Chapter (8)

Three Phase Induction Motors

Introduction

The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. In this chapter, we shall focus our attention on the general principles of 3-phase induction motors.

8.1 Three-Phase Induction Motor

Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a "transformer-type" a.c. machine in which electrical energy is converted into mechanical energy.

Advantages

(i) It has simple and rugged construction.
(ii) It is relatively cheap.
(iii) It requires little maintenance.
(iv) It has high efficiency and reasonably good power factor.
(v) It has self starting torque.

Disadvantages

(i) It is essentially a constant speed motor and its speed cannot be changed easily.
(ii) Its starting torque is inferior to d.c. shunt motor.

8.2 Construction

A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

1. Stator

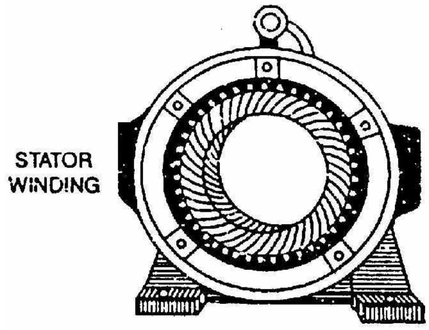

Fig.(8.1)

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations [See Fig. (8.1)]. The insulated connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field (See Sec. 8.3) of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

2. Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

(i) Squirrel cage type (ii) Wound type

(i) **Squirrel cage rotor**. It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. (8.2)]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.

Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.

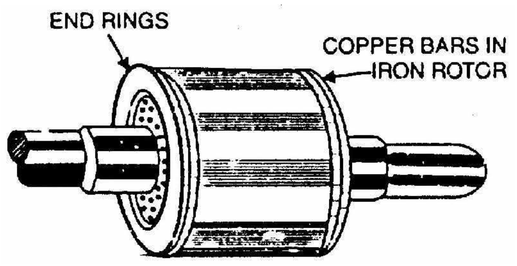

Fig.(8.2)

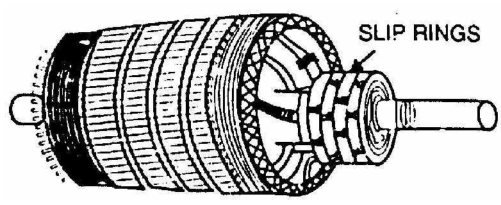

Fig.(8.3)

(ii) **Wound rotor**. It consists of a laminated cylindrical core and carries a 3-phase winding, similar to the one on the stator [See Fig. (8.3)]. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig. (8.4). At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed.

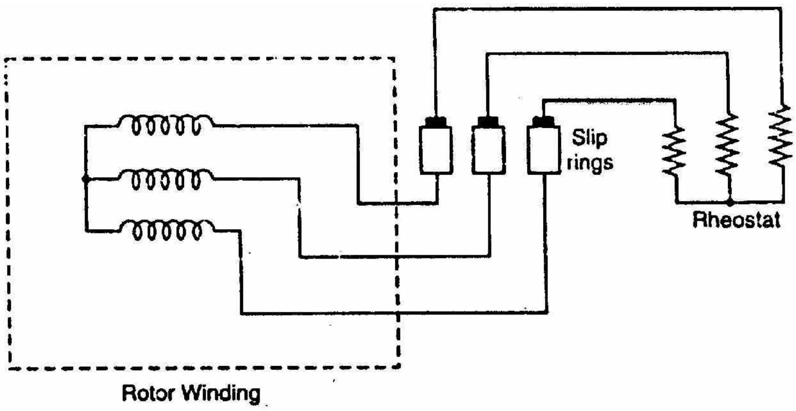

Fig.(8.4)

The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

8.3 Rotating Magnetic Field Due to 3-Phase Currents

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 ϕ_m where ϕ_m is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3i-phase winding as shown in Fig. (8.6 (i)). The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as I_x, I_y and I_z [See Fig. (8.6 (ii))]. Referring to Fig. (8.6 (ii)), the fluxes produced by these currents are given by:

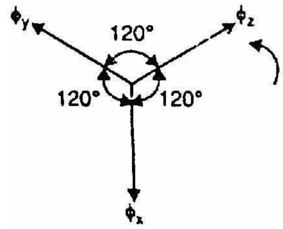

Fig.(8.5)

$$\phi_x = \phi_m \sin \omega t$$

$$\phi_y = \phi_m \sin (\omega t - 120°)$$

$$\phi_z = \phi_m \sin (\omega t - 240°)$$

Here ϕ_m is the maximum flux due to any phase. Fig. (8.5) shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5 ϕm.

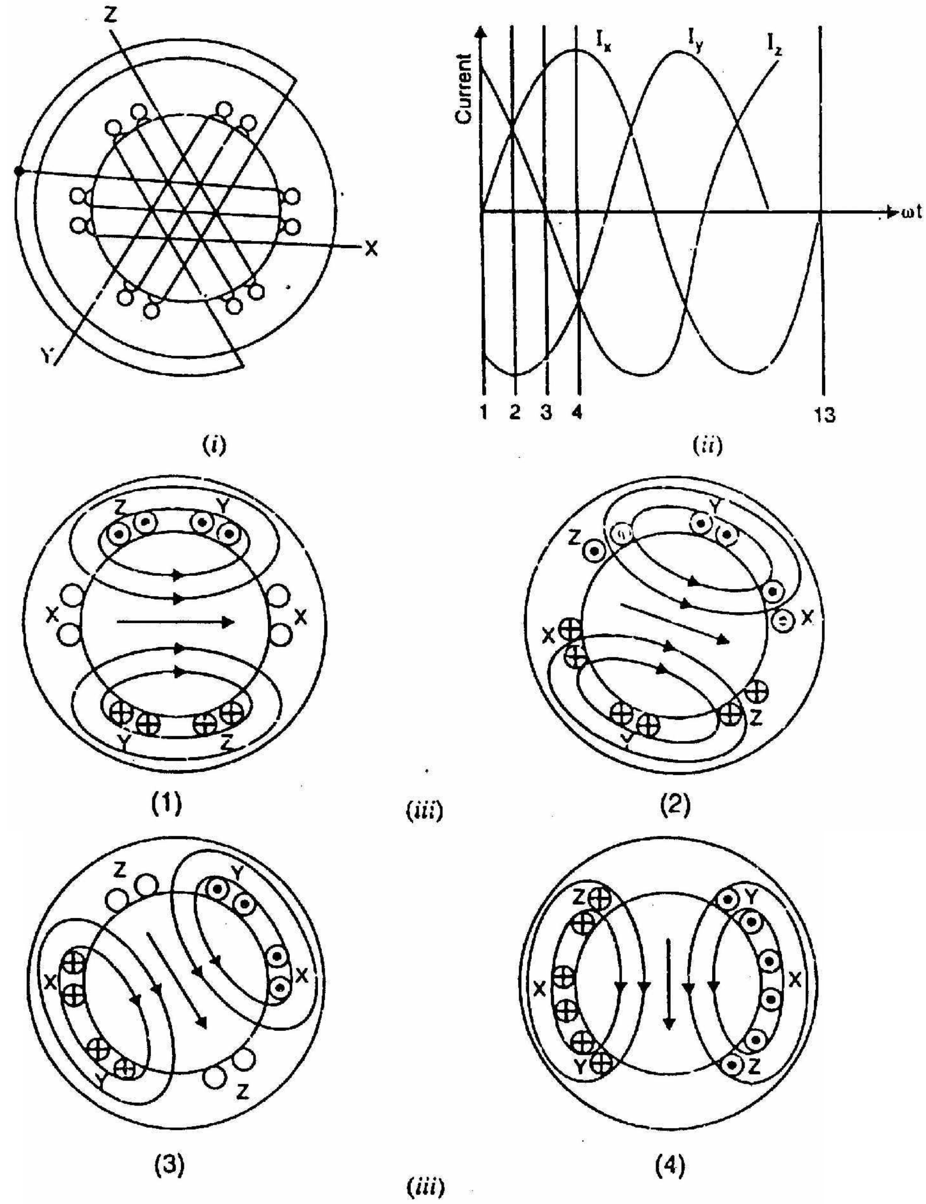

Fig.(8.6)

(i) At instant 1 [See Fig. (8.6 (ii)) and Fig. (8.6 (iii))], the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to 1.5 ϕ_m as proved under:

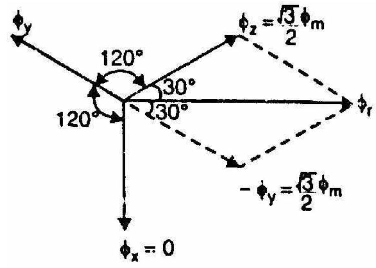

Fig.(8.7)

At instant 1, ωt = 0°. Therefore, the three fluxes are given by;

$$\phi_x = 0; \qquad \phi_y = \phi_m \sin(-120°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-240°) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of $-\phi_y$ and ϕ_z is the resultant flux ϕ_r [See Fig. (8.7)]. It is clear that:

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5\ \phi_m$$

(ii) At instant 2, the current is maximum (negative) in ϕ_y phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5 ϕ_m as proved under:

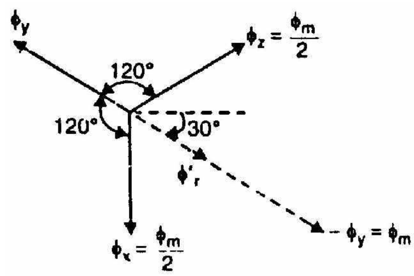

Fig.(8.8)

At instant 2, ωt = 30°. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 30° = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90°) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210°) = \frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

Phasor sum of ϕ_x and ϕ_z, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and $-\phi_y$, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

Note that resultant flux is displaced 30° clockwise from position 1.

(iii) At instant 3, current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y arc 0.866 × max. value). The magnitude of resultant flux is 1.5 ϕ_m as proved under:

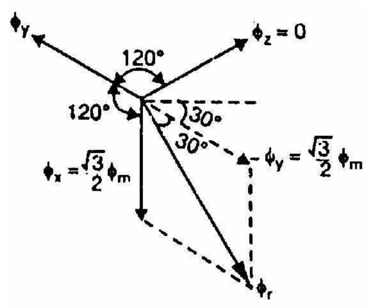

Fig.(8.9)

At instant 3, ωt = 60°. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 60° = \frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_y = \phi_m \sin(-60°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-180°) = 0$$

The resultant flux ϕ_r is the phasor sum of ϕ_x and $-\phi_y$ $(\because \ \phi_z = 0)$.

$$\phi_r = 2\times\frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 1.5\ \phi_m$$

Note that resultant flux is displaced 60° clockwise from position 1.

(iv) At instant 4, the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are 0.5 × max. value). This establishes a resultant flux downward as shown under:

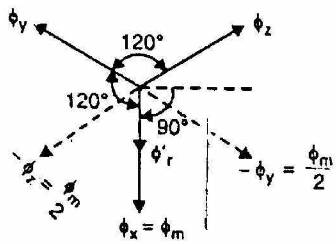

Fig.(7.10)

At instant 4, ωt = 90°. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90° = \phi_m$$

$$\phi_y = \phi_m \sin(-30°) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150°) = -\frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and $-\phi_z$ is the resultant flux ϕ_r

Phasor sum of $-\phi_z$ and $-\phi_y$, $\phi'_r = 2\times\frac{\phi_m}{2}\cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and ϕ_x, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= 1.5 ϕ_m, where ϕ_m is the maximum flux due to any phase).

