

Geometry and Trigonometry

Part I

Question 54

A train has a path of $y = 1$.

*a) On the number plane below, **graph** the path of the train.*

City “R” has coordinates of $(-10, 1)$ and city “Q” has coordinates of $(14, 1)$.

*b) On the same number plane **plot** the two cities.*

City “C” is 13 km away from both city R and city Q.

*c) **Plot** the two possible locations of city C.*

Cities A and B are on a different train path L2.

*d) **Write down** the equation of L2.*

A passenger wants to travel from B to Q.

*e) **Trace** the trip path that the passenger should take.*

*f) **Calculate** how the long will the passenger travel.*

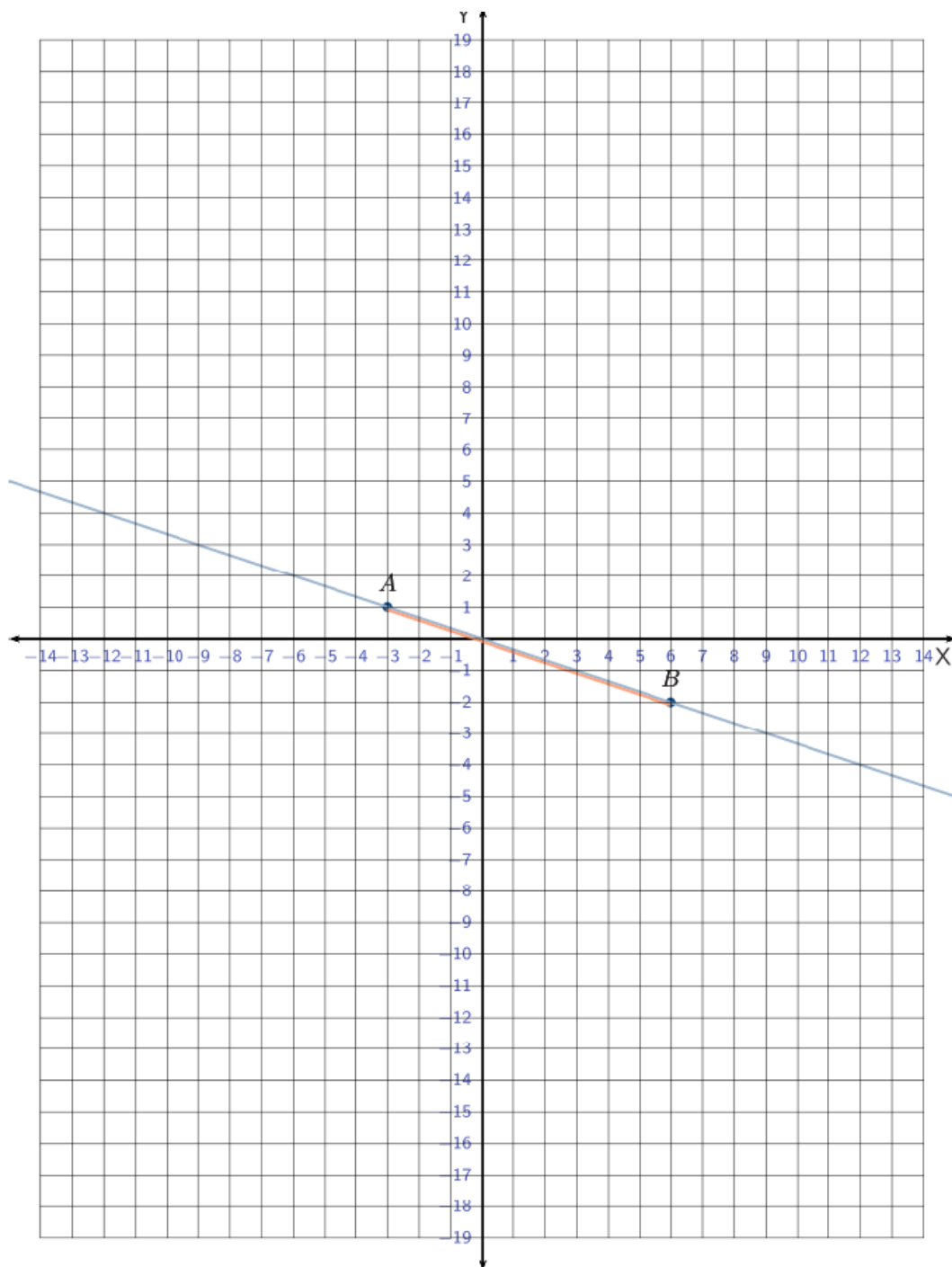
*g) **Select** all the cities that lie in the fourth quadrant.*

*h) **Calculate** the measure of $\angle BAQ$ to the nearest whole number.*

*i) **Describe** the behavior of a straight line that passes the first and third quadrants.*

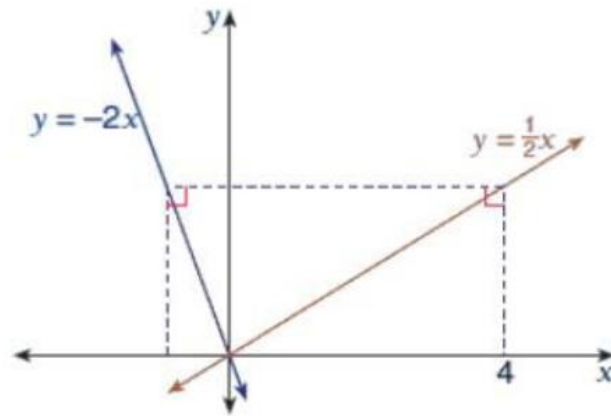
j) Cities G, A and R lie on the sides of a polygon. A lies on one of its vertices. Use your findings in part h, to identify the number of sides of the polygon.

*k) **Calculate** the distance between city Q and the path **AB**.*



Question 55

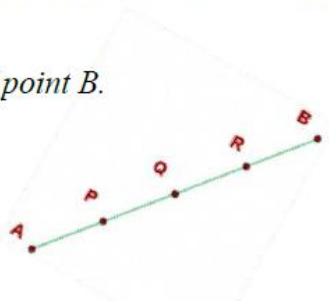
Use the information on the diagram below to find the area of the rectangle with dashed sides.



