P Control Design

- *P – Control Features & Design Basics*
- *P – Control Designs using Root Locus*
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Proportional Control Features

We know that **'P' control** can impact **performance** related to **all** aspects of closed loop **response.**

Therefore, in **view** of the fact that it is **also** the simplest **possible** control strategy, **'P' control** is commonly **employed** in many cases as the first **option.**

Proportional Control Impact

We see that **‘P’** control **impacts** closed loop **behaviour** on two **counts.**

Firstly, it **modifies** the overall loop **gain**, and hence, the **steady-state** behaviour.

Secondly, it **changes** dominant **pole location**, and thereby, both relative **stability** and **transient** response.

P – Control Design Basics

P – control, while the **simplest**, is also quite **restrictive**, as it provides only **one design** degree-of-freedom.

Thus, it is not **possible** to achieve a **wide** range of **performance** in the **closed loop.**

P – control is commonly used to **improve** the gain for **tracking**, though it can **also** benefit transient **response.**

‘P’ Design with Root Locus

Root Locus Design Steps

Convert specifications into the desired dominant pole.

Draw root locus of G(s), for K from 0 to ∞ and establish the existing pole location.

Superimpose closed loop performance parameters onto the root locus.

Use graphical technique to determine the total gain**.**

Ratio of total gain to plant gain gives P – control gain.

P – Control Design Example

Design a **'P' controller** using the **root locus** to achieve the following **performance** parameters in **closed loop**.

Ramp Error Constant ≥ 2.5

Peak Overshoot $\leq 20\%$

Settling Time ≤ 3.0 Seconds (2%)

$$G(s) = \frac{40}{s(s+4)(s+10)}$$

Specification Translation

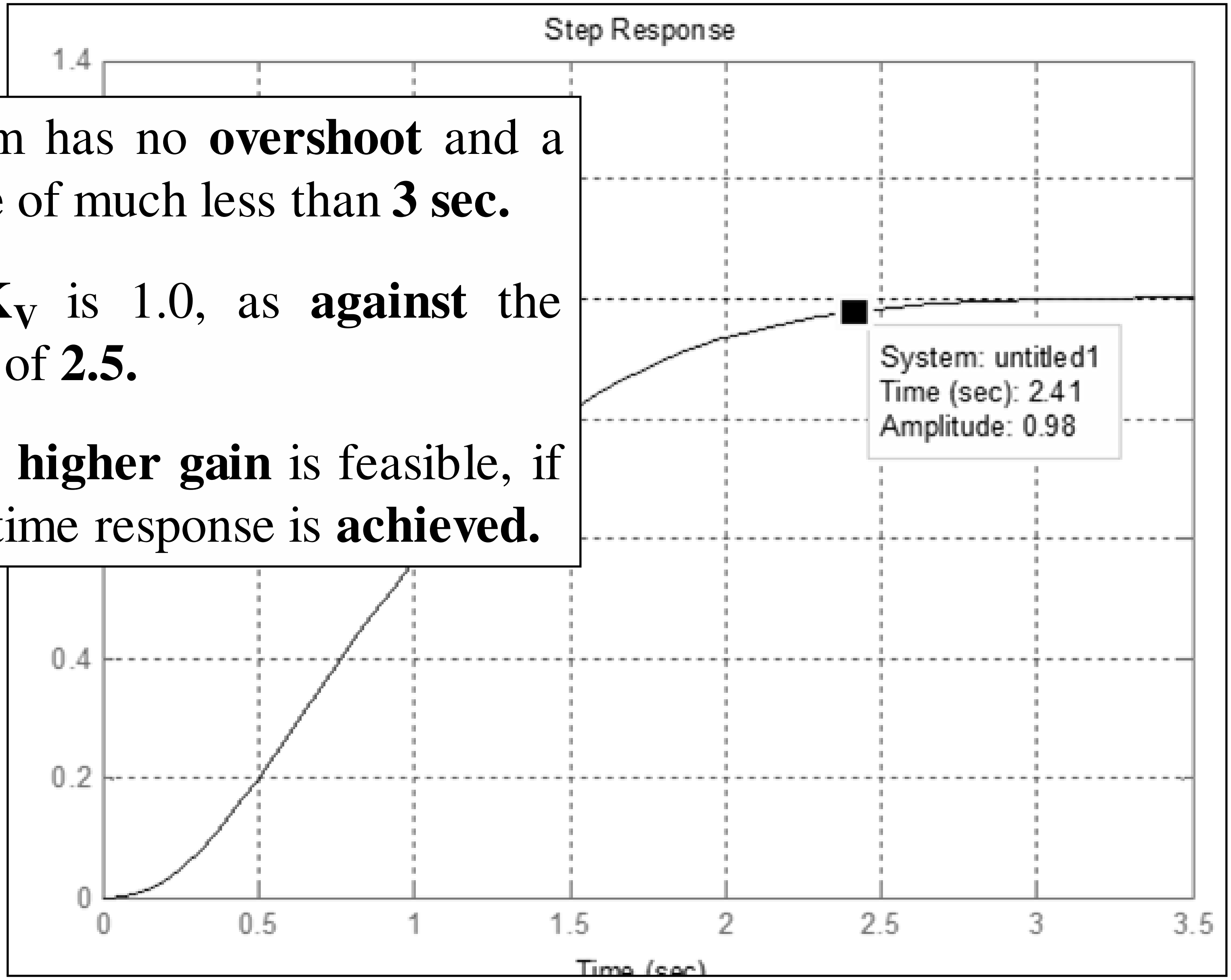
The **first task** is to convert **specifications** into pole.

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \le 0.20 \rightarrow \zeta \ge 0.456$$

$$T_s \cong \frac{4}{\sigma} \text{ (for 2\% Ripple) } \le 3.0 \rightarrow \sigma \ge 1.33$$

$$\omega_n = 2.92; \quad \omega_d = 2.59; \quad \text{Pole: Better than } -1.33 \pm j2.59$$

