Performance Characterization

- *System Performance Requirements*
- *Requirements on Stability*
- *Requirements on Tracking*
- *Requirements on Disturbance Rejection*

Performance Expectations

Dynamical systems are practically **useful** only if they **function** as desired around the **operating point**, for as long as it is **necessary.**

In addition, it is **expected** that systems should withstand **disturbances** of reasonable size, **without** significant **changes** to their **performance.**

Lastly, it is also **desired** that required changes though reference **inputs** should be as per the **specifications**.

Performance Attributes

These broad **expectations** are commonly **stated** in the form of **desirable** dynamical **properties** e.g. **stability**, reference **tracking** and disturbance **rejection.**

Stability pertains to its **ability** to return to its **operating** state, if **disturbed.**

Disturbance **rejection** pertains to the ability to **minimize** / eliminate the **impact** of a disturbance.

Tracking pertains to the ability to **follow** a desired **trajectory** with minimum or no **errors.**

Stability Concept

– *Definition of Stability of Dynamical Systems*

– *Natural & Forced Response Stability*

Concept of Stability

Stability (or instability) is **governed** by the **manner** in which the system **response** evolves over **time.**

Thus, we can **examine** the stability of any **system** by generating its time **response.**

Natural Response Based Stability

Consider the **following response** in 's' domain.

$$C(s) = \frac{N_0(s)}{D(s)} = \frac{A_1}{s - p_1} + \ldots.. + \frac{A_n}{s - p_n}$$

We can employ **partial fractions** method, as shown below.

$$A_k = \left[(s - p_k) \frac{N_0(s)}{D(s)} \right]_{s = p_k} \Rightarrow L^{-1}\left[\frac{A_k}{(s - p_k)} \right] = A_k e^{p_k t}$$

Natural Response Based Stability

We can obtain **c(t)** through **summation**, as shown below.

$$c(t) = \sum_{k=1}^{n} A_k e^{p_k t}$$

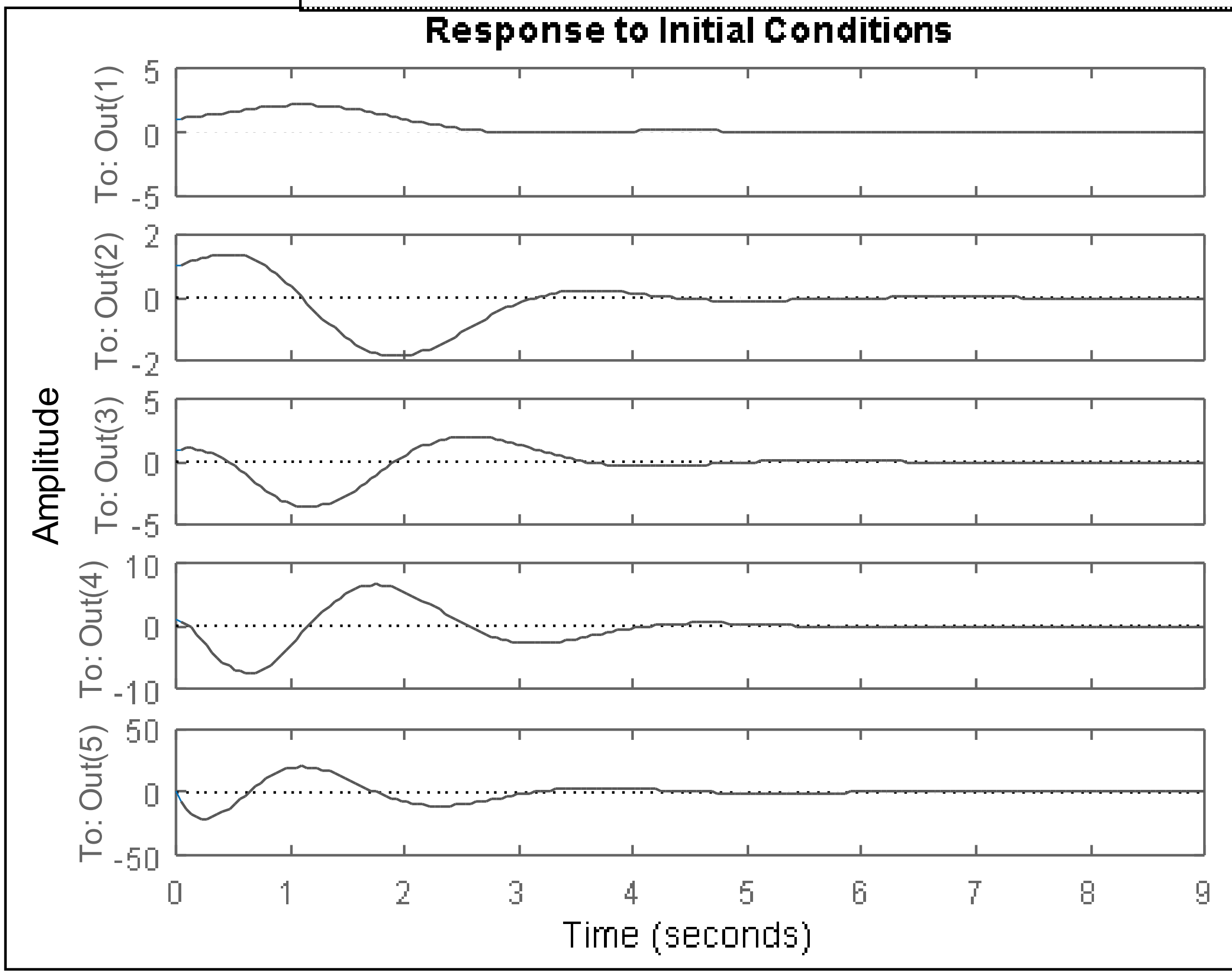
A **system** is said to be **stable** if every natural **response** decays to **zero** as **t** $\rightarrow \infty$.

