Tutorial-14

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Q1) Find z-transform of the following sequence & mention the RoC:

① $f(k) = (1/3)^k \quad k \geq 0$
$= 2^k \quad k < 0$

soln: By def ->

$$z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$z\{f(k)\} = \sum_{k=-\infty}^{-1} 2^k z^{-k} + \sum_{0}^{\infty} (1/3)^k z^{-k}$$

By standard sequence.

$$\Rightarrow \frac{z}{2-z} + \frac{z}{z-(1/3)}$$

$$\text{RoC} \Rightarrow |z| < |2| \quad |z| > |1/3|$$

$$= \left|\frac{1}{3}\right| < |z| < |2|$$

Q2) $f(k) = \cos\left(\frac{k\pi}{4} + \alpha\right) \quad k \geq 0$

$\rightarrow$ By Def $\Rightarrow$

$$Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$Z\left\{\cos\left(\frac{k\pi}{4} + \alpha\right)\right\} = \sum_{k=0}^{\infty} \left(\cos\frac{k\pi}{4}\cos\alpha - \sin\frac{k\pi}{4}\sin\alpha\right) z^{-k}$$

$$\Rightarrow \sum_{k=0}^{\infty}\left(\cos k\frac{\pi}{4}\cos\alpha\, z^{-k}\right) - \sum_{k=0}^{\infty}\left(\sin\frac{k\pi}{4}\sin\alpha\, z^{-k}\right)$$

Using standard sequence output

$$\Rightarrow \frac{\cos\alpha\, z(z - \cos\pi/4)}{z^2 - 2z\cos\left(\frac{\pi}{4}\right) + 1} - \frac{\sin\alpha\, z\sin(\pi/4)}{z^2 - 2z\cos(\pi/4) + 1}$$

$$\Rightarrow z\left[\frac{\cos\alpha\,(z - 1/\sqrt{2}) - \sin\alpha \times 1/\sqrt{2}}{z^2 - \sqrt{2}z + 1}\right]$$

ROC $\Rightarrow |z| > 1$

$\rightarrow$

Q3) $f(k) = e^{-3k} \sin 4k \quad k \geq 0$

soln -> By def ->

$$z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$z\{e^{-3k} \sin 4k\} = \sum_{k=0}^{\infty} e^{-3k} \sin 4k \, z^{-k}$$

By using property =>

Here $\alpha = 3$

$$z\{\sin 4k\} = \sum_{k=0}^{\infty} \sin 4k \, z^{-k}$$

$$\Rightarrow \frac{z \sin 4}{z^2 - 2z\cos 4 + 1}$$

ROC for $f(k) = \sin(4k) \Rightarrow |z| > 1$

$$\therefore z\{e^{-3k} \sin 4k\} = \sum_{k=0}^{\infty} e^{-3k} \sin 4k \, z^{-k}$$

Ans => $\dfrac{e^3 z \sin 4}{e^6 z^2 - 2e^3 z \cos 4 + 1}$

ROC => $|ze^3| > 1$

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(4) $F(k) = 2^k \cos(3k+2) \quad k \geq 0$

soln → By defn →

$$Z\{F(k)\} = \sum_{\cancel{k=\infty}\ k=-\infty}^{\infty} F(k) z^{-k}$$

$$Z\{2^k \cos(3k+2)\} = \sum_{k=0}^{\infty} 2^k \cos(3k+2) z^{-k}$$

Here we will use property

$$\rightarrow Z\{a^k f(k)\} = F(z/a)$$

Here $a=2$

$$Z\{\cos(3k+2)\} = \sum_{k=0}^{\infty} (\cos 3k \cos 2 z^{-k} - \sin 3k \sin 2 z^{-k})$$

$$\Rightarrow \frac{\cos 2(z \cos 3)}{z^2 - 2z\cos 3 + 1} - \frac{\sin 2 z \sin 3}{z^2 - 2z\cos 3 + 1}$$

$$\Rightarrow z\left[\frac{\cos 2(z - \cos 3) - \sin 2 \sin 3}{z^2 - 2z\cos 3 + 1}\right]$$

Applying property

$$\Rightarrow \frac{z/2\left[\cos 2(z/2 - \cos 3) - \sin 2 \sin 3\right]}{\frac{z^2}{4} - 4z\cos 3 + 4}$$

$$\Rightarrow \frac{2z\left[\cos 2(z/2 - \cos 3) - \sin 2 \sin 3\right]}{z^2 - 4z\cos 3 + 4}$$

ROC ⇒ $|z/2| > 1 \quad |z| > 2$

⑤ $f(k) = \sin\left(k\frac{\pi}{2} + \frac{\pi}{4}\right) \quad k \geq 0$

$\Rightarrow$ By defn -

$$\rightarrow z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

$$z\left\{\left(\sin k\frac{\pi}{2} + \frac{\pi}{4}\right\}_{k\geq 0} = \sum_{k=0}^{\infty} \sin\left(\frac{k\pi}{2} + \frac{\pi}{4}\right) z^{-k}$$

$$\Rightarrow \sum_{k=0}^{\infty} \left(\sin k\frac{\pi}{2} \cos\frac{\pi}{4} + \cos k\frac{\pi}{2} z^{-k}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\frac{z \sin \pi/2}{z^2 - 2z\cos \pi/2 + 1} + \frac{z(z - \cos \pi/2)}{z^2 - 2z\cos \pi/2 + 1}\right]$$

$$\rightarrow \frac{1}{\sqrt{2}}\left[\frac{z + z^2}{z^2 + 1}\right]$$

$$\Rightarrow \frac{z}{\sqrt{2}}$$

ROC $\Rightarrow |z| > 1$