Uncompensated CL Features

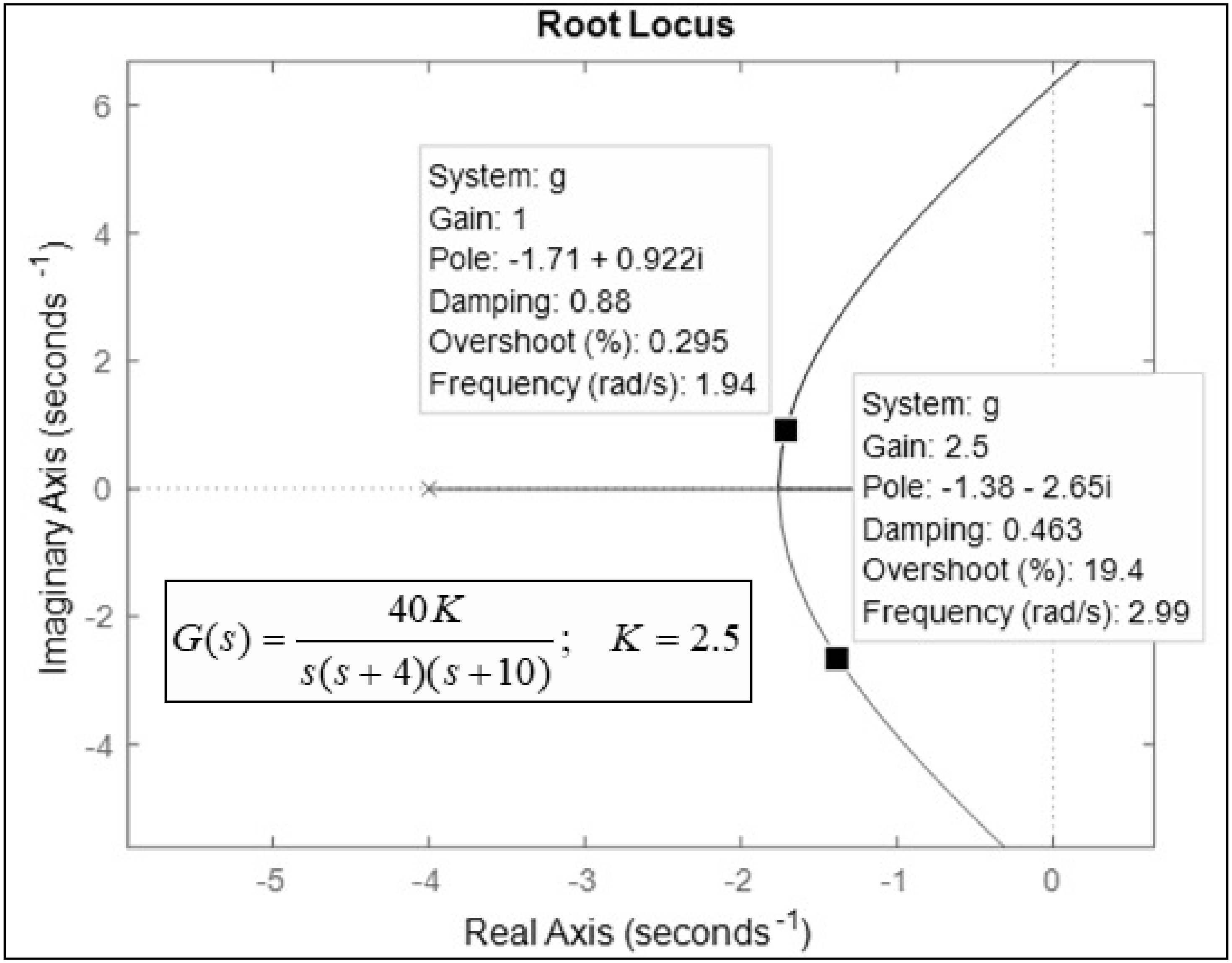


Basic system has no **overshoot** and a settling **time** of much less than **3 sec.**

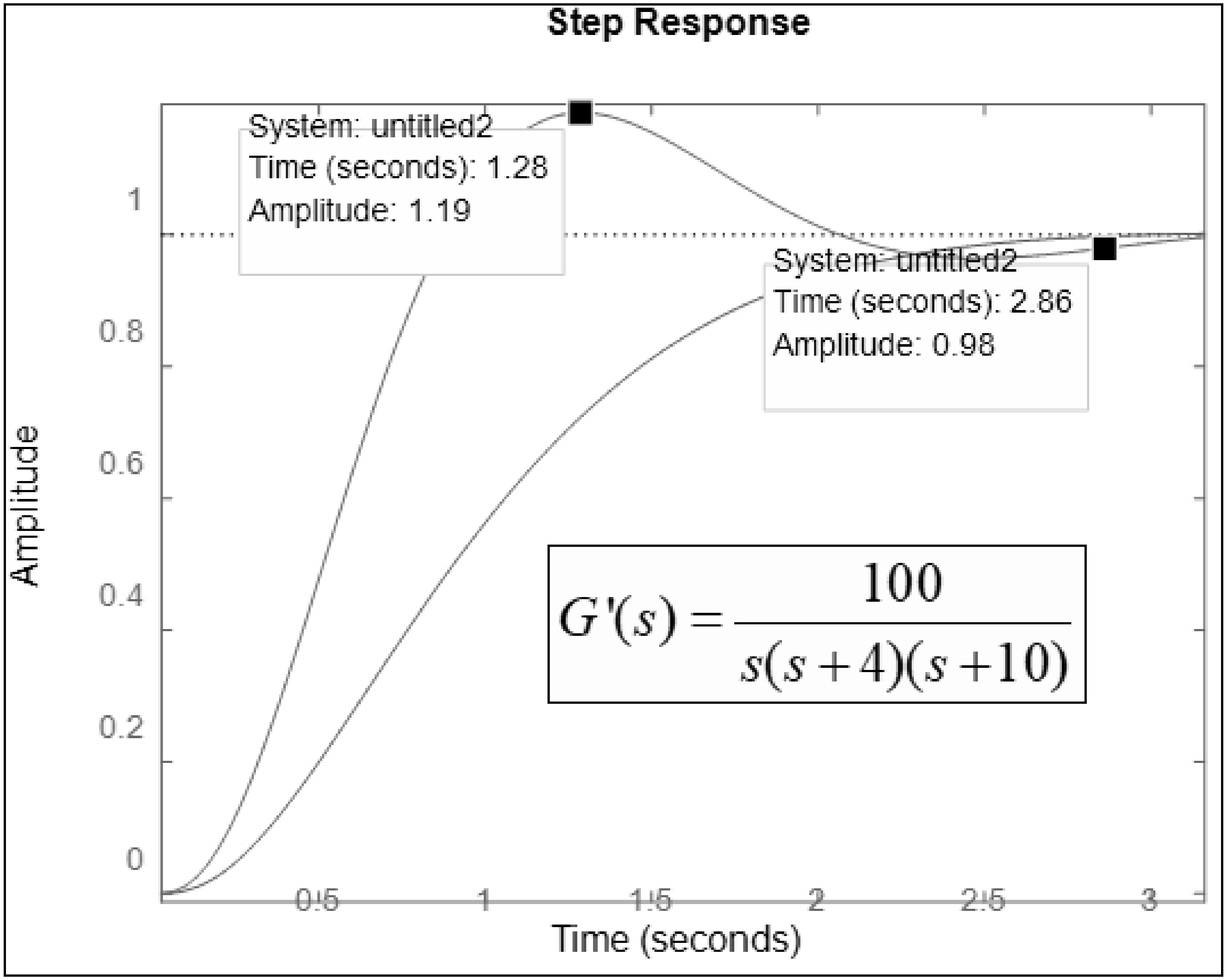
However, $\mathbf{K_V}$ is 1.0, as **against** the requirement of **2.5.**

Therefore, a **higher gain** is feasible, if the **desired** time response is **achieved.**

Design Visualization – Root Locus



Design Verification – Step Response



$$G'(s) = \frac{100}{s(s+4)(s+10)}$$

Design Solution Analysis

We find that **by** satisfying a **single** specification of **ramp** error constant, it is **possible** to also **meet** specifications on **transient** response.

Further, while **root** locus predicts a **19.8%** overshoot, step response shows **only 19%** overshoot, **indicating** that we could **increase** gain marginally.

