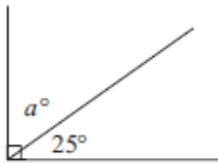


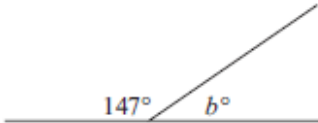
Angle Properties and Triangle properties

Find the value of the unknown in:

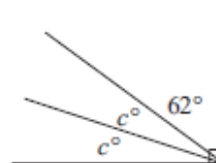
a



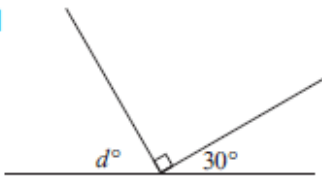
b



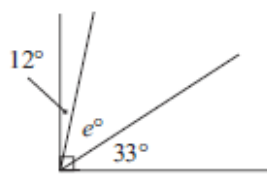
c



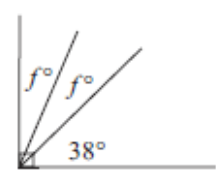
d



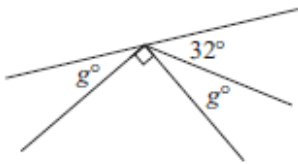
e



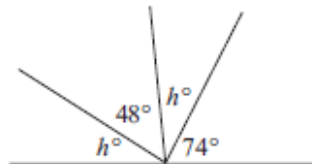
f



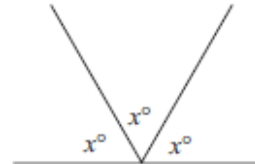
g



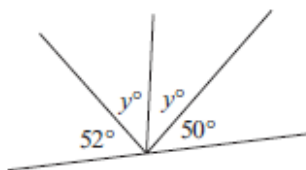
h



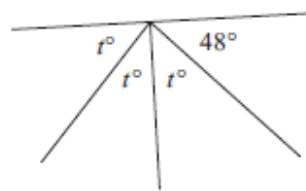
i



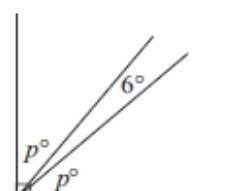
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k

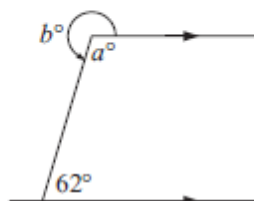


l

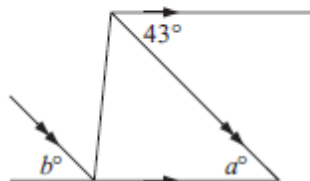


Find, giving brief reasons, the values of the unknowns in alphabetical order:

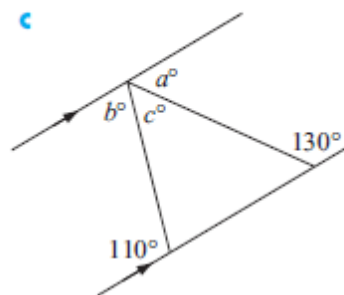
a



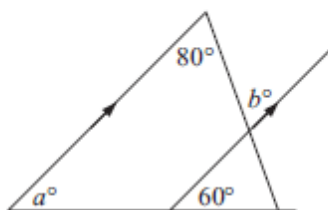
b



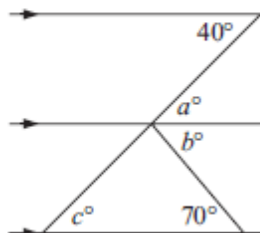
c



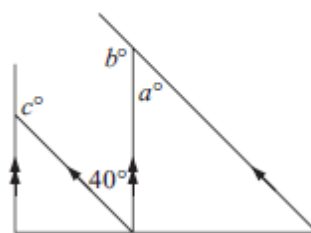
d



e

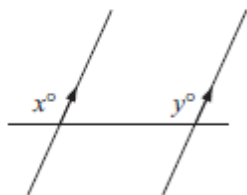


f

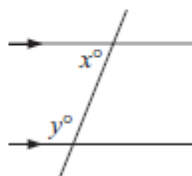


Write a statement connecting the unknowns, giving a brief reason:

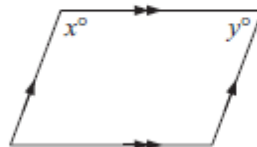
a



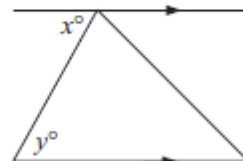
b



c

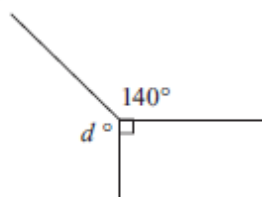


d

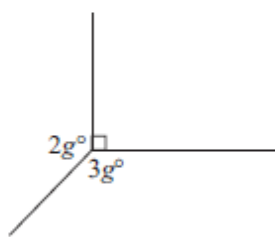


Find, giving brief reasons, the value of the unknown in:

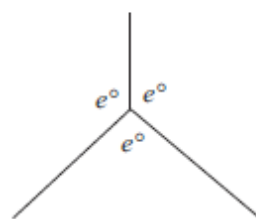
a



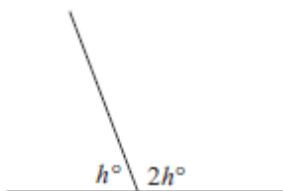
b



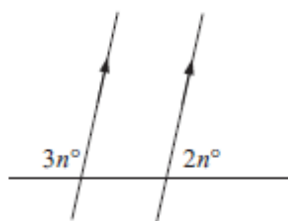
c



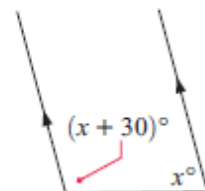
d



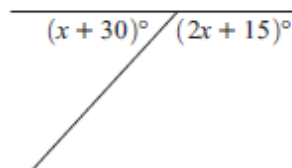
e



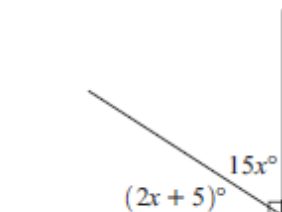
f



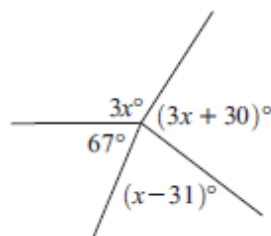
g



h

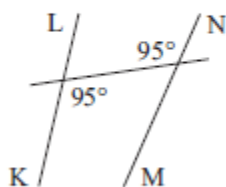


i

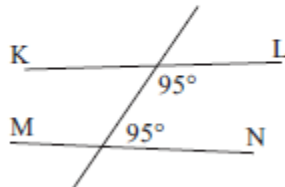


State whether [KL] is parallel to [MN], giving a brief reason for your answer. These diagrams are sketches only and have not been drawn to scale.

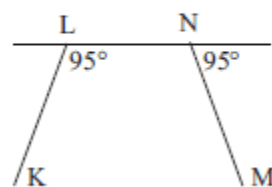
a



b

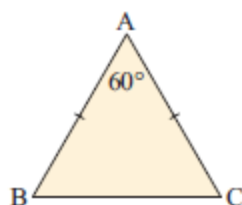


c

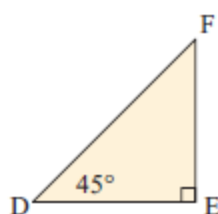


The following triangles are not drawn to scale. Classify them as equilateral, isosceles or scalene. The information marked on them is correct.

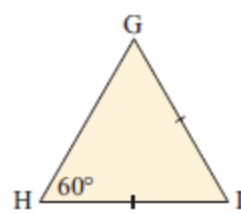
a



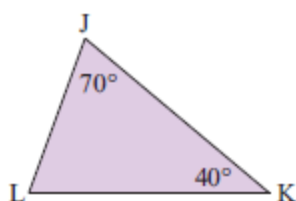
b



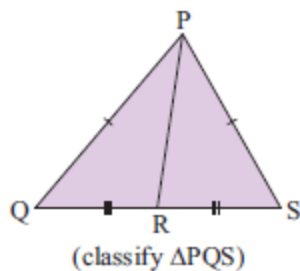
c



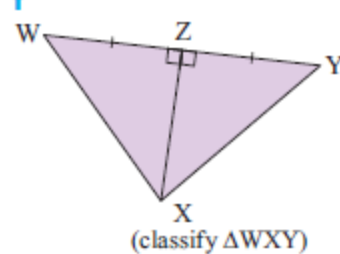
d



e

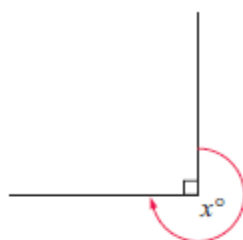


f

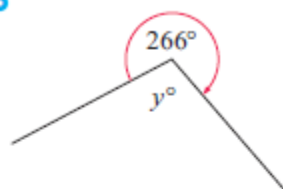


Find the sizes of the unknown angles:

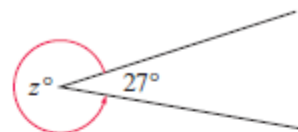
a



b

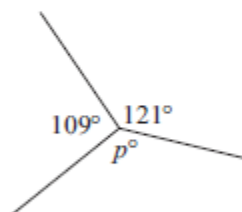


c

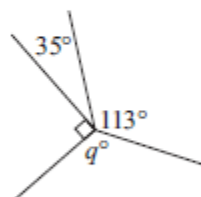


Find the values of the unknowns in:

