Control Specifications

<u>Stability Related Specifications</u>

<u>Tracking Performance Specifications</u>

<u>Disturbance Rejection Specifications</u>

Planned Control Elements

Among the **various control elements** presented earlier, we generally employ the **following options.**

P **–** :Generic & Most of Benign Control Tasks

I & PI **–** : Mainly Tracking

PD **–** : Relative Stability, Transient Response

PID **–** : All Objectives

Control Task Description

As was mentioned earlier, **control tasks** fall in three **broad categories** of Stability, Tracking & Transient response control (or disturbance rejection).

In the **practical** context, there are specific **requirements** related to each of **these tasks** that are put on the **closed loop** system behaviour.

In general, these **requirements** relate to the system **response** and are captured through specific features called '**figures of merit**'.

Figures of Merit for Stability

Stability is the desirable **property** of any system to **return** to its earlier state, when **disturbed** from it.

In the context of control, the **stability related** objectives can either be (a) to **provide stability** to unstable systems or (b) to **improve the level** of stability of stable systems.

In general, **stability margins** are employed as figures of merit, which are related to the **behaviour** of the closed loop **poles**, in s-plane.

Tracking Control Requirements

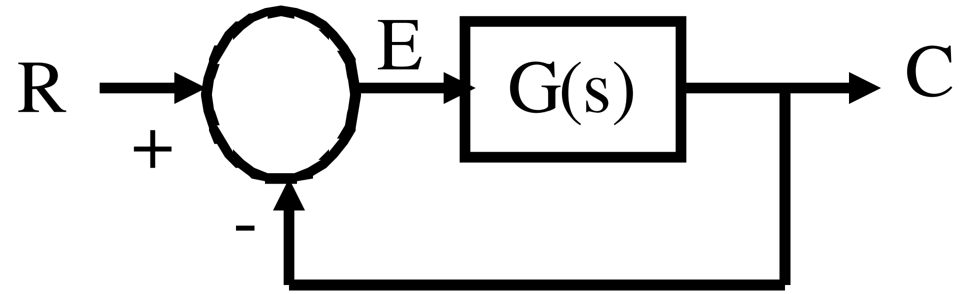
Tracking control task involves ensuring that the **desired output** is equal to the reference input as $\mathbf{t \rightarrow \infty}$**.**

Therefore, **requirements** for tracking are arrived at by quantifying the **deficiency** in steady-state performance of the **plant** in a unity feedback closed loop **configuration.**

In this context, we make use of **final value theorem**, to arrive at the tracking **deficiency.**

Tracking Error Characterization

Consider a **unity feedback uncompensated** closed loop system, as shown below.



We can write the **error in tracking** as well as its **final value** in steady-state as,

$$E(s) = R - C = \left(1 - \frac{C}{R}\right) \times R = \frac{R}{1+G}$$

$$e_{ss}(t) = \lim_{t \to \infty} e(t) = \lim_{s \to o} sE(s) = \lim_{s \to 0} \frac{sR}{1+G(s)}$$

Tracking Error Formulation

It is seen that **error** depends on both **R & G(0)** and that **non-zero** error indicates the **inability of the plant** to follow a given input **exactly** in the closed loop.

This enables us to define **quantitative measures** for assessing the closed loop **tracking error**, which are called **error constants** and are defined as **figures of merit.**

Error constants are nothing but limiting **values** of G(s), sG(s) and $s^2G(s)$ for step, ramp, parabolic inputs, as $\mathbf{s \rightarrow 0}$.

It should be noted that **system type** plays an important **role** in deciding the above **error constants.**

Position Error & Constant

Position (**or step**) error is defined for a **unit step input** as,

$$\boxed{\lim_{s\to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1+G(s)}\right] = \frac{1}{1+G(0)} = \frac{1}{1+K_p}}$$

