3. Analysis of Transient Response in Circuits.

Whenever a n/w containing energy storage elements such as inductor, or capacitor is switched from one condition to another, either by change ~~of~~ in applied source or change in n/w elements, the response current and voltage change from one state to other state. The time taken to change from an initial steady state to the final steady state is known as the transient period. This response is known as transient response or transients. The response of the n/w after it attains a final steady value is independent of time and is called as the steady state response. The complete response of the n/w is determined with the help of a differential eq^n.

Initial Conditions

While solving differential eq^{n}s, we get some arbitrary constants. Initial conditions are used to determine these arbitrary constants. It helps us to know behaviour of elements at the instant of switching.

To differentiate betn the time immediately before and after the switching, the signs '−' and '+' are used. The conditions existing just before switching are denoted as $i(0^-)$, $v(0^-)$, etc. conditions just after switching are denoted as $i(0^+)$, $v(0^+)$, etc.

Sometimes conditions at $t = \infty$ are used in evaluation of arbitrary constants. These are known as final conditions.

So, the time period is divided as follows –

1. Just before switching (from $t = -\infty$ to $t = 0^-$)
2. Just after switching ($t = 0^+$)
3. After switching ($t > 0$)

Initial conditions for Resistor – for a resistor, current & voltage relation is given by

$$v(t) = R \cdot i(t)$$

The current in the resistor will change instantaneously, ~~similarly~~, if the vltg. changes instantaneously. Similarly, the voltage will change if current changes instantaneously.

Initial conditions for Inductor – For a inductor, the relation betn vltg. & current is given by,

$$v(t) = L \frac{di}{dt}$$

voltage across the inductor is proportional to the rate of change of current. An inductor does not allow an abrupt change in current through it. The current through inductor is given by

$$i(t) = \frac{1}{L}\int_0^t v(t)\,dt + i(0)$$

where $i(0)$ is the initial current through inductor.

If there is no current flowing through the inductor at $t=0^-$, the inductor will act as an open circuit at $t=0^+$.

If a current of value I_0 flows through inductor at $t=0^-$, the inductor can be regarded as a current source of I_0 ampere at $t=0^+$

<u>Initial conditions for the capacitor</u> - Current and voltage relation for a capacitor is given by

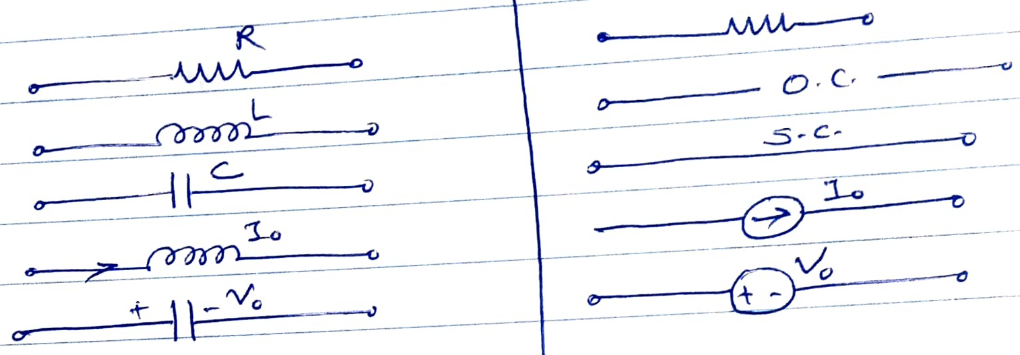
$$v(t) = \frac{1}{C}\int_0^t i(t)\,dt + v(0)$$

where $v(0)$ is the initial vltg. across capacitor. If there is no voltage across the capacitor at $t=0^-$, the capacitor will act as short circuit at $t=0^+$.

If the capacitor is charged to a voltage V_0 at $t=0^-$, then it can be regarded as a voltage source of V_0 at $t=0^+$.

