MYP
REVISION BOOKLET
2019 -20

SUBJECT - Mathematics Standard

Part 4

Index

Unit 1: NUMBERS - PERCENTAGES

Mixed practice

1 Three times the reciprocal of a number equals nine times the reciprocal of 6. **Find** the number.

2 Angles x and $\frac{3}{4}x$ are complementary. **Find** x.

3 The denominator of a fraction is 1 less than twice the numerator. When 7 is added to both the numerator and denominator, the resulting fraction is equivalent to $\frac{7}{10}$. **Find** the original fraction.

4 Sophie leaves school and travels east at an average speed of 40 km/h. Jenny leaves school one hour later, and travels west at an average speed of 50 km/h. **Determine** how many hours after Jenny leaves school they will be 400 km apart.

5 Kim and Tom both cycle to work from their homes, at the same speed. Kim lives 20 km away from work, and Tom lives only 15 km away from work. It takes Kim 1 hour more to cycle to work than Tom. **Determine** how long Tom takes to get to work.

6 **Determine** how many milligrams of a metal containing 45% nickel must be combined with 6 milligrams of pure nickel to form an alloy containing 78% nickel.

7 Two different fruit juices are mixed together. 7 liters of the first fruit juice contains 60% apple juice, and 5 liters of the second fruit juice contains 30% apple juice. **Find** the percentage of the new mix that is apple juice.

8 Chloe can type a story in 6 hours. If Molly helps her, they finish typing the story in 4 hours. **Find** how long it would take Molly to type the story on her own.

9 Mary takes 8 days to do a full check of the library's online catalog. Her assistant takes 12 days. If they work together, **determine** the fraction of the catalog that is checked after 4 days.

10 The number of hamsters in a pet store doubles each month. Today, there are 20 hamsters.

a Supposing that no hamster dies and no hamster gets sold, **determine** in how many months the store will have 640 hamsters.

b If 10 hamsters are sold at the end of each month, after the number of hamsters double, **find** how many hamsters are left after 5 months.

11 Sacha picked some apples from a tree in her garden. She then gave some to her friends. To her first friend, she gave half the apples she had, plus 2 more. To her second friend, she gave half the apples she had left, plus 2 more. And she did the same thing for her third friend. After giving away all these apples, she had 1 apple left. **Find** how many apples she originally pick from the tree.

12 A house and a piece of land are sold for €850 000. The house is sold for one and a half times as much as the land. **Find** how much the house sold for.

Review in context

Scientific and technical innovation

Many people enjoy mathematical puzzles, not just mathematicians. Some puzzles have led to interesting discoveries in mathematical theory, or other fields entirely. Use problem-solving strategies to solve these famous historical puzzles.

1 Rice on a chessboard

Sissa ben Dahir, Grand Vizier to the Indian King Shirham, invented the game of chess. Shirham liked the game so much, that he asked Sissa what he wanted as a reward. Sissa replied: 'I would like to cover the chessboard with 1 grain of rice on the first square, 2 grains on the second, 4 grains on the third, 8 grains on the fourth, and so on.' The King accepted immediately, thinking he had himself a bargain.

How much rice would there be on the chessboard?

Answer these questions to help work it out.

a **Find** the number of grains of rice on:

i the 8th square

ii the 16th square

iii the 64th square.

How does it help to know that each square has double the grains of rice as the previous square?

b About 50 grains of rice fit in 1 cm^2.
Leaving your answer correct to 2 d.p., **find** how many cm^2 of rice are on:

i the 8th square

ii the 16th square

iii the 24th square.

c 10 000 cm^2 = 1 m^2.

Find how many m^2 of rice are on the 24th square.

d 1 000 000 m^2 = 1 km^2.

Find how many km^2 of rice are on the 64th square. Leave your answer correct to 2 d.p.

e Challenge: **Find** the total number of grains of rice in:

i the first row

ii the first two rows

iii the first three rows

iv the entire chessboard.

2 The Tower of Hanoi

French mathematician Edouard Lucas invented this puzzle in 1883.

This stack of disks in order from the largest to the smallest is called a tower. The aim of the puzzle is to move a tower from one rod to another, obeying the following rules:

Rule 1 Only one disk can be moved at a time.

Rule 2 One move consists of taking the top disk from one stack and placing it on top of a stack on another rod.

Rule 3 A disk can be placed only on top of a larger disk. No disk can be placed on top of a smaller disk.

The minimum number of disks in a tower is 3. The original problem had 64 disks.

a **Find** the minimum number of moves needed to move a tower of:

i 3 disks

ii 4 disks

iii 5 disks.

You could use different sized coins or counters to model the tower.

b Suppose each move takes 1 second. **Find** the minimum time it would take to move a tower of:

i 3 disks

ii 5 disks

iii 6 disks.

c The minimum number of moves for a tower of n disks is $2^n - 1$. **Find** the minimum time it would take to move a tower of:

i 10 disks

ii 20 disks

3 Rope around the Earth

The English philosopher and mathematician William Whiston first posed this in 1702.

Part 1: Wrap a rope around the equator of a basketball. What length of rope would need to be added to make the rope hover 1 inch away from the basketball's equator at all points?

Part 2: Wrap a very, very long rope around the equator of the earth. What length of rope would need to be added to make the rope hover 1 inch above the earth's equator at all points?

You may find this information useful.

- the diameter of a basketball is about 24 cm.
- the radius of the earth at the equator is 6378 km.

a Before calculating the amount of rope to be added in each part of the problem, **suggest** an amount that seems reasonable to you.

b Write down an expression to represent the circumference of a sphere with radius r.

c Write down an expression to represent the circumference of a sphere with radius (r + 1in). Simplify this expression.

d Hence, **find** how much string needs to be added to the circumference when adding 1in to the radius of a sphere.

e **Determine** whether the size of the sphere matters in part **d**. Justify your answer. Now you can answer the riddle. Are you surprised?

4 24 Game

In 1988, inventor Robert Sun created the 24 Game to make mathematics appealing and accessible to children. The rules of the game are:

Rule 1 Choose four numbers between 1 and 13 at random.

Rule 2 Use all four numbers with mathematica operations and brackets, to make a total of 24.

Use problem-solving strategies to make 24 from each set:

a 7, 2, 1, 1

b 2, 3, 2, 4

c 2, 3, 4, 6

d 2, 2, 7, 12

UNIT: SETS

Mixed practice

1 **Write down** the elements of each set in list form.

a A is the set of names of all continents.

b B is the set of names of the three tallest mountains in the world.

c C is the set of names of the three tallest buildings in the world.

d D is the set of positive integers between 20 and 50, including 20 and 50, that are multiples of 5.

2 **State** whether each set is finite or infinite. If the set is finite, **state** its cardinality.

a $\{x \mid x \in \mathbb{N}, x \text{ is a multiple of } 4\}$

b $\{x \mid x \in \mathbb{R}, -2 \leq x \leq 5\}$

c $\{y \mid y \in \mathbb{Z}, -3 < y \leq 2\}$

d $\{b \mid b \in \mathbb{N}, b < 7\}$

3 **Write down** these sets using set builder notation:

a F is the set of integers greater than zero

b $G = \{3, 6, 9, \ldots, 21\}$

c $H = \{1, 4, 9, 16, \ldots\}$

d $J = \{1, 8, 27, \ldots, 1000\}$

4 $U = \{x \mid x \in \mathbb{N}\}$,
$A = \{x \mid x \in \mathbb{N}, x \text{ is a multiple of } 7\}$,
$B = \{1, 3, 5, 7, 9, \ldots, 19\}$ and
$C = \{x \mid x \in \text{primes } 5 < x \leq 20\}$.

Find:

a $A \cap B$

b $A' \cap C$

c $A \cap B'$

d $(A \cap C)'$

5 **Create** Venn diagrams to represent the sets:

a $A \cap B'$

b $A' \cup B$

6 The table shows the facilities at three hotels in a resort.

	Hotel A	Hotel B	Hotel C
Pool	✓		✓
Bar	✓	✓	✓
Jacuzzi		✓	
Sauna	✓		
Tennis		✓	✓
Gym			✓

Draw a Venn diagram to represent this information.

7 **Create** a Venn diagram to illustrate the relationships of the following number sets: $\mathbb{R} = \{\text{real numbers}\}$, $\mathbb{Z} = \{\text{integers}\}$, $\mathbb{Q} = \{\text{rational numbers}\}$, $I = \{\text{irrational numbers}\}$, $\mathbb{N} = \{\text{natural numbers}\}$, and $P = \{\text{prime numbers}\}$.

8 **Create** a Venn diagram illustrating the relationships of the following geometric figures: $U = \{\text{all quadrilaterals}\}$, $P = \{\text{parallelograms}\}$, $R = \{\text{rectangles}\}$, $S = \{\text{squares}\}$, $K = \{\text{kites}\}$, $T = \{\text{trapezoids}\}$.

9 If $U = \{-10, -9, \ldots, 9, 10\}$, $A = \{0, 1, 2, \ldots, 9\}$, $B = \{-9, -8, \ldots, 0\}$ and $C = \{-5, -4, \ldots, 4, 5\}$, **list** the elements of the following sets:

a $A \cap B$ **b** $(B \cup C)'$

c $(A \cup B) \cap C$ **d** $A' \cap (B \cup C)$

e $(A \cap B) \cup (A \cap C)$

10 **Determine** whether the following statements are true or false. If false, explain why.

a $0 \in \mathbb{Q}$

b $\{\text{primes}\} \subseteq \{\text{odd integers}\}$

c If $U = \mathbb{N}$, then $(\mathbb{Z}^+ \cap \mathbb{N})' = \{0\}$

d $2 \subseteq \{\text{primes}\}$

Problem solving

11 In a school sports day, medals were awarded as follows: 36 in running, 12 in high jump, and 18 in discus. The medals were awarded to a total of 45 students, and only 4 students received medals in all three events. **Determine** how many students received medals in exactly two out of three events.

12 A survey of 39 university students found:

- 10 worked part-time while studying
- 18 received financial help from home
- 19 withdrew money from their savings as needed
- 2 financed themselves from all three sources
- 12 received financial help from home only
- 5 received financial help from home and withdrew money from savings
- 6 financed themselves only by working part-time and withdrawing money from savings.

Determine how many students:

a did not finance themselves using any of the three resources

b worked part-time and received money from home

c received financial help from home and withdrew money from their savings

d financed themselves using only one of the three ways surveyed.

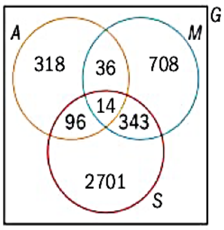
Review in context

Scientific and technical innovation

1 Below is a Venn diagram used by medical researchers showing the genes associated with different brain diseases.

The sets represent the number of genes associated with:

A = {Alzheimer's disease}
M = {multiple sclerosis}
S = {stroke}
G = {brain diseases}



Using this diagram:

a **Calculate** how many genes in total are associated with each of the three diseases: Alzheimer's, multiple sclerosis, and stroke.

b **State** how many genes all three diseases share in common.

c **Explain** how this diagram might be useful to medical researchers.

Problem solving

2 Venn diagrams are widely used to show 'overlapping' concepts. Here are two fields from Sociology and Environmental Science where a Venn diagram can be used to illustrate the interrelation of key concepts.

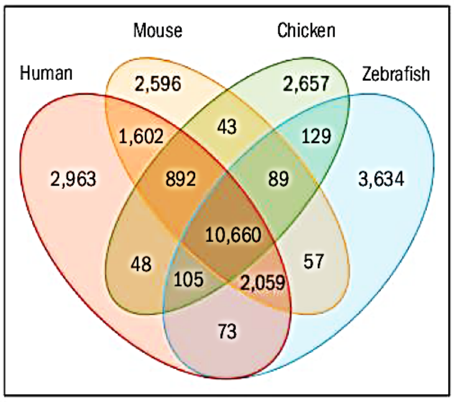
In each example, **identify** sets and **draw** a Venn diagram to illustrate the information given.

a According to Plato there are many propositions. Some of these are true and some are beliefs (some may be neither). Only true beliefs can be justified and those he defines as knowledge.

b There are three main types of development: environmental, social and economic. Development that is both social and economic is equitable, that which is economic and environmental is viable, and that which is environmental and social is bearable. Development that is all three of these is sustainable.

3 Scientists have studied the genome of a number of organisms. The genome contains all the information used to build and maintain that organism. For medical researchers, knowing that different species share particular genes will enable them to unlock the secrets of countless diseases.

The Venn diagram shows the genes of four species: human, mouse, chicken, and zebrafish.



Using the diagram:

a **Determine** how many genes are shared by all four species.

b **Determine** how many genes does a human have in total.

c **Determine** how many genes are shared by human and mouse, by human and chicken, and by human and zebrafish.

d **Determine** the percentage of genes shared by the human and each of these three species.

e If you were a medical scientist and you wanted to conduct research into a species that was genetically closest to the human, which of the three – mouse, chicken or zebrafish – would you choose?

UNIT 2: ALGEBRA – FUNCTIONS

Summary

In an **ordered pair** (x, y), the first term represents an object from a first set and the second term represents an object from a second set.

A **relation** is a set of ordered pairs: $\{(x, y) \mid x \in A, y \in B\}$.

It has three components:

- a relation or rule that maps x onto y for each ordered pair in the relation
- a set A that contains all the x elements of each ordered pair
- a set B that contains all the y elements.

A relation maps set A onto set B.

A **mapping diagram** shows how the elements in a relation are paired. Each set is represented by an oval, and lines or arrows are drawn from elements in the first set to elements in the second set for each ordered pair in the relation.

A **function** is a relation where each element in set A maps to one and only one element in set B.

A function can be written as $f(x) = y$, where:

- x is the input value, $x \in A$
- y is the output value, $y \in B$
- f is the name of the function that maps x to y.

$f(x) = y$ is read 'the function f of x is y' or just 'f of x is y'.

Another way of writing $f(x) = y$ is $f : x \mapsto y$.

The **domain** of a function is the set of input values that the function can take.

The **range** of a function is the set of all the output values that the function generates. The range is also called the set of images of the elements in the domain.

A **natural** domain is the largest possible set of values that a function can take.

A **restricted** domain is a subset of the natural domain of the function.

The **vertical line test**: if no vertical line intersects a graph at more than one point, the relation is a function. If a vertical line intersects the graph at more than one point, then the relation is not a function.

A function is **defined** by a mathematical expression that specifies the relationship between a domain (input values) and range (output values).

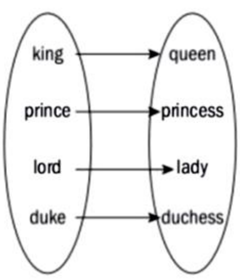
Evaluating a function means finding the element of the range that corresponds to a given element in the domain.

Solving an equation that is a function means finding the element of the domain that corresponds to a given element in the range.

Mixed practice

1 **Determine** whether each diagram represents a relation or a function. **Justify** your answer.

a



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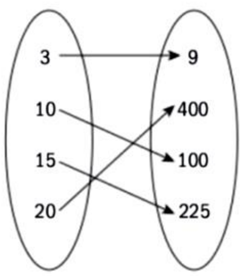

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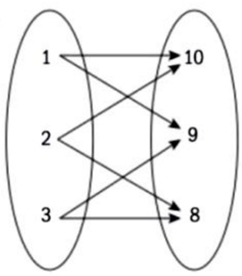
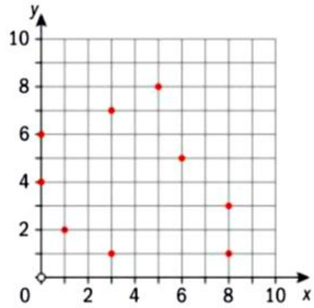


Determine whether each diagram represents a relation or a function. **Justify** your answer.

d



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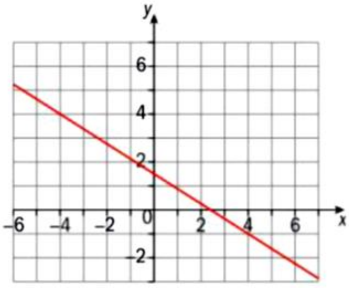

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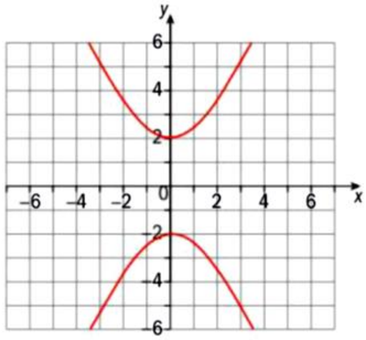

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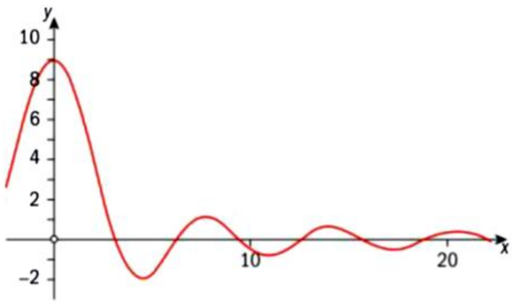

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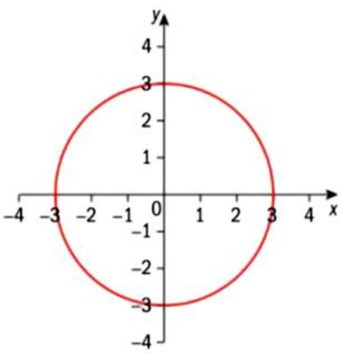

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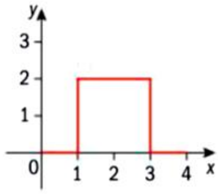

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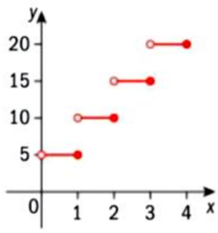


2 **State** the domain and range:

a $\{(1, 1), (2, 4), (-2, -4)\}$

b $\{(3, 5), (4, 5), (5, 6)\}$

c $\{(-2, -3), (-3, -2), (-2, 5)\}$

d $\{(1, 1), (2, 3), (5, 8)\}$

3 **Find** the largest possible domain and range:

a $f(x) = 3x - 8$ **b** $f(x) = 3\sqrt{x}$

c $f(x) = \frac{x}{4}$ **d** $f(x) = \frac{4}{x}$

e $f(x) = \frac{4}{x-4}$ **f** $f(x) = x^2 - 1$

4 **Evaluate** each function at the given values:

a $a(x) = 2 - x^2$

i $a(2)$ **ii** $a(0)$ **iii** $a(-2)$

b $b(x) = 8x - 4$

i $b(4)$ **ii** $b(3)$ **iii** $b(1)$

c $c(x) = \sqrt{x-1}$

i $c(1)$ **ii** $c(5)$ **iii** $c(0)$

d $d(x) = 2x + 3$

i $d(4)$ **ii** $d(4x)$ **iii** $d(2 - x)$

5 **Solve** each function at the given values:

a $f(x)=4x-2$

i $f(x)=18$ **ii** $f(x)=0$ **iii** $f(x)=2$

b $f(x)=\frac{1}{x}$

i $f(x)=\frac{1}{5}$ **ii** $f(x)=-\frac{1}{4}$ **iii** $f(x)=2$

c $f(x)=\sqrt{x}$

i $f(x)=5$ **ii** $f(x)=1$ **iii** $f(x)=9$

6 A computer program asks you for any word and returns the number of letters in that word.

a **Justify** why this computer program is a function.

b **Write down** what this computer program does in function notation, where the function is called 'p' for program.

c **Evaluate**:

i p(horse) **ii** p(Mississippi) **iii** p(hi)

d **Find** x such that

i $p(x)=3$ **ii** $p(x)=8$

e What is p(three)?

f For what numbers $0 \le x \le 10$ does $p(x)=x$?

7 A hospital has 9 floors above ground (including ground floor) and 3 floors below ground. $T(n)$ gives the average number of times the elevator stops at the nth floor each day.

a **Justify** why $T(n)$ is a function.

b **Find** the largest possible domain of $T(n)$.

c For each floor in the domain, **suggest** the number of times the elevator stops at that floor during 24 hours. Write your answer as a list of ordered pairs.

d Hence, **suggest** a possible range of $T(n)$. **Justify** your choice.

8 The table shows postage rates for letters and parcels.

Letters	
Weight less than	Price
50 g	£0.65
100 g	£0.92
200 g	£1.20
Small parcels	
Weight less than	Price
100 g	£1.90
200 g	£2.35
500 g	£3.40
750 g	£4.50
1 kg	£5.00

a **Draw** a mapping diagram to represent the weights of items and the delivery charge.

b Decide whether the weight determines the price or the price determines the weight. Add arrows to your mapping diagram to show this.

c **Determine** whether this relation is a function.

Review in context

Fairness and development

1 Many universities use a test in their admissions decisions. The results of one such test were paired with family income as reported by test-takers, shown in the table here:

Family income ($)	Total test score (max. 2400)
10 000	1326
30 000	1402
50 000	1461
70 000	1497
90 000	1535
110 000	1569
130 000	1581
150 000	1604
180 000	1625
More than 180 000	1714

a **Draw** a mapping diagram to represent the family income and the total test score.

b **Determine** whether or not this relation is a function.

c **Suggest** reasons why this relationship may exist.

d Should universities use this information in their admission decisions? If so, how?

2 The table shows the percentage of world population in different countries and their use of natural resources.

Country	Percentage of world population	Percentage of world's resources used
Brazil	3	2
China	19	20
India	17	5
Indonesia	4	1
Japan	3	4
Pakistan	3	0.5
Russian Federation	3	6
United States	5	18

a **State** the input variable and the output variable and the domain and range of this relation.

b Is this relation a function? **Explain**.

c **Draw** a mapping diagram of this relation.

d Does this demonstrate an equitable or fair relationship? **Explain** your reasoning.

e What action, if any, do you think should be taken?

3 Car insurance rates vary greatly depending on your age, the kind of car you drive and other factors. For example, male drivers under the age of 22 in one city can calculate their averag annual rate by multiplying their age by $1000 and subtracting that from $24 000.

a **Write down** the equation for the average annual cost of insurance (C) for a young m based on his age (a).

b **State** what $C(18)$ represents. Then find $C(1$

c For males under the age of 22, is the annua average cost of car insurance a function of age?

d Is charging different rates depending on the age of the driver a fair practice? **Explain**.

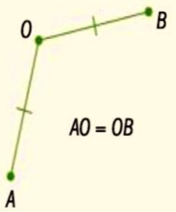
e **State** what $C(21)$ represents. Then find $C(2$

f **Suggest** reasons why this function is only valid for drivers under the age of 24.

UNIT 3: GEOMETRY & TRIGONOMETRY- COORDINATE GEOMETRY

Summary

- **Deductive reasoning** is the process of taking previously known or proven facts and putting them together to make new ones.
- *A* and *B* are **equidistant** from *O* if the distances *OA* and *OB* are equal.



