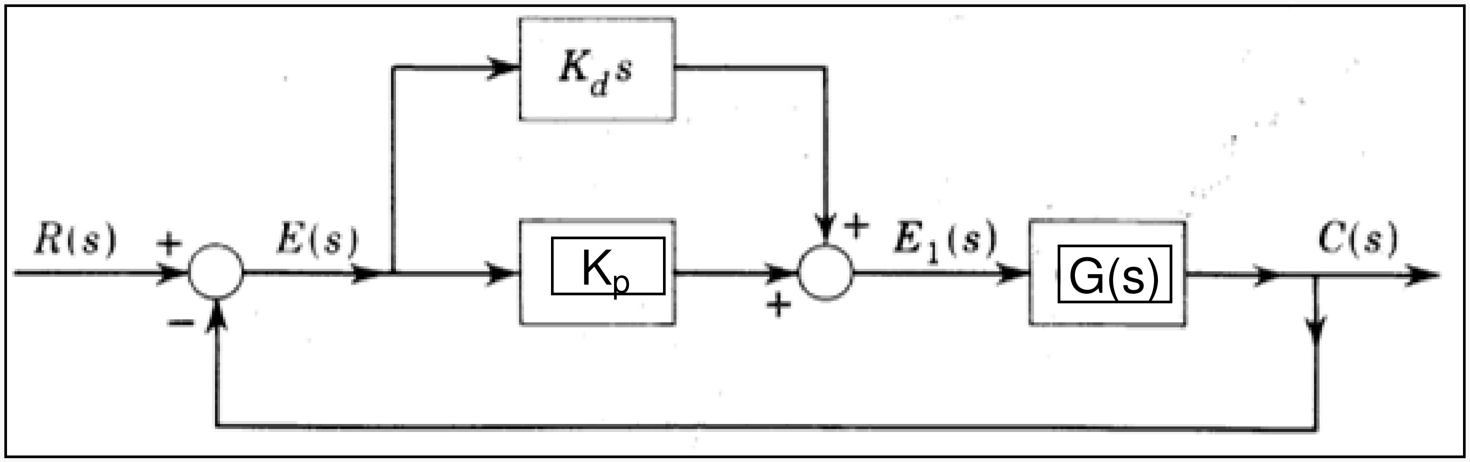
PD Control Analysis & Design

- *Damping Improvement with PD Control*
- *PD Control Design with Root Locus*
- *PD Control Design with Bode Plot*
- *Lead Compensator Concept*

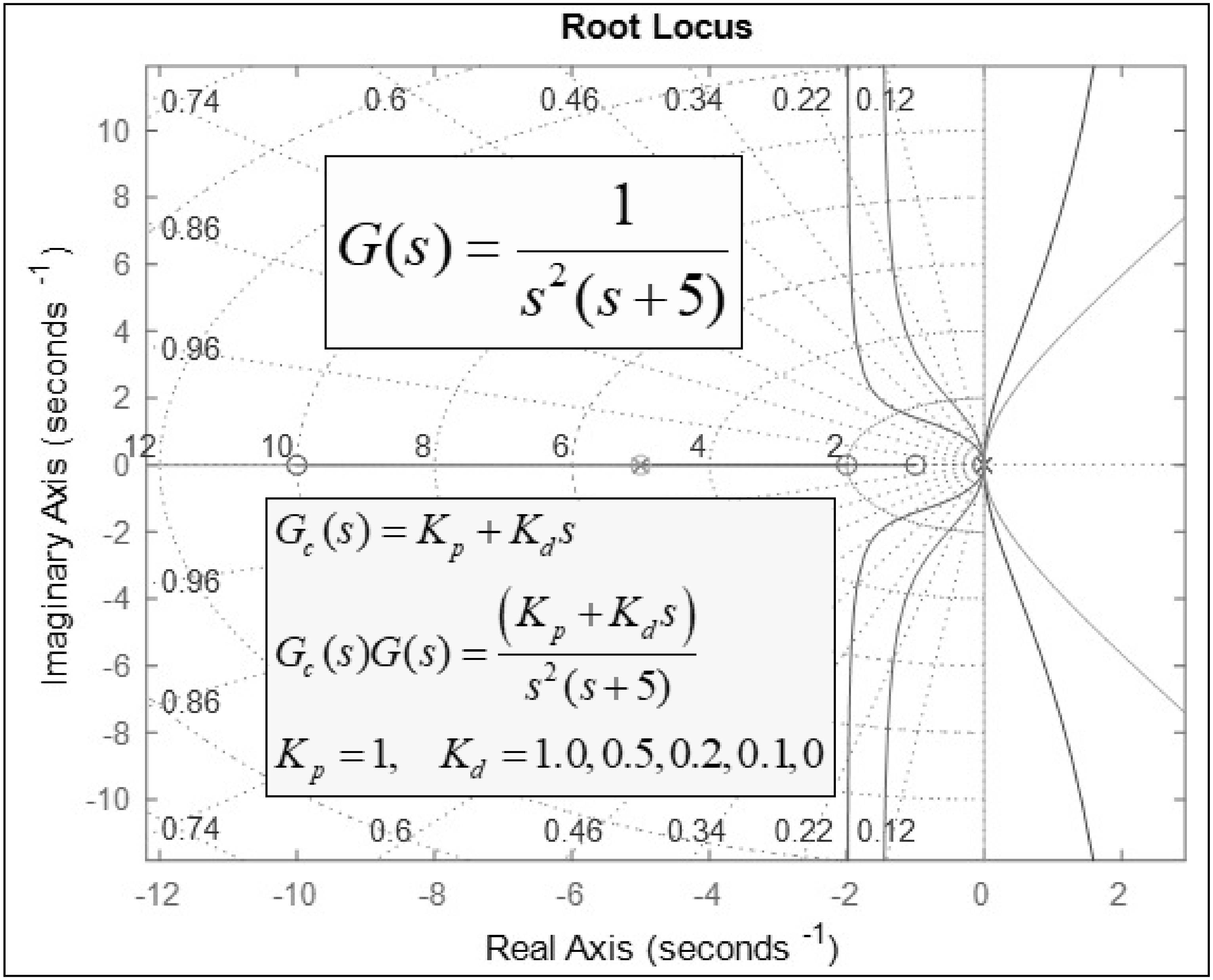
PD Control Concept

Typical structure with **PD control** is as shown below.

