Fuzzy Inference Systems

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Outline

Introduction

Mamdani fuzzy inference systems

Sugeno fuzzy inference systems

Tsukamoto fuzzy inference systems

Fuzzy modeling

Fuzzy Inference Systems

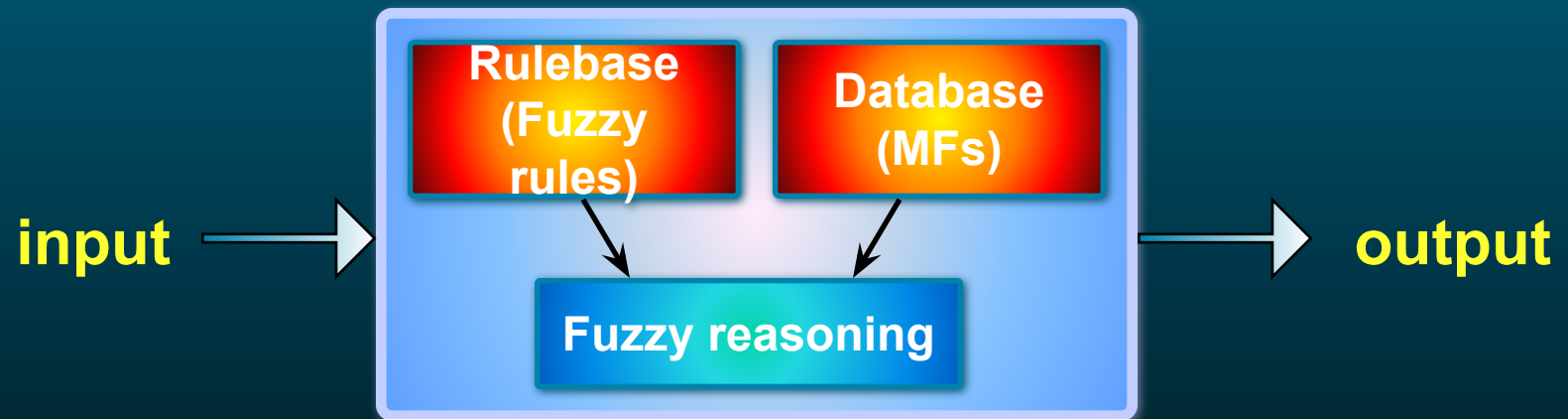**What is a fuzzy inference system (FIS)?**

> A nonlinear mapping that derives its output based on fuzzy reasoning and a set of fuzzy if-then rules. The domain and range of the mapping could be fuzzy sets or points in a multidimensional spaces.

Also known as

- **Fuzzy models**
- **Fuzzy associate memory**
- **Fuzzy-rule-based systems**
- **Fuzzy expert systems**

Fuzzy Inference Systems

Schematic diagram

Fuzzy Inference Systems

Operating block diagram

Max-Star Composition

Max-product composition:

$$\mu_{R_1 \cdot R_2}(x, z) = \bigvee_y [\mu_{R_1}(x, y)\mu_{R_2}(y, z)]$$

In general, we have max-* composition:

$$\mu_{R_1 \cdot R_2}(x, z) = \bigvee_y [\mu_{R_1}(x, y) * \mu_{R_2}(y, z)]$$

where * is a T-norm operator.

