# D. C. Circuits

## 2.1 Introduction

In practice, the electrical circuits may consist of one or more sources of energy and number of electrical parameters, connected in different ways. The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements alongwith various sources of energy gives rise to complicated electrical circuits, generally referred as **networks**. The terms **circuit** and **network** are used synonymously in the electrical literature. The d.c. circuits consist of only resistances and d.c. sources of energy. And the circuit analysis means to find a current through or voltage across any branch of the circuit. This chapter includes various techniques of analysing d.c. circuits.

The chapter explains the basic terminology used in the network analysis and classification of networks. It explains Ohm's law, Kirchhoff's laws and various network simplification techniques such as series-parallel combinations, star-delta transformation, source transformation etc. These techniques are very basic and useful, which can be further applied to understand various network theorems. The network theorems such as Superposition, Thevenin's, Norton's and Maximum power transfer as applied to d.c. circuits are also included in this chapter.

## 2.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

### 2.2.1 Network

Any arrangement of the various electrical energy sources along with the different circuit elements is called an **electrical network.** Such a network is shown in the Fig. 2.1.

### 2.2.2 Network Element

Any individual circuit element with two terminals which can be connected to other circuit element, is called a **network element.**

Network elements can be either active elements or passive elements. Active elements are the elements which supply power or energy to the network. Voltage source and current source are the examples of active elements. Passive elements are the elements which either store energy or dissipate energy in the form of heat. Resistor, inductor and capacitor are the three basic passive elements. Inductors and capacitors can store energy and resistors dissipate energy in the form of heat.

### 2.2.3 Branch

A part of the network which connects the various points of the network with one another is called **a branch.** In the Fig. 2.1, AB, BC, CD, DA, DE, CF and EF are the various branches. A branch may consist more than one element.

### 2.2.4 Junction Point

A point where three or more branches meet is called a **junction point.** Point D and C are the junction points in the network shown in the Fig. 2.1.

### 2.2.5 Node

A point at which two or more elements are joined together is called **node.** The junction points are also the nodes of the network. In the network shown in the Fig. 2.1, A, B, C, D, E and F are the nodes of the network.

### 2.2.6 Mesh (or Loop)

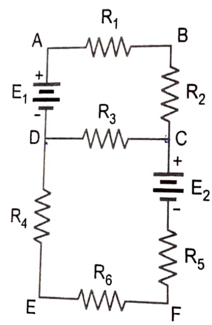



**Fig. 2.1 An electrical network**

**Mesh (or Loop)** is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes, without travelling through any node twice. In the Fig. 2.1 paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc. are the loops of the network.

In this chapter, the analysis of d.c. circuits consisting of pure resistors and d.c. sources is included.

## 2.3 Classification of Electrical Networks

The behaviour of the entire network depends on the behaviour and characteristics of its elements. Based on such characteristics electrical network can be classified as below :

**i) Linear Network :** A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network**. The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

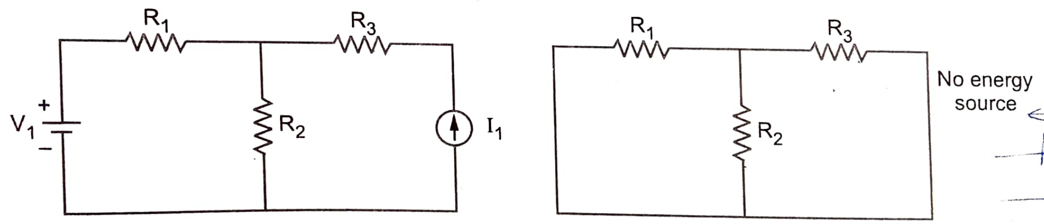
**ii) Non linear Network :** A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network** . The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

**iii) Bilateral Network :** A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network.** Network consisting only resistances is good example of bilateral network.

**iv) Unilateral Network :** A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network**. Circuit consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

**v) Active Network :** A circuit which contains at least one source of energy is called **active.** An energy source may be a voltage or current source.

**vi) Passive Network :** A circuit which contains no energy source is called **passive circuit.** This is shown in the Fig. 2.2.




(a) Active network

(b) Passive network

**Fig. 2.2**

**vii) Lumped Network :** A network in which all the network elements are physically separable is known as **lumped network**. Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

**viii) Distributed Network :** A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network**. The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, any where in the circuit.

The classification of networks can be shown as,

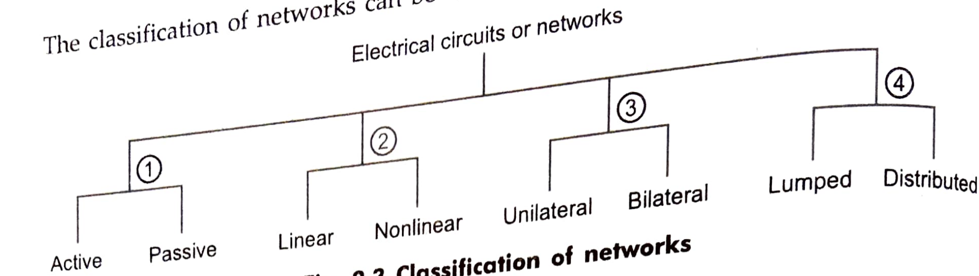



**Fig. 2.3 Classification of networks**

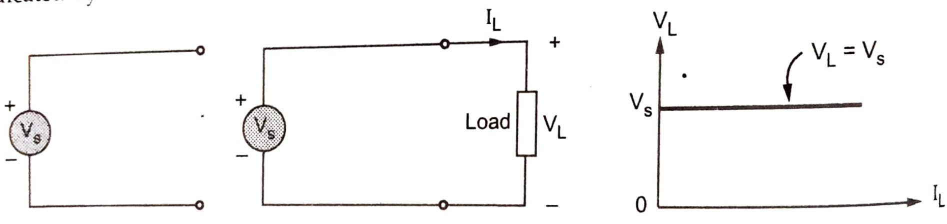
## 2.4 Energy Sources

There are basically two types of energy sources ; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

### 2.4.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 2.4 (a). This is connected to the load as shown in Fig. 2.4 (b). At any time the value of voltage at load terminals remains same. This is indicated by V- I characteristics shown in the Fig. 2.4 (c).

