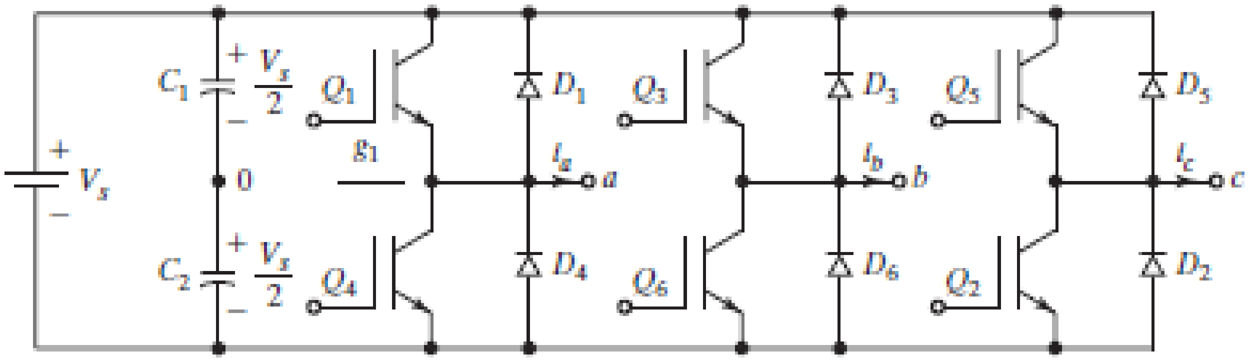
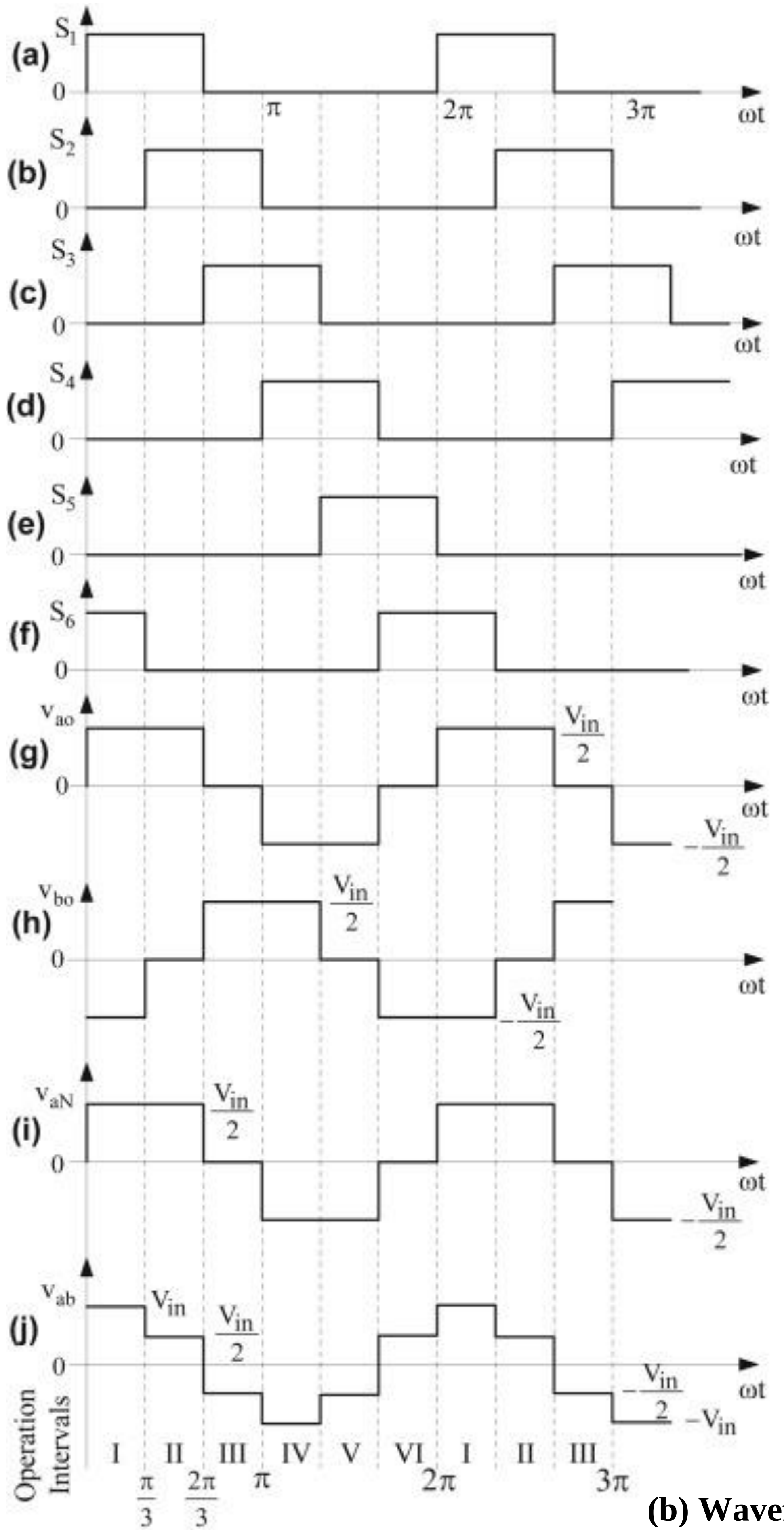
## 120-Degree Conduction

