Numerical Computing using Python
Lecture 6

2020-07-30

Optimization $\longleftrightarrow$ minimization of a function

~~maximization?~~

"best"

— cost function
— objective function

City
$B \to f(x) = |s_1| + |s_2| + |s_3| + \cdots + |s_N|$

City
A

s_1 s_2

P1

optimal path

$f(x) = s_1^2 + s_2^2 + s_3^2 + \cdots + s_N^2$

shortest distance   ✓

shortest time $\begin{cases} \text{distance} & ✓ \\ \text{traffic} & ✓ \end{cases}$

Calculus

f | Convex optimization
$f(a)$ $f(b)$

$x = a$ b x

hard to solve!

f non-convex optimization

x

$f(x_1, x_2)$

x_2

x_1

f

x

$-f$

x

Convex:
if $f(a) < f(x) < f(b)$
then $a < x < b$

$f(a)$
$f(b)$
$f(x)$
$f(a)$

a b

$y = x^2$

y

x

$$\underline{\text{rational number}} = \frac{\text{integer}}{\text{integer}} \qquad \text{e.g.} \; \boxed{\frac{1}{2}}$$

$$\frac{1}{2} + \frac{1}{3}$$

↙ floating pt. (F.P.)

error ≃ machine accuray

32-bit F.P.	64-bit F.P.
$\sim 10^{-8}$	$\sim 10^{-16}$

Numerically: $1/2 \rightarrow 0.5$ $\Big\}$ $0.833333\cdots$

$1|3 \rightarrow 0.33333\cdots$

Symbolically $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \boxed{\frac{5}{6}}$ ← rational

No error

(exactly)

$$f(x) = \sqrt{x_1^2 + x_2^2}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \left(x_1^2 + x_2^2\right)^{\frac{1}{2}}$$

$$\frac{\partial f}{\partial x_1} = \frac{1}{2}\left(x_1^2 + x_2^2\right)^{-\frac{1}{2}} \cdot \left(2x_1 + 0\right)$$

$$= \frac{x_1}{\left(x_1^2 + x_2^2\right)^{\frac{1}{2}}} = \frac{x_1}{f}$$

$$f(x) = \log(x^2 + 1)$$

$$\frac{df}{dx} = \frac{1}{x^2 + 1} \cdot 2x$$