Subject - LADC
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Tutorial - 6

Q1) Fill in the blanks

① nth derivative of $\frac{1}{x^3+6x^2+11x+6}$ = ______

$$y_n = (-1)^n n! \left[\frac{1}{x(x+1)^{n+1}} - \frac{1}{(x+3)^{n+1}} + \frac{1}{2(x+3)^{n+1}} \right]$$

② If $v = (x^2-y^2) f(xy)$ then $v_{xx} + v_{yy} =$ ______

$$v_{xx} + v_{yy} = (x^4 - y^4) f''(xy)$$

③ If $x = r\cos\theta$, $y = r\sin\theta$ then $\left(\frac{dy}{dr}\right)_x =$ ______

$$\left(\frac{dy}{dr}\right)_x = \frac{r}{\sqrt{r^2-x^2}}$$

④ If $x = \tan(\log y)$ then the relationship between y_1 & y ______

$$y_1 = \frac{1}{(1+x^2)} y$$

$$\therefore \frac{1}{(1+x^2)} y - y_1 = 0$$

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Q2) Solve -

1) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then $2x\frac{du}{dx} + 2y\frac{du}{dy} = ?$

Ans> If $f(x,y) = u \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$$f(u) = f(x,y) = \sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$$

$$f(tx,ty) = \sin u = \frac{t(x+y)}{\sqrt{t}(\sqrt{x}+\sqrt{y})}$$

$$= t^{1/2}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$f(tx,ty) = t^{1/2} f(x,y)$$

Hence it is a homogenous function of degree .

Now,

According to corollary II

$$x\cdot\frac{du}{dx} + y\cdot\frac{du}{dy} = \frac{nf(u)}{f'(u)}$$

$$x\cdot\frac{du}{dx} + y\frac{du}{dy} = \frac{1}{2}\left(\frac{\sin u}{\cos u}\right) \quad -①$$

Multiply eqn ① by ② we get

$$2x\cdot\frac{du}{dx} + 2y\frac{du}{dy} = \tan u$$

Ans - $2x\frac{du}{dx} + 2y\frac{du}{dy} = \tan\left[\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)\right]$

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Q2) If $y = e^{\tan^{-1}(x)}$ then PT $(x^2+1)y_{n+2} + (2(n+1)x - 1)y_{n+1} + n(n+1)y_n = 0$

Ans→ $y = e^{\tan^{-1}x}$

differentiate

$$y_1 = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$(1+x^2)y_1 = e^{\tan^{-1}x}$$

$$(1+x^2)y_1 = y$$

again differentiate both side.

$$(2x)y_1 + (1+x^2)y_2 = y_1$$

$$(1+x^2)y_2 + (2x-1)y_1 = 0$$

Now use lebinitz Theorem we get.

$${}^nC_0(1+x^2)y_{n+2} + {}^nC_1(2x)y_{n+1} + {}^nC_2(2)y_n + {}^nC_0(2x-1)y_{n+1} + {}^nC_1(2)y_n = 0$$

$$= (1+x^2)y_{n+1} + 2xn\,y_{n+1} + n(n-1)y_n + (2x-1)y_{n+1} + 2ny_n = 0$$

$$= (1+x^2)y_{n+1} + (2(n+1)(x-1))y_{n+1} + (n^2 n + 2n)y_n = 0$$

$$= (1+x^2)y_{n+2} + [2(n+1)(x-1)]y_{n+1} + [n(n+1)]y_n = 0$$

Hence proved.

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Q3) nth derivative of $e^x(2x+3)^3 = ?$

Ans $\rightarrow$ $y = e^x(2x+3)^3$

using lebnitz theorem ($u \rightarrow e^x$, $v \rightarrow (2x+3)^3$)

$$y_n = {}^nC_0 u_n v + {}^nC_1 u_{n-1} v_1 + \cdots {}^nC_n u v_n$$

$$y_n = {}^nC_0 e^x(2x+3)^3 + {}^nC_1 e^x(3(2x+3)^2) + {}^nC_2 e^x [6(2)(2x+3)(2)] + {}^nC_3 e^x[24(3)]$$

$$y_n = e^x(2x+3)^3 + 6n e^x(2x+3)^2 + n(n+1)e^x \, 12(2+3) + n(n+1)(n+2)(8)e^x$$

Ans $\rightarrow$ $y_n = e^x[(2x+3)^3 + 6n(2x+3)^2 + 12n(n+1)(2x+3) \, 8n(n+1)(n+2)]$