1\,rms}\, I_{o1\,rms} \cos\theta_1 = I^2_{1\,rms} \times R \qquad \text{...(3.16)}$$

$V_{o1\,rms}$ = RMS value of fundamental output voltage

$I_{o1\,rms}$ = RMS value of fundamental output current.

$\theta_1 = \tan^{-1}(\omega L / R)$

But

$$I_{o1\,rms} = \frac{2V}{\sqrt{2}\pi\sqrt{R^2+(\omega L)^2}} = \frac{2\sqrt{2}}{\pi\sqrt{R^2+(\omega L)^2}}\,\frac{V}{2} \qquad \text{...(3.17)}$$

and

$$P_{o1\,rms} = I^2_{o1\,rms} \times R$$

$$\therefore \quad P_{o1\,rms} = \left[\frac{2V}{\pi\sqrt{2}\sqrt{R^2+(\omega L)^2}}\right]^2 \times R \qquad \text{...(3.18)}$$

$$\therefore \quad P_{o1\,rms} = \left[\frac{4V^2 R}{2\pi^2 (R^2+\omega^2 L^2)}\right] = \left[\frac{2V^2 R}{\pi^2 (R^2+\omega^2 L^2)}\right] = \left[\frac{2\sqrt{2}}{\pi\sqrt{R^2+(\omega L)^2}}\,\frac{V}{2}\right]^2 \times R \qquad \text{...(3.19)}$$

Importance of fundamental power is that in many applications such as electric motor drives, the output power due to fundamental current only is generally the useful power and the power due to harmonic currents is dissipated as heat and increases the load dissipation.

Performance Parameters of Inverters :

The output of practical inverters contains harmonics and the quality of an inverter is normally judged in terms of following performance parameters :

- Harmonic factor of n^{th} harmonic.
- Total harmonic distortion.
- Distortion factor.
- Lowest order harmonic.

Harmonic Factor of n^{th} Harmonics HF_n :

- The harmonic factor is a measure of contribution of individual harmonics.
- It is defined as the ratio of the rms voltage of a particular harmonic component to the rms value of fundamental component.

$$\therefore \qquad HF_n = \frac{V_{on\,rms}}{V_{o1\,rms}} \qquad \text{...(3.20)}$$

where $V_{on\,rms}$ = RMS value of the n^{th} harmonic of output voltage.

and $V_{o1\,rms}$ = RMS value of the fundamental component.

Total Harmonic Distortion (THD) :

- The total harmonic distortion is a measure of the total amplitude of all the harmonics present in the output of inverter except the fundamental component.
- In other words it is the measure of closeness in shape between a waveform and its fundamental component.

The THD defined as,

$$\text{THD} = \frac{1}{V_{o1\,rms}} \left(\sum_{n=2,3,\ldots}^{\infty} V_{on\,rms}^{2} \right)^{1/2} \qquad \ldots(3.21)$$

$$= \frac{1}{V_{o1\,rms}} \left[V_2^2 + V_3^2 + V_4^2 + \ldots \right]^{1/2}$$

$$= \frac{1}{V_{o1\,rms}} \left[V_{o\,rms}^2 - V_{o1\,rms}^2 \right]^{1/2}$$

$$= \sqrt{\left(\frac{V_{o\,rms}}{V_{o1\,rms}} \right)^2 - 1} \qquad \ldots(3.22)$$

where, V_2, V_3 .... are the rms voltages at second, third harmonic frequencies. THD thus gives the total harmonic content.