This means that **real part** of all **poles** must be **negative** or that all **poles** must lie in the **left** half of s – **plane**.

This is also known as **Asymptotic** Stability.

Asymptotically Stable System

$$C(s) = \frac{1}{(s^5 + 6s^4 + 24s^3 + 50s^2 + 71s + 40)}$$



Poles:

-1.5000 + 2.3979i

-1.5000 - 2.3979i

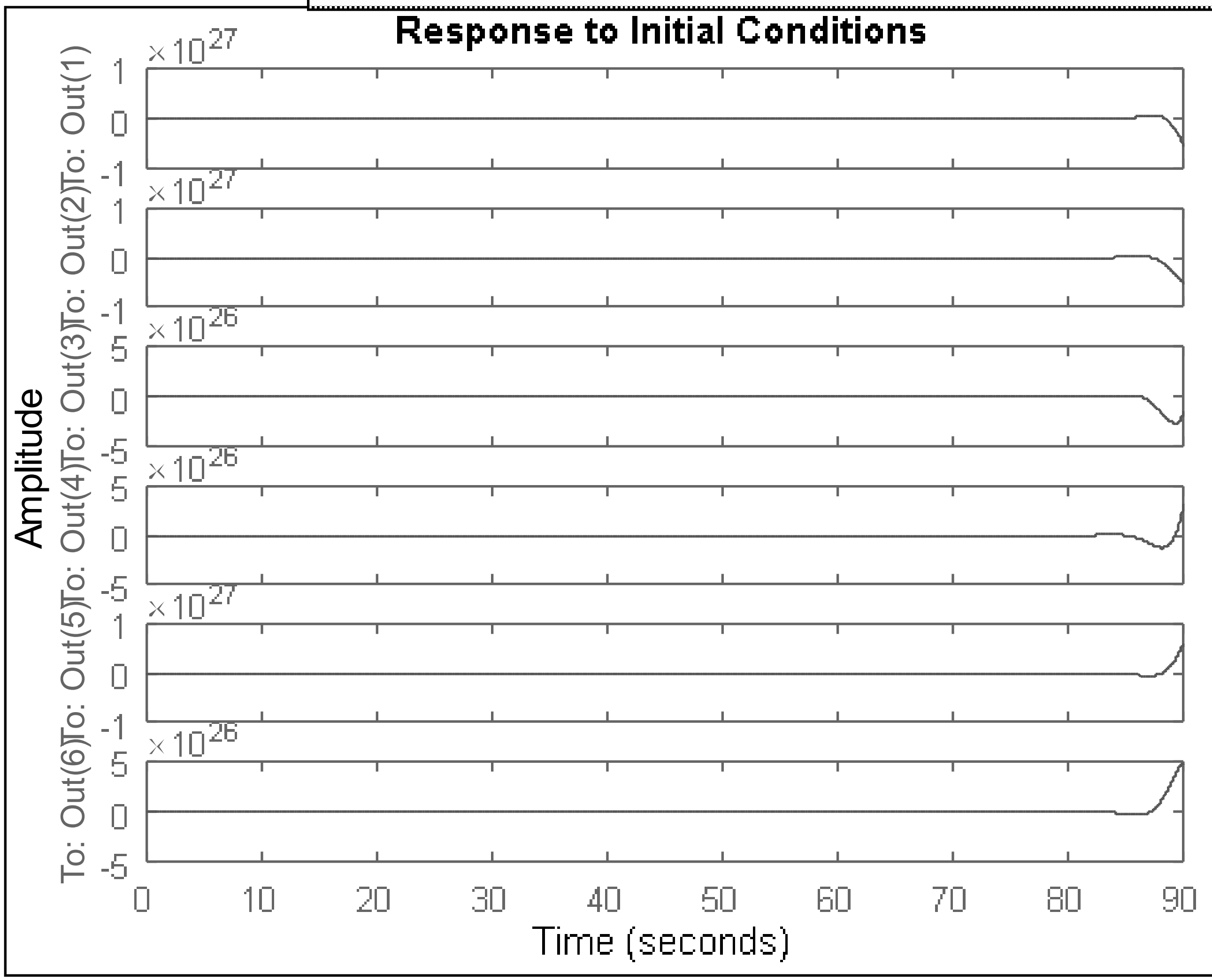
-1.0000 + 2.0000i

-1.0000 - 2.0000i

-1.0000

Unstable System

$$C(s) = \frac{1}{(s^6 + 4s^5 + 3s^4 + 2s^3 + s^2 + 4s + 4)}$$



Poles:

-3.2644

$0.6797 \pm 0.7488i$

$-0.6046 \pm 0.9935i$

-0.8858

Forced Response Based Stability

