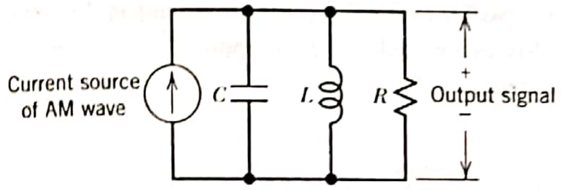

Figure P3.1

3.2 For a *p-n* junction diode, the current i through the diode and the voltage v across it are related by

$$i = I_0\left[\exp\left(-\frac{v}{V_T}\right) - 1\right]$$

where I_0 is the reverse saturation current, and V_T is the volt-equivalent of temperature defined by

$$V_T = \frac{kT}{e}$$

where k is Boltzmann's constant in joules per degree Kelvin, T is the absolute temperature in degrees Kelvin, and e is the charge of an electron. At room temperature, $V_T = 0.026$ volts.

(a) Expand i as a power series in v, retaining terms up to v^3.

(b) Let

$$v = 0.01\cos(2\pi f_m t) + 0.01\cos(2\pi f_c t) \text{ volts}$$

where $f_m = 1$ kHz and $f_c = 100$ kHz. Determine the spectrum of the resulting diode current i.

(c) Specify the band-pass filter required to extract from the diode current an AM signal with carrier frequency f_c.

(d) What is the percentage modulation of this AM signal?

3.3 Suppose that nonlinear devices are available for which the output current i_o and input voltage v_i are related by

$$i_o = a_1 v_1 + a_3 v_i^3$$

where a_1 and a_3 are constants. Explain how these devices may be used to provide: (a) a product modulator and (b) an amplitude modulator.

3.4 Figure P3.4 shows the circuit diagram of a *square-law modulator*. The signal applied to the nonlinear device is relatively weak, such that it can be represented by a square law:

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where a_1 and a_2 are constants, $v_1(t)$ is the input voltage, and $v_2(t)$ is the output voltage. The input voltage is defined by

$$v_1(t) = A_c\cos(2\pi f_c t) + m(t)$$

where $m(t)$ is a message signal and $A_c\cos(2\pi f_c t)$ is the carrier wave.

(a) Evaluate the output voltage $v_2(t)$.

(b) Specify the frequency response that the tuned circuit in Figure P3.4 must satisfy in order to generate an AM signal with f_c as the carrier frequency.

(c) What is the amplitude sensitivity of this AM signal?

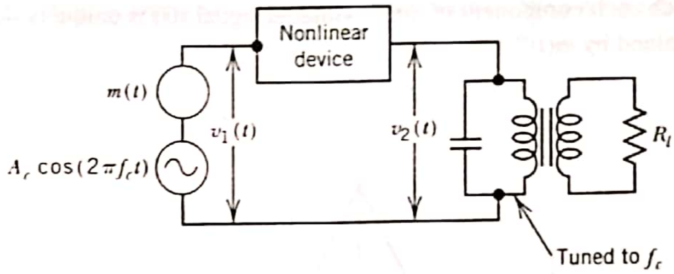

Figure P3.4

3.5 Consider the AM signal

$$s(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

produced by a sinusoidal modulating signal of frequency f_m. Assume that the modulation factor is $\mu = 2$, and the carrier frequency f_c is much greater than f_m. The AM signal $s(t)$ is applied to an ideal envelope detector, producing the output $v(t)$.

(a) Determine the Fourier series representation of $v(t)$.

(b) What is the ratio of second-harmonic amplitude to fundamental amplitude in $v(t)$?

3.6 Consider a *square-law detector*, using a nonlinear device whose transfer characteristic is defined by

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where a_1 and a_2 are constants, $v_1(t)$ is the input, and $v_2(t)$ is the output. The input consists of the AM wave

$$v_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

(a) Evaluate the output $v_2(t)$.

(b) Find the conditions for which the message signal $m(t)$ may be recovered from $v_2(t)$.

3.7 The AM signal

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

is applied to the system shown in Figure P3.7. Assuming that $|k_a m(t)| < 1$ for all t and the message signal $m(t)$ is limited to the interval $-W \le f \le W$, and that the carrier frequency $f_c > 2W$, show that $m(t)$ can be obtained from the square-rooter output $v_3(t)$.

