Set -

Collection of well defined objects implies set.

N ⇒ Natural numbers set.
R ⇒ Real numbers set.
Z ⇒ Integer set.
$\mathbb{C}$ ⇒ Complex no set.
Q = Rational number set.

Defⁿ ⇒ $A = \{x / x = \text{Student of SY-EE}\}$

such that ⇒ $x \in EE$

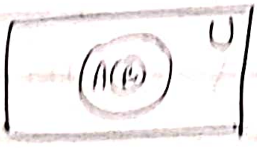
U ⇒ Universal set (relative concept)

sub set ⇒ $B \subseteq A$ ⇒ Elements of B must be in A.

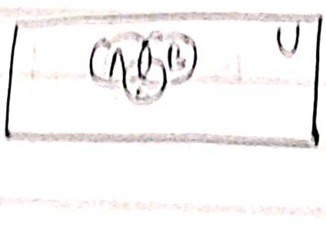
$B = \{x / x \in A\}$

Note ⇒ Pictorial presentation of a set by Venn diagram ⇒

Draw ⇒ rectangle as a universal set.
Draw ⇒ circles inside rectangle as the sets.



$B \subseteq A$



P.T.O

(1.) Union of 2 sets ⇒

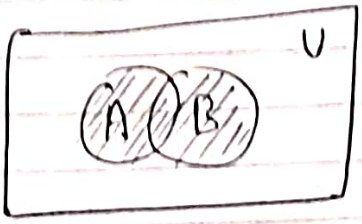
$A \cup B \Rightarrow \{x / x \in A \text{ or } x \in A\}$

Ex ⇒ $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 4, 6\}$

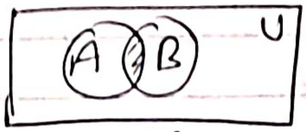
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$

$A \cup B$:



(2.) Intersection of 2 sets ⇒

$A \cap B = \{x / x \in A \ \& \ x \in B\}$



$A \cap B$

$A = \{x \mid 3 < x < 8\}$, $x \in Z^+$

$\Rightarrow \{4, 5, 6, 7\}$

$B = \{y / y > 5\}$, $y \in Z^+$ $= \{6, 7, 8 \text{ ---}\}$

$A \cap B = \{6, 7\}$

Property ⇒

$A \cap \phi = \phi$

$A \cap U = A$

$A \cup \phi = A$

3. Compliment of a set ⇒

$\bar{A} = \{x \mid x \notin A\}$

Ex ⇒ $A = \{x \mid x \in N, x > 5\}$

$\Rightarrow \{6, 7, 8 \ldots\}$

$\bar{A} = \{x \mid x \in N, x \leq 5\} = \{1, 2, 3, 4, 5\}$

★ $\bar{\bar{A}} = A$

Difference of sets ⇒

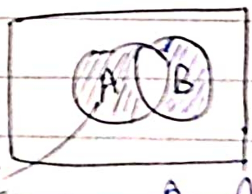
$A - B \Rightarrow \{x \mid x \in A \ \& \ x \notin B\}$

$B - A = \{x \mid x \in B \ \& \ x \notin A\}$

Ex ⇒ $A = \{1, 3, 5, 7, 9, 11\}$

$B = \{0, 2, 3, 4, 5\}$

$A - B = \{1, 7, 9, 11\}$ $B - A = \{0, 2, 4\}$



A-B B-A

$A - A = \phi$

$A - \bar{A} = A$

$\bar{A} - A = \bar{A}$

$A - \phi = A$

if $A - B = B - A \Rightarrow A = B$

$A - B = A \cap \overline{B}$

③ Symmetric difference ⇒

$A \oplus B = \{x / x \in A - B \text{ or } x \in B - A\}$

$= x\{x / x \in (A - B) \cup (B - A)\}$

Ex ⇒ $A = \{2, 4, 5, 9\}$ $B = \{x \in Z^+ / z^2 \le 16\}$

$= \{0, 1, 2, 3, 4\}$

Find

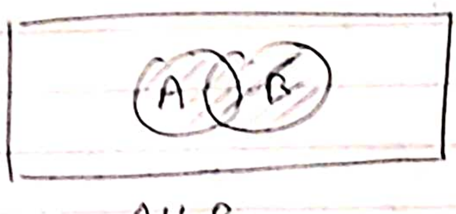
$A \oplus B = \{0, 5, 9, 1, 3\}$

HW Verify by Venn diagram ⇒

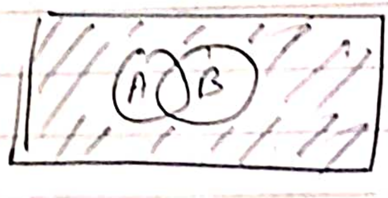
① $\overline{A \cup B} = \overline{A} \cap \overline{B}$

② $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

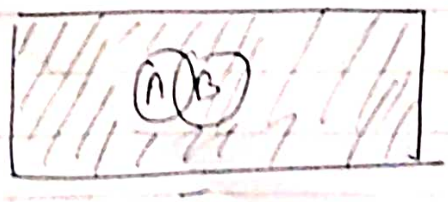
Ans ① ⇒



$A \cup B$



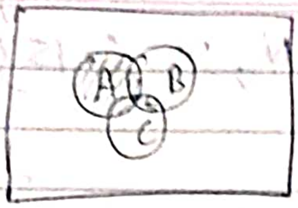
$\overline{A \cup B}$



$\overline{A}$

$\overline{B}$

(2.)



can be written as

$A \cup (B \cap C)$ " $(A \cup B) \cap (A \cup C)$

(1.) Algebra of set operations

(1.) $A \cup B = B \cup A$
$A \cap B = B \cap A$ } commutative Property

(2.) $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$ } Associative

(3.) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ } Distributive

(4.) $A \cup A = A$
$A \cap A = A$ } idempotent

(5.) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
&
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ } Demorgan's law

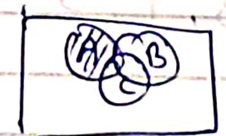
Prove with Venn diagram ⇒

$(A - B) - C = A - (B \cup C)$

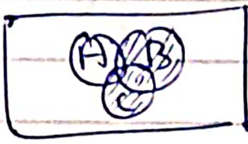
$\because A - B = \{x / x \in A$ & $x \notin B\}$

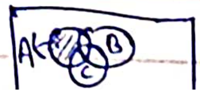
$B \cup C = \{x \mid x \in B$ or $x \in C\}$.

A - B



(A-B)-C





Ex⇒ If $U = \{x \mid x \in N, x \leq 15\}$

find $\bar{A} - \bar{B}$, $\bar{C} - \bar{A}$ & $\bar{B} - C$

$A = \{x \mid x \in N, 4 < x < 12\}$

$B = \{x \mid x \in N, 8 < x < 15\}$

$C = \{x \mid x \in N, 5 < x < 10\}$

Sol⇒

$A = [5, 6, 7, 8, 9, 10, 11]$

$\bar{A} = [1, 2, 3, 4, 12, 13, 14, 15]$

$B = [9, 10, 11, 12, 13, 14]$

$\bar{B} = [1, 2, 3, 4, 5, 6, 7, 8, 15]$

$C = [6, 7, 8, 9]$

$\bar{C} = [1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15]$

$\bar{A} - \bar{B} = [12, 13, 14]$

$\bar{C} - \bar{A} = [5, 10, 11]$

$\bar{B} - C = [1, 2, 3, 4, 5, 15]$

Q ① $\overline{A} \cup B$

② $\overline{B} \cap \overline{C}$

③ $A \oplus B$

① $\overline{A} \cup B = \{1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15\}$

② $\overline{B} \cap \overline{C} = \{1, 2, 3, 4, 5, 15\}$

③ $A \oplus B = (A-B) \cup (B-A)$

$A - B = \{5, 6, 7\}$

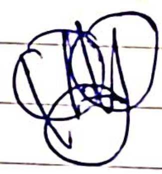
$B - A = \{12, 13, 14\}$

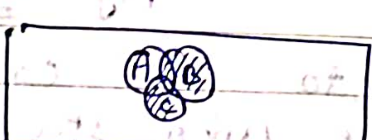
$\Rightarrow \{5, 6, 7, 12, 13, 14\}$

Q Prove

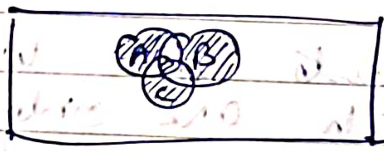
$\because A \oplus B = (A-B) \cup (B-A)$

$A \oplus (B \oplus C) = (A \oplus B) \oplus C$

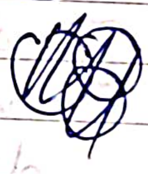




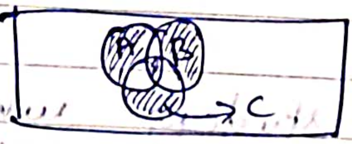
$B \oplus C$



$A \oplus (B \oplus C)$



$A \oplus B$



$(A \oplus B) \oplus C$

Q. ... in set notation

A = set of student studying maths.

B = set of student studying Physics
C = set of student " data structure
D = " " " CS
E = " " " live in hostel
F = " " " who went to watch cricket match last monday.

Q. Express in set notation:

(i) All the students who study neither maths nor Physics went to watch cricket match last monday.

$\boxed{(A \cup B)^c \subseteq F}$ $(A \cap B)^c \subseteq F$

My answer

(2) Students who went to see cricket match are only those who study data structure or maths.

$F \subseteq (C \cup A)$

(3) No student who is studying maths went to see cricket match.

$\overline{A} \cap F$ or $F \subseteq \overline{A}$

(iv) All ... went to ...

$F = A$ ...

$F \subseteq A \cup$ ...

Cardinality of ...

If A is finite ...

If $A = \emptyset$ ...

Note:

If A = set ...

E.g. $A = \{1$ ...

Cardinality ...

E: (a, b, c, d)

Q:) Express in set notations

If A = Set of Students studying Maths.

B = Set of Students studying Physics
C = Set of Students " Data Structure
D = " " " " C.S.
E = " " " Stay in hostel
F = " " " who went to watch cricket match last monday.

Q// Express in set notations =>

(1.) All the hostelites who study neither maths nor Physics ~~who~~ ~~either~~ went to watch cricket match last monday.

$\boxed{E \subseteq (\overline{A \cap B}) \cap F}$ $(\overline{A \cap B}) \cap E \subseteq F$

My answer

(2.) Students who went to see cricket match are only those who study data structure or maths.

