Chapter (1)

D.C. Generators

Introduction

Although a far greater percentage of the electrical machines in service are a.c. machines, the d.c. machines are of considerable industrial importance. The principal advantage of the d.c. machine, particularly the d.c. motor, is that it provides a fine control of speed. Such an advantage is not claimed by any a.c. motor. However, d.c. generators are not as common as they used to be, because direct current, when required, is mainly obtained from an a.c. supply by the use of rectifiers. Nevertheless, an understanding of d.c. generator is important because it represents a logical introduction to the behaviour of d.c. motors. Indeed many d.c. motors in industry actually operate as d.c. generators for a brief period. In this chapter, we shall deal with various aspects of d.c. generators.

1.1 Generator Principle

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f. (and hence current) is given by Fleming's right hand rule. Therefore, the essential components of a generator are:

(a) a magnetic field
(b) conductor or a group of conductors
(c) motion of conductor w.r.t. magnetic field.

1.2 Simple Loop Generator

Consider a single turn loop ABCD rotating clockwise in a uniform magnetic field with a constant speed as shown in Fig.(1.1). As the loop rotates, the flux linking the coil sides AB and CD changes continuously. Hence the e.m.f. induced in these coil sides also changes but the e.m.f. induced in one coil side adds to that induced in the other.

(i) When the loop is in position no. 1 [See Fig. 1.1], the generated e.m.f. is zero because the coil sides (AB and CD) are cutting no flux but are moving parallel to it

(ii) When the loop is in position no. 2, the coil sides are moving at an angle to the flux and, therefore, a low e.m.f. is generated as indicated by point 2 in Fig. (1.2).

(iii) When the loop is in position no. 3, the coil sides (AB and CD) are at right angle to the flux and are, therefore, cutting the flux at a maximum rate. Hence at this instant, the generated e.m.f. is maximum as indicated by point 3 in Fig. (1.2).

(iv) At position 4, the generated e.m.f. is less because the coil sides are cutting the flux at an angle.

(v) At position 5, no magnetic lines are cut and hence induced e.m.f. is zero as indicated by point 5 in Fig. (1.2).

(vi) At position 6, the coil sides move under a pole of opposite polarity and hence the direction of generated e.m.f. is reversed. The maximum e.m.f. in this direction (i.e., reverse direction, See Fig. 1.2) will be when the loop is at position 7 and zero when at position 1. This cycle repeats with each revolution of the coil.

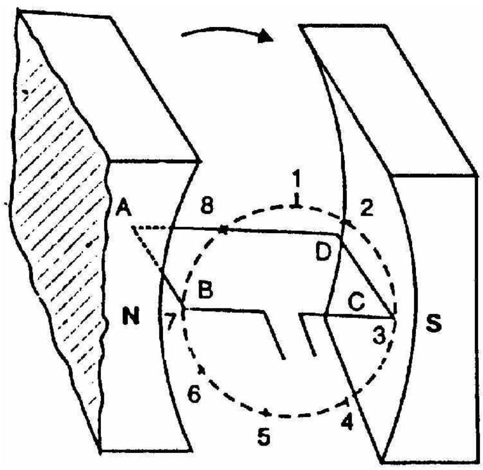


Fig. (1.1)

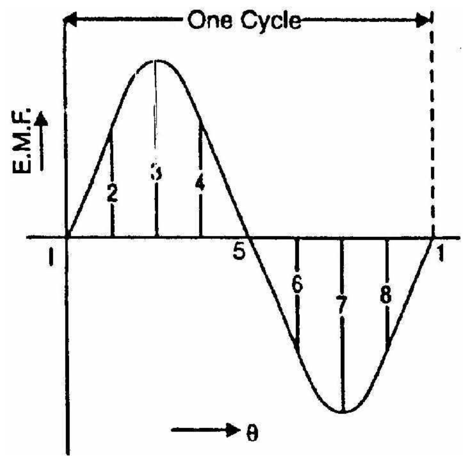


Fig. (1.2)

Note that e.m.f. generated in the loop is alternating one. It is because any coil side, say AB has e.m.f. in one direction when under the influence of N-pole and in the other direction when under the influence of S-pole. If a load is connected across the ends of the loop, then alternating current will flow through the load. The alternating voltage generated in the loop can be converted into direct voltage by a device called commutator. We then have the d.c. generator. In fact, a commutator is a mechanical rectifier.

1.3 Action Of Commutator

If, somehow, connection of the coil side to the external load is reversed at the same instant the current in the coil side reverses, the current through the load will be direct current. This is what a commutator does. Fig. (1.3) shows a commutator having two segments C_1 and C_2. It consists of a cylindrical metal ring cut into two halves or segments C_1 and C_2 respectively separated by a thin sheet of mica. The commutator is mounted on but insulated from the rotor shaft. The ends of coil sides AB and CD are connected to the segments C_1 and C_2 respectively as shown in Fig. (1.4). Two stationary carbon brushes rest on the commutator and lead current to the external load. With this arrangement, the commutator at all times connects the coil side under S-pole to the +ve brush and that under N-pole to the −ve brush.

(i) In Fig. (1.4), the coil sides AB and CD are under N-pole and S-pole respectively. Note that segment C_1 connects the coil side AB to point P of the load resistance R and the segment C_2 connects the coil side CD to point Q of the load. Also note the direction of current through load. It is from Q to P.

(ii) After half a revolution of the loop (i.e., 180° rotation), the coil side AB is under S-pole and the coil side CD under N-pole as shown in Fig. (1.5). The currents in the coil sides now flow in the reverse direction but the segments C_1 and C_2 have also moved through 180° i.e., segment C_1 is now in contact with +ve brush and segment C_2 in contact with −ve brush. Note that commutator has reversed the coil connections to the load i.e., coil side AB is now connected to point Q of the load and coil side CD to the point P of the load. Also note the direction of current through the load. It is again from Q to P.

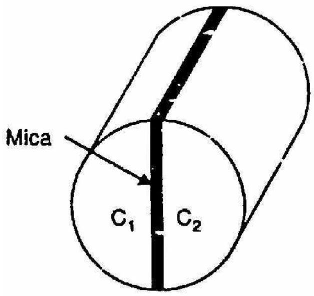


Fig.(1.3)

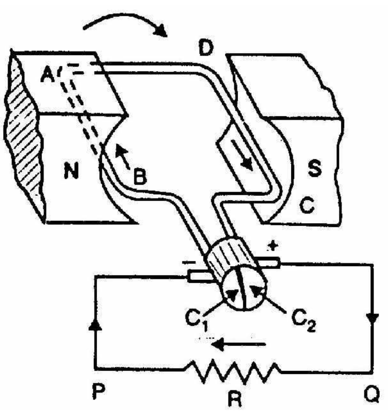


Fig.(1.4)

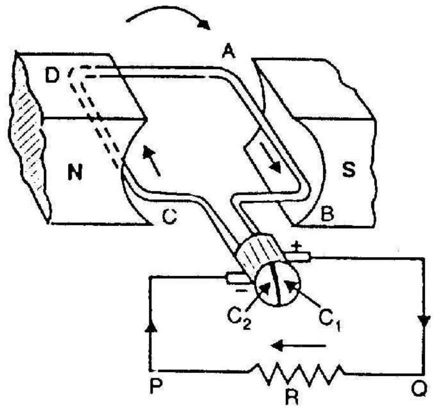


Fig.(1.5)

Thus the alternating voltage generated in the loop will appear as direct voltage across the brushes. The reader may note that e.m.f. generated in the armature winding of a d.c. generator is alternating one. It is by the use of commutator that we convert the generated alternating e.m.f. into direct voltage. The purpose of brushes is simply to lead current from the rotating loop or winding to the external stationary load.

The variation of voltage across the brushes with the angular displacement of the loop will be as shown in Fig. (1.6). This is not a steady direct voltage but has a pulsating character. It is because the voltage appearing across the brushes varies from zero to maximum value and back to zero twice for each revolution of the loop. A pulsating direct voltage such as is produced by a single loop is not suitable for many commercial uses. What we require is the steady direct voltage. This can be achieved by using a large number of coils connected in series. The resulting arrangement is known as armature winding.

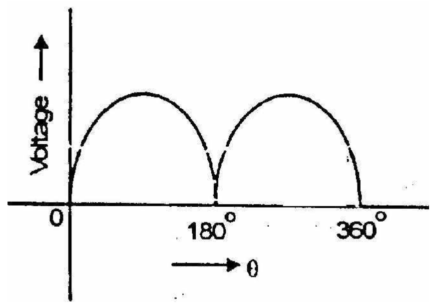


Fig. (1.6)

1.4 Construction of d.c. Generator

The d.c. generators and d.c. motors have the same general construction. In fact, when the machine is being assembled, the workmen usually do not know whether it is a d.c. generator or motor. Any d.c. generator can be run as a d.c. motor and vice-versa. All d.c. machines have five principal components viz., (i) field system (ii) armature core (iii) armature winding (iv) commutator (v) brushes [See Fig. 1.7].

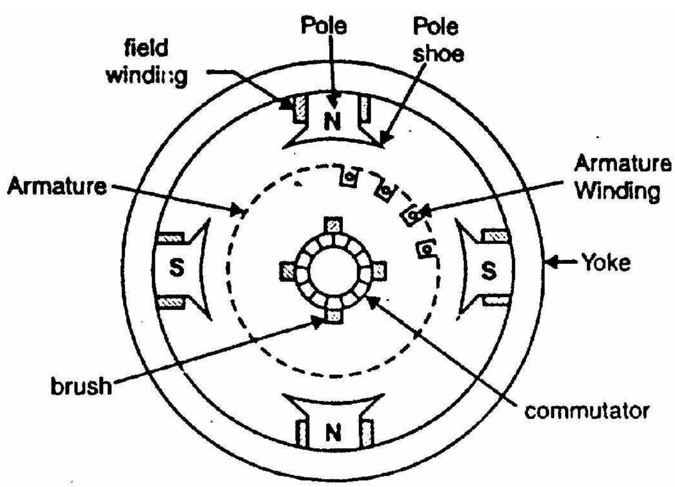


Fig. (1.7)

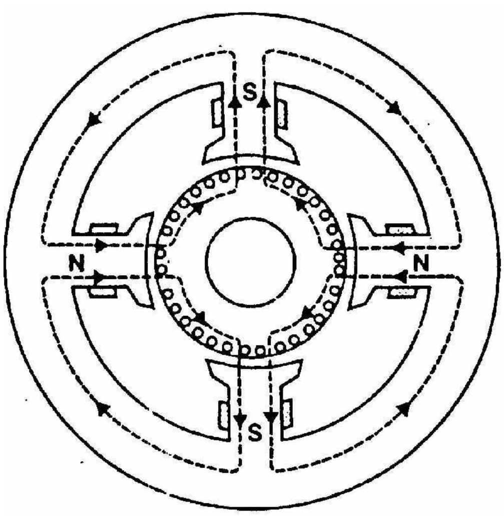


Fig. (1.8)

(i) Field system

The function of the field system is to produce uniform magnetic field within which the armature rotates. It consists of a number of salient poles (of course, even number) bolted to the inside of circular frame (generally called yoke). The yoke is usually made of solid cast steel whereas the pole pieces are composed of stacked laminations. Field coils are mounted on the poles and carry the d.c. exciting current. The field coils are connected in such a way that adjacent poles have opposite polarity.

The m.m.f. developed by the field coils produces a magnetic flux that passes through the pole pieces, the air gap, the armature and the frame (See Fig. 1.8). Practical d.c. machines have air gaps ranging from 0.5 mm to 1.5 mm. Since armature and field systems are composed of materials that have high permeability, most of the m.m.f. of field coils is required to set up flux in the air gap. By reducing the length of air gap, we can reduce the size of field coils (i.e. number of turns).

(ii) Armature core

The armature core is keyed to the machine shaft and rotates between the field poles. It consists of slotted soft-iron laminations (about 0.4 to 0.6 mm thick) that are stacked to form a cylindrical core as shown in Fig (1.9). The laminations (See Fig. 1.10) are individually coated with a thin insulating film so that they do not come in electrical contact with each other. The purpose of laminating the core is to reduce the eddy current loss. The laminations are slotted to accommodate and provide mechanical security to the armature winding and to give shorter air gap for the flux to cross between the pole face and the armature "teeth".

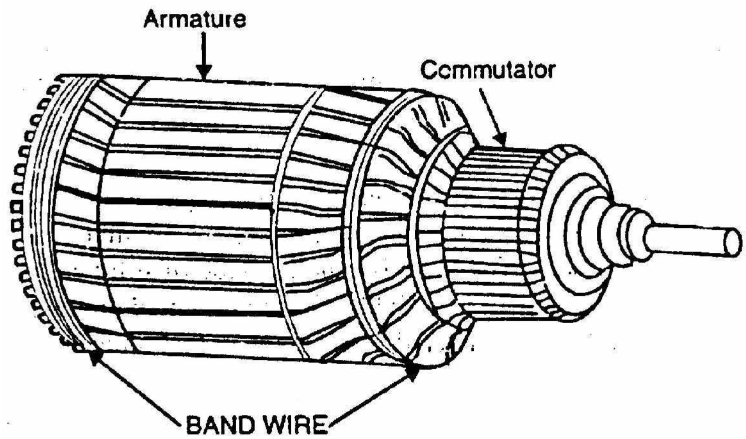


Fig. (1.9)

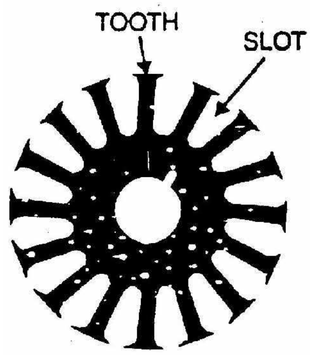


Fig. (1.10)

(iii) Armature winding

The slots of the armature core hold insulated conductors that are connected in a suitable manner. This is known as armature winding. This is the winding in which "working" e.m.f. is induced. The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current. The armature winding of a d.c. machine is a closed-circuit winding; the conductors being connected in a symmetrical manner forming a closed loop or series of closed loops.

(iv) Commutator

A commutator is a mechanical rectifier which converts the alternating voltage generated in the armature winding into direct voltage across the brushes. The commutator is made of copper segments insulated from each other by mica sheets and mounted on the shaft of the machine (See Fig 1.11). The armature conductors are soldered to the commutator segments in a suitable manner to give rise to the armature winding. Depending upon the manner in which the armature conductors are connected to the commutator segments, there are two types of armature winding in a d.c. machine viz., (a) lap winding (b) wave winding.

Great care is taken in building the commutator because any eccentricity will cause the brushes to bounce, producing unacceptable sparking. The sparks may bum the brushes and overheat and carbonise the commutator.

(v) Brushes

The purpose of brushes is to ensure electrical connections between the rotating commutator and stationary external load circuit. The brushes are made of carbon and rest on the commutator. The brush pressure is adjusted by means of adjustable springs (See Fig. 1.12). If the brush pressure is very large, the friction produces heating of the commutator and the brushes. On the other hand, if it is too weak, the imperfect contact with the commutator may produce sparking.

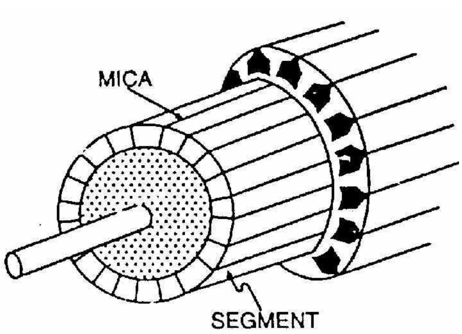


Fig. (1.11)

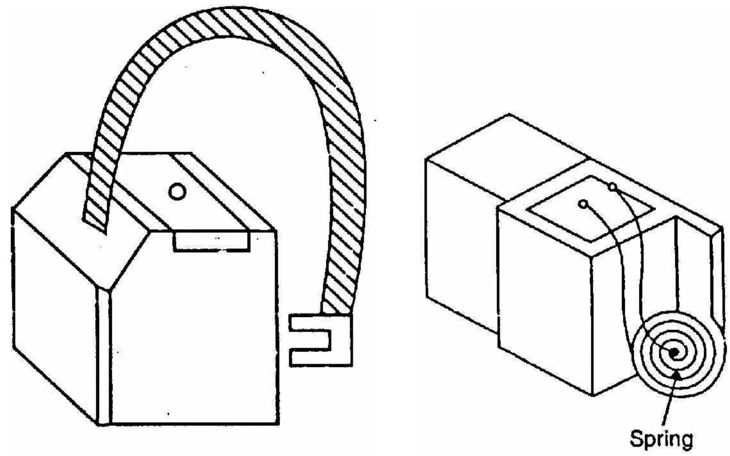


Fig. (1.12)

Multipole machines have as many brushes as they have poles. For example, a 4-pole machine has 4 brushes. As we go round the commutator, the successive brushes have positive and negative polarities. Brushes having the same polarity are connected together so that we have two terminals viz., the +ve terminal and the −ve terminal.

1.5 General Features OF D.C. Armature Windings

(i) A d.c. machine (generator or motor) generally employs windings distributed in slots over the circumference of the armature core. Each conductor lies at right angles to the magnetic flux and to the direction of its movement Therefore, the induced e.m.f. in the conductor is given by;

$$e = Bl\,v \quad \text{volts}$$

where B = magnetic flux density in Wb/m^2
l = length of the conductor in metres
v = velocity (in m/s) of the conductor

(ii) The armature conductors are connected to form coils. The basic component of all types of armature windings is the armature coil. Fig. (1.13) (i) shows a single-turn coil. It has two conductors or coil sides connected at the back of the armature. Fig. 1.13 (ii) shows a 4-turn coil which has 8 conductors or coil sides.

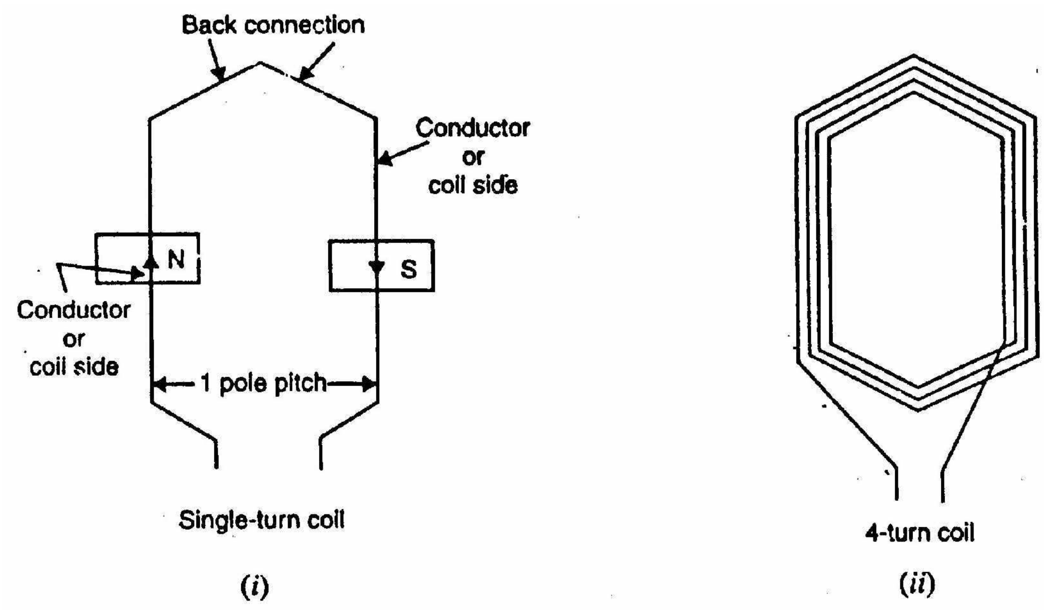


Fig. (1.13)

The coil sides of a coil are placed a pole span apart i.e., one coil side of the coil is under N-pole and the other coil side is under the next S-pole at the corresponding position as shown in Fig. 1.13 (i). Consequently the e.m.f.s of the coil sides add together. If the e.m.f. induced in one conductor is 2.5 volts, then the e.m.f. of a single-turn coil will be = 2 × 2.5 = 5 volts. For the same flux and speed, the e.m.f. of a 4-turn coil will be = 8 × 2.5 = 20 V.

(iii) Most of d.c. armature windings are double layer windings i.e., there are two coil sides per slot as shown in Fig. (1.14). One coil side of a coil lies at the top of a slot and the other coil side lies at the bottom of some other slot. The coil ends will then lie side by side. In two-layer winding, it is desirable to number the coil sides rather than the slots. The coil sides are numbered as indicated in Fig. (1.14). The coil sides at the top of slots are given odd numbers and those at the bottom are given even numbers. The coil sides are numbered in order round the armature.

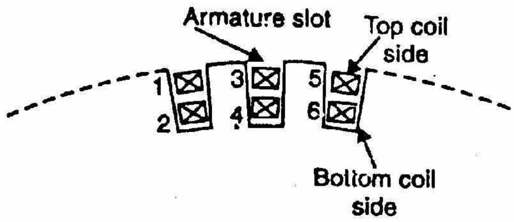


Fig. (1.14)

As discussed above, each coil has one side at the top of a slot and the other side at the bottom of another slot; the coil sides are nearly a pole pitch apart. In connecting the coils, it is ensured that top coil side is joined to the bottom coil side and vice-versa. This is illustrated in Fig. (1.15). The coil side 1 at the top of a slot is joined to coil side 10 at the bottom of another slot about a pole pitch apart. The coil side 12 at the bottom of a slot is joined to coil side 3 at the top of another slot. How coils are connected at the back of the armature and at the front (commutator end) will be discussed in later sections. It may be noted that as far as connecting the coils is concerned, the number of turns per coil is immaterial. For simplicity, then, the coils in winding diagrams will be represented as having only one turn (i.e., two conductors).

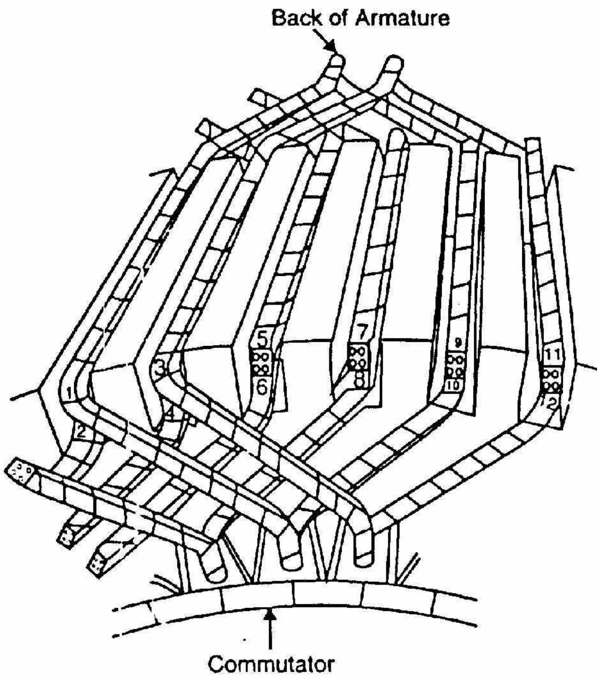


Fig. (1.15)

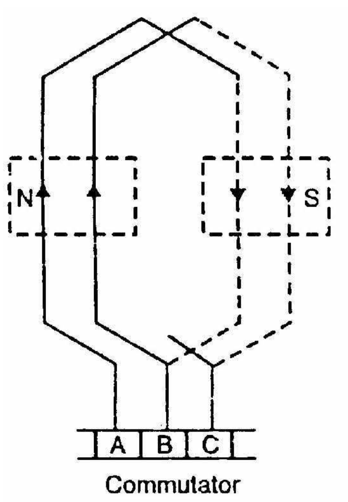


Fig. (1.16)

(iv) The coil sides are connected through commutator segments in such a manner as to form a series-parallel system; a number of conductors are connected in series so as to increase the voltage and two or more such series-connected paths in parallel to share the current. Fig. (1.16) shows how the two coils connected through commutator segments (A, R, C etc) have their e.m.f.s added together. If voltage induced in each conductor is 2-5 V, then voltage between segments A and C = 4 × 2.5 = 10 V. It may be noted here that in the conventional way of representing a developed armature winding, full lines represent top coil sides (i.e., coil sides lying at the top of a slot) and dotted lines represent the bottom coil sides (i.e., coil sides lying at the bottom of a slot).

(v) The d.c. armature winding is a closed circuit winding. In such a winding, if one starts at some point in the winding and traces through the winding, one will come back to the starting point without passing through any external connection. D.C. armature windings must be of the closed type in order to provide for the commutation of the coils.

1.6 Commutator Pitch (Y_C)

The commutator pitch is the number of commutator segments spanned by each coil of the winding. It is denoted by Y_C.

In Fig. (1.17), one side of the coil is connected to commutator segment 1 and the other side connected to commutator segment 2. Therefore, the number of commutator segments spanned by the coil is 1 i.e., $Y_C = 1$. In Fig. (1.18), one side of the coil is connected to commutator segment 1 and the other side to commutator segment 8. Therefore, the number of commutator segments spanned by the coil = 8 − 1 = 7 segments i.e., $Y_C = 7$. The commutator pitch of a winding is always a whole number. Since each coil has two ends and as two coil connections are joined at each commutator segment,

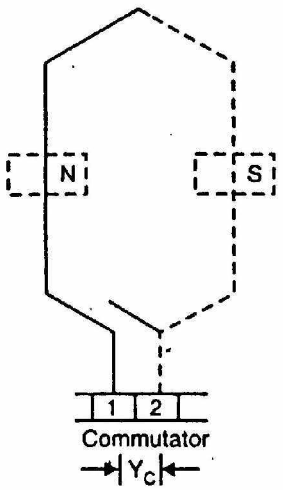


Fig. (1.17)

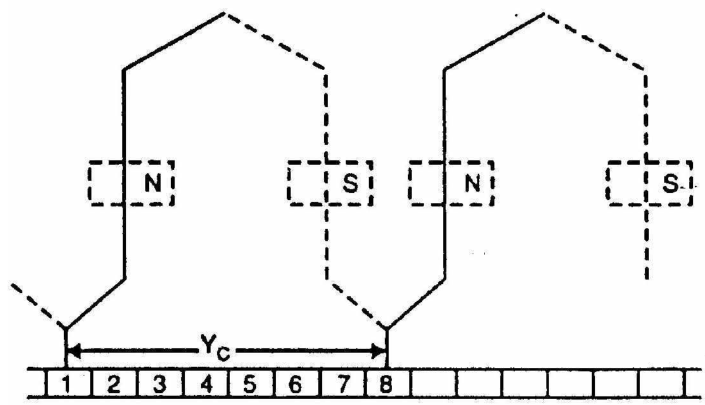


Fig. (1.18)

$$\therefore \qquad \text{Number of coils} = \text{Number of commutator segments}$$

For example, if an armature has 30 conductors, the number of coils will be 30/2 = 15. Therefore, number of commutator segments is also 15. Note that commutator pitch is the most important factor in determining the type of d.c. armature winding.

1.7 Pole-Pitch

It is the distance measured in terms of number of armature slots (or armature conductors) per pole. Thus if a 4-pole generator has 16 coils, then number of slots = 16.

$$\therefore \qquad \text{Pole pitch} = \frac{16}{4} = 4 \text{ slots}$$

Also $$\text{Pole pitch} = \frac{\text{No. of conductors}}{\text{No. of poles}} = \frac{16 \times 2}{4} = 8 \text{ conductors}$$

1.8 Coil Span or Coil Pitch (Y_S)

It is the distance measured in terms of the number of armature slots (or armature conductors) spanned by a coil. Thus if the coil span is 9 slots, it means one side of the coil is in slot 1 and the other side in slot 10.

1.9 Full-Pitched Coil

If the coil-span or coil pitch is equal to pole pitch, it is called full-pitched coil (See Fig. 1.19). In this case, the e.m.f.s in the coil sides are additive and have a phase difference of 0°. Therefore, e.m.f. induced in the coil is maximum. If e.m.f. induced in one coil side is 2-5 V, then e.m.f. across the coil terminals = 2 × 2.5 = 5 V. Therefore, coil span should always be one pole pitch unless there is a good reason for making it shorter.

Fractional pitched coil. If the coil span or coil pitch is less than the pole pitch, then it is called fractional pitched coil (See Fig. 1.20). In this case, the phase difference between the e.m.f.s in the two coil sides will not be zero so that the e.m.f. of the coil will be less compared to full-pitched coil. Fractional pitch winding requires less copper but if the pitch is too small, an appreciable reduction in the generated e.m.f. results.

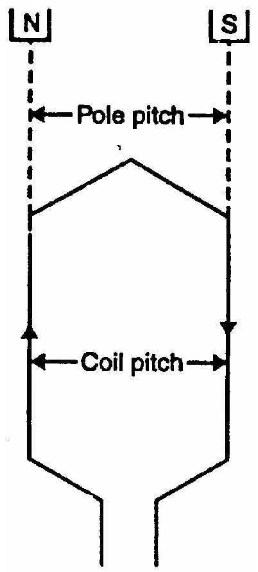


Fig. (1.19)

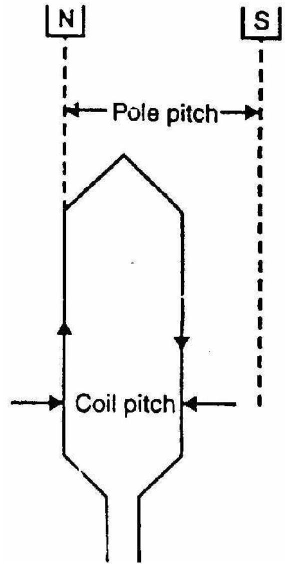


Fig. (1.20)

1.10 Types of D.C. Armature Windings

The different armature coils in a d.c. armature Winding must be connected in series with each other by means of end connections (back connection and front connection) in a manner so that the generated voltages of the respective coils will aid each other in the production of the terminal e.m.f. of the winding. Two basic methods of making these end connections are:

1. Simplex lap winding
2. Simplex wave winding

1. Simplex lap winding.

For a simplex lap winding, the commutator pitch Y_C = 1 and coil span $Y_S \simeq$ pole pitch. Thus the ends of any coil are brought out to adjacent commutator segments and the result of this method of connection is that all the coils of the armature .ire in sequence with the last coil connected to the first coil. Consequently, closed circuit winding results. This is illustrated in Fig. (1.21) where a part of the lap winding is shown. Only two coils are shown for simplicity. The name lap comes from the way in which successive coils overlap the preceding one.

2. Simplex wave winding

For a simplex wave winding, the commutator pitch $Y_C \simeq$ 2 pole pitches and coil span = pole pitch. The result is that the coils under consecutive pole pairs will be joined together in series thereby adding together their e.m.f.s [See Fig. 1.22]. After passing once around the armature, the winding falls in a slot to the left or right of the starting point and thus connecting up another circuit. Continuing in this way, all the conductors will be connected in a single closed winding. This winding is called wave winding from the appearance (wavy) of the end connections.

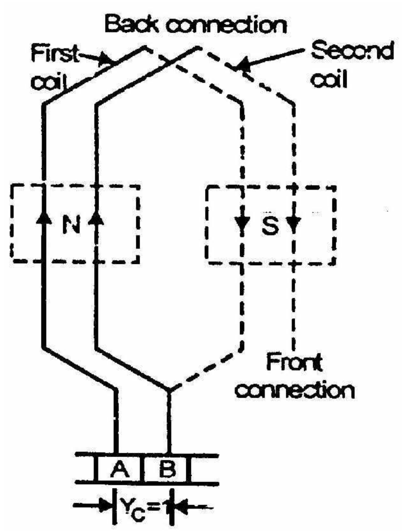


Fig. (1.21)

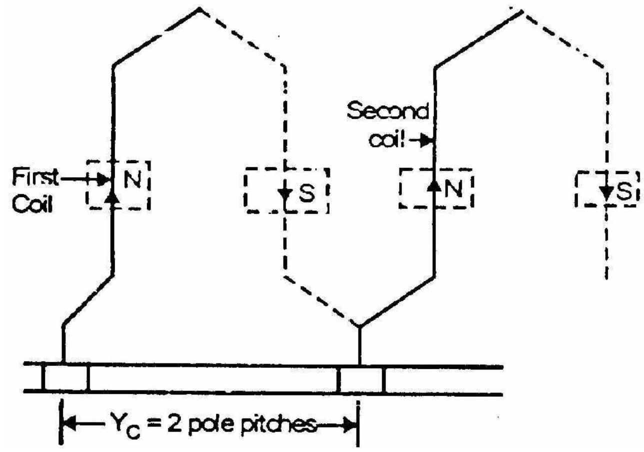


Fig. (1.22)

1.11 Further Armature Winding Terminology

Apart from the terms discussed earlier, the following terminology requires discussion:

(i) Back Pitch (Y_B)

It is the distance measured in terms of armature conductors between the two sides of a coil at the back of the armature (See Fig. 1.23). It is denoted by Y_B For example, if a coil is formed by connecting conductor 1 (upper conductor in a slot) to conductor 12 (bottom conductor in another slot) at the back of the armature, then back pitch is $Y_B = 12 - 1 = 11$ conductors.

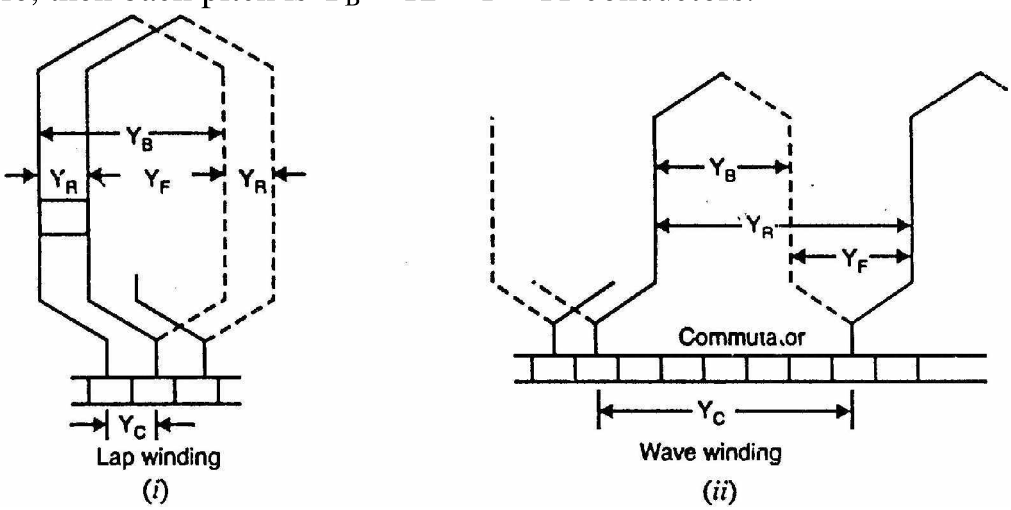


Fig. (1.23)

(ii) Front Pitch (Y_F)

It is the distance measured in terms of armature conductors between the coil sides attached to any one commutator segment [See Fig. 1.23]. It is denoted by Y_F For example, if coil side 12 and coil side 3 are connected to the same commutator segment, then front pitch is $Y_F = 12 - 3 = 9$ conductors.

(iii) Resultant Pitch (YR)

It is the distance (measured in terms of armature conductors) between the beginning of one coil and the beginning of the next coil to which it is connected (See Fig. 1.23). It is denoted by Y_R. Therefore, the resultant pitch is the algebraic sum of the back and front pitches.

(iv) Commutator Pitch (Y_C)

It is the number of commutator segments spanned by each coil of the armature winding.

For simplex lap winding, $Y_C = 1$

For simplex wave winding, $Y_C \simeq$ 2 pole pitches (segments)

(v) Progressive Winding

A progressive winding is one in which, as one traces through the winding, the connections to the commutator will progress around the machine in the same direction as is being traced along the path of each individual coil. Fig. (1.24) (i) shows progressive lap winding. Note that $Y_B > Y_F$ and $Y_C = +1$.

(vi) Retrogressive Winding

A retrogressive winding is one in which, as one traces through the winding, the connections to the commutator will progress around the machine in the opposite direction to that which is being traced along the path of each individual coil. Fig. (1.24) (ii) shows retrogressive lap winding. Note that $Y_F > Y_B$ and $Y_C = -1$. A retrogressive winding is seldom used because it requires more copper.

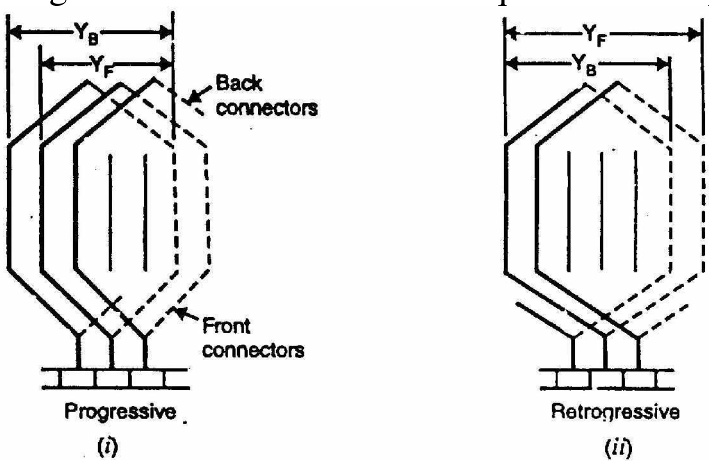


Fig. (1.24)

1.12 General Rules For D.C. Armature Windings

In the design of d.c. armature winding (lap or wave), the following rules may be followed:

(i) The back pitch (Y_B) as well as front pitch (Y_F) should be nearly equal to pole pitch. This will result in increased e.m.f. in the coils.

(ii) Both pitches (Y_B and Y_F) should be odd. This will permit all end connections (back as well as front connection) between a conductor at the top of a slot and one at the bottom of a slot.

(iii) The number of commutator segments is equal to the number of slots or coils (or half the number of conductors).

