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Number

1. Classify each of the following numbers as either rational or irrational. Justify your answer.

a $\frac{2}{3}$

b π

c 7.68

d $\sqrt{17}$

e $33.\overline{914}$

f 8.725555...

g $\sqrt{49}$

h 18

2. Represent each of the following rational numbers as a fraction in simplified form.

a 0.222...

b $11.\overline{68}$

c 3.1

d $2.\overline{4}$

e 2.4111...

f -0.862

g $3.\overline{043}$

h 7.40111...

3. Our current number system, the Hindu–Arabic numeral system, was developed because humans required an efficient way to represent the quantities they were working with. Several mathematicians from India are credited with the development of the place-value system and the introduction of the number zero in the 5th and 6th centuries. Represent each of the following place values as a power of 10.

BILLIONS			MILLIONS			THOUSANDS			ONES			DECIMALS				
hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten thousandths	hundred thousandths

Evaluate each of the following. Write your answer as either an integer or a simplified fraction.

$$\begin{array}{llll} \mathbf{a} \ 5^{-3} & \mathbf{b} \ -3^{-3} & \mathbf{c} \ (-27)^{\frac{1}{3}} & \mathbf{d} \ \left(\frac{3}{4}\right)^2 \\ \mathbf{e} \ \left(\frac{2}{5}\right)^{-1} & \mathbf{f} \ \left(\frac{4}{25}\right)^{-\frac{1}{2}} & \mathbf{g} \ \left(\frac{1}{64}\right)^{-\frac{1}{3}} & \mathbf{h} \ \left(\frac{6}{7}\right)^{-2} \end{array}$$

Simplify each of the following. Express your answers with positive exponents only.

$$\begin{array}{lll} \mathbf{a} \ a^3b^2 \times a^4b^{-5} & \mathbf{b} \ (28x^2y^{-3})^0 & \mathbf{c} \ -(14m^2n^3)(-2m^3n^2) \\ \mathbf{d} \ \frac{9a^2}{(-3a)^2} & \mathbf{e} \ (6x^5y^{-2})(3x^2y^3) & \mathbf{f} \ \frac{-14x^6y^7}{7x^{-3}y^9} \\ \mathbf{g} \ (6x^4y^3)(-4x^{-8}y^{-2}) & \mathbf{h} \ (15x^{4c})(7x^{-6c}) & \mathbf{i} \ \frac{20n^4m^{-3}}{8n^8m^{-5}} \end{array}$$

Find the following products and quotients. Write your answers using scientific notation.

$$\begin{array}{ll} \mathbf{a} \ (3 \times 10^5) \times (4 \times 10^8) & \mathbf{b} \ (2.4 \times 10^9) \div (1.2 \times 10^6) \\ \mathbf{c} \ (2.5 \times 10^{-4}) \times (3.1 \times 10^{-3}) & \mathbf{d} \ \frac{9.2 \times 10^3}{4.2 \times 10^6} \\ \mathbf{e} \ (8.1 \times 10^{-2}) \div (6.8 \times 10^{-7}) & \mathbf{f} \ (6.2 \times 10^{11}) \times (4.9 \times 10^{-13}) \end{array}$$

Our understanding of our own place in the universe has developed over time, often influenced by personal beliefs rather than scientific evidence. While Nicolaus Copernicus is credited with formulating the current model of our solar system, a Greek astronomer, Aristarchus, promoted the idea of the Sun being at the center of our universe about 1800 years earlier. Johannes Kepler was the first to propose the laws of planetary motion that are still in use today. The distance between the Earth and Mars is constantly changing as the planets rotate about the Sun. The smallest distance between them is 54.6 million km and the greatest distance between them is 401 million km. The fastest spaceship to leave Earth was NASA's New Horizons with a recorded speed of 58000 km/h. Use this information to answer the following questions.

a. Calculate the minimum number of days the spaceship would take to travel from Earth to Mars when the planets are at their closest to each other.

b. Calculate the minimum number of days the spaceship would take to travel from Earth to Mars when the planets

are at their furthest apart.

8. Find the following sums and differences. Write your answers using standard form.

a $(6 \times 10^8) - (2 \times 10^7)$

b $(5.5 \times 10^{-2}) + (3.1 \times 10^{-4})$

c $(7.3 \times 10^{-4}) - (8.6 \times 10^{-3})$

d $(6.27 \times 10^3) + (5 \times 10^4)$

e $(9.1 \times 10^{11}) + (4.4 \times 10^{13})$

f $(5 \times 10^9) - (2.7 \times 10^7)$

9. The Human Genome Project began in 1990 with the goal of discovering and recording the complete sequence of DNA base pairs in human genetic material. It was a 13-year project that was almost completely successful, mapping over 99% of the human genome. The table below gives the number of base pairs mapped during each three-year period from 1990 to 1999.

Years	Number of base pairs mapped in each period
1990–1993	2.0×10^6
1993–1996	1.4×10^8
1996–1999	4.7×10^9

- Find the total number of base pairs that were mapped between 1990 and 1999.
- If there are 3.2×10^{10} base pairs in the human genome, find the number of base pairs that were mapped in the final four years of the project.

10. Simplify each of the following. Express your answers with positive exponents only.

a $\left(\sqrt{49p^{26}q^{10}r^{-12}}\right)$ **b** $\frac{\left(2u^{\frac{2}{3}}v^{-2}\right)^{-6}}{(uv^3)^2}$ **c** $\frac{(a^5b^{-6})^{-2}}{(a^0b^7)^3}$

d $\left(\frac{x^2y^6}{a^{-2}b}\right)^2\left(\frac{x^3y^0}{ab}\right)^{-3}$ **e** $(16a^4b^{-2}c^6)^{\frac{1}{2}}(-27a^{-6}b^9c^3)^{\frac{1}{3}}$

f $\left(\frac{36m^3n^0}{121m^{-4}n^{-5}}\right)^{\frac{1}{2}}$ **g** $(25g^{-5}h^{-1}j^7)^{\frac{1}{2}}(-8g^4h^{-2}j^{-1})^{\frac{1}{3}}$

11. Neuroscience is the study of the nervous system, with neuroscientists focusing on the brain and its structure. It is a discipline that has developed since 500 BC, with major advances being made since the middle of the 20th century. While the brain has a high composition of water and fat, it also contains a large number of neurons that transmit and receive signals.

The cerebellum contains roughly 69 billion neurons. The cerebral cortex, made up of the frontal, parietal, occipital and temporal lobes, contains roughly 16 billion neurons. The remaining structures contain 690 million neurons.

- a. How many neurons are there in the brain in total? Express your calculations and answer in scientific notation.
- b. It has often been said that the number of neurons in the brain is the same as the number of stars in the universe. If there are 7.0×10^{22} stars in the universe, is this statement true? If not, which quantity is greater and by what factor?
- c. Each neuron has a length of approximately 1×10^{-4} meters. If they were placed end to end, what would be the length of all of the neurons in the brain?

12. The Richter Scale was devised by Charles Richter in 1940 to compare the intensities of earthquakes. The intensity of an earthquake is determined by the amount of ground motion measured on a seismometer. Each increase of 1 unit in magnitude on the Richter scale corresponds to an increase of 10 times the intensity measured on a seismometer.

- a. Using this ratio, how many times more intense was the 1556 earthquake in China with a magnitude of 8 compared with the 2010 earthquake in Haiti with a magnitude of 7?

By expressing the intensity (I) as an exponential function of the magnitude (M), you can compare the intensities of earthquakes that do not differ by a whole integer.

$$I = 10^M$$

- b. The world's most powerful earthquake was in Chile in 1960 and registered magnitude 9.5 on the Richter Scale. The deadliest recorded tsunami was caused by an earthquake which registered magnitude 9.1 on the Richter scale off the coast of Indonesia in 2004. Using the formula given above, how many times more intense was the earthquake in Chile than the one in Indonesia?
- c. The two most costly earthquakes both occurred in Japan: the earthquake of 2011 had a magnitude of 9.1; the earthquake of 1995 had a magnitude of 6.9. Using the formula given above, how many times more intense was the earthquake in 2011 compared with the one in 1995?

Answers:

- 1 a Rational b Irrational c Rational d Irrational
e Rational f Rational g Rational h Rational
- 2 a $\frac{2}{9}$ b $\frac{1157}{99}$ c $\frac{31}{10}$ d $\frac{22}{9}$
e $\frac{217}{90}$ f $-\frac{431}{500}$ g $\frac{3013}{990}$ h $\frac{6661}{900}$
- 3 $10^{11}, 10^{10}, 10^9, 10^8, 10^7, 10^6, 10^5, 10^4, 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}$
- 4 a $\frac{1}{125}$ b $-\frac{1}{27}$ c -3 d $\frac{9}{16}$
e $\frac{5}{2}$ f $\frac{5}{2}$ g 4 h $\frac{49}{36}$
- 5 a $\frac{a^7}{b^3}$ b 1 c $28m^5n^5$ d 1 e $18x^7y$
f $-\frac{2x^9}{y^2}$ g $-\frac{24y}{x^4}$ h $\frac{105}{x^{2c}}$ i $\frac{5m^2}{2n^4}$
- 6 a 1.2×10^{14} b 2×10^3 c 7.75×10^{-7} d 2.190476×10^{-3}
e 1.19117647×10^5 f 3.038×10^{-1}
- 7 a 39.2 days b 288 days
- 8 a 5.8×10^8 b 5.531×10^{-2} c -7.87×10^{-3} d 5.627×10^4
e 4.491×10^{13} f 4.973×10^9
- 9 a 4.842×10^9 b 2.7158×10^{10}
- 10 a 8.569×10^{10} b Stars greater, by factor 8.17×10^{11} c 8.569×10^6 m
- 11 a $\frac{7p^{13}q^5}{r^6}$ b $\frac{v^6}{64u^6}$ c $\frac{1}{a^{10}b^9}$ d $\frac{y^{12}a^7b}{x^5}$
e $-\frac{4a^4c^2}{3b^4}$ f $\frac{6m^{\frac{7}{2}}n^{\frac{5}{2}}}{11}$ g $-\frac{10j^{\frac{19}{6}}}{2g^{\frac{7}{6}}h^{\frac{6}{6}}}$
- 12 a 10 b $10^{0.4} (\approx 2.51)$ c $10^{2.2} (\approx 158)$

13. Mathematical puzzles have been developed over the centuries, dating as far back as Ancient Egypt. More contemporary versions include Rubik's Cube and Sudoku. The Tower of Hanoi is a puzzle invented by Edouard Lucas in 1883 that is based on a legend. According to the legend, there is a Hindu temple that contains three large posts and 64 different gold disks. All the disks are on one post, ordered in size with the largest disk at the bottom and the smallest on top. The temple priests must move the disks one at a time from one post to another, never placing a larger disk on top of a smaller one. The puzzle is completed (and the world ends!) when all of the disks are again stacked on another post in order from largest (on the bottom) to smallest (on top). How many moves would it take to complete the puzzle?

- What is the minimum number of moves you need to transfer three disks from the first post to the last, following the rule of never placing a larger disk on top of a smaller one?
- Gradually increase the number of disks as you solve the puzzle and record your results in a table like the one below.

Number of disks	Minimum number of moves
3	
4	
5	
6	

- Based on the pattern in your table, what is the minimum number of moves required for the priests to move all 64 of the disks to the last pole? Write your answer using an exponent.
- Use exponents to write a rule for the minimum number of moves required to solve the puzzle with n disks.
- If there were no disks, show that your rule correctly predicts the minimum number of moves necessary to solve the puzzle.

14. In 1800, a group of 25 astronomers calling themselves the Celestial Police were searching for a planet whose existence was predicted by the astronomer Johann Titius. Around the same time, another astronomer, Giuseppe Piazzi, discovered the first asteroid in a location now known as the asteroid belt. The asteroid belt is a ring of rocks and debris left over from the formation of the universe, located between Mars and Jupiter. Its average distance from the Sun is approximately 415 000 000 km.

- 415 000 000 can be represented in a variety of ways. Copy the table and fill in the missing values.

415 000 000 using products	415 000 000 using powers of 10
41 500 000 \times ____	41 500 000 $\times 10^1$
4 150 000 \times ____	
____ $\times 1000$	
____ $\times 10\,000$	
	4150×10^5
	415×10^6
$41.5 \times$ ____	
$4.15 \times$ ____	

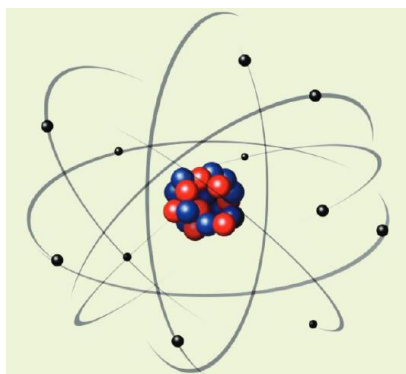
- b. Piazzzi named that first asteroid Ceres and it is now known to be the largest asteroid in the asteroid belt. It measures approximately 950 000 meters in diameter. Use the same procedure as in step a to represent this quantity in a variety of ways.

15. The idea that all matter is composed of atoms is an incredibly important development. Before atomic theory, people had a variety of beliefs, such as the idea that all objects were made of some combination of basic elements: earth, air, fire and water. Atomic theory helps to explain the

different phases of matter (solid, liquid, gas) and it allows you to predict how materials will react with each other. However, even the theory of the atom and its structure has developed over time, owing to important discoveries of the particles that make up an atom.

The current model of the atom includes three types of particle: the nucleus contains neutrons and protons and is surrounded by electrons that orbit around it.

Neutrons and protons have roughly equivalent masses that can be written as 1.66×10^{-27} kg. Using the same power of 10, electrons have a mass of 9.11×10^{-31} kg.



Atomic particles

- Represent each of these masses using correct scientific notation.
- Which particle has the smallest mass? Explain how you know.

Current elements

For each of the following questions, perform all of your calculations using scientific notation and show your working. Express your answers in correct scientific notation.

- How many times greater is the mass of a proton than the mass of an electron?
- Oxygen is an element that was discovered in 1772 by Carl Wilhelm Scheele. One atom of oxygen has 8 electrons, 8 protons and 8 neutrons. Calculate the mass, in kg, of one atom of oxygen (the atomic mass). Show your working.
- The element carbon-14, which is used in dating very old objects, was discovered in 1940 by Martin Kamen and Sam Ruben. Its atomic mass is 2.324728×10^{-26} kg. If one atom of carbon-14 has 6 electrons and 6 protons, find the number of neutrons in one atom of the element. Show your working.
- Because individual atoms are so small, they are often grouped together in a larger amount, called a mole. The number of atoms in a mole of a substance is 6.02×10^{23} ,

which is referred to as Avogadro's number. If one mole of water has a mass of 18g, find the number of atoms in 1000 grams (1 liter) of water.

- g. The distance from Earth to Mars is 54.6 million kilometres. A helium molecule has a length of approximately 280 picometers. Find how many helium molecules could fit between these two planets.

950 000 using products	950 000 using powers of 10
95 000 × ____	95 000 × 10 ⁷
9500 × ____	
	9.5 × 10 ⁷

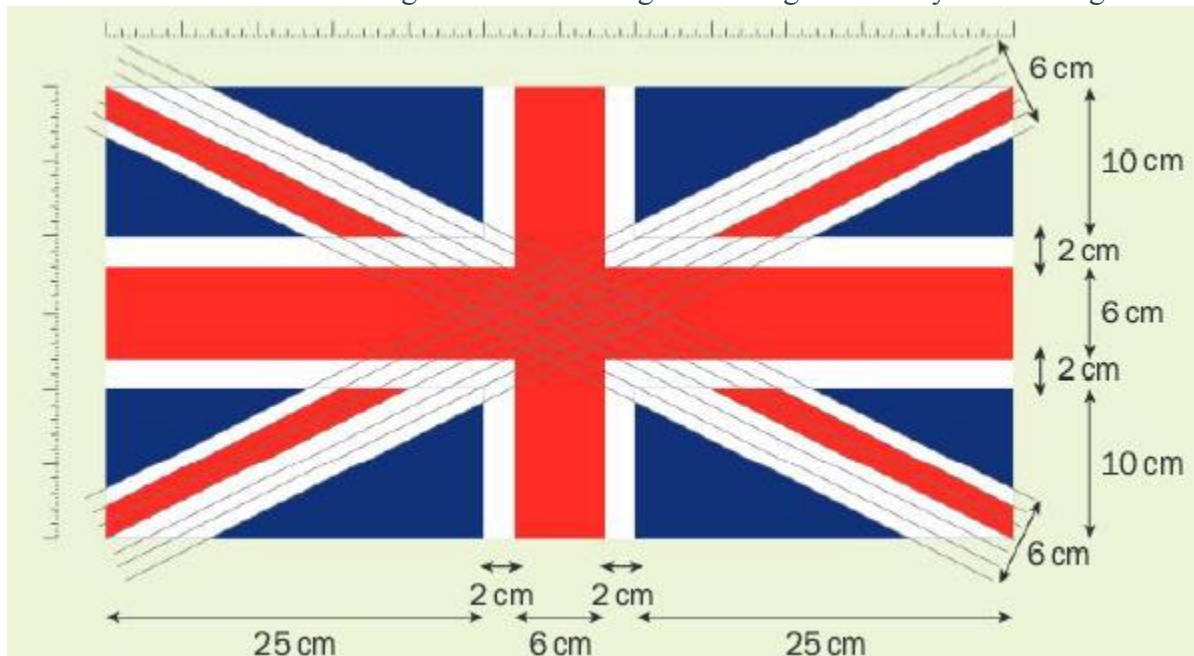
- c. Look at the powers of 10 representation in the last row of each table. These are both examples of scientific notation or standard form. Describe the components of a quantity represented in scientific notation.
- d. Write down a general rule for writing a large number in scientific notation.
- e. The Celestial Police discovered two large asteroids in the asteroid belt: Juno and Vesta. Vesta has a mass of 259 quintillion grams. At its closest, Juno is 297000000000 meters from the Sun. Verify your rule by writing each of these quantities in scientific notation.
- f. Justify why your rule works.

