

Topic	Skills
Standard and extended mathematics	
<p>Forms of numbers: integers, fractions, decimals, exponents, absolute value, standard form (scientific notation), recurring decimals and surds/radicals</p> <p>Number systems: set of positive integers and zero (N), integers (Z), rational numbers (Q), irrational numbers (Q'), and real numbers (R)</p>	<p>Ordering numbers</p> <p>Absolute value of a number</p> <p>Transformation between different forms of numbers</p> <p>Simplification of numerical expressions in the number systems and forms of number</p> <p>Recognizing and classifying numbers in different number systems, including recurring decimals</p>
<p>Sets</p> <p>Venn diagrams</p>	<p>Basic vocabulary (element, subset, null set, and so on)</p> <p>Performing operations</p> <p>Properties of sets (commutative, associative, distributive)</p> <p>Drawing and interpreting Venn diagrams</p> <p>Using Venn diagrams to solve problems in real-life contexts</p>
<p>The four number operations</p>	<p>Using the four number operations (addition, subtraction, multiplication and division) with integers, decimals and fractions</p>
<p>Prime numbers and factors, including greatest common divisor and least common multiple</p>	<p>Representing a number as the product of its prime factors and using this representation to find the greatest common divisor and least common multiple</p>
<p>Number lines</p>	<p>Expressing the solution set of a linear inequality on the number line (as well as set notation)</p>
<p>Estimation</p>	<p>Using different forms of rounding; decimal approximation and significant figures</p> <p>Using appropriate forms of rounding to estimate results</p>
<p>Units of measurement</p>	<p>Converting between different units of measurement and between different currencies</p>
<p>Ratio, percentage; direct and inverse proportion</p>	<p>Dividing a quantity in a given ratio</p> <p>Finding a constant of proportionality, setting up equations and graphing direct and inverse relationships</p>
<p>Number sequences</p>	<p>Predicting the next term in a number sequence (linear, quadratic, triangular, Fibonacci)</p>
<p>Integer exponents</p>	<p>Evaluating numbers with integer exponents</p>

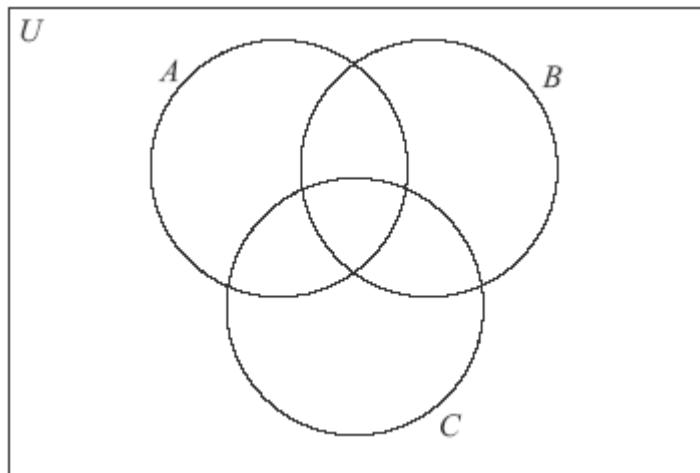
Topic	Skills
Extended mathematics	
Fractional exponents	Using the rules of indices to simplify numerical expressions involving radicals and exponents
Logarithms	Evaluating the logarithm of a number and simplifying numerical expressions
Number bases	Performing operations with numbers in different bases

NUMBER SYSTEMS – PRACTICE QUESTIONS

1. U is the set of all the **positive** integers less than or equal to 12.
 A , B and C are subsets of U .

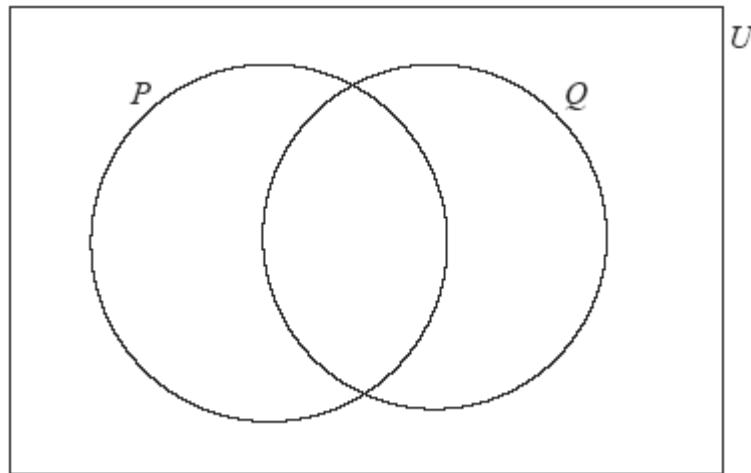
$$\begin{aligned} A &= \{1, 2, 3, 4, 6, 12\} \\ B &= \{\text{odd integers}\} \\ C &= \{5, 6, 8\} \end{aligned}$$

- (a) Write down the number of elements in $A \cap C$.
- (b) List the elements of B .
- (c) Complete the following Venn diagram with **all** the elements of U .



2. The sets P , Q and U are defined as

$$U = \{\text{Real Numbers}\}, P = \{\text{Positive Numbers}\} \text{ and } Q = \{\text{Rational Numbers}\}.$$

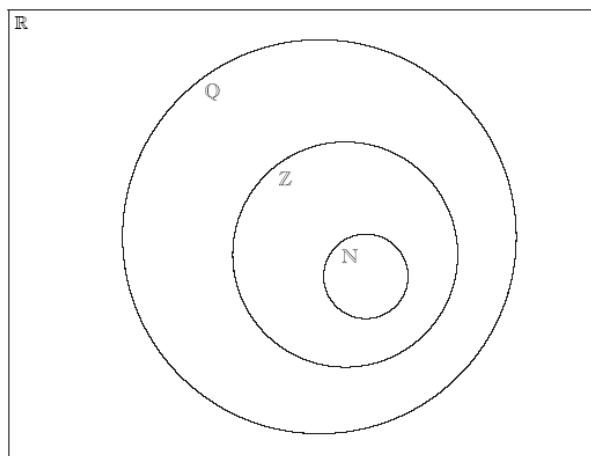


Write down in the correct region on the Venn diagram the numbers

$$\frac{22}{7}, \quad 5 \times 10^{-2}, \quad \sin(60^\circ), \quad 0, \quad \sqrt[3]{-8}, \quad -\pi$$

3. The Venn diagram shows the number sets \mathbb{N} , \mathbb{Z} , \mathbb{Q} and \mathbb{R} . Place each of the following numbers in the appropriate region of the Venn diagram.

$$\frac{1}{4}, -3, \pi, \cos 120^\circ, 2.7 \times 10^3, 3.4 \times 10^{-2}$$



4. Consider the numbers $\sqrt{3}$, 6, $2\frac{1}{2}$, \square , -5, and the sets \mathbb{N} , \mathbb{Z} , and \mathbb{Q} . Complete the following table by placing a tick in the appropriate box if the number is an element of the set.

	$\sqrt{3}$	6	$2\frac{1}{2}$	\square	-5
\mathbb{N}					
\mathbb{Z}					
\mathbb{Q}					

5. Given that $h = \sqrt{l^2 - \frac{d^2}{4}}$,
- (a) Calculate the **exact** value of h when $l = 0.03625$ and $d = 0.05$.
 - (b) Write down the answer to part (a) correct to three decimal places.
 - (c) Write down the answer to part (a) correct to three significant figures.
 - (d) Write down the answer to part (a) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.

6. (a) Write down the following numbers in increasing order.

$$3.5, 1.6 \times 10^{-19}, 60730, 6.073 \times 10^5, 0.006073 \times 10^6, \square, 9.8 \times 10^{-18}.$$

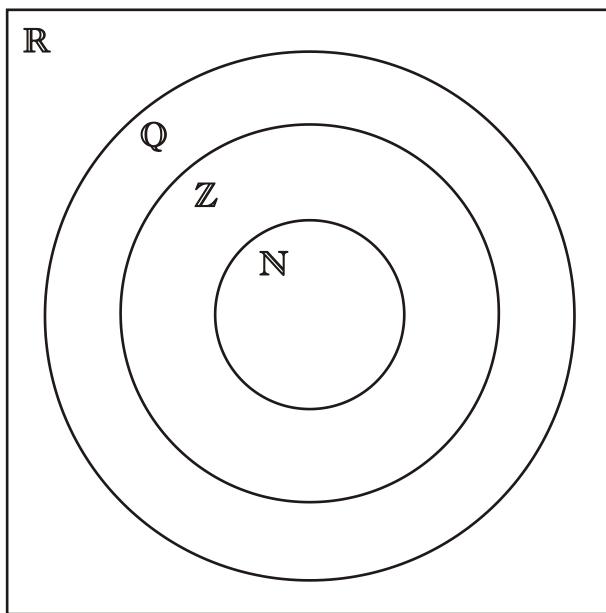
- (b) Write down the median of the numbers in part (a).
- (c) State which of the numbers in part (a) is irrational.

