Unit 1 - Control systems Modeling

* system - It is the inter connection of number of elements connected together to get a desire output.
  Each system has number of subsystem to get the desire task.

* control system -
  control system consists of different connected together to perform specific task and to get a desire response.
  eg - ① Generating system - To generate power.

* Two types of control system -

① open loop cs & closed loop cs

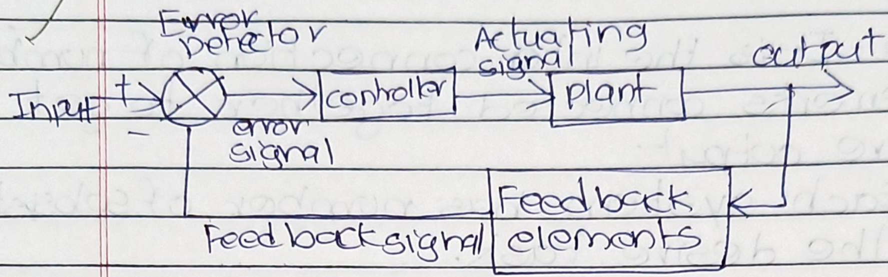
1) open loop cs → In open loop system output is independent of controlled input

Input parameter → controller → Actobtor → Process → output variable

① Output measurement not required.
② Feed back element absent
③ system is easy.

• Dis advantages -
① No feedback
② In case of disturbance output response may get change.
③ Accuracy is less
④ system is stable.

2) closed Loop system.



- Output Measurement is required.
- Feedback elements are present
- System is complicated.
- costly.

Advantages -
- Output of the system depends on the controlled input.
- High accuracy
- output remains the same due to external disturbance.

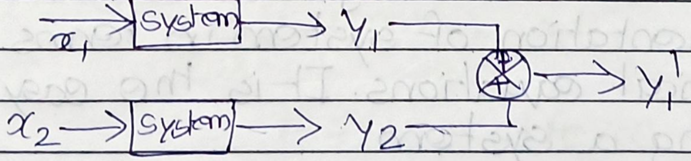
* Feedback control System -
A system in which output or proposition of output is feedback to input for comparing with the reference signal. Error signal is generated to drive the controller to get a desired response.

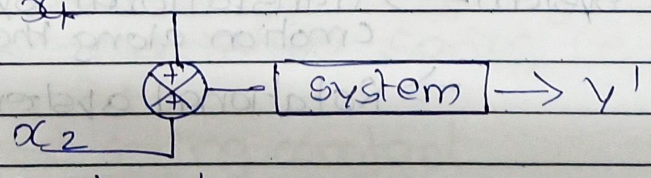
* Advantages –
- External disturbance & also due to internal parameter disturbance system remains stable & disturbance can be rejected from the system
- Internal Parameters becomes insensitive due to feedback elements.
- Due to over-correction of feedback elements system may become unstable.

* Types of control system –
- Natural control system.
- Manmade control system
- Combinational control system
- Time – " –

* Linear & Non linear systems ⇒

1) Linear systems – System which obeys superposition theorem & Homogeneity.





- If $y_1' = y'$
then, system is obeying superposition theorem.

Open loop	Closed loop
① Any change in output has no effect on the input ~~eg feedba~~	① Changes in output affects the input which is possible by use of feedback
② output measurement is not reaquired.	② output measurement is necessary.
③ Feedback element is absent	③ Feed back element is present.
④ error detector is absent	④ error detector is necessary.
⑤ It is inaccurate & unreliable	⑤ Highly accurate and reliable.
⑥ Highly sensitive to the disturbances	⑥ Less sensitive to the disturbances
⑦ simple to construct & cheap	⑧ Complicated to design and hence costly.
⑧ Highly affected by non-linearity	⑧ Redaced effect of nonlinearities.

* Types of control systems.

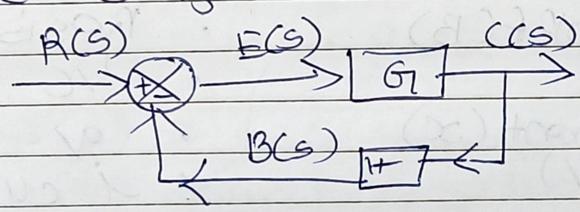
① Linear systems -
which obeys superposition theorem & Homogeneity.

② Non-Linear systems -
which donot follows superposition theorem & Homogeneity.

③ Time - variant system -
Parameters & output of the system donot changes with time.

④ Time - In variant system -
Parameters & output of the system changes with time

: Block diagram fundamentals.



$$\Rightarrow RS \rightarrow \boxed{\frac{G}{1 \pm GH}} \xrightarrow{C(S)}$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{G}{1 \pm GH}}$$

in general

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Page No.
Date

* Mathematical modelling of system-

(1) $\Rightarrow$ $$\boxed{F = \frac{Md^2x}{dt^2} + \frac{Bdx}{dt} + Kx}$$

* Mathematical modelling of electrical system

(2) $\Rightarrow$ $$\boxed{e = \frac{Ld^2q}{dt^2} + \frac{Rdq}{dt} + \frac{1}{C}q}$$

* Analogous parameters of mechanical & electrical systems:

Mech	Elec.
• Force	• Voltage
• Mass M	• Inductance L
• viscous coff (B)	• Resistance
• K	• 1/C
• Displacement (x)	• q
• velocity. (V)	• i current.

* current - force Analogy.

(3) $\Rightarrow$ $$\boxed{i = \frac{Cd^2\phi}{dt^2} + \frac{d\phi}{dt}\frac{1}{R} + \frac{1}{L}\phi}$$

* Laplace's Transform of a system.

$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Replace $\frac{dx}{dt} \rightarrow s$ $\quad \frac{d^2x}{dt^2} \rightarrow s^2$ $\quad x \rightarrow x(s)$

$$\therefore \boxed{f(s) = Ms^2x(s) + Bsx(s) + Kx(s)}$$

· Electrical system -

$$\boxed{e(s) = Ri(s) + Lsi(s) + \frac{1}{Cs}i(s)}$$

* Transfer function -

$$\xrightarrow{\delta(t)} \boxed{\text{System}} \longrightarrow g(t)$$

$$\underline{\mathcal{L}g(t) = G(s)}$$

- $\delta(t) \rightarrow$ unit impulse input
- $g(t) \rightarrow$ unit impulse response

Transfer function is defined as Laplace of impulse response of a system. Whenever transfer function of a system input is known then we can find out output or the response of system.

$$T.f = \frac{\text{Laplace of O/P}}{\text{Laplace of I/P}}$$

with ~~to~~ zero initial conditions

* T.F of mechanical system -

$F(s) = Ms^2 x(s) + Bs\cancel{x}(s) + Kx(s)$

i/p ↓ $F(s)$, o/p ↓ $x(s)$

$$\underset{i/p}{F(s)} = (Ms^2 + Bs + K)\underset{\to o/p}{x(s)}$$

$T.F = \frac{\mathcal{L}\, o/p}{\mathcal{L}\, i/p}$

$$\boxed{T.F = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}}$$

* Characteristics of T.F.

1. expresses in terms of S
2. Doesn't depend upon the type of input
3. expresses in terms of parameters of system
4. It doesn't give the info about the internal structure of system

* Poles & Zeros of a sys –

General T.f

$$T(s) = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^m + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$$T(s) = \frac{K \cdot (s-b_0)(s-b_1) \cdots (s-b_n)}{(s-a_0)(s-a_1) \cdots (s-a_n)}$$

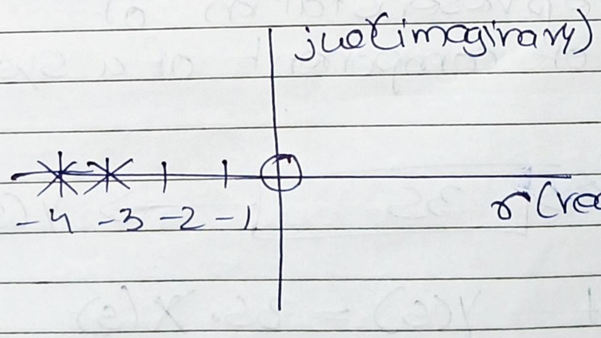
Zeros ⇒ These are ~~lovre~~ those valuse of s which make the T.f zero

$s = b_0, b_1, \ldots b_n$

Poles → These are those values of s which make the T.f infinity.

$s = a_0, a_1, \ldots a_n$.

* Plotting of Poles & Zeros.



$$T.F = \frac{1}{s^2+7s+12} = \frac{1}{(s+4)(s+3)}$$

Poles → ×

zeros → O

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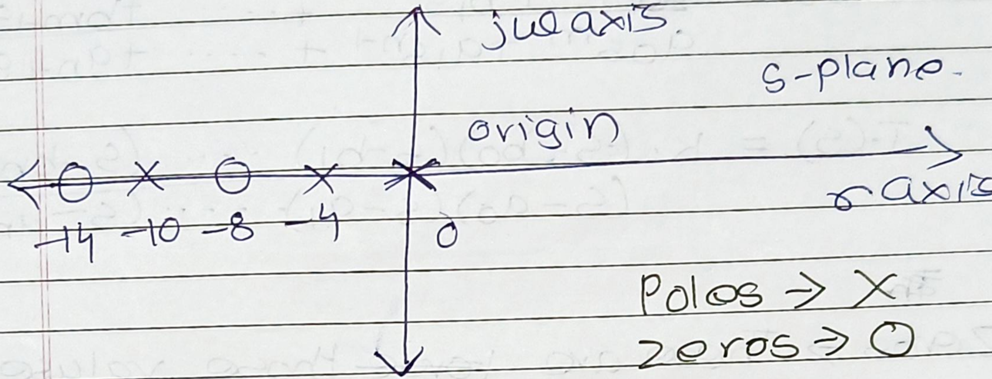
eg) $$G(s) = \frac{\overbrace{(s+8)(s+14)}^{\text{zeros}}}{\underbrace{s(s+4)(s+10)}_{\text{poles}}}$$ → zeros
→ Poles.



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* main elements of block diagram.

① Block.

block input → Block → output

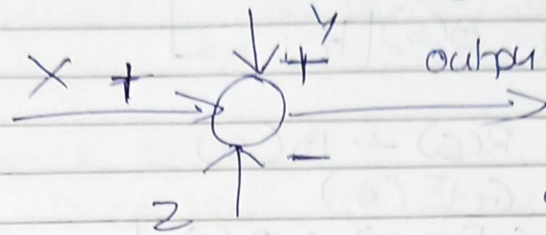
it is the representation of function of component of a system.

$X(s) \rightarrow \boxed{3s} \rightarrow Y(s)$

output $Y(s) = 3s \cdot X(s)$

(2) Summing Point –

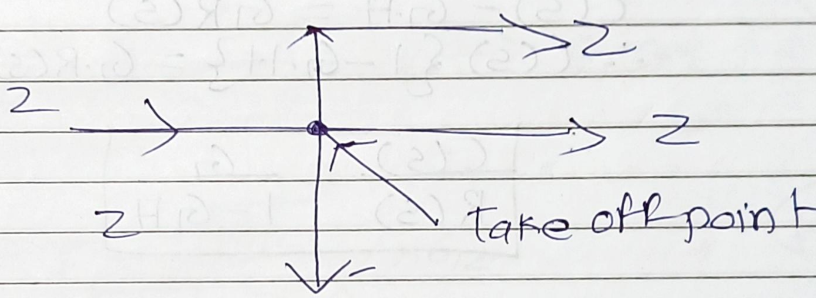
Two or more signals can be added / subtracted at summing point



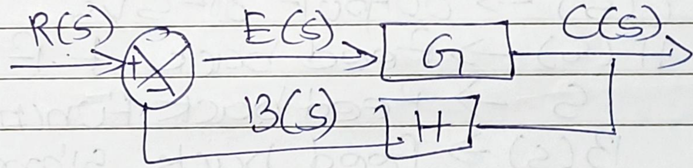
output $= x + y - z$

(3) Take off Point –

The output signal can be applied to two or more points from a take off point



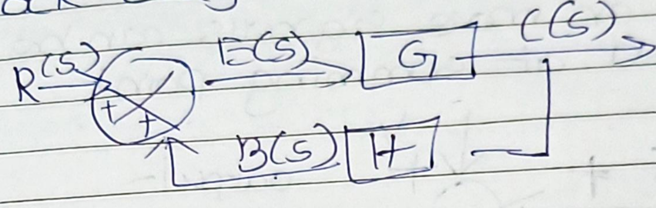
* Block Diag of Negative feed Back sys –



$$B(S) = C(S) \cdot H(S)$$
$$E(S) = R(S) - B(S)$$
$$C(S) = G(S) \cdot E(S)$$
$$= G(S)\ R(S) - B(S)$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)\,H(S)}}$$

* Block diag of Positive Feedback



$$E(S) = R(S) + B(S)$$

& $C(S) = G \cdot E(s)$

$= G[R(S) + B(S)]$

$= G R(S) + G B(s)$

but $B(s) = H \cdot C(s)$

$\therefore C(S) = G \cdot R(S) + G \cdot H C(S)$

$C(S) - G \cdot H = G R(S)$

$\therefore C(S)\{1 - G \cdot H\} = G \cdot R(S)$

$$\therefore \boxed{\frac{C(s)}{R(S)} = \frac{G}{1 - GH}}$$

Let $R(S) \Rightarrow$ reference input

$C(S) \rightarrow$ output of sys

$H(S) \rightarrow$ feed back T.F

$S \rightarrow$ feed back function

$B(S) \Rightarrow$ feed back signal

$E(S) \rightarrow$ Actuating lerror signal.

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in general

$$\frac{C(S)}{R(S)} = \frac{G}{1 \pm GH}$$

$+ \rightarrow$ Negative feedback

$- \rightarrow$ Positive feedback.

