



PI – Control with Root Locus

Consider the **plant** given below.

$$G(s) = \frac{75}{(s+5)(s+10)^2}$$

Design a **PI** using root locus to achieve a **K_V of 3**, while minimizing the **effect** on the closed loop system peak **overshoot** and 2% settling **time**.

Dominant Pole: $-5.97 \pm j2.89$; $\zeta = 0.9$

$M_p \approx 0.1\%$; $T_s = 0.78s$



PI Design

The following **PI** is synthesized.

$$G_{PI}(s) = \frac{K}{T_i s} (T_i s + 1); \quad \lim_{s \rightarrow 0} (s G_{PI}(s) G(s)) = K_V$$

$$0.15 \frac{K}{T_i} = 3 \rightarrow \frac{K}{T_i} = 20 \quad G_{PI}(s) = \frac{20}{s} (T_i s + 1)$$

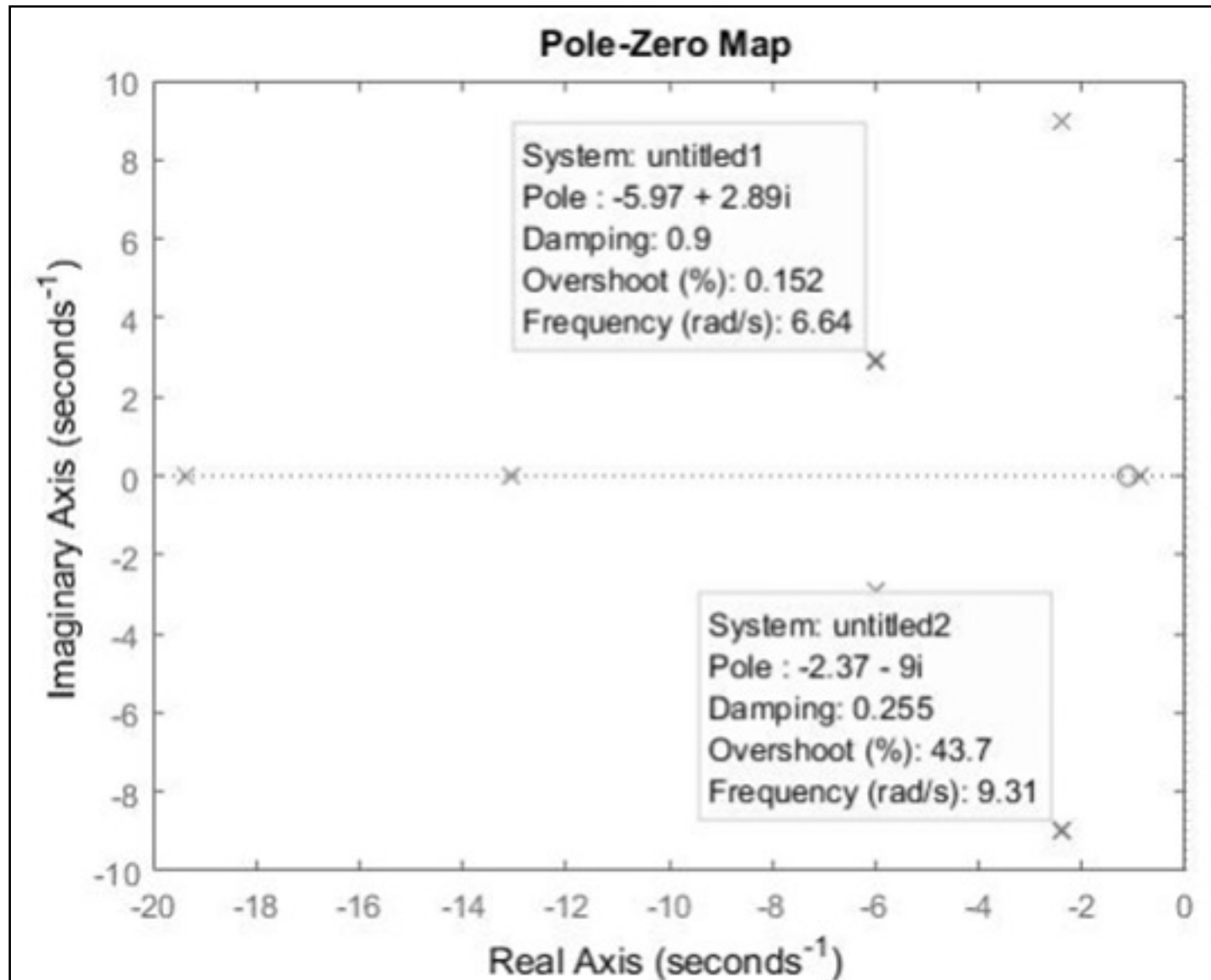
$$\angle G_{PI} = \angle (T_i s + 1) |_{s=-5.97 \pm 2.89j} - \angle s |_{s=-5.97 \pm 2.89j} = -5^\circ$$

