Normal Distribution

- The most often used continuous probability distribution is the normal distribution; it is also known as **Gaussian** distribution.

- Its graph called the normal curve is the bell-shaped curve.

- Such a curve approximately describes many phenomenon occur in nature, industry and research.

    - Physical measurement in areas such as meteorological experiments, rainfall studies and measurement of manufacturing parts are often more than adequately explained with normal distribution.

- A continuous random variable X having the bell-shaped distribution is called a normal random variable.

Normal Probability Distribution

- The normal probability distribution is the most important distribution for describing a continuous random variable.
- It is widely used in statistical inference.
- It has been used in a wide variety of applications including:
  - Heights of people
  - Rainfall amounts
  - Test scores
  - Scientific measurements
- Abraham de Moivre, a French mathematician, published *The Doctrine of Chances* in 1733.
- He derived the normal distribution.

Normal Probability Distribution

- Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where:

μ = mean

σ = standard deviation

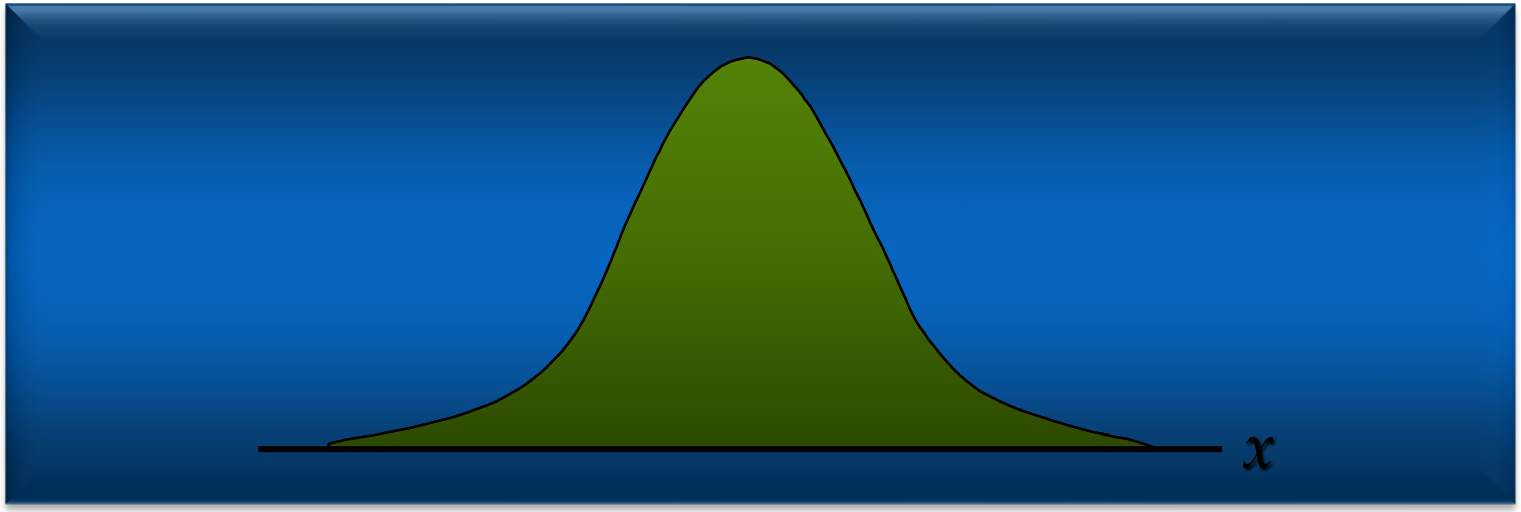
π = 3.14159

e = 2.71828

Normal Probability Distribution

- Characteristics

The distribution is symmetric; its skewness measure is zero.