7. The Venn diagram below shows the universal set of real numbers \mathbb{R} and some of its important subsets:

- \mathbb{Q} : the rational numbers,
- \mathbb{Z} : the integers,
- \mathbb{N} : the natural numbers.

Write the following numbers in the correct position in the diagram.

$$-1, 1, \frac{7}{16}, 3.33\dot{3}, \sqrt{3}.$$



ESTIMATION

To help find errors in a calculation, it is useful to be able to accurately estimate the answer. The estimate will tell us if the computed answer is **reasonable**.

When estimating we usually **round** each number to one significant figure and evaluate the result. We call this a **one figure approximation**.

Example 4

Self Tutor

Find the approximate value of 7235×591 .

We round each number to one significant figure.

$$\begin{aligned}7235 \times 591 &\approx 7000 \times 600 \\&\approx 4200000\end{aligned}$$

The estimate tells us the correct answer should have 7 places in it.

We expect the answer to be about 4 million.

PERCENTAGE CHANGE

If we compare the change to the original amount and express this as a percentage then we have calculated the **percentage change**.

$$\text{percentage change} = \frac{\text{change}}{\text{original}} \times 100\%$$

PERCENTAGE PROFIT OR LOSS

- percentage profit = $\frac{\text{profit}}{\text{cost price}} \times 100\%$
- percentage loss = $\frac{\text{loss}}{\text{cost price}} \times 100\%$

DISCOUNT

In order to attract customers or to clear old stock, many businesses reduce the price of items from the **marked price** shown on the price tag.

The amount of money by which the cost of the item is reduced is called **discount**. Discount is often stated as a percentage of the marked price or original selling price. It is thus a **percentage decrease**.



The price at which the item is sold is called the **selling price**.

$$\text{selling price} = \text{marked price} - \text{discount}$$

PRACTICE QUESTIONS

1. 80 matches were played in a football tournament. The following table shows the number of goals scored in all matches.

Number of goals	0	1	2	3	4	5
Number of matches	16	22	19	17	1	5

- (a) Find the mean number of goals scored per match.

- (b) Find the median number of goals scored per match.

A local newspaper claims that the mean number of goals scored per match is two.

- (c) Calculate the percentage error in the local newspaper's claim.

2. A manufacturer in England makes 16 000 garden statues. 12 % are defective and cannot be sold.

- (a) Find the number of statues that are non-defective.

The manufacturer sells each non-defective statue for 5.25 British pounds (GBP) to an American company. The exchange rate from GBP to US dollars (USD) is 1 GBP = 1.6407 USD.

- (b) Calculate the amount in USD paid by the American company for all the non-defective statues. Give your answer correct to **two decimal places**.

The American company sells one of the statues to an Australian customer for 12 USD.

The exchange rate from Australian dollars (AUD) to USD is 1 AUD = 0.8739 USD.

- (c) Calculate the amount that the Australian customer pays, in AUD, for this statue.

Give your answer correct to **two decimal places**.

3. A shipping container is a cuboid with dimensions 16 m, $1\frac{3}{4}$ m and $2\frac{2}{3}$ m.

- (a) Calculate the **exact** volume of the container. Give your answer as a fraction.

Jim estimates the dimensions of the container as 15 m, 2 m and 3 m and uses these to estimate the volume of the container.

- (b) Calculate the percentage error in Jim's estimated volume of the container.

RATIO & PROPORTION – PRACTICE QUESTIONS

My fortune of \$810 000 is to be divided in the ratio $4 : 3 : 2$. How much does each person receive?

An alloy is made from copper, zinc and tin in the ratio $17 : 2 : 1$. How much zinc is required to make 10 tonnes of the alloy?

When Michael makes pici pasta, he mixes semolina, “00” flour, and water in the ratio $6 : 3 : 2$. If he uses 150 g of “00” flour, what mass does he require of:

- a semolina b water?

Joe and Bob share the cost of a video game in the ratio $3 : 7$.

- a What fraction does each pay?
- b If the game costs £35, how much does each pay?
- c If Joe pays £12, how much does Bob pay?
- d If Bob pays £42, what is the price of the video game?

1 Find the missing numbers in the following proportions:

- | | | |
|----------------------------|----------------------------|----------------------------|
| a $3 : 4 = 6 : \square$ | b $3 : 6 = 12 : \square$ | c $2 : 5 = \square : 15$ |
| d $5 : 8 = \square : 40$ | e $1 : 3 = \square : 27$ | f $4 : 1 = 24 : \square$ |
| g $7 : 21 = \square : 33$ | h $15 : 25 = 30 : \square$ | i $\square : 18 = 32 : 48$ |
| j $5 : 100 = \square : 40$ | k $18 : 30 = 24 : \square$ | l $\square : 12 = 33 : 44$ |

2 A disaster relief team consists of engineers and doctors in the ratio of $2 : 5$.

- a If there are 18 engineers, find the number of doctors.
- b If there are 65 doctors, find the number of engineers.

3 The ratio of two angles in a triangle is $3 : 1$. Find the:

- a larger angle if the smaller is 18°
- b smaller angle if the larger is 63° .

4 The ratio of teachers to students in a school is $1 : 15$. If there are 675 students, how many teachers are there?

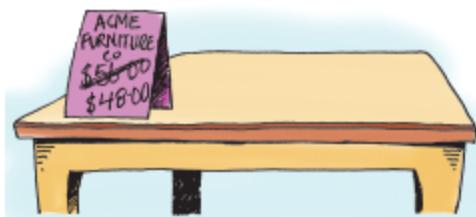
5 An MP3 player is bought for €240 and sold for €270. Find the ratio of the cost price to the selling price.

6 The maximum speeds of a boat and a car are in the ratio $2 : 7$. If the maximum speed of the boat is 30 km per hour, find the maximum speed of the car.

7 A farmer has sheep and cattle in the ratio $8 : 3$.

- a How many sheep has the farmer if he has 180 cattle?
- b Find the ratio of the number of sheep to the total number of animals.
- c Find the ratio of the total number of animals to the number of cattle.

- 8 The price of a table is reduced from \$56 to \$48. The set of chairs which go with the table was originally priced at \$140. If the price of the chairs is reduced in the same ratio as that of the table, find the new price of the chairs.



- 9 Sue invested money in stocks, shares, and property in the ratio $6 : 4 : 5$. If she invested €36 000 in property, how much did she invest in the other two areas?

SCALE DIAGRAMS

When designing a house it would be ridiculous for an architect to draw a full-size plan.

Instead the architect draws a smaller diagram in which all measurements have been divided by the same number or scale factor.

For house plans a scale factor of 100 would be suitable.



Similarly, a map of Brazil must preserve the shape of the country. All distances are therefore divided by the same scale factor. In this case the scale factor is 80 000 000.

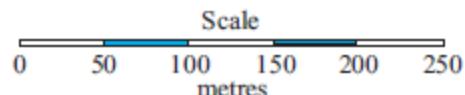
These diagrams are called **scale diagrams**.

In scale diagrams:

- All lengths have been changed by the same scale factor.
- All angles are unaltered.

To properly use a scale diagram we need to know the scale used.
Scales are commonly given in the following ways:

- *Scale:* 1 cm represents 50 m.
This tells us that 1 cm on the scale diagram represents 50 m on the real thing.
- A divided bar can be used to show the scale.
This scale tells us that 1 cm on the scale diagram represents 50 m on the real thing.
- *Scale:* 1 : 5000
This ratio tells us that 1 unit on the scale diagram represents 5000 of the same units on the real thing.
For example, 1 cm would represent 5000 cm or 50 m,
1 mm would represent 5000 mm or 5 m.



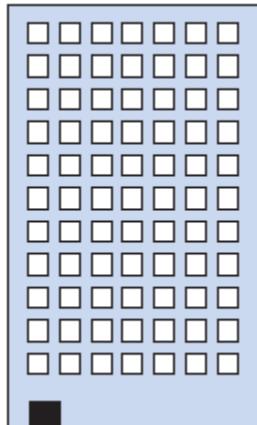
Scales are written in ratio form as drawn length : actual length.

