Subject - LADC
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Tutorial - 5

(Q1) Solve.

1) If $y=(x+\sqrt{x^2-1})^m$ show that $(x^2-1)y_{n+2} + (n^2-m^2)y_n = 0$

Ans→ $y = (x+\sqrt{x^2-1})^m$

$y_1 = m(x+\sqrt{x^2-1})^{m-1}\left(1+\frac{x}{\sqrt{x^2-1}}\right)$

$= m(x+\sqrt{x^2-1})^{m-1}\left(\frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}}\right)$

$y_1 = \frac{m}{\sqrt{x^2-1}}(x+\sqrt{x^2-1})^m$

$(\sqrt{x^2-1})y_1 = my$

Squaring both side;

$(x^2-1)y_1^2 = m^2y^2$ — ①

Differentiate eqn ① we get

$(\cancel{2}x)y_1^2 + (x^2-1)(\cancel{2}y_1y_2) = m^2(\cancel{2}yy_1)$

$xy_1 + (x^2-1)y_2 = m^2y$

$\therefore (x^2-1)y_2 + xy_1 - m^2y = 0$ — ②

Differentiating eq ② in time using Lebinits's theorem, we get

$(x^2-1)y_{n+2} + {}^nC_1(2x)y_{n+1} + {}^nC_2(2)y_n + (x)y_{n+1} + {}^nC_1 y_n - m^2y_n = 0$

$(x^2-1)y_{n+2} + n(2x)y_{n+1} + n(n-1)y_n + xy_{n+1} + ny_n - m^2y_n = 0$

$(x^2-1)y_{n+2} + x(2x+1)y_{n+1} + (n^2-m^2)y_n = 0$

Hence proved

Q2) $\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \frac{\pi}{2} - 3\left(x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right)$

Ans→ On solving RHS:

$$\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

put $x = \tan\theta$ we get,

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$$

$$\left\{ \because \tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} \right\}$$

$$\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \frac{\pi}{2} - \tan^{-1}(\tan 3\theta)$$

$$= \frac{\pi}{2} - 3\theta \quad \left\{ \because \tan\theta = x,\ \theta = \tan^{-1}x \right\}$$

$$= \frac{\pi}{2} - 3\tan^{-1}x$$

Expansion of $\tan^{-1}x$ is $x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots$

Hence,

$$\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \frac{\pi}{2} - 3\left(x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right)$$

Hence proved.

③ Find nth derivative of $x^2 e^{3x} \sin 4x$

Ans Using lebinitz theorem -

Let $v = x^2$

$u = e^{3x} \sin 4x$

then,

$y_n = {}^{x}C_0 u_n v + {}^{x}C_1 u_{n-1} v_1 + {}^{n}C_2 u_{n-2} v_2 + \dots$
$\dots + {}^{n}C_n u v_n$

$Y_n = C_0 (\sqrt{3^2+4^2})^n e^{3x} \sin(4x + n 53)(x^2) +$
${}^{n}C_1 (5)^{n-1} e^{3x} \sin(4x + 53(x-1)(2x)$
$+ {}^{n}C_2 (5)^{m-2} e^{3x} \sin(4x + 53(x-2)(2)$

$= 5^x e^{3x} \sin(4x + 53x)(x^2) + x(5)^{n-1} e^{3x} \sin$
$(4x + (x-1)53^\circ)(2x) + x(x-1)(5)^{x-2} e^{3x} \sin$
$(4x + 53^\circ(x-2))$

$= 5^{x-2} e^{3x} (25x^2 \sin(4x + 53x) + 10 m x \sin$
$(4x + (x-1)53^\circ) + x((x-1) \sin(4x + 53^\circ$
$(x-2))]$

$Y_x = 5^{x-2} e^{3x} [25x^2 \sin(4x + x 53^\circ) + 10x$
$x \sin(4x + (x-1)53^\circ) + x(x-1)$
$\sin(4x + (x-2)53^\circ)]$

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Q2)

1) If $y = \frac{x}{x+2}$ then nth derivative of y is

Ans → $y = x\left(\frac{1}{x+2}\right)$

Using Lebinitz thm

Let $u = \frac{1}{x+2}$ & $v = x$

$$u_x = {}^{x}C_0 u_x v + {}^{x}C_1 u_{x-1} v_1 + {}^{x}C_2 u_{x-2} v_3 + \dots$$

$$= {}^{x}C_0\left[\frac{(-1)^n n!}{(x+2)^{n+1}}\right]x + {}^{x}C_1\left[\frac{(-1)^{x-1}(x-1)!}{(x+2)^n}\right](1) + {}^{x}C_2\left[\frac{(-1)^{x-2}(x-2)!}{(x+2)^{x-1}}\right](0)$$

$$= \frac{(-1)^n n!}{(x+2)^{n+1}} \cdot (x) + \frac{x(-1)^{n-1}(x-1)!}{(x+2)^n}$$

$$y_n = \frac{(-1)^{x-1} n!}{(x+2)^n}\left[\frac{(-1)(x)}{(x+2)} + 1\right]$$

11) If $y = e^{m\cos^{-1}x}$ then find relation between y_1 & y.

→ $y_1 = e^{m\cos^{-1}x}\left(m\left(\frac{-1}{\sqrt{1-x^2}}\right)\right)$

$$y_1 = \frac{-m}{\sqrt{1-x^2}}y$$

$$my + \sqrt{1-x^2}\,y_1 = 0$$

iii) First two terms in the expansion of $e^x \sec x$ is

Ans $\Rightarrow$ $y = e^x \sec x$

using Maclausion thm

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{6x^6}{120} - \frac{4x^8}{640} \dots$

As $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$

Diffe , we get

$\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + \dots$

$\sec^2 x = \sqrt{1 + x^2 + \frac{2x^4}{3} + \frac{17}{45}x^6 + \dots}$

Let $y = x^2 + \frac{2}{3}x^4 + \frac{17}{45}x^6 + \dots$

$\sec x = \sqrt{1+y} = 1 + \frac{y}{2} - \frac{y^2}{8} + \frac{y^3}{16}$ — ①

put y in eqn ① we get,

$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} \frac{41x^8}{640} + \dots$

Expansion of $e^x \sec x$

$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)\left(1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \dots\right)$

$= (1+x)(1+x^2/2)$

$= 1 + \frac{x^2}{2} + x + \frac{x^3}{2}$

$= 1 + x + \frac{x^2}{2} + \frac{x^3}{2}$

Expansion of $e^x \sec x$ till first two term is $(1+x)$

IV) What substitution we need to simplify

$y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$

Ans $\rightarrow$ put $x = \tan\theta$

$$y = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4} + \theta$$

$$= \frac{\pi}{4} + \tan^{-1}x$$

$\therefore$ Hence, substitute $x = \tan\theta$ to simplify.