3. Transmission Line Parameters.

A transmission line has four parameters which affect its ability to fulfill its function as a part of a p.s. These parameters are resistance, inductance, capacitance and conductance.

Inductance of T.L.

When the current flows through the conductor, magnetic field is generated around it. Variation in the current causes a change in the number of lines of magnetic flux linking in the circuit. Any change in flux linking induces a voltage in the circuit and the induced voltage is proportional to the rate of change of flux. Inductance is the property of the circuit that relates the voltage induced by changing flux to the rate of change of current.

Voltage induced in the ckt. is given by

$$e = \frac{d\psi}{dt} \text{ V} \qquad \text{—— (1)}$$

where ψ represents the flux linkages (wb-T). This can be written as,

$$e = \frac{d\psi}{di} \cdot \frac{di}{dt} = L \cdot \frac{di}{dt} \text{ V.} \qquad \text{—— (2)}$$

where $L = \frac{d\psi}{dt}$ is defined as the inductance of the circuit in henry, which in general is function of i.

- In a linear magnetic ckt, ie ckt with const. permeability, flux linkages vary linearly with current, such that inductance is constant

$$\therefore L = \frac{\psi}{i} \text{ H} \qquad \text{—— (3)}$$

$$\text{OR} \quad \psi = Li \quad \text{wb-T}$$

• If the current is alternating, the above eqn can be written as,

$$\lambda = LI \quad \text{——} (4)$$

where, $\lambda \rightarrow$ rms value of flux linkages
$I \rightarrow$ rms value of current.

replacing $\frac{d}{dt}$ in eqn (1) by $j\omega$ and

replacing ψ by LI, we get

$$V = j\omega LI = j\omega\lambda \quad V. \quad \text{——} (5)$$

we get steady state AC vltg. drop due to alternating flux linkages as eqn (5).

Similarly, mutual inductance betn two ckts. is defined as flux linkages of one ckt. due to current in other i.e,

$$M_{12} = \frac{\lambda_{12}}{I_2} \text{ H} \quad \text{——} (6)$$

∴ The voltage in ckt. 1 due to current in ckt. 2 is,

$$V_1 = j\omega M_{12} I_2 = j\omega\lambda_{12} \quad \text{——} (7)$$

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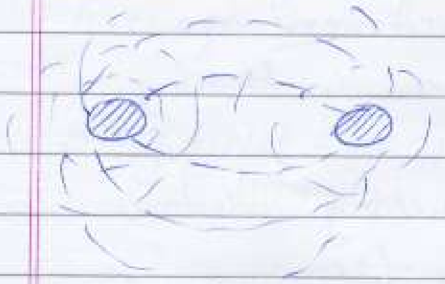
Partial flux linkages



Mag field associated with two wire line.

In this diag, only flux lines external to the conductors are shown.

Some of the magnetic field exists inside the conductors, although the amount of internal flux may be so small that it can be neglected at high frequencies. The changing lines of flux inside the conductors also contribute to the induced voltage of the ckt and therefore to the inductance. The correct value of inductance due to internal flux may be computed as the ratio of flux linkages to current, by taking into account the fact that each line of internal flux links only a fraction of total current.

The flux linkages of the internal flux in a tubular element are the product of flux in the element and the ratio of the current encircled by the tubular element to the total current in the conductor.

Thus a line of flux which encircles only half the current in a conductor contributes only half flux linkage.

Partial flux linkages are those linkages produced by flux which links only a part of the current. The total no. of flux linkages due to internal flux is the sum of of all partial linkages.

The sum of all partial flux linkages

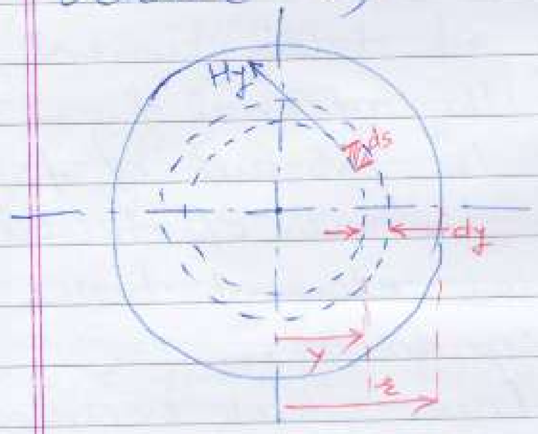
in weber-turns divided by the current in the ckt in amperes is the inductance in Henry due to internal flux.