$F \subseteq (C \cup A)$

(3.) No student who is studying maths went to see cricket match.

$\overline{A \cap F}$ or $F \subseteq \overline{A}$

(4.) All ~~Ex.~~ went to see cricket if $U = A \cup B \cup C \cup D \cup E$

$F = A \cup B \cup C \cup D \cup E$

if $F \subseteq A \cup B \cup C \cup D \cup E$

Cardinality of finite sets

If A is finite ⇒ Cardinality of $|A|$ = no of elements in A.

If $A = \phi = |A| = 0$

☆ Note ⇒

If A = set ⇒ P(A) = Power set = set of all subsets of A

Ex ⇒ $A = \{1,2,3\} = P(A) = \{\phi, A, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}\}$

$\Rightarrow |P(A)| = 8$

Cardinality of $|P(A)| = 2^n$

where n = no of elements in A.

Theorems :

1. If A & B are disjoint sets then

$$|A \cup B| = |A| + |B|$$

$$\Rightarrow |A_1 \cup A_2 \cup \ldots A_n| = |A_1| + |A_2| + |A_3| + \ldots |A_n|$$

2. If A = finite, B can be any

then $|A - B| = |A| - |A \cap B|$

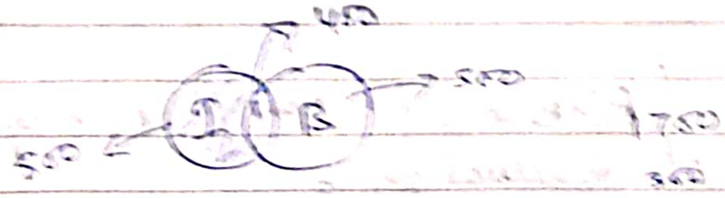
3. If A & B are finite 2 sets $\Rightarrow |A \cup B| = |A| + |B| - |A \cap B|$

(Principle of Inclusion exclusion)

4. 3 sets

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

Q) In the survey of 2000 people 1200 read india today 950 read business times & 450 read both find how many read at least 1 magazine & how many read neither.



Sol $\Rightarrow$

$|A| = 1200$ given $|U| = 2000$

$|B| = 900$

$|A \cap B| = 450$

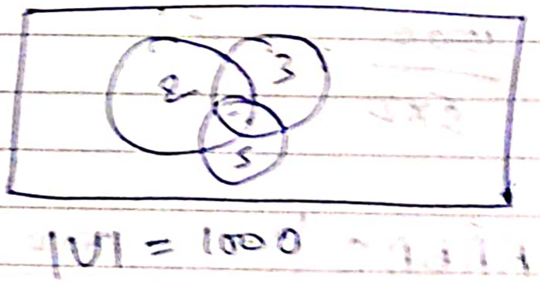
$|A \cup B| = |A| + |B| - |A \cap B|$

at least one magazine reader $= 1200 + 900 - 450 = 1750$

$\therefore$ neither magazines $= 2000 - 1700$

$\Rightarrow 300$

Ex How many integers between 1-1000 are divisible by either 2, 3 or 5?



Sol $\Rightarrow$ Here $|U| = 1000$

Let $|A|$ be set of integers divisible by $2 = \frac{1000}{2}$

$= 500$

$|B|$ i.e. set of integers divisible by $3 = \frac{1000}{3}$

$= 333$

$|C| =$ " " " " " $5 = \frac{1000}{5}$

$\Rightarrow 200$

To find $|A \cup B \cup C|$ using principle of inclusion & exclusion.

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B|$

$- |B \cap C| - |A \cap C|$

$+ |A \cap B \cap C|$

$\therefore |A \cap B| \Rightarrow$ set of integers divisible by 2 & 3 $\Rightarrow \frac{1000}{2 \times 3}$

$= 166$

$|B \cap C| = \frac{1000}{3 \times 5} = 66$

$\begin{array}{r} 166 \\ \times 2 \\ \hline 332 \end{array}$

$|C \cap A| = \frac{1000}{5 \times 2} = 100$

& $|A \cap B \cap C| = \frac{1000}{2 \times 3 \times 5} = 33$

$\therefore$ from ① $\Rightarrow |A \cup B \cup C| = 500 + 333 + 200 - 332$

$\Rightarrow$ 734

Q If $U = \{ n \in N \mid 1 \leq n \leq 9 \}$

$A = \{1, 2, 4, 6, 8\}$

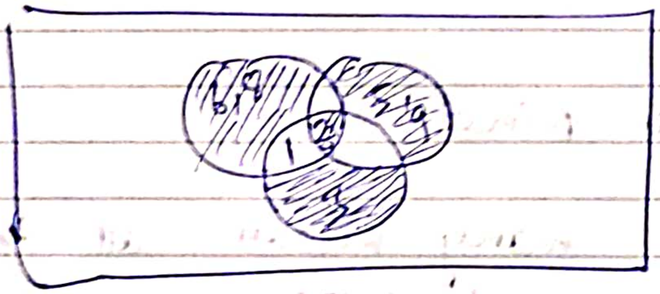
$B = \{2, 4, 5, 9\}$

$C = \{ x \in Z^+ \mid x^2 \leq 16 \}$ $C = \{1, 2, 3, 4\}$

Verify $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

$\longrightarrow$ Associative Property.

Sol $\Rightarrow$



$D = \begin{pmatrix} (A-B) \\ \cup (B-A) \end{pmatrix} = \{1, 6, 8, 5, 9\}$

$\begin{pmatrix} (D-C) \\ \cup \\ (C-A) \end{pmatrix} = \{6, 8, 5, 9, 2, 3, 4\}$

Similarly $\Rightarrow$

$(B \oplus C) = \{1, 5, 9, 3\}$

$\therefore A \oplus (B \oplus C)$

$\Rightarrow \{2, 3, 4, 5, 6, 8, 9\}$ — II

Q.) In the survey of 80 students studying Computer science.

50 → Cobol language

55 → C

46 → Pascal

37 → ~~students~~ study both C & Cobol

28 → both C & Pascal

25 → Pascal & Cobol

7 → Neither of the language

~~Ques~~ Find

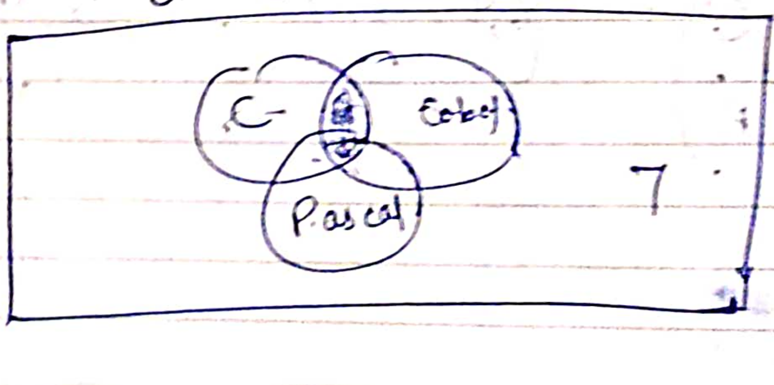
(1) How many knows all the three languages.

(2) exactly 2 languages

(3) Only one language.

73

Ry



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$73 \Rightarrow 55 + 50 + 46 - 28 - 25 - 37 + x$

(i) $x = 12$

(ii) $28 - 12 + 25 - 12 + 37 - 12 \Rightarrow 90 - 36$

$\Rightarrow 54$

(iii) $73 - 12 - 54 \Rightarrow 7$

By Maam $\Rightarrow$

$|A| = 50 \Rightarrow$ Cobol

$|B| = 55 \to C$

$|C| = 46 \Rightarrow$ Pascal

$|A \cup B \cup C| = 80 - 7$

$\Rightarrow 73$

(ii) To find $|A \cap B \cap \bar{C}| + |A \cap C \cap \bar{B}| + |B \cap C \cap \bar{A}|$

$\Rightarrow 25 + 13 + 16 \Rightarrow 54$

③ $|\overline{A}|_{\text{only}} = |A| - |A\cap B| - |A\cap C| + |A\cap B\cap C| = 0$

☆ Computer representation of sets ⇒

U = Universal set = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

If $A = \{1, 3, 5, 7, 9\}$ $B = \{2, 4, 6, 8, 10\}$

$\therefore A \subseteq U \Rightarrow A$ = String for A = 1010101010
$\rightarrow A \subseteq U$

if $x \in U \Rightarrow 1$
$x \notin U \Rightarrow 0$

B = 0101010101

$C = \{1, 3, 4, 5\} = 1011100000$

if $D = \{x \mid x \geq 5\} = \{5, 6, 7, 8, 9, 10\}$

String for D = 0000111111

And the union

Prop ⇒

1. Union − $A \cup B$

Write → 1 when union is for $1 \cup 1$ or $1 \cup 0$ or $0 \cup 1$

Write → 0 when union is for $0 \cup 0$.

Ex ⇒ $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 5, 7\}$ $B = \{1, 2, 6, 7, 8\}$

String $A = 1110101000$ $B = 1100011100$

$\therefore A \cup B = 1110111100$

Intersection ⇒
Write 1 when $1 \cap 1$ otherwise 0.

$\therefore A \cap B = 1100001000$

Ex ⇒ If $U = \{x \in N \mid 10 \le x \le 20\}$

Express following string as a set.

1. 11100010110 ⇒ $\{10, 11, 12, 16, 18, 19\}$
2. 10111110001 ⇒ $\{10, 12, 13, 14, 15, 16, 20\}$
3. 10000000000 ⇒ $\{10\}$

Multisets

In a set if the elements are repeated it becomes multiset &
the multiplicity of each element in the set is denoted by μ.

$A = [1, 1, 1, 2, 2, 3, 4] = $ m set

Multiplicity $\Rightarrow \mu$

$\mu(1) = 3 \quad \mu(2) = 2, \quad \mu(3) = 1$

$\mu(4) = 1$

Ex ⇒ (1) Garden of flowers
(2) Library books

Properties ⇒

(1) msets are equal if $\mu(x)$ is the same in sets.

if $A = [1, 1, 2, 2, 3]$
~~same~~
$B = [1, 2, 2, 1, 3]$ $\quad A = B$

if $A = [a, a, b]$ & $B = [a, b] \Rightarrow A \neq B$

(2) Subset $A \subseteq B$ if $\mu(x)$ in A is always less than or equal to $\mu(x)$ in B.

