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Q1) Solve the following

QA) Find eigen value & eigen vector corresponding to lowest eigen value.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Ans→ Polynomial is $\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

$S_1 = 0$

$$S_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$S_2 = -3$

$$|A| = 0 - (1)\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 2$$

$\therefore \lambda^3 - 3\lambda - 2 = 0$

$\therefore (-1)$ is the root

$\therefore (\lambda + 1)(\lambda^2 - \lambda - 2) = 0$

$(\lambda + 1)(\lambda - 2)(\lambda + 1) = 0$

Eigen values are $\lambda = -1, -1, 2$

Eigen vectors for $\lambda = -1$

$$A - \lambda I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigen value $\lambda = -1$

$$A + \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + x_2 + x_3 = 0$

$x_1 = -x_2 - x_3$

Eigen vector is $\begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix}$

Eigen vector corresponding to $\lambda = -1$

Let $x_2 = 1 \quad x_3 = 0$

$$V_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Let $x_2 = 0, \; x_3 = 1$

$$V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Now,
two vectors (-1,1,0) & (-1,0,1) are linearly independant

$$\begin{bmatrix} -x_2-x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Put $x_2 = 5 = 1$, we get Partial vector $= \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Q.B] Verify Cayley Hamilton theorem & find A^{-1} & A^{-2} using Cayley Hamilton theorem

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

Ans→ $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

$$[A-\lambda I] = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 1 & 1-\lambda \end{bmatrix}$$

$\therefore |A-\lambda I| = \lambda^2 - 2\lambda - 1 = 0$ .... Characteristic eqn

by putting A in eqn

$\therefore A^2 - 2A - 1 = 0$ — ①

$$\therefore \text{LHS} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}^2 - 2\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - 1I$$

$$= \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2-1 & 4-4-0 \\ 2-2-0 & 3-2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$= \text{R.H.S}$$

$\therefore$ From above calculation LHS = RHS
Hence, Cayley Hamilton theorem verified.

Now,
to find A^{-1} we multiply (1) by A^{-1}

$$\therefore A^2A^{-1} - 2AA^{-1} + A^{-1} = 0$$

$$\therefore A(AA^{-1}) - 2(AA^{-1}) = A^{-1}$$

$$\therefore AI - 2I = A^{-1}$$

$$\therefore A - 2 = A^{-1}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-0 \\ 1-0 & 1-2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

Now for A^{-2}

$$\therefore A^{-2}A^2 = 2AA^{-2} - IA^{-2} = 0$$

$$I - 2A^{-1} - A^{-2} = 0$$

$$A^{-2} = I - 2A^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 2 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$$

Q5C] Given a line segment starting at a point (0,0) ending point is (8,1). Rotate line by 45° & find new coordinate.

Ans> Old ending coordinates of the line
$= (x_{old}, y_{old}) = (8,1)$
Rotation angle = 45°

Let new ending coordinates of the line after rotation = (x_{new}, y_{new})

Applying the rotation equations, we have.

For x_{new},

$$= x_{old} \times \cos\theta - y_{old} \times \sin\theta$$
$$= 8 \times \cos(45°) - 1 \times \sin(45°)$$
$$= 8 \times 1/\sqrt{2} - 1 \times 1/\sqrt{2}$$
$$= \frac{8}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$
$$= \frac{7}{\sqrt{2}} = \frac{4.9}{}$$

For y new,

$= X_{old} \times \cos\theta + Y_{old} \times \sin\theta$

$= 8 \times \cos(45^\circ) + 1 \times \sin(45^\circ)$

$= 8 \times 1/\sqrt{2} + 1 \times 1/\sqrt{2}$

$= 8/\sqrt{2} + 1/\sqrt{2}$

$= 9/\sqrt{2}$

$= 6.3$

Thus, New ending coordinates of the line after rotation = (4.3, 6.3)

Q2) Fill in the blanks -

A) The given characteristic equation is $\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$. The algebraic multiplicity of each eigen value is & eigen value of A^{-2} is -

Ans-) Eigen values are - $\lambda = 3, 2$

Algebric multiplicity $= |A| = 6$

Eigen values of A^{-2} are $\frac{1}{4}$ & $\frac{1}{9}$

→

B) Given a square whose coordinates are given by $A \equiv (2,1)$ $B \equiv (3,1)$ $C \equiv (3,4)$ $D \equiv (2,4)$. Translate square by 7 units right & 6 units down. Find new coordinates.

Ans -> The new coordinates are:

$A = (9,-5)$ $\quad$ $C = (10,-2)$

$B = (10,-5)$ $\quad$ $D = (9,-2)$

c) The sum & product of eigen values of $A_{n \times n}$ matrix is if A have $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$ are eigen values of A.

Ans -> The sum of eigen values is the sum of principle elements of matrix and product of eigen values is the determinant value of matrix.

D) If A is orthogonal matrix if 6 is one eigen value then other eigen value is

Ans => -6