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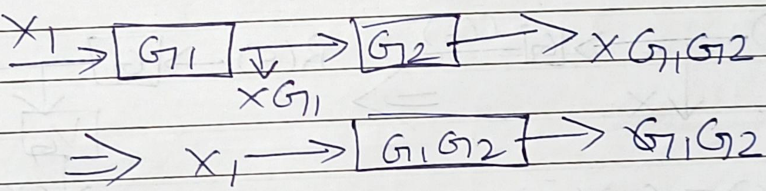
* Block diagram Reduction Rules –

- forward Path –
  direction of flow of signal is from input to output

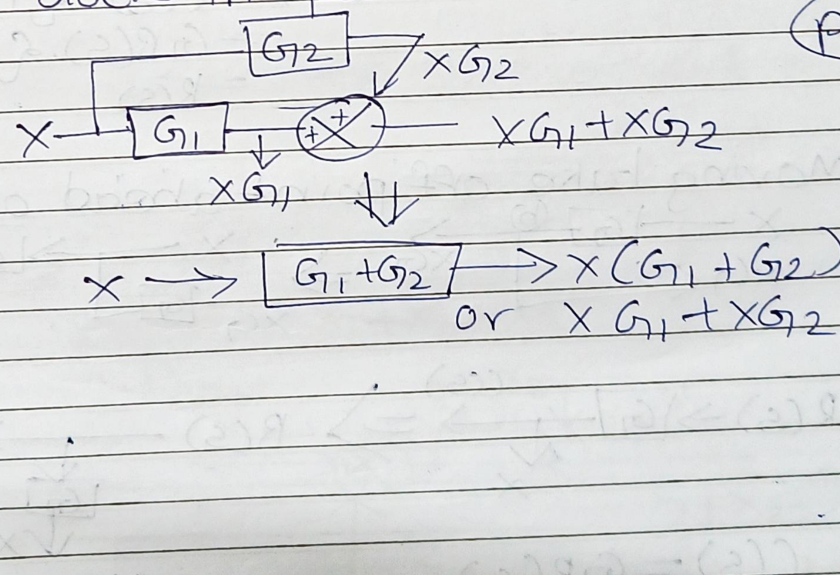
- feedback Path –
  Direction of flow of signal is from output to input.

(1) Blocks in Cascade → Can combine blocks



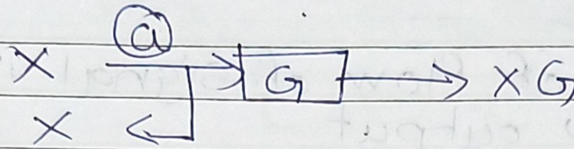
series
×

(2) Two blocks in parrallel –

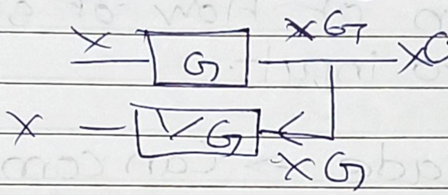


parallel
+

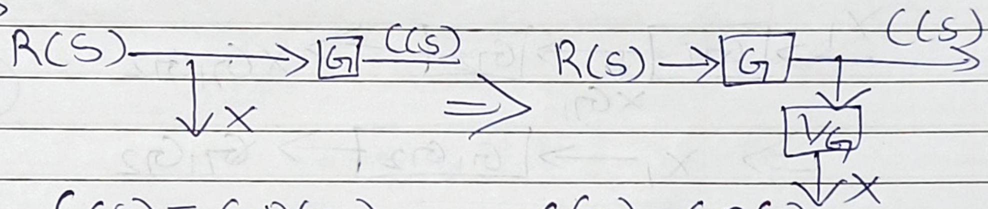
③ Moving take off point after the block -



moving take off point a after the block, block dig becomes ->



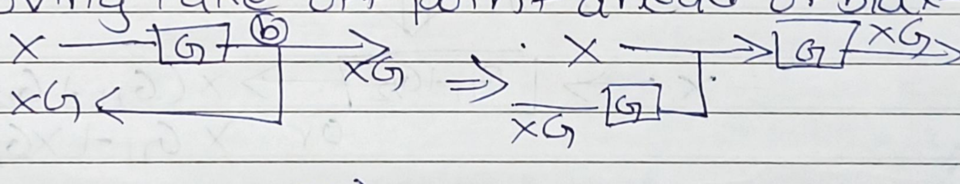
or =>



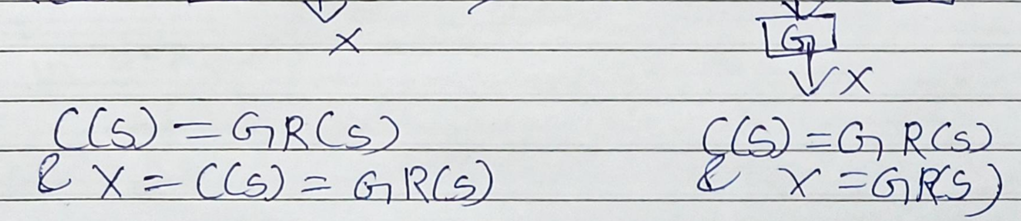
$C(S) = GR(S)$
$\&\ x = R(S)$

$C(S) = GR(S)$
$\&\ x = C(S) \cdot \{1/G\}$
$= GR(S) \cdot \{1/G\}$
$= R(S)$

④ Moving take off point ahead of block -



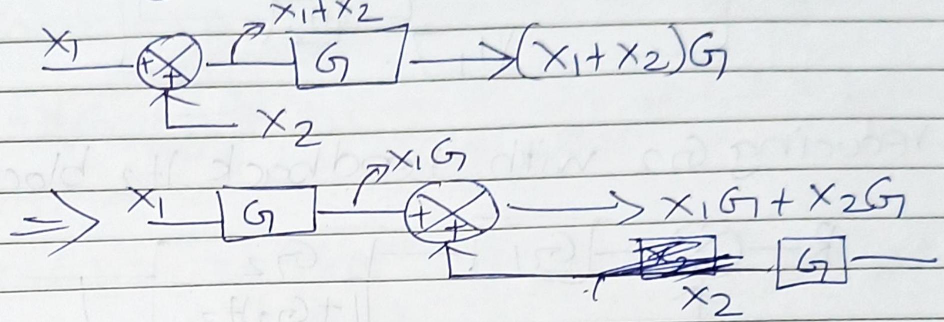
Or.



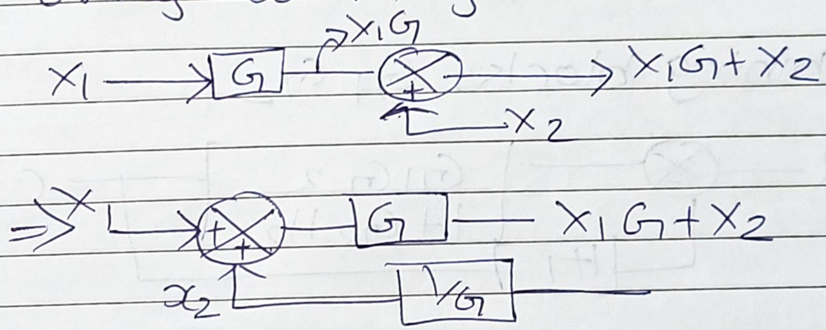
$C(S) = GR(S)$
$\&\ x = C(S) = GR(S)$

$C(S) = G\,R(S)$
$\&\ x = GR(S)$

5) Moving summing point after the block.

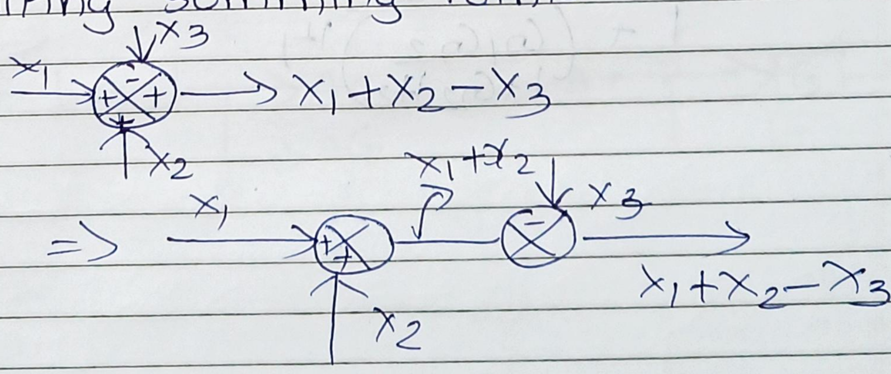


6) Moving summing point ahead of block –



$$\left(x_1+\frac{x_2}{G}\right)G \rightarrow x_1G+x_2$$

7) Spliting summing points –



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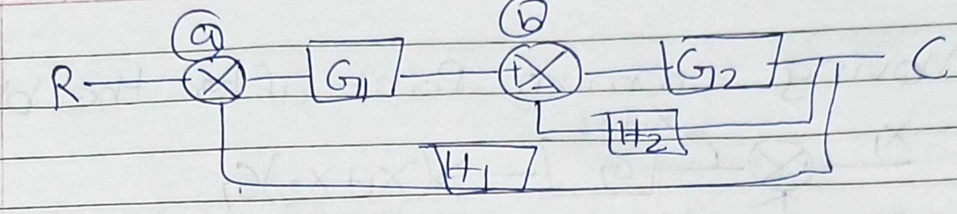
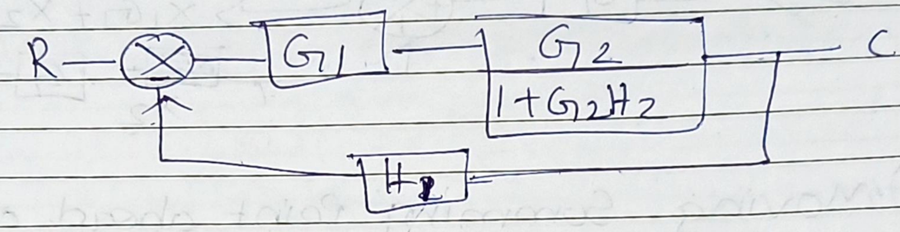
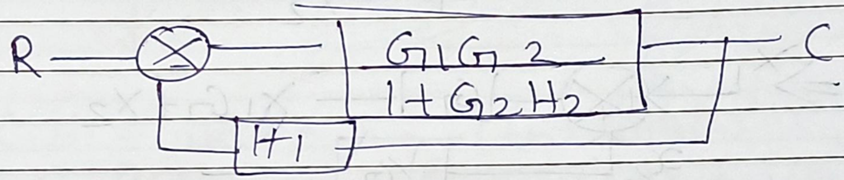
(5) 

i) reducing G_2 with feedback H_2 blockdig

→ 

ii) combining blocks G_1 G_2

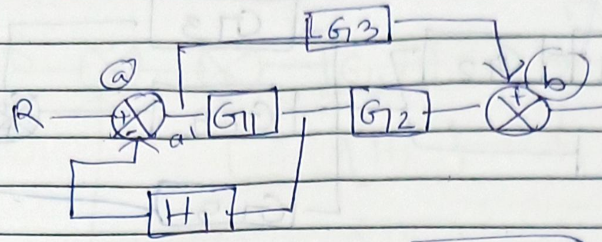


iii) Block diag with feedback H_1 is reduced to.

NOW $$\frac{\dfrac{G_1G_2}{1+G_2H_2}}{1+\left(\dfrac{G_1G_2}{1+G_2H_2}\right)H_1}$$

Page No.
Date 16/8/22

①



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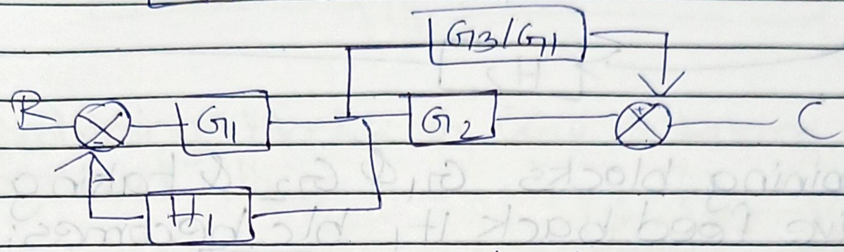
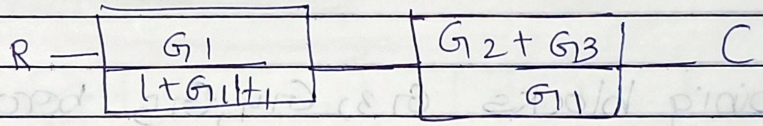


→ Moving take off point a' after the block G_1, block diagram becomes

→ Combining G_1 with H_1 negative feedback and combining G_2 & G_3/G_1:

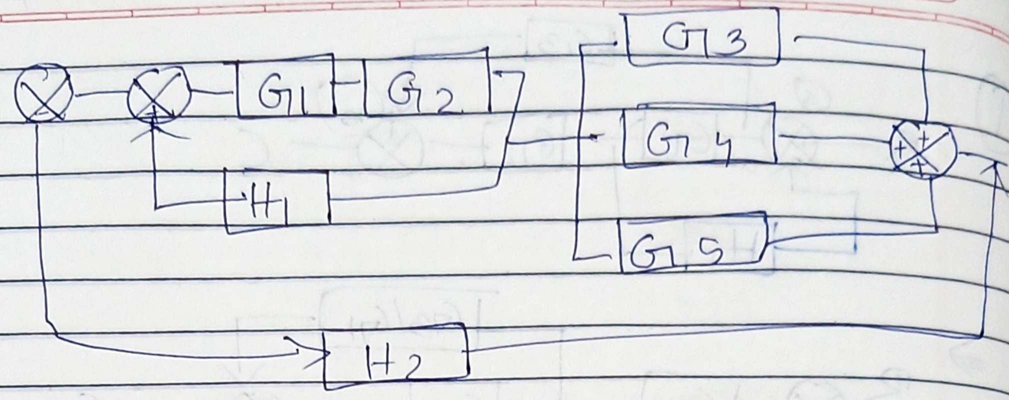


$$\frac{C(s)}{R(s)} = \frac{G_1\left(G_2 + \frac{G_3}{G_1}\right)}{(1+G_1H_1)}$$

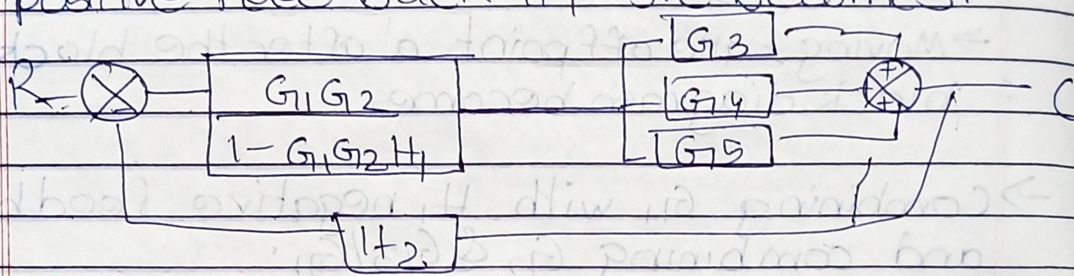
$$\frac{C(s)}{R(s)} = \frac{G_1G_2 + G_3}{1+G_1H_1}$$

Page No.
Date

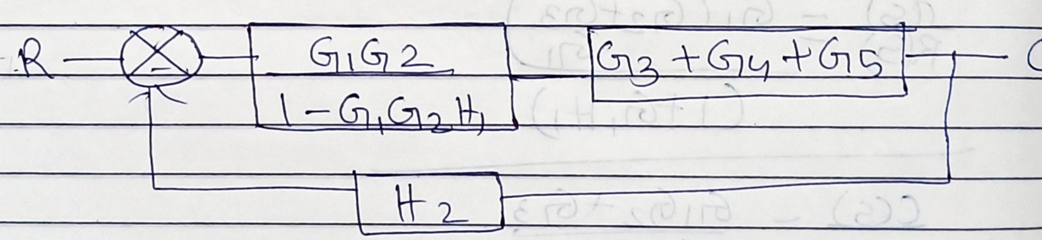
(2)



i) → Combining blocks G_1 & G_2 & taking positive feed back H_1 b/c becomes:-



ii) → combinig blocks G_3, G_4, G_5 because they are parallel.

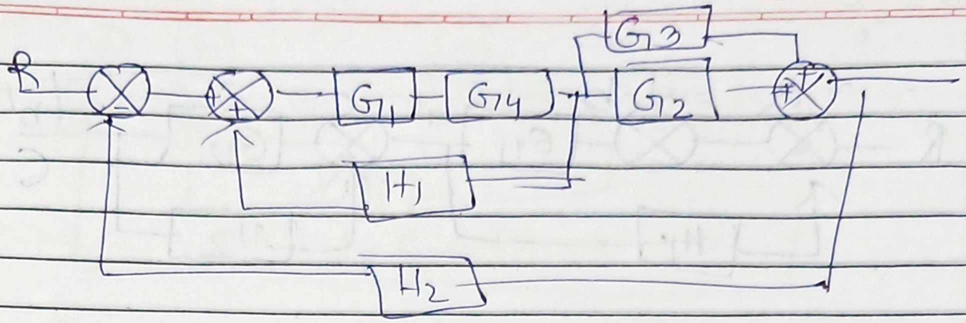


iii) combining two blocks and with negative feed back H_2

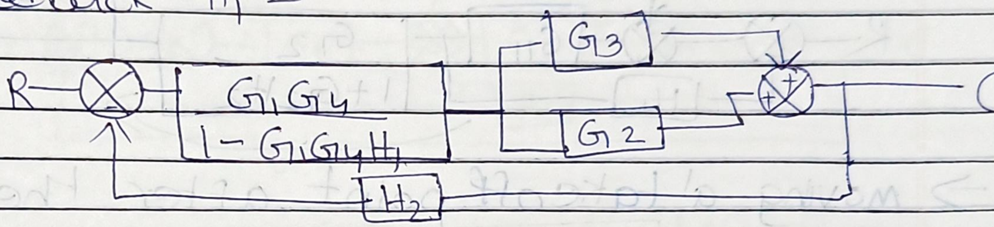
$$= \frac{\left(\frac{G_1G_2}{1-G_1G_2H_1}\right)(G_3+G_4+G_5)}{1+\left(\frac{G_1G_2}{1-G_1G_2H_1}\right)(G_3+G_4+G_5)\cdot H_2}$$

$$\frac{C}{R} = \frac{(G_1G_2)(G_3+G_4+G_5)}{(1-G_1G_2H_2)+(G_2G_1)(G_3+G_4+G_5)(H_2)}$$

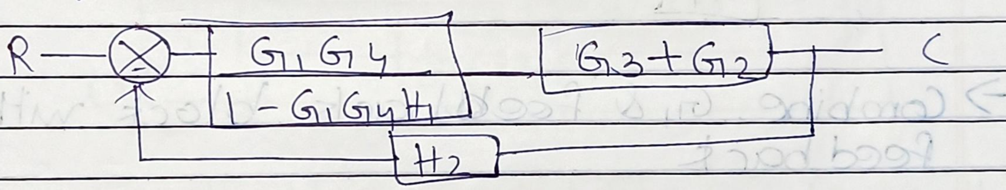
(3)



i) Combining blocks G_1 & G_4 & taking positive feedback H_1 –



ii) Combining blocks G_3 & G_2 because they are parallel.



iii) Combining two blocks –

$$= \frac{\left(\frac{G_1G_4}{1-G_1G_4H_1}\right)(G_3+G_2)}{1+\left(\frac{G_1G_4}{1-G_1G_4H_1}\right)(G_3+G_2)\cdot H_2}$$

$$\frac{C(s)}{R(s)} = \frac{(G_1G_4)(G_3+G_2)}{1+(G_1G_4)(G_3+G_2)\cdot H_2}$$

Page No. Date

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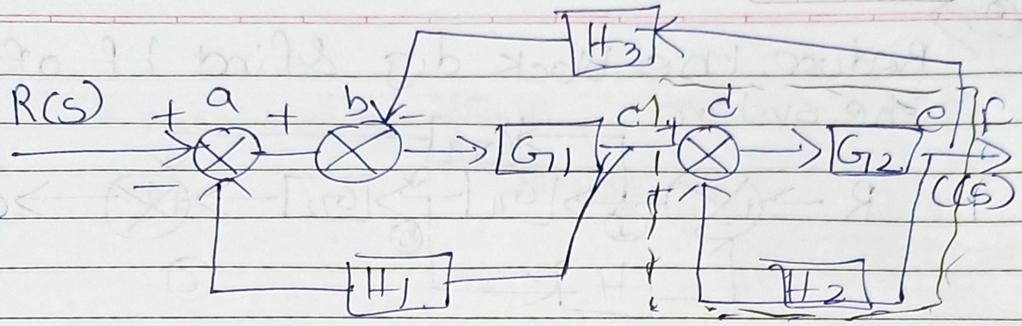
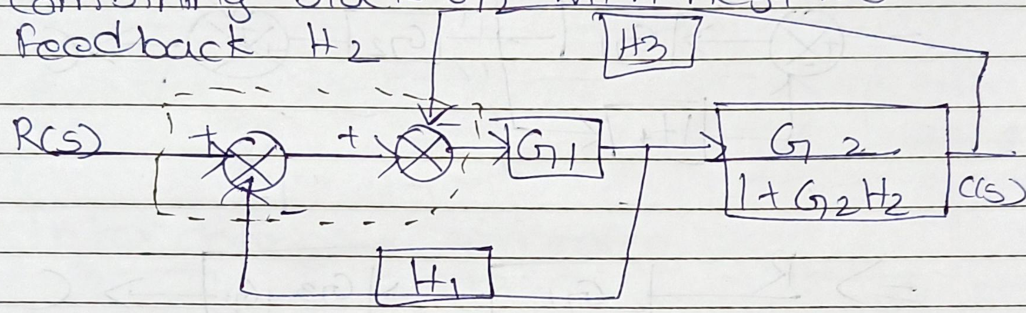
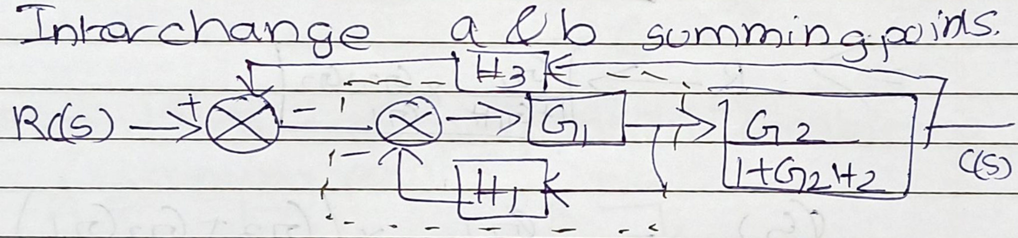


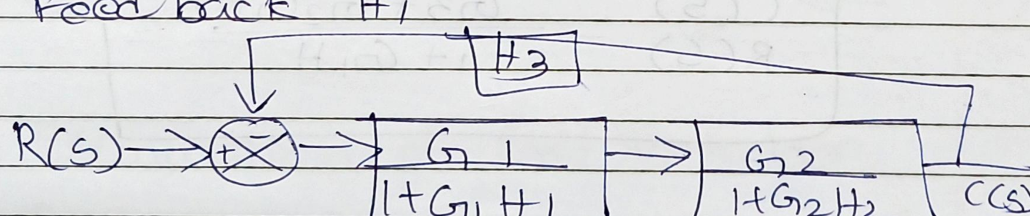
→ Combining block G_2 with negative
(1) feedback H_2



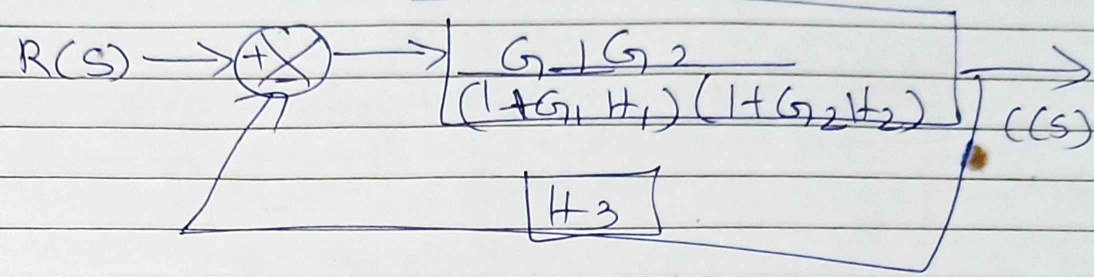
(2) Interchange a & b summing points.



(3) Combining block G1 with negative feedback H_1



(4) Combining the two blocks.



(5) Combining the block with −negative feedback H_3

$$\frac{\frac{G_1 G_2}{(1+G_1H_1)(1+G_2H_2)}}{1+\left(\frac{G_1 G_2}{(1+G_1H_1)(1+G_2H_2)}\right)\cdot H_3}$$

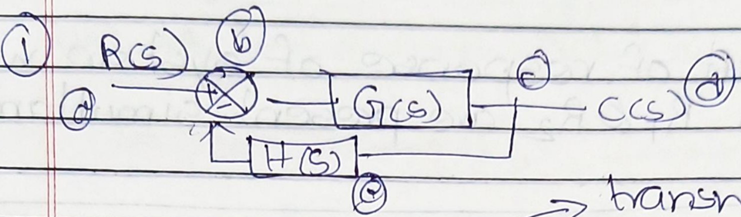
$$\Rightarrow \frac{\frac{G_1 G_2}{(1+G_1H_1)(1+G_2H_2)}}{(1+G_1H_1)(1+G_2H_2)+(G_1G_2)H_3}$$

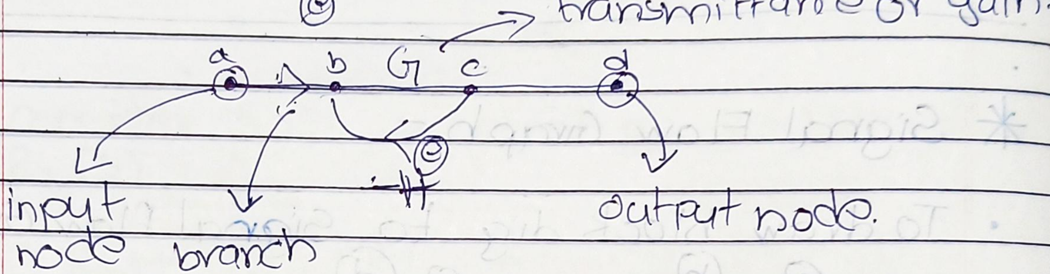
$$\frac{C(S)}{R(S)} \Rightarrow \frac{G_1 G_2}{(1+G_1H_1)(1+G_2H_2)+G_1G_2H_3}$$

Page No.
Date 22/8/22

Signal Flow Graph

It is the graphical representation of block diagram simplest method to analysize the system.





nodes - a, b, c, d.
branch - ab, bc, cd, cb

G is the transmittance of branch bc

- Input node has only outgoing signals.
- output node has only incomming signals.
- Mixed node has both incomming and outgoing signals.