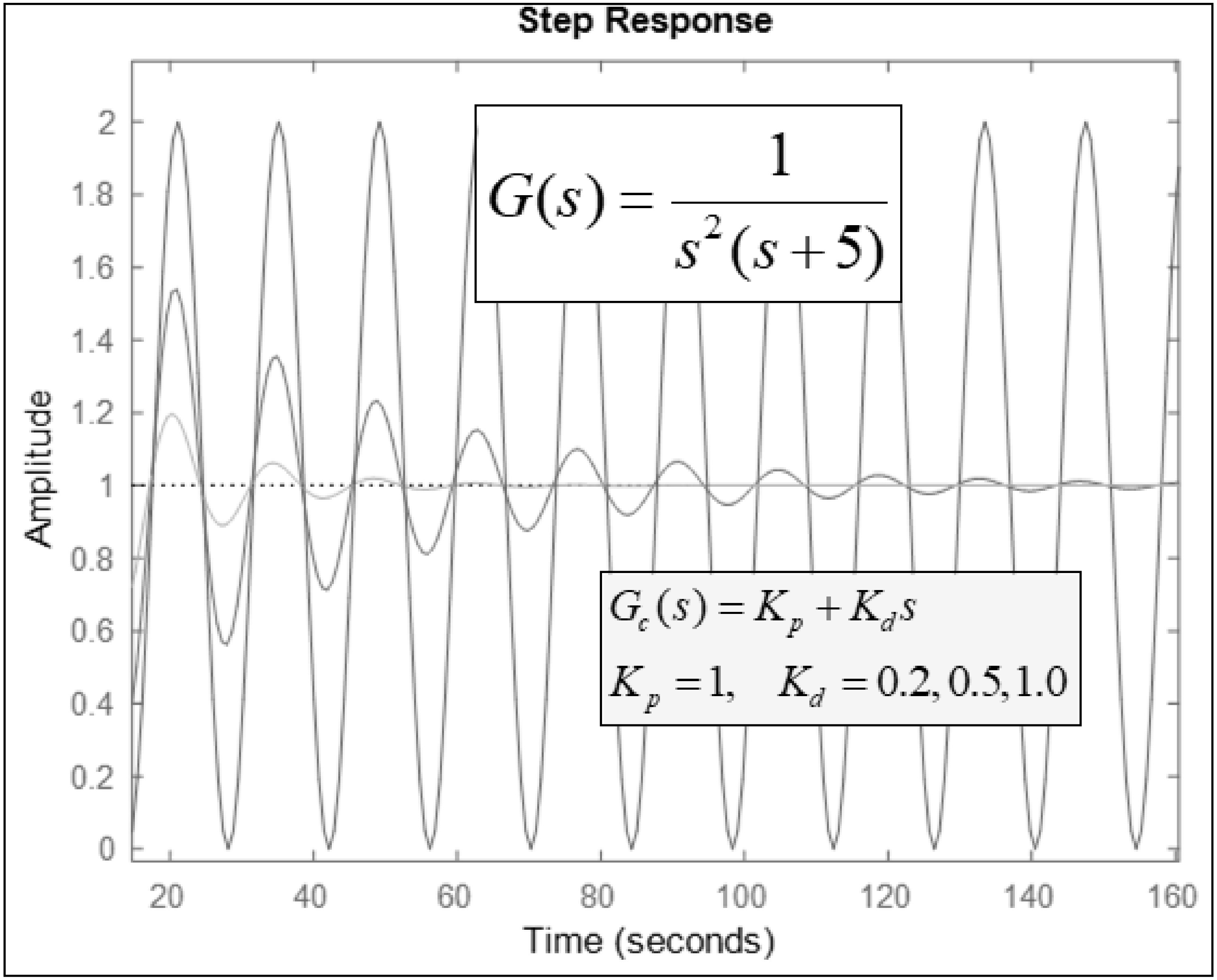


Here, $\mathbf{K_d}$ is the derivative gain, which adds a **pure zero** to the plant at $\mathbf{-(K_p/K_d)}$, while $\mathbf{K_p}$ is the proportional gain.

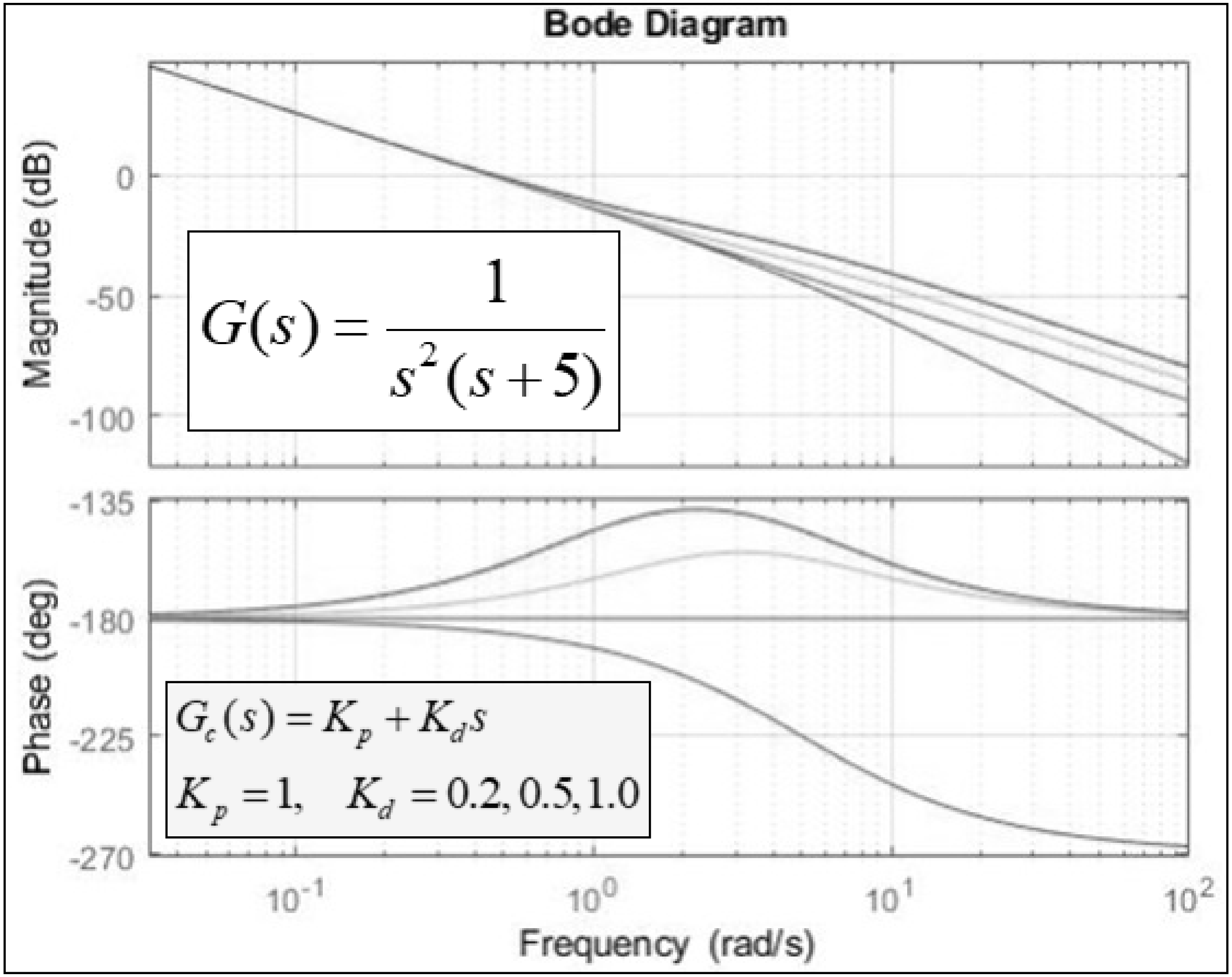
Effect of Derivative Gain on Root Locus



Effect of Derivative Gain on Response



Effect of Derivative Gain on Bode



Summary

PD controllers are used to improve the **damping** of dominant system **behaviour**, as well as the speed of response in **terms of rise time.**

PD Control Design with Root Locus

PD Control Design with Root Locus

Design of **PD** controllers is mainly concerned with **determining** the location of **‘zero’**, based on the closed loop **transient response** specifications.

In this method, we attempt to **modify** the root locus such that it **passes** through the desired dominant **closed loop poles.**

The **procedure** makes use of **angle** and **magnitude** conditions, commonly used for **drawing** the root locus.

PD Control Design Methodology

This is done by first calculating the **angle deficiency** at the required **dominant poles**, which is used to set the **zero location.**

Next, **gain** is determined from the **magnitude** condition.

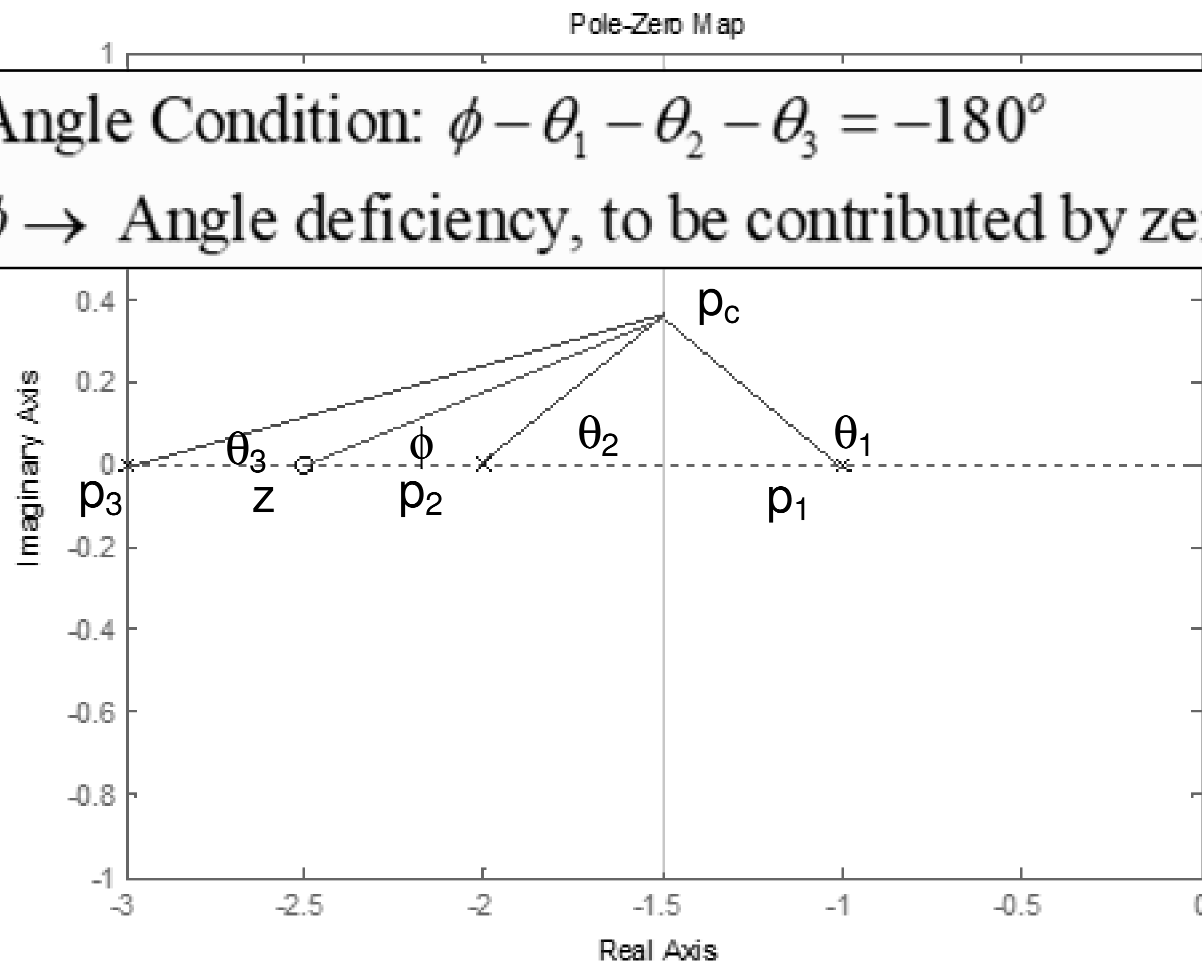
However, there is also **a requirement** of not tampering too much with the steady state **error** of the basic system. This requires the **DC gain** of PD controller to be **close to 1.**

