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MIT WORLD PEACE UNIVERSITY | PUNE
TECHNOLOGY, RESEARCH, SOCIAL INNOVATION & PARTNERSHIPS

Basics of Electrical and Electronics Engineering

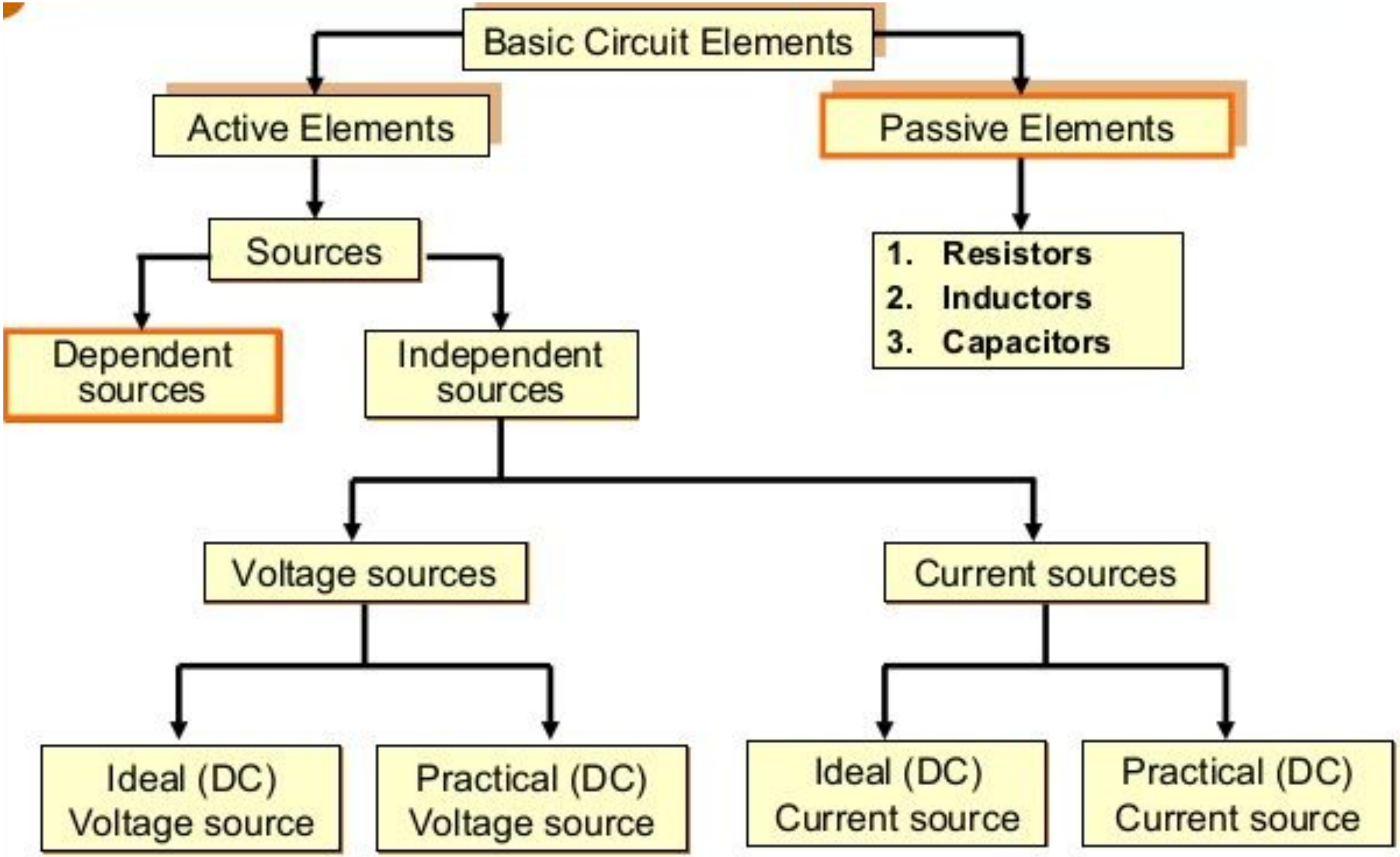
ECE1022A

UNIT V - D.C. CIRCUITS

Learning Resources

- DC series ccts- https://youtu.be/VV6tZ3Aqfuc
- Dc parallel ccts- https://youtu.be/5uyJezQNSHw

Circuit Elements



Basic Passive Circuit Elements

- A passive element is an electrical component that *does not generate power.*

instead

- dissipates, stores, and/or releases it
- Passive elements include *resistances*, *capacitors*, and *coils* (inductors).
- In most circuits, they are connected to active elements, typically semiconductor devices such as amplifiers and digital logic chips.

1. Resistance

- Resistance (R) : Property of an electric circuit tending to prevent the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.

$R = \rho l/A$

l - length in meters

ρ – Resistivity in ohms-meters

R-Resistance in ohms

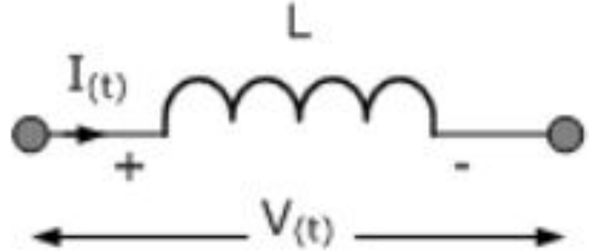
A- cross-sectional area in square meters

Conductance (G): reciprocal of resistance, indication of ease with which current can flow through the material, measured in unit Siemens

Denoted as $G = 1/R$

2. Inductor

- Inductance which has the symbol "L" and is measured in Henries (H), is the element used for the storage of energy in the form of an electromagnetic field.
- An inductor is a passive device that can store or deliver energy but cannot generate it.
- An ideal inductor is lossless, meaning that it can store energy indefinitely as no energy is lost as heat.
- Inductors present a low impedance path to DC current and a high impedance path to AC current.
- The impedance of an inductor called inductive reactance varies with frequency and in an ideal inductor the current of the AC sine wave lags the voltage by 90^{o}.

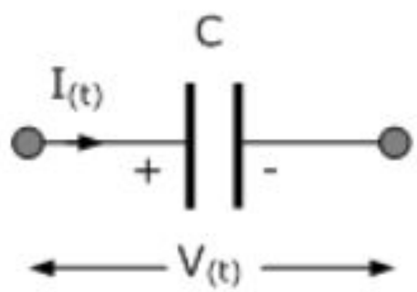


$$V_{(t)} = L\frac{di_{(t)}}{dt}$$

3. Capacitor

- A capacitor stores its energy electrostatically as a charge across its plates.
- A capacitor is made up of two or more conducting plates which are separated by a dielectric material.
- Capacitance, "C" is the property of a capacitor which opposes any changes in the voltage across it as defined by the constant of proportionality as the current flowing through it is proportional to the rate of change of voltage across it with respect to time.
- Capacitors present a low impedance path to AC signals but will block DC.
- The impedance of a capacitor called capacitive reactance varies with frequency and in an ideal capacitor the voltage of the AC sine wave lags the current by 90^{o}.
- Capacitance is always a positive value.

$$C=Q/V$$



$$I_{(t)} = C\frac{dv_{(t)}}{dt}$$

Basic Passive components

Resistor	Capacitor	Inductor
 R	 C	 L
Resistance	Capacitive reactance	Inductive reactance
$V_R / I = R$	$V_C / I = X_C = \frac{1}{\omega C}$	$V_L / I = X_L = \omega L$
V and I in phase	V lags I by $\pi/2$	V leads I by $\pi/2$

Dependent and Independent sources

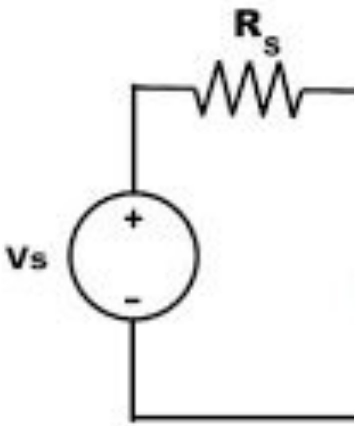
- *Sources* can be either *independent* or *dependent* of/upon some other quantities.

- Independent voltage (or current) source maintains a voltage (or current) which is not affected by any other quantity.

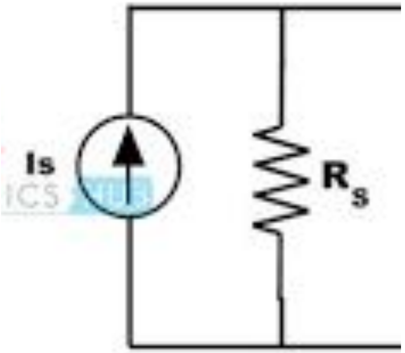
There are two principal types of sources, namely *voltage source and current source.*

Independent sources

Independent voltage source



Independent current source



Ideal Voltage Source

- An ideal voltage source has no internal resistance.
- It can produce as much current as is needed to provide power to the rest of the circuit.
- The voltage generated by the source never fluctuates and is not affected by the amount of current drawn by the circuit.

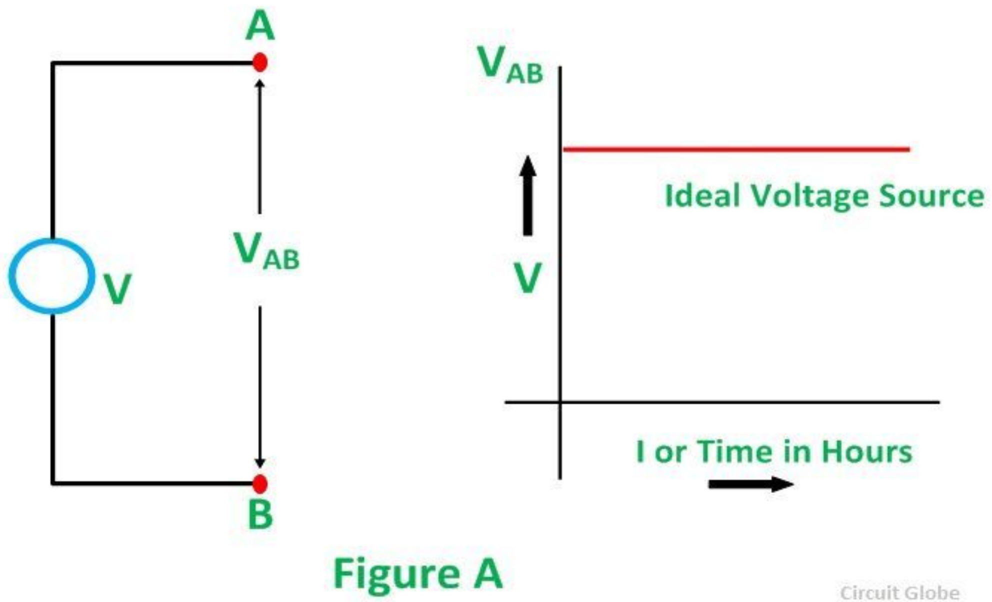


Figure A

Practical Voltage Source

Practical Voltage source:

- A practical voltage source has internal resistance
- Due to this internal resistance; voltage drop takes place, and it causes the terminal voltage to reduce.

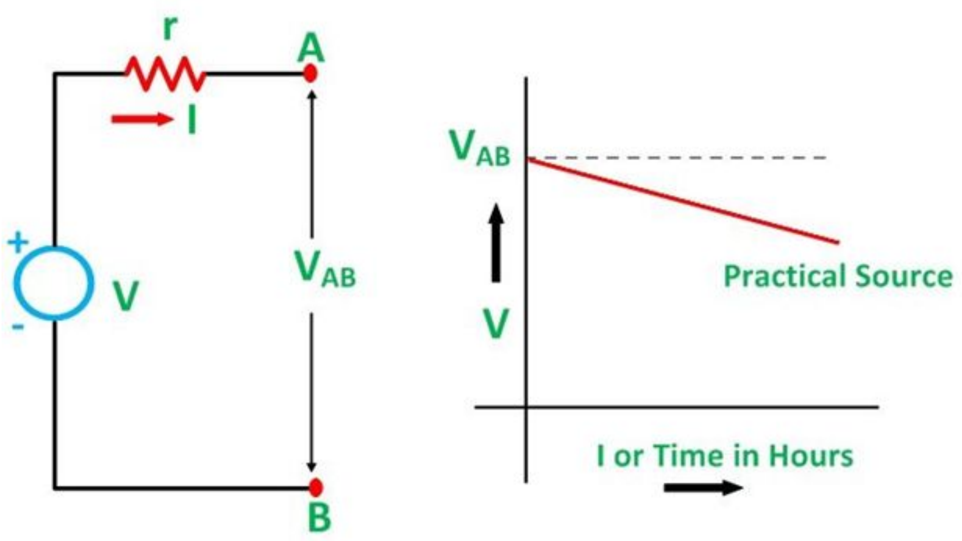


Figure B

Ideal and Practical current sources

- An Ideal current source in Figure C, is a two-terminal circuit element which supplies the same current to any load resistance connected across its terminals.
- An ideal current source always generates its exact rated current and is not affected by the characteristics of the circuit to which it is connected.
- A practical current source in Figure D, has large value of internal resistance. Current varies with respect to the voltage across the element.

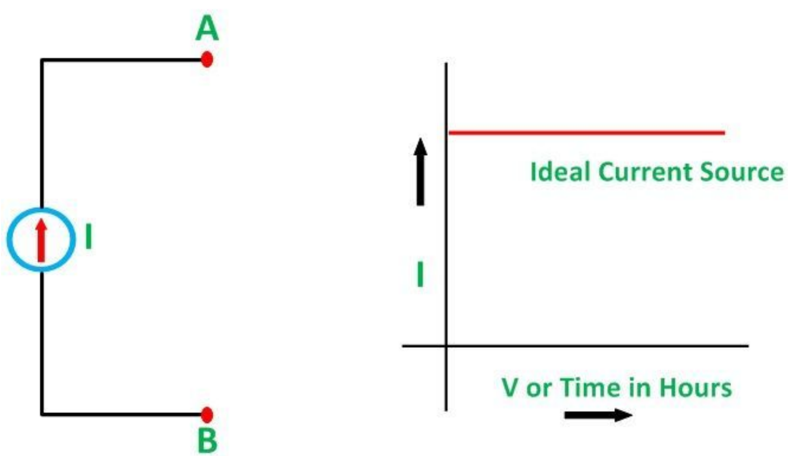


Figure C

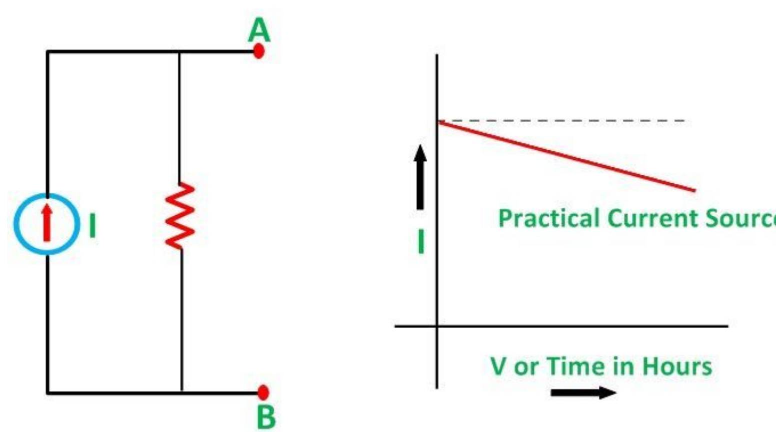
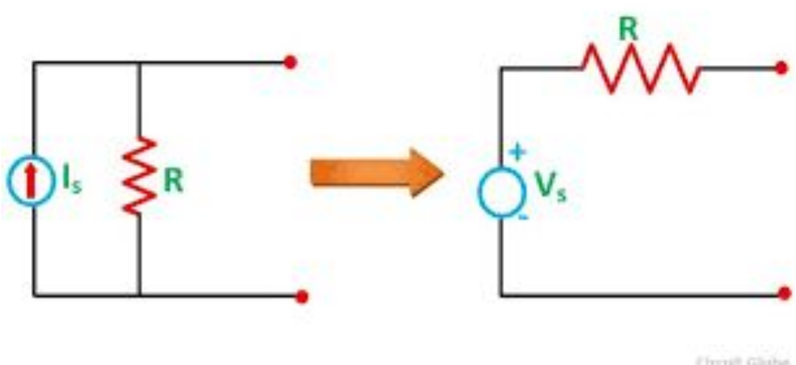


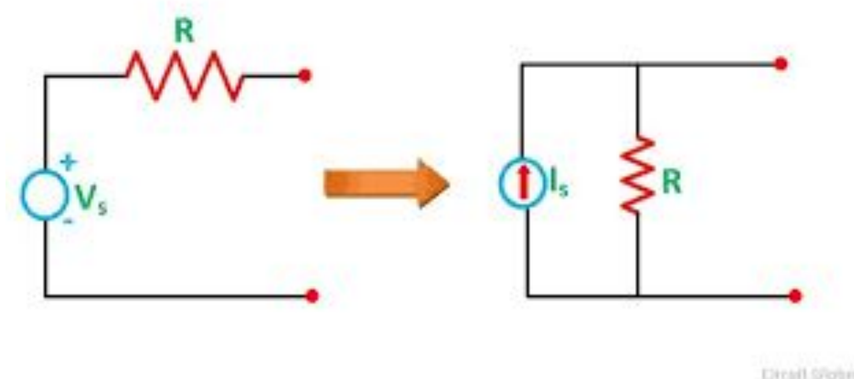
Figure D

Source Transformation

- Current to Voltage and vice versa
- Using Ohm's Law



Current to Voltage transformation



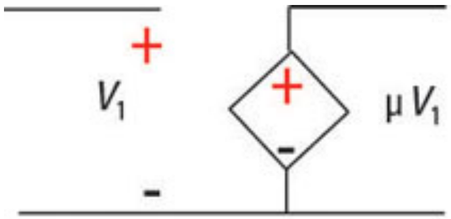
Voltage to Current transformation

Dependent sources

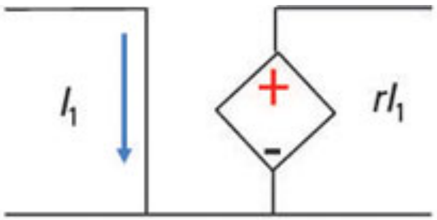
- A *dependent source* is a voltage source or a current source whose value depends on a voltage or current somewhere else in the network.
- In general, dependent source is represented by a diamond shaped symbol as not to confuse it with an independent source. One can classify dependent voltage and current sources into four types of sources as shown in fig.
- **Examples:** A *bipolar junction transistor (BJT)* can be modeled as a dependent current source whose magnitude depends on the magnitude of the current fed into its controlling base terminal.
- An *operational amplifier (OpAmp)* can be described as a voltage source dependent on the differential input voltage between its input terminals

Dependent sources

- **Voltage-controlled voltage source (VCVS)**:
- The source delivers the voltage as per the voltage of the dependent element
- In the VCVS dependent source μ is voltage gain because it's the ratio of the voltage output to the voltage input.
- **Current-controlled voltage source (CCVS):**
- The source delivers the voltage as per the current of the dependent element
- In the CCVS dependent source, the proportionality constant r is called the transresistance because its input-output relationship takes the form of Ohm's law: $v = iR$. $V2/V1= \mu$ Or $V2= r\,I1$



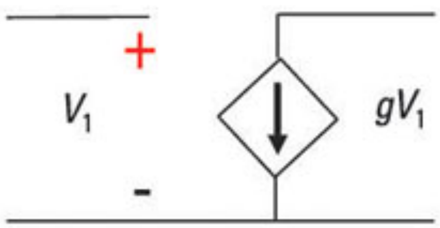
Voltage-controlled voltage source (VCVS)



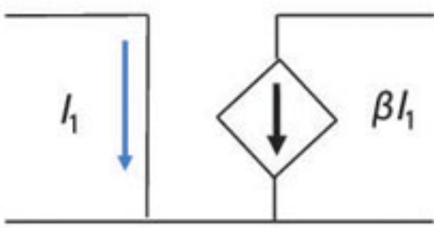
Current-controlled voltage source (CCVS)

Dependent sources

- **Voltage-controlled current source(VCCS):**
- The source delivers the current as per the voltage of the dependent element
- Similarly, the VCCS dependent source has a proportionality constant g, called the transconductance, following a variation of Ohm's law: i = Gv (where the conductance G = 1/R).
- **Current-controlled current source(CCCS):**
- The source delivers the current as per the current of the dependent element
- For the CCCS dependent source, you can think of the proportionality constant β as the current gain because it's the ratio of current output to current input.



