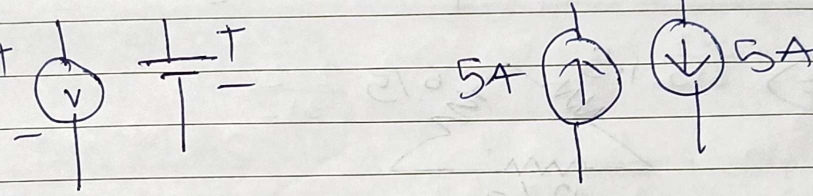
Unit - 1
Circuit Simplification Techniques.

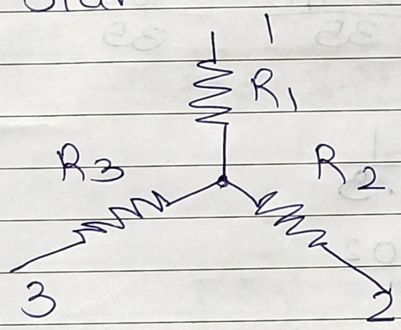
* Networks -
① Active / Passive ~~n/w~~ n/w
② Linear / Non-linear n/w
③ Unilateral / Bi-lateral n/w.

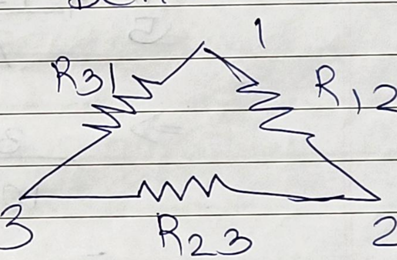
Sources -

voltage — current



Star - Delta





S to D

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

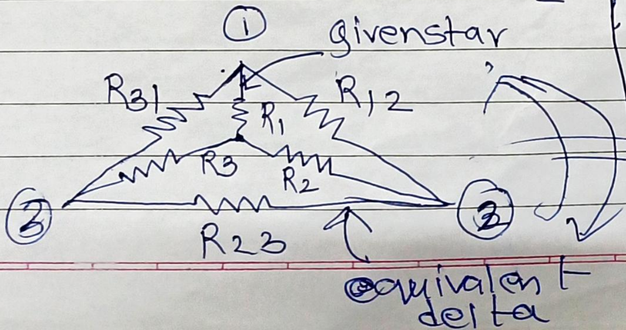
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

D to S

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$



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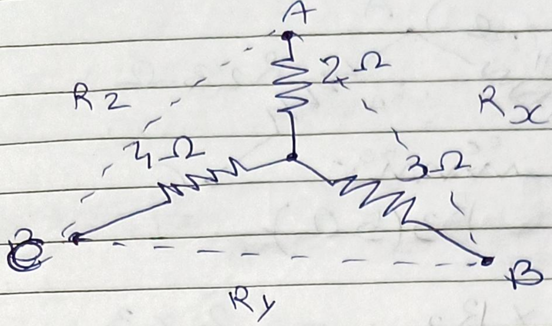
③ Star to delta



Using formula.

$$R_x = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$= 2 + 3 + \frac{2 \times 3}{4}$$

$$\boxed{R_x = 6.5}$$

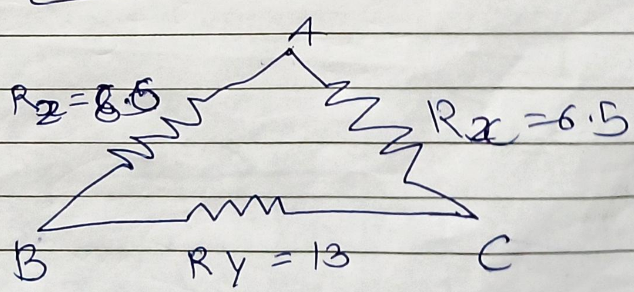
Simly - $R_y = 3 + 4 + \frac{3 \times 4}{2}$

$$\boxed{R_y = 13}$$

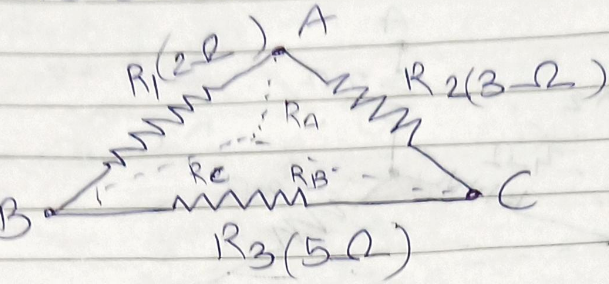
Simly - $R_2 = 2 + 4 + \frac{2 \times 4}{3}$

$$\boxed{R_2 = 8.6}$$

⇒



(3) Delta to Star

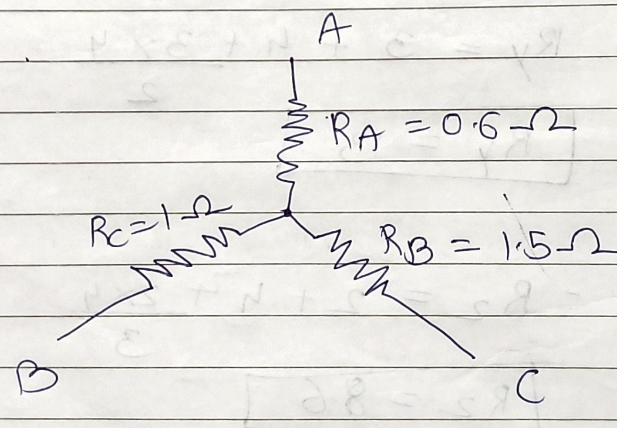


$$\therefore R_A = \frac{R_1 \times R_2}{R_1 + R_2 + R_3} = \frac{2 \times 3}{2+3+5} = 0.6\,\Omega$$

$$R_B = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} = \frac{3 \times 5}{2+3+5} = 1.5\,\Omega$$

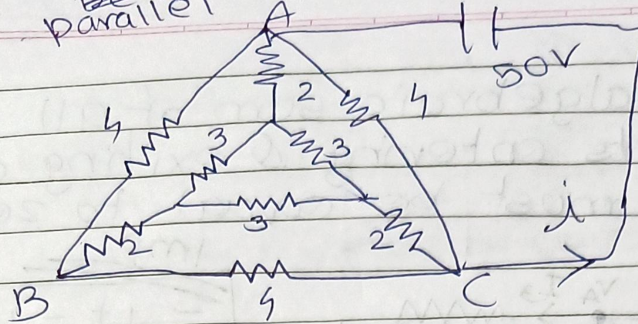
$$R_C = \frac{R_1 \times R_3}{R_1 + R_2 + R_3} = \frac{2 \times 5}{2+3+5} = 1\,\Omega$$

$\Rightarrow$



Series + ~~Delta~~ parallel: ~~$\frac{a\times b}{a+b}$~~ $a^{-1}+b^{-1}=ans^{-1}=$

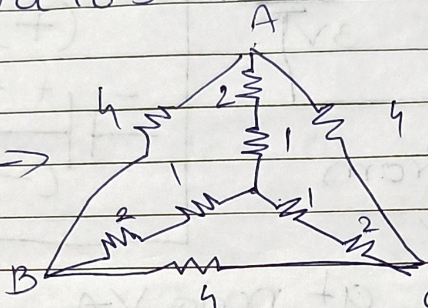
Q)



Delta to star.

$\Rightarrow$

series $\rightarrow$

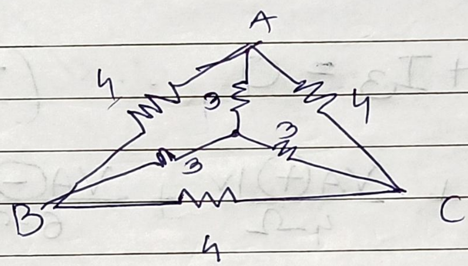


$R_1 = \frac{3\times 3}{3+3+3}$, $R_2 = \frac{3\times 3}{3+3+3}$

$R_3 = \frac{3\times 3}{3+3+3}$

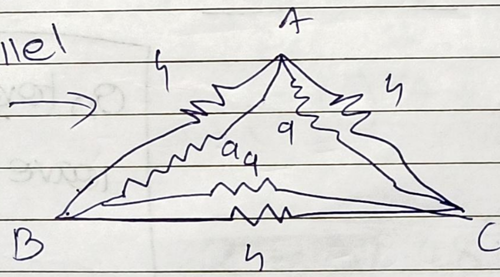
series $\rightarrow$ $a+b$
Parallel $\rightarrow$ $a^{-1}+b^{-1}$ $= Ans^{-1}$ $=$

$\Rightarrow$



Star to Delta.

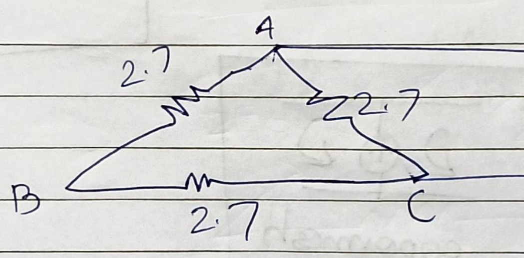
parallel $\rightarrow$



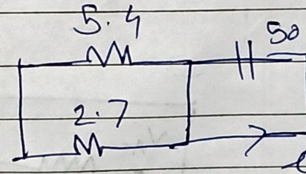
$R_{A} = R_1 + R_2 + \frac{R_1R_2}{R_3} = 9$

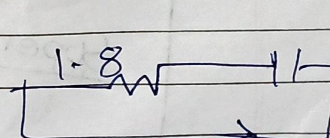
$R_B = 3+3+\frac{3\times 3}{3} = 9$

$R_C = \quad = 9$



$\Rightarrow$





$I = \frac{V}{R} = \frac{50}{1.8}$

$I = \underline{\underline{27.77}}$ Amp

Nodal analysis → KCL

KCL

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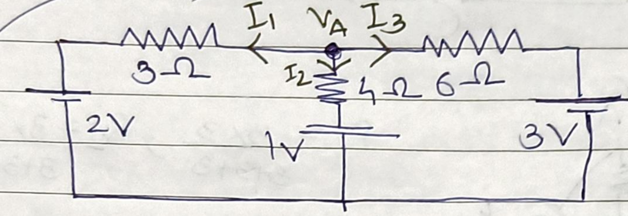
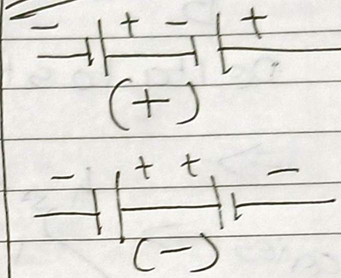
KCL - The algebraic sum of all currents entering & exiting a node must be equal to zero

Eg)



imp



assume away from jun.

Applying KCL at node V_A

assume V_A at higher potential

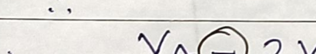
$$I_1 + I_2 + I_3 = 0 \quad \ldots \left(I = \frac{V}{R}\right)$$

$$\therefore \frac{V_A - 2V}{3\Omega} + \frac{V_A + 1V}{4\Omega} + \frac{V_A - 3V}{6\Omega} = 0$$

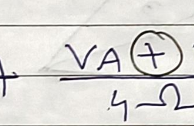
$$\therefore V_A = \_\_\_\_\_\_$$

entering → −ve

leave → +ve



Supernode



supermesh

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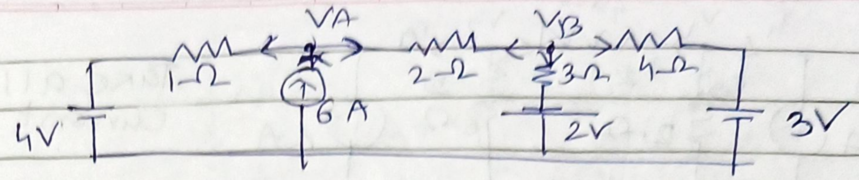
Q)



Apply KCL at node VA

$$\frac{V_A - 4}{1} + \frac{V_A - V_B}{2} = 6$$

Apply KCL at node VB

$$\frac{V_B - V_A}{2} + \frac{V_B + 2}{3} + \frac{V_B - 3}{4} = 0$$

$$V_A - 4 + \frac{V_A}{2} - \frac{V_B}{2} = 6$$

$$\left(1 - \frac{1}{2}\right) V_A - \left(\frac{1}{2}\right) V_B = 6 + 4 \quad \text{— (I)}$$

$$\frac{V_B}{2} - \frac{V_A}{2} + \frac{V_B}{3} + \frac{2}{3} + \frac{V_B}{4} - \frac{3}{4} = 0$$

$$\left(-\frac{1}{2}\right) V_A + \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) V_B = -\frac{2}{3} + \frac{3}{4} \quad \text{— (II)}$$

$x = 37.28 \qquad y = 17.28$

i.e. $V_A = 37.28V$, $V_B = 17.28$

entry → −ve
leave → +ve

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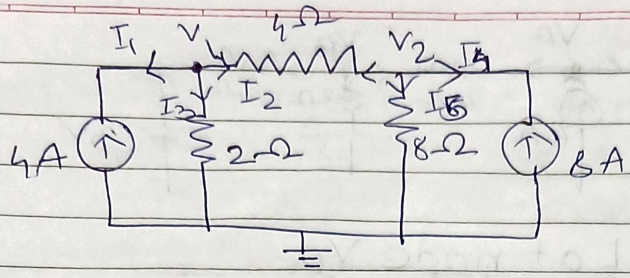
(8)



Take all current leave

Applying KCL at node V_1

$$I_1 + I_2 + I_3$$

$$-4 + \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2} = 0$$

$$-4 + \frac{V_1}{4} - \frac{V_2}{4} + \frac{V_1 - 0}{2} = 0$$

$$\left(\frac{1}{4} + \frac{1}{2}\right)V_1 - \left(\frac{1}{4}\right)V_2 = 4 \quad -(1)$$

Applying KCL at node V_2

$$I_2 + I_4 + I_5$$

$$\frac{V_2 - V_1}{4} + \frac{V_2 - 0}{8} + -8 = 0$$

$$\frac{V_2}{4} - \frac{V_1}{4} + \frac{V_2}{8} = 8$$

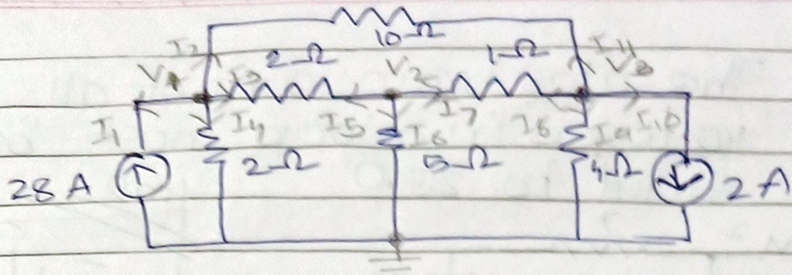
$$\left(\frac{1}{4} + \frac{1}{8}\right)V_2 - \left(\frac{1}{4}\right)V_1 = 8 \quad -(1)$$

solving (1) & (2)

$$x = 16, \quad y = 32$$

$$\therefore V_1 = 16V, \quad V_2 = 32$$

8) For the circuit shown below find the current flowing through 5ohm resistance by nodal analysis



