$(D^3 - 5D^2 + 8D - 4)\, y = e^{2x} + 2e^x + 3e^{-x} + 2$

Ans. $y = c_1 e^x + (c_2 + c_3 x)\, e^{2x} + \frac{e^{2x} x^2}{2} + 2xe^x - \frac{e^{-x}}{6} - \frac{1}{2}$

$(D^4 - 4D^3 + 6D^2 - 4D + 1)\, y = e^x + 2^x + \frac{1}{3}.$

Ans. $y = (c_1 x^3 + c_2 x^2 + c_3 x + c_4)\, e^x + \frac{x^4}{24} e^x + \frac{1}{(\log 2 - 1)^4} 2^x + \frac{1}{3}$

$\frac{d^2y}{dx^2} + 4y = \cos x \cdot \cos 2x \cdot \cos 3x$

Ans. $y = A \cos 2x + B \sin 2x + \frac{1}{16} + \frac{x \sin 2x}{16} - \frac{1}{48} \cos 4x - \frac{1}{128} \cos 6x$

$(D^5 - D^4 + 2D^3 - 2D^2 + D - 1)\, y = \cos x$

Ans. $y = c_1 e^x + (c_2 x + c_3) \cos x + (c_4 x + c_5) \sin x + \frac{1}{16} [(x^2 + 2x) \cos x - x^2 \sin x]$

$(D^4 - m^4)\, y = \sin mx$ **(May 2011)**

Ans. $y = c_1 e^{mx} + c_2 e^{-mx} + c_3 \cos mx + c_4 \sin mx + \frac{x}{4m^3} \cos mx$

$(D^3 + D)\, y = \cos x$ **(Dec. 2008)** **Ans.** $c_1 + c_2 \cos x + c_3 \sin x - \frac{x \cos x}{2}$

$\operatorname{cosec} x \frac{d^4y}{dx^4} + y \operatorname{cosec} x = \sin 2x$

Ans. $y = e^{\frac{x}{\sqrt{2}}} \left[c_1 \cos \frac{x}{\sqrt{2}} + c_2 \sin \frac{x}{\sqrt{2}} \right] + e^{-\frac{x}{\sqrt{2}}} \left[c_3 \cos \frac{x}{\sqrt{2}} + c_4 \sin \frac{x}{\sqrt{2}} \right] + \frac{1}{2} \left(\frac{\cos x}{2} - \frac{\cos 3x}{82} \right)$

$\frac{d^2x}{dt^2} + 9x = 4 \cos \left(\frac{\pi}{3} + t \right)$, given that $x = 0$ at $t = 0$ and $x = 2$ at $t = \frac{\pi}{6}$.

Ans. $x = \frac{1}{4} \cos 3t + 2 \sin 3t + \frac{1}{2} \cos \left(\frac{\pi}{3} + t \right)$

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = \sin^2 t$

Ans. $y = e^{-t} [A \cos 2t + b \sin 2t] + \frac{1}{10} - \frac{1}{34} [4 \sin 2t + \cos 2t]$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sin 2x - 2 \cos 2x$, given that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

Ans. $y = e^{-x} \sin x - \frac{1}{2} \sin 2x$

$\frac{d^2y}{dx^2} + n^2 y = h \sin px$, where h, p and n are constants satisfying the condition $y = 0$, $\frac{dy}{dx} = b$ for $x = 0$. **Ans.** $y = a \cos nx + \left[\frac{b}{n} - \frac{ph}{n(n^2 - p^2)} \right] \sin nx + \frac{h \sin px}{(n^2 - p^2)}$

- $(D^3 + 1)\, y = \cos(2x - 1) - \cos^2 \frac{x}{2}$

Ans. $y = c_1 e^{-x} + e^{x/2}\left[c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x\right.$
$+ \frac{1}{65}[\cos(2x - 1) - 8 \sin(2x - 1)] - \frac{1}{2} - \frac{1}{4}(\cos x - \sin x)$

- $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10 \sin x.$ **(Dec. 201**

Ans. $y = e^x (A \cos x + B \sin x) + 2 \sin x + \cos x$

- $(D^4 + 10D^2 + 9)\, y = 96 \sin 2x \cos x$
Given that at $x = 0$, $y = 0$, $y' = -2$, $y'' = -8$, $y''' = -18$.

Ans. $y = \cos 3x - \cos x + x(\cos 3x - 3 \cos x)$

- $(D^4 + 6D^2 + 8)\, y = \sin^2 x \cos 2x$

Ans. $y = c_1 \cos 2x + c_2 \sin 2x + c_3 \cos \sqrt{2}\, x + c_4 \sin \sqrt{2}\, x - \frac{x \sin 2x}{16} - \frac{1}{32} - \frac{\cos 4x}{672}$

- $(D^3 + 3D)\, y = \cosh 2x \sinh 3x.$

Ans. $y = c_1 + \left(c_2 \cos \sqrt{3}\, x + c_3 \sin \sqrt{3}\, x\right) + \frac{\cosh 5x}{280} + \frac{\cosh x}{8}$

