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1

Ratios and proportions

Competition and cooperation

Unit review



Key to Unit review question levels:

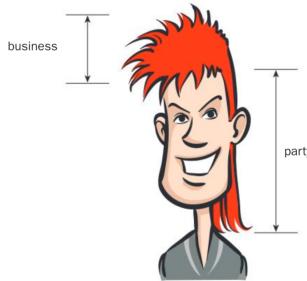
Level 1–2

Level 3–4

Level 5–6

Level 7–8

- 1** Simplify the following ratios where possible.
a 6 : 8 **b** 10 : 2 **c** 11 : 33
d 6 : 7 **e** 12 : 8 **f** 24 : 18
- 2** Write two equivalent ratios for each of the following ratios.
a 1 : 2 **b** 4 : 9 **c** 3 : 6
d 1 : 6 **e** 4 : 1 **f** 20 : 30
- 3** At the Iowa State Fair, contestants can compete in a Pigtail, Ponytail, Braid, Mullet and Mohawk competition. Competitors vie for the Blue Ribbon Award in one of the four hairstyle contests.
A mullet is a hairstyle where the front and sides are short and the back is long. Called the ‘Hunnic’ look in the 6th century, mullets have been described as ‘business in the front, party in the back’.



Some people suggest that the ratio of the length of the hair in back (the party) to the length of the hair in front (the business) would be a good measure of ‘mulletness’.

A large ratio of party : business is desired.

- a** **Describe** what that means in terms of which mullets are likely to win.
- b** Determine who wins the mullet contest given the following ratios. Show your working.

12 : 5

18 : 7

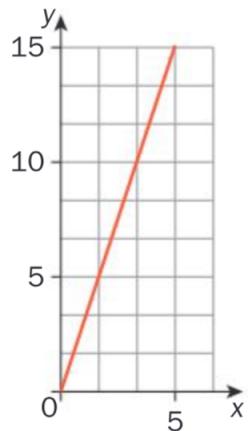
6 : 2

10 : 4

8 : 3

- 4** Despite heavy competition, the work force is made up of 36% college graduates and 34% high-school graduates. The remaining are neither college graduates nor high-school graduates.
- a** Rewrite the percentage of college graduates, high-school graduates and neither as simplified fractions.
 - b** Find the ratio of college graduates : high-school graduates : neither.
 - c** Find the simplified ratio of college graduate : high-school graduates : neither.

- 5** **a** Is the relationship shown in the graph proportional? **Explain.**
- b** What is the constant of proportionality?
- c** Find the value of y when x is 24.
- d** Find the value of x when y is 108.



- 6** In 2016, the International Olympic Committee created the Refugee Olympic Team (ROT) to draw attention to the refugee crisis occurring worldwide and to honour those who ‘have no home, no team, no flag, no national anthem’. In any other Olympics, the 10 athletes would have been on competing national teams, but this year they would cooperate as a unified team. The athletes and their country of origin are listed below.

Name	Country of origin
Rami Anis	Syria
Yiech Pur Biel	South Sudan
James Nyang Chiengjiek	South Sudan
Yonas Kinde	Ethiopia
Anjelina Nada Lohalith	South Sudan
Rose Nathike Lokonyen	South Sudan
Paulo Amotun Lokoro	South Sudan
Yolande Bukasa Mabika	Democratic Republic of the Congo
Yusra Mardini	Syria
Popole Misenga	Democratic Republic of the Congo

- a Write the ratio of the number of team members from each country of origin to the total number of team members.

- b** Paulo Lokoro ran the 1500 m race in 4 minutes and 4 seconds, while Yiech Biel ran the 800 m race in 115 seconds. If they ran a 400 m race against each other, is it possible to tell who would win? Why or why not?
- c** As a refugee, Yusra Mardini had to swim and push the dinghy that she used to flee from Syria. At the Olympics she swam the 100 m freestyle event in 69 seconds. Anjelina Lohalith ran the 1500 m race in 4 minutes 47 seconds. Who had the faster speed? **Show** your working.

- 7** Use proportional reasoning to solve for the variable. Round to the nearest tenth where necessary.

$$\begin{array}{lll} \mathbf{a} \quad 2 : 9 = 10 : p & \mathbf{b} \quad 36 : 12 = 54 : m & \mathbf{c} \quad 11 : 33 = y : 11 \\ \mathbf{d} \quad \frac{3}{t} = \frac{5}{11} & \mathbf{e} \quad \frac{8}{21} = \frac{f}{4} & \mathbf{f} \quad \frac{2.4}{5.7} = \frac{9.2}{k} \end{array}$$

- 8** Out of 2000 competitors at the World Iron Man triathlon, a total of 1815 completed the race.
- a** Assuming that male and female competitors dropped out at the same rate, find the number of male and female finishers given the ratio of male competitors : female competitors is 20 : 13.
- b** Find the simplified ratio of those who finished the race to those who did not.
- 9** A recent study found that the pay ratio of college graduates to non-college graduates over a lifetime is 184 : 100. On average, it is found that a person without a college degree will make \$1.3 million (\$1 300 000) in a lifetime. Given this ratio, how much would the average college graduate make in a lifetime?



11 In California, there is a ‘wildfire season’, when wildfires are most common. These can be incredibly destructive events, with some burning as many as 1300 km^2 . In 2007, the Moonlight Fire, caused by a lightning strike, affected people in three US states, requiring firefighters from across the United States to work together to get it under control. When the fire had burned 110 km^2 , 1900 firefighters were battling the blaze. By the time it was under control, 2300 firefighters were putting out the fire that burned a total of 263 km^2 . Because of this amazing cooperation, nobody perished and only two buildings were destroyed in the Moonlight Fire.

- a** Represent the ratio of firefighters to area burned at each stage of the fire in three different ways. Show your working.
- b** If the ratio of firefighters to area burned was maintained, how many firefighters should have been fighting the fire by the time it was under control? **Show** your working.
- c** **Suggest** reasons why the number of firefighters was less than the amount you calculated in part **b**.

- 12** At the 1996 Olympics, Michael Johnson won the 200 m race with a time of 19.32 seconds. Donovan Bailey won the 100 m race with a time of 9.84 seconds. Both men claimed to be “The World’s Fastest Man”.
- a** Who had the faster speed? **Show** your working.
The media reported that Johnson was a faster runner than Bailey.
- b** **Explain** why this interpretation makes sense.
- c** Bailey said that this is not a fair comparison. **Explain** why he is justified in saying so.
- d** In order to see who the ‘fastest man’ was, the two ran a race of 150 m in 1997. Using their results from the Olympics, **predict** the time for each in the 150 m race.
- e** Bailey won the race in 14.99 seconds. How does this compare to his speed for the 100 m race? **Show** your working.

- 13** The Human Genome Project is an international collaboration to map the three billion base pairs in the human genome, the complex set of genetic instructions. Over 1000 scientists from six different countries collaborated on the project, which took over 13 years to complete. Genome size (measured in picograms) is the amount of DNA contained in a single genome. One picogram (pg) contains 978 million base pairs.



- a** **Construct** a table with the number of picograms in one column and the corresponding number of base pairs (in millions) in the other. Show at least four rows.
- b** Is this a proportional relationship? **Justify** your answer.
- c** **Write** an equation for the data in your table.
- d** Find the number of picograms if there are 11 247 million base pairs. Show your working using both a proportion and your equation.

Answers

Unit Review

- 1** **a** 3:4 **b** 5:1 **c** 1:3 **d** Already simplified **e** 3:2 **f** 4:3
- 2** Individual response, example answers include:
a 2:4, 3:6 **b** 8:18, 12:27 **c** 1:2, 2:4 **d** 2:12, 3:18 **e** 8:2, 12:3 **f** 2:3, 4:6
- 3** **a** Individual response, which suggests that mullets with the largest ‘party length’ to ‘business length’ are most likely to win
b 6:2
- 4** **a** College graduates: $\frac{9}{25}$
 High-school graduates: $\frac{17}{50}$
 Neither: $\frac{3}{10}$
- 5** **a** Yes **b** 3 **c** 72 **d** 36
- 6** **a** Syria 1:5
 South Sudan 1:2
 Ethiopia 1:10
 Democratic Republic of Congo: 1:5
- b** No, it would be possible to make a prediction but, as the relationship is not necessarily proportional, it would not be possible to accurately determine the winner.
c Yusra
- 7** **a** $p = 45$ **b** $m = 18$ **c** $y = 3.7$ **d** $t = 6.6$ **e** $f = 1.5$ **f** $k = 21.9$
- 8** **a** Male: 1100, Female: 715 **b** 363:37
- 9** \$2.392 million (\$2 392 000)
- 10** 5:6
- 11** **a** $1900:11, \frac{190}{11}, 17.27$ **b** 4543 **c** Individual response (e.g. shortage of firefighters)
 $2300:263, \frac{2300}{263}, 8.75$
- 12** **a** Michael Johnson
- b** Individual response (Based on average speed alone, Johnson is quicker)
- c** Individual response (There was no race over the same distance from which a direct comparison could be made. A more sophisticated response may also indicate that the acceleration phase of a race is likely to bring the average speed over 100 m down more than the average speed over 200 m).
- d** Johnson: 14.49s
 Bailey: 14.76s
- e** This average speed slower than his average speed for the 100m race
- 13** **a** Individual response, for example:
- b** Yes
- c** $P = \frac{B}{978}$ where P is the number of picograms and B is the number of base pairs in millions
- d** 11.5
- | Number of Picograms | Number of Base Pairs (millions) |
|---------------------|---------------------------------|
| 1 | 978 |
| 2 | 1956 |
| 3 | 2934 |
| 4 | 3912 |



2

Probability

Games and play

Unit review



Key to Unit review question levels:

Level 1–2

Level 3–4

Level 5–6

Level 7–8

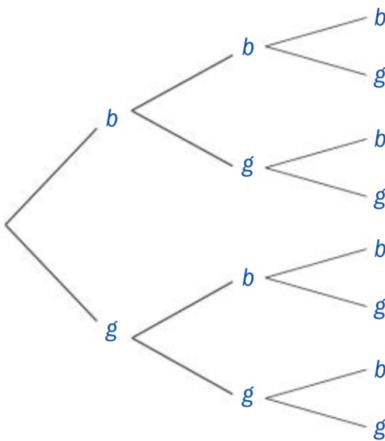
ATL2

- 1 Make another sample space representation for:

- a a list of outcomes for spinning a spinner and flipping a coin
- b a table for rolling two dice

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

- c a tree diagram for three rounds of drawing a blue or green marble from a bag.