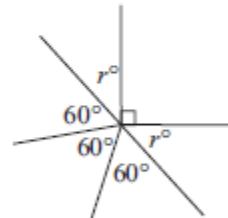
a



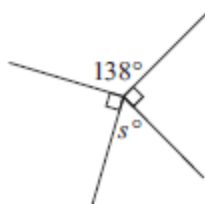
b



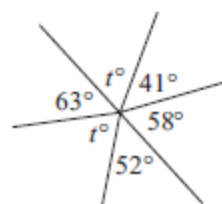
c



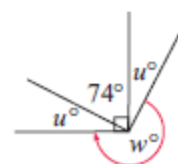
d



e

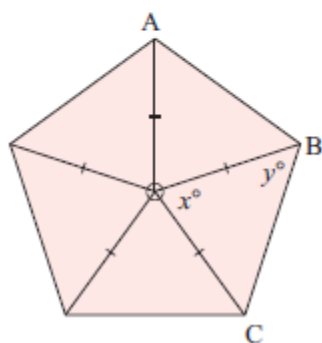
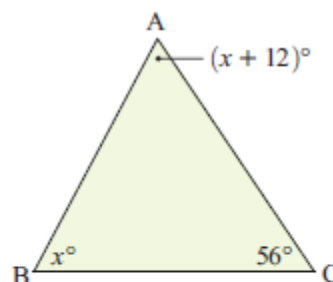


f



The figure alongside has not been drawn to scale:

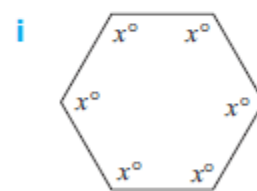
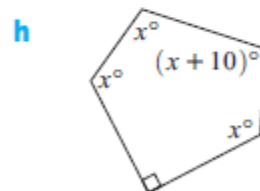
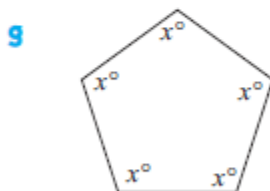
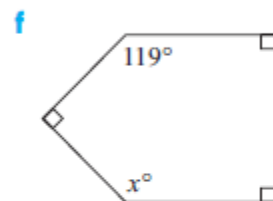
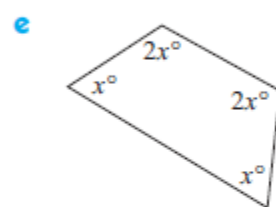
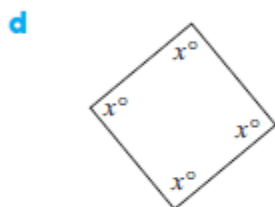
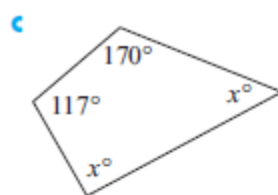
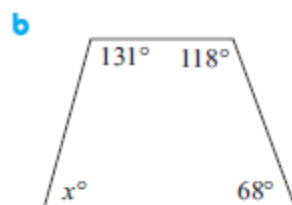
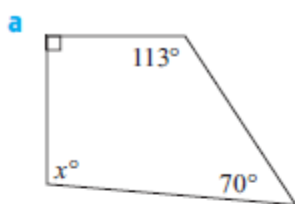
- a Find x .
- b What can be deduced about the triangle?



Because of its symmetry, a regular pentagon can be made up of five isosceles triangles.

- a Find the size of x , the angle at the centre of the pentagon.
- b Hence, find y .
- c Hence, find the measure of one interior angle such as \widehat{ABC} .

Find the value of x :



The sum of the angles of a polygon is 1980° . How many angles has the polygon?

Juan claims to have found a polygon which has angles with a sum of 2500° . Comment on Juan's finding.

TESTS FOR TRIANGLE CONGRUENCE

Two triangles are congruent if one of the following is true:

- All corresponding sides are equal in length. (SSS)



- Two sides and the **included** angle are equal. (SAS)



- Two angles and a pair of **corresponding** sides are equal. (AAcorS)

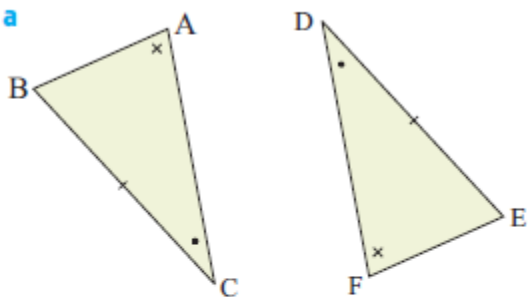


- For right angled triangles, the hypotenuses and one pair of sides are equal (RHS).

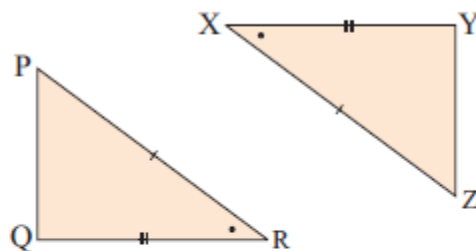


Are the following pairs of triangles congruent? If so, state the congruence relationship and give a brief reason.

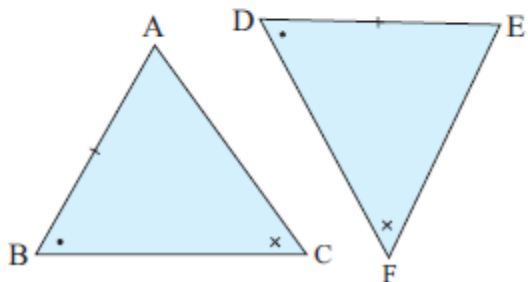
a



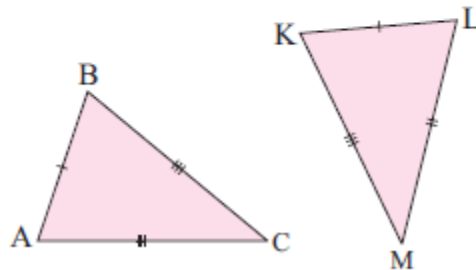
b

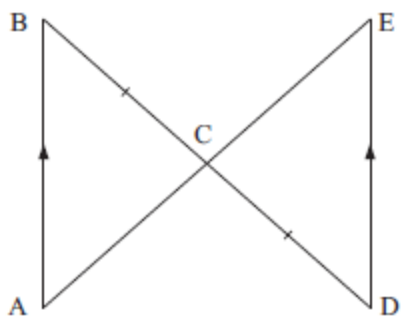


c



d

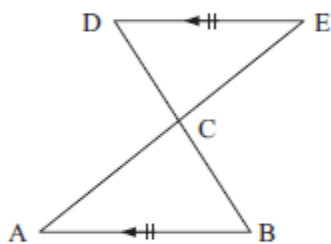
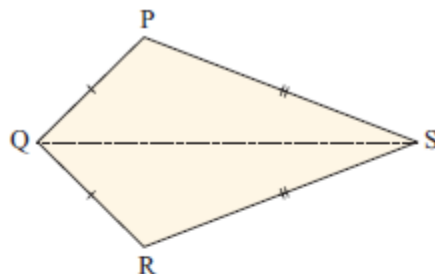




- a** Explain why triangles ABC and EDC are congruent.
- b** If $AC = 5 \text{ cm}$ and $\angle BAC = 37^\circ$, find:
 - i** the length of CE
 - ii** the size of $\angle DEC$.

PQRS is a kite and QS is added to the figure.

- a** Show that the triangles are congruent.
- b** What other facts can then be deduced about the figure?

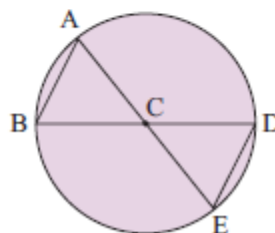


In the given figure DE is parallel to AB and $DE = AB$.

- a** Show that the triangles are congruent.
- b** What other facts can then be deduced about the figure?

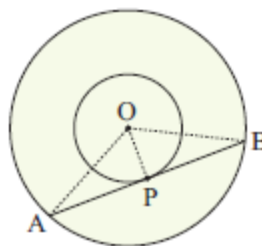
C is the centre of the circle.

- a** Show that the figure contains congruent triangles.
- b** What other facts can then be deduced about the figure?

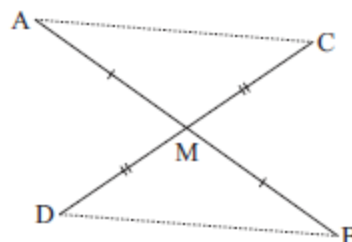


Two concentric circles are drawn. At P on the inner circle a tangent is drawn and it meets the other circle at A and B.

Use triangle congruence to prove that P is the midpoint of AB.

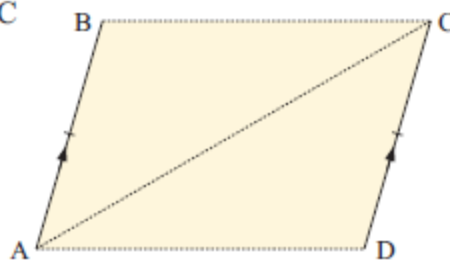


- a Prove that triangles AMC and BMD are congruent.
- b Deduce that AC and DB are parallel and equal in length.
- c What can be deduced about the quadrilateral ACBD?



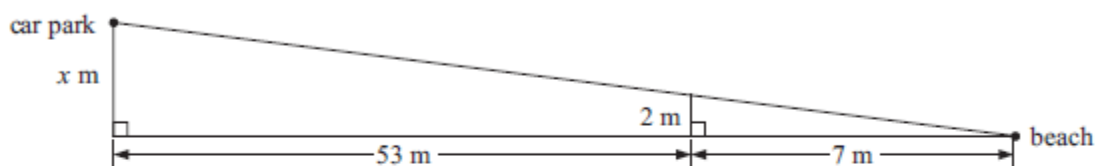
You are given that AB and DC are parallel and equal in length.