The above principle for computing inductance is applicable to inductance resulting from external as well as internal flux.

* Inductance of a conductor due to internal flux

In order to obtain an accurate value for the inductance of transmission line, it is necessary to consider the flux inside each conductor as well as external flux.

Consider a long, cylindrical conductor shown below (c.s.)



Let's assume that the return path for the current in this conductor is so far away that it doesnot appreciably affect the magnetic field of the conductor shown. Then the lines of flux are concentric with the conductor.

Prerequisite formulae.

- Mag field strength, or intensity $H = \frac{I}{2\pi R}$ A/m. —— (1)

- Flux density, $B = \mu H$ wb/m² (T). —— (2)

~~ϕ~~ Flux $\phi = B.A.\cos\theta$ (wb) —— (3)

$B \rightarrow$ flux density

$A \rightarrow$ surface area through which flux lines are passing

$\theta \rightarrow$ angle which normal of that surface makes with flux lines.

- Flux Linkages, $\lambda = N\phi$ --- wb-turns. —— (4)

~~N~~ $N \rightarrow$ No. of turns. (or no. of times the current is linking)

- $L = \frac{\lambda}{I}$ H. —— (5)

Let → • $r \rightarrow$ radius of conductor.

- Consider a section of radius y with thickness dy.
- $ds \rightarrow$ small length in the section under consideration.

Let's first calculate magnetic field intensity ~~d~~ H_y at a small length ds.

~~$\oint H_y \cdot ds = I_x I_y$~~

Since, dy is a small section, current in this small section will not be all the current ~~in the c~~ I in the conductor. It will be proportional to the thickness dy

Let that small fraction of current is I_y

Also magnetic field strength will be a fraction of all H, ie H_y

$\therefore$ From eq^n (1),

$$H_y = \frac{I_y}{2\pi y} \quad \text{——— (6)}$$

Value of I_y is different than I.

Current is directly prop. to C.S. area.

$$\therefore \frac{I_y}{I} = \frac{\pi y^2}{\pi r^2}$$

$$\therefore I_y = \left(\frac{y^2}{r^2}\right) \cdot I \quad \text{——— (7)}$$

Substitute eq^n (7) in (6)

$$\therefore H_y = \frac{y^2}{r^2} \cdot I \cdot \frac{1}{2\pi y}$$

$$\therefore H_y = \frac{y \cdot I}{2\pi r^2} \quad \text{——— (8)} \quad AT/m$$

To calculate, B_y,

$$B_y = \mu H_y$$

$$\therefore B_y = \frac{\mu \cdot y I}{2\pi r^2} \quad Wb/m^2 \quad \text{——— (9)}$$

$$d\phi = B_y \text{(c.s area)}$$

In the tubular element of thickness dy, the flux $d\phi$ is B_y times the c.s. area of the element normal to the flux lines (area is dy times the axial length),

The flux per meter length is,

$$d\phi = \frac{\mu y I}{2\pi r^2} \cdot dy \quad \text{wb/m} \quad \text{--- (10)}$$

$\therefore$ Flux linkages, $d\lambda$ per meter of length are caused by the flux in the tubular element, are ~~$d\lambda$~~ the product of the flux per meter of length and the fraction of the current linked.

