

ECAUnit 4 - Circuit simplification techniques.

Networks:

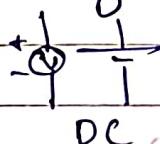
1. Active / Passive
2. Linear / Non-linear
3. Unilateral / Bi-lateral.

Main comp. - Resistance, Inductance, Capacitance, sources.

i) Sources - giving energy

1. Voltage
2. Current

Voltage:

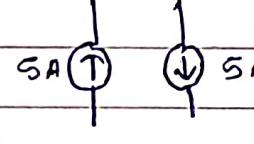


DC



AC.

Current:



SA ↑

SA ↓

x) Dependent voltage current.

→ value is not dependent on anything.

ii) Dependent

Vltg. dependent vltg source

$$v = kv_1$$

Current

$$v = ki_1$$

Vltg " "

current " "

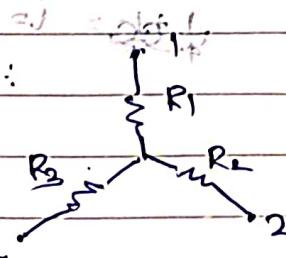
$$i = kv_1$$

Current " "

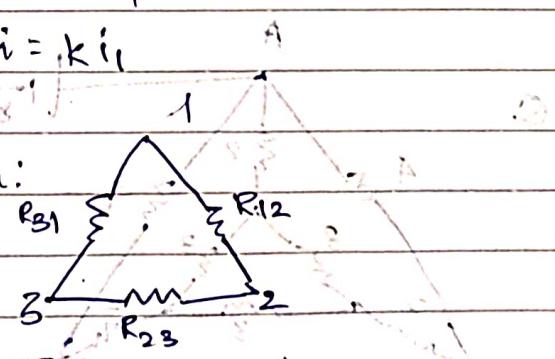
" "

$$i = ki_1$$

Star :



Delta:



$$S \text{ to } D - R_{12} = \frac{R_1 + R_2 + R_1 R_2}{R_3}$$

$$\text{PTOS} :: R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$Q_3: \text{Rechteck } A B C D \text{ mit } A(0|0), B(1|0), C(1|1), D(0|1) \text{ und } E(0.5|0.5)$$

$$B \begin{array}{c} \swarrow \\ M \\ \searrow \end{array} C = \frac{13}{9} = \frac{\frac{1}{6} + \frac{2}{6}}{\frac{6}{6}} \text{ (charakteristisch)} \\ = \frac{13}{9} = \frac{12+1}{12} \text{ (positiv)}$$

A *Inabot-ia* | *Inabot-ia*

$$\frac{6}{5} + \frac{12}{7} = \frac{42+60}{35} = \frac{102}{35}$$

$$\begin{array}{r} \text{faster} \\ \hline 102 \\ \hline 102 \end{array} \quad \begin{array}{r} 102 \\ \hline 102 \end{array} \quad \begin{array}{r} 102 \\ \hline 102 \end{array}$$

$$= \frac{52.5 + 102}{102.0} \cdot \text{faktorial} \quad | \begin{array}{l} 52.5 \\ 102.0 \end{array}$$

1st = 11 2nd = 153 3rd = 154.5

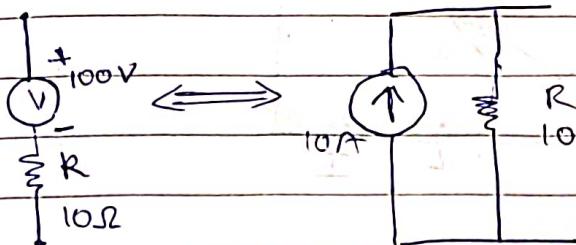
1830 32 10' distant
1845 10' 10'

1845 " ab 000

A just as u s w o f recued

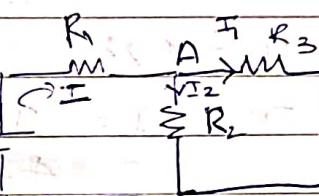
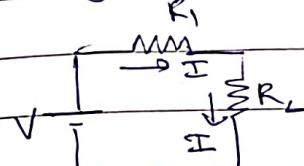
$$\frac{1}{4} \cdot 16 = i = ?$$

Source Transformation



$$V = IR \quad \text{Ohm's law} \quad I = \frac{V}{R}$$

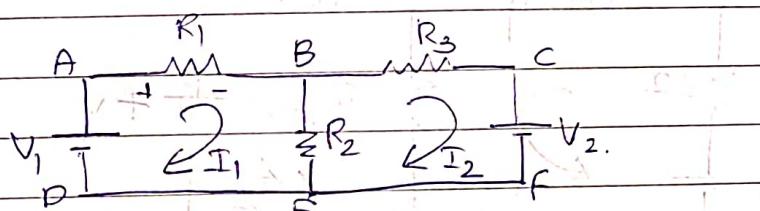
x) KVL & KCL



$$V = IR_1 + IR_2 \quad I = I_1 + I_2$$

Mesh analysis

Nodal analysis



Let 1) assume currents in the loops

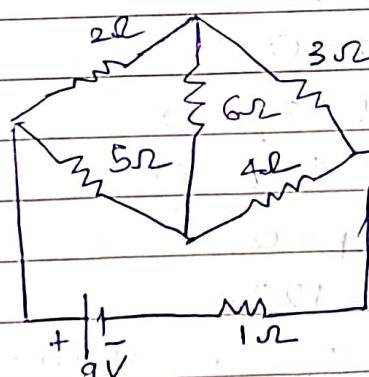
2) No. of currents assumed = no. of loops

For loop ABED.

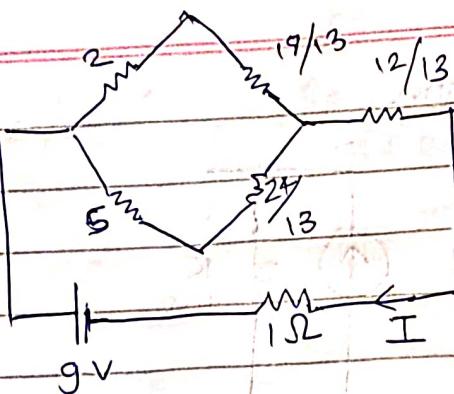
$$-I_1 R_1 - R_2 (I_1 - I_2) + V_1 = 0 \quad \text{--- (1)}$$

For loop BCFE

$$-I_2 R_3 - V_2 - R_2 (I_2 - I_1) = 0 \quad \text{--- (2)}$$



Using star-Delta conversion calculate current in 1Ω resistor.



$$3.38 \quad 6.84$$

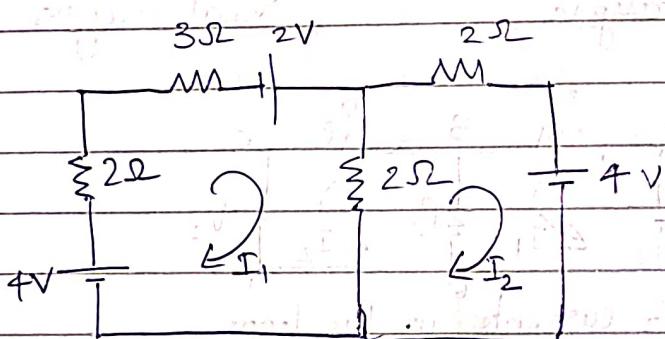
$$0.29 + 0.14 = 0.43$$

$$= 2.32.$$

$$3.24 + 1 = 4.24$$

$$V = IR \Rightarrow I = \frac{9}{4.24}$$

$$= 2.12 \text{ A.}$$



Calculate current in 3 ohm series using KVL eqn.

$$4 - 2I_1 - 3I_1 + 2 - 2(I_1 - I_2) = 0 \quad \text{①}$$

$$- 2I_2 - 4 - 2(I_2 - I_1) = 0 \quad \text{②}$$

$$4 - 7I_1 + 2 + 2I_2 = 0 \quad \text{③} \quad - 4 - 4I_2 + 2I_1 = 0$$

$$6 - 7I_1 + 2I_2 = 0$$

$$- 4I_2 - 6 + 2I_1 = 0$$

$$12 - 14I_1 + 4I_2 = 0$$

$$- 6 + 2I_1 - 4I_2 = 0$$

$$6 - 12I_1 = 0$$

$$6 = 12I_1$$

$$I_1 = \frac{1}{2} = 0.5 \text{ A.}$$

$$6 - 7I_1 + 2I_2 = 0$$

$$6 = 7I_1 - 2I_2$$

$$4 = 2I_1 - 4I_2$$

$$12 = 14I_1 - 8I_2$$

$$-4 = -2I_1 + 4I_2$$

$$8 = 12I_1$$

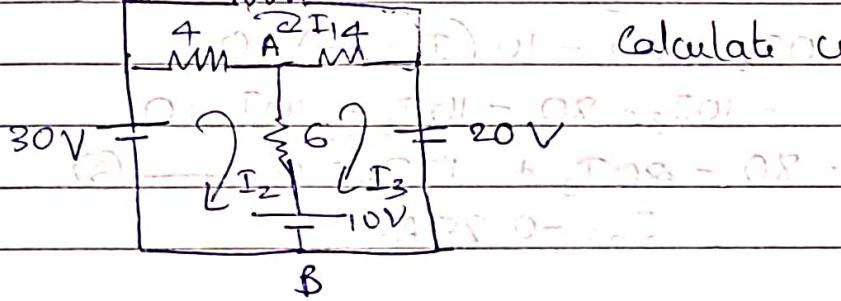
$$I_1 = \frac{8}{12} = \frac{2}{3} A$$

$$I_1 = 2 A$$

$$2 = \frac{2}{3} A \Rightarrow 2 \times 3 = 6 A$$

$$2 = 6 A \Rightarrow 2 \div 6 = \frac{1}{3} A$$

$$\textcircled{1} - \textcircled{2} \Rightarrow 2 = 6A - 2A \Rightarrow 4A = 2A$$



Calculate current in $\textcircled{1}$.

$$-5I_1 - 4(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$-13I_1 - 4I_3 - 4I_2 = 0$$

$$30 - 4(I_2 - I_1) - 6(I_2 - I_3) - 10 = 0$$

$$30 - 10I_2 - 4I_1 - 6I_3 - 10 = 0$$

$$20 - 10I_2 - 4I_1 - 6I_3 = 0$$

$$-4(I_3 - I_1) - 20 + 10 - 6(I_3 - I_2) = 0$$

$$-10I_3 - 4I_1 - 10 - 6I_2 = 0$$

$$10 = -4I_1 - 6I_2 - 10I_3$$

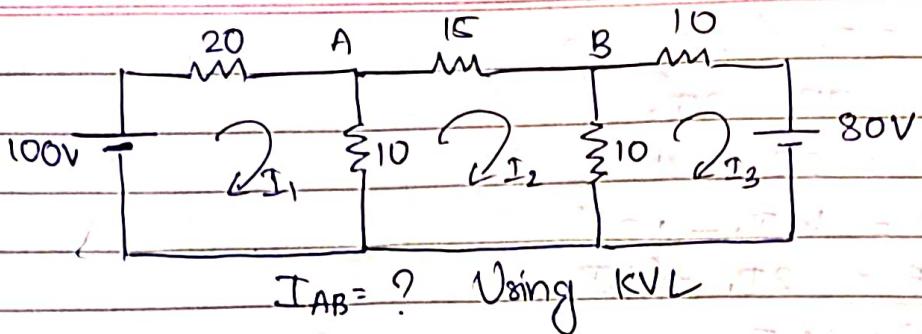
$$20 = 4I_1 + 10I_2 + 6I_3$$

$$30 = 4I_2 - 4I_3$$

$$7.5 = I_2 - I_3$$

$$I_2 = 7.5 + I_3$$

$$\text{Ans: } 1.86 A.$$



$$100 - 20I_1 - 10(I_1 - I_2) = 0$$

$$100 - 20I_1 - 10I_1 + 10I_2 = 0$$

$$100 - 30I_1 + 10I_2 = 0 \quad \text{--- (1)}$$

$$-15I_2 - 10(I_2 - I_3) - 10(I_2 - I_1) = 0$$

$$-15I_2 - 10I_2 + 10I_3 - 10I_2 + 10I_1 = 0$$

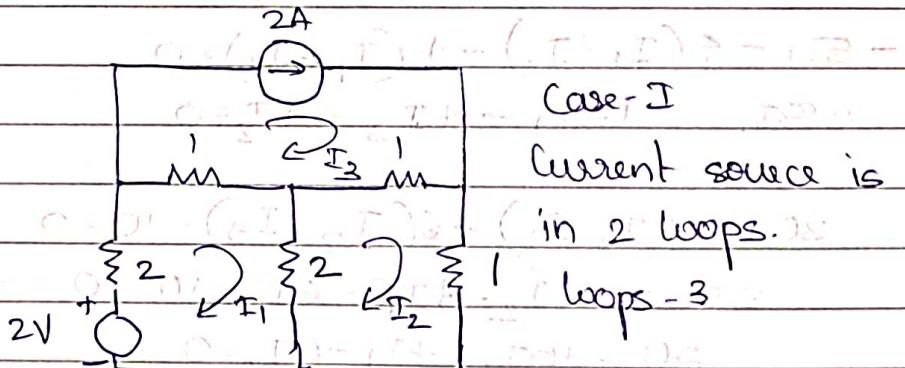
$$-10I_3 + 10I_1 - 35I_2 = 0 \quad \text{--- (2)}$$

$$-10I_3 - 80 - 10(I_3 - I_2) = 0$$

$$-10I_3 - 80 - 10I_3 + 10I_2 = 0$$

$$-80 - 20I_3 + 10I_2 = 0 \quad \text{--- (3)}$$

$$I_2 = -0.25 \text{ A}$$



$$-2I_1 - (I_1 - I_2) 2(I_1 - I_2) + 2 = 0$$

$$-5I_1 + 2I_2 + 2 = 0 \quad \text{--- (1)} \quad -5I_1 + 2I_2 + I_3 + 2 = 0$$

$$-I_2 - (I_2 - I_3) 2(I_2 - I_1) = 0$$

$$-4I_2 + 2I_1 = 0 \quad \text{--- (2)}$$

$$-(I_3 - I_2) = -(I_3 - I_1) \quad I_3 = 2 \text{ A.}$$

$$-5I_1 + 2I_2 + 4 = 0$$

$$-10I_2 + 2I_2 + 4 = 0 \quad \text{BQQA.}$$

$$-8I_2 + 4 = 0$$

$$\therefore 4 = 8I_2$$

$$I_1 = 1 \text{ A. } 1.25 \text{ A}$$

$$-\underline{I_2} - (I_2 - I_3) - 2(I_2 - I_1) = 0$$

$$-4T_{g2} + T_3 + 2T_1 = 0$$

$$\cancel{2I_2} - 4I_2 + 2I_1 + 2 = 0$$

$$2I_2 - 5I_1 + 4 = 0$$

$$4I_2 - 10I_1 + 8 = 0$$

$$-4x_1 + 2x_1 + 2 = 0$$

$$-8I_1 + 10 = 0$$

$$8I_1 = 10$$

7-18^s

884

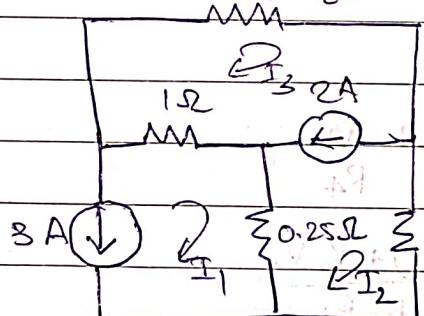
$$\underline{I} = 1.25 \text{ A.}$$

$$-4T_2 + 2.5 + 2 = 0$$

$$-4I_2 + 4.5 = 0$$

$$4I_3 = 4.5$$

$$I_2 = 1.125 \text{ A}$$



$$F_1 = -3A$$

$$I_3 - I_2 = 2, \quad I_3 = 2 + I_2$$

Removing a weak source Bigger loop - supermesh

$$-0.5I_3 - 0.33I_2 - 0.25(I_2 - I_1) - (I_3 - I_1) = 0$$

$$-0.5I_3 - 0.33I_2 - 0.25I_2 - 0.75 - I_3 - 3 = 0$$

$$-1.5\mathbb{I}_3 + 0.58\mathbb{I}_2 - 3.75 = 0$$

$$-1.5(2+I_2) + 0.58I_2 - 3.75 = 0$$

$$-3 + 1.5T, -0.58T = 3.75$$

$$0.92I_1 = 0.75 - 2.08I_1 = 6.75$$

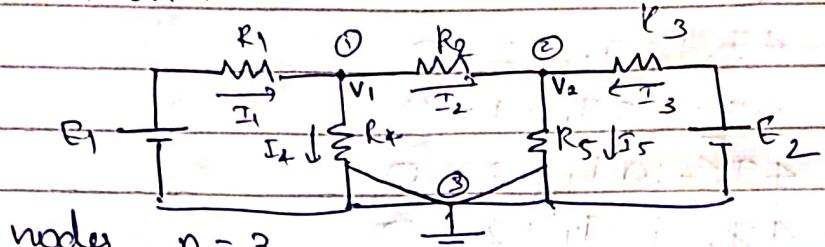
$$\therefore \frac{1}{2} = 0.87A$$

$$I_2 = -3.24 \text{ A.}$$

$$I_3 = 2 + 0.81 \\ = 2.81 \text{ A}$$

7) Nodal Analysis

→ Based on KCL

nodes, $n = 3$

$$\text{No. of eqns} = n-1 = 2$$

∴ One node made as reference/datum/zero potential
At node 1

$$I_1 = I_2 + I_4 \quad \text{--- (1)}$$

$$I_1 R_1 = E_1 - V_1$$

$$I_1 = \frac{E_1 - V_1}{R_1}$$

$$I_2 = \frac{V_1 - V_2}{R_2}$$

$$I_4 = \frac{V_1}{R_4}$$

$$\frac{E_1 - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_4}$$

$$\frac{E_1 - V_1}{R_1} = \frac{R_4(V_1 - V_2) + R_2 V_1}{R_2 R_4}$$

$$\frac{R_4(V_1 - V_2) + R_2 V_1}{R_2 R_4} - \frac{E_1 - V_1}{R_1} = 0$$

$$R_1 R_4 (V_1 - V_2) + R_1 R_2 V_1 - R_2 R_4 (E_1 - V_1) = 0$$

$$V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) - \frac{V_2}{R_2} - \frac{E_1}{R_1} = 0$$

At node 2

$$I_5 = I_2 + I_3 \quad \text{--- (2)}$$

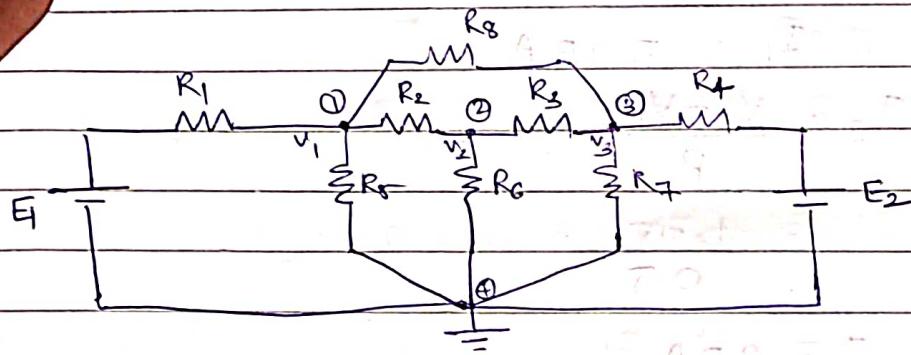
$$I_3 = \frac{E_2 - V_2}{R_3}$$

$$I_5 = \frac{V_2}{R_5}$$

$$\frac{V_2}{R_5} = \frac{V_1 - V_2 + E_2 - V_2}{R_2 + R_8}$$

$$V_2 \left(\frac{-1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) + \frac{V_1}{R_2} + \frac{E_2}{R_8} = 0.$$

$$V_2 \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) - \frac{V_1}{R_2} - \frac{E_2}{R_8} = 0.$$



$$n = 4$$

$$eq^{ns} = 3$$

Node 1: $V_1 + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4} - \frac{E_1}{R_1} - \frac{V_2}{R_5} = 0$