Question 56

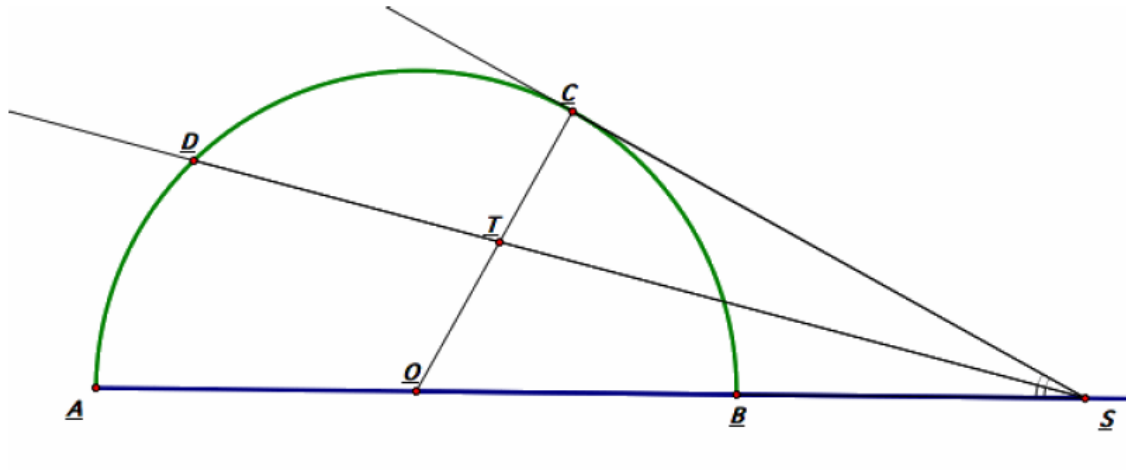
In the drawing below (not to scale) points P , Q and R divide the line segment AB into four equal segments. Points A and P have coordinates $(1,3)$ and $(5,6)$ respectively.

Calculate the coordinates of point B .



Question 57

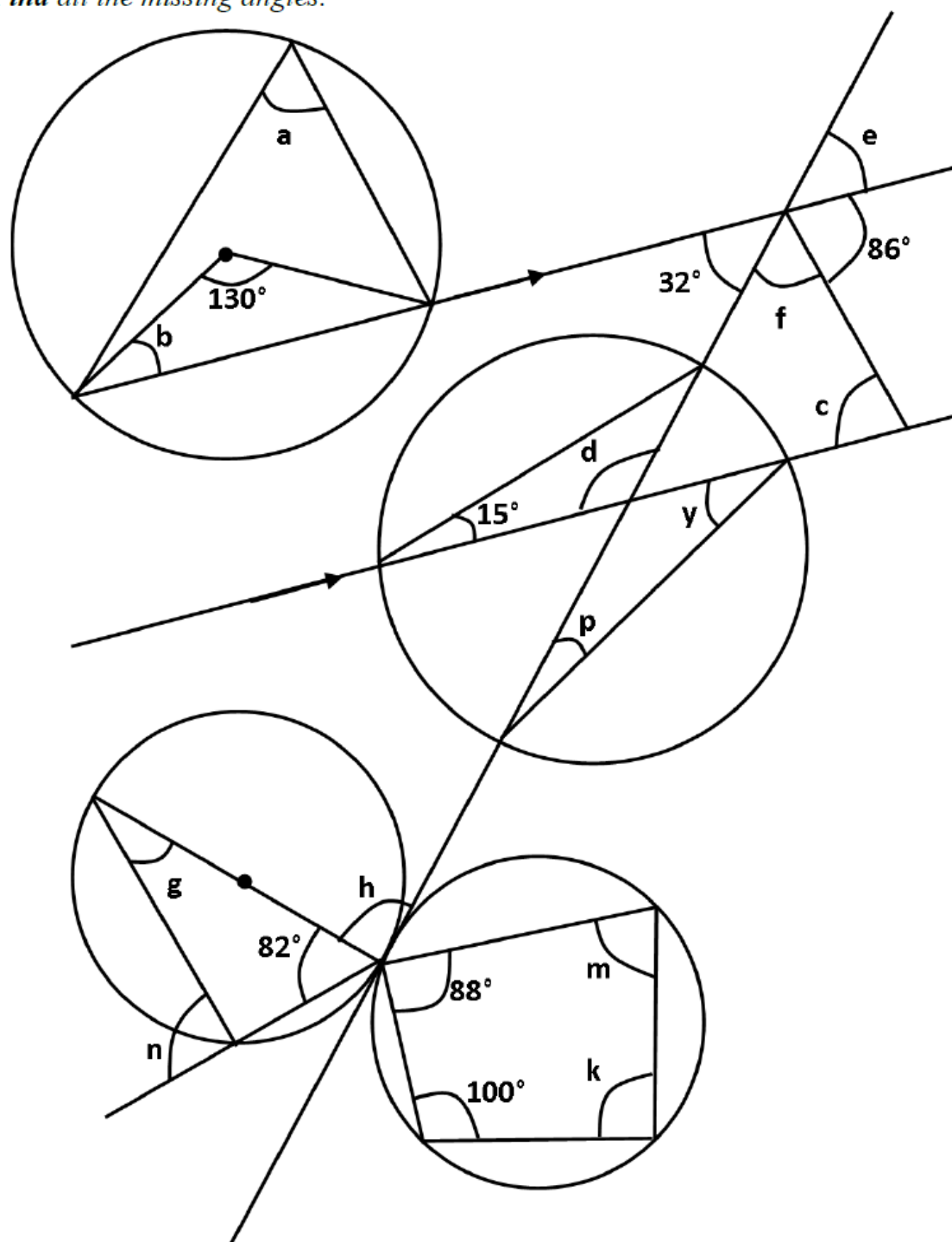
In the diagram below SC is tangent to the semi-circle and OC is the radius. $BS = 8$ cm, and $OC = 4$ cm. (Diagram not to scale)



- Find** the exact length of SC in two different methods.
- Find** the size of angle $\angle SOC$ to the nearest degree.
- Calculate** the area of the sector AOC.
- Point T is the midpoint of the line segment OC. **Calculate** the length of the line segment TS.

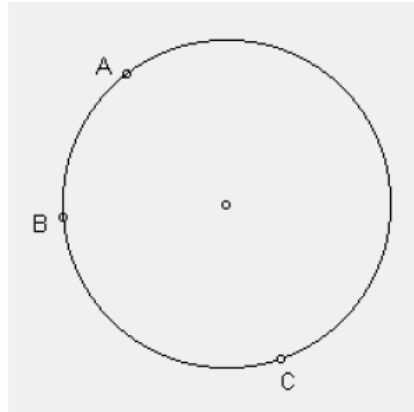
Question 58

Find all the missing angles.



Question 59

In the diagram below (not drawn to scale) the ratio of the arcs AB , BC and CA are $2:3:7$ respectively. **Calculate** the measure of angle $\angle ABC$.



