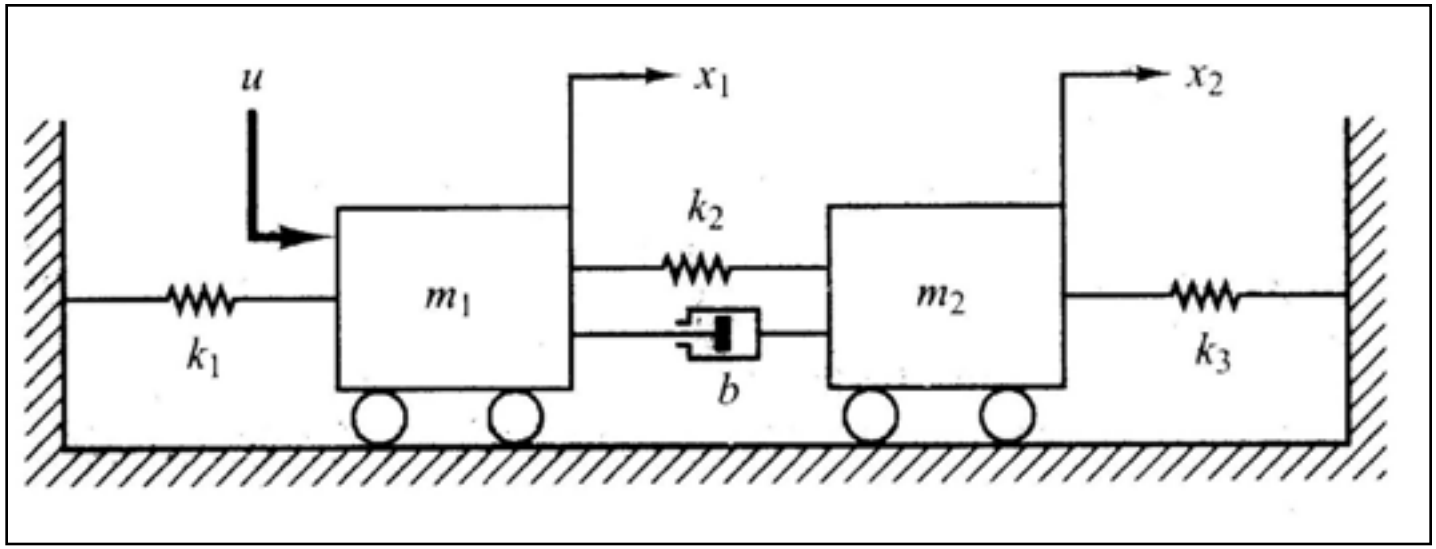
Spring-Mass-Damper Example

A 2-DOF **spring-mass-damper** system is shown below.



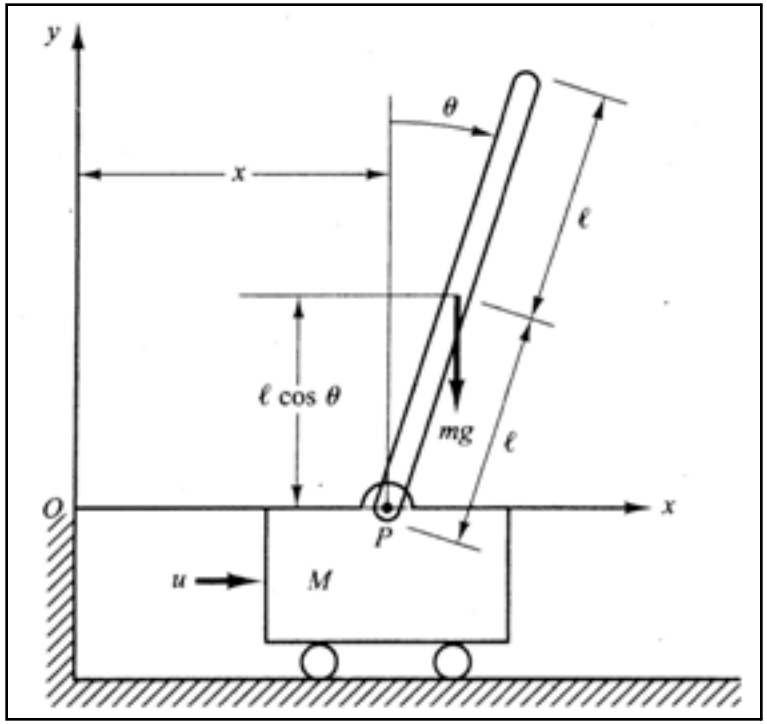
Obtain the **coupled equations** governing the motion of masses, $\mathbf{m_1}$ **&** $\mathbf{m_2}$, in terms of the **force, f.**

