



# *General LTI Systems*

- *LTI System Definition*
- *Basic Response Definition*



# ***Analysis & Design Methodology***

In general, all control **problems** are **stated** in terms of achieving the **desired system performance**.

These requirements are **translated** into **desired system behaviour** (or response) under **operating conditions**.

Thus, analysis and design **techniques** make extensive use of **system responses** for solving control problems.



## ***Role of Response in Control Studies***

**Control** analysis & design involves arriving at **control element** parameters, for a given system and **objectives**, which are specified in terms of **desired output features**.

This necessitates **characterization** of the response and, in this context, **linear** description of the **dynamical** systems (LTI) is considered to be **adequate**.



# *LTI Systems & Their Response*



## ***LTI System Description***

**LTI** (or Linear Time-invariant) systems are those that are described by **linear differential equations** having constant coefficients. **General form** of such systems is as below.

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y = b_0 \frac{d^m u}{dt^m} + \cdots + b_m u$$

Here, ‘**y**’ is the output and ‘**u**’ is the input. Terms, ‘**a<sub>1</sub>**’ to ‘**a<sub>n</sub>**’ and ‘**b<sub>1</sub>**’ to ‘**b<sub>m</sub>**’ are constants.

Further, ‘**n**’, which is the **highest degree** of derivative of ‘**y**’, is called the **order** of the system.



# *Linearity and Time Invariance*

- Linear

- Satisfies superposition and homogeneity
- Coefficients independent of output, input and their derivatives

Addition, Scaling

- Time invariant

- Delayed input produces same response with same delay
- Coefficients independent of  $t$



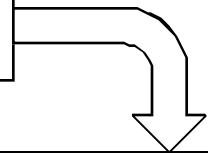
## *LTI System Response – Natural*

- Response when input = 0
- Determined solely by initial conditions

$$y(0), \frac{dy}{dt}(0), \frac{d^2y}{dt^2}(0), \dots, \frac{d^{n-1}y}{dt^{n-1}}(0)$$

- Satisfies

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_n y = 0$$



$$\frac{dy}{dt} + a_1 y = 0; \quad y(t) = Ae^{\lambda t}; \quad Ae^{\lambda t} (\lambda + a_1) = 0$$

$$\lambda = -a_1; \quad y(t) = Ae^{-a_1 t}; \quad y(t=0) = y_0; \quad y(t) = y_0 e^{-a_1 t}$$



# *LTI System Response - Forced*

- Response when initial conditions = 0

- Satisfies

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_n y = b_0 \frac{d^m u}{dt^m} + \dots + b_m u$$

$$y(0) = 0, \frac{dy}{dt}(0) = 0, \frac{d^2 y}{dt^2}(0) = 0, \dots, \frac{d^{n-1} y}{dt^{n-1}}(0) = 0$$

$$\frac{dy}{dt} + a_1 y = u(t); \quad \frac{d}{dt} \left( e^{a_1 t} y(t) \right) = e^{a_1 t} \left( \frac{dy}{dt} + a_1 y(t) \right)$$

$$y(t) = e^{-a_1 t} \left( \int u(\tau) e^{a_1 \tau} d\tau \right) \rightarrow \text{Depends on nature of } u(t)$$





## *Summary*

**LTI systems** are useful in arriving at system **behaviour** in a simple manner and also **help in synthesizing** acceptable control elements for **practical cases**.