# *Two Variable Linearization*

**Linearize** following nonlinear **algebraic relation**, by choosing **centre point** of the specified domain as the **operating point**, and determine its **accuracy** on the domain boundary.

$$z = x^2 + 8xy + 3y^2; \quad 2 \le x \le 4; \quad 10 \le y \le 12$$

$$z - z_0 = \frac{\partial z}{\partial x}\Big|_{3,11}(x-3) + \frac{\partial z}{\partial y}\Big|_{3,11}(y-11); \quad z_0 = 636; \quad \frac{\partial z}{\partial x}\Big|_{3,11} = 94$$

$$\frac{\partial z}{\partial y}\Big|_{3,11} = 90; \quad z - 636 = 94(x-3) + 90(y-11) \to z = 94x + 90y - 636$$

$$(x, y) = (2,10) \to z_{actual} = 464, \quad z_{linear} = 452$$

$$(x, y) = (4,12); \quad z_{actual} = 832, \quad z_{linear} = 820$$

# *Pendulum-cart Linearization*

**Linearize** the following pendulum-cart **model**.

$$\left(I + ml^2\right)\ddot{\theta} + ml\cos\theta\ddot{x} = mgl\sin\theta$$
$$\left(M + m\right)\ddot{x} + ml\cos\theta\ddot{\theta} = u$$

$$\left(I + ml^2\right)\delta\ddot{\theta} + ml\cos\theta_0\delta\ddot{x} - ml\sin\theta_0\ddot{x}_0\delta\theta - mgl\cos\theta_0\delta\theta = 0$$
$$\left(M + m\right)\delta\ddot{x} + ml\cos\theta_0\delta\ddot{\theta} - ml\sin\theta_0\ddot{\theta}_0\delta\ddot{\theta} = \delta u$$

# *Pendulum-cart Model Verification*

Let us assume **x = 0**, so that we have the following **nonlinear and linearized** equations for pendulum **alone.**

$$\left(I+ml^2\right)\ddot{\theta}-mgl\sin\theta=0$$

$$\left(I+ml^2\right)\delta\ddot{\theta}-mgl\cos\theta_0\delta\theta=0$$

We see that for $\boldsymbol{\theta_0}$ **= 0, and** $\boldsymbol{\theta = \delta\theta}$, both nonlinear and linear equations **are the same.**

Further, we note that we get **different linear models** for different values of ‘$\boldsymbol{\theta_0}$’, indicating that **nonlinear** effects are **adequately** captured in the **linearized** model.