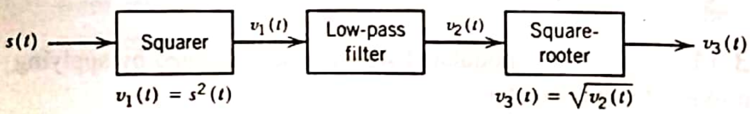

Figure P3.7

3.8 Consider a message signal $m(t)$ with the spectrum shown in Figure P3.8. The message bandwidth $W = 1$ kHz. This signal is applied to a product modulator, together with a carrier wave $A_c\cos(2\pi f_c t)$, producing the DSB-SC modulated signal $s(t)$. The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector, determine the spectrum of the detector output when: (a) the carrier frequency $f_c = 1.25$ kHz and (b) the carrier frequency $f_c = 0.75$ kHz. What is the lowest carrier frequency for

3.16 The single tone modulating signal $m(t) = A_m \cos(2\pi f_m t)$ is used to generate the VSB signal

$$s(t) = \frac{1}{2} a A_m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_m A_c (1 - a)\cos[2\pi(f_c - f_m)t]$$

where a is a constant, less than unity, representing the attenuation of the upper side frequency.

(a) If we represent this VSB signal as a quadrature carrier multiplex

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

What is $m_2(t)$?

(b) The VSB signal, plus the carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced by the quadrature component, $m_2(t)$.

(c) What is the value of constant a for which this distortion reaches its worst possible condition?

3.17 Using the message signal

$$m(t) = \frac{1}{1 + t^2}$$

determine and sketch the modulated waves for the following methods of modulation:

(a) Amplitude modulation with 50 percent modulation.

(b) Double sideband-suppressed carrier modulation.

3.18 The local oscillator used for the demodulation of an SSB signal $s(t)$ has a frequency error Δf measured with respect to the carrier frequency f_c used to generate $s(t)$. Otherwise, there is perfect synchronism between this oscillator in the receiver and the oscillator supplying the carrier wave in the transmitter. Evaluate the demodulated signal for the following two situations:

(a) The SSB signal $s(t)$ consists of the upper sideband only.

(b) The SSB signal $s(t)$ consists of the lower sideband only.

3.19 Figure P3.19 shows the block diagram of *Weaver's method* for generating SSB modulated waves. The message (modulating) signal $m(t)$ is limited to the frequency band $f_a \le |f| \le f_b$. The auxiliary carrier applied to the first pair of product modulators has a frequency f_0, which lies at the center of this band, as shown by

$$f_0 = \frac{f_a + f_b}{2}$$

The low-pass filters in the upper and lower branches are identical, each with a cutoff frequency equal to $(f_b - f_a)/2$. The carrier applied to the second pair of product modulators has a frequency f_c that is greater than $(f_b - f_a)/2$. Sketch the spectra at the various points in the modulator of Figure P3.19, and hence show that:

(a) For the lower sideband, the contributions of the upper and lower branches are of opposite polarity, and by adding them at the modulator output, the lower sideband is suppressed.

(b) For the upper sideband, the contributions of the upper and lower branches are of the same polarity, and by adding them, the upper sideband is transmitted.

(c) How would you modify the modulator of Figure P3.19, so that only the lower sideband is transmitted?

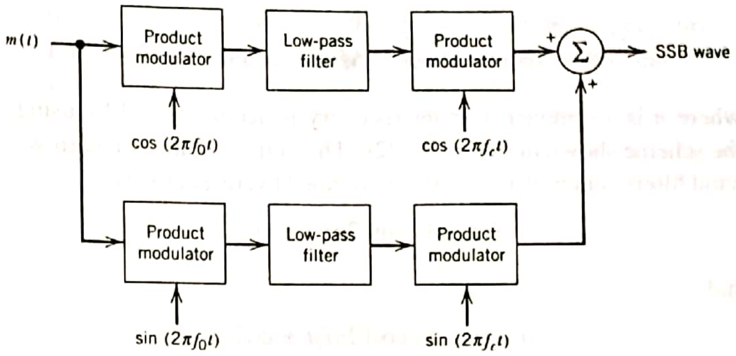

Figure P3.19

3.20

(a) Consider a message signal $m(t)$ containing frequency components at 100, 200, and 400 Hz. This signal is applied to an SSB modulator together with a carrier at 100 kHz, with only the upper sideband retained. In the coherent detector used to recover $m(t)$, the local oscillator supplies a sine wave of frequency 100.02 kHz. Determine the frequency components of the detector output.

(b) Repeat your analysis, assuming that only the lower sideband is transmitted.