- 2 *Who Wants to be a Millionaire?* is a British television game show in which players answer multiple choice questions and could win up to one million British pounds. Over 100 different international variations of the show have appeared since the show first began in 1998.
- a Each question is multiple choice with four possible answers (A, B, C, D). What is the probability of guessing an answer correctly?

- b** Play stops as soon as a question is answered incorrectly.
Represent the sample space for the outcomes of up to three questions answered in a row using a tree diagram and a list.
- c** If a player doesn't know the answer to a question, he can use one of three 'lifelines'. One of those is called 50/50, where two of the wrong answers are eliminated. What is $P(\text{correct answer})$ after the 50/50 lifeline option is used?
- 3** *Dungeons and Dragons* is a very popular role-playing game where players are given a character and embark on adventures in a fantasy land. A Dungeon Master controls the game, creating scenarios and challenges for the players. Many of the actions that are taken in *Dungeons and Dragons* rely on rolling dice that have 4, 6, 8, 10, 12 or 20 sides. Suppose you are rolling an 8-sided die.

a Calculate:

- i** $P(2)$
- ii** $P(2, 3 \text{ or } 4)$
- iii** $P(\text{even})$
- iv** $P(<5)$
- v** $P(9)$

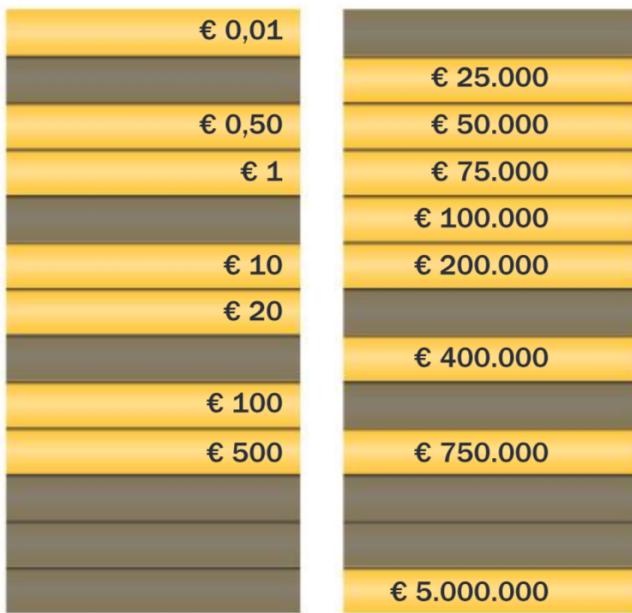
- b Show** that $P(1, 3, 4, 7, 8) + P(2, 5, 6) = 1$

- c** Provide a different example of complementary events and provide calculations to justify your example.
 - d** Given probability values you've calculated in parts **a** and **b**, determine which events are impossible, unlikely, equally likely as unlikely, likely and certain.
- 4** The game *Miljoenenjacht*, which originally began in the Netherlands, is more widely known as *Deal or No Deal*. In the game, there are 26 boxes, each with a different amount of money in it. At the beginning of the game, the player chooses one box which he/she hopes has the highest prize, €5 million. During the rest of the game, the player chooses boxes from those that remain. With each selection, the player knocks that box and the amount inside of it off the game board. The amounts in the boxes are shown in the table at the top of the next page.

€ 0,01	€ 10.000
€ 0,20	€ 25.000
€ 0,50	€ 50.000
€ 1	€ 75.000
€ 5	€ 100.000
€ 10	€ 200.000
€ 20	€ 300.000
€ 50	€ 400.000
€ 100	€ 500.000
€ 500	€ 750.000
€ 1.000	€ 1.000.000
€ 2.500	€ 2.500.000
€ 5.000	€ 5.000.000

- a** With a contestant's initial selection of a box, find the following probabilities:
- i $P(\text{€}5 \text{ million})$
 - ii $P(\text{amount} > \text{€}100\,000)$
 - iii $P(\text{amount} < \text{€}75\,000)$
 - iv $P(\text{at least } \text{€}1 \text{ million})$
 - v $P(\text{an amount in the thousands})$
 - vi $P(\text{€}0)$

- b** After a few rounds, only the boxes below remain.



Find the following probabilities related to the next box selected:

- i** $P(\text{the } €5 \text{ million is left on the board})$
 - ii** $P(\text{the } €5 \text{ million box is opened})$
 - iii** $P(\text{the box} < €20)$
 - iv** $P(\text{the box} > €100\,000)$
- c** After the few rounds, is the contestant in a better position to win the 5 million? Justify your response.

- 5** In the game of *Risk*, players set up armies on a map of the world that acts as the game board.



Players can add to their armies or wage war against armies in regions that are beside them. In these wars, each player rolls one or more six-sided dice, with each die representing a battle. With each battle, if the person initiating the war (the attacker) has a higher number, then he/she wins the battle. If the defender rolls a number that is equal to or greater than the attacker, then the defender wins the battle.

The number of dice rolled depends on the number of armies you have. When more than one die is rolled, whoever wins the most battles, wins the war. If each player wins the same number of battles, the defender wins the war.

- a Suppose each player rolls just one die. Find the probability that the defender wins the war if the attacker rolls a:

- i 3 ii 1 iii 5 iv 6

- b** Suppose each player rolls two dice. Each player's highest number is compared with their opponent's, with the winner of each battle being the higher number for that battle. The lower numbers are then compared in the same way. Write the sample space for all possible outcomes of the defender's roll when the attacker has rolled a 2 and a 5. Create a table like the one started below, for example. And remember, if each player wins one battle then the defender wins the war.

Attacker rolls	Defender rolls	Battle 1 won by	Battle 2 won by	War won by
2, 5	1, 1	Attacker	Attacker	Attacker
	1, 2	Attacker	Attacker	Attacker
	1, 3	Attacker	Attacker	Attacker
	1, 4	Attacker	Attacker	Attacker
	1, 5	Defender	Attacker	Defender

- c Find the probability of the defender winning the war (by winning one or both battles)
 - d What attacker rolls will result in the defender's probability of winning both battles be 1?
- 6 In the UK quiz show *Who Wants to be a Millionaire?* contestants continue to answer questions until they get one wrong.

The value of each question is shown here. If a contestant answers a question correctly, she moves on to the next question. If she answers incorrectly, she falls back to the amount in yellow. For example, if she is on question 9 and answers it incorrectly, she wins only £1000 and the game is over. If she answers any of the first four questions incorrectly, she wins nothing. A player can stop answering at any time and take the money she has earned.

15	£1 Million
14	£500,000
13	£250,000
12	£125,000
11	£64,000
10	£32,000
9	£16,000
8	£8,000
7	£4,000
6	£2,000
5	£1,000
4	£500
3	£300
2	£200
1	£100

- a Knowing that each question has four answer choices, and assuming a contestant does not use a single lifeline, design a simulation to determine the average winnings a player can expect if she just guesses each answer.
- b Record the data from your simulation in a table, showing the amount of money won each time. Average these over 20 trials.

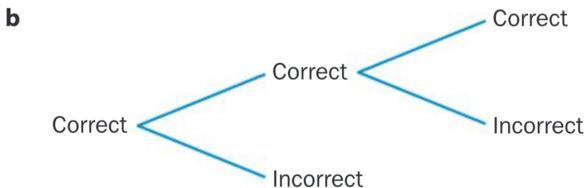
- c** What is the experimental probability of $P(> £16\,000)$?
- d** What is the experimental probability of $P(£1\,000\,000)$? Do you think anyone ever won that much money? If so, how?
- e** Suppose you have used all of your lifelines and both the audience and the friend you phoned think the answer to a question is ‘c’. You aren’t sure they are correct. What is the highest dollar level where you would choose answer ‘c’ instead of stopping and taking the money you have earned? **Justify** your choice.

Answers

Unit Review

- 1 **a** Individual response (Tree diagram or table)
b Individual response (List or tree diagram)
c Individual response (List or table)

2 **a** $\frac{1}{4}$



Incorrect

List: C = Correct, I = Incorrect
I, CI, CCI, CCC

c $\frac{1}{2}$

3 **a** **i** $\frac{1}{8}$

ii $\frac{3}{8}$

iii $\frac{1}{2}$

iv $\frac{1}{2}$

v $\frac{1}{8}$

b $\frac{5}{8} + \frac{3}{8} = 1$

c Individual response

- d** **(a)** (i): unlikely, (ii): unlikely, (iii): equally likely as unlikely, (iv): equally likely as unlikely, (v): unlikely
(b) certain

- | | | | | | | | |
|----------|----------|---------------------|-----------------|-----------|----------------|------------|----------------|
| 4 | a | i | $\frac{1}{26}$ | ii | $\frac{4}{13}$ | iii | $\frac{8}{13}$ |
| | | | | v | $\frac{8}{13}$ | vi | 0 |
| | b | i | $\frac{14}{15}$ | ii | $\frac{1}{15}$ | iii | $\frac{4}{15}$ |
| | c | Individual response | | | | iv | $\frac{4}{15}$ |
| 5 | a | i | $\frac{2}{3}$ | ii | 1 | iii | $\frac{1}{3}$ |
| | | | | | | iv | $\frac{1}{6}$ |

b Sample space:

Attacker rolls	Defender rolls	Battle 1 won by	Battle 2 won by	War won by
2, 5	1, 1	Attacker	Attacker	Attacker
	1, 2	Attacker	Attacker	Attacker
	1, 3	Attacker	Attacker	Attacker
	1, 4	Attacker	Attacker	Attacker
	1, 5	Defender	Attacker	Defender
	1, 6	Defender	Attacker	Defender
	2, 1	Attacker	Attacker	Attacker
	2, 2	Attacker	Defender	Defender
	2, 3	Attacker	Defender	Defender
	2, 4	Attacker	Defender	Defender
	2, 5	Defender	Defender	Defender
	2, 6	Defender	Defender	Defender
	3, 1	Attacker	Attacker	Attacker
	3, 2	Attacker	Defender	Defender
	3, 3	Attacker	Defender	Defender
	3, 4	Attacker	Defender	Defender
	3, 5	Defender	Defender	Defender
	3, 6	Defender	Defender	Defender
	4, 1	Attacker	Attacker	Attacker
	4, 2	Attacker	Defender	Defender
	4, 3	Attacker	Defender	Defender
	4, 4	Attacker	Defender	Defender
	4, 5	Defender	Defender	Defender
	4, 6	Defender	Defender	Defender
	5, 1	Defender	Attacker	Defender
	5, 2	Defender	Defender	Defender
	5, 3	Defender	Defender	Defender
	5, 4	Defender	Defender	Defender
	5, 5	Defender	Defender	Defender
	6, 1	Defender	Attacker	Defender
	6, 2	Defender	Defender	Defender
	6, 3	Defender	Defender	Defender
	6, 4	Defender	Defender	Defender
	6, 5	Defender	Defender	Defender
	6, 6	Defender	Defender	Defender

c $\frac{29}{36}$

- d** If the attacker rolls a 1 and any other number then the defender will either win both battles, or win one and lose one, and in the event of such a tie the defender wins.

