UNIT 1: THEORY OF MATRICES

1. Find the rank of the Matrices using Echelon form — Solutions

I. $\begin{bmatrix} 4 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 2 & -2 & 0 \end{bmatrix}$ — 2

II. $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ — 2

III. $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ — 3

2. Find the rank of the Matrices using normal form

I. $\begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ — 4

II. $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ — 3

III. $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ — 2

3. Examine for consistency and if consistent then solve it.

I. $\begin{cases} 4x - 2y + 6z = 8 \\ x + y - 3z = -1 \\ 15x - 3y + 9z = 21 \end{cases}$ — $\begin{cases} x = 1 \\ y = 3t - 2 \\ z = t \end{cases}$

II. $\begin{cases} 2x + z = 4 \\ x - 2y + 2z = 7 \\ 3x + 2y = 1 \end{cases}$ — $\begin{cases} x = 2 - \frac{t}{2} \\ y = -\frac{5}{2} + \frac{3t}{4} \\ z = t \end{cases}$

III. $\begin{cases} 2x_1 + x_2 - 5x_3 + x_4 = 8 \\ x_1 + 3x_2 - 6x_4 = -15 \\ 2x_2 - x_3 + 2x_4 = -5 \\ x_1 + 4x_2 - 7x_3 + 6x_4 = 0 \end{cases}$ $\begin{cases} x_1 = 3 \\ x_2 = -4 \\ x_3 = -1 \\ x_4 = 1 \end{cases}$

IV. $\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 5 \\ 2x_1 + 3x_2 - x_3 - 2x_4 = 2 \\ 4x_1 + 5x_2 + 3x_3 = 7 \end{cases}$ Inconsistent

V. $\begin{cases} x + 2y + 3z = 0 \\ 2x + 3y + z = 0 \\ 4x + 5y + 4z = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$

VI. $\begin{cases} 2x - y + 3z = 0 \\ 3x + 2y + z = 0 \\ x - 4y + 5z = 0 \end{cases}$ $\begin{cases} x = t \\ y = -t \\ z = t \end{cases}$

4. Investigate for what values of a & b, the system of simultaneous equation

$$2x - y + 3z = 2$$

$$x + y + 2z = 2$$

$$5x - y + az = b$$

Have (1) No solution (2) A unique solution (3) An infinite number of solutions.

(**<u>Solutions</u>** (1) $a = 8, b \neq 6$ (2) $a \neq 8, b \in R$ (3) $a = 8, b = 6$)

5. Investigate for what values of k the equations

$$x + y + z = 1$$

$$2x + y + 4z = k$$

$$4x + y + 10z = k^2$$

Have infinite number of solutions? Hence, find solutions.

<u>(Solutions</u> : $k = 1,2$)

6. Examine for Linear dependence or independence the following system of vectors. If dependent, find the relation between them

I. $x_1 = (1, -1, 1),\ x_2 = (2,1,1),\ x_3 = (3,0,2)$ Dependent $x_1 + x_2 = x_3$

II. $x_1 = (1,1,1,3),\ x_2 = (1,2,3,4),\ x_3 = (2,3,4,7)$ Dependent $x_1 + x_2 = x_3$

III. $x_1 = (3,1,-4),\ x_2 = (2,2,-3),\ \ x_3 = (0,-4,1)$ Dependent

$2x_1 = 3x_2 + x_3$

IV. $x_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}, x_2 = \begin{pmatrix}3\\-2\\1\end{pmatrix}, x_3 = \begin{pmatrix}1\\-6\\-5\end{pmatrix}$ Dependent

$2x_1 + x_3 = x_2$

7. Given the transformation $= \begin{bmatrix}1 & -2 & 3\\2 & 0 & -3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$. Find the co-ordinates (x_1, x_2, x_3) corresponding to $(2,3,0)$ in Y.

