09/13/2019.

1.1 CD ROM focusing Servo System uses compliant mechanism because

(A) (C) It gives frictionless and backlash - free motion

It is because high resolution and precision is required for CD ROM focusing. Hence the compliant mechanism should be frictionless and backlash free.

1.2 With reference to standard optical encoder (as a position sensor) with signals A and B as digital output signals the following statements are true

(Ans) (a) signals A and B are phase shifted by 90°

(c) Combined effect of A and B determines the direction of rotation of encoder

State	A	B
1	0	0
2	0	1
3	1	1
4	1	0

— Clockwise direction

State	A	B
1	0	0
2	1	0
3	1	1
4	0	1

→ Count Clock direct

Two signals which are 90° out of phase and their combination is required to sense the direction of rotation.

1.3 Match the following interfaces of microcontroller with corresponding applications.

1) Digital I/O
2) Analog to Digital Conversion (ADC)
3) QEI
4) PWM output
5) Digital to Analog Conversion (DAC)

P) Permanent Magnet DC servomotor
Q) Potentiometer
R) Dimmer LED
S) stepper motor
T) Optical encoder

(Ans) c 4-R, 5-R, 2-Q, 3-T, 4-P

It is because DAC cannot be interfaced with stepper motor, digital I/O cannot be interfaced with optical encoder, and QEI cannot be interfaced with dimmer LED.

1.4 Hard disc drive arm is designed with aluminium alloy as material and tapered construction because (2)

(Ans) (b) - It reduces the size of magnetic actuator to obtain high performance

(d) - Non magnetic aluminium eliminates undue magnetic interference effects when driver coil is carrying current (d)

This is because fast and accurate positioning is needed.

1.5 An arrangement with a spring loaded nut and its cage in high precision lead screw driven motion stage is provided in order to

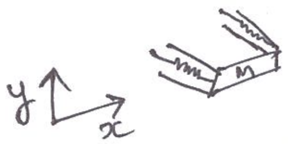
(Ans) (a) Reduce backlash & (d) To take care of any misalignment between axis of guide and lead screw.

This is because the spring Preload reduce the backlash. The clearance in the cage will allow for lateral motion of the nut.

1.6 Compliant mechanism in CD ROM drive from control perspective can be best modeled as.

(Ans) (a) A 2-DOF spring mass system with two springs and single mass

(b) A black box model found using system identification techniques



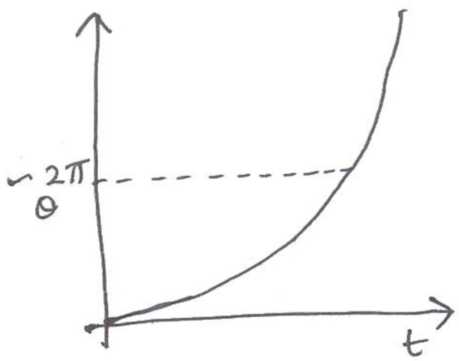
Two equations that could be used for 2-DOF spring & 1 mass system

$$m\ddot{x} + k_x x = F_x$$

$$m\ddot{y} + k_y y = F_y$$

1.7 The following figure shows a motor connected to a link in horizontal plane. A constant voltage V is applied to the motor from initial stationary condition with $\theta = 0$ at $t=0$. Which of the following trajectories represent correct evolution of motor angle θ w.r.t time

Ans (d)



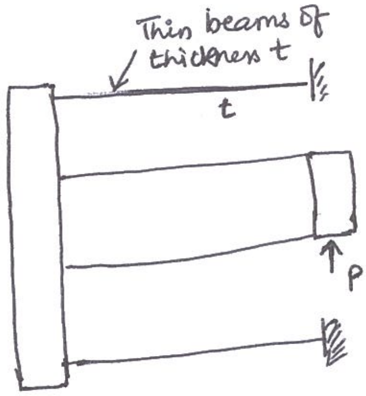
The governing equation of motor is given by

$$J\ddot{\theta} + B\dot{\theta} = T$$

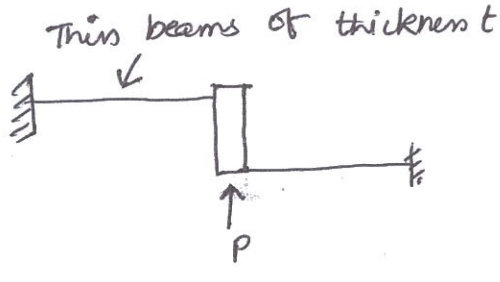
solving the above differential equation for t, with initial conditions for $t=0$. Integrating the differential equation will lead to a quadratic equation in t for θ.