No. of commutator segments = No. of slots = No. of coils

It is because each coil has two ends and two coil connections are joined at each commutator segment

(iv) The winding must close upon itself i.e. it should be a closed circuit winding.

1.13 Relations between Pitches for Simplex Lap Winding

In a simplex lap winding, the various pitches should have the following relation:

(i) The back and front pitches are odd and are of opposite signs. They differ numerically by 2,

$$\therefore \quad Y_B = Y_B = Y_F \pm 2$$

$$Y_B = Y_F + 2 \qquad \text{for progressive winding}$$

$$Y_B = Y_F - 2 \qquad \text{for retrogressive winding}$$

(ii) Both Y_B and Y_F should be nearly equal to pole pitch.

(iii) Average pitch $=(Y_B + Y_F)/2$. It equals pole pitch (= Z/P).

(iv) Commutator pitch, $Y_C = \pm 1$

$Y_C = +1$ for progressive winding

$Y_C = -1$ for retrogressive winding

(v) The resultant pitch (Y_B) is even, being the arithmetical difference of two odd numbers viz., Y_B and Y_F.

(vi) If Z = number of armature conductors and P = number of poles, then,

$$\text{Polr - pitch} = \frac{Z}{P}$$

Since Y_B and Y_F both must be. about one pole pitch and differ numerically by 2,

$$\left.\begin{aligned} Y_B &= \frac{Z}{P} + 1 \\ Y_F &= \frac{Z}{P} - 1 \end{aligned}\right\} \quad \text{For progressive winding}$$

$$\left.\begin{aligned} Y_B &= \frac{Z}{P} - 1 \\ Y_F &= \frac{Z}{P} + 1 \end{aligned}\right\} \quad \text{For retrogressive winding}$$

It is clear that Z/P must be an even number to make the winding possible.

Developed diagram

Developed diagram is obtained by imagining the cylindrical surface of the armature to be cut by an axial plane and then flattened out. Fig. (1.25) (i) shows the developed diagram of the winding. Note that full lines represent the top coil sides (or conductors) and dotted lines represent the bottom coil sides (or conductors).

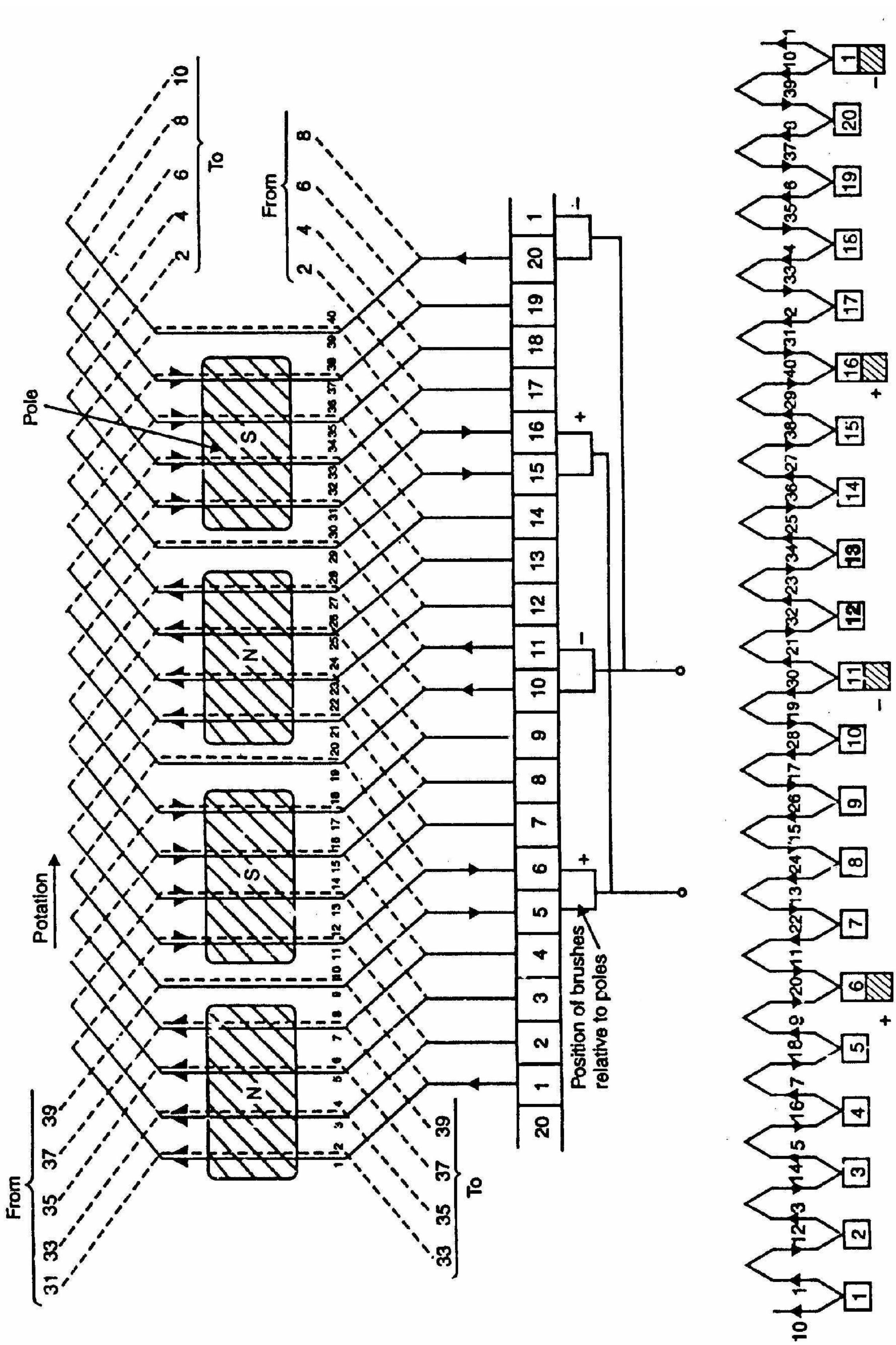
The winding goes from commutator segment 1 by conductor 1 across the back to conductor 12 and at the front to commutator segment 2, thus forming a coil. Then from commutator segment 2, through conductors 3 and 14 back to commutator segment 3 and so on till the winding returns to commutator segment 1 after using all the 40 conductors.

Position and number of brushes

We now turn to find the position and the number of brushes required. The brushes, like field poles, remain fixed in space as the commutator and winding revolve. It is very important that brushes are in correct position relative to the field poles. The arrowhead marked “rotation” in Fig. (1.25) (i) shows the direction of motion of the conductors. By right-hand rule, the direction of e.m.f. in each conductor will be as shown.

In order to find the position of brushes, the ring diagram shown in Fig. (1.25) (ii) is quite helpful. A positive brush will be placed on that commutator segment where the currents in the coils are meeting to flow out of the segment. A negative brush will be placed on that commutator segment where the currents in the coils are meeting to flow in. Referring to Fig. (1.25) (i), there are four brushes—two positive and two negative. Therefore, we arrive at a very important conclusion that in a simplex lap winding, the number of brushes is equal to the number of poles. If the brushes of the same polarity are connected together, then all the armature conductors are connected in four parallel paths; each path containing an equal number of conductors in series. This is illustrated in Fig. (1.26).

Since segments 6 and 16 are connected together through positive brushes and segments 11 and 1 are connected together through negative brushes, there are four parallel paths, each containing 10 conductors in series. Therefore, in a simplex lap winding, the number of parallel paths is equal to the number of poles.


Rotation
Pole
From
To
N
S
Position of brushes relative to poles

(i) (ii)

Fig. (1.25)

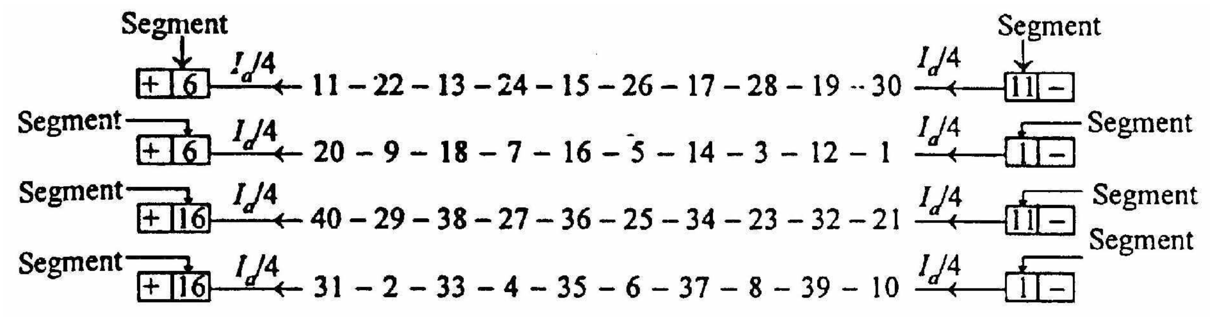


Fig. (1.26)

Conclusions

From the above discussion, the following conclusions can be drawn:

(i) The total number of brushes is equal to the number of poles.

(ii) The armature winding is divided into as many parallel paths as the number of poles. If the total number of armature conductors is Z and P is the number of poles, then,

Number of conductors/path = Z/P

In the present case, there are 40 armature conductors and 4 poles. Therefore, the armature winding has 4 parallel paths, each consisting of 10 conductors in series.

(iii) E.M.F. generated = E.M.F. per parallel path

$$= \text{average e.m.f. per conductor} \times \frac{Z}{P}$$

(iv) Total armature current, $I_a = P \times$ current per parallel path

(v) The armature resistance can be found as under:

Let l = length of each conductor; a = cross-sectional area

A = number of parallel paths = P for simplex lap winding

$$\text{Resistance of whole winding, } R = \frac{\rho l}{a} \times Z$$

$$\text{Resistance per parallel path } = \frac{R}{A} = \frac{\rho l Z}{a \times A}$$

Since there are A (= P) parallel paths, armature resistance R_a is given by:

$$R_a = \frac{\text{Resistance per parallel path}}{A} = \frac{1}{A}\left(\frac{\rho l Z}{a \times A}\right)$$

$$\therefore \quad R_a = \frac{\rho l Z}{a A^2}$$

1.14 Simplex Wave Winding

The essential difference between a lap winding and a wave winding is in the commutator connections. In a simplex lap winding, the coils approximately pole pitch apart are connected in series and the commutator pitch $Y_C = \pm 1$ segment. As a result, the coil voltages add. This is illustrated in Fig. (1.27). In a simplex wave winding, the coils approximately pole pitch apart are connected in series and the commutator pitch $Y_C \simeq 2$ pole pitches (segments). Thus in a wave winding, successive coils "wave" forward under successive poles instead of "lapping" back on themselves as in the lap winding. This is illustrated in Fig. (1.28).

The simplex wave winding must not close after it passes once around the armature but it must connect to a commutator segment adjacent to the first and the next coil must be adjacent to the first as indicated in Fig. (1.28). This is repeated each time around until connections are made to all the commutator segments and all the slots are occupied after which the winding automatically returns to the starting point. If, after passing once around the armature, the winding connects to a segment to the left of the starting point, the winding is retrogressive [See Fig. 1.28 (i)]. If it connects to a segment to the right of the starting point, it is progressive [See Fig. 1.28 (ii)]. This type of winding is called wave winding because it passes around the armature in a wave-like form.

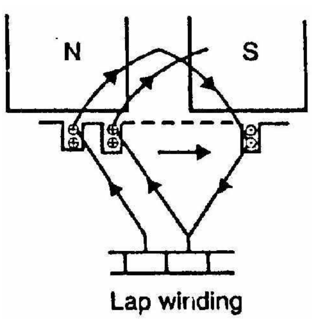


Fig. (1.27)

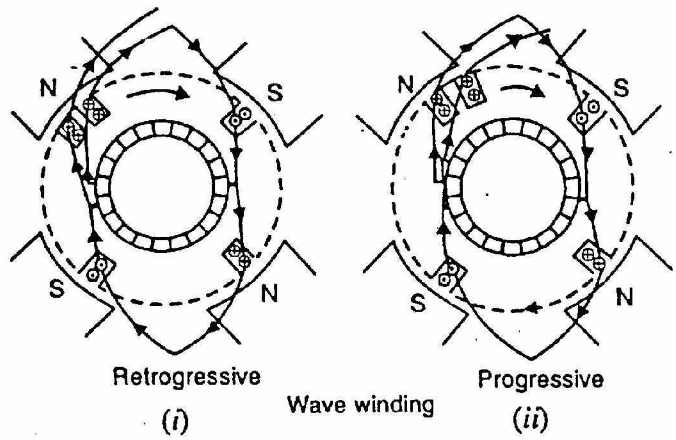


Fig. (1.28)

Various pitches

The various pitches in a wave winding are defined in a manner similar to lap winding.

(i) The distance measured in terms of armature conductors between the two sides of a coil at the back of the armature is called back pitch Y_B (See Fig. 1.29). The Y_B must be an odd integer so that a top conductor and a bottom conductor will be joined.

(ii) The distance measured in terms of armature conductors between the coil sides attached to any one commutator segment is called front pitch Y_B (See Fig. 1.29). The Y_B must be an odd integer so that a top conductor and a bottom conductor will be joined.

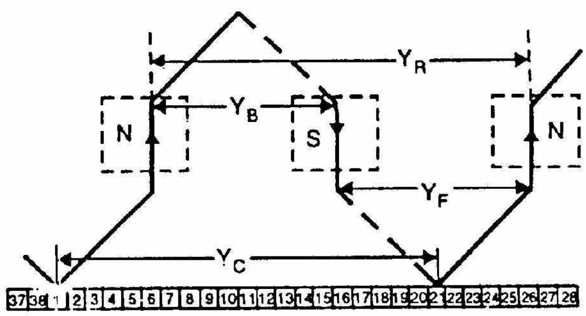


Fig. (1.29))

(iii) Resultant pitch, $Y_R = Y_B + Y_F$ (See Fig. 1.29)
The resultant pitch must be an even integer since Y_B and Y_F are odd. Further Y_R is approximately two pole pitches because Y_B as well as Y_F is approximately one pole pitch.

(iv) Average pitch, $Y_A = \frac{Y_B + Y_F}{2}$. When one tour of armature has been completed, the winding should connect to the next top conductor (progressive) or to the preceding top conductor (retrogressive). In either case, the difference will be of 2 conductors or one slot. If P is the number of poles and Z is the total number of armature conductors, then,

$$P \times Y_A = Z \pm 2$$

or $$Y_A = \frac{Z+2}{P} \qquad \text{(i)}$$

Since P is always even and $Z = PY_A \pm 2$, Z must be even. It means that Z ± 2/P must be an integer. In Eq.(i), plus sign will give progressive winding and the negative sign retrogressive winding.

(v) The number of commutator segments spanned by a coil is called commutator pitch (Y_C) (See Fig. 1.29). Suppose in a simplex wave winding,
P = Number of poles; N_C = Number of commutator segments;
Y_C = Commutator pitch.

∴ Number of pair of poles = P/2

If $Y_C \times P/2 = N_C$, then the winding will close on itself in passing once around the armature. In order to connect to the adjacent conductor and permit the winding to proceed,

$$Y_C \times \frac{P}{2} = N_C \pm 1$$

or $$Y_C = \frac{2N_C \pm 2}{P} = \frac{N_C \pm 1}{P/2} = \frac{\text{No. of commutator seg.} \pm 1}{\text{Number of pair of poles}}$$

Now $$Y_C = \frac{2N_C \pm 2}{P} = \frac{Z \pm 2}{P} = Y_A \qquad (\because\ 2N_C = Z)$$

$$\therefore \quad \text{Commutator pitch, } \ Y_C = Y_A = \frac{Y_B + Y_F}{2}$$

In a simplex wave winding Y_B, Y_F and Y_C may be equal. Note that Y_B, Y_F and Y_B are in terms of armature conductors whereas Y_C is in terms of commutator segments.

1.15 Design of Simplex Wave Winding

In the design of simplex wave winding, the following points may be kept in mind:

(i) Both pitches Y_B and Y_F are odd and are of the same sign.

(ii) Average pitch, $$Y_A = \frac{Z \pm 2}{P} \qquad \text{(i)}$$

(iii) Both Y_B and Y_F are nearly equal to pole pitch and may be equal or differ by 2. If they differ by 2, they are one more and one less than Y_A.

(iv) Commutator pitch is given by;

$$Y_C = Y_A = \frac{\text{Number of commutator segments} \pm 1}{\text{Number of pair of poles}}$$

The plus sign for progressive winding and negative for retrogressive winding.

(v) $$Y_A = \frac{Z \pm 2}{P}$$

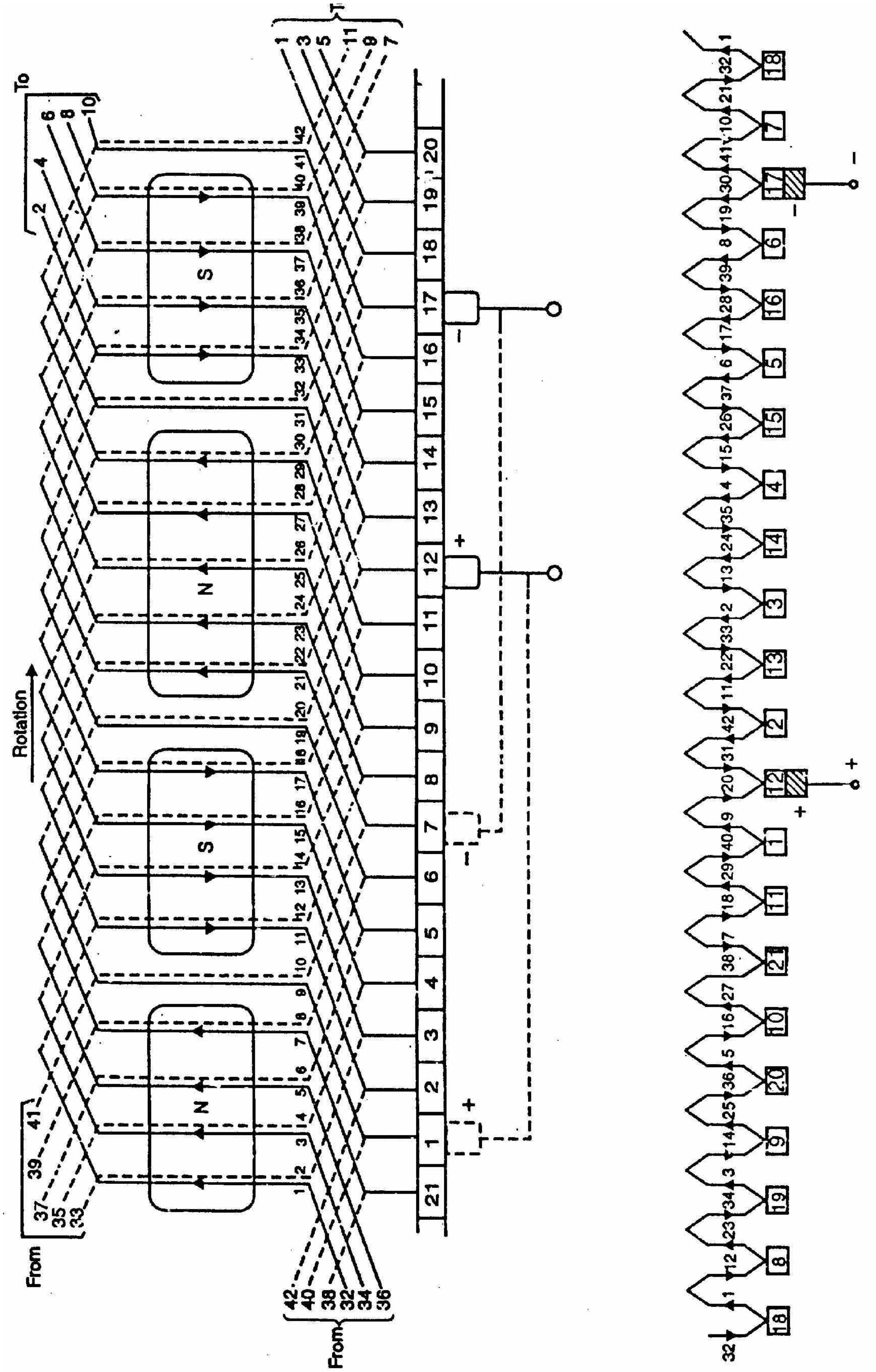
Since Y_A must be a whole number, there is a restriction on the value of Z. With Z = 180, this winding is impossible for a 4-pole machine because Y_A is not a whole number.

(vi) $$Z = P\,Y_A \pm 2$$

$$\therefore \quad \text{Numnber of coils } = \frac{Z}{2} = \frac{PY_A \pm 2}{2}$$

Developed diagram

Fig. (1.30) (i) shows the developed diagram for the winding. Note that full lines represent the top coil sides (or conductors) and dotted lines represent the bottom coil sides (or conductors). The two conductors which lie in the same slot are drawn nearer to each other than to those in the other slots.



(i) (ii)

Fig. (1.30)

Referring to Fig. (1.30) (i), conductor 1 connects at the back to conductor 12(1 + 11) which in turn connects at the front to conductor 23 (12 + 11) and so on round the armature until the winding is complete. Note that the commutator pitch Y_C = 11 segments. This means that the number of commutator segments spanned between the start end and finish end of any coil is 11 segments.

Position and number of brushes

We now turn to find the position and the number of brushes. The arrowhead marked "rotation" in Fig. (1.30) (i) shows the direction of motion of the conductors. By right hand rule, the direction of e.m.f. in each conductor will be as shown.

In order to find the position of brushes, the ring diagram shown in Fig. (1.30) (ii) is quite helpful. It is clear that only two brushes—one positive and one negative—are required (though two positive and two negative brushes can also be used). We find that there are two parallel paths between the positive brush and the negative brush. Thus is illustrated in Fig. (1.31).

Therefore, we arrive at a very important conclusion that in a simplex wave winding, the number of parallel paths is two irrespective of the number of poles. Note that the first parallel path has 11 coils (or 22 conductors) while the second parallel path has 10 coils (or 20 conductors). This fact is not important as it may appear at first glance. The coils m the smaller group should supply less current to the external circuit. But the identity of the coils in either parallel path is rapidly changing from moment to moment. Therefore, the average value of current through any particular coil is the same.

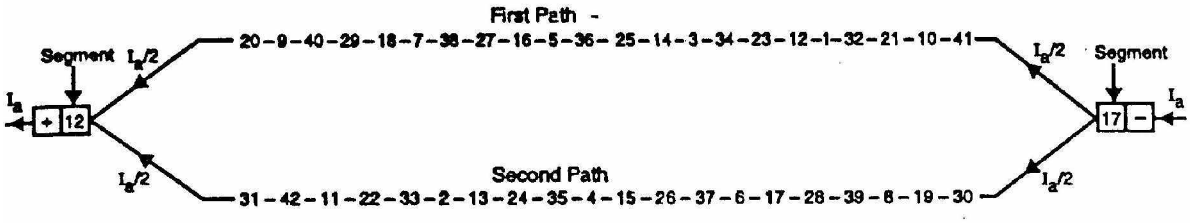


Fig. (1.31)

Conclusions

From the above discussion, the following conclusions can be drawn:

(i) Only two brushes are necessary but as many brushes as there are poles may be used.

(ii) The armature winding is divided into two parallel paths irrespective of the number of poles. If the total number of armature conductors is Z and P is the number of poles, then,

$$\text{Number of conductors/path} = \frac{Z}{2}$$

(iii) E.M.F. generated = E.M.F. per parallel path
= Average e.m.f. per conductor x —

(iv) Total armature current, $I_a = 2 \times$ current per parallel path

(v) The armature can be wave-wound if Y_A or Y_C is a whole number.

1.16 Dummy Coils

In a simplex wave winding, the average pitch Y_A (or commutator pitch Y_C) should be a whole number. Sometimes the standard armature punchings available in the market have slots that do not satisfy the above requirement so that more coils (usually only one more) are provided than can be utilized. These extra coils are called dummy or dead coils. The dummy coil is inserted into the slots in the same way as the others to make the armature dynamically balanced but it is not a part of the armature winding.

Let us illustrate the use of dummy coils with a numerical example. Suppose the number of slots is 22 and each slot contains 2 conductors. The number of poles is 4. For simplex wave wound armature,

$$Y_A = \frac{Z \pm 2}{P} = \frac{2 \times 22 \pm 2}{4} = \frac{44 \pm 2}{4} = 11\frac{1}{2} \text{ or } 10\frac{1}{2}$$

Since the results are not whole numbers, the number of coils (and hence segments) must be reduced. If we make one coil dummy, we have 42 conductors and

$$Y_A = \frac{42 \pm 2}{4} = 11 \text{ or } 10$$

This means that armature can be wound only if we use 21 coils and 21 segments. The extra coil or dummy coil is put in the slot. One end of this coil is taped and the other end connected to the unused commutator segment (segment 22) for the sake of appearance. Since only 21 segments are required, the two (21 and 22 segments) are connected together and considered as one.

1.17 Applications of Lap and Wave Windings

In multipolar machines, for a given number of poles (P) and armature conductors (Z), a wave winding has a higher terminal voltage than a lap winding because it has more conductors in series. On the other hand, the lap winding carries more current than a wave winding because it has more parallel paths.

In small machines, the current-carrying capacity of the armature conductors is not critical and in order to achieve suitable voltages, wave windings are used. On the other hand, in large machines suitable voltages are easily obtained because of the availability of large number of armature conductors and the current carrying capacity is more critical. Hence in large machines, lap windings are used.

Note: In general, a high-current armature is lap-wound to provide a large number of parallel paths and a low-current armature is wave-wound to provide a small number of parallel paths.

1.18 Multiplex Windings

A simplex lap-wound armature has as many parallel paths as the number of poles. A simplex wave-wound armature has two parallel paths irrespective of the number of poles. In case of a 10-pole machine, using simplex windings, the designer is restricted to either two parallel circuits (wave) or ten parallel circuits (lap). Sometimes it is desirable to increase the number of parallel paths. For this purpose, multiplex windings are used. The sole purpose of multiplex windings is to increase the number of parallel paths enabling the armature to carry a large total current. The degree of multiplicity or plex determines the number of parallel paths in the following manner:

(i) A lap winding has pole times the degree of plex parallel paths.

Number of parallel paths, $A = P \times \text{plex}$

Thus a duplex lap winding has 2P parallel paths, triplex lap winding has 3P parallel paths and so on. If an armature is changed from simplex lap to duplex lap without making any other change, the number of parallel paths is doubled and each path has half as many coils. The armature will then supply twice as much current at half the voltage.

(ii) A wave winding has two times the degree of plex parallel paths.

Number of parallel paths, $A = 2 \times \text{plex}$

Note that the number of parallel paths in a multiplex wave winding depends upon the degree of plex and not on the number of poles. Thus a duplex wave winding has 4 parallel paths, triplex wave winding has 6 parallel paths and so on.

1.19 Function of Commutator and Brushes

The e.m.f. generated in the armature winding of a d.c. generator is alternating one. The commutator and brushes cause the alternating e.m.f. of the armature conductors to produce a p.d. always in the same direction between the terminals of the generator. In lap as well as wave winding, it will be observed that currents in the coils to a brush are either all directed towards the brush (positive brush) or all directed away from the brush (negative brush). Further, the direction of current in coil reverses as it passes the brush. Thus when the coil approaches the contact with the brush, the current through the coil is in one direction; when the coil leaves the contact with the brush, the current has been reversed. This reversal of current in the coil as the coil passes a brush is called commutation and fakes place while the coil is short-circuited by the brush. These changes occur in every coil in turn. If, at the instant when the brush breaks contact with the commutator segment connected to the coil undergoing commutation, the current in the coil has not been reversed, the result will be sparking between the commutator segments and the brush.

The criterion of good commutation is that it should be sparkless. In order to have sparkless commutation, the brushes on the commutator should be placed at points known as neutral point where no voltage exists between adjacent segments. The conductors connected to these segments lie between the poles in position of zero magnetic flux which is termed as magnetic neutral axis (M.N.A)

1.20 E.M.F. Equation of a D.C. Generator

We shall now derive an expression for the e.m.f. generated in a d.c. generator.

Let ϕ = flux/pole in Wb

Z = total number of armature conductors

P = number of poles

A = number of parallel paths = 2 ... for wave winding

= P ... for lap winding

N = speed of armature in r.p.m.

E_g = e.m.f. of the generator = e.m.f./parallel path

Flux cut by one conductor in one revolution of the armature,

$$d\phi = P\phi \text{ webers}$$

Time taken to complete one revolution,

$$dt = 60/N \text{ second}$$

e.m.f generated/conductor $= \dfrac{d\phi}{dt} = \dfrac{P\phi}{60/N} = \dfrac{P\phi N}{60}$ volts

e.m.f. of generator,

E_g = e.m.f. per parallel path

= (e.m.f/conductor) × No. of conductors in series per parallel path

$$= \frac{P\phi\, N}{60} \times \frac{Z}{A}$$

$$\therefore \quad E_g = \frac{P\phi\, ZN}{60\, A}$$

where $A = 2$ for-wave winding

$= P$ for lap winding

1.21 Armature Resistance (R_a)

The resistance offered by the armature circuit is known as armature resistance (R_a) and includes:

(i) resistance of armature winding
(ii) resistance of brushes

The armature resistance depends upon the construction of machine. Except for small machines, its value is generally less than 1Ω.

1.22 Types of D.C. Generators

The magnetic field in a d.c. generator is normally produced by electromagnets rather than permanent magnets. Generators are generally classified according to their methods of field excitation. On this basis, d.c. generators are divided into the following two classes:

(i) Separately excited d.c. generators
(ii) Self-excited d.c. generators

The behaviour of a d.c. generator on load depends upon the method of field excitation adopted.

1.23 Separately Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied from an independent external d.c. source (e.g., a battery etc.) is called a separately excited generator. Fig. (1.32) shows the connections of a separately excited generator. The voltage output depends upon the speed of rotation of armature and the field current ($E_g = P\phi\ ZN/60\ A$). The greater the speed and field current, greater is the generated e.m.f. It may be noted that separately excited d.c. generators are rarely used in practice. The d.c. generators are normally of self-excited type.

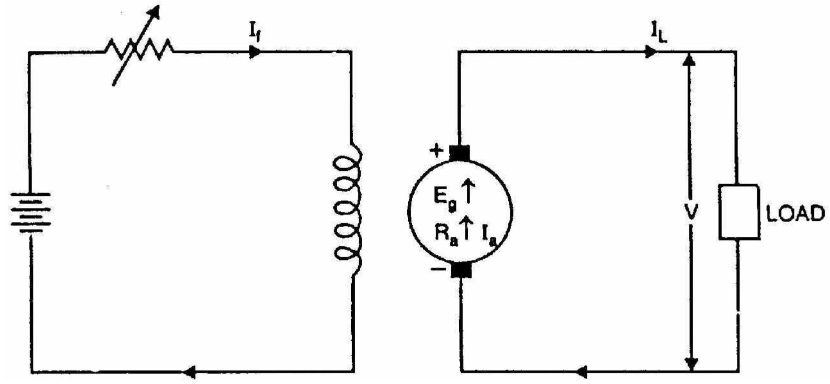


Fig. (1.32)

Armature current, $I_a = I_L$
Terminal voltage, $V = E_g - I_aR_a$
Electric power developed = E_gI_a
Power delivered to load = $E_gI_a - I_a^2R_a = I_a(E_g - I_aR_a) = VI_a$

1.24 Self-Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied current from the output of the generator itself is called a self-excited generator. There are three types of self-excited generators depending upon the manner in which the field winding is connected to the armature, namely;

(i) Series generator;
(ii) Shunt generator;
(iii) Compound generator

(i) Series generator

In a series wound generator, the field winding is connected in series with armature winding so that whole armature current flows through the field winding as well as the load. Fig. (1.33) shows the connections of a series wound generator. Since the field winding carries the whole of load current, it has a few turns of thick wire having low resistance. Series generators are rarely used except for special purposes e.g., as boosters.

Armature current, $I_a = I_{se} = I_L = I(\text{say})$
Terminal voltage, $V = E_G - I(R_a + R_{se})$
Power developed in armature = E_gI_a
Power delivered to load
$$= E_gI_a - I_a^2(R_a + R_{se}) = I_a[E_g - I_a(R_a - R_{se})] = VI_a \text{ or } VI_L$$

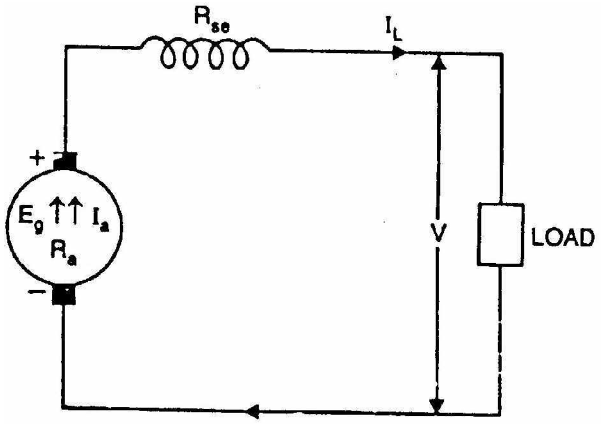


Fig. (1.33)

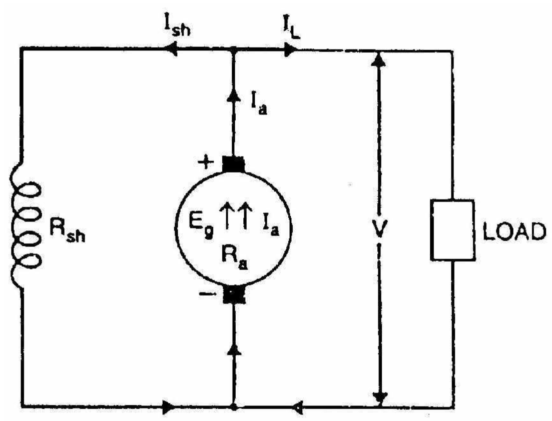


Fig. (1.34)

(ii) Shunt generator

In a shunt generator, the field winding is connected in parallel with the armature winding so that terminal voltage of the generator is applied across it. The shunt field winding has many turns of fine wire having high resistance. Therefore, only a part of armature current flows through shunt field winding and the rest flows through the load. Fig. (1.34) shows the connections of a shunt-wound generator.

Shunt field current, $I_{sh} = V/R_{sh}$
Armature current, $I_a = I_L + I_{sh}$
Terminal voltage, $V = E_g - I_aR_a$
Power developed in armature = E_gI_a
Power delivered to load = VI_L

(iii) Compound generator

In a compound-wound generator, there are two sets of field windings on each pole—one is in series and the other in parallel with the armature. A compound wound generator may be:

(a) Short Shunt in which only shunt field winding is in parallel with the armature winding [See Fig. 1.35 (i)].
(b) Long Shunt in which shunt field winding is in parallel with both series field and armature winding [See Fig. 1.35 (ii)].

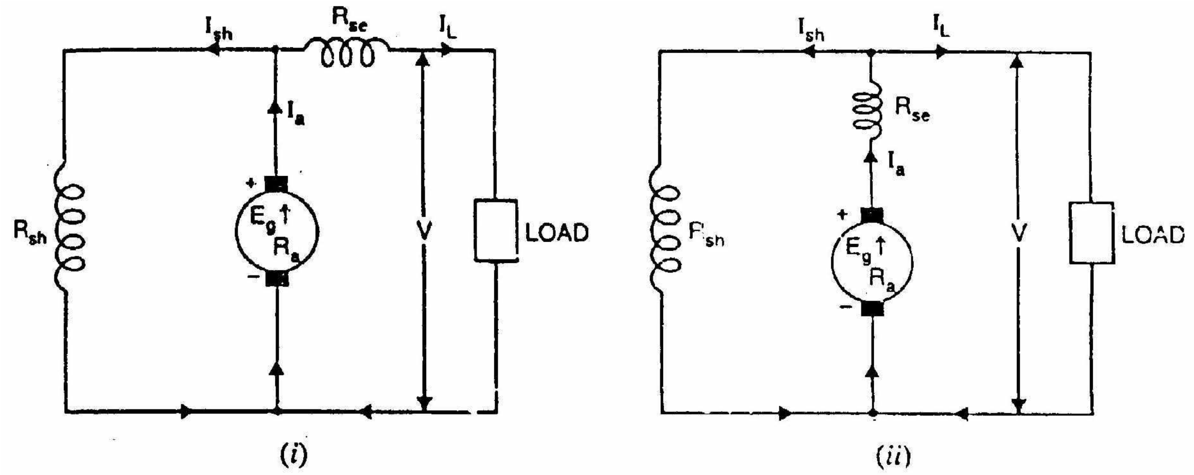


Fig. (1.35)

Short shunt

Series field current, $I_{se} = I_L$
Shunt field current, $I_{sh} = \dfrac{V + I_{se}R_{se}}{R_{sh}}$
Terminal voltage, $V = E_g - I_aR_a - I_{se}R_{se}$
Power developed in armature = E_gI_a
Power delivered to load = VI_L

Long shunt

Series field current, $I_{se} = I_a = I_L + I_{sh}$

Shunt field current, $I_{sh} = V/R_{sh}$

Terminal voltage, $V = E_g - I_a(R_a + R_{se})$

Power developed in armature = $E_g I_a$

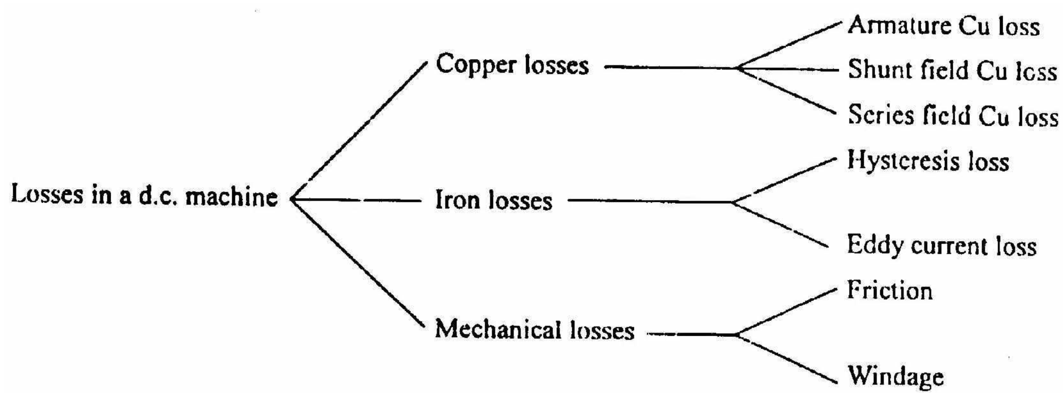
Power delivered to load = VI_L

1.25 Brush Contact Drop

It is the voltage drop over the brush contact resistance when current flows. Obviously, its value will depend upon the amount of current flowing and the value of contact resistance. This drop is generally small.