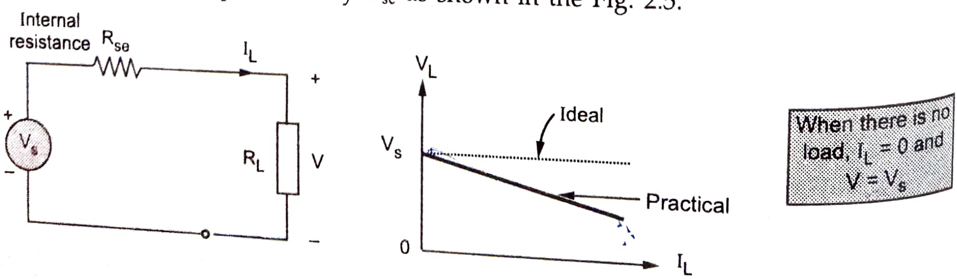



(a) Symbol (b) Circuit (c) Characteristics

**Fig. 2.4 Ideal voltage source**

**Practical voltage source :**

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by $R_{se}$ as shown in the Fig. 2.5.




(a) Circuit (b) Characteristics

**Fig. 2.5 Practical voltage source**

Because of the $R_{se}$, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$\boxed{V_L = -(R_{se})\, I_L + V_S = V_S - I_L R_{se}}$$

**Key Point**: *For ideal voltage source,*

$$R_{se} = 0$$

Voltage sources are further classified as follows,

**i) Time Invariant Sources :**

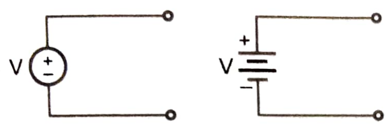



**Fig. 2.6 (a) D. C. source**

The sources in which voltage is not varying with time are known as **time invariant voltage sources** or **D.C. sources**. These are denoted by capital letters. Such a source is represented in the Fig. 2.6 (a).

**ii) Time Variant Sources :**

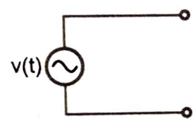
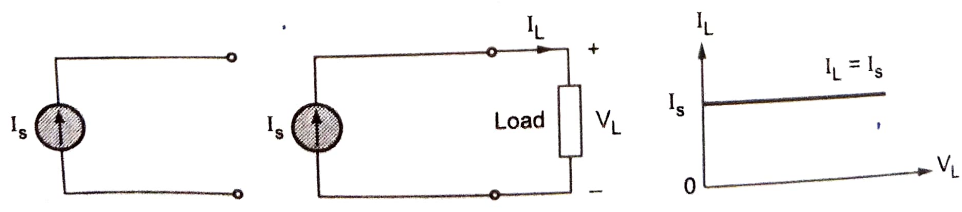



**Fig. 2.6 (b) A. C. source**

The sources in which voltage is varying with time are known as **time variant voltage sources** or **A.C. sources**. These are denoted by small letters. This is shown in the Fig. 2.6 (b).

## 2.4.2 Current Source

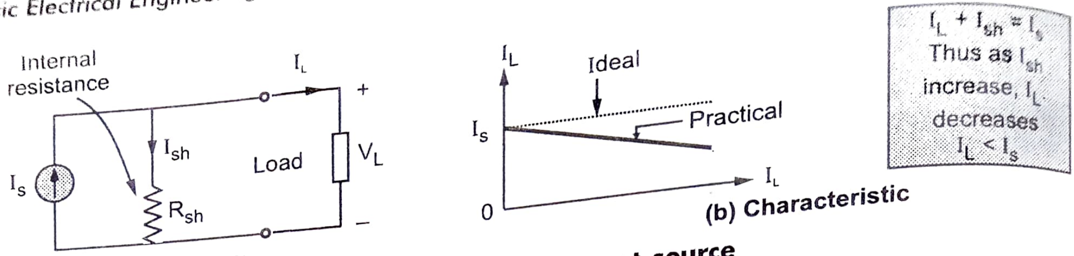
Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 2.7 (a). This is connected to the load as shown in the Fig. 2.7 (b). At any time, the value of the current flowing through load $I_L$ is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 2.7 (c).




**(a) Symbol** **(b) Circuit** **(c) Characteristics**

**Fig. 2.7 Ideal current source**

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by $R_{sh}$. This is shown in the Fig. 2.8.




(a) Circuit

(b) Characteristic

**Fig. 2.8 Practical current source**

Because of $R_{sh}$, current through its terminals decreases slightly with increase in voltage at its terminals.

**Key Point**: *For ideal current source, $R_{sh} = \infty$.*

Similar to voltage sources, current sources are classified as follows :

**i) Time Invariant Sources :**

The sources in which current is not varying with time are known as **time invarient current sources** or **D.C. sources.** These are denoted by capital letters.

Such a current source is represented in the Fig. 2.9 (a).

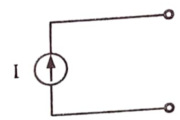



**Fig. 2.9 (a) D. C. source**

**ii) Time Variant Sources :**

The sources in which current is varying with time are known as **time variant current sources** or **A.C. sources.** These are denoted by small letters.

Such a source is represented in the Fig. 2.9 (b).

The sources which are discussed above are independent sources because these sources does not depend on other voltages or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside.

