Fundamentals of electromagnetics important for mechatronic actuators

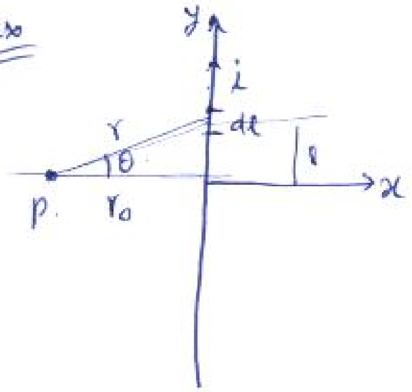
→ Biot Savart law

$$\int dB = \frac{\mu}{4\pi}\int \frac{I\, \vec{dl} \times \vec{e_r}}{r^2}$$

dl = elementary length of conductor carrying current I

$\vec{e_r}$ = unit vector from dl to P

ex



Field at point P can be obtained by BS law. Current carrying conductor placed at y axis is responsible for generating this magnetic field.

$$\vec{dl} = dl\,\vec{j}$$

$$\vec{e_r} = -\cos\theta\,\vec{i} - \sin\theta\,\vec{j}$$

$$\vec{dl} \times \vec{e_r} = [-\cos\theta(\vec{i}\times\vec{j}) - \sin\theta\,\vec{j}\times\vec{j}\,]\,dl$$

$$= -\cos\theta\,\vec{K}\,dl$$

$$\therefore \quad B \text{ at point } P = \frac{\mu I}{4\pi}\int_{-\infty}^{\infty} \frac{-\cos\theta\, dl}{r^2}$$

length of current carrying conductor is ∞. Hence $\theta = -\pi/2 \to \pi/2$.

$$r = r_0/\cos\theta \qquad r^2 = \frac{r_0^2}{(\cos\theta)^2}$$

$$l = r_0 \tan\theta$$

$$dl = r_0 \sec^2\theta\, d\theta$$

$$\therefore \quad B = \frac{\mu I}{4\pi}\int_{-\pi/2}^{\pi/2} \frac{-\cos\theta}{r_0^2}\cdot \cos^2\theta\,(r_0 \sec^2\theta\, d\theta)$$

$$= \frac{\mu I}{4\pi}\int_{-\pi/2}^{\pi/2} \frac{-\cos\theta}{r_0}\cdot d\theta$$

$$B = \frac{\mu I}{4\pi r_0}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

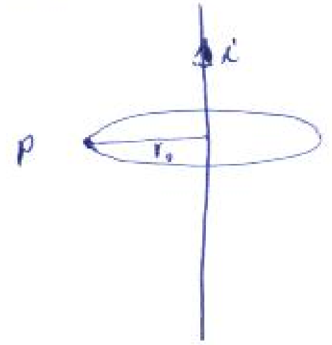
$$\boxed{B = \frac{\mu I}{2\pi r_0}}$$

Ampere's Law

Integral of a magnetic field over a closed path is equal to the current passing through the area covered by the closed path times the permeability of medium covered by the closed path of integration.

$$\oint_C \vec{B}.\vec{ds} = \mu . i$$

ex Previous case



B along a circle of radius r_0 will be constant (see fig). Hence we consider this circle as closed path.

$\therefore \quad \vec{B}.\vec{ds} = B.2\pi r$ since $\vec{ds}$ along the direction of B.

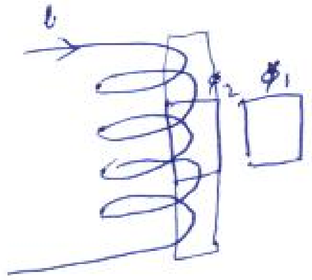
$$2\pi r_0 B = \mu i$$

$$\boxed{B = \frac{\mu I}{2\pi r_0}}$$ same as before.

⇒ Although both Ampere's law & Biot Savart law are always true (for steady currents), the use of one law as against the other should be made as per the convenience, ~~it~~ for ex. only for problems with symmetry (as above) B can be pulled out of the integral.

⇒ Also when path traced ~~goes through~~ passes through multiple media, we use the method of reluctance for analysing magnetic circuits.

ex



Coil carrying current i. consider loop ϕ_1 → No current passes through the area of loop → No B ~~along~~ along the loop. Considering many such loops outside the coil

⇒ flux is zero outside the coil.

Also by similar arguments

⇒ flux is uniform inside the coil.

