5.1 INTRODUCTION

In many industrial applications, it is required to convert a fixed-voltage dc source into a variable-voltage dc source. A dc–dc converter converts directly from dc to dc and is simply known as a dc converter. A dc converter can be considered as dc equivalent to an ac transformer with a continuously variable turns ratio. Like a transformer, it can be used to step down or step up a dc voltage source.

Dc converters are widely used for traction motor control in electric automobiles, trolley cars, marine hoists, forklift trucks, and mine haulers. They provide smooth acceleration control, high efficiency, and fast dynamic response. Dc converters can be used in regenerative braking of dc motors to return energy back into the supply, and this feature results in energy savings for transportation systems with frequent stops. Dc converters are used in dc voltage regulators; and also are used in conjunction with an inductor, to generate a dc current source, especially for the current source inverter. The dc–dc converters are integral parts of energy conversion in the evolving area of renewable energy technology.

5.2 PERFORMANCE PARAMETERS OF DC–DC CONVERTERS

Both the input and output voltages of a dc–dc converter are dc. This type of converter can produce a fixed or variable dc output voltage from a fixed or variable dc voltage as shown in Figure 5.1a. The output voltage and the input current should ideally be a pure dc, but the output voltage and the input current of a practical dc–dc converter contain harmonics or ripples as shown in Figures 5.1b and c. The converter draws current from the dc source only when the converter connects the load to the supply source and the input current is discontinuous.

The dc output power is

$$P_{dc} = I_a V_a \tag{5.1}$$

where V_a and I_a are the average load voltage and load current.

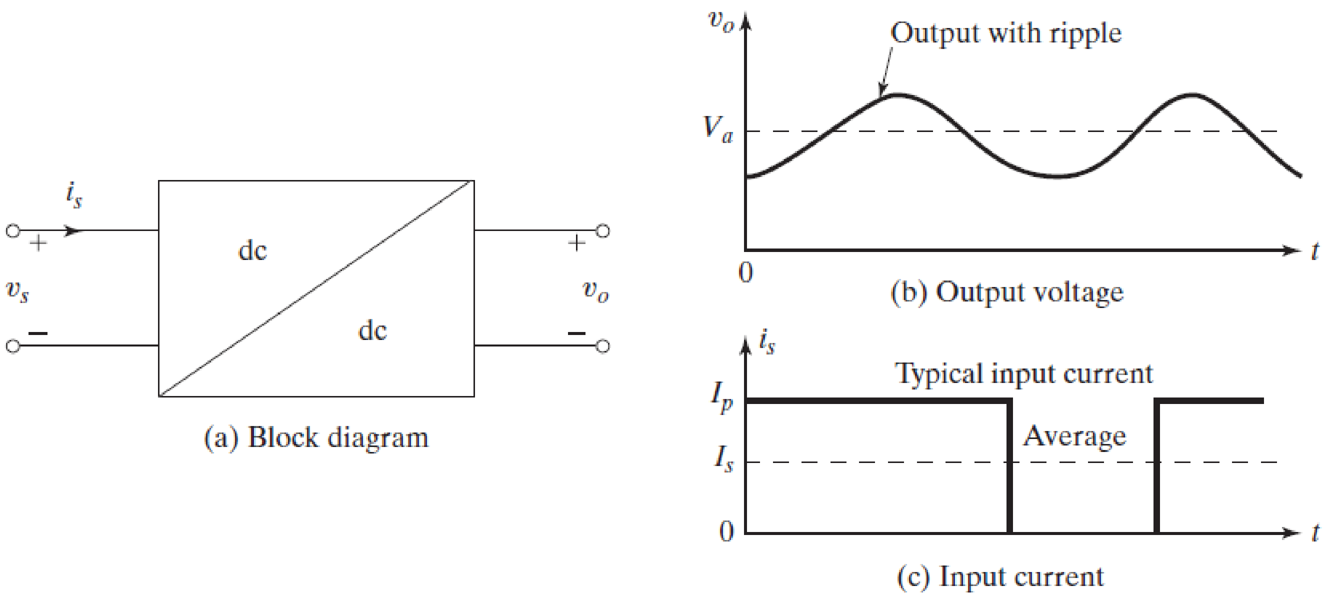


FIGURE 5.1

Input and output relationship of a dc–dc converter.

The ac output power is

$$P_{ac} = I_o V_o \tag{5.2}$$

where V_o and I_o are the rms load voltage and load current.

The converter efficiency (not the power efficiency) is

$$\eta_c = \frac{P_{dc}}{P_{ac}} \tag{5.3}$$

The rms ripple content of the output voltage is

$$V_r = \sqrt{V_o^2 - V_a^2} \tag{5.4}$$

The rms ripple content of the input current is

$$I_r = \sqrt{I_i^2 - I_s^2} \tag{5.5}$$

where I_i and I_s are the rms and average values of the dc supply current.

The ripple factor of the output voltage is

$$\mathrm{RF}_o = \frac{V_r}{V_a} \tag{5.6}$$

The ripple factor of the input current is

$$\mathrm{RF}_s = \frac{I_r}{I_s} \tag{5.7}$$

The power efficiency, which is the ratio of the output power to the input power, will depend on the switching losses, which in turn depend on the switching frequency of the converter. The switching frequency f should be high to reduce the values and sizes of capacitances and inductances. The designer has to compromise on these conflicting requirements. In general, f_s is higher than the audio frequency of 18 kHz.

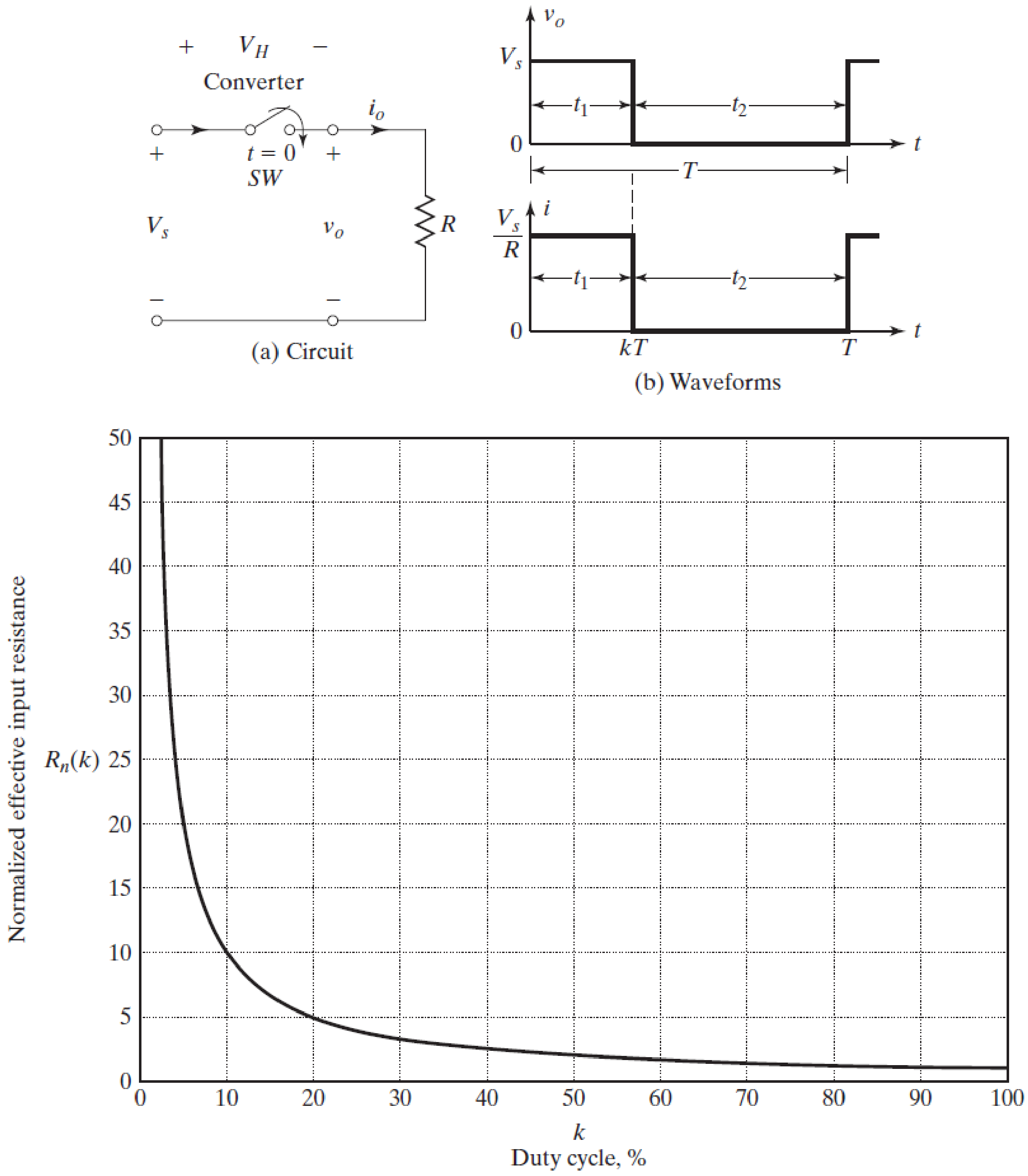
5.3 PRINCIPLE OF STEP-DOWN OPERATION

The principle of operation can be explained by Figure 5.2a. When switch SW, known as the chopper, is closed for a time t_1, the input voltage V_s appears across the load. If the switch remains off for a time t_2, the voltage across the load is zero. The waveforms for the output voltage and load current are also shown in Figure 5.2b. The converter switch can be implemented by using a (1) power bipolar junction transistor (BJT), (2) power metal oxide semiconductor field-effect transistor (MOSFET), (3) gate-turn-off thyristor (GTO), or (4) insulated-gate bipolar transistor (IGBT). The practical devices have a finite voltage drop ranging from 0.5 to 2 V, and for the sake of simplicity we shall neglect the voltage drops of these power semiconductor devices.

The average output voltage is given by

$$V_a = \frac{1}{T}\int_0^{t_1} v_0 \, dt = \frac{t_1}{T}V_s = ft_1 V_s = kV_s \tag{5.8}$$

and the average load current, $I_a = V_a/R = kV_s/R$,



(c) Effective input resistance against duty cycle

FIGURE 5.2

Step-down converter with resistive load.

where T is the chopping period;

$k = t_1/T$ is the duty cycle of chopper;

f is the chopping frequency.

The rms value of output voltage is found from

$$V_o = \left(\frac{1}{T}\int_0^{kT} v_0^2\,dt\right)^{1/2} = \sqrt{k}\,V_s \tag{5.9}$$

Assuming a lossless converter, the input power to the converter is the same as the output power and is given by

$$P_i = \frac{1}{T}\int_0^{kT} v_0 i \, dt = \frac{1}{T}\int_0^{kT} \frac{v_0^2}{R} dt = k\frac{V_s^2}{R} \tag{5.10}$$

The effective input resistance seen by the source is

$$R_i = \frac{V_s}{I_a} = \frac{V_s}{kV_s/R} = \frac{R}{k} \tag{5.11}$$

which indicates that the converter makes the input resistance R_i as a variable resistance of R/k. The variation of the normalized input resistance against the duty cycle is shown in Figure 5.2c. It should be noted that the switch in Figure 5.2 could be implemented by a BJT, a MOSFET, an IGBT, or a GTO.

The duty cycle k can be varied from 0 to 1 by varying t_1, T, or f. Therefore, the output voltage V_o can be varied from 0 to V_s by controlling k, and the power flow can be controlled.

1. *Constant-frequency operation*: The converter, or switching, frequency f (or chopping period T) is kept constant and the on-time t_1 is varied. The width of the pulse is varied and this type of control is known as *pulse-width-modulation* (PWM) control.
2. *Variable-frequency operation*: The chopping, or switching, frequency f is varied. Either on-time t_1 or off-time t_2 is kept constant. This is called *frequency modulation*. The frequency has to be varied over a wide range to obtain the full output voltage range. This type of control would generate harmonics at unpredictable frequencies and the filter design would be difficult.

Note: The efficiency calculation, which includes the conduction loss of the converter, does not take into account the switching loss due to turn-on and turn-off of practical converters. The efficiency of a practical converter varies between 92 and 99%.

Key Points of Section 5.3

- A step-down chopper, or dc converter, that acts as a variable resistance load can produce an output voltage from 0 to V_S.
- Although a dc converter can be operated either at a fixed or variable frequency, it is usually operated at a fixed frequency with a variable duty cycle.
- The output voltage contains harmonics and a dc filter is needed to smooth out the ripples.

5.3.1 Generation of Duty Cycle

The duty cycle k can be generated by comparing a dc reference signal v_r with a saw-tooth carrier signal v_{cr}. This is shown in Figure 5.3, where V_r is the peak value of v_r, and V_{cr} is the peak value of v_{cr}. The reference signal v_r is given by

$$v_r = \frac{V_r}{T} t \tag{5.17}$$

which must equal to the carrier signal $v_{cr} = V_{cr}$ at kT. That is,

$$V_{cr} = \frac{V_r}{T} kT$$

which gives the duty cycle k as

$$k = \frac{V_{cr}}{V_r} = M \tag{5.18}$$

where M is called the *modulation index*. By varying the carrier signal v_{cr} from 0 to V_{cr}, the duty cycle k can be varied from 0 to 1.

The algorithm to generate the gating signal is as follows:

1. Generate a triangular waveform of period T as the reference signal v_r and a dc carrier signal v_{cr}.
2. Compare these signals by a comparator to generate the difference $v_r - v_{cr}$ and then a hard limiter to obtain a square-wave gate pulse of width kT, which must be applied to the switching device through an isolating circuit.
3. Any variation in v_{cr} varies linearly with the duty cycle k.