Speed of rotating magnetic field

The speed at which the rotating magnetic field revolves is called the synchronous speed (N_s). Referring to Fig. (8.6 (ii)), the time instant 4 represents the completion of one-quarter cycle of alternating current I_x from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 or one complete cycle of current I_x from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

$$\therefore \qquad \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

or $$\text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}$$

Since revolutions per second is equal to the revolutions per minute (N_s) divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \qquad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

or $$N_s = \frac{120\,f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

Direction of rotating magnetic field

The phase sequence of the three-phase voltage applied to the stator winding in Fig. (8.6 (ii)) is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counterclockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

8.4 Alternate Mathematical Analysis for Rotating Magnetic Field

We shall now use another useful method to find the magnitude and speed of the resultant flux due to three-phase currents. The three-phase sinusoidal currents produce fluxes ϕ_1, ϕ_2 and ϕ_3 which vary sinusoidally. The resultant flux at any instant will be the vector sum of all the three at that instant. The fluxes are represented by three variable magnitude vectors [See Fig. (8.11)]. In Fig. (8.11), the individual flux directions arc fixed but their magnitudes vary sinusoidally as does the current that produces them. To find the magnitude of the resultant flux, resolve each flux into horizontal and vertical components and then find their vector sum.

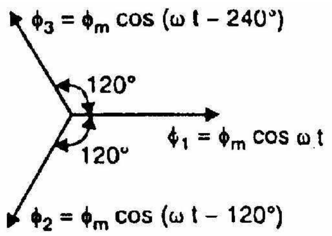

Fig.(8.11)

$$\phi_h = \phi_m \cos t\ \omega t - \phi_m \cos(\omega t - 120°)\cos 60° - \phi_m \cos(\omega t - 240°)\cos 60°$$
$$= \frac{3}{2}\phi_m \cos\omega t$$

$$\phi_v = 0 - \phi_m \cos(\omega t - 120°)\cos 60° + \phi_m \cos(\omega t - 240°)\cos 60° = \frac{3}{2}\phi_m \sin\omega t$$

The resultant flux is given by;

$$\phi_r = \sqrt{\phi_h^2 + \phi_v^2} = \frac{3}{2}\phi_m\left[\cos^2 \omega t + (-\sin\ \omega t)^2\right]^{1/2} = \frac{3}{2}\phi_m = 1.5\phi_m = \text{Constant}$$

Thus the resultant flux has constant magnitude (= 1.5 ϕ_m) and does not change with time. The angular displacement of ϕ_r relative to the OX axis is

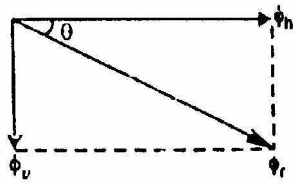

Fig.(8.12)

$$\tan\theta = \frac{\phi_v}{\phi_h} = \frac{\frac{3}{2}\phi_m \sin\omega t}{\frac{3}{2}\phi_m \cos\omega t} = \tan\ \omega t$$

$$\therefore \quad \theta = \omega t$$

Thus the resultant magnetic field rotates at constant angular velocity $\omega(= 2\ \pi f)$ rad/sec. For a P-pole machine, the rotation speed (ω_m) is

$$\omega_m = \frac{2}{P}\omega \text{ rad/sec}$$

or $$\frac{2\pi N_s}{60} = \frac{2}{P} \times 2\pi f$$ ... N_s is in r.p.m.

$$\therefore \quad N_s = \frac{120\,f}{P}$$

Thus the resultant flux due to three-phase currents is of constant value (= 1.5 ϕ_m where ϕ_m is the maximum flux in any phase) and this flux rotates around the stator winding at a synchronous speed of 120 f/P r.p.m.

For example, for a 6-pole, 50 Hz, 3-phase induction motor, N, = 120 × 50/6 = 1000 r.p.m. It means that flux rotates around the stator at a speed of 1000 r.p.m.

8.5 Principle of Operation

Consider a portion of 3-phase induction motor as shown in Fig. (8.13). The operation of the motor can be explained as under:

(i) When 3-phase stator winding is energized from a 3-phase supply, a rotating magnetic field is set up which rotates round the stator at synchronous speed N_s (= 120 f/P).

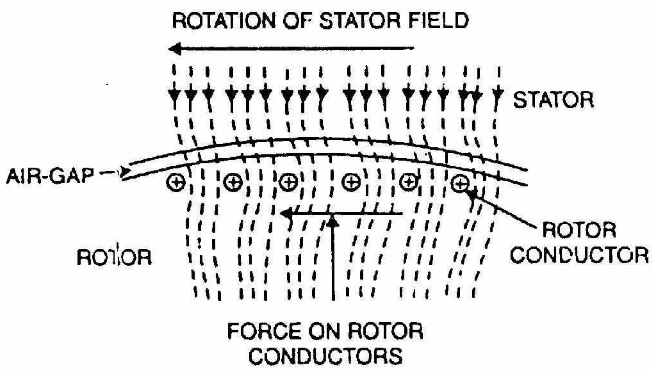

Fig.(1-)

(ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors.

(iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.

(iv) The fact that rotor is urged to follow the stator field (i.e., rotor moves in the direction of stator field) can be explained by Lenz's law. According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.

8.6 Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (N_s). This difference in speed depends upon load on the motor.

The difference between the synchronous speed N_s of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

$$\%\text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

(i) The quantity $N_s - N$ is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

8.7 Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula;

$$\text{Frequency } = \frac{NP}{120}$$

where N = Relative speed between magnetic field and the winding
P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is $N_s - N$. Consequently, the rotor current frequency f' is given by;

$$f' = \frac{(N_s - N)P}{120}$$

$$= \frac{sN_sP}{120} \qquad \left(\because\ s = \frac{N_s - N}{N_s}\right)$$

$$= sf \qquad \left(\because\ f = \frac{N_sP}{120}\right)$$

i.e., Rotor current frequency = Fractional slip x Supply frequency

(i) When the rotor is at standstill or stationary (i.e., s = 1), the frequency of rotor current is the same as that of supply frequency (f' = sf = 1× f = f).

(ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

Note. The relative speed between the rotating field and stator winding is $N_s - 0 = N_s$. Therefore, the frequency of induced current or voltage in the stator winding is $f = N_s P/120$—the supply frequency.

8.8 Effect of Slip on The Rotor Circuit

When the rotor is stationary, s = 1. Under these conditions, the per phase rotor e.m.f. E_2 has a frequency equal to that of supply frequency f. At any slip s, the relative speed between stator field and the rotor is decreased. Consequently, the rotor e.m.f. and frequency are reduced proportionally to sE_s and sf respectively. At the same time, per phase rotor reactance X_2, being frequency dependent, is reduced to sX_2.

Consider a 6-pole, 3-phase, 50 Hz induction motor. It has synchronous speed $N_s = 120\ f/P = 120 \times 50/6 = 1000$ r.p.m. At standsill, the relative speed between stator flux and rotor is 1000 r.p.m. and rotor e.m.f./phase = E_2(say). If the full-load speed of the motor is 960 r.p.m., then,

$$s = \frac{1000-960}{1000} = 0.04$$

(i) The relative speed between stator flux and the rotor is now only 40 r.p.m. Consequently, rotor e.m.f./phase is reduced to:

$$E_2 \times \frac{40}{1000} = 0.04 E_2 \qquad \text{or} \qquad sE_2$$

(ii) The frequency is also reduced in the same ratio to:

$$50 \times \frac{40}{1000} = 50 \times 0.04 \qquad \text{or} \qquad sf$$

(iii) The per phase rotor reactance X_2 is likewise reduced to:

$$X_2 \times \frac{40}{1000} = 0.04 X_2 \qquad \text{or} \qquad sX_2$$

Thus at any slip s,

Rotor e.m.f./phase = sE_2

Rotor reactance/phase = sX_2

Rotor frequency = sf

where E_2,X_2 and f are the corresponding values at standstill.

8.9 Rotor Current

Fig. (8.14) shows the circuit of a 3-phase induction motor at any slip s. The rotor is assumed to be of wound type and star connected. Note that rotor e.m.f./phase and rotor reactance/phase are s E_2 and sX_2 respectively. The rotor resistance/phase is R2 and is independent of frequency and, therefore, does not depend upon slip. Likewise, stator winding values R_1 and X_1 do not depend upon slip.

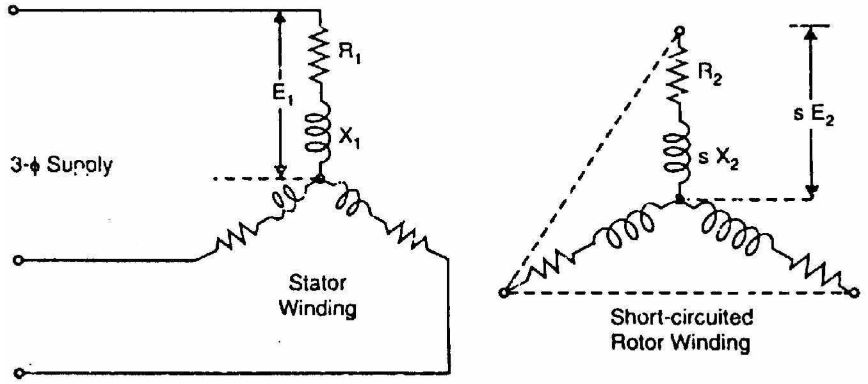

Fig.(8.14)

Since the motor represents a balanced 3-phase load, we need consider one phase only; the conditions in the other two phases being similar.

At standstill. Fig. (8.15 (i)) shows one phase of the rotor circuit at standstill.

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

$$\text{Rotor p.f., } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

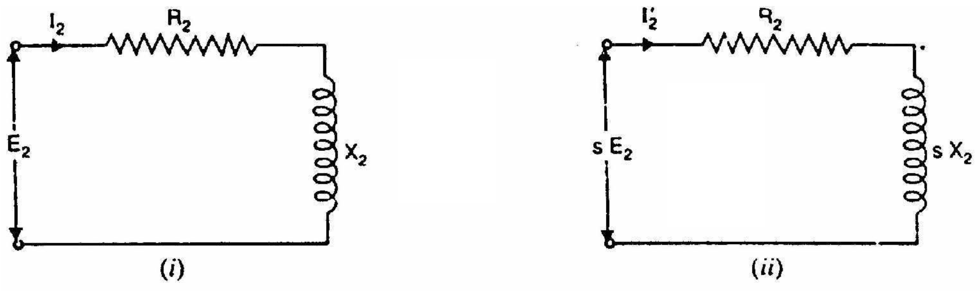

Fig.(8.15)

When running at slip s. Fig. (8.15 (ii)) shows one phase of the rotor circuit when the motor is running at slip s.

$$\text{Rotor current, } I'_2 = \frac{sE_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_2 = \frac{R_2}{Z'_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

8.10 Rotor Torque

The torque T developed by the rotor is directly proportional to:

(i) rotor current
(ii) rotor e.m.f.
(iii) power factor of the rotor circuit

$$\therefore \quad T \propto E_2 I_2 \cos\phi_2$$

or $$T = K E_2 I_2 \cos\phi_2$$

where I_2 = rotor current at standstill
E_2 = rotor e.m.f. at standstill
$\cos\phi_2$ = rotor p.f. at standstill

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

8.11 Starting Torque (T_s)

Let E_2 = rotor e.m.f. per phase at standstill
X_2 = rotor reactance per phase at standstill
R_2 = rotor resistance per phase

$$\text{Rotor impedance/phase, } Z_2 = \sqrt{R_2^2 + X_2^2} \qquad \text{...at standstill}$$

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \qquad \text{...at standstill}$$

$$\text{Rotor p.f., } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \qquad \text{...at standstill}$$

$$\therefore \quad \text{Starting torque, } T_s = K E_2 I_2 \cos\phi_2$$

$$= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Generally, the stator supply voltage V is constant so that flux per pole ϕ set up by the stator is also fixed. This in turn means that e.m.f. E_2 induced in the rotor will be constant.

$$\therefore \qquad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of R_2 and X_2 i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that K = 3/2 π N_s.

$$\therefore \qquad T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here N_s is in r.p.s.

