End-Semester Examination Solutions

Communication Systems (EE 308), Autumn'19

Question 1

This is problem 4.13 on p. 142, Haykin, which is part of Homework 4.

Question 2

This is problem 7.21 on p. 277, Haykin, which is part of Homework 8.

Question 3

Similar to the analysis on slide 25 of the lecture slides on "Noise in Analog Modulation", it can be shown that the desired ratio is:

$$I = \frac{(W/f_0)^3}{3c\left[\frac{W}{cf_0} - \tan^{-1}\left(\frac{W}{cf_0}\right)\right]}.$$

Question 4

(a) $f_c = 1800.5$ MHz, $f_{LO} = f_c + f_{IF} = 1800.5 + 250 = 2050.5$ MHz. The LO frequency can be synthesized by tuning the frequency synthesizer to 2050.5 MHz. The RF filter must pass the frequency band $1800 - 1801$ MHz and reject the band of width 1 MHz around the image frequency $f_c + 2f_{IF} = 1800.5 + 500 = 2300.5$ MHz. The IF filter must have a center frequency of 250 MHz, pass the band of width 1 MHz around 250 MHz and reject frequencies outside the band $(248.5 \text{ MHz}, 251.5 \text{ MHz})$.

(b) $f_c = 900.5$ MHz, $f_{LO} = f_c + f_{IF} = 900.5 + 250 = 1150.5$ MHz. The LO frequency can be synthesized by tuning the frequency synthesizer to $1150.5 \times 2 = 2301$ MHz and dividing the frequency by a factor of 2. The RF filter must pass the frequency band $900 - 901$ MHz and reject the band of width 1 MHz around the image frequency $f_c + 2f_{IF} = 900.5 + 500 = 1400.5$ MHz. The IF filter must have the same characteristics as the one in part (a).

Question 5

This is problem 7.17 on pp. 276-277, Haykin, which is part of Homework 8.

QUESTION 6

This is problem 4.73 on p. 216, Proakis.

QUESTION 7

(a) The random variable $X(t)$ is normal with zero mean and variance $E(X^2(t)) = R_X(0) = 4$, hence $P(X(t) \leq 3) = \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{3} e^{-x^2/8} dx$.

(b)

$$E\left\{[X(t+1) - X(t-1)]^2\right\} = 2(R_X(0) - R_X(2)) = 8(1 - e^{-4}).$$

QUESTION 8

$X(0) = 1$. So $E(X(0)) = 1$.

$$X\left(\frac{\pi}{\omega_{max}}\right) = \cos\left(\pi\left(\frac{\omega}{\omega_{max}}\right)\right).$$

Now, $\cos\left(\pi\left(\frac{\omega}{\omega_{max}}\right)\right) < 1$ for all $\omega \in (0, \omega_{max}]$. So $P\left(X\left(\frac{\pi}{\omega_{max}}\right) < 1\right) = 1$. Hence, $E\left(X\left(\frac{\pi}{\omega_{max}}\right)\right) < 1$. Thus, $E(X(t))$ is not a constant, and hence $X(t)$ is not WSS.

QUESTION 9

This is problem 7.2 on p. 275, Haykin, which is part of Homework 8.

QUESTION 10

The output range $[-1, 1]$ of the compressor is divided into $2^8 = 256$ steps. So the output step size is $\frac{2}{256} = \frac{1}{128}$ V. The smallest input step size occurs near $x = 0$, i.e., between $y_1 = 0$ and $y_2 = 1/128$. So $0 = \frac{\ln(1+255x_1)}{\ln(1+255)}$ giving $x_1 = 0$ and $\frac{1}{128} = \frac{\ln(1+255x_2)}{\ln(1+255)}$ giving $x_2 = 1.736 \times 10^{-4}$. So the smallest step size is $10(x_2 - x_1) = 1.736$ mV.

The largest input step size occurs between $y_1 = 1 - \frac{1}{128}$ and $y_2 = 1$. So $\frac{127}{128} = \frac{\ln(1+255x_1)}{\ln(1+255)}$ giving $x_1 = 0.9574$ and $1 = \frac{\ln(1+255x_2)}{\ln(1+255)}$ giving $x_2 = 1$. So the largest step size is $10(x_2 - x_1) = 0.4256$ V.

The step size for uniform quantization is $\frac{20}{256} = 0.078125$ V.

QUESTION 11

(a) The frequency-domain trapezoid corresponds to $p(t) = \text{sinc}(at)\text{sinc}(bt)$ in the time-domain, where $(a-b)/2 = 4$ and $(a+b)/2 = 10$. On solving, we obtain $a = 14$ MHz and $b = 6$ MHz. Thus, the time-domain pulse provides zeros at rates 14 MHz and 6 MHz; hence, it is indeed Nyquist at rate 14 Msymbols/s. The statement is therefore *true*.

(b) The symbol rate $1/T = 25$ Mbps/ (2 bits/symbol) = 12.5 Msymbols/s. This is not an integer multiple of either 14 MHz or 6 MHz, the rates at which zeros are provided by the two sinc factors. Thus, the Nyquist property does not hold, and the statement is *false*.

QUESTION 12

This is problem 3.17 on p. 135, Proakis.