3.21 The spectrum of a voice signal $m(t)$ is zero outside the interval $f_a \le |f| \le f_b$. In order to ensure communication privacy, this signal is applied to a *scrambler* that consists of the following cascade of components: a product modulator, a high-pass filter, a second product modulator, and a low-pass filter. The carrier wave applied to the first product modulator has a frequency equal to f_c, whereas that applied to the second product modulator has a frequency equal to $f_b + f_c$; both of them have unit amplitude. The high-pass and low-pass filters have the same cutoff frequency at f_c. Assume that $f_c > f_b$.

(a) Derive an expression for the scrambler output $s(t)$, and sketch its spectrum.

(b) Show that the original voice signal $m(t)$ may be recovered from $s(t)$ by using an *unscrambler* that is identical to the unit described above.

3.22 A method that is used for carrier recovery in SSB modulation systems involves transmitting two pilot frequencies that are appropriately positioned with respect to the transmitted sideband. This is illustrated in Figure P3.22*a* for the case when only the lower sideband is transmitted. In this case, the pilot frequencies f_1 and f_2 are defined by

$$f_1 = f_c - W - \Delta f$$

and

$$f_2 = f_c + \Delta f$$

where f_c is the carrier frequency and W is the message bandwidth. The Δf is chosen so as to satisfy the relation

$$n = \frac{W}{\Delta f}$$

where n is an integer. Carrier recovery is accomplished by using the scheme shown in Figure P3.22b. The outputs of the two narrow-band filters centered at f_1 and f_2 are defined by, respectively,

$$v_1(t) = A_1 \cos(2\pi f_1 t + \phi_1)$$

and

$$v_2(t) = A_2 \cos(2\pi f_2 t + \phi_2)$$

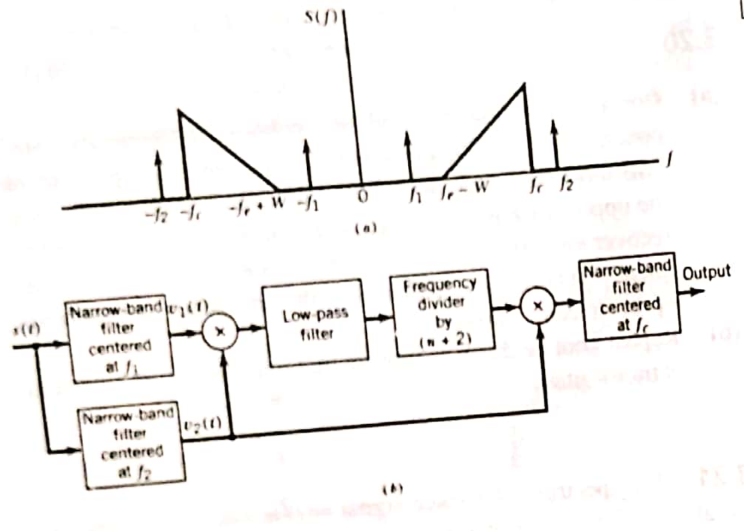

Figure P3.22

The low-pass filter is designed to select the difference frequency component of the first multiplier output due to $v_1(t)$ and $v_2(t)$.

(a) Show that the output signal of the circuit in Figure P3.22b is proportional to the carrier wave $A_c \cos(2\pi f_c t)$ if the phase angles ϕ_1 and ϕ_2 satisfy the relation

$$\phi_2 = -\frac{\phi_1}{1+n}$$

(b) For the case when only the upper sideband is transmitted, the two pilot frequencies are defined by

$$f_1 = f_c - \Delta f$$

and

$$f_2 = f_c + W + \Delta f$$

How would you modify the carrier recovery circuit of Figure P3.22b in order to deal with this case? What is the corresponding relation between ϕ_1 and ϕ_2 for the circuit output to be proportional to the carrier wave?

3.23 Figure P3.23 shows the block diagram of a *frequency synthesizer*, which enables the generation of many frequencies, each with the same high accuracy as the *master oscillator*. The master oscillator of frequency 1 MHz feeds two *spectrum generators*, one directly and the other through a *frequency divider*. Spectrum generator 1 produces a signal rich in the following harmonics: 1, 2, 3, 4, 5, 6, 7, 8, and 9 MHz. The frequency divider provides a 100-kHz output, in response to which spectrum generator 2 produces a second signal rich in the following harmonics: 100, 200, 300, 400, 500, 600, 700, 800, and 900 kHz. The harmonic selectors are designed to feed two signals into the mixer, one from spectrum generator 1 and the other from spectrum generator 2. Find the range of possible frequency outputs of this synthesizer and its resolution.