- a Join BC, AC and AD and show that Δ 's ABC and CDA are congruent.
- b Now show that ABCD is a parallelogram.
- c Copy and complete: *If a pair of opposite sides of a quadrilateral are parallel and equal in length then the quadrilateral is*

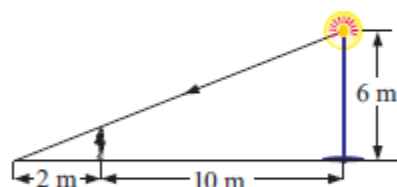


SIMILARITY

A path up to the car-park from the beach has a constant slope of 2 in 7. This means that for every 7 m horizontally the path rises 2 m. The car-park is 60 m horizontally from the beach end of the path. How high in the sand-dunes is the car-park?



An electric light has a globe which is 6 m above the ground. How tall is a girl who casts a 2 m long shadow when standing 10 m from the base of the pole?

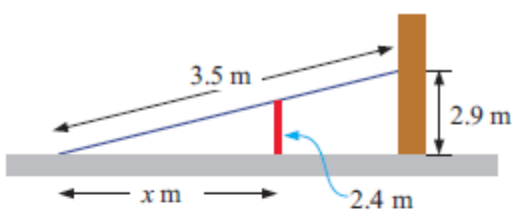


A picket fence surrounding a church is 150 cm high. At a certain time of the day it was noticed that the shadow of the fence was 250 cm long and the shadow of the church's steeple was 18 m long. How high is the steeple?

On a bright sunny day the shadow cast by a flagpole is 12 m long. At the same time the shadow cast by a 30 cm long ruler is 45 cm long. Find the height of the flag pole.

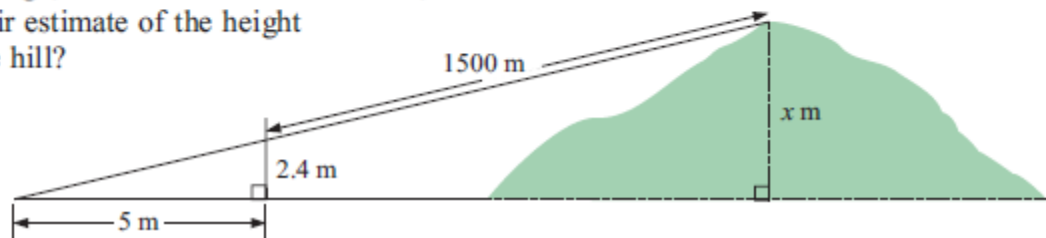
A 3.5 m ladder leans on a 2.4 m fence. One end is on the ground and the other end touches a vertical wall 2.9 m from the ground.

How far is the bottom of the ladder from the fence?



Two surveyors estimate the height of a nearby hill. One stands 5 m away from the other on horizontal ground holding a 3 m stick vertically. The other surveyor finds a "line of sight" to the top of the hill, and observes this line passes the vertical stick at 2.4 m. The distance from the stick to the top of the hill is 1500 m (as measured by laser equipment).

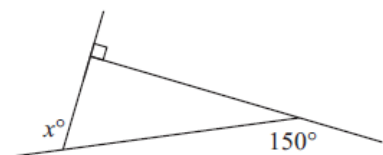
How high, correct to the nearest metre, is their estimate of the height of the hill?



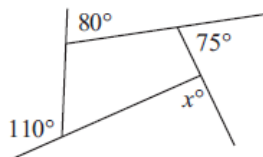
Exterior angles of a polygon

1 Solve for x :

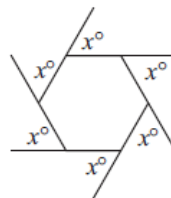
a



b



c



2 Calculate the size of each interior angle of these regular polygons:

a with 5 sides

b with 8 sides

c with 10 sides

d with 20 sides

e with 100 sides

f with n sides

3 Calculate the number of sides of a regular polygon given that an exterior angle is:

a 45°

b 15°

c 2°

d $\frac{1}{2}^\circ$

4 Calculate the number of sides of a regular polygon with an interior angle of:

a 120°

b 150°

c 175°

d 179°

PYTHAGORAS THEOREM

1 Determine if the following are Pythagorean triples:

a $\{8, 15, 17\}$

b $\{6, 8, 10\}$

c $\{5, 6, 7\}$

d $\{14, 48, 50\}$

e $\{1, 2, 3\}$

f $\{20, 48, 52\}$

2 Find k if the following are Pythagorean triples:

a $\{8, 15, k\}$

b $\{k, 24, 26\}$

c $\{14, k, 50\}$

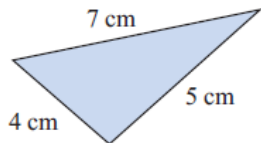
d $\{15, 20, k\}$

e $\{k, 45, 51\}$

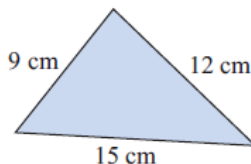
f $\{11, k, 61\}$

1 The following figures are not drawn to scale. Which of the triangles are right angled?

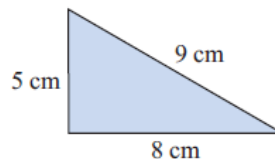
a



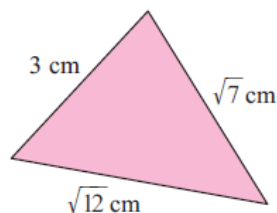
b



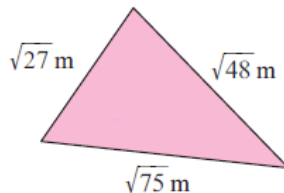
c



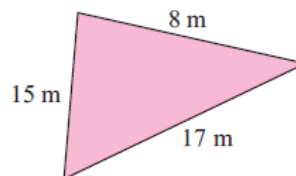
d



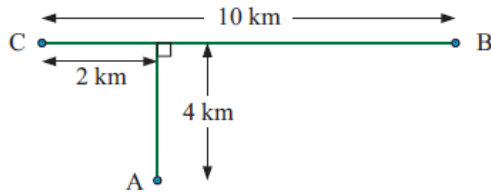
e



f



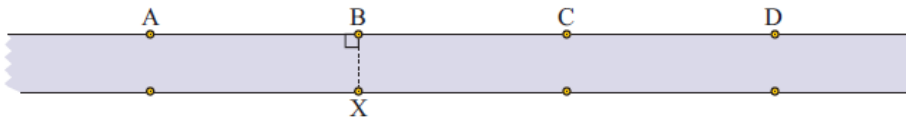
A yacht sails 5 km due west and then 8 km due south.
How far is it from its starting point?



A cyclist at C is travelling towards B. How far will he have to cycle before he is equidistant from A and B?

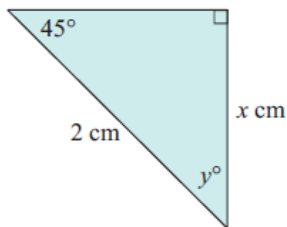
A street is 8 m wide, and there are street lights positioned either side of the street every 20 m.
How far is street light X from street light:

- a** A **b** B **c** C **d** D?

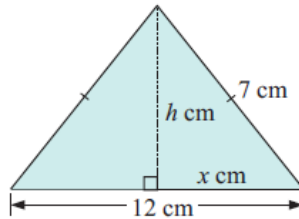


Find any unknowns in the following:

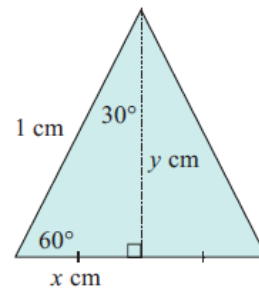
a



b



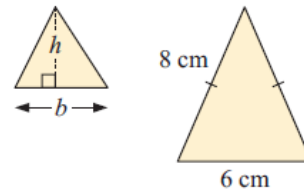
c



An equilateral triangle has sides of length 12 cm. Find the length of one of its altitudes.

The area of a triangle is given by the formula $A = \frac{1}{2}bh$.

- a** An isosceles triangle has equal sides of length 8 cm and a base of length 6 cm. Find the area of the triangle.
b An equilateral triangle has area $16\sqrt{3} \text{ cm}^2$. Find the length of its sides.

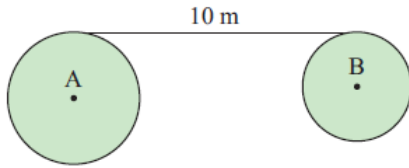
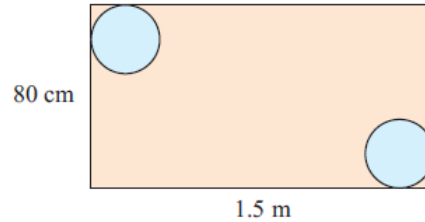


A chord is 5 cm from the centre of a circle of radius 8 cm. Find the length of the chord.

A circle has radius 3 cm. A tangent is drawn to the circle from point P which is 9 cm from O, the circle's centre. How long is the tangent? Leave your answer in surd form.

Find the radius of a circle if a tangent of length 12 cm has its end point 16 cm from the circle's centre.

