Table A-1
 Laplace Transform Pairs

	f(t)	F(s)
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \qquad (n=1,2,3,\dots)$	$\frac{1}{s^n}$
5	$t^n \qquad (n=1,2,3,\ldots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!}t^{n-1}e^{-at} \qquad (n=1,2,3,\dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at}$ $(n = 1, 2, 3,)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
11	cos ωt	$\frac{s}{s^2+\omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
14	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}\left(be^{-bt}-ae^{-at}\right)$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$

 Table A-1 (continued)

$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	to restrict the second		
$e^{-\omega} \sin \omega t \qquad \frac{\omega}{(s+a)^{2} + \omega^{2}}$ $21 \qquad e^{-\omega} \cos \omega t \qquad \frac{s+a}{(s+a)^{2} + \omega^{2}}$ $22 \qquad \frac{\omega_{n}}{\sqrt{1-\zeta^{2}}} e^{-\zeta \omega_{n}t} \sin \omega_{n} \sqrt{1-\zeta^{2}}t (0 < \zeta < 1) \qquad \frac{\omega_{n}^{2}}{s^{2} + 2\zeta \omega_{n}s + \omega_{n}^{2}}$ $-\frac{1}{\sqrt{1-\zeta^{2}}} e^{-\zeta \omega_{n}t} \sin(\omega_{n} \sqrt{1-\zeta^{2}}t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^{2}}}{\zeta} \qquad \frac{s}{s^{2} + 2\zeta \omega_{n}s + \omega_{n}^{2}}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$ $1 - \frac{1}{\sqrt{1-\zeta^{2}}} e^{-\zeta \omega_{n}t} \sin(\omega_{n} \sqrt{1-\zeta^{2}}t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^{2}}}{\zeta} \qquad \frac{\omega_{n}^{2}}{s(s^{2} + 2\zeta \omega_{n}s + \omega_{n}^{2})}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$ $1 - \cos \omega t \qquad \frac{\omega_{n}^{2}}{s(s^{2} + 2\zeta \omega_{n}s + \omega_{n}^{2})}$ $25 \qquad 1 - \cos \omega t \qquad \frac{\omega^{2}}{s(s^{2} + 2\zeta \omega_{n}s + \omega_{n}^{2})}$ $26 \qquad \omega t - \sin \omega t \qquad \frac{\omega^{3}}{s^{2}(s^{2} + \omega^{2})}$ $27 \qquad \sin \omega t - \omega t \cos \omega t \qquad \frac{2\omega^{3}}{(s^{2} + \omega^{2})^{2}}$ $28 \qquad \frac{1}{2\omega} t \sin \omega t \qquad \frac{s}{(s^{2} + \omega^{2})^{2}}$ $29 \qquad t \cos \omega t \qquad \frac{s^{2} - \omega^{2}}{(s^{2} + \omega^{2})^{2}}$ $\frac{1}{\omega^{2}_{2} - \omega_{1}^{2}} (\cos \omega_{1}t - \cos \omega_{2}t) (\omega_{1}^{2} \neq \omega_{2}^{2}) \qquad \frac{s}{(s^{2} + \omega_{1}^{2})(s^{2} + \omega_{2}^{2})}$	18	$\frac{1}{a^2}(1-e^{-at}-ate^{-at})$	$\frac{1}{s(s+a)^2}$
21 $e^{-\omega t} \cos \omega t$ $\frac{s+a}{(s+a)^2+\omega^2}$ 22 $\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}t$ $(0 < \zeta < 1)$ $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ 23 $e^{-t\omega_n t} \sin (\omega_n \sqrt{1-\zeta^2}t-\phi)$ $\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$ 24 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t-\phi)$ $\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$ 24 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$ $\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$ 25 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ 26 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ 27 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$ 28 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{\omega_n^2}{s(s^2+\omega^2)^2}$ 29 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{s}{(s^2+\omega^2)^2}$ 29 $e^{-(\omega_n t)} \sin (\omega_n \sqrt{1-\zeta^2}t+\phi)$ $\frac{s}{(s^2+\omega^2)^2}$ $\frac{s}{(s^2+\omega^2)^2}$	19	$\frac{1}{a^2}(at-1+e^{-at})$	$\frac{1}{s^2(s+a)}$