Distortion Factor DF :

- THD gives the total harmonic content but it does not indicate the level of each harmonic component.
- If a filter is used at the output of the inverter, the higher order harmonics would be attenuated more effectively. Therefore we must have information about both the frequency and the magnitude of each harmonic is important.
- The distortion factor indicates the amount of harmonic distortion that remains in a particular waveform after the harmonics of that waveform have been subjected to a second order attenuation. (i.e. divided by n^2).
- Thus the value of DF indicates the effectiveness with which the unwanted harmonics are reduced. DF is defined as

$$\text{DF} = \frac{1}{V_{o1\,rms}} \left(\sum_{n=2,3,\ldots}^{\infty} \left(V_{on\,rms} / n^2 \right)^2 \right)^{1/2} \qquad \ldots(3.33)$$

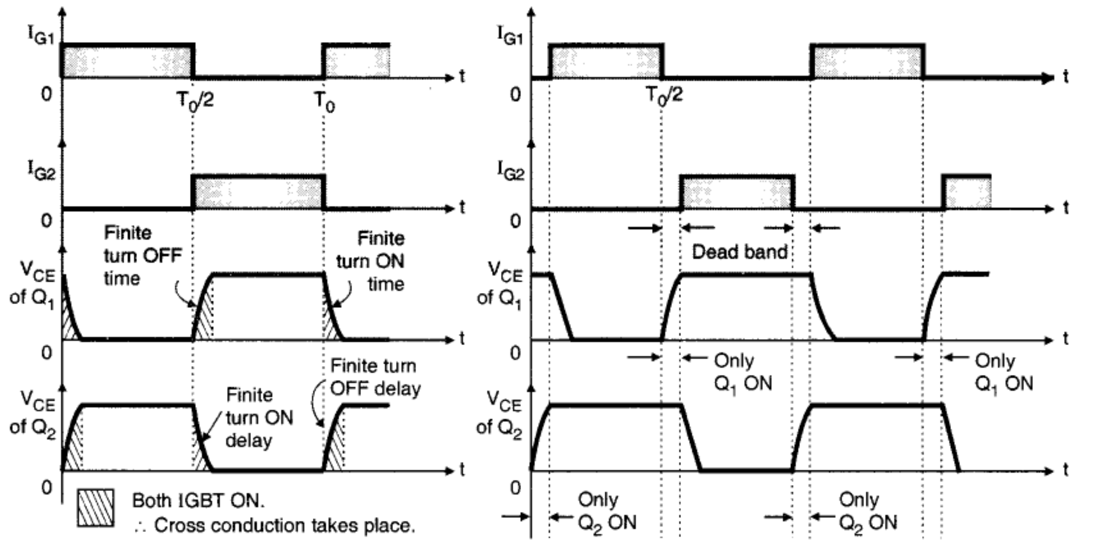
$$\text{DF} = \frac{1}{V_{o1\,rms}} \left[(V_2/2^2)^2 + (V_3/3^2)^2 + (V_4/4^2)^2 + \ldots \right]^{1/2} \qquad \ldots(3.34)$$

Lowest Order Harmonic LOH :

The lowest order harmonic is that harmonic component whose frequency is the closest to the fundamental one and its amplitude is greater than or equal to 3 % of the fundamental component.

Cross Conduction or Shoot through Fault :

- In the half bridge inverter circuit discussed above, each IGBT conducts for a period of "$T_o/2$" sec. or "π" radians.
- The gate driving waveforms for the two IGBTs is as shown in Fig. 3.8(a).
- At the instants 0, $T_o/2$, T_o etc., one of the IGBTs is turned ON and the other is turned OFF. However the outgoing IGBT does not turn OFF instantaneously due to its finite turn off delay.
- Due to this reason both the IGBTs (incoming and outgoing) conduct simultaneously for a short time. This is known as "CROSS CONDUCTION" or "SHOOT THROUGH" fault.



(a) Cross conduction due to finite turn on and turn off delays of the IGBTs

(b) Modified waveforms with introduction of dead band in order to avoid the cross conduction

Fig. 3.8

Effect of cross conduction :

- When both Q_1 and Q_2 conduct simultaneously the input dc supply is short circuited as shown in Fig. 3.9 and an unlimited current flows through the IGBT which may damage them.

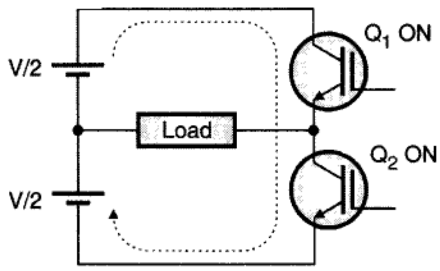


Fig. 3.9 : Effect of cross conduction

How to avoid cross conduction ?