PD Control Design Procedure

Consider the following **pole – zero map.**

Angle Condition: $\phi - \theta_1 - \theta_2 - \theta_3 = -180^o$

$\phi \rightarrow$ Angle deficiency, to be contributed by zero 'z'



Zero can be located from **angle deficiency**.

PD Control Design Example

Consider the following **plant**.

$$G(s) = \frac{20}{(s+1)(s+2)(s+3)}$$

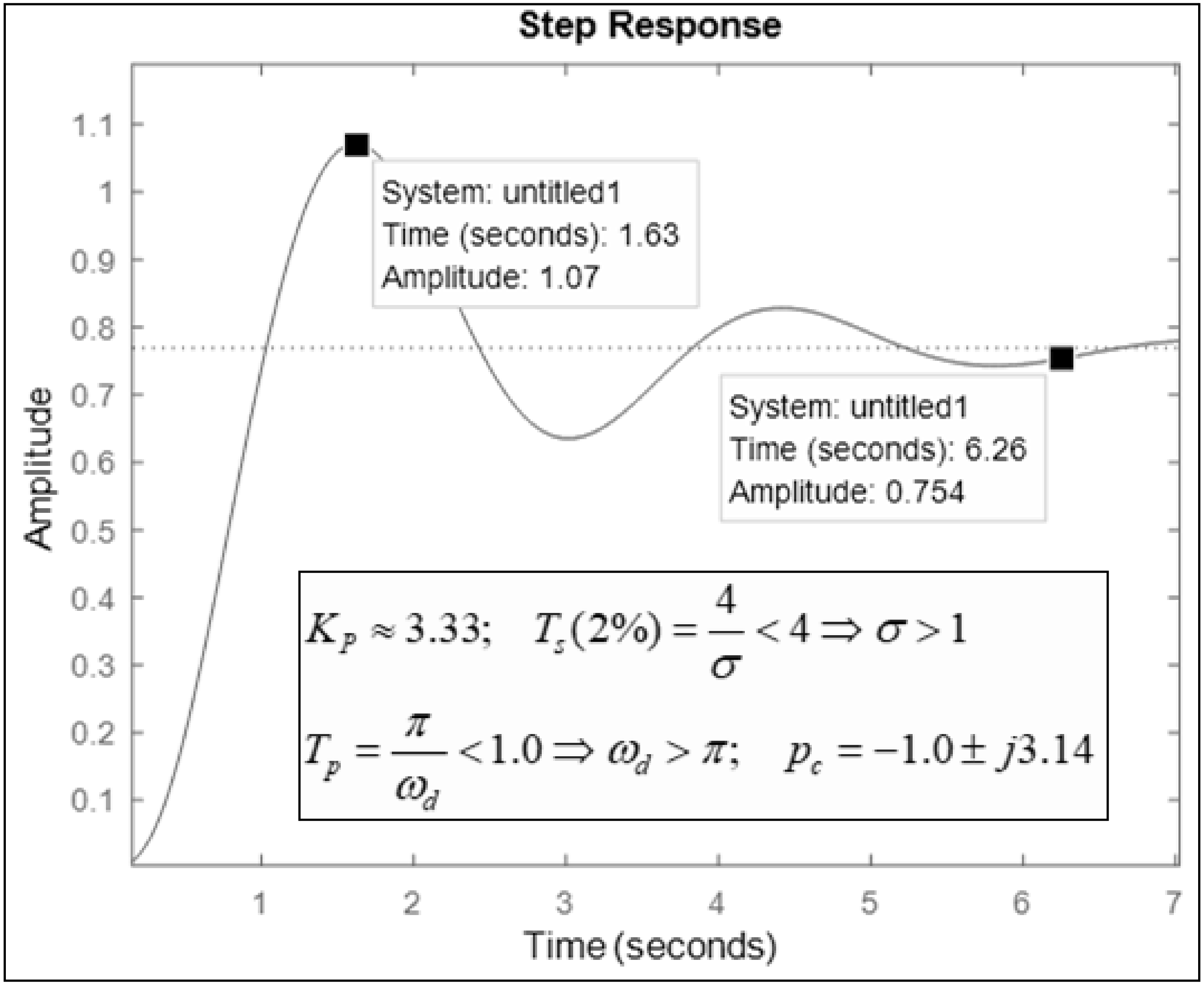
Design a PD controller to achieve **following** performance in the closed loop.

$\mathbf{K_P}$ to remain largely **unaffected.**

2% Settling Time $\leq$ **4.0 seconds**

Peak Time $\leq$ **1.0 second**

Requirements & Plant Features



$$K_p \approx 3.33; \quad T_s(2\%) = \frac{4}{\sigma} < 4 \Rightarrow \sigma > 1$$

$$T_p = \frac{\pi}{\omega_d} < 1.0 \Rightarrow \omega_d > \pi; \quad p_c = -1.0 \pm j3.14$$

PD Control Design Solution

The **design** solution **step – 1:**

Desired Dominant Closed Loop Pole: $p_c = -1 + j3.14$

$$\theta_1 = \tan^{-1}\left(\frac{3.14}{0}\right) = 90^o; \quad \theta_2 = \tan^{-1}\left(\frac{3.14}{1}\right) = 72.3^o$$

$$\theta_3 = \tan^{-1}\left(\frac{3.14}{2}\right) = 57.5^o; \quad \phi = \tan^{-1}\left(\frac{3.14}{z-1}\right)$$

Angle Condition: $\phi = -180^o + \theta_1 + \theta_2 + \theta_3 = 39.8^o$

$$\tan^{-1}\left(\frac{3.14}{z-1}\right) = 39.8^o \rightarrow z = 4.77; \quad G_{PD}(s) = K(0.2097s + 1)$$