Apply KCL at node 1

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$-28 + \frac{V_1 - V_3}{10} + \frac{V_1 - V_2}{2} + \frac{V_1 - 0}{2} = 0$$

$$\frac{V_1}{10} - \frac{V_3}{10} + \frac{V_1}{2} - \frac{V_2}{2} + \frac{V_1}{2} = 28$$

$$\left(\frac{1}{10} + \frac{1}{2} + \frac{1}{2}\right)V_1 - \left(\frac{1}{2}\right)V_2 - \left(\frac{1}{10}\right)V_3 = 28 \quad \text{—(I)}$$

Apply KCL at node 2

$$I_5 + I_6 + I_7 = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{5} + \frac{V_2 - V_3}{1} = 0$$

$$\frac{V_2}{2} - \frac{V_1}{2} + \frac{V_2}{5} + V_2 - V_3 = 0$$

$$\left(-\frac{1}{2}\right)V_1 + \left(1 + \frac{1}{2} + \frac{1}{5}\right)V_2 - (1)V_3 = 0 \quad \text{—(II)}$$

Apply KCL at node 3

$$I_8 + I_9 + I_{10} + I_{11} = 0$$

$$\frac{V_3 - V_2}{1} + \frac{V_3 - 0}{4} - 2 + \frac{V_3 - V_1}{10} = 0$$

$$V_3 - V_2 + \frac{V_3}{4} - 2 + \frac{V_3}{10} - \frac{V_1}{10} = 0$$

$$\left(-\frac{1}{10}\right)V_1 - V_2 + \left(1 + \frac{1}{4} + \frac{1}{10}\right)V_3 = 2 \quad \text{—(III)}$$

Solving (I), (II), (III)

$x = 38.76$, $y = 24.74$, $z = 22.68$

$\therefore$ current through $5\Omega = \frac{V_2 - 0}{5} = \frac{24.74 - 0}{5} = 4.948\,A$

mesh - smallest loop which cannot be further divided

KVL

mesh analysis → KVL

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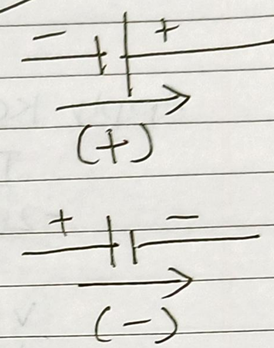
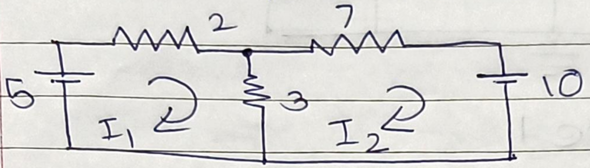
KVL - The algebraic sum of all voltages in a loop must be equal to zero

imp



whenever their is resistance their is voltage drop (−)

eg >



Applying KVL in loop ①

V=IR

$5 - 2I_1 - 3(I_1 - I_2) = 0$

$5 - 2I_1 - 3I_1 - 3I_2 = 0$

$5 - 5I_1 + 3I_2 = 0$

$\underline{5I_1 + 3I_2 = 5}$ — ①

Applying KVL in loop ②

$-3(I_2 - I_1) - 7I_2 - 10 = 0$

$-3I_2 + 3I_1 - 7I_2 - 10 = 0$

$\underline{10I_2 - 3I_1 = -10}$ — ②

solving ① & ② simultaneously (by Crammer rule.

$x = \frac{-1}{3} = -0.33$ $y = \frac{20}{9} = 2.22$

Shortcut
add R

loop 1

$3 + 2 = 5$

$5I_1 - 3I_2 = 5$

(−)

loop 2

$7 + 3 = 10$

$10I_2 \cancel{+} - 3I_1 = -10$

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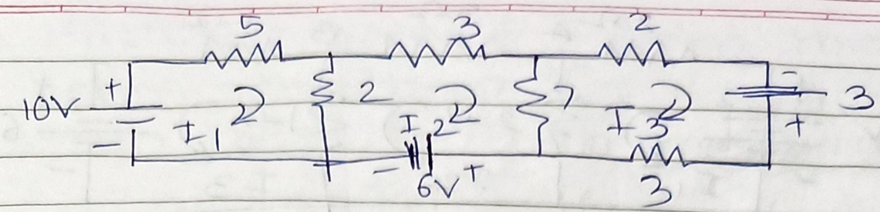
9)



Applying KVL in loop 1

$7I_1 - 2I_2 = 10\,V$ — (I)

Applying KVL in loop 2

$12 I_2 - 2 I_1 - 7 I_3 = -6\,V$ — (II)

Applying KVL in loop 3

$12 I_3 - 7 I_2 = 3V$ — (III)

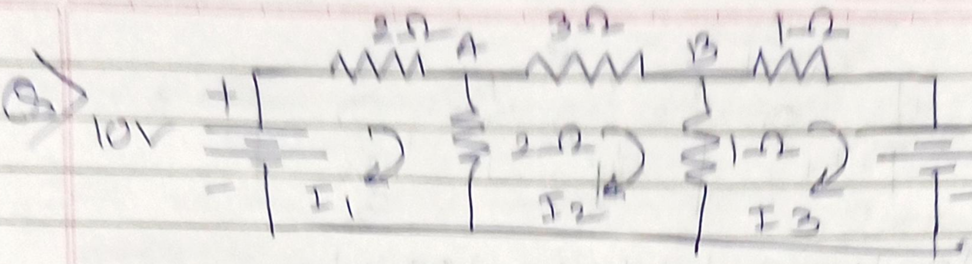
Solvin simultaneously

$x = \frac{848}{617} = 1.37$ $\therefore I_1 = 1.37$

$y = \frac{117}{617} = 0.189$ $\therefore I_2 = 0.189$

$z = \frac{86}{617} = 0.139$ $\therefore I_3 = 0.139$

Find current in AB or flowing thro A-B & power across 3Ω

(3) 

Applying KVL in loop (1)

$$-10 + 2I_1 + 2(I_1 - I_2) = 0$$
$$-10 + 2I_1 + 2I_1 - 2I_2 = 0$$

$$4I_1 - 2I_2 = 10 \quad -(1)$$

loop (2)

$$3I_2 + 1(I_2 - I_3) + 2(I_2 - I_1) = 0$$
$$3I_2 + I_2 - I_3 + 2I_2 - 2I_1 = 0$$

$$6I_2 - 2I_1 - I_3 = 0 \quad -(11)$$

loop (3)

$$6 + 1(I_3 - I) + I_3 = 0$$
$$6 + I_3 - I_2 + I_3 = 0$$

$$2I_3 - I_2 = -6 \quad -(111)$$

solving simultaneously.

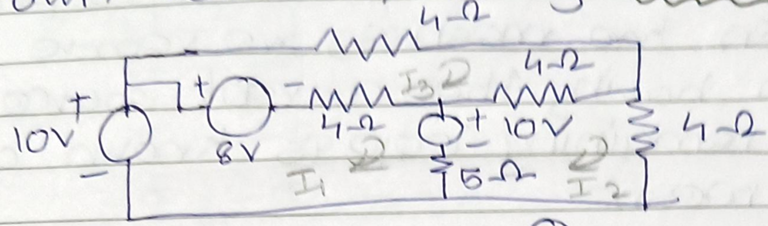
$x = \frac{49}{18} = 2.72$, $y = \frac{4}{9} = 0.44$, $z = \frac{-25}{9} = 2.77$

i.e. $I_1 = 2.72$, $I_2 = 0.44$, $I_3 = 2.77$

$$\boxed{I_2 = 0.44 A}$$

$$\boxed{\begin{aligned} P &= I^2R = (0.44)^2 \times 3 \\ P &= 0.581 \, W \end{aligned}}$$

Q) Find the current flowing through 5ohm resistance for the network shown below by using mesh analysis.



Applying KVL in loop ①

$9I_1 - 4I_3 - 5I_2 = 10 + -8 - 10$

$9I_1 - 5I_2 - 4I_3 = -8$ — (i)

Applying KVL in loop ②

$13I_2 - 5I_1 - 4I_3 = 10$

$-5I_1 + 13I_2 - 4I_3 = 10$ — (ii)

Applying KVL in loop ③

$12I_3 - 4I_1 - 4I_2 = +8$

$-4I_1 - 4I_2 + 12I_3 = 8$ — (iii)

Solving (i), (ii), (iii) simultaneously.

$x = 0.36$, $y = 1.28$, $z = 1.21$

$\therefore I_1 = 0.36A$, $I_2 = 1.28A$, $I_3 = 1.21A$

$\therefore$ The current flowing through 5Ω i.e.

Total current $I_{5\Omega} = I_2 - I_1$

$= 1.28 - 0.36$

$\boxed{I_{5\Omega} = 0.92A}$

Mesh Analysis

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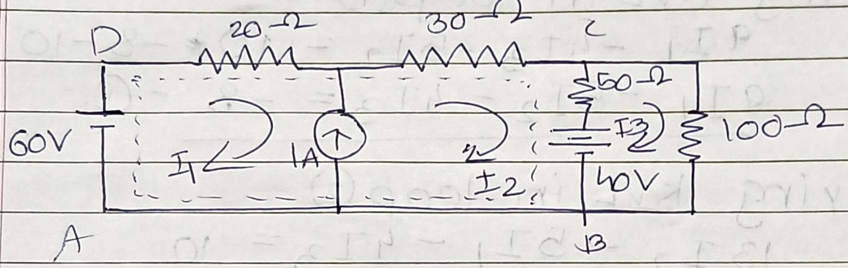
* Supermesh -
When a current source is present between two meshes, we remove the branch having the current source and then remaining loop is known as supermesh.

Q)


Apply KVL in loop 3

$$0 I_1 - 50 I_2 + 150 I_3 = 40V \quad - (3)$$

Apply supermesh.

$$I_2 - I_1 = 1 \quad - (2)$$

Applying K.V.L to complete loop of supermesh.

$$20 I_1 + 50 I_2 - 50 I_3 = 60 - 40$$

$$20 I_1 + 80 I_2 - 50 I_3 = 20 \quad - (2)$$

Solving (1), (2), (3) s——

$I_1 =$ ____, $I_2 =$ ____ $I_3 =$ ____

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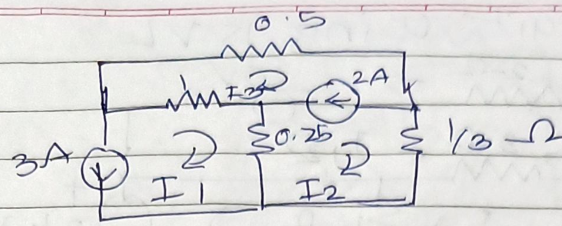
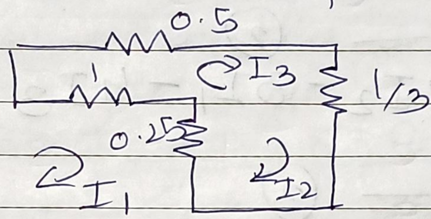
(Q)



$I_1 = -3\,A$ — (i)

$I_3 - I_2 = 2A$ — (ii)

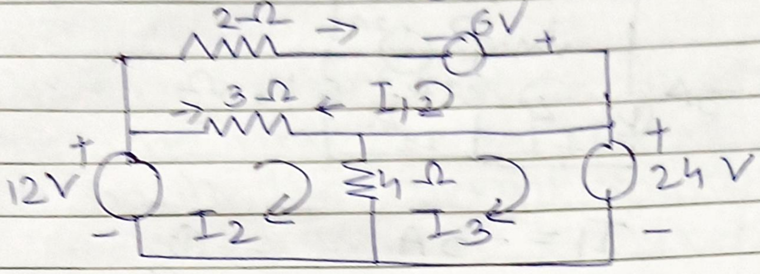
for eqn (3) remove 2A current source & write eqn for supermesh.



$$-0.5I_3 - 0.33I_2 - 0.25(I_2 - I_1) - (I_3 - I_1) = 0$$
$$-0.5I_3 - 0.33I_2 - 0.25(I_2 + 3) - (I_3 + 3) = 0$$
$$-0.58\,I_2 - 1.5I_3 = 3.75$$
put $I_3 = I_2 + 2$

$$\therefore \boxed{\begin{aligned} I_2 &= -3.24\,A \\ I_1 &= -1.24\,A \end{aligned}}$$

3) Find I_1, I_2 & I_3 for given circuit using KVL



App KVL in loop ①

$$5I_1 - 3I_2 = 6V \quad \text{— (i)}$$

②

$$7I_2 - 3I_1 - 4I_3 = 12V \quad \text{— (ii)}$$

③

$$4I_3 - 4I_2 = -24 \quad \text{— (iii)}$$

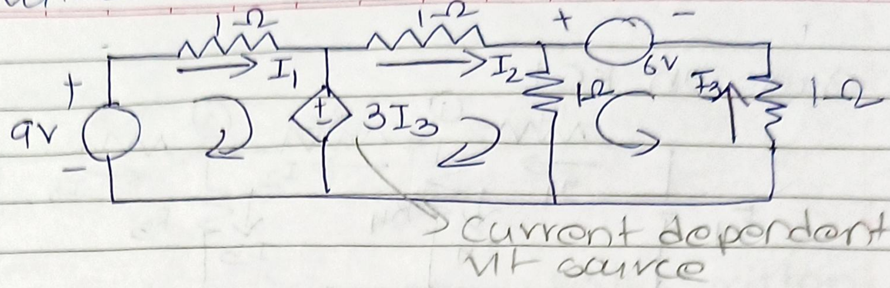
$x = -3A \quad y = -7A \quad z = -13\ A$

i.e. $I_1 = -3A$

$I_2 = -7A$

$I_3 = -13A$

Q) For the network shown in fig determine the values of I_1, I_2 & I_3



applying KVL in loop ①

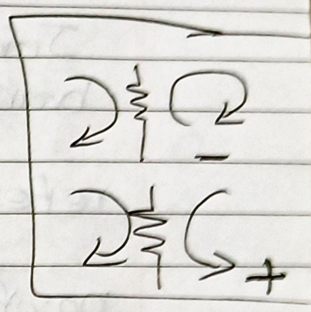
$$1I_1 + 3I_3 = 9 \quad — ①$$

Applying KVL in loop ②

$$2I_2 - 3I_3 + I_3 = 0$$

$$\cancel{2I_2 - 3I_3}$$

$$2I_2 - 2I_3 = 0 \quad — ②$$



Applying KVL in loop ③

$$2I_3 + 1I_2 = 6V$$

$$I_2 + 2I_3 = 6\,V \quad — ③$$

$x = 3, \; y = 2, \; z = 2.$

i.e. $I_1 = 3A, \; I_2 = 2A, \; I_3 = 2A$

Nodal Analysis

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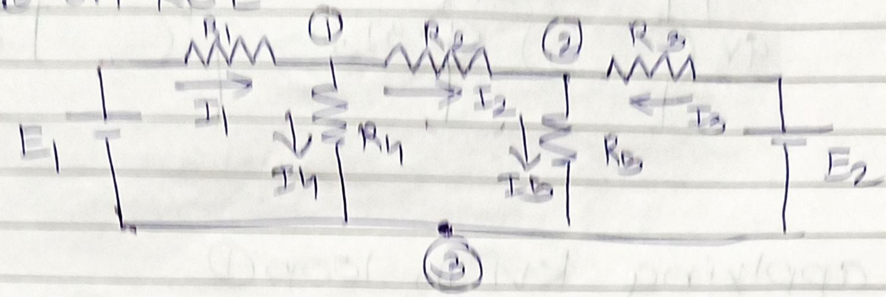
Base on KCL