- The shortest distance between a point and a line is measured along a line perpendicular to the line.
- A proof by contradiction involves applying a logical process to a hypothesis and producing a result that is obviously false, or contradicts your assumption. If this occurs, then the original assumption must have been incorrect.
- The midpoint of (a, b) and (c, d) is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.
- The process of justifying a claim by providing step-by-step reasons is known as **deduction**.
- A **deductive argument** is any proof or collection of reasoning that uses deduction to draw a conclusion.
- The distance formula states that the distance, d, between two points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Mixed practice

1 Find the distance between each pair of points.

a (0, 0) and (5, 12)

b (1, 4) and (4, 8)

c (5, 7) and (11, 7)

d (−3, −12) and (4, 12)

2 Find the distance between each pair of points, giving your answer correct to 3 significant figures.

a (4, 8) and (10, 13)

b (14, 8) and (17, −3)

c (1.5, 4.6) and (2.3, −1.8)

3 Find the distance between each pair of points, giving your answer in the form $\sqrt{n}$, where n is a whole number.

a (2, 7) and (6, 15)

b (−3, 9) and (5, −2)

c (−3, −5) and (−9, −14)

4 Find the midpoint of each pair of points.

a (4, 2) and (6, 12)

b (−3, 9) and (5, −2)

c (−3, −5) and (−8, −14)

d (2.3, 5.9) and (−3.6, 11.2)

e $\left(\frac{1}{4}, \frac{1}{3}\right)$ and $\left(\frac{1}{3}, \frac{1}{2}\right)$

5 Prove that the points (0, 0), (6, 0), and (3, 4) form an isosceles triangle.

6 Three points have coordinates $A(-4, -1)$, $B(1, 4)$, and $C(-5, 6)$.

Show that the points form an isosceles triangle.

7 Point A has coordinates (3, 5). **Construct** the line through A that is perpendicular to the line $y = -2x + 3$.

Find the shortest distance from A to the line $y = -2x + 3$. Give your answer to 1 dp.

8 A has coordinates $(-1, 3)$ and B has coordinates (4, 1).

Determine which point is closer to the line $y = x$.

9 Given the points $A(a, b)$ and $B(c, d)$, **prove** that the point $\left(\frac{3a+c}{4}, \frac{3b+d}{4}\right)$ divides the line segment AB in the ratio 1 : 3.

10 Point A has coordinates (a, b) and point B has coordinates (c, d).

M is the midpoint of AB.

a **Find** the coordinates of M.

b **Find** the distance AB.

c **Find** the distance AM.

d Hence, **prove** that $AM = \frac{1}{2}AB$

11 ABC is a triangle with vertices $A(1, 0)$, $B(5, 9)$ and $C(9, 0)$.

a **Show** that triangle ABC is isosceles.

b **Find** the coordinates of the midpoints of the three sides of ABC.

c **Show** that the midpoints of the three sides of ABC are the vertices of an isosceles triangle.

d **Show** that the isosceles triangle formed from the midpoints of the sides of ABC has area 8 square units.

12 In a Sudoku puzzle, players work with a set of digits. Most puzzles use the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Every row, every column, and every medium-sized square must contain all of the digits in the set.

The Sudoku puzzles in this question use the set $\{1, 2, 3, 4\}$.

a By considering the positions of the 1s and 2s **show** that it is not possible to complete this puzzle:

1	4	2	
	2	1	
3			
2	1		

b Copy this grid:

1	4	3	
			4
2	1		

i **Explain** how you can be certain that the top right-hand corner must contain a 2.

ii Hence **determine** the possible positions of the other two 2s in the grid.

Justify your answer.

iii **Deduce** the digits that should appear in the grids remaining cells.

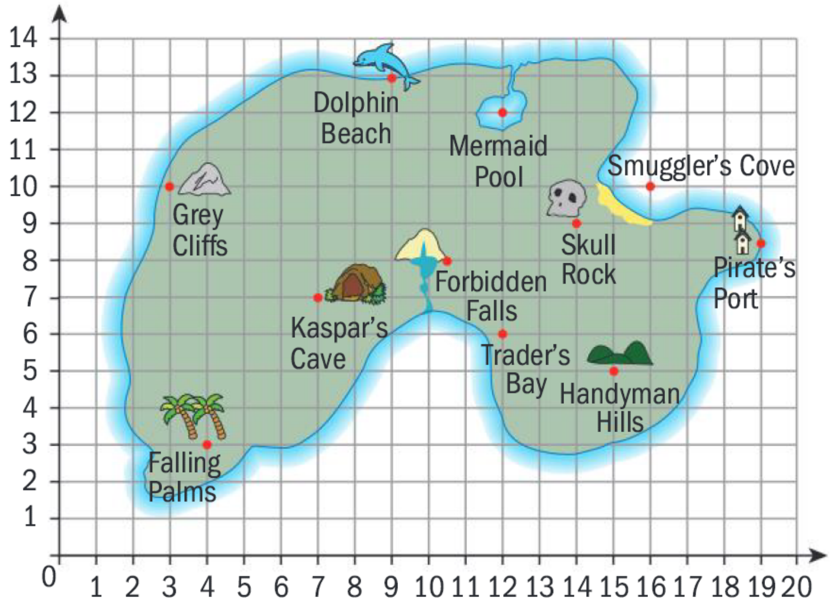
Problem solving

13 Point X has coordinates (4, 7) and is the midpoint of AB. Given that A has coordinates $(-3, 11)$, **find** the coordinates of B.

Review in context

Orientation in space and time

You are lucky enough to have found the only surviving copy of a treasure map left by the notorious pira Short James Platinum - scourge of the seven seas. He has buried many pieces of treasure on the island - ea in a different location. Can you find them all? (On the map, one unit equals one league.)



1 The Tsarina's Tiara is buried due west of Skull Rock. It is as far from Skull Rock to the Tiara as it is from Falling Palms to Kaspar's Cave.

Find the coordinates of the place where the Tsarina's Tiara is buried.

2 The wreck of the Purple Porpoise, a ship that sank carrying 200 gold doubloons, is exactly halfway between Falling Palms and Handyman Hills.

Find the coordinates of the Purple Porpoise.

3 If Skull Rock is closer to Trader's Bay than Forbidden Falls, then my prized Silver Cutlass is buried two leagues east of Mermaid Pool. If not, it is buried two leagues west of Mermaid Pool.

Determine the position of the Silver Cutlass.

4 The Emerald Crown, which I looted from an ancient tomb, has been hidden in a ruined building at one of the marked sites on the map. If you can find the correct site, you will find the

5 To find the Cursed Medal of Caracas, send your best navigator to the point halfway from Kaspar's Cave to the Grey Cliffs. From there, travel halfway to Pirate's Port, and there the medal is to be found.

Find the coordinates of the Cursed Medal of Caracas. Justify your answer.

6 The Bounty from Belize is buried deep - so be sure to pick the right place.

On the route straight from Falling Palms to tł Grey Cliffs, stop at the point that is closest to Kaspar's Cave. Head one league west and dig deep!

a **Find** the distance from Falling Palms to Kaspar's Cave.

b **Find** the distance from the Grey Cliffs to Kaspar's Cave.

c Hence **show** that Grey Cliffs, Kaspar's Ca

UNIT: STATISTICS AND PROBABILITY -1

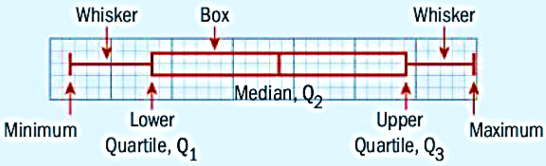
Summary

- In a stem-and-leaf diagram:
  - the **stem** represents the **category** figure
  - the **leaves** represent the **final** digit(s) of each data point
  - the **key** tells you how to read the values.

stem	leaf
0	0 0 0 1 1 1 4 6 7
1	0 0 1 2 3 4 5 7 7
2	0 2 6
3	9

Key: 1|0 represents 10

- For any set of data there are two categories of **summary statistics**:
  - **Measures of central tendency (location)** describe where most data lies. They answer the question 'What is an average data value?'
  - **Measures of dispersion (spread)** describe how spread out the data is. They answer the question 'How much variation is there between the values?'
- The **lower/first quartile (Q_1)** is the median of the observations to the left of the median in an arranged set of observations.
- The **upper/third quartile (Q_3)** is the median of the observations to the right of the median in an arranged set of observations.
- The **interquartile range (IQR)** is the difference between the lower quartile and the upper quartile ($Q_3 - Q_1$).
- The median is another name for the second quartile, Q_2.
- A five-point summary of a data set is:
  - the minimum data point (min)
  - the lower quartile (Q_1)
  - the median (Q_2)
  - the upper quartile (Q_3)
  - the maximum data point (max).
- A box-and-whisker diagram represents the five-point summary:



- **Qualitative data** – describes a certain characteristic using words (color, animal).
- **Quantitative data** – has a numerical value.

Quantitative data has a numerical value. There are two types:

  - **Discrete data** – can be counted (number of goals scored) or can only take certain values (shoe size)
  - **Continuous data** – can be measured (weight, height) and can take any value.

- The **distribution** of a data set describes the behavior of all the data points in the set.
- An **outlier** is a member of the data set which does not fit with the general pattern of the rest of the data. It could be an anomaly in the data or an inaccurate reading.
- A data point is an outlier if it is less than $1.5 \times$ IQR below Q_1, or greater than $1.5 \times$ IQR above Q_3.

Mixed practice

1 **Write down** whether these are qualitative or quantitative data.

- **a** The color of flowers
- **b** The prices of graphic calculators
- **c** The areas of public parks
- **d** The numbers of people at a store

2 **Write down** whether these are discrete or continuous data.

- **a** The time taken to run a marathon
- **b** The number of carriages on a train
- **c** The heights of Humboldt penguins
- **d** The temperature at midday in Iceland

3 The masses (in kilograms) of a group of people are given below.

46, 52, 64, 60, 82, 48, 72, 61, 70, 75, 59

- **a** **Construct** a stem-and-leaf diagram to represent the data.
- **b** **Write down** the number of people who weigh less than 60 kg.
- **c** **Find** the median mass.
- **d** **Find** the range.

4 During a one-hour study break, the amount of time students spent on social media sites was recorded. Their times, in minutes, were:

20, 45, 37, 29, 48, 52, 41, 32, 26, 50, 32, 44

- **a** **Calculate** the median time.
- **b** **Find** the interquartile range.
- **c** **Find** the range.

Problem solving

5 Sebastian collected data on the number of pairs of shoes owned by every student in his class. The values are in order.

5, 6, 7, 7, 9, 9, r, 10, s, 13, 13, t

He calculated that the median of the data set was 9.5 and the upper quartile Q_3 was 13.

- **a** **Write down** the value of r and s.
- **b** The mean of the data set is 10.

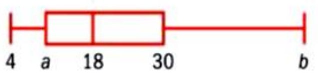
  Find the value of t.

6 All the dogs attending a veterinary surgery were weighed. Their masses, in kilograms, were:

26, 18, 54, 32, 30, 25, 6, 32, 43,
90, 16, 5, 27, 18, 3, 23, 27

- **a** **Find** a five-point summary for the data.
- **b** **Construct** a box-and-whisker diagram for the data.

7 This box-and-whisker diagram represents the results from a survey that asked: 'How old were you when you got your first smartphone?'

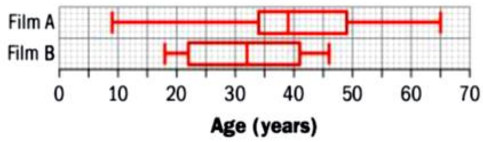


The interquartile range is 20 and the range is 40.

- **a** **Write down** the median value.
- **b** **Find** the values of a and b.
- **c** 160 people were surveyed. **Find** the number of people who were 30 or older.

Problem solving

8 The box-and-whisker diagrams show the ages of people watching two different films at a cinema.



- **a** **Compare** the ages.
- **b** **Deduce** which film is categorized as a family film. Give reasons for your answer.

9 The profits made by a small business during the last 11 years are as follows:

\$45 000, \$560 000, \$1000, \$85 000, \$160 000, \$170 000, \$62 000, \$250 000, \$3100, \$120 000, \$38 000

- **a** **Find** the interquartile range.
- **b** **Determine** if any of the data values can be considered outliers.
- **c** **Find** the range. Give a reason why any outliers have been included or excluded in your calculation.
- **d** **Find** the measure of central tendency which is most appropriate in this case. **Justify** your choice.

Problem solving

10 The table shows the masses (in kilograms) of Blackface sheep on two farms.

Farm A	63	48	60	55	45	49	19	55
Farm B	54	71	68	57	62	70	54	49
Farm A	65	49	57	56	64	43	64	48
Farm B	66	68	72	50	56	49	70	64

- **a** **Construct** a back-to-back stem-and-leaf diagram to represent the data.
- **b** **Compare** the masses of the Blackface sheep on the two farms.
- **c** The table shows the average adult bodyweight of Blackface sheep for different grazing conditions.

Poor hill	45–50 kg
Average/good hill	50–65 kg
Upland	70 kg

Deduce the grazing conditions available to the sheep on the two farms. Give reasons for your answer.

Review in context

Globalization and sustainability

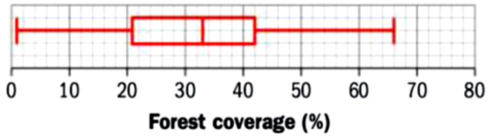
1 The table shows the percentage of land covered by forest in 19 European Union states in 2010.

Austria	46.7%
Belgium	22.0%
Czech Republic	34.3%
Denmark	11.8%
Estonia	53.9%
Finland	73.9%
France	28.3%
Germany	31.7%
Greece	29.1%
Hungary	21.5%
Ireland	9.7%
Italy	33.9%
Lithuania	33.5%
Netherlands	10.8%
Poland	30.0%
Portugal	41.3%
Slovakia	40.1%
Sweden	66.9%
United Kingdom	11.8%

a **Construct** a box-and-whisker diagram to represent the data.

b **Determine** if there are any outliers in this data.

2 The box-and-whisker diagram shows the forest coverage for the same EU states in 2012.



Compare the data for 2010 and 2012 and **identify** any changes between the two years.

3 **Use** the UNECE (United Nations Economic Commission for Europe) website to find more forestry data. **Use** an appropriate form of representation and prepare a short report to present to your classmates, and show how you can move between different forms of representation. **Explain** the conclusions you can draw from your diagram.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

How quantities are represented can help to establish underlying relationships and trends in a population.

UNIT: STATISTICS AND PROBABILITY -2

Mixed practice

1 The masses of 35 desert hedgehogs are listed here. All masses are to the nearest gram.

290	455	342	465	480	400	500
325	460	328	284	436	280	370
450	368	295	310	390	435	450
315	505	510	495	310	400	375
347	450	474	298	380	463	360

a **Construct** a grouped frequency table to represent the data.

b **State** the modal class.

c **Determine** the class interval that contains the median.

d **Calculate** an estimate for the mean mass.

2 The table shows learner drivers' marks in a hazard perception test.

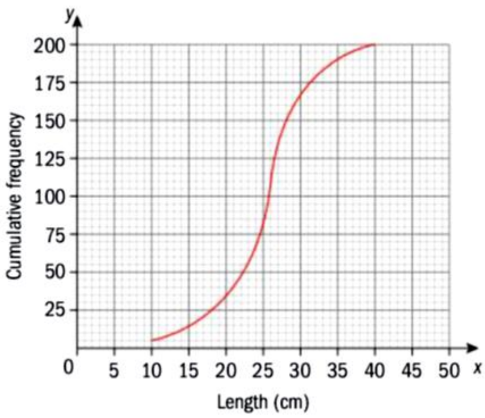
Mark, m	Frequency
$16 \le m \le 27$	9
$28 \le m \le 39$	21
$40 \le m \le 51$	18
$52 \le m \le 63$	23
$64 \le m \le 75$	19

a **Estimate** the range of the marks.

b **Construct** a cumulative frequency table.

c **Construct** a cumulative frequency curve.

3 The tails of a random sample of 200 adult foxes were measured in cm. The results are represented in the cumulative frequency curve.

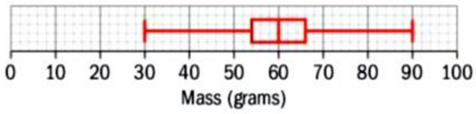


a **Estimate** the median length of fox tail in the sample.

b **Estimate** the interquartile range for the length of fox tails in the sample.

c Given that the shortest length was 11 cm and the longest 37 cm, **draw** and label a box-and-whisker plot for the data.

4 The box-and-whisker diagram shows the masses of 80 frogs at a zoo. **Construct** a cumulative frequency curve for the data.



Problem solving

5 The speeds of cars passing a speed camera on a highway are recorded in this table.

Speed, v (km/h)	Number of cars
$v \le 60$	0
$60 < v \le 70$	10
$70 < v \le 80$	22
$80 < v \le 90$	61
$90 < v \le 100$	74
$100 < v \le 110$	71
$110 < v \le 120$	39
$120 < v \le 130$	17
$130 < v \le 140$	6

a **Calculate** an estimate for the mean speed of the cars.

b Here is a cumulative frequency table for the same data.

Speed, v (km/h)	Number of cars	Cumulative frequency
$v \le 60$	0	0
$v \le 70$	10	10
$v \le 80$	22	32
$v \le 90$	61	93
$v \le 100$	74	a
$v \le 110$	71	238
$v \le 120$	39	b
$v \le 130$	17	294
$v \le 140$	6	300

Write down the values of a and b.

c On graph paper, **construct** a cumulative frequency curve to represent this information.

d 25% of cars exceed the speed limit. By calculating the upper quartile, **estimate** the speed limit on this stretch of highway.

6 The cumulative frequency curve shows the weights, in grams, of a selection of oranges.

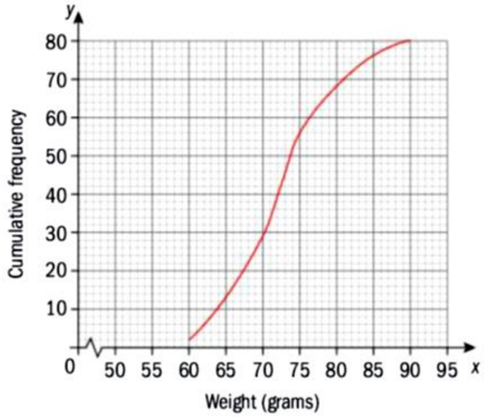
a Use the graph to **estimate**:

i the median

ii the upper quartile.

Give your answers to the nearest gram.

b 10% of the oranges weigh more than x grams. **Find** x.

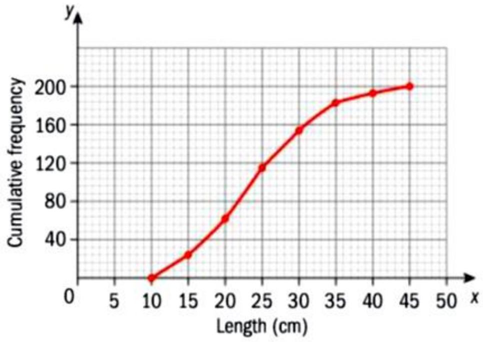


7 A fisherman catches 200 halibut. The table shows the lengths of these halibut to the nearest cm.

Length, x (cm)	Frequency
$10 < x \leq 15$	24
$15 < x \leq 20$	38
$20 < x \leq 25$	53
$25 < x \leq 30$	39
$30 < x \leq 35$	29
$35 < x \leq 40$	10
$40 < x \leq 45$	7

a **Calculate** an estimate for the mean length of the halibut.

b The cumulative frequency diagram shows the lengths of the halibut.



Estimate the interquartile range.

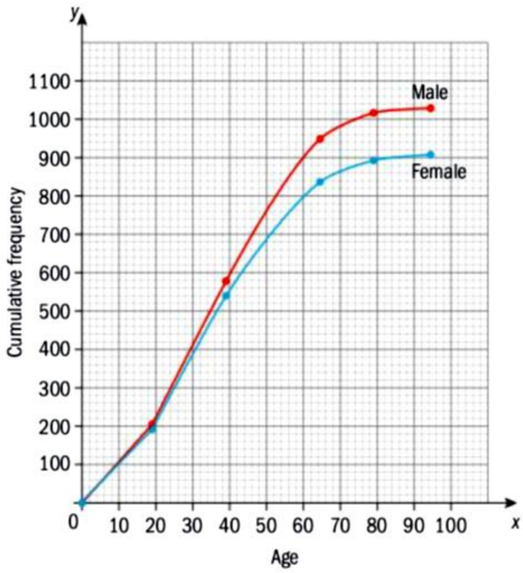
c The fisherman returns any fish smaller than 20 cm to the river. **Calculate** an estimate for the number of fish he returns.

d Fish greater than or equal to 20 cm but less than 28 cm are classified as small fish. Fish that are 28 cm or longer are classified as large fish.

Estimate the number of fish in each category.

e The fisherman sells small fish for \$6 and large fish for \$10. **Estimate** how much money he will earn if he sells all the fish.

8 The cumulative frequency curves show the ages of foreign male and foreign female residents in Switzerland in 2013.



By constructing a double box-and-whisker plot, **compare** the ages of the foreign male and foreign female residents.

ATL Review in context

Globalization and sustainability

1 A marine biologist is studying rainbow trout in a local river. To the nearest centimeter, she records the lengths of a sample of 100 rainbow trout from the river.

Rainbow trout	
Length, x (cm)	Frequency
$25 < x \leq 27$	1
$27 < x \leq 29$	5
$29 < x \leq 31$	9
$31 < x \leq 33$	21
$33 < x \leq 35$	29
$35 < x \leq 37$	19
$37 < x \leq 39$	16

a **Construct** a cumulative frequency table for this data.

b **Draw** a cumulative frequency curve.

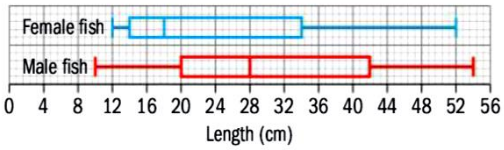
c **Use** the cumulative frequency curve to write a five-point summary for the data.

d Copy and complete this statement:

75% of the rainbow trout are over ____ cm in length.

e The rainbow trout feed on smaller fish of other species, so the biologist records the lengths of 100 female fish and 100 male fish of other species, from the same river.

These are shown in the box-and-whisker diagrams.



i **Estimate** the number of female fish that are smaller than 75% of the rainbow trout in the river.

ii **Estimate** the number of male fish that are smaller than 75% of the rainbow trout.

iii Hence **estimate** the percentage of the fish of other species in the river that 75% of the rainbow trout can feed on.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

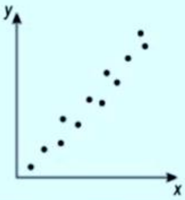
How quantities are represented can help to establish underlying relationships and trends in a population.

UNIT: STATISTICS AND PROBABILITY - 3

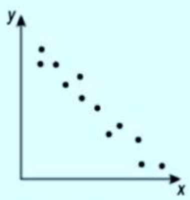
Summary

A scatter diagram shows the relationship between two quantitative variables. The independent variable is represented on the x-axis and the dependent variable on the y-axis. Data from two variables is called **bivariate data**.

Correlation is a measure of the association between two variables. When one variable increases as the other increases, there is positive correlation. When one variable decreases as the other increases, there is negative correlation.

