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S. Y. B. Tech   Mathematics-III

Practice Questions (Z- Transforms)

Q-1] Find the Z-transform of:

i) $f(k) = \sin\left(\dfrac{k\pi}{4} + \alpha\right), k \geq 0$

ii) $f(k) = \dfrac{\sin ak}{k}, \quad k > 0$

iii) $f(k) = \left(\dfrac{1}{2}\right)^{|k|}, \forall k$

iv) $f(k) = 2^k \cos(3k + 2)$

v) $f(k) = k^2 5^k, (k \geq 0)$

vi) $f(k) = (k+1)a^k, (k \geq 0)$

vii) $f(k) = 4^k + 5^k, (k \geq 0)$

viii) $f(k) = \dfrac{\sin ak}{k}, , k > 0$

ix) $\quad f(k) = \begin{cases} 2^k, k < 0 \\ \left(\dfrac{1}{2}\right)^k, k = 0,2,4,6,.... \\ \left(\dfrac{1}{3}\right)^k, k = 1,3,5,...... \end{cases}$

x) $\quad f(k) = \left(\dfrac{1}{5}\right)^{|k|}, \text{ for all } k$

xi) $\quad f(k) = k \quad , k \geq 0$

xii) $\quad f(k) = \dfrac{3^k}{k!} \quad , k \geq 0$

xiii) $\quad f(k) = k\, 2^{k-1} U(k-1) \quad , k \geq 0$

xiv) $\quad f(k) = \cosh\left(\dfrac{k\pi}{2}\right) \quad , k \geq 0$

xv) $\quad f(k) = e^{2k} \quad , k \geq 0$

xvi) $\quad f(k) = \dfrac{\cos 2k}{k} \quad , k \geq 0$

xvii) $\quad f(k) = \sin 4k \quad , k \geq 0$

xviii) $f(k) = e^{3k} \cos(5k+2) \quad , k \geq 0$

xix) $\quad f(k) = k \quad , k \geq 0$

xx) $\quad f(k) = 3^k + 5^{-k} \quad , k \geq 0$

Q-2] Find the inverse Z-transform of:

i) $F(z) = \dfrac{z^2}{z^2 + 4}, |z| > 2$

ii) $F(z) = \dfrac{z^3}{(z-3)(z-2)}, |z| > 3.$

iii) $F(z) = \dfrac{z^2}{\left(z-\dfrac{1}{4}\right)\left(z-\dfrac{1}{5}\right)}, \dfrac{1}{5} < |z| < \dfrac{1}{4}$

iv) $F(z) = \dfrac{z}{(z-3)(z-2)}, |z| > 3.$

v) $F(z) = \dfrac{z(z+2)}{z^2-2z+1}, \quad |z| > 1$

vi) $F(z) = \dfrac{z^2+z}{z^2-2z+1}, \quad |z| > 1$

vii) $F(z) = \dfrac{z^3}{(z-1)(z-\frac{1}{2})}, \quad |z| > \dfrac{1}{2}$

viii) $F(z) = \dfrac{3z^2+2z}{z^2-3z+2}, \quad 1 < |z| < 2$

ix) $F(z) = \dfrac{z}{(z-\frac{1}{4})(z-\frac{1}{5})}, \quad |z| > \dfrac{1}{4}$

x) $F(z) = \dfrac{z}{(z-1)(z-2)}, |z| > 2.$

xi) $F(z) = \dfrac{z^3}{(z-3)(z-2)^2}, \quad |z| > 3$

xii) $F(z) = \frac{z^2}{z^2+a^2}$, $|z| > |a|$

Q-3]   Find inverse Z-transforms by inversion integral method

a) $F(z) = \frac{z(z+1)}{(z-1)(z^2+z+1)}$

b) $F(z) = \frac{1}{(z-1)(z-2)}$

c) $F(z) = \frac{10z}{(z-1)(z-2)}$

d) $F(z) = \frac{z}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}$

e) $F(z) = \frac{2z^2+3z}{(z^2+z+1)}$

Q-4] Show that $Z^{-1}\left\{\frac{1}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}\right\} = \{x_k\}$ *for* $|z| > \frac{1}{2}$ where

$x_k = 6\left[\left(\frac{1}{2}\right)^{k-1} - \left(\frac{1}{3}\right)^{k-1}\right], k \geq 1.$

Q-5] Show that $Z^{-1}\left\{\dfrac{1}{(z-2)(z-3)}\right\}=\{x_k\}$ *for* $|z|<2,$ where

$$x_k=\left[2^{k-1}-3^{k-1}\right]\ ,k\le 0.$$

Q-6] Solve the difference equation:

1. $f(k+1)-f(k)=1, f(0)=0$

2. $f(k+2)+3f(k+1)+2f(k)=0, f(0)=0, f(1)=2, k\ge 0$

3. $f(k+1)+\dfrac{1}{2}f(k)=\left(\dfrac{1}{2}\right)^{k}, f(0)=0, k\ge 0$

4. $6f(k+2)-5f(k+1)+f(k)=0, f(0)=0, f(1)=3, k\ge 0$

5. $f(k+2)-3f(k+1)+2f(k)=U(k)\ ,f(0)=0, f(1)=0, k\ge 0$