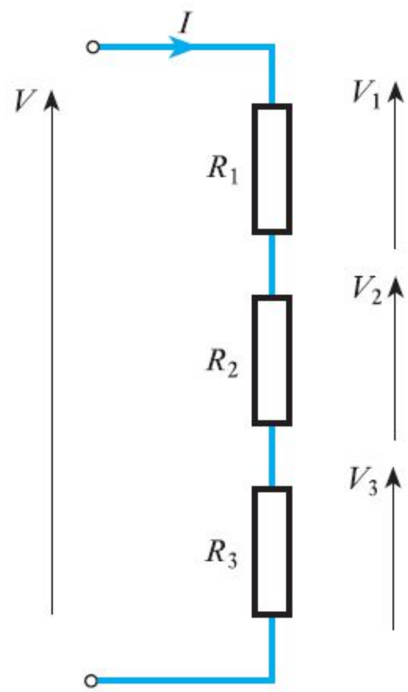
Voltage-controlled current source (VCCS)



Current-controlled current source (CCCS)

Series Networks

Volt drops in a series circuit:



$$V = V_1 + V_2 + V_3$$

Since, in general, $V = IR$, then $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$, the current I being the same in each resistor. Substituting in equation

$$V = IR_1 + IR_2 + IR_3$$

For the complete circuit, the effective resistance of the load R represents the

ratio of the supply voltage to the circuit current whence

$$V = IR$$

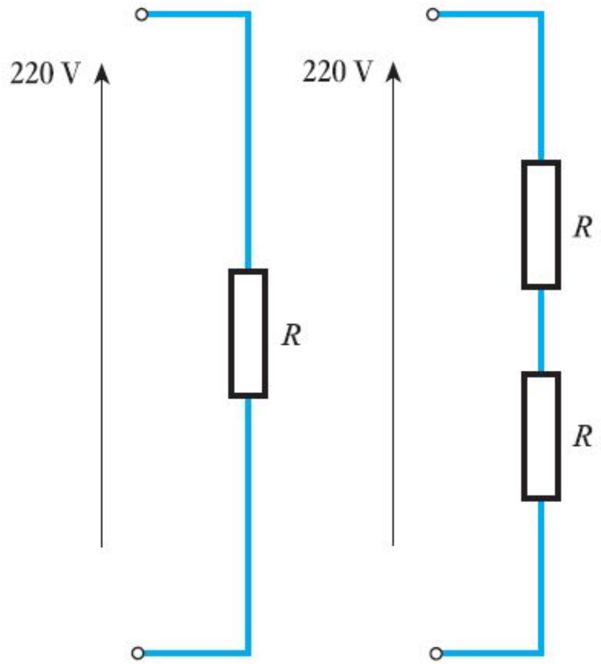
but $\quad V = IR_1 + IR_2 + IR_3$

hence $\quad IR = IR_1 + IR_2 + IR_3$

and $\quad R = R_1 + R_2 + R_3$

Series Networks

- Calculate for each of the circuits shown in Fig. the current flowing in the circuit given that R = 3 kΩ.



In the first case

$$I = \frac{V}{R} = \frac{220}{3 \times 10^3} = 0.073\text{ A} = \mathbf{73\text{ mA}}$$

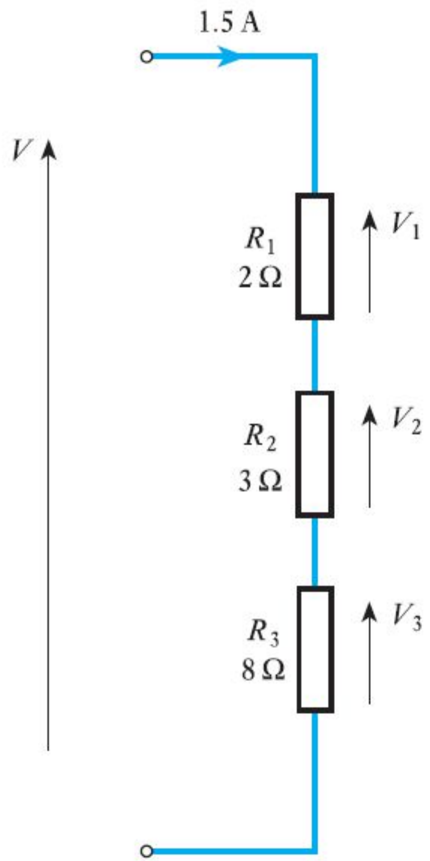
In the second case the circuit resistance is given by

$$R = R_1 + R_2 = 3 \times 10^3 + 3 \times 10^3 = 6000\ \Omega$$

$$I = \frac{V}{R} = \frac{220}{6000} = 0.037\text{ A} = \mathbf{37\text{ mA}}$$

Series Networks

- Calculate the voltage across each of the resistors shown in Fig. and hence calculate the supply voltage V.



$V_1 = IR_1 = 1.5 \times 2 = 3.0\text{ V}$

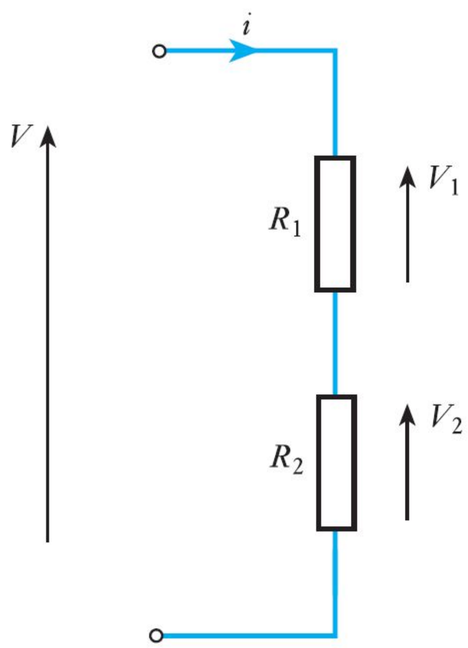
$V_2 = IR_2 = 1.5 \times 3 = 4.5\text{ V}$

$V_3 = IR_3 = 1.5 \times 8 = 12.0\text{ V}$

$V = V_1 + V_2 + V_3 = 3.0 + 4.5 + 12.0 = 19.5\text{ V}$

Series Networks

Voltage division between two resistors:



The total resistance of the circuit is

$$R = R_1 + R_2$$

and therefore the current in the circuit is

$$I = \frac{V}{R_1 + R_2}$$

The volt drop across R_1 is given by

$$IR_1 = \frac{V}{R_1 + R_2} \times R_1 = V_1$$

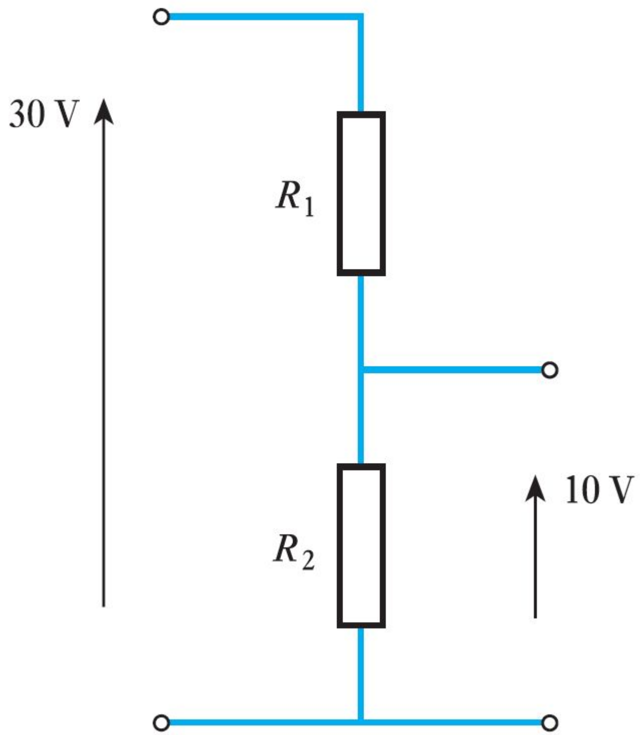
whence

$$\boxed{\frac{V_1}{V} = \frac{R_1}{R_1 + R_2}}$$

The ratio of the voltages therefore depends on the ratio of the resistances. This permits a rapid determination of the division of volt drops in a simple series circuit and the arrangement is called a voltage divider.

Series Networks

- A voltage divider is to give an output voltage of 10 V from an input voltage of 30 V as indicated in Fig. Given that R2 = 100 Ω, calculate the resistance of R1.



$$\frac{V_2}{V} = \frac{R_2}{R_1 + R_2}$$

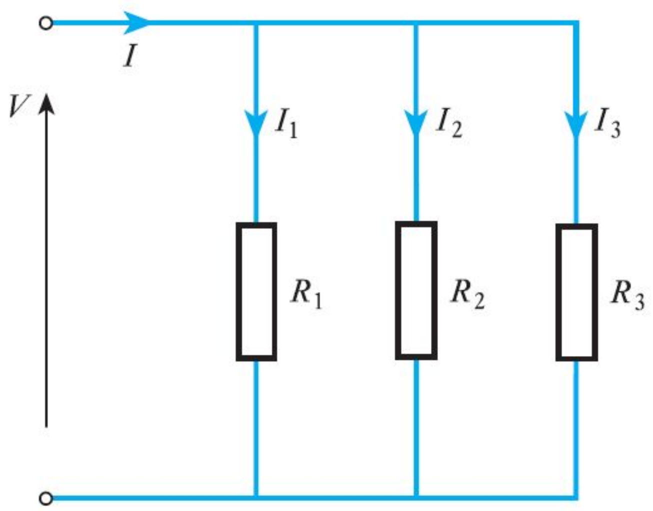
$$\frac{10}{30} = \frac{100}{R_1 + 100}$$

$$R_1 + 100 = 3 \times 100 = 300$$

$$R_1 = 200\ \Omega$$

Parallel networks

Currents in a parallel network:



$$\boxed{I = I_1 + I_2 + I_3}$$

Since in general

$$I = \frac{V}{R}, \quad \text{then } I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2} \quad \text{and} \quad I_3 = \frac{V}{R_3}$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = \frac{V}{R}$$

but $$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

hence

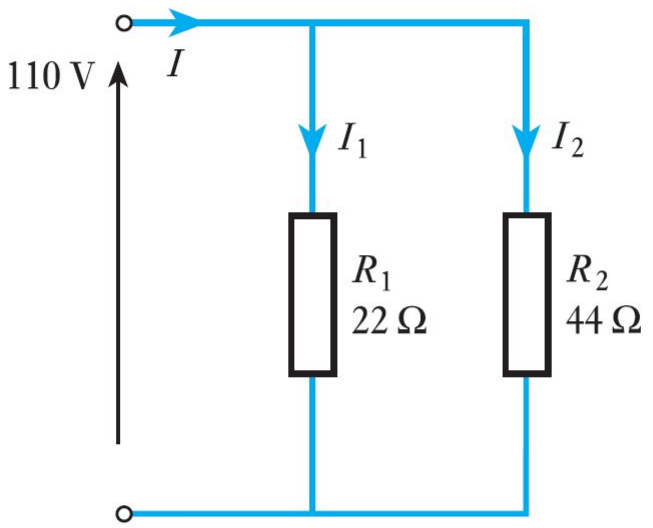
$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

and

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Parallel networks

- Calculate the supply current to the network shown in Fig.



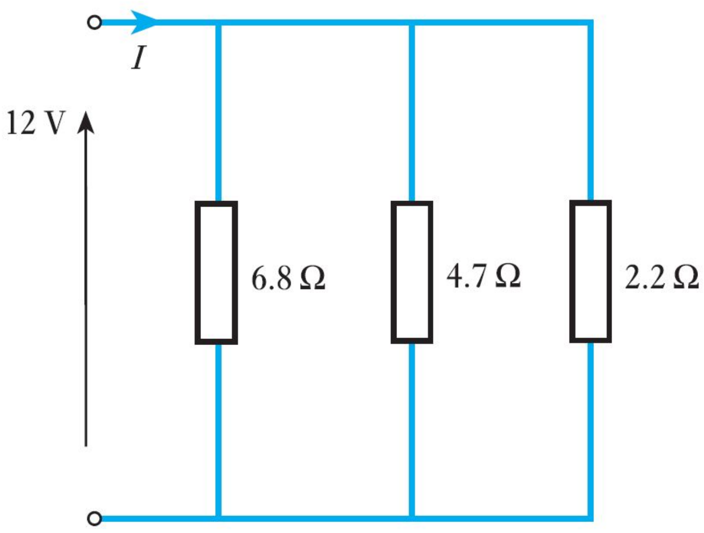
$$I_1 = \frac{V}{R_1} = \frac{110}{22} = 5.0\ \text{A}$$

$$I_2 = \frac{V}{R_2} = \frac{110}{44} = 2.5\ \text{A}$$

$$I = I_1 + I_2 = 5.0 + 2.5 = \mathbf{7.5\ A}$$

Parallel networks

- For the network shown in Fig. 3.20, calculate the effective resistance and hence the supply current.



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.8} + \frac{1}{4.7} + \frac{1}{2.2}$$

$$= 0.147 + 0.213 + 0.455 = 0.815$$

hence

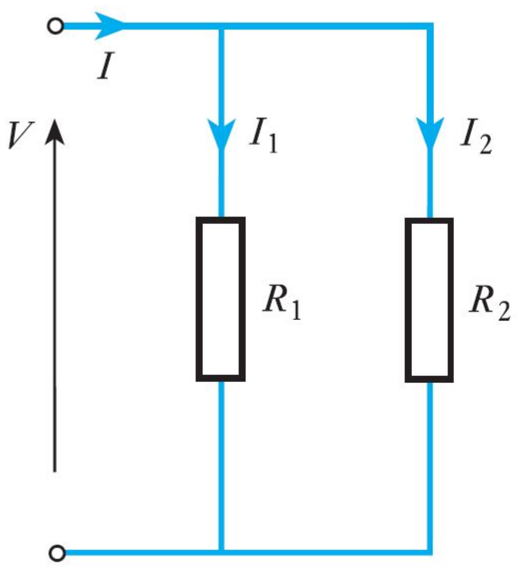
$$R = \frac{1}{0.815} = 1.23\ \Omega$$

$$I = \frac{V}{R} = \frac{12}{1.23} = 9.76\ \text{A}$$

Parallel networks

For the combination of two resistors in parallel, as shown in Fig. the effective resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$



hence

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Two parallel resistors share a supply current.

$$V = IR = I\frac{R_1 R_2}{R_1 + R_2}$$

also $V = I_1 R_1$, hence

$$\boxed{I_1 = I\frac{R_2}{R_1 + R_2}}$$

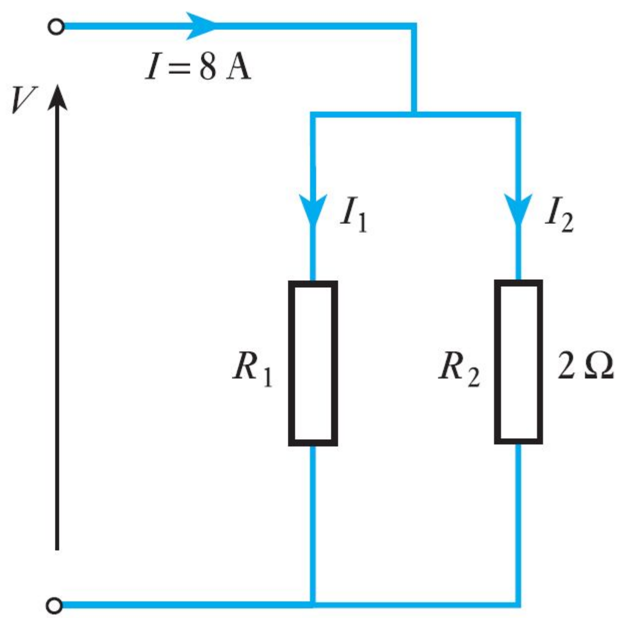
The current in one resistor is that portion of the total given by the ratio of the other resistance to the sum of the resistances.

Parallel networks

- A current of 8 A is shared between two resistors in the network shown in Fig. Calculate the current in the 2 Ω resistor, given that

  (a) R1 = 2 Ω;

  (b) R1 = 4 Ω.



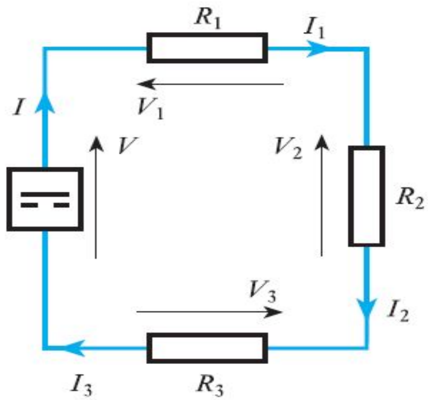
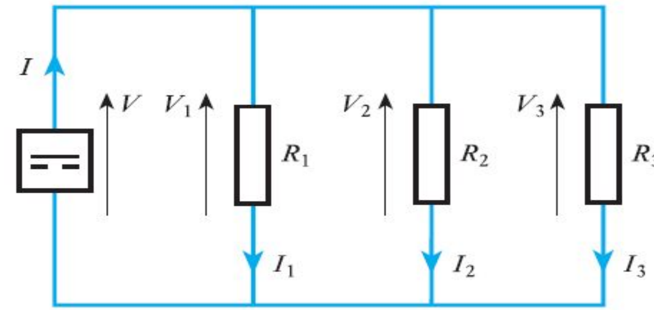
(a) $$I_2 = I\frac{R_1}{R_1 + R_2} = 8 \times \frac{2}{2+2} = \mathbf{4.0\ A}$$

From this, it is seen that equal resistances share the current equally.

(b) $$I_2 = I\frac{R_1}{R_1 + R_2} = 8 \times \frac{4}{4+2} = \mathbf{5.3\ A}$$

The lesser resistance which takes the greater part of the supply current.

Series and Parallel network Summery

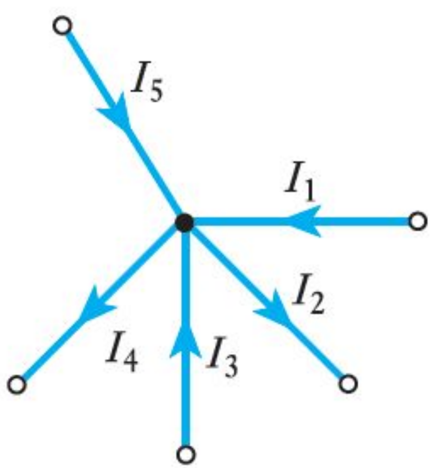
	Series network	Parallel network
		
Current	The current is the same in all parts of the circuit $I = I1 = I2 = I3$	
Voltage	The total voltage equals the sum of the voltages across the different parts of the circuit $V = V1 + V2 + V3$	
Resistance	The total resistance equals the sum of the separate resistances $R = R1 + R2 + R3$	

Kirchhoff's Current(first) Law

- Kirchhoff's Current Law states that: 'the algebraic sum of currents at a node is zero'.

Or

- The sum of currents entering the junction is thus equal to the sum of currents leaving



$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

a) A 'node' is the technical term for a junction in a circuit, where two or more branches are joined together.

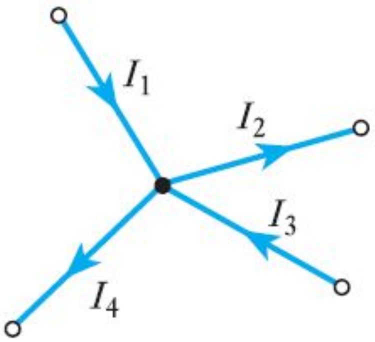
b) The phrase 'algebraic sum' reminds us that we have to take account of the current direction, as well as magnitude, when applying Kirchhoff's Current Law.

Kirchhoff's Current Law

For the network junction shown in Fig., calculate the current I3, given that I1 = 3 A, I2 = 4 A and I4 = 2 A.

$$I_1 - I_2 + I_3 - I_4 = 0$$

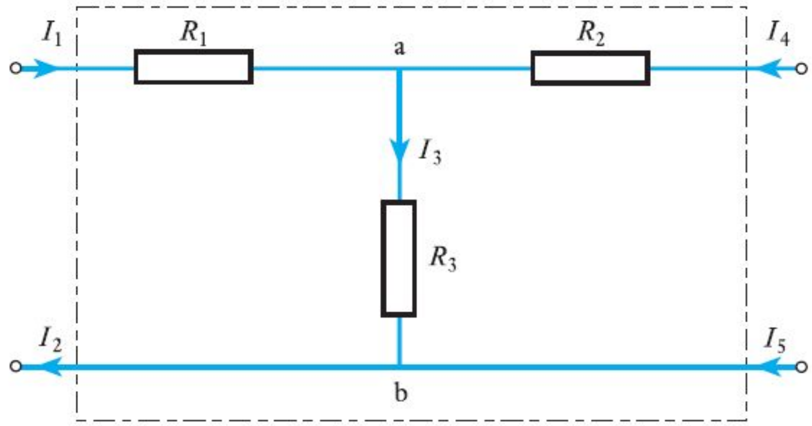
$$I_3 = -I_1 + I_2 + I_4 = -3 + 4 + 2 = \mathbf{3\ A}$$



$$I_1 - I_2 + I_3 - I_4 = 0$$

Kirchhoff's Current Law

- With reference to the network shown in Fig. determine the relationship between the currents I1, I2, I4 and I5.



For junction a:

$$I_1 + I_4 - I_3 = 0$$

hence $\quad I_3 = I_1 + I_4$

For junction b:

$$I_3 + I_5 - I_2 = 0$$

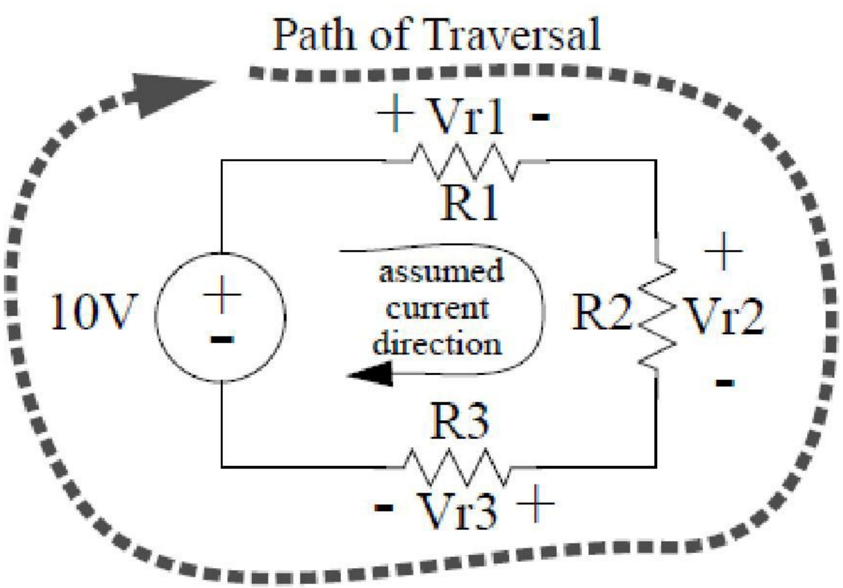
$\therefore \quad I_3 = I_2 - I_5$

$$I_1 + I_4 = I_2 - I_5 \quad \text{and} \quad I_1 - I_2 + I_4 + I_5 = 0$$

Kirchhoff 's Current (first) law need not only apply to a junction but may also apply to a section of a network.

