LINEAR INEQUALITIES

1 Solve the following inequalities and show the solutions on separate number lines:

a $a+4>6$ b $3b \leqslant -9$ c $\frac{c}{3}<2$

d $s-4<2$ e $-4b>16$ f $5+t>0$

g $-\frac{m}{5} \geqslant 12$ h $3x+12 \geqslant -3$ i $5-3b \leqslant 7$

j $3a-2 \geqslant 8$ k $4a+9<1$ l $7-2b<-3$

m $16+7s>2$ n $2a-4 \leqslant 0$ o $12-5b>-3$

p $3b-1 \geqslant 0$ q $5n+7<-3$ r $11-4b \leqslant 4$

2 Solve the following inequalities and show the solutions on separate number lines:

a $\frac{x}{3}+1>4$ b $\frac{b}{5}-2 \leqslant -3$ c $\frac{c}{4}+4 \geqslant 8$

d $\frac{2x}{3}-2<4$ e $\frac{3x}{2}+5 \geqslant -2$ f $1-\frac{3x}{2}<4$

g $2-\frac{x}{4} \geqslant -3$ h $3-\frac{2x}{5}<2$ i $5-\frac{3x}{4}>-2$

3 Solve the following inequalities and show the solutions on separate number lines:

a $\frac{a-3}{2}<6$ b $\frac{b+5}{4} \geqslant 1$ c $\frac{4c+3}{5} \leqslant -1$

d $\frac{4-a}{2}>5$ e $\frac{5-3x}{2} \leqslant -6$ f $\frac{3-2x}{3} \geqslant -1$

Extension:

4 Solve the following inequalities by first clearing the brackets:

a $3(c+1)>8$ b $5(1+3a) \leqslant -4$

c $3(1-2a) \geqslant -5$ d $2(2a+1)-3 \leqslant -5$

e $2(3-4a)+3<7$ f $4(2a+5)-12>9$

5 Solve the following inequalities by first interchanging RHS and LHS:

a $5>4+x$ b $7 \leqslant \frac{a}{3}$ c $-4>2b$

d $2 \leqslant 6c+14$ e $3<\frac{d-2}{4}$ f $6 \geqslant 3(p+2)-11$

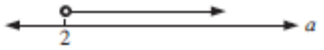
6 Solve for x and show the solution on a number line:

a $5x-3>3x+1$ b $2x+1 \geqslant 4x+7$ c $8x+6<3x+1$

d $2x+7>7x+3$ e $6x+2 \leqslant 3x-7$ f $x-11 \leqslant 6x-1$

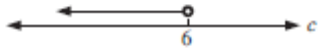
ANSWERS

1 **a** $a > 2$

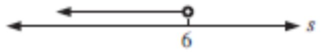


b $b \leqslant -3$

c $c < 6$

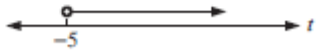


d $s < 6$



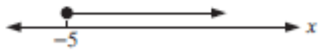
e $b < -4$

f $t > -5$

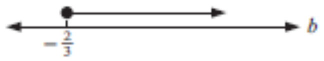


g $m \leqslant -60$

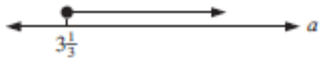
h $x \geqslant -5$



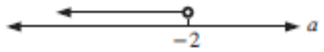
i $b \geqslant -\frac{2}{3}$



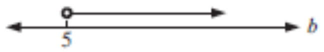
j $a \geqslant 3\frac{1}{3}$



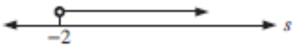
k $a < -2$



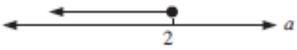
l $b > 5$



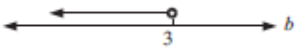
m $s > -2$



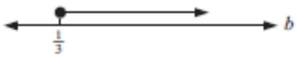
n $a \leqslant 2$



o $b < 3$

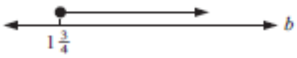


p $b \geqslant \frac{1}{3}$

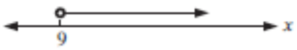


q $n < -2$

r $b \geqslant 1\frac{3}{4}$

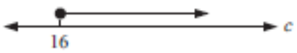


2 **a** $x > 9$

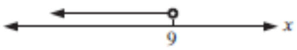


b $b \leqslant -5$

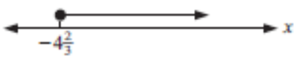
c $c \geqslant 16$



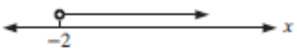
d $x < 9$



e $x \geqslant -4\frac{2}{3}$

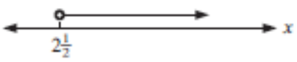


f $x > -2$

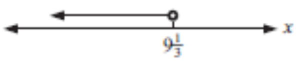


g $x \leqslant 20$

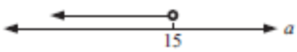
h $x > 2\frac{1}{2}$



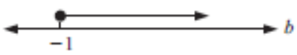
i $x < 9\frac{1}{3}$



3 **a** $a < 15$



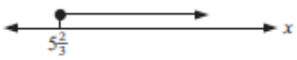
b $b \geqslant -1$



c $c \leqslant -2$

d $a < -6$

e $x \geqslant 5\frac{2}{3}$



f $x \leqslant 3$

4 **a** $c > 1\frac{2}{3}$ **b** $a \leqslant -\frac{3}{5}$ **c** $a \leqslant 1\frac{1}{3}$
d $a \leqslant -1$ **e** $a > \frac{1}{4}$ **f** $a > \frac{1}{8}$

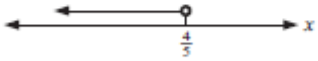
5 **a** $x < 1$ **b** $a \geqslant 21$ **c** $b < -2$
d $c \geqslant -2$ **e** $d > 14$ **f** $p \leqslant 3\frac{2}{3}$

6 **a** $x > 2$

b $x \leqslant -3$

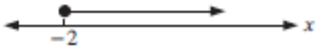
c $x < -1$

d $x < \frac{4}{5}$

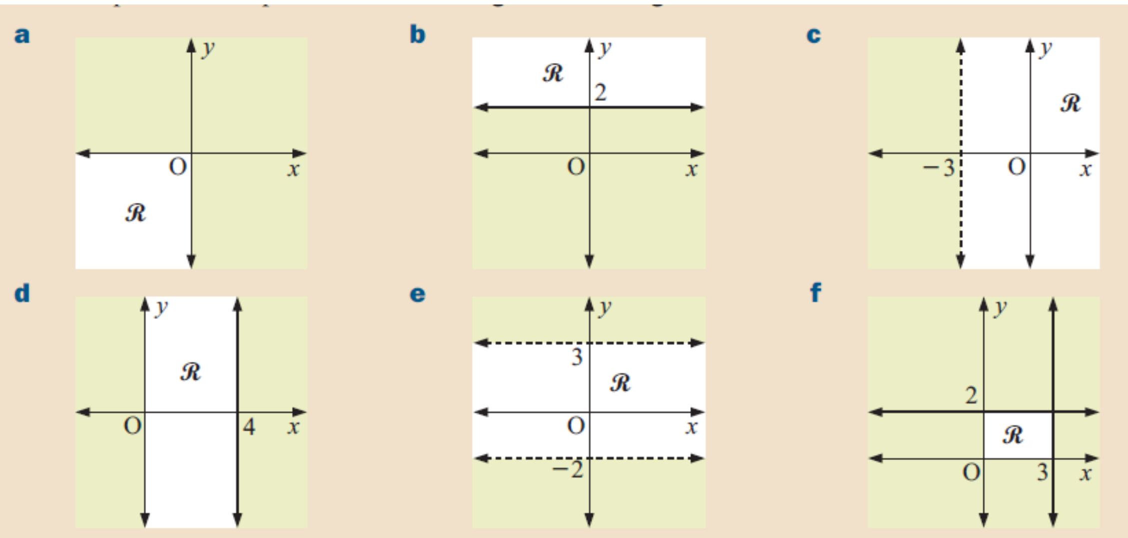


e $x \leqslant -3$

f $x \geqslant -2$

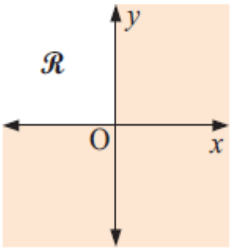


Write the inequalities to represent the following unshaded regions:

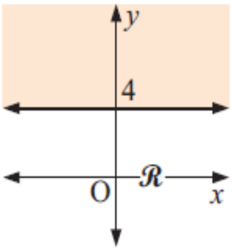


1 Write inequalities to represent the following unshaded regions, $\mathcal{R}$:

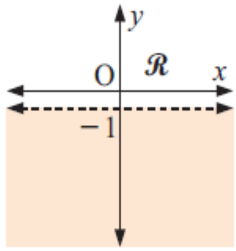
a



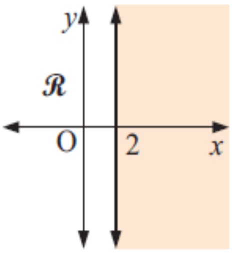
b



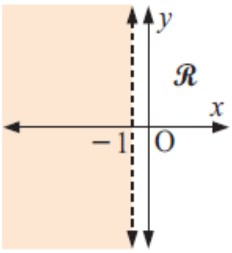
c



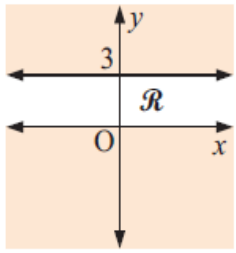
d



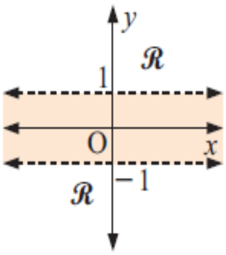
e



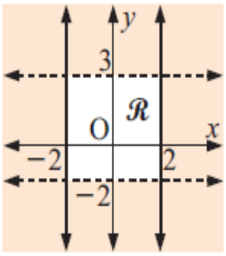
f



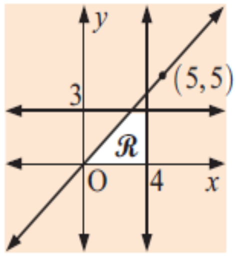
g



h



i



2 Graph the regions defined by:

a $x < 4$ **b** $x \geqslant -1$ **c** $y \geqslant 2$

d $y < -1$ **e** $0 < x < 3$ **f** $-1 \leqslant y \leqslant 2$

g $-2 < x \leqslant 3$ **h** $-1 \leqslant y < 4$ **i** $0 \leqslant x \leqslant 3$ and $1 \leqslant y \leqslant 4$

3 Graph the regions defined by:

a $x + 2y \leqslant 4$ **b** $2x + y > 5$ **c** $3x + 2y < 6$

d $2x + 3y \geqslant 6$ **e** $2x - 3y < 12$ **f** $5x + 2y > 10$

g $2x - 5y < -10$ **h** $4x + 3y < 6$ **i** $4x - 3y \geqslant 12$

j $y \geqslant x$ **k** $y < -x$ **l** $2x + 5y \geqslant 0$

4 Graph the regions defined by:

a $x \geqslant 0$ and $y \geqslant 2$

b $x \leqslant -2$ and $y \geqslant 4$

c $x \geqslant 0$, $y \geqslant 0$ and $x + y \leqslant 4$

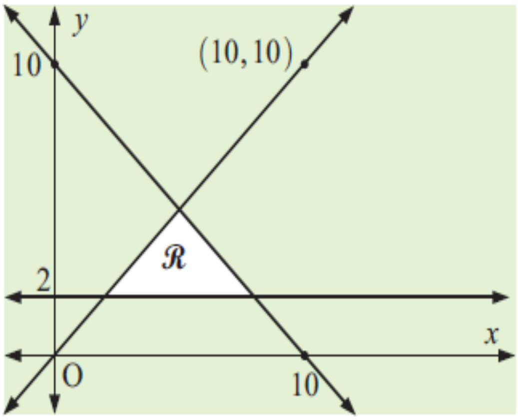
d $x \geqslant 0$, $y \geqslant 0$ and $2x + y < 6$

e $x \geqslant 2$, $y \geqslant 0$ and $x + y \geqslant 6$

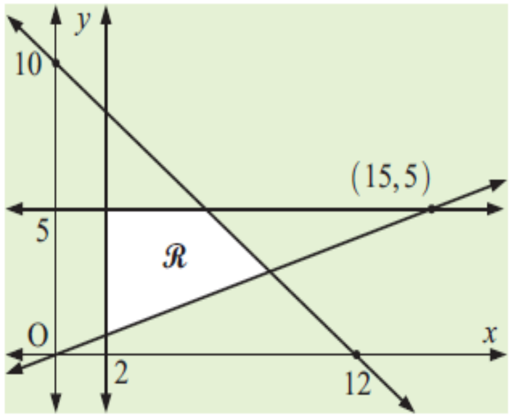
f $x \geqslant 0$, $y \geqslant 3$ and $2x + 3y \geqslant 12$

5 Write down inequalities which represent the unshaded region $\mathcal{R}$ of:

a



b



LINEAR PROGRAMMING

There are x girls and y boys in a school choir.

(a) **(i)** The number of girls is more than 1.5 times the number of boys in the choir.

Show that $y < \frac{2x}{3}$.

(ii) There are more than 12 girls in the choir.

There are more than 5 boys in the choir.

The maximum number of children in the choir is 35.

Write down three more inequalities.

(b) **(i)** Using a scale of 2 cm to represent 5 children on each axis, draw an x-axis for $0 \leqslant x \leqslant 40$ and a y-axis for $0 \leqslant y \leqslant 40$.

(ii) Draw 4 lines on your graph to represent the inequalities in **part (a)**.