That fraction of current is,

$$\frac{\pi y^2}{\pi r^2}$$

$\therefore$ flux linkage,

$$d\lambda = \frac{\pi y^2}{\pi r^2} \cdot d\phi$$

$$= \frac{\pi y^2}{\pi r^2} \cdot \frac{\mu y I}{2\pi r^2} \cdot dy$$

$$d\lambda = \frac{\mu I y^3}{2\pi r^4} dy \quad \text{wb-T/m} \quad \text{--- (11)}$$

$\therefore$ Flux linkages for whole conductor of radius r,

$$\lambda_{int} = \int_0^r \frac{\mu I y^3}{2\pi r^4} \cdot dy$$

$$= \frac{\mu I}{2\pi r^4} \cdot \frac{r^4}{4} = \frac{\mu I}{8\pi}$$

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∴ Internal flux linkages per m. length,

$$\lambda_{int} = \frac{\mu I}{8\pi} \quad \text{—— (12)}$$

$\mu = \mu_r \cdot \mu_0$

For conductor, ie, non-magnetic material like copper, aluminium, $\mu_r = 1$

$$\therefore \lambda_{int} = \frac{\mu_0 I}{8\pi} \qquad \mu_0 = 4\pi \times 10^{-7}$$

$$= \frac{4\pi \times 10^{-7}\, I}{8\pi}$$

$$\boxed{\lambda_{int} = 0.5\, I \times 10^{-7} \quad wb\text{-}T/m} \quad \text{—— (13)}$$

$$= I/2 \times 10^{-7}$$

∴ Internal inductance,

$$L_{int} = \frac{\lambda_{int}}{I}$$

$$\therefore \boxed{L_{int} = \frac{1}{2} \times 10^{-7} \quad henry/m} \quad \text{—— (14)}$$

∴ Internal inductance is independent of the radius of conductor & depends only on length.

$* \; B_y = \mu H_y = \frac{\mu \cdot I}{2\pi y}$

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* Flux linkages due to flux between two points external to conductor.

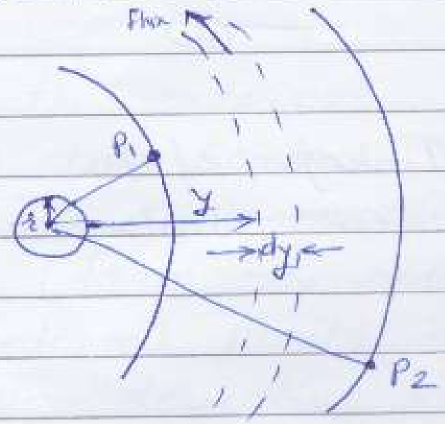

Figure shows points P_1 & P_2 at distances D_1 & D_2 from a conductor which carries a current of I amp.

- Assume that the conductor is far away from the return current path, the magnetic field external to the conductor is concentric circles around the conductor & therefore all the flux betn P_1 & P_2 lies within the concentric cylindrical surfaces that are passing through P_1 & P_2

Magnetic field intensity at distance y from the conductor is,

$$H_y = \frac{I}{2\pi y} \quad AT/m \quad \text{——— (1)}$$

The flux $d\phi$ contained in the tubular element of thickness dy is,

$$* \quad d\phi = \frac{\mu I}{2\pi y} \cdot dy \quad wb/m. \quad \text{——— (2)}$$

The flux $d\phi$ being external to the conductor, links all the current in the conductor which together with the return conductor at infinity forms a single return, such that the flux linkages are given by,

I is for all the current
I is linking

$$d\lambda = I \times d\phi$$

$$= 1 \times \frac{\mu I}{2\pi y} dy \quad \longleftarrow (3)$$

Hence, the total flux linkages of the conductor due to flux between points P_1 & P_2 is,

$$\lambda_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi y} dy$$

$$= \frac{\mu I}{2\pi} \int_{D_1}^{D_2} \frac{1}{y} dy$$

$$= \frac{\mu I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)$$

$\because \mu_r = 1$, & $\mu_0 = 4\pi \times 10^{-7}$

$$\therefore \boxed{\lambda_{12} = 2 \times 10^{-7} I \ \ln\left(\frac{D_2}{D_1}\right) \quad Wb\text{-}T/m} \quad \longleftarrow (4)$$

Hence, the inductance of the conductor contributed by the flux betn points P_1 & P_2 is,

$$L_{12} = \frac{\lambda_{12}}{I}$$

$$= \frac{2 \times 10^{-7} \times I}{I} \cdot \ln\left(\frac{D_2}{D_1}\right)$$

$$\boxed{L_{12} = 2 \times 10^{-7} \cdot \ln\left(\frac{D_2}{D_1}\right) \quad H/m} \quad \longleftarrow (5)$$

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* Flux linkages & hence inductance due to flux upto an external point.