The circuit diagram for 120º mode is shown in Fig. 3.23 (a) which is the same as Fig. 3.20 (a) for 180 º mode. Here, in this type of control, each switch conducts for 120º, with only two switches remaining on in any of the 6 sub-intervals of 60º. The gating signals are shown in Fig. 3.23 (b) from which it can be inferred that the conduction sequence of the switches is 61, 12, 23, 34, 45, 56 in intervals I, II, III, IV, V & VI respectively.




**(a) Circuit**

**Fig. 3.23 Three phase 6 step 120 mode bridge inverter**




**(b) Waveforms**



**Fig. 3.23 Three phase 6 step 120 mode bridge inverter**

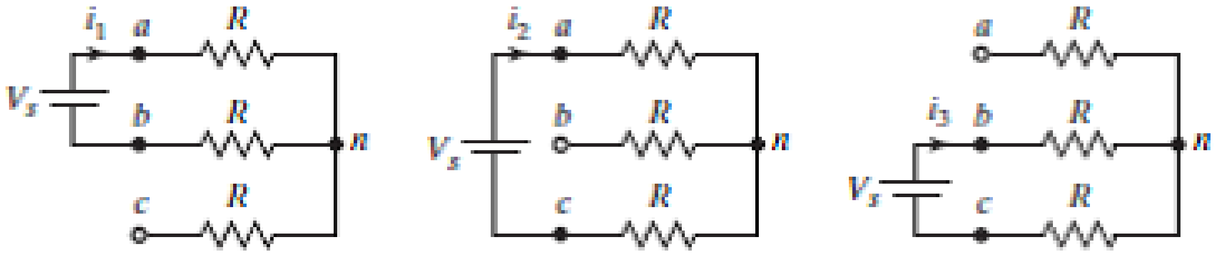
There are six modes of operation in each cycle, with Modes 1, 2 & 3 in the positive half-cycle and Modes 4, 5 & 6 in the negative half-cycle. The equivalent circuits in each of Modes 1, 2 & 3 are shown in Fig. 3.24

During mode 1 for $0 \leq \omega t \leq \pi/3$, transistors 1 and 6 conduct.

$$v_{an} = \frac{V_s}{2} \quad v_{bn} = -\frac{V_s}{2} \quad v_{cn} = 0 \quad v_{ab} = V_s \quad v_{bc} = -\frac{V_s}{2} \quad v_{ca} = -\frac{V_s}{2}$$

During mode 2 for $\pi/3 \leq \omega t \leq 2\pi/3$, transistors 1 and 2 conduct.

$$v_{an} = \frac{V_s}{2} \quad v_{bn} = 0 \quad v_{cn} = -\frac{V_s}{2} \quad v_{ab} = \frac{V_s}{2} \quad v_{bc} = \frac{V_s}{2} \quad v_{ca} = -V_s$$



(a) Mode 1 (b) Mode 2 (c) Mode 3

FIGURE 3.24

Equivalent circuits for Y-connected resistive load.

During mode 3 for $2\pi/3 \leq \omega t \leq 3\pi/3$, transistors 2 and 3 conduct.

$$v_{an} = 0 \quad v_{bn} = \frac{V_s}{2} \quad v_{cn} = -\frac{V_s}{2} \quad v_{ab} = -\frac{V_s}{2} \quad v_{bc} = V_s \quad v_{ca} = -\frac{V_s}{2}$$

The equivalent circuits for Modes 4, 5 & 6 can be obtained from Modes 1, 2 & 3, respectively, by reversing the polarity of the supply $V_S$. Accordingly, all phase and line voltages for Modes 4, 5 & 6 will be the negative of the corresponding voltages for Modes 1, 2 & 3.