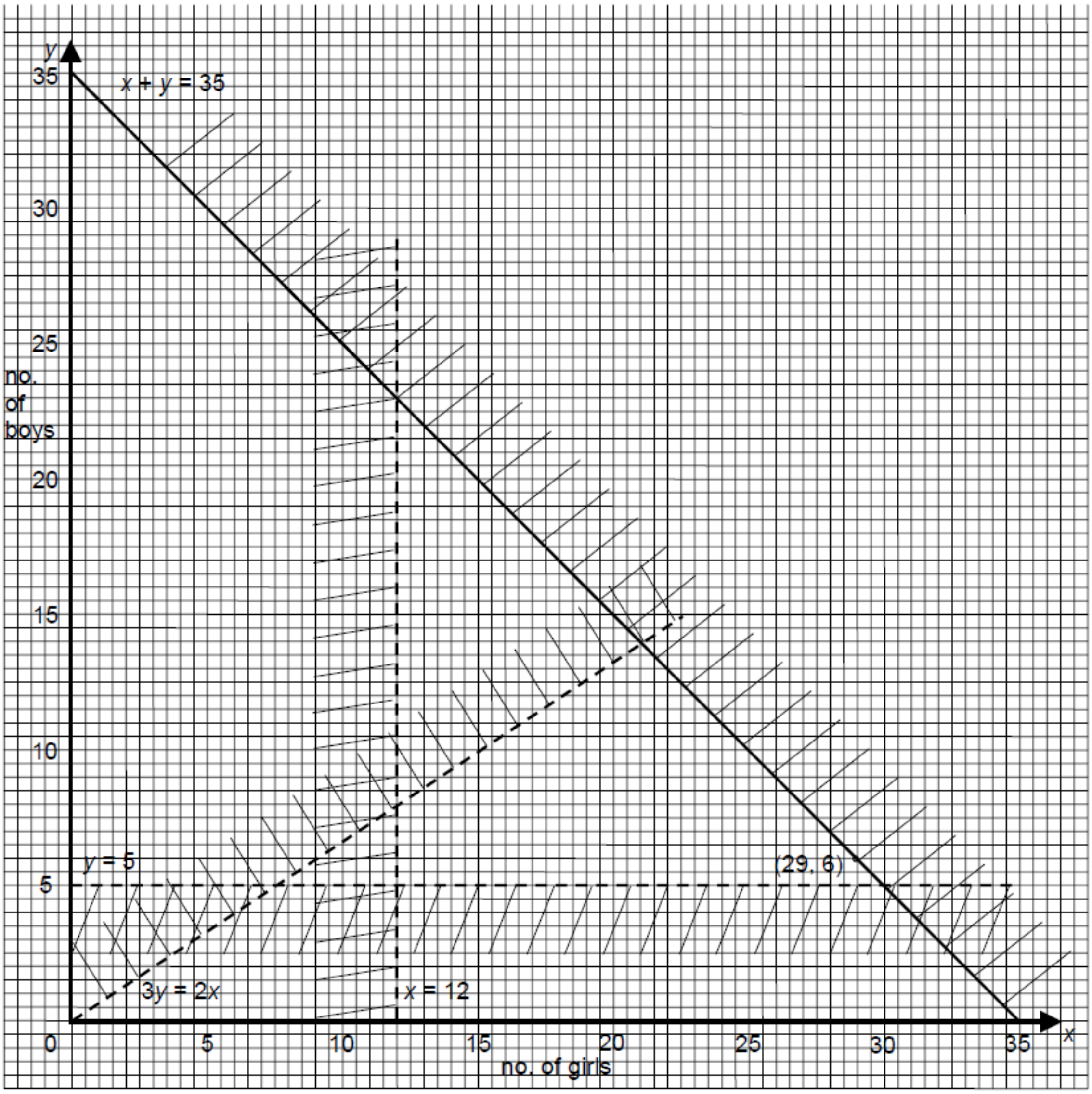
Shade the **unwanted** parts of the grid.

(c) The school buys a uniform for each choir member.

A girl's uniform costs \$25. A boy's uniform costs \$20.

Find the maximum possible cost for the choir uniforms. Mark clearly the point P on your graph which you use to calculate this cost.

ANSWERS



(a)	**(i)**	$x > \frac{3y}{2} \quad \left(\therefore y < \frac{2x}{3}\right)$
	(ii)	$x > 12$ or $x \geq 13$ $y > 5$ or $y \geq 6$ $x + y \leq 35$ or < 36

(b)		Scales correct Line $3y=2x$ from (0,0) to (21, 14) ** Line $y=5$ from (0, 5) to (30, 5) **Line $x=12$ from (12, 0) to (12, 8) line $x + y = 35$ from (20, 15) to (30, 5) 3 correct dotted/solid for their inequalities Shading correct for all 4
(c)		(29, 6) marked on graph $845

A taxi company has "SUPER" taxis and "MINI" taxis.
One morning a group of 45 people needs taxis.
For this group the taxi company uses x "SUPER" taxis and y "MINI" taxis.
A "SUPER" taxi can carry 5 passengers and a "MINI" taxi can carry 3 passengers.
So $5x + 3y \geqslant 45$.

(a) The taxi company has 12 taxis.
Write down **another** inequality in x and y to show this information. [1]

(b) The taxi company always uses at least 4 "MINI" taxis.
Write down an inequality in y to show this information. [1]

(c) Draw x and y axes from 0 to 15 using 1 cm to represent 1 unit on each axis. [1]

(d) Draw three lines on your graph to show the inequality $5x + 3y \geqslant 45$ **and** the inequalities from **parts (a) and (b)**.
Shade the **unwanted** regions. [6]

(e) The cost to the taxi company of using a "SUPER" taxi is \$20 and the cost of using a "MINI" taxi is \$10.
The taxi company wants to find the cheapest way of providing "SUPER" and "MINI" taxis for this group of people.
Find the **two** ways in which this can be done. [3]

(f) The taxi company decides to use 11 taxis for this group.

(i) The taxi company charges \$30 for the use of each "SUPER" taxi and \$16 for the use of each "MINI" taxi.
Find the two possible **total** charges. [3]

(ii) Find the largest possible **profit** the company can make, using 11 taxis. [1]

(a)	$x + y \le 12$	o.e
(b)	$y \ge 4$	o.e
(c)	Scales correct – full length	
(d)	$x + y = 12$ ruled and long enough $y = 4$ ruled and long enough $5x + 3y = 45$ ruled and long enough (1 mm at (9, 0) and (0, 15) if extended) <u>Unwanted</u> regions shaded	
(e)	6 super, 5 mini <u>and</u> 5 super, 7 mini (no extras) Can write as (6, 5) and (5, 7)	
(f)(i)	(7, 4) or (6, 5) ($) 274 ($) 260	s.o.i.
(f)(ii)	($) 94	c.a.o.

Tiago does some work during the school holidays.
In one week he spends x hours cleaning cars and y hours repairing cycles.
The time he spends repairing cycles is at least equal to the time he spends cleaning cars.
This can be written as $y \geqslant x$.

He spends no more than 12 hours working.
He spends at least 4 hours cleaning cars.

(a) Write down two more inequalities in x and/or y to show this information.

(b) Draw x and y axes from 0 to 12, using a scale of 1 cm to represent 1 unit on each axis.

(c) Draw three lines to show the three inequalities. Shade the **unwanted** regions.

(d) Tiago receives \$3 each hour for cleaning cars and \$1.50 each hour for repairing cycles.

(i) What is the least amount he could receive?

(ii) What is the largest amount he could receive?

9(a)	$x + y(\ldots.)12$	
	$x(\ldots.)4$	
	both inequality signs correct	$\leq$ $\geq$
(b)	Correct scales	
(c)	$x + y = 12$ ruled, sufficiently long	
	$x = 4$ ruled, sufficiently long	
	$y = x$ ruled, sufficiently long	
	Correct shading out of three regions	cao
(d)(i)	from (4, 4)	
	18	cao
(ii)	from (6, 6)	
	27	cao

ANSWERS

Two tins of race-horse food A and B are analysed and it is found that:
A contains 25 units of carbohydrate, 10 units of protein and 15 units of fat
B contains 50 units of carbohydrate, 10 units of protein and 9 units of fat.
Each day, a race-horse is to receive at least 225 units of carbohydrate, 80 units of protein and 90 units of fat. Tin A costs \$6 and B costs \$3.

a Find the combination of A and B which provides the cheapest food.

b For the optimal combination found, discuss the race-horse's diet.

ANSWERS

a *Step 1:* Let x be the number of tins of A used each day and
y be the number of tins of B used each day.

Step 2:

Type	*Carbohydrate*	*Protein*	*Fat*	*Number of tins*	*Costing*
1 tin of A	25 units	10 units	15 units	x	\$6
1 tin of B	50 units	10 units	9 units	y	\$3

Step 3: We are to minimise $(6x + 3y)$ dollars.

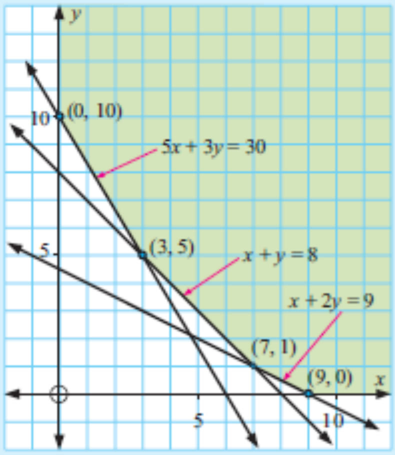
Step 4: $x \geqslant 0$ and $y \geqslant 0$

$25x + 50y \geqslant 225$, i.e., $x + 2y \geqslant 9$ {dividing each term by 25}

$10x + 10y \geqslant 80$, i.e., $x + y \geqslant 8$ {dividing each term by 10}

$15x + 9y \geqslant 90$, i.e., $5x + 3y \geqslant 30$ {dividing each term by 3}

Step 5: The simplex is:



Step 6:

Vertices	$6x + 3y$ dollars
(0, 10)	\$30 ← smallest or min. value
(3, 5)	\$33
(7, 1)	\$45
(9, 0)	\$54

$\therefore$ the cost is minimised when we use 0 tins of A and 10 tins of B.

b If the race-horse is fed 10 tins of B, then it receives the minimum 90 units of fat, but receives 500 units of carbohydrate (more than double the minimum) and 100 units of protein (20 units more than the minimum).

A chemical factory makes two different chemicals A and B and can sell all that it is able to produce. It has orders for at least 200 kg of A and 100 kg of B, but can produce at most 700 kg of the chemicals due to limited resources. If the profit on A is \$300 per kg and \$400 per kg on B, how many kg of each should be produced in order to maximise profit?

2 The vitamin content of two foods X and Y is shown (in units per kg) in the table alongside. A blend of X and Y is to be made which must contain at least 8 units of A, 30 units of B and 30 units of vitamin C.

Food	A	B	C
X	1	6	10
Y	4	6	5

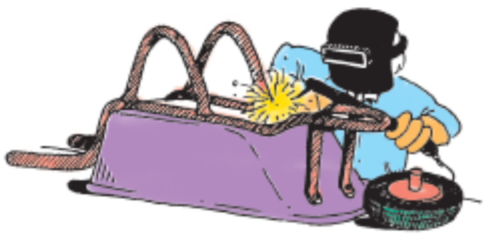
If X costs \$2.00 per kg and Y costs \$1.50 per kg, find the minimum cost of the food mixture, and detail the vitamin content it contains.

3 A manufacturer produces two kinds of table-tennis sets.

Set A contains 2 bats and 3 balls,
Set B contains 2 bats, 5 balls and 1 net.

In one hour the factory can produce at most 56 bats, 108 balls and 18 nets. If Set A earns a profit of \$3 and Set B earns a profit of \$5, determine the number of each of the sets that the manufacturer should produce per hour to maximise his profit. Which component is under-utilised?

4 A manufacturer of wheelbarrows makes two models, Deluxe and Standard. For the Deluxe model he requires the use of machine A for 2 minutes and machine B for 2 minutes.



For the Standard model he requires the use of machine A for 3 minutes and machine B for 1 minute. Machine A is available for at most 48 minutes and machine B for 20 minutes every hour.

He knows from past experience that he will sell at least twice as many of the Standard model as the Deluxe model.

If the Deluxe model earns him \$25 profit and the Standard model earns \$20 profit, how many of each should he produce per hour in order to maximise his profits? Which machine is fully used?

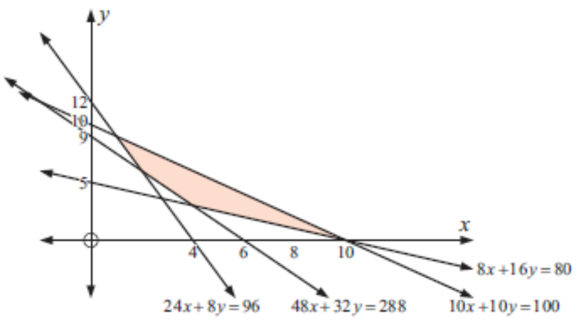
ANSWERS

1 200 kg of A, 500 kg of B **2** \$8 (1 kg of X, 4 kg of Y)

3 *Either* 6A and 18B (making of bats is under-utilised),
or 11A and 15B (making of bats and nets is under-utilised),
or 16A and 12B (making of nets is under-utilised)

4 3 deluxe, 14 standard. Both machines are fully used.

5 $x \geqslant 0$, $y \geqslant 0$, $24x + 8y \geqslant 96$, $8x + 16y \geqslant 80$, $48x + 32y \geqslant 288$, $10x + 10y \leqslant 100$



a 2 scoops of Foodo, 6 of Petmix
b **i** Yes, 10 scoops/day **ii** No

NON LINEAR INEQUALITIES

Quadratic inequalities are inequalities with the highest power of the unknown being two.

Examples.
$3x^2 - 2x + 1 > 0$ and $4x^2 + 5x < 7$ are quadratic inequalities.

The **critical values** of an inequality are the solutions of the quadratic equation formed by replacing the inequality symbol by "=".

Example.
Find the critical values of the inequality

$$x^2 - 3x \le 10.$$

$$x^2 - 3x = 10 \qquad [-10]$$
$$\Rightarrow x^2 - 3x - 10 = 0$$
$$\Rightarrow (x+2)(x-5) = 0$$

Either $x = -2$ or $x = 5$.

-2 and 5 are the critical values.

Exercise.
Find the critical values of the following inequalities.

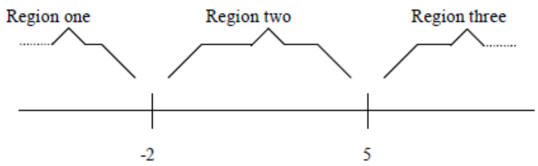
1. $x^2 - 4 > 0.$
2. $x^2 - 6x < 16.$
3. $x^2 - 6x \le 0.$
4. $2x^2 + 5x \le -2.$
5. $\dfrac{2}{x} + x \le 3.$

(Answers: -2, 2; 8, -2; 0, 6; -1/2, -2; 2, 1.)

The critical values split the number line into three possible solution sets.

Example.
Find the possible solution sets of $x^2 - 3x \le 10.$

The critical values of the inequality are -2 and 5.



The possible solution sets are:

$$x \le -2, \ -2 \le x \le 5, \ 5 \le x.$$

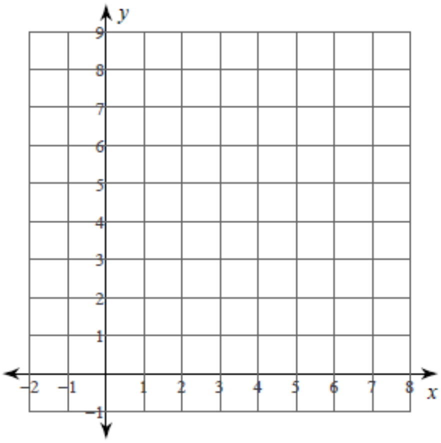
Notice that we include the "or equal to" part of the inequality sign in the possible solution sets as the "or equal to" part was allowed in the original inequality. Otherwise we wouldn't have included it.

