PD Design with Root Locus

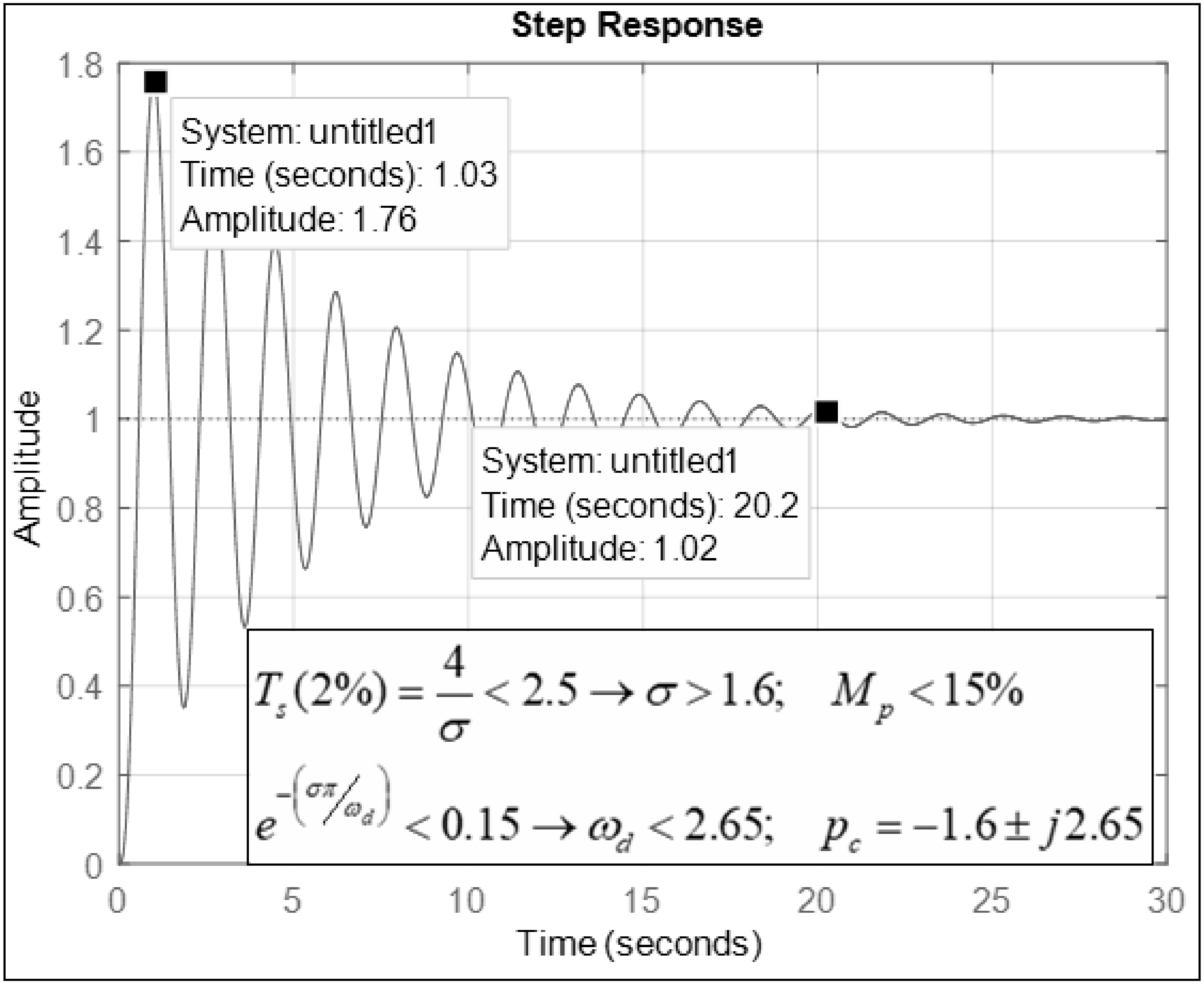
Consider the **plant** of a unity negative feedback system, as given **below.**

$$G(s) = \frac{100}{s(s+4)^2}$$

Design a **PD Controller** so that closed loop **step response** has $\mathbf{M_p < 15\%}$ and 2% $T_s < 2.5$ sec.

For the above design, determine the change in $\mathbf{T_r}$ **and** $\mathbf{K_V}$ due to the **PD Controller** designed.