- b & c are mixed nodes.
  a is i/p node & d is o/p node.

* Forward path-
Path traverse from input to output following the directions of arrows in forward direction.

- Forward path → abcd

* Loop - Path traversed starting from one node & reaching the same node also known as feedback loop

- Loop - bcob

* Forward path gain -
It is the product of all gains in that particular forward path.

- Forward path gain = $1 \times G \times 1 = G$
- Feedback loop gain = $-GH$

*

Transfer Function = $\boxed{\frac{\sum M_K \Delta_K}{\Delta}}$
(Mason's formula)

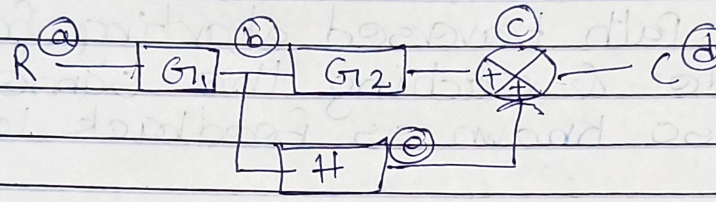
where, M_K - forward path, it is the forward gain k^{th} forward path

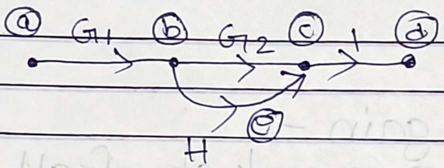
Δ_K - No. of non touching loops to k^{th} forward path.

• ~~Mason's Formula~~ –

$\Delta_K = 1 - \left(\begin{array}{l}\text{Sum of gains of} \\ \text{all individual loop}\end{array}\right) + \left(\begin{array}{l}\text{sum of product of gains of all} \\ \text{possible combination} \\ \text{of two non-touching} \\ \text{loop}\end{array}\right)$

$- (- \text{ " } - 3 \text{ none-touching loop})$

(1) 

SFG → 

i) Forward Path 1 → abcd gain (M_1) → $G_1 G_2$

Forward Path 2 → ~~GH~~ abecd gain (M_2) – $G_1 H$

ii) loops → 0

iii) since there are no loops which are not touching to forward path 1 & 2 :

$\therefore \Delta_1 = 1 \quad \& \Delta_2 = 1$

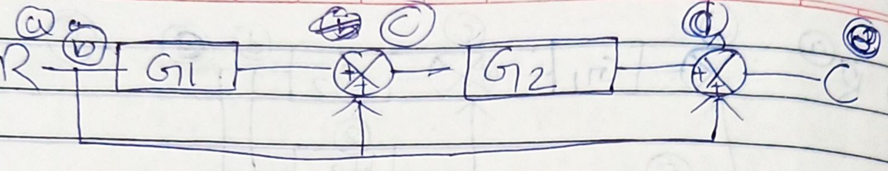
iv) $\Delta_K = 1 - 0 = 1$

$\therefore$ T.F will be $= \dfrac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$

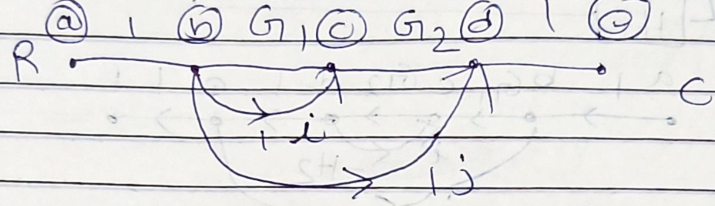
$= \dfrac{G_1 G_2 (1) + G_1 H (1)}{1}$

$= G_1 G_2 + G_1 H$

③



SFG →



i) Forward path 1 → abcde - gain - G_1G_2
Forward path 2 → ab i cde gain - G_2
Forward path 3 → abjde - gain - 1

ii) Loop → 0

iii) $\Delta = 1 - 0 = 1$

$\Delta_1 = 1$
$\Delta_2 = 1$
$\Delta_3 = 1$
} (since, No loops)

iv) $$T.F = \frac{\sum_K M_K \Delta_K}{\Delta}$$

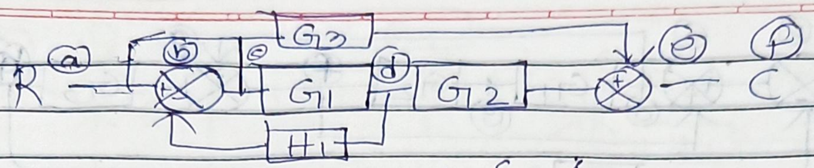
$$= \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta}$$

$$= \frac{G_1G_2(1) + G_2(1) + (1)(1)}{1}$$

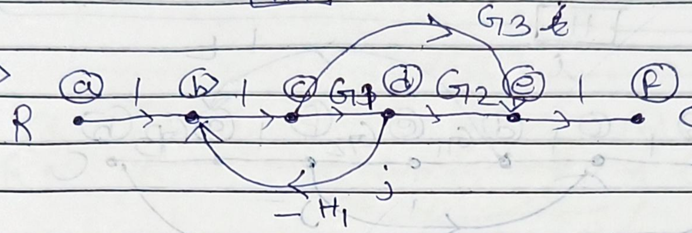
$$= G_1G_2 + G_2 + 1$$

Page No.
Date 23/8/22

(4)



SFG ⇒



i) Forward path 1 → abcdef Gain ⇒ $G_1 G_2$

Forward path 2 → abcef Gain → G_3

ii) Loop 1 → bcdb Gain → $-G_1 H_1$

iii) loop 1 is touching to forward path 1

∴ There is no loop which is non-touching to forward path-1

$\therefore \Delta_1 = 1$

$\Delta_2 = 1$

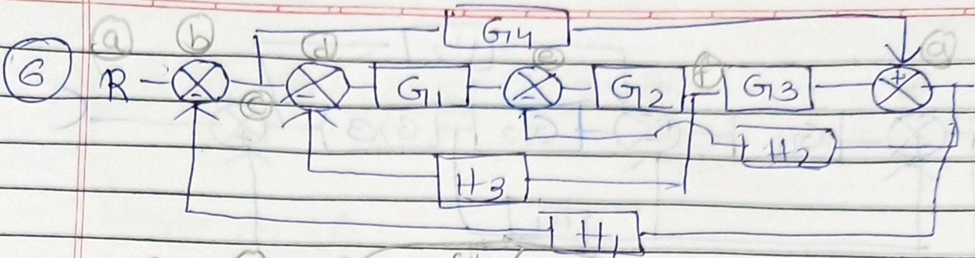
$\therefore \Delta = 1 - (-G_1 H_1) = 1 + G_1 H_1$

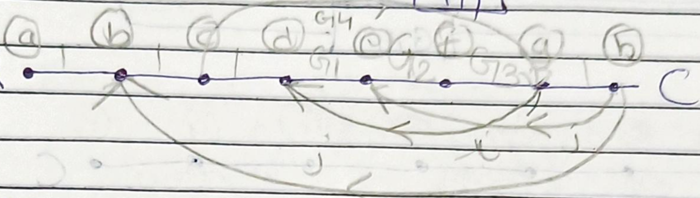
$\therefore T.F = \dfrac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$

$= \dfrac{G_1 G_2 (1) + G_3 (1)}{1 + G_1 H_1}$

$= \dfrac{G_1 G_2 + G_3}{1 + G_1 H_1}$

Page No.
Date





1) F.P - F.P.1) ⇒ abcdefghi   Gain ⇒ $G_1G_2G_3$

F.P.2) ⇒ abcghi   Gain ⇒ G_4

2) Loops ⇒ L_1 ⇒ bcghb   gain ⇒ $-G_4H_1$

L_2 ⇒ defd   gain ⇒ $-G_1G_2H_3$

L_3 ⇒ efghe   gain ⇒ $-G_2G_3H_2$

L_4 ⇒ bcdefghb   gain ⇒ $-G_1G_2G_3H_1$

$\Delta_1 = 1$ $\Delta_2 = 1 - (defd)$

$= 1 - (-G_1G_2H_3)$

$= 1 + G_1G_2H_3$

4) $\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$

$$T.F = \frac{\sum_K M_K \Delta_K}{\Delta} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_4}{1 + G_4H_1 + G_1G_2H_3 + G_2G_3H_2 + G_1G_2G_3H_1}$$

Eqn to SFG.

Let →

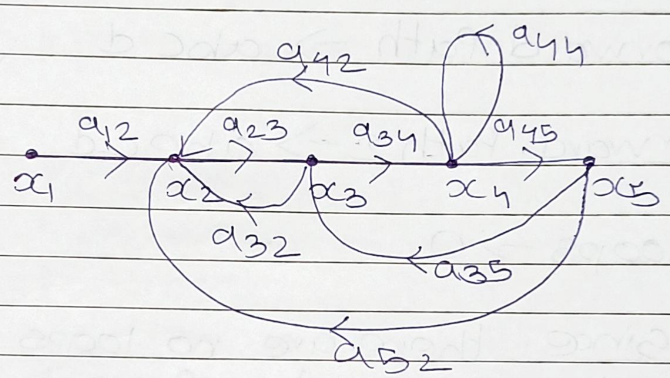
imp

$x_2 = a_{12}x_1 + a_{32}x_3 + a_{42}x_4 + a_{52}x_5$

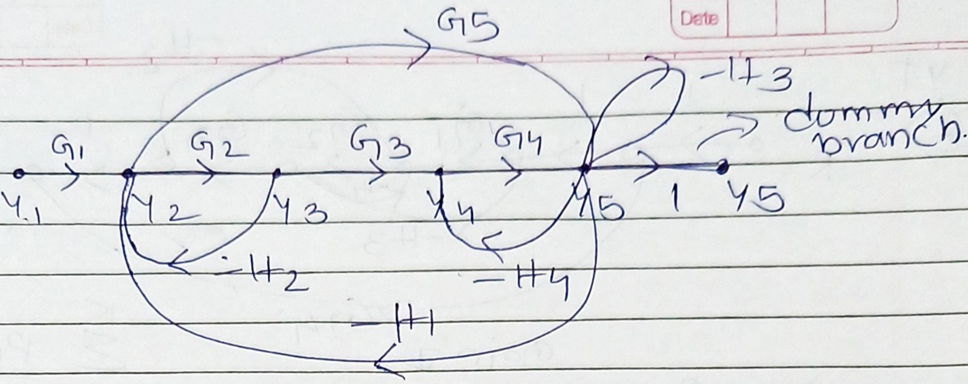
$x_3 = a_{23}x_2$

$x_4 = a_{34}x_3 + a_{44}x_4$

$x_5 = a_{35}x_3 + a_{45}x_4$



Q)



1) Forward Paths :-

$P1 = G_1 G_2 G_3 G_4$

$P2 = G_1 G_5$

2) Individual loops -

$I_1 = -G_2 H_2$

$I_2 = -G_4 H_4$

$I_3 = -H_3$

$I_4 = -G_2 G_3 G_4 H_1$

$I_5 = -G_5 H_1$

3) Two Non-touching loops -

$L_1 = (G_2 H_2 G_4 H_4)$

$L_2 = (G_2 H_2 H_3)$

Three Non-touching loops → NA

4)

$$\Delta = 1 - \begin{bmatrix} -G_2 H_2 - G_4 H_4 \\ - H_3 - G_5 H_1 \\ - G_2 G_3 G_4 H_1 \end{bmatrix} + \begin{bmatrix} G_2 G_4 H_2 H_4 \\ + G_2 H_2 H_3 \end{bmatrix}$$

Page No.
Date

3) Δ_1 & Δ_2

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Rightarrow \sum_{k=1}^{k} \frac{P_k \Delta_k}{\Delta}$

P1 $\quad \Delta 1 \quad$ P2 $\quad \Delta 2$

$$\frac{C(S)}{R(S)} \; \frac{Y5}{Y1} = \frac{G_1G_2G_3G_4(1) + G_1G_5(1)}{1 - \begin{bmatrix} -G_2H_2 - G_4H_4 - H_3 \\ -G_2G_3G_4H_1 - G_5H_1 \end{bmatrix} + \begin{bmatrix} G_2G_4H_2H_4 \\ +G_2H_2H_3 \end{bmatrix}}$$

Δ_1

Unit - 2

Output & the transient response of the system

* $c(t) = c_t(t) + c_{ss}(t)$

$\downarrow$ transient $\downarrow$ steady state.

* Transient Response → nature of response & speed of response

It is that part of output response which dies out after large time 't'.

$\Rightarrow \lim_{t \to \infty} c_t(t) = 0$

* steady state Response

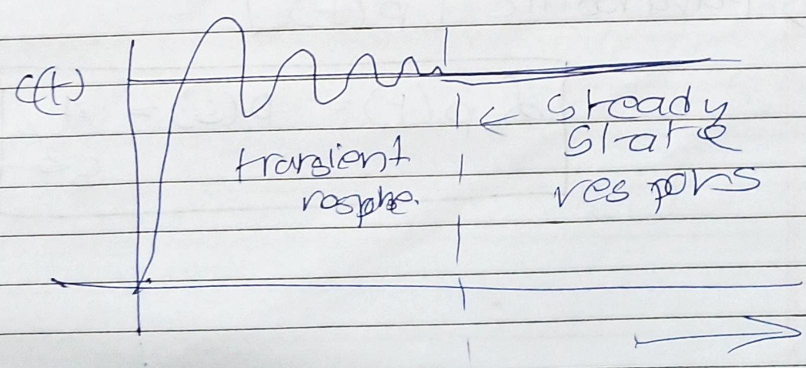
It is that part of output responce which remains after dying out of transient responce.

poles → values that make T.F infinity

• IMP -

① Transient response depends on no. of poles of system & not the type of input.

