MIT-WPU

॥ विश्वशान्तिर्ध्रुवं ध्रुवा ॥

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MIT WORLD PEACE UNIVERSITY | PUNE

TECHNOLOGY, RESEARCH, SOCIAL INNOVATION & PARTNERSHIPS

CET207A Data Structure-I

S. Y. B. Tech CSE **Trimester – V**

SCHOOL OF COMPUTER ENGINEERING AND TECHNOLOGY

Stack

Topics to be Covered

- Stack as an Abstract Data Type
- Representation of Stack Using Sequential Organization
- Applications of Stack- Expression Conversion and Evaluation
- Recursion

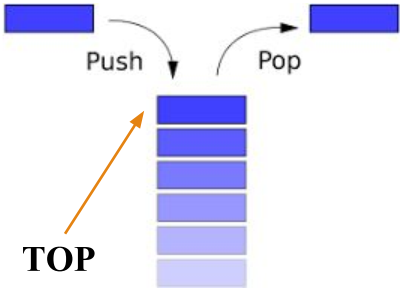
Unit-III

Stacks

Real life Applications of Stack

Stack

- Stack : Special case of ordered list also called as restricted/controlled list where insertion and deletion happens at only one end called as top of stack (homogeneous collection of elements.)
- Elements are added to and removed from the top of the stack *(the most recently added items are at the top of the stack).*
- The last element to be added is the first to be removed (**LIFO**: Last In, First Out).

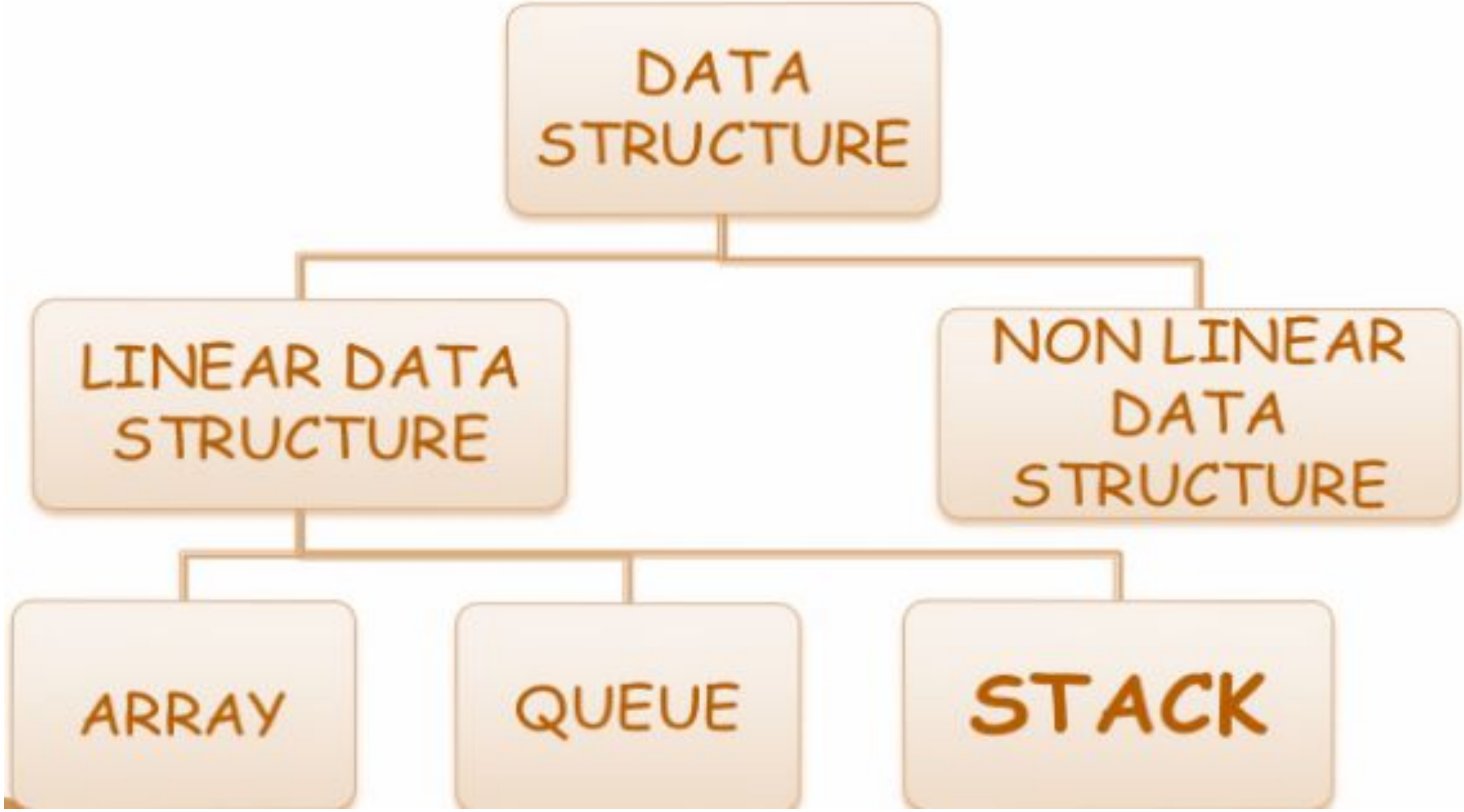


•Only access to the stack is the top element

- consider trays in a cafeteria

--to get the bottom tray out, you must first remove all of the elements above

Where stack resides in data structure family?



Stack ADT

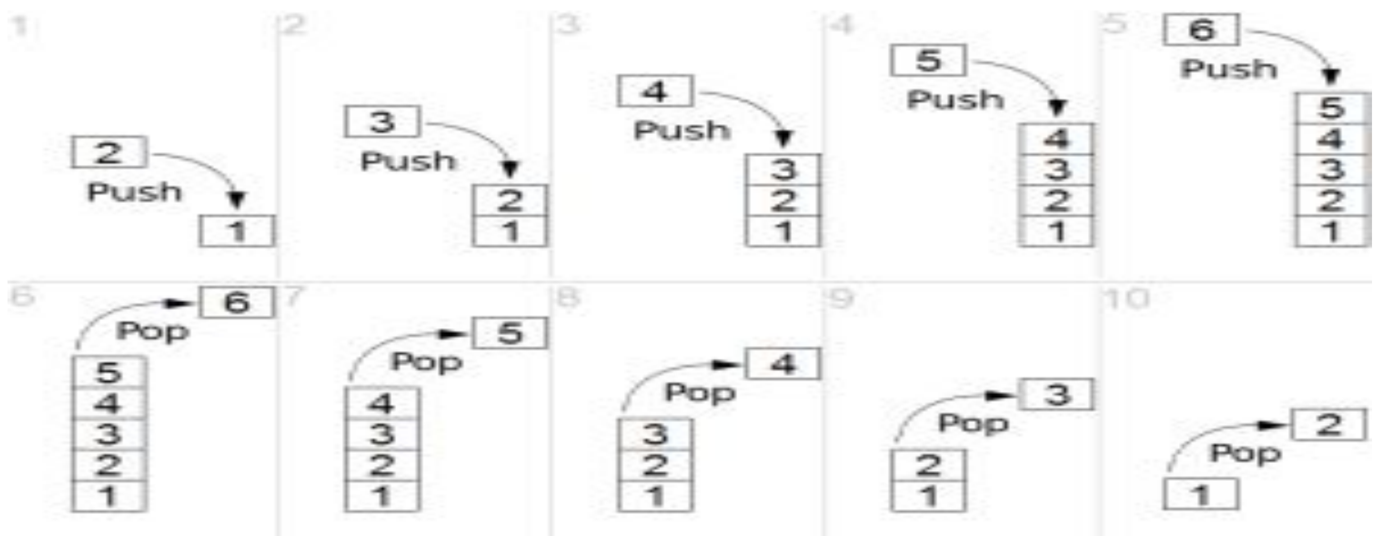
```
structure STACK (item)
 declare CREATE() -> stack
         ADD(item, stack) -> stack
         DELETE(stack) -> stack
         TOP(stack) -> item
         ISEMPS(stack) -> boolean;
 for all S ∈ stack,  i ∈ item let
     ISEMPS(CREATE) ::= true
     ISEMPS(ADD(i,S)) ::= false
     DELETE(CREATE) ::= error
     DELETE(ADD(i,S)) ::= S
     TOP(CREATE)
     TOP(ADD(i,S)) ::=  i
 end

 end STACK
```

Basic Stack Operations

Operations-

- ➢ isEmpty()-Checking stack is empty
- ➢ IsFull()-Checking stack is full
- ➢ push()-Pushing Element on top of stack
- ➢ pop() – Popping top element from stack



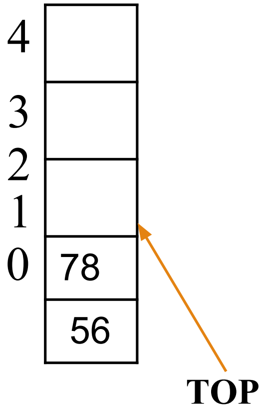
Representation of stack

Stack can be represented(implemented) using two data structures

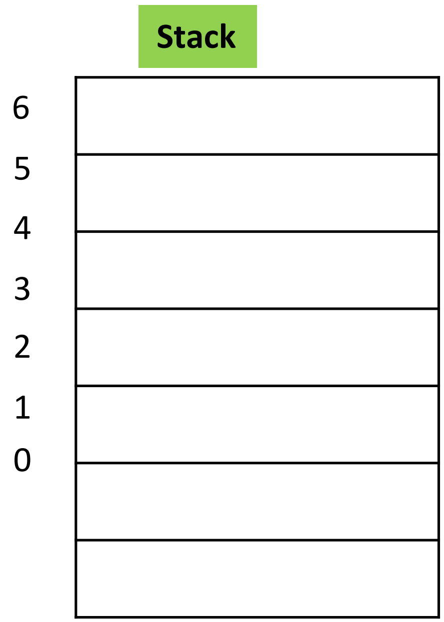
- **Array (Sequential Organization)**
- **Linked List (Linked Organization)**

Representation of Stack using Sequential Organization

- Allocate an array of some size (pre-defined)
  - Maximum N elements in stack
- Index of bottom most element of stack is 0
- Increment *top* when one element is pushed, decrement after pop
- Index of the most recently added item is given by *top*



Stack Operations using Array : Example



Stack Max Size=07

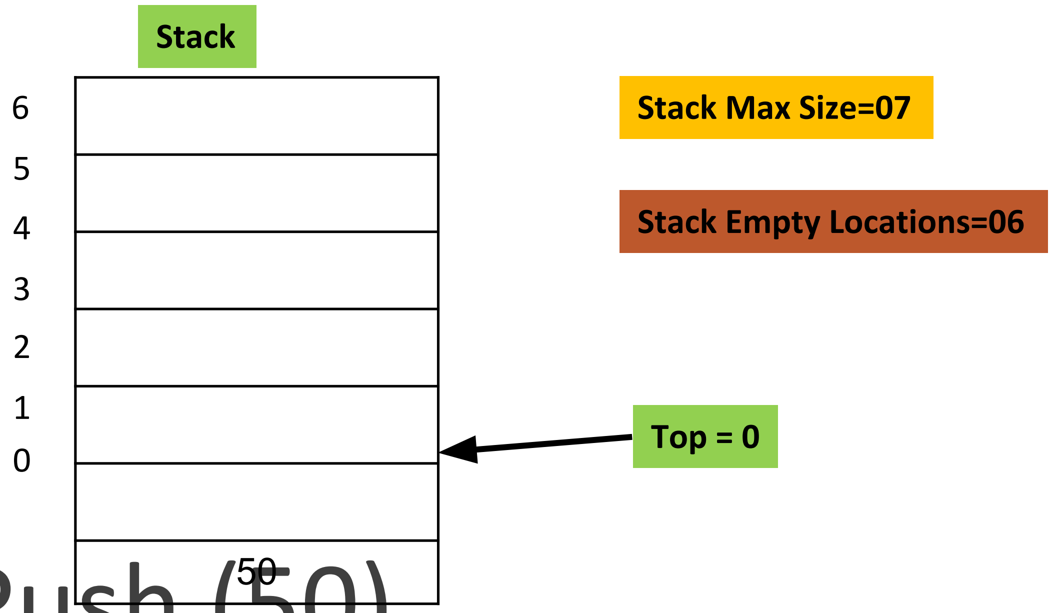
Stack Empty Locations=07

Top = -1

Stack Empty

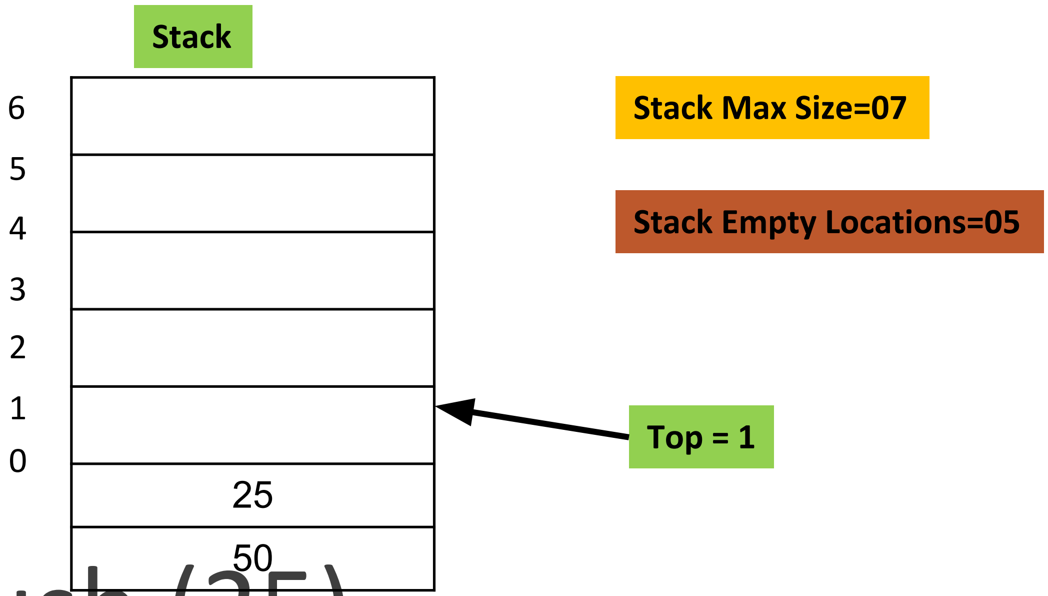
Empty Stack

Stack Operations using Array : Example



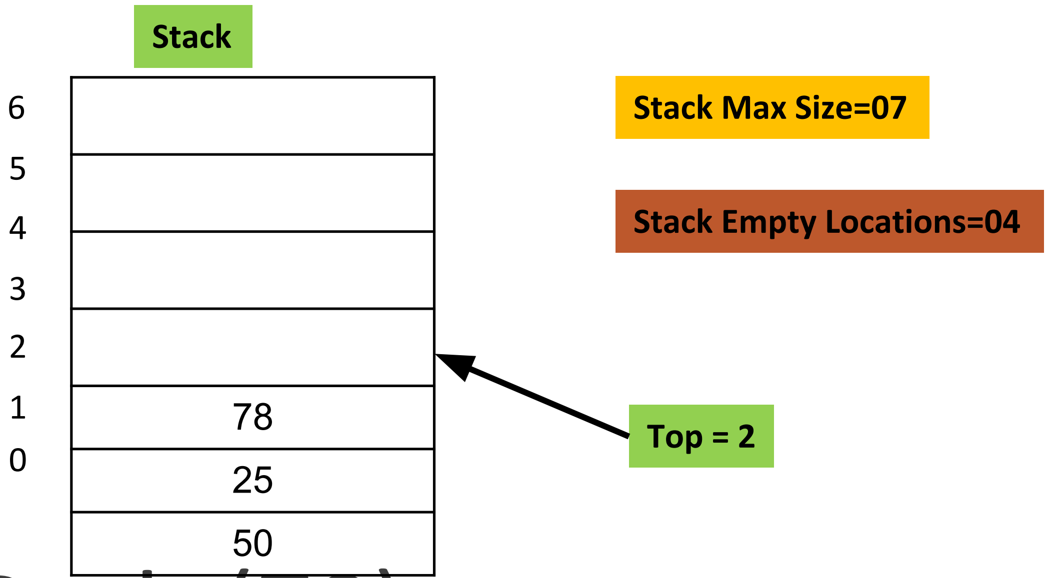
Push (50)