Kirchhoff's Voltage(second) Law

- 'The algebraic sum of voltages around a closed circuit loop is zero'.
- *There's the phrase 'algebraic sum' again, so we must* recognize that the direction of voltages matters when using Kirchhoff's Voltage Law.

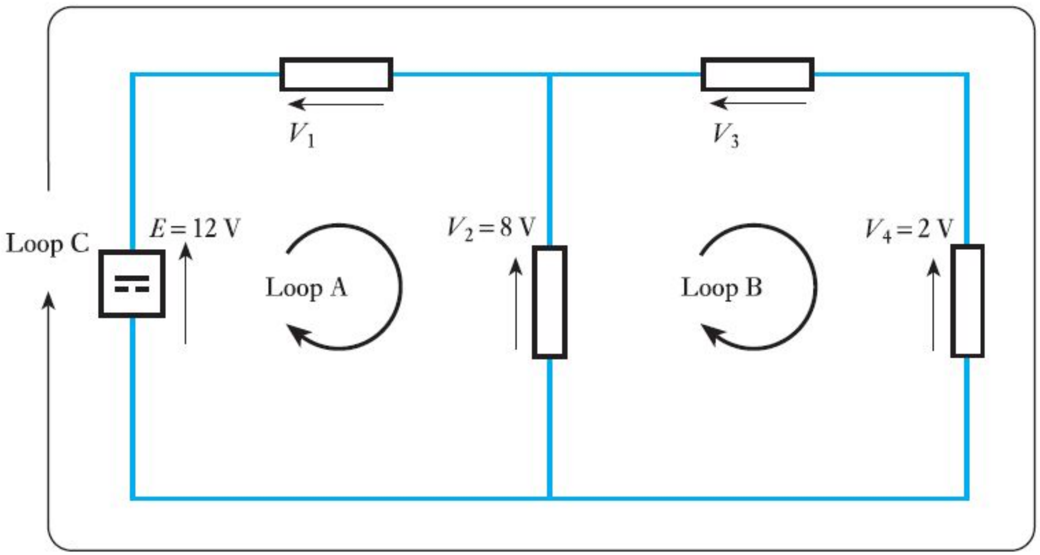


Apply KVL :

$$10 - V_{r1} - V_{r2} - V_{r3} = 0$$

Kirchhoff's Voltage Law

- For the network shown in Fig. determine the voltages V1 and V3.



For loop A:

$$E = V_1 + V_2$$

$$V_1 = E - V_2 = 12 - 8 = \mathbf{4\ V}$$

For loop B:

$$0 = -V_2 + V_3 + V_4$$

$$V_3 = V_2 - V_4 = 8 - 2 = \mathbf{6\ V}$$

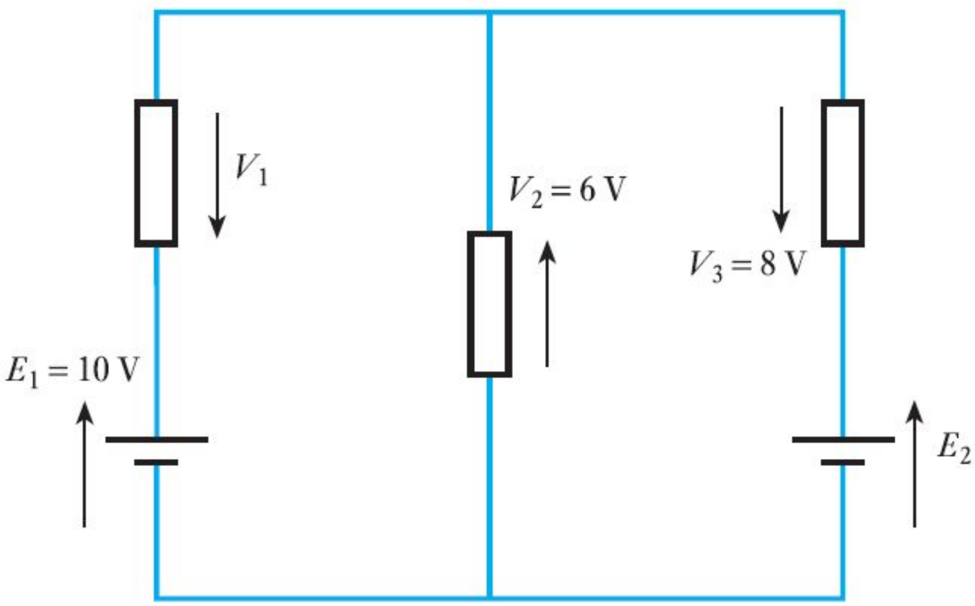
For loop C:

$$E = V_1 + V_3 + V_4$$

$$12 = 4 + 6 + 2 = 12$$

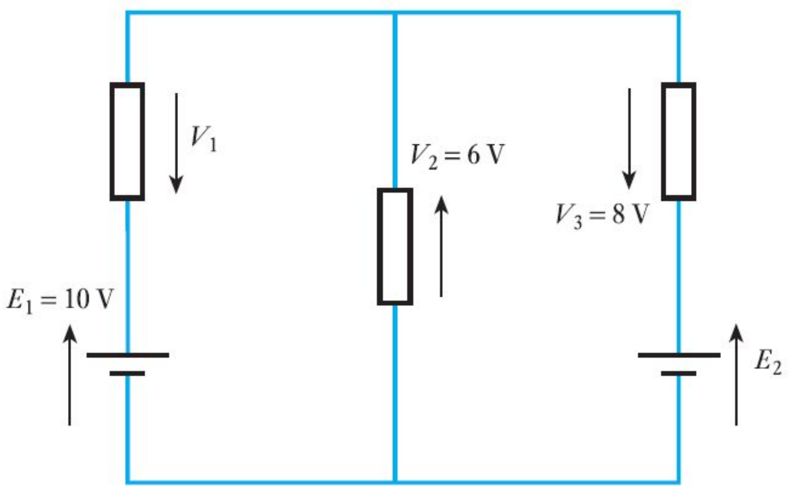
Kirchhoff's Voltage Law

- Figure shows a network with two sources of e.m.f. Calculate the voltage V1 and the e.m.f. E2.



Kirchhoff's Voltage Law

- Figure shows a network with two sources of e.m.f. Calculate the voltage V1 and the e.m.f. E2.



Applying Kirchhoff's second law to the left-hand loop,

E1 = V1 + V2

V1 = E1 − V2 = 10 − 6 = 4 V

The right-hand loop gives

−E2 = −V2 − V3

E2 = V2 + V3 = 6 + 8 = 14 V

These results may be checked by considering the outside loop

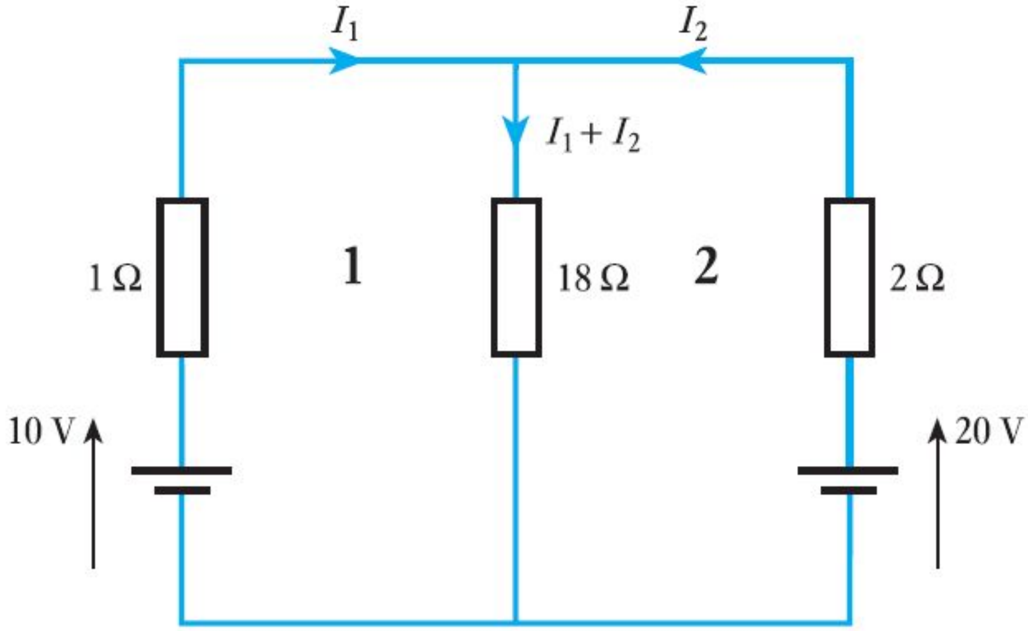
E1 − E2 = V1 − V3

10 − 14 = 4 − 8

which confirms the earlier results.

Numerical 1

- Calculate the currents in the network shown in Figure.



Numerical 1

Applying Kirchhoff's second law to loop 1:

$$10 = 1I_1 + 18(I_1 + I_2)$$

$$10 = 19I_1 + 18I_2 \qquad \text{(a)}$$

Applying Kirchhoff's second law to loop 2:

$$20 = 2I_2 + 18(I_1 + I_2)$$

$$20 = 18I_1 + 20I_2 \qquad \text{(b)}$$

(a) × 10: $100 = 190I_1 + 180I_2$ (c)

(b) × 9: $180 = 162I_1 + 180I_2$ (d)

(d) − (c): $80 = -28I_1$

$$I_1 = -2.85\ \text{A}$$

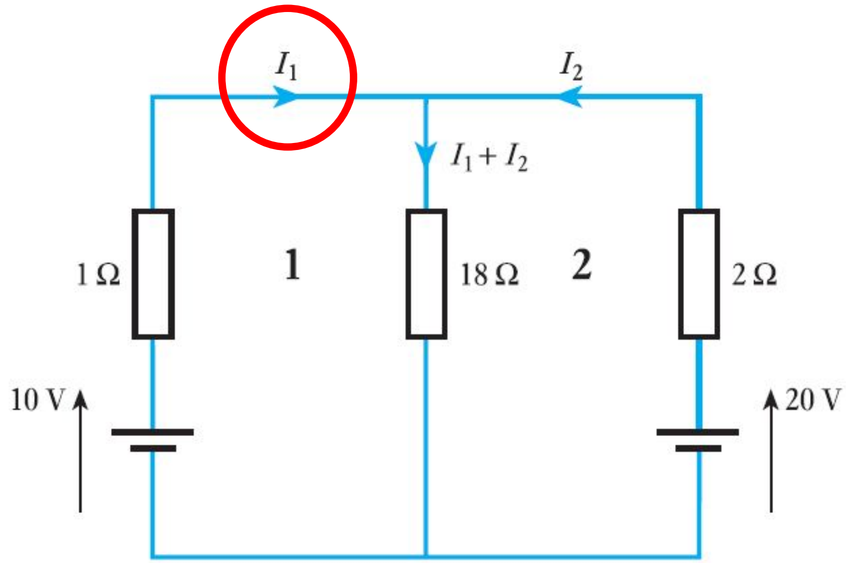
Substituting in (a)

$$10 = -54.34 + 18I_2$$

$$I_2 = 3.57\ \text{A}$$

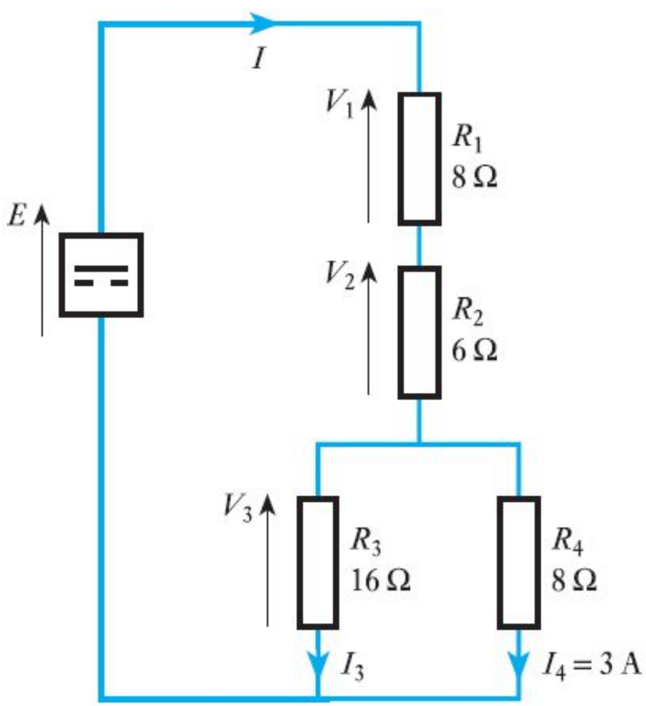
Current in 18 Ω resistor is

$$3.57 - 2.85 = 0.72\ \text{A}$$



Numerical 2

- For the network shown in Fig. determine the supply current and the source e.m.f.



Since R_3 and R_4 are in parallel

$$V_3 = I_4R_4 = 3 \times 8 = 24\text{ V} = I_3R_3 = I_3 \times 16$$

$$I_3 = 24/16 = 1.5\text{ A}$$

By Kirchhoff's first law

$$I = I_3 + I_4 = 1.5 + 3 = \mathbf{4.5\ A}$$

Also $V_1 = IR_1 = 4.5 \times 8 = 36$ V

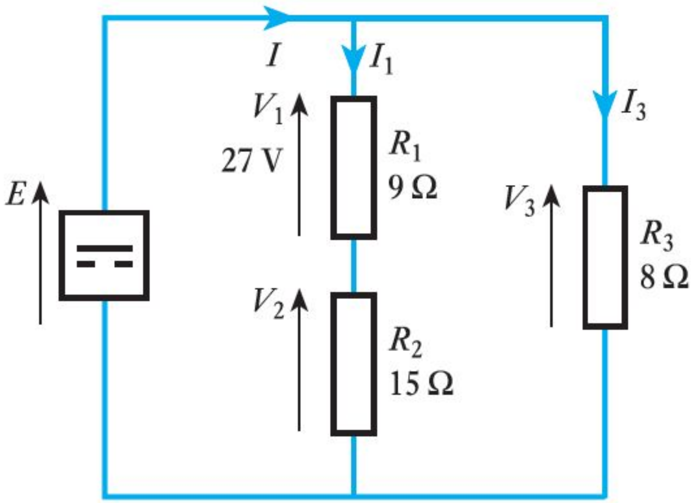
$$V_2 = IR_2 = 4.5 \times 6 = 27\text{ V}$$

By Kirchhoff's second law

$$E = V_1 + V_2 + V_3 = 36 + 27 + 24 = \mathbf{87\ V}$$

Numerical 3

- Given the network shown in Fig. determine I1, E, I3 and I.



$$I_1 = \frac{V_1}{R_1} = \frac{27}{9} = 3\ \text{A}$$

$$V_2 = I_1 R_2 = 3 \times 15 = 45\ \text{V}$$

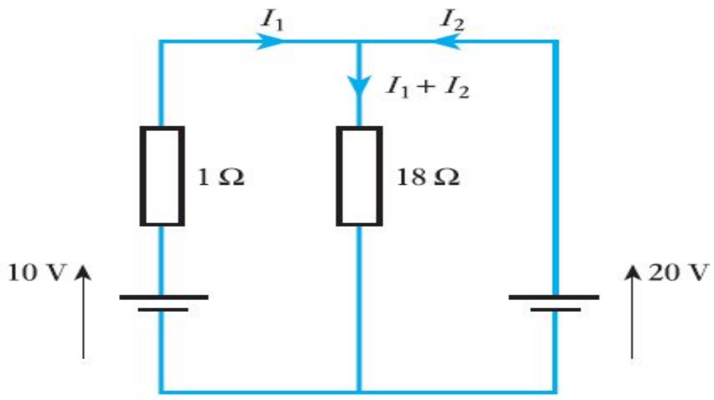
$$E = V = V_1 + V_2 = 27 + 45 = 72\ \text{V}$$

$$I_3 = \frac{V}{R_3} = \frac{72}{8} = 9\ \text{A}$$

$$I = I_1 + I_3 = 3 + 9 = 12\ \text{A}$$

Numerical 4

- Calculate the currents in the network shown in Fig.



Current in 18 Ω resistor is

$20/18 = 1.1$ A

Applying Kirchhoff's second law to the outside loop:

$20 - 10 = -I1 \times 1$

$I1 = -10$ A

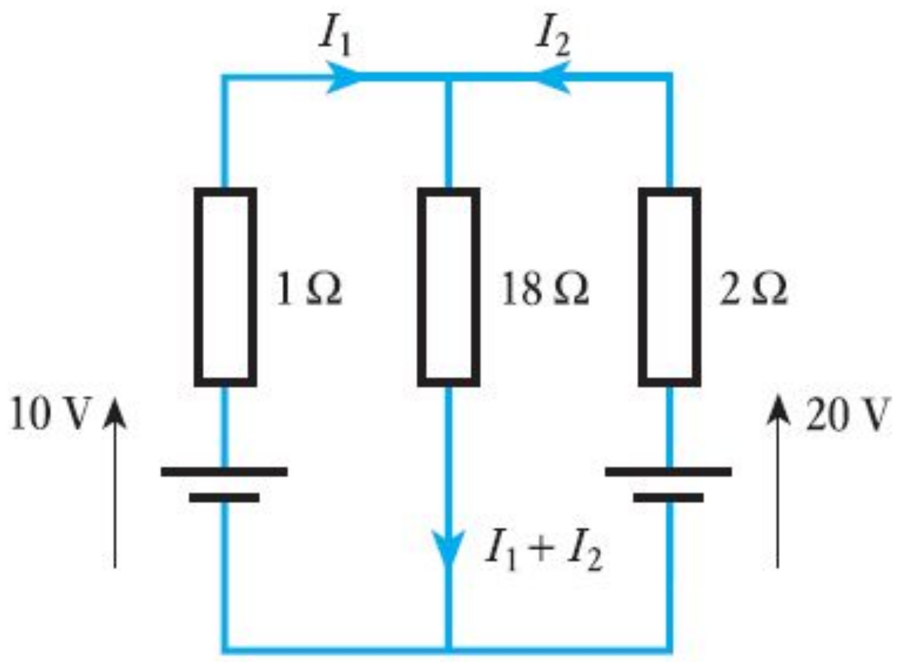
$I2 = -(-10) + 1.1 = 11.1$ A

Superposition theorem

- The Superposition theorem states that, the response in a particular branch of the network when multiple independent sources are acting at the same time is equivalent to the sum of the responses due to each independent source acting at a time.
- The current in, or the potential difference across, any branch can be found by considering each source separately and adding their effects
- Omitted sources of e.m.f. are replaced by resistances equal to their internal resistances.

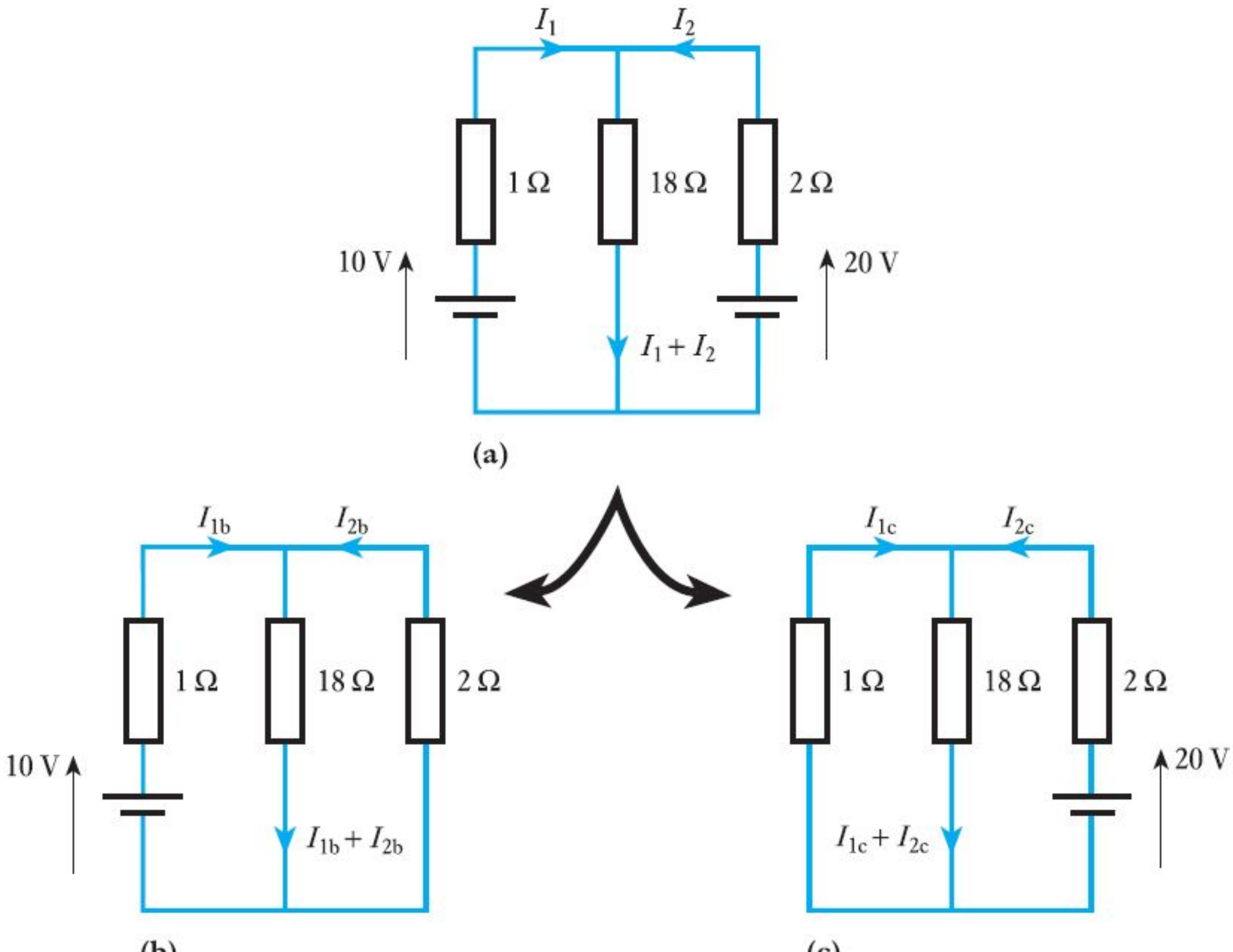
Superposition theorem

- By means of the Superposition theorem, calculate the currents in the network shown in Figure.