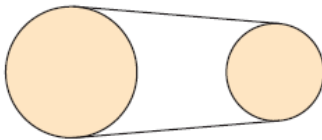
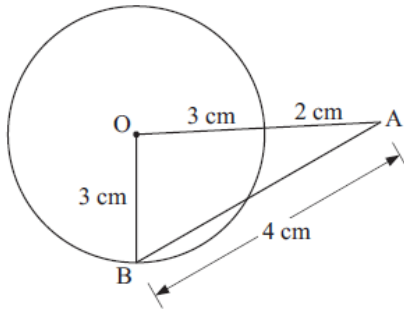
Two circular plates of radius 15 cm are placed in opposite corners of a rectangular table as shown. Find the distance between the centres of the plates.



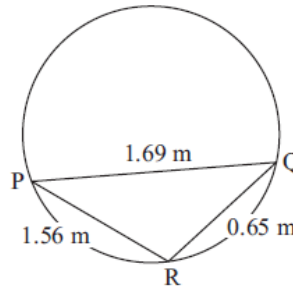
A and B are the centres of two circles with radii 4 m and 3 m respectively. The illustrated common tangent has length 10 m. Find the distance between the centres correct to 2 decimal places.

The following figures have not been drawn to scale, but the information marked on them is correct. What can you deduce from each figure?

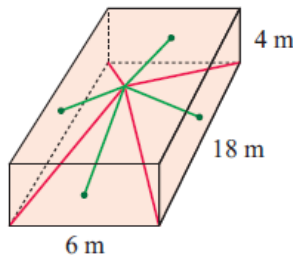
a



b



Any two circles which do not intersect have two common external tangents as illustrated. The larger circle has radius b and the smaller one has radius a . The circles are $2a$ units apart. Show that each common tangent has length $\sqrt{8a(a+b)}$ units.



A 6 m by 18 m by 4 m hall is to be decorated with streamers for a party.

4 streamers are attached to the corners of the floor, and 4 streamers are attached to the centres of the walls as illustrated. All 8 streamers are then attached to the centre of the ceiling. Find the total length of streamers required.

A room is 5 m by 3 m and has a height of 3.5 m. Find the distance from a corner point on the floor to the opposite corner of the ceiling.

Determine the length of the longest metal rod which could be stored in a rectangular box 20 cm by 50 cm by 30 cm.

A tree is 8 m north and 6 m east of another tree. One of the trees is 12 m tall, and the other tree is 17 m tall. Find the distance between:

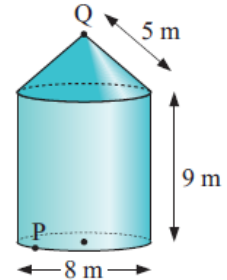
a the trunks of the trees

b the tops of the trees.

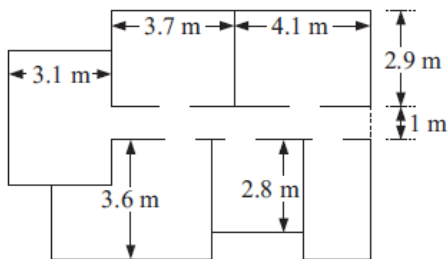
A rainwater tank is cylindrical with a conical top.

The slant height of the top is 5 m, and the height of the cylinder is 9 m.

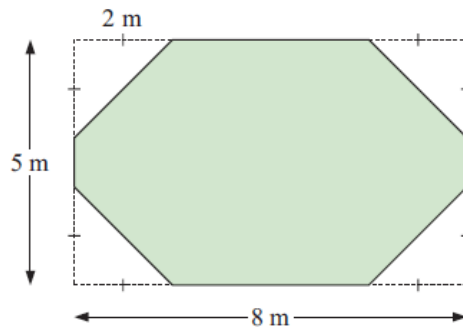
Find the distance between P and Q, to the nearest cm.



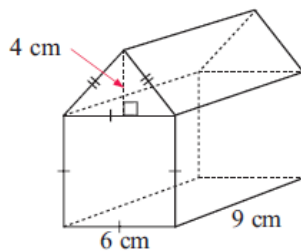
MENSURATION



Find the perimeter of the house in the plan alongside.



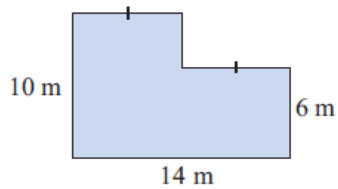
An octagonal area of lawn is created by removing 2 m by 2 m corners from a rectangular area. Find the new perimeter of the lawn.



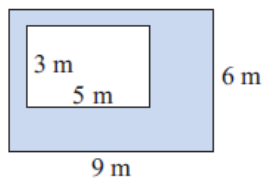
Calculate the length of wire required to construct the frame for the model house illustrated alongside.

Find the area shaded:

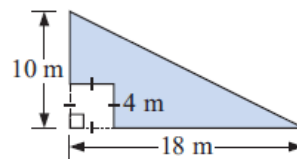
a



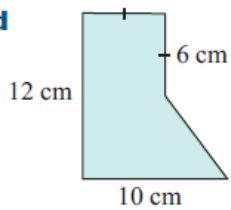
b



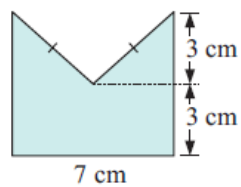
c



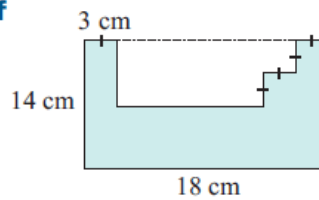
d



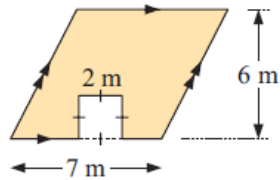
e



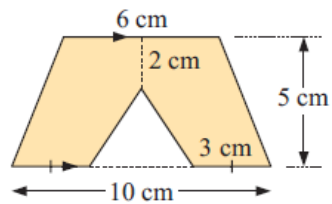
f



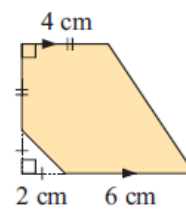
g



h



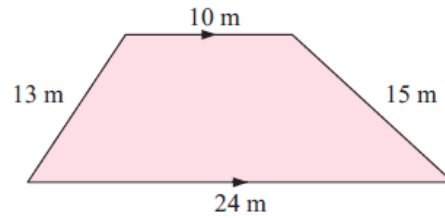
i



- a** A kite has diagonals of length 16 cm and 10 cm. Find its area.
- b** Find the area of a kite with diagonals of length a cm and b cm.

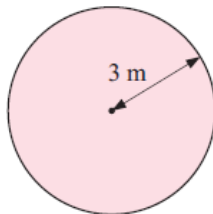
Parallelogram ABCD has $AB = 10$ cm and diagonal $DB = 15$ cm. If the shortest distance from C to line AB is 8 cm, find the shortest distance from A to DB.

Find the area of this trapezium:

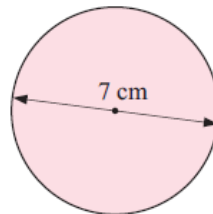


Find the area shaded:

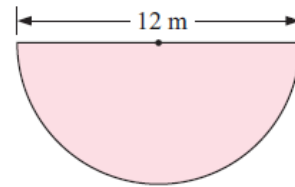
a



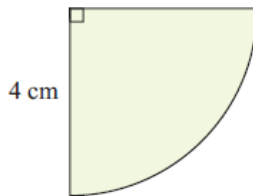
b



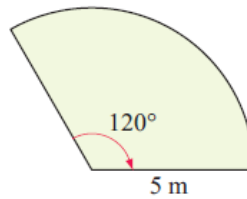
c



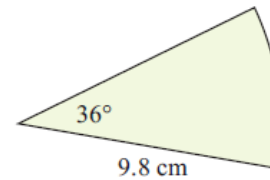
d



e

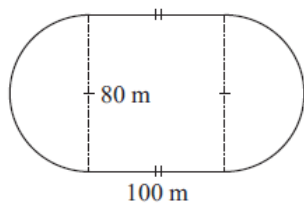


f

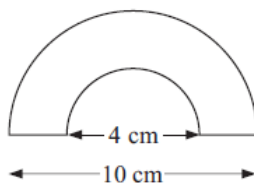


Find the perimeter and area of the following shapes:

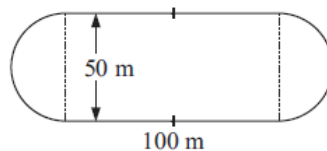
a



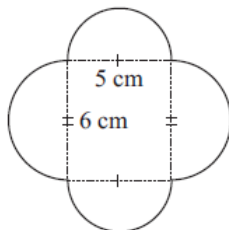
b



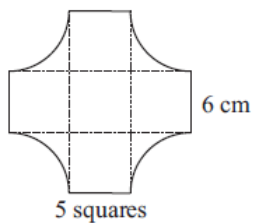
c



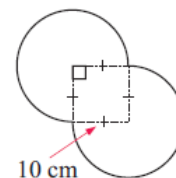
d



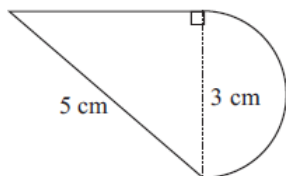
e



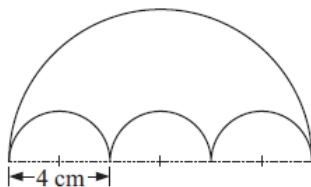
f



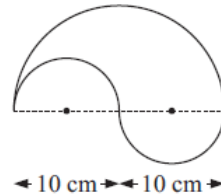
g



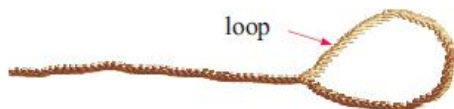
h



i



Find the radius of a trundle wheel with circumference 1 m.