$$\tan^{-1} \left(\frac{2.89 T_i}{1 - 5.97 T_i} \right) = \tan^{-1} \left(\frac{2.89}{-5.97} \right) - 5^\circ = 154.2^\circ$$

$$\frac{2.89 T_i}{1 - 5.97 T_i} = -0.596 \rightarrow T_i \approx 0.89; \quad G_{PI}(s) = \frac{20}{s} (0.89 s + 1)$$

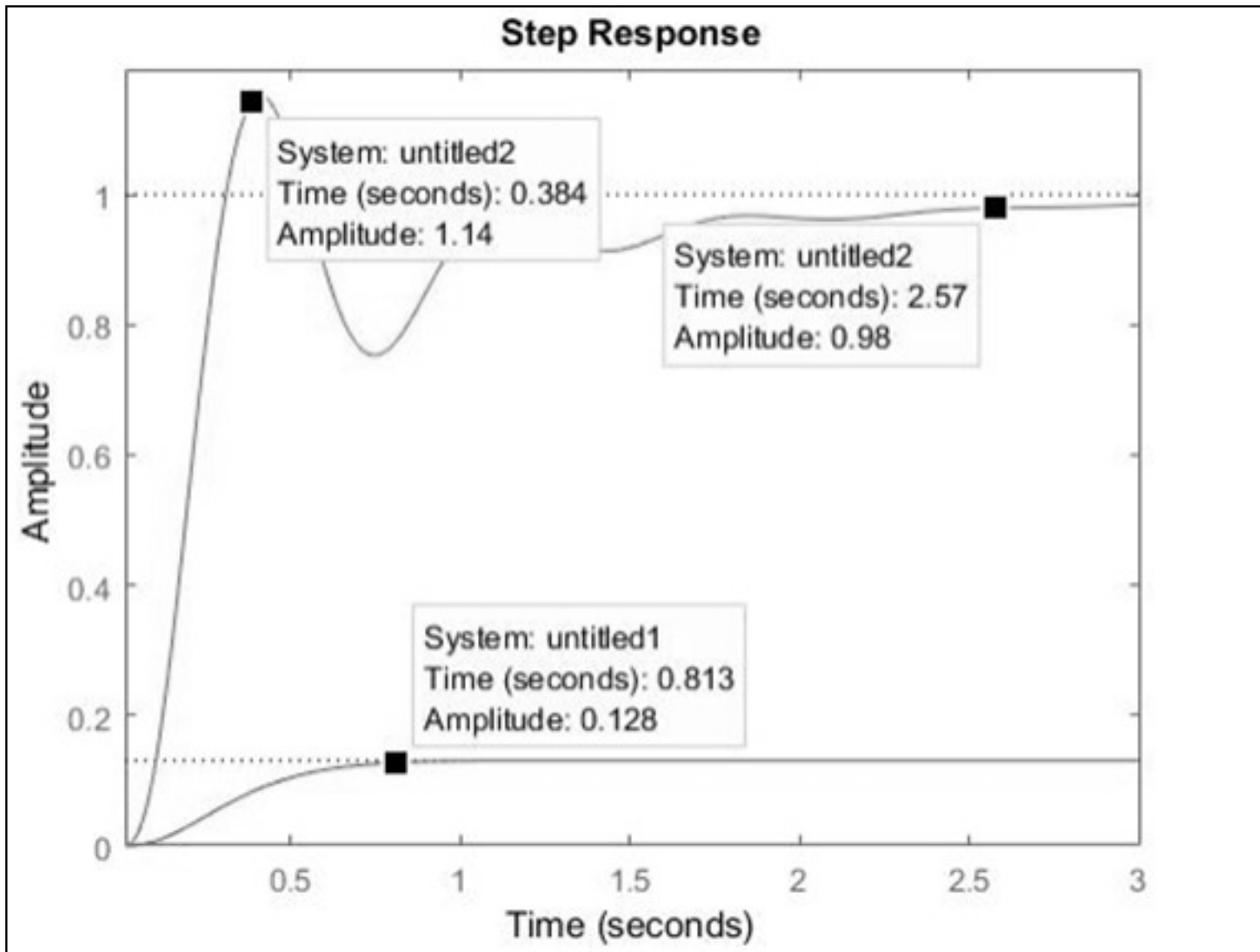


PI Compensated Roots





PI Step Response





PI – Control with Bode Plot

Consider the **plant** given below.