$ \frac{\omega_{n}}{\sqrt{1-\zeta^{2}}}e^{-\zeta\omega_{n}t}\sin\omega_{n}\sqrt{1-\zeta^{2}}t (0<\zeta<1) \qquad \frac{\omega_{n}^{2}}{s^{2}+2\zeta\omega_{n}s+\omega_{n}^{2}} $ $ -\frac{1}{\sqrt{1-\zeta^{2}}}e^{-\zeta\omega_{n}t}\sin(\omega_{n}\sqrt{1-\zeta^{2}}t-\phi) $ $ \phi = \tan^{-1}\frac{\sqrt{1-\zeta^{2}}}{\zeta} $ $ (0<\zeta<1, 0<\phi<\pi/2) $ $ 1-\frac{1}{\sqrt{1-\zeta^{2}}}e^{-\zeta\omega_{n}t}\sin(\omega_{n}\sqrt{1-\zeta^{2}}t+\phi) $ $ \phi = \tan^{-1}\frac{\sqrt{1-\zeta^{2}}}{\zeta} $ $ (0<\zeta<1, 0<\phi<\pi/2) $ $ 1-\frac{1}{\sqrt{1-\zeta^{2}}}e^{-\zeta\omega_{n}t}\sin(\omega_{n}\sqrt{1-\zeta^{2}}t+\phi) $ $ \phi = \tan^{-1}\frac{\sqrt{1-\zeta^{2}}}{\zeta} $ $ (0<\zeta<1, 0<\phi<\pi/2) $ $ 1-\cos\omega t $ $ \frac{\omega_{n}^{2}}{s(s^{2}+2\zeta\omega_{n}s+\omega_{n}^{2})} $ $ 25 \qquad 1-\cos\omega t $ $ \frac{\omega^{2}}{s(s^{2}+2\zeta\omega_{n}s+\omega_{n}^{2})} $ $ 26 \qquad \omega t-\sin\omega t $ $ \frac{\omega^{3}}{s^{2}(s^{2}+\omega^{2})} $ $ 27 \qquad \sin\omega t-\omega t\cos\omega t $ $ \frac{\omega}{s^{2}(s^{2}+\omega^{2})^{2}} $ $ 28 \qquad \frac{1}{2\omega}t\sin\omega t $ $ \frac{s}{(s^{2}+\omega^{2})^{2}} $ $ 29 \qquad t\cos\omega t $ $ \frac{s^{2}-\omega^{2}}{(s^{2}+\omega^{2})^{2}} $ $ 30 \qquad \frac{1}{\omega_{2}^{2}-\omega_{1}^{2}}(\cos\omega_{1}t-\cos\omega_{2}t) (\omega_{1}^{2}\neq\omega_{2}^{2}) $ $ \frac{s}{(s^{2}+\omega_{1}^{2})(s^{2}+\omega_{2}^{2})} $	20	e ^{at} sin ωt	$\frac{\omega}{(s+a)^2+\omega^2}$
23 $ \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t - \phi) $ $ \phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} $ $ (0 < \zeta < 1, 0 < \phi < \pi/2) $ $ 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi) $ $ \phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} $ $ (0 < \zeta < 1, 0 < \phi < \pi/2) $ $ 25 \qquad 1 - \cos \omega t \qquad \frac{\omega_n^2}{s(s^2 + 2\zeta \omega_n s + \omega_n^2)} $ $ 26 \qquad \omega t - \sin \omega t \qquad \frac{\omega^3}{s^2(s^2 + \omega^2)} $ $ 27 \qquad \sin \omega t - \omega t \cos \omega t \qquad \frac{2\omega^3}{(s^2 + \omega^2)^2} $ $ 28 \qquad \frac{1}{2\omega} t \sin \omega t \qquad \frac{s}{(s^2 + \omega^2)^2} $ $ 29 \qquad t \cos \omega t \qquad \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2} $ $ 30 \qquad \frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) (\omega_1^2 \neq \omega_2^2) \qquad \frac{s}{(s^2 + \omega_1^2)^2 (s^2 + \omega_2^2)} $	21	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