Superposition theorem

Because there are two sources of e.m.f. in the network, then two separate networks need to be considered, each having one source of e.m.f.



Superposition theorem

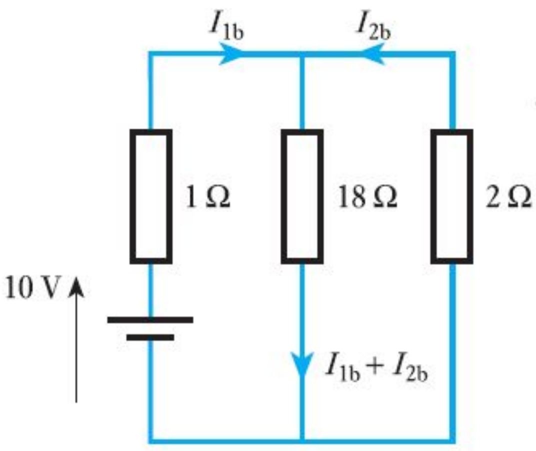
For the (b) arrangement, the total resistance is

$$1 + \frac{2 \times 18}{2 + 18} = 2.8\ \Omega$$

thus $\quad I_{1b} = \frac{10}{2.8} = 3.57\ \text{A}$

and $\quad I_{2b} = -\frac{18}{2 + 18} \times 3.57 = -3.21\ \text{A}$

also $I_{1b} + I_{2b} = 3.57 - 3.21 = 0.36\ \text{A}$



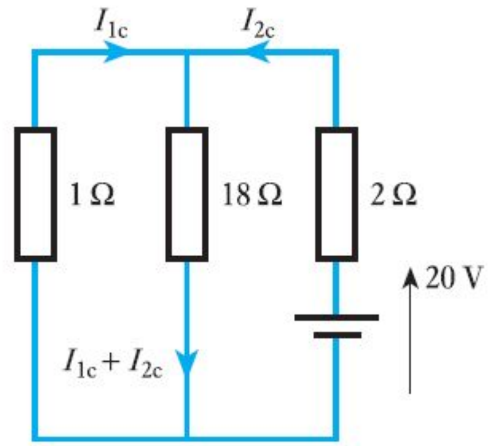
(b)

For the (c) arrangement, the total resistance is

$$2 + \frac{1 \times 18}{1 + 18} = 2.95\ \Omega$$

thus $\quad I_{2c} = \frac{20}{2.95} = 6.78\ \text{A}$

and $\quad I_{1c} = -\frac{18}{1 + 18} \times 6.78 = -6.42\ \text{A}$



(c)

Superposition theorem

$$I_{2c} + I_{1c} = 6.78 - 6.42 = 0.36\ \text{A}$$

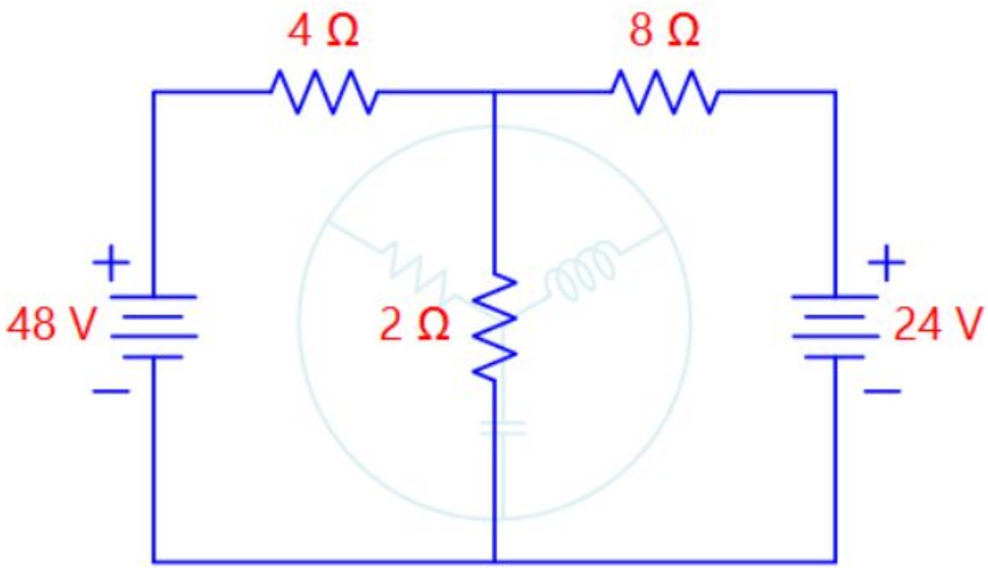
Thus $$I_1 = I_{1b} + I_{1c} = 3.57 - 6.42 = -2.85\ \text{A}$$

and $$I_2 = I_{2b} + I_{2c} = -3.21 + 6.78 = 3.57\ \text{A}$$

also $$I_1 + I_2 = -2.85 + 3.57 = 0.72\ \text{A}$$

Numerical 2

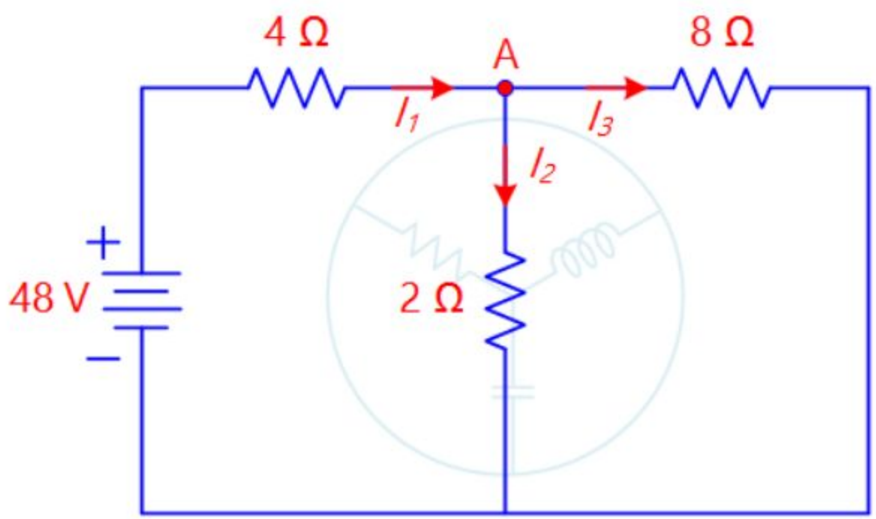
- Find the current through 2Ω resistor using superposition theorem in the given circuit



Numerical 2

Step 1

- At first, find the current through 2Ω resistor with 48V source acting alone. Hence replace the 24 V source by a short circuit.
- To find the current I2, find the total current supplied by the source (I1) with its total resistance. Then apply current division rule and find the current through 2Ω resistor with 48V source acting alone.
- Calculations for this step is as follows:



$$R_{eq} = 4 + \frac{8 \times 2}{8+2} = 5.6\ \Omega$$

$$\text{thus } I_1 = \frac{48}{5.6} = 8.57\text{ A}$$

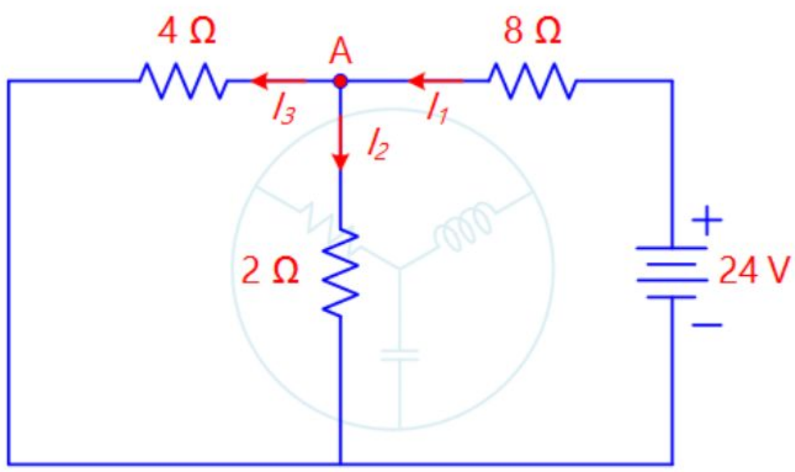
$$\text{and } I_2 = \frac{8}{8+2} \times 8.57 = 6.86\text{ A}$$

- Here, the current supplied by the 48V source is 6.86 Amperes.

Numerical 2

Step 2

- Now consider the 24V source alone and replace 48 V source by a short circuit.
- Now find the total resistance of the circuit and find the total current supplied by the source.
- Then apply current division rule at node ‘A’ and by find the current through 2 Ω resistor while 24V source acting alone.



$$R_{eq} = 8 + \frac{4 \times 2}{4+2} = 9.33\ \Omega$$

$$\text{thus} \quad I_1 = \frac{24}{9.33} = 2.57\text{ A}$$

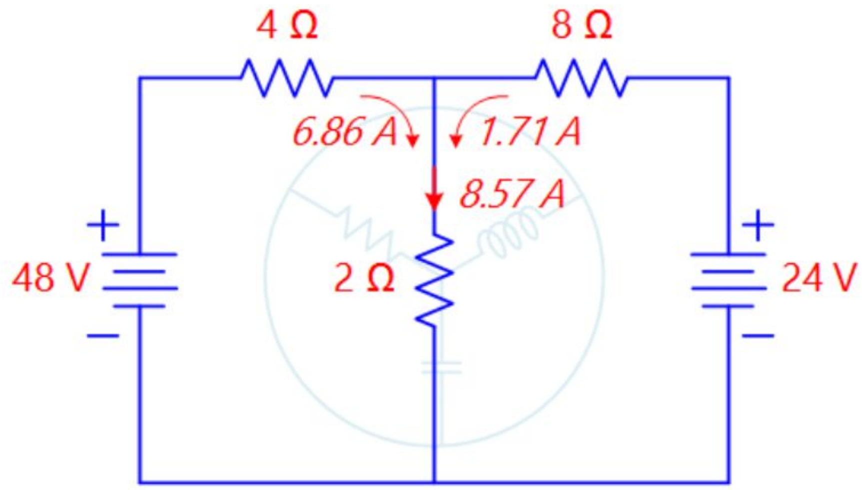
$$\text{and} \quad I_2 = \frac{4}{4+2} \times 2.57 = 1.71\text{ A}$$

- Here, the current supplied by the 24V source is 1.71 Amperes.

Numerical 2

Step 3

- Finally, add the two currents considering their direction.
- Here the two currents are flowing into the 2Ω resistor with the same direction. So the total current flowing through 2Ω will be the algebraic sum of I_2 due to 48V and I_2 due to 24V .

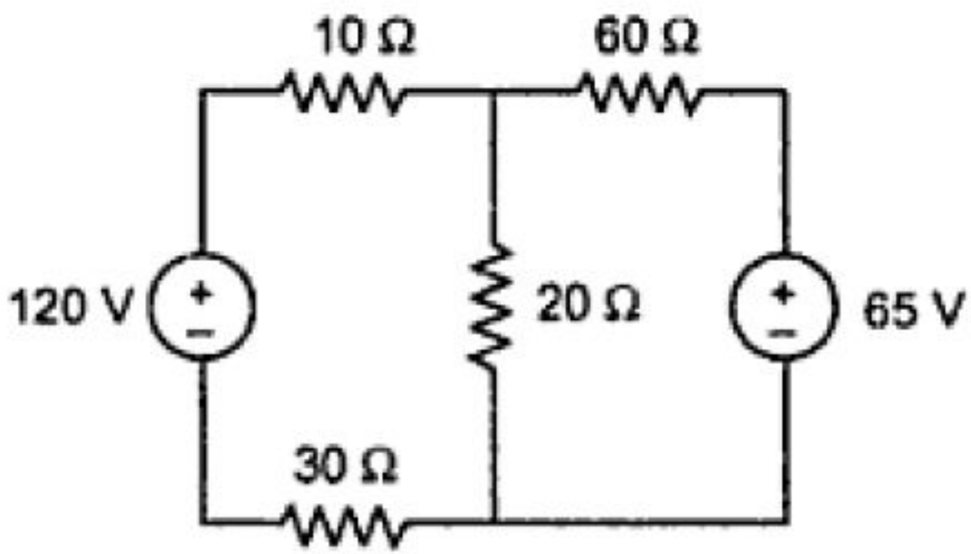


$$I_2 = I_{2(48V)} + I_{2(24V)} = 6.86 + 1.71 = 8.57\text{ A}$$

- Finally, the current through the 2Ω resistor is 8.57 amperes.

Numerical 3

- Find the current through 20Ω resistor using superposition theorem.



Numerical 3

Solution : Step 1 : Consider 120 V battery alone, shorting 65 V battery.

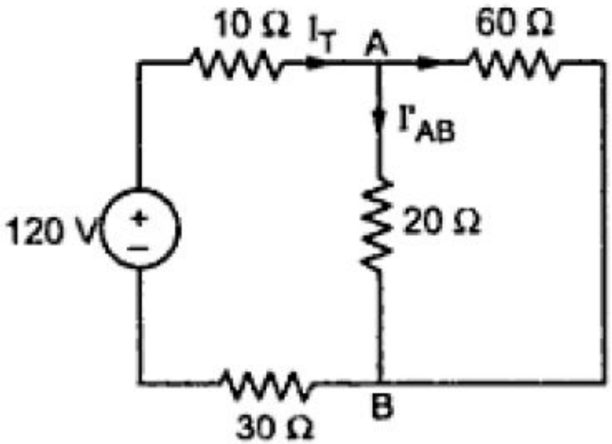


Fig. 2.66 (a)

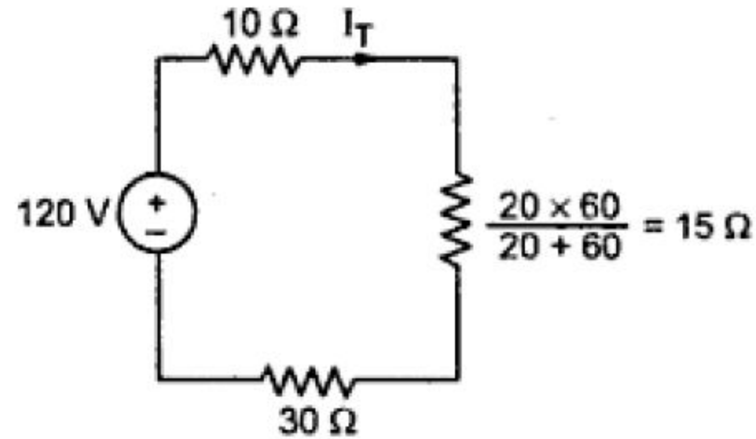


Fig. 2.66 (b)

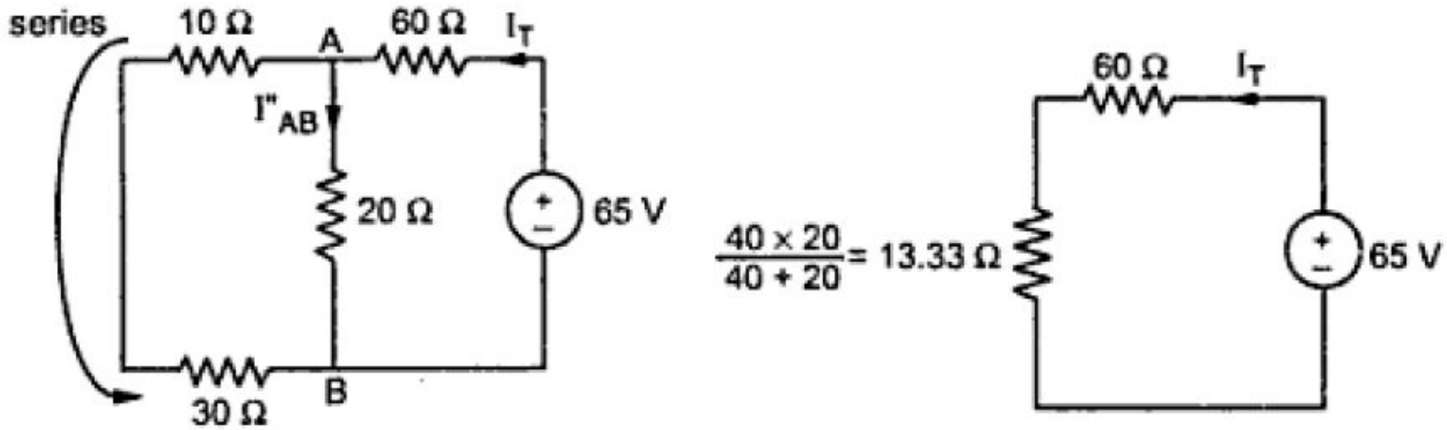
$$\therefore \quad I_T = \frac{120}{10+15+30} = 2.1818 \text{ A}$$

$$\therefore \quad I'_{AB} = I_T \times \frac{60}{20+60} \qquad \text{... Current division rule}$$

$$= \mathbf{1.6363 \text{ A}} \text{ due to 120 V battery} \downarrow$$

Numerical 3

Step 2 : Consider 65 V battery alone, shorting 120 V battery.



$$I_T = \frac{65}{73.33} = 0.8863\ \text{A}$$

$$\therefore \quad I''_{AB} = I_T \times \frac{40}{20+40} = \mathbf{0.5909\ A\ due\ to\ 65\ V\ battery} \downarrow$$

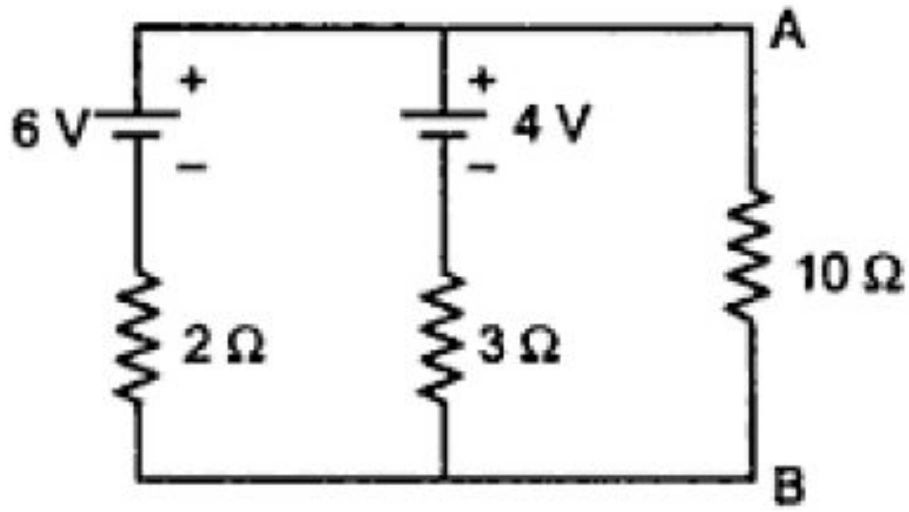
$\therefore$ Total current through 20 Ω resistance, according to Superposition theorem is,

$$I_{20\Omega} = 1.6363 + 0.5909 \text{ both in same direction}$$

$$= \mathbf{2.2272\ A} \downarrow$$

Numerical 4

- Find the current through branch AB using superposition theorem.



Numerical 4

Solution : Step 1 : Consider 6 V source alone

Now, resistances 10 Ω and 3 Ω are in parallel. Hence total current, I is

$$I = \frac{6}{2+(3||10)} = \frac{6}{2+\left(\frac{3\times10}{3+10}\right)} = \frac{6}{2+2.307}$$

$$\therefore \qquad I = 1.3928 \text{ A}$$

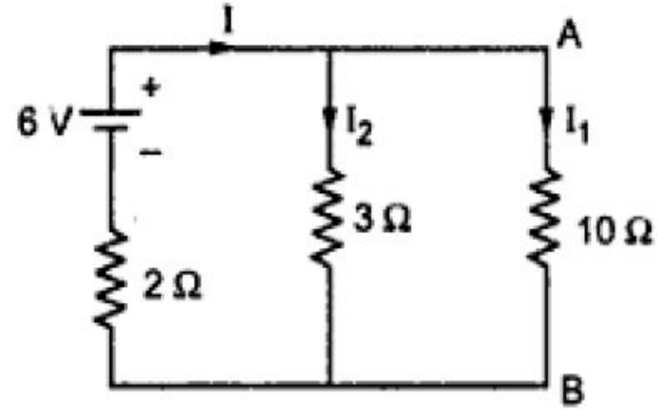


Fig. 2.67 (a)

As per current distribution in parallel branches,

$$I_1 = I\times\frac{3}{3+10} = \frac{1.3928\times3}{13} = 0.3214 \text{ A}\downarrow \quad \text{... (6 V alone)}$$

This is I_{AB} due to 6 V battery alone.

