Ex. 1. The energy of photon is 5.28×10^{-19} J. Calculate frequency and wavelength.

Soln :

$E = h\upsilon$

υ

Again

υ

$$\mathbf{= 3768\ A^0}$$

Ex. 2. Electrons moving with a speed of 7.3×10^7 m\s have wavelength of 0.1 A^0. Calculate Plank's constant.

Soln :

Given : $\lambda = 0.1\ A^0 = 0.1 \times 10^{-10}$m, $v = 7.3 \times 10^7$ m/s

Formula required :

$$h = \lambda \cdot mv$$
$$= 0.1 \times 10^{-10} \times 9.1 \times 10^{-31} \times 7.3 \times 10^7$$
$$\mathbf{= 6.643 \times 10^{-34}\ J.s}$$

Ex. 3. Find the De Broglie wavelength of 10 KeV electrons.

Soln :

$E = 10\ \text{KeV} = 10 \times 10^3\ \text{eV} = 10 \times 10^3 \times 1.6 \times 10^{-19}$ J

Formula required :

$$= 1.227 \times 10^{-11}\text{m}$$
$$\mathbf{= 0.1227\ A^0}$$

Ex. 4. De Broglie wavelength of electron in monochromatic beam is 7.2×10^{-11} meters. Calculate the momentum and energy of electrons in the beam in electron volts.

Soln :

Given : $\lambda = 7.2 \times 10^{-11}$ m

Formula required : Momentum p = , Energy E =

P =

E =

$E = 0.0291 \times 10^4$ eV

Determine the velocity and kinetic energy of a neutron having De Broglie wavelength 1.0 A0

Ex. 5. (mass of neutron is 1.67×10^{-27} kg).

Soln :

We have,

$\lambda =$

$v = 3.97 \times 10^3$ m/s

Again we can write,

$\rightarrow$ E =

E =

$= 13.16 \times 10^{-21}$ J

Ex. 6. A proton and an α particle are accelerated by the same potential difference. Show that the ratio of the De Broglie wavelengths associated with them is 2. Assume the mass of alpha particle to be 4 times the mass of proton.

Soln :

Given : $m_\alpha = 4m_p$,

m_α = mass of α – particle, m_p = mass of proton,

q_α = charge of particle, q_p = charge of proton

$q_\alpha = 2\ q_p$

Let $V \longrightarrow$ Accelerating potential for the particles.

Formula required :

λ =

λp = and $\lambda \alpha$ =

=

=

Ex. 7. What accelerating potential would be required for a proton with zero velocity to acquire a velocity corresponding to De Broglie's wavelength of 10^{-14} m ?
$h = 6.62 \times 10^{-34}$ Js, Mass of proton = 1.67×10^{-27} kg, $e = 1.6 \times 10^{-19}$ c.

Soln :

Given : $\lambda = 10^{-14}$ m , $h = 6.62 \times 10^{-34}$ Js,

$M_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ c.

Formula required :

Accelerating potential of proton should be

V = 8.199 Mev

Ex. 8. Find the De Broglie wavelength of

i) An electron accelerated through a potential difference of 182 volts and

ii) 1 kg object moving with a speed of 1m/sec.

Comparing the results, explain why the wave nature of matter is not more apparent in daily observations.

Soln :

Given : V = 182 V, m = 1 kg, v = 1 m/s.

Formula required :

i) De Broglie wavelength of the electron accelerated through a potential V volts is,

ii) De Broglie wavelength of a body of mass m moving with velocity v is,

From the results (i) and (ii) we see that the wavelength for electron is measurable.
But for the body of mass 1 kg the wavelength is too small to be detected. Hence the wave nature of matter is not more apparent in daily observations.

Ex. 9. What potential difference must be applied to an electron microscope to obtain electrons of wavelength 0.3 A^0 ?

Soln :

Ex. 10. Calculate the velocity and De-Broglie wavelength of an α-particle of energy 1 Kev.

Soln :

$\lambda\alpha$

= 0.0045 A^0

$\lambda\alpha$

$v\alpha$

$v\alpha$

$\mathbf{v\alpha}$

Ex. 11. An electron has kinetic energy equal to its rest mass energy. Calculate De-Broglie's wavelength associated with it.

Soln :

Data : $h = 6.63 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg.

$c = 3 \times 10^8$ m/s, $\lambda = ?$, $E = m_0c^2$

Formula :

$\lambda =$

$\lambda =$ $(E = mc^2)$

$\lambda =$

$\boldsymbol{\lambda}$

Ex. 12. Which has a shortest wavelength 1eV photon or 1 eV electron ? Calculate the value and explain.

Soln :

For photon,

$E = h\upsilon =$

λ_{ph}

For electron,

λ_e

λ_e

$\lambda_{ph} > \lambda_e$, So, frequency of electron is more, hence more effective.

Ex. 13. Calculate the minimum uncertainty in the velocity of an electron confined to a box of length 10 A^0.