Stack Operations using Array : Example



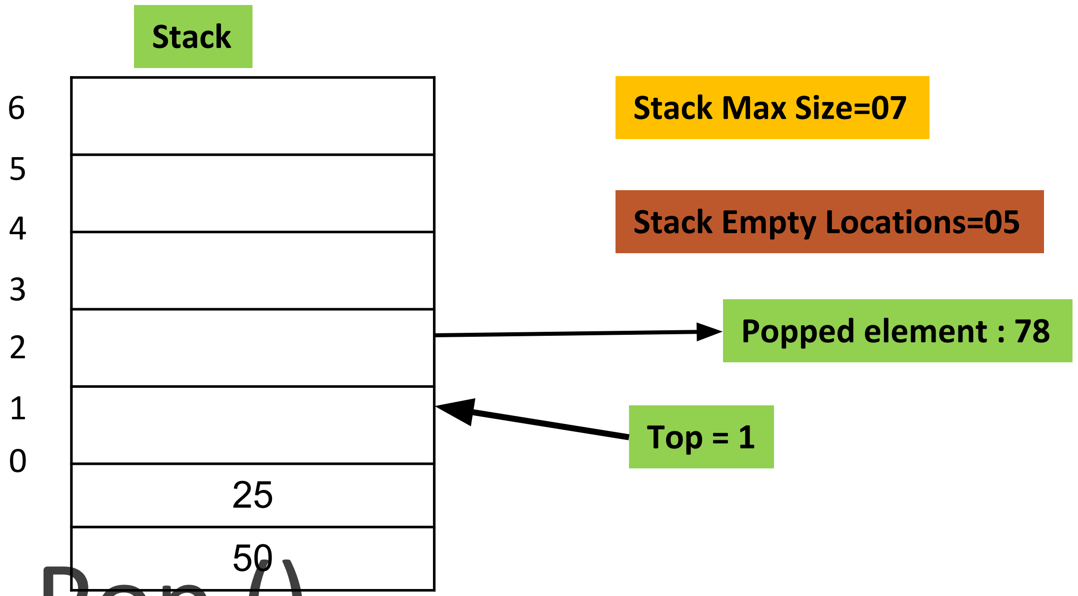
Push (25)

Stack Operations using Array : Example



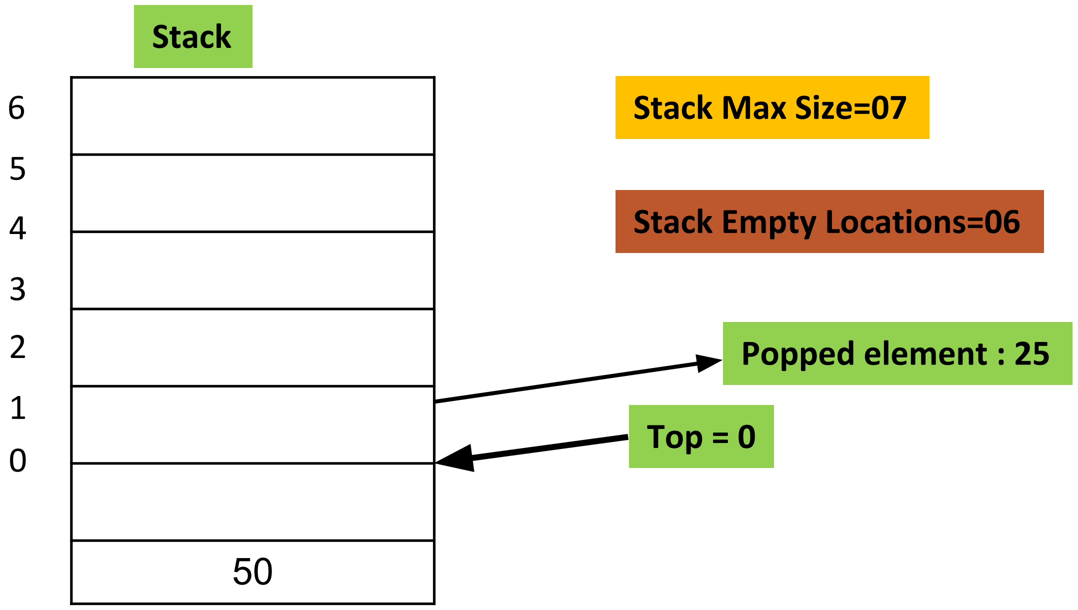
Push (78)

Stack Operations using Array : Example



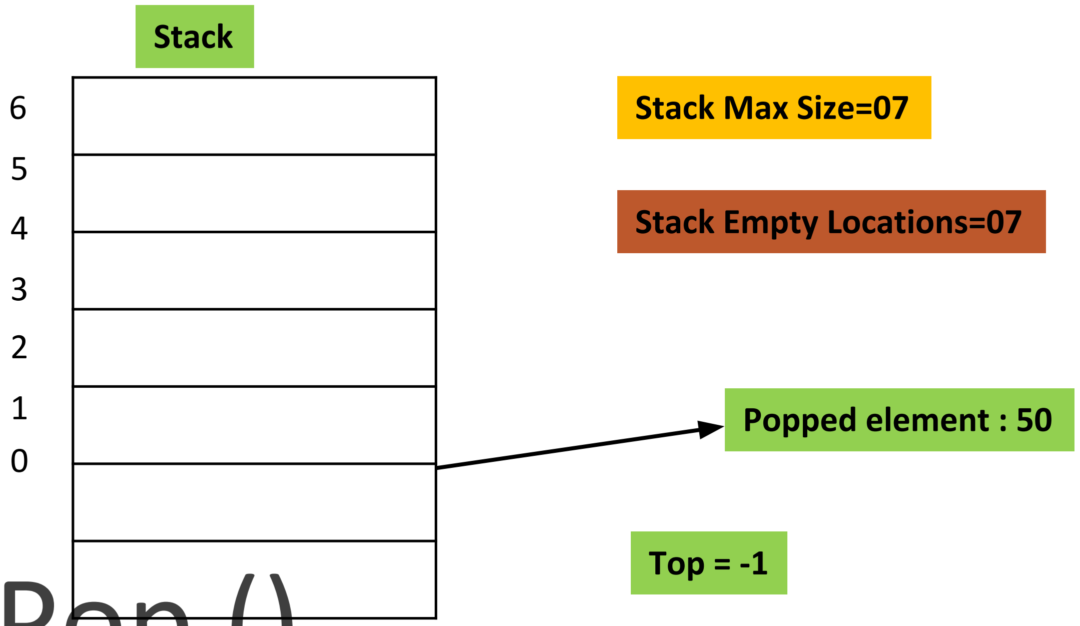
Pop ()

Stack Operations using Array : Example



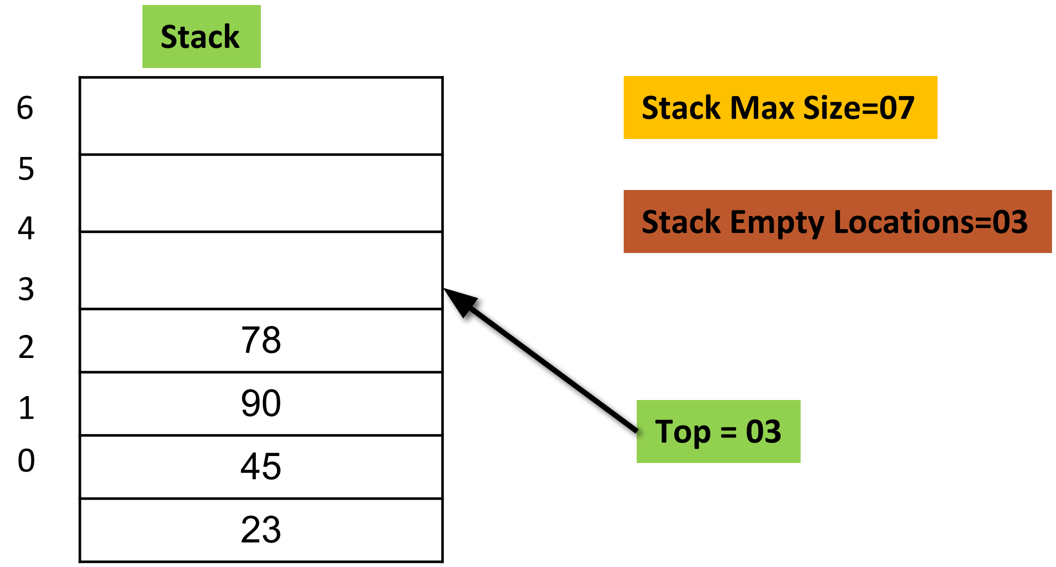
Pop ()

Stack Operations using Array : Example



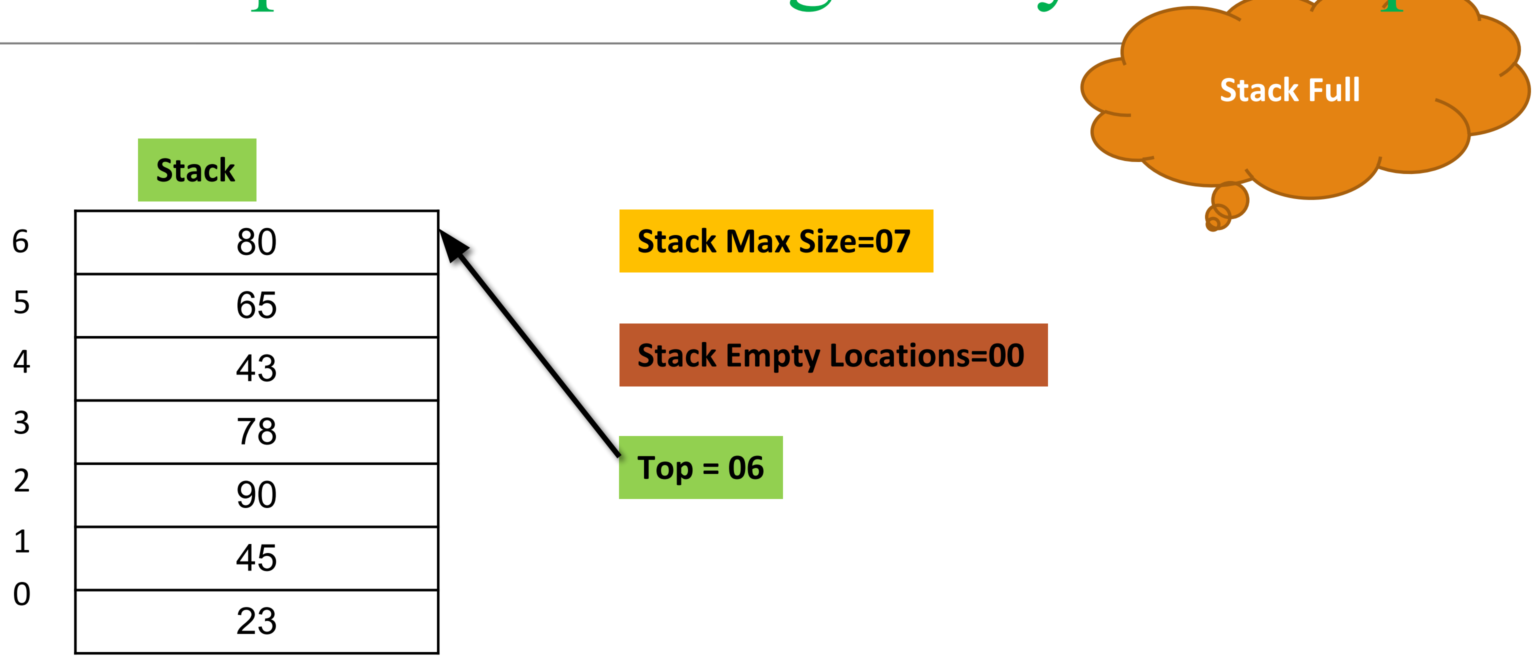
Pop ()

Stack Operations using Array : Example



Push (23),Push(45),Push(90),Push(78)

Stack Operations using Array : Example

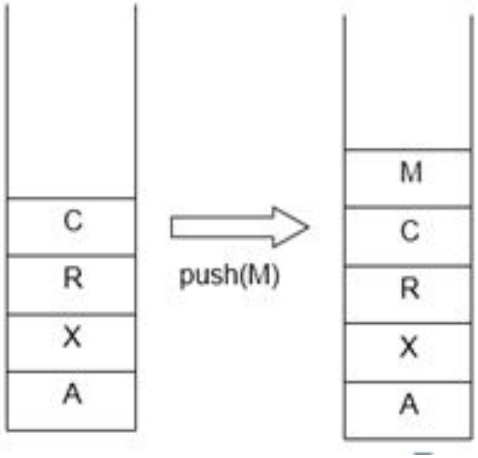


Push (43),Push(65),Push(80)

Operations on Stack (Algorithm and Pseudo Code)

Push (ItemType newItem)

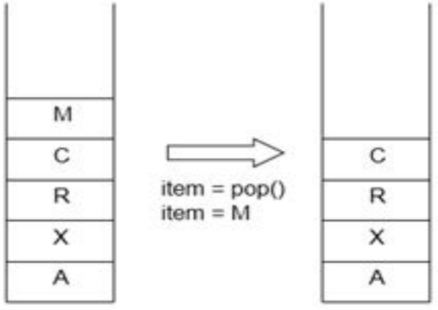
- ***Function***: Adds newItem to the top of the stack.
- ***Preconditions***: Stack has been initialized and is not full
- ***Postconditions***: newItem is at the top of the stack.



Operations on Stack (Algorithm and Pseudo Code)

Pop ()

- ***Function***: Removes top Item from stack and returns it .
- ***Preconditions***: Stack has been initialized and is not empty.
- ***Postconditions***: Top element has been removed from stack



Operations on Stack(Algorithm and Pseudo Code)

isFull()

Stack overflow

The condition resulting from trying to push an element onto a full stack.

Pseudo of isFull() function −

```
Algorithm isFull()
  {
     if (top ==MAXSIZE-1)
     return true ;
     else
     return false ;
  }
```

Operations on Stack (Algorithm and Pseudo Code)

Pseudo Code for Push

```
Algorithm push (stack, item)
  {
   if ( ! isFull ())
  {
  top=top+1
  stack[top]=item;
  }
}
```

Operations on Stack (Algorithm and Pseudo Code)

isEmpty()

Stack underflow (check if stack is empty.)

The condition resulting from trying to pop an empty stack.