Ex ⇒ $[1,1,1,2,2] \subseteq [1,1,1,1,2,2,2,3]$

$[1,1,2] \not\subseteq [1,2,3]$

3. Union of sets ⇒

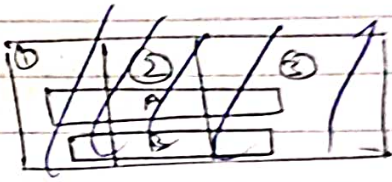
for each $x \in A \cup B$

$\mu(x) = \max\{\mu_A(x), \mu_B(x)\}$

Ex ⇒ $A = [1,1,2,2,3]$

$B = [1,2,3,3,4]$

$A \cup B = [1,1,2,2,3,3,4]$



4. Intersection for each $x \in A \cap B$

$\mu(x) = \min[\mu_A(x), \mu_B(x)]$

$A \cap B = [1,2,3]$

5. Sum of m-sets ⇒

$A + B = \{x \mid \mu(x) = \mu_A(x) + \mu_B(x)\}$

Ex ⇒ $A = [1,2,2,3,3,4]$ $B = [1,3,3,4]$

$A + B = [1,1,2,2,3,3,3,3,4,4]$

6. Difference of m-sets ⇒

$A - B = [2,2]$

(7.) Cardinality of m-set ⇒ no. of elements in m-sets.

$\therefore |A \cup B| = |A| + |B| - |A \cap B|$

Ex ⇒ If $A = [1,1,1,1]$ & $B = [1,2,2,3]$

then $|A \cup B| = [1,1,1,1,2,2,3]$

$|A \cup B| = \underline{\underline{7}}$

Polynomial representation of m-set ⇒

$[\;] = x^0$

$[x] = x^1$

$[x, x] = x^2$

$[x\,y] = xy$

Ex ⇒ Set of multi sets

$\{[\;], [x], [x,], [x, x]\}$

$\Rightarrow 1 + 2x + x^2$

Note ⇒ In general ⇒

$$(1+x)^n = \sum_{k=0}^{n} {}^nC_k\, x^{n-k}$$

(2.) $(1-x)^{-1} = 1 + x + x^2 + \text{----}$ infinite in sets

$= \{[\,], [x], [x, x], [x, x, x] \text{---}\}$

3. similarly $\Rightarrow [1-x]^{-2} = 1 + 2x + 3x^2 + 4x^3 \text{---}$

$\Rightarrow [[\,], [x][x], [x, x][x, x][x, x],$

$\text{-----}]$

Ex 2)

(1.) If $A = [a, b]$ $B = [a, b, c]$ find $\{A \cup B, A \cap B$.

(2.) $A = [a, b, b]$ $B = [a, b, a, b]$ find $|A \cup B| =$

(3.) $A = [1, 1, 3, 3, 3, 4]$ $B = [1, 2, 2, 4, 5, 5]$

find $A + B$ & $A - B$

(4.) find multiset that solves the eq^n

$A \cup [a, b, b, c] = [a, a, b, b, c, c, d]$

$A \cap [a, b, b, c, d] = [a, b, c, d]$

(1.) $A \cup B = [a, b, c]$ $\quad A \cap B = [a, b]$

(2.) $A \cup B = [a, a, b, b]$ $\quad |A \cup B| = 4$

(3.) $A + B = [1 1 1 2 2 3 3 3 4 4 5 5]$ $\quad A - B = [1 \; 3 3 3]$

Cartesian Product between sets

$$A \times B = \{ (a,b) \mid a \in A \ \& \ b \in B \}$$

Ex ⇒ $A = \{1,2,3\}$, $B = \{a,b\}$

$$A \times B = \{ (1,a)\ (1,b)\ (2,a)\ (2,b)\ (3,a)\ (3,b) \}$$

$$B \times A = \{ (a,1)\ (a,2)\ (a,3)\ (b,1)\ (b,2)\ (b,3) \}$$

$\therefore\ A \times B \neq B \times A$

$|A \times B| = |B \times A|$

Note ⇒ $A \times A = A^2$

$A \times A \times A = A^3$ - - -

$$A^n = \{ (a_1, a_2 - - - a_n) \mid a_i \in A \}$$

$i = 1, 2, - - -$

Ex ⇒ If $A = \{0,1\}$

$B = \{1, 2\}$ $\quad C = \{0,1,2\}$

$A \times (B \times C) = \{\cancel{(0,1,0)}$

$(B \times C) = \{ (1,0)\ (1,1)\ (1,2)\ (2,0)\ (2,1)\ (2,2) \}$

$A \times (B \times C) = \{ (0,1,0)\ (0,1,1)\ (0,1,2)\ (0,2,0)$

$(0,2,1)\ (0,2,2)\ (1,1,0)$

$(1,1,1)\ (1,1,2)\ (1,2,0)\ (1,2,1)\ (1,2,2) \}$

0 , 1   1 2   0 1 2

$(A \times B) \times C =$ $(A \times B) \Rightarrow \{(0,1)\ (0,2)$
$(1,1)\ (1,2)\}$

$\{(0\,1,0)$
$(0\,2\,0)\ (0\,1\,1)$
$(1\,1\,0)\ (0\,2\,1)$
$(1\,2\,0)\ (1,1,1)\ (1\,2\,1)$
$(0\,1\,2)\ (0\,2\,2)\ (1\,1\,2)$
$(1\,2\,2)\}$

$\Rightarrow A \times (B \times C)$

Ex $\Rightarrow$ If $A = \{1, 2\}$

find A^2 & A^3

$A^2 = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\}$

$A^3 = \{(1,1,1)\ (1,1,2)$
$(1,2,1)\ (1,2,2)$
$(2,1,1)\ (2,1,2)$
$(2,2,1)\ (2,2,2)\}$

Relation ⇒ A relation is an association b/w the components of set.

for ex ⇒

(a) x is father of y.
(b) y is a teacher for student z of class u in class room t.

∴ Relations are ordered n-tuples.

If $n = 2$ ⇒ ordered pair
$n = 3$ ⇒ " triplet.

If A & B are non-empty sets

then $A \times B$ ⇒ rel^n ⇒ universal rel

Ex ⇒ $A = \{1, 2\}$ $B = \{x, y\}$ ⇒ $A \times B = \{(1, x) (1, y) (2, x) (2, y)\}$

(2) If R → set of real nos then $R \times R$ is set of points in Co-ordinate Planes ⇒

Properties for the cartesian Product :-

(1) $A \times B \neq B \times A$

(2) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(3) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(4.) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(5.) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Q(6) If $A = \{1\}$ $B = \{a, b\}$ $C = \{2, 3\}$

find $A \times B \times C$, A^2, $B^2 \times A$, C^3

$A \times B \times C = A \times (B \times C) \Rightarrow (B \times C) = \{(a, 2) (a, 3)$
$(b, 2) (b, 3)\}$

$A \times B \times C = \{(1, a, 2) (1, a, 3)$
$(1, b, 2) \quad (1, b, 3)\}$

$A^2 = \{(1, 1)\}$

$B^2 \times A = \{(a, a, 1) (a, b, 1) \cdot (b, a, 1)$
$(b, b, 1)\}$

$C^3 = \{(2, 2, 2) \quad (2, 2, 3) \quad (2, 3, 2)$
$(2, 3, 3)$
$(3, 2, 2) \quad (3, 2, 3) \quad (3, 3, 2)$
$(3, 3, 3)\}$

Note $\Rightarrow$

If $\{A_1, A_2, A_3 \ldots A_n\}$ is a collection of non empty sets, then subset R of $A_1 \times A_2 \times A_3 \ldots \times A_n$ is an n-array relation on $A_1, A_2 \ldots A_n$.

If $n = 1 \Rightarrow R \rightarrow$ void relation
$n=2 \Rightarrow R \rightarrow$ binary relation.
$n=3 \Rightarrow R \Rightarrow$ tertiary relation.

$\therefore R = A_1 \times A_2 \times \ldots A_n$ is universal relation.

Ex $\Rightarrow$ If Z = set of integers,

then $R = \{x / x$ is an even integer $\} \Rightarrow$ void relation.

$\Rightarrow \{2, 4, 6 \ldots\}$

$R = \{(x, y) / x$ is divisible by $y\}$

$= \{(4,4) (2,2) (1,1) (4,2) \ldots\}$

Ex $\Rightarrow$ If $A = \{2, 3, 4\}$

& $R \Rightarrow x+y$ is divisible by z

find $R = \{(2,1,3) (2,2,4) (2,2,2) (4,2,3) (3,3,3)$
$(3,3,2) (4,4,2) (2\ 4\ 2)$
$(4,4,4) (4,2,2)\}$

HW

find ⇒

If $R \Rightarrow : x > y$ ⇒

$\{(2,3)\ (2,4)\ (3,4)\}$

Properties of Relations ⇒

If A & B are non-empty sets relation defined on A & B as $A \times B \Rightarrow R = \{(a,b) / a \in A$ & $b \in B\}$ is the binary relation

then Domain set of $R \Rightarrow D(R) = \{a \mid (a,b) \in R\}$

& Range set of $R \Rightarrow R_n(R) = \{b / (a,b) \in R\}$

Ex ⇒ $A = \{1,2,3\} \Rightarrow R \Rightarrow a < b \Rightarrow R = \{(1,2)\ (2,3)\ (1,3)\}$

$\therefore D(R) = \{1,2\} \quad R_n(R) = \{2,3\}$

(1.) Compliment of R ⇒

$\bar{R} = \{(a,b) / (a,b) \notin R\}$

(2.) Union & intersection of Relations ⇒

$R \cup S = \{(a,b) / (a,b) \in R \text{ or } (a,b) \in S\}$

$R \cap S = \{(a,b) \mid (a,b) \in R \text{ \& } (a,b) \in S\}$

(2) Converse of Relation (inverse) ⇒

$$R^{c} = \{(b,a) \mid (a,b) \in R\}$$

Properties ⇒ If R & S are relations ⇒

(1) Demorgan's law ⇒ $\overline{R \cup S} \Rightarrow \overline{R} \cap \overline{S}$

$\overline{R \cap S} \Rightarrow \overline{R} \cup \overline{S}$.