Triangles

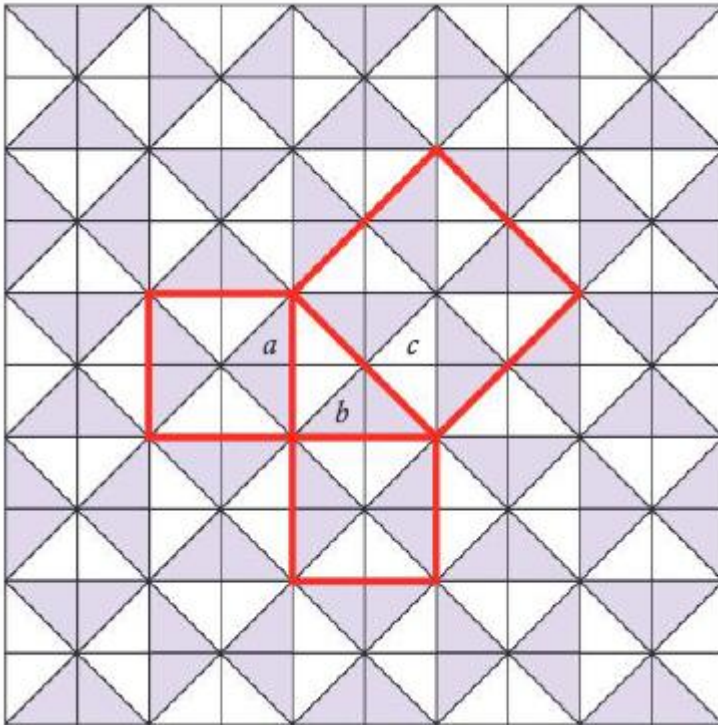
Practice Question: How do you solve the problem of creating a flag for three countries that already have their own individual flags? Add them all together! The Union Jack combines the flags of three of the countries that came together to form the United Kingdom: the English Cross of St George, the Scottish Cross of St Andrew and the Irish Cross of St Patrick. (The Welsh flag was not included, as Wales was considered a principality rather than a separate kingdom.)



Using your knowledge of similar triangles, Pythagoras' theorem and trigonometry, find the measurements of the blue triangles. Give side lengths and angles. Show your working.

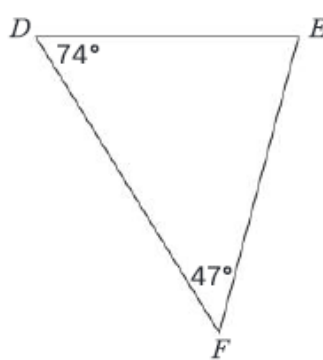
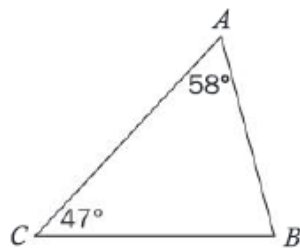


1. Explain how the diagram below can be used to prove Pythagoras' theorem.

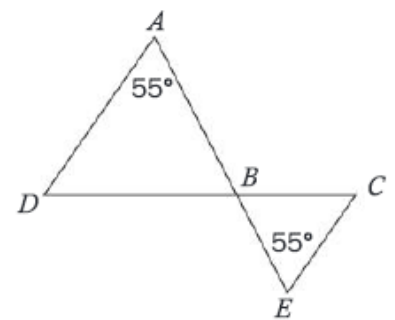


2. Are the following pairs of triangles similar? If they are, write down the similar triangles and the similarity postulate used.

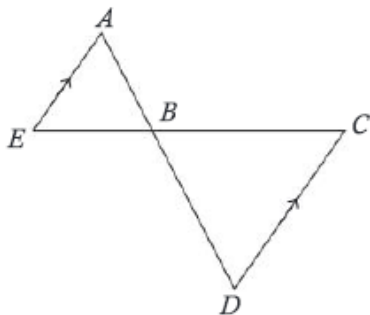
a



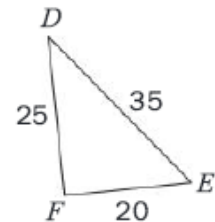
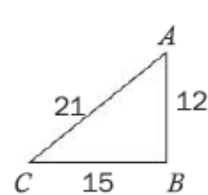
b



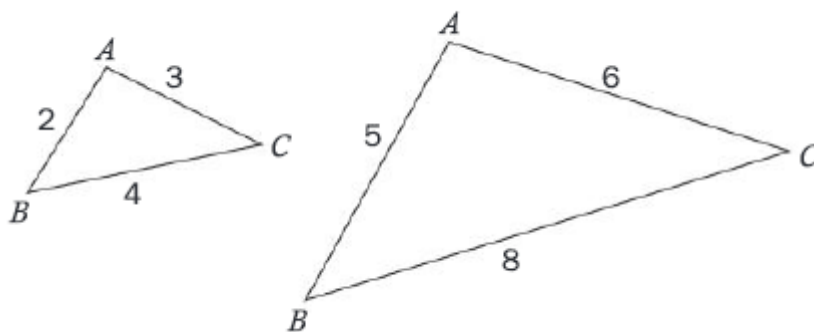
c



d



e



3. Find the value of the following trigonometric ratios. Round your answers to the nearest hundredth.

a $\tan 50^\circ$

b $\sin 38^\circ$

c $\cos 75^\circ$

d $\sin 8^\circ$

e $\cos 71^\circ$

f $\tan 26^\circ$

4. Find the measure of the angle with the given trigonometric ratio. Round your answers to the nearest tenth of a degree.

a $\cos A = \frac{5}{8}$

b $\tan A = \frac{4}{3}$

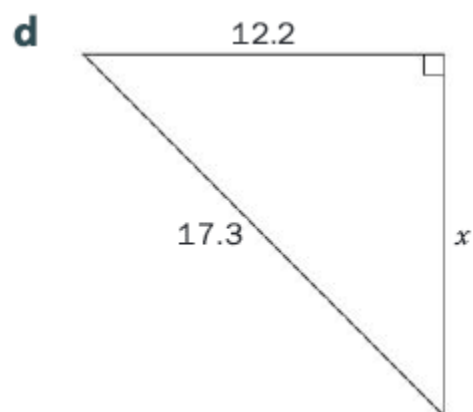
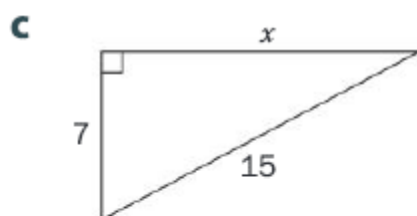
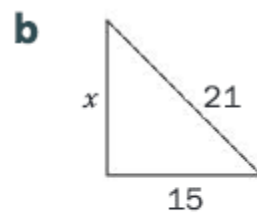
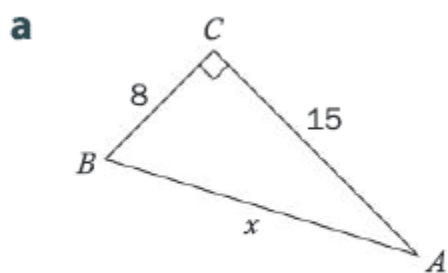
c $\sin A = \frac{4}{9}$

d $\tan A = 1.15$

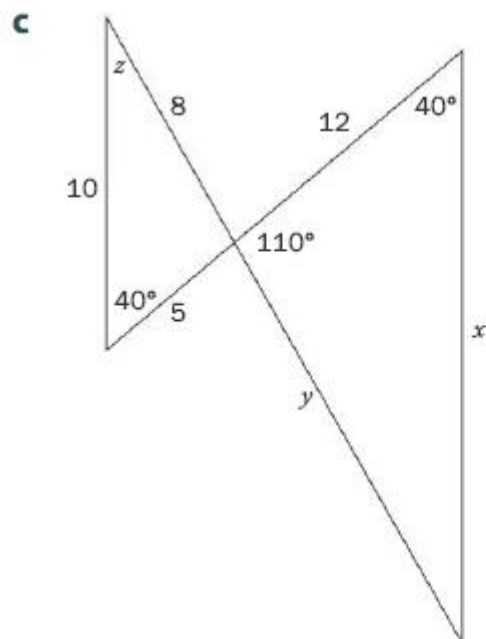
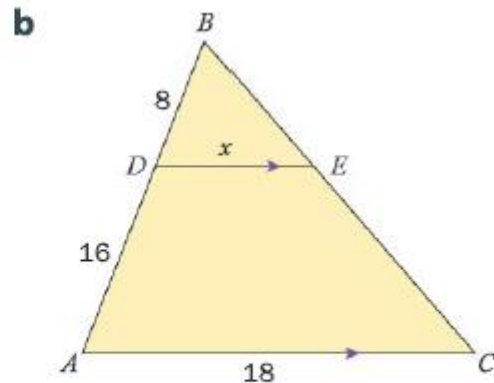
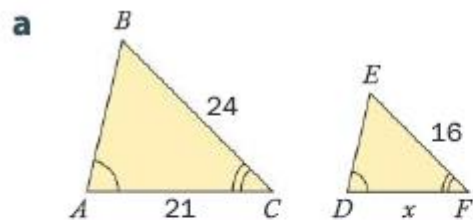
e $\sin A = 0.89$

f $\cos A = 0.71$

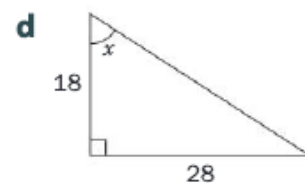
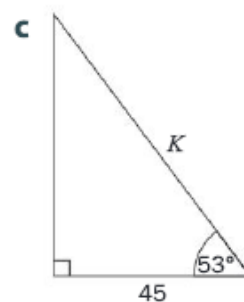
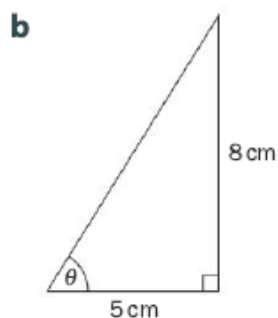
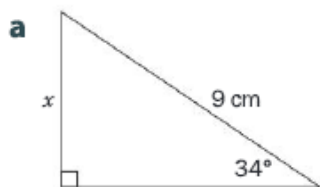
5. Find the size of the unknown side in each triangle.



6. Write down the similarity postulate that proves the triangles in each pair are similar. Then find the missing measurement. Round your answers to 3 s.f.



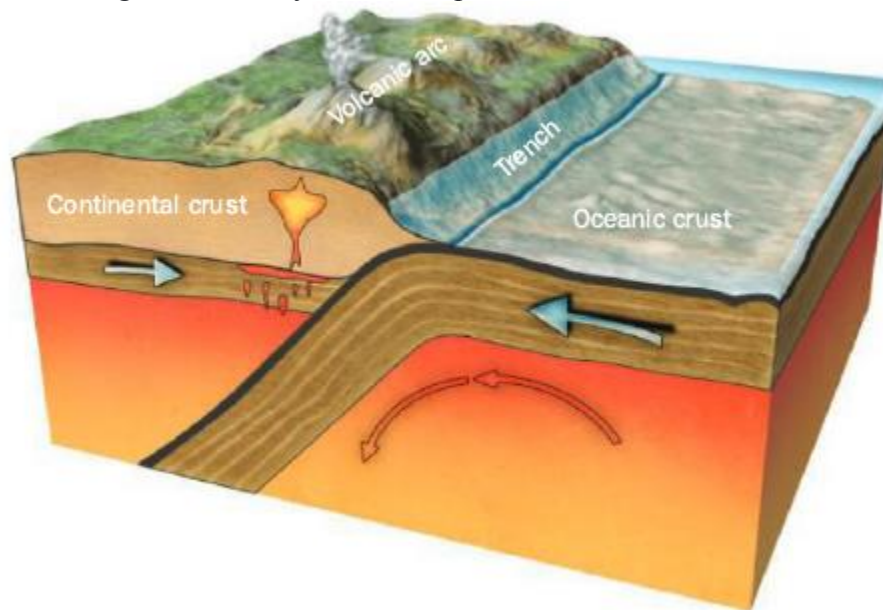
7. Find the value of the indicated measure in each of the following right triangles.



8. In ancient Egypt, when the Nile flooded, it often erased the boundaries between landowners. An accurate surveying process was necessary in order to re-establish property lines. Land surveyors used Pythagoras' theorem to verify that property lines were correctly drawn by calculating how long they should be.

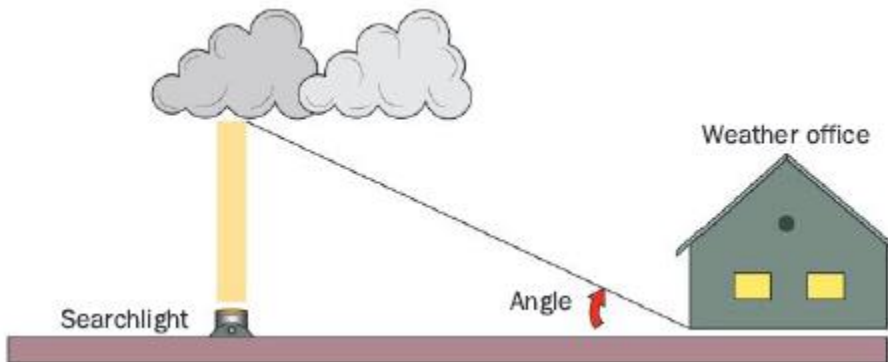
Two farmers occupy a property that is in the shape of a rectangle. Their property line is the diagonal of the rectangle. Find the length of the property line if the sides of the property measure 48m and 55m.

9. Find the distance between each pair of points.
 - a. $(0, -2)$ and $(-3, 6)$ c. $(-5, 1)$ and $(3, -8)$
 - b. $(4, 4)$ and $(-2, 8)$ d. $(-10, -3)$ and $(-7, 1)$
10. Subduction is a process where a heavier oceanic plate plunges underneath a lighter continental plate. The process can be either deep or shallow, depending on the angle of subduction. In very old subduction zones, a mountain range, called a volcanic arc, is formed directly above the point where the heavier plate reaches a depth of 100km. A deep trench along the boundary of the two plates is also formed.



- A volcanic arc is located a horizontal distance of 250km from the trench.
- a. Draw a diagram and indicate any relevant information.
 - b. Find the angle of subduction. Round your answer to the nearest tenth of a degree.
11. An isosceles triangle has a base of 6cm and its equal sides are 5cm long. Calculate:
 - a. the perpendicular height of the triangle
 - b. the area of the triangle.
 12. The owners of a house want to replace the steps leading upto their back porch with a ramp. The porch is 1 meter off the ground, and to comply with building regulations, the ramp must make an angle of 4.8 degrees with the ground.
 - a. Find how long the slope of the ramp will be. Show your working.
 - b. Find the horizontal distance between the porch and the start of the ramp. Show your working.

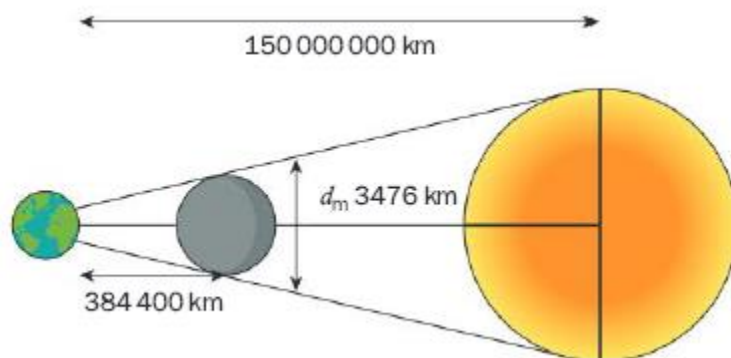
13. Airport meteorologists keep an eye on the weather to ensure the safety of airplanes. One thing they watch is the cloud ceiling. The cloud ceiling is the lowest altitude at which solid cloud is visible. If the cloud ceiling is too low, the planes are not allowed to take off or land.



One process a meteorologist can use to find the cloud ceiling at night is to shine a searchlight that is located a fixed distance from her office vertically into the clouds. Then she measures the angle from the office to the spot of light on the cloud. The searchlight is located 40m from the office.

- If the angle to the spot of light on the cloud is 38 degrees, how high is the cloud ceiling?
- The minimum height of the cloud ceiling for planes to safely take off or land is 305m. What is the angle to the spot of light on the cloud when the cloud ceiling is at the minimum height?

14. During a solar eclipse, the Moon passes between the Sun and Earth and covers the Sun almost entirely, despite being considerably smaller. How is this possible?



- Label the triangles. Write down the triangles that are similar and indicate the similarity postulate that you used.
- On average, the Earth is 384400km from the Moon and 150000000km from the Sun. The diameter of the Moon is 3476km while the diameter of the Sun is 1400000km. Use this information to determine why the Moon covers almost all of the Sun during an eclipse.

15. A television manufacturer made the following claim in an advertisement:
 “Our 80-inch AQUOS TV delivers more than double the screen area of a 55-inch TV, for an amazing viewing experience.”

- a. Show that an 80-inch television can deliver more than double the screen area of a 55-inch television.
- b. Show that this claim is true even if both televisions have an aspect ratio of 16:9. (Aspect ratio is length : width)



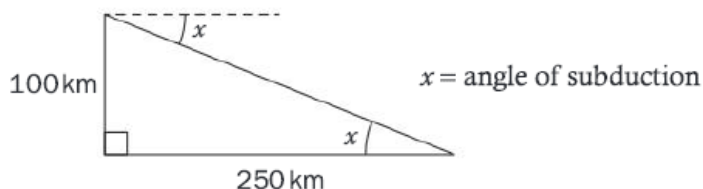
16. In order to describe the locations of celestial bodies, astronomers use a three-dimensional coordinate system with our Sun at the origin $(0, 0, 0)$. All other objects are given a coordinate in the form (x, y, z) .
 - a. Write down the distance formula for a pair of 3-dimensional coordinates.
 - b. Find the distance between our Sun and each of the following stars (distances given in light years). Round your answers to the nearest tenth.
 - i. Polaris $(99.6, 28.2, 376.0)$
 - ii. Alpha Centauri $(-1.8, 0.0, 3.9)$
 - iii. Sirius $(-3.4, -3.1, 7.3)$
 - c. Find the distance from Polaris to Alpha Centauri.

