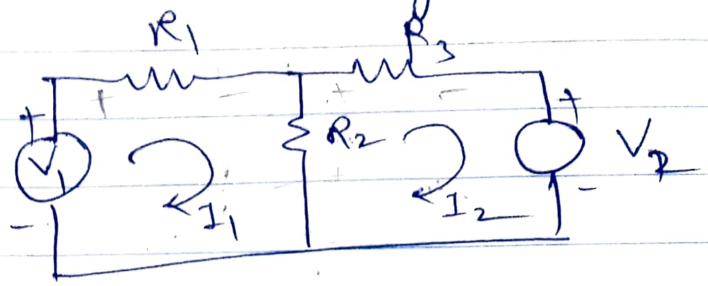
Mesh analysis.



N/w containing independent dc voltage source

$$V_1 - I_1R_1 - R_2(I_1 - I_2) = 0$$

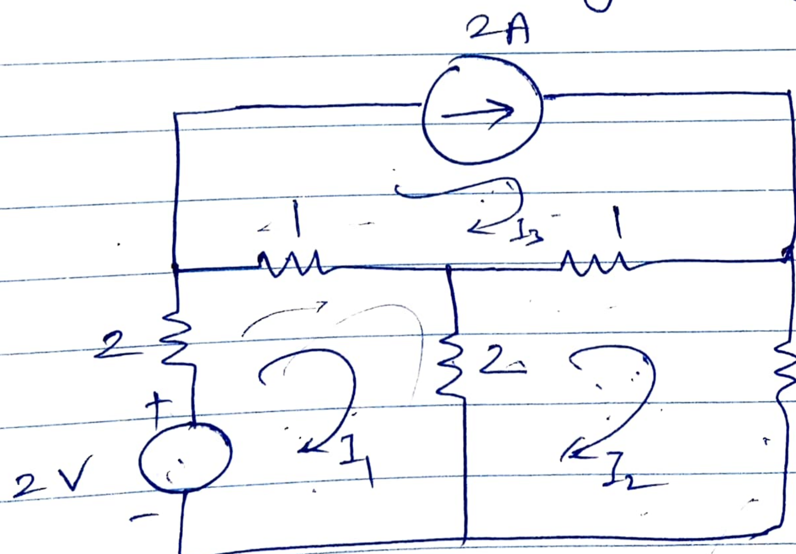
$$-V_2 - R_3I_2 - R_2(I_2 - I_1) = 0$$

Calculate I_1 & I_2

N/w containing dc current source.

Case 1.

Current source in a branch which is not shared by any loops.



Loop-1

$2 - 2I_1 - 1(I_1 - I_3) - 2(I_1 - I_2)$

$-5I_1 + 2I_2 + I_3 = -2$

Loop-2 $-5I_1 + 2I_2 = -4$

$-2(I_2 - I_1) - 1(I_2 - I_3) - I_2$

$2I_1 - 4I_2 + I_3 = 0$

$2I_1 - 4I_2 = -2$

Loop3

$I_3 = 2$

loop1 $\cancel{-4I_1 + I_2 + I_3 = -2}$

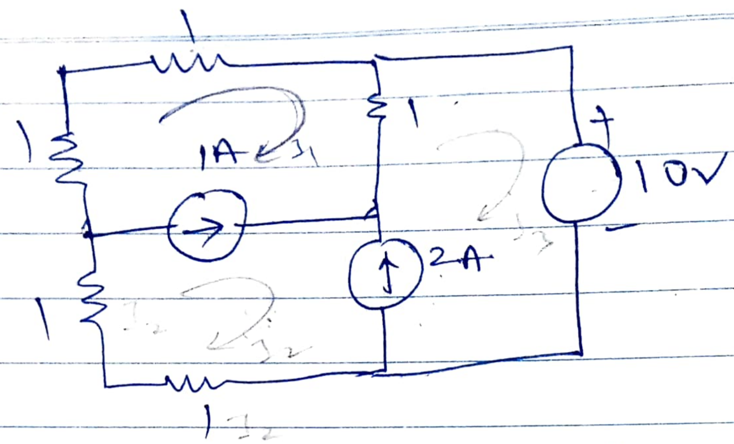
loop2 $\cancel{I_1 - 3I_2 + I_3 = 0}$

loop3 $I_3 = 2\,A$.

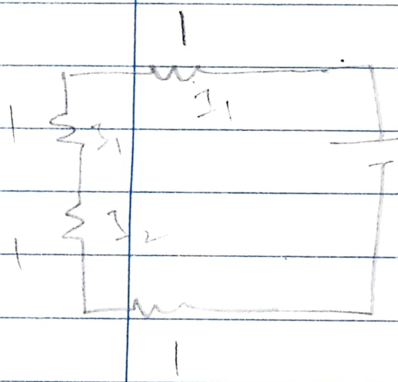
$I_1 = 1.25$ A

$I_2 = 1.125$ A

Supermesh Analysis.



