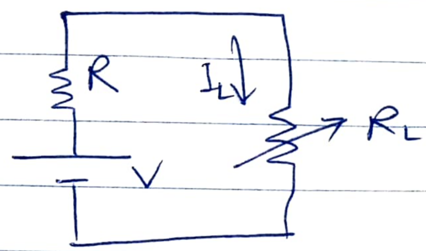
Maximum Power Transfer Theorem



Consider a ckt. with DC vltg. source V and resistance R Ω connected to variable load resistance R_L.

Load current can be calculated here as,

$$I_L = \frac{V}{R+R_L}$$

The power consumed by the load resistance R_L is,

$$P = I_L^2 R_L$$

$$= \left(\frac{V}{R+R_L}\right)^2 \cdot R_L$$

If R_L is changed, I_L will also change and at a particular value of R_L, the power transferred to the load will be max. Hence the power depends on the value of R_L. To get the ~~max~~ value of R_L at which power will be max, let's differentiate above eqn of power w.r.t. R_L & equate to zero.

$$\therefore \frac{dP}{dR_L} = 0$$

$$\therefore \frac{d}{dR_L}\left[\frac{V}{R+R_L}\right]^2 \cdot R_L = 0$$

$$V^2 \cdot \frac{d}{dR_L}\left[\frac{R_L}{(R+R_L)^2}\right] = 0$$

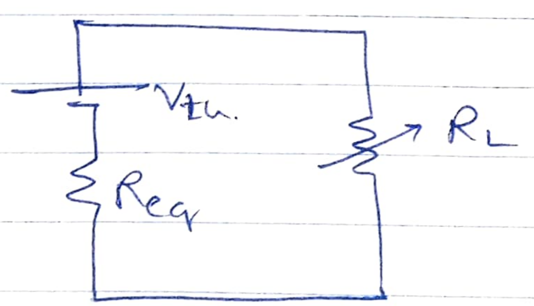
after solving this, we get;

$$R + R_L - 2R_L = 0$$

$$\therefore \boxed{R_L = R}$$

Thus when load resistance is equal to ~~internal~~ resis. of the ckt; max. power transfer takes place.

We represent the complex n/w with Thevenin's equi. ckt



If we compare ~~to~~ this ckt. with above drawn ckt., ~~we~~ we get;

$$\boxed{R_L = R_{eq}}$$ for max. power transfer.

And max. power will be calculated as,

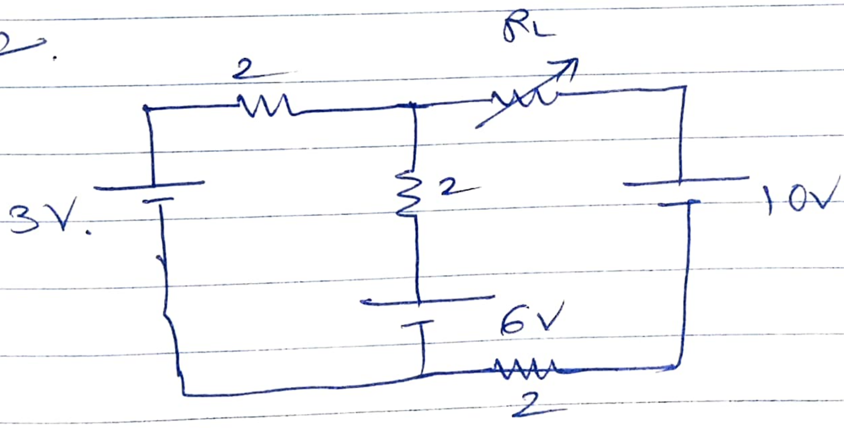
$$P_m = \left[\frac{V_{th}}{R_{eq} + R_L}\right]^2 \cdot R_L$$

with $R_L = R_{th}$;

$$P_m = \left[\frac{V_{th}}{2R_{eq}}\right]^2 \cdot R_{eq}$$

$$\boxed{P_m = \frac{V_{th}^2}{4R_{eq}} \text{ watts.}}$$

Examples.



$V_{th} = 5.5V$

$R_{eq} = 3\Omega$.