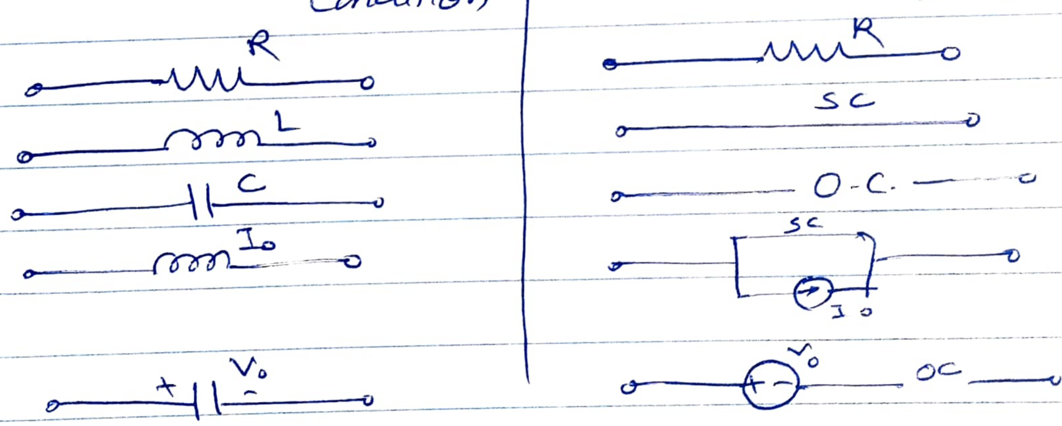
These conditions are summarized as,

Element with initial conditions	Equivalent ckt at $t = 0^+$



Similarly we can draw chart for final conditions.

Element with initial condition	Equivalent ckt at $t = \infty$



Procedure for evaluating initial conditions.

1. Draw the equi. ckt at $t=0^-$. Before switching take place (ie for $t=-\infty$ to $t=0^-$) the n/w is under steady state condn. Hence find current flowing through inductors $i_L(0^-)$ and vltg across capacitor $v_c(0^-)$

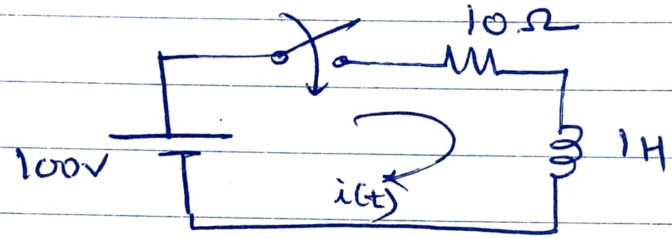
2. Draw the equi. n/w at $t=0^+$, ie, immediately after switching. Replace all inductors with O.C or with current sources $i_L(0^+)$ and replace all capacitors by S-C or vltg-sources $V_c(0^+)$. Keep resistors as it is.

3. Initial vltgs or currents are determined from the equi. n/w at $t=0^+$.

4. Initial conditions, ie, $\frac{di}{dt}(0^+)$, $\frac{dv}{dt}(0^+)$, $\frac{d^2i}{dt^2}(0^+)$, $\frac{d^2v}{dt^2}(0^+)$ are determined by writing integro-differential eqns for the n/w for $t>0$, ie after the switching action by making use of initial condn.

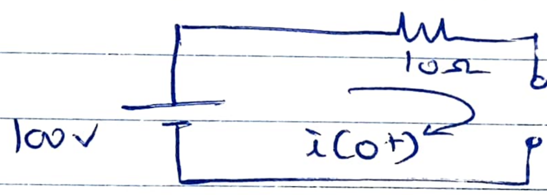
Examples

1. In the given n/w the switch is closed at $t=0$. With zero current in the inductor, find i, $\frac{di}{dt}$, and $\frac{d^2i}{dt}$ at $t=0^+$



Sol^n. at $t=0^-$, no current flows through the inductor. $\therefore$ $i(0^-)=0$

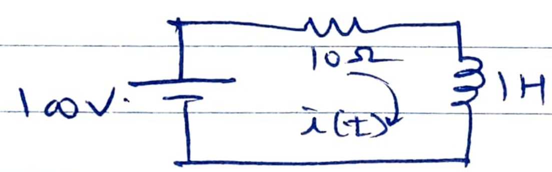
at $t=0^+$ the n/w is,



at $t=0^+$ inductor acts as O.C.