Forced response or Bounded Input Bounded Output (**BIBO**) Stability is ensured if and only if the response is bounded **as t** $\rightarrow \infty$, for an bounded input.

This can be **formulated** through the **convolution** approach as follows.

$$c(t) = \int_0^t g(t-\tau)u(\tau)d\tau; \quad |u(\tau)| \le M \text{ for any } t \ge 0$$

$$M \rightarrow \text{ Any real number } < \infty$$

Forced Response Based Stability

If the system is **stable**, then **c(t)** must also be **bounded**, as stated below.

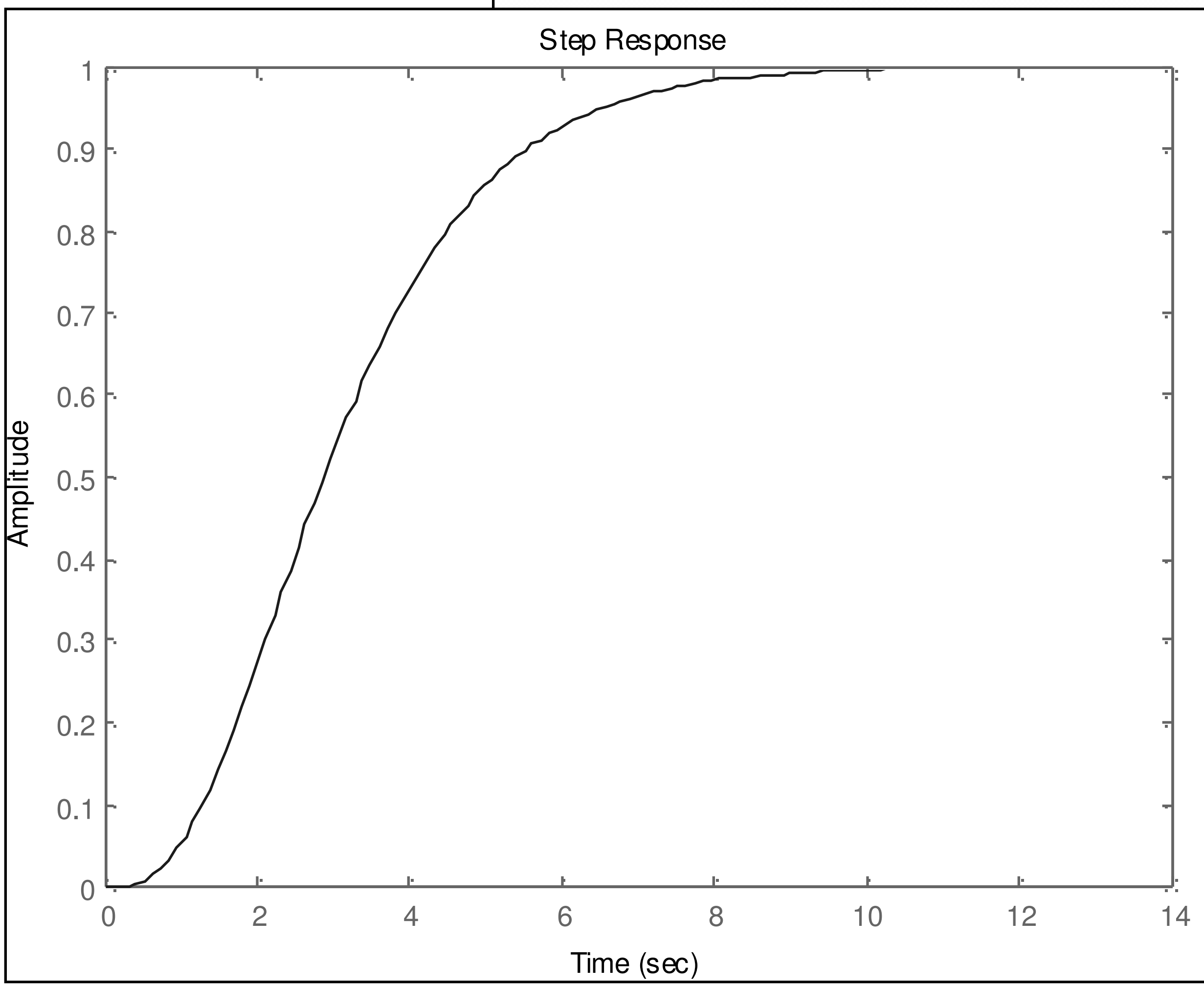
$$|c(t)| \leq |C| \rightarrow M \times \left| \int_0^t g(t-\tau)\,d\tau \right| \leq C \rightarrow \left| \int_0^t g(t-\tau)\,d\tau \right| \leq \frac{C}{M}$$