Junction where three or more branches meet is called as node

One of these is assumed as reference/ datum/ zero potential node

If n no. of total nodes are there no. of equations will be (n-1)

Node ③ in reference node

at node ①

$$I_1 = I_2 + I_4 \quad — Ⓐ$$

$$I_1 R_1 = E_1 - V_A$$

$$\therefore I_1 = \frac{E_1 - V_A}{R_1} \quad — ①$$

$$I_4 = \frac{V_A}{R_4}$$

$$I_2 R_2 = V_A - V_B$$

$$\therefore I_2 = \frac{V_A - V_B}{R_2}$$

put I_2 & I_4 in eqn Ⓐ

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0$$

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At node 2

$$I_5 = I_2 + I_3 \quad \text{— (B)}$$

$$I_5 = \frac{V_B}{R_5}$$

$$I_2 = \frac{V_A - V_B}{R_2} \quad \text{... already written}$$

$$I_3 = \frac{E_2 - V_B}{R_3}$$

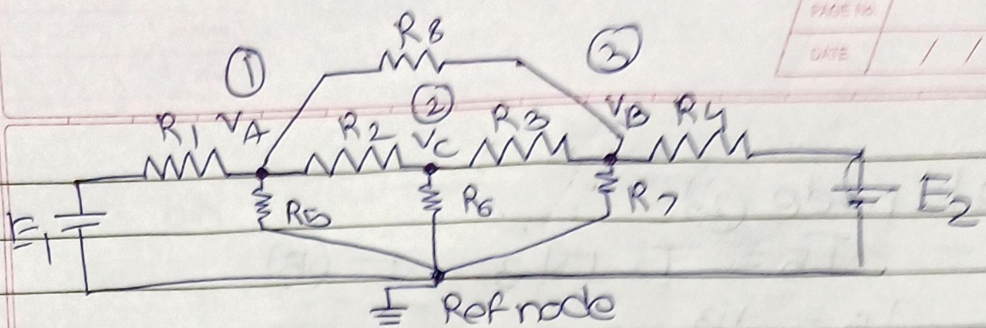
put these values in eqn (B)

$$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0$$

In short -

1. Product of node potential V_A & $(1/R_1 + 1/R_2 + 1/R_4)$ i.e, sum of the reciprocals of the branch resistances connected to this node
2. Minus the ratio of adjacent potential V_B & the interconnecting resistance R_2
3. Minus ratio of adjacent battery voltage E_1 & interconnecting resistance R_1.
4. All above equal to zero.

→

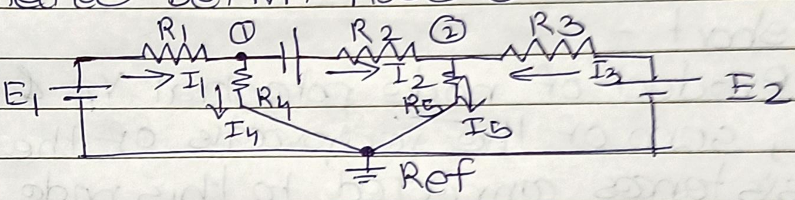


$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_8}\right)-\frac{V_C}{R_2}-\frac{V_B}{R_8}-\frac{E_1}{R_1}=0$$

$$V_B\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_7}+\frac{1}{R_8}\right)-\frac{V_A}{R_8}-\frac{V_C}{R_3}-\frac{E_2}{R_4}=0$$

$$V_C\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)-\frac{V_A}{R_2}-\frac{V_B}{R_3}=0$$

case 2:> when a third battery of emf E_3 is connected betwn node (1) & (2)



E_3 is connected betwn node (1) & (2). As we travel from (1) to (2), we go from -ve terminal of E_2 to +ve terminal. Hence acc to sign convention it must be taken +ve & vice versa for when we travel from (2) i.e. $-E_3$

at node (1),

$I_1 = I_2 + I_4$ $\quad I_2 = \frac{V_A + E_3 - V_B}{R_2}$ $\quad I_4 = \frac{V_A}{R_4}$

$I_1 = \frac{E_1 - V_A}{R_1}$

$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)-\frac{E_1}{R_1}-\frac{V_B}{R_2}+\frac{E_3}{R_2}=0$$

at node (2),

$I_2 + I_3 = I_5$

$I_2 = \frac{V_A + E_3 - V_B}{R_2}$ $\quad I_3 = \frac{E_2 - V_B}{R_3}$ $\quad I_5 = \frac{V_B}{R_5}$

$$V_B\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)-\frac{E_2}{R_3}-\frac{V_A}{R_2}-\frac{E_3}{R_2}=0$$

Supernode - V/I source between two nodes

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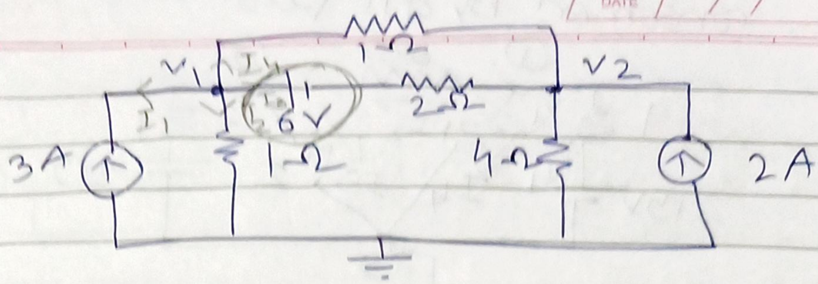
Q)



App K.C.L at node V_1

$$-3 + \frac{V_1 - V_2}{1} + \frac{V_1 - 6 - V_2}{2} = 0$$

$$5V_1 - 3V_2 = 12 \quad -①$$

App K.C.L at node V_2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 0}{4} + \frac{V_2 + 6 - V_1}{2} = 2$$

$$-6V_1 + 7V_2 = -4 \quad -②$$

Solvin ① & ②

$V_1 = 4.23V$, $V_2 = 3.05V$

Calculate total power consumed by all passive elements

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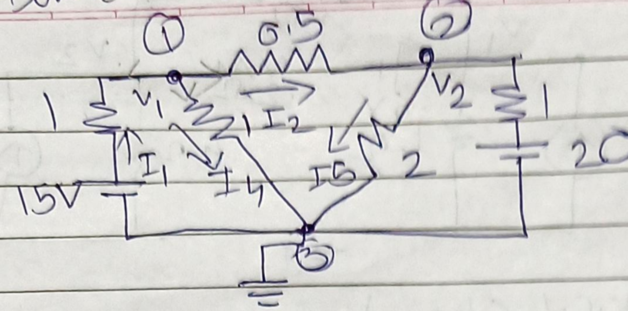
Q)



Applying KCL at node 1

$$\frac{v_1 - 15}{1} + \frac{v_1 - v_2}{0.5} + \frac{v_1 - 0}{1}$$

$$v_1 - 15 + \frac{v_1}{0.5} - \frac{v_2}{0.5} + v_1 = 0$$

$$\left(2 + \frac{1}{0.5}\right) v_1 - \left(\frac{1}{0.5}\right) v_2 = 15 \quad (1)$$

Applying KCL at node 2

$$\frac{v_2 - v_1}{0.5} + \frac{v_2 - 0}{2} + \frac{v_2 - 20}{1}$$

$$\frac{v_2}{0.5} - \frac{v_1}{0.5} + \frac{v_2}{2} + v_2 - 20 = 0$$

$$\left(\frac{1}{0.5}\right) v_1 + \left(1 + \frac{1}{2} + \frac{1}{0.5}\right) v_2 = 20 \quad (11)$$

$x = \frac{185}{36}$, $y = \frac{25}{9}$

$= 5.13$ $\quad y = 2.77$

$v_1 = 5.13$ $\quad v_2 = 2.77$

~~$I = V/R = 5.13$~~

$I_1 = \frac{5.13 - 15}{1}$

$=$

$I_2 = \frac{5.13 - 2.77}{0.5}$

$=$

$I_3 = \frac{5.13 - 0}{1}$

$=$

$I_4 = \frac{2.77 - 0}{2}$

$=$

$I_5 = \frac{2.77 - 20}{1}$

Total $P = I_1^2 R + I_2^2 R + I_3^2 R + I_4^2 R + I_5^2 R$

$I_{ab}, I_{bc}, I_{bd} = ?$

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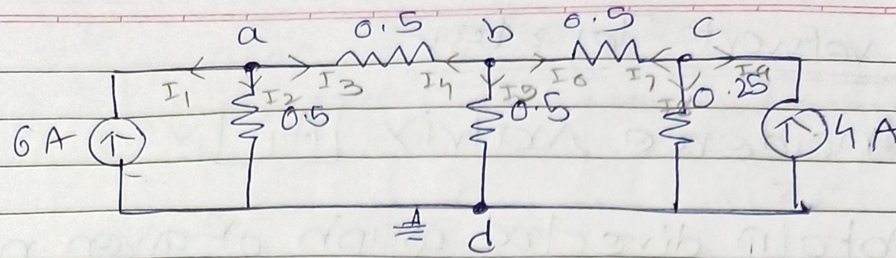


Appliy KCL at node a

$$I_1 + I_2 + I_3 = 0$$

$$-6 + \frac{V_a - 0}{0.5} + \frac{V_a - V_b}{0.5} = 0$$

$$\frac{V_a}{0.5} + \frac{V_a}{0.5} + \frac{V_b}{0.5} = 6$$

$$\underbrace{\left(\frac{1}{0.5} + \frac{1}{0.5}\right)}_{4} V_a + \underbrace{\left(\frac{1}{0.5}\right)}_{2} V_b = 6 \quad \text{—(I)}$$

Applying KCL at node b.

$$I_4 + I_5 + I_6 = 0$$

$$\frac{V_b - V_a}{0.5} + \frac{V_b - V_a}{0.5} + \frac{V_b - V_c}{0.5} = 0$$

$$\underbrace{\left(\frac{-1}{0.5}\right)}_{-2} V_a + \underbrace{\left(\frac{1}{0.5} + \frac{1}{0.5} + \frac{1}{0.5}\right)}_{6} V_a - \underbrace{\left(\frac{1}{0.5}\right)}_{-2} V_c = 0 \quad \text{(II)}$$

Applying KCL at node c

$$I_7 + I_8 + I_9 = 0$$

$$\frac{V_c - V_b}{0.5} + \frac{V_c - 0}{0.25} + -4 = 0$$

$$\overset{-2}{\left(-\frac{1}{0.5}\right)} V_b + \overset{6}{\left(\frac{1}{0.5} + \frac{1}{0.25}\right)} V_c = 4 \quad \text{(III)}$$

$$V_a = x = \frac{13}{12} = 1.08$$

$$V_b = y = \frac{5}{6} = 0.83$$

$$V_c = z = \frac{17}{12} = 1.41$$

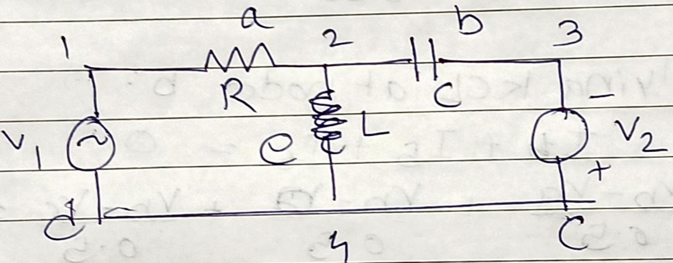
$$\frac{V_a - V_b}{0.5} = \frac{1.08 - 0.83}{0.5} =$$

* Network Topology.

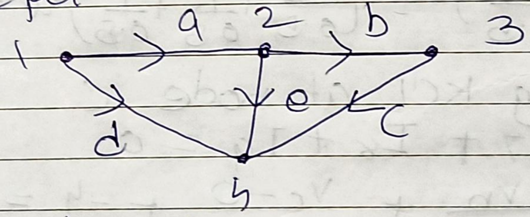
* Incidence Matrix $[A_i]$

1. obtain directed graph of given n/w
2. Assign '+1' in the matrix if the arrow of a branch is oriented away from that node.
3. Assign '(-1)' ... oriented towards that node
4. Assign '0' in the matrix if the branch is not connected to a node

Q1)



Directed path



branches →

nodes ↓	a	b	c	d	e
1	1	0	0	1	0
2	−1	1	0	0	1
3	0	−1	1	0	0
4	0	0	−1	−1	−1

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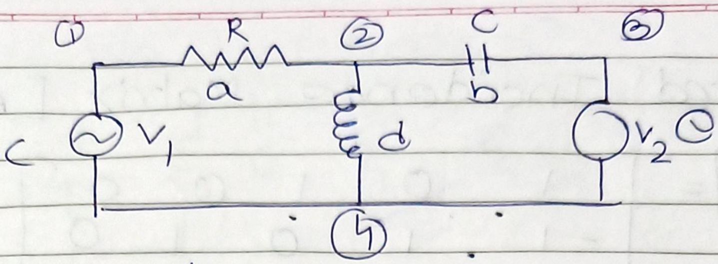
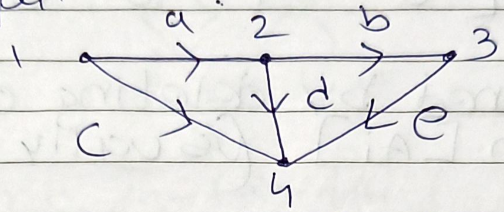
(2)



Directed part



incidence matrix $[A_i]$

branches → a b c d e
nodes ↓

nodes	a	b	c	d	e
1	1	0	1	0	0
2	-1	1	0	1	0
3	0	-1	0	0	1
4	0	0	-1	-1	-1

check (1) sum of columns is zero
(2) Det is zero

* Reduced Incidence Matrix $[A_r]$

$$[A_r] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}_{3\times5}$$

obtained by deleting one row from $[A_i]$ (usually last row)

No. of possible trees $= \det\{[A_r][A_r]^T\}$

$$\therefore [A_r] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}_{3\times5}$$

$$[A_r] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{5\times3}$$

$$\det\{[A_r]\cdot[A_r]^T\}$$

$$\det = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$D = 8$$

$\therefore$ No. of trees $= 8$

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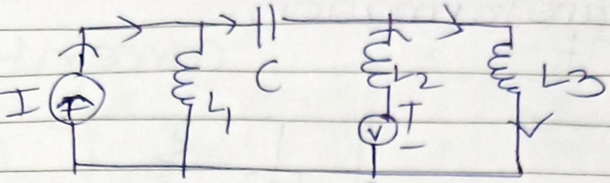
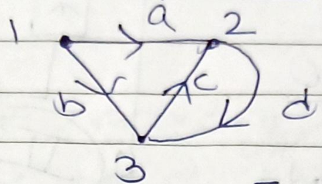
(Q) Possible no. of trees = ?