$i(0^+)=0$

For $t>0$, the n/w is,



writing KVL eq^n for $t>0$

$$100 - 10i - 1\frac{di}{dt} = 0 \qquad \text{——— (1)}$$

$$\frac{di}{dt} = 100 - 10i \qquad \text{——— (2)}$$

at $t=0^+$,

~~di~~

$$\frac{di}{dt}(0^+) = 100 - 10\,i(0^+)$$

$$= 100 - 10(0)$$

$$= 100 \text{ A/s.}$$

Differentiating eqn. (2),

$$\frac{d^2i}{dt^2} = -10\frac{di}{dt}$$

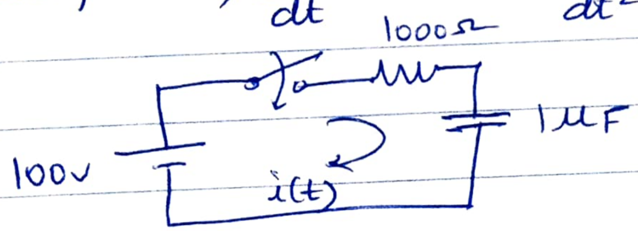
at $t = 0^+$,

$$\frac{d^2i}{dt^2}(0^+) = -10\frac{di}{dt}(0^+)$$

$$= -10(100)$$

$$= -1000 \text{ A/s}^2$$

2. In the n/w below the switch is closed at $t=0$. With the capacitor uncharged find value for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$



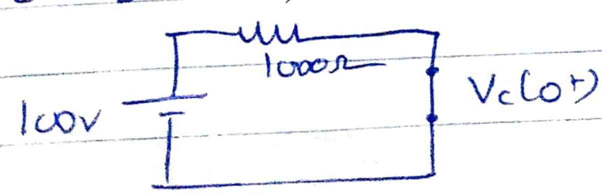
Soln.

At $t = 0^-$, the capacitor is uncharged

$$V_c(0^-) = 0$$

$$i(0^-) = 0$$

At $t = 0^+$, the n/w is as shown below

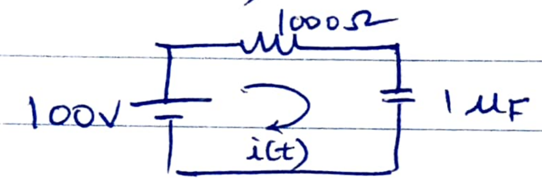


At $t = 0^+$, the capacitor acts as s.c.

$$V_c(0^+) = 0$$

$$i(0^+) = \frac{100}{1000} = 0.1\,A$$

For $t > 0$, the n/w is,



Writing KVL eqⁿ for $t > 0$,

$$100 - 1000i - \frac{1}{1\times10^{-6}}\int_0^t i\,dt = 0 \quad \text{—— (1)}$$

Differentiating the eqⁿ (1)

$$0 - 1000\frac{di}{dt} - 10^6 i = 0$$

$$\frac{di}{dt} = -\frac{10^6}{1000}\,i \quad \text{—— (2)}$$

At $t = 0^+$,

$$\frac{di}{dt}(0^+) = -\frac{10^6}{1000}\,i(0^+)$$

$$= \frac{-10^6}{1000}(0.1)$$

$$= \cancel{0.1\,A/s}\ -100\ A/s.$$

Differentiating eqⁿ (2),

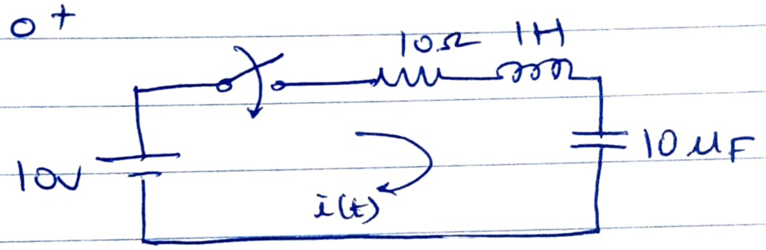
$$\frac{d^2i}{dt^2} = \frac{-10^6}{1000}\cdot\frac{di}{dt}$$

$$\frac{d^2i}{dt^2}(0^+) = \frac{-10^6}{1000}\,\frac{di}{dt}(0^+)$$

$$\therefore \frac{d^2 i}{dt^2}(0^+) = \frac{-10^6}{1000}(-100)$$

$$= 10^5 \ A/s^2.$$

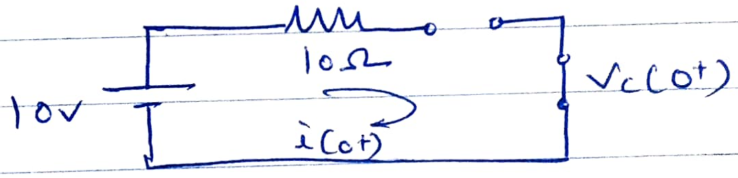
3. In the n/w shown below, assuming all initial conditions as zero find i, $\frac{di}{dt}$ and $\frac{d^2 i}{dt^2}$ at $t = 0^+$



