Topic	Skills
Standard and extended mathematics	
Addition, subtraction, multiplication and division of algebraic terms	Expanding and simplifying algebraic expressions
Factorization of algebraic expressions	Factorizing linear and quadratic expressions
Substitution	Using substitution to evaluate expressions
Rearranging algebraic expressions	Changing the subject of the formula
Algebraic fractions	Solving equations involving algebraic fractions
Integer and fractional exponents (including negative number exponents)	Using the laws of exponents
Patterns and sequences	Finding and justifying or proving general rules/ formulae for sequences

Algorithms	Analysing and using well-defined procedures for solving complex problems
Functions • Types of functions: linear, quadratic, exponential, sine and cosine • Domain and range • Transformations	The linear function, $f(x) = mx + c$, its graph, gradient and y-intercept Parallel and perpendicular lines and the relationships between their gradients Describing transformed linear, quadratic, exponential, and sine and cosine functions Example: $f(x) = a\,(x - h)^2 + k$ Note: Sine and cosine functions are limited to the form $f(x) = a \sin(bx) + c$ Graphing different types of functions and understanding their characteristics Determining the range, given the domain Translating, reflecting and dilating functions
Equations: • Linear • Quadratic • Simultaneous	Solving equations algebraically and graphically
Inequalities	Solving and graphing linear inequalities Linear programming

Extended mathematics	
Logarithms with different base number (including natural logarithms)	Using the laws of logarithms
Functions and graphs • Sine and cosine, logarithmic and rational (of the form $f(x) = 1/x$) functions • Inverse and composite functions	Graphing different types of functions and understanding their characteristics Addition and subtraction of functions Determining inverse and composite functions and their graphs Solving equations algebraically and graphically
Inequalities	Solving non-linear inequalities
Transformations of functions	Describing and analysing transformed logarithmic, rational (of the form $f(x) = 1/x$), and sine and cosine functions Example: $f(x) = a \sin(bx - c) + d$
Arithmetic and geometric series	Developing, and justifying or proving, general rules/ formulae for sequences Finding the sum of the series, including infinite series

ALGEBRAIC SUBSTITUTION – PRACTICE QUESTIONS

If $w=3$, $x=1$ and $y=-2$, evaluate:

a $\dfrac{y}{w}$ b $\dfrac{y+w}{x}$ c $\dfrac{3x-y}{w}$ d $\dfrac{5w-2x}{y-x}$

e $\dfrac{y-x+w}{2(y-w)}$ f $\dfrac{xy+w}{y-x}$ g $\dfrac{x-wy}{y+w-2x}$ h $\dfrac{y}{x}-w$

If $a=-3$, $b=-4$ and $c=-1$, evaluate:

a c^2 b b^3 c a^2+b^2 d $(a+b)^2$

e b^3+c^3 f $(b+c)^3$ g $(2a)^2$ h $2a^2$

If $p=4$, $q=-1$ and $r=2$, evaluate:

a $\sqrt{p}+q$ b $\sqrt{p+q}$ c $\sqrt{r-q}$ d $\sqrt{p-pq}$

e $\sqrt{pr-q}$ f $\sqrt{p^2+q^2}$ g $\sqrt{p+r+2q}$ h $\sqrt{2q-5r}$

LINEAR EQUATIONS – PRACTICE QUESTIONS

Solve for x:

a $\dfrac{5}{x}=\dfrac{2}{3}$ b $\dfrac{6}{x}=\dfrac{3}{5}$ c $\dfrac{4}{3}=\dfrac{5}{x}$ d $\dfrac{3}{2x}=\dfrac{7}{6}$

e $\dfrac{3}{2x}=\dfrac{7}{3}$ f $\dfrac{7}{3x}=-\dfrac{1}{6}$ g $\dfrac{5}{4x}=-\dfrac{1}{12}$ h $\dfrac{4}{7x}=\dfrac{3}{2x}$

Solve for x:

a $\dfrac{2x+3}{x+1}=\dfrac{5}{3}$ b $\dfrac{x+1}{1-2x}=\dfrac{2}{5}$ c $\dfrac{2x-1}{4-3x}=-\dfrac{3}{4}$

d $\dfrac{x+3}{2x-1}=\dfrac{1}{3}$ e $\dfrac{4x+3}{2x-1}=3$ f $\dfrac{3x-2}{x+4}=-3$

g $\dfrac{6x-1}{3-2x}=5$ h $\dfrac{5x+1}{x+4}=4$ i $2+\dfrac{2x+5}{x-1}=-3$

Solve for x:

a $\dfrac{x}{2}-\dfrac{x}{6}=4$ b $\dfrac{x}{4}-3=\dfrac{2x}{3}$

c $\dfrac{x}{8}+\dfrac{x+2}{2}=-1$ d $\dfrac{x+2}{3}+\dfrac{x-3}{4}=1$

e $\dfrac{2x-1}{3}-\dfrac{5x-6}{6}=-2$ f $\dfrac{x}{4}=4-\dfrac{x+2}{3}$

g $\dfrac{2x-7}{3}-1=\dfrac{x-4}{6}$ h $\dfrac{x+1}{3}-\dfrac{x}{6}=\dfrac{2x-3}{2}$

i $\dfrac{x}{5}-\dfrac{2x-5}{3}=\dfrac{3}{4}$ j $\dfrac{x+1}{3}+\dfrac{x-2}{6}=\dfrac{x+4}{12}$

k $\dfrac{x-6}{5}-\dfrac{2x-1}{10}=\dfrac{x-1}{2}$ l $\dfrac{2x+1}{4}-\dfrac{1-4x}{2}=\dfrac{3x+7}{6}$

Solve for x:

a $\frac{x}{3} = \frac{7}{x}$ **b** $\frac{x}{3} = \frac{3}{x}$ **c** $\frac{1}{x} = \frac{x}{2}$ **d** $\frac{x}{9} = \frac{9}{x}$

e $\frac{4}{x} = \frac{x}{5}$ **f** $\frac{7}{x} = \frac{x}{10}$ **g** $\frac{x}{2} = \frac{8}{x}$ **h** $\frac{x}{3} = \frac{-2}{x}$

MIXED QUESTIONS – BASIC ALGEBRAIC MANIPULATION

1. When $x = -3$ find the value of

$$x^3 + 2x^2.$$

2. Make s the subject of the formula

$p = st - q.$

3. a) Expand the bracket and simplify the expression

$$7x + 5 - 3(x - 4).$$

b) Simplify and write your answer as a fraction in its simplest form.

i) $\frac{x+2}{x} - \frac{x}{x+2}$. ii) $\frac{5}{5x+1} - \frac{2}{2x-3}$.

4. Factorise

a) $5x^2 - 7x$.

b) $12ax^3 + 18xa^3$.

c) $4x^2 - 9$,

d) $4x^2 - 9x$,

e) $4x^2 - 9x + 2$.

5. (i) On Monday a shop receives \$60.30 by selling bottles of water at 45 cents each. How many bottles are sold?

(ii) On Tuesday the shop receives x **cents** by selling bottles of water at 45 cents each. In terms of x, how many bottles are sold?

(iii) On Wednesday the shop receives $(x - 75)$ **cents** by selling bottles of water at 48 cents each.
In terms of x, how many bottles are sold?

(iv) The number of bottles sold on Tuesday was 7 more than the number of bottles sold on Wednesday.
Write down an equation in x and solve your equation.

6. (a) $4^p \times 4^5 = 4^{15}$. Find the value of p.

(b) $2^7 \div 2^q = 2^4$. Find the value of q.

(c) $5^r = \frac{1}{25}$. Find the value of r.

7. A packet of sweets contains chocolates and toffees.

(a) There are x chocolates which have a total mass of 105 grams.

Write down, in terms of x, the mean mass of a chocolate.

(b) There are $x + 4$ toffees which have a total mass of 105 grams.

Write down, in terms of x, the mean mass of a toffee.

(c) The difference between the two mean masses in **parts** (**a**) and (**b**) is 0.8 grams.

Write down an equation in x and show that it simplifies to $x^2 + 4x - 525 = 0$.

8. Amira takes 9 hours 25 minutes to complete a long walk.

(i) Show that the time of 9 hours 25 minutes can be written as $\frac{113}{12}$ hours.

(ii) She walks $(3y + 2)$ kilometres at 3 km/h and then a further $(y + 4)$ kilometres at 2 km/h.

Show that the total time taken is $\frac{9y+16}{6}$ hours

(iii) Solve the equation $\frac{9y+16}{6} = \frac{113}{12}$.

(iv) Calculate Amira's average speed, in kilometres per hour, for the whole walk.

9. Factorise completely:

(a) $1-16m^2$

(b) $\frac{12}{a^2}-\frac{4p}{a^2}$

(c) $121a^2+225-330a$

(d) x^4-$16y^4$

(e) $(a-b)^2-(x-y)^2$

(f) $\frac{1}{144}+36k^2+k$

(g) $10x^2-73x+21$

(h) $20ac-4ad-15kc+3kd$

10. Substitute $\frac{2a^2-3b^2}{a+2b}$ for x in $\frac{x+2a}{x-3b}$ and reduce it to the simplest form.

11. Divide $\frac{(a+b)^2-c^2}{a^2-(b+c)^2}$ by $\frac{b^2-(c+a)^2}{c^2-(a+b)^2}$

---

12. Change the subject to a:

(a) $\frac{1}{1-a}=\frac{2}{b-3}$

(b) $x=-b+\sqrt{b^2-4ac}$

---

13. Express as single fraction:

(c) $\frac{1}{2x^2-18}-\frac{1}{x^2-x-6}$

(d) $\frac{x^2+5x+4}{2x^2+5x-12}-\frac{1}{9-12x+4x^2}$

(e) $\frac{5x}{2x-y}+\frac{y}{3x-y}$

(f) $\frac{1}{2x-3}-\frac{2}{x+2}-\frac{2x-x^2}{2x^2+x-6}$

(g) $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

---

14. Solve:

(h) $6x^2-x-2=0$. (By factorization method)

(i) $5-y^2=0$. Give your answer in surd form.

(j) $8+2x-3x^2=0$

15. Solve $(5t-2)(t+1)=3t(3t-1)$.

16. Solve $\sqrt{2x+9} = 13 - x$, using completing square method.

17. Solve $3a^2x^2 - abx - 2b^2 = 0$, using the formula.

18. Solve $\dfrac{2}{x+3} - \dfrac{3}{3x-1} = 2$.

19. Solve $3x^2 - 5x + 1 = 0$ (Give your answer correct to 2 d.p.)

---

20. Solve the equation $3x^2 - 19x - 7 = 0$.

21. Solve $\dfrac{4}{x-1} - \dfrac{5}{x+2} = \dfrac{3}{x}$.

---

22. Solve $abx^2 - (a^2 + b^2)x + ab = 0$ for x in terms of a and b using quadratic formula.

QUADRATIC EQUATIONS

PROBLEM SOLVING METHOD

- Carefully **read the question** until you **understand** it. A **rough sketch** may be useful.
- Decide on the **unknown** quantity, calling it x, say.
- Find an **equation** which connects x and the information you are given.
- **Solve** the equation using factorisation and the **Null Factor law**.
- **Check** that any solutions satisfy the original problem.
- Write your answer to the question in **sentence form**.