② Steady state response depends on the type of input



* characteristics of input -

① sudden shock -> (impulse input)
② sudden change -> (step input)
③ Constant velocity -> (Ramp input)
④ Constant acceleration -> (Parabolic input)

Refer N.B

1> Impulse input. [$\delta(t)$]

- if $A = 1$, it is known as unit impulse input

$$\boxed{\mathcal{L}\,\delta(t) = \delta(s) = A}$$

2> Step Input [$u(t)$]

— // —

$$\boxed{\mathcal{L}\,u(t) = u(s) = \frac{A}{s}}$$

— // —

3> Ramp Input [$r(t)$]

$$\boxed{\mathcal{L}\,r(t) = R(s) = \frac{A}{s^2}}$$

— // —

4> Parabollic [$p(t)$]

$$\boxed{\mathcal{L}\,p(t) = p(s) = \frac{A}{s^3}}$$

Imp

Input		Laplace	
Impulse	$\int$ A	A	d/dt
step	$\int$ A	A/s	d/dt
Ramp	$\int$ At	A/s^2	d/dt
Parabolic	$\int At^2/2$	A/s^3	d/dt

* First order system.

$$T.F = \frac{K}{T.s+1}$$

General eqn -

$$\boxed{\mathcal{L}^{-1}\frac{1}{s+a} = e^{-at}}$$

* Transient Response of first order system for diff types of input

a) Step input.

$$\boxed{c(t) = 1 - e^{-t/T}}$$

$t\uparrow$ $e^{-t/T}\downarrow$ $c(t)\uparrow$

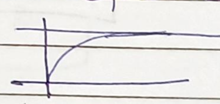
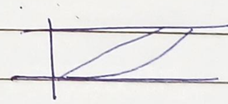
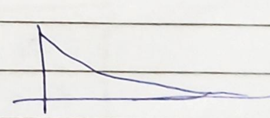
b) Ramp input

$$\boxed{c(t) = t - T + T e^{-t/T}}$$

- $c(t)\uparrow \rightarrow e^{-t/T}\downarrow \rightarrow c(t)\uparrow$
- c(t) increases exponentially

c) Impulse input

$$\boxed{c(t) = \frac{1}{T} e^{-t/T}}$$

input	c(t)	response.
step	$1 - e^{-t/T}$	
ramp	$t - T + Te^{-t/T}$	
impulse.	$\frac{1}{T} e^{-t/T}$	

* Speed of response -
Speed of response is defined as time required to reach particular percentage of final value.

* Error signal -
The dif between reference input $r(t)$ & output response $c(t)$

$$e(t) = r(t) - c(t)$$

* steady state error -
It is that value of error $[e(t)]$ when t reaches infinity.

$$\therefore e_{ss} = \lim_{t \to \infty} e(t)$$

1) For step input.

$$\boxed{\begin{aligned} e_{ss} &= \lim_{t \to \infty} e^{-t/T} \\ e_{ss} &= 0 \end{aligned}}$$

2) For Ramp input

$$\boxed{\begin{aligned} e_{ss} &= \lim_{t \to \infty} T - (Te^{-t/T}) \\ e_{ss} &= T \end{aligned}}$$

3) For Impulse input

$$\boxed{\begin{aligned} e_{ss} &= \lim_{t \to \infty} -1/T\, e^{-t/T} \\ e_{ss} &= 0 \end{aligned}}$$

* Time response of second order system.

* Damping -

it is defined as ability of system to supress or reduce the oscillations.

* Damping ratio / Damping factor [zeta → (ξ)]

it is the ratio of actual damping to critical damping.

* Natural frequency of system (ω_n)

unit rad/sec

it is the frequency of oscillations of system without damping.

- consider a second order system whose closed loop transfer function is given by.

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

- characteristic eqn.

$$\boxed{s^2 + 2\xi\omega_n s + \omega_n^2 = 0}$$

- Roots of ch eqn -

$$\boxed{\begin{aligned} s_1 &= -\xi\omega_n + j\omega_n\sqrt{1-\xi^2} \\ s_2 &= -\xi\omega_n - j\omega_n\sqrt{1-\xi^2} \end{aligned}}$$

roots depend on. ω_n & ξ

	ξ	roots / poles	location	response
undamped system	0	$-j\omega_n, +j\omega_n$	lie on imaginary axis only.	oscillatory, not stable.
underdamped system	$0<\xi<1$	$-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}$ $-\xi\omega_n - j\omega_n\sqrt{1-\xi^2}$	lie on left side half plane	oscillatory but stable at $t\to\infty$
critical damped system	$\xi=1$	$-\omega_n, -\omega_n$	both roots are negative real, equal lie on left side of s plane on real axis	No oscillations stable
overdamped system	$\xi>1$	real & unequal	Negative side of real axis	Non ossilatory but sluggish.

as ξ changes poles move in circular path.

- when ξ is changed from zero or increases, poles get shifted on left half of the plane in circular manner.

- Radius of that circle is ω_n

$$\boxed{\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}}$$

$\Rightarrow$

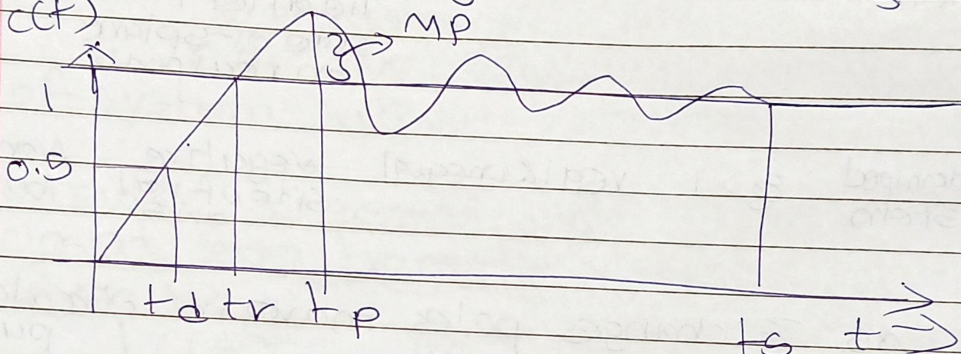
* Transient Response of second order system to the step input.

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left[\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$

- error signal -

$$e(t) = \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}}\left[\cos \omega_d t + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t\right]$$

imp



- Delay time (td)
  time required for the response to reach 50% of final value.

- Rise time (tr)
  time required for response to reach final value for first time.

- Peak time (tp)
  time required for response to reach overshoot mag of response

- Peak overshoot goes on decreasing when ξ goes on increasing

* → Derivations —

① Rise time.

$$t_r = \frac{\pi - \theta}{\omega_d}$$

② Peak time

$$t_p = \frac{\pi}{\omega_d}$$

③ $M_p = e^{-\pi \xi / \sqrt{1-\xi^2}}$

④ % Peak overshoot

$= M_p \times 100$

⑤ $t_s = \frac{4}{\xi \omega_n}$ 2% } criterium or tolerance

$= \frac{3}{\xi \omega_n}$ 5%

* Steady state Error & Error constants for a given transfer function / system.

$$\boxed{E(s) = R(s)\left[\frac{1}{1+G(s)}\right]}$$

error signal

* steady state error -

$$\boxed{e_{ss} = \lim_{s \to \infty} e(t)}$$

steady state error

- For step input.

$$e_{ss} = \frac{1}{1+K_p}$$

where $K_p = \lim_{s \to 0} G(s)$

where K_p is position error constant

- For Ramp input

$$e_{ss} = \frac{1}{K_v}$$

where $K_v = \lim_{s \to 0} s G(s)$

where K_v is velocity error constant

- For Parabolic Input.

$$e_{ss} = \frac{1}{K_a}$$

where, $K_a = \lim_{s \to 0} s^2 G(s)$

where K_a is acceleration error constant

$$\boxed{K_p = \lim_{s \to 0} G(s) \qquad e_{ss} = \frac{1}{1+K_p}}$$ } step

$$\boxed{K_v = \lim_{s \to 0} s\,G(s) \qquad e_{ss} = \frac{1}{K_v}}$$ } ramp

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s) \qquad e_{ss} = \frac{1}{K_a}}$$ } parabolla

---

* Types of systems –
  Systems can be classified according to no. of poles present at origin.

- Type 0 – There are no poles at origin.
- Type 1 – One pole at origin
- Type 2 – Two poles at origin

Steady state error shortcut.

	Type 0	Type 1	Type 2
step	$\frac{1}{1+K_p}$	0	0
ramp	∞	$\frac{1}{K_v}$	0
Parabolic	∞	∞	$\frac{1}{K_a}$

* DC Gain (K)

ratio of steady state o/p to i/p

$$\text{D.C gain }(K) = \frac{\text{steady state o/p}}{\text{I/p}}$$

* Time constant - T (τ)

it is the time required to reach 63% of final value.

Page No.
Date 13/9/22

Numericals -

Q1) Impulse response of first order system is given by $3e^{-0.5t}$ find ① time const, ② DC gain ③ T.F ④ step response

soln: Given. $c(t) = 3e^{-0.5t}$

① Taking Laplace of above system -

$$C(s) = 3\frac{1}{s+0.5}$$

Note
$$\boxed{e^{-at} = \frac{1}{s+a}}$$

$$= \frac{3}{0.5\left(\frac{s}{0.5}+1\right)}$$

$$C(s) = \frac{6}{2s+1}$$

comparing with basic eqn.

$$\frac{K}{Ts+1}$$

$K = 6$ & $T = 2\,sec$

② T.F = ? $\quad R(s) =$

input is unit impluse input

$$\frac{C(s)}{\delta(s)} = \frac{6}{2s+1}$$

$$\frac{C(s)}{R(s)} = \frac{6}{2s+1}$$

for step response.

$$R(s) = \frac{1}{s}$$

Page No.
Date

$$\therefore C(s) = \frac{6}{2s+1} \quad \frac{1}{s}$$

solving by Partial fraction

$$C(s) = \frac{A}{2s+1} + \frac{B}{s}$$

$$= \frac{2A}{2\left(s+\frac{1}{2}\right)} + \frac{B}{s}$$

$$B = s \cdot C(s) \Big|_{s=0}$$

$$= \cancel{s} \cdot \frac{6}{2(s+1/2)} \cdot \frac{1}{\cancel{s}} \Big|_{s=0}$$

$$= 6$$

$$A = (s+1/2)\, C(s) \Big|_{s=-1/2}$$

$$\underline{\underline{A = -6}}$$

$$\therefore C(s) = \frac{6}{s} - \frac{6}{(s+1/2)}$$

laplace

$$\frac{1}{s+a} \rightarrow e^{-at}$$

Taking Laplace inverse:

$$\boxed{C(t) = -6e^{-0.5t} + 6}$$

Q) A system has transfer function

$$\frac{C(s)}{R(s)} = \frac{20}{s+10}$$

Determine it's response for unit step and unit ramp input

→ a) step input

$$R(s) = \frac{1}{s}$$

soln: $C(s) = R(s) \frac{20}{s+10}$

$$= 1/s \cdot \frac{20}{(s+10)}$$

$$C(s) = \frac{20}{s(s+10)} \quad \text{— (1)}$$

$$= \frac{A}{s} + \frac{B}{s+10}$$

$$A = s \cdot C(s) \big|_{s=0}$$

$$\boxed{A=2}$$

$$B = (s+10) \cdot C(s) \big|_{s=-10}$$

$$\boxed{B=-2}$$

Substituting values of A & B in eqn (1)

$$C(s) = \frac{2}{s} - \frac{2}{s+10}$$

Taking laplace inverse

$$c(t) = \mathcal{L}^{-1} \frac{2}{s} - \mathcal{L}^{-1} \frac{2}{s+10}$$

$$\boxed{c(t) = 2 - 2e^{-10t}}$$

b) Ramp

$$R(s) = \frac{1}{s^2}$$

$$C(s) = R(s)\frac{20}{s+10}$$

$$C(s) = \frac{20}{s^2(s+10)} \quad \text{--- (1)}$$

$$C(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+10}$$

$$C(s) = \frac{A(s+10 + Bs(s+10) + Cs^2}{s^2(s+10)}$$

$$\frac{20}{s^2(s+10)} = \frac{A(s.10) \cdot B(s+10) + Cs^2}{s^2(s+10)}$$

$$20 = s^2(B+C + s(A+10B) + 10A$$

$$B+C=0$$

$$A+10B=0$$

$$\boxed{A=2}$$

$$\boxed{B=-0.2}$$

$$\boxed{C=0.2}$$

$$C(s) = \frac{2}{s^2} - \frac{0.2}{s} + \frac{0.2}{s+10}$$

taking laplace inv

$$c(t) = \mathcal{L}^{-1}\frac{2}{s^2} - \mathcal{L}^{-1}\frac{0.2}{s} + \mathcal{L}^{-1}\frac{0.2}{s+10}$$

$$\boxed{C(t) = 2t - 0.2 + e^{-10t}}$$

Q) Time response of a given system is given by

$$c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$$

i) obtain expression for closed loop T.f of a sys.
ii) Determine undamped natural Freq & damping ratio for step input.

Soln: i) $c(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$

taking laplace on both sides.

$$C(S) = \mathcal{L}1 + \mathcal{L}0.2e^{-60t} - \mathcal{L}1.2e^{-10t}$$

$$C(S) = 1/S + \frac{0.2}{S+60} - \frac{1.2}{S+10}$$

$$C(S) = \frac{(S+60)(S+10) + 0.2S(S+10) - 1.2S(S+60)}{S(S+60)(S+10)}$$

$$C(S) = \frac{S^2 + 10S + 60S + 600 + 0.2S^2 + 2S - 1.2S^2 - 72S}{S(S+60)(S+10)}$$

$$C(S) = \frac{600}{S(S+60)(S+100)}$$

$$\frac{C(S)}{R(S)} = \frac{\frac{600}{S(S+60)(S+10)}}{1/S}$$

R(S) - step i/p

$$\frac{C(S)}{R(S)} = \frac{600}{(S+60)(S+10)}$$

T.F of second order system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\frac{C(s)}{R(s)} = \frac{600}{s^2+70s+600}$$

comparing these two eqns.

$$\omega_n^2 = 600$$

$$2\xi\omega_n = 70$$

$$\omega_n = \sqrt{600}$$

$$\omega_n = 24.49 \text{ rad/s}$$

$$\xi = \frac{70}{2\times 24.49}$$

$$\boxed{\xi = 1.42}$$

Page No.
Date

Unit 3

Stability Analysis of Systems.

