AE 308/775 Mid-semester Examination Date: 16th September, 2019
Time: 1830-2030 Venue: LH 302 Weightage: 20%
Note: Closed notes. Only calculator permitted.
Model Solutions

1. Determine the inverse Laplace transform of the given function F(s), using partial fractions approach. (3)

$$F(s)=\frac{3s+2}{(s+2)^2(s+5)}$$ (Hint: Use the relations given below as applicable).

$$L^{-1}\left(\frac{1}{s+a}\right)=e^{-at};\quad L^{-1}\left(\frac{1}{\{s+a\}^2}\right)=te^{-at};\quad \frac{1}{(s-p_1)^2}=\frac{A_1}{s-p_1}+\frac{A_2}{(s-p_1)^2}$$

$$A_j=\frac{1}{(k-j)!}\frac{d^{k-j}}{ds^{k-j}}\left[(s-p_1)^k\,G(s)\right]|_{s=p_1};\quad j=1,2;\quad k=2$$

$$F(s)=\frac{3s+2}{(s+2)^2(s+5)}=\frac{A_1}{(s+2)}+\frac{A_2}{(s+2)^2}+\frac{A_3}{(s+5)}$$

$$A_3=\left(\frac{3s+2}{(s+2)^2}\right)_{s=-5}=-\frac{13}{9};\quad A_2=\left(\frac{3s+2}{(s+5)}\right)_{s=-2}=-\frac{4}{3}$$

$$A_1=\left(\frac{3s+2-3(s+5)}{(s+5)^2}\right)_{s=-2}=-\frac{13}{9};\quad f(t)=L^{-1}\left[\frac{\left(-13/9\right)}{(s+2)}+\frac{\left(-4/3\right)}{(s+2)^2}+\frac{\left(-13/9\right)}{(s+5)}\right]$$

$$f(t)=\left(-13/9\right)e^{-2t}+\left(-4/3\right)te^{-2t}+\left(-13/9\right)e^{-5t}$$

2. Consider the following characteristic equation and determine the number of roots lying in the RH s-plane. Also, determine the roots that are symmetrically located about the origin and correlate with the Routh's array 1st column entries. (2)

$$s^5+s^4+5s^3+5s^2+4s+4=0$$

1	5	4
1	5	4
0(4)	0(10)	0
2.5	4	0
3.6	0	
4		

A zero row indicates roots symmetric about the origin. However, as there are no sign changes, there are no roots in RH s-plane. The auxiliary polynomial and the symmetric roots are as follows.

$$s^4+5s^2+4=0\rightarrow\left(s^2+4\right)\times\left(s^2+1\right)=0\rightarrow s=\pm j2;\quad \pm j1$$

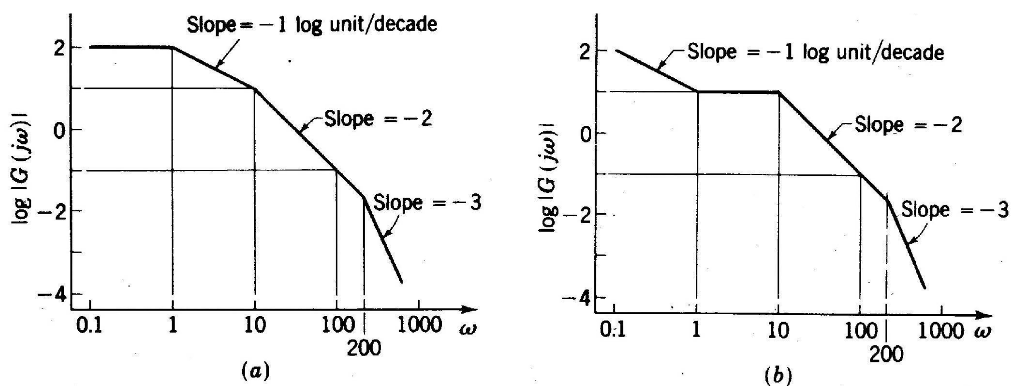
3. Sketch the straight line bode magnitude plot for the following system and mark pole, zero locations, ω = 0 intercept & slope, ω = ∞ slope and slope of line between ω = 1 & ω = 2. Also what is the phase angle for ω = 2? (3)