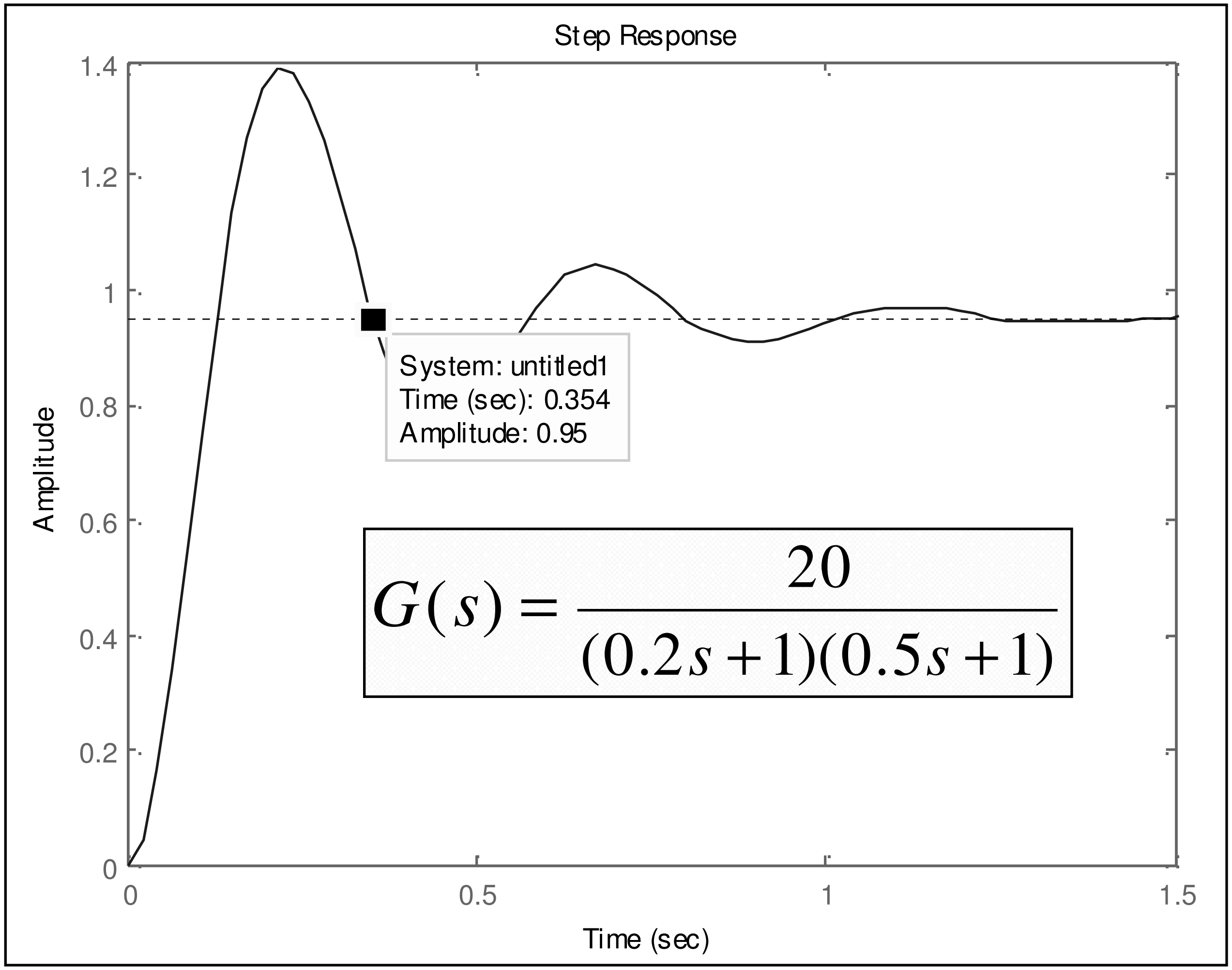
$G(0) = K_p$ – Position Error Constant

$K_p = K$; $e_{ss} = 1 / (1 + K)$; for type '0' Systems

$K_p = \infty$; $e_{ss} = 0$; for type $\geq$ '1' Systems

Position Error Example

Determine the **tracking error** for the following plant. to **unit step input** and verify it.



$$G(s) = \frac{20}{(0.2s+1)(0.5s+1)}$$

$$G(0) = K_p = 20$$

$$e_{ss} = \frac{1}{21} = 0.0476$$

$$c_{ss} = r(t) - e_{ss} = 0.952$$

Velocity Error & Constant

Velocity (**or ramp**) error is defined for a **unit ramp input** as,

$$\boxed{\lim_{s\to 0}\left[\frac{s\left(\frac{1}{s^2}\right)}{1+G(s)}\right]=\lim_{s\to 0}\left[\frac{1}{s\{1+G(s)\}}\right]=\lim_{s\to 0}\left[\frac{1}{sG(s)}\right]=\frac{1}{K_v}}$$

