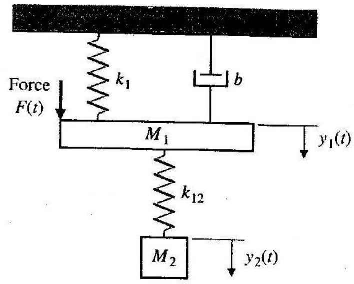
AE775/308 Quiz No. 1 Date: Monday 26th Aug, 2019 Time: 1405-1520
Weightage: 10% Closed Notes (Only calculator is allowed.)
Model Solutions

1 Obtain the applicable differential equation relating output 'y_1' and input 'F(t)', for the dynamic vibration absorber given below. (Hint: use d/dt ≡ D along with algebraic manipulations to find the equation) (2)



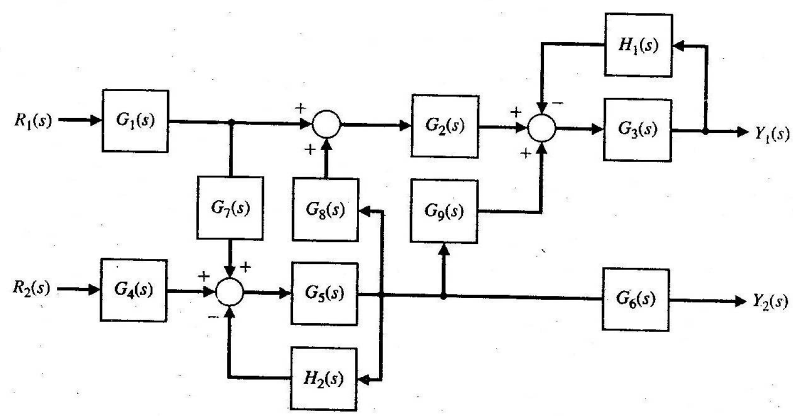
$$\text{Mass } M_1: \quad M_1\ddot{y}_1 + b\dot{y}_1 + k_1 y_1 + k_{12}\left(y_1 - y_2\right) = F; \quad \text{Mass } M_2: \quad M_2\ddot{y}_2 + k_{12}\left(y_2 - y_1\right) = 0$$

$$y_2 = -\left(\frac{k_{12}}{M_2 D^2 + k_{12}}\right) y_1; \quad \left(M_1 D^2 + bD + k_1 + k_{12}\right) y_1 + \left(\frac{k_{12}^2}{M_2 D^2 + k_{12}}\right) y_1 = F$$

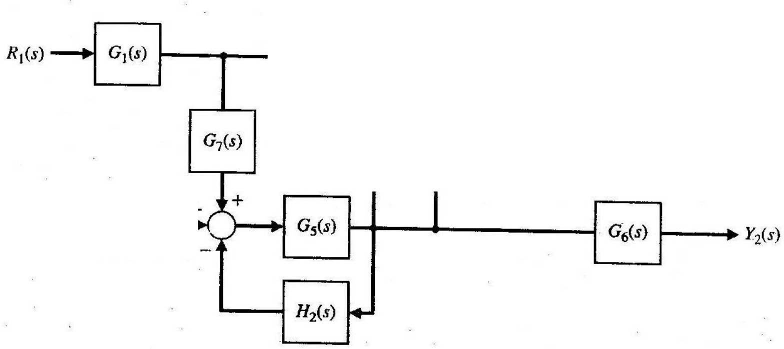
$$\left[\left(M_1 D^2 + bD + k_1 + k_{12}\right) \times \left(M_2 D^2 + k_{12}\right) + k_{12}^2\right] y_1 = \left(M_2 D^2 + k_{12}\right) F$$

$$M_1 M_2 \ddddot{y}_1 + M_2 b\dddot{y}_1 + \left(M_1 k_{12} + M_2\left\{k_1 + k_{12}\right\}\right)\ddot{y}_1 + bk_{12}\dot{y}_1 + \left(k_1 k_{12} + 2k_{12}^2\right) y_1 = M_2\ddot{F} + k_{12}F$$

2. Reduce following block diagram and obtain Y_2/R_1. (Hint: Assume $R_2 = 0$). (2)



Step-1: $R_2 = 0$, remove G_4 branch from the summing junction.
Step-2: Split the summing junction of step-1 & reduce the block formed by G_5 & H_2.
Step-3: Reduce the block containing G_1, G_7, $G_5/(1+G_5H_2)$ & G_6.