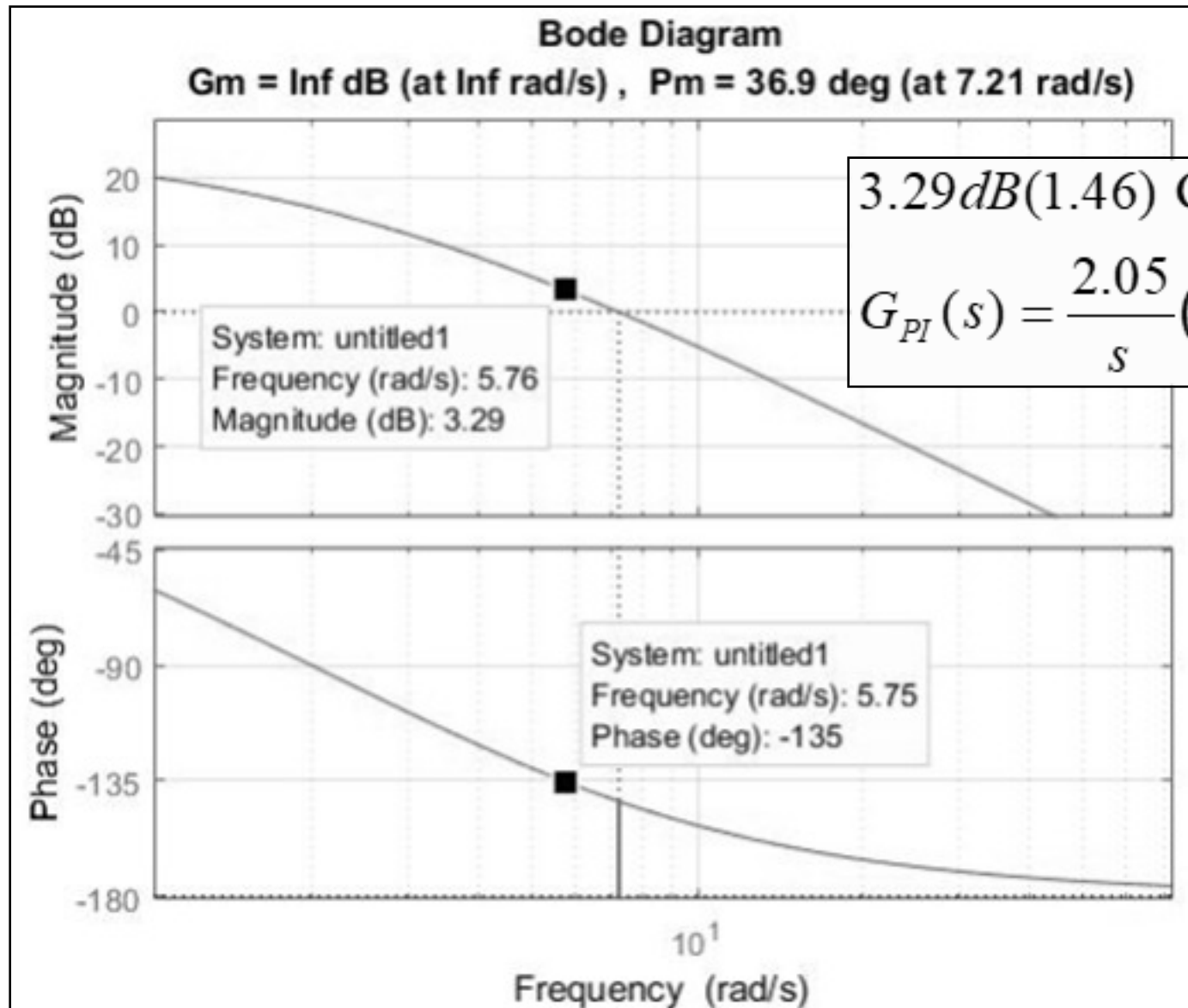
$$G(s) = \frac{5}{(s+1)(0.25s+1)} = \frac{20}{(s+1)(s+4)}$$

