USEFUL FORMULAE

TRIGONOMETRY

$\sin 2\theta = 2\sin\theta\cos\theta$,

Angle	0°	30°	45°	60°	90°	180°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

$\cos 2\theta = 2\cos^2\theta - 1$, $\cos 2\theta = 1 - 2\sin^2\theta$

$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

$\sin(90+\theta) = \cos\theta$ (change)

$\sin(\pi-\theta) = \sin\theta$ (No change)

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \cosh^2 x - \sinh^2 x = 1$$

$$\frac{d}{dx}(\sinh x) = \cosh x, \frac{d}{dx}(\cosh x) = \sinh x,$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}, \cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$\sinh ix = i\sin x$, $i\sinh x = \sin ix$, cash $ix = \cos x$, $\cosh x = \cos ix$

Binomial Theorem $(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2 + \frac{n(n-1)(n-2)}{\lfloor 3}x^3 + \ldots$

Polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $\theta = \tan^{-1}\frac{y}{x}$

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos i x$

Median is the line joining the vertex to the mid point of the opposite side of a triangle.
Centroid or C.G. is the point of intersection of the medians of a triangle.
Incentre is the point of intersection of the bisectors of the angles of a triangle.
Circumcentre is the point of intersection of the perpendicular bisectors of the sides of a triangle.
Orthocentre is the point of intersection of the perpendiculars drawn from vertex to the opposite sides of a triangle.
Asymptote is the tangent to a curve at infinity.

DIFFERENTIAL CALCULUS

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\frac{d}{dx}(\sin x) = \cos x$,

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

$\frac{d}{dx}(\sec x) = \sec x\tan x$

$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x\cot x$

$\frac{d}{dx}(a^x) = a^x\log_e a$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\quad \frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

TEGRAL CALCULUS

$\int x^n dx = \frac{x^{n+1}}{n+1}$ $\quad \int \frac{1}{x} dx = \log_e x$ $\quad \int e^x dx = e^x$

$\int a^x dx = a^x \log_a e$ $\quad \int \sin x dx = -\cos x$ $\quad \int \cos x dx = \sin x$

$\int \tan x dx = \log \sec x$ $\quad \int \cot x dx = \log \sin x$ $\quad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x}$

$\int \sec x dx = \log \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = \log(\sec x + \tan x)$ $\quad \int \sinh x dx = \cosh x$

$\int \operatorname{cosec} x dx = \log \tan \frac{x}{2} = \log(\operatorname{cosec} x - \cot x)$ $\quad \int \operatorname{cosech}^2 x dx = -\coth x$

$\int \sec x \tan x dx = \sec x$ $\quad \int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$ $\quad \int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a}$ $\quad \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a}$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$ $\quad \int \frac{-dx}{x^2 + a^2} = \frac{1}{a} \cot^{-1} \frac{x}{a}$ $\quad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a}$

$\int \frac{-dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{cosec}^{-1} \frac{x}{a}$ $\quad \int \cosh x dx = \sinh x$ $\quad \int \operatorname{sech}^2 x dx = \tanh x$

$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x$ $\quad \int \operatorname{cosech} x \coth x dx = -\operatorname{cosech} x$

$\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$ $\quad \int \sqrt{a^2 + x^2} dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} x$

$\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 \ldots$ (General formula for integration by parts)

LAPLACE TRANSFORMATION

1. $L(1) = \frac{1}{s}$ 2. $L(t^n) = \frac{\lfloor n}{s^{n+1}}$ 3. $L(e^{at}) = \frac{1}{s-a}$

4. $L(\cosh at) = \frac{s}{s^2 - a^2}$ 5. $L(\sinh at) = \frac{a}{s^2 - a^2}$ 6. $L(\sin at) = \frac{a}{s^2 + a^2}$

7. $L(\cos at) = \frac{s}{s^2 + a^2}$ 8. $Le^{at} f(t) = F(s-a)$ 9. $L^{-1} f'(t) = sLf(t) - f(0)$

10. $Lf''(t) = s^2 Lf(t) - sf(0) - f'(0)$ 11. $L\left[\int_0^1 f(t)dt\right] = \frac{1}{s}F(s)$ 12. $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[F(s)]$

13. $L\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)ds$ 14. $u(t-a) = \begin{cases} 0 \text{ when } t < a \\ 1 \text{ when } t > a \end{cases}$ 15. $L[u(t-a)] = \frac{e^{-ax}}{s}$

16. $L[f(t-a).u(t-a) = e^{-as}F(s)]$ 17. $L\delta(t-1) = \frac{1}{\in}$ 18. $L\delta(t-a) = e^{-ax}$

19. $Lf(t) = \frac{\int_0^T e^{-st} f(t)dt}{1 - e^{-sT}}$ 20. $L\frac{t}{2a}\sin at = \frac{s}{(s^2+a^2)^2}$ 21. $Lt\cos at = \frac{s^2 - a^2}{(s^2+a^2)^2}$

22. $L\frac{1}{2a^3}(\sin at - at\cos at) = \frac{1}{(s^2+a^2)^2}$ 23. $L\frac{1}{2a}(\sin at - at\cos at) = \frac{s^2}{(s^2+a^2)^2}$

CONVOLUTION THEOREM $L\left[\int_0^t f_1(x) f_2(t-x)dx\right] = F_1(s) * F_2(s)$

INVERSE LAPLACE TRANSFORM

1. $L^{-1}\left(\frac{1}{s}\right) = 1$ 2. $L^{-1}\frac{1}{s^n} = \frac{t^{n-1}}{n-1}$ 3. $L^{-1}\frac{1}{s-a} = e^{at}$

4. $L^{-1}\frac{s}{s^2-a^2} = \cosh at$ 5. $L^{-1}\frac{1}{s^2-a^2} = \frac{1}{a}\sinh at$ 6. $L^{-1}\frac{1}{s^2+a^2} = \frac{1}{a}\sin at$

7. $L^{-1}\frac{s}{s^2+a^2} = \cos at$ 8. $L^{-1}F(s-a) = e^{at} f(t)$

9. $L^{-1}\frac{1}{(s^2+a^2)^2} = \frac{1}{2a^3}(\sin at - at\cos at)$

10. $L^{-1}\frac{s}{(s^2+a^2)^2} = \frac{1}{2a}t\sin at$ 11. $L^{-1}\frac{s^2-a^2}{(s^2+a^2)^2} = t\cos at$

12. $L^{-1}\frac{s^2}{(s^2+a^2)^2} = \frac{1}{2a}(\sin at + at\cos at)$

13. $L^{-1}[sF(s)] = \frac{d}{dt}f(t) + f(0)$ 14. $L^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t f(t)dt$

15. $L^{-1}F(s+a) = e^{-at} f(t)$ 16. $L^{-1}\left[e^{-as}F(s)\right] = f(t-a)u(t-a)$

17. $L^{-1}\left[\frac{d}{ds}F(s)\right] = -t f(t)$ 18. $L^{-1}\left[\int_s^\infty F(s)ds\right] = \frac{f(t)}{t}$

19. $L^{-1}\int_0^s f_1(x) f_2(t-x)dx = F_1(s).F_2(s)$ 20. $f(t)$ = sum of the residues of $e^{st} F(s)$ at the poles of $F(s)$

21. $L^{-1}\left[\frac{F(s)}{G(s)}\right] = \sum_{i=1}^{n} \frac{F(\alpha_i)}{G'(\alpha_i)} e^{\alpha_i t}$