PRACTICE QUESTIONS

A company making surfboards finds that its profit $\$P$ per hour, is given by the formula $P = 60x - 3x^2$ where x is the number of surfboards made per hour.
When does the company make: **a** $\$0$ profit per hour **b** $\$225$ profit per hour?

When a cricket ball is hit directly upwards its height above the ground is given by $h = 40t - 5t^2$ metres, where t is the time after the ball is hit (in seconds).
When is the ball at a height of: **a** 0 m above the ground **b** 75 m?

2 **a** The sum of a positive integer and its square is 90. Find the number.

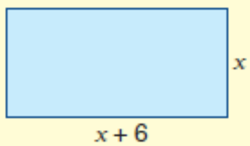
b The sum of a positive integer and its square is 132. Find the number.

c The difference between a positive integer and its square is 56. Find the number.

d The square of a number is equal to 5 times the number. What are the two possible answers?

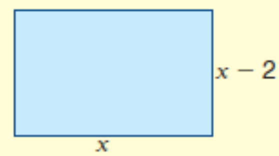
e When a number is subtracted from its square, the result is 42. Find the two possible solutions.

3 **a**

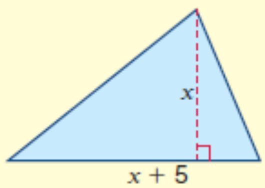


Find the dimensions of this rectangle if the length is 6 cm longer than the breadth and its area is 40 cm^2.

b The width of a rectangular room is 2 metres shorter than its length. If the area of the room is 255 m^2, find the dimensions of the room.

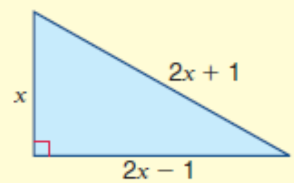


c



The base of a triangle is 5 cm longer than its height. If the area of the triangle is 7 cm^2, find the length of the base.

d A right-angled triangle is drawn so that the hypotenuse is twice the shortest side plus 1 cm, and the other side is twice the shortest side less 1 cm. Find the length of the hypotenuse.



4 **a** Michelle threw a ball vertically upwards, with its height h, in metres, after a time of t seconds, being given by the formula:

$$h = 8t - t^2$$

Find after what time the ball is first at a height of 12 m.

b The sum, S, of the first n positive integers is given by the formula

$$S = \frac{n}{2}(n+1)$$

Find the number of positive integers needed to give a total of 78.

c For the formula $s = ut + \frac{1}{2}at^2$, find the values of t if:

i $s = 18,\quad u = 7,\quad a = 2$

ii $s = 6,\quad u = 11,\quad a = 4$

iii $s = 7,\quad u = 1,\quad a = 6$

5 An n-sided polygon has $\frac{1}{2}n(n-3)$ diagonals. How many sides has a figure if it has 90 diagonals?

6 Jenny is y^2 years old and her daughter Allyson is y years old. If Jenny lives to the age of $13y$, Allyson will be y^2 years old. How old is Allyson now? (*Note:* the difference in ages must remain constant.)

7 Kylie bought an item for \$$x$ and sold it for \$10.56. If Kylie incurred a loss of x per cent, find x.

8 A relationship that is used to approximate car stopping distances (d) in ideal road and weather conditions is:

$$d = t_r v + kv^2$$

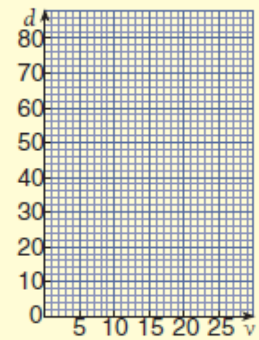
where t_r is the driver's reaction time, v is the velocity and k is a constant.

a Stirling's reaction time was measured to be 0·8 seconds. The distance it took him to stop while travelling at 20 m/s (72 km/h) was 51 metres. Substitute this information into the formula to find the value of k.

b If, for these particular conditions, Stirling's breaking distance is given by

$$d = 0{\cdot}8v + 0{\cdot}0875v^2$$

complete the table below, finding d correct to the nearest metre in each case.



v (in m/s)	0	5	10	15	20	25
d (in m)						

c Graph d against v using the number plane shown on the right. What kind of curve is produced?

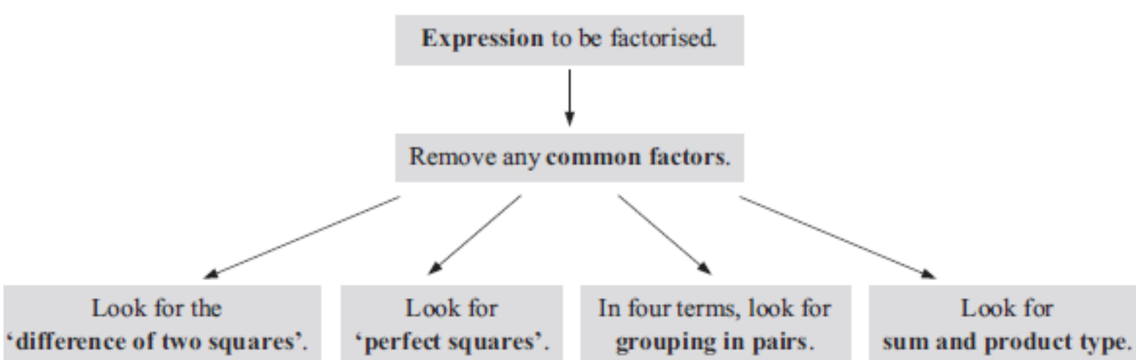
d Use your graph to find the velocity (in m/s) that would produce a stopping distance of 40 metres. Check your accuracy by solving the equation

$40 = 0{\cdot}8v + 0{\cdot}0875v^2$ using the formula $v = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

e What factors would determine the safe car separation distance in traffic?

FACTORIZATION

The following flowchart may prove useful:



PRACTICE QUESTIONS

1 Fully factorise:

a $3x^2 + 2x$	**b** $x^2 - 81$	**c** $2p^2 + 8$
d $3b^2 - 75$	**e** $2x^2 - 32$	**f** $n^4 - 4n^2$
g $x^2 - 8x - 9$	**h** $d^2 + 6d - 7$	**i** $x^2 + 8x - 9$
j $4t + 8t^2$	**k** $3x^2 - 108$	**l** $2g^2 - 12g - 110$
m $4a^2 - 9d^2$	**n** $5a^2 - 5a - 10$	**o** $2c^2 - 8c + 6$
p $x^4 - x^2$	**q** $d^4 + 2d^3 - 3d^2$	**r** $x^3 + 4x^2 + 4x$

2 Find the pattern in the following expressions and factorise:

a $x^2 - 6x + 9$	**b** $x^2 - 121$	**c** $x^2 - 2x + 1$
d $y^2 + 10y + 25$	**e** $x^2 + 22x + 121$	**f** $x^2 - 2xy + y^2$
g $1 - x^2$	**h** $25y^2 - 1$	**i** $49y^2 - 36z^2$
j $4d^2 + 28d + 49$	**k** $4ab^2 - ac^2$	**l** $2\pi R^2 - 2\pi r^2$

3 Fully factorise:

a $ab + ac - 2a$	**b** $a^2b^2 - 2ab$	**c** $18x - 2x^3$
d $x^2 + 14x + 49$	**e** $4a^3 - 4ab^2$	**f** $x^3y - 4xy$
g $4x^4 - 4x^2$	**h** $(x-2)y - (x-2)z$	**i** $(x+1)a + (x+1)b$
j $(x-y)a + (x-y)$	**k** $x(x+2) + 3(x+2)$	**l** $x^3 + x^2 + x + 1$

4 Factorise completely:

a $7x - 35y$	**b** $2g^2 - 8$	**c** $-5x^2 - 10x$
d $m^2 + 3mp$	**e** $a^2 + 8a + 15$	**f** $m^2 - 6m + 9$
g $5x^2 + 5xy - 5x^2y$	**h** $xy + 2x + 2y + 4$	**i** $y^2 + 5y - 9y - 45$
j $2x^2 + 10x + x + 5$	**k** $3y^2 - 147$	**l** $3p^2 - 3q^2$
m $4c^2 - 1$	**n** $3x^2 + 3x - 36$	**o** $2bx - 6b + 10x - 30$

5 Fully factorise:

a $12 - 11x - x^2$	**b** $-2x^2 - 6 + 8x$	**c** $14 - x^2 - 5x$
d $4x^2 - 2x^3 - 2x$	**e** $(a+b)^2 - 9$	**f** $(x+2)^2 - 4$

ALGEBRAIC FRACTIONS

The process is:

Algebraic fraction → Express both numerator and denominator as a **product of factors**. →

- If there are common factors, simplify by **cancelling**.
- If there are no common factors, you **cannot simplify**.

Simplify:

a $\dfrac{x^2-1}{x-1}$ b $\dfrac{x^2-1}{x+1}$ c $\dfrac{x+2}{x^2-4}$ d $\dfrac{a^2-b^2}{a+b}$

e $\dfrac{2x+2}{x^2-1}$ f $\dfrac{9-x^2}{3x-x^2}$ g $\dfrac{3x^2-3y^2}{2xy-2y^2}$ h $\dfrac{4xy-y^2}{16x^2-y^2}$

2 Simplify by factorising and cancelling common factors:

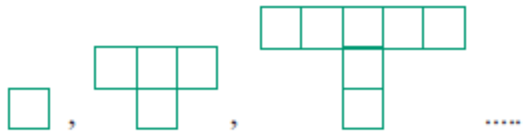
a $\dfrac{x^2-x-2}{x-2}$ b $\dfrac{x+3}{x^2-2x-15}$ c $\dfrac{x^2+4x+4}{x+2}$

d $\dfrac{x^2-6x+9}{x-3}$ e $\dfrac{2x^2+2x}{x^2-4x-5}$ f $\dfrac{x^2-2x}{x^2+x-6}$

g $\dfrac{x^2-4}{x^2+4x+4}$ h $\dfrac{x^2-x-12}{x^2-5x+4}$ i $\dfrac{2x^2-4x}{x^2+3x-10}$

PATTERNS & SEQUENCES – PRACTICE QUESTIONS

Consider the pattern:



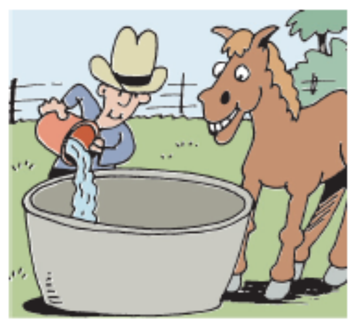
- **a** Continue the pattern for 2 more figures.
- **b** How many matchsticks are required to make each of the first five figures?
- **c** Copy and complete:

Number of figure (n)	1	2	3	4	5	8
Number of matchsticks (M)						

- **d** Write a description of the pattern.
- **e** Write a general rule for the connection between M and n.
- **f** Predict the number of matchsticks needed for the 30th figure.