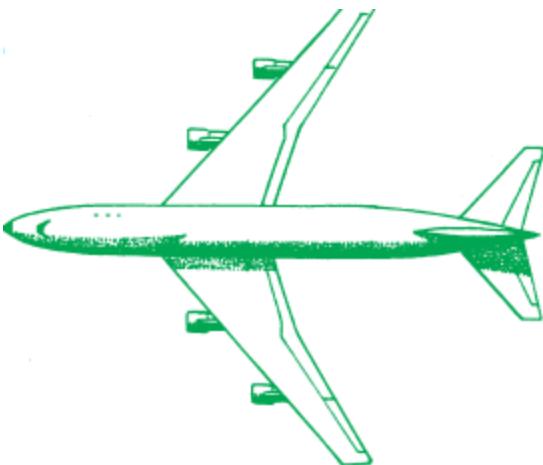
We usually simplify the scale to an equal ratio of the form 1 : the scale factor.

PRACTICE QUESTIONS

A scale diagram of a building is shown with scale 1 : 1000.

- a If the height is 5 cm and width is 3 cm on the drawing, find the actual height and width of the building in metres.
- b If the height of the windows on the drawing are 2.5 mm, how high are the actual windows?
- c If the actual height of the entrance door is 3.2 m, what is its height on the scale drawing?





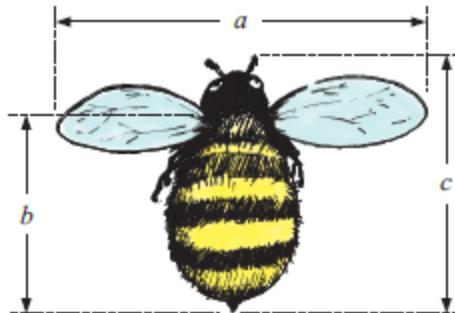
The actual length of the aeroplane shown in the scale drawing is 64 m.

Find:

- the scale used in the drawing
- the actual wingspan of the aeroplane
- the actual width of the fuselage.

The diagram given is of an *enlarged* bee, drawn to a scale of $1 : 0.25$. Find the actual length of the dimensions marked:

- wing span, a
- body length, b
- total length, c .



SCIENTIFIC NOTATION

Scientific notation or standard form involves writing any given number as a *number between 1 inclusive and 10*, multiplied by a *power of 10*, i.e., $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

Example 16

Self Tutor

Write in scientific notation:

a 37 600 **b** 0.000 86

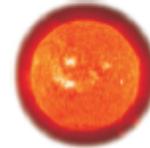
a $3\text{7}\overset{3}{6}\overset{0}{0} = 3.76 \times 10\,000$ {shift decimal point 4 places to the left and $\times 10\,000$ }
 $= 3.76 \times 10^4$

b $0.\overset{3}{0}\overset{0}{0}\overset{3}{8}\overset{6}{6} = 8.6 \div 10^4$ {shift decimal point 4 places to the right and $\div 10\,000$ }
 $= 8.6 \times 10^{-4}$

- Notice that:
- If the original number is > 10 , the power of 10 is **positive (+)**.
 - If the original number is < 1 , the power of 10 is **negative (-)**.
 - If the original number is between 1 and 10, we write the number as it is and multiply it by 10^0 , which is really just 1.

Express the following in scientific notation:

- a The distance from the earth to the sun is 149 500 000 000 m.
- b Bacteria are single cell organisms, some of which have a diameter of 0.0003 mm.
- c A speck of dust is smaller than 0.001 mm.
- d The probability that your six numbers will be selected for Lotto on Monday night is 0.000 000 141 62.
- e The central temperature of the sun is 15 million degrees Celsius.
- f A single red blood cell lives for about four months and during this time it will circulate around the body 300 000 times.



INDEX LAWS

If the bases a and b are both positive and the indices m and n are integers then:

$$a^m \times a^n = a^{m+n}$$
 To multiply numbers with the same base, keep the base and add the indices.

$$\frac{a^m}{a^n} = a^{m-n}$$
 To divide numbers with the same base, keep the base and subtract the indices.

$$(a^m)^n = a^{mn}$$
 When raising a power to a power, keep the base and multiply the indices.

$$(ab)^n = a^n b^n$$
 The power of a product is the product of the powers.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$
 The power of a quotient is the quotient of the powers.

$$a^0 = 1, \quad a \neq 0$$
 Any non-zero number raised to the power of zero is 1.

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n \quad \text{and in particular} \quad a^{-1} = \frac{1}{a}.$$

C

EXPONENTIAL EQUATIONS

An exponential equation is an equation in which the unknown occurs as part of the index or exponent.

For example: $3^x = 9$ and $3^2 \times 4^x = 8$ are both exponential equations.

Notice that if $3^x = 9$ then $3^x = 3^2$. Thus $x = 2$ is a solution to the exponential equation $3^x = 9$, and it is in fact the only solution to the equation.

In general:

If $a^x = a^k$ then $x = k$.

If the base numbers are the same, we can equate indices.

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$ where $\sqrt[n]{a}$ is called the ‘ n th root of a ’.

PRACTICE QUESTIONS

Evaluate.

1. $25^{1/2}$

2. $1000^{2/3}$

3. $121^{3/2}$

4. $\left(\frac{4}{9}\right)^{-1/2}$

5. $16^{-5/2}$

6. $(-216)^{-1/3}$

7. $\left(\frac{1}{125}\right)^{-1/3}$

8. $49^{-1/2}$

9. $32^{3/5}$

10. $0.04^{1/2}$

Simplify.

$$11. \sqrt[3]{a^7}$$

$$12. \sqrt[5]{z^{11}}$$

$$13. \sqrt[4]{16c^5d^8}$$

$$14. \sqrt[3]{27a^3b^5}$$

$$15. \sqrt[5]{243x^{11}y^8z^{20}}$$

$$16. \sqrt[3]{8x^5y^6}$$

$$17. \sqrt[6]{a^4b^8c^{12}}$$

$$18. \sqrt[3]{40d^5e^6f^4}$$

$$19. \sqrt[3]{m^{13}n^8p^9}$$

$$20. \sqrt[5]{64r^{13}s^{15}t^{12}}$$

Use the rules of exponents to simplify the following expressions.

$$11. a^{\frac{1}{3}} \cdot a^{\frac{1}{4}}$$

$$12. \frac{x^{\frac{2}{1}}}{x^{\frac{1}{4}}}$$

$$13. \left(x^{\frac{2}{5}} \right)^{\frac{3}{4}}$$

$$14. (3x^2y)^{\frac{1}{3}}$$

$$15. 6a^{\frac{2}{5}} \cdot 2a^{\frac{3}{4}}$$

$$16. \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}$$

$$17. \left(16x^{\frac{1}{6}} \right)^{\frac{2}{5}}$$

$$18. (4x^{-2}y^{-8})^{\frac{2}{9}}$$

PROPORTION - PRACTICE QUESTIONS

1. If the cost of one chair is Rs.3954, find the cost of 75 chairs.
2. If the cost of 12 apples is Rs.195, find the cost of one apple.
3. Find the price of one book, when 123 books were purchased for Rs.59778.

4. The populations of two cities A and C are 36000 and 64000 respectively. If the ratio of the population of B to A is the same as the ratio of the population of C to B, then find the population of B.
5. Twelve bottles of claret cost £ 33.96. How many bottles can be purchased for £ 76.41?
6. A vehicle can travel 84 km.on 7 litres of petrol.
- How far could the vehicle travel on 43 litres of petrol?
 - How many litres of petrol would be required to travel a distance of 534 km?
7. If it costs \$234 to lay a drain pipe of length 36m, determine :
- the cost of laying a pipe of length 72m
 - the length of pipe that could be laid for a cost of \$754.
8. Ms.X can embroider a saree in 5 days. What portion of the saree will she be able to embroider in (i) one day (ii) 3 days?
9. Four people can clean an office in 6 hours. How many people would be needed to clean the office in 4 hours?
10. A and B can separately do a piece of work in 20 and 30 days respectively. They work together for some time and then B stops. If A completes the rest of the work in 10 days, then find the number of days B has worked for.
11. A sum of \$1300 is divided among P,Q,R and S such that $P:Q = Q:R = R:S$. What is P's share?