Power supplied by the voltage source?



$I_1 + I_2 = -5$ —— (1)

$I_1 + (1 + I_1) = -5$

$I_2 - I_1 = 1$ —— (2) $I_2 = (1 + I_1)$

$I_3 - I_2 = 2$ —— (3)

$-I_1 - 10 - I_2 - I_1 = \ldots = 0$

$-2I_1 - 2I_2 = 10$

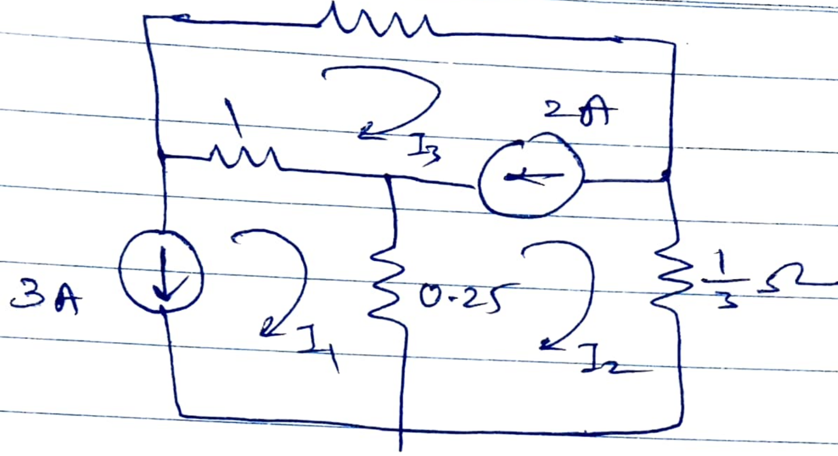
$I_1 + I_2 = -5$

$I_3 = 0$

$\therefore$ Power $= 0$.

* Supermesh – When a current source is present betn 2 meshes, we remove the branch having the current source and then the remaining loop is known as supermesh.

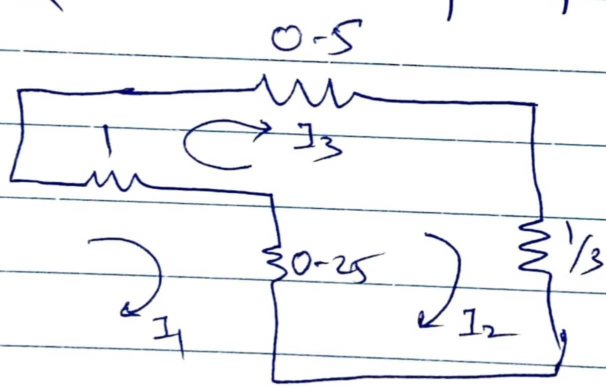
3



$I_1 = -3A$ —— (1)

$I_3 - I_2 = 2A$ —— (2)

For eqn (3) remove 2A current source & write eqn for supermesh



$$-0.5 I_3 - 0.33 I_2 - 0.25(I_2 - I_1) - (I_3 - I_1) = 0$$

$$-0.5 I_3 - 0.33 I_2 - 0.25(I_2 + 3) - (I_3 + 3) = 0$$

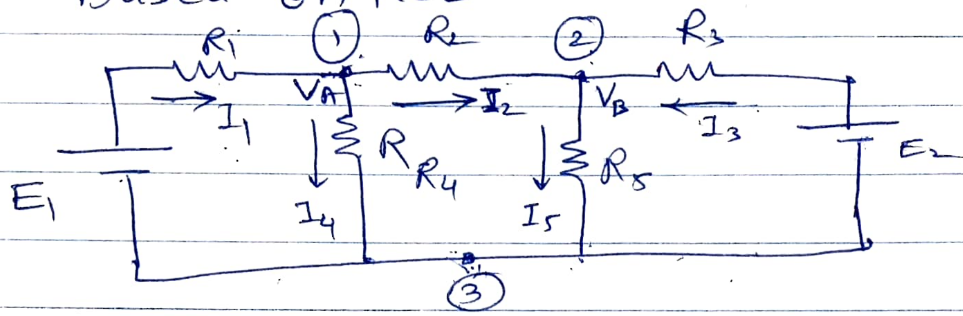
$$-0.58 I_2 - 1.5 I_3 = 3.75$$

Put $I_3 = I_2 + 2$

$\therefore$ $\boxed{I_2 = -3.24 A \quad I_1 = -1.24 A}$

Nodal Analysis.

Based on KCL.



Junction where three or more branches meet is called as node.

One of these is assumed as reference/ datum / zero potential node.

∴ If n no of total nodes are there, no. of ~~simul~~ equations will be $(n-1)$

Node 3 is reference node.

at node 1,

$$I_1 = I_2 + I_4 \quad \text{——— (A)}$$

$$I_1 R_1 = E_1 - V_A$$

$$\therefore I_1 = \frac{E_1 - V_A}{R_1} \quad \text{——— (1)}$$

$$I_4 = \frac{V_A}{R_4}$$

$$I_2 R_2 = V_A - V_B$$

$$\therefore I_2 = \frac{V_A - V_B}{R_2}$$

Put I_2 & I_4 in eqn (A).

