Homework 5 Solution Outlines

October 10, 2019

Problem 4.1

$$p(t) = sin(\pi t) I_{[0,1]}(t) \tag{1}$$

1. Simply taking Fourier Transform

$$\begin{aligned} P(f) &= \int_0^1 sin(\pi t) e^{-j2\pi f t} dt \\ &= \frac{2cos(\pi f) e^{-j\pi f}}{\pi(1-4f^2)} \end{aligned} \tag{2}$$

2. We have to find B such that with $\gamma = 0.95$, the following equation should be satisfied.

$$\int_{-B/2}^{B/2} S(f)df = \gamma \int_{-\infty}^{\infty} S(f)df \tag{3}$$

where $S(f) = |P(f)|^2 \frac{\sigma_b^2}{T}$ (from class notes). Note that, energy of the pulse $p(t)$ is 1/2. Using parseval's theorem and solving (3) we get

$$\int_0^{B/2} \frac{8cos^2(\pi f)}{\pi^2(1-4f^2)^2} df = \frac{0.95}{2} \tag{4}$$

A closed form doesn't exist for the left side term. However, using MATLAB integral function, it can be evaluated. B comes out to be 1.823 in normalized system. Since a pulse duration is 1 microsecond, the bandwidth in the original system is 1.823 MHz.

Problem 4.2

1. The time domain response is a trapezium, with the parallel sides of lengths 1 and $1-2a$. Fourier Transform:

$$\begin{aligned} P(f) &= \int_0^a \frac{t}{a} e^{-j2\pi f t} dt + \int_a^{1-a} e^{-j2\pi f t} dt + \int_{1-a}^1 \frac{1-t}{a} e^{-j2\pi f t} dt \\ &= -\frac{(1 - e^{-2j\pi f a} - e^{-j2\pi f(1-a)} + e^{-j2\pi f})}{4\pi^2 a f^2} \end{aligned} \tag{5}$$

2. Power Spectral Density

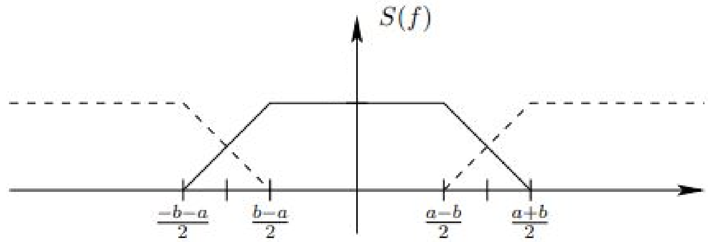
$$S_u(f) = \frac{\mid P(f) \mid^2}{T} \sigma_b^2 \tag{6}$$

In our case, $T = 1$ and $\sigma_b^2 = \frac{1+1+9+9}{4} = 5$

$$\begin{aligned} S_u(f) &= 5 \mid P(f) \mid^2 \\ &= 5 \mid -\frac{(1 - e^{-2j\pi f a} - e^{-j2\pi f(1-a)} + e^{-j2\pi f})}{4\pi^2 a f^2} \mid^2 \end{aligned} \tag{7}$$

Problem 4.3

a. The time domain multiplication of sinc functions will be convolution of rect function in frequency domain. The resulting frequency domain transform:



b.

$$\frac{1}{T} = \frac{R}{log_2(M)} = 600 symbols/s$$

Hence sampling frequency is 600 Hz.

$$\frac{a+b}{2} = 400Hz$$

For the pulse to be Nyquist, we need $s(mT) = 0$ for integer values of m. This is satisfied if either $a = \frac{1}{T}$ or $b = \frac{1}{T}$, i.e., if $a = 600$ or $b = 600$. However, since $a + b = 800$ and $a \geq b$, $b = 600$ is not possible.

Hence a = 600Hz and b = 200 Hz.

c.

$$\frac{log_2(64)}{T} = 60$$

So $T = 0.1$ microseconds.

$a + b = 20$ MHz.

$a = \frac{1}{T} = 10$ MHz. So $b = 10$ MHz.

d. Note that $s(t) = \frac{\sin(\pi at)}{\pi at}\frac{\sin(\pi bt)}{\pi bt}$. Consider the signal:

$$u(t) = \sum_n b[n]s(t - nT).$$

For the above pulse $s(t)$, the fact that at every fixed instant t, $u(t)$ is finite follows from the fact that the series $\sum_{j=1}^{\infty} \frac{1}{j^2}$ converges, i.e., $\sum_{j=1}^{\infty} \frac{1}{j^2} < \infty$.

Problem 4.4

(The following solution is for the modified version of Problem 4.4 in which the given expression for $P(f)$ is scaled by a factor T.)

(a) The Bit Rate of M-ary signalling is given by

$$\frac{log_2 M}{T} = BitRate$$

Now for 8-PSK signalling M=8 and BitRate given is 3 Mbps. Putting this in the equation we get,

$$\frac{1}{T} = SamplingFrequency = 1MHz$$

Now when we shift the frequency spectrum by 1 MHz and add it, we will observe some distortion in the frequency domain spectrum. That is, the following condition, which is necessary and sufficient for the pulse $p(t)$ to be Nyquist, is not satisfied:

$$\frac{1}{T}\sum_{k=-\infty}^{\infty} P\left(f + \frac{k}{T}\right) = 1, \ \forall f \tag{8}$$

So the answer is False.

(b) Similar to previous part, put the values in the equation. We get

$$\frac{1}{T} = SamplingFrequency = 1.5MHz \tag{9}$$

Now when we shift and add the Frequency spectrum, we will get a flat line (constant). In particular, the condition in (8) is satisfied. So the answer is True.

Problem 4.5

(a) Clearly, $p(0) = 1$ and $p(mT) = 0$ for $m \neq 0$. So the answer is True.

(b) The time-domain pulse corresponding to $|P(f)|^2$ is $p(t) * p(t)$. It is easy to check that $p(t) * p(t)$ does not vanish at $t = T$.

Thus, the answer is False.