8.12 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

Now $$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \qquad \text{(i)}$$

Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1\left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{\left(R_2^2 + X_2^2\right)^2}\right] = 0$$

or $$R_2^2 + X_2^2 = 2R_2^2$$

or $$R_2 = X_2$$

Hence starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

Under the condition of maximum starting torque, ϕ_2 = 45° and rotor power factor is 0.707 lagging [See Fig. (8.16 (ii))].

Fig. (8.16 (i)) shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when $R_2 = X_2$. If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

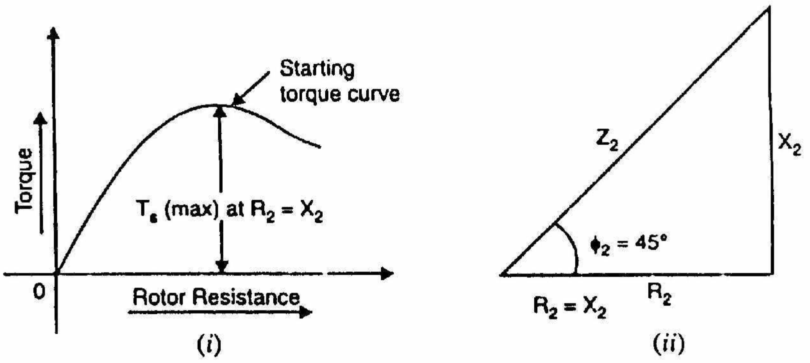

Fig.(8.16)

8.13 Effect of Change of Supply Voltage

$$T_s = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Since $E_2 \propto$ Supply voltage V

$$\therefore \quad T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

where K_2 is another constant.

$$\therefore \quad T_s \propto V^2$$

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20%. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plus friction torque.

8.14 Starting Torque of 3-Phase Induction Motors

The rotor circuit of an induction motor has low resistance and high inductance. At starting, the rotor frequency is equal to the stator frequency (i.e., 50 Hz) so that rotor reactance is large compared with rotor resistance. Therefore, rotor current lags the rotor e.m.f. by a large angle, the power factor is low and consequently the starting torque is small. When resistance is added to the rotor circuit, the rotor power factor is improved which results in improved starting torque. This, of course, increases the rotor impedance and, therefore, decreases the value of rotor current but the effect of improved power factor predominates and the starting torque is increased.

(i) **Squirrel-cage motors**. Since the rotor bars are permanently short-circuited, it is not possible to add any external resistance in the rotor circuit at starting. Consequently, the stalling torque of such motors is low. Squirrel cage motors have starting torque of 1.5 to 2 times the full-load value with starting current of 5 to 9 times the full-load current.

(ii) **Wound rotor motors**. The resistance of the rotor circuit of such motors can be increased through the addition of external resistance. By inserting the proper value of external resistance (so that $R_2 = X_2$), maximum starting torque can be obtained. As the motor accelerates, the external resistance is gradually cut out until the rotor circuit is short-circuited on itself for running conditions.

8.15 Motor Under Load

Let us now discuss the behaviour of 3-phase induction motor on load.

(i) When we apply mechanical load to the shaft of the motor, it will begin to slow down and the rotating flux will cut the rotor conductors at a higher and higher rate. The induced voltage and resulting current in rotor conductors will increase progressively, producing greater and greater torque.

(ii) The motor and mechanical load will soon reach a state of equilibrium when the motor torque is exactly equal to the load torque. When this state is reached, the speed will cease to drop any more and the motor will run at the new speed at a constant rate.

(iii) The drop in speed of the induction motor on increased load is small. It is because the rotor impedance is low and a small decrease in speed produces a large rotor current. The increased rotor current produces a higher torque to meet the increased load on the motor. This is why induction motors are considered to be constant-speed machines. However, because they never actually turn at synchronous speed, they are sometimes called asynchronous machines.
Note that change in load on the induction motor is met through the adjustment of slip. When load on the motor increases, the slip increases slightly (i.e., motor speed decreases slightly). This results in greater relative speed between the rotating flux and rotor conductors. Consequently, rotor current is increased, producing a higher torque to meet the increased load. Reverse happens should the load on the motor decrease.

(iv) With increasing load, the increased load currents I'_2 are in such a direction so as to decrease the stator flux (Lenz's law), thereby decreasing the counter e.m.f. in the stator windings. The decreased counter e.m.f. allows motor stator current (I_1) to increase, thereby increasing the power input to the motor. It may be noted that action of the induction motor in adjusting its stator or primary current with changes of current in the rotor or secondary is very much similar to the changes occurring in transformer with changes in load.

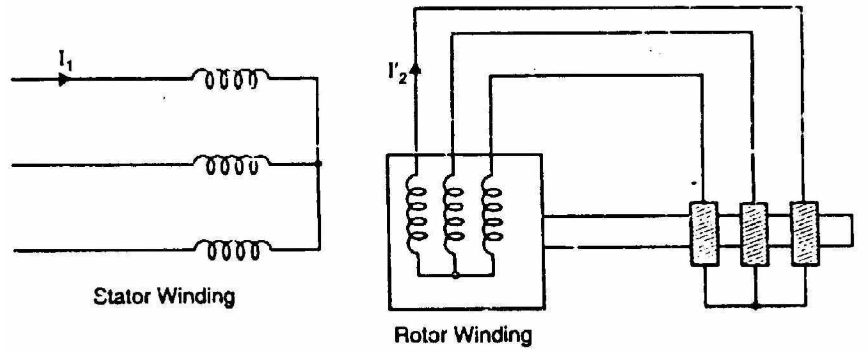

Fig.(8.17)

8.16 Torque Under Running Conditions

Let the rotor at standstill have per phase induced e.m.f. E_2, reactance X_2 and resistance R_2. Then under running conditions at slip s,

$$\text{Rotor e.m.f./phase, } E'_2 = sE_2$$

$$\text{Rotor reactance/phase, } X'_2 = sX_2$$

$$\text{Rotor impedance/phase, } Z'_2 = \sqrt{R_2^2 + (sX_2)^2}$$

$$\text{Rotor current/phase, } I'_2 = \frac{E'_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_m = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

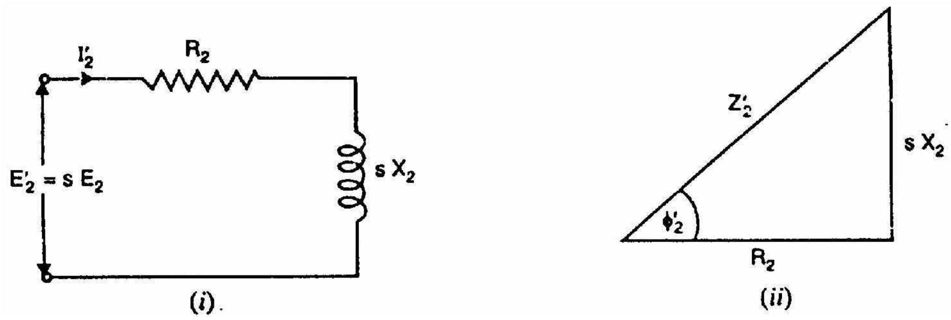

Fig.(8.18)

$$\text{Running Torque, } T_r \propto E'_2 I'_2 \cos\phi'_2$$

$$\propto \phi I'_2 \cos\phi'_2 \qquad (\because E'_2 \propto \phi)$$

$$\propto \phi \times \frac{s\,E_2}{\sqrt{R_2^2+(s\,X_2)^2}} \times \frac{R_2}{\sqrt{R_2^2+(s\,X_2)^2}}$$

$$\propto \frac{\phi\, s\,E_2\,R_2}{R_2^2+(s\,X_2)^2}$$

$$= \frac{K\,\phi\, s\,E_2\,R_2}{R_2^2+(s\,X_2)^2}$$

$$= \frac{K_1\, s\,E_2^2\,R_2}{R_2^2+(s\,X_2)^2} \qquad (\because E_2 \propto \phi)$$

If the stator supply voltage V is constant, then stator flux and hence E_2 will be constant.

$$\therefore \quad T_r = \frac{K_2\, s\,R_2}{R_2^2+(s\,X_2)^2}$$

where K_2 is another constant.

It may be seen that running torque is:

(i) directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.

(ii) directly proportional to square of supply voltage $(\because E_2 \propto V)$.

It can be shown that value of $K_1 = 3/2\,\pi\,N_s$ where N_s is in r.p.s.

$$\therefore \quad T_r = \frac{3}{2\pi N_s} \cdot \frac{s\,E_2^2\,R_2}{R_2^2+(s\,X_2)^2} = \frac{3}{2\pi N_s} \cdot \frac{s\,E_2^2\,R_2}{(Z'_2)^2}$$

At starting, s = 1 so that starting torque is

$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2\,R_2}{R_2^2+X_2^2}$$

8.17 Maximum Torque under Running Conditions

$$T_r = \frac{K_2\, s\,R_2}{R_2^2+s^2\,X_2^2} \qquad \text{(i)}$$

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t. s and equate the result to zero i.e.,

$$\frac{dT_r}{ds} = \frac{K_2\left[R_2\left(R_2^2+s^2\,X_2^2\right)-2s\,X_2^2(s\,R_2)\right]}{\left(R_2^2+s^2\,X_2^2\right)^2} = 0$$

or $\left(R_2^2 + s^2 X_2^2\right) - 2sX_2^2 = 0$

or $R_2^2 = s^2 X_2^2$

or $R_2 = s X_2$

Thus for maximum torque (T_m) under running conditions :

Rotor resistance/phase = Fractional slip × Standstill rotor reactance/phase

Now $$T_r \propto \frac{s R_2}{R_2^2 + s^2 X_2^2}$$ … from exp. (i) above

For maximum torque, $R_2 = s X_2$. Putting $R_2 = s X_2$ in the above expression, the maximum torque T_m is given by;

$$T_m \propto \frac{1}{2 X_2}$$

Slip corresponding to maximum torque, $s = R_2/X_2$.

It can be shown that:

$$T_m = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2 X_2} \text{ N - m}$$

It is evident from the above equations that:

(i) The value of rotor resistance does not alter the value of the maximum torque but only the value of the slip at which it occurs.
(ii) The maximum torque varies inversely as the standstill reactance. Therefore, it should be kept as small as possible.
(iii) The maximum torque varies directly with the square of the applied voltage.
(iv) To obtain maximum torque at starting (s = 1), the rotor resistance must be made equal to rotor reactance at standstill.

8.18 Torque-Slip Characteristics

As shown in Sec. 8.16, the motor torque under running conditions is given by;

$$T = \frac{K_2 \, s R_2}{R_2^2 + s^2 X_2^2}$$

If a curve is drawn between the torque and slip for a particular value of rotor resistance R_2, the graph thus obtained is called torque-slip characteristic. Fig. (8.19) shows a family of torque-slip characteristics for a slip-range from s = 0 to s = 1 for various values of rotor resistance.

