Date: 4/11/22

Tut-15

Name: Shreerang Mhatre
Roll no: 29
Batch: A2

Q1) Find z-transform of the following-

a) $f(k) = k$
$k \geq 0$

$z\{f(k)\} = F(z)$

$z\{k(f(k)\} = (-z\, d/dz) F(z)$

$$z\{1\} = \sum_{k=0}^{\infty} z^{-k}$$

$$\Rightarrow 1 + \frac{1}{z} + \frac{1}{z^2} + \cdots$$

So $z \quad \dfrac{1}{1-1/z}$ ROC - $\left|\dfrac{1}{z}\right| < 1$

$$\therefore z\{k\} = -z \frac{d}{dz}\left(\frac{z}{z-1}\right)$$ ROC $k z$

$$= -z\left[\frac{(z-1)-z}{(z-1)^2}\right]$$

$$\Rightarrow +\frac{z}{(z-1)^2}$$

$\Rightarrow$

(b) $f(k) = k5^k$
$k \geq 0$

Soln:- $Z\{5^k\} = \frac{z}{z-5} \quad \ldots$ for $|z| > |5|)$

$Z\{k5^k\} = -z \frac{d}{dz}\left(\frac{z}{z-5}\right)$

$\Rightarrow -z\left(\frac{(z-5)-z}{(z+5)^2}\right)$

$\Rightarrow \frac{5z}{(z-5)^2}$ for $|z| > 5$

(c) $f(k) = \frac{2^k}{k} \quad (k \geq 1)$

$z^1\{2^k\}$
$k \geq 1$

$\sum_{k=1}^{\infty} 2^k z^{-k}$

$\Rightarrow \left[\frac{2}{z} + \frac{2^2}{z^2} + \frac{2^3}{z^3} + \cdots\right]$

$|2/z| < 1$
$2 < |z| \Rightarrow S_\infty = \frac{2/z}{1 - 2/z}$

$S_\infty \Rightarrow \frac{2}{z-2}$

$$Z\left\{\frac{z^k}{k}\right\}_{k\geq 1} = \int_{z=2}^{\infty} z^{-1}\frac{z}{z-2}\,dz$$

$$\Rightarrow \int_{z=2}^{\infty} \frac{2}{z(z-2)}\,dz$$

$$\frac{2}{z(z-2)} \Rightarrow \frac{A}{z} + \frac{B}{z-2}$$

$$0z + 2 \Rightarrow Az - 2A + Bz$$

$$A + B = 0 \qquad 2 = -2A$$

$$\boxed{B = 1} \qquad A = -1$$

$$\Rightarrow \int_{z=2}^{\infty} -\frac{1}{z}\,dz + \int_{z=2}^{\infty} \frac{1}{z-2}\,dz$$

$$\Rightarrow \left[-\ln(z)\right]_2^{\infty} + \left[\ln(z-2)\right]_2^{\infty}$$

$$\Rightarrow \left[\ln\left(\frac{z-2}{z}\right)\right]_2^{\infty}$$

$$\Rightarrow \left[\ln\left(1-\frac{2}{z}\right)\right]_2^{\infty}$$

$$\Rightarrow \ln(1) - \ln(1-2/z)$$

$$\Rightarrow \ln\left(\frac{1}{1-2/z}\right) \Rightarrow \ln\left(\frac{z}{z-2}\right)$$

Q2) Find inverse z-transfor of F(z) given bellow →

a) $F(z) = \dfrac{z^2}{(z-1/4)(z-1/5)}$ for $|z| < -1/5$

$\Rightarrow \dfrac{F(z)}{z} = \dfrac{z}{(z-1/4)(z-1/5)} \Rightarrow \dfrac{A}{z-1/4} + \dfrac{B}{z-1/5}$

$z = Az + Bz$

$= \dfrac{A}{5} - \dfrac{B}{4} = 0$

$4A + 5B = 0$

$A + B = 1$

$\boxed{B = -4}$

$4A - 20 = 0$

$4A = 20$

$\boxed{A = 5}$

$F(z) = \dfrac{5z}{z-1/4} - \dfrac{4z}{z-1/5}$

$z^{-1}\left[\dfrac{5z}{z-1/4} - \dfrac{4z}{4-1/5}\right] \Rightarrow z^{-1}\left[\dfrac{5z}{z-1/4}\right] - z^{-1}\left[\dfrac{4z}{z-1/5}\right]$