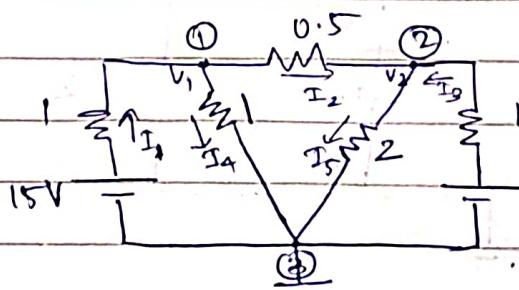
$$V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) - \frac{V_2}{R_2} - \frac{E_1}{R_1} - \frac{V_3}{R_3} - \frac{V_4}{R_4} = 0.$$

Node 2:

$$V_2 \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_5} \right) - \frac{V_1}{R_2} - \frac{V_3}{R_3} = 0$$

Node 3:

$$V_3 \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_7} + \frac{1}{R_8} \right) - \frac{V_2}{R_3} - \frac{E_2}{R_4} - \frac{V_1}{R_8} = 0.$$



Calculate total power consumed by all passive elements.

$$\text{Node 1: } V_1 \left(1 + 1 + 2 \right) - 2V_2 - 15 = 0$$

$$+V_1 - 2V_2 - 15 = 0$$

$$\text{Node 2: } V_2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{0.5} \right) - 20 - 2V_1 = 0$$

$$V_2 (3.5) - 20 - 2V_1 = 0$$

$$3.5V_2 - 2V_1 - 20 = 0.$$

$$V_1 = \frac{37}{4}, \quad V_2 = 11$$

$$I_1 = \frac{E_1 - V_1}{R_1} = \frac{15 - 3.75}{1} = 11.25 \text{ A}$$

$$I_1 = 5.75 \text{ A}$$

$$I_2 = \frac{V_2 - V_1}{R_2}$$

$$= \frac{1.75}{0.5}$$

$$I_2 = 3.5 \text{ A}$$

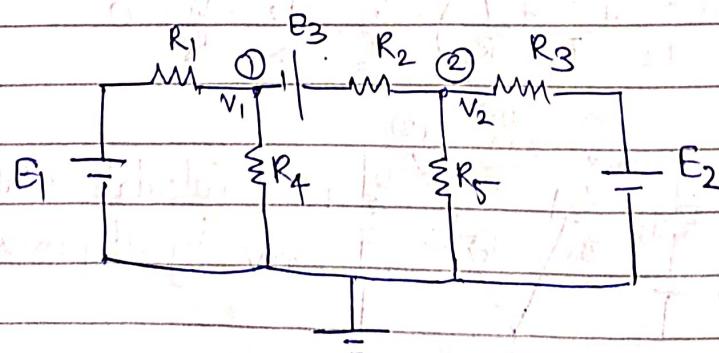
$$I_3 = \frac{E_2 - V_2}{R_4} = \frac{20 - 11}{1} = 9 \text{ A.}$$

$$I_4 = \frac{37}{4} = 9.25 \text{ A} \quad I_5 = \frac{V_2}{R_2} = \frac{11}{0.5} = 22 \text{ A.}$$

$$I = 33 \text{ A.}$$

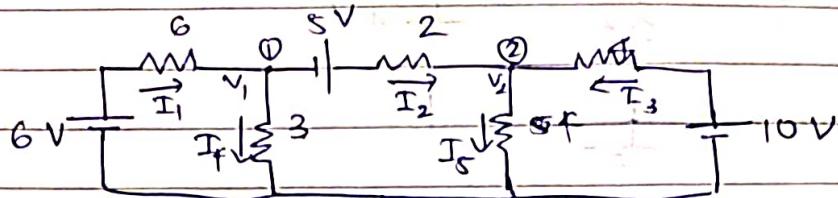
$$\begin{aligned} P &= (I_1^2 \cdot R_1) + I_2^2 (0.5) + I_3^2 \cdot R_4 + \frac{I_4^2 \cdot R_2}{4} + I_5^2 \cdot R_2 \\ &= 12.25 + 6.125 + 81 + 30.25 + \\ &= 33.0625 + 6.125 + 81 + 85.5625 = 160.5 \\ &= 266.25 \end{aligned}$$

Q.



$$\text{Node 1: } V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) - \frac{V_2}{R_2} - \frac{E_1}{R_1} + \frac{E_2}{R_2} = 0$$

Node 1: $V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{V_1}{R_1} - \frac{E_2}{R_3} - \frac{E_3}{R_2} = 0.$



Find all branch currents.

Node 1: $V_1 \left(\frac{1}{6} + \frac{1}{3} + \frac{1}{2} \right) - \frac{V_2}{2} - \frac{6}{6} + \frac{5}{2} = 0$

$$V_1 \left(\frac{1+2+3}{6} \right) - \frac{V_2}{2} - 1 + 2.5 = 0$$

$$V_1 - 0.5V_2 + 1.5 = 0$$

Node 2: $V_2 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{45} \right) - \frac{V_1}{2} - \frac{10}{4} - \frac{5}{2} = 0$

$$V_2 \left(\frac{10+5*4}{20} \right) - 2.5 - 2.5 - \frac{V_1}{2} = 0$$

$$0.95V_2 - 0.5V_1 - 5 = 0$$

$$-0.5V_1 + 0.95V_2 - 5 = 0.$$

$$V_1 = \frac{43}{28}$$

$$V_2 = \frac{85}{14}$$

$$V_1 = 1.53$$

$$V_2 = 6.07$$

$$I_5 = 1.415 A.$$

$$V_2 \left(\frac{2+1+1}{4} \right) - 0.5V_1 - 5 = 0$$

$$V_2 - 0.5V_1 - 5 = 0$$

$$-0.5V_1 + V_2 - 5 = 0$$

$$V_1 = \frac{4}{3}$$

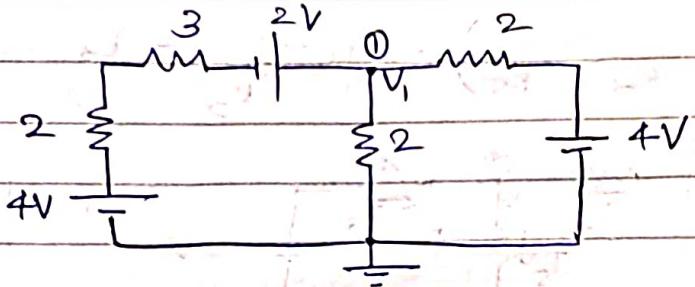
$$V_2 = \frac{17}{3}$$

$$I_1 = \frac{6 - \frac{4}{3}}{6} = \frac{4.67}{6} = 0.77 A$$

$$I_3 = \frac{10 - 5 - 6.67}{4} = 1.085 A$$

$$I_2 = \frac{\frac{4}{3} + 5 - 1.73}{2} = 0.835 A \quad I_4 = \frac{4}{9} A.$$

Q.

Find current in 3Ω

$$V_1 \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{2} \right) - \frac{4}{2} - \frac{2}{3} = 0$$

$$V_1 \left(\frac{1}{5} + \frac{1}{2} + \frac{1}{2} \right) = \frac{4}{5} - \frac{2}{5} - \cancel{\frac{2}{2}} - \frac{4}{2} = 0$$

~~$$V_1 \left(\frac{12}{10} \right) - 4 - \frac{2}{3} = 0$$~~

~~$$1.2V_1 - 4 - 0.66 = 0$$~~

~~$$V_1 = 4.66$$~~

$$\frac{12}{10} V_1 = \frac{14}{3}$$

~~$$V_1 = \frac{1.2}{(1.2 + 0.8)} V_1 = \frac{1.2}{2} V_1 = \frac{7}{10} \times 10$$~~

~~$$V_1 = 3.88 \text{ V}$$~~

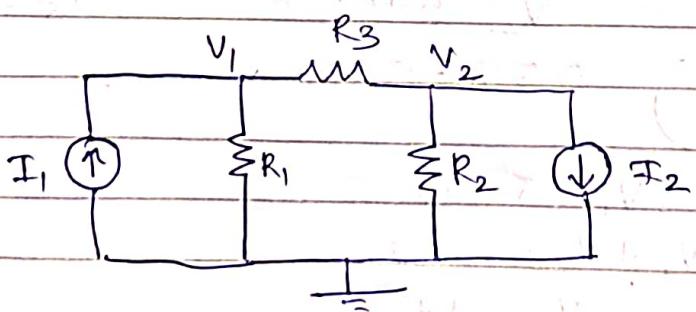
~~$$1.2V_1 - 4 - 0.66 = 0$$~~

~~$$V_1 = \frac{3.2}{1.2}$$~~

~~$$V_1 = 2.67 \text{ V}$$~~

$$I_1 = \frac{4 + 2 - 2.67}{5} = 0.667 \text{ A}$$

Q.

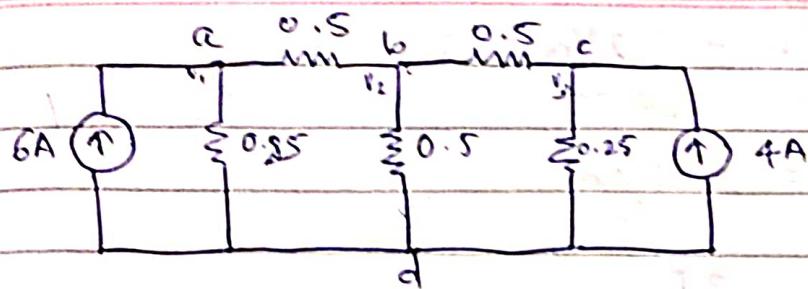


Node 1

$$V_1 \left(\frac{1}{R_1} + \frac{1}{R_3} \right) - \frac{V_2}{R_3} = I_1$$

Node 2

$$V_2 \left(\frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{V_1}{R_3} = -I_2$$



$$I_{ab}, I_{bc}, I_{cd} = ?$$

$$V_1(2+4) - 2V_2 = 6$$

$$6V_1 - 2V_2 = 6 \quad \text{--- (1)}$$

$$V_2(2+2+2) - 2V_1 - 2V_3 = 0$$

$$6V_2 - 2V_1 - 2V_3 = 0 \quad \text{--- (2)}$$

$$V_3(2+4) - 2V_2 = 4$$

$$6V_3 - 2V_2 = 4 \quad \text{--- (3)}$$

$$V_a = \frac{26}{21} \quad V_b = \frac{5}{7} \quad V_c = \frac{19}{21}$$

$$I_{ab} = (V_1 - V_2) / Z = (1.23 - 0.73) / 2$$

$$= (1.23 - 0.73) / 2 = 0.25 \text{ A}$$

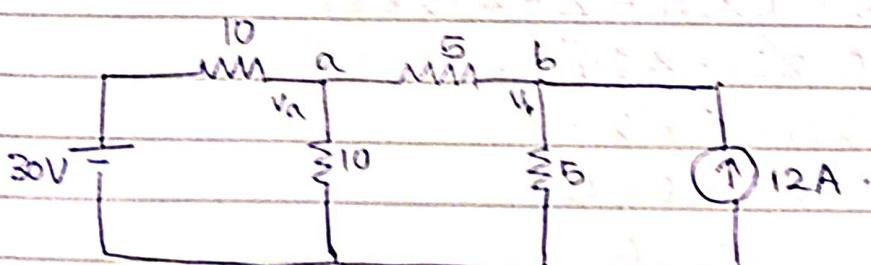
$$I_{bc} = (V_2 - V_3) / Z$$

$$= (0.73 - 0.90) / 2$$

$$I_{bc} = -0.38 \text{ A}$$

$$I_{cd} = (V_3 - V_1) / Z$$

$$I_{cd} = 1.42 \text{ A}$$



$$I_{ab} = ?$$

$$30 \left(\frac{1}{10} + \frac{1}{10} \right) - \frac{V_a}{5} + \frac{30}{10} = 0$$

$$0.2V_a - 0.2V_b + 30 = 0 \quad \text{--- (1)}$$

$$V_b \left(\frac{1}{5} + \frac{1}{5} \right) - \frac{V_b}{5} + 12 = 12$$

$$0.4V_b - 0.2V_a = 12$$

$$V_a = 30 \quad V_b = 75$$

$$I_{ab} = \frac{V_a - V_b}{5}$$

$$I_{ab} = 3 \text{ A}$$

$$V_a \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10S} \right) - \frac{V_b}{5} - \frac{30}{10} = 0$$

$$0.3V_a - 0.2V_b - 3 = 0$$

$$0.4V_a - 0.2V_b = 3 \quad \text{(1)}$$

~~$$V_a = 30 \quad V_b = 75$$~~

~~$$\Delta = 30 - (75) = -45 \text{ V}$$~~

~~$$I_{ab} = \frac{45}{5} = 9 \text{ A}$$~~

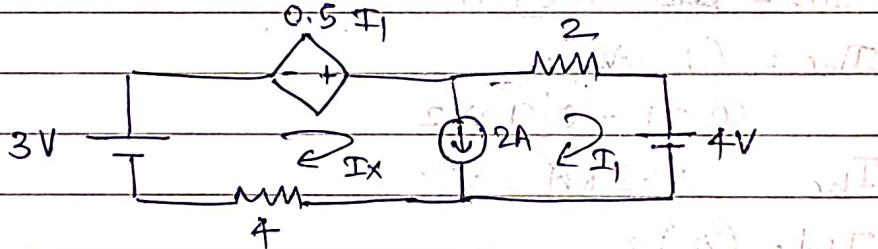
~~$$P = 5 \text{ W} \quad \alpha = -1 \quad \beta = 1 \text{ V}$$~~

~~$$I_{ab} = 0.15 \text{ A}$$~~

~~$$V_a = 30 \quad V_b = 45 \quad (30 - 45) = -15 \text{ V}$$~~

~~$$I_{ab} = -3 \text{ A} \quad (15 - 45) = -30 \text{ V}$$~~

Q.



Calculate using I_1 using KVL eqns.

Soln:

~~$$3 + 0.5I_1 + 2 - 4I_x = 0$$~~

~~$$0.5I_1 - 4I_x + 5 = 0 \quad \text{(1)}$$~~

~~$$-2I_1 - 4 - 2 = 0$$~~

~~$$-2I_1 - 6 = 0$$~~

~~$$-2I_1 = 6$$~~

~~$$I_1 = -3 \text{ A}$$~~

~~$$0.5(-3) - 4I_x + 5 = 0$$~~

~~$$-1.5 + 5 - 4I_x = 0$$~~

~~$$-3.5 - 4I_x = 0$$~~

~~$$I_x = -0.875 \text{ A}$$~~

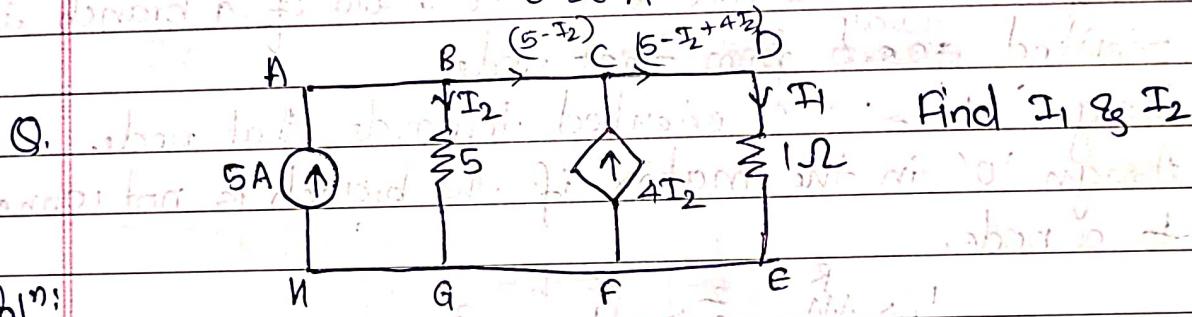
$$I_x = I_1 + 2 \quad I_1 - I_x + 2 = 0 \rightarrow \textcircled{1}$$

$$3 + 0.5I_1 - 2I_1 - 4 - 4I_x = 0$$

$$-1.5I_1 - 4I_x - 1 = 0 \rightarrow \textcircled{2}$$

$$I_1 = \frac{-18}{11} A \approx -1.64 A$$

$$I_x = \frac{4}{11} A$$



$$I_1 - 3I_2 = 5$$

loop BDEGBD

$$-I_1 + 5I_2 = 0$$

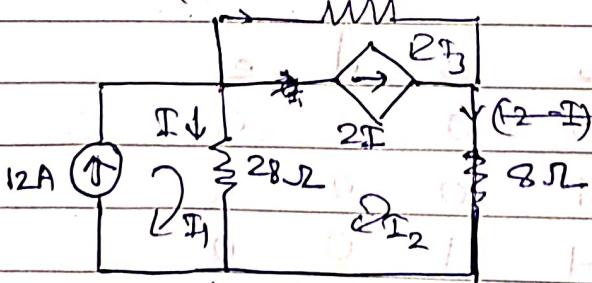
$$-(5 + 3I_2) + 5I_2 = 0$$

$$2I_2 - 5 = 0$$

$$I_2 = 2.5 A$$

$$I_1 = 5 + 3 \times 2.5$$

$$I_1 = 12.5 A$$



Calculate I

~~$$I_1 + I_2 + I_3 = 12 A$$~~

$$I_1 - I_2 = 7$$

$$I_1 = 12 A$$

~~$$I_2 - I_3 = 2I$$~~

$$12 - I_2 - I = 0$$

$$-4I_3 - 8I_2 + 28I = 0$$

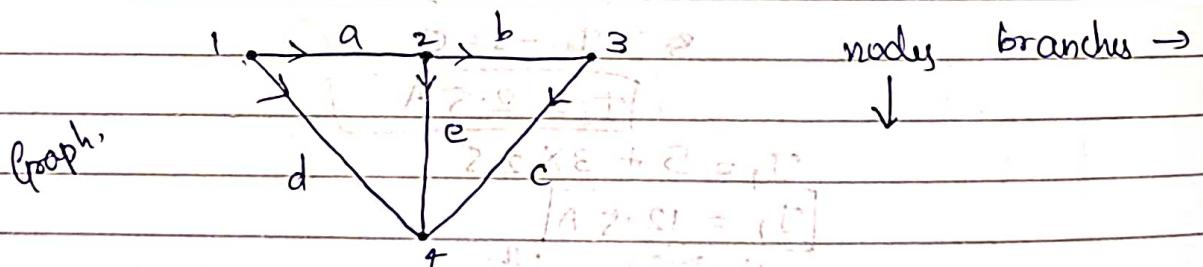
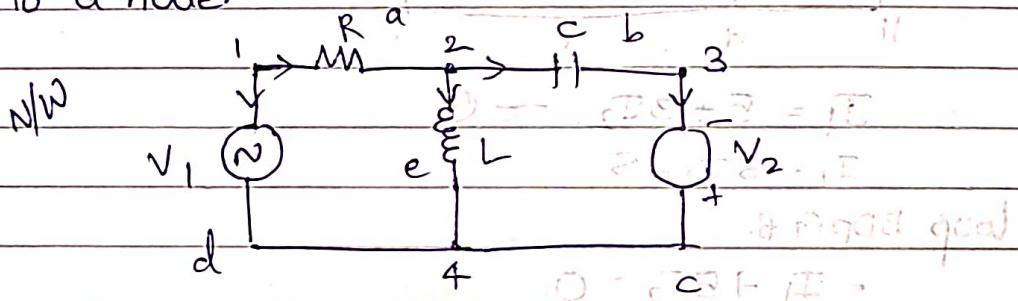
$$I_2 = -9 A \quad I_3 = -3 \quad I = -3 A$$

*) Incidence Matrix $[A_i]$

relation betⁿ no. of branches & no. of nodes from the direction graph.