Requirements & Plant Features



$$T_s(2\%) = \frac{4}{\sigma} < 2.5 \rightarrow \sigma > 1.6; \quad M_p < 15\%$$

$$e^{-\left(\sigma\pi/\omega_d\right)} < 0.15 \rightarrow \omega_d < 2.65; \quad p_c = -1.6 \pm j2.65$$

Design Solution

Desired Dominant Closed Loop Pole: $p_c = -1.6 + j2.65$

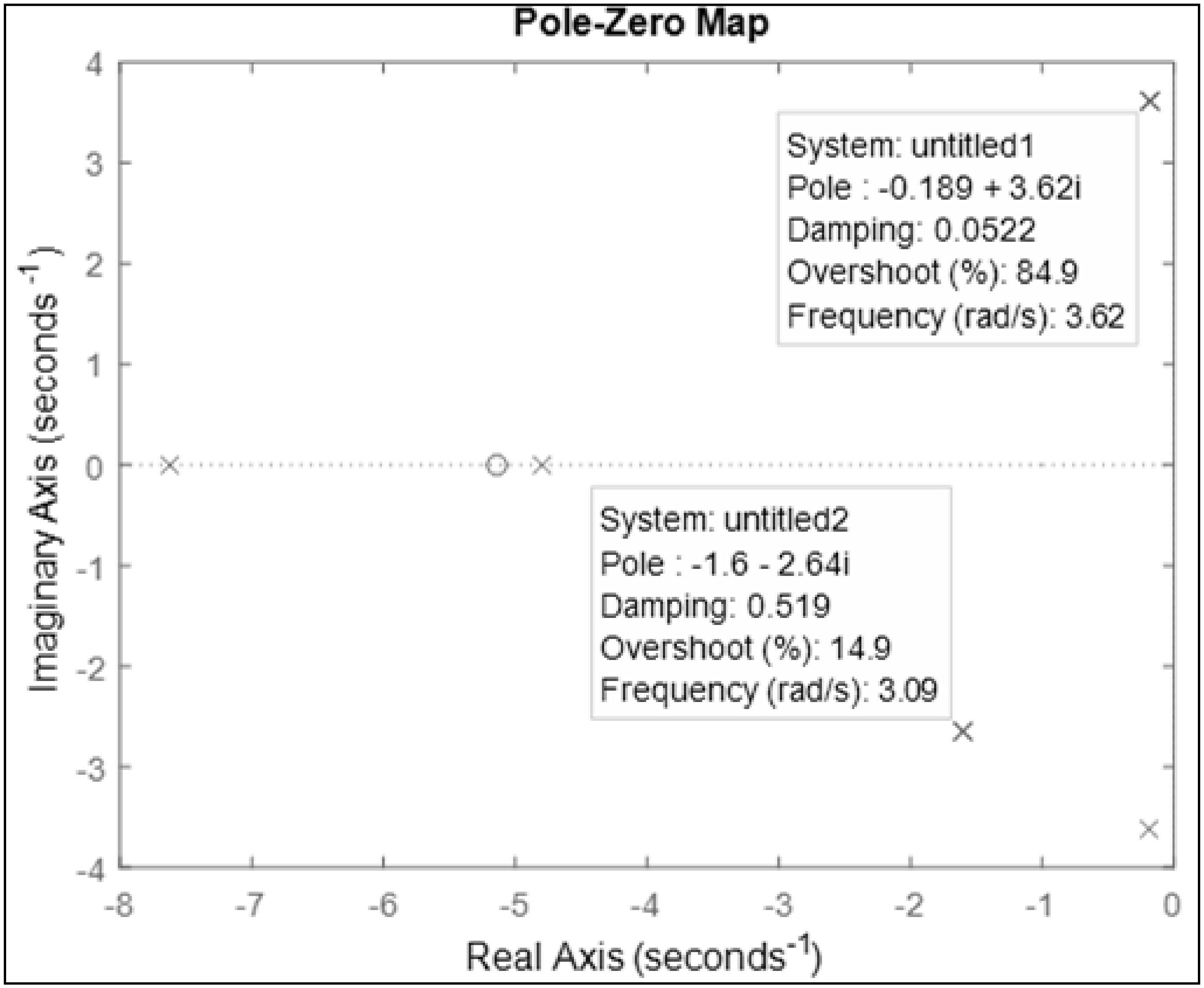
$$\theta_1 = 90^o + \tan^{-1}\left(\frac{1.6}{2.65}\right) = 121.1^o; \quad \theta_2 = \theta_3 = \tan^{-1}\left(\frac{2.65}{2.4}\right) = 47.8^o$$

$$\phi = \tan^{-1}\left(\frac{2.65}{z-1.6}\right) = -180 + 121.1 + 95.7 = 36.8^o \rightarrow z = 5.14$$

