Subject - LADC
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Div - 11

Tutorial - 4

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Q1) Solve.

A) Find n^{th} derivative of $\frac{x^4}{(x-1)(x-2)}$

Ans. $\frac{x^4-16+16}{(x-1)(x-2)}$

$\frac{(x^4-16)}{(x-1)(x-2)} + \frac{16}{(x-1)(x-2)}$

$\Rightarrow \frac{(x+2)(x-2)(x^2+4)}{(x-1)(x-2)} + \frac{16}{(x-1)(x-2)}$

$\Rightarrow \frac{(x-1+3)(x^2+4)}{(x-1)} + \frac{16}{(x-1)(x-2)}$

$\Rightarrow \frac{(x-1)(x^2+4)}{(x-1)} + \frac{3(x^2+4)}{(x-1)} + 16\left(\frac{1}{(x-2)} - \frac{1}{(x-1)}\right)$

$\Rightarrow x^2+4+\frac{3(x^2+5)}{(x-1)} + \frac{16}{x-2} - \frac{16}{x-1}$

$\Rightarrow x^2+4+\frac{3(x^2-1)}{(x-1)} + \frac{15}{(x-1)} + \frac{16}{(x-2)} + \frac{16}{(x-1)}$

$\Rightarrow x^2+4+3(x^3+1) + \frac{16}{(x-2)} + \frac{(15-16)}{(x-1)}$

$\Rightarrow x^2+4+3x+3+\frac{16}{(x-2)} - \frac{1}{(x-1)}$

$\Rightarrow x^2+3x+7-\frac{1}{(x-1)} + \frac{16}{(x-2)}$

differentiating w.r.t. x

$\Rightarrow \frac{d^n(x^2+3x+7)}{dx^n} - \frac{(-1)^n n!}{(x-1)^{n+1}} + \frac{16(-1)^n n!}{(x-2)^{n+1}}$

B) Find n^{th} derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Ans- Let $y = \tan^{-1}\frac{2x}{1-x^2}$

or $y = 2\tan^{-1}(n)$

$$y_1 = \frac{2}{1+n^2} = \frac{2}{(x+i)(x-i)} = \frac{1}{i}\left\{\frac{(x+i)-(x-i)}{(x+i)(n-i)}\right\}$$

$$= \frac{1}{i}\left\{\frac{1}{(x-i)} - \frac{1}{(x+i)}\right\}$$

$$y_{n+1} = \frac{1}{i}\left\{\frac{(-1)^n n!}{(x-i)^{n+1}} - \frac{(-1)^n . n!}{(x+i)^{n+1}}\right\}$$

$$= \frac{(-1)^n . n!}{i}\left\{\frac{1}{(x-i)^{n+1}} - \frac{\cancel{(-1)^n n!}}{(n+i)^{n+1}}\right\}$$

Puting $x = r\cos\theta$ & $1 = r\sin\theta$ & $r = \sqrt{n^2+1}$

& $\theta = \tan^{-1}\left(\frac{1}{x}\right)$

$$y_{n+1} = \frac{(-1)^n n!}{i r^{n+1}}\left\{\frac{1}{(\cos\theta - i\sin\theta)^{n+1}} - \frac{1}{(\cos\theta + i\sin\theta)^{n+1}}\right\}$$

$$= \frac{(-1)^n . n!}{i r^{n+1}}\left\{(\cos\theta - i\sin\theta)^{-(n+1)} - (\cos\theta + i\sin\theta)^{-(n+1)}\right\}$$

$$= \frac{(-1)^2 . n!}{i r^{n+1}}\left\{\cancel{\cos(n+1)\theta} + i\sin(n+1)\theta - \cancel{\cos(n+1)\theta} + i\sin(n+1)\theta\right\}$$

$y_{n+1} = \frac{2(-1)^n n!\ 2\cancel{i}\sin(n+1)\theta}{\cancel{i} r^{n+1}}$ & $\frac{1}{r} = \sin\theta$.

So $y_{n+1} = 2(-1)^n . n! \sin(n+)\theta \cdot \sin^{n+1}\theta$

So, $\boxed{y_n = 2(-1)^{n-1} . (n-1)!\ \sin n\theta . \sin^n\theta}$

where $\theta = \tan^{-1}(1/n)$

Q c) As the rotation is about y-axis & the translation is (u, v, w) rotation & translation together in matrix is given by.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & -u \\ 0 & 1 & 0 & -v \\ \sin\theta & 0 & \cos\theta & -w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

we have, $(u, v, w) = (-10, -5, 2)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(30) & 0 & -\sin(30) & 10 \\ 0 & 1 & 0 & 5 \\ \sin(30) & 0 & \cos(30) & -2 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 & 10 \\ 0 & 1 & 0 & 5 \\ 1/2 & 0 & \sqrt{3}/2 & -2 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 1 \end{bmatrix}$$

$\therefore x = \frac{\sqrt{3}}{2} \times 100 + 0 - \frac{100}{2} + 10$

$= 50\sqrt{3} - 50 + 10$

$= 50\sqrt{3} - 40$

$\boxed{x = 46.6}$

$y = 50 + 100 + 0 + 5$

$\boxed{y = 105}$

$z = \frac{100}{2} + 0 + \frac{\sqrt{3}}{2} \times 100 - 2$

$= 50 + \sqrt{3} + 50 - 2$

$= 50 \times \sqrt{3} + 48$

$\boxed{= 134.6}$

$\therefore$ The center of the arc of the circle in new coordinate system is.

$(x, y, z) = (46.6, 105, 134.6)$

Q2) Fill in the blanks -

A) Given a square whose coordinates are given by $A=(2,1)$ $B=(3,1)$, $C=(3,4)$, $D=(2,4)$
Translate square by 7 units right & 6 units down. Find new coordinates

Ans- The new coordinates are-
$A(9,-5)$ $B(10,-5)$ $C(10,-2)$ $D(9,-2)$

B) Given the line segment starting at a point $(0,0)$ ending point is $(8,1)$. Rotate line by 45 degree and find new coordinate.

Ans- New coordinates are. $(4.9, 6.3)$

C) n^{th} Derivative of $y=\sin^3(x)$ is

Ans- $y_n = \frac{1}{4}\left(3\sin\left(\frac{n\pi}{2}+x\right) - 3^n\sin\left(\frac{n\pi}{2}+3x\right)\right]$

D) n^{th} Derivative of $y=e^{2x}\cos(3x+4)$

Ans- $y_n = (13)^{n/2} \cdot e^{2x} \cdot \cos\left[3x+4+n \cdot \tan^{-1}\left(\frac{3}{2}\right)\right]$