To solve a quadratic inequality we should:

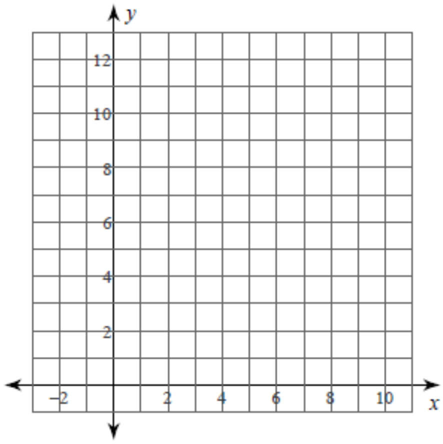
- Rearrange to obtain 0 as one side of the inequality, as with an equation.
- Find the critical values.
- Describe the possible solution sets.
- Identify the correct solution set by testing a point in each possible set to see whether it satisfies the inequality.

Sketch the graph of each function.

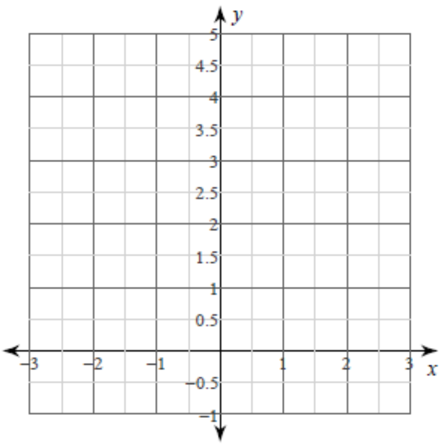
1) $y \geq 2x^2$



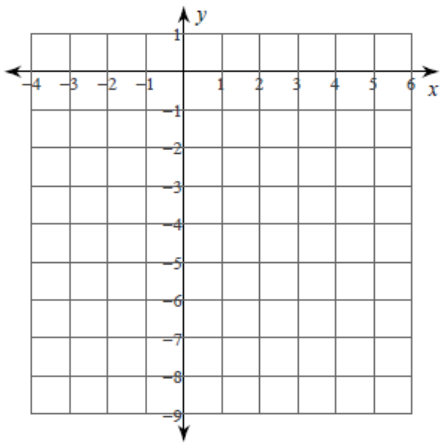
2) $y > 3x^2$



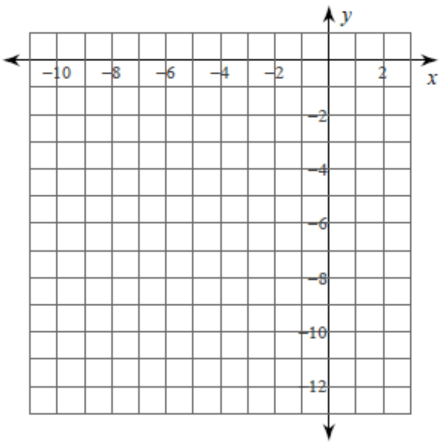
3) $y > x^2$



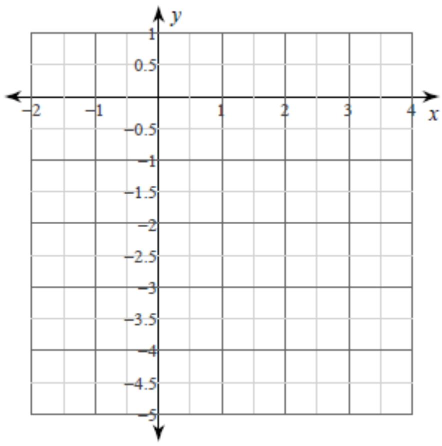
4) $y < -2x^2$



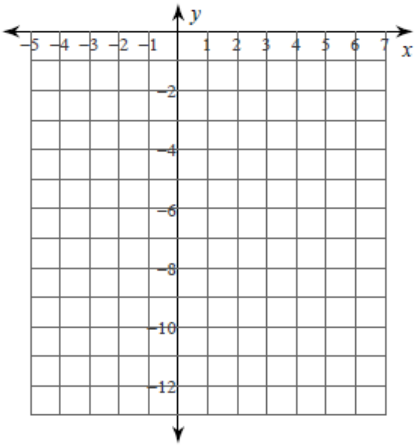
5) $y \geq -3x^2$



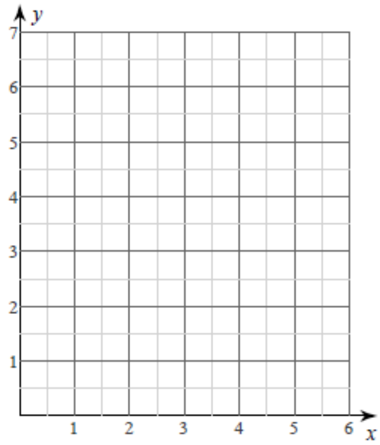
6) $y \leq -x^2$



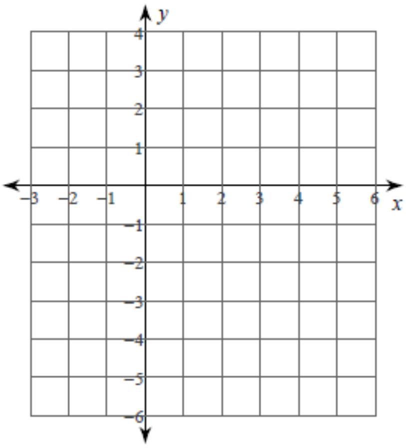
7) $y < -2x^2 - 8x - 12$



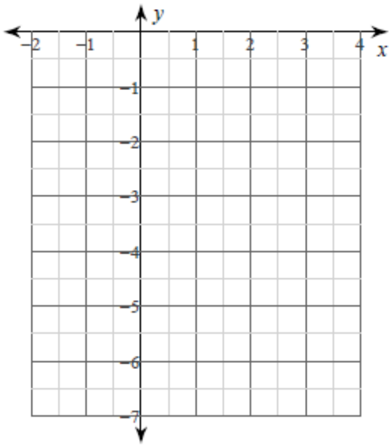
8) $y \leq x^2 - 6x + 11$



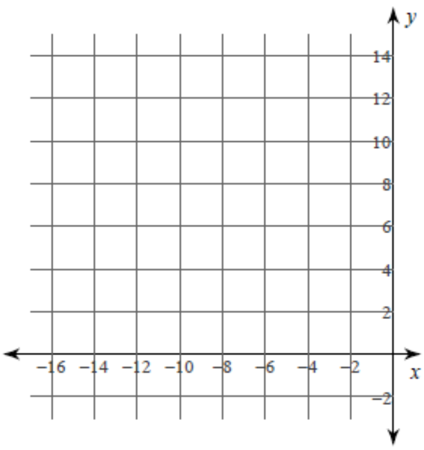
9) $y \geq -2x^2 + 16x - 29$



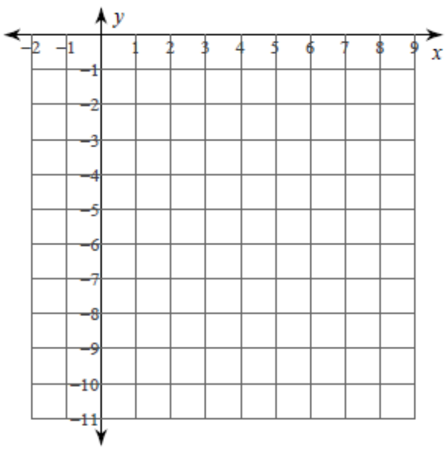
10) $y > -x^2 + 4x - 6$



11) $y \leq 4x^2 + 32x + 62$



12) $y > -2x^2 + 16x - 34$



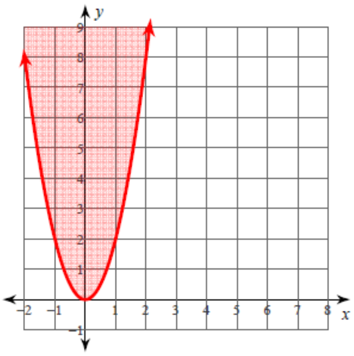
Critical thinking questions:

13) Name one solution to:
$y > x^2 + 6x + 5$

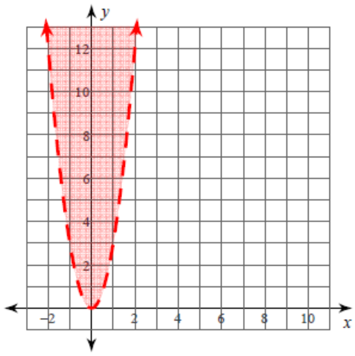
14) Name one solution to the system:
$y \geq x^2 - 2x + 2$
$y = x + 1$

Sketch the graph of each function.

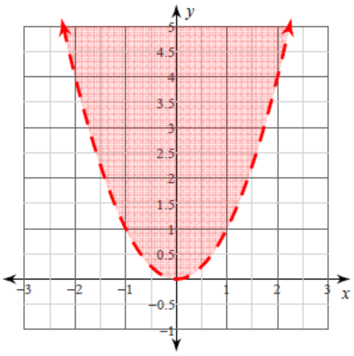
1) $y \geq 2x^2$



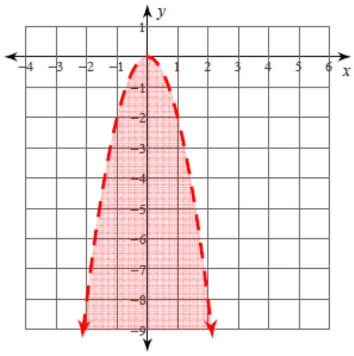
2) $y > 3x^2$



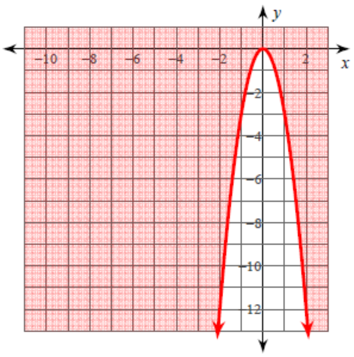
3) $y > x^2$



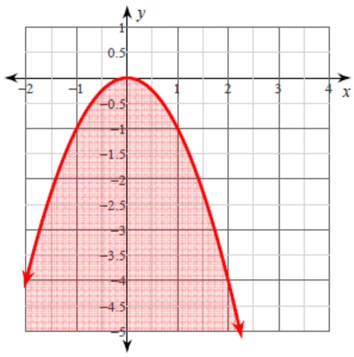
4) $y < -2x^2$



5) $y \geq -3x^2$

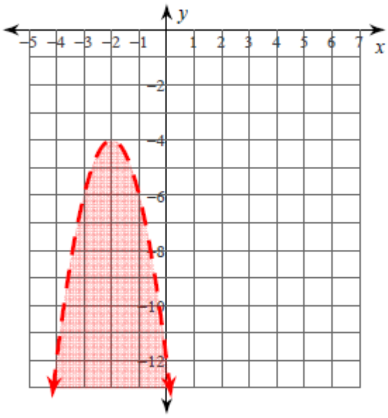


6) $y \leq -x^2$

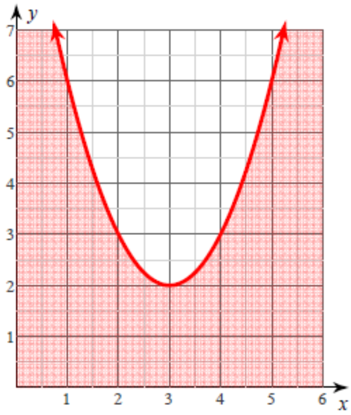


<u>ANSWERS</u>

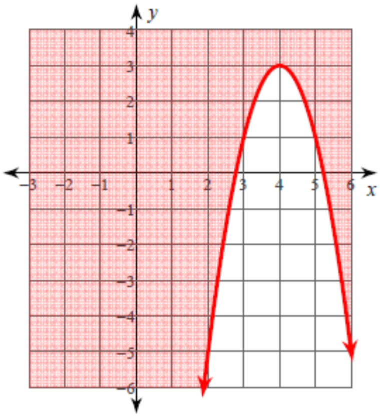
7) $y < -2x^2 - 8x - 12$



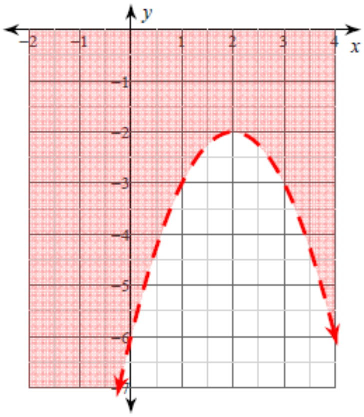
8) $y \le x^2 - 6x + 11$



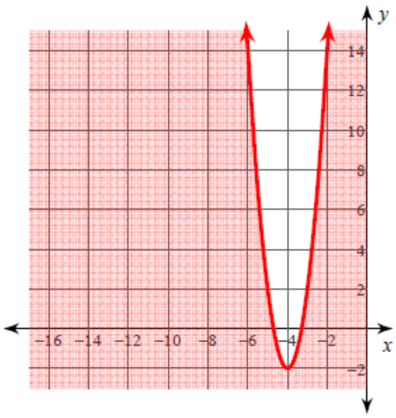
9) $y \ge -2x^2 + 16x - 29$



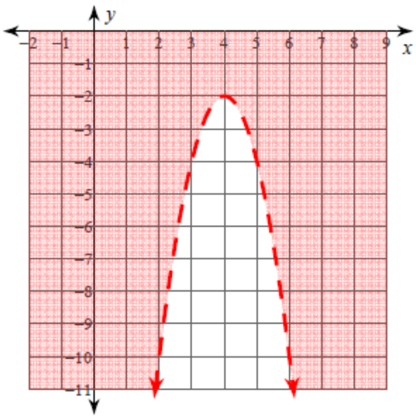
10) $y > -x^2 + 4x - 6$



11) $y \le 4x^2 + 32x + 62$



12) $y > -2x^2 + 16x - 34$



Critical thinking questions:

13) Name one solution to:
$y > x^2 + 6x + 5$

Many answers. Ex: $(-3, 0)$

14) Name one solution to the system:
$y \ge x^2 - 2x + 2$
$y = x + 1$

Many answers. Ex: $(1, 2)$, or $(2, 3)$

FURTHER FUNCTIONS – TRANSFORMATIONS IN FUNCTIONS

TRIGONOMETRIC FUNCTIONS

> The functions $y = a\sin b(x+c)+d$ and $y = a\cos b(x+c)+d$ have
>
> - amplitude a
> - period $\frac{2\pi}{b}$
> - minimum value $d-a$ and maximum value $d+a$

Note that the value of d is always half-way between the minimum and maximum values; in other words, $d = \frac{min+max}{2}$.
The amplitude is half the difference between the minimum and maximum values: $a = \frac{max-min}{2}$.
The value of c in the above equation determines the horizontal translation of the graph; therefore it affects the position of the maximum and minimum points. The following example shows how to work out these positions.