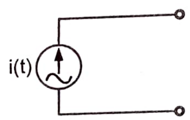



**Fig. 2.9 (b) A. C. source**

## 2.4.3 Dependent Sources

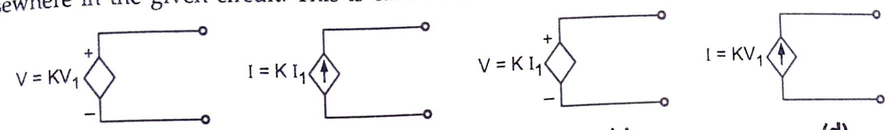
Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 2.10 and further classified as,

**i) Voltage Dependent Voltage Source :** It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS.** It is shown in the Fig. 2.10 (a).

**ii) Current Dependent Current Source :** It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. 2.10 (b).

**iii) Current Dependent Voltage Source :** It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. 2.10 (c).

**iv) Voltage Dependent Current Source :** It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 2.10 (d).




(a) (b) (c) (d)

**Fig. 2.10**

K is constant and $V_1$ and $I_1$ are the voltage and current respectively, present elsewhere in the given circuit. The dependent sources are also known as controlled sources.

In this chapter, d.c. circuits consisting of independent d.c. voltage and current sources are analysed.

## 2.5 Ohm's Law

This law gives relationship between the potential difference (V), the current (I) and the resistance (R) of a d.c. circuit. **Dr. Ohm** in 1827 discovered a law called **Ohm's Law.** It states,

**Ohm's Law :** *The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.*

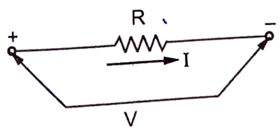



**Fig. 2.11 Ohm's law**

Mathematically, $\boxed{I \propto \dfrac{V}{R}}$

Where I is the current flowing in amperes, the V is the voltage applied and R is the resistance of the conductor, as shown in the Fig. 2.11.

Now $\quad I = \dfrac{V}{R}$

The unit of potential difference is defined in such a way that the constant of proportionality is unity.

$$\boxed{\begin{aligned} \text{Ohm's Law is, } I &= \frac{V}{R} \quad \text{amperes} \\ V &= I\,R \quad \text{volts} \\ \frac{V}{I} &= \text{constant} = R \ \text{ohms} \end{aligned}}$$

The Ohm's law can be defined as,

The ratio of potential difference (V) between any two points of a conductor to the current (I) flowing between them is constant, provided that the temperature of the [illegible] remains constant.

circuit. If it ... should be ...
entire circuit should be ...
then the resistance of that part and poten...

### 2.5.1 Limitations of Ohm's Law

The limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators etc.

2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$V = k\, I^{m}$ where k, m are constants.

## 2.6 Series Circuit

A **series** circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade** connection. There is onl one path for the flow of current.

Consider the resistances shown in the Fig. 2.12.

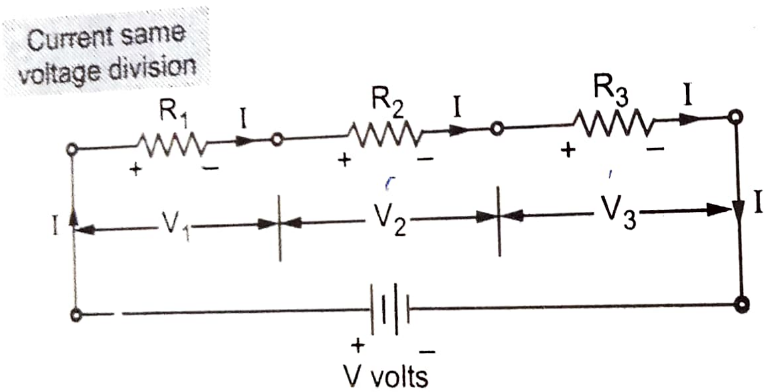



**Fig. 2.12 A series circuit**

The resistance $R_1$, $R_2$ and $R_3$ a said to be in series. The combination connected across a source of voltage volts. Naturally the current flowi through all of them is same indica as I amperes. e.g. the chain of sm lights, used for the decorat purposes is good example of se combination.

Now let us study the vol **distribution.**

Let $V_1, V_2$ and $V_3$ be the voltages across the terminals of resistances $R_1$, $R_2$ an respectively

Then,
$$V = V_1 + V_2 + V_3$$

Now according to Ohm's law,
$$V_1 = I R_1,\ V_2 = I R_2,\ V_3 = I R_3$$

Current through all of them is same i.e. I

$\therefore$
$$V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I\, R_{eq}$$

$$R = R_1 + R_2 + R_3 + \ldots\ldots + R_n$$

## .1 Characteristics of Series Circuits

1) The same current flows through each resistance.

2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots\ldots + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances.

i.e $\quad R > R_1 \; , \; R > R_2 \; , \; \ldots\ldots \; R > R_n$

## .7 Parallel Circuit

Voltage same current division

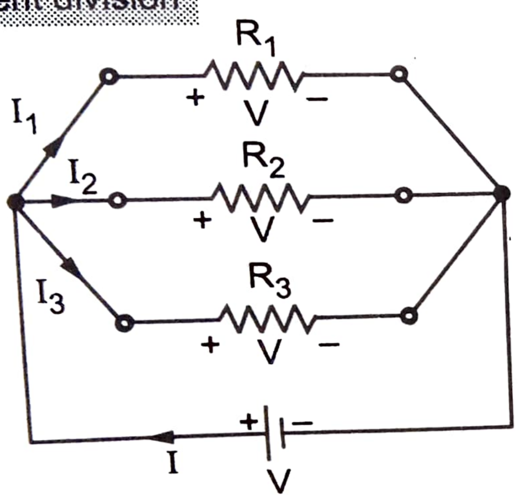



Fig. 2.13 A parallel circuit

The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the Fig. 2.13.