Pseudo of isEmpty() function −

```
Algorithm isEmpty( )
    {
    if (top = = -1)
    return true ;
    else
    return false ;
    }
```

Operations on Stack (Algorithm and Pseudo Code)

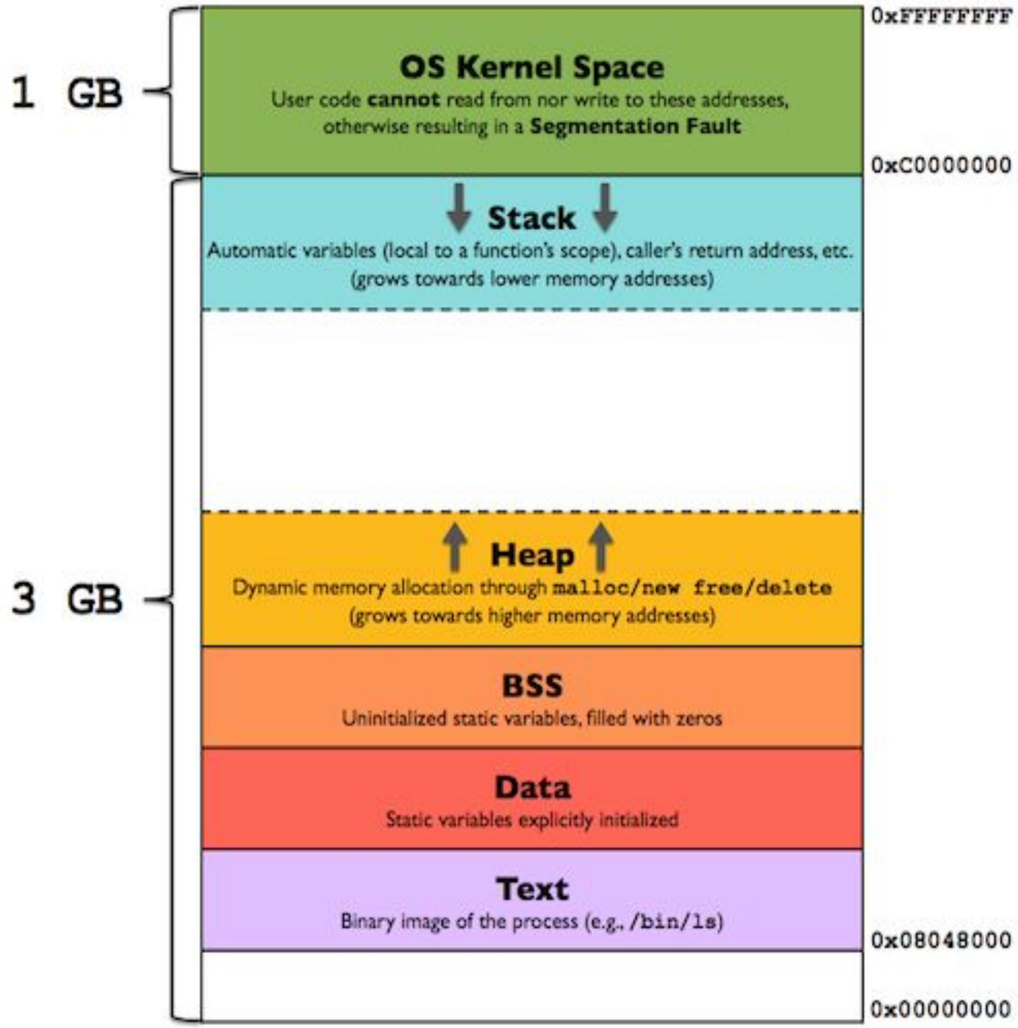
Pseudo Code for Pop

```
Algorithm  pop ()
  {
   if ( ! isEmpty())
   {
   temp=stack[top];
   top=top-1;
   return (temp);
   }
}
```

Applications of Stacks in Computer Science

- Process Function Calls
- Recursive Functions Calls
- Converting Expressions
- Evaluating expressions
- String Reverse
- Number Conversion
- Backtracking
- Parenthesis Checking

Program/Process in memory

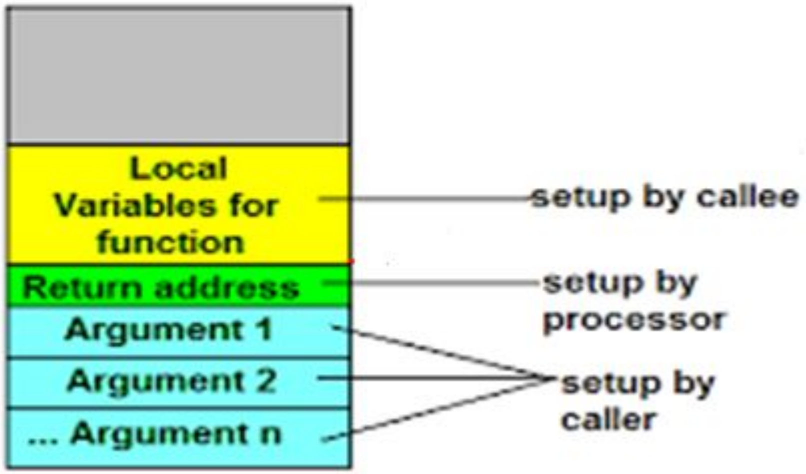


Applications of Stacks *(cont'd)*

1. **Processing Function calls:**

A stack is useful for the compiler/operating system to store local variables used inside a function block, so that they can be discarded once the control comes out of the function block.

When function execution completes, it is popped from stack



Applications of Stacks *(cont’d)*

```
main()
{
        int  a=10,b=15;
        float avg;
        avg = average(a,b,2);
        printf(“%f”,avg);
}
```

```
float average(int x,y,n)
{       float avg;
        int sum=add(x,y);
        avg = sum/n;
        return avg;
}
```

```
int add(int x,int  y)
{
        int sum=x+y;
        return sum;
}
```

Stack
Local variable of add
return address of add
Parameters of add
Local variables of average
return address of average
Parameters of average
Local variables
Return Address of Main()

Pushing add() Call

Pushing average() Call

Applications of Stacks *(cont'd)*

2. Recursive functions:

The stack is very much useful while implementing recursive functions.

The return values and addresses of the function will be pushed into the stack and the lastly invoked function will first return the value by popping the stack.

```
factorial(int x)
{
If(x==1)
return(1);
else
return(x* factorial(x-1));
}
```

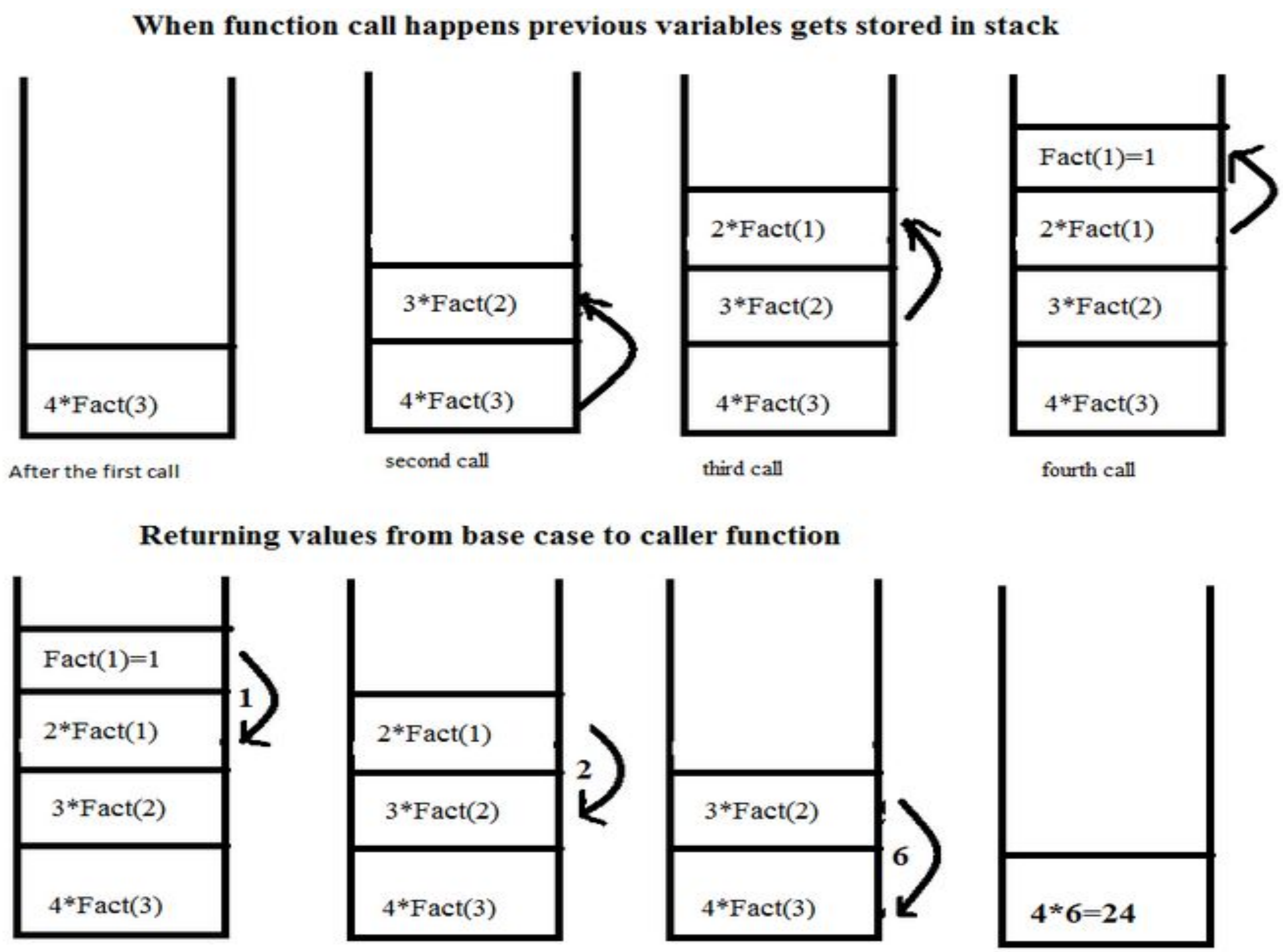
```
main()
{
factorial(4);
}
```

factorial(4)=
4*factorial(3);
4*3*factorial(2);
4*3*2*factorial(1);
4*3*2*1

factorial(1)
factorial(2)
factorial(3)
factorial(4)
main()

Will return value 1 and exit

Applications of Stacks *(cont'd)*



Applications of Stacks *(cont'd)*

Expression Conversion: There are different types of Expressions

1) Infix expression :- It is the general notation used for representing expressions.

"In this expression the operator is fixed in between the operands"

Ex: a + b*c

2) Postfix fix expression :- (Reverse polish notation)

"In this expression the operator is placed after the operands".

Ex : abc*+

3) Prefix expression :- (Polish notation)

"In this expression the operators are followed by operands i.e the operators are fixed before the operands"

Ex : *+abc

All the infix expression will be converted into post fix or prefix notation with the help of stack in any program.

Expression Conversion

Why to use PREFIX and POSTFIX notations when we have simple INFIX notation?

- INFIX notations are not as simple as they seem specially while evaluating them.

- To evaluate an infix expression we need to consider Operators' Priority and Associative Property

E.g. exp

To solve this problem Precedence or Priority of the operators were defined

Expression Conversion *(cont'd)*

Operator Precedence

- Operator precedence governs evaluation order. An operator with higher precedence is applied before an operator with lower precedence.

Rank	Operator
1	^
2	* / %
3	+ - (Binary)

Expression Conversion *(cont'd)*

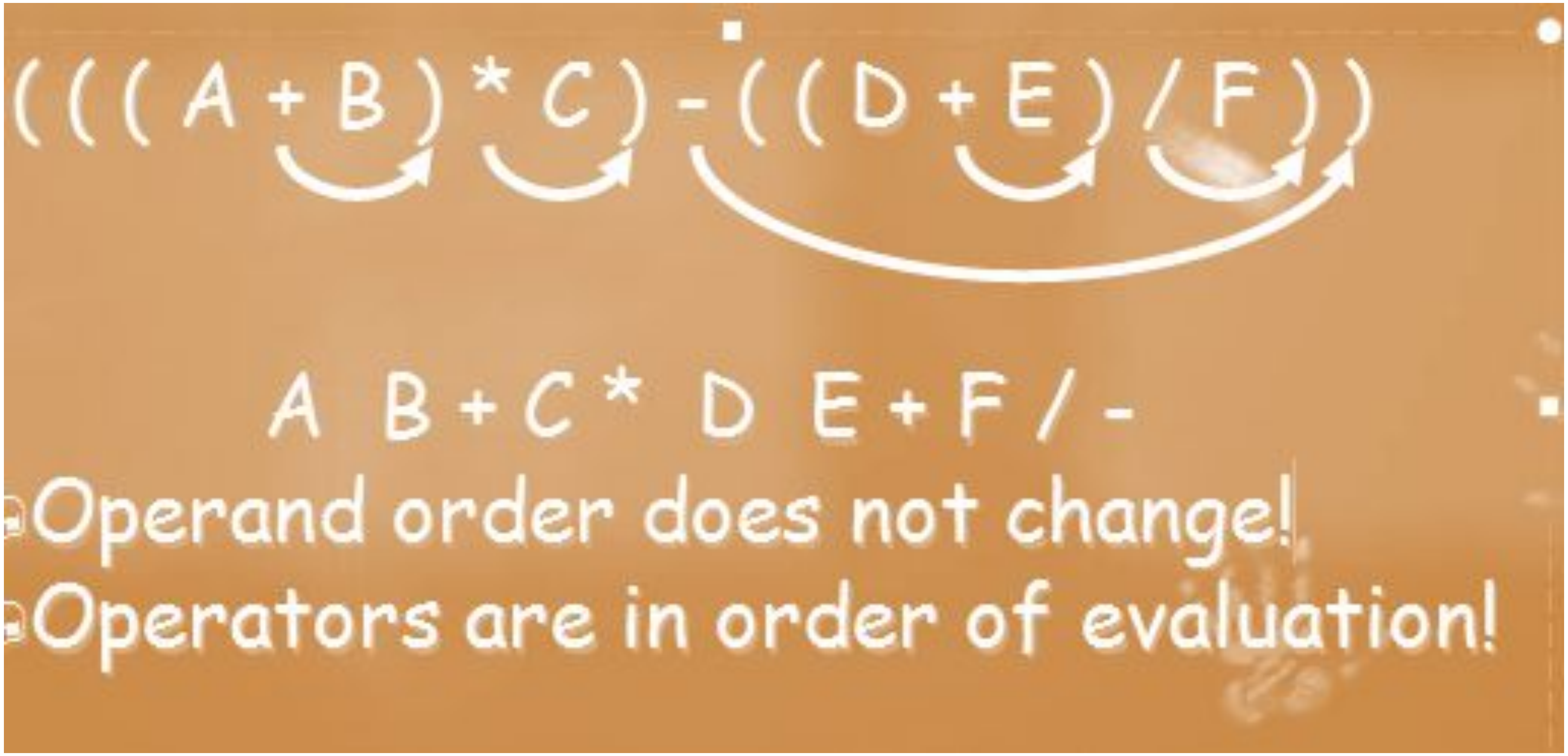
Expression Conversion Forms

Infix	Postfix	Prefix
A+B	AB+	+AB
(A+B) * (C + D)	AB+CD+*	*+AB+CD
A-B/(C*D^E)	ABCDE^*/-	-A/B*C^DE

We can convert any expression to any other two forms using Stack Data Structure

Infix to Postfix

Expression Conversion Infix to Postfix Example



Infix to Postfix

Operator Precedence (In stack and Incoming precedence)

- Operator precedence governs evaluation order. An operator with higher precedence is applied before an operator with lower precedence.