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0.$$

At node (2)

$$I_5 = I_2 + I_3 \quad \text{———— (B)}$$

$$I_5 = \frac{V_B}{R_5}$$

$$I_2 = \frac{V_A - V_B}{R_2} \quad \text{--- already written}$$

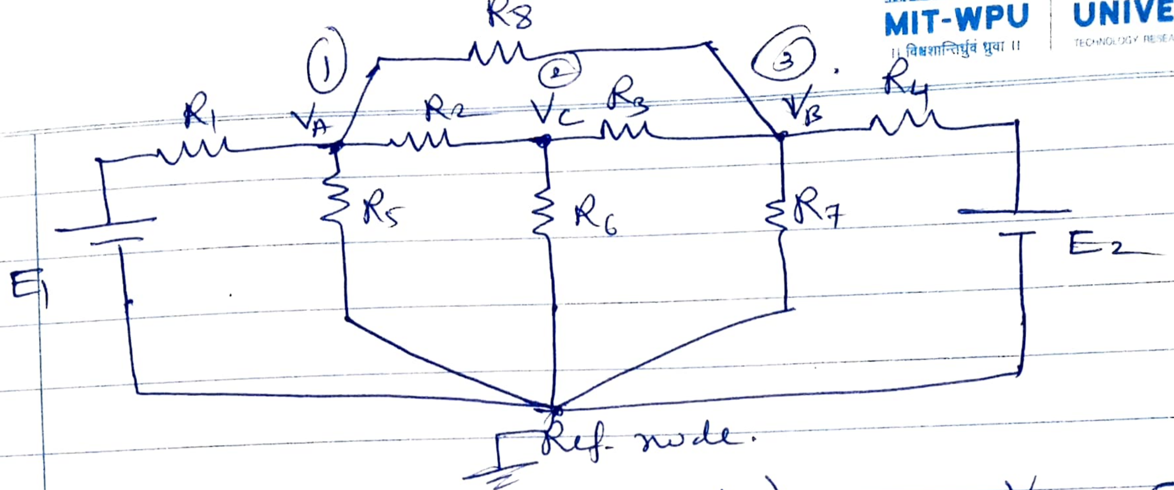
$$I_3 = \frac{E_2 - V_B}{R_3}$$

Put these values in eqn (B)

$$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0$$

In short,

1. Product of node potential V_A and $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right)$ ie, sum of the reciprocals of the branch resistances connected to this node
2. Minus the ratio of adjacent potential V_B and the interconnecting resis. R_2
3. Minus ratio of adjacent battery voltage E_1 & interconnecting resis R_1
4. All above equal to zero.



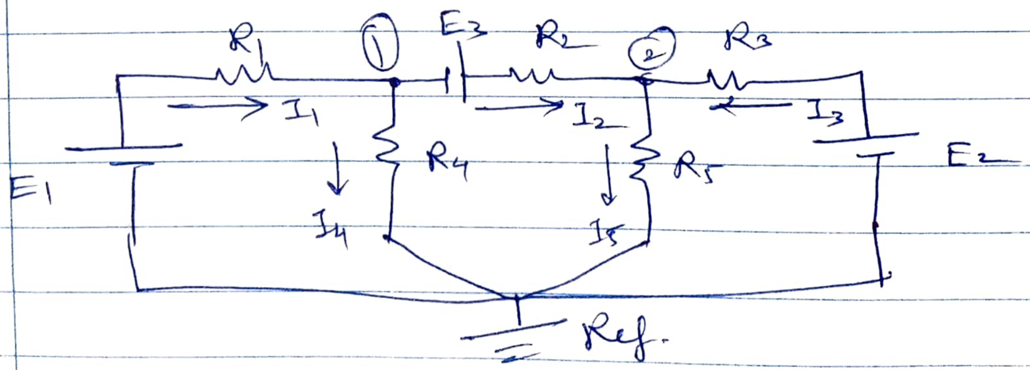
$$V_A\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_8}\right)-\frac{V_C}{R_2}-\frac{V_B}{R_8}-\frac{E_1}{R_1}=0$$

$$V_B\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_7}+\frac{1}{R_8}\right)-\frac{V_A}{R_8}-\frac{V_C}{R_3}-\frac{E_2}{R_4}=0$$

$$V_C\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_6}\right)-\frac{V_A}{R_2}-\frac{V_B}{R_3}=0$$

Case 2

When a third battery of emf E_3 is connected betn node (1) & (2).



E_3 is connected betn node (1) & (2)
As we travel from (1) to (2), we go from −ve terminal of E_3 to +ve terminal. Hence acc. to sign convention, it must be taken as +ve.

