

1 Hilbert Transform

Intuitively, the Hilbert transform of a signal applies a phase change of 90 degrees. Formally the Hilbert transform $\hat{x}(t)$ of a signal $x(t)$ is the output corresponding to the input $x(t)$ of the LSI system defined by the following frequency response.

$$H(f) = \begin{cases} -j & f > 0 \\ 0 & f = 0 \\ j & f < 0 \end{cases}$$

Convince yourself that the Hilbert transform corresponding to $\cos(t)$ is $\sin(t)$.

Q: What is the Fourier transform of $x(t) + j\hat{x}(t)$?

Q: What is the Fourier transform of $x(t) - j\hat{x}(t)$?

2 Hilbert Transform in GNURadio

The Hilbert Transform block in GNURadio generates the complex output $x(t) + j\hat{x}(t)$ corresponding to the input $x(t)$. In other words, for input real $x(t)$, the imaginary part of the block output is the actual Hilbert transform of $x(t)$.

Q: Is the Hilbert transform of a real signal guaranteed to be real?