Modeling and Simulation

ITC21

System

A system is defined to be a collection of entities, e.g., people or machines, that act and interact together towards the accomplishment of some logical end.

Examples of a System:

- Banking System
- Healthcare System
- Manufacturing System

In practice, what is meant by " the system" depends on the objectives of a particular study. The collection of entities that compose a system for one study might be only a subset of the overall system for another

Components of a System

Entity: It is used to denote an object of interest in a system.

Attribute: It denotes a property of an entity.

Activity: Any process that causes changes in the system is called as an activity.

Banking System

- Entity-> Customers
- Activity->Making Deposits
- Attribute->Balance

State of a System

Collection of variables necessary to describe a system at a particular time, relative to the objectives of a study.

In a Banking System, examples of possible state variables are:

- the number of busy tellers,
- the number of customers in the bank,
- the time of arrival of each customer in the bank.

System Environment

A system is often affected by changes occurring outside the system. Such changes occurring outside the system are said to occur in the system environment.

An important step in modelling system is to decide upon the boundary between the system and its environment.

Discrete and Continuous Systems

We categorize systems to be of two types, discrete and· continuous.

Discrete Systems

A Discrete System is one for which the state variables change instantaneously at separated points in time .

A bank is an example of a discrete system, since state variables--e .g., the number of customers in the bank- changes only when a customer arrives or departs from the bank.

Discrete and Continuous Systems

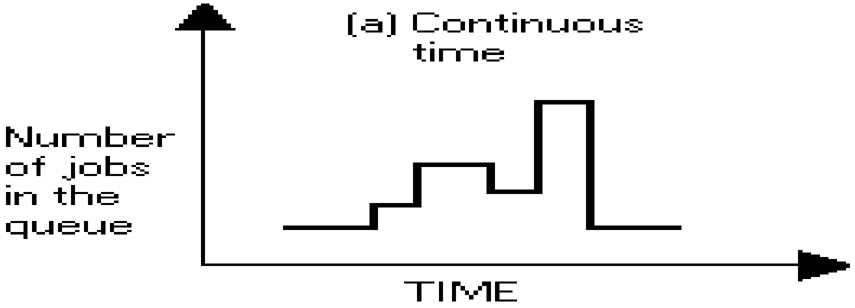
Continuous Systems

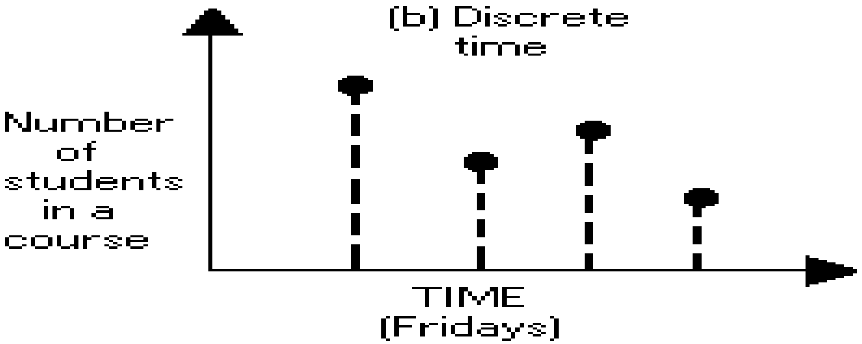
A Continuous System is one for which the state variables change continuously with respect to time. An airplane moving through the air is an example of a continuous system, since state variables such as position and velocity can change continuously with respect to time.