$x_1 = \frac{21}{19}$

$x_2 = -\frac{16}{19}$

$x_3 = -\frac{5}{19}$

8. Express each of the transformation $\begin{matrix}x_1 = 3y_1 + 5y_2\\x_2 = -y_1 + 7y_2\end{matrix}$ and

$y_1 = z_1 + 3z_2$
$y_2 = 4z_1$

In the matrix form and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2.

$x_1 = 23z_1 + 9z_2$
$x_2 = 27z_1 - 3z_2$

9. Verify whether the following matrices are orthogonal or not, if so write A^{-1} :

I. $A = \begin{bmatrix}\frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}}\\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\end{bmatrix}$ Yes

II. $A = \frac{1}{3}\begin{bmatrix}2 & 2 & 1\\-2 & 1 & 2\\1 & -1 & 2\end{bmatrix}$ No

10. If $A = \begin{bmatrix}\frac{1}{3} & \frac{2}{3} & a\\ \frac{2}{3} & \frac{1}{3} & b\\ \frac{2}{3} & -\frac{2}{3} & c\end{bmatrix}$ is orthogonal, Find a,b,c.

$a = \pm\frac{2}{3}$

$b = \mp\frac{2}{3}$

$c = \pm\frac{1}{3}$

11. Find the Eigen values and corresponding Eigen vectors for the following matrices

I. $A = \begin{bmatrix}9 & -1 & 9\\3 & -1 & 3\\-7 & 1 & -7\end{bmatrix}$

(**Solution** :- $-1,0,2$ *and* $\begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}4\\1\\-3\end{bmatrix}$)

II. $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

(**Solution** :- $0, 2, -2$ *and* $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$)

III. $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(**Solution** :- $5, -3, -3$ *and* $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$)

IV. $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

(**Solution** :- $1, 1, 1$ *and* $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$)

V. $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

(**Solution** :- $3, 2, 2$ *and* $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -5 \end{bmatrix}$)

12. Verify Cayley-Hamilton theorem for the following matrix and use it find Inverse:

I. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

II. $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

13. Find A^4 with the help of Cayley Hamilton theorem

If $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

(**Solution** :- $\begin{bmatrix} -49 & -50 & -40 \\ 65 & 66 & 40 \\ 130 & 130 & 81 \end{bmatrix}$)

14. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, then express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ in terms of A.

(**Solution** :- $A + 5I$)

15. Consider the triangle with Vertices $A(1,4), B(5,3)$ *and* $C(1,1)$ then

I. Rotate the triangle 90^0 clockwise.
(Solution: A'(1,-1),B'(3,-5),C'(4,-1))

II. Rotate the triangle 90^0 counter clockwise.
(Solution: A'(-1,1),B'(-3,5),C'(-4,1))

III. Take the reflection about X-axis
(Solution: A'(1,-1),B'(5,-3),C'(1,-4))

IV. Take its reflection about Y- axis
(Solution: A'(-1,1),B'(-5,3),C'(-1,4))

V. Translate the triangle 6 units right and 5 units down
(Solution: A'(7,-4),B'(11,-2),C'(7,-1))

16. Centre of the arc of the circle in a given coordinate system is $(100,100,100)$. Origin is shifted to the point $(-10,-5,-2)$.Rotation is carried out about Y axis through an angle of 30^0. Find the centre of the arc of the circle in new coordinate system. (46.66,105,134.66)

Unit 2: Differential Calculus

Q.1) Find n^th derivatives of the following functions $a)y = \frac{x}{(x+1)^4}$, $b)y = \frac{2x+3}{5x+7}$, $c)y = \frac{x}{(3x-5)(1-4x^2)}$, $d)y = \frac{x^4}{(x-1)(x-2)}$

$e)\ y = \frac{x}{1+x+x^2+x^3}$, $f)\ y = \frac{x^2}{(x-1)(x-2)}$, $g)\ y = sin^{-1}\left(\frac{2x}{1+x^2}\right)$ $h)y = sin2x\ cos3x$

$i)y = \frac{x}{(x+2)(2x+3)}$ $j)\ y = cos^{-1}\left[\frac{x-x^{-1}}{x+x^{-1}}\right]$, $k)\ y = tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