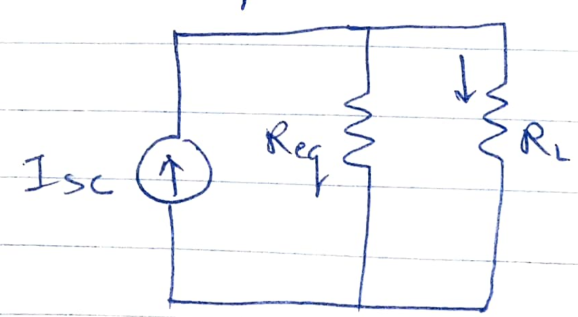
$P_m = 2.5208W$.

Norton's Theorem

This theorem is an alternative to the Thevenin's theorem. Actually, it is the dual of Thevenin's ~~theven~~ theorem. Norton's theorem replaces the network by an equivalent current source and a parallel resistance.

The Norton's theorem can be stated as, "Any two-terminal active network containing voltage sources and ~~res~~ resistances, when viewed from its output terminals, is equivalent to the constant current source and a parallel resistance. The constant current source is equal to the current which would flow in a short circuit placed across the terminals; and the parallel resistance is the resistance of the network when viewed from these open circuited terminals after all voltage & current sources have been removed and replaced by their ~~int~~ internal resistances."

Norton's equivalent circuit is,



$$\therefore I_L = I_{ss} \times \frac{R_{eq}}{R_{eq} + R_L}$$

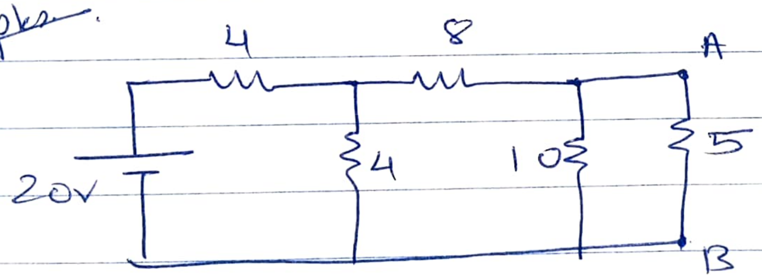
Steps.

1. Remove R_L and short circuit the terminals.
2. Calculate current through short circuit (I_{sc})
3. Remove s.c., replace sources and calculate R_{eq} as done in Thevenin's theorem.
4. Draw the Norton's equivalent circuit with I_{sc} & R_{eq} parallel with each other.
5. Connect R_L in parallel with the equi. ckt.
6. Calculate I_L using formula,

$$I_L = I_{sc} \times \frac{R_{eq}}{R_{eq} + R_L} \text{ A.}$$

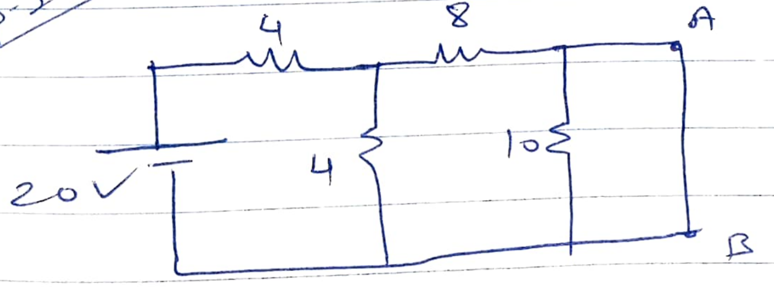
Examples.

1.



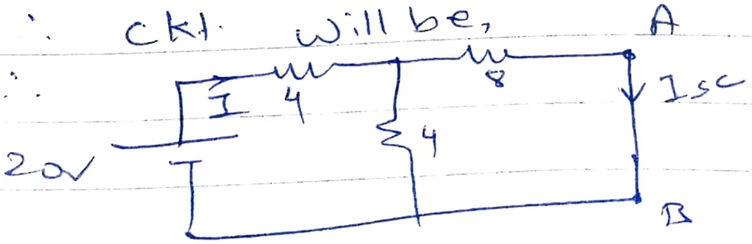
Calculate current in AB using Norton's.

Step-I



Now 10Ω has become redundant.

