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Q1.)
A) Determine the value of α for which the equations are consistent
$x + 2y + z = 3$, $x + y + z = \alpha$, $3x + y + 3z = \alpha^2$

Find the solution for one of the value of α

Ans ⇒ $[A][X] = [B]$ form,

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ \alpha \\ \alpha^2 \end{bmatrix}$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & 1 & 1 & \alpha \\ 3 & 1 & 3 & \alpha^2 \end{array}\right]$$

Now we can apply row transformation.
$R_1 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & \alpha - 3 \\ 0 & -5 & 0 & \alpha^2 - 9 \end{array}\right] \quad \rho(A) = 3 = \rho(A/B)$$

Hence there is unique solution to system and it is consistent. from $\sim(A/B)$

$$x + 2y + z = 3$$
$$-y = \alpha - 3$$
$$-5y = \alpha^2 - 9$$
$$5 = \alpha + 3$$
$$\underline{\alpha = 2}$$

This is the unique solution for α.

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B) Examine for linear dependence or independence of $x = (3,1,-4)$ $y = (2,2,-3)$ $z = (0,-4,1)$. If dependent then find relation between them.

Ans ⇒ $x_1 = (3,1,-4)$
$x_2 = (2,2,-3)$
$x_3 = (0,-4,1)$
Let $c_1x_1 + c_2x_2 + c_3x_3 = (0,0,0)$
$c_1(3,1,-4) + c_2(2,2,-3) + c_3(0,-4,1) = (0,0,0)$

$$\begin{bmatrix} 3c_1 + 2c_2 + 0c_3 \\ c_1 + 2c_2 - 4c_3 \\ -4c_1 - 3c_2 + c_3 \end{bmatrix}$$ ∵ 3 equation 3 variables.

In matrix form.

$$\begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, $[A/0]$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & -4 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_2 \sim \begin{bmatrix} \boxed{1} & 2 & -4 \\ 3 & 2 & 0 \\ -4 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 + 4R_1$ $\sim \begin{bmatrix} 1 & 2 & -4 \\ 0 & \boxed{-4} & 12 \\ 0 & 5 & -15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_3 \to R_3 + \frac{5}{4} R_3 \sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -4 & 12 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore \rho(A) = \rho(A/O) = 2 < 3$

$\therefore$ System has in finet solutions

Now,

$c_1 - 2c_2 - 4c_3 = 0$ — ①

$-4c_2 + 12c_3 = 0$

$\Rightarrow c_2 - 3c_3 = 0$

$\therefore$ Free variable $= 3 - 2 = 1$

Put $c_3 = t$, $c_2 = 3t$ & $c_1 = -2t$

$\because c_1 x_1 + c_2 x_2 + c_3 x_3 = 0$

$\Rightarrow -2t x_1 + 3t x_2 + 3t x_3 = 0$

$\Rightarrow t(-2x_1 + 3x_2 + x_3) = 0$

$\therefore t = 1$ given.

$-2x_1 = 3x_2 - x_3$

$\therefore 2x_1 = 3x + x_3$

$\therefore 2x_1 = 3x + x_3$ is the answer.

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c) Determine for what values of a, b, c the system have non trivial solution.

$ax + by + cz = 0, \quad bx + cz + az = 0 \quad cx + ay + bz = 0$

Ans ->
$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For a non-trial sol^n $|A| = 0$

$$|A| = a\begin{vmatrix} c & a \\ a & b \end{vmatrix} - b\begin{vmatrix} b & a \\ c & b \end{vmatrix} + c\begin{vmatrix} b & c \\ c & a \end{vmatrix}$$

$$0 = a(cb - a^2) - b(b^2 - ac) + c(ab - c^2)$$
$$0 = abc - a^3 - b^3 + abc + abc - c^3$$
$$a^3 + b^3 + c^3 - 3abc = 0$$
$$\therefore a^3 + b^3 + c^3 = 0$$
$$\approx a + b + c = 0$$

if $a + b + c = 0$ a non-trival soln must exist

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Q2)

A) For an orthogonal matrix A

$$= \begin{pmatrix} \cos\phi\cos\theta & \sin\phi & \cos\phi\sin\theta \\ -\sin\phi\cos\theta & \cos\phi & -\sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} A^{-1}$$

A^{-1} is $= \begin{bmatrix} \cos\phi\cos\theta & -\sin\phi\cos\theta & -\sin\theta \\ \sin\phi & \cos\phi & 0 \\ \cos\phi\sin\theta & -\sin\phi\sin\theta & \cos\theta \end{bmatrix}$

B) Is the transformation $y_1 = x + y + z$, $y_2 = 2x + 3y + 4z$, $y_3 = x - y + z$ regular? If yes explain.

→ Yes since every component of $f(x, y, z)$ repeals linearly.

C) For what value of k the matrix $\begin{pmatrix} 1/2 & k \\ -k & 1/2 \end{pmatrix}$ is orthogonal.

→ For ~~what~~ value of k the matrix $\begin{bmatrix} 1/2 & k \\ -k & -1/2 \end{bmatrix}$ is $\underline{k = 0}$

D) An $n \times n$ non-homogenous system of linear equations $AX = B$ with A is non-singular matrix have $\underline{n}$ soutions