$\Rightarrow 5 \times (-(1/4)^k) - 4(-(1/5)^k)$

$|z| < 1/4$ $\qquad |z| < 1/5$

$\Rightarrow -5(1/4)^k + 4(1/5)^k$

$|z| < 1/5$

b) $F(z) = \dfrac{z}{(z-1)(z-2)}$ $\quad |z| > 2$

$$\frac{F(z)}{z} \Rightarrow \frac{1}{(z-1)(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-2)}$$

$$0 = A + B$$
$$1 = -2A - B$$
$$1 = -A$$
$$A = -1$$
$$B = 1$$

$$\therefore F(z) = \frac{-z}{z-1} + \frac{z}{z-2}$$

$$z^{-1}(F(z)) = z^{-1}\left[\frac{-z}{z-1}\right] + z^{-1}\left[\frac{z}{z-2}\right]$$

$$\text{Ans} \Rightarrow \underset{|z|>1}{-1^k} + \underset{|z|>2}{2^k} \Rightarrow -1^k + 2^k$$

$$\Rightarrow \boxed{|z| > 2}$$

Q3)

a) $f(k+2) + 3f(k+1) + 2f(k) = 0 \quad k \geq 0$

$f(0) = 0, \; f(1) = 1$

soln:-> $Z\{f(k+2)\} + 3Z\{f(k+1)\} + 2Z\{f(k)\} = 0$

$$z^2 F(z) - z^2 f(0) - z f(1) + 3(zF(z) - \underbrace{zf(0)}_{0}) + 2F(z) = 0$$

$$z^2 F(z) - z + 3zF(z) + 2F(z) = 0$$

$$F(z)[z^2 + 3z + 2] = z$$

$$F(z) = \left[\frac{z}{z^2 + 3z + 2}\right]$$

$$F(z) = \frac{z}{z^2 + z + 2z + 2} = \frac{z}{z(z+1) + 2(z+1)}$$

$$\Rightarrow \frac{z}{(z+2)(z+1)}$$

$$\frac{F(z)}{z} = \frac{1}{(z+2)(z+1)} = \frac{A}{z+2} + \frac{B}{z+1}$$

$\Rightarrow A + B = 0$

$A + 2B = 1$

$1 - 2B + B = 0$

$1 = B$

$A = -1$

$\Rightarrow$

$$\therefore F(z) = \frac{-z}{z+2} + \frac{z}{z+1}$$

$$z^{-1}[F(z)] = z^{-1}\left[\frac{-z}{z+2}\right] + z^{-1}\left[\frac{z}{z+1}\right]$$

$$\Rightarrow -(-2)^k + (-1)^k$$

$$|z| > |-2| \qquad |z| > (-1)$$

$$\Rightarrow \boxed{|z| > 2}$$

b) $f(k) - \frac{5}{6} f(k-1) + \frac{1}{6} f(k-2) = \left(\frac{1}{2}\right)^k \; k \geq 0$

$$z\{f(k)\}_{k\geq 0} - \frac{5}{6} z\{f(k-1)\}_{k\geq 0} + \frac{1}{6} z\{f(k-2)\}_{k\geq 0}$$

$$= z\{(1/2)^k\}_{k\geq 0}$$

$$f(z) - \frac{5}{6}[z^{-1}f(z)] + \frac{1}{6}[z^{-2}f(z)] \Rightarrow \frac{z}{z-1/2}$$

$$F(z)\left[\frac{6z^2}{6z^2} - \frac{5z}{6z^2} + \frac{1}{6z^2}\right] = \frac{2z}{2z-1}$$

$$f(z) = \left[\frac{2z}{(2z-1)}\right]\left[\frac{6z^2}{6z^2-5z+1}\right]$$

$\rightarrow$

Page No.

Date

$$\frac{F(z)}{z} = \frac{12z^2}{(2z-1)(2z-1)(3z-1)}$$

$$\Rightarrow \frac{12z^2}{(3z-1)(2z-1)^2} = \frac{A}{2z-1} + \frac{B}{2z-1} + \frac{C}{3z-1}$$

$$\Rightarrow A(2z-1)(3z-1) + B(2z-1)(3z-1) + C(2z-1)(2z-1)$$

$$\Rightarrow A(6z^2-5z+1) + B(6z^2-5z+1) + C(4z^2+1-4z)$$

$$\Rightarrow 6Az^2 - 5Az + A + 6Bz^2 - 5Bz + B + 4Cz^2 + C - 4Cz$$

$$6A + 6B + 4C = 12$$
$$-5A - 5B - 4C = 0$$
$$A + B + C = 0$$
$$5A + 5B + 4C = 0$$
$$3A + 3B + 2C = 6$$