STATICS
Lecture Notes

By

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Chapter 1

Introduction to Engineering Mechanics

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1) Introduction to Engineering Mechanics:

Engineering Mechanics is the study of the effects that forces produce on bodies. Mechanics is that branch of physical science which deals with the state of rest or motion of bodies under the action of forces. Modern research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and molecular, atomic and subatomic behaviour are highly dependant upon the basic principle of mechanics.

Mechanics has two major subdivisions:

Statics which deals with the conditions of equilibrium of bodies acted upon by forces.
Dynamics which deals with bodies that are in motion when acted upon by forces.

Statics is one of the beginning courses in the fields of aeronautical, civil and mechanical engineering. A through understanding of its fundamental principle is a prerequisite for further study in dynamics, strength of materials, structural engineering, stress analysis and mechanical design and analysis.

The principles of dynamics has direct useful application in itself and is a prerequisite for further study in vibrations, dynamics of machinery and mechanical design and analysis. The basic principles of mechanics are relatively few in number but they have infinitely wide applications and the methods employed in mechanics carryover into many fields of engineering endeavor.

2) Basic Concepts:

Space: The region occupied by the bodies is called as space. The position of the body in the space can be defined by linear and angular measurements w.r.t. a co-ordinate system. For 3-dimensional problems the space requires 3 independents co-ordinates and for 2 dimensional problems we require 2 independent co-ordinates.

Time: Time is the measure of succession of events. It is the basic quantity in dynamics but it is not involved in statics.

Mass: Mass is a measure of inertia of the body. It is the property of every body by virtue of which each body is attracted by the other. Inertia is that property of every body by virtue of which every body resists the change in its state.

Force: Action of one body on the other which changes or tends to changes the state of the body is called as a force. It is that action exerted by one body over another which is at rest which tries to change the state of the other body. This is called as external force. The effect of the external force on the body at rest is to produce internal reactions and deformations. (sometimes the deformations being too small to cause any change in the geometrical dimensions of the body.)

There are many kinds of forces such as
1) gravity forces exerted by our earth on the bodies in the world
2) simple push or pull that we can exert upon a body with our hands.
3) Gravitational attraction between the sun and the planets.
4) Tractive force of a locomotive or an automobile
5) The force of magnetic attraction
6) Steam or gas pressure in a cylinder.
7) Wind pressure
8) Atmospheric pressure
9) Frictional resistance between the surfaces in contact

Force is a vector quantity. There are four characteristics of a force:

1) Magnitude of a force: It represents the numerical value of fix force.

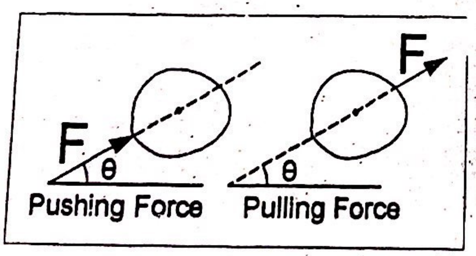
2) Unit of a force: In SI system of units, the unit of a force is 'N' (i.e. Newton)

1N(force) = 1 kg (mass) x 1 m/s^2 (acceleration)

Thus one Newton is the force required to produce an acceleration of 1 m/s^2 on a mass of 1 kg.

3)The direction and sense of a force:

The direction of a force is represented by the angle made by the line of action of the force with horizontal. The arrowhead represents the sense of the force.



4) Point of application:

The point of application of a force acting upon a body is that point in the body at which the force can be assumed to be concentrated. Physically it will be impossible to concentrate a force at a single point. There are some forces which can be explained completely without the point of application. These forces are called as **free vectors**. But there are some forces which can not be explained completely without their point of application. Hence point of application is must for them. These forces are called as **localized vectors.**

Graphical Representation of force:

Force is a vector quantity. It can be represented graphically by drawing an arrow. The length of the arrow is in proportion with the magnitude of the force. The direction of the force is represented by the inclination of the arrow line with the horizontal. The sense of the force is represented by the arrow head. For graphical representation of any force system we require to select a suitable scale for the forces as well as for the dimensions of the body under consideration.