-

Lars notices that the water in his horse's drinking trough is only 2 cm deep. The trough is cylindrical. Each time Lars tips a bucket of water into the trough, the water level rises 1.5 cm.



- **a** Find how much the water level rises if b buckets of water are tipped into the trough.
- **b** What depth D cm of water is in the trough if b buckets of water have been emptied into it?
- **c** How deep is the water in the trough if Lars tips:
  - **i** 5 buckets **ii** 18 buckets of water into it?

Students fill 600 mL bottles from a water cooler containing 50 litres of water.

- **a** If b bottles are filled, how much water is used?
- **b** How much water W litres is left in the cooler if b bottles have been filled?
- **c** How much water is left in the cooler if:
  - **i** 15 bottles **ii** 37 bottles have been filled?

1 Use the method of differences to find the general term u_n of:

a $1, 5, 9, 13, 17, 21, \ldots\ldots$

b $17, 14, 11, 8, 5, 2, \ldots\ldots$

c $2, 6, 12, 20, 30, 42, \ldots\ldots$

d $0, 6, 14, 24, 36, 50, \ldots\ldots$

e $6, 13, 32, 69, 130, 221, 348, \ldots\ldots$

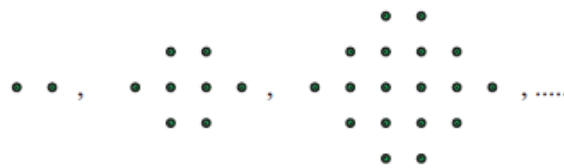
f $2, 7, 18, 38, 70, 117, 182, \ldots\ldots$

2 Consider the sequence: $2, 12, 30, 56, 90, 132, \ldots\ldots$

a Use the difference method to find the general term u_n.

b Suggest an alternative formula for u_n by considering $u_1 = 1 \times 2$, $u_2 = \ldots\ldots \times \ldots\ldots$, $u_3 = \ldots\ldots \times \ldots\ldots$, and so on.

3 Consider the dot pattern:



a Find u_n for $n = 1, 2, 3, 4, 5, 6$ and 7.

b Find a formula for the general term u_n.

c How many dots are needed to make up the 30th figure in the pattern?

1 Write down a rule for the sequence and find its next two terms:

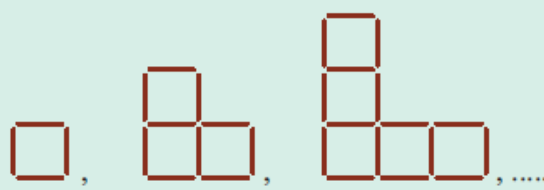
a $6, 10, 14, 18, 22, \ldots\ldots$

b $810, 270, 90, 30, \ldots\ldots$

2 Draw the next *two* matchstick figures in the pattern and write down the number of matchsticks used as a number sequence:

a

b



3 Find the first four terms of the sequence with nth term:

a $u_n = 6n - 1$

b $u_n = n^2 + 5n - 2$

4 a Find a formula for the general term u_n of the sequence: $4, 8, 12, 16, \ldots\ldots$

b *Hence* find u_n for:

i $1, 5, 9, 13, \ldots\ldots$

ii $\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \ldots\ldots$

c Find the 20th term for each of the sequences in **b**.

1) Four sequences, A, B, C and D, are defined by the following formulae:

A $u_n = 8n + 2$; B $u_n = 7n - 3$

C $u_n = 3n + 1$; D $u_n = 100 - 6n$

a) Which sequences have 4 as their first term?
b) Which sequence is decreasing?

c) Which sequence has a difference of 7 between terms?
d) Which sequence has 301 as its 100th term?

2) Determine the formula for the general term of each of the following sequences:
a) 4, 13, 22, 31, 40, …..
b) 1, 20, 39, 58, 77, …..
c) - 18, - 16, - 14, - 12, -10, ….
d) 8, 1, -6, -13, -20, ……

3) A sequence is listed below: 2, 9, 20, 35, 54, 77, ….
a) Calculate the second differences for this sequence.
b) Form a simpler sequence by subtracting $2n^2$ from each term.
c) Determine a formula for the general term of the simpler sequence.
d) Determine a formula for the general term of the original sequence.

4) Determine the formula for the general term of each of the following sequences:
a) 3, 17, 39, 69, 107, ….
b) 9, 23, 45, 75, 113, ….
c) – 4, 12, 38, 74, 120, ….

5) Consider the sequence 1, 4, 16, 64, 256, ….
a) Calculate the next 3 terms?
b) Determine a formula for the nth term of the sequence.

6) Determine the formula for the general term of each of the following sequences:
a) 1, 3, 9, 27, 81, …..
b) 20, 200, 2000, 20 000, 200 000, ….
c) 4, 28, 196, 1372, 9604, …..

7)
a) Determine the general formula for the terms of the sequence,
1, 7, 13, 19, 25, 31, …..

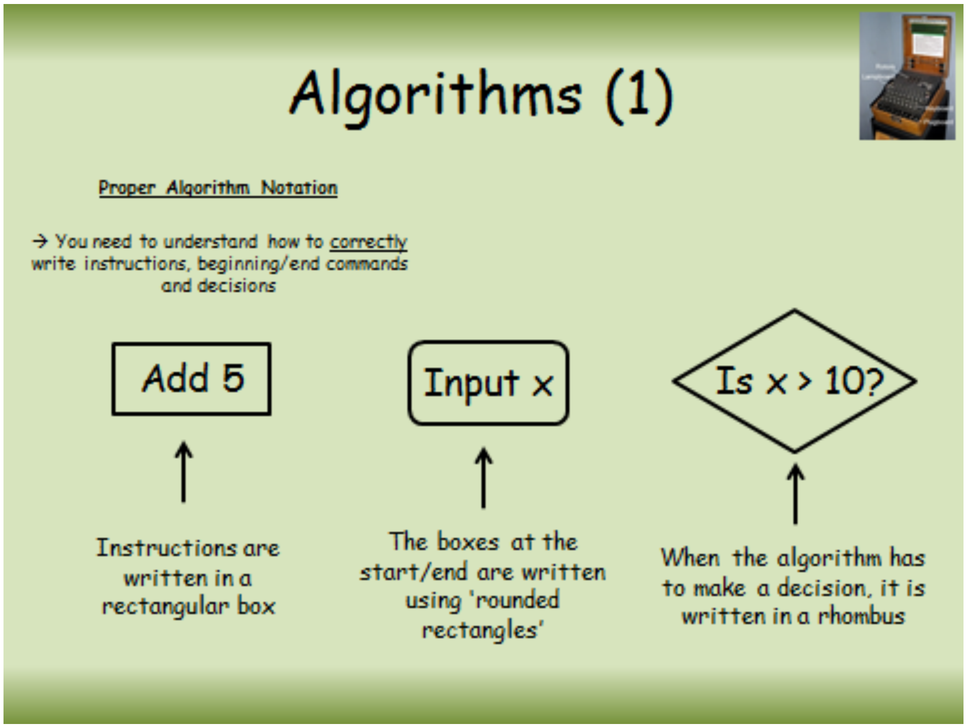
b) Determine the general formula for the terms of the sequence,
2, 10, 18, 26, 34, 42, ….

c) Determine the general formula for the terms of the sequence,
$$\frac{1}{2},\frac{7}{10},\frac{13}{18},\frac{19}{26},\frac{25}{34}....$$

8) Determine the general formula for the terms of each of the following sequences:
a) $\frac{1}{4},\frac{2}{5},\frac{3}{6},\frac{4}{7},....$
b) $\frac{1}{3},\frac{3}{11},\frac{5}{19},\frac{7}{27},....$
c) $\frac{1}{10},\frac{7}{20},\frac{13}{40},\frac{19}{80},....$

9) Find the next term and the nth term in each of the sequences.
a) 6,12, 24, 48, 96, ….
b) – 1, 0, 3, 8, 15, ….
c) 0, 3, 8, 15, 24, ….

ALGORITHMS – PRACTICE QUESTIONS



1.

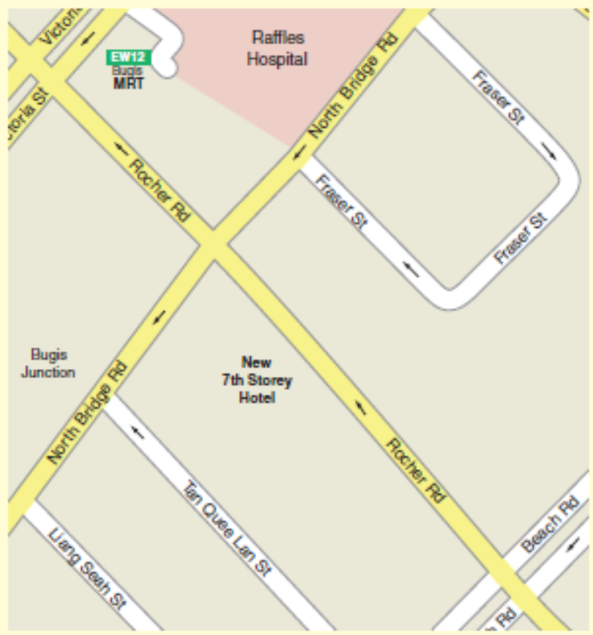
Here is an algorithm for how Michael, who lives in Singapore at Block 357 Clementi Avenue 2, travels to his friend Theo who is staying at the New 7th Storey Hotel, in another part of Singapore. Michael is using the Singapore MRT train.

Leave apartment 2305 on 23rd floor. Turn left. Catch lift to ground floor.
Turn right. Leave apartment block 357. Walk to Clementi Avenue 2.
Turn left. Walk to Commonwealth Avenue West.
Turn right. Walk straight ahead.
Enter Clementi MRT train station.
Buy a ticket to Bugis. Board east bound train.
Get off train at Bugis. Exit station. Walk to Rocher Rd.
Cross Rocher Rd. Turn left. Cross North Bridge Road.
Walk straight ahead.
Stop at the New 7th Story Hotel. Turn right and enter the hotel.

a Use the maps shown to write a similar algorithm for how Theo could travel to Michael's house.

b Would this be the only algorithm for this journey?

