Frequency Response Basics

– *Frequency Response Concept*

– *Frequency Response Definition*

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Frequency Response Concept

Apart from **impulse**, step, ramp and **parabolic** inputs, which are used as **test signals**, dynamical systems also experience, **harmonic inputs** quite frequently.

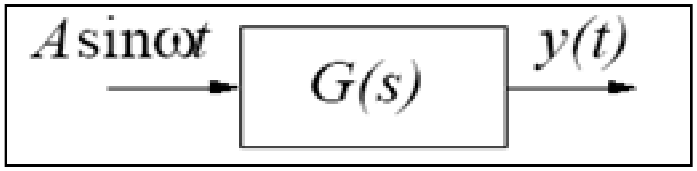
For example, excitations arising from **reciprocating** engines, **rotating** machines, ground/airborne **vibrations** etc. create **harmonic forces.**

Frequency response concept aims to characterize the **behaviour** of LTI systems to such **inputs.**

Frequency Response Definition

Frequency Response Definition

Consider a **system** acted upon by a **sinusoidal input**, as shown below.



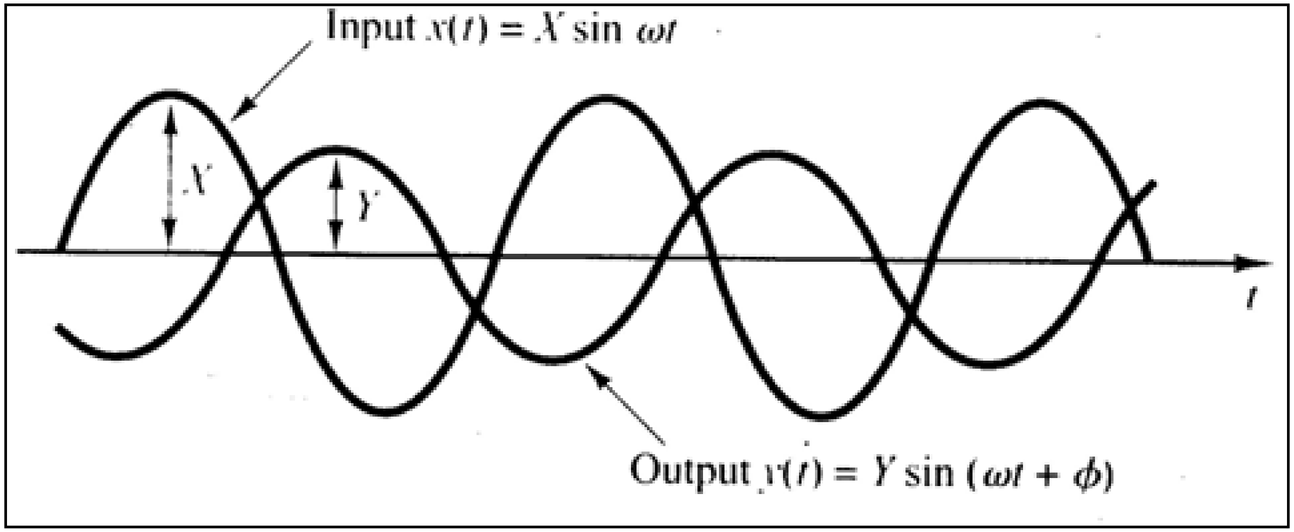
We can **arrive** at the forced **response**, as follows.

$$Y(s) = G(s)\frac{\omega A}{s^2+\omega^2}; \quad G(s) = \frac{N(s)}{(s-p_1)(s-p_2).....(s-p_n)}$$

$$Y(s) = \frac{a}{s+j\omega} + \frac{\bar{a}}{s-j\omega} + \sum_{j=1}^{n}\frac{b_j}{s-p_j}; \quad y(t) = ae^{-j\omega t} + \bar{a}e^{j\omega t} + \sum_{j=1}^{n} b_j e^{p_j t}$$

Steady-state Response Definition

The steady-state **response** is as shown below.



