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Tutorial 10 & 11

Page No.
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Q1) Find inverse Laplace Transform of the following -

a) $L^{-1}\left[\frac{1}{s^2+4s+5}\right]$

$$L^{-1}\left[\frac{1}{(s+2)^2+1^2}\right]$$

using property

$$L^{-1}[f(s+a)] = e^{-a} f(t)$$

$$L^{-1}\left[\frac{a}{s^2+a^2}\right] = \frac{\sin at}{a}$$

$$\therefore L^{-1}\left[\frac{1}{(s+2)^2+1^2}\right]$$

$$= e^{-2}\sin t$$

b) $L^{-1}\left[\frac{1}{s^2(s+3)}\right]$ use convolution property.

$$F(s) = \frac{1}{s^2} \quad G(s) = \frac{1}{s+3}$$

$$f(t) = L^{-1}\left[\frac{1}{s^2}\right] \quad g(t) = L^{-1}\left[\frac{1}{s+3}\right]$$

$f(t) = t \qquad g(t) = e^{-3t}$

$f(u) = u \qquad g(t-u) = e^{-3(t-u)}$

$$\therefore L^{-1}\left[\frac{1}{s^2(s+3)}\right] = \int_0^t u\, e^{-3(t-u)}\, du$$

$$= e^{-3t}\int_0^t u \cdot e^{3u}\, du$$

$$= e^{-3t}\left[\frac{ue^{3u}}{3} - \frac{e^{3u}}{9}\right]_0^t$$

$$= e^{-3t}\left[\frac{te^{3t}}{3} - \frac{e^{3t}}{9} + \frac{1}{9}\right]$$

$$= \frac{t}{3} - \frac{1}{9} + \frac{e^{-3t}}{9}$$

c) $L^{-1}\left[\frac{2s^2+15s+7}{(s+1)^2(s-2)}\right]$ use partial fraction form

$$\frac{2s^2+15s+7}{(s+1)^2(s-2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-2}$$

$$2s^2+15s+7 = A(s+1)(s-2) + B(s-2) + C(s+1)^2$$

putting $s=-1$

$-3B = -6$

$B = 2$

putting $s = 2$

$9C = 45$

$C = 5$

$\Rightarrow$

putting $s=0$

$$7 = -2A - 4 + 5$$
$$A = -3$$

$$L^{-1}\left[\frac{-1}{(s+1)} + \frac{2}{(s+1)^2} + \frac{5}{(s-2)}\right]$$

$$= -e^{-t} + 5e^{2t} + 2te^{-t}$$

Q2) Solve following diff eqn by L.T method

a) $y' - y = e^{3t}$, $y(0) = 2$

take L.T of the equation

$$L[y'(t)] - L[y(t)] - e^{3t}$$

$$sy(s) - y(0) - y(s) - \frac{1}{s-3} = 0$$

$$y(s)(s-1) - 2 - \frac{1}{s-3} = 0$$

$$y(s) = \left(2 + \frac{1}{(s-3)}\right)\frac{1}{s-1}$$

$$\therefore y(s) = \frac{2}{s-1} + \frac{1}{(s-1)(s-3)}$$

taking laplace inverse

$$L^{-1}[y(s)] = L^{-1}\left[\frac{2}{s-1}\right] + L^{-1}\left[\frac{1}{(s-1)(s-3)}\right]$$

$$\left[\frac{1}{(s-1)(s-3)}\right] = \frac{A}{s-1} + \frac{B}{s-3}$$

$$1 = A(s-3) + B(s-1)$$

putting $s = 3$

we get $2B = 1$

$B = 1/2$

putting $s = 1$

$A = -1/2$

$$\therefore L^{-1}\left[\frac{1}{(s-1)(s-3)}\right] = \frac{1}{2}\left(-\frac{1}{s-1} + \frac{1}{s-3}\right)$$

$$= \frac{1}{2}(e^{3t} - e^{t})$$

$$\therefore y(t) = 2e^{t} + \frac{1}{2}(e^{3t} - e^{t})$$

b) $s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0)$
$+9Y(s) = 0$

$\Rightarrow Y(s)(s^2-6s+9) - 2s - 9 + 12 = 0$

$$Y(s) = \frac{2s-9+12}{s^2+6s+9} = \frac{2s+3}{(s-3)^2}$$

$$\frac{2s+3}{(s-3)^2} = \frac{A}{(s-3)^2} + \frac{B}{s-3}$$

$\Rightarrow 2s+3 = A + B(s-3)$

$s=3$	$s=0$
$9=A \quad \therefore A=9$	$3 = 9 + B(-3)$
	$-6/-3 = B = 2$

$$y(t) = L^{-1}[Y(s)] = L^{-1}\left[\frac{9}{(s-3)^2} + \frac{2}{s-3}\right]$$

$= te^{3t} + 2e^{3t}$

$\therefore y(t) = e^{3t}(t+2)$

c) $s^2Y(s) - sy(0) - y(0) + 7[sY(s) - y(0)]$
$+10Y(s) = \frac{4}{s+3}$

$$Y(s)[s^2+7s+10] - 1 = \frac{4}{s+3}$$

$$Y(s) = \frac{4-s-3}{(s+3)(s^2+7s+10)} = \frac{1-s}{(s+3)(s+2)(3+s)}$$

$\rightarrow$

$$\frac{1-s}{(s+3)(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s+5}$$

$$1-s = A(s+3)(s+5) + B(s+2)(s+5) + C(s+2)(s+3)$$

$s = -2$

$3 = A(1)(3)$

$\therefore A = 1$

$s = -3$

$4 = (-1)(2)B$

$\therefore B = -2$

$s = -5$

$6 = C(-3)(-2)$

$C = -1$

$$\therefore L^{-1}\left[\frac{1}{s+2} - \frac{2}{s+3} - \frac{1}{s+5}\right] = y(t)$$

$$= e^{-2t} - 2e^{-3t} - e^{-5t}$$

$$\therefore y(t) = e^{-2t} - 2e^{-3t} - e^{-5t}$$