MIXED PERCENTAGES, RATIO AND PROPORTION - PRACTICE QUESTIONS

1. There are 565 sheets of paper in a book.
- How many sheets of paper are there in 2000 of these books?
Give your answer in standard form.
 - A pile of 565 sheets of paper is 25 millimetres high.
Calculate the thickness of 1 sheet of paper.
Give your answer in standard form.
-

2) In 2004 Colin had a salary of \$7200.

(a) This was an increase of 20% on his salary in 2002.
Calculate his salary in 2002.

(b) In 2006 his salary increased to \$8100.
Calculate the percentage increase from 2004 to 2006.

3) Marguerite earns \$336 per month.

She divides her earnings between bills, food, savings and personal spending.

(a) Her bills take $\frac{2}{7}$ of her earnings.

Show that \$240 is left for her other items.

(b) She divides the \$240 between food, savings and personal spending in the ratio 5 : 3 : 4.
Calculate how much she spends on food.

(c) She saves the same amount each month.

Show that she saves \$720 in one year.

(d) Marguerite invests \$720 in a bank which pays 6% per year **compound** interest. How
much will this be worth after 2 years?

4) Marcus uses 80% of \$150 to buy some clothes.

(i) Calculate the cost of the clothes.

(ii) The money remaining from the \$150 is $37\frac{1}{2}\%$ of the cost of a day trip to Cairo.

Calculate the cost of the trip.

5) (a) One day Amit works from 08 00 until 17 00

The time he spends on filing, computing, writing and having lunch is in the ratio

Filing: Computing : Writing : Lunch = 2:5:4:1

Calculate the time he spends

- (i) Writing,
- (ii) having lunch, giving this answer in minutes,
- (b) The amount earned by Amit, Bernard and Chris is in the ratio 2:5:3
 Bernard earns \$855 per week.
 Calculate how much
- (i) Amit earns each week
- (ii) Chris earns each week
- (c) After 52 weeks Bernard has saved \$2964.

What fraction of his earnings has he saved?

Give your answers in its lowest terms

- (d) Chris earns each week \$3500 this year. This is 40% more than he saved last year. Calculate how much he saved last year.

DIRECT AND INVERSE VARIATION - PRACTICE QUESTIONS

1. Find the Missing Variable:

- a) y varies directly with x. If $y = -4$ when $x = 2$, find y when $x = -6$.
- b) y varies inversely with x. If $y = 40$ when $x = 16$, find x when $y = -5$.
- c) y varies directly with x. If $y = 15$ when $x = -18$, find y when $x = 1.6$.
- d) y varies directly with x. If $y = 75$ when $x = 25$, find x when $y = 25$.

2. Classify the following as Direct / Inverse giving reason:

a) $m = -5p$	b) $c = \frac{e}{-4}$	c) $r = \frac{9}{t}$
d) $n = \frac{1}{2}f$	e) $d = 4t$	f) $z = \frac{-2}{t}$

3. Answer the following questions:

- a) The volume V of gas varies inversely to the pressure P . The volume of a gas is 200 cm^3 under pressure of 32 kg/cm^2 . What will be its volume under pressure of 40 kg/cm^2 ?
- b) The number of kilograms of water in a person's body varies directly as the person's mass. A person with a mass of 90 kg contains 60 kg of water. How many kilograms of water are in a person with a mass of 50 kg ?
- c) On a map, distance in km and distance in cm varies directly, and 25 km are represented by 2cm . If two cities are 7cm apart on the map, what is the actual distance between them?
- d) The time it takes to fly from Los Angeles to New York varies inversely as the speed of the plane. If the trip takes 6 hours at 900 km/h , how long would it take at 800 km/h ?

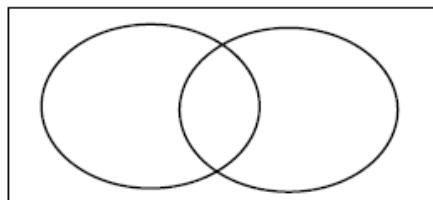
SET THEORY - PRACTICE QUESTIONS

1)

There are 50 people on a tour.

One day, 26 people went on the morning cruise and 29 to the evening barbecue.
Using Venn diagrams, or otherwise, answer the following questions.

- (a) It was thought that 4 people went to both events and 1 person to neither.
Explain why this was not possible.



- (b) Find the least number and the greatest number of people who could have gone to both events.

2)

- (a) In a group of language students,
24 studied Spanish, 23 studied French and 15 studied German,
12 studied Spanish and French,
10 studied German and French,
6 studied Spanish and German,
4 studied all three languages.
By drawing a Venn diagram, or otherwise, calculate the number of students who studied
- (i) both Spanish and French, but not German,
(ii) only one language.
- (b) The set A consists of the points whose coordinates (x, y) are given by
$$A = \{(x, y) : y = 2x + 1\}.$$
The points in set B are given by
$$B = \{(0, 0), (0, 1), (1, 2), (2, 5), (3, 6)\}.$$

Find

- (i) $n(B)$,
(ii) $A \cap B$.

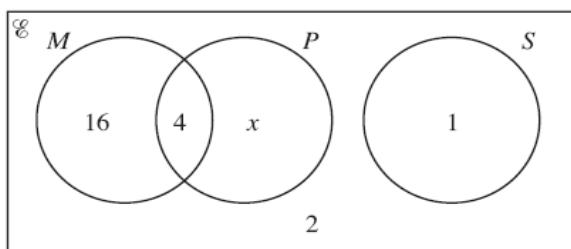
- (a) The results of a survey of 31 students are shown in the Venn diagram.

$$\mathcal{E} = \{\text{students questioned in the survey}\}$$

$$M = \{\text{students who study Mathematics}\}$$

$$P = \{\text{students who study Physics}\}$$

$$S = \{\text{students who study Spanish}\}$$



- (i) Write down the value of

- (a) x ,
(b) $n(M \cap P)$,
(c) $n(M \cup S)$,
(d) $n(P')$.

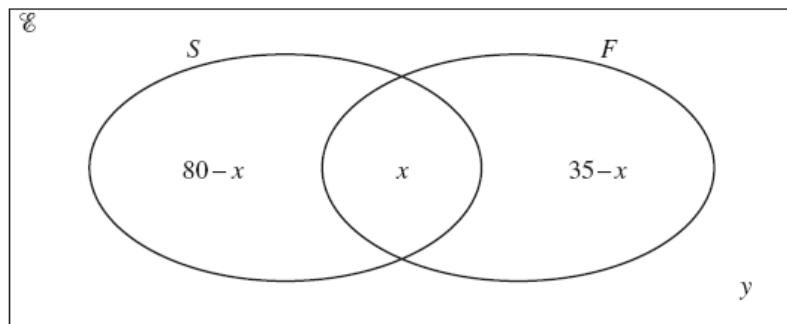
3. (ii) Write down a description, in words, of the set that has 16 members.

In a group of 100 students, 80 study Spanish and 35 study French.

x students study Spanish and French.

y students study neither Spanish nor French.

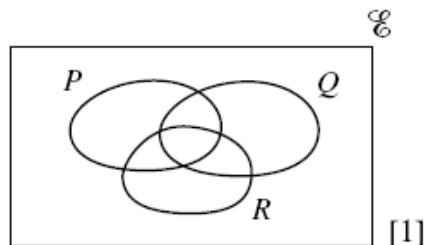
The Venn diagram illustrates this information.



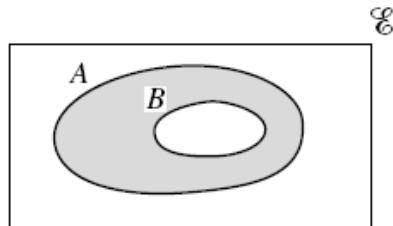
- (a) Expressed in set notation, the value of x is $n(S \cap F)$.
Express the value of y in set notation.
- (b) Find, in its simplest form, an expression for y in terms of x .
- (c) Find
- the least possible value of x ,
 - the greatest possible value of y .
- 4) (a) In the Venn diagram in the answer space, shade the region $(A \cup B) \cap C$.
- Answer (a)*
-
- A Venn diagram with three overlapping circles labeled A, B, and C. The region where circles A and B overlap but are within circle C is shaded.
- (b) All 30 students in a class study **at least one** of the two subjects History and Geography.
Twice as many study History as Geography.
8 students study **only** Geography.
By drawing a Venn diagram, or otherwise, find the number of students who study both History and Geography.
- 5)

- (a) On the Venn Diagram shown in the answer space, shade the set $(P \cap Q) \cup R$.