Vice versa for when we travel from (2) to (1)
ie − E_3

at node 1,

$$I_1 = I_2 + I_4$$

$$I_1 = \frac{E_1 - V_A}{R_1}$$

$$I_2 = \frac{V_A + E_3 - V_B}{R_2}$$

$$I_4 = \frac{V_A}{R_4}$$

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_3}{R_2} = 0$$

at node 2,

$$I_2 + I_3 = I_5$$

$$I_2 = \frac{V_A + E_3 - V_B}{R_2}$$

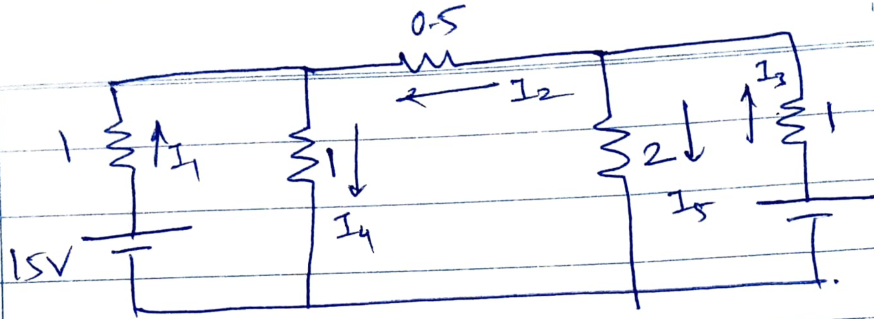
$$I_3 = \frac{E_2 - V_B}{R_3}$$

$$I_5 = \frac{V_B}{R_5}$$

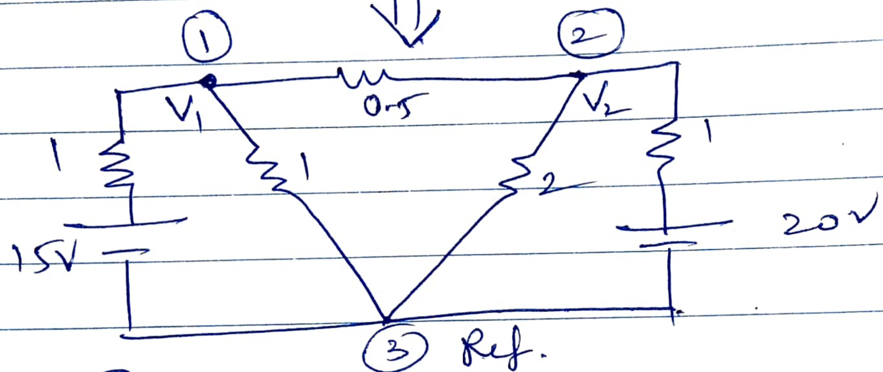
$$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0$$

Example

①



⇓



Total power consumed by all passive elements.

at ①

$$V_1\left(\frac{1}{1}+1+\frac{1}{0.5}\right)-\frac{V_2}{0.5}-\frac{15}{1}=0$$

at node ②,

$$V_2\left(1+\frac{1}{2}+\frac{1}{0.5}\right)-\frac{V_1}{0.5}-\frac{20}{1}$$

$V_2 = 11\,V$, $V_1 = 37/4\,V$.

$I_1 = 5.75\,A$

$I_2 = 3.5\,A$

$I_3 = 9\,A$,

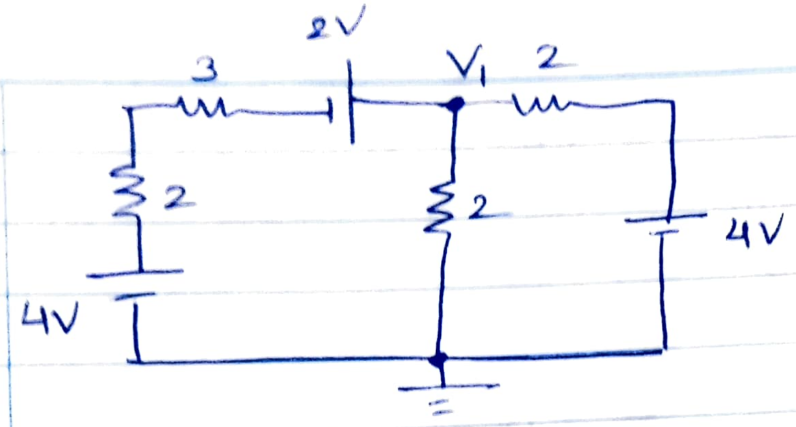
$I_4 = 9.25$

$I_5 = 5.5\,A$

Total power consumed =

$$= (5.75^2\times 1)+(3.5^2\times 0.5)+(9^2\times 1)+(9.25^2\times 1)+(5.5^2\times 2)$$

(2)



Find current in 3Ω resis.