$\oint_{\phi_2} \vec{B}.\vec{ds} = \mu i \qquad B.l = \mu N i \qquad \boxed{B = \frac{\mu N i}{l}}$

Lorentz force

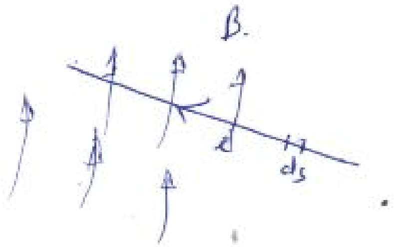
Current carrying conductor placed in magnetic field.

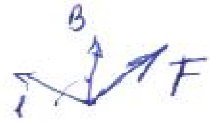
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$= q\frac{d\vec{s}}{dt} \times \vec{B}$$

$$= \vec{s}\,\frac{dq}{dt} \times \vec{B}$$

$$\boxed{\vec{F} = i \cdot \vec{s} \times \vec{B}}$$





$\Rightarrow$ Fundamental principle of motors.

Electromagnetic Circuits

Reluctance $\mathcal{R} = \dfrac{l}{\mu A}$ Similar to concept of resistance $R = \dfrac{\rho l}{A}$.

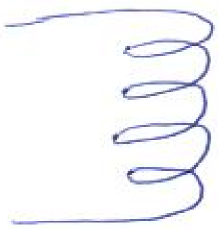
Flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$ (weber or Tesla.m^2)

$H = \dfrac{\vec{B}}{\mu}$ Magnetomotive force $= H.l$

Magnetic field strength.

$MMF = \dfrac{\vec{B}}{\mu}.l$ — equivalent to voltage.

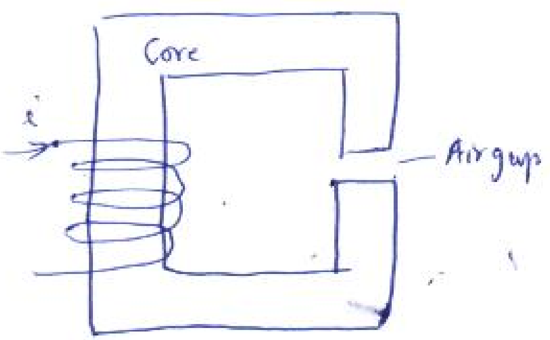
For example for coils we saw that $B = \dfrac{\mu N i}{l}$ → inside coil.



$MMF = \dfrac{\mu N i}{\cancel{l}\,\cancel{\mu}}.\cancel{l} = \underline{\underline{Ni}}$

example magnetic circuit

$\mathcal{R}_{core} = \dfrac{l_c}{\mu_c A_c}$ $\mathcal{R}_g = \dfrac{l_g}{\mu_0 A_g}$

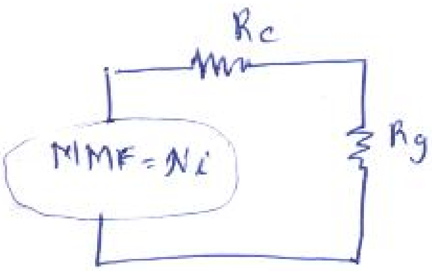


$\Phi_B = \dfrac{Ni}{\mathcal{R}_{total}}$ $\mathcal{R}_{total} = \mathcal{R}_c + \mathcal{R}_g$

$= \dfrac{l_c}{\mu_c A_c} + \dfrac{l_g}{\mu_0 A_g}$

$\mu_c >>> \mu_0$

$\mathcal{R}_{total} = \dfrac{l_g}{\mu_0 A_g}$



$\Phi_B = \dfrac{Ni}{l_g}.\mu_0 A_g$

$$\boxed{\Phi_B = \frac{N i \mu_0 A_g}{l_g}}$$ — Flux ~~through~~ through the circuit.

Flux linkage

When a conductor is placed in magnetic field ~~typically~~ flux links with the conductor flux linkage = $N\phi_B$. for N turns coils.