The line-to-neutral voltages that are shown in Figure 3.23 (b) can be expressed in Fourier series as

$$v_{an} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\frac{n\pi}{3} \sin n\left(\omega t + \frac{\pi}{6}\right) \tag{3.72a}$$

$$v_{bn} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{\pi}{2}\right) \tag{3.72b}$$

$$v_{cn} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{7\pi}{6}\right) \tag{3.72c}$$

The line $a$-to-$b$ voltage is $v_{ab} = \sqrt{3}v_{an}$ with a phase advance of 30° for a positive sequence, $n = 1, 7, 13, 19, \ldots$, and a phase delay of 30° for a negative sequence, $n = 5, 11, 17, 23, \ldots$ This phase shift is independent of the harmonic order. Therefore, the instantaneous line-to-line voltages (for a Y-connected load) are

$$v_{ab} = \sum_{n=1}^{\infty} \frac{2\sqrt{3}V_S}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin\left[n\left(\omega t + \frac{\pi}{6}\right) \pm \frac{\pi}{6}\right] \tag{3.73a}$$

$$v_{bc} = \sum_{n=1}^{\infty} \frac{2\sqrt{3}V_S}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin\left[n\left(\omega t - \frac{\pi}{2}\right) \pm \frac{\pi}{6}\right] \tag{3.73b}$$

$$v_{ca} = \sum_{n=1}^{\infty} \frac{2\sqrt{3}V_S}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{3}\right) \sin\left[n\left(\omega t - \frac{7\pi}{6}\right) \pm \frac{\pi}{6}\right] \tag{3.73c}$$

There is a delay of $\pi/6$ between turning off $Q_1$ and turning on $Q_4$. Thus, there should be no short circuit of the dc supply through one upper and one lower transistors. At any time, two load terminals are connected to the dc supply and the third one remains open. The potential of this open terminal depends on the load characteristics and would be unpredictable. Because one transistor conducts for 120°, the transistors are less utilized as compared with those of 180° conduction for the same load condition. Thus, the 180° conduction is preferred and it is generally used in three-phase inverters.

The rms value of the line-to-neutral output voltage can be obtained as:

$$V_P = \left[\sqrt{\frac{2}{2\pi}\int_o^{2\pi/3} (V_s/2)^2 d(\omega t)}\right] = \frac{V_s}{\sqrt{6}} = 0.4082V_S \tag{3.74}$$

The rms value of the fundamental line-to-neutral output voltage can be obtained from Eq. (3.72) as:

$$V_{P1} = \sqrt{\frac{3}{2}}\frac{V_S}{\pi} \tag{3.75}$$

The rms value of the line-to-line output voltage can be obtained as:

$$V_L = \left[\sqrt{\frac{2}{2\pi}\left((V_S/2)^2 + V_S^2 + (V_S/2)^2\right)\frac{\pi}{3}}\right]$$

$$= \frac{V_S}{\sqrt{2}} = 0.7071V_S \tag{3.76}$$

The rms value of the fundamental line-to-line output voltage is:

$$V_{L1} = \frac{3}{\sqrt{2}}\frac{V_S}{\pi} = 0.6752V_S \tag{3.77}$$

Thus it is seen that for 120º mode, the **line-to-line** output voltage is a **six step** waveform with amplitudes of +/- $V_S/2$ and +/-$V_S$, while the **phase** output voltage is a **quasi-square wave** with amplitude of +/- $V_S/2$ and pulse width of 120º.

This can be compared with 180º mode where it is seen that the **phase** output voltage is a **six step** waveform with amplitudes of +/- $V_S/3$ and +/-$2V_S/3$, while the **line-to-line** output voltage is a **quasi-square wave** with amplitude of +/- $V_S$ and pulse width of 120º.

# Three phase 6 step 180 & 120 mode bridge inverters

## A. Utility Factor, output current, switch current and output power

The comparison of the two schemes is done using a figure of merit termed as utility factor ($UF$) and is defined as

$$UF = P_0/P_T \tag{3.78}$$

where $P_0$ is the rated output power of the inverter and $P_T$ is the measure of total power handling capability of the thyristors employed in the inverter, and is defined as

$$P_T = NV_{\text{DRM}} I_{\text{rms}} \tag{3.79}$$

where $N$ = number of thyristors

$V_{\text{DRM}}$ = repetitive peak forward voltage

and $I_{\text{rms}}$ = rated RMS forward current

It is usual to calculate utility factor for ideal resistive loads for the purpose of comparison of the two control schemes.