Linguistic Variables

A numerical variables takes numerical values:
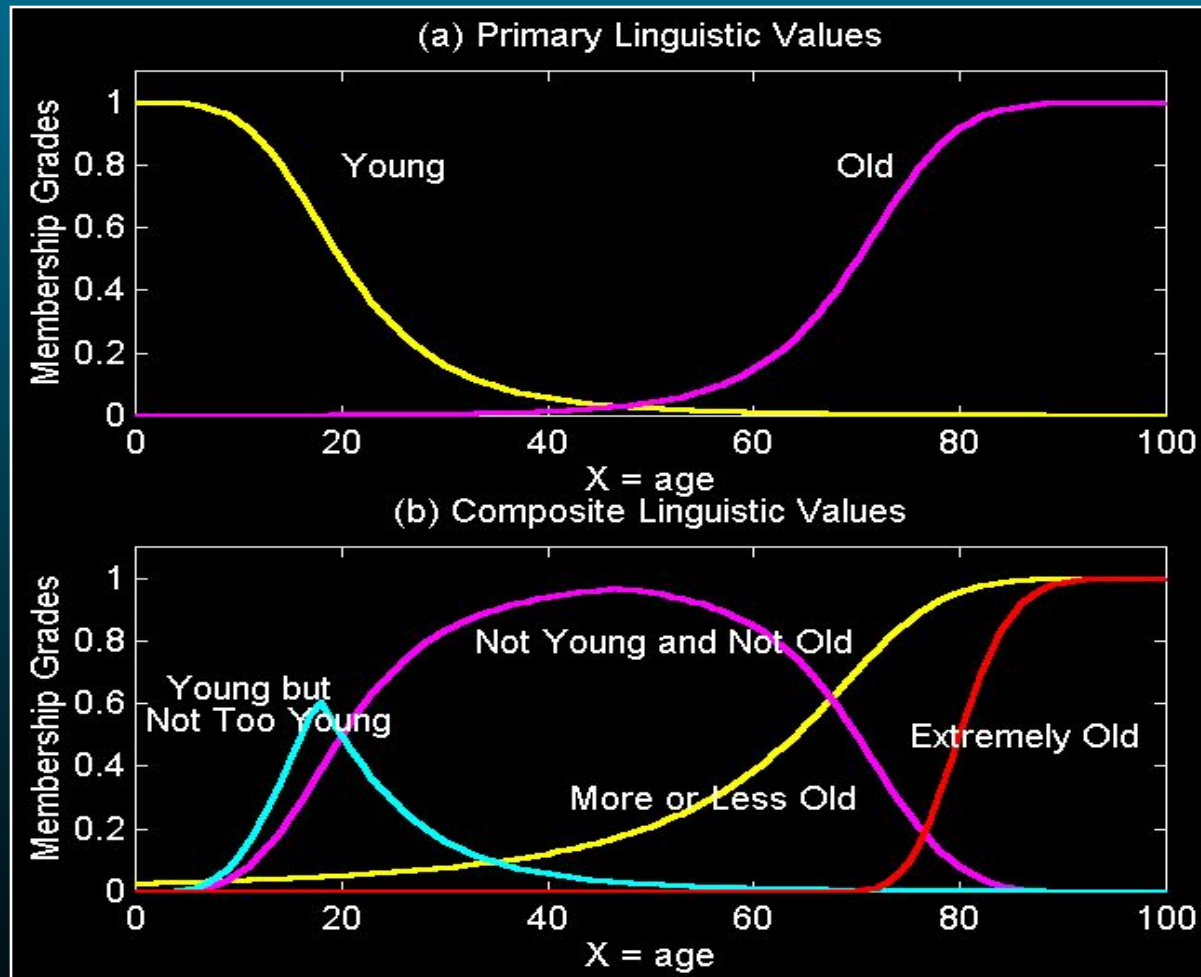
Age = 65

A linguistic variables takes linguistic values:

Age is old

A linguistic values is a fuzzy set.

All linguistic values form a term set:

T(age) = {young, not young, very young, ...

middle aged, not middle aged, ...

old, not old, very old, more or less old, ...

not very yound and not very old, ...}
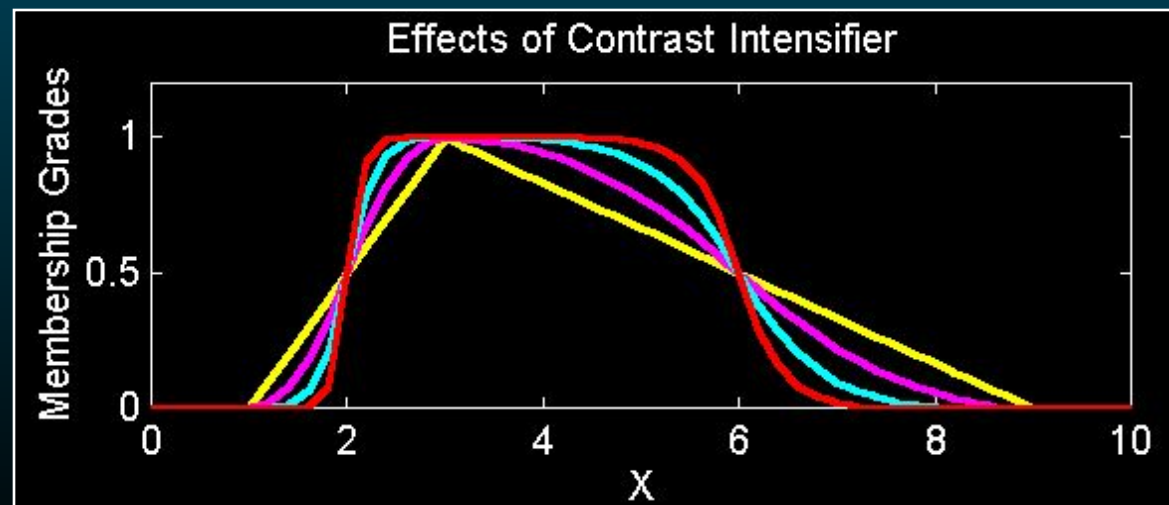
Linguistic Values (Terms)

(a) Primary Linguistic Values

(b) Composite Linguistic Values

complv.m

Operations on Linguistic Values

Concentration: $\longrightarrow$ $$CON(A) = A^2$$

Dilation: $\longrightarrow$ $$DIL(A) = A^{0.5}$$

Contrast intensification: $\longrightarrow$
$$INT(A) = \begin{cases} 2A^2, & 0 \leq \mu_A(x) \leq 0.5 \\ \neg 2(\neg A)^2, & 0.5 \leq \mu_A(x) \leq 1 \end{cases}$$

Effects of Contrast Intensifier

intensif.m

Fuzzy If-Then Rules

General format:

> **If x is A then y is B**

Examples:

- **If pressure is high, then volume is small.**
- **If the road is slippery, then driving is dangerous.**
- **If a tomato is red, then it is ripe.**
- **If the speed is high, then apply the brake a little.**

Fuzzy If-Then Rules

Two ways to interpret "If x is A then y is B":