3) Idealization of bodies in Engineering Mechanics:

1) **Particle Body:**
There are some situations in engineering Mechanics in which we do not consider the dimensions of the body under consideration. We consider the mass of the body and the forces acting on that body can be considered as a particle body. Particle is a dimensionless mass. The forces acting on a particle are always central forces. Due to this a particle body is subjected to only motion of translation. A particle can not rotate.

2) **Rigid Body:**
There are some situations in Engineering Mechanics in which we do consider the dimensions of the body in addition to its mass and the forces acting on it. In these situations the body is said to be a rigid body. Thus, the rigid body is the one which is having mass as well as dimensions and it is considered as a non deformable body. The bodies with which we deal with in engineering design of structures and machine parts are never absolutely rigid but deform slightly under the action of loads which thy have to carry. The forces acting on a rigid body can be central or eccentric also. Eccentricity of a force gives rise to rotation. Due to the this the rigid bodies are subjected to motion of translation as well as rotation.

4) Basic Units:

Mechanics deals with four fundamental quantities – length, mass, force and time. The units used to measure these quantities cannot all be chosen independently because they must be consistent with Newton's second law of motion. The international System of Units (SI) is termed as an absolute system. Since the measurement of the base quantity mass is independent of its environment (i.e. position w.r.t. sea level, longitude and latitude etc.)

Quantity	Dimensional symbol	Unit in SI system	Symbol
Mass	M	Kilogram	Kg
Length	L	Meter	M
Time	T	Second	T
Force	F	Newton	N

Scalars and Vectors:

In engineering mechanics we come across two types of quantities – scalars and vectors. Quantities which can be explained completely by only magnitude and unit are called as scalars. For example time, volume, density, speed, energy and mass etc. Quantities which can be explained completely by magnitude unit and direction and sense are called as vector quantities. For example force, moment, momentum, displacement, velocity and acceleration etc. Vectors can be classified into three categories.

Free Vectors:

Those vectors whose action is not confined to a unique line in space is called as free vectors.

Sliding vectors:

Those vectors whose action is confined to a unique line in space along which the quantity acts are called as sliding vectors.

Fixed vectors:

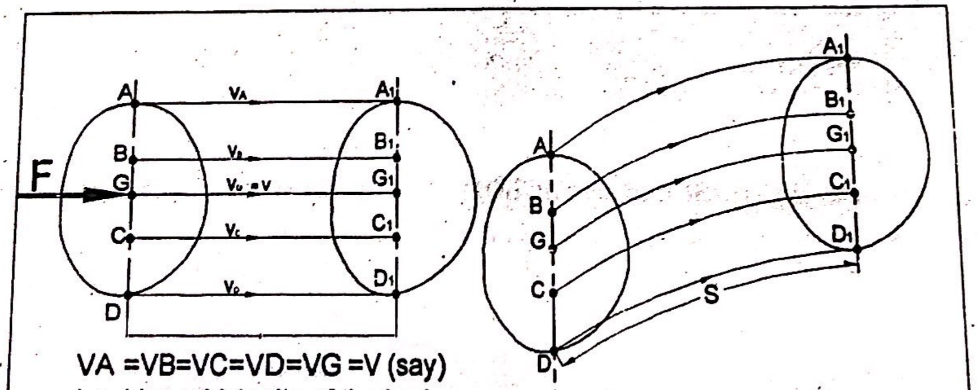
Those vectors whose action is confined to a unique point of application in space re called as fixed vectors or localized vectors.

Effect of forces on bodies:

A) External Effects :

1) Motion of Translation:

Under the action of external forces the body gets translated from one position to the other. The position co-ordinates of the body changes in this case. The paths of various particles of the body are parallel to each other. The magnitudes of linear velocities of all the particles are equal. Thus there is a common linear velocity to the body. If the body is translating along a rectilinear path, it is called as rectilinear translation. If the body is translating along a curvilinear path, it is called as curvilinear translation (plane curve or space curve).



