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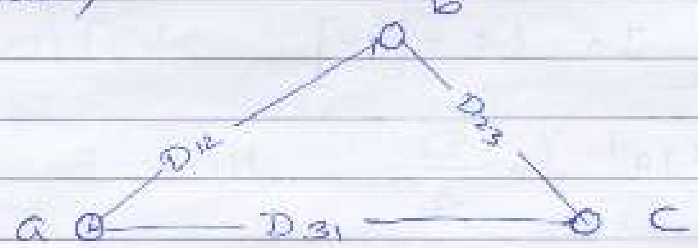
Inductance of three phase Lines.

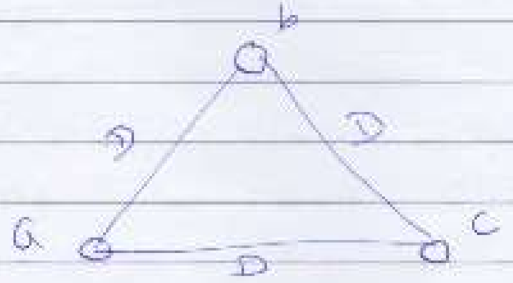
The basic eqⁿs developed for single phase lines can be easily used for the calculation of inductance of three phase lines.
Fig. below shows the conductors of three phase line with unsymmetrical spacing.
Assuming that there is no neutral wire, so that,



$I_a + I_b + I_c = 0$

Unsymmetrical spacing causes different flux linkages & hence different inductance of each phase resulting in unbalanced receiving end voltages, even when sending end voltages & line currents are balanced.

i) Equilateral spacing -



Conductors a, b, & c are spaced at the corners of an equilateral triangle having each side D, the conductors are each of radius r.

The currents in conductors are I_a, I_b & I_c such that $I_a + I_b + I_c = 0$

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Flux linkages of conductor a are due to currents I_a, I_b & I_c and can be written as,

$$\lambda_a = 2\times10^{-7}\left[I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right]$$

Since $I_b + I_c = -I_a$

$$\lambda_a = 2\times10^{-7}\left[I_a \ln \frac{1}{r'} - I_a \ln\left(\frac{1}{D}\right)\right]$$

$$\lambda_a = 2\times10^{-7}\left[I_a \ln \frac{D}{r'}\right] \quad \text{Wb-T/m.}$$

$$\therefore L_a = 2\times10^{-7} \ln \frac{D}{r'} \quad \text{H/m.}$$

$L = \frac{\lambda}{I}$

The eqn above is the same in form ~~for~~ as for single phase line.

$X = \omega L\,\Omega$

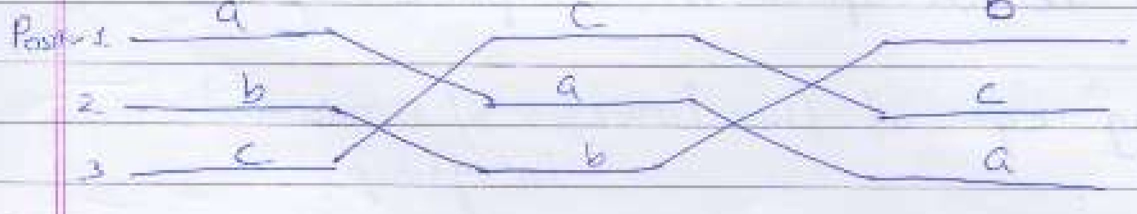
For standard conductors, D_s replaces r' in the eqn. Because of symmetry, the inductance of conductors b & c are the same as that of a. Since each phase consists of one conductor, above eqn give inductance per phase of the three phase line.

Inductance of threephase lines with Unsymmetrical spacing

When the conductors of a three-phase line are not spaced equilaterally, the flux linkages & inductance of each phase are not the same.

A different inductance in each phase results in an unbalanced circuit and in induced voltages in adjacent communication lines even when the phase current are balanced.

These undesirable characteristics can be overcome by exchanging the positions of the conductors at regular intervals along the line, so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of conductor positions is called transposition.



Transposition results in each conductor having the same average inductance over the whole cycle.

- If an untransposed telephone line parallels an untransposed power line, the flux produced by the power line induces a voltage of power-line frequency in the telephone line.
- Transposition of the power line without

transposition of the telephone line, ~~elce~~ eliminates interference of the power line with ~~the~~ the telephone line except for unbalanced load cases.