Answers:

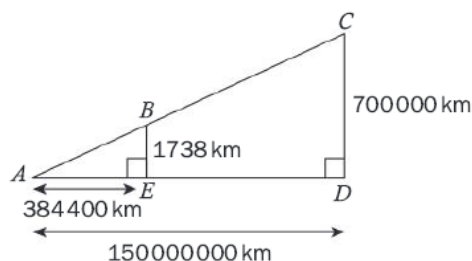
- 1 Individual response
- 2
 - a No, the triangles are not similar
 - b Yes, the triangles are similar (corresponding angles are equal)
 - c Yes, the triangles are similar (corresponding angles are equal)
 - d Yes, the triangles are similar (corresponding sides are proportional)
 - e No, the triangles are not similar
- 3

a 1.19	b 0.62	c 0.26	d 0.14	e 0.33	f 0.49
--------	--------	--------	--------	--------	--------

- 4 a 51.3° b 53.1° c 26.4° d 49.0°
e 62.9° f 44.8°
- 5 a 17 b $6\sqrt{6}$ c 13.3 (to 3 s.f.) d 12.3 (to 3s.f.)
- 6 a Postulate AAA 14 b Postulate AAA 6 c Postulate AAA
 $x = 15$
 $y = 7.5$
 $z = 30$
- 7 a 5.03 cm to 3s.f. b 58.0° to 3s.f. c 74.8 cm to 3s.f. d 57.3° to 3s.f.
- 8 73 m
- 9 a $\sqrt{73}$ b $\sqrt{145}$ c $2\sqrt{13}$ d 5
- 10 a b 21.8°

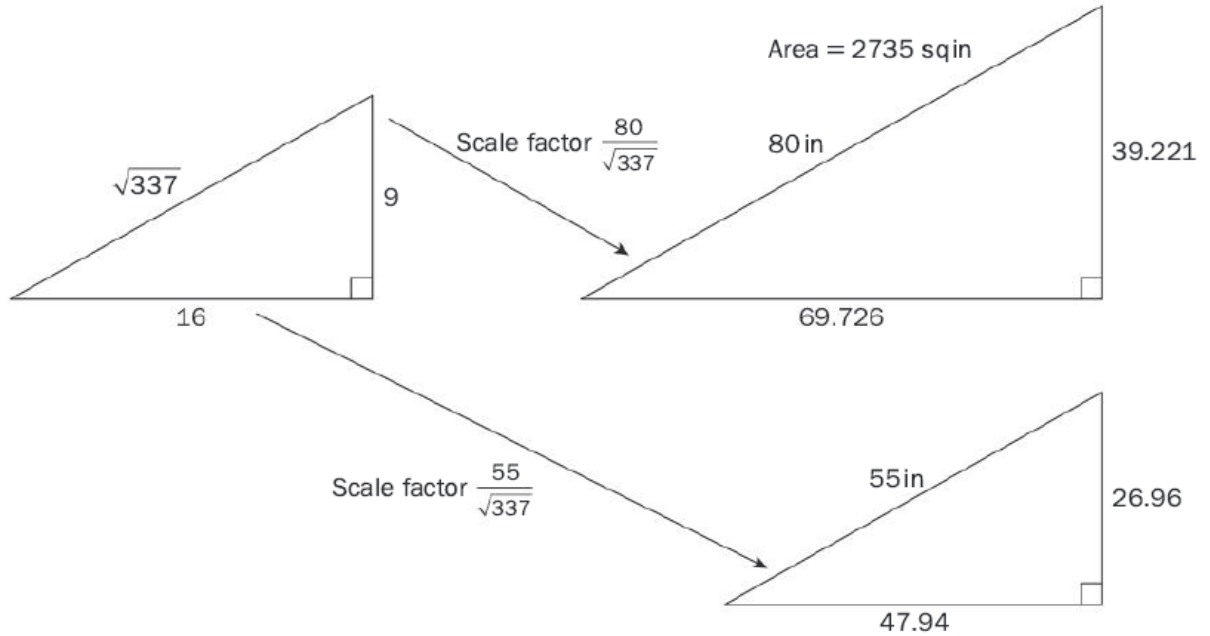


- 11 a 4 cm b 12 cm^2
- 12 a 12.0 m to 3s.f. b 11.9 m to 3s.f.
- 13 a 31.3 m to 3s.f. b 82.5° to 3s.f.
- 14 a



- $\triangle ABE \sim \triangle ACD$; AA postulate;
- b The angle enclosed by the lines from the top of the Sun to the earth and the Earth the centre of the Sun is 0.53° to the nearest hundredth. The corresponding angle for the Moon is 0.52° . Therefore, the majority of the sunlight will be blocked by the Moon
- 15 a Individual response (any suitable rectangle with diagonals measuring 80 and 55, such that the area of the larger rectangle is more than double the area of the smaller rectangle)

b



So the 80-inch TV has an area that is more than double the area of the 55-inch TV

16 a Let the pair of points be (x_1, y_1, z_1) and (x_2, y_2, z_2)

Then the distance between the points is

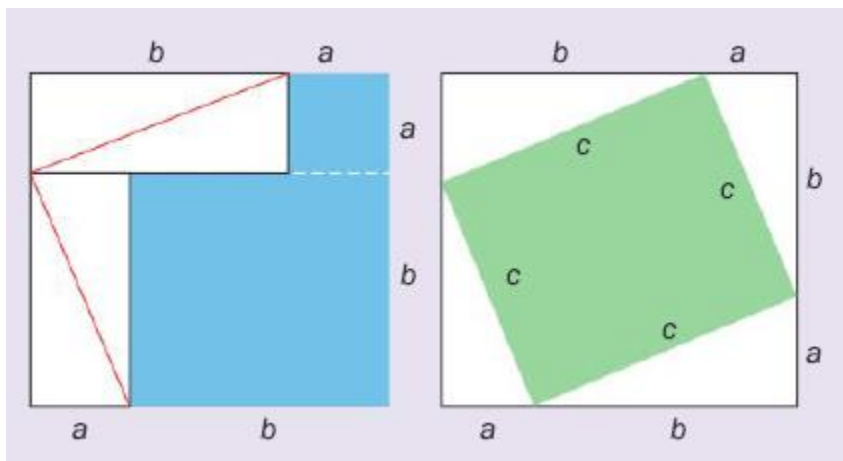
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

b i 390.0 light years **ii** 4.3 light years

iii 8.6 light years

c 386.7 light years

17. For the diagram below:



- Explain how you know that both of the large squares have the same area.
- Write down an expression to calculate the area of the green square in the diagram on the right.

- c. Write down an expression to calculate the area of each of the two blue squares in the diagram on the left.
- d. How do you know that the sum of the areas of the blue squares equals the area of the green square? Explain.
- e. How does this prove Pythagoras' theorem? (The theorem states, "In any right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the remaining two sides.")

Linear Relationships

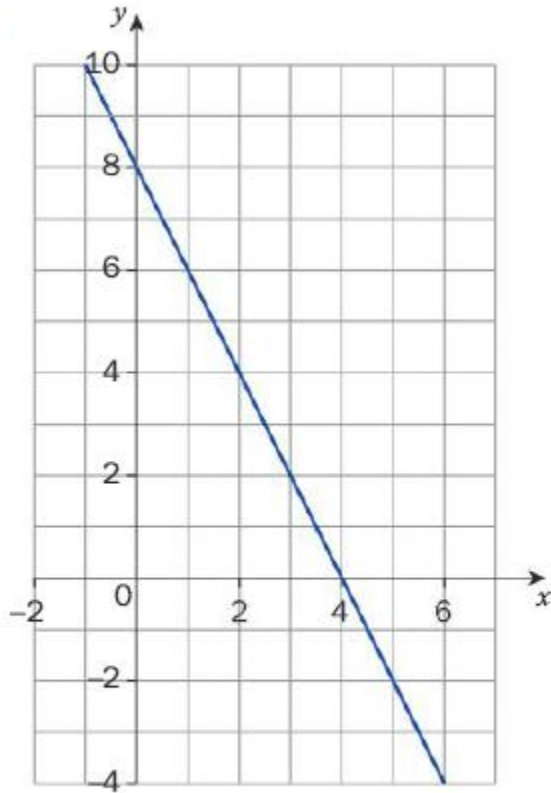
1. State the slope and the y-intercept of each of the following lines.

a $y = 6x - 12$

b $y = -\frac{2}{7}x + 23$

2. Determine the x- and y-intercepts of the following lines.

a



b $5x - 4y = 20$

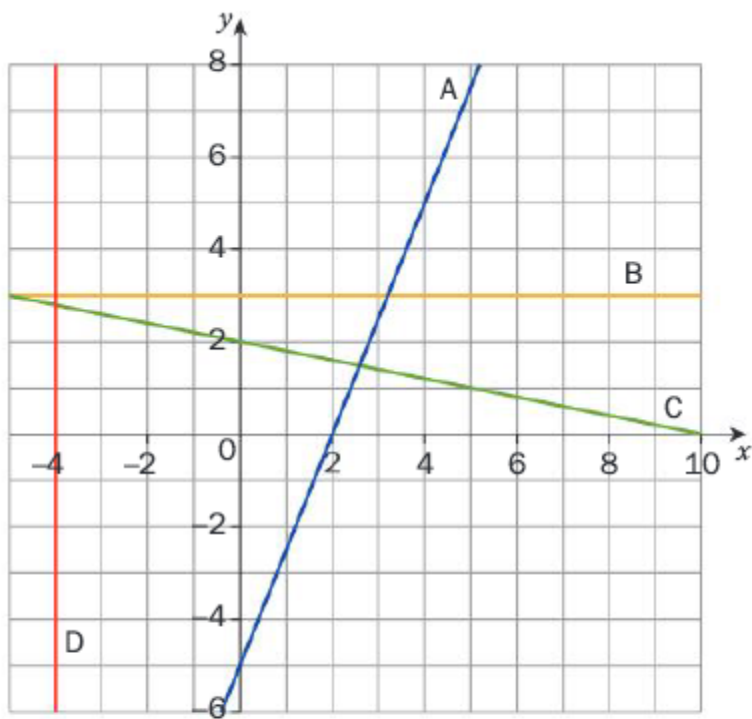
c $7x + 3y = 42$

3. Plot the graph of each of the following lines using a method of your choice.

a $y = -2x + 8$

b $y = x - 7$

4. Copy and complete the table for lines A, B, C and D.



Line	Slope	y-intercept	x-intercept	Equation
A				
B				
C				
D				

5. Represent each of the following equations using gradient– intercept form.

a $3y = \frac{1}{2}x - 12$

b $5x = 10y - 20$

c $x = 8y$

d $\frac{1}{2}x = 3y - 2$

e $\frac{2}{3}y - 8x = -6$

6. Find the gradient of the line L that passes through each pair of points.

a. A(1, 5) and B(1, -2) b. A(-1, 6) and B(2, 9)

c. A(0, 4) and B(2, 8) d. A(3, 2) and B(5, 4)

7. A tank has a slow leak in it. The water level starts at 100cm and falls 0.5cm a day.

a. Is this a constant increase or constant decrease situation?

b. Write down an equation showing the relationship between day, d, and water level, L.

c. After how many days will the tank be empty?

8. Graph each of the following lines using a method of your choice.

a $6y + 3x = 12$

b $10y - x = -50$

c $2y + \frac{1}{3}x = 10$

d $2y + 3 = \frac{x}{4}$

e $0 = 24 + 8x + 3y$

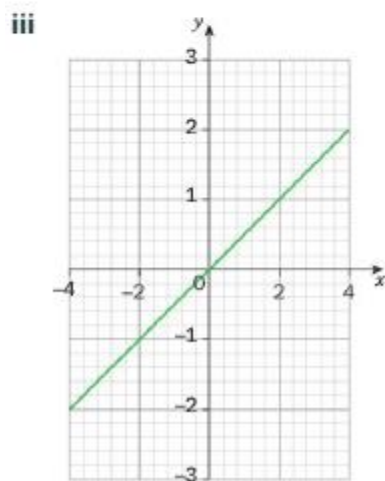
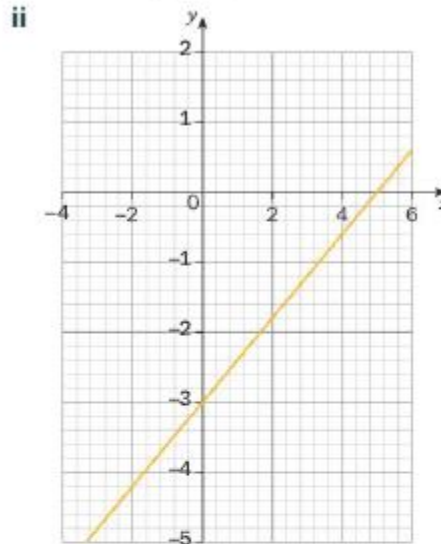
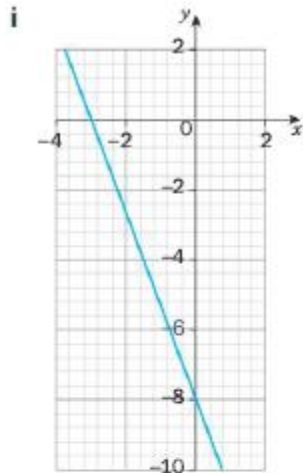
f $4x = \frac{10 + 2y}{3}$

9. Match each of the following equations to its graph.

a $3x - 5y = 15$

b $y = \frac{1}{2}x$

c $\frac{4}{3}x + \frac{1}{2}y + 4 = 0$



10. The line through $(2, -4)$ and $(c, 1)$ has a slope of -5 . Determine the value of c .

11. Write the equation of the line parallel to $3x + y - 4 = 0$ that passes through the point $(2, -5)$.

12. Find the equation of a line that goes through $(-3, 9)$ and $(9, 1)$.
13. Write the equation of the line perpendicular to $3x + y - 4 = 0$ that passes through the point $(2, -5)$.
14. Write the equation of the line parallel to $x = -3$ passes through $(6, -7)$.
15. Determine the equation of the line perpendicular to $y = 4$ that passes through $(-1, 6)$.
16. In 2016, there were a total of 36.63 million people already living with HIV, and there were 1.8 million new HIV infections in that year. The rate of new infections is decreasing by approximately 300000 people per year due to advancements in drug technology and the support of global agencies and local governments.
 - a. Write a linear equation that models the number of people in millions who have been newly infected with HIV since 2016.
 - b. If the rate of infections continues to follow this linear trend, in what year will there be no new infections?
 - c. Is this a realistic model to use for such an epidemic? Why or why not?
17. According to the National Academy of Sciences of the United States of America, there were 7100 cheetahs left in the wild in 2016. The number had decreased significantly from an estimated 14000 in 1975, when the last comprehensive count was done in Africa. The decision to convert wilderness areas to agricultural or livestock farms has caused the loss of habitat that is the main cause of this decline.
 - a. What is the average decrease in the number of cheetahs each year?
 - b. Assuming this is still the average rate of decline, what is the cheetah population this year?
 - c. Assuming this average rate of decline continues, when will the cheetah become extinct?
18. Determine the equation of the line perpendicular to the line $4x + 2y - 7 = 0$ that has the same x-intercept as the line $2x + 3y - 12 = 0$.
19. Find the value of k if the lines $3x - 2y - 5 = 0$ and $kx - 6y + 1 = 0$ are:
 - a. parallel
 - b. perpendicular.
20. Triangle ABC has vertices $A(3, -1)$, $B(-3, -5)$ and $C(-1, 5)$. Determine whether the triangle is a right triangle. Fully justify your answer.
21. In 1991, 18.6% of the world population was undernourished. In 2015, 10.9% of the population was undernourished.
 - a. What was the average percentage change per year?
 - b. Assuming this rate continues, what is the percentage of the population that is undernourished this year?
 - c. Determine an equation to represent this scenario.
 - d. Assuming this rate continues, when will there be no undernourished people in the world? Do you think this is actually possible?
 - e. What decisions at local, country and world level could be driving this change?
- (Set 1975 to be year 0 and determine the equation.)
22. The decision of which car to buy is no longer just about color and style; it may also involve choosing one that is environmentally friendly. A cost comparison of cars is now a necessity when shopping around for a new car. In the United States, the Ford Focus sells for \$19000 and costs 8 cents per mile to run. The zero-emission electric Focus sells for \$21600 and costs only 3 cents per mile to run.

- a. Determine the equation that represents the relationship between the total cost of each car (including both buying and running costs) and the miles traveled. What do the gradient and y-intercept represent in these equations?
 - b. What is the total cost of each car at 30000 miles? If you were planning to sell the car once it reached 30000 miles, which car would be the cheapest option?
 - c. What is the total cost of each car at 60000 miles? If you were planning to sell the car once it reached 60000 miles, which car would be the cheapest option?
 - d. Which characteristic of these linear relationships (gradient or y-intercept) would be affected if
 - i. the price of gasoline in the US increases?
 - ii. the US government offers a larger rebate (a partial refund) on the purchase of new electric cars?
 - e. When is the total cost of each car exactly the same? Write a general statement relating the mileage of the cars to when each model is the cheapest, using your original equations.
 - f. What other factors apart from cost might go into your decision of which car to buy?
23. On average, 3.85 kilograms of feed are needed for every kilogram of meat we consume (this is called the feed conversion ratio).
- a. Represent this information in a table showing the relationship between kg of feed and kg of animal meat. Your table must have at least 6 rows of data.
 - b. Sketch this linear model, with kg of feed on the x-axis and kg of animal meat on the y-axis. Even if your data doesn't go that high, make sure your x-axis goes to 130kg.