VA =VB=VC=VD=VG =V (say)
i.e. Linear Velocity of the body
(Rectilinear Translation)

(Curvillinear Translation)

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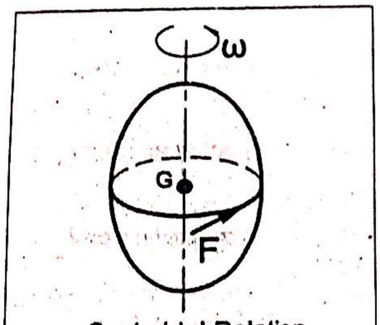
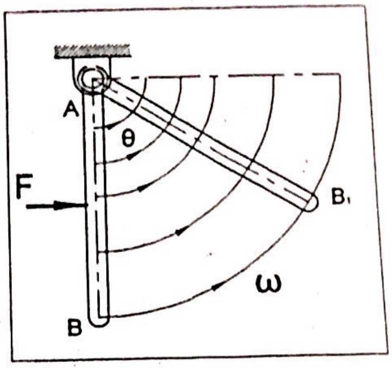
2) Motion of Rotation:

Sometimes due to external forces, instead of changing the position the body starts rotating at its position about an axis passing through the body. The paths of the particles of the body are arcs of concentric circles. There is a common angular velocity to the body. If the axis of rotation is passing through the body it is called as centroidal rotation otherwise it is called as non-centroidal rotation. Since, the axis of rotation is fixed in both the cases it is called as 'rotation about a fixed axis'.

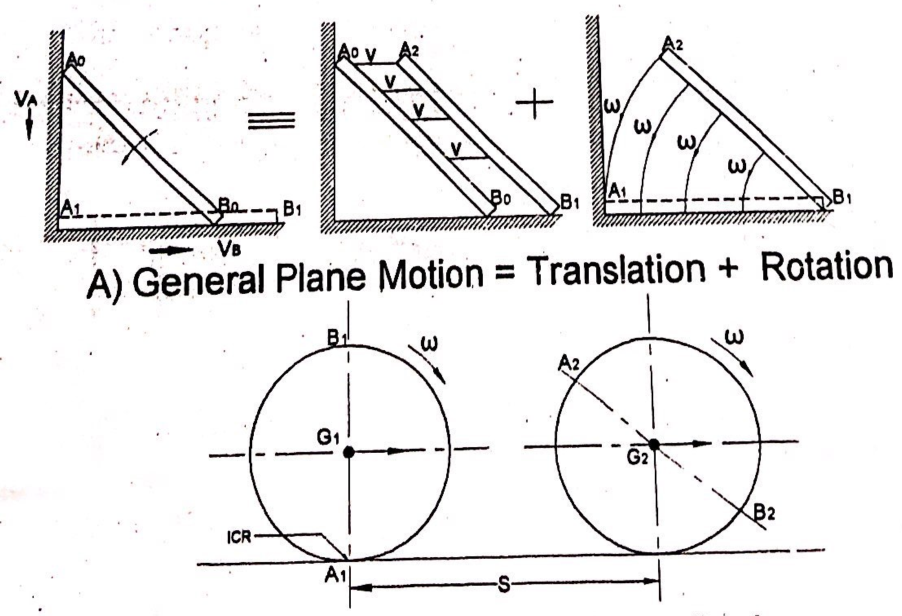


Centroidal Rotation



3) General Plane motion:

Under the action of external forces, sometimes the body is subjected to a motion which can be analyzed as combination of translation and rotation. This is called as general plane motion. In this motion the particles of the body move in one plane during the entire motion.
For example, i) a ladder sliding against a vertical wall
ii) Rolling of a wheel without sleeping



A) General Plane Motion = Translation + Rotation

B) Rolling of a wheel without slipping

(V= Linear velocity of the centre & ω = Angular Velocity of the wheel)

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4) Equilibrium:

Under the action of external forces, sometimes, the body is neither subjected to translation nor subjected to rotation, but it continues to be in the state of rest or the state of uniform rectilinear motion. This is achieved by offering equal and opposite balancing reactions by the body. Thus to develop the balancing relations under the action of external forces is also the property of the bodies in some situations. This is called as equilibrium.

Study of equilibrium is called as statics.

Study of bodies in motion or the study of bodies in motion or the study of the motion of the body is called as Dynamics.