$n = 2$

$\therefore$ Eq^{n}s $= 1$

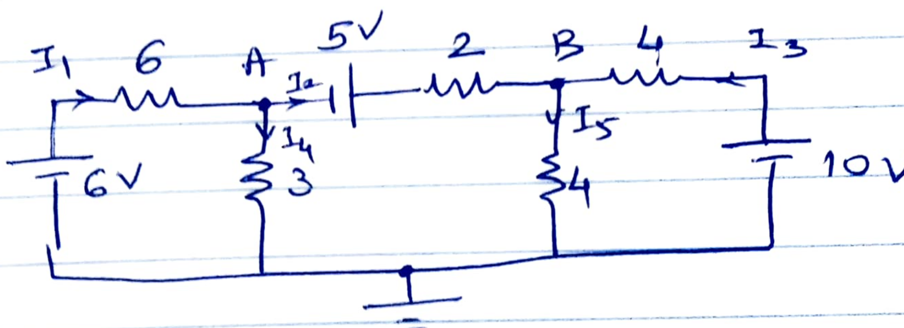
$$V_1\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right) - 4/2 - \frac{6}{5} = 0$$

$$V_1 = 8/3 \text{ V}$$

$$\text{Current in } 3\Omega \text{ resis.} = \frac{6-\frac{8}{3}}{3+2}$$

$$= 2/3 \text{ A.}$$

(3)



Find branch currents using nodal analy

at node A,

$$V_A\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{2}\right) - \frac{V_B}{2} - \frac{6}{6} + \frac{5}{2} = 0$$

$$V_B\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right) - \frac{10}{4} - \frac{V_A}{2} - \frac{5}{2} = 0$$

$$V_A = \frac{4}{3} \text{ V}$$

$$V_B = \frac{17}{3} \text{ V.}$$

$$I_1 = \frac{E_1 - V_A}{6} = 7/9\,A$$

$$I_2 = \frac{V_A + E_3 - V_B}{2} = 1/3\,A$$

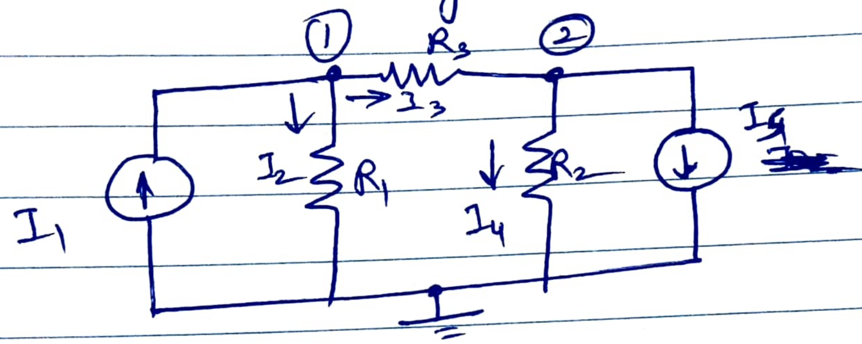
$$I_3 = \frac{E_2 - V_B}{4} = 13/12\,A$$

$$I_4 = \frac{V_A}{2} = 4/9\,A.$$

$$I_5 = \frac{V_B}{4} = 17/12\,A$$

Case – III

Nodal analysis with current sources.



Node 1

$$I_1 = I_2 + I_3$$

$$I_2 = \frac{V_1}{R_1}$$

$$I_3 = \frac{V_1 - V_2}{R_3}$$

$$\therefore I_1 = \frac{V_1}{R_1} + \frac{(V_1 - V_2)}{R_3}$$

$$\boxed{I_1 = V_1\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_2}{R_3}}$$

Node 2.

$\cancel{I_3 + I_4 = I_5}$ $I_3 = I_4 + I_5$

$$I_3 \rightarrow \text{from above} = \frac{V_1 - V_2}{R_3}$$

$$I_4 = \frac{V_2}{R_2}$$

$$\cancel{\frac{V_1 - V_2}{R_3} + \frac{V_2}{R_2} \; I_5}$$

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॥ विश्वशान्तिर्ध्रुवं ध्रुवा ॥
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TECHNOLOGY, RESEARCH, SOCIAL INNOVATION & PARTNERSHIPS

$$\frac{V_1 - V_2}{R_3} = \frac{V_2}{R_2} + I_S$$

$$\frac{V_1 - V_2}{R_3} - \frac{V_2}{R_2} = I_S$$

$$\frac{V_1}{R_3} - V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = I_S$$

$$\boxed{V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{V_1}{R_3} = -I_S}$$