Formation of $[A_i]$

1. Obtain directed graph of given network.
2. Assign '+1' in the matrix if the arrow of a branch is oriented away from that node.
3. Assign '-1' oriented towards that node.
4. Assign '0' in the matrix if the branch is not connected to a node.



	a	b	c	d	e
1	1	1	0	0	1
2	-1	1	0	0	0
3	0	-1	1	0	0
4	0	0	-1	-1	0

- check \rightarrow
- 1) sum of individual columns is zero.
 - 2) Det. is zero.

a) Reduced Incidence Matrix $[A_r]$.

$$[A_r] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Obtained by deleting one row from $[A_i]$.

No. of possible trees $\equiv \det \{ [A_r] [A_r]^T \}$.

$$[A_r]^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

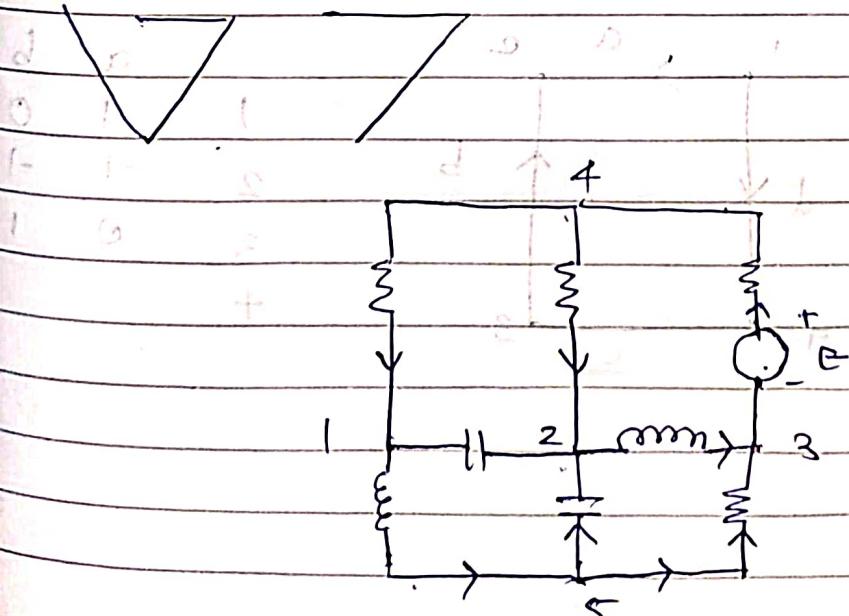
$$\{ [A_r] [A_r]^T \} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

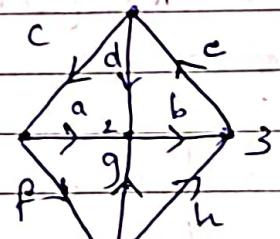
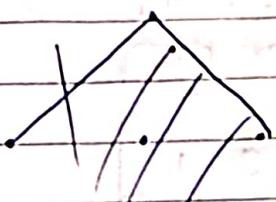
$$\begin{array}{l} \text{Row } 1: 1-0=1 \\ \text{Row } 2: -1+1=0 \\ \text{Row } 3: 0-0=0 \end{array} \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Det} \equiv 2(5) + 1(-2)$$

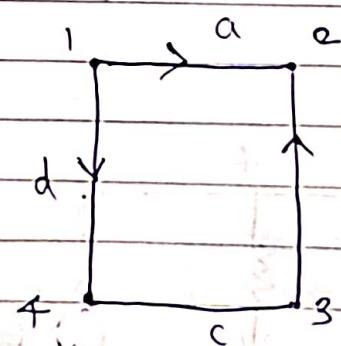
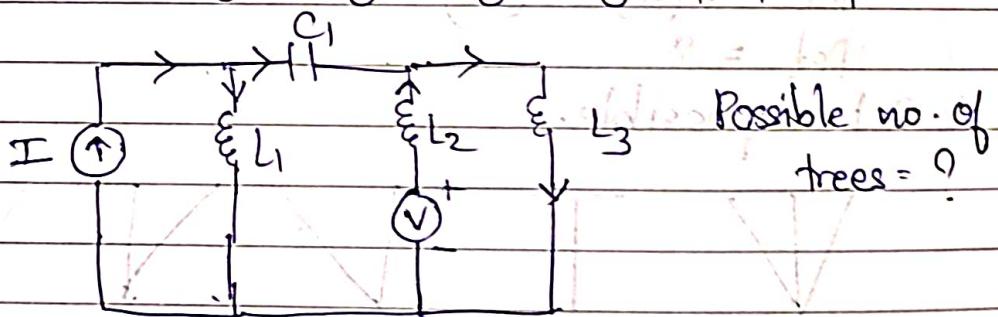
$$\text{Det.} = 8$$

$\therefore 8$ trees possible.





	a	b	c	d	e	f	g	h
1	1	1	0	-1	0	0	1	0
2	-1	1	0	0	-1	0	0	-1
3	0	-1	0	0	0	1	0	0
4	0	0	0	1	-1	0	-1	0
5	0	0	0	(0)	0	-1	1	0

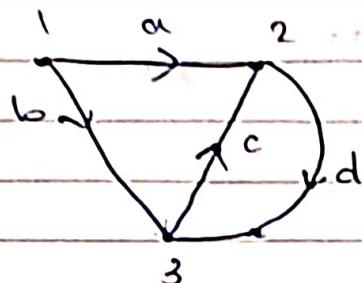


	a	b	c	d
1	1	0	0	1
2	-1	-1	0	0
3	0	1	0	0
4	0	0	0	0

discrete

time

lego



$$\begin{array}{c} \text{a}, \text{b}, \text{c}, \text{d} \\ \begin{matrix} 1 & -1 & 0 & 0 \\ 2 & -1 & 0 & -1 \\ 3 & 0 & -1 & 1 \end{matrix} \end{array}$$

$$[A_r] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \end{bmatrix}$$

$$[A_r]^T = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$

$$[A_r] [A_r]^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\text{Det} = 6 - 1 = 5$$

29/09/22

classmate

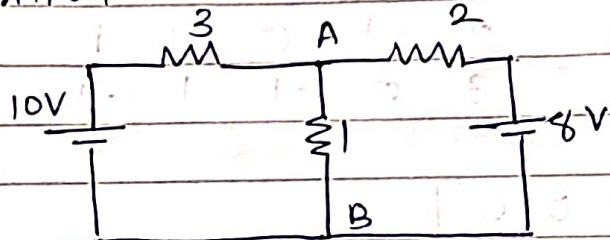
Date _____

Page _____

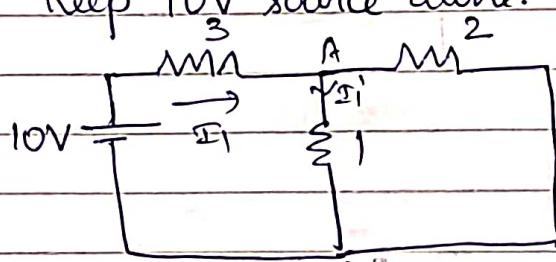
Unit 2

Network Theorems

Superposition Theorem.



Step 1: Keep 10V source alone.



$$1 \parallel 2 = \frac{2}{3}$$

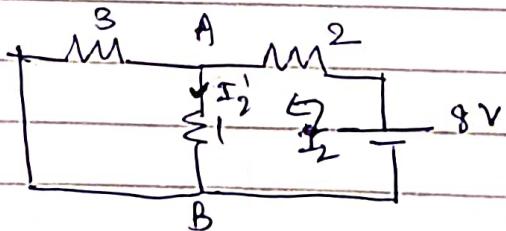
$$\begin{aligned} I_1' &= \frac{10}{3 + \frac{2}{3}} \\ &= \frac{30}{11} \end{aligned}$$

$$I_1 = 2.72A$$

$$I_1' = -2.72 \times \frac{2}{2+1}$$

$$= 1.81A$$

Step 2: Connect 8V source



$$\begin{aligned} I_{AB} &= I_1' + I_2' \\ &= 1.81 + 2.18 \\ &= 3.99A \end{aligned}$$

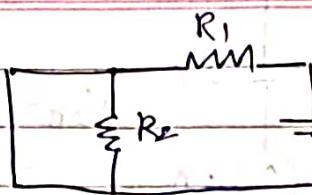
$$3 \parallel 1 = \frac{3}{4}$$

$$I_2 = \frac{8}{2 + \frac{3}{4}}$$

$$I_2 = 2.909 A$$

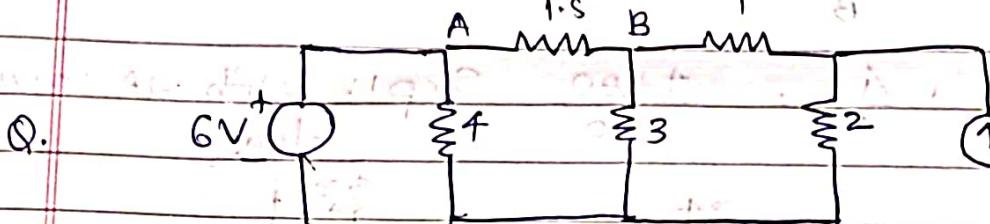
$$I_2' = \frac{2.909 \times 3}{3+1}$$

$$I_2' = 2.181 A \downarrow$$



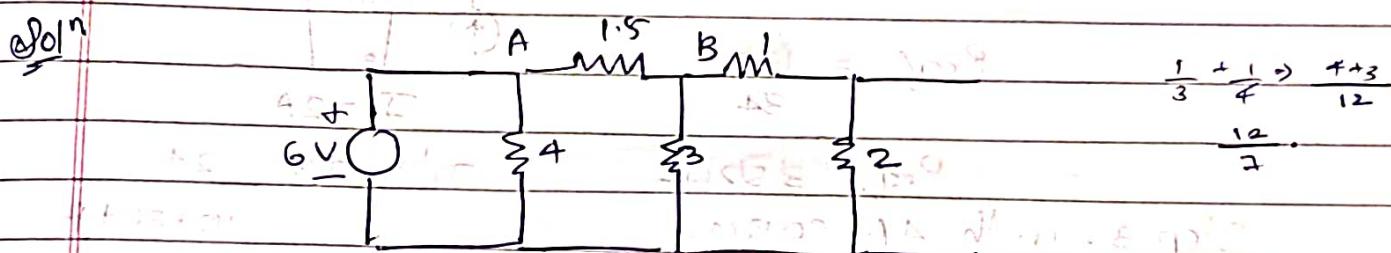
$$S.C. = R_2 I_3 = 0.$$

R_2 = Redundant branch.



$$I_{AB} = ?$$

By superposition Thm.



$$I_1' = \frac{6}{12/7}$$

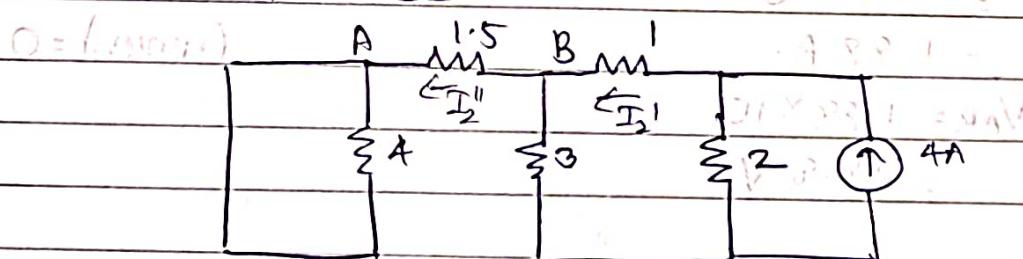
$$= 3.5A.$$

$$I_{AB} = 2 - 1.33$$

$$I_1' = \frac{3.5 \times 10}{4+1.5+1.5}$$

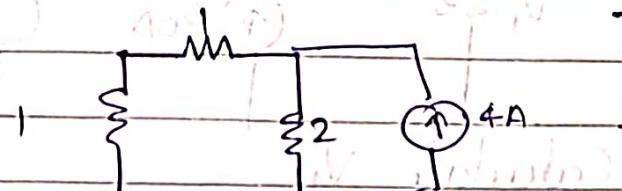
$$= 0.66A.$$

$$I_1' = 2.0A \rightarrow \text{Ans}$$



- 4Ω is redundant.

$$3 \parallel 1.5 = 1\Omega$$



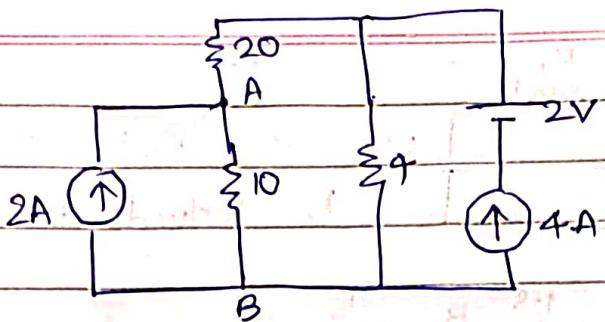
$$I_2 = 4A$$

$$I_2' = 4 \times \frac{2}{1+1+2}.$$

$$I_2'' = I_2' \times \frac{3}{3+1.5}$$

$$I_2'' = 1.33A \leftarrow$$

Q.



$V_{AB} = ?$ using superposition theorem?

Sol'n:

$$\frac{1}{30} + \frac{1}{4} - \frac{4}{120}$$

$$= \frac{34}{120}$$

$$\text{Reqf} = \frac{120}{34}$$

$$\text{Reqf} = 3.52 \Omega$$

Step 1: With 2A source alone

$$I_1 = 2A$$

$$I_1' = 2 \times \frac{24}{10+20+4}$$

Step 3: with 4A source.

$$I_2 = 4A$$

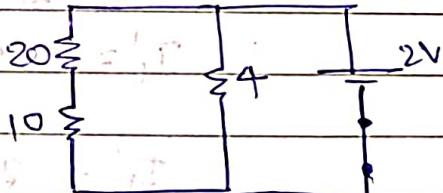
$$I_2' = 4 \times \frac{4}{10+20+4}$$

$$I_1' = 1.41A \downarrow$$

Step 2: 2V alone

$$I_2' = 0.44A \downarrow$$

$$\begin{aligned} \therefore I_{AB} &= I_1' + I_2' \\ &= 1.41 + 0.44 \\ &= 1.88 A. \end{aligned}$$

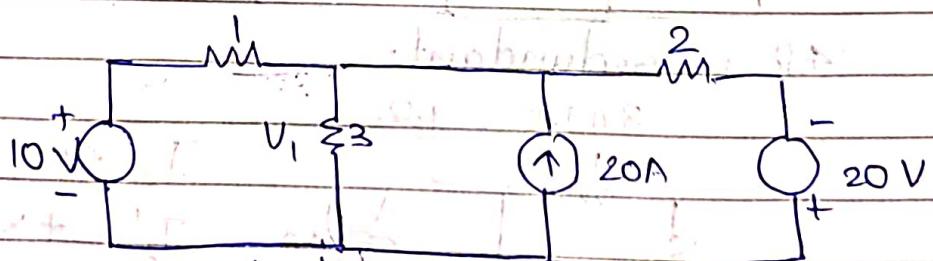


Current = 0

$$V_{AB} = 1.88 \times 10.$$

$$= 18.8 V.$$

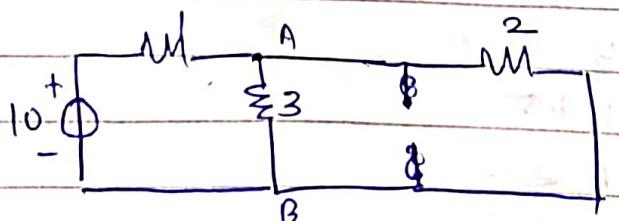
Q.



Calculate V_1

Sol'n:

with 10V alone



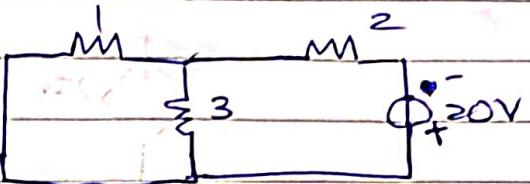
$$\frac{1}{3} + \frac{1}{2} \Rightarrow \frac{2+3}{6} \Rightarrow \frac{6}{5}$$

$$\frac{6}{5} + 1 = \frac{11}{5}$$

$$I_1 = \frac{10}{11}$$

$$I_1 = \frac{50}{11}$$

with 20V alone



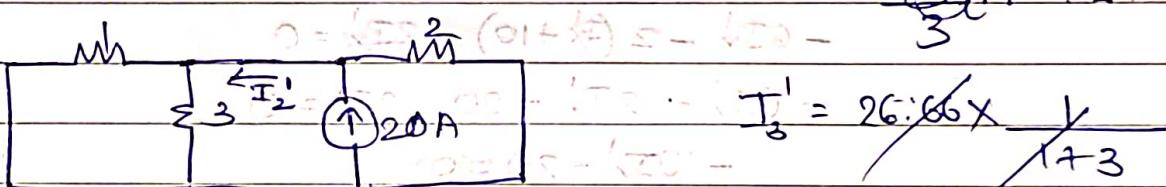
$$I_1 = 4.54 \text{ A}$$

$$I_1' = 4.54 \times \frac{2}{3+2}$$

$$I_1' = 4.54 \times \frac{2}{5}$$

$$I_1' = 1.81 \text{ A} \downarrow$$

With 20A alone

~~I₁ & 20A & 02.~~

$$03 = I_1 + I_2 = 6.66 \text{ A} \uparrow$$

$$3111 \Rightarrow \frac{3}{4} \cdot \frac{1}{2} = 7.27 \text{ A}$$

$$I_3' = 7.27 \times \frac{1}{1+3}$$



$$I_{AB} = 1.81 + 3.63 - 1.81$$

$$I_2 = 20A - 03 + 7.27 = 3.63A$$

$$I_2' = 20 \times \frac{2}{2+3/4} = 14.54 \text{ A}$$

$$= 14.54 \text{ A}$$

$$I_2'' = 14.54 \times \frac{1}{1+3}$$

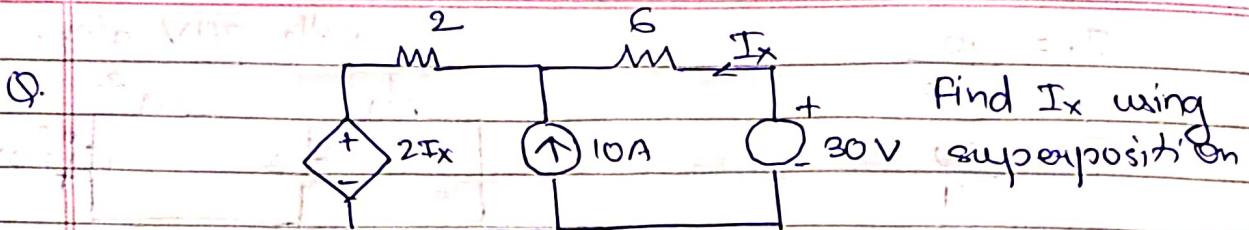
$$= 3.63 \text{ A} \downarrow$$

$$I_{AB} = 1.81 + 3.63 - 6.66$$

$$= 1.22 \text{ A} \uparrow$$

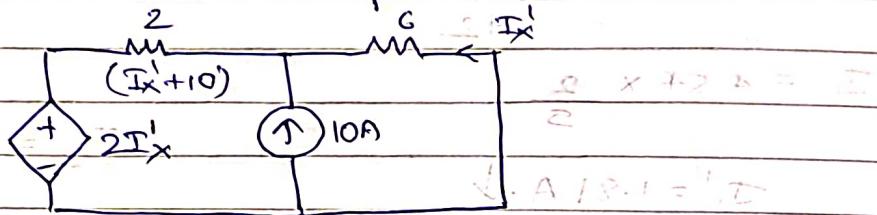
$$V_{AB} = 1.22 \times 3$$

$$= 3.66 \text{ V}$$



* Dependent sources are kept as it is.