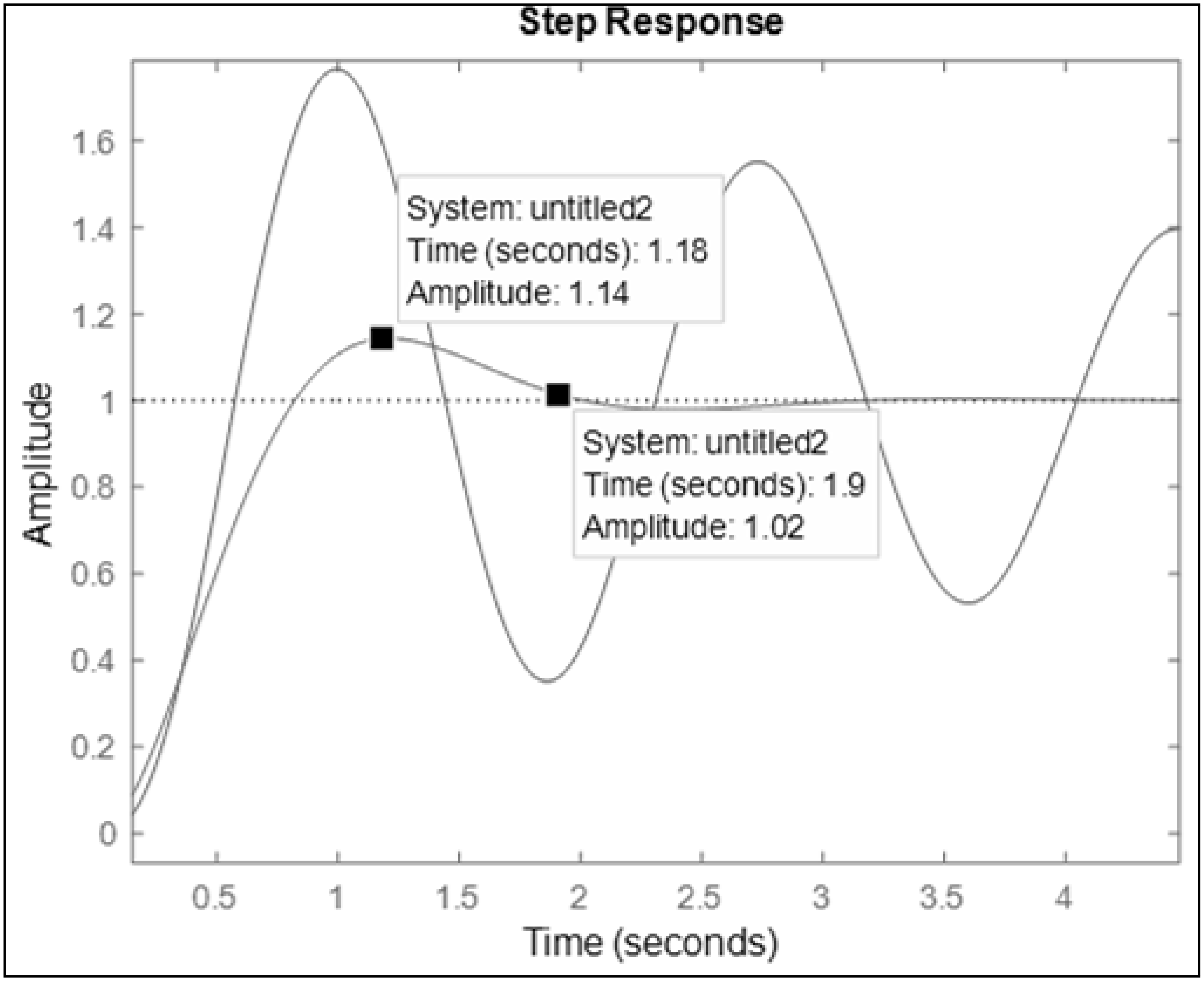
$$G_{PD}(s) = K(0.194s+1); \quad \frac{K\left|0.69 + j0.51\right| \times 100}{\left|-1.6 + j2.65\right| \times \left|2.4 + j2.65\right|^2} = 1$$

$$K = 0.459; \quad G_{PD}(s) = 0.459(0.194s+1) = 0.089(\text{s}+5.14)$$

Pole-zero Map Comparison



Step Response Comparison



Design is **fine,** from **transient** point of view, but we find that **K_v has reduced** by a factor of **0.459.**

Further, we find that **rise time** increases from 0.576s to 0.826s.