Question 60

The bearing of a ship from a lighthouse is 050° . **Write down** the bearing of the lighthouse from the ship.

Question 61

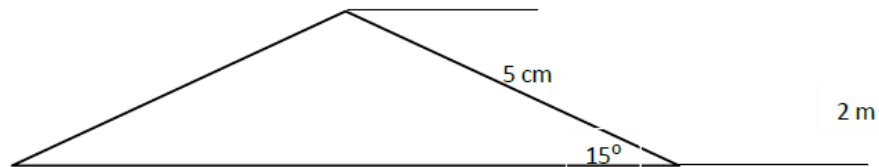
Discuss how angles of elevation or depression could be used to measure the height of a building.

Question 62

Explain why similarity would be a more appropriate method to measure the height of a building rather than the angles of elevation or depression when performed with simple tools.

Question 63

A factory imports aluminum in beams that have an isosceles triangular cross section as shown in the diagram (not to scale).



- a) Calculate the volume of aluminum in each beam.*
- b) Calculate the total surface area of each beam.*

The factory melts the aluminum beams and casts them in the form of cuboids.

- c) Find the dimensions of each cuboid given that one beam is melted into a cuboid. Use appropriate degree of rounding.*
- d) Discuss the benefits of melting the beams into cuboids based on what you learned in other MYP subjects. (Suggest at least two benefits)*

Question 64

A company produces tennis balls have two methods for packing tennis balls.

Method one: packing 4 balls in cylindrical container.

Method two: packing 8 balls in cubic container.

In both cases, the balls just fit in the containers touching the interior edges of the containers.

- a) **Identify** in which packing method the balls fit better into the container.*
- b) Both containers are of a transparent plastic sheets that cost JD 5 /squared meter. **Calculate** the cost of producing packs for 200 balls using both methods.*
- c) **State** two advantages and two disadvantages for each packing method.*
- d) **Suggest** a third packing method that would be desirable over the previous two packing methods. Justify your answer.*

Question 65

***Derive** a rule to find the area of regular hexagon whose side is “2x”.*

Hint: Use interior angle of hexagon and diagonal properties.

Question 66

A tank is to be designed. The capacity of the tank is to be 2 cubic meters.

It could either be a spherical, cylindrical, rectangular prism or cuboid.

- Suggest** dimensions for Cylinder 3 and Rectangular prism 3 in the table below.
- Verify** whether you can suggest different dimensions for spheres or cuboids that would yield the same volume.
- Use** the following table to help you investigate which shape will have the least surface area.

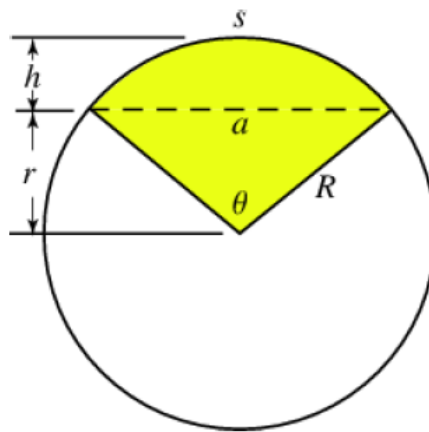
Shape	volume	Dimensions		Surface area
Sphere	2	Radius=0.782		
Cuboid	2	Side= 1.260		
Cylinder 1	2	Radius=1.000	Height= 0.637	
Cylinder 2	2	Radius=0.799	Height= 1.000	
Cylinder 3	2	Radius=	Height=	
Rectangular P 1	2	L= 1.000 W= 1.000 H= 2.000		
Rectangular P 2	2	L= 1.415 W= 1.415 H= 1.000		
Rectangular P 3	2	L= W= H=		

- Comment** on your findings.

Question 67

Samira cuts a wedge out of circular disks of silver of radius 0.79 cm to make jewelry. If the weight of the disks is uniformly distributed, and the weights of the disk and the wedge are 0.8g and 0.488g respectively.

- a) On the diagram below (not to scale) **label** each of the following:
- A major sector.
 - An obtuse central angle.
 - A minor segment.
 - A chord.



- b) **Write down** the value of θ to the nearest tenth.
- c) **Calculate** the value of h to an appropriate degree of accuracy.
- d) **Justify** the degree of accuracy you used in part c.
- e) **Calculate** the area of the minor segment to a suitable degree of accuracy.
- f) **Explain** how related are the degrees of accuracy in parts c and e.

Question 68

A model of cylindrical tank whose capacity is 0.005 cubic meter is 90 cm height. **Calculate** the capacity of the original tank if its height is 5.4 meters.

Question 69

A frustum of a cone was created by topping off (horizontally) the upper 3cm of a right cone whose radius and height are 10cm and 12cm respectively.

- a) **Sketch** a diagram of the frustum.
- b) **Find** the ratio of the area of the top end to the area of the bottom end of the frustum.
- c) To paint the lateral area of the cone, each cone cost JD1.3 **calculate** the cost of painting the lateral area of the frustum.

Question 70

Spheres have the least surface area when compared to other shapes having the same capacity. **Suggest** three applications where the use of spheres is favorable and **explain** your answer.

Question 71

Write down the transformations that will yield congruent images.

Question 72

Write down the transformations that will yield similar images.

Question 73

A triangle with coordinates $A(0,0)$, $B(3,0)$ and $C(3,4)$ was enlarged by a scale factor of 2 into triangle $A'B'C'$, then it was rotated -90° into triangle $A''B''C''$.

- a) **Draw** the triangle $A'B'C'$ on the same number plane.*
- b) **Calculate** the length of the side $A''C''$.*
- c) **Write down** the coordinates of A'' , B'' and C'' .*

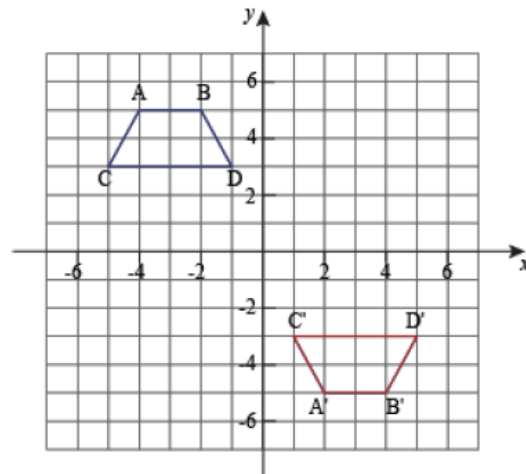
Question 74

The line segment joining the points $A(2,3)$ and $B(4,4)$ is reflected on the line $2y - 6 = x$.