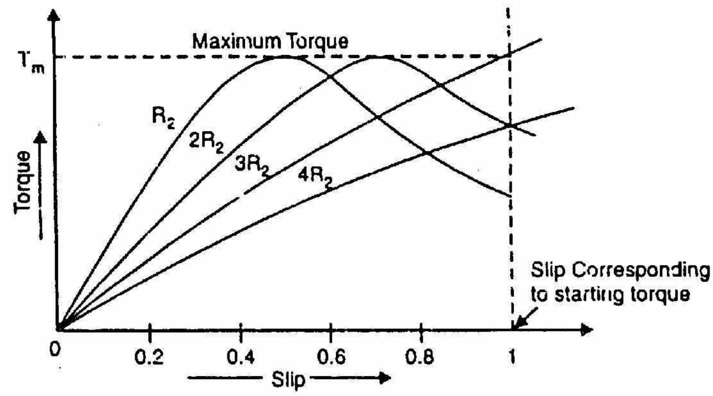

Fig.(8.19)

The following points may be noted carefully:

(i) At s = 0, T = 0 so that torque-slip curve starts from the origin.

(ii) At normal speed, slip is small so that s X_2 is negligible as compared to R_2.

$$\therefore \quad T \propto s/R_2$$

$$\propto s \qquad \text{... as } R_2 \text{ is constant}$$

Hence torque slip curve is a straight line from zero slip to a slip that corresponds to full-load.

(iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at $s = R_2/X_2$. This maximum torque in an induction motor is called pull-out torque or break-down torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.

(iv) to maximum torque, the term $s^2X_2^2$ increases very rapidly so that R_2^2 may be neglected as compared to $s^2X_2^2$.

$$\therefore \quad T \propto s/s^2X_2^2$$

$$\propto 1/s \qquad \text{... as } X_2 \text{ is constant}$$

Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.

(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.

8.19 Full-Load, Starting and Maximum Torques

$$T_f \propto \frac{s\,R_2}{R_2^2 + (s\,X_2)^2}$$

$$T_s \propto \frac{R_2}{R_2^2 + X_2^2}$$

$$T_m \propto \frac{1}{2\,X_2}$$

Note that s corresponds to full-load slip.

(i) $\therefore$ $$\frac{T_m}{T_f} = \frac{R_2^2 + (s\,X_2)^2}{2s\,R_2\,X_2}$$

Dividing the numerator and denominator on R.H.S. by X_2^2, we get,

$$\frac{T_m}{T_f} = \frac{(R_2/X_2)^2 + s^2}{2s(R_2/X_2)} = \frac{a^2 + s^2}{2as}$$

where $$a = \frac{R_2}{X_2} = \frac{\text{Rotor resistance/phase}}{\text{Standstill rotor reactance/phase}}$$

(ii) $$\frac{T_m}{T_s} = \frac{R_2^2 + X_2^2}{2R_2\,X_2}$$

Dividing the numerator and denominator on R.H.S. by X_2^2, we get,

$$\frac{T_m}{T_f} = \frac{(R_2/X_2)^2 + 1}{2(R_2/X_2)} = \frac{a^2 + 1}{2a}$$

where $$a = \frac{R_2}{X_2} = \frac{\text{Rotor resistance/phase}}{\text{Standstill rotor reactance/phase}}$$

8.20 Induction Motor and Transformer Compared

An induction motor may be considered to be a transformer with a rotating short-circuited secondary. The stator winding corresponds to transformer primary and rotor winding to transformer secondary. However, the following differences between the two are worth noting:

(i) Unlike a transformer, the magnetic circuit of a 3-phase induction motor has an air gap. Therefore, the magnetizing current in a 3-phase induction motor is much larger than that of the transformer. For example, in an induction motor, it may be as high as 30-50 % of rated current whereas it is only 1-5% of rated current in a transformer.

(ii) In an induction motor, there is an air gap and the stator and rotor windings are distributed along the periphery of the air gap rather than concentrated on a core as in a transformer. Therefore, the leakage reactances of stator and rotor windings are quite large compared to that of a transformer.

(iii) In an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. However, in a transformer, input as well as output is electrical.

(iv) The main difference between the induction motor and transformer lies in the fact that the rotor voltage and its frequency are both proportional to slip s. If f is the stator frequency, E_2 is the per phase rotor e.m.f. at standstill and X_2 is the standstill rotor reactance/phase, then at any slip s, these values are:

Rotor e.m.f./phase, $E'_2 = s\,E_2$

Rotor reactance/phase, $X'_2 = sX_2$

Rotor frequency, $f' = sf$

8.21 Speed Regulation of Induction Motors

Like any other electrical motor, the speed regulation of an induction motor is given by:

$$\%\text{ age speed regulation} = \frac{N_0 - N_{F.L.}}{N_{F.L.}} \times 100$$

where N_0 = no-load speed of the motor
$N_{F.L.}$ = full-load speed of the motor

If the no-load speed of the motor is 800 r.p.m. and its fall-load speed in 780 r.p.m., then change in speed is 800 − 780 = 20 r.p.m. and percentage speed regulation = 20 × 100/780 = 2.56%.

At no load, only a small torque is required to overcome the small mechanical losses and hence motor slip is small i.e., about 1%. When the motor is fully loaded, the slip increases slightly i.e., motor speed decreases slightly. It is because rotor impedance is low and a small decrease in speed produces a large rotor current. The increased rotor current produces a high torque to meet the full load on the motor. For this reason, the change in speed of the motor from no-load to full-load is small i.e., the speed regulation of an induction motor is low. The speed regulation of an induction motor is 3% to 5%. Although the motor speed does decrease slightly with increased load, the speed regulation is low enough that the induction motor is classed as a constant-speed motor.

8.22 Speed Control of 3-Phase Induction Motors

$$N = (1 - s)N_s = (1 - s)\frac{120\,f}{P} \qquad \text{(i)}$$

An inspection of eq. (i) reveals that the speed N of an induction motor can be varied by changing (i) supply frequency f (ii) number of poles P on the stator and (iii) slip s. The change of frequency is generally not possible because the commercial supplies have constant frequency. Therefore, the practical methods of speed control are either to change the number of stator poles or the motor slip.

1. Squirrel cage motors

The speed of a squirrel cage motor is changed by changing the number of stator poles. Only two or four speeds are possible by this method. Two-speed motor has one stator winding that may be switched through suitable control equipment to provide two speeds, one of which is half of the other. For instance, the winding may be connected for either 4 or 8 poles, giving synchronous speeds of 1500 and 750 r.p.m. Four-speed motors are equipped with two separate stator windings each of which provides two speeds. The disadvantages of this method are:

(i) It is not possible to obtain gradual continuous speed control.

(ii) Because of the complications in the design and switching of the interconnections of the stator winding, this method can provide a maximum of four different synchronous speeds for any one motor.

2. Wound rotor motors

The speed of wound rotor motors is changed by changing the motor slip. This can be achieved by;

(i) varying the stator line voltage

(ii) varying the resistance of the rotor circuit

(iii) inserting and varying a foreign voltage in the rotor circuit

8.23 Power Factor of Induction Motor

Like any other a.c. machine, the power factor of an induction motor is given by;

$$\text{Power factor, } \cos\phi = \frac{\text{Active component of current } (I\cos\phi)}{\text{Total current } (I)}$$

The presence of air-gap between the stator and rotor of an induction motor greatly increases the reluctance of the magnetic circuit. Consequently, an induction motor draws a large magnetizing current (I_m) to produce the required flux in the air-gap.

(i) At no load, an induction motor draws a large magnetizing current and a small active component to meet the no-load losses. Therefore, the induction motor takes a high no-load current lagging the applied voltage by a large angle. Hence the power factor of an induction motor on no load is low i.e., about 0.1 lagging.

(ii) When an induction motor is loaded, the active component of current increases while the magnetizing component remains about the same. Consequently, the power factor of the motor is increased. However, because of the large value of magnetizing current, which is present regardless of load, the power factor of an induction motor even at full-load seldom exceeds 0.9 lagging.

8.24 Power Stages in an Induction Motor

The input electric power fed to the stator of the motor is converted into mechanical power at the shaft of the motor. The various losses during the energy conversion are:

1. Fixed losses

(i) Stator iron loss
(ii) Friction and windage loss

The rotor iron loss is negligible because the frequency of rotor currents under normal running condition is small.

2. Variable losses

(i) Stator copper loss
(ii) Rotor copper loss

Fig. (8.20) shows how electric power fed to the stator of an induction motor suffers losses and finally converted into mechanical power.

The following points may be noted from the above diagram:

(i) Stator input, P_i = Stator output + Stator losses
= Stator output + Stator Iron loss + Stator Cu loss

(ii) Rotor input, P_r = Stator output
It is because stator output is entirely transferred to the rotor through air-gap by electromagnetic induction.

(iii) Mechanical power available, $P_m = P_r$ – Rotor Cu loss
This mechanical power available is the gross rotor output and will produce a gross torque T_g.

(iv) Mechanical power at shaft, $P_{out} = P_m$ – Friction and windage loss
Mechanical power available at the shaft produces a shaft torque T_{sh}.

Clearly, $P_m - P_{out}$ = Friction and windage loss

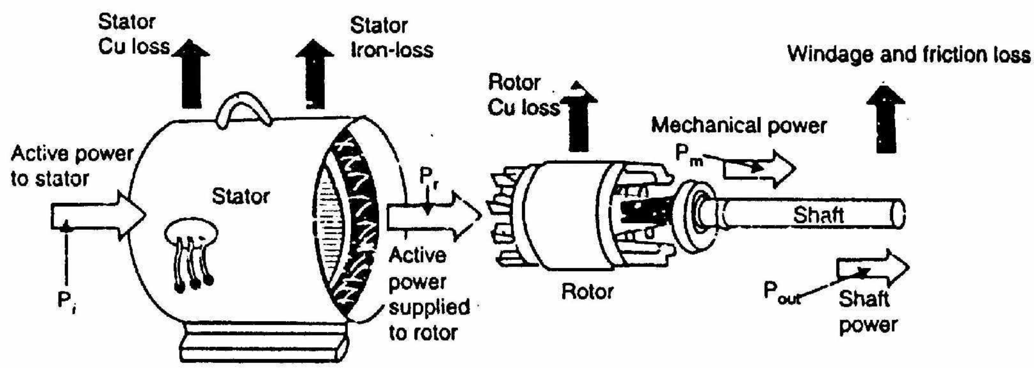

Fig.(8.20)

8.25 Induction Motor Torque

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi N T}{60} \quad \text{watts}$$

where N = speed of the motor in r.p.m.
T = torque developed in N-m

$$\therefore \quad T = \frac{60}{2\pi}\frac{P}{N} = 9.55\frac{P}{N} \text{ N - m}$$

If the gross output of the rotor of an induction motor is P_m and its speed is N r.p.m., then gross torque T developed is given by:

$$T_g = 9.55\frac{P_m}{N} \text{ N - m}$$

Similarly, $$T_{sh} = 9.55\frac{P_{out}}{N} \text{ N - m}$$

Note. Since windage and friction loss is small, $T_g = T_{sh}$,. This assumption hardly leads to any significant error.

8.26 Rotor Output

If T_g newton-metre is the gross torque developed and N r.p.m. is the speed of the rotor, then,

$$\text{Gross rotor output} = \frac{2\pi N T_g}{60} \quad \text{watts}$$

If there were no copper losses in the rotor, the output would equal rotor input and the rotor would run at synchronous speed N_s.

