Tutorial - 1.

1. (a) $J = -10^6 z^{1.5}\, a_z \; A/m^2$; $0 \le \rho \le 20\ \mu m$.

$\quad = 0$; $\rho \geq 20\ \mu m$.

$$I = \int_S J \cdot dS$$

$$= \int_0^{20}\int_0^{2\pi} -10^6 (0.1)^{1.5}\, \hat{a}_z \cdot \rho\, d\phi\, d\rho\, \hat{a}_n$$

$$= (-10^6)(0.1)^{1.5}\, 2\pi\, \frac{\rho^2}{2}\Big|_0^{20}$$

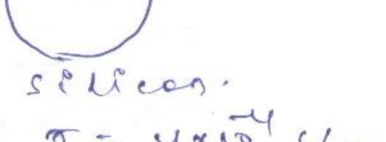
$$= -39.7\ \mu A.$$

(b) $u = 2\times10^6\ m/s \Big|_{z=0.1}$

$$J = \rho_v u.$$

$$\Rightarrow \rho_v = \frac{-10^6 (0.1)^{1.5}}{2\times10^6} = -15.8\ mC/m^3$$

2. $\rho_0 = 10^{-6}\ C/m^3 \Big|_{t=0}$



silicon.
$\sigma = 4\times10^{-4}\ S/m$.
$\epsilon_r = 12$

$$\nabla \cdot J = \frac{\partial \rho_v}{\partial t}.$$

$$\Rightarrow \sigma(\nabla \cdot E) = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma \cdot \frac{\rho_v}{\epsilon} = \frac{-\partial \rho_v}{\partial t}$$

$$\Rightarrow \frac{\partial \rho_v}{\partial t} + \sigma \frac{\rho_v}{\epsilon} = 0.$$

$$\cancel{\partial \rho_v} \int \frac{d\rho_v}{\rho_v} = -\int \frac{\sigma}{\epsilon} dt.$$

$$\Rightarrow \quad \ln \rho_v = -\frac{\sigma}{\epsilon} t + c.$$

$$\Rightarrow \quad \rho_v = c_1 e^{-\sigma/\epsilon t}$$

$$\rho_0 = c_1 = 10^{-6}$$

$$\Rightarrow \quad \rho_v(t) = 10^{-6} e^{-\frac{\sigma}{\epsilon} t} \quad c/m^3$$

$$Q = \rho_v(t) \cdot \frac{4}{3}\pi r^3 = 10^{-6} \cdot e^{-\frac{\sigma}{\epsilon} t} \cdot \frac{4}{3}\pi (0.1)^3.$$

$$I(t) = \frac{-dQ}{dt}. = -\frac{4}{3}\pi 10^{-6} (0.1)^3 \frac{d}{dt} e^{-\sigma/\epsilon t}$$

$$= -\frac{4}{3}\pi 10^{-6} (0.1)^3 \left(-\frac{\sigma}{\epsilon}\right) e^{-\sigma/\epsilon t}$$

$$= 15.77 e^{-\sigma/\epsilon t} \text{ mA}.$$

(b) time constant of charge decay i.e the relaxation time.

$$\rho_v = \rho_0 e^{-t/\tau}.$$

$$\tau = \epsilon/\sigma = 2.65 \times 10^{-7} \text{ s}.$$

(c) $$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} = 3.78 \times \cancel{10}\, e^{-\frac{\sigma}{\epsilon} t}. \quad A/m^3$$