P – Control Design Example

Consider the following **plant** transfer function.

$$G(s) = \frac{1}{(s+1)^3}$$

Design a **'P' gain** to achieve a **peak time** of around 3.4s, and 2% **settling** time of around 7s.

In case the **design** is not **feasible**, give the **best** possible solution.

Specification Translation

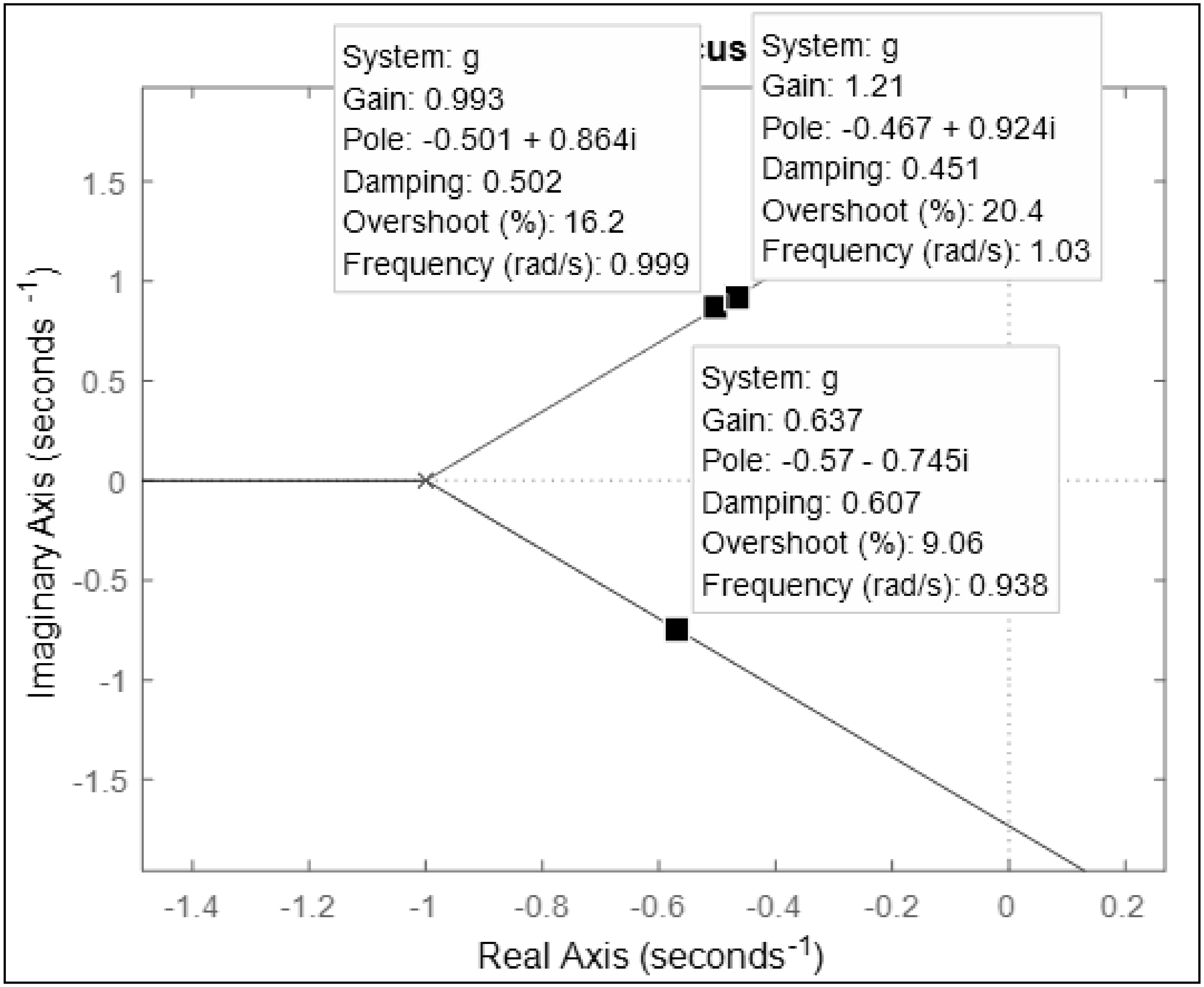
The **first** task is to **convert** given **specifications** into the desired **pole** location.

$$T_s \cong \frac{4}{\sigma} \text{ (for 2\% Ripple) } \sim 7.0 \rightarrow \sigma \sim 0.571$$

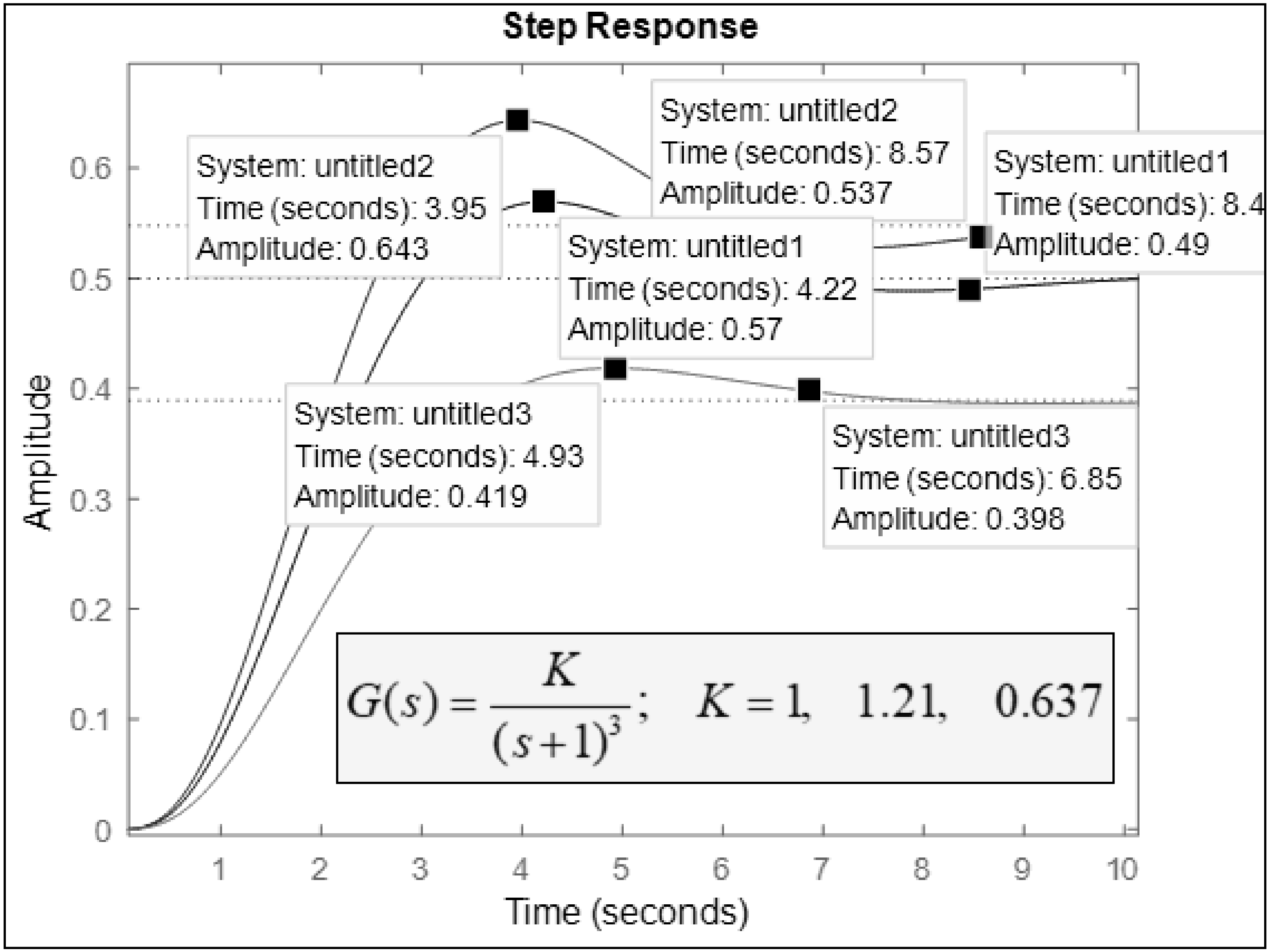
$$T_p = \frac{\pi}{\omega_d} \sim 3.4 \rightarrow \omega_d \sim 0.924; \quad \text{Pole: } -0.571 \pm j0.924$$