(2) $(R^{c})^{c} = R$

(3) $(R \cup S)^{c} = R^{c} \cup S^{c}$

$(R \cap S)^{c} = R^{c} \cap S^{c}$

Ex ⇒ If $A = \{1,2,3,4\}$ $B = \{a,b,c\}$

& $R = \{(1,a)\ (1,b)\ (2,a)\ (3,c)\}$.

& $S = \{(2,b)\ (3,c)\ (1,c)\ (4,a)\}$

find (1) $\overline{R}$ & $\overline{S}$ & (2) Verify D-morgan's law ⇒

Let ⇒ $A \times B$ = Universal Relation ⇒

$\{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,b)\ (2,c)$
$(3,a)\ (3,b)\ (3,c)\ (4,a)\ (4,b)\ (4,c)\}$

$\overline{R} = \{(1,c)\ (1,b)\ (2,c)\ (3,a)\ (3,b)\ (4,a)\ (4,b)$
$(4,c)\}$

$\overline{S} = \{(1,a)\ (1,b)\ (2,a)\ (2,c)\ (3,a)\ (3,b)$
$(4,b)\ (4,c)\}$

$\overline{R \cap S} = \{(1,a)(1,b)(1,c)\ (2,a)\ (2,b)\ (2,c)$
$(3,a)\ (3,b)\ (4,a)\ (4,b)\ (4,c)\}$

$\overline{R} \cup \overline{S} = \{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,b)$
$(2,c)\ (3,a)$
$(3,b)\ (4,a)$
$(4,b)\ (4,c)\}$

$\therefore \overline{R \cap S} = \overline{R} \cup \overline{S}$

$\overline{R \cup S} = \{(2,c)\ (3,a)\ (3,b)\ (4,b)(4,c)\}$
$\overline{R} \cap \overline{S} = \{(2,a)\ (2,c)\ (3,b)\ (4,b)(4,c)\}$
$\overline{R \cup S} = \overline{R} \cap \overline{S}$

Q: Let $A = \{2,3,4,6\}$ & $R = \{(a,b) \mid a = b+1$ or $b = 2a\}$

$S = \{(a,b) \mid a$ divides $b\}$

Find $D(R)$, $R_n(S)$, R^c, $(R \cap S)^c$

Sol: $R = \{(2,2)\ (4,3)\ (2,4)(3,6)\}$

$S = \{(2,4)\ (2,2)\ (3,6)\ (2,6)(3,3)(4,4)(6,6)\}$

(1.) $D(R) = \{3, 4, 2, 6\}$

(2.) $R_n(S) = \{4, 2, 6, 3\}$

(4.) $R \cap S = \{(2,4)\ (3,6)\}$ $(R \cap S)^c = \{(4,2)\ (6,3)\}$

(5.) $R^c = \{(2,3)(3,4)\ (4,2)(6,3)\}$

* Composite Relation ⇒

If R_1 is relation b/w A & B
R_2 is relation b/w B & C

then $R_1 \circ R_2 = R_1 R_2$ ⇒ Relation b/w A & C.

$= \{(a,c) \mid (a,b) \in R_1\ \&\ (b,c) \in R_2\}$

Properties ⇒

(1.) Associative ⇒ $R_1(R_2 R_3) = (R_1 R_2) R_3$

(2.) $(R_1 R_2)^C = R_2^C R_1^C$

Ex ⇒ If $A = \{2, 3, 4, 5, 6\}$ $R_1 = \{(a,b) \mid a - b = 2\}$

& $R_2 = \{(a,b) \mid a+1 = b \text{ or } a = 2b\}$

Find (1) $R_1 R_2$ (2.) $R_1 R_2 R_1$ (3) $R_1 R_2^2$ (4) $R_2 R_1$

soln ⇒ $R_1 = \{(4,2)\ (6,4)\ (5,3)\}$ $R_2 = \{(2,3)\ (3,4)$
$(4,5)\ (5,6)\ (4,2)\ (6,3)\}$

(1.) $R_1 R_2 = \{(4,3)\ (6,5)\ (6,2)\ (5,4)\}$

(2.) $R_2 R_1 = \{(3,2)\ (4,3)\ (5,4)\}$

(3.) $R_1 R_2 R_1 = \{(6,3)(5,2)\}$

(4.) $R_1 R_2^2 \Rightarrow \{(4,4)\ (6,6)\ (6,3)\ (5,5)\ (5,2)\}$

Matrix presentation for relation

If $A = \{a_1, a_2, ---- a_n\}$ & $B = \{b_1, b_2 ---- b_n\}$

$R \subseteq A \times B$ in matrix form,

$$M_{ij} = 1 \quad \text{for } (a_i, b_j) \in R$$
$$= 0 \quad \text{for } (a_i, b_j) \notin R$$

Ex: $A = \{a, b, c, d\}$ $\quad B = \{1, 2, 3\}$

& $R = \{(a,1)\ (a,2)\ (b,1)\ (c,2)\ (d,1)\}$

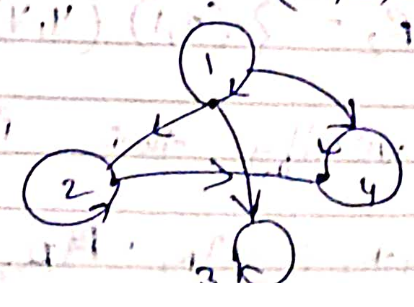
$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

In the graph $\rightarrow$ diagraph

Let $A = \{1, 2, 3, 4\}$

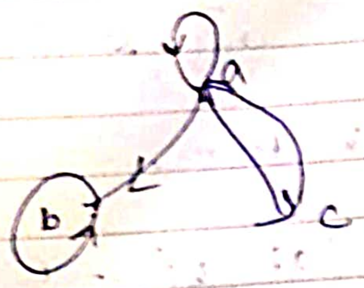
$R = \{(a,b) / a \text{ divides } b\}$

$R = \{(1,1)\ (2,2)\ (3,3)(4,4)\ (1,3)\ (1,2)\ (1,4)\ (2,4)\}$



Note ⇒ If A & B are not empty set with matrices
$A=[a_{ij}]_{m\times n}$ $B=[b_{ij}]_{m\times n}$ → are $m\times n$ matrices
then $A+B=[C_{ij}]$ where $C_{ij}=1$ if a_{ij} or $b_{ij}=1$
$=0$ if a_{ij} & $b_{ij}=0$

Ex ⇒ Diagraph is for $A=\{a,b,c\}$



$R=\{(a,a)\ (b,b)\ (a,b)\ (a,c)\ (c,a)\}$

$$M_R=\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Matrix Relation Properties / operations ⇒
If R_1 & R_2 are relations defined on the sets $\underbrace{A \text{ to } B}_{R_1}$ & $\underbrace{B \text{ to } C}_{R_2}$.

(1) $M_{R_1\cdot R_2}=M_{R_1}\cdot M_{R_2}$

(2) $M_{(R_1\cdot R_2)^c}=M_{(R_2^c\cdot R_1^c)}=M_{R_2^c}\cdot M_{R_1^c}$

Ex ⇒ $A=\{1,2,3,4\}$ $R_1=\{(1,1)\ (1,2)\ (2,3)\ (2,4)\ (3,4)\ (4,1)\ (4,2)\}$

$R_2=\{(3,1)\ (4,4)\ (2,3)\ (2,4)\ (1,1)\ (1,4)\}$

verify the Property ⇒

(1) $M_{R_1\cdot R_2}=M_{R_1}\cdot M_{R_2}$

& $A \cdot B = [d_{ij}]_{m \times r}$ $A = [a_{ij}]_{m \times n}$ $B = [b_{ij}]_{n \times r}$

where $d_{ij} = 1$ if $a_{ij} = b_{ij} = 1$

$= 0$ if $a_{ij} = 0$ or $b_{ij} = 0$

(2) $M_{(R_1 \cdot R_2)^c} = M_{(R_2^c \cdot R_1^c)} = M_{R_2^c} \cdot M_{R_1^c}$

Sol $R_1 \cdot R_2 = \{(1,1)\ (1,3)\ (1,4)\ (2,1)\ (2,4)$

$(3,4)\ (4,1)\ (4,4)\ (4,3)\}$

$$\therefore M_{R_1 \cdot R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array} \quad \& \quad M_{R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$M_{R_1} \cdot M_{R_2} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

(2) If $A = \{1, 2, 3, 4\}$ $R_1 = \{(1,1)\ (1,2)\ (2,3)$
$(2,4)\ (3,4)\ (4,1)\ (4,2)\}$

$R_2 = \{(3,1)\ (4,4)\ (2,3)$
$(2,4)(1,1)\ (1,4)\}$

PT $\Rightarrow M_{(R_1 \cdot R_2)} = M_{R_1} \cdot M_{R_2}$

$M_{(R_1 \cdot R_2)^c} = M_{R_2^c} \cdot M_{R_1^c}$

$(R_1 \cdot R_2)^c = \{(1,1)\ (3,1)\ (4,1)\ (1,2)\ (4,2)\ (4,3)\ (1,4)$
$(4,4)\ (3,4)\}$

$$M_{(R_1 \cdot R_2)^c} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & & & \\ 3 & & & & \\ 4 & & & & \end{array} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$R_1^c = \{(1,1)\ (2,1)\ (3,2)\ (4,2)\ (4,3)\ (1,4)\ (2,4)\}$

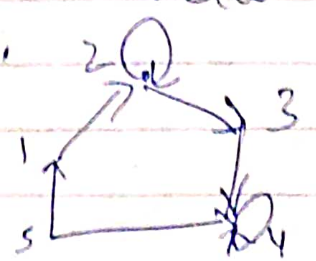
$R_2^c = \{(1,3)\ (4,4)\ (3,2)\ (4,2)\ (1,1)\ (4,1)\}$

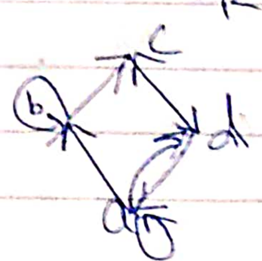
$$M_{R_2^c} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 1 & 1 & 0 & 1 \end{array}$$

$$M_{R_1^c} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 \end{array} = (M_{R_1})'$$

$$M_{R_2^c} \cdot M_{R_1^c} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Q Find the relation R & M_R from the diagraph.