PD Design with Bode

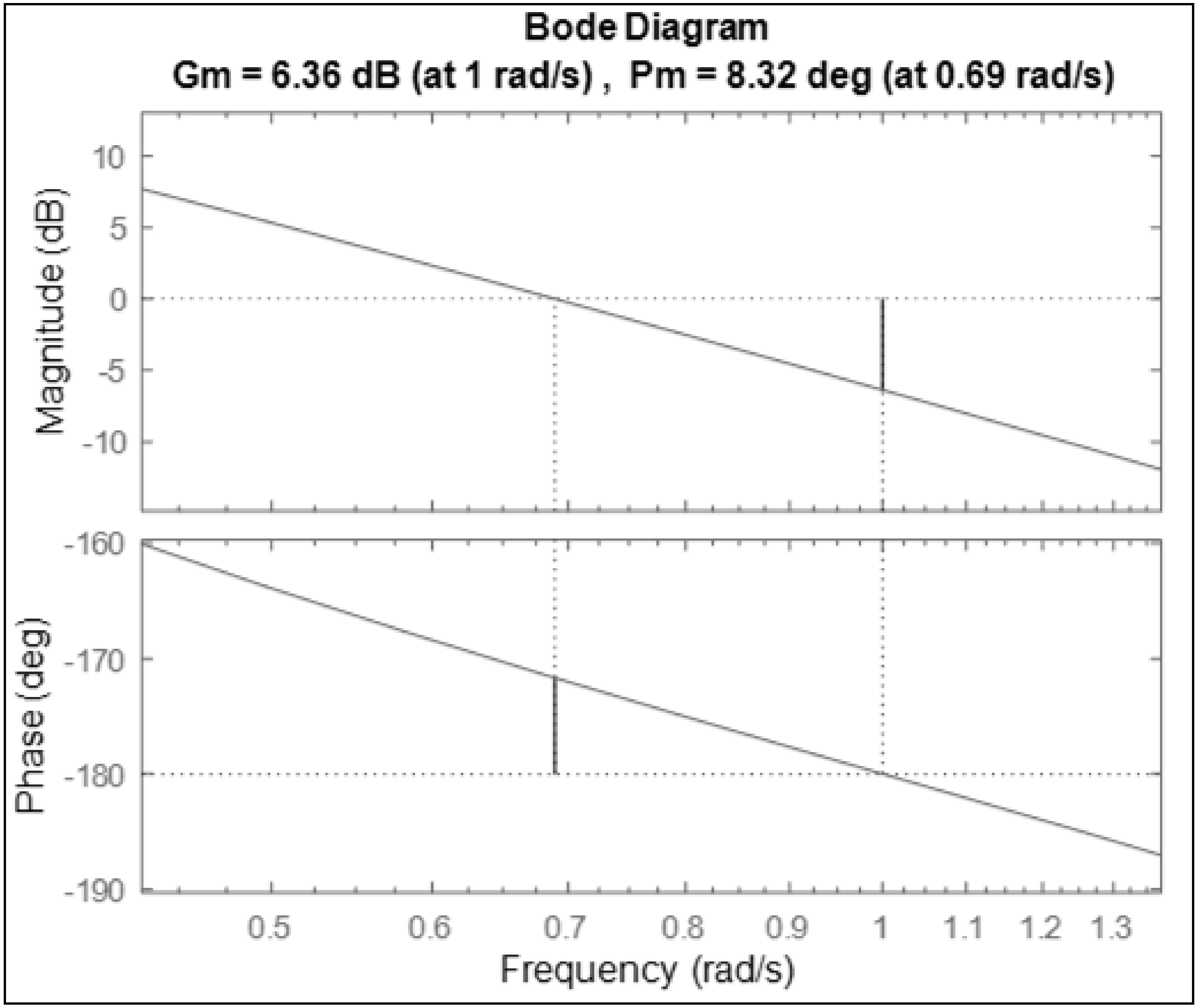
Consider the **plant** given below.

$$G(s) = \frac{2.5}{s(s+0.2)(s+5)}$$

Design a **PD controller** so that the resulting **PM** is greater than **20^o.**

Determine the **new GM**, closed loop **step response** and compare these with that of the **uncompensated system.**