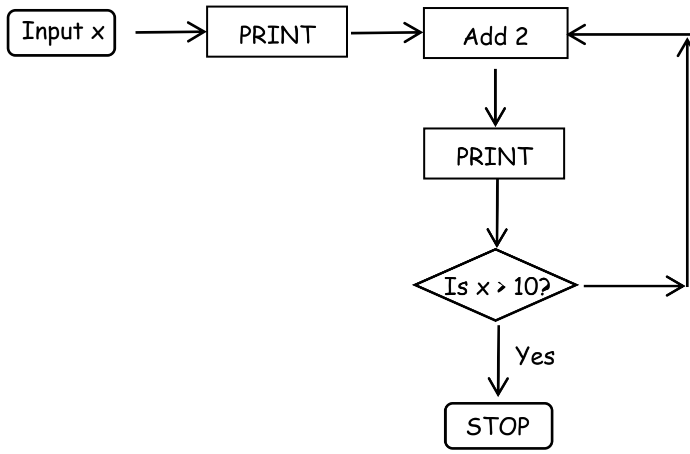
c Do a search on the internet to find another algorithm for Michael's trip.



For each question, use the algorithm to generate a sequence of numbers, based on the input given.

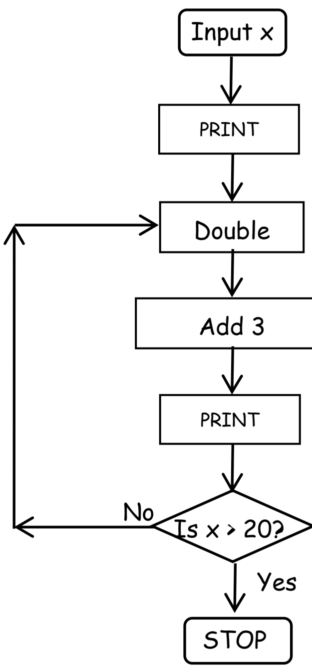
1a) $x = 5$
b) $x = 0$

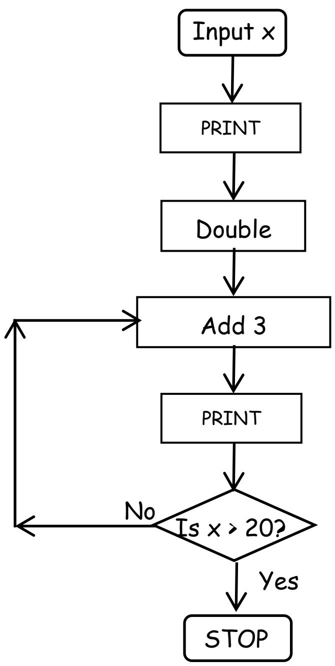
Write down the sequences you get by inputting the numbers indicated



2a) $x = 1$
b) $x = 2$
c) $x = 4$

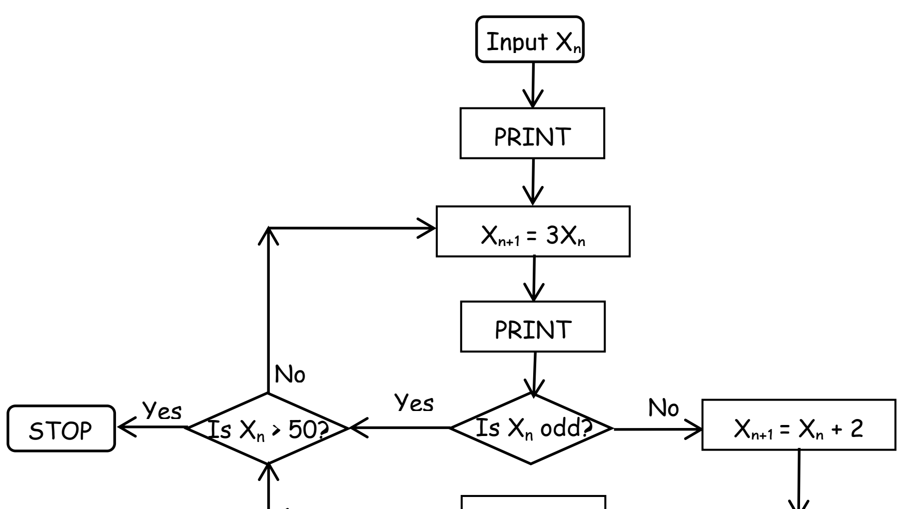
For this question compare the different sequences you get, based on the change in the algorithm!





3a) $x = 2$
b) $x = 3$
c) $x = 4$
d) $x = 5$

Write down the sequences you get by inputting the numbers indicated. $X_{n+1} = 3X_n$ means to get the next number (X_{n+1}), multiply the previous number by 3 ($3X_n$)



For each sequence, write an algorithm to generate it. <u>Remember to include the 'print' and 'stop' commands!</u> You will also need to state the input value!

1a) 5, 6, 7, 8, 9, 10, 11

b) 2, -2, 2, -2, 2, -2, 2, -2

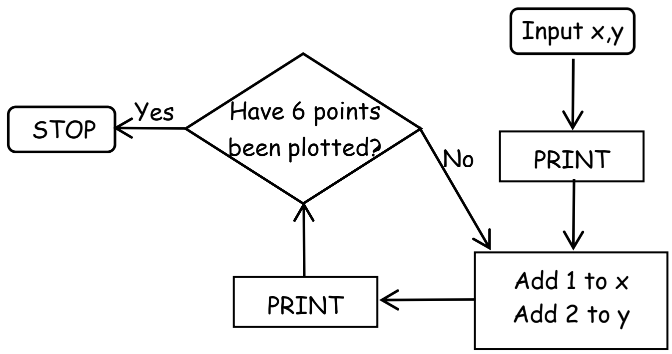
c) 2, 6, 7, 21, 22, 66, 67

d) 15, 13, 11, 9, 7, 5

e) 3, 7, 15, 31, 63

For each question, calculate the coordinates which will be created. Every 'PRINT' will plot the coordinate which exists at that time.

1a) x = 1, y = 1
b) x = 1, y = 2
c) x = 2, y = 2
d) x = 5, y = 1



Write an algorithm to generate the following coordinate sequences. Remember to include both x and y input values! For the later ones you should pay attention to where the 'rules' change!
<u>Also Remember to include the 'print' and 'stop' commands@</u>

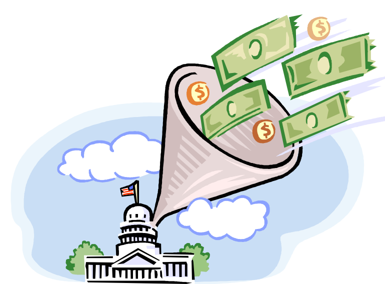
a) (1,1), (2,2), (3,3), (4,4), (5,5)

b) (0,2), (2,3), (4,4), (6,5), (8,6), (10,7)

c) (1,8), (2,6), (4,4), (8,2), (16,0)

d) (3,5), (4,6), (5,7), (6,8), (4,5), (2,2), (0,-1)

e) (1,1), (3,2), (7,5), (15,14), (7,5), (3,2),



SOLUTION BY SUBSTITUTION

The method of **solution by substitution** is used when at least one equation is given with either x or y as the **subject** of the formula. We substitute an expression for this variable into the other equation.

Example 1 Self Tutor

Solve simultaneously by substitution: $y = x + 5$
$3x - y = 1$

$y = x + 5$ ......(1)
$3x - y = 1$ ......(2)

Since $y = x + 5$, $3x - (x + 5) = 1$
$\therefore\ 3x - x - 5 = 1$
$\therefore\ 2x = 6$
$\therefore\ x = 3$

When $x = 3$, $y = 3 + 5$ {substituting $x = 3$ into (1)}
$\therefore\ y = 8$

The solution is $x = 3$, $y = 8$.

Check: (1) $8 = 3 + 5$ ✓
(2) $3(3) - 8 = 9 - 8 = 1$ ✓

Notice that $(x + 5)$ is substituted for y in the other equation.

SOLUTION BY ELIMINATION

In many problems which require the simultaneous solution of linear equations, each equation will be of the form $ax + by = c$. Solution by substitution is often tedious in such situations and the method of **elimination** of one of the variables is preferred.

In this method, we make the coefficients of x (or y) the **same size** but **opposite in sign** and then **add** the equations. This has the effect of **eliminating** one of the variables.

The method of elimination uses the fact that: if $a = b$ and $c = d$ then $a + c = b + d$.

Example 3 Self Tutor

Solve simultaneously, by elimination:

$$3x + 2y = 5 \quad \text{...... (1)}$$
$$x - 2y = 3 \quad \text{...... (2)}$$

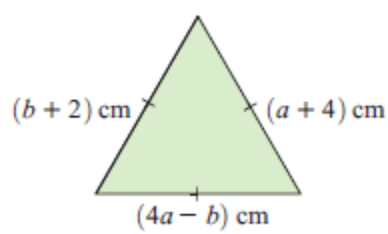
Notice that the coefficients of y are the same size but opposite in sign.

We **add** the LHSs and the RHSs to get an equation which contains x only.

$$\begin{array}{rrl} & 3x + 2y = 5 & \\ + & x - 2y = 3 & \\ \hline \therefore & 4x \quad\quad = 8 & \{\text{adding the equations}\} \\ & \therefore \quad x = 2 & \{\text{dividing both sides by } 4\} \end{array}$$

Margarine is sold in either 250 g or 400 g packs. A hotel ordered 19.6 kg of margarine and received 58 packs. How many of each type did the hotel receive?

Given that the triangle alongside is equilateral, find a and b.



A rectangle has perimeter 32 cm. If 3 cm is taken from the length and added to the width, the rectangle becomes a square. Find the dimensions of the original rectangle.

A motor boat travels at 12 km h^{-1} upstream against the current and 18 km h^{-1} downstream with the current. Find the speed of the current and the speed of the motor boat in still water.

A jet plane made a 4000 km trip downwind in 4 hours, but required 5 hours to make the return trip. Assuming the speed of the wind was constant throughout the entire journey, find the speed of the wind and the average speed of the plane in still air.

A man on foot covers the 25 km between two towns in $3\frac{3}{4}$ hours. He walks at 4 km h^{-1} for the first part of the journey and runs at 12 km h^{-1} for the remaining part.

a How far did he run? **b** For how long was he running?

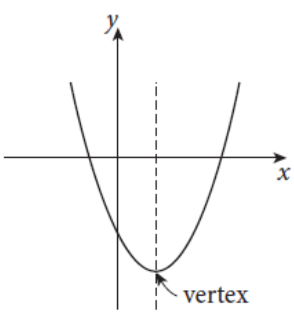
a Explain why any two digit number can be written in the form $10a + b$.

b A number consists of two digits which add up to 9. When the digits are reversed, the original number is decreased by 45. What was the original number?

FUNCTIONS

Common curves

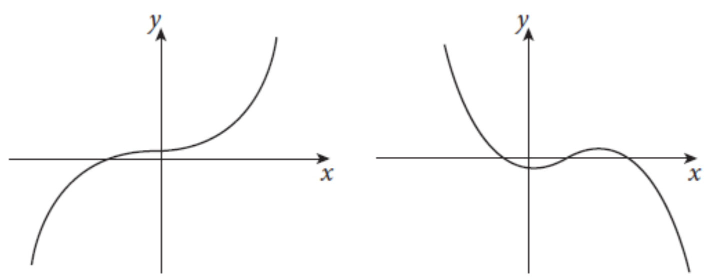
$y = ax^2 + bx + c$



The curve is symmetrical about a line parallel to the y axis which passes through the lowest point on the curve (the vertex).