There are no **requirements** on steady-state **response**, which indicates that we are **required** to maintain **existing** tracking **performance**, as far as possible.

Root Locus Based Design Domain



Design Visualization – Step Response



Design Solution Analysis

We find that **P-control** is **unable** to meet the requirements **as stated.**

In this context, a **compromise** is necessary, which can be arrived at, by **bringing** in additional information.

E.g., if **settling time** is treated as a **soft** requirement, we can use a **higher gain** to improve tracking.

Similarly, if **settling time** is a **hard** requirement, we can **reduce gain** to also improve the **peak overshoot.**

‘P’ Design with Bode Plot

Bode Plot Based P-Design Steps

Draw Bode' plot of G(s) and mark existing features e.g. GCO, PCO, GM & PM.

Superimpose closed loop performance parameters.

Assess shift required in magnitude plot for achieving the performance.

Antilog of shift in gain gives the gain of **'P' controller.**

P – Control Design Example

Design a **‘P’ controller** using **bode’ plot** to achieve the following **performance** for the compensated **plant.**

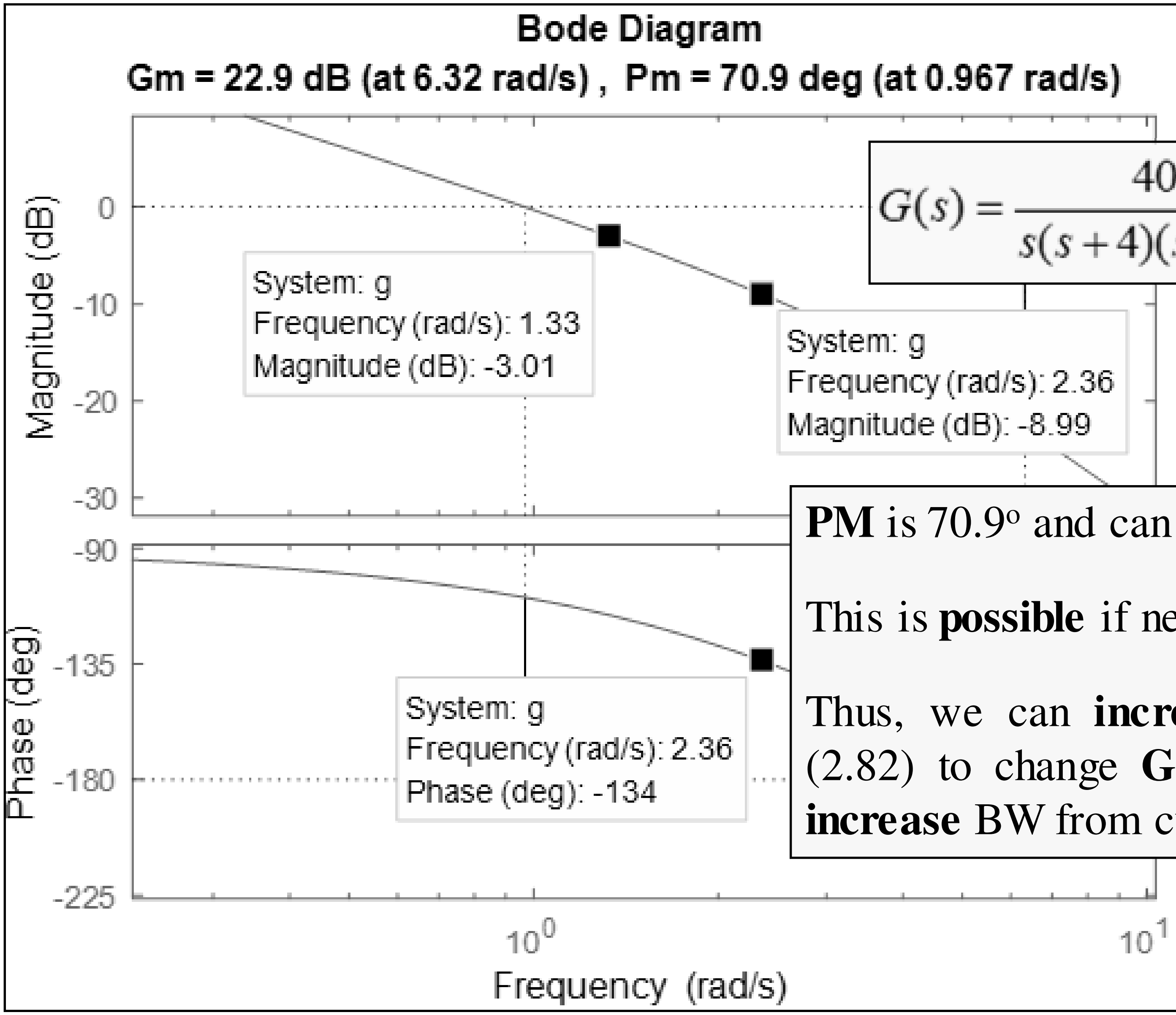
Phase Margin $\geq 45.6^{o}$

Bandwidth ≥ 3.83

$$\boxed{G(s) = \frac{40}{s(s+4)(s+10)}}$$

Also **commen**t on change in **gain margin.**

Uncompensated Bode' Plot



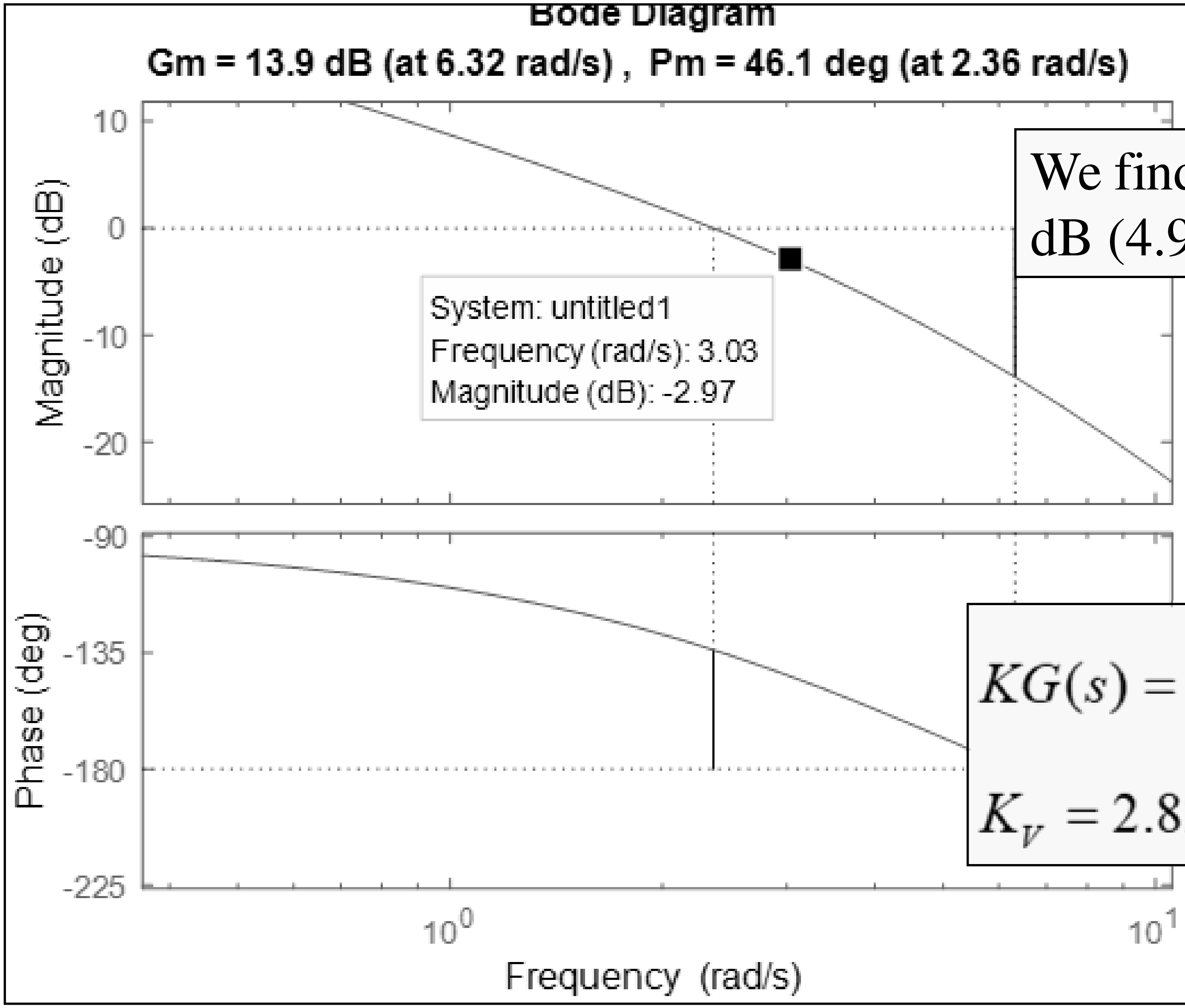
$$G(s) = \frac{40}{s(s+4)(s+10)}$$

PM is 70.9^o and can be **reduced** by 25.3^o.

This is **possible** if new **GCO** is ~ 2.36.

Thus, we can **increase** gain by **~9 dB** (2.82) to change **GCO,** which will also **increase** BW from current **1.47.**

Compensated Bode' Plot

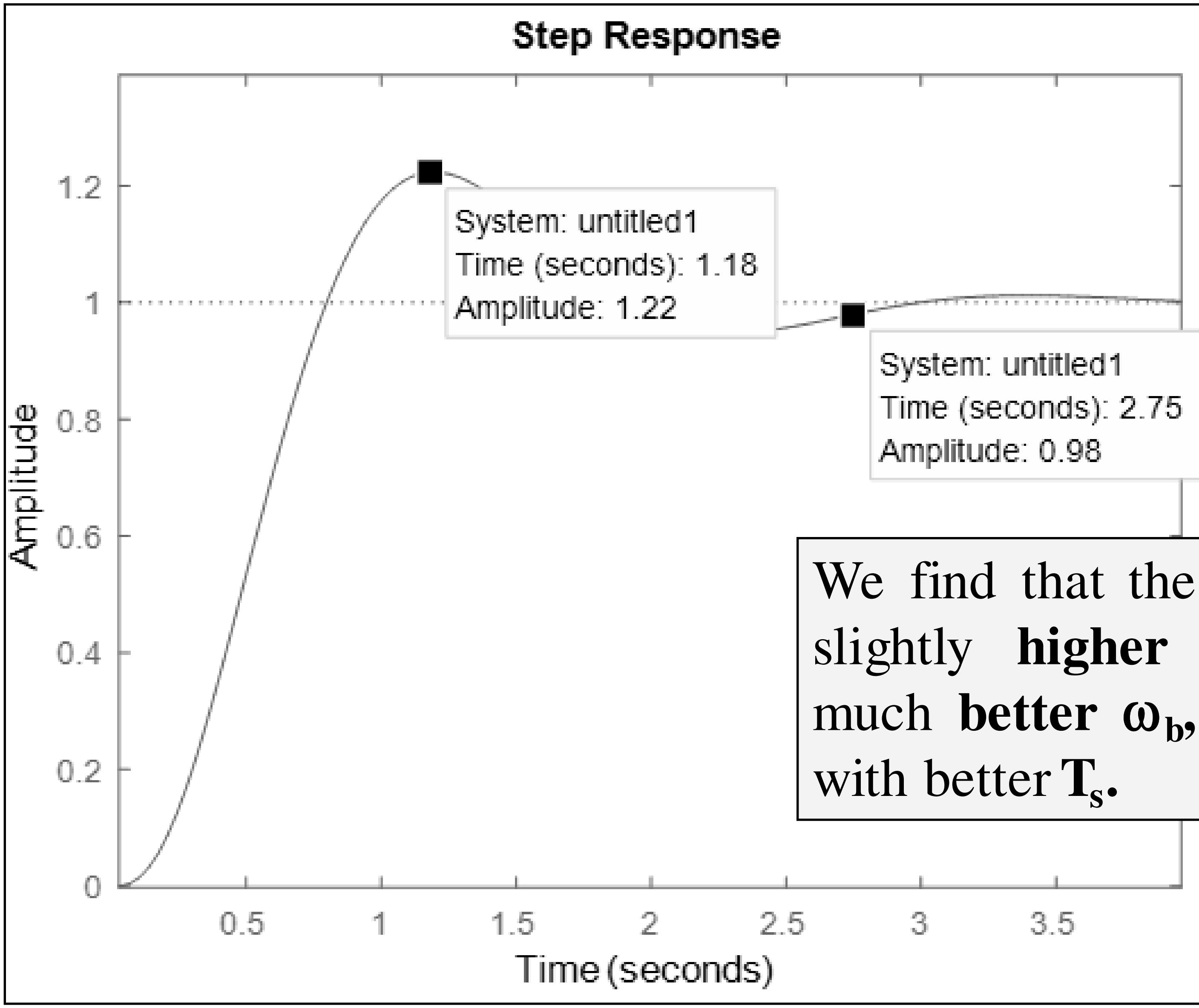


We find that **GM = 13.9** dB (4.95)**.**

$$KG(s) = \frac{112.8}{s(s+4)(s+10)}$$

$$K_V = 2.82; \quad \omega_b = 4.14$$

Compensated Step Response



We find that the **design** gives a slightly **higher M_p** but gives much **better ω_b, and K_V,** along with better **T_s.**