1.) $R_b = \{(2,2)\ (2,3)\ (3,4)\ (4,4)\ (5,4)\ (5,1)\ (1,2)\}$

$$M_R = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

2.) $R = \{(a,a)(a,d)(b,b)\ (a,b)(b,c)\ (c,d)\ (d,a)\ (c,c)\}$

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Q. If $A = \{1, 2, 3, 4\}$ & $R = \{(a,b) / (a-b)$ is a multiple of $2\}$
$S = \{(a,b) / (a-b)$ is a multiple of $3\}$
find $R \cup S$, $R \cap S$, draw M_R & M_S with diagraph?

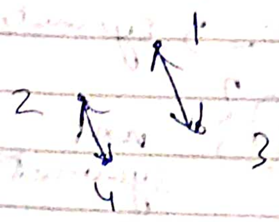
Sol $\Rightarrow$

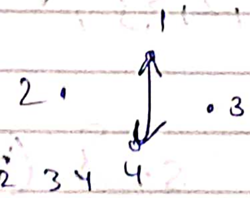
$R = \{(4,2)\ (3,1)\ (1,3)\ (2,4)\}$

$S = \{(4,1)\ (1,4)\}$

$R \cup S = \{(2,4)\ (4,2)\ (1,3)\ (3,1)\ (4,1), (1,4)\}$

$R \cap S = \phi.$





$$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}
\qquad
M_S = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Properties of Relation ⇒

(1.) Reflexive ⇒
If A is any set & R is relation on A then for every $a \in A$
$\Rightarrow (a,a) \in R$
or aRa

then R is Reflexive.

Ex ⇒ $A = \{1, 2, 3\}$
$R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\}$
is reflexive.
$S = \{(1,1)\ (2,2)\ (1,3)\}$ not reflexive
as $(3,3) \notin R$

$R \in$ irreflexive
if ~~no element~~ for every
$a \in A$, $(a,a) \notin R$.

Ex ⇒ $T = \{(1,2)\ (3,2)\}$ is irreflexive

(2.) Symmetric : if $(a,b) \in R \Rightarrow (b,a) \in R$

Ex ⇒ $A = \{1,2,3\}$, $R = \{(1,1)\ (1,2)\ (2,1)\}$
→ Symmetric

$S = \{(2,2)\ (2,3)\}$ not symmetric

Assymmetric ⇒ if $(a,b) \in R \Rightarrow aR_b$ then
S is assymmetric $(b,a) \notin R$.

Antisymmetric $\Rightarrow$ if $(a,b) \in R$ & $(b,a) \in R \Rightarrow$ $a=b$

ex $\Rightarrow$ $T = \{(1,1)(2,2)\}$ anti symmetric

(3.) Transitive $\Rightarrow$ if $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$

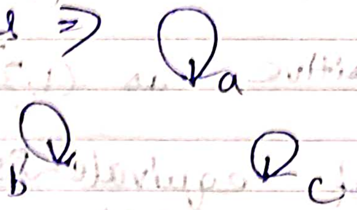
$A = \{1,2,3\}$ $R = \{(1,1)(2,2)(1,3)(3,2)(1,2)\}$ → transitive

$S = \{(3,3)(2,3)(3,1)\}$ not transitive as $(2,1) \notin R$

b is interior vertex.

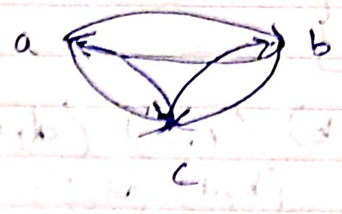
Diagraphs $\Rightarrow$

(1) Reflexive $\Rightarrow$

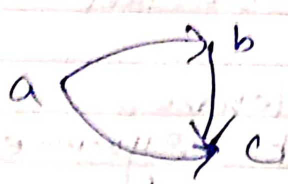


$A = \{a,b,c\}$

(2.) Symmetric



(3.) Transitive $\Rightarrow$



Equivalence relation ⇒ If A is any set & R is relation which is reflexive, symmetric & transitive, then R is equivalence relation.

Ex ⇒ $A = \{1,2,3,4\}$

$R = \{(1,1)(2,2)(1,2)(2,1)(3,3)(2,3)\}$

R is not reflexive as $(4,4) \notin R$

R is not symmetric as $(3,2) \notin R$

R is not transitive as $(1,3) \notin R$

So R is not equivalence relation.

Ex ⇒ $A = \{a,b,c,d\}$

$R = \{(a,a)(b,b)(c,c)(d,d)(b,a)(d,c)\}$

determine whether R is equivalence relation?

Ans ⇒ R is reflexive.
R is not symmetric as $(a,b) \notin R$, $(c,d) \notin R$

R is transitive

Not equivalence.

Q. If $A = N =$ natural no. set.

& $R = \{(a,b) / a,b \in N$ & $a+b$ is odd no$\}$

Determine whether R is eq. relⁿ or not.

Ans ⇒ R is symmetric
R is not reflexive
R is not transitive

Not equivalence Relation.

* Equivalence class ⇒

for every $a \in A$, R is equivalence Relation.

Let $[a]_R = \{x \in A / (x,a) \in R\}$
$(a,x) \in$

Ex ⇒ $A = \{1,2,3\}$ $R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\}$
eq. rel.

$[1]_R = \{1,2\}$

$[2]_R = \{1,2\} = [1]_R$

$[3]_R = \{3\}$

$\bigcup_{a \in A} [a]_R = A$

~~No of distinct classes define the rank of R.~~

Rank of R ⇒ no. of distinct eq. classes.

Note ⇒ R ⇒ equivalence Relation.

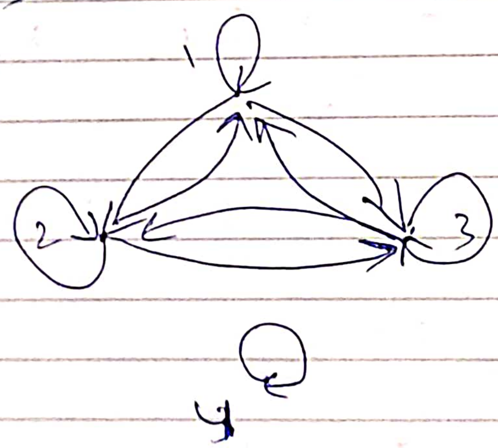
$\forall a,b \in A$ either (i) $[a]_R = [b]_R$ or
$[a]_R \cap [b]_R = \phi$

(2) $\bigcup_{a \in A} [a] = A$

Ex. If $A = \{1, 2, 3, 4\}$, $R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (2,3)\ (1,3)\ (3,2)\ (2,1)\ (3,1)\}$

S.T. R is equivalent relation & hence find equivalent classes & rank.

Diagraph ⇒



$[1]_R = \{1, 2, 3\}$

$[4]_R = \{4\}$

$[2]_R = \{1, 2, 3\} = [1]_R$

$[3]_R = \{1, 2, 3\} = [1]_R$

∴ 2 distinct classes

rank = 2

Partition of A : If $A_1, A_2 - - - A_n$ is collection of non empty sets such that $\bigcup_{i=1}^{n} A_i = A$

& (2) $A_i \cap A_j = \phi \quad i \neq j$

then Partition $\pi = \{A_1, A_2 - - - A_n\}$

Partition is not unique.

Ex ⇒ $A = \{1,2,3\}$ then $\pi_1 = \{\{1,2\}\{3\}\}$

$\pi_2 = \{\{1\}\{2\}\{3\}\}$

$\pi_3 = \{\{1\}\{2,3\}\}$ - - - etc

Note ⇒ ① if A is non-empty set, R is equivalence relation on A then set of all equivalence classes constitute the Partition.

② If π is a Partition on A then π induces an equivalence relation on the set A.

Ex ⇒ If $A = \{1,2,3\}$ & $R = \{(1,1)(2,2)(3,3)(1,3)(3,1)\}$

∴ eq classes $[1]_R = \{1,3\}$ ∴ set of all distinct equivalence classes $= \{\{1,3\}\{2\}\}$ ⇒ Partition ⇒ π

$[2]_R = \{2\}$

$[3]_R = \{3,1\} = [1]_R$

if $\pi = \{\{1,3\}\{2\}\}$ then $R = \{(1,1)(2,2)(3,3)(1,3)(3,1)\}$

Q.) if $A = \{1, 3, 5, 7\}$

Find Partitions of A s.t. π_1 & π_2

induces equivalence on A.

Sol:) $\pi_1 = \{\{1\}, \{3\} \{5\} \{7\}\}$

$\pi_2 = \{\{1, 3\} \{5\} \{7\}\}$

$R_1 = \{(1,1) \ (3,3) \ (5,5) \ (7,7)\}$

$R_2 = \{(1,1) \ (1,3) \ (3,1) (3,3) \ (5,5) (7,7)\}$

Transitive closure → It is the smallest transitive relation on set A. for that

$A = \{ \quad \}$, $R = \{ \quad \}$

To find the transitive closure:—

$R^* = R \cup R^2 \cup R^3 \ldots \cup R^n$ where n = no. of elements in A.

$R^2 = R \cdot R$ → Composite Relⁿ (transitivity)

$R^3 = R^2 \cdot R$ →

— — —

Let $A = \{1, 2, 3, 4\}$ & $R = \{(1,2)(2,3)(3,4)\}$

$R^* = R \cup R^2 \cup R^3 \cup R^4$

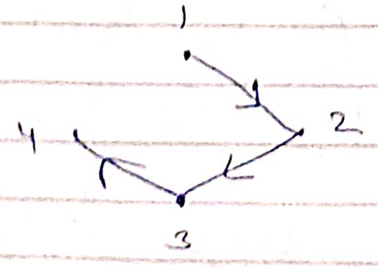
$R \cdot R = \{(1,3)(2,4)\} = R^2$

$R^2 \cdot R = \{(1,4)\}$

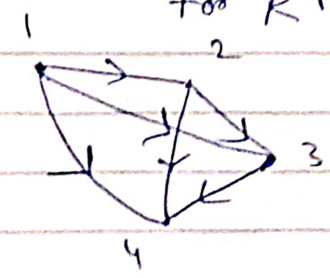
$R^3 \cdot R = \{ \; \} = \phi$

$R^* = \{(1,2)(2,3)(3,4)(1,3)(2,4)(1,4)\}$

for R



for R^*



Ex ⇒ if $A = \{a, b, c, d\}$

$R_1 = \{(a,a)\ (b,b)\ (c,c)\ (a,b)\}$

$R_2 = \{(a,a)\ (b,d)\ (d,c)\}$

find $(R_1 \cup R_2)^*$

Sol ⇒ let $(R_1 \cup R_2) = \{(a,a)\ (b,b)\ (c,c)\ (a,b)$
$(b,d)\ (d,c)\}$
$= R$

∴ $R \cdot R = \{(a,b)\ (b,d)\ (a,d)$
$(b,c)\}$

$R^2 \cdot R = \{(a,b)\ (a,d)\ (b,c)$
$(a,c)\}$

$R^3 \cdot R = \{(a,b)\ (a,d)\ (a,c)$
$(b,c)\}$

$(R_1 \cup R_2)^* = R \cup R^2 \cup R^3 \cup R^4 = \{(a,a)\ (b,b)$
$(c,c)\ (a,b)$
$(b,d)\ (d,c)$
$(a,d)\ (b,c)\ (a,c)\}$

Warshall's Algorithm ⇒

Find the transitive closure in the algorithm form warshall has defined step-wise procedure while considering the relationship matrix M_R on the set A where transitive closure is matrix $= W_K$ when K is no. of elements in A.

