Design Strategy & Spaces

- *Exact Design Procedure*
- *Design Space Concept*

Control Design Strategy

Control Design Procedures

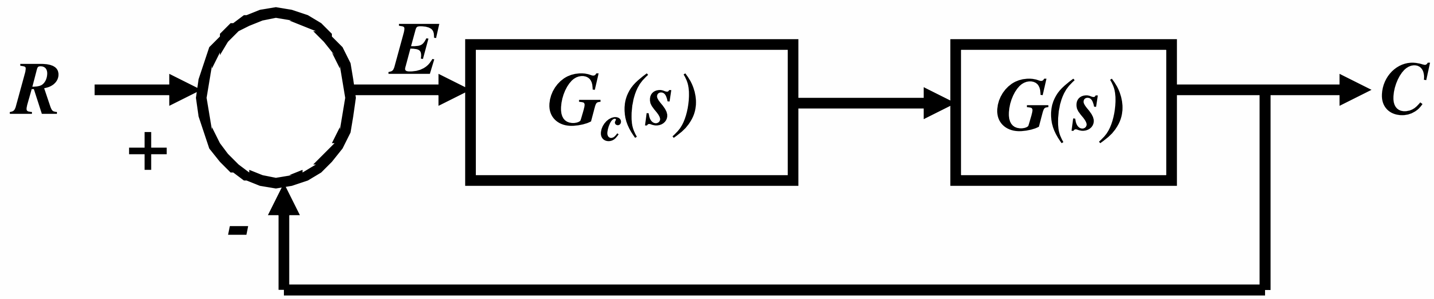
Control design procedures **aim** to arrive at the **controller**, $G_c(s)$, for given closed loop **specifications.**

However, as **specifications** can be in different **domains,** the procedures to **arrive** at controllers also **differ.**

In this **regard**, let us first **understand** the design **task.**

Exact Design Procedure

If we **wish** for closed loop performance **exactly** as specified by **benchmark** 2nd order transfer **function**, we can adopt the **following** approach.



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)} = \frac{G_c G}{1 + G_c G}; \quad G(s) = \frac{N(s)}{D(s)}$$

$$G_c = \frac{C(s)}{G[R(s) - C(s)]} = \frac{\omega_n^2 D(s)}{s\left(s + 2\zeta\omega_n\right)N(s)}$$

Exact Solution Limitations

We see that **resulting** $G_c(s)$ is **fairly complex**, whose order depends on **N(s)** and realizability on **D(s).**

Further, if plant is **unstable**, it results in **non-minimum** phase **controller**, while if the plant is **non-minimum** phase, controller is **unstable.**