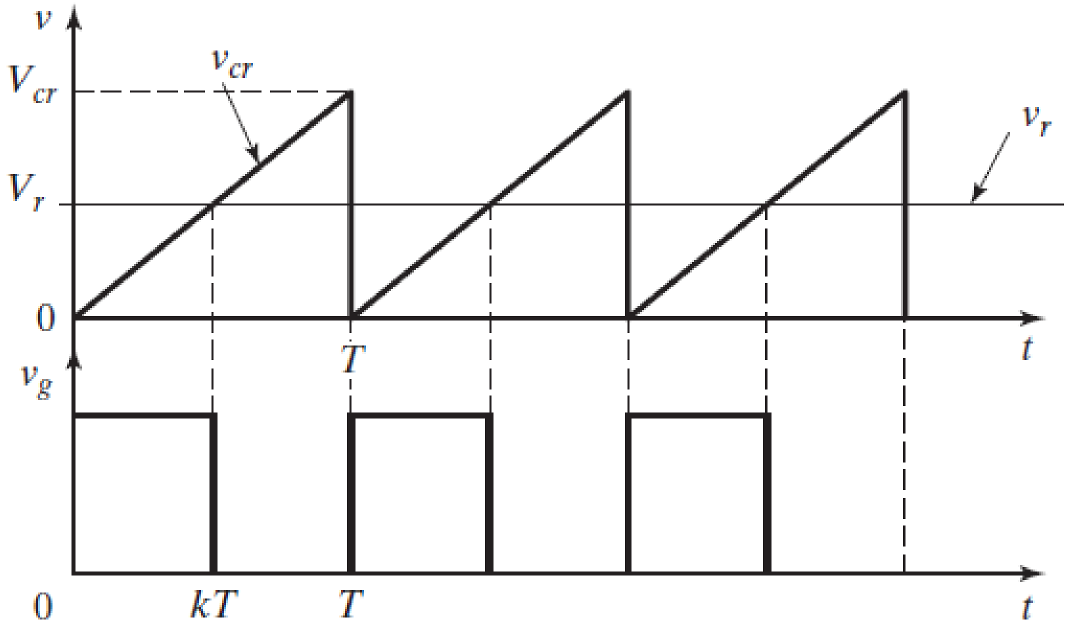


FIGURE 5.3

Comparing a reference signal with a carrier signal.

5.4 STEP-DOWN CONVERTER WITH *RL* LOAD

A converter [1] with an *RL* load is shown in Figure 5.4. The operation of the converter can be divided into two modes. During mode 1, the converter is switched on and the current flows from the supply to the load. During mode 2, the converter is switched off and the load current continues to flow through freewheeling diode D_m. The equivalent circuits for these modes are shown in Figure 5.5a. The load current and output voltage waveforms are shown in Figure 5.5b with the assumption that the load current rises linearly. However, the current flowing through an *RL* load rises or falls exponentially with a time constant. The load time constant $(\tau = L/R)$ is generally much higher than the switching period T. Thus, the linear approximation is valid for many circuit conditions and simplified expressions can be derived within reasonable accuracies.

The load current for mode 1 can be found from

$$V_s = Ri_1 + L\frac{di_1}{dt} + E$$

which with initial current $i_1(t = 0) = I_1$ gives the load current as

$$i_1(t) = I_1 e^{-tR/L} + \frac{V_s - E}{R}\left(1 - e^{-tR/L}\right) \tag{5.19}$$

This mode is valid $0 \le t \le t_1\,(=kT)$; and at the end of this mode, the load current becomes

$$i_1(t = t_1 = kT) = I_2 \tag{5.20}$$

The load current for mode 2 can be found from

$$0 = Ri_2 + L\frac{di_2}{dt} + E$$

With initial current $i_2(t = 0) = I_2$ and redefining the time origin (i.e., $t = 0$) at the beginning of mode 2, we have

$$i_2(t) = I_2 e^{-tR/L} - \frac{E}{R}\left(1 - e^{-tR/L}\right) \tag{5.21}$$

This mode is valid for $0 \le t \le t_2\,[=(1 - k)T]$. At the end of this mode, the load current becomes

$$i_2(t = t_2) = I_3 \tag{5.22}$$

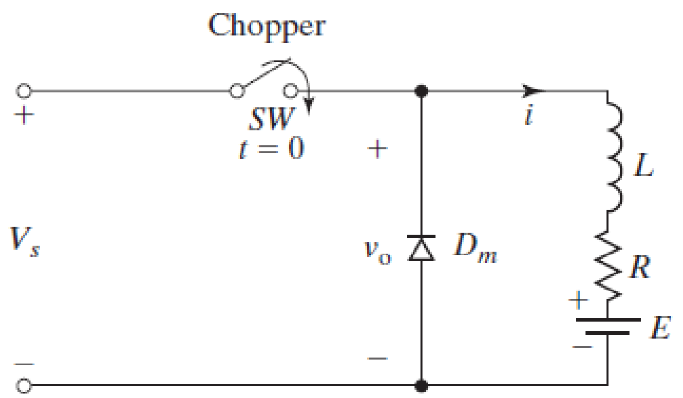


FIGURE 5.4
Dc converter with *RL* loads.

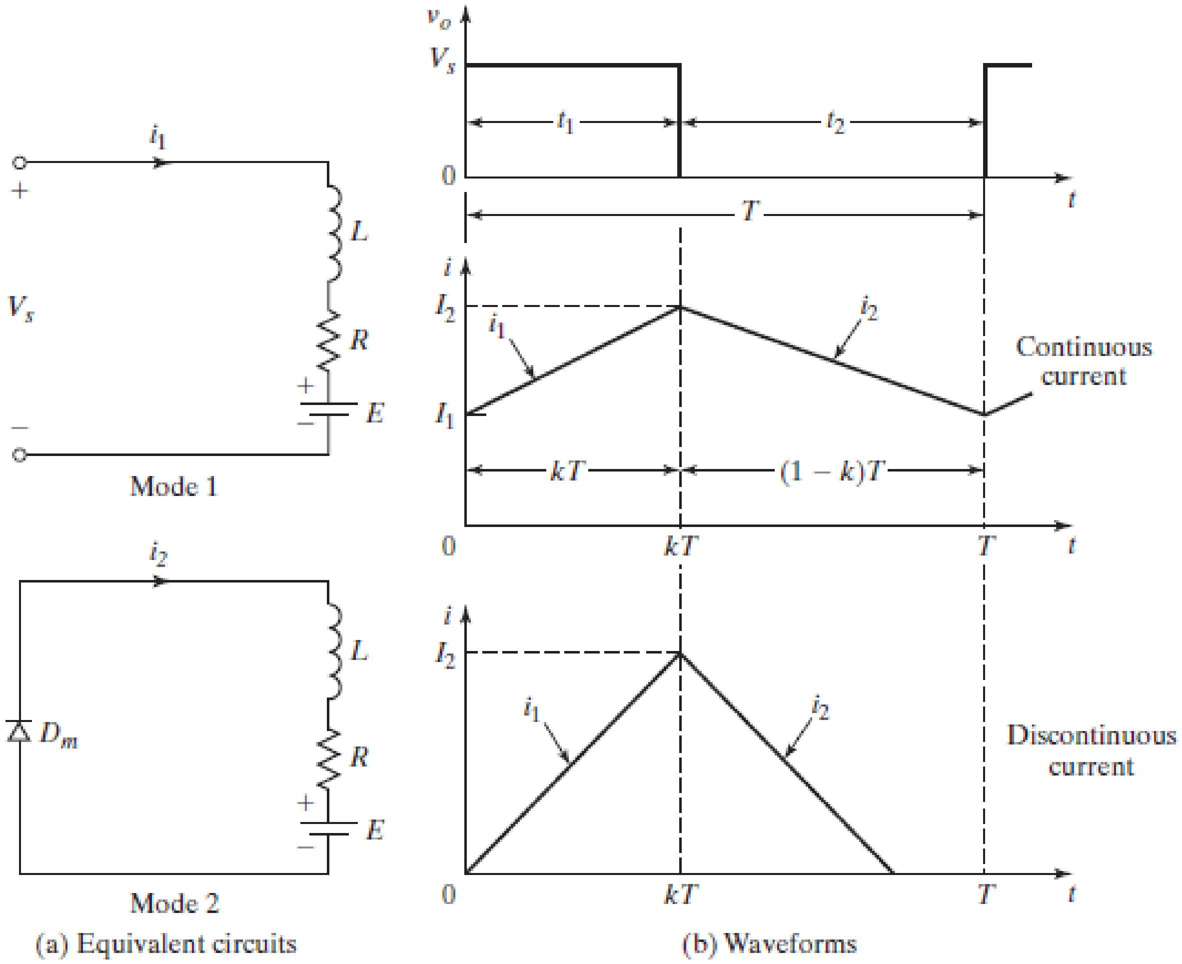


FIGURE 5.5

Equivalent circuits and waveforms for RL loads.

At the end of mode 2, the converter is turned on again in the next cycle after time, $T = 1/f = t_1 + t_2$.

Under steady-state conditions, $I_1 = I_3$. The peak-to-peak load ripple current can be determined from Eqs. (5.19) to (5.22). From Eqs. (5.19) and (5.20), I_2 is given by

$$I_2 = I_1 e^{-kTR/L} + \frac{V_s - E}{R}\left(1 - e^{-kTR/L}\right) \tag{5.23}$$

From Eqs. (5.21) and (5.22), I_3 is given by

$$I_3 = I_1 = I_2 e^{-(1-k)TR/L} - \frac{E}{R}\left(1 - e^{-(1-k)TR/L}\right) \tag{5.24}$$

Solving for I_1 and I_2 we get

$$I_1 = \frac{V_S}{R}\left(\frac{e^{kz} - 1}{e^{z} - 1}\right) - \frac{E}{R} \tag{5.25}$$

where $z = \dfrac{TR}{L}$ is the ratio of the chopping or switching period to the load time constant.

$$I_2 = \frac{V_s}{R}\left(\frac{e^{-kz} - 1}{e^{-z} - 1}\right) - \frac{E}{R} \tag{5.26}$$

The peak-to-peak ripple current is

$$\Delta I = I_2 - I_1$$

which after simplifications becomes

$$\Delta I = \frac{V_s}{R}\frac{1 - e^{-kz} + e^{-z} - e^{-(1-k)z}}{1 - e^{-z}} \tag{5.27}$$

The condition for maximum ripple,

$$\frac{d(\Delta I)}{dk} = 0 \tag{5.28}$$

gives $e^{-kz} - e^{-(1-k)z} = 0$ or $-k = -(1 - k)$ or $k = 0.5$. The maximum peak-to-peak ripple current (at $k = 0.5$) is

$$\Delta I_{\max} = \frac{V_s}{R}\tanh\frac{R}{4fL} \tag{5.29}$$

For $4fL \gg R$, $\tanh\theta \approx \theta$ and the maximum ripple current can be approximated to

$$\Delta I_{\max} = \frac{V_s}{4fL} \tag{5.30}$$

Note: Equations (5.19) to (5.30) are valid only for continuous current flow. For a large off-time, particularly at low-frequency and low-output voltage, the load current may be discontinuous. The load current would be continuous if $L/R \gg T$ or $Lf \gg R$. In case of discontinuous load current, $I_1 = 0$ and Eq. (5.19) becomes

$$i_1(t) = \frac{V_s - E}{R}(1 - e^{-tR/L})$$

and Eq. (5.21) is valid for $0 \leq t \leq t_2$ such that $i_2(t = t_2) = I_3 = I_1 = 0$, which gives

$$t_2 = \frac{L}{R}\ln\left(1 + \frac{RI_2}{E}\right)$$

Because at $t = kT$, we get

$$i_1(t) = I_2 = \frac{V_s - E}{R}\left(1 - e^{-kz}\right)$$

which after substituting for I_2 becomes

$$t_2 = \frac{L}{R}\ln\left[1 + \left(\frac{V_s - E}{E}\right)\left(1 - e^{-kz}\right)\right]$$

Condition for continuous current: For $I_1 \geq 0$, Eq. (5.25) gives

$$\left(\frac{e^{kz} - 1}{e^{z} - 1} - \frac{E}{V_s}\right) \geq 0$$

which gives the value of the load electromotive force (emf) ratio $x = E/V_s$ as

$$x = \frac{E}{V_s} \leq \frac{e^{kz} - 1}{e^{z} - 1} \tag{5.31}$$

5.5 PRINCIPLE OF STEP-UP OPERATION

A converter can be used to step up a dc voltage and an arrangement for step-up operation is shown in Figure 5.7a. When switch SW is closed for time t_1, the inductor current rises and energy is stored in the inductor L. If the switch is opened for time t_2, the energy stored in the inductor is transferred to load through diode D_1 and the inductor current falls. Assuming a continuous current flow, the waveform for the inductor current is shown in Figure 5.7b.