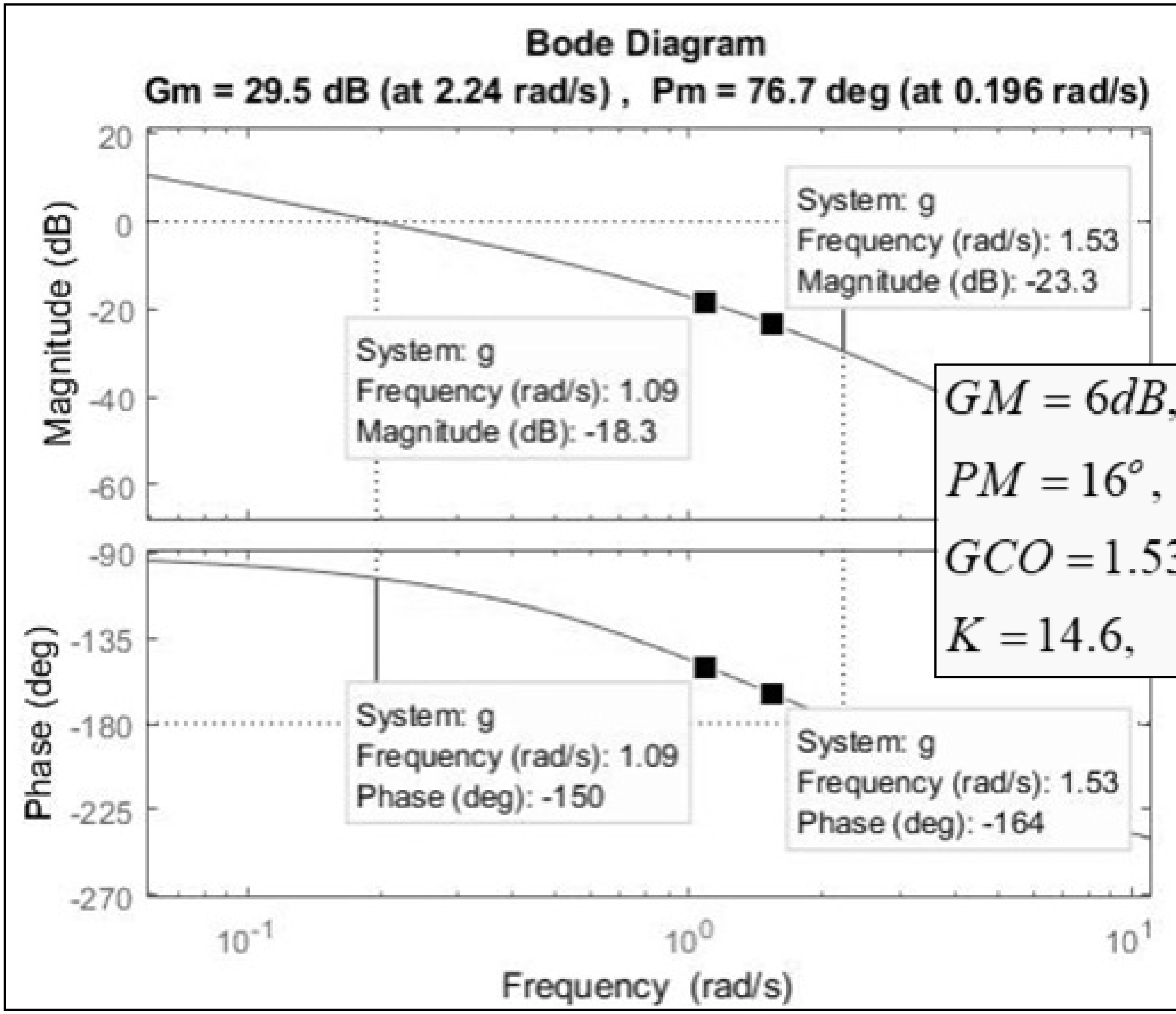
P – Control Design Example

Consider the following **plant.**

$$\boxed{G(s) = \frac{1}{s(s+1)(s+5)}}$$

Determine **maximum** increase possible in ramp error constant to maintain a **GM >** 6 dB & **PM >** 30^o, and give the **dominant pole** and closed loop **damping ratio.**