Thus, we need **methods** that give a **reasonable** $\mathbf{G_c(s)}$**.**

For this, we **need** to map the various design **spaces.**

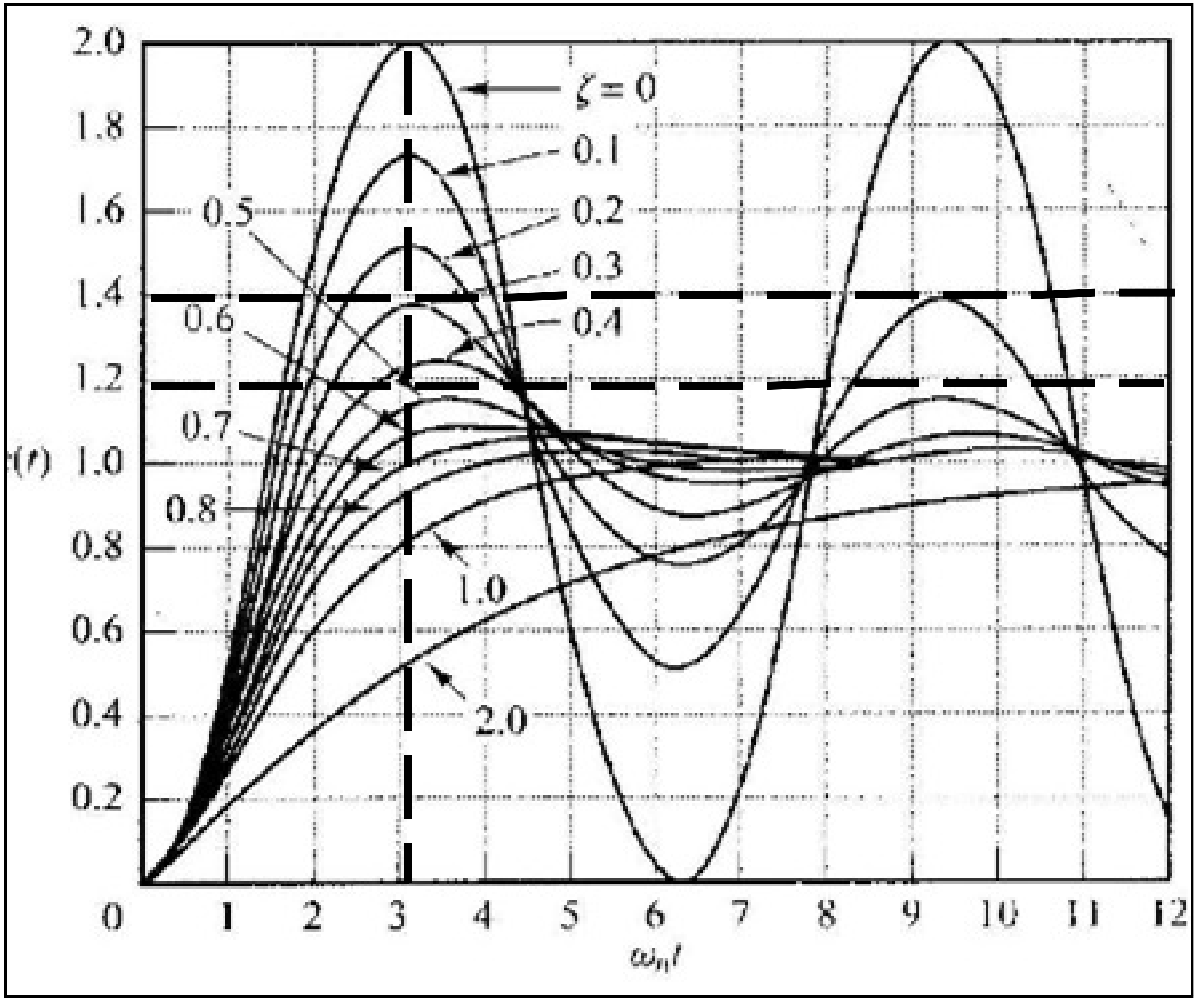
Design Space Formulation

Time Domain Design Space

