Mid-Semester Examination Solutions

Communication Systems (EE 308), Autumn'19

QUESTION 1

(a) The spectrum of $u(t)$ is:

$$\begin{aligned} U(f) &= 10[\delta(f-f_c)+\delta(f+f_c)] \\ &\quad +0.5[\delta(f-f_c-1500)+\delta(f-f_c+1500)+\delta(f+f_c-1500)+\delta(f+f_c+1500)] \\ &\quad +2.5[\delta(f-f_c-3000)+\delta(f-f_c+3000)+\delta(f+f_c-3000)+\delta(f+f_c+3000)] \end{aligned}$$

(b) The power at the frequency f_c is $\frac{400}{2}=200$, the power at frequency f_c+1500 is the same as the power at f_c-1500 and equals 0.5, whereas the power at each of the frequencies f_c+3000 and f_c-3000 is $\frac{25}{2}=12.5$.

(c) The power in the sidebands is:

$$P_{sidebands} = 0.5+0.5+12.5+12.5 = 26$$

and the total power is $P_{total} = P_{carrier} + P_{sidebands} = 200+26 = 226$. So the ratio of the sidebands power to the total power is $\frac{26}{226}$.

QUESTION 2

Let

$$\Pi(f) = \begin{cases} 1, & |f| < \frac{1}{2}, \\ 0, & \text{else.} \end{cases}$$

and

$$\Lambda(f) = \begin{cases} f+1, & -1 \le f < 0, \\ -f+1, & 0 \le f < 1, \\ 0, & \text{else.} \end{cases}$$

The modulated signal is:

$$u(t) = m(t)c(t) = A(\text{sinc}(t)+\text{sinc}^2(t))\cos(2\pi f_c t).$$

Taking Fourier transforms on both sides:

$$U(f) = \frac{A}{2}[\Pi(f-f_c)+\Lambda(f-f_c)+\Pi(f+f_c)+\Lambda(f+f_c)]$$

So the bandwidth of the modulated signal is 2.

QUESTION 3

The instantaneous frequency is:

$$f_i(t) = f_c + \frac{k}{\pi} m(t) \frac{dm(t)}{dt}.$$

So $s(t)$ is an FM signal with modulating signal $\tilde{m}(t) = m(t)\frac{dm(t)}{dt}$ and frequency sensitivity $k_f = \frac{k}{\pi}$. The bandwidth of the signal $\tilde{m}(t)$ is $2B$. Assume that:

$$-m_p \leq m(t)\frac{dm(t)}{dt} \leq m_p,$$

where m_p is a constant. The frequency deviation is:

$$\Delta f = k_f m_p = \frac{k m_p}{\pi}.$$

So by Carson's rule, an estimate for the bandwidth of $s(t)$ is:

$$B_T \approx \frac{2km_p}{\pi} + 4B.$$

QUESTION 4

The image frequency $f_c' = f_c + 2f_{IF}$. So we require that:

$$f_c + 2f_{IF} \geq 108, \;\; \forall f_c \in [88, 108].$$

So $f_{IF} \geq 10$ MHz.

Now, $f_{LO} = f_c + f_{IF}$. So f_{LO} varies between $88 + 10 = 98$ MHz and $108 + 10 = 118$ MHz.

QUESTION 5

This is problem 2.21 on p. 72, Haykin and Moher, which is part of Homework 1.

QUESTION 6 (2.5 MARKS)

The total amount of frequency multiplication required is:

$$\frac{75}{1.5} = 50.$$

If the narrowband FM signal is input to a frequency multiplier with factor 50, then the carrier frequency gets converted to $0.1 \times 50 = 5$ MHz. So at the output of the frequency multiplier, we need a mixer, which shifts the carrier frequency by 99 MHz to 104 MHz.

QUESTION 7

This is problem 3.4 on p. 97, Haykin and Moher, which is part of Homework 4.

QUESTION 8

This is problem 4.16 on p. 142, Haykin and Moher, which is part of Homework 4.

QUESTION 9

(a) The Fourier transform of the complex envelope of the impulse response of the bandpass filter is:

$$\tilde{H}(f) = \begin{cases} \frac{2f}{W} + 1, & |f| \leq \frac{W}{2}, \\ 2, & \frac{W}{2} < f \leq W, \\ 0, & \text{else.} \end{cases}$$

Taking the inverse Fourier transform and simplifying, we get:

$$\tilde{h}(t) = \frac{j}{\pi t}[\text{sinc}(Wt) - e^{j2\pi Wt}] - W\text{sinc}(Wt).$$

(b) An expression for the modulated signal $u(t)$ is obtained as follows:

$$\begin{aligned} u(t) &= Re[(\frac{1}{2}Am(t) * \tilde{h}(t))e^{j2\pi f_c t}] \\ &= \frac{1}{2}Am(t)\cos(2\pi f_c t) - \frac{1}{2}Am(t) * \left(\frac{1}{\pi t}\text{sinc}(Wt)\right)\sin(2\pi f_c t) \\ &\quad -\frac{1}{2}\left[Am(t) * W\text{sinc}(Wt)\right]\cos(2\pi f_c t). \end{aligned}$$

To obtain the above, we have used the fact that:

$$\mathcal{F}\left[m(t) * \frac{1}{j\pi t}e^{j2\pi Wt}\right] = -M(f)sgn(f-W) = M(f),$$

since $sgn(f-W) = -1$ for $f < W$.