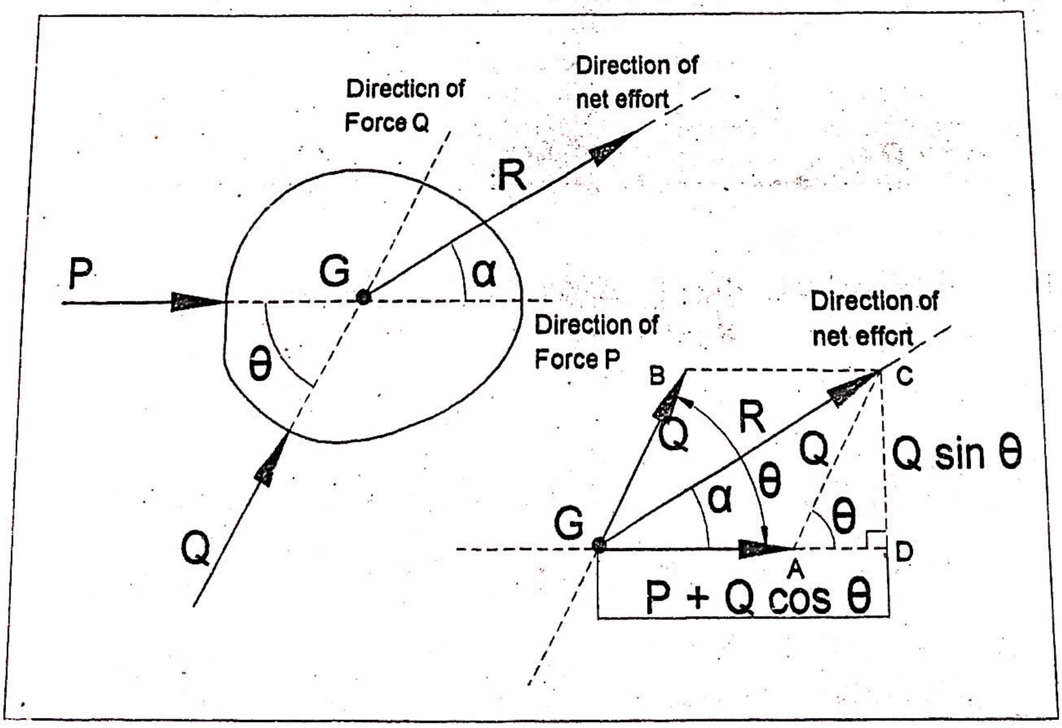
B) Internal Effects:

Sometimes, under the action of external forces, even though the body is in equilibrium, it changes its shape and size. This is called as deformation. If the load is within the elastic limits, this deformation gets cancelled on removal of the loads. The deformation due to external forces is called as strain. Resistance offered by the body to the deformation is called as stress. The study of relationship between stresses and strains is called as mechanics of deformable bodies or strength of materials.

Axioms in Engineering Mechanics:

1) The law of parallelogram of forces:

If a body is simultaneously acted upon by two forces 'P' and 'Q' then the net effect of these forces is given by a force represented by the diagonal of a parallelogram passing through the body whose adjacent sides are the given two forces.



The net effect of the two forces 'P' and 'Q' acting simultaneously on a body is also a force, called as 'Resultant force' (R). The two forces 'P' and 'Q' are called as the components of force R,
Thus,
Assuming the statement of the parallelogram law to be true, the geometrical construction is drawn.

