Laplace Transforms

QB

① Find Laplace trans. of the following

(a) $f(t) = t e^{-2t} \sin t$ (b) $f(t) = \int_0^t e^{-t} \sin 2t \, dt$

(c) $f(t) = \int_0^t \frac{\sin 3t}{t} dt$ (d) $f(t) = \frac{1-\cos t}{t}$

② Find inverse L.T. of the following

(a) $\bar{f}(s) = \frac{1}{s^2-5s+6}$ (b) $\bar{f}(s) = \log\left(1+\frac{a^2}{s^2}\right)$

(c) $\bar{f}(s) = \frac{2s-5}{4s^2-25}$ (d) $\bar{f}(s) = \frac{e^{-3s}}{(s-2)^4}$

③ Evaluate using L.T. $\int_0^\infty t\, e^{-t} \sin t \, dt$

④ Solve by L.T. method

(a) $y'' - 3y' = 9$, $y(0) = y'(0) = 0$

(b) $y'' + 9y = 18t$, $y(0) = 0$ & $y(\frac{\pi}{2}) = 0$

P.D.E. - QB

① Solve heat eqⁿ $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ if $u(0,t) = 0 = u(l,t)$
$u(x,0) = 2x \quad 0 \le x \le l$

② A string is stretched & fastened betⁿ 2 pts. l apart. Motion is started by displacing the string in the form $u = K(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement $u(x,t)$ from one end using wave eqⁿ $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

③ Find the deflection $u(x,t)$ of a vibrating string of length π, both ends fixed, where deflection is $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$