• For balanced three phase currents in a transposed power line, the magnetic field linking ~~in~~ an ~~adjust~~ adjacent telephone line is shifted 120° in time phase with each rotation of the conductor positions in the transposition cycle. Over the length of one transposition cycle of the power line, the net voltage induced in the telephone line is zero, because it is the sum of three induced voltages of equal in magnitude and displaced 120° from ~~each~~ ~~oth~~ each other.

$L = \frac{\lambda}{I}$

$X = 2\pi f L$

To find the average inductance of each conductor of a transposed line, the flux linkages of the conductor are found for each position it occupies in transposed cycle.

Applying eqn of flux linkages of conductor in group;

$$(\text{i.e., } \lambda_x = 2\times10^{-7}\left(I_1 \ln\frac{1}{D_{x1}} + I_2 \ln\frac{1}{D_{x2}} \cdots I_n \ln\frac{1}{D_{xn}}\right)$$

For section 1 of transposition cycle where a is in position 1, b in 2 & c in 3, we get,

$$\lambda_{a1} = 2\times10^{-7}\left[I_a \ln\frac{1}{r_a'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{3}}\right.$$

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for the 2nd section,

$$\lambda_{a2} = 2\times10^{-7}\left[I_a \ln\frac{1}{r_a'} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{D_{12}}\right]$$

& for the third section,

$$\lambda_{a3} = 2\times10^{-7}\left[I_a \ln\frac{1}{r_a'} + I_b \ln\frac{1}{D_{13}} + I_c \ln\frac{1}{D_{23}}\right]$$

Average flux linkages of conductor a are,

$$\lambda_a = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

$$= 2\times10^{-7}\left[I_a \frac{1}{r_a'} + I_b \ln\frac{1}{(D_{12}\cdot D_{23}\cdot D_{31})^{1/3}} + I_c \ln\frac{1}{(D_{12}\, D_{23}\, D_{31})^{1/3}}\right]$$

but $I_b + I_c = -I_a$, hence,

$$\lambda_a = 2\times10^{-7}\, I_a \ln\frac{(D_{12}\cdot D_{23}\cdot D_{31})^{1/3}}{r_a'}$$

Let $D_{eq} = (D_{12}\, D_{23}\, D_{31})^{1/3}$ = equivalent equilateral spacing.

Then, $L_a = 2\times10^{-7} \ln\dfrac{D_{eq}}{r_a'}$

This is the same relation as, $L_A = 2\times10^{-7} \ln\dfrac{D_m}{D_{sA}}$

Where, $D_m = D_{eq}$, the mutual GMD betn the three ph. ~~conductor~~ conductors.

If $r_a = r_b = r_c$ we have,

$L_a = L_b = L_c$

Presently, it is not the practice to transpose the power lines at regular intervals. However, an interchange in the position of conductors is made at switching stations to balance the inductance of the phases.

If the spacing is equilateral,

then, $D_{eq} = D$.

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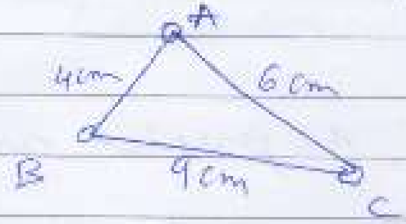
Problems

1. A three ph. T.L. has conductor dia. of 1.8cm each, Conductors being spaced as shown below. The loads are balanced & the line is transposed. Find inductance of line per ph. per km.



$$D_{eq} = \sqrt[3]{4 \times 6 \times 9} = 6$$

$$r = \cancel{1.8cm}\ \frac{1.8cm}{2}$$

$$r' = 0.7788 \times \frac{1.8}{2} = 0.70092$$

$$\therefore L \text{ per ph.} = 2 \times 10^{-4} \ln \frac{D_{eq}}{r'} \quad \frac{6}{0.70092}$$

$$= \cancel{1.3..mH/ph./km.}$$

4.29×10^{-4} H/ph/km

2. A three phase, 80 km long T.L. has its conductors of 1 cm dia, spaced at the corners of the equilateral Δ of 100 cm sides. Find inductance per ph. for the entire sys.

$$r = \frac{1}{2} = 0.5 \text{ cm}$$

$$r' = 0.7788 \times 0.5$$

$$D = 100 \text{ cm}$$

$$L = 2 \times 10^{-7} \ln \frac{D}{r'} \quad H/m$$

$$\therefore L \text{ for entire sys.} = 2 \times 10^{-7} \times 80 \times 1000 \times \ln \frac{D}{r'}$$

$$= 0.08275 \text{ H.}$$

3. Calculate the ~~loops~~ loop inductance per km of a single phase T.L. comprising of two parallel conductors 1 m apart & 1.25 cm in dia. Also calculate reactance of the T.L. if the freq. is 50 Hz.