1.26 Losses in a D.C. Machine

The losses in a d.c. machine (generator or motor) may be divided into three classes viz (i) copper losses (ii) iron or core losses and (iii) mechanical losses. All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine.



1. Copper losses

These losses occur due to currents in the various windings of the machine.

(i) Armature copper loss = $I_a^2 R_a$

(ii) Shunt field copper loss = $I_{sh}^2 R_{sh}$

(iii) Series field copper loss = $I_{se}^2 R_{se}$

Note. There is also brush contact loss due to brush contact resistance (i.e., resistance between the surface of brush and surface of commutator). This loss is generally included in armature copper loss.

2. Iron or Core losses

These losses occur in the armature of a d.c. machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz., (i) hysteresis loss (ii) eddy current loss.

(i) **Hysteresis loss**

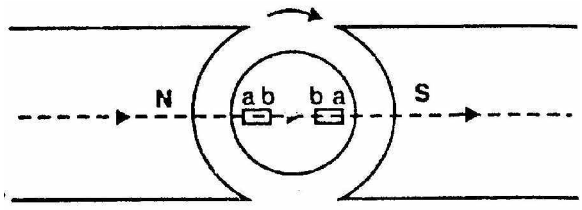


Fig. (1.36)

Hysteresis loss occurs in the armature of the d.c. machine since any given part of the armature is subjected to magnetic field reversals as it passes under successive poles. Fig. (1.36) shows an armature rotating in two-pole machine. Consider a small piece ab of the armature. When the piece ab is under N-pole, the magnetic lines pass from a to b. Half a revolution later, the same piece of iron is under S-pole and magnetic lines pass from b to a so that magnetism in the iron is reversed. In order to reverse continuously the molecular magnets in the armature core, some amount of power has to be spent which is called hysteresis loss. It is given by Steinmetz formula. This formula is

$$\text{Hysteresis loss, } P_h = \eta B_{max}^{16} f V \quad \text{watts}$$

where B_{max} = Maximum flux density in armature
f = Frequency of magnetic reversals
= NP/120 where N is in r.p.m.
V = Volume of armature in m^3
η = Steinmetz hysteresis co-efficient

In order to reduce this loss in a d.c. machine, armature core is made of such materials which have a low value of Steinmetz hysteresis co-efficient e.g., silicon steel.

(ii) **Eddy current loss**

In addition to the voltages induced in the armature conductors, there are also voltages induced in the armature core. These voltages produce circulating currents in the armature core as shown in Fig. (1.37). These are called eddy currents and power loss due to their flow is called eddy current loss. The eddy current loss appears as heat which raises the temperature of the machine and lowers its efficiency.

If a continuous solid iron core is used, the resistance to eddy current path will be small due to large cross-sectional area of the core. Consequently, the magnitude of eddy current and hence eddy current loss will be large. The magnitude of eddy current can be reduced by making core resistance as high as practical. The core resistance can be greatly increased by constructing the core of thin, round iron sheets called laminations [See Fig. 1.38]. The laminations are insulated from each other with a coating of varnish. The insulating coating has a high resistance, so very little current flows from one lamination to the other. Also, because each lamination is very thin, the resistance to current flowing through the width of a lamination is also quite large. Thus laminating a core increases the core resistance which decreases the eddy current and hence the eddy current loss.

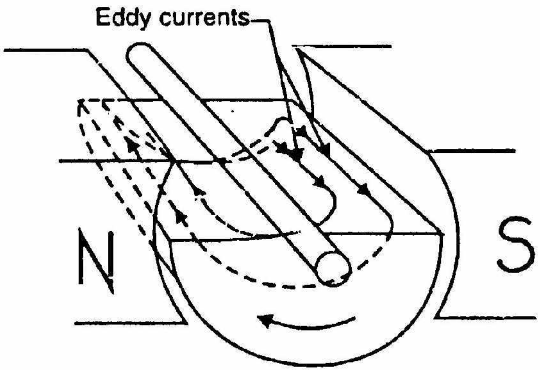


Fig. (1.37)

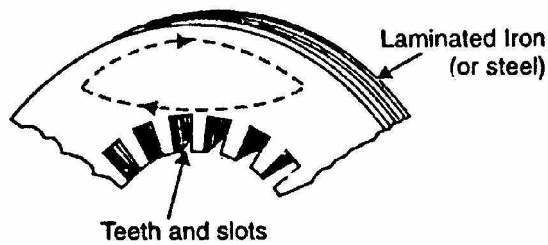


Fig. (1.38)

$$\text{Eddy current loss, } P_e = K_e B_{max}^2 f^2 t^2 V \quad \text{watts}$$

where K_e = Constant depending upon the electrical resistance of core and system of units used

B_{max} = Maximum flux density in Wb/m2

f = Frequency of magnetic reversals in Hz

t = Thickness of lamination in m

V = Volume of core in m^3

It may be noted that eddy current loss depends upon the square of lamination thickness. For this reason, lamination thickness should be kept as small as possible.

3. Mechanical losses

These losses are due to friction and windage.

(i) friction loss e.g., bearing friction, brush friction etc.

(ii) windage loss i.e., air friction of rotating armature.

These losses depend upon the speed of the machine. But for a given speed, they are practically constant.

Note. Iron losses and mechanical losses together are called stray losses.

1.27 Constant and Variable Losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (i) constant losses (ii) variable losses.

(i) **Constant losses**

Those losses in a d.c. generator which remain constant at all loads are known as constant losses. The constant losses in a d.c. generator are:

- (a) iron losses
- (b) mechanical losses
- (c) shunt field losses

(ii) Variable losses

Those losses in a d.c. generator which vary with load are called variable losses. The variable losses in a d.c. generator are:

(a) Copper loss in armature winding ($I_a^2 R_a$)

(b) Copper loss in series field winding ($I_{se}^2 R_{se}$)

Total losses = Constant losses + Variable losses

Note. Field Cu loss is constant for shunt and compound generators.

1.28 Power Stages

The various power stages in a d.c. generator are represented diagrammatically in Fig. (1.39).

A – B = Iron and friction losses

B – C = Copper losses

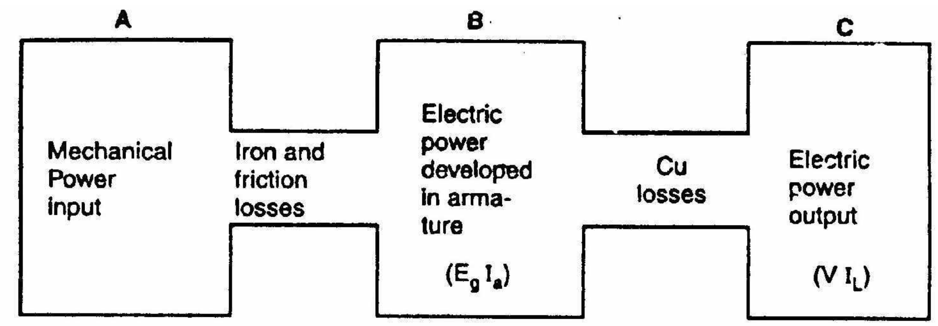


Fig. (1.39)

(i) Mechanical efficiency

$$\eta_m = \frac{B}{A} = \frac{E_g I_a}{\text{Mechanical power input}}$$

(ii) Electrical efficiency

$$\eta_e = \frac{C}{B} = \frac{V\ I_L}{E_g\ I_a}$$

(iii) Commercial or overall efficiency

$$\eta_c = \frac{C}{A} = \frac{V\ I_L}{\text{Mechanical power input}}$$

Clearly $\quad \eta_c = \eta_m \times \eta_e$

Unless otherwise stated, commercial efficiency is always understood.

Now, $\quad$ commercial efficiency, $\eta_c = \dfrac{C}{A} = \dfrac{\text{output}}{\text{input}} = \dfrac{\text{input} - \text{losses}}{\text{input}}$

1.29 Condition for Maximum Efficiency

The efficiency of a d.c. generator is not constant but varies with load. Consider a shunt generator delivering a load current I_L at a terminal voltage V.

$$\begin{aligned}
\text{Generator output} &= V\ I_L \\
\text{Generator input} &= \text{Output} + \text{Losses} \\
&= V\ I_L + \text{Variable losses} + \text{Constant losses} \\
&= VI_L + I_a^2 R_a + W_C \\
&= VI_L + (I_L + I_{sh})^2 R_a + W_C \qquad [\because\ I_a + I_L + I_{sh}]
\end{aligned}$$

The shunt field current I_{sh} is generally small as compared to I_L and, therefore, can be neglected.

$\therefore \quad$ Generator input $= VI_L + I_L^2 R_a + W_C$

Now $\quad \eta = \dfrac{\text{output}}{\text{input}} = \dfrac{VI_L}{VI_L + I_L^2 R_a + W_C}$

$$= \frac{1}{1 + \left(\dfrac{I_L R_a}{V} + \dfrac{W_C}{VI_L}\right)} \qquad \text{(i)}$$

The efficiency will be maximum when the denominator of Eq.(i) is minimum i.e.,

$$\frac{d}{dI_L}\left(\frac{I_L R_a}{V} + \frac{W_C}{VI_L}\right) = 0$$

or $\quad \dfrac{R_a}{V} - \dfrac{W_C}{VI_L^2} = 0$

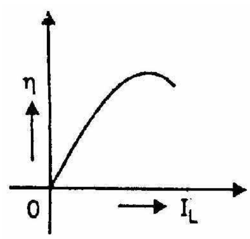


Fig. (1.40)

or $$\frac{R_a}{V} = \frac{W_C}{VI_L^2}$$

or $$I_L^2 R_a = W_C$$

i.e. Variable loss = Constant loss $\left(\because \; I_L \simeq I_a\right)$

The load current corresponding to maximum efficiency is given by;

$$I_L = \sqrt{\frac{W_C}{R_a}}$$

Hence, the efficiency of a d.c. generator will be maximum when the load current is such that variable loss is equal to the constant loss. Fig (1.40) shows the variation of η with load current.

Chapter (2)

Armature Reaction and Commutation

Introduction

In a d.c. generator, the purpose of field winding is to produce magnetic field (called main flux) whereas the purpose of armature winding is to carry armature current. Although the armature winding is not provided for the purpose of producing a magnetic field, nevertheless the current in the armature winding will also produce magnetic flux (called armature flux). The armature flux distorts and weakens the main flux posing problems for the proper operation of the d.c. generator. The action of armature flux on the main flux is called armature reaction.

In the previous chapter (Sec 1.19), it was hinted that current in the coil is reversed as the coil passes a brush. This phenomenon is termed as commutation. The criterion for good commutation is that it should be sparkless. In order to have sparkless commutation, the brushes should lie along magnetic neutral axis. In this chapter, we shall discuss the various aspects of armature reaction and commutation in a d.c. generator.

2.1 Armature Reaction

So far we have assumed that the only flux acting in a d.c. machine is that due to the main poles called main flux. However, current flowing through armature conductors also creates a magnetic flux (called armature flux) that distorts and weakens the flux coming from the poles. This distortion and field weakening takes place in both generators and motors. The action of armature flux on the main flux is known as armature reaction.

The phenomenon of armature reaction in a d.c. generator is shown in Fig. (2.1). Only one pole is shown for clarity. When the generator is on no-load, a smal1 current flowing in the armature does not appreciably affect the main flux ϕ_1 coming from the pole [See Fig 2.1 (i)]. When the generator is loaded, the current flowing through armature conductors sets up flux ϕ_1. Fig. (2.1) (ii) shows flux due to armature current alone. By superimposing ϕ_1 and ϕ_2, we obtain the resulting flux ϕ_3 as shown in Fig. (2.1) (iii). Referring to Fig (2.1) (iii), it is clear that flux density at; the trailing pole tip (point B) is increased while at the leading pole tip (point A) it is decreased. This unequal field distribution produces the following two effects:

(i) The main flux is distorted.

(ii) Due to higher flux density at pole tip B, saturation sets in. Consequently, the increase in flux at pole tip B is less than the decrease in flux under pole tip A. Flux ϕ_3 at full load is, therefore, less than flux ϕ_1 at no load. As we shall see, the weakening of flux due to armature reaction depends upon the position of brushes.

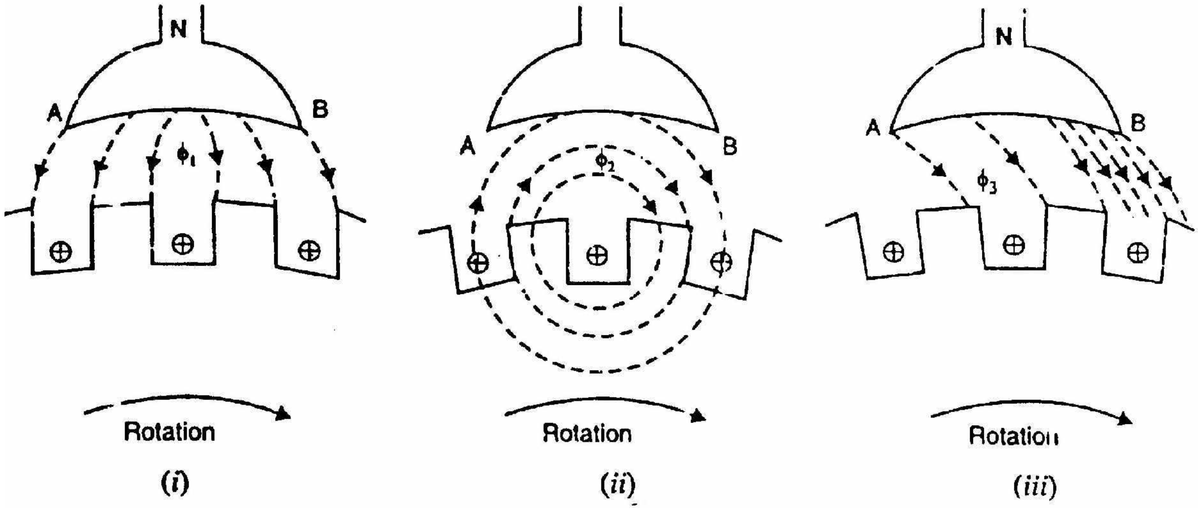


Fig. (2.1)

2.2 Geometrical and Magnetic Neutral Axes

(i) The geometrical neutral axis (G.N.A.) is the axis that bisects the angle between the centre line of adjacent poles [See Fig. 2.2 (i)]. Clearly, it is the axis of symmetry between two adjacent poles.

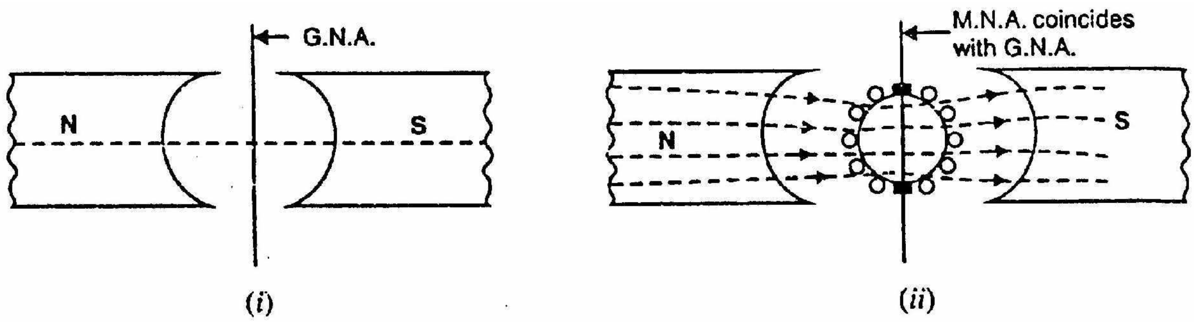


Fig. (2.1)

(ii) The magnetic neutral axis (M. N. A.) is the axis drawn perpendicular to the mean direction of the flux passing through the centre of the armature. Clearly, no e.m.f. is produced in the armature conductors along this axis because then they cut no flux. With no current in the armature conductors, the M.N.A. coincides with G, N. A. as shown in Fig. (2.2).

(ii). In order to achieve sparkless commutation, the brushes must lie along M.N.A.

2.3 Explanation of Armature Reaction

With no current in armature conductors, the M.N.A. coincides with G.N.A. However, when current flows in armature conductors, the combined action of main flux and armature flux shifts the M.N.A. from G.N.A. In case of a generator, the M.N.A. is shifted in the direction of rotation of the machine. In order to achieve sparkless commutation, the brushes have to be moved along the new M.N.A. Under such a condition, the armature reaction produces the following two effects:

1. It demagnetizes or weakens the main flux.
2. It cross-magnetizes or distorts the main flux.

Let us discuss these effects of armature reaction by considering a 2-pole generator (though the following remarks also hold good for a multipolar generator).

(i) Fig. (2.3) (i) shows the flux due to main poles (main flux) when the armature conductors carry no current. The flux across the air gap is uniform. The m.m.f. producing the main flux is represented in magnitude and direction by the vector OF_m in Fig. (2.3) (i). Note that OF_m is perpendicular to G.N.A.

(ii) Fig. (2.3) (ii) shows the flux due to current flowing in armature conductors alone (main poles unexcited). The armature conductors to the left of G.N.A. carry current “in” (×) and those to the right carry current “out” (•). The direction of magnetic lines of force can be found by cork screw rule. It is clear that armature flux is directed downward parallel to the brush axis. The m.m.f. producing the armature flux is represented in magnitude and direction by the vector OF_A in Fig. (2.3) (ii).

(iii) Fig. (2.3) (iii) shows the flux due to the main poles and that due to current in armature conductors acting together. The resultant m.m.f. OF is the vector sum of OF_m and OF_A as shown in Fig. (2.3) (iii). Since M.N.A. is always perpendicular to the resultant m.m.f., the M.N.A. is shifted through an angle θ. Note that M.N.A. is shifted in the direction of rotation of the generator.

(iv) In order to achieve sparkless commutation, the brushes must lie along the M.N.A. Consequently, the brushes are shifted through an angle θ so as to lie along the new M.N.A. as shown in Fig. (2.3) (iv). Due to brush shift, the m.m.f. F_A of the armature is also rotated through the same angle θ. It is because some of the conductors which were earlier under N-pole now come under S-pole and vice-versa. The result is that armature m.m.f. F_A will no longer be vertically downward but will be rotated in the direction of rotation through an angle θ as shown in Fig. (2.3) (iv). Now F_A can be resolved into rectangular components F_c and F_d.

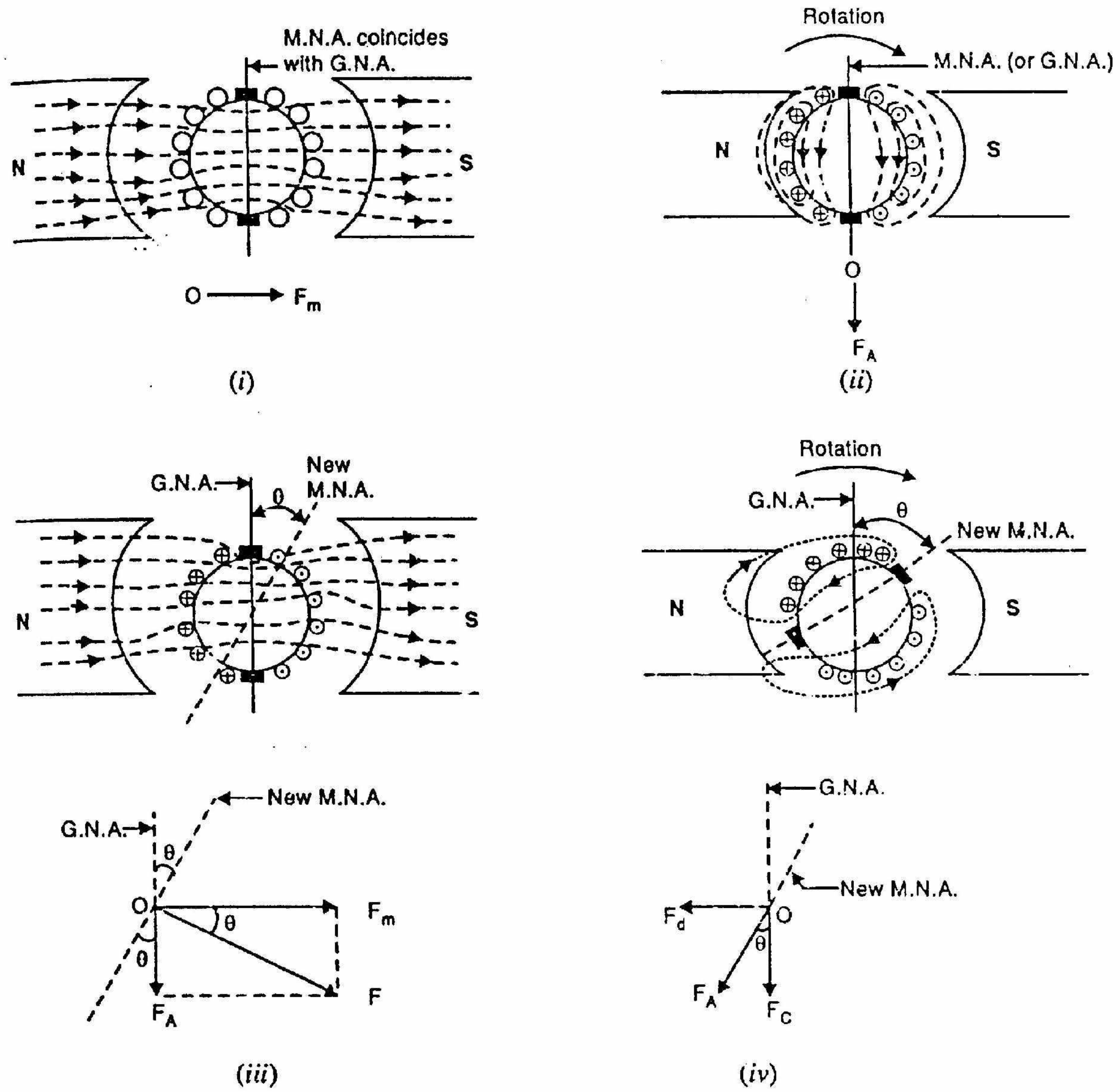


Fig. (2.3)

(a) The component F_d is in direct opposition to the m.m.f. OF_m due to main poles. It has a demagnetizing effect on the flux due to main poles. For this reason, it is called the demagnetizing or weakening component of armature reaction.

(b) The component F_c is at right angles to the m.m.f. OF_m due to main poles. It distorts the main field. For this reason, it is called the cross-magnetizing or distorting component of armature reaction.

It may be noted that with the increase of armature current, both demagnetizing and distorting effects will increase.

Conclusions

(i) With brushes located along G.N.A. (i.e., $\theta = 0°$), there is no demagnetizing component of armature reaction ($F_d = 0$). There is only distorting or cross-magnetizing effect of armature reaction.

(ii) With the brushes shifted from G.N.A., armature reaction will have both demagnetizing and distorting effects. Their relative magnitudes depend on the amount of shift. This shift is directly proportional to the armature current.

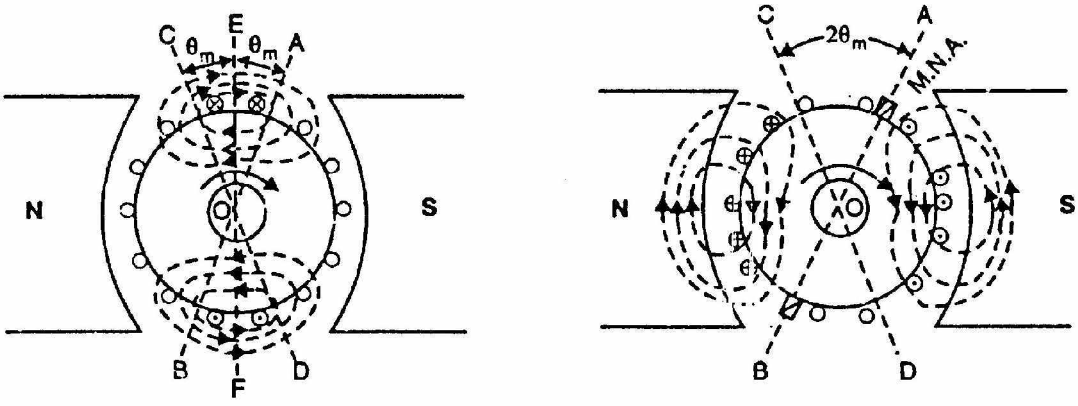
(iii) The demagnetizing component of armature reaction weakens the main flux. On the other hand, the distorting component of armature reaction distorts the main flux.

(iv) The demagnetizing effect leads to reduced generated voltage while cross-magnetizing effect leads to sparking at the brushes.

2.4 Demagnetizing and Cross-Magnetizing Conductors

With the brushes in the G.N.A. position, there is only cross-magnetizing effect of armature reaction. However, when the brushes are shifted from the G.N.A. position, the armature reaction will have both demagnetizing and cross-magnetizing effects. Consider a 2-pole generator with brushes shifted (lead) θ_m mechanical degrees from G.N.A. We shall identify the armature conductors that produce demagnetizing effect and those that produce cross-magnetizing effect.

(i) The armature conductors θ_m^c on either side of G.N.A. produce flux in direct opposition to main flux as shown in Fig. (2.4) (i). Thus the conductors lying within angles AOC = BOD = $2\theta_m$ at the top and bottom of the armature produce demagnetizing effect. These are called demagnetizing armature conductors and constitute the demagnetizing ampere-turns of armature reaction (Remember two conductors constitute a turn).



(i) (ii)

Fig.(2.4)

(ii) The axis of magnetization of the remaining armature conductors lying between angles AOD and COB is at right angles to the main flux as shown in Fig. (2.4) (ii). These conductors produce the cross-magnetizing (or distorting) effect i.e., they produce uneven flux distribution on each pole. Therefore, they are called cross-magnetizing conductors and constitute the cross-magnetizing ampere-turns of armature reaction.

2.5 Calculation of Demagnetizing Ampere-Turns Per Pole (AT_d/Pole)

It is sometimes desirable to neutralize the demagnetizing ampere-turns of armature reaction. This is achieved by adding extra ampere-turns to the main field winding. We shall now calculate the demagnetizing ampere-turns per pole (AT_d/pole).

Let Z = total number of armature conductors

I = current in each armature conductor

$= I_a/2$ ... for simplex wave winding

$= I_a/P$ ... for simplex lap winding

θ_m = forward lead in mechanical degrees

Referring to Fig. (2.4) (i) above, we have,
Total demagnetizing armature conductors

$$= \text{Conductors in angles AOC and BOD} = \frac{4\theta_m}{360} \times Z$$

Since two conductors constitute one turn,

$$\therefore \quad \text{Total demagnetizing ampere-turns} = \frac{1}{2}\left[\frac{4\theta_m}{360} \times Z\right] \times I = \frac{2\theta_m}{360} \times ZI$$

These demagnetizing ampere-turns are due to a pair of poles.

$$\therefore \quad \text{Demagnetizing ampere-turns/pole} = \frac{\theta_m}{360} \times ZI$$

i.e.,
$$AT_d\,/\,\text{pole} = \frac{\theta_m}{360} \times ZI$$

As mentioned above, the demagnetizing ampere-turns of armature reaction can be neutralized by putting extra turns on each pole of the generator.

$$\therefore \quad \text{No. of extra turns/pole} = \frac{AT_d}{I_{sh}} \qquad \text{for a shunt generator}$$

$$= \frac{AT_d}{I_a} \qquad \text{for a series generator}$$

Note. When a conductor passes a pair of poles, one cycle of voltage is generated. We say one cycle contains 360 electrical degrees. Suppose there are P poles in a generator. In one revolution, there are 360 mechanical degrees and $360 \times P/2$ electrical degrees.

$$\therefore \quad 360° \text{ mechanical} = 360 \times \frac{P}{2} \text{ electrical degrees}$$

or

$$1° \text{ Mechanical} = \frac{P}{2} \text{ electrical degrees}$$

$$\therefore \quad \theta \text{ (mechanical)} = \frac{\theta(\text{electrical})}{\text{Pair of pols}}$$

or

$$\theta_m = \frac{\theta_e}{P/2} \qquad \therefore \quad \theta_m = \frac{2\theta_e}{P}$$

2.6 Cross-Magnetizing Ampere-Turns Per Pole (AT_c/Pole)

We now calculate the cross-magnetizing ampere-turns per pole (AT_c/pole).

Total armature reaction ampere-turns per pole

$$= \frac{Z/2}{P} \times I = \frac{Z}{2P} \times I \qquad (\because \text{ two conductors make one turn})$$

Demagnetizing ampere-turns per pole is given by;

$$AT_d / \text{pole} = \frac{\theta_m}{360} \times ZI$$

(found above)

$\therefore$ Cross-magnetizing ampere-turns/pole are

$$AT_d\,/\,pole = \frac{Z}{2P} \times I - \frac{\theta_m}{360} \times ZI = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

$$\therefore \quad AT_d\,/\,pole = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

2.7 Compensating Windings

The cross-magnetizing effect of armature reaction may cause trouble in d.c. machines subjected to large fluctuations in load. In order to neutralize the cross-magnetizing effect of armature reaction, a compensating winding is used.

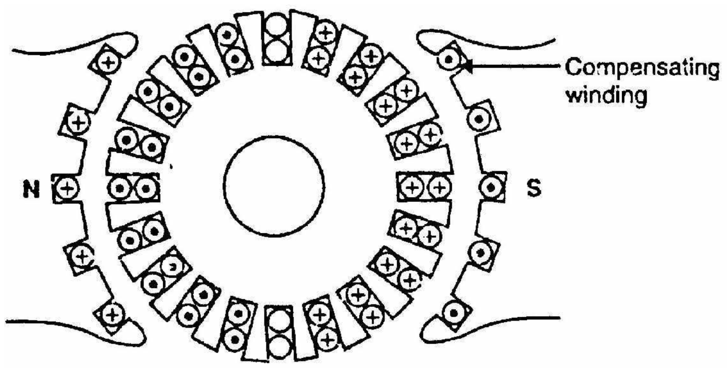


Fig. (2.5)

A compensating winding is an auxiliary winding embedded in slots in the pole faces as shown in Fig. (2.5). It is connected in series with armature in a manner so that the direction of current through the compensating conductors in any one pole face will be opposite to the direction of the current through the adjacent armature conductors [See Fig. 2.5]. Let us now calculate the number of compensating conductors/ pole face. In calculating the conductors per pole face required for the compensating winding, it should be remembered that the current in the compensating conductors is the armature current I_a whereas the current in armature conductors is I_a/A where A is the number of parallel paths.

Let Z_c = No. of compensating conductors/pole face

Z_a = No. of active armature conductors

I_a = Total armature current

I_a/A = Current in each armature conductor

$$\therefore \quad Z_c I_a = Z_a \times \frac{I_a}{A}$$

or $$Z_c = \frac{Z_a}{A}$$

The use of a compensating winding considerably increases the cost of a machine and is justified only for machines intended for severe service e.g., for high speed and high voltage machines.

2.8 AT/Pole for Compensating Winding

Only the cross-magnetizing ampere-turns produced by conductors under the pole face are effective in producing the distortion in the pole cores. If Z is the total number of armature conductors and P is the number of poles, then,

$$\text{No. of armature conductors/pole} = \frac{Z}{P}$$

$$\text{No. of armature turns/pole} = \frac{Z}{2P}$$

$$\text{No. of armature turns under pole face} = \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

If I is the current through each armature conductor, then,

$$\text{AT/pole required for compensating winding} = \frac{ZI}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

$$= \text{Armature AT/pole} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

2.9 Commutation

Fig. (2.6) shows the schematic diagram of 2-pole lap-wound generator. There are two parallel paths between the brushes. Therefore, each coil of the winding carries one half ($I_a/2$ in this case) of the total current (I_a) entering or leaving the armature.

Note that the currents in the coils connected to a brush are either all towards the brush (positive brush) or all directed away from the brush (negative brush). Therefore, current in a coil will reverse as the coil passes a brush. This reversal of current as the coil passes & brush is called commutation.

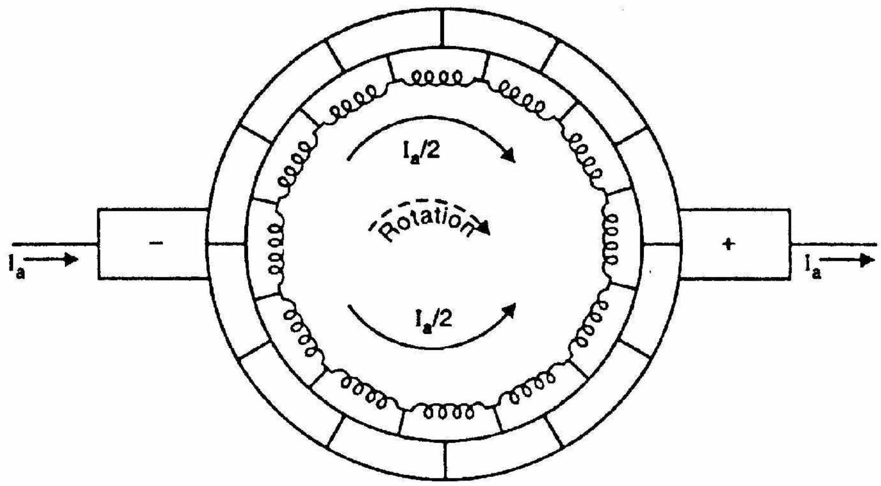


Fig. (2.6)

The reversal of current in a coil as the coil passes the brush axis is called commutation.

When commutation takes place, the coil undergoing commutation is short-circuited by the brush. The brief period during which the coil remains short-circuited is known as commutation period T_c. If the current reversal is completed by the end of commutation period, it is called ideal commutation. If the current reversal is not completed by that time, then sparking occurs between the brush and the commutator which results in progressive damage to both.

Ideal commutation

Let us discuss the phenomenon of ideal commutation (i.e., coil has no inductance) in one coil in the armature winding shown in Fig. (2.6) above. For this purpose, we consider the coil A. The brush width is equal to the width of one commutator segment and one mica insulation. Suppose the total armature current is 40 A. Since there are two parallel paths, each coil carries a current of 20 A.

(i) In Fig. (2.7) (i), the brush is in contact with segment 1 of the commutator. The commutator segment 1 conducts a current of 40 A to the brush; 20 A from coil A and 20 A from the adjacent coil as shown. The coil A has yet to undergo commutation.

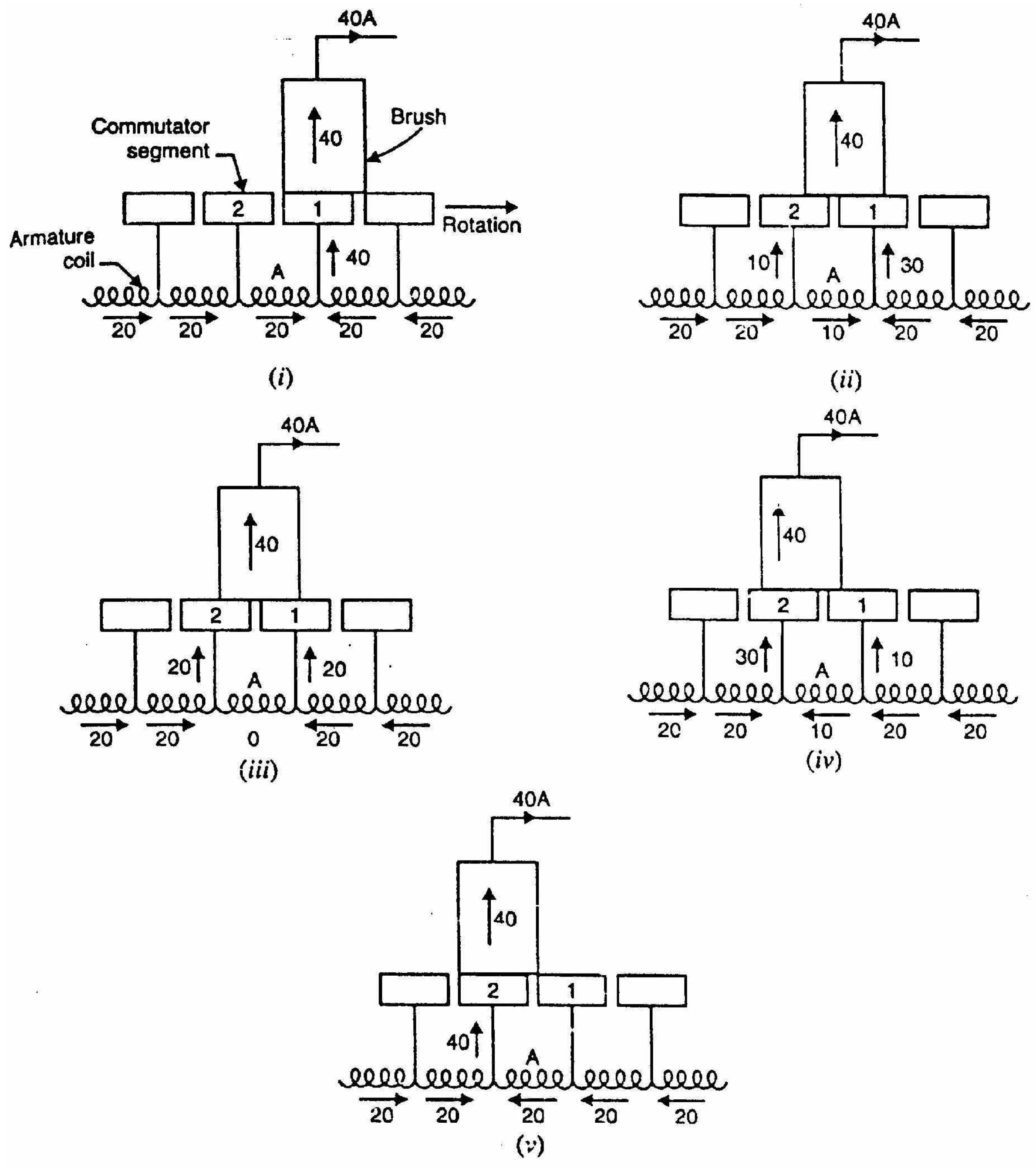


Fig. (2.7)

(ii) As the armature rotates, the brush will make contact with segment 2 and thus short-circuits the coil A as shown in Fig. (2.7) (ii). There are now two parallel paths into the brush as long as the short-circuit of coil A exists. Fig. (2.7) (ii) shows the instant when the brush is one-fourth on segment 2 and three-fourth on segment 1. For this condition, the resistance of the path through segment 2 is three times the resistance of the path through segment 1 (Q contact resistance varies inversely as the area of contact of brush with the segment). The brush again conducts a current of 40 A; 30 A through segment 1 and 10 A through segment 2. Note that current in coil A (the coil undergoing commutation) is reduced from 20 A to 10 A.

