CPE/EE 695: Applied Machine Learning

Lecture 10-1: Instance-Based Learning

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Instance-Based Learning

Key idea: just store all training examples $\langle x_i, f(x_i)\rangle$

Nearest neighbor:

- Given query instance x_q, first locate nearest training example x_n, then estimate $\hat{f}(x_q) \leftarrow f(x_n)$

k-Nearest neighbor:

- Given x_q, take vote among its k nearest nbrs (if discrete-valued target function)
- take mean of f values of k nearest nbrs (if real-valued)

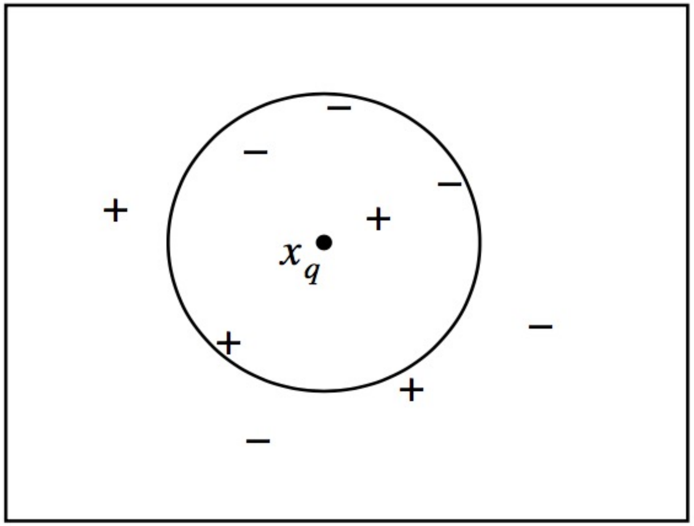
$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^{k} f(x_i)}{k}$$

k Nearest Neighbor (kNN)

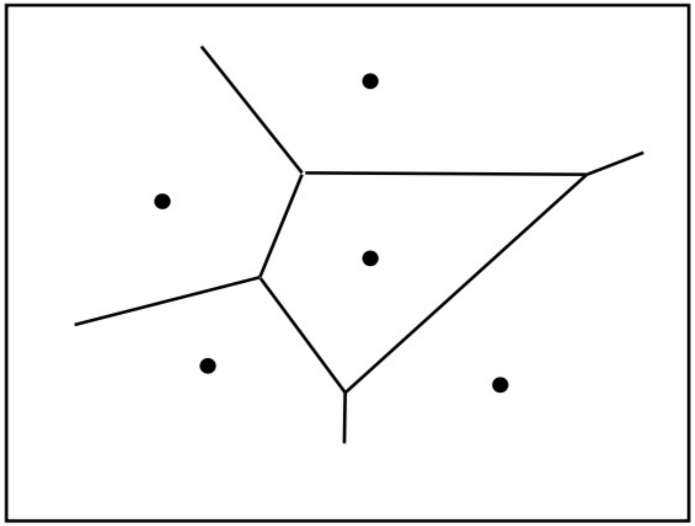
Euclidean distance between two instances is calculated:

$$d(x_i, x_j) \equiv \sqrt{\sum_{r=1}^{n} (a_r(x_i) - a_r(x_j))^2}$$

where an instance is described by the feature vector: $\langle a_1(x), a_2(x), \ldots a_n(x) \rangle$



5-NN



1-NN (Voronoi Diagram)

When To Consider kNN

- Instances map to points in $\Re^n$
- Less than 20 attributes per instance
- Lots of training data

Advantages:

- Training is very fast
- Learn complex target functions
- Don't lose information

Disadvantages:

- Slow at query time
- Easily fooled by irrelevant attributes

Distance Weighted kNN

Might want weight nearer neighbors more heavily...

$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^{k} w_i f(x_i)}{\sum_{i=1}^{k} w_i}$$

where

$$w_i \equiv \frac{1}{d(x_q, x_i)^2}$$

and $d(x_q, x_i)$ is distance between x_q and x_i

Note now it makes sense to use *all* training examples instead of just k

$\rightarrow$ Shepard's method

Curse of Dimensionality

Imagine instances described by 20 attributes, but only 2 are relevant to target function

Curse of dimensionality: nearest nbr is easily mislead when high-dimensional X

One approach:

- Stretch jth axis by weight z_j, where $z_1, \ldots, z_n$ chosen to minimize prediction error
- Use cross-validation to automatically choose weights $z_1, \ldots, z_n$
- Note setting z_j to zero eliminates this dimension altogether

Locally Weighted Regression

Note kNN forms local approximation to f for each query point x_q

Why not form an explicit approximation $\hat{f}(x)$ for region surrounding x_q

- Fit linear function to k nearest neighbors
- Fit quadratic, ...
- Produces "piecewise approximation" to f

Several choices of error to minimize:

- Squared error over k nearest neighbors

$$E_1(x_q) \equiv \frac{1}{2} \sum_{x \in\ k\ nearest\ nbrs\ of\ x_q} (f(x) - \hat{f}(x))^2$$

- Distance-weighted squared error over all nbrs

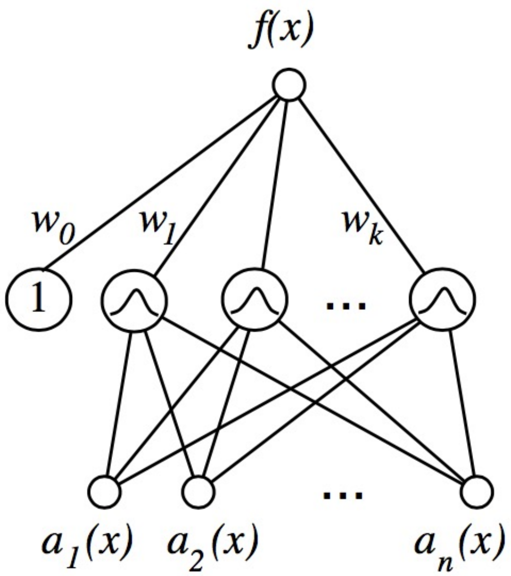
$$E_2(x_q) \equiv \frac{1}{2} \sum_{x \in D} (f(x) - \hat{f}(x))^2\ K(d(x_q, x))$$

- ...

Radial Basis Functions

- Global approximation to target function, in terms of linear combination of local approximations
- Used, e.g., for image classification
- A different kind of neural network
- Closely related to distance-weighted regression, but "eager" instead of "lazy"

Radial Basis Function Network



where $a_i(x)$ are the attributes describing instance x, and

$$f(x) = w_0 + \sum_{u=1}^{k} w_u K_u(d(x_u, x))$$

One common choice for $K_u(d(x_u, x))$ is

$$K_u(d(x_u, x)) = e^{-\frac{1}{2\sigma_u^2} d^2(x_u, x)}$$

Lazy versus Eager Learning

Lazy Learing

Defers decision to generalize beyond training data until each new query instance is encountered

examples include k-nearest neighbor, locally-weighted regression, and case-based reasoning

Eager Learning

Generalizes beyond the training data before observing the new query

examples include Decision Tree Learning algorithms such as ID3 and ANNs

Reference

The lecture notes in this lecture are mainly based on the following textbooks:

T. M. Mitchell, Machine Learning, McGraw Hill, 1997. ISBN: 978-0-07-042807-2

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