incidence Matrix $[A_i]$

$$\begin{array}{c|cccc} n \downarrow \; br \rightarrow & a & b & c & d \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 0 & -1 & 1 \\ 3 & 0 & -1 & 1 & -1 \end{array}$$

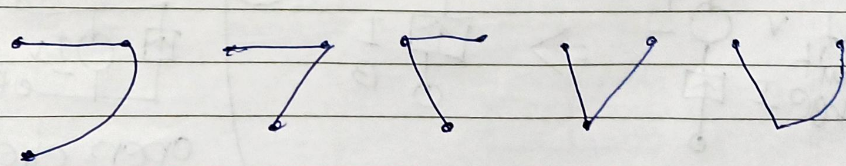
$$A_r = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \end{bmatrix} \quad [A_r]^T = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$

$$[A_r][A_r]^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\times \; = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\text{Det} \Rightarrow \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix}$$

$$= 5$$



Ohms law $\boxed{V = IR}$

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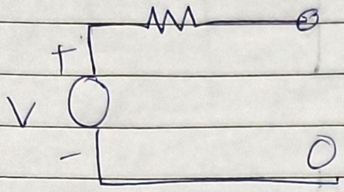
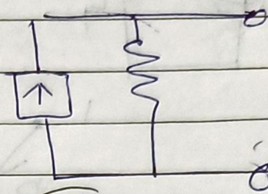
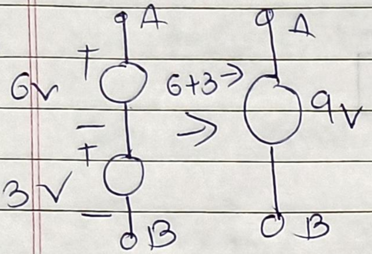
* Source Transformation

volt | current



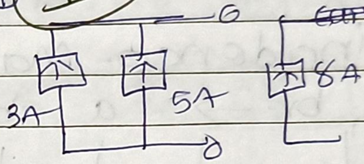


① I



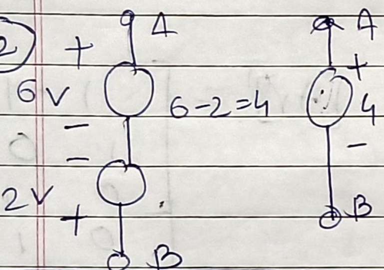
add when series with unequal sign

① II



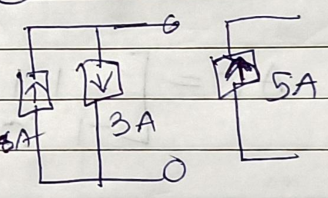
Add current sources parallel with same direction

②



sub when series with equal sign

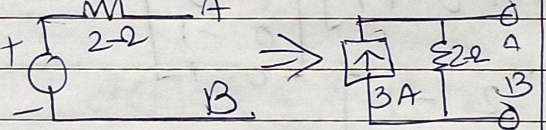
②



sub current sources parallel with opp direc

III Transformation.

①



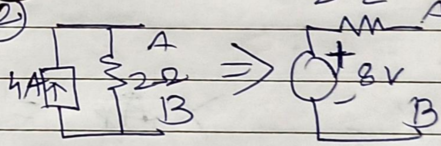
$V = 6$

$R = 2$

$I = V/R$

$I = 6/2 = 3$

IV ②



$I = 4A$

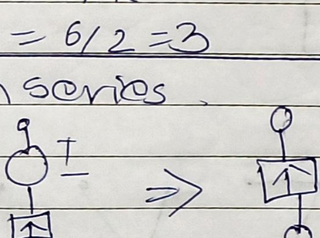
$R = 2$

$V = IR$

$= 4 \times 2$

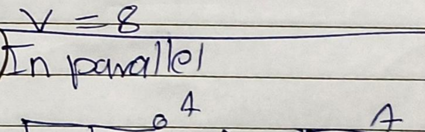
$V = 8$

V in series



short circuit voltage

VI In parallel



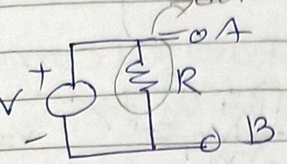
open circuit voltage.

Voltage source always in series
current source always in parallel

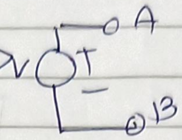
VII

① Volt & R in parallel

dummy element

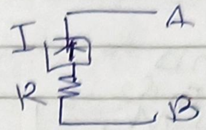


⇒

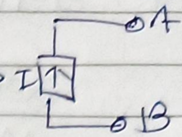


VIII

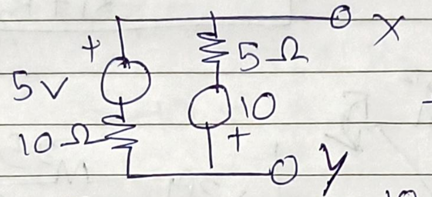
② I & R in series



⇒

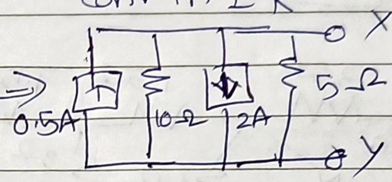


eg ⇒ Reduce the network shown in figure to a single voltage source in series with single element.



conv in I R

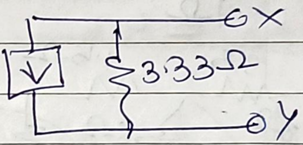
⇒



$V = 5V$
$R = 10$
$I = V/R = 5/10 = 0.5A$

$V = 10$
$R = 5$
$I = 2A$

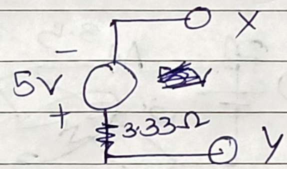
⇓ sub current so



$\frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 5}{10+5}$
$= 3.33\Omega$

$2 - 0.5$
$= 1.5A$

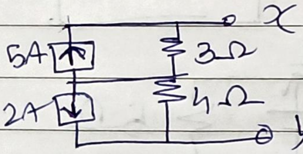
⇐



⇐

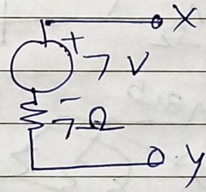
$V = IR$
$= 1.5A \times 3.33\Omega$
$= 4.99 \approx 5V$

Q ⇒ Convert given combination into a single current source in parallel with a single element

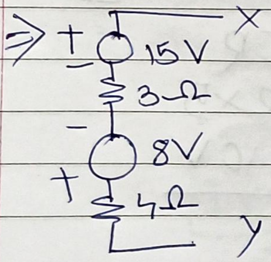


⇒ $V = IR$

⇒



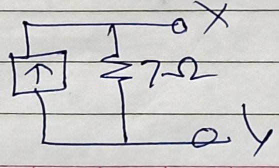
⇒



$15 - 8 = 7V$

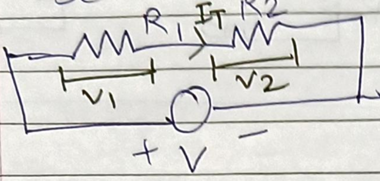
$I = \frac{V}{R} = \frac{7}{7} = 1A$

⇒

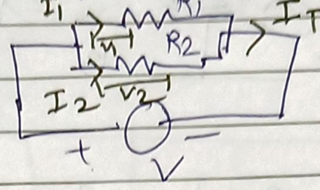


* Series Parallel combination

Ser comb



Parallel com.



$v = v_1 \neq v_2$

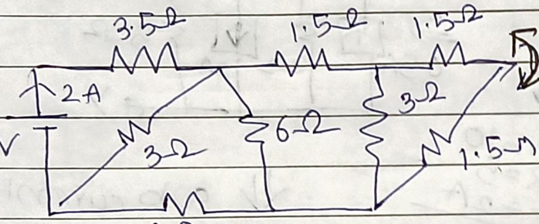
$R_T = R_1 + R_2 + \cdots R_n$

$$\boxed{R_T = R_1 + R_2}$$

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}}$$

Q)



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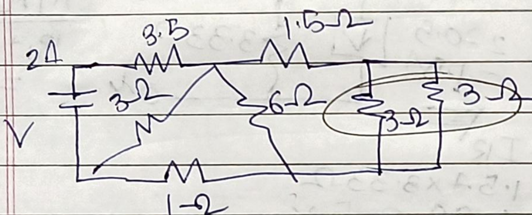

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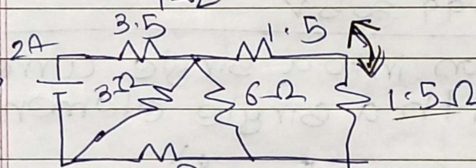
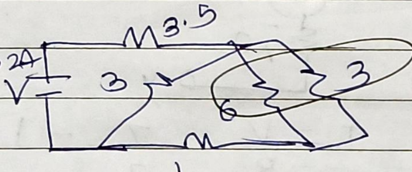
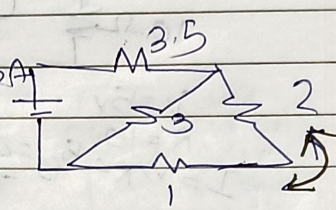


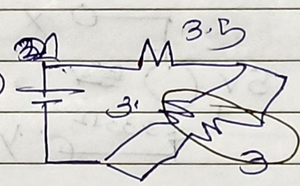
⇒



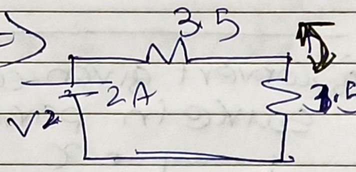
⇒



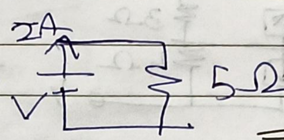
⇒



⇒



⇒

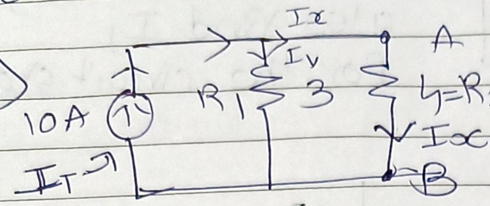


$v = IR$

$v = 2 \times 5$

$V = 10V$

* Current Division Formula –

Q) 

calculate the current through branch AB using current division formula in circuit.

Current-Division formula
$I_x = I_T \times \dfrac{\text{Res}^r \text{ of opp branch}}{\text{Res of opp branch} + \text{Res}^r \text{ of current branch}}$
imp

① first find total current $[I_T]$

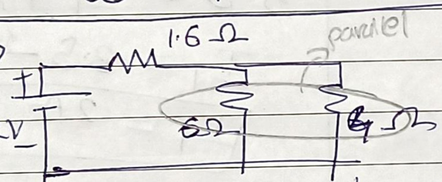
by current division formula

$$I_x = I_T \times \frac{R_1}{R_1 + R_2}$$

$$= 10 \times \frac{3}{3+4}$$

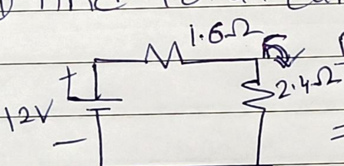
$$= 10 \times \frac{3}{7}$$ i.e. $I_{AB} = 4.28A$

$$I_x = 4.28\ A$$

Q2) 

find the current through 4Ω resistor for the circuit shown.

① find total current

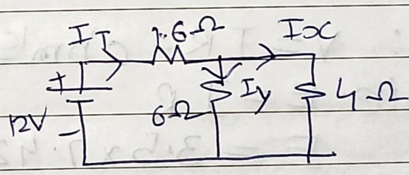


$$\frac{R_1 \times R_2}{R_1 + R_2} = \frac{6 \times 4}{6+4} = 2.4\Omega$$

$\therefore$ Total $R_T = 1.6 + 2.4 = 4\,\Omega$

$$\therefore I_T = V \cancel{/} R_T = \frac{12}{4} = 3A$$

$$\boxed{I_T = 3A}$$



by current div formula.

$$I_x = I_T \times \frac{6}{6+4}$$

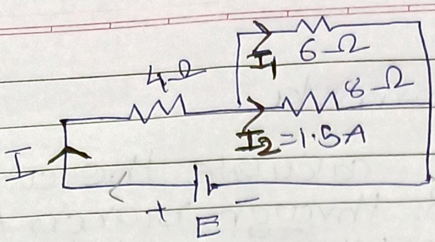
$$= 3 \times \frac{6}{6+4}$$

$$= 3 \times 6/10$$

$$I_x = 1.8A$$

i.e. current flowing through 4Ω R is 1.8A

Q)



Find the value of battery voltage, also find I_1 for the circuit shown

by CDR

$$I_2 = I \times \frac{6}{6+8}$$

$$1.5 = I \times \frac{6}{6+8}$$

$$\frac{1.5 \times 14}{6} = I$$

$$\boxed{I = 3.5A}$$

Total Resistance R_T

$$R_T = 4 + \frac{6 \times 8}{6+8}$$

$$\boxed{R_T = 7.428\,\Omega}$$

$\therefore$ $V = IR$ - ohm's law

i.e. $E = IR$

$$E = 3.5 \times 7.428$$

$$E = 25.998\,V$$

$$\boxed{E \approx 26\,V}$$

By CDR

$$I_1 = I \times \frac{8}{8+6}$$

$$= 3.5 \times \frac{8}{14}$$

$$\boxed{I_1 = 2A}$$

or

$$I = I_1 + I_2$$

$$3.5 = I_1 + 1.5A$$

$$I_1 = 3.5 - 1.5A$$

$$\boxed{I_1 = 2A}$$

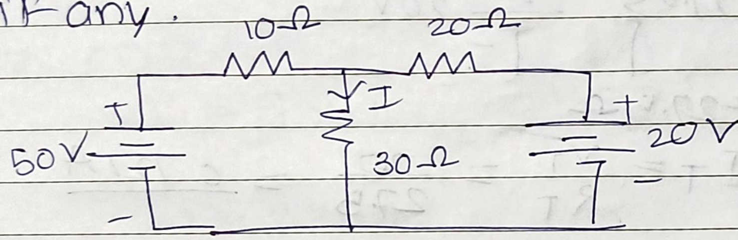
$\therefore$ $\boxed{\begin{array}{l} E = 26\,V \\ I_1 = 2A \end{array}}$

Unit II
Network Theorems

* Superposition Theorem.

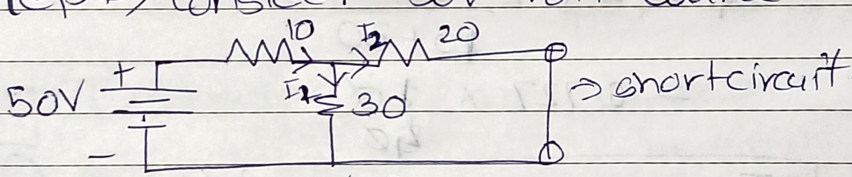
In any multisource linear, bilateral network, voltage across or current through any given element is algebraic sum of individual voltage or current considering one source at a time and replacing all remaining sources (only independent) by their internal resistances if any.