When the converter is turned on, the voltage across the inductor is

$$v_L = L\frac{di}{dt}$$

and this gives the peak-to-peak ripple current in the inductor as

$$\Delta I = \frac{V_s}{L}t_1 \tag{5.34}$$

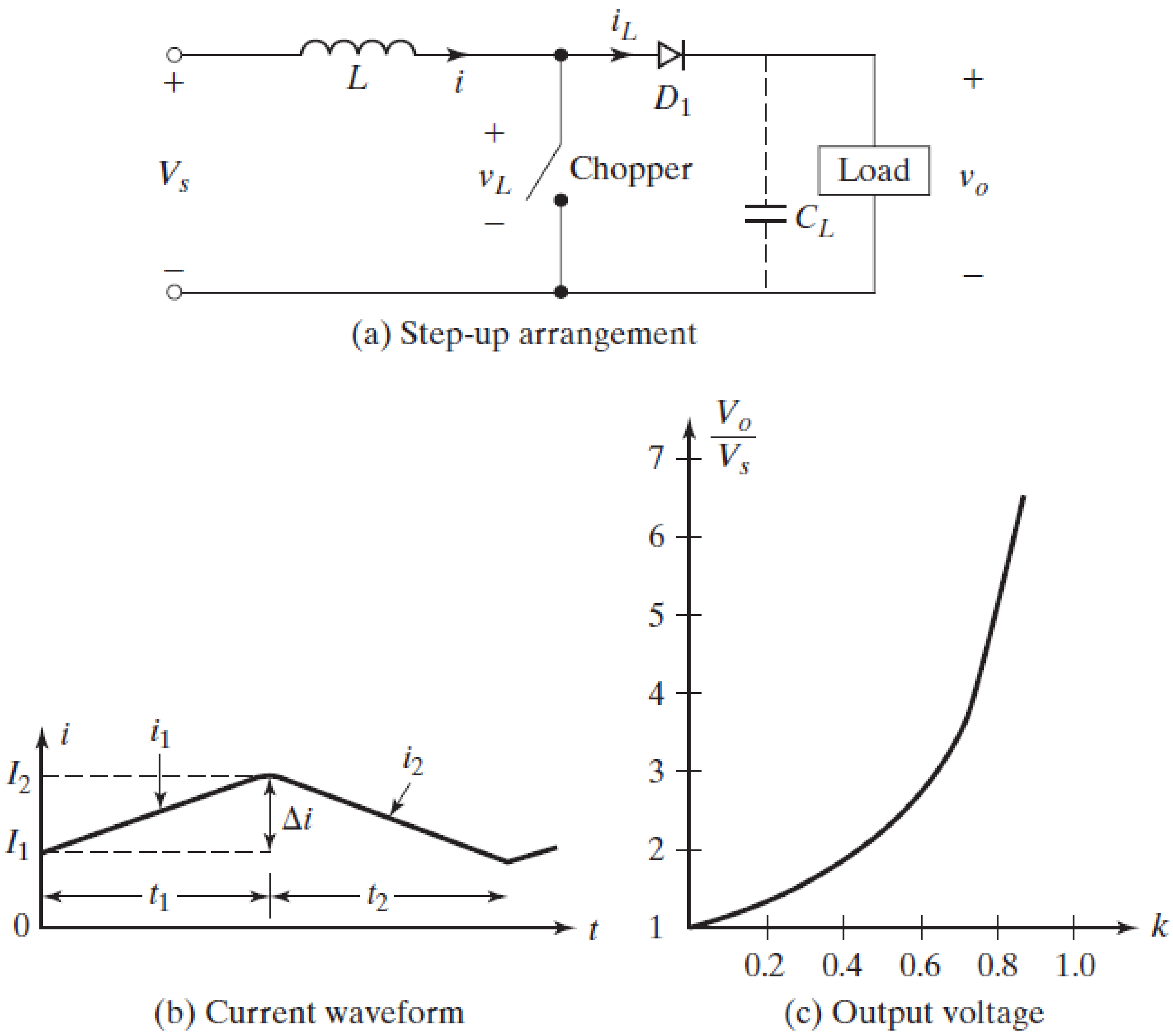


FIGURE 5.7

Arrangement for step-up operation.

The average output voltage is

$$v_o = V_s + L\frac{\Delta I}{t_2} = V_s\left(1 + \frac{t_1}{t_2}\right) = V_s\frac{1}{1 - k} \tag{5.35}$$

If a large capacitor C_L is connected across the load as shown by dashed lines in Figure 5.7a, the output voltage is continuous and v_o becomes the average value V_a. We can notice from Eq. (5.35) that the voltage across the load can be stepped up by varying the duty cycle k and the minimum output voltage is V_s when $k = 0$. However, the converter cannot be switched on continuously such that $k = 1$. For values of k tending to unity, the output voltage becomes very large and is very sensitive to changes in k, as shown in Figure 5.7c.

This principle can be applied to transfer energy from one voltage source to another as shown in Figure 5.8a. The equivalent circuits for the modes of operation are shown in Figure 5.8b and the current waveforms in Figure 5.8c. The inductor current for mode 1 is given by

$$V_s = L\frac{di_1}{dt}$$

and is expressed as

$$i_1(t) = \frac{V_s}{L}t + I_1 \tag{5.36}$$

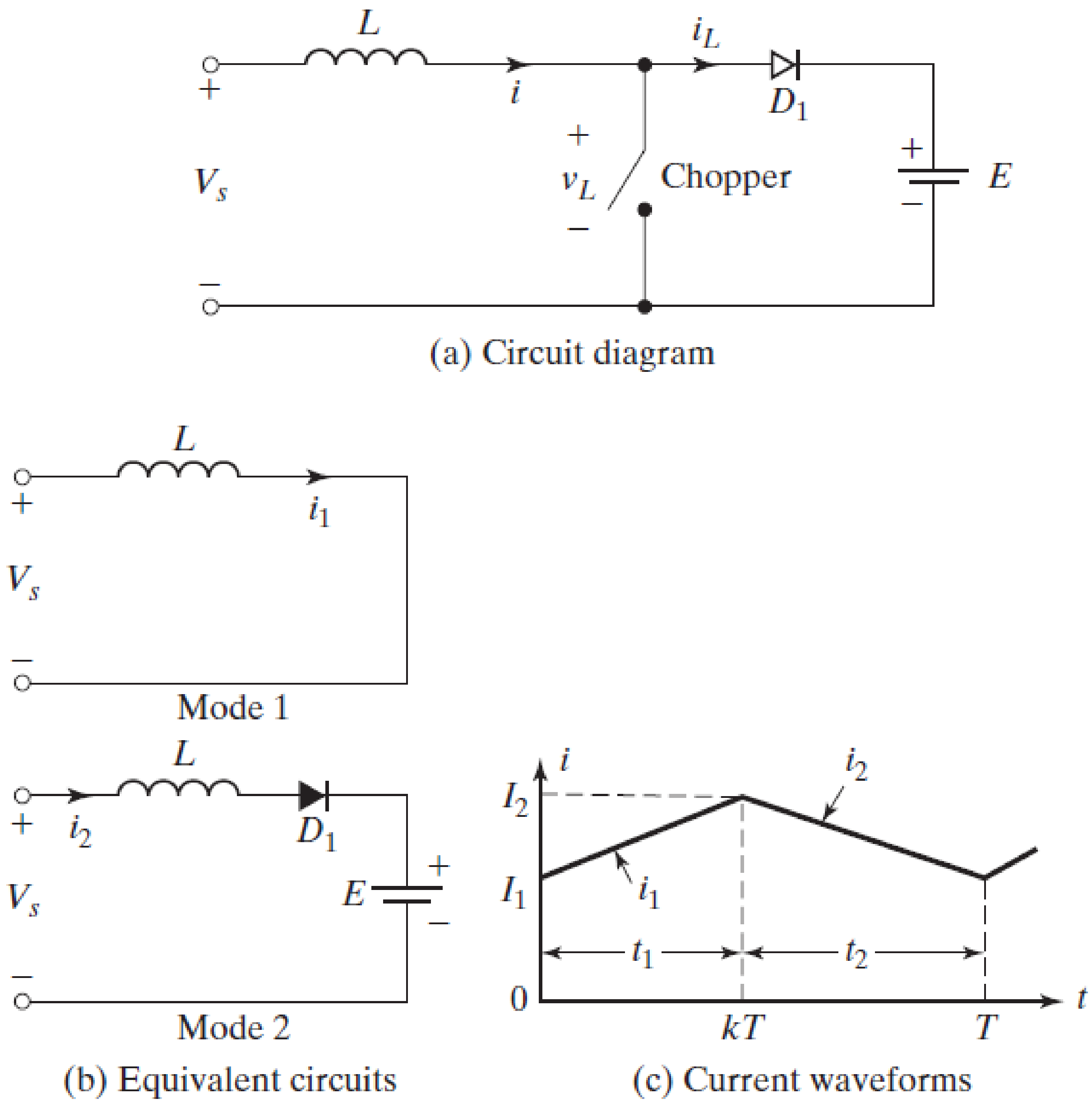


FIGURE 5.8

Arrangement for transfer of energy.

where I_1 is the initial current for mode 1. During mode 1, the current must rise and the necessary condition,

$$\frac{di_1}{dt} > 0 \quad \text{or} \quad V_s > 0$$

The current for mode 2 is given by

$$V_s = L\frac{di_2}{dt} + E$$

and is solved as

$$i_2(t) = \frac{V_s - E}{L}t + I_2 \tag{5.37}$$

where I_2 is the initial current for mode 2. For a stable system, the current must fall and the condition is

$$\frac{di_2}{dt} < 0 \quad \text{or} \quad V_s < E$$

If this condition is not satisfied, the inductor current continues to rise and an unstable situation occurs. Therefore, the conditions for controllable power transfer are

$$0 < V_s < E \tag{5.38}$$

Equation (5.38) indicates that the source voltage V_s must be less than the voltage E to permit transfer of power from a fixed (or variable) source to a fixed dc voltage. In electric braking of dc motors, where the motors operate as dc generators, terminal voltage falls as the machine speed decreases. The converter permits transfer of power to a fixed dc source or a rheostat.

When the converter is turned on, the energy is transferred from the source V_s to inductor L. If the converter is then turned off, a part of the energy stored in the inductor is forced to battery E.

Note: Without the chopping action, v_s must be greater than E for transferring power from V_s to E.

Key Points of Section 5.5

- A step-up dc converter can produce an output voltage that is higher than the input. The input current can be transferred to a voltage source higher than the input voltage.

5.7 FREQUENCY LIMITING PARAMETERS

The power semiconductor devices require a minimum time to turn on and turn off. Therefore, the duty cycle k can only be controlled between a minimum value k_{min} and a maximum value k_{max}, thereby limiting the minimum and maximum value of output voltage. The switching frequency of the converter is also limited. It can be noticed from Eq. (5.30) that the load ripple current depends inversely on the chopping frequency f. The frequency should be as high as possible to reduce the load ripple current and to minimize the size of any additional series inductor in the load circuit.

The frequency limiting parameters of the step-up and step-down converters are as follows:

Ripple current of the inductor, ΔI_L;
Maximum switching frequency, f_{max};
Condition for continuous or discontinuous inductor current;
Minimum value of inductor to maintain continuous inductor current;

Ripple content of the output voltage and output current, also known as the total harmonic content THD;
Ripple content of the input current, THD.

5.8 CONVERTER CLASSIFICATION

The step-down converter in Figure 5.2a only allows power to flow from the supply to the load, and is referred to as the first quadrant converter. Connecting an antiparallel diode across a transistor switch allows bidirectional current flow operating in two quadrants. Reversing the polarity of the voltage across the load allows bidirectional voltage. Depending on the directions of current and voltage flows, dc converters can be classified into five types:

1. First quadrant converter
2. Second quadrant converter
3. First and second quadrant converter
4. Third and fourth quadrant converter
5. Four-quadrant converter

First quadrant converter. The load current flows into the load. Both the load voltage and the load current are positive, as shown in Figure 5.11a. This is

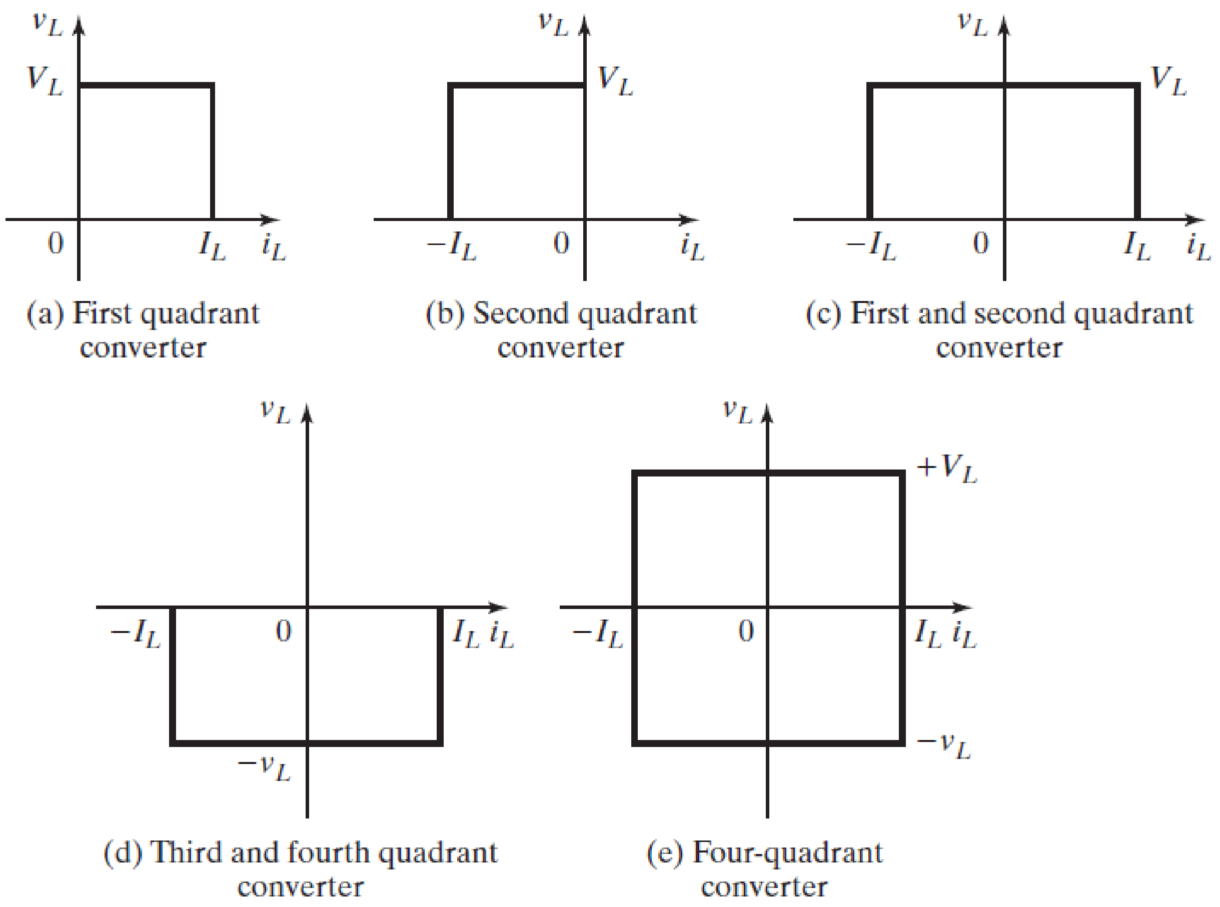


FIGURE 5.11

Dc converter classification.