- Cross conduction can be avoided by allowing the outgoing IGBT to turn off completely first and then applying the base drive to the incoming IGBT as shown in Fig. 3.8 (b).
- In order to do so the gate driving waveforms are modified as shown in Fig. 3.8 (b). A "Dead Band" or "Delay" is introduced between the trailing edge of the base drive of outgoing IGBT and the leading edge of the base drive of incoming IGBT. Thus during the "Dead Band" interval no IGBT receivers base drive.
- The "Dead Band" should be longer than the turn off time of the power devices used in the inverter circuit. e.g. for MOSFETs the Dead Band can be as short as 10 to 15 μS whereas for thyristors it can be as long as 100 μS.

Single Phase Full Bridge Inverter (Square Wave Output) :

- A single phase bridge inverter is shown in Fig. **3.10**. It consists of four IGBTs. These IGBTs are turned on and off in pairs of Q_1 Q_2 and Q_3 Q_4.
- In order to develop a positive voltage + V across the load, the IGBTs Q_1 and Q_2 are turned on simultaneously whereas to have a negative voltage – V across the load we need to turn on the devices Q_3 and Q_4.
- Diodes D_1, D_2, D_3, D_4 are known as the **feedback diodes**, because energy feedback takes place through these diodes when the load is inductive.

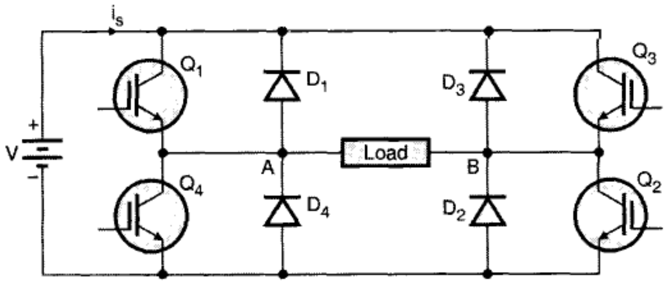


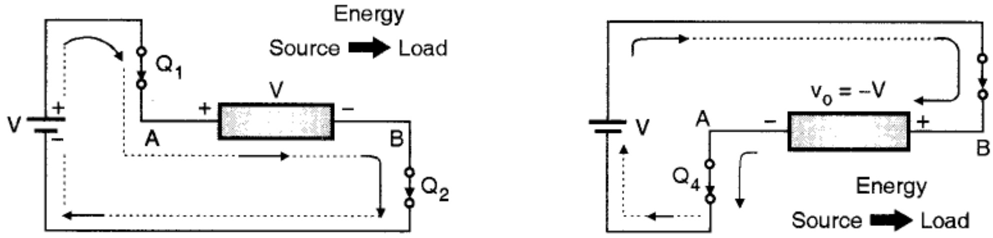
Fig. 3.10 : Single phase full bridge inverter

Operation with Resistive Load :

- With the purely resistive load the bridge inverter operates in two different intervals in one cycle of the output.

Mode I ($0 - T_o/2$) :

- The IGBTs Q_1 and Q_2 conduct simultaneously in this mode. The load voltage is + V and load current flows from A to B.
- The equivalent circuit for mode I is as shown in Fig. **3.11** (a). At t = $T_o/2$, Q_1 and Q_2 are turned off and Q_3 and Q_4 are turned on.



