General LTI Systems

- *LTI System Definition*
- *Basic Response Definition*

Analysis & Design Methodology

In general, all control **problems** are **stated** in terms of achieving the **desired system performance**.

These requirements are **translated** into **desired system behaviour** (or response) under **operating conditions**.

Thus, analysis and design **techniques** make extensive use of **system responses** for solving control problems.

Role of Response in Control Studies

Control analysis & design involves arriving at **control element** parameters, for a given system and **objectives**, which are specified in terms of **desired output features**.

This necessitates **characterization** of the response and, in this context, **linear** description of the **dynamical** systems (LTI) is considered to be **adequate.**

LTI Systems & Their Response

LTI System Description

LTI (or Linear Time-invariant) systems are those that are described by **linear differential equations** having constant coefficients. **General form** of such systems is as below.

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y \;=\; b_0 \frac{d^m u}{dt^m} + \cdots + b_m u$$

Here, '**y**' is the output and '**u**' is the input. Terms, '$\mathbf{a_1}$' to '$\mathbf{a_n}$' and '$\mathbf{b_1}$' to '$\mathbf{b_m}$' are constants.

Further, '**n**', which is the **highest degree** of derivative of 'y', is called the **order** of the system.

Linearity and Time Invariance

- Linear
  - Satisfies superposition and homogeneity $\boxed{\text{Addition, Scaling}}$
  - Coefficients independent of output, input and their derivatives
- Time invariant
  - Delayed input produces same response with same delay
  - Coefficients independent of t

LTI System Response – Natural

- Response when input = 0
- Determined solely by initial conditions

$$y(0), \frac{dy}{dt}(0), \frac{d^2y}{dt^2}(0), \cdots, \frac{d^{n-1}y}{dt^{n-1}}(0)$$

- Satisfies

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_n y = 0$$

$$\frac{dy}{dt} + a_1 y = 0; \quad y(t) = Ae^{\lambda t}; \quad Ae^{\lambda t}(\lambda + a_1) = 0$$

$$\lambda = -a_1; \quad y(t) = Ae^{-a_1 t}; \quad y(t=0) = y_0; \quad y(t) = y_0 e^{-a_1 t}$$

LTI System Response - Forced

- Response when initial conditions = 0
- Satisfies

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y = b_0 \frac{d^m u}{dt^m} + \cdots + b_m u$$

$$y(0) = 0, \frac{dy}{dt}(0) = 0, \frac{d^2 y}{dt^2}(0) = 0, \cdots, \frac{d^{n-1} y}{dt^{n-1}}(0) = 0$$

$$\frac{dy}{dt} + a_1 y = u(t); \quad \frac{d}{dt}\left(e^{a_1 t} y(t)\right) = e^{a_1 t}\left(\frac{dy}{dt} + a_1 y(t)\right)$$

$$y(t) = e^{-a_1 t}\left(\int u(\tau) e^{a_1 \tau} d\tau\right) \rightarrow \text{ Depends on nature of } u(t)$$

Summary

LTI systems are useful in arriving at system **behaviour** in a simple manner and also **help in synthesizing** acceptable control elements for **practical cases.**