Quiz 1 Solutions

Communication Systems (EE 308), Autumn'19

QUESTION 1

(a) The average transmitted power is $\frac{A_c^2}{2} = \frac{(100)^2}{2} = 5000$.

(b) The peak phase deviation is $\max|4\sin(2000\pi t)| = 4$.

(c) The instantaneous frequency is:

$$f_i = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_c + 4000\cos(2000\pi t).$$

So the peak frequency deviation is $\max|f_i - f_c| = 4000$.

(d) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with phase sensitivity $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with frequency sensitivity $k_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$.

QUESTION 2

(a) The Hilbert transform of $\cos(2\pi 1000t)$ is $\sin(2\pi 1000t)$, whereas the Hilbert transform of $\sin(2\pi 1000t)$ is $-\cos(2\pi 1000t)$. Thus:

$$\hat{m}(t) = \sin(2\pi 1000t) - 2\cos(2\pi 1000t).$$

(b) The expression for the LSSB AM signal is:

$$u_l(t) = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t).$$

Substituting the values of A_c, $m(t)$ and $\hat{m}(t)$ and simplifying, we get:

$$u_l(t) = 100\cos(2\pi(f_c - 1000)t) - 200\sin(2\pi(f_c - 1000)t).$$

(c) The Fourier transform of $u_l(t)$ is:

$$U_l(f) = (50 + 100j)\delta(f - f_c + 1000) + (50 - 100j)\delta(f + f_c - 1000).$$

Hence, the magnitude spectrum is:

$$|U_l(f)| = 10\sqrt{125}(\delta(f - f_c + 1000) + \delta(f + f_c - 1000)).$$

QUESTION 3

This is problem 4.12 on p. 141, Haykin, which is part of Homework 3.

QUESTION 4

(a) The modulated signal is:

$$\begin{aligned} u(t) &= 10\cos\left(2\pi f_c t + 2\pi k_f \int_0^t \cos(20\pi\tau)d\tau\right) \\ &= 10\cos\left(2\pi f_c t + 5\sin(20\pi t)\right). \end{aligned}$$

(b) The modulation index is given by:

$$\beta = k_f \frac{\max(|m(t)|)}{f_m} = 5.$$

So the FM-modulated signal can be written as:

$$u(t) = \sum_{n=-\infty}^{\infty} 10 J_n(5)\cos(2\pi(f_c + 10n)t).$$

To ensure that at least 98% of the total power is within the transmission bandwidth, we have to choose k large enough such that:

$$50\left[J_0^2(5) + 2\sum_{n=1}^{k} J_n^2(5)\right] \geq 50 \times 0.98.$$

Using the table of Bessel functions, we find that we need to choose $k = 6$. So the transmission bandwith is $2 \times 10 \times 6 = 120$ Hz.

QUESTION 5

(a) The VSB signal $y(t)$ is obtained by first generating a DSB signal $x(t)$ and then passing it through the VSB filter.

$$x(t) = 1.5\left[\cos(2\pi(f_c + f_1)t) + \cos(2\pi(f_c - f_1)t)\right] + 2.5\left[\cos(2\pi(f_c + f_2)t) + \cos(2\pi(f_c - f_2)t)\right],$$

where $f_1 = 3$ kHz and $f_2 = 8$ kHz.

The frequency components in the DSB signal are $f_c - f_2 = 92$ kHz, $f_c - f_1 = 97$ kHz, $f_c + f_1 = 103$ kHz and $f_c + f_2 = 108$ kHz.

The VSB filter stops the 92 kHz component. The other three frequency components are multiplied with the filter gains at those frequencies. Hence, the output of the VSB filter is:

$$y(t) = 1.5\left[H(f_c + f_1)\cos(2\pi(f_c + f_1)t) + H(f_c - f_1)\cos(2\pi(f_c - f_1)t)\right] + 2.5\left[H(f_c + f_2)\cos(2\pi(f_c + f_2)t)\right].$$

Using the given expression for $H(f)$: $H(f_c - f_1) = 0.2$, $H(f_c + f_1) = 0.8$ and $H(f_c + f_2) = 1.0$. Hence:

$$y(t) = 0.3\cos(194\pi t) + 1.2\cos(206\pi t) + 2.5\cos(216\pi t).$$

(b) The power in $y(t)$ is:

$$\frac{(0.3)^2}{2} + \frac{(1.2)^2}{2} + \frac{(2.5)^2}{2} = 3.89W.$$