For single phase line, 2 parallel cond's,

$$L = 4 \times 10^{-7} \ln \frac{D}{r'} \quad H/m$$

$$= 4 \times 10^{-4} \ln \frac{D}{r'} \quad H/km$$

$$r = \frac{1.25}{2}$$

$$\therefore r' = 0.7788 \times \frac{1.25}{2} = 0.4875 \text{ cm}$$

$$D = 1 \text{ m} = 100 \text{ cm}$$

$$\therefore L = 4 \times 10^{-4} \ln \frac{100}{0.4875}$$

$$= 2.219 \times 10^{-3} \quad H/km$$

$$\therefore X_L = 2\pi f L$$

$$= 2 \times \pi \times 50 \times 2.219 \times 10^{-3}$$

$$= 0.6689 \ \Omega/km$$

Resistance of T.L.

The resistance of T.L. conductors is the most imp. cause of power loss in a T.L.

The effective a.c. resistance of the conductor is,

$$R = \frac{\text{Power loss in conductor}}{I^2} \ \Omega \quad \text{——} (1)$$

where I is rms current in amp. & power is in watts.

Ohmic DC resistance is given by,

$$R_0 = \frac{\rho l}{A} \quad \text{——} (2)$$

ρ = resistivity of cond.
l = length
A = c-s area.

The effective resistance given in eqn (1) is equal to the DC resis. of the cond. given by eqn (2) only if the current distribution is uniform throughout the cond.

For small changes in temp. the resis increases ~~is~~ with temperature in accordance with the relation,

$$R_t = R_0(1 + \alpha_0 t) \quad \text{——} (3)$$

Where, R_0 → resistance at temp. 0°C
α_0 → temp. coeff. at 0°C

Eqn (3) can be used to find the resis. R_{t2} at a temp t_2 if resis R_{t1} at temp t_1 is known.

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$$\therefore \frac{R_{t2}}{R_{t1}} = \frac{1/\alpha_0 + t_2}{1/\alpha_0 + t_1}$$

Skin effect & Proximity effect.

Uniform distribution of current throughout the C.S. of a conductor exists only for Direct current. When A.C. flowing through the conductor, the current is non-uniformly distributed over the C.S. in a manner that current density is higher at the surface of the conductor compared to the current density at its centre. This effect ~~a~~ becomes more pronounced as frequency increases. This phenomenon is called skin effect.

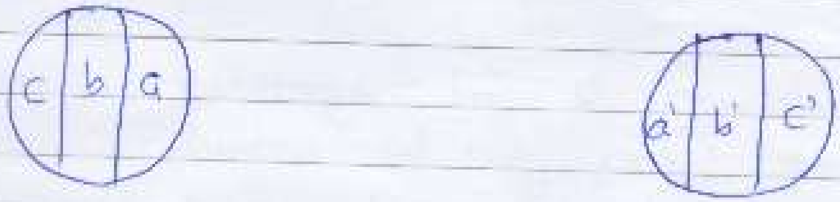
It causes larger power loss ~~at~~ for a given rms value of alternating current than the loss when same value of D.C. is flowing through the conductor. Consequently, the effective conductor resis. is more for AC than DC.

Imagine a solid round conductor made ~~of~~ up of round filaments of equal C.S area. The flux linking with the filaments progressively decreases as we move towards the outer filaments for a simple reason that flux inside a filament does not link it; hence flux linkages of a filament ~~at~~ near the surface are less than those of filament near the interior. The inductive reactance of the imaginary filaments therefore decreases outwards with the result that the outer

Reduces effective C.S. area of cond^r & therefore effective resis. of cond^r is reduced. This increases power loss

filaments conduct more AC than filaments at interior. With the increase in frequency, the non-uniformity of inductive reactance of filaments becomes more pronounced, so also the non-uniformity of the current distribution. For large solid conductors, the skin effect is quite significant even at 50 Hz.

Apart from skin effect, non-uniformity of current distribution is also caused by proximity effect. Consider a two wire line shown below.



Each conductor is divided into sections of equal C.S. area. Pairs aa', bb', & cc' form three loops in parallel.