Examples Of Both Type Models

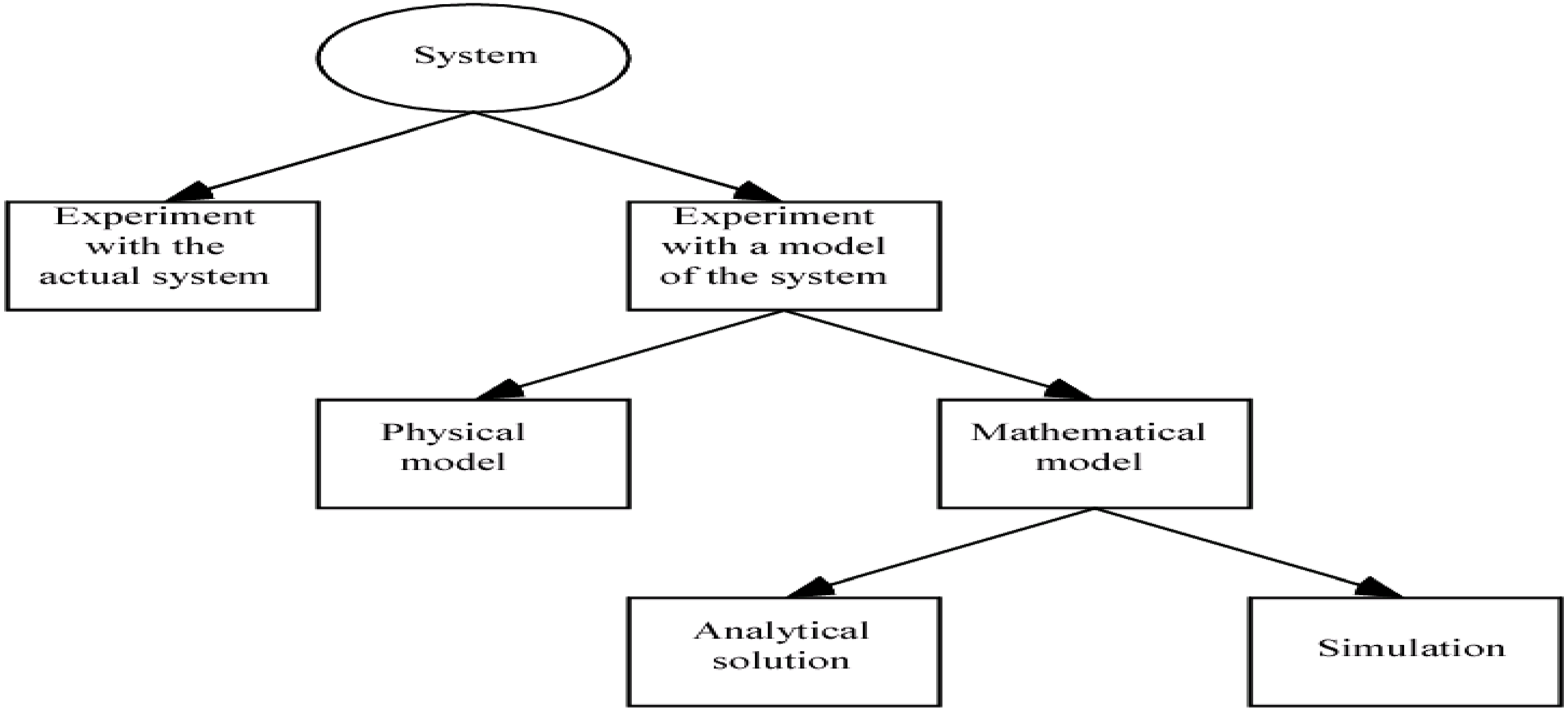
Continuous Time and Discrete Time Models:

- CPU scheduling model vs. number of students attending the class.