$$\boxed{\lim_{s\to 0}\left[sG(s)\right]=K_v}$$

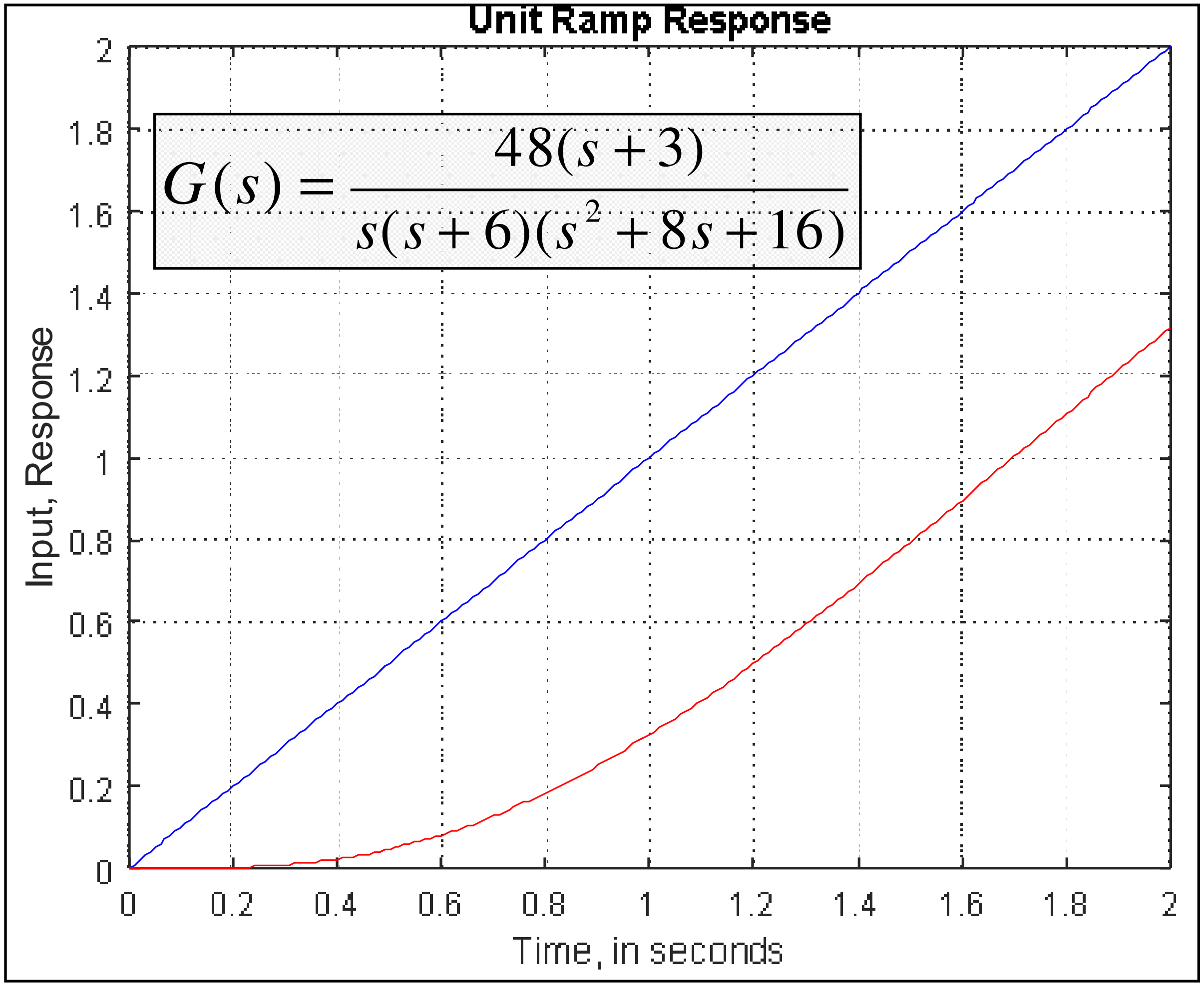
$K_v = 0$; $e_{ss} = \infty$; for type '0' Systems

$K_v = K$; $e_{ss} = 1 / K$; for type '1' Systems

$K_v = \infty$; $e_{ss} = 0$; for type $\geq$ '2' Systems

Velocity Error Example

Determine the **tracking error** to unit ramp input for the following **plant** and verify it.