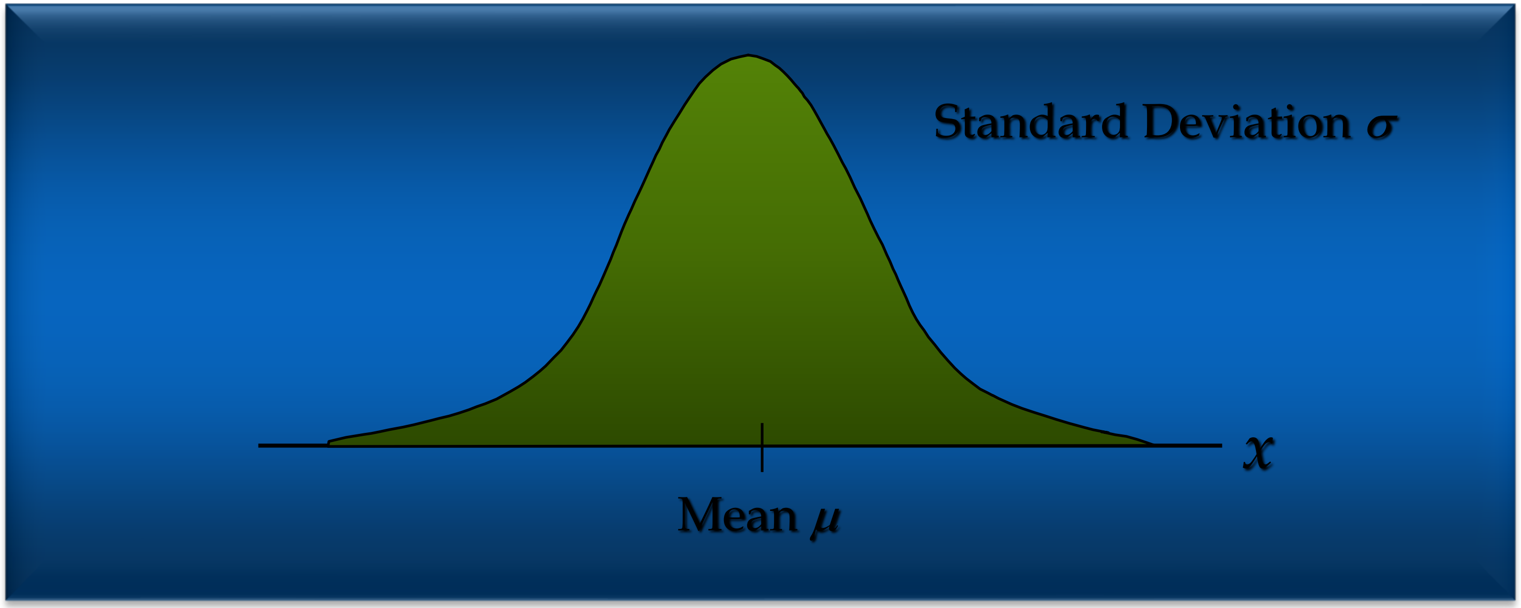


The random variable x can take any value from $-\infty$ to ∞.

Normal Probability Distribution

- Characteristics

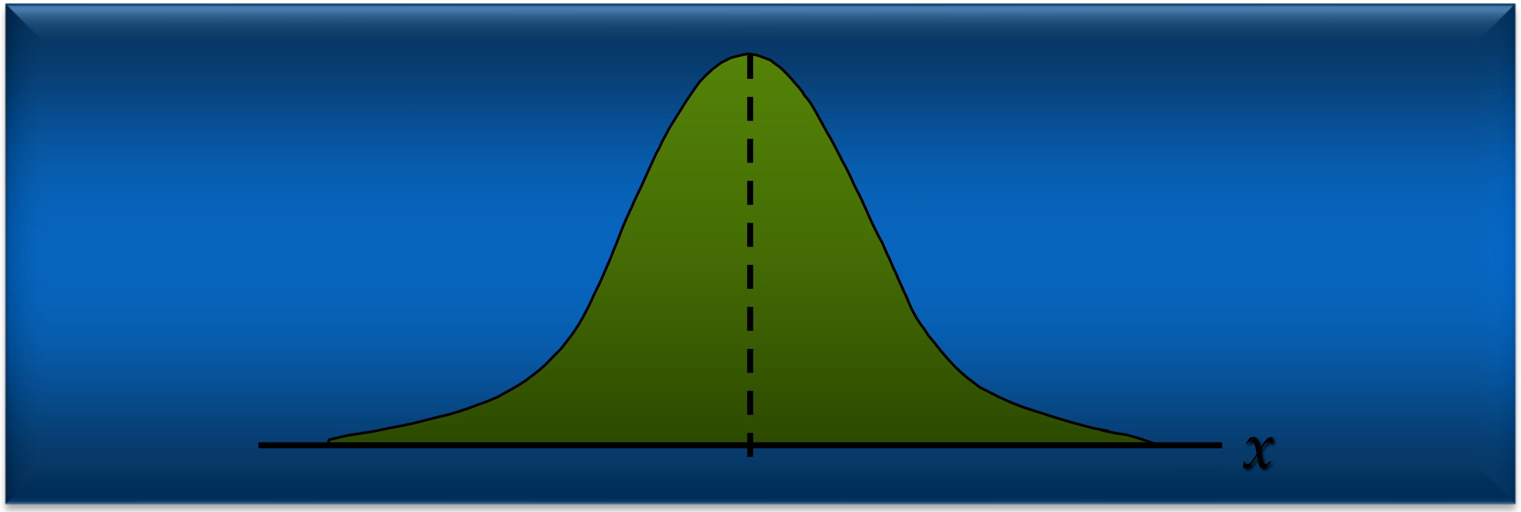
The entire family of normal probability distributions is defined by its mean μ and its standard deviation σ.