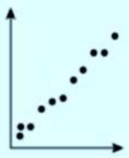


Positive correlation



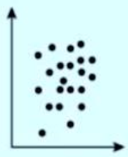
Negative correlation

The closer the points are to a straight line, the stronger the correlation between the variables.



Strong correlation

Moderate correlation



No linear correlation

A **line of best fit** is a straight line drawn through the middle of a set of data points so that they are equally distributed on either side of the line. The line of best fit passes through the point $(\bar{x}, \bar{y})$ where $\bar{x}$ is the mean of the x-values and $\bar{y}$ is the mean of the y-values.

To draw a line of best fit by eye:

- Calculate the mean of x $(\bar{x})$ and the mean of y $(\bar{y})$.
- Plot the point $M(\bar{x}, \bar{y})$ on the scatter diagram.
- Draw a line through M and through the middle of the other points. The points should be evenly distributed above and below the line.

An **outlier** is an anomaly in the data. To identify outliers in bivariate data, look at the distance of the data point from the line of best fit.

Predicting values inside the range of the given data is called **interpolation**. Predicting values outside the range of the given data is called **extrapolation**. In general, interpolation is more likely to give an accurate prediction than extrapolation.

Mixed practice

1 The table gives the length and width of 10 oak leaves that fell to the ground.

Length (mm)	Width (mm)
103	38
146	44
119	38
149	53
89	36
135	38
151	51
147	43
123	33
128	42

a Represent the data on a scatter diagram, with length on the x-axis and width on the y-axis.

b **Comment** on the correlation between length and width of the leaves.

c By finding the mean length and mean width, **draw** a line of best fit.

Problem solving

2 Two of these statements about correlation contain a mistake. **Explain** what the mistakes are and how the third statement could be correct.

a 'There is a high correlation between the income of workers and their gender.'

b 'There is a strong correlation between the age of Miss America and the number of murders using hot objects between 1999 and 2009.'

c 'There is a positive correlation between the average speed and the time taken by a train between two stations.'

Objective: **D.** Applying mathematics in real-life contexts **ii.** select appropriate mathematical strategies when solving authentic real-life situations
In this review you will have to use the mathematical strategies of drawing scatter diagrams, using technology to extract information and determining the reasons for outliers, in real-life situations.

Review in context

Identities and relationships

1 Florence's parents were concerned that she seemed short for her age and they recorded her height over a 36-month period:

Age (months)	36	48	51	58	66	72
Height (cm)	86	90	91	93	94	95

a **Draw** a scatter diagram of this data.

b **Use** this scatter diagram to **estimate** her height at age 42 months.

c Could you use this data and the scatter diagram to **predict** her height at age 18?

2 There are 12 students training for a char
10 km run. The table shows the average
of hours of training per week and the tin
to complete the run.

Training time (hours)	Time to compl (minute
10	56
9	54
13	53
4	62
26	42
7	66
11	54
6	66
7	68
22	39
5	70
10	67

a **Comment** on the correlation betweer training time and the time to complet the run.

b **Estimate** how long the run would tak student who trains 18 hours per week

c **Determine** whether you could use th to **predict** how long the run would ta student who trained 50 hours per wee

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Generalizing and representing relationships can help to clarify trends among individuals.

UNIT: STATISTICS AND PROBABILITY - 4

Summary

The sample space S, is a representation of the complete set of all possible outcomes from an experiment. It can be a list, a table or a diagram.

A single **event** is a subset of the possible outcomes listed in the sample space.

P(A) represents the probability of event A occurring. P(A') is the probability of A not occuring.

$$P(A) + P(A') = 1$$

The probability of an event happening is the proportion of times the event would occur in a large number of trials.

$$\text{Probability of event } A = P(A) = \frac{\text{number of ways event } A \text{ can occur}}{\text{total number of possible outcomes}} = \frac{n(A)}{n(S)}$$

Mixed practice

1 Miloš is taking two summer classes at the local college. One course is 'pass/fail' (those are the only two grades) while the other has a grading system of A, B, C, F (with F being the only failing grade). Assume that the probability of each course is equal.

a **Write** the sample space as a list and as a table.

b **Use** your diagrams to **find** the probability that:

i Miloš passes both classes

ii Miloš passes exactly one class

iii Miloš fails both classes.

2 There are four main blood types: A, B, AB and O. These are paired with something called a Rhesus factor, which is either '+' or '–'. For example, your blood type could be B+.

a **Write** the sample space for the different blood types that are possible.

If all blood types are equally likely, what is the probability that you have:

b type AB– blood

c type O blood

d a blood type other than A or B

e a 'positive' blood type?

3 At the school picnic, one of the coolers contains 12 cans of juice and 10 cans of soda. Rhona reaches into the cooler to grab a drink for herself and one for her friend Marco.

Draw a tree diagram and **calculate** the probability that:

a she grabs two juices

b she grabs two drinks that are the same

c she grabs two drinks that are different

d neither person gets a juice.

4 Olivia rolls two 6-sided dice at the same time. One die has three red sides and three black sides. The other die has the sides numbered from 1 to 6. By means of a tree diagram, table of outcomes or otherwise:

a **Find** how many different possible combinations she can roll.

b **Calculate** the probability that she will roll a red and an even number.

c **Calculate** the probability that she will roll a red or black and a 5.

d **Calculate** the probability that she will roll a number less than 2.

5 Ann has a bag containing 3 blue whistles, 4 red whistles and 1 green whistle.

Simon has a bag containing 2 blue whistles and 3 red whistles.

The whistles are identical except for the color.

Ann chooses a whistle at random from her bag and Simon chooses a whistle at random from his bag.

a **Draw** a tree diagram to represent this information and **write down** the probability of each of the events on the branches of the tree diagram.

b **Calculate** the probability that both Ann and Simon will choose a blue whistle.

c **Calculate** the probability that the whistle chosen by Ann will be a different color to the one chosen by Simon.

6 A recent study of 24 sodas revealed that 8 have high amounts of caffeine, 12 have high amounts of sugar and 6 have both.

a **Draw** a Venn diagram to represent this information.

What is the probability that a soda picked at random from the group in the study:

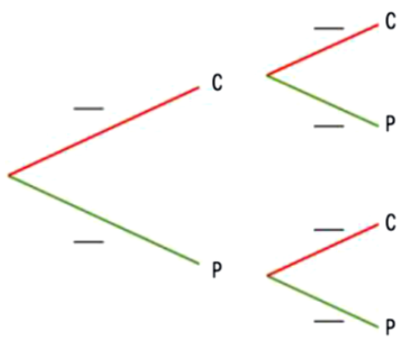
b is high in sugar but not in caffeine

c is high in caffeine only

d is not high in caffeine or sugar?

7 A bag contains four calculators (C) and six protractors (P). One item is taken from the bag at random and *not replaced*. A second item is then taken at random.

a Complete the tree diagram by writing probabilities in the spaces provided.



b **Calculate** the probability that one protractor and one calculator are taken from the bag.

8 Repeat **Q7**, but this time the item *is* replaced.

Calculate how the probabilities change.

9 A typical teenager should consume approximately 2000 calories per day. A survey of 120 students, three-fifths of which were male, revealed the following:

- 80 students ate more than the recommended amount.
- Half of the girls ate less than the recommended amount.
- Five-sixths of the boys ate more than the recommended amount.
- The same number of boys as girls ate the recommended amount of calories.

a Complete a two-way table to represent this information.

Calculate the probability that a student selected at random:

b ate the recommended amount of calories

c is male and ate less than the recommended amount

d ate more than the recommended amount given that they are female.

10 In a group of 50 people, 10 are healthy and the rest have either high blood pressure, high cholesterol or both; 23 people have high blood pressure and 28 have high cholesterol. **Find** the probability that a person selected at random:

a has high blood pressure

b has high blood pressure and high cholesterol

c has high blood pressure or high cholesterol

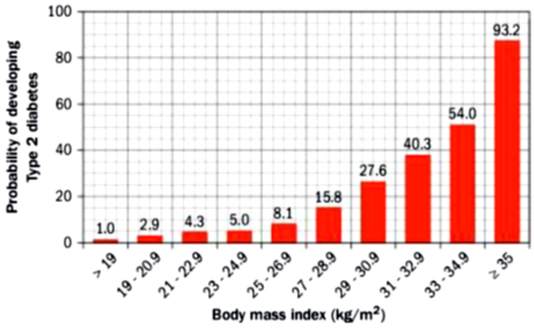
d has high cholesterol only.

Review in context

Identities and relationships

Problem solving

1 The chart shows the risk of developing Type 2 diabetes for different body mass index (BMI) values. BMI is a way of measuring the amount of fat in the body. For adults, a healthy BMI is between 18.5 and 25.



- **a** An adult has BMI 28. What is their risk of developing Type 2 diabetes?
- **b** **Describe** how the risk of developing diabetes changes as BMI increases.
- **c** An adult has BMI 31. He loses weight and has BMI 26. How many times smaller is his risk of Type 2 diabetes now?
- **d** **Write** some advice for reducing the risk of developing Type 2 diabetes. **Use** probabilities to justify your comments.

2 Vincent knows that there is a 15% chance of his children inheriting a disease that runs in his family. Suppose he has three children. **Find** the probability of:

- **a** all three of his children having the disease
- **b** at least two of his children having the disease
- **c** none of his children having the disease.

3 Probability can be used in genetics to predict the likelihood of children inheriting a condition from their parents. Suppose the gene for normal sight is represented by 'M' while that of short sightedness (myopia) is represented by 'm'. A child inherits one allele (gene) from each parent. For instance, if the parents are MM and Mm, then the child would receive an 'M' from the first parent and an 'M' or 'm' from the second.

- **a** Copy and complete the sample space for the different outcomes.

			Mother					
			MM		Mm		mm	
			M	M	M	m	m	m
Father	MM	M						
		M						
	Mm	M						
		m						
	mm	m						
		m						

- **b** If a child inherits myopia only when they inherit 'mm', calculate the probability that a child born will have myopia.
- **c** **Calculate** the probability that the child will carry the myopia gene (m).

Suppose two parents, both of whom are Mm, have two children.

- **d** **Calculate** the probability that the first child will have myopia (mm).
- **e** **Calculate** the probability that only one child will have myopia.
- **f** **Calculate** the probability that neither child will have myopia.
- **g** **Determine** which genes the parents would have to have in order for the probability that their first child has myopia to be $\frac{1}{2}$.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Understanding health and making healthier choices result from using logical representations and systems.

UNIT: NUMBERS – CURRENCY CONVERSION

Summary

A **rate of exchange**, or **exchange rate**, gives the value of one currency in terms of another currency.

The **commission** is a fee that foreign-exchange providers charge for exchanging one currency into another. The commission is charged in the currency that they buy and sell.

The **buying** and **selling rates** are from the point of view of the bank, who is buying or selling currencies.

Mixed practice

1 1 EUR = 1.10 USD.

a Convert 950 EUR into USD.

b Convert 2750 USD into EUR.

2 1 Mexican Peso (MXN) = 0.055 US Dollars (USD).

a Convert 4400 MXN into USD.

b A commission of 1% is charged.

Find the cost of the commission in USD.

c **Calculate** the final number of USD received.

3 1 CHF = 0.72 GBP. A commission of 2% is charged. Convert 1600 CHF into GBP.

4

	Buy RUB	Sell RUB
1 GBP	107.30	105.80

a Jasmine is going on holiday.

She wants to exchange 1400 GBP for Russian Rubles (RUB).

Calculate how many RUB she will receive.

b Kali has returned from holiday.

She wants to change 1420 RUB back to GBP.

Calculate how many GBP she will receive.

5 The table below show part of a currency conversion chart. **Find** the value of p and q.

	EUR	GBP	AUD
EUR	1	p	1.53
GBP	1.27	1	1.92
AUD	q	0.52	1

6 The exchange rate from US Dollars (USD) to Thai Baht (THB) is 1 USD = 33.45 THB. Give the answers to the following correct to 4 s.f.

a **Find** the value of 115 US Dollars in THB.

b **Calculate** the value of 1 THB in USD.

c Alexis receives 600 New Zealand Dollars (NZD) for 14 670 THB. **Calculate** the value of 1 THB in NZD.

d **Calculate** the value of the US Dollar in New Zealand Dollars.

7 A bank buys 1 Australian Dollar (AUD) for 0.72 Euros (EUR) and sells 1 AUD for 0.70 EUR. Frida wants to exchange 800 AUD for EUR.

a **Find** how many Euros Frida will receive.

b Frida has to cancel her trip and changes her money back later when the rates are 'buys 1 AUD = 0.73 EUR, sells 1 AUD = 0.69 EUR'. **Find** how many Australian Dollars she receives.

c **Determine** how many Australian Dollars she has lost on the transaction.

8 Ed had to change British Pounds (GBP) into Swiss Francs (CHF) at a bank. The exchange rate was 1 GBP = 1.4 CHF. The bank charged 2% commission.

a **Determine** how many Swiss Francs Ed bought with 200 GBP.

b Ed has 100 CHF of his initial amount left. **Find** how many British Pounds he could buy.

9 Santiago lives in Mexico. He wants to change 67 000 Mexican Pesos (MXN) into Euros (EUR).

A bank buys 1 MXN for 0.051 EUR and sells 1 MXN for 0.0531 EUR.

An exchange center has an exchange rate of 1 MXN = 0.054 EUR and charges a 2% commission.

Find the difference between the number of EUR he would receive from the bank and the exchange center.

10 Pavla travels from the Czech Republic to Thailand. She changes 29 000 Czech Koruna (CZK) to Thai Baht. The exchange rate is 1 CZK = 1.34 THB.

a **Calculate** the number of THB Pavla buys.

Pavla leaves Thailand and travels to New Zealand. She has 20 000 THB and uses these to buy New Zealand Dollars (NZD). The exchange rate is 24 450 THB = 1000 NZD.

b **Calculate** the total number of New Zealand Dollars Pavla receives.

c **Find** an approximate exchange rate between New Zealand Dollars and Czech Koruna.

Give your answer in the form 1 NZD = x CZK, correct to 2 d.p.

Review in context

Globalization and sustainability

1 The headquarters of Jim's company are in the UK. He is expanding and starting a company in Singapore. He will be exchanging British Pounds (GBP) into Singaporean Dollars (SGD). The exchange rate is 1 GBP = 2.07 SGD, and there is a bank charge of 10 GBP for each transaction.

a **Determine** how many SGD Jim could buy with 1330 GBP.

b Let s be the number of SGD received in exchange for b GBP. Express s in terms of b.

c At the end of the second year, Jim needs to transfer 15 000 SGD from Singapore to his UK bank account. He has two ways of converting SGD to GBP. He can use the British bank (the exchange rate is 1 GBP = 2.06 SGD, and there is a bank charge of 10 GBP for each transaction), or he can use the Singaporean bank (the exchange rate is 1 SGD = 0.49 GBP, and there is a bank charge of 2% for each transaction).

Decide which is the better option.

2 Andy, Neil and Jelena each had 3000 USD to invest in 2005.

Andy used his 3000 USD to buy Swiss Francs.

Neil used his 3000 USD to buy Brazil Reals.

Jelena used his 3000 USD to buy Swedish Krone.

The table shows the exchange rates per 1 USD at the time of conversion and at five-year intervals.

2005	2010	2015
1.32 CHF	0.97 CHF	0.85 CHF
2.50 BRL	1.52 BRL	3.29 BRL
7.25 SEK	6.40 SEK	9.30 SEK

Ignoring any bank buying and selling costs:

a **Calculate** the amount each person received in their chosen currency in 2005.

b **Calculate** the amount in USD each person would receive if they chose to sell in 2010.

c **Calculate** the amount each person would receive if they chose to convert it back to USD in 2015.

Reflect and discuss 3

How have you explored the statement of inquiry? Give specific examples.

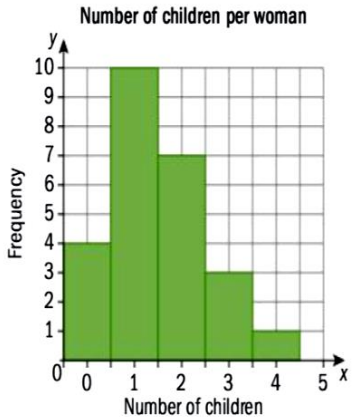
Statement of Inquiry:

Quantities and measurements illustrate the relationships between human-made systems and communities.

Review in context

Globalization and sustainability

1 Twenty-five women from Mexico were asked how many children they had. The results are shown in the frequency histogram.



a **Show that** the mean number of children per woman is 1.48.

b A group of 25 women from Australia were asked the same question. The results are given in this table.

Number of children	0	1	2	3	4	5
Frequency	4	8	5	4	5	2

Use the results from parts **a** and **b** to **describe** and **compare** the distribution of the numbers of children per woman in Australia and Mexico.

2 The amount of land covered by rainforest in the Amazon region has been rapidly declining over the past decades. This frequency table show the percentage of deforestation for 20 South American countries.

0.4%	0.3%	1.6%	2.1%	0.3%
2.7%	2.2%	3.5%	2.0%	9.2%
3.2%	0%	3.0%	3.3%	4.0%
0.6%	0.5%	4.0%	1.7%	2.6%

a **Sketch** a frequency histogram for this data.

b **Comment** on the distribution.

c **Determine** any data that could be outliers.

Reflect and discuss 6

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

How quantities are represented can help to establish underlying relationships and trends in a population.

Review in context

Globalization and sustainability

1 The most recent census in China was in July 2015 and the population was reported in official government statistics to be 1 376 048 943.

In two separate articles, this figure was quoted to be 1 376 049 000 and 1.38 billion.

a **Calculate** the absolute error in each of these figures compared to the official figure.

b **Calculate** the percentage errors.

c **Discuss** whether it is better to use absolute or percentage errors.

d Government sources suggest that, due to the reliability of the way it is measured, the official figure may be inaccurate by plus or minus 1.8%.

Calculate what this is as an absolute error.

e **Calculate** the possible range for the population.

f **Discuss** what might be a reliable figure to use to report the population of China.

g Here are the populations of some countries, also given in 2015:

United States	321 442 019
Germany	80 688 545
Malaysia	30 331 007
Australia	23 968 973
Monaco	37 731

If the figures could also be inaccurate by 1.8%, **find** the absolute errors. Use these to give reliable values to quote for these populations, and **justify** why the values are reliable.

2 **UK population grows by half a million in a year**

Between 2013 and 2014 the UK population rose from 64 106 779 to 64 510 376.

a **Find** the exact change in population.

b **Find** the percentage error in the headline figure of half a million.

c In 2014, 17.7% of the population were aged 65 and over. In 2013 the percentage was 17.4%.

Find the actual change in the number of people aged 65 and over between 2013 and 2014.

d Is this figure an increase or a decrease?

e **Suggest** how these figures could be useful for planning healthcare in the UK.

3 Tobias went grocery shopping for himself and his roommate Felix. They usually split the cost of groceries in half, and since they never have exact change, they usually round the amount paid before splitting it in half. Today, Tobias paid exactly €95.74, and he rounded the amount up to an even €100 before telling Felix that he owed Tobias €50.

a **Calculate** the percentage error between the true amount Tobias paid and the rounded amount.

b **Discuss** whether or not it is reasonable for Felix to pay €50.

c **Calculate** the exact amount Felix should have reimbursed Tobias. Then **calculate** the percentage error between what Felix did pay and what he should have paid.

d **Discuss** what you notice about the two percentage errors. Would it have made sense to just calculate absolute errors in this case?

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Quantities and measurements illustrate the relationships between human-made systems and communities.

Review in context

Globalization and sustainability

1 The Golden Gate Bridge in San Francisco opened for vehicular traffic in 1937.

a Below are some facts about the bridge. **Find** the equivalent measurements in metric units.

i The towers are 746 feet tall.

ii Each of the two main cables is 7650 feet long.

iii The total length of wire used in both main cables is 80000 miles.

b Carbon steel is used for the main cables. 1 cm^3 of carbon steel has a mass of about 7.85 g. **Find** the density of carbon steel in kg/m^3.

c When the bridge was built, 389000 cubic yards of concrete was used.

1 yard = 0.9144 m

After the original concrete roadway deck was replaced, there was 6.4% less concrete. **Calculate** the total quantity of concrete in the refurbished bridge in m^3.

d A bolt of diameter 2.125 inches was used in the construction.

i **Find** the diameter of the bolt in mm.

ii Imagine that a bolt of diameter 2.125 cm was mistakenly used. **Calculate** the error in mm.

2 Railway tracks in different countries have different gauges (spacing between the rails).

a Approximately 60% of the world's railways today use the standard gauge, which is 4 ft $8\frac{1}{2}$ in. **Find** this measurement in metric units, correct to the nearest mm.

b Originally, English trains used a 4 ft 8 in gauge for coal transport trains. Half an inch was added to the gauge for passenger trains, as this made the trains run more smoothly on the curves.

i **Find** the length of the coal-mining gauge in metric units, to the nearest mm.

ii **Find** the difference between the two gauge sizes, in mm.

iii **Determine** whether your answer in part **ii** corresponds to $\frac{1}{2}$ in. If not, find the percentage error.

c Some countries use the meter gauge, which is exactly 1000 mm. **Find** this measurement in imperial units, in feet and inches.

d Until 2011, Spain used a 1668 mm gauge, and France used the standard gauge. **Suggest** why Spain modified its railway tracks and engines.

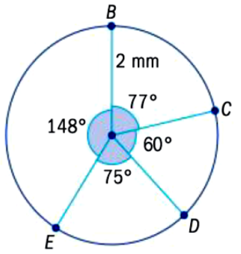
UNIT: GEOMETRY – ARC LENGTH AND SECTOR

Mixed practice

1 a Find the arc length of sector *DC*.

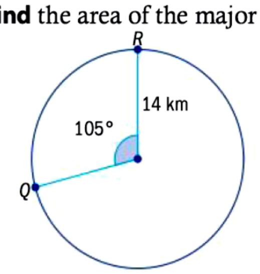
b Find the perimeter of sector *BD*.

c Find the area of sector *EB*.



2 a Find the length of the major arc *QR*.

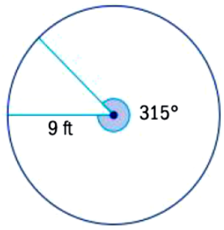
b Find the area of the major sector.



3 Find the perimeter and area of:

a the minor sector

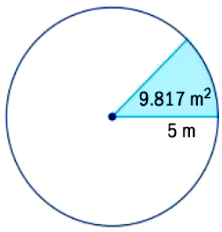
b the major sector.



4 a Find the length of the minor arc.

b Find the perimeter of the minor sector.

c Find the area of the major sector.

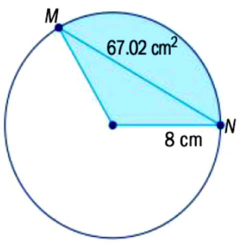


5 a Find the measure of the central angle.

b Find the length of the minor arc.

c Find the perimeter of the shaded sector.