Uncompensated Margins



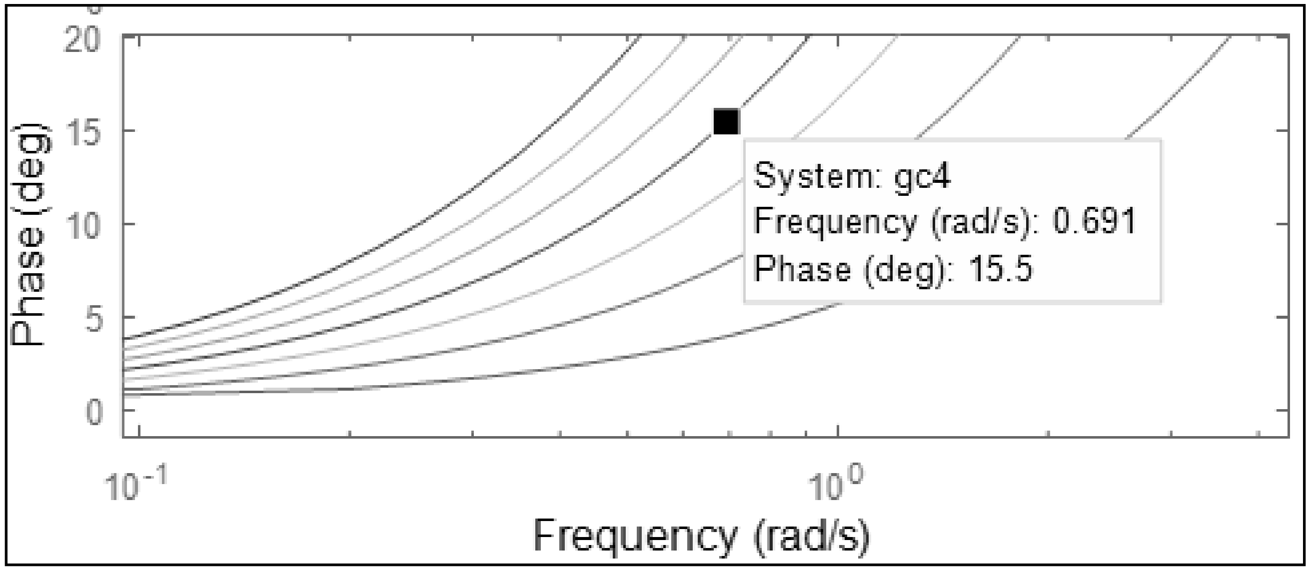
We see that **PM** is to be increased by **~12^{o} @ 0.69**.