$$G(s) = \frac{48(s+3)}{s(s+6)(s^2+8s+16)}$$

$$K_v = 1.5$$

$$e_{ss} = 1/1.5 = 0.667$$

$$c_{ss} = t - 0.667$$

Acceleration Error & Constant

Acceleration (**or parabolic**) error is defined for a **parabolic input** as,

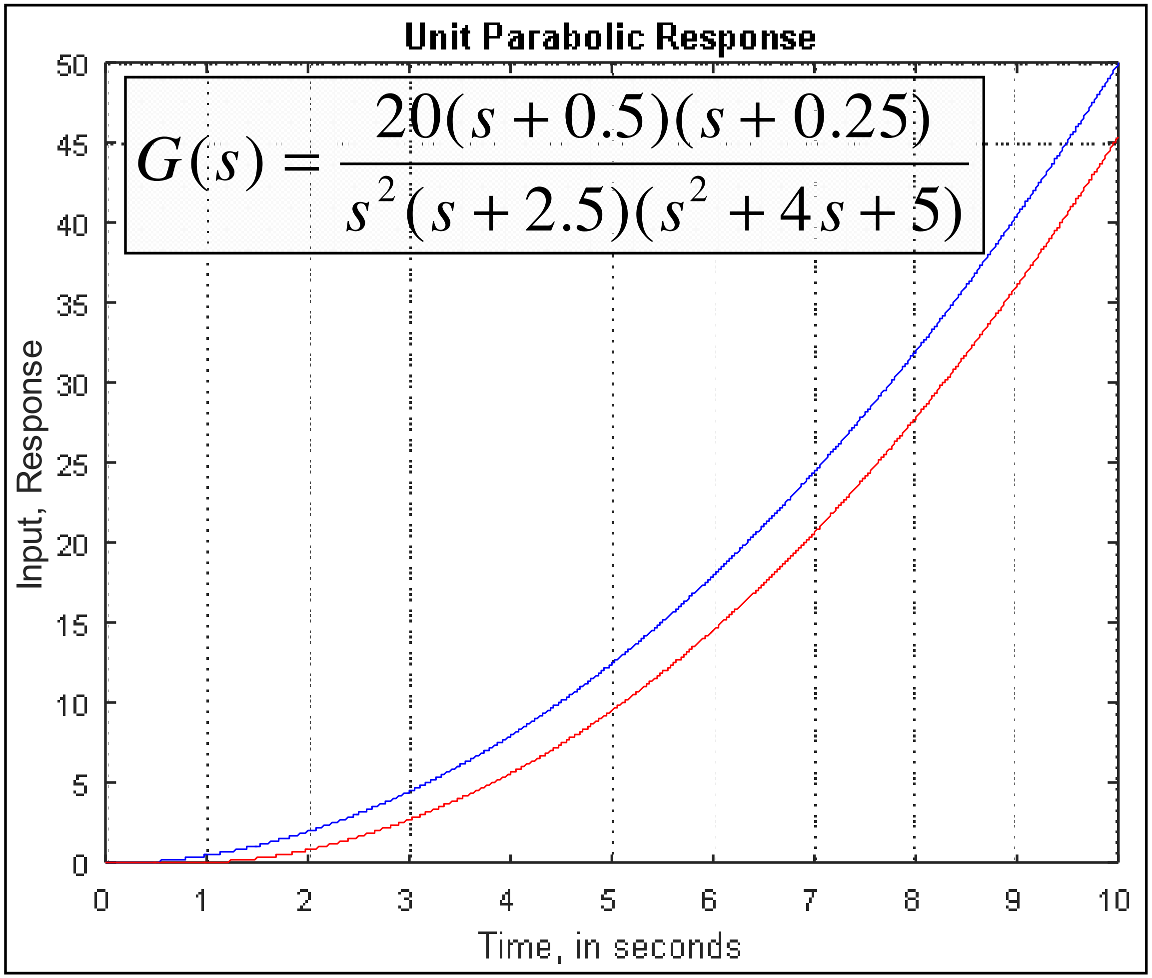
$$\lim_{s\to 0}\left[\frac{s\left(\frac{1}{s^3}\right)}{1+G(s)}\right] = \lim_{s\to 0}\left[\frac{1}{s^2\{1+G(s)\}}\right] = \lim_{s\to 0}\left[\frac{1}{s^2 G(s)}\right] = \frac{1}{K_a}$$

$$\lim_{s\to 0}\left[s^2 G(s)\right] = K_a$$

$K_a = 0;$	$e_{ss} = \infty;$	for type '0/1' Systems
$K_a = K;$	$e_{ss} = 1 / K;$	for type '2' Systems
$K_a = \infty;$	$e_{ss} = 0;$	for type $\geq$ '3' Systems