We find that while the **output waveform** has the same **frequency** 'ω', its **magnitude**, 'Y' is '**X|G(jω)|**', and its **phase** is shifted by an angle ϕ **= ∠G(jω).**

Frequency Response Features

Thus, we see that by **taking the ratio** of two waveforms, we can get **$|G(+j\omega)|$** and by **measuring** the phase difference, we can get **$\angle G(+j\omega)$.**

These are **nothing** but the **magnitude and phase** of the system **TF**, G(s), evaluated at **$s = +j\omega$.**

Further, it can be **shown** that substitution of **$s = -j\omega$** generates the **conjugate** of the frequency **response**.

Therefore, complex function, **$G(\pm j\omega)$**, is the system unit **impulse response** as seen along the imaginary **$(\pm j\omega)$** axis.

1^{st} Order System Frequency Response

Consider the **following system.**

$$G(s) = \frac{K}{s+p}$$

Obtain the **frequency response** in terms of **|G(jω)| and ∠G(jω).**

$$G(j\omega) = \frac{K}{j\omega + p} \rightarrow |G(j\omega)| = \frac{K}{|j\omega + p|} = \frac{K}{\sqrt{\omega^2 + p^2}}$$

$$\angle G(j\omega) = \angle K - \angle(j\omega + p) = 0 - \tan^{-1}\left(\omega/p\right) = -\tan^{-1}\left(\omega/p\right)$$

1^{st} *Order System Response Analysis*

It can be seen that **|G(jω)|** is (**K/p**) for $\omega = 0$, while for $\omega = \pm\infty$, it is **zero.**

Thus, for systems with **poles** lying on the origin (**type ≥ 1**), the magnitude becomes **infinite for $\omega = 0$**.

However, in case of **∠G(jω),** it is '0' for **$\omega = 0$**, -90^o for **$\omega = +\infty$** and **$+90^o$** for **$\omega = -\infty$,** resulting in the **conjugate** part of the frequency **response.**

2^{nd} Order System Frequency Response

Consider the **following system.**

$$G(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Obtain its **frequency response.**

$$\left|G(j\omega)\right| = \frac{K}{\left|(j\omega)^2 + j2\zeta\omega_n\omega + \omega_n^2\right|}; \quad \left|G(j\omega)\right| = \frac{K}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + \left(2\zeta\omega_n\omega\right)^2}}$$

$$\angle G(j\omega) = -\angle\left\{(j\omega)^2 + j2\zeta\omega_n\omega + \omega_n^2\right\}; \quad \angle G(j\omega) = -\tan^{-1}\left(\frac{2\zeta\omega_n\omega}{\left(\omega_n^2 - \omega^2\right)}\right)$$

2^{nd} *Order System Response Analysis*

It can be seen that $|G(j\omega)|$ is $(\mathbf{K/2\zeta\omega_n^2})$ for $\omega = \omega_n$ and is $(\mathbf{K/\omega_n^2})$ for $\omega = 0$. We also see that for $\boldsymbol{\omega} = \infty$, $|G(j\omega)|$ becomes **zero.**

However, in case of $\angle\mathbf{G(j\omega)}$**,** we find that it is '0' for $\boldsymbol{\omega} = \mathbf{0}$, while it approaches -90^o as $\boldsymbol{\omega \rightarrow \omega_n}$**.**

Lastly, as $\boldsymbol{\omega} \rightarrow \infty$**,** $\angle G(j\omega) \rightarrow \mathbf{-180^o}$**.**

Frequency Response Representation

We see from **preceding examples** that frequency response **features** not only depend on **G(s)**, but also show varied **characteristics** over the applicable **frequency range.**

Thus, it appears **logical to explore** the variation of **$G(j\omega)$** over the complete **range** of 'ω', i.e. **from '0' to '∞'.**

In this context, it is **worth noting** that, while we can get **analytical** expressions for **$|G(j\omega)|$ & $\angle G(j\omega)$**, these tend to become **unwieldy** as system order **increases.**

In view of the above, **graphical representations** are employed for **better overall view** of the response.

Summary

Frequency response is the **steady state response** of a system to **sinusoidal inputs**.

Frequency response contains the plant **characteristics** as well as their **impact** on the system