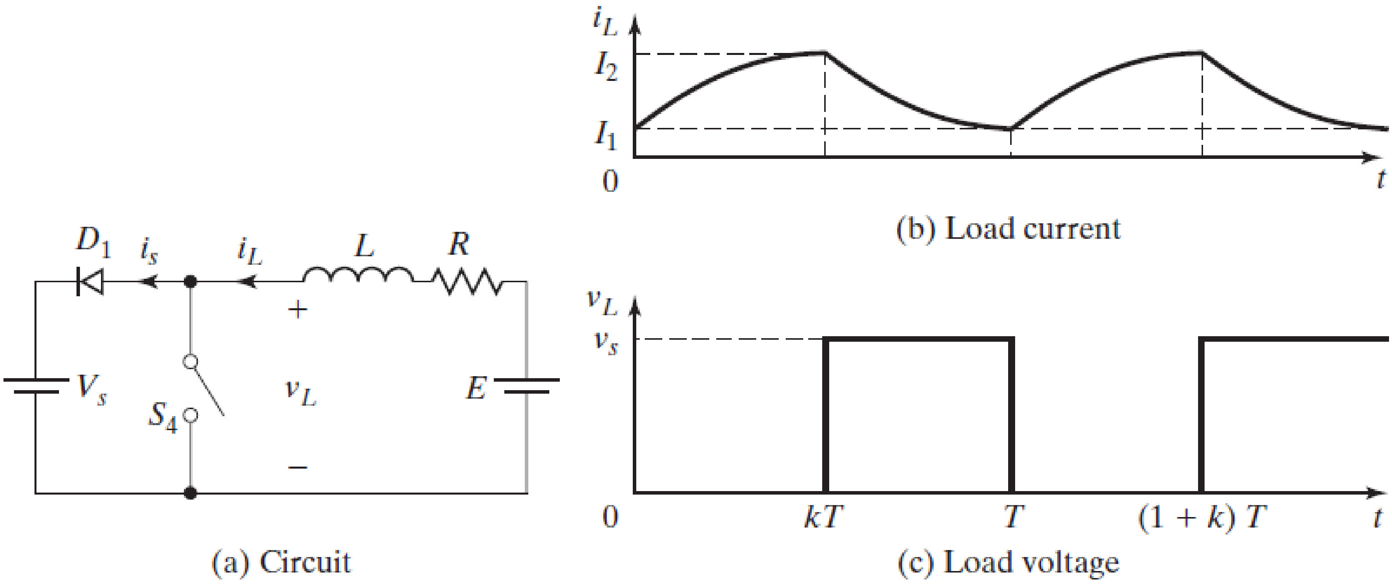


FIGURE 5.12

Second quadrant converter.

a single-quadrant converter and is said to be operated as a rectifier. Equations in Sections 5.3 and 5.4 can be applied to evaluate the performance of a first quadrant converter.

Second quadrant converter. The load current flows out of the load. The load voltage is positive, but the load current is negative, as shown in Figure 5.11b. This is also a single-quadrant converter, but operates in the second quadrant and is said to be operated as an inverter. A second quadrant converter is shown in Figure 5.12a, where the battery E is a part of the load and may be the back emf of a dc motor.

When switch S_4 is turned on, the voltage E drives current through inductor L and load voltage v_L becomes zero. The instantaneous load voltage v_L and load current i_L are shown in Figure 5.12b and 5.12c, respectively. The current i_L, which rises, is described by

$$0 = L\frac{di_L}{dt} + Ri_L - E$$

which, with initial condition $i_L(t = 0) = I_1$, gives

$$i_L = I_1 e^{-(R/L)t} + \frac{E}{R}\left(1 - e^{-(R/L)t}\right) \quad \text{for } 0 \le t \le kT \tag{5.46}$$

At $t = t_1$,

$$i_L(t = t_1 = kT) = I_2 \tag{5.47}$$

When switch S_4 is turned off, a magnitude of the energy stored in inductor L is returned to the supply V_s via diode D_1. The load current i_L falls. Redefining the time origin $t = 0$, the load current i_L is described by

$$-V_s = L\frac{di_L}{dt} + Ri_L - E$$

which, with initial condition $i(t = t_2) = I_2$, gives

$$i_L = I_2 e^{-(R/L)t} + \frac{-V_s + E}{R}\left(1 - e^{-(R/L)t}\right) \quad \text{for } 0 \le t \le t_2 \tag{5.48}$$

where $t_2 = (1 - k)T$. At $t = t_2$,

$$\begin{aligned} i_L(t = t_2) &= I_1 \quad \text{for steady-state continuous current} \\ &= 0 \quad \text{for steady-state discontinuous current} \end{aligned} \tag{5.49}$$

Using the boundary conditions in Eqs. (5.47) and (5.49), we can solve for I_1 and I_2 as

$$I_1 = \frac{-V_S}{R}\left[\frac{1 - e^{-(1-k)z}}{1 - e^{-z}}\right] + \frac{E}{R} \tag{5.50}$$

$$I_2 = \frac{-V_S}{R}\left(\frac{e^{-kz} - e^{-z}}{1 - e^{-z}}\right) + \frac{E}{R} \tag{5.51}$$

where $z = TR/L$.

First and second quadrant converter. The load current is either positive or negative, as shown in Figure 5.11c. The load voltage is always positive. This is known as a *two-quadrant converter*. The first and second quadrant converters can be combined to form this converter, as shown in Figure 5.13. S_1 and D_4 operate as a first quadrant converter. S_4 and D_1 operate as a second quadrant converter. Care must be taken to ensure that the two switches are not fired together; otherwise, the supply V_s becomes short-circuited. This type of converter can operate either as a rectifier or as an inverter.

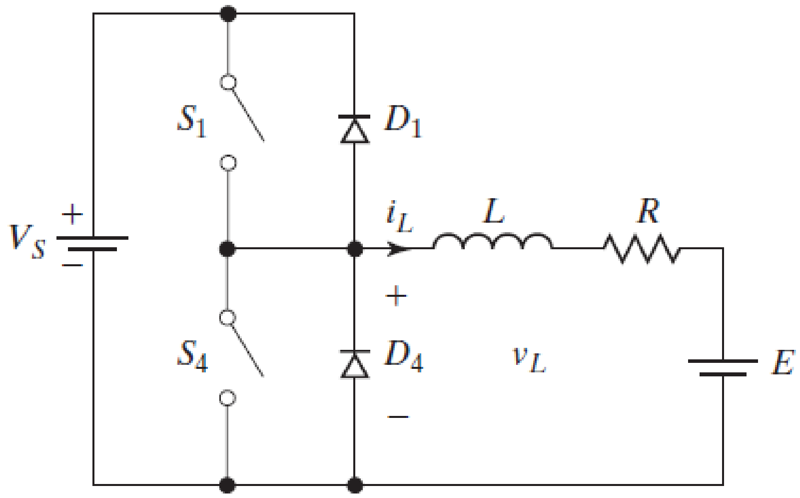
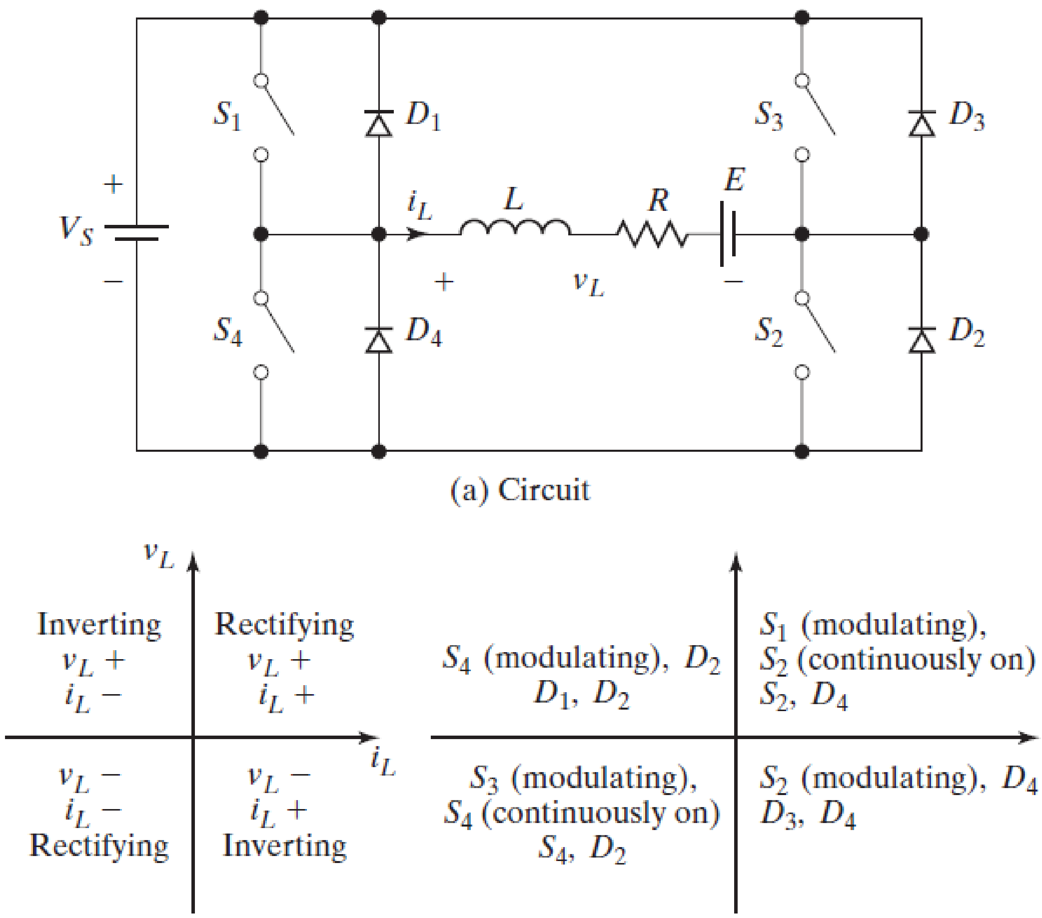


FIGURE 5.13

First and second quadrant converter.

Four-quadrant converter [2]. The load current is either positive or negative, as shown in Figure 5.11e. The load voltage is also either positive or negative. One first and second quadrant converter and one third and fourth quadrant converter can be combined to form the four-quadrant converter, as shown in Figure 5.15a. The polarities of the load voltage and load currents are shown in Figure 5.15b. The devices that are operative in different quadrants are shown in Figure 5.15c. For operation in the fourth quadrant, the direction of the battery E must be reversed. This converter forms the basis for the single-phase full-bridge inverter in Section 6.4.

For an inductive load with an emf (E) such as a dc motor, the four-quadrant converter can control the power flow and the motor speed in the forward direction



(a) Circuit

(b) Polarities

(c) Conducting devices

FIGURE 5.15

Four-quadrant converter.

(v_L positive and i_L positive), forward regenerative braking (v_L positive and i_L negative), reverse direction (v_L negative and i_L negative), and reverse regenerative braking (v_L negative and i_L positive).

Key Points of Section 5.8

- With proper switch control, the four-quadrant converter can operate and control flow in any of the four quadrants. For operation in the third and fourth quadrants, the direction of the load emf E must be reversed internally.

5.9 SWITCHING-MODE REGULATORS

Dc converters can be used as switching-mode regulators to convert a dc voltage, normally unregulated, to a regulated dc output voltage. The regulation is normally achieved by PWM at a fixed frequency and the switching device is normally BJT, MOSFET, or IGBT. The elements of switching-mode regulators are shown in Figure 5.16. We can notice from Figure 5.2b that the output of dc converters with resistive load is discontinuous and contains harmonics. The ripple content is normally reduced by an *LC* filter.