$l)\frac{x^2+x+1}{x^3-6x^2+11x-6}$, $m)y = \frac{x^2}{(x+2)(2x+3)}$

Q.2) Prove that $\frac{d^n}{dx^n}(x^{n-1}\text{log}x) = \frac{(n-1)!}{x}$

Q.3) If $y = xlog(x+1)$ then prove that $y_n = \frac{(-1)^{n-1}(n-2)!(x+n)}{(x+1)^n}$

Q.4) If $y = \frac{ax+b}{cx+d}$ $then\ prove\ that\ y_1y_3 = 3{y_2}^2$

Q.5) If $x = sint, y = sinpt$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

Q.6) If $f(x) = tanx, then\ prove\ that$

$$f^n(0) - n_{C_2}f^{n-2}(0) + n_{C_4}f^{n-4}(0) + -----= \sin(\frac{n\pi}{2})$$

Q.7) Find nth derivative of $y = tan^{-1}x$. Hence prove that the value of $D^n(tan^{-1}x)\ at\ x = 0\ is\ 0,$

$(n-1)!\ or\ -(n-1)!$ according as n is of the form $2p, (4p+1)\ or\ (4p+3)$ respectively.

Q.8) State Leibnitz's theorem and find the n^{th} derivatives of following functions:

a) $x^2e^x cosx,\quad b)\ x^2e^{3x}cos4x,\quad c)\ e^x(2x+3)^3,\quad d)\ x^2e^x$

Q.9) $y = e^{mcos^{-1}x}, prove\ that\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$

Hence evaluate $(y_n)_0$.

Q.10) $y = sin2\theta, x = sin\theta, show\ that\ (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-4)y_n = 0.$

Q.11) $y = coswt, x = sint, show\ that\ i)\ (1-x^2){y_1}^2 = w^2(1-y^2)$

$ii)(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-w^2)y_n = 0.$

Q.12) $If\ x = \tan(logy), prove\ that\ (1+x^2)y_{n+1} + (2nx+1)y_n - n(n-1)y_{n-1} = 0.$

Q.13) $If\ y = \left[x+\sqrt{x^2+1}\right]^m, prove\ that\ (x^2-1)y_{n+2} + (2n+1)xy_{n+1} - (n^2-m^2)y_n = 0$

Q.14) Expand the following functions:

(a) $(1+x)^x$ *in a series up to a term in* x^4.

(b) $\text{Log}(1+x+x^2+x^3)$ *upto* x^8.

Q.15) Prove that

a) $\log(secx) = \frac{x^2}{2} + \frac{1}{3}\frac{x^4}{4} + \frac{2}{15}\frac{x^6}{6} + \cdots.$

b) $log(1+sinx) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$

c) $e^{e^x} = e\left[1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + \cdots\right]$

d) $\sqrt{1+sinx} = 1 + \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{48} + \frac{x^4}{348} - \cdots$

e) $xcosecx = 1 + \frac{x^2}{6} - \frac{7}{360}x^4 + \ldots$

Q.16) Using Taylor's theorem, find the expansion of following functions in ascending powers of x

a) $\tan\left[x+\frac{\pi}{4}\right]$ up to terms in x^4 and find the approximately value of $\tan(43^0)$

b) $\log\cos(x+\frac{\pi}{4})$, hence find the value of $logcos(48^0)$ upto three decimal places.

Q.17) Expand

$a)\ (x-2)^4 - 3(x-2)^3 + 4(x-2)^2 + 5\ in\ \ powers\ of\ x$

b) $2x^3 + 7x^2 + x - 6$ *in ascending powers of* $(x - 2)$.

c) $49 + 69x + 42x^2 + 11x^3 + x^4$ *in powers of* $(x+2)$.

d) If $x = (1 - y)(1 - 2y)$, then show that $y = 1 + x - 2x^3 + \ldots$

e) If $x^3 + 2xy^2 - y^3 + x = 1$, obtain the expansion of y in ascending powers of x

Unit 3 : Partial Differentiation

1) If $u = x^2 \tan^{-1}(\frac{y}{x}) - y^2 \tan^{-1}(\frac{x}{y})$, find u_{xy}. Ans : $u_{xy} = \frac{x^2 - y^2}{x^2 + y^2}$

2) If $u = x^2 - y^2 - f(xy)$ then show that $u_{xx} + u_{yy} = (x^4 - y^4)\, f''(xy)$.