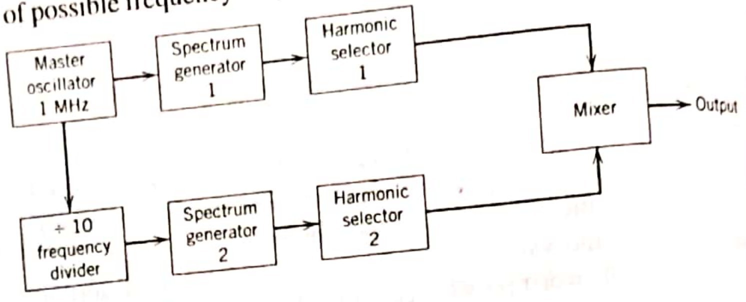

Figure P3.23

3.24 Consider a multiplex system in which four input signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$, are respectively multiplied by the carrier waves

$$[\cos(2\pi f_a t) + \cos(2\pi f_b t)]$$
$$[\cos(2\pi f_a t + \alpha_1) + \cos(2\pi f_b t + \beta_1)]$$
$$[\cos(2\pi f_a t + \alpha_2) + \cos(2\pi f_b t + \beta_2)]$$
$$[\cos(2\pi f_a t + \beta_3) + \cos(2\pi f_b t + \beta_3)]$$

and the resulting DSB-SC signals are summed and then transmitted over a common channel. In the receiver, demodulation is achieved by multiplying the sum of the DSB-SC signals by the four carrier waves separately and then using filtering to remove the unwanted components.

(a) Determine the conditions that the phase angles α_1, α_2, α_3 and β_1, β_2, β_3 must satisfy in order that the output of the kth demodulator is $m_k(t)$, where $k = 1, 2, 3, 4$.

(b) Determine the minimum separation of carrier frequencies f_a and f_b in relation to the bandwidth of the input signals so as to ensure a satisfactory operation of the system.

Computer Problems

3.25 In this computer experiment, we simulate the modulation and demodulation of an AM wave.

(a) Develop a Matlab script to simulate the modulation of a kHz carrier with a 0.4 kHz modulating wave. Use a modulation index of 50 percent and a sampling rate of 160 kHz.

(b) An envelope detector is assumed to have a forward resistance r_f of 25 Ω and a capacitance of 0.01 μF. The source resistance is 75 Ω and the load resistance is 10 kΩ.

(i) What is the charging time constant? Compare this constant to one period of the modulating wave, and comment on how well it should track the envelope.

(ii) What is the capacitor discharge time constant? Compare this to one period of the carrier wave. Using a linear approximation, what fraction of the capacitor voltage decays during one sample period?

4.13 An FM signal with a frequency deviation of 10 kHz at a modulation frequency of 5 kHz is applied to two frequency multipliers connected in cascade. The first multiplier doubles the frequency and the second multiplier triples the frequency. Determine the frequency deviation and the modulation index of the FM signal obtained at the second multiplier output. What is the frequency separation of the adjacent side frequencies of this FM signal?

4.14 An FM signal is applied to a square-law device with output voltage v_2 related to input voltage v_1 by

$$v_2 = av_1^2$$

where a is a constant. Explain how such a device can be used to obtain an FM signal with a greater frequency deviation than that available at the input.

4.15 Figure P4.15 shows the frequency-determining network of a voltage-controlled oscillator. Frequency modulation is produced by applying the modulating signal $A_m \sin(2\pi f_m t)$ plus a bias V_b to a pair of varactor diodes connected across the parallel combination of a 200-μH inductor and 100-pF capacitor. The capacitor of each varactor diode is related to the voltage V (in volts) applied across its electrodes by

$$C = 100V^{-1/2}\text{pF}$$

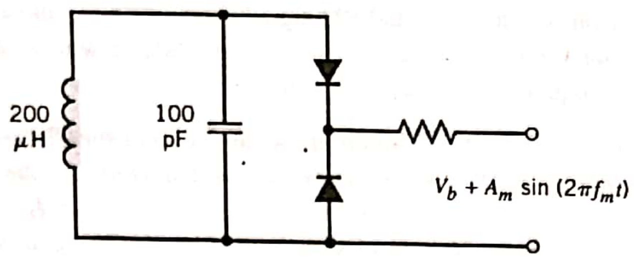

Figure P4.15

The unmodulated frequency of oscillation is 1 MHz. The VCO output is applied to a frequency multiplier to produce an FM signal with a carrier frequency of 64 MHz and a modulation index of 5. Determine (a) the magnitude of the bias voltage V_b and (b) the amplitude A_m of the modulating wave, given that f_m = 10 kHz.