However, as **GCO increases,** we need to keep a **buffer for PM.**

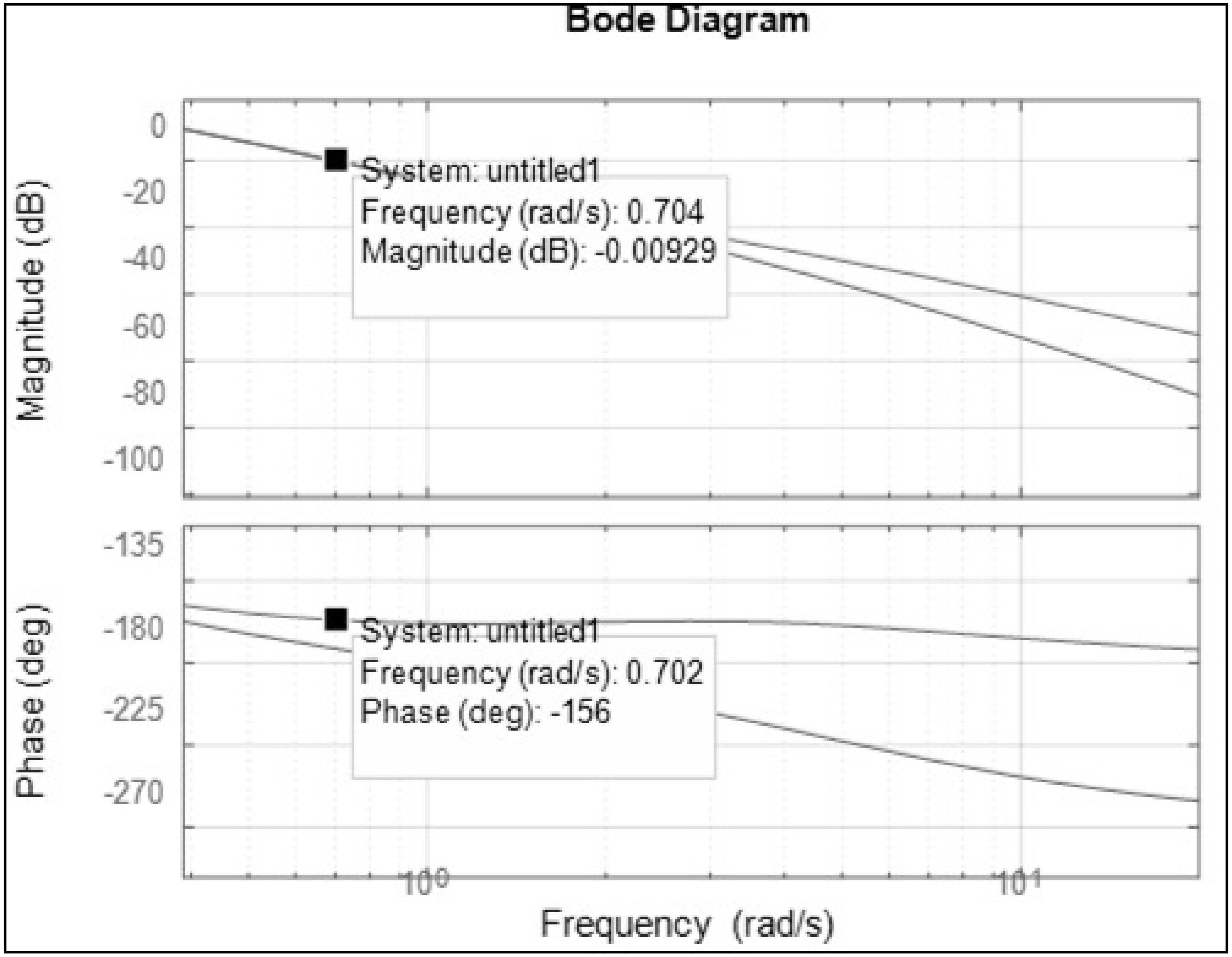
PD Design Space Exploration

Desired $\mathbf{T_d}$ is taken from **phase plot**, of a large number of **PD controllers**, as shown alongside.

It is seen that **4th curve** satisfies the **requirement,** which corresponds to a **PD** as **(0.4s+1).**



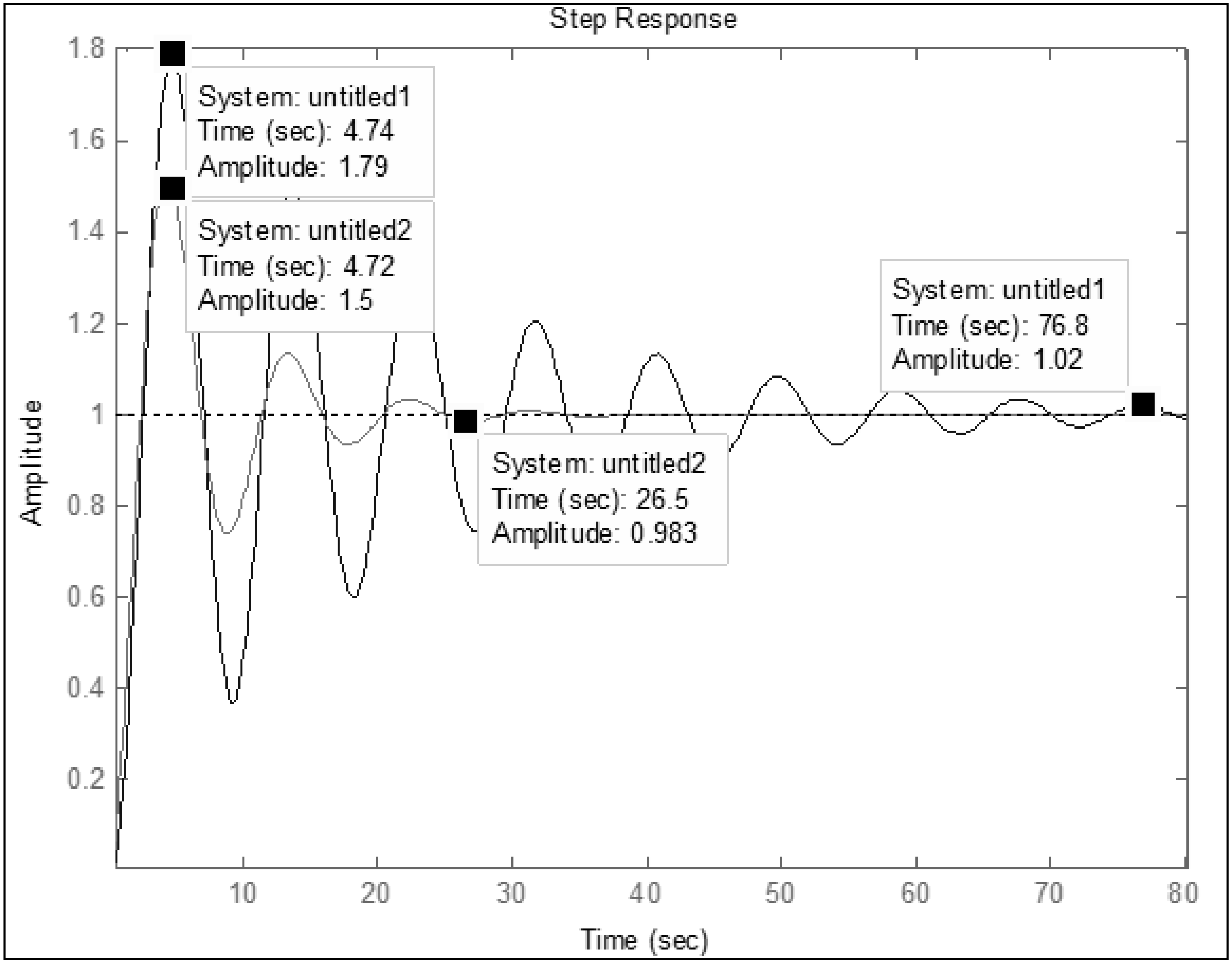
Comparison of Margins



We see that **GCO increases** only by a **small** amount so that there is **no need** for buffer phase.