$$G(s) = \frac{5(s+1)}{(s+2)(s+0.2)}$$

Corner frequencies: 0.2, 1, 2
ω = 0 intercept: 21.9 dB
ω = 0 slope: 0 dB / decade
ω = ∞ slope: -20 dB / decade
Slope between ω = 1 & ω = 2: 0 dB / decade

$$\phi|_{\omega=2} = \tan^{-1}2 - \tan^{-1}1 - \tan^{-1}10 = 63.4^o - 45^o - 84.3^o = -65.9^o$$

4. Consider the two straight line log-magnitude plots for stable minimum phase plants, as given below. Determine the corresponding transfer functions. (Hint: 1 log unit = 20 dB). (3)



Plot (a) Features: G(0) = 40 dB; dG(0)/dω = 0 dB/decade so type 0; slope change by -20 dB/decade at each corner frequency so all poles with multiplicity of 1; 3 Corner Frequencies = 1, 10 200 (all poles, no zeros), dG(∞)/dω = -60 dB/decade; K = 2×10^5.

$$G_a(s) = \frac{2\times10^5}{(s+1)(s+10)(s+200)}$$

Plot (b) Features: G(0) = ∞ dB; dG(0)/dω = -20 dB/decade so type 1; Slope change from -20 dB/decade to 0 dB/decade indicating a zero at corner frequency 1; Slope change from 0 dB/decade to -40 dB/decade at corner frequency of 10 indicating double pole at -10; next slope change by -20 dB/decade at corner frequency 200; 5 Corner Frequencies = 0, 1, 10, 10, 200 (4 poles, one zeros), dG(∞)/dω = -60 dB/decade; G(0.1) = 40 dB; K = 2×10^5.

$$G_b(s) = \frac{2\times10^5(s+1)}{s(s+10)^2(s+200)}$$

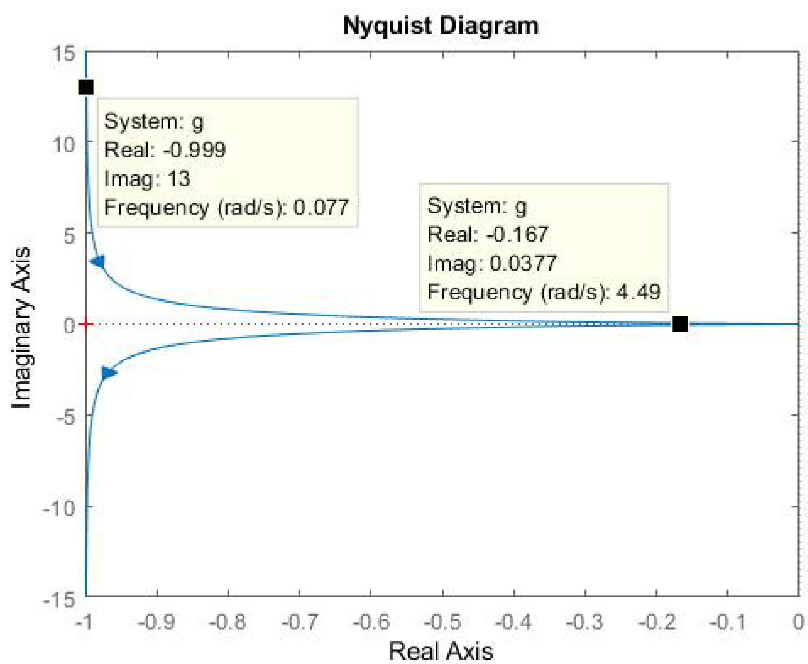
5. Sketch the positive half of the Nyquist plot for the following plant and mark the point (both magnitude & frequency) at which it crosses the imaginary axis. (3)

