① $I_0 dl = 100 \times 0.1\lambda$

$\lambda = \frac{c}{10 \times 10^6} = 30m$

$\therefore I_0 dl = 300$

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{30} = 0.209$ rad/m

E_r, E_θ and H_ϕ are given by,

$$H_\phi = j\beta I_0 \frac{dl}{4\pi r} \sin\theta \left[1 - \frac{j}{\beta r}\right] e^{-j\beta r}$$

$$E_r = \eta \frac{I_0 dl}{2\pi r^2} \cos\theta \left\{1 - \frac{j}{\beta r}\right\} e^{-j\beta r}$$

$$E_\theta = \eta \frac{I_0 dl}{4\pi r} j\beta \sin\theta \left\{1 - \frac{j}{\beta r} - \frac{1}{(\beta r)^2}\right\} e^{-j\beta r}$$

$\theta = 0^\circ \Rightarrow$ along z-axis

$r =$	10 m	100 m	10 km
E_r	$\frac{-81.75 - j56.57}{2}$	$\frac{-0.452 - j0.775}{2}$	$\frac{(-59.77 + j66.86) \times 10^{-6}}{2}$
E_θ	0	0	0
H_ϕ	0	0	0

$\theta = 90^\circ \Rightarrow$ In xy plane

$r =$	10 m	100 m	10 km
E_r	0	0	0
E_θ	$81.26 - j150.11$	~~$15.84 - j10.09$~~ $16.23 - j9.45$	~~$-0.14 - j0.125$~~ $-(3.72 + j3.32) \times 10^{-4}$
H_ϕ	$0.81 - j0.454$	$0.043 - j0.025$	

$$E_\theta = \eta \frac{I_0 dl}{4\pi} \sin\theta \left\{ \underbrace{\frac{j\beta}{r}}_{\text{radiation field}} + \underbrace{\frac{1}{r^2} - \frac{j}{\beta r^3}}_{\text{reactive field}} \right\}$$

$$\frac{1}{r^2} = 0.1\left(\frac{\beta}{r}\right) \rightarrow \text{As } \left(\frac{1}{r^3}\right) \text{ fields diminish rapidly}$$

$$r = \frac{1}{0.1\beta}$$

$$= \frac{\lambda}{0.1 \times 2\pi}$$

$$\therefore r = \frac{300}{0.1 \times 2\pi} = 477.46 \text{ m}$$

5. $E(\theta) = \cos\theta \cdot \cos 2\theta \cdot \quad 0 < \theta < 90^\circ$

6

(a) $E(\theta)$ at half power $= 0.707$

$$0.707 = \cos\theta \cdot \cos 2\theta = 1/\sqrt{2}$$

$$\Rightarrow \cos 2\theta = \frac{1}{\sqrt{2}\cdot\cos\theta}$$

$$\Rightarrow 2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$$

$$\Rightarrow \theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right)$$

Iterating with $\theta' = 0 \Rightarrow \theta = 22.5^\circ$

$\theta' = 22.5^\circ \Rightarrow \theta = 20.03^\circ$

$\theta' = 20.47^\circ \Rightarrow \theta = 20.5^\circ$

$$HPBW = 2\theta = 41^\circ$$

(b) $\cos\theta \cdot \cos 2\theta = 0.$

$\Rightarrow \theta = 45^\circ$

$FNBW = 90^\circ$

6. (3) $d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$

$$\Rightarrow \Omega = \int_{30^\circ}^{70^\circ} d\phi \int_{20^\circ}^{40^\circ} d\theta \sin\theta$$

$$= \frac{40}{360} 2\pi \left[-\cos\theta\right]_{20}^{40^\circ}$$

$$= 0.22\pi \times 0.173 = 0.121 \text{ sr.}$$

7. Directivity $= \dfrac{U_{max.}}{U_{avg.}}$

$$= \frac{4\pi E_{max.}^2}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} |E(\theta,\phi)|^2 \sin\theta \, d\theta \, d\phi}.$$

$$E = \frac{1}{2}\cos^4\theta.$$

$$E_{max} = \frac{1}{2}, \quad |E_{max}|^2 = \frac{1}{4}$$

$$\int_0^{\pi/2}\int_0^{2\pi} \frac{1}{4}\cos^8\theta \cdot \sin\theta \, d\theta \cdot d\phi = \frac{1}{4}(2\pi)\cdot\left[\frac{-\cos^9\theta}{9}\right]_0^{\pi/2}$$

$$= -\frac{1}{2}(2\pi)\left(\frac{-1}{9}\right)$$

$$= \frac{\pi}{18}$$

$$D = \frac{4\pi \cdot 1/4}{\pi/18} = 18$$

8. $W_{rad} = \hat{a}_r W_r = \hat{a}_r \dfrac{A_0 \sin\theta}{r^2}$ (W/m^2).

$$P_{rad} = \oint_S W_{rad} \cdot \hat{n} \, da$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\hat{a}_r \frac{A_0 \sin\theta}{r^2}\right) \cdot \left(\hat{a}_r r^2 \sin\theta \, d\theta \, d\phi\right)$$

$$= \pi^2 A_0 \cdot (W)$$

Alternatively w.r.to radiation intensity.

$$U = r^2 W_{rad} = A_0 \sin\theta.$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

$$= A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = \pi^2 A_0.$$

3. $$E_\theta = j60 I_m \frac{e^{-j\beta R}}{R} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}.$$