Normal Probability Distribution

- Characteristics

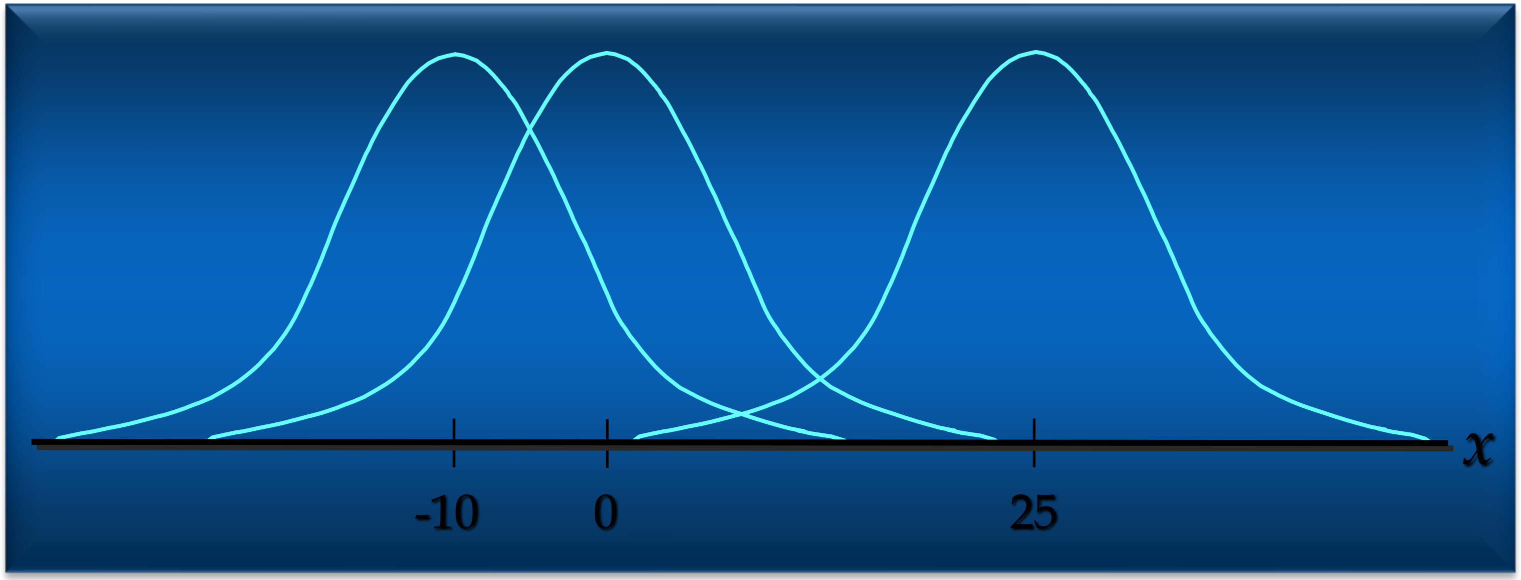
The highest point on the normal curve is at the mean, which is also the median and mode.



Normal Probability Distribution

- Characteristics

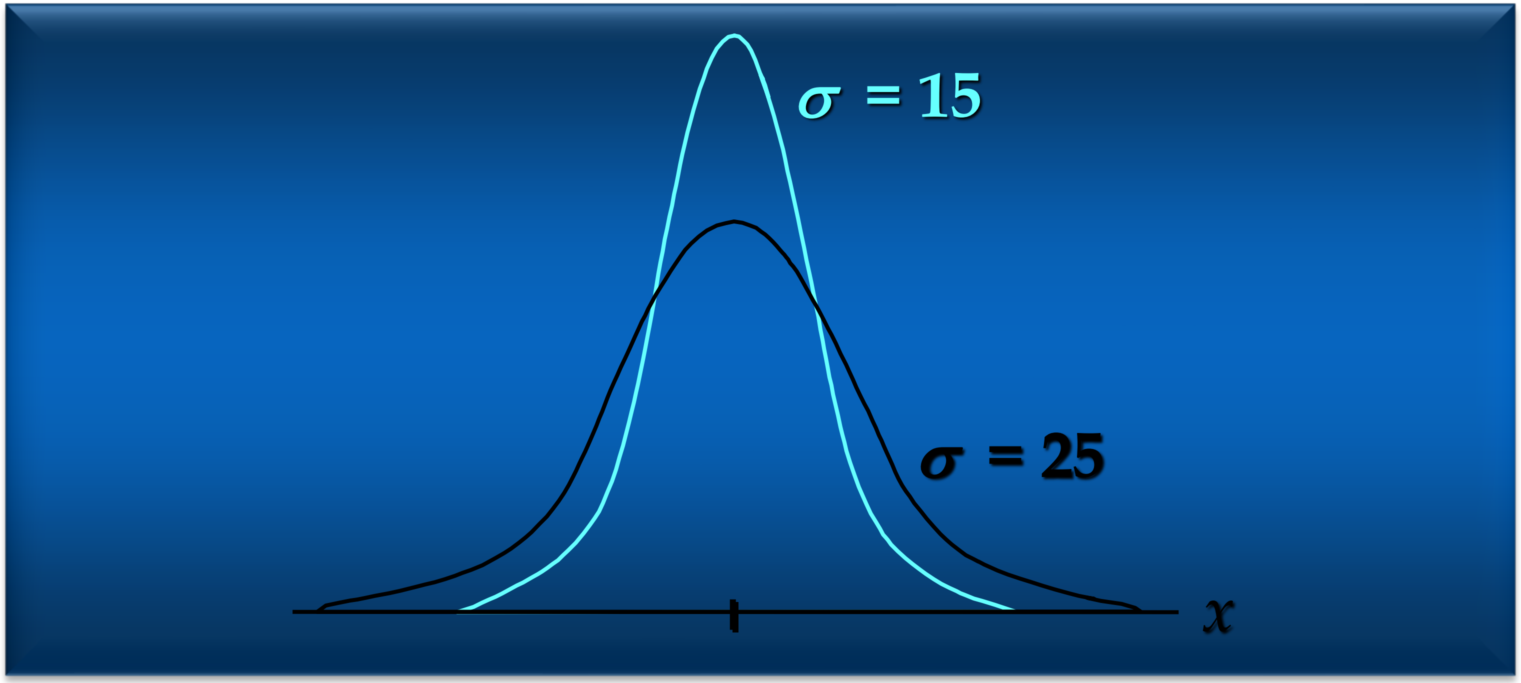
The mean can be any numerical value: negative, zero, or positive.