1.8 A compliant mechanism (structure with flexible links) is used to get precision straight line motion over a small range. Which one or more of the following mechanism in the horizontal plane would give a perfect straight line motion upon application of the force shown

Ans (c)

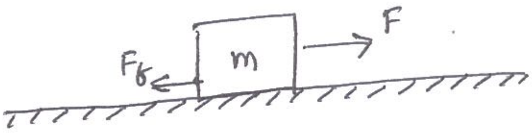


(d)

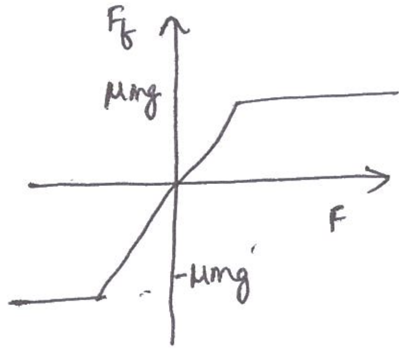


The configuration (c) is used for large range and configuration (d) is used for small range. There will be a slight rotation is involved.

(1.9) A block of mass m is kept on horizontal platform as shown in the figure below. Coefficient of friction between the block and platform is μ. A horizontal force F is applied on the block. Force of friction is denoted by F_f as shown below. (4)



(Ans) (a)

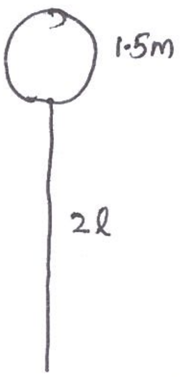


Before sliding happens. The friction force = Applied force. At the time of sliding we have

Applied force > μmg.

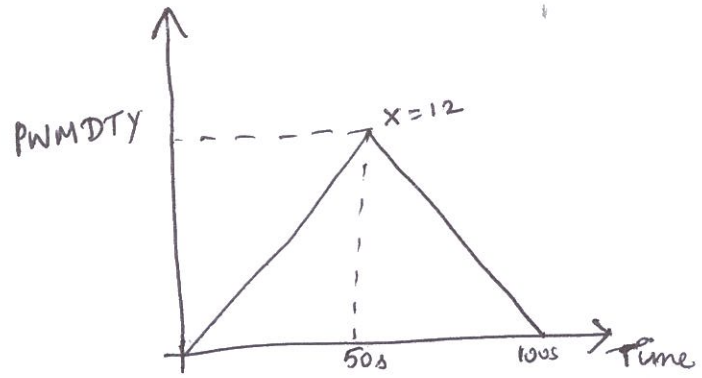
(1.10) Which one of the following sticks (mass less) with indicated point mass on the top is easy to balance on hand?

