

Tutorial 6, 7

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Q1) Find Laplace transform of the following functions

$$\textcircled{1} F(t) = 4e^{5t} + 6t^3 - 3\sin 4t + 2\cos 2t$$

$$L[F(t)] = \frac{4}{s-5} + 6\left(\frac{6}{s^4}\right) - \frac{3(4)}{s^2+16} + \frac{2s}{s^2+4}$$

$$F(s) = \frac{4}{s-5} + \frac{36}{s^4} - \frac{12}{s^2+16} + \frac{2s}{s^2+4}$$

$$\textcircled{2} F(t) = \cos^2 2t + (e^t - 1)^2$$
$$\cos^2(2t) = \frac{\cos 4t + 1}{2}$$

$$\therefore F(t) = \frac{1}{2} + \frac{\cos 4t}{2} + e^{2t} - 2e^t + 1$$

$$\therefore L[F(t)] = \frac{1}{2s} + \frac{1}{2}\left(\frac{s}{s^2+16}\right) + \frac{1}{s-2} - 2\left(\frac{1}{s-1}\right) + \frac{1}{s}$$

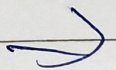
$$\therefore F(s) = \frac{1}{2s} + \frac{s}{2s^2+32} + \frac{1}{s-2} - \frac{2}{s-1} + \frac{1}{s}$$



$$\begin{aligned}
 (3) \quad F(t) &= \cos 3t \cdot \sin t \cdot \cos 2t \\
 &= \cos\left(\frac{4t+2t}{2}\right) \cdot \sin\left(\frac{4t-2t}{2}\right) \cdot \cos 2t \\
 &= \left(\frac{\sin 4t - \sin 2t}{2}\right) \cos 2t \\
 &= \frac{1}{2} (\sin 4t \cdot \cos 2t - \sin 2t \cdot \cos 2t) \\
 &= \frac{1}{2} \left(\sin\left(\frac{6t+2t}{2}\right) \cdot \cos\left(\frac{6t-2t}{2}\right) - \sin 2t \cos 2t \right) \\
 &= \frac{1}{2} (\sin 6t + \sin 2t - \sin 2t \cdot \cos 2t) \\
 &= \frac{1}{2} \left(\frac{\sin 6t + \sin 2t}{2} - \frac{\sin 4t}{2} \right) \\
 &= \frac{1}{4} (\sin 6t + \sin 2t - \sin 4t)
 \end{aligned}$$

$$\therefore L[F(t)] = \frac{1}{4} \left(\frac{6}{s^2+36} + \frac{2}{s^2+4} - \frac{4}{s^2+16} \right)$$

$$\therefore F(s) = \frac{3}{2s^2+72} + \frac{1}{2s^2+8} - \frac{1}{s^2+16}$$



$$\textcircled{4} f(t) = [3t^5 - 2e^{-5t} - 3\sin 6t]e^{2t}$$

$$L[f(t)] = ?$$

$$f(t) = 3e^{2t} \cdot t^5 - 2e^{-3t} - 3\sin 6t \cdot e^{2t}$$

$$L[f(t)] = 3(-1) \frac{d^5}{ds^5} \left(\frac{1}{s-2} \right) - \frac{2}{s+3} - 3L[\sin 6t e^{2t}]$$

$$= +3 \left(\frac{120}{(s-2)^6} \right) - \frac{2}{s+3} - 3L[\sin 6t e^{2t}]$$

$$= \frac{360}{(s-2)^6} - \frac{2}{s+3} - 3 \left(\frac{6}{(s-2)^2 + 36} \right)$$

$$\therefore F(s) = \frac{360}{(s-2)^6} - \frac{2}{s+3} - \frac{18}{(s-2)^2 + 36}$$

$$\textcircled{5} f(t) = \begin{cases} 0 & 0 < t < 1 \\ t & 1 < t < 4 \\ 0 & t > 4 \end{cases}$$

$$L[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$$

$$\therefore L[f(t)] = \int_0^1 e^{-st} f(t) dt + \int_1^4 e^{-st} f(t) dt + \int_4^{\infty} e^{-st} f(t) dt$$

$$= \int_0^1 e^{-st} (0) dt + \int_1^4 e^{-st} (t) dt + \int_4^{\infty} e^{-st} (0) dt$$

$$= \int_1^4 e^{-st} t dt + 0 + 0$$

→

$$= \int_1^4 e^{-st} dt$$

$$= t \int_1^4 e^{-st} dt + \int_1^4 (1) \frac{e^{-st}}{s} dt$$

~~$$= -\frac{t}{s} (4)$$~~

$$= -1 \left[\frac{e^{-st}}{s} \cdot t \right]_1^4 - \frac{1}{s} \left[\frac{e^{-st}}{s} \right]_1^4$$

$$= -\frac{1}{s} (4e^{-4s} - e^{-s}) - \frac{1}{s^2} (e^{-4s} - e^{-s})$$

$$\therefore F(s) = \frac{e^{-s}}{s} - \frac{4e^{-4s}}{s} + \frac{e^{-s}}{s^2} - \frac{e^{-4s}}{s^2}$$