More flux linkage more resistance to current change will be created in the coil. according to Faraday's law, For change of flux

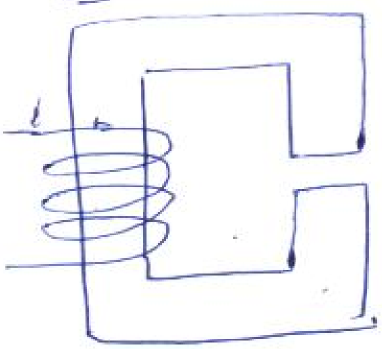
$$\frac{d\phi}{dt} = L \cdot \frac{di}{dt} = \text{Emf induced in coil.}$$

$$N\frac{d\phi_B}{dt} = L\frac{di}{dt}$$

$$N\phi_B = Li$$

~~for~~

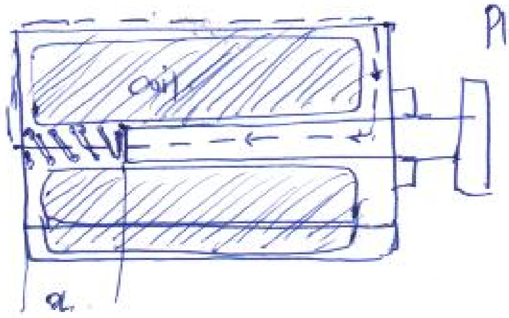
Ex previous magnetic circuit for coil



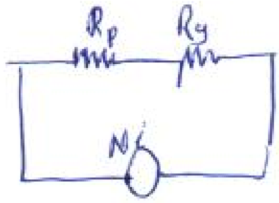
$$\phi_B = \frac{N i \mu_0 A}{l_g}$$

$$L\cancel{i} = \frac{N^2 \cancel{i} \mu_0 A}{l_g} \quad \boxed{L = \frac{N^2 \mu_0 A_g}{l_g}}$$ self inductance

Application to solenoid case :



Plunger.



equivalent magnetic circuit ~~equivalent~~

$$R_p = \frac{l_p}{\mu_p A_p} \qquad R_g = \frac{l_g}{\mu_0 A_g}$$

$\mu_p >>> \mu_0 \Rightarrow R_p = 0.$

$l_g = x$

$$\Phi_B = \frac{NI}{R_t} = \frac{NI\mu_0 A}{x}$$

flux linkange to coil $= N\phi_B$

$$\lambda = \frac{N^2 I \mu_0 A}{x}$$

$$\lambda = Li \qquad \boxed{L = \frac{N^2 \mu_0 A}{x}}$$

Lagrange formulation for solenoid with spring loaded plunger

$$KE = \frac{1}{2} Li^2 + \frac{1}{2} m\dot{x}^2$$

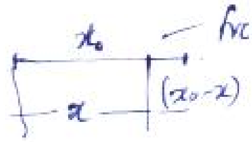
m = mass of plunger

$\dot{x}$ = velocity of plunger.

$$T = KE = \frac{1}{2} \frac{c_L}{x} i^2 + \frac{1}{2} m\dot{x}^2$$

$c_L = N^2 \mu_0 A$. $\quad L_x = \frac{c_L}{x}$

$$V = PE = \frac{1}{2} kx_1^2$$
$$= \frac{1}{2} k(x_0 - x)^2$$



x_0 — free length of spring.

$$\mathcal{L} = T - V$$
$$= \left(\frac{1}{2} \frac{c_L}{x} i^2 + \frac{1}{2} m\dot{x}^2\right) - \left(\frac{1}{2} k(x_0 - x)^2\right) \qquad i = \dot{q}$$

Generalised coordinates q & x. $\quad i = \dot{q}$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\frac{1}{2}\frac{c_L}{x}(2\dot{q})\right) = \frac{c_L}{x}\ddot{q} + \frac{c_L \dot{q}}{x^2}(-\dot{x})$$

$$\frac{\partial \mathcal{L}}{\partial q} = 0.$$

V_1 = external force in direction of q

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = V_1$$

$$\therefore \quad \frac{C_L}{x}\ddot q + \frac{C_L \dot q}{x^2}(-\dot x) = V_1$$

$$\therefore \quad \frac{C_L}{x}\frac{di}{dt} - \frac{C_L i}{x^2}\dot x = V_1$$

$$\boxed{V_1 = L_x \frac{di}{dt} - \frac{L_x}{x} i \dot x}$$

Neglecting resistive loss in circuit.

$$\boxed{V_1 = L_x \frac{di}{dt} - \frac{L_x}{x} i \dot x + Ri}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot x}\right) = \frac{d}{dt}\left(\frac{1}{2}m(2\dot x)\right) = m\ddot x$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2}\cdot\frac{C_L}{x^2}(-1)i^2 - \frac{1}{\cancel{2}} K\cancel{2}(x_0 - x)\cdot(-1)$$

$$= -\frac{1}{2}\frac{C_L i^2}{x^2} + K(x_0 - x)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot x}\right) - \frac{\partial \mathcal{L}}{\partial x} = 0.$$

$$m\ddot x + \frac{1}{2}\frac{C_L i^2}{x^2} - K(x_0 - x) = 0$$

$$\boxed{m\ddot x + Kx + \frac{C_L i^2}{2x^2} - Kx_0 = 0.}$$

Lagrange formulation for motor (PMDC)