Soln. At $t = 0^-$,

$$i(0^-) = 0$$
$$V_c(0^-) = 0$$

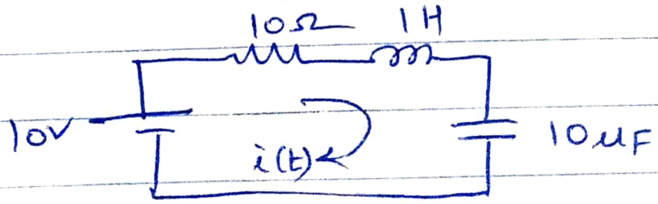
- At $t = 0^+$, the n/w is as below,



At $t = 0^+$, the inductor acts as o.c & capacitor acts as s.c.

$$\therefore i(0^+) = 0$$
$$V_c(0^+) = 0$$

- At $t > 0$, the n/w is shown below;



writing KVL eqn for $t > 0$,

$$10 - 10i - 1\frac{di}{dt} - \frac{1}{10\times10^{-6}}\int_0^t i\,dt = 0 \quad \text{--- (1)}$$

At $t = 0^+$,

$$10 - 10i(0^+) - \frac{di}{dt}(0^+) - 0 = 0$$

$$\therefore \frac{di}{dt}(0^+) = 10 \text{ A/s.}$$

Differentiating eqn (1),

$$0 - 10\frac{di}{dt} - \frac{d^2i}{dt^2} - \frac{1}{10\times10^{-6}}\,i = 0$$

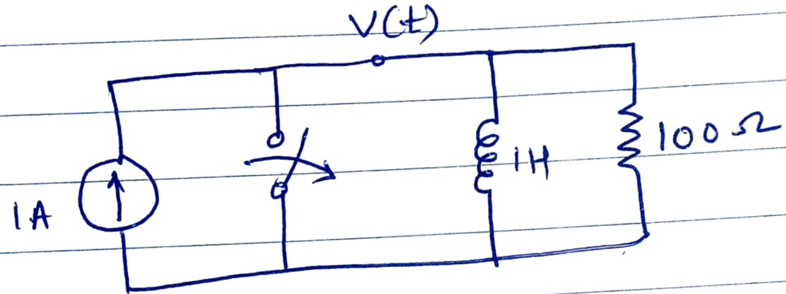
At $t = 0^+$,

$$0 - 10\frac{di}{dt}(0^+) - \frac{d^2i}{dt^2}(0^+) - \frac{1}{10^{-5}}\,i(0^+) = 0$$

$$-10\times10 - \frac{d^2i}{dt^2}(0^+) = 0$$

$$\boxed{\therefore \frac{d^2i}{dt^2}(0^+) = -100 \text{ A/m}^2}$$

4. In the n/w shown below, the switch is opened at $t=0$. Calculate v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t=0^+$.

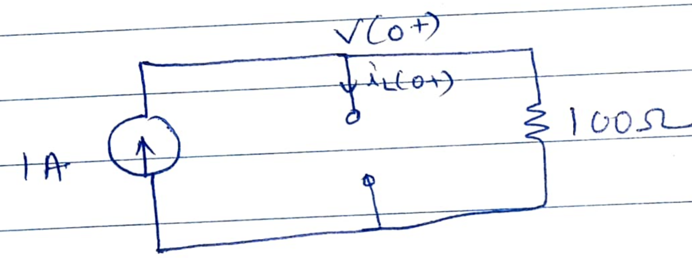


Sol^n

At $t=0^-$ the switch is closed. Hence no current flows through the inductor

$\therefore\ i_L(0^-) = 0$

- At $t=0^+$, the n/w is,

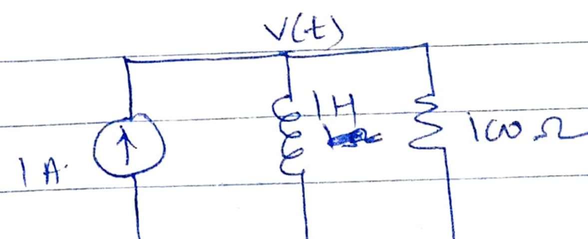


at $t=0^+$ the inductor acts as O.C.