In the parallel connection shown, the three resistances $R_1$ , $R_2$ and $R_3$ are connected in parallel and combination is connected across a source of voltage 'V '.

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through $R_1$, second through $R_2$ and third through $R_3$. Depending upon the values of $R_1$ , $R_2$ and $R_3$ the appropriate fraction of total current passes through them. These individual currents are shown as $I_1$ , $I_2$ and $I_3$. While the voltage across the two ends of each resistances $R_1$ , $R_2$ and $R_3$ is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1 \; , \; V = I_2 R_2 \; , \; V = I_3 R_3$$

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \ldots$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and
$$I = \frac{V}{R_{eq}} \quad \ldots$$

where $R_{eq}$ = Total or **equivalent resistance** of the circuit

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}}$$

**Conductance (G) :**

It is known that, $\frac{1}{R} = G$ (conductance) hence,

$$\therefore \quad \boxed{G = G_1 + G_2 + G_3 + \ldots\ldots + G_n} \quad \ldots \text{For parallel c}$$

**Important result :**

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad \boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

This formula is directly used hereafter for ...

## 2.14 Kirchhoff's Laws

In 1847, a German Physicist, **Kirchhoff,** formulated two fundamental laws of electricity. These laws are of tremendous importance from network simplification point of view.

### 2.14.1 Kirchhoff's Current Law (KCL)

Consider a junction point in a complex network as shown in the Fig. 2.33.

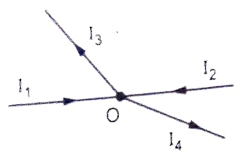



**Fig. 2.33 Junction point**

At this junction point if $I_1 = 2A$, $I_2 = 4A$ and $I_3 = 1A$ then to determine $I_4$ we write, total current entering is $2 + 4 = 6A$ while total current leaving is $1 + I_4$ A

And hence, $I_4 = 5$ A.

This analysis of currents entering and leaving is nothing but the application of Kirchhoff's Current Law. The law can be stated as,

***The total current flowing towards a junction point is equal to the total current flowing away from that junction point.***

Another way to state the law is,

***The algebraic sum of all the current meeting at a junction point is always zero.***

The word algebraic means considering the signs of various currents.

$$\boxed{\sum I \text{ at junction point } = 0}$$

***Sign convention : Currents flowing towards a junction point are assumed to be positive while currents flowing away from a junction point assumed to be negative.***

e.g. Refer to Fig. 2.33, currents $I_1$ and $I_2$ are positive while $I_3$ and $I_4$ are negative.

Applying KCL, $\sum I$ at junction O $= 0$

$$I_1 + I_2 - I_3 - I_4 = 0 \text{ i.e. } I_1 + I_2 = I_3 + I_4$$

The law is very helpful in network simplification.

### 2.14.2 Kirchhoff's Voltage Law (KVL)

***"In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f s in the path"***

In other words, "the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

$$\boxed{\text{Around a closed path } \sum V = 0}$$

The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction till the starting point is reached again, he must be at the same potential with which he started tracing a closed path.

Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.

This law is very useful in loop analysis of the network.

### 2.14.3 Sign Conventions to be Followed while Applying KVL

When current flows through a resistance, the voltage drop occurs across the resistance. The polarity of this voltage drop always depends on direction of the current. The current always flows from higher potential to lower potential.

In the Fig. 2.34 (a), current I is flowing from right to left, hence point B is at higher potential than point A, as shown.

Fig. 2.34

In the Fig. 2.34 (b), current I is flowing from left to right, hence point A is at higher potential than point B, as shown.

Once all such polarities are marked in the given circuit, we can apply KVL to any closed path in the circuit.

Now while tracing a closed path, if we go from – ve marked terminal to + ve marked terminal, that voltage must be taken as positive. This is called **potential rise**.

For example, if the branch AB is traced from A to B then the drop across it must be considered as rise and must be taken as + IR while writing the equations.

While tracing a closed path, if we go from +ve marked terminal to – ve marked terminal, that voltage must be taken as negative. This is called **potential drop**.

For example, in the Fig. 2.34 (a) only, if the branch is traced from B to A then it should be taken as negative, as – IR while writing the equations.

Similarly in the Fig. 2.34 (b), if branch is traced from A to B then there is a voltage drop and term must be written negative as – IR while writing the equation. If the branch is traced from B to A, it becomes a rise in voltage and term must be written positive as IR while writing the equation.

**Key Point**:

1) ***Potential rise*** *i.e. travelling from negative to positively marked terminal, must be considered as* ***Positive****.*
2) ***Potential drop*** *i.e. travelling from positive to negatively marked terminal, must be considered as* ***Negative****.*
3) *While tracing a closed path, select any one direction clockwise or anticlockwise. This selection is totally independent of the directions of currents and voltages of various branches of that closed path.*

### Let us see what is Star connection ?

If the three resistances are connected in such a manner that one end of each is connected together to form a junction point called **Star point**, the resistances are said to be connected in **Star**.

The Fig. 2.38 (a) and (b) show star connected resistances. The star point is indicated as S. Both the connections Fig. 2.38 (a) and (b) are exactly identical. The Fig. 2.38 (b) can be redrawn as Fig. 2.38 (a) or vice-versa, in the circuit from simplification point of view.