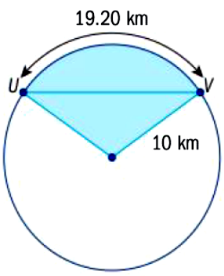
d Find the length of the chord *MN*.



6 a Find the measure of the central angle.

b Find the perimeter of the shaded sector.

c Find the length of the chord *UV*.



7 Toni has a rectangular lawn 8 m by 10 m. Her lawn sprinkler sprays water up to 5 m.

a Draw and label a scale diagram of the lawn.

b By using compasses to draw the region watered by the sprinkler to scale, **determine** the best position for the sprinkler so that it waters as much of the lawn as possible.

c Divide the circle that lies inside the garden boundary into two sectors and two isosceles triangles. **Hence, calculate** the area of the lawn watered by the sprinkler.

d Calculate the percentage of the lawn that is not watered by the sprinkler.

Problem solving

8 A regular hexagon is constructed in a circle of radius 5 cm. **Calculate** the perimeter of the hexagon. Give your answer to a suitable degree of accuracy.

Review in context

Personal and cultural expression

1 'Pendulum dowsing' as a means of finding water, gold or even answers to questions has been used throughout history and has been recorded as far back as the time of the pharaohs in Egypt. In one version, a pendulum is held above a cloth with 'yes' or 'no' written on it and the degree to which the pendulum sways to one side or the other is an indication of how likely or unlikely the event in question is going to happen.

   a If the pendulum has a total length of 30 cm, **calculate** the distance travelled if it moved a total of 45 degrees.

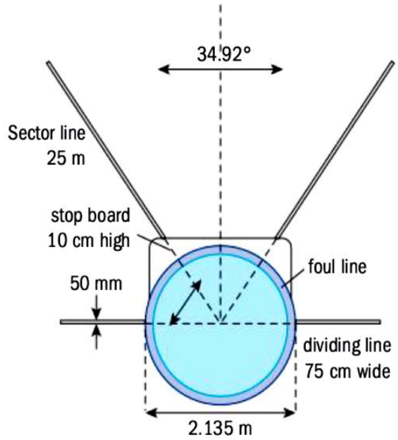
   b If the pendulum starts facing straight down and it travels an arc of 10 cm, **calculate** the turned angle.

2 The diagram shows the landing area for the shot put. The throwing circle has a diameter of 2.135 m, and the landing area is a sector with angle 34.92° and sector lines 25 m long, starting from the center of the circle. The distance thrown is measured from the circumference of the throwing circle to the imprint made in the soil by the shot in the landing area.

   a **Find** the area enclosed by the entire sector.

   b **Find** the area enclosed by the sector inside the throwing circle.

   c Hence, **find** the area of the landing area.



3 'Medicine wheels' were used by native Americans for a variety of rituals. The Big Horn Medicine Wheel is a circle of stones with a diameter of about 24 meters. It is divided into 28 approximately equal sections, likely to represent the 28 days of the lunar calendar.

   a **Calculate** the approximate central angle of each sector.

   b **Calculate** the area of each sector.

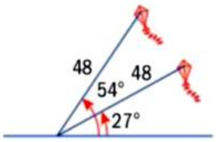
   c Different spots along the circle have been found to correspond to specific astronomical events (winter solstice, rising of stars). Two spots related to the rising of the star Sirius are 20 meters apart. **Calculate** their approximate distance apart along the circle.

   d The rising of the star Aldebaran happens between two points whose central angle measures 80 degrees. **Calculate** the shortest distance between them.

UNIT: TRIGONOMETRY

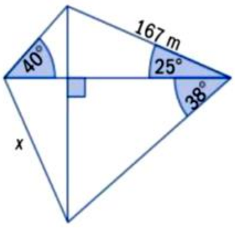
Mixed practice

1 A kite string is 48 m long. As the wind blew, the angle between the kite and the ground went from 27 degrees to 54 degrees. **Determine** the increase in the vertical height of the kite above the ground.

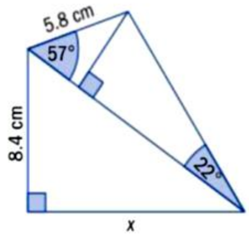


2 **Find** length x in each diagram.

a



b



3 $ABCD$ is a square, where $AP = 5$ cm, $QC = 7$ cm and $PB = 12$ cm.

Calculate:

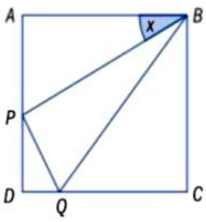
a the size of the angle marked x

b the length of AB

c the length of DQ

d the length of PD

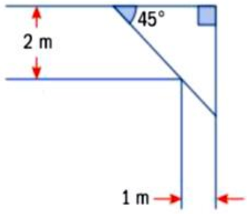
e the size of $\angle BQD$.



4 In a house that is being renovated, two corridors meet at right angles, as shown in the diagram. Workmen attempting to take a ladder around the corner get the ladder stuck when it is at an angle of 45° to each wall.

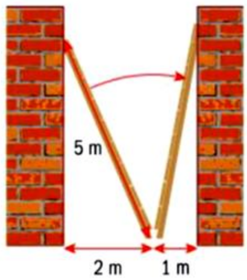
a **Determine** the length of the ladder.

b **Determine** the length of a 2nd ladder that got stuck when it was at an angle of 60° with the wall of the 1 m corridor.



Problem solving

5 A 5 m ladder is resting against a vertical wall. The base of the ladder is 2 m from the wall. Keeping its base fixed, the ladder is rotated so that it now rests against the opposite wall which is 1 m away. **Determine** the angle through which the ladder has been rotated.



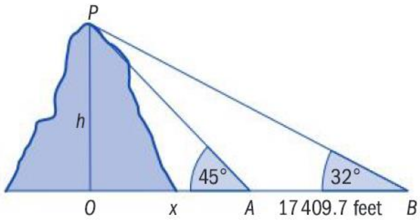
6 A coast guard attendant atop a cliff 120 m high observes two boats in line with him at angles of depression of 45 degrees and 69 degrees.

Determine how far apart the boats are.

Review in context

Scientific and technical innovation

1 In 1852, an Indian mathematician, Radhanath Sikdar, used measurements and trigonometry to calculate the height of Peak XV in the Himalayas (later to be named Mount Everest). This required the use of a device that measured angles from the ground to the top of an object. By measuring the angle to the top of Peak XV from two different spots, and knowing the distance between these spots, Sikdar may have come up with a drawing like the following:



The angle of elevation to the top of the peak (from point B) was measured to be 32°. From a point 17 409.7 feet closer (point A), the angle of elevation was 45°.

Find the height Sikdar calculated for Peak XV.

Legend has it that the value Sikdar calculated was so perfect that he thought nobody would believe him. Supposedly, he added 2 feet to make it more believable! History books record his calculated height to be 29 002 feet.

2 You are the pilot of a light aircraft, returning to the airfield after a parachuting trip. Your altitude is 3050 m and you are heading towards the landing strip which is currently 14 km away. What angle of depression should you use so that you touch down on the landing strip at the closest point?

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Mixed practice

1 **Find** a formula for the nth term of each sequence.

a 6, 10, 14, 18, 22, 26, …

b 42, 35, 28, 21, …

c 2, 5, 10, 17, 26, 37, …

d 3, 8, 15, 24, 35, …

e 2.5, 7, 11.5, 16, 20.5, …

f −0.5, 4, 13.5, 28, 47.5, …

g 9, 13, 13, 9, 1, …

h 6, 7, 7, 6, 4, …

2 A linear sequence begins 14, 17, 20, …

Find its hundredth term.

3 A sequence has formula $u_n = u_{n-1} - 5$ and $u_1 = 12$.

a **Write down** the first five terms.

b **Find** a formula for the nth term of the sequence.

Problem solving

4 A sequence has formula $u_{n+1} = u_n + 7$ and $u_0 = 15$.

a **Write down** the first five terms of the sequence.

b **Show that** 85 is a member of the sequence.

5 A linear sequence begins 18, 33, 48, ...

Find the first term greater than 1000.

6 A quadratic sequence begins 4, 10, 18, 28, …

a **Find** a formula for the nth term.

b **Write down** the value of the seventh term.

c **Show that** 460 is a member of the sequence and find its term number.

7 A quadratic sequence begins 2, 4, $\frac{20}{3}$, 10, 14, …

a **Find** a formula for u_n, the nth term of the sequence.

b **Find** the value of u_{20}.

Problem solving

8 A linear sequence has terms $u_3 = 6$ and $u_6 = 27$.

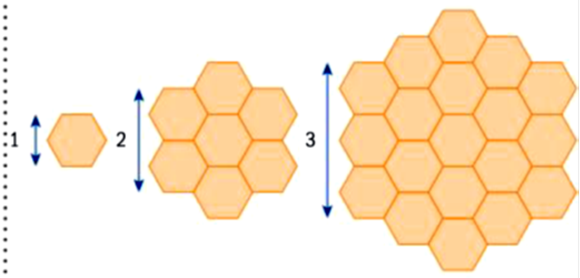
a **Find** the difference between successive terms.

b **Write down** the value of u_7.

9 A quadratic sequence begins 3, x, 15, 25.5, …

Find the value of x.

10 Consider the honeycomb patterns below, which have side length 1, 2 and 3 hexagons respectively.



Find the side length of the first pattern that contains over 1000 hexagons.

11 In a football stadium, the first row of seats has 610 seats. The second row has 620 seats, the third has 630 seats, and so on. The stadium sells seats from the middle of a row outwards. Each row must be completely sold before they sell tickets for the next row.

a **Find** the number of seats that can be sold if only the front two rows are used.

b **Find** the number of seats that can be sold if only the front three rows are used.

c **Create** a formula for the total number of seats that can be sold if the front n rows are used.

d Hence **find** the number of seats that can be sold if the front 30 rows are used.

e **Find** the number of rows needed for the first 10 000 tickets.

12 A 12-storey tall building has an elevator that stops at every floor.

The elevator takes 8 seconds to travel one floor, 14 seconds to travel two floors, 20 seconds to travel three floors and 26 seconds to travel four floors.

Let T_n be the time taken to travel n floors.

a **Show that** T_n forms a linear sequence.

b **Find** a formula for T_n.

c **Find** the greatest number of floors that the elevator could travel in under a minute.

Review in context

Scientific and technical innovation

1 Engineers are laying signalling cable alongside a railway track. They place fixed lengths of cable separated by junction boxes.

The total length of cable for 1, 2, 3 or 4 junction boxes is 12 m, 18 m, 24 m or 30 m.

Let u_n be the cable length for n junction boxes.

a **Find** a formula for u_n.

b **Use** your formula to **predict** the length of cable needed for 30 junction boxes.

2 The number of passengers, P_n, who will comfortably fit into an n-carriage train is given by the following table for small values of n:

n	1	2	3	4
P_n	40	110	180	250

a **Show** that P_n forms a linear sequence.

b **Find** a formula for P_n.

c **Predict** the number of passengers that an eight-carriage train could comfortably hold.

d **Suggest** a reason why a two-carriage train might hold more than twice the number of passengers of a one-carriage train.

3 In a subway network of stations, every journey consists of a start point and a different end point.

If you consider just two of the stations, call them A and B, then the only two possible journeys are AB and BA.

If you add in a third station, C, there are six possible journeys.

a **List** the six journeys that are possible with three stations, A, B and C.

b **List** the journeys that would be possible with four stations, A, B, C and D.

Hence **write down** the total number of possible journeys with four stations.

c **List** the journeys that would be possible with five stations, A, B, C, D and E.

Hence **write down** the total number of possible journeys with five stations.

d **Create** a formula linking the number of stations on the network to the total number of possible journeys.

The *Metropolitano de Lisboa*, Lisbon's underground railway, has 55 stations.

e **Predict** the total number of possible journeys.

f **Predict** the number of additional possible journeys that there would be if two extra stations were built.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Using different forms to generalize and justify patterns can help improve products, processes and solutions.

UNIT: ALGEBRA – FACTORIZATION

Mixed practice

1 Factorize:

a x^2+4x+4 **b** $x^2-13x+36$

c $x^2+5x-14$ **d** $x^2+18x+81$

e x^2-7x-8 **f** $x^2-11x+24$

g $x^2-20x+100$ **h** $x^2-15x-100$

i x^2+4x+3 **j** $x^2+11x-42$

k $x^2+4x-96$ **l** x^2+7x+6

m $x^2-15x+56$ **n** $x^2-11x-60$

2 Factorize fully:

a x^2-49 **b** x^2-169

c $64-x^2$ **d** $25x^2-4$

e $144x^2-81$ **f** $256x^2-169$

g $4x^2-y^2$ **h** $16x^2-9y^2$

i $25x^2-289y^2$ **j** x^4-16

k $16x^4-1$ **l** $9x^4-729y^4$

3 Factorize completely:

a $2x^2+8x+6$ **b** $2x^2+14x+20$

c $2x^2-12x+10$ **d** $3x^2-9x+6$

e $2x^2+8x-24$ **f** $3x^2+21x-24$

g $4x^2-8x-32$ **h** $2x^2-2x-40$

4 Factorize:

a $2x^2+5x+2$ **b** $2x^2+13x+20$

c $2x^2-11x+12$ **d** $3x^2-8x-3$

e $5x^2-12x+4$ **f** $7x^2+38x-24$

g $6x^2+31x+5$ **h** $6x^2-17x+5$

i $8x^2+18x+9$ **j** $8x^2-41x+5$

k $9x^2+12x-32$ **l** $10x^2-13x-30$

5 Factorize:

a $4a^2-3a$ **b** $7b^3-35b^2-168b$

6 Factorize:

a $a^2-5a-36$ **b** $16b^2-9$

c $c^2+11c+24$ **d** $4d^2-17d-15$

e $4e^2-3e$ **f** $f^2+2f-48$

g $3g^2-23g+14$ **h** $16h^2-1$

Problem solving

7 Ornella thinks of a whole number, adds four to it, squares the result, and subtracts 9.

a By letting the original number be n, **write down** an expression for this process.

b Hence **show** the number that Ornella obtains can always be written as the product of two integers with a difference of 6.

8 Copy and complete:

a $x^2-\square x+15\equiv(x-3)(x-\square)$

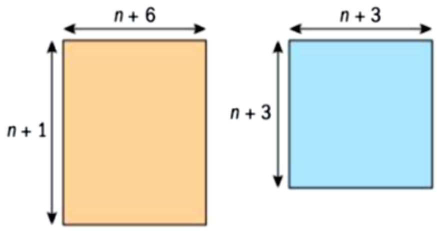
b $x^2\square\square x+20\equiv(x+4)(x\square\square)$

c $x^2-8x\square\square\equiv(x+\square)(x-11)$

d $x^2\square 11x+\square\equiv(x\square 5)(x-\square)$

9 Dagmar thinks of an even number, squares it, and subtracts 1. By expressing his original number in the form $2n$, or otherwise, **show** that the result of this process can be written as the product of two consecutive odd integers.

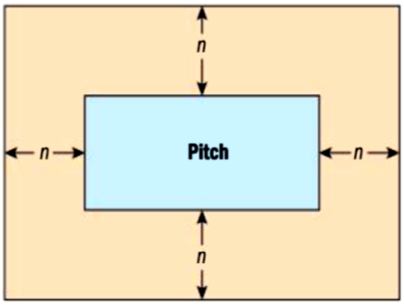
10 You have two rectangular grids of 1 cm³ cubes, one measuring $n+1$ by $n+6$ cm, and the other measuring $n+3$ by $n+3$ cm, where n is a positive integer.



a **Find** and simplify an expression for the total number of 1 cm³ cubes in the two rectangular grids combined.

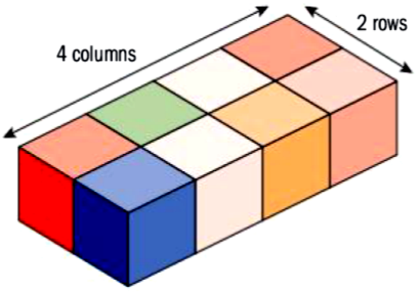
b **Show** that if you take apart the rectangles and recombine the 1 cm³ cubes you will always be able to form a rectangle with no cubes left over (where the rectangle will not simply be a straight line of cubes).

11 A rectangular sports pitch has a border of n meters on each side. The total area occupied by the pitch and its border is $4n^2 + 28n + 45$.



Find the dimensions of the pitch.

12 Start with a rectangular grid of 8 cubes:



Somebody else then adds some extra rows and columns to the grid, but doesn't tell you how many they've added. They do tell you that the grid now contains 45 cubes.

They repeat the process, adding the same number of rows as before and the same number of columns as before. Now the grid contains 112 cubes.

They repeat the process a third time, and the resulting grid contains 209 cubes. After a final repetition, there are 336 cubes in the grid.

a **Construct** a difference diagram showing the sequence 8, 45, 112, 209, 336. Analyze the differences and **show** that these numbers follow a quadratic sequence.

b Letting u_n be the number of cubes in the nth grid (so $u_1 = 8$), **find** a formula for u_n in the form $an^2 + bn + c$.

c Factorize your formula for u_n.

d Hence **determine** the number of rows and columns being added each time.

Reflect and discuss

How have you explored the statement of conceptual understanding? Give specific examples.

Conceptual understanding:

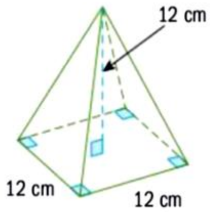
Patterns can be represented in equivalent forms.

UNIT: GEOMETRY AND TRIGONOMETRY- MENSURATION

Mixed practice

1 **Find** the surface area and volume of each 3D solid.

a



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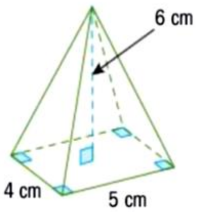

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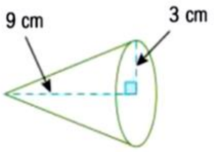

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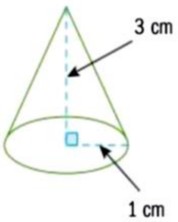

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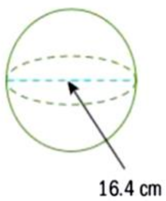

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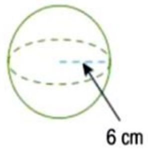
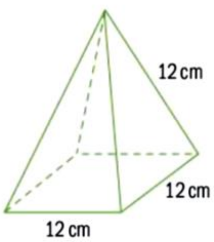


Problem solving

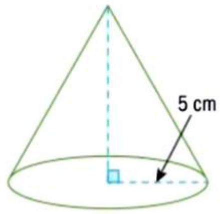
2 All eight edges of a square-based pyramid are 12 cm long.



a **Find** its surface area and its volume.

b **Find** the surface area and volume of a mathematically similar pyramid, with base a square of side 9 cm.

3 A cone has surface area 204.2 cm^2, and base radius 5 cm.



a **Find** the slant height of the cone.

b **Find** the volume of the cone.

4 The volume of a soccer ball is 5.6 dm^3.

According to the regulations, the circumference of the ball must be between 68 cm and 70 cm. **Determine** whether this soccer ball satisfies the regulations. **Justify** your answer.

5 A spherical scoop of ice cream is cut in half, and covered in chocolate (including the flat face). The total area of chocolate is 85 cm^2.

a **Find** the radius of the spherical scoop of ice cream.

b Hence, **find** the volume of the ice cream covered in chocolate.

6 Two paper cones are mathematically similar. The capacity of the larger cone is 8 times the capacity of the smaller cone. The surface area of the smaller cone is 50 cm^2.

Find the surface area of the larger cone.

Objective: D. Applying mathematics in real-life contexts
ii. select appropriate mathematical strategies when solving authentic real-life situations

In these real-life situations, select the strategies you have learned for finding the volume and surface area of 3D shapes to answer the questions.

Review in context

Personal and cultural expression

1 The Egyptians built square-based pyramids as part of the burial ritual for queens and pharaohs. The three largest and best preserved pyramids are located in the town of Giza, near Cairo. Many believe that the builders were influenced by the Golden Ratio (1.62).

a Each of the sides of the base of the Great Pyramid measures 230.4 meters while the height measures 145.5 meters. How close is the ratio of side length to height to the Golden Ratio?

b The entire pyramid was originally covered with a layer of white limestone. **Calculate** the area of limestone used.

c **Compare** the total surface area of the sides to the area of the square base. How does this relate to the Golden Ratio?

d **Calculate** the volume of the pyramid.

e **Calculate** the perimeter of the base of the pyramid as well as the circumference of a circle with a radius equal to the height of the pyramid. **Describe** what you notice about these results.

2 Near Mexico City is The Temple of the Feathered Serpent, one of hundreds of pyramids in the Mesoamerican city of Teotihuacan. Archaeologists used a robot and found hundreds of spheres with circumferences ranging from about 3.5 cm to 12.5 cm. The spheres were covered in a yellow material called jarosite.

a **Find** the amount of jarosite needed to paint:
i the smallest ball **ii** the largest ball.

b **Find** the volume of clay needed for each size ball.

3 Potentially the earliest team sport, ballgame, was played in Mesoamerica. A very large court is still visible at the archaeological site of Chichen Itza in Mexico. Players could end the game by getting a heavy ball through a ring mounted vertically on the wall of the court (no hands allowed). If the circumference of the ring was 95 cm and the surface area of the ball was 2830 cm^2, **justify** why putting the ball through the hoop resulted in ending the game.

4 The Christmas tradition of decorating evergreen trees began in Germany in the 16th century. With their roughly conical shape, it was possible to decorate them and enjoy their beauty from all sides. (No decorations were put underneath the circular face.)

a **Determine** which tree would have more area for decorations: one with a height of 215 cm and a maximum circumference of 200 cm or a tree with a height of 180 cm and a circumference of 210 cm? **Justify** your answer.

b A tree located in Rockefeller Center in New York City is lit every year in early December. The largest tree measured 100 feet tall and had a volume equivalent to 65 500 cubic feet. **Find** the area was available for decorations.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

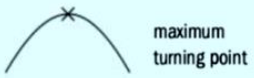
Statement of Inquiry:

Generalizing relationships between measurements enables the construction and analysis of activities for ritual and play.

UNIT: ALGEBRA – QUADRATIC EQUATION

Summary

- The **standard form** of a quadratic function is $y = ax^2 + bx + c$ where a, b and c are real numbers, and $a \neq 0$.
- The **factorized form** of a quadratic function is $y = a(x-p)(x-q)$, $a \neq 0$.
- The **vertex form** of a quadratic function is $y = a(x-h)^2 + k$, $a \neq 0$, where (h, k) is the vertex.
- The **degree** of a polynomial function is the value of its largest exponent of x. A linear function is a polynomial function of degree 1. A quadratic function is a polynomial function of degree 2. A constant function has degree 0.
- A concave down parabola has a maximum turning point. $y = ax^2 + bx + c$, $a < 0$



- A concave up parabola has a minimum turning point. $y = ax^2 + bx + c$, $a > 0$

- For a quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$:
  - the x-coordinate of the vertex is $-\frac{b}{2a}$
  - the equation of its axis of symmetry is $x = -\frac{b}{2a}$
  - the coordinates of its vertex are $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
  - the y-intercept is c
- For a quadratic function $f(x)$ with x-intercepts x_1 and x_2:
  - the x-coordinate of the vertex is $x_v = \frac{x_2 + x_2}{2}$
  - the y-coordinate of the vertex is $f(x_v)$

Mixed practice

1 For each quadratic function:

i **find** the vertex, x-intercepts and y-intercept

ii **state** the axis of symmetry, and whether it is concave up or concave down

iii **sketch** the graph.

a $y = x^2 + x - 12$ **b** $y = x^2 + 7x + 12$

c $y = 2x^2 - x - 3$ **d** $y = 2 - x - 3x^2$

e $y = -6x^2 + 5x - 1$ **f** $y = 2x^2 - 9x - 5$

2 These quadratic functions are in standard form. Change each one to vertex form, and **state** the coordinates of the vertex.