Uncompensated Bode' Plot



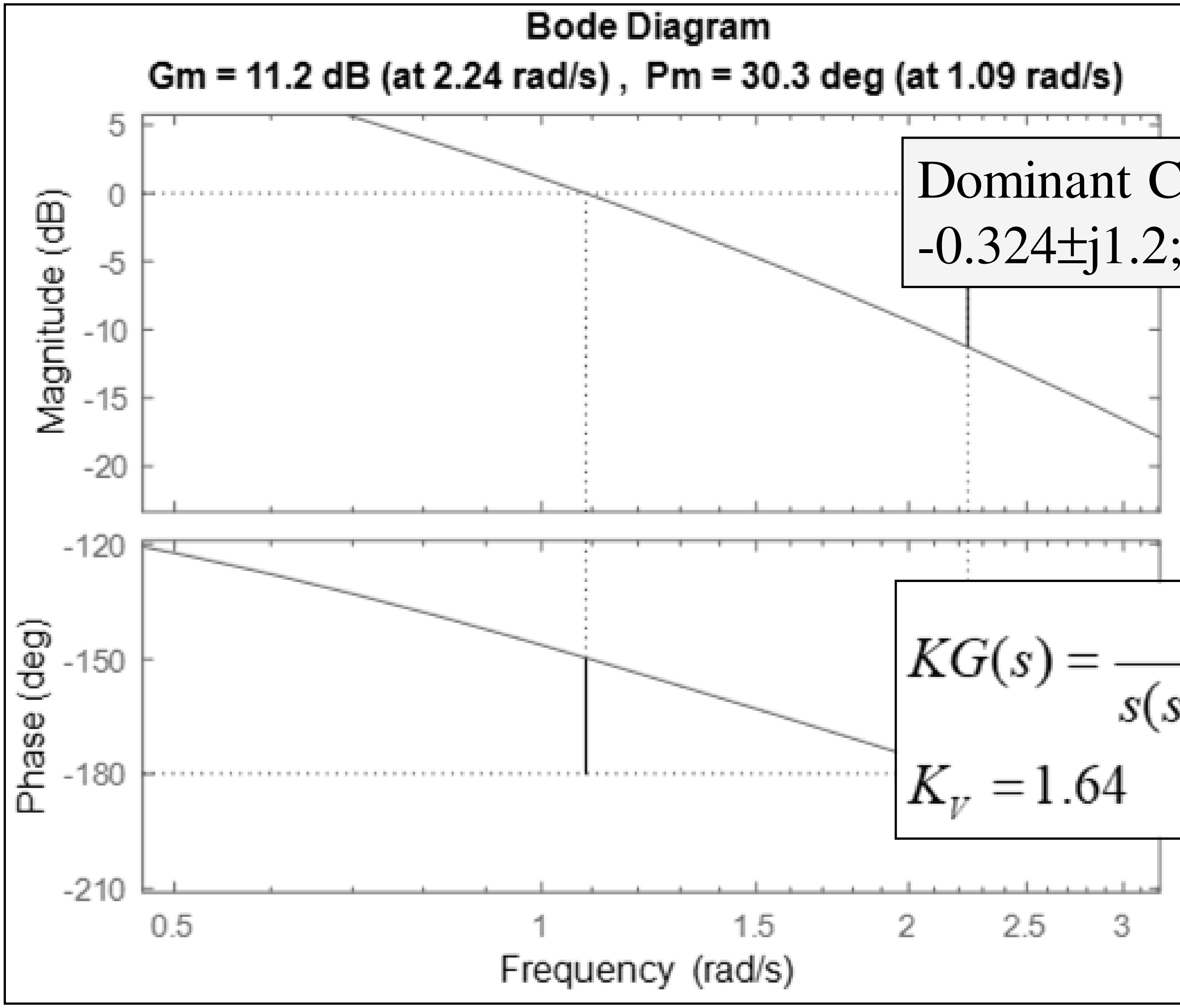
$$GM = 6dB, \quad 11dB$$

$$PM = 16^o, \quad 30^o$$

$$GCO = 1.53, \quad 1.09$$

$$K = 14.6, \quad 8.4$$

Compensated Bode' Plot

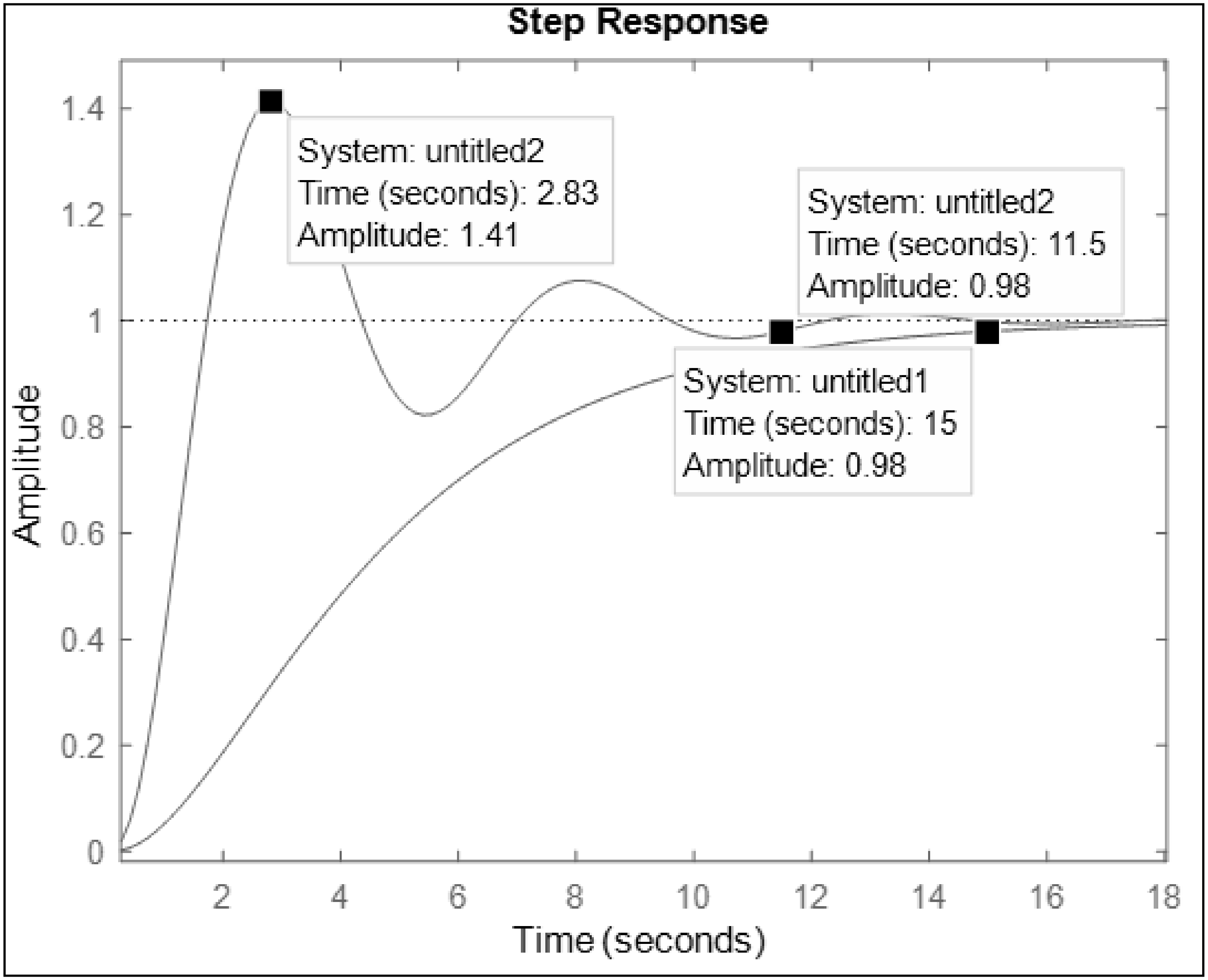


Dominant Closed Loop Pole
$-0.324 \pm j1.2$; $\zeta = 0.26$

$$KG(s) = \frac{8.2}{s(s+1)(s+5)}$$

$$K_V = 1.64$$

Compensated Step Response



'P' Design with Nyquist Plot

Nyquist Plot Based P-Design Steps

Draw Nyquist plot of G(s).

Generate desired M-circle on the Nyquist plot corresponding to the resonant peak specification.

Determine the gain required through trial & error for the Nyquist plot to become tangent to the M-circle.