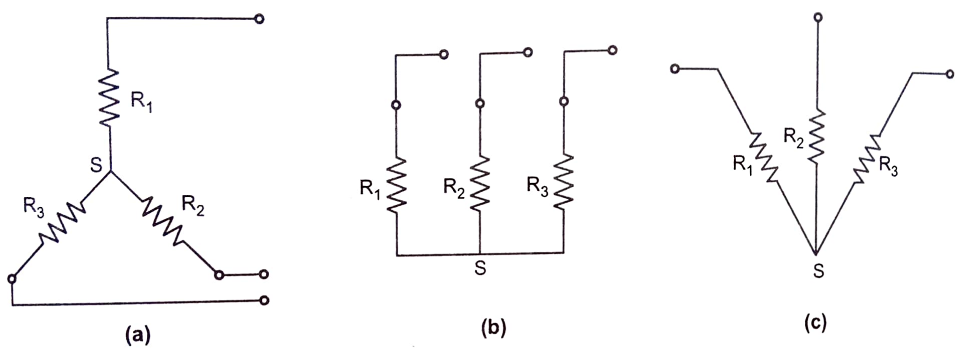



**Fig. 2.38 Star connection of three resistances**

### Let us see what is delta connection ?

If the three resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in **Delta**.

**Key Point**: *Delta connection always forms a loop, closed path.*

The Fig. 2.39 (a) and (b) show delta connection of three resistances. The Fig. 2.39 (a) and (b) are exactly identical.

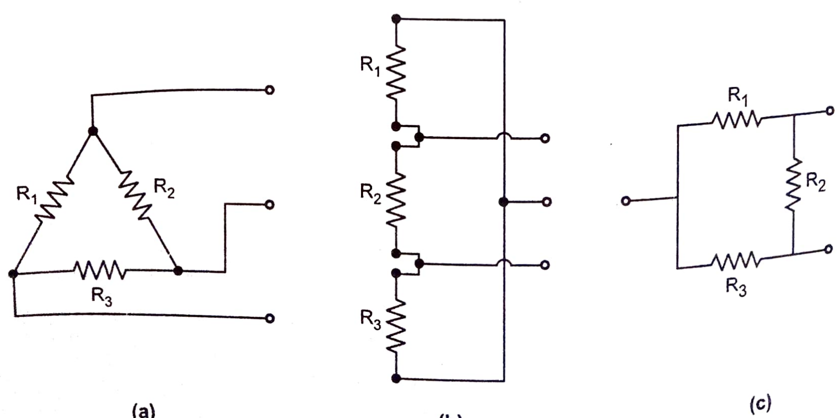



**Fig. 2.39 Delta connection of three resistances**

### 2.16.1 Delta-Star Transformation

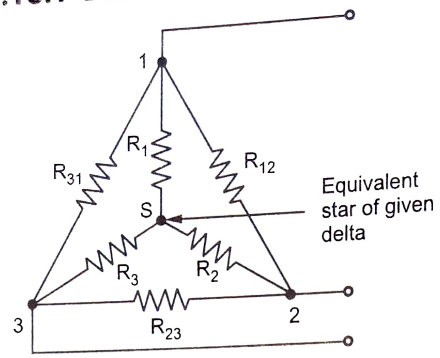


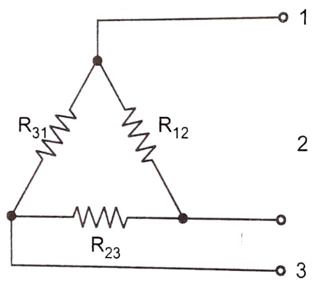

**Fig. 2.40 Delta and equivalent Star**

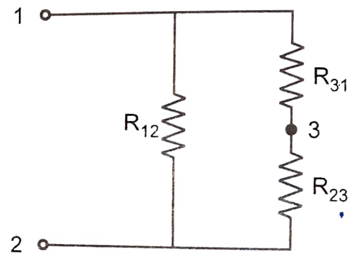
Consider the three resistances $R_{12}, R_{23}, R_{31}$ connected in Delta as shown in the Fig. 2.40. The terminals between which these are connected in Delta are named as 1, 2 and 3.

Now it is always possible to replace these Delta connected resistances by three equivalent Star connected resistances $R_1, R_2, R_3$ between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 2.40 which is called **equivalent Star of Delta connected resistances.**

**Key Point** : *Now to call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.*

Let us analyse Delta connection first, shown in the Fig. 2.40 (a).




**(a) Given Delta** **Fig. 2.40** **(b) Equivalent between 1 and 2**

Now consider the terminals (1) and (2). Let us find equivalent resistance between (1) and (2). We can redraw the network as viewed from the terminals (1) and (2), without considering terminal (3). This is shown in the Fig. 2.40(b).

Now terminal '3' we are not considering, so between terminals (1) and (2) we get the combination as,

$R_{12}$ parallel with $(R_{31} + R_{23})$ as $R_{31}$ and $R_{23}$ are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31} + R_{23})}{R_{12} + (R_{31} + R_{23})} \quad \text{...(a)}$$

[using $\frac{R_1 R_2}{R_1 + R_2}$ for parallel combination]

Now consider the same two terminals of equivalent Star connection shown in the Fig. 2.41.

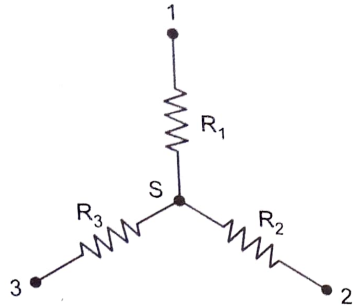



**Fig. 2.41 Star connection**

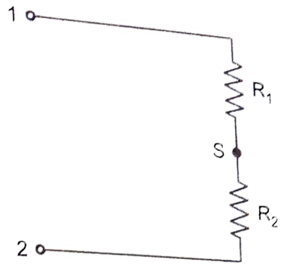



**Fig. 2.42 Equivalent between 1 and 2**

Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).

And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 2.42.

$\therefore$ Between (1) and (2) theresistance is $= R_1 + R_2$ ... (b)

This is because, two of them found to be in series across the terminals 1 and 2 while 3 found to be open.