3) If $u = \log(x^3 + y^3 - x^2 y - xy^2)$, then prove that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y})^2 u = \frac{-4}{(x+y)^2}$

4) If $u = x^y$ then verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

5) Let $v = \tan^{-1}(\frac{x}{y})$, find $\frac{\partial^2 v}{\partial x \partial y}$ *and* $\frac{\partial^2 v}{\partial y \partial x}$. Is $\frac{\partial^2 v}{\partial x \partial y} = \frac{\partial^2 v}{\partial y \partial x}$?

6) If $u = 3xy - y^3 + (y^2 - 2x)^{1/2}$ then verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

7) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$ prove that $u_{xx} + u_{yy} = f''(r) + \frac{1}{r} f'(r)$

8) If $u = \log \sqrt{x^2 + y^2 + z^2}$, show that $(x^2 + y^2 + z^2)(u_{xx} + u_{yy} + u_{zz}) = 1$ (Hint: Consider $r^2 = x^2 + y^2 + z^2$ hence $u = \log r$)

9)If $u = ax + by$, $v = bx - ay$ find the value of $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$

Ans:1

10) If $x = u \tan v$, $y = u \sec v$ then prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial v}{\partial x}\right)_y = \left(\frac{\partial u}{\partial y}\right)_x \left(\frac{\partial v}{\partial y}\right)_x$

11)If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that

$$x^2 \left(\frac{\partial^2 u}{\partial x^2}\right) + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \left(\frac{\partial^2 u}{\partial y^2}\right) = \frac{1}{4}(\tan^3 u - \tan u)$$

12) If $u = \frac{x^3 + y^3}{y\sqrt{x}}$,find the value $x^2 \left(\frac{\partial^2 u}{\partial x^2}\right) + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \left(\frac{\partial^2 u}{\partial y^2}\right) + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ at point (1,2)

13) If x = e^u tanv, y= e^u secv find the value of $\left(x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y}\right)\left(x.\frac{\partial v}{\partial x} + y.\frac{\partial v}{\partial y}\right)$. Ans:0

14) If u = $x^2 + y^2$ where x = s + 3t, y =2s - t ,prove that $\frac{\partial^2 u}{\partial t^2} = 2\frac{\partial^2 u}{\partial s^2}$)

15)If z = f(x,y) where x = e^u+e^{-v}, y = $e^{-u} - e^v$ then prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

16) Find $\frac{du}{dx}$ given that u = x log xy and $x^3 + y^3$= -3xy

17) If Φ(x,y,z)=0 then prove that $\left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial y}{\partial x}\right)_z$ = -1

18) If $(\cos x)^y = (\sin y)^x$ then find $\frac{dy}{dx}$.

19) If u.x + v.y = 0 and $\frac{u}{x} + \frac{v}{y} = 1$ then prove that

$\frac{u}{x}\left(\frac{\partial x}{\partial u}\right)_v + \frac{v}{y}\left(\frac{\partial y}{\partial v}\right)_u$ =0

20) If $x = r\cos\theta$, $y = r\sin\theta$ then show that

a) $\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1$ b) $\left(\frac{\partial y}{\partial r}\right)_x \left(\frac{\partial y}{\partial r}\right)_\theta = 1$

Unit 4: Application of Partial Differentiation

1) If $x = ar\sin\theta\cos\emptyset$, $y = br\sin\theta\sin\emptyset$, $z = cr\cos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = abcr^2\sin\theta$
2) If $ux = yz, vy = zx, wz = xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ Answer = 4
3) If $u = x + 2y^2 - z^3, v = x^2yz,\ w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ $at\ (1,-1,0)$