- $(D^3 - 25D)\, y = \cosh 2x \sinh 3x.$ **(May 2010)**

Ans. $y = c_1 + c_2 e^{5x} + c_3 e^{-5x} + \frac{x}{100} \sinh 5x - \frac{1}{48} \cosh x$

- $(D^4 - 1)\, y = \cosh x \sinh x$ **Ans.** $y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x + \frac{1}{30} \sinh 2x$

- $(D^2 + 13D + 36)\, y = e^{-4x} + \sinh x.$

Ans. $y = c_1 e^{-9x} + c_2 e^{-4x} + \frac{x}{5} e^{-4x} - \frac{1}{1200}(13 \cosh x - 37 \sinh x)$

- $(D^3 + 1)\, y = \sin(2x + 3) + e^{-x} + 2^x.$ **(May 2012)**

Ans. $y = c_1 e^{-x} + e^{(1/2)x}[c_2 \cos(\sqrt{3}/2)\, x + c_3 \sin(\sqrt{3}/2)\, x]$
$+ \frac{1}{65}[\sin(2x + 3) + 8 \cos(2x + 3)] + \frac{x}{3} e^{-x} + \frac{2^x}{(\log 2)^3 + 1}$

- $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 50x$ with $y = 0$, $\frac{dy}{dx} = 1$ at $x = 0$

Ans. $y = 5x - 3 + e^{-3x}(3 \cos x + 5 \sin x)$

- $(D^2 - 2D + 5)\, y = 25x^2.$ **Ans.** $y = e^x [c_1 \cos 2x + c_2 \sin 2x] + 5x^2 + 4x - \frac{2}{5}$

- $(D^4 + D^2 + 1)\, y = 53x^2 + 17$

ns. $y = e^{-x/2}\left[c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x\right] + e^{x/2}\left[c_3 \cos \frac{\sqrt{3}}{2} x + c_4 \sin \frac{\sqrt{3}}{2} x\right] + 53x^2 - 89$

- $(D^2 + 5D + 4)\, y = x^2 + 7x + 9.$

Ans. $y = c_1 e^{-4x} + c_2 e^{-x} + \frac{1}{(}$

$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} \sin 2x + 4x^2 e^x$ **(May 2011)**

Ans. $y = c_1 e^{-2x} + c_2 e^{-3x} - \frac{e^{-2x}}{10}(\cos 2x + 2 \sin 2x) + \frac{e^x}{3}\left(x^2 - \frac{7}{6}x + \frac{37}{72}\right)$

$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 3x + x e^x.$ **Ans.** $y = c_1 + c_2 x + c_3 e^x - 2x e^x + \frac{x^2 e^x}{2} - \frac{x^3}{2} - \frac{3x^2}{2}$

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x e^{3x} + \sin 2x.$

Ans. $y = c_2 e^x + c_1 e^{2x} + e^{3x}\left(\frac{x}{2} - \frac{3}{4}\right) + \frac{1}{20}(3 \cos 2x - \sin 2x)$

$(D^2 - 6D + 13)\, y = 8 e^{3x} \sin 4x + 2^x$ **(Dec. 2004)**

Ans. $y = e^{3x}(A \cos 2x + B \sin 2x) - \frac{2e^{3x} \sin 4x}{3} + \frac{2^x}{(\log 2)^2 - 6 \log 2 + 13}$

$(D^4 + D^2 + 1)\, y = ax^2 + be^{-x} \sin 2x.$

Ans. $y = e^{(-1/2)x}[c_1 \cos(\sqrt{3}/2)\, x + c_2 \sin(\sqrt{3}/2)\, x]$

$e^{(1/2)x}[c_3 \cos(\sqrt{3}/2)\, x + c_4 \sin(\sqrt{3}/2\, x] + a(x^2 - 2) - \frac{b}{481} e^{-x}(20 \cos 2x + 9 \sin 2x)$

$(D^2 - 4)\, y = x \sinh x$ **(May 11) Ans.** $y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{3}[x \sinh x + \frac{2}{3} \cosh x]$

$(D^2 - 20D + 1)\, y = x^2 e^x \sin x$. **Ans.** $y = (c_1 x + c_2) e^x - e^x [4x \cos x + (x^2 - 6) \sin x]$

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2 \cdot e^{2x} \sin 2x.$ **(May 2014)**

Ans. $y = e^{2x}[c_1 + c_2 x + 3 \sin 2x - 2x^2 \sin 2x - 4x \cos 2x]$

$(D^2 + 2D + 1)\, y = x \cos x$ **(M-11) Ans.** $y = (c_1 x + c_2) e^{-x} + \frac{1}{2}(x \sin x + \cos x - \sin x)$

$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x \sin 2x$

Ans. $y = c_1 e^{-2x} + c_2 e^{-x} + \left(\frac{7 - 30x}{200}\right) \cos 2x + \left(\frac{12 - 5x}{100}\right) \sin 2x$

$(D^4 + 2D^2 + 1)\, y = x \cos x.$ **(May 2011)**

Ans. $y = (c_1 x + c_2) \cos x + (c_3 x + c_4) \sin x - \frac{x^3}{24} \cos x + \frac{x^2}{2} \sin x$

$(D^2 + 1)^2\, y = 24x \cos x.$

Ans. $y = (c_1 x + c_2) \cos x + (c_3 x + c_4) \sin x - x^3 \cos x + 3x^2 \sin x$

$(D^2 + 2D + 5)^2\, y = x e^{-x} \cos 2x.$

ns. $y = e^{-x}[(c_1 x + c_2) \cos 2x + (c_3 x + c_4) \sin 2x] - \frac{e^{-x}}{32}\left[(x^3 - x^2) \cos 2x - \frac{2}{3} x^3 \sin 2x\right]$

Solve the following differential equations by the method of variation of parameters.