Step-I ⇒ Write W_0 = initial matrix $= M_R$ (Given relⁿ)

Step-II ⇒ for the 1st element in A ⇒ $K=1$ ⇒ find W_1 (matrix) by identifing the transitive connection ~~from~~ where $K=1$ is an interior vertex, Put ~~that~~ as 1 in place of transitive connection.

(1,2) (2,3)

interior vertex

Step-III ⇒ For the interior vertex $K=2$ find the ~~the~~ matrix W_2. if $K=2$ is not the interior vertex for any pair then ~~that~~

$$W_1 = W_2$$

Step-IV ⇒ for $K=3$ → identify W_3 →

— — — — —

Continue Procedure for W_K where

$$W_K = R^* = M_R^*$$

Ex ⇒ Let $A = \{1,2,3,4\}$

$R = \{(1,2)\ (2,4)\ (1,3)\ (3,2)\}$

Find M_R^* ?

Sol ⇒ $R^* = R \cup R^2 \cup R^3 \cup R^4 = \{(1,2)$
$(2,4)\ (1,3)\ (3,2)$
$(3,4)\ (1,4)\}$

By warshall's Algorithm ⇒

Let $W_0 = M_R =$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \Rightarrow W_1 = W_0$$

for $k=1$ → interior vertex ⇒ not any pair

for $k=2$ → is interior vertex for
$(1,2)\ (2,4) \Rightarrow (1,4)$
also for $(3,2)$ & $(2,4) \Rightarrow (3,4)$
Put (1) ⇒

$$\therefore W_2 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

which is already Present

for $K=3$ $\rightarrow W_2 = W_3 \Rightarrow$ interior vertex 3 for $(1,3)(3,2)$

& $(1,3)$ & $(3,4) \Rightarrow (1,4)$ Present

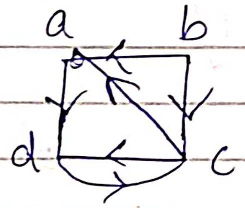
for $K=4$ $\Rightarrow$ not interior vertex for any pair

~~Warshall's Algorithm~~

$$\therefore W_4 = W_3 = W_2 = M_R^{*} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Transitive closure

Using the diagraph, find M_R & M_R^{*} on $A=\{a,b,c,d\}$



$R = \{(a,d)(b,a)(c,a)(c,d)(d,c)(b,c)\}$

$$M_R = W_0 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

for $K=a \Rightarrow$

it is interior vertex for $\Rightarrow (b,a)(a,d)$
& $(c,a)(a,d)$

$\therefore \Rightarrow (b,d)\ (c,d)$

$$W_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

for ~~C~~ $K = b \Rightarrow$

It is interior vertex for no pair $\Rightarrow$

$W_1 = W_2$

(d,b)

for $\Rightarrow K = C \Rightarrow$ interior vertex for d to b $\Rightarrow$

Put 1

also d to a $\Rightarrow$ Put 1

(d, a)

$$W_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

(a,d) (d, a)

$\Rightarrow$ (a, a)

for $\Rightarrow K = d \Rightarrow$

interior vertex for ~~(c,d)(d,c)~~

(c,d) (d,c) $\Rightarrow$ (c,c)

$\therefore$ (a,d) (d,c)

$\Rightarrow$ (a, c)

$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Ans

Q.) If $A = \{a_1, a_2, a_3, a_4, a_5\}$ find M_R^*

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$R = \{(a_1, a_1), (a_1, a_4), (a_2, a_2), (a_3, a_4), (a_3, a_5), (a_4, a_1), (a_5, a_2), (a_5, a_5)\}$

(a_4, a_4)

$$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$W_3 = W_1 = W_2$

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a_3, a_1)

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a_3, a_2)

$\Rightarrow M_R^*$

Hasse
Partial order Rel^n, ~~Hasse~~ diagram & lattice ⇒

If A is any non-empty set & R is rel^n on A which is reflexive, antisymmetric & transitive than (A, R) is called partial order rel^n set or PO set & R is partial order rel^n.

notation ⇒ $\leq$ (POR)

Reflexive → aRa for every $a \in A$
antisymmetric ⇒ aRb but $b \not R a$ if $a \neq b$

Transitive → aRb & bRc ⇒ aRc

Hasse Diagram ⇒

1) No arrow heads, no loops.
2) Transitivity arc (or edge) is not drawn ⇒ a to b are present if there is no c between a & b.
3) Arc pointing upwards is drawn from a to b if $a \neq b$ & aRb.

Ex ⇒ $A = \{1, 2, 3, 4\}$

$R = \{ (1,1) (2,2) (3,3) (4,4) (1,2) (2,4) (1,3) (3,4) (1,4) \}$

Is R a POR? if so draw Hasse diagram.
Ans ⇒ R is reflexive as $(1,1) (2,2) (3,3) (4,4) \in R$

R is antisymmetric as (1,2) but $(2,1) \notin R$
(2,4) but $(4,2) \notin R$

$(1,2)\ (3,4)\ (1,4) \in R$

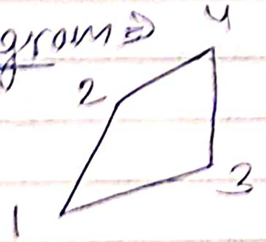
But $(2,1)\ (4,3)\ (4,1) \notin R$

R is transitive as $(1,3)\ (3,4) \in R$ $(1,4) \in R$

& $(1,2)\ (2,4) \in R \Rightarrow (1,4) \in R$

$\therefore$ R is partial order Relation.

Hasse diagram $\Rightarrow$



Q.) If $A = \{2, 3, 4, 6\}$

& $R = \{(a,b) \,/\, a \text{ divides } b\}$

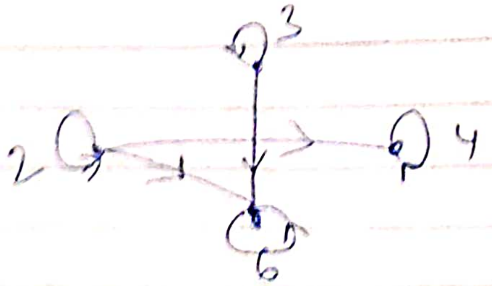
Is (A,R) a poset? draw diagraph $\Rightarrow$

$R = \{(2,4)(2,6)(3,6)\ (2,2)\ (3,3)$
$(4,4)\ (6,6)\}$

It is reflexive $\Rightarrow$

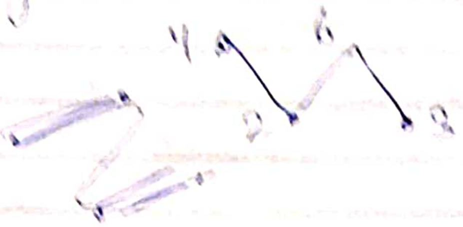
It is anti symmetric.

It is transitive.



Diagraph.

Hasse diagram ⇒



Chain & antichains (Using Hasse diagram)

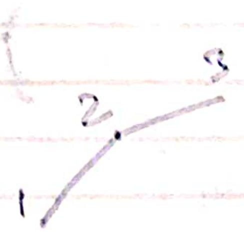
If $(A, \leq)$ is a Poset then subset of A is chain if every pair of subset are related.

A subset of A becomes anti chain if no 2 distinct elements are connected from subset.

∴ length of chain = no. of elements in chain

Q) If $A = \{1, 2, 3\}$ & $R = \{(a,b) / a \leq b\}$ find the chains & antichains if (A;R) is Poset.

Sol ⇒ $R = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (1,3)\}$



chains ⇒ subsets of A

$\{1, 2, 3\} = 3$ $\{1, 2\}$, $\{2, 3\}$

⇒ length = 2

anti chain = $\{1, 3\} \subseteq A$

Q) If $A = \{a,b\}$

ST $(P(A), \subseteq)$ is a Poset.

Find chains & antichains.

$P(A) = \{\emptyset, \{a\} \{b\} \{a,b\}\}$

$\therefore (P(A), \subseteq) \rightarrow POR$

$\Rightarrow \emptyset \subseteq \emptyset$

$\{b\} \subseteq \{b\} \Rightarrow (\{b\}, \{b\})$

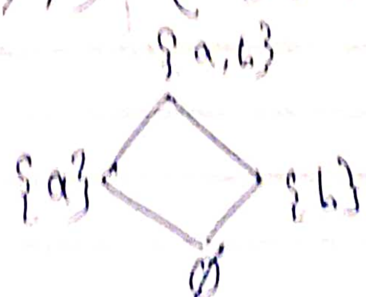
$\{a\} \subseteq \{a\}$

$\{a,b\} \subseteq \{a,b\}$

$\Rightarrow$ reflexive

antisymmetric $\Rightarrow aR_b \Rightarrow b \not R_a$ unless $a=b$

$R = \{ (\{a\}\{a\}) (\{b\}\{b\}) (\{a,b\}\{a,b\}) (\{a\}, \{a,b\}) (\emptyset, \{a\}) (\emptyset, \{b\}) (\emptyset, \{a,b\}) (\emptyset, \emptyset) (\{b\}\{a,b\}) \}$



chain ⇒ $\{\emptyset, \{a\}, \{a,b\}\}$

length = 3

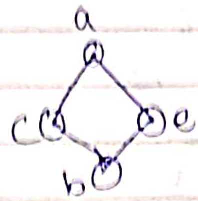
A Poset & lattice

A set with Partial order relation ($\leq$) is called Poset.

