Physical Layer Design for a Narrow Band Communication System

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***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/SiddharthPh/
    Summer2020/trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/gadepall/school/trunk/
    ncert/geometry/figs
```

1 Specifications

1.1 Modulation

1.1.1. See Fig.1.1.1.1 for the constellation diagram. The transmitted symbol set is given by

$$\mathbf{s}_m = \begin{pmatrix} \cos\frac{2m\pi}{8} \\ \sin\frac{2m\pi}{8} \end{pmatrix}, \quad m \in \{0, 1, \dots, 3\}. \tag{1.1.1.1}$$

The numerical values for $\mathbf{s}_m$ are listed in Table 1.1.2

1.1.2. See Table 1.1.2 for the encoding scheme.

Symbol	**Grey Code**	**Co-ordinates**
s_0	00	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
s_1	01	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
s_2	11	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$
s_3	10	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

TABLE 1.1.2

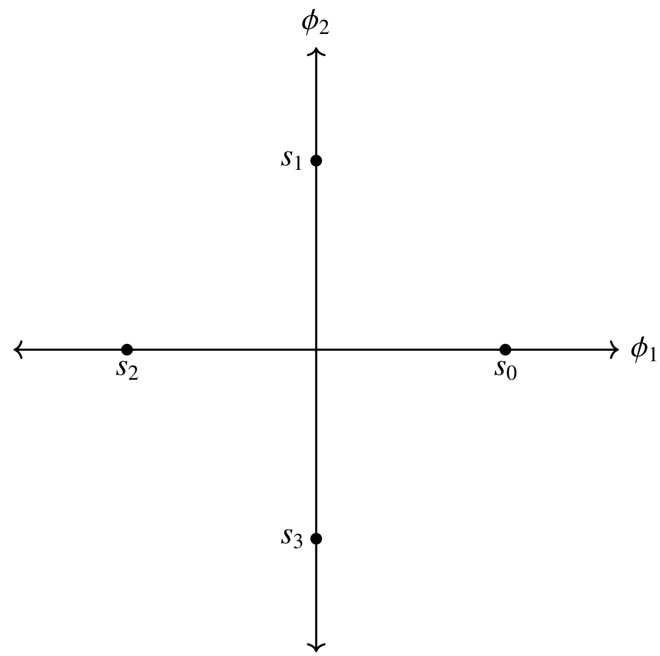

Fig. 1.1.1.1: constellation diagram

1.2 Demodulation

1.2.1. The received symbol is then obtained as

$$\mathbf{y} = \sqrt{E_s}\mathbf{s} + \mathbf{n} \tag{1.2.1.1}$$

where E_s is the symbol energy and

$$\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}, \frac{N_0}{2}\mathbf{I}\right) \tag{1.2.1.2}$$

$$\mathbf{s} \in \{\mathbf{s}_m\}_{m=0}^{3} \tag{1.2.1.3}$$

1.2.2. The decision rule is given by Fig.1.2.2.1 and can be expressed as
Minimum distance Criterion:

$$\hat{s} = min \left\|\mathbf{y} - \mathbf{s}\right\| \tag{1.2.2.1}$$

From eq.1.2.2.1,s_0 is chosen if

$$\left\|\mathbf{y} - \mathbf{s}_0\right\|^2 < \left\|\mathbf{y} - \mathbf{s}_1\right\|^2 \tag{1.2.2.2}$$

$$\left\|\mathbf{y} - \mathbf{s}_0\right\|^2 < \left\|\mathbf{y} - \mathbf{s}_2\right\|^2 \tag{1.2.2.3}$$

$$\left\|\mathbf{y} - \mathbf{s}_0\right\|^2 < \left\|\mathbf{y} - \mathbf{s}_3\right\|^2 \tag{1.2.2.4}$$

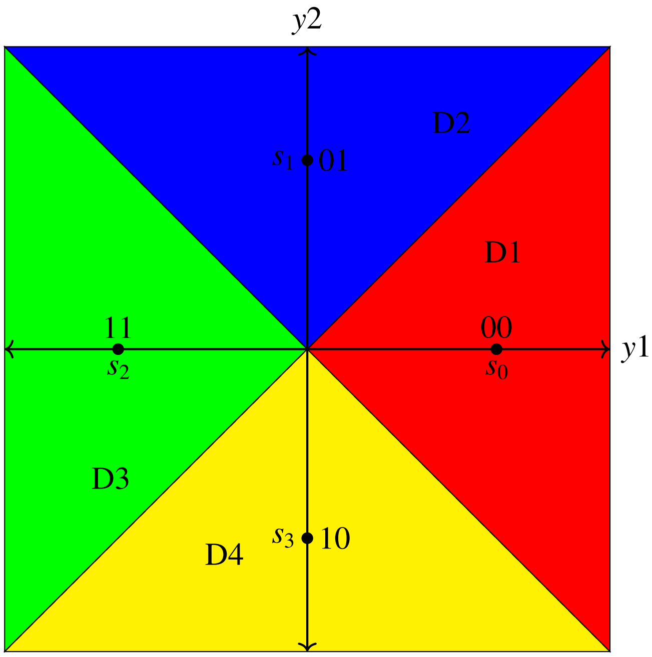

Fig. 1.2.2.1: decision regions

The above conditions can be simplified to obtain the region

$$(\mathbf{s}_0 - \mathbf{s}_1)^T \mathbf{y} > 0 \tag{1.2.2.5}$$

$$(\mathbf{s}_0 - \mathbf{s}_2)^T \mathbf{y} > 0 \tag{1.2.2.6}$$

$$(\mathbf{s}_0 - \mathbf{s}_3)^T \mathbf{y} > 0 \tag{1.2.2.7}$$

Substituting the values of $s_0, s_1, ...s_7$ in the above

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}^T \mathbf{y} > 0 \tag{1.2.2.8}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{y} > 0 \tag{1.2.2.9}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \mathbf{y}^T > 0 \tag{1.2.2.10}$$

yielding $|y_2| <$ y1 i.e D1 region (red) is detected at the receiver.
Similarly, from eq.1.2.2.1
For detecting s_1, $y_1 > -y_2$ and $y_1 < y_2$ i.e D2 region (blue) is detected at the receiver.
For detecting s_2, $y_1 < -y_2$ and $y_1 < y_2$ i.e D3 region (green) is detected at the receiver.
For detecting s_3, $y_1 < -y_2$ and $y_1 > y_2$ i.e D4 region (yellow) is detected at the receiver.

1.2.3. The following code has simulation of QPSk.

```
codes/qpsk.py
```

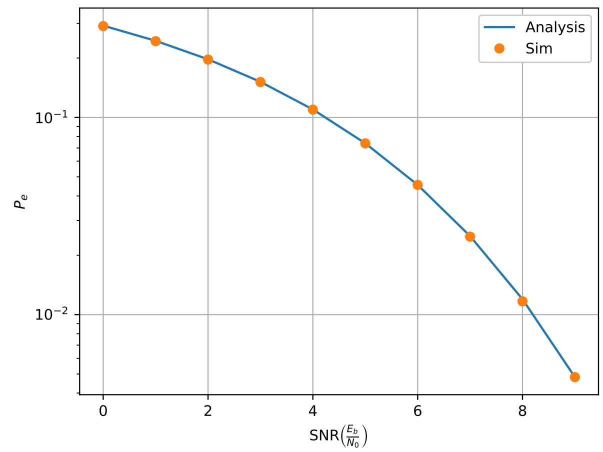

Fig. 1.2.3.1: Result from Simulation