Non-minimum Phase Systems

- *Non-minimum Phase Systems*
- *Pade's Approximation*
- *Role of RH s-plane Zeros*

Minimum Vs. Non – minimum Phase

Systems which have all their **zeros and poles** in the LH **s-plane** are termed '**minimum phase**' system due to their specific **phase** characteristics.

However, there are **quite a few** systems which **violate** the above **criteria** and have either a **pole or a zero** in RH s-plane.

In all **such** systems an **additional phase** of 180^o (per pole /zero) gets **added** to the phase of the basic **TF.**

Non – minimum Phase Systems

All such systems are called **non – minimum** phase systems as their **phase** characteristics are **significantly** different.

A **common** source of such **non-minimum** phase is the presence of transport **lag** and computational time **delays**, which add an **exponential** factor to the **TF.**

Non-minimum Phase Situations

E.g., **fluids and gases** flowing through pipes take a **finite time** to reach the **destination.**

Similarly, **heat flux** takes finite time to **reach** the object to be **heated / cooled.**

In **communication** systems, signals take **small,** but finite, **time** to generate triggers.

Non-minimum Phase Impact

We know from **Laplace transform** that transfer function of such systems **gets modified**, as shown below.

$$\boxed{G'(s) = e^{-\tau_d s} \cdot G(s); \quad \tau_d \rightarrow \text{ Time delay}}$$

We can see that the **system** acquires additional **dynamics** that needs to be **included** in the analysis.

However, as '$e^{-\tau ds}$' in **irrational**, we first need to **convert** it into its **rational** form, as required by **TF** representation.

Approximation of Exponential Term

In **mathematics**, Pade's series is commonly **employed** for arriving at the **approximate** rational representation of **irrational** functions.

Therefore, we also **use** the same for **approximating** the time **delay term** to rationalize the **TF.**

The **Pade's series** is a ratio of **two infinite series**, which is normally **truncated** to a specific **order** depending on the **accuracy** requirements.

Approximation of Exponential TErm

Given below is **1st order** approximation, which is **most commonly** employed for representing the **time delay.**

$$\text{Pade's 1st Order Form: } e^{-\tau_d s} \approx \frac{1-\left(\tau_d/2\right)s}{1+\left(\tau_d/2\right)s}$$

It is seen that the representation results in a **zero** in right half of **s-plane.**

In some cases, a **2nd order** representation is also **used.**

Time Delay Example

Obtain unit **impulse response** of the given plant.

$$G(s)=\frac{1}{(s+1)}\times e^{-s}$$

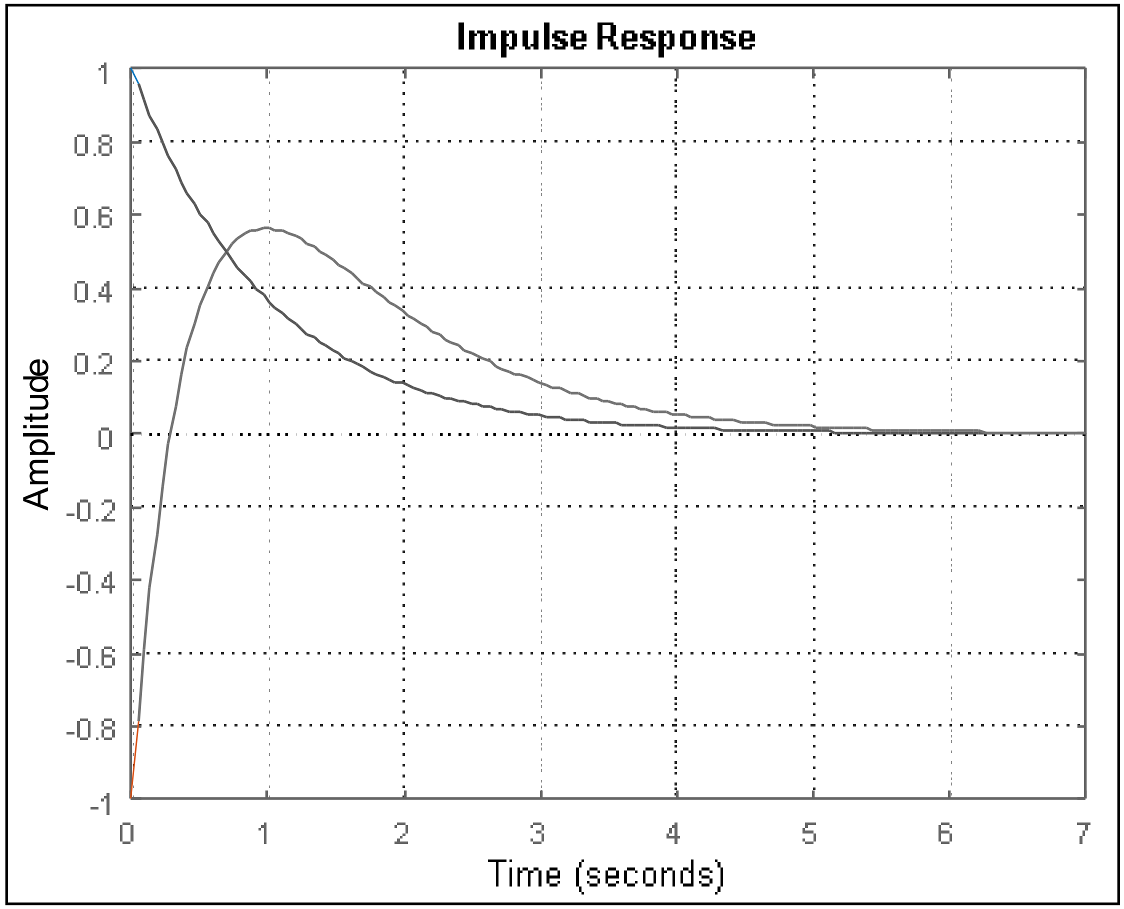
$$G(s)=\frac{2}{(s+1)}\times\frac{1-0.5s}{(s+2)}$$

$$G(s)=\frac{A_1}{s+1}+\frac{A_2}{s+2}$$

$$A_1=2; \quad A_2=-4$$

$$G(s)=\frac{3}{s+1}-\frac{4}{s+2}$$

$$g(t)=3e^{-t}-4e^{-2t}$$



Summary

Non-minimum phase systems arise due to transport **lag** and computational **delays** in systems.

Zeros in right half of **s-plane** impact the system phase adversely, thereby, **impacting dynamic** characteristics.