PD Control Design Solution

The design **solution step-2:**

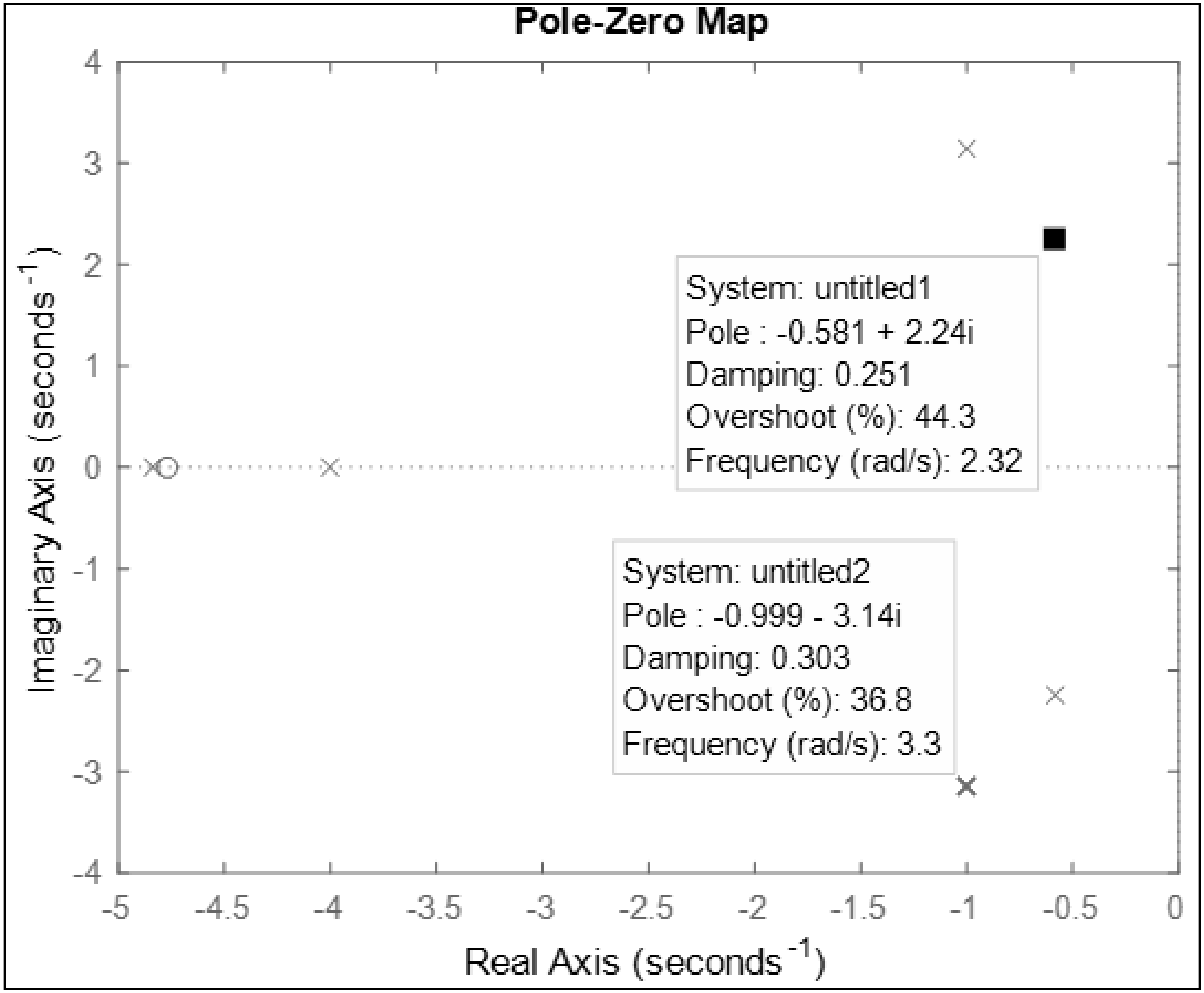
$$\left|G_{PD}\cdot G\right|=1\rightarrow K=\frac{\left|p_c+1\right|\times\left|p_c+2\right|\times\left|p_c+3\right|}{20\left|0.2097p_c+1\right|}$$

$$=\frac{3.14\times3.297\times3.724}{20\times1.029}=1.874$$

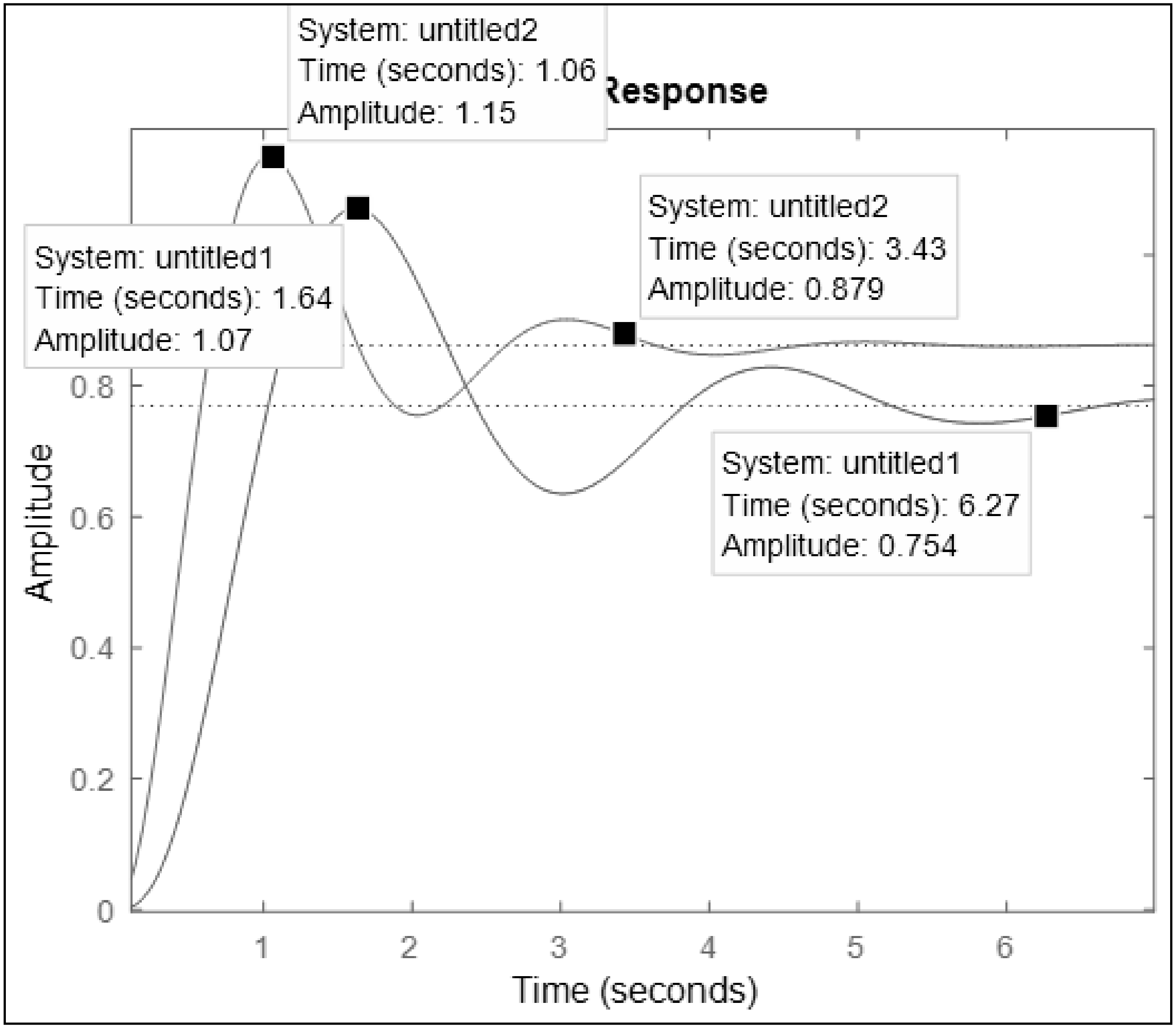
PD Controller: $G_{PD}(s)=1.874(0.2097s+1)$