(iii) Fig. (2.7) (iii) shows the instant when the brush is one-half on segment 2 and one-half on segment 1. The brush again conducts 40 A; 20 A through segment 1 and 20 A through segment 2 (Q now the resistances of the two parallel paths are equal). Note that now. current in coil A is zero.

(iv) Fig. (2.7) (iv) shows the instant when the brush is three-fourth on segment 2 and one-fourth on segment 1. The brush conducts a current of 40 A; 30 A through segment 2 and 10 A through segment 1. Note that current in coil A is 10 A but in the reverse direction to that before the start of commutation. The reader may see the action of the commutator in reversing the current in a coil as the coil passes the brush axis.

(v) Fig. (2.7) (v) shows the instant when the brush is in contact only with segment 2. The brush again conducts 40 A; 20 A from coil A and 20 A from the adjacent coil to coil A. Note that now current in coil A is 20 A but in the reverse direction. Thus the coil A has undergone commutation. Each coil undergoes commutation in this way as it passes the brush axis. Note that during commutation, the coil under consideration remains short-circuited by the brush.

Fig. (2.8) shows the current-time graph for the coil A undergoing commutation. The horizontal line AB represents a constant current of 20 A upto the beginning of commutation. From the finish of commutation, it is represented by another horizontal line CD on the opposite side of the zero line and the same distance from it as AB i.e., the current has exactly reversed (– 20 A). The way in which current changes from B to C depends upon the conditions under which the coil undergoes commutation. If the current changes at a uniform rate (i.e., BC is a straight line), then it is called ideal commutation as shown in Fig. (2.8). Under such conditions, no sparking will take place between the brush and the commutator.

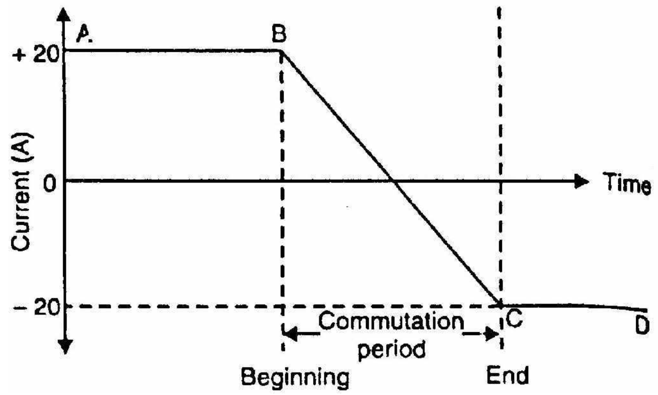


Fig. (2.8)

Practical difficulties

The ideal commutation (i.e., straight line change of current) cannot be attained in practice. This is mainly due to the fact that the armature coils have appreciable inductance. When the current in the coil undergoing commutation changes, self-induced e.m.f. is produced in the coil. This is generally called reactance voltage. This reactance voltage opposes the change of current in the coil undergoing commutation. The result is that the change of current in the coil undergoing commutation occurs more slowly than it would be under ideal commutation. This is illustrated in Fig. (2.9). The straight line RC represents the ideal commutation whereas the curve BE represents the change in current when self-inductance of the coil is taken into account. Note that current CE (= 8A in Fig. 2.9) is flowing from the commutator segment 1 to the brush at the instant when they part company. This results in sparking just as when any other current-carrying circuit is broken. The sparking results in overheating of commutator-brush contact and causing damage to both.

Fig. (2.10) illustrates how sparking takes place between the commutator segment and the brush. At the end of commutation or short-circuit period, the current in coil A is reversed to a value of 12 A (instead of 20 A) due to inductance of the coil. When the brush breaks contact with segment 1, the remaining 8 A current jumps from segment 1 to the brush through air causing sparking between segment 1 and the brush.

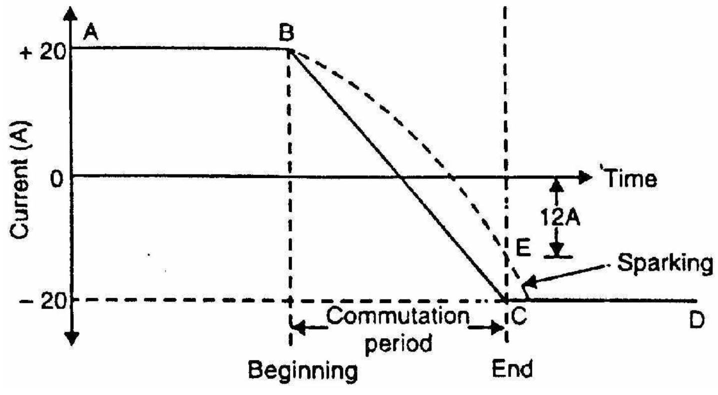


Fig. (2.9)

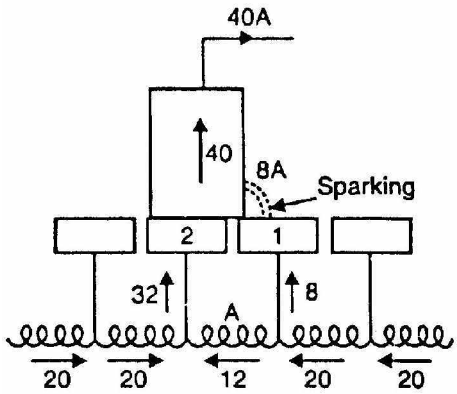


Fig. (2.10)

2.10 Calculation of Reactance Voltage

Reactance voltage = Coefficient of self-inductance × Rate of change of current

When a coil undergoes commutation, two commutator segments remain short-circuited by the brush. Therefore, the time of short circuit (or commutation period T_c) is equal to the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating strip of mica.

Let W_b = brush width in cm; W_m = mica thickness in cm

v = peripheral speed of commutator in cm/s

$$\therefore \quad \text{Commutation period, } T_c = \frac{W_b - W_m}{v} \quad \text{seconds}$$

The commutation period is very small, say of the order of 1/500 second.

Let the current in the coil undergoing commutation change from + I to − I (amperes) during the commutation. If L is the inductance of the coil, then reactance voltage is given by;

Reactance voltage,

$$E_R = L \times \frac{2I}{T_c}$$

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2.11 Methods of Improving Commutation

Improving commutation means to make current reversal in the short-circuited coil as sparkless as possible. The following are the two principal methods of improving commutation:

(i) Resistance commutation
(ii) E.M.F. commutation

We shall discuss each method in turn.

2.12 Resistance Commutation

The reversal of current in a coil (i.e., commutation) takes place while the coil is short-circuited by the brush. Therefore, there are two parallel paths for the current as long as the short circuit exists. If the contact resistance between the brush and the commutator is made large, then current would divide in the inverse ratio of contact resistances (as for any two resistances in parallel). This is the key point in improving commutation. This is achieved by using carbon brushes (instead of Cu brushes) which have high contact resistance. This method of improving commutation is called resistance commutation.

Figs. (2.11) and (2.12) illustrates how high contact resistance of carbon brush improves commutation (i.e., reversal of current) in coil A. In Fig. (2.11) (i), the brush is entirely on segment 1 and, therefore, the current in coil A is 20 A. The coil A is yet to undergo commutation. As the armature rotates, the brush short-circuits the coil A and there are two parallel paths for the current into the brush. Fig. (2.11) (ii) shows the instant when the brush is one-fourth on segment 2 and three-fourth on segment 1. The equivalent electric circuit is shown in Fig. (2.11) (iii) where R_1 and R_2 represent the brush contact resistances on segments 1 and 2. A resistor is not shown for coil A since it is assumed that the coil resistance is

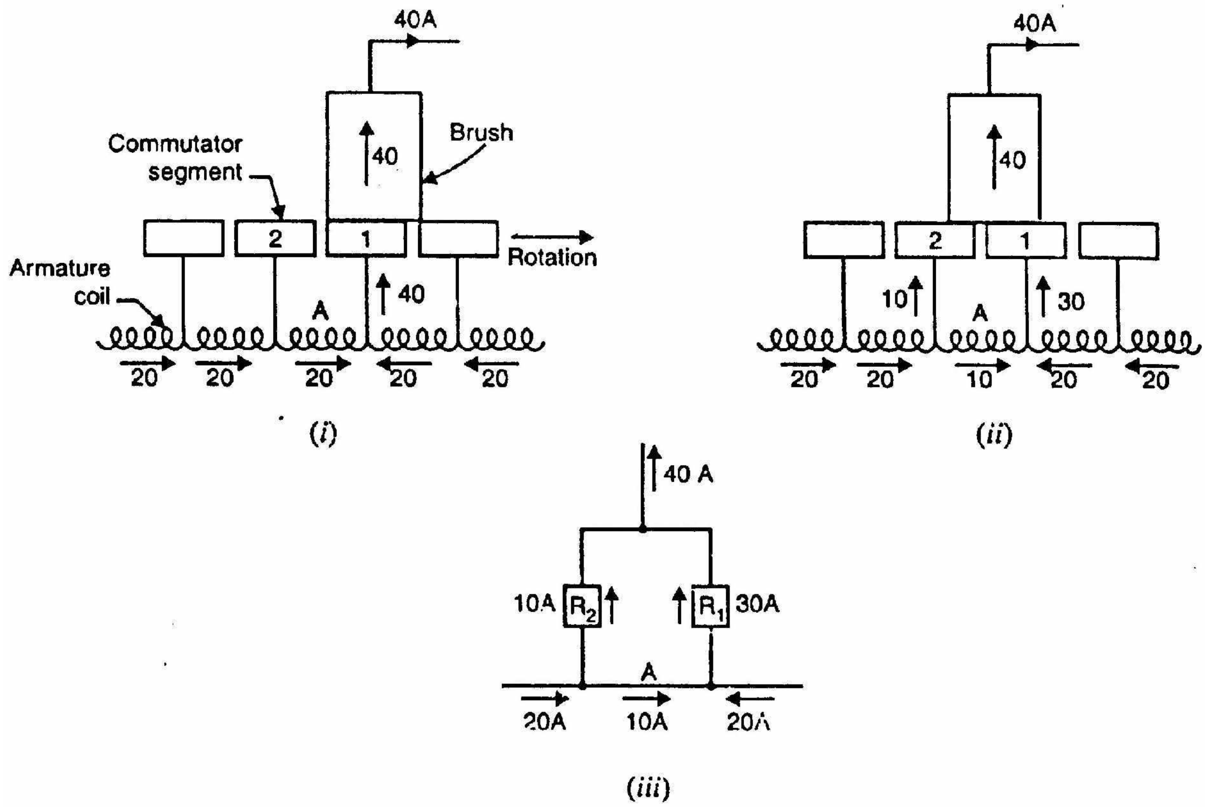

Fig. (2.11)

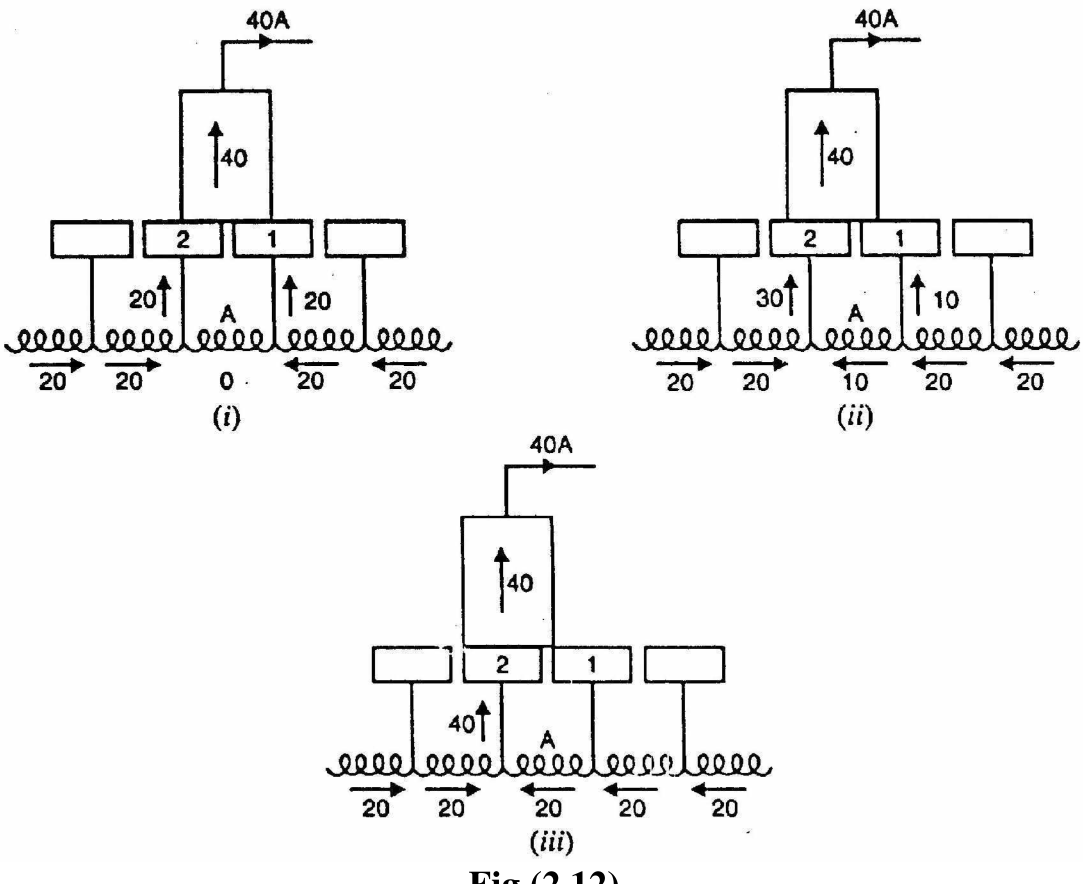

Fig.(2.12)

negligible as compared to the brush contact resistance. The values of current in the parallel paths of the equivalent circuit are determined by the respective resistances of the paths. For the condition shown in Fig. (2.11) (ii), resistor R_2 has three times the resistance of resistor R_1. Therefore, the current distribution in the paths will be as shown. Note that current in coil A is reduced from 20 A to 10 A due to division of current in (he inverse ratio of contact resistances. If the Cu brush is used (which has low contact resistance), R_1 R_2 and the current in coil A would not have reduced to 10 A.

As the carbon brush passes over the commutator, the contact area with segment 2 increases and that with segment 1 decreases i.e., R_2 decreases and R_1 increases. Therefore, more and more current passes to the brush through segment 2. This is illustrated in Figs. (2.12) (i) and (2.12) (ii), When the break between the brush and the segment 1 finally occurs [See Fig. 2.12 (iii)], the current in the coil is reversed and commutation is achieved.

It may be noted that the main cause of sparking during commutation is the production of reactance voltage and carbon brushes cannot prevent it. Nevertheless, the carbon brushes do help in improving commutation. The other minor advantages of carbon brushes are:

(i) The carbon lubricates and polishes the commutator.
(ii) If sparking occurs, it damages the commutator less than with copper brushes and the damage to the brush itself is of little importance.

2.13 E.M.F. Commutation

In this method, an arrangement is made to neutralize the reactance voltage by producing a reversing voltage in the coil undergoing commutation. The reversing voltage acts in opposition to the reactance voltage and neutralizes it to some extent. If the reversing voltage is equal to the reactance voltage, the effect of the latter is completely wiped out and we get sparkless commutation. The reversing voltage may be produced in the following two ways:

(i) By brush shifting
(ii) By using interpoles or compoles

(i) By brush shifting

In this method, the brushes are given sufficient forward lead (for a generator) to bring the short-circuited coil (i.e., coil undergoing commutation) under the influence of the next pole of opposite polarity. Since the short-circuited coil is now in the reversing field, the reversing voltage produced cancels the reactance voltage. This method suffers from the following drawbacks:

(a) The reactance voltage depends upon armature current. Therefore, the brush shift will depend on the magnitude of armature current which keeps on changing. This necessitates frequent shifting of brushes.
(b) The greater the armature current, the greater must be the forward lead for a generator. This increases the demagnetizing effect of armature reaction and further weakens the main field.

(ii) By using interpoles or compotes

The best method of neutralizing reactance voltage is by, using interpoles or compoles. This method is discussed in Sec. (2.14).

2.14 Interpoles or Compoles

The best way to produce reversing voltage to neutralize the reactance voltage is by using interpoles or compoles. These are small poles fixed to the yoke and spaced mid-way between the main poles (See Fig. 2.13). They are wound with comparatively few turns and connected in series with the armature so that they carry armature current. Their polarity is the same as the next main pole ahead in the direction of rotation for a generator (See Fig. 2.13). Connections for a d.c. generator with interpoles is shown in Fig. (2.14).

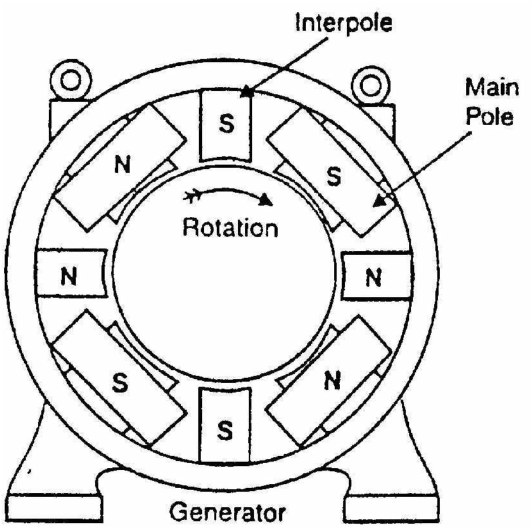

Fig. (2.13)

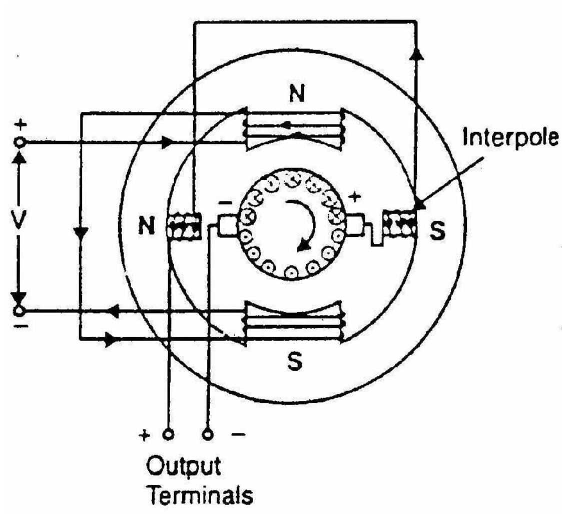

Fig. (2.14)

Functions of Interpoles

The machines fitted with interpoles have their brushes set on geometrical neutral axis (no lead). The interpoles perform the following two functions:
(i) As their polarity is the same as the main pole ahead (for a generator), they induce an e.m.f. in the coil (undergoing commutation) which opposes

reactance voltage. This leads to sparkless commutation. The e.m.f. induced by compoles is known as commutating or reversing e.m.f. Since the interpoles carry the armature current and the reactance voltage is also proportional to armature current, the neutralization of reactance voltage is automatic.

(ii) The m.m.f. of the compoles neutralizes the cross-magnetizing effect of armature reaction in small region in the space between the main poles. It is because the two m.m.f.s oppose each other in this region.

Fig. (2.15) shows the circuit diagram of a shunt generator with commutating winding and compensating winding. Both these windings are connected in series with the armature and so they carry the armature current. However, the functions they perform must be understood clearly. The main function of commutating winding is to produce reversing (or commutating) e.m.f. in order to cancel the reactance voltage. In addition to this, the m.m.f. of the commutating winding neutralizes the cross-magnetizing ampere-turns in the space between the main poles. The compensating winding neutralizes the cross-magnetizing effect of armature reaction under the pole faces.

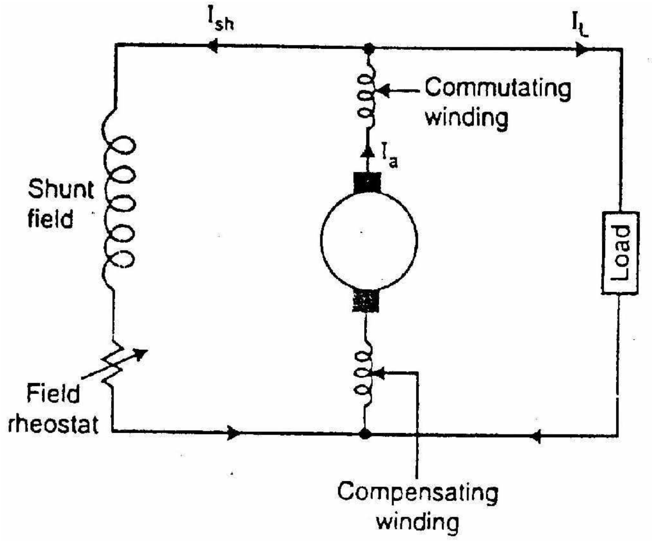

Fig. (2.15)

2.15 Equalizing Connections

We know that the armature circuit in lap winding of a multipolar machine has as many parallel paths as the number of poles. Because of wear in the bearings, and for other reasons, the air gaps in a generator become unequal and, therefore, the flux in some poles becomes greater than in others. This causes the voltages of the different paths to be unequal. With unequal voltages in these parallel paths, circulating current will flow even if no current is supplied to an external load. If these currents are large, some of the brushes will be required to carry a greater current at full load than they were designed to carry and this will cause sparking. To relieve the brushes of these circulating currents, points on the armature that are at the same potential are connected together by means of copper bars called equalizer rings. This is achieved by connecting to the same equalizer ring the coils that occupy the same positions relative to the poles (See Fig. 2.16). Thus referring to Fig. (2.16), the coil consisting of conductor 1 and conductor 8 occupies the same position relative to the poles as the coil consisting of conductors 13 and 20. Therefore, the two coils are connected to the same equalizer ring. The equalizers provide a low resistance path for the circulating current. As a result, the circulating current due to the slight differences in the voltages of the various parallel paths passes through the equalizer rings instead of passing through the brushes. This reduces sparking.

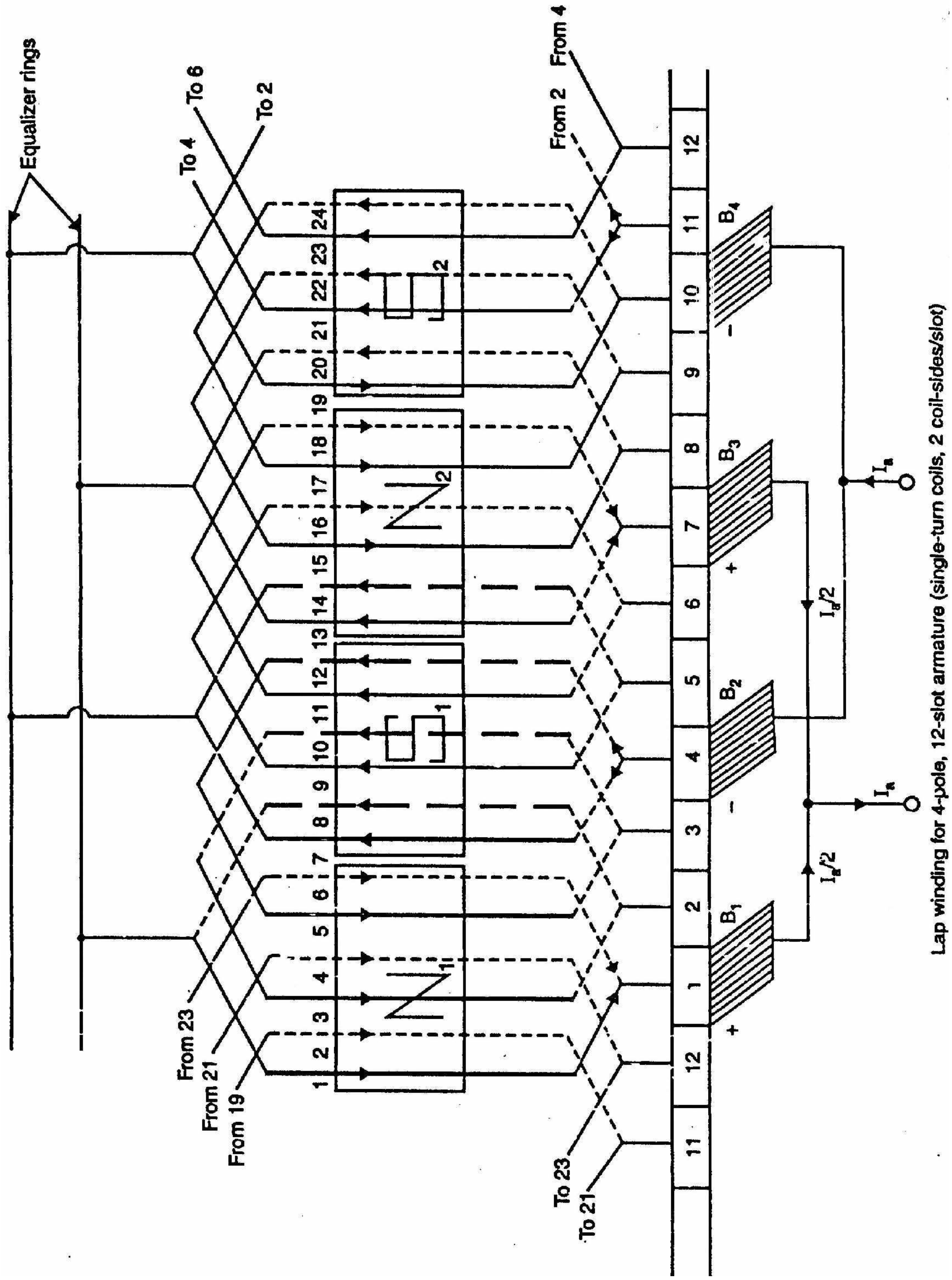

Fig. (2.16)

Equalizer rings should be used only on windings in which the number of coils is a multiple of the number of poles. For best results, each coil should be connected to an equalizer ring but this is seldom done. Satisfactory results are obtained by connecting about every third coil to an equalizer ring. In order to distribute the connections to the equalizer rings equally, the number of coils per pole must be divisible by the connection pitch.

Note. Equalizer rings are not used in wave winding because there is no imbalance in the voltages of the two parallel paths. This is due to the fact that conductors in each of the two paths pass under all N and S poles successively (unlike a lap winding where all conductors in any parallel path lie under one pair of poles). Therefore, even if there are inequalities in pole flux, they will affect each path equally.

Chapter (3)

D.C. Generator Characteristics

Introduction

The speed of a d.c. machine operated as a generator is fixed by the prime mover. For general-purpose operation, the prime mover is equipped with a speed governor so that the speed of the generator is practically constant. Under such condition, the generator performance deals primarily with the relation between excitation, terminal voltage and load. These relations can be best exhibited graphically by means of curves known as generator characteristics. These characteristics show at a glance the behaviour of the generator under different load conditions.

3.1 D.C. Generator Characteristics

The following are the three most important characteristics of a d.c. generator:

1. Open Circuit Characteristic (O.C.C.)

This curve shows the relation between the generated e.m.f. at no-load (E_0) and the field current (I_f) at constant speed. It is also known as magnetic characteristic or no-load saturation curve. Its shape is practically the same for all generators whether separately or self-excited. The data for O.C.C. curve are obtained experimentally by operating the generator at no load and constant speed and recording the change in terminal voltage as the field current is varied.

2. Internal or Total characteristic (E/I_a)

This curve shows the relation between the generated e.m.f. on load (E) and the armature current (I_a). The e.m.f. E is less than E_0 due to the demagnetizing effect of armature reaction. Therefore, this curve will lie below the open circuit characteristic (O.C.C.). The internal characteristic is of interest chiefly to the designer. It cannot be obtained directly by experiment. It is because a voltmeter cannot read the e.m.f. generated on load due to the voltage drop in armature resistance. The internal characteristic can be obtained from external characteristic if winding resistances are known because armature reaction effect is included in both characteristics.

3. External characteristic (V/I_L)

This curve shows the relation between the terminal voltage (V) and load current (I_L). The terminal voltage V will be less than E due to voltage drop in the armature circuit. Therefore, this curve will lie below the internal characteristic. This characteristic is very important in determining the suitability of a generator for a given purpose. It can be obtained by making simultaneous measurements of terminal voltage and load current (with voltmeter and ammeter) of a loaded generator.

3.2 Open Circuit Characteristic of a D.C. Generator

The O.C.C. for a d.c. generator is determined as follows. The field winding of the d.c. generator (series or shunt) is disconnected from the machine and is separately excited from an external d.c. source as shown in Fig. (3.1) (ii). The generator is run at fixed speed (i.e., normal speed). The field current (I_f) is increased from zero in steps and the corresponding values of generated e.m.f. (E_0) read off on a voltmeter connected across the armature terminals. On plotting the relation between E_0 and I_f, we get the open circuit characteristic as shown in Fig. (3.1) (i).

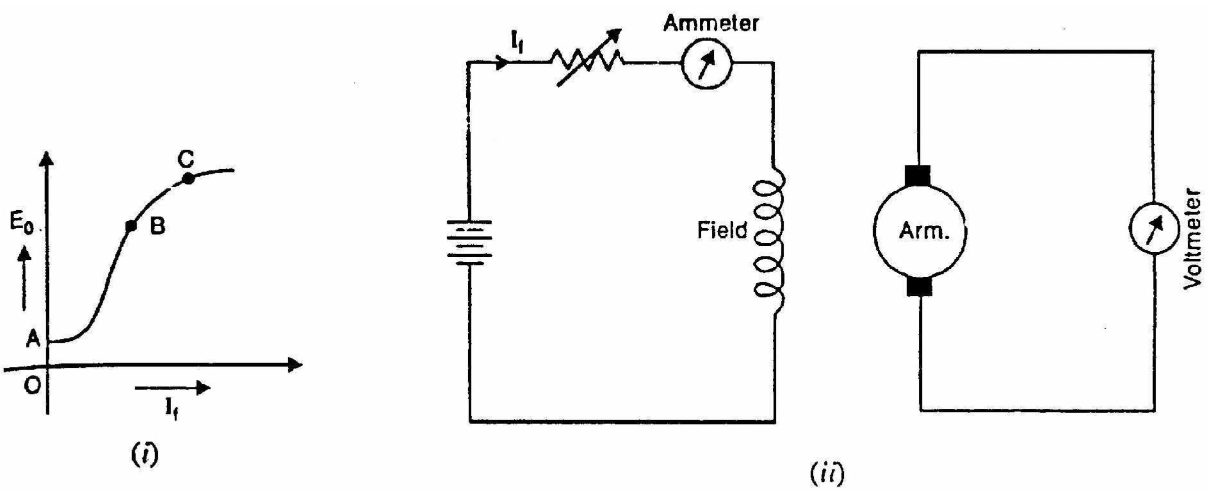

Fig. (3.1)

The following points may be noted from O.C.C.:

(i) When the field current is zero, there is some generated e.m.f. OA. This is due to the residual magnetism in the field poles.

(ii) Over a fairly wide range of field current (upto point B in the curve), the curve is linear. It is because in this range, reluctance of iron is negligible as compared with that of air gap. The air gap reluctance is constant and hence linear relationship.

(iii) After point B on the curve, the reluctance of iron also comes into picture. It is because at higher flux densities, μ_r for iron decreases and reluctance of iron is no longer negligible. Consequently, the curve deviates from linear relationship.

(iv) After point C on the curve, the magnetic saturation of poles begins and E_0 tends to level off.

The reader may note that the O.C.C. of even self-excited generator is obtained by running it as a separately excited generator.

3.3 Characteristics of a Separately Excited D.C. Generator

The obvious disadvantage of a separately excited d.c. generator is that we require an external d.c. source for excitation. But since the output voltage may be controlled more easily and over a wide range (from zero to a maximum), this type of excitation finds many applications.

(i) Open circuit characteristic.

The O.C.C. of a separately excited generator is determined in a manner described in Sec. (3.2). Fig. (3.2) shows the variation of generated e.m f. on no load with field current for various fixed speeds. Note that if the value of constant speed is increased, the steepness of the curve also increases. When the field current is zero, the residual magnetism in the poles will give rise to the small initial e.m.f. as shown.

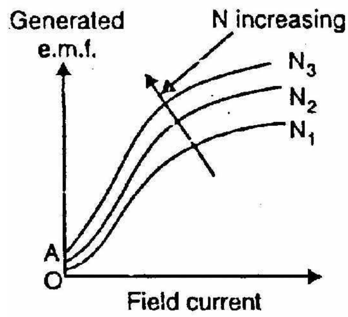

Fig. (3.2)

(ii) Internal and External Characteristics

The external characteristic of a separately excited generator is the curve between the terminal voltage (V) and the load current I_L (which is the same as armature current in this case). In order to determine the external characteristic, the circuit set up is as shown in Fig. (3.3) (i). As the load current increases, the terminal voltage falls due to two reasons:

(a) The armature reaction weakens the main flux so that actual e.m.f. generated E on load is less than that generated (E_0) on no load.

(b) There is voltage drop across armature resistance ($= I_LR_a = I_aR_a$).

Due to these reasons, the external characteristic is a drooping curve [curve 3 in Fig. 3.3 (ii)]. Note that in the absence of armature reaction and armature drop, the generated e.m.f. would have been E_0 (curve 1).

The internal characteristic can be determined from external characteristic by adding I_LR_a drop to the external characteristic. It is because armature reaction drop is included in the external characteristic. Curve 2 is the internal characteristic of the generator and should obviously lie above the external characteristic.

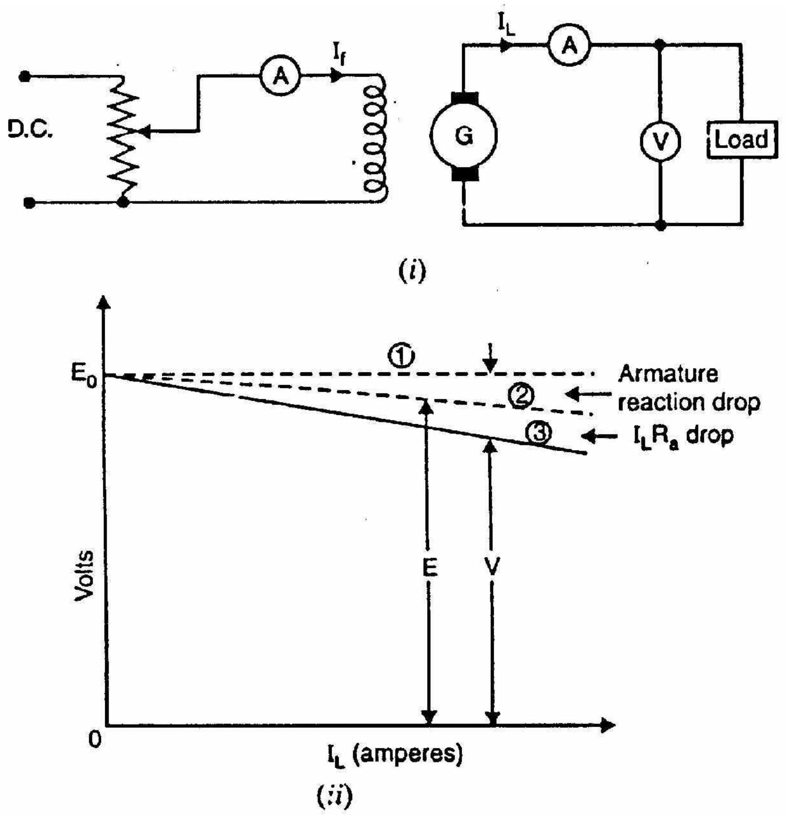

Fig. (3.3)

3.4 Voltage Build-Up in a Self-Excited Generator

Let us see how voltage builds up in a self-excited generator.

(i) Shunt generator

Consider a shunt generator. If the generator is run at a constant speed, some e.m.f. will be generated due to residual magnetism in the main poles. This small e.m.f. circulates a field current which in turn produces additional flux to reinforce the original residual flux (provided field winding connections are correct). This process continues and the generator builds up the normal generated voltage following the O.C.C. shown in Fig. (3.4) (i).

The field resistance R_f can be represented by a straight line passing through the origin as shown in Fig. (3.4) (ii). The two curves can be shown on the same diagram as they have the same ordinate [See Fig. 3.4 (iii)].