6 Individual response

3

Integers

Human explorations

Unit review



Key to Unit review question levels:

Level 1–2

Level 3–4

Level 5–6

Level 7–8

- 1** Insert the appropriate symbol ($<$ or $>$) between each pair of quantities to indicate which one is greater.

a $-8 \square -3$

b $-53 \square -62$

c $-7 \square -11$

d $-1 \square 0$

e $|-5| \square |-2|$

f $|-11| \square |-12|$

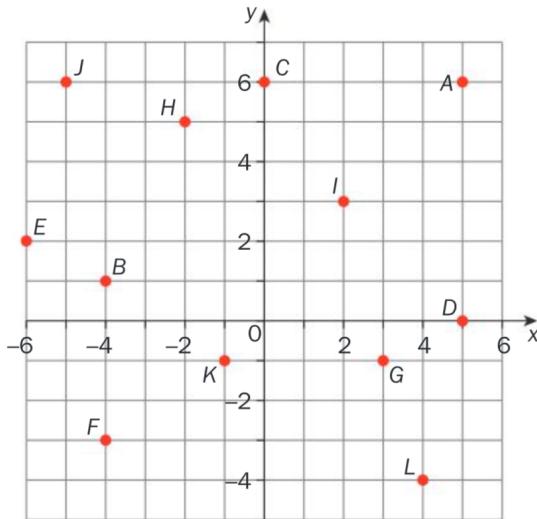
- 2** Insert the appropriate symbol ('+' or '-') in order to make the statement true.

a $\square 6 < -2$ b $\square 1 < 1$ c $4 > \square 6$ d $8 < \square 10$ e $-2 < \square 5$ f $\square 10 > -9$

- 3** Write down the following quantities in order from least to greatest.

a $0, -3, 2, -1, -7$ b $9, -12, 5, -6, 7, -2, 1$ c $|-2|, |8|, |-4|, |-10|, |0|, |-1|$

- 4** State the ordered pair (x, y) for each marked point on the grid below:



- 5** On graph paper, **plot** and **label** these points on a grid and **label** the x - and y -axes.

$$E(5, 1) \quad F(0, 4) \quad G(-2, 6) \quad H(-4, -3) \quad I(4, -5)$$

$$J(2, 7) \quad K(-5, 1) \quad L(0, -3) \quad M(6, 0) \quad N(1, -2)$$

$$O(0,0) \quad P(-3, 5) \quad Q(-2, -6)$$

- 6** As seen previously, the magnitude of celestial objects uses positive and negative values, like those in the table below.

Object	Stellar magnitude
Our sun	-27
The full moon	-13
Venus	-4
Jupiter	-2
Sirius (star)	-1
Halley's comet	+2
North Star (Polaris)	+2
The Andromeda galaxy	+3

- a** When their magnitude is as bright or brighter than that of Venus (-4), meteors are called ‘fireballs’. Are fireballs ‘meteors with a magnitude greater than -4’ or ‘meteors with a magnitude less than -4’? **Explain**.
- b** **Explain** how the concept of absolute value makes this system more sensible.
- 7** The time of day at any one instant is different depending on where you are in the world. Earth is split up into different time zones, with Coordinated Universal Time (UTC) being considered zero. Locations to the east of UTC are *ahead* of UTC (positive) and locations west of UTC are *behind* UTC (negative). Some major cities and their time zones relative to UTC are given below:

Location	Time zone relative to UTC
Madrid, Spain	+2
Auckland, New Zealand	+12
Montreal, Canada	-4
San Diego, USA	-7
Tokyo, Japan	+9
Reykjavik, Iceland	0
Islamabad, Pakistan	+5
Istanbul, Turkey	+3
Rio de Janeiro, Brazil	-3

- a Write down the cities as they would appear on a number line, based on their time zone relative to UTC.
- b Which two cities have relative time zones with the same absolute value? What does this mean?
- c Write down the cities in order of the absolute value of their relative time zone, from least to greatest.
- d Do some research and find cities for which the relative time zone has an absolute value between that of Montreal and San Diego.

8 Evaluate.

a $(-3)(-5)$

b $-4 - 7$

c $(-5) + (+2)$

d $8(-11)$

e $2 - (+9)$

f $-3 - (-1)$

g $-24 \div 6$

h $-21 + (-12)$

i $(-4)(-2)(3)$

j $-18 \div -2$

k $8 - (-10)$

l $(+6)(-7)$

m $-2 - 8$

n $-10 \div -16$

o $(-3)(2)$

p $35 \div -7$

q $-14 + 10$

r $17 - 23$

s $-3(7)$

t $(+8) - (+13)$

u $(-3) + (-8) - (-5)$

v $(-2)(3)(-5)$

w $-12 - (-4) + (-17) - (+8)$

x $(-24) \div (-2) \div (-3)$

y $-7 + (-5) - (-6) - (+10)$

- 9** Explain the difference between $(3, -5)$ and $(-5, 3)$ and describe the way you would plot both points on a grid.

Suppose you visited EarthCaches at the following locations.

Location	Latitude (degrees)	Longitude (degrees)
Blue Hole, Belize	+17	-88
Stonehenge, England	+51	-2
Angkor Wat, Cambodia	+13	+104
Victoria Falls, Zambia	-18	+26
Chichen Itza, Mexico	+21	-89
Machu Picchu, Peru	-13	-73
Petra, Jordan	+30	+35
Great Barrier Reef, Australia	-18	+148
Taj Mahal, India	+27	+78

- a** Write down an itinerary to visit these destinations in order of their latitudes. Show your itinerary on a number line.
- b** Find the largest degree difference in latitude. Show your working.
- c** Find the largest degree difference in longitude. Show your working.
- d** If you traveled from Victoria Falls to the Blue Hole, find your change in latitude and longitude in degrees.
- e** Suppose you traveled from location to location in the order given in the table. Show that the difference (in degrees) in latitude and longitude from the Blue Hole to the Taj Mahal is equivalent to the sum of the differences between each location. Show your working.

15

a
$$\frac{(-75) - (-40) \div (-4) + (-5)}{[(-45) + 90] \div (-5)}$$

b
$$\frac{81 \div (-9) + 30 - (-19)}{(-6) \times (-5) + (-10)} + \frac{100 + (-60) \div (-12) - (-5)}{(80 + 10) \div (-9)}$$

c
$$\frac{(-8)(+11) + (-6)(+7)}{(-39) - (-52)} - \frac{(-12)(5) + (-6)}{(+7)(-3) - (-10)}$$

- 10** On a trip from 1903 to 1906, Roald Amundsen became the first person to sail through the Northwest Passage, the route across the Arctic that joins the Atlantic and Pacific oceans. His route is summarized in the table below and shown in the map that follows.

Location	Date	Average low temperature that month (°C)
Christiana, Norway	June 1903	+11
Godhaven, Greenland	July and August 1903	+2
Beechey Island, Nunavut	August 1903	0
King William Island, Nunavut	Winter 1903 – summer 1905	-37 (winter) +3 (summer)
Victoria Island, Nunavut	August 1905	+4
King Point, Yukon	December 1905	-28
Nome, Alaska	August 1906	+7

- a** What is the difference between summer and winter temperatures on King William Island? **Show** your working.
- b** Find which is colder, King Point or King William Island (winter) and by how many degrees. **Show** your working.
- c** Find the largest temperature difference between two points on the trip. **Show** your working.
- d** What special equipment or precautions do you think Amundsen took in order to be prepared for such a large temperature swing? **Explain**, using at least two examples.



11 Evaluate these calculations that involve multiple operations.

a $-5 + 2(-3)$

c $(-3)^2 - (-18) \div (-3)$

e $-16 \div (-8) + (-5)(-3)$

g $-6 - 4 - 2(-3)^2$

i $\frac{-6(4) + (-18) \div (+2) - 12}{2 - 9(3) + 10}$

k $[(-5)(-12)] \div (+6) + (-5)$

b $10 \div -2 - (-12) \div 6$

d $-24 \div (-2)^3 - (-8)$

f $\frac{-10 + (-3)(6)}{2 + 3(4)}$

h $\frac{-2(5^2)}{-5 - (-3) + (-2)(4)}$

j $(-4)^2 - (-2)^3 + (-10)(3)$

12 By the 18th century, there were at least 35 different temperature scales in existence across the planet. For explorers, that potentially meant having to convert from one system to another, just to make sure they had the correct gear. In order to convert a temperature from degrees Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$), you use the following formula.

$$\text{C} = 5(\text{F} - 32) \div 9$$

Find the Celsius temperature that is equivalent to each of the following Fahrenheit temperatures.

a 50°F

b 5°F

c -4°F

d -13°F

e -40°F

13 Create an expression using integers and at least four different operations from BEDMAS (brackets, exponents, division/multiplication, addition/subtraction) where the answer is:

- a** 0 **b** 3 **c** -2 **d** -12

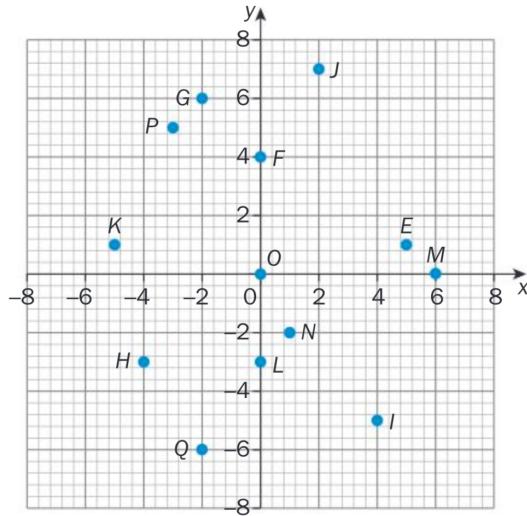
14 If you want to explore your neighborhood, city, country or even the entire planet, you can try either geocache or EarthCache. On a device, you receive the GPS coordinates of a location where someone has hidden some form of ‘treasure’ which could be anything from items to trade, a logbook or information about the unique geological features of the site. If you visit a large number of EarthCaches, you can earn awards for passing certain milestones (50 sites, 100 sites, etc.). You can even create your own caches for others to find.

