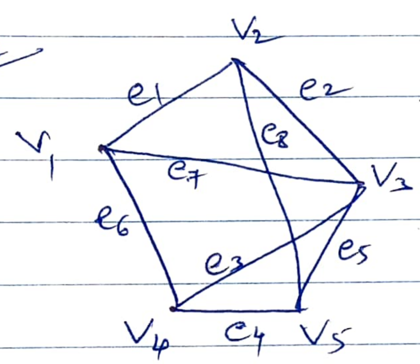
Subgraph

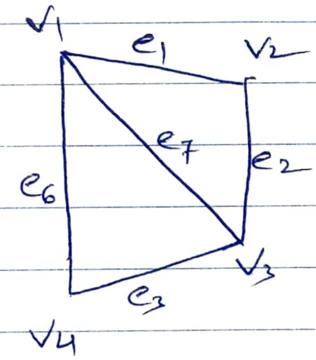
Given two graphs $G(V, E)$ and $G_1(V_1, E_1)$, we say that G_1 is a subgraph of G if following conditions are satisfied –

i) All the vertices and all the edges of G_1 are in G

ii) each edge of G_1 has the same end vertices in G. ~~as in~~

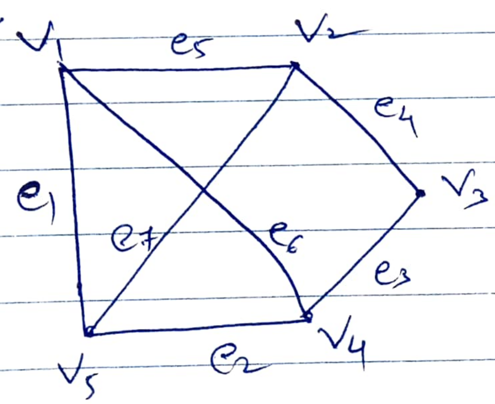
Ex. 1

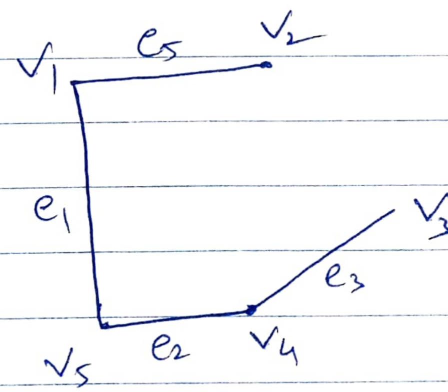




graph G(V, E) — subgraph $G_1(V_1, E_1)$

Ex. 2



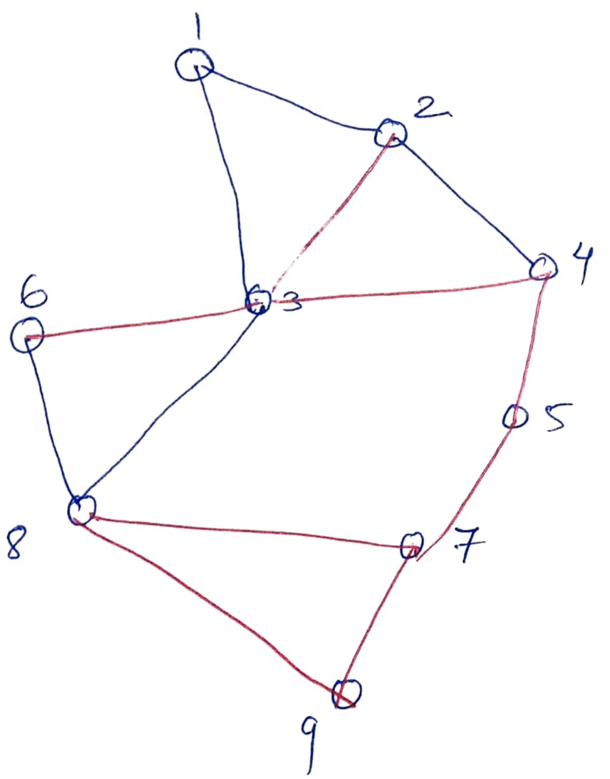


Graph G(V, E) — Sub-graph $G_1(V_1, E_1)$

1. Every graph is a ~~sb~~ subgraph of itself
2. ~~Every~~ If G_1 is subgraph of G_2 and G_2 is a subgraph of G, then G_1 is subgraph of G