TRANSLATIONS

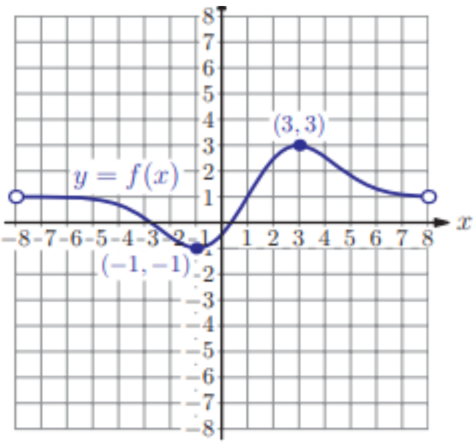
1. Given the graph of $y = f(x)$, sketch the graph of the following functions, indicating the positions of the minimum and maximum points.

   (a) (i) $y = f(x)+3$ (ii) $y = f(x)+5$

   (b) (i) $y = f(x)-7$ (ii) $y = f(x)-0.5$

   (c) (i) $y = f(x+2)$ (ii) $y = f(x+4)$

   (d) (i) $y = f(x-1.5)$ (ii) $y = f(x-2)$



2. Find the equation of the graph after the given transformation is applied.

   (a) (i) $y = 3x^2$ after a translation of 3 units vertically up

   (ii) $y = 9x^3$ after a translation of 7 units vertically down

   (b) (i) $y = 7x^3 - 3x + 6$ after a translation of 2 units down

   (ii) $y = 8x^2 - 7x + 1$ after a translation of 5 units up

   (c) (i) $y = 4x^2$ after a translation of 5 units to the right

   (ii) $y = 7x^2$ after a translation of 3 units to the left

   (d) (i) $y = 3x^3 - 5x^2 + 4$ after a translation of 4 units to the left

   (ii) $y = x^3 + 6x + 2$ after a translation of 3 units to the right

3. Find the required translations.

(a) (i) Transforming the graph of $y = x^2 + 3x + 7$ to the graph of $y = x^2 + 3x + 2$

(ii) Transforming the graph of $y = x^3 - 5x$ to the graph of $y = x^3 - 5x - 4$

(b) (i) Transforming the graph of $y = x^2 + 2x + 7$ to the graph of $y = (x+1)^2 + 2(x+1) + 7$

(ii) Transforming the graph of $y = x^2 + 5x - 2$ to the graph of $y = (x+5)^2 + 5(x+5) - 2$

(c) (i) Transforming the graph of $y = e^x + x^2$ to the graph of $y = e^{x-4} + (x-4)^2$

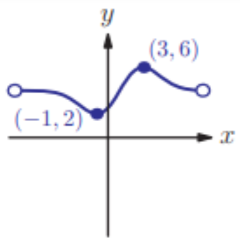
(ii) Transforming the graph of $y = \log(3x) - \sqrt{4x}$ to the graph of $y = \log(3(x-5)) - \sqrt{4(x-5)}$

(d) (i) Transforming the graph of $y = \ln(4x)$ to the graph of $y = \ln(4x + 12)$

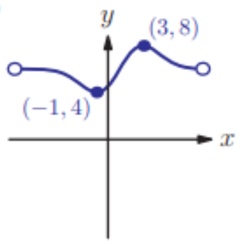
(ii) Transforming the graph of $y = \sqrt{2x+1}$ to the graph of $y = \sqrt{2x-3}$

ANSWERS

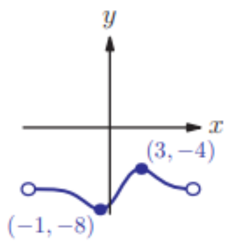
1. (a) (i)



(ii)



(b) (i)



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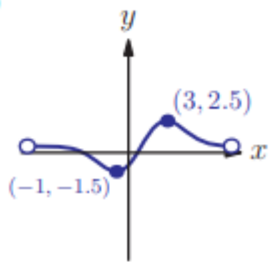
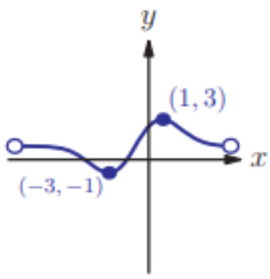
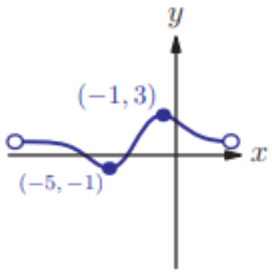


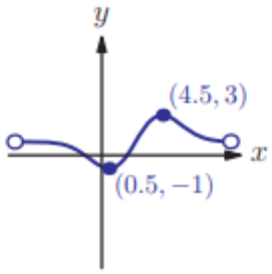
(c) (i)



(ii)



(d) (i)



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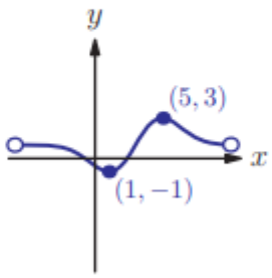


2. (a) (i) $y = 3x^2 + 3$ (ii) $y = 9x^3 - 7$
(b) (i) $y = 7x^3 - 3x + 4$ (ii) $y = 8x^2 - 7x + 6$
(c) (i) $y = 4(x-5)^2$ (ii) $y = 7(x+3)^2$
(d) (i) $y = 3(x+4)^3 - 5(x+4)^2 + 4$
(ii) $y = (x-3)^3 + 6(x-3) + 2$

3. (a) (i) Vertically down 5 units
(ii) Vertically down 4 units
(b) (i) Left 1 unit (ii) Left 5 units
(c) (i) Right 4 units (ii) Right 5 units
(d) (i) Left 3 units (ii) Right 2 units

STRETCHES/DILATION

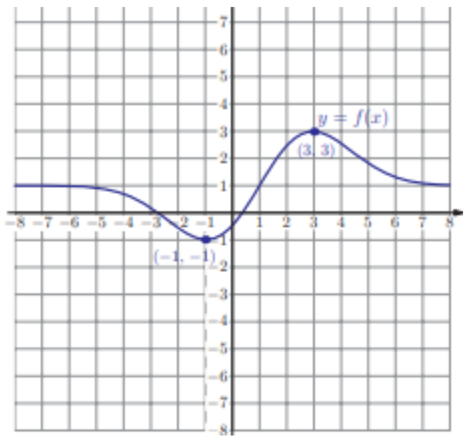
1. Given the graph of $y = f(x)$, sketch the graph of the following functions, indicating the positions of the minimum and maximum points.

(a) (i) $y = 3f(x)$ (ii) $y = 5f(x)$

(b) (i) $y = \dfrac{f(x)}{4}$ (ii) $y = \dfrac{f(x)}{2}$

(c) (i) $y = f(2x)$ (ii) $y = f(6x)$

(d) (i) $y = f\left(\dfrac{2x}{3}\right)$ (ii) $y = f\left(\dfrac{5x}{6}\right)$



2. Find the equation of the graph after the given transformation is applied.

(a) (i) $y = 3x^2$ after a vertical stretch 7 relative to the x-axis

(ii) $y = 9x^3$ after a vertical stretch 2 relative to the x-axis

(b) (i) $y = 7x^3 - 3x + 6$ after a vertical stretch factor $\frac{1}{3}$ relative to the x-axis

(ii) $y = 8x^2 - 7x + 1$ after a vertical stretch factor $\frac{4}{5}$ relative to the x-axis

(c) (i) $y = 4x^2$ after a horizontal stretch factor 2 relative to the y-axis

(ii) $y = 7x^2$ after a horizontal stretch factor 5 relative to the y-axis

(d) (i) $y = 3x^3 - 5x^2 + 4$ after a horizontal stretch factor $\frac{1}{2}$ relative to the y-axis

(ii) $y = x^3 + 6x + 2$ after a horizontal stretch factor $\frac{2}{3}$ relative to the y-axis

3. Describe the following stretches.

(a) (i) Transforming the graph of $y = x^2 + 3x + 7$ to the graph of $y = 4x^2 + 12x + 28$

(ii) Transforming the graph of $y = x^3 - 5x$ to the graph of $y = 6x^3 - 30x$

(b) (i) Transforming the graph of $y = x^2 + 2x + 7$ to the graph of $y = (3x)^2 + 2(3x) + 7$

(ii) Transforming the graph of $y = x^2 + 5x - 2$ to the graph of $y = (4x)^2 + 5(4x) - 2$

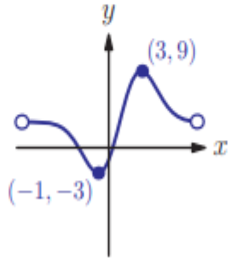
(c) (i) Transforming the graph of $y = e^x + x^2$ to the graph of $y = e^{\frac{x}{2}} + \left(\frac{x}{2}\right)^2$

(ii) Transforming the graph of $y = \log(3x) - \sqrt{4x}$ to the graph of $y = \log\left(\frac{3x}{5}\right) - \sqrt{\frac{4x}{5}}$

(d) (i) Transforming the graph of $y = \ln(4x)$ to the graph of $y = \ln(12x)$

(ii) Transforming the graph of $y = \sqrt{2x+1}$ to the graph of $y = \sqrt{x+1}$

1. (a) (i)



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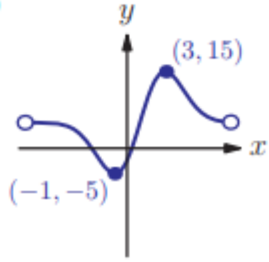
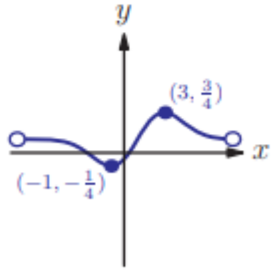


(b) (i)



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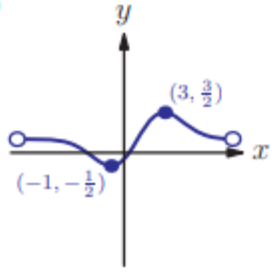
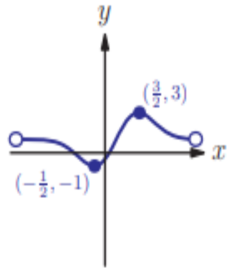


(c) (i)



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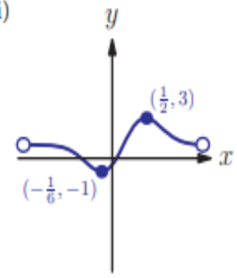
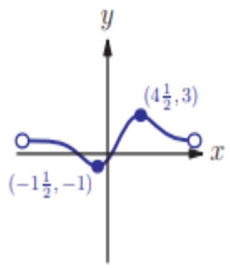
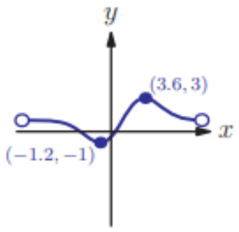


(d) (i)



(ii)



2. (a) (i) $y = 21x^2$ (ii) $y = 18x^3$

(b) (i) $y = \frac{1}{3}(7x^3 - 3x + 6)$ (ii) $y = \frac{4}{5}(8x^2 - 7x + 1)$

(c) (i) $y = 4\left(\frac{x}{2}\right)^2$ (ii) $y = 7\left(\frac{x}{5}\right)^2$

(d) (i) $y = 3(2x)^3 - 5(2x)^2 + 4$

(ii) $y = \left(\frac{3x}{2}\right)^3 + 6\left(\frac{3x}{2}\right) + 2$

3. (a) (i) Vertical stretch, scale factor 4
(ii) Vertical stretch, scale factor 6

(b) (i) Horizontal stretch, scale factor $\frac{1}{3}$

(ii) Horizontal stretch, scale factor $\frac{1}{4}$

(c) (i) Horizontal stretch, scale factor 2
(ii) Horizontal stretch, scale factor 5

REFLECTIONS

2. Find the equation of the graph after the given transformation is applied.

   (a) (i) $y = 3x^2$ after reflection in the x-axis

   (ii) $y = 9x^3$ after reflection in the x-axis

   (b) (i) $y = 7x^3 - 3x + 6$ after reflection in the x-axis

   (ii) $y = 8x^2 - 7x + 1$ after reflection in the x-axis

   (c) (i) $y = 4x^2$ after reflection in the y-axis

   (ii) $y = 7x^3$ after reflection in the y-axis

   (d) (i) $y = 3x^3 - 5x^2 + 4$ after reflection in the y-axis

   (ii) $y = x^3 + 6x + 2$ after reflection in the y-axis

3. Describe the following transformations

   (a) (i) Transforming the graph of $y = x^2 + 3x + 7$ to the graph of $y = -x^2 - 3x - 7$

   (ii) Transforming the graph of $y = x^3 - 5x$ to the graph of $y = 5x - x^3$

   (b) (i) Transforming the graph of $y = x^2 + 2x + 7$ to the graph of $y = x^2 - 2x + 7$

   (ii) Transforming the graph of $y = x^2 - 5x - 2$ to the graph of $y = x^2 + 5x - 2$

   (c) (i) Transforming the graph of $y = e^x + x^2$ to the graph of $y = e^{-x} + x^2$

   (ii) Transforming the graph of $y = \log(3x) - \sqrt{4x}$ to the graph of $y = \sqrt{4x} - \log(3x)$

   (d) (i) Transforming the graph of $y = \ln(4x)$ to the graph of $y = \ln(-4x)$

   (ii) Transforming the graph of $y = \sqrt{2x - 1}$ to the graph of $y = \sqrt{-1 - 2x}$

2. (a) (i) $y=-3x^2$ (ii) $y=-9x^3$
(b) (i) $y=-7x^3+3x-6$ (ii) $y=-8x^2+7x-1$
(c) (i) $y=4x^2$ (ii) $y=-7x^3$
(d) (i) $y=-3x^3-5x^2+4$ (ii) $y=-x^3-6x+2$

3. (a) (i) Reflection in the x-axis
(ii) Reflection in the x-axis
(b) (i) Reflection in the y-axis
(ii) Reflection in the y-axis
(c) (i) Reflection in the y-axis
(ii) Reflection in the x-axis
(d) (i) Reflection in the y-axis
(ii) Reflection in the y-axis

COMBINED TRANSFORMATIONS

3. If $f(x)=x^2$, express each of the following functions as $af(x)+b$ and hence describe the transformation(s) mapping $f(x)$ to the given function.
(a) (i) $k(x)=2x^2-6$ (ii) $k(x)=5x^2+4$
(b) (i) $h(x)=5-3x^2$ (ii) $h(x)=4-8x^2$

4. If $f(x)=2x^2-4$, write down the function $g(x)$ which gives the graph of $f(x)$ after:
(a) (i) translation $\begin{pmatrix}0\\2\end{pmatrix}$, followed by a vertical stretch of scale factor 3
(ii) translation $\begin{pmatrix}0\\6\end{pmatrix}$, followed by a vertical stretch of scale factor $\frac{1}{2}$
(b) (i) vertical stretch of scale factor $\frac{1}{2}$, followed by a translation $\begin{pmatrix}0\\6\end{pmatrix}$
(ii) vertical stretch of scale factor $\frac{7}{2}$, followed by a translation $\begin{pmatrix}0\\10\end{pmatrix}$

(d) (i) reflection through the horizontal axis and vertical stretch of scale factor $\frac{1}{2}$, followed by a translation $\begin{pmatrix}0\\3\end{pmatrix}$
(ii) reflection through the horizontal axis followed by a translation $\begin{pmatrix}0\\-6\end{pmatrix}$ followed by a vertical stretch, scale factor $\frac{3}{2}$

5. If $f(x)=x^2$, express each of the following functions as $f(ax+b)$ and hence describe the transformation(s) mapping $f(x)$ to the given function.

