Solved Examples
Polarization

Ex 1. A ray of light is incident on the surface of a glass plate of refractive index 1.732 at the polarizing angle. Calculate the angle of refraction of the ray.

Soln :

Data :

$\mu = 1.732$

Formula :

$\mu = \tan i_p$, i_p = polarizing angle

$1.732 = \tan i_p$

$i_p = \tan^{-1}(1.732)$

$i_p = 60^0$

Now

$i_p + r_p = 90^0$

Angle of refraction is $r_p = 90^0 - i_p$

$= 90^0 - i_p$

$= 30^0$

Ex 2. At what angle of incidence should a beam of sodium light be directed upon the surface of diamond to produce most complete polarization ? Critical angle for diamond is 24.5^0

Soln :

Data :

Ic = 24.50 = critical angle

Formula :

$\mu = \tan i_p$,

The refractive index of diamond is given by

$\mu = \ = \ = \ = 2.414$

Now

$\mu = \tan i_p$

Angle of incidence required to produce complete polarization is the polarizing angle

$i_p = \tan^{-1}(\mu)$

$= \tan^{-1}(2.414)$

$= 67.47^0$

$i_p = 67^0 28'$

Ex 3. Polarizer and analyzer are set with their polarizing directions parallel, so that the intensity of transmitted light is maximum. Through what angle should either be turned, so that the intensity be reduced to and 25 % of the maximum intensity.

Soln :

Data :

(i)

(ii)

Formula :

$I = I_m \cos^2\theta$

(i) $\cos^2\theta$

$\cos\theta = \pm$

(ii)

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Ex 4. At what angle light should be incident on a glass plate (μ = 1.5697) to get a plane polarized light by reflection ?

Soln :

Data :

$\mu = 1.5697$

Formula :

$\mu = \tan i_p$

The angle if incidence on the glass plate to get the plane polarized light by reflection is the polarizing angle

$i_p = \tan^{-1} (\mu)$

$i_p = \tan^{-1} (1.5697)$

$\mathbf{i_p = 57^0\ 30'}$

Ex 5. A polarizer and analyzer are oriented so that the amount of light transmitted is maximum. To what fraction of its maximum value is the intensity of transmitted light reduced when the analyzer is roteted through (1) 45^0 and (2) 90^0 ?

Soln :

Data :

(1) $\theta = 45^0$ and (2) $\theta = 90^0$

Formula :

$I = I_m \cos^2 \theta$

(2) =

Ex 6. Two polarizing plates have polarizing directions parallel so as to transmit maximum intensity of light. Through what angle must either plates be turned if the intensity of the transmitted beam is one third the intensity of the incident beam.

Soln :

Formula :

$I = I_m \cos^2 \theta$

Data given :

So, $\cos^2 \theta =$

$\cos \theta = \pm$

$\mathbf{\theta = \pm 54.73^0}$ **and** $\mathbf{\pm 125.26^0}$

Ex 7. The critical angle of light in a certain substance is 40^0. What is the polarizing angle ?

Soln :

Given :

$i_c = 40^0$, $i_p = ?$

We have refractive index of air w. r. t. substance.

${}_s\mu_a =$

Since for i = ic, r = 90^0, hence ${}_s\mu_a = \sin i_c$

But, ${}_a\mu_s = \ =$

Also according to Brewster's law,

${}_a\mu_s = \tan i_p$ Hence $\tan i_p$

Hence $\tan i_p$ Hence $\tan i_p = 1.555$

Hence $i_p = \tan^{-1} (1.555)$

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$\mathbf{i_p = 57^0 25'}$

Ex 8. At what angle should light be incident on a glass plate having refractive index of 1.5697 to get plane polarized light by reflection. Also find the angle of refraction.

Soln :

$\mu = 1.5697 = \tan i_p$ Hence $1.5697 = \tan i_p$

$i_p = \tan^{-1}(1.5697)$ Hence $i_p = 57.50^0$

As $i_p + r_p = 90^0$ Hence $r_p = 90^0 - i_p = 90^0 - 57.5^0$

Or $\mathbf{r_p = 32.5^0}$

Ex 9. A beam of linearly polarized light is converted into circularly polarized light by passing it through a slice of crystal of 0.003 cm thickness. Calculate birefringence of the crystal. Give wavelength of light used = 6000 A^0

Soln :

The plate that convert linearly polarized light into circularly polarized light is QWP.