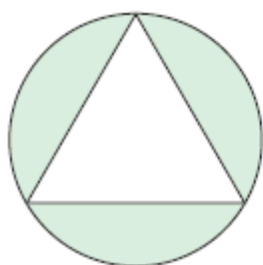
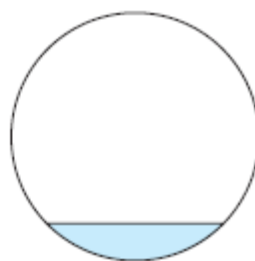
A lasso is made from a rope that is 10 m long. The loop of the lasso has a radius of 0.6 m when circular. Find the length of the rope that is not part of the loop.

How far is a chord of length 8 cm from the centre of a circle of radius 5 cm?

How far is a chord of length 9 cm from the centre of a circle of radius 6 cm?

The diagram on the right shows the cross-section of a cylindrical pipe with water lying in the bottom.

- a) If the maximum depth of the water is 2 cm and the radius of the pipe is 7 cm, find the area shaded.
- b) What is the *volume* of water in a length of 30 cm?



An equilateral triangle is inscribed in a circle of radius 10 cm. Find:

- a) the area of the triangle
- b) the area shaded green.

A solid cylinder of radius 4 cm and length 8 cm is melted down and recast into a solid cube. Find the side of the cube.

A solid rectangular block of copper 5 cm by 4 cm by 2 cm is drawn out to make a cylindrical wire of diameter 2 mm. Calculate the length of the wire.

Water flows through a circular pipe of internal diameter 3 cm at a speed of 10 cm/s. If the pipe is full, how much water flows from the pipe in one minute? (Answer in litres.)

Water flows from a hose-pipe of internal diameter 1 cm at a rate of 5 litres per minute. At what speed is the water flowing through the pipe?

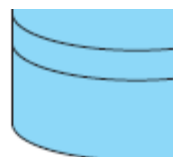
A cylindrical metal pipe has external diameter of 6 cm and internal diameter of 4 cm. Calculate the volume of metal in a pipe of length 1 m. If 1 cm^3 of the metal has a mass of 8 g, find the mass of the pipe.

For two cylinders A and B, the ratio of lengths is 3 : 1 and the ratio of diameters is 1 : 2. Calculate the ratio of their volumes.

A prism has volume 100 cm^3 and length 8 cm. If the cross-section is an equilateral triangle, find the length of a side of the triangle.



A business buys paint at \$30 000 for 20 m^3 . It puts the paint in tins of capacity 0.8 litres and sells them at \$5.95 each. Work out the profit.



Natalia decided to build a garage and began by calculating the number of bricks required. The garage was to be 6 m by 4 m and 2.5 m in height. Each brick measures 22 cm by 10 cm by 7 cm. Natalia estimated that she would need about 40 000 bricks. Is this a reasonable estimate?

A cylindrical can of internal radius 20 cm stands upright on a flat surface. It contains water to a depth of 20 cm. Calculate the rise in the level of the water when a brick of volume 1500 cm^3 is immersed in the water.

A cylindrical tin of height 15 cm and radius 4 cm is filled with sand from a rectangular box. How many times can the tin be filled if the dimensions of the box are 50 cm by 40 cm by 20 cm?

A rectangular piece of card 12 cm by 20 cm is rolled up to make a tube (with no overlap). Find the volume of the tube if

- the long sides are joined
- the short sides are joined.

Rain which falls onto a flat rectangular surface of length 6 m and width 4 m is collected in a cylinder of internal radius 20 cm. What is the depth of water in the cylinder after a storm in which 1 cm of rain fell?

A cone is attached to a hemisphere of radius 4 cm. If the total height of the object is 10 cm, find its volume.

A toy consists of a cylinder of diameter 6 cm 'sandwiched' between a hemisphere and a cone of the same diameter. If the cone is of height 8 cm and the cylinder is of height 10 cm, find the total volume of the toy.

Find the height of a pyramid of volume 20 m^3 and base area 12 m^2 .

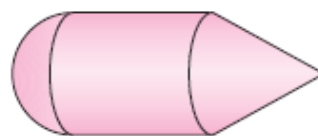
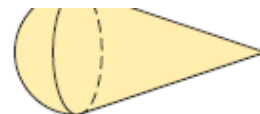
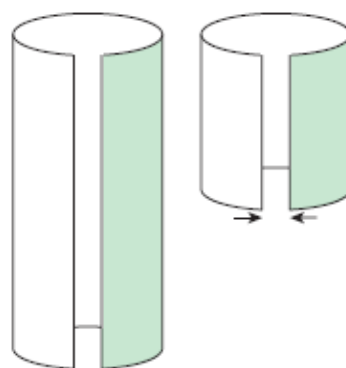
Find the radius of a sphere of volume 60 cm^3 .

Find the height of a cone of volume 2.5 litres and radius 10 cm.

Six square-based pyramids fit exactly onto the six faces of a cube of side 4 cm. If the volume of the object formed is 256 cm^3 , find the height of each of the pyramids.

A solid metal cube of side 6 cm is recast into a solid sphere. Find the radius of the sphere.

A hollow spherical vessel has internal and external radii of 6 cm and 6.4 cm respectively. Calculate the mass of the vessel if it is made of metal of density 10 g/cm^3 .

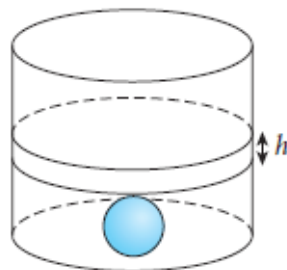


A solid metal sphere is recast into many smaller spheres.
Calculate the number of the smaller spheres if the initial and final radii are as follows

- Initial radius = 10 cm, final radius = 2 cm
- Initial radius = 7 cm, final radius = $\frac{1}{2}$ cm
- Initial radius = 1 m, final radius = $\frac{1}{3}$ cm

A spherical ball is immersed in water contained in a vertical cylinder.
Assuming the water covers the ball, calculate the rise in the water level if:

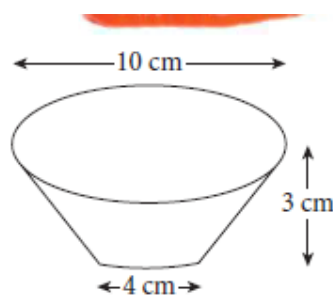
- sphere radius = 3 cm, cylinder radius = 10 cm
- sphere radius = 2 cm, cylinder radius = 5 cm.



A spherical ball is immersed in water contained in a vertical cylinder. The rise in water level is measured in order to calculate the radius of the spherical ball. Calculate the radius of the ball in the following cases.

- Cylinder of radius 10 cm, water level rises 4 cm
- Cylinder of radius 100 cm, water level rises 8 cm

A **frustum** is a cone with 'the end chopped off'. A bucket in the shape of a frustum as shown has diameters of 10 cm and 4 cm at its ends and a depth of 3 cm. Calculate the volume of the bucket.

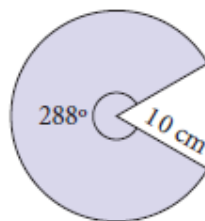


Find the volume of a frustum with end diameters of 60 cm and 20 cm and a depth of 40 cm.

The diagram shows a sector of a circle of radius 10 cm.

- Find, as a multiple of π , the arc length of the sector.
- The straight edges are brought together to make a cone. Calculate:

- the radius of the base of the cone,
- the vertical height of the cone.



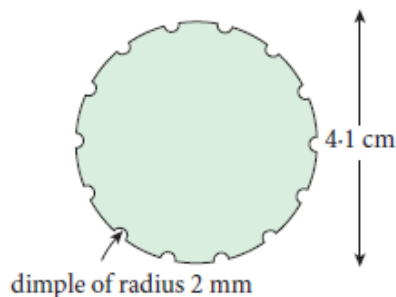
A solid wooden cylinder of height 8 cm and radius 3 cm is standing vertically. It is then cut in two along a vertical plane of symmetry. Calculate the total surface area of the two pieces.

A tin of paint covers a surface area of 60 m^2 and costs \$4.50. Find the cost of painting the outside surface of a hemispherical dome of radius 50 m. (Just the curved part.)

A solid cylinder of height 10 cm and radius 4 cm is to be plated with material costing \$11 per cm^2 . Find the cost of the plating.

Find the volume of a sphere of surface area 100 cm^2 .

- i. Find the surface area of a sphere of volume 28 cm^3 .
- . An inverted cone of vertical height 12 cm and base radius 9 cm contains water to a depth of 4 cm. Find the area of the interior surface of the cone not in contact with the water.
- . A circular piece of paper of radius 20 cm is cut in half and each half is made into a hollow cone by joining the straight edges. Find the slant height and base radius of each cone.
- . A golf ball has a diameter of 4.1 cm and the surface has 150 dimples of radius 2 mm. Calculate the total surface area which is exposed to the surroundings. (Assume the dimples are hemispherical.)
- . A cone of radius 3 cm and slant height 6 cm is cut into four identical pieces. Calculate the total surface area of the four pieces.

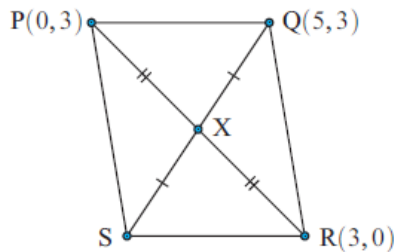


COORDINATE GEOMETRY

AB is the diameter of a circle with centre C. If A is $(3, -2)$ and B is $(-1, -4)$, find the coordinates of C.