(a) Mode I equivalent circuit **(b) Mode II equivalent circuit**

Fig. 3.11

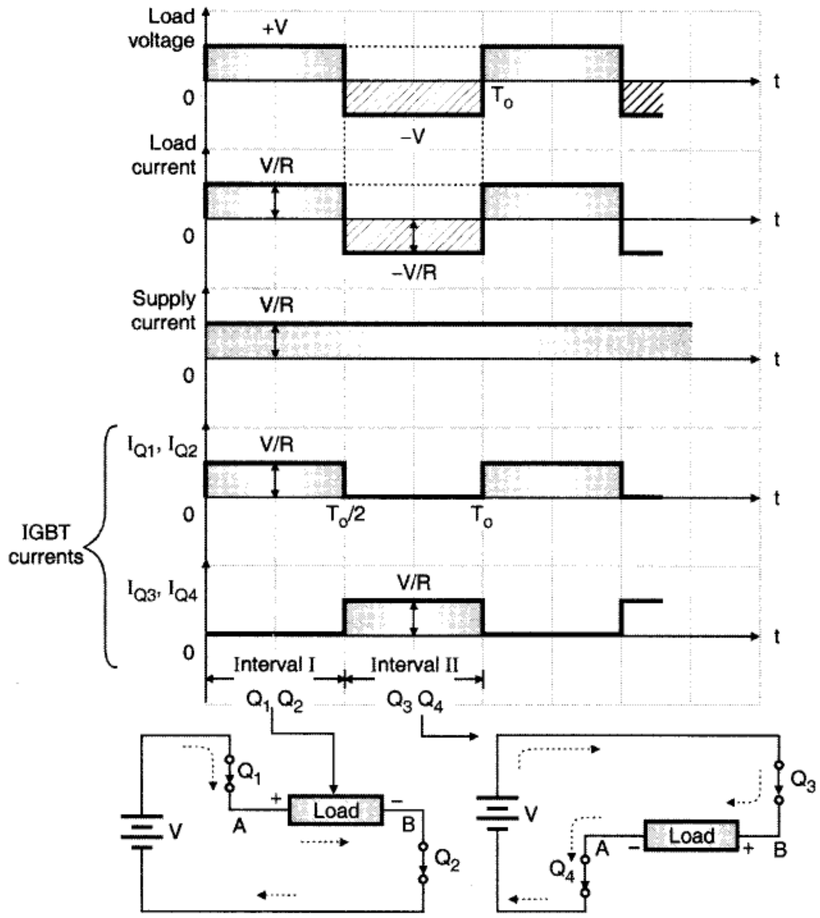


Fig. 3.12 : Voltage and current waveforms with resistive load

Mode II ($T_o/2 - T_o$) :

- At $t = T_o/2$, Q_3 and Q_4 are turned on and Q_1 and Q_2 are turned off. The load voltage is – V and load current flows from B to A. The equivalent circuit for mode II is as shown in Fig. 3.11(b). At $t = T_o$, Q_3 and Q_4 are turned off and Q_1 and Q_2 are turned on again.
- As the load is resistive it does not store any energy. Therefore the feedback diodes are not effective here.
- The voltage and current waveforms with resistive load are as shown in Fig. 3.12

The important observations from the waveforms of Fig. 3.12 are as follows :

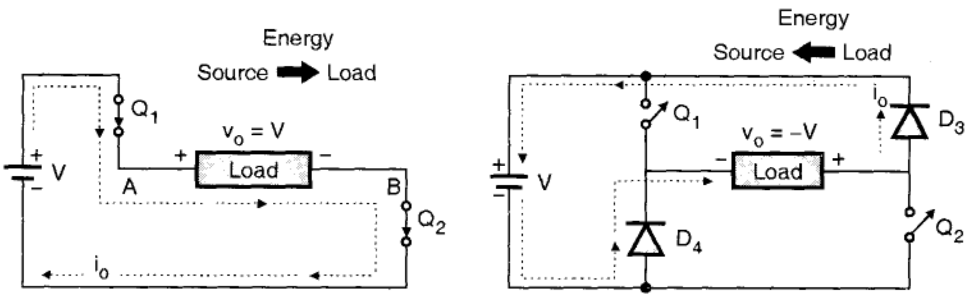
1. The load current is in phase with the load voltage .
2. The conduction period for each IGBT is π radians or 180°.
3. The peak load voltage = V
4. The rms load voltage = V
5. The peak load current = V/R
6. The rms load current = V/R
5. The peak switch current = V/R
6. The rms switch current = $V/\sqrt{2}R$
7. The average switch current = V/2R
8. The peak off-state switch voltage = V
5. The peak supply current = V/R
6. The rms supply current = V/R
7. The average supply current = V/R

Single Phase Bridge Inverter with RL Load :

The operation of the circuit can be divided into four intervals or modes. The waveforms are as shown in Fig. 6.5.6.

