EE301 - Tutorial 4

1. Find the Poynting vector on the surface of a long straight conducting wire (of radius 'a' and conductivity σ) that carries a direct current I. Assume current flow in positive z direction.
2. A metallic conductor has a circular cross section of radius 1 cm and $\sigma = 2 \times 10^7$ S/m. The conductor carries uniformly distributed current of 100 Amp d.c. in the $\overline{a_z}$ direction. Calculate:
   i. Resistance R of one meter length and use I^2R to find dc power loss in that length.
   ii. Calculate $\bar{J}$, $\bar{E}$, $\bar{H}$ and Poynting vector within the conductor.
   iii. Integrate $\bar{P}$ over the cylindrical surface enclosing one meter length of conductor and show that the answer is same as in part (i).
3. In free space $\bar{H} = 0.2\cos(\omega t - \beta x)\overline{a_z}$ (A/m). Find the total average power passing through
   i. A square plate of side 10 cm on plane $x + z = 1$ and
   ii. A circular disc of radius 5 cm on plane $x = 1$.
4. Electric field of an electromagnetic wave propagating in a medium in $+\overline{a_z}$ direction is given by
$$\overline{E_s} = E_0(\overline{a_y} - j\overline{a_z})e^{-j\beta x}$$
Determine the polarization of the wave.
5. An electromagnetic wave has the electric field intensity in the phasor form given by
$$\overline{E_s} = 4(\overline{a_z} - j\overline{a_x})e^{-j\beta y}$$
The EM wave is incident on a perfect conductor located at $y = 0$. What will be the polarization of the reflected wave?
6. The electric field of an electromagnetic wave propagation in the positive direction is given by $\bar{E} = \overline{a_x}\sin(\omega t - \beta z) + \overline{a_y}\sin(\omega t - \beta z + \pi/2)$. Determine the polarization of the wave.
7. Two plane waves propagate in positive z direction. Both waves are at the same frequency and have equal amplitudes. Wave A is polarized linearly in the x direction, and wave B is polarized in the direction $\overline{a_x} + \overline{a_y}$. In addition, wave B lags behind wave A by a small angle θ. What is the polarization of the sum of the two waves?

Review Questions

1. Does the Poynting theorem apply to static fields? Explain
2. Define complex permittivity. Is this quantity only a convenient notation or is it a physically measurable quantity? If so, what are the meanings of its real and imaginary parts?
3. Comment on how the dispersion will change with frequency in a lossy dielectric material.

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EE301

T4

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1. (a)  $\longrightarrow \hat{a}_z$

Assuming small $a \Rightarrow$ insignificant skin effect

$$\Rightarrow \vec{J} = \left(\frac{I}{\pi a^2}\right)\hat{a}_z$$

$$\vec{E} = \frac{\vec{J}}{\sigma} = \left(\frac{I}{\pi\sigma a^2}\right)\hat{a}_z$$

Now,

$$\oint \vec{H}\cdot\vec{dl} = i + \int \frac{\partial \vec{D}}{\partial t}\cdot\vec{ds}$$

Also $\frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t} = 0$

$$\Rightarrow \oint \vec{H}\cdot\vec{dl} = i$$

$$\Rightarrow H\cdot 2\pi r = \left(\frac{I}{\pi a^2}\right)(\pi r^2) \Rightarrow \vec{H} = \left(\frac{Ir}{2\pi a^2}\right)\hat{a}_\phi$$

At the surface, $r = a$

$$\Rightarrow \vec{H} = \left(\frac{I}{2\pi a}\right)\hat{a}_\phi$$

$$\therefore \vec{P} = \vec{E}\times\vec{H} = \left(\frac{I}{\pi\sigma a^2}\right)\left(\frac{I}{2\pi a}\right)(\hat{a}_z\times\hat{a}_\phi) = \underline{\left(\frac{-I^2}{2\sigma\pi^2 a^3}\right)\hat{a}_r}$$

2. (i) $R = \frac{l}{\sigma A} \Rightarrow \left(\frac{R}{l}\right) = \frac{1}{\sigma A} = \frac{1}{2\times10^7\times\pi\times(0.01)^2} = \underline{1.59\times10^{-4}\,\Omega m^{-1}}$

$$P = I^2R = (100)^2(1.59)(10^{-4}) = \underline{\underline{1.59\text{ W}}}$$

(ii) $\vec{J} = \frac{I}{A} \Rightarrow \vec{E} = \frac{I}{A\sigma} = \underline{\left(\frac{I}{A\sigma}\right)\hat{a}_z}$

