Homework 1

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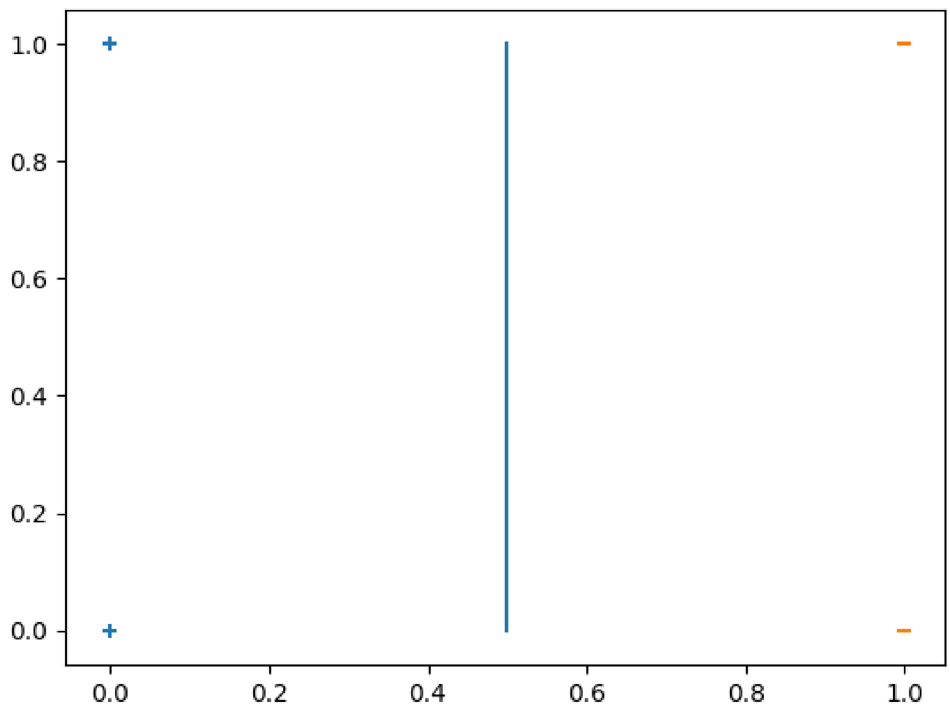
1 Perceptron Algorithm and Convergence Analysis

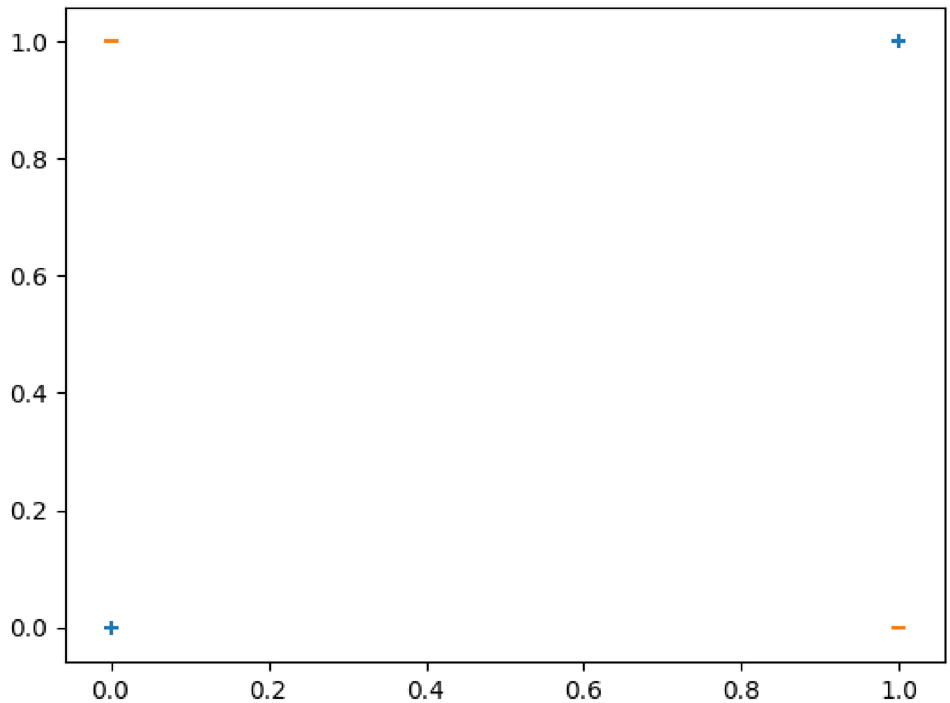
1. (a) $y = f(x_1, x_2) = x_1$



(b) A truth table for this function could be:

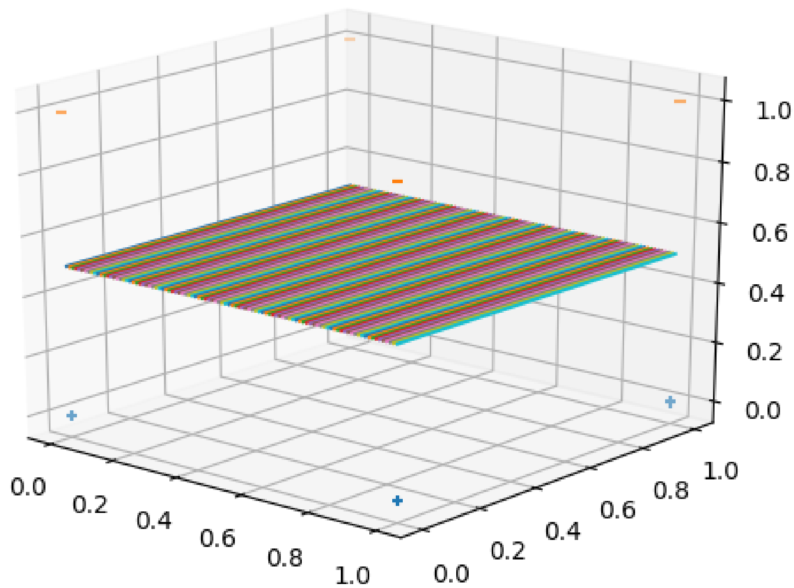
x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

We can plot those points in a figure.



The perceptron algorithm gives us a weight vector which corresponds to its orthogonal hyperplane to divide points into 2 parts. In this figure since we can't find a hyperplane to divide the 2 kinds, this function can't be separated by single perceptron.

(c) $y = f(x_1, x_2, x_3) = x_3$



2. β is a weight vector perpendicular to the hyperplane the signed Euclidean Distance of the point x to the hyperplane is given by:

$$d = \frac{(x - x_0) \cdot \beta y}{\|\beta\|_2}$$

where

$$x_0 \in \{x | f(x) = 0\}$$

$$\because f(x_0) = \beta_0 + \beta^T x_0$$

$$\therefore \beta^T x_0 = -\beta_0$$

$$
\begin{aligned}
d &= \frac{y(x \cdot \beta - x_0 \cdot \beta)}{\|\beta\|_2} \\
&= \frac{y(\beta^T x - \beta^T x_0)}{\|\beta\|_2} \\
&= \frac{y(\beta^T x + \beta_0)}{\|\beta\|_2} \\
&= \frac{y f(x)}{\|\beta\|_2}
\end{aligned} \tag{1}
$$

3. By perceptron convergence theorem we get that the perceptron algorithm makes at most $\frac{1}{\delta^2}$ mistakes where $\frac{1}{\delta^2}$ is the lower bound for $y_i(w^{sep^T} x_i)$ and w^{sep} is the "good separator":

$$T \leq \frac{1}{\delta^2}$$

where

$$y_i(w^{sep} x_i) \geq \delta$$

Since

$$y_i w^{sep} x_i = 1$$

we have

$$T \leq 1$$

Since $w^0 = 0$, $\|w^{sep}\| = 1$, we have:

$$\|w^{(0)} - w^{sep}\|_2^2 = \|w^{sep}\|_2^2 = 1$$

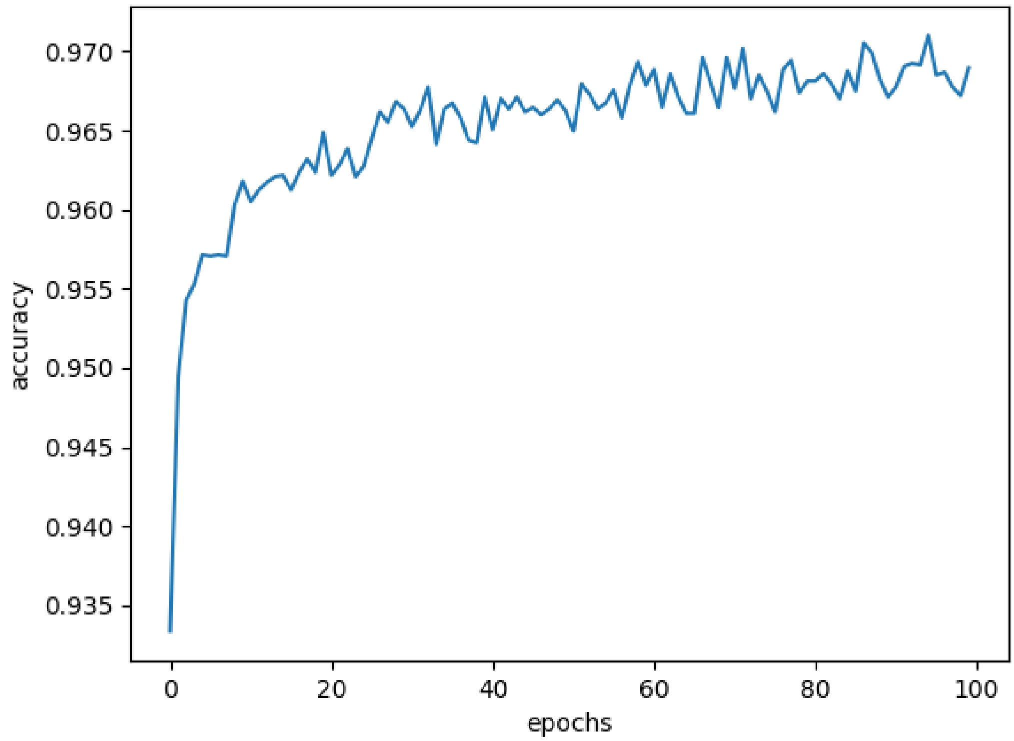
Therefore,

$$T \leq \|w^{(0)} - w^{sep}\|_2^2$$

2 Programming Assignment

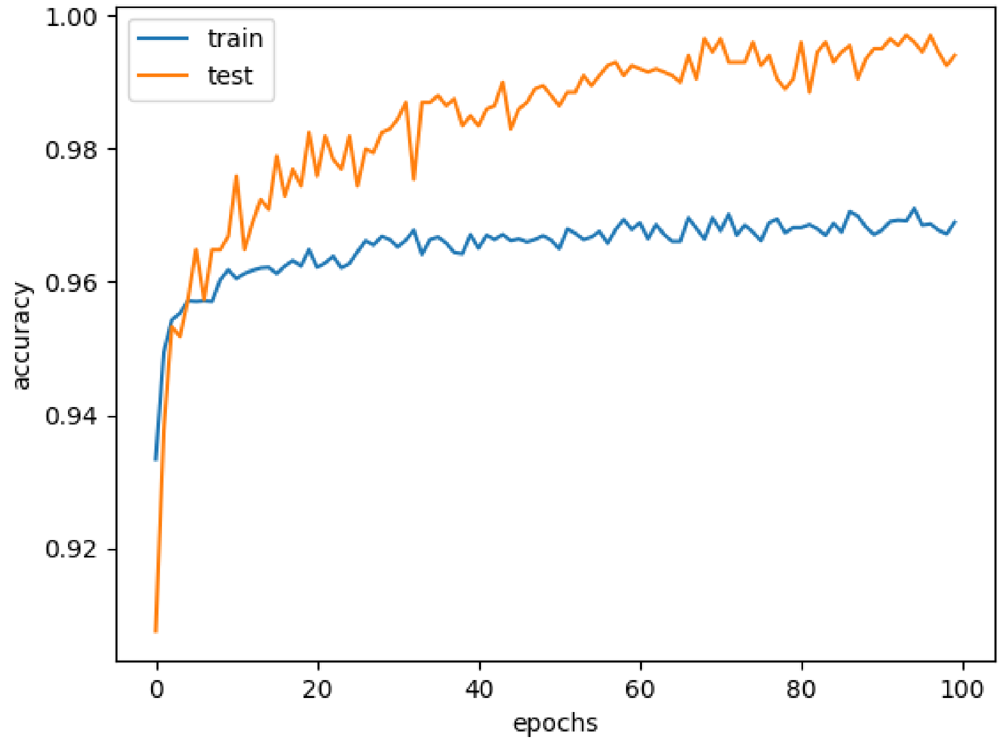
1. Please refer code in "perceptron.py"