Overall **time** domain design **space** is shown alongside**.**

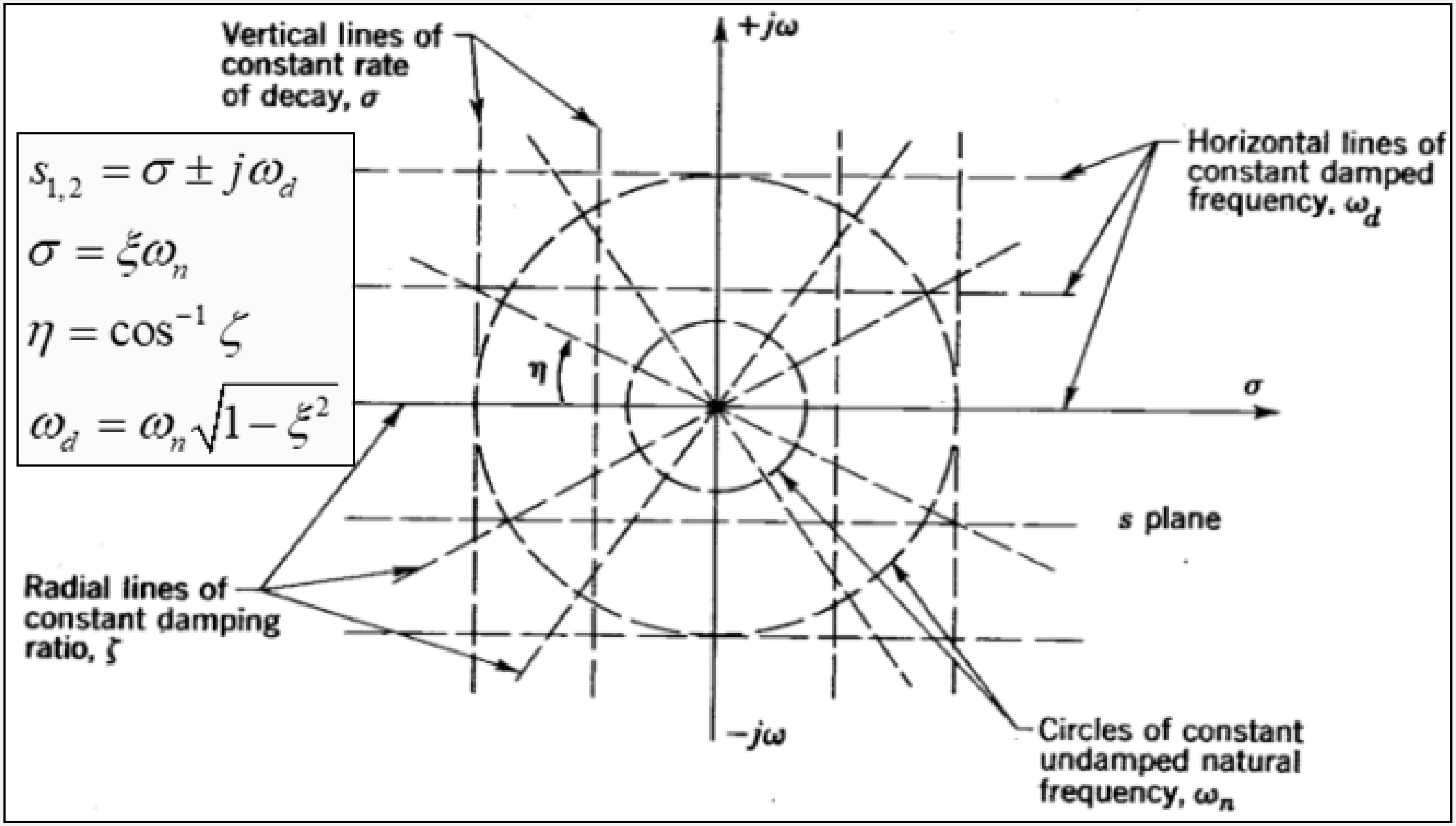
We can **choose** non-dimensional frequency, **'ω_nt'** and damping ratio, 'ζ', depending on the desired **attributes.**