Step 1:



$$-6I_x' - 2(I_x' + 10) - 2I_x' = 0$$

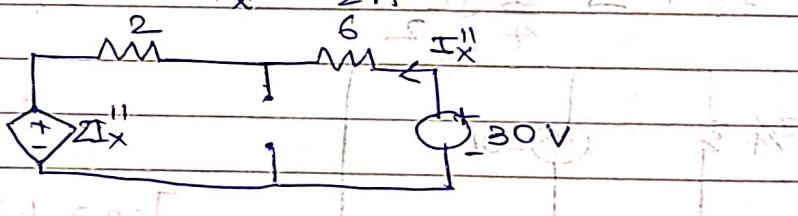
$$-6I_x' - 2I_x' - 20 - 2I_x' = 0$$

$$-10I_x' - 20 = 0$$

$$-10I_x' = 20$$

$$I_x' = -2 \text{ A.}$$

Step 2:



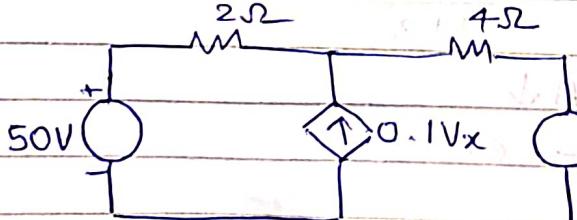
$$-6I_x'' - 2I_x'' - 2I_x'' + 30 = 0$$

$$-10I_x'' = -30$$

$$I_x'' = 3 \text{ A.}$$

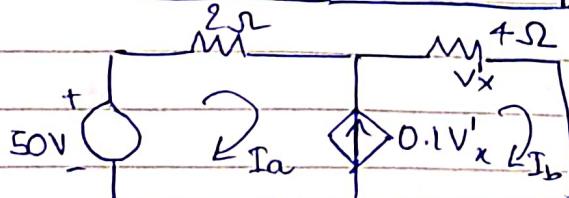
$$I_x = I_x'' + I_x'$$

$$I_x = 1 \text{ A}$$



Calculate V_x .
Using superposition.

Ans:



$$I_b - I_a = 0.1 V_x' \quad \text{--- (1)}$$

Writing eqⁿ for outer loop,

$$-2I_a - 4I_b + 50 = 0$$

$$2I_a + 4I_b = 50$$

$$I_a + 2I_b = 25 \quad \text{--- (2)}$$

$$-4I_b = V_x' \quad \text{from 1ckt diag. --- (3)}$$

$$I_a - I_b - I_a = 0.1 V_x'$$

$$I_b - I_a = 0.1 (-4I_b) \quad \text{--- (4)}$$

$$I_b - I_a = -0.4I_b$$

$$1.4I_b - I_a = 0$$

$$I_a = 10.29$$

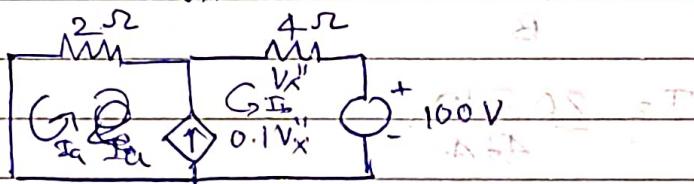
$$I_b = 7.35 \text{ A}$$

$$V_x' = -4I_b$$

$$= -4 \times 7.35$$

$$V_x' = -29.4 \text{ V}$$

Step 2:



$$I_a + I_b = -0.1 V_x''$$

$$I_a - I_b = 0.1 V_x''$$

$$-4I_b - 2I_a + 100 = 0 \quad \text{--- (1)}$$

$$2I_a + 4I_b = 100$$

$$I_a + 2I_b = 50 \quad \text{--- (1)}$$

$$-4I_b = V_x''$$

$$I_a - I_b = 0.1 (+4I_b)$$

$$I_a - I_b = 0.4I_b$$

$$I_a - 0.4I_b = 0 \quad \text{--- (2)}$$

$$I_a = 14.70 \quad I_b = 14.70$$

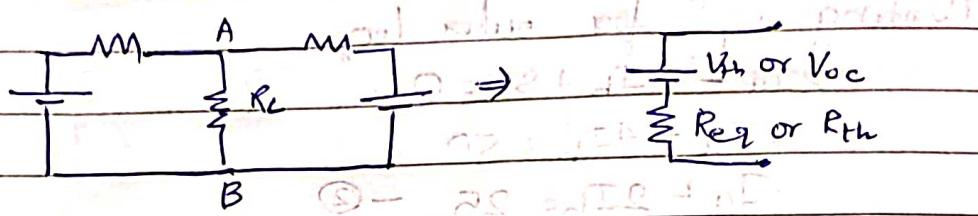
$$V_x'' = 4 \times 14.70$$

$$V_x'' = 58.8 \text{ V}$$

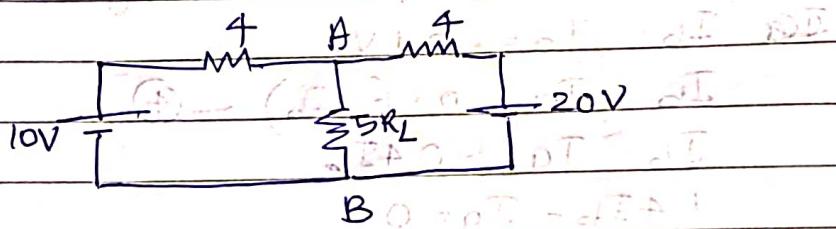
$$V_x = V_x' + V_x''$$

$$= 29.4 \text{ V}$$

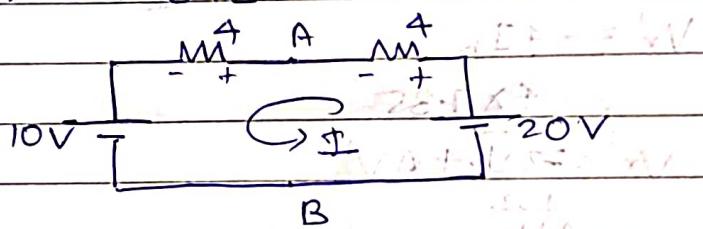
A) Thevenin's Theorem:



\Rightarrow ~~open~~ Load resistance (R_L)



Step 1: Remove R_L .



$$I = \frac{20 - 10}{4 + 4}$$

$$= \frac{10}{8} \text{ A}$$

$$I = 1.25 \text{ A}$$

$$V_{th} = 20 - (4 \times 1.25)$$

$$= 15 \text{ V}$$

$$A \text{ open } \rightarrow V_{th} = 15 \text{ V}$$

$$B \text{ open } \rightarrow V_{th} = 10 + (4 \times 1.25)$$

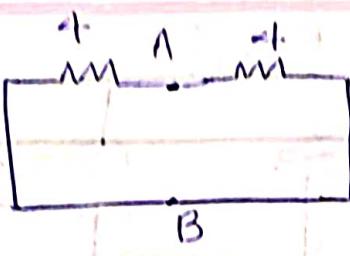
$$= 15 \text{ V}$$

$$V_{th} = 15 \text{ V}$$

$$V_{th} = 15 \text{ V}$$

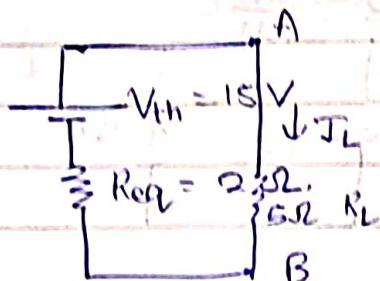
$$\begin{matrix} 1 \times \frac{1}{6} \\ 3 \\ 2 \times \frac{1}{6} \\ 6 \end{matrix}$$

Step 2:



$$Req = 1/|1| = 2\Omega$$

Step 5:

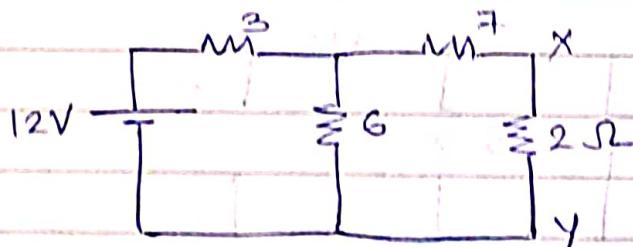


Equivalent circuit.

$$I_L = \frac{V_{th}}{Req + RL}$$

$$= \frac{15}{2 + 5}$$

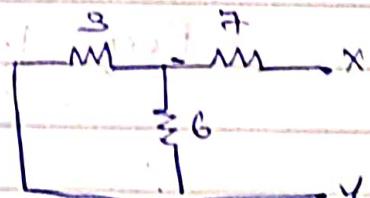
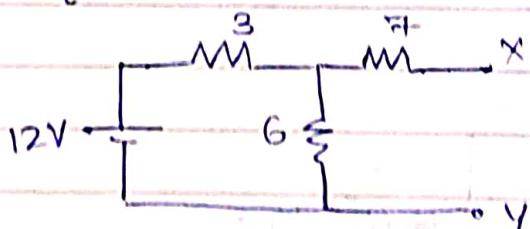
$$I_L = 2.14 \text{ A.}$$



$I_{xy} = ?$ using
Thevenin's thm.

Soln:

$$RL = 2\Omega.$$



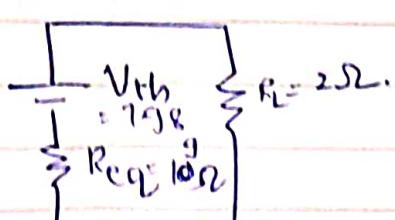
$$\text{Req B. } Req = 10\Omega$$

$$I = \frac{12}{3 + 6}$$

$$I = 1.33 \text{ A}$$

$$V_{th} = 1.33 \times 6 + 7 \times 0$$

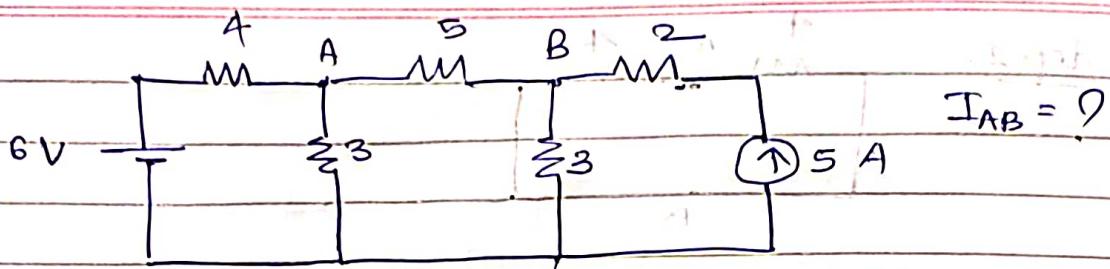
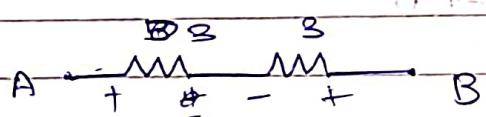
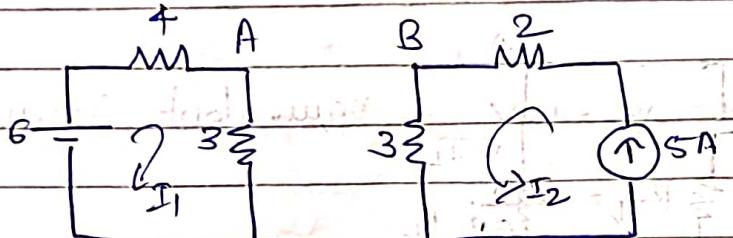
$$= 7.98 \text{ V}$$



$$I_{xy} = \frac{7.98}{10 + 2}$$

$$I_{xy} = 0.72 \text{ A}$$

Q.

Soln!

$$V_{th} = 3 \times 5 = 15 \text{ V}$$

$$I = \frac{6}{7} \text{ A}$$

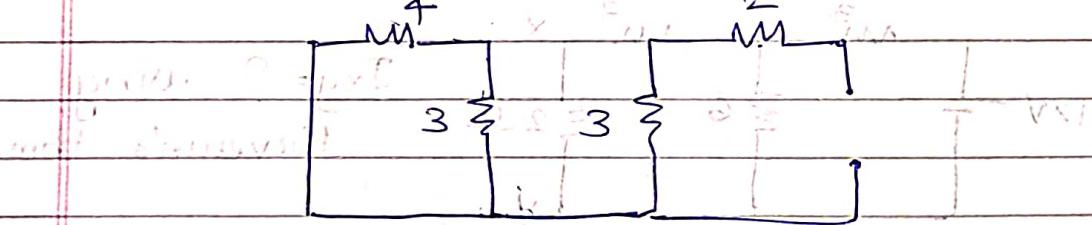
$$= 15 + 0.85 \times 3$$

$$= 12.45 \text{ V}$$

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\frac{4+3}{12}$$

$$\frac{12}{7}$$



$$R_{eq} = 1.71 + 3 + 2$$

$$= 6.71 \Omega$$

$$V_{th} = 12.45 \text{ V}$$

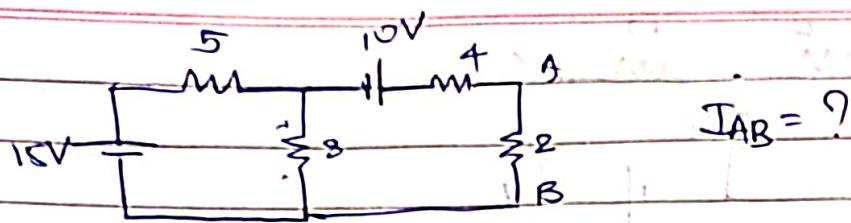
$$R_{eq} = 6.71 \Omega$$

$$5\Omega = R_L$$

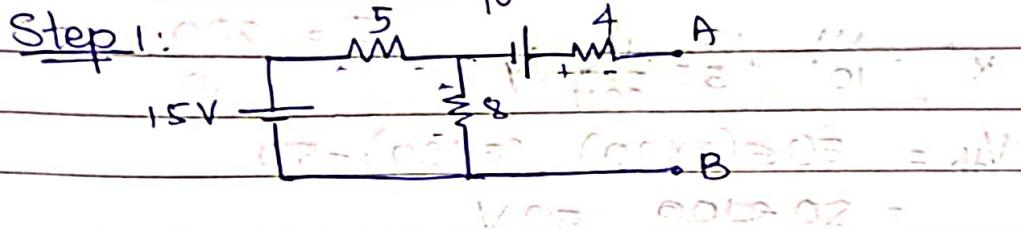
$$I_{AB} = \frac{12.45}{6.71 + 5} = 1.06 \text{ A}$$

$$I_{AB} = 1.06 \text{ A}$$

Q.



Sol'n:

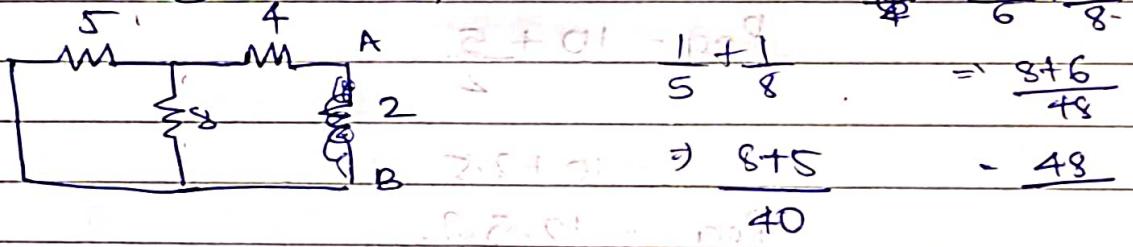


$$A - \frac{10}{5+1} + B = 10 \quad \text{V}_{AB} = 10 \times 0.2 = 2V$$

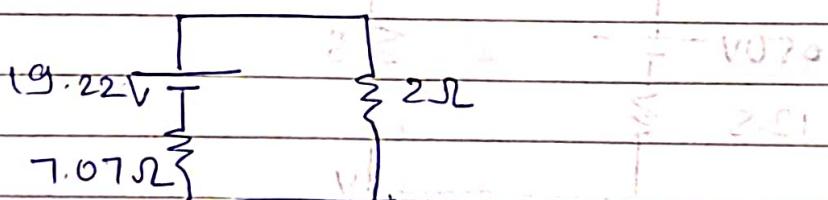
$$\text{V}_{AB} = \frac{10 \times 0.2}{1.15} = 1.15A$$

$$V_{th} = 10 + 8 \times 1.15 \\ = 10 + 9.2 \\ = 19.2V$$

$$= 19.2V$$



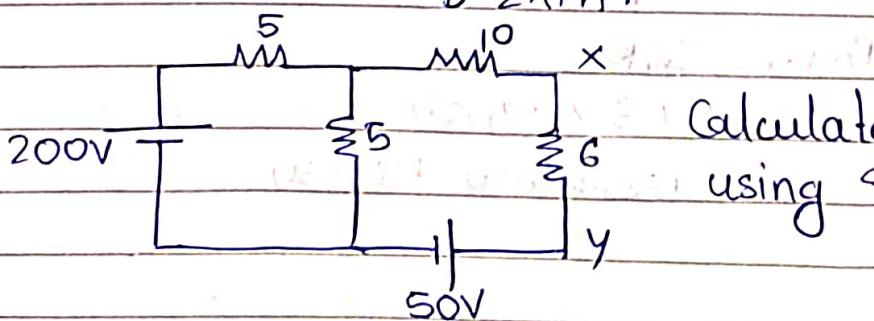
$$R_{eq} = 1.125\Omega$$



$$I_{AB} = \frac{19.2}{7.07+2}$$

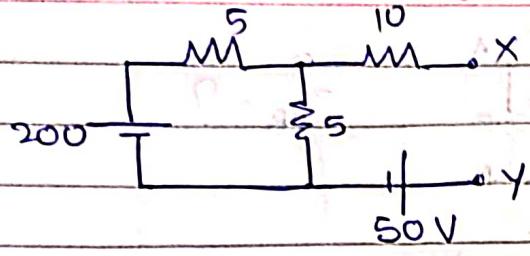
$$= 2.11A$$

Q.



Calculate power in XY using Thevenin's thm.