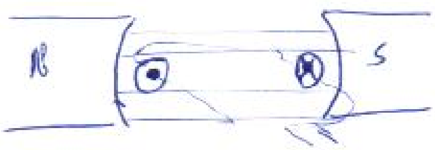
$$KE = \frac{1}{2} L i^2 + \frac{1}{2} J \dot{\theta}^2$$

J = inertia of motor

L = inductance $= L_p + L_s$.

Now inductance is result of flux linkages. There are two kinds of flux linkages = (1) from self magnetic field - Self inductance - L_s.

(2) from permanent magnet of motor - L_p.



Flux ~~linkage~~ Linkage due to permanent magnet $= 2\Phi_p N = \lambda_p$.
(Both sides of coil are in magnetic field)

Inductance due to this λ_p is L_p.

$\Phi_p = \theta \propto B$

$$\lambda_p = 2\Phi_p N = L_p i$$

$$L_p = \frac{2\Phi_p N}{i} \qquad i = \dot{q} \qquad q = \text{charge}$$

$$T = \frac{1}{2}\left(L_s + \frac{2\Phi_p N}{\dot{q}}\right)\dot{q}^2 + \frac{1}{2} J\dot{\theta}^2 = \frac{1}{2} L_s \dot{q}^2 + \theta\cdot\beta\,\dot{q}$$

losses $= \frac{1}{2} B\dot{\theta}^2$ damping = c

$V = 0$.

$L = T - V = T$ Generalised coordinates = q & θ

β = const.

$N\phi_p = \theta \propto B = \theta\cdot\beta$

$N\phi_p = NB.A$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \cancel{\frac{d}{dt}\left[\frac{1}{2}\, 2\left(L_s + \frac{2\phi_p N}{\dot{q}}\right)\cdot\left(\frac{-2\phi_p N}{\dot{q}^2}\right)\right]}$$

$$= \frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}}\left[\frac{1}{2} L_s \dot{q}^2 + \frac{2\phi_p N}{2}\dot{q}\right]\right)$$

$$= \frac{d}{dt}\left(\frac{1}{2} L_s (2\dot{q}) + \theta\beta\right)$$

$$= L_s \ddot{q} + \beta\cdot\dot{\theta}$$

Back emf, losses, Voltage applied

$$L_s \frac{di}{dt} + \beta\dot{\theta} + R i = V_1$$

$$\frac{\partial L}{\partial q} = 0.$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta}\right) = \frac{d}{dt}\left(\frac{1}{2}J\,2\dot\theta\right) = J\ddot\theta$$

$$\left(\frac{\partial \mathcal{L}}{\partial \theta}\right) = \beta \dot q$$

damping

$$\therefore \quad J\ddot\theta + \underbrace{c\dot\theta}_{\text{damping}} - \beta\dot q = 0.$$

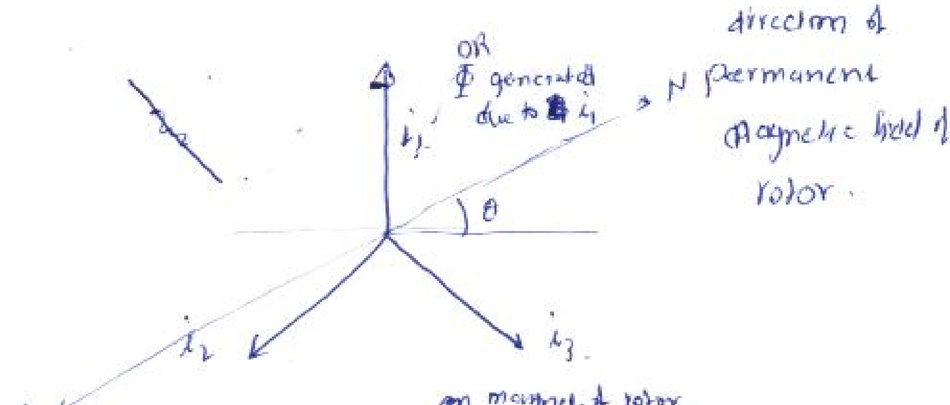
$$J\ddot\theta + c\dot\theta = \beta\dot q = \beta i \qquad \beta = k_t = k_b.$$