1. $\frac{d^2y}{dx^2} + 4y = \tan 2x$ **Ans.** $y = A \cos 2x + B \sin 2x - \frac{1}{4} \cos 2x \log (\sec 2x + \tan 2x)$ **(Dec. 2010, 2013)**

2. $\frac{d^2y}{dx^2} + y = x \sin x.$ **Ans.** $y = A \cos x + B \sin x + \frac{x}{2} \sin x - \frac{x^2}{4} \cos x$

3. $(D^2 + 3D + 2)\, y = \sin e^x$ **(May 2010) Ans.** $y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x} \sin e^x$

4. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \cdot \sin x$ **(May 2011) Ans.** $y = A + B\, e^{2x} - \frac{e^x}{2} \sin x$

5. $(D^2 + 4)\, y = 4 \sec^2 2x.$ **(May 2005, 2012)**
Ans. $y = A \cos 2x + B \sin 2x - 1 + \sin 2x \log (\sec 2x + \tan 2x)$

6. $(D^2 - 1)\, y = (1 + e^{-x})^{-2}$ **(Dec. 2008) Ans.** $y = A\, e^x + B\, e^{-x} - 1 + e^{-x} \log (1 + e^x)$

7. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ **(Dec. 2005) Ans.** $y = Ae^{-x} + B\, e^{-2x} + e^{-2x} e^{e^x}$

8. $(D^2 + 1)\, y = 3x - 8 \cot x.$
Ans. $y = c_1 \cos x + c_2 \sin x + 3x - 8 \sin x \log (\operatorname{cosec} x - \cot x)$

9. $(D^2 - 4D + 4)\, y = e^{2x} \sec^2 x$ **Ans.** $y = [c_1 + c_2 x + \log (\sec x)]\, e^{2x}$ **(Dec. 2007, 2010)**

10. $\frac{d^2y}{dx^2} + y = \tan x.$ **(May 06, 11) Ans.** $y = A \cos x + B \sin x - \cos x \log (\sec x + \tan x)$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin(\log x^2)$$

Ans. $y = c_1\cos(\log x) + c_2\sin(\log x) - \frac{1}{3}\sin(\log x^2)$

$$x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x).$$

Ans. $y = c_1x^{-2} + x\,(c_2\cos\sqrt{(3)}\log x + c_3\sin(\sqrt{3})\log x) - \sin(\log x) + 8\cos(\log x)$

$$(x^2D^2 + 5xD + 3)\,y = \left(1 + \frac{1}{x}\right)^2\log x$$ **(Dec. 2007)**

$$(x^2D^2 - 3xD + 1)\,y = \log x\left[\frac{\sin(\log x) + 1}{x}\right]$$

$$\left(D^3 - \frac{4}{x}D^2 + \frac{5}{x^2}D - \frac{2}{x^3}\right)y = 1$$

Ans. $y = c_1x^2 + c_2\,x^{\left(\frac{5-\sqrt{21}}{2}\right)} + c_3\,x^{\left(\frac{5-\sqrt{21}}{2}\right)} - \frac{x^3}{5}$

$$(x^2D^2 - 4xD + 6)\,y = -x^4\sin x$$

Ans. $y = c_1x^2 + c_2x^3 + x^2\sin x$

$$(2x + 3)^2\frac{d^2y}{dx^2} - 2(2x + 3)\frac{dy}{dx} - 12y = 6x$$ **(May 2008)**

Ans. $y = c_1(2x + 3)^3 + c_2(2x + 3)^{-1} - \frac{3}{16}(2x + 3) + \frac{3}{4}$

$$(x + a)^2\frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$$ **(Dec. 2010)**

(May 2006) Ans. $y = A(x + a)^3 + B(x + a)^2 + \frac{3x + 2a}{6}$

$$7(2 + x)^2\frac{d^2y}{dx^2} + 8(2 + x)\frac{dy}{dx} + y = 4\cos[\log(2 + x)]$$

$$(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4\cos[\log(1 + x)]$$ **(May 2011)**

Ans. $y = c_1\cos[\log(x + 1)] + c_2\sin[\log(1 + x)]$

$$(x + 2)^2\frac{d^2y}{dx^2} - (x + 2)\frac{dy}{dx} + y = 3x + 4$$

Ans. $y = (x + 2)[c_1 + c_2\log(x + 2)] + \frac{3}{2}(x + 2)[\log(x + 2)]^2 - 2$