Sol'n:

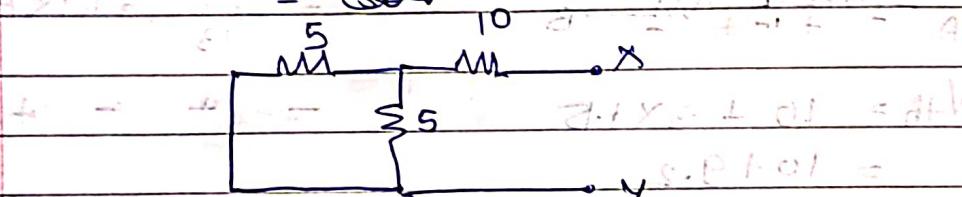


$$I = \frac{200}{10} = 20A.$$

$$V_{th} = 50 \times (5 \times 20) - 50$$

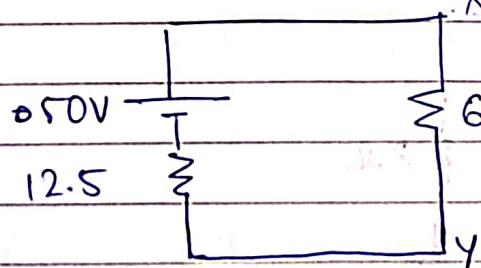
$$= 50 \times 100 - 50V$$

$$= 450V$$



$$Req = 10 + \frac{5}{2}$$

$$Req = 12.5\Omega$$



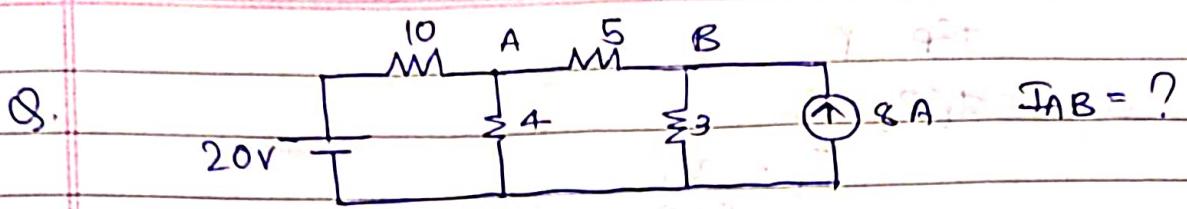
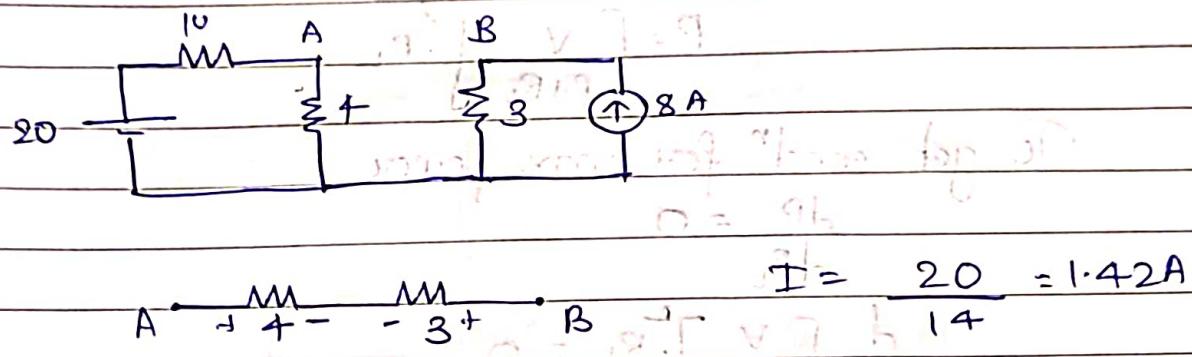
$$I_{xy} = \frac{50}{12.5 + 6}$$

$$= 2.702A.$$

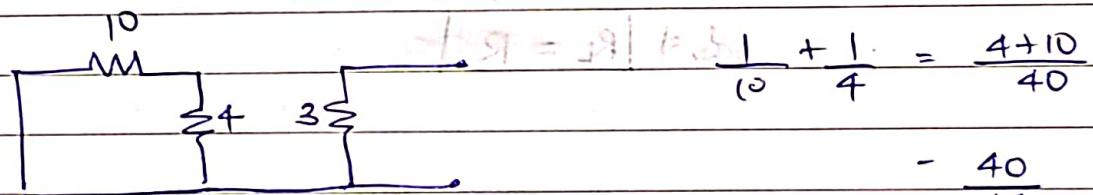
$$Power = I^2 R$$

$$= 7.3 \times 12.5 \times 6$$

$$= 1350W. 43.8W$$

Soln:

$$V_{th} = 4 \times 1.42 - 24 \\ = -18.32 \text{ V}$$



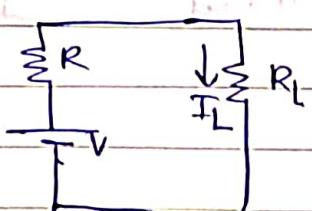
$$R_{eq} = 5.85 \Omega$$



$$I_{AB} = \frac{18.32}{5.85}$$

$$= 1.68 \text{ A}$$

* Max. Power transfer theorem.



$$I^2 R = P$$

$$I_L^2 R_L = P_m$$

$$I_L = \frac{V}{R+R_L}$$

$$P = \left[\frac{V}{R+R_L} \right]^2 \cdot R_L$$

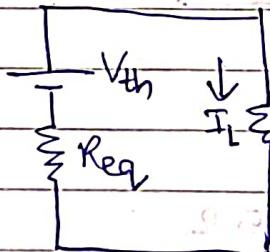
To get condⁿ for max. power

$$\frac{dP}{dR_L} = 0$$

$$\frac{d}{dR_L} \left[\frac{V}{R+R_L} \right]^2 \cdot R_L = 0$$

$$V^2 \frac{d}{dR_L} \left[\frac{R_L}{(R+R_L)^2} \right] = 0$$

$$\Rightarrow R_L = R$$



Condⁿ for max. power transfer,

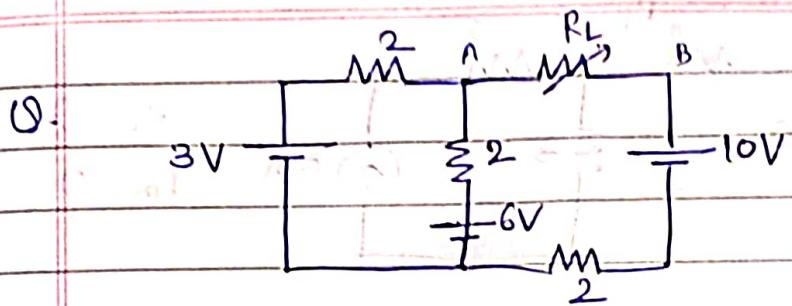
$$R_L = \text{Req.}$$

$$P_m = \left[\frac{V_{th}}{R_L + \text{Req.}} \right]^2 \cdot R_L$$

P_m will be max at $R_L = \text{Req.}$

$$P_m = \left[\frac{V_{th}}{\frac{R_L + \text{Req.}}{2} + \text{Req.}} \right]^2 \cdot \text{Req.}$$

$$P_m = \frac{V_{th}^2}{4 \cdot \text{Req.}} \text{ Watts}$$

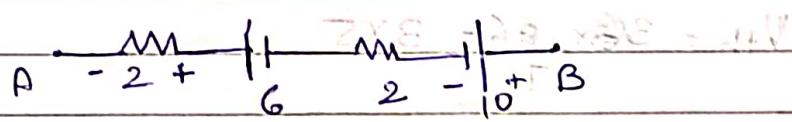


Calculate R_L so the max power is transferred.

Soln:

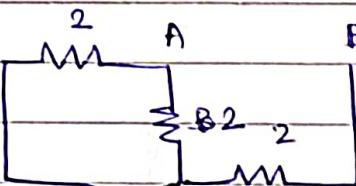
$$I_1 = \frac{6-3}{4} = \frac{3}{4} \text{ A}$$

$$I_2 = \frac{10}{2} = 5 \text{ A}$$



$$V_{th} = 2 \times \frac{3}{4} + 10 - 6$$

$$= 5.5 \text{ V}$$



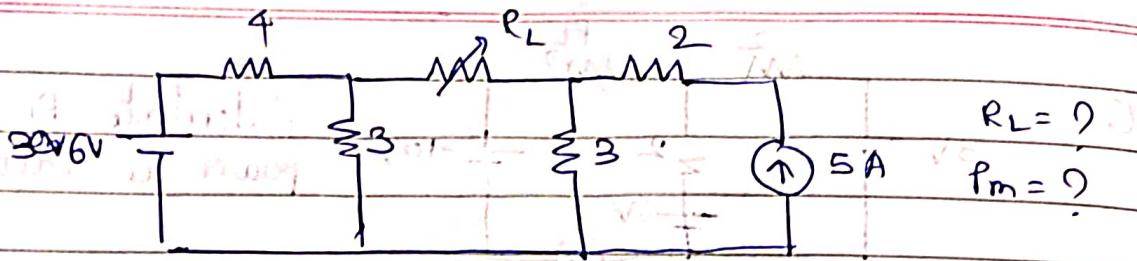
$$V_{th} = 1 + 2 = 3 \Omega$$

$$R_{eq} = 3 \Omega \therefore R_L = 3 \Omega$$

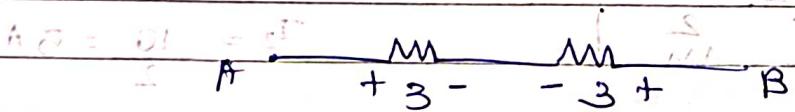
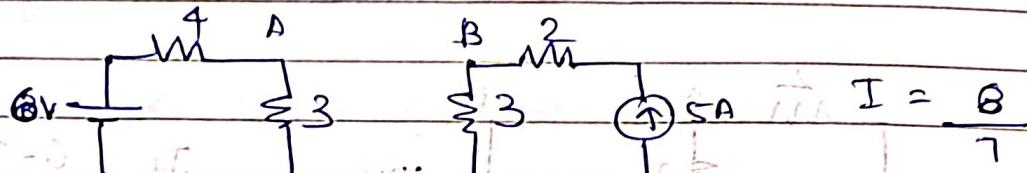


$$P = \frac{(5.5)^2}{4 \times 3} = \frac{30.25}{12} = 2.52 \text{ W}$$

Q.

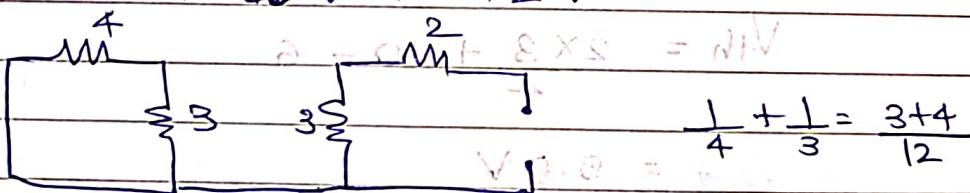


Soln:



$$V_{th} = 30 - 3 \times 5$$

$$= 12.42 \text{ V}$$



$$\text{Req} = \frac{12}{7} + 5$$

$$= 8.71 \Omega$$

$$R_L = 6.71 \Omega$$

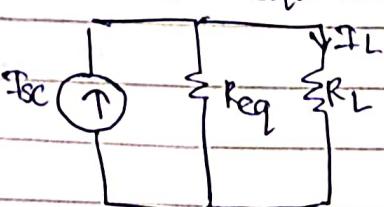
$$P_m = \frac{(12.42)^2}{4 \times 6.71} = \frac{154.25}{26.84} = 5.74 \text{ W}$$

*) Norton's Theorem:

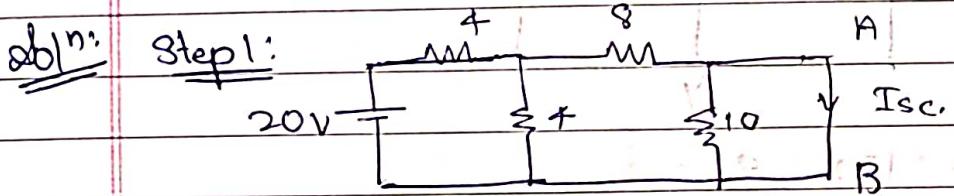
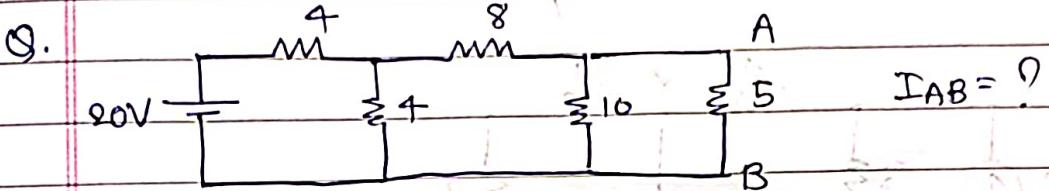
(Dual of Thevenin's)



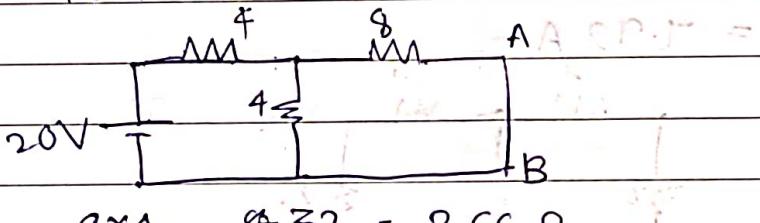
Norton's Equickt.



$$I_L = I_{SC} \times \frac{R_{eq}}{R_{eq} + R_L}$$



10Ω branch is redundant



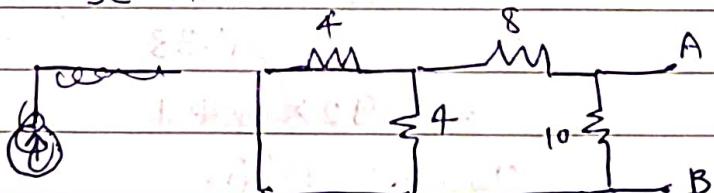
$$R_{eq} = 2.66 + 4 = 6.66 \Omega$$

$$\frac{V}{R_{eq}} = \frac{20}{6.66} = 3A$$

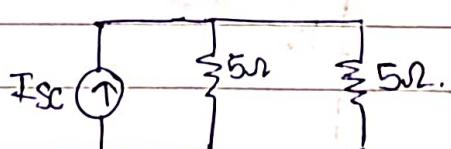
$$I = 3A$$

$$I_{SC} = 3 \times \frac{4}{4+8}$$

$$I_{SC} = 1A$$

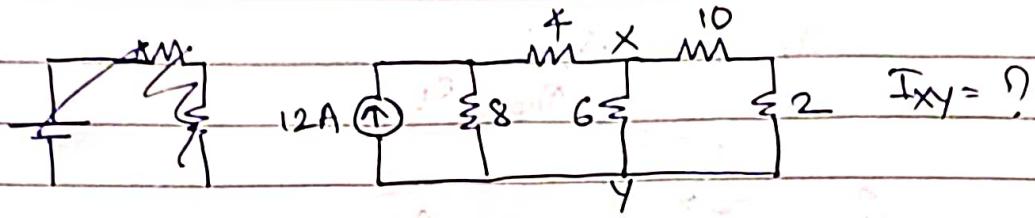
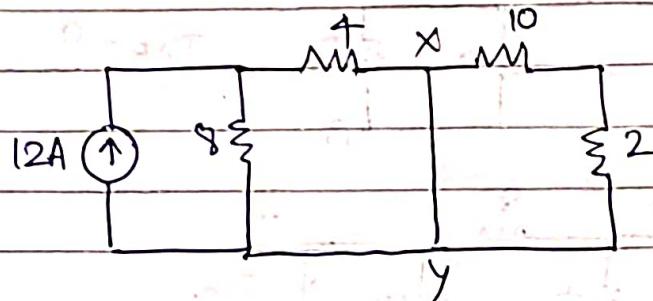


$$R_{eq} = 5 \Omega$$



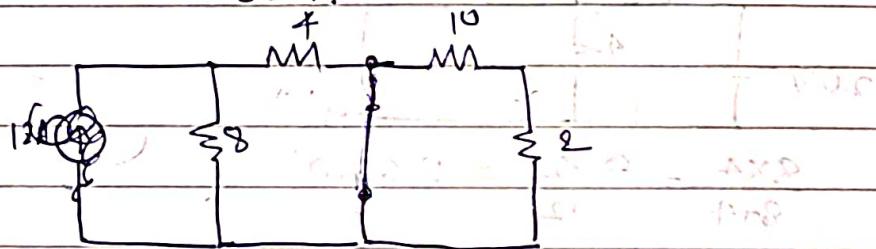
$$I_L = 0.5A$$

Q.

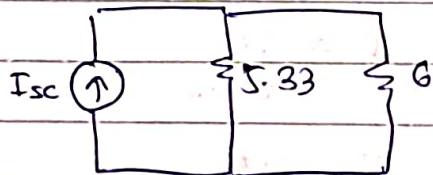
ddn:

$$I_{sc} = 12 \times \frac{8}{4+8}$$

$$= 7.92 \text{ A}$$



$$Req = \frac{8 \times 16}{8+16} = \frac{128}{24} = 5.33 \Omega \quad Req = 6 \Omega$$



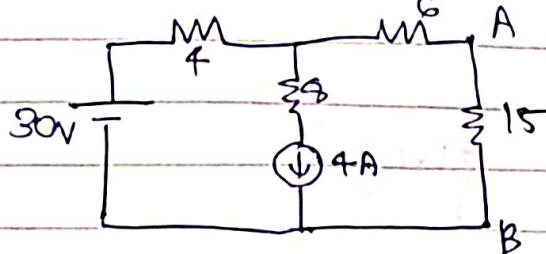
$$I_{xy} = 7.92 \times \frac{5.33}{5.33+6}$$

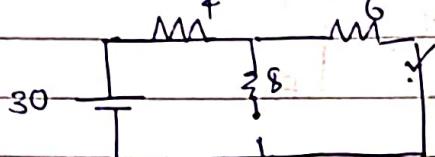
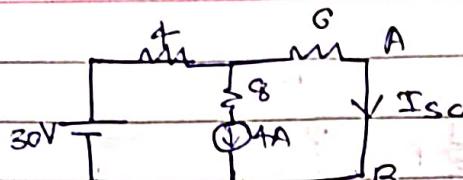
$$= 7.92 \times \frac{5.33}{11.33} \quad I_{xy} = 3.96 \text{ A.}$$

$$= 7.92 \times 0.47$$

$$I_{xy} = 3.72 \text{ A.}$$

Q.





$$I_1 = \frac{30}{10} = 3A.$$

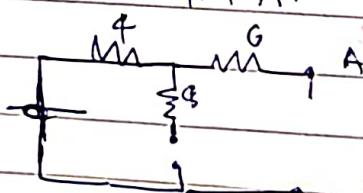
$$\frac{4 \times 6}{4+6} = \frac{48}{10} = 4.8A.$$

$$I_2 = 4 \times \frac{8}{6+8} = 1.6A.$$

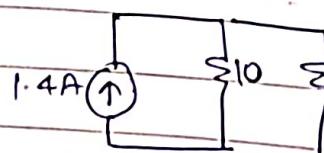
$$I_2 = 4 \times \frac{4}{4+6}$$

$$I_2 = 1.6A.$$

$$I_{SC} = 3 - 1.6 \\ = 1.4A.$$

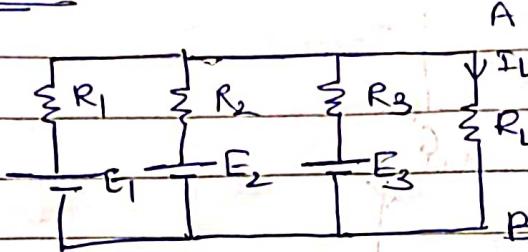


$$Req = 4+6 = 10\Omega.$$



$$I_L = 1.4 \times \frac{10}{10+15}$$

$$I_L = 0.56A.$$

*) Millman's Theorem:

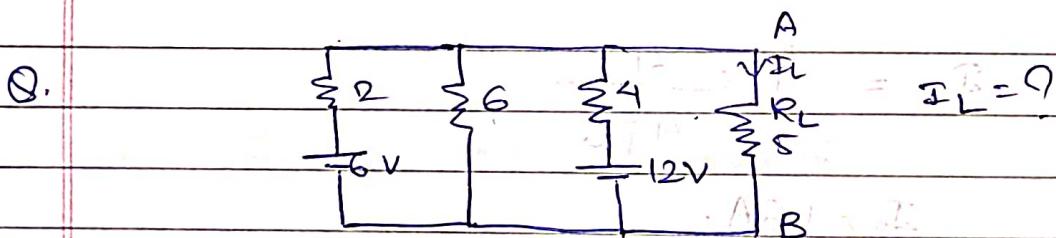
$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{R_1 + R_2 + R_3}$$

$$V_{AB} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G}$$

$$= V_{th}$$

Req \rightarrow same way as Thevenin's.