23 $\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^{2}}}{\zeta}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$ $1 - \frac{1}{\sqrt{1 - \zeta^{2}}} e^{-\zeta \omega_{n} t} \sin(\omega_{n} \sqrt{1 - \zeta^{2}} t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^{2}}}{\zeta}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$ $1 - \cos \omega t$ $\frac{\omega^{2}}{s(s^{2} + 2\zeta \omega_{n} s + \omega_{n}^{2})}$ $25 \qquad 1 - \cos \omega t$ $\frac{\omega^{2}}{s(s^{2} + 2\zeta \omega_{n} s + \omega_{n}^{2})}$ $26 \qquad \omega t - \sin \omega t$ $\frac{\omega^{3}}{s^{2}(s^{2} + \omega^{2})}$ $27 \qquad \sin \omega t - \omega t \cos \omega t$ $\frac{2\omega^{3}}{(s^{2} + \omega^{2})^{2}}$ $28 \qquad \frac{1}{2\omega} t \sin \omega t$ $\frac{s}{(s^{2} + \omega^{2})^{2}}$ $29 \qquad t \cos \omega t$ $\frac{s^{2} - \omega^{2}}{(s^{2} + \omega^{2})^{2}}$ $30 \qquad \frac{1}{\omega^{2}_{2} - \omega_{1}^{2}} (\cos \omega_{1} t - \cos \omega_{2} t) (\omega_{1}^{2} \neq \omega_{2}^{2})$ $\frac{s}{(s^{2} + \omega^{2})^{2}(s^{2} + \omega^{2})^{2}}$	22	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t (0<\zeta<1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
$(0 < \zeta < 1, 0 < \phi < \pi/2)$ $1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$ $(0 < \zeta < 1, 0 < \phi < \pi/2)$ $25 \qquad 1 - \cos \omega t \qquad \frac{\omega^2}{s(s^2 + 2\zeta \omega_n s + \omega_n^2)}$ $26 \qquad \omega t - \sin \omega t \qquad \frac{\omega^3}{s^2(s^2 + \omega^2)}$ $27 \qquad \sin \omega t - \omega t \cos \omega t \qquad \frac{2\omega^3}{(s^2 + \omega^2)^2}$ $28 \qquad \frac{1}{2\omega} t \sin \omega t \qquad \frac{s}{(s^2 + \omega^2)^2}$ $29 \qquad t \cos \omega t \qquad \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ $30 \qquad \frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) (\omega_1^2 \neq \omega_2^2) \qquad \frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$	23	, - ,	<u>s</u>
$ \frac{\omega_{n}^{2}}{s(s^{2} + 2\zeta\omega_{n}s + \omega_{n}^{2})} = \frac{\omega_{n}^{2}}{s(s^{2} + 2\zeta\omega_{n}s + \omega_{n}^{2})} $ $ \frac{\omega_{n}^{2}}{s(s^{2} + \omega_{n}^{2})} $ $ $		9 .	$s^2 + 2\zeta \omega_n s + \omega_n^2$
$(0 < \zeta < 1, 0 < \phi < \pi/2)$ $1 - \cos \omega t$ $\frac{\omega^2}{s(s^2 + \omega^2)}$ 26 $\omega t - \sin \omega t$ $\frac{\omega^3}{s^2(s^2 + \omega^2)}$ 27 $\sin \omega t - \omega t \cos \omega t$ $\frac{2\omega^3}{(s^2 + \omega^2)^2}$ 28 $\frac{1}{2\omega} t \sin \omega t$ $\frac{s}{(s^2 + \omega^2)^2}$ 29 $t \cos \omega t$ $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$ 30 $\frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) (\omega_1^2 \neq \omega_2^2)$ $\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$	24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi)$	2
25		$\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
$ \frac{1 - \cos \omega t}{s(s^2 + \omega^2)} $ $ 26 \qquad \omega t - \sin \omega t \qquad \frac{\omega^3}{s^2(s^2 + \omega^2)} $ $ 27 \qquad \sin \omega t - \omega t \cos \omega t \qquad \frac{2\omega^3}{(s^2 + \omega^2)^2} $ $ 28 \qquad \frac{1}{2\omega} t \sin \omega t \qquad \frac{s}{(s^2 + \omega^2)^2} $ $ 29 \qquad t \cos \omega t \qquad \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2} $ $ 30 \qquad \frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) (\omega_1^2 \neq \omega_2^2) \qquad \frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)} $		$(0 < \zeta < 1, \ \ 0 < \phi < \pi/2)$	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	25	$1-\cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