Answers

Unit Review

- 1** **a** $<$ **b** $>$ **c** $>$ **d** $<$ **e** $>$ **f** $<$
2 **a** $-$ **b** $-$ **c** $-$ **d** $+$ **e** $+$ **f** $+$
3 **a** $-7, -3, -1, 0, 2$ **b** $-12, -6, -2, 1, 5, 7, 9$ **c** $|0|, |-1|, |-2|, |-4|, |8|, |-10|$
4 $A(5, 6); B(-4, 1); C(0, 6); D(5, 0); E(-6, 2); F(-4, -3); G(3, -1); H(-2, 5); I(2, 3); J(-5, 6); K(-1, -1); L(4, -4)$

5



- 6** **a** Individual response **b** Individual response

7 **a**



b Istanbul, Rio de Janeiro

c Reykjavik, Madrid, Istanbul & Rio de Janeiro, Montreal, Islamabad, San Diego, Tokyo, Auckland

d Individual response

8 a 15	b -11	c -3	d -88	e -7
f -2	g -4	h -33	i 24	j 9
k 18	l -42	m -10	n $\frac{5}{8}$	o -6
p -5	q -4	r -6	s -21	t -5
u -6	v 30	w -33	x -4	y -16

- 9** (3, -5) lies in the fourth quadrant with x -coordinate 3 and y -coordinate -5 whereas (-5, 3) lies in the second quadrant with x -coordinate -5 and y -coordinate 3

10 a 40 Degrees Celsius

c Christiana and King William Island by 48 Degrees Celsius

b King William Island by 9 Degrees Celsius

d Individual response

11 a -11	b -3	c 3	d 11	e 17	f -2
g -28	h 5	i 3	j -6	k 5	

- 12 a** 10 Degrees Celsius **b** -15 Degrees Celsius **c** -20 Degrees Celsius
d -25 Degrees Celsius **e** -40 Degrees Celsius

13 Individual response

- 14 a** Great Barrier Reef & Victoria Falls, Machu Picchu, Angkor Wat, Blue Hole, Chichen Itza, Petra, Stonehenge



b 69 degrees

d Change in latitude: +35 degrees
Change in longitude: -114 degrees

c 237 degrees

e Latitude: $+10 = +34 - 38 - 31 + 39 - 34 + 43 - 48 + 45$
Longitude: $+166 = +86 + 106 - 78 - 115 + 16 + 108 + 113 - 70$

15 a 10

b -9

c -16

4

Algebraic expressions and equations

Puzzles and tricks

Unit review



Key to Unit review question levels:

Level 1–2

Level 3–4

Level 5–6

Level 7–8

- 1** Classify each polynomial in as many ways as possible (number of terms, degree, constant/linear/quadratic/cubic/quartic/quintic).
a $3t^2 + 5t - 2$ **b** $4w - 9$
c $m^2y^6 + my^5 - 3m^2y^2 + 8my^4$ **d** $2x^3$
- 2** Simplify these expressions.
a $-4w^3y^4 + 7w^2y + 9w^2y - w^3y^4$
b $8m^3 - 7m + 2 + 10m - 11m^3 - 6$
c $4gh^2 + 8g^2h - 6gh^2 - gh^2 - 9g^2h$

3 Solve these equations.

a $3z + 5 = 14$

b $d - 3 = 2d + 1$

c $6w = -12$

d $2 - z = 14$

4 Solve these equations.

a $3w + 5 = 2w + 7$

b $-6t + 5t + 3 = 4 + t$

c $9r + 3 - 6r - 2 = -2r + 8$

5 Solve these inequalities and represent each solution on a number line.

a $v + 6 > -1$

b $3g \leq -6$

c $2 \leq x - 4$

d $5 > \frac{x}{2}$

6 Use algebra to justify why the following number trick always works.

- Take an integer and add to it the next consecutive integer.
- Add 7 to this result.
- Divide your new number by 2.

- Subtract the original number.
- The result is 4.

7 Solve these equations.

a $\frac{2b}{5} + 10 = 8$

b $\frac{2a}{3} + 3 = 13 - a$

8 Solve the inequalities and represent each solution on a separate number line.

a $3t - 4 < 6t + 5$

b $-2g + 7 \leq 11$

c $2a - 12 > \frac{a}{2}$

9 Justify the following number tricks using algebra.

- a** Take any three consecutive integers and add the highest and lowest numbers. Show that the sum is always double the middle number.
- b** Take any four consecutive integers and add the highest and lowest numbers. Show that the sum is always equal to the sum of the two middle numbers.
- c** Take any five consecutive integers and add the highest and lowest numbers. Show that the sum is always double the middle number and the same as the sum of the other two middle values.

10 Find two numbers that differ by 5, where the sum of the smaller one and four times the larger one is 100.

11 Solve these equations.

a $5(g - 3) + 2 = 3(g + 1)$

b $\frac{4(x - 4)}{3} - 3x = x$

12 Use algebra to justify why the following number tricks always work.

- a** If you take a two-digit number and subtract the number created when you reverse its digits, you always get nine times the difference in the digits.
- b** Show that the mean of a set containing an odd number of consecutive integers is always the middle integer in the set.

13 Find three consecutive numbers such that three times the lowest one plus five times the middle one is equal to 39 less than ten times the largest one.

14 The Atbash cipher was used around 500 BC and maps the alphabet in reverse.

Plain	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A

- a** Find an encryption function for the Atbash cipher.
- b** Write down a message and encode it with this cipher.
- c** Find the inverse of the encryption function. Show that the inverse function decodes your message.

★ 15 You are creating a number trick of your own. You will ask a volunteer to select a number and then you will:

- ask them to *multiply the number by two*.
- choose an even number and ask your volunteer to *add this number to the one in your head*.
- say *divide the result by 2 and subtract your original number*.

What is the result? Justify your answer using algebra.

★ 16 In pairs, try the following card trick to see if you and your partner can figure out how it works using algebra.

Using a standard deck of 52 playing cards, turn the top card of the deck face up.

Begin counting from the face value of this card, turning cards up from the deck until you have counted to 14.

- For the picture cards: you count jacks as 11, queens as 12 and kings as 13. Aces count as 1.
- For example, if you turned over a 6 first, you would turn over eight more cards to put in that pile ($14 - 6 = 8$).

Next, you form three more piles using the exact same process.

Then turn all four piles face down.

Discard leftover cards in a separate pile.

Have a peer pick up two of the piles and put them in the discard pile – without you seeing which piles they removed so you do not know the cards.

Then have your peer turn up the top card of one of the two piles remaining.

Deal 22 cards from the discard pile and place off to the side.

Remove the number of cards from the discard pile that corresponds to the face value of the card your peer turned up.

Count the number of cards left in the discard pile. It will equal the face value of the card on the top of the last pile.

Answers

Unit Review

- 1 a** Three terms, degree 3, quadratic
c Four terms, degree 8, polynomial

2 a $-5w^3y^4 + 16w^2y$

b $-3m^3 + 3m - 4$

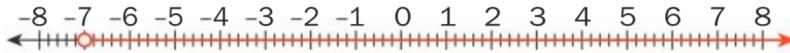
3 a $z = 3$

b $d = -4$

4 a $w = 2$

b $t = -\frac{1}{2}$

5 a $v > -7$



b $g \leq -2$



c $x \geq 6$



d $x < 10$



6 n

$$n + (n+1) = 2n + 1$$

$$2n + 1 + 7 = 2n + 8$$

$$(2n + 8) \div 2 = n + 4$$

$$n + 4 - n = 4$$

7 a $b = -5$

b $a = 6$

8 a $t > -3$

b $g \geq -2$

c $a > 8$

9 a $n + (n+2) = 2(n+1)$

b $n + (n+3) = 2n+3 = (n+1)+(n+2)$

c $n + (n+4) = 2n+4 = 2(n+2) = (n+1)+(n+3)$

10 16, 21

11 a $g = 8$

b $x = -2$

12 a Let the original number be $10a+b$ where $0 \leq a, b \leq 9$

$$10a+b - (10b+a) = 9a - 9b = 9(a-b)$$

b Let there be $2m+1$ consecutive integers $\{n, n+1, \dots, n+2m\}$ so that the middle integer is $n+m$. Then the mean of these numbers is

$$\frac{n+(n+1)+\dots+(n+2m)}{2m+1} = \frac{(2m+1)n + \frac{1}{2}(2m)(2m+1)}{2m+1} = n+m$$

as required

13 12, 13, 14

14 Individual response

15 Half of the even number

16 Individual response

5

2D and 3D geometry Human and natural landscapes

Unit review



Key to Unit review question levels:

Level 1–2

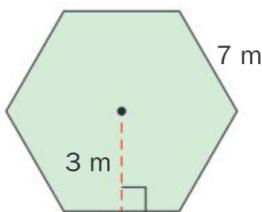
Level 3–4

Level 5–6

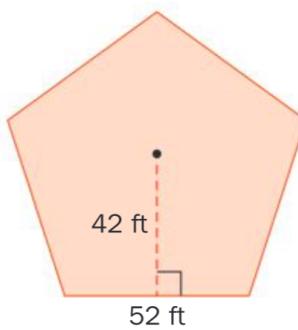
Level 7–8

- 1 Find the perimeter and area of the following shapes. Round answers to the nearest tenth where necessary.