Ways to study a system



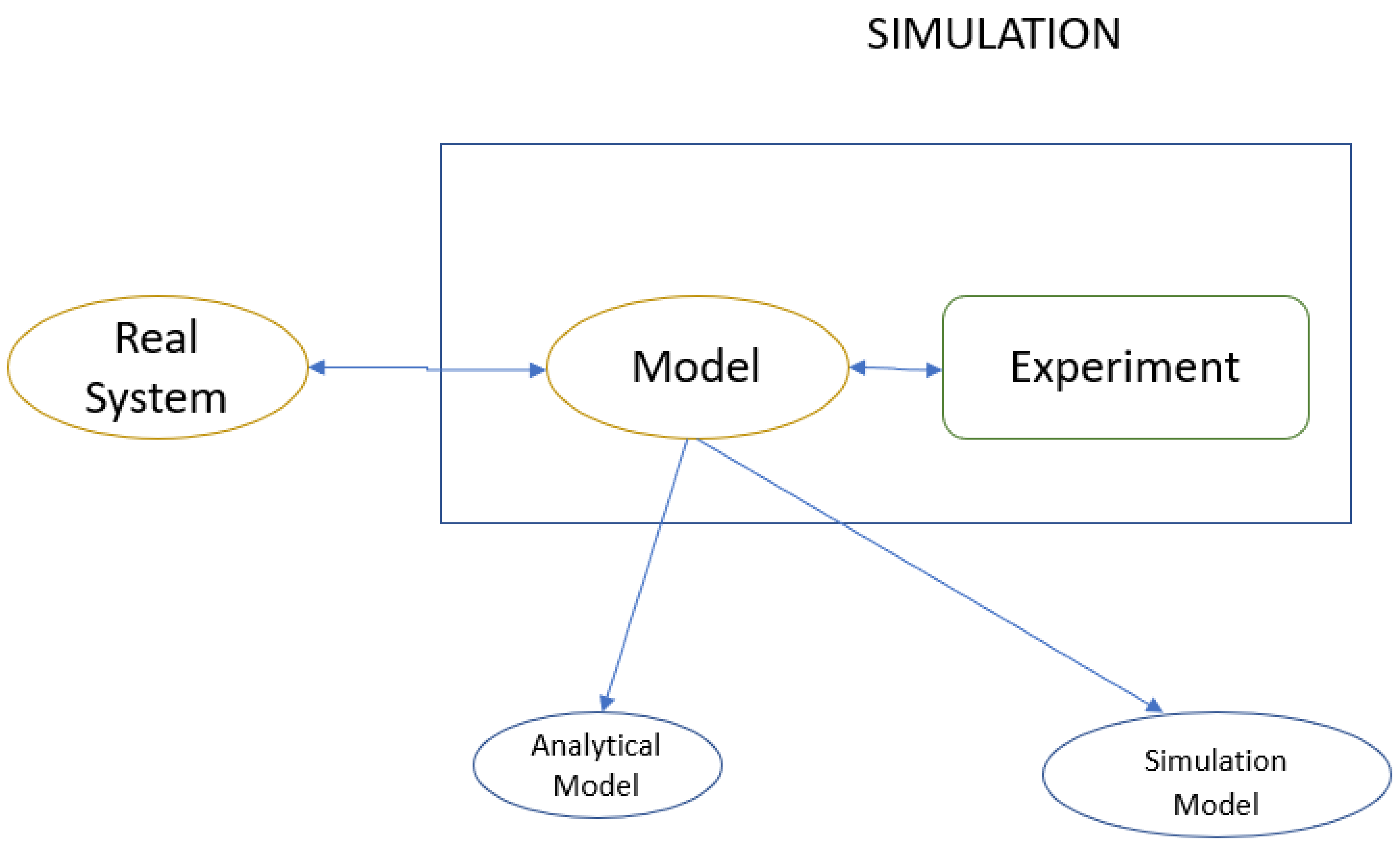
Simulation

- Simulation is the process of using a model to study the performance of a system. It is an act of using a model for simulation.
- Simulation of a system is the operation of a model in terms of time or space, which helps analyze the performance of an existing or a proposed system.
- In its broadest sense, simulation is a tool to evaluate the performance of a system, existing or proposed, under different configurations of interest and over long periods of real time.
- It is used before an existing system is altered or a new system built, to reduce the chances of failure to meet specifications, to eliminate unforeseen bottlenecks, to prevent under or overutilization of resources, and to optimize system performance.

Modelling

- Modelling is creating a model which represents a system including their properties. It is an act of building a model.
- Modelling is the process of representing a model which includes its construction and working.
- A model is similar to a real system, which helps the analyst predict the effect of changes to the system.

Diagrammatic Representation of a Simulation System



System Models

A simulation model is a mathematical model that calculates the impact of uncertain inputs and decisions we make on outcomes that we care about, such as profit and loss, investment returns, etc.

- A simulation model will include:

 Model inputs that are uncertain numbers/ uncertain variables

 Intermediate calculations as required

 Model outputs that depend on the inputs -- These are uncertain functions

Types of System Models

- Static vs. Dynamic Simulation Models
- Deterministic vs. Stochastic Activities
- Continuous vs. Discrete system

Static vs. Dynamic Simulation Models

Static Simulation Model

A static simulation model is a representation of a system at a particular time, or one that may be used to represent a system in which time simply plays no role.

- *Example:* Monte Carlo models

Dynamic Simulation Model

A dynamic simulation model represents a system as it evolves over time.

- *Example*: Banking System.

Deterministic vs. Stochastic Activities

Deterministic

An activity is said to be deterministic where the outcome of an activity can be described completely in term of its input.

- *Example*: AND, OR, NOT operations.

Stochastic

An activity is said to be stochastic where the effects of the activity vary randomly over various possible outcomes.

- *Example*: Throwing a dice or tossing a coin.

Continuous vs. Discrete system

Continuous system

Systems in which the changes are predominantly smooth are called continuous system.

- *Example*: The movement of the aircraft occurs smoothly so aircraft system is a continuous system.

Discrete system

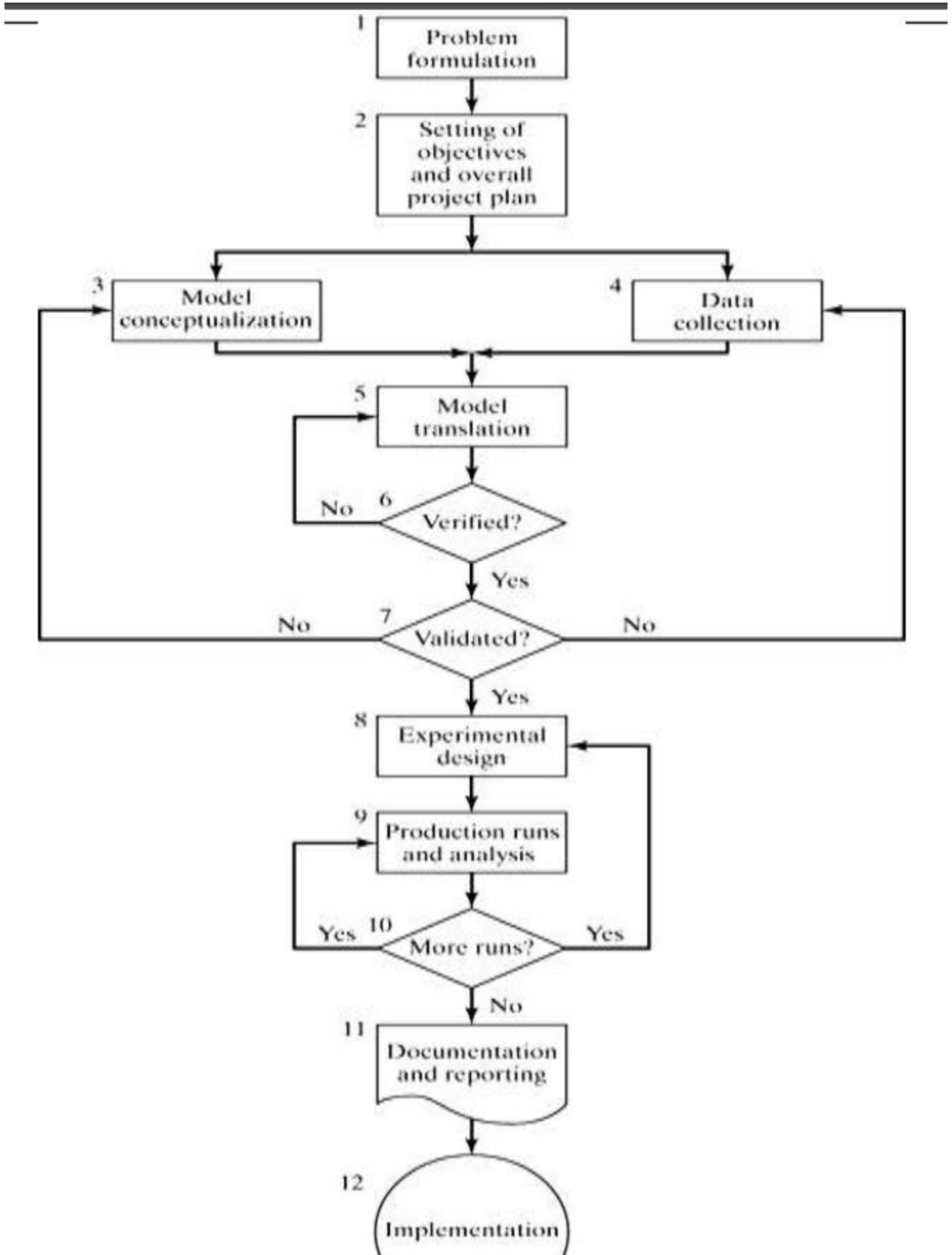
Systems in which the changes are predominantly discontinuous are called discrete system.