* Types of stability -
  1. Stable system
  2. Unstable system
  3. Critically or marginally stable system
  4. Conditionally stable system.

(1) Absolute stable system (Absolute stability)
It gives the information whether the system is stable or not.

(2) Relative stability -
It gives degree of stability, It gives information about the stability of one system with another.

* Factors affecting stability of system
  1. Type of input
  2. Parameters of system
  3. Initial conditions -

Note - If a small changes in input, parameters, conditions in the system if output tends to zero than a system has good stability.

* Diff methods for analysing stability of sys -
  1. BIBO
  2. Location of poles on ($j\omega$) axis.
  3. examining the sign of coefficients of characteristic eqn.
  4. Rowth's criterion
  5. Root locus.

1) BIBO (Bounded input bounded output)

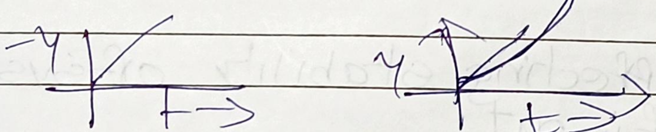
- Bounded signal -
  - confined to particular values
  - Have some limits.
  - Maxima & minima of the signal should have a finite value

eg - ① $y(t) = 6$ dc value.

② $y = \sin t$ $\sin(t)$ has range of -1 to 1, has finite value.

③ $y = \cos(t)$

- examble of unbounded signal



* Determination of stability of sys by examining input & output.

$y(t) = o/p$ $x(t) = i/p$

① $y(t) = t\,x(t)$

where $x(t) = u(t)$ → o/p

$x(t) \to$ syst $\to y(t)$

i/p $u(t)$ $= t\,x(t)$

✓ bounded i/p

- for bounded input is output is unboundes system is unstable

② $y(t) = x(t) + 2$

$x(t) = u(t)$ ↘ const 2

step i/p

bonded.

for bounded input if output is also bounded it is known as stable sys.

* Critically or Marginally stable system

-> For bounded input if output oscillates with constant amplitude & frequency it is known as critically or marginally stable sys.

* Conditionally stable sys -
for a bounded input sys is stable for a particular ranges of parameters.

* output response of a sys

o/p response = zero state response + zero input response.

zero state response - output response due to input only.

zero input response - Response due to initial conditions only.

Stability -

* A system is stable if its output is bounded for any bounded input

* A system is asymptotically stable if in the absence of input, the output tends towards zero irrespective of initial conditions.

* A system is unstable if its output is unbounded for any bounded input.

* stability of the system depends upon poles.

* If all the poles are located in left half of S-plane, then the system is stable.

* As pole approaches origin then stability decreases

* when poles are located on imaginary axis then the system is marginally stable

* If poles located on imaginary axis are repeated then system is unstable

* If poles are located in right half of S-plane then system is unstable.

* The poles which are close to origin are called dominant.

Page No. | Date

Q) Different ch eqn are given, investigate whether the sys is stable or not

① $s^5 + 4s^4 + s^2 + s + 2 = 0$
→ s^3 term is missing
∴ system is unstable

② $5s^4 - 2s^3 + 3s^2 + 2s + 16 = 0$
→ Negative sign of s^3, all coeff are not of same sign
∴ system is unstable.

③ $s^4 + 2s^3 + 5s^2 + s = 0$
→ s^0 term is missing
∴ System is unstable

Q) Determine if the system is stable, unstable or marginally stable from given ch eqn

① $(s+4)(s+5) = 0$

→ poles $s = -4$, $s = -5$
both the poles lie on the left hand side of $j\omega$ axis or s plane.
∴ System is stable.

2) $(s-2)(s-4)=0$

$\Rightarrow \therefore s = 2, 4$

both poles lie on RHS of s plane

$\therefore$ sys is unstable.

3) $(s+2+j3)(s+2-j3)=0$

$s_1 = -2-j3$

$s_2 = -2+j3$

both poles are on LHS

$\therefore$ sys is stable

4) $(s-2-4j)(s-2+j4)=0$

$s_1 = 2+4j$

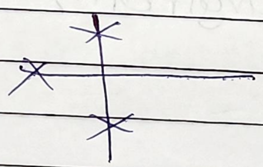
$s_2 = 2-j4$

both on RHS of s plane

$\therefore$ sys is unstable.

5) $(s+5)(s^2+2)=0$

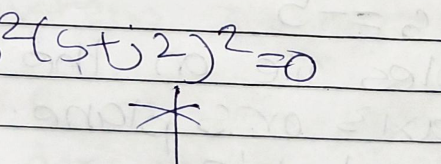
$\Rightarrow (s+5)(s+j\sqrt{2})(s-j\sqrt{2})=0$



$\therefore$ The two roots are on $j\omega$ axis, sys is marginally stable.

6) $(s-j2)^2(s+j2)^2=0$

$\Rightarrow$



$\therefore$ repeated roots on $j\omega$ axis, sys is unstable.

imp

* Routh's Criteria –

For a system to be stable it is necessary & sufficient that each turn of first column of Routh array must be +ve if $a_0 > 0$ if this condition is not met, the system is unstable & the number of sign changes in the first column corresponds to the number of roots of characteristic equation in the right half of s-plane.

$$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_{n-1} s + a_n = 0$$

routh array –

s^n	$a_0 \quad a_2 \quad a_4 \quad a_6 \cdots$	where,
	$\cancel{a_1}$	$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$
s^{n-1}	$a_1 \quad a_3 \quad a_5 \cdots$	$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$
s^{n-2}	$b_1 \quad b_2 \quad b_3 \cdots$	$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$
s^{n-3}	$c_1 \quad c_2 \quad c_3 \cdots$	$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$
$\vdots$		
s^0	a_n	

Note – For sys to be stable, all the elements of first column should be positive

imp – If there is a sign change, system becomes unstable

– No. of sign changes in the first column, gives the no. of roots of RHS of s-plane

Q> Using routh array, determine the stability of the system represented by the characteristic equation $s^4+8s^3+18s^2+16s+5=0$

→

s^4	1	18	5	
s^3	8	16	0	
s^2	$\frac{8\cdot18-16}{8}=16$	$\frac{8\cdot5-0}{8}$	0	
s^1	$\frac{16\cdot16-8\cdot5}{10}$	0	0	
s^0	5	0	0	

constant

As all elements in first column are positive system is stable.

Q> Using routh array, determine the stability of the system represented by the characteristic eqn $3s^4+10s^3+5s^2+5s+2=0$

s^4	3	5	2	
s^3	10	5	0	
s^2	$\cancel{\frac{3\cdot5-...}{10}}$ 3.5 $\left(\frac{10\cdot5-3\cdot5}{10}\right)$	2 $\left(\frac{10\cdot2-3\cdot0}{10}\right)$	0	
s^1	$\frac{3.5\cdot5-10\cdot2}{3.5}$ → -ve	0	0	
s^0	2			

constant

As one element in first column is negative system is unstable

There is 2 sign change.

① s^2 to s^1 & ② s^1 to s^0

∴ There are two roots on RHS of s plane

Page No.
Date

Difficulty - 1

When the first term in any row of the routh array is zero while rest of the row has atleast one nonzero term.

$$s^4 + 2s^3 + 10s^2 + 20s + 5 = 0$$

s^4	1	10	5
s^3	2	20	0
s^2	0	5	0

zero element has appeared as the first element in s^2 row

Difficulty 1 arises

Replace zero by small variable a formulating the new Routh's table.

s^4	1	10	5	
s^3	2	20	0	
s^2	a	5	0	
s^1	$\frac{2a-10}{a}$	0	0	
s^0	$\frac{\frac{(20a-10)}{a} 5 - 0}{\frac{(20a-10)}{a}} = 5$		0	0

If a is very very small first element of s^1 will be negative.

There will be two sign changes

① from s^2 to s^1 row

② from s^1 to s^0 row

Hence system is unstable with two roots on RHS of splane.

Page No.
Date

Difficulty 2
All the elements of any row is zero

eg -> $s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4$

s^6	1	5	8	4
s^5	3	9	6	0
s^4	2	6	4	0
s^3	0	0	0	0

-> All the elements of s^3 row are zero difficulty arises.
-> whenever difficulty ~~2~~ 2 ariese auxillary equation has to be made.
-> Auxillarly eqn is made from a row just above the row which has all zero elements.

s^3 - all elements zero
s^4 - auxillary eqn using s^4 row

$$A(s) = 2s^4 + 6s^2 + 4 = 0$$

ii) $$\frac{dA(s)}{d(s)} = d/ds\,(2s^4 + 6s^2 + 4) = 8s^3 + 12s$$

Again formulating the routh's table

s^6	1	5	8	4
~~s^5~~	~~3~~	~~9~~	~~6~~	0
s^4	2	6	4	0
s^3	8	12	0	0
s^2	3	4	0	
s^1	4/3	0		
s^0	4			

~~All elements of first column are positive & no sign change in first column.~~

~~As there is no sign change in first column system is stable.~~

Elements of first colom of Routh's table are positive still system is unstable due to presence of all zero's in ~~1st~~ s^3 row.

Since all elements of first row are positive no roots are present on RHS of s plane.

Page No.
Date

* Determination of Range of K using Routh's criteria.

① $s^4 + 4s^3 + 13s^2 + 36s + K = 0$
find range of K for the stability of the system using Routh's criteria.

s^4	1	13	K
s^3	4	36	0
s^2	4	K	
s^1	36−K	0	
s^0	K		

For system to be stable first column of Routh's table should be positive.

i.e. $36 - K > 0$

$36 > K$

$K < 36$

$K > 0$

∴ Range of K for system to be stable is $\boxed{0 < K < 36}$

For finding out marginal K

$36 - K = 0$

$\boxed{K = 36}$

$36 - K_{marg} \Rightarrow$

$\boxed{K_{marg} = 36}$

Page No.
Date

Q) A unity feed back control system has open loop transfor fuction has

$$G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$$

using Routh's critoria calculate
a) Range of k for system to be stable.
b) find marginal value of k
c) Location of poles on imaginary axis
d) And frequency of oscillations.

=> : For closed loop transfor function.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{\underbrace{1+G(s)H(s)}_{\text{charc-eqn}}}$$

=> since unity feedback $H(s) = 1$

∴ charac eqn will be.

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+13)}{s(s+3)(s+7)} = 0$$

$$\Rightarrow \frac{K(s+13)}{s(s+3)(s+7)} \quad \left[\frac{\cancel{s(s+3)(s+7)}}{s(s+3)(s+7) + K(s+13)}\right]$$

$$\Rightarrow \frac{K(s+13)}{(s^2+3s)(s+7) + K(s+13)}$$

$$\Rightarrow \frac{K(s+13)}{s^3 + 7s^2 + 3s^2 + 21s + Ks + K}$$

$$\Rightarrow \frac{K(s+13)}{s^3 + 10s^2 + (21+K)s + 13K}$$

s^3 1   ~~4~~   21+K

s^2 10   13K → aux' eqn

s^1 $\frac{210-3K}{10}$ 0

s^0 13K

For system to be stable first column of routh's table should be positive.

$\therefore \quad 13K > 0$

$\boxed{K > 0}$

also $\frac{210-3K}{10} > 0$

$210 - 3K > 0$

$210 > 3K$

$70 > K$

$\boxed{K < 70}$

For finding out marginal K

$\frac{210 - 3K_{marginal}}{10} = 0$

$\therefore \boxed{K_{mar} = 70}$

For finding out roots on immaginary axis substitute K marginal in auxillary eqn

$A(s) = 10s^2 + 13K_m$

$= 10s^2 + 13(70)$

$= 10s^2 + 910$

$10s^2 + 910 = 0$

$s^2 = \frac{-910}{10}$

$s = \pm j9.53$

∴ System is marginally systre & oscillates at 9.53 rad/s

Rules for Root locus

① find location of poles & zero's

② find no. of poles & zero's

③ Locate poles & zero's on splane

④ find no. of branches / segments for root locus $\left[\begin{array}{l} \text{if } P > Z \text{ no. of branches} = P \\ \text{if } Z > P \text{ no. of branches} = Z \end{array}\right]$

⑤ find no. of branches tending / terminating to infinity $(p-z)$

⑥ find angles of asymtotes

$$\theta = \frac{(2q+1)180}{p-z} \quad \text{where } q = 0, 1, 2 \cdots$$

⑦ find centroid

$$\sigma = \frac{\sum \text{real parts of Pole} - \sum \text{real parts of zero}}{p-z}$$

break away points are between two poles

⑧ find break points → break out for poles → break in for zeros.

① consider ch eqn
② write that eqn in terms of $k = \cdots$
③ $\frac{dk}{ds} = 0$
④ find roots
⑤ substitute the values of roots in cheqn

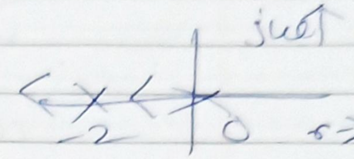
⑨ find k

⑩ Break point = value of s for which k is positive.