$\therefore\ i_L(0^+) = 0$

$V(0^+) = 100 \times 1\,A = 100\,V$

- For $t>0$ the n/w is,



Writing KCL eq^n for, $t > 0$,

$$\frac{V}{100} + \frac{1}{1}\int_0 v\,dt = 1 \quad \text{—— (1)}$$

→ current value of current source

Differentiating eq^n (1),

$$\frac{1}{100}\frac{dv}{dt} + v = 0 \quad \text{—— (2)}$$

at $t = 0^+$

$$\frac{dv}{dt}(0^+) = -100 \cdot v(0^+)$$

$$\boxed{\frac{dv}{dt}(0^+) = -100 \times 100 = -10000 \text{ V/s.}}$$

Differentiating eq^n (2),

$$\frac{1}{100}\frac{d^2v}{dt^2} + \frac{dv}{dt} = 0$$

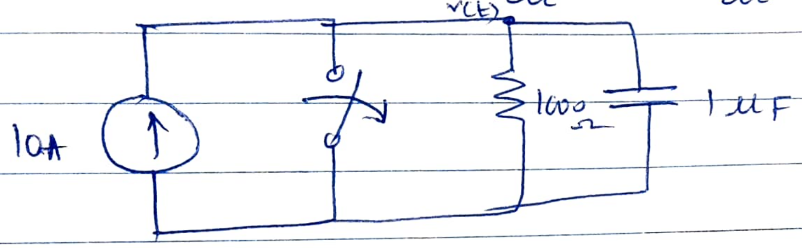
at $t = 0^+$

$$\frac{d^2v}{dt^2}(0^+) = -100\frac{dv}{dt}(0^+)$$

$$= -100(-10000)$$

$$\boxed{\frac{d^2v}{dt^2} = 10^6 \text{ V/s}^2.}$$

5. In the n/w below, the switch is opend at $t=0$. Solve for v, $\frac{dv}{dt}$ & $\frac{d^2v}{dt^2}$ at $t=0^+$

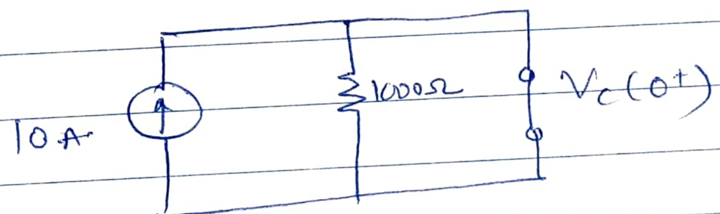


Soln.

At $t=0^-$, switch is closed, hence the voltage across the capacitor is zero.

$$\therefore V(0^-) = V_c(0^-) = 0$$

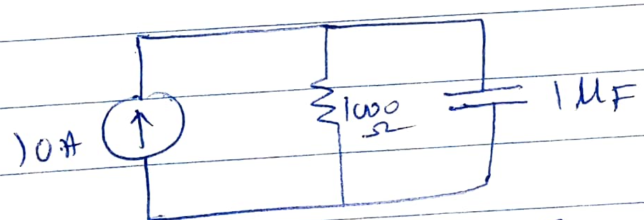
• At $t=0^+$, the n/w is,



at $t=0^+$, the capacitor acts as S.C.