Thickness of plate is $t =$

Birefringence of the crystal is

$4(\mu_e - \mu_0) =$

$4(\mu_e - \mu_0) =$

$\mathbf{(\mu_e - \mu_0) = 0.005}$

Ex 10. A plane polarized light beam of wavelength 6000 A. U. is incident on a thin quartz plate. Calculate the minimum thickness of the plate for which the O-ray and E-ray will combine to produce plane polarized light.

Given $\mu_0 = 1.544$ and $\mu_E = 1.553$.

Soln : Given :

$\mu_0 = 1.544$, $\mu_E = 1.553$, $\lambda = 6000 = 6000 \times 10^{-8}$ cm

To produced plane polarized light, the phase difference between the O-ray and E-ray on emergence should be 180^0. Hence the plate should be a HWP.

Thickness $t =$

$t = \quad = \quad = 3.33 \times 10^{-3}$cm.

$\mathbf{t = 3.33 \times 10^{-3}}$**cm**

Ex 11. Two Nicols are oriented with their principal planes making an angle od 60^0. What percentage of the incident unpolarised light will pass through the system ?

Soln : Given :

$\theta = 60^0$

$I = I_0 \cos^2\theta$

$I = I_0(\cos 60)2$

$= I_0$

$= I_0$

$=$

Percentage of incident unpolarised light transmitted through the system is **25 %**

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Ex 12. A glass plate of refractive index 1.5 is to be used as a polarizer. What is the angle of polarization and angle or refraction ?

Soln :

Given :

$\mu = 1.5, \quad i_p = ?$

Formulae :

$\mu = \tan i_p$

$i_p = \tan^{-1} \mu = \tan^{-1} (1.5) = 56.30^0$

Angle of refraction : $90^0 - 56.30^0$

$= \mathbf{33.69^0}$

Ex 13. A quarter wave plate of thickness 2.275×10^{-3} is cut with its faces parallel to the optic axis. The emergent beam of light is elliptically polarized. Find the wavelength of monochromatic light made incident normally on the plate.
Given : $\mu_0 = 1.586$; $\mu_e = 1.592$.

Soln :

Formulae :

t =

Given :

$t = 2.275 \times 10^{-3}$

$\mu_0 = 1.586$; $\mu_e = 1.592$

So, $\lambda = 4 \times 2.275 \times 10^{-3} (1.592 - 1.586)$

$= 5.46 \times 10^{-5}$cm

$\boldsymbol{\lambda} = \mathbf{5460\ A^0}$

Ex 14. Calculate the thickness of a quarter wave plate of quartz for sodium light of wavelength 5893 A0. The ordinary and extraordinary refractive indices for sodium are 1.54425 and 1.55336 respectively.

Soln :

We have, thickness of quarter wave plate

t =

Given :

$\mu_0 = 1.54425$, $\mu_e = 1.55336$, $\lambda = 5.893 \times 10^{-5}$cm

t =

$\mathbf{t = 1.62 \times 10^{-3}\ cm}$

Ex 15. Find the thickness of a quarter wave plate for the wavelength of light of 589 nm and $\mu_0 = 1.55$, $\mu_e = 1.54$

Soln :

We have, thickness of quarter wave plate for negative crystal

t =

Given :

$\mu_0 = 1.55$, $\mu_e = 1.54$, $\lambda = 589$ nm

t =

$t = \times 10^{-5}$cm

$t = 147.25\ \ 10^2$ nm

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Ex 16. The value of μ_e and μ_0 for quartz are 1.5508 and 1.5418 respectively. Calculate the phase retardation for $\lambda = 5000\ A^0$ when the plate thickness is 0.032 mm.