$\therefore$ equations of motor

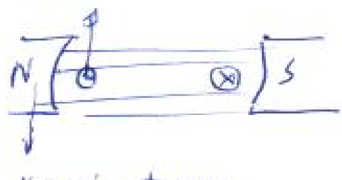
$$J\ddot\theta + c\dot\theta = k_t i \quad \text{No external load} \rightarrow \text{mechanical eq}^n$$

$$L_s\frac{di}{dt} + k_b\dot\theta + Ri = V_1 \rightarrow \text{electrical eq}^n$$

Equations of motion for BLDC motor. $\rightarrow$ Case similar to PMDC motor now coils are 3 and all fixed



Recall



Reaction Torque generated, for current i_1 through phase 1

$$T_1 = i_1 \cdot k_t \cdot \sin\theta.$$

Since physically coils are located 120° to each other

$$T_2 = i_2 k_t \cdot \sin(\theta + 120)$$

&

$$T_3 = i_3 k_t \sin(\theta + 240).$$

total torque $= T_1 + T_2 + T_3$.

We see that for $i_1 = i_2 = i_3 = i$

$T_1 + T_2 + T_3 = 0$. Thus flowing constant current through stator coils will not work.

BLDC motor

Commutation in current

So Normally $i_1 = i \sin\theta$

$i_2 = i \sin\theta + 120$

$i_3 = i \sin \theta + 240$

$\therefore$ $T_1 = K_t \, i \, \sin^2\theta$

$T_2 = k_t \, i \, \sin^2(\theta + 120)$

$T_3 = k_t \, i \, \sin^2(\theta + 240)$

$\therefore$ $T = T_1 + T_2 + T_3$

$= K_t i \left(\sin^2\theta + \sin^2(\theta + 120) + \sin^2(\theta + 240) \right)$

$= k_t i \left(\frac{1 - \cos 2\theta}{2} + \frac{1 - \cos 2(\theta + 120)}{2} + \frac{1 - \cos 2(\theta + 240)}{2} \right)$

$= \frac{K_t i}{2} \left(3 - \left[\cos 2\theta + \cos 2(\theta + 120) + \cos 2(\theta + 240) \right] \right)$

$\boxed{T = \frac{3}{2} K_t i}$ $\rightarrow$ gives torque proportional to i

Case of misalignment

Normally hall effect sensors are used to sense directions θ, $\theta + 120$, $\theta + 240$.

If consider overall misalignment of α. (In placement of hall effect sensors based on which current commutation is done)

$T_1 = i_1 \, k_t \sin\theta$

$T_2 = i_2 \, k_t \sin \theta + 120$

$T_3 = i_3 \, k_t \sin \theta + 240$

$i_1 = i \sin(\theta + \alpha)$

$i_2 = i \sin(\theta + \alpha + 120)$

$i_3 = i \sin(\theta + \alpha + 240)$.

$$T_1 = i \sin(\theta+\alpha) \sin\theta \, k_t$$

$$T_2 = i \, k_t \sin(\theta+\alpha+120) \sin(\theta+120)$$

$$T_3 = i \, k_t \sin(\theta+\alpha+240) \sin(\theta+240)$$

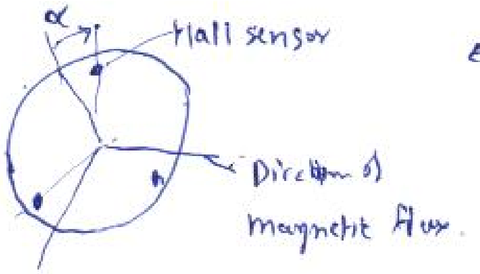
$$T = T_1 + T_2 + T_3$$

$$= i k_t \left[\sin(\theta+\alpha)\sin\theta + \sin(\theta+\alpha+120)\sin(\theta+120) + \sin(\theta+\alpha+240)\sin(\theta+240) \right]$$

$$= \frac{i k_t}{2} \left[\cos\alpha - \cos(2\theta+\alpha) + \cos\alpha - \cos[2(\theta+120)+\alpha] + \cos\alpha - \cos[2(\theta+240)+\alpha] \right]$$

$$\boxed{T = \frac{i k_t}{2} \, 3\cos\alpha}$$

$\Rightarrow$ Torque is now lesser by factor of $\cos\alpha$. Hence alignment of hall effect sensors w.r.t the direction of magnetic field in respective coils is very important



← Misalignment ~~in~~ in mounting Hall sensors