Interval I ($t_1 - t_2$) :

- At instant t_1, the pair of IGBTs Q_1 and Q_2 is turned on. The IGBTs are assumed to be ideal switches. Therefore point A gets connected to positive point of dc source V through Q_1 and point B gets connected to negative point of input supply.
- The output voltage v_o = + V as shown in Fig. 6.5.4(a). The load current starts increasing exponentially due to the inductive nature of the load.
- The instantaneous current through Q_1 and Q_2 is equal to the instantaneous load current. The energy is stored into the inductive load during this interval of operation.



(a) Interval I ($t_1 - t_2$) **(b) Interval II ($t_2 - t_3$)**

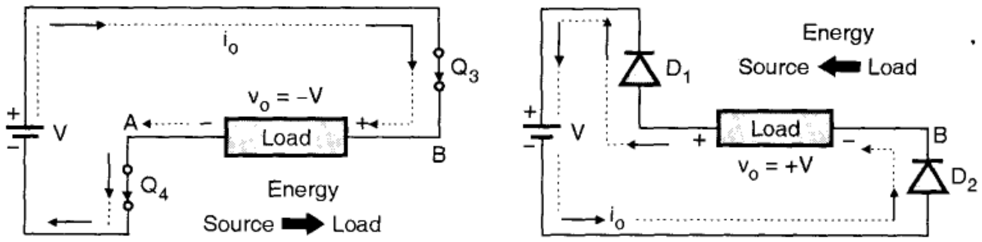
Fig. 3.13 : Equivalent circuit

Interval II ($t_2 - t_3$) :

- At instant t_2, both the IGBTs Q_1 and Q_2 are turned off. But the load current does not reduce to 0 instantaneously, due to its inductive nature.
- So in order to maintain the flow of current in the same direction there is a self induced voltage across the load. The polarity of this voltage is exactly opposite to that in the previous mode.
- Thus output voltage becomes negative equal to – V. But the load current continues to flow in the same direction, through D_3 and D_4 as shown in Fig. **3.13** (b).
- Thus the stored energy in the load inductance is returned back to the source in this mode. The diodes D_1 to D_4 are therefore known as the feedback diodes.
- The load current decreases exponentially and goes to 0 at instant t_3 when all the energy stored in the load is returned back to supply. D_3 and D_4 are turned off at t_3.

Interval III ($t_3 - t_4$) :

- At instant t_3, Q_3 and Q_4 are turned on simultaneously. The load voltage remains negative equal to – V but the direction of load current will reverse and become negative.
- The current increases exponentially in the negative direction. And the load again stores energy in this mode of operation. This is as shown in Fig. **3.14** (a).



(a) Interval III ($t_3 - t_4$) **(b) Interval IV (t_4 to t_5) or ($t_0 - t_1$)**

Fig. 3.14 : Equivalent circuits

Interval IV (t_4 to t_5) or ($t_0 - t_1$) :

- At instant t_4 or t_0 the IGBTs Q_3 and Q_4 are turned off. The load inductance tries to maintain the load current in the same direction, by inducing a positive load voltage.
- This will forward bias the diodes D_1 and D_2. The load stored energy is returned back to the input dc supply. The load voltage v_o = + V but the load current remains negative and decreases exponentially towards 0. This is as shown in Fig. **3.14** (b).
- At t_5 or t_1 the load current goes to zero and IGBTs Q_1 and Q_2 can be turned on again.

Conduction period of devices :

- The conduction period with a very highly inductive load, will be $T_o/4$ or 90° for all the IGBTs as well as the diodes. (see Fig. **3.15**).
- The conduction period of IGBTs will increase towards $T_o/2$ or 180° with increase in the load power factor. (i.e. as the load becomes more and more resistive).