- a) **Find** the coordinates of the of the points A and B when reflected on the line $2y - 6 = x$
- b) **Verify** whether the two line segments are parallel.

Question 75

Describe the transformation(s) which transforms the blue figure into the red figure.



Question 76

Under enlargement, the image of a point $P(-3, 5)$ is $P'(3, -7)$. If the enlargement factor is 3, **find** the center of enlargement.

Question 77

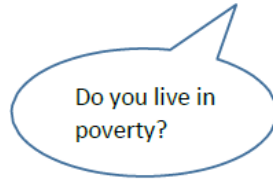
IB grades of nine students are shown below:



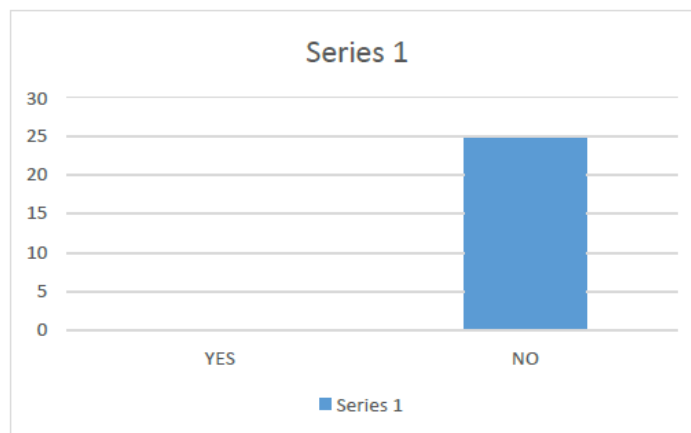
- a) **State** a grade that can be added that will leave the mean, mode and median unchanged.
- b) One student's grade was missing. When this tenth grade was added the new mean is 4.9 **Calculate** the missing grade.
- c) The grades available are members of the set $\{1, 2, 3, 4, 5, 6, 7\}$. An eleventh grade is added. **Calculate** the difference between the maximum possible mean and the minimum possible mean. Give your answer to two significant figures.

Question 78

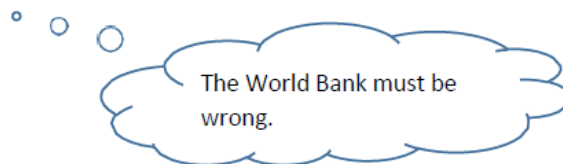
The World Bank estimates that around 20% of the Bulgarian population are living in poverty. As part of a school project, a student in Bulgaria visits her local shopping center at 15:00 on a Saturday afternoon. She asks 25 people.



The bar chart below shows the responses to her question.



The student concludes



Discuss the student's conclusion by considering her sampling method and provide four possible reasons that may have affected her responses.

Question 79

A charitable organization provides support to two different areas in Rio de Janeiro, Brazil. The following fact file provides information about the two communities that the organization supported that year.



Community A

<i>Total community size served</i>	<i>3600</i>
<i>Number of people registered for food bank</i>	<i>1800</i>
<i>Average visits to the food bank per month per person</i>	<i>2.4</i>
<i>Poverty rate for this community</i>	<i>72%</i>
<i>Total funding received for food banks</i>	<i>2500 BRL</i>
<i>Average meal cost per person</i>	<i>0.68 BRL</i>



Community B

<i>Total community size served</i>	<i>4500</i>
<i>Number of people registered for food bank</i>	<i>3500</i>
<i>Average visits to the food bank per month per person</i>	<i>5.2</i>
<i>Poverty rate for this community</i>	<i>81%</i>
<i>Total funding received for food banks</i>	<i>8000 BRL</i>
<i>Average meal cost per person</i>	<i>0.62 BRL</i>

- a) **Write down** the probability that a randomly selected resident of community A lives in poverty.
- b) Two people are randomly selected from community A. **Find** the probability that they both live in poverty.
- c) A service learning project in Singapore has raised a sum of money to fund food banks. It was decided that the project will fund communities A and B in Rio de Janeiro with the amount of 10 000 Brazilian Real (BRL) between communities A and B.

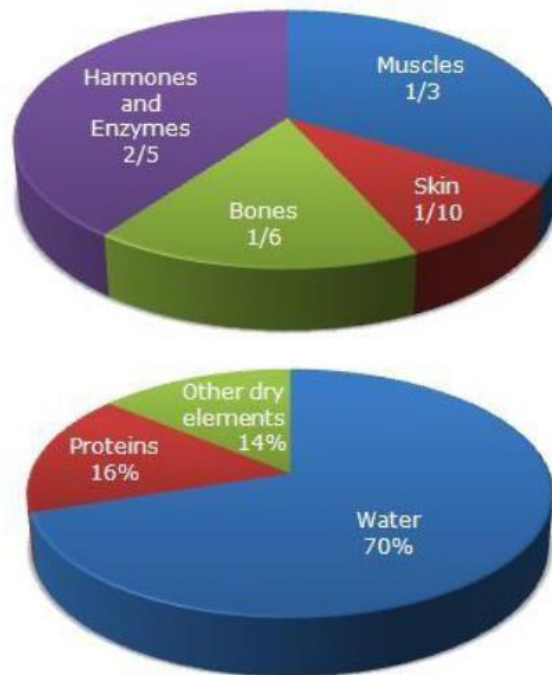
Using the information presented in the fact file, **justify** how you would divide the money between the two communities. In your answer you should:

- Consider which data are the most relevant to the problem of dividing the money.
- Consider whether it is appropriate to use a combination of data.
- Show how you have used mathematics to reach your conclusion.
- Explain and demonstrate how your conclusion divides the money fairly.

Question 80

Study the following pie-diagrams carefully and answer the questions given below it:

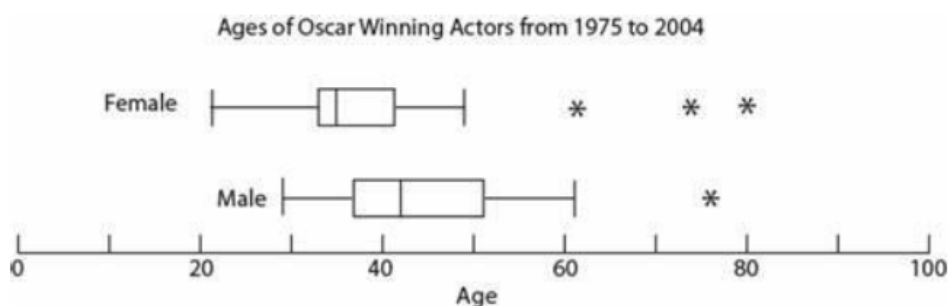
Percentage Composition of Human Body



- Write down** what percent of the total weight of human body is equivalent to the weight of the proteins in skin in human body.
- Calculate** the quantity of water in the body of a person weighing 50 kg.
- Write down** the ratio of the distribution of proteins in the muscles to that of the distribution of proteins in the bones.
- Find** the angle at the center that should subtend the arc of the circle which shows the distribution of bones in the human body.
- In the human body, **state** what part is made of neither bones nor skin?