$$I_L = \frac{V_{AB}}{Req + R_L}$$



Soln:

$$V_{th} = \frac{\frac{6}{2} + \frac{12}{4}}{\frac{1}{2} + \frac{1}{6} + \frac{1}{4}}$$

$$= \frac{3+3}{6+2+3}$$

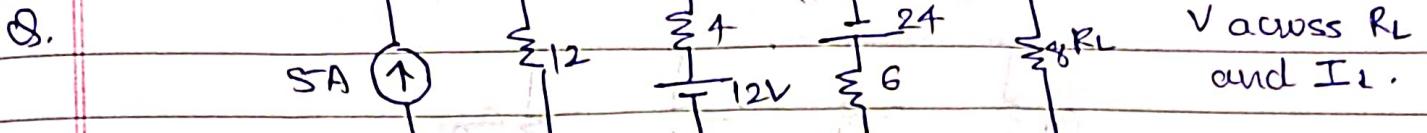
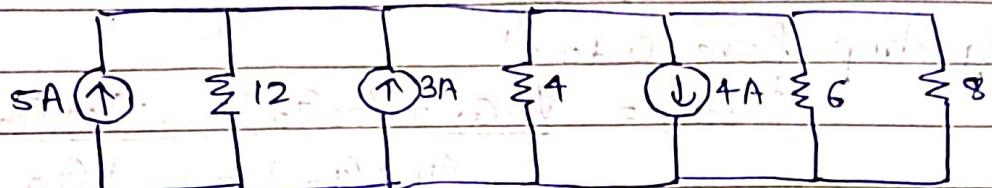
$$= \frac{6 \times 12}{11}$$

$$V_{AB} = 6.54 \text{ V}$$

$$\frac{2 \times 6}{6+2} = \frac{12}{8} = \frac{1.5 \times 4}{1.5 + 4} = \frac{6}{5.5} = 1.09 \Omega$$

$$I_L = \frac{6.54}{1.09 + 9} = 0.63 \text{ A}$$

$$I_L = 1.09 \text{ A}$$

Soln:

$$V_{AB} = 5 + 3 - 4$$

$$= \frac{1}{12} + \frac{1}{4} + \frac{1}{6}$$

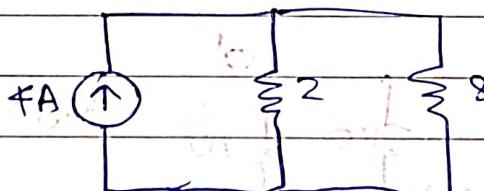
$$= \frac{1+3+2}{12}$$

$$= \frac{4 \times 12^2}{8} = 48 \text{ V}$$

$$V_{AB} = 8 \text{ V}$$

$$R_{eq} = \frac{12 \times 4}{12+4} = \frac{3 \times 6}{6+3} = \frac{18}{9} = 2 \Omega$$

$$I = \frac{8}{2+8} = \frac{2}{10} = 0.2 \text{ A}$$



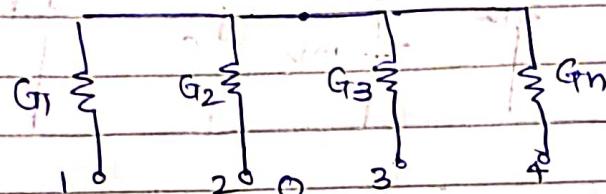
$$I_L = 4 \times \frac{2}{2+8}$$

$$= 0.8 \text{ A}$$

$$V_L = 0.8 \times 8$$

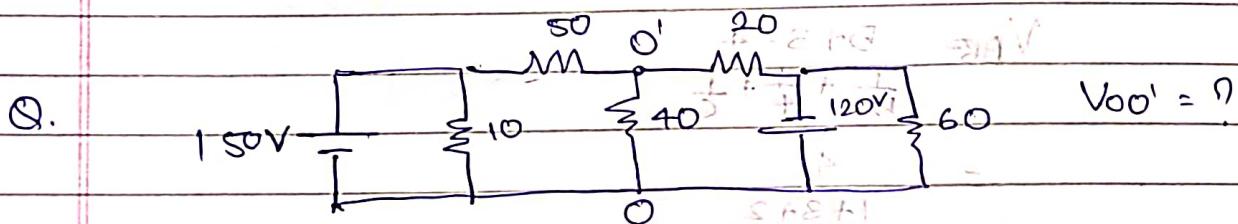
$$= 6.4 \text{ V}$$

* Generalised form of Millman's theorem:



Voltage drop from 0 to 0'

$$(V_{OO'}) = \frac{V_{O1}G_1 + V_{O2}G_2 + \dots + V_{On}G_n}{G_1 + G_2 + \dots + G_n}$$



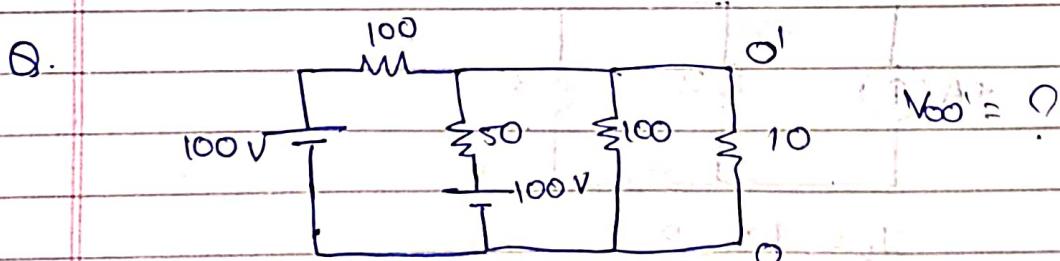
Sol'n:

$$V_{OO'} = \frac{-\frac{150}{50} + \frac{120}{20}}{\frac{1}{50} + \frac{1}{40} + \frac{1}{20}}$$

$$= \frac{-3 + 6}{0.02 + 0.025 + 0.05}$$

$$= \frac{3}{0.095} = 31.5V$$

$$V_{OO'} = 31.5V.$$



Sol'n:

$$V_{OO'} = \frac{-\frac{100}{100} + \frac{100}{50}}{\frac{1}{100} + \frac{1}{50} + \frac{1}{10}}$$

$$= \frac{-1 + 2}{1 + 2 + 10} = -\frac{1}{13}$$

$$= -\frac{3 \times 100}{13} = -21.4V.$$

9/11/22

Unit-3 Analysis of Transient Response in Circuits

Initial condⁿ:

Time periods $\begin{matrix} 1^- \\ + \end{matrix}$ 0^+

1. Just before switching ($t = -\infty$ to $t = 0^-$)
2. Just after switching ($t = 0^+$)
3. After switching ($t > 0$).

1. Initial condⁿ for R

$$V = \cancel{IR} iR$$

2. Initial condⁿ for L

$$V = L \cdot \frac{di}{dt}$$

$$i(t) = \frac{1}{L} \int_0^t v(t) dt + i(0)$$

$i(0)$ is initial current through L

If no current through L at $t = 0^-$

L will act as O.C.

If current I_0 is flowing through L at $t = 0^-$,
L will act as current source of I_0 A.

3. Capacitor

$$V = \frac{1}{C} \int_0^t i(t) dt + V(0)$$

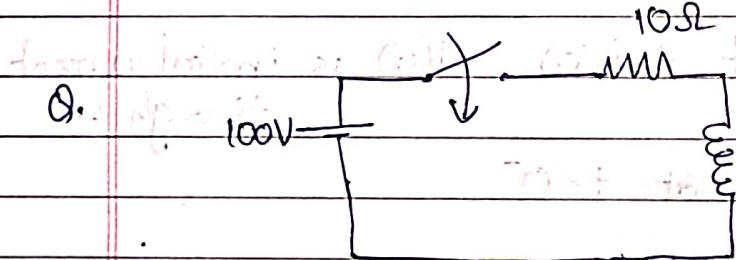
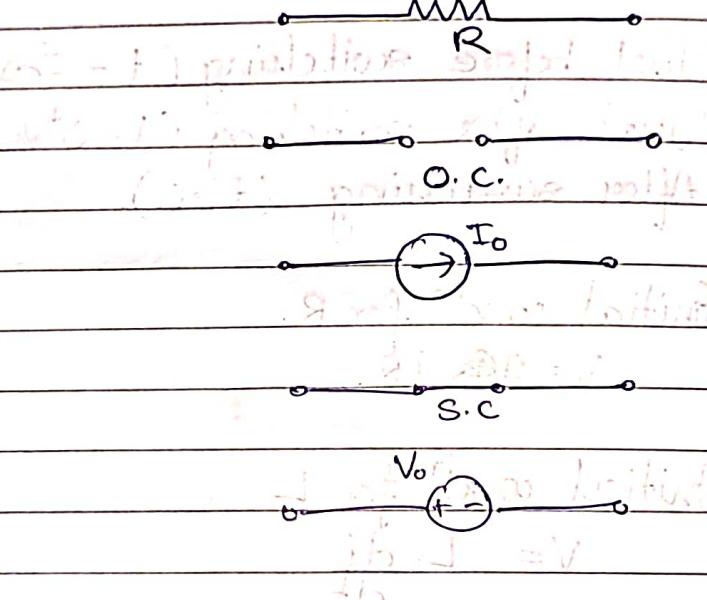
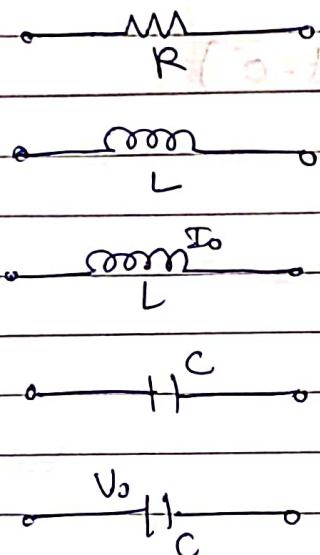
$V(0)$ is initial voltage across C.

If there is no voltage across cap. at $t = 0^-$
C will act as short circuit.

If initially the capacitor is charged to v₀ at $t = 0^-$
C will act as voltage source of v_0 .

Element with initial condⁿ

Equi. ckt. at $t=0^+$

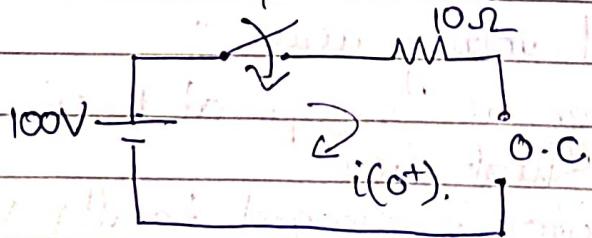


In the given network switch is closed at $t=0$, with 0 current in Inductor, find i , $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$

Solⁿ: at $t=0^-$, no current flow through L

$$\therefore i(0^-) = 0$$

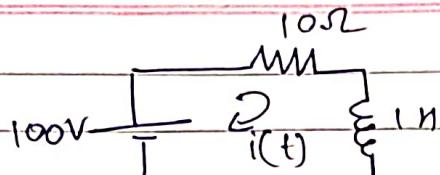
At $t=0^+$ the n/w will be



Inductor acts as O.C.

$$\therefore i(0^+) = 0.$$

At $t>0$



$$100 - 10i - \frac{di}{dt} = 0 \quad \textcircled{1}$$

$$\frac{di}{dt} = 100 - 10i \quad \textcircled{2}$$

At $t = 0^+$

$$\frac{di}{dt}(0^+) = 100 - 10i(0^+)$$

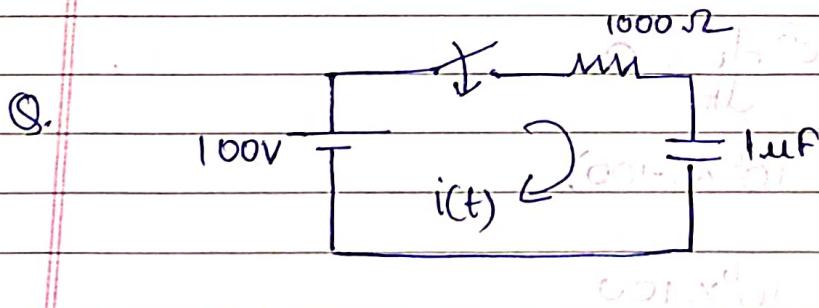
$$\frac{di}{dt}(0^+) = 100 \text{ A/s.}$$

Diff. eqn $\textcircled{2}$

$$\frac{d^2i}{dt^2} = -10 \frac{di}{dt}$$

$$\frac{d^2i}{dt^2}(0^+) = -10 \frac{di}{dt}(0^+)$$

$$= -1000 \text{ A/s}^2$$

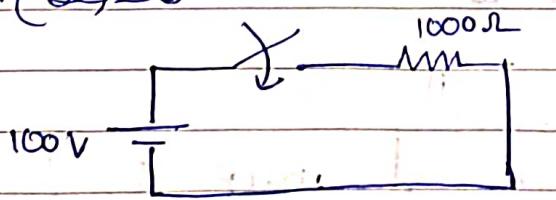


switch closed at $t = 0$
with capacitor uncharged
 $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t = 0^+$

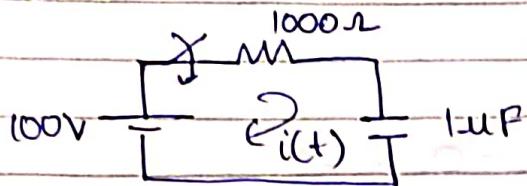
Sol'n: At $t = 0^+$, no ~~case~~ voltage through C

$$i(0^+) = 0$$

$$V(0) = 0 \\ \therefore i(0) = 0.$$



$$i(0^+) = \frac{100}{1000} = 0.1 \text{ A.}$$

At $t > 0$ 

$$100 - 1000i - 10^6 \int_0^t i(t) dt = 0$$

$$10^6 \int_0^t i(t) dt = 1000i - 100$$

$$i = 0 \rightarrow 1000 \frac{di}{dt} - 10^6 i = 0$$

At $t = 0^+$

$$0 - 1000 \frac{di}{dt} - 10^6 i = 0$$

$$\frac{di}{dt} = \frac{10^6 i}{1000}$$

$$= \frac{0.1 \times 10^6}{1000}$$

$$\frac{di(0)}{dt} = -100 \text{ A/S}$$

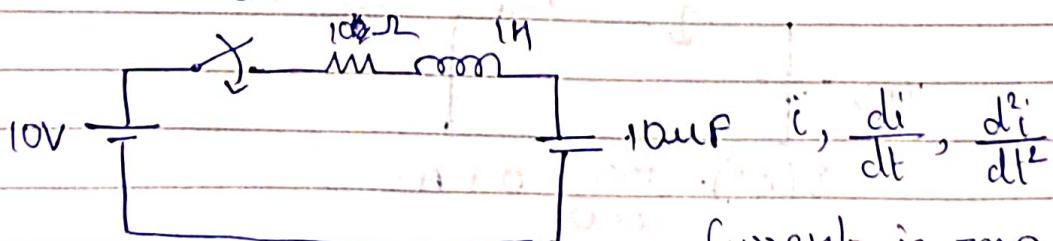
$$-1000 \frac{d^2i}{dt^2} - 10^6 \frac{di}{dt} = 0$$

$$-1000 \frac{d^2i}{dt^2} - 10^6 \frac{di}{dt} = 10^6 \times (-100)$$

$$\frac{d^2i}{dt^2} = \frac{10^6 \times 100}{1000}$$

$$\frac{d^2i}{dt^2} = 10^5 \text{ A}$$

Q.



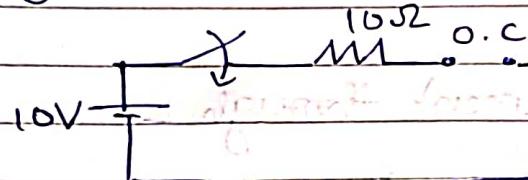
Current is zero
as Capacitor is uncharged
 $t = 0^-$

$$\text{Soln: } A \ L = 0^-$$

$$i(0^-) = 0$$

$$V_o(0^-) = 0$$

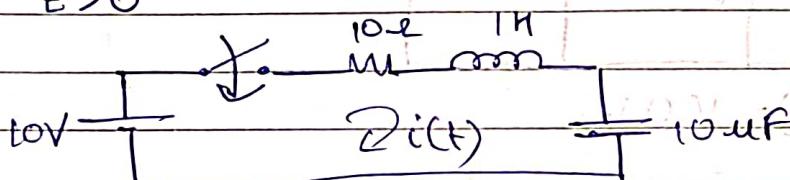
$$A \ L = 0^+ \text{ (initial)}$$



Ans

$$i(0^+) = 0$$

$$\text{At } t > 0$$



$$10 - 10i(t) - \frac{di}{dt} - \frac{1}{10^5} \int_{0^+}^t i(t) dt = 0.$$

$$10 - 10i(t) - \frac{di}{dt} = 0 \quad \text{--- (1)}$$

$$\frac{di}{dt} = 10 - 10i(t)$$

$$\text{At } t = 0^+$$

$$\frac{di}{dt}(0^+) = 10 - 10i(0^+) \quad \text{(initial)}.$$

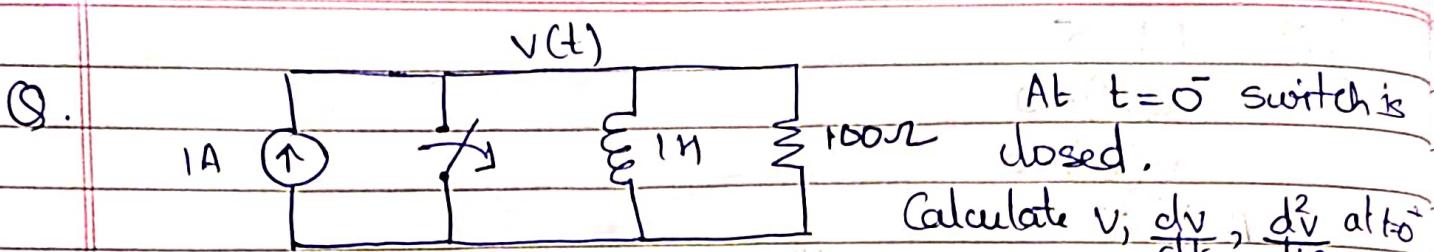
$$\frac{di}{dt}(0^+) = 10 \text{ A/s.} \quad \text{initial}$$

$$\text{Ab} \quad 10 - 10 \frac{di}{dt} - \frac{d^2i}{dt^2} = 0 \quad \text{Ab}$$

$$\frac{d^2i}{dt^2} = 10 - 10 \frac{di}{dt}$$

$$\text{At } t = 0^+$$

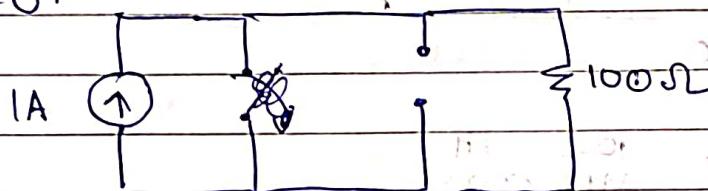
$$\frac{d^2i}{dt^2}(0^+) = -10 \times 10 \\ = -100.$$



doln: At $t = 0^-$

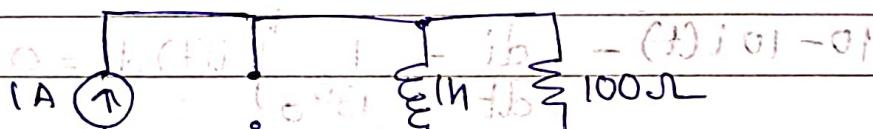
$$i(0^-) = 0A \cdot \text{No current through } L$$

At $t = 0^+$



$$v(0^+) = 100V$$

At $t > 0$



$$I_1 = \frac{1}{L} \int v dt$$

$$I_2 = \frac{V}{100}$$

$$\frac{1}{L} \int v dt + \frac{V}{100} = 1 - 10t - (t_0) \frac{i_b}{100}$$

$$\frac{1}{L} V + \frac{1}{100} \frac{dv}{dt} = 100 \cdot 0 + (t_0) \frac{ib}{100}$$

$$\frac{dv}{dt} = -\frac{V}{L} \times 100 + \frac{(t_0) ib}{100}$$

$$= \frac{1}{100} \frac{100V}{1}$$

$$\frac{dv}{dt} = -100V$$

At $t = 0^+$

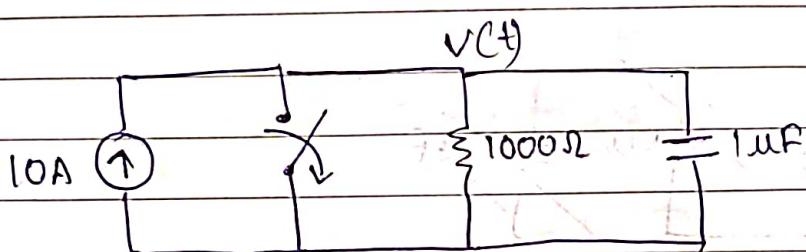
$$\frac{dv(0^+)}{dt} = -10^7 \text{ V/s}$$

$$\frac{d^2v}{dt^2} = -100 \frac{dv}{dt}$$

$$A1 = 0^+$$

$$\begin{aligned}\frac{d^3v}{dt^3}(0^+) &= -100 \times (-10^7) \\ &= 10^6 \text{ V/s}^2\end{aligned}$$

Q.