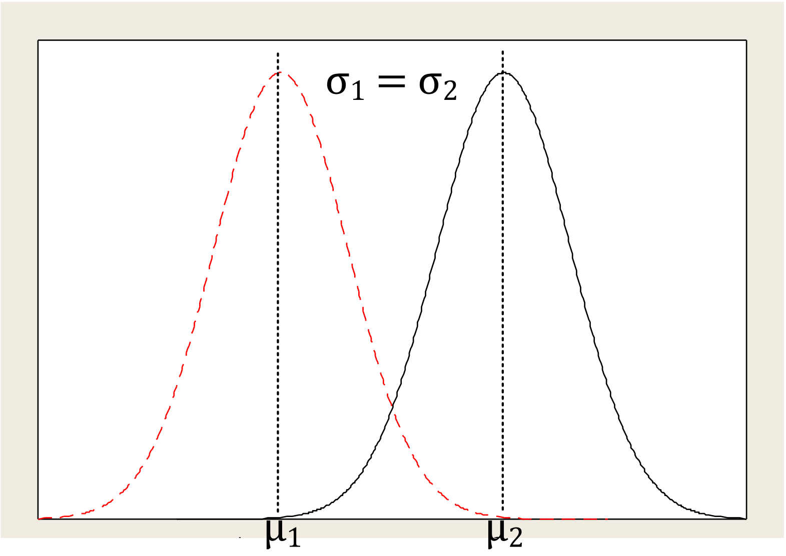
Normal Probability Distribution

- Characteristics

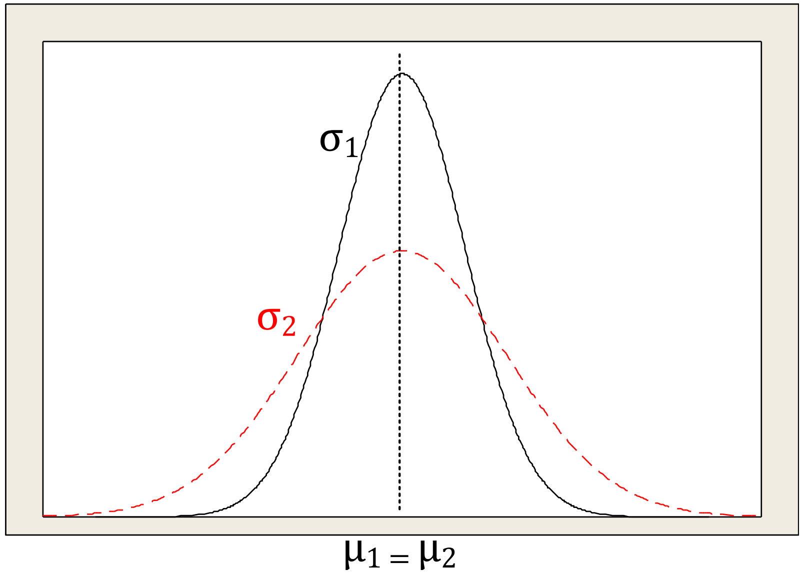
The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



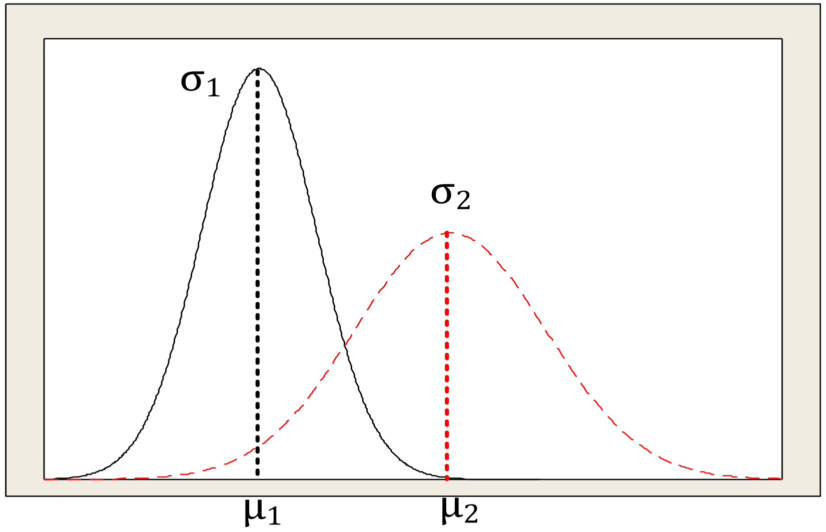
Normal Distribution



Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$



Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$

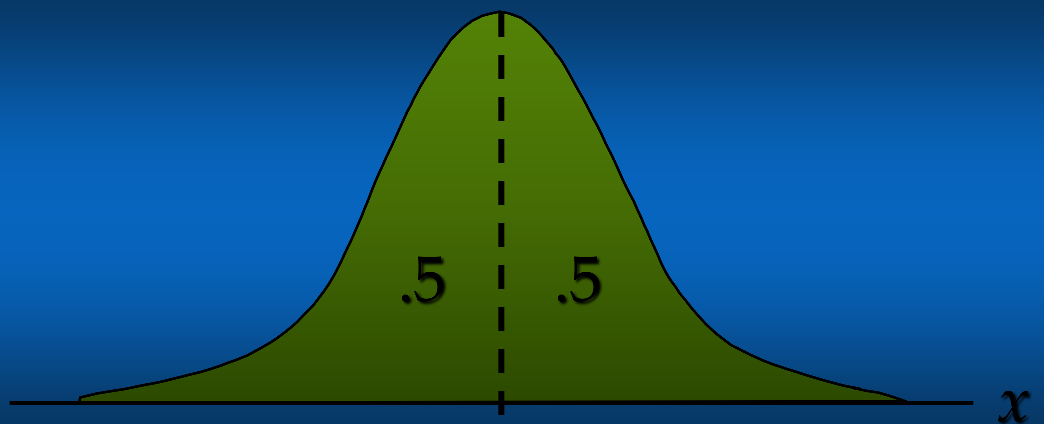


Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 < \sigma_2$

Normal Probability Distribution

- Characteristics

Probabilities for the normal random variable are given by <u>areas under the curve</u>. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).