This means that **all** unit impulse **responses** must be **bounded** for all times **t > 0.**

As we know that **g(t)** is related to **g(0+),** it follows that all **natural responses** also must be **bounded.**

BIBO Stable System

$$C(s) = \frac{5}{s^4 + 8s^3 + 18s^2 + 16s + 5}$$



Poles:

-1

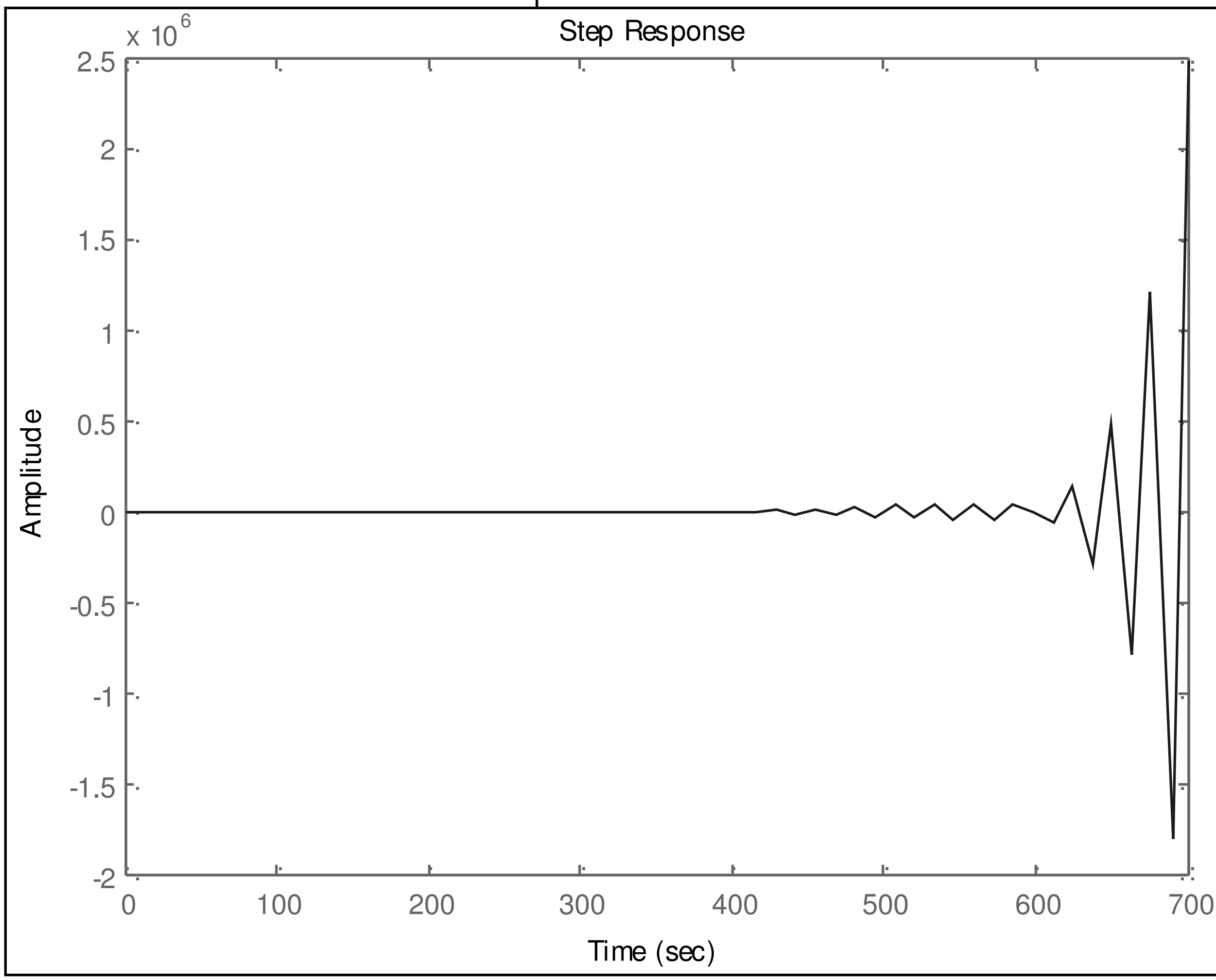
-1

-1

-5

BIBO Unstable System

$$C(s) = \frac{2}{3s^4 + 10s^3 + 5s^2 + 5s + 2}$$



Poles:
-2.93
$0.022 \pm j0.714$
-0.445

Summary

Stability is an important **attribute** of the system, which is indicated by its **behaviour** in terms of pole **locations.**

Concept of Tracking