Now to call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence equating equations (a) and (b),

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1 + R_2 \quad \text{...(c)}$$

Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2 + R_3 \quad \text{...(d)}$$

Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3 + R_1 \quad \text{...(e)}$$

Now we are interested in calculating what are the values of $R_1, R_2, R_3$ interms of known values $R_{12}, R_{23}$, and $R_{31}$.

Subtracting (d) from (c),

$$\frac{R_{12}(R_{31}+R_{23}) - R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1 + R_2 - R_2 - R_3$$

$$\therefore \quad R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \quad \ldots (f)$$

Adding (f) and (e),

$$\frac{R_{12} R_{31} - R_{23} R_{31} + R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} = R_1 + R_3 + R_1 - R_3$$

$$\therefore \quad \frac{R_{12} R_{31} - R_{23} R_{31} + R_{31} R_{12} + R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = 2R_1$$

$$2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$\boxed{R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (c), (d) and (e) we can get,

$$\boxed{R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}}$$

and

$$\boxed{R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}}$$

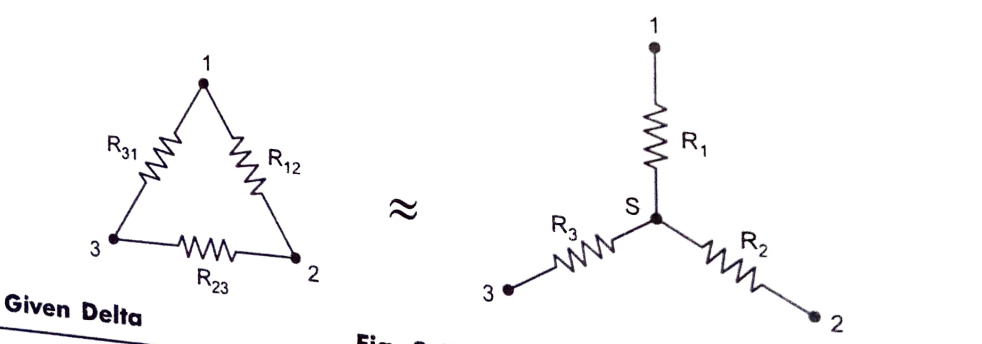



Fig. 2.43

**Easy way of remembering the result :**

*The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.*

So if we want equivalent resistance between terminal (2) and star point i.e. $R_2$ then it is the product of two resistances in delta which are connected to same terminal i.e. terminal (2) which are $R_{12}$ and $R_{23}$ divided by sum of all delta connected resistances i.e. $R_{12}$, $R_{23}$ and $R_{31}$.

## 2.16.2 Star-Delta Transformation

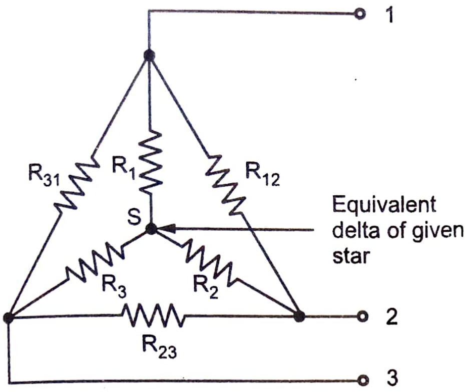



Fig. 2.44 Star and equivalent Delta

Consider the three resistances $R_1, R_2$ and $R_3$ connected in Star as shown in Fig. 2.44.

Now by Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances $R_{12}, R_{23}$ and $R_{31}$, between the same terminals. This is called **equivalent Delta of the given star.**

Now we are interested in finding out values of $R_{12}, R_{23}$ and $R_{31}$ interms of $R_1, R_2$ and $R_3$.

For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12}\, R_{31}}{R_{12}+R_{23}+R_{31}} \quad \ldots(g)$$

$$R_2 = \frac{R_{12}\, R_{23}}{R_{12}+R_{23}+R_{31}} \quad \ldots(h)$$

$$R_3 = \frac{R_{23}\, R_{31}}{R_{12}+R_{23}+R_{31}} \quad \ldots(i)$$

Now multiply (g) and (h), (h) and (i), (i) and (g) to get following three equations.

$$R_1 R_2 = \frac{R_{12}^2\, R_{31}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \ldots(j)$$

$$\therefore \quad R_2 R_3 = \frac{R_{23}^2\, R_{12}\, R_{31}}{(R_{12}+R_{23}+R_{31})^2} \quad \ldots(k)$$

$$R_3 R_1 = \frac{R_{31}^2\, R_{12}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \ldots(l)$$

Now add (j), (k) and (*l*)

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{31} R_{23} + R_{23}^2 R_{12} R_{31} + R_{31}^2 R_{12}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\therefore \quad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{31} R_{23}(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{31}\, R_{23}}{R_{12}+R_{23}+R_{31}}$$

But $$\frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} = R_1 \quad \text{From equation (g)}$$

$\therefore$ Substituting in above in R.H.S. we get,

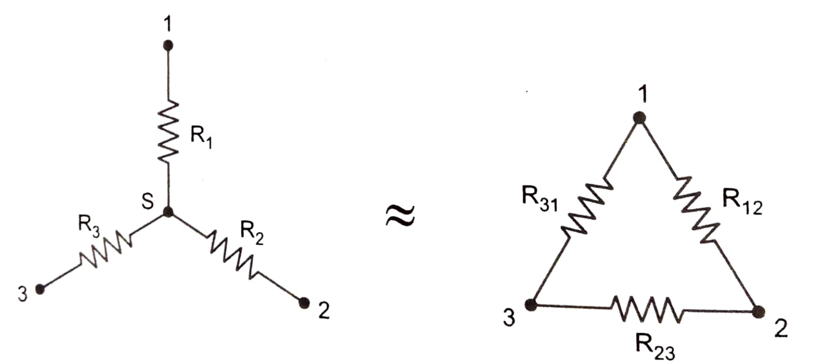
$$R_1R_2+R_2R_3+R_3R_1 = R_1 R_{23}$$

$\therefore$ $$\boxed{R_{23} = R_2+R_3+\frac{R_2 R_3}{R_1}}$$

$\therefore$ Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1+R_2+\frac{R_1 R_2}{R_3}}$$

