Chapter (7)

Transformer

Introduction

The transformer is probably one of the most useful electrical devices ever invented. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency—as high as 99%. In this chapter, we shall study some of the basic properties of transformers.

7.1 Transformer

A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. (7.1). The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V_1 whose magnitude is to be changed is applied to the primary. Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a step up-transformer. On the other hand, if $V_2 < V_1$, it is called a step-down transformer.

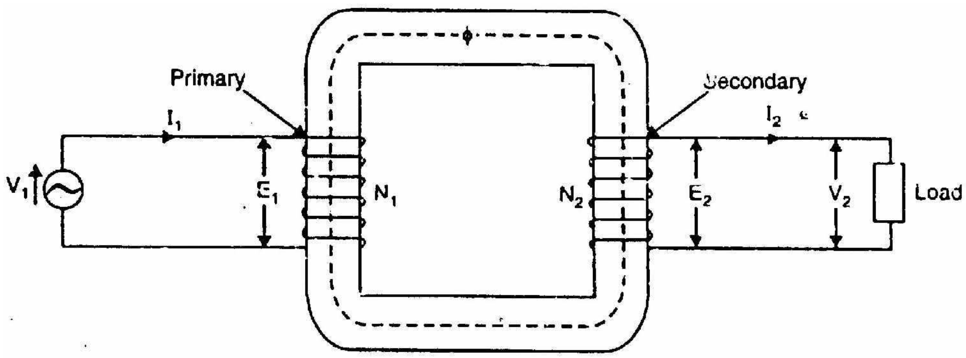

Fig.(7.1)

Working

When an alternating voltage V_1 is applied to the primary, an alternating flux ϕ is set up in the core. This alternating flux links both the windings and induces e.m.f.s E_1 and E_2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E_1 is termed as primary e.m.f. and e.m.f. E_2 is termed as secondary e.m.f.

Clearly, $$E_1 = -N_1 \frac{d\phi}{dt}$$

and $$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively. If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer. On the other hand, if $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f. E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

The following points may be noted carefully:

(i) The transformer action is based on the laws of electromagnetic induction.

(ii) There is no electrical connection between the primary and secondary. The a.c. power is transferred from primary to secondary through magnetic flux.

(iii) There is no change in frequency i.e., output power has the same frequency as the input power.

(iv) The losses that occur in a transformer are:

  (a) core losses—eddy current and hysteresis losses

  (b) copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

7.2 Theory of an Ideal Transformer

An ideal transformer is one that has

(i) no winding resistance

(ii) no leakage flux i.e., the same flux links both the windings

(iii) no iron losses (i.e., eddy current and hysteresis losses) in the core

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.

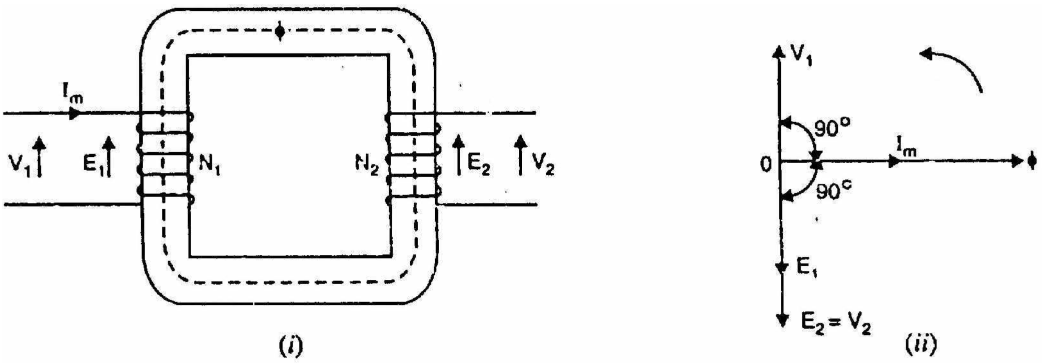

Fig.(7.2)

Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in Fig. (7.2 (i)). Under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage V_1 is applied to the primary, it draws a small magnetizing current I_m which lags behind the applied voltage by 90°. This alternating current I_m produces an alternating flux ϕ which is proportional to and in phase with it. The alternating flux ϕ links both the windings and induces e.m.f. E_1 in the primary and e.m.f. E_2 in the secondary. The primary e.m.f. E_1 is, at every instant, equal to and in opposition to V_1 (Lenz's law). Both e.m.f.s E_1 and E_2 lag behind flux ϕ by 90° (See Sec. 7.3). However, their magnitudes depend upon the number of primary and secondary turns.

Fig. (7.2 (ii)) shows the phasor diagram of an ideal transformer on no load. Since flux ϕ is common to both the windings, it has been taken as the reference phasor. As shown in Sec. 7.3, the primary e.m.f. E_1 and secondary e.m.f. E_2 lag behind the flux ϕ by 90°. Note that E_1 and E_2 are in phase. But E_1 is equal to V_1 and 180° out of phase with it.

7.3 E.M.F. Equation of a Transformer

Consider that an alternating voltage V_1 of frequency f is applied to the primary as shown in Fig. (7.2 (i)). The sinusoidal flux ϕ produced by the primary can be represented as:

$$\phi = \phi_m \sin\omega t$$

The instantaneous e.m.f. e_1 induced in the primary is

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin \omega t)$$
$$= -\omega N_1 \phi_m \cos \omega t = -2\pi f N_1 \phi_m \cos \omega t$$
$$= 2\pi f N_1 \phi_m \sin(\omega t - 90°) \qquad \text{(i)}$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value E^ of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}}$$

or $\quad E_1 = 4.44 f N_1 \phi_m$

Similarly $\quad E_2 = 4.44 f N_2 \phi_m$

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$.

Note. It is clear from exp. (i) above that e.m.f. E_1 induced in the primary lags behind the flux ϕ by 90°. Likewise, e.m.f. E_2 induced in the secondary lags behind flux ϕ by 90°.

7.4 Voltage Transformation Ratio (K)

From the above equations of induced e.m.f., we have (See Fig. 7.3),

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The constant K is called *voltage transformation ratio*. Thus if K = 5 (i.e. $N_2/N_1 = 5$), then $E_2 = 5 E_1$.

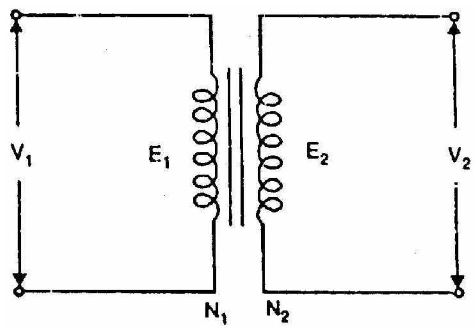

Fig.(7.3)

For an ideal transformer;

(i) $E_1 = V_1$ and $E_2 = V_2$ as there is no voltage drop in the windings.

$$\therefore \quad \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(ii) there are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$V_1 I_1 = V_2 I_2$$

or $$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of voltage transformation ratio. This simply means that if we raise the voltage, there is a corresponding decrease of current.

7.5 Practical Transformer

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) magnetic leakage, giving rise to leakage reactances.

(i) **Iron losses**. Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.

(ii) **Winding resistances**. Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R_1 and secondary resistance R_2 act in series with the respective windings as shown in Fig. (7.4). When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and E_1 will be less than V_1 while V_2 will be less than E_2.

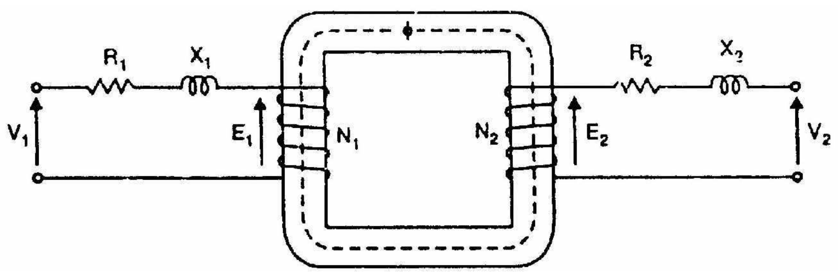

Fig.(7.4)

(iii) **Leakage reactances**. Both primary and secondary currents produce flux. The flux ϕ which links both the windings is the useful flux and is called mutual flux. However, primary current would produce some flux ϕ which would not link the secondary winding (See Fig. 7.5). Similarly, secondary current would produce some flux ϕ that would not link the primary winding. The flux such as ϕ_1 or ϕ_2 which links only one winding is called leakage flux. The leakage flux paths are mainly through the air. The effect of these leakage fluxes would be the same as though inductive reactance were connected in series with each winding of transformer that had no leakage flux as shown in Fig. (7.4). In other words, the effect of primary leakage flux ϕ_1 is to introduce an inductive reactance X_1 in series with the primary winding as shown in Fig. (7.4). Similarly, the secondary leakage flux ϕ_2 introduces an inductive reactance X_2 in series with the secondary winding. There will be no power loss due to leakage reactance. However, the presence of leakage reactance in the windings changes the power factor as well as there is voltage loss due to IX drop.

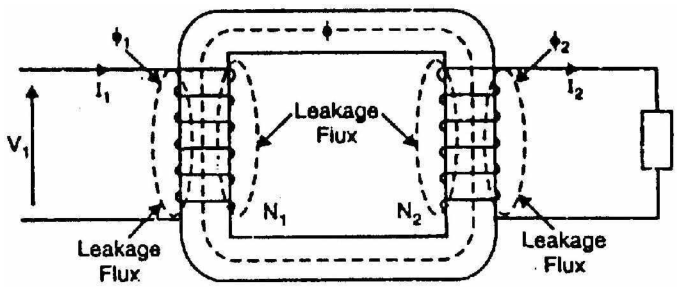

Fig.(7.5)

Note. Although leakage flux in a transformer is quite small (about 5% of ϕ) compared to the mutual flux ϕ, yet it cannot be ignored. It is because leakage flux paths are through air of high reluctance and hence require considerable e.m.f. It may be noted that energy is conveyed from the primary winding to the secondary winding by mutual flux ϕ which links both the windings.

7.6 Practical Transformer on No Load

Consider a practical transformer on no load i.e., secondary on open-circuit as shown in Fig. (7.6 (i)). The primary will draw a small current I_0 to supply (i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current I_0 is not 90° behind the applied voltage V_1 but lags it by an angle $\phi_0 < 90°$ as shown in the phasor diagram in Fig. (7.6 (ii)).

No load input power, $W_0 = V_1 I_0 \cos \phi_0$

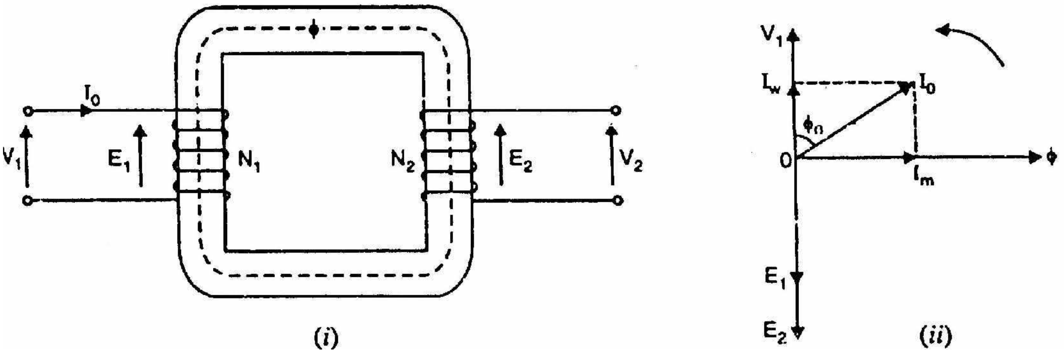

Fig.(7.6)

As seen from the phasor diagram in Fig. (7.6 (ii)), the no-load primary current I_0 can be resolved into two rectangular components viz.

(i) The component I_W in phase with the applied voltage V_1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_W = I_0 \cos \phi_0$$

(b) The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is this component which produces the mutual flux ϕ in the core.

$$I_m = I_0 \sin \phi_0$$

Clearly, I_0 is phasor sum of I_m and I_W,

$$\therefore \quad I_0 = \sqrt{I_m^2 + I_W^2}$$

No load p.f., $\cos\phi_0 = \dfrac{I_W}{I_0}$

It is emphasized here that no load primary copper loss (i.e. $I_0^2 R_1$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

No load input power, W_0 = Iron loss

Note. At no load, there is no current in the secondary so that $V_2 = E_2$. On the primary side, the drops in R_1 and X_1, due to I_0 are also very small because of the smallness of I_0. Hence, we can say that at no load, $V_1 = E_1$.

7.7 Ideal Transformer on Load

Let us connect a load Z_L across the secondary of an ideal transformer as shown in Fig. (7.7 (i)). The secondary e.m.f. E_2 will cause a current I_2 to flow through the load.

$$I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$$

The angle at which I_2 leads or lags V_2 (or E_2) depends upon the resistance and reactance of the load. In the present case, we have considered inductive load so that current I_2 lags behind V_2 (or E_2) by ϕ_2.

The secondary current I_2 sets up an m.m.f. N_2I_2 which produces a flux in the opposite direction to the flux ϕ originally set up in the primary by the magnetizing current. This will change the flux in the core from the original value. However, the flux in the core should not change from the original value.

In order to fulfill this condition, the primary must develop an m.m.f. which exactly counterbalances the secondary m.m.f. N_2I_2. Hence a primary current I_1 must flow such that:

$$N_1I_1 = N_2I_2$$

or $$I_1 = \frac{N_2}{N_1}I_2 = KI_2$$

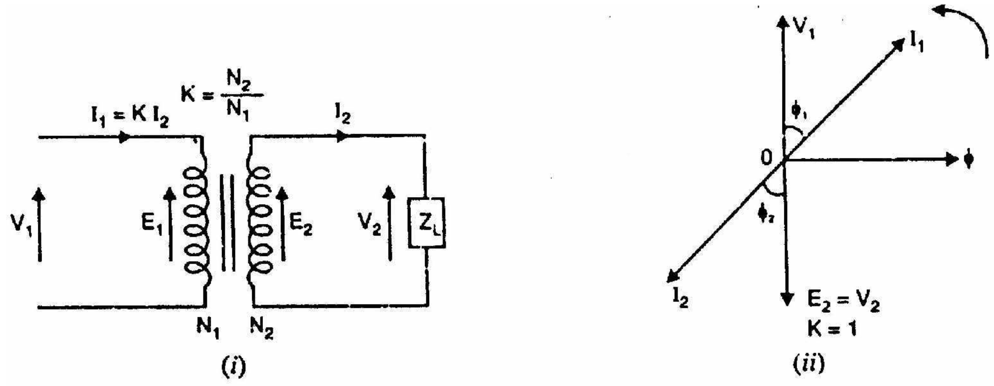

Fig.(7.7)

Thus when a transformer is loaded and carries a secondary current I_2, then a current I_1, (= K I_2) must flow in the primary to maintain the m.m.f. balance. In other words, the primary must draw enough current to neutralize the demagnetizing effect of secondary current so that mutual flux ϕ remains constant. Thus as the secondary current increases, the primary current I_1 (= KI_2) increases in unison and keeps the mutual flux ϕ constant. The power input, therefore, automatically increases with the output. For example if K = 2 and I_2 = 2A, then primary will draw a current $I_1 = KI_2 = 2 \times 2 = 4$A. If secondary current is increased to 4A, then primary current will become $I_1 = KI_2 = 2 \times 4 = 8$A.

Phaser diagram: Fig. (7.7 (ii)) shows the phasor diagram of an ideal transformer on load. Note that in drawing the phasor diagram, the value of K has been assumed unity so that primary phasors are equal to secondary phasors. The secondary current I_2 lags behind V_2 (or E_2) by ϕ_2. It causes a primary current $I_2 = KI_2 = 1 \times I_2$ which is in antiphase with it.

(i) $$\phi_1 = \phi_2$$

or $$\cos\phi_1 = \cos\phi_2$$

Thus, power factor on the primary side is equal to the power factor on the secondary side.

(ii) Since there are no losses in an ideal transformer, input primary power is equal to the secondary output power i.e.,

$$V_1I_1\cos\phi_1 = V_2I_2\cos\phi_2$$

7.8 Practical Transformer on Load

We shall consider two cases (i) when such a transformer is assumed to have no winding resistance and leakage flux (ii) when the transformer has winding resistance and leakage flux.

(i) No winding resistance and leakage flux

Fig. (7.8) shows a practical transformer with the assumption that resistances and leakage reactances of the windings are negligible. With this assumption, $V_2 = E_2$ and $V_1 = E_1$. Let us take the usual case of inductive load which causes the secondary current I_2 to lag the secondary voltage V_2 by ϕ_2. The total primary current I_1 must meet two requirements viz.

(a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I'_0 to counteract the demagnetizing effect of secondary currently I_2. The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$

or $$I'_2 = \frac{N_2}{N_1} I_2 = KI_2$$

The total primary current I_1 is the phasor sum of I'_2 and I_0 i.e.,

$$I_1 = I'_2 + I_0$$

where $$I'_2 = -KI_2$$

Note that I'_2 is 180° out of phase with I_2.

