Convolution Application

Solve the following **differential** equation using **convolution** integral approach.

$$\ddot{y} + 4\dot{y} + 4y = 3\dot{u} + 2u; \quad y(0) = \dot{y}(0) = 0; \quad u(t) = e^{-3t} \text{ for } t \geq 0$$

$$g(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t; \quad m = 1, \quad \zeta\omega_n = 2, \quad \omega_n = 2, \quad \omega_d = 0$$

$$g(t) = \lim_{\omega_d \to 0} \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t = \frac{1}{m} e^{-\zeta\omega_n t} \times \lim_{\omega_d \to 0} \frac{\sin\omega_d t}{\omega_d} = te^{-2t}$$

$$y(t) = \int_0^t (t-\tau)e^{-2(t-\tau)}\left[-7e^{-3\tau}\right]d\tau = -7\int_0^t \left[te^{-2t}.e^{-\tau} - \tau e^{-2t}.e^{-\tau}\right]d\tau$$

$$y(t) = -7te^{-2t}\left[-e^{-\tau}\right]_0^t - 7e^{-2t}\left[\tau e^{-\tau} + e^{-\tau}\right]_0^t = 7te^{-2t} - 7e^{-3t} + 7e^{-2t}$$