Switching regulators are commercially available as integrated circuits. The designer can select the switching frequency by choosing the values of *R* and *C* of frequency oscillator. As a rule of thumb, to maximize efficiency, the minimum oscillator period should be about 100 times longer than the transistor switching time; for example, if a transistor has a switching time of 0.5 μs, the oscillator period would be 50 μs, which gives the maximum oscillator frequency of 20 kHz. This limitation is due to a switching loss in the transistor. The transistor switching loss increases with the switching frequency and as a result the efficiency decreases. In addition, the core loss of inductors limits the high-frequency operation. Control voltage v_c is obtained by comparing the output voltage with its desired value. The v_{cr} can be compared with a sawtooth voltage v_r to generate the PWM control signal for the dc converter. There are four basic topologies of switching regulators [33, 34]:

1. Buck regulators
2. Boost regulators
3. Buck–boost regulators
4. Cúk regulators

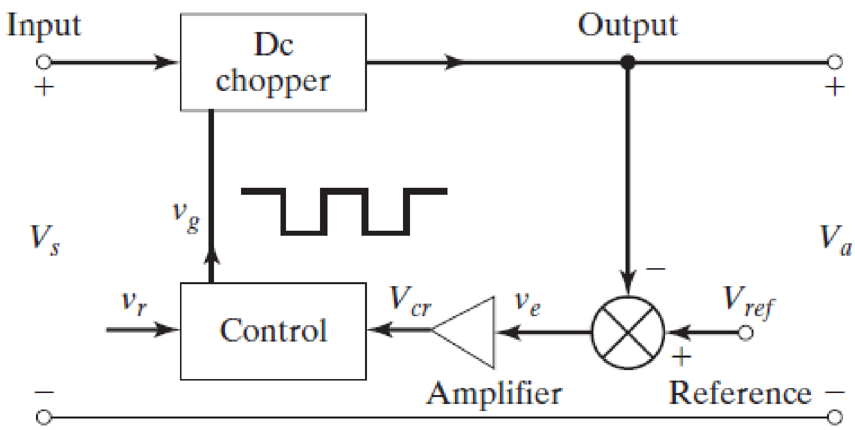


FIGURE 5.16

Elements of switching-mode regulators.

5.9.1 Buck Regulators

In a buck regulator, the average output voltage V_a is less than the input voltage, V_s—hence the name "buck," a very popular regulator [6, 7]. The circuit diagram of a buck regulator using a power BJT is shown in Figure 5.17a, and this is like a step-down converter. Transistor Q_1 acts as a controlled switch and diode D_m is an uncontrolled switch. They operate as two single-pole-single-through (SPST) bidirectional switches. The circuit in Figure 5.17a is often represented by two switches as shown in Figure 5.17b. The circuit operation can be divided into two modes. Mode 1 begins when transistor Q_1 is switched on at $t = 0$. The input current, which rises, flows through filter inductor L, filter capacitor C, and load resistor R. Mode 2 begins when transistor Q_1 is switched off at $t = t_1$. The freewheeling diode D_m conducts due to energy stored in the inductor, and the inductor current continues to flow through L, C, load, and diode D_m. The inductor current falls until transistor Q_1 is switched on again in the next cycle. The equivalent circuits for the modes of operation are shown in Figure 5.17c. The waveforms for the voltages and currents are shown in Figure 5.17d for a continuous current flow in the inductor L. It is assumed that the current rises and falls linearly. In practical circuits, the switch has a finite, nonlinear resistance. Its effect can generally be negligible in most applications. Depending on the switching frequency, filter inductance, and capacitance, the inductor current could be discontinuous.

The voltage across the inductor L is, in general,

$$e_L = L\frac{di}{dt}$$

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s - V_a = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{5.52}$$

or

$$t_1 = \frac{\Delta I\, L}{V_s - V_a} \tag{5.53}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

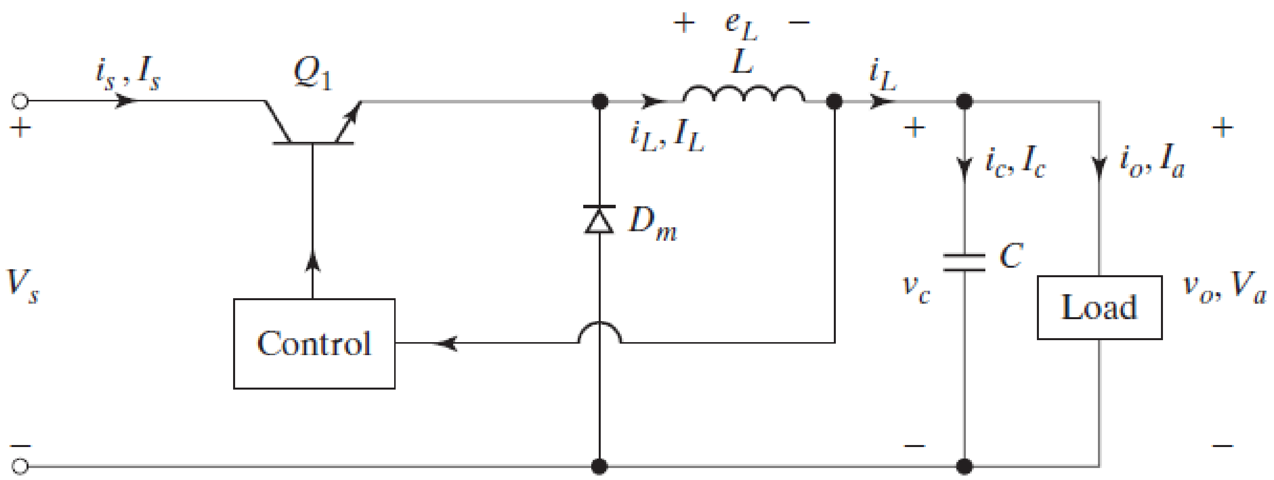
$$-V_a = -L\frac{\Delta I}{t_2} \tag{5.54}$$

or

$$t_2 = \frac{\Delta I\, L}{V_a} \tag{5.55}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of the inductor L. Equating the value of ΔI in Eqs. (5.52) and (5.54) gives

$$\Delta I = \frac{(V_s - V_a)t_1}{L} = \frac{V_a t_2}{L}$$



(a) Circuit diagram

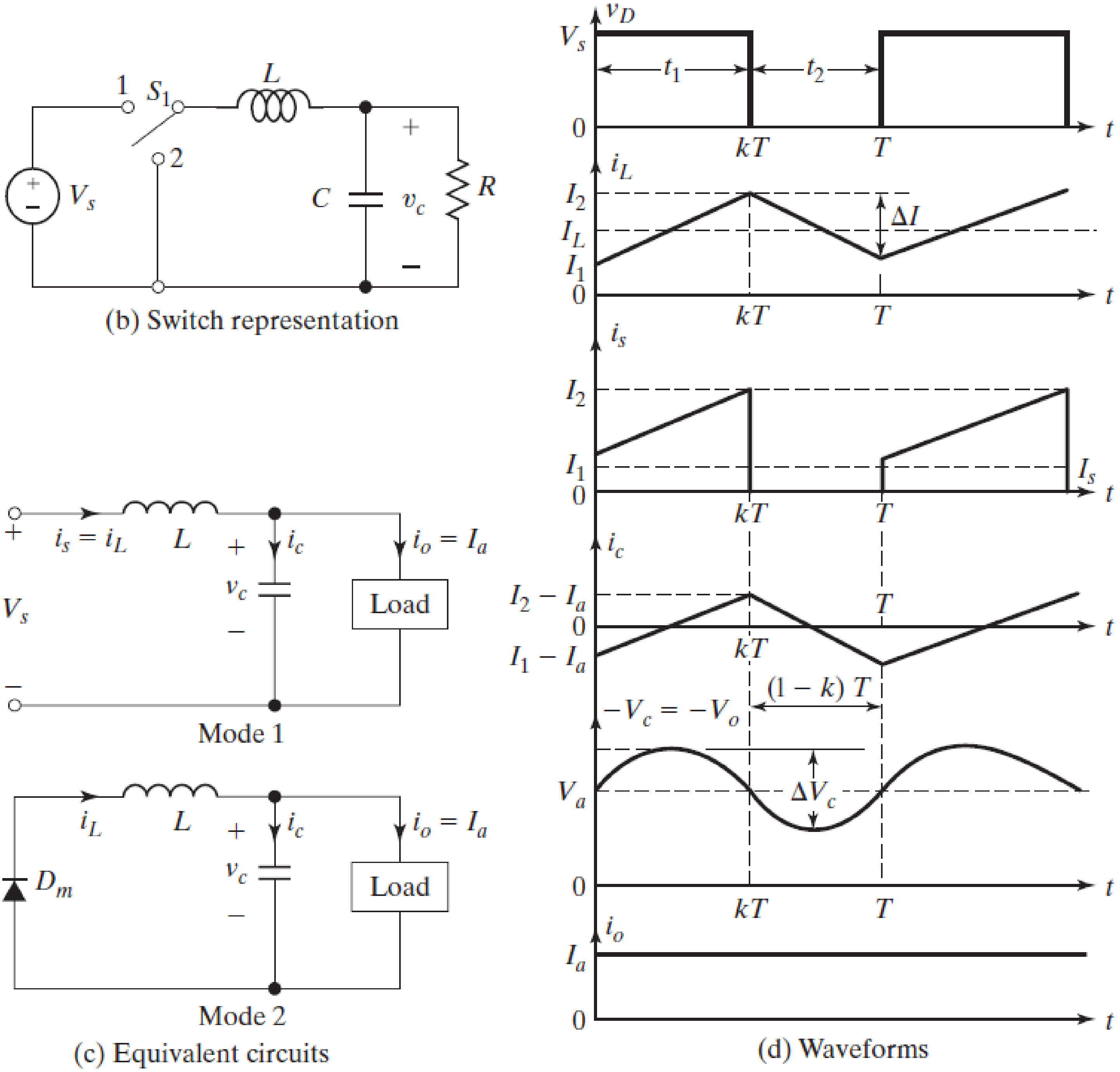


FIGURE 5.17

Buck regulator with continuous i_L.

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ yields the average output voltage as

$$V_a = V_s \frac{t_1}{T} = kV_s \tag{5.56}$$

Assuming a lossless circuit, $V_s I_s = V_a I_a = kV_s I_a$ and the average input current

$$I_s = kI_a \tag{5.57}$$

Peak-to-peak inductor ripple current. The switching period T can be expressed as

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I\, L}{V_s - V_a} + \frac{\Delta I\, L}{V_a} = \frac{\Delta I\, LV_s}{V_a(V_s - V_a)} \tag{5.58}$$

which gives the peak-to-peak ripple current as

$$\Delta I = \frac{V_a(V_s - V_a)}{fLV_s} \tag{5.59}$$

or

$$\Delta I = \frac{V_s k(1 - k)}{fL} \tag{5.60}$$

Peak-to-peak capacitor ripple voltage. Using Kirchhoff's current law, we can write the inductor current i_L as

$$i_L = i_c + i_o$$

If we assume that the load ripple current Δi_o is very small and negligible, $\Delta i_L = \Delta i_c$. The average capacitor current, which flows into for $t_1/2 + t_2/2 = T/2$, is

$$I_c = \frac{\Delta I}{4}$$

The capacitor voltage is expressed as

$$v_c = \frac{1}{C}\int i_c\, dt + v_c(t = 0)$$

and the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c(t=0) = \frac{1}{C}\int_0^{T/2} \frac{\Delta I}{4}\, dt = \frac{\Delta I\, T}{8C} = \frac{\Delta I}{8fC} \tag{5.61}$$

Substituting the value of ΔI from Eq. (5.59) or (5.60) in Eq. (5.61) yields

$$\Delta V_c = \frac{V_a(V_s - V_a)}{8LCf^2 V_s} \tag{5.62}$$

or

$$\Delta V_c = \frac{V_s k(1-k)}{8LCf^2} \tag{5.63}$$

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, the inductor ripple current $\Delta I = 2I_L$.

Using Eqs. (5.56) and (5.60), we get

$$\frac{V_S(1-k)k}{fL} = 2I_L = 2I_a = \frac{2kV_s}{R}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{(1-k)R}{2f} \tag{5.64}$$

If V_c is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$. Using Eqs. (5.56) and (5.63), we get

$$\frac{V_s(1-k)k}{8LCf^2} = 2V_a = 2kV_s$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{1-k}{16Lf^2} \tag{5.65}$$

The buck regulator requires only one transistor, is simple, and has high efficiency greater than 90%. The *di/dt* of the load current is limited by inductor L. However, the input current is discontinuous and a smoothing input filter is normally required. It provides one polarity of output voltage and unidirectional output current. It requires a protection circuit in case of possible short circuit across the diode path.

5.9.2 Boost Regulators

In a boost regulator [8, 9] the output voltage is greater than the input voltage—hence the name "boost." A boost regulator using a power MOSFET is shown in Figure 5.18a. Transistor M_1 acts as a controlled switch and diode D_m is an uncontrolled switch. The circuit in Figure 5.18a is often represented by two switches as shown in Figure 5.18b. The circuit operation can be divided into two modes. Mode 1 begins when transistor. M_1 is switched on at $t = 0$. The input current, which rises, flows through inductor L and transistor Q_1. Mode 2 begins when transistor M_1 is switched off at $t = t_1$. The current that was flowing through the transistor would now flow through L, C, load, and diode D_m. The inductor current falls until transistor M_1 is turned on again in the next cycle. The energy stored in inductor L is transferred to the load. The equivalent circuits for the modes of operation are shown in Figure 5.18c. The waveforms for voltages and currents are shown in Figure 5.18d for continuous load current, assuming that the current rises or falls linearly.