Let the external point be at distance D from the centre of the conductor.

Flux linkages of the conductor due to external flux (from the surface of the conductor upto the external point) is obtained from eqn (4) in earlier derivation, by substituting $D_1 = r$ and $D_2 = D$, ie,

$$\lambda_{ext} = 2\times10^{-7} \cdot I \cdot \ln\left(\frac{D}{r}\right) \quad \text{—— (6)}$$

Total flux linkages of the conductor due to internal & external flux are,

$$\lambda = \lambda_{int} + \lambda_{ext}$$

$$= \frac{I}{2}\times10^{-7} + 2\times10^{-7} \cdot I \cdot \ln\left(\frac{D}{r}\right)$$

$$= 2\times10^{-7} I\left(\frac{1}{4} + \ln\left(\frac{D}{r}\right)\right)$$

$$= 2\times10^{-7} I \ \ln\left(\frac{D}{r \cdot e^{-1/4}}\right)$$

Let $r' = r \cdot e^{-1/4} = 0.7788r$

$$\therefore \lambda = 2\times10^{-7} I \ln\left(\frac{D}{r'}\right) \text{ wb-T/m.} \quad \text{—— (7)}$$

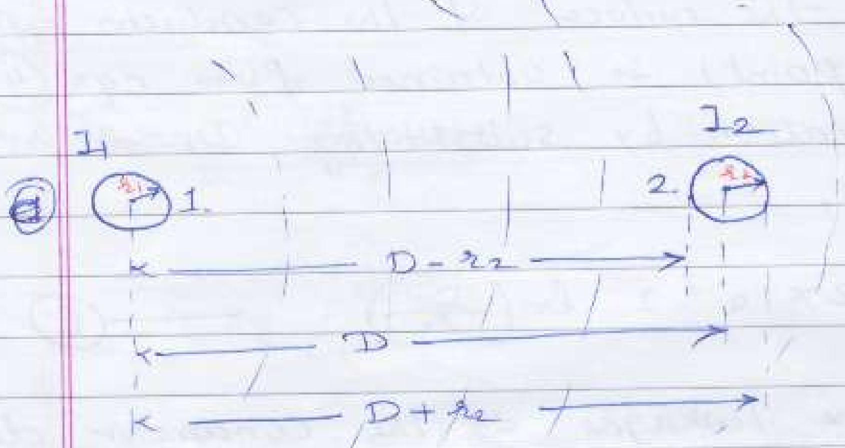
$\therefore$ Inductance of the conductor due to flux upto an external point is,

$$L = 2\times10^{-7} \ln\left(\frac{D}{r'}\right) \quad H/m$$

• Here r' can be regarded as the radius of a fictitious conductor with no internal inductance but same total inductance as the actual conductor.

Inductance of a single-phase two wire line.

Consider a simple two-wire line composed of solid round conductors carrying currents I_1 & I_2 as shown below,



In a single phase line,

$$I_1 + I_2 = 0$$

ie, $I_2 = -I_1$

Firstly, consider only the flux linkages of circuit caused by the current in conductor 1.

We consider three flux linkages –

i) External flux from r_1 to $(D-r_2)$ links all the current I_1 in conductor 1.

ii) External flux from $(D-r_2)$ to $(D+r_2)$ links a current whose magnitude progressively reduces from I_1 to zero along this distance, because of the effect of negative current flowing in conductor 2.

iii) Flux beyond $(D+r_2)$ links a net current of zero.

For calculating the total inductance due to current in conductor 1, a simple assumption is made. If D is much greater than r_1 & r_2, it can be assumed that the flux from $(D-r_2)$ to the centre of conductor 2 links all the current I_1 & the flux from the centre of conductor 2 to $(D+r_2)$ links zero current.