$\Downarrow$

Hasse diagram

Lattice $\Rightarrow$ If A is any set with Partial order Relation then from Hasse diagram of the poset the set L for the PO relation is called lattice in which every subset ~~(a, b)~~ $\{a, b\}$ of L has lub (least upper bound) & glb (greatest lower bound).



$\Rightarrow$ Lattice

$\{c, e\} \longrightarrow$ lub $\Rightarrow a$

glb $\Rightarrow b$

& $\{a, c\}$ = lub = a

glb = b

$\{a, c\}$ = glb = b

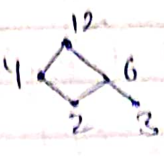
lub = a

Ex? Identify whether lattice

If $A = \{2, 3, 4, 6, 12\}$ & $R = \{(a, b) / a \text{ divides } b\}$

$R = \{(2,2)\ (2,4)\ (2,6)\ (2,12)\ (3,3)\ (3,6)\ (3,12)$
$(4,4)\ (4,12)\ (6,6)\ (6,12)\ (12,12)\}$

Hasse diagram $\Rightarrow$



$\therefore (L, \leq)$ is not Lattice as $\{2, 3\}$ does not contain glb

Ex:- If $A = \{1, 2, 3, 4, 6, 12\}$ $R = \{(a,b) / a \text{ divides } b\}$

Is it lattice?

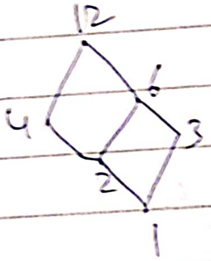
$R = \{(1,1) (1,2) (1,3) (1,4) (1,6) (1,12)$

$(2,2) (2,4) (2,6) (2,12) (3,6) (3,3)$

$(3,12) (4,4) (4,12) (6,6) (6,12)$

$(12,12)\}$

Hasse diagram ⇒



It is a lattice.

Q. If $A = \{a, b, c\}$, Is $(P(A), \subseteq)$ a lattice?

Here $P(A) = \{\phi, \{a\} \{b\} \{c\}$

$\{a,b\} \{b,c\} \{c,a\}$

$\{a,b,c\}\}$

$R = \{(\phi,\phi) (\phi, \{a\}) (\phi, \{b\}) (\phi, \{c\})$

$(\phi, \{a,b\}) (\phi, \{b,c\}) (\phi, \{c,a\}) (\{a\}, \{a\}$

$(\phi, \{a,b,c\}) (\{a\}, \{a,b\}) (\{a\}, \{c$

$(\{a\}, \{a,b,c\})$ $(\{b\}, \{b\})$ $(\{b\}, \{a,b\})$ $(\{b\}, \{b,c\})$

$(\{b\}, \{a,b,c\})$ $(\{c\}, \{c\})$ $(\{c\}, \{b,c\})$

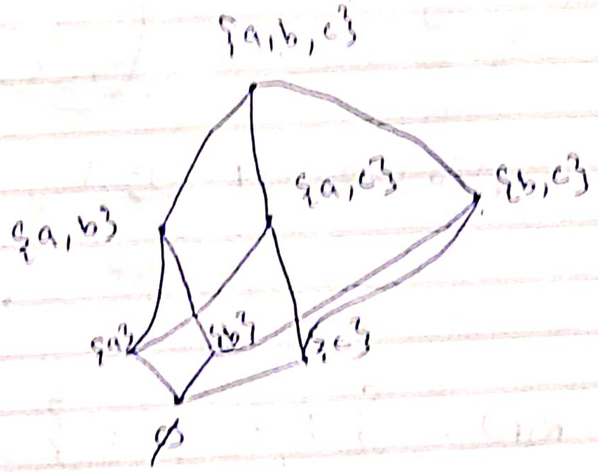
$(\{c\}, \{c,a\})$ $(\{c\}, \{a,b,c\})$

$(\{a,b\}, \{a,b\})$ $(\{a,b\}, \{a,b,c\})$

$(\{b,c\}, \{b,c\})$ $(\{b,c\}, \{a,b,c\})$

$(\{c,a\}, \{c,a\})$ $(\{c,a\}, \{a,b,c\})$

$(\{a,b,c\}, \{a,b,c\})$



Functions

If A & B are non-empty sets then a function A defined from A to B is a particular relation where every element of A has unique image in B denoted by $f(a) = b$

$A \Rightarrow$ domain of $f \Rightarrow D(f)$

$B \rightarrow$ co-domain of f.

& $f(a) \Rightarrow$ is range of $f \rightarrow R(f) \subseteq B$

Ex ⇒

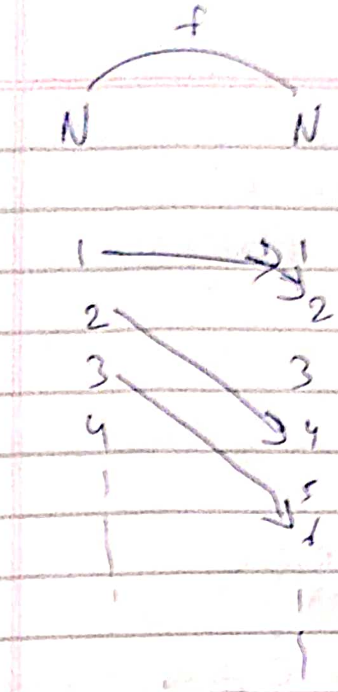
$f(a)=2$, $f(b)=1$, $f(c)=1$, $f(d)=4$, $f(e)=5$

$R(f) = \{1, 2, 4, 5\} \subseteq B$

★ Types of f ⇒

① $A \rightarrow A$ is identity function.

Ex ⇒ $f: N \rightarrow N$ defined by $f(x) = 2x$

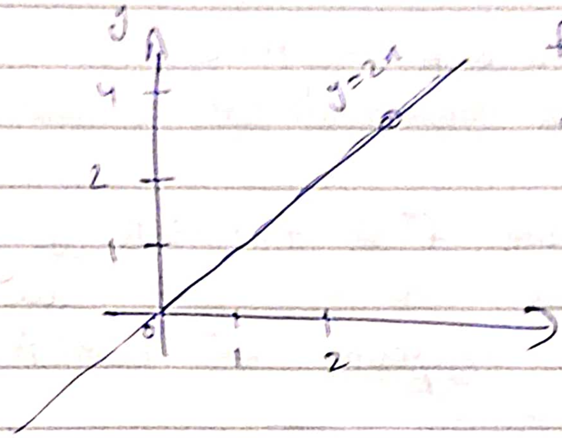


$\therefore R(f) =$ Even no set

$= \{2, 4, 6, 8 \ldots\}$

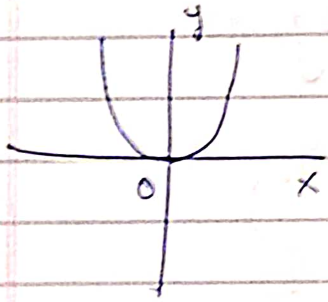
$f(x) = y = 2x$

straight line



Ex ⇒ If $f : N \to N$

$f(x) = x^2 = R(f) = \{1, 4, 9, 16, \ldots\}$



for computer programming function is a procedure which gives unique output for the suitable input.

Ex ⇒ $f(x) = \sqrt{x}$ & $f : Z \to Z$

is not f^n as $f(4) = \sqrt{4} = \pm 2$ #

Note ⇒ If f^n is not defined for particular argument $(a \in A)$

⇒ Partial function.

Ex ⇒ If $f : Z \to Z$ & $f(x) = 1/x \Rightarrow \infty$ when $x = 0$

$\therefore$ f is partial function.

Ex ⇒ $f : R \to R$ & $f(x) = \sqrt{x}$ ⇒ Partial function

⇒ for negative values of x.

Equivalent f^n ⇒ If $f: A \to B$ & $g: C \to D$ are f^ns.

then f & g are equivalent if $f(a) = g(a)$

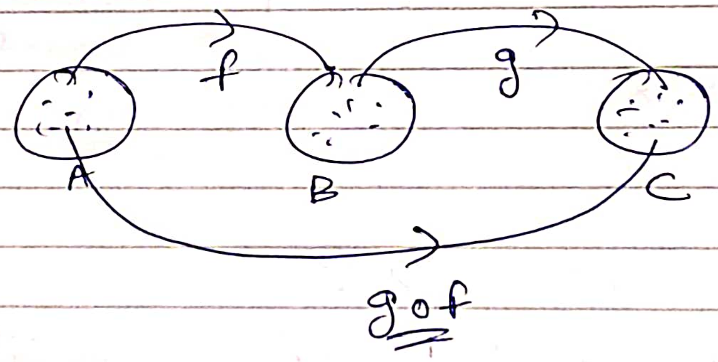
$\forall a \in A$ ⇒ $A = C$
$B = D$

Composite f^n ⇒ $f: A \to B$ & $g: B \to C$ are f^n.

then composite f^n from A to C ⇒ $g \circ f: A \to C$ is f^n.

& if $\left.\begin{array}{l} f: f(x) \\ g: g(x) \end{array}\right\}$ then $g \circ f = g[f(x)]$ = composite f^n.