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{5.66}$$

or

$$t_1 = \frac{\Delta I L}{V_s} \tag{5.67}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_s - V_a = -L\frac{\Delta I}{t_2} \tag{5.68}$$

or

$$t_2 = \frac{\Delta I L}{V_a - V_s} \tag{5.69}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L. From Eqs. (5.66) and (5.68),

$$\Delta I = \frac{V_s t_1}{L} = \frac{(V_a - V_s)t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ yields the average output voltage,

$$V_a = V_s\frac{T}{t_2} = \frac{V_s}{1 - k} \tag{5.70}$$

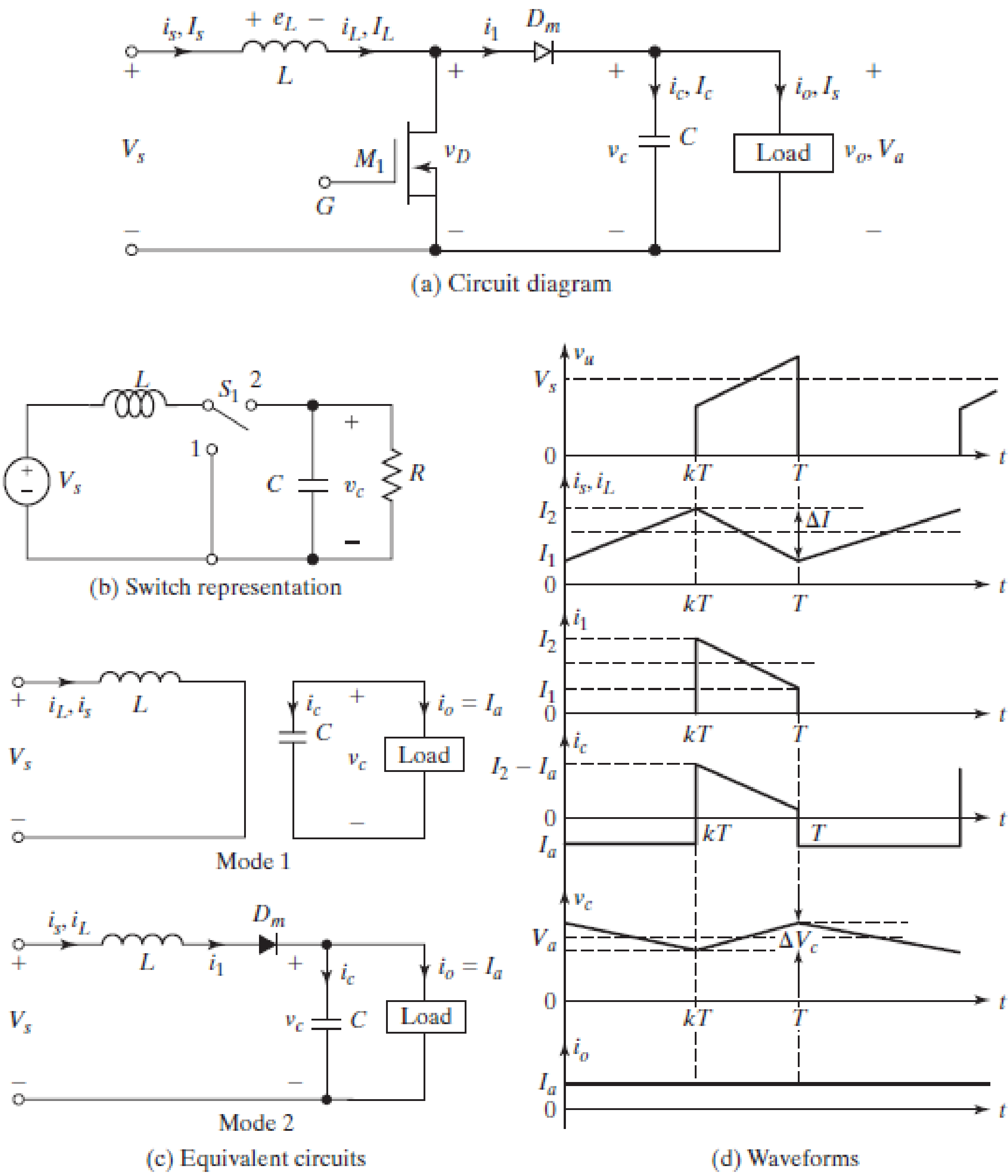


FIGURE 5.18

Boost regulator with continuous i_L.

which gives

$$(1 - k) = \frac{V_s}{V_a} \tag{5.71}$$

Substituting $k = t_1/T = t_1 f$ into Eq. (5.71) yields

$$t_1 = \frac{V_a - V_s}{V_a f} \tag{5.72}$$

Assuming a lossless circuit, $V_sI_s = V_aI_a = V_sI_a/(1 - k)$ and the average input current is

$$I_s = \frac{I_a}{1 - k} \tag{5.73}$$

Peak-to-peak inductor ripple current. The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta IL}{V_s} + \frac{\Delta IL}{V_a - V_s} = \frac{\Delta ILV_a}{V_s(V_a - V_s)} \tag{5.74}$$

and this gives the peak-to-peak ripple current:

$$\Delta I = \frac{V_s(V_a - V_s)}{fLV_a} \tag{5.75}$$

or

$$\Delta I = \frac{V_sk}{fL} \tag{5.76}$$

Peak-to-peak capacitor ripple voltage. When the transistor is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time t_1 is $I_c = I_a$ and the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c(t = 0) = \frac{1}{C}\int_0^{t_1} I_c\,dt = \frac{1}{C}\int_0^{t_1} I_a\,dt = \frac{I_at_1}{C} \tag{5.77}$$

Substituting $t_1 = (V_a - V_s)/(V_af)$ from Eq. (5.72) gives

$$\Delta V_c = \frac{I_a(V_a - V_s)}{V_afC} \tag{5.78}$$

or

$$\Delta V_c = \frac{I_ak}{fC} \tag{5.79}$$

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, at the critical condition for continuous conduction the inductor ripple current $\Delta I = 2I_L$.

Using Eqs. (5.70) and (5.76), we get

$$\frac{kV_s}{fL} = 2I_L = 2I_s = \frac{2V_s}{(1 - k)^2}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{k(1 - k)R}{2f} \tag{5.80}$$

If V_c is the average capacitor voltage, at the critical condition for continuous conduction the capacitor ripple voltage $\Delta V_c = 2V_a$. Using Eq. (5.79), we get

$$\frac{I_a k}{Cf} = 2V_a = 2I_a R$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{k}{2fR} \tag{5.81}$$

A boost regulator can step up the output voltage without a transformer. Due to a single transistor, it has a high efficiency. The input current is continuous. However, a high-peak current has to flow through the power transistor. The output voltage is very sensitive to changes in duty cycle k and it might be difficult to stabilize the regulator. The average output current is less than the average inductor current by a factor of $(1 - k)$, and a much higher rms current would flow through the filter capacitor, resulting in the use of a larger filter capacitor and a larger inductor than those of a buck regulator.

5.9.3 Buck–Boost Regulators

A buck–boost regulator provides an output voltage that may be less than or greater than the input voltage—hence the name "buck–boost"; the output voltage polarity is opposite to that of the input voltage. This regulator is also known as an *inverting regulator*. The circuit arrangement of a buck–boost regulator is shown in Figure 5.19a. Transistor Q_1 acts as a controlled switch and diode D_m is an uncontrolled switch. They operate as two SPST current-bidirectional switches. The circuit in Figure 5.19a is often represented by two switches as shown in Figure 5.19b.

The circuit operation can be divided into two modes. During mode 1, transistor Q_1 is turned on and diode D_m is reversed biased. The input current, which rises, flows through inductor L and transistor Q_1. During mode 2, transistor Q_1 is switched off and the current, which was flowing through inductor L, would flow through L, C, D_m, and the load. The energy stored in inductor L would be transferred to the load and the inductor current would fall until transistor Q_1 is switched on again in the next cycle. The equivalent circuits for the modes are shown in Figure 5.19c. The waveforms for steady-state voltages and currents of the buck–boost regulator are shown in Figure 5.19d for a continuous load current.

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{5.82}$$

or

$$t_1 = \frac{\Delta I L}{V_s} \tag{5.83}$$

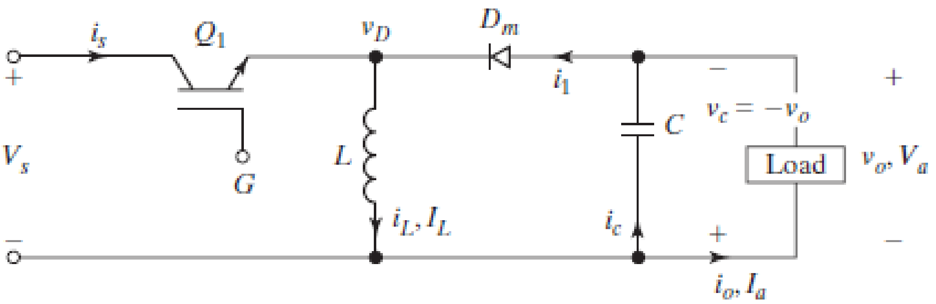
and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_a = -L\frac{\Delta I}{t_2} \tag{5.84}$$

or

$$t_2 = \frac{-\Delta I L}{V_a} \tag{5.85}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L. From Eqs. (5.82) and (5.84),



(a) Circuit diagram

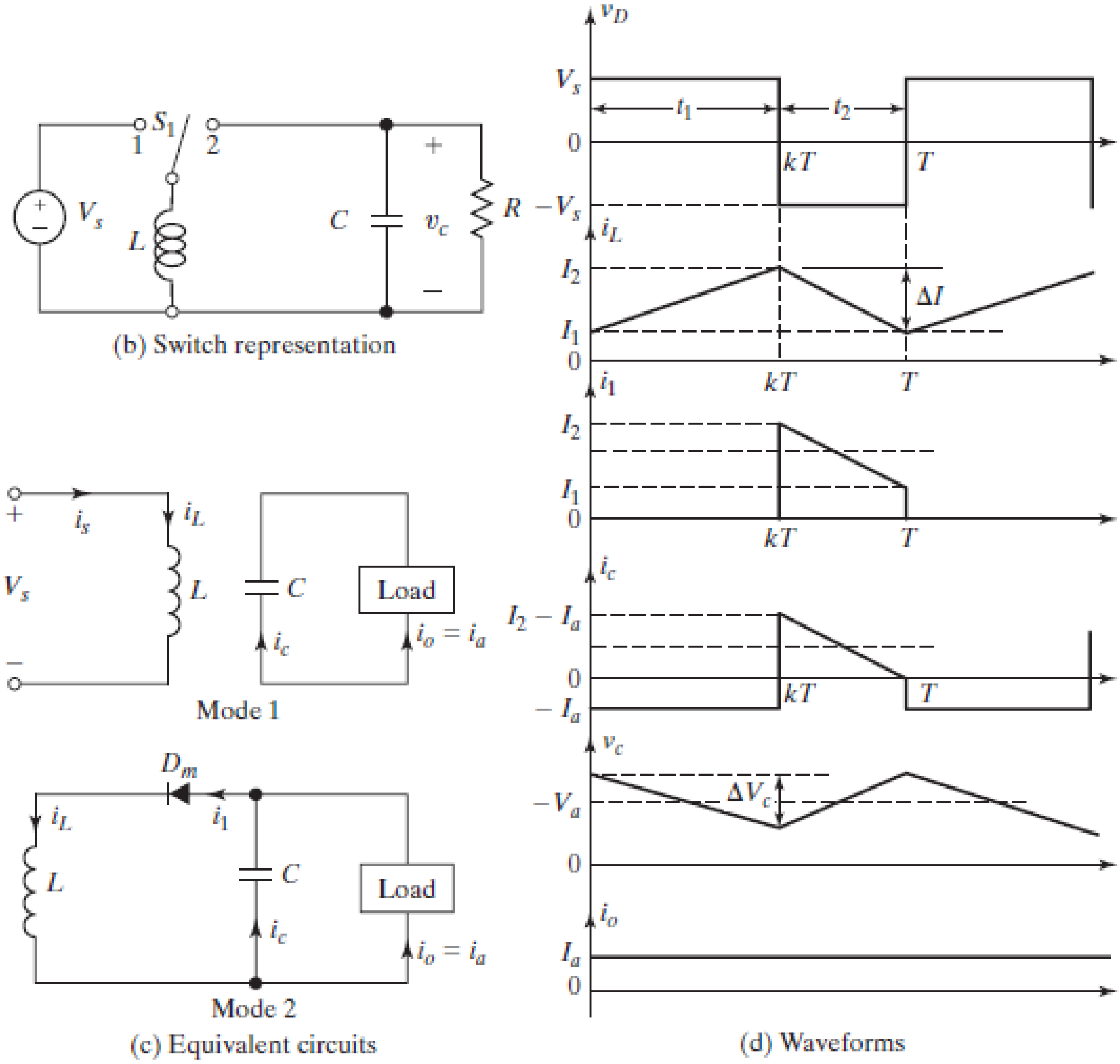


FIGURE 5.19

Buck–boost regulator with continuous i_L.