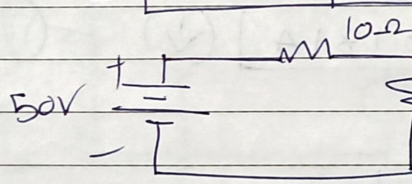
Q)



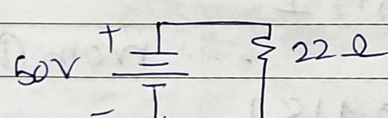
find I i.e current flowing through 30Ω

step I) Consider 50v volt source.





$\frac{20 \times 30}{20+30} = 12\Omega$



$I_T = \frac{V_T}{R_T} = \frac{50}{22} = 2.272A$

$\boxed{I_T = 2.272 A}$

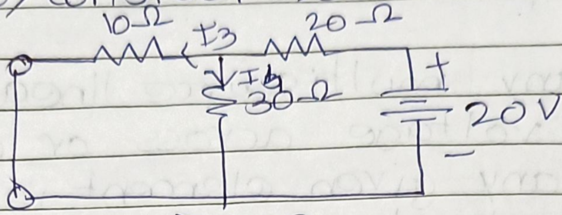
by CDR

$I_1 = 2.272 \times \frac{20}{20+30}$

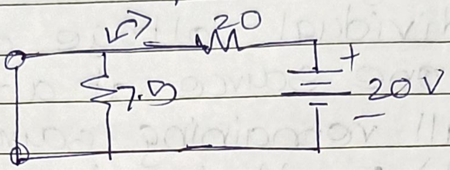
$\boxed{I_1 = 0.908 A}$ (↓) — ①

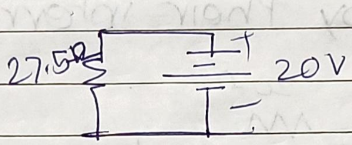
voltage source → short circuit
current source → open circuit

Step II) Consider 20V volt source.



$\frac{30 \times 10}{30+10} = \frac{300}{40} = 7.5\Omega$





$R_T = 27.5\Omega$

$I_T = \frac{V_T}{R_T} = \frac{20}{27.5} = 0.727A$

by CDR

$I_4 = I_T \times \frac{10}{10+30}$

$= 0.727 \times \frac{10}{40}$

$\boxed{I_4 = \cancel{0.2908}\ 0.181A}$ (↓) — (II)

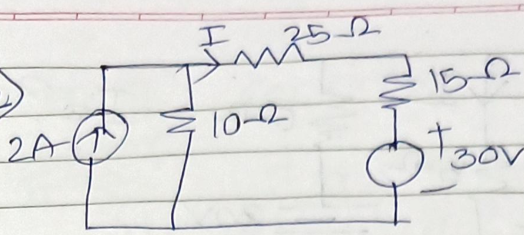
Step III) final current I

$I = I_1 + I_4$ ... from (I) & (II)

$I = 0.908 + 0.181$

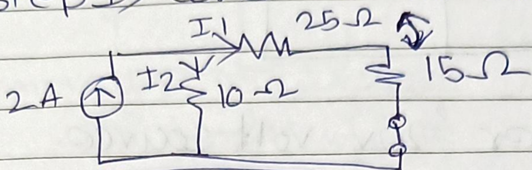
$\boxed{I_1 = 1.089 A \downarrow}$

(Q2)



find I

step I) consider 2A current source.



$\boxed{I_T = 2A}$)

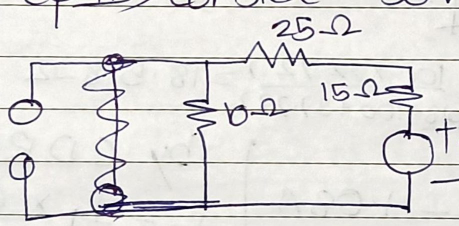
by CDR

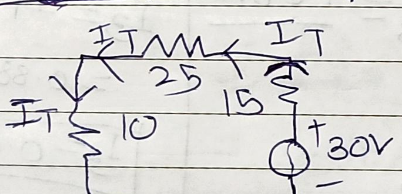
$I_1 = I \times \frac{10}{10 + 40}$ → current branch $= 25 + 15 = 40$

$I_1 = 2 \times \frac{10}{10+40}$

$\boxed{I_1 = 0.4A}$ (→)

Step II) consider 30V volt source.





$R_T = 25 + 15 + 10 = 50\Omega$

$V = 30V$

$I_T = \frac{30}{50} = 0.6A$

$I_2 = I_T$ $\boxed{I_T = 0.6A}$ ←

~~by CDR~~

No CDR because no current is divided.

Step3> I?

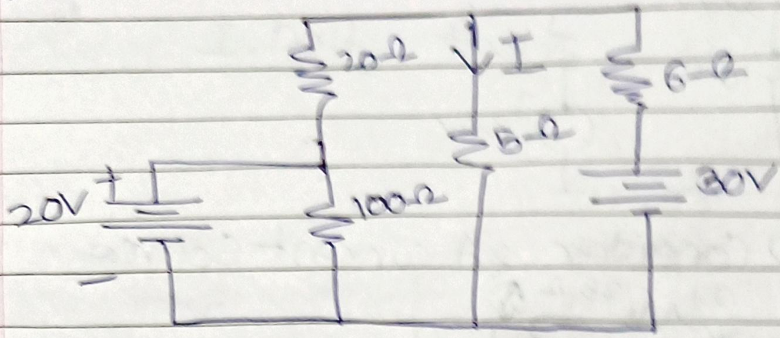
$\therefore I_{25\Omega} = I_2 - I_1$

$= 0.6 - 0.4$

$\boxed{I_{25} = 0.2A}$ (←)

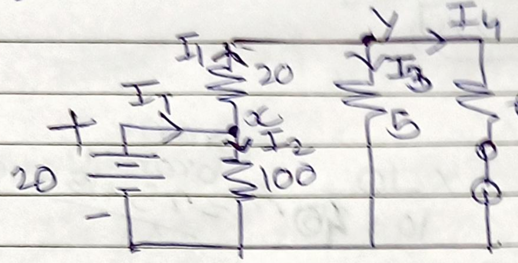
dummy branch pattern

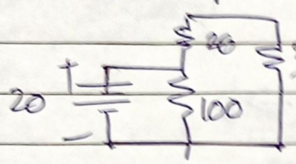
Q3)



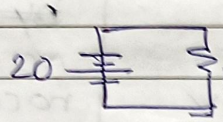
$I = ?$

step I > consider 20v volt source.





$\frac{5 \times 6}{5+6}$



$\frac{100 \times 22.727}{100+22.727} = 18.518\ \Omega$

$R_T = 18.518$

$V = 20$

$I = \frac{V_T}{R_T} = \frac{20}{18.518} = 1.08A$

by CDR at x

$I_1 = I_T \times \frac{100}{100+22.727}$

~~= 1.08 × 100/122 = 25.625~~

~~0.9~~

$\boxed{I_1 = 0.88A}$

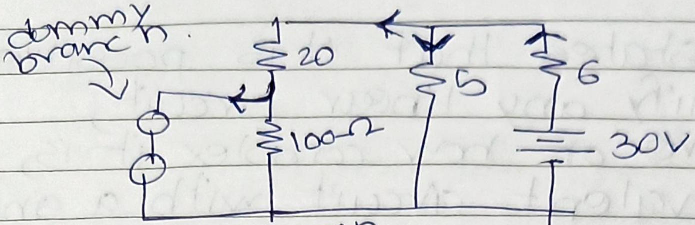
by CDR at y

$I_3 = I_1 \times \frac{6}{6+5}$

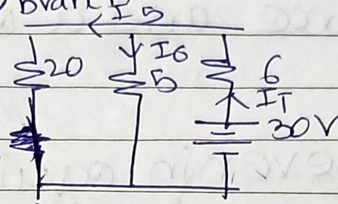
$= 0.88 \times \frac{6}{6+5}$

$\boxed{I_3 = 0.48}$ ↓

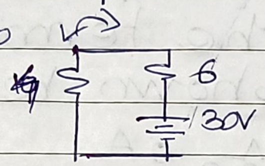
step II) consider 30 V volt source.



remove dummy branch
=>



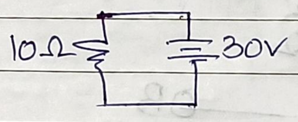
$\frac{20\times5}{20+5}=10$



$R_T = 4+6 = 10$

$V = 30V$

$I_T = \frac{30}{10} = 3A$



$\boxed{I_T = 3A}$

by CDR

$I_6 = I_T \times \left(\frac{20}{20+5}\right)$

$= \cancel{3\times}\ 3\times\frac{20}{20+5}$

$\boxed{I_6 = 2.4\ A}$ ↓

Step III) Total current (I) →

$\therefore I = I_3 + I_6$

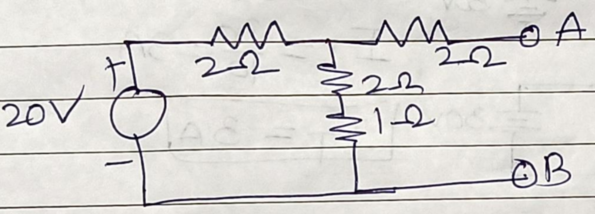
$= 0.48 + 2.4$

$\boxed{I = 2.88A}$ ↓

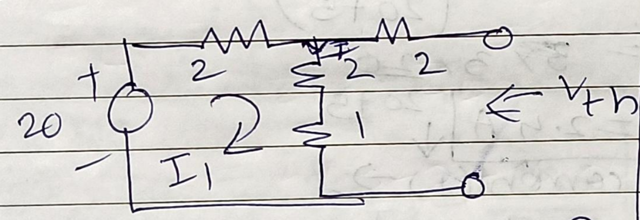
* Thevenin's Theorem

It states that it is possible to simplify any linear circuit, irrespective of how complex it is, to an equivalent circuit with a single voltage source and a series resistance.

Q1) Find the thevenin equivalent circuit between the terminals A & B for the circuit shown below.



Step I) Calculation of V_{th}



apply KVL in loop ①

$$5I_1 = 20$$

$$I_1 = 20/5$$

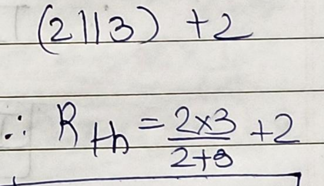
$$\boxed{I_1 = 4A}$$

$$\therefore V_{th} = I_1 \times (2+1)$$

$$= 4 \times 3$$

$$\boxed{V_{th} = 12V}$$

Step II) calc of R_{th}



$(2 \| 3) + 2$

$$\therefore R_{th} = \frac{2 \times 3}{2+3} + 2$$

$$\boxed{R_{th} = 3.2\Omega}$$

by deactivating all sources

Thevenin ⇒ ① V_{th}
② Z_{th}
③ equi ckt

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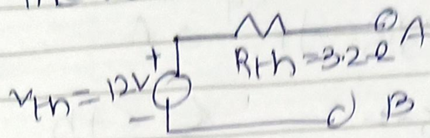
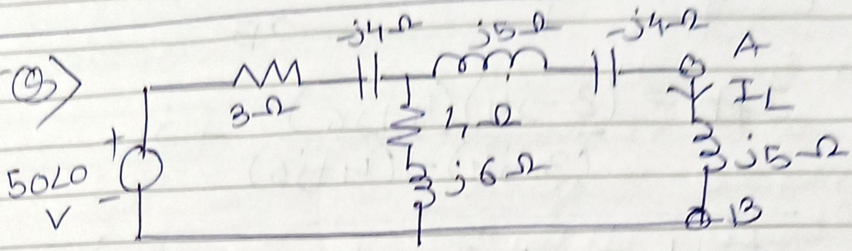
Refer

Thevenin's equivalent circuit

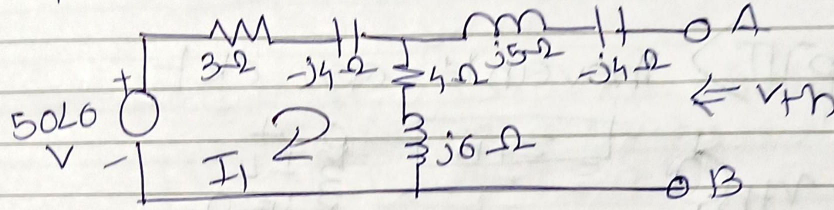


③>



$I_L = ?$

Step I) Calculation of V_{th}



by applying KVL in loop ①

$$-50\angle 0 + 3I_1 - j4I_1 + 4I_1 + j6I_1 = 0$$

$$(7+j2)I_1 = 50\angle 0$$

$$I_1 = \frac{50\angle 0}{7+j2}$$

$$\boxed{I_1 = 6.868\angle -15.945 A}$$

$$V_{th} = I_1 \times (4+j6)$$

$$= (6.868\angle -15.945) \times (4+j6)$$

$$\boxed{V_{th} = 49.525\angle 40.364 V}$$

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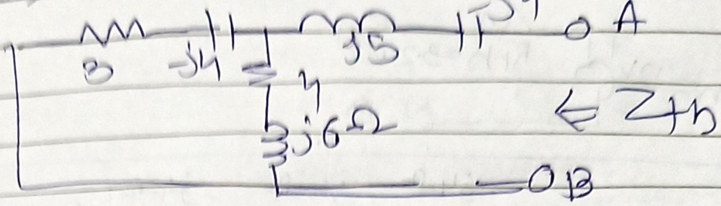
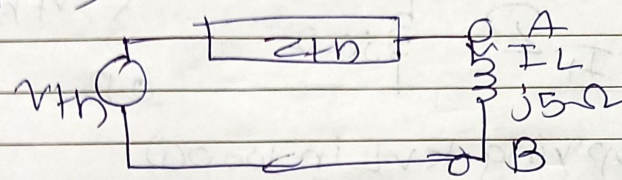
Step II) Calculation of Zth.



$$Z_{th} = \frac{(3-j4)(4+j6)}{(3-j4)+(4+j6)} + j1$$

$$\boxed{Z_{th} = 4.831 \angle -1.118\ \Omega}$$

Step III) Thevenin's eqv ckt



I_L = $\dfrac{V_{th}}{Z_{th} + Z_L}$
Load current

$$= \frac{49.525 \angle 40.364}{(4.831 \angle -1.118) + j5}$$

$$\boxed{I_L = 7.193 \angle -5.081\ A}$$

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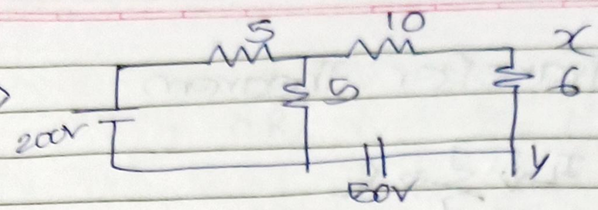
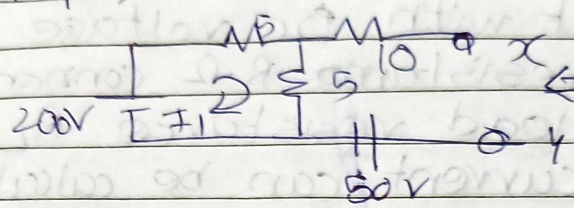
(Q3) 

calculate power in xy using Thevenin's theorem

Step I) Calculation of Vth.