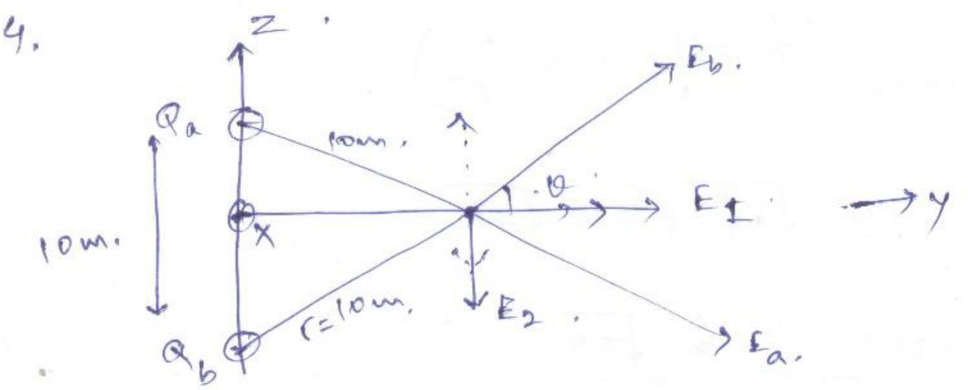
3. $$E = E_0 \sin \omega t$$

$$J_d = \frac{\partial D}{\partial t}$$

$$= \frac{\partial \epsilon E}{\partial t} = \epsilon E_0 \cos \omega t \cdot \omega$$

$$= 2 \times \epsilon_0 \times 100 \times 2\pi \times 2.45 \times 10^9 \cdot \cos \omega t \quad A/m^2.$$

4.



$Q_a = 10\cos\omega t$

$Q_b = -10\cos\omega t$.

$E_a = \dfrac{10\cos\omega t}{4\pi\epsilon_0 r}$, $E_b = \dfrac{-10\cos\omega t}{4\pi\epsilon_0 r}$.

$E_1 = E_{a_1} + E_{b_1} = 0$. where $E_{a1} = E_a\cos\theta$, $E_{b1} = +E_b\cos\theta$

$E_2 = \underbrace{E_{a_2}}_{E_a\sin\theta} + \underbrace{E_{b_2}}_{-E_b\sin\theta} = \dfrac{20\cos\omega t}{4\pi\epsilon_0 r}\sin\theta$.

$E = -\dfrac{10\cos\omega t}{4\pi\epsilon_0 r}\hat{z}$

$\nabla \times E = -\dfrac{\partial B}{\partial t} = -j\omega\mu_0 H$

$\Rightarrow \quad H = \dfrac{-1}{j\omega\mu_0}(\nabla \times E)$

$= \dfrac{-1}{j\omega\mu_0}\left(\hat{x}\dfrac{\partial E_z}{\partial y} - \hat{y}\dfrac{\partial E_z}{\partial x}\right)$

$= \dfrac{1}{j\omega\mu_0}\dfrac{10\cdot\cos\omega t}{4\pi\epsilon_0}\left(\hat{x}\dfrac{\partial}{\partial y}\cdot\dfrac{1}{r} - \hat{y}\dfrac{\partial}{\partial x}\dfrac{1}{r}\right)$

where $r = \sqrt{x^2+y^2+z^2}$

$\dfrac{\partial}{\partial y}\dfrac{1}{r} = \dfrac{-y}{r^3}$, $\dfrac{\partial}{\partial x}\dfrac{1}{r} = \dfrac{-x}{r^3}$

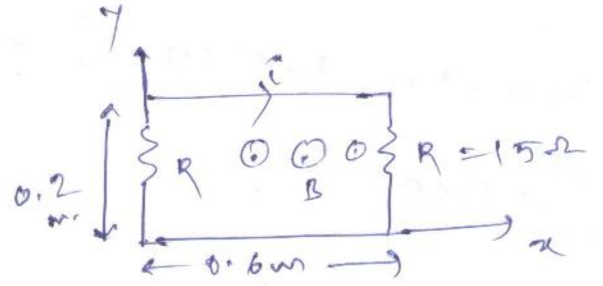
$$= \frac{10\cos\omega t}{j\omega\mu_0 \, 4\pi\epsilon_0}\left(\hat{x}\frac{-y}{r^3} + \hat{y}\frac{x}{r^3}\right)$$

at point p $\quad y = \sqrt{10^2 - 5^2} = \sqrt{75}$

$$H = \frac{j\,10\cos\omega t}{4\pi\mu_0\epsilon_0\,\omega}\,\hat{x}\,\frac{\sqrt{75}}{10^3}$$

$$= j\,\frac{6.89\cos\omega t}{\omega\mu_0\epsilon_0}\,\hat{x} \quad mA/m.$$

7.