$$\Delta I = \frac{V_s t_1}{L} = \frac{-V_a t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$, the average output voltage is

$$V_a = -\frac{V_s k}{1 - k} \tag{5.86}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ into Eq. (5.86) yields

$$(1 - k) = \frac{-V_s}{V_a - V_s} \tag{5.87}$$

Substituting $t_2 = (1 - k)T$, and $(1 - k)$ from Eq. (5.87) into Eq. (5.86) yields

$$t_1 = \frac{V_a}{(V_a - V_s)f} \tag{5.88}$$

Assuming a lossless circuit, $V_sI_s = -V_aI_a = V_sI_ak/(1 - k)$ and the average input current I_s is related to the average output current I_a by

$$I_s = \frac{I_ak}{1 - k} \tag{5.89}$$

Peak-to-peak inductor ripple current. The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta IL}{V_s} - \frac{\Delta IL}{V_a} = \frac{\Delta IL(V_a - V_s)}{V_sV_a} \tag{5.90}$$

and this gives the peak-to-peak ripple current,

$$\Delta I = \frac{V_sV_a}{fL(V_a - V_s)} \tag{5.91}$$

or

$$\Delta I = \frac{V_sk}{fL} \tag{5.92}$$

The average inductor current is given by

$$I_L = I_s + I_a = \frac{kI_a}{1 - k} + I_a = \frac{I_a}{1 - k} \tag{5.92a}$$

Peak-to-peak capacitor ripple voltage. When transistor Q_1 is on, the filter capacitor supplies the load current for $t = t_1$. The average discharging current of the capacitor is $I_c = -I_a$ and the peak-to-peak ripple voltage of the capacitor is

$$\Delta V_c = \frac{1}{C}\int_0^{t_1} -I_c\,dt = \frac{1}{C}\int_0^{t_1} I_a\,dt = \frac{I_at_1}{C} \tag{5.93}$$

Substituting $t_1 = V_a/[(V_a - V_s)f]$ from Eq. (5.88) becomes

$$\Delta V_c = \frac{I_aV_a}{(V_a - V_s)fC} \tag{5.94}$$

or

$$\Delta V_c = \frac{I_ak}{fC} \tag{5.95}$$

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, at the critical condition for continuous conduction the inductor ripple current $\Delta I = 2I_L$. Using Eqs. (5.86) and (5.92), we get

$$\frac{kV_s}{fL} = 2I_L = 2I_a = \frac{2kV_s}{(1-k)R}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{(1-k)R}{2f} \tag{5.96}$$

If V_c is the average capacitor voltage, at the critical condition for continuous conduction the capacitor ripple voltage $\Delta V_c = -2V_a$. Using Eq. (5.95), we get

$$-\frac{I_a k}{Cf} = -2V_a = -2I_a R$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{k}{2fR} \tag{5.97}$$

A buck–boost regulator provides output voltage polarity reversal without a transformer. It has high efficiency. Under a fault condition of the transistor, the *di/dt* of the fault current is limited by the inductor L and will be V_s/L. Output short-circuit protection would be easy to implement. However, the input current is discontinuous and a high peak current flows through transistor Q_1.

13.2.1 Switched-Mode Dc Power Supplies

The switching mode supplies have high efficiency and can supply a high-load current at a low voltage. There are five common configurations for the switched-mode or PWM operation of the inverter (or dc–ac converter) stage: flyback forward, push–pull, half-bridge, and full-bridge [1, 2]. The output of the inverter, which is varied by a PWM technique, is converted to a dc voltage by a diode rectifier. Because the inverter can operate at a very high frequency, the ripples on the dc output voltage can easily be filtered out with small filters. To select an appropriate topology for an application, it is necessary to understand the merits and drawbacks of each topology and the requirements of the application. Basically, most topologies can work for different applications [3, 9].

13.2.2 Flyback Converter

Figure 13.1a shows the circuit of a flyback converter. There are two modes of operation: (1) mode 1 when switch Q_1 is turned on, and (2) mode 2 when Q_1 is turned off. Figure 13.1b–f shows the steady-state waveforms under a *discontinuous-mode* operation. It is assumed that the output voltage as shown in Figure 13.1f is ripple free.

Mode 1. This mode begins when switch Q_1 is turned on and it is valid for $0 < t \leq kT$, where k is the duty-cycle ratio and T is the switching period. The voltage across the primary winding of the transformer is V_s. The primary current i_p starts to build up and stores energy in the primary winding. Due to the opposite polarity arrangement between the input and output windings of the transformer, diode D_1 is reverse biased. There is no energy transferred from the input to load R_L. The output filter capacitor C maintains the output voltage and supplies the load current i_L. The primary current i_p that increases linearly is given by

$$i_p = \frac{V_s t}{L_p} \tag{13.1}$$

where L_p is the primary magnetizing inductance. At the end of this mode at $t = kT$, the peak primary current reaches a value equal to $I_{p(pk)}$ as given by

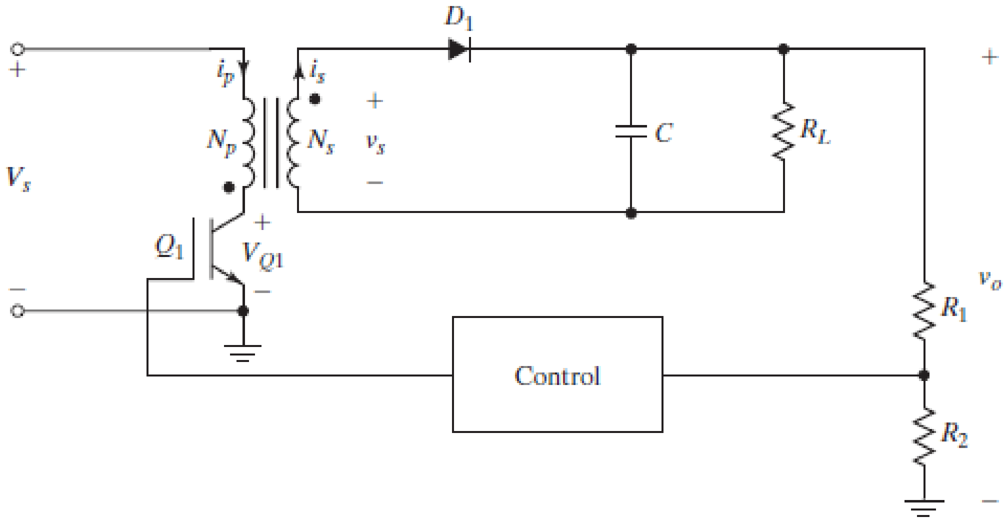
$$I_{p(pk)} = i_p(t = kT) = \frac{V_s kT}{L_p} \tag{13.2}$$

The peak secondary current $I_{se(pk)}$ is given by

$$I_{se(pk)} = \left(\frac{N_p}{N_s}\right) I_{p(pk)} \tag{13.3}$$

Mode 2. This mode begins when switch Q_1 is turned off. The polarity of the windings reverses due to the fact that i_p cannot change instantaneously. This causes diode D_1 to turn on and charges the output capacitor C and also delivers current to R_L. The secondary current that decreases linearly is given by

$$i_{se} = I_{se(pk)} - \frac{V_o}{L_s} t \tag{13.4}$$



(a)

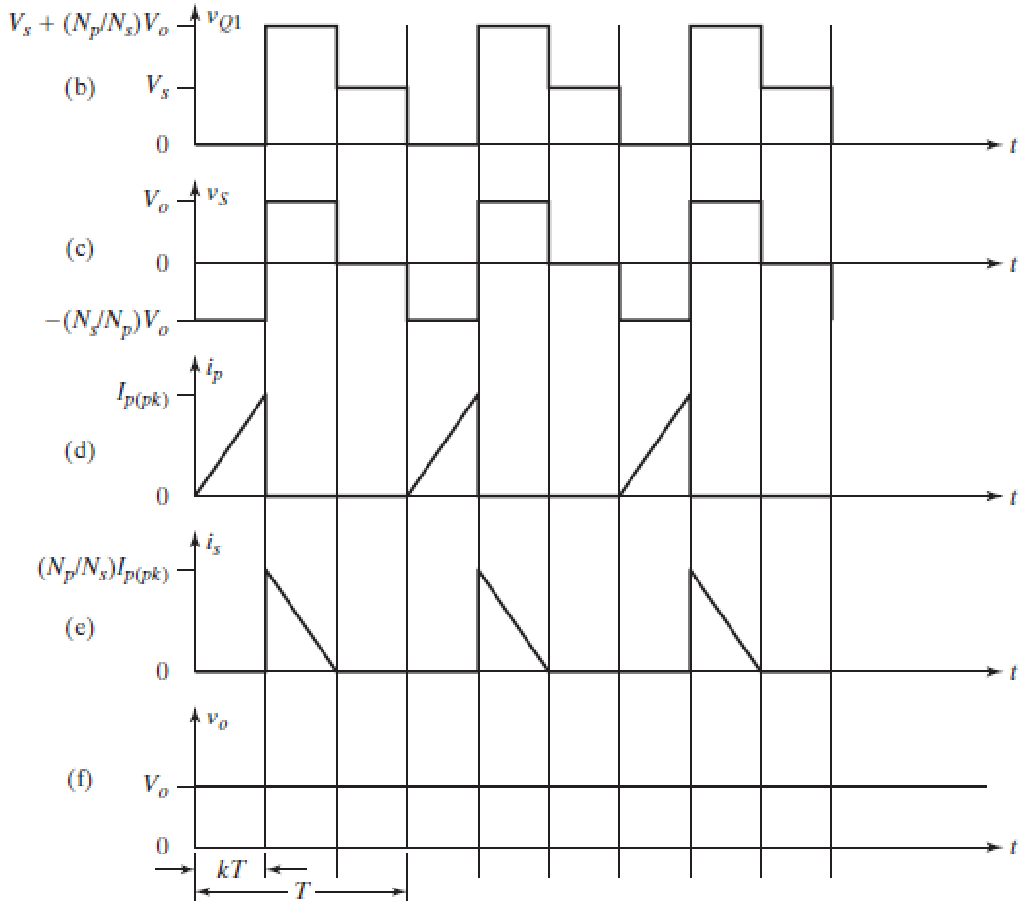


FIGURE 13.1

Flyback converter. (a) Circuit, (b) Transistor Q_1 voltage, (c) Secondary voltage, (d) Primary current, (e) Secondary current, and (f) Output voltage.

where L_s is the secondary magnetizing inductance. Under the discontinuous-mode operation, i_{se} decreases linearly to zero before the start of the next cycle.

Because energy is transferred from the source to the output during the time interval 0 to kT only, the input power is given by

$$P_i = \frac{\frac{1}{2}L_p I_{p(pk)}^2}{T} = \frac{(kV_s)^2}{2fL_p} \tag{13.5}$$

For an efficiency of η, the output power P_o can be found from

$$P_o = \eta P_i = \frac{\eta(V_s k)^2}{2fL_p} \tag{13.6}$$

which can be equated to $P_o = V_o^2/R_L$ so that we can find the output voltage V_o as

$$V_o = V_s k\sqrt{\frac{\eta R_L}{2fL_p}} \tag{13.7}$$

Thus, V_o can be maintained constant by keeping the product $V_s kT$ constant. Because the maximum duty cycle k_{max} occurs at minimum supply voltage $V_{s(min)}$, the allowable k_{max} for the discontinuous mode can be found from Eq. (13.7) as

$$k_{\max} = \frac{V_o}{V_{s(\min)}}\sqrt{\frac{2fL_p}{\eta R_L}} \tag{13.8}$$

Therefore, V_o at k_{max} is then given by

$$V_o = V_{s(\min)}k_{\max}\sqrt{\frac{\eta R_L}{2fL_p}} \tag{13.9}$$

Because the collector voltage V_{Q1} of Q_1 is maximum when V_s is maximum, the maximum collector voltage $V_{Q1(max)}$, as shown in Figure 13.1b, is given by

$$V_{Q1(\max)} = V_{s(\max)} + \left(\frac{N_p}{N_s}\right) V_o \tag{13.10}$$

The peak primary current $I_{p(pk)}$, which is the same as the maximum collector current $I_{C(max)}$ of the power switch Q_1, is given by

$$I_{C(\max)} = I_{p(pk)} = \frac{2P_i}{kV_s} = \frac{2P_o}{\eta V_s k} \tag{13.11}$$

The flyback converter is used mostly in applications below 100 W. It is widely used for high-output voltage at relatively low power. Its essential features are simplicity and low cost. The switching device must be capable of sustaining a voltage $V_{Q1(max)}$ in Eq. (13.10). If the voltage is too high, the double-ended flyback converter, as shown in Figure 13.2, may be used. The two devices are switched on or off simultaneously. Diodes D_1 and D_2 are used to limit the maximum switch voltage to V_s.