and $$\boxed{R_{31} = R_3+R_1+\frac{R_1 R_3}{R_2}}$$




Given Star Fig. 2.45 Equivalent Delta

**Easy way of remembering the result :**

*The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.*

So if we want equivalent delta resistance between terminals (3) and (1), then take sum of the two resistances connected between same two terminals (3) and (1) and star point respectively i.e. terminal (3) to star point $R_3$ and terminal (1) to star point i.e. $R_1$. Then to this sum of $R_1$ and $R_3$, add the term which is the product of the same two resistances i.e. $R_1$ and $R_3$ divided by the third star resistance which is $R_2$.

$\therefore$ We can write, $R_{31} = R_1+R_3+\frac{R_1R_3}{R_2}$ which is same as deriv...

**Result for equal ...**

i.e. **equivalent Star contains three equal resistances, each of magnitude one third of magnitude of the resistances connected in Delta.**

If all three resistances in a Star connection are of same magnitude say R, then equivalent Delta contains all resistances of same magnitude of ,

$$R_{12} = R_{31} = R_{23} = R+R+\frac{R\times R}{R} = 3R$$

i.e. **equivalent delta contains three resistances each of magnitude thrice the magnitude of resistances connected in Star.**

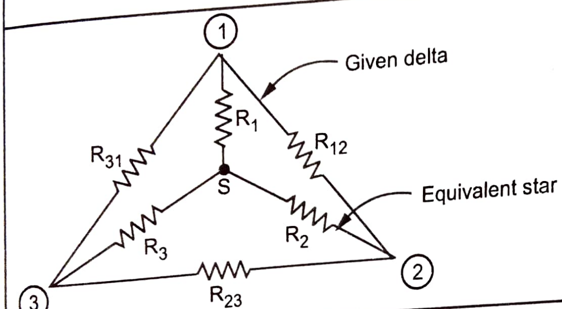
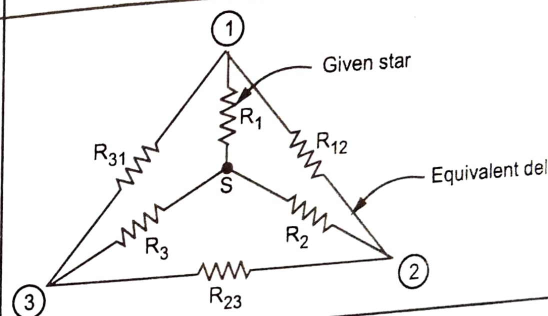
<table>
<tr><th>Delta-Star</th><th>Star-Delta</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$</td><td>$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$</td></tr>
<tr><td>$R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}$</td><td>$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$</td></tr>
<tr><td>$R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$</td><td>$R_{31} = R_3 + R_1 + \frac{R_1R_3}{R_2}$</td></tr>
</table>

**Table 2.1 Star-Delta and Delta-Star Transformations**

**Example 2.6 :** *Convert the given Delta in the Fig. 2.46 into equivalent Star.*

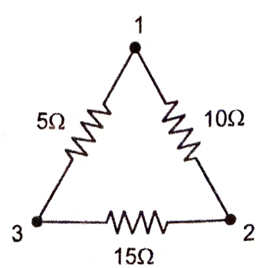



**Fig. 2.46**

**Example 2.8 :** *Find equivalent resistance between points A-B.*

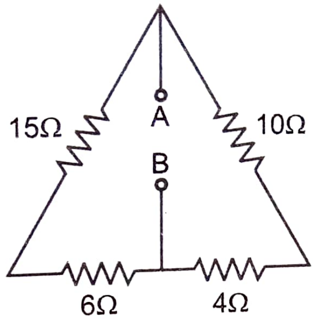



**Fig. 2.49**

**Solution :** Redrawing the circuit,

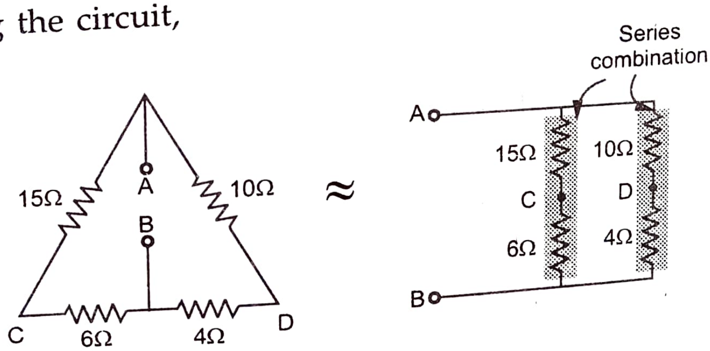



**Fig. 2.49 (a)**

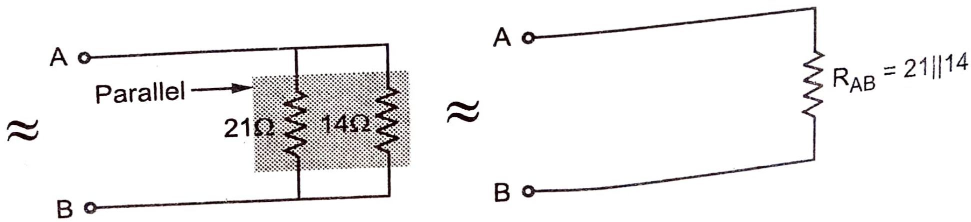
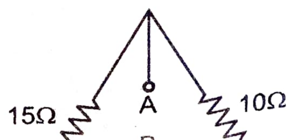