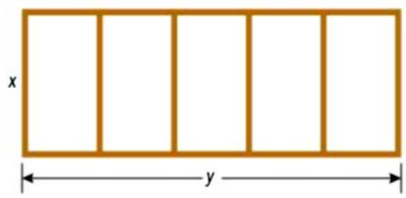
a $y = x^2 - 4x + 6$ **b** $y = x^2 + 6x + 8$

c $y = x^2 + 2x - 9$ **d** $y = x^2 - 2x + 7$

e $y = x^2 + x - 5$ **f** $y = x^2 - x + 7$

Problem solving

3 Kanye has 600 m of fencing to make five adjacent pens like this:



a Express the total area of the pens in terms of x.

b **Determine** the value of x that will maximize the total area.

c **Find** the maximum area.

4 Soraya has 40 m of fencing to make a rectangular play area. **Find** the dimensions for the maximum play area.

5 The population P of an animal species is modelled by the function $P(t) = -0.38t^2 + 134t + 1100$, where t is the time in months since the population was first observed. **Determine**:

a the number of months until the population is at its maximum

b the maximum population

c the number of months before the species disappears.

6 A ball is thrown up into the air, from 5 m above the ground. After 2 seconds, the ball reaches a maximum height of 9 m. It lands on the ground 5 seconds after it was thrown. **Find** a quadratic function that models this situation. Write it in standard and vertex form.

Review in context

Scientific and technical innovation

The area of a region formed by a parabola is $A=\frac{2}{3}bh$, where b is the length of the base of the parabola, and h is its height, i.e., the distance from its vertex to the base.

1 This rollercoaster is in Münich, Germany.

a Trace the outline of the parabolic section of the foreground section of track.

b **Draw** axes and use the map to determine a suitable scale.

c **Find** a suitable function to model the shape of section of track.

d **Use** the function to estimate the area under this section.

2 At right is the tunnel entrance that runs through Castle Hill in Budapest, Hungary.

a If the maximum height of the tunnel is the same as its maximum width, 9.8 meters, **determine** a quadratic model to represent the tunnel.

b **State** a geometric model that could be used to represent a truck passing through the tunnel.

c If a typical truck has a width of 2.62 m, determine the height limitation that should be put on these trucks if they are going to travel through the tunnel safely. **State** any assumptions that you made and be sure to show all of the steps in your solution.

d Because its length is almost 350 meters, a truck stopped in the tunnel would need enough room for the passenger door to open fully. Suppose a truck with a width of 2.6 m and height of 3.4 m is stuck in the right lane of the tunnel by the center line. The top of the driver's door is 0.2 m below the top of the truck, and the door opens 0.8 m to the side. **Show that** the door can open fully, showing all of your steps.

e **Determine** how close to the side of the tunnel the truck could be in **d** be and still be able to open the passenger door.

f **Describe** some issues/concerns that you think architects take into account when modelling a tunnel before its construction.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Representing patterns with equivalent forms can lead to better systems, models and methods.

UNIT: FUNCTIONS – TRANSFORMATION

Summary

Translations

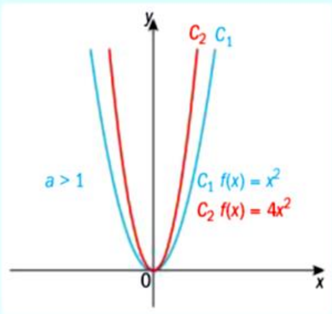
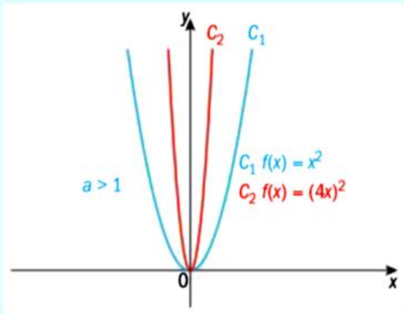
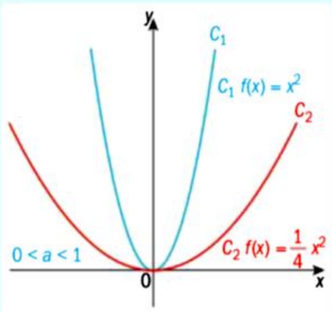
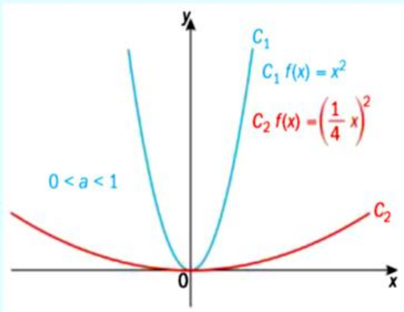
- $y = f(x - h)$ translates $y = f(x)$ by h units in the x-direction.
- $y = f(x) + k$ translates $y = f(x)$ by k units in the y-direction.
- $y = f(x - h) + k$ translates $y = f(x)$ by h units in the x-direction and k units in the y-direction.

Dilations

- $y = af(x)$ is a vertical dilation of $f(x)$, scale factor a, parallel to the y-axis.
- $y = f(ax)$ is a horizontal dilation of $f(x)$, scale factor $\frac{1}{a}$, parallel to the x-axis.

Reflections

- The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis.
- The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis.

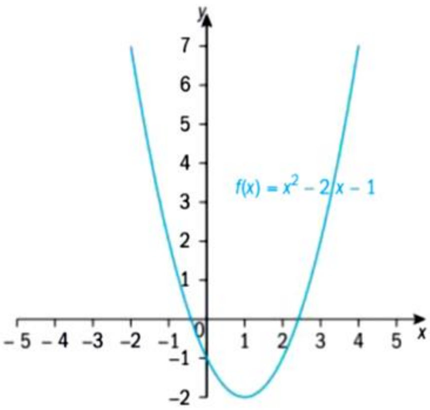
$af(x)$	$f(ax)$
 vertical dilation scale factor 4	 horizontal dilation scale factor $\frac{1}{4}$
 vertical dilation scale factor $\frac{1}{4}$	 horizontal dilation scale factor 4

Combinations of transformations

- Any linear function can be defined by one or more transformations applied one after the other to the function $y = x$.
- Any quadratic function can be defined by one or more transformations applied one after the other to the function $y = x^2$.

Mixed practice

1 Here is the graph of $f(x) = x^2 - 2x - 1$.



Use this graph to help you **sketch** the graph of each function and **write down** the coordinates of the new vertex.

a $f(x+2)$ **b** $f(x) - 4$

c $-f(x)$ **d** $f(-x)$

e $2f(x)$ **f** $f(-3x)$

g $f(x+1) - 3$ **h** $3 - f(x)$

i $-2f(x) - 1$ **j** $5f(-0.5x) + 4$

2 **State** which combination of transformations has been applied to $f(x)$ to get $g(x)$:

a $f(x) = x$, $g(x) = 2x - 8$

b $f(x) = -3x - 6$, $g(x) = x + 2$

c $f(x) = 12x + 5$, $g(x) = 6x + 6$

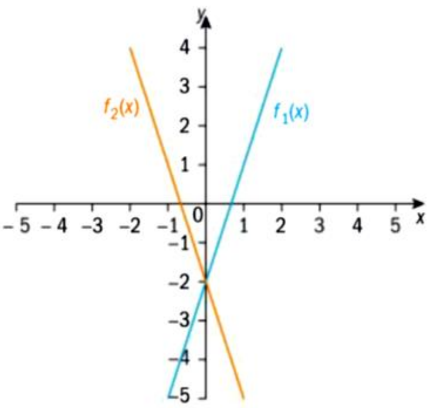
d $f(x) = x^2 + 4x + 4$, $g(x) = x^2 - 5$

e $f(x) = (x-2)^2 + 1$, $g(x) = (x+1)^2 - 2$

f $f(x) = 2x^2 + 8x + 4$, $g(x) = x^2 + 4x + 4$

3 For each pair of graphs, **find** the single transformation applied to $f_1(x)$ to get $f_2(x)$. **Write down** the function $f_2(x)$ as a transformation of $f_1(x)$.

a



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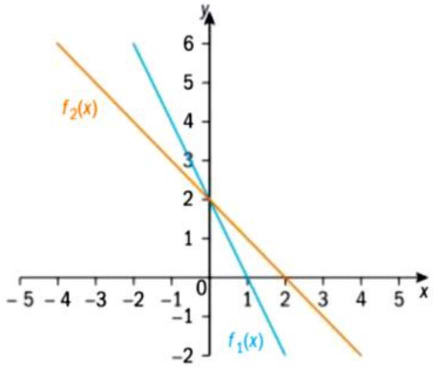

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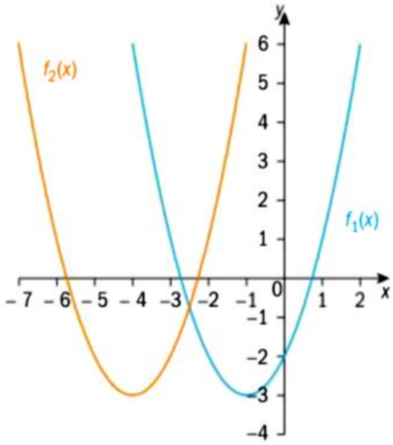

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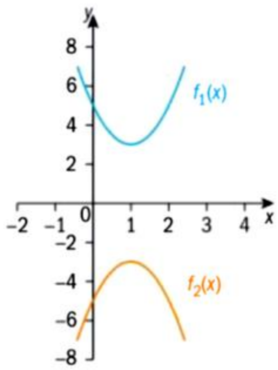

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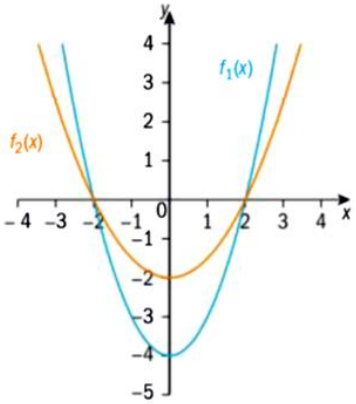

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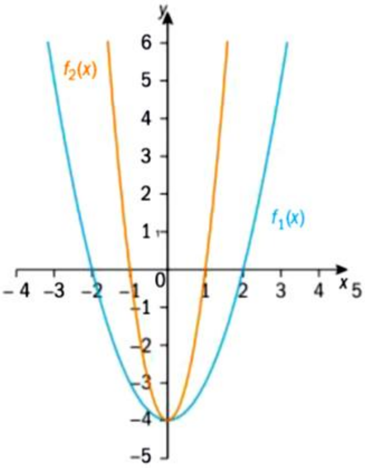
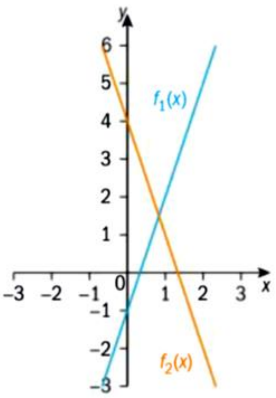


4 For each pair of graphs, **find** the transformations applied to $f_1(x)$ to get $f_2(x)$. **Write down** the function $f_2(x)$ as a transformation of $f_1(x)$.

a



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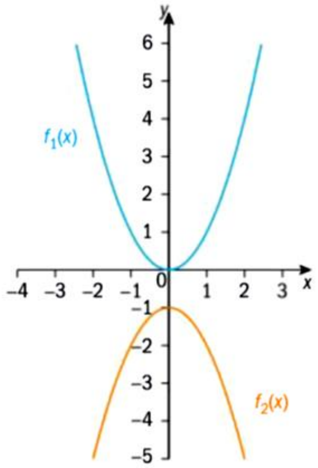

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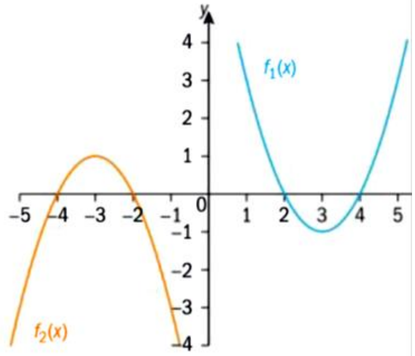

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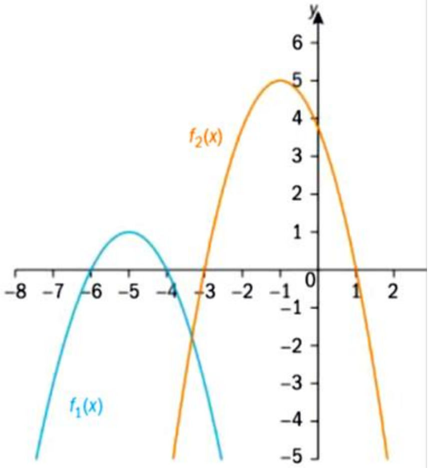

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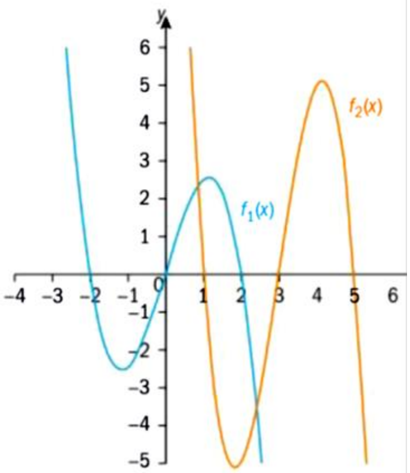

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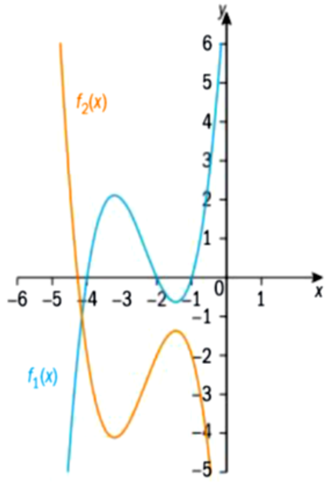
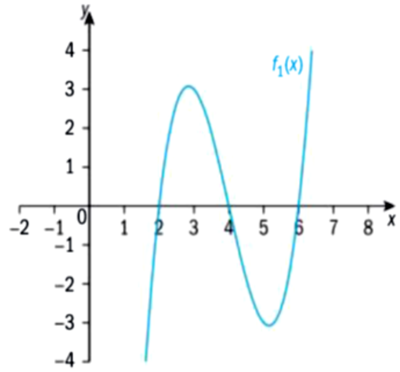


5 Given the graph of $f_1(x)$, **draw** the graph of $f_2(x)$.



a $f_2(x) = 2f_1(x+4)$ **b** $f_2(x) = f_1(2x) + 4$

c $f_2(x) = -f_1(2x)$ **d** $f_2(x) = 2f_1(-x)$

Review in context

Orientation in space and time

Whenever an object such as a ball, arrow, or dart flies freely through the air, its motion is affected by gravity. Any object which isn't self-propelled (like a bird or a rocket would be) follows a path given by a parabola. This is because gravity always acts directly downward and exerts a constant force.

The path that an object takes in flight is known as its *trajectory*. The equation of a trajectory usually gives the height (y) in terms of the horizontal distance travelled (x).

1 A frog leaps from a lily pad. Viewed from the side, its initial position is given by (0, 0). The x-axis is horizontal and the y-axis is vertical; 1 unit represents 1 meter.

The trajectory of its flight is given by $y = f(x)$ where $f(x) = \frac{3}{4}x - \frac{1}{4}x^2$.

a **Draw** a graph of its trajectory.

b **Find** the distance it travels before returning to its starting height.

c The frog can change its jump by modifying the angle and speed with which it launches off. **Describe** clearly how its jump would vary if its new trajectory were given by

i $y = f(2x)$ **ii** $y = 1.2f(0.7x)$ **iii** $y = f(-x)$

d **Explain** clearly why $y = -0.8f(x)$ does not model a trajectory for the frog's jump.

2 A student throws a ball from a window in a tall building.

Its trajectory is given by $y = h - 5\left(\frac{x}{u}\right)^2$, where $(0, h)$ is the position from which it is thrown and u is its initial horizontal velocity in m/s.

a **Describe** the effect of increasing the value of h on the graph of the trajectory.

b **Describe** the effect of increasing the value of u on the graph of the trajectory.

3 A baseball player is practicing her batting on an indoor range. Let (0, 0) be the point at which she hits the ball. The indoor range is about 5 m higher than the point at which she hits the ball.

a Her first hit follows a trajectory given by $y = -\frac{1}{20}(x-10)^2 + 5$.

i **Describe** the transformations that map a curve with equation $y = x^2$ onto a curve with equation $y = -\frac{1}{20}(x-10)^2 + 5$.

ii **Show that** this curve passes through the point (0, 0) as described.

iii Use the graph transformations to determine whether the maximum height of this curve is more than 4 meters above her hitting point and hence would hit the roof.

b Her second shot follows a trajectory given by $y = -\frac{1}{40}\left[(x-10)^2 - 100\right]$.

By considering graph transformations, find the coordinates of the maximum point of the curve. Hence **determine** whether or not the ball will hit the ceiling.

4 Car loans use simple interest so that, every month, the amount you owe on the loan (the loan balance) decreases by the same amount. (House loans or mortgages do not work this way.) Suppose you have $12 000 in car loans and you are making payments of $150 per month.

a **Write** an equation to represent the loan balance (b) as a function of the number of months that have gone by (t).

b **Draw** a graph of this relationship, $b(t)$.

c **Find** $b(50)$. **State** what this amount represents.

d **Find** how long it will take you to pay off the loan.

e **Suggest** what change(s) to the scenario could produce a parallel graph with a higher y-intercept.

f **Suggest** what change(s) to the scenario could produce a graph with a lower y-intercept but the same x-intercept.

g **State** how the graph will be transformed if you increase your original payment to $250 per month. **Write down** an equation for this new scenario.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Relationships model patterns of change that can help clarify and predict duration, frequency and variability.

UNIT: ALGEBRA – CHANGE OF SUBJECT OF FORMULA

Mixed practice

Rearrange the formulae to make the variable in brackets the subject.

1 $V = \frac{4}{3}\pi r^3$ [r]

2 $P = \frac{500T}{v}$ [T]

3 $F = G\frac{m_1 m_2}{r^2}$ [r]

4 $W = \frac{1}{2}CU^2$ [C]

5 $E = mc^2$ [c]

6 $v = v_0\sqrt{\frac{c-v}{c+v}}$ [c]

Simplify each rational expression. State any restrictions on the variables.

7 $\frac{12x^4y}{36xy}$

8 $\frac{5x^7 - 6x^5}{4x^3}$

9 $\frac{x^2 - 4x - 21}{x^2 - 8x - 33}$

10 $\frac{x+4}{x^2 - 16}$

11 $\frac{2}{x} + \frac{y}{5}$

12 $\frac{16x^3y^2}{5z^3} \times \frac{15z}{8xy}$

13 $\frac{x^3y^2 + x^2y^3}{x^3y^3} \div \frac{x^2 - y}{x^4y^4}$

Simplify these expressions. **State** the restrictions on the variable.

14 $\frac{x-2y}{4} - \frac{1+2xy}{x}$

15 $\frac{7}{x-7} + \frac{5}{x+5}$

16 $\frac{-3}{x+9} + \frac{-2}{x-8}$

17 $\frac{x+1}{x-1} - \frac{x-4}{x+4}$

18 $\frac{9}{x-1} - \frac{9x-4}{x^2-1}$

19 $\frac{3}{a} + \frac{5}{b} - \frac{a+b}{4}$

20 $\frac{x+1}{6} + \frac{2}{x} - \frac{x}{x-2}$

Review in context

Scientific and technical innovation

1 An important relationship in the study of motion is $V = \frac{d}{t}$, where V is the velocity of an object, d is distance and t is the time taken. An Airbus A333 has a cruising speed of 870 km/h. In this exercise, consider this speed to be the average speed for the entire flight.

a On a flight from Brussels to Montreal, it is flying with a headwind of 40 km/h. **Find** the actual speed of the A333.

b On the return flight, it is flying with a tailwind of 30 km/h. **Find** the actual speed of the A333.

c **Write down** an expression for the actual speed of the A333 with a headwind of x km/h.

d **Write down** an expression for the actual speed of the A333 with a headwind of y km/h.

The distance between Brussels and Montreal is 5560 km.

e Make t the subject of the formula.

f **Find** the time it takes to travel without any wind.

g **Find** the time it took on the flight described in part **a**.

h **Find** the time it took on the return flight described in part **b**.

i **Find** the wind speed and direction (headwind or tailwind) if the flight took 6 hours and 57 minutes.

j **Find** the wind speed and direction (headwind or tailwind) if the flight took 6 hours 2 minutes and 30 seconds.

2 When resistors are connected in parallel, the total resistance R_{tot} is given by $\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \quad = \frac{1}{R_{\text{to}}}$.

When resistors are connected in series, the total resistance is simply the sum of the individual resistances.

a **Write down** an expression for the total resistance of a circuit with 3 resistors, where one of them is 4 ohms larger than the smallest resistor and the largest resistor is 2 ohms larger than twice the smallest one. **Simplify** the expression into a single fraction.

b **Write down** an expression for the total resistance when a resistor the same as the largest one in **a** is connected in series to the circuit in **a**. Then **simplify** the expression into a single fraction.

3 Ohm's Law states that the voltage (V), in an electrical circuit with current (I) and two resistors, R and r connected in series, is given by $V = IR + Ir$.

a Make I the subject of the formula.

b Circuit A has a resistor (R) of 6 ohms and a smaller resistor with resistance r, connected in series. Circuit B has a resistor (R) of 5 ohms and a smaller resistor three times the smaller resistor in Circuit A. Circuit A and circuit B are connected in parallel, and have a voltage of 12 volts. **Find** an expression for the total current flowing in both circuits in terms of r.

Tip

The total current is divided between the circuits if they are connected in parallel.

Problem solving

4 Scientists calculate the gravitational force between *any* two masses (m_1 and m_2) using the formula $F = \frac{Gm_1m_2}{r^2}$, where r is the distance between them and G is the gravitational constant.

a **Determine** how many times larger the force of gravity is when the distance between the two masses is halved.

b One mass is doubled and the distance between the masses is increased by scale factor 4. **Determine** the effect on the gravitational force between them.

c One mass is doubled, the other is tripled and the distance between them is reduced by scale factor 4. **Determine** the effect on the gravitational force between them.

d One mass is halved and the other is made eight times larger. **Find** how the distance r needs to change to keep the gravitational force the same.