(Q) Draw root locus of a given open loop TF

$$G(s)H(s) = \frac{K}{s(s+2)}$$

① Poles:- $s=0,\ s=-2$
zeros: no zeros



② No. of poles $P = 2$
No. of zeros $Z = 0$

③ No. of branches $P - Z = 2 - 0 = 2$

④ Two branches originating from $s=0$ & $s=-2$ terminate to infinite.

⑤ Angle of Asymtotes.

$$\theta = \frac{(2q+1)180}{P-Z}$$

where ~~P-Z = 2~~ $q = 0, 1$

$\theta_1 = 90° \cdots q = 0$

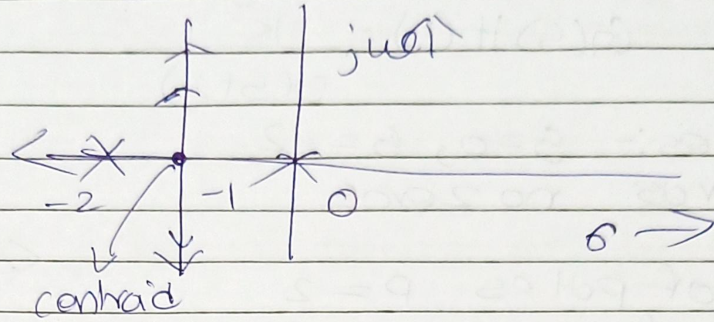
$\theta_2 = 270° \cdots q = 1$

⑥ centroid. - It is the point on real axis where asymtote meet.

$$\sigma = \frac{\Sigma \text{ rp of pole} - \Sigma \text{ rp of zero}}{P-Z}$$

$$\sigma = \frac{0 - 2 - (0)}{2}$$

$$= -1$$



(8) Break away point :-

ch eqn

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+2)} = 0$$

$$s^2 + 2s + K = 0$$

$$K = -s^2 - 2s$$

$$\frac{dK}{ds} = -2s - 2 = 0$$

$$= -2s - 2 = 0$$

$$2s - 2 = 0$$

$$\boxed{s = -1}$$

$$\therefore K = (-1)^2 - 2(-1)$$

$$= 1 + 2$$

$$K = 3$$

Page No.			
Date			

Unit-IV

Frequency Response Analysis.

Frequency response is the steady state output of a system to the sinusoidal input.

If frequency response input is sinusidal & system is analysied for this -

Let i/p ⇒ $r(t) = A\sin\omega t$

mag → phase.

o/p ⇒ $c(t) = B\sin(\omega t + \phi)$

- when sinousidal input is given to the system output is also sinusiodal in nature. Mag & phase can be different.

by changing angular frequency (ω) output calculated & system is analysized.

- Correlation between time response & frequency response of a second order system.

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Sub $s = j\omega$

$$= \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{\omega_n^2}{-\omega^2 + 2j\xi\omega_n\omega + \omega_n^2}$$

$$= \frac{\cancel{\omega_n^2}}{\cancel{\omega_n^2}\left(1 - \frac{\omega^2}{\omega_n^2} + \frac{2j\xi\omega}{\omega_n}\right)}$$

Let $\frac{\omega}{\omega_n} = x$

$$T(s) = T(j\omega) = \frac{1}{1-x^2+2j\xi x}$$

$$= \frac{1}{(1-x^2)+j2\xi x}$$

Magnitude

$$|M(j\omega)| = |T(j\omega)|$$

$$\left|\frac{1}{(1-x^2)+j2\xi x}\right| = \frac{1}{\sqrt{(1-x^2)^2+(2\xi x)^2}}$$

i.e. Magnitude

$$\boxed{M\left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}}}$$

phase.

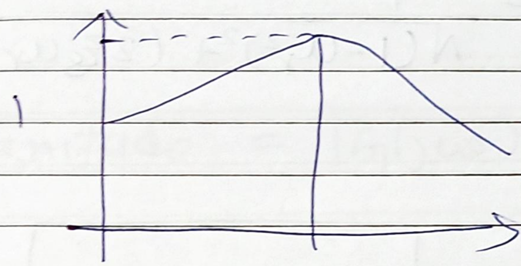
$$\angle T(j\omega) = -\tan^{-1}\left(\frac{2\xi u}{1-u^2}\right)$$

$\rightarrow$

* Resonant Peak -
It is the peak of magnitude in magnitude plot

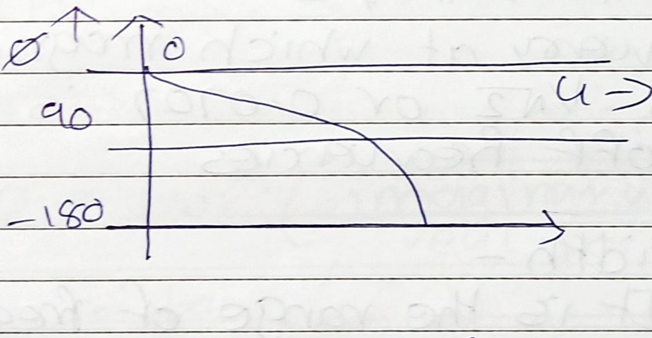
* Resonant frequency - (ω_r)
The frequency at which resonant peak occurs is known as resonant frequency. it is the frequency at which maximum magnitude occurs

- Magnitude plot -

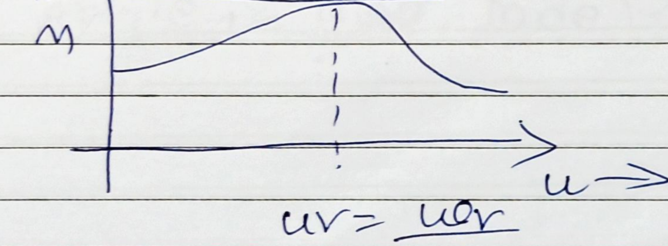


$u_r = \frac{\omega_r}{\omega_n}$

- Phase plot -



- Resonant frequency & Resonant peak



$\xi = 0$

$\xi = 1$

$u_r = \frac{\omega_r}{\omega_n}$

* Expression for resonant frequency & Resonant Peak

$$\boxed{\omega_r = \omega_n \sqrt{1-2\xi_s^2}}$$

after sub value of ω_r & ω_n in magnitude eqn.

i.e. $M_r = \dfrac{1}{\sqrt{(1-u_r^2)^2 + (2\xi_s u_r)^2}}$

we get

$$\boxed{M_r = \frac{1}{2\xi_s\sqrt{1-\xi_s^2}}}$$

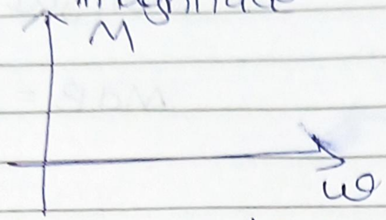
* cutt off frequency –
frequency at which magnitude becomes $1/\sqrt{2}$ or 0.0707 is known as cut off frequencies.

* Band width –
It is the range of frequencies where magnitude is $1/\sqrt{2}$ or greater than that is known as band width.

* Bode plot

magnitude plot

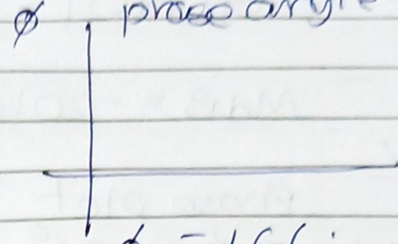
magnitude



$M = 20 \log |G(j\omega)|$

phase plot

phase angle



$\phi = \angle G(j\omega)$

1) For constant K

Magnitude $= |G(j\omega)| = K$

$$\boxed{M_{dB} = 20 \log K}$$

for $K < 1$
$M_{dB} = -ve$
for $K > 1$
$M_{dB} = +ve$

for constant K mag in dB is independent of ω

phase plot.

$$\phi = \tan^{-1}\left(\frac{\text{imaginary part}}{\text{real part}}\right)$$

$$= \tan^{-1}\left(\frac{0}{K}\right)$$

$$\phi = 0^{\circ}$$

Angle is also independent of ω.

Bode plot

(1)

$$G(s) = \frac{10}{s(s+1)(s+5)}$$

→ ① Time constant form

$\left((1+sT) \; \omega_c = 1/T \right)$

$$G(s) = \frac{10}{s(1+s) \cdot 5\left(1+\frac{s}{5}\right)}$$

$$= \frac{2}{s(1+s)\left(1+\frac{s}{5}\right)}$$

② factors -

i) constant $K = 2$

Mag in dB $= 20\log 2 = 6.02\,dB$

ii) Pole at origin. $- 1/s$

iii) First order pole. $\rightarrow 1/1+s$ $\quad \omega_c = \frac{1}{T}$

$\omega_c = 1/1$

iv) First order pole $\rightarrow \dfrac{1}{1+\frac{s}{5}}$

Table.

factors	corner freq	slope	resultant slope	start point	end
constant K $K=2$	–	no slope but mag in dB $=6.02$	–	–	∞
② Pole at origin	–	-20 dB/dec	-20 dB/dec	–	1
③ 1st order pole $(1+s)$	$\omega_c = 1$	-20 dB/dec	-40 dB/dec	1	5
④ $1+s/5$	$\omega_c = 5$	-20 dB/dec	-60 dB/dec	5	–

phase plot –

$$\phi = \frac{\tan^{-1}\left(\frac{0}{2}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega}{1}\right)\tan^{-1}\left(\frac{\omega}{5}\right)}$$

$$= \frac{0^\circ}{90^\circ \tan^{-1}\left(\frac{\omega}{1}\right)\tan^{-1}\left(\frac{\omega}{5}\right)}$$

$$\boxed{\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{5}}$$

ω	ϕ
0.1	-96.8
1	-146.3
10	-213
100	-237
	-266.56

(2) $G(s) = \frac{100(s+3)}{s(s+1)(s+5)}$

(1) Time constant form $(1+sT)$

$\therefore G(s) = \frac{\overset{60}{\cancel{100 \times 3}}(1+s/3)}{s(s+1)\cancel{5}(1+s/5)}$

$\therefore G(s) = \frac{60(1+s/3)}{s(s+1)(1+s/5)}$

(2) factors -

a) constant $K = 60$

b) Pole at origin. $\omega_c = 0$

c) 1st order pole $\rightarrow (1+s)$ $\omega_c = 1$

d) 1st order zero $\rightarrow (1+s/3)$ $\omega_c = 3$

e) 1st order pole $\rightarrow (1+s/5)$ $\omega_c = 5$

$\Rightarrow$

factors	Corner freq	slope	resl Slope	star P	e p
constant K $K=60$	—	$20\log 60 = 35.5dB$	—	—	∞
② pole at origin.	—	-20dB/de	-20dB/de	—	1
③ 1st order pole (1+s)	$\omega_c = 1$	+20dB/de	-20dB/de	3	5
⑤ $\left(1+\frac{s}{5}\right)$	$\omega_c = 5$	-20dB/de	-40dB/de	5	∞

phase plot

$$\phi = \tan^{-1}(0/60) \tan^{-1}(\omega/3)$$
$$\tan^{-1}\left(\frac{\omega}{0}\right) \tan^{-1}\left(\frac{\omega}{1}\right) \tan^{-1}\left(\frac{\omega}{5}\right)$$

$$\phi = -90 + \tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

ω	ϕ
0.1	-94.9
1	-127.9
3	-147.5
5	-154.8
10	-164.4
100	-178.2
500	-179.6

w_{gc} -> Gain crossover frequency
freq at which mag plot crosses 0dB

w_{pc} -> Phase crossover freq
freq at which phase plot crosss -180 degrees.

PM -> Phase Margin.
calculate ~~PM~~ at w_{gc} $PM = \phi + 180°$

GM -> Gain Margin.
calculate at ~~Phase~~ w_{pc}

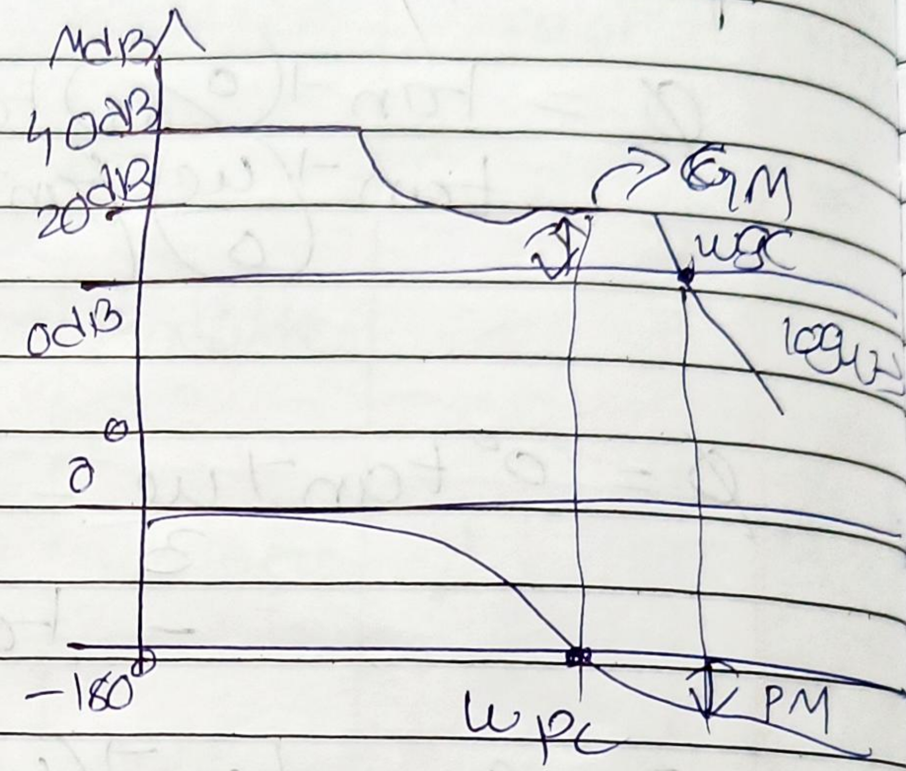
Page No.
Date 23/11/22

* Gain margin (GM)

Phase margin (PM)

Gain Cross over frequency (ω_{gc})

Phase crossover frequency (ω_{pc})



$$PM = \phi + 180^{\circ}$$
$$GM = 0 - ($$

For a stable system, GM & PM must be positive & $\omega_{pc} > \omega_{gc}$

Unit 5

State Space Analysis

Transfer function approach	State space approach
(1) It is the classical method	(1) It is a modern method.
(2) can be used for linear time invariant systems	(2) can be used for linear non-linear time variant, time invariant systems.
(3) single input single output calculations are easier	(3) Multiple input multiple output calculations are complex
(4) Takes into consideration only input & output	(4) Takes into consideration state variables or the system variables.
(5) Analysis is done from I/O & error signals	(5) Analysis done by conside diff state variable or parameters
(6) Only output is feedback for controlling & anlysis	(6) Internal parameters can be feedback
(7) Output has to be measurable	(7) Som of the outputs may not be measurable
(8) Do not take into consideration initial conditions	(8) Initial conditions are considered.