$\therefore$ Rotor input = $\dfrac{2\pi N_s T_g}{60}$ watts

$\therefore$ Rotor Cu loss = Rotor input − Rotor output

$$= \frac{2\pi T_g}{60}(N_s - N)$$

(i) $$\frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s - N}{N_s} = s$$

$\therefore$ Rotor Cu loss = s × Rotor input

(ii) Gross rotor output, P_m = Rotor input − Rotor Cu loss

= Rotor input − s × Rotor input

$\therefore$ P_m = Rotor input (1 − s)

(iii) $$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s}$$

(iv) $$\frac{\text{Rotor Cu loss}}{\text{Gross rotor output}} = \frac{s}{1-s}$$

It is clear that if the input power to rotor is P_r then s P_r is lost as rotor Cu loss and the remaining $(1 - s)P_r$ is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

Note.

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses arc neglected, then,

Gross rotor output = Useful output

Rotor input = Stator input

$$\therefore \quad \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is 1 − s. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is = 1 − 0.125 = 0.875 or 87.5%.

8.27 Induction Motor Torque Equation

The gross torque T_g developed by an induction motor is given by;

$$T_g = \frac{\text{Rotor input}}{2\pi N_s} \quad \text{... } N_s \text{ is r.p.s.}$$

$$= \frac{60 \times \text{Rotor input}}{2\pi N_s} \quad \text{(See Sec. 8.26)} \quad \text{... } N_s \text{ is r.p.s.}$$

Now $\quad$ Rotor input $= \dfrac{\text{Rotor Cu loss}}{s} = \dfrac{3(I'_2)^2 R_2}{s}$ $\quad$ (i)

As shown in Sec. 8.16, under running conditions,

$$I'_2 = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} = \frac{s K E_1}{\sqrt{R_2^2 + (s X_2)^2}}$$

where $\quad K = \text{Transformation ratio} = \dfrac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \quad \text{Rotor input} = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of I'_2 in eq.(i))

Also $\quad \text{Rotor input} = 3 \times \dfrac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \times \dfrac{1}{s} = \dfrac{3 s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2}$

(Putting me value of I'_2 in eq.(i))

$$\therefore \quad T_g = \frac{\text{Rotor input}}{2\pi N_s} = \frac{3}{2\pi N_s} \times \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \quad \text{...in terms of } E_2$$

$$= \frac{3}{2\pi N_s} \times \frac{s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \quad \text{...in terms of } E_1$$

Note that in the above expressions of T_g, the values E_1, E_2, R_2 and X_2 represent the phase values.

8.28 Performance Curves of Squirrel-Cage Motor

The performance curves of a 3-phase induction motor indicate the variations of speed, power factor, efficiency, stator current and torque for different values of load. However, before giving the performance curves in one graph, it is desirable to discuss the variation of torque, and stator current with slip.

(i) Variation of torque and stator current with slip

Fig. (8.21) shows the variation of torque and stator current with slip for a standard squirrel-cage motor. Generally, the rotor resistance is low so that full-load current occurs at low slip. Then even at full-load f' (= sf) and. therefore, X'_2 (= 2π f' L_2) are low. Between zero and full-load, rotor power factor (= cos ϕ'_2) and rotor impedance (= Z'_2) remain practically constant. Therefore, rotor current $I'_2(E'_2/Z'_2)$ and, therefore, torque (T_r) increase directly with the slip. Now stator current I_1 increases in proportion to I'_2. This is shown in Fig. (8.21) where T_r and I_1 are indicated as straight lines from no-load to full-load. As load and slip are increased beyond full-load, the increase in rotor reactance becomes appreciable. The increasing value of rotor impedance not only decreases the rotor power factor cos ϕ'_2 (= R_2/Z'_2) but also lowers the rate of increase of rotor current. As a result, the torque T_r and stator current I_1 do not increase directly with slip as indicated in Fig. (8.21). With the decreasing power factor and the lowered rate of increase in rotor current, the stator current I_1 and torque T_r increase at a lower rate. Finally, torque T_r reaches the maximum value at about 25% slip in the standard squirrel cage motor. This maximum value of torque is called the pullout torque or breakdown torque. If the load is increased beyond the breakdown point, the decrease in rotor power factor is greater than the increase in rotor current, resulting in a decreasing torque. The result is that motor slows down quickly and comes to a stop.

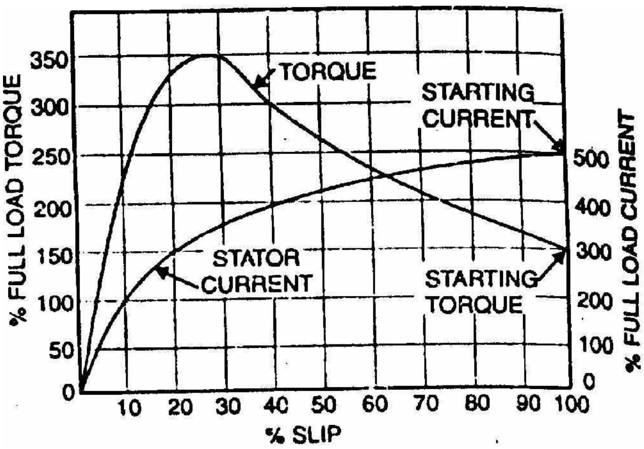

Fig.(8.21)

In Fig. (8.21), the value of torque at starting (i.e., s = 100%) is 1.5 times the full-load torque. The starting current is about five times the full-load current. The motor is essentially a constant-speed machine having speed characteristics about the same as a d.c. shunt motor.

(ii) Performance curves

Fig. (8.22) shows the performance curves of 3-phase squirrel cage induction motor.

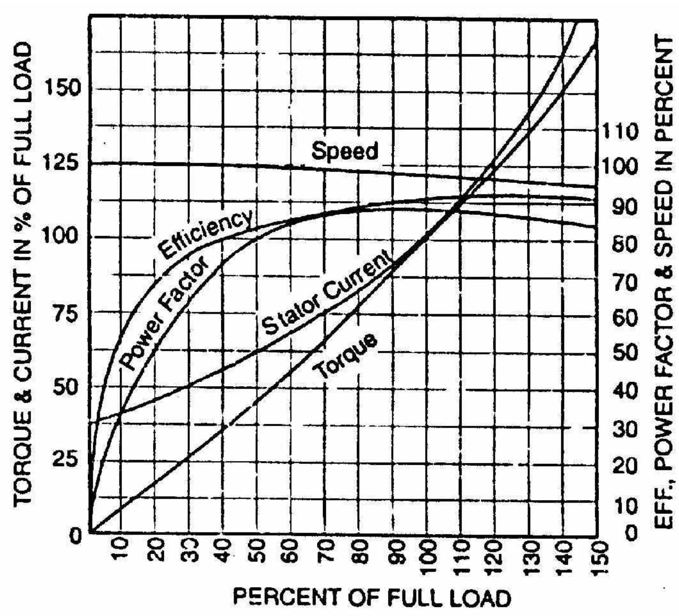

Fig.(8.22)

The following points may be noted:

(a) At no-load, the rotor lags behind the stator flux by only a small amount, since the only torque required is that needed to overcome the no-load losses. As mechanical load is added, the rotor speed decreases. A decrease in rotor speed allows the constant-speed rotating field to sweep across the rotor conductors at a faster rate, thereby inducing large rotor currents. This results in a larger torque output at a slightly reduced speed. This explains for speed-load curve in Fig. (8.22).

(b) At no-load, the current drawn by an induction motor is largely a magnetizing current; the no-load current lagging the applied voltage by a large angle. Thus the power factor of a lightly loaded induction motor is very low. Because of the air gap, the reluctance of the magnetic circuit is high, resulting in a large value of no-load current as compared with a transformer. As load is added, the active or power component of current increases, resulting in a higher power factor. However, because of the large value of magnetizing current, which is present regardless of load, the power factor of an induction motor even at full-load seldom exceeds 90%. Fig. (8.22) shows the variation of power factor with load of a typical squirrel-cage induction motor.

(c) $\text{Efficiency} = \dfrac{\text{Output}}{\text{Output} + \text{Losses}}$

The losses occurring in a 3-phase induction motor are Cu losses in stator and rotor windings, iron losses in stator and rotor core and friction and windage losses. The iron losses and friction and windage losses are almost independent of load. Had I^2R been constant, the efficiency of the motor would have increased with load. But I^2R loss depends upon load.

Therefore, the efficiency of the motor increases with load but the curve is dropping at high loads.

(d) At no-load, the only torque required is that needed to overcome no-load losses. Therefore, stator draws a small current from the supply. As mechanical load is added, the rotor speed decreases. A decrease in rotor speed allows the constant-speed rotating field to sweep across the rotor conductors at a faster rate, thereby inducing larger rotor currents. With increasing loads, the increased rotor currents are in such a direction so as to decrease the staler flux, thereby temporarily decreasing the counter e.m.f. in the stator winding. The decreased counter e.m.f. allows more stator current to flow.

(e) Output = Torque × Speed

Since the speed of the motor does not change appreciably with load, the torque increases with increase in load.

8.29 Equivalent Circuit of 3-Phase Induction Motor at Any Slip

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor finding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. (8.23) shows the equivalent circuit (though not the only one) per phase for an induction motor. Let us discuss the stator and rotor circuits separately.

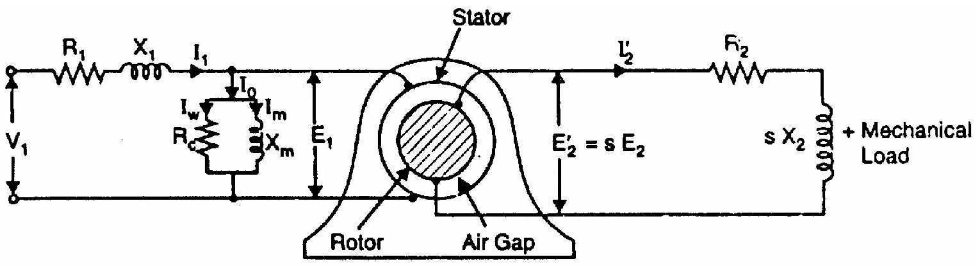

Fig.(8.23)

Stator circuit. In the stator, the events are very similar to those in the transformer primal y. The applied voltage per phase to the stator is V_1 and R_1 and X_1 are the stator resistance and leakage reactance per phase respectively. The applied voltage V_1 produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-induced e.m.f. E_1 is induced in the stator winding and mutually induced e.m.f. E'_2(= s E_2 = s K E_1 where K is transformation ratio) is induced in the rotor winding. The flow of stator current I_1 causes voltage drops in R_1 and X_1.

$$\therefore \quad V_1 = -E_1 + I_1(R_1 + j\,X_1) \qquad \text{...phasor sum}$$

When the motor is at no-load, the stator winding draws a current I_0. It has two components viz., (i) which supplies the no-load motor losses and (ii) magnetizing component I_m which sets up magnetic flux in the core and the air-gap. The parallel combination of R_c and X_m, therefore, represents the no-load motor losses and the production of magnetic flux respectively.

$$I_0 = I_w + I_m$$

Rotor circuit. Here R_2 and X_2 represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip s, the rotor reactance will be s X_2 The induced voltage/phase in the rotor is E'_2 = s E_2 = s K E_1. Since the rotor winding is short-circuited, the whole of e.m.f. E'_2 is used up in circulating the rotor current I'_2.

$$\therefore \quad E'_2 = I'_2\,(R_2 + j\,s\,X_2)$$

The rotor current I'_2 is reflected as I''_2 (= K I'_2) in the stator. The phasor sum of I''_2 and I_0 gives the stator current I_1.