Acceleration Error Example

Determine the **tracking error** to unit parabolic input for the following **plant** and verify it.



$$K_a = 0.2$$

$$e_{ss} = 1/0.2 = 5.0$$

$$c_{ss} = \frac{t^2}{2} - 5.0$$

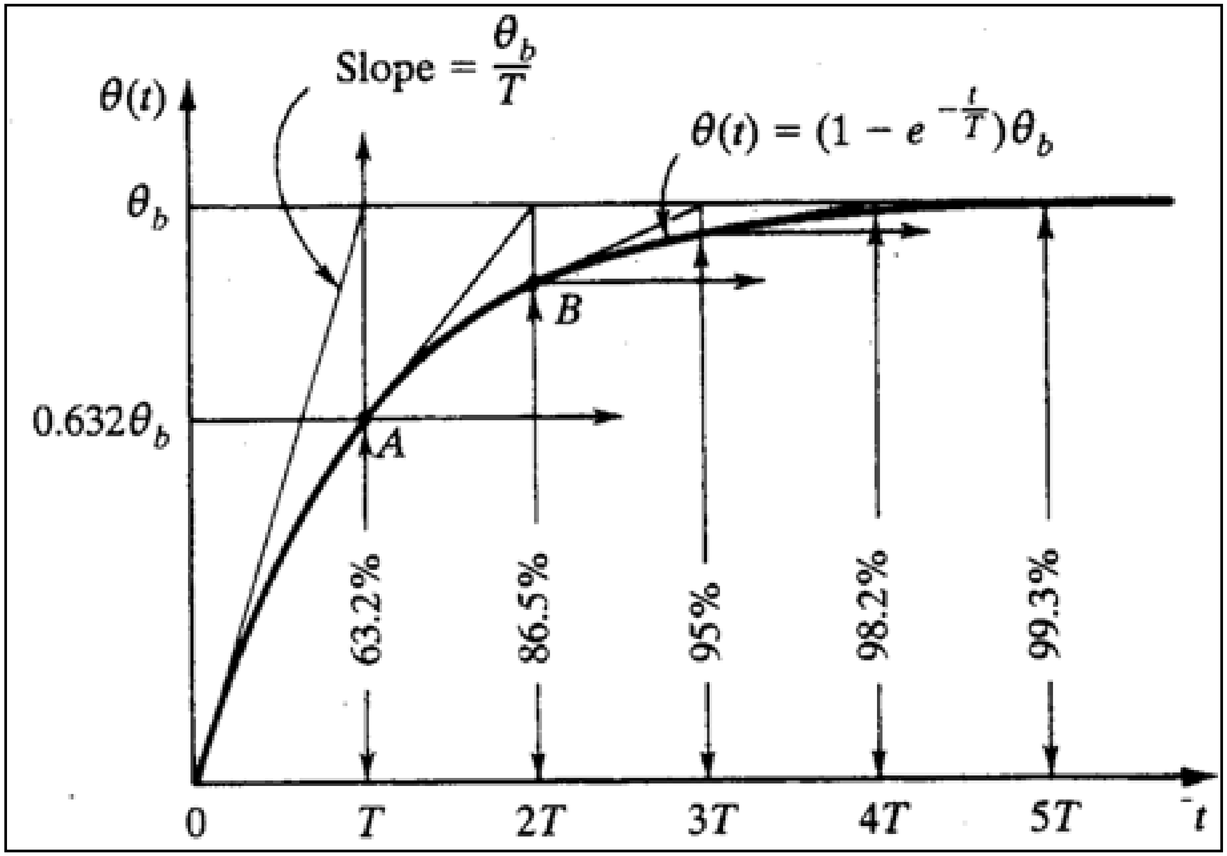
Disturbance Rejection Requirements

In all cases of **disturbance input**, it is desired that these are **rejected quickly**, and without **much impact.**

Further, we know that systems have **characteristics** that decide how **quickly** it reaches the steady-state & how **much error** it shows while achieving the **steady-state.**

These attributes are typically called '**transient response**' characteristics and **can be extracted** from the generic features of **1st and 2nd order systems**, which are also the basic **building blocks** of the system response.