∴ ckt. will be,



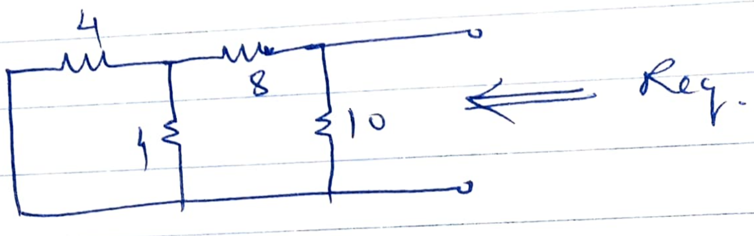
$\therefore (8 \| 4) + 4 = \frac{32}{12} + 4 = 6.667 \ \Omega$

$\therefore I = \frac{20}{6.667} = 3 A.$

$\therefore I_{sc} = I \times \frac{4}{4+8}$

$= 3 \times \frac{4}{12} = 1 A.$

step II To calculate Req.,

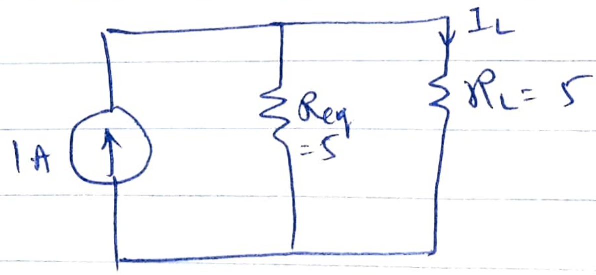


$(4 \| 4) = 2$

$2 + 8 = 10$

$(10 \| 10) = 5 = R_{eq}.$

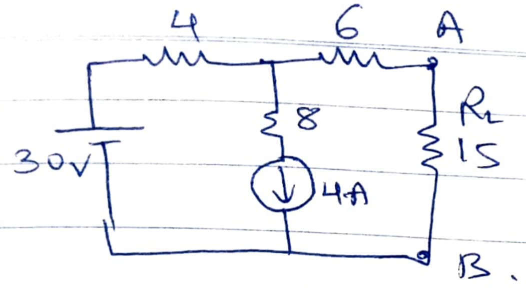
step III Equivalent circuit is,



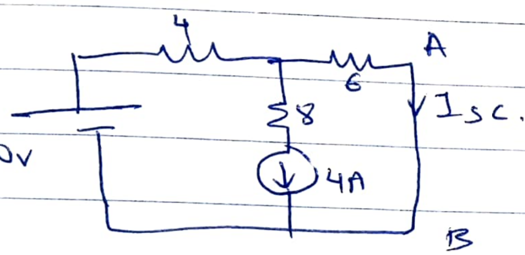
$\therefore I_L = 1 \times \frac{5}{5+5}$

$= \underline{\underline{0.5 A}}$

2.



Calculate current in 15Ω



step.I * Calculate I_{sc} by using superposition

- I_{sc}' with current source only.

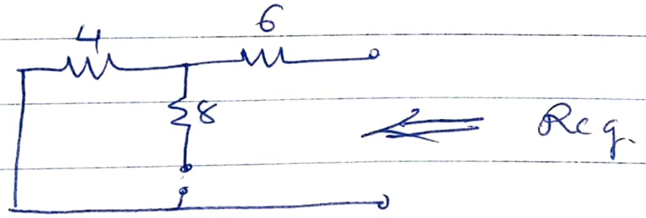
$$I_{sc}' = 4 \times \frac{4}{10} = 1.6\,A \uparrow$$

- I_{sc}'' with vltg. source only

$$I_{sc}'' = \frac{30}{10} = 3\,A \downarrow$$

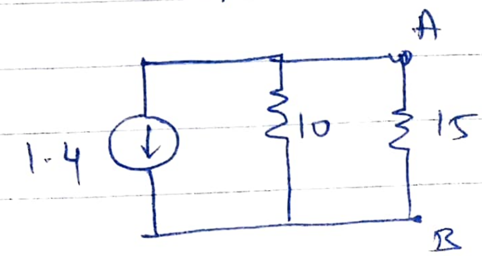
$\therefore$ $I_{sc} = I_{sc}'' - I_{sc}' = 3 - 1.6 = 1.4\,A$.

step.II To calculate Req,



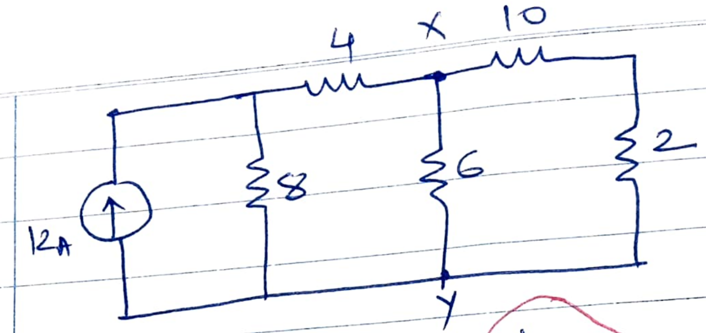
$\therefore$ $R_{eq} = 4 + 6 = 10\,\Omega$

step III Equi. ckt.