It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.

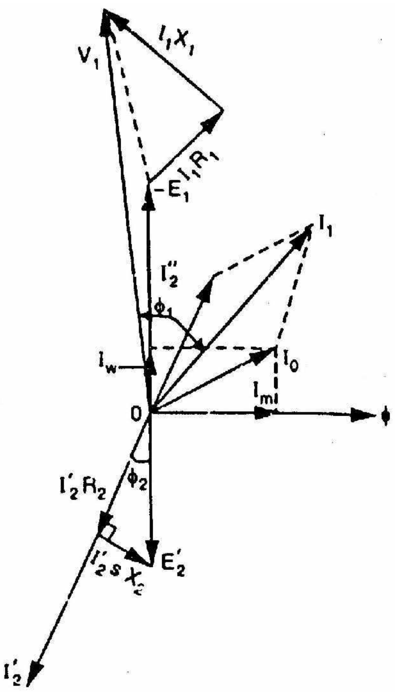

Fig.(8.24)

It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed N_s. The stator currents produce a magnetic flux which rotates at a speed N_s. At slip s, the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$= \frac{120\,f'}{P} = \frac{120\,s\,f}{P} = s\,N_s$$

But the rotor is revolving at a speed of N relative to the stator core. Therefore, the speed of rotor field relative to stator core

$$= sN_s + N = (N_s - N) + N = N_s$$

Thus no matter what the value of slip s, the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings.

Fig. (8.24) shows the phasor diagram of induction motor.

8.30 Equivalent Circuit of the Rotor

We shall now see how mechanical load of the motor is replaced by the equivalent electrical load. Fig. (8.25 (i)) shows the equivalent circuit per phase of the rotor at slip s. The rotor phase current is given by;

$$I'_2 = \frac{s\,E_2}{\sqrt{R_2^2 + (s\,X_2)^2}}$$

Mathematically, this value is unaltered by writing it as:

$$I'_2 = \frac{E_2}{\sqrt{(R_2/s)^2 + (X_2)^2}}$$

As shown in Fig. (8.25 (ii)), we now have a rotor circuit that has a fixed reactance X_2 connected in series with a variable resistance R_2/s and supplied with constant voltage E_2. Note that Fig. (8.25 (ii)) transfers the variable to the resistance without altering power or power factor conditions.

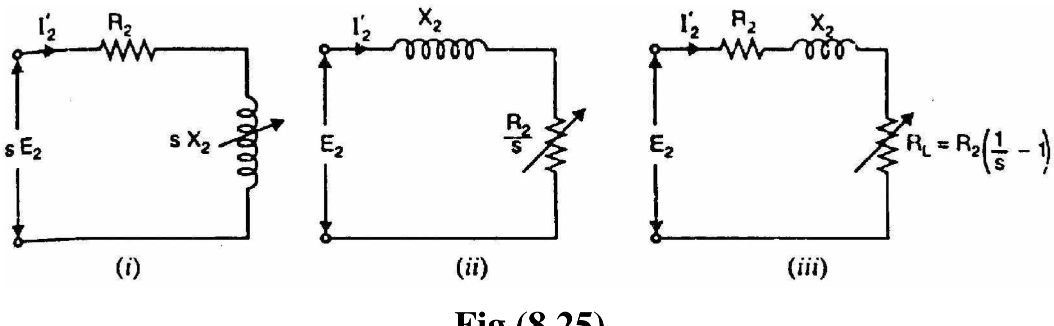

Fig.(8.25)

The quantity R_2/s is greater than R_2 since s is a fraction. Therefore, R_2/s can be divided into a fixed part R_2 and a variable part $(R_2/s - R_2)$ i.e.,

$$\frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right)$$

(i) The first part R_2 is the rotor resistance/phase, and represents the rotor Cu loss.

(ii) The second part $R_2\left(\frac{1}{s}-1\right)$ is a variable-resistance load. The power delivered to this load represents the total mechanical power developed in the rotor. Thus mechanical load on the induction motor can be replaced by a variable-resistance load of value $R_2\left(\frac{1}{s}-1\right)$. This is

$$\therefore \quad R_L = R_2\left(\frac{1}{s}-1\right)$$

Fig. (8.25 (iii)) shows the equivalent rotor circuit along with load resistance R_L.

8.31 Transformer Equivalent Circuit of Induction Motor

Fig. (8.26) shows the equivalent circuit per phase of a 3-phase induction motor. Note that mechanical load on the motor has been replaced by an equivalent electrical resistance R_L given by;

$$R_L = R_2\left(\frac{1}{s}-1\right) \qquad \text{(i)}$$

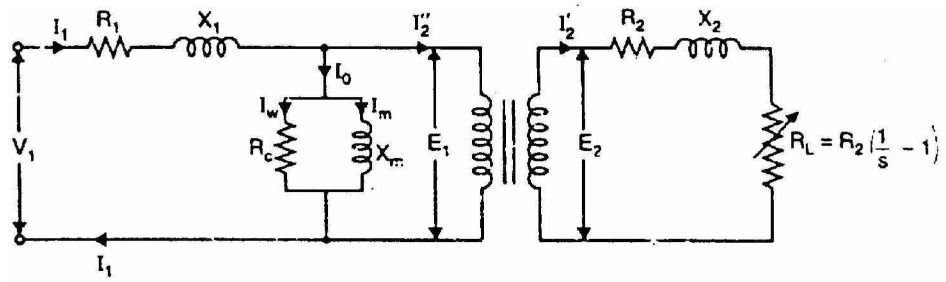

Fig.(8.26)

Note that circuit shown in Fig. (8.26) is similar to the equivalent circuit of a transformer with secondary load equal to R_2 given by eq. (i). The rotor e.m.f. in the equivalent circuit now depends only on the transformation ratio K (= E_2/E_1).

Therefore; induction motor can be represented as an equivalent transformer connected to a variable-resistance load R_L given by eq. (i). The power delivered to R_L represents the total mechanical power developed in the rotor. Since the equivalent circuit of Fig. (8.26) is that of a transformer, the secondary (i.e., rotor) values can be transferred to primary (i.e., stator) through the appropriate use of transformation ratio K. Recall that when shifting resistance/reactance from secondary to primary, it should be divided by K^2 whereas current should be multiplied by K. The equivalent circuit of an induction motor referred to primary is shown in Fig. (8.27).

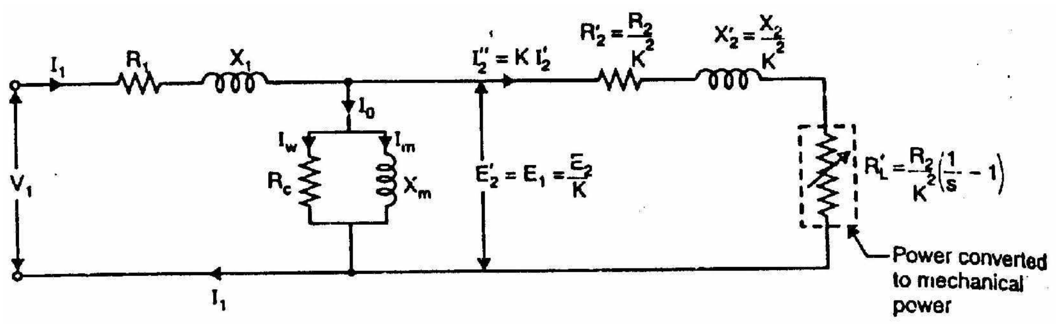

Fig.(8.27)

Note that the element (i.e., R'_L) enclosed in the dotted box is the equivalent electrical resistance related to the mechanical load on the motor. The following points may be noted from the equivalent circuit of the induction motor:

(i) At no-load, the slip is practically zero and the load R'_L is infinite. This condition resembles that in a transformer whose secondary winding is open-circuited.

(ii) At standstill, the slip is unity and the load R'_L is zero. This condition resembles that in a transformer whose secondary winding is short-circuited.

(iii) When the motor is running under load, the value of R'_L will depend upon the value of the slip s. This condition resembles that in a transformer whose secondary is supplying variable and purely resistive load.

(iv) The equivalent electrical resistance R'_L related to mechanical load is slip or speed dependent. If the slip s increases, the load R'_L decreases and the rotor current increases and motor will develop more mechanical power. This is expected because the slip of the motor increases with the increase of load on the motor shaft.

8.32 Power Relations

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. (8.28) shows the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).

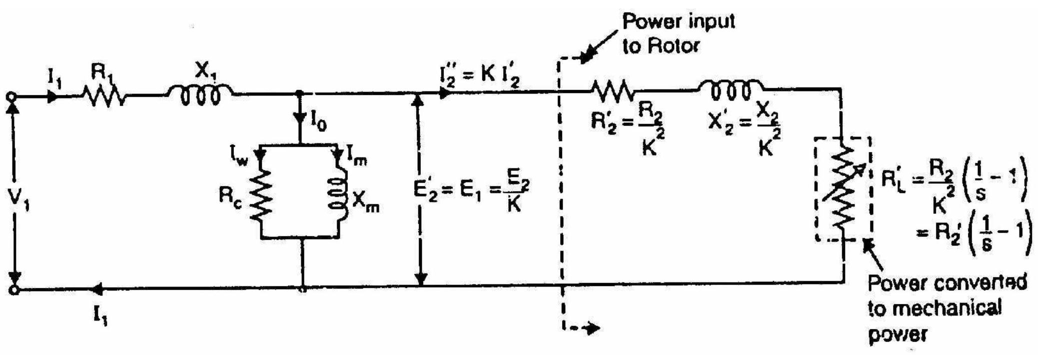

Fig.(8.28)

(i) Total electrical load $= R'_2\left(\frac{1}{s}-1\right)+R'_2=\frac{R'_2}{s}$

Power input to stator = $3V_1\, I_1 \cos\phi_1$

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

(ii) Rotor input = $\dfrac{3(I''_2)^2 R'_2}{s}$

Rotor Cu loss = $3(I''_2)^2 R'_2$

Total mechanical power developed by the rotor is

$$P_m = \text{Rotor input} - \text{Rotor Cu loss}$$

$$= \frac{3(I''_2)^2 R'_2}{s} 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2\left(\frac{1}{s}-1\right)$$

This is quite apparent from the equivalent circuit shown in Fig. (8.28).

(iii) If T_g is the gross torque developed by the rotor, then,

$$P_m = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2\left(\frac{1}{s}-1\right)=\frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2\left(\frac{1-s}{s}\right)=\frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2\left(\frac{1-s}{s}\right)=\frac{2\pi N_s(1-s)\, T_g}{60} \qquad [\because N = N_s(1-s)]$$

$\therefore$ $$T_g = \frac{3(I''_2)^2 R'_2/s}{2\pi N_s/60} \quad \text{N - m}$$

or $$T_g = 9.55\frac{3(I''_2)^2 R'_2/s}{N_s} \quad \text{N - m}$$

Note that shaft torque T_{sh} will be less than T_g by the torque required to meet windage and frictional losses.

8.33 Approximate Equivalent Circuit of Induction Motor

As in case of a transformer, the approximate equivalent circuit of an induction motor is obtained by shifting the shunt branch ($R_c - X_m$) to the input terminals as shown in Fig. (8.29). This step has been taken on the assumption that voltage drop in R_1 and X_1 is small and the terminal voltage V_1 does not appreciably differ from the induced voltage E_1. Fig. (8.29) shows the approximate equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).