$$\frac{Y_2}{R_1} = \frac{G_1 G_5 G_6 G_7}{1 + G_5 H_2}$$

3. Obtain y(t) of the following 2^{nd} order system, using convolution integral, for given initial conditions. (2)

$$\ddot{y} + 4y = 3u(t), \quad u(t) = 1(t); \quad y(0) = 0; \quad \dot{y}(0) = 1$$

Hint: $y(t) = y(0) + \int_0^t g(t-\tau)u(\tau)d\tau$; $g(t) = \frac{1}{m\omega_d} e^{-\sigma t} \sin \omega_d t \rightarrow$ Unit Impulse Response; $1(t) \rightarrow$ Unit Step

$2\sigma = \frac{c}{m}$; $\omega_n^2 = \frac{k}{m}$; Interpret m, c, k, ω_d as per their standard meaning for a 2^{nd} order LTI system.

$$\ddot{y} + 4y = 3u(t), \quad m = 1; \quad c = 0; \quad k = 4 \rightarrow \sigma = 0; \quad \omega_d = \omega_n = 2; \quad g(t) = \frac{1}{m\omega_d} e^{-\sigma t} \sin \omega_d t = \frac{1}{2} \sin 2t$$

$$y_h(t) = g(t) = \frac{1}{2} \sin 2t \quad \{g(t) = g(0+) \text{ as unit impulse same as unit velocity as initial condition}\}$$

$$y(t) = y_h(t) + \frac{1}{2}\int_0^t \sin 2(t-\tau) \times 3d\tau; \quad y(t) = \frac{1}{2}\sin 2t + \frac{3}{2}\int_0^t (\sin 2t \cos 2\tau - \cos 2t \sin 2\tau)\, d\tau$$

$$y(t) = \frac{1}{2}\sin 2t + \frac{3}{2}\sin 2t \int_0^t \cos 2\tau d\tau - \frac{3}{2}\cos 2t \int_0^t \sin 2\tau d\tau$$

$$y(t) = \frac{1}{2}\sin 2t + \frac{3}{2}\sin 2t \times \left[\frac{1}{2}\sin 2\tau\right]_0^t - \frac{3}{2}\cos 2t \times \left[-\frac{1}{2}\cos 2\tau\right]_0^t$$

$$y(t) = \frac{1}{2}\sin 2t + \frac{3}{4}\sin^2 2t + \frac{3}{4}\cos 2t \times (\cos 2t - 1) = \frac{3}{4} + \frac{1}{2}\sin 2t - \frac{3}{4}\cos 2t$$

4. Consider the system transfer function given below. (3)

$$G(s) = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

Obtain the unit step response y(t) using the partial fractions decomposition approach. (Hint: Use the following rules as applicable).

$$A_i = \left[(s - p_i)Y(s)\right]|_{s=p_i}; \quad L^{-1}\left[\frac{A_i}{s + p_i}\right] = A_i e^{-p_i t}; \quad \text{One of the roots is -3.}$$

$$Y(s) = U(s)G(s) = \frac{24}{s\left(s^3 + 9s^2 + 26s + 24\right)}$$

$$Y(s) = \frac{24}{s(s+3)\left(s^2 + as + b\right)} = \frac{24}{s(s+3)\left(s^2 + 6s + 8\right)} = \frac{24}{s(s+3)(s+2)(s+4)}$$

$$Y(s) = \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+3} + \frac{A_4}{s+4}; \quad A_1 = 1; \quad A_2 = -6; \quad A_3 = 8; \quad A_4 = -3$$

$$y(t) = L^{-1}(Y(s)) = 1 - 6e^{-2t} + 8e^{-3t} - 3e^{-4t}$$

5. Obtain the expressions for magnitude and phase (in degrees) of the following system, subjected to the sinusoidal input and give their values for ω = 1.99, 2.01. (1)

$$G(s) = \frac{4}{s^2 + 4}; \quad u(t) = 2\sin \omega t; \quad s = j\omega$$

$$G(s)=\frac{4}{s^2+4}; \quad G(j\omega)=\frac{4}{(j\omega)^2+4}=\frac{4}{4-\omega^2}$$

$$\left|G(j\omega)\right|=\left|\frac{4}{4-\omega^2}\right|=\frac{4}{\left|4-\omega^2\right|}; \quad \left|G(j1.99)\right|=100.25; \quad \left|G(j2.01)\right|=99.75$$

$$\angle G(j\omega)=\angle\frac{4}{4-\omega^2}=\tan^{-1}\frac{0}{4}\times\left(4-\omega^2\right); \quad \angle G(j1.99)=\tan^{-1}\frac{0}{4}\times 0.04=0^o$$

$$\angle G(j2.01)=\tan^{-1}\frac{0}{4}\times(-0.04)=180^o$$

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