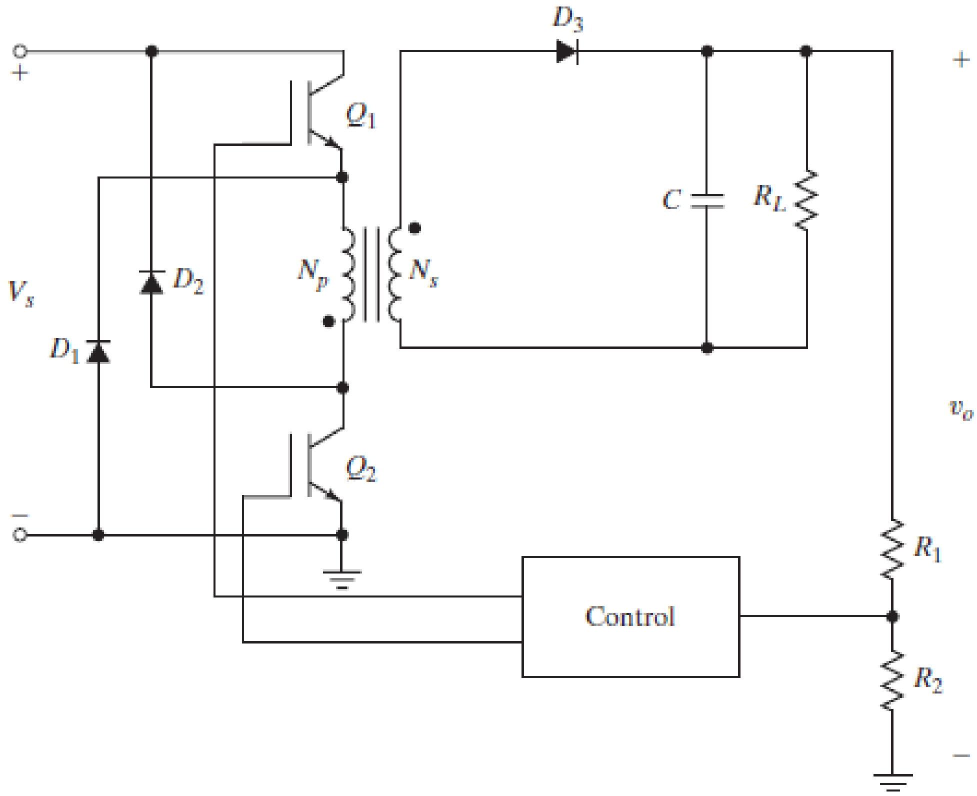


FIGURE 13.2

Double-ended flyback converter.

Continuous versus discontinuous mode of operation. In a continuous mode of operation, switch Q_1 is turned on before the secondary current falls to zero. The continuous mode can provide higher power capability for the same value of peak current $I_{p(pk)}$. It means that, for the same output power, the peak currents in the discontinuous mode are much higher than those in continuous mode. As a result, a more expensive power transistor with a higher current rating is needed. Moreover, the higher secondary peak currents in the discontinuous mode can have a larger transient spike at the instant of turn-off. However, despite all these problems, the discontinuous mode is still more preferred than the continuous mode. There are two main reasons. First, the inherently smaller magnetizing inductance in the discontinuous mode has a quicker response and a lower transient output voltage spike to sudden change in load current or input voltage. Second, the continuous mode has a right-half-plane zero in its transfer function, thereby making the feedback control circuit more difficult to design [10, 11].

13.2.5 Half-Bridge Converter

Figure 13.7a shows the basic configuration of a half-bridge converter. This converter can be viewed as two back-to-back forward converters that are fed by the same input voltage, each delivering power to the load at each alternate half-cycle. The capacitors C_1 and C_2 are placed across the input terminals such that the voltage across the primary winding always is half of the input voltage $V_s/2$.

There are four modes of operation: (1) mode 1 when switch Q_1 is turned on and switch Q_2 is off, (2) mode 2 when both Q_1 and Q_2 are off, (3) mode 3 when switch Q_1 is off and switch Q_2 is turned on, and (4) mode 4 when both Q_1 and Q_2 are off again. Switches Q_1 and Q_2 are switched on and off accordingly to produce a square-wave ac at the primary side of the transformer. This square wave is either stepped down or up by the isolation transformer and then rectified by diodes D_1 and D_2. The rectified voltage is, subsequently, filtered to produce the output voltage V_o. Figure 13.7b–g shows the steady-state waveforms in the continuous mode operation.

Mode 1. During this mode Q_1 is on and Q_2 is off, D_1 conducts and D_2 is reverse biased. The primary voltage V_p is $V_s/2$. The primary current i_p starts to build up and stores energy in the primary winding. This energy is forward transferred to the secondary and onto the L_1C filter and the load R_L through the rectifier diode D_1.

The voltage across the secondary winding is given by

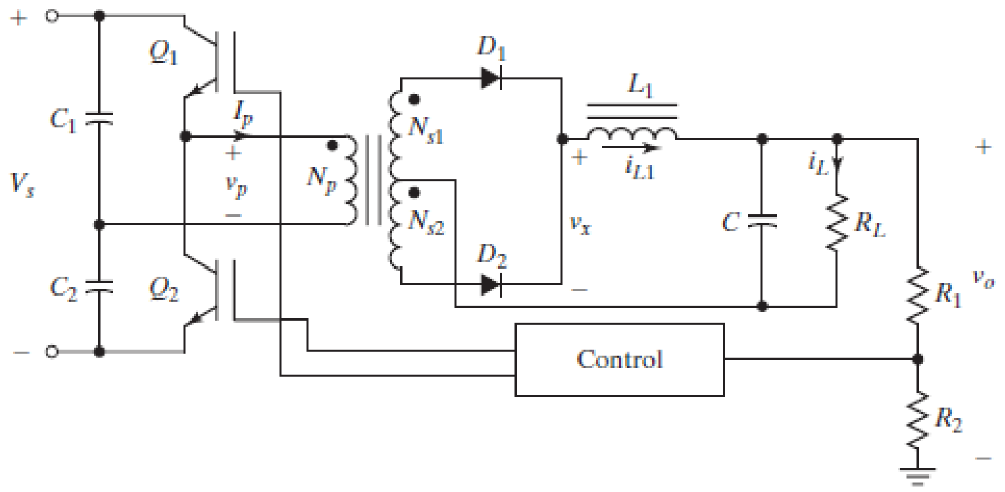
$$V_{se} = \frac{N_{s1}}{N_p}\left(\frac{V_s}{2}\right) \tag{13.26}$$

The voltage across the output inductor is then given by

$$v_{L1} = \frac{N_{s1}}{N_p}\left(\frac{V_s}{2}\right) - V_o \tag{13.27}$$

The inductor current i_{L1} increases linearly at a rate of

$$\frac{di_{L1}}{dt} = \frac{v_{L1}}{L_1} = \frac{1}{L_1}\left[\frac{N_{s1}}{N_p}\left(\frac{V_s}{2}\right) - V_o\right]$$



+
Q_1
C_1
I_p
D_1
L_1
N_{s1}
+
v_p
−
N_p
V_s
i_{L1}
i_L
+
N_{s2}
v_x
C
R_L
D_2
C_2
Q_2
−
R_1
v_o
−
Control
R_2
−

(a)

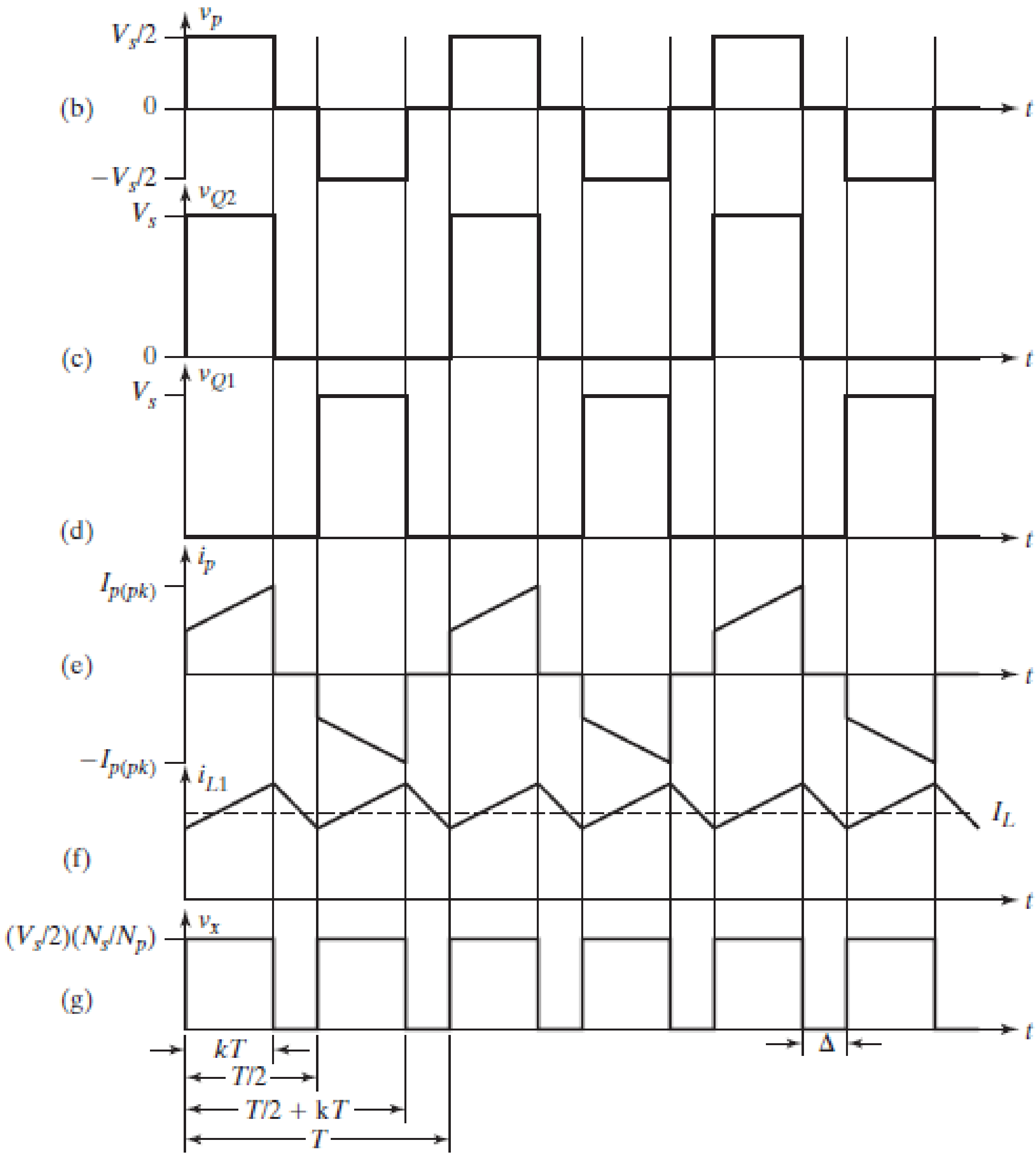


FIGURE 13.7

Half-bridge converter. (a) Circuit, (b) Primary voltage, (c) Transistor Q_2 voltage, (d) Transistor Q_1 voltage, (e) Primary current,(f) Inductor L_1 current, and (g) Rectifier output voltage.

which gives the peak inductor current $I_{L1(pk)}$ at the end of this mode at $t = kT$ as given by

$$I_{L1(pk)} = I_{L1}(0) + \frac{1}{L_1}\left[\frac{N_{s1}}{N_p}\left(\frac{V_s}{2}\right) - V_o\right]kT \tag{13.28}$$

Mode 2. This mode is valid for $kT < t \leq T/2$. During this mode both Q_1 and Q_2 are off, and D_1 and D_2 are forced to conduct the magnetizing current that resulted during mode 1. Redefining the time origin at the beginning of this mode, the rate of fall of i_{L1} is given by

$$\frac{di_{L1}}{dt} = -\frac{V_o}{L_1} \qquad \text{for } 0 < t \leq (0.5 - k)T \tag{13.29}$$

which gives $I_{L1}(0) = i_{L1}[t = (0.5 - k)T] = I_{L1(pk)} - V_o(0.5 - k)T/L_1$.

Modes 3 and 4. During mode 3, Q_2 is on and Q_1 off, D_1 is reverse biased, and D_2 conducts. The primary voltage V_p is now $-V_s/2$. The circuit operates in the same manner as in mode 1 followed by mode 4 that is similar to mode 2.

The output voltage V_o can be found from the time integral of the inductor voltage v_{L1} over the switching period T. That is,

$$V_o = 2 \times \frac{1}{T}\left[\int_0^{kT}\left(\frac{N_{s1}}{N_p}\left(\frac{V_s}{2}\right) - V_o\right)dt + \int_{T/2}^{T/2+kT} -V_o dt\right]$$

which gives V_o as

$$V_o = \frac{N_{s1}}{N_p} V_s k \tag{13.30}$$

The output power P_o is given by

$$P_o = V_o I_L = \eta P_i = \eta \frac{V_s I_{p(\mathrm{avg})} k}{2}$$

which gives

$$I_{p(\mathrm{avg})} = \frac{2P_o}{\eta V_s k} \tag{13.31}$$

where $I_{p(\mathrm{avg})}$ is the average primary current. Assuming that the secondary load current reflected to the primary side is much greater than the magnetizing current, the maximum collector currents for Q_1 and Q_2 are given by

$$I_{C(\max)} = I_{p(\mathrm{avg})} = \frac{2P_o}{\eta V_s k_{\max}} \tag{13.32}$$

The maximum collector voltages for Q_1 and Q_2 during turn-off are given by

$$V_{C(\max)} = V_{s(\max)} \tag{13.33}$$

The maximum duty cycle k can never be greater than 50%. The half-bridge converter is widely used for medium-power applications. Because of its core-balancing feature, the half-bridge converter becomes the predominant choice for output power ranging from 200 to 400 W.

Forward versus half-bridge converter. In a half-bridge converter, the voltage stress imposed on the power transistors is subject to only the input voltage and is only half of that in a forward converter. Thus, the output power of a half-bridge is double to that of a forward converter for the same semiconductor devices and magnetic core. Because the half-bridge is more complex, for application below 200 W, the flyback or forward converter is considered to be a better choice and more cost-effective. Above 400 W, the primary and switch currents of the half-bridge become very high. Thus, it becomes unsuitable for high-power applications.

Note: The emitter of Q_1 is not at ground level, but is at a high-ac level. Thus, the gate driver circuit must be isolated from the ground through transformers or other coupling devices.