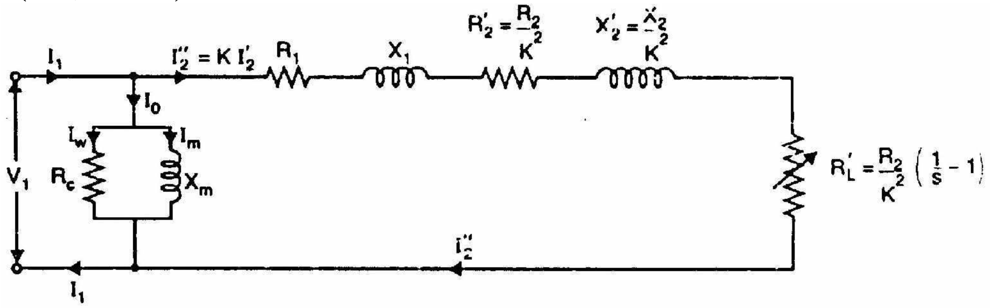

Fig.(8.29)

The above approximate circuit of induction motor is not so readily justified as with the transformer. This is due to the following reasons:

(i) Unlike that of a power transformer, the magnetic circuit of the induction motor has an air-gap. Therefore, the exciting current of induction motor (30 to 40% of full-load current) is much higher than that of the power transformer. Consequently, the exact equivalent circuit must be used for accurate results.

(ii) The relative values of X_1 and X_2 in an induction motor are larger than the corresponding ones to be found in the transformer. This fact does not justify the use of approximate equivalent circuit

(iii) In a transformer, the windings are concentrated whereas in an induction motor, the windings are distributed. This affects the transformation ratio.

In spite of the above drawbacks of approximate equivalent circuit, it yields results that are satisfactory for large motors. However, approximate equivalent circuit is not justified for small motors.

8.34 Starting of 3-Phase Induction Motors

The induction motor is fundamentally a transformer in which the stator is the primary and the rotor is short-circuited secondary. At starting, the voltage induced in the induction motor rotor is maximum (Q s = 1). Since the rotor impedance is low, the rotor current is excessively large. This large rotor current is reflected in the stator because of transformer action. This results in high starting current (4 to 10 times the full-load current) in the stator at low power factor and consequently the value of starting torque is low. Because of the short duration, this value of large current does not harm the motor if the motor accelerates normally. However, this large starting current will produce large line-voltage drop. This will adversely affect the operation of other electrical equipment connected to the same lines. Therefore, it is desirable and necessary to reduce the magnitude of stator current at starting and several methods are available for this purpose.

8.35 Methods of Starting 3-Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

(i) Direct-on-line starting
(ii) Stator resistance starting
(iii) Autotransformer starting
(iv) Star-delta starting
(v) Rotor resistance starting

Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon ,the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

8.36 Methods of Starting Squirrel-Cage Motors

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

(i) Direct-on-line starting

This method of starting in just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

Relation between starling and F.L. torques. We know that:

$$\text{Rotor input} = 2\pi N_s T = kT$$

But $\quad \text{Rotor Cu loss} = s \times \text{Rotor input}$

$$\therefore \quad 3(I'_2)^2 R_2 = s \times kT$$

or $\quad T \propto (I'_2)^2 / s$

or $\quad T \propto I_1^2 / s \qquad (\because I'_2 \propto I_1)$

If I_{st} is the starting current, then starting torque (T_{st}) is

$$T \propto I_{st}^2 \qquad (\because \text{ at starting } s = 1)$$

If I_f is the full-load current and s_f is the full-load slip, then,

$$T_f \propto I_f^2 / s_f$$

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current I_{sc}.

$$\therefore \quad \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Let us illustrate the above relation with a numerical example. Suppose I_{sc} = 5 I_f and full-load slip s_f =0.04. Then,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f = \left(\frac{5\ I_f}{I_f}\right)^2 \times 0.04 = (5)^2 \times 0.04 = 1$$

$$\therefore \quad T_{st} = T_f$$

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

(ii) Stator resistance starting

In this method, external resistances are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor.

This method suffers from two drawbacks. First, the reduced voltage applied to the motor during the starting period lowers the starting torque and hence increases the accelerating time. Secondly, a lot of power is wasted in the starting resistances.

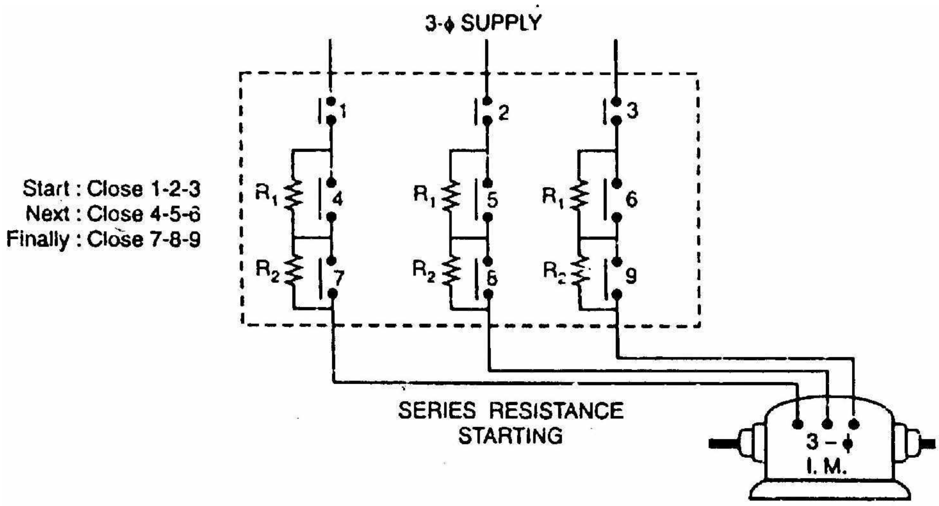

Fig.(8.30)

Relation between starting and F.L. torques. Let V be the rated voltage/phase. If the voltage is reduced by a fraction x by the insertion of resistors in the line, then voltage applied to the motor per phase will be xV.

$$I_{st} = x\, I_{sc}$$

Now $$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

or $$\frac{T_{st}}{T_f} = x^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Thus while the starting current reduces by a fraction x of the rated-voltage starting current (I_{sc}), the starting torque is reduced by a fraction x^2 of that obtained by direct switching. The reduced voltage applied to the motor during the starting period lowers the starting current but at the same time increases the accelerating time because of the reduced value of the starting torque. Therefore, this method is used for starting small motors only.

(iii) Autotransformer starting

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed. Fig. (8.31) shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

At the instant of starting, the change-over switch is thrown to “start” position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value. When the motor attains about 80% of normal speed, the changeover switch is thrown to "run" position. This takes out the autotransformer from the circuit and puts the motor to full line voltage. Autotransformer starting has several advantages viz low power loss, low starting current and less radiated heat. For large machines (over 25 H.P.), this method of starting is often used. This method can be used for both star and delta connected motors.

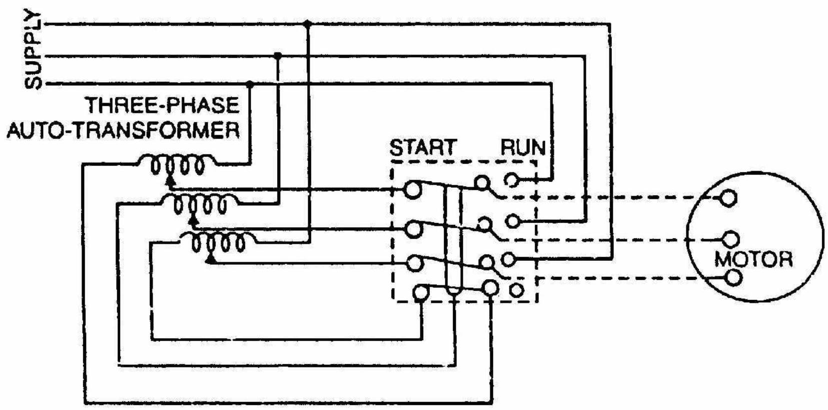

Fig.(8.31)

Relation between starting And F.L. torques. Consider a star-connected squirrel-cage induction motor. If V is the line voltage, then voltage across motor phase on direct switching is $V/\sqrt{3}$ and starting current is $I_{st} = I_{sc}$. In case of autotransformer, if a tapping of transformation ratio K (a fraction) is used, then phase voltage across motor is $KV/\sqrt{3}$ and $I_{st} = K\,I_{sc}$,

Now $$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f = \left(\frac{K\,I_{sc}}{I_f}\right)^2 \times s_f = K^2\left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

$$\therefore \quad \frac{T_{st}}{T_f} = K^2\left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

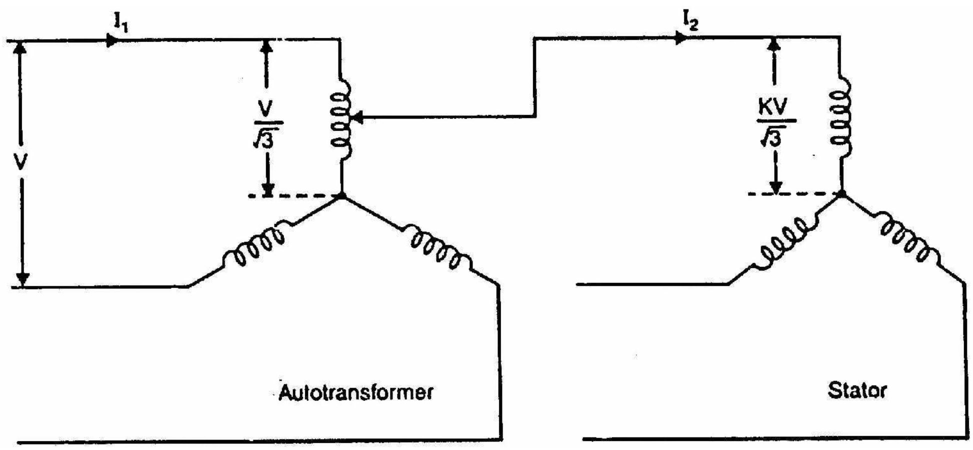

Fig.(8.32)

The current taken from the supply or by autotransformer is $I_1 = KI_2 = K^2I_{sc}$. Note that motor current is K times, the supply line current is K^2 times and the starting torque is K^2 times the value it would have been on direct-on-line starting.

(iv) Star-delta starting

The stator winding of the motor is designed for delta operation and is connected in star during the starting period. When the machine is up to speed, the connections are changed to delta. The circuit arrangement for star-delta starting is shown in Fig. (8.33).

The six leads of the stator windings are connected to the changeover switch as shown. At the instant of starting, the changeover switch is thrown to "Start" position which connects the stator windings in star. Therefore, each stator phase gets $V/\sqrt{3}$ volts where V is the line voltage. This reduces the starting current. When the motor picks up speed, the changeover switch is thrown to "Run" position which connects the stator windings in delta. Now each stator phase gets full line voltage V. The disadvantages of this method are:

(a) With star-connection during starting, stator phase voltage is $1/\sqrt{3}$ times the line voltage. Consequently, starting torque is $\left(1/\sqrt{3}\right)^2$ or 1/3 times the value it would have with Δ-connection. This is rather a large reduction in starting torque.

(b) The reduction in voltage is fixed.

This method of starting is used for medium-size machines (upto about 25 H.P.).

