Page No.
Date

Subject - LADC
Name - Shreerang P Mhatre
Rollno - 111056
Div - 11
Batch - K3

Tutorial - 7

Page No.
Date

Tutorial-7

Q1) Find max & min value of $x^3+3xy^2-3x^2-3y^2+4$

Ans $\Rightarrow$ $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$

$f_x = 3x^2+3y^2-6x=0 \Rightarrow x^2+y^2=2x$ -①

$f_y = 6xy-6y=0 \Rightarrow xy=y$ -②

from eqn ② we get,

$y=0 \quad x=1,$

Hence, value of x wrt '$y=0$'

$x^2-2x=0$

$x(x-2)=0$

$x=0 \,\&\, 2$

value of y wrt ($x=1$)

$y^2=2x-x^2$

$y^2=2-1=1$

$y=\pm 1$

Hence critical points are

$(0,0);(2,0);(1,1);(1,-1)$

Now, $f_{xx}=6x-6$

$f_{xy}=6y$

$f_{yy}=6x-6$

For pt $(0,0)$

$r=f_{xx}(0,0)=-6$

$s=f_{xy}(0,0)=0$

$t=f_{yy}(0,0)=-6 \quad ; \quad r<0$

$\Rightarrow$

Page No.
Date

Now $rt - s^2 = 0$
$36 > 0$
Hence, at $(0,0)$ fonction will be maximum

and minimum value of fonction.
$f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$
$f(0,0) = 4$

Now, for point $(2,0)$
$r = f_{xx}(2,0) = 6$
$s = f_{xy}(2,0) = 0$
$t = f_{yy}(2,0) = 6$

$r > 0$ & ~~rt~~ $rt - s^2 = 36 > 0$

Hence at $(2,0)$ fonction will be minimum and minimum value of function.

$f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$
$f(2,0) = 8 - 12 + 4$
$f(2,0) = 0$

Hence,
maximum value of fonction is 4
& minimum value of fonction is 0

Q2) If $u^3+v^3+w^3 = x+y+z$
$u^2+v^2+w^2 = x^3+y^3+z^3$
$u+v+w = x^2+y^2+z^2$
show that
$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

Ans $u^3+v^3+w^3 = x+y+z$
$$f_1 = u^3+v^3+w^3-x-y-z = 0 \quad -(1)$$

$u^2+v^2+w^2 = x^3+y^3+z^3$
$$f_2 = u^2+v^2+w^2-x^3-y^3-z^3 = 0 \quad -(2)$$

$u+v+w = x^2+y^2+z^2$
$$f_3 = u+v+w-x^2-y^2-z^2 = 0 \quad -(3)$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \left[\frac{\partial(f_1,f_2,f_3)/\partial(x,y,z)}{\partial(f_1,f_2,f_3)/\partial(u,v,w)}\right]$$

$$\frac{\partial(f_1,f_2,f_3)}{\partial(x,y,z)} = \begin{vmatrix} f_{1x} & f_{1y} & f_{1z} \\ f_{2x} & f_{2y} & f_{2z} \\ f_{3x} & f_{3y} & f_{3z} \end{vmatrix} = \begin{vmatrix} -1 & -1 & -1 \\ -3x^2 & -3y^2 & -3z^2 \\ -2x & -2y & -2z \end{vmatrix}$$

$$= (-1)^3 \begin{vmatrix} 1 & 1 & 1 \\ 3x^2 & 3y^2 & 3z^2 \\ 2x & 2y & 2z \end{vmatrix}$$

$$= (-1)^3 (3)(2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}$$

Page No.
Date

$$= (-1)^3 (3)(2)\left[1(y^2z - yz^2) - 1(x^2z - z^2x) + 1(x^2y - y^2x)\right]$$

$$= -6\left[y^2z - yz^2 - x^2z + z^2y + x^2y - xy^2\right]$$

$$= -6\left[z^2(x-y) + x^2(y-z) + y^2(z-x)\right]$$

$$\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)} = \begin{vmatrix} 2u & 2v & 2w \\ 1 & 1 & 1 \\ 3u^2 & 3v^2 & 3w^2 \end{vmatrix}$$

$$= (1)(2)(3)\begin{vmatrix} u & v & w \\ 1 & 1 & 1 \\ u^2 & v^2 & w^2 \end{vmatrix}$$

$$= 6\left[u(w^2 - v^2) - v(w^2 - u^2) + w(v^2 - u^2)\right]$$

$$= 6\left[uw^2 - uv^2 - vw^2 + vu^2 + wv^2 - wu^2\right]$$

$$= 6\left[w^2(u-v) + v^2(w-u) + u^2(v-w)\right]$$

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = (-1)\left[\frac{-6\left[z^2(x-y) + x^2(y-z) + y^2(z-x)\right]}{6\left[w^2(u-v) + v^2(w-u) + u^2(v-w)\right]}\right]$$