4.16 The FM signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

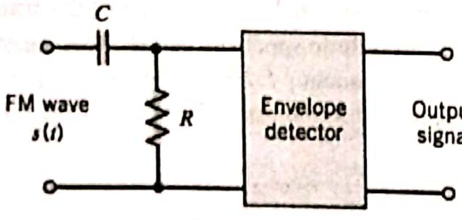

Figure P4.16

is applied to the system shown in Figure P4.16 consisting of a high-pass RC filter and an envelope detector. Assume that (a) the resistance R is small compared with the reactance of the capacitor C for all significant frequency components of $s(t)$ and (b) the envelope detector does not load the filter. Determine the resulting signal at the envelope detector output, assuming that $k_f|m(t)| < f_c$ for all t.

4.17 In the frequency discriminator of Figure 4.14, let the frequency separation between the resonant frequencies of the two parallel-tuned LC filters be denoted by $2kB$, where $2B$ is the 3-dB bandwidth of either filter and k is a scaling factor. Assume that both filters have a high Q-factor.

(a) Show that the total response of both filters has a slope equal to $2k/B(1 + k^2)^{3/2}$ at the center frequency f_c.

(b) Let D denote the deviation of the total response measured with respect to a straight line passing through $f = f_c$ with this slope. Plot D versus δ for k = 1.5 and $-kB \le \delta \le kB$ where $\delta = f - f_c$.

4.18 Consider the frequency demodulation scheme shown in Figure P4.18 in which the incoming FM signal $s(t)$ is passed through a delay line that produces a phase-shift of $\pi/2$ radians at the carrier frequency f_c. The delay-line output is subtracted from the incoming FM signal, and the resulting composite signal is then envelope-detected. This demodulator finds application in demodulating microwave FM signals. Assuming that

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right]$$

analyze the operation of this demodulator when the modulation index β is less than unity and the delay T produced by the delay line is sufficiently small to justify making the approximations

$$\cos(2\pi f_m T) \simeq 1$$

and

$$\sin(2\pi f_m T) \simeq 2\pi f_m T$$

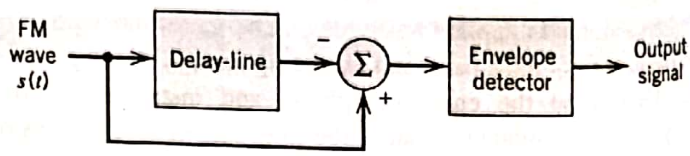

Figure P4.18

4.19 Figure P4.19 shows the block diagram of a *zero-crossing detector* for demodulating an FM signal. It consists of a limiter, a pulse generator for producing a short pulse at each zero-crossing of the input, and a low-pass filter for extracting the modulating wave.

(a) Show that the instantaneous frequency of the input FM signal is proportional to the number of zero crossings in the time interval $t - (T_1/2)$ to $t + (T_1/2)$, divided by T_1. Assume that the modulating signal is essentially constant during this time interval.

(b) Illustrate the operation of this demodulator, using the sawtooth wave of Figure P4.1 as the modulating wave.

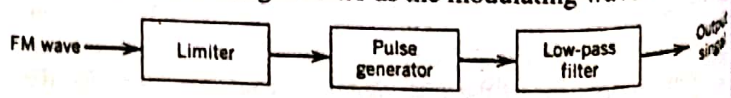

Figure P4.19

4.20 Suppose that the received signal in an FM system contains some residual amplitude modulation of positive amplitude $a(t)$, as shown by

$$s(t) = a(t)\cos[2\pi f_c t + \phi(t)]$$

where f_c is the carrier frequency. The phase $\phi(t)$ is related to the modulating signal $m(t)$ by

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

where k_f is a constant. Assume that the signal $s(t)$ is restricted to a frequency band of width B_T, centered at f_c, where B_T is the transmission bandwidth of the FM signal in the absence of amplitude modulation, and that the amplitude modulation is slowly varying compared with $\phi(t)$. Show that the output of an ideal frequency discriminator produced by $s(t)$ is proportional to $a(t)m(t)$. *Hint:* Use the complex notation described in Chapter 2 to represent the modulated wave $s(t)$.