Based on above assumption, the flux linkages of the circuit caused by current in conductor 1 as per eq^n (7) of earlier page, are,

$$\lambda_1 = 2\times10^{-7} I_1 \ln\left(\frac{D}{r_1'}\right) \qquad (1)$$

The inductance of the conductor due to current in conductor 1 is then,

$$L_1 = 2\times10^{-7} \ln\left(\frac{D}{r_1'}\right) \qquad (2)$$

Similarly, the inductance of the circuit due to current in conductor 2 is,

$$L_2 = 2\times10^{-7} \ln\left(\frac{D}{r_2'}\right) \qquad (3)$$

Using superposition theorem, the flux linkages & hence inductances of the circuit caused by current in each conductor considered separately may be added to obtain the total circuit inductance,

$$L = L_1 + L_2 \qquad (4)$$

$$= 4\times10^{-7} \ln\left(\frac{D}{r_1' + r_2'}\right) \quad H/m$$

If $r_1' = r_2' = r'$, then,

$$L = 4 \times 10^{-7} \ln\left(\frac{D}{r'}\right) \ H/m$$

$$= 0.921 \log\left(\frac{D}{r'}\right) \ H/m.$$

Bundled Conductors.

* already studied.

Total no. of strands,

$$N = 3n^2 - 3n + 1$$

Where, n = no. of layers.

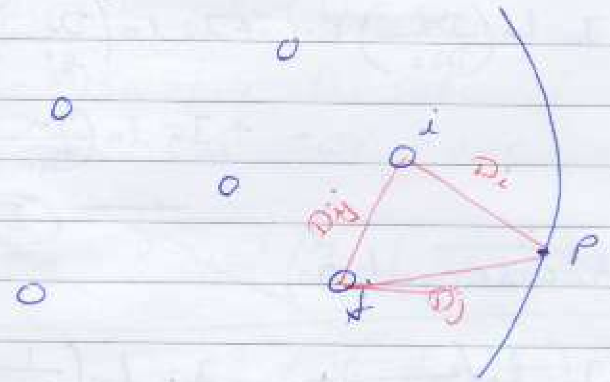
∴ Overall diameter,

$$D = (2n-1) \times d$$

where d = diameter of each strand.

Flux linkages of one conductor in group.

Consider a group of n parallel round conductors carrying currents $I_1, I_2, \ldots I_n$ whose sum is zero.



Distances of these conductors from a remote point P are indicated as $D_1, D_2, D_3 \ldots D_n$

Here we are going to ~~obj~~ derive an expression for total flux linkages of the i^{th} conductor of the group considering flux upto point P only.

- The flux linkages of i^{th} conductor due to its own current I_i (self linkages) are,

$$\lambda_{ii} = 2\times10^{-7} I_i \ln\left(\frac{D_i}{r_i'}\right) \quad \text{wb-T/m.} \quad \text{(already derived)} \quad \text{--- (1)}$$

- Similarly, flux linkages of conductor i due to current in conductor j is,

$$\lambda_{ij} = 2\times10^{-7} I_j \ln\left(\frac{D_j}{D_{ij}}\right) \quad \ldots \text{wb-T/m} \quad \text{--- (2)}$$

Where D_{ij} is distance of i^{th} conductor from j^{th} conductor carrying current I_j

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From eqⁿ (1) and by repeated use of eqⁿ (2), the total flux linkages of conductor i due to flux upto point P are,

$$\lambda_i = \lambda_{i1} + \lambda_{i2} + \lambda_{i3} + \dots \lambda_{ii} + \dots \lambda_{in}$$

$$= 2\times10^{-7}\left[\left(I_1 \ln\left(\frac{D_1}{D_{i1}}\right) + I_2 \ln\left(\frac{D_2}{D_{i2}}\right) + \dots + I_i \ln\left(\frac{D_i}{r_i'}\right) + \dots + I_n \ln\left(\frac{D_n}{D_{in}}\right)\right]$$ — (3)

above eqⁿ can be rearranged as,

$$\lambda_i = 2\times10^{-7}\left[\left(I_1 \ln\left(\frac{1}{D_{i1}}\right) + I_2 \ln\left(\frac{1}{D_{i2}}\right) + \dots I_i \ln\left(\frac{1}{r_i'}\right) + \dots I_n \ln\left(\frac{1}{D_{in}}\right)\right) + \left(I_1 \ln D_1 + I_2 \ln D_2 + \dots I_i \ln D_i + \dots + I_n \ln D_n\right)\right]$$ — (4)