$$\therefore V(0^+) = V_c(0^+) = 0$$

• For $t>0$, the n/w is,



writing KCL eqn. for $t>0$,

$$\frac{V}{1000} + 10^{-6}\frac{dv}{dt} = 10 \quad \text{--- (1)}$$

At $t = 0^+$,

$$\frac{v(0^+)}{1000} + 10^{-6}\frac{dv(0^+)}{dt} = 10$$

$\overset{\shortparallel}{0}$

$$\therefore \frac{dv}{dt}(0^+) = 10 \times 10^6$$

$$\frac{dv}{dt} = 10^7 \text{ V/s.}$$

• Differentiating eqn ①,

$$\cancel{\frac{1}{1000}\frac{dv}{dt}(0^+)}$$

$$\frac{1}{1000}\frac{dv}{dt} + 10^{-6}\frac{d^2v}{dt^2} = 0$$

at $t = 0^+$

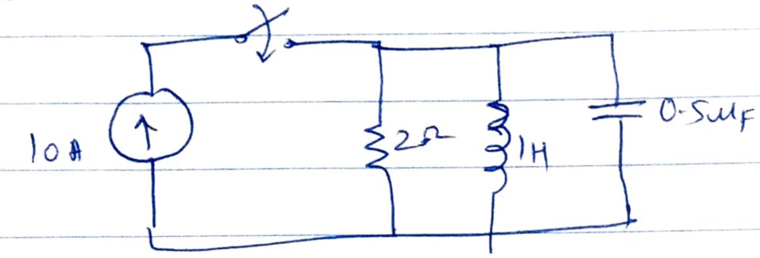
$$\therefore \frac{1}{1000}\frac{dv}{dt}(0^+) + 10^{-6}\frac{d^2v}{dt^2}(0^+) = 0$$

$$\therefore 10^{-6}\frac{d^2v}{dt^2}(0^+) = -\frac{1}{1000} \times 10^7$$

$$\therefore \frac{d^2v}{dt^2}(0^+) = -\frac{1}{1000} \times 10^7 \times 10^6$$

$$= -10^{10} \text{ V/s}^2$$

6. For the n/w shown below, the switch is closed at $t=0$, determine $v, \frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ at $t=0^+$



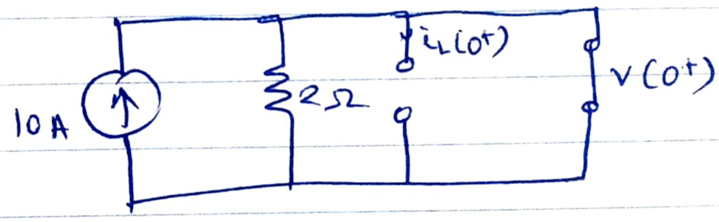
Soln.

at $t=0^-$ no current flows through inductor

$$\therefore \; i_L(0^-) = 0$$

$$v(0^-) = 0$$

at $t=0^+$ the n/w is,

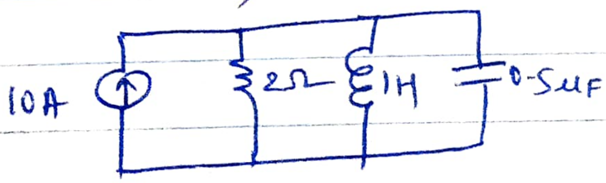


at $t=0^+$, inductor acts as o.c. & capacitor as s.c.

$$\therefore \; i_L(0^+) = 0$$

$$v(0^+) = 0$$

for $t>0$,



Writing KCL eqn for $t>0$

$$\frac{V}{2} + \frac{1}{1}\int_0^t v\,dt + 0.5\times10^{-6}\frac{dv}{dt} = 10 \quad \text{——} (1)$$

at $t = 0^+$

$$\underbrace{\frac{v(0^+)}{2}}_{0} + \underbrace{\int_0^t v(0^+)\,dt}_{0} + (0.5\times10^{-6})\frac{dv(0^+)}{dt} = 10$$

$$\therefore \boxed{\frac{dv}{dt} = \frac{10}{0.5\times10^{-6}} = 20\times10^{6}\ V/s.}$$

Differentiating eqn (1),

$$\frac{1}{2}\cdot\frac{dv}{dt} + V + (0.5\times10^{-6})\frac{d^2V}{dt^2} = 0$$

at $t = 0^+$

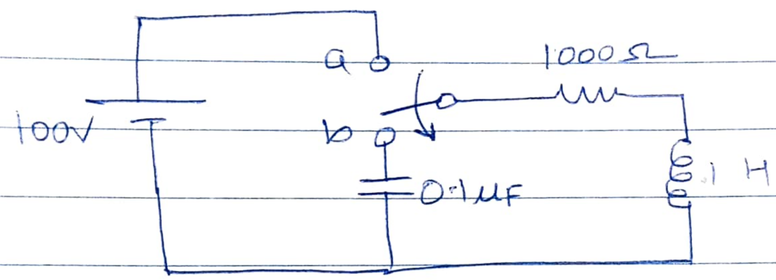
$$\frac{1}{2}\frac{dv}{dt}(0^+) + \underbrace{v(0^+)}_{0} + (0.5\times10^{-6})\cdot\frac{d^2v}{dt}(0^+) = 0$$