(a) (i) $g(x)=x^2+2x+1$ (ii) $g(x)=x^2-6x+9$

(b) (i) $h(x)=4x^2$ (ii) $h(x)=\dfrac{x^2}{9}$

(c) (i) $k(x)=4x^2+8x+4$ (ii) $k(x)=9x^2-6x+1$

6. If $f(x)=2x^2-4$, write down the function $g(x)$ which gives the graph of $f(x)$ after:

(a) (i) translation $\begin{pmatrix}1\\0\end{pmatrix}$ followed by a horizontal stretch of scale factor $\frac{1}{4}$

(ii) translation $\begin{pmatrix}-2\\0\end{pmatrix}$, followed by a horizontal stretch of scale factor $\frac{1}{2}$

(b) (i) horizontal stretch of scale factor $\frac{1}{2}$, followed by a translation $\begin{pmatrix}-4\\0\end{pmatrix}$

(ii) horizontal stretch of scale factor $\frac{2}{3}$, followed by a translation $\begin{pmatrix}1\\0\end{pmatrix}$

(c) (i) translation $\begin{pmatrix}-3\\0\end{pmatrix}$ followed by a reflection through the vertical axis

(ii) reflection through the vertical axis followed by a translation $\begin{pmatrix}-3\\0\end{pmatrix}$

7. For each of the following functions $f(x)$ and $g(x)$, express $g(x)$ in the form a: $f(x+b)+c$ for some values a,b and c, and hence describe a sequence of horizontal and vertical transformations which map $f(x)$ to $g(x)$.

(a) (i) $f(x)=x^2,\ g(x)=2x^2+4x$

(ii) $f(x)=x^2,\ g(x)=3x^2-24x+8$

(b) (i) $f(x)=x^2+3,\ g(x)=x^2-6x+8$

(ii) $f(x)=x^2-2,\ g(x)=2+8x-4x^2$

8. If $f(x)=2^x+x$, give in simplest terms the formula for $h(x)$, which is obtained from transforming $f(x)$ by the following sequence of transformations:

- vertical stretch, scale factor 8 relative to $y=0$
- translation by $\begin{pmatrix}1\\4\end{pmatrix}$
- horizontal stretch, scale factor $\frac{1}{2}$ relative to $x=0$ *[6 marks]*

9. Sketch the following graphs. In each case, indicate clearly the positions of the vertical asymptote and the x-intercept.

(a) $y=\ln x$

(b) $y=3\ln(x+2)$

(c) $y=\ln(2x-1)$ *[6 marks]*

10. (a) The graph of the function $f(x)=ax+b$ is transformed by the following sequence:

- translation by $\begin{pmatrix}1\\2\end{pmatrix}$
- reflection in $y=0$
- horizontal stretch, scale factor $\frac{1}{3}$ relative to $x=0$

The resultant function is $g(x)=4-15x$. Find a and b.

ANSWERS

ANSWER HINT (3)

There are alternative answers; for example, (a)(i) could be translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ followed by vertical sketch with s.f.2.

3. (a) (i) $k(x) = 2f(x) - 6$; vertical stretch with scale factor 2 and then translation $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$

(ii) $k(x) = 5f(x) + 4$; vertical stretch with scale factor 5 and then translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(b) (i) $h(x) = 5 - 3f(x)$; vertical stretch with scale factor 3, reflection in x-axis and translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

(ii) $h(x) = 4 - 8f(x)$; vertical stretch with scale factor 8, reflection in x-axis and translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

4. (a) (i) $g(x) = 6x^2 - 6$ (ii) $g(x) = x^2 + 1$

(b) (i) $g(x) = x^2 + 4$ (ii) $g(x) = 7x^2 - 4$

(c) (i) $g(x) = 4 - 2x^2$ (ii) $g(x) = 6 - 2x^2$

(d) (i) $g(x) = 5 - x^2$ (ii) $g(x) = -3 - 3x^2$

5. (a) (i) $f(x+1)$; translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

(ii) $f(x-3)$; translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

(b) (i) $f(2x)$; horizontal stretch with scale factor $\frac{1}{2}$; or $4f(x)$; vertical sketch with scale factor 4

(ii) $f\left(\frac{x}{3}\right)$; horizontal stretch with scale factor 3; or $\frac{1}{9}f(x)$; vertical sketch with

(c) (i) $f(2x+2)$; translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and then horizontal stretch with scale factor $\frac{1}{2}$ *or* horizontal stretch with scale factor $\frac{1}{2}$ and then translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

(ii) $f(3x-1)$; translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ then horizontal stretch with scale factor $\frac{1}{3}$ *or* horizontal stretch with scale factor $\frac{1}{3}$ and then translation $\begin{pmatrix} 1/3 \\ 0 \end{pmatrix}$

5. (a) (i) $g(x) = 32x^2 - 16x - 2$

(ii) $g(x) = 8x^2 + 16x + 4$

(b) (i) $g(x) = 8x^2 + 64x + 124$

(ii) $g(x) = \frac{9x^2}{2} - 9x + \frac{1}{2}$

(c) (i) $g(x) = 2x^2 - 12x + 14$

(ii) $g(x) = 2x^2 + 12x + 14$

7. (a) (i) $g(x) = 2f(x+1) - 2$; translation by $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$, then vertical stretch with scale factor 2

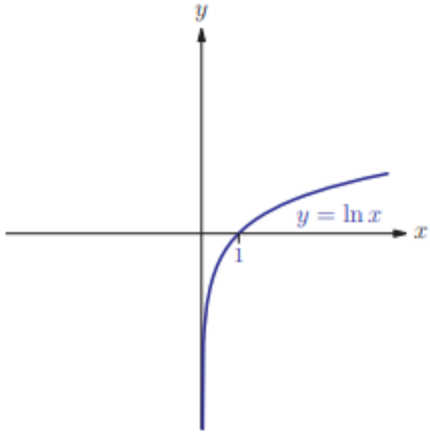
(ii) $g(x) = 3f(x-4) - 40$; vertical stretch with scale factor 3, then translation by $\begin{pmatrix} 4 \\ -40 \end{pmatrix}$

(b) (i) $g(x) = f(x-3) - 4$; translation by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

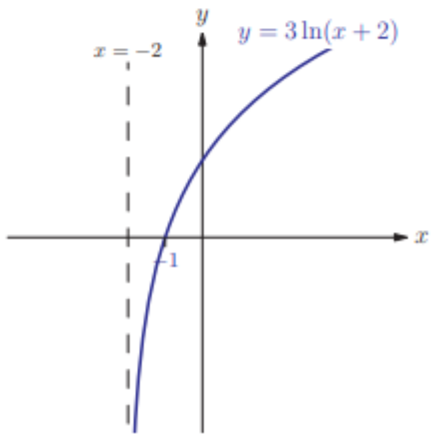
(ii) $g(x) = -4f(x-1) + 8$; vertical stretch with scale factor 4, then translation by $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$

8. $h(x) = 4^{x+1} + 16x - 4$

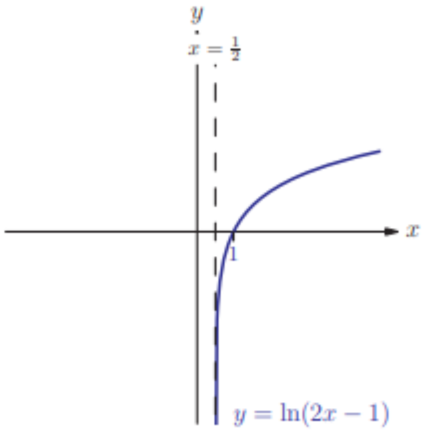
9. (a)



(b)



(c)



10. (i) $a = 5$, $b = -1$
(ii) $a = 16$, $b = 0$, $c = -25$

ADDITION AND SUBTRACTION OF FUNCTIONS

Perform the indicated operation.

1) $g(x) = 3x + 4$
$h(x) = -3x^3 - 2x$
Find $(g + h)(x)$

2) $h(x) = x + 3$
$g(x) = x^3 + 2x^2$
Find $(h - g)(x)$

3) $f(x) = 2x + 4$
$g(x) = x + 3$
Find $(f - g)(x)$

4) $g(t) = t^2 + 5t$
$f(t) = -t - 3$
Find $(g + f)(t)$

5) $f(a) = -2a^2 + 5a$
$g(a) = 4a - 2$
Find $(f + g)(a)$

6) $f(a) = 2a - 4$
$g(a) = a^2 + 1$
Find $(f + g)(a)$

7) $g(x) = x^2 + 3x$
$h(x) = 4x - 3$
Find $(g - h)(x)$

8) $f(n) = 3n + 2$
$g(n) = -2n^2 + n$
Find $(f + g)(n)$

9) $g(n) = -2n - 4$
$f(n) = 2n + 1$
Find $(g - f)(2)$

10) $g(a) = -4a + 4$
$h(a) = -2a^2 - 4$
Find $(g - h)(8)$

11) $h(x) = 2x - 4$
$g(x) = x^3 - 3$
Find $(h + g)(4)$

12) $f(n) = n^3 - 2n^2$
$g(n) = -2n - 1$
Find $(f - g)(3)$

13) $g(x) = x^2 + 3$
$h(x) = 4x - 3$
Find $(g - h)(1)$

14) $f(n) = n^3 + 2$
$g(n) = 4n - 1$
Find $(f + g)(1)$

15) $g(x) = 3x + 5$
$f(x) = x^2 + 2$
Find $(g + f)(8)$

16) $g(a) = a^2 - a$
$f(a) = 3a - 4$
Find $(g - f)(-7)$

17) $f(x) = 4x - 2$
$g(x) = -x^3 + 4$
Find $(f - g)(3x)$

18) $h(x) = 4x - 1$
$g(x) = x^2 + 4$
Find $(h - g)(2x)$

19) $g(a) = a - 2$
$f(a) = 2a^2 - 5a$
Find $(g - f)\left(\frac{a}{2}\right)$

20) $f(a) = 2a - 2$
$g(a) = a^2 + 2$
Find $(f - g)\left(\frac{a}{4}\right)$

21) $f(x) = x^3 - 2$
$g(x) = x + 5$
Find $(f - g)(x + 1)$

22) $h(t) = -t - 1$
$g(t) = -2t^3 - 1$
Find $(h - g)(-4x)$

Answers to Functions and Their Operations Adding and Subtracting (FATOAS)

1) $-3x^3 + x + 4$	2) $-x^3 - 2x^2 + x + 3$	3) $x + 1$	4) $t^2 + 4t - 3$
5) $-2a^2 + 9a - 2$	6) $a^2 + 2a - 3$	7) $x^2 - x + 3$	8) $-2n^2 + 4n + 2$
9) -13	10) 104	11) 65	12) 16
13) 3	14) 6	15) 95	16) 81
17) $27x^3 + 12x - 6$	18) $-4x^2 + 8x - 5$	19) $\frac{-4 + 6a - a^2}{2}$	20) $\frac{-64 + 8a - a^2}{16}$
21) $x^3 + 3x^2 + 2x - 7$	22) $-128x^3 + 4x$	23) $-2a^6 + 3$	24) $-3n + 3$

LINEAR, QUADRATIC AND SIMULTANEOUS EQUATIONS – APPLICATION

Solve the simultaneous equations

$$\tfrac{1}{2}x + y = 5,$$
$$x - 2y = 6.$$

Answer $x =$ .... $y =$ .... [3]

Solve the equation

$$x^2 + 4x - 22 = 0.$$

Give your answers correct to 2 decimal places.
Show all your working.

Answer $x =$ .... or $x =$ .... [4]

Angharad had an operation costing \$500.
She was in hospital for x days.
The cost of nursing care was \$170 for each day she was in hospital.

(a) Write down, in terms of x, an expression for the total cost of her operation and nursing care.

Answer(a)\$ .... [1]

(b) The total cost of her operation and nursing care was \$2370.
Work out how many days Angharad was in hospital.

Answer(b) .... [2]

(a) **(i)** The cost of a book is \$$x$.
Write down an expression in terms of x for the number of these books which are bought for \$40. [1]

(ii) The cost of each book is increased by \$2.
The number of books which are bought for \$40 is now one less than before.
Write down an equation in x and show that it simplifies to $x^2 + 2x - 80 = 0$. [4]

(iii) Solve the equation $x^2 + 2x - 80 = 0$. [2]

(iv) Find the original cost of one book. [1]

(b) Magazines cost \$$m$ each and newspapers cost \$$n$ each.
One magazine costs \$2.55 more than one newspaper.
The cost of two magazines is the same as the cost of five newspapers.

(i) Write down two equations in m and n to show this information. [2]

(ii) Find the values of m and n. [3]

---

1) A trader bought some paraffin for \$500. He paid \$x for each litre of paraffin.

a) Find, in terms of x, an expression for the number of litres he bought. [1]

b) Due to a leak, he lost 3 litres of paraffin.
He sold the remainder of the paraffin for \$1 per litre more than he paid for it.

Write down an expression, in terms of x, for the sum of money he received. [2]

c) He made a profit of \$20.

i) Write down an equation in x to represent this information, and show that it reduces to $3x^2 + 23x - 500 = 0$. [3]

ii) Solve the equation $3x^2 + 23x - 500 = 0$, giving both answers correct to one decimal place. [4]

d) Find, correct to the nearest whole number, how many litres of paraffin he sold. [2]

---

2) A road tanker holds 24 tonnes of oil.

a) In cold weather it can pump out *x* tonnes of oil per minute.
Write down an expression, in terms of *x*, for the number of minutes it takes to empty the tanker in cold weather. [1]

b) In hot weather it can pump out (*x* + 0.5) tonnes of oil per minute.
Write down an expression, in terms of *x*, for the number of minutes it takes to empty the tanker in hot weather. [1]

c) It takes 2 minutes longer to empty the tanker in cold weather than in hot weather. Write down an equation in *x*, and show that it simplifies to $2x^2 + x - 12 = 0$ [3]

d) Solve the equation $2x^2 + x - 12 = 0$, giving the solutions **correct to 3 decimal places**. [4]

e) Find the time taken, in minutes and seconds, correct to the nearest second, to empty the tanker in cold weather. [2]

---

3)

a) Express as a single fraction in its simplest form

$$\frac{200}{x} - \frac{200}{x+4}.$$

. [2]

b) When driven in town, a car runs x kilometres on each litre of petrol.

i) Find, in terms of x, the number of litres of petrol used when the car is driven 200 km in town. [1]

ii) When driven out of town, the car runs $(x + 4)$ kilometres on each litre of petrol.
It uses 5 litres less petrol to go 200 km out of town than to go 200 km in town.