*Inverter with 180° conduction*: If resistance per phase is $R$, then for a star connected load,

$$P_0 = 3\left(1/\pi \int_0^{\pi} \frac{v_{AN}^2}{R}\, d(\omega t)\right) \tag{3.80}$$

Substituting values for $v_{AN}$ gives

$$P_0 = 2V^2/3R \tag{3.81}$$

Thyristor rating will be chosen such that the rms current flowing through it will be equal to its rms current rating $I_{rms}$. During a cycle, current of any phase is shared between two thyristors in series, for example for phase $A$, thyristor $T_1$ carries load current during the positive half cycle and thyristor $T_4$ during the negative half cycle. Therefore the RMS value of load phase current under the rated conditions, $I_p$ is given by

$$I_p = \sqrt{2}\, I_{rms} \tag{3.82}$$

Load power $P_0$ can also be written as

$$P_0 = 3I_p^2 R = 6I_{rms}^2 R \tag{3.83}$$

From eqns. (3.81) and (3.83)

$$I_{rms} = V/3R \quad I_p = \sqrt{2} V/3R \tag{3.84}$$

The repetitive peak forward voltage rating that a thyristor should have for this load, $V_{DRM} = V$. Thus,

$$P_T = 6V\frac{V}{3R} = \frac{2V^2}{R} = 6VI_{rms} \tag{3.85}$$

From eqns. (3.78), (3.81) and (3.85)

$$UF = \frac{P_0}{P_T} = \frac{1}{3} = 33.33 \text{ per cent} \tag{3.86}$$

This indicates that the rated power output of the inverter in this case is only 33.33 per cent of the combined maximum power rating of the six thyristors.

*Inverter with* 120° *conduction*: For a star connected load with a per phase resistance of $R$, $P_0$ will be given by eqn.(3.80). Substituting values for $v_{AN}$ in eqn. (3.80) yields

$$P_0 = \frac{V^2}{2R} = 3I_p^2 R = 6I_{rms}^2 R \tag{3.87}$$

Here also the relation between $I_{rms}$ and $I_p$ is given by eqn. (3.82) and $P_0$ in terms of $I_{rms}$ is given by eqn. (3.83). From eqns. (3.83) and (3.87)

$$I_{rms} = \frac{1}{2\sqrt{3}}\frac{V}{R} \quad I_p = V/\sqrt{6}R \tag{3.88}$$

Now from eqn. (3.79)

$$P_T = 6V\frac{1}{2\sqrt{3}}\frac{V}{R} = \sqrt{3}\frac{V^2}{R} \tag{3.89}$$

and from eqn. (3.78)

$$UF \simeq \frac{1}{2\sqrt{3}} \simeq 28.9 \text{ per cent} \tag{3.90}$$

In this case the rated inverter output is 28.9 per cent of the combined maximum power rating of 6 thyristors.

Comparison of eqns. (3.86) and (3.90) will show that the utility factor is lower in the case of 120° conduction scheme.

**B. Summary of expressions for 180° and 120° modes**

| Sr. No. | Parameter | 180° | 120° |
|---|---|---|---|
| 1 | Line-to-line waveform | Quasi-square, amplitude $V_S$, pulse width 120° | 6 step, step amplitudes $V_S/3$, $2V_S/3$ |
| 2 | Line-to-neutral waveform | 6 step, step amplitudes $V_S/2$, $V_S$ | Quasi-square, amplitude $V_S/2$, pulse width 120° |
| 3 | $V_L$ rms | $\sqrt{\frac{2}{3}}V_S$ | $\frac{1}{\sqrt{2}}V_S$ |
| 4 | $V_{L1}$ rms | $\frac{\sqrt{6}}{\pi}V_S$ | $\frac{3}{\pi\sqrt{2}}V_S$ |
| 5 | $V_P$ rms | $\frac{\sqrt{2}}{3}V_S$ | $\frac{1}{\sqrt{6}}V_S$ |
| 6 | $V_{P1}$ rms | $\frac{\sqrt{2}}{\pi}V_S$ | $\frac{1}{\pi}\sqrt{\frac{3}{2}}V_S$ |

| Sr. No. | Parameter | 180° | 120° |
|---|---|---|---|
| 7 | $I_P$ rms = $I_L$ rms (for balanced star R load) | $V_P/R$ | $V_P/R$ |
| 8 | $I_{P1}$ rms = $I_{L1}$ rms (for balanced star R load) | $V_{P1}/R$ | $V_{P1}/R$ |
| 9 | $I_{SW}$ rms | $V_P/(\sqrt{2}R)$ | $V_P/(\sqrt{2}R)$ |
| 10 | $P_O$ average | $\frac{2V_s^2}{3R}$ | $\frac{\sqrt{3}V_s^2}{R}$ |