**Fig. 2.49 (b)**

$$R_{AB} = \frac{21 \times 14}{21 + 14} = 8.4\ \Omega$$

**Example 2.9 :** *Find equivalent resistance between points A-B.*



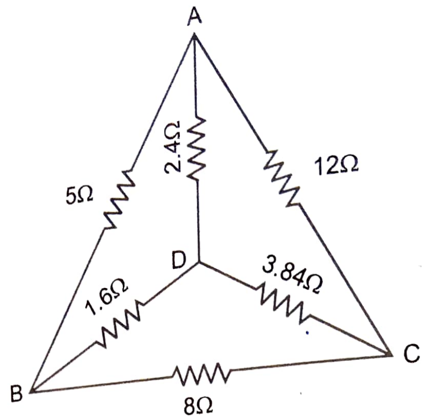



**Fig. 2.54**

**Solution :** Converting star ADCB to delta ACB.

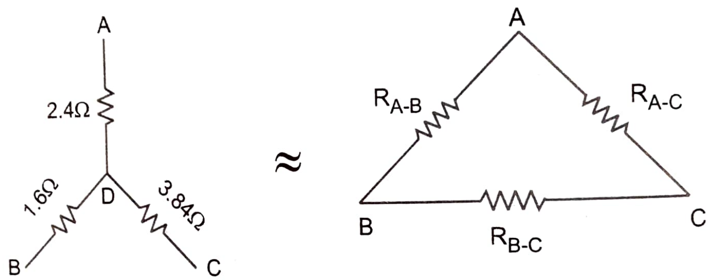



**Fig. 2.54 (a)**

$$R_{AB} = 2.4 + 1.6 + \frac{2.4 \times 1.6}{3.84} = 5\ \Omega$$

$$R_{AC} = 2.4 + 3.84 + \frac{2.4 \times 3.84}{1.6} = 12\ \Omega$$

$$R_{BC} = 1.6 + 3.84 + \frac{1.6 \times 3.84}{2.4} = 8\ \Omega$$

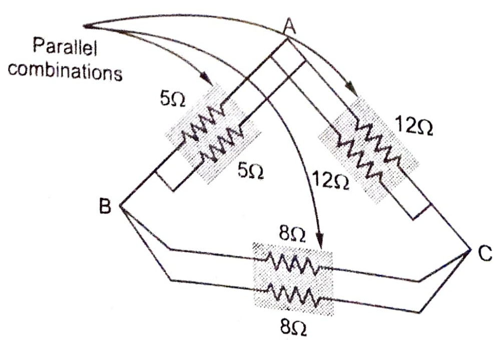
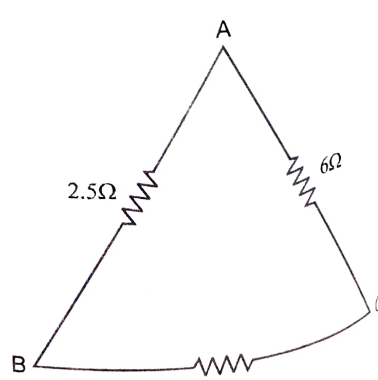



**Fig. 2.54 (b)**

≈



$$R_1 = \frac{5\times5}{5+5} = 2.5\ \Omega,\quad R_2 = \frac{12\times12}{12+12} = 6\ \Omega,\quad R_3 = \frac{8\times8}{8+8} = 4\ \Omega$$

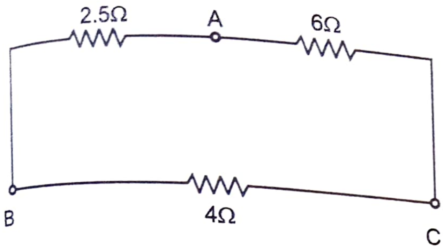



Fig. 2.54 (d)

≈

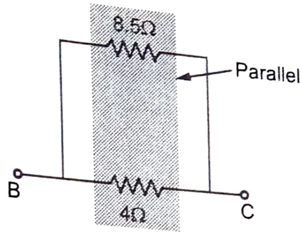



Fig. 2.54 (e)

$$R_{BC} = \frac{4\times8.5}{4+8.5} = 2.72\ \Omega$$

**Example 2.13 :** *Using Kirchhoff's laws, calculate the current delivered by the battery shown in Fig. 2.55.* **(May - 99)**

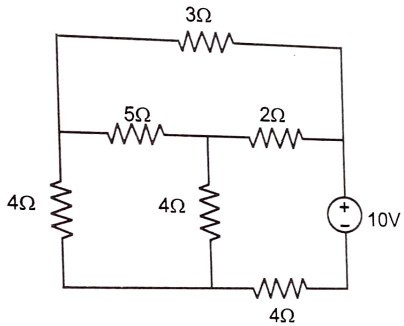



**Fig. 2.55**

**Solution :** The various branch currents are shown in the Fig. 2.55 (a).

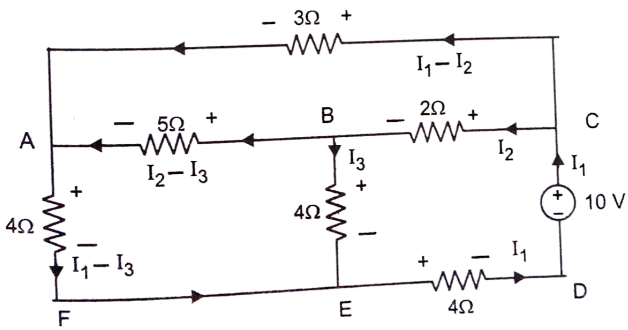



**Fig. 2.55 (a)**