Output voltage waveform :

The output voltage waveform remains a square wave independent of whether the load is resistive or reactive.

Interval	$(t_0 - t_1)$	$(t_1 - t_2)$	$(t_2 - t_3)$	$(t_3 - t_4)$
Mode	IV	I	II	III
Conducting Devices	D_1 D_2	Q_1 Q_2	D_3 D_4	Q_3 Q_4
Output Voltage	+ V	+ V	– V	– V
Output Current	Negative	Positive	Positive	Negative
Power Flow	Load to source	Source to Load	Load to source	Source to Load

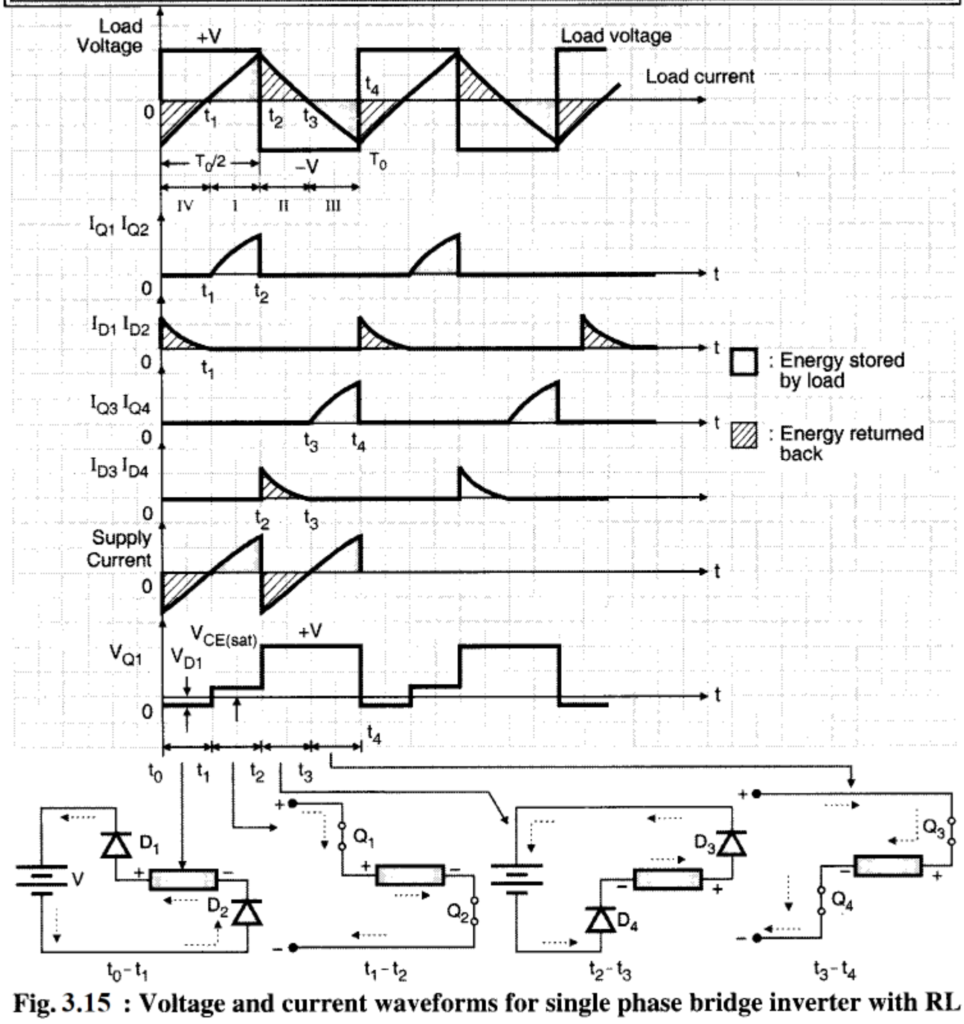


Fig. 3.15 : Voltage and current waveforms for single phase bridge inverter with RL load

Analysis of Bridge Inverter :

The output voltage waveform is as shown in Fig. **3.15** .