The minimum speed required for an object (usually a rocket) to break free of Earth's gravitational field is called 'escape velocity'. The equation for the escape velocity v_e of an object is given by: $v_e = \sqrt{\frac{2Gm_E}{r}}$ where G is the gravitational constant, m_E is Earth's mass, and r is the distance between Earth's center of mass and the object.

The escape velocity for objects leaving the Earth is approximately 25 000 miles per hour.

Now, rockets require a tremendous amount of fuel to break away from Earth's gravitational pull. This fuel adds considerable weight to the rocket, and thus it takes more thrust to lift it. But to create more thrust, you need more fuel. It's a vicious circle that scientists hope to overcome by building lighter vehicles, discovering more efficient fuels and new methods of propulsion.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Representing change and equivalence in a variety of forms has helped humans apply their understanding of scientific principles.

UNIT: ALGEBRA - PROPORTION

Summary

<table>
<tr><th>Proportional relationships</th><th>General shape of the graph</th></tr>
<tr><td>

- Two variables are said to be in **direct proportion** if, and only if, their ratio is a constant for all values of each variable.
- $y \propto x$ means 'y varies directly as x' or 'y is directly proportional to x'.
- $y \propto x$ means that $y = kx$ for a constant k, where $k \neq 0$.
  k is called the **proportionality constant**, or **constant of variation**.
- The function $y = kx$ is called a **linear variation function**.

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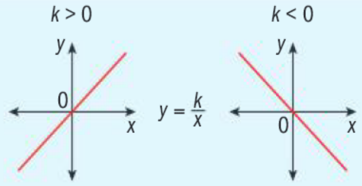

direct linear proportion

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- Two variables x and y are **inversely proportional** if multiplying one of them by a non-zero number results in the other variable being divided by the same non-zero number.
- If x and y are in an inverse linear proportion, you can say 'y varies inversely as x' or 'y is inversely proportional to x', and you can write $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.
- If y is inversely proportional to x, then y is directly proportional to $\frac{1}{x}$.
- An equation $y = k \times \frac{1}{x}$ or $y = \frac{k}{x}$ represents a relationship of inverse proportion or a **reciprocal relationship**.

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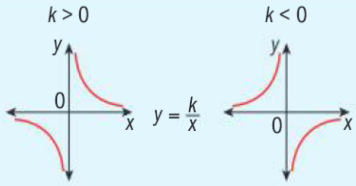

inverse linear proportion

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- Two variables x and y are in **direct non-linear proportion** if y is proportional to a power of x, or $y \propto x^n$, $n > 0$.
- The variation function is $y = kx^n$, $k \neq 0$ and $n > 0$.
- y varies directly as x^n or y is in direct proportion to x^n.

- $y = kx^n$ is a relationship of direct variation. When x is multiplied by a constant c, then y is multiplied by c^n.

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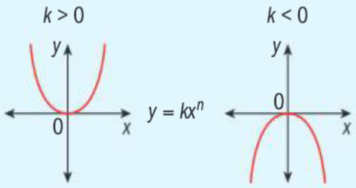
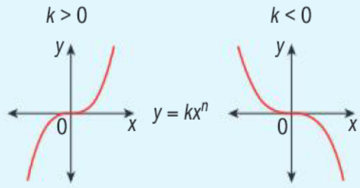


direct non-linear proportion, when n is even



direct non-linear proportion, when n is odd

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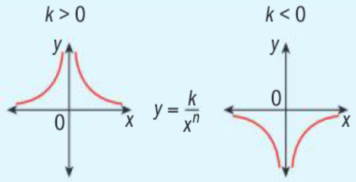
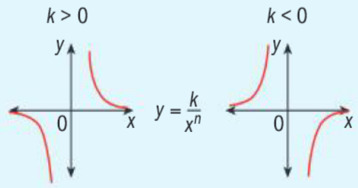
</table>

- Two variables x and y are in an **inverse non-linear proportion** if y is proportional to a power of $\frac{1}{x}$, or $y \propto \frac{1}{x^n}, n > 0$. You can also write this as $y \propto x^{-n}, n > 0$.
- The variation function is $y = \frac{k}{x^n}, k \neq 0$ and $n > 0$.

  y varies inversely as x^n or y is inversely proportional to x^n
- $y = \frac{k}{x^n}$ is a relationship of inverse variation. When x is multiplied by a constant c, then y is divided by c^n.



indirect non-linear proportion, when n is even



indirect non-linear proportion, when n is odd

Mixed practice

1 **Sketch** the graph of each variation function and **determine** whether it is a direct or inverse relationship.

a $y = 5x$ **b** $y = \frac{x}{3}$

c $y = 1.4x^2$ **d** $y = \frac{2}{x^2}$

e $y = \frac{7}{x}$ **f** $y = \frac{1}{3x}$

g $y = \frac{3x^2}{5}$ **h** $y = \frac{3}{5x^2}$

2 The variable x is directly proportional to the variable t. When $t = 4.6$, $x = 3.45$

a **Find** the value of the constant of proportionality.

b **Write down** the function relating t and x.

c Find x when $t = 8.2$

3 P is inversely proportional to y. When $P = 15$, $y = 0.2$. **Find** P when $y = 1.5$

4 M varies directly as the square of c. When $M = 12.6$, $c = 3$. **Find** c when $M = 17.15$

5 p is inversely proportional to the square of q. **Find** the missing values in this table:

p	100		1
q		0.4	2

6 **Write down** the function connecting x and y for each table of values:

a

x	−1	3	5	8
y	0.25	2.25	6.25	16

b

x	−2	−1	0.5	4
y	2.5	5	−10	−1.25

7 A supermarket sells 300ml bottles of ketchup for £1.30 a bottle.

a **Explain** why total cost of ketchup is in direct proportion to the number of bottles of ketchup bought.

b In a special offer, if you buy two bottles you get one free.

Is the total cost still in direct proportion to the number of bottles bought? **Justify** your answer.

Problem solving

8 The force needed to break a board varies inversely with the length of the board. It takes a force of 120 Newtons to break a board 60 cm long. **Find** the force needed to break a board 20 cm long.

9 The number of tennis balls you can pack in a box varies inversely with the volume of the balls. **Write down** the variation function that expresses the relationship between the number of balls that can fit in a box and the radius of the balls.

10 A giraffe's weight varies directly with the cube of the animal's height. An adult giraffe is 5 m tall and weighs 1.1 metric tonnes. **Find** the weight of a 2 m tall baby giraffe.

11 During freefall, the distance an object falls is directly proportional to the square of the time spent falling. An object falls 40.8 m in 20 s. **Find** how far it falls in 1 minute.

12 The shutter speed of a camera varies inversely with the square of the aperture setting. When the aperture setting is 8, the shutter speed is 125. **Find** the shutter speed when the aperture setting is 4.

13 The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

- **a** If the cylinder has a fixed height, **write down** the variation function between V and r. Hence, **write down** the proportionality constant in this case.
- **b** If the cylinder has a fixed radius, **write down** the variation function between V and h. Hence, **write down** the proportionality constant in this case.

Review in context

Globalization and sustainability

Problem solving

1 In order to reduce overcrowding in their cities, Boomcity and Alphatown have begun pricing office real estate in a way that they hope will encourage developers to construct buildings outside the city center.

In Boomcity, the price of a building plot varies inversely with the distance of the plot from the city's center.

In Alphatown, the price of a plot varies inversely with the square of the distance from the center.

In both Alphatown and Boomcity, a plot 5 km from the center costs \$250 000.

Find the cost of a plot 10 km from the center of each city.

2 The amount of water that flows through a water pipe is directly proportional to the square of the diameter of the pipe. A pipe of diameter 10 cm can serve 50 houses.

- **a** Make a table of the number of houses served by water pipes of diameter 10 cm, 20 cm, 30 cm, 40 cm and 50 cm.
- **b** **Use** your table to **estimate** the number of houses that can be served by a water pipe with diameter 25 cm.
- **c** **Draw** a graph of the variation function that represents the relationship between the diameter of the water pipe and the number of houses served.
- **d** Now **use** your graph to **estimate** the number of houses that can be served by a water pipe with diameter 25 cm.
- **e** **Use** the equation of the variation function to **find** the actual number of houses that can be served by a water pipe with diameter 25 cm.
- **f** **Discuss** how far off your estimations were from the actual number of houses.
- **g** Water conservation is a priority in many parts of the world, yet humans want and need water for so many of our activities. **Discuss** what the trade-offs might be between using larger diameter of pipes and our need to conserve water.

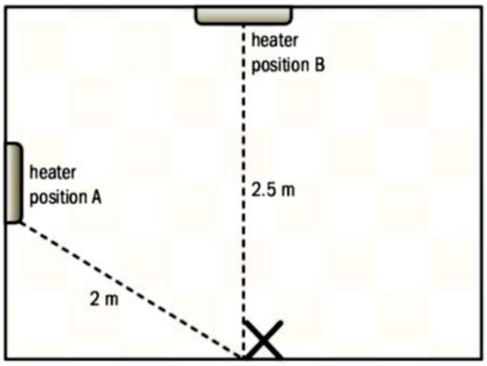
The average water consumption per head varies widely between the northern and southern hemispheres. But there is also great variation between European countries. Luxembourg's average of 80 cubic meters per person per year is 5 times less than Germany's average of 400 cubic meters per person.

Problem solving

3 Generating electricity, whether through hydroelectricity, fossil fuels, wind or nuclear energy, can have serious negative effects on the environment. We should all do our part to decrease the amount of energy we consume; something that will also save money.

In the construction of an office building, a heating engineer tries to place a heater in the spot where it will be most efficient. The heat received at X is inversely proportional to the square of the distance from the heat source. The engineer plans to put the heater at position A, however, the company's manager prefers position B.

Determine how much moving the heater to B will reduce the heat received at X. Give your answer as a percentage.



4 The 'urban heat island effect' occurs when buildings and parking lots are constructed, covering grass and dirt with concrete. Temperatures in these areas tend to be higher than normal, leading to health issues. To combat this, some cities now have ordinances that require the planting of trees to provide shade and decrease these temperature changes.

Suppose the temperature change varies directly with the cube of the diameter of the trees planted. Trees with a diameter of 1.5 meters produce an average temperature change of -0.3°C.

a **Find** the temperature change that would be expected for a tree with a diameter of 2 meters.

b If a temperature change of -1°C was desired, **determine** what size trees should be used.

5 Companies use a variety of ways to try to entice consumers to buy their products. One of those is by setting a price that will encourage people to purchase their goods. A clothing company makes jeans in a variety of price ranges. The Bootcut jeans sell for \$25 while the Classic jeans sell for \$160. The demand for this company's jeans is inversely proportional to the square of the price.

a If 50 000 pairs of the Bootcut jeans were bought, **find** how many of the Classic jeans were bought.

b If the company wanted to double the number of pairs of Classic jeans bought, **determine** at what price they should be sold. Your answer should be to the nearest dollar.

6 Since the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, you would think that the price of ice cream should vary directly with the cube of the radius of the scoop. However, the price of one small scoop of ice cream at Dave's Ice Cream Shop (radius = 3 cm) is \$2.50 and the price of a large scoop of ice cream (radius = 5 cm) is \$5.00.

a **Show that** this does not represent a direct cube variation.

b If this were a direct cube variation, **determine** the price of one large scoop.

c **Suggest** reasons why the price does not seem to vary based on the volume of the ice cream.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Changing to simplified forms can help analyze the effects of consumption and conservation.

Objective: A. Knowing and understanding
iii. solve problems correctly in a variety of contexts

For the mathematical model that describes each real-world problem, select the most efficient solution method for solving systems of equations.

Review in context

Identities and relationships

1 You decide that instead of taking vitamin and mineral supplements you will get your calcium and vitamin A by drinking milk and orange juice. An ounce of milk contains 38 mg of calcium and 56 µg (micrograms) of vitamin A; an ounce of orange juice contains 5 mg of calcium and 60 µg of vitamin A. **Determine** how many ounces of milk and orange juice you would need to drink daily in order to meet your minimum requirement of 550 mg of calcium and 1200 µg of vitamin A. Decide whether or not this a realistic amount for you to drink.

2 A chemist has been asked to make a solution of 8 liters containing 20% acid. The problem is that he has only two acidic solutions, one containing 12% acid and the other containing 32% acid. **Determine** how many liters of each he should use in order to create the solution he has been asked to make.

3 A coffee distributor has two types of coffee. The premium blend sells for $10.50 per kilogram, and the standard blend sells for $8.25 per kilogram. The distributor wishes to create 20 kilograms of a mixture containing these two blends to sell at $9 per kilogram. **Determine** how many kilograms of each blend should be in the mixture.

4 You've decided to buy a printer and have narrowed it down to two choices. The laser printer costs $150 but the average cost of each page is just 1.5 cents. The other option is an inkjet printer, which costs $30, but each page has an average cost of 6 cents. **Determine** the conditions (e.g. the number of pages) where each printer is the better buy.

Coffee is the world's second most valuable traded commodity, surpassed only by petroleum.

Since Brazil produces around 40% of the world's coffee, the single most influential factor in world coffee prices is the weather in Brazil.

In 1991, a group of Cambridge University scientists aimed a fixed camera on their department's coffee pot, streaming the live footage on the web so that they could tell if the pot was empty or not. This made it the world's first live webcam.

Before coffee caught on in the US in the 1700s, the preferred breakfast drink of Americans was beer.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

UNIT: ALGEBRA – FACTORISATION 2

Mixed practice

1 Solve each quadratic by factorizing, and check your answers graphically. Leave your answers exact.

a $x^2 - 2x - 24 = 0$ **b** $x^2 - 6x = 27$

c $6x^2 = 5x + 4$ **d** $17x - 2x^2 = 21$

e $-5x^2 + 7x = 2$ **f** $2x^2 = 8x - 6$

2 Solve, if possible, by completing the square, and check your answers graphically. Leave your answers exact.

a $x^2 + 6x - 59 = 0$ **b** $x^2 + 12x = -23$

c $x^2 - 10x + 26 = 8$ **d** $3x^2 + 6x = 7$

e $2x^2 = 8x - 3$ **f** $2x^2 - 5x - 3 = 0$

3 Solve, if possible, using the quadratic formula, giving answers to 2 d.p., and check your answers graphically.

a $x^2 + 10x + 13 = 0$ **b** $x^2 - 5x = 7$

c $3x^2 = 1 + 3x$ **d** $x^2 - \frac{1}{3}x = 2$

e $4x^2 + 8x = 1$ **f** $-3x^2 + 2x = 12$

4 Solve, if possible, using the most appropriate method. Leave your answer exact and completely simplified. Check your answers graphically.

a $(x+3)^2 + x^2 - 9x = 8$ **b** $\frac{x^2}{2} + \frac{5}{2} = -x$

c $(x-2)^2 - 11 = 0$ **d** $2x(x-1) - 5 = -x^2$

e $(2x-6)^2 = 12$

f $(2x+5)(x-1) = (x-3)(x+8)$

Solve these next problems using the most efficient method. Round your answers appropriately.

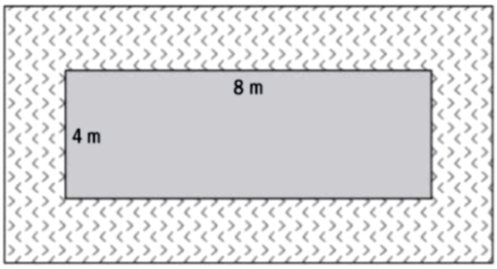
Problem solving

5 The length of a rectangle is 3 cm greater than its width. Its area is 108 cm². **Find** the dimensions of the rectangle.

6 The length of a rectangle is 2 cm more than 3 times its width. Its area is 85 cm². **Find** the dimensions of the rectangle.

7 The length of a rectangle is 1 cm more than its width. If the length of the rectangle is doubled, the area of the rectangle increases by 30 cm². **Find** the dimensions of the original rectangle.

8 A rectangular lawn measures 8 m by 4 m and is surrounded by a border of uniform width. The combined area of the lawn and border is 165 m². **Find** the width of the border.



9 The height of a right-angled triangle is 5 cm less than its base. The area of the triangle is 42 cm². **Find** its base and height.

10 One side of a triangle is 2 cm shorter than the hypotenuse and 7 cm longer than the third side. **Find** the side lengths of the triangle.

11 Find two consecutive odd integers whose product is 99.

12 The square of a number exceeds the number itself by 72. **Find** the number.

13 Find two consecutive positive integers such that the square of the first less 17 equals 4 times the second.

14 The height h in meters of a football kicked into the air can be modeled by the function $h(t) = -4.9t^2 + 24.5t + 1$ where t is in seconds.

a Find how long after being kicked it takes the ball to hit the ground.

b Determine how long it takes the object to reach a height of 20 meters.

c For how long was it above this height?

15 The profits of an international ticket agency can be modeled by the function $P(t) = -37t^2 + 1258t - 7700$, where t is the number of tickets sold and P is in dollars. **Determine** the ticket price that would leave the agency no profit or loss.

16 A square piece of cardboard is to be formed into a box. After 5 cm squares are cut from each corner and the sides are folded up, the box will have a volume of 400 cm³. **Find** the length of a side of the original piece of cardboard.

Objective: D. Applying mathematics in real-life contexts
ii. select appropriate mathematical strategies when solving authentic real-life situations

In the Review in context section, draw and label suitable diagrams, create equations, and select the most effective method to solve them.

Review in context

Scientific and technical innovation

Quadratic equations can model the path of a projectile through space, and its height above the ground at different times. A projectile is any object that is propelled with force through the air, such as kicking a soccer ball, doing the high jump or even launching fireworks.

Quadratic models of projectile motion take into account three factors:

- the initial height off the ground
- the initial velocity with which the object moves
- the acceleration due to gravity that affects all falling objects.

The formula for calculating the height above ground is:

$$h(t)=\frac{1}{2}(-9.8)t^2+V_it+h_i$$

where the acceleration due to gravity (on Earth) is -9.8 m/s^2, V_i is the initial velocity, and h_i is the initial height.

Problem solving

1 A model rocket is launched from 2.5 m above the ground with initial velocity 49 m/s. **Use** the quadratic model $h(t)=\frac{1}{2}(-9.8)t^2+49t+2.5$ to **determine** how long it takes for it to land.

2 An object is launched directly upward at 24 m/s from a platform 30 m high.

a Write the quadratic model for this object.

b **Determine:**

i the maximum height the object reaches

ii how long the object takes to reach this height

iii how long the object takes to strike the ground again.

3 A projectile is launched from ground level directly upward at 39.2 m/s. **Determine** how long the projectile's altitude is 34.3 m or above.

4 Suppose NASA wants to launch a probe on the surface of the moon, which has one sixth the gravitational pull of the Earth (and therefore one sixth the acceleration due to gravity). The height of the launcher is 0.5 m.

a On the moon, how long would it take the probe to reach a height of 60 m if its initial velocity is 15 m/s?

b If the initial velocity is 24.5 m/s, how much longer will it be in the air on the moon compared to a similar launch on Earth? (It is launched from the surface in both places.)

c **Explain** how you think scientists test their hypotheses about the behavior of objects on the moon (or Mars) when the force of gravity there is so different than on Earth.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Review in context

Scientific and technical innovation

1 The water in the Danube Delta flows at about 2 km/h. On a kayaking trip Max paddles upstream for 15 km, takes a half-hour lunch break, and then returns to his original starting point. His entire trip takes 3.5 hours (including his lunch break).

Let x km/h represent the kayak's average speed. Then the kayak's average speed upstream is $(x - 2)$ km/h and downstream is $(x + 2)$ km/h.

Average speed = $\frac{\text{total distance}}{\text{total time}}$

a **Justify** the expressions for the average speed of the kayak going upstream and downstream.

b **Write** an expression for the time taken for the kayak to travel:

i upstream

ii downstream.

c **Use** your expressions from part **b** to write an equation representing the whole journey.

d **Find** the average speed of the kayak in still water.

2 When resistors are connected in parallel, the total resistance R_{tot} is given by:

$$\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\dots=\frac{1}{R_{tot}}.$$

Find the resistances of two resistors connected in parallel where:

a One resistance is three times as large as the other. Total resistance is 12 ohms.

b One resistance is 3 ohms greater than the other. Total resistance is 2 ohms.

c **Find** the resistances of three resistors connected in parallel where the resistance of one is 2 ohms greater than the smallest one, and the other has resistance twice as large as the smallest one.

The total resistance is $\frac{10}{7}$ ohms.

d An engineer wants to build a circuit with total resistance of 3 ohms. **Show** how he can do this with two resistors, where the resistance of one is 8 ohms greater than the other.

3 Lucy makes an 8-hour round trip of 45 km upstream and 45 km back downstream on a motorboat travelling at an average speed of 12 km/h relative to the river. **Determine** the speed of the current.

'speed 12 km/h relative to the river' means the speed travelling upstream is $12 - x$, where x is the speed of the current.

4 Lenses are used to look at objects, and to project images on to a screen.

The focal length of a lens is the distance between the lens and the point where parallel rays of light passing through the lens would converge.

The distance between a lens and the image produced d_i is related to the focal length f and the distance between the lens and the object d_o by the formula $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$.

a The focal length of a lens is 15 cm. The image appears to be twice as far away as the original object. **Find** how far the object is from the lens.

b An object is placed in front of a lens, at a distance 2 cm more than the focal length. The image appears to be 4 cm from the lens. **Find** the focal length of the lens.

c The human eye contains a lens that focuses an image on the retina for it to be seen clearly.

An object is at a distance from the eye that is 20 cm longer than the focal length. The distance from the eye to the retina is –20 mm (on the other side of the lens from the object). **Find** the focal length of this eye.

5 An airplane travels 910 miles with a tailwind in the same time that it travels 660 miles with headwind. The speed of the airplane is 305 m/h in still air. **Determine** the wind's speed.

UNIT: ALGEBRA – SEQUENCES

Summary

- In an **arithmetic sequence** the difference between consecutive terms is constant.
- An arithmetic sequence with first term a and common difference d has recursive formula $u_{n+1} = u_n + d$, $u_1 = a$, and explicit formula $u_n = a + (n - 1)d$ for the nth term.
- In a **geometric sequence** the ratio between consecutive terms is constant.
- A geometric sequence with first term a and common ratio r has recursive formula $u_{n+1} = ru_n$, with $u_1 = a$ and explicit formula $u_n = ar^{n-1}$ for the nth term.
- In a geometric sequence, the common ratio, r, cannot be equal to 0.

Mixed practice

1 An arithmetic sequence begins 7, 25, 43, …

a **Find** the common difference.

b **Write down** a recursive formula linking the nth term to the $(n + 1)$th term.

c **Write down** an explicit formula for the nth term.

d **Find** the value of the 15th term.

e **Find** the term number of the term with value 367.

2 An arithmetic sequence has fourth term 253 and fifth term 291.

a **Write down** the value of the common difference.

b **Hence find** the first term.

c **Find** the sum of the first three terms.