But $I_n = -(I_1 + I_2 + \dots + I_{n-1})$

Substituting I_n value in 2^{nd} term of eqⁿ (4) we get,

$$\lambda_i = 2\times10^{-7}\left[\left(I_1 \left(\ln\frac{1}{D_{i1}}\right) + I_2 \ln\left(\frac{1}{D_{i2}}\right) + \dots + I_i \ln\left(\frac{1}{r_i'}\right) + \dots I_n \ln\left(\frac{1}{D_{in}}\right)\right) + \left(I_1 \ln\left(\frac{D_1}{D_n}\right) + I_2 \ln\left(\frac{D_2}{D_n}\right) + \dots I_i \ln\left(\frac{D_i}{D_n}\right) + \dots I_{n-1} \ln\left(\frac{D_{n-1}}{D_n}\right)\right)\right]$$ — (5)

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In order to account for total flux linkages of conductor i, let the point P now move to infinity.

$\therefore$ The terms such as $\ln\left(\frac{D_1}{D_n}\right)$, etc approach $\ln(1) = 0$.

Also for the sake of symmetry, denoting r_i' as D_{ii}, we have,

$$\cancel{\lambda_i = 2\times10^{-7}\left(I_1 \ln \frac{1}{D_{i1}}\right) + \left(I_2\right.}$$

$$\boxed{\lambda_i = 2\times10^{-7}\left(I_1 \ln\left(\frac{1}{D_{i1}}\right) + I_2 \ln\left(\frac{1}{D_{i2}}\right) + \dots I_i \ln\left(\frac{1}{D_{ii}}\right) + \dots I_n\left\{\ln\left(\frac{1}{D_{in}}\right)\right)\ \text{Wb-T/m.}}$$

— (6)

• Inductance of Composite Conductor Lines.

To study inductance of transmission lines composed of composite conductors.

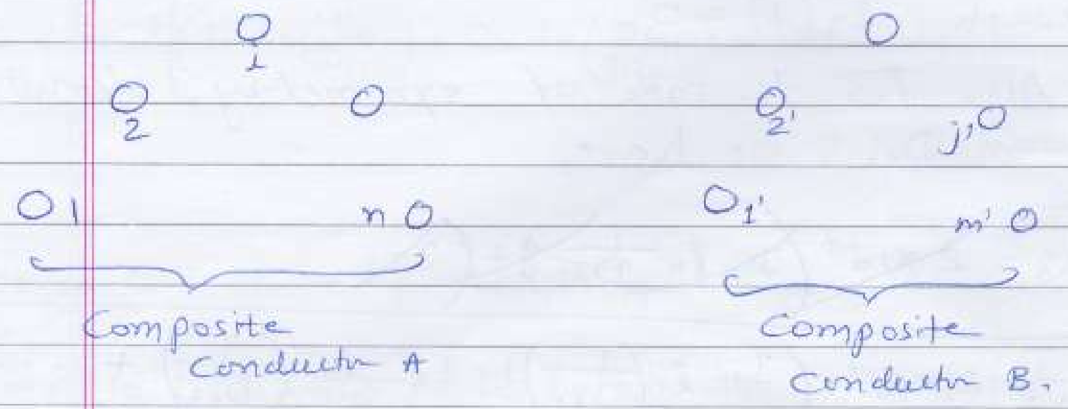

Fig. above shows a single phase line made up of composite conductors A & B having 'n' parallel filaments in A & m' parallel filaments in B.

It is assumed that current is equally divided among the filaments of each composite conductor. Therefore, each filament of A carries a current of I/n and each filament of B carries a return current of $-I/m'$.