Answer (a)



- (b) Express in set notation, as simply as possible, the set shaded in the Venn Diagram.



Answer (b) [1]

- (c) There are 34 children in a class.

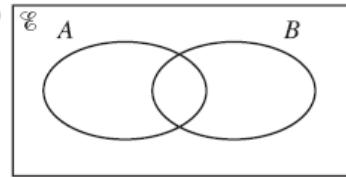
Of these, 22 take History, 19 take Geography and 5 take neither History nor Geography.

Using a Venn Diagram, or otherwise, find the number of children who take History but not Geography

6)

- (a) On the Venn Diagram in the answer space, shade the region $A \cup B'$.

Answer (a)



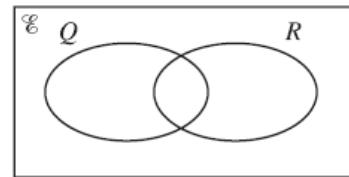
- (b) $E = \{\text{all polygons}\}$, $Q = \{\text{all quadrilaterals}\}$, $R = \{\text{all regular polygons}\}$.

(i) What is the special name of the polygons which belong to $Q \cap R$?

(ii) On the Venn Diagram in the answer space, show the set
 $T = \{\text{all equilateral triangles}\}$.

Answer (b)(i)

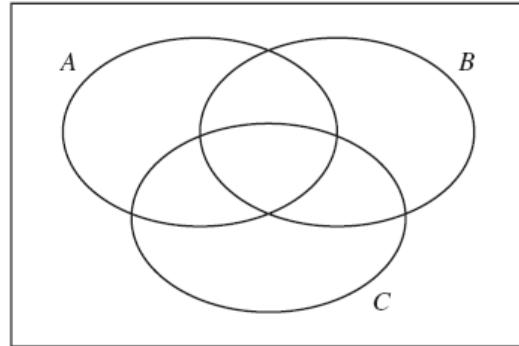
(ii)



7)

- (a) On the Venn Diagram in the answer space, shade the set $(A' \cup B') \cap C$.

Answer (a)



- (b) $E = \{x : x \text{ is an integer and } 4 \leq x \leq 16\}$

$$P = \{x : x \text{ is a prime number}\}$$

$$S = \{x : x \text{ is an odd number}\}$$

$$T = \{x : x \text{ is a multiple of 3}\}$$

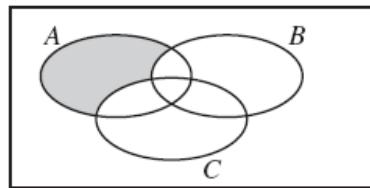
(i) List the members of the set $S \cap T$.

(ii) Describe, in words, the set S' .

(iii) Find $n(P \cup T)$.

8)

- (a) Express in set notation, as simply as possible, the subset shaded in the Venn diagram.

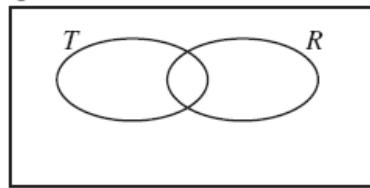


Answer (a)

- (b) $\mathcal{E} = \{\text{all polygons}\}$,
 $T = \{\text{all triangles}\}$,
 $R = \{\text{all regular polygons}\}$,
 $Q = \{\text{all quadrilaterals}\}$.

Add the set Q to the Venn diagram in the answer space.

Answer (b) \mathcal{E}



- (c) $\mathcal{E} = \{x : x \text{ is an integer and } 3 \leq x \leq 11\}$,
 $F = \{x : x \text{ is a factor of } 12\}$,
 $O = \{x : x \text{ is an odd number}\}$.

List the elements of the set $(F \cup O)'$.

Answer (c)

- (d) It is given that $n(\mathcal{E}) = 20$.
 P and S are sets such that $n(P) = 7$ and $n(S) = 16$.

Find the smallest possible value of $n(P \cap S)$.

9)

Answer (d)

LOGARITHMS – PRACTICE QUESTIONS

1 Write an equivalent logarithmic statement for:

a $2^2 = 4$

b $4^2 = 16$

c $3^2 = 9$

d $5^3 = 125$

e $10^4 = 10\,000$

f $7^{-1} = \frac{1}{7}$

g $3^{-3} = \frac{1}{27}$

h $27^{\frac{1}{3}} = 3$

i $5^{-2} = \frac{1}{25}$

j $2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}$

k $4\sqrt{2} = 2^{2.5}$

l $0.001 = 10^{-3}$

2 Write an equivalent exponential statement for:

a $\log_2 8 = 3$

b $\log_2 1 = 0$

c $\log_2 \left(\frac{1}{2}\right) = -1$

d $\log_2 \sqrt{2} = \frac{1}{2}$

e $\log_2 \left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$

f $\log_{\sqrt{2}} 2 = 2$

g $\log_{\sqrt{3}} 9 = 4$

h $\log_9 3 = \frac{1}{2}$

3 Without using a calculator, find the value of:

a $\log_{10} 100$

b $\log_2 8$

c $\log_3 3$

d $\log_4 1$

e $\log_5 125$

f $\log_5(0.2)$

g $\log_{10} 0.001$

h $\log_2 128$

i $\log_2 \left(\frac{1}{2}\right)$

j $\log_3 \left(\frac{1}{9}\right)$

k $\log_2(\sqrt{2})$

l $\log_2 (\sqrt{8})$

4 Rewrite as logarithmic equations:

a $y = 4^x$

b $y = 9^x$

c $y = a^x$

d $y = (\sqrt{3})^x$

e $y = 2^{x+1}$

f $y = 3^{2n}$

g $y = 2^{-x}$

h $y = 2 \times 3^a$

5 Rewrite as exponential equations:

a $y = \log_2 x$

b $y = \log_3 x$

c $y = \log_a x$

d $y = \log_b n$

e $y = \log_m b$

f $T = \log_5 \left(\frac{a}{2}\right)$

g $M = \frac{1}{2} \log_3 p$

h $G = 5 \log_b m$

i $P = \log_{\sqrt{b}} n$

6 Rewrite the following, making x the subject:

a $y = \log_7 x$

b $y = 3^x$

c $y = (0.5)^x$

d $z = 5^x$

e $t = \log_2 x$

f $y = 2^{3x}$

g $y = 5^{\frac{x}{2}}$

h $w = \log_3(2x)$

i $z = \frac{1}{2} \times 3^x$

j $y = \frac{1}{5} \times 4^x$

k $D = \frac{1}{10} \times 2^{-x}$

l $G = 3^{x+1}$

1 Write as a single logarithm:

a $\log_3 2 + \log_3 8$

b $\log_2 9 - \log_2 3$

c $3\log_5 2 + 2\log_5 3$

d $\log_3 8 + \log_3 7 - \log_3 4$

e $1 + \log_3 4$

f $2 + \log_3 5$

g $1 + \log_7 3$

h $1 + 2\log_4 3 - 3\log_4 5$

i $2\log_3 m + 7\log_3 n$

j $5\log_2 k - 3\log_2 n$

2 If $\log_2 7 = p$ and $\log_2 3 = q$, write in terms of p and q :

a $\log_2 21$

b $\log_2 \left(\frac{3}{7}\right)$

c $\log_2 49$

d $\log_2 27$

e $\log_2 \left(\frac{7}{9}\right)$

f $\log_2(63)$

g $\log_2 \left(\frac{56}{9}\right)$

h $\log_2(5.25)$

3 Write y in terms of u and v if:

a $\log_2 y = 3\log_2 u$

b $\log_3 y = 3\log_3 u - \log_3 v$

c $\log_5 y = 2\log_5 u + 3\log_5 v$

d $\log_2 y = u + v$

e $\log_2 y = u - \log_2 v$

f $\log_5 y = -\log_5 u$

g $\log_7 y = 1 + 2\log_7 v$

h $\log_2 y = \frac{1}{2}\log_2 v - 2\log_2 u$

i $\log_6 y = 2 - \frac{1}{3}\log_6 u$

j $\log_3 y = \frac{1}{2}\log_3 u + \log_3 v + 1$

4 Without using a calculator, simplify:

a $\frac{\log_2 16}{\log_2 4}$

b $\frac{\log_p 16}{\log_p 4}$

c $\frac{\log_5 25}{\log_5 \left(\frac{1}{5}\right)}$

d $\frac{\log_m 25}{\log_m \left(\frac{1}{5}\right)}$

Simplify: **a** $\log_2 7 - \frac{1}{2}\log_2 3 + \log_2 5$ **b** $3 - \log_2 5$

If $\log_3 5 = p$ and $\log_3 8 = q$, write in terms of p and q :

