DETT

Tutorial-II

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Q1) $(D^2-9D+18)y = e^{-e^{3x}}$

$D^2-9D+18=0$

$D^2-6D-3D+18=0$

$D(D-6)-3(D-6)=0$

$(D-3)(D-6)=0$

C.F is $C_1e^{6x}+C_2e^{3x}$

P.I $\Rightarrow \frac{1}{(D-6)(D-3)} e^{e^{-3x}}$

$= \frac{1}{D-6} e^{3x}\int e^{-3x}e^{e^{-3x}}dx$

$= \frac{1}{D-6}\frac{e^{3x}}{-3}\int e^t dt$ $\quad \left[\begin{array}{l} e^{-3x}=t \\ -3e^{-3x}dx=dt \end{array}\right)$

$= \frac{1}{D-6}\frac{e^{3x}}{-3}e^{e^{-3x}}$

$= \frac{e^{6x}}{-3}\int e^{-6x+3x}e^{e^{-3x}}dx$

$= \frac{e^{6x}}{-3}\int e^{-3x}\cdot e^{e^{-3x}}dx$

$\therefore$ P.I $= \frac{e^{6x}}{-3}e^{e^{-3x}}$

$\therefore$ Soln. is $y = \frac{-e^{6x}e^{e^{-3x}}}{3} + C_1e^{6x} + C_2e^{3x}$

Q2) $(D^3+D)y = \cos x$

$$P.I = \frac{1}{D^3+D}\cos x \quad a=1$$

put $D^2 = -1^2 = -1$

$$P.I = \frac{1}{D^2 \cdot D + D}\cos x$$

$$= \frac{1}{-D+D}\cos x$$

Now,

Diff D & multiply by x

$$= x\,\frac{1}{3D^2+1}\cdot\cos x$$

$\therefore$ put $D^2 = -1$

$$P.I = -\frac{x\cos x}{2}$$

$$C.F = C_1\cos x + C_2\sin x + C_3$$

$$C.F = D^3 + D = 0$$

$$D(D^2+1) = 0$$

$$D = 0, i, -i$$

$$G.I = CF + PI$$

$$= -\frac{x}{2}\cos x + C_1 + C_2\cos x + C_3\sin x$$

Q3) $(D^3-5D^2+8D-4)y = e^{2x}+2^{-x}+3$

$A.E = D^3-5D^2+8D-4=0$

$D=1$, $(D-1)$ is a factor

$D=1$	1	−5	8	−4
		1	−4	4
	1	−4	4	0

$D^2-4D+4=0$

$D^2-2D-2D+4=0$

$D(D-2)-2(D-2)=0$

$D=2 \quad D=2$

$CF = (C_1+C_2x)e^{2x}+C_3e^{x}$

$P.I = \frac{1}{D^3-5D^2+8D-4}e^{2x} + \frac{1}{D^3-5D^2+8D-4}e^{-x\log 2}$

$D=2$ $\qquad D=-\log 2$

D^r coming out to bezer $\qquad + \frac{1}{(D-1)(D+2)(D+2)}$

$\Rightarrow$ D^r & multiby by x $\qquad D=0$

$\frac{x}{3D^2-10D+8}e^{2x} + \frac{1}{(-\log 2)^3-5(-\log 2)^2+8(\log 2)-4}$

$\Rightarrow$ it is comming to be zero

diff again & multiply by x

$PI = \frac{x^2}{6D-10}e^{x} + \frac{2^{-x}}{(-\log 2)^3-5(-\log 2)^2+8(-\log 2)-4}$ $\left(-\frac{3}{4}\right)$

$GS = C_4e^{x}+(C_2+(3x)e^{2x}+PI$

Q4) $(D^3+8)y = x^4+2x+1$

$cf = 0$

$\Rightarrow D^3+8=0$

$D^3 = -8 \Rightarrow D = -1 \pm i\sqrt{3}, -2$

$\alpha = 1, \beta = \sqrt{3}$

$cf = e^{-x}(C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x) + C_3 e^{-2x}$