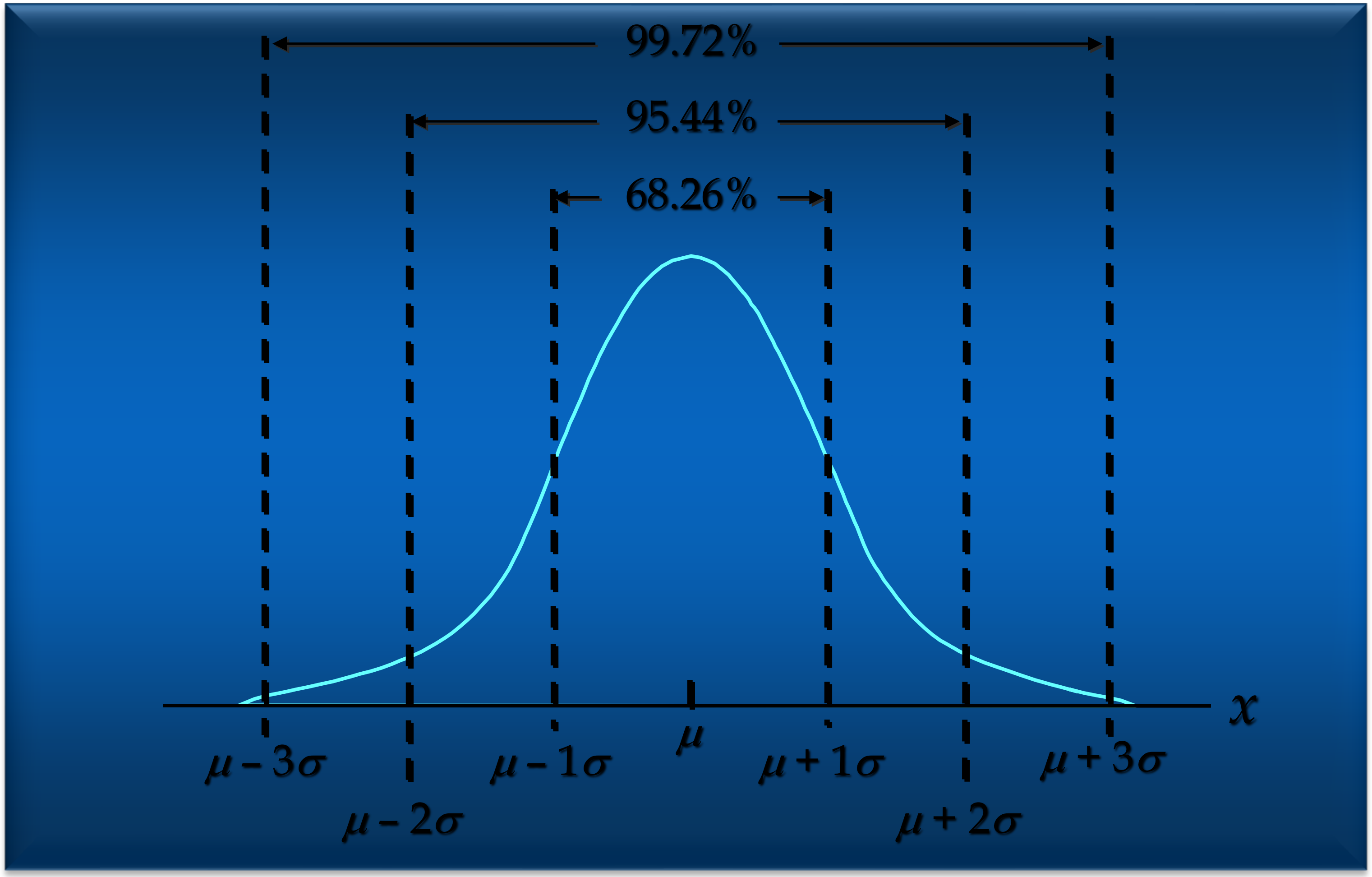
Normal Probability Distribution

- Characteristics (basis for the empirical rule)

  - 68.26% of values of a normal random variable are within +/- 1 standard deviation of its mean.
  - 95.44% of values of a normal random variable are within +/- 2 standard deviations of its mean.
  - 99.72% of values of a normal random variable are within +/- 3 standard deviations of its mean.