$In \Delta OCD,$

$OC^2 = OD^2 + DC^2$

$= (OA^2 + AD^2) + DC^2$

$= OA^2 + 2OA.OD + AD^2 + DC^2$

$= OA^2 + AC^2 + 2.P.Q.COS\theta$

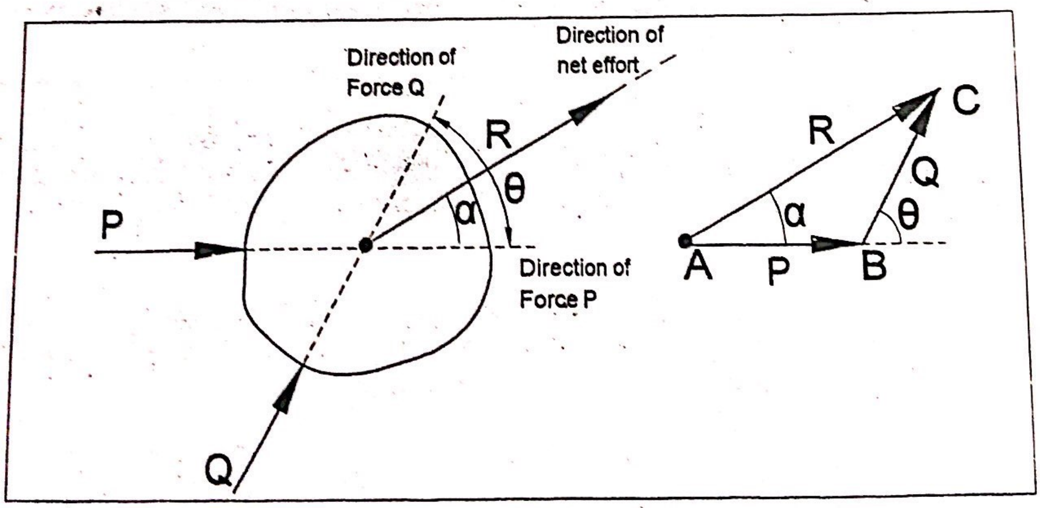
$\therefore R = \sqrt{P^2 + Q^2 + 2.P.Q.COS\theta}$ --------(1)

$\tan\alpha = \dfrac{CD}{OD} = \dfrac{CD}{OA + AD}$

$\tan\alpha = \dfrac{Q.\sin\theta}{P + Q.\cos\theta}$ ----------------(2)

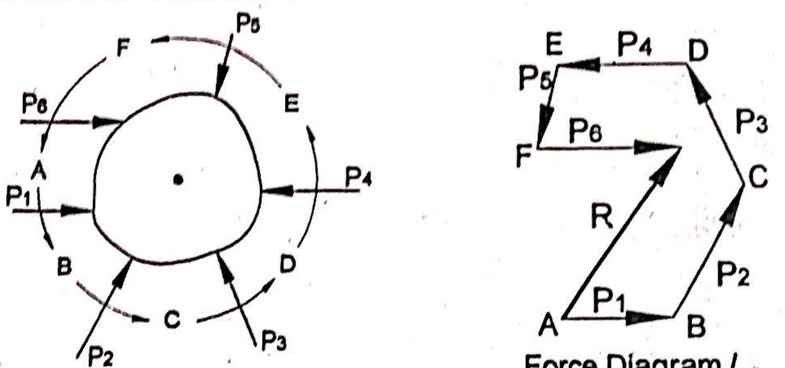
Equation (1) gives us the magnitude of the resultant force and equation (2) gives us the direction of the resultant force.

2) Triangle law of addition of forces:



This is a graphical representation of the law of parallelogram of forces. This is also called as a 'tip-tail' addition. In this case any one force is plotted graphically to some scale on paper and the second force is drawn parallel to itself at the tip of the first force. Thus at the tip of first force the tail of the second force joined graphically. Then the vector joining the tail of graphically. Then the vector joining the tail of the first force to the tip of the second force is representing the resultant of the two forces.

3) Polygon law of addition of forces:



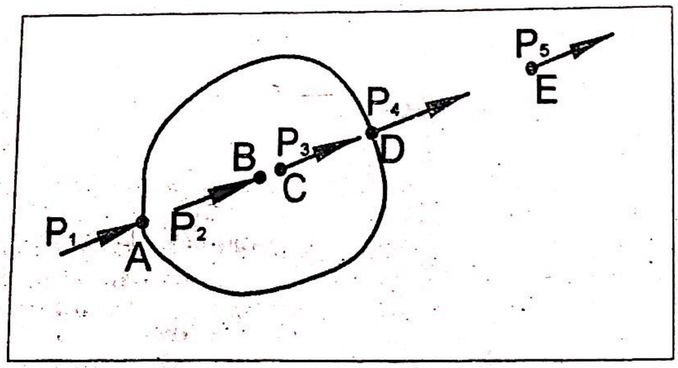
Space Diagram

Force Diagram / Bow's Notation

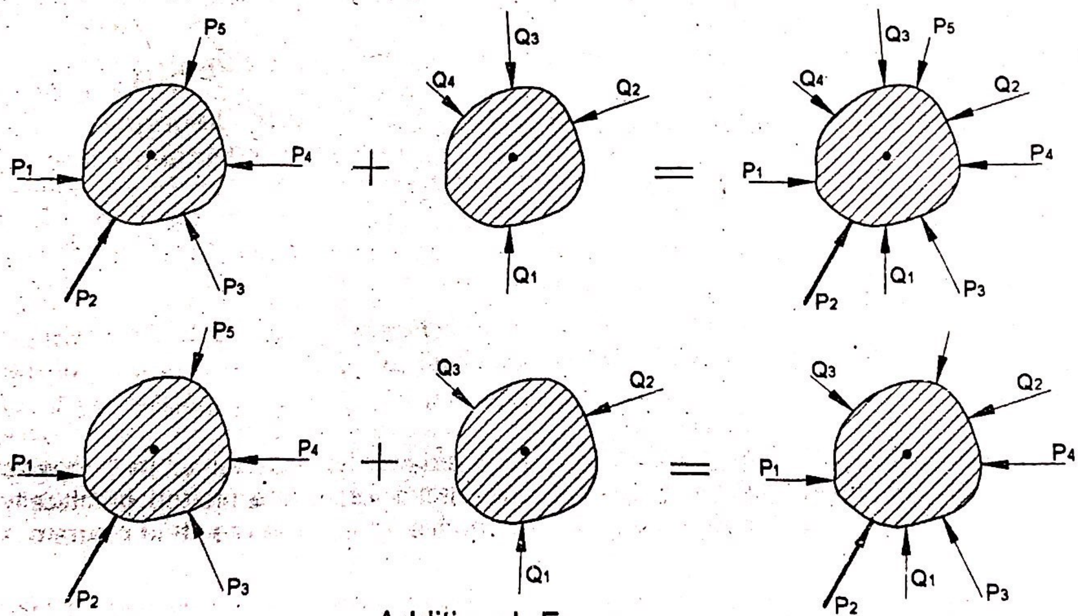
Extension of the triangle law to many coplanar forces is called as polygon law of addition of forces. In this case first we draw a neat sketch of the isolated body and the external forces acting on it at their respective points of application. The spaces between the forces are then named by using Bow's notation. For this purpose take a round around the body either in clockwise sense or in anticlockwise sense. Then all the forces are plotted graphically parallel to themselves as shown in the above figure. The resulting diagram is called as the force diagram. The force vector joining the tail of the first force to the tip of the last force is representing the resultant of the given force system acting on the body.

4) The principle of transmissibility of a force:

The effect of the force acting on the body remains unchanged even if it is translated along the same line of action on the body. In the above figure the forces P1, P2, P3 and P4 are having same magnitude, same line of action and same sense. They all are acting on the body at points A, B, C and D respectively. Because of this when they are acting independently, they produce the same effect on the body. But force P5 having same magnitude, same sense, same line of action but point of application not in contact with the body, then it will not have any effect on the body. Force P1, P2, P3 and P4 are called as 'sliding vectors'.



5) The principle of superposition of forces:



Initial Force System | Additional Force System in Equilibrium | Combined Force System

When a body acted upon by one force system is superimposed by another system of forces then their combined effect is the addition of their individual effects.

The effect of the force system acting on the body remains unchanged even if we superimpose another force system onto it which itself is in equilibrium.The above principle is used to develop the 'principle of parallel transfer of a force.'