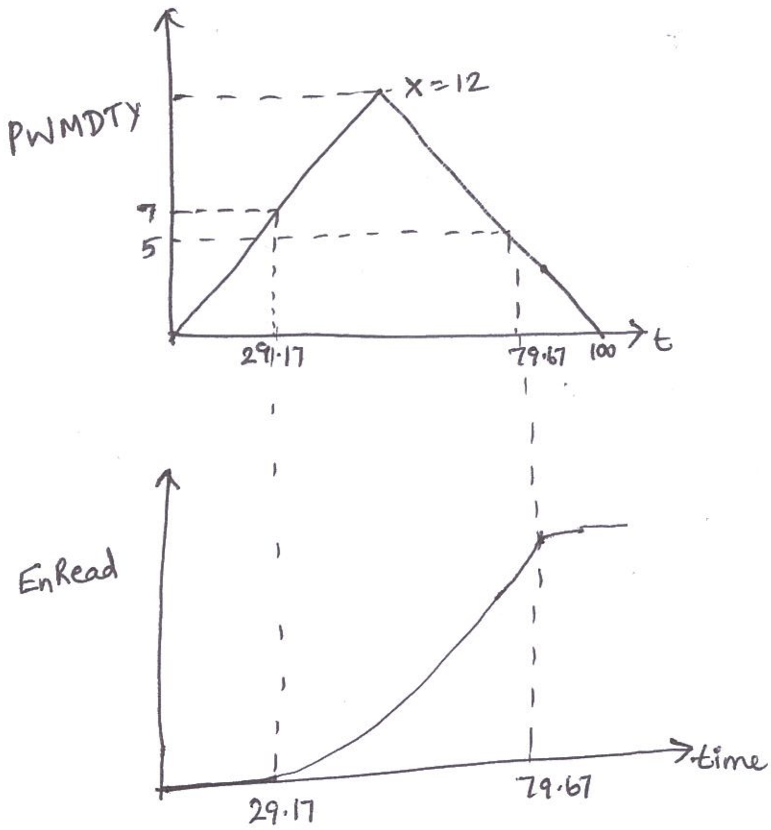
(Ans) (b)

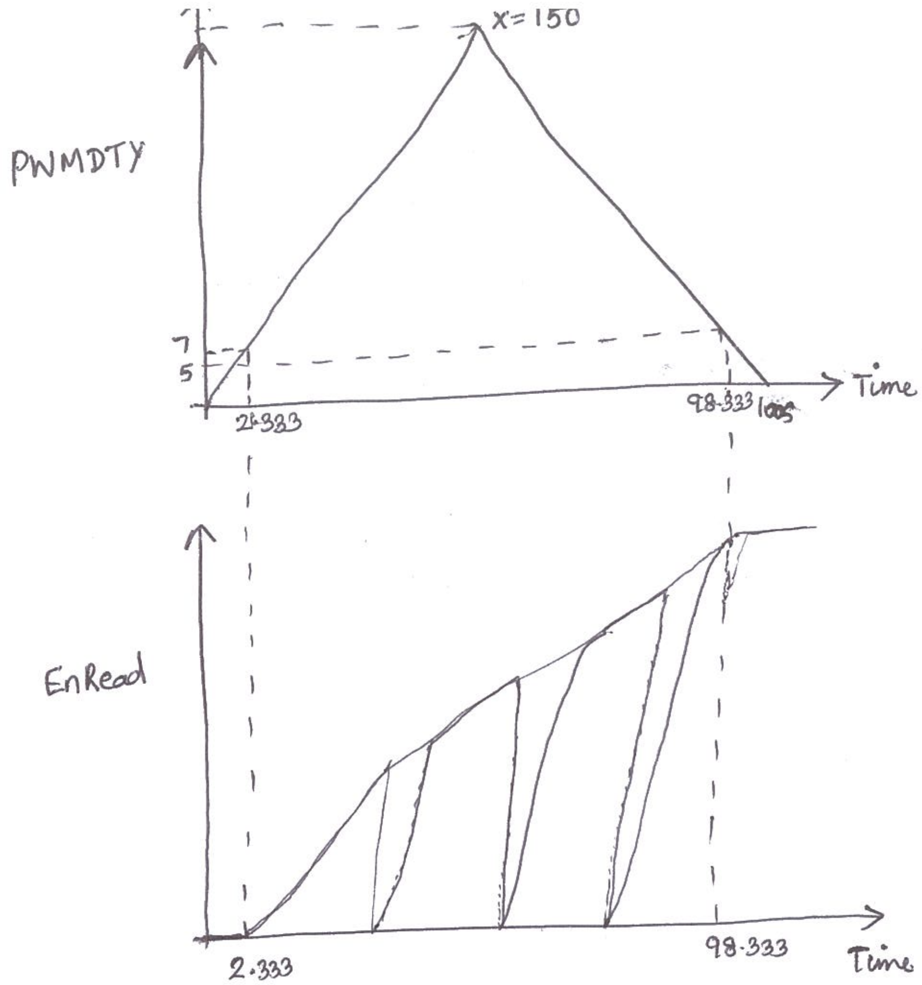


It takes more time to fall when the mass is more and the stick is longer. This long duration gives ample time to take control action and prevent the mass and stick toppling.