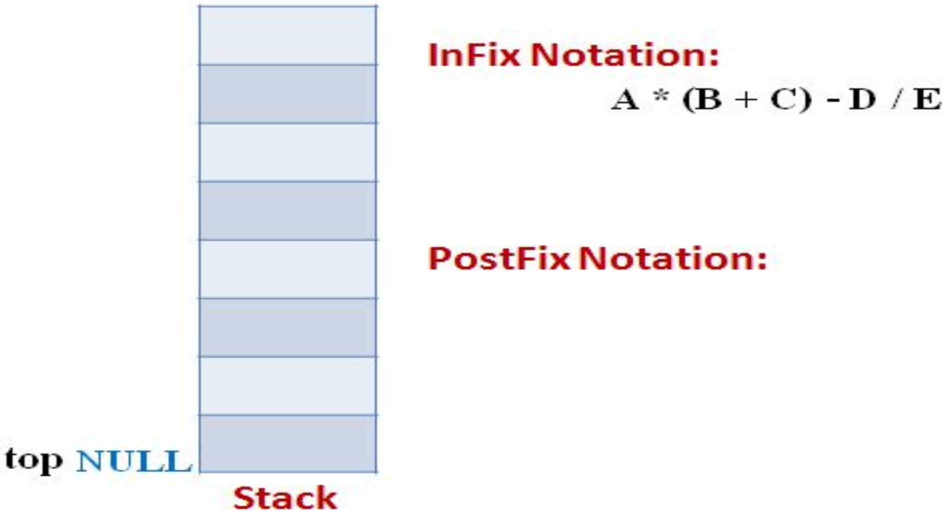
Operator	ICP	ISP
(	5	0
^	4	3
* / %	2	2
+ - (Binary)	1	1

Infix to Postfix : Example 1

★Let the incoming the Infix expression be:

A * (B + C) – D / E

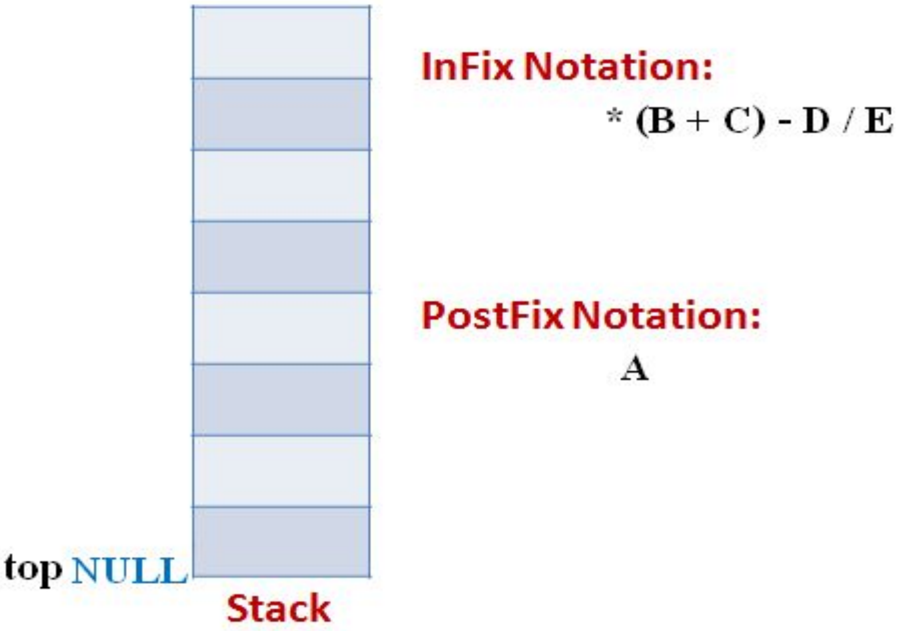
Stage 1: **Stack is empty** and we only have the Infix Expression.



Infix to Postfix : Example 1

Stage 2

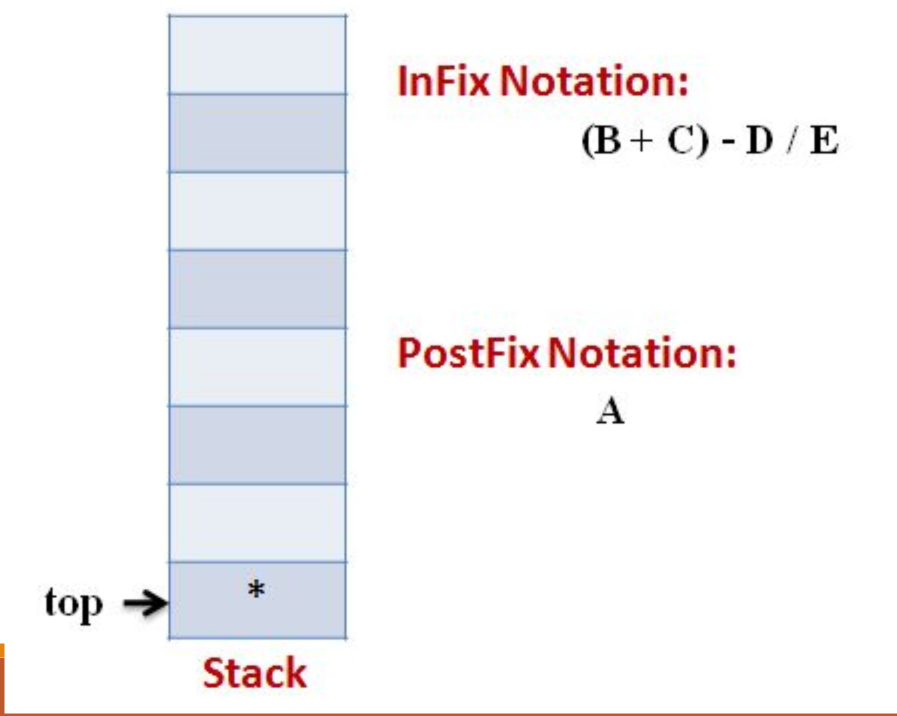
★The first token is **Operand A** Operands are Appended to the Output as it is.



Infix to Postfix : Example 1

Stage 3

★Next token is * Since **Stack is empty (top==-1)** it is **pushed into the Stack**

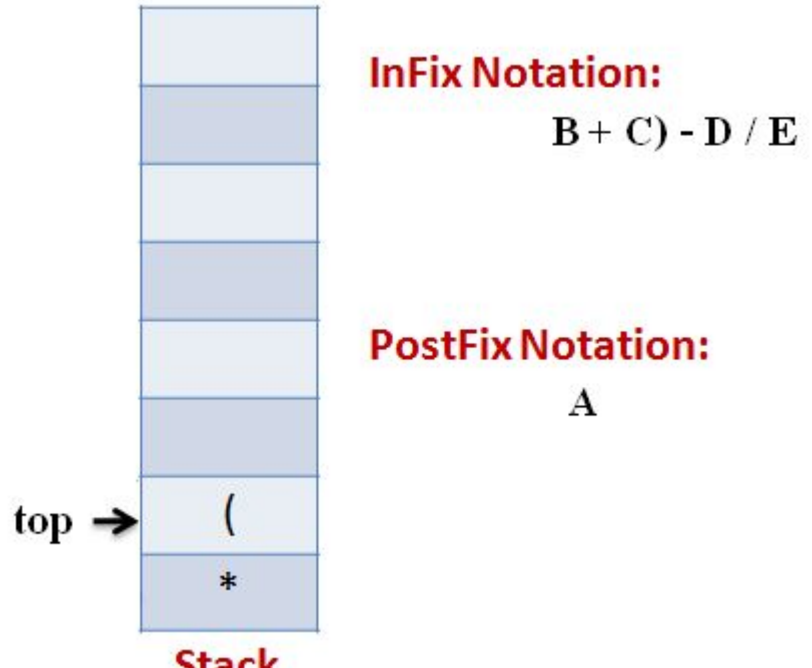


Infix to Postfix : Example 1

Stage 4

★~~Next token is **(** the precedence of open-parenthesis, when it is to go inside,~~ is maximum.

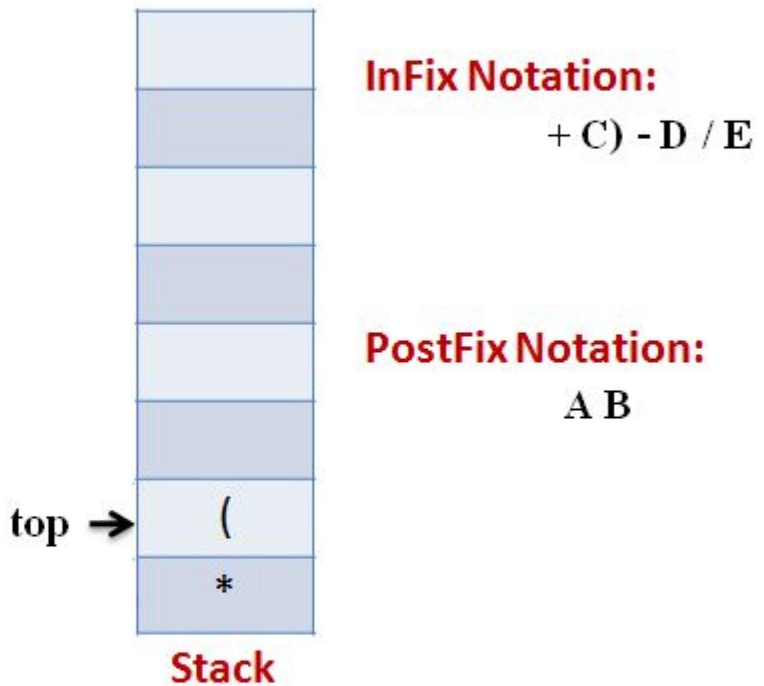
★But when another operator is to come on the top of **‘(‘** then its precedence is least.



Infix to Postfix : Example 1

Stage 5

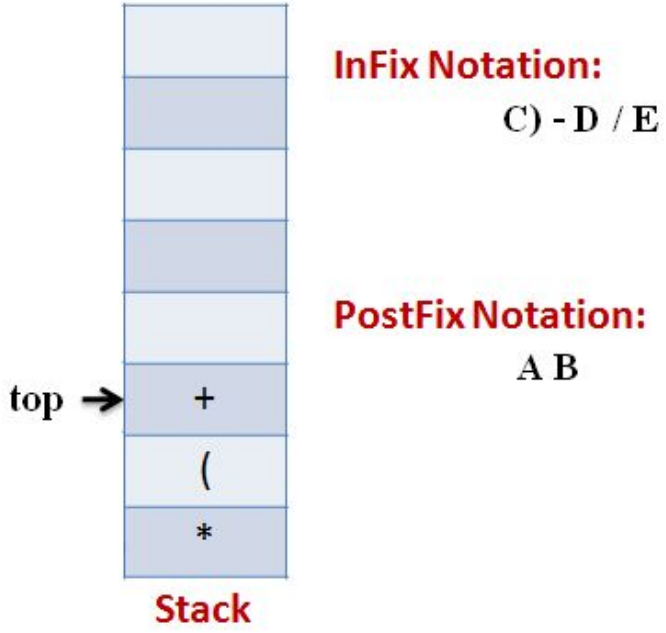
★Next token, **B** is an operand which will go to the Output expression as it is



Infix to Postfix : Example 1

Stage 6

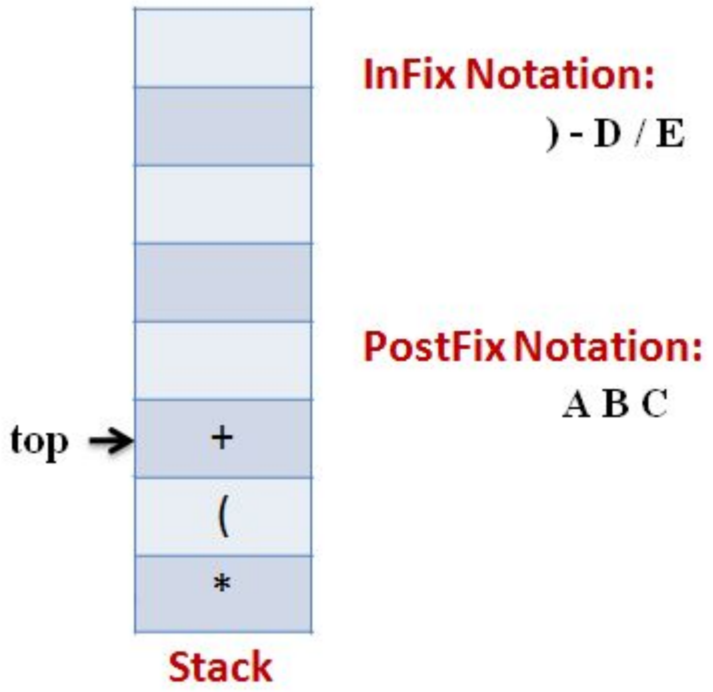
★Next token, **+** is operator, We consider the precedence of **top element in the Stack**, **‘(**‘. The outgoing precedence of open parenthesis is the least (refer point 4. Above). So **+** gets **pushed into the Stack**



Infix to Postfix : Example 1

Stage 7

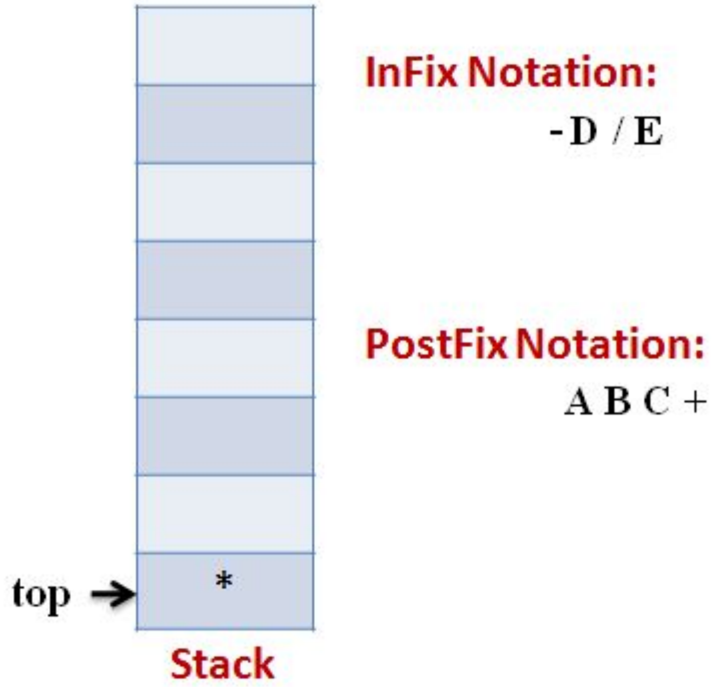
★ Next token, **C**, is appended to the output



Infix to Postfix : Example 1

Stage 8

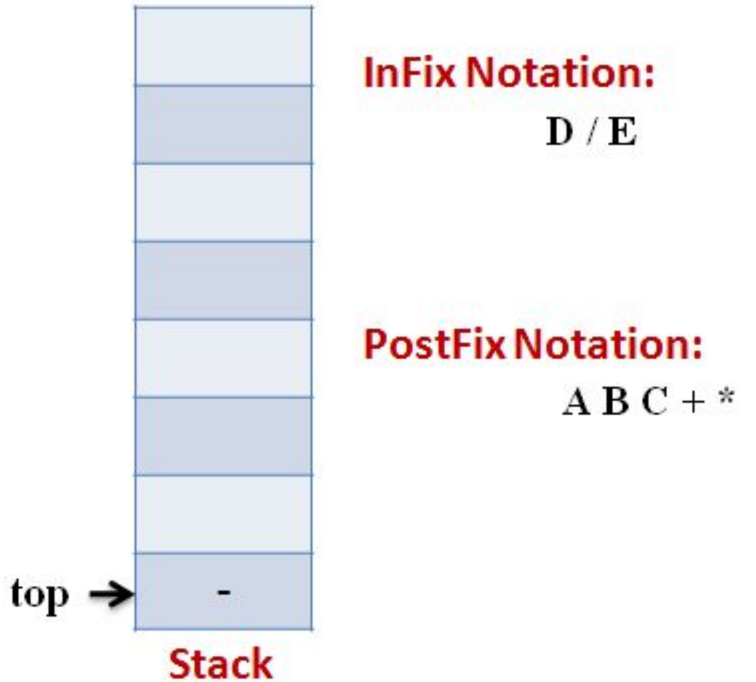
★Next token **)**, means that **pop all the elements from Stack** and **append them to the output** expression till we read an opening parenthesis.