Compensated $K_P=6.246,\quad e_{ss}=0.138$

PZ-Map Comparison



Step Response Comparison

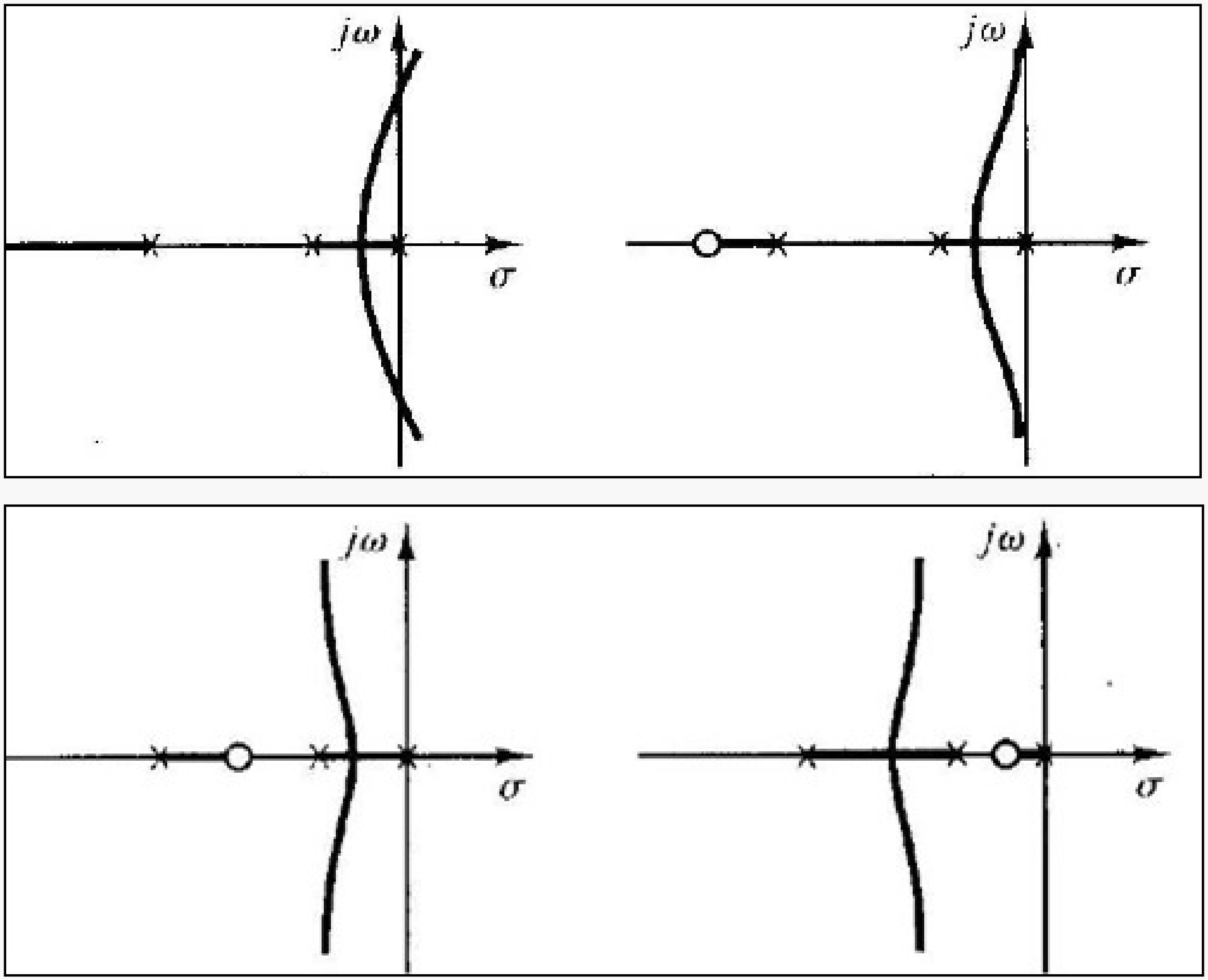


Design is **fine,** from **transient** point of view, as $\mathbf{T_s}$ **&** $\mathbf{T_p}$ are met.

As a **bonus, tracking** error & $\mathbf{M_p}$ are also **reduced.**

Generic Effects of PD Control

PD control action depends on **zero location.**



Summary

PD controllers are simpler to synthesize, but have the basic drawback of **non-causal** nature of transfer function.

Large improvements in transient **response** are possible with **PD controllers**.

The addition of **'zero'** changes the root locus **shape** and influences both **'σ' and 'ω_d',** so that all attributes of the **transient response** are influenced.

PD Control Design with Bode

PD Design in Frequency Domain

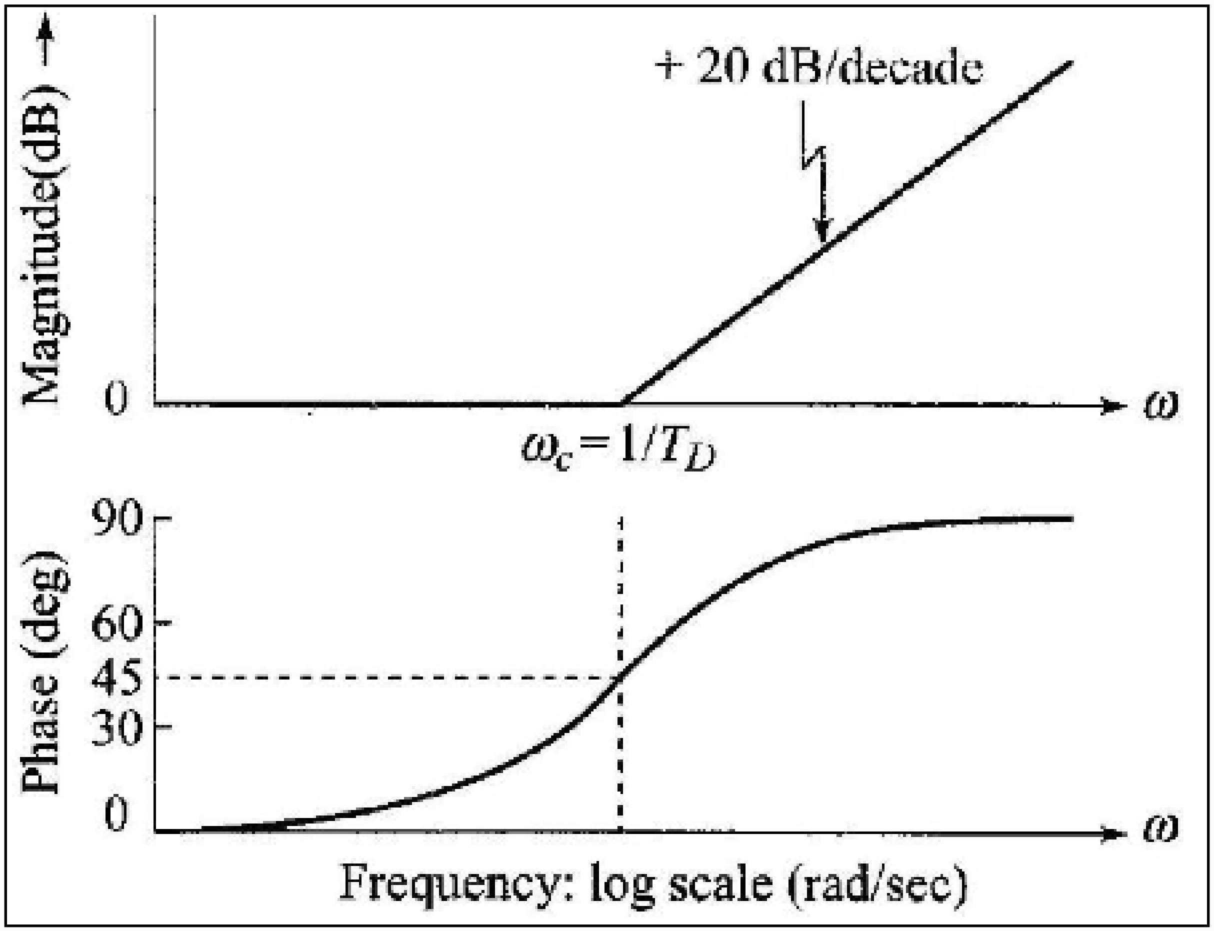
Design of **PD – controller** in frequency domain is primarily **governed** by the requirements on **PM.**

In addition, a **condition** is put that **DC gain** remains unchanged. Therefore, in **case** there are requirements also on **K_V,** these are **satisfied first**, before designing PD.

The **general form** of PD in this case is **K_P $(1+T_d s)$**, where corner frequency **'$1/T_d$'** is chosen such that the **positive phase** to be added, occurs close to the **GCO.**

PD Design in Frequency Domain

Given **below** is a typical **bode' plot** for PD.



PD Design in Frequency Domain

We see that at **frequencies > 1/T_D,** increase in phase is accompanied by an **increase in gain** as well.

This has the **effect** of pushing the **GCO** of the compensated system to a **higher value.**

Thus, the design of **PD controller** has to take this fact into account and add the **required** additional phase at the **new GCO**.

This also results in a kind of **iteration** as the additional phase is **actually calculated** at the **original** GCO.