Tracking relates to the act of **following** an input as **faithfully** as possible.

For example, **aircraft** after taking off, **follows a path** to destination, which is **based** on pre-decided **air routes.**

Similarly, a ground **robot** is given a **path** as input, which it is expected to **adhere**, in order to avoid **collisions.**

Thus, **tracking** is an important **attribute** of any **system.**

Systems with Poor Tracking

Consider **two plants**, subjected to **unit step input**, as given below.

$$G_1(s) = \frac{1}{s+5}; \quad G_2(s) = \frac{1}{s^2+2s+5}; \quad U(s) = \frac{1}{s}$$

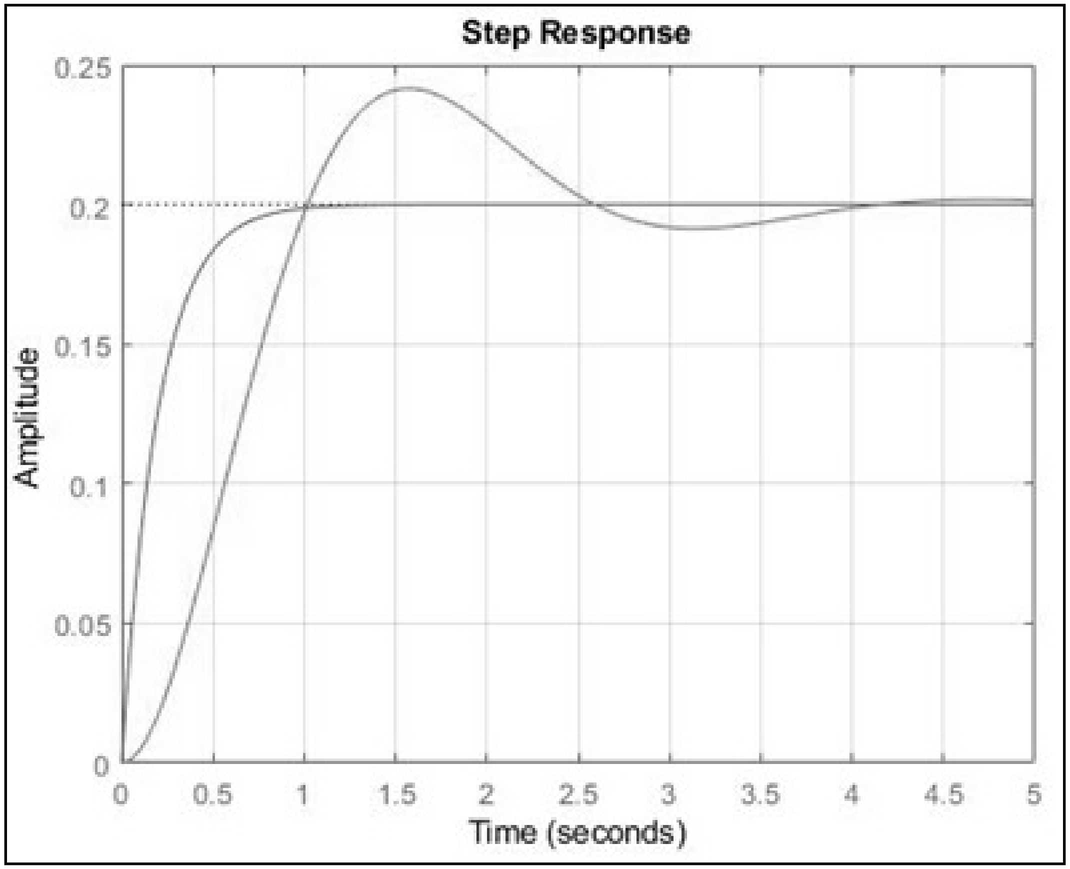
The corresponding **responses** are as follows.