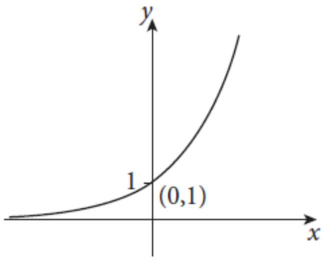
$y = ax^3 + bx^2 + cx + d$
$(a > 0)$

$y = ax^3 + bx^2 + cx + d$
$(a < 0)$



These two curves may cut or touch the x-axis at 1, 2 or 3 points.

$y = a^x$

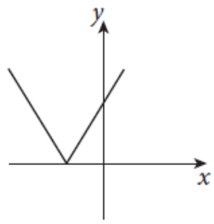


$y = a^x$ is known as an **exponential** function.

$y = 0$ is an asymptote.

The curve cuts the y axis at (0, 1).

$y = |ax + b|$



The absolute value function meets the x axis at $x = -\frac{b}{a}$

The graph is symmetrical about the line $x = -\frac{b}{a}$

The 'V' is symmetrical about a vertical axis.

QUADRATIC FUNCTION

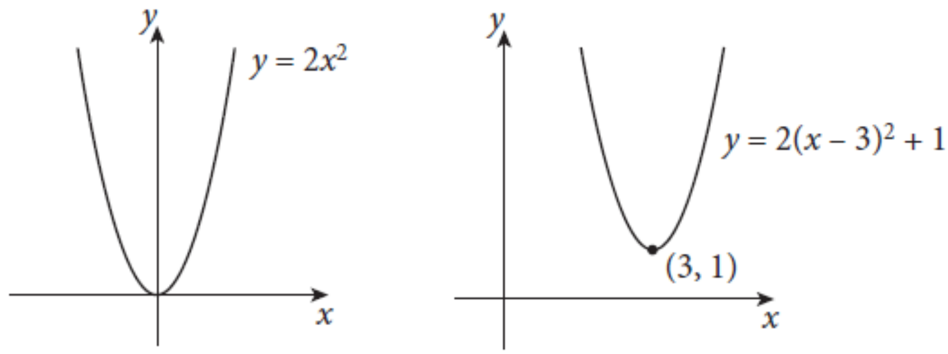
The general form of the quadratic function is $f(x) = ax^2 + bx + c$, where a, b and c are constants (numbers).

We can find the function if we know either

- the x-intercepts and a point,
- the vertex or x-intercepts with $a = 1$, or
- the vertex and another point.

Graphs of the form $y = a(x - h)^2 + k$

- Here are sketch graphs of $y = 2x^2$ and $y = 2(x - 3)^2 + 1$:



The shape of the two curves is the same and the second curve is obtained by the **translation** $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

In general graphs of the form $y = a(x - h)^2 + k$ have the same shape as the graph of $y = ax^2$ and are obtained by a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.

The vertex of the second curve has coordinates (h, k).

- Graphs of the form $y = x^2 + k$ have exactly the same shape as the graph of $y = x^2$. In fact, k is the **vertical translation** factor.
  Every point on the graph of $y = x^2$ is translated $\begin{pmatrix} 0 \\ k \end{pmatrix}$ to give the graph of $y = x^2 + k$.
- Graphs of the form $y = (x-h)^2$ have exactly the same shape as the graph of $y = x^2$. In fact, h is the **horizontal translation** factor.
  Every point on the graph of $y = x^2$ is translated $\begin{pmatrix} h \\ 0 \end{pmatrix}$ to give the graph of $y = (x-h)^2$.
- Graphs of the form $y = (x-h)^2 + k$ have the same shape as the graph of $y = x^2$ and can be obtained from $y = x^2$ by using a **horizontal shift** of h units and a **vertical shift** of k units. This is a **translation** of $\begin{pmatrix} h \\ k \end{pmatrix}$. The **vertex** is at (h, k).
- If $a > 0$, $y = ax^2$ opens upwards i.e., 

  If $a < 0$, $y = ax^2$ opens downwards i.e., 

If $a < -1$ or $a > 1$ then $y = ax^2$ is 'thinner' than $y = x^2$.
If $-1 < a < 1$, $a \neq 0$ then $y = ax^2$ is 'wider' than $y = x^2$.

$a > 0$ 
$a < 0$ 

vertical shift of k units:
if $k > 0$ it goes up
if $k < 0$ it goes down

$$\boldsymbol{y = a(x-h)^2 + k}$$

$a < -1$ or $a > 1$, thinner than $y = x^2$
$-1 < a < 1$, $a \neq 0$, wider than $y = x^2$

horizontal shift of h units:
if $h > 0$ it goes right
if $h < 0$ it goes left

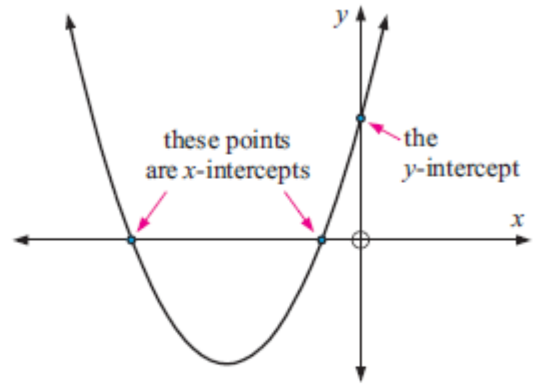
Given the equation of any curve:

An x-**intercept** is a value of x where the graph meets the x-axis,

A y-**intercept** is a value of y where the graph meets the y-axis.

x-intercepts are found by letting y be 0 in the equation of the curve.

y-intercepts are found by letting x be 0 in the equation of the curve.



THE y-INTERCEPT

You will have noticed that for a quadratic function of the form $y = ax^2 + bx + c$, the y-intercept is the constant term c. This is because any curve cuts the y-axis when $x = 0$.

For example, if $y = x^2 + 3x - 4$ and we let $x = 0$

then $y = 0^2 + 3(0) - 4$

$\therefore$ $y = -4$, which is the constant term.

THE x-INTERCEPTS

You should have noticed that for a quadratic function of the form $y = (x - \alpha)(x - \beta)$ or $y = -(x - \alpha)(x - \beta)$, the x-intercepts are α and β.

This is in fact true for all quadratic functions of the form $y = a(x - \alpha)(x - \beta)$, since any curve cuts the x-axis when $y = 0$.

If we substitute $y = 0$ into the function we get $a(x - \alpha)(x - \beta) = 0$

$\therefore$ $x = \alpha$ or β {by the Null Factor law}

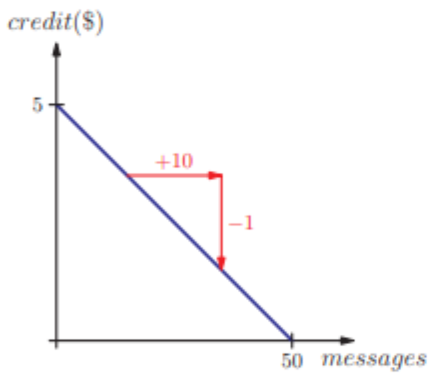
This suggests that x-intercepts are easy to find when the quadratic is in **factorised** form.

4E Rational functions

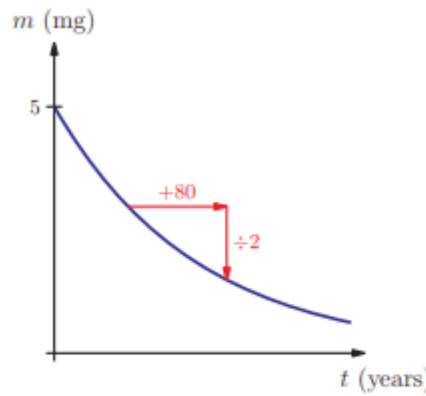
There are many situations where one quantity decreases as another increases.

For example, the amount of your phone credit decreases as the number of text messages you send increases; moreover, as the number of messages increases by a fixed number, the credit decreases by a fixed amount – this is called linear decay.

Another example is radioactive decay, where the amount of a radioactive substance halves in a fixed time period, called the half-life; in this case, as time increases by a fixed number, the amount of substance decreases by a fixed factor – this is called exponential decay.

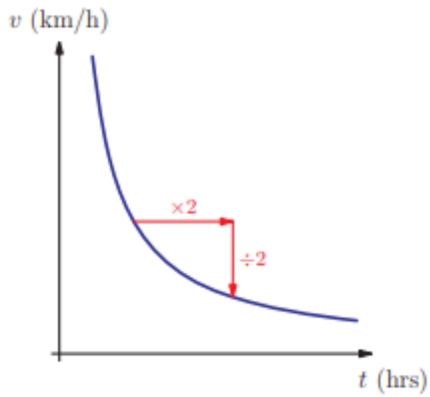


linear decay



exponential decay

In this section we look at a third type of decay, called inverse proportion, where as one quantity increases by a fixed factor, another decreases by the same factor. For example, if you double your speed, the amount of time it takes to travel a given distance will halve. If the total distance travelled is 12 km, then the equation for travel time (in hours) in terms of speed (in km/h) is $t = \frac{12}{v}$. This is an example of a **reciprocal function**, which has the general form $f(x) = \frac{k}{x}$.

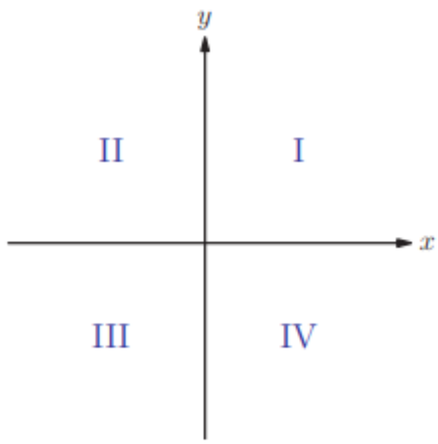


EXAM HINT

The reciprocal of a non-zero real number x is $\frac{1}{x}$.

For example, the reciprocal of -2 is $-\frac{1}{2}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

Graphs of reciprocal functions all have the same shape, called a **hyperbola**. A hyperbola is made up of two curves, with the axes as asymptotes. The function is not defined for $x = 0$ (the y-axis is a vertical asymptote), and as x gets very large (positive or negative) y approaches zero (the x-axis is a horizontal asymptote). This means that neither x nor y can equal zero. The two parts of the hyperbola can be either in the first and third **quadrants** or in the second and fourth quadrants, depending on the sign of k.