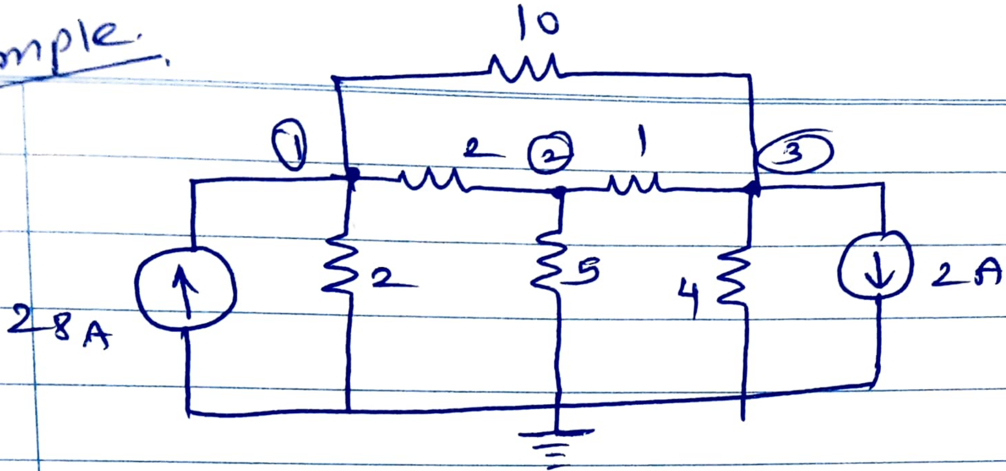
Above two eq^ns can be written by simple inspection. It includes,

i) product of potential V_1 and $\left(\frac{1}{R_1} + \frac{1}{R_3}\right)$ ie, sum of reciprocals of the branch resistances connected to this node

ii) minus the ratio of adjoining potential V_2 and interconnecting resistance R_3.

iii) All the above is equated to the current supplied by the current source connected to this node. This current is taken positive if flowing into the node & negative if flowing out of it.

Example.



Calculate diff. node voltages.

Node 1.

$$V_1\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{10}\right)-\frac{V_2}{2}-\frac{V_3}{10}=28$$

Node 2

$$V_2\left(\frac{1}{2}+\frac{1}{5}+1\right)-\frac{V_1}{2}-\frac{V_3}{1}=0$$

Node 3.

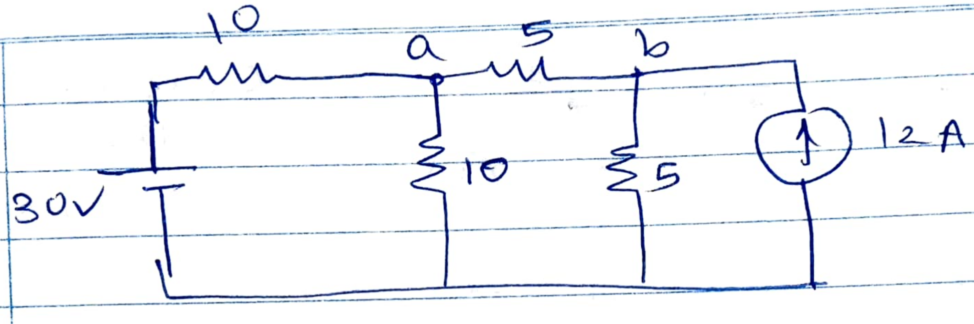
$$V_3\left(1-\frac{1}{4}-\frac{1}{10}\right)-\frac{V_2}{1}-\frac{V_1}{10}=-2$$

$\therefore$ $V_1 = 36\,V$

$V_2 = 20\,V$

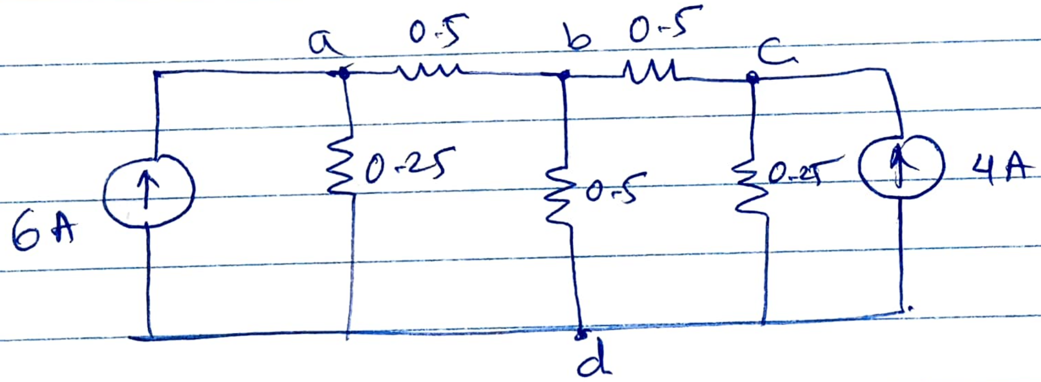
$V_3 = 16\,V.$

2



$I_{ab} = -3A$

3



Calculate I_{ab}, I_{bd}, I_{bc}.