Soln :

The path difference between O-ray and E-ray for quartz crystal is given by,

$\mu_0 = 1.5418$, $\mu_e = 1.5508$, $\lambda = 5000 \times 10^{-8}$cm, t = 0.032 mm = 0.0032 cm

Phase retardation

Phase retardation = 3.617 radians

Ex 17. At a certain temperature the critical angle of incidence of water is 48^0 for certain wavelength. What is the polarizing angle and the angle of refraction for light incident on the water at an angle that gives maximum polarization of the reflected light ?
(Given $\sin 48^0 = 0.731$)

Soln :

We have,

$\mu =$

$\mu = \ = \ = 1.345$

For maximum polarization, the light must be incident at polarizing angle.

We have, Brewster's Law

$\mu = \tan i_p$

$i_p = \tan^{-1}(1.345) = 53^0\ 22'$

If r is the angle of refraction,

$i_p + r = 90^0$

$r = 90^0 - i_p$

$= 90 - 53^0\ 22'$

$\mathbf{r = 36^0\ 38'}$

Ex 18. Plane polarized light is incident on a plate of quartz cut with faces parallel to optic axis. Calculate the thickness for which the phase difference between the two rays is 60^0.
Where $\mu_0 = 1.5442$, $\mu_e = 1.5583$, $\lambda = 5000A^0$

Soln :

For quartz plate the path difference between O-rays and E-rays is

$\Delta = (\mu_e - \mu_0)\ t$

And phase difference

$\delta = (\mu_e - \mu_0)\ t$

Given,

$\delta = 60^0 = 60 \times \ = \ \text{radians}$

$\mu_0 = 1.5442$, $\mu_e = 1.5583$, $\lambda = 5000 \times 10^{-8}$cm

So,

$=$

t = 0.00091 cm

Ex 19. Calculate thickness of a mica plate required to make a quarter wave plate and a half wave plate for light of wavelength 5890 A^0. (Given $\mu_0 = 1.586$, $\mu_e = 1.592$)

Soln :

Thickness of QWP t = $\ =$

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Thickness of HWP t = =

Ex 20. A 20 cm long tube containing 48 cm^3 of sugar solution rotates the plane of polarization by 11^0. If the specific rotation of sugar is 66^0 calculate the mass of sugar in the solution.

Soln :

We have specific rotation,

S = →

Given,

S = 66^0, l = 20 cm = 2 decimeters, θ = 11^0

→

1 cm^3 of sugar solution contains 1/12 × 48 = 4 gm

Hence, the mass of sugar in solution is 4 gm.

Ex 21. 80 gm of impure sugar is dissolved in a litre of water. The solution gives an optical rotation of 9.9^0 when placed in a tube of length 20 cm. If the specific rotation of pure sugar solution is 66^0 dm^{-1} $(gm/cc)^{-1}$, find the percentage of purity of the sugar sample.

Soln :

We have, specific rotation,

S =

→

Given,

S = 66^0, c = ?, θ = 9.9^0, l = 2 decimeter

= 0.075 gm/cc

c = 75 gm/lit

In one litre of water 80 gm of impure sugar is dissolved which contains 75 gm of pure sugar.

Hence, the purity of sugar sample

=

= 93.75 %

Ex 22. A sugar solution in a tube of length 20 cm produces optical rotation of 13^0. The solution is then diluted to one third of its previous concentration. Find optical rotation produced by 30 cm long tube containing the diluted solution.

Soln :

We have specific rotation,

S =

If θ is the optical rotation produced by solution of length l decimeter, when its concentration is change to c', then we have

S =

So we can write ,

S = =

θ' =

Given,

l = 20 cm = 2.0 dm, θ = 130, c' = c/3, l' = 3dm, θ' = ?

So,

θ' =

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$\theta' = 6.5^0$

Ex 23. Calculate the specific rotation of sugar solution from the following data : Length of the tube containing the solution = 22 cm volume of the solution = 88 cc, amount of sugar in the solution = 6 gm and angle of rotation = 9^0 54'.

Soln :

We have specific rotation,

S =

Given,

l = 2.2 dm, θ = 9.90, c =

S =

S = 660