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Q1) Solve the following

QA) Find eigen value & eigen vector corresponding to lowest eigen value.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Ans→ Polynomial is $\lambda^3 - S_1\lambda^2 + S_2\lambda - |A| = 0$

$S_1 = 0$

$$S_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$S_2 = -3$

$$|A| = 0 - (1)\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 2$$

$\therefore \lambda^3 - 3\lambda - 2 = 0$

$\therefore$ (-1) is the root

$\therefore (\lambda + 1)(\lambda^2 - \lambda - 2) = 0$

$(\lambda + 1)(\lambda - 2)(\lambda + 1) = 0$

Eigen values are $\lambda = -1, -1, 2$

Eigen vectors for $\lambda = -1$

$$A - \lambda I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigen value $\lambda = -1$

$$A + \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + x_2 + x_3 = 0$

$x_1 = -x_2 - x_3$

Eigen vector is $\begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix}$

Eigen vector correponding to $\lambda = -1$

Let $x_2 = 1 \quad x_3 = 0$

$$v_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Let $x_2 = 0, x_3 = 1$

$$v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Now,
two vectors $(-1,1,0)$ & $(-1,0,1)$ are linearly independent

$$\begin{bmatrix} -x_2-x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Put $\cancel{x} = 5 = 1$, we get partial vector $= \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Q.B] Verify Cayley Hamilton theorem & find A^{-1} & A^{-2} using Cayley Hamilton theorem

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

Ans $\rightarrow$ $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

$$[A-\lambda I] = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 1 & 1-\lambda \end{bmatrix}$$

$\therefore |A-\lambda I| = \lambda^2 - 2\lambda - 1 = 0$ .... (characteristic eqn)

by putting A in eqn

$\therefore A^2 - 2A - I = 0$ — ①

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$$\therefore LHS = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}^2 - 2\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - 1I$$

$$= \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2-1 & 4-4-0 \\ 2-2-0 & 3-2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$= R.H.S$

$\therefore$ From above calculation LHS = RHS
Hence, cayley Hamilton theorem verified.

Now.
to find A^{-1} we multiply (1) by A^{-1}

$$\therefore A^2A^{-1} - 2AA^{-1} - A^{-1} = 0$$

$$\therefore A(AA^{-1}) - 2(AA^{-1}) = A^{-1}$$

$$\therefore AI - 2I = A^{-1}$$

$$\therefore A - 2 = A^{-1}$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2 & 2-0 \\ 1-0 & 1-2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

Now for A^{-2}

$$\therefore A^{-2}A^{2} = 2AA^{-2} - IA^{-2} = 0$$

$$I - 2A^{-1} - A^{-2} = 0$$

$$A^{-2} = I - 2A^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 2 & -2 \end{bmatrix}$$

$$A^{2} = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$$

Q.C] Given a line segment starting at a point (0,0) ending point is (8,1). Rotate line by 45° & find new coordinate.

Ans) Old ending coordinates of the line

$= (x_{old}, y_{old}) = (8,1)$

Rotation angle $= 45^{\circ}$

Let new ending coordinates of the line after rotation $= (x_{new}, y_{new})$

Applying the rotation equations, we have.

For X_{new},

$$= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$$

$$= 8 \times \cos(45^{\circ}) - 1 \times \sin(45^{\circ})$$

$$= 8 \times 1/\sqrt{2} - 1 \times 1/\sqrt{2}$$

$$= \frac{8}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$= \frac{7}{\sqrt{2}} = 4.9$$

For y new,

$= X_{old} \times \cos\theta + Y_{old} \times \sin\theta$

$= 8 \times \cos(45^\circ) + 1 \times \sin(45^\circ)$

$= 8 \times 1/\sqrt{2} + 1 \times 1/\sqrt{2}$

$= 8/\sqrt{2} + 1/\sqrt{2}$

$= 9/\sqrt{2}$

$= 6.3$

Thus, New ending coordinates of the line after rotation = (4.3, 6.3

Q2). Fill in the blanks -

A) The given characteristic equation is $\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$. The algebraic multiplicity of each eigen value is & eigen value of A^{-2} is -

Ans-) Eigen values are - $\lambda = 3, 2$

Algebric multiplicity $= |A| = 6$

Eigen values of A^{-2} are $\frac{1}{4}$ & $\frac{1}{9}$

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B) Given a square whose coordinates are given by $A \equiv (2,1)$ $B \equiv (3,1)$ $C \equiv (3,4)$ $D \equiv (2,4)$. Translate square by 7 units right & 6 units down. Find new coordinates.

Ans → The new coordinates are:

$A = (9,-5)$ $C = (10,-2)$

$B = (10,-5)$ $D = (9,-2)$

c) The sum & product of eigen values of $A_{n \times n}$ matrix is if A have $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$ are eigen values of A.

Ans → The sum of eigen values is the sum of principle elements of matrix and product of eigen values is the determinant value of matrix.

D) If A is orthogonal matrix if 6 is one eigen value then other eigen value is

Ans → -6