apply KVL.

$$I_1 = \frac{200}{5+5} = \frac{200}{10} = 20A$$

$$V_{th} = I_1 \times R$$

$$= \cancel{20} \; 50 - (20 \times 5)$$

$$\boxed{V_{th} = -50V}$$

Step II) Calculate Rth.

$$R_{th} = (5 \| 5) + 10$$

$$= \frac{5 \times 5}{5+5} + 10$$

$$= \frac{25}{10} + 10$$

$$= 2.5 + 10 = 12.5\Omega$$

$$\boxed{R_{th} = 12.5\Omega}$$

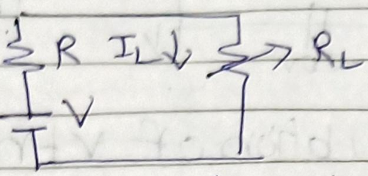
Step3) $I_L = \frac{-50V}{12.5+6} = \frac{-50}{18.5} = 2.702$

$$\text{Power} = I^2 R$$

$$= (-2.702)^2 \times 6$$

$$= 43.74$$

* Max Power Transfer Theorem



imp theorem

Consider a ckt with DC voltage source V & resistance R Ω connected to variable load resistance R_L

Load current can be calculated here as

$$I_L = \frac{V}{R+R_L}$$

The power consumed by the load resistance R_L is,

$$P = I_L^2 R_L$$

$$= \left(\frac{V}{R+R_L}\right)^2 R_L$$

If R_L is changed, I_L will also change and at a particular value of R_L, the power transferred to the load will be maximum. Hence the power depends on the value of R_L. To get the value of R_L at which power will be max; let's differentiate above eqn of power w.r.t R_L & equate to zero

$$\therefore \frac{dP}{dR_L} = 0$$

$$\therefore \frac{d}{dR_L}\left[\frac{V}{R+R_L}\right]^2 \cdot R_L = 0$$

$$V^2 \frac{d}{dR_L}\left[\frac{R_L}{(R+R_L)^2}\right] = 0$$

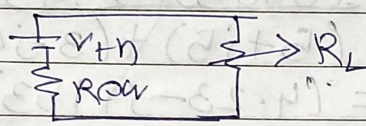
after solving this we get,

$$R + R_L - 2R_L = 0$$

$$\therefore \boxed{R_L = R}$$

Thus when load resistance is equal to rests of the ckt, max power transfer takes place.

we represent the complex network with Thevenin's eqv ckt



If we compare this ckt with above ckt diagram we get,

$\boxed{R_L = Req}$ ... for max power transfer

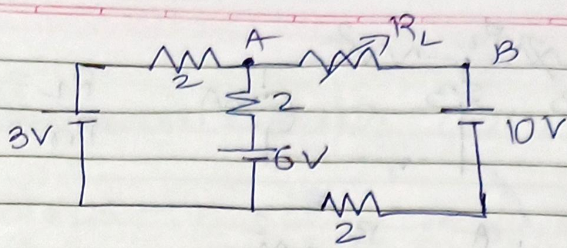
And max. power will be calculated as:-

$$P_m = \left[\frac{V_{th}}{Req + R_L}\right]^2 \cdot R_L$$

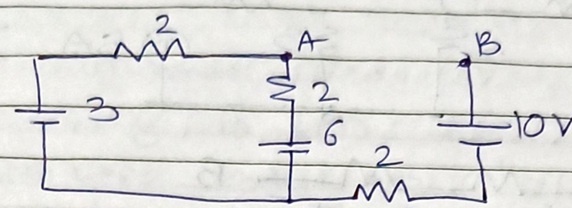
with $R_L = R_{th}$

$$P_m = \left[\frac{V_{th}}{2Req}\right]^2 \cdot Req$$

$$\boxed{P_m = \frac{V_{th}^2}{4Req} \text{ watts.}}$$

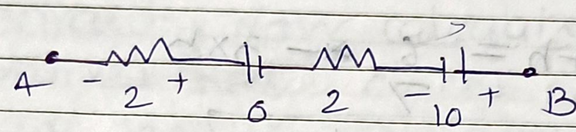
Q1) 

calculate R_L so the max power is transforred

-> 

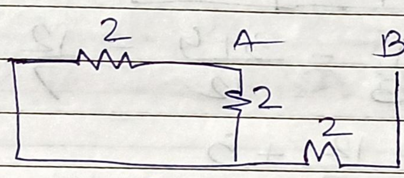
$$I_1 = \frac{6-3}{4} = \frac{3}{4}\ A$$

$$I_2 = \frac{10}{2} = 5A$$



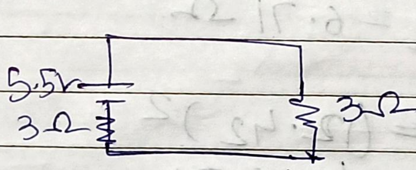
$$V_{th} = 2 \times \frac{3}{4} + 10 - 6$$

$$\boxed{V_{th} = 5.5\ V}$$

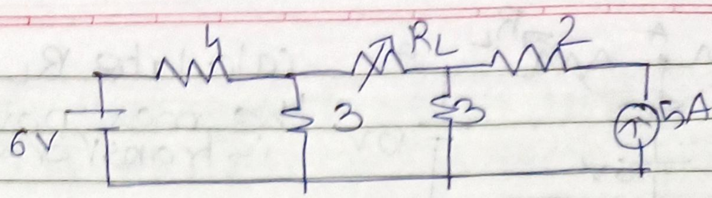


$$R_{eq} = 3\,\Omega$$

$$\therefore R_L = 3\,\Omega$$



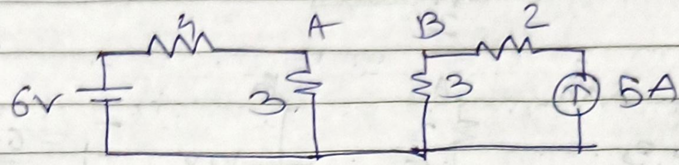
$$P = \frac{(5.5)^2}{4 \times 3} = \frac{30.25}{12} = \underline{2.52\,W}$$

Q) 

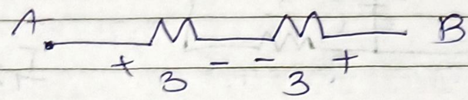
$R_L = ?$

$P_m = ?$

Soln:

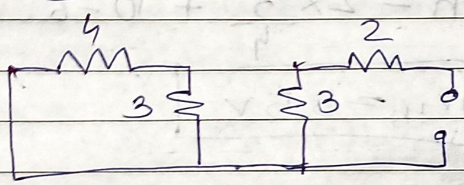


$I = \frac{6}{7}$



$$V_{th} = \frac{6}{7} \cancel{\times} - 3\times 5$$

$$\boxed{V_{th} = -12.42\,V}$$



$$\frac{1}{4} + \frac{1}{3} = \frac{3+4}{12} = \frac{12}{7}$$

$$R_{eq} = \frac{12}{7} + 5$$

$$= 6.71\,\Omega$$

$$R_L = 6.71\,\Omega$$

$$P_m = \frac{(12.42)^2}{4\times 6.71}$$

$$= \frac{154.25}{26.84}$$

$$= 5.74\,W$$

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* Thevenin's Rule.

Steps –

1. Remove load resistance & calculate voltage at open circuited terminals This vltg is V_{th}.
2. Remove sources also. Replace vltg source by s.c & current source by opencircuit. Now calculate equivalent resistance (R_{eq}) of the circk as if you are looking into the ckt from open circuited terminals.
3. Draw Thevenin's eq ckt
4. Connect R_L to eq ckt
5. Calculate load current by using

$$I_L = \frac{V_{th}}{R_{eq} + R_L}$$

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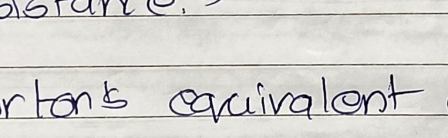
* Norton's Theorem.

Any two terminal active network containing voltage source and resistances, when viewed from its output terminals, is equivalent to the constant current source and a parallel resistance. The constant current source is equal to the current which would flow in a short circuit placed across the terminals; and the parallel resistance is the resistance of the network when viewed from those open circuited terminals after all voltage & current sources have been removed and replaced by their internal resistance."

Norton's equivalent circuit is –



$$\therefore I_L = I_{sc} \times \frac{R_{eq}}{R_{eq} + R_L}$$

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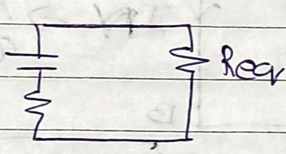
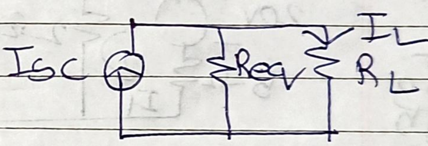
* Steps -

① Remove R_L and short circuit the terminals.
② Calculate current through short circuit (I_{sc})
③ Remove S.C, replace sources and calculate R_{eq} as done in Thevenin's theorem.
④ Draw the Norton's equivalent circuit with I_{sc} & R_{eq} parallel with each other
⑤ Connect R_L in parallel with the eqv ckt
⑥ Calculate I_L using formula,

$$I_L = I_{sc} \times \frac{R_{eq}}{R_{eq} + R_L} \text{ A}$$

Dual of Thevenins: Norton's eqv ckt





=> "Any linear circuit containing several energy sources and resistances can be replaced by a single constant current generator in parallel with a single Resistor."

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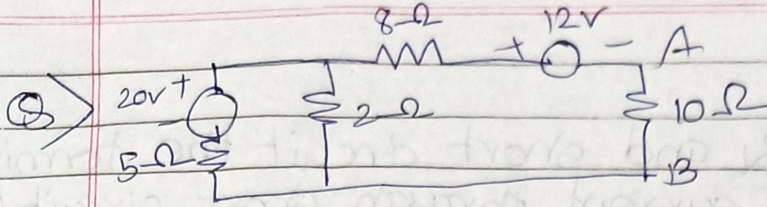
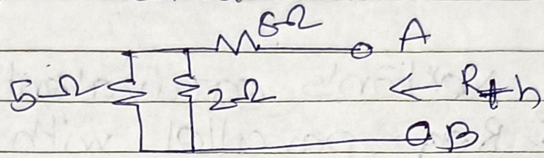
Q)



Step I) calculation of R_{th}

'calculate R_{th} at AB by replacing all vltg & current sources



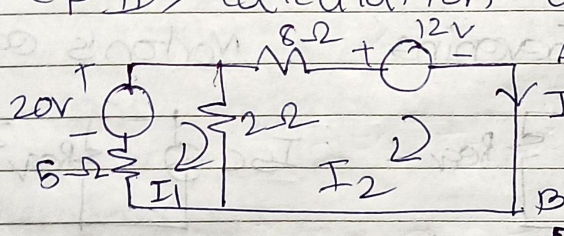
$$(5 \| 2) + 8$$

$$R_{th} = \frac{5 \times 2}{5+2} + 8 =$$

$$\boxed{R_{th} = 9.4285}$$

Step II) calculation of I_N

Remove load & short AB & current flowing through it is I_N



Apply KVL in loop ①

$$7I_1 - 2I_2 = 20V \quad \text{—①}$$

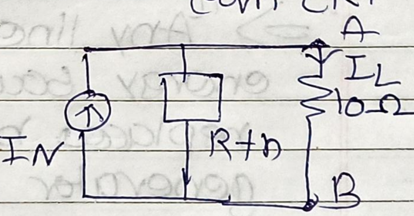
apply KVL in loop ②

$$10I_2 - 2I_1 = -12 \quad \text{—①}$$

$$\therefore \; I_1 = 2.66 \qquad I_2 = -0.66$$

$$\therefore \boxed{I_N = I_2 = -0.66\,A}$$

Step III) Norton eqn ckt



$\therefore$ by CDR

$$I_L = I_N \times \frac{R_{th}}{R_{th} + 10}$$

$$= -0.66 \times \frac{9.428}{9.42 + 10}$$

$$\boxed{I_L = -0.320}\; A \downarrow$$

$$\boxed{I_L = 0.320} \uparrow$$

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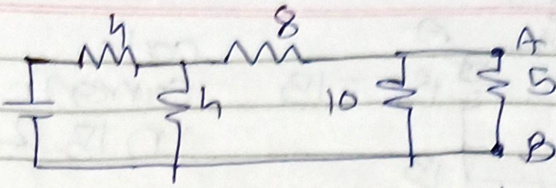
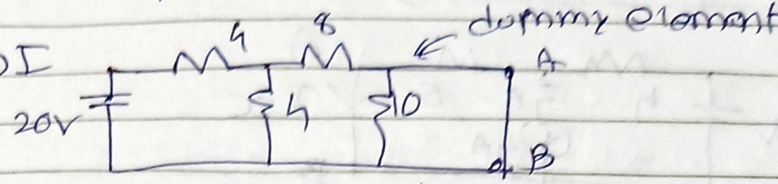
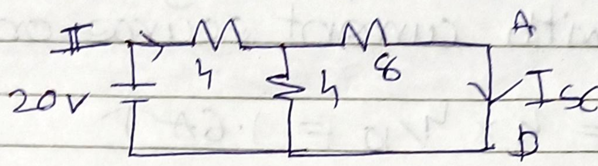
Q1) 

Calculate current in AB using Norton's.

step I 

Now 10 Ω has become redundant
∴ ckt will be –



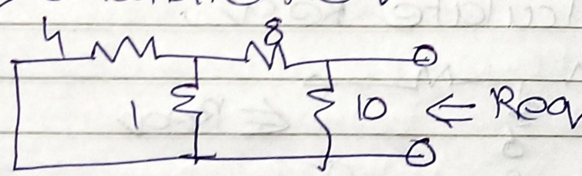
$(8 \| 4) + 4 = \frac{32}{12} + 4 = 6.667\,\Omega$

$\therefore I = \frac{V}{R} = \frac{20}{6.667} = 3A$

by CDR

$\therefore I_{sc} = I \times \frac{4}{4+8} = \frac{3 \times 4}{12} = 1A$

step II) calculate Req

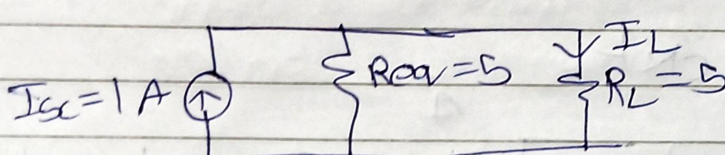


$(4 \| 4) = 2$

$2 + 8 = 10$

$(10 \| 10) = \boxed{5 = Req}$

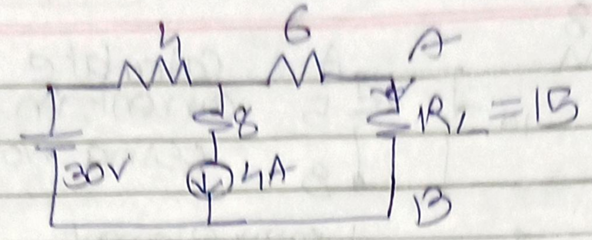
step III) equivalent circuit is –



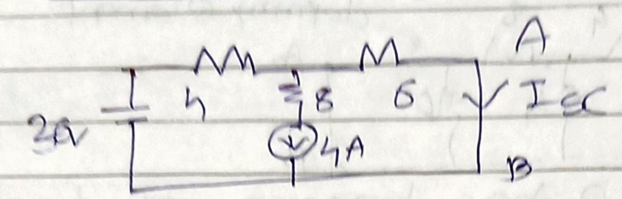
$\therefore I_L = 1 \times \frac{5}{5+5}$

$\boxed{I_L = 0.5A}$

②



calc current in 15 Ω



step I) calculate Isc by using superposition

· I_{sc}' with current source only.