Answers:

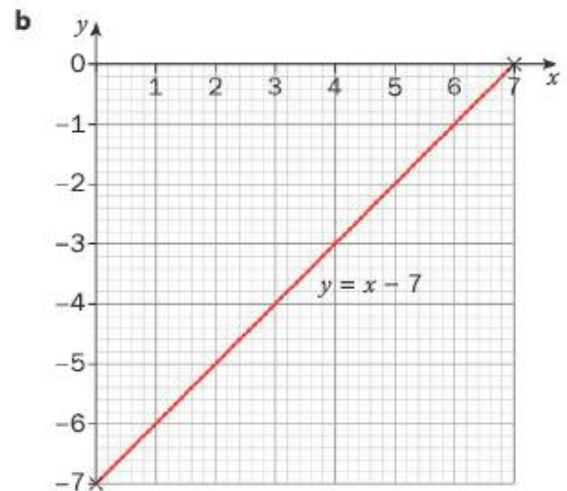
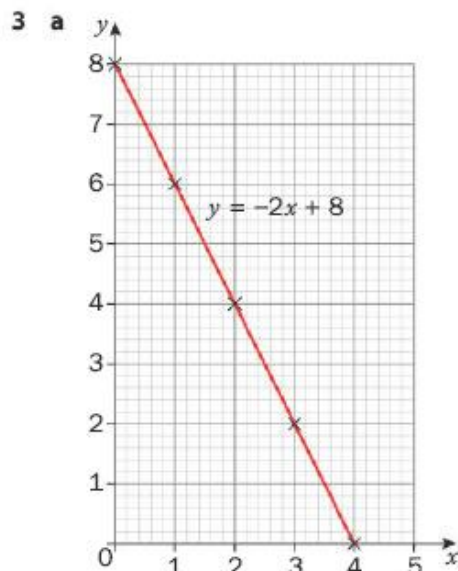
1 a $6, -12$

b $-\frac{2}{7}, 23$

2 a $x = 4, y = 8$

b $x = 4, y = -5$

c $x = 6, y = 14$



4

Line	Slope	y-intercept	x-intercept	Equation
A	$\frac{5}{2}$	-5	2	$y = \frac{5}{2}x - 5$
B	0	3	None	$y = 3$
C	$-\frac{1}{5}$	2	10	$y = -\frac{1}{5}x + 2$
D	Undefined	None	-4	$x = -4$

5 a $y = \frac{1}{6}x - 4$

b $y = \frac{1}{2}x + 2$

c $y = \frac{1}{8}x$

d $y = \frac{1}{6}x + \frac{2}{3}$

e $y = 12x - 9$

6 a Undefined

b 1

c 2

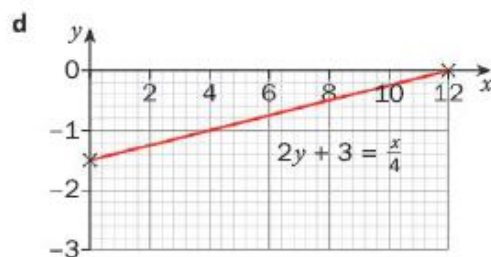
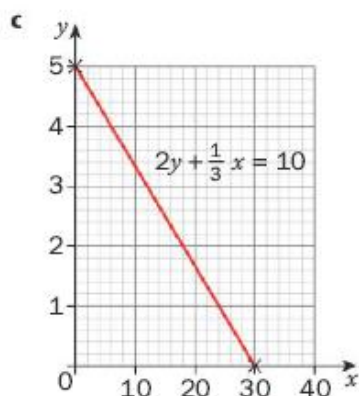
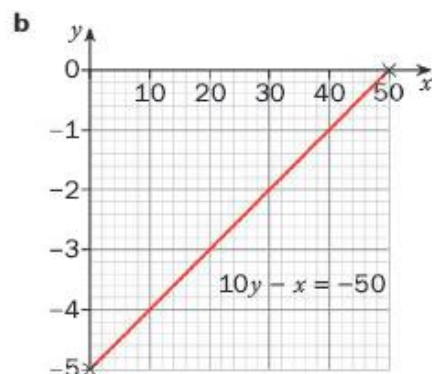
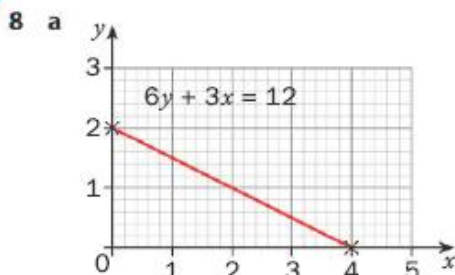
d 1

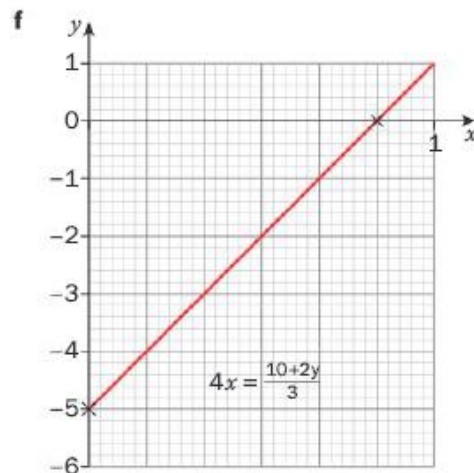
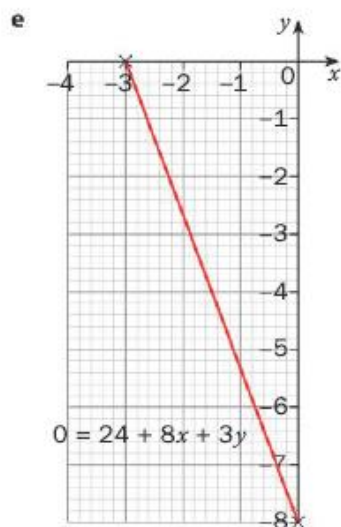
7 a Constant decrease

b $L = -0.5d + 100$

c 200

8





9 a ii

b iii

c i

10 i

11 $y = -3x + 1$

12 $y = -\frac{2}{3}x + 7$

13 $y = \frac{1}{3}x - \frac{17}{3}$

14 $x = 6$

15 $x = -1$

16 a $y = -0.3x + 1.8$

b 2022

c Individual response: trend not likely to be linear

17 a $\frac{6900}{41} \approx 168$

b Individual response according to year: equation is $y = -\frac{6900}{41}x + 14000$ where x is years since 1975 and y is number of cheetahs

c 2058

18 $y = \frac{1}{2}x - 3$

19 a 9

b -4

20 Yes. Gradient AB = $\frac{2}{3}$; gradient BC = $-\frac{3}{2}$ so perpendicular because $\frac{2}{3} \times -\frac{3}{2} = -1$

21 a -0.321%/year

b Individual response according to year

c $y = -0.321x + 18.6$ where x is years since 1991 and y is number of people who are undernourished.

d 2049

e Individual response

22 a $y = 0.08x + 19\,000$; $y = 0.03x + 21\,600$

b \$21400; \$22500

c \$23 800; \$23 400

d Individual response

e 52 000 miles If the number miles is less than 52 000 it is cheaper to drive the Ford Focus, but if the number of miles driven is more than 52 000 it is cheaper to drive a zero-emission electric Focus.

f Individual response

23 a Individual response: values on table should satisfy equation $y = 3.85x$ where x is animal meat in kg and y is feed in kg.

24. A leaking faucet may not seem like a huge waste of water, but it certainly can add up if you leave it unfixed for a period of time. On average, a leaking faucet drips 15 milliliters of water per minute.

a. Represent this pattern with a table of values. Create a table like the one below, using 1-minute intervals for the x values up to a maximum of 10 minutes.

x (minutes)	0	1	2	3	4	5
y (milliliters)	0	15				

b. Represent the pattern as a set of coordinates.

c. Represent the pattern as a graph for these values of x and y :

$0 \leq x \leq 10$, $0 \leq y \leq 200$ (Please use a ruler when connecting the points.)

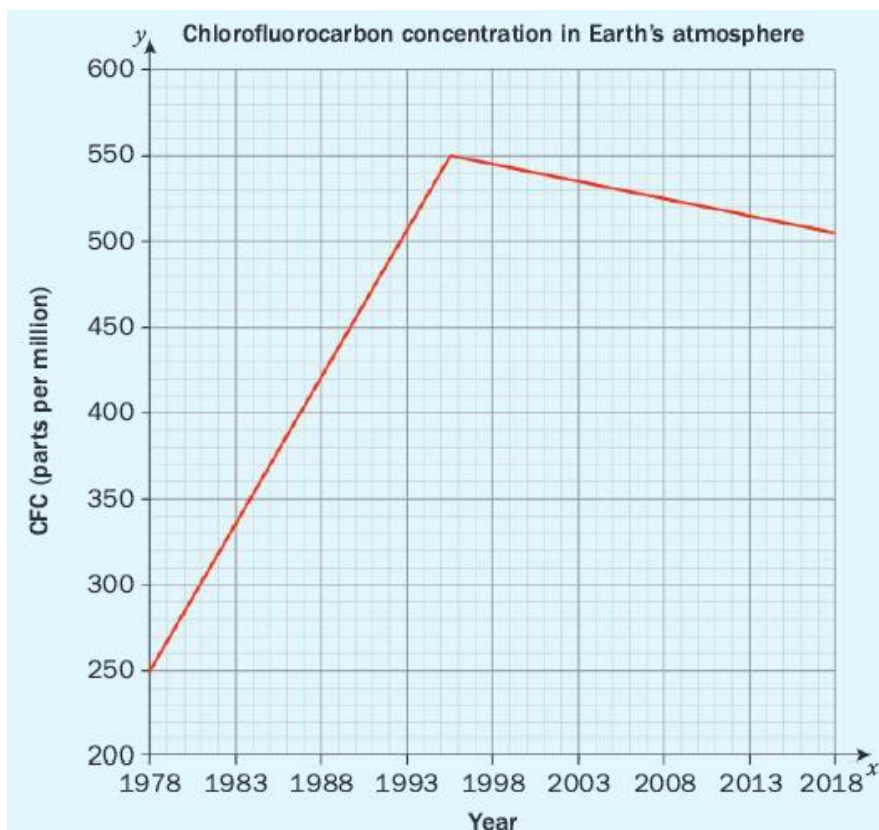
d. Represent the pattern as a verbal description of the relationship between the amount of water wasted (in millilitres) and the time (measured in intervals of 10 minutes).

e. Show that each representation will give the same amount of wasted water after 1 hour.

f. Set up a table showing the same linear relationship but using weeks as the time interval. How much water would be wasted in the course of a year?

g. Most people use about 110 liters of water in a bath. How many baths could be filled using the water from a faucet that has been leaking for one year? Show your working.

25. Chlorofluorocarbons (CFCs) were created in 1928 and were used in products such as aerosol propellants, cleaning solvents and refrigerants. It has since been proved that CFCs contribute to the destruction of the ozone layer. They continued to accumulate in our atmosphere until a global decision was made to address this potentially catastrophic issue.



A graph that contains two sets of trends, like this one, can be referred to as a piecewise relationship.

- Use the graph to determine whether the global agreement to ban CFCs was successful. Explain your answer.
- According to the graph, what was the rate of change in CFC concentrations in the atmosphere leading up to the global ban? Verify your answer using different points on the graph. Discuss any discrepancies you may have.
- According to the graph, what was the rate of change in CFC concentrations in the atmosphere after the global effort started? Verify your answer using different points on the graph. Discuss any discrepancies you may have.
- Although much has been accomplished, CFCs decompose very slowly, so even when all CFC production has stopped (which is not yet the case), some concentration of CFCs will remain in the atmosphere for over 100 years. According to the decreasing trend, in what year will there be zero CFCs left in the atmosphere?
- Write down two facts that you have learned from this case study and its graph.

26. Reducing the emission of greenhouse gases has been a major focus of governments across the globe since the mid-1990s. In 1997, most countries signed the Kyoto Protocol, agreeing to reduce greenhouse gases and their effect on global temperatures. The following data represent the CO₂-equivalent concentration of all greenhouse gases in the atmosphere in parts per million over a span of 20 years.

1995	427
2000	441
2005	455
2010	469
2015	483

- Plot these points on a graph, making sure you start your x-axis at 1990 (as that was the benchmark year agreed in the Kyoto Protocol). Plot the years from 1990 to 2030.
- According to your linear model, what was the concentration of the mix of greenhouse gases in 1990?
- According to your linear model, what is the change per year in the concentration of greenhouse gases in the atmosphere?
- Use your graph to determine the equation of the line, using 1990 as the initial condition. Write the equation in the three different forms.
- Verify your answer to step d using algebra.
- According to your model, what is the concentration of greenhouse gases this year?
- If the Kyoto Protocol threshold is 500 ppm for the concentration of greenhouse gases, in what year will the concentration reach that amount if current trends continue?
- Draw this threshold on your graph as a dotted horizontal line. Continue your line to show the point at which it intersects the threshold. Verify that your graphical results are the same.

27. By 2030, it is estimated that we will need to feed a world population of 8.5 billion people. How can we do that in a way that is environmentally sustainable....

Part 1 – Worldwide meat consumption

There has been increasing pressure on farmers to produce meat (beef, pork, lamb, chicken and other types of meat that humans eat), which has undesirable effects on the environment. According to the Food and Agriculture Organization of the United Nations, the annual world meat consumption in 1965 was 24.2kg per capita (per person). This had increased to 41.3kg per capita in 2015.

- Assuming a linear model, determine the equation for worldwide meat consumption per capita since 1965.
- Use your model to predict meat consumption per capita this year.
- Predict the meat consumption per capita in 2030.
- Discuss whether you think this is a realistic model.
- Do you think this increase in meat consumption is sustainable? Explain.

To put this in perspective, take a look at the following table, which compares meat consumption in developed and developing countries

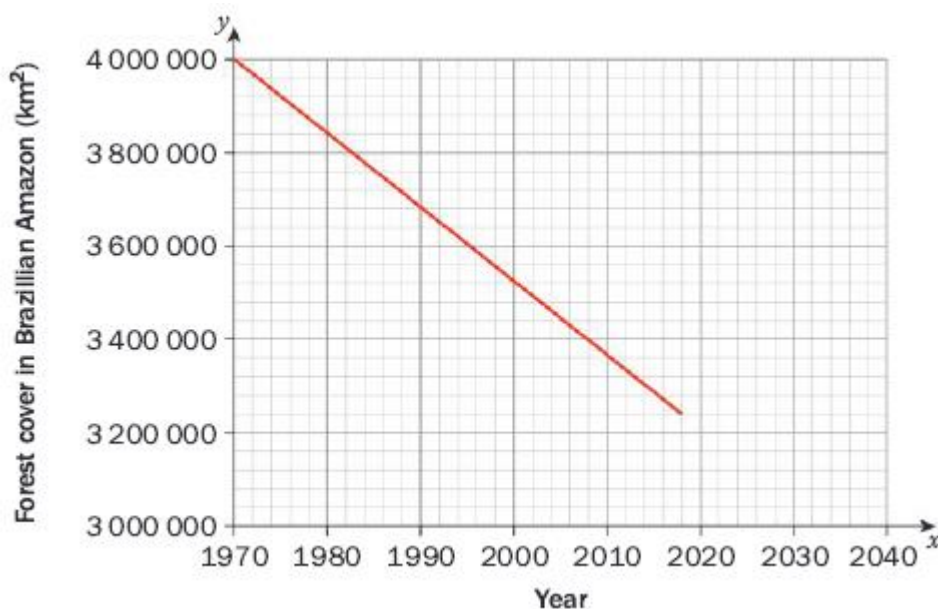
	1997 meat consumption (million tonnes)	Average annual increase in meat consumption since 1997 (million tonnes)
Developed countries	98	0.8
Developing countries	111	4.6

- Graph each set of data on the same axes. Assume a linear model for each.
- Determine the equation of each line.
- What does the slope of each line represent? Which slope is more concerning? Explain.
- Will there ever be a time when the meat consumption of developing countries is equal to that of developed countries? Explain. If it is possible, use your equations to try to find out approximately when this will happen.

Part 2 – The Brazilian rainforest

Approximately 70% of clearcutting (cutting down all or most trees in an area) in the Brazilian rainforest is to provide land for cattle ranches and farming. This accounts for almost 15% of the world's total annual deforestation, making it the largest cause of deforestation worldwide.

The graph below represents the forest cover in the Brazilian Rainforest since 1970.

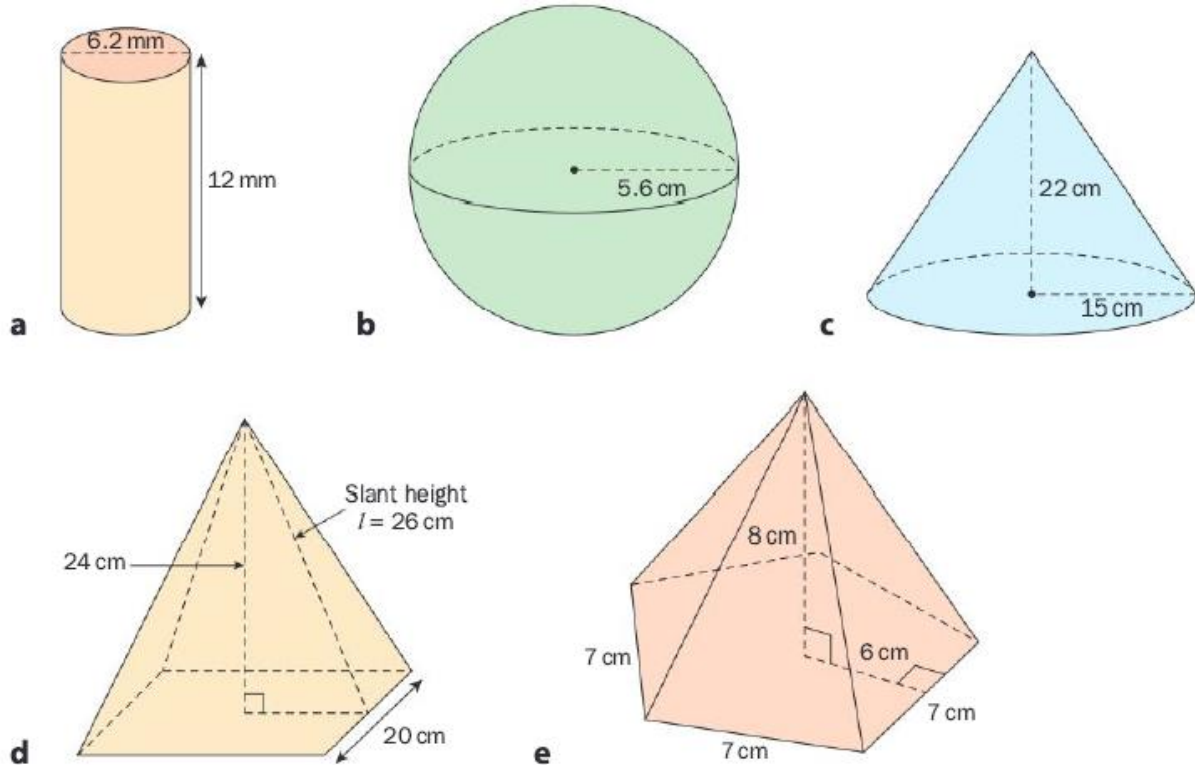


- Determine a linear equation that represents the relationship between the amount of forest cover in the Brazilian Amazon and the year. What does the y-intercept represent? What does the gradient represent?
- Assuming that the deforestation rate is constant, calculate the approximate area of forest that is cleared annually.
- Some scientists warn that the rainforest cannot deplete in size to less than 75% of its 1970 size without catastrophic irreversible consequences to the rainforest ecosystem. If

clearcutting continues at the constant rate represented in the graph, in what year will that occur?

3D Shapes

1. Calculate the volume and surface area of each of the following shapes. Round answers to the nearest tenth.



2. In order to help reduce employees' stress levels and give them a place to get away from their office cubicle, Amazon has built three giant spheres in downtown Seattle, USA. Inside the spheres, the climate is very similar to that of Costa Rica, providing a nice break from the sometimes cold and rainy Seattle weather. The spheres also contain waterfalls, a river and tropical gardens. The largest sphere measures 40m in diameter.



- a. Find the volume of space available in the largest sphere. Round your answer to the nearest tenth.
- b. If the spheres are covered in glass, find the area of glass needed to cover the largest sphere.

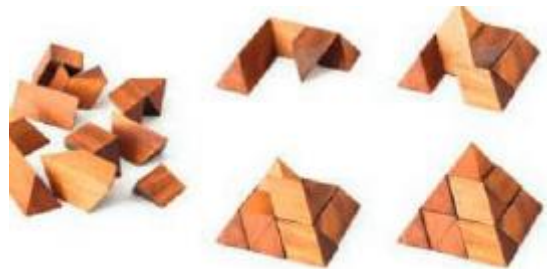
3. Airbags became widely used in cars in the 1980s and 1990s as a method of making them safer. Although seat belts had become a standard safety feature, injuries still persisted as the driver and passengers impacted the inside of the car. Airbags are installed so that, in a crash, they will deploy and create a soft barrier between the driver/passenger and the car and its windows. A car can have multiple airbags, each positioned to protect a specific passenger. The driver's airbag is located in the steering wheel of the car, and when fully inflated can be approximated by a cylinder. If the cylinder has a height of 25cm and a radius of 28cm, find:
- the volume of air in the airbag when fully inflated
 - the amount of material required to make the airbag.