Use this information to write down an equation involving x, and show that it simplifies to
$x^2 + 4x - 160 = 0$. [3]

c) Solve the equation $x^2 + 4x - 160 = 0$, giving both answers correct to two decimal places. [4]

d) Calculate the **total** volume of petrol used when the car is driven 40 km in town and then 120 km out of town. [2]

---

4) A company was contracted to make 840 vans in 90 days. After they made 540, the manager worked out that, if they could increases this average by 1 van per day, they could fulfill the contract in exactly 90 days.
Let the average for the first 540 be x vans per day.

a) Write down an equation in x and show that it simplifies to $\frac{18}{x} + \frac{10}{x+1} = 3$.

b) Use algebra to solve the equation. Hence find the average production of the first 540 vans. [2+3]

SEQUENCES

1 Write down a rule to describe the sequence and hence find its next *two* terms:

a 5, 8, 11, 14, 17, ... **b** 2, 9, 16, 23, 30, ... **c** 8, 19, 30, 41, 52, ...

d 38, 34, 30, 26, 22, ... **e** 3, -2, -7, -12, -17, ... **f** $\frac{1}{2}$, 2, $3\frac{1}{2}$, 5, $6\frac{1}{2}$, ...

2 Write down a rule to describe the sequence and hence find its next *two* terms:

a 3, 6, 12, 24, 48, ... **b** 1, 2, 4, 8, 16, ... **c** 2, 10, 50, 250, ...

d 36, 18, 9, $4\frac{1}{2}$, ... **e** 162, 54, 18, 6, ... **f** 405, 135, 45, 15, ...

3 Find the next *two* terms of:

a 0, 1, 4, 9, 16, ... **b** 1, 4, 9, 16, 25, ... **c** 0, 1, 8, 27, 64, ...

d $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, ... **e** 1, 2, 4, 7, 11, ... **f** 2, 6, 12, 20, 30, ...

4 Write down a rule to describe the sequences and hence find its next *three* terms:

a 1, 1, 2, 3, 5, 8, ... **b** 1, 3, 4, 7, 11, ... **c** 5, 8, 12, 18, 24, 30, ...

5 Draw the next *two* matchstick figures in these sequences and write the number of matchsticks used as a number sequence:

a

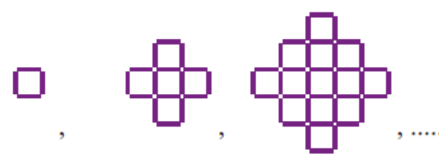
b

c

d

e

f



6 Draw the next *two* figures in these sequences and write the number of dots used as a number sequence:

a

b

c

d

6 **a** Find a general formula for: 1, 2, 3, 4, 5, 6, ...

b *Hence* find a general formula for:

i 2, 3, 4, 5, 6, 7, ...

ii 3, 4, 5, 6, 7, 8, ...

iii $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, ...

iv $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$, ...

v $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$, ...

vi $\frac{3}{1}, \frac{4}{2}, \frac{5}{3}, \frac{6}{4}, \frac{7}{5}, \frac{8}{6}$, ...

vii $1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5$, ...

viii $2 \times 3, 3 \times 4, 4 \times 5, 5 \times 6$, ...

ix $1 \times 3, 2 \times 4, 3 \times 5, 4 \times 6$, ...

x $\frac{1}{3}, \frac{4}{6}, \frac{7}{9}, \frac{10}{12}$, ...

7 **a** Find a general formula for: 1, 4, 9, 16, 25, ...

b *Hence* find a general formula for:

i 4, 9, 16, 25, 36, ...

ii 0, 3, 8, 15, 24, ...

iii $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$, ...

iv $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}$, ...

8 Find a general formula for:

a 1, 8, 27, 64, 125, ...

b 0, 7, 26, 63, 124, ...

c 3, 6, 12, 24, 48, ...

d 24, 12, 6, 3, $1\frac{1}{2}$, ...

1 List the first *five* terms of the geometric sequence defined by:

a $u_n = 3 \times 2^n$

b $u_n = 3 \times 2^{n-1}$

c $u_n = 3 \times 2^{n+1}$

d $u_n = 5 \times 2^{n-1}$

e $u_n = 24 \times (\frac{1}{2})^{n-1}$

f $u_n = 36 \times (\frac{1}{3})^{n-1}$

g $u_n = 24 \times (-2)^n$

h $u_n = 8(-1)^{n-1}$

i $u_n = 8(-\frac{1}{2})^n$

2 Find the next *two* terms and a formula for the nth term of:

a 1, −1 , 1 , −1 , 1 , ...

b −1, 1 , −1 , 1 , −1 , ...

c 2, 4, 8, 16, 32, ...

d 2, −4, 8, −16, 32, ...

e 6, 18, 54, 162, ...

f 6, −18, 54, −162, ...

g 4, 12, 36, 108, ...

h 2, −14, 98, −686, ...

i 3, −6, 12, −24, ...

j −16, 8, −4, 2, ...

1 Use the method of differences to find the general term u_n of:

a 1, 5, 9, 13, 17, 21, ...

b 17, 14, 11, 8, 5, 2, ...

c 2, 6, 12, 20, 30, 42, ...

d 0, 6, 14, 24, 36, 50, ...

e 6, 13, 32, 69, 130, 221, 348, ...

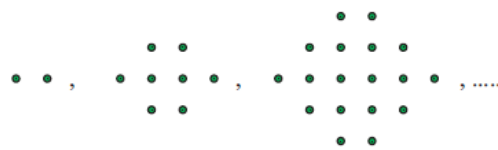
f 2, 7, 18, 38, 70, 117, 182, ...

2 Consider the sequence: 2, 12, 30, 56, 90, 132, ...

a Use the difference method to find the general term u_n.

b Suggest an alternative formula for u_n by considering $u_1 = 1 \times 2$, $u_2 = ... \times ......$, $u_3 = ... \times ......$, and so on.

3 Consider the dot pattern:



a Find u_n for $n = 1, 2, 3, 4, 5, 6$ and 7.

b Find a formula for the general term u_n.

c How many dots are needed to make up the 30th figure in the pattern?

3. An arithmetic sequence has 5 and 13 as its first term and second term, respectively.

(a) Write down, in terms of n, an expression for the nth term a_n.

(b) Find the number of terms of the sequence which are less than 400. *[8 marks]*

4. The 10th term of an arithmetic sequence is 61 and the 13th term is 79. Find the value of the 20th term. *[4 marks]*

5. The 8th term of an arithmetic sequence is 74 and the 15th term is 137. Which term has the value 227? *[4 marks]*

6. The heights above ground of the rungs in a ladder form an arithmetic sequence. The third rung is 70 cm above the ground and the tenth rung is 210 cm above the ground. If the top rung is 350 cm above the ground, how many rungs does the ladder have? *[5 marks]*

7. The first four terms of an arithmetic sequence are 2, $a - b$, $2a + b + 7$ and $a - 3b$, where a and b are constants. Find a and b. *[5 marks]*

ANSWERS

2. (a) (i) 33 (ii) 29
   (b) (i) 100 (ii) 226

3. (a) $a_n = 5 + 8(n-1)$ (b) 50

4. 121

5. 25th

6. 17

7. $a = 2, b = -3$

8. (b) 456

1. Find the sum of the following arithmetic sequences.

   (a) (i) $12, 33, 54, \ldots$ (17 terms)

   (ii) $-100, -85, -70, \ldots$ (23 terms)

   (b) (i) $3, 15, \ldots, 459$ (ii) $2, 11, \ldots, 650$

   (c) (i) $28, 23, \ldots, -52$ (ii) $100, 97, \ldots, 40$

   (d) (i) $15, 15.5, \ldots, 29.5$ (ii) $\frac{1}{12}, \frac{1}{6}, \ldots, 1.5$

2. An arithmetic sequence has first term 4 and common difference 8. Find the number of terms required to get a sum of:

   (a) (i) 676 (ii) 4096 (iii) 11236

   (b) x^2, $x > 0$

3. The second term of an arithmetic sequence is 7. The sum of the first four terms of the sequence is 12. Find the first term, a, and the common difference, d, of the sequence. *[5 marks]*

4. Consider the arithmetic series $2 + 5 + 8 + \ldots$

   (a) Find an expression for S_n, the sum of the first n terms.

   (b) Find the value of n for which $S_n = 1365$. *[5 marks]*

5. The sum of the first n terms of a series is given by $S_n = 2n^2 - n$, where $n \in \mathbb{Z}^+$.

(a) Find the first three terms of the series.

(b) Find an expression for the nth term of the series, giving your answer in terms of n. *[7 marks]*

6. Find the sum of the positive terms of the arithmetic sequence 85, 78, 71, ... *[6 marks]*

7. The second term of an arithmetic sequence is 6, and the sum of the first four terms of the sequence is 8. Find the first term, a, and the common difference, d, of the sequence. *[6 marks]*

8. Consider the arithmetic series $-6+1+8+15+\cdots$

Find the least number of terms so that the sum of the series is greater than 10 000. *[6 marks]*

9. The sum of the first n terms of an arithmetic sequence is $S_n = 3n^2 - 2n$. Find the nth term, u_n. *[6 marks]*

10. A circular disc is cut into twelve sectors whose angles are in an arithmetic sequence. The angle of the largest sector is twice the angle of the smallest sector. Find the size of the angle of the smallest sector. *[6 marks]*

ANSWERS

1. (a) (i) 3060 (ii) 1495
(b) (i) 9009 (ii) 23798
(c) (i) −204 (ii) 1470
(d) (i) 667.5 (ii) 14.25

2. (a) (i) 13 (ii) 32 (iii) 53

(b) $\frac{x}{2}$

3. $a = 15,\ d = -8$

4. (a) $S_n = \frac{n}{2}(3n+1)$ (b) 30

5. (a) 1, 5, 9 (b) $u_n = 4n - 3$

6. 559

7. $a = 14,\ d = -8$

8. 55

9. $u_n = 6n - 5$

10. 20°

2. Find the value of the common ratio if

(a) (i) the first term is 11, sum of the first 12 terms is 2 922 920

(ii) the first term is 1, sum of the first 6 terms is 1.24992

(b) (i) the first term is 12, sum of the first 6 terms is −79 980

(ii) the first term is 10, sum of the first 4 terms is 1

3. The nth term, u_n, of a geometric sequence is given by $u_n = 3\times 5^{n+2}$.

(a) Find the common ratio r.

(b) Hence or otherwise find S_n, the sum of the first n terms of this sequence. *[5 marks]*

4. The sum of the first three terms of a geometric sequence is $23\frac{3}{4}$, and the sum of the first four terms is $40\frac{5}{8}$. Find the first term and the common ratio. *[6 marks]*

5. The first term of a geometric series is 6, and the sum of the first 15 terms is 29. Find the common ratio. *[5 marks]*

ANSWERS

2. (a) (i) 3 (ii) 0.2
(b) (i) −6 (ii) −0.947

3. (a) 5 (b) $S_n = \frac{375(5^n - 1)}{4}$

4. $a = 5$, $r = 1.5$

5. 0.8 or −1.16

INFINITE GEOMETRIC SERIES

If we keep adding together terms of an arithmetic sequence, the sum will grow (or decrease) without limit, and is said to be **divergent**. This can occur with some geometric series, too, but it could also happen that the sum gets closer and closer to and 'settles down' to a finite number; in this case we say that the geometric series is **convergent**.

The graph shows the values of S_n for a geometric series with first term $u_1 = 4$ and common ratio $r = 0.2$. As n increases, the value of S_n seems to be getting closer and closer to 5; thus we say that the series converges to 5.

Not all geometric series converge. To determine which ones do, we need to look at the formula for a geometric series:

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

We want to know what happens to S_n as n gets large, so we focus on the r^n term. With most numbers, when you raise the number to a larger power the result gets bigger; for example, $1.2^{20} = 38.3$ and $1.2^{30} = 237$. The exception is when r is a number between -1 and 1. In this case, r^n gets smaller as n increases – in fact, it approaches zero; for example, $0.2^2 = 0.04, 0.2^3 = 0.008$ and $0.2^{20} = 1.05\times10^{-14}$. This means that for $-1 < r < 1$, as n increases the value of S_n will get closer and closer to $\frac{u_1}{1-r}$.

KEY POINT 6.7

As n increases, the sum of a geometric series converges to

$$S_\infty = \frac{u_1}{1-r} \text{ if } |r| < 1.$$

This is called the **sum to infinity of the series**.

$|r|$ is the modulus, or absolute value, of r. The modulus leaves positive values unchanged but reverses the sign of negative values. So, $|8| = 8$ and $|-8| = 8$.

The sum to infinity of a geometric sequence is 5. The second term is $-\frac{6}{5}$. Find the common ratio.

The geometric series $(2-x)+(2-x)^2+(2-x)^3+\cdots$ converges. What values can x take?

1. Find the value of each of the following infinite geometric series, or state that the series is divergent.

(a) (i) $9+3+1+\frac{1}{3}+\ldots$ (ii) $56+8+1\frac{1}{7}+\ldots$

(b) (i) $0.3+0.03+0.003+\ldots$

(ii) $0.78+0.0078+0.000078+\ldots$

(c) (i) $0.01+0.02+0.04+\ldots$ (ii) $\frac{19}{10000}+\frac{19}{1000}+\frac{19}{100}+\ldots$

(d) (i) $10-2+0.4+\ldots$ (ii) $6-4+\frac{8}{3}+\ldots$

(e) (i) $10-40+160+\ldots$ (ii) $4.2-3.36+2.688+\ldots$

2. Find the values of x which allow the following geometric series to converge.

(a) (i) $9+9x+9x^2+\ldots$ (ii) $-2-2x-2x^2+\ldots$

(b) (i) $1+3x+9x^2+\ldots$ (ii) $1+10x+100x^2+\ldots$

(c) (i) $-2-10x-50x^2+\ldots$ (ii) $8+24x+72x^2+\ldots$

(d) (i) $40+10x+2.5x^2+\ldots$ (ii) $144+12x+x^2+\ldots$

(e) (i) $243-81x+27x^2+\ldots$ (ii) $1-\frac{5}{4}x+\frac{25}{16}x^2+\ldots$

(f) (i) $3-\frac{6}{x}+\frac{12}{x^2}+\ldots$ (ii) $18-\frac{9}{x}+\frac{1}{x^2}+\ldots$

(g) (i) $5+5(3-2x)+5(3-2x)^2+\ldots$

(ii) $7+\frac{7(2-x)}{2}+\frac{7(2-x)^2}{4}+\ldots$

(h) (i) $1+\left(3-\frac{2}{x}\right)+\left(3-\frac{2}{x}\right)^2+\ldots$ (ii) $1+\frac{1+x}{x}+\frac{(1+x)^2}{x^2}+\ldots$

(i) (i) $7+7x^2+7x^4+\ldots$ (ii) $12-48x^3+192x^6+\ldots$

3. Find the sum to infinity of the geometric sequence $-18, 12, -8, \ldots$. *[4 marks]*

4. The first and fourth terms of a geometric sequence are 18 and $-\frac{2}{3}$ respectively.

(a) Find the sum of the first n terms of the sequence.