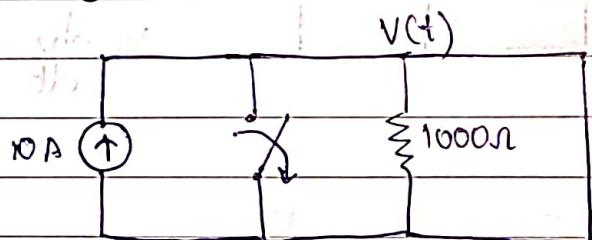


Switch opened at $t=0$
 $\frac{dv}{dt}, \frac{d^2v}{dt^2}$ at $t=0^+$

soln: At $t=0^-$

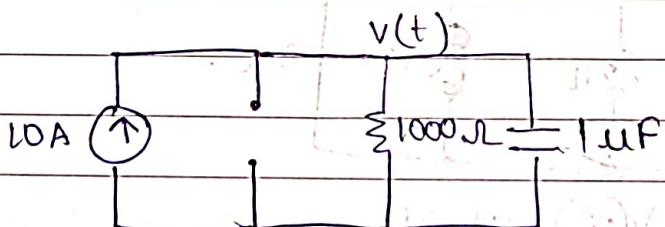
$$v(0^-) = 0$$

At $t=0^+$



$$v(0^+) = 10^6 \text{ V}$$

At $t>0$



$$\frac{v}{1000} + C \frac{dv}{dt} = 10$$

$$C \frac{dv}{dt} = 10 - \frac{v}{1000}$$

$$\frac{dv}{dt} = \frac{1000}{C} \frac{10 - v}{1000}$$

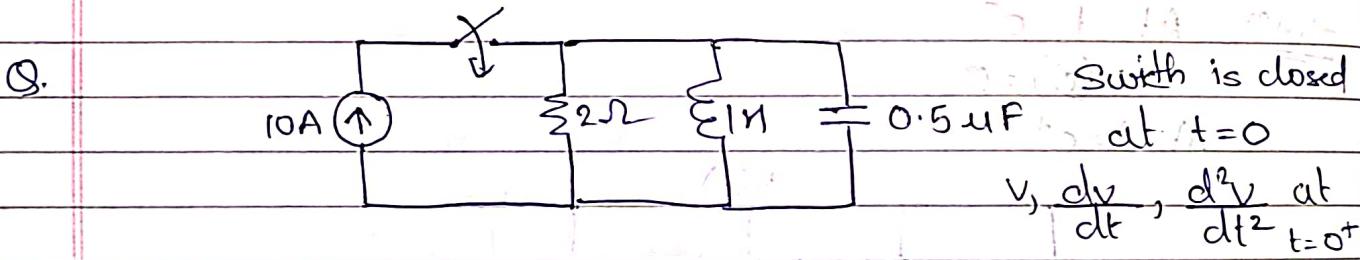
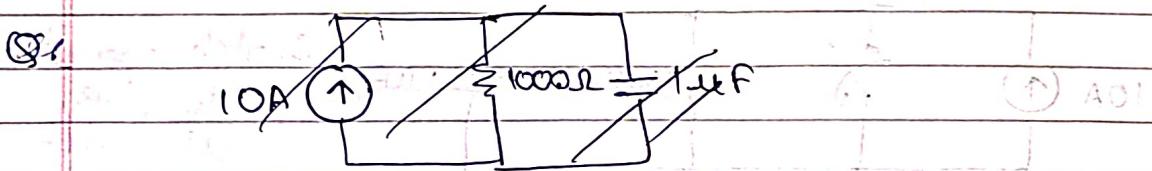
$$= 10^7 \text{ V/s}$$

$$\frac{1}{1000} \frac{dv}{dt} + C \frac{d^2v}{dt^2} = 0$$

$$10^4 + 16 \frac{d^2v}{dt^2} = 0$$

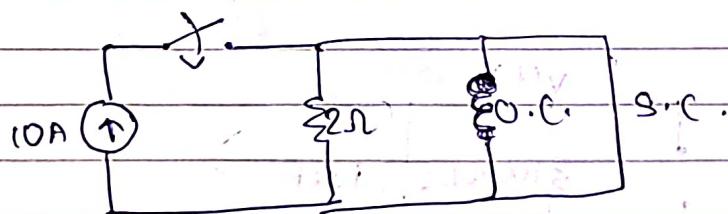
$$10^6 \frac{d^2v}{dt^2} = -10^4$$

$$\frac{d^2v}{dt^2} = -10^{10} \text{ V/s}^2$$



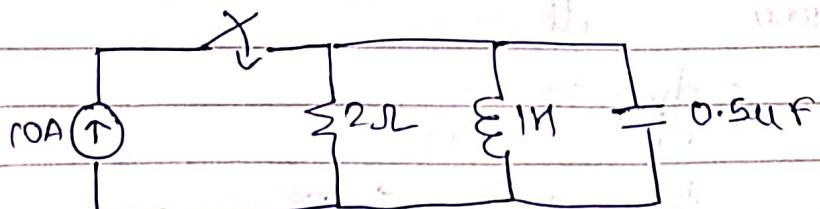
Sol'n: At $t=0^-$
 $v(0^-) = 0$

At $t=0^+$



$$V(0^+) = 40 \text{ V}$$

At $t > 0$



$$\frac{1}{2} I^2 + 10 + \int v dt + C \frac{dv}{dt} = 10$$

At $t = 0^+$

$$\frac{v(0^+)}{2} + \frac{1}{L} \int v(0^+) dt + C \frac{dv}{dt} = 10$$

$$\frac{dv}{dt} = \frac{10}{0.5 \times 10^6}$$

$$\frac{dv}{dt} = 2 \times 10^7 \text{ V/s}$$

$$\frac{1}{2} \frac{dv}{dt} + \frac{1}{L} v + C \frac{d^2v}{dt^2} = 0$$

$$\frac{1}{2} 2 \times 10^7 + \frac{1}{L} v(0^+) + C \frac{d^2v}{dt^2} = 0$$

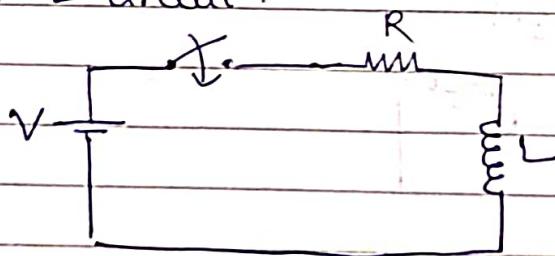
$$10^7 + 0.5 \times 10^{-6} \frac{d^2v}{dt^2} = 0$$

$$0.5 \times 10^{-6} \frac{d^2v}{dt^2} = -10^7$$

$$\frac{d^2v}{dt^2} = -2 \times 10^{13} \text{ V/s}^2$$

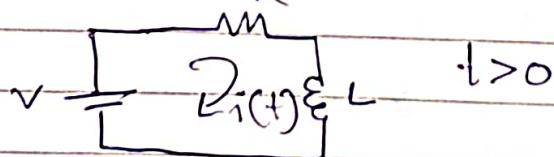
1) Transient Response:

- R-L circuit:



Switch is closed at $t = 0$.

\therefore Current in inductor at $t = 0^- = 0$.



KVL eqn at $t > 0$

$$V - R i(t) - L \frac{di}{dt} = 0 \quad - \text{Linear D.E. order 1.}$$

If variables are separated

$$(V - R_i) dt = L di$$

$$\frac{L}{V - R_i} di = dt$$

Integrating both sides

$$-\frac{L}{R} \ln(V - R_i) = t + k \quad \text{--- (1)}$$

To get value of k , at $t=0, i=0$

$$-\frac{L}{R} \ln V = k \quad \text{--- (2)}$$

Put (2) in eqn (1)

$$-\frac{L}{R} (\ln V - R_i) = t - \frac{L}{R} \ln V$$

$$t = R_i t - \frac{L}{R} \ln(V - R_i)$$

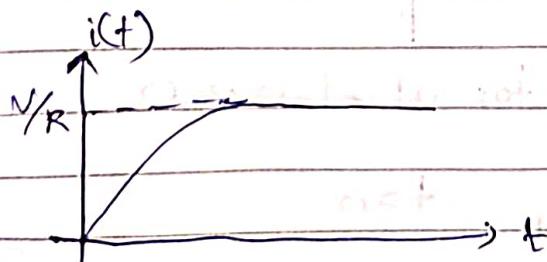
$$\cancel{V - R_i}$$

$$= -\frac{L}{R} [\ln(V - R_i) - \ln V] = t$$

$$\frac{V - R_i}{V} = e^{-\frac{R_i t}{L}}$$

$$V - R_i = V \cdot e^{-\frac{R_i t}{L}}$$

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R_i t}{L}}$$



Voltage across R

$$V_R = R \times \frac{V}{R} \left(1 - e^{-\frac{R_i t}{L}}\right)$$

$$V_R = V \left(1 - e^{-\frac{R_i t}{L}}\right) \text{ for } t > 0$$

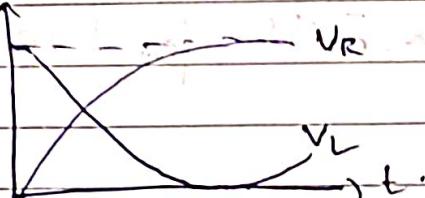
Voltage across L

$$V_L = L \cdot \frac{di}{dt}$$

$$= L \cdot \frac{v}{R} \cdot \frac{d}{dt} (1 - e^{-\frac{R}{L} \cdot t})$$

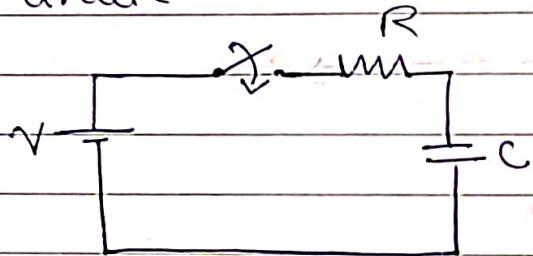
$$V_L = V \cdot e^{-\frac{R}{L} \cdot t} \quad \text{for } t > 0.$$

$v(t)$



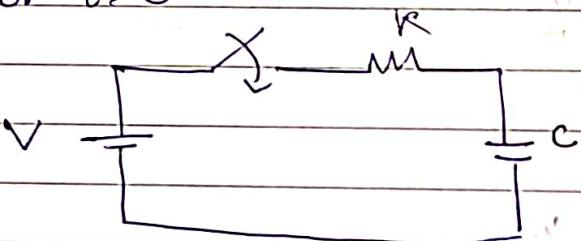
$\frac{L}{R}$ → time constant of R-L circuit.

- R-C circuit.



At $t=0$ switch is closed

∴ Capacitor is uncharged initially
for $t > 0$



$$V - Ri - \frac{1}{C} \int_0^t i dt = 0$$

$$0 - R \frac{di}{dt} - \frac{i}{C} = 0$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0.$$

$$\frac{di}{i} = -\frac{dt}{RC}$$

Int. both sides.

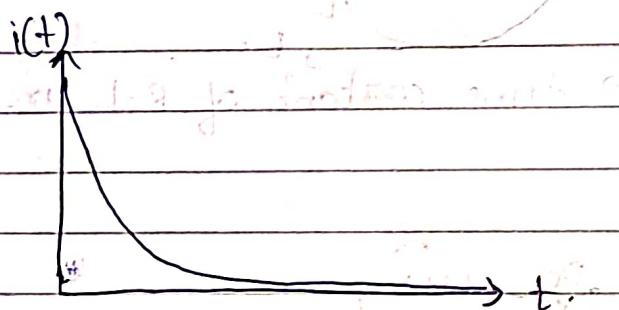
$$\ln i = -\frac{1}{RC} t + k.$$

At $t=0^+$ $i = V/R$.

$$\therefore \ln\left(\frac{V}{R}\right) = k.$$

$$\therefore \ln i = -\frac{1}{RC} t + \ln\left(\frac{V}{R}\right).$$

$$i = \frac{V}{R} \cdot e^{-t/RC} \text{ for } t > 0.$$



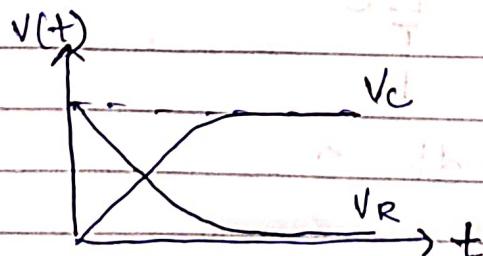
Voltage across R.

$$V_R = V \cdot e^{-t/RC}$$

Voltage across C

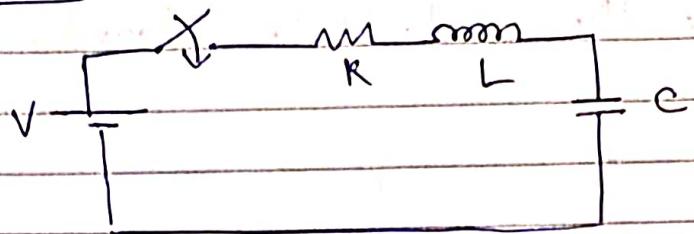
$$V_C = \frac{1}{C} \int_0^t \frac{V}{R} e^{-t'/RC} dt.$$

$$V_C = -V e^{-t/RC}$$



$RC \rightarrow$ time constant of RC ckt. (T).

i) RLC-Circuit:

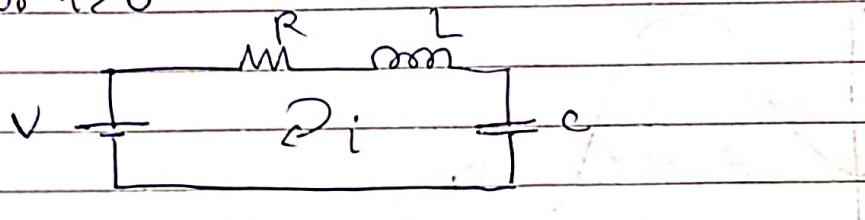


switch is closed at $t = 0$.

Inductor current $i = 0$.

Capacitor voltage V_C

For $t > 0$



$$V - Ri - L \frac{di}{dt} - \frac{1}{C} \int i dt = 0.$$

Differentiate

$$0 - R \frac{di}{dt} - L \frac{d^2i}{dt^2} - \frac{i}{C} = 0.$$

$$\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0.$$

2nd order D.E.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{1}{LC}}$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha + \beta$$

$$s_2 = -\alpha - \beta$$

$$\alpha = \frac{R}{2L}$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$

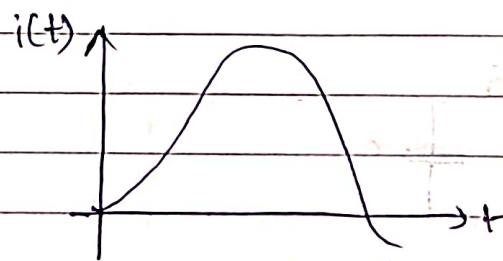
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$
 k_1, k_2 are the const. to be determined.

I] $\alpha > \omega_0$

i.e. $R > \frac{1}{2L \sqrt{LC}}$

Roots are real & unequal.
 Overdamped condn.



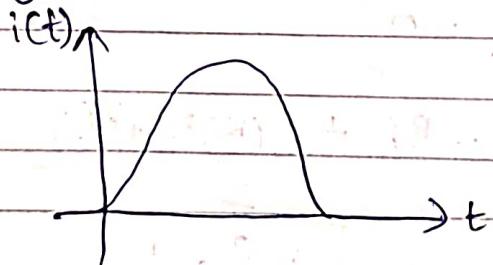
solⁿ is $i = e^{-\alpha t} (k_1 e^{\beta t} + k_2 e^{-\beta t})$
 Or $i = k_1 e^{s_1 t} + k_2 e^{s_2 t} \mid_{t \geq 0}$

II] $\alpha = \omega_0$

i.e. $R = \frac{1}{2L \sqrt{LC}}$

Roots are real & equal.

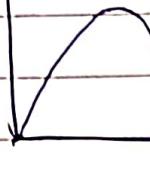
Critically damped.



solⁿ is $i = e^{-\alpha t} (k_1 + k_2 t) \mid_{t \geq 0}$

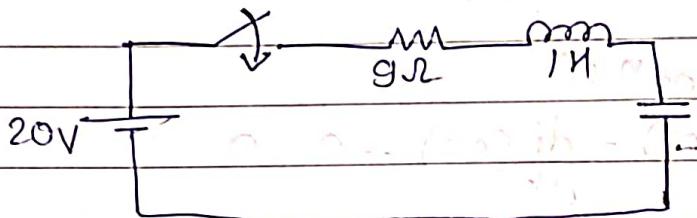
III] $\alpha < \omega_0$

Roots are complex conjugate
 Underdamped.

$i(t)$ 

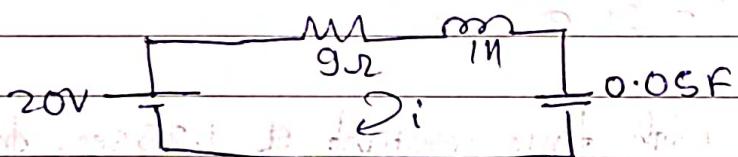
$$i = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

Q.

switch is closed at $t=0$.Obtain $i(t)$ at $t > 0$.obtn: Inductor current $i = 0$, $i(0) = 0$ Capacitor voltage $V_C = 0$.At $t \rightarrow 0$, $t = 0^+$

$$i = 0$$

$$V_C = 0$$

At $t > 0$ 

$$20 - 9i - 1 \cdot di - \frac{1}{0.05} \int i dt = 0 \quad \text{--- (1)}$$

$$-9 \frac{di}{dt} - \frac{d^2i}{dt^2} - 20i = 0 \quad \text{in steady state}$$

$$\frac{d^2i}{dt^2} + 9 \frac{di}{dt} + 20i = 0$$

$$s^2 + 9s + 20 = 0,$$

$$s_1 = -4 \quad s_2 = -5$$

$$i = k_1 e^{-4t} + k_2 e^{-5t} \quad \text{--- (2)}$$

 $k_1 + k_2 = 0$. Diff. eqn (2)

$$\frac{di}{dt} = -4k_1 e^{-4t} - 5k_2 e^{-5t} \quad \text{--- (3)}$$

At $t=0$, $i=0$

from eqⁿ (2)

$$k_1 + k_2 = 0 \quad \text{--- (4)}$$

from eqⁿ (3)

$$-4k_1 - 5k_2 = \frac{d}{dt} i(0^+)$$

Put $t=0$ in eqⁿ (1)

$$20 - 9i(0^+) - \frac{di(0^+)}{dt} = 0 = 0$$

$$\frac{di(0^+)}{dt} = 20 \text{ A/s}$$

$$-4k_1 - 5k_2 = 20$$

$$-4k_1 + 5k_1 = 20$$

$$k_1 = 20.$$

$$k_2 = -20.$$

$$\therefore i = 20e^{-4t} - 20e^{-5t}$$

Q. In a R-L circuit with time constant of 1.25 sec, Inductor current increases from the initial value of zero to final value 1.2A.

a) Inductor current at $t=0.4$ s, $t=0.8$ s, $t=1.2$ s.