4. In order to solve problems of manoeuvrability and stability, inventor James Dyson inserted a ball into his vacuum cleaner design. The motor could be moved to fit inside the ball, which lowered the center of gravity and increased the stability of the machine. At the same time, the ball allows the machine to be turned around corners with ease. If the motor for a particular vacuum cleaner has a volume of 4200cm^3 , find the radius of the smallest ball that could be used to hold it.



5. Three-dimensional puzzles are popular toys for adolescents, especially since the Rubik's cube. The wooden puzzle shown here has only 10 pieces but is very difficult to solve. The puzzle comes with the pieces arranged in the shape of a cube, and the aim is to rearrange them to make a square pyramid. If the puzzle contains 9 cm^3 of wood, find the dimensions of the pyramid.



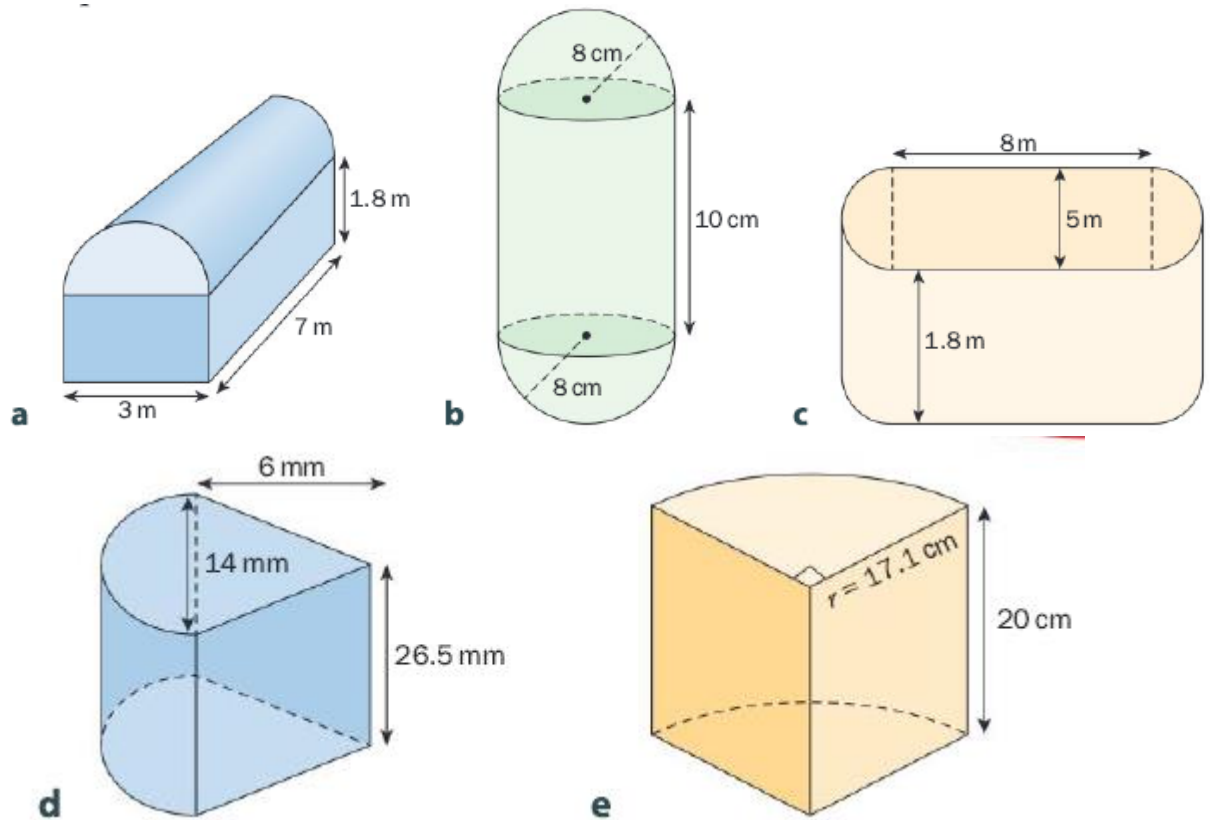
6. The LifeStraw personal water filter removes 99.9999% of waterborne bacteria. It can turn up to 1000 liters of contaminated water into safe drinking water. If the length of the straw is 9 inches and the diameter is 1 inch, calculate the volume of water that the straw can hold.



7. New Zealand, which is known for its adventurous activities, is the home of “zorbing”. This involves rolling down a hill in an inflatable ball. The zorb is actually a sphere within a sphere, with a layer of air in between to help absorb shocks. Zorbs can hold up to three people and zorbers can even choose the “aqua option”, where the inner sphere is partly filled with water. This makes for a refreshing ride down the hill on a hot day. If the outer sphere has a diameter of 3m and the inner sphere has a diameter of 2m, find the volume of the air layer between the inner and outer spheres. (The plastic material for each sphere is approximately 0.8mm thick, which can be ignored in these calculations.) Round your answer to 3 s.f.



8. Find the volume and surface area of each of the following 3D shapes.



9. Specialized paint has been designed to protect storage silos against corrosion and to prolong their life. First the silo is painted with a layer of rust-resistant primers and then with two layers of the specialized paint. The height of the cylindrical part of one agricultural silo is 17m and the total height of the silo is 20m. The diameter of the silo is 15m. Find out how much paint and primer are needed to protect the five silos shown here.



10. The Morning Glory cloud formation (shown in the photo) occurs very

predictably on the north coast of Australia. Although its causes are not known precisely, it forms mostly in October over the Gulf of Carpentaria during a process when sea breezes from the west meet breezes from the east.

To study such phenomena, scientists use a tool called a radiosonde, attached to a weather balloon (which is spherical!). These cylindrical cloud formations can be 1000km long with a radius of 150m. Moving at speeds up to 60km/h, there is a lot of cloud moving overhead.

- Find the surface area of the Morning Glory cloud formation. Round your answer to the nearest hundredth.
- If the cloud formation were only half as long, by what factor would that change the surface area? Justify your answer with calculations.
- If the cloud formation had only half the radius, by what factor would that change the surface area? Justify your answer with calculations.



11. A common product found in many restaurants and kitchens is the pepper grinder. It allows users to grind their own pepper, keeping it as fresh as possible until used. Pepper grinders can come in a wide range of shapes and sizes. One product is in the shape of a cylinder with a height of 14cm and a radius of 5cm. Another is in shape of a ball (sphere). Find the radius of a ball pepper grinder if it has to contain the same volume of pepper as the cylinder.



12. Access to clean water is a problem in many developing countries, especially as clean water sources are often located far away from the people who need the water. The Q Drum water transporter was developed to solve this problem. Its design allows it to transport a large volume of water over long distances simply by rolling the drum. This means that anyone can use the device, including children. The Q Drum holds 50000cm^3 of water. Its height is 36cm and the diameter of the base is 50cm. Find the radius of the inner hole through the center of the drum.



13. The traditional igloo, with its approximately hemispherical dome, is traditionally built by Canadian Inuit and the people of Greenland. It is made of compacted snow, which is a poor conductor of heat, so any heat generated inside the structure will stay inside.
- If the radius inside an igloo is 1.8m, find the volume of air contained inside.
 - If the thickness of the walls of compacted snow is 0.5m and the inner radius of the entrance is 0.8m, find the surface area of the outside of the igloo.



14. The two-piece hard capsule, patented in 1847, was designed both to ensure the correct dosage of a medication and to make the medicine easy to swallow, without having to taste the medicine. The pharmaceutical manufacturer fills the hollow gelatin capsule with medicines such as antibiotics. Capsules come in many sizes, but a common one has a total length of 11mm and a diameter of 5mm.
- Find the volume of medicine that a capsule of this size can hold.
 - Find the amount of gelatin needed to produce one capsule.



Answers:

- | | | |
|---|---|---|
| 1 a Surface area: 294.1 mm^2
Volume: 362.3 mm^3 | b Surface area: 394.1 cm^2
Volume: 735.6 cm^3 | c Surface area: 1961.6 cm^2
Volume: 5183.6 cm^3 |
| d Surface area: 1440 cm^2
Volume: 3200 cm^3 | e Surface area: 280 cm^2
Volume: 280 cm^3 | |
| 2 a 33510.3 m^3 | b 5026.5 m^2 | |
| 3 a 61575.2 cm^3 | b 9324.2 cm^2 | |

4 10.0cm

5 Base: 3cm by 3 cm; height: 3cm

6 7.07 inches³ to the nearest hundredth

7 79.6m³

8 a Surface Area: 97.1 m^2
Volume: 62.5 m^3

b Surface Area: 1306.9 m^2
Volume: 4155.3 m^3

c Surface Area: 176.3 m^2
Volume: 107.3 m^3

d Surface Area: 1309.3
Volume: 3152.7 cm^3

e Surface Area: 1680.5 cm^2
Volume: 4593.2 cm^3

9 Enough to cover 4957.1 m^2

10 a 942.62 km^2

b Approximately 2

c Approximately 2

11 6.40cm to the nearest hundredth

12 13.5cm

13 a 12.2 m^3 to the nearest tenth

b 33.2 m^2

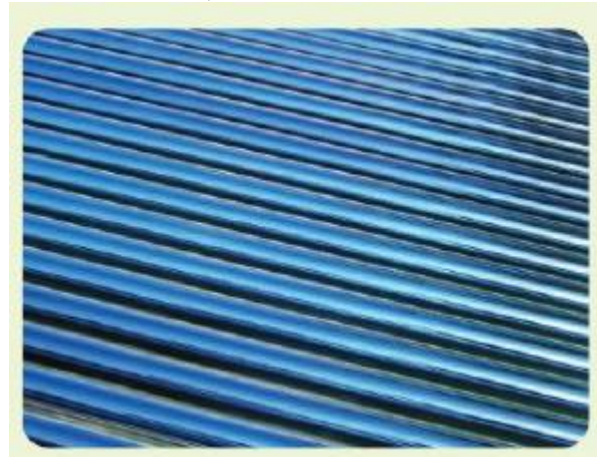
14 Volume: 183.3 mm^3
Area: 172.8 mm^3

15. Solar energy is becoming increasingly important as one potential solution for the planet's energy needs. However, transforming the energy of the sun into electricity requires the use of photovoltaic cells, which must be mounted in areas that receive a lot of sunlight. Mounting these cells has become a problem on its own, and a wide range of solutions have been proposed. In this task, you will analyze products that have been created to try to improve the effectiveness of solar power. Please show your working throughout the task.

Cylindrical solar cells

The traditional solar panel is a rectangle. However, manufacturers of the cylindrical solar panel claim that it is a much more efficient way of collecting solar energy, especially on larger buildings. The cylinder shape allows absorption of the Sun's rays from just about every angle as it passes overhead. (A traditional panel is most efficient when the Sun is directly overhead.)

When mounted on a roof that has been painted white (such as many businesses have today), even the underside of the panel will absorb sunlight that is reflected off the roof. The shape also reduces wind resistance, which makes these solar cells easier and less expensive to install.



The height of each cylinder that is covered by photovoltaic cells is 100cm. The diameter of each cylinder is 22mm.

- a. Find the total surface area that is exposed to the Sun of 200 of these solar cells are installed on the roof of a building.
- b. Find the amount of space (volume) that the 200 cells take up.

Spin cell cones

Spin cell cones use some of the solar energy that they generate to rotate, but their manufacturer explains that the rotation allows them to collect the Sun's rays from every angle without overheating. Most of the cone is covered in photovoltaic cells, with the exception of its base.

- c. If the radius of one cone is 0.55m and its height is 0.82m, find the area of the cone covered in solar cells.
- d. Find the volume occupied by an array of 50 of these spin cell cones.

You have a section of a flat white roof with dimensions of 10m by 6m.

- e. A typical rectangular solar panel measures 1.65m by 1m. Calculate the area that they will cover.
- f. Cylindrical solar cells are installed side by side and need at least 20mm between them. Calculate the total surface area of these cylindrical panels that will be exposed to the Sun.
- g. Spin cell cones can be installed so that they touch each other. How many cones will you be able to place on the roof? Calculate the total surface area of the cones that will be exposed to the Sun.

- h. What factor(s) would influence your choice of solar panels to install? Explain.

16. Solar pyramid

The solar pyramid was designed to help solve the problem of the solar cells overheating. This device is made for much smaller applications than the cylindrical cell, such as camping or charging tools at a construction site. The square base measures 25 cm on each side and has a height of 35 cm.

- Assuming the pyramid sits on the ground, find the surface area available for solar cells. (Assume that they can be placed all the way to the top.)
- If each pyramid is filled with air, find the volume of air inside each pyramid.



Rawlemon

Inspired by his daughter's toy marbles, German architect Andre Brossel created the Rawlemon. It combines the benefits of a magnifying glass with the potential of solar energy in order to produce up to 70% more energy than a typical photovoltaic panel.

Rawlemon works with both sunlight and moonlight and is capable of following the movement of the Sun or Moon thanks to its motorized base. The large crystal ball is filled with water to help magnify the light rays on the solar cells.

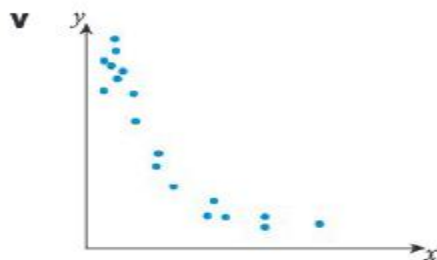
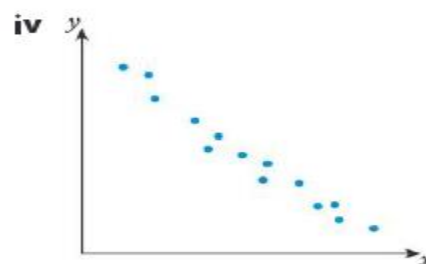
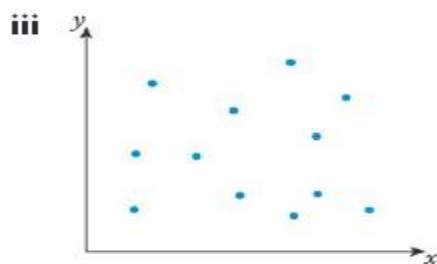
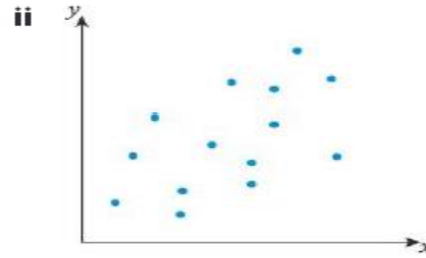
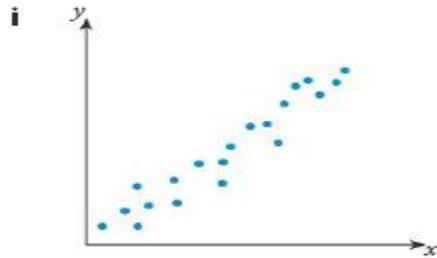
The Rawlemon comes in a variety of sizes.

- Find the area of the sphere that is exposed to sunlight if it has a diameter of 1.8m. (Assume that the whole of the sphere's surface area is exposed to sunlight.)
- Find the volume of water in a sphere with a diameter of 1.8m.



Bivariate Data

1. For each of the following scatter plots:
 - a. describe the form, direction and strength
 - b. suggest two variables whose relationship might be represented by the scatter plot.



2. Copy and complete this table.

Variables	Independent variable	Dependent variable	Sketch of scatter plot
1 Number of times you laugh in a day 2 Age			
1 Outside temperature 2 Inside temperature			
1 Numbers of languages learned 2 Number of nationalities in family tree			
1 Son's height 2 Father's height			
1 Reaction time 2 Age			

3. Represent each of the following data sets with a scatter plot.

a

x	y
10	12
13	13
15	17
19	19
21	15
24	18
26	20
31	21
25	17

b

x	y
2	44
3	51
6	58
4	31
3	39
5	42
7	27

c

x	y
95	35
87	41
92	40
81	44
83	42
104	22
93	31
90	39

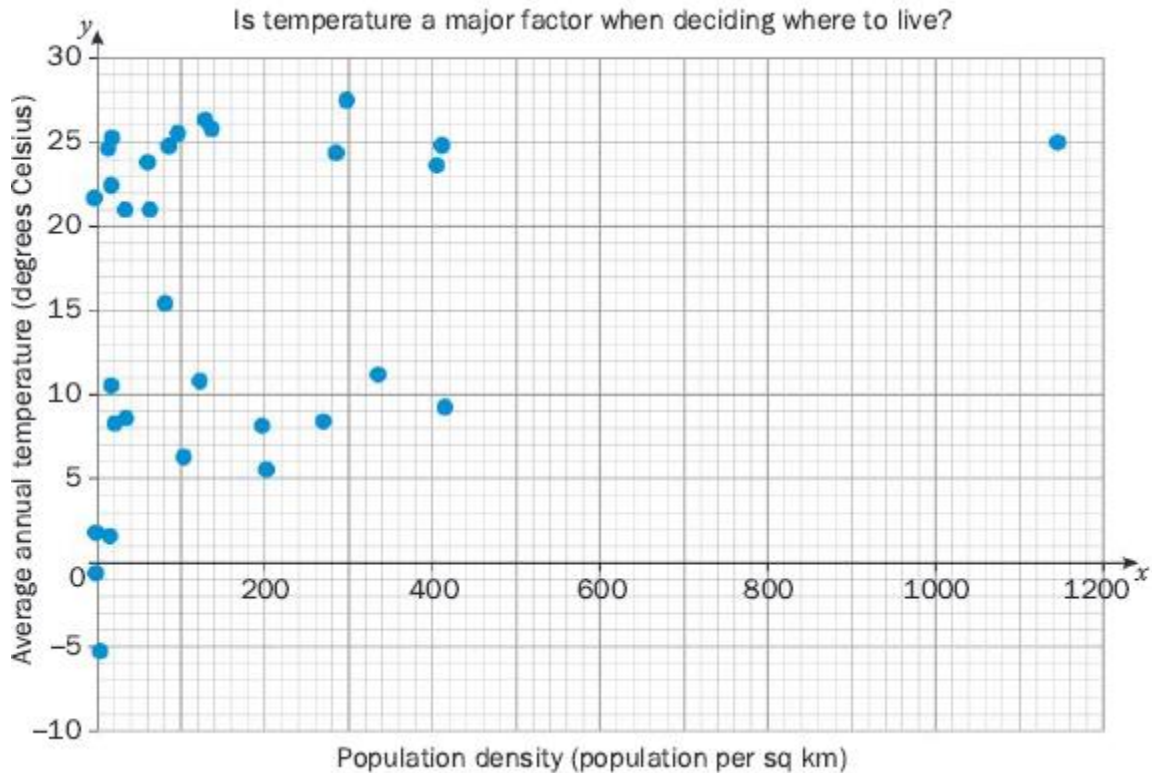
d

x	y
27	77
21	71
22	62
33	51
24	70
28	85
38	56
30	61
20	90

4. The countries with the highest life expectancy for males and females are given in the table below.

Country	Female life expectancy (years)	Male life expectancy (years)
Japan	86.8	80.5
Switzerland	85.3	81.3
Singapore	86.1	80.0
Australia	84.8	80.9
Spain	85.5	80.1
Iceland	84.1	81.2
Italy	84.8	80.5
Israel	84.3	80.6
Sweden	84.0	80.7
France	85.4	79.4
South Korea	85.5	78.8
Canada	84.1	80.2

- Represent the data in a scatter plot. Be sure to include a key.
 - What trend(s) do you see in the data? Explain how the scatter plot makes the trend(s) more obvious than the table.
 - What aspects of the data are more visible in the table than in the scatter plot? Explain
5. Unlike most other species, humans live just about everywhere on the planet. We have been able to adapt to a wide range of surroundings, so much so that almost any location is habitable. Why do we choose to live where we do? Does the temperature help people decide where to live? Below is a scatter plot representing data for countries from all around the world in all continents (except Antarctica).