Infix to Postfix : Example 1

Stage 9

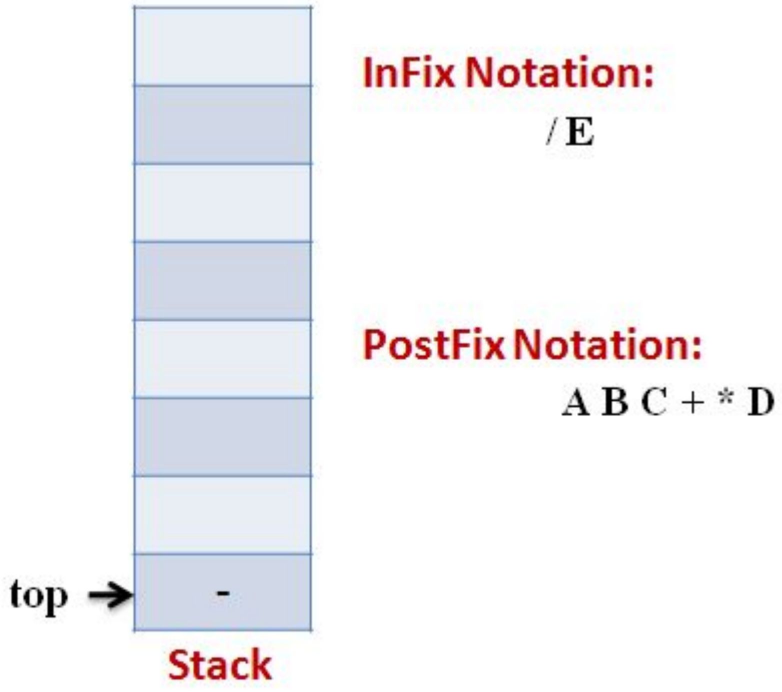
★Next token, **-**, is an operator. The precedence of operator on the top of Stack **'*'** is more than that of Minus. So we **pop multiply** and **append it to output** expression. Then **push minus in the Stack**.



Infix to Postfix : Example 1

Stage 10

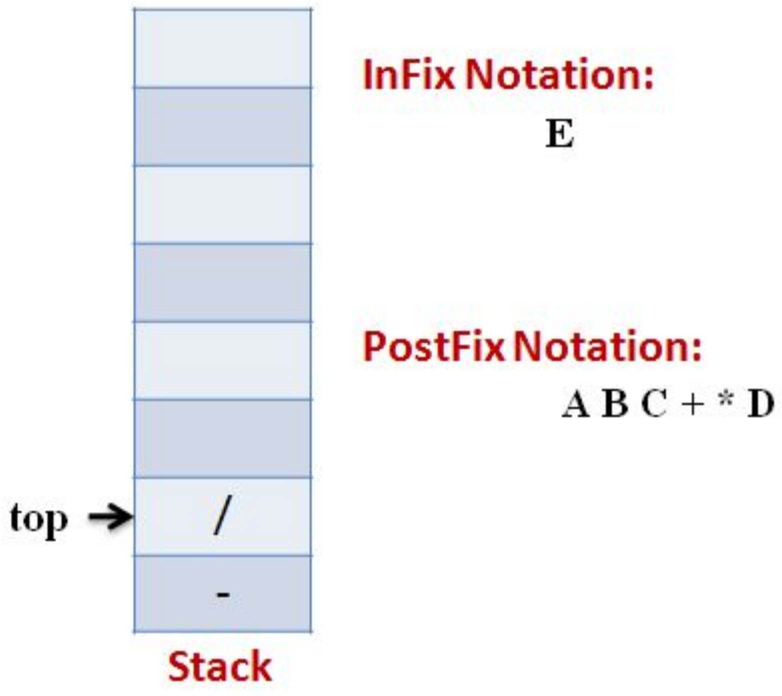
★Next, Operand '**D**' gets **appended to the output**.



Infix to Postfix : Example 1

Stage 11

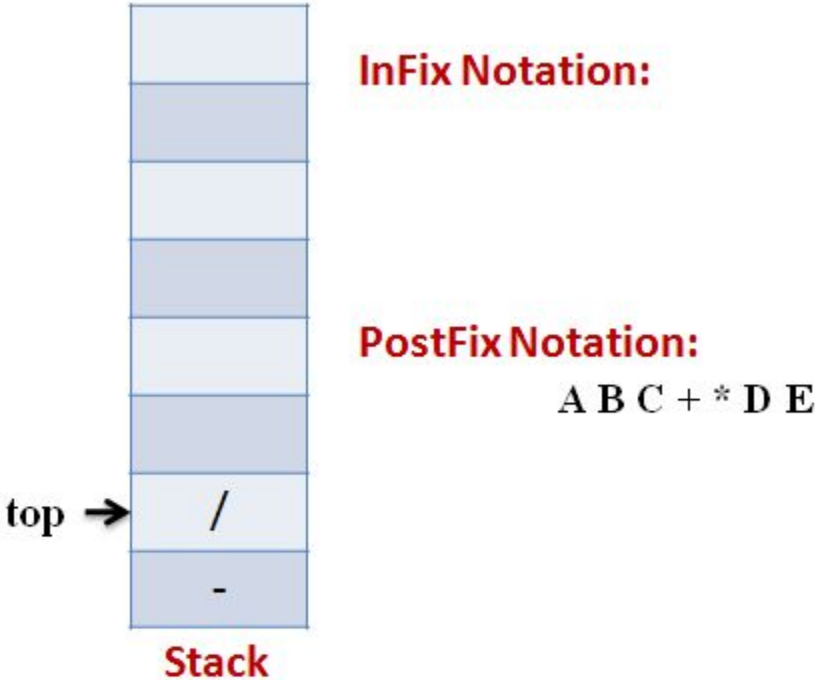
★Next, we will insert the **division** operator into the Stack because its precedence is more than that of minus.



Infix to Postfix : Example 1

Stage 12

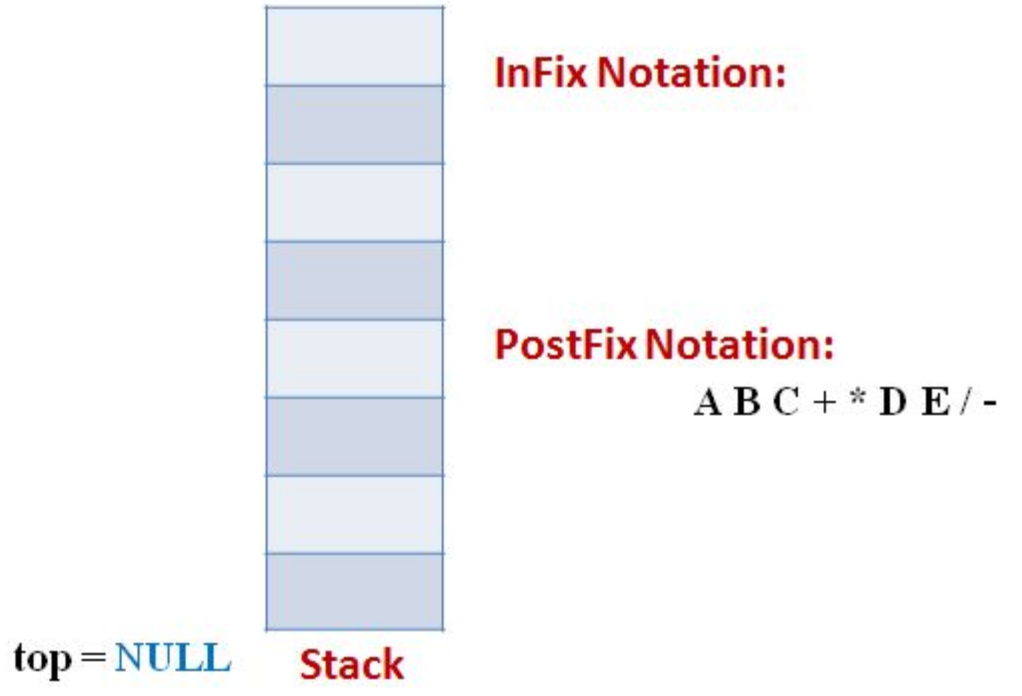
★The last token, **E**, is an operand, so we **insert it to the output** Expression as it is.



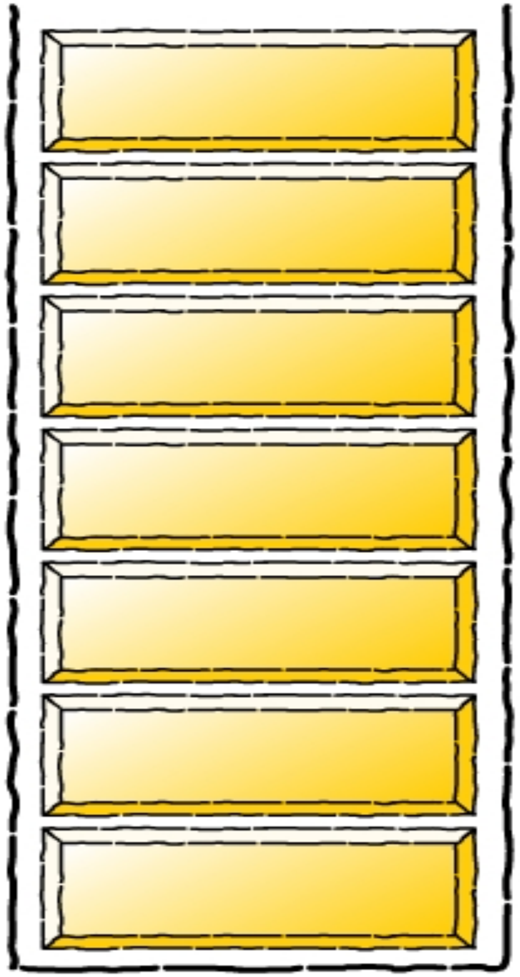
Infix to Postfix : Example 1

Stage 13

★The input Expression is complete now. So we **pop the Stack** and **Append it to the Output Expression** as we pop it.



Infix to Postfix : Example 2



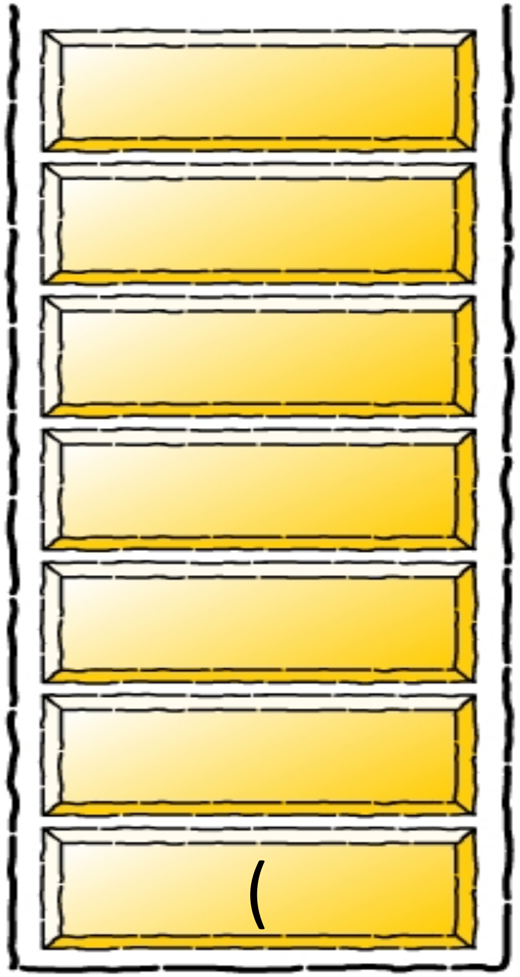
infixVect

(a + b - c) * d – (e + f)

postfixVect

Infix to Postfix : Example 2

stackVect



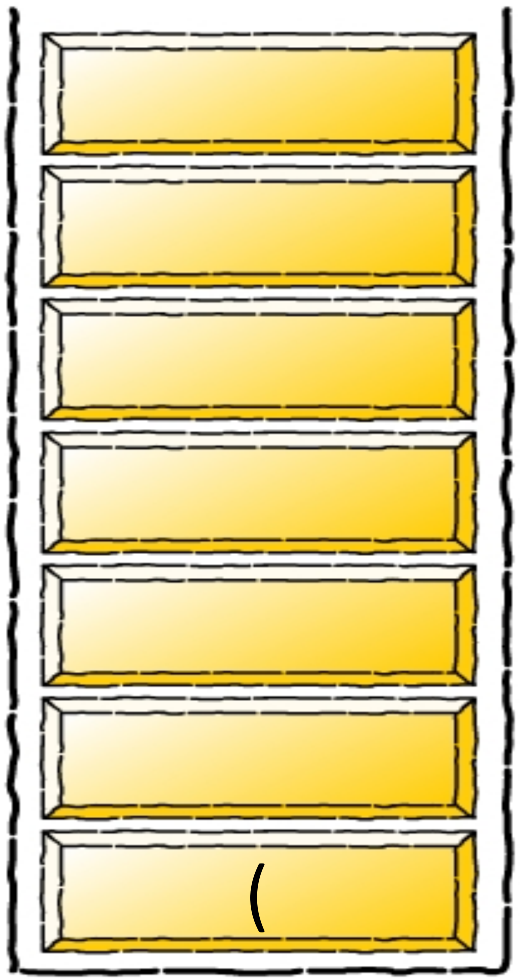
infixVect

a + b - c) * d – (e + f)