$\because D^3 + (2)^3$

$= (a+b)(a^2-ab+b^2)$

$= (D+2)(D^2-2D+(2)^2)$

$PI = \frac{1}{D^3+8}(x^4+2x+1)$

$= \frac{1}{8}\left(1+\frac{D^3}{2}\right)^{-1}(x^4+2x+1)$

$= \frac{1}{8}\left(1-\frac{D^3}{8}\right) * (x^4+2x+1)$

$D(f(x)) = 4x^3+2$

$= 12x^2$

$= 24x$

$D(fx) = 24$

∴ The f(x) can be diff 4 time hence. we will discard D^5 onwards.

$P.I = 2 - 24x(x^4+2x-1)$

$= x^4+2x-1-24x^5-48x^2+24x$

$= -24x^5 + x^4 - 48x^2 + 26x - 1$

$GS = cf + PI$

$= e^{-x}(C_1 \cos\sqrt{3}x + C_2 \sin\sqrt{3}x) + C_3 e^{-2x}$

$-24x^5 + x^4 - 48x^2 + 26x + 7$

→

9) $(D^2+2D+1)y = \frac{e^{-x}}{x+2}$

AE $= D^2+2D-1$

roots are: $\frac{-2 \pm \sqrt{4-4}}{2a}$

$\therefore -1, -1$

$C.F = (c_1 + c_2 x)e^{-x}$

$P.I = \frac{1}{D^2+2D+1}\left(\frac{e^{-x}}{x+2}\right)$

put $D = D-1$

$= e^{-x} \frac{1}{(D-1)^2+2(D-1)+1} \cdot \frac{1}{x+2}$

$= e^{-x} \frac{1}{D^2-2D+1+2D-2+1} \cdot \frac{1}{x+2}$

$= e^{-x} \frac{1}{D^2} \cdot \frac{1}{x+2}$

$= e^{-x} \frac{1}{D^2} \frac{1}{x+2}$

$= e^{-x} \frac{1}{D} \int \frac{1}{x+2}$

$= e^{-x} \frac{1}{D} \log|x+2) + c_3$

$= e^x \int \log(x+2) + c_3$

$P.I = e^{-x}[x+2)\log(x+2) - 2x + \log(x+2)] + c_4$

6) $(D^2+3D+2)y = x\sin 2x$

$\rightarrow$ AE $\Rightarrow D^2+2D+D+2=0$

$\Rightarrow D(D+2)+1(D+2)=0$

$(D+1)(D+2)=0$

$D=-1, D=-2$

$CF = C_1e^{-x} + C_2e^{-2x}$

$P.I \Rightarrow y = \frac{1}{D^2+3D+2} \cdot x\sin 2x$

$= \left[x - \frac{2D+3}{D^2+3D+2}\right] \frac{1}{D^2+3D+2} \sin 2x$

$D^2 = -4$

$= \left[x - \frac{2D+3}{-4+3D+2}\right] \frac{1}{-4+3D+2} \sin 2x$

$= \left[x - \frac{2D+3}{3D-2}\right] \frac{1}{3D-2} \sin 2x$

multiply N^r & D^r by conjugate.

$= \left[x - \frac{2D+3}{3D-2}\right] \frac{3D+2}{9D^2-4} \sin 2x$

$= \left[x - \frac{2D+3}{3D-2}\right] \frac{3D+2}{-4D}$

$= +\frac{3D+2}{4D}x + \frac{(2D+3)(3D+2)}{4D(3D-2)}$

$= -\frac{3D+2}{4D}x + \frac{6D^2+13D+6}{4D(3D-2)}$

$= \frac{-3D+2}{4D}x + \frac{-24+13D+6}{12D-8D}$

$= -\frac{3D+2}{4D}x + \frac{13D-18}{12D-8D}$