Numerical Computing using Python

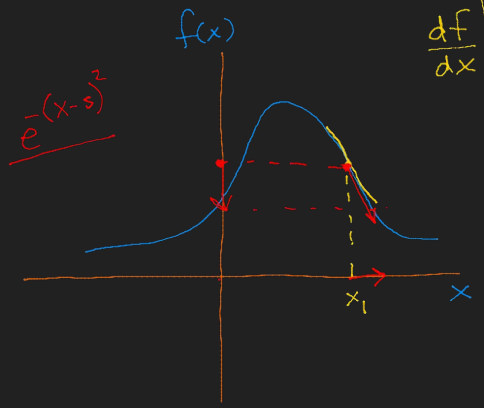
Lecture 7

2020-08-06

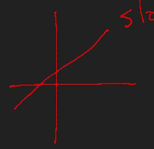
Differentiation

"derivative"

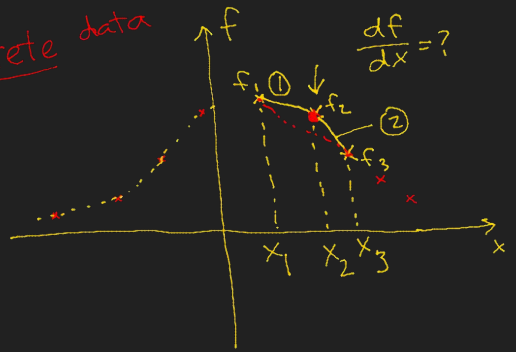
$$\begin{cases} f(x) = x^3 \\ f'(x) = \dfrac{df}{dx} = 3x^2 \end{cases}$$



$$\left.\frac{df}{dx}\right|_{x=x_1} = \text{slope of } f(x) \text{ at } x = x_1$$



discrete data

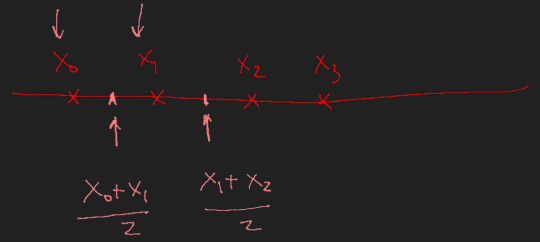


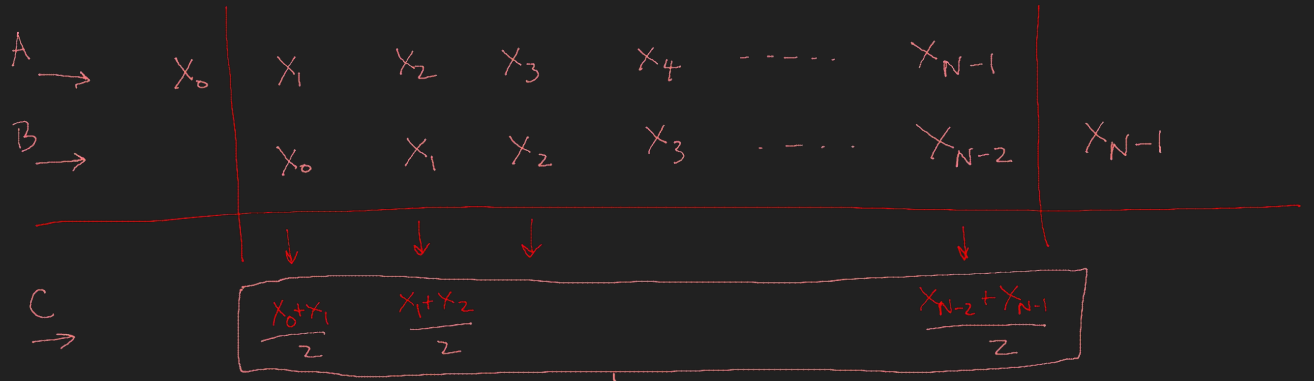
$$\text{slope}\,① = \frac{f_2 - f_1}{x_2 - x_1} \quad \text{(straight line)}$$