postfixVect

Infix to Postfix : Example 2

stackVect



infixVect

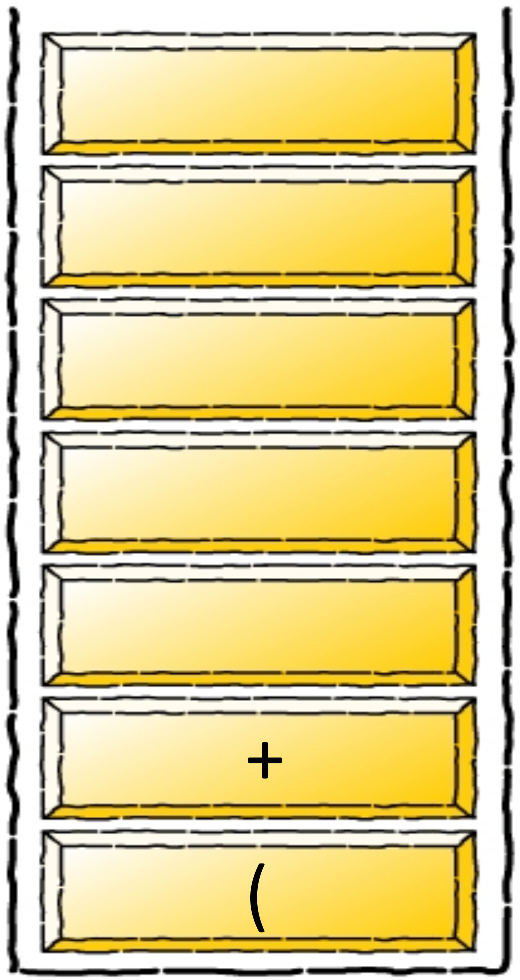
+ b - c) * d – (e + f)

postfixVect

a

Infix to Postfix : Example 2

stackVect



infixVect

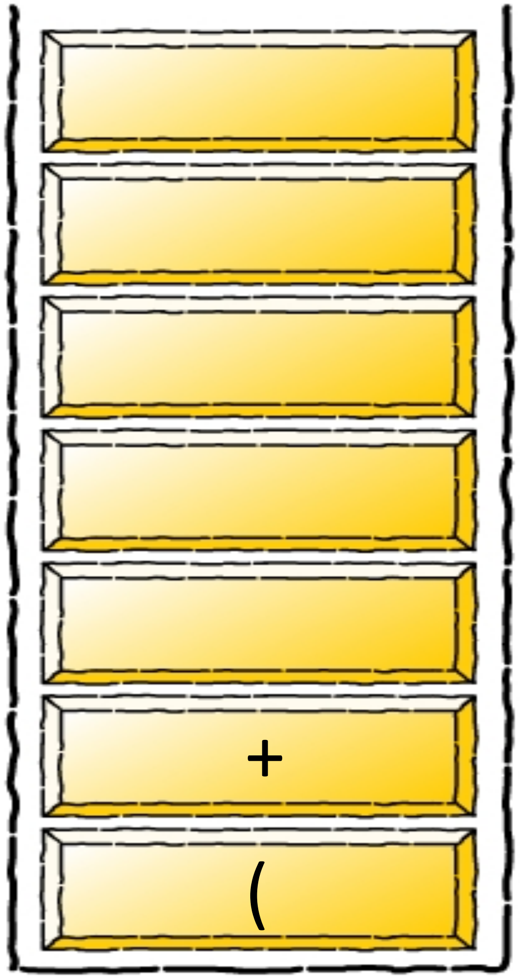
b - c) * d – (e + f)

postfixVect

a

Infix to Postfix : Example 2

stackVect



infixVect

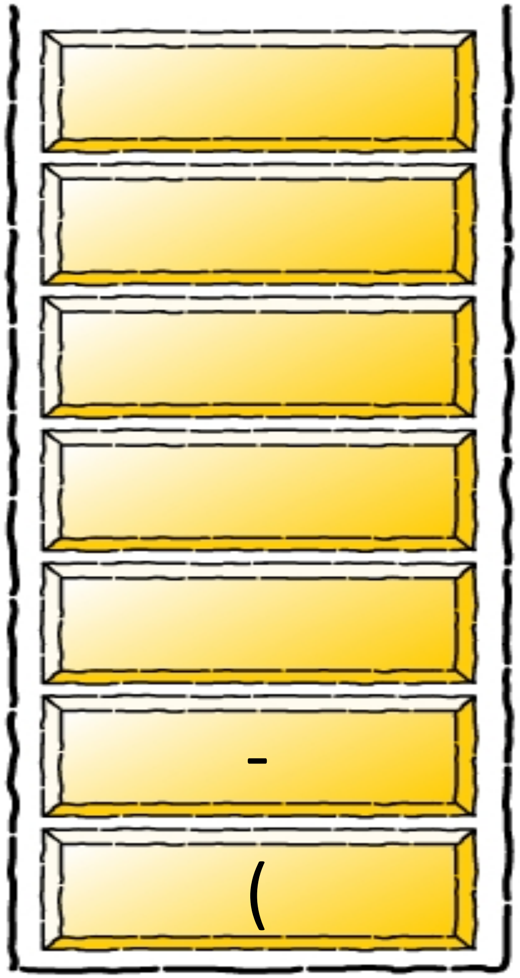
- c) * d – (e + f)

postfixVect

a b

Infix to Postfix : Example 2

stackVect



infixVect

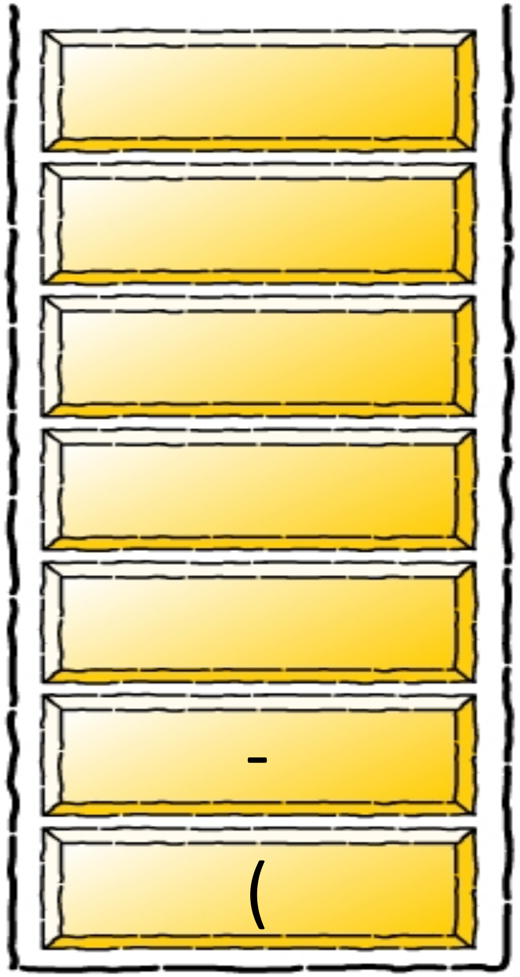
c) * d – (e + f)

postfixVect

a b +

Infix to Postfix : Example 2

stackVect



infixVect

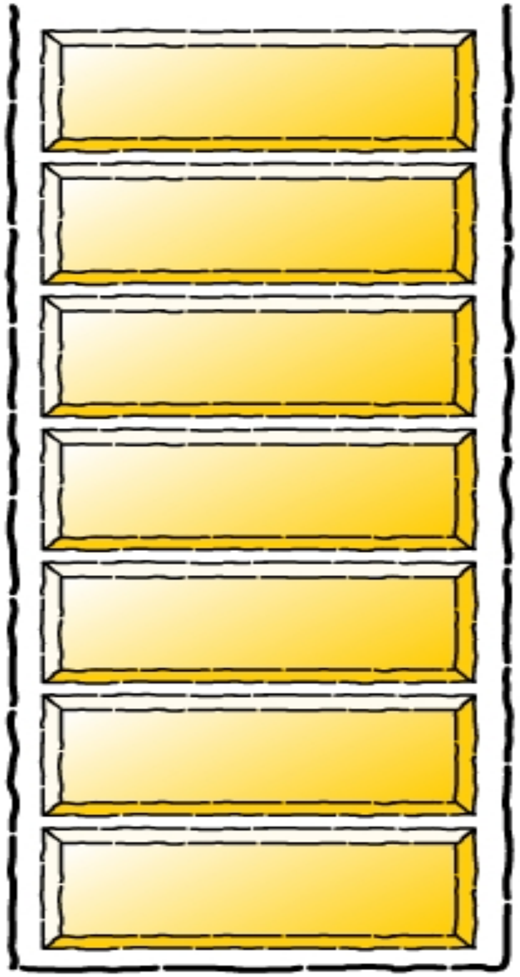
) * d – (e + f)

postfixVect

a b + c

Infix to Postfix : Example 2

stackVect



infixVect

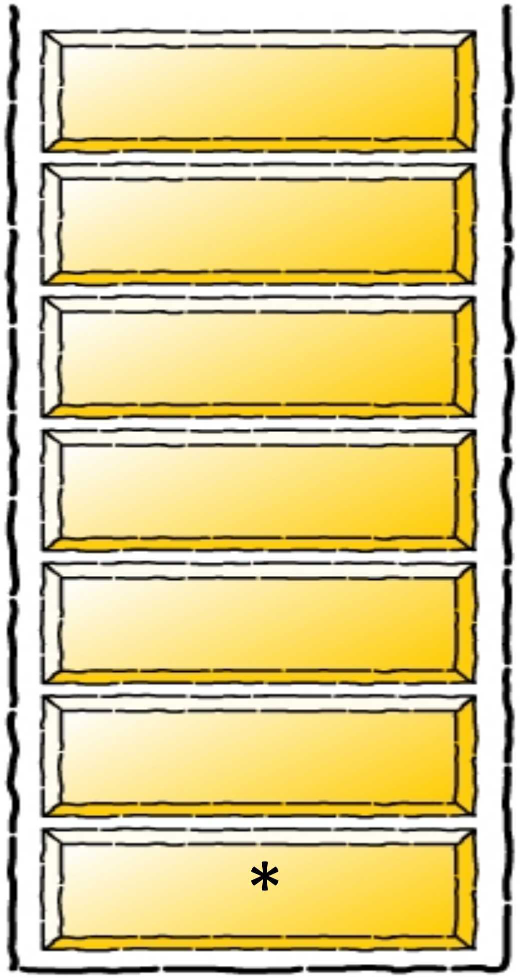
* d – (e + f)

postfixVect

a b + c -

Infix to Postfix : Example 2

stackVect



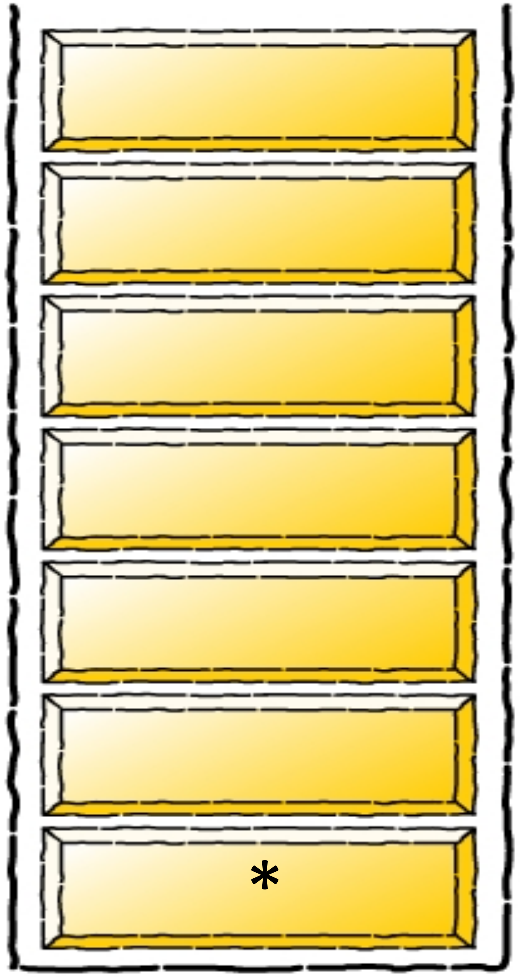
infixVect

d – (e + f)

postfixVect

a b + c -

Infix to Postfix : Example 2



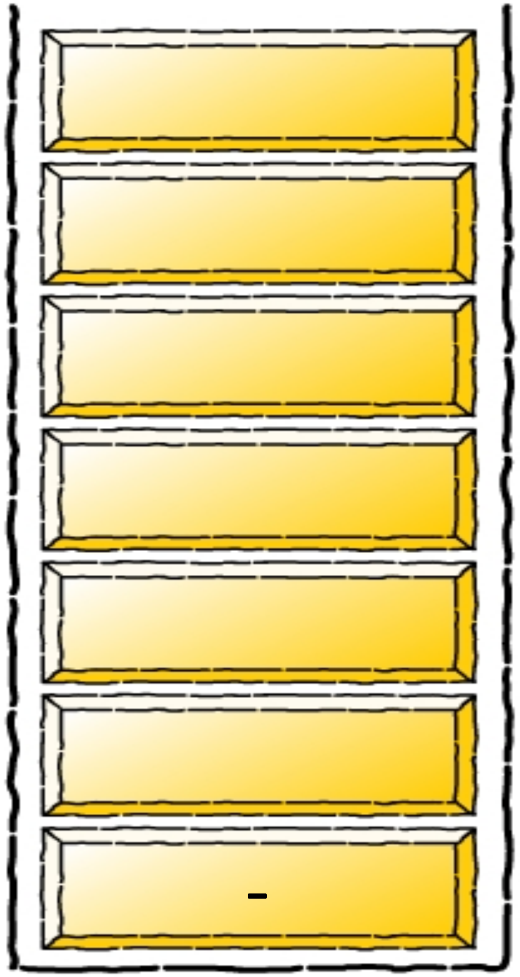
infixVect

– (e + f)

postfixVect

a b + c - d

Infix to Postfix : Example 2



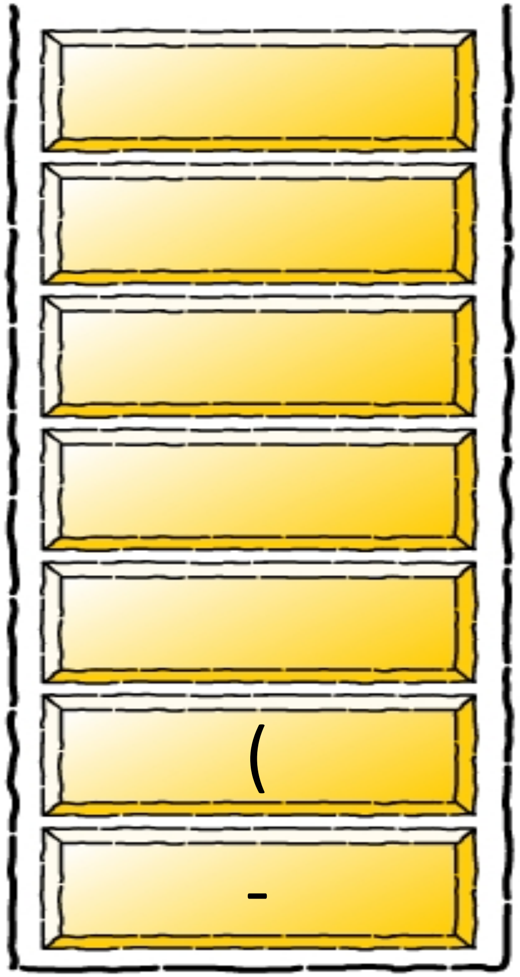
infixVect

(e + f)