$$m_1\ddot{x}_1 + b\dot{x}_1 + (k_1 + k_2)x_1 = b\dot{x}_2 + k_2x_2 + u$$

$$m_2\ddot{x}_2 + b\dot{x}_2 + (k_2 + k_3)x_2 = b\dot{x}_1 + k_2x_1$$

Inverted Pendulum Example

An **inverted pendulum**, mounted on a motor-driven **cart**, is shown below.



I – Moment of Inertia about centroid

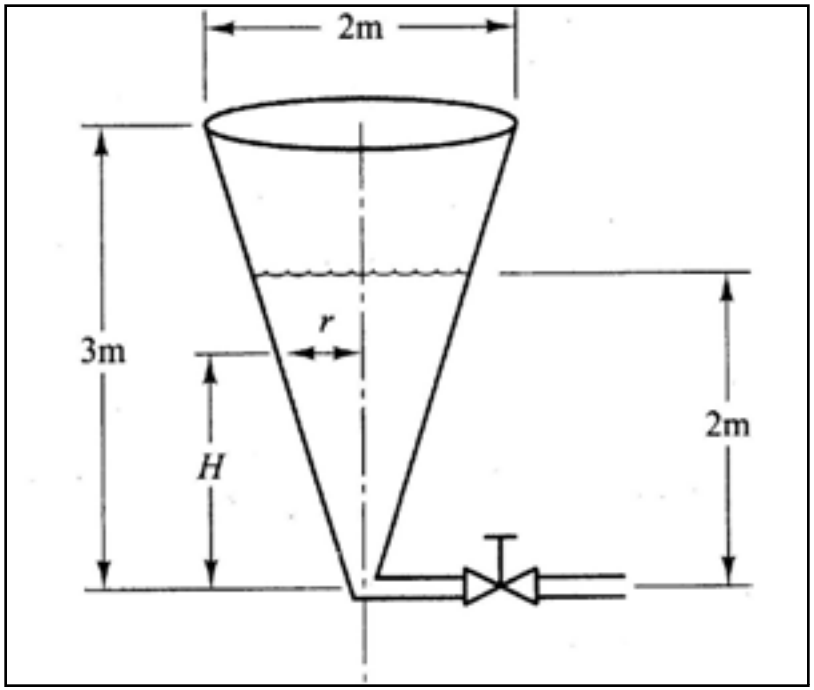
$$\left(I + ml^2\right)\ddot{\theta} + ml\cos\theta\ddot{x} = mgl\sin\theta$$

$$\left(M + m\right)\ddot{x} + ml\cos\theta\ddot{\theta} = u$$

Obtain **coupled equations** of rotational motion of **pendulum** & linear motion of **cart**, in terms of **force 'u'.**

Liquid Storage Tank Example

Consider a **conical tank**, along with model of **tap**, as shown below.



$$Q = 0.005\sqrt{H}$$

$$V = \frac{\pi}{3} \times \left(\frac{H}{3}\right)^2 H; \quad Q = 0.005\sqrt{H}$$

$$\frac{dV}{dt} = \frac{\pi}{9} H^2 \frac{dH}{dt} = -Q = -0.005\sqrt{H}$$

$$\frac{dH}{dt} + \frac{0.045\sqrt{H}}{\pi H^2} = 0$$

Obtain the **differential equation** for liquid height 'H'.