Soln :

Given : $\Delta x = 10\ A^0 = 10 \times 10^{-10}$ m

Formula required :

$$(\Delta x) \cdot (\Delta px) = h$$

$$(\Delta x)_{max} \cdot (\Delta px)_{min} = h$$

Or $(\Delta x)_{max} \cdot (m \cdot \Delta vx)_{min} = h$

$(\Delta Vx)_{min}$

Ex. 14. An electron has a speed of 600 m/s with an accuracy of 0.005 %. Calculate the uncertainty with which we can locate the position of the electron.

Soln :

Given : $v = 600$ m/s, $\Delta v = 0.005$ % of v

Formula required :

$$(\Delta x) \cdot (\Delta px) = h$$

Now the uncertainty in velocity is $\Delta vx =$

The uncertainty in the location of electron is

Ex. 15. The position and momentum of 1 KeV electron are simultaneously measured. If its position is located to within 1 A^0, find the percentage of uncertainty in its momentum.
Rest mass of electron = 9.1×10^{-31} kg

Soln :

Given : $\Delta x = 1\ A^0 = 10^{-10}$ m

Formula required :

$$(\Delta x) \cdot (\Delta px) = h$$

$$(\Delta px) =$$

$$(\Delta px) = 6.62 \times 10^{-24} \text{ kg.m.s}$$

Given energy of electron is 1 KeV or 10^3 eV which is less than the rest mass energy ($m_0c^2 = 511$ KeV) of the electron.

Hence momentum of the electron can be determined as p =

Hence Px =

$= 1.7 \times 10^{-23}$ kg.m.s

Percentage of uncertainty in the momentum of electron is,

$\times 100 =$ **38.94 %**

Ex. 16. Compute the minimum uncertainty in the location of a 2gm. Mass moving with a speed of 1.5 m/s and the minimum uncertainty in the location of an electron moving with a speed of 0.5×10^8 m/s. Given that the uncertainty in the momentum is $\Delta p = 10^{-3}$ p for both.

Soln :

Given :

$\Delta p = 10^{-3}$ p, Speed = 1.5 m/s, mass = 2gm

Formula required :

$$(\Delta x) \cdot (\Delta p) = h$$

$(\Delta p) = 10^{-3}\, p = 10^{-3}\, (mv)$ $\qquad$ $(p = m \cdot v)$

(i) For the body

$(\Delta x) \cdot (\Delta p) = h$

Hence, $\Delta x =$

(ii) For the electron

$\Delta x =$

Solved Problems (Wave Equation)

Ex. 1. Find the lowest energy level and momentum of an electron of an electron in one dimensional potential well of width $1A^0$.

Soln :

Given : $L = 1A^0 = 10^{-10}$ m.

Formula required :

En =

The energy of an electron in a potential well of width L is

En =

The lowest energy level corresponds to n = 1. Hence it is E_1

Hence, $E_1 =$

$= 6.038 \times 10^{-18}$ J

Ex. 2. Compare the lowest three energy states for (i) an electron confined in an infinite potential well of width $10A^0$ and (ii) a grain of dust with mass 10^{-6} gm in an infinite potential well of width 0.1 mm. What can you conclude from this comparison ?

Soln :

Given : $L = 10A^0$

Formula required :

En =

The energy levels of the particle in an infinite potential well of width L is given by,

En =

(i) For the electron , m = 9.1 × 10-31 kg

L = 10A0 = 10-9 m

Hence En J

Or En

The lowest three energy states for the electron are

$E_1 = 0.377 \times (1)^2 = 0.377$ eV

$E_2 = 0.377 \times (2)^2 = 1.508$ eV

$E_3 = 0.377 \times (3)^2 = 3.393$ eV

(ii) For the grain of dust

$M = 10^{-6}$ gm $= 10^{-9}$ kg

$L = 0.1$ mm $= 10^{-4}$ m

Hence En

Or En

The lowest three energy states for the grain of dust are

$E_1 =$ eV

$E_1 =$

$E_1 =$

From the energy values in cases (i) and (ii) we see that the energy levels are discrete for the electron. But for the grain of dust, the difference between the energy levels being very small, they cannot be identified as discrete and can be treated as almost continuous. Hence quantization of energy is observable for the microscopic particles only.

Ex. 3. An electron trapped in a rigid box of width $2A^0$. Find its lowest energy level and momentum. Hence find energy of the 3^{rd} energy level.

Soln :

Formulae required:

$$E_n =$$

For lowest energy level n = 1

$$L = 2\ A^0 = 2 \times 10^{-10}\ m$$

$$E_1 =$$

$$= \ eV$$

$$F_1 = 9.5\ eV$$

$$P_1 =$$

$$= 1.663 \times 10^{-24}\ kg\text{-}m/s$$

and $E_1 = 9 \times 9.5\ eV = 85.5\ eV$ (as n = 3)

Solved Problems (Wave Particle Duality)