4.21

(a) Let the modulated wave $s(t)$ in Problem 4.20 be applied to an ideal amplitude *limiter*, whose output $z(t)$ is defined by

$$z(t) = \mathrm{sgn}[s(t)] = \begin{cases} +1, & s(t) > 0 \\ -1, & s(t) < 0 \end{cases}$$

Show that the limiter output may be expressed in the form of a Fourier series as follows:

$$z(t) = \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\cos[2\pi f_c t(2n+1) + (2n+1)\phi(t)]$$

(b) Suppose that the limiter output is applied to a band-pass filter with a passband amplitude response of one and bandwidth B_T centered about the carrier frequency f_c, where B_T is the transmission bandwidth of the FM signal in the absence of amplitude modulation. Assuming that f_c is much greater than B_T, show that the resulting filter output equals

$$y(t) = \frac{4}{\pi}\cos[2\pi f_c t + \phi(t)]$$

By comparing this output with the original modulated signal $s(t)$ defined in Problem 4.20, comment on the practical usefulness of the result.

4.22 In this problem we study the idea of mixing in a superheterodyne receiver. To be specific, consider the block diagram of the *mixer* shown in Figure P4.22 that consists of a product modulator with a local oscillator of *variable frequency* f_l, followed by a band-pass filter. The input signal is an AM wave of bandwidth 10 kHz and carrier frequency that may lie anywhere in the range 0.535–1.605 MHz; these parameters are typical of AM radio broadcasting. It is required to translate this signal to a frequency band centered at a fixed *intermediate frequency* (IF) of 0.455 MHz. Find the range of tuning that must be provided in the local oscillator in order to achieve this requirement.

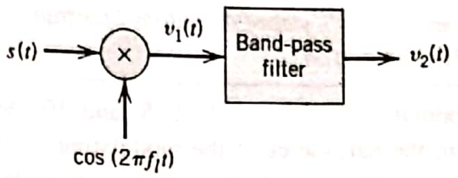

Figure P4.22

4.23 Figure P4.23 shows the block diagram of a *heterodyne spectrum analyzer*. It consists of a variable-frequency oscillator, multiplier, band-pass filter, and root mean-square (rms) meter. The oscillator has an amplitude A and operates over the range f_0 to $f_0 + W$, where f_0 is the mid-band frequency of the filter and W is the signal bandwidth. Assume that $f_0 = 2W$, the filter bandwidth Δf is small compared with f_0, and that the passband amplitude response of the filter is one. Determine the value of the rms meter output for a low-pass input signal $g(t)$.

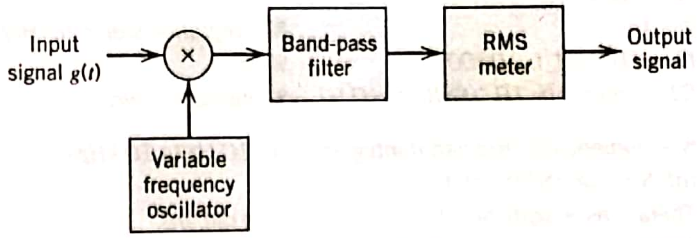

Figure P4.23

4.24 What is the analytical description of the amplitude spectrum corresponding to the Gaussian pulse of the Theme Example in Section 4.7? Justify your answer.

4.25 What are the Carson rule and 1 percent FM bandwidth for GSM? Justify your answer.

Computer Problems

4.26 In this problem, we simulate the spectrum produced by an FM modulator with input $A_m \sin(2\pi f_m t)$. It is suggested that the following Matlab script be used to simulate the FM modulator.

```
fc    = 100;                      % Carrier frequency (kHz)
Fs    = 1024;                     % Sampling rate (kHz)
fm    = 1;                        % Modulating frequency (kHz)
Ts    = 1/Fs;                     % Sample period (ms)
t     = [0:Ts:120];               % Observation period (ms)
m     = cos(2*pi*fm*t);           % modulating signal
beta  = 1.0;                      % modulation index
theta = 2*pi*fc*t+ 2*pi*beta*     % integrate signal
        cumsum(m)*Ts;
s     = cos(theta);
FFTsize = 4096;
S     = spectrum(s,FFTsize);
```