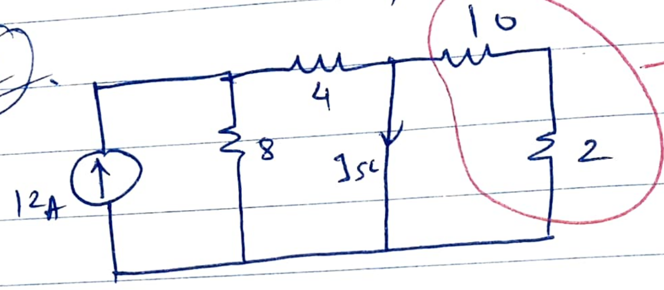


$$I_L = 1.4 \times \frac{10}{25} =$$

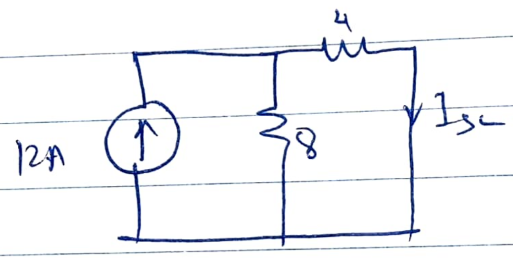
$$= 0.56\,A$$

3. 

$I_{xy} = ?$

I. 

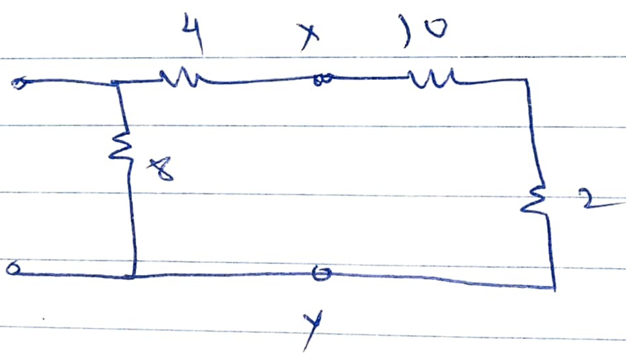
Redundant due to S.C.



$$\therefore I_{sc} = 12 \times \frac{8}{8+4}$$

$$= 8A$$

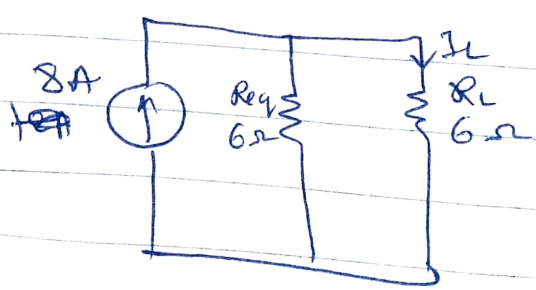
II.



$$\therefore R_{eq} = (8+4) \parallel (10+2)$$

$$= 6\ \Omega$$

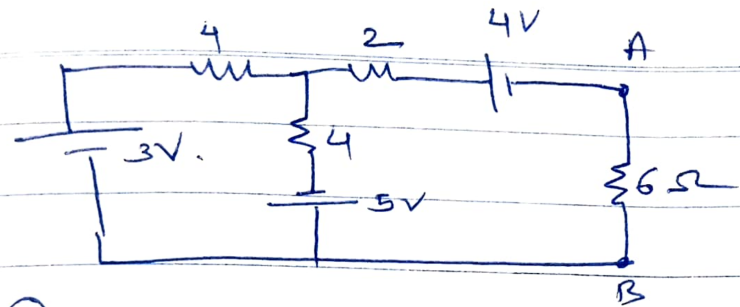
Equi. ckt



$$\therefore I_L = 8 \times \frac{6}{6+6}$$

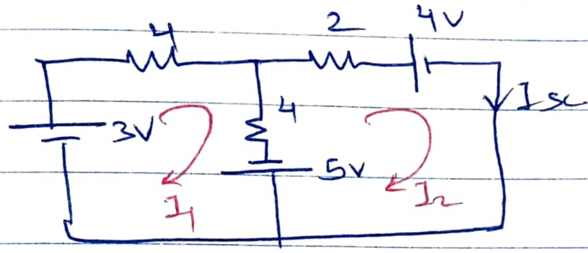
$$= 4A$$

4.