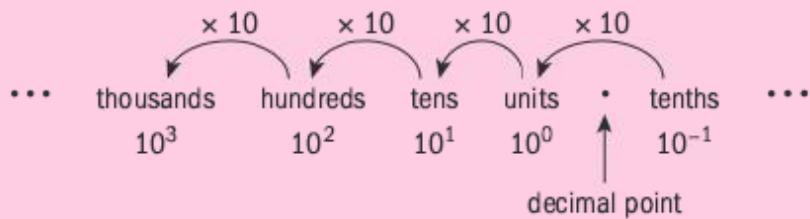
a $\log_3 40$

b $\log_3 25$

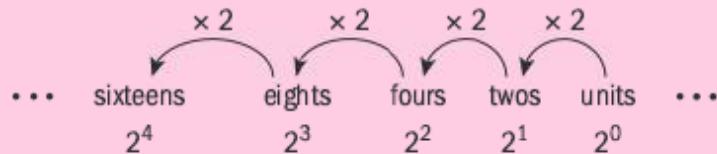
c $\log_3 \left(\frac{64}{125}\right)$

NUMBERS WITH DIFFERENT BASES

Our number system is known as decimal (also as base ten, or denary) because the value of each place value column is ten times the value of the column to its right.



You can make a place value system with any natural number base. In binary (base two) each column is worth twice the column to its right.



The binary number 10111_2 represents a set containing:

sixteens	eights	fours	twos	units
1	0	1	1	1
				

The subscript 2 means that the number 10111_2 is written in base 2.

Computer logical systems use base 2. This is why you see powers of 2 in lots of contexts relating to computers. For example, SD cards which store 8 GB (gigabytes), 16 GB, 32 GB, 64 GB and so on, rather than 10 GB, 20 GB, etc. Also, whereas the prefix *kilo* usually means 1000 (e.g. there are 1000 meters in a kilometer), in computing, the term *kilo* means 1024 (2^{10}), so there are 1024 bytes in a kilobyte.

Tip

- Base 3 - ternary, or trinary
- Base 4 - quaternary
- Base 8 - octal
- Base 12 - duodecimal
- Base 16 - hexadecimal (widely used in computing)

The base of a number system tells you how many unique symbols the number system has. Base 10 has ten unique number symbols, 0 through 9. Base 2 has two unique symbols, 0 and 1.

Example 1

Find the value of 12011_3 in base 10.

3^4	3^3	3^2	3^1	3^0
81	27	9	3	1
1	2	0	1	1

Write the powers of 3 above the digits, starting with $3^0 = 1$ at the right-hand side.

$$81 + 2 \times 27 + 3 + 1 = 139$$

Add up the parts that make up the number.

$$12011_3 = 139_{10}$$

Use subscripts to show the base.

Practice 1

1 Find the value of each binary number in base 10.

a 10111_2

b 11001_2

c 1101101_2

2 Convert each number to base 10.

a 21002_3

b 22101_3

c 412_5

d 332_5

e 64_8

f 77_9

Problem solving

3 Write these numbers in ascending order:

$$1010_4 \quad 100011_2 \quad 1011_3 \quad 1111_5$$

4 The number $1101011000_2 = 856_{10}$.

a Describe the relationship between 1101011000_2 and 110101100_2 .

b Find the value of 1101011_2 .

The following algorithm produces the digits of a number n in base b , starting with the units digit and then working to the left.

- Start with the number, n . Divide n by b and find the quotient q , and the remainder r .
- Write down the value of r .
- If $q > 0$, replace n with the value of q ; otherwise stop.
- Repeat from the beginning with the new value of n , and record new remainders to the left of any you have already written down.

4 Here the algorithm is used to convert 1038_{10} into base 3.

$n \div b$		
n	q	r
1038	346	0
346	115	1
115	38	1
38	12	2
12	4	0
4	1	1
1	0	1
Read the number in base b upwards from here: 1 102 110		

1038 \div 3 = 346 r 0. The units digit is 0. Change n to 346.

346 \div 3 = 115 r 1. The 3s digit is 1. Change n to 115.

115 \div 3 = 38 r 1. The 3^2 digit is 1. Change n to 38.

38 \div 3 = 12 r 2. The 3^3 digit is 2.

12 \div 3 = 4 r 0. The 3^4 digit is 0.

4 \div 3 = 1 r 1. The 3^5 digit is 1.

1 \div 3 = 0 r 1. The 3^6 digit is 0. Stop because $q=0$.

Use the algorithm to convert:

a 1000_{10} to base 2

b 513_{10} to base 3

c 673_{10} to base 4.

PRACTICE 2

- 1 Convert 999_{10} to:
 - a base 2
 - b base 3
 - c base 4
 - d base 5
- 2
 - a Find the value of 472_8 in base 10.
 - b Hence find the value of 472_8 in base 5.
- 3 By first converting to base 10, find the value of these numbers in the given base.
 - a 223_5 in base 7
 - b 431_8 in base 2
 - c 214_6 in base 2
 - d 1011_2 in base 6
 - e 110111_2 in base 8
 - f 110213_4 in base 9
 - g 8868_9 in base 3
 - h 101101_2 in base 4
 - i 2468_9 in base 8

Problem solving

- 4 Four students write the same number in different bases:

Alberto: 1331_a Benito: 2061_b Claudio: 3213_c Donatello: 1000_d

- a Determine which of the four students used the largest base. Explain how you know.
- b Use your answer to part a, and any other information you can gain from the students' numbers, to list the numbers a , b , c and d in ascending order.
- c Determine the minimum possible value for b .
- d Find values of a , b , c and d such that $1331_a = 2061_b = 3213_c = 1000_d$.
- e Use your answer to d to find the value of the number in base 10.

Base 10 (decimal) uses ten different symbols to describe the whole numbers:

0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Similarly, base 2 uses two symbols: 0 and 1.

Base 12 (duodecimal) requires twelve symbols, but you cannot use '10' or '11' because these both involve two digits.

When you write numbers in bases greater than 10, letters are used for the extra symbols needed.

In base 12, the symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B.

Sometimes lowercase letters a and b are used instead of uppercase A and B.

Example 2

Find the value of $A3B_{12}$ in base 10.

12^2	12^1	12^0
144	12	1
A	3	B

The letter A represents 10, and B represents 11.

$$\begin{aligned}A3B_{12} &= (10 \times 144 + 3 \times 12 + 11 \times 1)_{10} \\&= 1487_{10}\end{aligned}$$

Example 3

Find the value of 500 in base 16.

n	$n \div b$	
	q	r
500	31	4
31	1	F
1	0	1

$$500_{10} = 1F4_{16}$$

$500 \div 16 = 31 \text{ r } 4$. The units digit is 4. Change n to 31.

$31 \div 16 = 1 \text{ r } 15$. The symbol for 15 is F. Change n to 1.

$1 \div 16 = 0 \text{ r } 1$. The 16^2 digit is 1. Stop because $q = 0$.

Practice 3

1 Convert to base 10:

a 210_{12}

b 301_{16}

c BAA_{12}

d $G0_{20}$

2 Find the value in base 16:

a 190_{10}

b 2766_{10}

c 47806_{10}

d 48879_{10}

e 51966_{10}

f 64206_{10}

Problem solving

3 Hexadecimal (base 16) codes are often used as passwords for home Wi-Fi hubs and other digital services.

a Explain why there are 256_{10} possible 2-character passwords in hexadecimal.

b Find the number of possible 10-character hexadeciml passcodes.

: Give your answer to a suitable degree of accuracy.

You can add numbers in other bases in the same way as decimal numbers, ‘carrying’ when a number is greater than or equal to the base number.

Example 4

Find the value of $465_7 + 326_7$.

$$\begin{array}{r} 465 \\ +326 \\ \hline 14 \end{array}$$

Units column: $5 + 6 = 11_{10}$. In base 7 this is 1 seven and 4 units, or 14_7 . Write 4 in the units column and carry a 1 into the sevens column.

$$\begin{array}{r} 465 \\ +326 \\ \hline 124 \end{array}$$

Sevens column: $6 + 2 + 1 = 9_{10} = 12_7$.

$$\begin{array}{r} 465 \\ +326 \\ \hline 1124 \end{array}$$

7^2 column: $4 + 3 + 1 = 8_{10} = 11_7$.