b) Find a time at which the current reaches 0.3 A, 0.6 A, 0.9 A

Solⁿ:

$$i_L = \frac{V}{R} (1 - e^{-\frac{R}{L}t})$$

$$T = 1.25$$

$$\frac{L}{R} = 1.25$$

$$\frac{R}{L} = 0.8$$

$$i_L = 1.2 \left(1 - e^{-0.8 \times 0.4} \right)$$

$$= 1.2 \left(1 - e^{-0.32} \right)$$

$$i_L = 1.2 \left(1 - 0.726 \right)$$

$$= 1.2 \times 0.274$$

$$i_L = 0.32 \text{ at } t = 0.4 \text{ s.}$$

$$i_L = 1.2 \left(1 - e^{-0.8 \times 0.8} \right)$$

$$= 1.2 \left(1 - e^{-0.64} \right)$$

$$= 1.2 \left(1 - 0.527 \right)$$

$$= 1.2 \times 0.473 = 0.567 \text{ A}$$

$$i_L = 0.567 \text{ at } t = 0.8 \text{ s.}$$

$$i_L = 1.2 \left(1 - e^{-0.8 \times 2} \right)$$

$$= 1.2 \left(1 - e^{-1.6} \right)$$

$$= 1.2 \left(1 - 0.201 \right)$$

$$= 1.2 \times 0.799 = 0.958 \text{ A}$$

$$i_L = 0.958 \text{ at } t = 2 \text{ s.}$$

$$0.3 = 1.2 \left(1 - e^{-0.8t} \right)$$

$$0.3 = 1.2 - 1.2 e^{-0.8t}$$

$$1.2 e^{-0.8t} = 1.2 - 0.3$$

$$1.2 e^{-0.8t} = 0.9$$

$$e^{-0.8t} = 0.75$$

$$-0.8t = -0.28$$

$$t = 0.35 \text{ s when } i = 0.3 \text{ A.}$$

$$0.6 = 1.2 - 1.2 e^{-0.8t}$$

$$1.2 e^{-0.8t} = 0.6$$

$$e^{-0.8t} = 0.5$$

$$-0.8t = -0.693$$

$$t = 0.866 \text{ s when } i = 0.6 \text{ A}$$

$$0.9 = 1.2 - 1.2 e^{-0.8t}$$

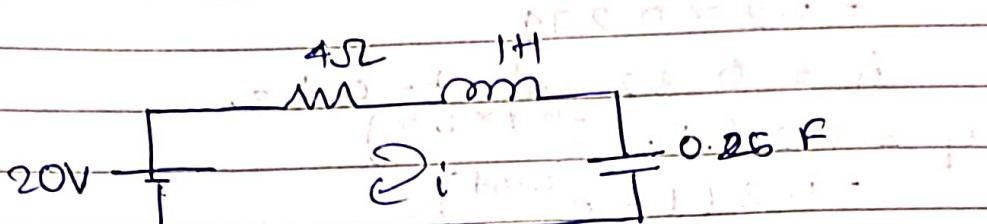
$$1.2 e^{-0.8t} = 0.3$$

$$e^{-0.8t} = 0.25$$

$$-0.8t = -1.386$$

$$t = 1.73 \text{ s when } i = 0.9 \text{ A.}$$

- Q. Find exp. for current in series RLC ckt fed by a DC voltage of 20V with $R = 4\Omega$, $L = 1H$, $C = \frac{1}{4}F$. Assume initial condns to be zero.

soln:

$$20 - 4i - \frac{di}{dt} - 4 \int i dt = 0$$

$$-4 \frac{di}{dt} - \frac{d^2i}{dt^2} - 4i = 0$$

$$s^2 + 4s + 4 = 0$$

$$s_1, s_2 = -2$$

$$i = k_1 e^{-2t} + k_2 t e^{-2t}$$

$$\text{At } t=0, i=0$$

$$k_1 + k_2 \cdot 0 = 0 \quad k_1 = 0$$

$$20 - 4i(0^+) - \frac{di(0^+)}{dt} = 0$$

$$20 - \frac{di(0^+)}{dt} = 0$$

$$\frac{di(0^+)}{dt} = 20 \text{ A/s}$$

$$\frac{di}{dt} = -2k_1 e^{-2t} - 2k_2 t e^{-2t}$$

$$20 = -2k_1 \cancel{e^{-2t}} (k_1 + k_2 t) \quad 20 = .$$

$$-10 = k_1 + k_2 t$$

$$-10 = k_1 + 0 \cdot t \quad k_1 = -10$$

$$20 = -2k_1 +$$

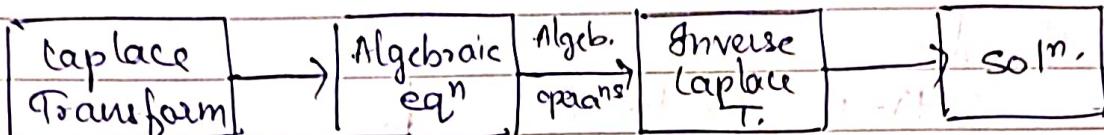
$$\frac{di}{dt} = -2k_1 e^{-2t} + k_2 (t(-2)e^{-2t} + e^{-2t})$$

$$20 = k_2 e^{-2t}$$

$$\therefore i = 20 e^{-2t} A.$$

f) Analysis of Transient Response in ckt. using Laplace T.

Integro
P.E.



Properties of L.T.

i) Linearity

$$\mathcal{L}\{f_1(t) + f_2(t) + \dots + f_n(t)\} = F_1(s) + F_2(s) + \dots + F_n(s).$$

$$\mathcal{L}\{a_1 f_1(t) + a_2 f_2(t) + \dots\} = a_1 F_1(s) + a_2 F_2(s) + \dots$$

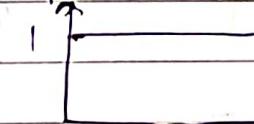
$a_1, a_2 \rightarrow \text{const.}$

Step funcⁿ

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

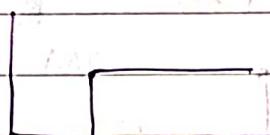
$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$f(t) = u(t)$$



If step is shifted by T

$$f(t) = u(t-T).$$

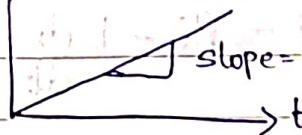


Ramp funcⁿ

$$x(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

$$\mathcal{L}\{t + u(t)\} = \frac{1}{s^2}$$

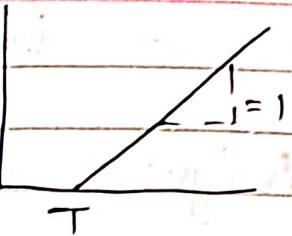
$$f(t) = x(t)$$



If slope is A

$$= \frac{A}{s^2}$$

$$\mathcal{L}\{x(t-T)\} = \frac{e^{-Ts}}{s^2}$$

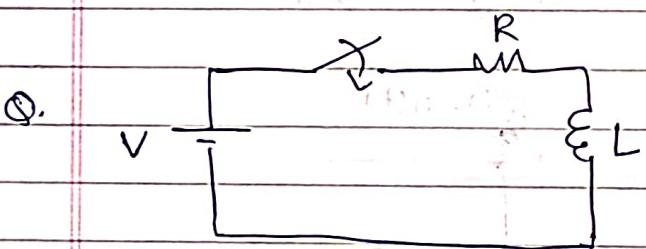
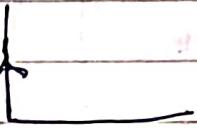


Impulse

$$s(t) = 1 \quad \text{for } t=0 \\ = 0 \quad \text{for } t \neq 0$$

$$\mathcal{L}\{s(t)\} =$$

$$\mathcal{L}\{s(t-T)\} = e^{-Ts}$$

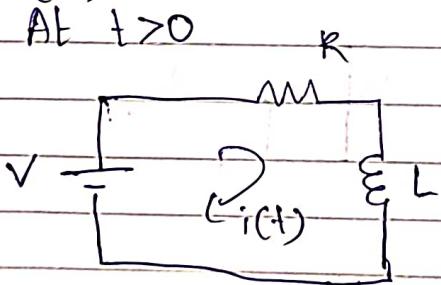


switch is closed at $t=0$. Find the eqn of i .

Ans:

$$i(0^-) = 0 = i_L$$

$$i(0^+) = 0$$

At $t > 0$ 

$$V - Ri - L \frac{di}{dt} = 0. \quad \text{where } i = i(t).$$

$V = \text{const}$ as it is dc

$$V = Ri(t) + L \frac{di(t)}{dt}$$

Taking L.T. of above eqn.

$$I_s R + L [s \cdot I_s - i(0^-)] = \frac{V}{s}$$

$$I_s R + L [s I_s - i(0^-)] = \frac{V}{s}$$

$$I_s [R + sL] = \frac{V}{s}$$

$$I_s = \frac{V}{s(R+SL)}$$

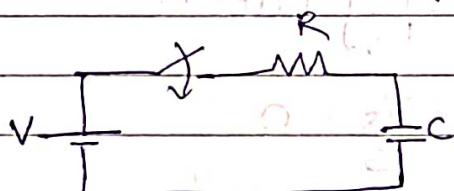
$$I_s = \frac{V/L}{s(s+R/L)}$$

$$I^{-1}[I_s] = \frac{V}{L} I^{-1}\left[\frac{1}{s(s+R/L)}\right]$$

$$= \frac{V}{L} I^{-1}\left[\frac{s+R/L - s}{s(s+R/L)}\right] \times \frac{R}{R} = \frac{R}{s+R/L}$$

$$= \frac{V}{R} I^{-1}\left[\frac{1}{s} - \frac{R}{s+R/L}\right]$$

$$= \frac{V}{R} \left[\frac{1}{s} - e^{-Rt/L} \right]$$

R-C

$$V - Ri - \frac{1}{C} \int_0^t i dt = 0$$

$$V = Ri + \frac{1}{C} \int_0^t i dt$$

Taking Laplace

$$\frac{V}{s} = I_s R + \frac{1}{C} \frac{I_s}{s}$$

$$\frac{V}{s} = I_s \left(R + \frac{1}{Cs} \right)$$

$$I_s = \frac{RCs + \frac{V}{Cs}}{Rs + (RCs + 1)s}$$

$$I_s = \frac{V_c}{RCS + 1}$$

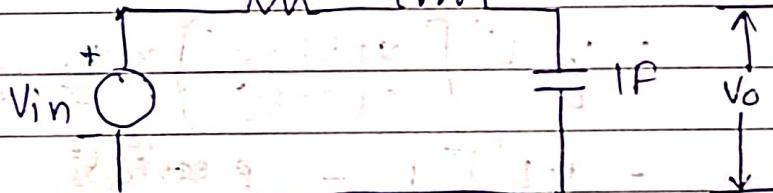
~~$$\frac{V_c}{RCS + 1} = \frac{A}{RCS + 1}$$~~

$$I_s = \frac{V_c}{R\left(\frac{1}{RC} + s\right)}$$

→ ~~ead~~ Taking Lap. Inv

~~$$i(t) = \frac{V_c}{R} e^{-\frac{t}{RC}}$$~~

* RLC Ckt



Determine the response in time domain for unit impulse input.

doln

$$V_{in} - 2i - I \cdot \frac{di}{dt} - \frac{1}{C} \int_0^t i dt = 0$$

~~$$V_{in} - 2I_s - S I_s - \frac{I_s}{S} = 0$$~~

$$I_s \left[-2 - S - \frac{1}{S} \right] = -V$$

$$I_s = \frac{V}{(-2 - S - \frac{1}{S})}$$

$$= \frac{V}{S(-2S - S^2 - 1)}$$

$$I_s = \frac{VS}{-2S - S^2 - 1}$$

$$I_s = \frac{-VS}{2S + S^2 + 1}$$

while, o/p voltage

$$V_o(s) = \frac{I_s}{s}$$

$$I(s) = S \cdot V_o(s)$$

$$S \cdot V_o(s) \left(\frac{s^2 + 2s + 1}{s} \right) = V_{in}(s)$$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

Input is unit impulse

$$V_{in}(s) = L[s(t)]^2 = 1$$

$$V_o(s) = \frac{1}{(s+1)^2}$$

$$V_o(s) = 1 = \frac{A}{(s+1)^2} + \frac{B}{(s+1)}$$

$$1 = A + B(s+1)$$

$$\text{Simplifying: } 1 = A + Bs + B$$

$$A=0 \quad A+B=1 \quad Bs=0$$

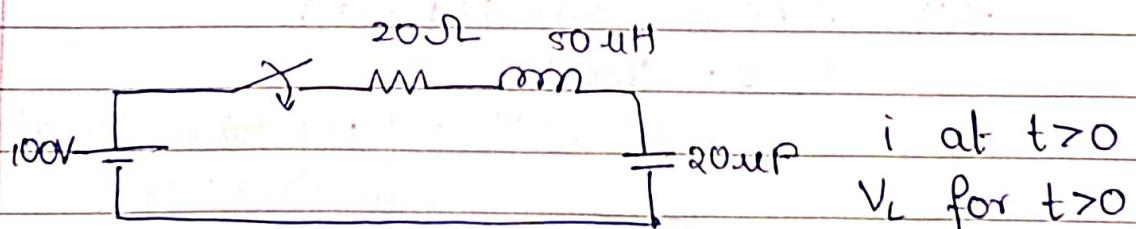
$$A=1 \quad B=0$$

Taking L. Inv.

$$V_o(t) = L^{-1} \left[\frac{1}{(s+1)^2} \right]$$

$$= t \cdot e^{-t}$$

Q.



Soln:

$$i(0^-) = 0$$

$$v(0^-) = 0$$

$$100 - 20i - 50 \times 10^{-6} \frac{di}{dt} - 20 \times 10^{-6} \int i dt = 0$$

$$\frac{100 - 20I_s - 50 \times 10^{-6} [s I_s]}{s} - \frac{1}{20 \times 10^{-6}} \times \frac{I_s}{s} = 0$$

$$S^2 + 400S + (200)^2$$

$$40000.$$

$$10^6.$$

$$e^{\int^n} \frac{1}{(S+200)^2} = e^{-t}$$

$$\frac{1}{S^2} = t$$

classmate

Date _____

Page _____

$$\frac{100}{S} = 20I_s + 50 \times 10^3 S I_s + 0.05 \times 10^6 \frac{I_s}{S}$$

$$\frac{100}{S} = I_s \left[20 + 50 \times 10^3 S - \frac{5 \times 10^4}{S} \right]$$

$$\frac{100}{S} = I_s \left[\frac{20S + 50 \times 10^3 S^2 + 5 \times 10^4}{S} \right]$$

$$I_s = \frac{100}{20S + 50 \times 10^3 S^2 + 5 \times 10^4}$$

$$I_s = \frac{100}{0.05S^2 + 20S + 5 \times 10^4}$$

$$I_s = \frac{2000}{S^2 + 400S + 10^6}$$

~~$$\frac{2000}{S^2 + 400S + 10^6} = \frac{A}{S + 200} + \frac{B}{S + 200}$$~~

~~$$As + B = 2000$$~~

~~$$B = 2000 \Rightarrow A = 0.$$~~

$$I_s = \frac{2000}{S^2 + 400S + 10^6}$$

$$I_s = \frac{2000}{(S+200)^2 + (979.79)^2}$$

$$I_s = \frac{2000}{(S+200)^2} + \frac{2000}{979.79}$$

$$i(t) = 2.04 e^{200t} \sin(979.79t)$$

$$V_L(s) = I(s) L s$$

$$\approx 100 e^{200t} \cos(979.79t) - 20.41 e^{200t} \sin(979.79t)$$



Cap. is charged to 1 V initially.
Find $i(t)$

$$\text{Soln: } i(0) = 0$$

$$v(0) = 0$$

$$i(0^+) = 0$$

$$v(0^+) = 1 \text{ V}$$

At $t > 0$

$$-2i - 1 \frac{di}{dt} - 2 \int i dt = 0$$

$$1 - 2I_s - sI_s - \frac{2I_s}{s} = 0$$

$$I_s \left[-2 - s - \frac{2}{s} \right] = 0 \Rightarrow -1$$

$$I_s = \frac{1}{2 + s + \frac{2}{s}} = \frac{s}{2s + s^2 + 2}$$

$$= \frac{s}{s^2 + 2s + 2} = \frac{s}{(s+1)^2 + 1}$$

$$I_s = s \frac{1}{\sqrt{2}}$$

$$\frac{1}{s} - 2I_s - sI_s - \frac{2I_s}{s} = 0$$

$$I_s = \left(2 + s + \frac{1}{s} \right) = \frac{1}{s}$$

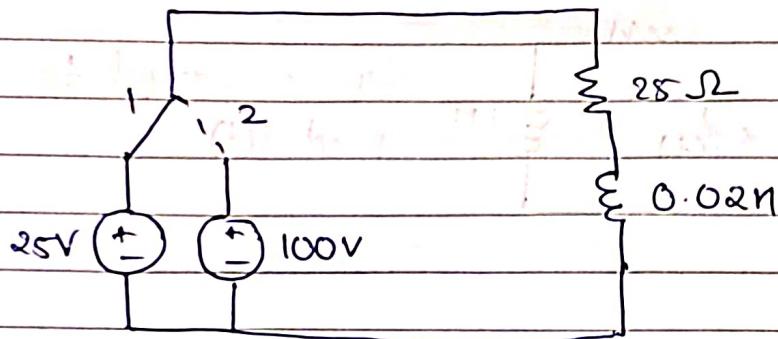
$$I_s = \left(\frac{2s + s^2 + 1}{s} \right) = \frac{1}{s(s+1)^2 + 1}$$

$$I_s = \frac{1}{2s + s^2 + 2}$$

$$I_s = \frac{1}{(s+1)^2 + 1}$$

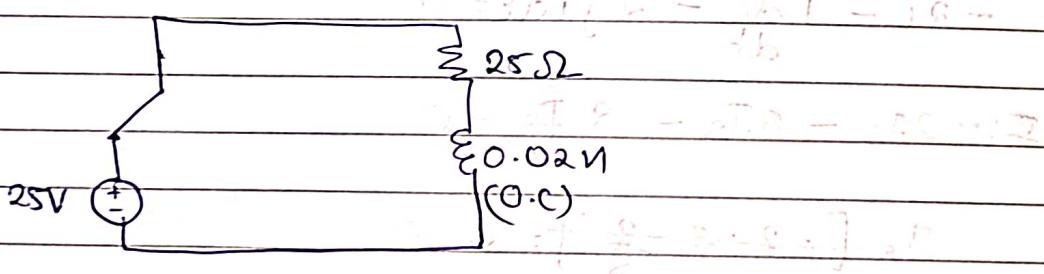
Taking laplace
 $i(t) = e^{-t} \sin t$.

Q.



In the circuit shown, the switch is in position 1 for a long time enough to establish a steady state condn. At $t=0$, the switch is thrown to position 2.

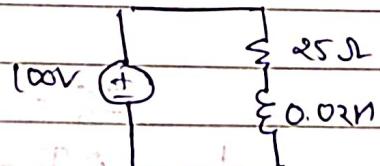
Ans: At $t=0^+$



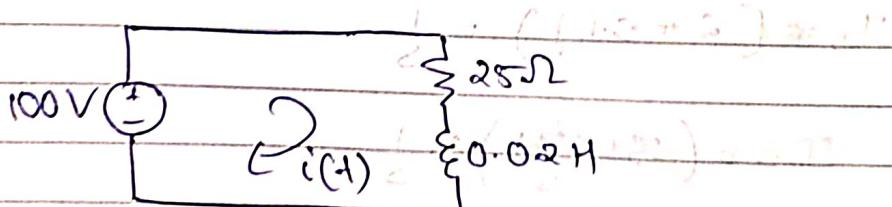
$$i = \frac{25}{25} = 1A. \quad V = 25V$$

At $t=0^+$

$$i = \frac{100}{25} = 4A \quad V = 100V$$



At $t>0$



$$100 - 25i - 0.02 \frac{di}{dt} = 0$$

$$\frac{100}{S} - \frac{25I_S}{S} - \frac{0.02sI_S}{S} = 0$$

$$I_S \cdot (25 + 0.02s) = \frac{100}{S}$$

$$I_s = \frac{100}{s(25 + 0.02s)}$$

$$I_s = \frac{100}{0.02s^2 + 25}$$

$$\cancel{100 - 25i} = 0.02 \frac{di}{dt}$$

$$\cancel{\frac{100 - 25i}{0.02}} = \frac{di}{dt}$$

$$5000 - 1250i dt = di$$

Integrating

$$5000i - \cancel{1250i^2}$$

$$0.02sI_s - 0.02i(0) + 25I(s) = \frac{100}{s}$$

$$I(s) = \frac{100 + 0.02s}{s(25 + 0.02s)}$$

$$= \frac{s + 5000}{s(s + 1250)}$$

$$= \frac{A}{s} + \frac{B}{s + 1250}$$

$$s + 5000 = A(s + 1250) + B(s)$$

$$A + B = 1 \quad 1250A = 5000$$

$$B = -3 \quad A = 4$$

$$I_s = \frac{4}{s} - \frac{3}{s + 1250}$$

Taking $i(t)$.

$$i(t) = 4 - 3e^{-1250t}$$