Numerical 4

Step 2 : Consider 4 V battery alone.

Now, the resistances 2 Ω and 10 Ω are in parallel. Hence, current I can be obtained as,

$$I = \frac{4}{3+(21||0)} = \frac{4}{3+\left(\frac{2\times 10}{2+10}\right)}$$

$$= \frac{4}{3+1.67} = 0.8571 \text{ A}$$

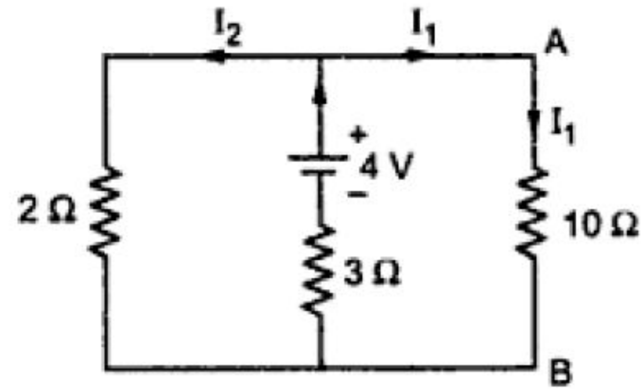


Fig. 2.67 (b)

According to current distribution in parallel branches,

$$I_1 = I \times \frac{2}{(2+10)} = 0.8571 \times \frac{2}{12}$$

$$= \mathbf{0.1428\ A} \downarrow \qquad \text{... (4 V alone)}$$

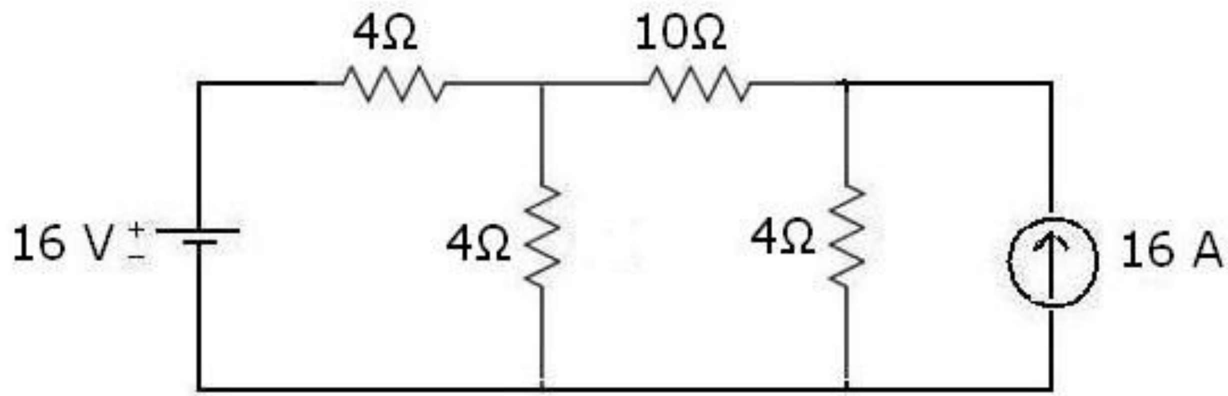
This is I_{AB} due to 4 V battery alone.

According to Superposition theorem,

$$\text{Total } I_{AB} = 0.3214 \text{ A} \downarrow + 0.1428 \text{ A} \downarrow = \mathbf{0.4642\ A} \downarrow$$

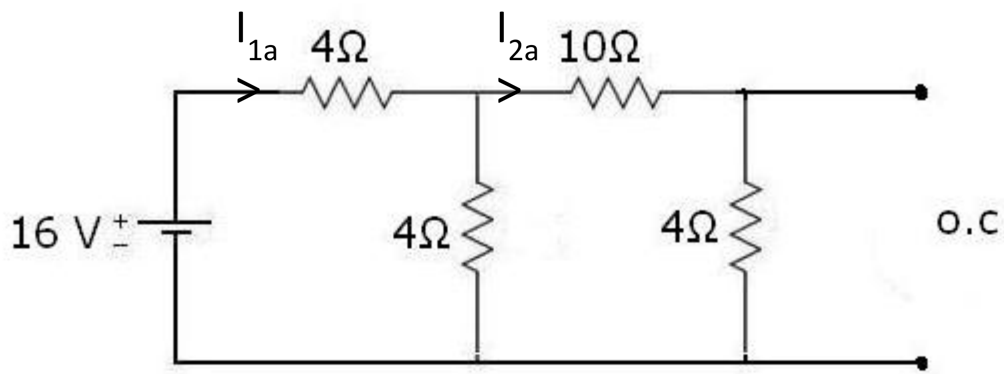
Numerical 5

- Find the current through 10 Ω resistance in the given network by using superposition theorem?



Numerical 5

Step 1: Activating '16V' source at a time, other will be deactivated.



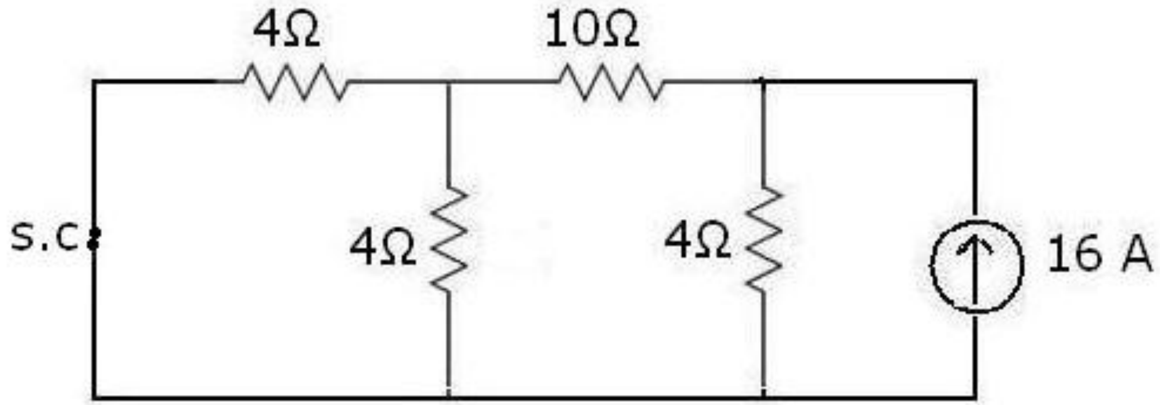
$$R_{eq} = 4 + \frac{4 \times 14}{4 + 14} = 7.11\ \Omega$$

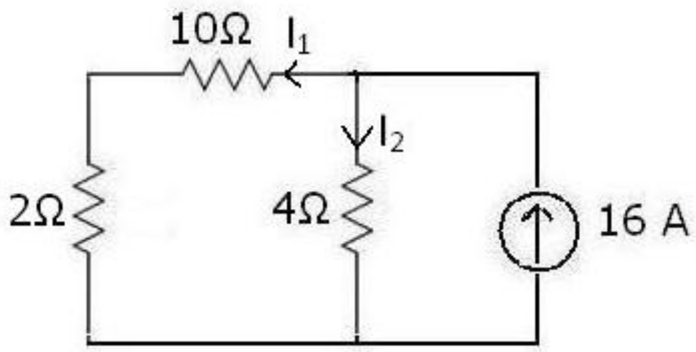
thus $I_{1a} = \frac{16}{7.11} = 2.25\text{ A}$

and $I_{2a} = \frac{4}{4 + 14} \times 2.25 = 0.5\text{ A}$

Numerical 5

Step 2:After deactivation of '16V' voltage source by short circui





By current division rule,

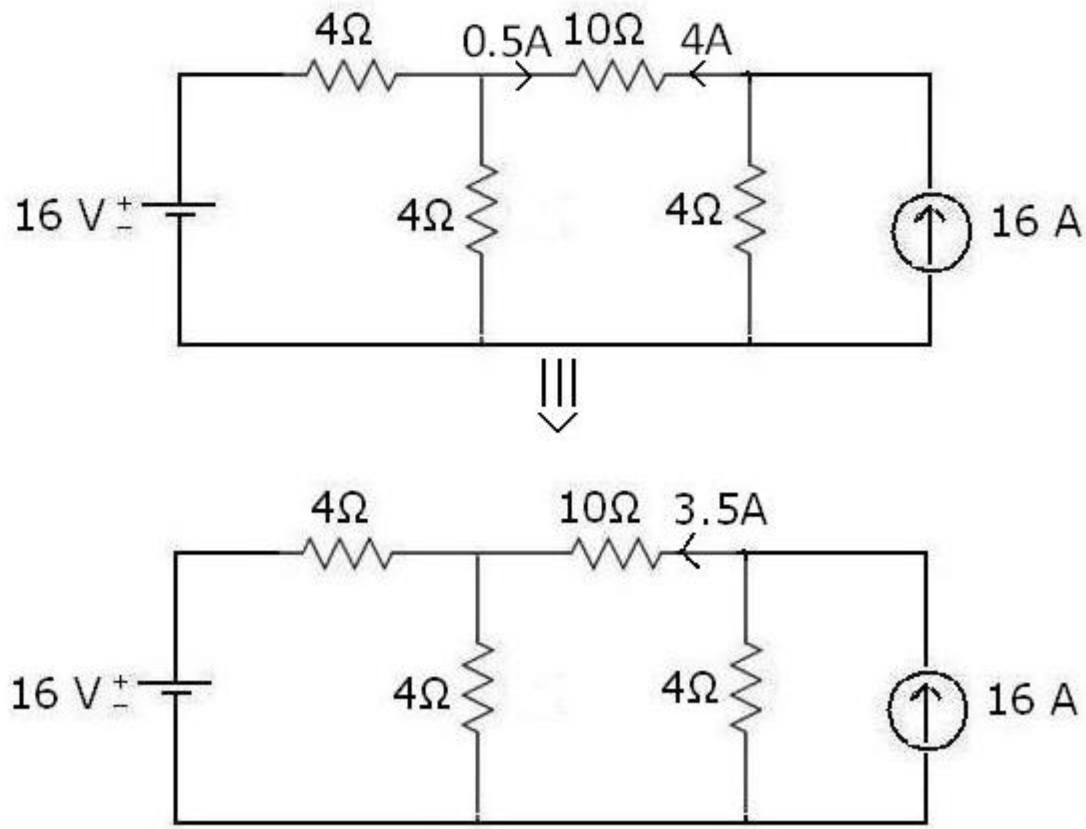
$$I_1 = \frac{4}{4+12} \times 16 = 4\ \text{A}$$

and $$I_2 = \frac{12}{4+12} \times 16 = 12\ \text{A}$$

So, current in 10Ω resistor- $I_1 = 4$ A

Numerical 5

- Direction of current when single source is active and other is deactivated through 10Ω resistance will be shown with their value, as shown below



- Finally, the current through 10Ω resistance is 3.5A

Thevenin’s theorem

- Thevenin’s Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load.

- Thevenin’s Theorem makes this easy by temporarily removing the load resistance from the original circuit and reducing what’s left to an equivalent circuit composed of a single voltage source and series resistance. The load resistance can then be re-connected to this “Thevenin equivalent circuit” and calculations carried out as if the whole network were nothing but a simple series circuit.

Thevenin's theorem

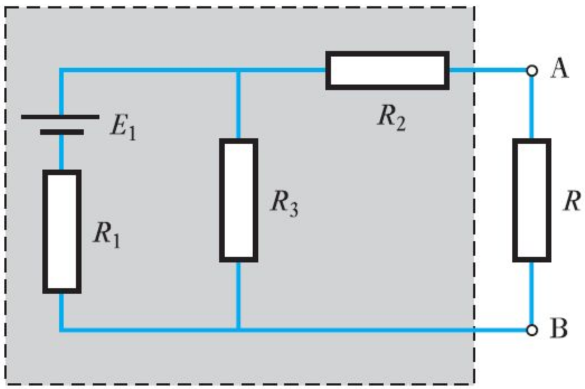
- The current through a resistor R connected across any two points A and B of an active linear network [i.e. a network containing one or more sources of e.m.f.] is obtained by dividing the potential difference between A and B, with R disconnected, by (R + r), where r is the resistance of the network measured between points A and B with R disconnected and the sources of e.m.f. replaced by their internal resistances.

OR

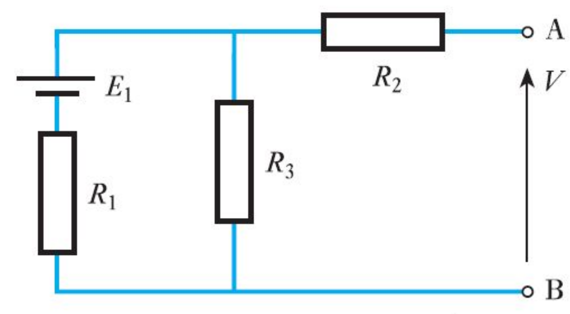
- An active network having two terminals A and B can be replaced by a constant-voltage source having an e.m.f. E and an internal resistance r.
- The value of E is equal to the open-circuit potential difference between A and B, and r is the resistance of the network measured between A and B with the load disconnected and the sources of e.m.f. replaced by their internal resistances.

Thevenin's theorem

- Suppose A and B in Fig. (a) to be the two terminals of a network consisting of resistors having resistances *R*2 and *R*3 and a battery having an e.m.f. *E*1 and an internal resistance *R*1. It is required to determine the current through a load of resistance *R* connected across AB. With the load disconnected as in Fig. (b),



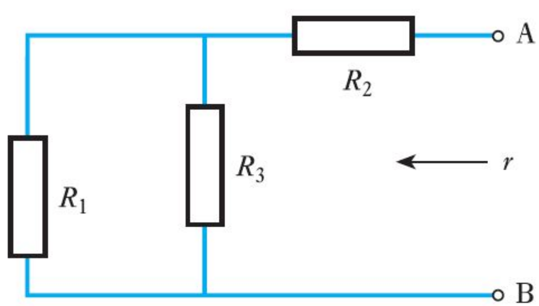
(a)



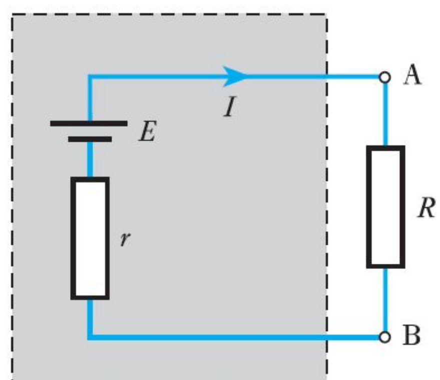
(b)

$$\text{Current through } R_3 = \frac{E_1}{R_1 + R_3}$$

$$\text{PD across } R_3 = \frac{E_1 R_3}{R_1 + R_3}$$



(c)



(d)

Thevenin’s theorem

Since there is no current through R2, potential difference across AB is

$$V = \frac{E_1 R_3}{R_1 + R_3}$$

Figure (c) shows the network with the load disconnected and the battery replaced by its internal resistance R1. Resistance of network between A and B is

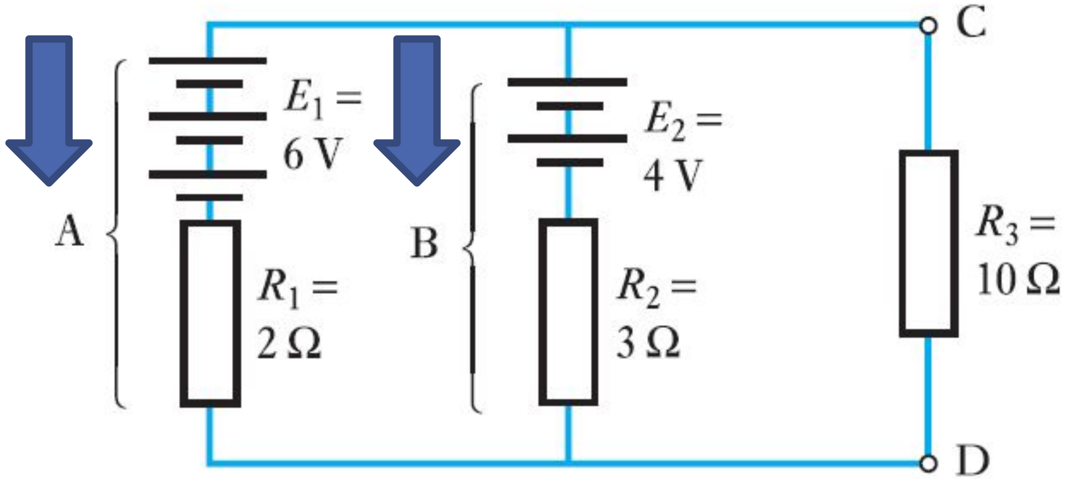
$$r = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Thevenin’s theorem merely states that the active network enclosed by the dotted line in Fig. (a) can be replaced by the very simple circuit enclosed by the dotted line in Fig. (d) and consisting of a source having an e.m.f. E equal to the open-circuit potential difference V between A and B, and an internal resistance r, where V and r have the values determined above. Hence

$$\text{Current through } R = I = \frac{E}{r + R}$$

Numerical 1
(Same as Numerical 4 with Superposition Thm, slide 54)

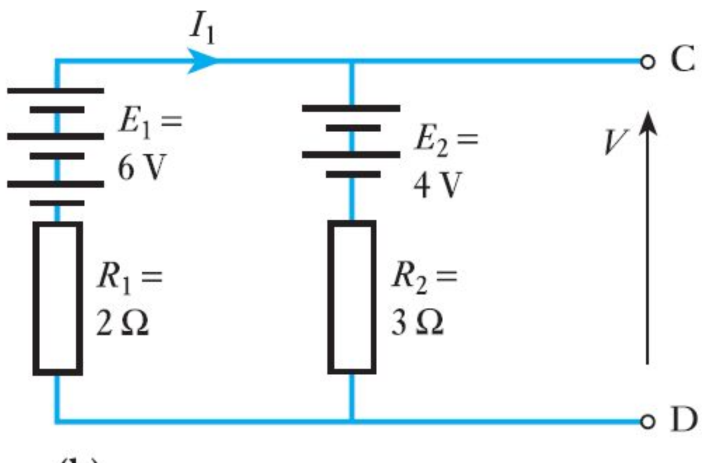
- In Fig. (a) C and D represent the two terminals of an active network. Calculate the current through R3 using Thevenin's theorem



(a)

Numerical 1 Solution- Step 1

- With R3 disconnected, as in Fig. (b),



(b)

E1-E2-I1*R2 – I1*R1=0

$$I_1 = \frac{6-4}{2+3} = 0.4\ \text{A}$$

and p.d. across CD is $E_1 - I_1R_1$,

i.e. $\quad E = 6 - (0.4 \times 2) = 5.2\ \text{V}$

OR $\quad$ E = E2 + (0.4 * 3) = 5.2V

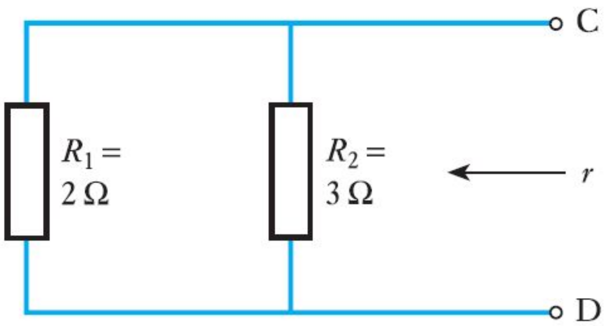
Numerical 1 Solution –Step 2

- When the e.m.f.s are removed, as in Fig. (c), total resistance between C and D is

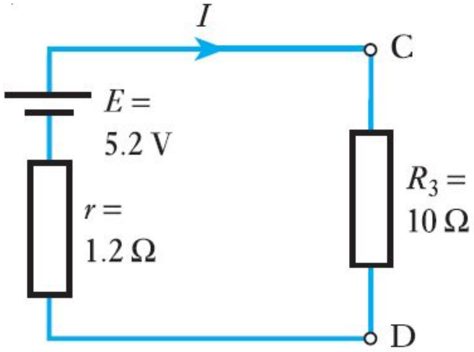
$$\frac{2 \times 3}{2 + 3}, \quad \text{i.e.} \quad r = 1.2\ \Omega$$

- Hence the network in Fig. (a) can be replaced by a single source having an e.m.f. of 5.2 V and an internal resistance of 1.2 Ω, as in Fig. (d); consequently the current thru' R3 is

$$I = \frac{5.2}{1.2 + 10} = \mathbf{0.46\ A}$$



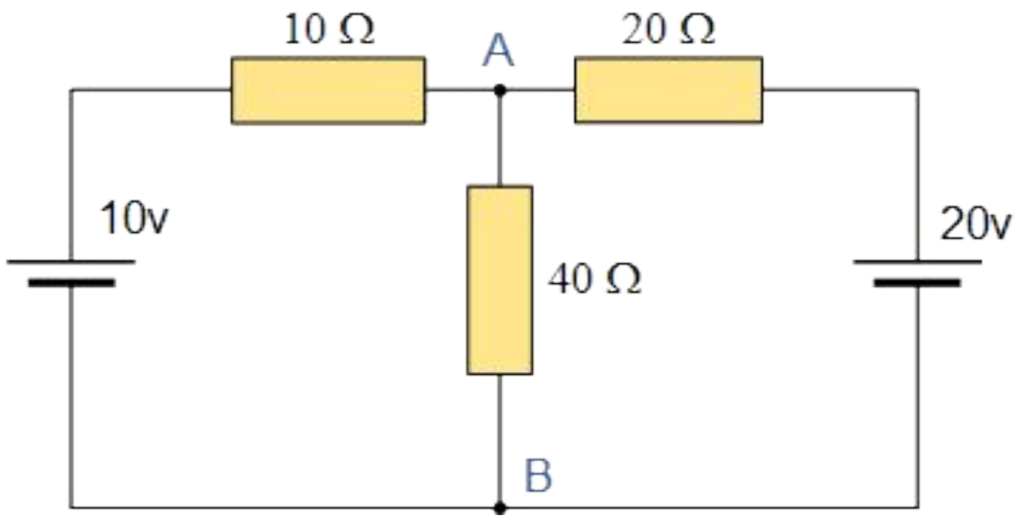
(c)



(d)

Numerical 2

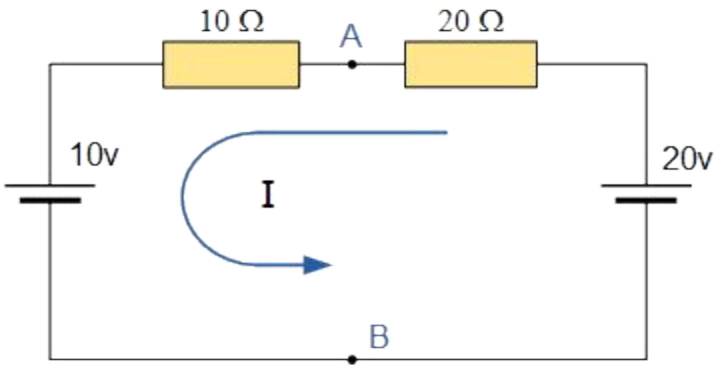
- Calculate the current through 40Ω resistor using Thevenin's theorem



Numerical 2 -Solution- step 1

Find the Equivalent Voltage:

- Remove the center 40Ω load resistor connected across the terminals A-B



Applying KVL

20- I* 20 – I * 10 – 10 =0

$$I = \frac{V}{R} = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\ amps$$

- The voltage drop across the 20Ω resistor or the 10Ω resistor can be calculated as:

VAB = 20 – (20Ω x 0.33amps) = 13.33 volts.