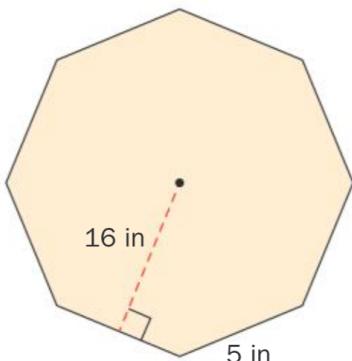
a



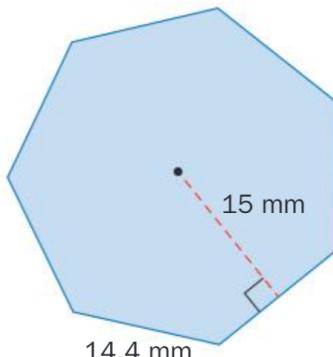
b

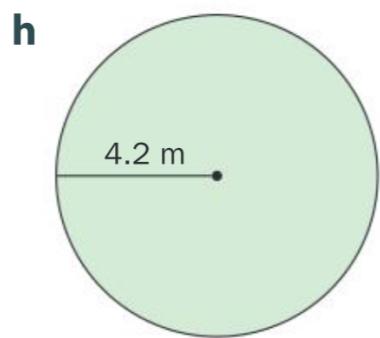
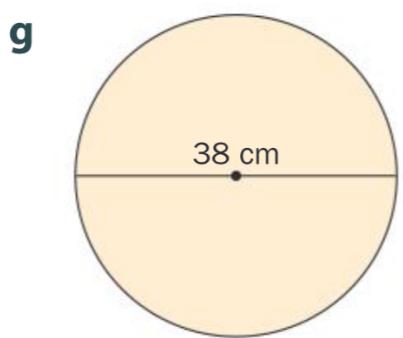
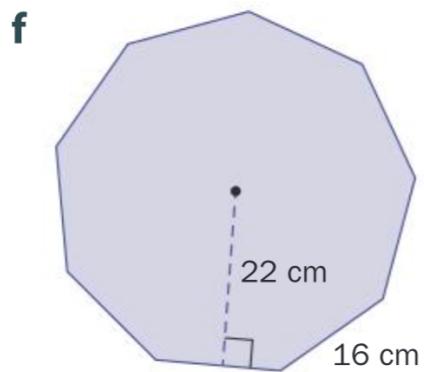
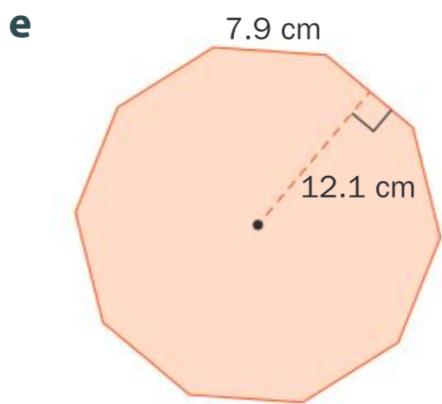


c

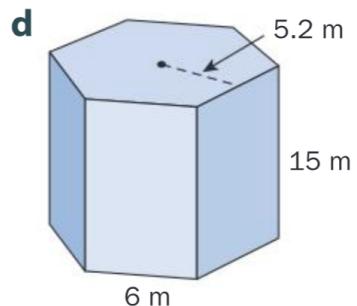
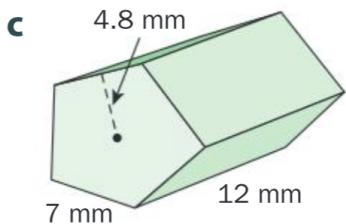
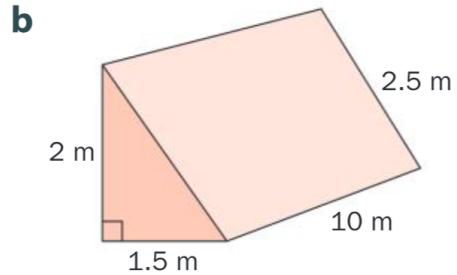
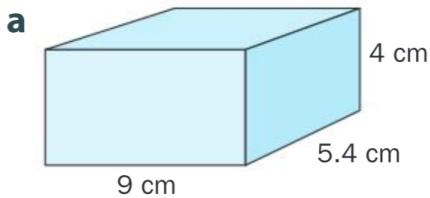


d

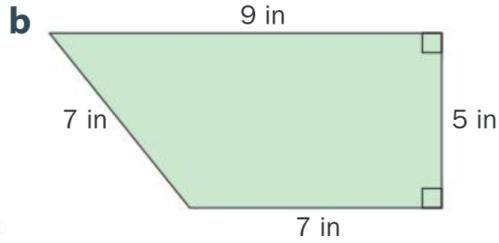
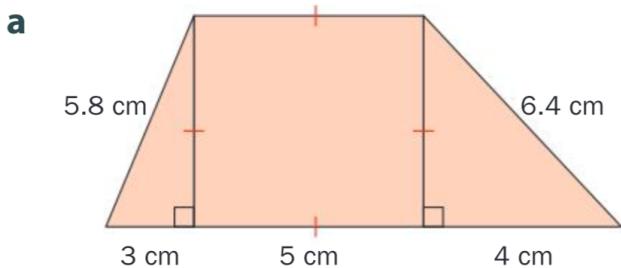




2 Find the volume and surface area of the following shapes.



3 Find the perimeter and area of these shapes.



- 4 Assuming each figure is a regular polygon, find the missing values in the following table. Round answers to the nearest tenth where necessary.

Polygon	Side length (cm)	Apothem (cm)	Area (cm ²)
Pentagon	9		92.3
Hexagon		10	144
Octagon	24		672.5
Decagon		8	1200

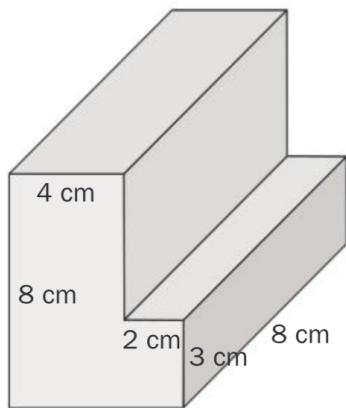
- 5 Antonio Gaudí was one of the most celebrated architects in Spain, designing houses, churches and even sidewalks. In many parts of Barcelona, you will find sidewalks that he designed in 1904 to represent what it would be like to walk on the bottom of the ocean, complete with shells, starfish and seaweed.

Each of Gaudí's tiles is a regular hexagon with sides measuring 20 cm and an apothem of 17.3 cm.

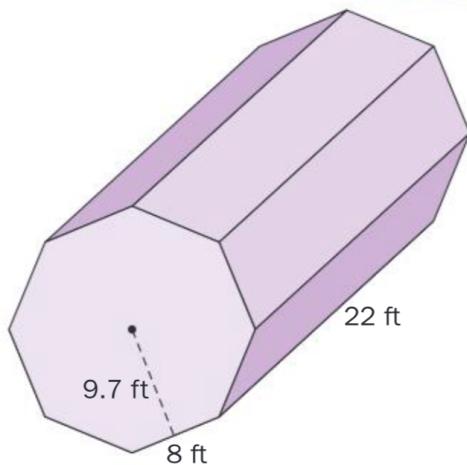


Find the area of a single tile. Show your working.

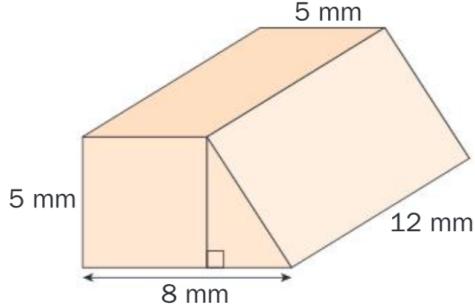
6 a



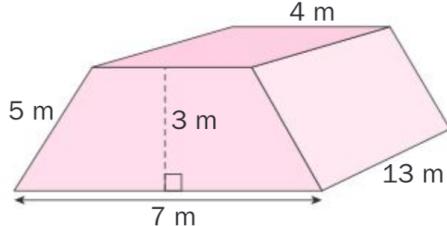
b



c



d



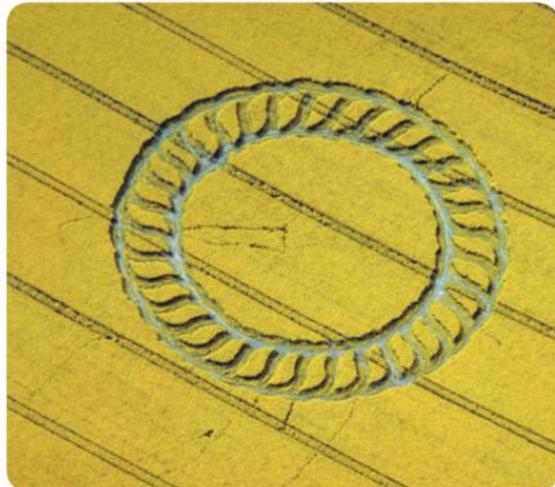
- 7** In a housing development, land is often broken into sections or parcels. These parcels can be kept together or sold individually for other people to build homes or otherwise develop. An example of parcels of land is given on the right.

- a Parcel A has two parallel sides that measure 20 m and 35 m. The perpendicular distance between these sides is 50 m. Find the area of the parcel.
- b Parcel B has the same area as parcel A but its parallel sides measure 23 m and 27 m. Find the perpendicular distance between these parallel sides.
- c Parcel C has parallel sides that measure 20 m and 29 m. The perpendicular distance between them is 47 m. Parcel C is being sold for 75 000 euros. Parcel D has the same perpendicular distance and price in total and per square meter, but one of its parallel sides measures 31 m. Find the measure of the other parallel side of Parcel D.



- 8** Crop circles are phenomena that began appearing in fields in the 1970s. At first attributed to aliens by some people, many have come forward to demonstrate how they are, in fact, a human landscape. The crop circle below was created overnight in a field in Wiltshire, England.

- a If the diameter of the outermost circle measures 105 meters, find the area occupied by the whole crop circle.
- b Find the distance around the crop circle if you walked on the outermost circle.
- c Find the radius of the inner circle if its area is half the area of the whole crop circle.



- 9** Stonehenge in England is one of the most mysterious human landscapes ever constructed. Nobody is exactly sure how the stones (some of which weighed as much as 23 metric tonnes) were moved, much less lifted, to create the structure. An outer circle of sarsen stones surrounds a smaller horseshoe of bluestones, with the structure seeming to point in the direction of the sunrise during the summer solstice. The stones are arranged in a circle with the diameter of 33 metres. It is believed that Stonehenge was built on sacred ground and that among other things it was a burial site. Despite the many theories, scientists are still trying to unravel the mystery of Stonehenge.