- *Example*: Changes in the factory occur discontinuously so factory system is a discrete system.

Steps in Simulation Study

Simulation models consist of the following components: system entities, input variables, performance measures, and functional relationships. Following are the steps to develop a simulation model.

- **Step 1** − Identify the problem with an existing system or set requirements of a proposed system.
- **Step 2** − Design the problem while taking care of the existing system factors and limitations.
- **Step 3** − Collect and start processing the system data, observing its performance and result.
- **Step 4** − Develop the model using network diagrams and verify it using various verifications techniques.
- **Step 5** − Validate the model by comparing its performance under various conditions with the real system.
- **Step 6** − Create a document of the model for future use, which includes objectives, assumptions, input variables and performance in detail.
- **Step 7** − Select an appropriate experimental design as per requirement.
- **Step 8** − Induce experimental conditions on the model and observe the result.



Steps in Simulation Study

Simulation Techniques

Simulation techniques can be used to assist management decision-making, where analytical methods are either not available or inappropriate.

- Typical business problems where simulation could be used to aid management decision-making are
- Inventory control.
- Queuing problems.
- Production planning.

Areas of application

- Manufacturing Applications
- Semiconductor Manufacturing
- Construction Engineering and project management
- Military application
- Logistics, Supply chain and distribution application
- Transportation modes and Traffic
- Business Process Simulation
- Health Care
- Automated Material Handling System (AMHS)(Test beds for functional testing of control-system software)
- Risk analysis(Insurance, portfolio,...)
- Computer Simulation(CPU, Memory,…)
- Network simulation(Internet backbone, LAN (Switch/Router), Wireless, PSTN (call center),...)

Advantages of Simulation

- New policies, operating procedures, information flows and son on can be explored without disrupting ongoing operation of the real system.
- New hardware designs, physical layouts, transportation systems and … can be tested without committing resources for their acquisition.
- Time can be compressed or expanded to allow for a speedup or slow-down of the phenomenon(clock is self-control).
- Insight can be obtained about interaction of variables and important variables to the performance.
- Bottleneck analysis can be performed to discover where work in process, the system is delayed.
- A simulation study can help in understanding how the system operates.
- “What if” questions can be answered.

Disadvantages of simulation

- Model building requires special training.- Vendors of simulation software have been actively developing packages that contain models that only need input (templates).
- Simulation results can be difficult to interpret.
- Simulation modeling and analysis can be time consuming and expensive. - Many simulation software have output-analysis.

Monte Carlo Simulation

- Monte Carlo simulation is a computerized mathematical technique to generate random sample data based on some known distribution for numerical experiments.
- This method is applied to risk quantitative analysis and decision making problems.
- It is used by the professionals of various profiles such as finance, project management, energy, manufacturing, engineering, research & development, insurance, oil & gas, transportation, etc.
- It is a method of approximating things using samples.

Monte Carlo Simulation

- The underlying concept of Monte Carlo Technique is to use the concept of “randomness” to solve problems.
- They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches.
- Monte Carlo methods are mainly used in three problem classes: optimization, numerical integration, and generating draws from a probability distribution.

Monte Carlo Simulation — Important Characteristics

Following are the three important characteristics of Monte-Carlo method −

- Its output must generate random samples.
- Its input distribution must be known.
- Its result must be known while performing an experiment.

Monte Carlo Simulation Technique

The principle behind the Monte Carlo technique is representative of the given system:

- Setting up a probability distribution to be analyzed**.**
- Building a cumulative probability distribution for a random variable.
- Generate random numbers . Assign an appropriate set of random numbers to represents value or range(interval) of values each random variable.

Monte Carlo Simulation – Advantages

- Easy to implement.
- Provides statistical sampling for numerical experiments using the computer.
- Provides approximate solution to mathematical problems.
- Can be used for both stochastic and deterministic problems.
- Probabilistic Results: Results show not only what could happen, but how likely each outcome is.
- Graphical Results: In Monte Carlo simulation , it’s easy to create graphs of different outcomes and their chances of occurrence.