$I_{ab} = 22/21$

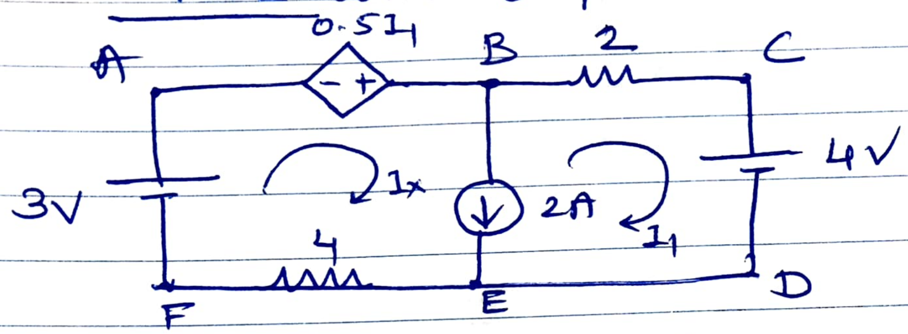
$I_{bd} = 10/7$

$I_{bc} = -8/21$

Mesh analysis.

KVL with dependent sources.

1]



Calculate current I_1

Applying KCL at node B,

$$I_x = I_1 + 2 \qquad \text{--- (1)}$$

Applying KVL to loop ABCDEFA,

$$3 + 0.5 I_1 - 2 I_1 - 4 - 4 I_x = 0$$

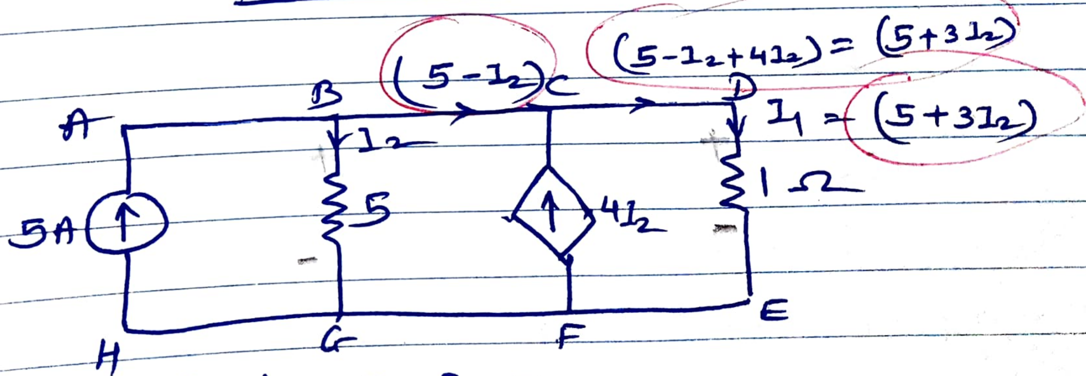
$$-1.5 I_1 - 4 I_x = 1 \qquad \text{--- (2)}$$

substituting value of I_x in (2) from (1)

$$-1.5 I_1 - 4(I_1 + 2) = 1$$

$$\therefore \boxed{I_1 = -1.6363\ A}$$

2]



Find I_1 & I_2

From above diag,

$$I_1 = 5 + 3I_2$$

Writing KVL eq^n for, BCDEFGB

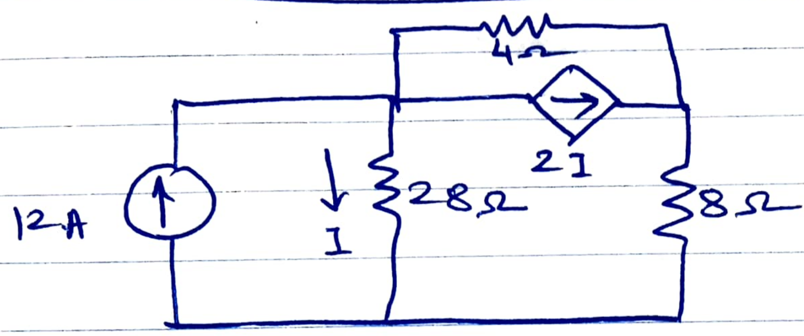
$$-1(5+3I_2)+5I_2=0$$

$$2I_2=5$$

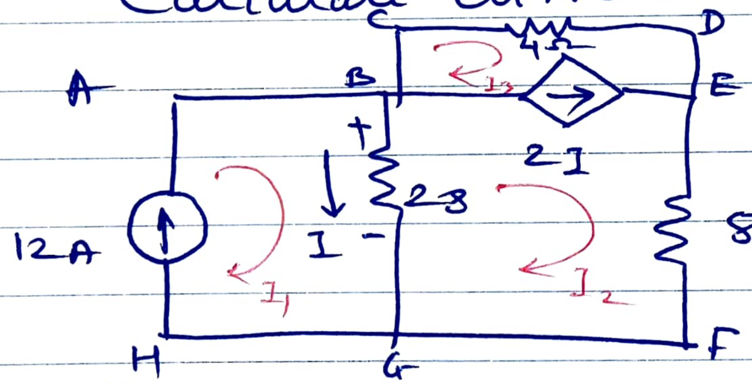
$$I_2=2.5\text{ A}$$

$$\therefore I_1=12.5\text{ A}$$

3]