$I_{AB} = ?$

I



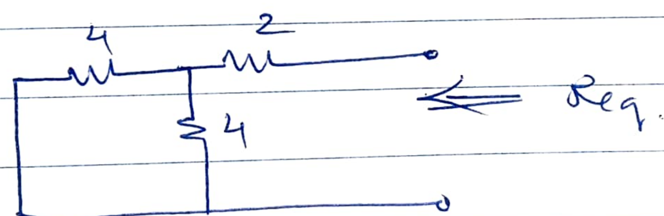
I_2 is I_{sc}

$$3 - 4I_1 - 4(I_1 - I_2) + 5 = 0$$

$$-2I_2 - 4 - 5 - 4(I_2 - I_1) = 0$$

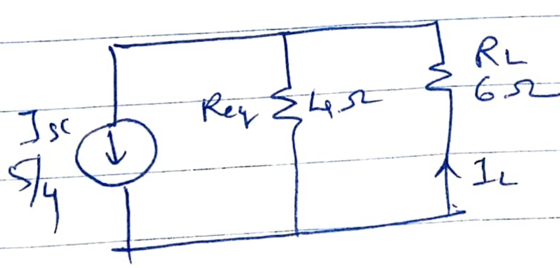
$$\therefore I_2 = -5/4 \text{ A}$$

II



$$R_{eq} = (4 \| 4) + 2 = 4\Omega$$

Equi. ckt.



$$\therefore I_L = \frac{5}{4} \times \frac{4}{4+6}$$

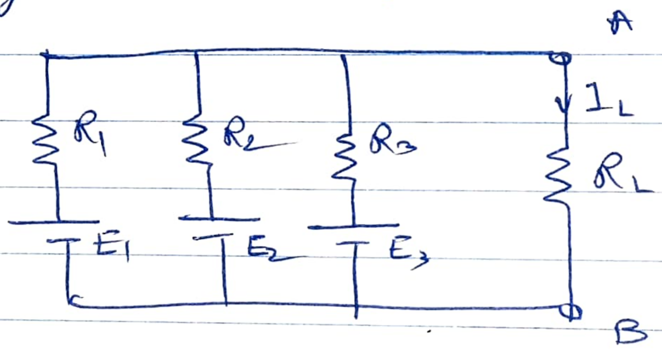
$$= 0.5 \text{ A} \uparrow$$

Millman's Theorem

This theorem can be stated either in-terms of voltage sources or current sources or both.

a] **As applicable to voltage sources:-**

This theorem is a combination of Thevenin's & Norton's theorems. It is used for finding common voltage across any n/w which contains a no. of parallel voltage sources as shown below:-



The common voltage V_{AB} which appears across the o/p terminals A & B is affected the voltage sources E_1, E_2 and E_3. The value of the voltage is given by,

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G}$$

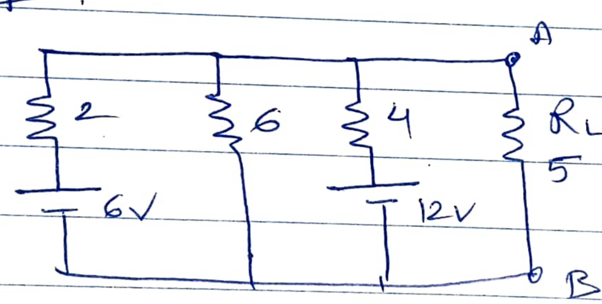
This voltage V_{AB} represents the Thevenin's voltage V_{th}:

The resistance R_{th} or R_{eq} can be calculated as usual, by replacing each vltg. source by a s.c.. If there is a load resistance R_L across terminal ~~AB~~ A & B, the load current I_L is given by,

$$I_L = \frac{V_{th}}{R_{eq} + R_L}$$

* if a branch does not contain a vltg. source, then the vltg. is equated to zero.