Monte Carlo Simulation ─ Disadvantages

- Time consuming as there is a need to generate large number of sampling to get the desired output.
- The results of this method are only the approximation of true values, not the exact.

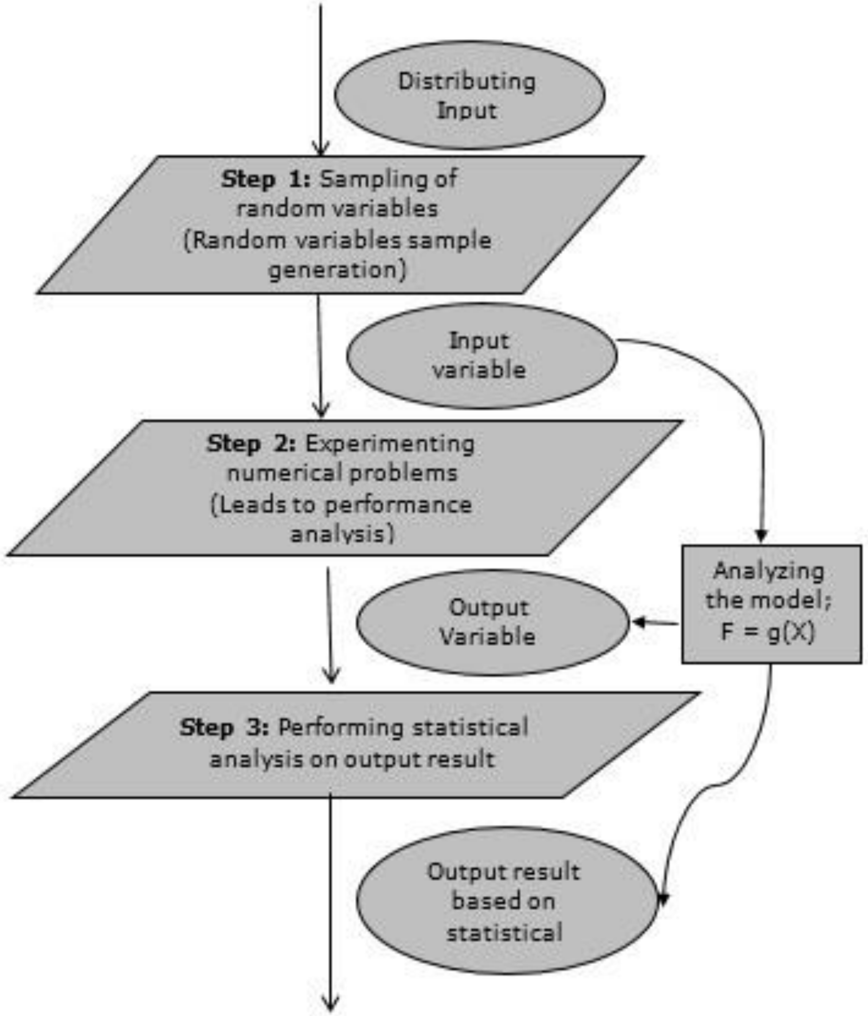
Monte Carlo Simulation Software

- Various options are available to use Monte Carlo simulations In computers.
- High-level programming language like C, C++, Java, or one of the .NET programming language, to develop a computer program for generating uniform random numbers,
- This program will possibly be tailor-made for specific situations.
- Various software libraries are also available in most of these high level programming languages, to facilitate the development of Monte Carlo simulation code.

Monte Carlo Simulation Software

- Stand-alone software packages can be used for MC simulations.
- These are general purpose simulation software packages, which can be used to model an industry-specific problem, generate random numbers, and perform output analysis.
- Monte Carlo simulations can also be performed using add-ins to popular spreadsheet software like MicrosoftR Excel.
- Using these software, one typically starts by developing a deterministic model for the problem, and then defines distributions for the input variables which contain uncertainty.
- These add-ins are capable of generating charts and graphs of the output parameters for further analysis.
- Crystal Ball from OracleR @RISK from Palisade, and the Solver add-in from Frontline Systems are a few examples of this type of software.