As obtained in Q1, $\vec{H} = \underline{\left(\frac{Ir}{2\pi R^2}\right)\hat{a}_\phi}$

$$\vec{P} = \vec{E}\times\vec{H} = \underline{\underline{\left(\frac{-I^2 r}{2\pi^2\sigma R^3}\right)\hat{a}_r}}$$

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On calculating, $\vec{J} = \frac{100}{\pi(0.01)^2}\hat{a}_z = 3.183 \times 10^5 \hat{a}_z \ (A\,m^{-2})$

$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{3.183\times10^5}{2\times10^7} = 1.59\times10^{-2}\hat{a}_z \ (V m^{-1})$

$\vec{H} = \frac{Jr}{2}\hat{a}_\phi = (1.59\times10^5) r \,\hat{a}_\phi \ (A m^{-1})$

$\vec{P} = \vec{E}\times\vec{H} = (-2533\,r)\,\hat{a}_r \ (W m^{-2})$

(ii) $\int P dv = 2(2\pi)(1)\int_0^{0.01}(-2533r)\,dr = -31830.9\left[\frac{r^2}{2}\right]_0^{0.01}$

$= -1.59\ W$

3. $\vec{H} = 0.2\cos(\omega t - \beta x)\,\hat{a}_z \ (A m^{-1})$

Direction of propagation $= \hat{a}_x$

(i) $\vec{P}_{avg} = \frac{1}{2}\frac{E_0^2}{\eta}\hat{a}_x = \frac{1}{2}\eta H_0^2 \hat{a}_x$

In free space, $\eta \approx 120\pi$

Now, $P = \int \vec{P}_{avg}\cdot\vec{dS} = \int \vec{P}_{avg}\cdot(dS\,\hat{a}_n)$

For the eqn. of plane, $Ax + By + Cz + D = 0$,

$\hat{a}_n = \frac{A\hat{a}_x + B\hat{a}_y + C\hat{a}_z}{\sqrt{A^2+B^2+C^2}}$

Here, $\hat{a}_n = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

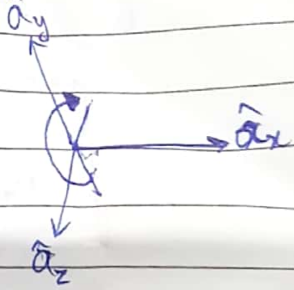
$\Rightarrow \vec{P} = \left[\vec{P}_{avg}\cdot\left[(0.1)^2\left(\frac{\hat{a}_x+\hat{a}_y}{\sqrt{2}}\right)\right]\right] = \frac{1}{2}(120\pi)(0.2)^2\frac{(0.1)^2}{\sqrt{2}}(\hat{a}_x\cdot\hat{a}_x)$

$= 53.31\ mW$

(ii) Now the dir. of propagation is $\perp$ to the plane of interest.

$$\Rightarrow P = \int \vec{P}_{avg} \cdot \vec{dS} = \frac{1}{2}(120\pi)(0.2)^2(\pi)(0.05)^2 = \underline{\underline{59.21 \text{ mW}}}$$

4. $(-j) \Rightarrow$ phase shift of -90°



Looking from $\hat{a}_x$, the $\vec{E}_s$ vector moves clockwise. $\Rightarrow$ Right-handed wrt receiver

Now,

$$\vec{E}_s = E_0 e^{-j\beta x}\,\hat{a}_y - jE_0 e^{-j\beta x}\,\hat{a}_z$$

$$= E_0(\cos(\beta x) - j\sin(\beta x))\,\hat{a}_y - jE_0(\cos(\beta x) - j\sin(\beta x))\,\hat{a}_z$$

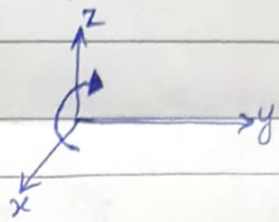
$$= (E_0\cos(\beta x)\,\hat{a}_y - E_0\sin(\beta x)\,\hat{a}_z) - j(E_0\sin(\beta x)\,\hat{a}_y + E_0\cos(\beta x)\,\hat{a}_z)$$

As is visible, $\vec{E}_s$ presumes a sinusoidal nature with respect to x.

$\therefore$ The wave is right-handed circular polarized with respect to the receiver.

5. $(-j) \Rightarrow$ phase diff. of 90° for $\hat{a}_z$ wrt. $\hat{a}_x$

$\Longrightarrow$ Wave is right handed circ. pol.

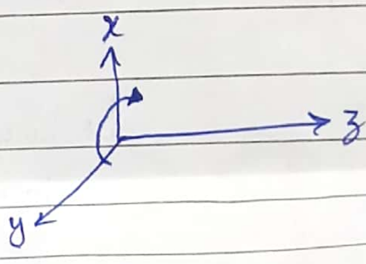


But, when the wave is incident on a perfect conductor, the dir. of the wave changes to $-\hat{a}_y$ but the electric field is in the same dir.