It can also be **seen** that for such a **closed loop,** K_V is **'$\omega_n/(2\zeta)$'.**



s – Domain Design Space

Once, **'ω_n' & 'ζ'** are picked up from **t-domain** space, the **design** normally proceeds in **s-domain**, as shown below.



GCO, PM, ω_b Space

Following **relations,** derived earlier, provide a **basis** for developing the **design space** in frequency domain.

$$\omega_{GCO} = \omega_n \sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}; \quad PM^o = \zeta \times 100^o$$

$$\omega_b = \omega_n \sqrt{\sqrt{\left(1-2\zeta^2\right)^2 + 1} + \left(1-2\zeta^2\right)}$$

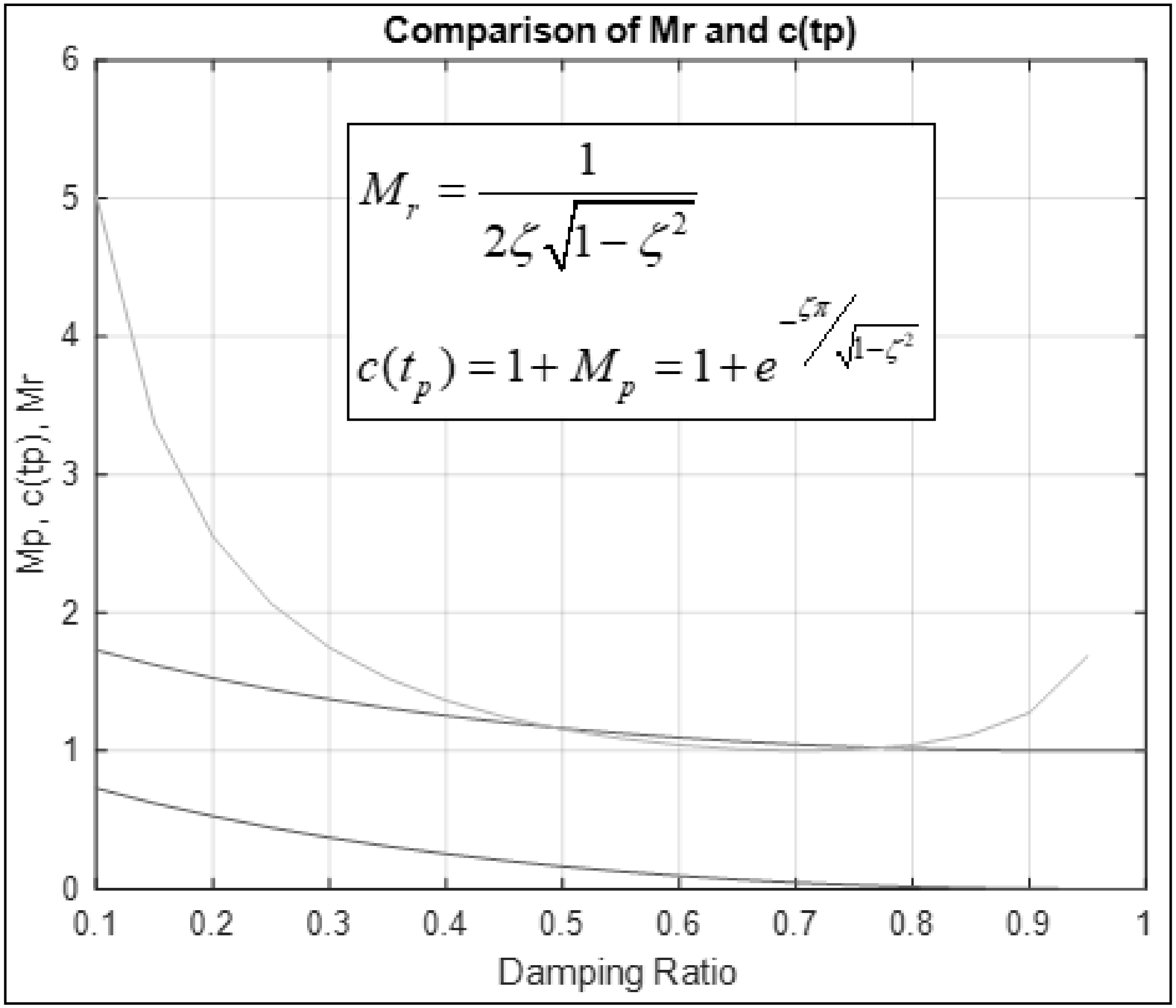
M_r as Design Specification

M_r provides **indirect** route to **time** domain design **space** through **M_p** correlation.

Figure along side shows **mapping** between ‘**M_r**’ & ‘**M_p**’, for different ‘ζ’.

It is seen that **nature** of two plots is **nearly same** for **ζ > 0.4** & **ζ < 0.8.**

Thus, **given M_p**, we can arrive at **M_r** or vice versa.



M_r Design Space

In **many** cases, we can **directly use M_r** for design, along with ω_b, in the **frequency domain**, in order to avoid **errors** due to **mapping** in time domain.

This is **enabled** through the concept of **M-circles** as contours in closed loop **design space.**

We can **apply** this directly in the **G(jω)** plane to arrive at **desired** controller **configurations** for specified M_r values.

M – Circle Formulation

M – circle can be **formulated** as shown below.

$$G(j\omega)=X(\omega)+jY(\omega); \quad \frac{C(j\omega)}{R(j\omega)}=\frac{G(j\omega)}{1+G(j\omega)}=\frac{X+jY}{(1+X)+jY}$$

$$M=\left|\frac{C(j\omega)}{R(j\omega)}\right|=\left|\frac{X+jY}{(1+X)+jY}\right|=\frac{|X+jY|}{|(1+X)+jY|}\rightarrow M^2=\frac{X^2+Y^2}{(1+X)^2+Y^2}$$

$$M\neq 1\rightarrow X^2\left(1-M^2\right)-2M^2X-M^2+\left(1-M^2\right)Y^2=0$$

$$X^2+\frac{2M^2}{M^2-1}X+\frac{M^2}{M^2-1}+Y^2=0\rightarrow\left(X+\frac{M^2}{M^2-1}\right)^2+Y^2=\frac{M^2}{\left(M^2-1\right)^2}$$

