Page No.
Date 13/10/22

Tutorial 6, 7

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Q1) Find Laplace transform of the following functions

① $f(t) = 4e^{5t} + 6t^3 - 3\sin 4t + 2\cos 2t$

$$L[f(t)] = \frac{4}{s-5} + 6\left(\frac{6}{s^4}\right) - \frac{3(4)}{s^2+16} + \frac{2s}{s^2+4}$$

$$F(s) = \frac{4}{s-5} + \frac{36}{s^4} - \frac{12}{s^2+16} + \frac{2s}{s^2+4}$$

② $f(t) = \cos^2 2t + (e^t - 1)^2$

$$\cos^2(2t) = \frac{\cos 4t + 1}{2}$$

$$\therefore f(t) = \frac{1}{2} + \frac{\cos 4t}{2} + e^{2t} - 2e^t + 1$$

$$\therefore L[F(t)] = \frac{1}{2s} + \frac{1}{2}\left(\frac{s}{s^2+16}\right) + \frac{1}{s-2} - 2\left(\frac{1}{s-1}\right) + 1/s$$

$$\therefore F(s) = \frac{1}{2s} + \frac{s}{2s^2+32} + \frac{1}{s-2} - \frac{2}{s-1} + \frac{1}{s}$$

→

(3) $F(t) = \cos 3t \cdot \sin t \cdot \cos 2t$

$$= \cos\left(\frac{4t+2t}{2}\right) \cdot \sin\left(\frac{4t-2t}{2}\right) \cdot \cos 2t$$

$$= \left(\frac{\sin 4t - \sin 2t}{2}\right) \cos 2t$$

$$= \frac{1}{2}\left(\sin 4t \cdot \cos 2t - \sin 2t \cdot \cos 2t\right)$$

$$= \frac{1}{2}\left(\sin\left(\frac{6t+2t}{2}\right) \cdot \cos\left(\frac{6t-2t}{2}\right) - \sin 2t \cos 2t\right)$$

$$= \frac{1}{2}\left(\frac{\sin 6t + \sin 2t}{6} - \sin 2t \cdot \cos 2t\right)$$

$$= \frac{1}{2}\left(\frac{\sin 6t + \sin 2t}{2} - \frac{\sin 4t}{2}\right)$$

$$= \frac{1}{4}\left((\sin 6t + \sin 2t) - \sin 4t\right)$$

$$\therefore L[F(t)] = \frac{1}{4}\left(\frac{6}{s^2+36} + \frac{2}{s^2+4} - \frac{4}{s^2+16}\right)$$

$$\therefore F(s) = \frac{3}{2s^2+72} + \frac{1}{2s^2+8} - \frac{1}{s^2+16}$$

Page No.
Date

(4) $f(t) = [3t^5 - 2e^{-5t} - 3\sin 6t]\, e^{2t}$

$\cancel{L[f(t)] = 3}$

$f(t) = 3e^{2t} \cdot t^5 - 2e^{-3t} - 3\sin 6t \cdot e^{2t}$

$$L[f(t)] = 3(-1)\frac{d^5}{ds^5}\left(\frac{1}{s-2}\right) - \frac{2}{s+3} - 3L[\sin 6t\, e^{2t}]$$

$$= +3\left(\frac{120}{(s-2)^6}\right) - \frac{2}{s+3} - 3L[\sin 6t\, e^{2t}]$$

$$= \frac{360}{(s-2)^6} - \frac{2}{s+3} - 3\left(\frac{6}{(s-2)^2+36}\right)$$

$$\therefore F(s) = \frac{360}{(s-2)^6} - \frac{2}{s+3} - \frac{18}{(s-2)^2+36}$$

(5) $f(t) = 0 \quad 0 < t < 1$

$= t \quad 1 < t < 4$

$= 0 \quad t > 4$

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

$$\therefore L[f(t)] = \int_0^1 e^{-st} f(t)\, dt + \int_1^4 e^{-st} f(t)\, dt + \int_4^\infty e^{-st} f(t)\, dt$$

$$= \int_0^1 e^{-st}(0)\, dt + \int_1^4 e^{-st}(t)\, dt + \int_4^\infty e^{-st}(0)\, dt$$

$$= \int_1^4 e^{-st}\, t\, dt + 0 + 0$$

$\Rightarrow$

$$= \int_1^4 e^{-st}\, t\, dt$$

$$= t \int_1^4 e^{-st}\, dt + \int_1^4 (1)\, \frac{e^{-st}}{s}\, dt$$

$$\cancel{= \frac{-t}{s}(4}$$

$$= -1 \left[\frac{e^{-st}}{s} \cdot t\right]_1^4 - \frac{1}{s}\left[\frac{e^{-st}}{s}\right]_1^4$$

$$= -\frac{1}{s}\left(4e^{-4s} - e^{-s}\right) - \frac{1}{s^2}\left(e^{-4s} - e^{-s}\right)$$

$$\therefore F(s) = \frac{e^{-s}}{s} - \frac{4e^{-4s}}{s} + \frac{e^{-s}}{s^2} - \frac{e^{-4s}}{s^2}$$