Calculate current I



$I_1=12 \longrightarrow$ loop 1 —— (1)

$I_2-I_3=2I \longrightarrow$ at branch BE —— (2)

$I_1-I_2=I \longrightarrow$ at branch BG

$12-I_2=I$ —— (3)

Writing KVL eq^n for BCDEFGB

$$-4I_3-8I_2-28(I_2-I_1)=0$$

$$28I_1-36I_2-4I_3=0 \quad \text{—— (4)}$$

From eqn (2)

$$I_2 = 2I + I_3$$

$$= 2(12 - I_2) + I_3$$

$$\therefore I_2 = 24 - 2I_2 + I_3$$

$$\therefore 3I_2 = 24 + I_3$$

$$\therefore I_2 = \frac{24 + I_3}{3} \qquad \text{--- (5)}$$

Substituting eqn (1) & (5) in (4)

$$(28 \times 12) - 36\left(\frac{24 + I_3}{3}\right) - 4I_3 = 0$$

$$\therefore I_3 = 3A$$

$$I_2 = \frac{24 + 3}{3} = 9A$$

$$I = I_1 - I_2$$

$$= 12 - 9 = 3A.$$

Nodal analysis with dependent sources.

Network Topology

Network topology is a generic name that refers to all properties arising from the structure or geometry of a n/w. The topology is a branch of science which deals with the study of geometrical properties and special relations unaffected by continuous change of shape or size of figures. Since the topological properties of an electrical n/w are independent of the types of elements or components (R, L, C) which constitute the branches, it is convenient to replace each n/w element by a simple line segment without caring for VI relation of the component. The resulting structure therefore consists of line segments and nodes. By replacing the elements at different places and increasing the size of wire, the same circuit can be drawn in diff. ways. As long as the relationship betn the nodes and branches are maintained, the circuit response will be same.

Terms related to n/w topology

1. **Graph of the n/w** – When all the elements (R, L, C) in a n/w are replaced by lines with circles or dots at both ends, the configuration is called graph of the n/w. The graph of n/w is drawn by keeping all points of intersection of two or more branches and representing the n/w elements by lines, voltage and current sources by their internal resistances.

   Internal impedance of an ideal voltage source is zero & hence it is to be replaced by a short circuit. Internal impedance of an ideal current source is infinite & hence it is to be replaced by an open ckt.

2. **Directed or oriented graph** – A graph with directions or orientations represented for each branch is known as directed or oriented graph.

3. **Node** – point of intersection of 2 or more branches.

4. **Branch** – A line joins one n/w element or combination of element betn 2 point. A branch does not indicate anything about the type of the elements.

5. **Degree of a node** – It is the no. of branches connected to a node.

6. **Tree** – A tree is a part of directed graph containing all the nodes and without any closed path. Properties –

① it consists of all nodes of graph.

② For an 'n' node graph, a tree has (n-1) branches.

③ There cannot be any closed path in a tree.

7. **Twig** – A branch of a tree is known as twig. A twig is represented by a thick line.

8. **Co-tree** – The part of a directed graph that is not covered by the tree is known as a co-tree. A co-tree is represented by a dotted configuration

9. **Link or chord** – The branch of a co-tree is known as link or chord and is represented by a dotted line.

No of branches in a directed graph = No. of twigs + No. of links.

10. Sub-graph – A graph G_s is said to be sub-graph of graph G if every node of G_s is a node of G and every branch of G_s is also a branch of G.

11. Connected graph – When at least one path along branches exists betn every pair of nodes of a graph, it is known as connected graph.

12. Path – An ordered sequence of branches traversing from one node to another is called a path in a graph.

13. Tree complement – it is the totality of Tree links.

No. of branches, $b = n - 1 + L$

where, L = ~~no of~~ total no. of links,

n = no. of nodes

14. Incidence Matrix ~~(Ai)~~ [Ai] – A matrix representing the relation betn no. of branches and no. of nodes in a direction graph is known as incidence matrix.

(a) Determinant of incidence matrix of a closed loop is zero.

(b) Algebraic sum of the column entries of an incidence matrix is zero.

15. Reduced incidence matrix ~~[A]~~ [A_r] - A matrix obtained by deleting a row of the incidence matrix is known as reduced incidence matrix.

16. Tieset matrix [M] - A matrix representing the relation betn no. of link currents and no. of branch currents in a direction graph is known as tieset matrix.
Each branch current is the algebraic sum of link currents below the column of that branch number in the tieset matrix.
Tieset is a set of branches contained in a loop such that ~~t~~ each loop contains only one link and rest are tree branches.

17. Cutset matrix - A matrix representing the relation betn cutsets and no. of branches in a directed graph. It is that set of elements which separates two main parts of n/w such that replacing any one element will destroy this property.
Each cutset contains one and only ~~only~~ one tree branch, the remaining elements being tree links.

18. Isomorphism - The property betn two graphs so that both have got same incidence matrix is known as isomorphism.