Example.



$$V_{AB} = \frac{6/2 + 0/6 + 12/4}{1/2 + 1/6 + 1/4} = 6.55V = V_{th}$$

$$R_{th} = 2 \| 6 \| 4 = 12/11 \ \Omega.$$

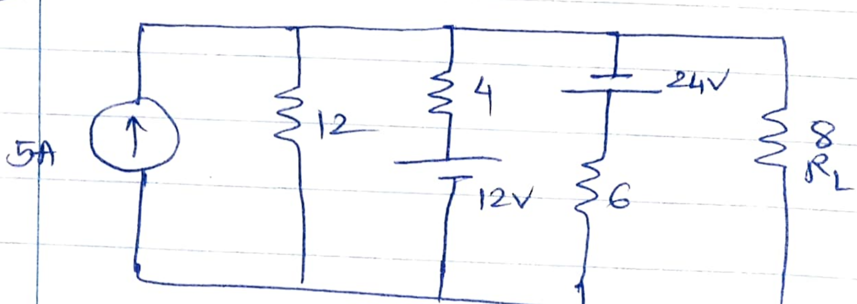
$$\therefore I_L = \frac{6.55}{(12/11) + 5} = 1.07 \ A.$$

b) As applicable to current sources.

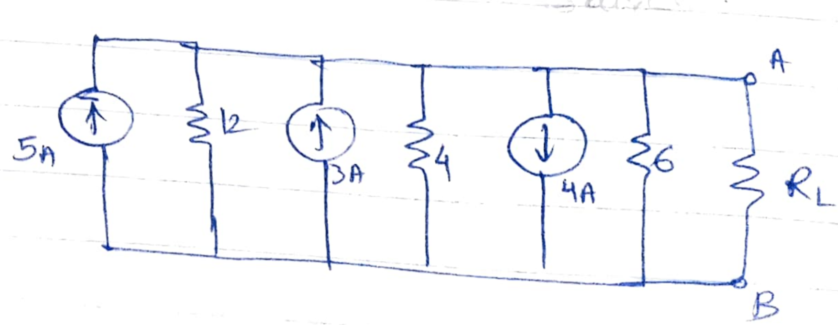
This theorem is applicable to a mixture of parallel voltage & current sources that are reduced to a single final equivalent source which is either a constant current or a constant voltage source. The theorem can be stated as, "Any no. of constant current sources which are directly connected in parallel can be converted into a single current source whose value is algebraic sum of the individual source currents and whose total internal resistance equals the combined individual source resistances in parallel."

Ex:

Calculate voltage across & current in R_L



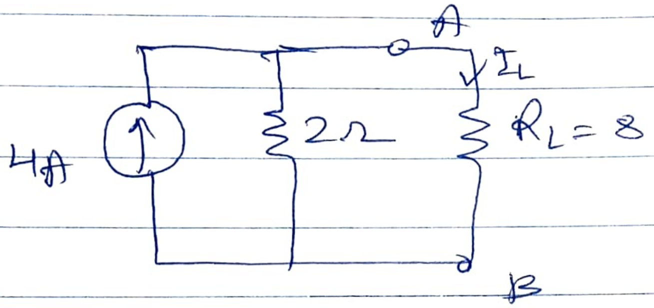
Converting to current sources



$\therefore$ Net current = $5+3-4 = 4A \uparrow$

Combined resis. = $12 \| 4 \| 6 = 2\Omega$

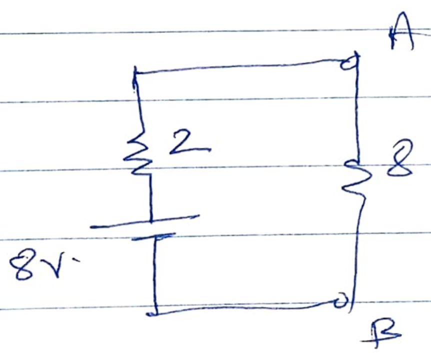
$\therefore$ Simplified circuit is,



$$\therefore I_L = 4 \times \frac{2}{2+8} = 0.8A.$$

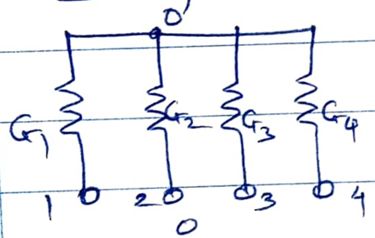
$$V_L = 8 \times 0.8 = 6.4V.$$

OR convert it into vltg. source



$$I = \frac{8}{2+8} = 0.8A.$$

Generalized form of Millman's theorem



Consider no. of admittances $G_1, G_2, G_3, \ldots G_n$ terminate at common point O'. The other ends of admittances are named as $1, 2, 3, \ldots n$.