A coupled with B
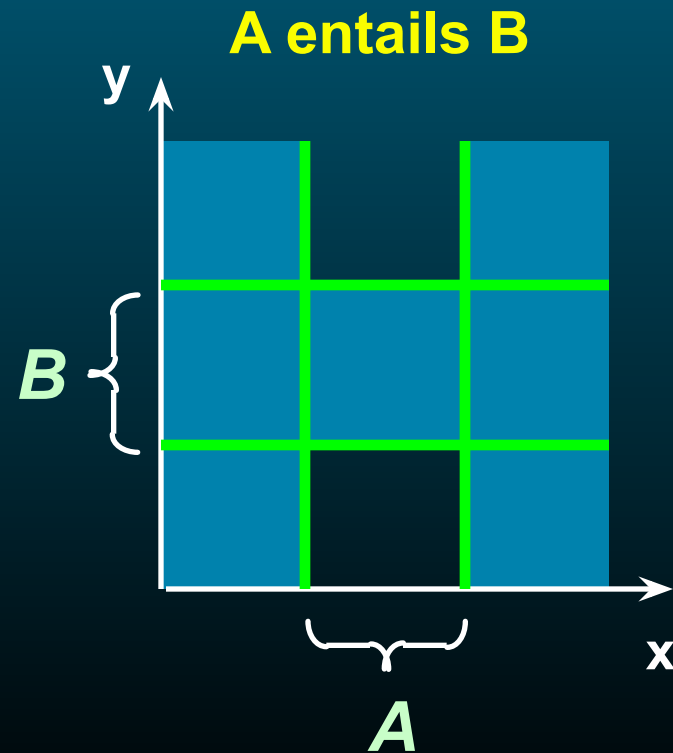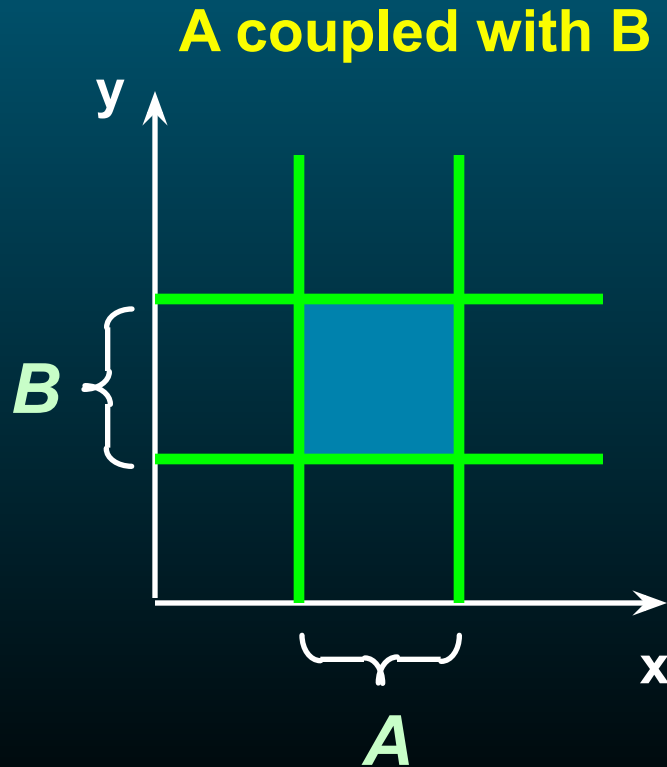
A entails B

Fuzzy If-Then Rules

Two ways to interpret "If x is A then y is B":

- **A coupled with B: (*A and B*)**

$$R = A \rightarrow B = A \times B = \int \mu_A(x) \overset{\sim}{*} \mu_B(y) | (x, y)$$

- **A entails B: (*not A or B*)**

    - Material implication

    - Propositional calculus

    - Extended propositional calculus

    - Generalization of modus ponens

Fuzzy If-Then Rules

Fuzzy implication function:

$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y)) = f(a, b)$$

A coupled with B
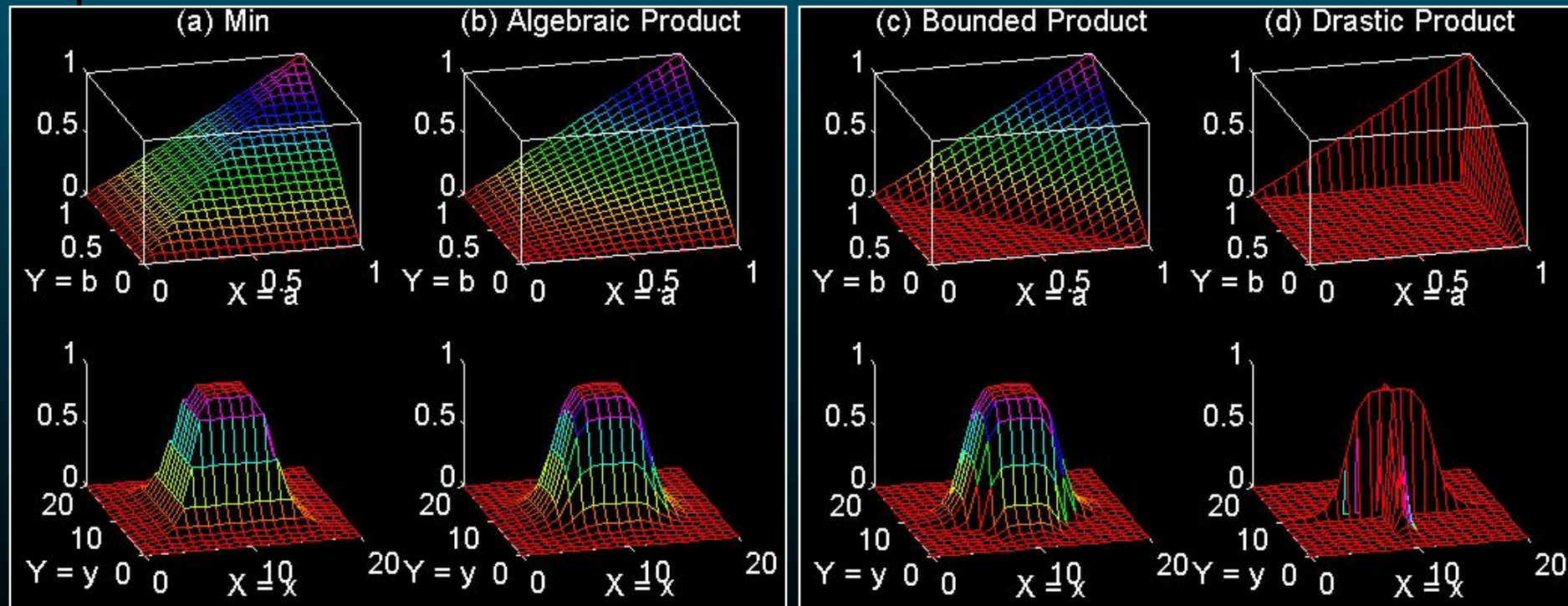
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Fuzzy If-Then Rules

A entails B
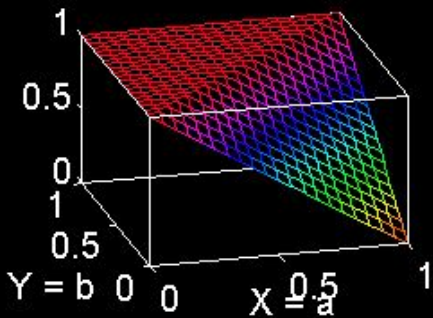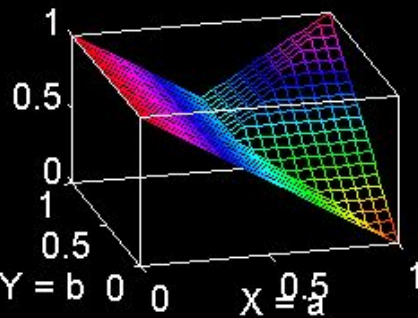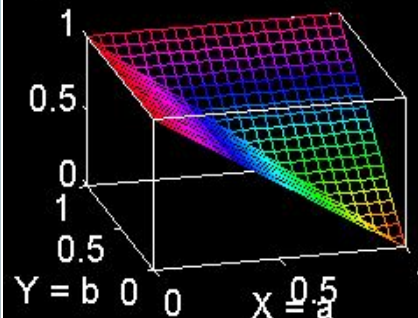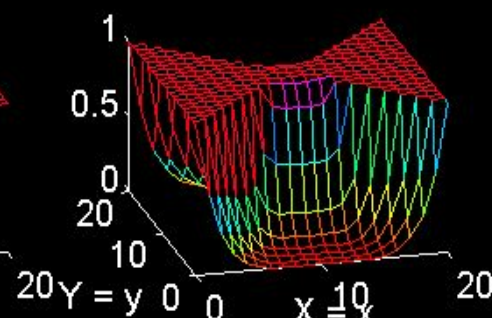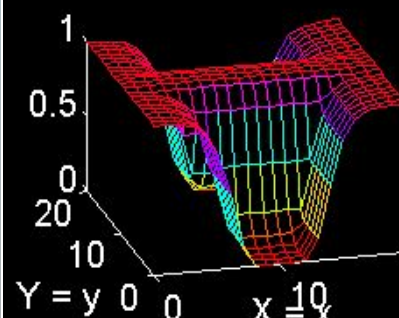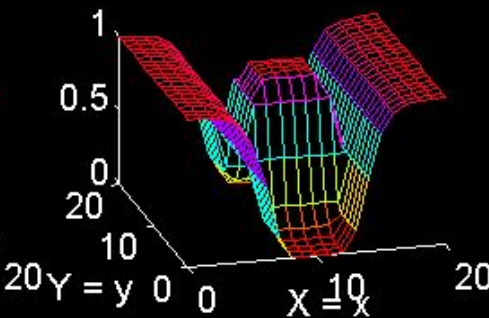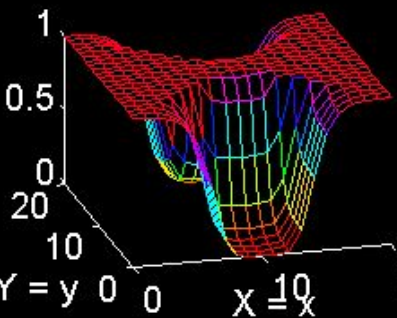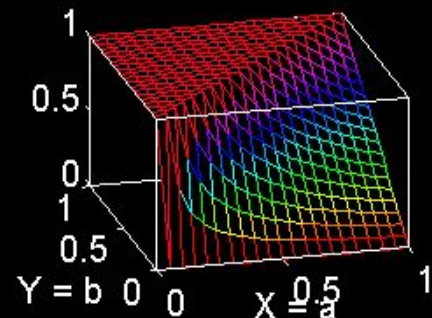
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Compositional Rule of Inference