The sarsen stones in the outer circle can be modeled by rectangular prisms, with a base measuring 2.1 m by 1.1 m and a height of 4.1 m. It is thought that there were 30 such stones in the outer circle.

- a** What area does Stonehenge cover?
- b** Find the total volume and surface area of the 30 sarsen stones.

- 10** One of the most famous buildings in New York City is the Flatiron building shown here. It was designed to make the most of a small triangular lot, but because of its shape as a triangular prism with a height of 285 feet, most people initially thought it would eventually fall down. Over 115 years later, it stands as one of the most iconic structures in the Big Apple. The base of the building is a right-angled triangle with a height of 173 feet and a base measuring 87 feet.

- a** The building was designed to house offices for the company that built it. Find the maximum volume of office space that could be expected.
- b** **Explain** why the actual available space for offices is less than the value you calculated in **a**.

Because all sides of the building face streets, occupants have offices that are very well lit. The middle 12 stories (with a total height of 156 feet) of the building have windows that measure 7 feet by 2 feet. The length of the long side of the building (triangle) is 193.6 m.

- c** What percentage of the surface area of the middle 12 stories is covered in windows if there are a total of 528 windows in these 12 stories?



11 The Marianas Trench is the deepest point on Earth, located in the Pacific Ocean, east of the Philippines. The trench was formed by the process of *subduction*, when one tectonic plate dives under another (in this case, the Pacific plate dives under the Philippine plate). The trench can be modeled by a triangular prism with a base width of 69 km and a base height of 11 km (the deepest part of the ocean). The trench has a length of 2500 km.

- a** Sketch a diagram of the trench and indicate its dimensions.
- b** Find the volume of water contained in the trench if the sides were perfectly smooth.

12 With the invention of the elevator in 1853, it became possible to imagine creating tall buildings where large numbers of people could work or live. City centers were created with these ‘skyscrapers’, often in the shape of rectangular prisms. Regulations on these buildings have evolved so that they now require a specific amount of fresh air, q , to be supplied to the building, where q is measured in m^3/h .

The formula for calculating this fresh air supply is given by:

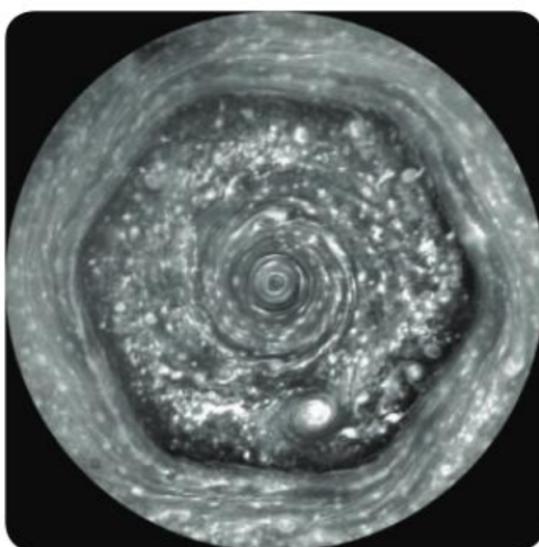
$q = nV$, where n is the number of air changes per hour and V is the volume of the building measured in m^3 .

- a** If the number of air changes per hour needs to be 4, find the fresh air supply for a building in the shape of a rectangular prism with a length of 40 m, a width of 45 m and a height of 250 m.
- b** The length of an office building is 30 m and its width is 50 m. Find the maximum height of the building if it needs 4 air changes per hour and the ventilation system will provide fresh air at a rate of $19\ 500 \text{ m}^3/\text{h}$.

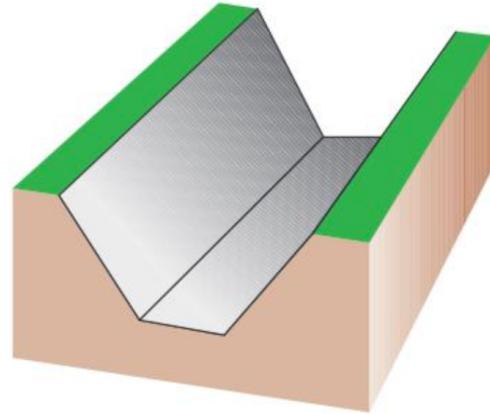


- 13** In 1988, scientists discovered a hexagonal storm pattern on the north pole of the planet Saturn. The sides of the hexagon measure 13 800 km while its apothem measures approximately 12 000 km. Readings from NASA's Cassini mission suggest that the structure has a depth of roughly 100 km.

At the center of the hexagon storm is a swirling circular storm (called a cyclone) that measures approximately 3 000 km in diameter.



- a** Find the area of the hexagonal storm at the north pole.
Show your working.
 - b** Find the percentage of the hexagonal storm's area that is occupied by the central circular cyclone. **Show** your working.
 - c** Find the volume of the hexagonal prism formed by the storm structure. **Show** your working.
 - d** Calculate the percentage of the hexagonal prism's volume that is occupied by the central cylindrical structure.
- 14** On average, Los Angeles, California, receives less than 15 inches of rain every year, but it is home to approximately 4 million inhabitants. Supplying water to this many people requires transporting it from far away through the use of aqueducts, like those shown below.



These channels, which extend over 80 km of land, were built with a bottom width of 25 meters while the width at the top is 45 meters.

These channels, which extend over 80 km of land, were built with a bottom width of 25 meters while the width at the top is 45 meters.

- a** If the vertical height of the aqueduct is 5 m, what is the maximum volume of water the aqueduct could hold over a length of 1 km?
- b** Explain why you think this shape was used instead of a rectangular prism.
- 15** For what radius value will the circumference of a circle be numerically equal to its area?

Answers

Unit Review

- | Polygon | Side length (cm) | Apothem (cm) | Area (cm²) |
|----------------|-------------------------|---------------------|------------------------------|
| Pentagon | 9 | 4.1 | 92.3 |
| Hexagon | 4.8 | 10 | 144 |
| Octagon | 24 | 7.0 | 672.5 |
| Decagon | 30 | 8 | 1200 |
- 1** **a** Perimeter: 42 m
Area: 63 m²
e Perimeter: 79 cm
Area: 477.95 cm²
- 2** **a** Surface Area: 212.4 cm²
Volume: 194.4 cm³
c Surface Area: 588 mm²
Volume: 1008 mm³
- 3** **a** Perimeter: 29.2 cm
Area: 42.5 cm²
- 4** **a** Perimeter: 260 ft
Area: 5460 ft²
f Perimeter: 144 cm
Area: 1584 cm²
- 5** 1038 cm²
- 6** **a** Surface Area: 300 cm²
Volume: 304 cm³
c Surface Area: 351.0 mm²
Volume: 390 mm³
- 7** **a** 1375 m²
b 55 m
- c** Perimeter: 40 in
Area: 320 in²
g Perimeter: 38π cm
Area: 361π cm²
- d** Perimeter: 100.8 mm
Area: 756 mm²
h Perimeter: 8.4 m
Area: 17.64π m²
- b** Surface Area: 63 m²
Volume: 15 m³
d Surface Area: 727.2 m²
Volume: 234 m³
- b** Perimeter: 28 in
Area: 40 in²
- b** Surface Area: 2028.8 ft²
Volume: 6828.8 ft³
d Surface Area: 306 m²
Volume: 214.5 m³
- c** 18 m

- 8 a** 8659.0 m^2 **b** 329.9 m **c** 53.0 m
- 9 a** 855.3 m^2 **b** Surface Area: 30.9 m^2
Volume: 284.1 m^3
- 10 a** 2 144 767.5 feet³
b Individual response (e.g. flooring/ceiling on each floor takes up volume, need for corridors etc.)
c 10.45% (accurate to 2 decimal places)
- 11 a** Triangular prism with base width of 69 km, base height of 11 km, and length of 2 500 km
b $948\,750 \text{ km}^3$
- 12 a** $1\,800\,000 \text{ m}^3/\text{h}$ **b** 3.25 m
- 13 a** $496\,800\,000 \text{ km}^2$ **b** 1.42% **c** $4.968 \times 10^{10} \text{ km}^3$ **d** 1.42%
- 14 a** $175\,000 \text{ km}^3$ **b** Individual response
- 15** 2



6

Rates

Interconnectedness of human-made systems

Unit review



Key to Unit review question levels:

Level 1–2

Level 3–4

Level 5–6

Level 7–8

- 1** Convert the following measurements.
 - a 300 inches = _____ meters
 - b 26.2 miles = _____ km
 - c 30 m = _____ yards
 - d 5 tablespoons = _____ liters
 - e $\frac{3}{4}$ cup = _____ mL
 - f 500 mL = _____ tablespoons
- 2** Find the unit rate for the following situations.
 - a 80 miles per 4 hours
 - b 120 kilometers per 8 hours
 - c 20 inches per 30 days
 - d 15 miles per 40 minutes
- 3** Determine whether the following situations represent a constant rate of change or not.

- a** Marta saves 30 euros a day for 20 days.
 - b** Harry runs for 8 kilometers. He runs the first kilometer in 6 minutes, then runs the rest of the distance in 40 minutes.
 - c** While driving her car, Zara uses cruise control and drives 50 miles per hour for 9 hours.
 - d** The snow melts one inch per hour on the first warm day of spring.
- 4** The fuel economy or gas consumption of a car is given as a rate of distance per volume. A car is advertised as getting 32 miles per gallon in the United States. How would this rate be expressed in Canada (in kilometers per liter) if one gallon is approximately equal to 3.8 liters?
- 5** The track at a local school is known to be 400 meters long. If Cynthia wants to train for a 3.5-mile race, how many laps around the track should she run? **Show** your working.

- 6** Use this table to convert the currencies listed below it.

