Fourier Transforms

QB

(1) Use Fourier Integral representation to show that $\int_0^\infty \frac{\lambda^3 \sin \lambda x}{\lambda^4 + 4}\, d\lambda = \frac{\pi}{2} e^{-x} \cos x, \quad x > 0$

(2) Show that Fourier trans. of $e^{-|x|}$ is $\frac{2}{1+\lambda^2}$

(3) If $f(x) = e^{-x} + e^{-2x}$ $(x > 0)$ then S.T. Fourier cosine trans of $f(x)$ is $\frac{6 + 3\lambda^2}{4 + 5\lambda^2 + \lambda^4}$

(4) Solve the integral eqn $\int_0^\infty f(x) \cos \lambda x\, dx = 1 - \lambda \quad 0 \le \lambda \le 1$
$= 0 \quad \lambda \ge 1$

& hence show that $\int_0^\infty \frac{\sin^2 z}{z^2}\, dz = \frac{\pi}{2}$

(5) Find Fourier sine trans of $f(x) = e^{-x}$
& hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi}{2} e^{-m}$

(6) Find Fourier trans. of $f(x) = x \quad |x| \le a$
$= 0 \quad |x| > a$

(7) Find the finite Fourier sine & cosine trans. of $f(x) = x^2 \quad 0 \le x \le 2$

(8) Find $f(x)$ if $F_S[f(n)] = \frac{1}{n^2} \sin \frac{n\pi}{3}, \quad 0 \le x \le \pi$.