Monte Carlo Simulation Method — Flow Diagram



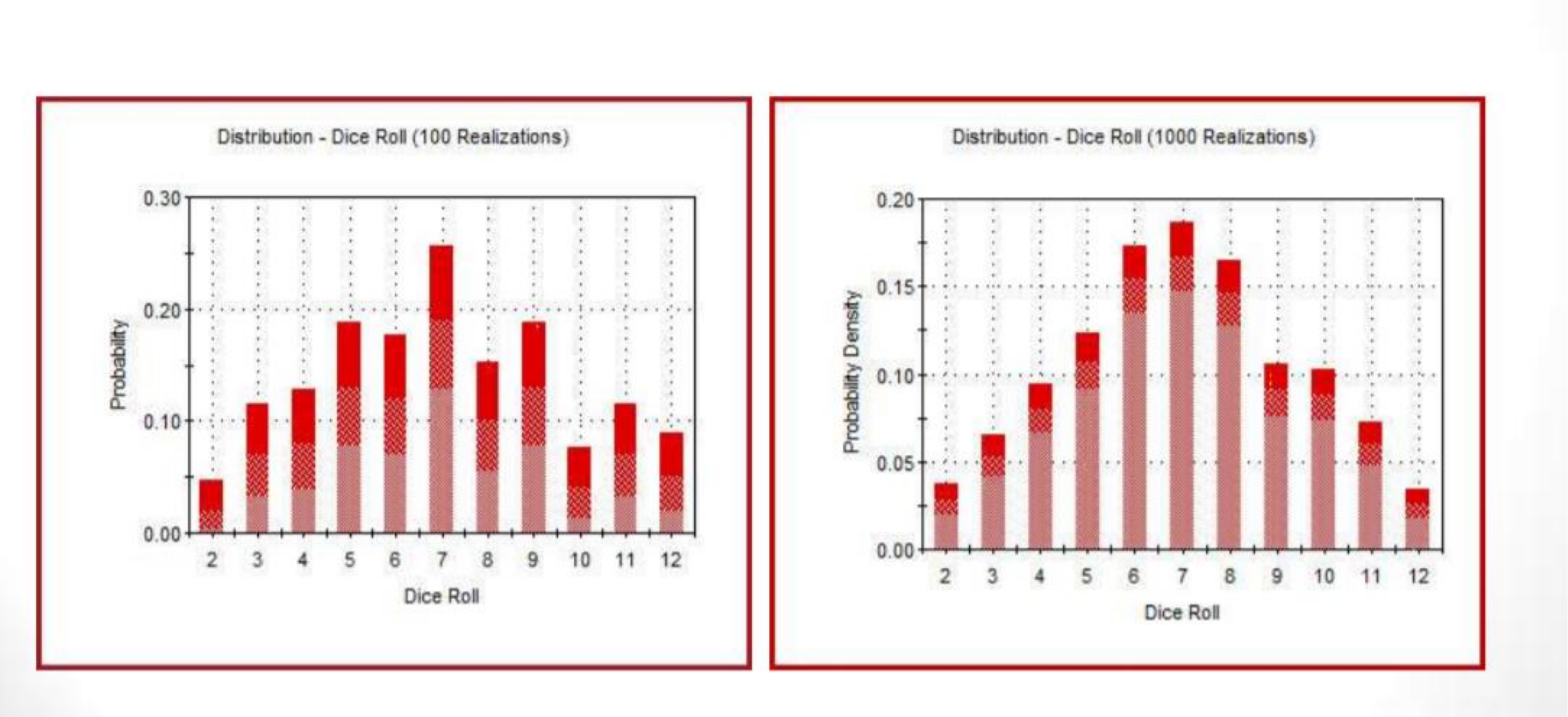
EXAMPLE: ROLLING DICE

- As a simple example of a Monte Carlo simulation, consider calculating the probability of a particular sum of the throw of two dice (with each die having values one through six). In this particular case, there are 36 combinations of dice rolls.
- Based on this, We can manually compute the probability of a particular outcome. For example, there are six different ways that the dice could sum to seven. Hence, the probability of rolling seven is equal to 6 divided by 36 = 0.167.

- Without computer, we could throw the dice a hundred times and record how many times each outcome occurs. If the dice totaled seven 18 times (out of 100 rolls), we would conclude that the probability of rolling seven is approximately 0.18 (18%).
- But, better than rolling dice a hundred times, we can easily use a computer to simulate rolling the dice 10,000 times (or more).

HOW THE RESULTS ARE ACCURATE

The accuracy of a Monte Carlo simulation is a function of the number of realizations.



A simple example of Monte Carlo- Estimating Pi

Can we determine the value of π using a Monte Carlo method ?

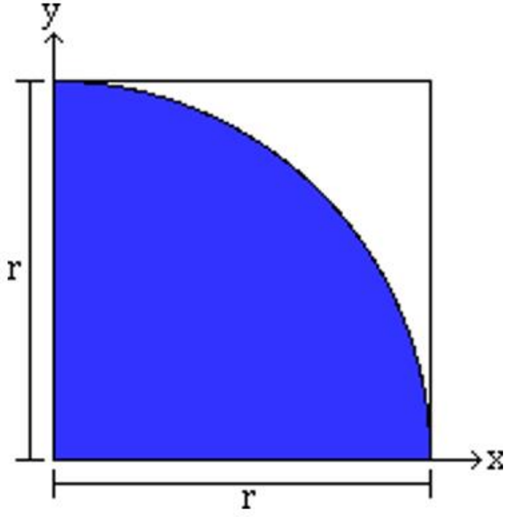
- Draw a square on the ground, then inscribe a circle within it.
- Uniformly scatter some objects of uniform size (grains of rice, sand or throw darts) over the square.
- Count the number of objects inside the circle and the total number of objects.
- The ratio of the two counts is an estimate of the ratio of the two areas, which is $\pi/4$.
- Multiply the result by 4 to estimate π.