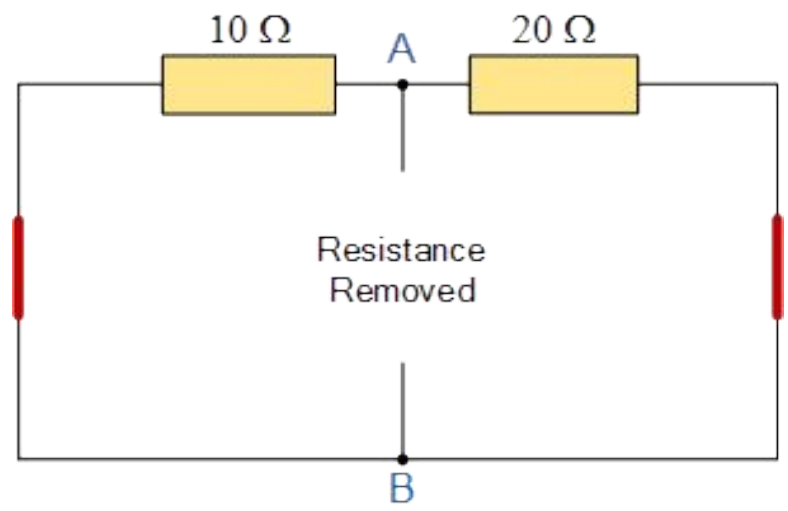
or

VAB = 10 + (10Ω x 0.33amps) = 13.33 volts, the same.

Numerical 2 Solution- Step 2

Find the Equivalent Resistance:

- Remove any internal resistance associated with the voltage source(s).

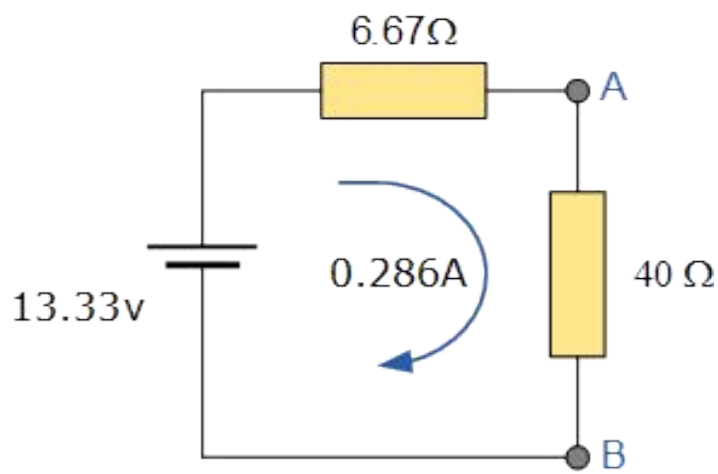


10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

Numerical 2 Solution- Step 3

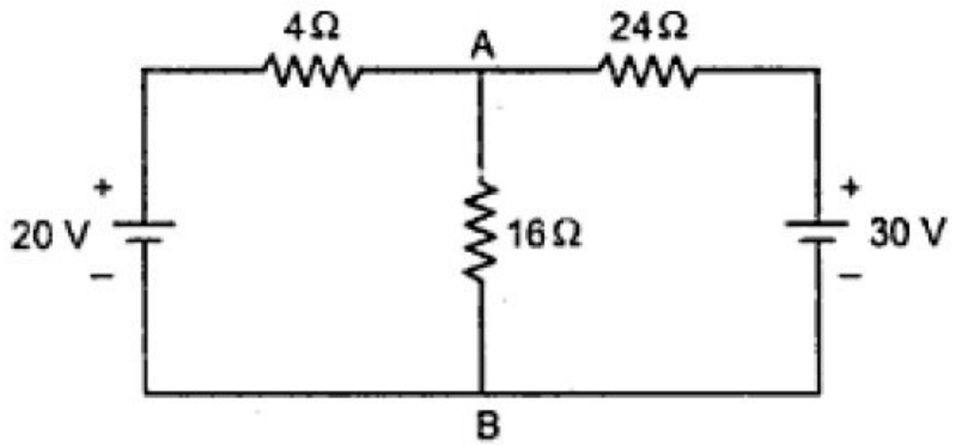
- Then the Thevenin's Equivalent circuit would consist or a series resistance of 6.67Ω and a voltage source of 13.33 V. With the 40Ω resistor connected back into the circuit we get:



$$I = \frac{V}{R} = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286\ amps$$

Numerical 3

- Calculate the current through 16Ω resistor using Thevenin's theorem



Numerical 3- Solution

Solution : Step 1 : Remove 16 Ω resistance.

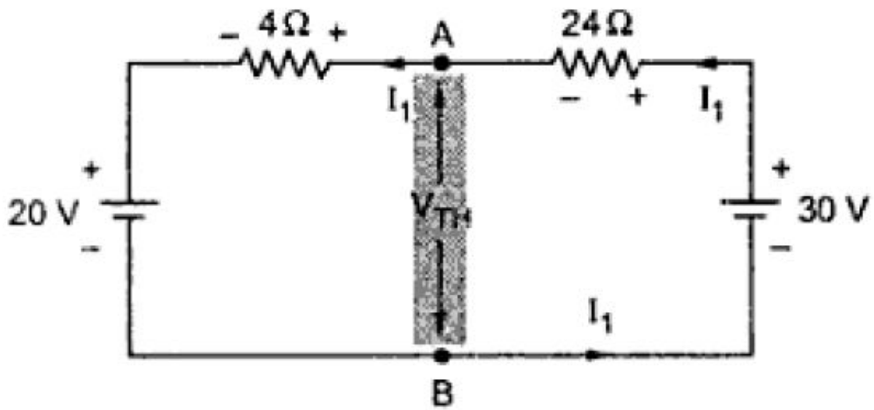


Fig. 2.70 (a)

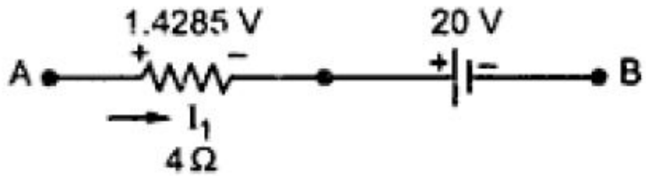


Fig. 2.70 (b)

Step 2 : Find open circuit voltage V_{TH}.

$$\therefore -24\ I_1 - 4I_1 - 20 + 30 = 0$$

$$\therefore\ 28\ I_1 = 10$$

$$\therefore\quad I_1 = \frac{10}{28}\ A$$

$$\therefore \text{Drop across } 4\ \Omega \text{ is } = \frac{10}{28} \times 4$$

$$= 1.4285\ V$$

Trace the path from A to B and arrange the voltage drops as shown in the Fig. 2.70 (b).

$$\therefore\ V_{AB} = V_{TH} = 20 + 1.4285$$

$$= \mathbf{21.4285\ V\ with\ A\ positive}$$

Numerical 3 Solution

Step 3 : Calculate R_{eq} shorting both the voltage sources.

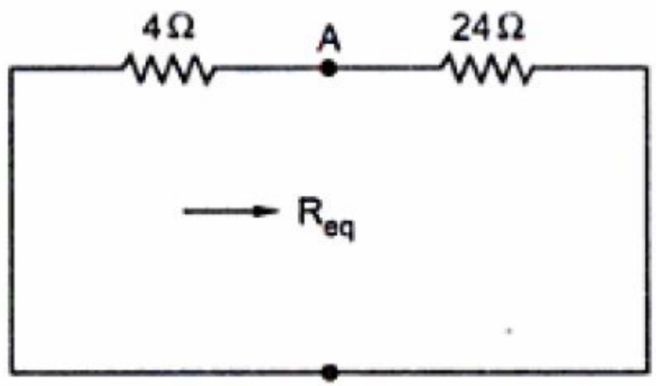


Fig. 2.70 (c)

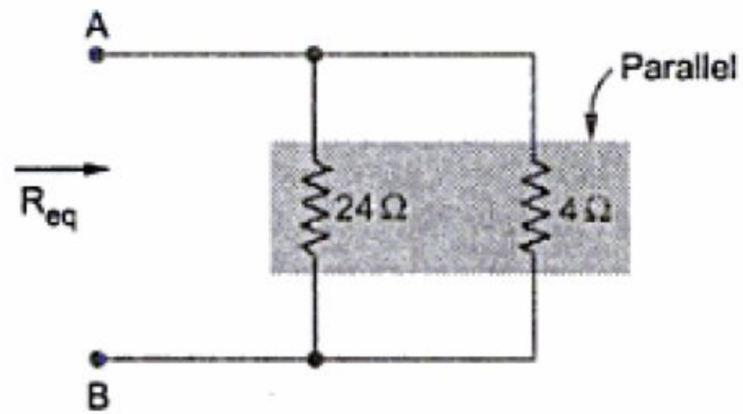
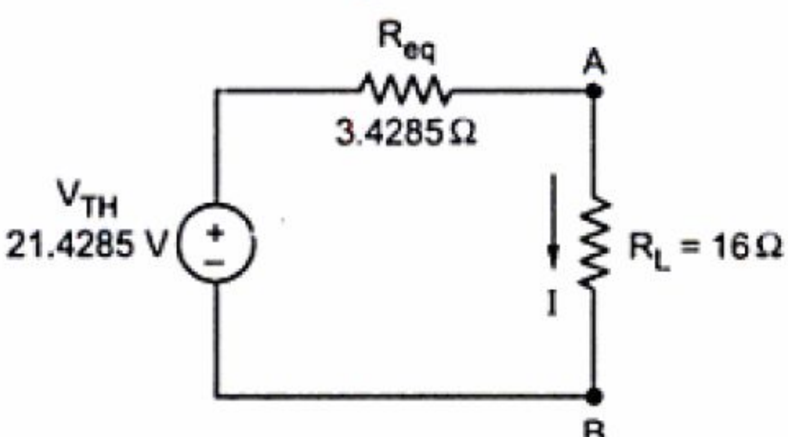


Fig. 2.70 (d)

$$\therefore \quad R_{eq} = R_{AB} = 24 \parallel 4 = 3.4285\ \Omega$$



Step 4 : Thevenin's equivalent is shown in the Fig. 2.70 (e).

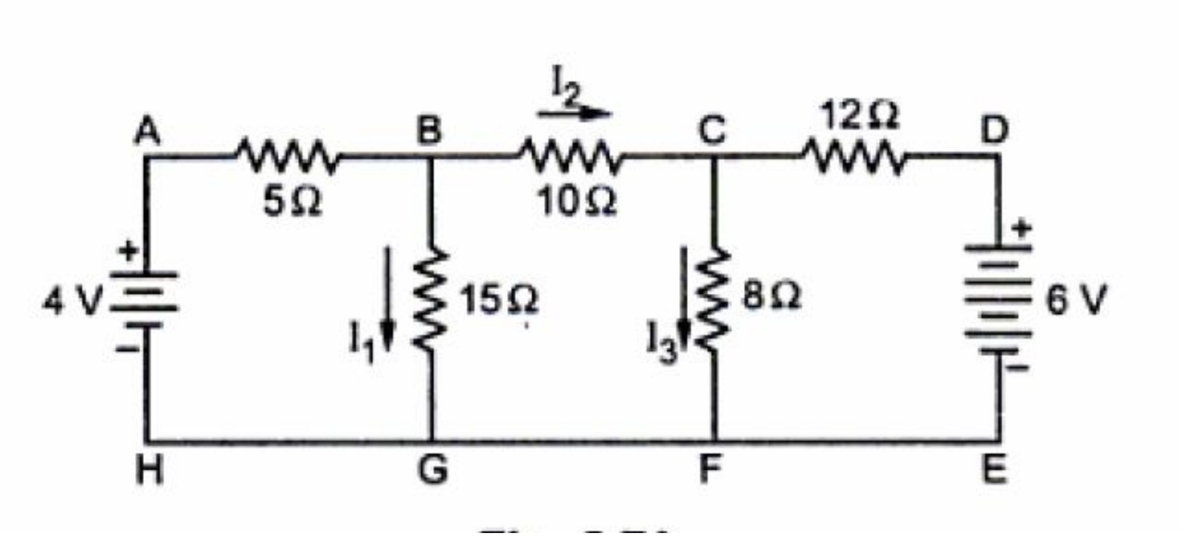
Step 5 : Hence current I is,

$$I = \frac{V_{TH}}{R_{eq} + R_L} = \frac{21.4285}{3.4285 + 16}$$

$$= 1.1029\ A \downarrow$$

Numerical 4

- Calculate the current through 10Ω resistor using Thevenin's theorem



Numerical 4 Solution

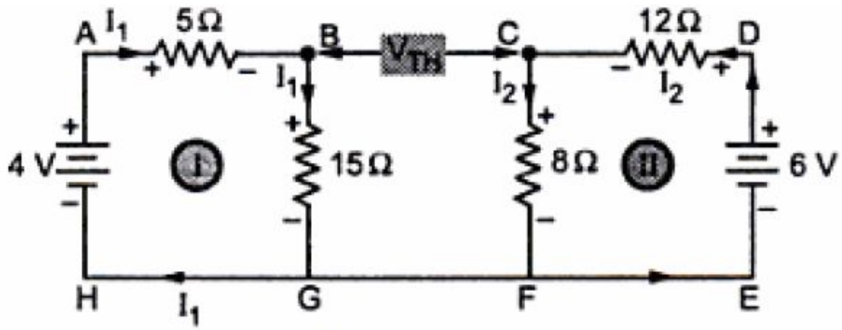


Fig. 2.71 (a)

Applying KVL to the two loops,

$$-5I_1 - 15I_1 + 4 = 0 \quad \text{i.e.} \quad I_1 = \frac{4}{20} = 0.2 \text{ A}$$

$$\therefore \qquad \text{Drop } V_{BG} = 15 \times 0.2 = 3 \text{ V}$$

$$-12I_2 - 8I_2 + 6 = 0 \quad \text{i.e.} \quad I_2 = \frac{6}{20} = 0.3 \text{ A}$$

$$\therefore \qquad \text{Drop } V_{CF} = 8 \times 0.3 = 2.4 \text{ V}$$

Trace path from B to C as shown in the Fig. 2.71 (b), showing drops with proper polarities.

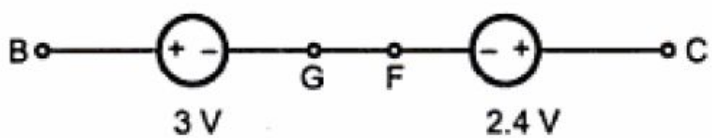


Fig. 2.71 (b)

Both drops are in opposite direction.

$$\therefore \qquad V_{TH} = V_{BC} = 3 - 2.4 = 0.6 \text{ V with B positive}$$

Numerical 4

Step 3 : Find $R_{eq} = R_{BC}$ with voltage sources replaced by short circuits.

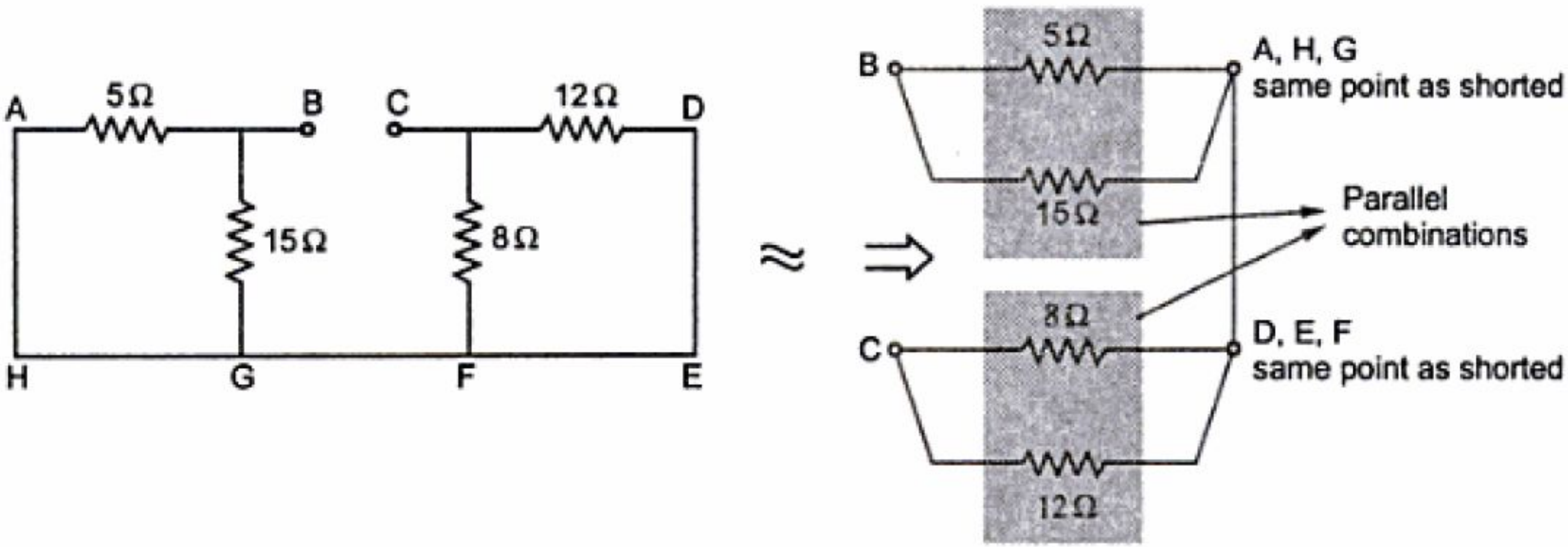


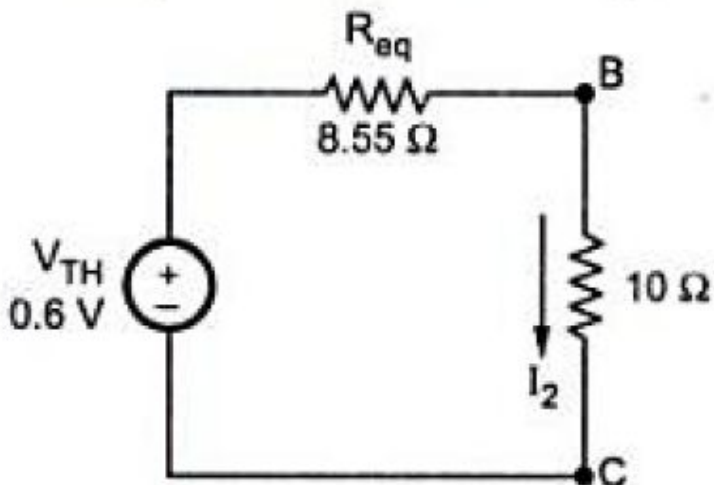
Fig. 2.71 (c)

$$\therefore \quad R_{BC} = (5 \parallel 15) + (8 \parallel 12) = 8.55\ \text{W} = R_{eq}$$

Step 4 : Thevenin's equivalent is shown in the Fig. 2.71 (d).