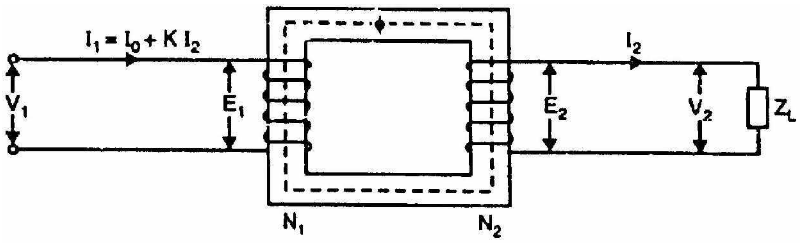

Fig.(7.8)

Phasor diagram. Fig. (7.9) shows the phasor diagram for the usual case of inductive load. Both E_1 and E_2 lag behind the mutual flux ϕ by 90°. The current I'_2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2. Now $I'_2 = K\ I_2$ and is antiphase with I_2. I_0 is the no-load current of the transformer. The phasor sum of I'_2 and I_0 gives the total primary current I_1. Note that in drawing the phasor diagram, the value of K is assumed to be unity so that primary phasors are equal to secondary phasors.

Primary p.f. = $\cos \phi_1$

Secondary p.f. = $\cos \phi_2$
Primary input power = $V_1 I_1 \cos \phi_1$
Secondary output power = $V_1 I_2 \cos \phi_2$

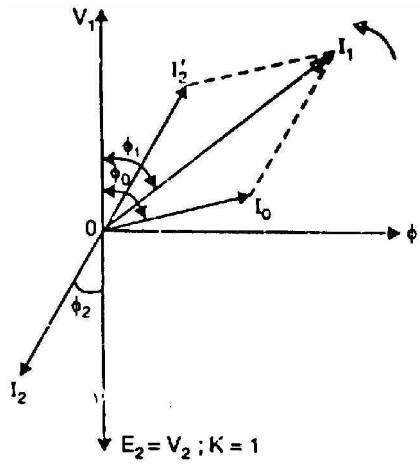

Fig.(7-9)

(ii) Transformer with resistance and leakage reactance

Fig. (7.10) shows a practical transformer having winding resistances and leakage reactances. These are the actual conditions that exist in a transformer. There is voltage drop in R_1 and X_1 so that primary e.m.f. E_1 is less than the applied voltage V_1. Similarly, there is voltage drop in R_2 and X_2 so that secondary terminal voltage V_2 is less than the secondary e.m.f. E_2. Let us take the usual case of inductive load which causes the secondary current I_2 to lag behind the secondary voltage V_2 by ϕ_2. The total primary current I_1 must meet two requirements viz.

(a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I'_2 to counteract the demagnetizing effect of secondary current I_2. The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$

or $$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

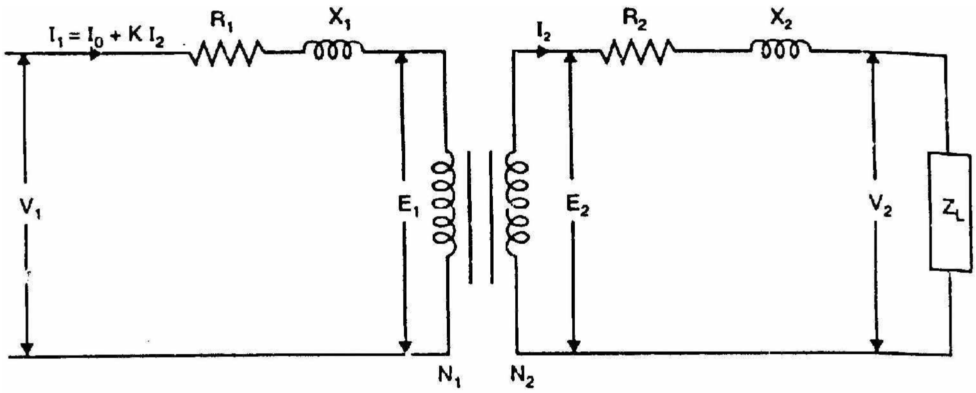

Fig.(7.10)

The total primary current I_1 will be the phasor sum of I'_2 and I_0 i.e.,

$$I_1 = I'_2 + I_0 \qquad \text{where} \qquad I'_2 = -KI_2$$

$$V_1 = -E_1 + I_1(R_1 + jX_1) \qquad \text{where} \qquad I_1 = I_0 + (-KI_2)$$

$$= -E_1 + I_1 Z_1$$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$
$$= E_2 - I_2Z_2$$

Phasor diagram. Fig. (7.11) shows the phasor diagram of a practical transformer for the usual case of inductive load. Both E_1 and E_2 lag the mutual flux ϕ by 90°. The current I'_2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2. Now $I'_2 = K\ I_2$ and is opposite to I_2. Also I_0 is the no-load current of the transformer. The phasor sum of I'_2 and I_0 gives the total primary current I_1.

Note that counter e.m.f. that opposes the applied voltage V_1 is $-E_1$. Therefore, if we add I_1R_1 (in phase with I_1) and $I_1\ X_1$ (90° ahead of I_1) to $-E_1$, we get the applied primary voltage V_1. The phasor E_2 represents the induced e.m.f. in the secondary by the mutual flux ϕ. The secondary terminal voltage V_2 will be what is left over after subtracting I_2R_2 and I_2X_2 from E_2.

Load power factor = $\cos \phi_2$

Primary power factor = $\cos \phi_1$

Input power to transformer, $P_1 = V_1I_1 \cos \phi_1$

Output power of transformer, $P_2 = V_2I_2 \cos \phi_2$

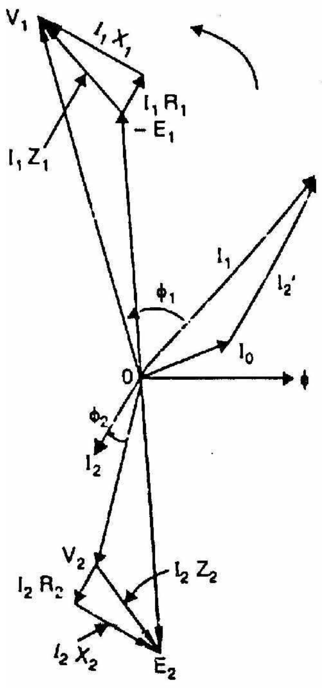

Fig.(7-11)

Note: The reader may draw the phasor diagram of a loaded transformer for (i) unity p.f. and (ii) leading p.f. as an exercise.

7.9 Impedance Ratio

Consider a transformer having impedance Z_2 in the secondary as shown in Fig. (7.12).

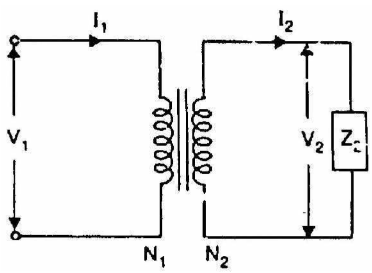

Fig.(7.12)

$$Z_2 = \frac{V_2}{I_2}$$

$$Z_1 = \frac{V_1}{I_1}$$

$$\therefore \quad \frac{Z_2}{Z_1} = \left(\frac{V_2}{V_1}\right) \times \left(\frac{I_1}{I_2}\right)$$

or $$\frac{Z_2}{Z_1} = K^2$$

i.e., impedance ratio (Z_2/Z_1) is equal to the square of voltage transformation ratio. In other words, an impedance Z_2 in secondary becomes Z_2/K^2 when transferred to primary. Likewise, an impedance Z_1 in the primary becomes $K^2 Z_1$ when transferred to the secondary.

Similarly, $\frac{R_2}{R_1} = K^2$ and $\frac{X_2}{X_1} = K^2$

Note the importance of above relations. We can transfer the parameters from one winding to the other. Thus:

(i) A resistance R_1 in the primary becomes $K^2 R_1$ when transferred to the secondary.
(ii) A resistance R_2 in the secondary becomes R_2/K^2 when transferred to the primary.
(iii) A reactance X_1 in the primary becomes $K^2 X_1$ when transferred to the secondary.
(iv) A reactance X_2 in the secondary becomes X_2/K^2 when transferred to the primary.

Note: It is important to remember that:

(i) When transferring resistance or reactance from primary to secondary, multiply it by K^2.
(ii) When transferring resistance or reactance from secondary to primary, divide it by K^2.
(iii) When transferring voltage or current from one winding to the other, only K is used.

7.10 Shifting Impedances in A Transformer

Fig. (7.13) shows a transformer where resistances and reactances are shown external to the windings. The resistance and reactance of one winding can be transferred to the other by appropriately using the factor K^2. This makes the analysis of the transformer a simple affair because then we have to work in one winding only.

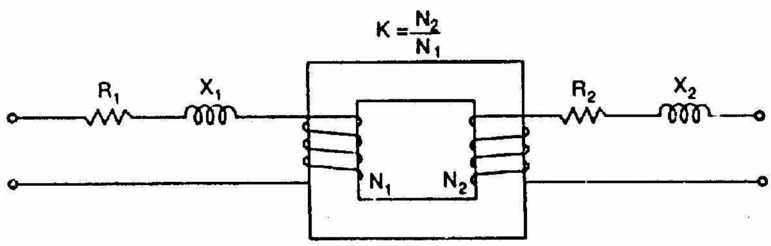

Fig.(7.13)

(i) Referred to primary

When secondary resistance or reactance is transferred to the primary, it is divided by K^2. It is then called equivalent secondary resistance or reactance referred to primary and is denoted by R'_2 or X'_2.

Equivalent resistance of transformer referred to primary

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}$$

Equivalent reactance of transformer referred to primary

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2}$$

Equivalent impedance of transformer referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

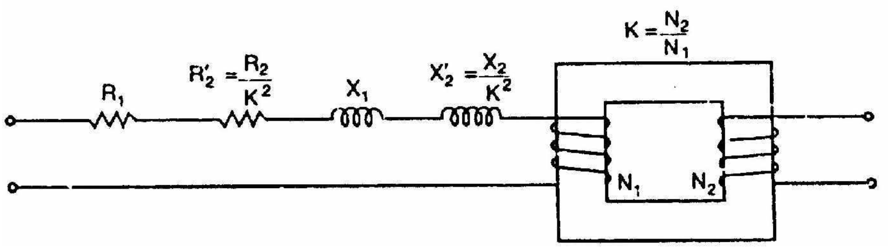

Fig. (7.14)

Fig. (7.14) shows the resistance and reactance of the secondary referred to the primary. Note that secondary now has no resistance or reactance.

(ii) Referred to secondary

When primary resistance or reactance is transferred to the secondary, it is multiplied by K^2. It is then called equivalent primary resistance or reactance referred to the secondary and is denoted by R'_1 or X'_1.

Equivalent resistance of transformer referred to secondary

$$R_{02} = R_2 + R'_1 = R_2 + K^2 R_1$$

Equivalent reactance of transformer referred to secondary

$$X_{02} = X_2 + X'_1 = X_2 + K^2 X_1$$

Equivalent impedance of transformer referred to secondary

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

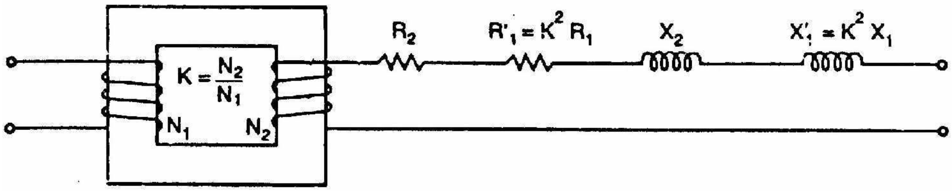

Fig. (7.15)

Fig. (7.15) shows the resistance and reactance of the primary referred to the secondary. Note that primary now has no resistance or reactance.

7.11 Importance of Shifting Impedances

If we shift all the impedances from one winding to the other, the transformer is eliminated and we get an equivalent electrical circuit. Various voltages and currents can be readily obtained by solving this electrical circuit.

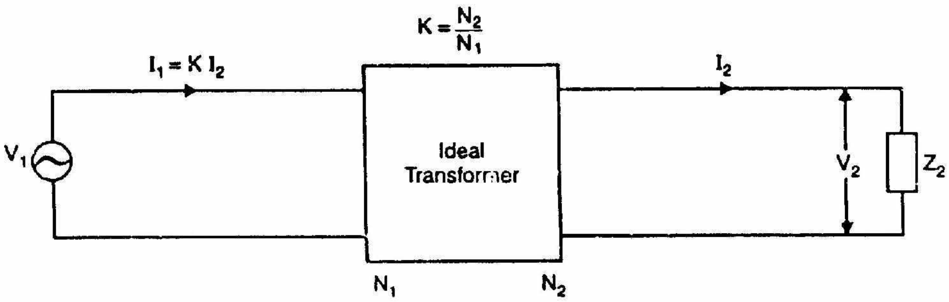

Fig.(7.16)

Consider an ideal transformer having an impedance Z_2 in the secondary as shown in Fig. (7.16).

(i) Referred to primary

When impedance Z_2 in the secondary is transferred to the primary, it becomes Z_2/K^2 as shown in Fig. (7.17 (i)). Note that in Fig. (7.17 (i)), the secondary of the ideal transformer is on open-circuit. Consequently, both primary and secondary currents are zero. We can, therefore, remove the transformer, yielding the equivalent circuit shown in Fig. (7.17 (ii)). The primary current can now be readily found out.

$$I_1 = \frac{V_1}{\left(Z_2 / K^2\right)}$$

The circuits of Fig. (7.16) and Fig. (7.17 (ii)) are electrically equivalent. Thus referring to Fig. (7.16),

$$I_1 = KI_2$$

Also if we refer to Fig. (7.17 (ii)). we have,

$$I_1 = \frac{V_1}{(Z_2 / K^2)} = \frac{K^2 V_1}{Z_2} = \frac{K(K\ V_1)}{Z_2}$$

$$= K\frac{V_2}{Z_2} = KI_2 \qquad \left(\because\ I_2 = \frac{V_2}{Z_2}\right)$$

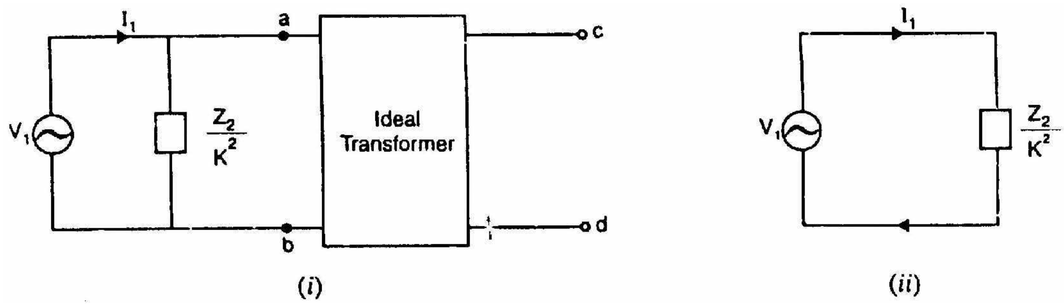

Fig. (7.17)

Thus the value of primary current I_1 is the same whether we use Fig. (7.16) or Fig. (7.17 (ii)). Obviously, it is easier to use Fig. (7.17 (ii)) as it contains no transformer.

(ii) Referred to secondary

Refer back to Fig. (7.16). There is no impedance on the primary side. However, voltage V_1 in the primary when transferred to the secondary becomes K V_1 as shown in Fig. (7.18 (i)). Note that in Fig. (7.18 (ii)), the primary of the transformer is on open circuit. Consequently, both primary and secondary currents are zero. As before, we can remove the transformer yielding the equivalent circuit shown in Fig. (7.18 (ii)). The secondary current I_2 can be readily found out as:

$$I_2 = \frac{K\ V_1}{Z_2}$$

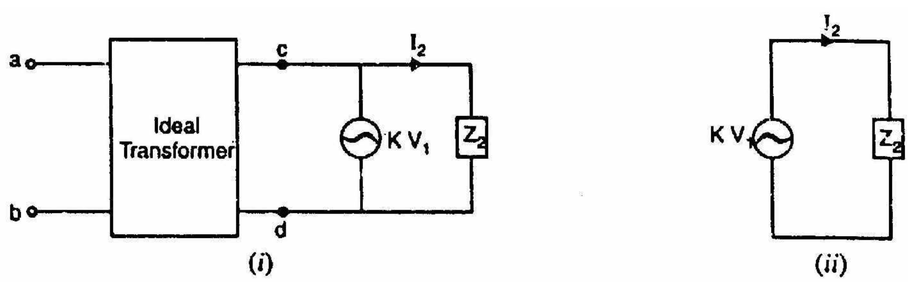

Fig. (7.18)

The circuits of Fig. (7.16) and Fig. (7.18 (ii)) are electrically equivalent. Thus referring back to Fig. (7.16), we have,

$$I_2 = \frac{V_2}{Z_2}$$

Also if we refer to Fig. (7.18 (ii)), we have,

$$I_1 = \frac{K\ V_1}{Z_2} = \frac{K(V_2/K)}{Z_2} = \frac{V_2}{Z_2}$$

Thus the value of secondary current I_2 is the same whether we use Fig. (7.16) or Fig. (7.18 (ii)). Obviously, it is easier to use Fig. (7.18 (ii) as it contains no transformer.