$$= \frac{z^2(x-y) + x^2(y-z) + y^2(z-x)}{w^2(u-v) + v^2(w-u) + u^2(v-w)}$$

$$= \frac{z^2x - z^2y + x^2y - x^2z + zy^2 - y^2x}{w^2u - w^2v + v^2w - uv^2 + u^2v - u^2w}$$

$$= \frac{y(x^2 - z^2 - zy - xy) + z^2x - x^2w}{((u-w)(u+w) + v(w-u)) + uw(w-u)}$$

$$= \frac{y(x-z)(x+z-y) + xz(z-x)}{v(u-w)(u+w-v) + uw(w-u)}$$

$$= \frac{(x-z)\left[y(x+z-y) \div xz\right]}{\cancel{uw}(u-w)\,\cancel{(u+w}\cdot\left[v(u+w-v) - uw\right]}$$

$$= \frac{(x-z)\left[y\cdot(x-y) + z(y-x)\right]}{(u-w)\left[v(u-v) + w(v-u)\right]}$$

$$= \frac{(x-z)(x-y)(y-z)}{(u-w)(w-v)(v-w)} = \frac{-(x-y)(y-z)(z-x)}{-(u-v)(v-w)(w-u)}$$

$$\Rightarrow \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

Page No.
Date

Q3) Find point on the surface $z^2 = xy+1$ nearest to the origin by using Lagrange method

Ans-> nearest point on the origin let it be (x,y,z) at distance (d) then

$$d = \sqrt{x^2+y^2+z^2}$$

$$f(x,y,z) \Rightarrow d^2 = x^2+y^2+z^2$$
$$\phi(x,y,z) = z^2-xy-1=0 \quad -① $$
$$F(x,y,z) = (x^2+y^2+z^2)+\lambda(z^2-xy-1)=0$$

$$\frac{\partial F}{\partial x} = 2x-y\lambda=0 \quad -②$$
$$\frac{\partial F}{\partial y} = 2y-x\lambda=0 \quad -③$$
$$\frac{\partial F}{\partial z} = 2z+2z\lambda=0 \Rightarrow \lambda=-1$$

Now put $\lambda=-1$ in eqn ② & ③ we get

$$2x+y=0 \quad -④$$
$$2y+x=0 \quad -⑤$$

on solving eqn ④ & ⑤ we get

$$x=0 \quad \& \quad y=0.$$

Put value of x & y in eqn ① we get

$$z^2-1=0$$
$$z^2=1$$
$$z=\pm1$$

pts $(0,0,1)$ and $(0,0,-1)$ are mid strawed from origin. Hence minimum distance from origin on the surface $z^2=xy+1$ is 1 unit and at pts $(0,0,1)$ and $(0,0,-1)$.

Q2) Fill ups -

① The ~~cr~~ critical points of $x^2+y^2+6x+12$ are

$$f(x,y) = x^2+y^2+6x+12$$
$$f_x = 2x+6 = 0$$
$$\Rightarrow x = -3$$
$$f_y = 2y = 0.$$
$$y = 0$$
critical pt is $(-3,0)$.

② $\frac{1}{v} - \frac{1}{u} = \frac{2}{f}$ - ①

$100 \frac{du}{u} = 2 \quad 100 \frac{dv}{v} = 2$

differential eqn ① we get

$$\frac{-1}{v^2} dv + \frac{1}{u^2} du = \frac{-2}{f^2} \cdot df \text{ - ②}$$

multiply eqn ② by 100 we get

$$\frac{100\, dv}{v}\left(\frac{-1}{v}\right) + \frac{100\, du}{u}\left(\frac{1}{u}\right) = \frac{100\, df}{f}\left(\frac{-2}{f}\right)$$

$$\frac{-2}{v} + \frac{2}{u} = 100 \frac{df}{f}\left(\frac{-2}{f}\right)$$

$$+\frac{1}{v} - \frac{1}{u} = 100 \frac{df}{f}\left(\frac{1}{f}\right)$$

$$\frac{2}{f} = 100 \frac{df}{f}\left(\frac{1}{f}\right)$$

% error in f will be 2%

Page No. Date

③ $u = \frac{x+y}{z}$, $v = \frac{y+z}{x}$; $w = \frac{y(x+y+w)}{xz}$

Relation between u, v and w is $w = uv - 1$

Proof, $w = uv - 1$

$$= \left(\frac{x+y}{z}\right)\left(\frac{y+z}{x}\right) - 1$$

$$= \frac{xy + xz + y^2 + yz - 1}{xz}$$

$$= \frac{xy + y^2 + yz + yz - xz}{xz}$$

$$= \frac{y(x+y+z)}{xz} = w$$

④ $x = u\cos v$ & $y = u\sin v$.

$$\frac{\partial(u,v)}{\partial(x,y)} = J' \qquad J \cdot J' = 1$$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} \cos v & -u\sin v \\ \sin v & u\cos v \end{vmatrix}$$

$$= u\cos^2 v + u\sin^2 v = u.$$

$$J \cdot J' = 1$$

$$J' = \frac{1}{u} = \frac{\partial(u,v)}{\partial(x,y)}$$