Question 81

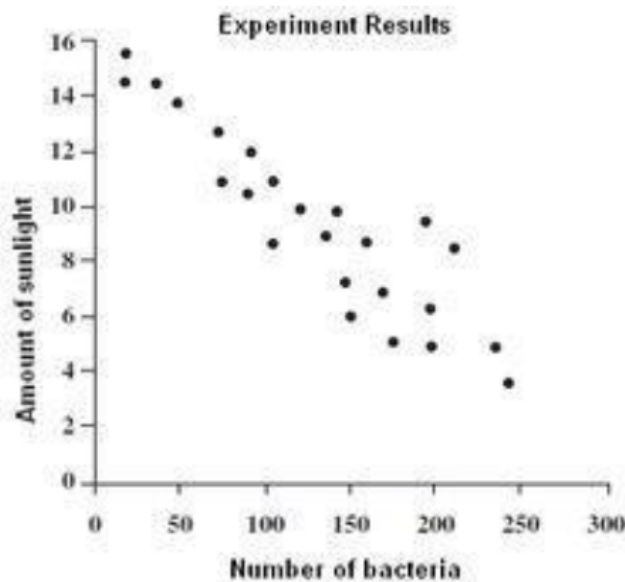
The whisker plot below shows the age distribution of Oscar winning actors.



- Write down** the range of ages for both male and female winning actors.
- State** any outliers for male winning actors.
- Write down** the percentage of male winning actors who are less than 42 years old.
- An actress was chosen at random, **write down** the probability that she is less than 20 years old.
- An actor was chosen at random, **write down** the probability that he is more than 51 years old.
- Calculate** the interquartile range for both plots.
- Comment** on your findings in part f in relation to age and gender.

Question 82

The scatter plot below shows the correlation between the amount of sunlight (in hours) and the number of bacteria in an experiment.



- Comment** on the relationship between the amount of sunlight and the number of bacteria.
- Draw** a line of best fit.
- Write down** the equation of the line of best fit.
- Comment** on the value of the slope of the line.
- A new sample is to be exposed to 10 hours of sunlight, **predict** the number of bacteria that will be present at the end of the 10 hours.
- Comment** on the validity of predicting the amount of bacteria when exposing the sample to 20 hours of sunlight.
- Calculate** the average amount of sunlight of all samples.

Question 83

Given that x and y are integers such that $y > x$, $y > 0$
A set of data is given in **ascending** order as follows:

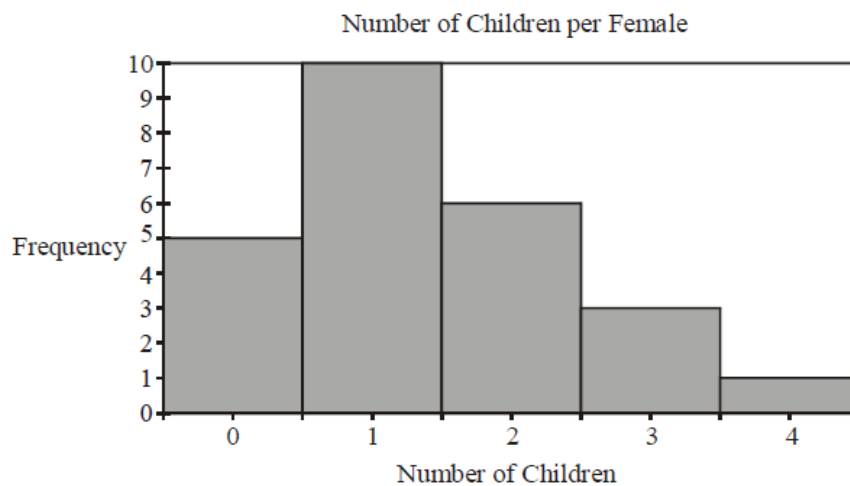
$$y - x, \quad -1, \quad -1, \quad y, \quad y^2 - 5y$$

Given that the range of the set is x and the mean is 1.6

Find the value of x and the value of y .

Question 84

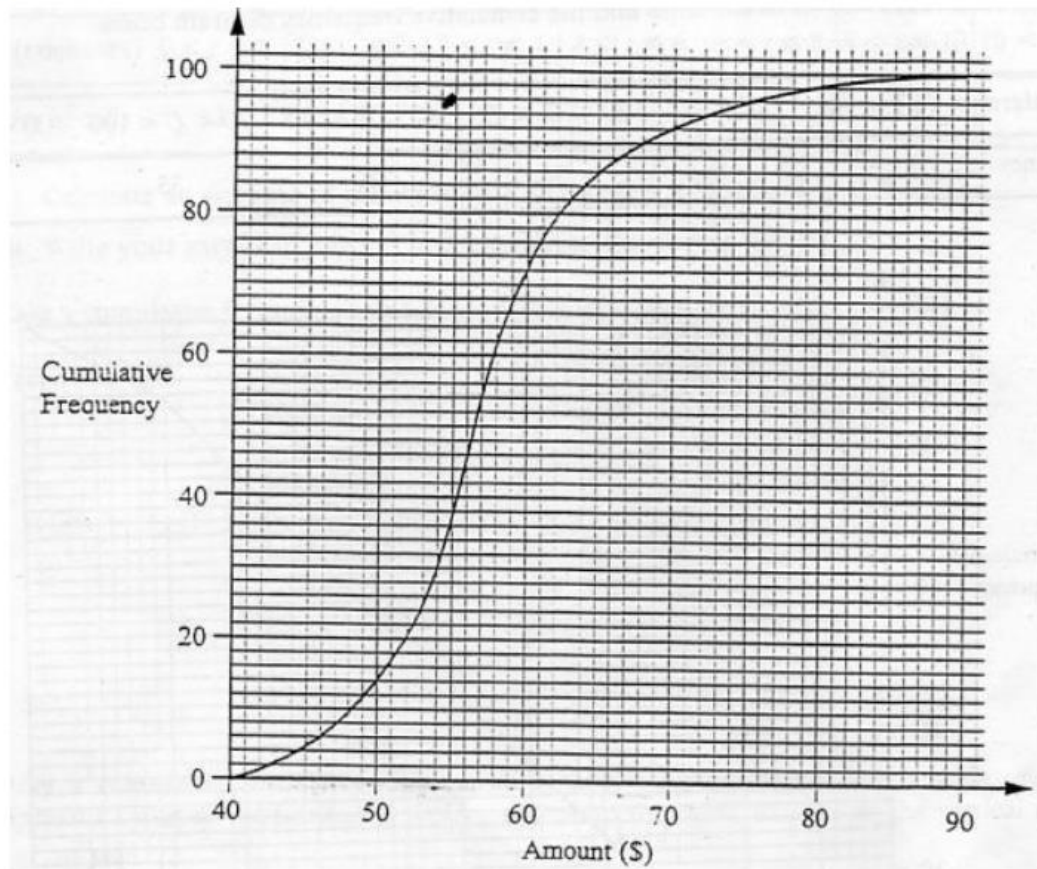
A group of 25 females were asked how many children they each had. The results are shown in the histogram below.



