Closed Loop Relative Stability

– *Relative Stability Concept*

– *Modified Routh's Technique*

Absolute Stability Features

Absolute stability analysis, using **Routh's** method, provides i**nformation** about number of closed loop **poles** lying in the **RH s-plane.**

In some **special** cases, we can also **determine** the poles lying on the **imaginary axis.**

Absolute Stability Limitation

Though, we obtain **same** information through **Nyq**uist stability criterion, in **both** methods, their actual **location** with respect to **imaginary** axis in **unknown.**

Further, in case of **stable** closed loop, both the **methods** are not **directly** useful for closed loop relative **stability.**

Concept of Relative Stability

Thus, we need to **employ** a methodology for **quantifying** the real part of the closed loop **poles** lying in the **vicinity** of the **imaginary** axis (either to the left or to the right).

This is typically **enabled** through the concept of **relative stability** which provides the **location** of poles, **closest** to the **imaginary** axis ,for a **closed loop** system.

Relative Stability From Routh's

We can **arrive at** this information from **Routh's** method in the following **manner.**

We make **use** of the conditions that result in **poles** lying on the **imaginary axis** to quantify the **closest** poles.

This is **possible** through generation of a '**zero**' in the 1^{st} **column** of the Routh's **array.**

Modification to Routh's Method

Modified Routh's method provides a way of **getting** this information in an **approximate** way.

The modification is **based** on the **premise** that we normally have a **requirement** on desired stability **level,** which is **translated** into a stability **margin, 'σ'.**

Modification to Routh's Method

This is achieved by **relocating** the imaginary **axis** to the left (or right) and **re-designating** the plane as '**z-plane**',

Next, characteristic **polynomial**, D(s) is transformed into **D(z)**, by substituting **'(z - σ)'** for 's' and examining **D(z)** for its absolute **stability.**

If there are **no unstable** roots for **modified** polynomial, then the system is **supposed** to have a **minimum** stability margin of 'σ'.

Minimum Stability Margin Example

Consider the following **polynomial** and determine if the **system** has a minimum stability **margin of 1.**

$$\boxed{s^3 + 7s^2 + 25s + 39 = 0}$$

Let s = z – 1.

$$\boxed{z^3 + 4z^2 + 14z + 20 = 0}$$

Z^3	1	14
Z^2	4	20
Z^1	9	0
Z^0	20	

Complete polynomial with all coefficients having same sign.

No sign changes, so no pole in right half z – plane. System has desired stability margin.

-2.0000 + 3.0000i
-2.0000 - 3.0000i
-3.0000

Actual Stability Margin Example

Consider the following **polynomial** and determine if the **system** has a minimum stability **margin of 1.**

$$s^3 + 7s^2 + 25s + 39 = 0$$

Let s = z – σ.

$$z^3 + (7 - 3\sigma)z^2 + (3\sigma^2 - 14\sigma + 25)z$$

$$+ (7\sigma^2 - 25\sigma + 39 - \sigma^3) = 0$$

Real pole: $-\sigma^3 + 7\sigma^2 - 25\sigma + 39 = 0 \rightarrow \sigma = 3$

Imaginary poles: $(7 - 3\sigma)\times(3\sigma^2 - 14\sigma + 25)$

$-(7\sigma^2 - 25\sigma + 39 - \sigma^3) = 0 \rightarrow \sigma = 2$

Modified Routh's Features

Modified **Routh's** technique has the **potential** to extract the **roots** of a characteristic equation, by **successively shifting** the 'jω' **axis.**

However, we **also know** that the method is **quite tedious** and can become **unwieldy** for higher order systems.

Therefore, we need a **methodology** that provides **reasonably accurate** quantification of stability **margins** while keeping the **numerical effort** reasonable.

Summary

Relative stability analysis requires **quantification** of real part of **dominant** poles.

Modified Routh's method shifts the imaginary **axis** to quantify the dominant **pole** through **absolute** stability analysis **concepts.**