$$\frac{1}{2}\times20\times10^{6} + 0.5\times10^{-6}\cdot\frac{d^2v}{dt^2}(0^+) = 0$$

$$\therefore \frac{d^2v}{dt^2}(0^+) = \frac{-10\times10^{6}}{0.5\times10^{-6}}$$

$$\therefore \boxed{\frac{d^2v}{dt^2}(0^+) = -20\times10^{12}\ V/s^2}$$

In the n/w shown below, the switch is changed from position 'a' to 'b' at $t=0$. Solve for i, $\frac{di}{dt}$ & $\frac{d^2i}{dt^2}$ at $t=0^+$

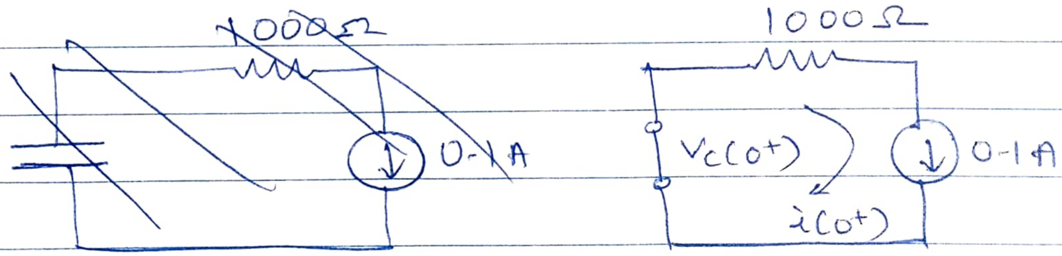


• At $t=0^-$, the n/w attains steady condn. Hence inductor acts as s.c.

$$\therefore i(0^-) = \frac{100}{1000} = 0.1\,A$$

$$V_c(0^-) = 0\ V.$$

• At $t=0^+$, the n/w is,

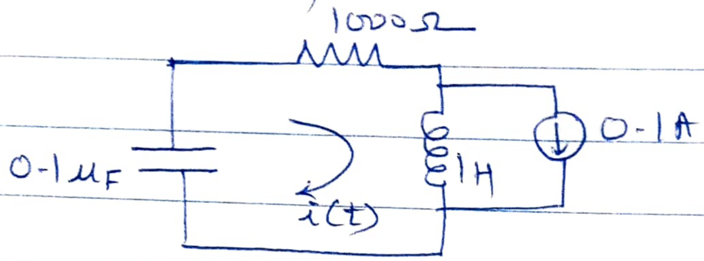


At $t=0^+$, the inductor acts as a current source of 0.1A & the capacitor acts as S.C.

$$\therefore i(0^+) = 0.1A$$

$$V_c(0^+) = 0$$

For $t > 0$, the n/w is



Writing KVL eqn for $t > 0$

$$-\frac{1}{0.1 \times 10^{-6}} \int_0^t i\, dt - 1000 i - 1 \frac{di}{dt} = 0 \quad \text{——— (1)}$$

at $t = 0^+$,

$$0 - 1000\, i(0^+) - \frac{di}{dt}(0^+) = 0$$

$$\therefore \frac{di}{dt}(0^+) = -1000(0.1)$$

$$= -100 \text{ A/s}$$

Differentiating eqn (1),

$$-\frac{1}{10^{-7}} i - 1000 \frac{di}{dt} - \frac{d^2 i}{dt^2} = 0$$

at $t = 0^+$

$$-10^{+7} i(0^+) - 1000 \frac{di}{dt}(0^+) - \frac{d^2 i}{dt^2}(0^+) = 0$$

$$\therefore \frac{d^2 i}{dt^2}(0^+) = -10^7 (0.1) - 1000(-100)$$

$$= -9 \times 10^5 \text{ A/s}^2.$$