Show that the mean number of children per female is 1.4.

Question 85

The cumulative frequency diagram shows the weekly amount spent on food by 100 families.



- (a) Use the diagram to **find**, to the nearest dollar.
- the median;
 - the upper and lower quartiles;
 - the 60th percentile.
- (b) i. **Work out** the inter-quartile range.
- ii. The range is \$50. **Write down** the inter-quartile range as a percentage of the range.

(c) i. Complete.

<i>Weekly amount (\$x)</i>	<i>Frequency (f)</i>
$40 < x \leq 50$	-----
$50 < x \leq 60$	-----
-----	-----

ii. **Write down** the modal class?

iii. **Calculate** an estimate of the mean weekly amount spent on food.
Show all your working.

iv. **Explain** how to get more accurate estimate of the mean, using only data displayed in the diagram.

Question 86

In a history test, Andrew got 60%. For the whole class, the mean mark was 60% and the median mark was 57%. **Explain** which 'average' tells Andrew he is in the 'top' half or the 'bottom' half of the class?

Question 87

A survey of couples in a city found the following probabilities:

The probability that the husband is employed is 0.85.

The probability that the wife is employed is 0.60.

The probability that both are employed is 0.55.

*A couple is selected at random. **Find** the probability that*

- a) At least one of them is employed.*
- b) Neither is employed.*

Question 88

*A grade 12 student applied to Surrey and Bath universities. The probability of being accepted at Surrey is 0.35, from Bath is 0.48, and from both is 0.15. **Find** the probability of receiving an offer from either universities but not both.*

Question 89

A survey was developed to estimate the average number of hours per week that adults spend exercising. Every third person entering a large fitness complex was surveyed, with a total of 2,500 people being surveyed. The data showed that the mean number of hours spent exercising was 3.2 hours per week.

- a) **Explain** whether the outcomes of the experiment can be trusted or not.*
- b) **Suggest** how to increase the reliability of the outcomes of the experiment.*

Question 90

A bag contains 12 red Skittles, 12 blue Skittles, and 12 green Skittles candies. What is the probability of drawing two candies of the same color in a row, when the first candy drawn is looked at and eaten?

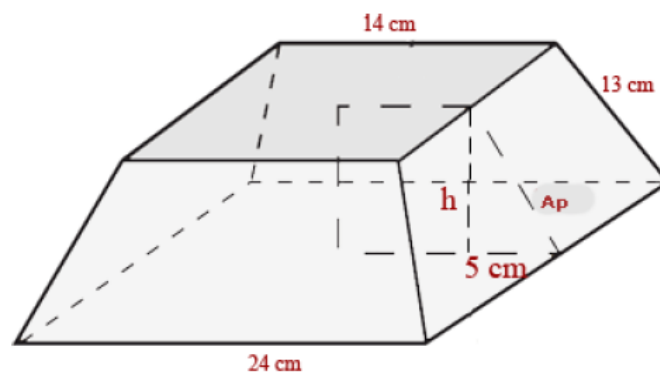
Question 91

The design of the traditional pyramids, such as those at Giza, were built with flat faces of white sandstone. The shape of the pyramid apparently was associated with the sun and the Sun god, Re. According to traditional beliefs, the sun would strengthen its rays when a Pharaoh died. This was supposedly so that they deceased ruler would have an easier time ascending into heaven. Accordingly, the sides of the pyramids symbolized the extending sunrays. Furthermore, the casing stones were white, so it is believed that the pyramids actually glowed in the sunlight.

Egyptian also believed that the spirit of the pharaoh ascends into the heavens climbing on the stairs of the pyramid.

Pyramids are still used in modern art like in *The Pyramids of Indianapolis* of the famed architect Kevin Roche.

You work for a design company. The diagram below shows a model of a solid pyramid fulcrum that will serve as a base for a modern artistic piece. The fulcrum is to be made out of copper.



- a) The top side of the model is a square with a side length of 14cm. If there was 0.01cm margin of error in measurement. **Calculate** the maximum possible area of the top side.

- b) The exact height of the model is 10.908 cm. **Calculate** the lateral surface area of the model itself.
- c) The model is made to a scale of 1:50. **Calculate** the actual cost of painting the real fulcrum at JD12/meters squares.

The original design of the pyramid with its dimensions was lost.

- d) **Calculate** the height of the original pyramid before cutting off the top part.
- e) **Calculate** the volume of the base.
- f) **Find** the cost of the pyramid at USD 4600/tonne.
- g) **Suggest** what can be made to reduce the amount of copper used to create the base.
- h) Based on what you learned in your MYP studies, **Discuss** how geometry could be used as personal and cultural expression.

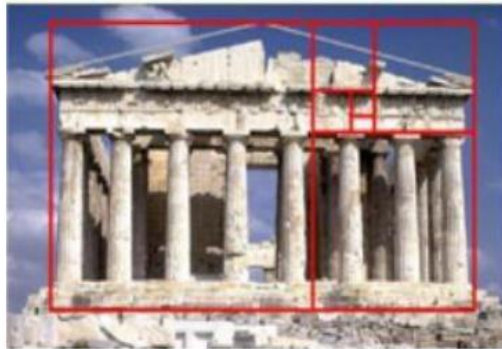
Question 92

<https://www.youtube.com/watch?v=SiSHVDfXHQ4>

The Golden Rectangle is considered to be one of the most pleasing and beautiful shapes to look at, which is why many artists have used it in their work. The two artists who perhaps the most famous for their use of the golden ratio are Leonardo Da Vinci and Piet Mondrian.