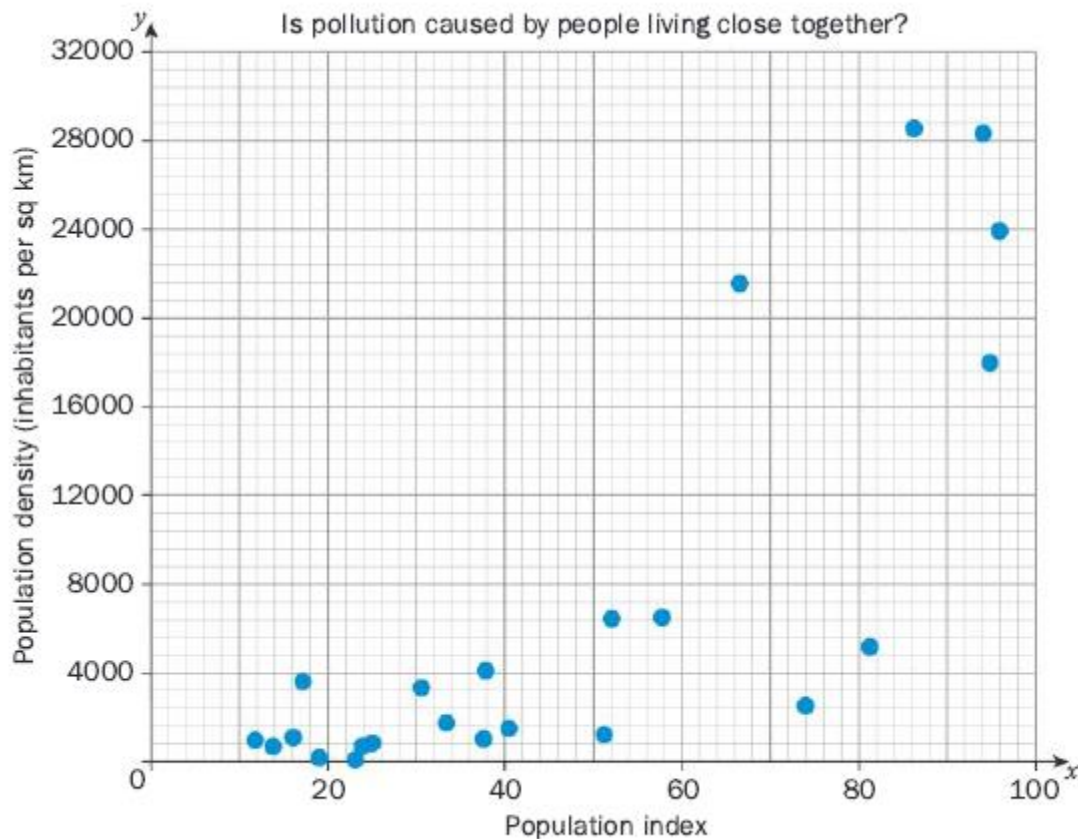
- a. What does population density tell us?
 - b. What does the average annual temperature of a country tell us? What do you think would be an ideal average annual temperature?
 - c. The data point to the far right is Bangladesh with a population density of 1138 people per square kilometre and an average annual temperature of 25°C. Would you consider this point an outlier? Justify your answer.
 - d. Describe the relationship between population density and average annual temperature.
 - e. Is temperature a major factor when deciding where to live? Suggest three factors that you think might be more important.
 - f. Are these characteristics good indicators to look at when addressing such a question? What would you alter if you wanted to investigate this question further?
6. One of our most basic human needs is sleep. We spend about a third of our lives doing it! Research suggests that this is because brain cells build connections with other parts of the brain while we are awake and need time to strengthen these connections (and dispose of unimportant ones), which is mostly done while sleeping. As a result, there are many negative health implications when we do not get enough sleep. The table below lists the amount of sleep people, of various ages between 1 and 65, had in a 24-hour period.

Age (years)	Amount of sleep (hours)
2	14
3	13
4	13
4	14
5	12
11	11
12	10
19	9
25	8
27	10
33	8
38	8
42	8
44	7
51	7
60	7
65	6

- Represent the data with a scatter plot. Describe the correlation in terms of form, direction and strength.
- Find the equation of the line of best fit using technology.
- What does the slope represent? What does the y-intercept represent?
- Use the equation to predict how much sleep a person of your age would have based on this data. How many hours of sleep do you get in a 24-hour period? How far are you from the predicted result?
- According to this data, predict how much sleep an 80-year old would have? Do you think your prediction is accurate?
- How much sleep would you need if you reached the age of 120? Does this answer make sense? Discuss.

- Over half of the world's population lives in cities. We are the only species that urbanize! We see images of big cities and sometimes these cities look highly polluted with dismal air quality. Is that because there are so many people living in such close proximity to each other?

The scatter plot below is a comparison of the pollution index and the population density of cities ranging in size from every continent (except Antarctica). The pollution index is an estimate of the overall pollution in the city (air, water, etc.), with higher numbers indicating more pollution.



- Was this result what you would have predicted?
- Describe the correlation in terms of form, direction and strength.
- Assuming a linear relationship, find the equation of the line.
- Manila is the most densely populated city in the world, with over 71000 people per square kilometer. What pollution index would you expect this city to have? Does this result make sense?
- In general, you cannot use the scatter plot nor the line of best fit for values that are outside the range of data on the graph. Explain why this is.
- Tokyo, Japan, has a pollution index of 46.87. What population density would you expect Tokyo to have?
- Suggest three other factors that would affect the pollution of a city.

Answers:

- 1 i a Form: Linear
Direction: Positive
Strength: Moderate
b Individual response
iv a Form: Linear
Direction: Negative
Strength: Strong
b Individual response

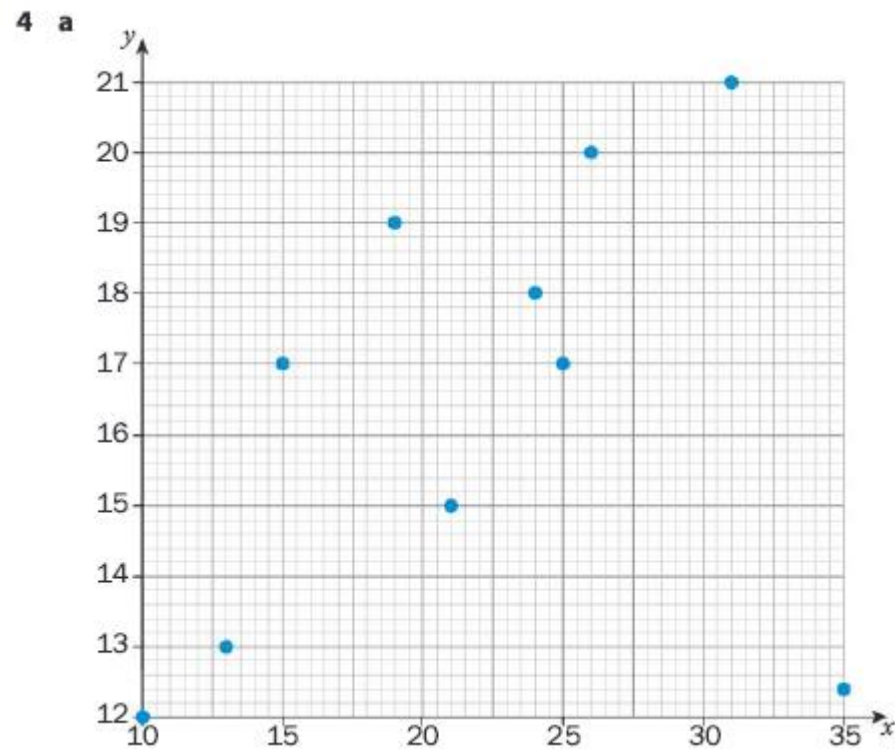
- ii a Form: Linear
Direction: Positive
Strength: Weak
b Individual response
v a Form: Non-linear
Direction: Negative
Strength: Strong
b Individual response

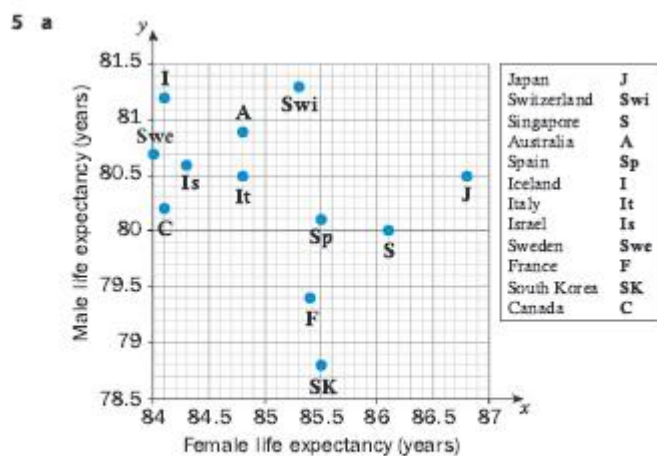
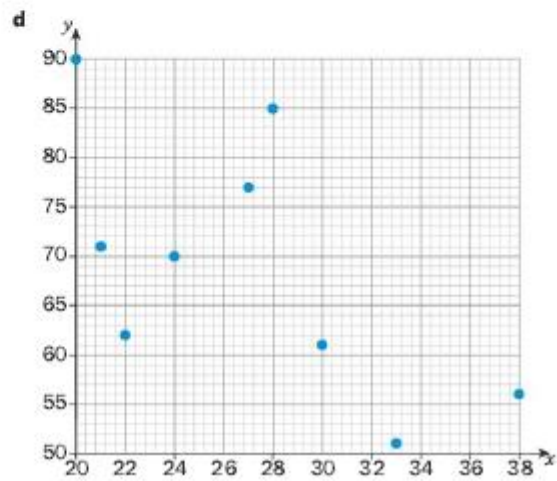
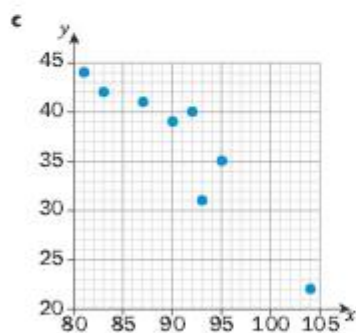
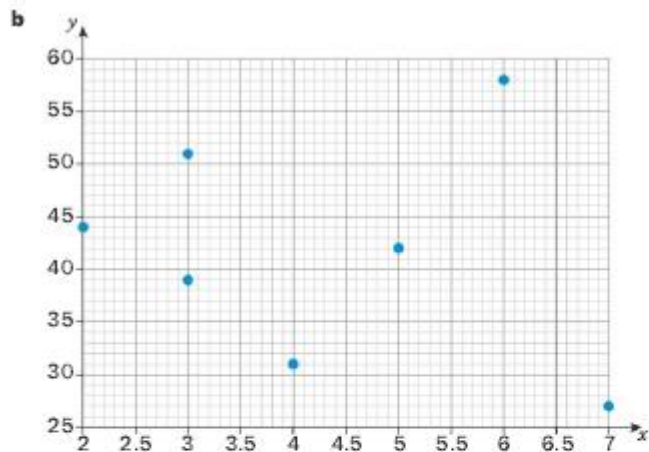
- iii a No correlation
b Individual response

2 a

Variables	Independent Variable	Dependent Variable	Sketch of scatter plot
1 Number of times you laugh in a day 2 Age	2	1	Individual response with adequate justification
1 Outside temperature 2 Inside temperature	1	2	Individual response with adequate justification
1 Number of languages learnt 2 Number of nationalities in family tree	2	1	Individual response with adequate justification
1 Son's height 2 Father's height	2	1	Individual response with adequate justification
1 Reaction time 2 Age	2	1	Individual response with adequate justification

- b Individual response



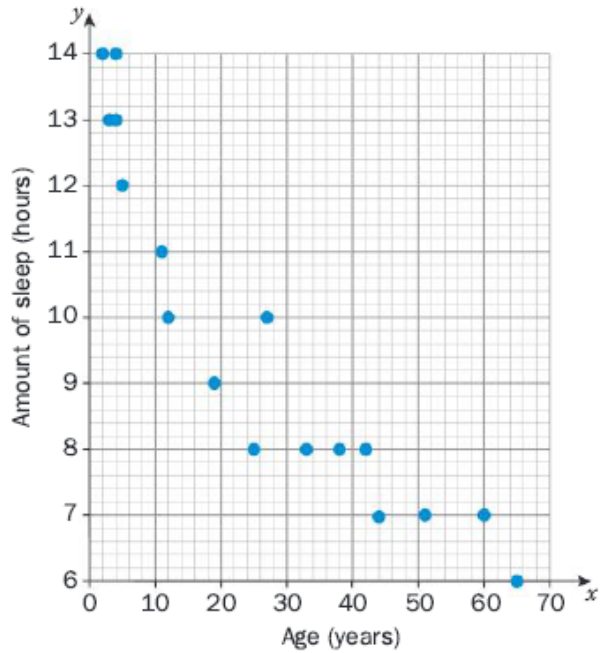


b No correlation

c Comparisons between countries are easier to make using the table

- 8 a How many inhabitants there are in a given area of land
 b The sum of the daily temperatures divided by the number of days in a year
 Individual response
 c Yes, it has a much higher population density than the other countries that have a similar average annual temperature
 d No correlation
 e No
 Individual response e.g. people don't always choose where they live; it is where they are born
 f Individual response e.g. the cost of living, healthcare, political stability

9 a



Form: Non-linear
 Direction: Negative
 Strength: moderate

- b** Approximating this relationship using a line of best fit,
 $y = -0.117x + 12.758$
- c** Slope of the line represents the average change in number of hours of sleep per increase in age of one year
 y-intercept represents the hypothetical average number of hours of sleep at age zero
- d** Individual response e.g. the equation predicts that a 15-year-old person would have 11 hours sleep
- e** 3.4 hours to 1 d.p. no
- f** -1.3 hours to 1d.p.
 This would not make sense, and hence supports the idea that this is a non-linear relationship

10 a Individual response (should be yes)

- b** Form: Non-linear
 Direction: Positive
 Strength: Moderate
- c** $y = 230x - 3400$
- d** 323; no, this would be too high so the relationship is likely to be non-linear
- e** Individual response (e.g. extrapolation likely to be inaccurate)
- f** Line of best fit predicts approximately 7400; the actual population is about 6000
- g** Individual response e.g. the amount of traffic, the amount of industry, the climate

(Note: As the question numbers and answer numbers may not be in order, you may cross-check with your teacher for clarity.)

Geometric Transformations

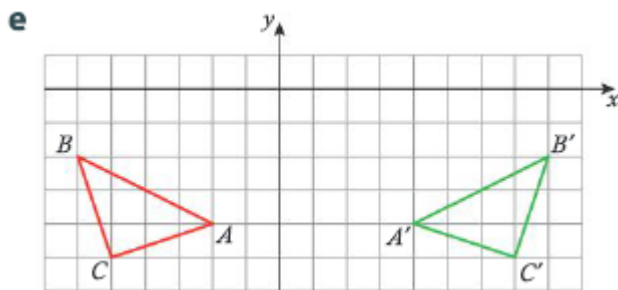
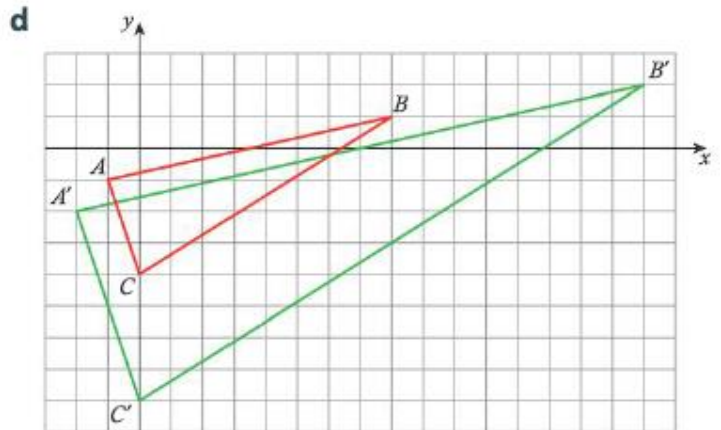
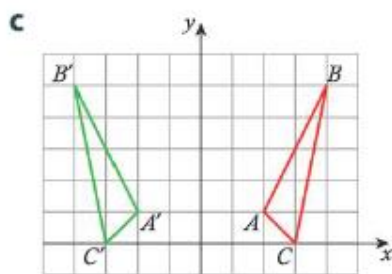
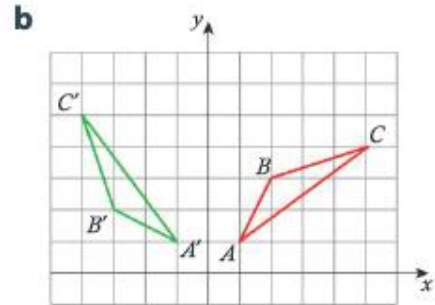
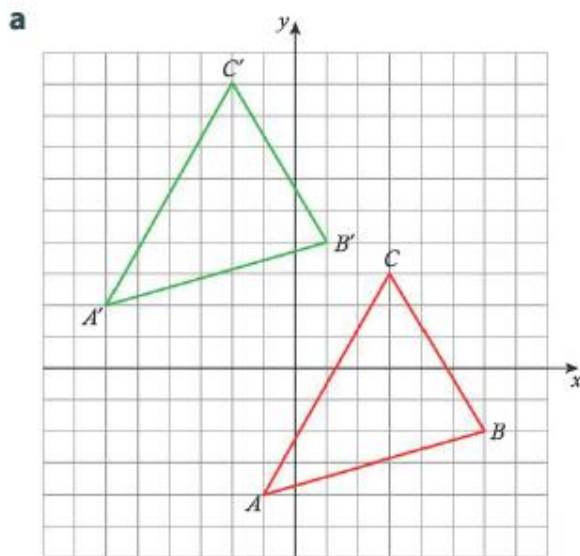
1. Copy and complete the table below.