Design a **PI** using bode plot to achieve a **K_V** of 10, and **phase margin** of more than 45° .

$$G_{PI}(s) = \frac{3}{s}(s+z)$$



Gain Adjusted Bode Plot

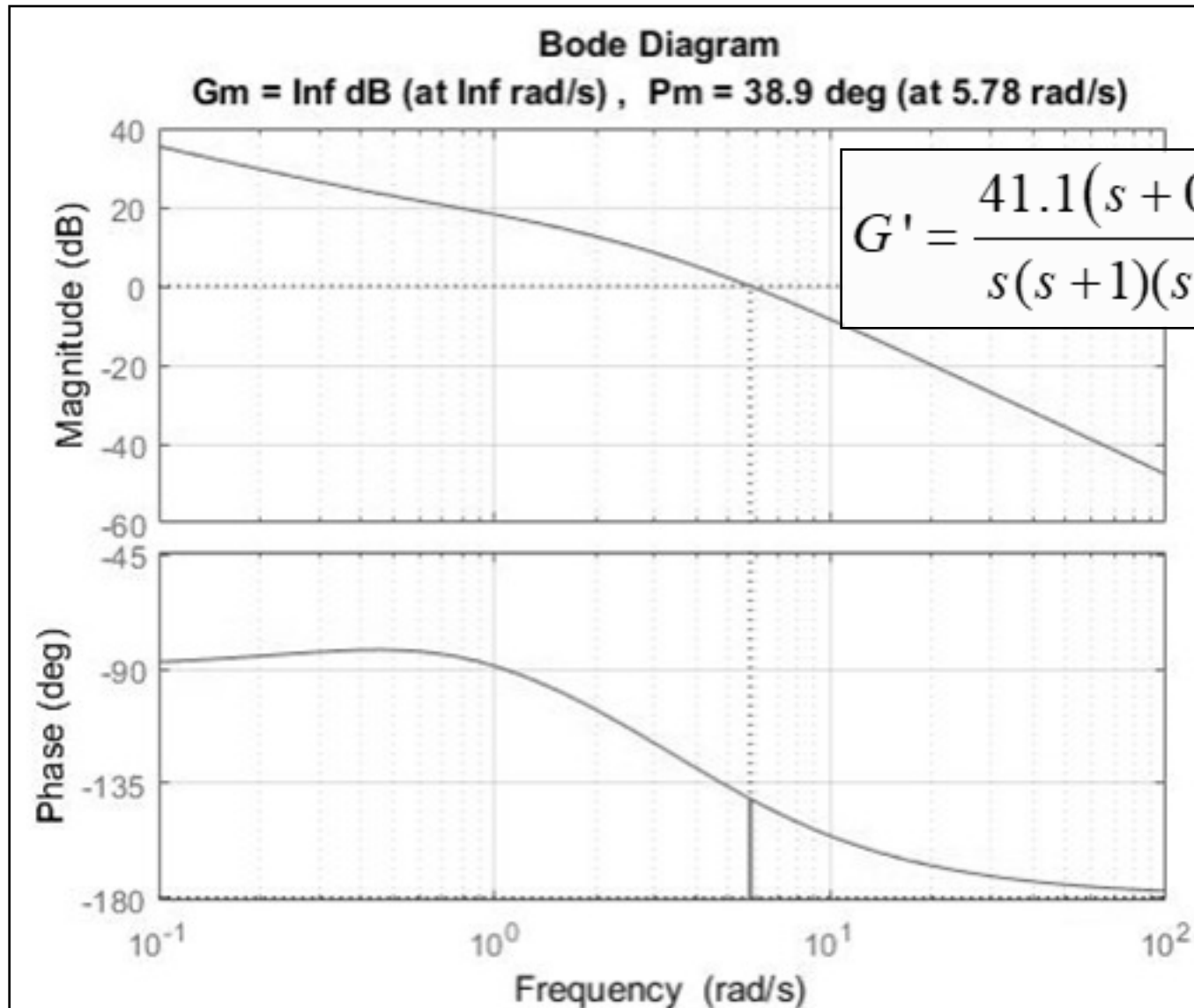


3.29dB(1.46) Gain Reduction

$$G_{PI}(s) = \frac{2.05}{s}(s + 0.57)$$



PI Compensated Bode Plot



$$G' = \frac{41.1(s + 0.57)}{s(s + 1)(s + 4)}; \quad K_V = 5.8$$