This **means** that we could have **used** a smaller $\mathbf{T_d}$ (larger corner frequency.).

Step Response Comparison



PD Design with Bode

Consider the following **plant.**

$$G(s) = \frac{K}{s(s+4)(s+6)}$$

Design a **PD controller** to satisfy **following** specifications in the **closed loop.**

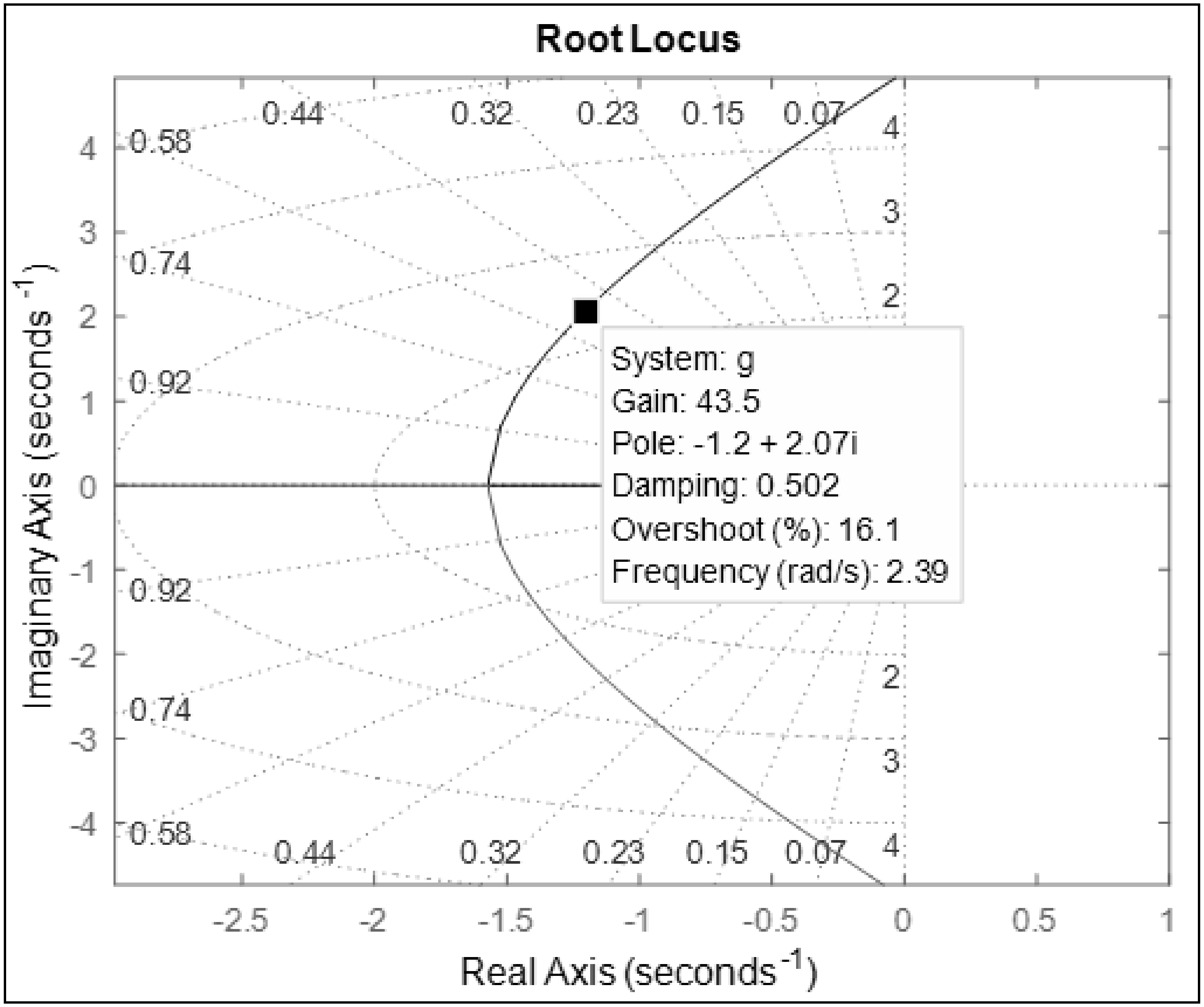
$\zeta \geq 0.5$, PM > 65°.