$$y_1(t) = \frac{1}{5}\left(1 - e^{-5t}\right)$$

$$y_2(t) = \frac{1}{5}\left[1 - \frac{2}{\sqrt{5}} e^{-t} \sin\left(2t + 1.11\right)\right]$$

Systems with Poor Tracking

The **tracking** performance is as given **below.**



We see that output is **1/5th** of the input.

Systems with Good Tracking

Consider **two plants**, subjected to **unit step input**, as given below.

$$G_1(s) = \frac{4}{s+5}; \quad G_2(s) = \frac{4}{s^2+2s+5}; \quad U(s) = \frac{1}{s}$$

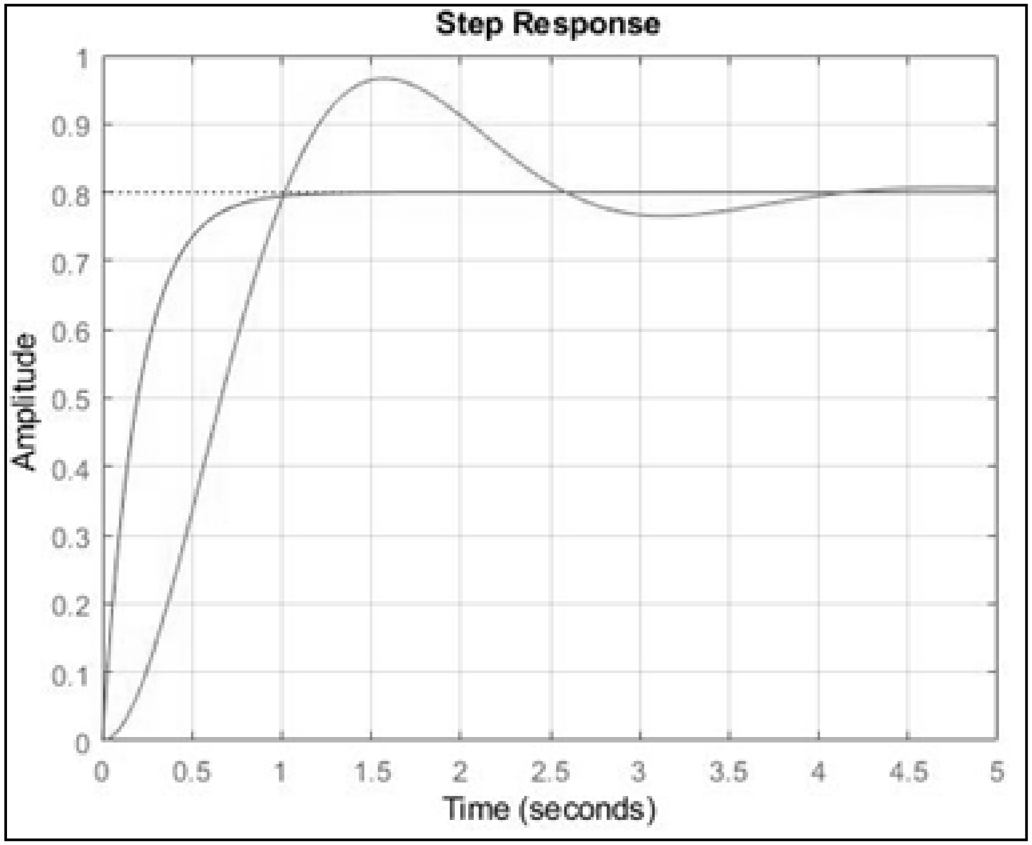
The corresponding **responses** are as follows.

$$y_1(t) = \frac{4}{5}\left(1 - e^{-5t}\right)$$

$$y_2(t) = \frac{4}{5}\left[1 - \frac{2}{\sqrt{5}} e^{-t} \sin\left(2t + 1.11\right)\right]$$

Systems with Good Tracking

The **tracking** performance is as given **below.**



We see that output is 4/**5**$^{\mathbf{th}}$ of the input.