Since the field circuit is inductive, there is a delay in the increase in current upon closing the field circuit switch The rate at which the current increases depends upon the voltage available for increasing it. Suppose at any instant, the field current is i (= OA) and is increasing at the rate di/dt. Then,

$$E_0 = i\,R_f + L\frac{di}{dt}$$

where R_f = total field circuit resistance
L = inductance of field circuit

At the considered instant, the total e.m.f. available is AC [See Fig. 3.4 (iii)]. An amount AB of the c.m.f. AC is absorbed by the voltage drop iR_f and the remainder part BC is available to overcome L di/dt. Since this surplus voltage is available, it is possible for the field current to increase above the value OA. However, at point D, the available voltage is OM and is all absorbed by i R_f drop. Consequently, the field current cannot increase further and the generator build up stops.

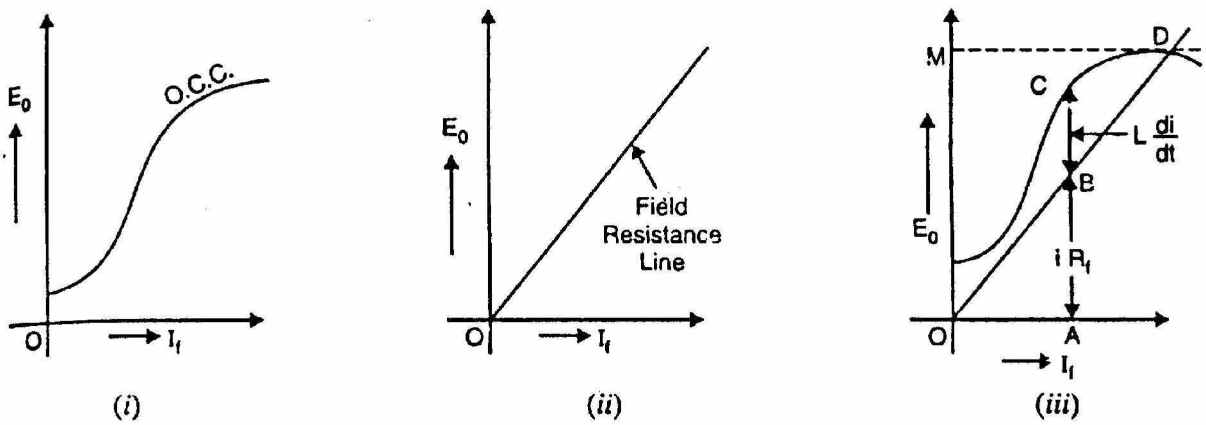

Fig. (3.4)

We arrive at a very important conclusion that the voltage build up of the generator is given by the point of intersection of O.C.C. and field resistance line. Thus in Fig. (3.4) (iii), D is point of intersection of the two curves. Hence the generator will build up a voltage OM.

(ii) Series generator

During initial operation, with no current yet flowing, a residual voltage will be generated exactly as in the case of a shunt generator. The residual voltage will cause a current to flow through the whole series circuit when the circuit is closed. There will then be voltage build up to an equilibrium point exactly analogous to the build up of a shunt generator. The voltage build up graph will be similar to that of shunt generator except that now load current (instead of field current for shunt generator) will be taken along x-axis.

(iii) Compound generator

When a compound generator has its series field flux aiding its shunt field flux, the machine is said to be cumulative compound. When the series field is connected in reverse so that its field flux opposes the shunt field flux, the generator is then differential compound.

The easiest way to build up voltage in a compound generator is to start under no load conditions. At no load, only the shunt field is effective. When no-load voltage build up is achieved, the generator is loaded. If under load, the voltage rises, the series field connection is cumulative. If the voltage drops significantly, the connection is differential compound.

3.5 Critical Field Resistance for a Shunt Generator

We have seen above that voltage build up in a shunt generator depends upon field circuit resistance. If the field circuit resistance is R_1 (line OA), then generator will build up a voltage OM as shown in Fig. (3.5). If the field circuit resistance is increased to R_2 (tine OB), the generator will build up a voltage OL, slightly less than OM. As the field circuit resistance is increased, the slope of resistance line also increases. When the field resistance line becomes tangent (line OC) to O.C.C., the generator would just excite. If the field circuit resistance is increased beyond this point (say line OD), the generator will fail to excite. The field circuit resistance represented by line OC (tangent to O.C.C.) is called critical field resistance R_C for the shunt generator. It may be defined as under:

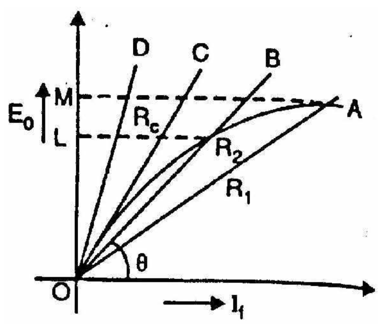

Fig. (3.5)

The maximum field circuit resistance (for a given speed) with which the shunt generator would just excite is known as its critical field resistance.

It should be noted that shunt generator will build up voltage only if field circuit resistance is less than critical field resistance.

3.6 Critical Resistance for a Series Generator

Fig. (3.6) shows the voltage build up in a series generator. Here R_1, R_2 etc. represent the total circuit resistance (load resistance and field winding resistance). If the total circuit resistance is R_1, then series generator will build up a voltage OL. The

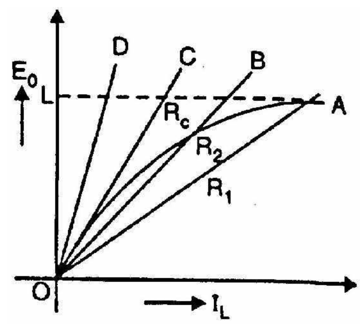

Fig. (3.6)

line OC is tangent to O.C.C. and represents the critical resistance R_C for a series generator. If the total resistance of the circuit is more than R_C (say line OD), the generator will fail to build up voltage. Note that Fig. (3.6) is similar to Fig. (3.5) with the following differences:

(i) In Fig. (3.5), R_1, R_2 etc. represent the total field circuit resistance. However, R_1, R_2 etc. in Fig. (3.6) represent the total circuit resistance (load resistance and series field winding resistance etc.).

(ii) In Fig (3.5), field current alone is represented along X-axis. However, in Fig. (3.6) load current I_L is represented along Y-axis. Note that in a series generator, field current = load current I_L.

3.7 Characteristics of Series Generator

Fig. (3.7) (i) shows the connections of a series wound generator. Since there is only one current (that which flows through the whole machine), the load current is the same as the exciting current.

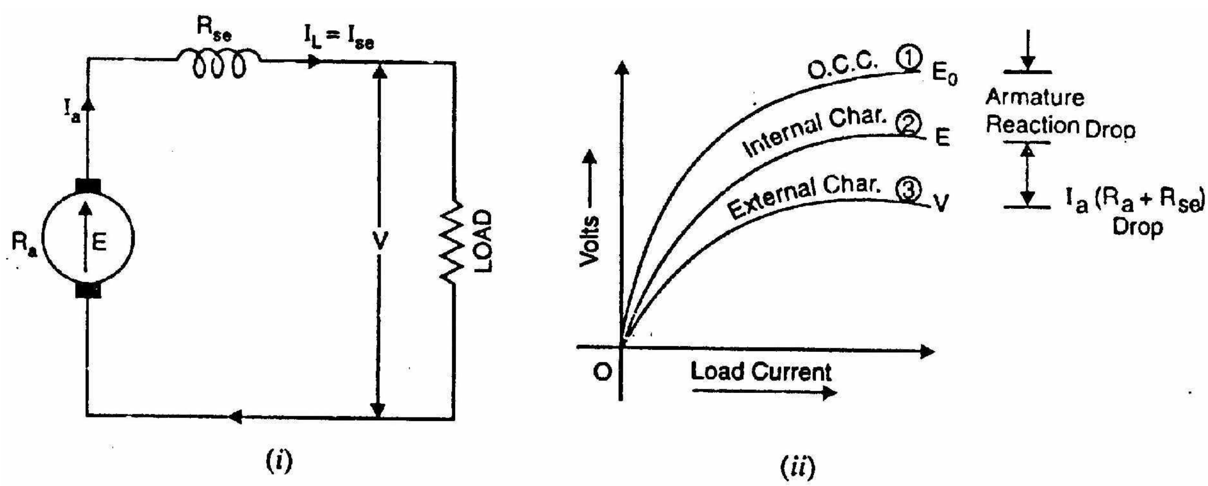

Fig. (3.7)

(i) O.C.C.

Curve 1 shows the open circuit characteristic (O.C.C.) of a series generator. It can be obtained experimentally by disconnecting the field winding from the machine and exciting it from a separate d.c. source as discussed in Sec. (3.2).

(ii) Internal characteristic

Curve 2 shows the total or internal characteristic of a series generator. It gives the relation between the generated e.m.f. E. on load and armature current. Due to armature reaction, the flux in the machine will be less than the flux at no load. Hence, e.m.f. E generated under load conditions will be less than the e.m.f. E_0 generated under no load conditions. Consequently, internal characteristic curve lies below the O.C.C. curve; the difference between them representing the effect of armature reaction [See Fig. 3.7 (ii)].

(iii) External characteristic

Curve 3 shows the external characteristic of a series generator. It gives the relation between terminal voltage and load current I_L.

$$V = E - I_a(R_a + R_{se})$$

Therefore, external characteristic curve will lie below internal characteristic curve by an amount equal to ohmic drop [i.e., $I_a(R_a + R_{se})$] in the machine as shown in Fig. (3.7) (ii).

The internal and external characteristics of a d.c. series generator can be plotted from one another as shown in Fig. (3.8). Suppose we are given the internal characteristic of the generator. Let the line OC represent the resistance of the whole machine i.e. $R_a + R_{se}$. If the load current is OB, drop in the machine is AB i.e.

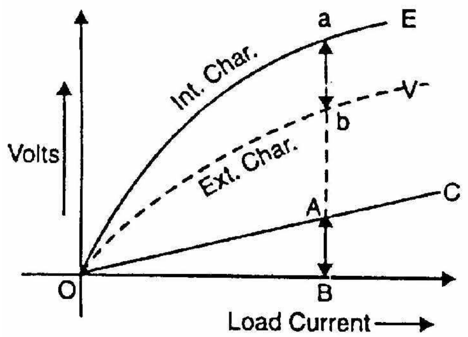

Fig. (3.8)

$$AB = \text{Ohmic drop in the machine} = OB(R_a + R_{se})$$

Now raise a perpendicular from point B and mark a point b on this line such that ab = AB. Then point b will lie on the external characteristic of the generator. Following similar procedure, other points of external characteristic can be located. It is easy to see that we can also plot internal characteristic from the external characteristic.

3.8 Characteristics of a Shunt Generator

Fig (3.9) (i) shows the connections of a shunt wound generator. The armature current I_a splits up into two parts; a small fraction I_{sh} flowing through shunt field winding while the major part I_L goes to the external load.

(i) O.C.C.

The O.C.C. of a shunt generator is similar in shape to that of a series generator as shown in Fig. (3.9) (ii). The line OA represents the shunt field circuit resistance. When the generator is run at normal speed, it will build up a voltage OM. At no-load, the terminal voltage of the generator will be constant (= OM) represented by the horizontal dotted line MC.

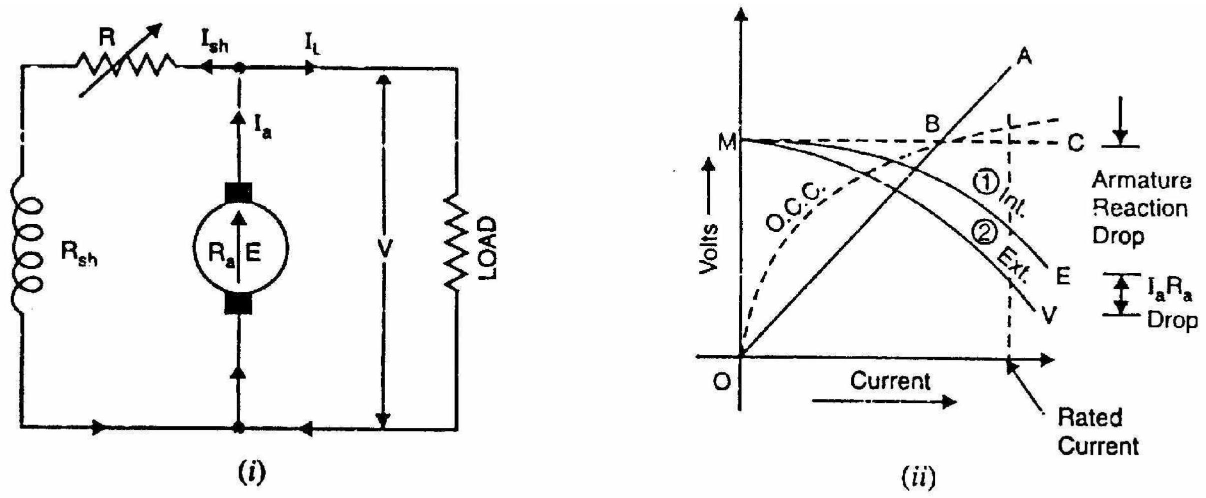

Fig. (3.9)

(ii) Internal characteristic

When the generator is loaded, flux per pole is reduced due to armature reaction. Therefore, e.m.f. E generated on load is less than the e.m.f. generated at no load. As a result, the internal characteristic (E/I_a) drops down slightly as shown in Fig. (3.9) (ii).

(iii) External characteristic

Curve 2 shows the external characteristic of a shunt generator. It gives the relation between terminal voltage V and load current I_L.

$$V = E - I_a R_a = E - (I_L + I_{sh}) R_a$$

Therefore, external characteristic curve will lie below the internal characteristic curve by an amount equal to drop in the armature circuit [i.e., $(I_L + I_{sh})R_a$] as shown in Fig. (3.9) (ii).

Note. It may be seen from the external characteristic that change in terminal voltage from no-load to full load is small. The terminal voltage can always be maintained constant by adjusting the field rheostat R automatically

3.9 Critical External Resistance for Shunt Generator

If the load resistance across the terminals of a shunt generator is decreased, then load current increase? However, there is a limit to the increase in load current with the decrease of load resistance. Any decrease of load resistance beyond this point, instead of increasing the current, ultimately results in

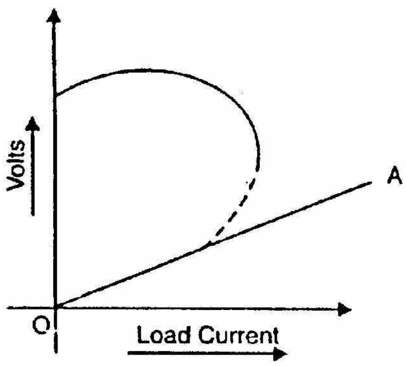

Fig. (3.10)

reduced current. Consequently, the external characteristic turns back (dotted curve) as shown in Fig. (3.10). The tangent OA to the curve represents the minimum external resistance required to excite the shunt generator on load and is called critical external resistance. If the resistance of the external circuit is less than the critical external resistance (represented by tangent OA in Fig. 3.10), the machine will refuse to excite or will de-excite if already running This means that external resistance is so low as virtually to short circuit the machine and so doing away with its excitation.

Note. There are two critical resistances for a shunt generator viz., (i) critical field resistance (ii) critical external resistance. For the shunt generator to build up voltage, the former should not be exceeded and the latter must not be gone below.

3.10 How to Draw O.C.C. at Different Speeds?

If we are given O.C.C. of a generator at a constant speed N_1, then we can easily draw the O.C.C. at any other constant speed N_2. Fig (3.11) illustrates the procedure. Here we are given O.C.C. at a constant speed N_1. It is desired to find the O.C.C. at constant speed N_2 (it is assumed that $n_1 < N_2$). For constant excitation, $E \propto N$.

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

or $$E_2 = E_1 \times \frac{N_2}{N_1}$$

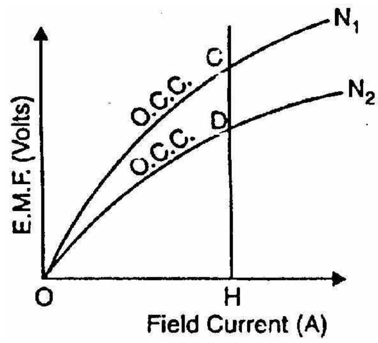

Fig. (3.11)

As shown in Fig. (3.11), for I_f = OH, E_1 = HC. Therefore, the new value of e.m.f. (E_2) for the same I_f but at N_2 i

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

This locates the point D on the new O.C.C. at N_2. Similarly, other points can be located taking different values of I_f. The locus of these points will be the O.C.C. at N_2.

3.11 Critical Speed (N_C)

The critical speed of a shunt generator is the minimum speed below which it fails to excite. Clearly, it is the speed for which the given shunt field resistance represents the critical resistance. In Fig. (3.12), curve 2 corresponds to critical speed because the shunt field resistance (R_{sh}) line is tangential to it. If the generator runs at full speed N, the new O.C.C. moves upward and the R'_{sh} line represents critical resistance for this speed.

$\therefore$ Speed $\propto$ Critical resistance

In order to find critical speed, take any convenient point C on excitation axis and erect a perpendicular so as to cut R_{sh} and R'_{sh} lines at points B and A respectively. Then,

$$\frac{BC}{AC} = \frac{N_C}{N}$$

or $$N_C = N \times \frac{BC}{AC}$$

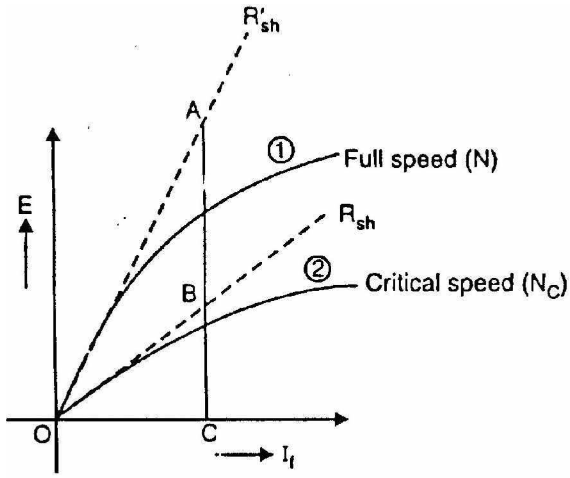

Fig. (3.12)

3.12 Conditions for Voltage Build-Up of a Shunt Generator

The necessary conditions for voltage build-up in a shunt generator are:

(i) There must be some residual magnetism in generator poles.
(ii) The connections of the field winding should be such that the field current strengthens the residual magnetism.
(iii) The resistance of the field circuit should be less than the critical resistance. In other words, the speed of the generator should be higher than the critical speed.

3.13 Compound Generator Characteristics

In a compound generator, both series and shunt excitation are combined as shown in Fig. (3.13). The shunt winding can be connected either across the armature only (short-shunt connection S) or across armature plus series field (long-shunt connection G). The compound generator can be cumulatively compounded or differentially compounded generator. The latter is rarely used in practice. Therefore, we shall discuss the characteristics of cumulatively-compounded generator. It may be noted that external characteristics of long and short shunt compound generators are almost identical.

External characteristic

Fig. (3.14) shows the external characteristics of a cumulatively compounded generator. The series excitation aids the shunt excitation. The degree of compounding depends upon the increase in series excitation with the increase in load current.

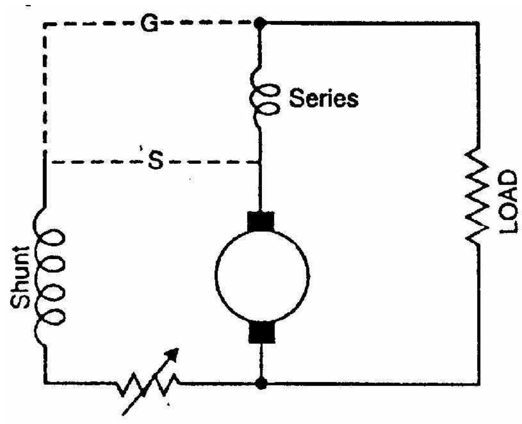

Fig. (3.13)

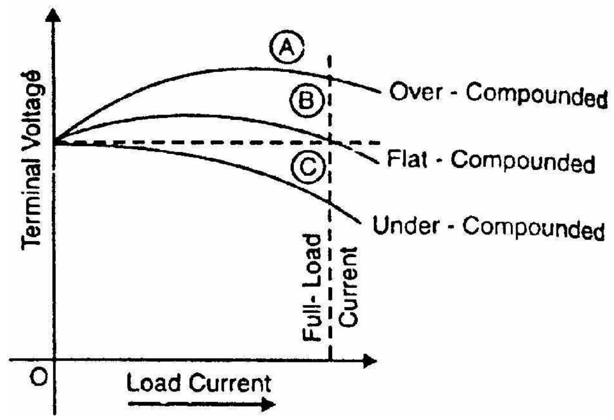

Fig. (3.14)

(i) If series winding turns are so adjusted that with the increase in load current the terminal voltage increases, it is called over-compounded generator. In such a case, as the load current increases, the series field m.m.f. increases and tends to increase the flux and hence the generated voltage. The increase in generated voltage is greater than the I_aR_a drop so that instead of decreasing, the terminal voltage increases as shown by curve A in Fig. (3.14).

(ii) If series winding turns are so adjusted that with the increase in load current, the terminal voltage substantially remains constant, it is called flat-compounded generator. The series winding of such a machine has lesser number of turns than the one in over-compounded machine and, therefore, does not increase the flux as much for a given load current. Consequently, the full-load voltage is nearly equal to the no-load voltage as indicated by curve B in Fig (3.14).

(iii) If series field winding has lesser number of turns than for a flat-compounded machine, the terminal voltage falls with increase in load current as indicated by curve C m Fig. (3.14). Such a machine is called under-compounded generator.

3.14 Voltage Regulation

The change in terminal voltage of a generator between full and no load (at constant speed) is called the voltage regulation, usually expressed as a percentage of the voltage at full-load.

$$\%\,\text{Voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

where V_{NL} = Terminal voltage of generator at no load

V_{FL} = Terminal voltage of generator at full load

Note that voltage regulation of a generator is determined with field circuit and speed held constant. If the voltage regulation of a generator is 10%, it means that terminal voltage increases 10% as the load is changed from full load to no load.

3.15 Parallel Operation of D.C. Generators

In a d.c. power plant, power is usually supplied from several generators of small ratings connected in parallel instead of from one large generator. This is due to the following reasons:

(i) Continuity of service

If a single large generator is used in the power plant, then in case of its breakdown, the whole plant will be shut down. However, if power is supplied from a number of small units operating in parallel, then in case of failure of one unit, the continuity of supply can be maintained by other healthy units.

(ii) Efficiency

Generators run most efficiently when loaded to their rated capacity. Electric power costs less per kWh when the generator producing it is efficiently loaded. Therefore, when load demand on power plant decreases, one or more generators can be shut down and the remaining units can be efficiently loaded.

(iii) Maintenance and repair

Generators generally require routine-maintenance and repair. Therefore, if generators are operated in parallel, the routine or emergency operations can be performed by isolating the affected generator while load is being supplied by other units. This leads to both safety and economy.

(iv) Increasing plant capacity

In the modern world of increasing population, the use of electricity is continuously increasing. When added capacity is required, the new unit can be simply paralleled with the old units.

(v) Non-availability of single large unit

In many situations, a single unit of desired large capacity may not be available. In that case a number of smaller units can be operated in parallel to meet the load requirement. Generally a single large unit is more expensive.

2.16 Connecting Shunt Generators in Parallel

The generators in a power plant are connected in parallel through bus-bars. The bus-bars are heavy thick copper bars and they act as +ve and −ve terminals. The positive terminals of the generators are .connected to the +ve side of bus-bars and negative terminals to the negative side of bus-bars.

Fig. (3.15) shows shunt generator 1 connected to the bus-bars and supplying load. When the load on the power plant increases beyond the capacity of this generator, the second shunt generator 2 is connected in parallel wish the first to meet the increased load demand. The procedure for paralleling generator 2 with generator 1 is as under:

(i) The prime mover of generator 2 is brought up to the rated speed. Now switch S_4 in the field circuit of the generator 2 is closed.

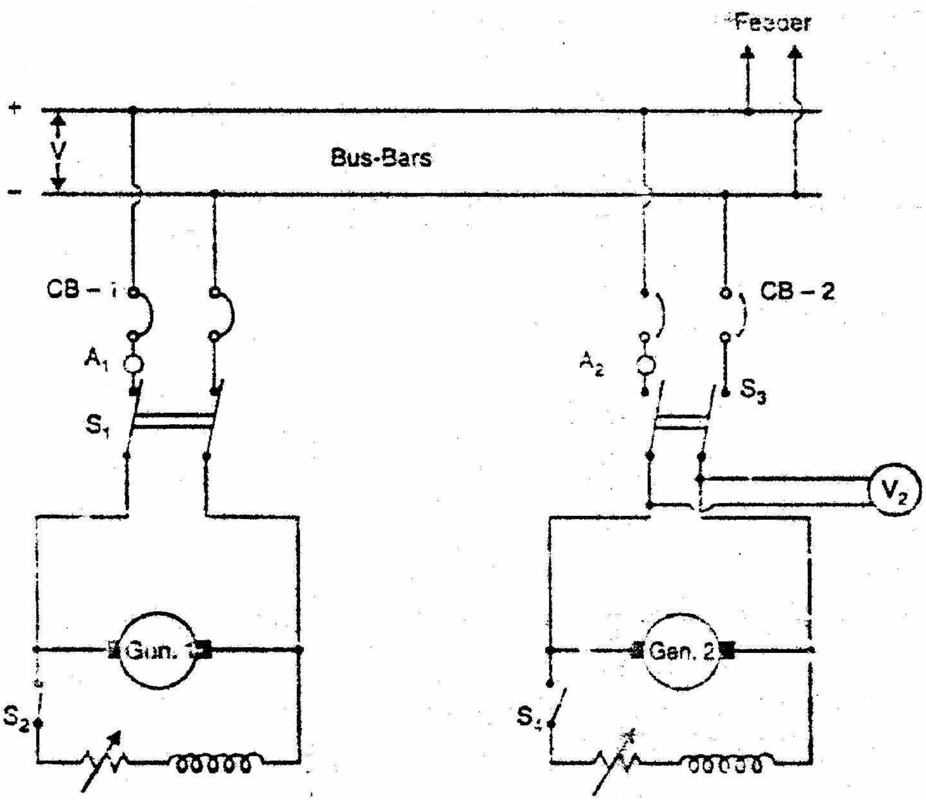

Fig. (3.15)

(ii) Next circuit breaker CB-2 is closed and the excitation of generator 2 is adjusted till it generates voltage equal to the bus-bars voltage. This is indicated by voltmeter V_2.

(iii) Now the generator 2 is ready to be paralleled with generator 1. The main switch S_3, is closed, thus putting generator 2 in parallel with generator 1. Note that generator 2 is not supplying any load because its generated e.m.f. is equal to bus-bars voltage. The generator is said to be “floating” (i.e., not supplying any load) on the bus-bars.

(iv) If generator 2 is to deliver any current, then its generated voltage E should be greater than the bus-bars voltage V. In that case, current supplied by it is $I = (E - V)/R_a$ where R_a is the resistance of the armature circuit. By increasing the field current (and hence induced e.m.f. E), the generator 2 can be made to supply proper amount of load.

(v) The load may be shifted from one shunt generator to another merely by adjusting the field excitation. Thus if generator 1 is to be shut down, the whole load can be shifted onto generator 2 provided it has the capacity to supply that load. In that case, reduce the current supplied by generator 1 to zero (This will be indicated by ammeter A_1) open C.B.-1 and then open the main switch S_1.

3.17 Load Sharing

The load sharing between shunt generators in parallel can be easily regulated because of their drooping characteristics. The load may be shifted from one generator to another merely by adjusting the field excitation. Let us discuss the load sharing of two generators which have unequal no-load voltages.

Let E_1, E_2 = no-load voltages of the two generators

R_1, R_2 = their armature resistances

V = common terminal voltage (Bus-bars voltage)

Then $$I_1 = \frac{E_1 - V}{R_1} \quad \text{and} \quad I_2 = \frac{E_2 - V}{R_2}$$

Thus current output of the generators depends upon the values of E_1 and E_3. These values may be changed by field rheostats. The common terminal voltage (or bus-bars voltage) will depend upon (i) the e.m.f.s of individual generators and (ii) the total load current supplied. It is generally desired to keep the bus-bars voltage constant. This can be achieved by adjusting the field excitations of the generators operating in parallel.

3.18 Compound Generators in Parallel

Under-compounded generators also operate satisfactorily in parallel but over-compounded generators will not operate satisfactorily unless their series fields are paralleled. This is achieved by connecting two negative brushes together as shown in Fig. (3.16) (i). The conductor used to connect these brushes is generally called equalizer bar. Suppose that an attempt is made to operate the two generators in Fig. (3.16) (ii) in parallel without an equalizer bar. If, for any reason, the current supplied by generator 1 increases slightly, the current in its series field will increase and raise the generated voltage. This will cause generator 1 to take more load. Since total load supplied to the system is constant, the current in generator 2 must decrease and as a result its series field is weakened. Since this effect is cumulative, the generator 1 will take the entire load and drive generator 2 as a motor. Under such conditions, the current in the two machines will be in the direction shown in Fig. (3.16) (ii). After machine 2 changes from a generator to a motor, the current in the shunt field will remain in the same direction, but the current in the armature and series field will reverse. Thus the magnetizing action, of the series field opposes that of the shunt field. As the current taken by the machine 2 increases, the demagnetizing action of series field becomes greater and the resultant field becomes weaker. The resultant field will finally become zero and at that time machine 2 will short-circuit machine 1, opening the breaker of either or both machines.

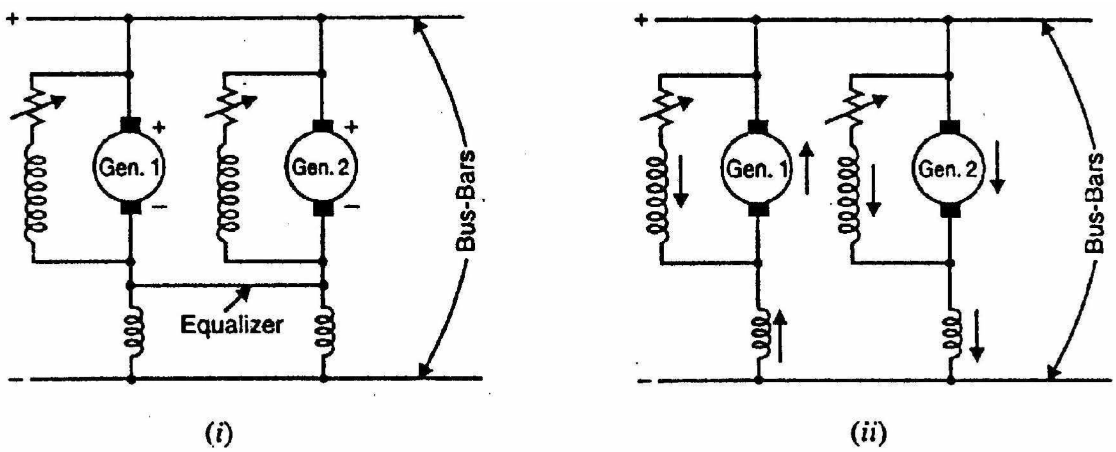

Fig. (3.16)

When the equalizer bar is used, a stabilizing action exist? and neither machine tends to take all the load. To consider this, suppose that current delivered by generator 1 increases [See Fig. 3.16 (i)]. The increased current will not only pass through the series field of generator 1 but also through the equalizer bar and series field of generator 2. Therefore, the voltage of both the machines increases and the generator 2 will take a part of the load.

Chapter (4)

D.C. Motors

Introduction

D. C. motors are seldom used in ordinary applications because all electric supply companies furnish alternating current However, for special applications such as in steel mills, mines and electric trains, it is advantageous to convert alternating current into direct current in order to use d.c. motors. The reason is that speed/torque characteristics of d.c. motors are much more superior to that of a.c. motors. Therefore, it is not surprising to note that for industrial drives, d.c. motors are as popular as 3-phase induction motors. Like d.c. generators, d.c. motors are also of three types viz., series-wound, shunt-wound and compound-wound. The use of a particular motor depends upon the mechanical load it has to drive.

4.1 D.C. Motor Principle

A machine that converts d.c. power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by;

$$F = BIl \quad \text{newtons}$$

Basically, there is no constructional difference between a d.c. motor and a d.c. generator. The same d.c. machine can be run as a generator or motor.

4.2 Working of D.C. Motor

Consider a part of a multipolar d.c. motor as shown in Fig. (4.1). When the terminals of the motor are connected to an external source of d.c. supply:

(i) the field magnets are excited developing alternate N and S poles;
(ii) the armature conductors carry ^currents. All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction.

Suppose the conductors under N-pole carry currents into the plane of the paper and those under S-pole carry currents out of the plane of the paper as shown in Fig.(4.1). Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. Referring to Fig. (4.1) and applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating. When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

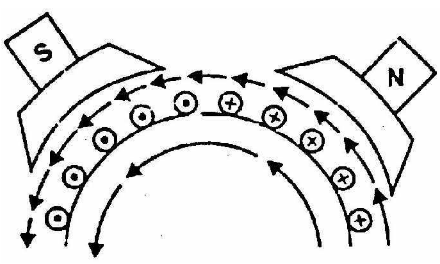

Fig. (4.1)

4.3 Back or Counter E.M.F.

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f. is induced in them as in a generator The induced e.m.f. acts in opposite direction to the applied voltage V(Lenz's law) and in known as back or counter e.m.f. E_b. The back e.m.f. E_b(= P ϕ ZN/60 A) is always less than the applied voltage V, although this difference is small when the motor is running under normal conditions.

Consider a shunt wound motor shown in Fig. (4.2). When d.c. voltage V is applied across the motor terminals, the field magnets are excited and armature conductors are supplied with current. Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back e.m.f. E_b is induced which opposes the applied voltage V. The applied voltage V has to force current through the armature against the back e.m.f. E_b. The electric work done in overcoming and causing the current to flow against E_b is converted into mechanical energy developed in the armature. It follows, therefore, that energy conversion in a d.c. motor is only possible due to the production of back e.m.f. E_b.

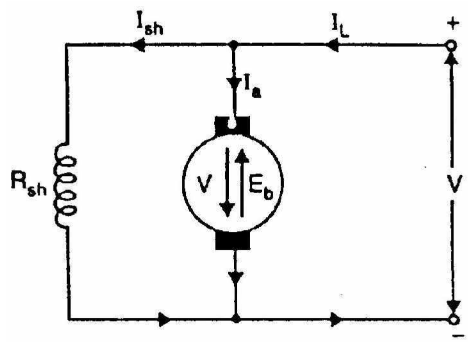

Fig. (4.2)

Net voltage across armature circuit = V − E_b

If R_a is the armature circuit resistance, then, $I_a = \dfrac{V - E_b}{R_a}$

Since V and R_a are usually fixed, the value of E_b will determine the current drawn by the motor. If the speed of the motor is high, then back e.m.f. E_b (= P ϕ

ZN/60 A) is large and hence the motor will draw less armature current and vice-versa.

4.4 Significance of Back E.M.F.

The presence of back e.m.f. makes the d.c. motor a self-regulating machine i.e., it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load.

$$\text{Armature current, } I_a = \frac{V - E_b}{R_a}$$

(i) When the motor is running on no load, small torque is required to overcome the friction and windage losses. Therefore, the armature current I_a is small and the back e.m.f. is nearly equal to the applied voltage.

(ii) If the motor is suddenly loaded, the first effect is to cause the armature to slow down. Therefore, the speed at which the armature conductors move through the field is reduced and hence the back e.m.f. E_b falls. The decreased back e.m.f. allows a larger current to flow through the armature and larger current means increased driving torque. Thus, the driving torque increases as the motor slows down. The motor will stop slowing down when the armature current is just sufficient to produce the increased torque required by the load.

(iii) If the load on the motor is decreased, the driving torque is momentarily in excess of the requirement so that armature is accelerated. As the armature speed increases, the back e.m.f. E_b also increases and causes the armature current I_a to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load.

It follows, therefore, that back e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

4.5 Voltage Equation of D.C. Motor

Let in a d.c. motor (See Fig. 4.3),

V = applied voltage
E_b = back e.m.f.
R_a = armature resistance
I_a = armature current

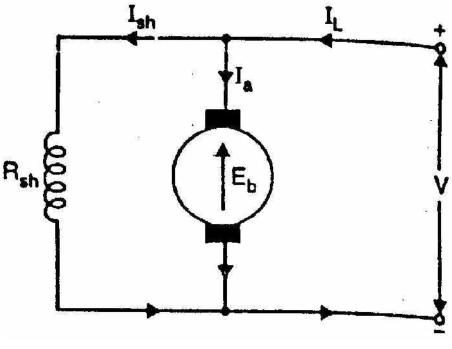

Fig. (4.3)

Since back e.m.f. E_b acts in opposition to the applied voltage V, the net voltage across the armature circuit is $V - E_b$. The armature current I_a is given by;

$$I_a = \frac{V - E_b}{R_a}$$

or $$V = E_b + I_a R_a \qquad \text{(i)}$$

This is known as voltage equation of the d.c. motor.

4.6 Power Equation

If Eq.(i) above is multiplied by ly throughout, we get,

$$VI_a = E_b I_a + I_a^2 R_a$$

This is known as power equation of the d.c. motor.

VI_a = electric power supplied to armature (armature input)

$E_b I_a$ = power developed by armature (armature output)

$I_a^2 R_a$ = electric power wasted in armature (armature Cu loss)

Thus out of the armature input, a small portion (about 5%) is wasted as $I_a^2 R_a$ and the remaining portion $E_b I_a$ is converted into mechanical power within the armature.