$$B = a_z 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right)\mu T$$

$$\Phi = \oint B.ds = \int_0^{0.2}\int_0^{0.6} 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right)10^{-6}.\,dx\,dy$$

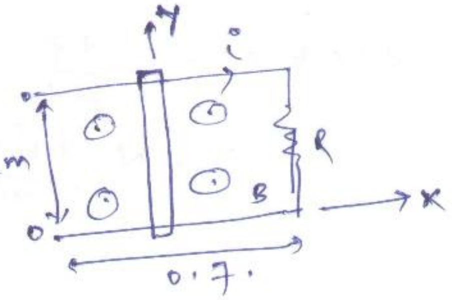
$$= -\frac{0.9}{\pi}\left[\sin\left(5\pi 10^7 t - \frac{2}{3}\pi(0.6)\right) - \sin\left(5\pi 10^7 t\right)\right].\times 10^{-6}$$

$$emf = -\frac{d\Phi}{dt} = \frac{0.9}{\pi}\left[\cos\left(5\pi 10^7 t - \frac{2}{3}0.4\pi\right) - \cos\left(5\pi 10^7 t\right)\right].\times 10^{-6}\times 5\pi\times 10^7.$$

$$i = \frac{emf}{2R} = \frac{0.9}{30\pi}\times 50\pi\left[\cos\left(5\pi 10^7 t - 0.4\pi\right) - \cos\left(5\pi 10^7 t\right)\right] A.$$

$$=$$

8. $B = a_z 5 \cos\omega t \ \text{mT}$



$\phi = \oint B \cdot ds$

$= \int_0^{0.2} \int_{(0.7-x)}^{0.7} 5 \cos\omega t \times 10^{-3} \, dx \, dy.$

$= 5 \times 10^{-3} \cos\omega t \cdot (0.7 - 0.7 + x) \ 0.2$

$= \quad 0.35 (1 - \cos\omega t) . \cos\omega t \times 10^{-3}$

$\text{emf} = -\frac{d\phi}{dt} = 0. -\frac{d}{dt}(\cos\omega t - \cos^2\omega t) \ 0.35 \times 10^{-3}.$

$i = \frac{\text{emf}}{0.2\Omega}.$

9. $B = \hat{a}_z B_0 \cos\left(\frac{\pi r}{2b}\right) \cdot \sin\omega t$

$\phi = \int_S B \cdot ds$

$= \int_0^b a_z B_0 \cos\left(\frac{\pi r}{2b}\right) \cdot \sin\omega t \ . \ a_z 2\pi r \, dr.$

$= 2\pi B_0 \sin\omega t \int_0^b r \cos\left(\frac{\pi r}{2b}\right) dr.$

$= 2\pi B_0 \sin\omega t \left[r \sin\left(\frac{\pi r}{2b}\right)\left(\frac{2b}{\pi}\right) + \cos\left(\frac{\pi r}{2b}\right)\left(\frac{2b}{\pi}\right)^2 \right]_0^b.$

$= 2\pi B_0 \sin\omega t \left[b\left(\frac{2b}{\pi}\right) - \left(\frac{2b}{\pi}\right)^2 \right]$

$= \quad 2\pi B_0 \cdot \sin\omega t \cdot \frac{4b^2}{\pi^2} \left(\frac{\pi}{2} - 1\right).$

$= \frac{8 b^2 B_0}{\pi} \sin\omega t \left(\frac{\pi}{2} - 1\right)$

$$emf = -N\frac{d\phi}{dt}$$

$$= -N\frac{8b^2B_0}{\pi}\left(\frac{\pi}{2}-1\right)\cdot\cos\omega t\cdot\omega \quad (N)$$

10 $E = (kx - 100t)\,a_y$

$H = (x + 20t)\,a_z$

$\mu = \cancel{0.2H}\ 0.25\ H/m.$

$\epsilon = 0.01\ F/m.$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{\partial E}{\partial x}\cdot\hat{a}_z - \hat{a}_x\frac{\partial E}{\partial z} = -\frac{\partial}{\partial t}\mu\cdot H$$

$$= -\mu\cdot\frac{\partial H}{\partial t}.$$

$$\Rightarrow \quad = -20\times 0.25$$

$$k = -5.$$

10.