Tracking Performance Analysis

We see that **modified** system shows **better** tracking of the unit **step input.**

Therefore, we **conclude** that the **earlier** plant had **deficiency** with respect to the **tracking task.**

In these **cases** we see that a **simple** multiplication to the numerator **constant** is able to **reduce** the error with respect to the **desired** performance.

However, in a **general** case, we would **need** a structured **methodology** to achieve the desired **tracking.**

Summary

Tracking task involves **following** a desired **input** and its **quality** is rated based on the **error** with respect to the **desired** reference input.

It is **primarily** a steady-state response **attribute.**

Concept of Rejection

Disturbance **rejection** relates to the **ability** to nullify the **impact** of a disturbance in **shortest** possible time and with **least** possible departure from equilibrium.

For example, **aircraft** during **cruise** may experience a **gust**, but it should not **generate** significant departure from its **speed and altitude.**

Thus, disturbance **rejection** is an important **attribute.**

Systems with Poor Rejection

Consider **two plants**, subjected to **unit** impulse (as disturbance), as given below.

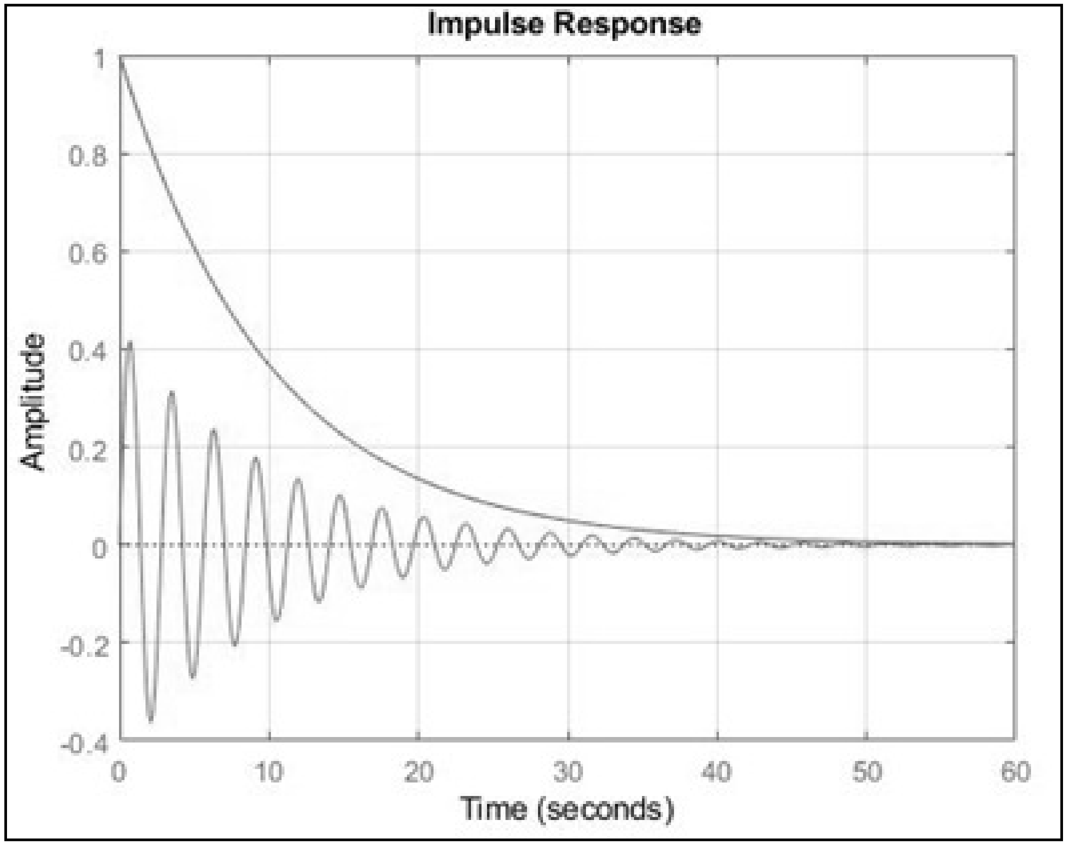
$$G_1(s) = \frac{1}{s + 0.1}; \quad G_2(s) = \frac{1}{s^2 + 0.2s + 5}; \quad U(s) = 1$$

The corresponding **responses** are as follows.

$$y_1(t) = e^{-0.1t}; \quad y_2(t) = \frac{1}{2.23} e^{-0.1t} \sin\left(2.23t + 1.53\right)$$

Systems with Poor Rejection

The **rejection** performance is as given **below.**



We see that **output** takes almost **40s** before **settling.**

Systems with Good Rejection

Consider **two plants**, subjected to **unit impulse**, as given below.