Q2 One of the ways to measure friction in TEP100 (5) based DC motor control setup is to use triangular waveform with very low frequency (0.01 Hz) as input PWMDTY and monitor when motor starts and stops. Sketch output of motor measured by encoder (EnRead counts) when such triangular waveform input is given to PWMDTY as shown below under two cases $X = 12$ and $X = 150$. Ass value of Coulomb friction to correspond to PWMDTY = 5 and static friction to correspond to PWMDTY = 7 and encoder output is 16 bit digital number. What is the correct choice of X for accurate estimation of friction?







The correct choice of static and coulomb friction occurs when X=12. This is due to the fact that in second case fluctuations ~~too~~ occur at the friction evaluation points.

Q3 For x positioning stage driven by a ball screw with pitch diameter 15mm and pitch of 2mm. Closed loop positioning accuracy of 0.005mm is required. The ball screw is directly driven by motor with encoder mounted on the motor. If we are interested in the range of 50 mm, determine minimum bit accuracy (number of bits for encoder digital output without

$$\text{Min bit}\underset{\text{resolution}}{\text{accuracy}} = \frac{\text{Range}}{\text{accuracy}}$$

$$= \frac{50}{0.005}$$

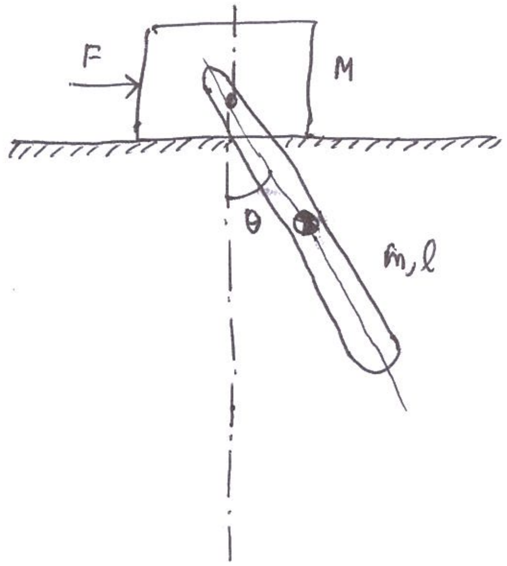
Min bit resolution $= 10000$

Expressing them in terms of bits we need 14 bit.

$2 | 10000$
$2 | 5000 - 0$
$2 | 2500 - 0$
$2 | 1250 - 0$
$2 | 625 - 0$
$2 | 312 - 1$
$2 | 156 - 0$
$2 | 78 - 0$
$2 | 39 - 0$
$2 | 19 - 1$
$2 | 9 - 1$
$2 | 4 - 1$
$2 | 2 - 0$
$1 - 0$

Minimum bit accuracy required = 14 bit

Q4 Given the rigid bar pendulum (mass m and length l, cg at $l/2$) mounted on cart with unactuated hinge (no motor at hinge) as shown in the figure below. Control input is the force on cart and Other parameters are specified in the figure.



a) How many degrees of freedom this system has and what are they?

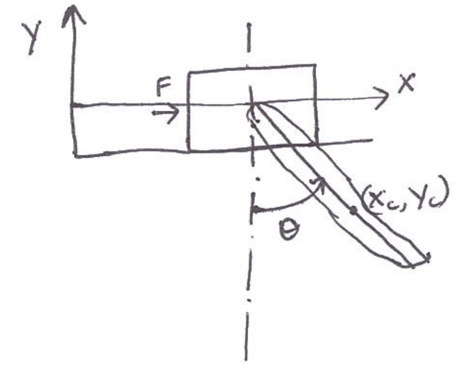
b) Using Kinematics find velocity of CG of bar and total KE of the systems in inertial reference frame?

c) Mentioning suitable reference find potential energy of the system.

d) Derive the dynamic equations governing the system using Lagrange formulation equation : $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau_q$. Is the system nonlinear?

e) Consider now acceleration of mass M as input (instead of Force F considered as input in case mentioned above) and rewrite the equation of motion governing pendulum angle DoF. How many degrees of freedom this system has?