(b) Find the sum to infinity of the sequence. *[5 marks]*

ANSWERS

1. (a) (i) $u_n = 9+3(n-1)$ (ii) $u_n = 57+0.2(n-1)$

(b) (i) $u_n = 12-(n-1)$ (ii) $u_n = 18-\frac{1}{2}(n-1)$

(c) (i) $u_n = 1+3(n-1)$ (ii) $u_n = 9+10(n-1)$

(d) (i) $u_n = 4-4(n-1)$ (ii) $u_n = 27-7(n-1)$

(e) (i) $u_n = -17+11(n-1)$

(ii) $u_n = -32+10(n-1)$

2. (a) (i) 33 (ii) 29

(b) (i) 100 (ii) 226

3. (a) $a_n = 5+8(n-1)$ (b) 50

4. 121

LOGARITHMS

1 Write as a single logarithm or as an integer:

a $\log 8 + \log 2$
b $\log 4 + \log 5$
c $\log 40 - \log 5$
d $\log p - \log m$
e $\log_4 8 - \log_4 2$
f $\log 5 + \log(0.4)$
g $\log 2 + \log 3 + \log 4$
h $1 + \log_2 3$
i $\log 4 - 1$
j $\log 5 + \log 4 - \log 2$
k $2 + \log 2$
l $t + \log w$
m $\log_m 40 - 2$
n $\log_3 6 - \log_3 2 - \log_3 3$
o $\log 50 - 4$
p $3 - \log_5 50$
q $\log_5 100 - \log_5 4$
r $\log\left(\frac{4}{3}\right) + \log 3 + \log 7$

2 Write as a single logarithm or integer:

a $5\log 2 + \log 3$
b $2\log 3 + 3\log 2$
c $3\log 4 - \log 8$
d $2\log_3 5 - 3\log_3 2$
e $\frac{1}{2}\log_6 4 + \log_6 3$
f $\frac{1}{3}\log\left(\frac{1}{8}\right)$
g $3 - \log 2 - 2\log 5$
h $1 - 3\log 2 + \log 20$
i $2 - \frac{1}{2}\log_n 4 - \log_n 5$

4 Show that:

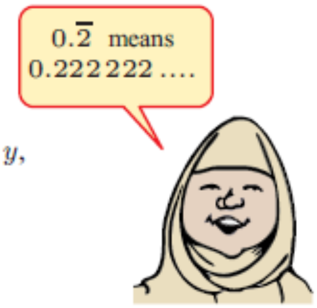
a $\log 9 = 2\log 3$ **b** $\log \sqrt{2} = \frac{1}{2}\log 2$ **c** $\log\left(\frac{1}{8}\right) = -3\log 2$

d $\log\left(\frac{1}{5}\right) = -\log 5$ **e** $\log 5 = 1 - \log 2$ **f** $\log 5000 = 4 - \log 2$

5 If $p = \log_b 2$, $q = \log_b 3$, and $r = \log_b 5$ write in terms of p, q, and r:

a $\log_b 6$ **b** $\log_b 45$ **c** $\log_b 108$

d $\log_b\left(\frac{5\sqrt{3}}{2}\right)$ **e** $\log_b\left(\frac{5}{32}\right)$ **f** $\log_b(0.\overline{2})$



6 If $\log_2 P = x$, $\log_2 Q = y$, and $\log_2 R = z$ write in terms of x, y, and z:

a $\log_2(PR)$ **b** $\log_2(RQ^2)$ **c** $\log_2\left(\frac{PR}{Q}\right)$

d $\log_2(P^2\sqrt{Q})$ **e** $\log_2\left(\frac{Q^3}{\sqrt{R}}\right)$ **f** $\log_2\left(\frac{R^2\sqrt{Q}}{P^3}\right)$

7 If $\log_t M = 1.29$ and $\log_t N^2 = 1.72$ find:

a $\log_t N$ **b** $\log_t(MN)$ **c** $\log_t\left(\frac{N^2}{\sqrt{M}}\right)$

ANSWERS

1 **a** $\log 16$ **b** $\log 20$ **c** $\log 8$ **d** $\log \frac{p}{m}$
e 1 **f** $\log 2$ **g** $\log 24$ **h** $\log_2 6$
i $\log 0.4$ **j** 1 **k** $\log 200$
l $\log(10^t \times w)$ **m** $\log_m\left(\frac{40}{m^2}\right)$ **n** 0
o $\log(0.005)$ **p** $\log_5\left(\frac{5}{2}\right)$ **q** 2 **r** $\log 28$

2 **a** $\log 96$ **b** $\log 72$ **c** $\log 8$ **d** $\log_3\left(\frac{25}{8}\right)$
e 1 **f** $\log \frac{1}{2}$ **g** $\log 20$ **h** $\log 25$
i $\log_n\left(\frac{n^2}{10}\right)$

3 **a** 2 **b** $\frac{3}{2}$ **c** 3 **d** $\frac{1}{2}$ **e** -2 **f** $-\frac{3}{2}$

4 For example, for **a**, $\log 9 = \log 3^2 = 2\log 3$

5 **a** $p+q$ **b** $2q+r$ **c** $2p+3q$ **d** $r+\frac{1}{2}q-p$
e $r-5p$ **f** $p-2q$

6 **a** $x+z$ **b** $z+2y$ **c** $x+z-y$ **d** $2x+\frac{1}{2}y$
e $3y-\frac{1}{2}z$ **f** $2z+\frac{1}{2}y-3x$

7 **a** 0.86 **b** 2.15 **c** 1.075

MYP E ASSESSMENT TYPE QUESTIONS

1.

A gardener has a fence that is 900 meters long. She wants to use it to fence a rectangular garden with the maximum possible area to plant. ***Apply*** *your knowledge in Algebra and geometry to find the dimensions of the garden that would give the maximum possible area.*

Comment *on your findings.*

2.

In a specific community "A", people believe that Christmas cake is best served any day between day 12 and day 25 of the production of the cake. In community "B" people believe that the cake is best served anytime between day 18 and day 40.

a) ***Write down*** *two inequalities that represent the optimal range of time for serving the cake in each community.*
b) ***Draw*** *each range suggested by the communities on separate number lines.*
c) *Some members of both communities were gathered for Christmas celebrations on 24^{th} of December. On a third number line* ***demonstrate*** *the possible production dates for a cake served that evening.*

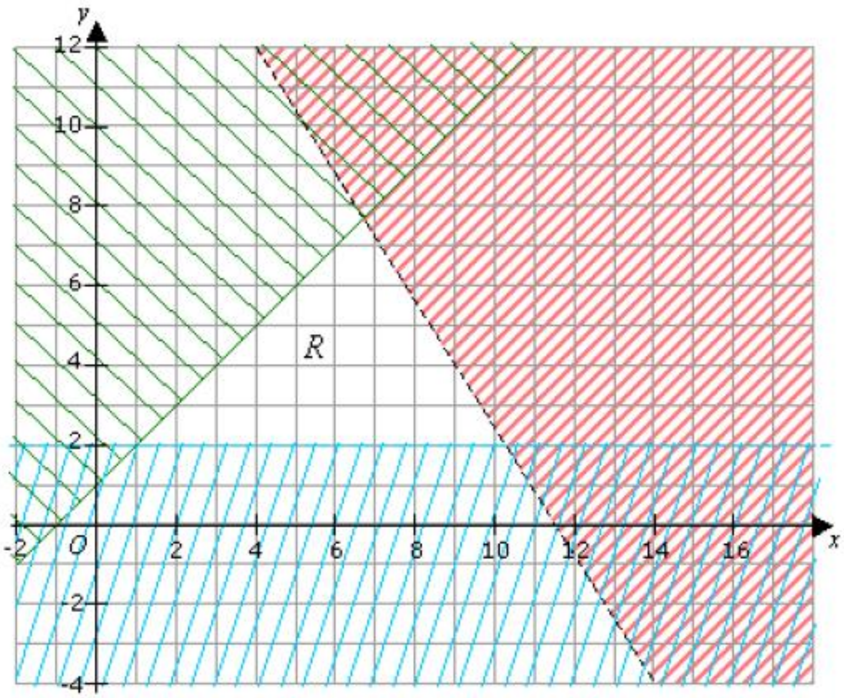
3.

On an appropriate plane:

c) Draw *the line $3x + 4y = 12$.*
d) Identify *the region of $3x + 4y \geq 12$ to show the appropriate solution set.*

4.

Identify *the solution set for the region by writing appropriate inequalities.*



5.

Trees in urban areas help keep air fresh by absorbing carbon dioxide. A city has \$2100 to spend on planting spruce and maple trees. The land available for planting is 45,000 square feet. Spruce trees cost \$30 to plant and require 600 square feet of space. Maple trees cost \$40 to plant and require 900 square feet of space.

a) ***Write down*** *a system of inequalities to represent the information given in the question.*

b) ***Draw*** *the system of inequalities on the same number plane and* ***label*** *the region that satisfies all inequalities.*

c) Spruce trees absorb 650 lb/yr of carbon dioxide and maple trees absorb 300 lb/yr of carbon dioxide. ***Calculate*** *how many of each tree should the city plant to maximize carbon dioxide absorption.*

State the domain of the following functions:

a) $y = x^3 - x^{-1}$

b) $(x) = 2(3^x) - 5$

c) $(x) = \dfrac{\pi}{\sin x}$

d) $y = \dfrac{5}{\sqrt{x-5}}$

6.

7.

The world population at the beginning of 1990 was 5.3 billion. Assume that the population continues to grow at the rate of approximately 2% per year.

a) ***Use*** *the information given above to complete the following table.*

Year	***World population***
1990	*5.3 billion*
1995	
2000	
2005	

b) ***Suggest*** *a general formula to predict the world population in a given year.*

c) ***Verify*** *your rule.*

d) ***Predict*** *the world population in the year 2016.*

e) *The actual world population of 2016 is 7.4 billion. Percentage error can be calculated using the formula:*

$$percentage\ error = \left|\frac{actual\ value - rounded\ value}{actual\ value}\right| \times 100\%$$

Calculate *the percentage error for the rounded value in 2016.* ***Write*** *your answer correct to decimal places.*

f) ***Comment*** *on the validity of predictions made in the extrapolation region.* ***Justify*** *your answer.*

g) ***Comment*** *on how the trend suggested by the general rule in part "a" will help decision makers to plan for the future.*

8.

You entered your room at night. You tried to switch on the light but it did not work. ***Construct*** *a flow chart (algorithm) of your analysis of the problem suggesting solutions.*

9.

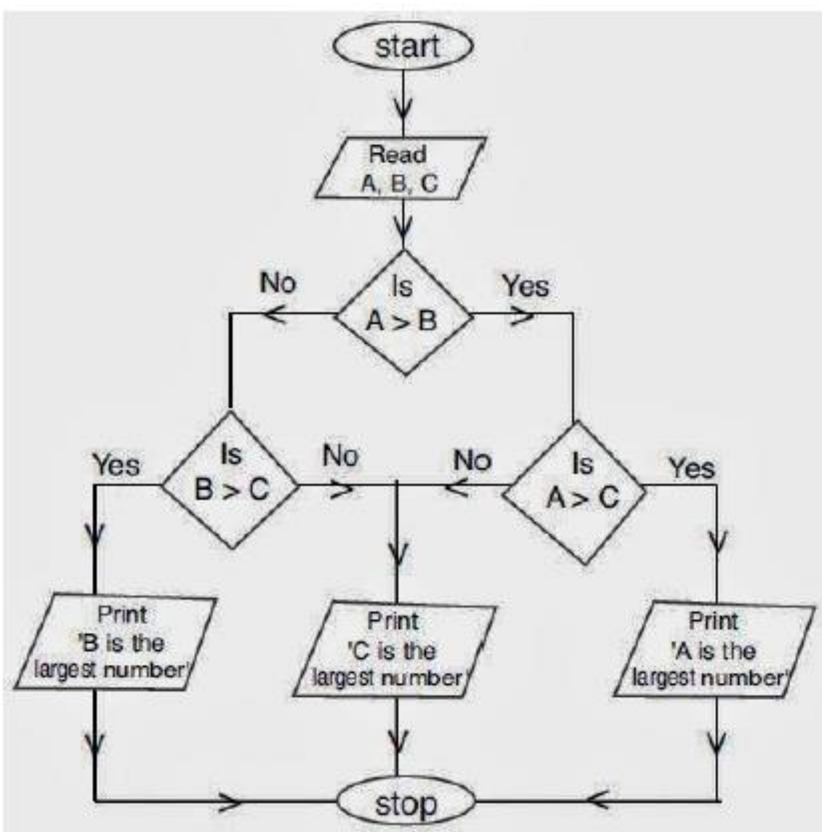
You bought a new train toy for your brother. When you unpacked the train it did not start. ***Construct*** *a flow chart (algorithm) of your analysis of the problem suggesting solutions.*

10.

The formula for the area of the circle is $A = \pi r^2$. *The circumference is* $C = 2\pi r$.
Derive *a formula for the circumference in terms of A.*

Trace *the following algorithm when A, B and C have values of -10, 2 and 7 Respectively.*

11.



12.
Derive *the following equation in terms of X. (make X the subject of the equation).*

$$y = \frac{x+1}{3x-2}$$

13.
Rearrange the following equation so as A becomes the subject of the equation.

$$r = \sqrt{\frac{A}{\pi}}$$

14.
Find *the largest possible rectangular area you can enclose, assuming you have 128 meters of fencing.* ***Comment*** *on the (geometric) significance of the dimensions of this largest possible enclosure?*

15.
You have to make a square-bottomed, unlidded box with a height of three centimeters and a volume of approximately 42 cubic cm. You will be taking a piece of cardboard, cutting three-cm squares from each corner, scoring between the corners, and folding up the edges.

a) ***Write down*** *an expression for the volume of the box in terms of the length of the cardboard.*
b) ***Calculate*** *the dimensions of the cardboard, to the nearest cm?*
c) *Your factory produces a number of boxes that ranges from 2000-2500 boxes every week.* ***Identify*** *two factors why you production should not drop under 2000 boxes a week and two factors why it should not exceed 2500 boxes.*

16.