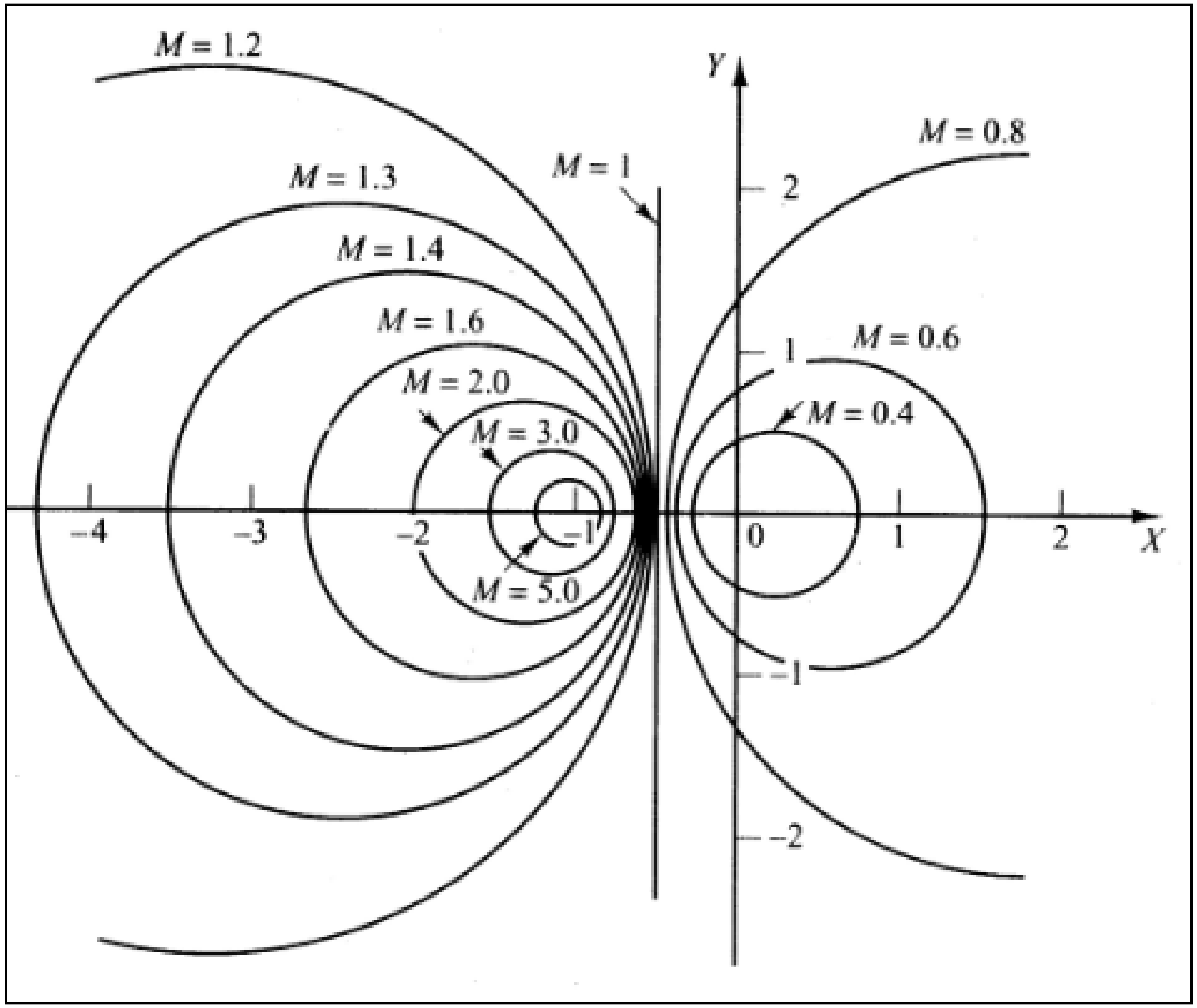
$$\text{Circle with } R=\frac{M}{\left(M^2-1\right)}; \quad \text{Centre: }\left(-\frac{M^2}{M^2-1},0\right); \quad M=1\rightarrow X=-\frac{1}{2}$$

M – circle Plots

Given **alongside** is **family** of **M-circles**, in **G(jω) plane.**

We find that for **no resonance**, M-circle is a **straight line.**

We also see that as **M → ∞**, circle becomes **-1+j0.**



M – circle Application

As **Nyquist** plot is created in **G(jω)** plane, we realize that there will be **intersections** of Nyquist plot with **M-circles** of different **magnitudes.**

These **intersection** points **indicate** the closed loop **resonant peaks** & frequencies corresponding to **G(jω).**

Thus, we can **design** a Nyquist plot of **G(jω)H(jω),** based on a **desired** $\mathbf{M_r}$ for the closed loop system.

Control Design Procedure

As we see, the **design** can be carried out in **any domain.**

However, to do it in **time domain**, we need to generate a **large number** of time responses for **different** control **options**, which is quite **tedious.**

Therefore, we **carry** out the **design** in either **s- or ω-**domain and **verify** it in time domain.

However, **specifications** can be in **any** domain.

Summary

Exact design procedure generates a **controller** which may not be **realizable** or of very high order / **unstable.**

Time domain design **space** is in terms of $\mathbf{M_p}$ **&** $\mathbf{T_s}$, while **s-domain** design **space** makes use of, $\boldsymbol{\sigma}$**,** $\boldsymbol{\zeta}$**,** $\boldsymbol{\omega_n}$**, &** $\boldsymbol{\omega_d}$**.**

PM & $\boldsymbol{\omega_b}$ are commonly used in **bode** and nyquist domain, while $\mathbf{M_r}$. maps help in **control design** with nyquist plot.

The **design** is carried out in **s- or** $\boldsymbol{\omega}$**-** domain.