So, $465_7 + 326_7 = 1124_7$

For subtraction, you can ‘borrow’ the base number, in the same way that you ‘borrow’ a 10 in base 10.

Example 5

Calculate $783_9 - 267_9$.

$$\begin{array}{r} 783 \\ -267 \\ \hline \end{array}$$

$7 > 3$, so you cannot subtract 7 from 3. Rewrite 783 as 77 13 , because 3 units, 8 nines and 7 eighty-ones is the same as 13 units, 7 nines and 7 eighty-ones.

$$\begin{array}{r} 7713 \\ -267 \\ \hline 5 \end{array}$$

Find $13_9 - 7_9$ by writing it in base 10:
 $13_9 - 7_9 = 12_{10} - 7_{10} = 5_{10} = 5_9$

$$\begin{array}{r} 77^{13} \\ - 267 \\ \hline 15 \end{array}$$

$$7 - 6 = 1$$

$$\begin{array}{r} 77^{13} \\ - 267 \\ \hline 515 \end{array}$$

So, $783_9 - 267_9 = 515_9$

Practice 4

1 Calculate:

a $124_8 + 321_8$

b $77_8 + 261_8$

c $563_8 + 241_8 + 757_8$

2 Calculate:

a $453_6 - 231_6$

b $341_6 - 153_6$

c $1231_6 - 402_6$

3 Calculate:

a $115_8 - 23_8 + 45_8$

b $2231_7 - 125_7 - 216_7$

c $8463_9 + 728_9 - 541_9 + 18_9$

4 Calculate:

a $92A1_{12} + 4436_{12}$

b $10000_{12} - 123_{12}$

Problem solving

- 5 Vorbelar the alien does not count in base 10, but in order to make things easy for us to understand, she uses our base 10 symbols in the usual order. She calculates $216 + 165$ and obtains the answer 403. Find the value of $216 - 165$, giving your answer using Vorbelar's base.

Calculate, giving your answers in the base used in the question:

- a $11011_2 - 101_2$
- b $111011_2 - 11101_2$
- c $10110_2 - 111_2$
- d $2210_3 - 102_3$
- e $3011_4 - 231_4$
- f $44125_6 - 5150_6$
- g $50112_6 - 1045_6$
- h $B791_{12} - 89A0_{12}$
- i $\text{FEED}_{16} - \text{BEEF}_{16}$

If the current time was 10:45, **use** addition and subtraction to **find** the time that is:

- a 35 minutes from now
- b $2\frac{1}{2}$ hours from now
- c 3 hours and 40 minutes from now
- d 1 hour and 19 minutes ago
- e 6 hours and 50 minutes ago

A congressional committee decides to streamline the US monetary system. All coins will be phased out except for 1 cent, 5 cents and 25 cents. All existing bills (bank notes) will be phased out and replaced with bills valued \$1, \$5, \$25, \$125 and \$625.

- a** A mathematician suggests that the dollar should be revalued to be worth 125 cents, not 100 cents as it is currently.
 - i** **Suggest** reasons why the mathematician thinks this might be a good idea.
 - ii** **Suggest** reasons why it might not be a good idea.
- b** **Find** 540_{10} in base 5.
Hence find the smallest number of notes that you would need to give somebody \$540 under the new system.
- c** **Find** 732_{10} in base 5 and 246_{10} in base 5.
- d** Rania owes \$732 to her credit card company. She makes a payment of \$246.
Find the total amount outstanding (still owed), and the least number of notes she would need to repay the amount.
- e** In real life, monetary systems do not use regular bases, but have combinations of notes that follow irregular patterns. For example, in the UK, there are £1 and £2 coins, then notes valued £5, £10, £20 and £50.
Suggest reasons why it would not be convenient to use a perfectly regular base as suggested above.

3 The scientists also encoded some information about our solar system. They used information about quantities that can be counted, rather than measured, because measurements require units whereas counting does not. One quantity that can be counted is the number of ‘days’ in each planet’s year. Earth has (roughly) 365 Earth days per year because that is how many times it rotates around its own axis while orbiting the sun. The table gives some information about the planets in our solar system:

Planet	Year length	Day length
Mercury	87.96 Earth days	1408 Earth hours
Venus	224.68 Earth days	5832 Earth hours
Earth	365.26 Earth days	24 Earth hours
Mars	686.98 Earth days	25 Earth hours
Jupiter	11.862 Earth years	10 Earth hours
Saturn	29.456 Earth years	11 Earth hours
Uranus	84.07 Earth years	17 Earth hours
Neptune	164.81 Earth years	16 Earth hours

- a Calculate** the number of Earth hours in a Mars year. Give your answer in denary.
- b Hence find** the number of Mars days in a Mars year. Give your answer in denary.
- c Find** (to the nearest whole number) the number of Mars days in a Mars year in binary.
- d Find**, in binary, the number of days in a year for each of the other outer planets (Jupiter to Neptune).
- e Describe** any problems you would encounter when trying to perform a similar calculation for Venus.

MYP E – ASSESSMENT TYPE QUESTIONS

Question 1

Evaluate:



$$\frac{(6 - \sqrt[3]{64})^{-1}}{(-0.02)^2}$$

Question 2

Organize the following numbers from smallest to greatest:



$$0.314, \quad \frac{314}{999}, \quad \frac{\pi}{10}, \quad 0.314, \quad 3.145\%$$

Question 3

Boutique “A” is offering 12.5% discount on all its items. Boutique “B” is offering 2 items at the marked price and the third item for JD 2.00. (The third item should be the least expensive)

- a) *Compare the amounts that you would pay in both shops if you need a pair of trousers marked JD 25.00, and two shirts marked JD 20.00 and JD 6.00 respectively.*
- b) *You are a sales person working in boutique “A”. State two different reasons why should customers be interested in your offer over that of boutique “B”.*
- c) *Two smart sisters; Lara and Nadeen, visit boutique “B”. They decide to buy the following items:*

Lara buys three items that cost JD36, JD10, and JD35

Nadeen buys three items that cost JD 35, JD15, and JD 5.

When they get to the cashier they think of a smart way to save money. Suggest a way where the two sisters can save money.

Suggest how they would share the money they saved by following your way between them. And justify your answer.

Question 4

The Roman Empire has grown dramatically in a few centuries. It was born on the Palatine hill according to the legend, but in about three centuries it extended its dominion to most of Mediterranean countries in Europe and Africa and to the Middle East.

The population of the Roman Empire according to the roman census in different years are tabulated below:

Year	Population
70/69 BC	910,000
28 BC (Augustus)	4,063,000
8 BC (Augustus)	4,233,000
14 AD (Augustus)	4,937,000
47 AD	6,941,000

- a) **Outline the trend over the mentioned years.**
- b) **Calculate the percentage increase from 28 BC to 14 AD.**
- c) **Estimate the roman population at birth of Christ.**
- d) **Suggest two different reasons for the increase from 70 BC to 28 BC.**
- e) **Discuss whether it would be trustable to predict the number of the roman population in the year 100 AD? Justify your answer.**

Question 5

A Bank in USA offers the following exchange rate between American dollars (USD) and Euros (EUR). The bank sells 1 USD for 0.92 EUR and buys 1 USD for 0.88 EUR.

A customer wishes to exchange 800 US dollars for Euros.

- a) *Find how many Euros the customer will receive to the nearest whole number.*
- b) *The customer has to cancel his trip and exchanges his money back later when the rates are “sells 1 USD = 0.915 EUR, buys 1 USD = 0.879 EUR. Calculate how many US dollars has he lost on the transaction.*
- c) *A humanitarian society in US finances a project in Romania. The society donates USD 256,853,000. A newspaper wants to report the size of American donation in USD. Write down your answer to a degree of accuracy appropriate for reporting in the newspaper. Justify the degree of accuracy you have selected.*

Question 6

Calculate the following:



$$3 \times 10^{12} - 2 \times 10^5 \div 1 \times 10^{-8}$$

Question 7

An international organization supports Refugee Camps with a monthly sum of \$4,540,000.

- a) *Write down the donation in scientific notation.*
- b) *Write down the donation to 1 significant figure.*
- c) *The total expenses of the camps during winter months were $\$1.84 \times 10^7$. Calculate the camps deficit during winter in standard form.*

Question 8

You have the option to invest JD10,000 in one of the 3 accounts below.