   (a) Epoch number is set to 100. The figure is shown below:



The accuracy increases drastically before $I \approx 15$. Then it rises shakingly and slowly.
By increasing/decreasing I the shape of the curve won't change but just reveals more/less of the curve in the figure.

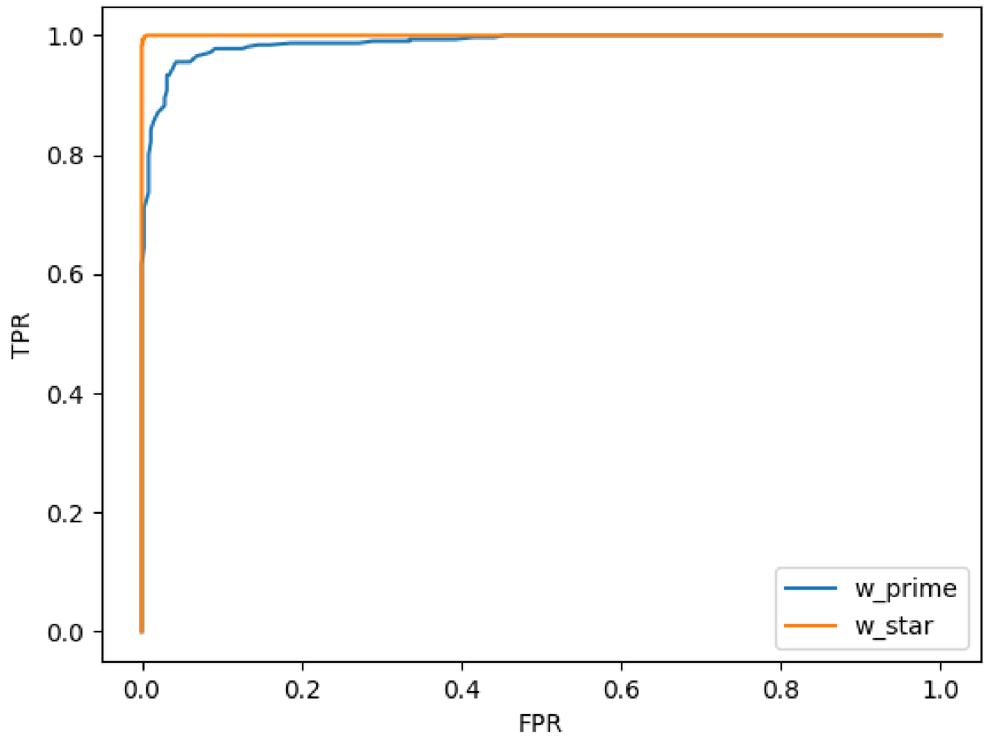
(b) Epoch number is set to 100. The figure is shown below:



Red one is for training data, while blue one is for testing data.
For testing data, the result is even better. As the accuracy continues to rise(shakingly).

(c) accuracy: 0.99397287779507785
confusion matrix:

$$\begin{bmatrix} 1009 & 40 \\ 0 & 942 \end{bmatrix}$$



(d)

(e) AUC(w') = 0.9877518533678995
AUC(w^*) = 0.9999666948582919 The greater the AUC value, the bigger area that is under the ROC curve, the 'upper' that the ROC curve goes.

2. (a)

   (b)