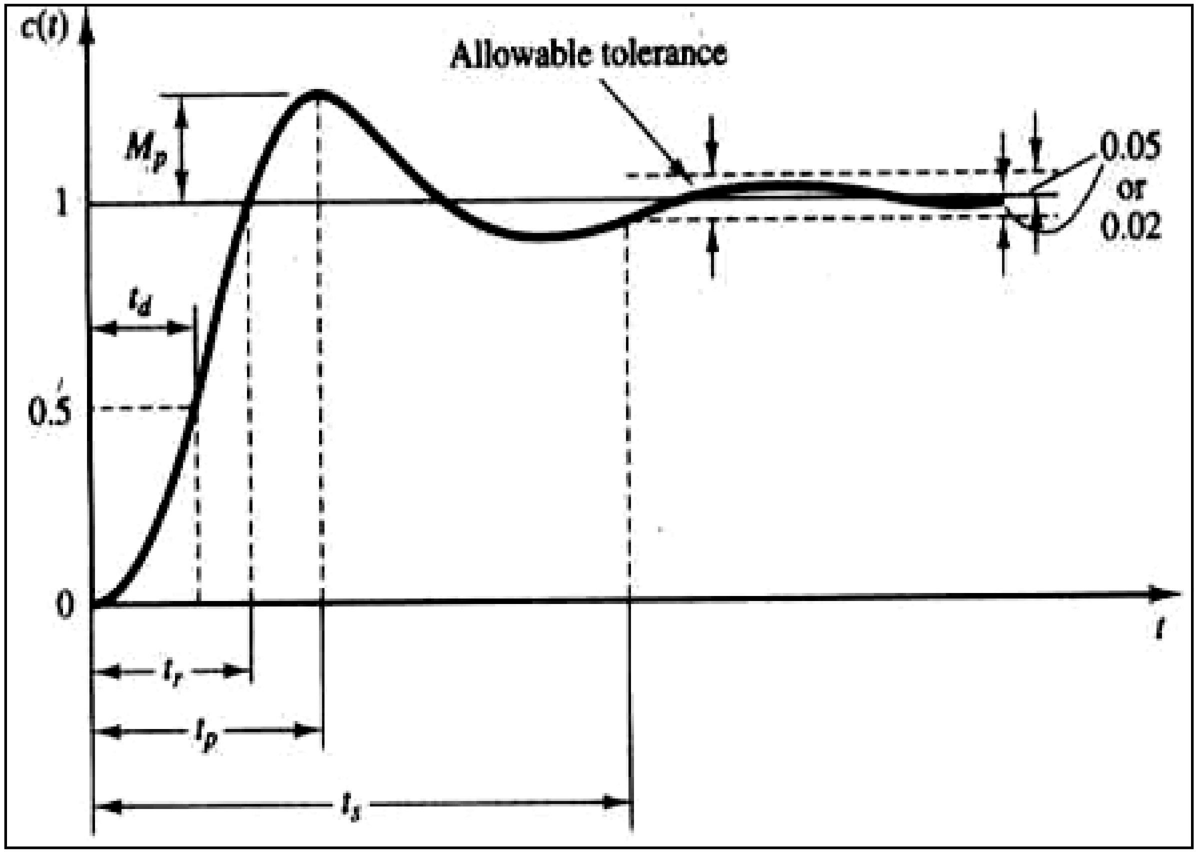
1^{st} Order System Features



In the context of 1^{st} **Order** systems, time constant, '**T**' is the basic figure of merit **for characterizing** transient (disturbance rejection) performance **measure.**

Further, **settling time**, 'T_s', defined with respect to **an acceptable error**, is also **used** as the desirable **feature.**

2^{nd} *Order System Features*



In the context of **2^{nd} Order** systems, there are **many features** that capture the nature of the system **behaviour.**

These include, rise time, ‘$\mathbf{t_r}$’, peak time, ‘$\mathbf{t_p}$’, peak overshoot, ‘$\mathbf{M_p}$’, and settling time, ‘$\mathbf{t_s}$’, for an acceptable error system response.

Summary

Generally, **specifications** for the closed loop systems are expressed in terms of **error constants** and **transient response features.**