Account 1	Account 2	Account 3
<i>Simple interest</i>	<i>Compounded annually</i>	<i>Compounded monthly</i>
Rate = 2%	Rate = 1.2%	Rate = 1.2%

- a) **Predict** which account will yield the highest income.
- b) **Find** which account will give the highest amount after 2 years to the nearest 100 Jordanian Dinars.
- c) **Solve** for the number of years where the investment in account 2 will first exceed the investment in account 1. **Verify** your conclusion.
- d) **Identify** three factors that affect the future value of your investment.

Question 9

For the following arithmetic sequence:



$$\frac{\sqrt{27}}{\sqrt{3}}, \quad 3 + \sqrt{3}, \quad 3 + 12^{\frac{1}{2}}, \quad \dots$$

- a) **Write down** the following three terms.
- b) Hence, or otherwise, **deduce** the general formula for the sequence.
- c) **Explain** why $\sqrt{3} \neq 1.732050807$

Question 10

Write down the following in the form of $a + b\sqrt{c}$ in its simplest form.



$$(\sqrt{3} + 2\sqrt{2})^2 + 3\sqrt{24}$$

Question 11

Jim has just retired. He has the option of receiving a monthly payment of €3667 or €330 000 as a ‘lump sum’ (a final single payment).

- a) **Find** the yearly superannuation payment.
- b) **Find** the yearly income that would result if the lump sum could be invested at 12% p.a.?
- c) **State** which option seems most attractive and **explain** why?
- d) **Calculate** his yearly income if he elected to receive 30% of the monthly payment, and 70% of the lump sum of which he invested 40% at 12% p.a.?

Question 12

The sum (S_n) of n terms of an arithmetic sequence is given by the rule:

$$S_n = \frac{n}{2}[2a + (n - 1)]$$

Find the term at which the sum of the following sequence will first exceed 205.

$$2, 7, 12, 17, \dots \dots \dots$$

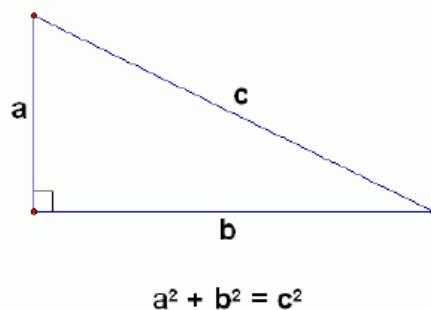
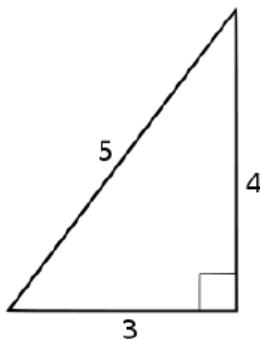
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Question 13

You will have noticed that the sum of two square numbers is also a square number. These three numbers can then form the sides of a right angled triangle. The ancient Egyptians are known to have used the most common example: the 3, 4, 5 triangle. Which symbolized their divinity.



Three numbers like 3, 4, 5 which satisfy the equation $a^2 + b^2 = c^2$ and which form the sides of a right-angled triangle are known as Pythagorean triples. There are an infinite number of Pythagorean triples. Here are some triples arranged in a table.

Row	<i>a</i>	<i>b</i>	<i>c</i>
1	3	4	5
2	5	12	13
3	7	24	25
4	9	40	41
5	11	60	
6	13		
7			

- a) **Explore** further by completing rows 5, 6 and 7 of the table above.
- b) **Show** that 5, 12, 13 is a Pythagorean triple.
- c) **Describe** in your words, two patterns you have discovered from the table in part a.
- d) **Find** as a general rule any two patterns you have discovered from the table in part a.
- e) **Verify** your general rule from part d.

Question 14

For the following sequences:

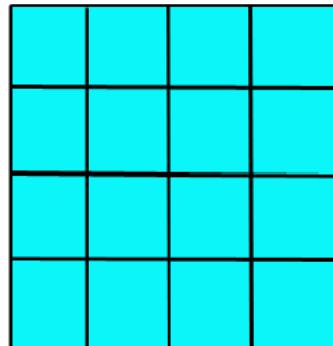
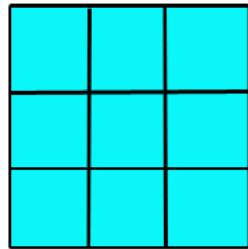
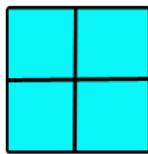
$$-4, \quad 3, \quad 22, \quad 59, \dots$$

$$1, \quad 3, \quad 6, \quad 10, \dots$$

- a) **Write down** the next two terms in each of these sequence.
- b) **Describe** patterns in your own words.
- c) **Find** a general rule for the sequence of numbers.
- d) **Verify** your general rule.

Question 15

Can you see there are five squares in this figure? There are four small squares with side one unit long and one big square with side length of two units.



- a) **Find** the total number of squares in the third and fourth figures below.
- b) **State** the next number in the pattern.

Question 16

Athletes at the training session chose their own activities. Possible activity options were running, cycling and swimming.

The athletes' choices are listed below:

- Everyone chose at least one activity.
- 4 athletes chose every event.
- 12 athletes chose swimming only.
- 15 athletes chose running only.
- 3 athletes chose cycling and swimming but not running.
- Everyone who chose cycling also chose another activity.
- 50 athletes did NOT choose cycling.
- 45 athletes choose running.

a) Show this information in a Venn diagram.

b) Determine the number of athletes who attended the training session.

Question 17

$$\text{Set } A = \{x; x \in \mathbb{N}\}$$

$$\text{Set } B = \{|x|; x \in \mathbb{Z}\}$$

$$\text{Set } C = \{x; x \text{ is irrational number}\}$$

$$\text{Set } D = \{\pi, 0, 1, \sqrt{2}, 4, 6\}$$

$$\text{Set } E = \{x; x \text{ is any number that can be written in the form of } \frac{a}{b} \text{ where } b \neq 0\}$$

a) Find

$$i) A \cap E$$

$$ii) (B \cap C) \cup D$$

$$iii) C'$$

$$iv) D \cap C$$

b) State whether:

$$i) \{-5\} \in B$$

$$ii) A \subset E$$

$$iii) E \subset A$$

Question 18

You jog three times a week, walk five times every fortnight and swim three times a month. Today you did all three sports, how many days will pass until you will be able to do all three sports together again. (Consider a month to be 4 weeks).

Question 19

To make this large abstract outdoor statue a 10 meter by 25 cm rectangular metal stripe was used.

- a) If there is 0.015 cm error margin in measurements, calculate the maximum possible area of the rectangular stripe to a 8 significant figures.
- b) A model was made for this statue using a scale of 25:1. Find the dimensions of the stripe in the model.
- c) The cost of production of one statue is \$320.00. Because of bad finishing and poor transportation and storing conditions the store offers the statues for 55% off. Calculate how much was lost for each statue.
- d) As a researcher you were asked to investigate how to reduce the losses. Trace a plan for your research showing different levels of importance, and justify your choices.



Question 20

The Fibonacci sequence is named for Leonardo Pisano, an Italian mathematician who lived from 1170-1250. The sequence starts as follows:

$$1, 1, 2, 3, 5, 8, 13, \dots$$

Part I

- a) **Describe** the term to term(s) relationship.
- b) **Investigate** the ratio of any term to the next term. **Use** the table below to help you investigate.

<i>Previous Term</i>	<i>Current Term</i>	$\frac{\text{Previous}}{\text{Current}}$	<i>Answer correct to 3 decimal places</i>
1	1		
1	2		
2	3		
3	5		

- c) **Use** your findings in part “b” to find the ratio between the 100th and 101st terms.

Part II

Fibonacci can be extended backwards, like this:

=	n	...	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	...
$x_n =$...						-1	1	0	1	1	2	3	5	8	...

- a) **Complete** the table above.
- b) **Write down** your conclusions
- c) **Write down** a general rule for Fibonacci when extended backwards
- d) **Verify** your general rule.

Question 21

Al Rahma Restaurant was inaugurated in December 2015 in Amman Jabal Webdeh and helped the following numbers of people to have a daily meal.

Year	Number of people helped
2015	257
2016	1964
2017	2133

The restaurant's manager wants you predict how many people they can help in 2019. **Predict** the number of people the restaurant will help in 2019 using a strategy used in your MYP studies. You should include details, for example:

- A prediction of the number of people that will be helped.
- Details of the method used with clear lines of reasoning.
- A justification of whether this prediction is a reasonable estimate for 2018.