RMS output voltage :

- The rms output voltage can be found from the output voltage waveform of Fig. **3.15** as

$$V_{o\,rms} = \left[\frac{1}{T_o/2}\int_0^{T_o/2} V^2 dt\right]^{1/2} \qquad \textbf{...(3.35)}$$

$$\therefore \quad V_{o\,rms} = \left[\frac{2V^2}{T_o}\left(\frac{T_o}{2}-0\right)\right]^{1/2} = V \text{ volts} \qquad \textbf{...(3.36)}$$

Fourier series representation :

- Fourier series for output voltage of full bridge inverter is found in a similar manner as that of a half bridge inverter discussed in the previous section.
- The shape of the load voltage waveform of a bridge inverter is same as that of a half bridge circuit, except for the value of peak output voltage.
- The peak output voltage is "V" volts here therefore the expression for the output voltage in terms of Fourier series is expressed on the same lines, i.e. substitute the value of instantaneous output voltage as + V instead of + V/2.
- The instantaneous output voltage can be expressed in Fourier series as follows :

$$v_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi} \sin n\omega t \qquad \textbf{...(3.37)}$$

That means

$$v_o(\omega t) = \left(\frac{4V}{\pi}\sin \omega t + \frac{4V}{3\pi}\sin 3\,\omega t + \frac{4V}{5\pi}\sin 5\omega t + \ldots\right) \qquad \textbf{...(3.38)}$$

Conclusion :

This equation indicates following things :

1. The output voltage waveform contains only the odd order harmonic components i.e. 3,5,7..... . The even order harmonics (i.e. n = 2,4,6...) are automatically cancelled.
2. For n = 1 the Equation (**3.38**) gives the rms value of fundamental component as,

$$V_{o1\,rms} = \frac{4V}{\sqrt{2}\pi} = 0.9003V \text{ volts} \qquad \textbf{...(3.39)}$$

Instantaneous output current with RL load :

- For RL load the equation for the instantaneous current i_o can be found using the Equation (**3.37**).

$$i_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V}{n\pi\sqrt{R^2+(n\omega L)^2}} \sin(n\omega t - \theta_n) \qquad \textbf{...(3.40)}$$

- In this equation $Z_n = \sqrt{R^2+(n\omega L)^2}$ is the impedance offered by the load to the n^{th} harmonic component and $\frac{4V}{n\pi}$ is the peak amplitude of n^{th} harmonic voltage.

and $\theta_n = \tan^{-1}(n\omega L/R)$ **...(3.41)**

Harmonic Factor of n^{th} Harmonic HF_n :

$$HF_n = \frac{V_{on\ rms}}{V_{o1\ rms}} = \frac{1}{n}$$

Total Harmonic Distortion (THD) :

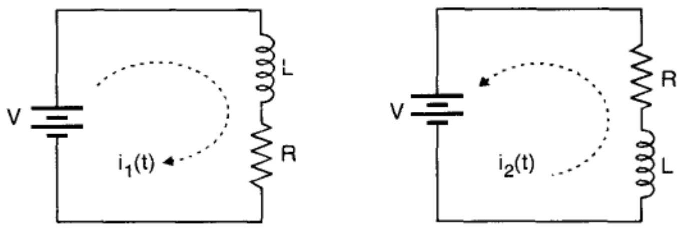
$$THD = \sqrt{\left(\frac{V_{o\,rms}}{V_{o1\_rms}}\right)^2 - 1} = \sqrt{\left(1/\frac{4}{\sqrt{2}\pi}\right)^2 - 1} = \sqrt{\left(\frac{\pi^2}{8}\right) - 1} = 0.4834 = 48.34\%$$

Comparison of Half Bridge and Full Bridge Inverters :

