Tutorial 3 Solution

1) Speed of propagation $= \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} \simeq 3\times10^{8}$ m/s

$\omega = 2\pi f = 10^5$

$\therefore$ Frequency $= 15.91$ kHz

Amplitude $= 50$ V/m

Direction of propagation is $\hat{z}$

2) From Maxwell's Equation

$$\nabla \times D = -\mu\varepsilon \frac{\partial H}{\partial t}$$

Taking curl on both sides

$$\nabla \times \nabla \times D = -\mu\varepsilon \frac{\partial}{\partial t}(\nabla \times H)$$

$$-\nabla^2 D + \nabla(\nabla \cdot D) = -\mu\varepsilon \frac{\partial}{\partial t}\left(J + \frac{\partial D}{\partial t}\right)$$

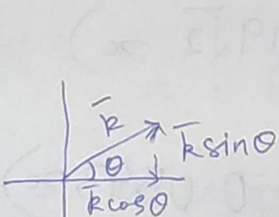
$$-\nabla^2 D + \nabla \rho_v = -\mu\varepsilon \frac{\partial J}{\partial t} - \mu\varepsilon \frac{\partial^2 D}{\partial t^2}$$

For source free wave equation

$\rho_v = 0 \qquad J = 0$

$$\therefore \nabla^2 D = \mu\varepsilon \frac{\partial^2 D}{\partial t^2}$$

3)



$$|k| = \frac{\omega\sqrt{\mu_r\varepsilon_r}}{c} = \frac{\omega\sqrt{9}}{c} = \frac{2\pi f \times 3}{c}$$

$|k| = 3.14$

$$\therefore \vec{k} = 3.14\left(\cos 30^\circ\, \hat{a}_x + \sin 30^\circ\, \hat{a}_y\right)$$

$$= 2.72\,\hat{a}_x + 1.57\,\hat{a}_y$$

$$\vec{r} = x\hat{a}_x + y\hat{a}_y$$

$$E = E_0 e^{-i(\vec{k}\cdot\vec{r})} = 10\, e^{i(2.7x + 1.6y)} \text{ mV/m}$$

4) Assuming electric field in z-direction and wave along x-direction

a) $E(x) = \hat{z} E_0 e^{-jk_0 x}$

Magnetic field comes to be in $-y$ direction

$H(x) = -\hat{y} \frac{E_0}{\eta_0} e^{-jk_0 x}$

$k_0 = \frac{\omega}{c} = \frac{2\pi f}{c} = 0.0419\ m^{-1}$

$\eta_0 = 377\ \Omega$

$H(x) = -\hat{y} \frac{10}{377} e^{-j0.0419x} = -\hat{y}\, 0.0265 e^{-j0.0419x}$

b) Assuming zero initial phase

$E(x,t) = \hat{z}\, Re[E_0 e^{-jk_0 x} e^{j\omega t}] = \hat{z}\, 10 \cos(4\pi \times 10^6 t - 0.0419x)$

Similarly $H(x,t) = -\hat{y}\, 0.0265 \cos(4\pi \times 10^6 t - 0.0419x)$

c) $V_p = \frac{\omega}{k}$ $\quad k = \omega\sqrt{\mu\varepsilon}$

$\therefore V_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{2}} = 2.12 \times 10^8\ ms^{-1}$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2}} = 266.58\ \Omega$

$H(x,t) = -\hat{y} \frac{10}{266.58} \cos(4\pi \times 10^6 t - 0.0419\sqrt{2}\, x)$

$H(x,t) = -\hat{y}\, 0.0375 \cos(4\pi \times 10^6 t - 0.0593x)$

5) Given $\omega = 10^6\pi$ $\mu = \mu_0$ $\varepsilon = \varepsilon_0$ $\sigma = 1.5\times10^{-5}$ S/m

$$\eta = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\varepsilon_0}} = 342.69 + j86.5\ \Omega$$

$$\alpha = \omega\sqrt{\frac{\mu_0\varepsilon_0}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon_0}\right)^2} - 1\right]} = 2.73\times10^{-3}\ \frac{Np}{m}$$

$$\beta = \omega\sqrt{\frac{\mu_0\varepsilon_0}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon_0}\right)^2} + 1\right]} = 0.0108\ \frac{rad}{m}$$

At origin $H = \hat{y}\,\frac{8}{\eta}\cos(10^6\pi t)$

At distance z $H(z) = \hat{y}\,\frac{8}{\eta}\,e^{-\alpha z}\cos(10^6\pi t - \beta z)$

$$H(z=1000\,m) = \hat{y}(0.0014 - j0.0004)e^{-2.73}\cos(10^6\pi t - 10.8)\ A/m$$

b) Check low-loss condition

$$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-6}}{2\pi\times30\times10^9\times8.854\times1.5\times10^{-12}}$$

$$= 3.995\times10^{-7} \ll 1$$

$\therefore$ Low-loss approximation applies

a) Phase velocity in free space $\Rightarrow c = 3\times10^8\ ms^{-1}$

Atmosphere $\frac{c}{\sqrt{\varepsilon_r}} = 2.45\times10^8\ ms^{-1}$

b) $\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}} = \frac{\sigma\eta_0}{2\sqrt{\varepsilon_r}} = 1.539\times10^{-4}\ \frac{Np}{m}$