Normal Probability Distribution

- Characteristics (basis for the empirical rule)



Standard Normal Probability Distribution

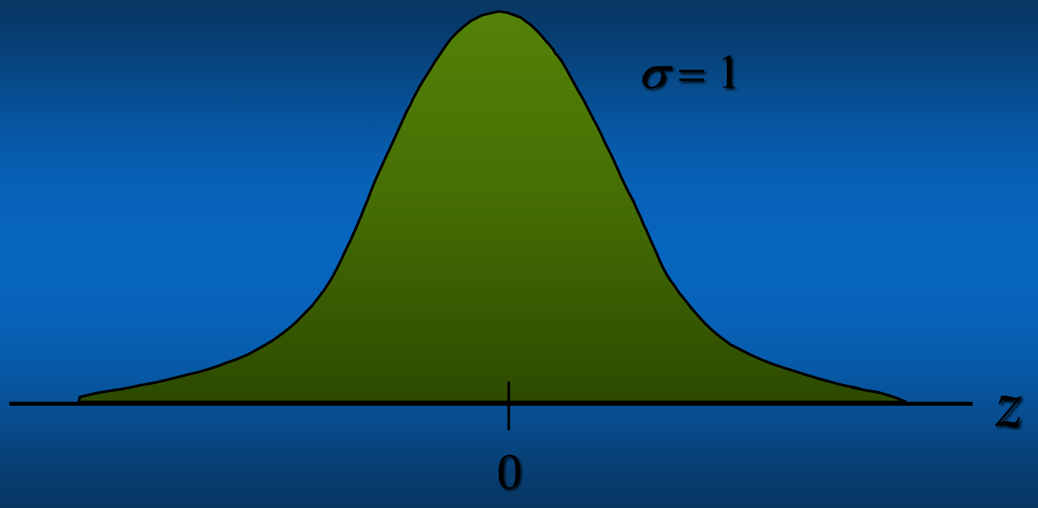
- Characteristics

A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

Standard Normal Probability Distribution

- Characteristics

The letter *z* is used to designate the standard normal random variable.



Standard Normal Probability Distribution

- Converting to the Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

We can think of z as a measure of the number of standard deviations x is from μ.

Standard Normal Probability Distribution

- Example: Pep Zone

Pep Zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallons, a replenishment order is placed.

The store manager is concerned that sales are being lost due to stockouts while waiting for a replenishment order.

Standard Normal Probability Distribution

- Example: Pep Zone

  It has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

  The manager would like to know the probability of a stockout during replenishment lead-time. In other words, what is the probability that demand during lead-time will exceed 20 gallons?

$$P(x > 20) = ?$$

Standard Normal Probability Distribution

- Solving for the Stockout Probability

Step 1: Convert x to the standard normal distribution.

$$z = (x - \mu)/\sigma$$
$$= (20 - 15)/6$$
$$= .83$$

Step 2: Find the area under the standard normal curve to the left of $z = .83$.

see next slide

Standard Normal Probability Distribution

- Cumulative Probability Table for the Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
.	.	.	.	.	.	.	.	.	.	.

$P(z \leq .83)$

Standard Normal Probability Distribution

- Solving for the Stockout Probability

Step 3: Compute the area under the standard normal curve to the right of $z = .83$.

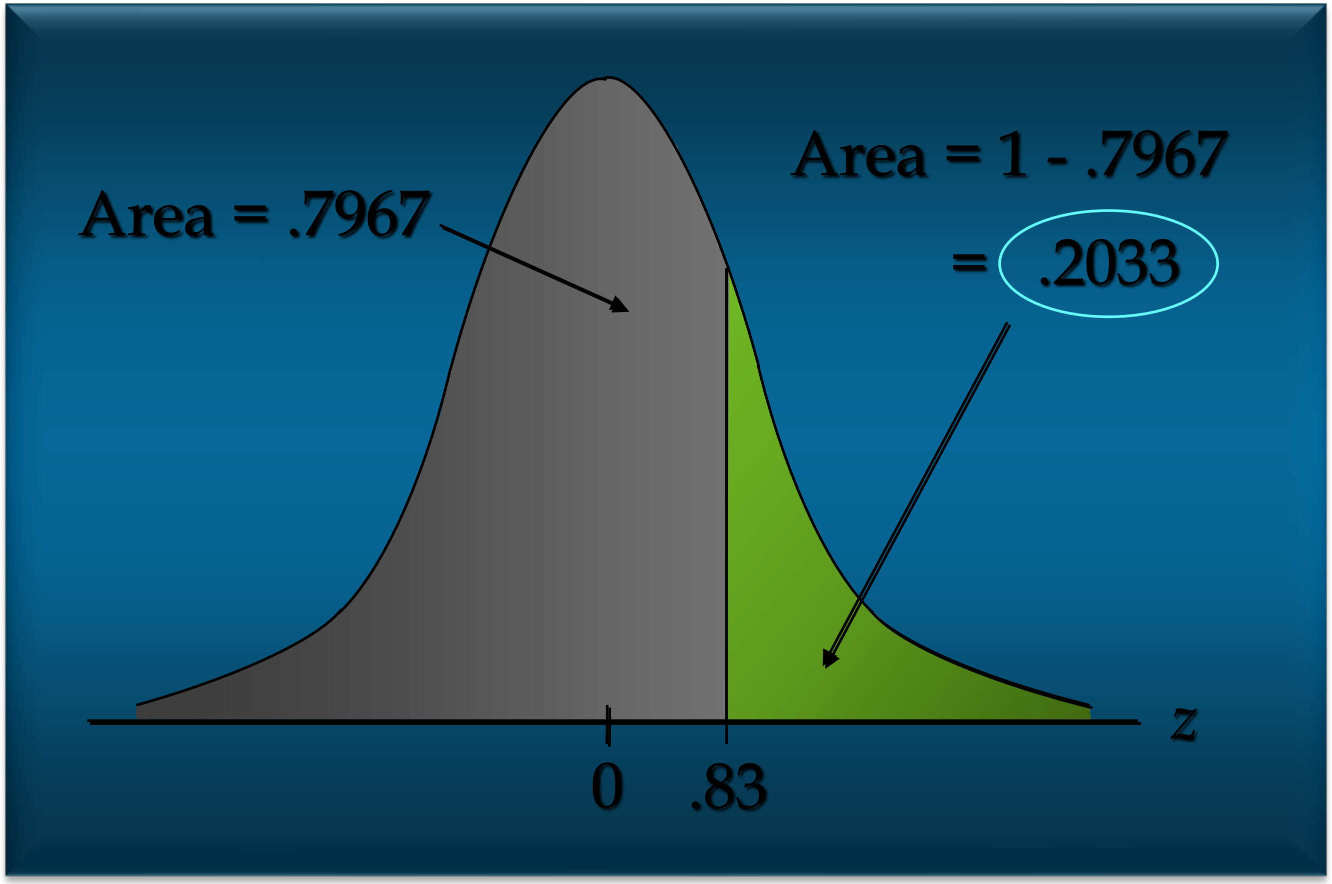
$$P(z > .83) = 1 - P(z \leq .83)$$
$$= 1 - .7967$$
$$= .2033$$