Relation between starting and F.L. torques. In direct delta starting,

Starting current/phase, $I_{sc} = V/Z_{sc}$ where V = line voltage

Starting line current = $\sqrt{3}\, I_{sc}$

In star starting, we have,

$$\text{Starting current/phase, } I_{st} = \frac{V/\sqrt{3}}{Z_{sc}} = \frac{1}{\sqrt{3}} I_{sc}$$

Now
$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f = \left(\frac{I_{sc}}{\sqrt{3} \times I_f}\right)^2 \times s_f$$

or
$$\frac{T_{st}}{T_f} = \frac{1}{3}\left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

where I_{sc} = starting phase current (delta)
I_f = F.L. phase current (delta)

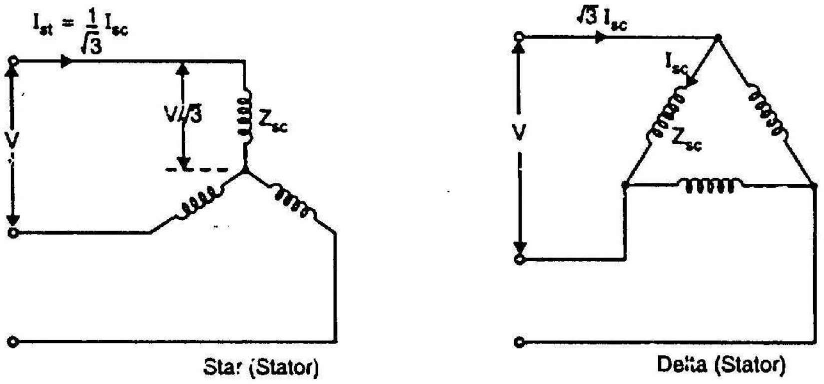

Fig.(8.33)

Note that in star-delta starting, the starting line current is reduced to one-third as compared to starting with the winding delta connected. Further, starting torque is reduced to one-third of that obtainable by direct delta starting. This method is cheap but limited to applications where high starting torque is not necessary e.g., machine tools, pumps etc.

8.37 Starting of Slip-Ring Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in Fig. (8.34).

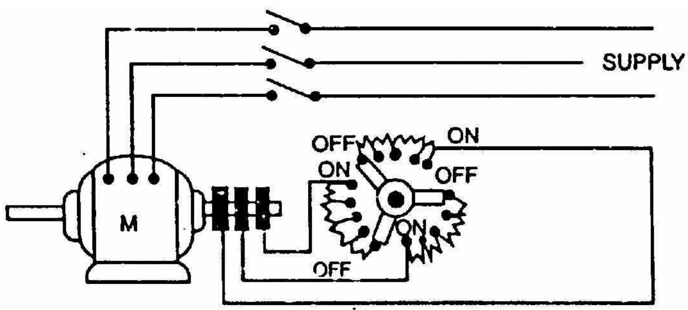

Fig.(8.34)

(i) At starting, the handle of rheostat is set in the OFF position so that maximum resistance is placed in each phase of the rotor circuit. This reduces the starting current and at the same time starting torque is increased.

(ii) As the motor picks up speed, the handle of rheostat is gradually moved in clockwise direction and cuts out the external resistance in each phase of the rotor circuit. When the motor attains normal speed, the change-over switch is in the ON position and the whole external resistance is cut out from the rotor circuit.

8.38 Slip-Ring Motors Versus Squirrel Cage Motors

The slip-ring induction motors have the following advantages over the squirrel cage motors:

(i) High starting torque with low starting current.
(ii) Smooth acceleration under heavy loads.
(iii) No abnormal heating during starting.
(iv) Good running characteristics after external rotor resistances are cut out.
(v) Adjustable speed.

The disadvantages of slip-ring motors are:

(i) The initial and maintenance costs are greater than those of squirrel cage motors.
(ii) The speed regulation is poor when run with resistance in the rotor circuit

8.39 Induction Motor Rating

The nameplate of a 3-phase induction motor provides the following information:

(i) Horsepower	(ii) Line voltage	(iii) Line current
(iv) Speed	(v) Frequency	(vi) Temperature rise

The horsepower rating is the mechanical output of the motor when it is operated at rated line voltage, rated frequency and rated speed. Under these conditions, the line current is that specified on the nameplate and the temperature rise does not exceed that specified.

The speed given on the nameplate is the actual speed of the motor at rated full-load; it is not the synchronous speed. Thus, the nameplate speed of the induction motor might be 1710 r.p.m. It is the rated full-load speed.

8.40 Double Squirrel-Cage Motors

One of the advantages of the slip-ring motor is that resistance may be inserted in the rotor circuit to obtain high starting torque (at low starting current) and then cut out to obtain optimum running conditions. However, such a procedure cannot be adopted for a squirrel cage motor because its cage is permanently short-circuited. In order to provide high starting torque at low starting current, double-cage construction is used.

Construction

As the name suggests, the rotor of this motor has two squirrel-cage windings located one above the other as shown in Fig. (8.35 (i)).

(i) **The outer winding** consists of bars of smaller cross-section short-circuited by end rings. Therefore, the resistance of this winding is high. Since the outer winding has relatively open slots and a poorer flux path around its bars [See Fig. (8.35 (ii))], it has a low inductance. Thus the resistance of the outer squirrel-cage winding is high and its inductance is low.

(ii) **The inner winding** consists of bars of greater cross-section short-circuited by end rings. Therefore, the resistance of this winding is low. Since the bars of the inner winding are thoroughly buried in iron, it has a high inductance [See Fig. (8.35 (ii))]. Thus the resistance of the inner squirrel-cage winding is low and its inductance is high.

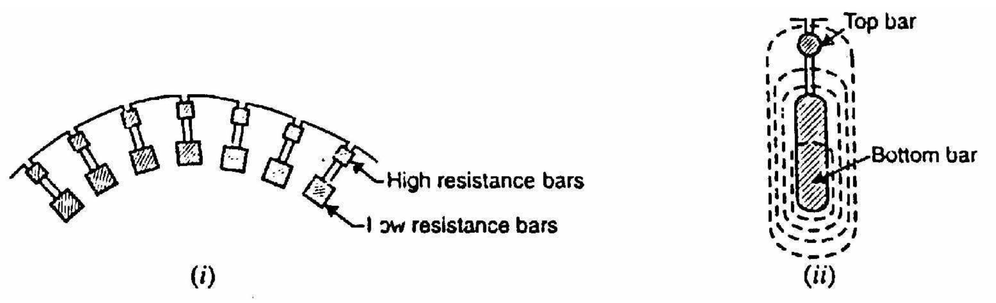

Fig.(8.35)

Working

When a rotating magnetic field sweeps across the two windings, equal e.m.f.s are induced in each.

(i) At starting, the rotor frequency is the same as that of the line (i.e., 50 Hz), making the reactance of the lower winding much higher than that of the upper winding. Because of the high reactance of the lower winding, nearly all the rotor current flows in the high-resistance outer cage winding. This provides the good starting characteristics of a high-resistance cage winding. Thus the outer winding gives high starting torque at low starting current.

(ii) As the motor accelerates, the rotor frequency decreases, thereby lowering the reactance of the inner winding, allowing it to carry a larger proportion of the total rotor current At the normal operating speed of the motor, the rotor frequency is so low (2 to 3 Hz) that nearly all the rotor current flows in the low-resistance inner cage winding. This results in good operating efficiency and speed regulation.

Fig. (8.36) shows the operating characteristics of double squirrel-cage motor. The starting torque of this motor ranges from 200 to 250 percent of full-load torque with a starting current of 4 to 6 times the full-load value. It is classed as a high-torque, low starting current motor.

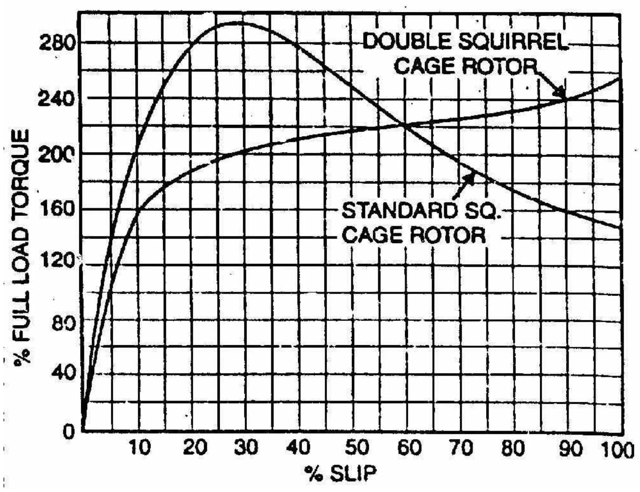

Fig.(8.36)

8.41 Equivalent Circuit of Double Squirrel-Cage Motor

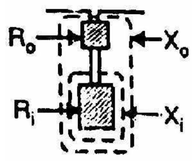

Fig.(8.37)

Fig. (8.37) shows a section of the double squirrel cage motor. Here R_o and R_i are the per phase resistances of the outer cage winding and inner cage winding whereas X_o and X_i are the corresponding per phase standstill reactances. For the outer cage, the resistance is made intentionally high, giving a high starting torque. For the inner cage winding, the resistance is low and the leakage reactance is high, giving a low starting torque but high efficiency on load. Note that in a double squirrel cage motor, the outer winding produces the high starting and accelerating torque while the inner winding provides the running torque at good efficiency.

Fig. (8.38 (i)) shows the equivalent circuit for one phase of double cage motor referred to stator. The two cage impedances are effectively in parallel. The resistances and reactances of the outer and inner rotors are referred to the stator. The exciting circuit is accounted for as in a single cage motor. If the magnetizing current (I_0) is neglected, then the circuit is simplified to that shown in Fig. (8.38 (ii)).

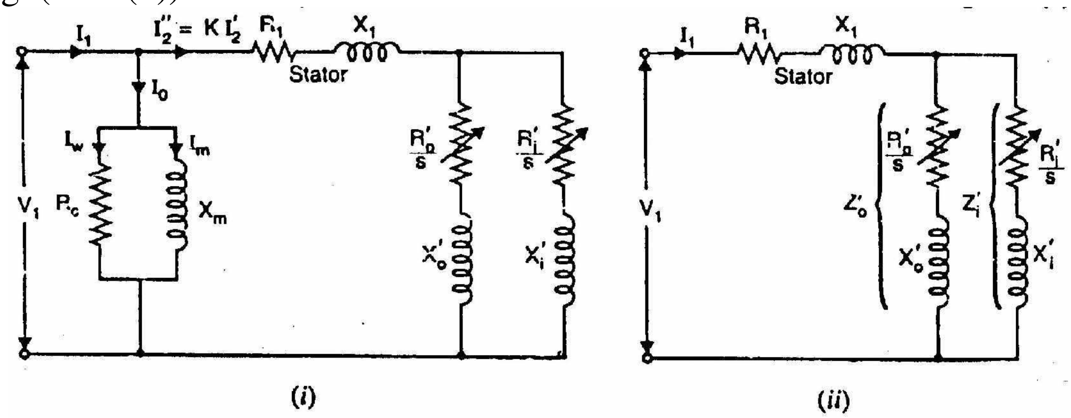

Fig.(8.38)

From the equivalent circuit, the performance of the motor can be predicted.

Total impedance as referred to stator is

$$Z_{o1} = R_1 + j\,X_1 + \frac{1}{1/Z'_i + 1/Z'_o} = R_1 + j\,X_1 + \frac{Z'_i\,Z'_o}{Z'_i + Z'_o}$$