7.12 Exact Equivalent Circuit of a Loaded Transformer

Fig. (7.19) shows the exact equivalent circuit of a transformer on load. Here R_1 is the primary winding resistance and R_2 is the secondary winding resistance. Similarly, X_1 is the leakage reactance of primary winding and X_2 is the leakage reactance of the secondary winding. The parallel circuit $R_0 - X_0$ is the no-load equivalent circuit of the transformer. The resistance R_0 represents the core losses (hysteresis and eddy current losses) so that current I_W which supplies the core losses is shown passing through R_0. The inductive reactance X_0 represents a loss-free coil which passes the magnetizing current I_m. The phasor sum of I_W and I_m is the no-load current I_0 of the transformer.

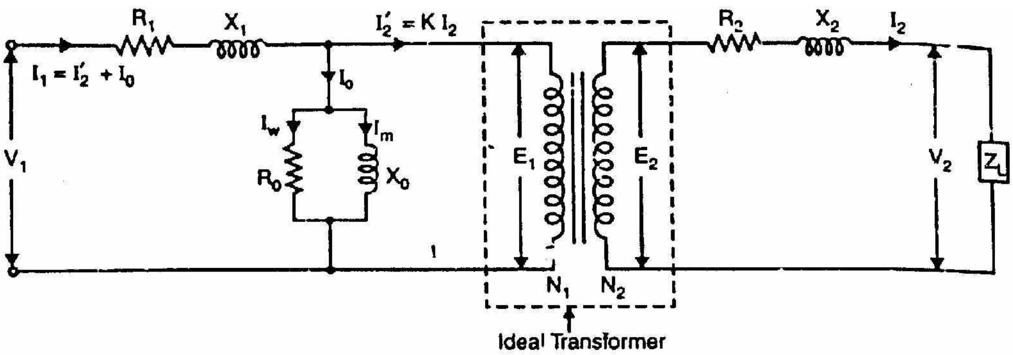

Fig. (7.19)

Note that in the equivalent circuit shown in Fig. (7.19), the imperfections of the transformer have been taken into account by various circuit elements. Therefore, the transformer is now the ideal one. Note that equivalent circuit has created two normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I'_2}{I_2}$$

The following points may be noted from the equivalent circuit:

(i) When the transformer is on no-load (i.e., secondary terminals are open-circuited), there is no current in the secondary winding. However, the primary draws a small no-load current I_0. The no-load primary current I_0

is composed of (a) magnetizing current (I_m) to create magnetic flux in the core and (b) the current I_W required to supply the core losses.

(ii) When the secondary circuit of a transformer is closed through some external load Z_L, the voltage E_2 induced in the secondary by mutual flux will produce a secondary current I_2. There will be $I_2 R_2$ and $I_2 X_2$ drops in the secondary winding so that load voltage V_2 will be less than E_2.

$$V_2 = E_2 - I_2(R_2 + j X_2) = E_2 - I_2 Z_2$$

(iii) When the transformer is loaded to carry the secondary current I_2, the primary current consists of two components:

(a) The no-load current I_0 to provide magnetizing current and the current required to supply the core losses.

(b) The primary current I'_2 (= K I_2) required to supply the load connected to the secondary.

$$\therefore \quad \text{Total primary current } I_1 = I_0 + (-KI_2)$$

(iv) Since the transformer in Fig. (7.19) in now ideal, the primary induced voltage E_1 can be calculated from the relation:

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

If we add I_1R_1 and I_1X_1 drops to E_1, we get the primary input voltage V_1

$$V_1 = -E_1 + I_1(R_1 + j X_1) = -E_1 + I_1 Z_1$$

or $$V_1 = -E_1 + I_1 Z_1$$

7.13 Simplified Equivalent Circuit of a Loaded Transformer

The no-load current I_0 of a transformer is small as compared to the rated primary current. Therefore, voltage drops in R_1 and X_1 due to I_0 are negligible. The equivalent circuit shown in Fig. (7.19) above can, therefore, be simplified by transferring the shunt circuit $R_0 - X_0$ to the input terminals as shown in Fig. (7.20). This modification leads to only slight loss of accuracy.

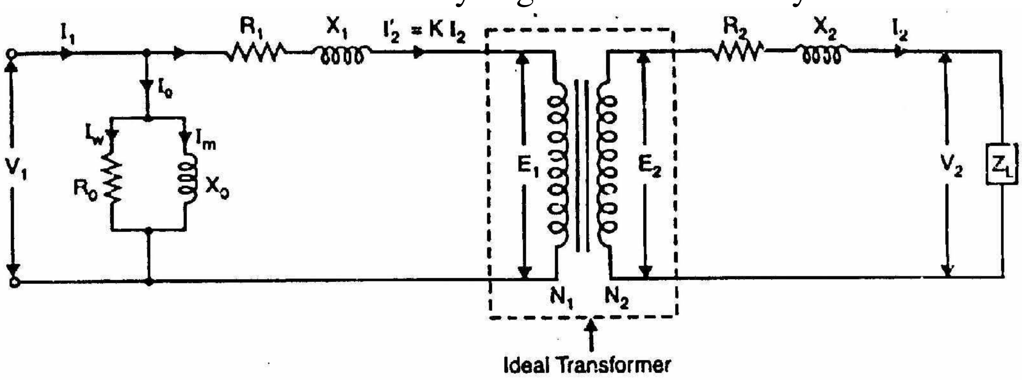

Fig.(7.20)

(i) Equivalent circuit referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in Fig. (7.21 (i)). This further reduces to Fig. (7.21 (ii)). Note that when secondary quantities are referred to primary, resistances/reactances/impedances are divided by K^2, voltages are divided by K and currents are multiplied by K.

$$\therefore \quad K'_2 = \frac{R_2}{K^2}; \quad X'_2 = \frac{X_2}{K^2}; \quad Z'_L = \frac{Z_L}{K^2}; \quad V'_2 = \frac{V_2}{K}; \quad I'_2 = K\,I_2$$

$$Z_{01} = R_{01} + j\,X_{01}$$

where $\quad R_{01} = R_1 + R'_2; \quad X_{01} = X_1 + X'_2$

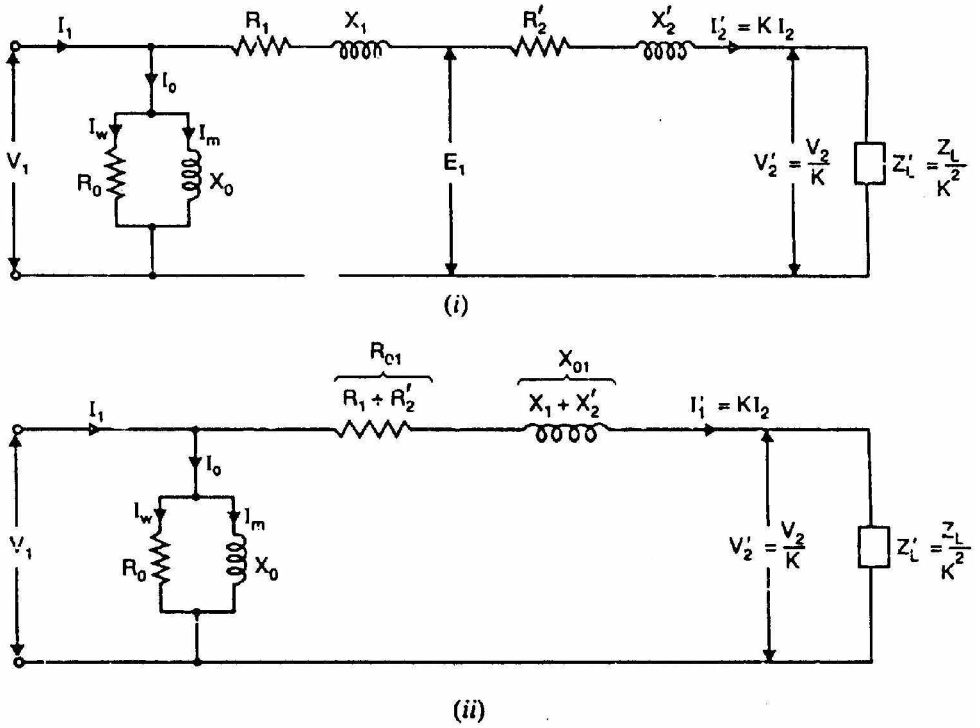

Fig. (7.21)

Phasor diagram. Fig. (7.22) shows the phasor diagram corresponding to the equivalent circuit shown in Fig. (7.21 (ii)). The referred value of load voltage V'_2 is chosen as the reference phasor. The referred value of load current I'_0 is shown lagging V'_2 by phase angle ϕ_2. For a given value of V'_2 both I'_2 and ϕ_2 are determined by the load. The voltage drop $I'_2\,R_{01}$

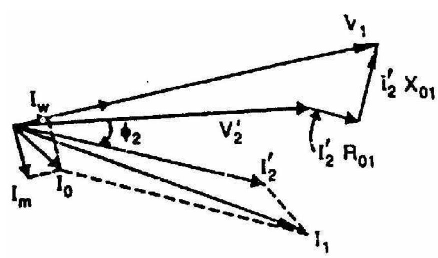

Fig.(7.22)

is in phase with I'_2 and the voltage drop $I'_2 X_{01}$, leads I'_2 by 90°. When these voltage drops are added to V'_2, we get the input voltage V_1.

The current I_W is in phase with V_1 while the magnetization current I_m lags behind V_1 by 90°. The phasor sum of I_W and I_m is the no-load current I_0. The phasor sum of I_0 and I'_2 is the input current I_1.

(ii) Equivalent circuit referred to secondary.

If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown in Fig. (7.23 (i)). This further reduces to Fig. (7.23 (ii)). Note that when primary quantities are referred to secondary resistances/reactances/impedances are multiplied by K^2, voltages are multiplied by K. and currents are divided by K.

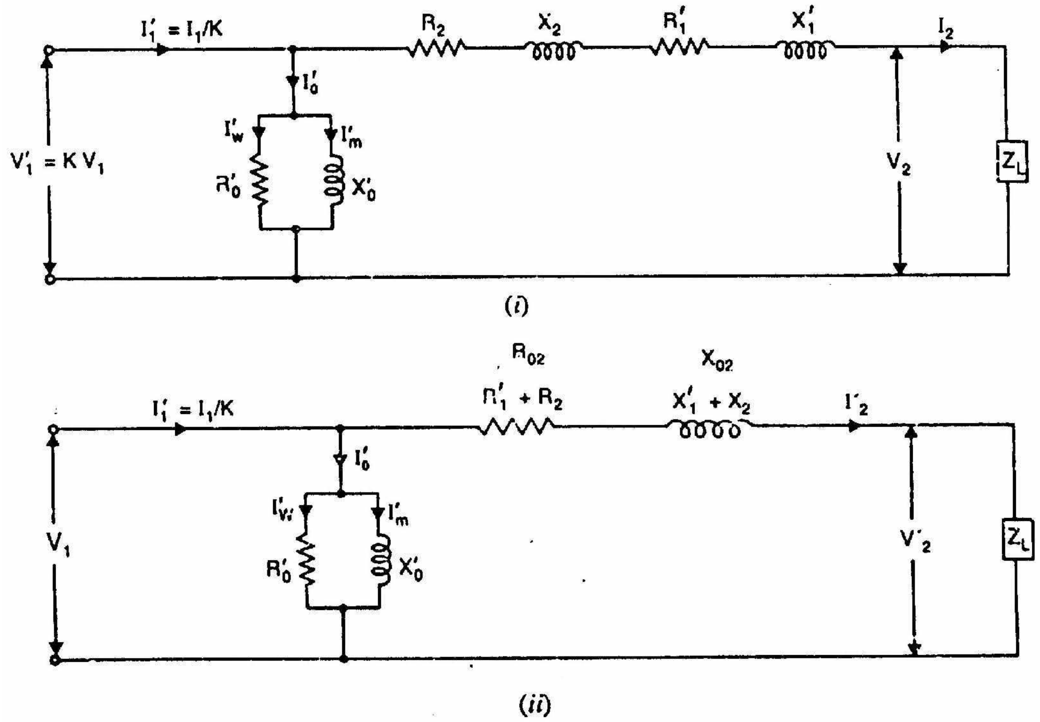

Fig. (7.23)

$$\therefore \quad R'_1 = K^2 R_1; \qquad X'_1 = K^2 X_1; \qquad V'_2 = K\, V_1;\ I'_1 = \frac{I_1}{K}$$

$$Z_{02} = R_{02} + j\, X_{02}$$

where $\quad R_{02} = R_2 + R'_1; \quad X_{02} = X_2 + X'_1$

Phasor diagram. Fig. (7.24) shows the phasor diagram of the equivalent circuit shown in Fig. (7.23 (ii)). The load voltage V_2 is chosen as the

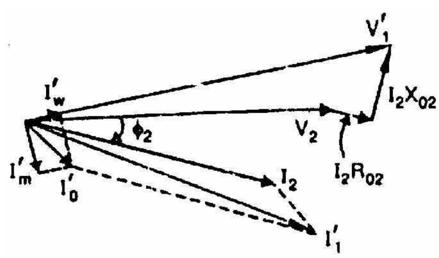

Fig.(7-24)

reference phasor. The load current I_2 is shown lagging the load voltage V_2 by phase angle ϕ_2. The voltage drop $I_2 R_{02}$ is in phase with I_2 and the voltage drop $I_2 X_{02}$ leads I_2 by 90°. When these voltage drop are added to V_2, we get the referred primary voltage V'_1 (= KV_1).

The current I'_W is in phase with V'_1 while the magnetizing current I'_m lags behind V'_1 by 90°. The phasor sum of I'_W and I'_m gives the referred value of no-load current I'_0. The phasor sum of I'_0 and load current I_2 gives the referred primary current I'_1 (= I_1/K).

7.14 Approximate Equivalent Circuit of a Loaded Transformer

The no-load current I_0 in a transformer is only 1-3% of the rated primary current and may be neglected without any serious error. The transformer can then be shown as in Fig. (7.25). This is an approximate representation because no-load current has been neglected. Note that all the circuit elements have been shown external so that the transformer is an ideal one.

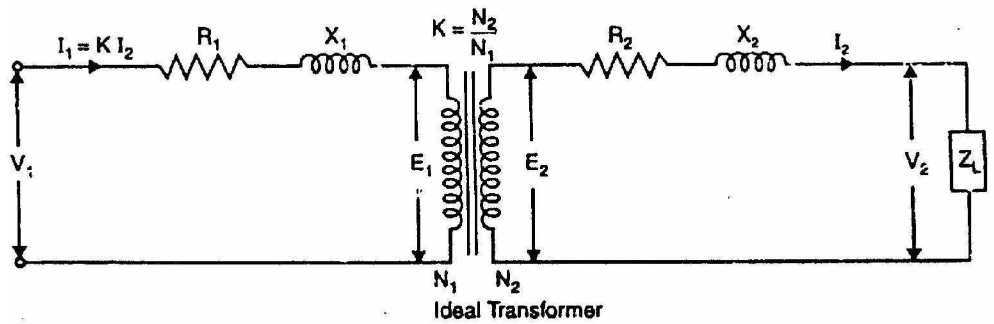

Fig. (7.25)

As shown in Sec. 7.11, if we refer all the quantities to one side (primary or secondary), the ideal transformer stands removed and we get the equivalent circuit.

(i) Equivalent circuit of transformer referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to primary as shown in Fig. (7.26). Note that when secondary quantities are referred to primary, resistances/reactances are divided by K^2, voltages are divided by K and currents are multiplied by K.

The equivalent circuit shown in Fig. (7.26) is an electrical circuit and can be solved for various currents and voltages. Thus if we find V'_2 and I'_2, then actual secondary values can be determined as under:

Actual secondary voltage, $V_2 = K V'_2$

Actual secondary current, $I_2 = I'_2/K$

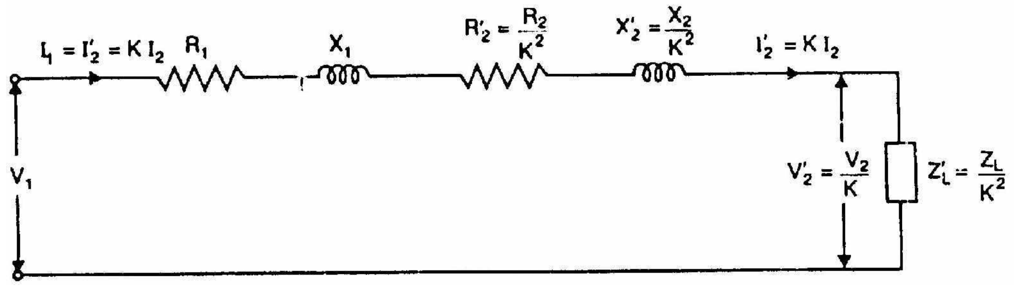

Fig.(7.26)

(ii) Equivalent circuit of transformer referred to secondary

If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown in Fig. (7.27). Note that when primary quantities are referred to secondary, resistances/reactances are multiplied by K^2, voltages are multiplied by K and currents are divided by K.

