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Tutorial III

Q1) $(D^2-1)y = x^3$

$\cancel{Py = 1}$

$\rightarrow P.I = \frac{1}{(D^2-1)} x^3$

$= \frac{1}{-(1-D^2)} x^3$

$= -(1-D^2)^{-1} x^3$

here, $f(x) = x^3$

$\rightarrow D(x^3) = 3x^2$

$D^2(x^3) = 6x$

$D^3(x^3) = 6$

$D^4(x^3) = 0$

by using binomial th -

$\rightarrow -(1-D^2)^{-1} x^2 = -(1+D^2+D^4 \dots) x^3$

$= -(x^3 + x^3 D^2)$

$= -(x^3 + D^2 x^3)$

$= -(x^3+6x)$

$= (-x^3-6x)$

Ans $\rightarrow$ D

Q2) $(D^2+1)y = \tan x$

$C.F = [c_1 \cos x + c_2 \sin x]$ $\therefore y_1 = \cos x$
$y_2 = \sin x$

$P.I = u\cos x + v \sin x$ — ①

Here,

$u = \int -\frac{y_2 f(x)}{w} dx$

$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$v = \int \frac{y_1 f(x)}{w} dx$

$\Rightarrow W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$\Rightarrow W = (\cos x \cos x) - (\sin x - \sin x)$

$\Rightarrow W = \cos^2 x + \sin^2 x$

$\Rightarrow W = 1$

$\rightarrow$

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$\therefore v = \int \frac{y f(x)}{w} = \int \frac{\cos x \cdot (\tan x)}{1} dx$

$= \int \cancel{(\cos x)} \times \frac{\sin x}{\cancel{\cos x}} dx$

$= \int (\sin x) dx$

Ans - a $\quad = -\cos x .$

Q3) $D(D-1)y - 3Dy + 5y = e^{2z} \sin z$ $\quad (z = \log x)$

$D^2 - y - Dy - 3Dy + 5y = e^{2z} \sin z$

$D^2 y - 4Dy + 5y = e^{2z} \sin z$

$\therefore (D^2 - 4D + 5) y = e^{2z} \sin z$

Ans - a

Q4) The differential eqn $(4x+1)^2 \frac{d^2y}{dx^2} + 2(4x+1)\frac{dy}{dx} + 2y$

$= 2x+1$ on putting $4x+1 = e^z$ & using $D = \frac{d}{dz}$ -

$\Rightarrow (4^2 D(D-1) + 2(4)(D) + 2) y = 2\left(\frac{e^z - 1}{4}\right) + 1$

$(16D^2 - 16D + 8D + 2) y = \frac{e^z}{2} + \frac{1}{2}$

$\therefore (16D^2 - 8D + 2) y = \frac{e^z + 1}{2}$

Ans - b

$\rightarrow$

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Q5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$

$\to$ $D^2 - 2D + 1 = 0$

$(D-1)(D-1) = 0$

$\frac{1}{(D-1)^2} xe^x \sin x$

$\left[\left(x - \frac{\phi'(D)}{\phi(D)}\right] \frac{1}{\phi(D)} V\right]$

$= \left[x - \frac{2D-2}{D^2-2D+1}\right] \frac{1}{(D-1)^2} e^x \sin x$

$= \left[x - \frac{2D-2}{D^2-2D+1}\right] \frac{1}{(D-1)} e^x \int e^{-x} e^x \sin x \, dx$

$= \left[x - \frac{2(D-1)}{(D-1)^2}\right] e^x \int e^{-x} e^x (-\cos x) dx$

$= -\left[x - \frac{2}{D-1}\right] e^x \sin x$

$= -e^x x \sin x + \frac{2}{D-1} e^x \sin x$

$= -x \frac{(\sin x)}{e^x} + 2e^x \int e^{-x} e^x \sin x \, dx$

$= -xe^x \sin x - 2\cos x e^x$

$= -e^x(x \sin x + 2\cos x)$

~~Q6.~~ Ans Q

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Q36) $(D-1)^3 y = e^x \sqrt{x}$ is