Atmosphere

$\beta \simeq \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}\sqrt{\varepsilon_r} = 769.53\ \frac{rad}{m}$

In free space $\alpha = 0$ $\beta = \frac{\omega}{c} = 628.32\ \frac{rad}{m}$

$\therefore$ Propagation constant in free space $\gamma = j\beta = j628.32$

Atmosphere $\gamma = \alpha + j\beta$

$= 1.539\times10^{-4} + j769$

Intrinsic Impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

Free space = $377\ \Omega$

Atmosphere = $\frac{377}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{1.5}}$

$= 307.82\ \Omega$

b) $E = E_0 e^{-\alpha d}$, $d = 2 \times 15$ km $= 30000$ m

$10 \times 10^{-3} = E_0 e^{-1.539 \times 10^{-4} \times 3 \times 10^{4}}$

$\therefore E_0 = 1.012$ V/m

7) For conductor ~~copper~~ aluminiom

$\alpha = \sqrt{\pi f \mu_0 \sigma} = \sqrt{\pi \times 10^6 \times 4\pi \times 10^{-7} \times 3.7 \times 10^7} = 1.21 \times 10^4\ \frac{Np}{m}$

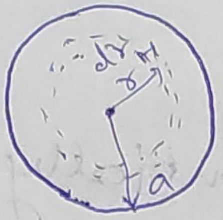
$10^{-6} E_0 = E_0 e^{-\alpha d}$

$d = 1.142$ mm of aluminiom

Iron

$\alpha = \sqrt{\pi f \mu_r \mu_0 \sigma} = \sqrt{\pi \times 10^6 \times 100 \times 4\pi \times 10^{-7} \times 10^7} = 6.28 \times 10^4\ \frac{Np}{m}$

$d = 0.22$ mm of iron

8) 

$\alpha = \sqrt{\pi f \mu \sigma}$

$= \sqrt{\pi \times 100 \times 20 \times 4\pi \times 10^{-7} \times 10^{7}}$

$= 281$ Np/m

Current density decays exponentially from surface

$\therefore \; J(r) = J_{max} \, e^{-\alpha(a-r)}$

$J_{max} = 100 \, A/mm^2 = 10^8 \, A/m^2$ $a = 0.1m$

$\therefore \; J(r) = 10^8 \, e^{-281(0.1-r)}$

At center $J(r=0) = 10^8 e^{-28.1} = 6.256 \times 10^{-5} \, A/m^2$

$dI = J(r) \, 2\pi r \, dr$

$I_{total} = \int_0^a J_{max} e^{-\alpha(a-r)} \, 2\pi r \, dr$

$= 2\pi J_{max} e^{-\alpha a} \int r e^{\alpha r} dr$

$= 2\pi J_{max} e^{-\alpha a} \left[\frac{e^{\alpha r}}{\alpha^2} (\alpha r - 1) \right]_0^a$

$I_{total} = 2\pi J_{max} e^{-\alpha a} \left[\frac{e^{\alpha a}}{\alpha^2} (\alpha a - 1) + \frac{1}{\alpha^2} \right]$

$I_{total} = 215.64$ kA

9) a) $k_x = 2m^{-1}$ $k_y = -4m^{-1}$ $k_z = 4m^{-1}$

$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = 6m^{-1}$ $\lambda = \frac{2\pi}{k} = 1.05m$

$f = \frac{c}{\lambda} = 2.857 \times 10^8$ Hz

b) $\hat{k} = \frac{1}{3}(\hat{x} - 2\hat{y} + 2\hat{z})$

c) $\because \hat{k} \cdot \hat{n} = 0$ wave is transverse

10) For seawater $\frac{\sigma}{\omega\varepsilon} = \frac{9.98 \times 10^8}{f}$ For $f = 10MHz$ $\frac{\sigma}{\omega\varepsilon} = 99.8 >> 1$

For $f = 100$ Hz $\frac{\sigma}{\omega\varepsilon} = 9.98 \times 10^6 >> 1$

$\therefore$ Seawater is highly lossy in both case

10MHz

$\alpha = \sqrt{\pi \mu f \sigma}$

$\alpha = 12.566$ Np/m

$10^{-12} E_0 = E_0 e^{-\alpha d}$

$d = 2.199$ m

100 Hz

$\alpha = 0.0397$ Np/m

$d = 695.995$ m

$\therefore$ Range is greater for lower frequency.