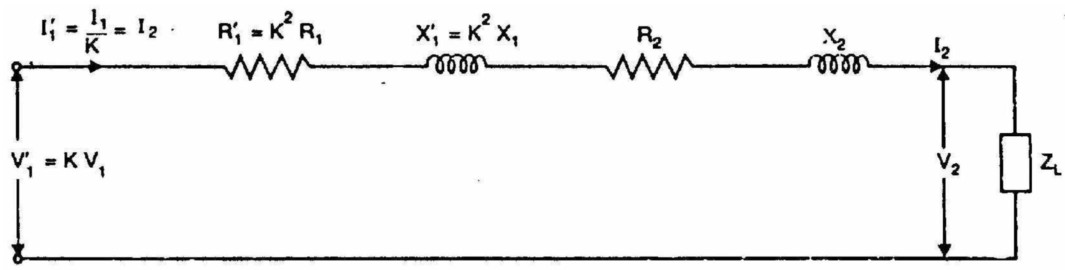

Fig. (7.27)

The equivalent circuit shown in Fig. (7.27) is an electrical circlet and can be solved for various voltages and currents. Thus is we find V'_1 and I'_1, then actual primary values can be determined as under:

Actual primary voltage, $V_1 = V'_1/K$

Actual primary current, $I_1 = KI'_1$

Note: The same final answers will be obtained whether we use the equivalent circuit referred to primary or secondary. The use of a particular equivalent circuit would depend upon the conditions of the problem.

7.15 Approximate Voltage Drop in a Transformer

The approximate equivalent circuit of transformer referred to secondary is shown in Fig. (7.28). At no-load, the secondary voltage is K V_1. When a load having a lagging p.f. cos ϕ_2 is applied, the secondary carries a current I_2 and voltage drops occur in $(R_2 + K^2 R_1)$ and $(X_2 + K^2 X_1)$. Consequently, the secondary voltage falls from K V_1 to V_2. Referring to Fig. (7.28), we have,

$$V_2 = KV_1 - I_2\left[\left(R_2 + K^2R_1\right) + j\left(X_2 + K^2X_1\right)\right]$$
$$= KV_1 - I_2\left(R_{02} + j\,X_{02}\right)$$
$$= KV_1 - V_2 = I_2 Z_{02}$$

Drop in secondary voltage $= KV_1 - V_2 = I_2 Z_{02}$

The phasor diagram is shown in Fig. (7.29). It is clear from the phasor diagram that drop in secondary voltage is AC = $I_2\ Z_{02}$. It can be found as follows. With O as centre and OC as radius, draw an arc cutting OA produced at M. Then AC = AM = AN. From B, draw BD perpendicular to OA produced. Draw CN perpendicular to OM and draw BL || OM.

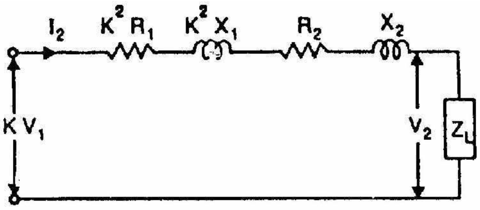

Fig.(7.28)

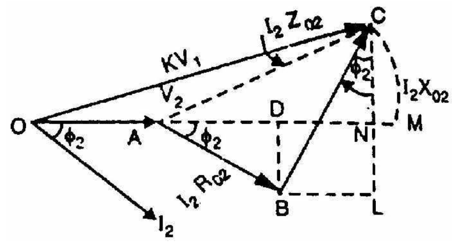

Fig.(7.29)

Approximate drop in secondary voltage

$$= AN = AD + DN$$
$$= AD + BL \qquad (\because\ BL = DN)$$
$$= I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$$

For a load having a leading p.f. cos ϕ_2, we have,

Approximate voltage drop $= I_2R_{02}\cos\phi_2 - I_2X_{02}\sin\phi_2$

Note: If the circuit is referred to primary, then it can be easily established that:

Approximate voltage drop $= I_1R_{01}\cos\phi_2 \pm I_1X_{01}\sin\phi_2$

7.16 Voltage Regulation

The voltage regulation of a transformer is the arithmetic difference (not phasor difference) between the no-load secondary voltage (${}_0V_2$) and the secondary voltage V_2 on load expressed as percentage of no-load voltage i.e.

$$\%\text{age voltage regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$$

where ${}_0V_2$ = No-load secondary voltage = K V_1

V_2 = Secondary voltage on load

As shown in Sec. 7.15

$$ {}_0V_2 - V_2 = I_2R_{02}\cos\phi_2 \pm I_2X_{02}\sin\phi_2 $$

The +ve sign is for lagging p.f. and −ve sign for leading p.f.

It may be noted that %age voltage regulation of the transformer will be the same whether primary or secondary side is considered.

7.17 Transformer Tests

The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests (i) open-circuit test and (ii) short-circuit lest. These tests are very convenient as they provide the required information without actually loading the transformer. Further, the power required to carry out these tests is very small as compared with full-load output of the transformer. These tests consist of measuring the input voltage, current and power to the primary first with secondary open-circuited (open-circuit test) and then with the secondary short-circuited (short circuit test).

7.18 Open-Circuit or No-Load Test

This test is conducted to determine the iron losses (or core losses) and parameters R_0 and X_0 of the transformer. In this test, the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open-circuited. The applied primary voltage V_1 is measured by the voltmeter, the no-load current I_0 by ammeter and no-load input power W_0 by wattmeter as shown in Fig. (7.30 (i)). As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current I_0 is very small (usually 2-10 % of rated current). Cu losses in the primary under no-load condition are negligible as compared with iron losses. Hence, wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads. Fig. (7.30 (ii)) shows the equivalent circuit of transformer on no-load.

Iron losses, P_i = Wattmeter reading = W_0

No load current = Ammeter reading = I_0

Applied voltage = Voltmeter reading = V_1

Input power, $W_0 = V_1 I_0 \cos\phi_0$

$$\therefore \quad \text{No - load p.f., } \cos\phi_0 = \frac{W_0}{V_1} I_0$$

$$I_W = I_0 \cos\phi_0; \qquad I_m = I_0 \sin\phi_0$$

$$R_0 = \frac{V_1}{I_W} \quad \text{and} \quad X_0 = \frac{V_1}{I_m}$$

Thus open-circuit test enables us to determine iron losses and parameters R_0 and X_0 of the transformer.

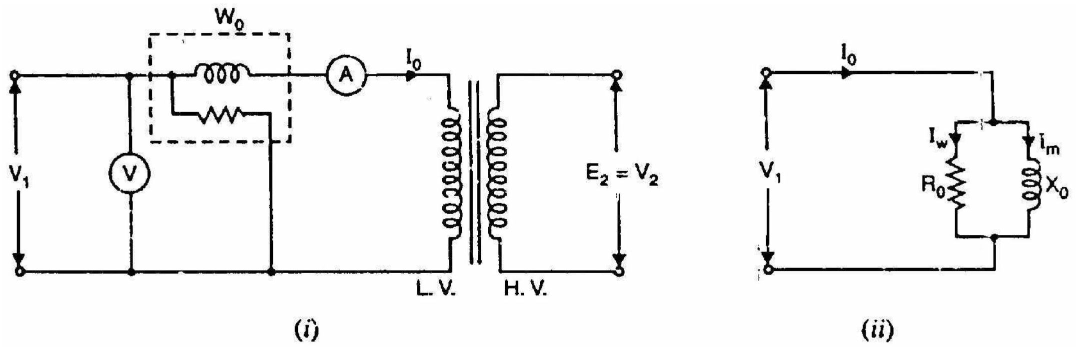

Fig.(7.30)

7.19 Short-Circuit or Impedance Test

This test is conducted to determine R_{01} (or R_{02}), X_{01} (or X_{02}) and full-load copper losses of the transformer. In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in Fig. (7.31 (i)). The low input voltage is gradually raised till at voltage V_{SC}, full-load current I_1 flows in the primary. Then I_2 in the secondary also has full-load value since $I_1/I_2 = N_2/N_1$. Under such conditions, the copper loss in the windings is the same as that on full load.

There is no output from the transformer under short-circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss in the core is negligibly small since the voltage V_{SC} is very small. Hence, the wattmeter will practically register the full-load copper losses in the transformer windings. Fig. (7.31 (ii)) shows the equivalent circuit of a transformer on short circuit as referred to primary; the no-load current being neglected due to its smallness.

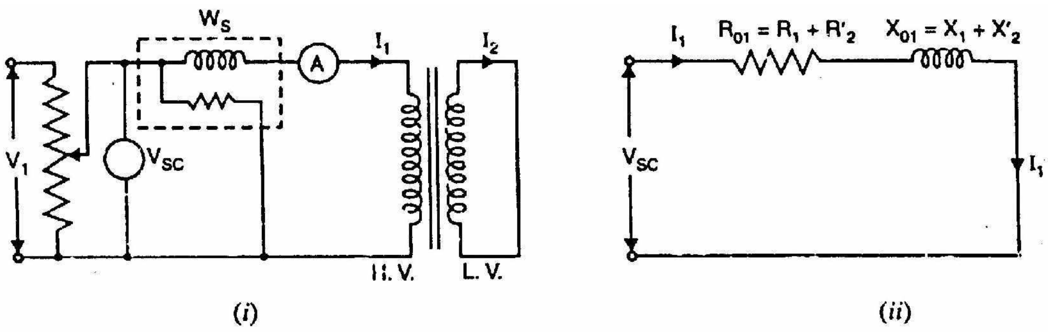

Fig. (7.31)

Full load Cu loss, P_C = Wattmeter reading = W_S
Applied voltage = Voltmeter reading = V_{SC}
F.L. primary current = Ammeter reading = I_1

$$P_C = I_1^2 R_1 + I_1^2 R'_2 = I_1^2 R_{01}$$

$$\therefore \quad R_{01} = \frac{P_C}{I_1^2}$$

where R_{01} is the total resistance of transformer referred to primary.

Total impedance referred to primary, $Z_{01} = \frac{V_{SC}}{I_1}$

Total leakage reactance referred to primary, $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$

Short-circuit p.f, $\cos\phi_2 = \frac{P_C}{V_{SC} I_1}$

Thus short-circuit lest gives full-load Cu loss, R_{01} and X_{01}.

Note: The short-circuit test will give full-load Cu loss only if the applied voltage V_{SC} is such so as to circulate full-load currents in the windings. If in a short-circuit test, current value is other than full-load value, the Cu loss will be corresponding to that current value.

7.20 Advantages of Transformer Tests

The above two simple transformer tests offer the following advantages:

(i) The power required to carry out these tests is very small as compared to the full-load output of the transformer. In case of open-circuit lest, power required is equal to the iron loss whereas for a short-circuit test, power required is equal to full-load copper loss.

(ii) These tests enable us to determine the efficiency of the transformer accurately at any load and p.f. without actually loading the transformer.

(iii) The short-circuit test enables us to determine R_{01} and X01 (or R_{02} and X_{02}). We can thus find the total voltage drop in the transformer as referred to primary or secondary. This permits us to calculate voltage regulation of the transformer.

7.21 Separation of Components of Core Losses

The core losses (or iron losses) consist of hysteresis loss and eddy current loss. Sometimes it is desirable to find the hysteresis loss component and eddy current loss component in the total core losses.

$$\text{Hysteresis loss, } P_h = k_h \ f \ B_m^{1.6} \qquad \text{watts/m}^3$$

$$\text{Eddy current loss, } P_e = k_e \ f^2 \ B_m^2 t^2 \qquad \text{watts/m}^3$$

where B_m = maximum flux density; f = frequency; k_h, k_e = constants

For a given a.c. machine and maximum flux density (B_m),

$$P_h \propto f \qquad \text{and} \qquad P_e \propto f^2$$

or

$$P_h = a\,f \qquad \text{and} \qquad P_e = b\,f^2$$

where a and b are constants.

Total core loss, $P_i = af + bf^2$

Hence if the total core loss for given B_m is known at two frequencies, the constants a and b can be calculated. Knowing the values of a and b, the hysteresis loss component and eddy current loss component of the core loss can be determined.

P_t/f and f curve

$$P_i = af + bf^2$$

or

$$\frac{P_i}{f} = a + bf$$

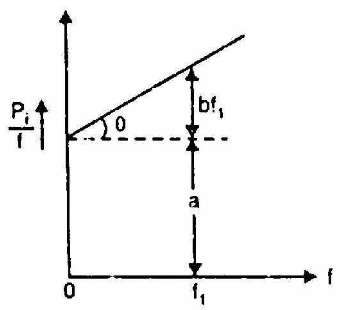

Fig.(7.32)

The total core losses are measured at various frequencies while the other factors upon which core losses depend are maintained constant. If a graph is plotted between P_i/f and f, it will be a straight line with slope tan θ = b (See Fig. 7.32). Therefore, constants a and b can be evaluated. Hence the hysteresis and eddy current losses at a given frequency (say f_1) can be found out.

7.22 Why Transformer Rating in kVA?

An important factor in the design and operation of electrical machines is the relation between the life of the insulation and operating temperature of the machine. Therefore, temperature rise resulting from the losses is a determining factor in the rating of a machine. We know that copper loss in a transformer depends on current and iron loss depends on voltage. Therefore, the total loss in a transformer depends on the volt-ampere product only and not on the phase angle between voltage and current i.e., it is independent of load power factor. For this reason, the rating of a transformer is in kVA and not kW.

7.23 Sumpner or Back-to-Back Test

This test is conducted simultaneously on two identical transformers and provides data for finding the efficiency, regulation and temperature rise. The main advantage of this test is that the transformers are tested under full-load conditions without much expenditure of power. The power required to conduct this test is equal to the losses of the two transformers. It may be noted that two identical transformers are needed to carry out this test.

Circuit

Fig. (7.33) shows the connections for back-to-back test on two identical transformers T_1 and T_2. The primaries of the two transformers are connected in parallel across the rated voltage V_1 while the two secondaries are connected in phase opposition. Therefore, there will be no circulating current in the loop formed by the secondaries because their induced e.m.f.s are equal and in opposition. There is an auxiliary low-voltage transformer which can be adjusted to give a variable voltage and hence current in the secondary loop circuit. A wattmeter W_1, an ammeter A_1 and voltmeter V_1 are connected to the input side. A wattmeter W_2 and ammeter A_2 are connected in the secondary circuit.

Operation

(i) The secondaries of the transformers are in phase opposition. With switch S_1 closed and switch S_2 open (i.e., regulating transformer not in the circuit), there will be no circulating current ($I_2 = 0$) in the secondary loop circuit. It is because the induced e.m.f.s in the secondaries are equal and in opposition. This situation is just like an open-circuit test. Therefore, the current drawn from the supply is $2\ I_0$ where I_0 is the no-load current of each transformer. The reading of wattmeter W_1 will be equal to the core losses of the two transformers.

$$W_1 = \text{Core losses of the two transformers}$$

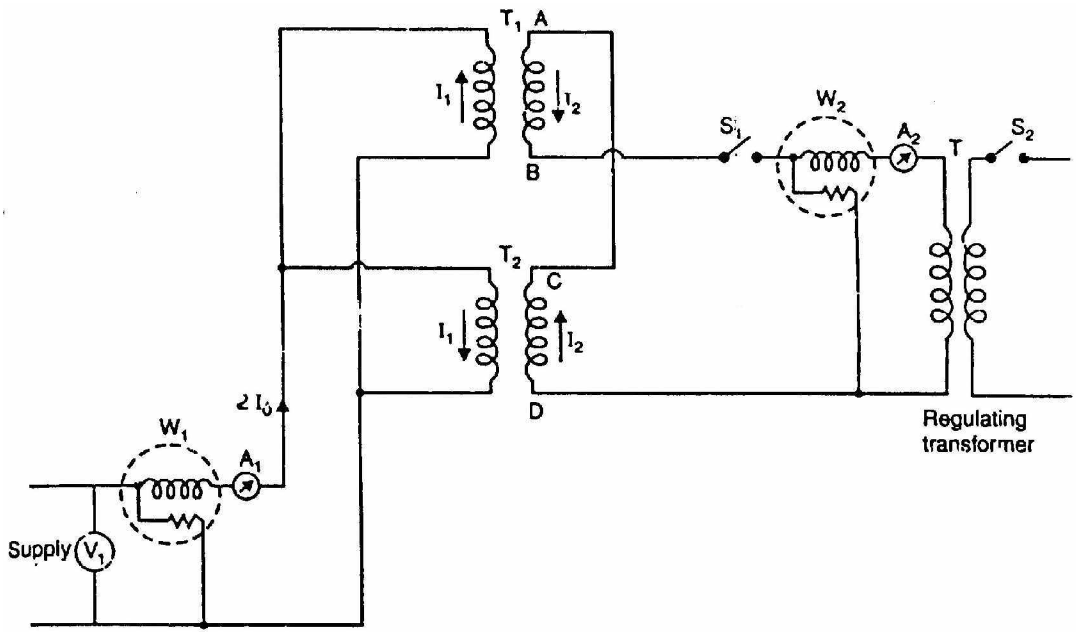

Fig.(7.33)

(ii) Now switch S_2 is also closed and output voltage of the regulating transformer is adjusted till full-load current I_2 flows in the secondary loop circuit. The full-load secondary current will cause full-load current I_1 (= K I_2) in the primary circuit. The primary current I_1 circulates in the primary winding only and will not pass through W_1. Note that full-load currents are flowing through the primary and secondary windings. Therefore, reading of wattmeter W_2 will be equal to the full-load copper losses of the two transformers.