Applying eq^n (6) to filament i of conductor A, we get the flux linkages as,

$$\lambda_i = 2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{D_{i1}} + \ln\frac{1}{D_{i2}} + \dots + \ln\frac{1}{D_{ii}} + \dots + \ln\frac{1}{D_{in}}\right) -$$

$$2\times10^{-7}\frac{I}{m'}\left(\ln\frac{1}{D_{i1'}} + \ln\frac{1}{D_{i2'}} + \dots \ln\frac{1}{D_{im'}}\right) \quad \text{--- (1)}$$

$$= 2\times10^{-7} I \ln\left(\frac{(D_{i1'} \cdot D_{i2'} \cdot \dots D_{im'})^{1/m'}}{(D_{i1} \cdot D_{i2} \cdot \dots D_{ii} \cdot \dots D_{in})^{1/n}}\right) \text{ Wb-T/m} \quad \text{--- (2)}$$

The inductance of filament ~~i~~ i is then,

$$L_i = \frac{\lambda_i}{I/n}$$

$$= 2n \times 10^{-7} \ln \frac{(D_{ii'} * D_{i2'} * \ldots D_{im'})^{1/m'}}{(D_{i1} * D_{i2} * \ldots D_{in})^{1/n}} \ H/m$$ —— (3)

The average inductance of the filaments of composite conductor A is,

$$L_{avg} = \frac{L_1 + L_2 + \ldots L_n}{n}$$ —— (4)

Since conductor A is composed of n filaments electrically in parallel, its inductance is 1/n times the avg inductance.

$$L_A = \frac{L_{avg}}{n} = \frac{L_1 + L_2 + \ldots L_n}{n^2}$$ —— (5)

Using the expression for filament inductance from eqn (3) in eqn (5), we get,

$$L_A = 2 \times 10^{-7} \ln \frac{\left[(D_{11'} * \ldots D_{1j'} * \ldots D_{1m'}) \ldots (D_{i1'} * \ldots D_{ij'} * \ldots D_{im'}) \ldots (D_{n1'} * D_{n2'} * \ldots D_{nj'} * \ldots D_{nm'})\right]^{1/mn}}{\left[(D_{11} * \ldots D_{1i} * \ldots D_{1n}) \ldots (D_{i1} * \ldots D_{ii} * \ldots D_{in}) \ldots (D_{n1} * D_{n2} * \ldots D_{ni} * \ldots D_{nn})\right]^{1/n^2}}$$

H/m —— (6)

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The numerator of ~~eqn 6~~ logarithm of eqn 6 above is the m'th root of the m'n terms, which are the products of all possible mutual distances from the n filaments of conductor A to m' filaments of conductor B. It is called mutual geometric mean distance (mutual GMD) betn conductors A & B and is abbreviated as Dm.

Similarly, the denominator of the logarithm is the n^2th root of n^2 product terms (n sets of n product terms each). Each set of n product term pertains to a filament and consists of r' (Dii) ~~for that~~ for that filament and (n-1) distances from that filament to every other filament in conductor A. The denominator is defined as self geometric mean distance (self GMD) of conductor A and is abbreviated as ~~Dsa~~ D_{SA}. Sometimes self GMD is also called geometric mean radius (GMR).

In terms of above symbols, we can write eqn 6 as,

$$L_A = 2\times 10^{-7} \ln \frac{D_m}{D_{SA}} \quad H/m \qquad \text{—— (7)}$$

$$= 0.461 \log \frac{D_m}{D_{SA}} \quad H/m$$

The similarity of eqn 7 with eqn of inductance of the conductor due to current in conductor 1 only $\left(L_1 = 2\times 10^{-7} \ln \frac{D}{r'}\right)$ -- already derived gives the inductance of one conductor of a single phase line for the special case of two solid, round conductors. Where, r' is the self GMD of the single conductor and D is the mutual GMD of two single conductors.

The inductance of the composite conductor B is determined in a similar manner, and the total inductance of the line is,

$$L = L_A + L_B.$$