postfixVect

a b + c – d *

Infix to Postfix : Example 2



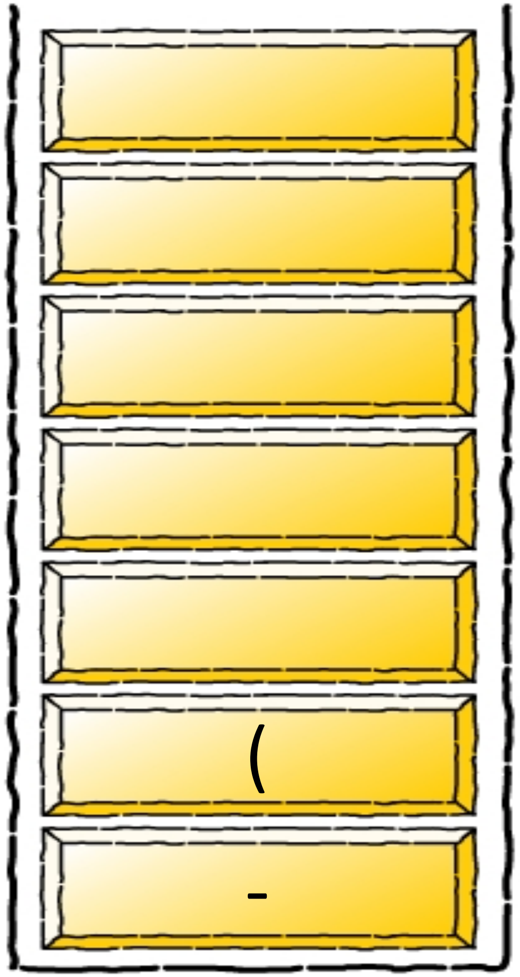
infixVect

e + f)

postfixVect

a b + c – d *

Infix to Postfix : Example 2



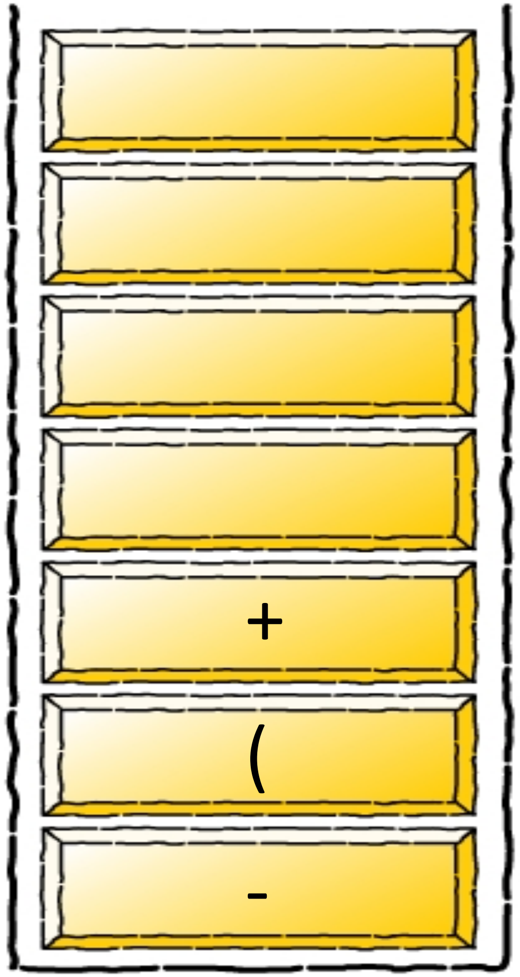
infixVect

+ f)

postfixVect

a b + c – d * e

Infix to Postfix : Example 2



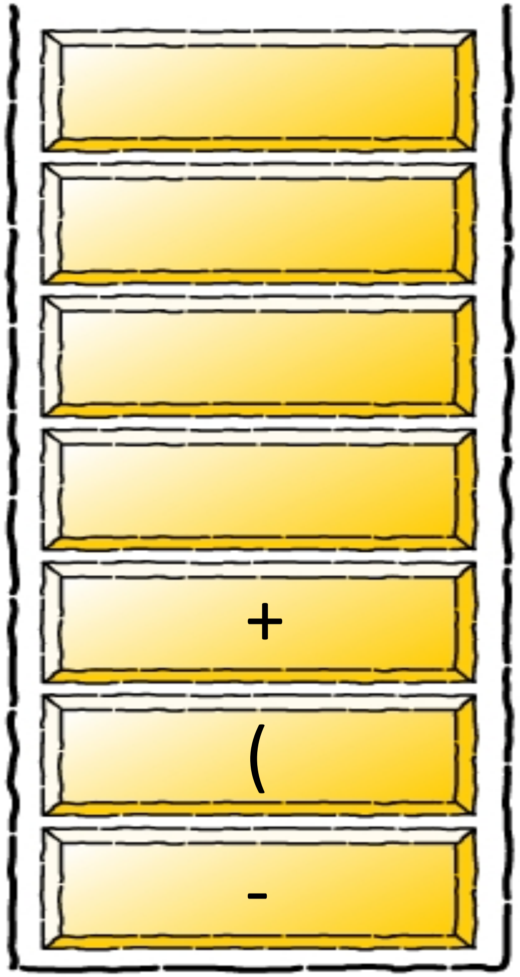
infixVect

f)

postfixVect

a b + c – d * e

Infix to Postfix : Example 2



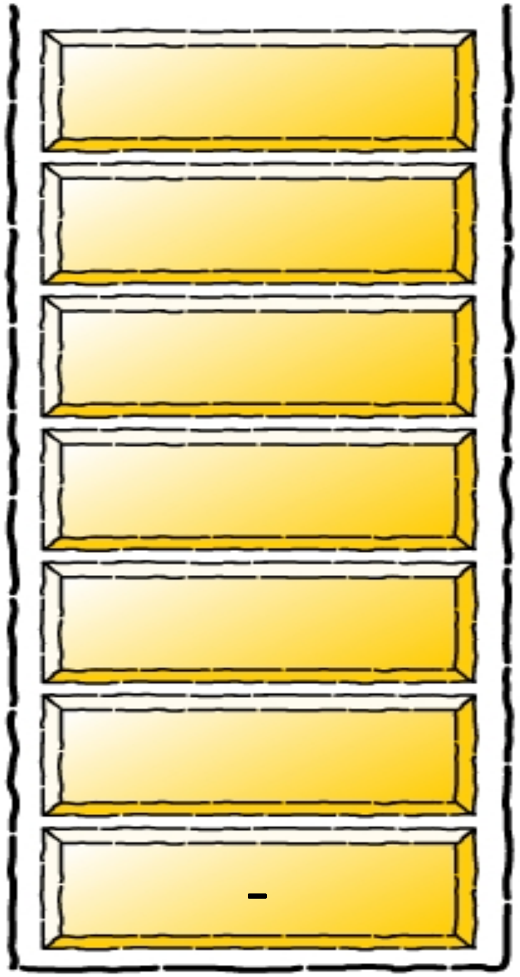
infixVect

)

postfixVect

a b + c – d * e f

Infix to Postfix : Example 2

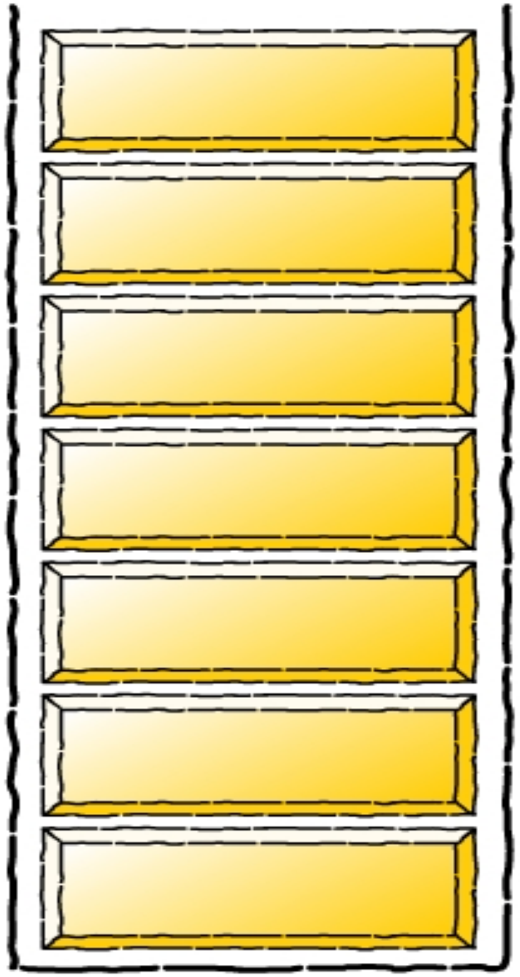


infixVect

postfixVect

a b + c – d * e f +

Infix to Postfix : Example 2



infixVect

postfixVect

a b + c – d * e f + -

Infix to Postfix : Example 3

Expression Conversion Infix to Postfix

Suppose we want to convert 2*3/(2-1)+5*3 into Postfix form

Expression	Stack	Output
2	Empty	2
*	*	2
3	*	23
/	/	23*
(	/(	23*
2	/(	23*2
-	/(-	23*2
1	/(-	23*21
)	/	23*21-
+	+	23*21-/
5	+	23*21-/5
*	+*	23*21-/53
3	+*	23*21-/53
	Empty	23*21-/53*+

In Stack and Incoming priorities functions (Infix→Postfix)

```
icp(ch)
{
 if(ch=='+' || ch=='-')
      return 1;
 if(ch=='*' || ch=='/')
      return 2;
if(ch=='^')
      return 4;
if(ch=='(')
      return 5;
  else
      return 0;
}
```

```
isp( ch)
{
  if(ch=='+' || ch=='-')
      return 1;
  if(ch=='*' || ch=='/')
      return 2;
  if(ch=='^')
      return 3;
  else
      return 0;
}
```

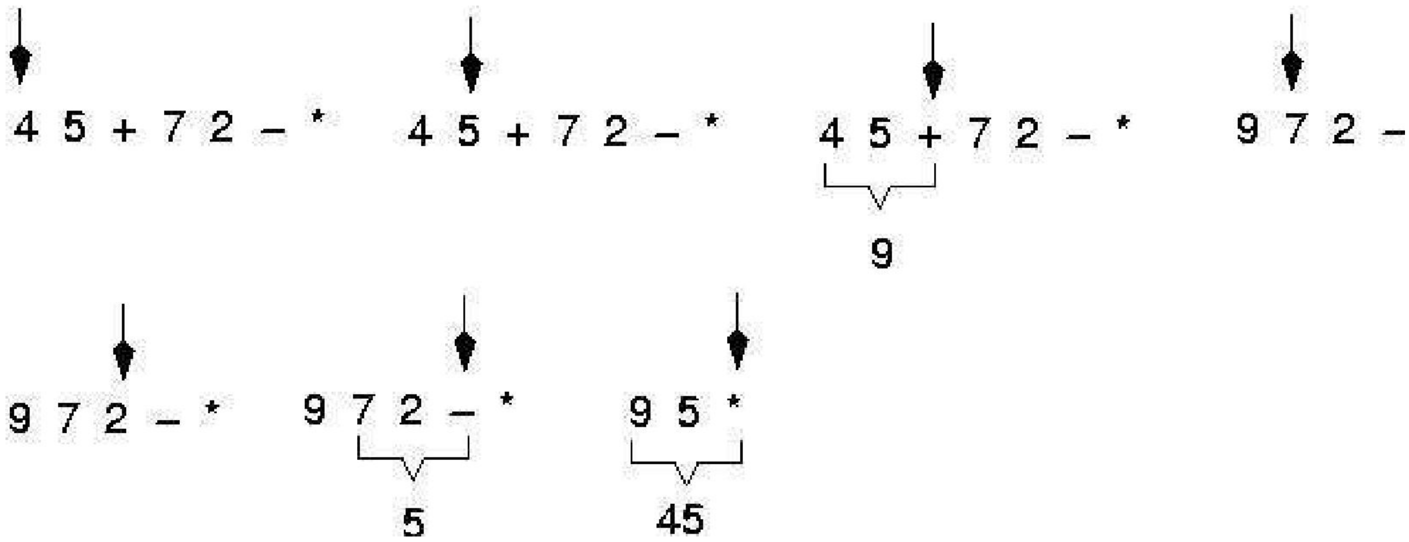
```
Algorithm in_post(inexp[ ])
{ // postexp[ ] has the postfix expression
   k=0; i=0;
  tkn=inexp[i];
  while ( tkn!='\0')
  {  if  tkn is an operand
     { postexp[k]=inexp[i];
       k++;
     }
     else //1st
     {  if tkn=='('  //open paranthesis
        {  push('(');  }
        else //2nd
        {
           if tkn==')'  //open paranthesis
           { while (tkn=pop()) !='('
              {   postexp[k]=tkn; k++; }
           }
           else //3rd
           { while (stack not empty &&
                   isp(stk[top]) >=   icp(tkn) )
               {  postexp[k]=pop(); k++;
               }
             push(tkn);
           } // end of  3rd else
        }//end of 2nd  else
     }// end of 1st else

     // read next token
        i++;
       tkn=inexp[i];
  }//end  of outer while
     while stack not empty
        { postexp[k]=pop(); k++ }
```

Applications of Stacks *(cont'd)*

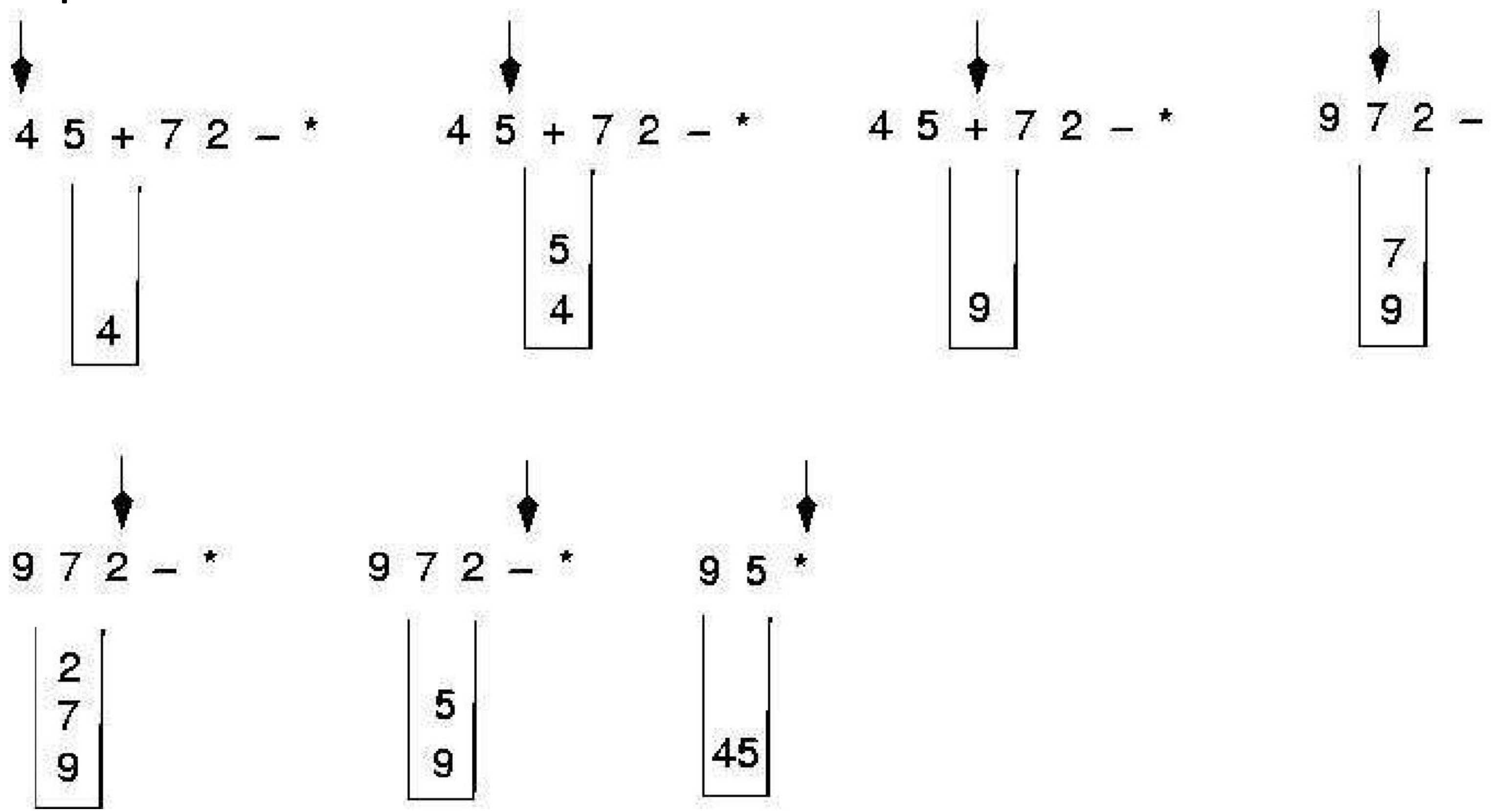
3. Expression Evaluation

- For evaluating expression ,it is converted into prefix or postfix form.
- Expression in postfix or prefix form can be easily evaluated by computer because no precedence of operators is required in this.



