

Subject Name - LADC (TUT)

Subject code -

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Division - II

Roll no - 111056

Batch - K3

Q1) Solve the following -

1) Find the rank of
$$\begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 2 & 3 \\ 0 & -1 & -3 & -2 \end{bmatrix}$$

Ans $\Rightarrow A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 2 & 3 \\ 0 & -1 & -3 & -2 \end{bmatrix}$

$R_1 \leftrightarrow R_2$
$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 2 & 3 & 1 & 4 \\ 0 & -1 & -3 & -2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S(A) = 2$$

\rightarrow

2) Find the rank of $\begin{bmatrix} 1 & 3 & 8 & 6 \\ 2 & 6 & -1 & 4 \\ 3 & 9 & 7 & 10 \end{bmatrix}$

Ans $\Rightarrow A = \begin{bmatrix} 1 & 3 & 8 & 6 \\ 2 & 6 & -1 & 4 \\ 3 & 9 & 7 & 10 \end{bmatrix}$

$$\begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array} \quad \begin{bmatrix} 1 & 3 & 8 & 6 \\ 0 & 0 & -17 & -8 \\ 0 & 0 & -17 & -8 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2 \quad \begin{bmatrix} 1 & 3 & 8 & 6 \\ 0 & 0 & -17 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 2}$$

Q.3) Find normal form of

$$\begin{bmatrix} 3 & 1 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

Ans \Rightarrow

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 2 & 0 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

 $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 3 & 2 \\ 3 & 1 & 2 & 1 \\ 3 & 1 & 4 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 3R_1$

$R_5 \rightarrow R_5 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -3 & -6 \\ 0 & -5 & -7 & -11 \\ 0 & -5 & -5 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \rightarrow C_2 - 2C_1$

$C_3 \rightarrow C_3 - 3C_1$

$C_4 \rightarrow C_4 - 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -4 & -3 & -6 \\ 0 & -5 & -7 & -11 \\ 0 & -5 & -5 & -11 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



$$\begin{array}{l}
 R_2 \rightarrow (-1)R_2 \\
 R_3 \rightarrow (-1)R_3 \\
 R_4 \rightarrow (-1)R_4
 \end{array}
 \begin{bmatrix}
 -1 & 0 & 0 & 0 \\
 0 & 4 & 3 & 6 \\
 0 & 5 & 7 & 11 \\
 0 & 5 & 5 & 11 \\
 0 & 0 & 0 & 0
 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3
 \begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & 4 & 3 & 6 \\
 0 & 5 & 7 & 11 \\
 0 & 0 & 2 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2
 \begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & 4 & 3 & 6 \\
 0 & 1 & 4 & 5 \\
 0 & 0 & 2 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3
 \begin{bmatrix}
 -1 & 0 & 0 & 0 \\
 0 & 1 & 4 & 5 \\
 0 & 4 & 3 & 6 \\
 0 & 0 & 2 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_4
 \begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 5 \\
 0 & 4 & 3 & 6 \\
 0 & 0 & 2 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 4 & 1 & 6 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_4 \rightarrow C_4 - C_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 4 & 1 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_4 \rightarrow C_4 - 4C_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & -14 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_4 \rightarrow \begin{pmatrix} -1 \\ 14 \end{pmatrix} C_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_2 \rightarrow C_2 - 4C_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_4 \rightarrow R_4 \times \frac{1}{2} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_4 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \leftrightarrow R_4 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} I_4 \\ 0 \end{bmatrix}$$

$$\boxed{\rho(A) = 4}$$

(Q2) Fill in the blanks -

1) If the matrix A have at least one minor of order 3 is non-zero & every minor of order 4 zero then rank of A is 3

2) By performing elementary operations if any non-zero matrix A of order 4×4 is reduced to the normal form $\begin{bmatrix} I_4 & 0 \\ 0 & 0 \end{bmatrix}$ then rank of A is 4

3) Normal form of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

4) When a system of equations have infinitely many solutions

\Rightarrow If $\rho(A) = \rho(A/B) < \text{no. of variables}$ and the matrix is consistent.