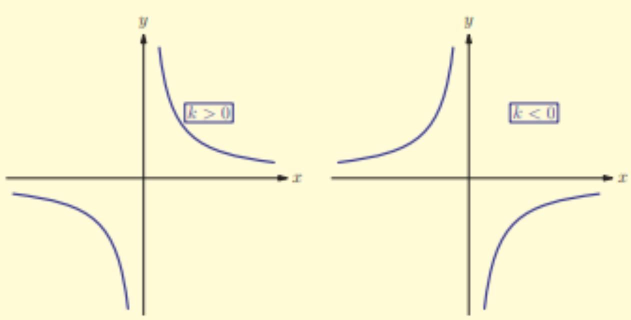
Quadrants

KEY POINT 4.8

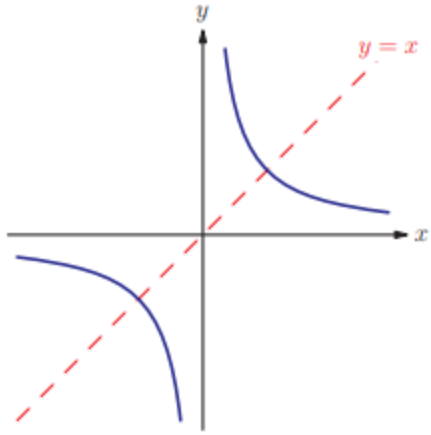
A reciprocal function has the form $f(x) = \frac{k}{x}$.

The domain of f is $x \neq 0$ and the range is $y \neq 0$.

The graph of $f(x)$ is a hyperbola.



What is the inverse of a reciprocal function? If $y = \frac{k}{x}$, then $xy = k$ and hence $x = \frac{k}{y}$. This means that $f^{-1}(x) = \frac{k}{x} = f(x)$, so the reciprocal function is its own inverse. We can also see this from the graph: a hyperbola is symmetrical about the line $y = x$, so its reflection in the line is the same as itself.



KEY POINT 4.9

The reciprocal function $f(x) = \dfrac{k}{x}$ is a **self-inverse function**; that is, $f^{-1}(x) = f(x)$.

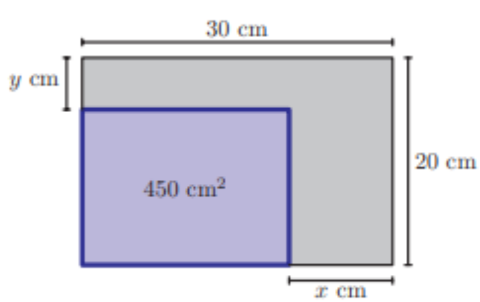
Related to reciprocal functions, **rational functions** are a ratio of two polynomials: $f(x) = \dfrac{p(x)}{q(x)}$ can be used to model a wider variety of situations where one quantity decreases as another increases. The following example illustrates one such situation.

Worked example 4.9

A rectangular piece of card has dimensions 30 cm by 20 cm. Strips of width x cm and y cm are cut off the ends, as shown in the diagram, so that the remaining card has area 450 cm^2.

(a) Find an expression for y in terms of x in the form $y = \dfrac{ax - b}{cx - d}$.

(b) Sketch the graph of y against x.



KEY POINT 4.10

The graph of a rational function of the form $f(x)=\dfrac{ax+b}{cx+d}$ is a hyperbola which has

- vertical asymptote $x=-\dfrac{d}{c}$ (where $cx+d=0$)
- horizontal asymptote $y=\dfrac{a}{c}$
- x-intercept at $x=-\dfrac{b}{a}$ (where $ax+b=0$)
- y-intercept at $y=\dfrac{b}{d}$ (where $x=0$)

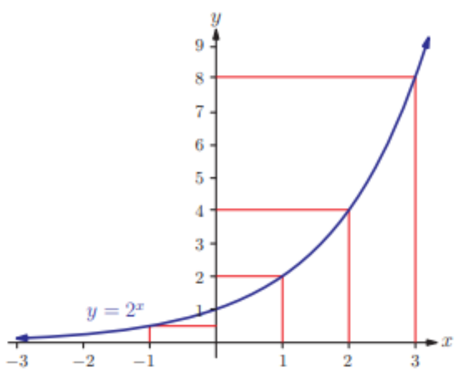
Knowing the position of the asymptotes tells you the domain and range of the function.

EXPONENTIAL FUNCTIONS

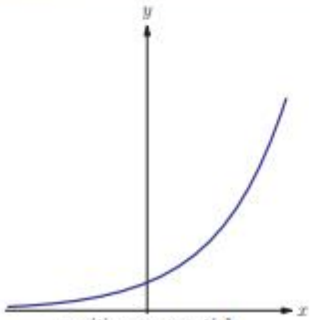
In an exponential function, the unknown appears in the exponent. The general form of a simple exponential function is $f(x)=a^x$.

We will only consider situations where the base a is positive, because otherwise some exponents cannot easily be defined (for example, we cannot square root a negative number).

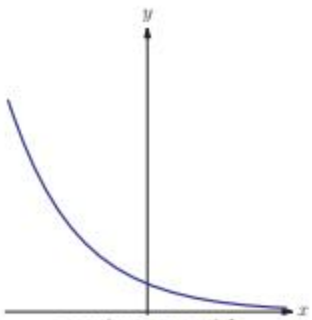
Here is the graph of $y=2^x$.



For large positive values of x, the y-value gets very large ('approaches infinity'). For large negative values of x, the y-value approaches (but never reaches) zero. A line that a graph gets increasingly close to (but never touches) is called an **asymptote**. In this case we would say that the x-axis is an asymptote to the graph.



positive exponential



negative exponential

EXAM HINT

Note that exponential decay be written

KEY POINT 2.10

For the graphs of $y = a^x$, where $a > 0$:

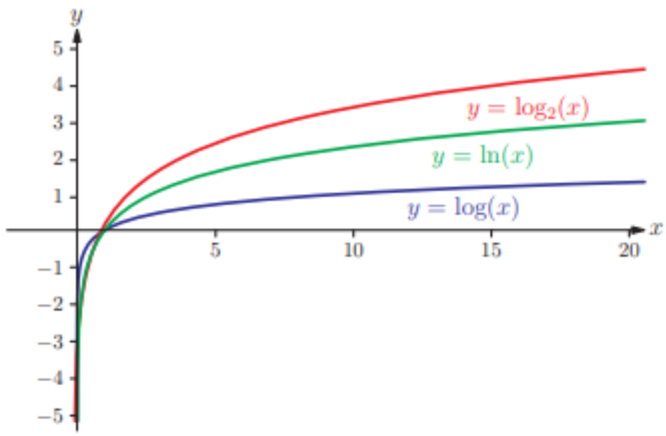
- The y-intercept is always (0, 1), because $a^0 = 1$.
- The graph of the function lies entirely above the x-axis, since $a^x > 0$ for all values of x.
- The x-axis is an asymptote.
- If $a > 1$, then as x increases, so does y. This is called a **positive exponential**.
- If $0 < a < 1$, then as x increases, y decreases. This is called a **negative exponential**.

Many mathematical models are based on the following property of the exponential function $N = a^t$: as time (t) increases by a *fixed amount*, the quantity we are interested in (N) will rise by a *fixed factor*, called the **growth factor**. Exponential functions can therefore be used to represent many physical, financial and biological forms of **exponential growth** (positive exponential models) and **exponential decay** (negative exponential models).

To model more complex situations we may need to include more constants in our exponential function. A form that is commonly used is

2F Graphs of logarithms

Let us now look at the graph of the logarithm function and the various properties of logarithms that we can deduce from it. Here are the graphs of $y = \log x, y = \log_2 x$ and $y = \ln x$.



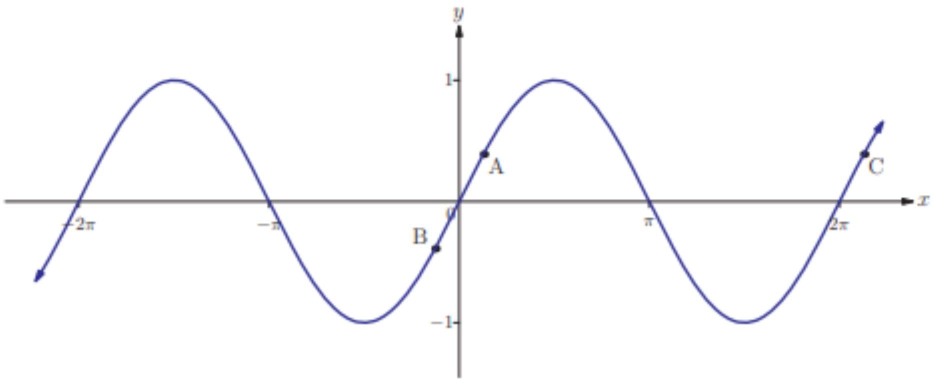
Given the change-of-base rule from section E (Key point 2.22), it is not surprising that these curves all have a similar shape: since $\log_2 x = \dfrac{\log x}{\log 2}$ and $\ln x = \dfrac{\log x}{\log e}$, each of the logarithm functions is a multiple of the common logarithm function.

KEY POINT 2.23

If $y = \log_a x$ (for any positive value of a), then

- the graph of y against x crosses the x-axis at (1, 0), because $\log_a 1 = 0$
- $\log x$ is negative for $0 < x < 1$ and positive for $x > 1$
- the graph lies entirely to the right of the y-axis, since the logarithm of a negative value of x is not a real number
- the graph increases (slopes upward from left to right) throughout, and as x tends to infinity so does y
- the y-axis is an asymptote to the curve.

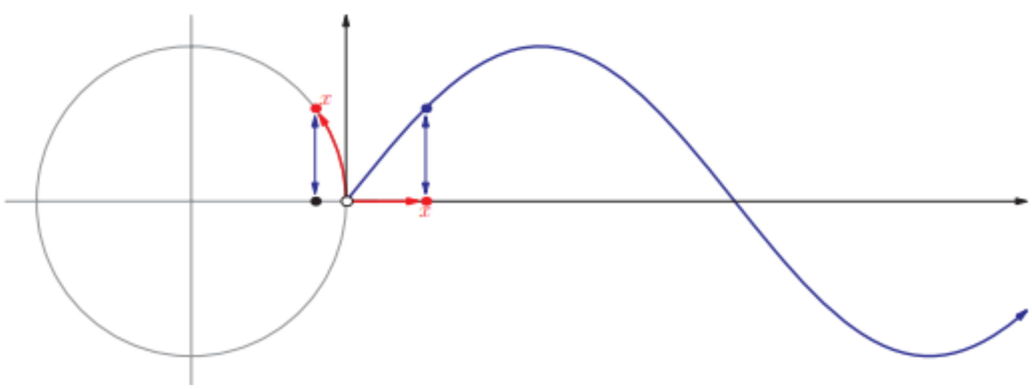
All of the properties of the sine function discussed above can be seen in its graph. For example, increasing x by 2π corresponds to making a full turn around the circle and returning to the same point; therefore, $\sin(x+2\pi)=\sin x$. We say that the sine function is **periodic** with **period** 2π. Looking at the graph below, by considering points A and B we can see that $\sin(-x)=-\sin x$. We can also see that the minimum possible value of $\sin x$ is -1 and the maximum value is 1. Thus we say that the sine function has **amplitude** 1.