Applications of Stacks *(cont'd)*

3. Expression Evaluation
Example



Algorithm for evaluating a postfix expression

```
WHILE more input items exist
{
 If symb is an operand
       then push (opndstk,symb)
 else        //symbol is an operator
 {
       Opnd2=pop(opndstk);
       Opnd1=pop(opndnstk);
       Value = result of applying symb to opnd1 & opnd2
       Push(opndstk,value);
 }           //End of else
} // end while
Result = pop (opndstk);
```

Question : Evaluate the following expression in postfix : 623+-382/+*2^3+

Final answer is

- 49
- 51
- 52
- 7
- None of these

Evaluate- 623+-382/+*2^3+

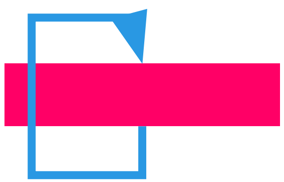
Symbol	opnd1	opnd2	value	opndstk
6				6
2				6,2
3				6,2,3
+	2	3	5	6,5
-	6	5	1	1
3	6	5	1	1,3
8	6	5	1	1,3,8
2	6	5	1	1,3,8,2
/	8	2	4	1,3,4
+	3	4	7	1,7
*	1	7	7	7
2				7,2
^	7	2	49	49
3				49,3
+	49	3	52	52

Concept of Recursion

- Sometimes, the best way to solve a problem is by solving a smaller version of the exact same problem first

- Recursion is a technique that solves a problem by solving a smaller problem of the same type

- A procedure that is defined in terms of itself



Concept of Recursion

What’s Behind this function ?

```
int f(int a){
        if (a==1)
          return(1);
        else
          return(a * f( a-1));
}
```

It computes f! (factorial)

Concept of Recursion

Factorial:

$$a! = 1 * 2 * 3 * ... * (a-1) * a$$

Note:

$$a! = a * (a-1)!$$

remember:

...splitting up the problem into a smaller problem of the same type...

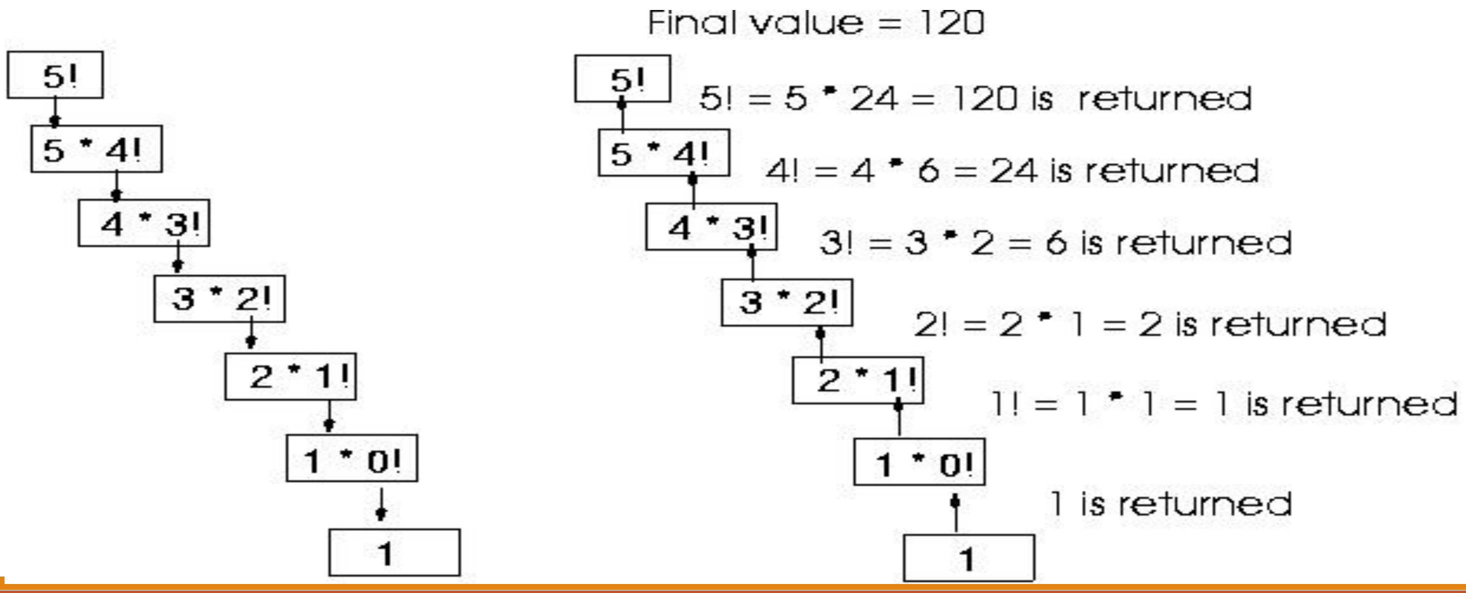
$$a!$$

$$a \quad * \quad (a-1)!$$

Concept of Recursion

```
int factorial(int a){
        if (a==0)
          return(1);
        else
          return(a * factorial( a-1));
}
```

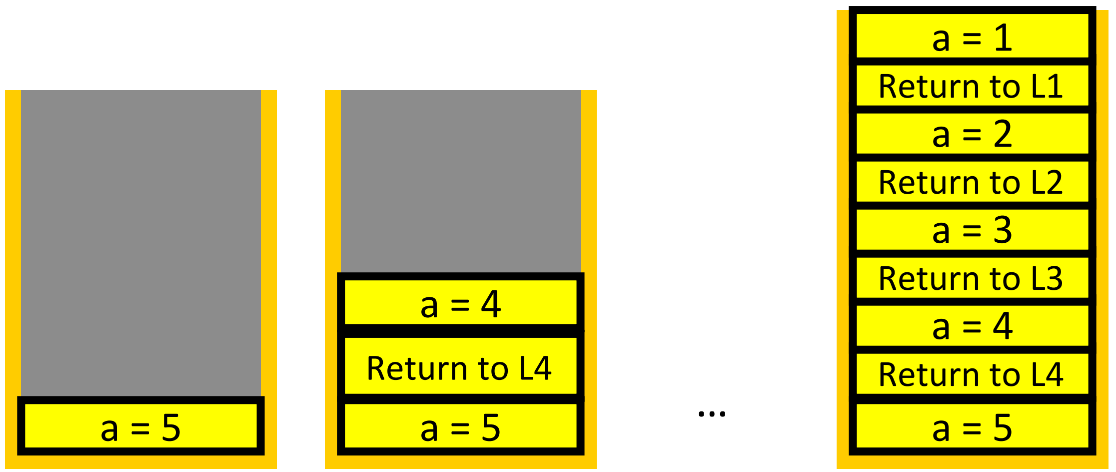
RECURSION !



Concept of Recursion

```
int factorial(int a){
            if (a==1)
              return(1);
            else
              return(a * factorial( a-1));
}
```

Watching the Stack



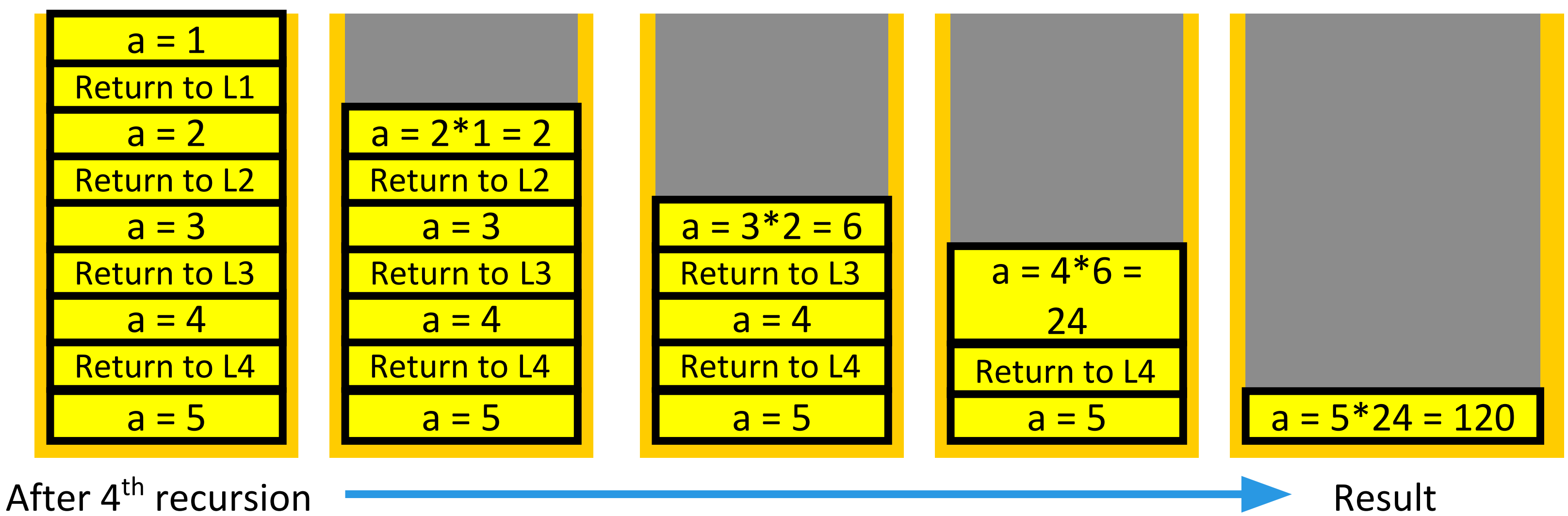
Initial   After 1 recursion   After 4^{th} recursion

Every call to the method creates a new set of local variables !

Concept of Recursion

```
int factorial(int a){
        if (a==1)
          return(1);
        else
          return(a * factorial( a-1));
}
```

Watching the Stack



Properties of Recursion

Problems that can be solved by recursion have these characteristics:

- One or more stopping cases have a simple, nonrecursive solution
- The other cases of the problem can be reduced (using recursion) to problems that are closer to stopping cases
- Eventually the problem can be reduced to only stopping cases, which are relatively easy to solve

Follow these steps to solve a recursive problem:

- Try to express the problem as a simpler version of itself
- Determine the stopping cases
- Determine the recursive steps

Concept of Recursion

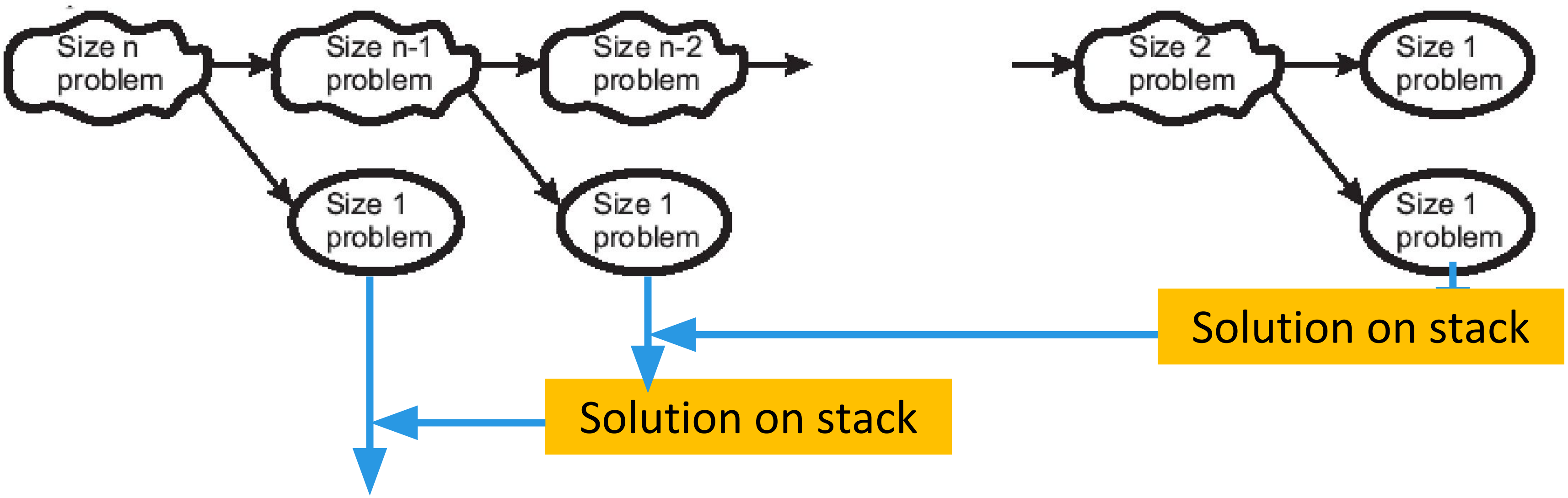
The recursive algorithms we write generally consist of an if statement:

IF

the stopping case is reached solve it

ELSE

split the problem into simpler cases using recursion



Concept of Recursion

Common Programming Error

Recursion does not terminate properly: Stack Overflow !

Exercise

Define a recursive solution for the following function:

$$f(x) = x^n$$

Recursion vs. Iteration

You could have written the power-function *iteratively,* i.e. using a loop construction

Where's the difference ?

Recursion vs. Iteration

- Iteration can be used in place of recursion
  - An iterative algorithm uses a *looping construct*
  - A recursive algorithm uses a *branching structure*

- Recursive solutions are often less efficient, in terms of both *time* and *space*, than iterative solutions

- Recursion can simplify the solution of a problem, often resulting in shorter, more easily understood source code

- (Nearly) every recursively defined problem can be solved iteratively iterative optimization can be implemented after recursive design

Deciding whether to use a Recursive Function

- When the depth of recursive calls is relatively "shallow"
- The recursive version does about the same amount of work as the non recursive version
- The recursive version is shorter and simpler than the non recursive solution

Takeaways

- ❑ Stack is Last in First Out(LIFO) Data Structure
- ❑ Primitive operations on Stack are push, pop, isEmpty and isFull
- ❑ Stack can be represented by using Array as well as linked list.
- ❑ Stack is commonly used in expression conversion and Evaluation.
- ❑ Recursion is one of the important application of stack

FAQS

1. Write an ADT for Stack
2. What are the primitive operations of stack.
3. Explain with example stack applications.

References

- Horowitz,Sahani,"Fundamentals of Data Structures", Galgotia Publication
- Gilberg, Forozen,"Data Structures :A Pseudo Code Approach with C"
- Maureen Sprankle,"Problem Solving and Programming Concepts"