(4)

$$E_{\theta max} = j60 \frac{I_m e^{-j\beta R}}{R}.$$

$$= j60 \cdot \frac{(10\sqrt{2})}{10^4} \cdot e^{-j\frac{2\pi}{0.3}\cdot 10} \quad V/m.$$

$$H_{\phi max} = \frac{E_{\theta max}}{\eta} = \frac{E_{\theta max}}{120\pi}.$$

(5) $$P_r = \frac{1}{2}\frac{|E_\theta|^2}{\eta} = \frac{1}{2}\left[\frac{1}{120\pi}\left(\frac{60 I_m \cos\left(\frac{\pi}{2}\cos\theta\right)}{R\sin\theta}\right)^2\right] \quad W/m^2.$$

total power radiated.

$$W_r = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_r \cdot R^2 \sin\theta \, d\theta \, d\phi$$

$$= 2\pi \int_{\theta=0}^{\pi} \frac{15}{\pi} I_m^2 \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \sin\theta \, d\theta \, d\phi.$$

$$= 30 I_m^2 \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta = 36.54 I_m^2$$

$$\therefore W = 36.54 I_m^2 = 36.54 (10\sqrt{2})^2 \ W.$$

$$= 73.08 \times 100$$

$$= 7.308 \ kW.$$

1.

$$E_\phi = \frac{120\pi^2 I_0}{r} \cdot \frac{s}{\lambda^2} \sin\theta \cdot e^{-j\beta r}$$

$$P_{avg} = \frac{1}{2} \frac{|E_\phi|^2}{\eta}$$

$$P_{rad} = \int \bar{P}_{avg} \, d\bar{s}$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left[\frac{120\pi^2 I_0}{r} \cdot \frac{s}{\lambda^2} \sin\theta \right]^2 \frac{1}{2} \cdot \frac{1}{120\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{\pi} \frac{1}{2} \frac{120\pi^3 s^2}{\lambda^4} \sin^3\theta \cdot 2\pi \cdot I_0^2 \, d\theta$$

$$= I_0^2 \cdot \frac{120\pi^4 s^2}{\lambda^4} \int_0^{\pi} \sin^3\theta \, d\theta = \frac{120\pi^4 s^2}{\lambda^4} \frac{4}{3} I_0^2$$

$$= \frac{160\pi^4 s^2 I_0^2}{\lambda^4}$$

$$R_{rad} = \frac{2 P_{rad}}{I_0^2} = \frac{320\pi^4 s^2 I_0^2}{\lambda^4}$$

for. $I_0 = 1$ amp.

$f = 1$ GHz.

$a = 4.5$ cm.

$R_{rad} = 15.78\ \Omega$.

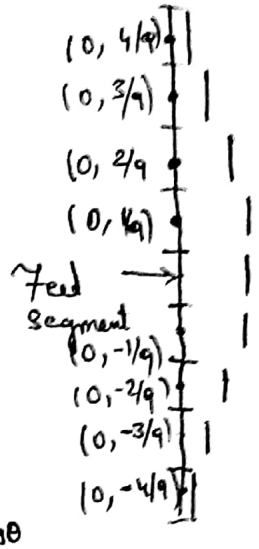
10.

Given $\lambda/2 = 1$

$\lambda = 2m$

$f = \frac{c}{\lambda} = \frac{3\times10^8}{2} = 1.5\times10^8 Hz.$

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi.$



$$E_\theta = j\eta \frac{\beta}{4\pi} \frac{e^{-j\beta R}}{R} \sin\theta$$

$$\Big[I_1 e^{j\beta \cos\theta (4/9)} + I_2 e^{j\beta (3/9)\cos\theta} + I_3 e^{j\beta (2/9)\cos\theta}$$

$$I_4 e^{j\beta \cos\theta (1/9)} + I_5 e^{j\beta (0)\cos\theta} + I_6 e^{j\beta (-1/9)\cos\theta}$$

$$I_7 e^{j\beta \cos (-2/9)} + I_8 e^{j\beta \cos\theta (-3/9)} + I_9 e^{j\beta (-4/9)\cos\theta} \Big].$$

$I_1 = I_9, \quad I_2 = I_8, \quad I_3 = I_7, \quad I_4 = I_6$

$$E_\theta = j(120\pi) \frac{\pi}{4\pi} \frac{e^{-j\pi 2}}{2} \sin(60) \Big[2I_1 \cos\left(\pi \tfrac{1}{2} \tfrac{4}{9}\right) + 2I_2 \cos\left(\pi \tfrac{2}{9} \tfrac{1}{2}\right)$$

$$+ 2I_3 \cos\left(\pi \tfrac{2}{9} \tfrac{1}{2}\right) + 2I_4 \cos\left(\pi \tfrac{1}{9} \tfrac{1}{2}\right) + I_5 \Big].$$

$$= j\,7.5\sqrt{3}\pi \Big[2(1m)\cos\left(\frac{\pi}{6}\right) + 2(3m)\cos\left(\frac{\pi}{9}\right) + 2(5m)\cos\left(\frac{\pi}{8}\right) + 5m \Big]$$

$$= j\,0.906\ V/m.$$

$$E_\theta = j\eta \frac{\beta}{4\pi} \frac{e^{-j\beta R}}{R} \sin\theta \sum_{i=1}^{n} I_i e^{j\beta z' \cos\theta}$$