Numerical 4 Solution

Step 5 : Hence current I_2 is,



$$I_2 = \frac{V_{TH}}{R_{eq}+10} = \frac{0.6}{8.55+10}$$

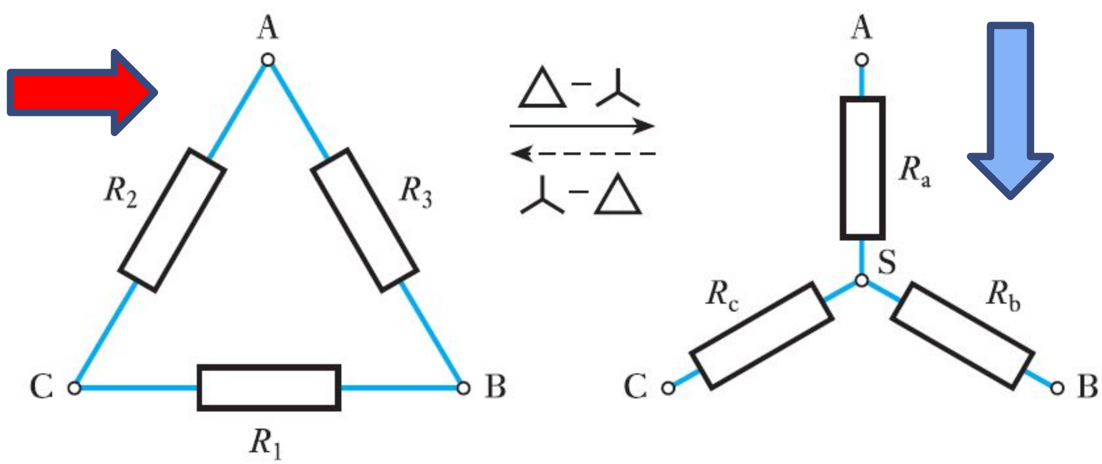
$$= 0.032345\ \text{A}$$

$$= \mathbf{32.345\ mA}$$

Delta–star transformation

- Figure (a) shows three resistors R1, R2 and R3 connected in a **closed mesh or delta** to three terminals A, B and C, their numerical subscripts 1, 2 and 3 being opposite to the terminals A, B and C respectively.

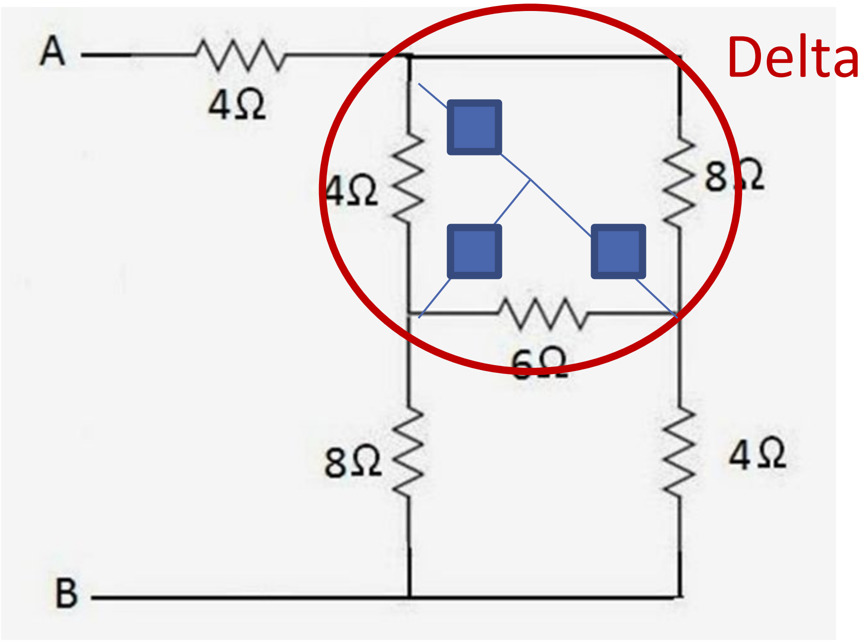
- It is possible to replace these delta-connected resistors by three resistors Ra, Rb and Rc connected respectively between the same terminals A, B and C and a common point S, as in Fig. (b). Such an arrangement is said to be **star-connected**.



(a) (b)

Why?

Calculation of equivalent resistance using delta to star (Wye) conversion



Delta–star transformation

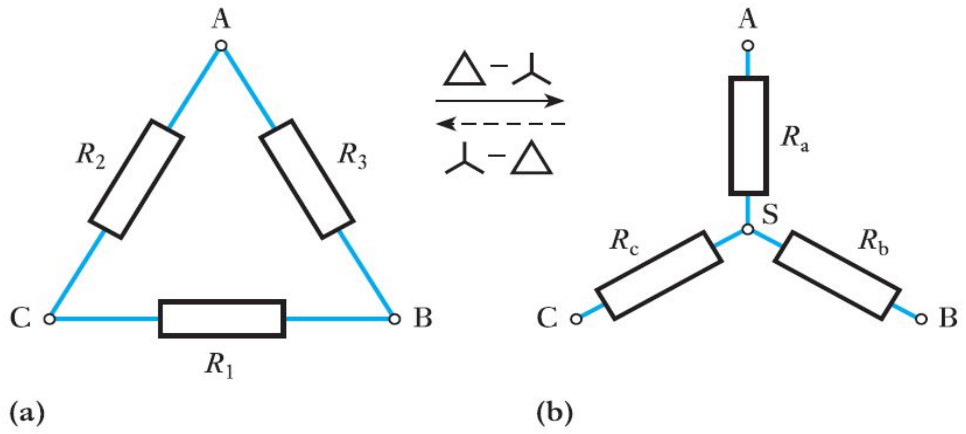
For Fig. (a), we have

$$R_{AB} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \qquad \text{----------- [1]}$$

For Fig. (b), we have

$$R_{AB} = R_a + R_b \qquad \text{----------- [2]}$$

In order that the networks of Fig. (a) and (b) may be equivalent to each other, the values of R_{AB} represented by expressions [1] and [2] must be equal.



Delta–star transformation

$$\therefore \quad R_a + R_b = \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3} \quad \text{----------- [3]}$$

Similarly

$$R_b + R_c = \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} \quad \text{----------- [4]}$$

and

$$R_a + R_c = \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3} \quad \text{----------- [5]}$$

Subtracting equation [4] from [3], we have

$$R_a - R_c = \frac{R_2R_3 - R_1R_2}{R_1 + R_2 + R_3} \quad \text{------- [6]}$$

Delta–star transformation

Adding equations [5] and [6] and dividing by 2, we have

$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3} \quad \text{----------- [7]}$$

Similarly

$$R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3} \quad \text{----------- [8]}$$

and

$$R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3} \quad \text{----------- [9]}$$

These relationships may be expressed thus: the equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.

Star–delta transformation

- Let us next consider how to replace the star-connected network of Fig. (b) by the equivalent delta-connected network of Fig. (a).

Dividing equation [7] by equation [8], we have

$$\frac{R_a}{R_b} = \frac{R_2}{R_1}$$

$$\therefore \qquad R_2 = \frac{R_1 R_a}{R_b}$$

Similarly, dividing equation [7] by equation [9], we have

$$\frac{R_a}{R_c} = \frac{R_3}{R_1}$$

$$\therefore \qquad R_3 = \frac{R_1 R_a}{R_c}$$

Star–delta transformation

Substituting for R2 and R3 in equation [7], we have

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a}$$ -- [10]

Similarly

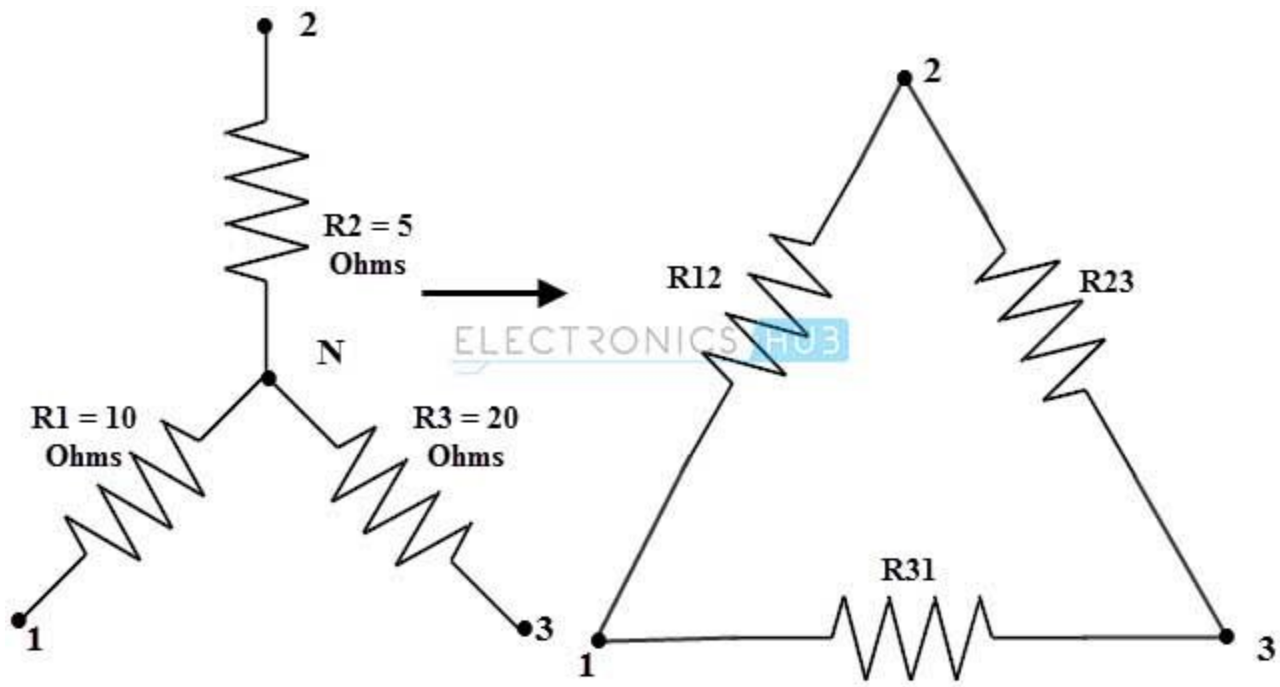
$$R_2 = R_c + R_a + \frac{R_c R_a}{R_b}$$ -- [11]

and

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c}$$ -- [12]

These relationships may be expressed thus as: the equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same two star resistances divided by the third star resistance.

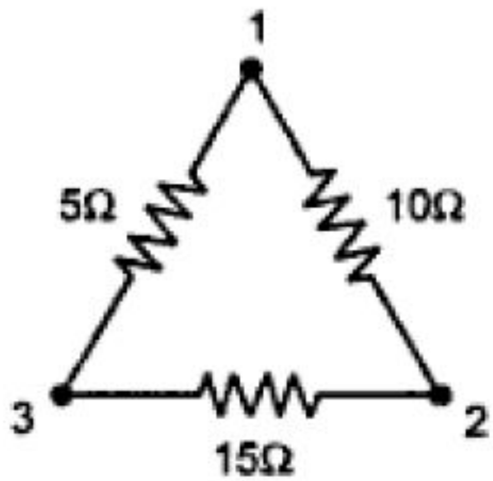
Star to Delta Transformation



R12= 17.5 Ohms
R31=70 Ohms
R23=35 ohms

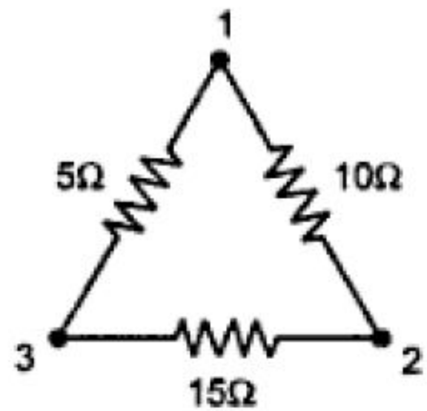
Numerical 1

- Convert given delta into star



Numerical 1-Solution

- Convert given delta into star



Solution : Its equivalent star is as shown in the Fig. 2.47.

where

$$R_1 = \frac{10\times5}{5+10+15} = 1.67\ \Omega$$

$$R_2 = \frac{15\times10}{5+10+15} = 5\ \Omega$$

$$R_3 = \frac{5\times15}{5+10+15} = 2.5\ \Omega$$

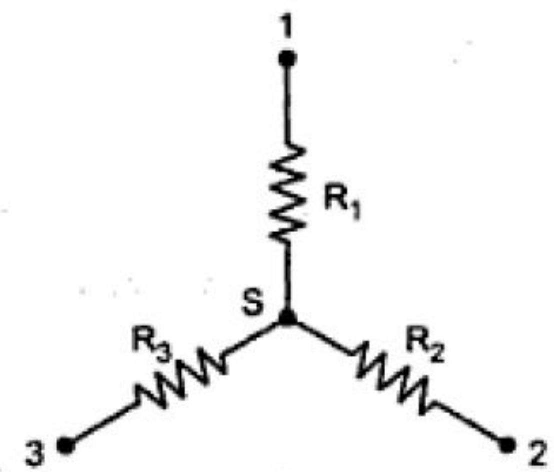
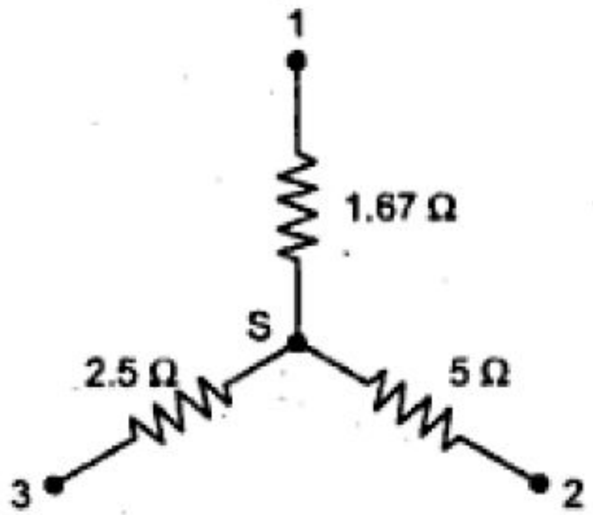


Fig. 2.47

Numerical 2

- Convert given star into its equivalent Delta



Numerical 2

- Convert given star into its equivalent Delta

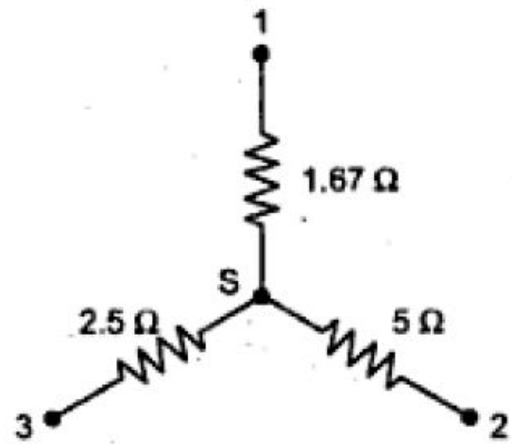
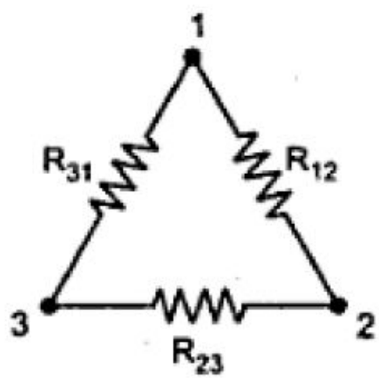


Fig. 2.48

Solution : Its equivalent delta is as shown in the Fig. 2.48 (a).



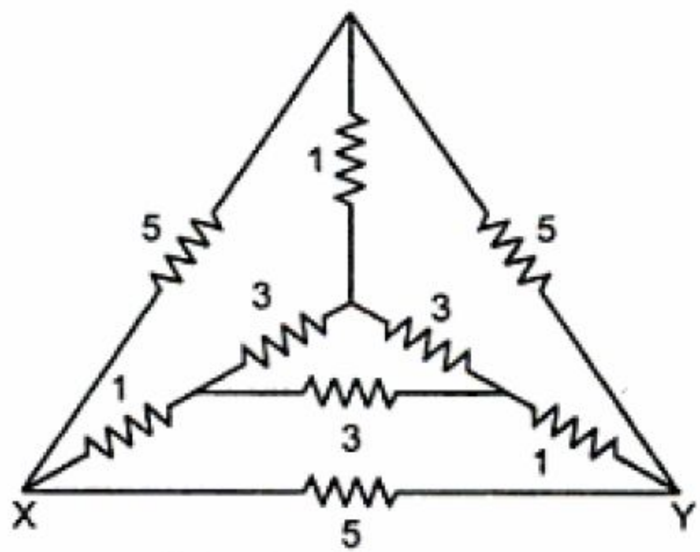
$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10\ \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15\ \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833 = 5\ \Omega$$

Numerical 3

- Determine the resistance between the terminals X and Y for the circuit shown below.



(All resistances in ohm)

Numerical 3 Solution

Solution : Converting inner delta to star.

$$\text{Each resistance} = \frac{3\times 3}{3+3+3} = 1\Omega$$

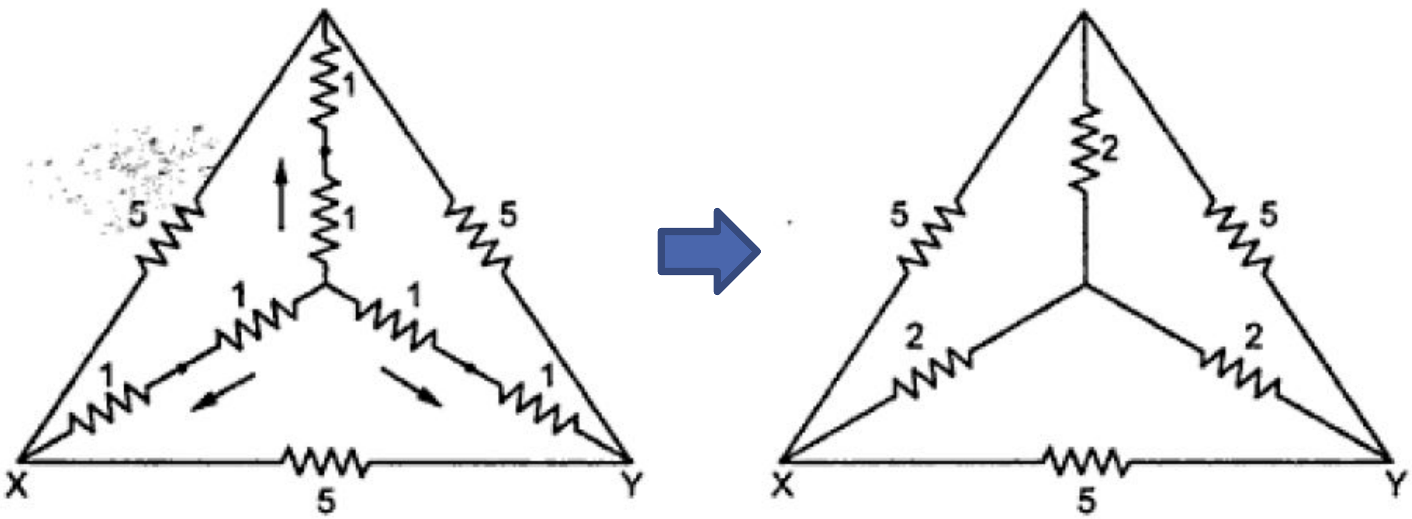
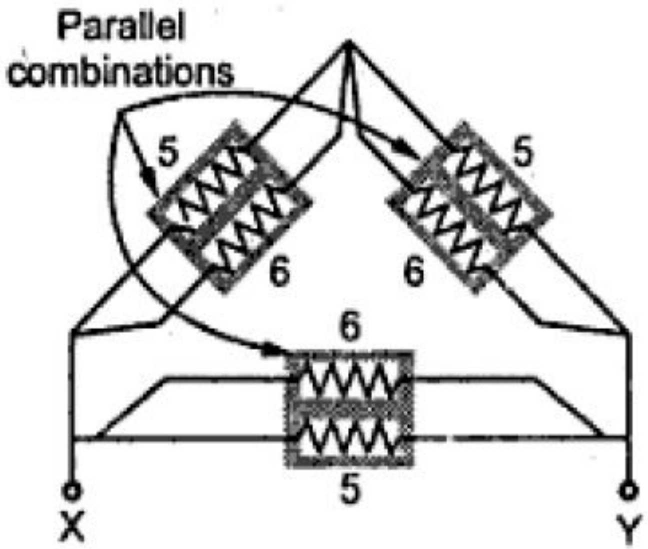


Fig. 2.82 (a)

Converting inner star to delta.

$$\text{Each resistance} = 2+2+\frac{2\times 2}{2} = 6\ \Omega$$

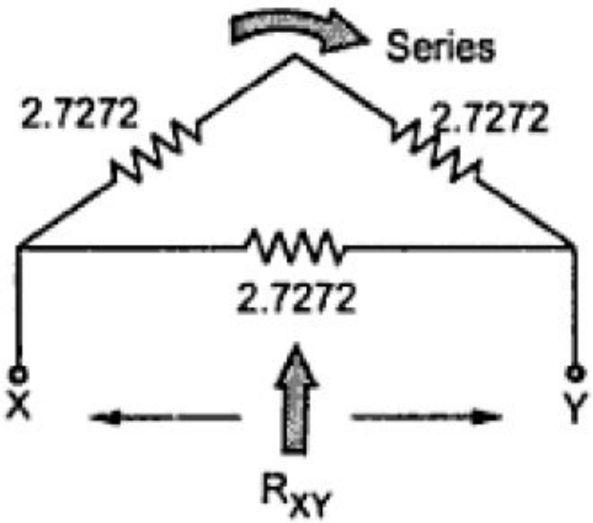
Numerical 3 Solution



All three parallel combinations,

$$5||6 = \frac{5\times 6}{5+6}$$

$$= 2.7272\ \Omega$$



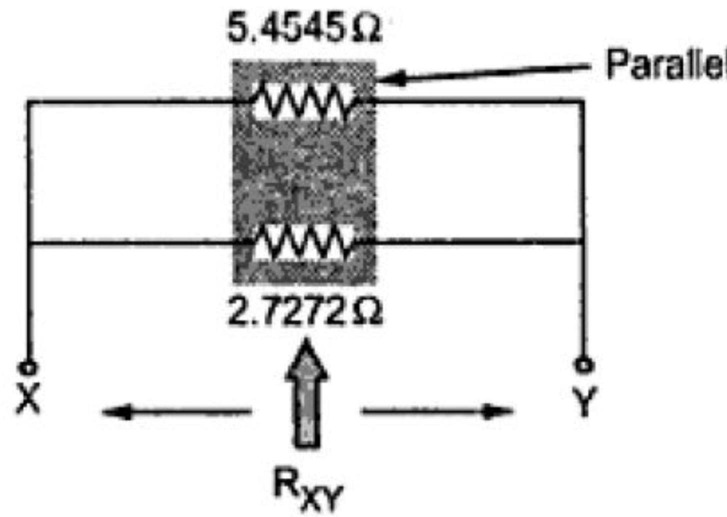
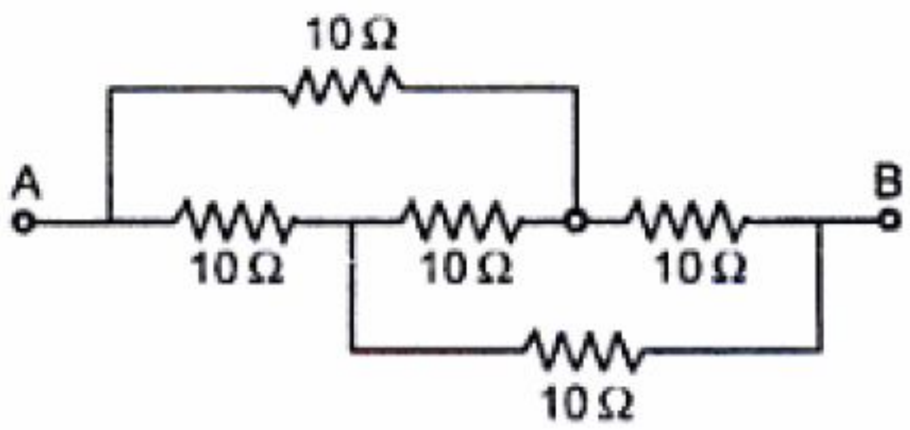