$\therefore$ The final wave is left-handed circular polarized wrt. the receiver.

6. The wave is evidently circular polarized because of the phase diff. of $\pi/2$

Now, $\vec{E}_y$ is 90° ahead of $\vec{E}_x$



$\therefore$ The wave is right-handed circular polarized wrt. the receiver.

7. $\vec{E}_A = (E_0 \cos(\omega t - \beta z))(\hat{a}_x)$

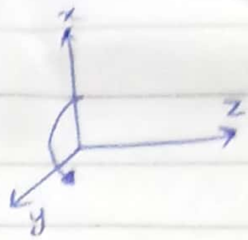
$\vec{E}_B = (E_0 \cos(\omega t - \beta z - \theta))\left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$

$$\vec{E} = \vec{E}_A + \vec{E}_B = E_0\left[\hat{a}_x\left(\cos(\omega t - \beta z) + \frac{1}{\sqrt{2}}\cos(\omega t - \beta z - \theta)\right) + \hat{a}_y\left(\frac{1}{\sqrt{2}}\cos(\omega t - \beta z - \theta\right.\right.$$

$$= E_0\left[\hat{a}_x \frac{\left(\cos(A)\cdot(1 + \frac{1}{\sqrt{2}}\cos\theta) + \sin(A)\cdot(\sin\theta/\sqrt{2})\right)}{\sqrt{3/2 + \sqrt{2}\cos\theta}} \cdot \sqrt{3/2 + \sqrt{2}\cos\theta} + \hat{a}_y\left(\frac{1}{\sqrt{2}}\right)\cdot\cos(\omega t - \beta z - \theta)\right]$$

$$= E_0\left[\hat{a}_x \cdot \left(\sqrt{3/2 + \sqrt{2}\cos\theta}\right)\cdot \cos\left(\omega t - \beta z - \sin^{-1}\left(\frac{\sin\theta}{\sqrt{2}}\right)\right) + \hat{a}_y \cdot (1/\sqrt{2})\cdot \cos(\omega t - \beta z - \theta)\right.$$

The amplitudes of $\vec{E}_x$ and $\vec{E}_y$ are different.
Also, $\vec{E}_x$ leads $\vec{E}_y$ by $\left(\theta - \sin^{-1}\left(\frac{\sin\theta}{\sqrt{2}}\right)\right)$



$\therefore$ It is an oblique elliptical polarized wave left-handed wrt the receiver.

Review Questions

1. The Poynting theorem is applicable to static fields as well. In such a situation, the outward flux remains constant and the work done by the field is dependent on what it acts on.

i.e. for

$$\nabla\cdot\vec{P} + \frac{\varepsilon}{2}\frac{d|\vec{E}|^2}{dt} + \frac{\mu}{2}\frac{d|\vec{B}|^2}{dt} + \sigma|\vec{E}|^2 = 0$$

$\frac{d|\vec{E}|^2}{dt} = 0$, $\frac{d|\vec{B}|^2}{dt}$ is also possibly zero.

or $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$\Rightarrow \nabla\cdot\vec{P} + \frac{\mu}{2}\frac{d|\vec{B}|^2}{dt} + \sigma|\vec{E}|^2 = 0$$

2. Complex permittivity $= \varepsilon_c = \varepsilon' - j\varepsilon''$

where,

$$\varepsilon' = (\varepsilon), \quad \varepsilon'' = \left(\frac{\sigma}{\omega\varepsilon}\right)$$

ε' represents the actual permittivity of the medium.
ε'' represents the effect of losses and, though it is not directly intuitive from the definition of complex permittivity, it can be used to model the losses in the electric field.

3. Dispersion is modelled using two parameters :- α and β, which together make up γ.

$$\gamma = \alpha + j\beta,$$

α = attenuation constant ($Np\,m^{-1}$)

β = phase constant ($rad\,m^{-1}$)

$$\vec{E} = E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z}.$$

Now, dispersion depends on the losses from the ~~wave~~ wave which can be understood by the ratio of $\vec{J}$ and $\frac{\partial \vec{D}}{\partial t}$

$$\frac{\vec{J}}{(\partial D/\partial t)} = \frac{\sigma \vec{E}}{\varepsilon\, \partial \vec{E}/\partial t} = \frac{\sigma}{\omega\varepsilon}$$

As a result, dispersion is inversely proportional to the frequency of the wave; i.e, as the frequency of the wave increases, the effect of dispersion decreases and vice versa.