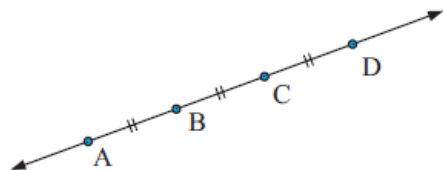
PQ is a diameter of a circle with centre $(3, -\frac{1}{2})$. Find the coordinates of P given that Q is $(-1, 2)$.

The diagonals of parallelogram PQRS bisect each other at X. Find the coordinates of S.



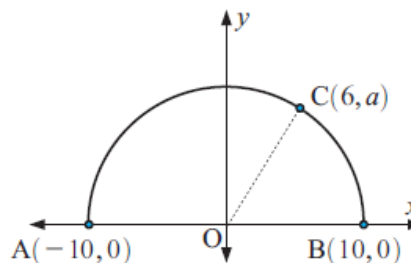
Triangle ABC has vertices $A(-1, 3)$, $B(1, -1)$, and $C(5, 2)$. Find the length of the line segment from A to the midpoint of BC.

A, B, C and D are four points on the same straight line. The distances between successive points are equal, as shown. If A is $(1, -3)$, C is $(4, a)$ and D is $(b, 5)$, find the values of a and b .



$S(6, a)$ lies on a semi-circle as shown.

- a** Find a .
- b** Using this value of a , find the gradient of:
 - i** AC
 - ii** CB.
- c** Use **b** to show that angle ACB is a right angle.



Find k if the line joining $X(2, -3)$ and $Y(-1, k)$ is:

- a** parallel to a line with gradient $\frac{1}{2}$
- b** perpendicular to a line with gradient $-\frac{1}{4}$.

Find b given that $A(-6, 2)$, $B(b, 0)$ and $C(3, -4)$ are collinear.

Given $A(-3, 1)$, $B(1, 4)$ and $C(4, 0)$:

- a** Show that triangle ABC is isosceles.
- b** Find the midpoint X of AC.
- c** Use gradients to verify that BX is perpendicular to AC.

- a** Find the midpoint of the line segment joining $A(-2, 3)$ to $B(-4, 3)$.
- b** Find the distance from $C(-3, -2)$ to $D(0, 5)$.
- c** Find the gradient of all lines perpendicular to a line with gradient $\frac{2}{3}$.

On different sets of axes, show all points with:

- a** x -coordinates equal to -3
- b** y -coordinates equal to 5
- c** positive x -coordinates and negative y -coordinates.

$K(-3, 2)$ and $L(3, m)$ are 9 units apart. Find m .

Find c if the line joining $A(5, 3)$ to $B(c, -2)$ is perpendicular to the line with gradient 3.

$A(-1, 2)$, $B(3, a)$ and $C(-3, 7)$ are collinear. Find a .

Given $A(-3, 2)$, $B(2, 3)$, $C(4, -1)$ and $D(-1, -2)$ are the vertices of quadrilateral ABCD:

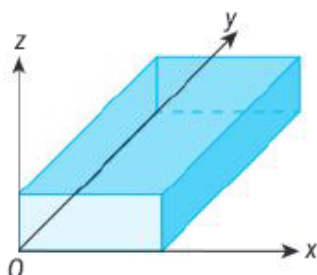
- a** Find the gradient of AB and DC.
- b** Find the gradient of AD and BC.
- c** What do you deduce from your answers to **a** and **b**?
- d** Find the midpoints of the diagonals of the quadrilateral. What property of parallelograms does this check?

3D COORDINATE GEOMETRY

three perpendicular measurements: width, depth and height.

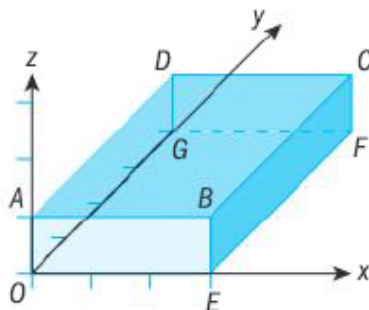
You can describe positions in 3D space using x , y and z coordinates.

The x -axis, y -axis and z -axis are perpendicular to each other.



A flat surface in 3D space is called a **plane**. The plane containing the x - and y -axes is the **x - y plane**, and the planes including the other pairs of axes are named similarly.

The cuboid below is drawn on a 3D coordinate grid.



From the origin O to point A is 0 units in the x -direction, 0 units in the y -direction, and 1 unit in the z -direction. The coordinates of A are $(0, 0, 1)$.

To find the point with coordinates $(3, 4, 1)$ start at the origin and move:

3 units in the positive x -direction to point E

4 units in the positive y -direction to point F

1 unit in the positive z -direction, to point C .

Example 1

Two points have coordinates $C(3, 4, 7)$ and $D(3, 4, -18)$.

Calculate the distance CD .

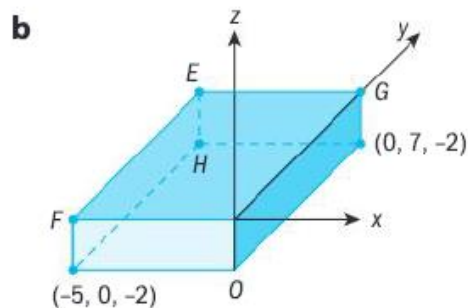
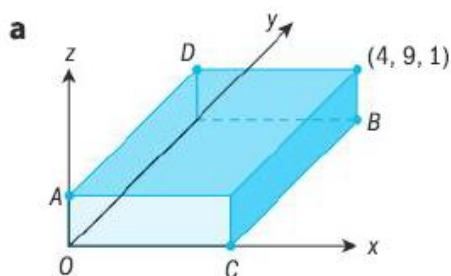
C has the same x and y coordinates as D ,
so it is vertically above D .

$$7 - (-18) = 25$$

$$CD = 25 \text{ units}$$

Practice 1

1 Write the (x, y, z) coordinates for each lettered vertex in these cuboids.



2 Find the distance between these pairs of points.

a $(1, 1, 4)$ and $(1, 1, 11)$

b $(3, 2, 0)$ and $(9, 2, 0)$

c $(-4, -2, 11)$ and $(-4, 11, 11)$

Practice 1

1 **a** $A(0, 0, 1)$, $B(4, 9, 0)$, $C(4, 0, 0)$, $D(0, 9, 1)$

b $E(-5, 7, 0)$, $F(-5, 0, 0)$, $G(0, 7, 0)$, $H(-5, 7, -2)$

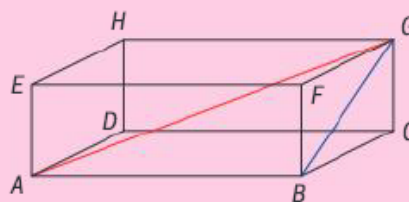
2 **a** 7

b 6

c 13

The **space diagonal** of a cuboid is the straight line joining one vertex to the opposite vertex, passing through the center of the cuboid.

A **face diagonal** is a straight line joining one vertex to another, passing through one of the faces of the shape.



AG is the space diagonal
BG is the face diagonal

In a cuboid with dimensions x , y and z , the space diagonal has length $\sqrt{x^2 + y^2 + z^2}$.

The space diagonal is also called the **volume diagonal**.

Practice 2

- Find the length of the space diagonal of these cuboids:
 - a cereal box measuring 9 cm by 12 cm by 20 cm
 - a room with floor measuring 4 m by 7 m, which is 4 m tall
 - a pencil box that is 2 cm by 7 cm by 26 cm.
- Find the length of the space diagonal of each cuboid, giving your answer to a suitable degree of accuracy:
 - a cardboard box measuring 50 cm by 15 cm by 22 cm
 - a smartphone whose dimensions are 123.8 mm by 58.6 mm by 7.6 mm
 - a storage box measuring 13 inches by 13 inches by 18 inches.
- Find the length of the space diagonal of a cube whose sides are 5 cm. Give your answer in an exact form.



Practice 2

- 25 cm
 - 9 m
 - 27 cm
- $56.648... \approx 57$ cm
 - 137.2 mm
 - $25.729 \approx 26$ inches
- $5\sqrt{3}$
- 16.6 cm

The distance formula

The distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Practice 3

- 1 Find the distance between each pair of points, giving your answer in exact form.
 - a $(0, 0, 0)$ and $(2, 3, 6)$
 - b $(5, 6, 8)$ and $(6, 8, 10)$
 - c $(23, 11, 18)$ and $(17, 17, 11)$
 - d $(-4, 3, -2)$ and $(0, 7, -9)$
 - e $(-3, 11, -16)$ and $(33, 3, -19)$
 - f $(5.5, 11, -4)$ and $(-4.5, 1, 13.5)$
- 2 a Find the distance OA , where O is the origin and A is the point $(4, 6, 12)$.
b Use a similar method to find the distance from the origin to $B(3, 14, 18)$.

Problem solving

- 3 Three points have coordinates $O(0, 0, 0)$, $A(3, 6, 6)$ and $B(4, 4, 7)$.
Show that OAB forms an isosceles triangle.
- 4 Three points have coordinates $P(0, 9, 1)$, $Q(1, 5, 9)$ and $R(5, 1, 2)$.
Show that PQR forms an isosceles triangle. Find the area of triangle PQR .