Your factory produces lemon-scented widgets. You know that each unit is cheaper, the more you produce. But you also know that costs will eventually go up if you make too many widgets, due to the costs of storage of the overstock. The guy in accounting says that your cost for producing x thousands of units a day can be approximated by the formula $C = 0.04x^2 - 8.504x + 25302$.

a) ***Find*** *the daily production level that will minimize your costs.*
b) ***Demonstrate*** *that your will have a cost of JD 25302 at two different production.*
c) *In the light of your findings in part b,* ***comment*** *why do factories stick to a production range.*
d) ***State*** *whether this equation has x-intercepts. If not, then* ***explain*** *what it would mean for the factory if the equation had x-intercepts. Would that make sense? Why, why not?*

Solve *the following system of equations:*

$$\begin{aligned} 3x - 2y &= -7 \\ 2x + 3y &= 17 \end{aligned}$$

17.

Solve *the following system of equations:*

$$\begin{aligned} x^2 + y^2 &= 1 \\ y &= x^2 \end{aligned}$$

18.

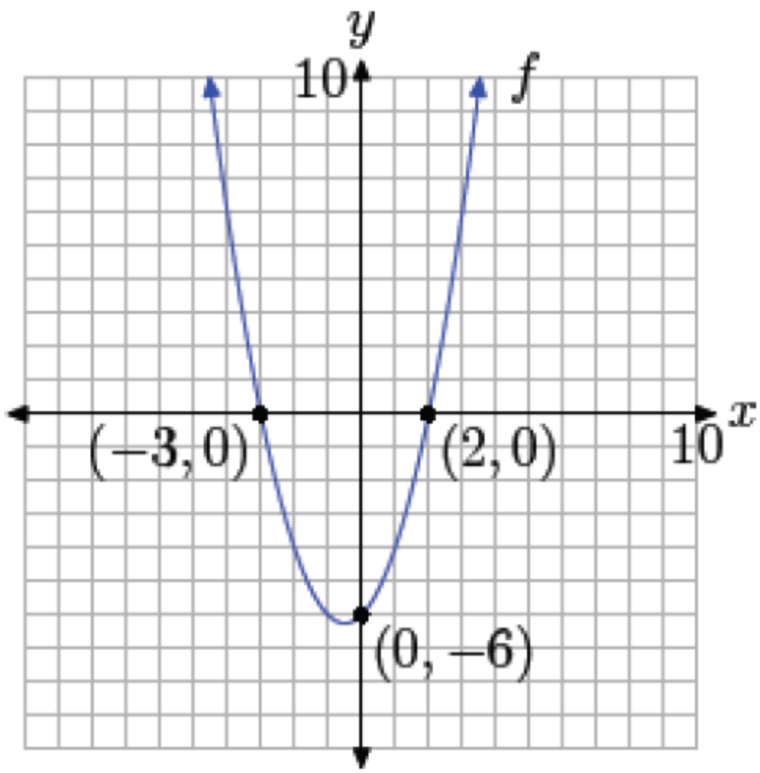
19.

Demonstrate *that the following system of equations have an infinite number of solutions.*

$$\begin{aligned} y &= 3x + 2 \\ 6x - 2y + 4 &= 0 \end{aligned}$$

20.

The graph of f(x) is shown below.



a) ***Describe*** *the behavior of the function f(x) in the interval [-3,2]*
b) ***Write down*** *the equation of f(x).*
c) ***Draw*** *the function g(x) = 2x – 4.*
d) ***Plot*** *the points where f(x) = 2x – 4.*

21.

The function f(x) is defined such that $(x) = 3x^3 - 5$.

a) ***Sketch*** *the function f(x).*
b) ***Label*** *the x-intercept(s)*
c) ***Describe*** *the behavior of the function f(x) in the interval [-2,2]*
d) ***Describe*** *the transformation required to transform f(x) into the function* $(x) = x^3$

22.

The functions f(x) and g(x) are defined such that

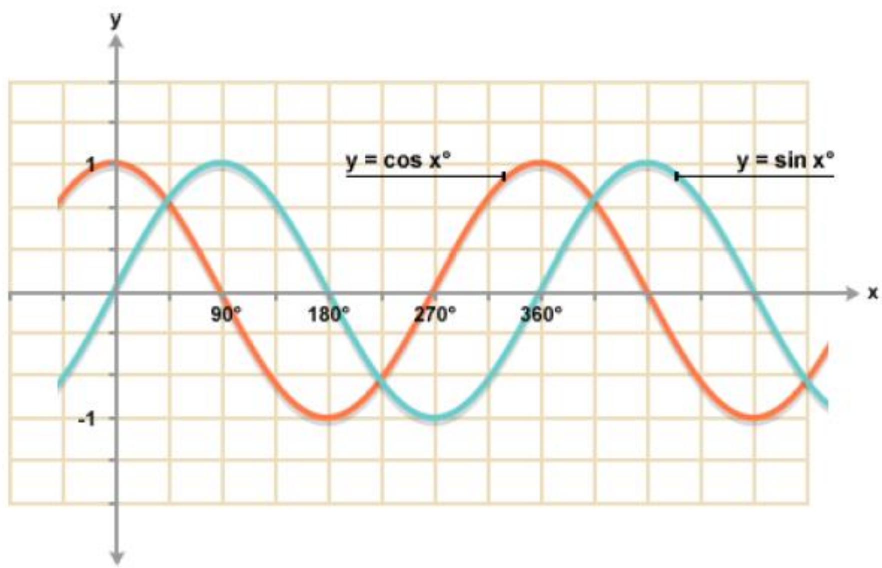
$$(x) = -4x^3,$$

$$(x) = 4x^2 - x - 1$$

a) ***Solve*** *algebraically for all values of x where f(x) = g(x).*
b) ***Calculate*** *the distance between the vertex of g(x) and the y-intercept of f(x). The distance between two points in the coordinate plane is given by the following formula:* $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

23.

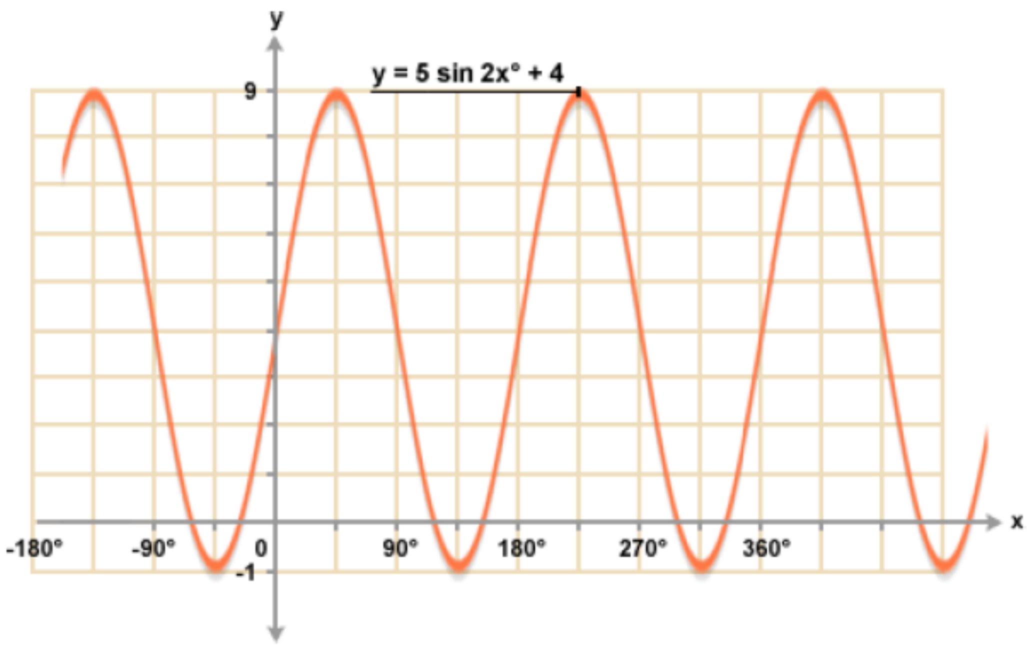
Referring to the graph below:



a) ***Write down*** *the range of f(x) = sin(x).*
b) ***Write down*** *the amplitude of g(x) = cos(x).*
c) *f(x) was transformed into h(x) where h(x) = g(x).* ***Use*** *your knowledge in trigonometric functions to write an expression for h(x). (hint: write h(x) as a transformation of f(x))*

24.

The graph below shows the function $(x) = 5f(x) + 4$
Where $(x) = \sin 2x$



a) ***Write down*** *the range of g(x).*
b) ***Describe*** *the transformation from f(x) to 5f(x) + 4.*
c) ***Determine*** *the range of 5f(x) + 1.*
d) ***Determine*** *the period of g(x).*
e) ***Write down*** *the amplitude of g(x).*

25.

The point A(12,48) lies on the curve of f(x). ***Calculate*** *the new coordinates of the point when mapped on g(x) = 3f(2x) – 10.*

26.

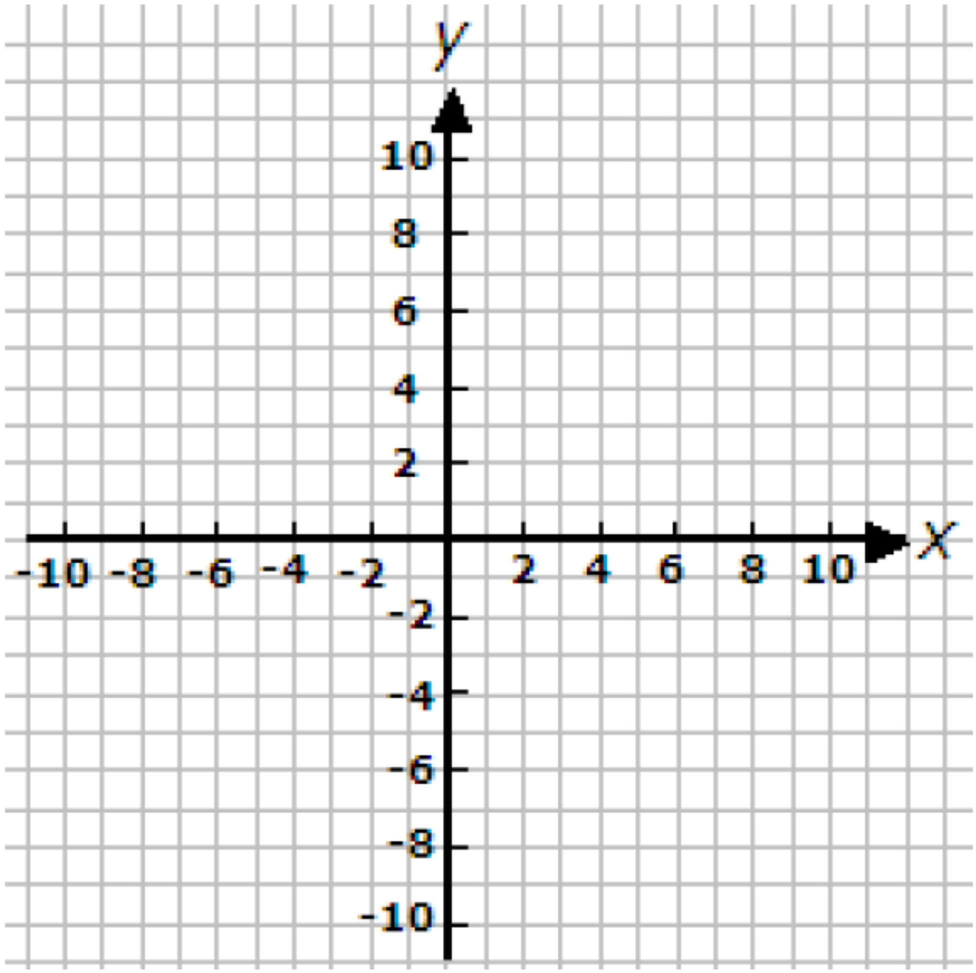
The two functions f(x) and g(x) are defined such that $(x) = -x^2 - 3$ *and* $g(x$

2.5. *On the grid below:*

a) ***Sketch*** *f(x).*

b) ***Draw*** *g(x).*

c) ***Label*** *the point(s) where g(x) = f(x) + 9*



Solve *the following equation:*

$$\frac{1}{4(x-1)} = \frac{1}{x+3} - \frac{2}{}$$

27.

28.

The income tax deduction I(s) is a function of the yearly salary (s).

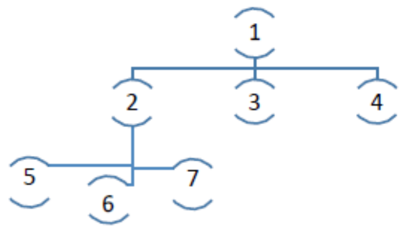
$$(s) = 0.07\,(s - 12000)$$

This function is defined on the interval $s \in [0,\ 24000)$.

***Write down** the Range of the function.*

29.

In the movie "pay it forward" the student suggested that each person does a favor to three different people who in turn do a favor to three different people each.
The diagram below shows part of the chain. The chain starts at level 1.
Watch the video link: https://www.youtube.com/watch?v=KxB43PxasGA



a) ***Write down** how many people will be involved in level 3.*
b) ***Determine** the number of people involved in level 4.*
c) ***Describe** the pattern of change in the number of the people involved in the chain.*
d) ***Suggest** a mathematical model that predicts the number of people involved at a certain level.*
e) *The sum of all people involved in the chain is given by the formula:*

$$S = \frac{3^n - 1}{2}$$

Where n is the level number in the chain,
***Find** at which level the number of people involved rises above 700.*

f) *Referring to your MYP studies, **explain** how mathematical sequences could be used as expression of personal and cultural existence and belief.*

30.

The following triangle is made up of odd numbers:

Row 1					1					
Row 2				3		5				
Row 3			7		9		11			
Row 4		13		15		17		19		
Row 5	21		23		25		27		29	
Row 6										

a) *Write down the numbers in row 6*

b) *Now you will investigate the difference between the first and the last numbers in each row. For example; the difference in row 1 is 0, since row1 starts and ends with the same number 1. The difference in row 2 is 2 since it starts with 3 and ends with a 5. 5 -3 = 2. The difference in row 3 is 11- 7 = 4. Calculate the difference for rows 4, 5, and 6.*

c) *Complete the following table:*

Row number (n)	*The difference between the first and last numbers in the row (D)*
1	0
2	2
3	4
4	
5	
6	

d) *Describe in your words, any pattern you notice.*

e) *Write down a general formula for the difference (D) in terms of the row number (n).*

f) *Row k starts with the number 91 and ends with the number 109. Find k.*

Now investigate the sum of ALL the numbers in any row, you might need to organize your work by using a table.

Write a report of your investigation to predict the sum of terms (S) in terms of the row number (n) and use it to predict the sum of the numbers in row 7. Verify your general rule.

31.

***Prove** that a sphere fits a cuboid better than a cuboid fits a sphere.*