Laplace Based Natural Response

Solve following **homogeneous** differential equation under **given initial** conditions, using **Laplace** transform method.

$$\dddot{y} + 4\ddot{y} + 6\dot{y} + 4y = 0; \quad y(0) = 1, \dot{y}(0) = 0, \ddot{y}(0) = -1$$

$$L\left(\dddot{y} + 4\ddot{y} + 6\dot{y} + 4y\right) = s^3Y(s) + 4s^2Y(s) + 6sY(s) + 4Y(s)$$

$$-6y(0) - 4sy(0) - 4\dot{y}(0) - s^2y(0) - s\dot{y}(0) - \ddot{y}(0) = 0$$

$$Y(s) = \frac{\left(s^2 + 4s + 6\right)y(0) + (s+4)\dot{y}(0) + \ddot{y}(0)}{\left(s^3 + 4s^2 + 6s + 4\right)} = \frac{s^2 + 4s + 5}{s^3 + 4s^2 + 6s + 4} = \frac{s^2 + 4s + 5}{(s+2)\left(s^2 + 2s + 2\right)}$$

$$Y(s) = \frac{A_1}{s+2} + \frac{A_2 s + A_3}{s^2 + 2s + 2} \rightarrow \left(A_1 + A_2\right)s^2 + \left(2A_1 + 2A_2 + A_3\right)s + \left(2A_1 + 2A_3\right) = s^2 + 4s + 5$$

$$A_1 + A_2 = 1, \quad 2A_1 + 2A_2 + A_3 = 4, \quad 2A_1 + 2A_3 = 5 \rightarrow A_3 = 2, \quad A_1 = \frac{1}{2}, \quad A_2 = \frac{1}{2}$$

$$y(t) = \frac{1}{2}e^{-2t} + 2e^{-t}\sin t - \frac{1}{\sqrt{2}}e^{-t}\sin\left(t - \pi/4\right)$$

TF Based Forced Response

Obtain **time response** for the following system using **TF approach**, assuming **zero initial** conditions.

$$\ddot{y} + 4\dot{y} + 8y = \dot{u} + 8u; \quad u(t) \rightarrow \text{ Unit step function}$$

$$G(s) = \frac{s+8}{s^2+4s+8}; \quad Y(s) = G(s)U(s) = \frac{s+8}{s\left(s^2+4s+8\right)}$$

$$y(t) = L^{-1}[Y(s)]; \quad Y(s) = \frac{A_1}{s} + \frac{A_2 s + A_3}{s^2+4s+8} = \frac{1}{s} - \frac{s+3}{s^2+4s+8}$$

$$y(t) = 1 - \frac{3}{8}e^{-2t}\sin 2t + \sqrt{2}e^{-2t}\sin(2t - \pi/4)$$