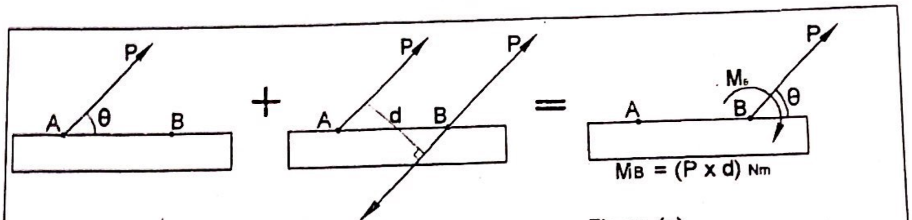


Figure (a) Figure (b) Figure (c)

In figure (a): A and B are two different points of a body. The given system consists of single force 'P' acting at point A.

In figure (b): On the force system in figure(a), we are adding another force system consisting of two equal and opposite collinear forces parallel to the force 'P' at A but acting at point B. Thus, we are adding a zero force system to the original force system.

In figure (c): We have got a force-couple system acting at B. Here, in this system the force at B is identical to the original force at A and it is accompanied by a couple of moment 'M_B' obtained by taking moment of force 'P' about point B. $M_B = (P \times d)$ Nm

Thus the three force system in the above three figures are having same effect on the body on which they are acting such force systems are called as 'equivalent systems'.

In the above example we have transferred the force 'P' acting at point A to point B on the same body without changing its original effect on the body. Thus, to transfer a force acting at one point on the body to the other point on the same body without changing its effect, it is transferred by a force-couple system.

6) Newton's Laws of Motion:

A) Newton's First Law:

A body continues to be in the state of rest or in the state of uniform rectilinear motion unless and until it is acted upon by an external unbalanced force.

B) Newton's Second Law:

The force acting on the body is directly proportional to the rate of change of linear momentum.

Where,

V = velocity of the body

a = acceleration of the body

m = mass of the body

$$\overline{F} \propto \frac{d}{dt}(m\,\overline{v})$$

In SI units,

$$\overline{F} = \frac{d}{dt}(m\,\overline{v})$$

$$\overline{F} = m \cdot \frac{d\overline{v}}{dt}$$

$$\overline{F} = m \cdot \overline{a}$$

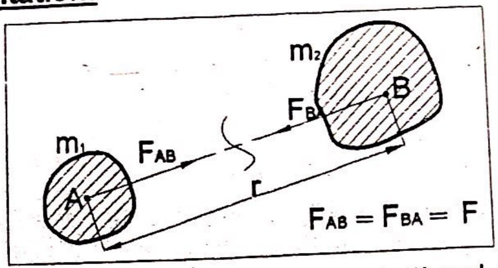
C) Newton's Third Law:

To every action, there is equal and opposite reaction.

7) Newton's Law Of Gravitation:

$$F \alpha \left(\frac{m_1.m_2}{r^2}\right)$$

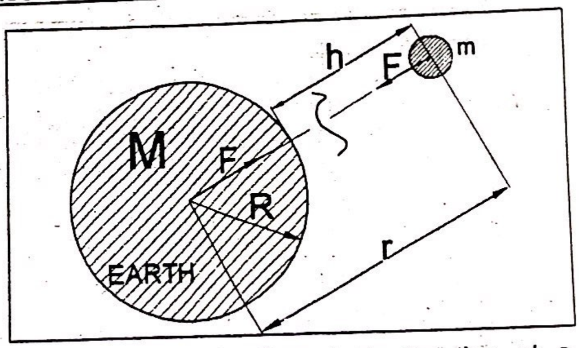
$$F = \left(\frac{G.m_1.m_2}{r^2}\right)$$



The force of attraction between two bodies of masses 'm1' and 'm2' separated by distance 'r' directly proportional to the product of their masses and inversely proportional to the square of the distance between them. 'G' is called as Universal Gravitational Constant'.

Concept of Gravitational Acceleration:

Consider a body of mass 'm' at a distance of 'h' from the surface of the earth.



Let M=mass of the earth,
R=radius of the earth

Then, $F = \frac{GMm}{(R+h)^2} = g'm$

Where $g' = \frac{GM}{(R+h)^2}$ is called as earth's gravitational acceleration at a distance of 'h' from its surface. When the body is on the surface of the earth, i.e. when h=0.

$$\text{gravitational acceleration} = g = \left(\frac{GM}{R^2}\right) = 9.81 m/s^2$$