Also, determine the **GM,** peak overshoot & **bandwidth** of compensated system and **compare** these with those of **uncompensated system.**

PD Design Strategy

As no **K_V requirement** is specified, we can determine, **'K' using 'ζ'**, through **root locus**, as shown along side.

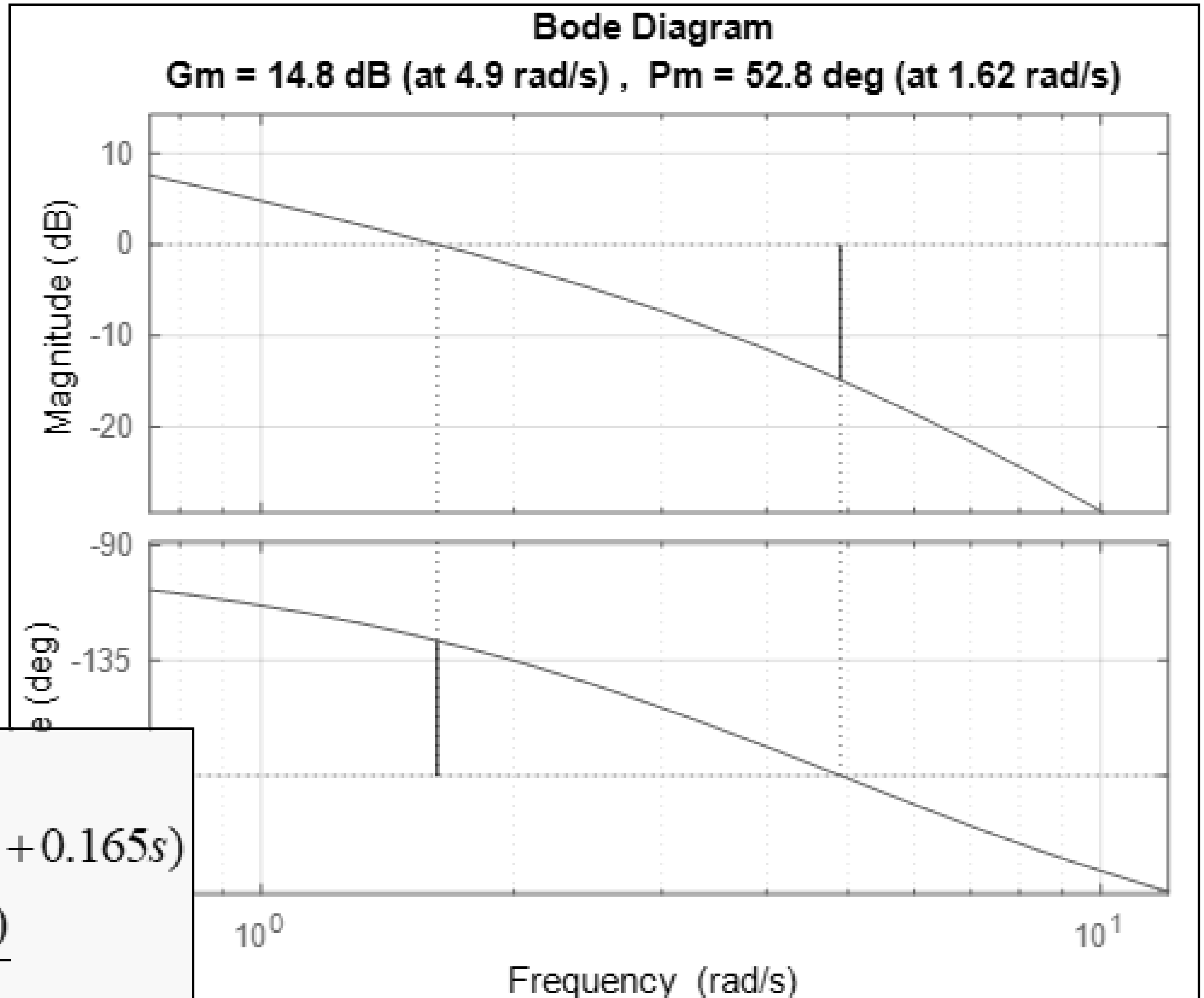
We find that **K = 43.5** meets 'ζ' requirement.



PD Design Strategy

We now **obtain the PM** of the gain **adjusted system** as shown alongside.

We find that we **need to augment** the PM by **~13°**, as shown below.



$$1.62T_d = \tan 15^o = 0.268$$

$$T_d = 0.165; \quad G_{PI}(s) = (1+0.165s)$$

$$G'''(s) = \frac{43.5(1+0.165s)}{s(s+4)(s+6)}$$

PD Design Strategy

We find that **PM** has changed only **marginally,** as shown alongside.

What seems to **be the issue?** How can this be **addressed?**