It can be found in art and architecture of ancient Greece and Rome, in works of renaissance period. Through to modern art of the 20th Century. The Golden Rectangles present in the following figures are quite obvious. However, various features of the Mona Lisa have Golden proportions too.

Further classic subdivisions of the rectangle align perfectly with major architectural features of the structure. An example is the pantheon below.



One property of rectangles is that their length (l) and width (w) are related according to the following relation.

$$\frac{l}{w} = \frac{l+w}{l}$$

- Use the rule above to **prove** that the ratio of length to width is 1.618.
- Based on your findings, **suggest** dimensions of a new mobile phone design that would look more appealing to people.

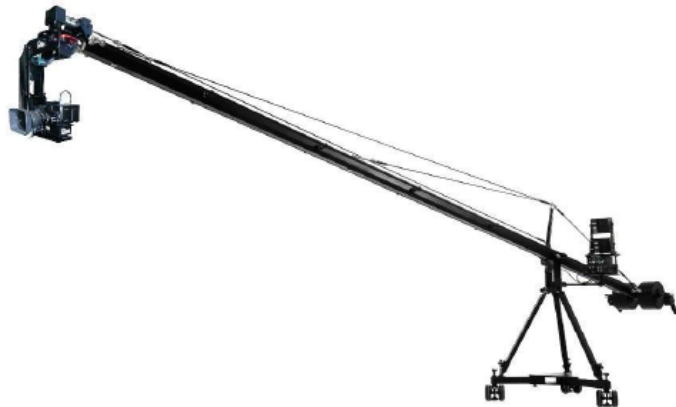
Question 93

Mathematics is widely used in filmmaking. A practical example is the use of angles of elevation and depression. Watch the following video:

<https://www.youtube.com/watch?v=0bJqkd74mCE>

For a specific shot, the camera crane below needs to start at an angle of depression of 5° then end up at an angle of depression of 60° . In both situations the camera travels in a vertical line upward.

- a) If the object filmed should be 3 meters west of the camera, **Calculate** the distance travelled by the camera.
- b) If the camera is 6 meters west of the base of the crane. And the crane's base is 60 cm high. **Calculate** the minimum crane length needed for the shooting.



- c) **State** three different applications of Mathematics that can be used in filmmaking. **Explain** how they are used.

Question 94

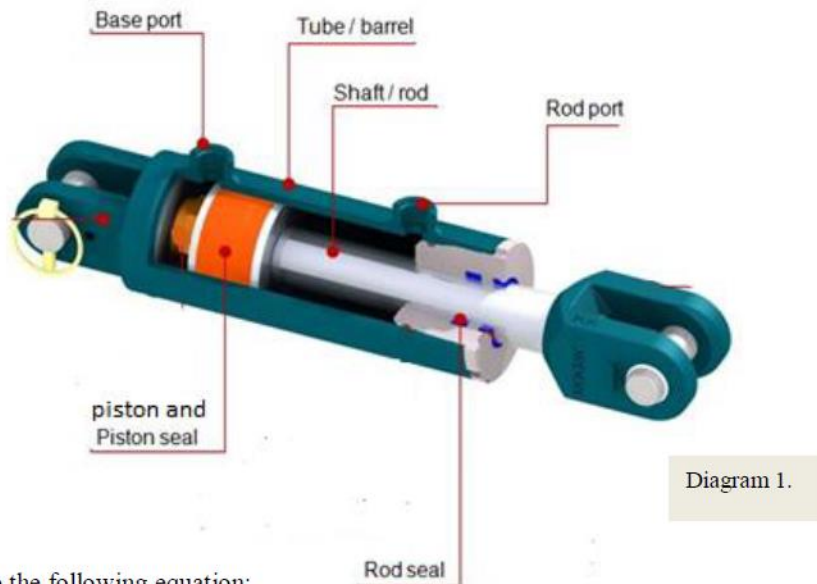
Visit the following link: <http://news.nationalgeographic.com/news/2013/09/130913-friday-luck-lucky-superstition-13/>

You are designing lottery cards. Each card has a three digit number.

- a) **Write down** how many possible combination of numbers are there.
- b) **Write down** the probability that you would get a lucky number of 7.
- c) **Write down** the probability that you would get a 666.
- d) Sami bought two cards, **calculate** the probability that he would get a triple twos and a triple threes
- e) **Explain** whether the number 666 has a different probability to show other than all other 3 digit numbers.
- f) **Suggest** a numbering system for the cards that would be more appealing to customers and increases the sales.
- g) Does your numbering system change the probability of certain numbers to happen?

Question 95 General Assessment A

Hydraulic systems are widely used in our daily lives. They utilize incompressible fluids like hydraulic oils to transfer pressure and create forces. Among the utilizations of hydraulics are hydraulic cylinders. A hydraulic cylinder is a completely sealed metal cylinder (Tube/Barrel) as shown in the diagram below. When oil enters from base port, the oil pressure increases and pushes the piston to the front. This causes oil to leave from the rod port. Once the operator gives a reverse order, the oil enters from the rod port and pushes the piston backwards. The oil then leaves from the base port back to the tanks.

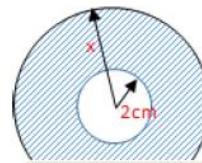
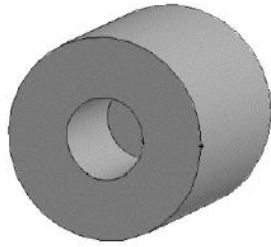


Question 1:

a) Solve the following equation:

$$x^2 - 16 = 0$$

- b) Below is a diagram and a front view of the piston. The piston is circular in shape and has a circular hole of radius 2cm at the center. **Calculate** the radius (x) of the larger circle if the shaded area is 188.4 cm². (Area of circle is $A = \pi r^2$. Assume $\pi = 3.14$)



Front View
Diagram not to scale

Question 2:

Under certain conditions, the pressure (P) [in psi units] of the oil in the cylinder is modelled as dependent variable with oil flow rate (t) according to the following relationship.

$$P = 0.004t^2 - 0.1 + 1.6$$

Calculate the flow rate, which would result in a pressure (P) of 20 psi.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 3:

The piston is sealed using a fiber seal as shown in diagram 1. Due to complaints about piston seals premature failures, (seals not serving long enough), the quality control department in the factory have collected data about the service life of seals in various locations and tabulated the following data in the table below:

Service-hour range	No. of failures	
$1000 \leq h < 1500$	239	
$1500 \leq h < 2500$	873	
$2500 \leq h < 3500$	1347	
$3500 \leq h < 4500$	311	

- a) **Write down** the modal class of seal failures.