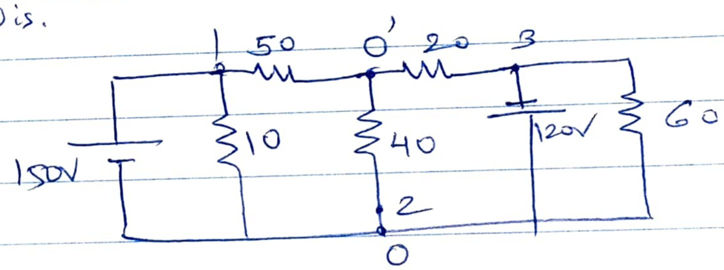
Let O be any other point in the n/w.

Acc. to this theorem, the vltg. drop from O to O' ($V_{OO'}$) is given by,

$$V_{OO'} = \frac{V_{O1}G_1 + V_{O2}G_2 + V_{O3}G_3 + \ldots V_{On}G_n}{G_1 + G_2 + G_3 + \ldots G_n}$$

**Note:- Only those resistances or admittances are taken into consideration which terminate at the common point. All those admittances are ignored which donot terminate at the common point even though they are connected in the circuit.

Ex. calculate vltg. developed across 40 Ω resis.



Let two ends of 40 Ω resistor be marked as O & O'.

The end points of the three resistors terminating at the common point O' have been marked 1, 2, and 3.

- 10 Ω & 60 Ω resistors will not come in calculation because they are not directly connected to common point O'.

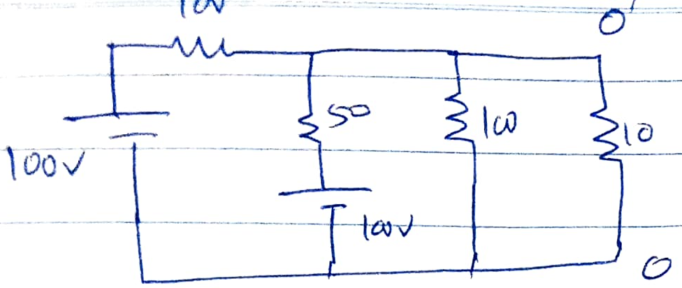
Here voltage $V_{01} = -150\,V$

$V_{02} = 0\,V$

$V_{03} = 120\,V$.

$$\therefore V_{00'} = \frac{(-150/50) + (0/40) + (120/20)}{1/50 + 1/40 + 1/20}$$

$= 31.6\,V$.



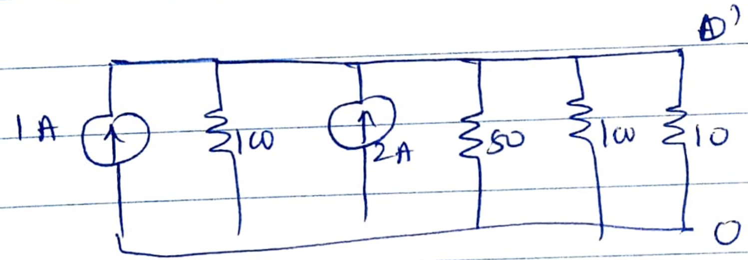
$$V_{o1} = -100, \quad V_{o2} = -100, \quad V_{o3} = 0, \quad V_{o4} = 0$$

$$\therefore V_{oo'} = \frac{(-100/100) + (-100/50) + (0/100) + (0/10)}{(1/100 + 1/50 + 1/100 + 1/10)}$$

$$= -21.4$$

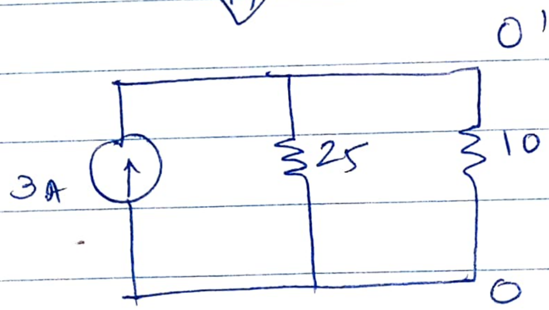
Method - II.

Use source conversion



$\Downarrow$

$100 \| 50 \| 100 = 25\Omega$



$$I_L = 3 \times \frac{25}{35} = 2.1428 \text{ A}$$

$$\therefore V_{oo'} = 10 \times 2.1428 = 21.4 \text{V}$$