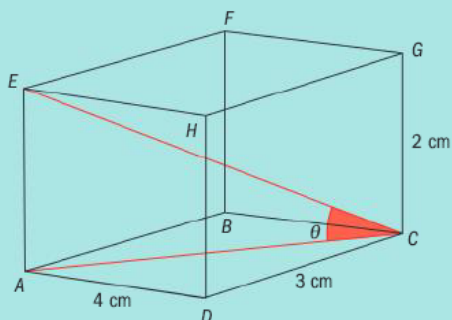
Practice 3

- 1 a 7 b 3 c 11
 d 9 e 37 f 22.5
- 2 a 14 b 23
- 3 $OA = 9$, $OB = 9$. So OAB is isosceles because it has two sides of equal length.
- 4 Area = 36.3 (3 s.f.)

Example 3



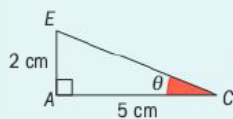
Find the angle between the diagonal CE and the plane $ABCD$ in this cuboid. Give your answer to the nearest degree.



$$AC^2 = 3^2 + 4^2 = 25$$

$$\Rightarrow AC = 5 \text{ cm}$$

Find the length AC.



Sketch the triangle ACE.

$$\tan \theta = \frac{2}{5}$$

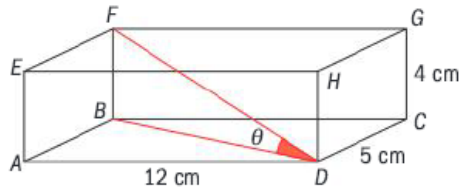
Use the tan ratio.

$$\theta = \tan^{-1}\left(\frac{2}{5}\right) = 21.801\dots^\circ$$

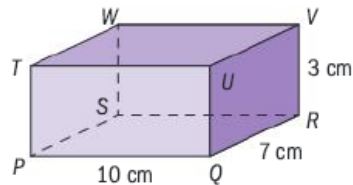
$$\theta \approx 22^\circ$$

Practice 4

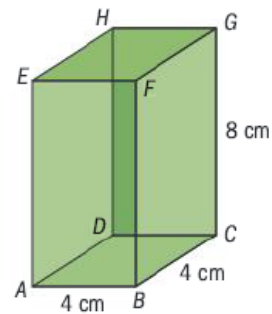
- 1 Find the angle between the diagonal FD and the plane $ABCD$ in this cuboid.
Give your answer to the nearest degree.



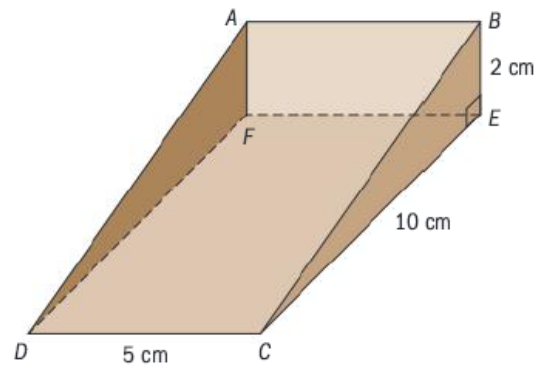
- 2 a In cuboid $PQRSTUW$, show that the length of PR is $\sqrt{149}$ cm.
b Find the angle between VP and the plane $PQRS$ correct to the nearest degree.



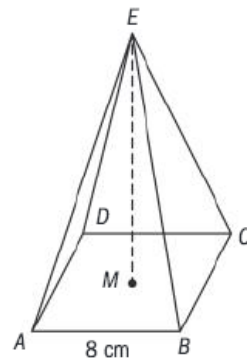
- 3 a In cuboid $ABCDEFGH$, find the angle between the diagonal BH and the plane $ABCD$ to 1 d.p.
b Hence find the angle between BH and the plane $EFHG$.



- 4 In this triangular prism, BCE and AFD are right-angled triangles. The other faces are rectangles.
a Find the length BC , correct to 3 s.f.
b Find the length DB , correct to 3 s.f.
c Find the angle that DB makes with the plane $CDEF$, correct to the nearest degree.
d Find angle BCE , the angle between planes $ABCD$ and $CDEF$, correct to the nearest degree.



- 5 $ABCDE$ is a square-based right pyramid, with $AB = 8$ cm. The point E is 8 cm directly above the point M , which is the midpoint of the base.
a Find BD .
b Hence find BM .
c Find EB .
d Find the angle between EB and the base $ABCD$. Give your answer to the nearest degree.



Practice 4

- 1 17° 2 **b** 14°
3 **a** 54.7° **b** 54.7°
4 **a** 10.2 cm **b** 11.4 cm **c** 10° **d** 11°
5 **a** $8\sqrt{2}$ **b** $4\sqrt{2}$ **c** $4\sqrt{6}$ **d** 55°
6 **a** 13.6 m **b** 4.8° 7 12.3 m

The midpoint M of the line segment joining $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given by $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$.

Practice 5

- 1 Find the midpoint of each pair of points:
a $A(0, 0, 0)$ and $B(4, 8, 16)$
b $C(1, 1, 1)$ and $D(25, 11, 7)$
c $E(-2, -6, -1)$ and $F(-4, 4, 23)$
d $G(1, 0, 8)$ and $H(8, 10, 8)$
e $I(-11, 32, 12)$ and $J(-4, -7, 5)$
f $K(1.7, -0.4, 11.8)$ and $L(2.3, 4.1, -12.1)$
- 2 Two points have coordinates $A(1, -1, 3)$ and $M(11, 3, 0.5)$. M is the midpoint of AB . Find the coordinates of B .
- 3 Three points have coordinates $P(a - 2, b, 2c)$, $Q(a + 2, b + 3, 3b)$ and $R(4a + 3, 3b, -3c)$. Q is the midpoint of PR . Find a , b and c .

Problem solving

- 4 Points A , B , C and D have coordinates $A(1, 3, 11)$, $B(-2, 11, -1)$, $C(7, 11, 7)$ and $D(10, 3, 19)$. Show that the midpoint of AC and midpoint of BD are the same. Describe the quadrilateral $ABCD$.

Practice 5

- 1** **a** (2, 4, 8) **b** (13, 6, 4) **c** (-3, -1, 11)
d (4.5, 5, 8) **e** (-7.5, 12.5, 8.5) **f** (2, 1.85, -0.15)
2 (21, 7, -2) **3** $a = 1, b = 3, c = -18$
4 $ABCD$ is a parallelogram.
5 $P(1, 7, 2), Q(3, 8, 4), R(5, 9, 6)$

The section formula

If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ where P_3 divides P_1P_2 in the ratio $m : n$, then

P_3 has coordinates $\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}, \frac{nz_1 + mz_2}{m+n} \right)$.

Example 6

Point T divides the line segment SU in the ratio 1 : 4. The coordinates of S and U are (4, -3, -8) and (24, -18, 21) respectively. Find the coordinates of T .

$$T = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}, \frac{nz_1 + mz_2}{m+n} \right)$$

$$= \left(\frac{4 \times 4 + 1 \times 24}{1+4}, \frac{4 \times (-3) + 1 \times (-18)}{1+4}, \frac{4 \times (-8) + 1 \times 21}{1+4} \right)$$

$$= (8, -6, -2.2)$$

Substitute the values into the formula.

Practice 6

- 1 Use the section formula to divide each line segment in the given ratio.
 - a Divide $A(5, 11)$ and $B(25, 27)$ in the ratio $1 : 3$.
 - b Divide $C(-4, 6)$ and $D(5, 0)$ in the ratio $1 : 2$.
 - c Divide $E(11, 4)$ and $F(1, 24)$ in the ratio $2 : 3$.
- 2 Use the section formula to divide each line segment in the given ratio.
 - a Divide $A(1, 3, -5)$ and $B(25, -9, 4)$ in the ratio $1 : 2$.
 - b Divide $C(3, -2, 6)$ and $D(15, 6, 0)$ in the ratio $3 : 1$.
 - c Divide $E(-2, -8, 3)$ and $F(12, -1, 24)$ in the ratio $4 : 3$.
- 3 Find the coordinates of a point three fifths of the way from the point $(4, 6, 7)$ to the point $(24, -4, 42)$.

Practice 6

- 1 a $(10, 15)$ b $(-1, 4)$ c $(7, 12)$
- 2 a $(9, -1, -2)$ b $(12, 4, 1.5)$ c $(6, -4, 15)$
- 3 $(16, 2, 28)$
- 4 $m : n = 10 : 15$ so let $m : n$ be $2 : 3$.
 $a = -4, b = 5$

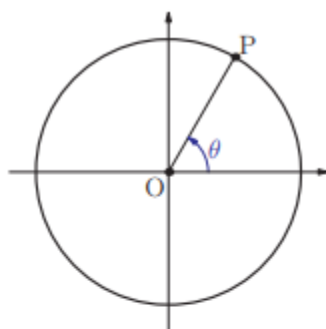
TRIGONOMETRY

3. Express the following angles in radians, giving your answers in terms of π :

- | | |
|---------------------|------------------|
| (a) (i) 135° | (ii) 45° |
| (b) (i) 90° | (ii) 270° |
| (c) (i) 120° | (ii) 150° |
| (d) (i) 50° | (ii) 80° |

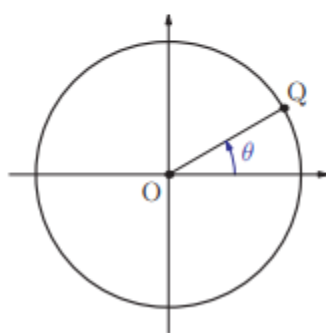
Express the following angles in degrees:

- | | |
|--------------------------|-----------------------|
| (a) (i) $\frac{\pi}{3}$ | (ii) $\frac{\pi}{4}$ |
| (b) (i) $\frac{5\pi}{6}$ | (ii) $\frac{2\pi}{3}$ |
| (c) (i) $\frac{3\pi}{2}$ | (ii) $\frac{5\pi}{3}$ |
| (d) (i) 1.22 | (ii) 4.63 |



6. The diagram shows point P on the unit circle corresponding to angle θ (measured in degrees). Copy the diagram and mark the points corresponding to the following angles.