- b) **Calculate** the average service life of seals. (Add new columns or rows to the table above if you need to do so).

- c) A quality control engineer claimed that the best way to calculate the 20th percentile of the data is by constructing a box and whisker. **Comment** on the validity of his statement.

- d) One customer reported that a seal have lasted more than 8000 hours. The quality control department have decided to discard this value from the study. **State** whether you agree or disagree with their decision and **state** the reason.

Question 4:

In order to be more specific in addressing the issue, the quality control department revised the class length the data in question 3 and produced the cumulative frequency curve below:



- a) **Find** an estimate of the median seal service life in hours.
- b) Given that the lower quartile is 2350, **Calculate** the interquartile range of the seal service life in hours

- c) **Comment** on the degree of accuracy of your findings.

Through analyzing competitors data collected from customers, the data analysts in the factory were able to draw the following competitor box and whisker.

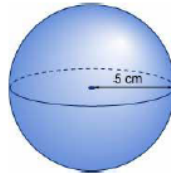
- d) On the same diagram, **draw** a parallel box and whisker for the data represented by the cumulative frequency curve above. The minimum service life was 1002 hours, and the maximum service life was 4434 hours.



Question 95 General Assessment B

The art of Packing

A company produces Christmas decorative spherical balls of radius 5 cm. Every four balls are packed in a cylindrical plastic container as shown below. The balls fit snugly in the container (They just touch the inside, the top and bottom of the cylindrical container).



One Christmas ball.

Question 1:

Write down the radius and the height of the cylindrical plastic container.

Radius of the cylindrical container =

Height =

Question 2:

Calculate the circumference of the base of the cylindrical container.

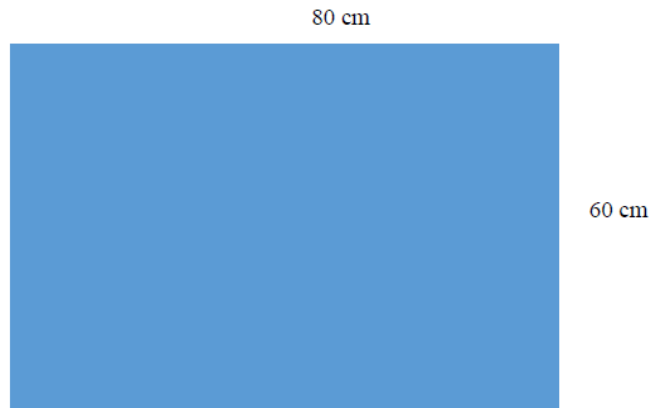
Question 3:

Calculate the lateral (curved) surface area of the cylindrical container.

Question 4:

The cylindrical plastic containers are made from plastic sheets that come in a standard size 80 cm × 60 cm, as shown below. The sheets are sold in packs of 50 and each pack costs JD2.5

You are a cost analyst and you need to study and **calculate** the minimum cost needed for packing 2000 Christmas balls. (Note: The drawings below are not to scale)



A sheet that will be cut to produce the cylindrical containers.

In your study, you should:

- Identify the relevant elements of the problem.
- Provide evidence through calculations.
- Comment on the accuracy of your results.
- Justify your reasoning.

Question 96 General Assessment C

In this task you will explore the consequences of sharing finite resources with other people and with other living things. This task focuses on criterion D (applying mathematics in real-life contexts) and criterion C (communicating).



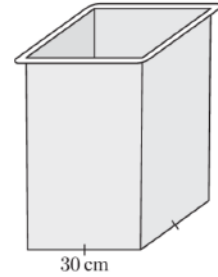
Almost everything we do creates rubbish. When we are finished with an item, or it is broken and cannot be repaired, we place it in the bin where it is collected and taken away. In some countries, the local governments provides a number of bins for households to place different kinds of rubbish in. The bins are then collected on a weekly or bi-weekly schedule, and the rubbish is taken away. However, where does it all go in the end?

Managing the rubbish produced on a daily basis is a huge task for the local and national governments around the world. In Jordan, 85% of rubbish ends its trip in a dumping site called landfill. In a landfill, the collected rubbish is compacted and buried in layers where it can be safely stored out of the sight for decades. Over time, these landfills are completely filled and no more rubbish can be stored. They are then closed and a new location needs to be found to put the rubbish. An area must be found that is not too expensive for the land, that does not affect any residents nearby, and that does not cause any problems for the local environment. These limitations make finding new sites difficult. Unfortunately, many items that which could have been reused, recycled or composed end up in the landfills, taking up valuable space.

Question 1

A household rubbish bin is approximately a square-based prism with base sides 30 cm. Calculate the volume of the bin if the height is:

- a) 60 cm.
- b) 70 cm.



Question 2

Rubbish bags used in the bin have a capacity of 56 L. which of the bins in question 1 would be the best fit for the bags. Justify your answer. (1L = 1000 cm^3)

Question 3

In Europe, an average of 1.37 kg of household rubbish is produced per person each day. From that 16% is composted, 22% is recycled, 20% is incinerated and the remaining rubbish is stored as landfill. This makes Europe one of the best places in the world for managing rubbish.

Assuming that the same amount of rubbish is produced by Jordanians, and knowing that the 85% of the amount ends up in the landfill. Calculate:

- a) The amount of household rubbish produced on average by a Jordanian during one week.

- b) How much of this is stored in the landfill?

Question 4

On average, each cubic centimeter of rubbish weighs approximately 0.33g. **Estimate**

- a) the weight of a full 56 L bag of rubbish.
- b) The volume of rubbish produced by a family of four in Jordan during one week.

Question 5

Rubbish that arrives in a landfill is compacted so that each gram has volume approximately 1.6 cm^3 . Estimate the volume of rubbish added to landfills in Jordan during one year, given that the population of Jordan is approximately 9 million people. (assume one year to be 365 days)

Question 6

A landfill in Rusayfa between Zarqa and Amman has approximately triangular shape as shown below. It lies on the skirts of the city with Angle ACB is 62° . The landfill is designed so as the depth of rubbish in the landfill can reach up to 10 meters.

You need to write an evaluative report on the design of this landfill, in your report you should:

- Write down the relevant information to this problem considering the information given above in the task.
- Evaluate the choice of the size of the landfill and its location. (give two ideas)
- Support your work with calculations and evidence.
- **Suggest** modifications to the presented model and **Justify** your choices.
- **Comment** on the degree of accuracy of the presented model and on the accuracy of your calculations.