   Answer = 6
4) If $x = u - v + w,\ y = u^2 - v^2 - w^2,\ z = u^3 v$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

   Answer = $6u^2(v + w) +$

   $2u + 2w$

5) If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

Answer = 0

6) If $x = a(u + v), y = b(u - v) where\ u = r^2 \cos 2\theta, v = r^2 \sin 2\theta$, $a\ and\ b$ being constant, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

Answer = $-8abr^3$

7) If $x = \sqrt{vw}, y = \sqrt{uw}, z = \sqrt{uv}$ and $= r sin\theta cos\emptyset, v = r sin\theta sin\emptyset, z = r cos\theta\ then\ find\ \frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$.

Answer= $\frac{1}{4}(r^2 sin\theta)$

8) $x = e^u \cos v, y = e^u sinv$ prove that $\frac{\partial(x,y)}{\partial(u,v)} \frac{\partial(u,v)}{\partial(x,y)} = 1$

9) $x = e^v \sec u, y = e^v tanu$ prove that $\frac{\partial(x,y)}{\partial(u,v)} \frac{\partial(u,v)}{\partial(x,y)} = 1$

10) If $x = u(1 - v), y = uv$ show that $JJ' = 1$

11) If $x = v^2 + w^2, y = w^2 + u^2, z = u^2 + v^2$ then prove that

$$JJ' = 1$$

12) Show that $JJ' = 1$ for the following

i) $x = uv, y = \frac{u}{v}$

ii) $u = xy, v = x + y$

13) Check whether the following functions are functionally dependent, if so find the relation between them, $u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$

14) Check whether the following functions are functionally dependent, if so find the relation between them, $u = \sin^{-1} x + \sin^{-1} y, v = x\sqrt{1 - y^2} + y\sqrt{1 - x^2}$

15) If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ examine whether the above functions are functionally dependent; if so find the relation between them.

16) Show that the function $u = x + y + z, v = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx, w = x^3$ are functionally related

17) Under which condition $u = a_1x + b_1y + c_1, and\ v = a_2x + b_2y + c_2$ are functionally dependent.

18) If $f(x, y) = (50 - x^2 - y^2)^{\frac{1}{2}}$ then find the approximate value of $f(3,4) - f(2.9,4.1)$.

Answer is 0.02

19) If the area of rectangular field is calculated by measuring its length and breadth. If there is an error of 2% in measuring the length and an error of 3% in measuring the breadth of the field, find the approximate % error in the calculated area of the field. Answer : 5%

20) The focal length of the mirror is found from the formula : $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$ find the percentage error in f, if u and v are both in error by 2% each.

Answer is 2%

21) Find Maximum and minimum value of following functions

1. $(x - y)(x^2 + y^2)(x + y - 1)$ Ans: No maxima , No Minima
2. $2(x^2 - y^2) - x^4 + y^4$ Ans: Max at $(\pm 1,0)$ minima at $(0, \pm 1)$
3. $(x^2 + y^2)^2 - 2(x^2 - y^2)$, Ans: Min value -1 at (1,0) and (-1,0)

22) Divide 24 into three parts such that the continue product of the first square of second and cube of third is maximum.

Ans: 4,8,12

23) Find three positive numbers whose sum is 100 and product is maximum

Ans : $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$

24) Find Minimum value of $x^2 + y^2 + z^2$, given $x + y + z = 3a$ uisng Lagrange's method. Ans: $3a^2$

25) Find the points on the surface $z^2 = xy + 1$nearest to the origin, by using Lagrange's Method Ans: $(0,0, \pm 1)$

26) Find Maximum and minimum distance of the point $(3,4,12)$from the sphere $x^2 + y^2 + z^2 = 1$, using Lagrange's Method

Ans: Maximum Distance = 14

Minimum Distance = 12