PD Control Design Example

Consider the following **plant.**

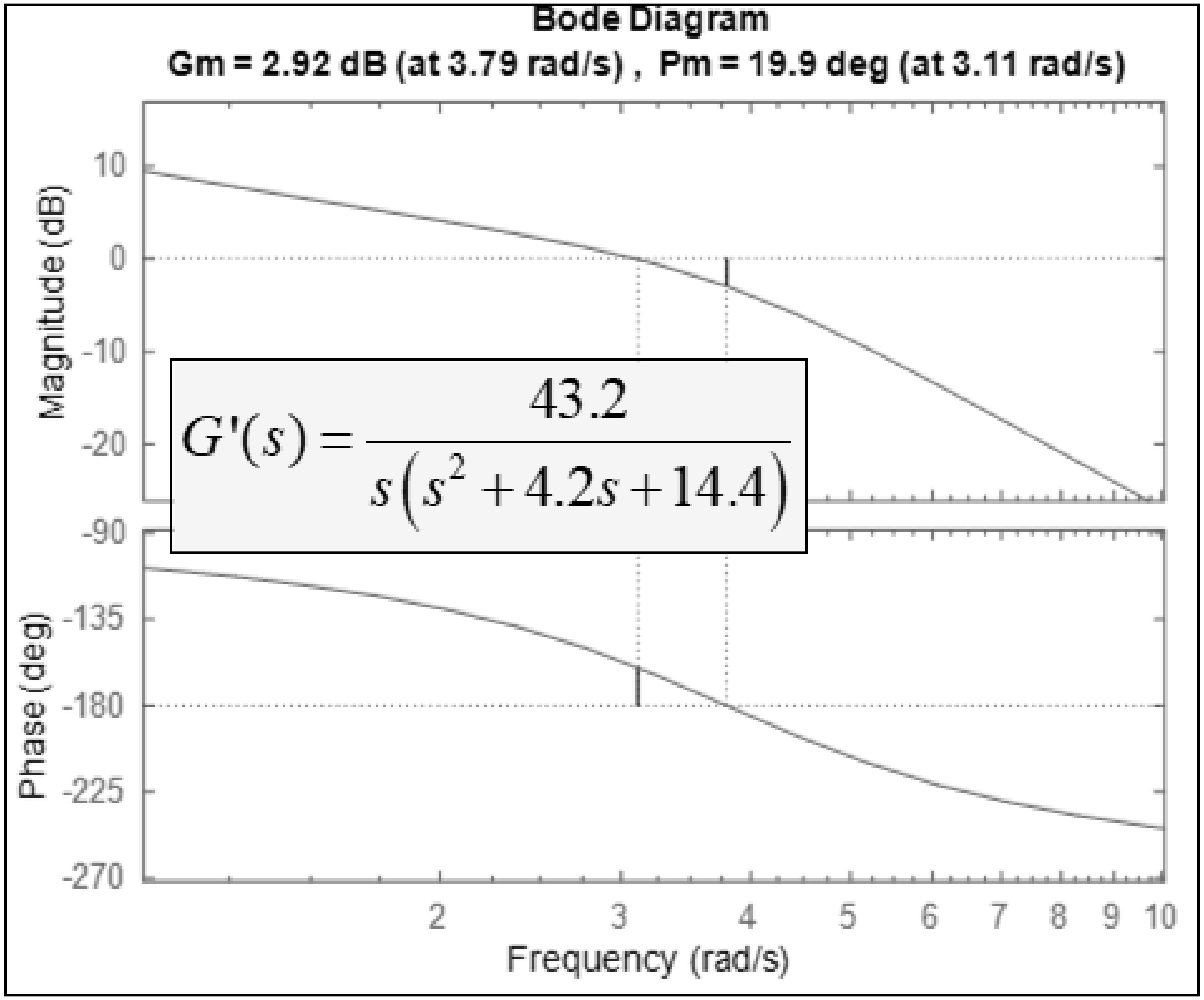
$$\boxed{G(s) = \frac{K_x}{s(s^2 + 4.2s + 14.4)}}$$

Design a **PD controller** to satisfy following **specifications** in the **closed loop.**

$K_v \geq 3.0$, GM > 6 dB, PM > 30^o.

First step is to achieve the **specified K_V** which can be done by making **K_x = 43.2**

Gain Adjusted Margins



$$G'(s) = \frac{43.2}{s\left(s^2 + 4.2s + 14.4\right)}$$

Next, we establish the GM, **PM** of the gain **augmented system**, as shown alongside.

We find that both **GM and PM** are below the **desired values.**

PD Design Space Exploration

PM is to be increased by **~10^o @ 3.11 rad/sec**. (Both GCO & PCO **increase** with PD)

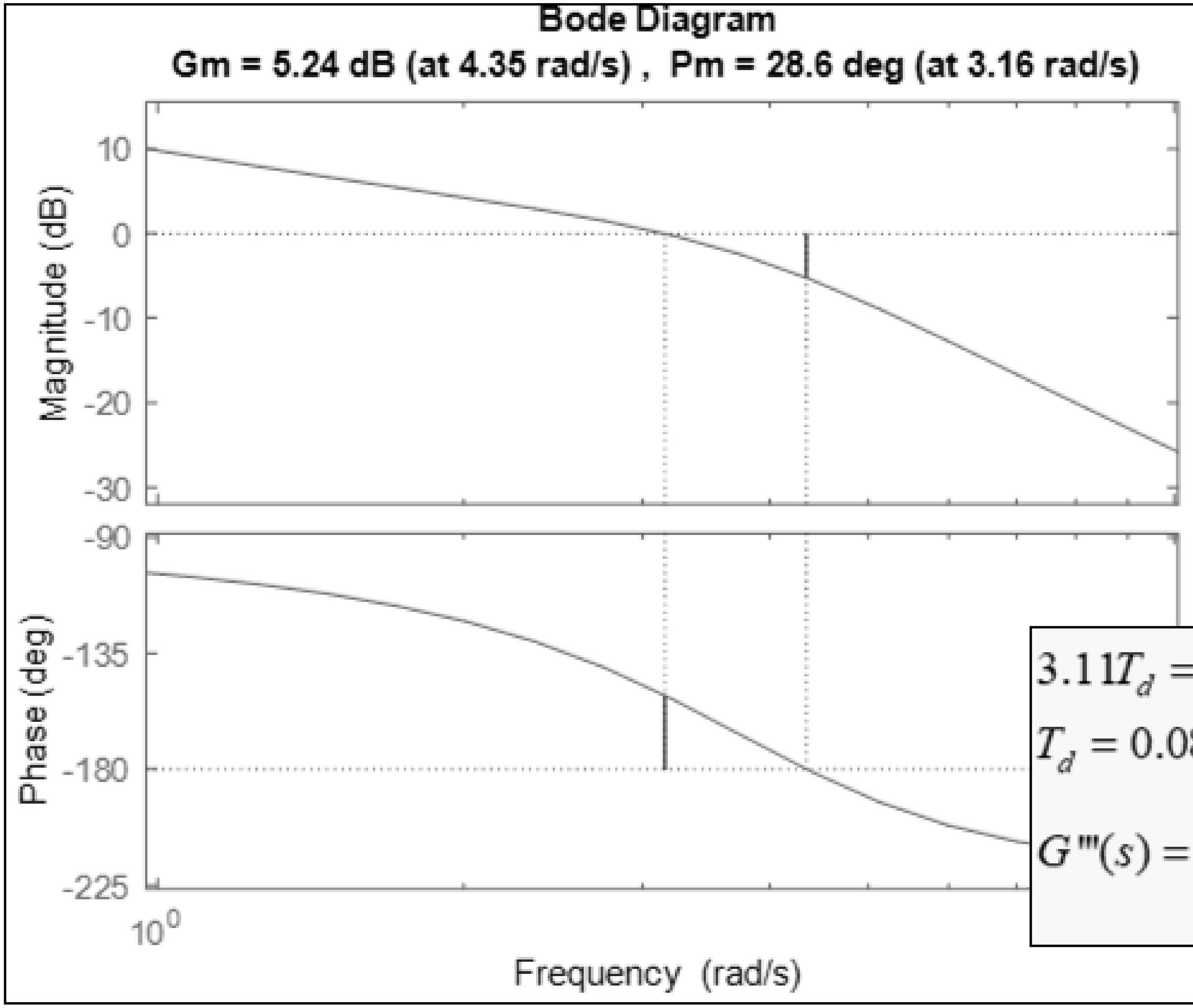
An **approximate** solution for **'T_d'** (with no PM buffer) and **PD controller** can be obtained **as follows.**

$$\angle(1+T_d s)\,|_{\omega=3.11} = 10^o \rightarrow 3.11T_d = \tan 10^o = 0.176$$