$\frac{1}{2\omega}t\sin\omega t - \omega t\cos\omega t$ $\frac{1}{2\omega}t\sin\omega t$ $\frac{1}{2\omega}t\sin\omega t$ $\frac{s}{(s^2 + \omega^2)^2}$ $\frac{1}{2\omega^2 - \omega^2}(\cos\omega_1 t - \cos\omega_2 t) (\omega_1^2 \neq \omega_2^2)$ $\frac{1}{2\omega^2 - \omega_1^2}(\cos\omega_1 t - \cos\omega_2 t) (\omega_1^2 \neq \omega_2^2)$ $\frac{1}{2\omega^2 - \omega_1^2}(\cos\omega_1 t - \cos\omega_2 t) (\omega_1^2 \neq \omega_2^2)$	26	$\omega t - \sin \omega t$	
$\frac{s^{2} - \omega^{2}}{(s^{2} + \omega^{2})^{2}}$ $\frac{1}{\omega_{2}^{2} - \omega_{1}^{2}}(\cos \omega_{1}t - \cos \omega_{2}t) (\omega_{1}^{2} \neq \omega_{2}^{2}) \qquad \frac{s}{(s^{2} + \omega_{1}^{2})(s^{2} + \omega_{2}^{2})}$	27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
$\frac{1}{\omega_{2}^{2} - \omega_{1}^{2}} (\cos \omega_{1} t - \cos \omega_{2} t) \qquad (\omega_{1}^{2} \neq \omega_{2}^{2}) \qquad \frac{s}{(s^{2} + \omega_{1}^{2})(s^{2} + \omega_{2}^{2})}$	28	$\frac{1}{2\omega}t\sin\omega t$	$\frac{s}{(s^2+\omega^2)^2}$
$\frac{1}{\boldsymbol{\omega}_2^2 - \boldsymbol{\omega}_1^2} (\cos \boldsymbol{\omega}_1 t - \cos \boldsymbol{\omega}_2 t) \qquad (\boldsymbol{\omega}_1^2 \neq \boldsymbol{\omega}_2^2) \qquad \frac{s}{(s^2 + \boldsymbol{\omega}_1^2)(s^2 + \boldsymbol{\omega}_2^2)}$	29	$t\cos\omega t$	$\frac{s^2-\omega^2}{\left(s^2+\omega^2\right)^2}$
$\frac{1}{2\omega}\left(\sin\omega t + \omega t\cos\omega t\right) \qquad \frac{s^2}{\left(s^2 + \omega^2\right)^2}$	30	$\frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) \qquad (\omega_1^2 \neq \omega_2^2)$	
	31	$\frac{1}{2\omega}\left(\sin\omega t + \omega t\cos\omega t\right)$	$\frac{s^2}{\left(s^2+\omega^2\right)^2}$

Table A-2	Properties of Laplace Transforms
1	$\mathscr{L}[Af(t)] = AF(s)$
2	$\mathscr{L}[f_1(t) \perp f_2(t)] = F_1(s) \perp F_2(s)$
3	$\mathscr{L}_{\pm}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathscr{L}_{\pm}\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0+) - \dot{f}(0\pm)$
5	$\mathcal{L}_{\pm}\left[\frac{d^n}{dt^n}f(t)\right] = s^n F(s) - \sum_{k=1}^n s^{n-k} f(0\pm)$
	where $f(t) = \frac{d^{k-1}}{dt^{k-1}} f(t)$
6	$\mathcal{L}_{=}\left[\int f(t) dt\right] = \frac{F(s)}{s} + \frac{1}{s} \left[\int f(t) dt\right]_{t=0\pm}$
7	$\mathscr{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^n \frac{1}{s^{n-k+1}} \left[\int \cdots \int f(t)(dt)^k\right]_{t=0}$
8	$\mathscr{L}\left[\int_0^t f(t)dt\right] = \frac{F(s)}{s}$
9	$\int_0^\infty f(t) dt = \lim_{s \to 0} F(s) \qquad \text{if } \int_0^\infty f(t) dt \text{ exists}$
10	$\mathscr{L}[e^{-\alpha t}f(t)] = F(s+a)$
- 11	$\mathscr{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s}F(s) \qquad \alpha \ge 0$
12	$\mathscr{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathscr{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
14	$\mathscr{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s) \qquad (n = 1, 2, 3, \dots)$
15	$\mathcal{L}\left[\frac{1}{t}f(t)\right] = \int_{s}^{\infty} F(s) ds \qquad \text{if } \lim_{t \to 0} \frac{1}{t} f(t) \text{ exists}$
16	$\mathscr{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$
17	$\mathscr{L}\left[\int_0^t f_1(t-\tau)f_2(\tau)d\tau\right] = F_1(s)F_2(s)$
18	$\mathscr{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s-p) dp$