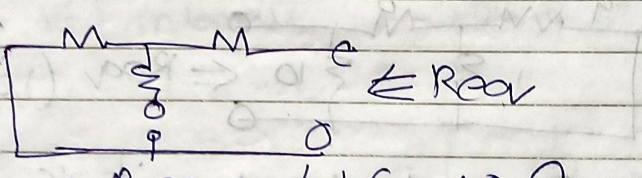
$$I_{sc}' = 4 \times 4/10 = 1.6A \uparrow$$

· I_{sc}'' with vltg source only

· $I_{sc}'' = 30/10 = 3A \downarrow$

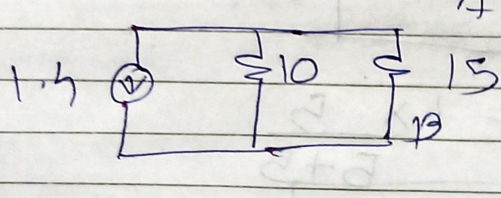
$$\therefore I_{CS} = I_{sc}'' - I_{sc}' = 3 - 1.6 = 1.4A$$

step II) To calculate Req,



$$\therefore R_{eq} = 4 + 6 = 10\,\Omega$$

step III) eq ckt



$$I_L = 1.4 \times \frac{10}{25}$$

$$= 0.56A$$

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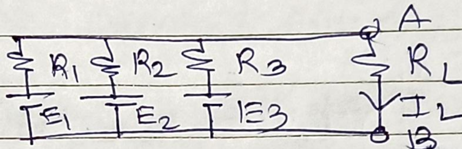
* Millman's Theorem.

a) As applicable to voltage source.

This theorem is a combination of Thevenin's & Norton's theorem. It is used for finding common voltage across any n/w which contains a no. of parallel voltage sources →



The common voltage V_{AB} which appears across the o/p terminals A&B is affected the voltage sources E_1, E_2 & E_3 the value of the voltage is given by.

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G}$$

The voltage V_{AB} represents the thevenin's voltage V_{th}

The resistance R_{th} or R_{eq} can be calculated as usual, by replacing each vltg source by a terminal A&B, this load current is given by

$$I_L = \frac{V_{th}}{R_{eq} + R_L}$$

→

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b) As applicable to current sources.

This theorem is applicable to a mixture of parallel voltage & current sources that are reduced to a single final equivalent source which is either a constant current or a constant voltage source. The theorem can be stated as -

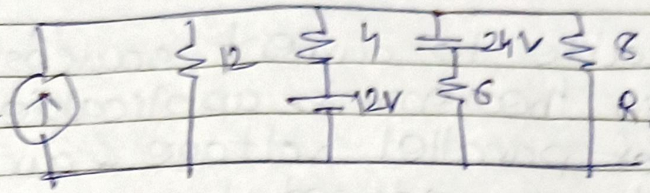
"Any no. of constant current sources which are directly connected in parallel can be converted into a single current source whose value is algebraic sum of the individual source currents and whose total internal resistance equals the combined individual source resistances in parallel".

$$\boxed{V_{AB} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\Sigma \frac{V}{R}}{\Sigma \frac{1}{R}}}$$

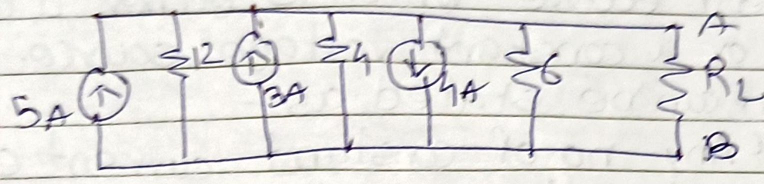
$$\boxed{I_L = \frac{V_{th}}{R_{eq} + R_L}}$$

$$R_{eq} = R_{th} = \frac{1}{R_1} + \frac{1}{R_2} \cdots$$

Ex-



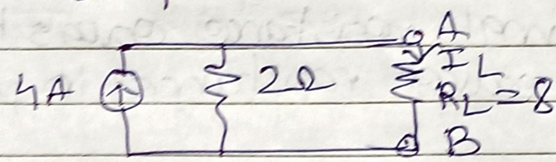
calculate voltage across & current in R_L



∴ Net current $= 5+3-4 = 4A \uparrow$

combined res $= 12 \| 4 \| 6 = 2\Omega$

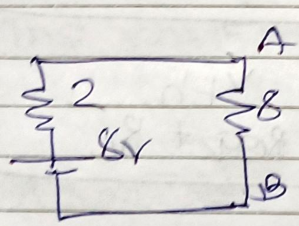
∴ simplified circuit is,



$$\therefore I_L = 4 \times \frac{2}{2+8} = 0.8A$$

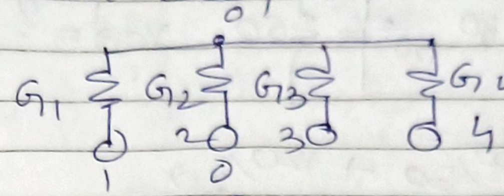
$$V_L = 8 \times 0.8 = 6.4V$$

OR convert it into vltg source.



$$I = \frac{8}{2+8} = 0.8A$$

* Generalized form of Millman's Th.



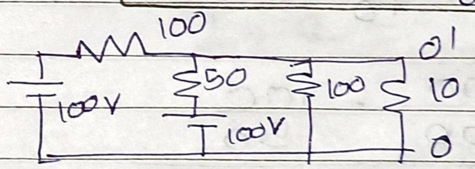
consider no. of admittances $G_1, G_2, G_3 \cdots G_n$ terminate at common point o'. The othe ends of admittances are named as $1, 2, 3 \cdots n$.

Let o be any other point in the network.

Acc to this theorem, the vltg. drop from o to o' ($V_{oo'}$) is given by -

$$\boxed{V_{oo'} = \frac{V_{o1}G_1 + V_{o2}G_2 + V_{o3}G_3 + \cdots V_{on}G_n}{G_1 + G_2 + G_3 + \cdots G_n}}$$

Q)

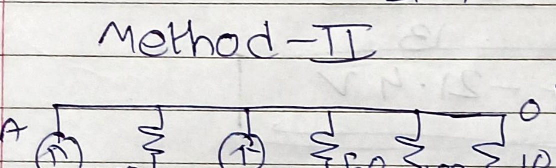


$V_{o1} = -100, \; V_{o2} = -100, \; V_{o3} = 0, \; V_{o4} = 0$

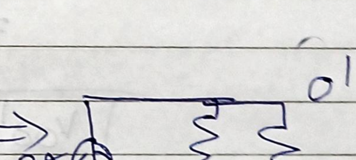
$$\therefore V_{oo'} = \frac{(-100/100) + (-100/50) + (0/100) + (0/10)}{(1/100 + 1/50 + 1/100 + 1/10)}$$

$= -21.4$

Method-II



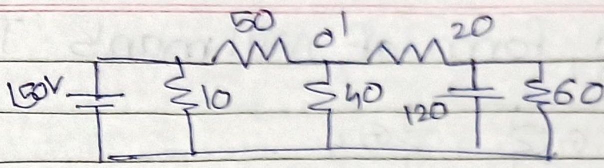
$\Rightarrow$



$100 \| 50 \| 100 = 25\Omega$

$$I_L = 3 \times \frac{25}{35} = 2.1428\,A$$

$\therefore V_{oo'} = 10 \times 2.1428 = 21.4V$

Q)


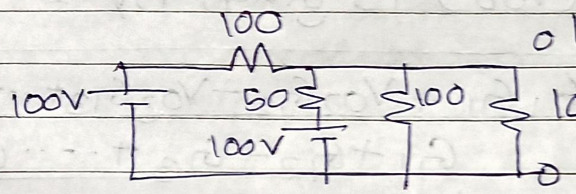
$V_{oo'} = ?$

$\rightarrow$ $$V_{oo'} = \frac{-150/50 + 120/20}{1/50 + 1/40 + 1/20}$$

$$= \frac{-3+6}{0.02+0.025+0.05}$$

$$= \frac{3}{0.095}$$

$$V_{oo'} = 31.5V$$

Q)


$V_{oo'} = ?$

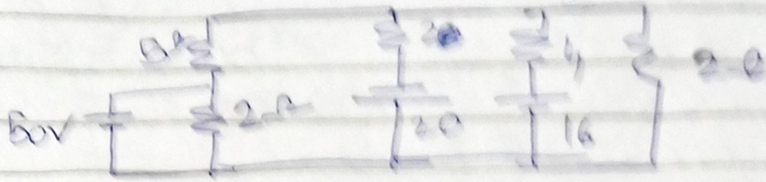
$$V_{oo'} = \frac{\frac{-100}{100} - \frac{100}{50}}{1/100 + 1/50 + 1/10}$$

$$= \frac{-1-2}{\frac{1+2+10}{100}}$$

$$= \frac{-3 \times 100}{13}$$

$$\boxed{V_{oo'} = -21.4V}$$

Q)



Find volt & current through 2Ω res —

$$V_E = \frac{\frac{50}{5} - \frac{20}{2} + \frac{16}{4}}{1/5 + 1/20 + 1/4}$$

$$= 26V$$

$$R_{Th} = \frac{1}{1/5 + 1/20 + 1/4} = 2\Omega$$

$$I_{2\Omega} = \frac{26}{2+2} = 6.5A$$

$$V_L = I_{2\Omega} \times 2 = 13V$$

Unit-3
Analysis of Transient Response in circuits.

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Initial cond^{ns}.

Time periods $(-)$ 0 $(+)$

① Just before switching ($t = -\infty$ to $t = 0^-$)
② Just after switching ($t = 0^+$)
③ After switching ($t > 0$)

1) Initial cond^n for R

$\Rightarrow \underline{V = iR}$

2) Initial cond^n for L

$\Rightarrow v = L \cdot \frac{di}{dt}$

$i(t) = \frac{1}{L}\int_0^t v(t)\,dt + i(0)$. $i(0)$ is initial current through L

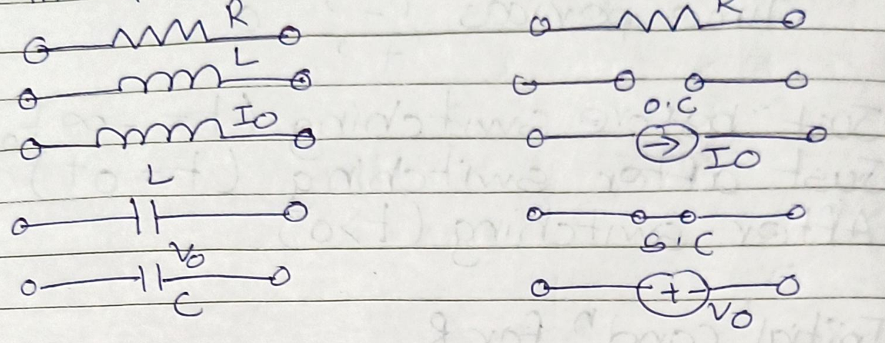
If no current through L at $t = 0^-$
L will act as O.C.
If current I_0 is flowing through L at $t = 0^-$,
L will act as current source of I_0 A.

3) Capacitor

$\Rightarrow v = \frac{1}{C}\int_0^t i(t)\,dt + v(0)$

$v(0)$ is initial voltage across C.
If there is no voltage across cap at $t = 0^-$
C will act as short circuit.
If initially the capacitor is charged to vltg V_0 at $t = 0^-$
C will act as voltage source of V_0

Element with initial condn | Equi ckt at $t=0^+$



* Procedure for evaluating initial condns.

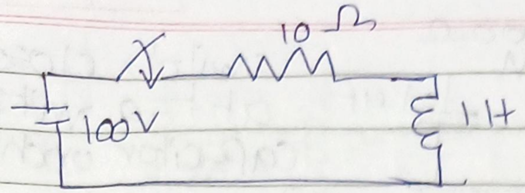
① Draw the equi ckt at $t=0^-$. Before switching take place (i.e. for $t=-\infty$ to $t=0^-$) the n/w is under steady state condn. Hence find current flowing through inductors $i_L(0)$ & vltg across capacitor $v_C(0^-)$

② Draw the equi n/w at $t=0^+$, i.e. imediately after switching. Replace all inductors with O.C or with current sources $i_L(0^+)$ & replace all capacitors by S.C or vltg sources $V_C(0^+)$. Keep resistors as it is.

③ Initial vltg or currents are determined from the equi n/w at $t=0^+$.

④ Initial conditions, i.e. $\frac{di}{dt}(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2i}{dt^2}(0^+)$, $\frac{d^2v}{dt^2}(0^+)$ are determined by writting inte-differential eqns for the n/w for $t>0$, i.e after the switching action by making use of initial condn.