1 Brazilian real = 9 Uruguayan pesos	1 Rwandan franc = 0.30 Yemeni rials
1 British pound = 1990 Lebanese pounds	1 Australian dollar = 79 Serbian dinars
1 Dominican peso = 0.05 Belize dollars	1 Barbadian dollar = 0.84 UAE dirhams

- a** 60 Brazilian reals to Uruguayan pesos
 - b** 900 Serbian dinars to Australian dollars
 - c** 45 British pounds to Lebanese pounds
 - d** 1500 Yemeni rials to Rwandan francs
 - e** 87 Dominican pesos to Belize dollars
 - f** 2000 UAE dirhams to Barbadian dollars
- 7** Use the table above to decide which quantity is higher.
- a** 700 Brazilian reals or 6500 Uruguayan pesos
 - b** 18900 Lebanese pounds or 12 British pounds
 - c** 150 UAE dirhams or 86 Barbadian dollars
 - d** 13 Rwandan francs or 50 Yemeni rials
- 8** In 1999, the Institute for Safe Medical Practices reported that a patient was given 0.5 grams of a drug instead of 0.5 *grains*, as was written on the label. The same dose was administered for three straight days before the error was found. If 2 grains are equivalent to 0.13 grams, how many grains did the patient receive over the course of the three days? **Show** your working.

- 9** For a school fundraiser, 15 students washed 40 cars in 4 hours.
- Find how long it would take 10 students to wash the same 40 cars. Show your working.
 - If the goal of the fundraiser was to raise \$500 and each driver donated \$10 to have their car washed, for how long will those 10 students be washing cars?
- 10** Joseph is offered a job in Brazil for 8 weeks in the summer, paying 1600 Brazilian reals per week.
- Create a table to represent the money he will make each week for 8 weeks.
 - Construct** a graph using the table you created.
 - Create an equation to represent how much money he will make in any given week.
 - Describe in words what the point on the graph (1, 1600) means in this situation.
- 11** In physics, force can be measured in either newtons (N) or pounds (lb).

In 1999, the Mars Climate Orbiter came too close to Mars and burned up in its atmosphere. Part of the problem was that the software used to control the orbiter's propulsion was calculating force in pounds, but a different piece of software using those calculations was calibrated to use newtons. The miscalculation caused the orbiter to be propelled too close to the surface of the planet where it was destroyed.

20 newtons is roughly 4.5 pounds of force.

- How many pounds of force is equivalent to 3000 newtons? **Show** your working.
- If the first piece of software calculated a value of 2900 pounds, how many newtons would that have been equivalent to?



- 12** Rebecca goes on vacation with 2000 dollars. She is on vacation for 20 days. She spends 100 dollars per day.

- a** Create a table to represent the total amount of money she has spent at the end of each day.
- b** **Construct** a graph using the table you created.
- c** Create an equation to represent how much money she will have spent after any given day.
- d** **Describe** in words what the point on the graph (20, 2000) means in this situation.

Answers

Unit Review

- | | | | | | | |
|----------|----------------------------|---------------------------------|-------------------------------------|---------------------|---------------------------------|---------------------------|
| 1 | a 7.62 meters | b 42.2 km | c 32.8 yards | d 0.1 litres | e 180 mL | f 33.3 tablespoons |
| 2 | a 20 miles per hour | b 15 kilometres per hour | c $\frac{2}{3}$ inch per day | | d 0.375 miles per minute | |

- 3** **a** Constant rate of change
c Constant rate of change
- b** Not a constant rate of change
d Constant rate of change
- 4** 13.6 km/liter
- 5** 14.1 laps
- 6** **a** 540 Uruguayan pesos
d 5000 Rwandan francs
- b** 11.39 Australian dollars
e 4.35 Belize dollars
- c** 89 550 Lebanese pounds
f 2380.95 Barbadian dollars
- 7** **a** 6500 Uruguayan pesos
c 150 UAE dirhams
- b** 12 British pounds
d 50 Yemeni rials
- 8** 23.1 grains
- 9** **a** 6 hours
b 3 hours 20 minutes

10 a

Time (Weeks)	Money earnt (Brazilian reals)
1	1600
2	3200
3	4800
4	6400
5	8000
6	9600
7	11200
8	12800

- b** Money earnt (in Brazilian reals) plotted against time (in weeks). Straight line passing through origin with gradient 1600
- c** Let the money earnt be represented by m and time in weeks be represented by w . Then, $m = 1600w$
- d** After 1 week, Joseph earnt 1600 Brazilian reals

11 a 675 pounds of force

b 12888.9 newtons

12 a

Day	Total spent (\$)
1	100
2	200
3	300
4	400
5	500
6	600
7	700
8	800
9	900
10	1000
11	1100
12	1200
13	1300
14	1400
15	1500
16	1600
17	1700
18	1800
19	1900
20	2000

b Dollars spent plotted against days on vacation. Straight line through origin, with gradient 100, from (0,0) to (20,2000).

c Let money spent be represented by m and days spent on vacation be represented by d . Then, $m = 100d$

d This point represents how much money in total Rebecca has spent after 20 days on her vacation.

7

Univariate data

Accessing equal opportunities

Unit review



Key to Unit review question levels:

Level 1–2

Level 3–4

Level 5–6

Level 7–8

- 1** Draw a stem-and-leaf plot (and key) for each of the given data sets.
 - a** 61, 72, 64, 70, 79, 83, 64, 61, 80, 65, 73, 79
 - b** 3.4, 5.1, 4.7, 5.5, 3.0, 3.9, 4.1, 5.5, 4.6, 4.9, 6.8
 - c** 1210, 1350, 1270, 1390, 1280, 1400, 1360, 1260, 1350, 1200, 1410, 1560, 1200
- 2** Find the measures of central tendency for the data sets in question 1. Show your working and give answers to the nearest tenth where necessary.
- 3** Find the range, quartiles and interquartile range for the data sets in question 1.

- 4** The low-income countries with the highest level of spending on education (as a percentage of overall government spending) are represented in the stem-and-leaf plot below.

18	0	6	6	9
19	0	7		
20	1	5	7	
21	1	6		
22	3			
23	4			
24	1			
25	5			

Key: 21 | 1 = 21.1

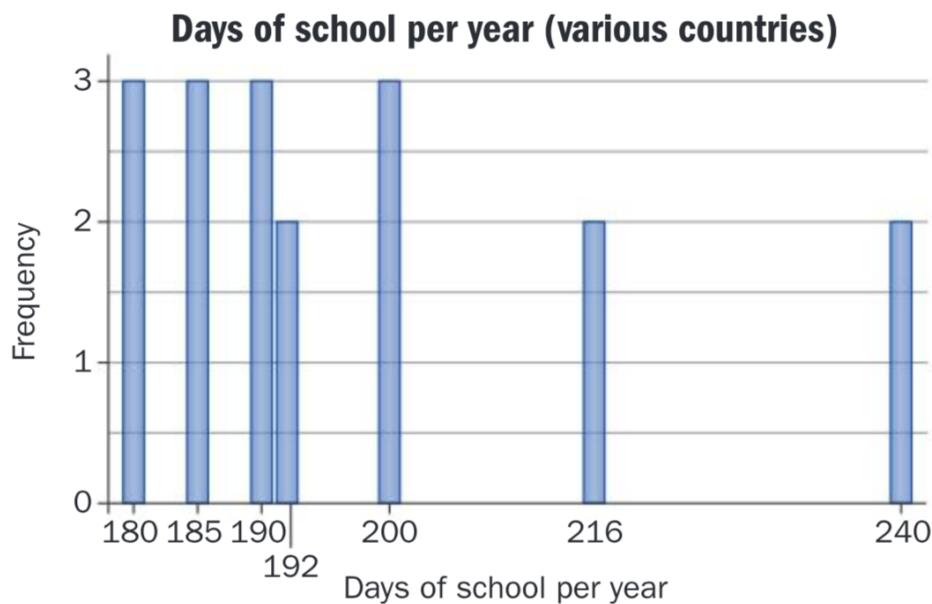
- a** What is the highest percentage of spending on education?
What is the lowest?
- b** Finland is one of the top developed countries in terms of spending on education, spending roughly 19% of its total government budget. How many countries spent more than Finland?
- c** Find the mean, median and mode of the data set. Show your working and give answers to the nearest tenth where necessary.

- 5** Literacy rates can be a measure of the quality of the educational system in a country. Data for the 15 high-income countries and 15 low-income countries with the lowest literacy rates are summarized below.

High-income countries (%)	Low income countries (%)
95	19
95	27
97	30
97	36
98	37
98	38
98	38
98	39
99	40
99	43
99	44
100	48
100	48
100	49
100	52

- a** Represent each set of data using a box-and-whisker plot.
Draw both plots on the same axis.

- b** Can you tell whether or not there is an unequal access to education in low and high-income countries? Which group seems to have better access? Use your representations to support your conclusion.
- 6** Students in high and low-income countries attend school for a specified number of days per year, as represented below.



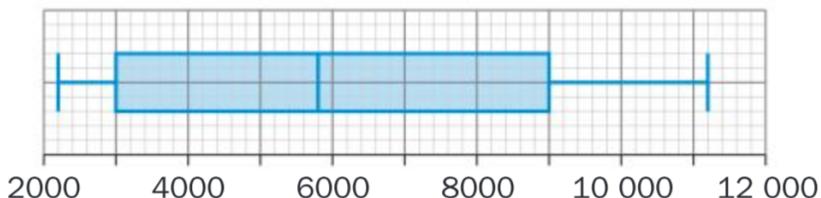
- a** Convert the data to a stem-and-leaf plot as well as a box-and-whisker plot.
- b** What are the advantages and disadvantages of each of the three representations? **Explain**.
- c** Two of the countries in the bar graph are low-income countries. How long do you think their school year is? **Explain**.
- d** What other information would be important to know when comparing how long students in different countries are in school? **Explain**.
- 7** The population density is a measure of how many people live in a given area. The highest population densities in the world are given here, measured in the number of people per square kilometer.

Lower-income countries	
Maldives	1392
Bangladesh	1252
West Bank and Gaza	756
Mauritius	622
Lebanon	587
Republic of Korea (North Korea)	526
Rwanda	483
India	445
Comoros	428

Higher-income countries	
Monaco	19 250
Singapore	7909
Gibraltar	3441
Bahrain	1848
Malta	1365
Bermuda	1307
St Maarten (Dutch)	1177
Channel Islands	866
Barbados	663

- a** Are there any outliers in the two data sets? **Explain.**
- b** **Find** the measures of central tendency with and without the outlier(s).
- c** Which measure of central tendency do you think best represents the data? **Explain.**
- d** Do your calculations indicate an inequality between high and low-income countries when it comes to population density? **Explain**, being sure to state which group of countries benefits the most.
- e** Propose a reason for the differences between high and low-income countries.
- f** What must it be like to live in a place with **(i)** a high population density, **(ii)** a low population density? Which would you prefer? **Explain.**

- 8** The following is a box-and-whisker plot of the average annual income (Australian dollars, AUD) per person per country in the Oceania region.