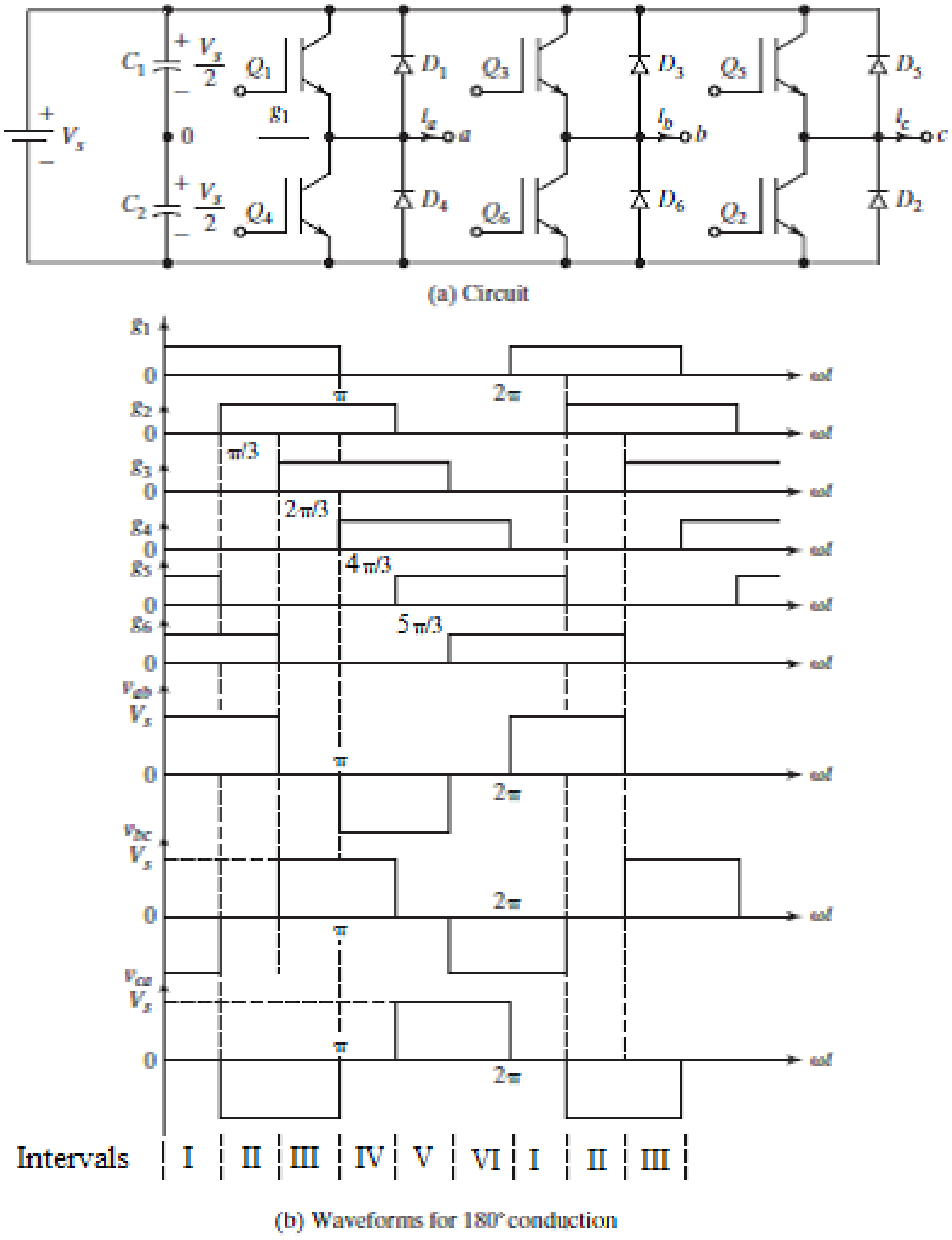



**Fig. 3.20 Three phase 6 step 180 mode bridge inverter**

### 180-Degree Conduction

Each transistor conducts for 180°. Three transistors remain on at any instant of time. When transistor $Q_1$ is switched on, terminal $a$ is connected to the positive terminal of the dc input voltage. When transistor $Q_4$ is switched on, terminal $a$ is brought to the negative terminal of the dc source. There are six modes of operation in a cycle and the duration of each mode is 60°. The transistors are numbered in the sequence of gating the transistors (e.g., 123, 234, 345, 456, 561, and 612). The gating signals shown in Figure 3.20 are shifted from each other by 60° to obtain three-phase balanced (fundamental) voltages.

The load may be connected in Y or delta as shown in Figure 3.21. The switches of any leg of the inverter ($S_1$ and $S_4$, $S_3$ and $S_6$, or $S_5$ and $S_2$) cannot be switched on simultaneously; this would