A simple example of Monte Carlo- Estimating Pi

If you are a very poor dart player, it is easy to imagine throwing darts randomly at the figure, and

it should be apparent that of the total number of darts that hit within the square,

the number of darts that hit the shaded part (circle quadrant) is proportional to the area of that part. In other words,



$$\frac{\text{\# darts hitting shaded area}}{\text{\# darts hitting inside square}} = \frac{\text{area of shaded area}}{\text{area of square}}$$

A simple example of Monte Carlo- Estimating Pi

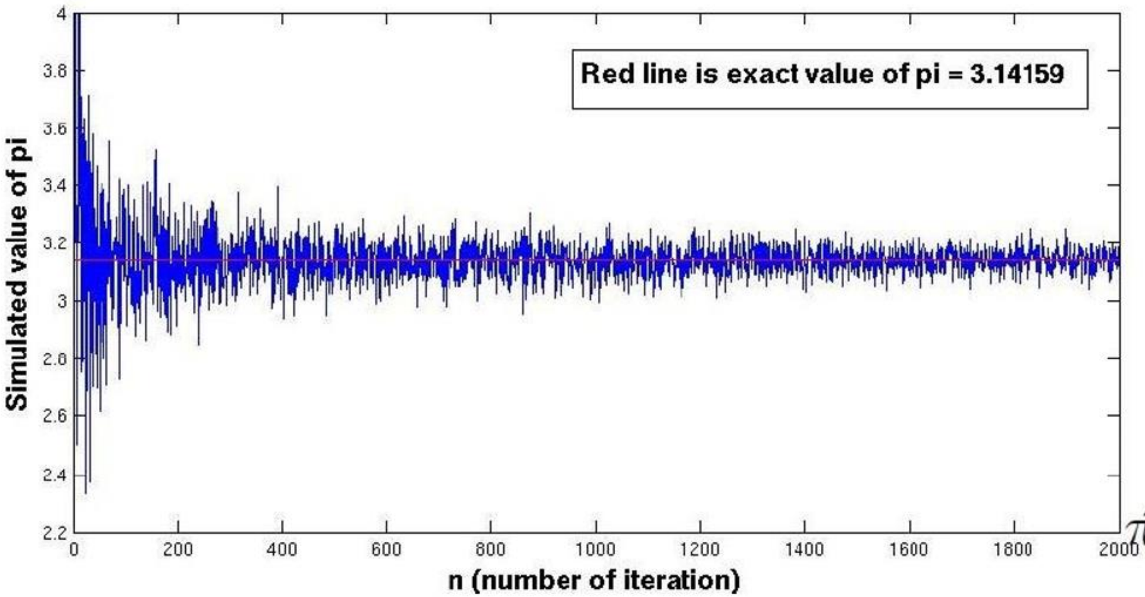
If you remember your geometry, it's easy to show that

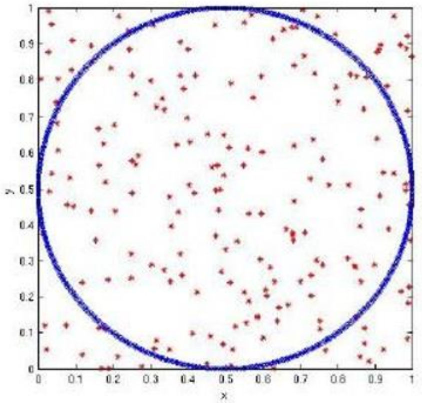
$$\frac{\#\ \text{darts hitting shaded area}}{\#\ \text{darts hitting inside square}} = \frac{\frac{1}{4}\pi r^2}{r^2} = \frac{1}{4}\pi$$

or

$$\pi = 4\,\frac{\#\ \text{darts hitting shaded area}}{\#\ \text{darts hitting inside square}}$$

A simple example of Monte Carlo- Estimating Pi





$$\hat{\pi}_n = 4 \times \frac{164}{200} = 3.28.$$

A Simple Integral

Consider the simple integral:

$$I = \int_b^a f(x)\,dx$$

This can be evaluated in the same way as the ***pi*** example.

By randomly tossing darts at a graph of the function and tallying the ratio of hits inside and outside the function.

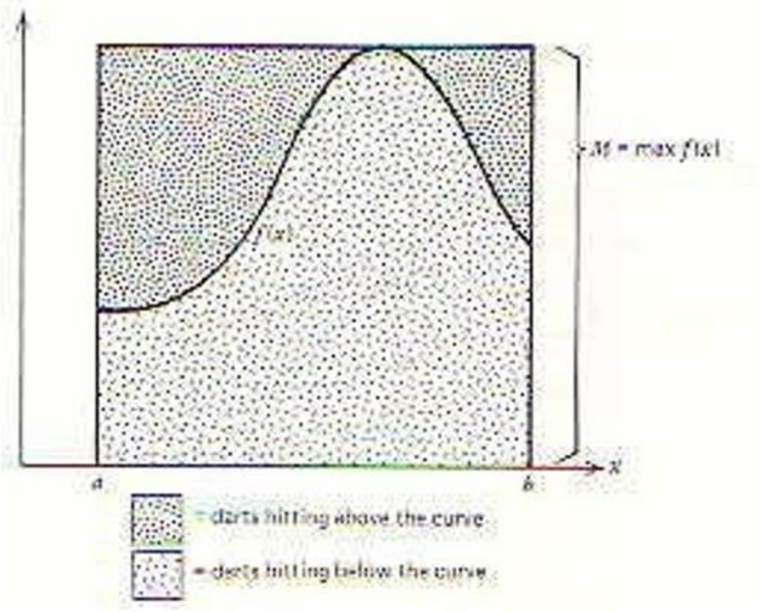
A Simple Integral (continued…)

$R = \{(x,y): a \le x \le b, 0 \le y \le \max f(x)\}$

Randomly tossing 100 or so darts we could approximate the integral…

*I = [fraction under $f(x)$] * (area of R)*

This assumes that the dart player is throwing the darts randomly, but not so random as to miss the square altogether.