- a** What is the minimum average annual income? What must it be like to live on that amount of money per year? **Explain**.
- b** 75% of all average annual incomes are below what amount?
- c** Two outliers were not included in the graph: one country with an average income of 50 000 AUD and the other with an average income of 58 000 AUD. Research the countries in the Oceania region. Which two countries do you think these outliers represent?
- d** If you created a new box-and-whisker plot to include the two countries in part **c**, **describe** how the new box-and-whisker plot would differ from the current one.
- e** Do you think it is preferable to have these two countries listed as outliers? **Explain**.

- 9 Access to improved sanitation facilities can have a large impact on the health of individuals and communities. The following tables show countries with the lowest percentage of the urban population that have access to improved sanitation.

Lower-income countries		Higher-income countries	
South Sudan	16%	Ireland	89%
Madagascar	18%	Guam	90%
Democratic Republic of Congo	20%	Latvia	91%
Ghana	20%	Trinidad and Tobago	92%
Sierra Leone	23%	The Bahamas	92%
Togo	25%	Cayman Islands	96%
Ethiopia	27%	Barbados	96%
Liberia	28%	Uruguay	97%
Uganda	29%	Lithuania	97%

-
- a** Find the five-number summary for each set of data.
 - b** Represent each data set using a box-and-whisker plot.
Plot the two graphs on the same axis.
 - c** What does the box-and-whisker plot indicate about the equality of access to improved sanitation in urban areas?
Explain.
 - d** Find the measures of central tendency for the two sets of data.
 - e** Which measure of central tendency do you think most accurately represents the data sets? **Explain.**
 - f** Does your chosen measure of central tendency reveal an unfairness in access to improved sanitation between high and low-income countries? **Explain.**
 - g** What other data or information might affect your decision as to whether or not there is inequality between these two groups? **Explain.**
 - h** What must it be like to live in a place where the access to improved sanitation is low, when you know that it is much higher in many other regions? **Explain.**

- 10** **a** Create a set of data with at least eight values that has a mean of 10, a median of 11 and mode of 8.
- b** Add an outlier that will increase the mean to 15.
- 11** **a** While doing research on developed countries and the percentage of homelessness, Mesfin found the average of ten countries to be 5%. After he included data on another country, the average jumped to 9%. What was the new data value that he found?
- b** With a mean value of 9% for eleven countries, Mesfin finds a country that has 3% homelessness. Find the mean value for the twelve countries.

- 12** GDP per capita generally shows the relative performance of countries, taking into account the cost of living within that country. In the Middle East, GDP per capita data (per thousand) are listed in the tables below.

Country	GDP per capita (per 1000)
Bahrain	30
Iran	18
Iraq	17
Israel	35
Jordan	11
Kuwait	71
Lebanon	19

Country	GDP per capita (per 1000)
Oman	44
Qatar	130
Saudi Arabia	54
Syria	3
Turkey	21
UAE	68
Yemen	3

- a** For the above data, **calculate** the mean GDP per capita for the Middle East region.
- b** Do you think Qatar is an outlier in this data set? **Justify** why or why not.
- c** Take Qatar out of the data and recalculate the mean GDP per capita.
- d** **Comment** on the effect that an extreme value has on the mean.
- e** Do some research and try to **explain** why Qatar has such a high value in comparison to the surrounding countries.

- f** Draw a box-and-whisker for this data set. For the purpose of this question, we will use a statistical definition of an outlier, which is, a data point that is either:

$3 \times$ the interquartile range or more *above* the third quartile,

or

$3 \times$ the interquartile range or more *below* the first quartile.

Use this rule to reassess whether or not Qatar would be considered an outlier.

Answers

Unit Review

1 a	Stem Leaf
	6 1 1 4 4 5
	7 0 2 3 9 9
	8 0 3

Key: 6 | 1 = 61

b	Stem	Leaf
	3	0 4 9
	4	1 6 7 9
	5	1 5 5
	6	8

Key: 3 | 0 = 3.0

C	Stem	Leaf
	12	00 00 10 60 70 80
	13	50 50 60 90
	14	00 10
	15	60

Key: 12 | 00 = 1200

- 2** **a** Mean: 70.9
Median: 71
Two modes: 61 and 79

b Mean: 4.7
Median: 4.7
Mode: 5.5

c Mean: 1326.2
Median: 1350
Two modes: 1200 and 1350

3 **a** Range: 22; Lower Quartile: 64; Upper Quartile: 79; Interquartile Range: 15
b Range: 3.8; Lower Quartile: 3.9; Upper Quartile: 5.5; Interquartile Range: 1.6
c Range: 360; Lower Quartile: 1235; Upper Quartile: 1395; Interquartile Range: 160

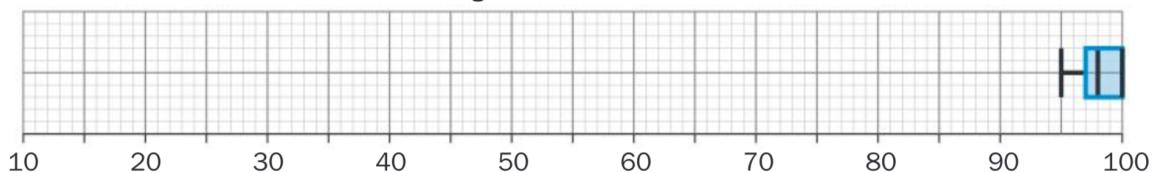
4 **a** Highest: 25.5%
Lowest: 18.0%

b 10

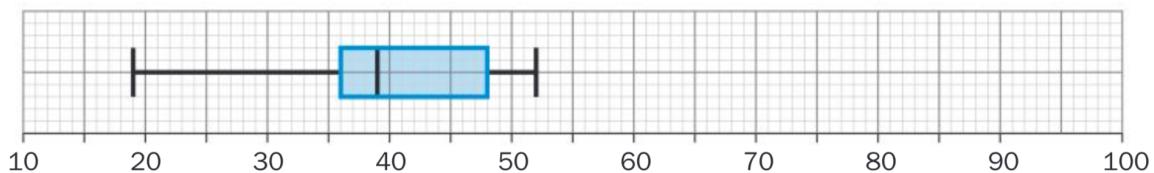
c Mean: 20.8
Median: 20.5
Mode: 18.6

5 a

High-income countries



Low-income countries

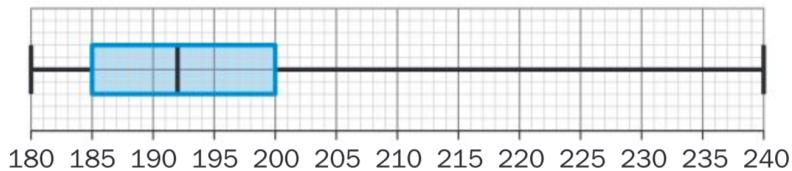


b Individual response

6 a

Stem	Leaf
18	0 0 0 5 5 5
19	0 0 0 2 2
20	0 0 0
21	6 6
22	
23	
24	0 0

Key: 18 | 0 = 180 days of school per year



7 a Low-income countries: Maldives and Bangladesh
High-income countries: Monaco

b Low-income countries with outliers: Mean: 721.2

Median: 587

Mode: None

Low-income countries without outliers: Mean: 549.6

Median: 526

Mode: None

High-income countries with outliers: Mean: 4202.9

Median: 1365

Mode: None

High-income countries without outliers: Mean: 2322

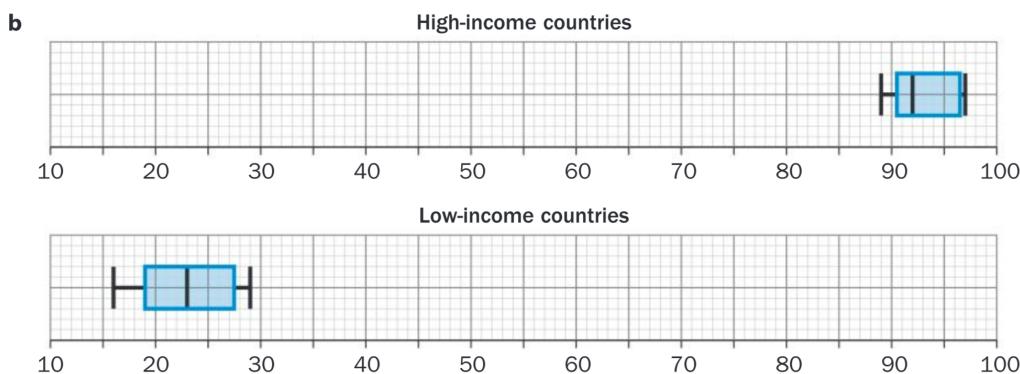
Median: 1336

Mode: None

c Individual response d Individual response e Individual response f Individual response

- 8** **a** 2200 AUD **b** 9000 AUD **c** Individual response
 Individual response
d The new box-and-whisker plot would have the same lower value, but the much higher new upper value would impact the median and the two quartiles.
e Individual response

- 9** **a** Low-income countries: Smallest value = 16%, Q1 = 19%, Median = 23%, Q3 = 27.5%, Largest value = 29%
 High-income countries: 89%, 90.5%, 92%, 96.5%, 97%



- c** Individual response (e.g. High-income countries have greater access to improved sanitation in urban areas than low-income countries)
d Low-income countries: Mean: 22.9% High-income countries: Mean: 93.3%
 Median: 23% Median: 92%
 Mode: 20% Mode: 92% and 96% and 97%
e Individual response
f Individual response (e.g. Yes the mean/median/mode of the high-income countries is a significantly higher percentage than the mean/median/mode for the low-income countries, suggesting an unfairness in access to improved sanitation)
g Individual response **h** Individual response

10 Individual response

- 11** **a** 49% **b** 8.5%
12 **a** 37.4 **b** Individual response **c** 30.3
d Individual response (e.g. an extreme value artificially shifts the mean towards the direction in which the value is extreme)
e Individual response

