Fermat's little thm.

Let p be a prime

If $p \nmid a$ then $a^{p-1} \equiv 1 (\text{mod} p)$, $a \in \mathbb{Z}$.

For every integer a, $a^p \equiv a (\text{mod } p)$

Ex :- Find the remainder of 8^{401}, when divided by 13.

→ Here $13 \nmid 8$ by Fermat's thm,

$$8^{13-1} \equiv 1 (\text{mod } 13)$$

$$8^{12} \equiv 1 (\text{mod } 13)$$

$$(8^{12})^{33} \equiv (1)^{33} (\text{mod } 13)$$

$$8^{396} \equiv 1 (\text{mod } 13) \quad \text{—} \; (1)$$

$$\begin{array}{r} 33 \\ 12 \overline{)401} \\ -36 \\ \hline 41 \\ -36 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 11 \\ 13 \overline{)144} \\ -13 \\ \hline 14 \\ -13 \\ \hline 1 \end{array}$$

Now remaining 8^5

$$8^2 = 64 \equiv 12 (\text{mod } 13)$$

$$8^2 \equiv 12 (\text{mod } 13)$$

$$(8^2)^2 \equiv (12)^2 (\text{mod } 13)$$

$$\equiv 1 (\text{mod } 13)$$

$$8^4 8 \equiv 8 (\text{mod } 13)$$

$$8^5 \equiv 8 (\text{mod } 13) \quad \text{—} \; (2)$$

From (1) & (2)

$$8^{396} 8^5 \equiv 8 (\text{mod } 13)$$

$$8^{401} \equiv 8 (\text{mod } 13)$$

Hence. Remainder is 8

Find x such that

$$x^{109} \equiv 8 \pmod{37}$$

$$\begin{array}{r} 36 \\ \times\ 3 \\ \hline 108 \end{array}$$

Using Fermat's thm,

$$\begin{array}{r} 3 \\ 36 \overline{)109} \\ -108 \\ \hline 1 \end{array}$$

$$x^{36} \equiv 1 \pmod{37}$$

$$(x^{36})^3 \equiv (1)^3 \pmod{37}$$

$$x^{108} \equiv 1 \pmod{37} \quad \text{— (1)}$$

Here remainder is 8

so multiply eq^n (1) by 8.

$$x^{108}\, 8 \equiv 8 \pmod{37}$$

$\therefore\ x = 8$

$$8^{109} \equiv 8 \pmod{37}$$