$$T_d = 0.057; \quad G_{PI}(s) = (1+0.057s)$$

$$G''(s) = \frac{43.2(1+0.057s)}{s\left(s^2+4.2s+14.4\right)}$$

Compensated Margins



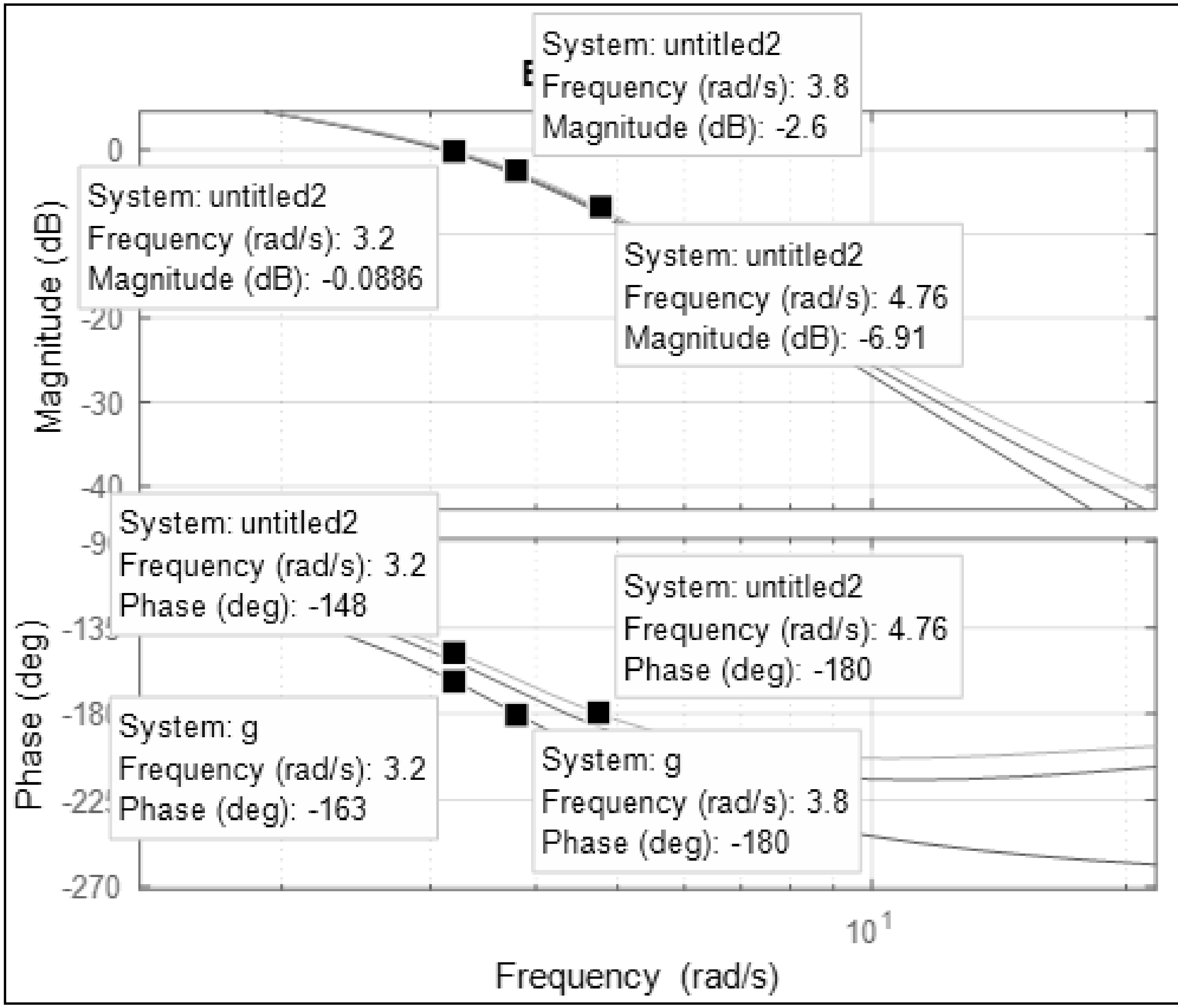
We find that both **GM & PM** are still less than **desired** values and hence we need to **employ buffer** for PM, as shown below.

$$3.11T_d = \tan 15^o = 0.268$$

$$T_d = 0.086; \quad G_{PI}(s) = (1 + 0.086s)$$

$$G'''(s) = \frac{43.2(1+0.086s)}{s\left(s^2+4.2s+14.4\right)}$$

Redesigned Margins



We see that **redesigned system** meets **all the requirements.**

Summary

In case of PD – controllers, **phase margin** is the primary **driver** of the closed loop **design.**

In this case, net **DC gain** of PD part is kept close **to unity**, while **gain** is related to **explicit K_V** requirements.

Lead Compensator Concept

PD Controller Drawbacks

PD controllers are improper transfer functions and hence, **reduce** relative degree **(n – m),** and may result in **unexpected changes** to root locus**.**

Further, we may also wish to **preserve (n – m)** in order to ensure a **desired slope** of high frequency **asymptote** in bode plot.

Therefore, we need an **alternative** to PD control to **ensure** (n – m), which is the **lead** compensator.

Lead Compensator Structure

Lead compensator structure is as **shown** below.

$$G_{Lead}(s) = K_c \frac{\alpha(Ts+1)}{\{\alpha Ts+1)} = K_c \frac{(s+1/T)}{\left(s+1/\alpha T\right)}; \quad \alpha < 1$$

Here, $\mathbf{K_c}$ is compensator gain, **T** is the compensator time constant and **(α)** is a parameter that **decides** the amount of **lead** added by the **compensator.**

We see that above **form** will preserve **relative degree.**

Lead Compensator Features

Lead compensator adds a **zero** at s = **-1/T** & a **pole** at s = **-1/ (αT)**, to the plant, so that (n – m) is **constant.**

Further, as a **bonus**, we also get **additional** design **degree of freedom**, to better achieve the **specifications.**

When $\alpha \rightarrow 0$, **pole** lies at **-∞,** resulting in **PD** controller. Also, if $\alpha \rightarrow 0$ **& T** $\rightarrow \infty$**,** the **zero** moves towards the **origin**, leading to a **pure D control**.

DC **gain** of lead compensator is $K_c\alpha$, and is usually **kept ~1.0**, which fixes K_C once 'α' is determined.

Effect of ‘α’