Q)

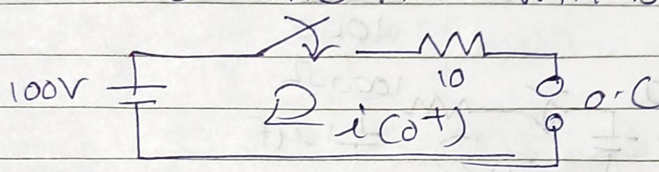


In the given n/w switch is closed at $t=0$ with 0 current in inductor, find $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$

→ at $t=0^-$, no current flow through L

$\therefore i(0^-)=0$

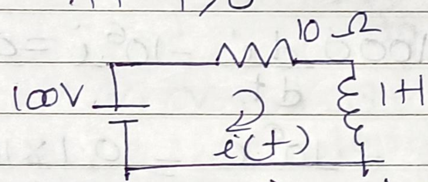
→ At $t=0^+$ the n/w will be



Inductor acts as O.C

$\therefore i(0^+)=0$

→ At $t>0$



$100 - 10i - 1\frac{di}{dt} = 0$ — ①

$di/dt = 100 - 10i$ — ②

→ At $t=0^+$

$\frac{di}{dt}(0^+) = 100 - 10i(0^+)$

$\boxed{\frac{di}{dt}(0^+) = 100 \; A/S}$

Diff eqn ②

$d^2i/dt^2 = -10\, di/dt$

$\frac{d^2i}{dt^2}(0^+) = -10\frac{di}{dt}(0^+) = -1000 \, A/S^2$

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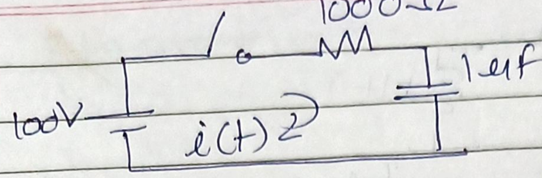
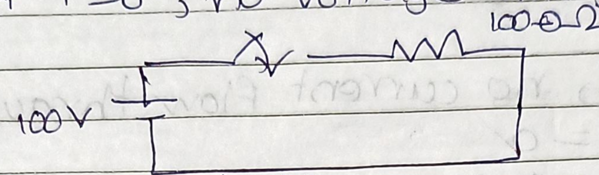
(Q) 

switch closed at $t=0$ with capacitor uncharged

$i, \frac{di}{dt}, \frac{d^2 i}{dt^2}$ at $t=0^+$

Soln: At $t=0^+$, no voltage through C

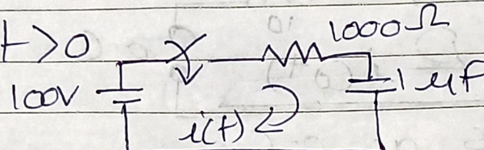


$V(0^-)=0$

$\therefore i(0^-)=0$

$$i(0^+) = \frac{100}{1000} = 0.1A$$

At $t>0$



$$100 - 1000\,i - 10^6 \int_0^t i(t)\,dt = 0$$

At $t=0^+$
$$0 - 1000\frac{di}{dt} - 10^6 i = 0$$

$$\frac{di}{dt} = \frac{10^6 i}{1000} = \frac{0.1\times 10^6}{1000}$$

$$\frac{di(0^+)}{dt} = -100 \text{ A/s}$$

$$-1000\frac{d^2 i}{dt} - 10^6\frac{di}{dt} = 0$$

$$-1000\frac{d^2 i}{dt^2} = 10^6 \times (-100)$$

$$\frac{d^2 i}{dt^2} = \frac{10^6 \times 100}{1000}$$

$$\frac{d^2 i}{dt^2} = 10^5$$

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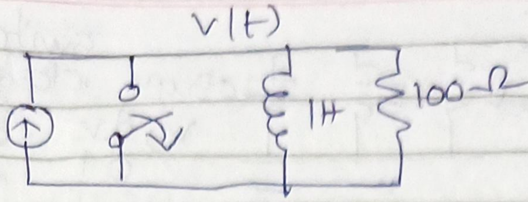
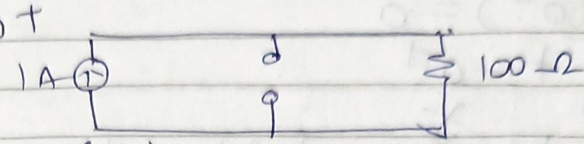


At $t = 0^-$ switch is closed
Calculate $v, \frac{dv}{dt}, \frac{d^2v}{dt^2}$ at $t = 0$

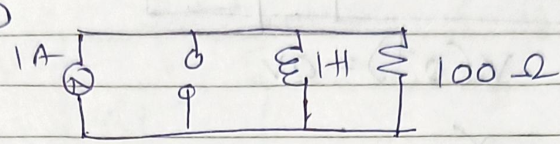
→ At $t = 0^-$ $i_L(0^-) = 0\,A$. No current through L

At $t = 0^+$



$v(0^+) = 100\,V$

At $t > 0$



$$I_1 = \frac{1}{L}\int v\,dt$$

$$I_2 = v/100$$

$$\frac{1}{L}\int v\,dt + \frac{v}{100} = 1$$

$$\frac{1}{L}v + \frac{1}{100}\frac{dv}{dt} = 0$$

$$\frac{dv}{dt} = -\frac{v}{L} \times 100$$

$$\frac{dv}{dt} = -100\,v$$

At $t = 0^+$

$$\frac{dv}{dt}(0^+) = -10^4\ V/S$$

$$\frac{d^2v}{dt^2} = -100\frac{dv}{dt}$$

At $t = 0^+$

$$\frac{d^2v}{dt^2}(0^+) = -100 \times (-10^4)$$
$$= 10^6\,V/S^2$$

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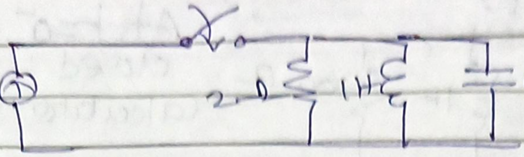


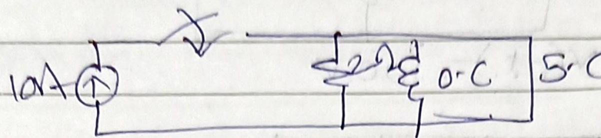
Switch is closed at $t=0$

$v, \frac{dv}{dt}, \frac{d^2v}{dt^2}$ at $t=0^+$

Soln → At $t=0^-$

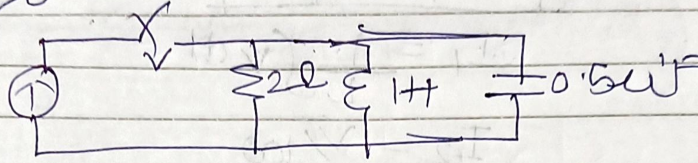
$v(0^-)=0$

At $t=0^+$



$v(0^+)=0V$

At $t>0$



$$v/2 + 1/L\int v\,dt + C\,dv/dt = 10$$

At $t=0^+$

$$\frac{v(0^+)}{2} + \frac{1}{L}\int v(0^+)dt + C\frac{dv}{dt} = 10$$

$$\frac{dv}{dt} = \frac{10}{0.5\times10^{-6}}$$

$$\frac{dv}{dt} = 2\times10^{7}\ V/S$$

$$1/2\,\frac{dv}{dt} + \frac{1}{L}v + \frac{C\,d^2v}{dt} = 0$$

$$1/2\ 2\times10^{7} + 1/L\,v(0^+) + 0.5\times10^{-6}\frac{d^2v}{dt^2} = 0$$

$$10^{7} + 0.5\times10^{-6}\,d^2v/dt^2 = -10^{7}$$

$$\frac{d^2v}{dt^2} = -2\times10^{13}\ V/S^2$$

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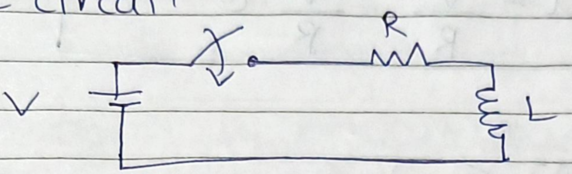
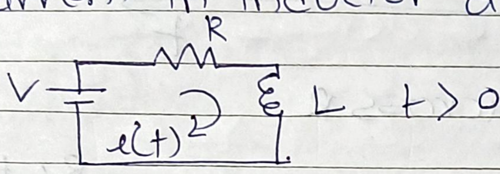
* Transient Response.

1) R-L circuit



switch is closed at $t=0$

$\therefore$ current in inductor at $t=0^{-} = 0$.



KVL eqn at $t>0$

$$V - Ri(t) - L\frac{di}{dt} = 0$$ - Linear D·E order 1

if variables are separated

$$(V-Ri)\,dt = L\,di$$

$$\frac{L}{V-Ri}\,di = dt$$

integrating both sides.

$$-\frac{L}{R}\ln(V-Ri) = t+K \quad \text{—①}$$

to get value of K, at $t=0$, $i=0$

$$-\frac{L}{R}\ln V = K \quad \text{—②}$$

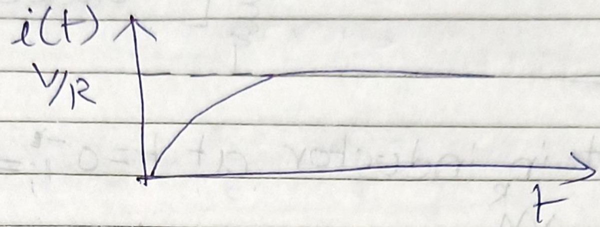
put ② in eq ①

$$-\frac{L}{R}(\ln V - Ri) = t - \frac{L}{R}\ln V.$$

→

$$v - Ri = v e^{-R/L\, t}$$

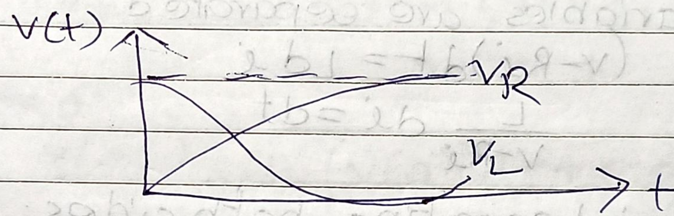
$$\boxed{i = \frac{V}{R} - \frac{V}{R} e^{-R/L\, t}}$$



Voltage across R

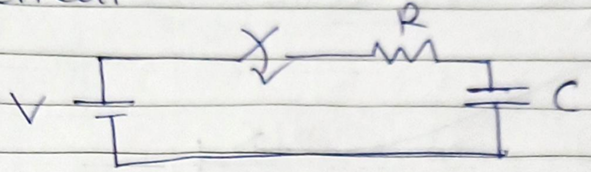
$$V_R = \cancel{V} R i = R \times \frac{V}{R} (1 - e^{-R/L\, t})$$

$$V_L = V \cdot e^{-R/L\, t} \quad \text{for } t > 0$$



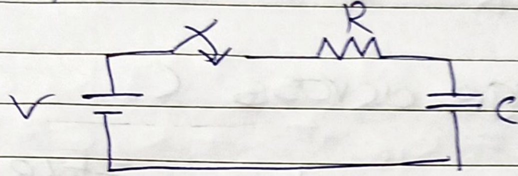
$\frac{L}{R}$ → time constant of R–L circuit.

2) R – C circuit



At $t=0$ switch is closed

∴ Capacitor is uncharged initially for $t>0$



$$V - Ri - \frac{1}{C}\int_0^t i\,dt = 0$$

$$0 - R\frac{di}{dt} - \frac{i}{C} = 0$$

$$\frac{di}{dt} + \frac{1}{RC}i = 0$$

Int both side

$$\ln i = -\frac{1}{RC}t + K$$

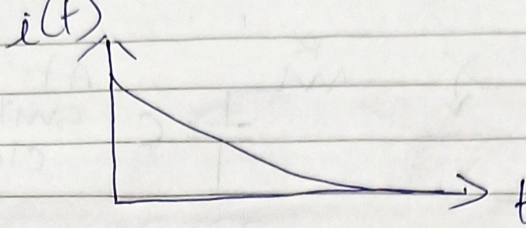
At $t=0$ $\quad i = V/R$

$$\therefore \ln\left(\frac{V}{R}\right) = K$$

$$\therefore \ln i = \frac{-t}{RC} + \ln\left(\frac{V}{R}\right)$$

$$i = \frac{V}{R} \cdot e^{-t/RC} \text{ for } t>0$$

→



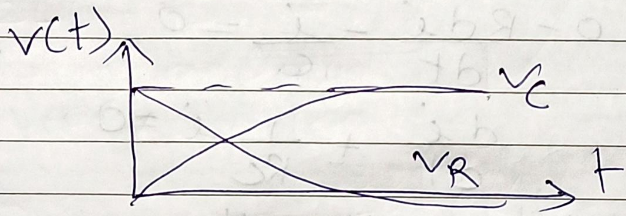
voltage across R

$$v_R = V \cdot e^{-t/RC}$$

voltage across C

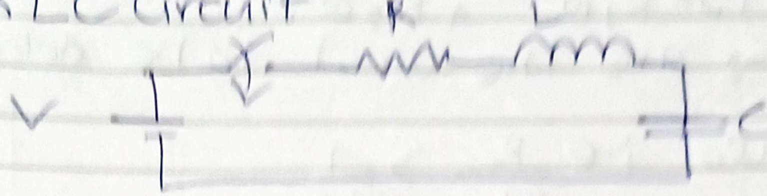
$$v_C = \frac{1}{C}\int_0^t \frac{V}{R} e^{-t/RC} dt$$

$$v_C = -Ve^{-t/RC}$$



RC → time constant of RC ckt (T).

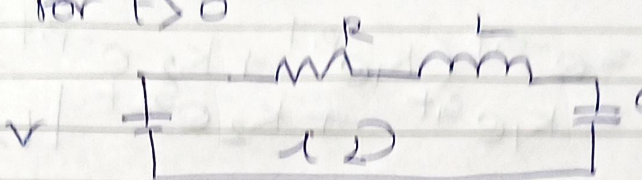
III) RLC Circuit



Switch is closed at $t=0$

Inductor current $\left.\right\}=0$
capacitor voltage

for $t>0$



$$V - Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,dt = 0$$

diff

$$0 - R\frac{di}{dt} - L\frac{d^2 i}{dt^2} - \frac{i}{C} = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

2nd order D.E

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s_1, s_2 = \frac{-R/L \pm \sqrt{(R/L)^2 - 4(1/LC)}}{2}$$

$$s_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha + \beta$$

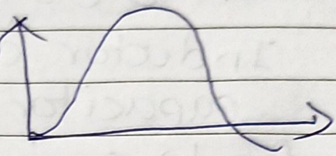
$$s_2 = -\alpha - \beta$$

$i(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

K_1, K_2 are the const to be determined.

I] $\alpha > \omega_0$ i.e. $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$

Roots are real & unequal
overdamped condn



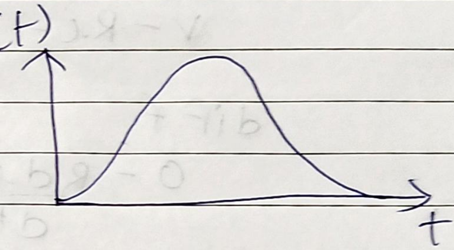
Soln is $i = e^{-\alpha t}(K_1 e^{\beta t} + K_2 e^{-\beta t})$

or $i = K_1 e^{s_1 t} + K_2 e^{s_2 t} \Big|_{t>0}$

II] $\alpha = \omega_0$

i.e. $\frac{R}{2L} = \frac{1}{\sqrt{LC}}$

Roots are real & equal
critically damped

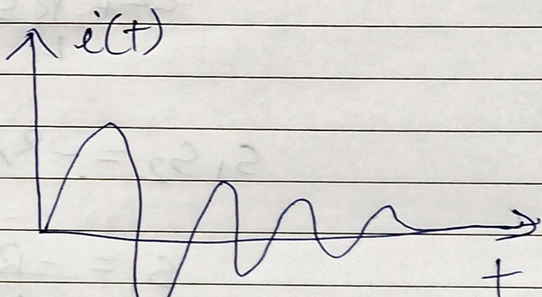


soln: is $i = e^{-\alpha t}(K_1 + K_2 t)$ $t > 0$

III] $\alpha < \omega_0$

i.e. $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$

Roots are complex conjugate.
underdamped.



$i = K_1 e^{s_1 t} + K_2 e^{s_2 t}$