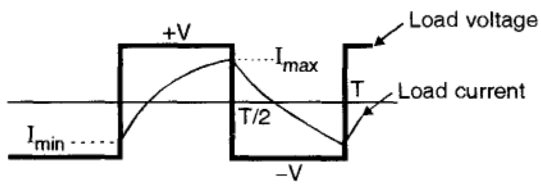
Sr. No.	Parameter	Half bridge	Full bridge
1.	Need of an output transformer	Not needed	Not needed
2.	Number of IGBTs required to be used.	Two	Four
3.	Efficiency	High	High
4.	Voltage across the nonconducting IGBT	V Volts	V Volts
5.	Output voltage waveform	Square, Quasi square or PWM	Square, Quasi square or PWM
6.	Current rating of power device	Equal to the load current	Equal to the load current
7.	Number of devices conducting simultaneously	One	Two
8.	Necessity of dead band to avoid cross conduction	Yes	Yes

Expression for peak load current for a full bridge square wave inverter operating with an inductive (RL) load

- Refer to the equivalent circuits shown in Fig. **3.16** (a) and (b) which have been drawn for the time durations 0 to T/2 and T/2 to T.



(a) Equivalent circuit for (0 ≤ t ≤ T/2) **(b) Equivalent circuit for (T/2 ≤ t ≤ T)**



(c) Load voltage and current waveforms for bridge inverter operating with RL load

Fig. 3.16

To find the equation for the peak value of load current following are the assumptions :

1. Positive peak i.e. I_{max} = Negative peak I_{min}.
2. As shown in Fig. **3.15** (c) the load voltage is positive from 0 to T/2 and negative from T/2 to T. The equivalent circuits for these periods are as shown in Fig. **3.15** (a) and Fig. **3.16** (b) respectively.

Refer to Fig. **3.16** (a), Applying KVL to this figure we can write,

$$L\frac{di_1(t)}{dt} + Ri_1(t) = V \qquad \text{...(3.42)}$$

Taking the Laplace transform and rearranging we can write,

$$I_1(S) = \frac{V}{L}\cdot\frac{1}{S}\cdot\frac{1}{\left[S+\frac{R}{L}\right]} + \frac{I_1(0)}{\left[S+\frac{R}{L}\right]} \qquad \text{...(3.43)}$$

$$\therefore \quad I_1(S) = \frac{V}{R}\left[\frac{1}{S} - \frac{1}{\left(S+\frac{R}{L}\right)}\right] + \frac{I_1(0)}{\left[S+\frac{R}{L}\right]}$$

Take the inverse Laplace we can write,

$$i_1(t) = \frac{V}{R}[1 - e^{-t/\tau}] + I_{1(0)}\,e^{-t/\tau} \qquad \text{...(3.44)}$$

where $\tau = \frac{L}{R}$ = Load time constant.

Now substitute the initial conditions as

$i_1(t) = I_{min}$ at t = 0 in Equation **3.44** we get,

$$I_1(0) = I_{min}$$

$$\therefore \quad i_1(t) = \frac{V}{R}[1 - e^{-t/\tau}] + I_{min}\,e^{-t/\tau} \qquad \text{...(3.45)}$$

Now substitute the final condition as

$i_1(t) = I_{max}$ at t = T/2 in Equation **3.45** we get

$$I_{max} = \frac{V}{R}[1 - e^{-T/2\tau}] + I_{min}\,e^{\,T/2\tau} \qquad \text{...(3.46)}$$

Since $I_{min} = -I_{max}$ we get

$$I_{max} = \frac{V}{R}\left[\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right] \qquad \text{...(3.47)}$$

and

$$I_{min} = \frac{-V}{R}\left[\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right] \qquad \text{...(3.48)}$$

thus $I_{max} = -\,I_{min}$

The equations for the instantaneous currents $i_1(t)$ and $i_2(t)$ for the two modes can be written as,

$$i_1(t) = \frac{V}{R}[1 - e^{-t/\tau}] + I_{min}\,e^{-t/\tau} \qquad \text{...(3.49)}$$

and

$$i_2(t) = \frac{-V}{R}[1 - e^{-(t-T/2)/\tau}] + I_{max}\,e^{-(t-T/2)/\tau} \qquad \text{...(3.50)}$$

Note : All the equations derived for full bridge inverter are applicable to the half bridge inverter except for the fact that "V " should be replaced by "V/2" in all the equations.