Let us consider the **following plant**, along with the **lead compensator.**

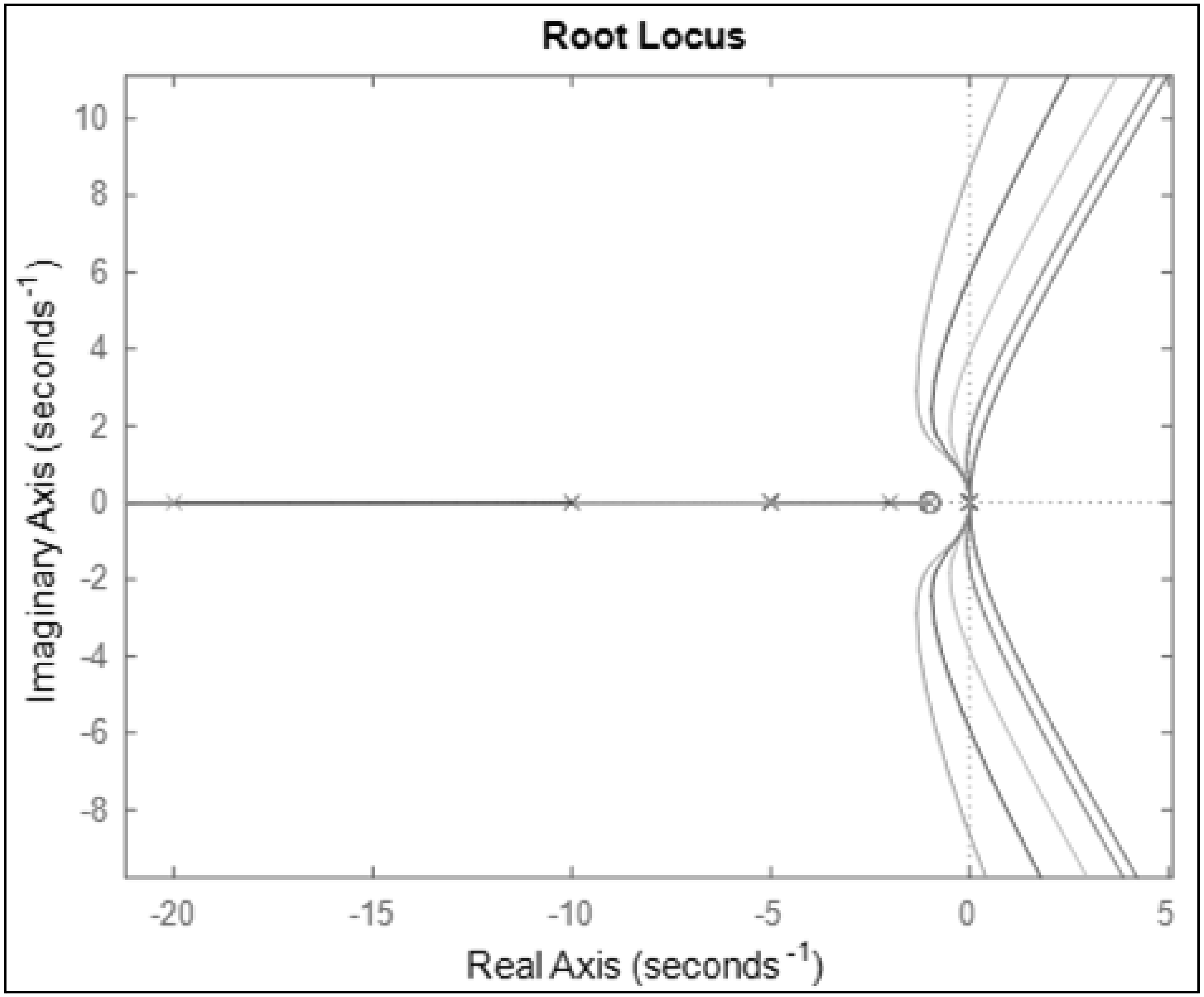
$$G = \frac{1}{s^2(s+5)}; \quad T = 1$$

$$G_c = \frac{K_c \alpha (1+s)}{(\alpha s + 1)}$$

$$K_c = 1.0, 2.0, 5.0, 10, 20$$

$$\alpha = 1.0, 0.5, 0.2, 0.1, 0.05$$

Root locus, shown alongside, **brings** out the effect of **α.**



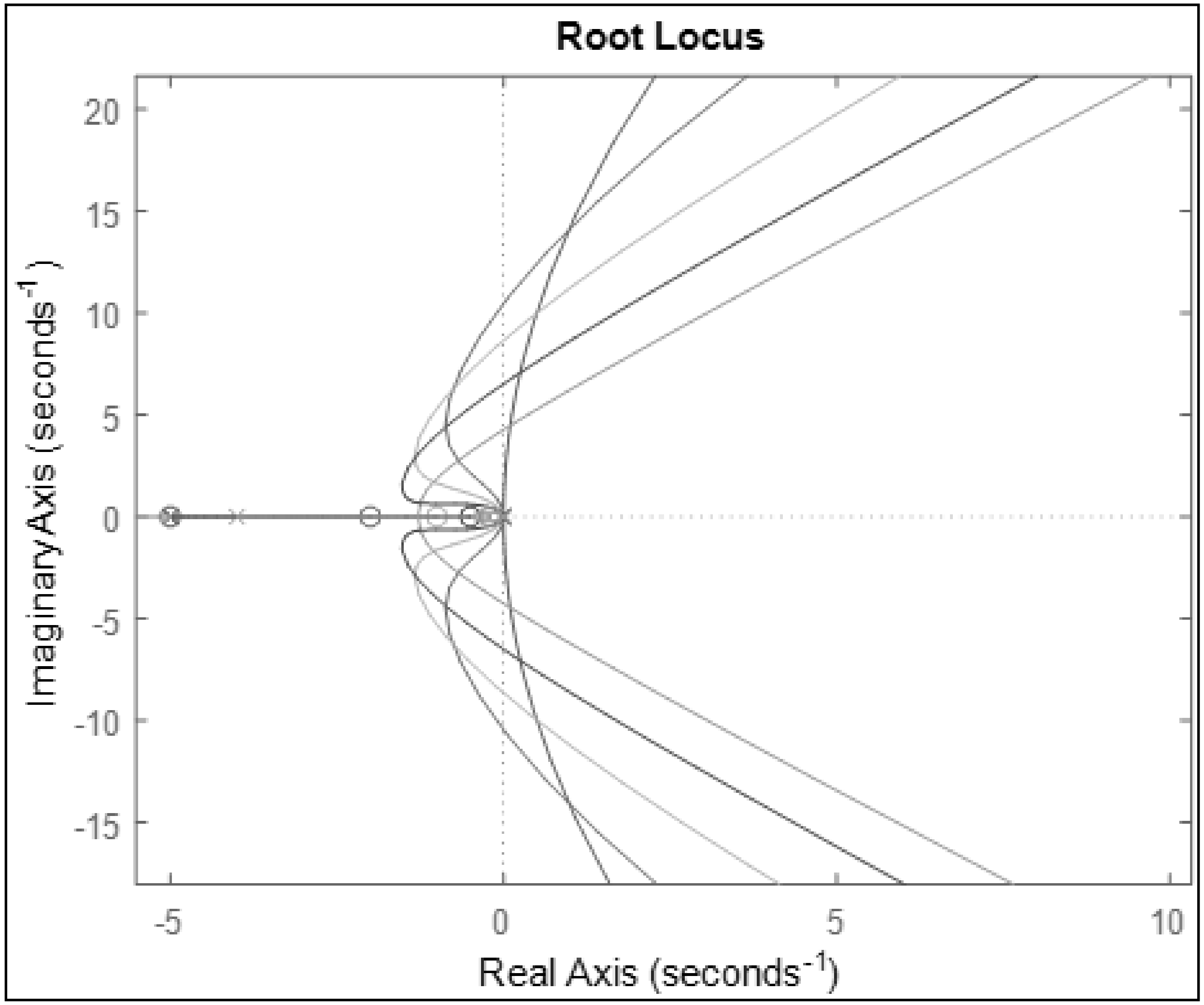
Effect of ‘T’

Let us now consider the **effect** of different values of ‘**T**’ on the **impact** of **lead** compensator.

$K_c = 20; \quad \alpha = 0.05$

$T = 0.2, 0.5, 1.0, 2.0, 5.0$

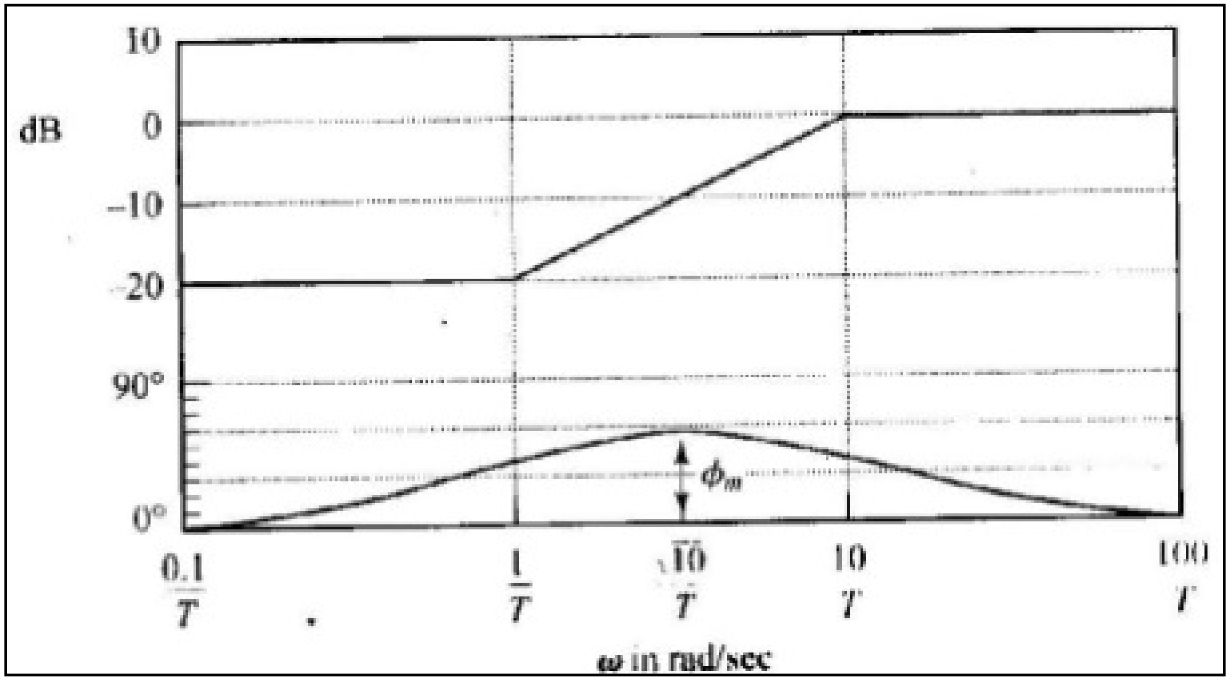
Root locus, shown alongside, **brings** out the effect of **T.**



Lead Compensator Bode Plot

Bode plot of the lead compensator with **K_c = 1 & α = 0.1**, is shown alongside.

We see that **peak positive phase** occurs at a **frequency** which is **related** to the **two corner frequencies**, as shown below.



$$\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T; \quad \frac{d\phi}{d\omega} = 0 \rightarrow \omega_m = \frac{1}{\sqrt{\alpha}\cdot T}$$

$$\tan\phi_m = \frac{1-\alpha}{2\sqrt{\alpha}} \rightarrow \sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

Summary

Lead compensator is a **better** alternative to **PD** controller but is **less effective** in terms of transient **improvement.**