W_2 = Full-load Cu losses of two transformers

$\therefore$ $W_1 + W_2$ = Total losses of two transforms at full load

The following points may be noted:

(a) The wattmeter W_1 gives the core losses of the two transformers while wattmeter W_2 gives the full-load copper losses (or at any other load current I_2) of the two transformers. Therefor, power required to conduct this test is equal to the total losses of the two transformers.

(b) Although transformers are not supplying any load, yet full iron loss and full-load copper losses are occurring in them.

(c) There are two voltage sources (supply voltage and regulating transformer) and there is no Interference between them. The supply voltage gives only $2I_0$ while regulating transformer supplies I_2 and hence I_1 (= K I_2).

Advantages

(i) The power required to carry out the test is small.

(ii) The transformers are tested under full-load conditions.

(iii) The iron losses and full-load copper losses are measured simultaneously.
(iv) The secondary current I_2 can be adjusted to any current value. Therefore, we can find the copper loss at full-load or at any other load.
(v) The temperature rise of the transformers can be noted.

7.24 Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core or Iron losses   2. Copper losses

These losses appear in the form of heat and produce (i) an increase in temperature and (ii) a drop in efficiency.

1. Core or Iron losses (P_i)

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test.

$$\text{Hysteresis loss, } = k_h \ f \ B_m^{1.6} \quad \text{watts}/\text{m}^3$$

$$\text{Eddy current loss, } = k_e \ f^2 \ B_m^2 t^2 \qquad \text{watts}/\text{m}^3$$

Both hysteresis and eddy current losses depend upon (i) maximum flux density B_m in the core and (ii) supply frequency f. Since transformers are connected to constant-frequency, constant voltage supply, both f and B_m are constant. Hence, core or iron losses are practically the same at all loads.

$$\text{Iron or Core losses, } P_i = \text{Hysteresis loss} + \text{Eddy current loss}$$
$$= \text{Constant losses}$$

The hysteresis loss can be minimized by using steel of high silicon content whereas eddy current loss can be reduced by using core of thin laminations.

2. Copper losses

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test.

Total Cu losses, $P_C = I_1^2 R_1 + I_2^2 R_2$

$$= I_1^2 R_{01} \text{ or } I_2^2 R_{02}$$

It is clear that copper losses vary as the square of load current Thus if copper losses are 400 W at a load current of 10 A, then they will be $(1/2)^2 \times 400 = 100$ W at a load current of 5A.

Total losses in a transformer = $P_1 + P_C$

= Constant losses + Variable losses

It may be noted that in a transformer, copper losses account for about 90% of the total losses.

7.25 Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.,

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

It may appear that efficiency can be determined by directly loading the transformer and measuring the input power and output power. However, this method has the following drawbacks:

(i) Since the efficiency of a transformer is very high, even 1% error in each wattmeter (output and input) may give ridiculous results. This test, for instance, may give efficiency higher than 100%.

(ii) Since the test is performed with transformer on load, considerable amount of power is wasted. For large transformers, the cost of power alone would be considerable.

(iii) It is generally difficult to have a device that is capable of absorbing all of the output power.

(iv) The test gives no information about the proportion of various losses.

Due to these drawbacks, direct loading method is seldom used to determine the efficiency of a transformer. In practice, open-circuit and short-circuit tests are carried out to find the efficiency.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

The losses can be determined by transformer tests.

7.26 Efficiency from Transformer Tests

F.L. Iron loss = P_1 ...from open-circuit test

F.L. Cu loss = P_C ...from short-circuit test

Total F.L. losses = $P_1 + P_C$

We can now find the full-load efficiency of the transformer at any p.f. without actually loading the transformer.

$$\text{F.L. efficiency, } \eta_{F.L.} = \frac{\text{Full - load VA} \times \text{p.f.}}{(\text{Full - load VA} \times \text{p.f.}) + P_i + P_C}$$

Also for any load equal to x x full-load,

Corresponding total losses = $P_i + x^2 P_C$

$$\text{Corresponding } \eta_x = \frac{(\text{xx Full - load VA}) \times \text{p.f.}}{(\text{xx Full - load VA} \times \text{p.f.}) + P_i + x^2 P_C}$$

Note that iron loss remains the same at all loads.

7.27 Condition for Maximum Efficiency

Output power = $V_2 I_2 \cos \phi_2$

If R_{02} is the total resistance of the transformer referred to secondary, then,

$$\text{Total Cu loss, } P_C = I_2^2 R_{02}$$

Total losses = $P_i + P_C$

$$\therefore \quad \text{Transformer } \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{02}}$$

$$= \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + P_i / I_2 + I_2 R_{02}} \qquad \text{(i)}$$

For a normal transformer, V_2 is approximately constant. Hence for a load of given p.f., efficiency depends upon load current I_2. It is clear from exp (i) above that numerator is constant and for the efficiency to be maximum, the denominator should be minimum i.e.,

$$\frac{d}{dI_2}(\text{denominator}) = 0$$

or

$$\frac{d}{dI_2}\left(V_2 \cos\phi_2 + P_i / I_2 + I_2 R_{02}\right) = 0$$

or $$0 - \frac{P_i}{I_2^2} + R_{02} = 0$$

or $$P_i = I_2^2 R_{02} \qquad \text{(ii)}$$

i.e., Iron losses = Copper losses

Hence efficiency of a transformer will be maximum when copper losses are equal to constant or iron losses.

From eq. (ii) above, the load current I_2 corresponding to maximum efficiency is given by;

$$I_2 = \sqrt{\frac{P_i}{R_{02}}}$$

The relative value of these losses is in the control of the designer of the transformer according to the relative amount of copper and iron he uses. A transformer which is to operate continuously on full-load would, therefore, be designed to have-maximum efficiency at full-load. However, distribution transformers operate for long periods on light load. Therefore, their point of maximum efficiency is usually arranged to be between three-quarter and half lull-load.

Note. In a transformer, iron losses are constant whereas copper losses are variable. In order to obtain maximum efficiency, the load current should be such that total Cu losses become equal to iron losses.

7.28 Output kVA Corresponding to Maximum Efficiency

Let P_C = Copper losses at full-load kVA

P_i = Iron losses

x = Fraction of full-load kVA at which efficiency is maximum

Total Cu losses = $x^2 P_C$

$x^2 P_C = P_i$ ...for maximum efficiency

or $$x = \sqrt{\frac{P_i}{P_C}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

∴ Output kVA corresponding to maximum efficiency

$$= \text{xx Full - load kVA} = \text{Full - load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

It may be noted that the value of kVA at which the efficiency is maximum is independent of p.f. of the load.

7.29 All-Day (or Energy) Efficiency

The ordinary or commercial efficiency of a transformer is defined as the ratio of output power to the input power i.e.,

$$\text{Commercial efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

There are certain types of transformers whose performance cannot be judged by this efficiency. For instance, distribution transformers used for supplying lighting loads have their primaries energized all the 24 hours in a day but the secondaries supply little or no load during the major portion of the day. It means that a constant loss (i.e., iron loss) occurs during the whole day but copper loss occurs only when the transformer is loaded and would depend upon the magnitude of load. Consequently, the copper loss varies considerably during the day and the commercial efficiency of such transformers will vary from a low value (or even zero) to a high value when the load is high. The performance of such transformers is judged on the basis of energy consumption during the whole day (i.e., 24 hours). This is known as all-day or energy efficiency.

The ratio of output in kWh to the input in kWh of a transformer over a 24-hour period is known as all-day efficiency i.e.,

$$\eta_{\text{all-day}} = \frac{\text{kWh output in 24 hours}}{\text{kWh input in 24 hours}}$$

All-day efficiency is of special importance for those transformers whose primaries are never open-circuited but the secondaries carry little or no load much of the time during the day. In the design of such transformers, efforts should be made to reduce the iron losses which continuously occur during the whole day.

Note. Efficiency of a transformer means commercial efficiency unless stated otherwise.

7.30 Construction of a Transformer

We usually design a power transformer so that it approaches the characteristics of an ideal transformer. To achieve this, following design features are incorporated:

(i) The core is made of silicon steel which has low hysteresis loss and high permeability. Further, core is laminated in order to reduce eddy current loss. These features considerably reduce the iron losses and the no-load current.

(ii) Instead of placing primary on one limb and secondary on the other, it is a usual practice to wind one-half of each winding on one limb. This ensures tight coupling between the two windings. Consequently, leakage flux is considerably reduced.

(iii) The winding resistances R_1 and R_2 are minimized to reduce I^2R loss and resulting rise in temperature and to ensure high efficiency.

7.31 Types of Transformers

Depending upon the manner in which the primary and secondary are wound on the core, transformers are of two types viz., (i) core-type transformer and (ii) shell-type transformer.

(i) **Core-type transformer**. In a core-type transformer, half of the primary winding and half of the secondary winding are placed round each limb as shown in Fig. (7.34). This reduces the leakage flux. It is a usual practice to place the low-voltage winding below the high-voltage winding for mechanical considerations.

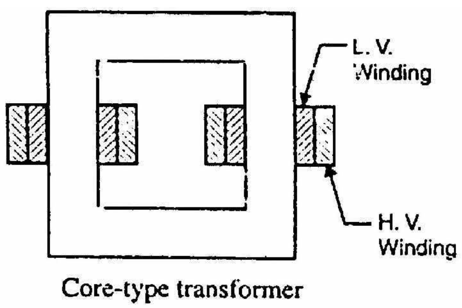

Fig.(7.34)

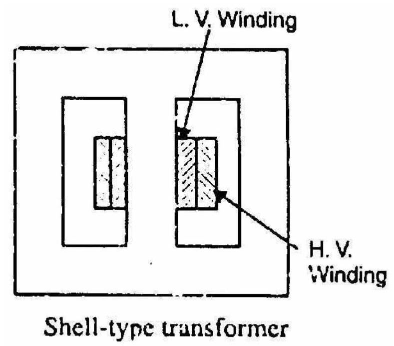

Fig.(7.35)

(ii) **Shell-type transformer**. This method of construction involves the use of a double magnetic circuit. Both the windings are placed round the central limb (See Fig. 7.35), the other two limbs acting simply as a low-reluctance flux path.

The choice of type (whether core or shell) will not greatly affect the efficiency of the transformer. The core type is generally more suitable for high voltage and small output while the shell-type is generally more suitable for low voltage and high output.

7.32 Cooling of Transformers

In all electrical machines, the losses produce heat and means must be provided to keep the temperature low. In generators and motors, the rotating unit serves as a fan causing air to circulate and carry away the heat. However, a transformer has no rotating parts. Therefore, some other methods of cooling must be used. Heat is produced in a transformer by the iron losses in the core and I^2R loss in the windings. To prevent undue temperature rise, this heat is removed by cooling.

(i) In small transformers (below 50 kVA), natural air cooling is employed i.e., the heat produced is carried away by the surrounding air.

(ii) Medium size power or distribution transformers are generally cooled by housing them in tanks filled with oil. The oil serves a double purpose, carrying the heat from the windings to the surface of the tank and insulating the primary from the secondary.

(iii) For large transformers, external radiators are added to increase the cooling surface of the oil filled tank. The oil circulates around the transformer and moves through the radiators where the heat is released to surrounding air. Sometimes cooling fans blow air over the radiators to accelerate the cooling process.

7.33 Autotransformer

An autotransformer has a single winding on an iron core and a part of winding is common to both the primary and secondary circuits. Fig. (7.36 (i)) shows the connections of a step-down autotransformer whereas Fig. (7.36 (ii)) shows the connections of a step-up autotransformer. In either case, the winding ab having N_1 turns is the primary winding and winding be having N_2 turns is the secondary winding. Note that the primary and secondary windings are connected electrically as well as magnetically. Therefore, power from the primary is transferred to the secondary conductively as well as inductively (transformer action). The voltage transformation ratio K of an ideal autotransformer is

$$K = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

Note that in an autotransformer, secondary and primary voltages are related in the same way as in a 2-winding transformer.

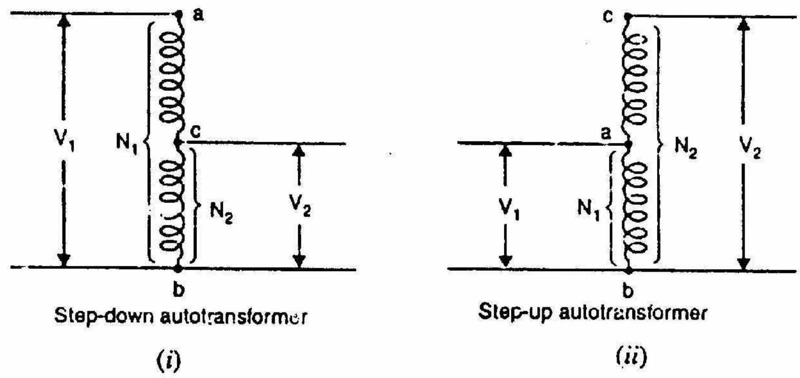

Fig.(7.36)

Fig. (7.37) shows the connections of a loaded step-down as well as step-up autotransformer. In each case, I_1 is the input current and I_2 is the output or load current. Regardless of autotransformer connection (step-up or step-down), the current in the portion of the winding that is common to both the primary and the secondary is the difference between these currents (I_1 and I_2). The relative direction of the current through the common portion of the winding depends upon the connections of the autotransformer. It is because the type of connection determines whether input current I_1 or output current I_2 is larger. For step-down autotransformer $I_2 > I_1$ (as for 2-winding transformer) so that $I_2 - I_1$ current flows through the common portion of the winding. For step-up autotransformer, $I_2 < I_1$. Therefore, $I_1 - I_2$ current flews in the common portion of the winding.

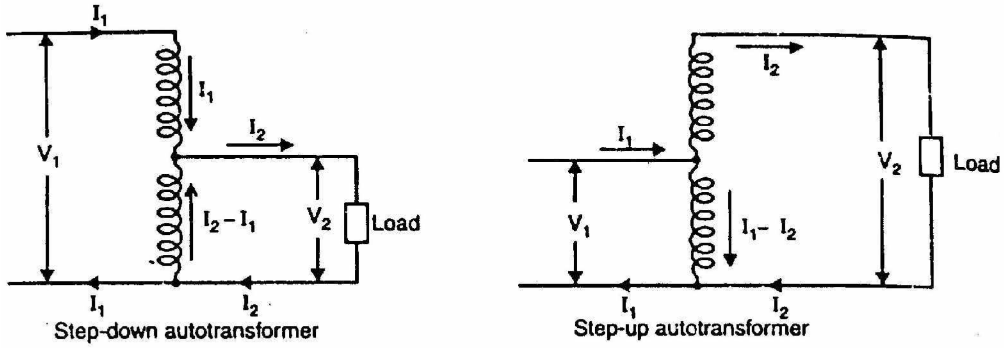

Fig.(7.37)

In an ideal autotransformer, exciting current and losses are neglected. For such an autotransformer, as K approaches 1, the value of current in the common portion ($I_2 - I_1$ or $I_1 - I_2$) of the winding approaches zero. Therefore, for value of K near unity, the common portion of the winding can be wound with wire of smaller cross-sectional area. For this reason, an autotransformer requires less copper.

7.34 Theory of Autotransformer

Fig. (7.38 (i)) shows an ideal step-down autotransformer on load. Here winding 1-3 having N_1 turns is the primary winding while winding 2-3 having N_2 turns is the secondary winding. The input current is I_1 while the output or load current is I_2. Note that portion 1-2 of the winding has $N_1 - N_2$ turns and voltage across this portion of the winding is $V_1 - V_2$. The current through the common portion of the winding is $I_2 - I_1$.

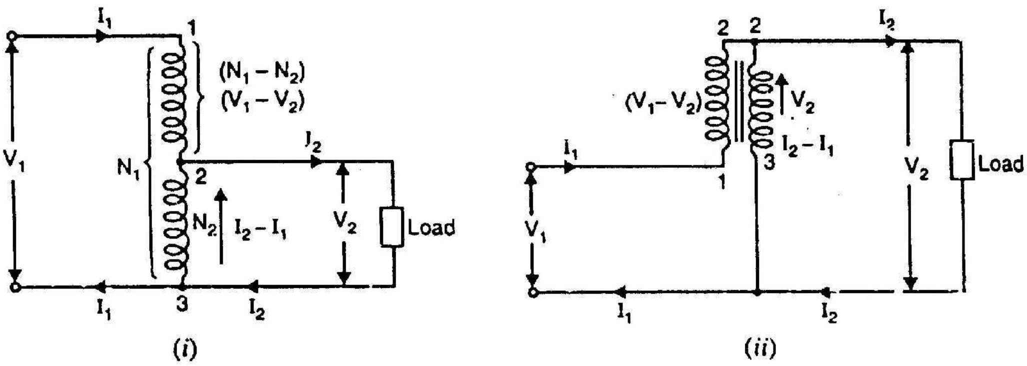

Fig.(7.38)

Fig. (7.38 (ii)) shows the equivalent circuit of the autotransformer. From this equivalent circuit, we have,

$$\frac{V_2}{V_1 - V_2} = \frac{N_2}{N_1 - N_2}$$

$$V_2(N_1 - N_2) = N_2(V_1 - V_2)$$

or $$V_2N_1 - V_2N_2 = N_2V_1 - N_2V_2$$

or $$V_2N_1 = N_2V_1$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{N_2}{N_1} = K \qquad \text{(i)}$$

Also $$(V_1 - V_2)I_1 = (I_2 - I$$

or $$V_1I_1 - V_2I_1 = V_2I_2 -$$

or $$V_1I_1 = V_2I_2$$

$$\therefore \quad \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

(
i

From i) eqs. (i) and (ii),

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

Also $V_1 I_1 = V_2 I_2$ (Input apparent power = Output apparent power)

Output

Since the primary and secondary windings of an autotransformer are connected magnetically as well as electrically, the power from primary is transferred to the secondary inductively (transformer action) as well as conductively (i.e., conducted directly from source to the load).