result in a short circuit across the dc-link voltage supply. Similarly, to avoid undefined states and thus undefined ac output line voltages, the switches of any leg of the inverter cannot be switched off simultaneously; this can result in voltages that depend on the respective line current polarity.

**$R_Y = R_\Delta/3$**

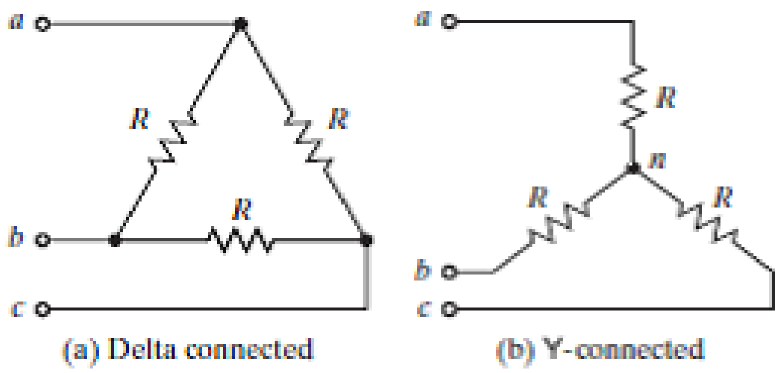



FIGURE 3.21 Delta- and Y-connected balanced resistive load.

For a delta-connected load, the phase currents can be obtained directly from the line-to-line voltages. Once the phase currents are known, the line currents can be determined. For a Y-connected load, the line-to-neutral voltages must be determined to find the line (or phase) currents. There are three modes of operation in a half-cycle and the equivalent circuits are shown in Figure 3.22a for a Y-connected load.

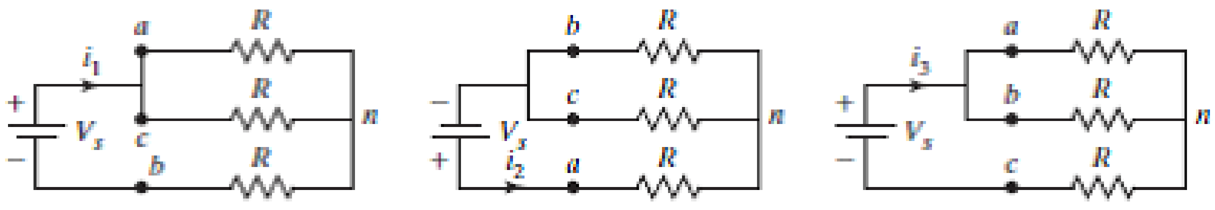
During mode 1 for $0 \le \omega t < \pi/3$, transistors $Q_1$, $Q_5$, and $Q_6$ conduct