$$\text{slope}\,② = \frac{f_3 - f_2}{x_3 - x_2} \quad "$$

$$\text{slope at } x_2 \sim \frac{f_3 - f_1}{x_3 - x_1}$$

$$X = [X_0, X_1, X_2, X_3, X_4 \ldots X_{N-1}]$$



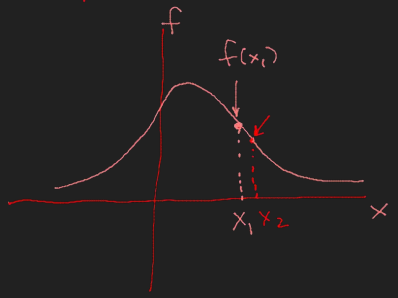


$A = X[1:]$

$B = X[:-1]$

$C = (A+B)/2$

Taylor series

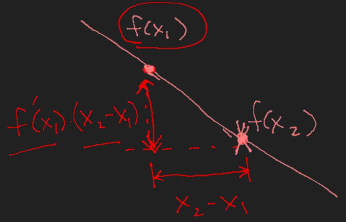


derivative

Know: $f(x_1)$, $f'(x_1)$, $f''(x_1)$, $f'''(x_1)$, ···· at x_1

→ can estimate function value at x_2

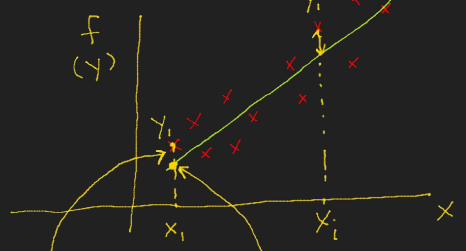
$$f(x_2) = \underbrace{f(x_1) + f'(x_1)\cdot(x_2-x_1) + \frac{(x_2-x_1)^2}{2!} f''(x_1) + \frac{(x_2-x_1)^3}{3!} f'''(x_1) + \frac{(x_2-x_1)^4}{4!} f''''(x_1) + \cdots}_{\text{info at } x_1}$$



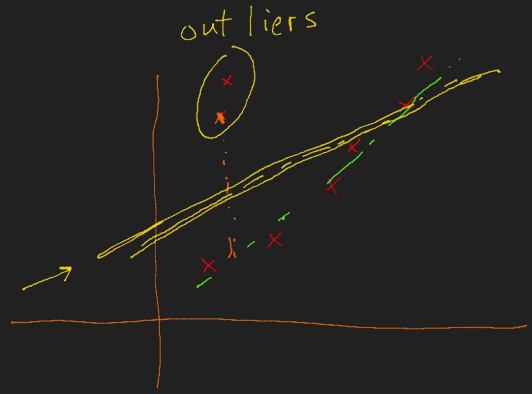
$3! = 1 \times 2 \times 3 = 6$

$n! = 1 \times 2 \times 3 \times \cdots \times n$

(LS)
Least squares



Given Given
$Y = aX + b$
parameters to estimated (computed)



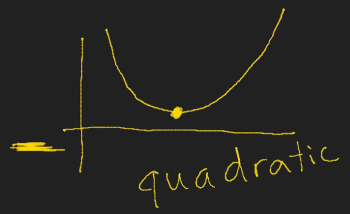
Minimize Error

"best" "optimal"

$$E = \left[Y_1 - (ax_1 + b)\right]^2 + \left[Y_2 - (ax_2 + b)\right]^2 + \cdots + \left[Y_N - (ax_N + b)\right]^2$$

data line

① (remove) delete outliers then compute LS problem



$$E_{RLS} = \left[Y_1 - (ax_1 + b)\right]^2 + \cdots + \left[Y_N - (ax_N + b)\right]^2 + \lambda \|a\|^2$$

② RLS = regularized LS

$\lambda = 0$: LS
$\lambda > 0$: RLS