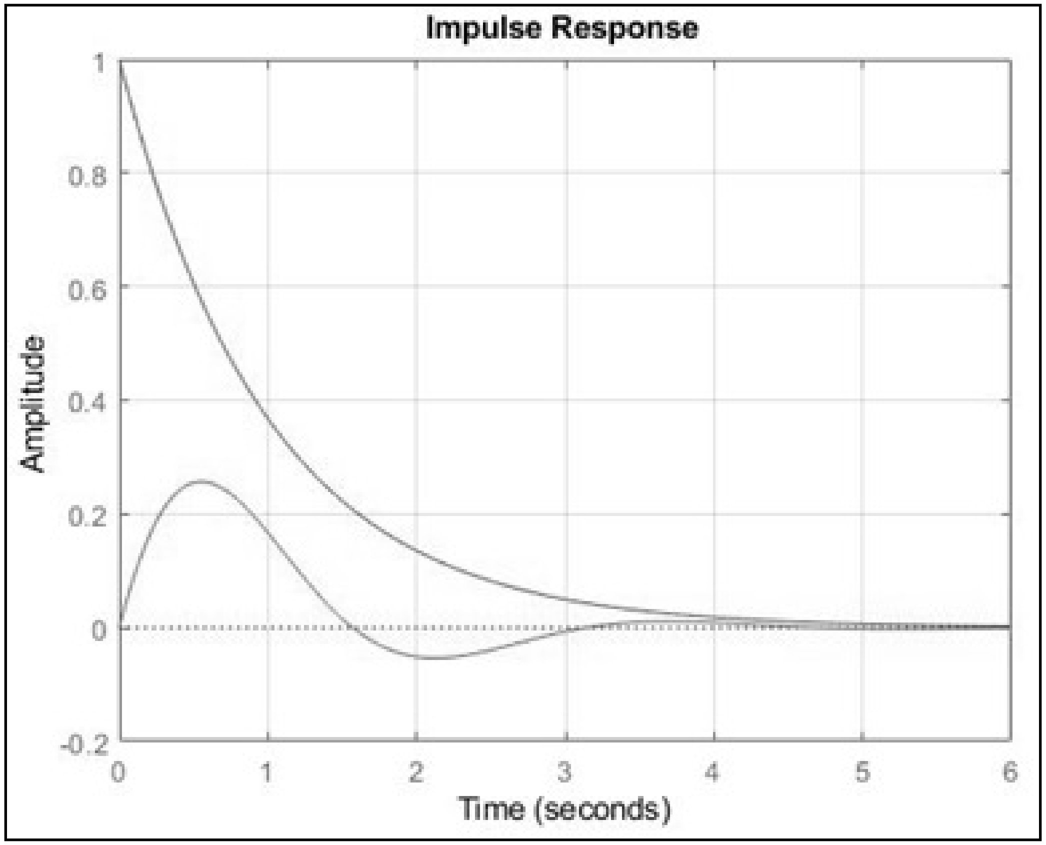
$$G_1(s) = \frac{1}{s+1}; \quad G_2(s) = \frac{1}{s^2+2s+5}; \quad U(s) = 1$$

The corresponding **responses** are as follows.

$$y_1(t) = e^{-t}; \quad y_2(t) = \frac{\sqrt{5}}{2} e^{-t} \sin\left(2t + 1.11\right)$$

Systems with Good Rejection

The **rejection** performance is as given **below.**



We see that **output** settles within **4s.**

Rejection Performance Analysis

We see that **modified** system shows **better** rejection of the unit **impulse.**

Therefore, we **conclude** that the **earlier** plant had **deficiency** with respect to the **rejection task.**

In these **cases** we see that a **more complicated** operation of the **denominator** is needed to **reduce** the departures as well as **settling time.**

However, in a **general** case, we would **need** a structured **methodology** to achieve the desired **rejection.**

Summary

Rejection task aims to **reduce** departures with minimal **time** to achieve the **steady-state** and is **essentially** a transient **response** attribute.