Probability of a stockout

$P(x > 20)$

Standard Normal Probability Distribution

- Solving for the Stockout Probability



Standard Normal Probability Distribution

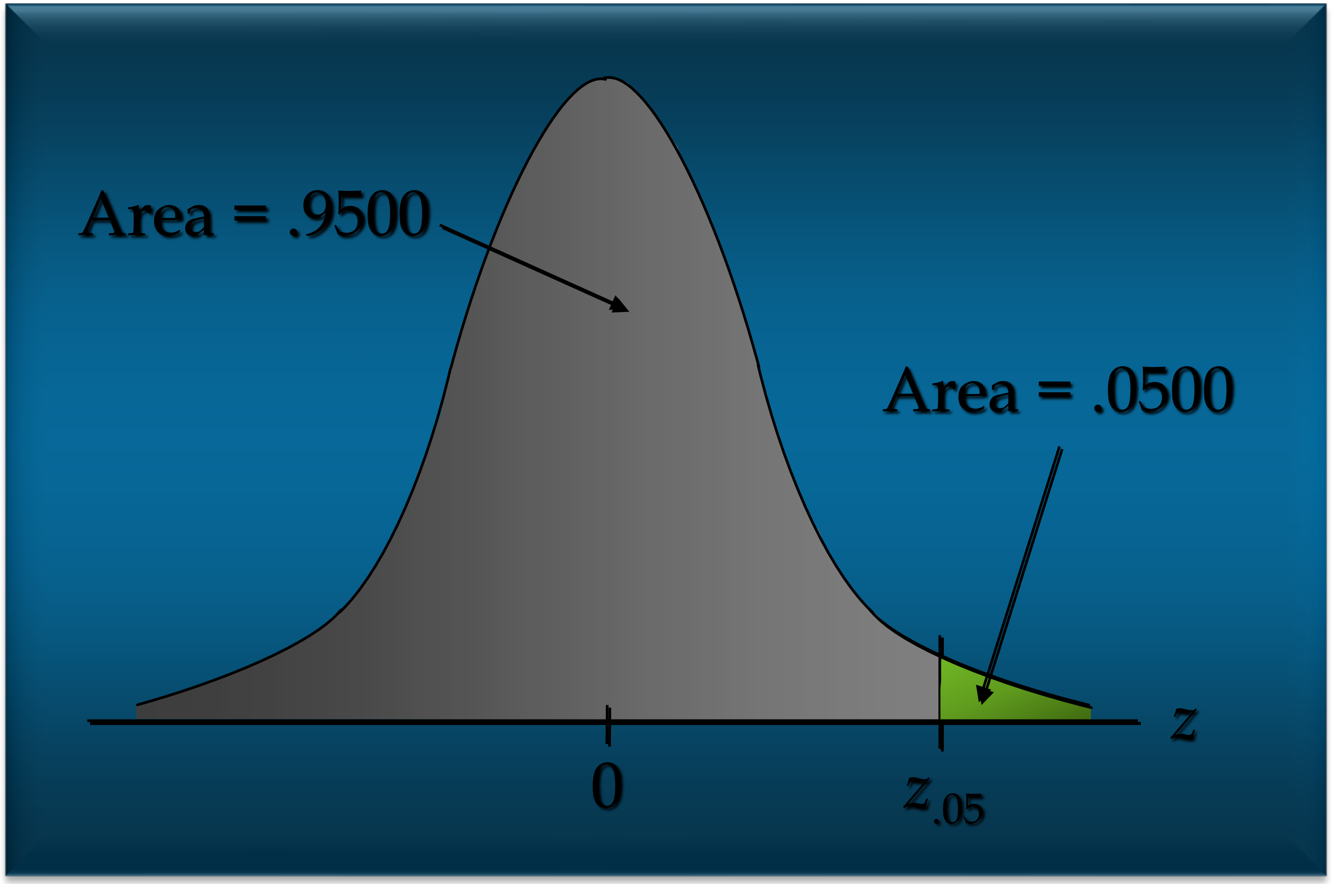
Standard Normal Probability Distribution

- If the manager of Pep Zone wants the probability of a stockout during replenishment lead-time to be no more than .05, what should the reorder point be?