Ans



ⓐ There are two degrees of freedom, they are displacement x and angle θ.

(b) KE of mass $M = \frac{1}{2} M \dot{x}^2$

Velocity of CG = ?

Position coordinates of CG

$$x_c = x + \frac{l}{2}\sin\theta$$

$$y_c = -\frac{l}{2}\cos\theta.$$

$$\dot{x}_c = \dot{x} + \frac{l}{2}\cos\theta\,\dot{\theta}$$

$$\dot{y}_c = \frac{l}{2}\sin\theta\,\dot{\theta}$$

$$V_{cg}^2 = \dot{x}_c^2 + \dot{y}_c^2$$

$$= \left(\dot{x} + \frac{l}{2}\dot{\theta}\cos\theta\right)^2 + \left(\frac{l}{2}\dot{\theta}\sin\theta\right)^2$$

$$= \dot{x}^2 + 2\dot{x}\frac{l}{2}\dot{\theta}\cos\theta + \left(\frac{l}{2}\dot{\theta}\right)^2$$

$$V_{cg}^2 = \dot{x}^2 + \dot{x}l\dot{\theta}\cos\theta + \left(\frac{l}{2}\dot{\theta}\right)^2$$

$$KE = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}l\dot{\theta}\cos\theta + \frac{1}{2}m\left(\frac{l}{2}\dot{\theta}\right)^2 + \frac{1}{2}I_{cg}\dot{\theta}^2$$

(c) PE of the systems $= mg\left(\frac{l}{2} - \frac{l}{2}\cos\theta\right)$ ref along CG in vertical position.

(d) $L = KE - PE$

$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}m\dot{x}l\dot{\theta}\cos\theta + \frac{1}{2}m\left(\frac{l}{2}\dot{\theta}\right)^2 + \frac{1}{2}I_{cg}\dot{\theta}^2 - mg\left(\frac{l}{2} - \frac{l}{2}\cos\theta\right)$$

$$\frac{\partial L}{\partial \dot{x}} = (M+m)\dot{x} + \frac{1}{2}ml\dot{\theta}\cos\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = (M+m)\ddot{x} + \frac{1}{2}ml\ddot{\theta}\cos\theta - \frac{1}{2}ml\dot{\theta}^2\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F$$

$$\boxed{(M+m)\ddot{x} + \frac{1}{2} m l \ddot{\theta} \cos\theta - \frac{1}{2} m l \dot{\theta}^2 \sin\theta = F}$$

For θ

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m \dot{x} l \cos\theta + m\left(\frac{l}{2}\right)^2 \dot{\theta} + I_{cg}\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{1}{2} m \ddot{x} l \cos\theta - \frac{1}{2} m \dot{x} l \sin\theta\, \dot{\theta} + m\left(\frac{l}{2}\right)^2 \ddot{\theta} + I_{cg}\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -\frac{1}{2} m \dot{x} l \dot{\theta} \sin\theta - mg\frac{l}{2}\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \frac{1}{2} m \ddot{x} l \cos\theta - \frac{1}{2} m \dot{x}(l\sin\theta)\dot{\theta} + m\left(\frac{l}{2}\right)^2 \ddot{\theta} + I_{cg}\ddot{\theta}$$

$$+ \frac{1}{2} m \dot{x}(l \sin\theta)\dot{\theta} + mg\frac{l}{2}\sin\theta = 0$$

$$\boxed{\left[m\left(\frac{l}{2}\right)^2 + I_{cg}\right]\ddot{\theta} + \frac{1}{2} m l \ddot{x}\cos\theta + \frac{mgl}{2}\sin\theta = 0}$$

The system is nonlinear.

(2) when the acceleration is used as input

$\ddot{x} = u$.

Now the system has only 1 (one) DOF

The modified equation of DOF θ

$$\boxed{\left[m\left(\frac{l}{2}\right)^2 + I_{cg}\right]\ddot{\theta} + \frac{mgl}{2}\sin\theta = \left(-\frac{1}{2} m l \cos\theta\right)u}$$