4.7 Condition For Maximum Power

The mechanical power developed by the motor is $P_m = E_b I_a$

Now $$P_m = VI_a - I_a^2 R_a$$

Since, V and R_a are fixed, power developed by the motor depends upon armature current. For maximum power, dP_m/dI_a should be zero.

$$\therefore \quad \frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

or $$I_a R_a = \frac{V}{2}$$

Now, $$V = E_b + I_a R_a = E_b + \frac{V}{2} \qquad \left[\therefore\ I_a R_a = \frac{V}{2}\right]$$

$$\therefore \quad E_b = \frac{V}{2}$$

Hence mechanical power developed by the motor is maximum when back e.m.f. is equal to half the applied voltage.

Limitations

In practice, we never aim at achieving maximum power due to the following reasons:

(i) The armature current under this condition is very large—much excess of rated current of the machine.

(ii) Half of the input power is wasted in the armature circuit. In fact, if we take into account other losses (iron and mechanical), the efficiency will be well below 50%.

4.8 Types of D.C. Motors

Like generators, there are three types of d.c. motors characterized by the connections of field winding in relation to the armature viz.:

(i) **Shunt-wound motor** in which the field winding is connected in parallel with the armature [See Fig. 4.4]. The current through the shunt field winding is not the same as the armature current. Shunt field windings are designed to produce the necessary m.m.f. by means of a relatively large number of turns of wire having high resistance. Therefore, shunt field current is relatively small compared with the armature current.

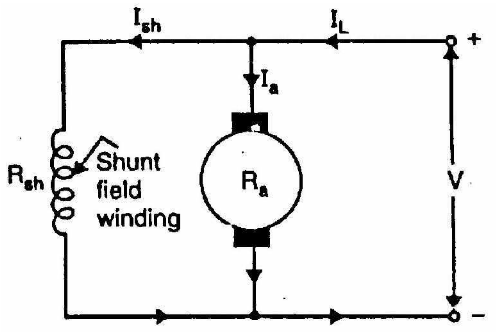

Fig. (4.4)

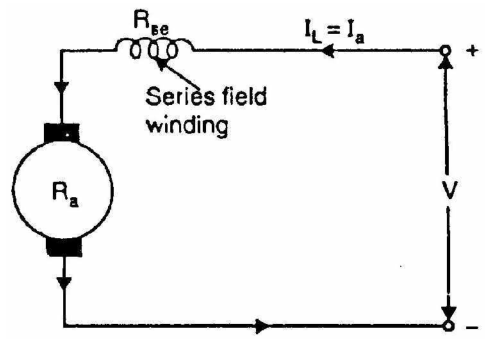

Fig. (4.5)

(ii) **Series-wound motor** in which the field winding is connected in series with the armature [See Fig. 4.5]. Therefore, series field winding carries the armature current. Since the current passing through a series field winding is the same as the armature current, series field windings must be designed with much fewer turns than shunt field windings for the same m.m.f. Therefore, a series field winding has a relatively small number of turns of thick wire and, therefore, will possess a low resistance.

(iii) **Compound-wound motor** which has two field windings; one connected in parallel with the armature and the other in series with it. There are two types of compound motor connections (like generators). When the shunt field winding is directly connected across the armature terminals [See Fig. 4.6], it is called short-shunt connection. When the shunt winding is so connected that it shunts the series combination of armature and series field [See Fig. 4.7], it is called long-shunt connection.

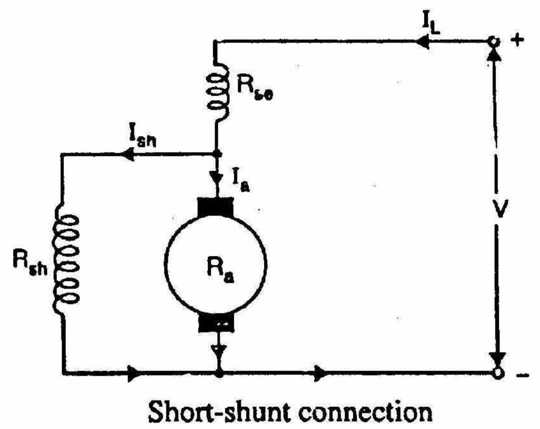

Fig. (4.6)

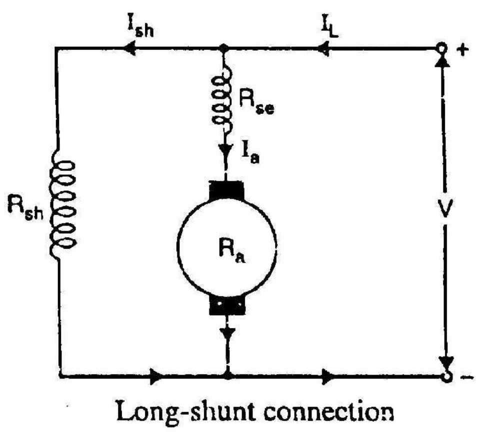

Fig. (4.7)

The compound machines (generators or motors) are always designed so that the flux produced by shunt field winding is considerably larger than the flux produced by the series field winding. Therefore, shunt field in compound machines is the basic dominant factor in the production of the magnetic field in the machine.

4.9 Armature Torque of D.C. Motor

Torque is the turning moment of a force about an axis and is measured by the product of force (F) and radius (r) at right angle to which the force acts i.e.
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$$T = F \times r$$

In a d.c. motor, each conductor is acted upon by a circumferential force F at a distance r, the radius of the armature (Fig. 4.8). Therefore, each conductor exerts a torque, tending to rotate the armature. The sum of the torques due to all armature conductors is known as gross or armature torque (T_a).

Let in a d.c. motor

r = average radius of armature in m
l = effective length of each conductor in m
Z = total number of armature conductors
A = number of parallel paths
i = current in each conductor = I_a/A
B = average flux density in Wb/m2
ϕ = flux per pole in Wb
P = number of poles

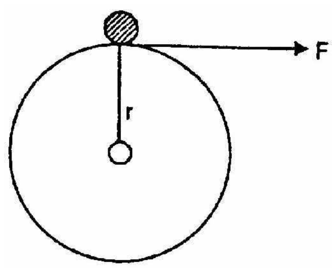

Fig. (4.8)

Force on each conductor, $F = B\, i\, l$ newtons

Torque due to one conductor = $F \times r$ newton- metre

Total armature torque, $T_a = Z\,F\,r$ newton-metre

$$= Z\,B\,i\,l\,r$$

Now $i = I_a/A$, $B = \phi/a$ where a is the x-sectional area of flux path per pole at radius r. Clearly, $a = 2\pi r l/P$.

$$\therefore \quad T_a = Z \times \left(\frac{\phi}{2}\right) \times \left(\frac{I_a}{A}\right) \times l \times r$$

$$= Z \times \frac{\phi}{2\pi\, r l / P} \times \frac{I_a}{A} \times l \times r = \frac{Z\phi I_a P}{2\pi\, A}\ \text{N - m}$$

or $$T_a = 0.159\, Z\phi I_a \left(\frac{P}{A}\right)\ \text{N - m} \qquad \text{(i)}$$

Since Z, P and A are fixed for a given machine,

$$\therefore \quad T_a \propto \phi I_a$$

Hence torque in a d.c. motor is directly proportional to flux per pole and armature current.

(i) For a shunt motor, flux ϕ is practically constant.

$$\therefore \quad T_a \propto I_a$$

(ii) For a series motor, flux ϕ is directly proportional to armature current I_a provided magnetic saturation does not take place.

$$\therefore \quad T_a \propto I_a^2$$

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Alternative expression for T_a

$$E_b = \frac{P\phi ZN}{60A}$$

$$\therefore \quad \frac{P\phi Z}{A} = \frac{60 \times E_b}{N}$$

From Eq.(i), we get the expression of T_a as:

$$T_a = 0.159 \times \left(\frac{60 \times E_b}{N} \right) \times I_a$$

or $$T_a = 9.55 \times \frac{E_b I_a}{N} \text{ N - m}$$

Note that developed torque or gross torque means armature torque T_a.

4.10 Shaft Torque (T_{sh})

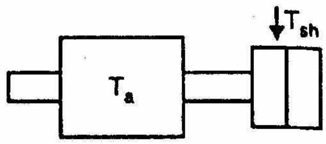


Fig. (4.9)

The torque which is available at the motor shaft for doing useful work is known as shaft torque. It is represented by T_{sh}. Fig. (4.9) illustrates the concept of shaft torque. The total or gross torque T_a developed in the armature of a motor is not available at the shaft because a part of it is lost in overcoming the iron and frictional losses in the motor. Therefore, shaft torque T_{sh} is somewhat less than the armature torque T_a. The difference $T_a - T_{sh}$ is called lost torque.

Clearly, $$T_a - T_{sh} = 9.55 \times \frac{\text{Iron and frictional losses}}{N}$$

For example, if the iron and frictional losses in a motor are 1600 W and the motor runs at 800 r.p.m., then,

$$T_a - T_{sh} = 9.55 \times \frac{1600}{800} = 19.1 \text{ N - m}$$

As stated above, it is the shaft torque T_{sh} that produces the useful output. If the speed of the motor is N r.p.m., then,

$$\text{Output in watts} = \frac{2\pi N T_{sh}}{60}$$

or $$T_{sh} = \frac{\text{Output in watts}}{2\pi\, N/60}\ \text{N - m}$$

or $$T_{sh} = 9.55 \times \frac{\text{Output in watts}}{N}\ \text{N - m} \qquad \left(\because\ \frac{60}{2\pi} = 9.55\right)$$

4.11 Brake Horse Power (B.H.P.)

The horse power developed by the shaft torque is known as brake horsepower (B.H.P.). If the motor is running at N r.p.m. and the shaft torque is T_{sh} newton-metres, then,

W.D./revolution = force x distance moved in 1 revolution

$$= F \times 2\pi\, r = 2\pi \times T_{sh}\ J$$

$$\text{W.D./minute} = 2\pi\, N\, T_{sh}\ J$$

$$\text{W.D./sec.} = \frac{2\pi\, N\, T_{sh}}{60}\, Js^{-1}\ \text{or watts} = \frac{2\pi\, N\, T_{sh}}{60 \times 746}\ \text{H.P.}$$

$$\therefore\quad \text{Useful output power} = \frac{2\pi\, N\, T_{sh}}{60 \times 746}\ \text{H.P.}$$

or $$\text{B.H.P.} = \frac{2\pi\, N\, T_{sh}}{60 \times 746}$$

4.12 Speed of a D.C. Motor

$$E_b = V - I_a R_a$$

But $$E_b = \frac{P\phi Z N}{60\ A}$$

$$\therefore\quad \frac{P\phi Z N}{60\ A} = V - I_a R_a$$

or $$N = \frac{(V - I_a R_a)}{\phi}\, \frac{60\ A}{P Z}$$

or $$N = K \frac{(V - I_a R_a)}{\phi} \qquad \text{where} \qquad K = \frac{60\, A}{PZ}$$

But $V - I_a R_a = E_a$

$$\therefore \quad N = K \frac{E_b}{\phi}$$

or $$N \propto \frac{E_b}{\phi}$$

Therefore, in a d.c. motor, speed is directly proportional to back e.m.f. E_b and inversely proportional to flux per pole ϕ.

4.13 Speed Relations

If a d.c. motor has initial values of speed, flux per pole and back e.m.f. as N_1, ϕ_1 and E_{b1} respectively and the corresponding final values are N_2, ϕ_2 and E_{b2}, then,

$$N_1 \propto \frac{E_{b1}}{\phi_1} \quad \text{and} \quad N_2 \propto \frac{E_{b2}}{\phi_2}$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

(i) For a shunt motor, flux practically remains constant so that $\phi_1 = \phi_2$.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

(ii) For a series motor, $\phi \propto I_a$ prior to saturation.

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

where I_{a1} = initial armature current
I_{a2} = final armature current

4.14 Speed Regulation

The speed regulation of a motor is the change in speed from full-load to no-loud and is expressed as a percentage of the speed at full-load i.e.

$$\%\ \text{Speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100$$

$$= \frac{N_0 - N}{N} \times 100$$

where N_0 = No - load .speed
N = Full - load speed

4.15 Torque and Speed of a D.C. Motor

For any motor, the torque and speed are very important factors. When the torque increases, the speed of a motor increases and vice-versa. We have seen that for a d.c. motor;

$$N = K\frac{(V - I_a R_a)}{\phi} = \frac{K\,E_b}{\phi} \qquad \text{(i)}$$

$$T_a \propto \phi I_a \qquad \text{(ii)}$$

If the flux decreases, from Eq.(i), the motor speed increases but from Eq.(ii) the motor torque decreases. This is not possible because the increase in motor speed must be the result of increased torque. Indeed, it is so in this case. When the flux decreases slightly, the armature current increases to a large value. As a result, in spite of the weakened field, the torque is momentarily increased to a high value and will exceed considerably the value corresponding to the load. The surplus torque available causes the motor to accelerate and back e.m.f. ($E_a = P\,\phi\,Z\,N/60\,A$) to rise. Steady conditions of speed will ultimately be achieved when back e.m.f. has risen to such a value that armature current [$I_a = (V - E_a)/R_a$] develops torque just sufficient to drive the load.

Illustration

Let us illustrate the above point with a numerical example. Suppose a 400 V shunt motor is running at 600 r.p.m., taking an armature current of 50 A. The armature resistance is 0.28 Ω. Let us see the effect of sudden reduction of flux by 5% on the motor.

Initially (prior to weakening of field), we have,

$$E_a = V - I_a R_a = 400 - 50 \times 0.28 = 386 \text{ volts}$$

We know that $E_b \propto \phi N$. If the flux is reduced suddenly, $E_b \propto \phi$ because inertia of heavy armature prevents any rapid change in speed. It follows that when the flux is reduced by 5%, the generated e.m.f. must follow suit. Thus at the instant of reduction of flux, $E'_b = 0.95 \times 386 = 366.7$ volts.

Instantaneous armature current is

$$I'_a = \frac{V - E'_b}{R_a} = \frac{400 - 366.7}{0.28} = 118.9 \text{ A}$$

Note that a sudden reduction of 5% in the flux has caused the armature current to increase about 2.5 times the initial value. This will result in the production of high value of torque. However, soon the steady conditions will prevail. This will depend on the system inertia; the more rapidly the motor can alter the speed, the sooner the e.m.f. rises and the armature current falls.

4.16 Armature Reaction in D.C. Motors

As in a d.c. generator, armature reaction also occurs in a d.c. motor. This is expected because when current flows through the armature conductors of a d.c. motor, it produces flux (armature flux) which lets on the flux produced by the main poles. For a motor with the same polarity and direction of rotation as is for generator, the direction of armature reaction field is reversed.

(i) In a generator, the armature current flows in the direction of the induced e.m.f. (i.e. generated e.m.f. E_g) whereas in a motor, the armature current flows against the induced e.m.f. (i.e. back e.m.f. E_b). Therefore, it should be expected that for the same direction of rotation and field polarity, the armature flux of the motor will be in the opposite direction to that of the generator. Hence instead of the main flux being distorted in the direction of rotation as in a generator, it is distorted opposite to the direction of rotation. We can conclude that:

Armature reaction in a d.c. generator weakens the jinx at leading pole tips and strengthens the flux at trailing pole tips while the armature reaction in a d. c. motor produces the opposite effect.

(ii) In case of a d.c. generator, with brushes along G.N.A. and no commutating poles used, the brushes must be shifted in the direction of rotation (forward lead) for satisfactory commutation. However, in case of a d.c. motor, the brushes are given a negative lead i.e., they are shifted against the direction of rotation.

With no commutating poles used, the brushes are given a forward lead in a d.c. generator and backward lead in a d.c. motor.

(iii) By using commutating poles (compoles), a d.c. machine can be operated with fixed brush positions for all conditions of load. Since commutating poles windings carry the armature current, then, when a machine changes from generator to motor (with consequent reversal of current), the polarities of commutating poles must be of opposite sign.

Therefore, in a d.c. motor, the commutating poles must have the same polarity as the main poles directly back of them. This is the opposite of the corresponding relation in a d.c. generator.

4.17 Commutation in D.C. Motors

Since the armature of a motor is the same as that of a generator, the current from the supply line must divide and pass through the paths of the armature windings.

In order to produce unidirectional force (or torque) on the armature conductors of a motor, the conductors under any pole must carry the current in the same direction at all times. This is illustrated in Fig. (4.10). In this case, the current flows away from the observer in the conductors under the N-pole and towards the observer in the conductors under the S-pole. Therefore, when a conductor moves from the influence of N-pole to that of S-pole, the direction of current in the conductor must be reversed. This is termed as commutation. The function of the commutator and the brush gear in a d.c. motor is to cause the reversal of current in a conductor as it moves from one side of a brush to the other. For good commutation, the following points may be noted:

(i) If a motor does not have commutating poles (compoles), the brushes must be given a negative lead i.e., they must be shifted from G.N.A. against the direction of rotation of, the motor.

(ii) By using interpoles, a d.c. motor can be operated with fixed brush positions for all conditions of load. For a d.c. motor, the commutating poles must have the same polarity as the main poles directly back of them. This is the opposite of the corresponding relation in a d.c. generator.

Note. A d.c. machine may be used as a motor or a generator without changing the commutating poles connections. When the operation of a d.c. machine changes from generator to motor, the direction of the armature current reverses. Since commutating poles winding carries armature current, the polarity of commutating pole reverses automatically to the correct polarity.

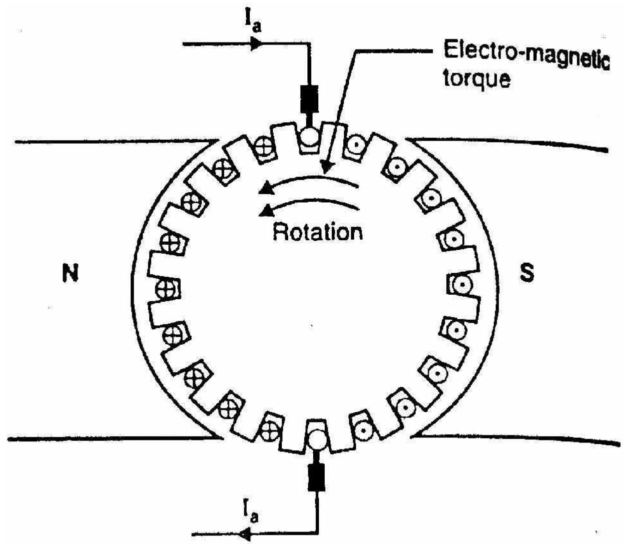


Fig. (4.10)

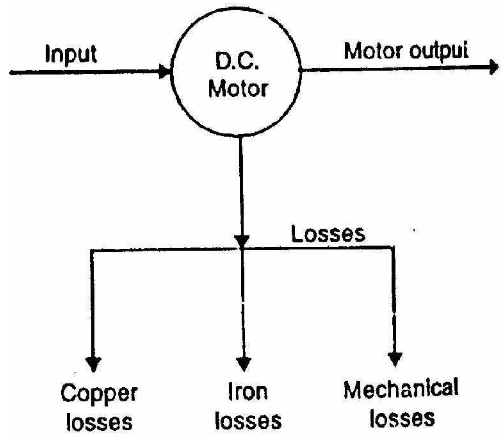


Fig. (4.11)

4.18 Losses in a D.C. Motor

The losses occurring in a d.c. motor are the same as in a d.c. generator [See Sec. 1.26]. These are [See Fig. 4.11]:

(i) copper losses (n) Iron losses or magnetic losses
(ii) mechanical losses

As in a generator, these losses cause (a) an increase of machine temperature and (b) reduction in the efficiency of the d.c. motor.

The following points may be noted:

(i) Apart from armature Cu loss, field Cu loss and brush contact loss, Cu losses also occur in interpoles (commutating poles) and compensating windings. Since these windings carry armature current (I_a),

  Loss in interpole winding = I_a^2 × Resistance of interpole winding

  Loss in compensating winding = I_a^2 × Resistance of compensating winding

(ii) Since d.c. machines (generators or motors) are generally operated at constant flux density and constant speed, the iron losses are nearly constant.
(iii) The mechanical losses (i.e. friction and windage) vary as the cube of the speed of rotation of the d.c. machine (generator or motor). Since d.c. machines are generally operated at constant speed, mechanical losses are considered to be constant.

4.19 Efficiency of a D.C. Motor

Like a d.c. generator, the efficiency of a d.c. motor is the ratio of output power to the input power i.e.

$$\text{Efficiency, } \eta = \frac{\text{output}}{\text{input}} \times 100 = \frac{\text{output}}{\text{output} + \text{losses}} \times 100$$

As for a generator (See Sec. 1.29), the efficiency of a d.c. motor will be maximum when:

Variable losses = Constant losses

Therefore, the efficiency curve of a d.c. motor is similar in shape to that of a d.c. generator.

4.20 Power Stages

The power stages in a d.c. motor are represented diagrammatically in Fig. (4.12).

A − B = Copper losses

B − C = Iron and friction losses

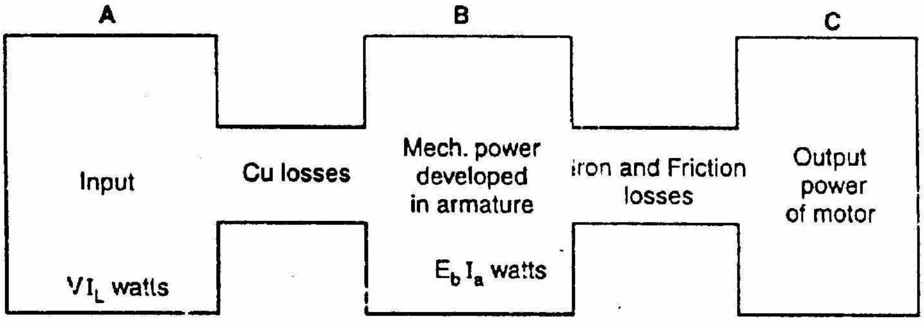


Fig. (4.12)

Overall efficiency, $\eta_c = C/A$
Electrical efficiency, $\eta_e = B/A$
Mechanical efficiency, $\eta_m = C/B$

4.21 D.C. Motor Characteristics

There are three principal types of d.c. motors viz., shunt motors, series motors and compound motors. Both shunt and series types have only one field winding wound on the core of each pole of the motor. The compound type has two separate field windings wound on the core of each pole. The performance of a d.c. motor can be judged from its characteristic curves known as motor characteristics, following are the three important characteristics of a d.c. motor:

(i) Torque and Armature current characteristic (T_a/I_a)

It is the curve between armature torque T_a and armature current I_a of a d.c. motor. It is also known as electrical characteristic of the motor.

(ii) Speed and armature current characteristic (N/i_a)

It is the curve between speed N and armature current I_a of a d.c. motor. It is very important characteristic as it is often the deciding factor in the selection of the motor for a particular application.

(iii) Speed and torque characteristic (N/T_a)

It is the curve between speed N and armature torque T_a of a d.c. motor. It is also known as mechanical characteristic.

4.22 Characteristics of Shunt Motors

Fig. (4.13) shows the connections of a d.c. shunt motor. The field current I_{sh} is constant since the field winding is directly connected to the supply voltage V which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant.

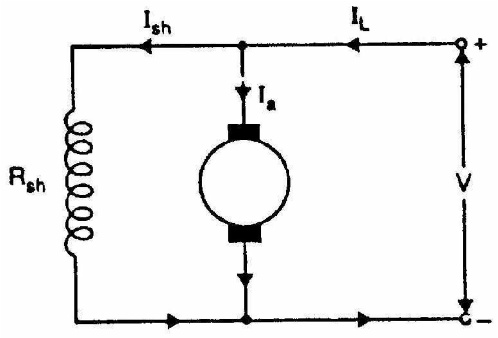


Fig. (4.13)

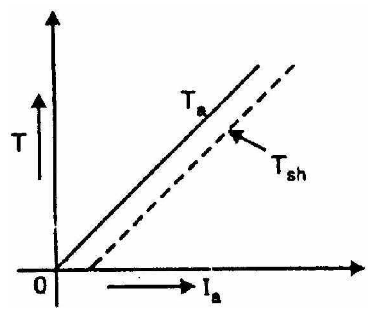


Fig. (4.14)

(i) T_a/I_a Characteristic. We know that in a d.c. motor,

$$T_a \propto \phi I_a$$

Since the motor is operating from a constant supply voltage, flux ϕ is constant (neglecting armature reaction).

$$\therefore \quad T_a \propto I_a$$

Hence T_a/I_a characteristic is a straight line passing through the origin as shown in Fig. (4.14). The shaft torque (T_{sh}) is less than T_a and is shown by a dotted line. It is clear from the curve that a very large current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

(ii) **N/I_a Characteristic.** The speed N of a. d.c. motor is given by;

$$N \propto \frac{E_b}{\phi}$$

The flux ϕ and back e.m.f. E_b in a shunt motor are almost constant under normal conditions. Therefore, speed of a shunt motor will remain constant as the armature current varies (dotted line AB in Fig. 4.15). Strictly speaking, when load is increased, E_b (= $V - I_aR_a$) and ϕ decrease due to the armature resistance drop and armature reaction respectively. However, E_b decreases slightly more than ϕ so that the speed of the motor decreases slightly with load (line AC).

(iii) **N/T_a Characteristic**. The curve is obtained by plotting the values of N and T_a for various armature currents (See Fig. 4.16). It may be seen that speed falls somewhat as the load torque increases.

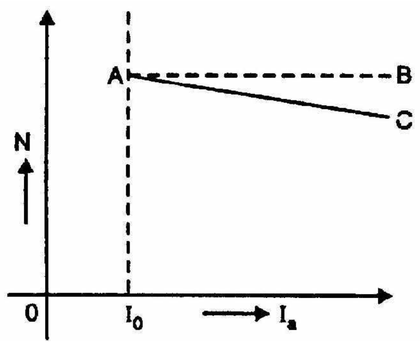


Fig. (4.15)

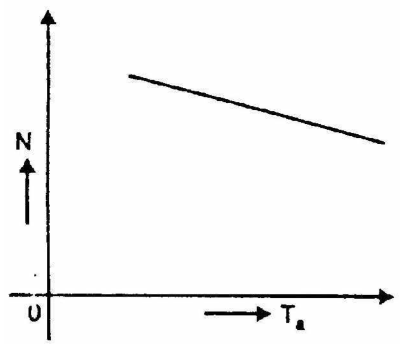


Fig. (4.16)

Conclusions

Following two important conclusions are drawn from the above characteristics:

(i) There is slight change in the speed of a shunt motor from no-load to full-load. Hence, it is essentially a constant-speed motor.

(ii) The starting torque is not high because $T_a \propto I_a$.

4.23 Characteristics of Series Motors

Fig. (4.17) shows the connections of a series motor. Note that current passing through the field winding is the same as that in the armature. If the mechanical load on the motor increases, the armature current also increases. Hence, the flux in a series motor increases with the increase in armature current and vice-versa.

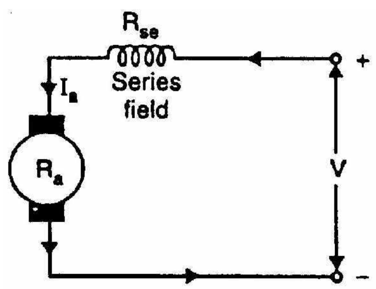


Fig. (4.17)

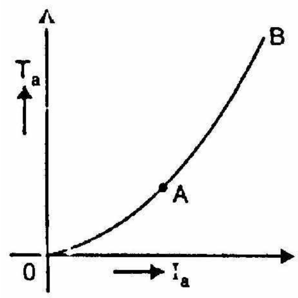


Fig. (4.18)

(i) **T_a/I_a Characteristic**. We know that:

$$T_a \propto \phi I_a$$

Upto magnetic saturation, $\phi \propto I_a$ so that $T_a \propto I_a^2$

After magnetic saturation, ϕ is constant so that $T_a \propto I_a$

Thus upto magnetic saturation, the armature torque is directly proportional to the square of armature current. If I_a is doubled, T_a is almost quadrupled.

Therefore, T_a/I_a curve upto magnetic saturation is a parabola (portion OA of the curve in Fig. 4.18). However, after magnetic saturation, torque is directly proportional to the armature current. Therefore, T_a/I_a curve after magnetic saturation is a straight line (portion AB of the curve).

It may be seen that in the initial portion of the curve (i.e. upto magnetic saturation), $T_a \propto I_a^2$. This means that starting torque of a d.c. series motor will be very high as compared to a shunt motor (where that $T_a \propto I_a$).

(ii) **N/I_a Characteristic.** The speed N of a series motor is given by;

$$N \propto \frac{E_b}{\phi} \qquad \text{where} \qquad E_b = V - I_a\left(R_a + R_{se}\right)$$

When the armature current increases, the back e.m.f. E_d decreases due to $I_a(R_a + R_{se})$ drop while the flux ϕ increases. However, $I_a(R_a + R_{se})$ drop is quite small under normal conditions and may be neglected.

$$\therefore \quad N \propto \frac{1}{\phi}$$

$$\propto \frac{1}{I_a} \quad \text{upto magnetic saturation}$$

Thus, upto magnetic saturation, the N/I_a curve follows the hyperbolic path as shown in Fig. (4.19). After saturation, the flux becomes constant and so does the speed.

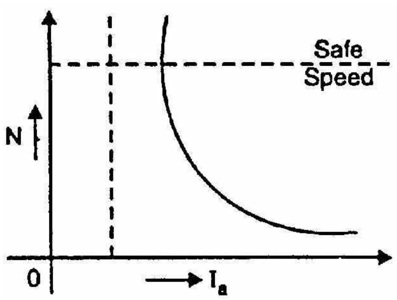


Fig. (4.19)

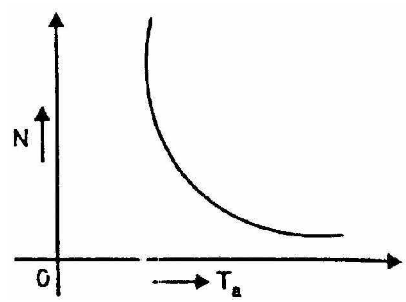


Fig. (4.20)

(iii) **N/T_a Characteristic**. The N/T_a characteristic of a series motor is shown in Fig. (4.20). It is clear that series motor develops high torque at low speed and vice-versa. It is because an increase in torque requires an increase in armature current, which is also the field current. The result is that flux is strengthened and hence the speed drops (Q $N \propto 1/\phi$). Reverse happens should the torque be low.

Conclusions

Following three important conclusions are drawn from the above characteristics of series motors:

(i) It has a high starting torque because initially $T_a \propto I_a^2$.

(ii) It is a variable speed motor (See N/I_a curve in Fig. 4.19) i.e., it automatically adjusts the speed as the load changes. Thus if the load decreases, its speed is automatically raised and vice-versa.

(iii) At no-load, the armature current is very small and so is the flux. Hence, the speed rises to an excessive high value (Q $N \propto 1/\phi$). This is dangerous for the machine which may be destroyed due to centrifugal forces set up in the rotating parts. Therefore, a series motor should never be started on no-load. However, to start a series motor, mechanical load is first put and then the motor is started.

Note. The minimum load on a d.c. series motor should be great enough to keep the speed within limits. If the speed becomes dangerously high, then motor must be disconnected from the supply.

4.24 Compound Motors

A compound motor has both series field and shunt field. The shunt field is always stronger than the series field. Compound motors are of two types:

(i) *Cumulative-compound motors* in which series field aids the shunt field.

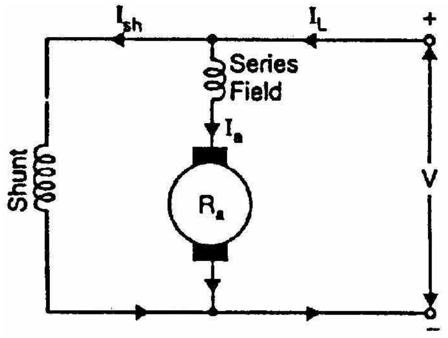
(ii) *Differential-compound motors* in which series field opposes the shunt field.

Differential compound motors are rarely used due to their poor torque characteristics at heavy loads.

4.25 Characteristics of Cumulative Compound Motors

Fig. (4.21) shows the connections of a cumulative-compound motor. Each pole carries a series as well as shunt field winding; the series field aiding the shunt field.

(i) **T_a/I_a Characteristic**. As the load increases, the series field increases but shunt field strength remains constant. Consequently, total flux is increased and hence the armature torque (Q $T_a \propto \phi I_a$). It may be noted that torque of a cumulative-compound motor is greater than that of shunt motor for a given armature current due to series field [See Fig. 4.22].



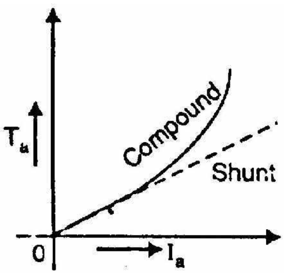
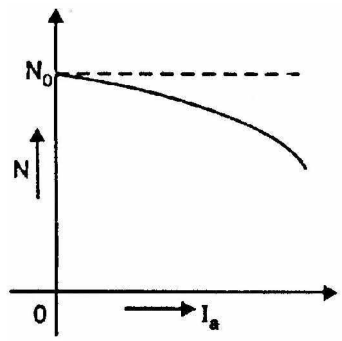


Fig. (4.21) Fig. (4.22)

(ii) **N/I_a Characteristic**. As explained above, as the lead increases, the flux per pole also increases. Consequently, the speed ($N \propto 1/\phi$) of the motor tails as the load increases (See Fig. 4.23). It may be noted that as the load is added, the increased amount of flux causes the speed to decrease more than does the speed of a shunt motor. Thus the speed regulation of a cumulative compound motor is poorer than that of a shunt motor.

Note: Due to shunt field, the motor has a definite no load speed and can be operated safely at no-load.



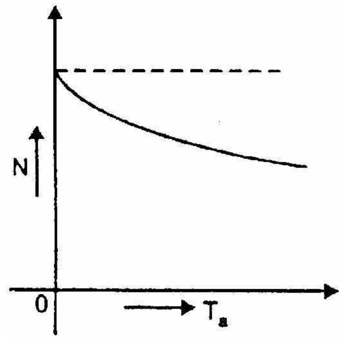


Fig. (4.23) Fig. (4.24)

(iii) **N/T_a Characteristic**. Fig. (4.24) shows N/T_a characteristic of a cumulative compound motor. For a given armature current, the torque of a cumulative compound motor is more than that of a shunt motor but less than that of a series motor.

Conclusions

A cumulative compound motor has characteristics intermediate between series and shunt motors.

(i) Due to the presence of shunt field, the motor is prevented from running away at no-load.

(ii) Due to the presence of series field, the starting torque is increased.

4.26 Comparison of Three Types of Motors

(i) The speed regulation of a shunt motor is better than that of a series motor.

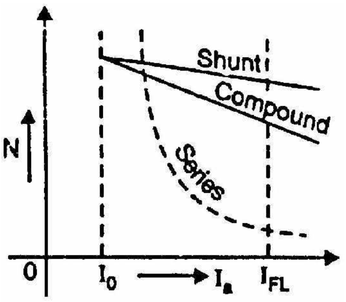


Fig. (4.25)

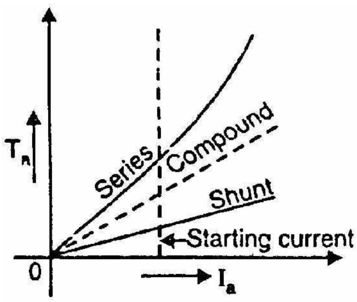


Fig. (4.26)

However, speed regulation of a cumulative compound motor lies between shunt and series motors (See Fig. 4.25).

(ii) For a given armature current, the starting torque of a series motor is more than that of a shunt motor. However, the starting torque of a cumulative compound motor lies between series and shunt motors (See Fig. 4.26).

(iii) Both shunt and cumulative compound motors have definite no-load speed. However, a series motor has dangerously high speed at no-load.

4.27 Applications of D.C. Motors

1. Shunt motors

The characteristics of a shunt motor reveal that it is an approximately constant speed motor. It is, therefore, used

(i) where the speed is required to remain almost constant from no-load to full-load

(ii) where the load has 10 be driven at a number of speeds and any one of which is required to remain nearly constant

Industrial use: Lathes, drills, boring mills, shapers, spinning and weaving machines etc.

2. Series motors

It is a variable speed motor i.e., speed is low at high torque and vice-versa. However, at light or no-load, the motor tends to attain dangerously high speed. The motor has a high starting torque. It is, therefore, used

(i) where large starting torque is required e.g., in elevators and electric traction

(ii) where the load is subjected to heavy fluctuations and the speed is automatically required to reduce at high torques and vice-versa

Industrial use: Electric traction, cranes, elevators, air compressors, vacuum cleaners, hair drier, sewing machines etc.

3. Compound motors

Differential-compound motors are rarely used because of their poor torque characteristics. However, cumulative-compound motors are used where a fairly constant speed is required with irregular loads or suddenly applied heavy loads.

Industrial use: Presses, shears, reciprocating machines etc.

4.28 Troubles in D.C. Motors

Several troubles may arise in a d.c. motor and a few of them are discussed below:

1. Failure to start

This may be due to (i) ground fault (ii) open or short-circuit fault (iii) wrong connections (iv) too low supply voltage (v) frozen bearing or (vi) excessive load.

2. Sparking at brushes

This may be due to (i) troubles in brushes (ii) troubles in commutator (iii) troubles in armature or (iv) excessive load.

(i) Brush troubles may arise due to insufficient contact surface, too short a brush, too little spring tension or wrong brush setting.

(ii) Commutator troubles may be due to dirt on the commutator, high mica, rough surface or eccentricity.

(iii) Armature troubles may be due to an open armature coil. An open armature coil will cause sparking each time the open coil passes the brush. The location of this open coil is noticeable by a burnt line between segments connecting the coil.