$$G(s) = \frac{10(s+2)}{(s+5)(s+1)}$$

$\omega = 0 \rightarrow |G(j0)| = 4.0; \angle G(j0) = 0$
$\omega = 1 \rightarrow |G(j1)| = 3.1; \angle G(j1) = -29.7^o$
$\omega = 2 \rightarrow |G(j2)| = 2.3; \angle G(j2) = -40.2^o$
$\omega = 3 \rightarrow |G(j3)| = 1.9; \angle G(j3) = -46.2^o$
$\omega = 5 \rightarrow |G(j5)| = 1.5; \angle G(j5) = -55.5^o$
$\omega = 10 \rightarrow |G(j10)| = 0.9; \angle G(j0) = -69.0^o$
$\omega = 20 \rightarrow |G(j10)| = 0.5; \angle G(j0) = -78.8^o$

The plot never crosses imaginary axis and becomes tangent to negative imaginary axis for $\omega = \infty$.

6. Given below is the Nyquist plot of a plant, which has one pole in RH s-plane. Using features in the plot, give the applicable complete picture and based on it, determine whether or not the closed loop system is stable. (2)



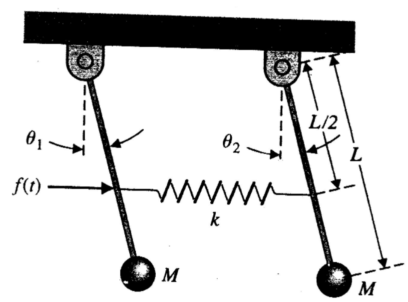
As the system has one pole in RH s-plane, it is a non-minimum phase system so that an additional 180^o phase is added and hence, while the $\omega = 0+$ asymptote is tangent to positive imaginary axis, we consider its counterpart i.e. $\omega = 0-$ asymptote, which is negative real axis. Therefore, the system is also type ‘1’. In view of the above, the remaining part of the Nyquist plot corresponding to the Nyquist curve segment from 0- to 0+ can be obtained as follows.

$$\lim_{s\to 0^-} G(s) \approx -\frac{K}{s}; \quad s = \varepsilon e^{j\theta} \quad -90 < \theta < +90; \quad \lim_{s\to 0^-} G(s) \approx -\frac{K}{\varepsilon} e^{-j\theta}$$

$$|G(j\omega)| \approx \infty; \quad \angle G(j\omega) = -90^o, \quad -180^o, \quad -270^o$$

Thus, the remaining segment of Nyquist plot starts from –ve imaginary axis at infinity and traces out a semi-circle of infinite radius in the clockwise direction from –ve imaginary axis to positive imaginary axis. In view of the above, we find that the Nyquist plot encircles the point -1+j0 once in clockwise direction so that N = 1. Further, as P = 1, we get Z = 2, so that the closed system has 2 poles in RH s-plane and hence, is unstable. Closed loop characteristic equation gives a pair of unstable complex conjugate poles as 0.794 ± j1.37 and a stable real pole as -1.59.

7. Consider the double pendulum system connected via a spring, as shown below.



(a) Assuming hinges to be frictionless, rods to be rigid and massless and both the angles to be small quantities, derive the applicable linear equations of motion. (Spring force is zero when $\theta_1 = \theta_2$). (1)

$$\theta_1 \text{ pendulum equilibrium:} \quad ML^2\ddot{\theta}_1 + k\times\frac{L}{2}(\theta_1-\theta_2)\times\frac{L}{2} + MgL\theta_1 = f(t)\frac{L}{2}$$

$$\theta_2 \text{ pendulum equilibrium:} \quad ML^2\ddot{\theta}_2 + k\times\frac{L}{2}(\theta_2-\theta_2)\times\frac{L}{2} + MgL\theta_2 = 0$$

(b) Next, create a block diagram by converting the above equations in Laplace domain and arrive at the transfer function between θ_2 and f by suitably reducing the block diagram. (3)

The Laplace form of equations is as follows.

$$\Theta_1 \text{ pendulum equilibrium: } ML^2s^2\Theta_1 + k\left(\frac{L}{2}\right)^2(\Theta_1-\Theta_2) + MgL\Theta_1 = F(s)\frac{L}{2}$$