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

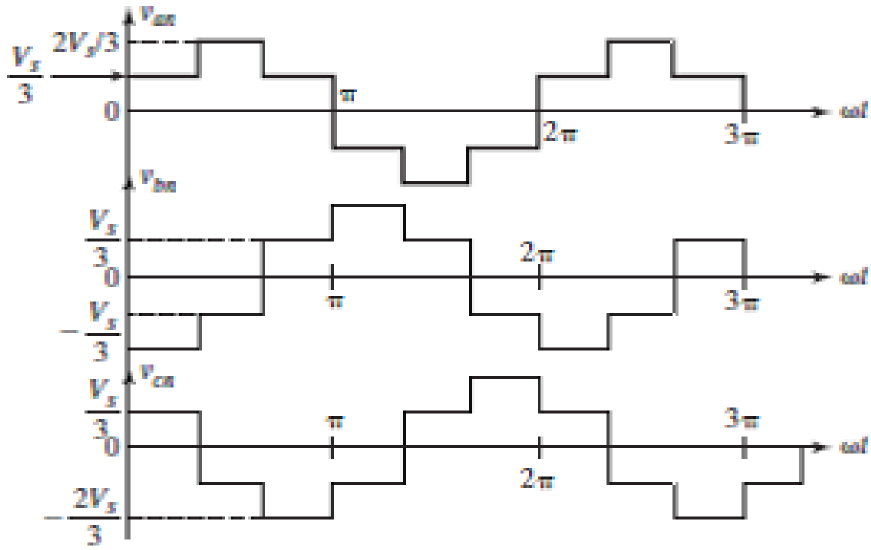
$$i_1 = \frac{V_s}{R_{eq}} = \frac{2V_s}{3R}$$

$$v_{an} = v_{cn} = \frac{i_1 R}{2} = \frac{V_s}{3}$$

$$v_{bn} = -i_1 R = \frac{-2V_s}{3}$$




(a) Equivalent circuits



(b) Phase voltages for 180° conduction

FIGURE 3.22

Equivalent circuits for Y-connected resistive load.

During mode 2 for $\pi/3 \le \omega t < 2\pi/3$, transistors $Q_1$, $Q_2$ and $Q_6$ conduct

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$i_2 = \frac{V_s}{R_{eq}} = \frac{2V_s}{3R}$$

$$v_{an} = i_2 R = \frac{2V_s}{3}$$

$$v_{bn} = v_{cn} = \frac{-i_2 R}{2} = \frac{-V_s}{3}$$

During mode 3 for $2\pi/3 \le \omega t < \pi$, transistors $Q_1$, $Q_2$, and $Q_3$ conduct

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$i_3 = \frac{V_s}{R_{eq}} = \frac{2V_s}{3R}$$

$$v_{an} = v_{bn} = \frac{i_3 R}{2} = \frac{V_s}{3}$$
$$v_{cn} = -i_3 R = \frac{-2V_s}{3}$$

The line-to-neutral voltages are shown in Figure 3.22b.The instantaneous line-to-line voltage $v_{ab}$ in Figure 3.20b can be expressed in a Fourier series,

$$v_{ab} = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

Due to the quarter-wave symmetry along the $x$-axis, both $a_0$ and $a_n$ are zero. Assuming symmetry along the $y$-axis at $\omega t = \pi/6$, we can write $b_n$ as

$$b_n = \frac{1}{\pi}\left[\int_{-5\pi/6}^{-\pi/6} -V_s \sin(n\omega t)\, d(\omega t) + \int_{\pi/6}^{5\pi/6} V_s \sin(n\omega t)\, d(\omega t)\right] = \frac{4V_s}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{3}\right)$$

which, recognizing that $v_{ab}$ is phase shifted by $\pi/6$ and the even harmonics are zero, gives the instantaneous line-to-line voltage $v_{ab}$ (for a Y-connected load) as

$$v_{ab} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\frac{n\pi}{3}\sin n\left(\omega t + \frac{\pi}{6}\right) \tag{3.63a}$$

Both $v_{bc}$ and $v_{ca}$ can be found from Eq. (3.63a) by phase shifting $v_{ab}$ by 120° and 240°, respectively,

$$v_{bc} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\frac{n\pi}{3}\sin n\left(\omega t - \frac{\pi}{2}\right) \tag{3.63b}$$

$$v_{ca} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi}\sin\left(\frac{n\pi}{2}\right)\sin\frac{n\pi}{3}\sin n\left(\omega t - \frac{7\pi}{6}\right) \tag{3.63c}$$

We can notice from Eqs. (3.63a) to (3.63c) that the triplen harmonics ($n = 3, 9, 15, \ldots$) would be zero in the line-to-line voltages.