- (Hint: Given a probability, we can use the standard normal table in an inverse fashion to find the corresponding *z* value.)

Standard Normal Probability Distribution

- Solving for the Reorder Point



Standard Normal Probability Distribution

- Solving for the Reorder Point

  - **Step 1: Find the *z*-value that cuts off an area of .05 in the right tail of the standard normal distribution.**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	[illegible]	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	[illegible]	[illegible]	[illegible]	[illegible]
1.9	.9713	.9719	.9726	.9732	.9738	.9744	[illegible]	[illegible]	[illegible]	[illegible]
.	.	.	.	.	.	.				

We look up
the complement
of the tail area
(1 - .05 = .95)

Standard Normal Probability Distribution

- Solving for the Reorder Point

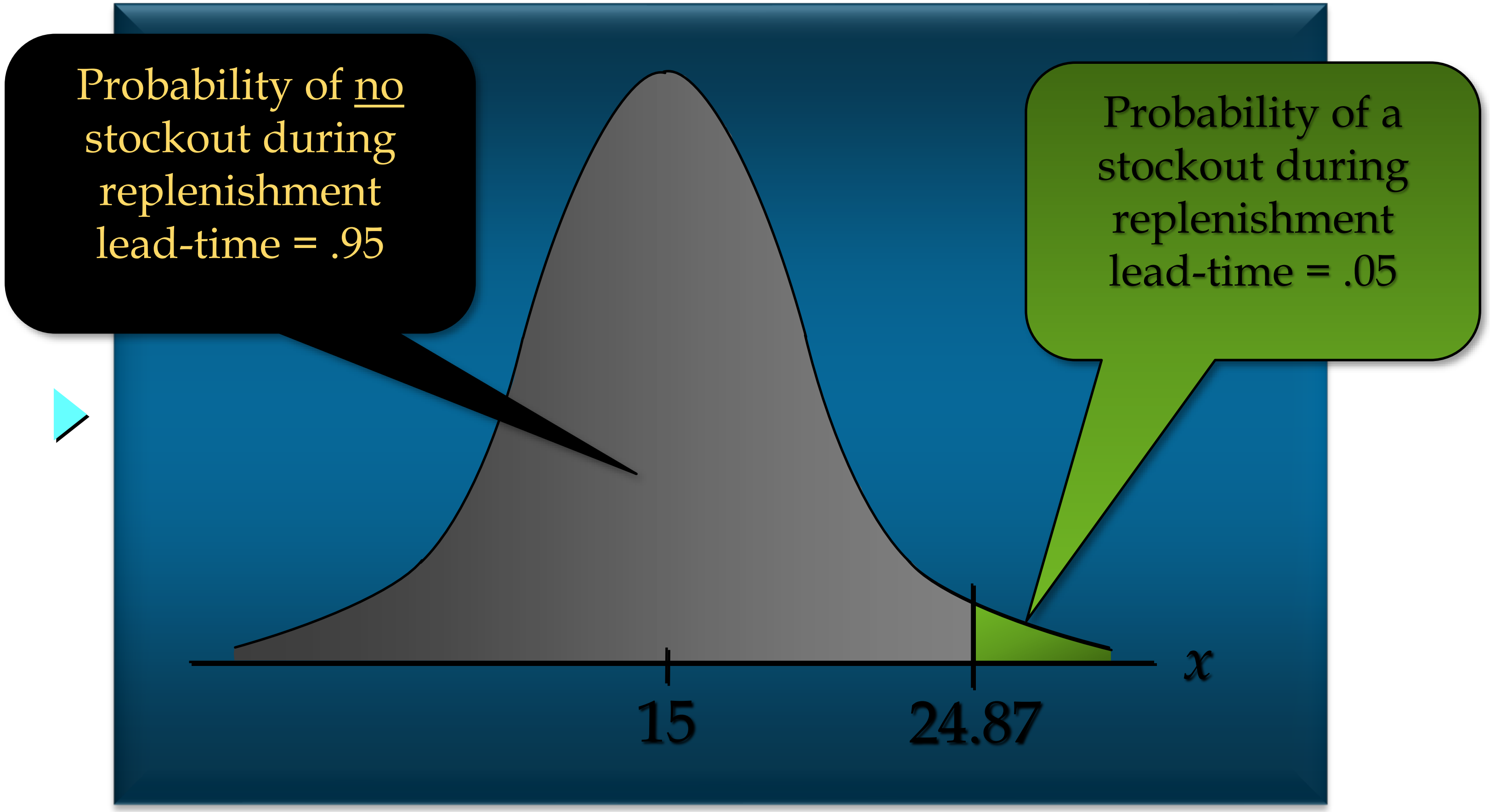
Step 2: Convert $z_{.05}$ to the corresponding value of x.

$$x = \mu + z_{.05}\sigma$$
$$= 15 + 1.645(6)$$
$$= 24.87 \text{ or } 25$$

A reorder point of 25 gallons will place the probability of a stockout during leadtime at (slightly less than) .05.

Normal Probability Distribution

- Solving for the Reorder Point



Standard Normal Probability Distribution

- Solving for the Reorder Point

- By raising the reorder point from 20 gallons to 25 gallons on hand, the probability of a stockout decreases from about .20 to .05.
  This is a significant decrease in the chance that Pep Zone will be out of stock and unable to meet a customer's desire to make a purchase.