The above gain is the gain of **'P' controller.**

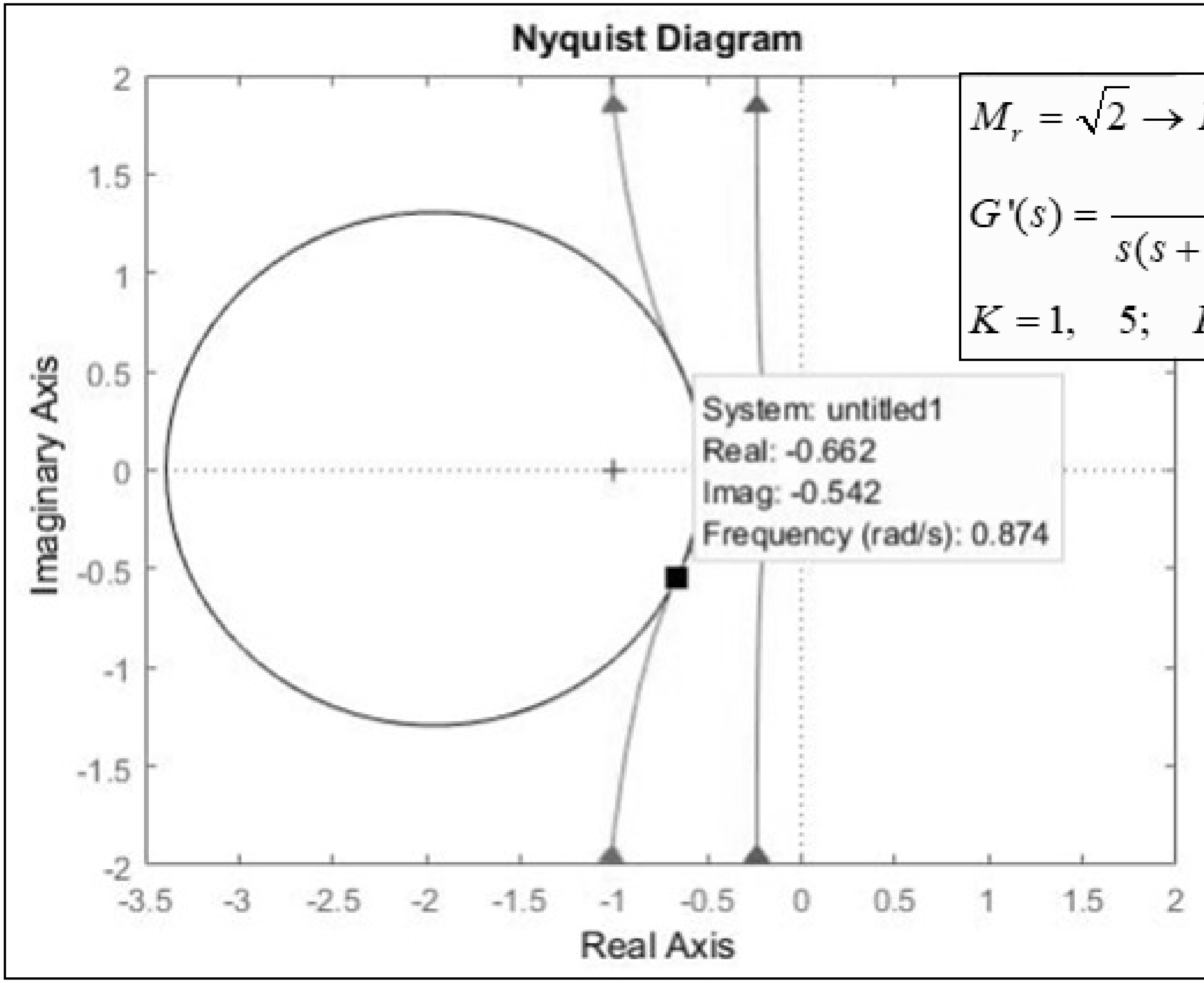
P – Control Design Example

Consider the following **plant.**

$$G(s) = \frac{1}{s(s+1)(s+5)}$$

Determine **maximum** increase possible in **ramp** error constant while **limiting** the resonant peak to **1.414.**

M-circle Based Design

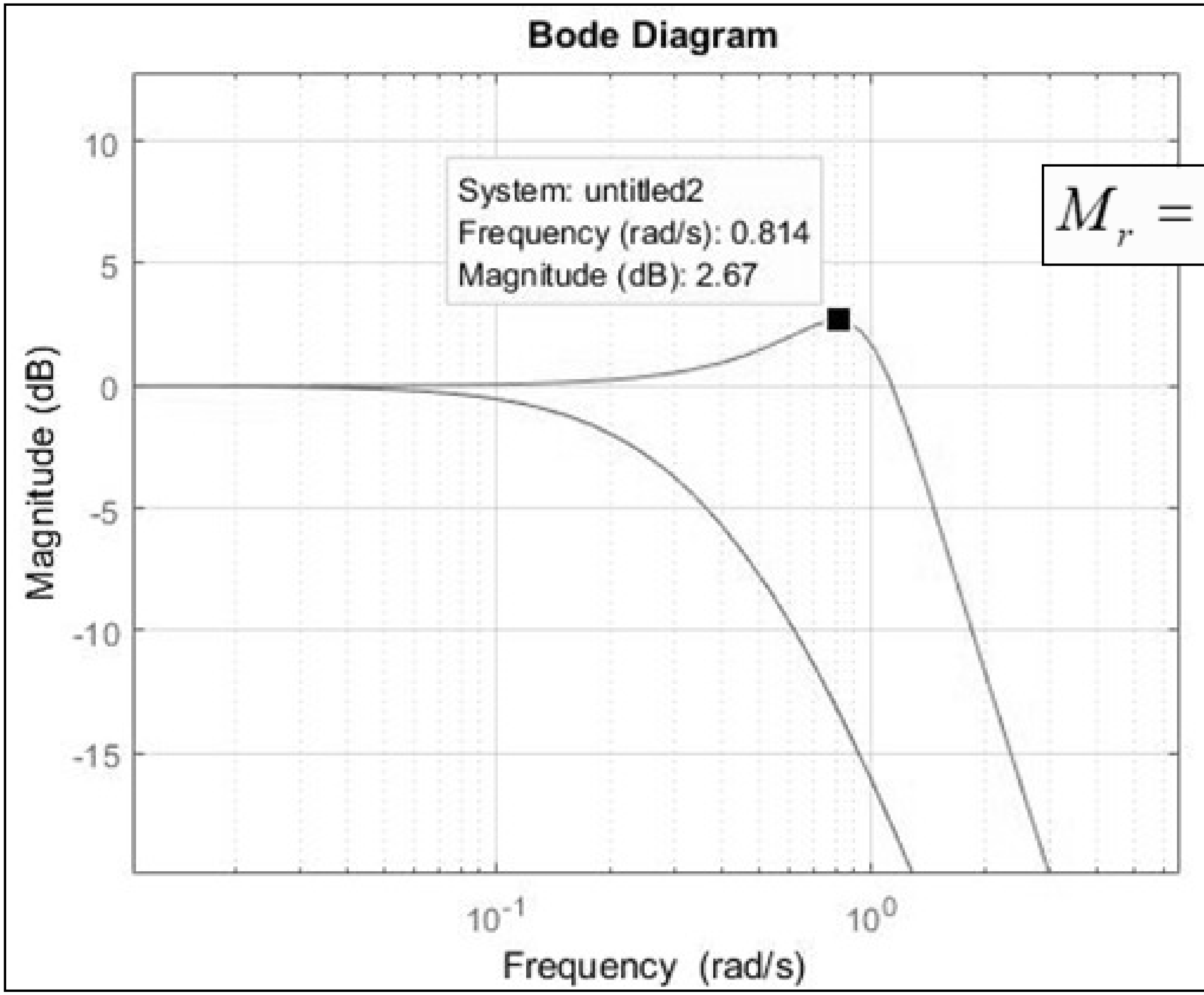


$$M_r = \sqrt{2} \rightarrow R = 2; \quad X_c = \sqrt{2}$$

$$G'(s) = \frac{K}{s(s+1)(s+5)}$$

$$K = 1, \quad 5; \quad K_V' = 1$$

M-circle Based Design Verification



$$M_r = 1.36$$

Summary

P-control is **simplest** & most **restrictive** controller.

Design with **root locus** is straightforward and **intuitive.**

In **frequency domain**, design is primarily driven by **phase margin** and is carried out using **Bode** plot.

P-control design with **M-circle** requires **iteration** and is driven mainly by **resonant peak** specification.