Derivation of _y = b_ from _x = a_ and _y = f(x)_:
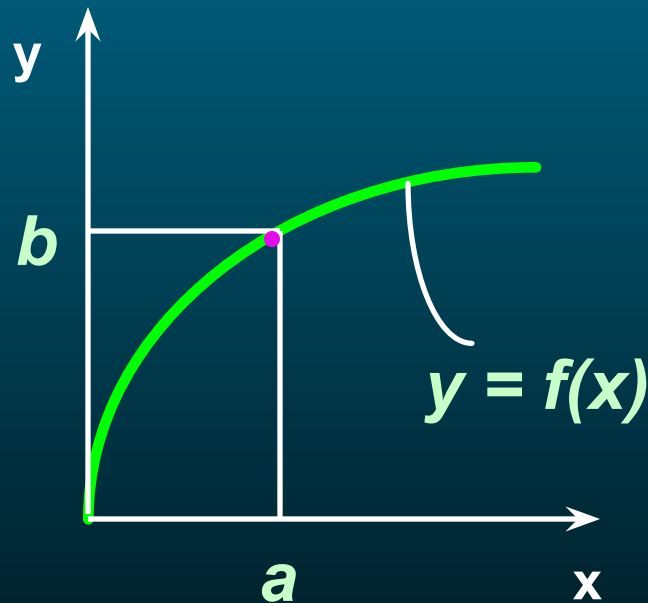
_a_ and _b_: points
_y = f(x)_ : a curve
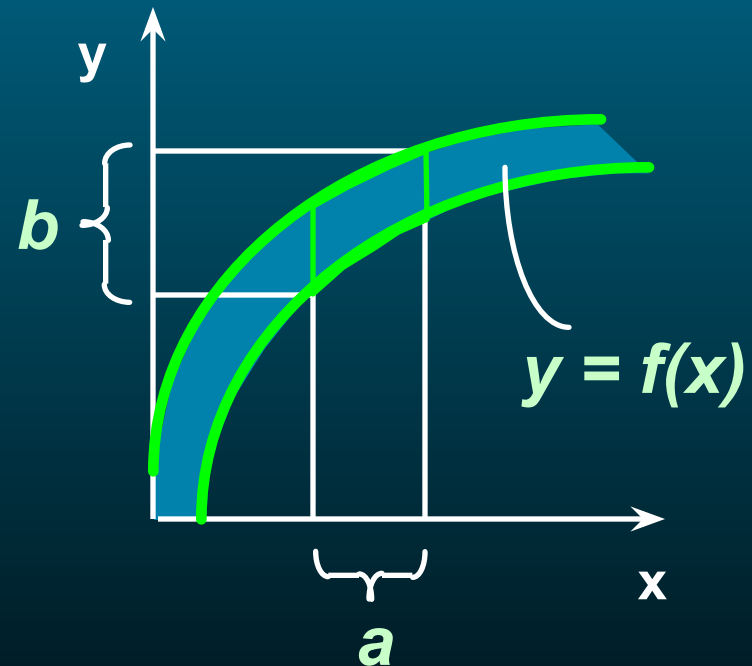
_a_ and _b_: intervals
_y = f(x)_ : an interval-valued
function

Compositional Rule of Inference

a is a fuzzy set and *y = f(x)* is a fuzzy relation:

cri.m
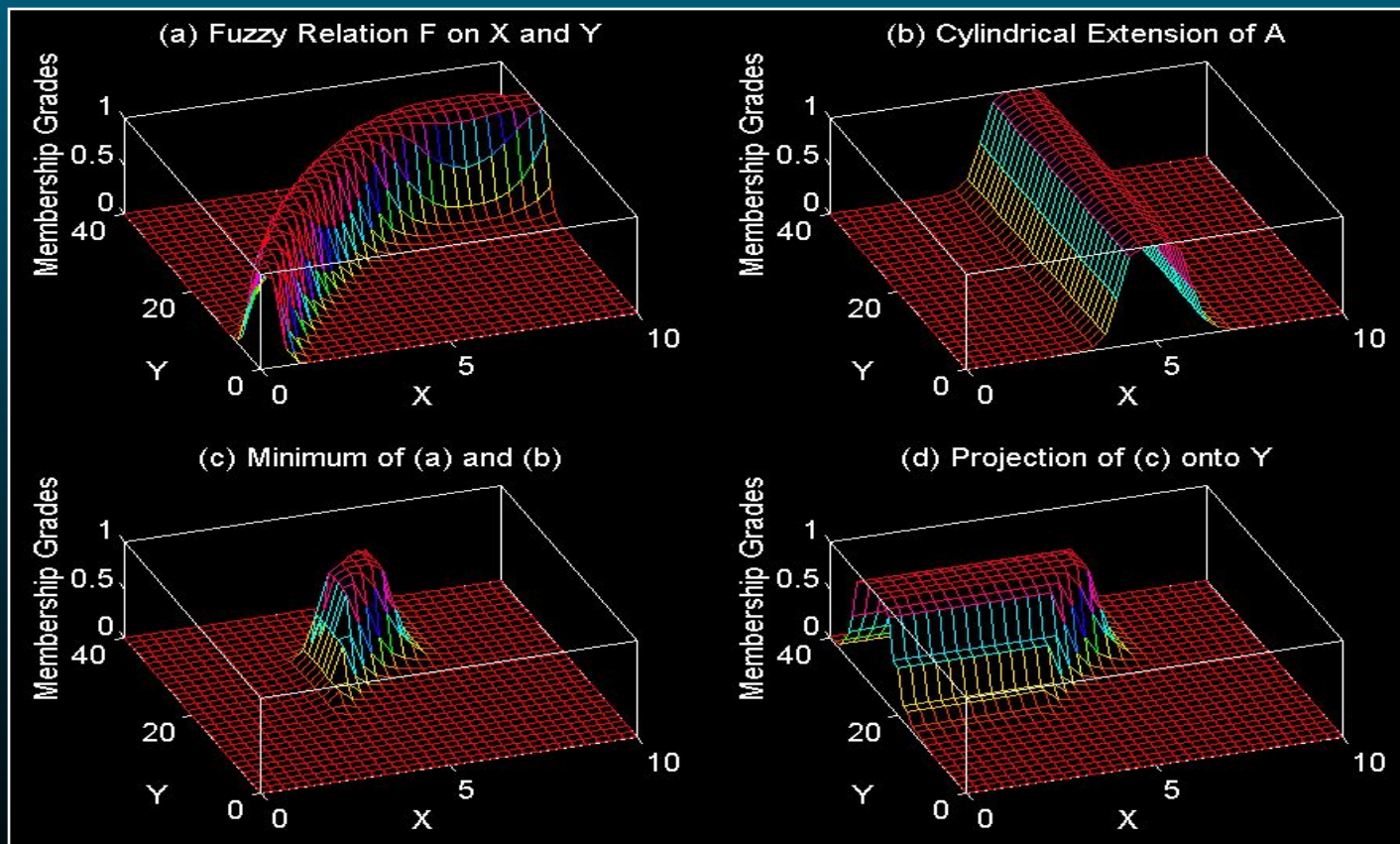
Fuzzy Reasoning

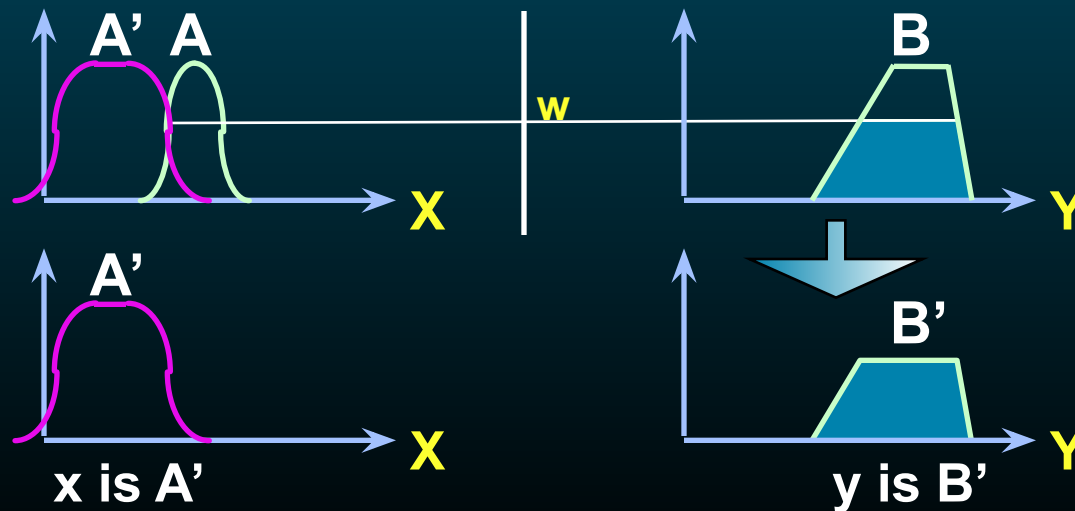**Single rule with single antecedent**

Rule: if x is A then y is B

Fact: x is A'

Conclusion: y is B'

Graphic Representation:

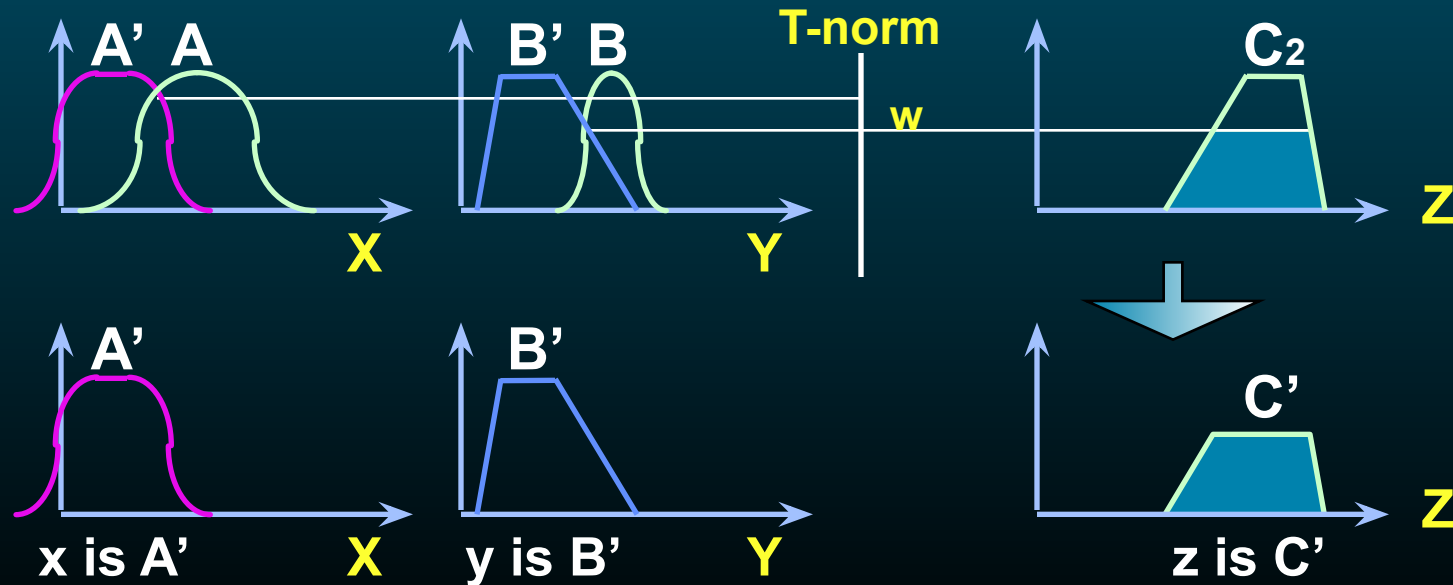A' A   w   B

x is A'

y is B'

Fuzzy Reasoning

Single rule with multiple antecedent

  Rule: if x is A and y is B then z is C

  Fact: x is A' and y is B'

  Conclusion: z is C'

Graphic Representation:

Fuzzy Reasoning

Multiple rules with multiple antecedent

> **Rule 1: if x is A_1 and y is B_1 then z is C_1**
>
> **Rule 2: if x is A_2 and y is B_2 then z is C_2**
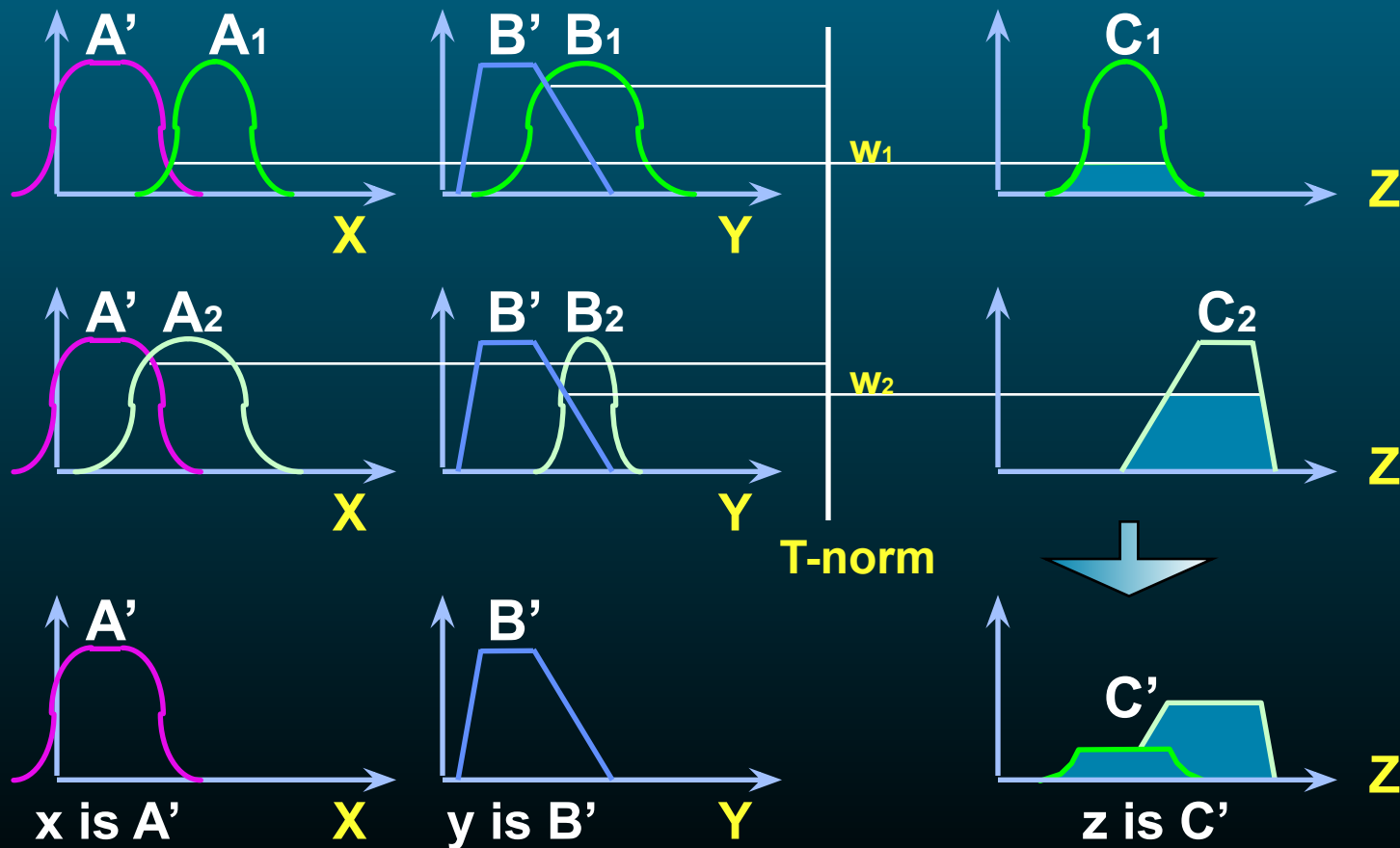>
> **Fact: x is A' and y is B'**
>
> **Conclusion: z is C'**

