MATRIX PROJECT

INTRODUCTION TO AI AND ML

EE1390

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Question
JEE MAIN 2007

The equation of a tangent to the parabola

$$y^2 = 8x \tag{1}$$

is

$$y = x + 2 \tag{2}$$

Find the point on this line from which the other tangent to the parabola is perpendicular to the given tangent.

Question
JEE MAIN 2007

The equation of a tangent to the parabola

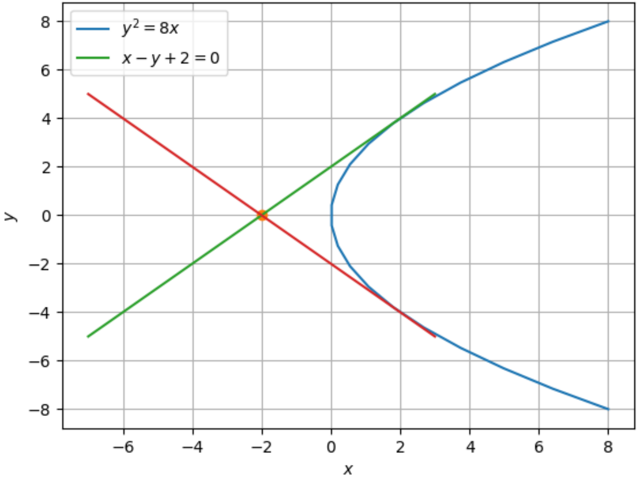
$$\mathbf{X}^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{X} + \begin{bmatrix} -8 & 0 \end{bmatrix} \mathbf{X} = 0 \tag{3}$$

is

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \mathbf{X} = -2 \tag{4}$$

Find the point on this line from which the other tangent to the parabola is perpendicular to the given tangent.

FIGURE



SOLUTION
METHOD-1

Given equation of the line in matrix form : $\begin{bmatrix} 1 & -1 \end{bmatrix}$ **X**= -2
Tangent to a standard parabola is of the form $\begin{bmatrix} m & -1 \end{bmatrix}$ **X**= (-a/m)
Normal vector of the line specified = **A** and given that the other tangent is perpendicular to this line. Let the normal vector of the other tangent be **B**. Now **A** and **B** are orthogonal i.e.. $\mathbf{A}^T\mathbf{B} = 0$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} m \\ -1 \end{bmatrix} = 0 \tag{5}$$

$$m = -1 \tag{6}$$

SOLUTION
METHOD-1

Equation of perpendicular tangent: $\begin{bmatrix} -1 & -1 \end{bmatrix}$ **X** = 2

$\begin{bmatrix} 1 & -1 \end{bmatrix}$ **X** = -2

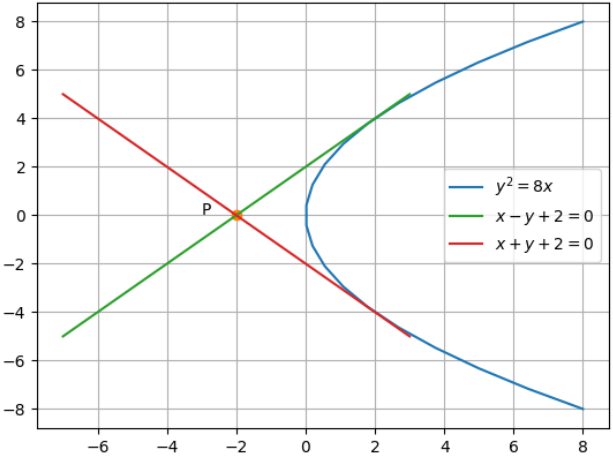
Point of intersection of two tangent:

$$\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = 1/2 \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

x = -2, y = 0

FIGURE FOR THE SOLUTION



SOLUTION

METHOD-2

From the properties of parabola ,we know that the locus of point of intersection of perpendicular tangents to the parabola is given by the directrix of the parabola. The equation to the directrix of the standard parabola is $[1\ 0]\mathbf{X} = -a$

SOLUTION
METHOD-2

For the given parabola $y^2 = 8x, a = 2$
Hence the equation of the directrix becomes [1 0]**X** = -2. Let the required point of intersection be [h k]. h = -2 as it lies on the directrix.

SOLUTION
METHOD-2

Now, since $\begin{bmatrix} -2 \\ k \end{bmatrix}$ lies on the line [1 -1]**X** = -2, substituting **X**= $\begin{bmatrix} -2 \\ k \end{bmatrix}$ gives us the equation, -2 + k = -2 i.e.. k = 0. Hence the point is (-2,0).

FIGURE FOR THE SOLUTION