Point	Translation up 3 units	Translation left 5 units	Translation down 2 units and right 7 units	Dilation by a scale factor of 4 about the origin
(0, 0)				
(6, 1)				
(3, -2)				
(-4, 7)				
(-8, -5)				

2. The photo shows a detail of the Serpent Bench in the Parc Güell, in Barcelona, in the Catalonia region of Spain. The bench winds around the park's perimeter and is covered in a mosaic of ceramic tiles. It was created by the artist Antoni Gaudí, who expressed his love of natural forms in his fantastical designs. Eusebi Güell, who commissioned the work, wanted Gaudí to represent the connection between ancient Greek symbols, Christian symbols and symbols of Catalonia. Is this design a tessellation? Explain why or why not.



3. Describe the transformations that have occurred in each diagram.



4. Using a dilation of scale factor 5, with the center at the origin, find the coordinates of the image L' of the point $L(12, -9)$?
5. What is the scale factor of the dilation (with center at the origin) if the image of point $T(6, -18)$ is $T'(2, -6)$?
6. Plot the image of the shape with vertices $(-5, 8)$, $(3, -6)$ and $(-3, 0)$ using the following translations.

a. $(x, y) \rightarrow (x + 4, y - 9)$

b. $(x, y) \rightarrow (x - 3, y + 6)$

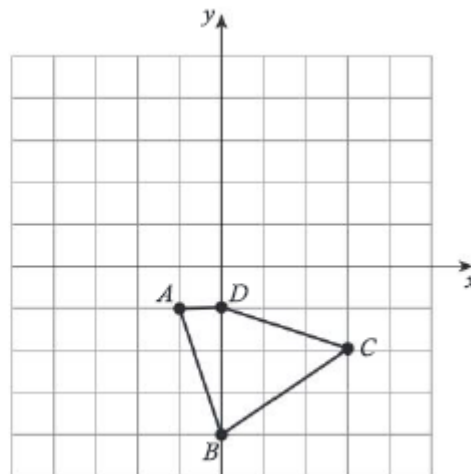
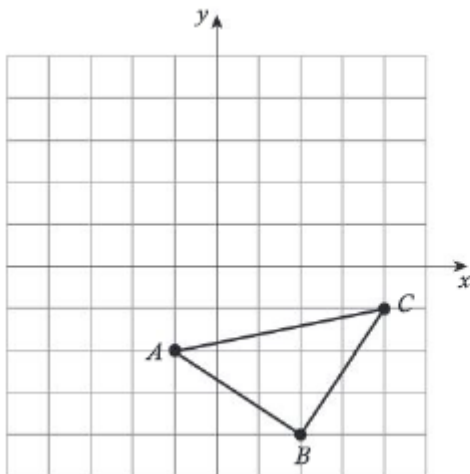
7. Copy and complete the table below.

Point	Reflection in x -axis	Reflection in y -axis	Reflection in $y = x$	Reflection in $y = -x$	Reflection in $y = 3$	Reflection in $x = -4$
$(0, 0)$						
$(8, 2)$						
$(7, -6)$						
$(-4, 1)$						
$(-5, -10)$						
$(0, -3)$						
$(9, 0)$						

8. Plot the image of each shape on your own copy of the Cartesian grid.

a. Rotation 90° clockwise about the origin

b. Rotation 180° about the origin



9. This is a folk pattern used in traditional Romanian embroidery. It is said that every form on a Romanian woman's blouse represents an aspect of the woman's history and family. Even the color of the pattern expresses the region where it was made and can indicate the age and social status of the woman wearing the blouse.

a. Identify the original shape(s) used to make this pattern.

b. Describe the transformations that have occurred to the original shape(s).

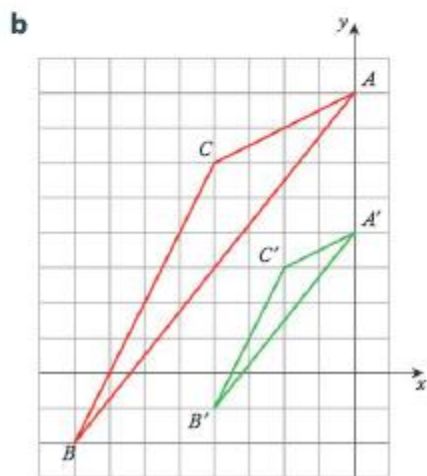
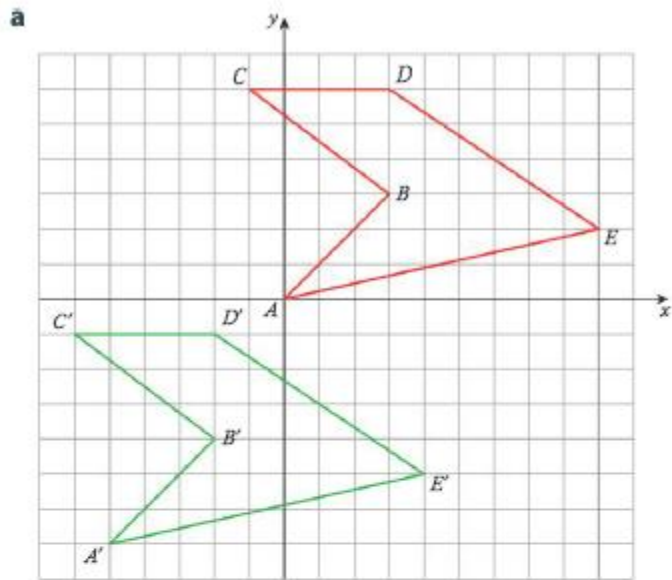
c. Explain how the shapes are tessellated.

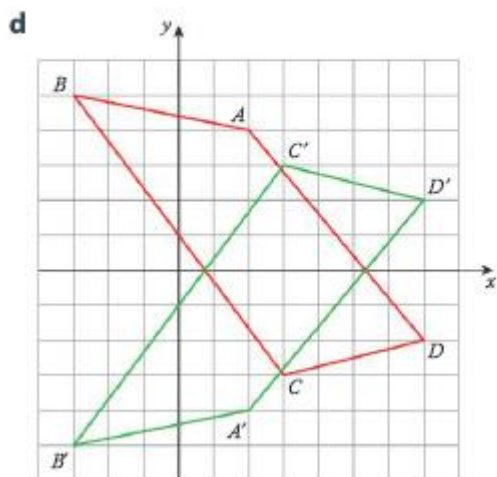
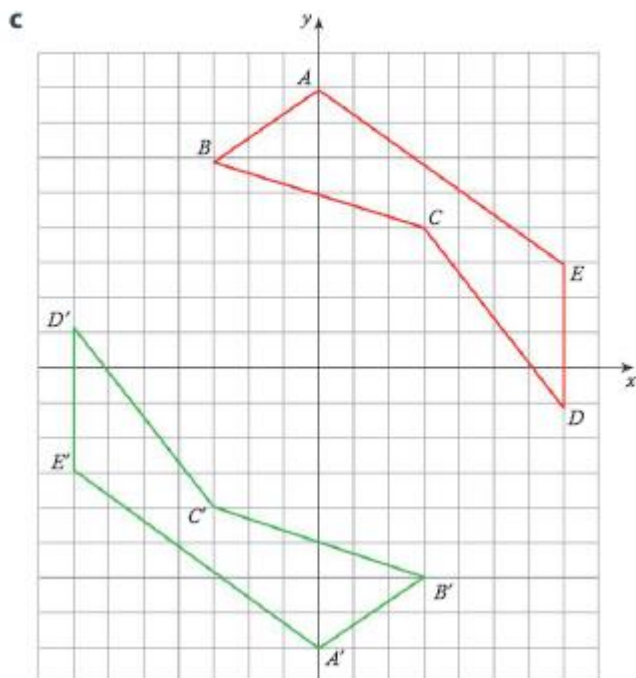


10. Copy and complete this table. All rotations are about the point $(0, 0)$.

Point	Rotation of 90 degrees clockwise	Rotation of 180 degrees clockwise	Rotation of 270 degrees clockwise	Rotation of 360 degrees clockwise	Rotation of -90 degrees clockwise
$(0, 0)$					
$(8, 2)$					
$(7, -6)$					
$(-4, 1)$					
$(-5, -10)$					
$(0, -3)$					
$(9, 0)$					

11. Describe the transformation that has been performed in each diagram and express it using correct arrow notation.





12. Given the rotation $(x, y) \rightarrow (y, -x)$, find the coordinates of the image of the given points and describe the rotation in words.
- $(7, 14)$
 - $(-12, -5)$
 - $(0, 3)$
13. Given the reflection $(x, y) \rightarrow (-y, -x)$, find the coordinates of the image of the given points and describe the reflection in words.
- $(0, -9)$
 - $(14, -4)$
 - $(3, 5)$
14. This door is in the Forbidden City, a palace in Beijing, China. Philosophy and religion are expressed in the art and architecture throughout the Forbidden City, as is the power of the

Chinese imperial family. Even colors express ideas and beliefs, with the color red symbolizing power, happiness, luck and wealth.

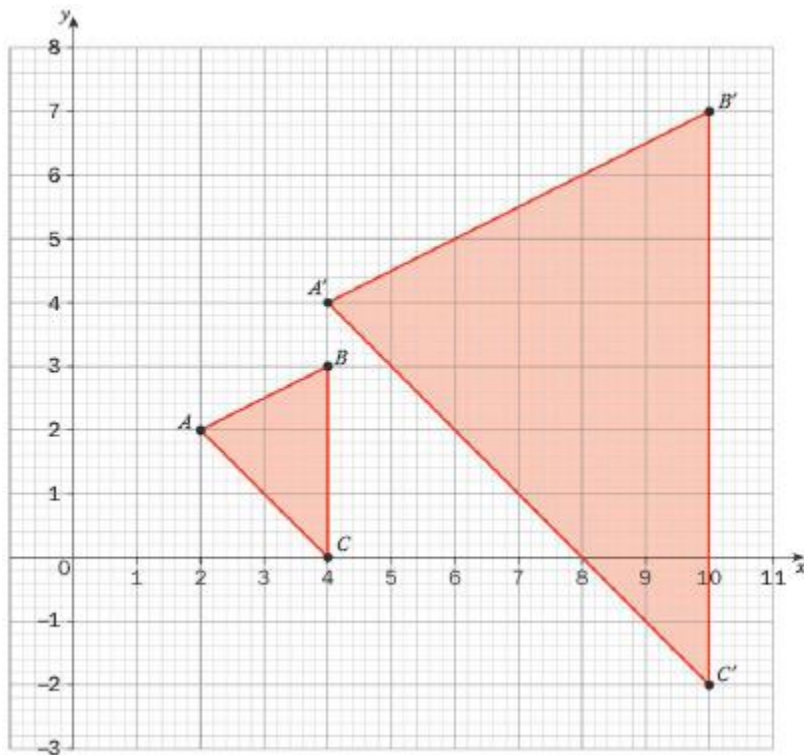
- a. Identify the original shape(s) in this design.
 - b. Describe the transformations that have occurred to the original shape(s).
 - c. Explain how the shapes are tessellated.
15. Is a reflection in the line $y = -x$ the same as a rotation of 180° ? Explain using arrow notation.
16. In ancient Rome, “tesserae” were small squarish tiles that artists used to make bigger pictures that covered floors and walls. This mosaic is a geometric mosaic found in the ancient city of Ephesus, Turkey. Mosaic floors such as this one were used to express wealth and power, since most people could not afford such a luxury. The symbols within mosaics had their own meaning. Some mosaics are meant to depict important events or people. A mosaic like this one is full of symbolism, such as the Solomon’s knot figure (the two intertwined bands) which often expressed a belief in immortality or eternity.
- a. Identify preimage shapes in the mosaic that have been transformed.
 - b. Describe the transformations that have occurred to these preimage shapes.
 - c. Explain how the shapes are tessellated.



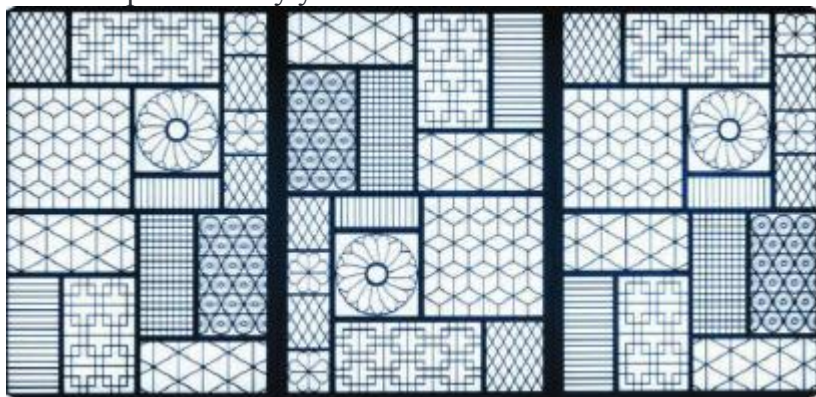
17. What single transformation gives the same result as each of the following?
- a. A reflection in the x -axis followed by a reflection in the y -axis
 - b. A reflection in the line $y = x$ followed by a 90° -degree clockwise rotation about the origin.
18. This Chinese pattern was used on cards, paper and fabric to depict the Chinese New Year of the Rooster in 2017.
- a. If the larger circles have a diameter of 5cm, identify the preimage shape(s) and describe the transformations that were used to create the pattern.
 - b. Describe another transformation that could have been used to create this design.



19. Describe the transformation that has occurred to create the image in the diagram below.

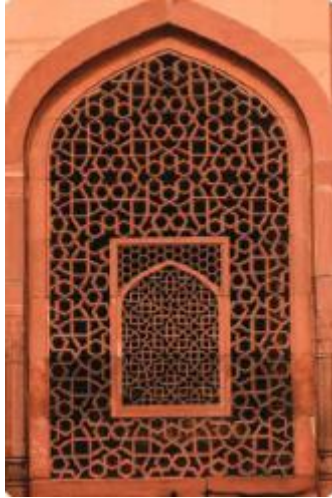


20. This is a wood panel at the Sejong Cultural Center in Seoul, Korea, made as a tribute to ancient panels that were used in traditional Korean houses. Many homes had such geometric designs as window frames and walls. Korean paper, made of mulberry bark, is very strong and was used instead of glass prior to the 19th century. The paper would be replaced every year or so.



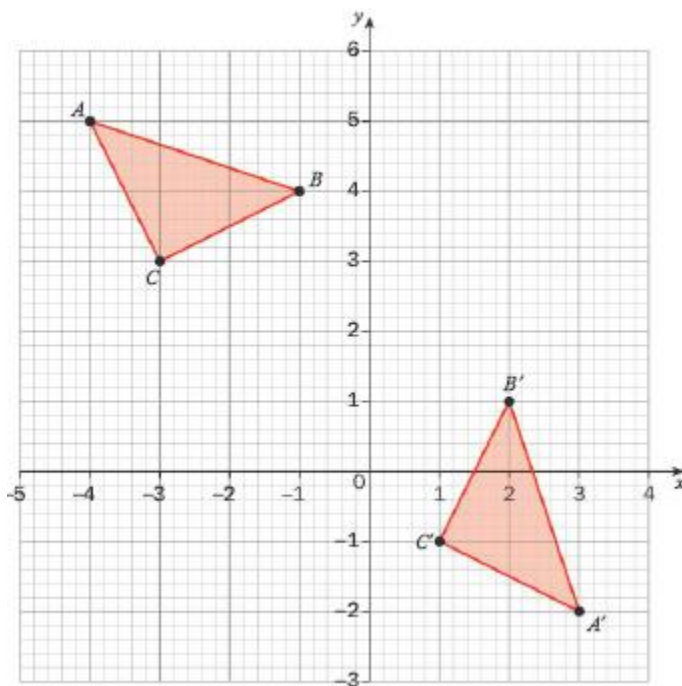
- Select two panels that contain a tessellation that could be created using two different types of transformation.
- Sketch the tessellation in each of your chosen panels. Annotate your sketch to show the original shape and the geometric transformation that has occurred to create each tessellation.

21. This window is built of red sandstone and is part of an ancient monument called Humayun's Tomb in New Delhi, northern India. Built to honor the Emperor Humayun, the window uses geometric shapes much like those used in other Muslim buildings. The six-pointed star (or hexagram) is often associated with Judaism, so it may seem a surprising symbol to see here, but it is likely that it was used to honor and express Humayun's interest in astrology.



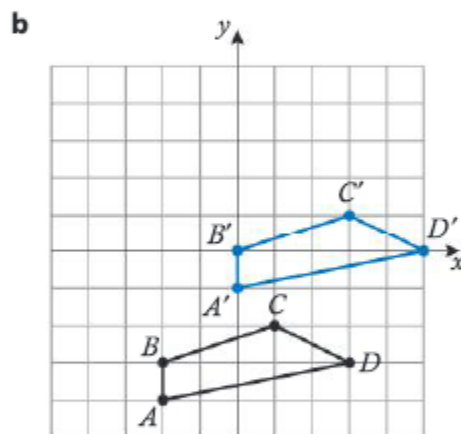
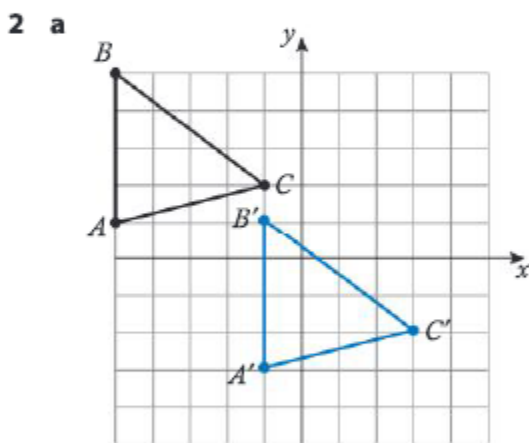
Describe a base shape in this tessellation. Then explain how this shape has been transformed and what other shapes have been added to create the tessellation.