Graphic Representation: (next slide)
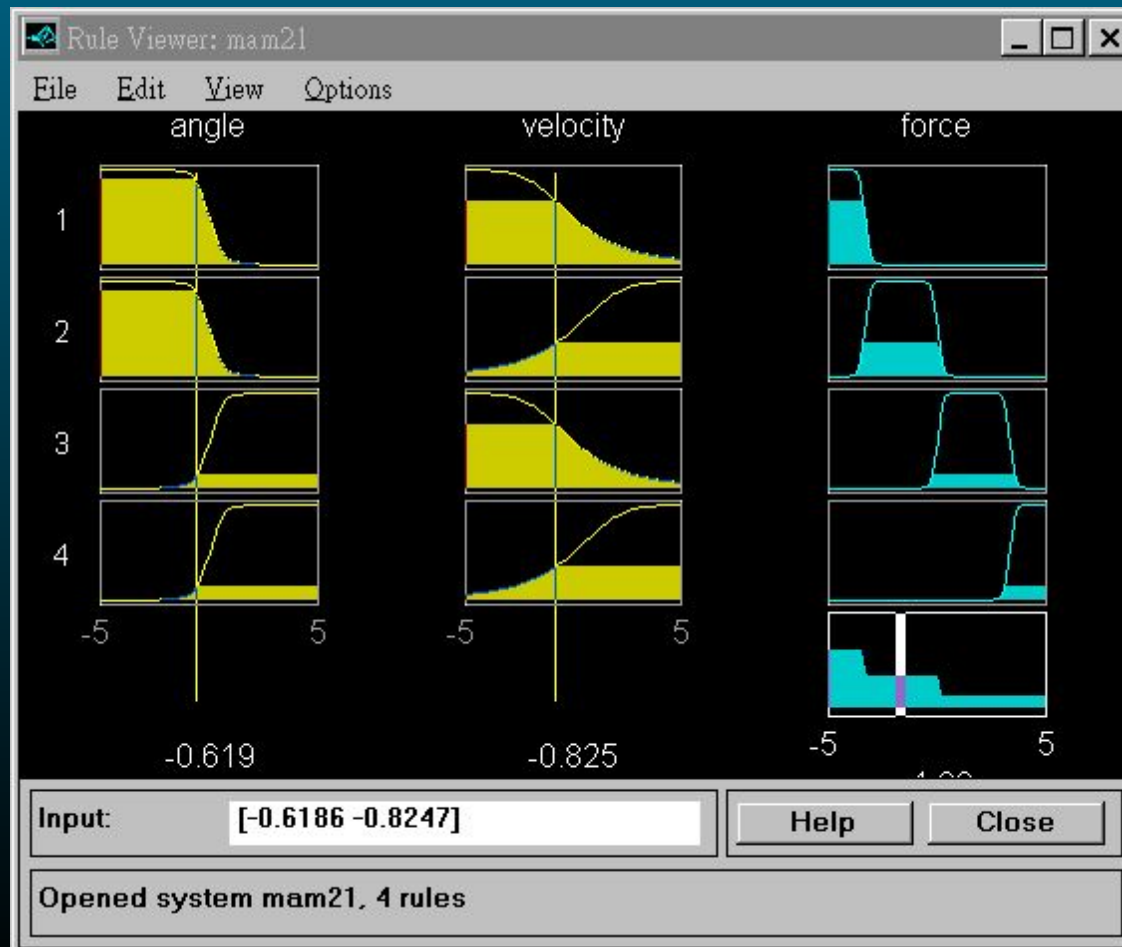
Fuzzy Reasoning

Graphics representation:

Fuzzy Reasoning: MATLAB Demo

>> ruleview mam21

Other Variants

Some terminology:

- **Degrees of compatibility (match)**
- **Firing strength**
- **Qualified (induced) MFs**
- **Overall output MF**