3 An arithmetic sequence has third term 31 and sixth term 52. Let the first term be a and the common difference be d.

a Form two simultaneous equations in terms of a and d.

b **Hence find** a, d and the value of the 10th term.

4 An arithmetic sequence has common difference 6. The product of the first two terms is 91. **Find** the value of the first term, given that it is negative.

5 The third term of an arithmetic sequence is twice the first term. The second term is 45.

a **Find** the value of the first term and the common difference.

b **Determine** whether or not 310 is a term of the sequence.

6 A geometric sequence begins 6, 42, 294, …

a **Write down** the value of the first term and the common ratio.

b **Write down** a recursive formula linking the nth term to the $(n + 1)$th term.

c **Write down** an explicit formula for the nth term.

d **Find** the value of the fifth term.

e **Find** the value of the first term to exceed one million.

7 A geometric sequence begins 6, 3, 1.5, …

a **Write down** the value of the first term and the common difference.

b **Write down** a recursive formula linking the nth term to the $(n + 1)$th term.

c **Write down** an explicit formula for the nth term.

d **Find** the value of the tenth term, correct to three significant figures.

8 A geometric sequence has common ratio 4. The second term is 9 more than the first term.

a **Write down** two different expressions for the second term in terms of a, the first term.

b **Hence** write an equation using this information and **find** the value of a.

c **Find** how many terms there are in the sequence that are less than 1000.

9 A geometric sequence has first term 12 and third term 48. The common ratio is r.

a **Show that** $r^2 = 4$.

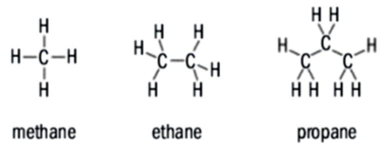
b **Hence find** two possible values for the common ratio.

c **Find** the possible values of the sixth term.

Review in context

Scientific and technical innovation

1 Consider the following diagram, which illustrates the chemical structure of methane, ethane and propane, three examples of chemicals known as alkanes.



a **Show that** the number of hydrogen atoms (represented by an H) in the three alkanes pictured forms an arithmetic sequence.

b Let u_n be the number of hydrogen atoms in an alkane with n carbon (C) atoms. **Write down** an explicit formula for u_n.

c **Find** the number of hydrogen atoms in an alkane with 20 carbon atoms.

d **Find** the number of carbon atoms in an alkane with 142 hydrogen atoms.

2 The developers of a new social media website think that its membership will grow by the same scale factor every month. At the end of the first month it has 20 000 members, and at the end of the second month it has 25 000 members.

a **Explain** which information in the statement above suggests that this can be modelled with a geometric sequence.

b **Determine** how many members this model would predict for the website to have at the end of the 12th month.

c **Determine** how many members this model predicts for the end of the second year.

3 A design company produces business cards. They charge a set fee for design, and then sell the cards in boxes of 100 cards. Each box costs the same amount.

The total cost (including the design fee) for 400 cards is \$108.

The total cost (including the design fee) for 600 cards is \$133.

Find the cost for 1000 cards, and **calculate** the design fee.

4 A machine produces a constant number of components per hour, except in the first hour of operation.

By the end of the second hour of operation, it has made 6000 components. By the end of the seventh hour, it has made 24 000 components.

a **Explain** why the number of components made by the nth hour forms an arithmetic sequence.

b **Find** the number of components made in the first hour.

c **Find** the number of components made in total over a nine-hour working day.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Using different forms to generalize and justify patterns can help improve products, processes and solutions.

UNIT: STATISTICS – SAMPLING

Summary

- A **population** is a set of objects under consideration in a statistical context. It could be a group of people, or a collection of objects.
- A **census** is a collection of data from every member of a population.
- A **sampling frame** is a list of every member of the population.
- A **sample** is a subset of a population.
  - A sample is **representative** if its properties reflect those of the population from which it comes.
  - A **simple random sample** is a sample such that every member of the population is equally likely to be included in the sample, independent of any other member of the population.
  - A **systematic sample** takes members of the population at regular intervals from the sampling frame.
  - A **stratified sample** identifies different sections of the population and ensures that those groups are represented in appropriate proportions in the sample.
  - A **quota sample** is used when the sampling frame is unknown, or does not exist. It involves collecting data until enough pieces of data are found.
- A sample is **biased** if it is not a fair representation of the population.
- **Inference** is the act of drawing conclusions about a population based on a sample.
  - You can use the sample mean to estimate the mean of a population.
  - Larger samples provide more reliable estimates.

Mixed practice

1 Tom and Jo are comparing their marks in recent English assessments. The IQR for Tom's marks is 13 and the IQR for Jo's marks is 18. Tom says that because his IQR is smaller he has performed better in the assessments. Jo says that because her IQR is larger she has performed better in the assessments.

Comment on their claims.

2 A school's ballet class has 11 students, with a mean height of 1.51 m and a range of heights of 21 cm. The school's chess club also has 11 students, with a mean height of 1.73 m and a range of heights of 43 cm.

a **Comment** on the differences between the heights of the members of the ballet class and the chess club.

b **Explain** why it would be preferable to know the interquartile range rather than the range of the data sets.

c Claire proposes to use the data to make predictions concerning adult ballet dancers and adult chess players. **Explain** why these

3 Paul wants to estimate the average length of the fish in a lake.

He knows that roughly 40% of the fish in the lake are salmon and 60% of the fish are trout. To measure the fish, he will catch them with a large net and measure them on the shore before returning them to the water. He plans to take a sample of size 50.

a **Suggest** a suitable sampling method for Paul to use.

b **Explain** how he would take such a sample.

4 A polling company would like to conduct a survey to determine the extent to which the residents of a city feel that they are subject to unwanted advertising. The company proposes to conduct a simple random sample using the local phone directory as a sampling frame.

Evaluate the company's plan.

Review in context

Identities and relationships

1 The average household income in two neighboring towns is assessed by taking a random sample of residents using a recent census as a sampling frame. The data obtained is as follows:

Income range ($)	Town A	Town B
$20\,000 \le x < 30\,000$	4	1
$30\,000 \le x < 40\,000$	7	4
$40\,000 \le x < 50\,000$	11	7
$50\,000 \le x < 60\,000$	15	6
$60\,000 \le x < 70\,000$	5	12
$70\,000 \le x < 80\,000$	4	9
$80\,000 \le x < 90\,000$	4	7
$90\,000 \le x < 100\,000$	0	3
$100\,000 \le x < 110\,000$	0	1

a **Present** this information using a pair of cumulative frequency graphs on the same axes.

b **Comment** on the differences between the earnings of households in towns A and B.

c The statistician conducting the survey claims that the samples are representative of the towns they came from. **Explain** what is meant by the word *representative* in this context.

2 To assess the impact of a new literacy strategy in 50 primary schools, a local education authority assesses the reading age of all 11-year-olds in three of the schools before and after implementation. The following table summarizes the data.

	Before	After
Minimum	8.4	8.2
Q_1	9.2	10.3
Q_2	10.1	10.8
Q_3	10.5	11.1
Maximum	13.2	12.8

a **Present** the data with over-and-under box-and-whisker diagrams.

b **Describe** the similarities and differences between the two sets of data.

c **Comment** on the effectiveness of the education authority's intervention.

d **Evaluate** the authority's sampling method.

3 You will be able to find data about the representation of women in national parliaments online from the Inter-Parliamentary Union (www.ipu.org) or from the World Bank (data.worldbank.org). You are to investigate the proposition that the percentage of women's representation varies in different continents.

a Take a random stratified sample of 5 countries from Africa, Asia and Europe. **Compare** the means of these samples.

b **Comment** on whether the samples are representative of the data in this case.

c **Describe** how you would take a *census* from this data to compare the average representation from the three continents.

4 Elderly people were surveyed about the number of hours, t, of social contact they have with volunteers after the introduction of a new assistance scheme.

Hours	Before	After
$0 \le t < 2$	4	0
$2 \le t < 4$	10	8
$4 \le t < 5$	7	9
$5 \le t < 6$	5	8
$6 \le t < 8$	6	5
$8 \le t < 10$	4	5
$10 \le t < 12$	3	2
$12 \le t < 16$	0	2
$16 \le t < 20$	1	1

a **Interpret** the data and **suggest** whether or not the scheme was successful.

The sample was obtained by collecting contact details for 40 elderly people. The researcher waited in the local bus depot and asked people who he thought were over 75 whether they would be interested in such a scheme and whether he could have their contact details to evaluate the success of the scheme.

b **Determine** the type of sampling used.

c **Evaluate** the researcher's methods.

d **Suggest** a more appropriate way of constructing the sample.

5 You want to find out if students in your school are more likely to donate to local, national or international causes.

a **Outline** difficulties you expect to encounter in obtaining data relevant to your inquiry.

You decide that the easiest way to collect reliable data is to give people forms to keep for a whole year, which they can fill in when they donate money to charity.

You give a form to every student in your school to take home.

You believe that donation trends will vary depending on whether the parents in the family grew up locally, grew up elsewhere in the country, or grew up in a different country.

Separate research suggests that 30% of families had parents that grew up locally, 45% grew up elsewhere in the country and 25% grew up in a different country.

b **Explain** how you would use this information to try to make your sample representative.

After a year, you receive 40 responses from families with parents that grew up locally, 75 from those that grew up elsewhere in the country and 50 from those that grew up in a different country.

c **Calculate** the number of responses you should use from each category of respondent in order to obtain the largest representative sample possible.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

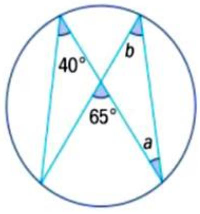
Statement of Inquiry:

Generalizing and representing relationships can help to clarify trends among individuals.

Mixed practice

1 **Find** the size of the marked angles.

a



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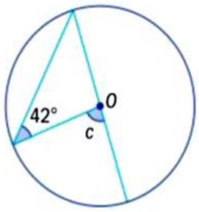
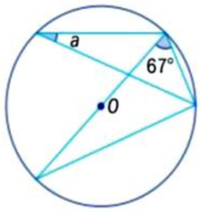


2 **Find** the size of the marked angle.
Justify your answers.

a



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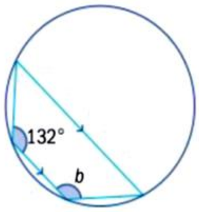


3 For each statement, **write down** the converse. Determine which statements, and which of their converses, are true.

a If $a = 7$ then $3a - 2 = 19$.

b If $b = a$ then $a - b = 0$.

c If it is a bird then it has wings.

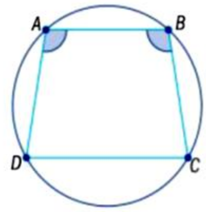
d If a polygon has four sides, then it is a square.

e If $c = 9$ then $c^2 = 81$.

f If a right-angled triangle is drawn with its vertices on the circumference of a circle then its hypotenuse is a diameter of the circle.

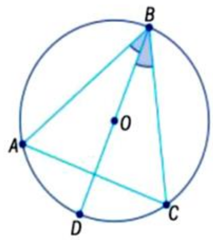
4 In the cyclic quadrilateral $ABCD$, $\angle DAB \cong \angle ABC$.

Prove that $AD = BC$.



5 A, B, C and D lie on a circle center O, and DOB bisects $\angle ABC$.

Prove that ΔABC is isosceles.

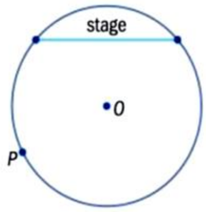


Review in context

Personal and cultural expression

1 A circular auditorium has a stage across the end of it. When a patron sits at O, the center of the auditorium, the stage occupies 88° of the patron's field of vision.

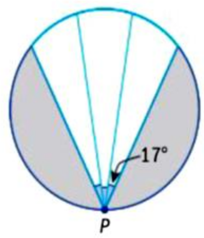
Find the amount of the field of vision that the stage would occupy when viewed from P, at the very edge of the auditorium.



2 Three projectors, used to display images on the wall of a circular performance space, are mounted on the opposite wall. The projectors each have a beam width of 17°, and are placed one above another at a single point P.

The circle has diameter 6 m.

Find the total length of the wall projected on when the projectors are positioned so their beams overlap by 20 cm.



Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Logic can justify generalizations that increase our appreciation of the aesthetic.

Objective: D. Applying mathematics in real-life contexts
ii. select appropriate mathematical strategies when solving authentic real-life situations

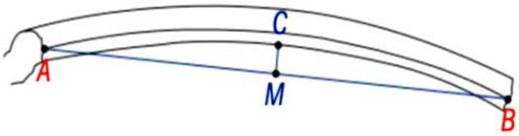
For each of the questions below, consider the geometry of the real-life situation and determine which version of the intersecting chords theorem you need to use.

Review in context

Personal and cultural expression

1 A fragment of a bowl, bearing an ancient inscription, dates from the 7th century BCE. Archaeologists believe the original bowl was circular.

Points A and B are chosen on the edge of the bowl and AB is measured to be 5 cm. M is the midpoint of AB, and MC is perpendicular to AB where C lies on the circumference of the bowl. MC is measured to be 0.25 cm.



a **Sketch** the complete bowl, including the line segments AB and MC.

b **Show** that the line segment MC passes through the center of the bowl when extended.

c **Use** the intersecting chords theorem to **find** the diameter of the bowl.

2 Huge stone circles have been discovered in Jordan in the Middle East. Archaeologists have been attempting to find out more about these circles. Measuring hundreds of meters across, one of the challenges has been to measure them accurately, especially as some of them are incomplete.

Two points, A and B are marked on the circle and the distance between them is measured. The mid-point of AB is found and the distance from this point to the nearest point on the circle is measured.

Surveyors found that AB was 114 m and the distance from the midpoint was 9 m.

a **Use** this information to **calculate** the diameter of the circle.

A second group of archaeologists use a different technique. From a point outside the circle they make a straight line which is a tangent to the circle and measure the distance to the point of contact. Then they measure the shortest distance from the original point to the circle.

The distances are 150 m and 52 m respectively.

b **Calculate** the diameter from these measurements and then **compare** the two results.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

UNIT: EXPONENTIAL FUNCTIONS

Summary

An exponential function is of the form $f(x) = a \times b^x$, where $a \neq 0$, $b > 0$, $b \neq 1$.

The independent variable x is the exponent.

In the standard form of the exponential function $y = a \times b^x$:

- the parameter a represents the initial amount
- the parameter b represents the growth or reduction (decay) factor.
- x is the independent variable (e.g. time)
- y is the dependent variable (e.g. amount at time x)

The exponential function $y = b^x$ does not have any x-intercepts.

For $b > 1$, as x gets less and less, the function approaches 0, but never equals 0.

For $0 < b < 1$, as x gets larger and larger, the function approaches 0, but never equals 0.

For any value of $b > 0$ the function $y = b^x$ has a horizontal asymptote at $y = 0$.

A horizontal asymptote is the line that the graph of $f(x)$ approaches as x gets larger and larger.

Exponential growth is modeled by $y = a(1 + r)^x$ where r is the growth rate in decimal form (the percentage expressed as a decimal). Exponential decay is modeled by $y = a(1 - r)^x$ where r is the decay rate in decimal form.

Transformations of exponential functions

Reflection: For the exponential function $f(x) = a \times b^x$:

The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis.

The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis.

Translation: For the exponential function $f(x) = a \times b^x$:

- $y = f(x - h)$ translates $y = f(x)$ by moving it h units in the x-direction.

  When $h > 0$, the graph moves in the positive x-direction, to the right.

  When $h < 0$, the graph moves in the negative x-direction, to the left.
- $y = f(x) + k$ translates $y = f(x)$ by moving it k units in the y-direction.

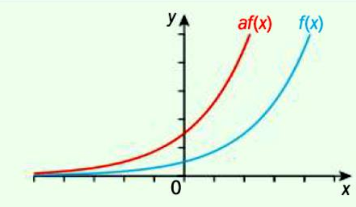
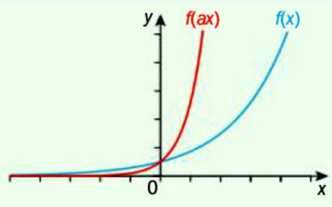
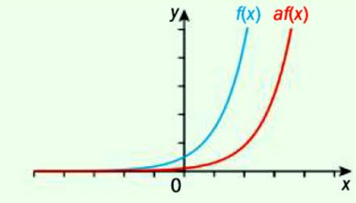
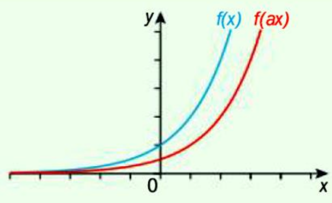
  When $k > 0$, the graph moves in the positive y-direction, up.

  When $k < 0$, the graph moves in the negative y-direction, down.
- $y = f(x - h) + k$ translates $y = f(x)$ by h units in the x-direction and k units in the y-direction.

Dilation: For the exponential function $f(x) = a \times b^x$:

$y = af(x)$ is a vertical dilation of $f(x)$, scale factor a, parallel to the y-axis.

$y = f(ax)$ is a horizontal dilation of $f(x)$, scale factor $\frac{1}{a}$, parallel to the x-axis.

	$af(x)$	$f(ax)$
$a > 1$	 vertical dilation	 horizontal dilation
$0 < a < 1$	 vertical dilation	 horizontal dilation

Objective: **D.** Applying mathematics in real-life contexts
iii. apply the selected mathematical strategies successfully to reach a solution

In this Review in context you will apply exponential functions to determine the age of fossils using carbon dating.

Review in context

Orientation in space and time

Ötzi was found on 19 September 1991 by two German tourists, Helmut and Erika Simon at an elevation of 3210 meters (10 530 ft) on the Austrian–Italian border. Because the body, clothing and tools were so well preserved, the Simons thought the man had died recently. Scientists at the University of Innsbruck in Austria used carbon dating to estimate that Ötzi died about 5300 years ago, making him the oldest, best preserved mummy in the world.

Carbon dating of fossils is based upon the decay of ^{14}C, a radioactive isotope of carbon with a relatively long half-life of about 5700 years.

All living organisms get ^{14}C from the atmosphere. When an organism dies, it stops absorbing ^{14}C, which begins to decay exponentially. Carbon dating compares the amount of ^{14}C in fossil remains with the amount in the atmosphere, to work out how much has decayed, and therefore how long ago the organism died.

1 Let N_0 = the initial amount of ^{14}C at the time of death.

Let x = the number of half-lives, where each half-life is 5700 years.

Let N = the amount present after x number of half-lives.

Write an exponential function relating N_0, x, and N.

You can use your function from **1** to help you solve the following problems.

Problem solving

2 When it dies, an organism contains 30 000 nuclei of ^{14}C. **Calculate** the number of ^{14}C nuclei in the organism after 11 400 years.

3 The amount of ^{14}C in a fossil is calculated to be 0.25 times the amount when the organism died. **Calculate** the approximate age of the fossil.

4 A fossil bone is approximately 16 500 years old. **Estimate** the fraction of ^{14}C still in the fossil.

5 Only 6% of the original amount of ^{14}C remains in a fossil bone. **Estimate** how many years ago it died.

6 **Calculate** the approximate percentage of ^{14}C left in a fossil bone sample after 35 000 years.

7 Analysis on an animal bone fossil at an archeological site reveals that the bone has lost between 90% and 95% of its ^{14}C. **Determine** an approximate interval for the possible ages of the bone.

8 Using your function, **explain** why fossils older than 50 000 years may have an undetectable amount of ^{14}C.

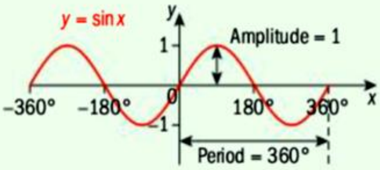
Reflect and discuss

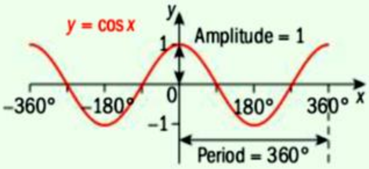
How have you explored the statement of inquiry? Give specific examples.

Summary

The **amplitude** is the height from the mean value of the function to its maximum or minimum value. The graphs of $y = \sin x$ and $y = \cos x$ have amplitude 1.

The **period** is the horizontal length of one complete cycle. The graphs of $y = \sin x$ and $y = \cos x$ have period 360°.





The graph of $y = a \sin x$ has amplitude a.
The graph of $y = a \cos x$ has amplitude a.

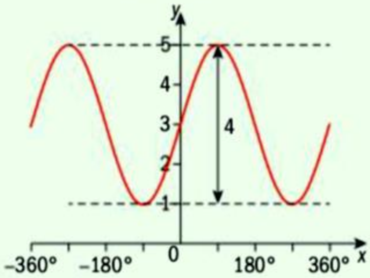
A periodic function repeats a pattern of y-values at regular intervals.

The period of the functions $y = a \sin bx$ and $y = a \cos bx$ is $\frac{360°}{b}$

Period $= \frac{360°}{\text{frequency}}$

One complete repetition of the pattern is called a **cycle**. The parameter b is the **frequency**, or number of cycles between 0° and 360°.

The amplitude is $(y_{max} - y_{min}) / 2$.

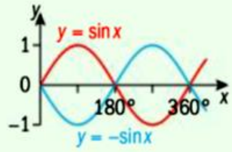


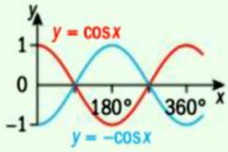
Amplitude $= \frac{4}{2} = 2$

A **sinusoidal function** is any function that can be obtained from a transformation of the sine function. The graph of $y = \cos x$ can be obtained by translating the graph of $y = \sin x$ 90° to the left. So $y = \cos x$ is a sinusoidal function.

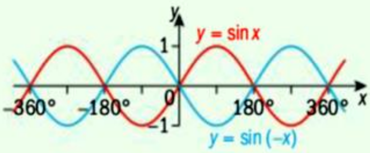
Reflection: For the sinusoidal functions $f(x) = \sin x$ and $f(x) = \cos x$.

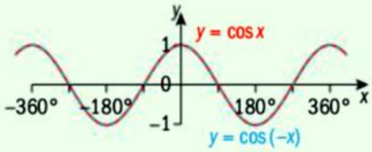
The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x-axis.





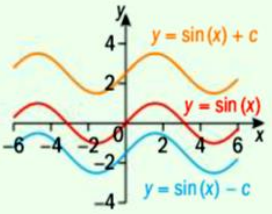
The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y-axis.





Translation: For the sinusoidal functions $f(x) = \sin x$ and $f(x) = \cos x$:

- $y = f(x) + d$ translates $y = f(x)$ by d units in the y-direction.
  When $d > 0$, the graph moves in the positive y-direction, up
  When $d < 0$, the graph moves in the negative y-direction, down.



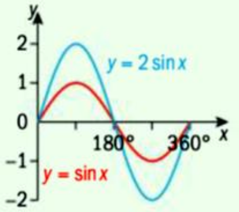
- $y = f(x - h)$ translates $y = f(x)$ by h units in the x-direction.
  When $h > 0$, the graph moves in the positive x-direction, to the right.
  When $h < 0$, the graph moves in the negative x-direction, to the left.