PAGE No.
DATE / /

* State space Model parameters -

① State -

state of a dynamic system is the smallest set of variables such that knowledge of these variables at $t=0$. Along with the knowledge of input for $t>0$ completely determined the behaviour of the system.

② State variable.

State variables of a dynamic system are the smallest of variables

③ State vector -

It is a vector whose components are stable variable is known as a state vector.

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

PAGE No.
DATE / /

* state space model for.

① Multiple input, multiple output

in general

$$\boxed{y(t) = Cx(t) + Du(t)}$$

where C & D are coefficients matrices for state variable & input

- state variable eqns:-

$$\boxed{x(t) = Ax(t) + Bu(t)}$$

* Diff methods of representing in state space model.

- electrical sys -

physical quantities are used as physical variables

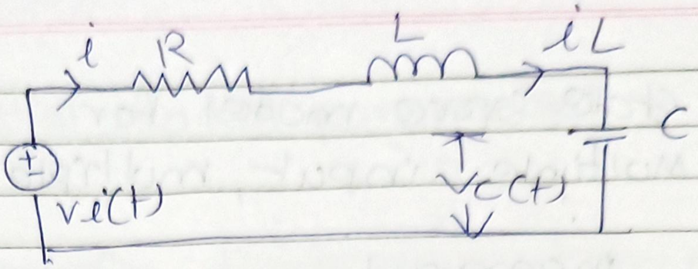
variables which stores energy are chosen as the state variable &

- Procedure for solving by physical variable.

① Assign all inductor currents & capacitor voltages as state variable.

② select a set of loop currents & write the relationship between the state variable & their derivatives in terms of these loop currents.

Q>



$i = i_L$

Step 1: choosing i_L & v_C as the state variables.

$v_i(t)$ - i/p to the system

$v_C(t)$ - o/p of the system

Applying KVL to the given loop.

$$Ri_L + L\frac{di_L}{dt} + \frac{1}{C}\int i_L\,dt = v_i$$

$$L\frac{di_L}{dt} = v_i - Ri_L - \frac{1}{C}\int i_L\,dt$$

$$\frac{di_L}{dt} = \frac{v_i}{L} - \frac{Ri_L}{L} - \frac{v_C}{L}$$

$$\frac{di_L}{dt} = \frac{-Ri_L}{L} - \frac{v_C}{L} + \frac{v_i}{L} \quad -(1)$$

$$v_C = \frac{1}{C}\int i_L\,dt$$

Differentiating both sides

$$\frac{dv_C}{dt} = \frac{i_L}{C} \quad -(2)$$

$$\text{o/p } y(t) = v_C(t) \quad -(3)$$

$\Rightarrow$

PAGE No.

DATE / /

$x_1(t) = i_L \qquad x_2(t) = v_C$

$\dot{x}_1(t) = \frac{di_L}{dt} \qquad \dot{x}_2(t) = \frac{dv_C}{dt}$

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} v_i(t) \end{bmatrix}$$

$y(t) = v_C(t)$

$y(t) = Cx(t) + Du(t)$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \ \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$$

* Phase variable form -

Phase variables are those state variables which are obtained from output variables and their derivatives -

state variables = o/p variables or derivatives of o/p variables

Note: $\boxed{\frac{5d^2y}{dt^2} = 5\ddot{y} \quad \frac{5dy}{dt} = 5\dot{y} \quad \frac{5d^2y}{dt^2} = 5s^2y(s)}$

Q) obtain the state model for the system

$$\frac{Y(s)}{X(s)} = \frac{1}{s^3+6s^2+10s+5}$$

① $x(s) = y(s)(s^3+6s^2+10s+5)$

Taking laplace inv

$$x = \frac{d^3y}{dt^3} + \frac{6d^2y}{dt^2} + 10\frac{dy}{dt} + 5y$$

$$x = \dddot{y} + 6\ddot{y} + 10\dot{y} + 5y \quad \text{—} ①$$

Let x_1 be state variable

$\therefore \quad x_1 = y$

$y = x_1$

$\dot{y} = \dot{x}_1 = x_2$

$\ddot{y} = \ddot{x}_1 = \dot{x}_2 = x_3$

$\dddot{y} = \dddot{x}_1 = \ddot{x}_2 = \dot{x}_3$

from ①

$$\dddot{y} = -6\ddot{y} - 10\dot{y} - 5y + x$$

$$\boxed{\dot{x}_3 = -6x_3 - 10x_2 - 5x_1 + x}$$

$$\boxed{\dot{x}_1 = x_2}$$

$$\boxed{\dot{x}_2 = x_3}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -10 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} x \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q) $\frac{Y(s)}{U(s)} = \frac{5}{s^3+6s+7}$

state space model of a given T.F.

-> $5U(s) = Y(s)(s^3+6s+7)$

Taking laplace Inv.

$5u = \frac{d^3y}{dt^3} + \frac{6dy}{dt} + 7y$

$5u = \dddot{y} + 6\dot{y} + 7y$ — (1)

Let x_1 be the state variable

$y = x_1$

$\dot{y} = \dot{x}_1 = x_2$

$\ddot{y} = \ddot{x}_1 = \dot{x}_2 = x_3$

$\dddot{y} = \dddot{x}_1 = \ddot{x}_2 = \dot{x}_3$

from eqn (1)

$\dddot{y} = -6\dot{y} - 7y + 5u$

$\dot{x}_3 = -6x_2 - 7x_1 + 5u$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} [u]$$

Q) $\ddddot{y} + 5\dddot{y} + 7\dot{y} + 9y = 8u$

$\Rightarrow$ Let x_1 be the state varible

$$y = x_1$$
$$\dot{y} = \dot{x}_1 = x_2$$
$$\ddot{y} = \ddot{x}_1 = \dot{x}_2 = x_3$$
$$\dddot{y} = \dddot{x}_1 = \ddot{x}_2 = \dot{x}_3 = x_4$$
$$\ddddot{y} = \ddddot{x}_1 = \dddot{x}_2 = \ddot{x}_3 = \dot{x}_4$$

From eqn (1)

$$\ddddot{y} + 5\dddot{y} + 7\dot{y} + 9y = 8u$$

$\cancel{x_4}$

$$\ddddot{y} = -5\dddot{y} - 7\dot{y} - 9y + 8u$$
$$\Rightarrow \dot{x}_4 = -5x_4 - 7x_2 - 9x_1 + 8u$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & -7 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 8 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

y

PAGE No.
DATE / /

(3) Direct Decomposition Method or signal flow method

Q) $$\frac{Y(s)}{U(s)} = \frac{3s+4}{s^2+5s+6}$$

step1) Divide Num & Den by highest order i.e. s^2

$$\frac{Y(s)}{U(s)} = \frac{\frac{3s+4}{s^2}}{\frac{s^2+5s+6}{s^2}}$$

$$= \frac{3/s + 4/s^2}{1+5/s+6/s^2}$$

step2) Multiply num & den by some arbitary i/p X_s

$$\frac{Y(s)}{U(s)} = \frac{(3/s+4/s^2)\,X(s)}{\left(1+\frac{5}{s}+\frac{6}{s^2}\right)X(s)}$$

$$Y(s) = \left(\frac{3}{s}+\frac{4}{s^2}\right)X(s) \quad -①$$

$$U(s) = \left(1+\frac{5}{s}+\frac{6}{s^2}\right)X(s) \quad -②$$

$$\boxed{Y(s) = \frac{3}{s}X(s) + \frac{4}{s^2}X(s)}$$

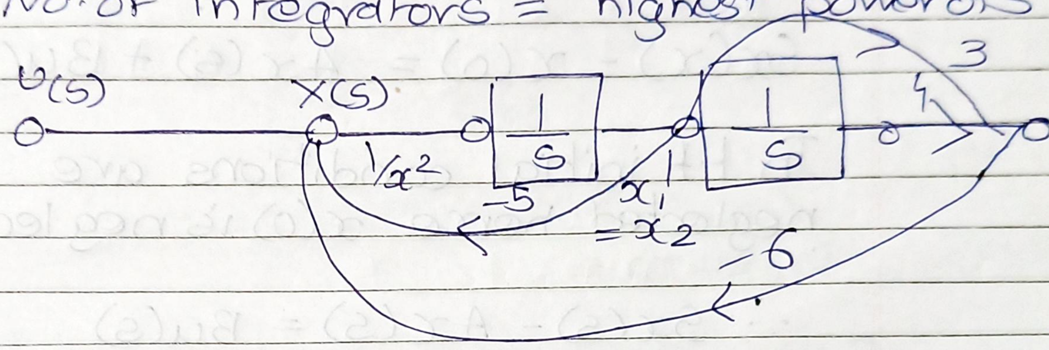
$$U(s) = \left(1+\frac{5}{s} + \frac{6}{s^2}\right) X(s)$$

$$= X(s) + \frac{5}{s}X(s) + \frac{6}{s^2}X(s)$$

$$\Rightarrow \boxed{X(s) = U(s) - \frac{5}{s}X(s) - \frac{6}{s^2}X(s)}$$

$1/s$ is used as & integrator

No. of integrators = highest power of s



$$x_1' = x_2$$

$$x_2' = -6x_1 - 5x_2 + U(s)$$

$$Y(s) = 4x_1 + 3x_2$$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

$$[y] = [4\ 3] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0][u]$$

PAGE No.
DATE / /

* Find T.F from State space Model

① State space form is given by

$$x(t) = Ax(t) + Bu(t) \quad -①$$
$$y(t) = Cx(t) + Du(t) \quad -②$$

Taking laplace of eqn ①

$$Sx(x) - x(0) = Ax(s) + Bu(s)$$

In T.f initial conditions are neglected hence $x(0)$ is neglected.

$$\therefore Sx(s) - Ax(s) = Bu(s)$$
$$x(s)[SI - A] = Bu(s)$$
$$x(s) = [SI - A]^{-1} = BU(S)$$

Similarly,

$$Y(s) = C[SI - A]^{-1} + D$$

Substituting value $X(s)$ in eqn ② with taking laplace of eqn ②

$$Y(s) = C[SI - A]^{-1} BU(s) + DU(s)$$
$$= U(s)\left[C[SI - A]^{-1} B + D\right]$$
$$\frac{Y(s)}{U(s)} = C[SI - A]^{-1} B + D$$

$$\boxed{T.f = C[SI - A]^{-1} B + D}$$

PAGE No.
DATE / /

Q) From the given state space equations

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 0$$

(1)

$$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$[SI-A]^{-1} = \frac{adj\,(SI-A)}{|SI-A|}$$

$$adj\,(SI-A) = \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$[SI-A]^{-1} = \frac{\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}}{S(S+3)+2}$$

$$T.F = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix} [1\ 0] \cdot \frac{1}{S^2+3S+2}$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} S+S & 1 \\ -2 & S \end{bmatrix}$$

$$\boxed{T.F = \frac{1}{S^2+3S+2}}$$

PAGE No.
DATE / /

* Controllability & observability.

Q1) Controllability -
verify the controllability of a control system which is represented by state equation

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$

$$= A \quad x + B \quad u$$

if $|Q| \neq 0$
sys is controllable
if $|Q_c| = 0$
sys is uncontrollable.

by Kalman's test.

$$Q_c = [B \quad AB \quad A^2B \cdots \cdots A^{n-1}B]$$

Given $A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$n = 2$ .... ($n \times n$ matrix size)

$\therefore Q_c = [B \quad AB] \cdots$

$\therefore Q_c = \left[\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right]$

$= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

Now $|Q_c| = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1 \cdots \neq 0$

$\therefore$ The sys is controllable.

PAGE No. DATE / /

* Observability -

Q-> Verify the observability of a control system which is represented in the state space model.

state equation -> $\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u]$

$\dot{x} = A \quad x + B \quad u$

output equation -> $y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$y = Cx + Du$

$\therefore$ from given: $A = \begin{bmatrix} -2 & -2 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$ $D = 0$

by Kalaman's test

① $Q_o = \begin{bmatrix} C^T & A^T C^T & (A^T)^2 C^T \cdots\cdots & (A^T)^{n-1} C^T \end{bmatrix}$

n ... 2x2

$\therefore Q_o = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$

$Q_o = \begin{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{bmatrix}$

$\begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+1 \\ -2+0 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$Q_o = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$

② $|Q_o| = \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = -2+1 = -1 \neq 0$

$\therefore$ The sys is observable.

$-2+1$

if $|Q_c| \neq 0$ -> sys observable

if $|Q_o| = 0$ -> sys is unobservable.