Output apparent power = $V_2 I_2$

Apparent power transferred inductively = $V_2(I_2 - I_1) = V_2(I_2 - K\,I_2)$

$= V_2 I_2(1 - K) = V_1 I_1(1 - K)$

$\therefore$ Power transferred inductively = Input $\times$ $(1 - K)$

$\therefore$ Power transferred conductively = Input $-$ Input $(1 - K)$

= Input $[1 - (1 - K)]$

= $K \times$ Input

Suppose the input power to an ideal autotransformer is 1000 W and its voltage transformation ratio K= 1/4. Then,

$$\text{Power transferred inductively} = \text{Input} \times (1 - K) = 1000\left(1 - \frac{1}{4}\right) = 750 \text{ W}$$

$$\text{Power transferred conductively} = K \times \text{Input} = \frac{1}{4} \times 1000 = 250 \text{ W}$$

Note that input power to the autotransformer is 1000 W. Out of this, 750 W is transferred to the secondary by transformer action (inductively) while 250 W is conducted directly from the source to the load (i.e., it is transferred conductively to the load).

7.35 Saving of Copper in Autotransformer

For the same output and voltage transformation ratio $K(N_2/N_1)$, an autotransformer requires less copper than an ordinary 2-winding transformer. Fig. (7.39 (i)) shows an ordinary 2-winding transformer whereas Fig. (7.39 (ii)) shows an autotransformer having the same output and voltage transformation ratio K.

The length of copper required in a winding is proportional to the number of turns and the area of cross-section of the winding wire is proportional to the current rating. Therefore, the volume and hence weight of copper required in a winding is proportional to current x turns i.e.,

$$\text{Weight of Cu required in a winding} \propto \text{current} \times \text{turns}$$

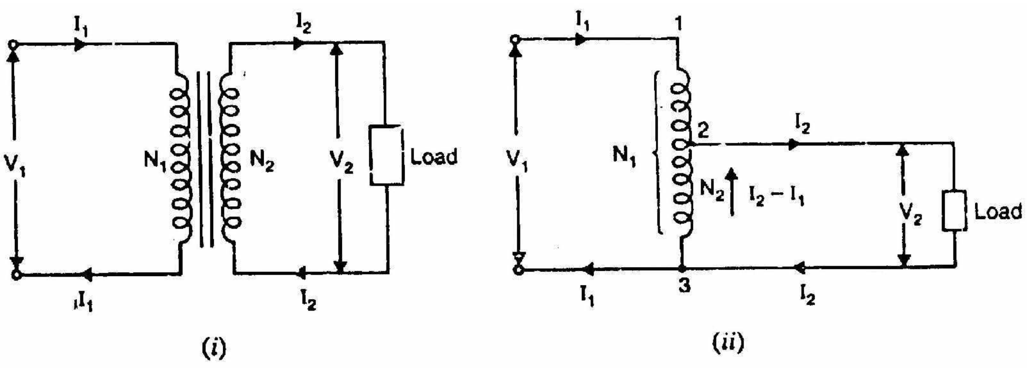

Fig.(7.39)

Winding transformer

$$\text{Weight of Cu required} \propto (I_1N_1 + I_2N_2)$$

Autotransformer

$$\text{Weight of Cu required in section 1-2} \propto I_1(N_1 - N_2)$$

$$\text{Weight of Cu required in section 2-3} \propto (I_2 - I_1)N_2$$

$$\therefore \quad \text{Total weight of Cu required} \propto I_1\,(N_1 - N_2) + (I_2 - I_1)N_2$$

$$\frac{\text{Weight of Cu in autotransformer}}{\text{Weight of Cu in ordinary transformer}} = \frac{I_1(N_1 - N_2) + (I_2 - I_1)N_2}{I_1N_1 + I_2N_2}$$

$$= \frac{N_1I_1 - N_2I_1 + N_2I_2 - N_2I_1}{N_1I_1 + N_2I_2}$$

$$= \frac{N_1I_1 + N_2I_2 - 2N_2I_1}{N_1I_1 + N_2I_2}$$

$$= 1 - \frac{2N_2I_1}{N_1I_1 + N_2I_2}$$

$$= 1 - \frac{2N_2I_1}{2N_1I_1} \qquad (\because \ N_2I_2 = N_1I_1)$$

$$= 1 - \frac{N_2}{N_1} = 1 - K$$

$\therefore$ Wt. of Cu in autotransformer (W_a)

= $(1 - K) \times$ Wt. in ordinary transformer (W_o)

or $\qquad W_a = (1 - K) \times W_o$

$\therefore \qquad$ Saving in Cu = $W_o - W_a = W_a - (1 - K)W_o = K\,W_o$

or $\qquad$ Saving in Cu = $K \times$ Wt. ofCu in ordinary transformer

Thus if K = 0.1, the saving of Cu is only 10% but if K = 0.9, saving of Cu is 90%. Therefore, the nearer the value of K of autotransformer is to 1, the greater is the saving of Cu.

7.36 Advantages and Disadvantages of autotransformers

Advantages

(i) An autotransformer requires less Cu than a two-winding transformer of similar rating.

(ii) An autotransformer operates at a higher efficiency than a two-winding transformer of similar rating.

(iii) An autotransformer has better voltage regulation than a two-winding transformer of the same rating.

(iv) An autotransformer has smaller size than a two-winding transformer of the same rating.

(v) An autotransformer requires smaller exciting current than a two-winding transformer of the same rating.

It may be noted that these advantages of the autotransformer decrease as the ratio of transformation increases. Therefore, an autotransformer has marked advantages only for relatively low values of transformation ratio (i.e. values approaching 1).

Disadvantages

(i) There is a direct connection between the primary and secondary. Therefore, the output is no longer d.c. isolated from the input.

(ii) An autotransformer is not safe for stepping down a high voltage to a low voltage. As an illustration, Fig. (7.40) shows 11000/230 V step-down autotransformer. If an open circuit develops in the common portion 2-3 of the winding, then full-primary voltage (i.e., 11000 V in this case) will appear across the load. In such a case, any one coming in contact with the secondary is subjected to high voltage. This could be dangerous to both the persons and equipment. For this reason, autotransformers are prohibited for general use.

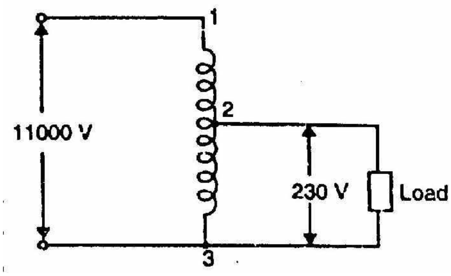

Fig.(7-40)

(iii) The short-circuit current is much larger than for the two-winding transformer of the same rating. It can be seen from Fig. (7.40) that a short-circuited secondary causes part of the primary also to be short-circuited. This reduces the effective resistance and reactance.

7.37 Applications of Autotransformers

(i) Autotransformers are used to compensate for voltage drops in transmission and distribution lines. When used for this purpose, they are known as booster transformers.

(ii) Autotransformers are used for reducing the voltage supplied to a.c. motors during the starting period.

(iii) Autotransformers are used for continuously variable supply.

7.38 Conversion of Two-Winding Transformer Into Autotransformer

A two-winding transformer can be converted into an autotransformer, either step-up or step-down. Consider a 10 kVA, 2300/230 V two-winding transformer shown in Fig. (7.41 (i)). If we want to convert it into autotransformer, the two windings of the transformer are connected in series. If we use the additive polarity as shown in Fig. (7.41 (ii)), we get step-up autotransformer. The voltage rating of the autotransformer is now 2300/2530 V. If we use subtractive polarity as shown in Fig. (7.41 (iii)), we get a step-down autotransformer. The voltage rating of the transformer is now 2300/2070 V.

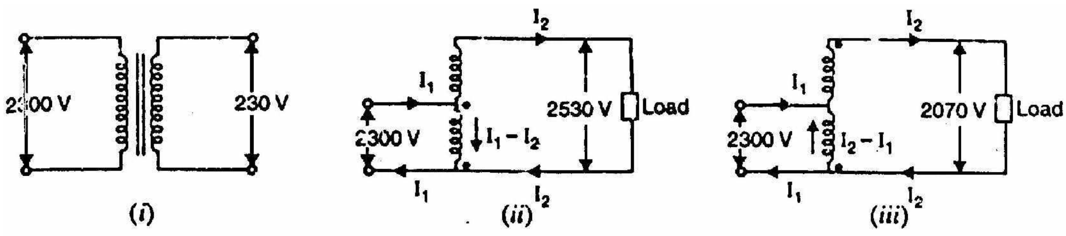

Fig.(7.41)

When a two-winding transformer is converted into autotransformer, the kVA rating of the resulting autotransformer is greatly increased. This higher rating results from the conduction connection.

7.39 Parallel Operation of Single-Phase Transformers

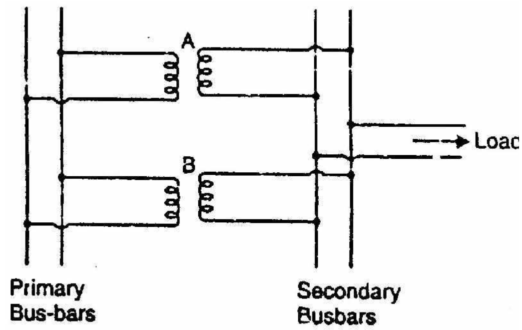

Fig.(7.42)

Two transformers are said to be connected in parallel if the primary windings are connected to supply busbars and secondary windings are connected to load busbars. Fig. (7.42) shows two transformers A and B in parallel. While connecting two or more than two transformers in parallel, it is essential that their terminals of similar polarities are joined to the same busbars as shown in Fig. (7.42). The wrong connections may result in a dead short-circuit and primary transformers may be damaged unless protected by fuses or circuit breakers. There are three principal reasons for connecting transformers in parallel. Firstly, if one transformer fails, the continuity of supply can be maintained through other transformers. Secondly, when the load on the sub-station becomes more than the capacity of the existing transformers, another transformer can be added in parallel. Thirdly, any transformer can be taken out of the circuit for repair/routine maintenance without interrupting supply to the consumers.

Conditions for satisfactory parallel operation

In order that the transformers work satisfactorily in parallel, the following conditions should be satisfied:

(i) Transformers should be properly connected with regard to their polarities.

(ii) The voltage ratings and voltage ratios of the transformers should be the same.

(iii) The per unit or percentage impedances of the transformers should be equal.

(iv) The reactance/resistance ratios of the transformers should be the same.

Condition (i)

Condition (i) is absolutely essential because wrong connections may result in dead short-circuit. Fig. (7.43 (i)) shows the correct method of connecting two single-phase transformers in parallel. It will be seen that round the loop formed by the secondaries, the two secondary e.m.f.s E_A and E_B oppose and there will be no circulating current.

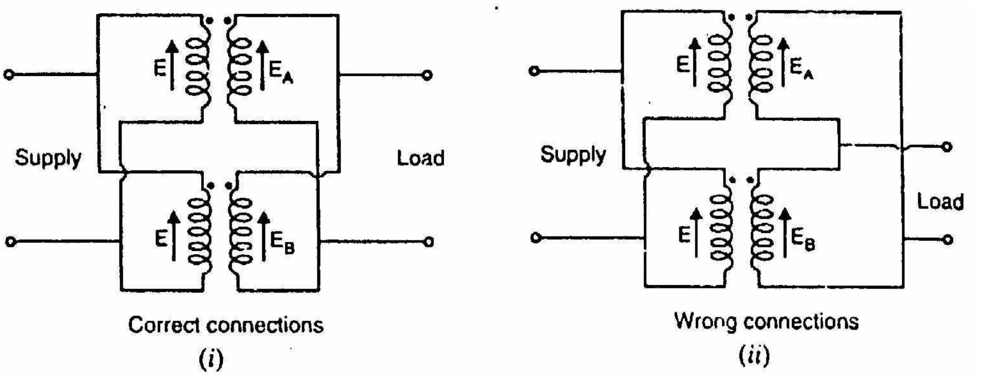

Fig.(7.43)

Fig. (7.43 (ii)) shows the wrong method of connecting two single-phase transformers is parallel. Here the two secondaries are so connected that their e.m.f.s E_A and E_B are additive. This may lead to short-circuit conditions and a very large circulating current will flow in the loop formed by the two secondaries. Such a condition may damage the transformers unless they are protected by fuses and circuit breakers.

Condition (ii)

This condition is desirable for the satisfactory parallel operation of transformers. If this condition is not met, the secondary e.m.f.s will not be equal and there will be circulating current in the loop formed by the secondaries. This will result in the unsatisfactory parallel operation of transformers. Let us illustrate this point. Consider two single-phase transformer A and B operating in parallel as shown in Fig. (7.44). Let E_A and E_B be their no-load secondary voltages and Z_A and Z_B be their impedances referred to the secondary. Then at no-load, the circulating current in the loop formed by the secondaries is

$$\text{Circulating current, } I_C = \frac{E_A - E_B}{Z_A + Z_B} \qquad \text{assuming } E_A > E_B$$

Even a small difference in the induced secondary voltages can cause a large circulating current in the secondary loop because impedances of the transformers

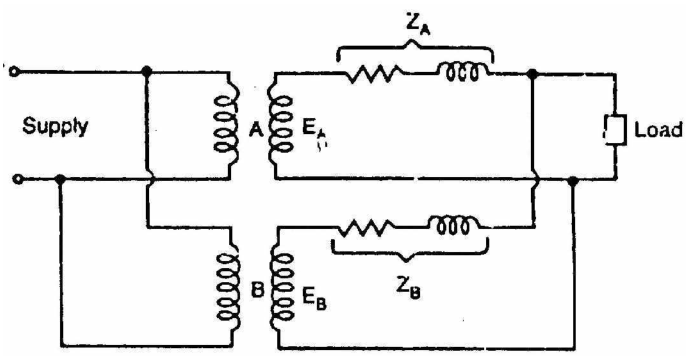

Fig.(7.44)

are small. This secondary circulating current will cause current to be drawn from the supply by the primary of each transformer. These currents will cause copper losses in both primary and secondary. This creates heating with no useful output. When load is connected to the system, this circulating current will tend to produce unequal loading conditions i.e., the transformers will not share the load according to their kVA ratings. It is because the circulating current will tend to make the terminal voltages of the same value for both transformers. Therefore, transformer with smaller voltage ratio will tend to carry more than its proper share of load. Thus, one transformer would tend to become overloaded than the other and the system could not be loaded to the summation of transformer ratings without overloading one transformer.

Condition (iii)
This condition is also desirable for proper parallel operation of transformers. If this condition is not met, the transformers will not share the load according to their kVA ratings. Sometimes this condition is not fulfilled by the design of the transformers. In that case, it can be corrected by inserting proper amount of resistance or reactance or both in series with either primary or secondary circuits of the transformers where the impedance is below the value required to fulfil condition (iii).

Condition (iv)
If the reactance/resistance ratios of the two transformers are not equal, the power factor of the load supplied by the transformers will not be equal. In other words, one transformer will be operating with a higher and the other with a lower power factor than that of the load. Condition (iii) is much more important than condition (iv). Considerable deviation from condition (iv) will result in only a small reduction in the satisfactory degree of operation. When desired, condition (iv) also may be improved by inserting external impedance of proper value.

7.40 Single-Phase Equal Voltage Ratio Transformers in Parallel

Fig. (7.45) shows two single-phase equal voltage ratio transformers A and B in parallel. The secondary e.m.f.s of the two transformers are equal (i.e., $E_A = E_B = E$) because they have the same turns ratio and have their primaries connected to the same supply.

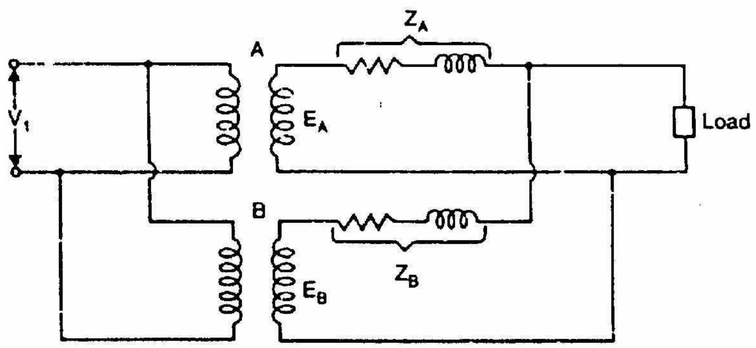

Fig.(7.45)

If the magnetizing current is ignored, the two transformers can be represented by their equivalent circuits referred to secondary as shown in Fig. (7.46). It is clear that the transformers will share total load in the same way as two impedances in parallel.