The line-to-line rms voltage can be found from

$$V_L = \left[ \frac{2}{2\pi} \int_0^{2\pi/3} V_s^2 \, d(\omega t) \right]^{1/2} = \sqrt{\frac{2}{3}} V_s = 0.8165 V_s \tag{3.64}$$

From Eq. (3.63a), the rms $n$th component of the line voltage is

$$V_{Ln} = \frac{4V_s}{\sqrt{2} n\pi} \sin \frac{n\pi}{3} \tag{3.65}$$

which, for $n = 1$, gives the rms fundamental line voltage.

$$V_{L1} = \frac{4V_s \sin 60^\circ}{\sqrt{2}\pi} = \frac{\sqrt{6}\, V_s}{\pi} = 0.7797 V_s \tag{3.66}$$

The line-to-neutral voltages in Figure 3.22 can be expressed in a Fourier series as:

$$v_{an} = \sum_{n=1,5,7\ldots}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t \tag{3.67}$$

$$v_{bn} = \sum_{n=1,5,7\ldots}^{\infty} \frac{2V_s}{n\pi} \sin \left( n \left( \omega t - \frac{2\pi}{3} \right) \right) \tag{3.68}$$

$$v_{cn} = \sum_{n=1,5,7\ldots}^{\infty} \frac{2V_s}{n\pi} \sin \left( n \left( \omega t + \frac{2\pi}{3} \right) \right) \tag{3.69}$$

The rms value of line-to-neutral voltages can be found from the line voltage,

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{\sqrt{2}\, V_s}{3} = 0.4714 V_s \tag{3.70}$$

The rms value of the fundamental line-to-neutral voltage can also be obtained from the corresponding line-to-line voltage as:

$$V_{P1} = \frac{\sqrt{2}V_S}{\pi} = 0.4502V_S \tag{3.71}$$

Thus it is seen that for 180° mode, the **line-to-line** output voltage is a **quasi-square wave** with amplitude of +/- $V_S$ and pulse width of 120°, while the **phase** output voltage is a **six step** waveform with amplitudes of +/- 1/3$V_S$ and +/- 2/3$V_S$.

This can be compared with 120° mode where it is seen that the **phase** output voltage is a **quasi-square wave** with amplitude of +/- $V_S$/2 and pulse width of 120°, while the **line-to-line** output voltage is a **six step** waveform with amplitudes of +/- $V_S$/2 and +/-$V_S$.

# Waveform symmetries for Fourier Analysis

## 1. Even symmetry:

$f(t) = f(-t)$ $\rightarrow b_n = 0$ for all n $\rightarrow$ **only DC and cosine terms present**

or $f(\omega t) = f(-\omega t)$

hence $a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\omega t)d(\omega t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\omega t)d(\omega t)$

$$\boldsymbol{a_0 = \frac{1}{\pi}\int_0^{\pi} f(\omega t)d(\omega t)} \qquad \textbf{(1.1)}$$

$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)\cos(n\omega t)\, d(\omega t)$

or $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\omega t)\cos(n\omega t)\, d(\omega t)$

$$\boldsymbol{a_n = \frac{2}{\pi}\int_0^{\pi} f(\omega t)\cos(n\omega t)\, d(\omega t)} \qquad \textbf{(1.2)}$$

$\boldsymbol{b_n = 0}$ **for all n** **(1.3)**

## 2. Odd symmetry:

$f(t) = -f(-t)$ → $a_n = 0$ for all n → **only sine terms present**

or $f(\omega t) = -f(-\omega t)$

hence $b_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)\sin(n\omega t)\, d(\omega t)$

or $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\omega t)\sin(n\omega t)\, d(\omega t)$

$$\mathbf{b_n = \frac{2}{\pi}\int_0^{\pi} f(\omega t)\sin(n\omega t)\, d(\omega t)} \qquad \textbf{(2.1)}$$

$$\mathbf{a_n = 0 \text{ for all } n} \qquad \textbf{(2.2)}$$

## 3. Half-wave symmetry:

$f(t) = -f(t \pm T/2)$ → $a_0$, $b_0 = 0$ for n = 0, 2, 4, 6 ...→ **only odd terms present**

or $f(\omega t) = -f(\omega t \pm \pi)$

### 3.1. Even half-wave symmetry: → only DC and even cosine terms present

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(\omega t)d(\omega t) \qquad (1.1)$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(\omega t)\cos(n\omega t)\, d(\omega t) \qquad (1.2)$$

**n is even (2, 4, 6…)**

$$b_n = 0 \text{ for all n} \qquad (1.3)$$

### 3.2. Odd half-wave symmetry: → only odd sine terms present

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(\omega t)\sin(n\omega t)\, d(\omega t) \qquad (2.1)$$

**n is odd (1, 3, 5…)**

$$a_n = 0 \text{ for all n} \qquad (2.2)$$