22. The original triangle has undergone three transformations (a rotation followed by a translation, followed by a reflection) to become the image triangle. Identify the three transformations and use appropriate notation to describe them.



Answers:

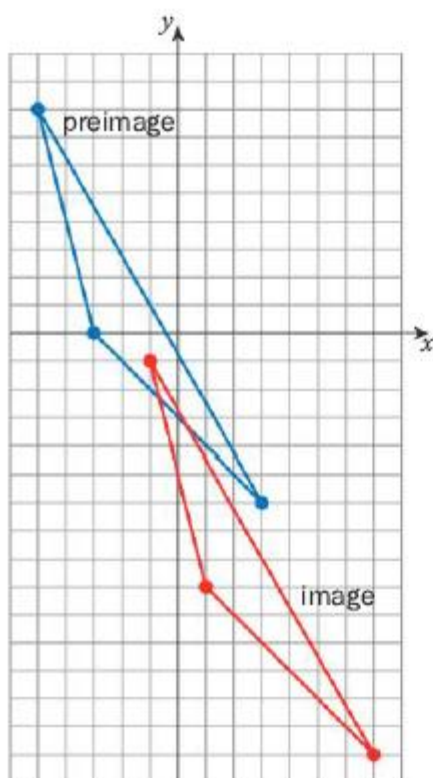
Point	Translation up 3 units	Translation left 5 units	Translation down 2 units and right 7 units	Dilation by a scale factor of 4 about the origin
(0, 0)	(0, 3)	(-5, 0)	(7, -2)	(0, 0)
(6, 1)	(6, 4)	(1, 1)	(13, -1)	(24, 4)
(3, -2)	(3, 1)	(-2, -2)	(10, -4)	(12, -8)
(-4, 7)	(-4, 10)	(-9, 7)	(3, 5)	(-16, 28)
(-8, -5)	(-8, -2)	(-13, -5)	(-1, -7)	(-32, -20)



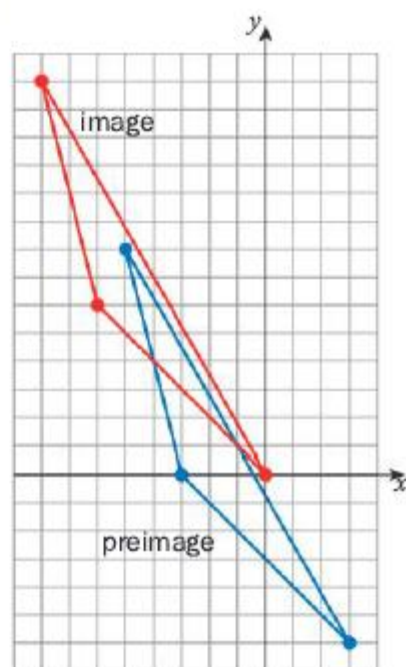
- 3** No, it does not consist of the same pattern repeated
- 4**
- a** Translation 5 units to the left, 6 units up
 - b** Rotation of 90° anticlockwise about the origin
 - c** Reflection in the line $x = 0$ (i.e. the y -axis)
 - d** Dilation of scale factor 2 centered about the origin
 - e** Reflection in the line $x = 1$
- 5** (60, -45)

6 Scale factor $\frac{1}{3}$

7 a



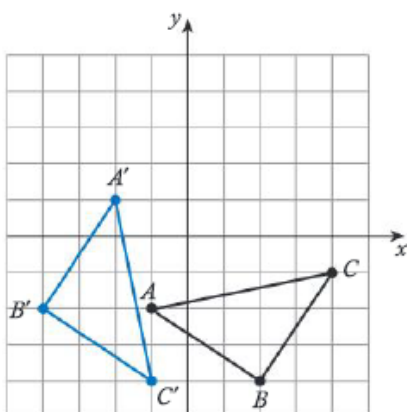
b



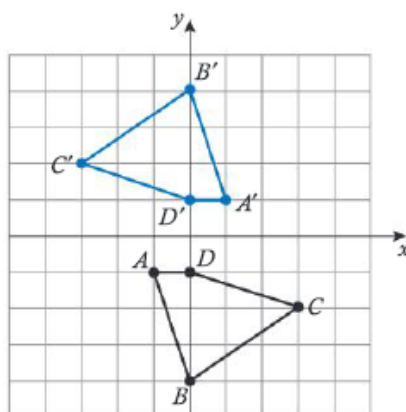
8

Point	Reflection in x-axis	Reflection in y-axis	Reflection in $y = x$	Reflection in $y = -x$	Reflection in $y = 3$	Reflection in $x = -4$
(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 6)	(-8, 0)
(8, 2)	(8, -2)	(-8, 2)	(2, 8)	(-2, -8)	(8, 4)	(-16, 2)
(7, -6)	(7, 6)	(-7, -6)	(-6, 7)	(6, -7)	(7, 12)	(-15, -6)
(-4, 1)	(-4, -1)	(4, 1)	(1, -4)	(-1, 4)	(-4, 5)	(-4, 1)
(-5, -10)	(-5, 10)	(5, -10)	(-10, -5)	(10, 5)	(-5, 16)	(-3, -10)
(0, -3)	(0, 3)	(0, -3)	(-3, 0)	(3, 0)	(0, 9)	(-8, -3)
(9, 0)	(9, 0)	(-9, 0)	(0, 9)	(0, -9)	(9, 6)	(-17, 0)

9 a



b



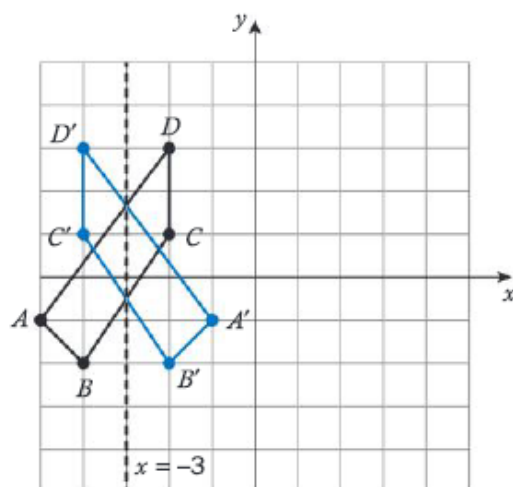
10 a Individual response

b Each square contains reflections and rotations

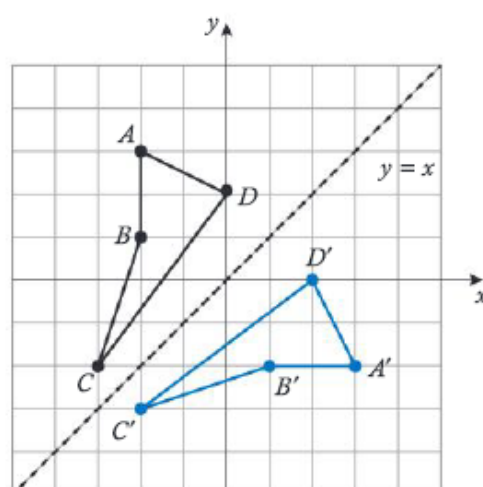
c Each square has been translated a distance equal to its diagonal horizontally and vertically

11	Point	Rotation of 90 degrees	Rotation of 180 degrees	Rotation of 270 degrees	Rotation of 360 degrees	Rotation of -90 degrees
	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	(8, 2)	(2, -8)	(-8, -2)	(-2, 8)	(8, 2)	(-2, 8)
	(7, -6)	(-6, -7)	(-7, 6)	(6, 7)	(7, -6)	(6, 7)
	(-4, 1)	(1, 4)	(4, -1)	(-1, -4)	(-4, 1)	(-1, -4)
	(-5, -10)	(-10, 5)	(5, 10)	(10, -5)	(-5, -10)	(10, -5)
	(0, -3)	(-3, 0)	(0, 3)	(3, 0)	(0, -3)	(3, 0)
	(9, 0)	(0, -9)	(-9, 0)	(0, 9)	(9, 0)	(0, 9)

12 a



b



13 a $(x, y) \mapsto (x-5, y-7)$ b $(x, y) \mapsto (0.5x, 0.5y)$ c $(x, y) \mapsto (-x, -y)$ d $(x, y) \mapsto (x, -y)$

14 a (14, -7) b (-5, 12)

c (3, 0)

Rotation of 90° clockwise about the origin

15 a (9, 0) b (4, -14)

c (-5, -3)

Rotation of 180° about the origin

16 a Individual response

b Reflections and rotations

c Individual response

17 No, the reflection is the transformation $(x, y) \mapsto (-y, -x)$ whereas the rotation is the transformation $(x, y) \mapsto (-x, -y)$

18 a Individual response b Individual response c Individual response

19 a Rotation of 180° about the origin

b Reflect in the line $y = 0$ i.e. the x -axis

20 a Individual response b Individual response

Linear Systems

1. Solve each equation and verify your answers.

a $2x + 7 = 11$

b $3(5 + 2x) = -21$

c $5(x + 4) = 3(x - 3) + 5$

d $\frac{x+3}{4} = \frac{x-1}{2}$

e $\frac{1}{2}y - 3 = \frac{2}{3}y + 4$

f $\frac{3}{4}(y+2) = 2y - 11$

2. Sketch the following graphs and write down the coordinates of each point of intersection.

a $y = -3x + 10$ and $y = 2x - 5$

b $y = 2x - 6$ and $y = \frac{3}{4}x - 6$

3. Find the exact intersection point of the following two linear systems using technology.

a $y = x - 8$ and $y = -\frac{1}{3}x - 4$

b $x = 5y - 75$ and $x + \frac{1}{2}y - 7 = 0$

4. Solve the following linear systems algebraically.

a $y = 6x + 7$ and $3y + 12x - 6 = 0$

b $3a + 2b = 16$ and $2a + 3b = 14$

c $14x + 21 = -21y$ and $2x + 3y = 12$

d $2y - 4 = x$ and $3y - 6x = -3$

e $6x = 2y - 8$ and $5y - 5x + 10 = 0$

5. Classify each of the following linear systems and justify your answers.

a $4x - 3y = 15$ and $8x - 9y = 15$

b $4x - 3y = 5$ and $8x - 6y = 10$

c $2x = 3y - 6$ and $4x - 6y = 24$

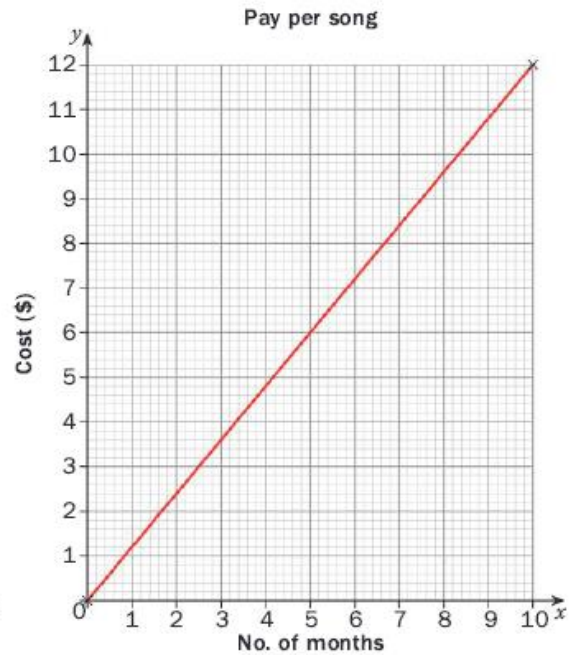
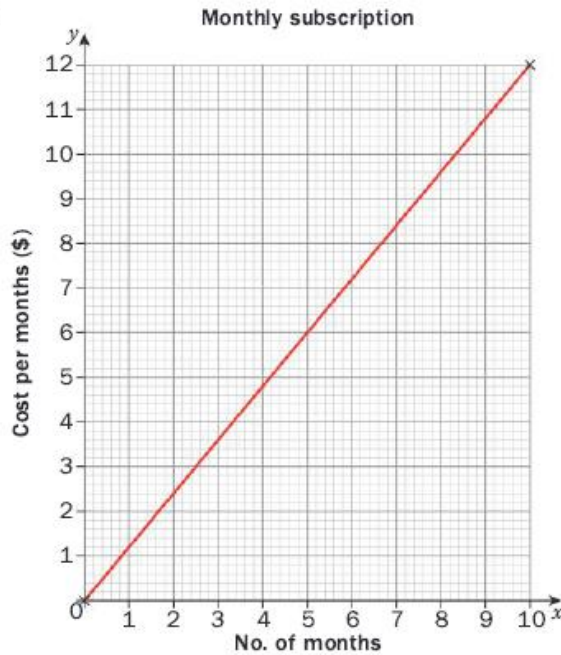
6. When the sum of four times a number and ten is divided by five, the result is negative 2. Find the number.
7. Two consecutive integers are both smaller than 20. If three times the smaller integer is equal to eight less than double the larger integer, find both integers.
8. A four-sided figure with a perimeter of 120cm has a width which is 30cm less than twice its length. What kind of quadrilateral is this? Justify your answer.
9. The difference between two numbers is divided by 2 and the result is 3. Find these two numbers if triple the smaller number is 4 less than double the larger number.
10. You have three dollars' worth of coins in your piggy bank consisting of nickels, dimes and quarters. The number of dimes is double the number of nickels. The number of quarters is one-third of the number of nickels. How many coins are in your piggy bank in total?
(A nickel is 5 cents, a dime is 10 cents and a quarter is 25 cents.)
11. You could use a music streaming service and pay \$9.99 a month or pay an average of \$1.14 for each song you download.
 - a. Graph this scenario using technology.
 - b. How many songs a month would you have to download to make the streaming service the cheaper option?
 - c. Can you think of why you would choose the pay-per-song option instead, even if you downloaded 20 songs a month?
12. Some teachers are taking a group of Year 3 students to a talk on sustainable supply chains at which some social entrepreneurs will speak. 42 tickets cost \$582 in total. If each student's ticket costs \$12 and each teacher's ticket costs \$25, use an algebraic method to find the number of teachers and students going to the talk.
13. A microlender invested AU\$20000 in a combination of two ventures, one with low risk yielding 3% per annum and one with higher risk yielding 8% per annum. If the interest after one year was AU\$850, calculate algebraically the amount invested in each venture.
14. In some communities in the United States, local food trucks have started giving out meals to the homeless. While the trucks still have delicious meals for sale, people who cannot afford them can get a meal for free. The cost to make each meal is \$2.50 and it is sold for \$8. The food truck costs \$3000 per month to operate. They must make a profit of at least \$2000 each month to cover the costs of the free meals. What is the least number of meals they need to make and sell in a month (assume 30 days) to cover the cost of the free meals?
15. In some areas of the world, you may be living on land where you could drill for your own gas. A farmer pays \$5000 for natural gas each year. If you drill yourself, it costs \$40000 to start up your own gas line and you could drill enough to produce \$7500 worth of gas each year and then sell what you don't use into the grid. How many years would it take to pay off the gas line costs? Would this be a good investment? What other factors would you have to consider?
16. You are boxing up care packages to send to developing countries. The first type contains bed nets to prevent mosquito bites and the spread of malaria (€10 each) and solar power kits (€60 each) to provide light in areas with restricted or no electricity. The second

contains medicine kits for mothers and babies (€20 each) and baby blankets and supplies (€15 each). There are 140 items in the first package. In the second package, the number of baby blankets is twice the number of medicine kits. If the package with the medicine kits and blankets cost €3000 and the package with the nets and solar kits cost €3400, calculate how many of each of the four items there are in the packages.

Answers:

- 1 **a** $x = 2$ **b** $x = -6$ **c** $x = -12$ **d** $x = 5$
e $y = -42$ **f** $y = 10$
- 2 **a** $x = 3, y = 1$ **b** $x = 0, y = -6$
- 3 **a** $x = 3, y = -5$ **b** $x = -\frac{5}{11}, y = \frac{164}{11}$
- 4 **a** $x = -\frac{1}{2}, y = 4$ **b** $a = 4, b = 2$ **c** No solution **d** $x = 2, y = 3$
e $x = -3, y = -5$
- 5 **a** One solution **b** Infinitely many solutions **c** No solution
- 6 $x = -5$
- 7 $-6, -5$
- 8 Square; all the sides are 30 cm long.
- 9 $x = 14, y = 8$
- 10 30 coins; 9 nickels, 18 dimes and 3 quarters

11 a



b 9

c Individual response

12 6 teachers, 36 students

13 AU\$15000 at 3%, AU\$5000 at 8%

14 910 meals that are paid for and 800 free meals so a total of 1710 meals.

15 16 years

16 100 nets, 40 solar kits, 60 medicine kits, 120 blankets.

17. More companies are starting corporate social responsibility (CSR) reporting, sometimes referred to as the triple bottom line (triple for “people, planet and profit”). This means that the companies take the initiative to ensure that their supply chains are ethically and environmentally responsible and that they have positive interactions with the communities in which they operate. Unfortunately, profit is still the driving force of many companies and in order to cut costs, employees, especially those in developing countries, may work in dangerous and unhealthy conditions. Pollution in manufacturing and shipping can be extensive and the Earth’s natural resources can be used in an unsustainable way.

Two premium denim jean factories in Asia have the following costs per month. The first one is a socially responsible factory and the second is a traditional factory of a type still common in Asia.

Cost	Socially responsible factory	Traditional factory
Materials per pair of jeans	\$12*	\$7
Wash/dye process per pair of jeans	\$2**	\$0.25
Labour cost per pair of jeans	\$1.50***	\$0.50
Rental of building, utilities, admin costs, programs, etc.	\$10000****	\$5800

* All raw materials that go into making the denim jeans meet the highest ethical and environmental standards.

**The factory uses organic cotton and a water recycling system to reduce the environmental impact and not contaminate local community water supplies. It also uses non-toxic organic vegetable dye to ensure that no harm is done to the workers, rather than the harmful chemical dyes used in traditional jean manufacturing.

*** Workers earn a living wage, which is 3 times the average of workers in the industry, allowing them to better support themselves and their families and even to save for the future.

**** Fixed costs are higher due to better ventilation, light, space and general facilities in the factory. English language courses and training courses in all components of jean production require more time and every worker is rotated to ensure they are skilled in every component of jean making.

Build a linear systems model:

- Derive a cost function for both factories.
- If the jeans are sold to the wholesaler for \$30 a pair, determine a revenue function.
- Graph the cost functions and the revenue functions of both factories on one set of axes.
- What is the break-even point for both factories? Comment on the difference between the two.
- Verify your answer by solving the system algebraically. Show your working.
- At what price would the socially responsible factory have to sell a pair of jeans if they wanted the same break-even point as the traditional factory? Round your answer appropriately and explain the degree of accuracy.
- How much more is this per pair as a percentage of the original price?