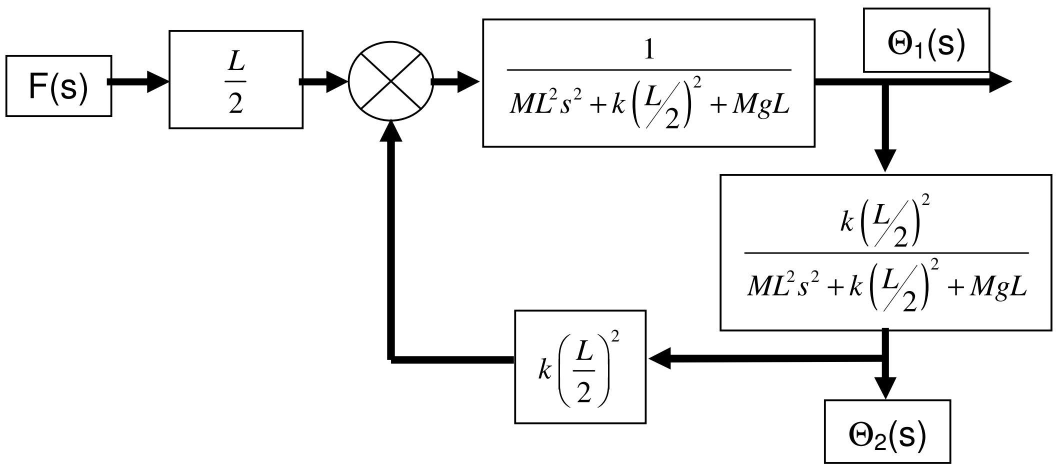
$$\Theta_2 \text{ pendulum equilibrium: } ML^2s^2\Theta_2 + k\left(\frac{L}{2}\right)^2(\Theta_2-\Theta_1) + MgL\Theta_2 = 0$$

The applicable block diagram can be created by re-writing the equations in the input – output form as follows.

$$\Theta_1 \text{ I/O form:} \quad \Theta_1 = \left[F(s) \times \left(L/2 \right) + k \left(L/2 \right)^2 \Theta_2 \right] \times \frac{1}{ML^2 s^2 + k \left(L/2 \right)^2 + MgL}$$

$$\Theta_2 \text{ I/O form:} \quad \Theta_2 = \left[\frac{k \left(L/2 \right)^2}{ML^2 s^2 + k \left(L/2 \right) + MgL} \right] \times \Theta_1$$

The corresponding block diagram representation is as follows.



The block diagram can be reduced by splitting the branch point, which generates a feedback loop to summing junction with TF in the feedback path and a branch from Θ_1 to Θ_2 through the TF.. Next, we can reduce the feedback loop using the standard formula and arrive at the relation between 'Θ_1' and 'F(s)'. Lastly, relation between 'Θ_2' and 'F(S)' can be obtained by multiplying Θ_1 transfer function with the TF value used earlier.

$$\Theta_1 = \left[F(s) \times \left(L/2 \right) \right] \times \frac{1 \Big/ \left(ML^2 s^2 + k \left(L/2 \right)^2 + MgL \right)}{1 - \left[k \left(L/2 \right)^2 \right]^2 \Big/ \left(ML^2 s^2 + k \left(L/2 \right)^2 + MgL \right)^2} = \frac{\left(L/2 \right) \times \left(ML^2 s^2 + k \left(L/2 \right)^2 + MgL \right)}{\left(ML^2 s^2 + k \left(L/2 \right)^2 + MgL \right)^2 - \left[k \left(L/2 \right)^2 \right]^2} F(s)$$

$$\Theta_2 = \left[k \left(L/2 \right)^2 \Big/ \left(ML^2 s^2 + k \left(L/2 \right)^2 + MgL \right) \right] \times \Theta_1 = \left[\frac{k \left(L^2/4 \right) \times \left(L/2 \right)}{\left(ML^2 s^2 + k \left(L/2 \right)^2 + MgL \right)^2 - \left[k \left(L/2 \right)^2 \right]^2} \right] F(s)$$

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