& $g \circ f \neq f \circ g$



Ex ⇒ If $f: N \to N$ defined as $f(x) = x + 1$

& $g: N \to N$ defined as $g(x) = 3x$

find $g \circ f$, $f \circ g$, $f \circ f$, $g \circ g$

Soln ⇒ $g \circ f = N \to N$ ⇒ $g[f(x)] = g[x+1]$

⇒ $3(x+1)$

$fog : N \Rightarrow N \Rightarrow f[g(x)] = f[3x]$

$= 3x+1$

$fof : N \to N \Rightarrow f[f(x)] = f[x+1]$

$\Rightarrow x+2$

$gog : N \to N \Rightarrow g[g(x)] = g[3x]$

$\Rightarrow 9x$

Q.) find composite f^n $f : Z \to R \Rightarrow$ by $f(x) = \frac{x+1}{2}$

$g : R \to R \Rightarrow g(x) = x^2$

Here composition is Posible gof $Z \to R$

$g[f(x)] \Rightarrow g\left[\frac{x+1}{2}\right] \Rightarrow \left(\frac{x+1}{2}\right)^2 \Rightarrow \frac{x^2+1+2x}{4}$

HW

I $f : N \to N \Rightarrow f(x) = x-1$ find gof, fog, $hogof$, gof

$g : N \to N \Rightarrow g(x) = x^2+1$

$h \Rightarrow N \to N \Rightarrow h(x) = 2x$

(1.) $g(x-1) = (x-1)^2+1$

(2.) $f(x^2+1) = x^2$

$2(x^2+1) \Rightarrow hog$

$2((x-1)^2+1) \Rightarrow hogof$

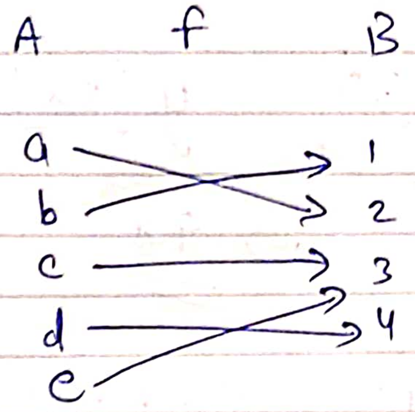
$(x^2+1)^2+1 \Rightarrow gog$

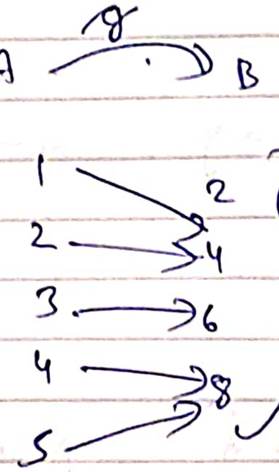
Special types ⇒

(1) Surjective (on to) $f^n \Rightarrow f : A \to B$ if
$f(A) = B$

(all elements of B set should be mapped

Ex ⇒



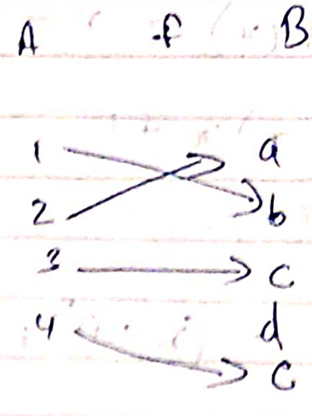


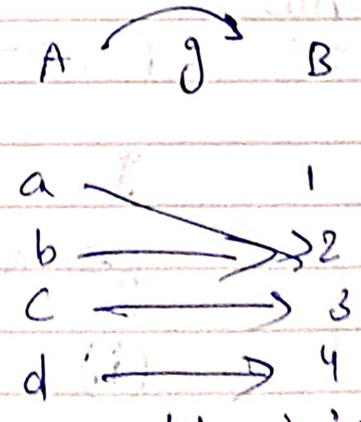
$f(a) = 2$
$f(b) = 1$
$f(c) = 3$
$f(d) = 4$
$f(e) = 3$ $f \Rightarrow$ is onto

→ g is not onto as 2 → not image of any element A.

(2) Injective (one – one) $f^n \Rightarrow$
if $f(a) = f(a') \Rightarrow a = a'$ or $f(a) \neq f(a') \Rightarrow a \neq a'$ where
$f : A \to B$

Ex ⇒





f is one-one

Not injective.

Note ⇒ A f^n is said to be bijective if it is one - one & on to.

2.) Identity f^n $I : A \to A$ is bijective.

Ex ⇒ Identify of whether one - one, on-to or bijective.

(1.) $f : R \to R$ as $f(x) = x+1$
bijective

(2) $f : R \to R^+$ as $f(x) = x^2$
on to
not one - one

(3.) $f : \underset{\text{even}}{E} \to \underset{\text{all integer}}{Z}$ as $f(x) = 2x$

one - one
not on-to

$R \xrightarrow{f} R^+$

$0 \quad 0$

$-1 \to 1$

$1 \to 1$

$2 \to 4$

$\frac{1}{3} \to \frac{1}{9}$

Properties ⇒

$f \Rightarrow A \to B$ & $g : B \to C$ are f^n

then (1.) f & g are onto ⇒ gof is onto

(2.) f & g are one - one ⇒ gof is one - one

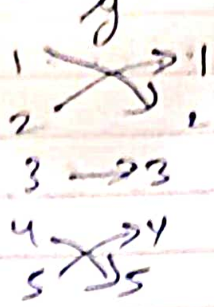
(3.) f & g " bijective ⇒ gof is bijective

conversely (1.) If gof is on to ⇒ g is on to

(2.) If gof is one - one ⇒ f is one - one

(3.) If gof is bijective ⇒ g is onto & f is one - one

Ex ⇒ If $A = \{1, 2, 3, 4, 5\}$ & $g : A \to A$ is shown by



Find gog, $go(gog)$. Identify whether one - one or onto

Sol ⇒ $g(1) = 2$

$g(2) = 1$

$g(3) = 3$

$g(4) = 5$

$g(5) = 4$

$\therefore gog(1) \Rightarrow g[g(1)] = g[2] = 1$

$gog(2) \Rightarrow g[g(2)] = g[1] = 2$

$gog(3) = g[g(3)] = g(3) = 3$

$gog(4) = g[g(4)] = g(5) = 4$

$gog(5) = g[g(5)] = g(4) = 5$

$\therefore$ gog is bijective

similarly $go[gog]$ bijective.

Ex ⇒ $f : N \to N$ (including 0)

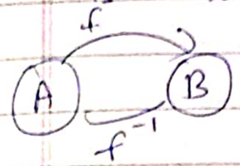
define as (1) $f(x) = x^2 + 2$

(2) $f(x) = 1$; x is odd

$= 0$; x is even

Inverse $f^n \Rightarrow$

$f : A \rightarrow B$ is a one-one f^n defined with $f(a) = b$ where $a \in A$ & $b \in B$ then inverse of f is defined as $f^{-1} : B \rightarrow A$ such that



$$f^{-1}(b) = a$$

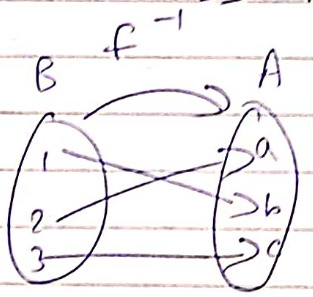
Ex $\Rightarrow$ (1) If $A = \{a, b, c\}$ & $B = \{1, 2, 3\}$ & f^n is defined by $f(a) = 2$, $f(b) = 1$, $f(c) = 3$ is inverse exist? if so find f^{-1}?

Soln $\Rightarrow$ As f is 1-1 $\Rightarrow f^{-1}$ exists & $f^{-1}(1) = b$

$f^{-1}(2) = a$

$f^{-1}(3) = c$

$f^{-1} \Rightarrow$



(2) f $R \rightarrow R$ by $f(x) = x^2$ Is f inversible?

$f(2) = 4$

$f(-2) = 4$

f^n is not one-one

f^{-1} does not exist

Note $\Rightarrow$ (1.) $f : A \rightarrow B \Rightarrow$ one-one $\Rightarrow f^{-1}$ exists $\Rightarrow$

$f(a) = b$ $a \in A$, $b \in B$

& $(f^{-1} \circ f)(a) = f^{-1}[f(a)] = f^{-1}[b] = a = I_A$

(identity mapping)

Similarly $(f \circ f^{-1})(b) = f[f^{-1}(b)] = f(a) = b = I_B$

(P) If $f: A \to B$ & $g: B \to C$ are 1-1 f
then $(gof)^{-1}(x) = f^{-1} o g^{-1}(x)$
(verify in ex)

Q. If $f: R \to R$ & $g: R \to R$ defined by
$f(a) = 2a+1$ & $g(b) = b/3$
verify $gof^{-1} = f^{-1} o g^{-1}$

~~$(gof) = g[2a+1] \Rightarrow \frac{2a+1}{3}$~~

~~$\frac{f(a)-1}{2} = a$ $3g(b) = b$~~

~~$f^{-1}[3g(b)](x) =$~~

Soln $\Rightarrow$ Consider $(gof)(a) = g[f(a)]$

$= g[2a+1] = \frac{2a+1}{3} \to y$ — (1)

$(gof)^{-1}(b) = \frac{3b-1}{2}$ — (4.) $a = \frac{3y-1}{2}$

Let $f(a) = b = 2a+1 \Rightarrow a = \frac{b-1}{2} \Rightarrow f^{-1}(b) = \frac{b-1}{2}$ -

Let $g(b) = \frac{b}{3} = a \Rightarrow b \Rightarrow 3a$ $\quad g^{-1}(a) = 3a$ — (3)

$(f^{-1} \circ g^{-1})(a) = f^{-1}[g^{-1}(a)] = f^{-1}[3a]$

$= \frac{3b-1}{2}$ — (5)

Ex: Find the inverse of $f(x) = x^3 + 1$

where $f: R \rightarrow R$

$f(x) = y = x^3 + 1$

$\sqrt[3]{y-1} = x$

$f^{-1}(y) = \sqrt[3]{y-1}$

fuzzy set

~~Ø~~ ~~is a function~~ The membership function $\mu_A(x)$ of a fuzzy set A is a function $\mu_A : x \rightarrow [0,1]$

So, every element in x in X has membership degree: $\mu_A(x) \in [0,1]$

A is completely determined by the set of tuples: $A = \{ (x, \mu_A(x)) \, x \in X \}$

☆ Properties of fuzzy set ⇒

~~Operations~~

☆ Operations on fuzzy set ⇒

1. Equal fuzzy sets ⇒

2. Complement of fuzzy set $A(x)$

$$\bar{A}(x) = 1 - A(x) \text{ for all } x \in X$$

3. Intersection of fuzzy sets ⇒

$$\mu_{(A \cap B)}(x) = \min \{ \mu_A(x), \mu_B(x) \}$$

4. Union of fuzzy sets ⇒

⑤ Algebraic product of fuzzy sets ⇒

$$A(x) \cdot B(x) = \{ (x, \mu_A(x) \cdot \mu_B(x)), x \in X \}$$

⑥ Multiplication of fuzzy set

⑦ Power of a fuzzy set ⇒

$$\mu_{A^P}(x) = \{ \mu_A(x) \}^P, x \in X \}$$

$P >= 1 \rightarrow A^P(x)$ is called concentration

$P < 1 \rightarrow A^P(x)$ is called dilation

⑧ Algebraic sum of 2 fuzzy sets ⇒

$$A(x) + B(x) = \{ (x, \mu_{A+B}(x), x \in X \}$$

where $\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$

⑨ Bounded sum of 2 fuzzy sets ⇒