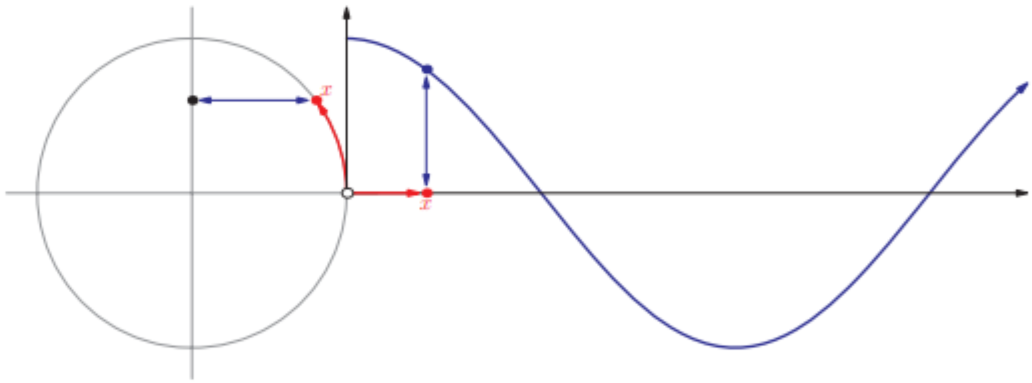
KEY POINT 8.6

A function is **periodic** if its pattern repeats regularly. The interval between the start of two consecutive repeating blocks is called the **period**.

SINE FUNCTION



COSINE FUNCTION

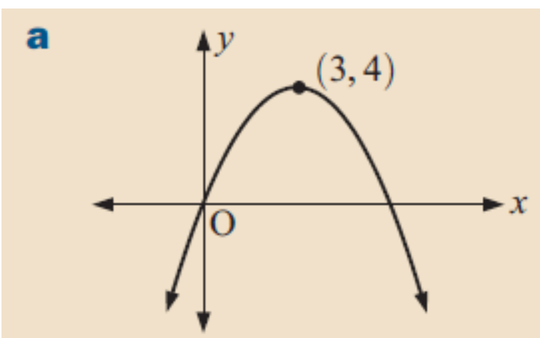


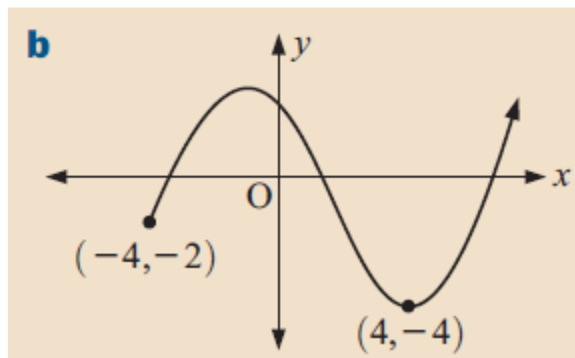
KEY POINT 8.7

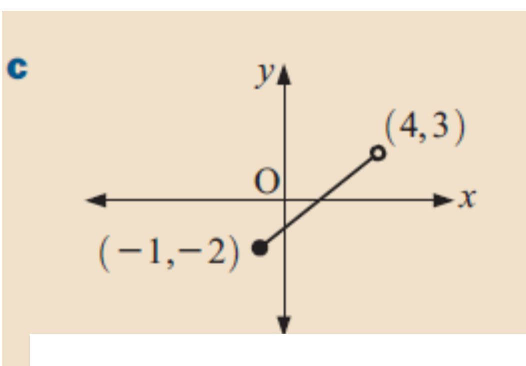
The sine and cosine functions are periodic with period 2π.

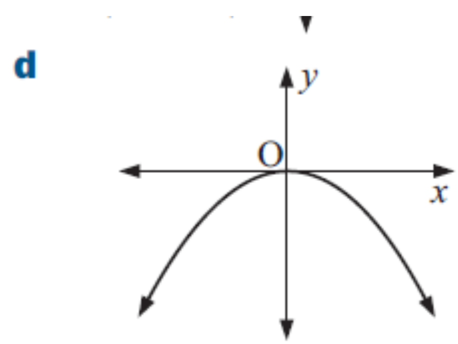
The sine and cosine functions have amplitude 1.

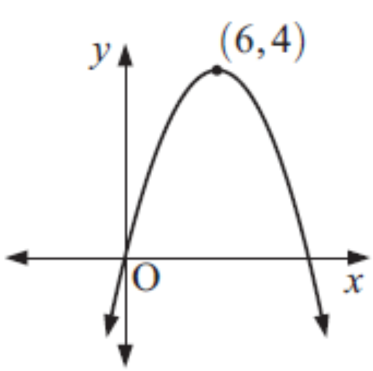
Find the domain and range of the following functions :-

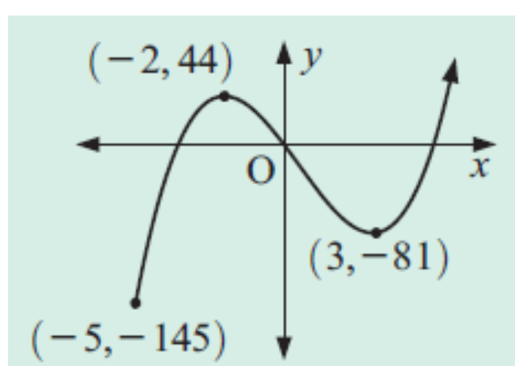












2. Draw the graph of the function $f(x) = 2x + 1$ and from the graph find :

a) Zero (x-intercept or Root)

b) y – intercept

c) Local Maximum

d) Local Minimum

e) Value of 'y' when x = 3

f) Value of 'x' when $y = -8$

3. Draw the graph of the function $f(x) = x^2 - 3x + 2$ in the domain $[-3,5]$ and from the graph find :

a) Zero (x-intercept or Root)

b) y – intercept

c) Local Maximum

d) Local Minimum

e) Value of 'y' when x = 10

f) Value of 'x' when y = 3

4. Draw the graph of the function $f(x) = x^2 - 2x + 1$ in the domain $-2 \leq x \leq 7$ and from the graph find :

a) Zero (x-intercept or Root)

b) y – intercept

c) Local Maximum

d) Local Minimum

e) Value of 'y' when $x = -5$

f) Value of 'x' when y = 10

5. Draw the graph of the function $f(x) = -x^2 + 3x + 10$ and from the graph find :

a) Zero (x-intercept or Root)

b) y – intercept

c) Local Maximum

d) Local Minimum

e) Value of 'y' when $x = 6$

f) Value of 'x' when $y = -4$

6. Draw the graph of the function $f(x) = x^2 - 5x + 10$ and from the graph find :

a) Zero (x-intercept or Root)

b) y – intercept

c) Local Maximum

d) Local Minimum

e) Value of 'y' when x = -5

f) Value of 'x' when y = 10

7. Draw the graph of the function $f(x) = x^3 - 2x^2 - x + 2$ in the domain $-3 \le x \le 4$ and from the graph find :

a) Zero (x-intercept or Root)

b) y – intercept

c) Local Maximum

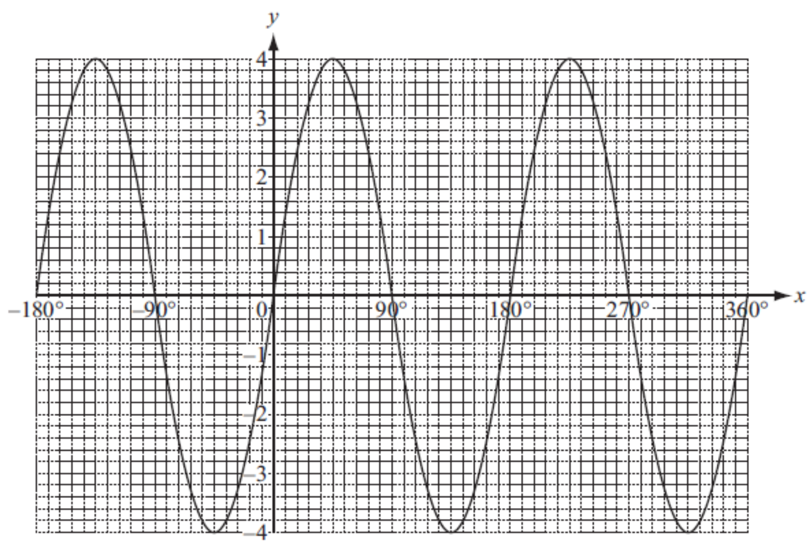
d) Local Minimum

e) Value of 'y' when x = -5

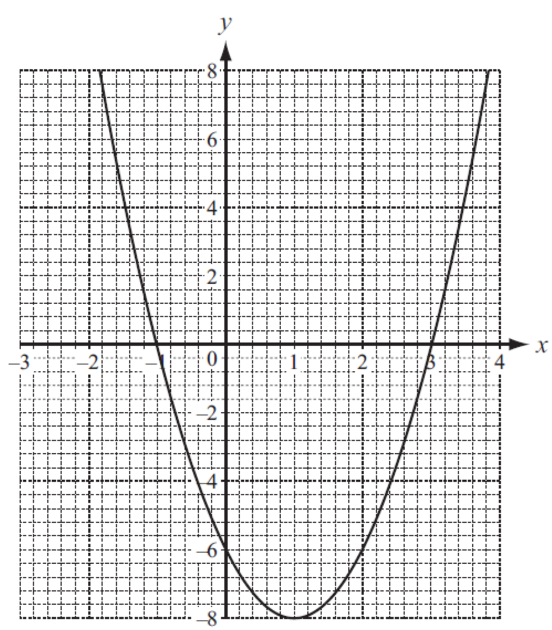
f) Value of 'x' when y = 10

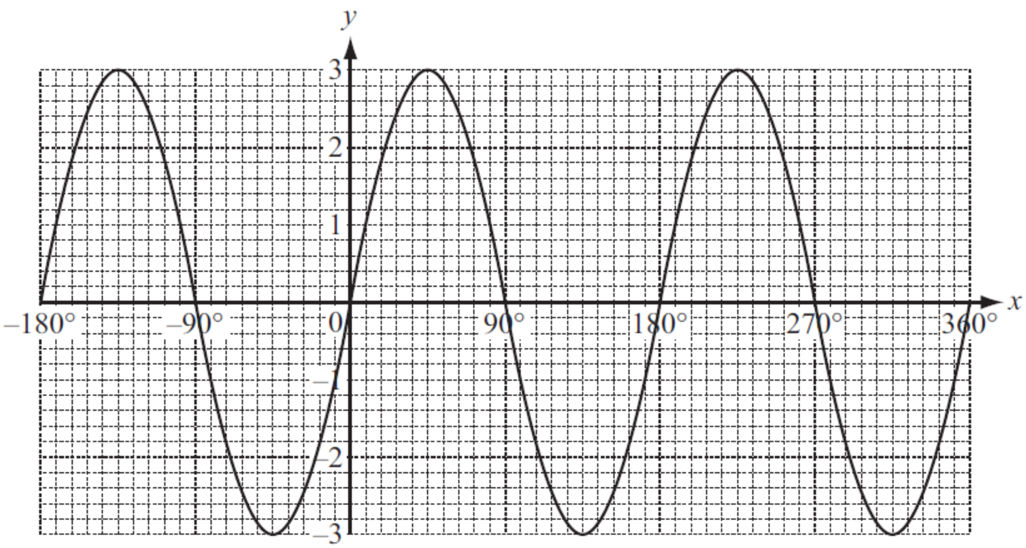
The diagram shows the graph of $y = f(x)$, where $f(x) = a\sin(bx)$

Find the values of a and b.