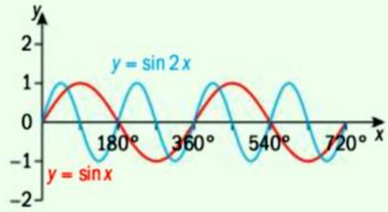
Horizontal translation of cosine and sine graphs is an extended topic, but here you can see that the rules are the same as for translations of all functions $f(x)$.

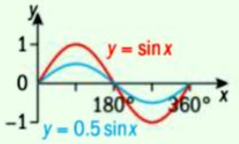
Dilation: For the sinusoidal functions $f(x) = a \sin bx$ and $f(x) = a \cos bx$:

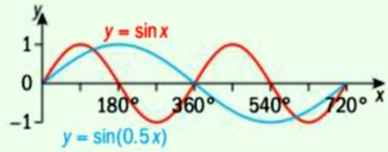
$y = af(x)$ is a vertical dilation of $f(x)$, scale factor a, parallel to the y-axis. a is the amplitude.

$y = f(bx)$ is a horizontal dilation of $f(x)$, scale factor $\frac{1}{b}$, parallel to the x-axis. b is the frequency.

<table>
<tr><th></th><th>af(x)</th><th></th><th>f(bx)</th></tr>
<tr><td>a > 1</td><td>

vertical stretch</td><td>b > 1</td><td>

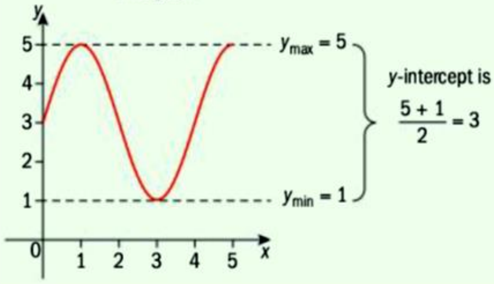
horizontal compression</td></tr>
<tr><td>0 < a < 1</td><td>

vertical compression</td><td>0 < b < 1</td><td>

horizontal stretch</td></tr>
</table>

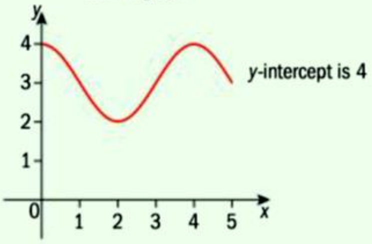
For $y = a\sin bx + d$, the y-intercept is the average value of the function: $\frac{y_{max} + y_{max}}{2}$

sine graph



For $y = a\cos(bx) + d$, the y-intercept is the maximum value of y.

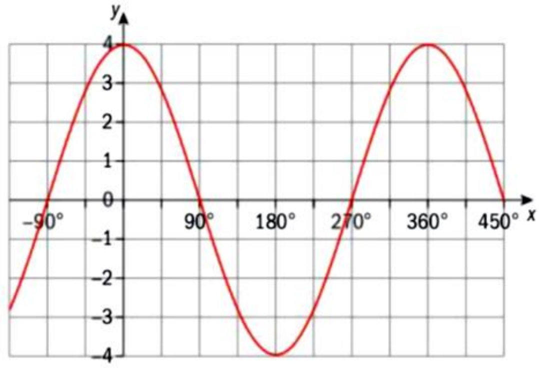
cosine graph



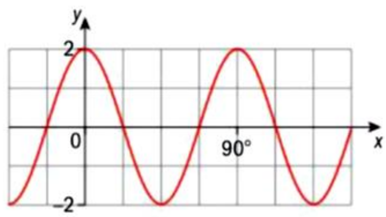
Mixed practice

1 For each of the following graphs, **find** the amplitude, frequency and period. Then, **write down** the equation the represents the function.

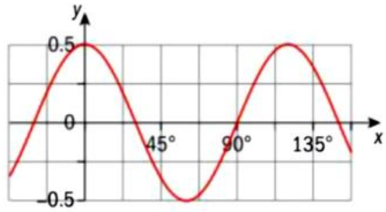
a



b



c



2 **Draw** the graph of each function showing at least two cycles. Be sure to clearly indicate the amplitude and period.

a $y = 0.4\sin(2x)$ **b** $y = 7\cos(5x)$

c $y = \frac{2}{3}\cos(8x)$ **d** $y = \frac{5}{2}\sin\left(\frac{-x}{3}\right)$

3 **State** the amplitude, frequency and period of each graph. **Sketch** the graph for $-360° \le x \le 360°$.

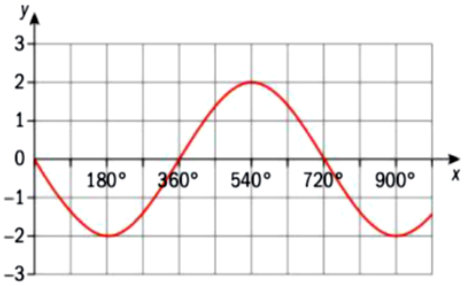
a $y = 4\cos(3x)$

b $y = -\sin(0.5x)$

c $y = 0.5\sin(2x) + 3$

d $y = 3\cos(0.5x) - 1$

4 This graph is a result of a transformation on $y = \sin x$ or $y = \cos x$.



Identify the transformation, and determine the function of the transformed graph.

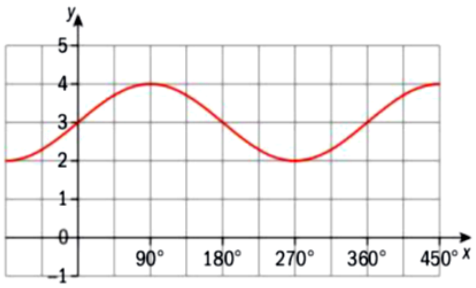
5 **Sketch** the graph of each of the following, indicating the amplitude and period, and showing at least two cycles:

a $y = \sin x + 12$ **b** $y = \sin(-x)$

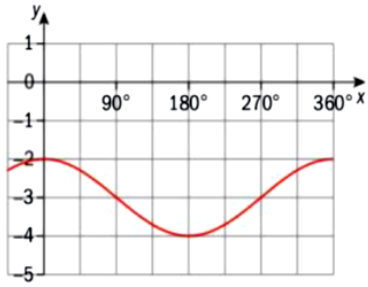
c $y = \cos x - 1$ **d** $y = \cos x + 6$

6 **Find** the amplitude, period and equation of the following graphs:

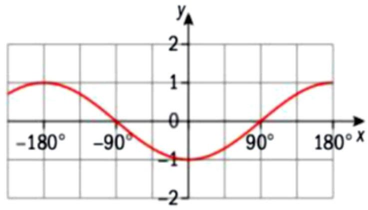
a



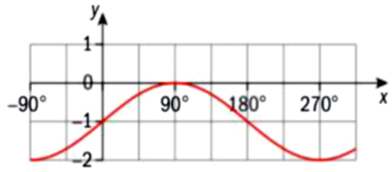
b



c



d



7 **Describe** in words how to transform the graph from $y = \sin x$ to:

a $y = 3\sin x + 2$ **b** $y = -2\sin(4x)$

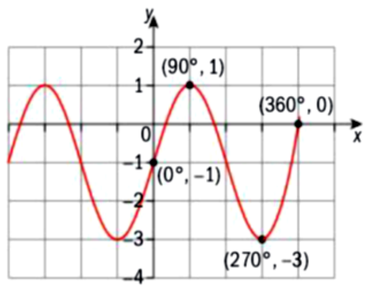
c $y = -\sin(2x) - 3$ **d** $y = \frac{1}{2}\sin\left(\frac{x}{3}\right) - 4$

e $y = -3\sin(3x) + 3$ **f** $y = \frac{3}{4}\sin\left(\frac{3x}{4}\right)$

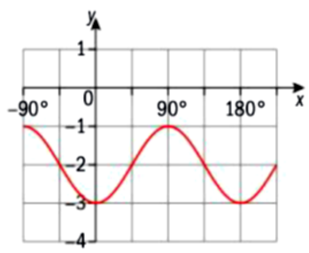
8 Each graph is a result of a combination of transformations on $y = \sin x$ or $y = \cos x$.

Identify the transformations, and **determine** the function of the transformed graph.

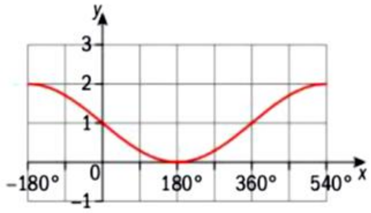
a



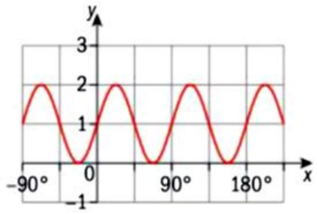
b



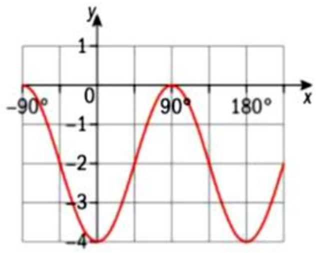
c



d



e



roblem solving

9 a A Ferris wheel has a radius of 10 meters. The bottom of the wheel is 2 meters above the ground. The wheel, rotating at a constant speed, takes 100 seconds to complete one revolution.

i Find a function that models this Ferris wheel.

ii Draw a graph of your function.

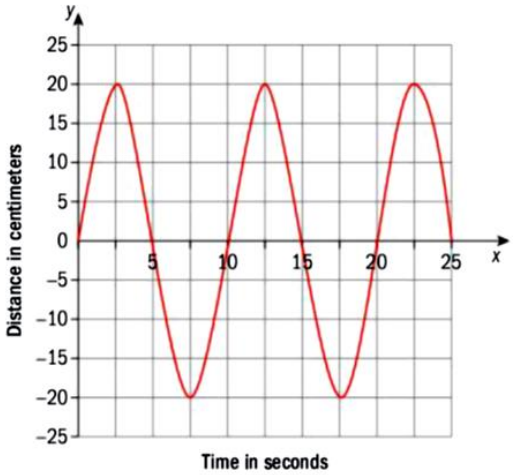
b Another Ferris wheel has a radius of 5 meters and is 1 meter above the ground. It takes 2 minutes to make one complete revolution.

i Find a function that models this Ferris wheel.

ii Draw a graph your function.

iii For which age group do you think this Ferris wheel has been designed?

10 The graph shows the motion of a tall building as it sways to and fro in the wind.

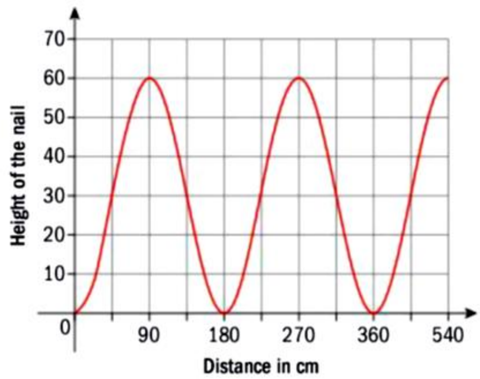


a Determine the period, and explain what it means in this problem.

b State the maximum number of centimeters that the tall building sways from its vertical position.

c Find a sinusoidal function to model this situation.

11 A nail is stuck in a car tire. The height of the nail above the ground varies as the wheel turns and can be modeled by this graph.



Find:

a the period of the graph, and explain what it means in this situation.

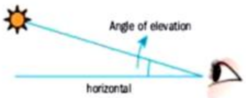
b the amplitude, and explain what it means in this situation.

c the radius of the wheel.

d a sinusoidal function to describe the relationship between the distance the tire travels and its height above the ground.

Problem solving

12 When the sun first rises, the angle of elevation increases rapidly at first, then more slowly, until the maximum angle is reached at about noon. The angle then decreases until sunset.



Assume that the relationship between the time of the day and the angle of elevation is sinusoidal.

- Let t be the number of hours since midnight.
- Let the amplitude be 60 degrees.
- The maximum angle of elevation occurs at noon.
- The period is 24 hours.
- The angle of elevation at midnight is −65 degrees.

a **Sketch** a graph of hours after midnight against angle of elevation.

b **Explain** the significance of the t-intercepts.

c **Explain** what the values below the t-axis mean.

d **Predict** the angle of elevation at 9:00 in the morning and 2:00 in the afternoon.

e **Predict** the time of sunrise.

Objective: D. Applying mathematics in real-life contexts **iii.** apply the selected mathematical strategies successfully to reach a solution
In this Review you will apply the strategies of interpreting amplitude, period and dilation to sketch the graphs and reach appropriate solutions.

Review in context

Scientific and technical innovation

1 A tsunami, or monster tidal wave, can have a period anywhere between ten minutes and 2 hours, with a wavelength well over 500 km. Because of its incredible destructive powers, warning systems have been developed in regions where tsunamis are most likely to happen. A particular tsunami to hit a coastal town in Japan had a period of about a quarter of an hour. The normal ocean depth in this town was 9 m, and a tsunami of amplitude 10 m hit the coast.

a Write a sinusoidal function to describe the relationship of the depth of water and time.

b Use your function to predict the depth of the water after **i** 1 minute, **ii** 5 minutes, and **iii** 10 minutes.

c Use your function to determine the minimum depth of the water.

d Interpret your answer to **c** in terms of the real world.

2 In 2005, the United States had one of the worst hurricane seasons on record. Computer models were continuously generated and updated in order to try to predict the path of oncoming storms. For almost five days, a portion of Hurricane Franklin's path could be approximated by a sinusoidal function, as seen in the following data:

Time (hours)	0	20	40	60	80	100	120
Longitude (degrees)	−78	−77	−74.5	−71.5	−69	−68	−69

a **Draw** a graph of the data and clearly indicate the amplitude and period.

b **Write down** the function representing this model.

c At what longitude was the hurricane at $t = 50$ hours?

d **Describe** how the graph of $y = \cos x$ was transformed to obtain this graph.

e Do you think the latitude of the hurricane followed a similar model? **Explain**.

3 The Bay of Fundy and Ungava Bay, both in Canada, have some of the highest tides in the world. Tides show periodic behavior, with their constant shift from high to low tide and back again. Suppose a low tide of 1.5 m occurs at 6 am and a high tide of 18.5 m occurs twelve hours later.

a **Draw** a graph to represent the height of the tide as a function of the time of day.

b **Write down** the equation for this model.

c **Find** the height of the tide at 3 am and at 3 pm.

d **Find** the time(s) at which the height of the tide is 10 m.

e **Explain** why harnessing electricity from this tidal energy is considered more reliable than wind energy.

4 A chemotherapy treatment is designed to kill cancer cells but it also kills red blood cells which are vital in the transport of oxygen in the body. The amount of red blood cells can be modelled by a sinusoidal function since it decreases after the treatment and then steadily increases until the next treatment. Suppose a patient's red blood cell count is 5 million cells per microliter on the day of treatment and hits a low of 2 million cells per microliter 10 days after treatment. If treatments occur every 20 days:

a **Draw** a graph to represent the patient's red blood cell count over the course of 30 days.

b From the graph, **state** the amplitude, period and frequency, and **write down** the function that models the situation.

c On day 15, does the patient have more or less than half of their red blood cell count back? **Justify** your answer.

d If the 20th day were a holiday, would you rather go in for the treatment before the holiday or after? **Explain**.

Problem solving

5 The electricity delivered to your home is called 'alternating current' because it alternates back and forth between positive and negative voltage. It does so with regularity and so it can be described by a sinusoidal function.

a The voltage in many European countries is 220 V with a frequency of 50 Hz (which means that 50 cycles occur in one second). **Draw** a graph of the amount of voltage over time, assuming at $t = 0$ you have 220 V.

> Draw the graph between $t = 0$ seconds and $t = 0.02$ seconds.

b **Write down** the period of your graph.

c **Write down** the equation of your graph.

d In North America, the typical voltage is 120 V at 60 Hz. Draw a graph of voltage as a function of time, assuming at $t = 0$ you have 120 V.

e **Write down** the period, amplitude and equation for your graph.

f **Describe** the transformations that occurred from the European model to the North American model.

Reflect and discuss 3

How have you explored the statement of inquiry? Give specific examples.

UNIT : PROBABILITY 5

Summary

- Axiom 1: For any event A, $P(A) \geq 0$
- Axiom 2: $P(S) = 1$
- Two events A and B are mutually exclusive if it is impossible for them to happen together $A \cap B = \varnothing$ and so $P(A \cap B) = 0$
- Axiom 3: If $\{A_1, A_2, A_3, \ldots\}$ is a set of mutually exclusive events then:
  $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$
- Proposition 1: Complementary events: if A is an event then $P(A') = 1 - P(A)$
- Corollary 1: $P(\varnothing) = 0$
- Corollary 2: $P(A) \leq 1$
- If the outcome of an event in one experiment does not affect the probability of the outcomes of the event in the second experiment, then the events are **independent**.
- Theorem 1: For any events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Theorem 2: For independent events A and B, $P(A \cap B) = P(A) \times P(B)$

Mixed practice

1 A group of 30 students were asked about their favourite way of eating eggs.

- 18 liked boiled eggs (B)
- 10 liked fried eggs (F)
- 6 liked neither

a **Find**

i $P(B)$ **ii** $P(B \cup F)$

iii $P(F)$ **iv** $P(B' \cup F')$

b Use Theorem 1 to **find** $P(B \cap F)$.

c Represent the information on a Venn diagram.

d A student chosen at random likes boiled eggs. **Find** the probability that the student also likes fried eggs.

Problem solving

2 A survey was carried out at an international airport. Travelers were asked their flight destinations and results are shown in the table.

Destination	Geneva	Vienna	Brussels
Number of males	45	62	37
Number of females	35	46	25

a **i** **Determine** whether or not the destinations are mutually exclusive.

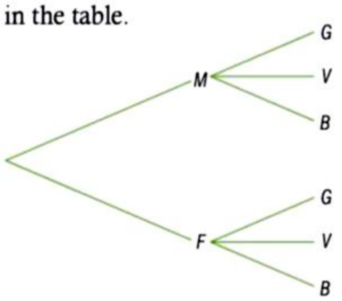
ii **Determine** whether or not gender is mutually exclusive.

b One traveller is chosen at random. **Find** the probability that this traveller is going to Vienna.

c One female traveller is chosen at random. **Find** the probability that she is going to Geneva.

d One traveller is chosen at random from those **not** going to Vienna. **Find** the probability that the chosen traveller is female.

e Copy and complete the tree diagram for the data in the table.



For this data, test whether or not destination and gender are independent.

3 Let $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$

a **Find** $P(A \cap B)$.

b **Draw** a tree diagram. **Find** the probability of getting B, given that A has occurred.

c **Determine** whether or not A and B are independent events. Give a reason for your answer.

4 100 students were asked if they liked various toast toppings.

- 56 like avocado
- 38 like marmalade
- 22 like soft cheese
- 16 like avocado and marmalade, but not soft cheese
- 8 like soft cheese and marmalade, but not avocado
- 3 like avocado and soft cheese, but not marmalade
- 4 like all three toppings

a **Draw** a Venn diagram to represent this information.

b **Find** the number of students who didn't like any topping.

c **Determine** if the toppings are mutually exclusive.

d A student is chosen at random from the group who like soft cheese. **Find** the probability that they also like marmalade.

5 Events A and B have probabilities $P(A) = 0.4$, $P(B) = 0.65$, and $P(A \cup B) = 0.85$.

a **Calculate** $P(A \cap B)$.

b **Determine** whether or not events A and B are independent.

c **Determine** whether or not events A and B are mutually exclusive.

6 Stefan rolls two 6-sided dice at the same time. One die has three green sides and three black sides. The other die has sides numbered from 1 to 6.

a **Draw** a tree diagram to represent this sequence of events.

b **Find** the probability that Stefan rolls a 5.

c **Find** the probability that he rolls a number less than 3.

d **Find** the probability that he gets green on the one die and an even number on the other.

e **Determine** if the events 'rolling green' and 'rolling an even number' are independent, and explain your answer.

7 The table below shows the number of left- and right-handed golf players in a sample of 50 males and females.

	Left-handed	Right-handed	Total
Male	3	29	32
Female	2	16	18
Total	5	45	50

A golf player is selected at random.

a **Find** the probability that the player is:

i male and left-handed

ii right-handed

iii right-handed, given that the player selected is female.

b **Determine** whether the events 'male' and 'left-handed' are:
i independent
ii mutually exclusive.

Review in context

Identities and relationships

Remember to use the probability axioms when answering these questions.

Problem solving

1 The table shows 60 students' choices of yoghurt.

Sugar levels	Yoghurt			
–	Strawberry	Chocolate	Vanilla	Total
Low sugar	3	8	14	25
High sugar	8	9	18	35
Total	11	17	32	60

a One student is selected at random.

i **Find** the probability that the student chose vanilla yoghurt.

ii **Find** the probability that the student chose a yoghurt that was not vanilla.

iii The student chose chocolate yoghurt. **Find** the probability that the student chose the low sugar one.

iv **Find** the probability that the student chose a high sugar or vanilla yoghurt.

v The student chose a low sugar yoghurt. **Find** the probability that the student chose strawberry.

b The 60 yoghurts were then classified according to fat content type: 15 of the yoghurts had high fat, 37 had medium fat and 8 had low fat content. Two yoghurts were randomly selected.
Find the probability that:

i both yoghurts had low fat content

ii neither of the yoghurts had medium fat content.

2 All human blood can be categorized into one of four types: A, B, AB or O. The distribution of the blood groups varies between races and genders. The table shows the distribution of the blood types for the US population.

Blood type	O	A	B	AB
Probability	0.42	0.43	0.11	x

a **Find** the probability that a person chosen at random:

i has blood type AB

ii does **not** have blood type AB.

b **Determine** if the two events 'have blood type AB' and 'do not have blood type AB' are mutually exclusive.

c **Find** the probability that a randomly selected person in the US has blood type A or B.

d Damon has blood type B, so he can safely receive blood from people with blood types O and B. **Find** the probability that a randomly chosen person can donate blood to Damon.

e Given that Damon received blood, **find** the probability that it was type O.

3 This table shows the distribution of blood type in the US by gender.

	Probability		
Blood type	Male	Female	Total
O	0.21	0.21	0.42
A	0.215	0.215	0.43
B	0.055	0.055	0.11
AB	0.02	0.02	0.04
Total	0.50	0.50	1.0

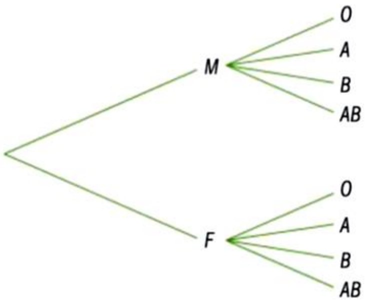
a **Determine** if the events 'gender' and 'blood type' are mutually exclusive.

Let M be the event 'selecting a male'.

Let F be the event 'selecting a female'.

b **Find i** $P(M)$ **ii** $P(F)$

c Copy and complete the tree diagram.



d Confirm that blood type and gender are independent events, using the multiplication rule for independent events.

Reflect and discuss

How have you explored the statement of inquiry? Give specific examples.

Statement of Inquiry:

Understanding health and making healthier choices result from using logical representations and systems.