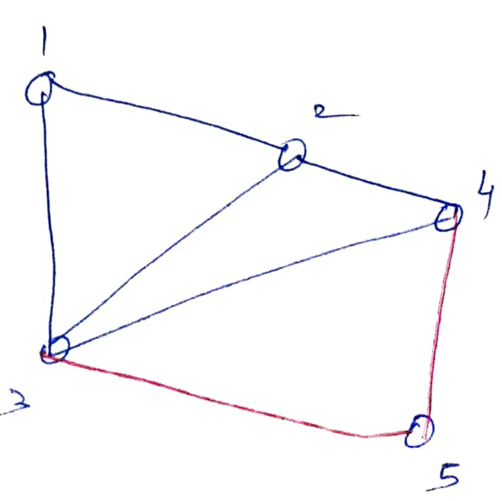
Path

- A path is a trail in which neither vertices nor edges are repeated



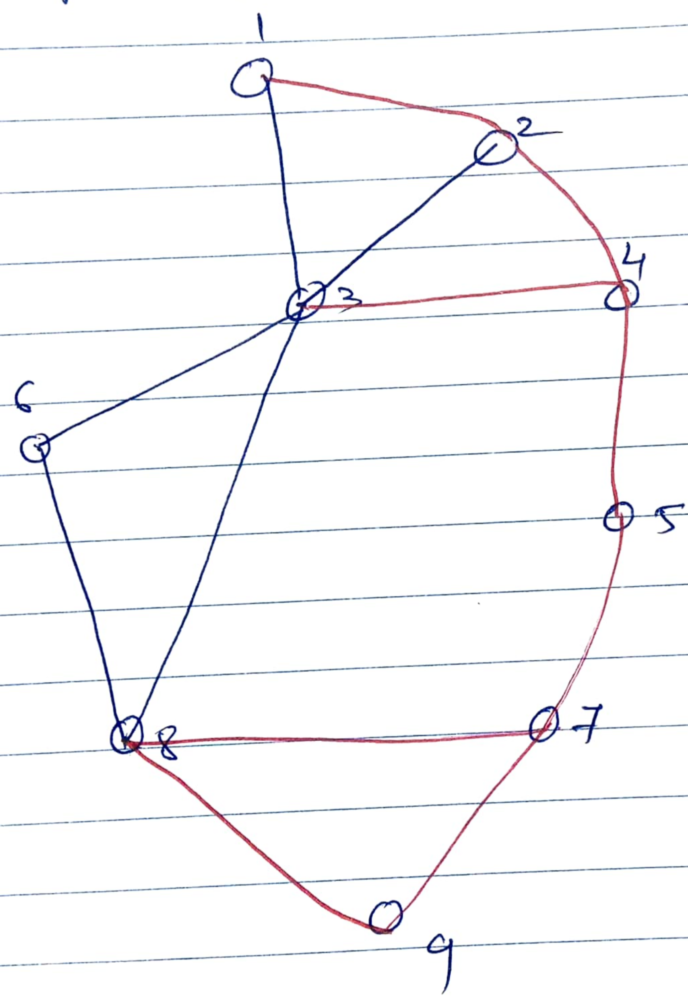
Here,
6-8-3-1-2-4 is a path

- walk - A walk is a sequence of vertices & edges of a graph; ie if we traverse a graph then we get a walk. Vertices & edges can be repeated.



1-2-3-4-2-1-3 is a walk.

Trail - a trail is an open walk in which no edge is repeated (vertex can be repeated)



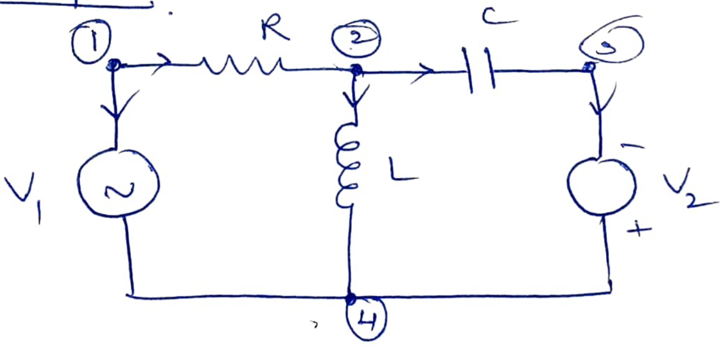
1-3-8-6-3-2 is a trail.

Formation of Incidence Matrix.

Procedure -

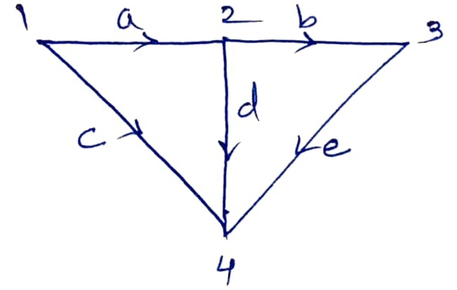
1. Obtain directed graph for the given n/w.
2. Assign '+1' in the matrix if the arrow of a branch is oriented away from the node.
3. Assign '-1' in the matrix if the arrow of a branch is oriented towards a node.
4. Assign '0' in the matrix if the branch is not connected to a node.

Example.



Given n/w consists of 4 nodes & 5 branches.

Let 4 nodes are - 1, 2, 3, 4.
5 branches are - a, b, c, d, e.



The directed graph is,

The directed graph is obtained by representing each n/w element (except current source) by a straight line with the arrows oriented in the same direction as given in n/w.

nodes ↓ branches →

	a	b	c	d	e
1	1	0	1	0	0
2	-1	1	0	1	0
3	0	-1	0	0	1
4	0	0	-1	-1	-1

node 1 → branches a & c are away from node 1
∴ entries are +1 in the matrix.
branches b, d, & e are not connected to node 1
∴ entries are 0 there.

node 2 → branch a is towards nod 2.
∴ entry is -1
b & d are away ∴ entry is +1
it is not connected to c & e ∴ entry is 0

Verification

1. Algebraic sum of each column in incidence matrix is zero.
2. Determinant of the incidence matrix is zero.

Reduced incidence Matrix [A_r]

The reduced incidence matrix is obtained by deleting any row in the incidence matrix.

For example, ~~in~~ from the above incidence matrix, row 4 is deleted, then the reduced incidence matrix is,

$$[A_r] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Note that,

$$\det [A_r][A_r]^T = \text{No. of possible trees.}$$

$$= \det \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}_{3\times5} \begin{bmatrix} +1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{5\times3}$$

$$= \det \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}_{3\times3}$$

$$= 2(6-1) + 1(-2) + 0$$

$$= 10 - 2$$

$$= 8$$

$\therefore$ $\boxed{\text{Possible trees are 8.}}$