3. Vibrations and pounding noises

These maybe due to (i) worn bearings (ii) loose parts (iii) rotating parts hitting stationary parts (iv) armature unbalanced (v) misalignment of machine (vi) loose coupling etc.

4. Overheating

The overheating of motor may be due to (i) overloads (ii) sparking at the brushes (iii) short-circuited armature or field coils (iv) too frequent starts or reversals (v) poor ventilation (vi) incorrect voltage.

Chapter (5)

Speed Control of D.C. Motors

Introduction

Although a far greater percentage of electric motors in service are a.c. motors, the d.c. motor is of considerable industrial importance. The principal advantage of a d.c. motor is that its speed can be changed over a wide range by a variety of simple methods. Such a fine speed control is generally not possible with a.c. motors. In fact, fine speed control is one of the reasons for the strong competitive position of d.c. motors in the modem industrial applications. In this chapter, we shall discuss the various methods of-speed control of d.c. motors.

5.1 Speed Control of D.C. Motors

The speed of a d.c. motor is given by:

$$N \propto \frac{E_b}{\phi}$$

or $$N = K\frac{(V - I_a R)}{\phi} \text{ r.p.m.} \qquad \text{(i)}$$

where $R = R_a$ for shunt motor

$= R_a + R_{se}$. for series motor

From exp. (i), it is clear that there are three main methods of controlling the speed of a d.c. motor, namely:

(i) By varying the flux per pole (ϕ). This is known as flux control method.

(ii) By varying the resistance in the armature circuit. This is known as armature control method.

(iii) By varying the applied voltage V. This is known as voltage control method.

5.2 Speed Control of D.C. Shunt Motors

The speed of a shunt motor can be changed by (i) flux control method (ii) armature control method (iii) voltage control method. The first method (i.e. flux control method) is frequently used because it is simple and inexpensive.

1. Flux control method

It is based on the fact that by varying the flux ϕ, the motor speed ($N \propto 1/\phi$) can be changed and hence the name flux control method. In this method, a variable resistance (known as shunt field rheostat) is placed in series with shunt field winding as shown in Fig. (5.1).

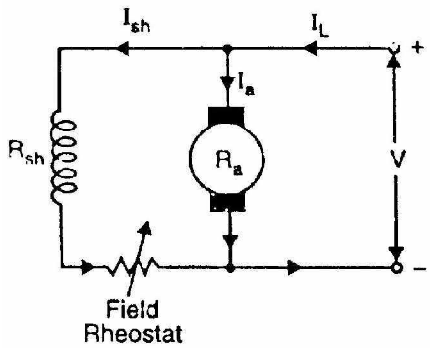


Fig. (5.1)

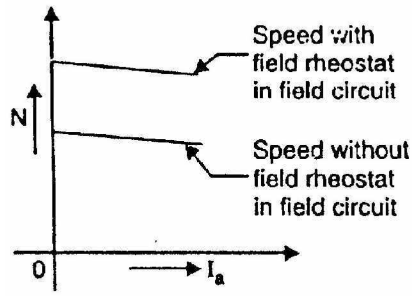


Fig. (5.2)

The shunt field rheostat reduces the shunt field current I_{sh} and hence the flux ϕ. Therefore, we can only raise the speed of the motor above the normal speed (See Fig. 5.2). Generally, this method permits to increase the speed in the ratio 3:1. Wider speed ranges tend to produce instability and poor commutation.

Advantages

(i) This is an easy and convenient method.
(ii) It is an inexpensive method since very little power is wasted in the shunt field rheostat due to relatively small value of I_{sh}.
(iii) The speed control exercised by this method is independent of load on the machine.

Disadvantages

(i) Only speeds higher than the normal speed can be obtained since the total field circuit resistance cannot be reduced below R_{sh}—the shunt field winding resistance.
(ii) There is a limit to the maximum speed obtainable by this method. It is because if the flux is too much weakened, commutation becomes poorer.

Note. The field of a shunt motor in operation should never be opened because its speed will increase to an extremely high value.

2. Armature control method

This method is based on the fact that by varying the voltage available across the armature, the back e.m.f and hence the speed of the motor can be changed. This is done by inserting a variable resistance R_C (known as controller resistance) in series with the armature as shown in Fig. (5.3).

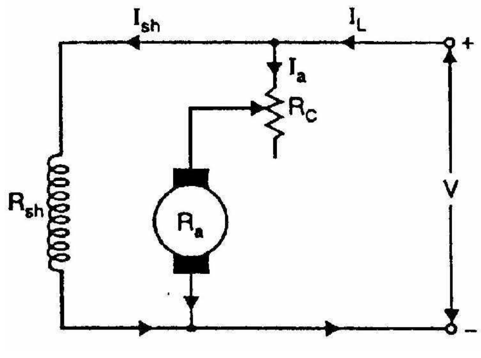


Fig. (5.3)

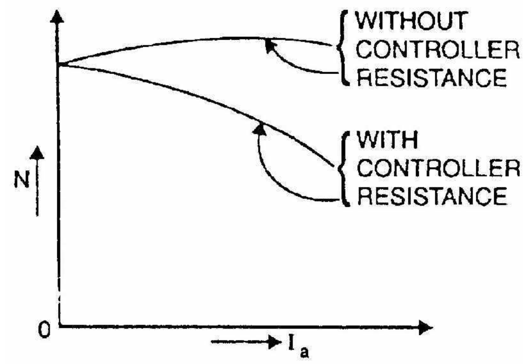


Fig. (5.4)

$$N \propto V - I_a(R_a + R_C)$$

where $\quad R_C$ = controller resistance

Due to voltage drop in the controller resistance, the back e.m.f. (E_b) is decreased. Since $N \propto E_b$, the speed of the motor is reduced. The highest speed obtainable is lhat corresponding to $R_C = 0$ i.e., normal speed. Hence, this method can only provide speeds below the normal speed (See Fig. 5.4).

Disadvantages

(i) A large amount of power is wasted in the controller resistance since it carries full armature current I_a.
(ii) The speed varies widely with load since the speed depends upon the voltage drop in the controller resistance and hence on the armature current demanded by the load.
(iii) The output and efficiency of the motor are reduced.
(iv) This method results in poor speed regulation.

Due to above disadvantages, this method is seldom used to control tie speed of shunt motors.

Note. The armature control method is a very common method for the speed control of d.c. series motors. The disadvantage of poor speed regulation is not important in a series motor which is used only where varying speed service is required.

3. Voltage control method

In this method, the voltage source supplying the field current is different from that which supplies the armature. This method avoids the disadvantages of poor speed regulation and low efficiency as in armature control method. However, it is quite expensive. Therefore, this method of speed control is employed for large size motors where efficiency is of great importance.

(i) **Multiple voltage control**. In this method, the shunt field of the motor is connected permanently across a-fixed voltage source. The armature can be connected across several different voltages through a suitable switchgear. In this way, voltage applied across the armature can be changed. The speed will be approximately proportional to the voltage applied across the armature. Intermediate speeds can be obtained by means of a shunt field regulator.

(ii) **Ward-Leonard system**. In this method, the adjustable voltage for the armature is obtained from an adjustable-voltage generator while the field circuit is supplied from a separate source. This is illustrated in Fig. (5.5). The armature of the shunt motor M (whose speed is to be controlled) is connected directly to a d.c. generator G driven by a constant-speed a.c. motor A. The field of the shunt motor is supplied from a constant-voltage exciter E. The field of the generator G is also supplied from the exciter E. The voltage of the generator G can be varied by means of its field regulator. By reversing the field current of generator G by controller FC, the voltage applied to the motor may be reversed. Sometimes, a field regulator is included in the field circuit of shunt motor M for additional speed adjustment. With this method, the motor may be operated at any speed upto its maximum speed.

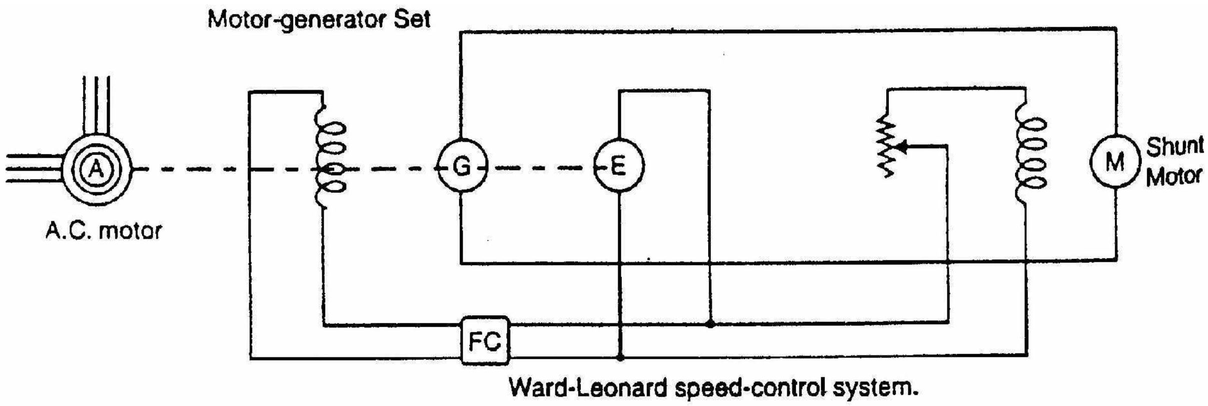


Fig. (5.5)

Advantages

(a) The speed of the motor can be adjusted through a wide range without resistance losses which results in high efficiency.

(b) The motor can be brought to a standstill quickly, simply by rapidly reducing the voltage of generator G. When the generator voltage is reduced below the back e.m.f. of the motor, this back e.m.f. sends current through the generator armature, establishing dynamic braking. While this takes place, the generator G operates as a motor driving motor A which returns power to the line.

(c) This method is used for the speed control of large motors when a d.c. supply is not available.

The disadvantage of the method is that a special motor-generator set is required for each motor and the losses in this set are high if the motor is operating under light loads for long periods.

5.3 Speed Control of D.C. Series Motors

The speed control of d.c. series motors can be obtained by (i) flux control method (ii) armature-resistance control method. The latter method is mostly used.

1. Flux control method

In this method, the flux produced by the series motor is varied and hence the speed. The variation of flux can be achieved in the following ways:

(i) **Field divertcrs**. In this method, a variable resistance (called field diverter) is connected in parallel with series field winding as shown in Fig. (5.6). Its effect is to shunt some portion of the line current from the series field winding, thus weakening the field and increasing the speed ($Q \quad N \propto 1/\phi$). The lowest speed obtainable is that corresponding to zero current in the diverter (i.e., diverter is open). Obviously, the lowest speed obtainable is the normal speed of the motor. Consequently, this method can only provide speeds above the normal speed. The series field diverter method is often employed in traction work.

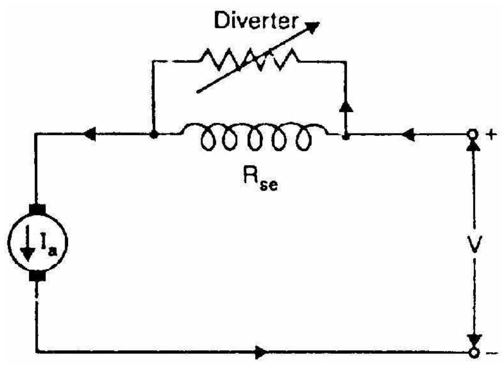


Fig. (5.6)

(ii) **Armature diverter**. In order to obtain speeds below the normal speed, a variable resistance (called armature diverter) is connected in parallel with the armature as shown in Fig. (5.7). The diverter shunts some of the line current, thus reducing the armature current. Now for a given load, if I_a is decreased, the flux ϕ must increase ($Q \quad T \propto \phi I_a$). Since $N \propto 1/\phi$, the motor speed is decreased. By adjusting the armature diverter, any speed lower than the normal speed can be obtained.

(iii) **Tapped field control**. In this method, the flux is reduced (and hence speed is increased) by decreasing the number of turns of the series field winding as shown in Fig. (5.8). The switch S can short circuit any part of the field winding, thus decreasing the flux and raising the speed. With full turns of the field winding, the motor runs at normal speed and as the field turns are cut out, speeds higher than normal speed are achieved.

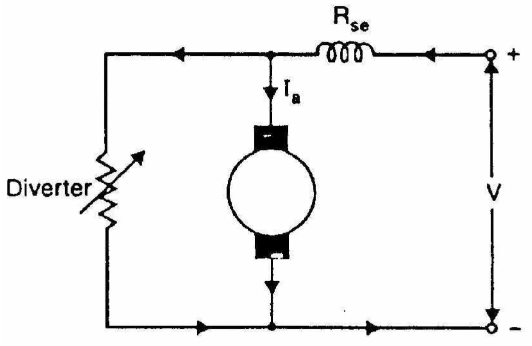


Fig. (5.7)

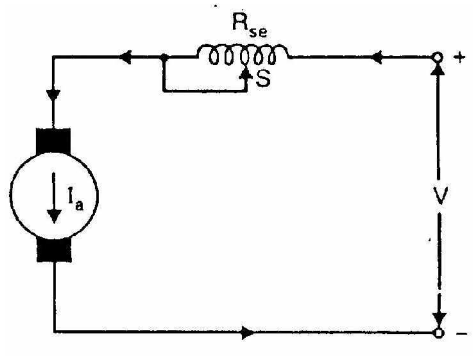


Fig. (5.8)

(iv) **Paralleling field coils**. This method is usually employed in the case of fan motors. By regrouping the field coils as shown in Fig. (5.9), several fixed speeds can be obtained.

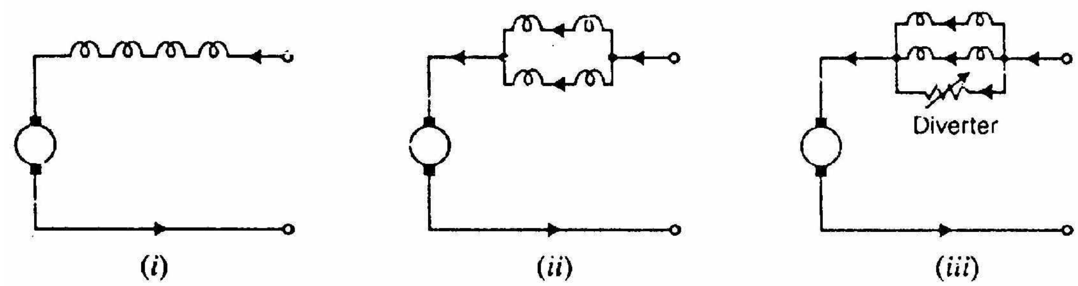


Fig. (5.9)

2. Armature-resistance control

In this method, a variable resistance is directly connected in series with the supply to the complete motor as shown in Fig. (5.10). This reduces the voltage available across the armature and hence the speed falls. By changing the value of variable resistance, any speed below the normal speed can be obtained. This is the most common method employed to control the speed of d.c. series motors. Although this method has poor speed regulation, this has no significance for series motors because they are used in varying speed applications. The loss of power in the series resistance for many applications of series motors is not too serious since in these applications,

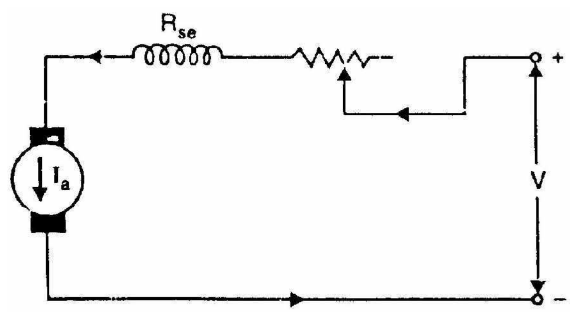


Fig. (5.10)

the control is utilized for a large portion of the time for reducing the speed under light-load conditions and is only used intermittently when the motor is carrying full-load.

5.4 Series-Parallel Control

Another method used for the speed control of d.c. series motors is the series-parallel method. In this system which is widely used in traction system, two (or more) similar d.c. series motors are mechanically coupled to the same load.

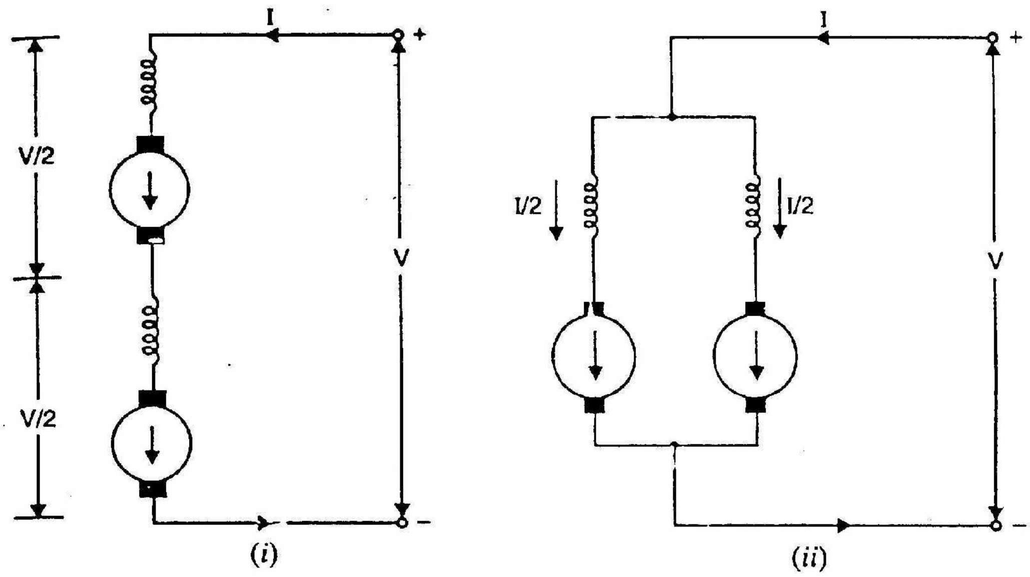


Fig. (5.11)

When the motors are connected in series [See Fig. 5.11 (i)], each motor armature will receive one-half the normal voltage. Therefore, the speed will be low. When the motors are connected in parallel, each motor armature receives the normal voltage and the speed is high [See Fig. 5.11 (ii)]. Thus we can obtain two speeds. Note that for the same load on the pair of motors, the system would run approximately four times the speed when the machines are in parallel as when they are in series.

Series-parallel and resistance control

In electric traction, series-parallel method is usually combined with resistance method of control. In the simplest case, two d.c. series motors are coupled mechanically and drive the same vehicle.

(i) At standstill, the motors are connected is series via a starting rheostat. The motors are started up in series with each other and starting resistance is cut out step by step to increase the speed. When all the resistance is cut out (See Fig. 5.12), the voltage applied to each motor is about one-half of the line voltage. The speed is then about one-half of what it would be if the full line voltage were applied to each motor.

(ii) To increase the speed further, the two motors are connected in parallel and at the same time the starting resistance is connected in series with the combination (See Fig. 5.12). The starting resistance is again cut out step by step until full speed is attained. Then field control is introduced.

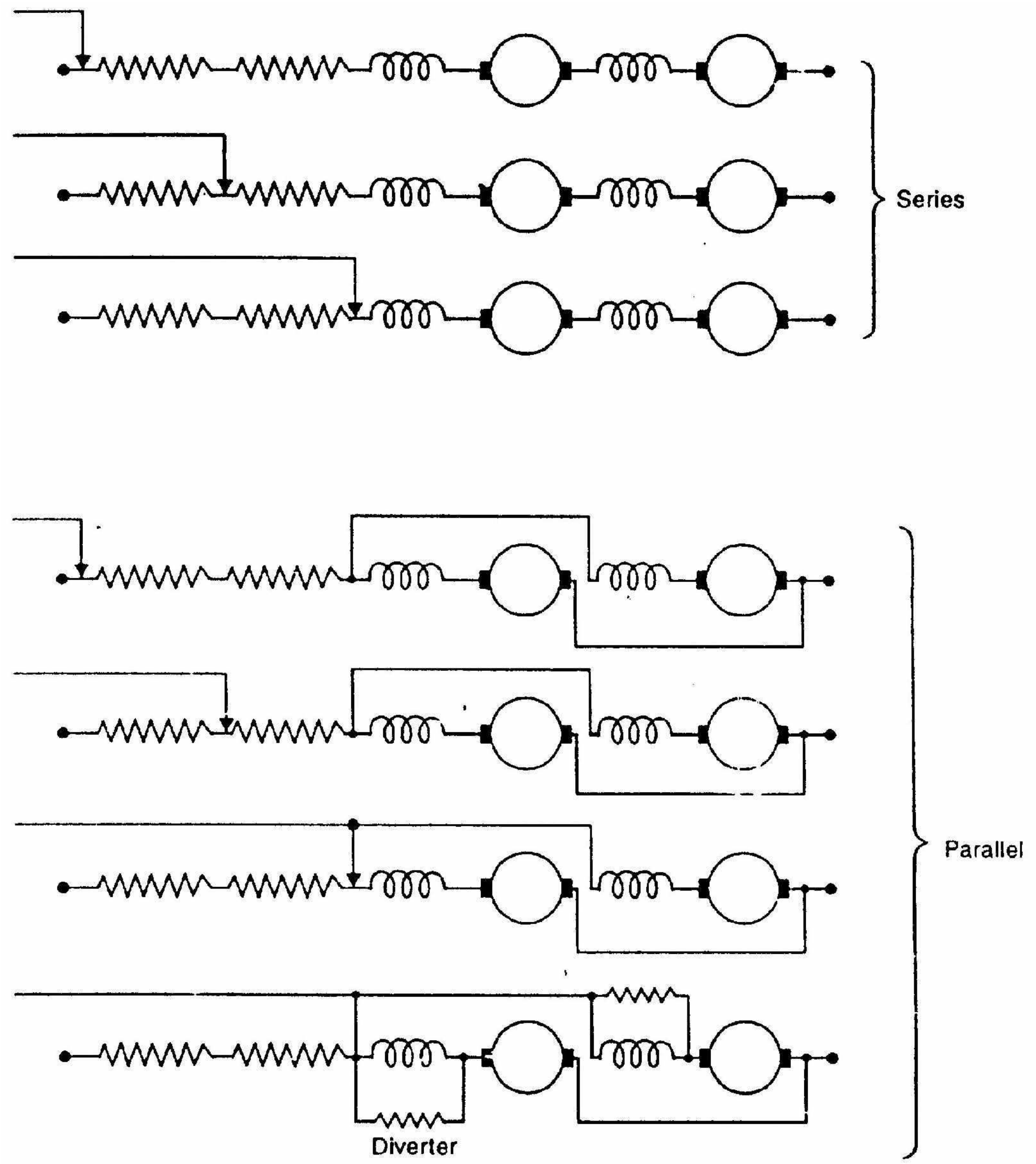


Fig. (5.12)

5.5 Electric Braking

Sometimes it is desirable to stop a d.c. motor quickly. This may be necessary in case of emergency or to save time if the motor is being used for frequently repeated operations. The motor and its load may be brought to rest by using either (i) mechanical (friction) braking or (ii) electric braking. In mechanical braking, the motor is stopped due to the friction between the moving parts of the motor and the brake shoe i.e. kinetic energy of the motor is dissipated as heat. Mechanical braking has several disadvantages including non-smooth stop and greater stopping time.

In electric braking, the kinetic energy of the moving parts (i.e., motor) is converted into electrical energy which is dissipated in a resistance as heat or alternativley, it is returned to the supply source (Regenerative braking). For d.c. shunt as well as series motors, the following three methods of electric braking are used:

(i) Rheostatic or Dynamic braking
(ii) Plugging
(iii) Regenerative braking

It may be noted that electric braking cannot hold the motor stationary and mechanical braking is necessary. However, the main advantage of using electric braking is that it reduces the wear and tear of mechanical brakes and cuts down the stopping time considerably due to high braking retardation.

(i) Rheostatic or Dynamic braking

In this method, the armature of the running motor is disconnected from the supply and is connected across a variable resistance R. However, the field winding is left connected to the supply. The armature, while slowing down, rotates in a strong magnetic field and, therefore, operates as a generator, sending a large current through resistance R. This causes the energy possessed by the rotating armature to be dissipated quickly as heat in the resistance. As a result, the motor is brought to standstill quickly.

Fig. (5.13) (i) shows dynamic braking of a shunt motor. The braking torque can be controlled by varying the resistance R. If the value of R is decreased as the motor speed decreases, the braking torque may be maintained at a high value. At a low value of speed, the braking torque becomes small and the final stopping of the motor is due to friction. This type of braking is used extensively in connection with the control of elevators and hoists and in other applications in which motors must be started, stopped and reversed frequently.

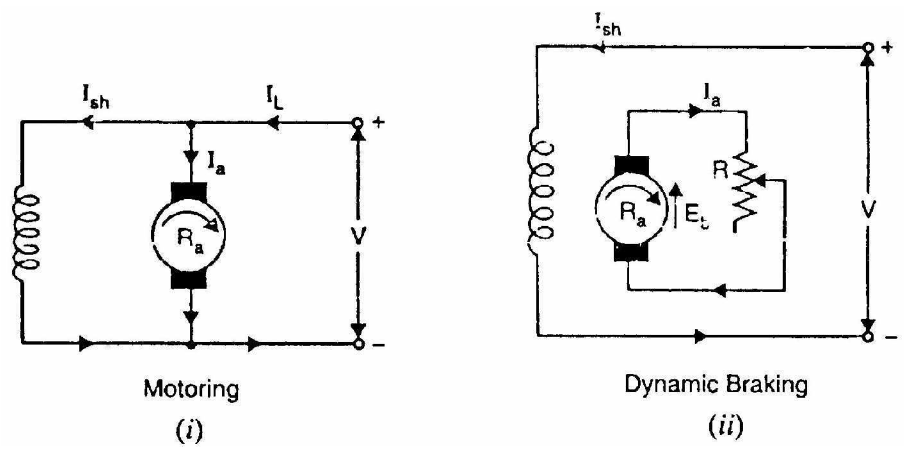


Fig. (5.13)

We now investigate how braking torque depends upon the speed of the motor. Referring to Fig. (5.13) (ii),

$$\text{Armature current, } I_a = \frac{E_b}{R + R_a} = \frac{k_1 N \phi}{R + R_a} \qquad (\because E_b \propto \phi N)$$

$$\text{Braking torque, } T_B = k_2 I_a \phi = k_2 \phi \left(\frac{k_1 N \phi}{R + R_a} \right) = k_3 N \phi^2$$

where k_2 and k_3 are constants

For a shunt motor, ϕ is constant.

$\therefore$ Braking torque, $T_B \propto N$

Therefore, braking torque decreases as the motor speed decreases.

(ii) Plugging

In this method, connections to the armature are reversed so that motor tends to rotate in the opposite direction, thus providing the necessary braking effect. When the motor comes to rest, the supply must be cut off otherwise the motor will start rotating in the opposite direction.

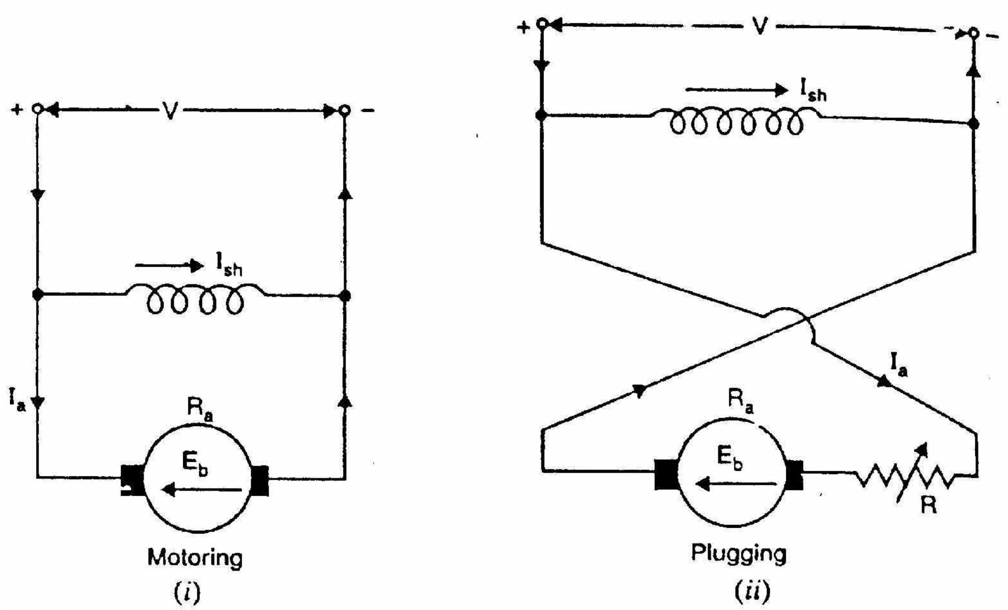


Fig. (5.14)

Fig. (5.14) (ii) shows plugging of a d.c. shunt motor. Note that armature connections are reversed while the connections of the field winding are kept the same. As a result the current in the armature reverses. During the normal running of the motor [See Fig. 5.14 (i)], the back e.m.f. E_b opposes the applied voltage V. However, when armature connections are reversed, back e.m.f. E_b and V act in the same direction around the circuit. Therefore, a voltage equal to

V + E_b is impressed across the armature circuit. Since $E_b \simeq V$, the impressed voltage is approximately 2V. In order 10 limit the current to safe value, a variable resistance R is inserted in the circuit at the time of changing armature connections.

We now investigate how braking torque depends upon the speed of the motor. Referring to Fig. (5.14) (ii),

$$\text{Armature current, } I_a = \frac{V+E_b}{R+R_a} = \frac{V}{R+R_a} + \frac{k_1 N\phi}{R+R_a} \qquad (\because E_b \propto \phi N)$$

$$\text{Braking torque, } T_B = k_2 I_a \phi = k_2 \phi \left(\frac{V}{R+R_a} + \frac{k_1 N\phi}{R+R_a} \right) = k_3\phi + k_4 N\phi^2$$

For a shunt motor, ϕ is constant.

$$\therefore \quad \text{Braking torque, } T_B = k_5 + k_6 N$$

Thus braking torque decreases as the motor slows down. Note that there is some braking torque ($T_B = k_5$) even when the motor speed is zero.

(iii) Regenerative braking

In the regenerative braking, the motor is run as a generator. As a result, the kinetic energy of the motor is converted into electrical energy and returned to the supply. Fig. (5.15) shows two methods of regenerative braking for a shunt motor.

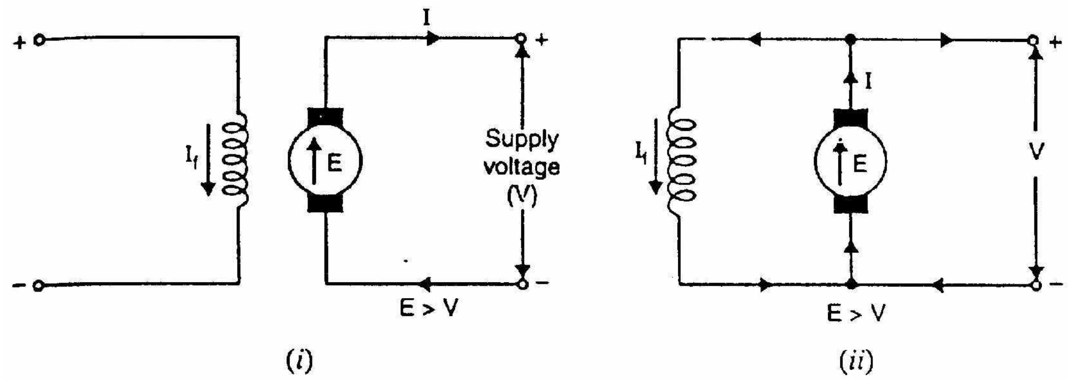


Fig. (5.15)

(a) In one method, field winding is disconnected from the supply and field current is increased by exciting it from another source [See Fig. 5.15 (i)]. As a result, induced e.m.f. E exceeds the supply voltage V and the machine feeds energy into the supply. Thus braking torque is provided upto the speed at which induced e.m.f. and supply voltage are equal. As the machine slows down, it is not possible to maintain induced e.m.f. at a higher value than the supply voltage. Therefore, this method is possible only for a limited range of speed.

(b) In a second method, the field excitation does not change but the load causes the motor to run above the normal speed (e.g., descending load on a crane). As a result, the induced e.m.f. E becomes greater than the supply voltage V [See Fig. 5.15 (ii)]. The direction of armature current I, therefore, reverses but the direction of shunt field current I_f remains unaltered. Hence the torque is reversed and the speed falls until E becomes less than V.

5.6 Speed Control of Compound Motors

Speed control of compound motors may be obtained by any one of the methods described for shunt motors. Speed control cannot be obtained through adjustment of the series field since such adjustment would radically change the performance characteristics of the motor.

5.7 Necessity of D.C. Motor Starter

At starting, when the motor is stationary, there is no back e.m.f. in the armature. Consequently, if the motor is directly switched on to the mains, the armature will draw a heavy current ($I_a = V/R_a$) because of small armature resistance. As an example, 5 H.P., 220 V shunt motor has a full-load current of 20 A and an armature resistance of about 0.5 Ω. If this motor is directly switched on to supply, it would take an armature current of 220/0.5 = 440 A which is 22 times the full-load current. This high starting current may result in:

(i) burning of armature due to excessive heating effect,
(ii) damaging the commutator and brushes due to heavy sparking,
(iii) excessive voltage drop in the line to which the motor is connected. The result is that the operation of other appliances connected to the line may be impaired and in particular cases, they may refuse to work.

In order to avoid excessive current at starting, a variable resistance (known as starting resistance) is inserted in series with the armature circuit. This resistance is gradually reduced as the motor gains speed (and hence E_b increases) and eventually it is cut out completely when the motor has attained full speed. The value of starting resistance is generally such that starting current is limited to 1.25 to 2 times the full-load current.

5.8 Types of D.C. Motor Starters

The stalling operation of a d.c. motor consists in the insertion of external resistance into the armature circuit to limit the starting current taken by the motor and the removal of this resistance in steps as the motor accelerates. When the motor attains the normal speed, this resistance is totally cut out of the armature circuit. It is very important and desirable to provide the starter with protective devices to enable the starter arm to return to OFF position

(i) when the supply fails, thus preventing the armature being directly across the mains when this voltage is restored. For this purpose, we use no-volt release coil.
(ii) when the motor becomes overloaded or develops a fault causing the motor to take an excessive current. For this purpose, we use overload release coil.

There are two principal types of d.c. motor starters viz., three-point starter and four-point starter. As we shall see, the two types of starters differ only in the manner in which the no-volt release coil is connected.

5.9 Three-Point Starter

This type of starter is widely used for starting shunt and compound motors.

Schematic diagram

Fig. (5.16) shows the schematic diagram of a three-point starter for a shunt motor with protective devices. It is so called because it has three terminals L, Z and A. The starter consists of starting resistance divided into several sections and connected in series with the armature. The tapping points of the starting resistance are brought out to a number of studs. The three terminals L, Z and A of the starter are connected respectively to the positive line terminal, shunt field terminal and armature terminal. The other terminals of the armature and shunt field windings are connected to the negative terminal of the supply. The no-volt release coil is connected in the shunt field circuit. One end of the handle is connected to the terminal L through the over-load release coil. The other end of the handle moves against a spiral spring and makes contact with each stud during starting operation, cutting out more and more starting resistance as it passes over each stud in clockwise direction.

Operation

(i) To start with, the d.c. supply is switched on with handle in the OFF position.
(ii) The handle is now moved clockwise to the first stud. As soon as it comes in contact with the first stud, the shunt field winding is directly connected across the supply, while the whole starting resistance is inserted in series with the armature circuit.
(iii) As the handle is gradually moved over to the final stud, the starting resistance is cut out of the armature circuit in steps. The handle is now held

magnetically by the no-volt release coil which is energized by shunt field current.

(iv) If the supply voltage is suddenly interrupted or if the field excitation is accidentally cut, the no-volt release coil is demagnetized and the handle goes back to the OFF position under the pull of the spring. If no-volt release coil were not used, then in case of failure of supply, the handle would remain on the final stud. If then supply is restored, the motor will be directly connected across the supply, resulting in an excessive armature current.

(v) If the motor is over-loaded (or a fault occurs), it will draw excessive current from the supply. This current will increase the ampere-turns of the over-load release coil and pull the armature C, thus short-circuiting the no-volt release coil. The no-volt coil is demagnetized and the handle is pulled to the OFF position by the spring. Thus, the motor is automatically disconnected from the supply.