The flux linking loop aa' is the least & it increases somewhat for loops bb' & cc'. Thus the density of alt. current flowing through conductors is highest at the inner edges (aa') of the conductors and is least at outer edges cc'. This non uniform distribution of alt. current becomes more pronounced as the dist bet the conductors is reduced.

Like skin effect, the non-uniformity of current distribution caused by proximity effect

also increases the effective conductor resistance.
For normal spacing of overhead lines, the effect is negligible. But for underground cables, where conductors are close to each other, the proximity effect causes appreciable increase in effective conductor res's.
Both these effects depend upon the conductor size, freq., dist betn conductors & permeability of conductor material.

Capacitance of T.L.

The potential difference betn the conductors of a T.L causes the conductors to be charged in the same manner as the plates of the capacitor are charged when there is a p.d. in between it.

The capacitance betn conductors is the charge per unit of p.d. Capacitance betn parallel conductors is a constant depending on the size & spacing of conductors. For power lines less than about 100km, the effect of capacitance is negligible. For longer lines it becomes increasingly important & has to be accounted for.

An alt. voltage in T.L. causes the charge on the conductors at any point to increase & decrease with the instantaneous value of voltage betn conductors at that point. The flow of charge is a current and the current caused by the alternate charging & discharging of the line due to an alternating voltage is called the charging current of the line. Charging current flows in a T.L. even when it is open circuited.

The electric field intensity at a distance y from the axis of the conductor

Capacitance of T-Lines.

1. Capacitance of a two wire line.

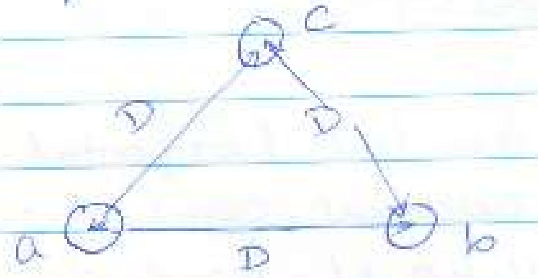
$$r_a = r_b = r.$$

Capacitance of each line to neutral is,

$$C_n = C_{an} = C_{bn} = 2C_{ab}$$

$$= \frac{0.0242}{\log(D/r)} \ \mu F/km$$

2. Capacitance of a 3 ph. line with equilateral spacing,



$$C_n = \frac{0.0242}{\log(D/r)} \ \mu F/km$$

3. Capacitance of a three ph. line with unsymmetrical spacing,

$$C_n = \frac{0.0242}{\log(D_{eq}/r)} \ \mu F/km$$

where, $D_{eq} = (D_{12} \cdot D_{23} \cdot D_{31})^{1/3}$

Problem:

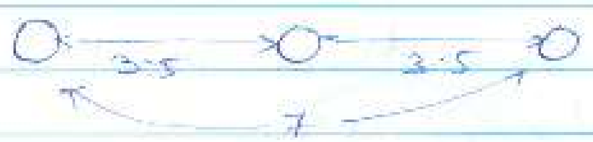
1. Calculate the capacitance to neutral/km of a single-phase line composed of two single strand conductors having radius 0.328 cm & placed 3 m apart.

$$C_n = \frac{0.0248}{\log(D/r)}$$

$$= \frac{0.0248}{\log\left(\frac{300}{0.328}\right)}$$

$$= 8.37 \times 10^{-3} \ \mu F/km$$

2. A 3 ph 50Hz T.L. has flat horizontal spacing with 3.5 m betn adjacent conductors. The conductors are made up of hard drawn copper having diameter 1.05 cm. The voltage of the line is 110kV. Find capacitance to neutral and the charging current per km of line.



$\therefore D_{12} = 3.5$

$D_{23} = 3.5$

$D_{31} = 7$

$\therefore \ D_{eq} = (3.5 \times 3.5 \times 7)^{1/3} = 4.4 \ m = 440 cm$

$r = \frac{1.05}{2} \ \cancel{\times 100} \ cm$

$C_n = \frac{0.0242}{\log(D_{eq}/r)}$

$= 0.00826 \ \mu F/km$

$X_n = \frac{1}{2\pi f C_n} = \frac{1}{2\pi 50 \times 0.00826 \times 10^{-6}}$

$= 0.384 \times 10^{6} \ \Omega/km$ to neut

Charging current $= \frac{V_n}{X_n}$

Given in line voltage converting it to V_{ph} ← $= \frac{110/\sqrt{3} \times 10^{3}}{0.384 \times 10^{6}}$

$= 0.17 \ A/km$