- | | |
|------------------------------|---------------------------|
| (a) (i) $180^\circ - \theta$ | (ii) $180^\circ + \theta$ |
| (b) (i) $\theta + 180^\circ$ | (ii) $\theta + 90^\circ$ |
| (c) (i) $90^\circ - \theta$ | (ii) $270^\circ - \theta$ |
| (d) (i) $\theta - 360^\circ$ | (ii) $\theta + 360^\circ$ |



7. The diagram shows point Q on the unit circle corresponding to the real number θ .

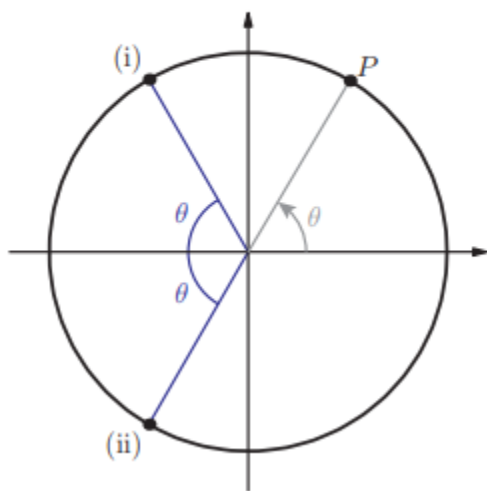
Copy the diagram and mark the points corresponding to the following real numbers.

- | | |
|----------------------------------|-------------------------------|
| (a) (i) $2\pi - \theta$ | (ii) $\pi - \theta$ |
| (b) (i) $\theta + \pi$ | (ii) $-\pi - \theta$ |
| (c) (i) $\frac{\pi}{2} + \theta$ | (ii) $\frac{\pi}{2} - \theta$ |
| (d) (i) $\theta - 2\pi$ | (ii) $\theta + 2\pi$ |

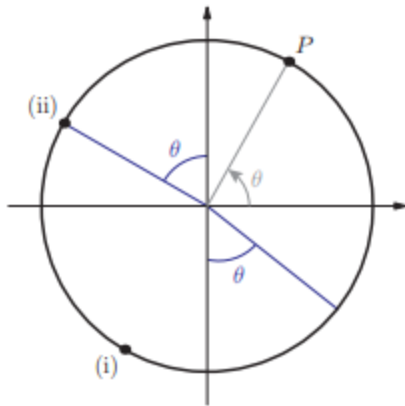
3. (a) (i) $\frac{3\pi}{4}$ (ii) $\frac{\pi}{4}$
 (b) (i) $\frac{\pi}{2}$ (ii) $\frac{3\pi}{2}$
 (c) (i) $\frac{2\pi}{3}$ (ii) $\frac{5\pi}{6}$
 (d) (i) $\frac{5\pi}{18}$ (ii) $\frac{4\pi}{9}$
4. (a) (i) 5.585 (ii) 0.349
 (b) (i) 4.712 (ii) 1.571
 (c) (i) 1.134 (ii) 2.531
 (d) (i) 1.745 (ii) 1.449
5. (a) (i) 60° (ii) 45°
 (b) (i) 150° (ii) 120°

- (c) (i) 270° (ii) 300°
 (d) (i) 69.9° (ii) 265.3°

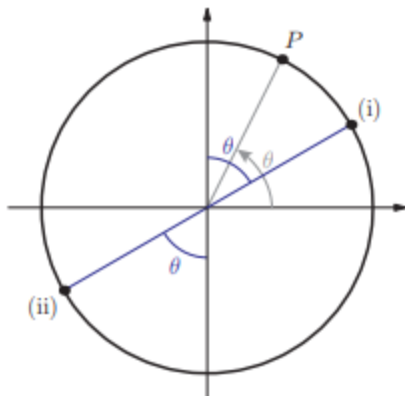
6. (a)



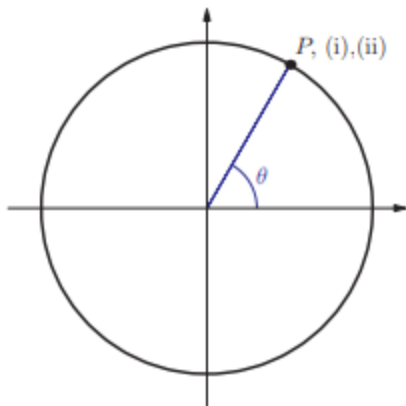
(b)



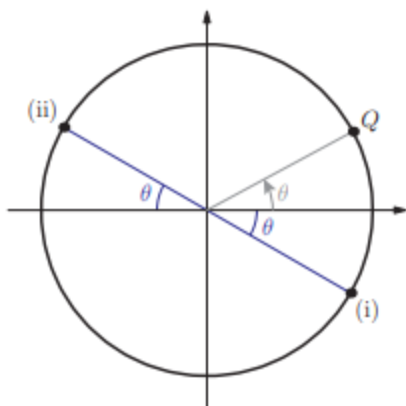
(c)

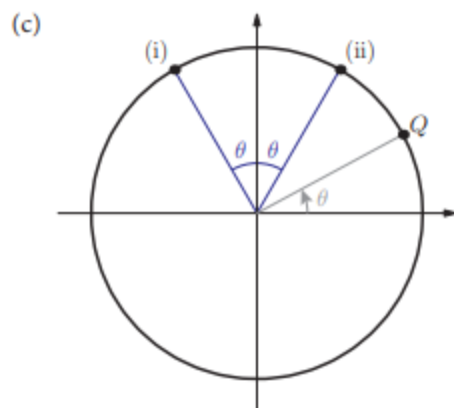
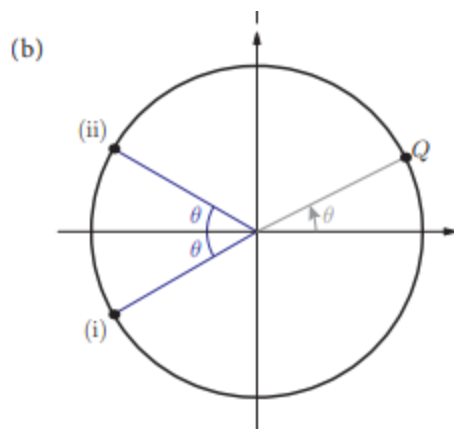


(d)



7. (a)





KEY POINT 8.5

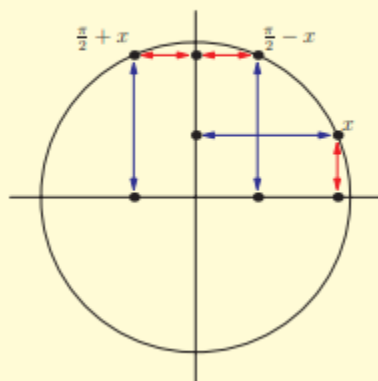
For any real number x :

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$



2. Use the unit circle to find the following values:

- (a) (i) $\sin \frac{\pi}{2}$ (ii) $\sin 2\pi$
(b) (i) $\cos 0$ (ii) $\cos(-\pi)$
(c) (i) $\sin\left(-\frac{\pi}{2}\right)$ (ii) $\cos \frac{5\pi}{2}$

3. Use the unit circle to find the following values:

- (a) (i) $\cos 90^\circ$ (ii) $\cos 180^\circ$
(b) (i) $\sin 270^\circ$ (ii) $\sin 90^\circ$
(c) (i) $\sin 720^\circ$ (ii) $\cos 450^\circ$

4. Given that $\cos \frac{\pi}{5} = 0.809$, find the value of:

- (a) $\cos \frac{4\pi}{5}$ (b) $\cos \frac{21\pi}{5}$
(c) $\cos \frac{9\pi}{5}$ (d) $\cos \frac{6\pi}{5}$

5. Given that $\sin \frac{2\pi}{3} = 0.866$, find the value of:

- (a) $\sin\left(\frac{-2\pi}{3}\right)$ (b) $\sin \frac{4\pi}{3}$
(c) $\sin \frac{10\pi}{3}$ (d) $\sin \frac{\pi}{3}$

6. Given that $\cos 40^\circ = 0.766$, find the value of:

- (a) $\cos 400^\circ$ (b) $\cos 320^\circ$
(c) $\cos(-220^\circ)$ (d) $\cos 140^\circ$

7. Given that $\sin 130^\circ = 0.766$, find the value of:

- (a) $\sin 490^\circ$ (b) $\sin 50^\circ$
(c) $\sin(-130^\circ)$ (d) $\sin 230^\circ$

Answers

- | | |
|---------------|------------|
| 2. (a) (i) 1 | (ii) 0 |
| (b) (i) 1 | (ii) -1 |
| (c) (i) -1 | (ii) 0 |
| 3. (a) (i) 0 | (ii) -1 |
| (b) (i) -1 | (ii) 1 |
| (c) (i) 0 | (ii) 0 |
| 4. (a) -0.809 | (b) 0.809 |
| (c) 0.809 | (d) -0.809 |
| 5. (a) -0.866 | (b) -0.866 |
| (c) -0.866 | (d) 0.866 |
| 6. (a) 0.766 | (b) 0.766 |
| (c) -0.766 | (d) -0.766 |
| 7. (a) 0.766 | (b) 0.766 |
| (c) -0.766 | (d) -0.766 |