Let Z_A, Z_B = Impedances of transformers referred to secondary

I_A,I_B = Their respective currents

V_2 = Common terminal voltage

I = Total load current

It is clear from Fig. (7.46) that:

$$I_A + I_B = I \qquad \text{(i)}$$

and $$I_A Z_A = I_B Z_B$$

$$\therefore \quad I_A = I_B \frac{Z_B}{Z_A}$$

$$\therefore \quad I_B \frac{Z_B}{Z_A} + I_B = I$$

or $$I_B\left(1 + \frac{Z_B}{Z_A}\right) = I$$

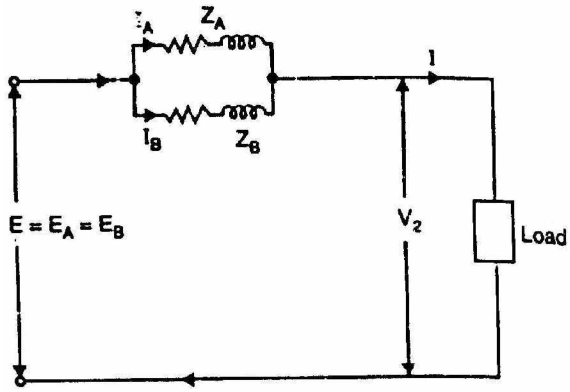

Fig.(7.46)

$$\therefore \quad I_B = I\frac{Z_A}{Z_A + Z} \qquad \text{(ii)}$$

Similarly, $I_A = I\frac{Z_B}{Z_A + Z_B}$ (iii)

Thus the way in which the load current I is shared by the transformers is independent of load impedance and depends only on the transformer impedances.

kVA carried by each transformer

Let S = total load kVA = $V_2\, I \times 10^{-3}$

S_A = kVA carried by transformer A

S_B = kVA carried by transformer B

$$\therefore \quad S_A = V_2 I_A \times 10^{-3} = V_2 I \times 10^{-3} \times \frac{Z_B}{Z_A + Z_B} = S\frac{Z_B}{Z_A + Z_B}$$

or $$S_A = S\frac{Z_B}{Z_A + Z_B}$$

Also $$S_B = V_2 I_B \times 10^{-3} = V_2 I \times 10^{-3} \times \frac{Z_A}{Z_A + Z_B} = S\frac{Z_A}{Z_A + Z_B}$$

or $$S_B = S\frac{Z_A}{Z_A + Z_B}$$

Therefore, S_A and S_B are obtained in magnitude as well as m phase from the above expressions. It may be noted that in these expressions, Z_A and Z_B can be expressed in ohms or in p.u. If p.u. values are to be used, they should be with respect to common base kVA.

7.41 Single- Phase Unequal Voltage Ratio Transformers in Parallel

Fig. (7.47) shows two single-phase unequal voltage ratio transformers A and B in parallel. Since the voltage ratios of the transformers are unequal, their no-load secondary voltages will also be unequal. We shall calculate how load current is shared between the two transformers.

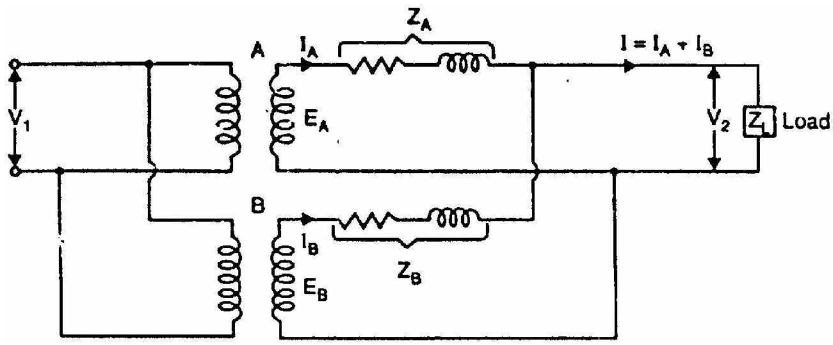

Fig.(7.47)

Fig. (7.48) shows the equivalent circuit of the transformers referred to secondary in a simplified way.

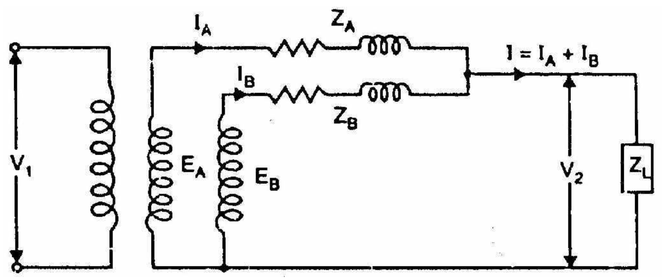

Fig.(7.48)

Let E_A, E_B = no-load secondary voltages of the two transformers. It is assumed that $E_A > E_B$.

I_A, I_B = their respective currents

I = load current

Z_A, Z_B = impedances of the transformers referred to secondary

Z_B = load impedance

V_2 = load voltage

At no-load, the circulating current I_C is

$$I_C = \frac{E_A - E_B}{Z_A + Z_B}$$

When the system is loaded, the load current I is shared by the two transformers. By kirchhof's oitage law,

$$E_A = V_2 + I_A Z_A$$

and $$E_B = V_2 + I_B Z_B$$

But $$V_2 = I\,Z_L = (I_A + I_B)Z_L$$

$$\therefore \quad E_A = (I_A + I_B)Z_L + I_A Z_A \qquad \text{(i)}$$

and $$E_B = (I_A + I_B)Z_L + I_B Z_B \qquad \text{(ii)}$$

Now $$E_A - E_B = I_A Z_A - I_B Z_B$$

or $$I_A = \frac{(E_A - E_B) + I_B Z_B}{Z_A}$$

Putting this value of I_A in eq. (ii), we get,

$$E_B = \left[\frac{(E_A - E_B) + I_B Z_B}{Z_A} + I_B\right] Z_L + I_B Z_B$$

On solving, $$I_B = \frac{E_B Z_A - (E_A - E_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)} \qquad \text{(iii)}$$

From the symmetry of the expression, we get,

$$I_A = \frac{E_A Z_B + (E_A - E_B)Z_L}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

Also $$I = I_A + I_B = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L(Z_A + Z_B)}$$

$$V_2 = I Z_L = \left[\frac{E_A Z_B + E_B Z_A}{Z_A Z_B + Z_L(Z_A + Z_B)}\right] Z_L$$

or $$V_2 = \frac{E_A Z_B + E_B Z_A}{\dfrac{Z_A Z_B}{Z_L} + Z_A + Z_B}$$

Since the transformers have a common primary voltage, E_A and E_B will be in phase with each other.

7.42 Three-Phase Transformer

A three-phase system in used to generate and transmit electric power. Three-phase voltages are raised or lowered by means of three-phase transformers. A three-phase transformer can be built in two ways viz., (i) by suitably connecting a bank of three single-phase transformers or (ii) by constructing a three-phase transformer on a common magnetic structure. In either case, the windings may be connected in Y – Y, Δ – Δ, Y – Δ or Δ – Y.

(i) Bank of three single-phase transformers

Three similar single-phase transformers can be connected to form a three-phase transformer. The primary and secondary windings may be connected in star (Y) or delta (Δ) arrangement. Fig. (7.49 (i)) shows a Y - Δ connection of a three-phase transformer. The primary windings are connected in star and the secondary windings are connected in delta. A more convenient way of showing this connection is illustrated in Fig. (7-49 (ii)). The primary and secondary windings shown parallel to each other belong to the same single-phase transformer. The ratio of secondary phase voltage to primary phase voltage is the phase transformation ratio K.

$$\text{Phase transformation ratio, K} = \frac{\text{Secondary phase voltage}}{\text{Primary phase voltage}} = \frac{N_2}{N_1}$$

Referring to Fig. (7.49 (ii)), primary line-to-line voltage is V and the primary line current is I. The phase transformation ratio is K (= N_2/N_1). The secondary line voltage and line current are also shown.

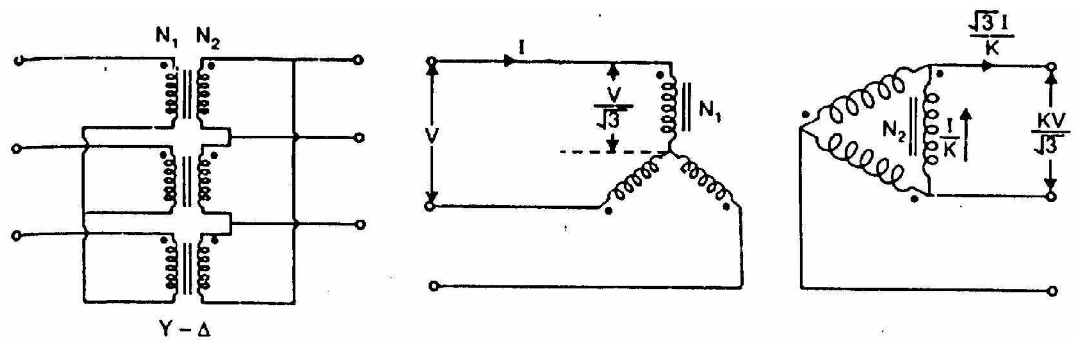

Fig.(7.49)

(ii) Three-phase transformer

A three-phase transformer can be constructed by having three primary and three secondary windings on a common magnetic circuit. The basic principle of a 3-phase transformer is illustrated in Fig. (7.50 (i)). The three single-phase core-type transformers, each with windings (primary and secondary) on only one leg have their unwound legs combined to provide a path for the returning flux. The primaries as well as secondaries may be connected in star or delta. If the primary is energized from a 3-phase supply, the central limb (i.e., unwound limb) carries the fluxes produced by the 3-phase primary windings. Since the phasor sum of three primary currents at any instant is zero, the sum of three fluxes passing through the central limb must be zero. Hence no flux exists in the central limb and it may, therefore, be eliminated. This modification gives a three leg core-type 3-phase transformer. In this case, any two legs will act as a return path for the flux in the third leg. For example, if flux is ϕ in one leg at some instant, then flux is $\phi/2$ in the opposite direction through the other two legs at the same instant. All the connections of a 3-phase transformer are made inside the case and for delta-connected winding three leads are brought out while for star-connected winding four leads are brought out.

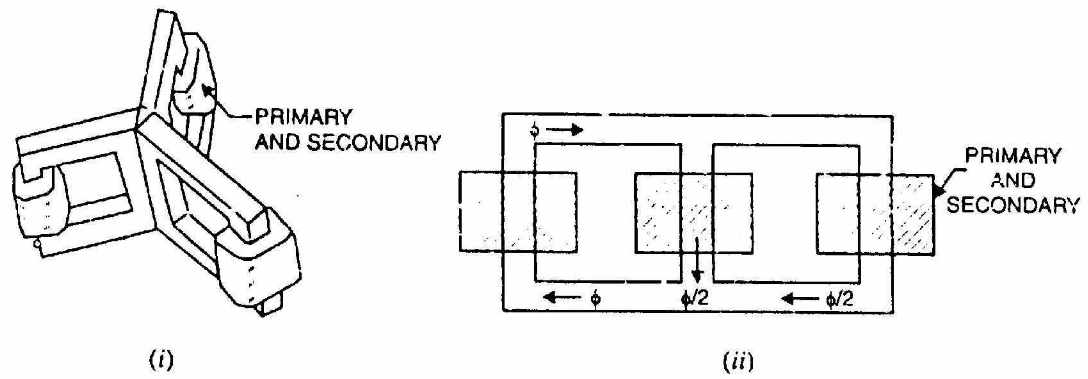

Fig.(7.50)

For the same capacity, a 3-phase transformer weighs less, occupies less space and costs about 20% less than a bank of three single-phase transformers. Because of these advantages, 3-phase transformers are in common use, especially for large power transformations.

A disadvantage of the three-phase transformer lies in the fact that when one phase becomes defective, the entire three-phase unit must be removed from service. When one transformer in a bank of three single-phase transformers becomes defective, it may be removed from service and the other two transformers may be reconnected to supply service on an emergency basis until repairs can be made.

7.43 Three-Phase Transformer Connections

A three-phase transformer can be built by suitably connecting a bank of three single-phase transformers or by one three-phase transformer. The primary or secondary windings may be connected in either star (Y) or delta (Δ) arrangement. The four most common connections are (i) Y-Y (ii) Δ-Δ (iii) Y-Δ and (iv) Δ-Y. These four connections are shown in Fig. (7.51). In this figure, the windings at the left are the primaries and those at the right are the secondaries. The primary and secondary voltages and currents are also shown. The primary line voltage is V and the primary line current is I. The phase transformation ratio K is given by;

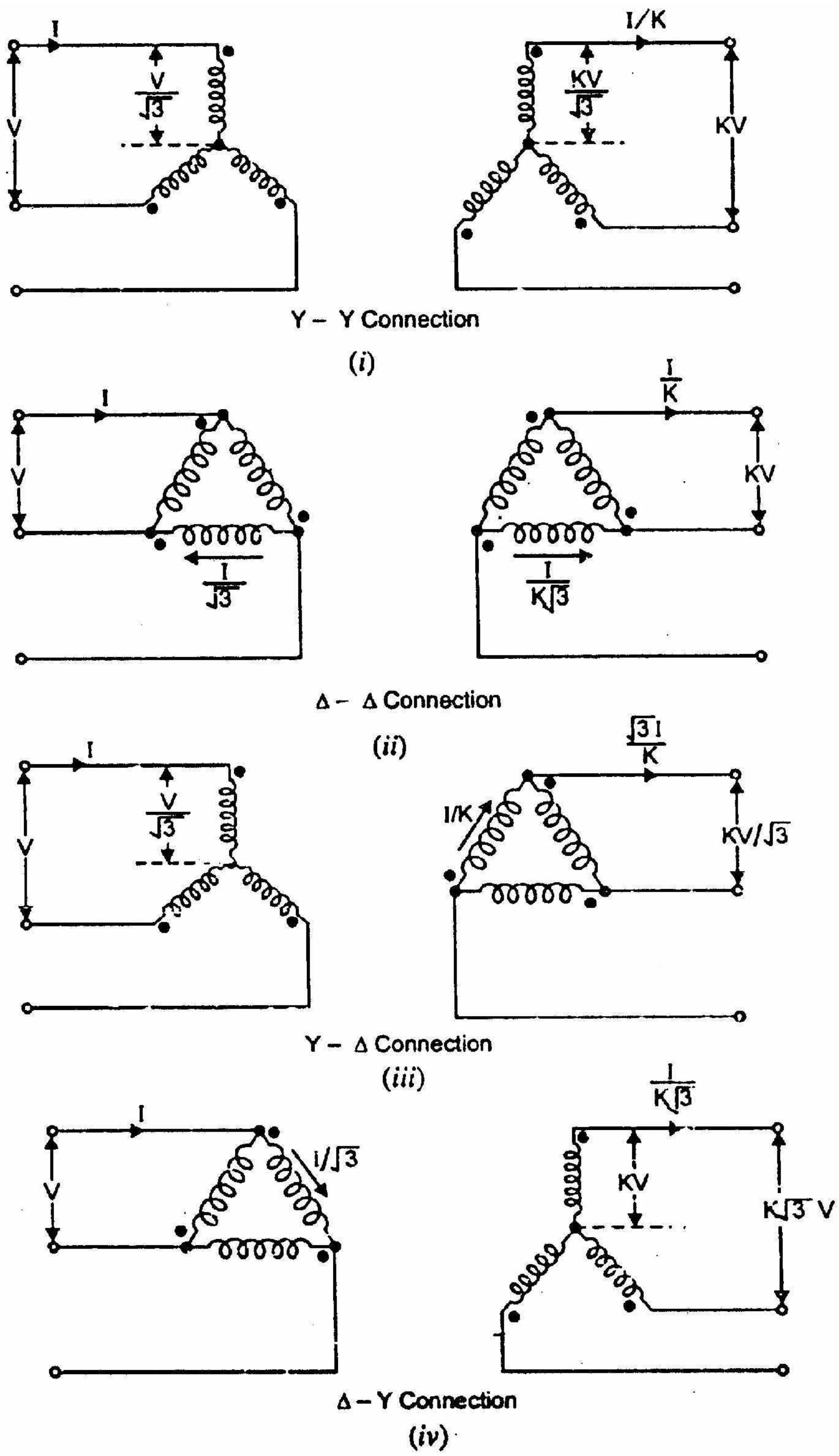

Fig.(7.51)

$$K = \frac{\text{Secondary phase voltage}}{\text{Primary phase voltage}} = \frac{N_2}{N_1}$$

(i) **Y-Y Connection**. In the Y-Y connection shown in Fig. (7.51 (i)), 57.7% (or $1/\sqrt{3}$) of the line voltage is impressed upon each winding but full line current flows in each winding. Power circuits supplied from a Y-Y bank often create serious disturbances in communication circuits in their immediate vicinity. Because of this and other disadvantages, the Y-Y connection is seldom used.