Fig. 2.82 (b)

$$\therefore \quad R_{XY} = 5.4545||2.7272 = \mathbf{1.8181\ \Omega.}$$

Numerical 4

- Calculate resistance between terminal AB



Numerical 4 Solution

Solution : Refer Fig. 2.81 (a),

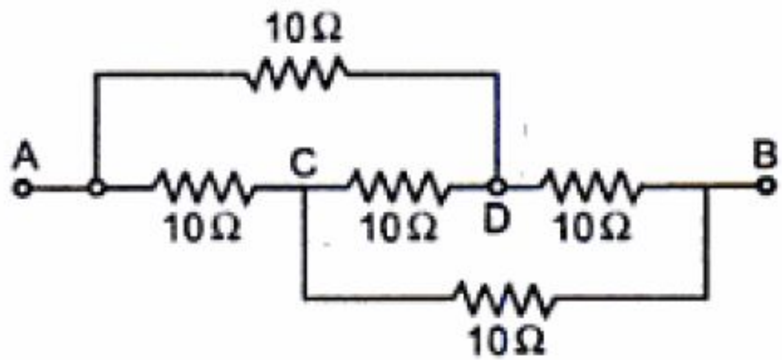
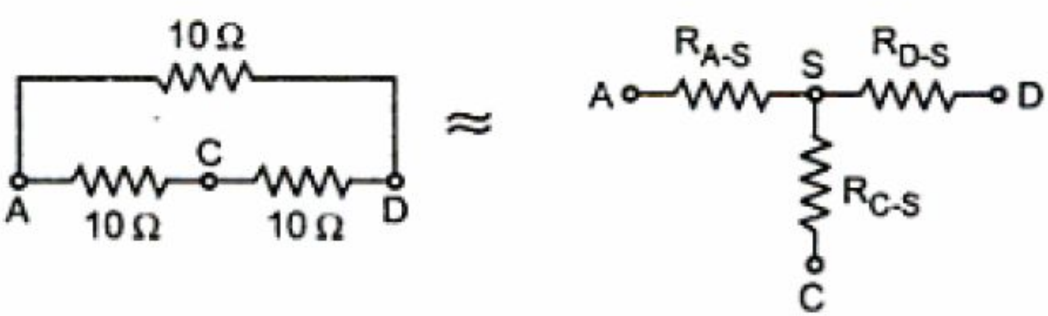


Fig. 2.81 (a)

Loop A-C-D forms Δ converting to Star,



$$R_{AS} = \frac{10 \times 10}{10+10+10}$$

$$= 3.33\ \Omega$$

$$= R_{CS} = R_{DS}$$

Numerical 4 Solution

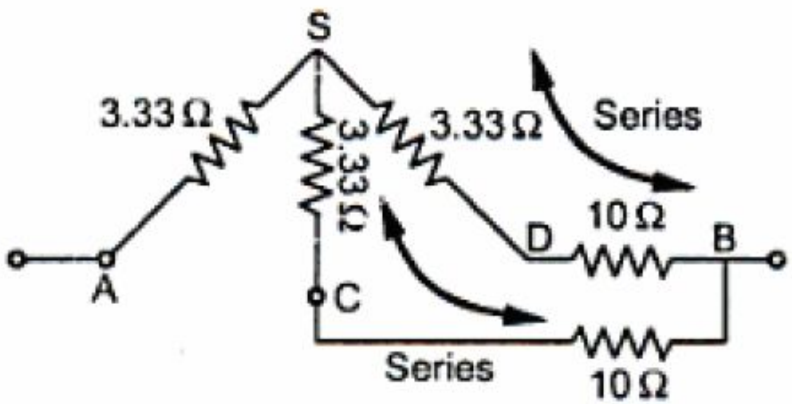


Fig. 2.81 (c)

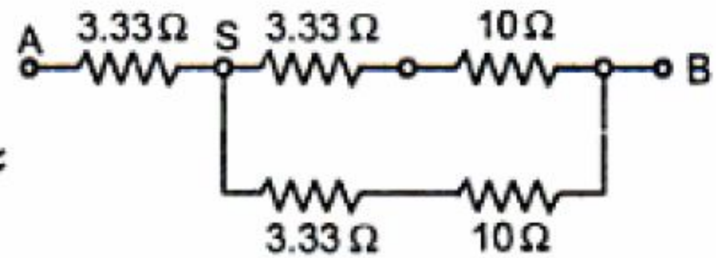


Fig. 2.81 (d)

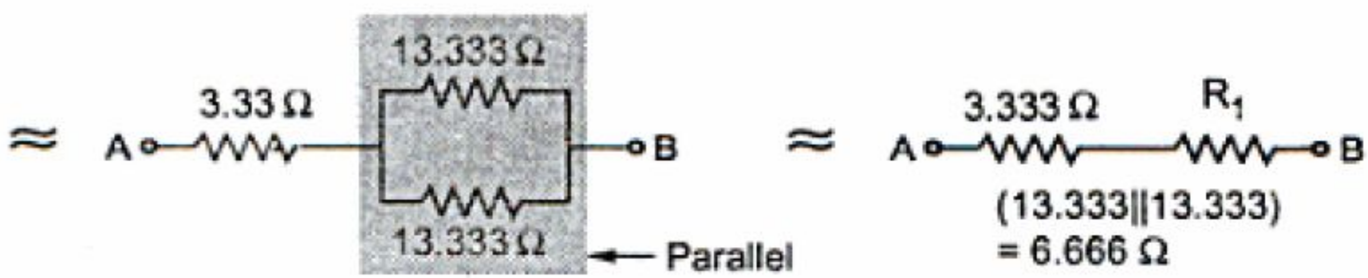
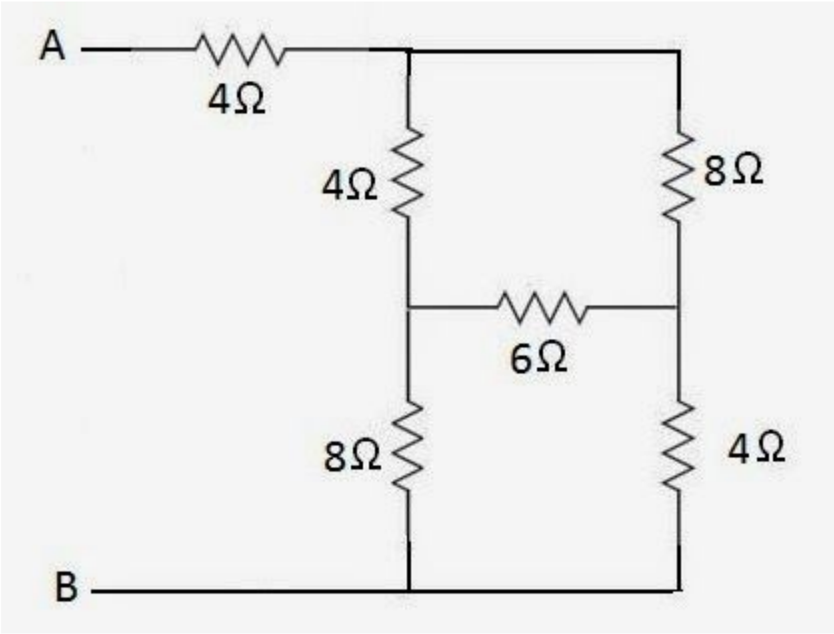


Fig. 2.81 (e)

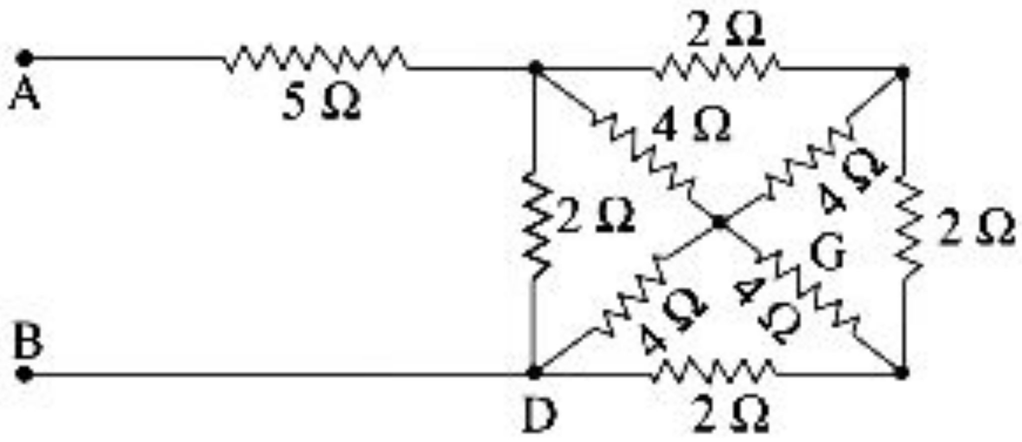
$$R_{AB} = 3.333 + 6.666 = 10\ \Omega$$

Find R between A and B



Ans:

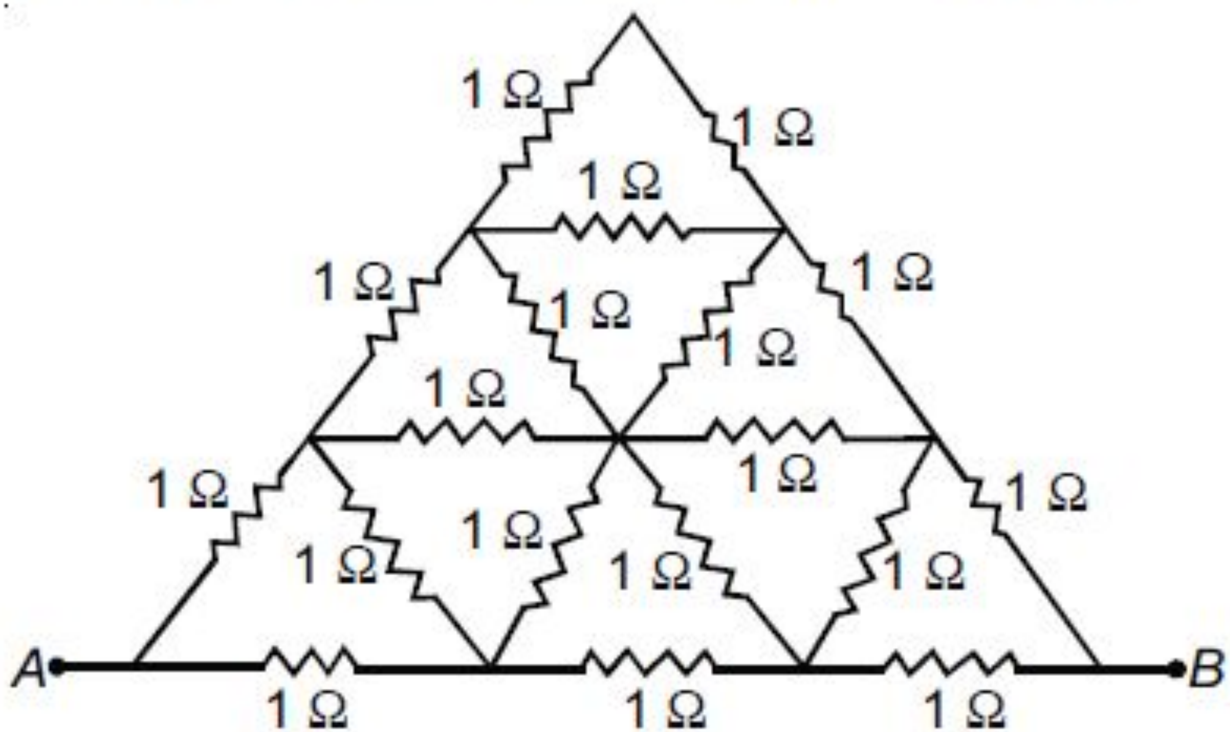
R_{AB}=9.07 Ohms

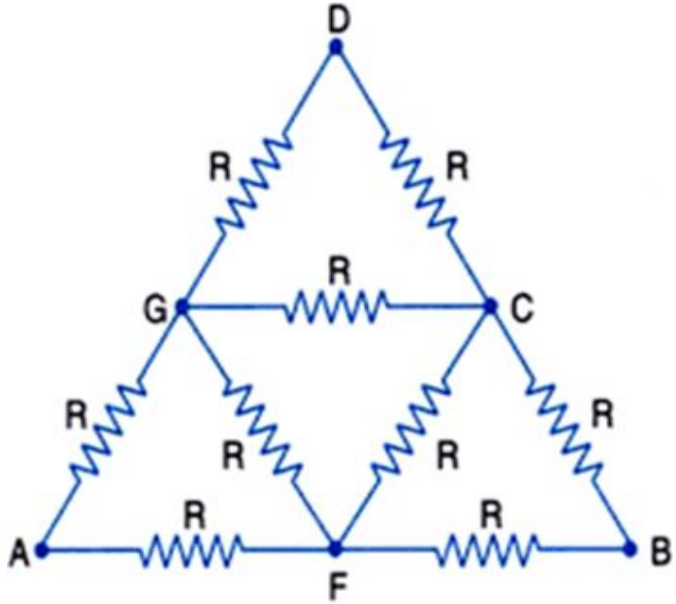


Ans:

R_{AB}=

The equivalent resistence between A and B is :

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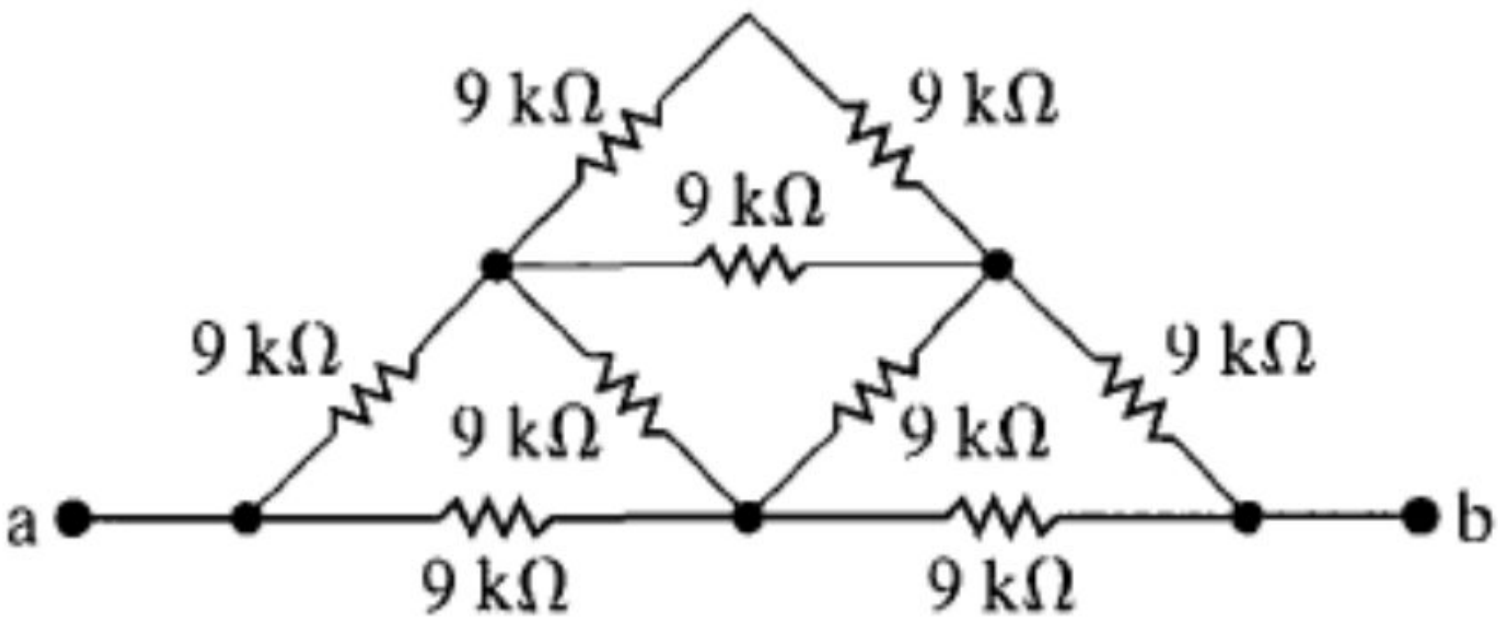
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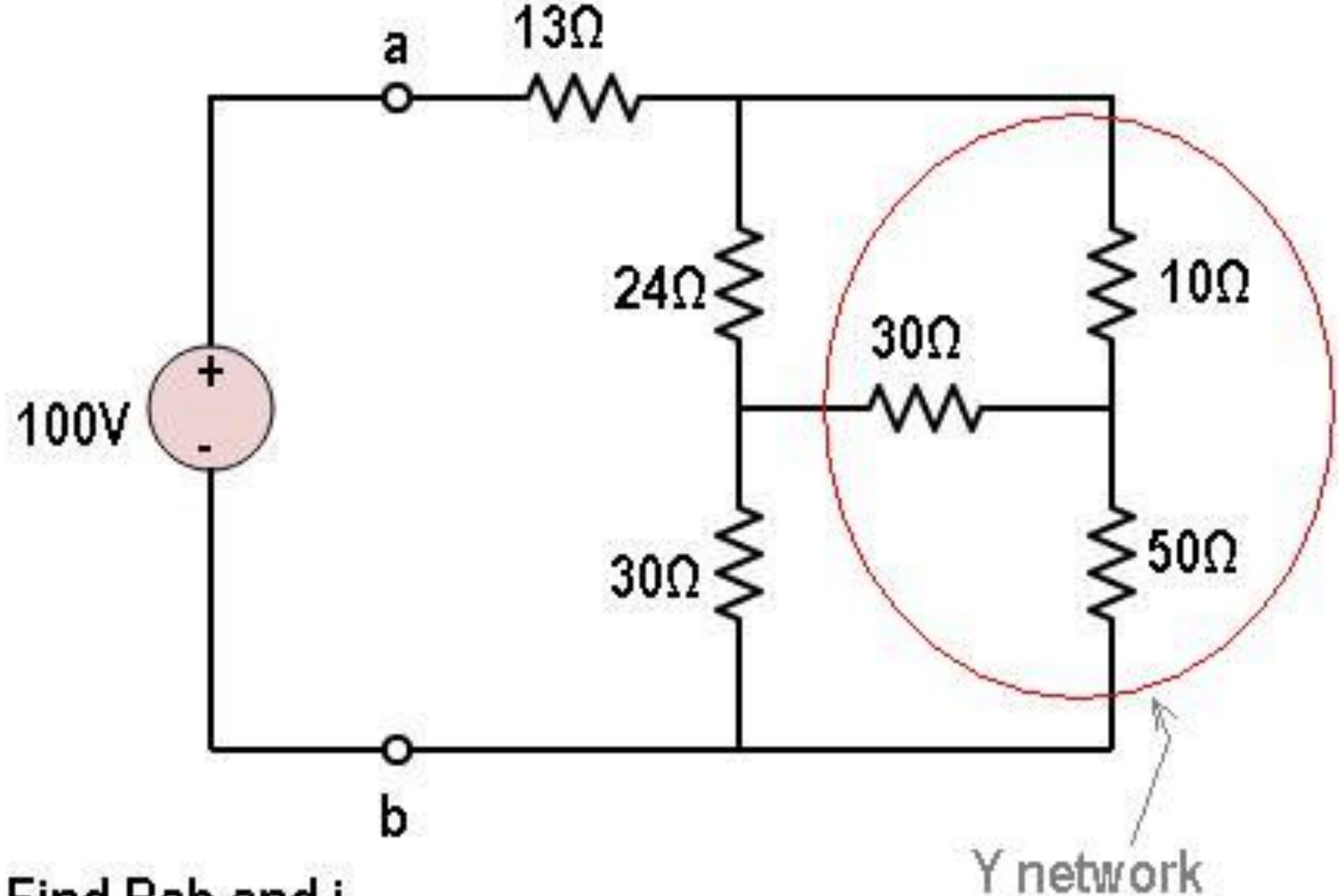

Find Rab and i

Extra Resources

- Multisim cct analysis

https://youtu.be/2SOyJYQQflU