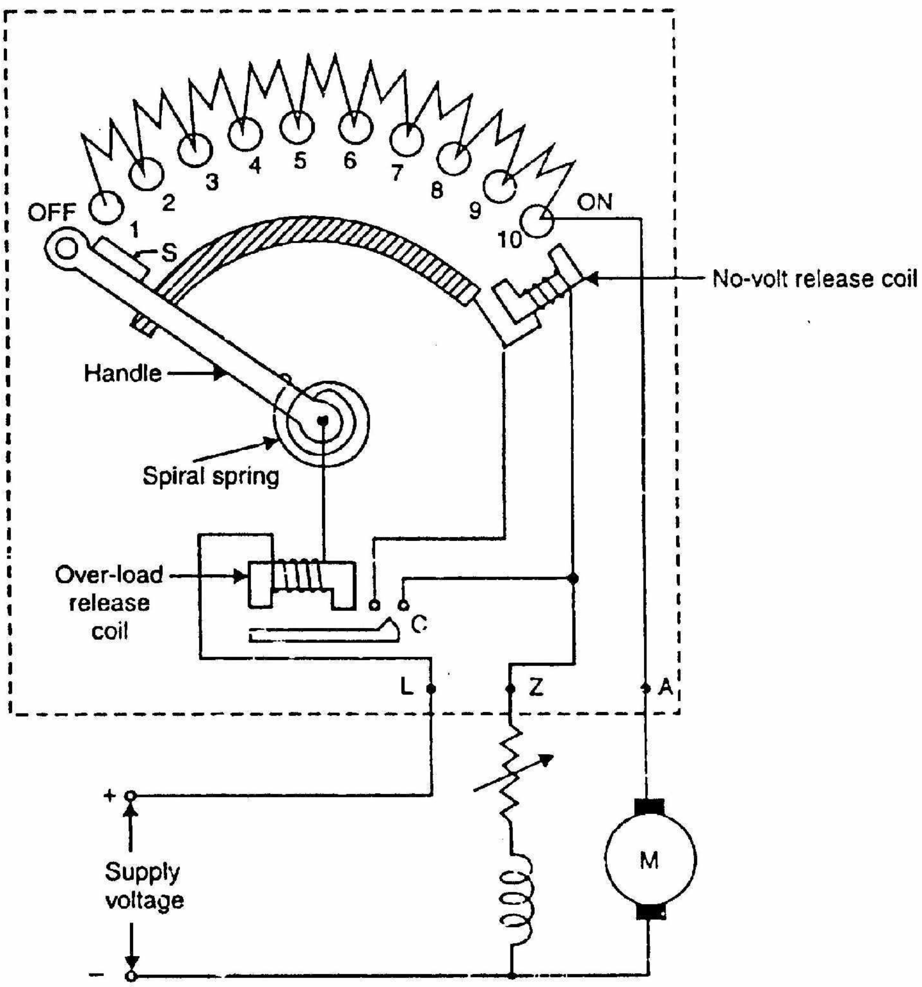


Fig. (5.16)

Drawback

In a three-point starter, the no-volt release coil is connected in series with the shunt field circuit so that it carries the shunt field current. While exercising speed control through field regulator, the field current may be weakened to such an extent that the no-volt release coil may not be able to keep the starter arm in the ON position. This may disconnect the motor from the supply when it is not desired. This drawback is overcome in the four point starter.

5.10 Four-Point Starter

In a four-point starter, the no-volt release coil is connected directly across the supply line through a protective resistance R. Fig. (5.17) shows the schematic diagram of a 4-point starter for a shunt motor (over-load release coil omitted for clarity of the figure). Now the no-volt release coil circuit is independent of the shunt field circuit. Therefore, proper speed control can be exercised without affecting the operation of no-volt release coil.

Note that the only difference between a three-point starter and a four-point starter is the manner in which no-volt release coil is connected. However, the working of the two starters is the same. It may be noted that the three-point starter also provides protection against an open-field circuit. This protection is not provided by the four-point starter.

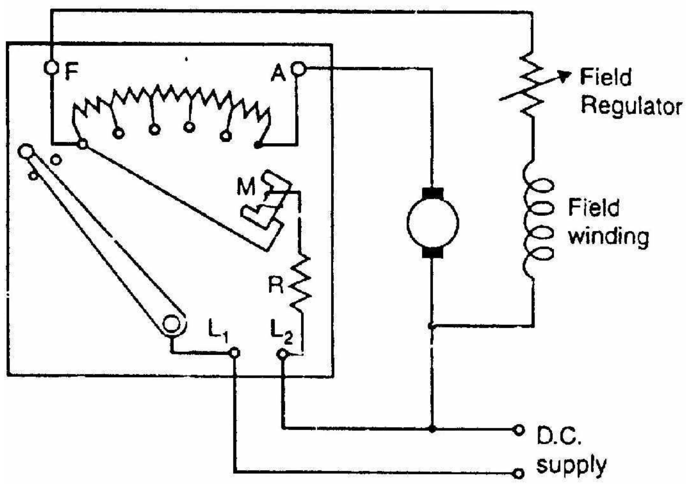


Fig. (5.17)

5.11 Grading of Starting Resistance—Shunt Motors

For starting the motor satisfactorily, the starting resistance is divided into a number of sections in such a way that current fluctuates between maximum (I_m) and minimum (I) values. The upper limit is that value established as the maximum permissible for the motor; it is generally 1.5 times the full-load current of the motor. The lower limit is the value set as a minimum for starting operation; it may be equal to full-load current of the motor or some predetermined value. Fig. (5.18) shows shunt-wound motor with starting resistance divided into three sections between four studs. The resistances of these sections should be so selected that current during starting remains between I_m and I as shown in Fig. (5.19).

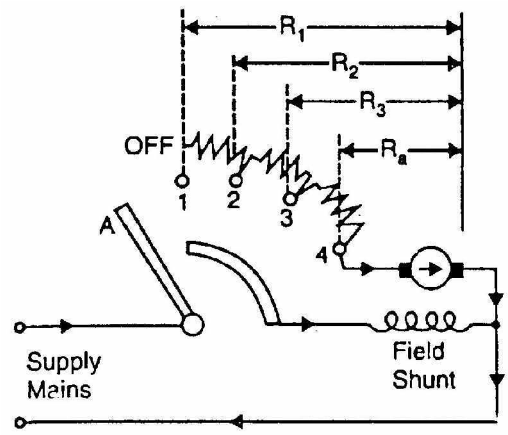


Fig. (5.18)

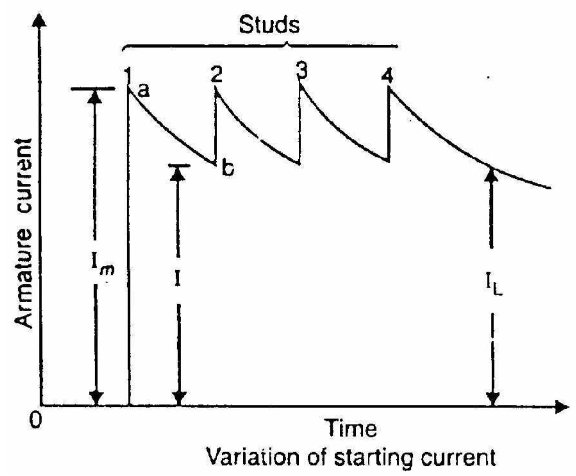


Fig. (5.19)

(i) When arm A is moved from OFF position to stud 1, field and armature circuits are energized and whole of the starting resistance is in series with the armature. The armature current jumps to maximum value given by;

$$I_m = \frac{V}{R_1}$$

where R_1 = Resistance of starter and armature

(ii) As the armature accelerates, the generated e.m.f. increases and the armature current decreases as indicated by curve ab. When the current has fallen to I, arm A is moved over to stud 2, cutting out sufficient resistance to allow the current to rise to I_m again. This operation is repeated until the arm A is on stud 4 and the whole of the starting resistance is cut out of the armature circuit.

(iii) Now the motor continues to accelerate and the current decreases until it settles down at some value I_L such that torque due to this current is just sufficient to meet the load requirement.

5.12 Starter Step Calculations for D.C. Shunt Motor

Fig. (5.20) shows a d.c. shunt motor starter with n resistance sections and (n + 1) studs.

Let R_1 = Total resistance in the armature circuit when the starter arm is on stud no. 1 (See Fig. 5.20)

R_2 = Total resistance in the armature circuit when the starter arm is on stud no. 2 and so on

I_m = Upper current limit
I = Lower current limit
n = Number of sections in the starter resistance
V = Applied voltage
R_a = Armature resistance

On stud 1. When the starter arm-moves to stud 1, the total resistance in the armature circuit is R_1 and the circuit current jumps to maximum values I_m given by;

$$I_m = \frac{V}{R_1} \qquad \text{(i)}$$

Since torque ∝ ϕI_a, it follows that the maximum torque acts on the armature to accelerate it. As the armature accelerates, the induced e.m.f. (back e.m.f.) increases and the armature current decreases When the current has fallen to the predetermined value I, the starter arm is moved over to stud 2. Let the value of back e.m.f. be E_{b1} at the instant the starter arm leaves the stud 1. Then I is given by;

$$I = \frac{V - E_{b1}}{R_1} \qquad \text{(ii)}$$

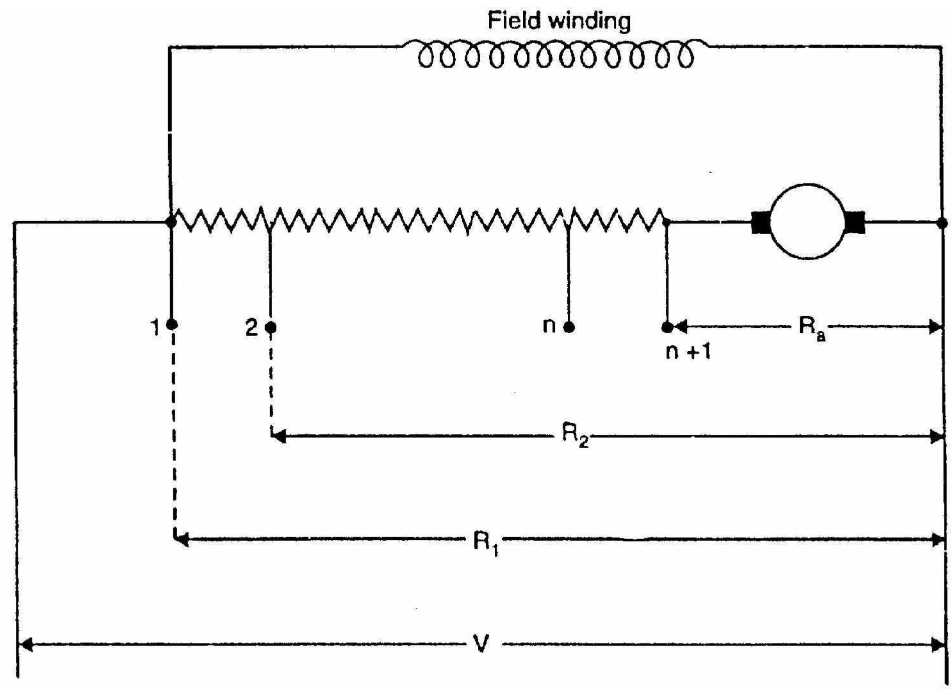


Fig. (5.20)

On stud 2. As the starter arm moves over to stud 2, sufficient resistance is cut out (now total circuit resistance is R_2) and current rises to maximum value I_m once again given by;

$$I_m = \frac{V - E_{b1}}{R_2} \qquad \text{(iii)}$$

The acceleration continues and the back e.m.f. increases and the armature current decreases. When the current has fallen to the predetermined value I, the starter arm is moved over to stud 3. Let E_{b2} be the value of back e.m.f. at the instant the starter arm leaves the stud 2. Then,

$$I = \frac{V - E_{b2}}{R_2} \qquad \text{(iv)}$$

On stud 3.

As the starter arm moves to stud 3, $I_m = \dfrac{V - E_{b2}}{R_3}$ (v)

As the starter arm leaves stud 3, $I = \dfrac{V - E_{b3}}{R_3}$ (vi)

On n<u>th</u> stud.

As the starter arm leaves n<u>th</u> stud, $I = \dfrac{V - E_{bn}}{R_n}$

On (n + 1)<u>th</u> stud. When the starter arm moves over to (n + 1)<u>th</u> stud, all the external starting resistance is cut out, leaving only the armature resistance R_a.

$$\therefore \quad I_m = \frac{V - E_{bn}}{R_a} \quad \text{and} \quad I = \frac{V - E_b}{R_a}$$

Dividing Eq.(ii) by Eq.(iii), we get,

$$\frac{I}{I_m} = \frac{R_2}{R_1}$$

Dividing Eq.(iv) by Eq. (v), we get,

$$\frac{I}{I_m} = \frac{R_3}{R_2}$$

Continuing these divisions, we Lave finally,

$$\frac{I}{I_m} = \frac{R_a}{R_n}$$

Let $\dfrac{I}{I_m} = k.$ Then $\dfrac{R_2}{R_1} = \dfrac{R_3}{R_2} = \dots = \dfrac{R_a}{R_n} = k$

If we multiply these n equal ratios together, then,

$$\frac{R_2}{R_1} \times \frac{R_3}{R_2} \times \frac{R_4}{R_3} \times \ldots \times \frac{R_a}{R_n} = k^n$$

$$\therefore \quad \frac{R_a}{R_1} = k^n$$

Thus we can calculate the values of R_2, R_3, R_4 etc. if the values of R_1, R_a and n are known.

Chapter (6)

Testing of D.C. Machines

Introduction

There are several tests that are conducted upon a d.c. machine (generator or motor) to judge its performance. One important test is performed to measure the efficiency of a d.c. machine. The efficiency of a d.c. machine depends upon its losses. The smaller the losses, the greater is the efficiency of the machine and vice-versa. The consideration of losses in a d.c. machine is important for two principal reasons. First, losses determine the efficiency of the machine and appreciably influence its operating cost. Secondly, losses determine the heating of the machine and hence the power output that may be obtained without undue deterioration of the insulation. In this chapter, we shall focus our attention on the various methods for the determination of the efficiency of a d.c. machine.

6.1 Efficiency of a D.C. Machine

The power that a d.c. machine receives is called the input and the power it gives out is called the output. Therefore, the efficiency of a d.c. machine, like that of any energy-transferring device, is given by;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \qquad \text{(i)}$$

$$\text{Output} = \text{Input} - \text{Losses} \qquad \text{and} \qquad \text{Input} = \text{Output} + \text{Losses}$$

Therefore, the efficiency of a d.c. machine can also be expressed in the following forms:

$$\text{Efficiency} = \frac{\text{Input} - \text{Losses}}{\text{Input}} \qquad \text{(ii)}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} \qquad \text{(iii)}$$

The most obvious method of determining the efficiency of a d.c. machine is to directly load it and measure the input power and output power. Then we can use Eq.(i) to determine the efficiency of the machine. This method suffers from three main drawbacks. First, this method requires the application of load on the machine. Secondly, for machines of large rating, the loads of the required sizes may not be available. Thirdly, even 'fit is possible to provide such loads, large power will be dissipated, making it an expensive method.

The most common method of measuring the efficiency of a d.c. machine is to determine its losses (instead of measuring the input and output on load). We can then use Eq.(ii) or Eq.(iii) to determine the efficiency of the machine. This method has the obvious advantage of convenience and economy.

6.2 Efficiency By Direct Loading

In this method, the d.c. machine is loaded and output and input are measured to find the efficiency. For this purpose, two simple methods can be used.

(i) Brake test

In this method, a brake is applied to a water-cooled pulley mounted on the motor shaft as shown in Fig. (6.1). One end of the rope is fixed to the floor via a spring balance S and a known mass is suspended at the other end. If the spring balance reading is S kg-Wt and the suspended mass has a weight of W kg-Wt, then,

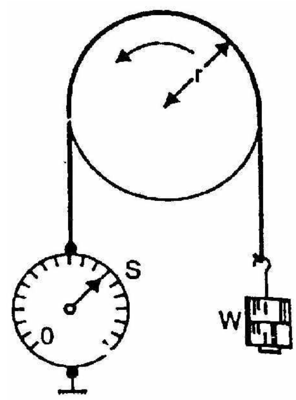


Fig. (6.1)

$$\text{Net pull on the rope} = (W - S)\text{ kg-Wt} = (W - S) \times 9.81 \text{ newtons}$$

If r is the radius of the pulley in metres, then the shaft torque T_{sh} developed by the motor is

$$T_{sh} = (W - S) \times 9.81 \times r \quad \text{N - m}$$

If the speed of the pulley is N r.p.m., then,

$$\text{Output power} = \frac{2\pi\, N\, T_{sh}}{60} = \frac{2\pi\, N \times (W - S) \times 9.81 \times r}{60} \quad \text{watts}$$

Let V = Supply voltage in volts
I = Current taken by the motor in amperes

$$\therefore \quad \text{Input to motor} = V\, I \quad \text{watts}$$

$$\therefore \quad \text{Efficiency} = \frac{2\pi\, N(W - S) \times r \times 9.81}{60 \times VI}$$

(ii) In another method, the motor drives a calibrated generator i.e. one whose efficiency is known at all loads. The output of the generator is measured with the help of an ammeter and voltmeter.

$$\therefore \quad \text{Output of motor} = \frac{\text{Generator output}}{\text{Generator efficiency}}$$

Let V = Supply voltage is volts

I = Current taken by the motor is amperes

Input to motor = VI

Thus efficiency of the motor can be determined.

Because of several disadvantages (See Sec. 6.1), direct loading method is used only for determining the efficiency of small machines.

6.3 Swinburne's Method for Determining Efficiency

In this method, the d.c. machine (generator or motor) is run as a motor at no-load and losses of the machine are determined. Once the losses of the machine are known, its efficiency at any desired load can be determined in advance. It may be noted that this method is applicable to those machines in which flux is practically constant at all loads e.g., shunt and compound machines. Let us see how the efficiency of a d.c. shunt machine (generator or motor) is determined by this method. The test insists of two steps:

(i) Determination of hot resistances of windings

The armature resistance and shunt field resistance are measured using a battery, voltmeter and ammeter. Since these resistances are measured when the machine is cold, they must be converted to values corresponding to the temperature at which the machine would work on full-load. Generally, these values are measured for a temperature rise of 40°C above the room temperature. Let the hot resistances of armature and shunt field be R_a and R_{sh} respectively.

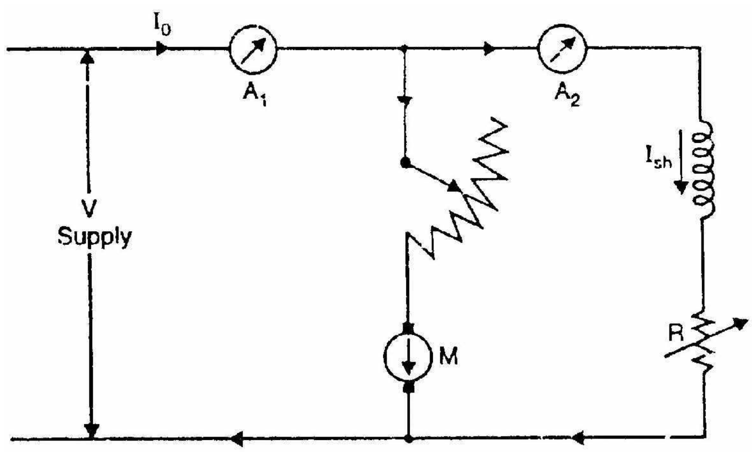


Fig. (6.2)

(ii) Determination of constant losses

The machine is run as a motor on no-load with supply voltage adjusted to the rated voltage i.e. voltage stamped on the nameplate. The speed of the motor is adjusted to the rated speed with the help of field regulator R as shown is Fig. (6.2).

Let V = Supply voltage

I_0 = No-load current read by ammeter A_1

I_{sh} = Shunt-field current read by ammeter A_2

$\therefore$ No-load armature current, $I_{a0} = I_0 - I_{sh}$

No-load input power to motor = $V\,I_0$

No-load power input to armature = $V\,I_{a0} = V(I_0 - I_{sh})$

Since the output of the motor is zero, the no-load input power to the armature supplies (a) iron losses in the core (b) friction loss (c) windage loss (d) armature Cu loss $\left[I_{a0}^2 R_a \text{ or } (I_0 - I_{sh})^2 R_a\right]$.

Constant losses, W_C = Input to motor − Armature Cu loss

or $$W_C = V\,I_0 - (I_0 - I_{sh})^2 R_a$$

Since constant losses are known, the efficiency of the machine at any other load can be determined. Suppose it is desired to determine the efficiency of the machine at load current I. Then,

Armature current, $I_a = I - I_{sh}$ ... if the machine is motoring

$= I + I_{sh}$ ... if the machine is generating

Efficiency when running as a motor

Input power to motor = VI

Armature Cu loss = $I_a^2 R_a = (I - I_{sh})^2 R_a$

Constant losses = W_C found above

Total losses = $(I - I_{sh})^2 R_a + W_C$

$$\therefore \quad \text{Motor efficiency, } \eta_m = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{VI - (I - I_{sh})^2 R_a - W_C}{VI}$$

Efficiency when running as a generator

Output of generator = VI

Armature Cu loss = $(I + I_{sh})^2 R_a$

Constant losses = W_C found above

$$\text{Total losses} = (I + I_{sh})^2 R_a + W_C$$

$$\therefore \quad \text{Generator efficiency, } \eta_g = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{VI}{VI + (I + I_{sh})^2 R_a + W_C}$$

Advantages of Swinburne's test

(i) The power required to carry out the test is small because it is a no-load test. Therefore, this method is quite economical.

(ii) The efficiency can be determined at any load because constant losses are known.

(iii) This test is very convenient.

Disadvantages of Swinburne's test

(i) It does not take into account the stray load losses that occur when the machine is loaded.

(ii) This test does not enable us to check the performance of the machine on full-load. For example, it does not indicate whether commutation on full-load is satisfactory and whether the temperature rise is within the specified limits.

(iii) This test does not give quite accurate efficiency of the machine. It is because iron losses under actual load are greater than those measured. This is mainly due to armature reaction distorting the field.

6.4 Regenerative or Hopkinson's-Test

This method of determining the efficiency cf a d.c. machine saves power and gives more accurate results. In order to carry out this test, we require two identical d.c. machines and a source of electrical power.

Principle

Two identical d.c. shunt machines are mechanically coupled and connected in parallel across the d.c. supply. By adjusting the field excitations of the machines, one is run as a motor and the other as a generator. The electric power from the generator and electrical power from the d.c. supply are fed to the motor. The electric power given to the motor is mostly converted into mechanical power, the rest going to the various motor losses. This mechanical power is given to the generator. The electrical power of the generator is given to the motor except that which is wasted as generator losses. Thus the electrical power taken from the d.c. supply is the sum of motor and generator losses and this can be measured directly by a voltmeter and an ammeter. Since the power input from the d.c. supply is equal to the power required to supply the losses of the two machines, this test can be carried out with a small amount of power. By adjusting the field strengths of the machines, any load can be put on the machines. Therefore, we can measure the total loss of the machines at any load. Since the machines can be tested under full-load conditions (of course at the expense of power equal to the losses in the two machines), the temperatures rise and commutation qualities of the machines can be observed.

Circuit

Fig. (6.3) shows the essential connections for Hopkinson's test. Two identical d.c. shunt machines are mechanically coupled and are connected in parallel across the d.c. supply. By adjusting the field strengths of the two machines, the machine M is made to run as a motor and machine G as a generator. The motor M draws current I_1 from the generator G and current I_2 from the d.c. supply so that input current to motor M is $(I_1 + I_2)$. Power taken from the d.c. supply is VI_2 and is equal to the total motor and generator losses. The field current of motor M is I_4 and that of generator G is I_3.

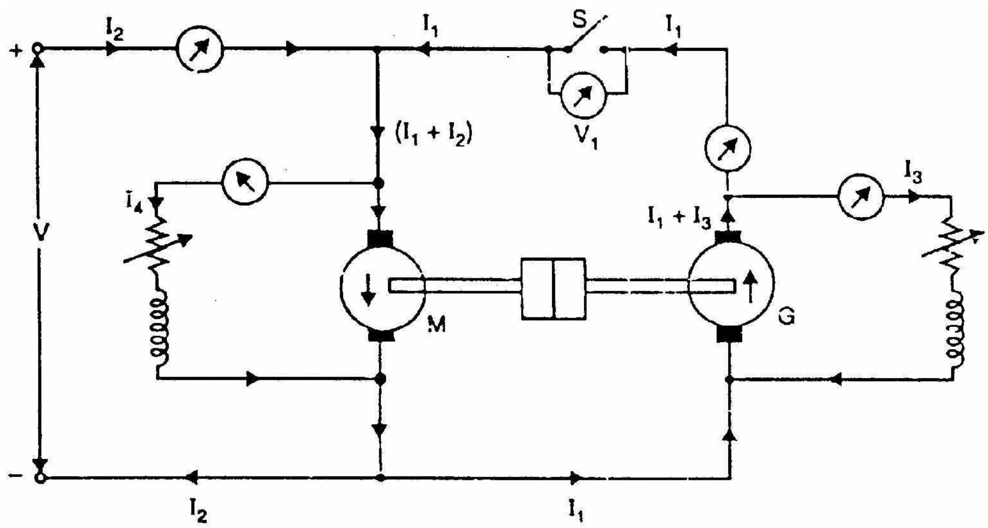


Fig. (6.3)

Calculations

If V be the supply voltage, then,

$$\text{Motor input} = V(I_1 + I_2)$$

$$\text{Generator output} = VI_1$$

We shall find the efficiencies of the machines considering two cases viz. (i) assuming that both machines have the same efficiency η (ii) assuming iron, friction and windage losses are the same in both machines.

(i) Assuming that both machines have the same efficiency η

Motor output = η × motor input = η $V(I_1 + I_2)$ = Generator input

Generator output = η × generator input = η × η $V(I_1 + I_2) = \eta^2 V(I_1 + I_2)$

But generator output is VI_1

$$\therefore \quad \eta^2 V(I_1 + I_2) = VI_1$$

or
$$\eta = \sqrt{\frac{I_1}{I_1 + I_2}}$$

This expression gives the value of efficiency sufficiently accurate for a rough test. However, if accuracy is required, the efficiencies of the two machines should be calculated separately as below.

(ii) Assuming that iron, friction and windage losses are same in both machines.

It is not to assume that the two machines have the same efficiency. It is because armature and field in the two machines are not the same. However, iron, friction and windage losses in the two machines will be the same because the machines are identical. On this assumption, we can find the of each machine as under:

Let R_a = armature resistance of each machine

I_3 = field current of generator G

I_4 = field current of motor M

Armature Cu loss in generator = $(I_1 + I_3)^2 R_a$

Armature Cu loss in motor = $(I_1 + I_2 - I_4)^2 R_a$

Shunt Cu loss in generator = $V I_3$

Shunt Cu loss in motor = $V I_4$

Power drawn from the d.c. supply is VI_2 and is equal to the total losses of the motor and generator

VI_2 = Total losses of motor and generator

If we subtract armature and shunt Cu losses of the two machines from VI_2, we get iron, friction windage losses of the two machines.

Iron, friction and windage losses of two machines (M and G)

$$= VI_2 - \left[(I_1 + I_3)^2 R_a + (I_1 + I_2 - I_4)^2 R_a + VI_3 + VI_4\right] = W \quad \text{(say)}$$

∴ Iron, friction and windage losses of each machine = W/2

For generator

Output of generator = VI_1

$$\text{Total losses} = \frac{W}{2} + (I_1 + I_3)^2 R_a + VI_3 = W_g \quad \text{(say)}$$

$$\therefore \quad \text{Generator efficiency, } \eta_g = \frac{VI_1}{VI_1 + W_g}$$

For motor

Input to motor = $V(I_1 + I_2)$

$$\text{Total losses} = (I_1 + I_2 - I_4)^2 R_a + VI_4 + \frac{W}{2} = W_m \quad \text{(say)}$$

$$\therefore \quad \text{Motor efficiency, } \eta_m = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{V(I_1 + I_2) - W_m}{V(I_1 + I_2)}$$

6.5 Alternate Connections for Hopkinson's Test

Fig. (6.4) shows the alternative connections for Hopkinson's test. The main difference is that now the shunt field windings are directly connected across the lines. Therefore, the input line current is I_1, excluding the field currents. The power VI_1 drawn from the d.c. supply is equal to the total losses of the two machines except the shunt field losses of the two machines i.e.,

VI_1 = Total losses o

f the two machines except shunt field losse s of the two machines

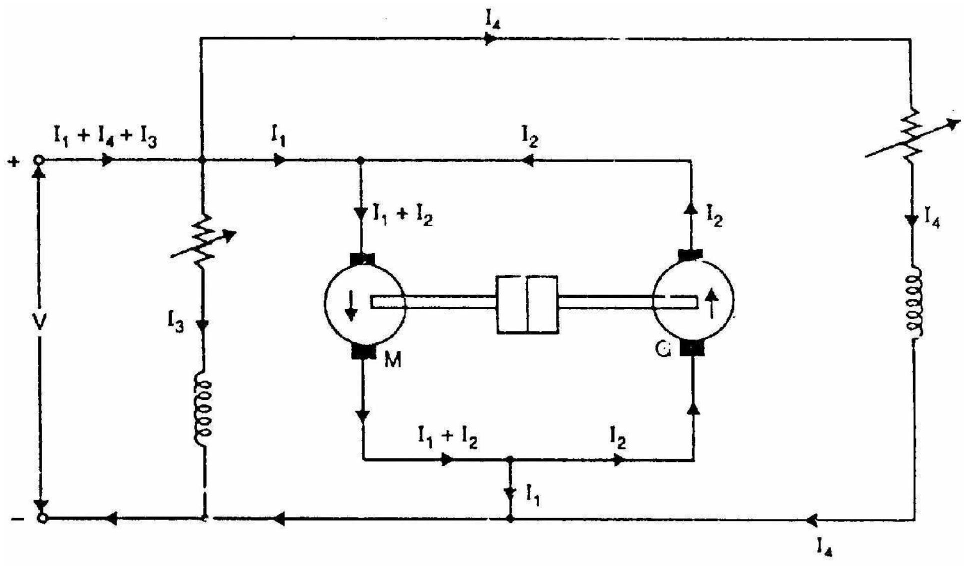


Fig. (6.4)

Motor armature Cu loss = $(I_1 + I_2)^2 R_a$

Generator armature Cu loss = $I_2^2 R_a$

Iron, friction and windage losses of the two machines are VI_1 minus armature Cu losses of the two machines i.e..

Iron, friction and windage losses of the two machines

$$= VI_1 - \left[(I_1 + I_2)^2 R_a + I_2^2 R_a\right] = W \quad \text{(say)}$$

Iron, friction and windage losses of each machine = W/2

Motor efficiency

Motor input, $P_i = V(I_1 + I_2 + I_3)$

Motor losses = $(I_1 + I_2)^2 R_a + VI_3 + \frac{W}{2} = W_m \quad \text{(say)}$

$$\therefore \quad \text{Motor efficiency, } \eta_m = \frac{\text{Motor input} - \text{Losses}}{\text{Motor input}} = \frac{P_i - W_m}{P_i}$$

Generator efficiency

Generator output = VI_2

Generator losses = $I_2^2 R_a + VI_4 + \frac{W}{2} = W_g \quad \text{(say)}$

$$\therefore \quad \text{Generator efficiency, } \eta_g = \frac{VI_2}{VI_2 + W_g}$$

6.6 Advantages of Hopkinson's Test

The advantages of Hopkinson's test are :

(i) The total power required to test the two machines is small compared with the full-load power of each machine.

(ii) The machines can be tested under full-load conditions so that commutation qualities and temperature rise can be checked.

(iii) It is more accurate to measure the loss directly than to measure it as the difference of the measured input and output.

(iv) All the measurements are electrical which are simpler and more accurate than mechanical measurements.

The main disadvantage is that two similar d.c. machines are required.

6.7 Retardation or Running down Test

This is the best and simplest method to find the efficiency of a constant-speed d.c. machine (e.g., shunt generator and motor). In this method, we find the mechanical (friction and windage) and iron losses of the machine. Then knowing the armature and shunt Cu losses at any load, the efficiency of the machine can be calculated at that load.

Principle

Consider a d.c. shunt motor running at no-load.

(i) If the supply to the armature is cut off but field remains normally excited, the motor slows down gradually and finally stops. The kinetic energy of the armature is used up to overcome friction, windage and iron losses.

(ii) If the supply to the armature as well as field excitation is cut off, the motor again slows down and finally stops. Now the kinetic energy of the armature is used up to overcome only the friction and windage losses. This is expected because in the absence of flux, there will be no iron losses.

By carrying out the first test, we can find out the friction, windage and iron losses and hence the efficiency of the machine. However, if we perform the second test also, we can separate friction and windage losses from the iron losses.

Theory of retardation test

In the retardation test, the d.c. machine is run as a motor at a speed just above the normal. Then the supply to the armature is cut off while the field is normally excited. The speed is allowed to fall to some value just below normal. The time taken for this fall of speed is noted. From these observations, the rotational losses (i.e., friction, windage and iron losses) and hence the efficiency of the machine can be determined.

Let N = normal speed in r.p.m.

ω = normal angular velocity in rad/s = 2π N/60

$\therefore$ Rotational losses, W = Rate of loss of K.E. of armature

or $$W = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\omega\frac{d\omega}{dt}$$

Here I is the moment of inertia of the armature. As $\omega = 2\pi$ N/60,

$$\therefore \quad W = I \times \frac{2\pi N}{60} \times \frac{d}{dt}\left(\frac{2\pi N}{60}\right) = \left(\frac{2\pi}{60}\right)^2 IN\frac{dN}{dt}$$

or $$W = 0.011\, IN\frac{dN}{dt}$$

Let us illustrate the application of retardation test with a numerical example. Suppose the normal speed of a d.c. machine is 1000 r.p.m. When retardation test is performed, the time taken for the speed to fall from 1030 r.p.m. to 970 r.p.m.

is 15 seconds with field normally excited. If the moment of inertia of the armature is 75 kg m , then,

$$\text{Rotational losses, } W = 0.011\, IN\frac{dN}{dt}$$

Here $\quad I = 75\ \text{kg}\,\text{m}^2; \qquad N = 1000\ \text{r.p.m}$

$$dN = 1030 - 970 = 60\ \text{r.pm.}; \qquad dt = 15\ \text{sec}$$

$$\therefore \quad W = 0.011 \times 75 \times 1000 \times \frac{60}{15} = 3300\ \text{ watts}$$

The main difficulty with this method is the accurate determination of the speed which is continuously changing.

6.8 Moment of Inertia (I) of the Armature

In retardation test, the rotational losses are given by;

$$W = 0.011\, IN\frac{dN}{dt}$$

In order to find W, the value of I must be known. It is difficult to determine I directly or by calculation. Therefore, we perform another experiment by which either I is calculated or it is eliminated from the above expression.

(i) First method

It is a fly-wheel method in which the value of I is calculated. First, retardation test is performed with armature alone and dN/dt_1 is determined. Next, a fly-wheel of known moment of inertia I_1 is keyed on to the shaft of the machine. For the same change in speed, dN/dt_2 is noted. Since the addition of fly-wheel will not materially affect the rotational losses in the two cases,

$$\therefore \quad \text{For the first case, } W = 0.011\, IN\frac{dN}{dt_1}$$

$$\text{For the second case, } W = 0.011\,(I + I_1)N\frac{dN}{dt_2}$$

$$\therefore \quad 0.011\, IN\frac{dN}{dt_1} = 0.011\,(I + I_1)N\frac{dN}{dt_2}$$

or

$$I\frac{dN}{dt_1} = (I + I_1)\frac{dN}{dt_2}$$

or

$$\frac{I + I_1}{I} = \frac{dN/dt_1}{dN/dt_2} = \frac{dt_2}{dt_1}$$

or $$\frac{I_1}{I} = \frac{dt_2 - dt_1}{dt_1} = \frac{t_2 - t_1}{t_1}$$

or $$I = I_1 \times \frac{t_2 - t_1}{t_1}$$

Since the values of I_1, t_1 and t_2 are known, the moment of inertia I of the armature can be determined.

(ii) Second method

In this method, I is eliminated from the expression by an experiment. First, retardation test is performed with armature alone. The rotational losses are given by;

$$W = 0.011\ IN\frac{dN}{dt_1}$$

Next the motor is loaded with a known amount of power W' with a brake. For the same change in speed, dN/dt_2 is noted. Then,

$$W + W' = 0.011\ IN\frac{dN}{dt_2}$$

$$\therefore \quad \frac{W + W'}{W} = \frac{dt}{dt_2} = \frac{t_1}{t_2}$$

or $$\frac{W'}{W} = \frac{t_1 - t_2}{t_2}$$

$$\therefore \quad W = W' \times \frac{t_1 - t_2}{t_2}$$

Since the values of W', t_1 and t_2 are known, the value of W can be determined.

6.9 Electric Loading in Retardation Test

In a retardation test, the rotational losses W are given by;

$$W = 0.011\ IN\frac{dN}{dt}$$

As discussed in Sec. (6.8), we can eliminate I (moment of inertia of armature) from the above expression by applying either mechanical or electric loading to the armature. The electric leading is preferred because of convenience and reliability. Fig. (6.5) illustrates how electric loading is applied to slow down the armature. The double throw switch S is thrown to the supply and the machine is brought to full-load speed. Then the switch S is thrown to the other side connecting a non-inductive resistance R across the armature. The supply now is cut off and the power dissipated in R acts as a retarding torque to slow down the armature.

Let V' = average voltage across R

I'_a = average current through R

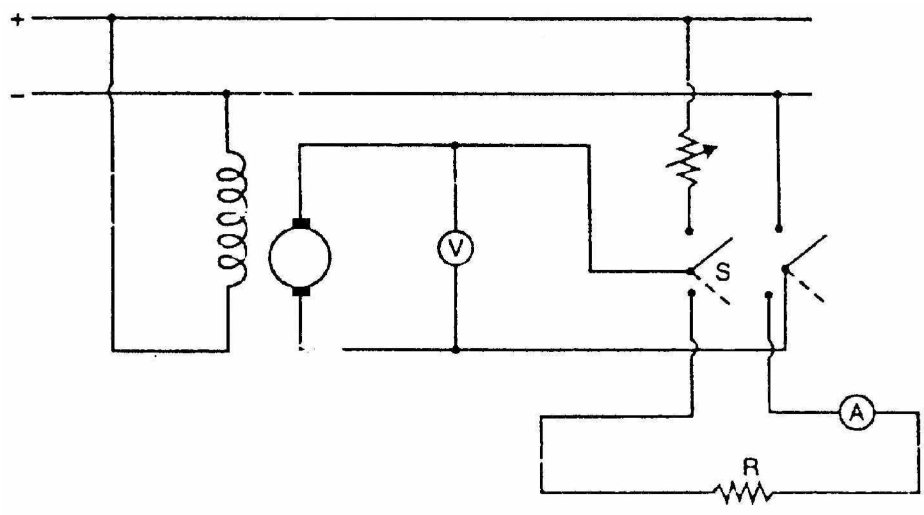


Fig. (6.5)

The electric loading W' (or extra power loss) is given by;

$$W' = \text{average voltage x average current} = V' I'_a$$