(ii) **Δ-Δ Connection**. The Δ-Δ connection shown in Fig. (7.51 (ii)) is often used for moderate voltages. An advantage of this connection is that if one transformer gets damaged or is removed from service, the remaining two can be operated in what is known as the open-delta or V-V connection. By being operated in this way, the bank still delivers three-phase currents and voltages in their correct phase relationships but the capacity of the bank is reduced to 57.7% of what it was with all three transformers in service.

(iii) **Y-Δ Connection**. The Y-Δ connection shown in Fig. (7.51(iii)) is suitable for stepping down a high voltage. In this case, the primaries are designed for 57.7% of the high-tension line voltages.

(iv) **Δ-Y Connection**. The Δ-Y connection shown in Fig. (7.51 (iv)) is commonly used for stepping up to a high voltage.

7.44 Three-Phase Transformation with Two Single-Phase Transformers

It is possible to transform three-phase power by using two single-phase transformers. Two methods of doing this are:

(i) the connection of two identical single-phase transformers in open delta (or V-V connection).

(ii) the T-T connection (or Scott connection) of two nonidehtical single-phase transformers.

Both of these methods of three-phase transformation result in slightly unbalanced output voltages under load because of unsymmetrical relations. The unbalance is negligible under usual conditions of operation.

7.45 Open-Delta or V-V Connection

If one transformer breaks down in a star-star connected system of 3 single-phase transformers, three-phase power cannot be supplied until the defective transformer has been replaced or repaired. To eliminate this undesirable condition, single-phase transformers are generally connected in Δ-Δ. In this case, if one transformer breaks down, it is possible to continue supplying three-phase power with the other two transformers because this arrangement maintains correct voltage and phase relations on the secondary. However, with two transformers, the capacity of the bank is reduced to 57.7% of what it was with all three transformers in service (i.e., complete Δ-Δ circuit).

Theory

If one transformer is removed in the Δ-Δ connection of three single-phase transformers, the resulting connection becomes open delta or V-V connection. In complete Δ-Δ connection, the voltage of any one phase is equal and opposite to the sum of the voltages of the other two phases. Therefore, under no-load conditions if one transformer is removed, the other two will maintain the same three line voltages on the secondary side. Under load conditions, the secondary line voltages will be slightly unbalanced because of the unsymmetrical relation of the impedance drops in the transformers.

Fig. (7.52 (i)) shows open delta (or V-V) connection; one transformer shown dotted is removed. For simplicity, the load is considered to be star connected. Fig. (7.52 (ii)) shows the phasor diagram for voltages and currents. Here V_{AB}, V_{BC} and V_{CA} represent the line-to-line voltages of the primary; V_{ab}, V_{bc} and V_{ca} represent line-to-line voltages of the secondary and V_{an}, V_{bn} and V_{cn} represent the phase voltages of the load. For inductive load, the load currents I_a, I_b and I_c will lag the corresponding voltages V_{an}, V_{bn} and V_{cn} by the load phase angle ϕ.

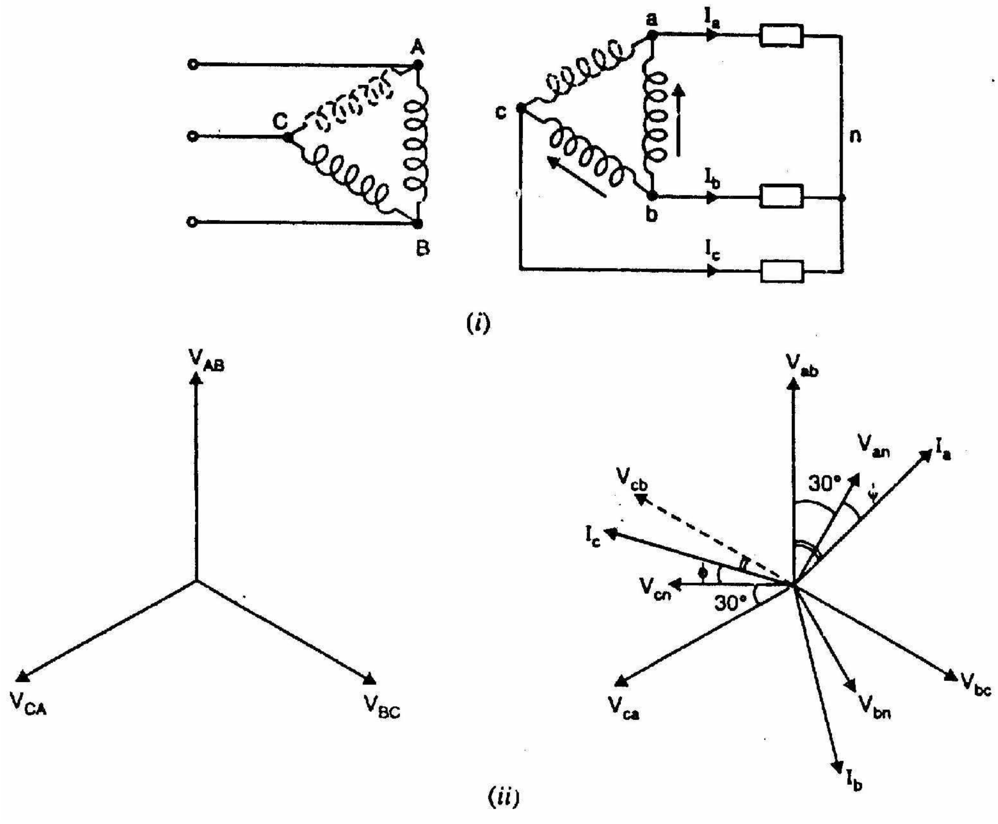

Fig.(7.52)

The transformer windings ab and bc will deliver power given by;

$$P_{ab} = V_{ab}\, I_a \cos(30° + \phi)$$

$$P_{bc} = V_{cb}\, I_c \cos(30° - \phi)$$

Let $V_{ab} = V_{cb} = V$, the voltage rating of transformer secondary winding

$I_a = I_c = I$, current rating of the transformer secondary winding

p.f. = 1 i.e. $\phi = 0°$ ... For resistive load

$\therefore$ Power delivered to the resistive load by V-V connection is

$$P_V = P_{ab} + P_{bc} = \text{vIU}\cos 30° + VI\cos 30° = 2VI\cos 30°$$

With all the three transformers connected in delta, the power delivered to the resistive load is

$$P_\Delta = 3VI$$

$$\therefore \quad \frac{P_V}{P_\Delta} = \frac{2VI\cos 30°}{3VI} = \frac{2\cos 30°}{3} = 0.577$$

Hence the power-handling capacity of a V-V circuit (without overheating the transformers) is 57.7% of the capacity of a complete Δ-Δ circuit of the same transformers.

In a V-V circuit, only 86.6% of the rated capacity of the two transformers is available. This can be readily proved.

$$\frac{\text{Operating capacity}}{\text{Available capacity}} = \frac{2VI\cos 30°}{2VI} = \cos 30° = \frac{\sqrt{3}}{2} = 0.866$$

Let us illustrate V-V connection with a numerical example. Suppose three identical single-phase transformers, each of capacity 10 kVA, are connected in Δ-Δ. The total rating of the three transformers is 30 kVA. When one transformer is removed, the system reverts to V-V circuit and can deliver 3-phase power to a 3-phase load. However, the kVA capacity of the V-V circuit is reduced to 30 × 0.577 = 17.3 kVA and not 20 kVA as might be expected. This reduced capacity can be determined in an alternate way. The available capacity of the two transformers is 20 kVA. When operating in V-V circuit, only 86.6% of the rated capacity is available i.e. 20 × 0.866 = 17.3 kVA.

7.46 Power Factor of Transformers in V-V Circuit

When V-V circuit is delivering 3-phase power, the power factor of the two transformers is not the same (except at unity p.f.). Therefore, the voltage regulation of the two transformers will not be the same. If the load power factor angle is ϕ, then,

p.f. of transformer 1 = cos (30° − ϕ)
p.f. of transformer 2 = cos (30° + ϕ)

(i) When load p.f. = 1 i.e. $\phi = 0°$
In this case, each transformer will have a power factor of 0.866.

(ii) When load p.f. = 0.866 i.e. $\phi = 30°$
In this case, one transformer will have a p.f. of cos (30° − 30°) = 1 and the other of cos (30° + 30°) = 0.5.

(iii) When load p.f = 0.5 i.e. $\phi = 60°$
In this case, one transformer will have a p.f. of cos (30° − 60°) = 0.866 and the other of cos (30° + 60°) = 0. Thus at a load p.f. of 0.5, one transformer delivers all the power at 0.866 p.f. and the other (although still necessary to be in the circuit) delivers no power at all.

7.47 Applications of Open Delta or V-V Connection

The V-V circuit has a number of features that arc advantageous. A few applications are given below by way of illustration:

(i) The circuit can be employed in an emergency situation when one transformer in a complete Δ-Δ circuit must be removed for repair and continuity of service is required.
(ii) Upon failure of the primary or secondary of one transformer of a complete Δ-Δ circuit, the system can be operated as V-V circuit and can deliver 3-phase power (with reduced capacity) to a 3-phase load.
(iii) A circuit is sometimes installed as V-V circuit with the understanding that its capacity may be increased by adding one more transformer to form complete Δ-Δ circuit. As shown earlier,

$$\frac{P_V}{P_\Delta} = \frac{1}{\sqrt{3}} = 0.577$$

$$\therefore \quad P_\Delta = \sqrt{3} \times P_V$$

Thus if a V-V circuit is changed to complete Δ-Δ circuit, the capacity is increased by a factor of $\sqrt{3} = (= 1.732)$.

7.48 Scott Connection or T-T Connection

Although there are now no 2-phase transmission and distribution systems, a 2-phase supply is sometimes required. We can convert 3-phase supply into 2-phase supply through scott or T-T connection of two single-phase transformers. One is called the main transformer M which has a centre-tapped primary; the centre-tap being C [See Fig. (7.53 (i))]. The primary of this transformer has N_1 turns and is connected between the terminals B and Y of the 3-phase supply. The other transformer is called teaser transformer T and its primary has 0.866 N_1 turns. One end of this primary is connected to centre-tap C and the other end to the terminal R of the 3-phase supply. The number of turns (N_2) of the secondary windings of the two transformers are equal. As we shall see, the voltages across the secondaries are equal in magnitude having a phase difference of 90°. Thus scott connection of two single-phase transformers enables us to convert 3-phase supply to 2-phase supply.

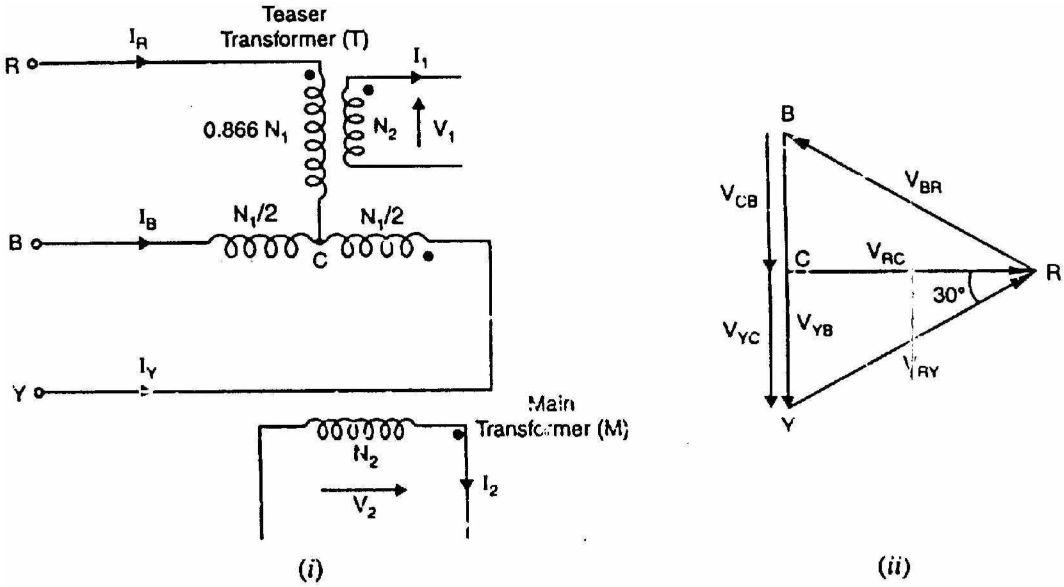

Fig.(7.53)

Theory

Referring to Fig. (7.53 (i)), the centre-tapped primary of the main transformer has line voltage V_{YB} applied to its terminals. The secondary terminal voltage V_2 of the main transformer is

$$V_2 = \frac{N_2}{N_1} V_{YB} = \frac{N_2}{N_1} V_L \qquad (V_L = \text{line voltage})$$

Fig. (7.53 (ii)) shows the relevant phasor diagram. The line voltages of the 3-phase system V_{RY}, V_{YB} and V_{BR} are balanced and are shown on the phasor diagram as a closed equilateral triangle. The voltages across the two halves of the centre tapped primary of the main transformer, V_{CB} and V_{YC} are equal and in phase with V_{YB}. Clearly, V_{RC} leads V_{YB} by 90°. This voltage (i.e., V_{RC}) is applied to the primary of the teaser transformer.

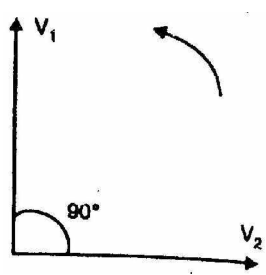

Fig.(7.54)

Therefore, the secondary voltage V_1 of the teaser transformer will lead the secondary voltage V_2 by 90° as shown in Fig. (7.54). We now show that magnitudes of V_2 and V_1 are equal,

$$V_{RC} = V_{RY} \cos 30° = \frac{\sqrt{3}}{2} V_L = 0.866\ V_L$$

$$V_1 = \frac{N_2}{0.866\ N_1} V_{RC} = \frac{N_2}{0.866\ N_1} \times 0.866\ V_L = \frac{N_2}{N_1} V_L = V_2$$

Thus voltages V_1 and V_2 constitute balanced 2-phase system consisting of two voltages of equal magnitude having a phase difference of 90°.

7.49 Applications of Transformers

There are four principal applications of transformers viz.

(i) power transformers (ii) distribution transformers
(iii) autotransformers (iv) instrument transformers

(i) **Power Transformers**. They are designed to operate with an almost constant load which is equal to their rating. The maximum efficiency is designed to be at full-load. This means that full-load winding copper losses must be equal to the core losses.

(ii) **Distribution Transformers**. These transformers have variable load which is usually considerably less than the full-load rating. Therefore, these are designed to have their maximum efficiency at between 1/2 and 3/4 of full-load.

(iii) **Autotransformers**. An autotransformer has only one winding and is used in cases where the ratio of transformation (K), either step-up or step down, differs little from 1. For the same output and voltage ratio, an autotransformer requires less copper than an ordinary 2-winding transformer. Autotransformers are used for starting induction motors (reducing applied voltage during starting) and in boosters for raising the voltage of feeders.

(iv) **Instrument transformers**. Current and voltage transformers are used to extend the range of a.c. instruments.

(a) Current transformer

A current transformer is a device that is used to measure high alternating current in a conductor. Fig. (7.55) illustrates the principle of a current transformer. The conductor carrying large current passes through a circular laminated iron core. The conductor constitutes a one-turn primary winding. The secondary winding consists of a large number of turns of much fine wire wrapped around the core as shown. Due to transformer action, the secondary current is transformed to a low value which can be measured by ordinary meters.

$$\text{Secondary current, } I_S = I_P \times \frac{N_P}{N_S}$$

For example, suppose that I_P = 100 A in Fig. (7.55) and the ammeter is capable of measuring a maximum of 1 A. Then,

$$N_S = N_P \times \frac{I_P}{I_S} = 1 \times \frac{100}{1} = 100$$

(b) Voltage transformer

It is a device that is used to measure high alternating voltage. It is essentially a step-down transformer having small number of secondary turns as shown in Fig. (7.56). The high alternating voltage to be measured is connected directly across the primary. The low voltage winding (secondary winding) is connected to the voltmeter. The power rating of a potential transformer is small (seldom exceeds 300 W) since voltmeter is the only load on the transformer.

$$V_P = V_S \times \frac{N_P}{N_S}$$

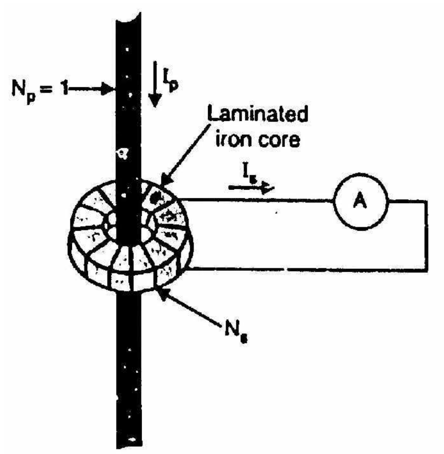

Fig.(7.55)

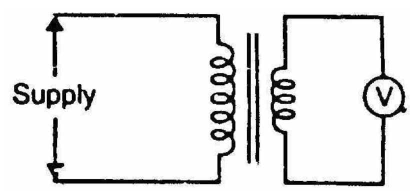

Fig.(7.56)