Homework 6

Communication Systems (EE 308), Autumn'19

1) The following problems from Haykin, Chapter 5: 5.4 to 5.24, 5.27, 5.29 to 5.32 on pp. 201-205 (ignore the question about ergodicity in 5.9 (d)).
2) The correlation coefficient of two random variables X and Y is defined to be:

$$\rho_{X,Y} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y},$$

where $\text{Cov}(X,Y)$ is the covariance of X and Y, and σ_X and σ_Y are the standard deviations of X and Y respectively. Show that:

(a) $|\rho_{X,Y}| \leq 1, \ \forall X, Y$.

(b) If X and Y are independent, then $\rho_{X,Y} = 0$.

(c) If $Y = aX + b$, where $a \neq 0$ and b are constants, then $|\rho_{X,Y}| = 1$. Also, $\rho_{X,Y} = 1$ if $a > 0$ and $\rho_{X,Y} = -1$ if $a < 0$.

(d) Note that parts (a), (b) and (c) show that intuitively, $\rho_{X,Y}$ is a measure of the amount of dependence between X and Y. However, $\rho_{X,Y}$ does not always work well as a measure of dependence. For example, suppose X is uniformly distributed in $[-1, 1]$ and $Y = X^2$. Show that $\rho_{X,Y} = 0$.