The diagram shows part of the graph of $y = f(x)$, where $f(x) = ax^2 + bx - 6$ .Find the values of a and b.





(a) Write down the equation of the graph.

Answer(a) .... [2]

(b) **On the same axes** above sketch the graph of $y = 2 \sin x$ for $-180° \leqslant x \leqslant 360°$. [2]

f: $x \mapsto 5 - 3x$.

(a) Find f(–1).

(b) Find $f^{-1}(x)$.

(c) Find $f f^{-1}(8)$.

10. The function f(x) is given by

$$f(x) = 3x - 1.$$

Find, in its simplest form,

(a) $f^{-1}f(x)$,

(b) $ff(x)$.

1. Given the functions $h(x) = x^2 + 1$ and $g(x) = 10x + 1$, find:

a) h(2), h(−3), h(0) **b)** g(2), g(10), g(−3)

In questions **2** to **15**, draw a flow diagram for each function.

2. $f(x) = 5x + 4$

3. $f(x) = 3(x - 4)$

4. $f(x) = (2x + 7)^2$

5. $f(x) = \left(\frac{9+5x}{4}\right)$

6. $f(x) = \frac{4-3x}{5}$

7. $f(x) = 2x^2 + 1$

8. $f(x) = \frac{3x^2}{2} + 5$

9. $f(x) = \sqrt{(4x-5)}$

10. $f(x) = 4\sqrt{(x^2+10)}$

11. $f(x) = (7 - 3x)^2$

12. $f(x) = 4(3x + 1)^2 + 5$

13. $f(x) = 5 - x^2$

14. $f(x) = \frac{10\sqrt{(x^2+1)}+6}{4}$

15. $f(x) = \left(\frac{x^3}{4}+1\right)^2 - 6$

In question **14**, $\sqrt{\ }$ means 'the positive square root'

For questions **16**, **17** and **18**, the functions f, g and h are defined as follows:

$f(x) = 1 - 2x$ $g(x) = \frac{x^3}{10}$ $h(x) = \frac{12}{x}$

16. Find:

a) f(5), f(−5), $f\left(\frac{1}{4}\right)$ **b)** g(2), g(−3), $g\left(\frac{1}{2}\right)$ **c)** h(3), h(10), $h\left(\frac{1}{3}\right)$

If $g : x \mapsto 2^x + 1$, find:

a) g(2) **b)** g(4) **c)** g(−1) **d)** the value of x if $g(x) = 9$

The function f is defined as $f : x \to ax + b$ where a and b are constants.

If f(1) = 8 and f(4) = 17, find the values of a and b.

The function g is defined as $g(x) = ax^2 + b$ where a and b are constants.

If g(2) = 3 and g(−3) = 13, find the values of a and b.

In questions **9** to **22**, find the inverse of each function in the form '$x \mapsto \ldots$'

9. $f: x \mapsto 5x - 2$

10. $f: x \mapsto 5(x-2)$

11. $f: x \mapsto 3(2x+4)$

12. $g: x \mapsto \dfrac{2x+1}{3}$

13. $f: x \mapsto \dfrac{3(x-1)}{4}$

14. $g: x \mapsto 2(3x+4)-6$

15. $h: x \mapsto \dfrac{1}{2}(4+5x)+10$

16. $k: x \mapsto -7x+3$

17. $j: x \mapsto \dfrac{12-5x}{3}$

A billiard table manufacturer finds that the cost per table of making x tables per month is given by $C = x^2 - 16x + 67$ hundred dollars.

a How many tables should be made per month to minimise the production cost of each?

b What is the minimum cost per table?

c What is the cost per table if 5 tables are made in a month?

A car driving along a city street has speed given by the function $s = -t^2 + 6t + 40 \text{ km h}^{-1}$ where $0 \leqslant t \leqslant 10$ seconds.

a How fast was the car travelling at time $t = 0$ seconds?

b After how many seconds did the speed of the car first reach 45 km h^{-1}?

c At what time did the speed reach 45 km h^{-1} again?

d What was the maximum speed of the car and at what time did it occur?

A company supplies stalls which sell drinks at a festival. The profit obtained from operating n stalls is given by $P = 160n - 1520 - n^2$ dollars.

a What number of stalls gives the maximum profit?

b What is the maximum profit?

c How much money is lost if there is a thunderstorm and the festival is cancelled at the last minute?

Functions g and h are defined by $g(x)=\sqrt{x}$ and $h(x)=\dfrac{2x-3}{x+1}$.

(a) Find the range of h.

(b) Solve the equation $h(x)=0$.

(c) Find the domain and range of $g \circ h$. *[6 marks]*

The function f is defined by $f: x \rightarrow x^3$. Find an expression for $g(x)$ in terms of x in each of the following cases:

(a) $(f \circ g)(x)=2x+3$

(b) $(g \circ f)(x)=2x+3$ *[6 marks]*

Functions f and g are defined by $f(x)=\sqrt{x^2-2x}$ and $g(x)=3x+4$. The composite function $f \circ g$ is *undefined* for $x \in \left]a,b\right[$.

(a) Find the value of a and the value of b.

(b) Find the range of $f \circ g$. *[7 marks]*

Define $f(x)=x-1$, $x>3$ and $g(x)=x^2$, $x \in \mathbb{R}$.

(a) Explain why $g \circ f$ exists but $f \circ g$ does not.

(b) Find the largest possible domain for g so that $f \circ g$ is defined. *[6 marks]*

Inverse functions

1. Find $f^{-1}(x)$ if

(a) (i) $f(x)=3x+1$ (ii) $f(x)=7x-3$

(b) (i) $f(x)=\dfrac{2x}{3x-2},\ x\neq\dfrac{2}{3}$ (ii) $f(x)=\dfrac{x}{2x+1},\ x\neq-\dfrac{1}{2}$

(c) (i) $f(x)=\dfrac{x-a}{x-b},\ x\neq b$ (ii) $f(x)=\dfrac{ax-1}{bx-1},\ x\neq\dfrac{1}{b}$

(d) (i) $f(a)=1-a$ (ii) $f(y)=3y+2$

(e) (i) $f(x)=\sqrt{3x-2},\ x\geq\dfrac{2}{3}$ (ii) $f(x)=\sqrt{2-5x},\ x\leq\dfrac{2}{5}$

(f) (i) $f(x)=\ln(1-5x),\ x<0.2$ (ii) $f(x)=\ln(2x+2),\ x>-1$

(g) (i) $f(x)=7e^{\frac{x}{2}}$ (ii) $f(x)=9e^{10x}$

(h) (i) $f(x)=x^2-10x+6,\ x<5$ (ii) $f(x)=x^2+6x-1,\ x>0$

3. The following table gives selected values of the function $f(x)$.

x	-1	0	1	2	3	4
$f(x)$	-4	-1	3	0	7	2

(a) Evaluate $ff(2)$.

(b) Evaluate $f^{-1}(3)$. *[4 marks]*

4. The function f is defined by $f:x\mapsto\sqrt{3-2x},\ x\leq\dfrac{3}{2}$.

Evaluate $f^{-1}(7)$. *[4 marks]*

5. Given that $f(x)=3e^{2x}$, find the inverse function $f^{-1}(x)$. *[4 marks]*

6. Given functions $f:x\mapsto 2x+3$ and $g:x\to x^3$, find the function $(f\circ g)^{-1}$. *[5 marks]*

7. The functions f and g are defined by $f:x\mapsto e^{2x}$ and $g:x\mapsto x+1$.

(a) Calculate $f^{-1}(3)\times g^{-1}(3)$.

(b) Show that $(f\circ g)^{-1}(3)=\ln\sqrt{3}-1$. *[6 marks]*

5. Find the equations of the asymptotes of the graph of $y = \frac{3x-1}{4-5x}$. *[3 marks]*

6. Let $f(x) = \frac{1}{x+3}$.

(a) Find the domain and range of $f(x)$.

(b) Find $f^{-1}(x)$. *[5 marks]*

Find the domain, range and inverse function of the following rational functions.

(a) (i) $f(x) = \frac{3}{x}$ (ii) $f(x) = \frac{7}{x}$

(b) (i) $f(x) = \frac{2}{x-3}$ (ii) $f(x) = \frac{5}{x+1}$

(c) (i) $f(x) = \frac{2x+1}{3x-1}$ (ii) $f(x) = \frac{4x-5}{2x+1}$

(d) (i) $f(x) = \frac{5-2x}{x+2}$ (ii) $f(x) = \frac{3x-1}{4x-3}$

2. An algal population on the surface of a pond grows by 10% every day. The area it covers can be modelled by the equation $y = k \times 1.1^t$, where t is measured in days, starting from 09:00 on Tuesday, when the algae covered $10\,\text{m}^2$. What area will it cover by 09:00 on Friday? *[4 marks]*

3. A tree branch is observed to bend as the fruit growing on it increase in size. By estimating the mass of the developing fruit and plotting the data over time, a student finds that the height h in metres of the branch end above the ground is closely approximated by the function

$$h = 2 - 0.2 \times 1.6^{0.2m}$$

where m is the estimated mass, in kilograms, of fruit on the branch.

(a) Sketch the graph of h against m.

(b) What height above ground is the branch without fruit?

(c) The total mass of fruit on the branch at harvest was 7.5 kg. Find the height of the branch immediately prior to harvest.

(d) The student wishes to estimate what mass of fruit would cause the branch end to touch the ground. Why might his model not be suitable to assess this? *[10 marks]*

3. In a yeast culture, cell numbers are given by $N = 100e^{1.03t}$, where t is measured in hours after the cells are introduced to the culture.

(a) What is the initial number of cells?

(b) How many cells will be present after 6 hours?

(c) How long will it take for the population to reach one thousand cells? *[4 marks]*

4. A rumour spreads exponentially through a school, so that the number of people who know it can be modelled by the equation $N = Ae^{kt}$, where t is the time, in minutes, after 9 a.m. When school begins (at 9 a.m.), 18 people know the rumour. By 10 a.m. 42 people know it.

(a) Write down the value of A.

(b) Show that $k = 0.0141$, correct to three significant figures.

(c) How many people know the rumour at 10:30 a.m.?

(d) There are 1200 people in the school. According to the exponential model, at what time will everyone know the rumour? *[6 marks]*