$$P.I = e^{ax} \frac{1}{\phi(D+a)} V$$

$$P.I = e^x \frac{1}{(D-1+1)^2} \sqrt{x} = e^x \frac{1}{D^3} \sqrt{x}$$

$$= e^x \frac{1}{D^2} \int \sqrt{x}\, dx = \frac{e^x\, x^{3/2}}{D^2\, 3/2}$$

$$= \frac{2}{3} e^x \frac{1}{D} \int x^{3/2} dx$$

$$= e^x \frac{2}{3} \cdot \frac{2}{5} \int x^{5/2} dx$$

$$= \frac{4}{15} \cdot \frac{2}{7} x^{7/2} e^x$$

$$= \frac{8}{105} e^x x^{7/2}$$

Ans - b

Q37) $\frac{1}{D+2} e^{-x} e^{e^x}$

$$= e^{-2x} \int e^{2x-x} e^{e^x} dx$$

$$= e^{-2x} \int e^x e^{e^x} dx$$

Let

$$e^x = t \rightarrow dt = e^x dx$$

$$= e^{-2x} \int e^t dt$$

$$= e^{-2x} e^t$$

$$= e^{-2x} e^{e^x}$$

Ans b

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Q8) $(D^3+D)y = \cos x$ –

$\rightarrow$ $\phi(a)=0$ $\quad \phi(D) = D^3 - D^2 + D - 1$

$\therefore \phi'(a) = 3D^2 - 2D + 1$

$\phi'(a) = 2$

$\therefore P.I = x \dfrac{1}{\phi'(a)} e^x$

$= \dfrac{1}{2} x e^x$ $\quad$ Ans C

Q10) $2\dfrac{d^2Q}{dtQ^2} + 4\dfrac{dQ}{dt} + \dfrac{Q}{0.05} = 100$

$\dfrac{d^2Q}{dt^2} + 2\dfrac{dQ}{dt} + 10Q = 50$

A.E $\rightarrow (D^2+2D+10) = 0$

$D = \dfrac{-2 \pm \sqrt{4-40}}{2} = \dfrac{-2 \pm \sqrt{36}}{2}$

$D = -1 \pm 3i$

$\therefore C.F = e^{-x}(c_1\cos 3x + c_2\sin 3x)$ $(x=t)$

$P.I = \dfrac{1}{D^2+2D+10} 50 = \dfrac{1}{D^2+2D+10} 50e^{0x}$

$= 50 \dfrac{1}{D^2+2D+10} e^{0x} = 50 \dfrac{1}{10} = 5$

$\rightarrow$

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$\therefore Q(t) = e^{-x}(c_1\cos 3x + c_2\sin 3x) + 5$ $(x=t)$

$I(t) = \frac{dQ(t)}{dt}$

$\therefore I(t) = -e^{-x}(c_1\cos 3x + c_2\sin 3x)$
$+ e^{-x}(-3c_1 e\sin 3x + 3c_2\cos 3x)$

at initial conditions, $Q=0$, $I=0$, $t=0$

$\therefore 0 = e^{-x}c_1 + 5$ (from Q)

$\therefore c_1 = -5$

$0 = -c_1 + 3c_2 + 5$ (from I)

$0 = -5 + 3c_2$

$\therefore c_2 = 5/3$

$\therefore Q(t) = e^{-x}(-5\cos 3x) + 5$ $(x=t)$

$\therefore I(t) = -e^{-t}(-5\cos 3t + 5/3\sin 3t)$
$+ e^{-t}(15\sin 3x - 5\cos 3x)$

$\therefore I(t) = e^{-t}(15\sin 3t - 5\cos 3t)$
$- e^{t}(\frac{5}{3}\sin 3t - 5\cos 3t)$

Ans =

Q9) $\phi(a) = 0$ $\quad \phi(D) = D^3 - D^2 + D - 1$

$\therefore \phi'(a) = 3D^2 - 2D + 1$

$\phi'(a) = 2$

$\therefore P.I = x \frac{1}{\phi'(a)} e^{x}$

$= \frac{1}{2} x e^{x}$

Ans C