Ways of Sampling

Some examples of Monte Carlo sampling methods include: direct sampling, importance sampling, and rejection sampling.

- **Direct Sampling**. Sampling the distribution directly without prior information.
- **Importance Sampling**. Sampling from a simpler approximation of the target distribution.
- **Rejection Sampling**. Sampling from a broader distribution and only considering samples within a region of the sampled distribution.

Monte Carlo Simulation-Methodology

1. **Static Model Generation**

- Every Monte Carlo simulation starts off with developing a deterministic model which closely resembles the real scenario.
- In this deterministic model, we use the most likely value (or the base case) of the input parameters.
- We apply mathematical relationships which use the values of the input variables, and transform them into the desired output.

Monte Carlo Simulation-Methodology

2. **Input Distribution Identification**

- When we are satisfied with the deterministic model, we add the risk components to the model.
- As mentioned before, since the risks originate from the stochastic nature of the input variables, we try to identify the underlying distributions, if any, which govern the input variables.
- This step needs historical data for the input variables. There are standard statistical procedures to identify input distributions,

Monte Carlo Simulation-Methodology

3. **Random Variable Generation**

- After identifying underlying distributions for the input variables, we generate a set of random numbers (also called random samples) from these distributions.
- One set of random numbers, consisting of one value for each of the input variables, will be used in the deterministic model, to provide one set of output values.
- This process is repeated by generating more sets of random numbers, one for each input distribution, and collect different sets of possible output values.
- This part is the core of Monte Carlo simulation.

Random Sampling

- In statistics, a finite subset of individuals from a population is called a sample.
- In random sampling, the samples are drawn at random from the population, which implies that each unit of population has an equal chance of being included in the sample.

Random Number

- What is random number ? Is 3 ?

  There is no such thing as single random number

- Random number

  A set of numbers that have nothing to do with the other numbers in the sequence

- In a uniform distribution of random numbers in the range [0,1] , every number has the same chance of turning up.
  - 0.00001 is just as likely as 0.5000

Random vs. Pseudo-Random

- **Random numbers** have no defined sequence or formulation. Thus, for any *n* random numbers, each appears with equal probability.
- If we restrict ourselves to the set of 32-bit integers, then our numbers will start to repeat after some very large *n*. The numbers thus clump within this range and around these integers.
- Due to this limitation, computer algorithms are restricted to generating what we call **pseudo-random numbers**.

Pseudo Random Number Generators

- A random number generator is designed to generate a sequence of numbers that appear to be independent draws from a population, and that also pass a series of statistical tests.

- Usually a uniform distribution in the range [0,1]

Monte Carlo Simulation-Methodology

4. **Analysis and Decision Making**

- After we have collected a sample of output values in from the simulation, we perform statistical analysis on those values.
- This step provides us with statistical confidence for the decisions which we might make after running the simulation.

Steps for Monte Carlo

- Establish Probability Distribution.
- Cumulative Probability Distribution
- Setting Random Numbers Interval
- Generating Random Numbers.
- Find the answer of a question asked during the above four steps.

Question: Simulate the dentist clinic for 4 hours where each patient takes around 30 minutes depending on the type of dental work to be done. Find out:
Average Waiting Time for patients?
Idleness of Doctor?
Random Nos: 40, 82, 11, 34, 25, 66, 17, 19

Category	Time Required (in minutes)	No. of Patients (Total 100)	Probability	Cumulative Probability	Random Number
Filling	45	40	0.40	0.40	00-39
Crown	60	15	0.15	0.55	40-54
Cleaning	15	15	0.15	0.70	55-69
Extracting	45	10	0.10	0.80	70-79
Check-up	15	20	0.20	1.0	80-99

Patient Serial No.	Scheduled Arrival	Random Numbers	Category	Service Time
1	8.00	40	Crown	60
2	8.30	82	Check-up	15
3	9.00	11	Filling	45
4	9.30	34	Filling	45
5	10.00	25	Filing	45
6	10.30	66	Cleaning	15
7	11.00	17	Filling	45
8	11.30	79	Extracting	45

Patient	Arrival	Service Starts	Service Duration (in minute)	Service Ends	Waiting Time (in minutes)	Idle Time
1	8.00	8.00	60	9.00	0	0
2	8.30	9.00	15	9.15	30	0
3	9.00	9.15	45	10.00	15	0
4	9.30	10.00	45	10.45	30	0
5	10.00	10.45	45	11.30	45	0
6	10.30	11.30	15	11.45	60	0
7	11.00	11.45	45	12.30	45	0
8	11.30	12.30	45	1.15	60	0

Average Waiting Time: 285/8 minutes
Average Idle Time: 0 minutes

Solve the following question:

Q A bakery keeps records of a popular brand of cake.

Daily Demand	0	15	25	35	45	50
Probability	0.01	0.15	0.20	0.50	0.12	0.02

Given set of Random Numbers: 21, 27, 47, 54, 60, 39, 43, 91, 25 and 20

(i) Using this sequence simulate demand for next 10 days.

(ii) Find stock simulation if owner make 30 cakes/day. Also estimate average daily demand for the cakes on the basis of simulated data.