PID from PI & PD

Consider a **plant**, as given below.

$$G(s) = \frac{K(s+8)}{(s+3)(s+6)(s+10)}$$

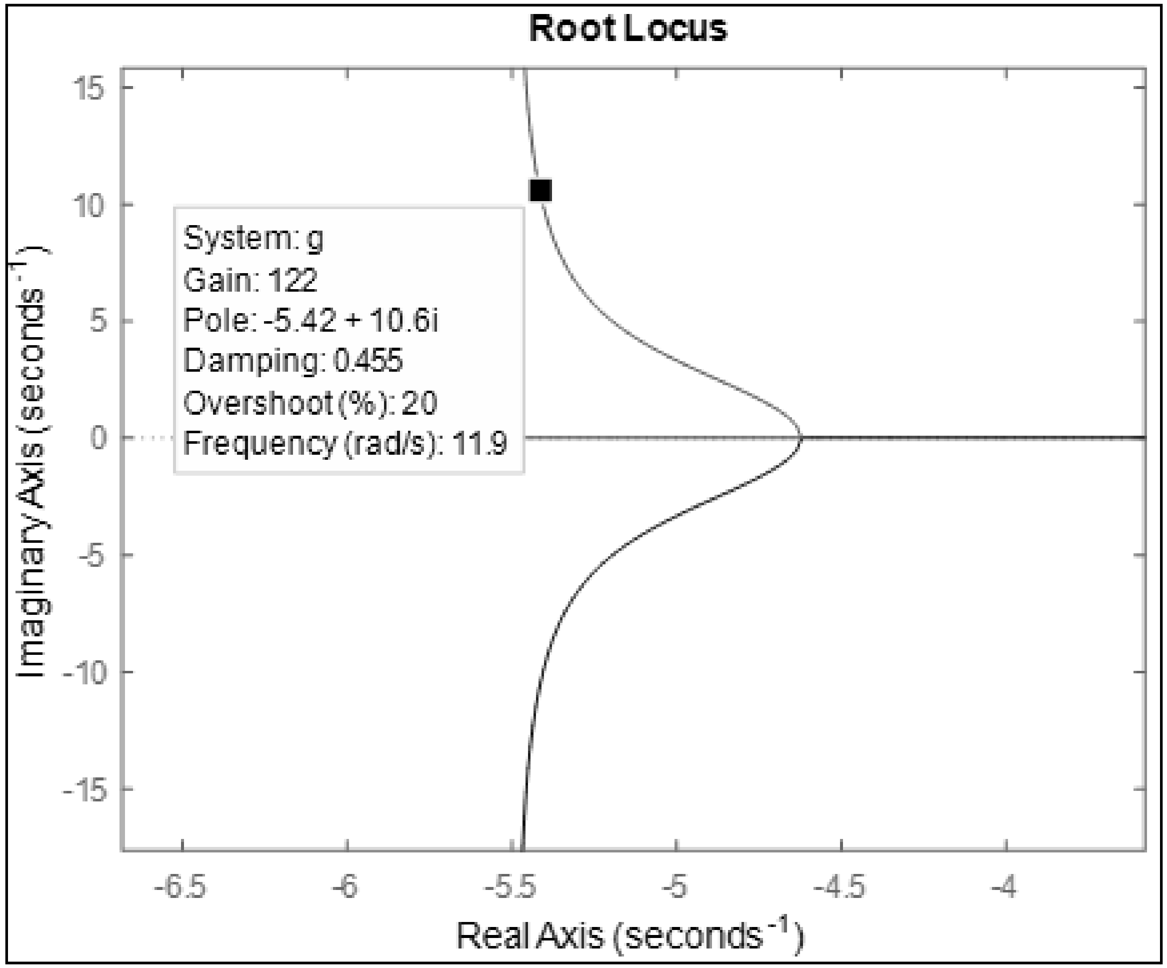
Design a **PID** (PI + PD) so that closed loop **system** has M_p **< 20%**, T_p = **0.2** sec and tracks the step input **exactly**.

Specification & Plant Features

Specifications indicate the following **closed** loop characteristics.

$$M_p < 20\% \rightarrow \zeta > 0.456;$$
$$T_p = 0.2 \rightarrow \omega_d = 15.92$$
$$p_c = -8.16 \pm j15.92;$$
$$K_P = \infty \text{ (i.e. Type '1' Plant)}$$

Here, 'ζ' specification can be **met** by keeping K = **122**, as shown alongside.



Design of PD Part of PID

We can design **PD** part, by **quantifying** angle **deficiency** at the dominant closed loop **pole**, as shown below.

$$\phi + \tan^{-1}\frac{15.92}{0.16} = -180 + \tan^{-1}\frac{15.92}{5.16} + \tan^{-1}\frac{15.92}{2.18} + \tan^{-1}\frac{15.92}{1.84}$$

$$\phi + 90.6 = -180 + 108 + 97.8 + 83.4 \rightarrow \phi = 18.6^{o}$$

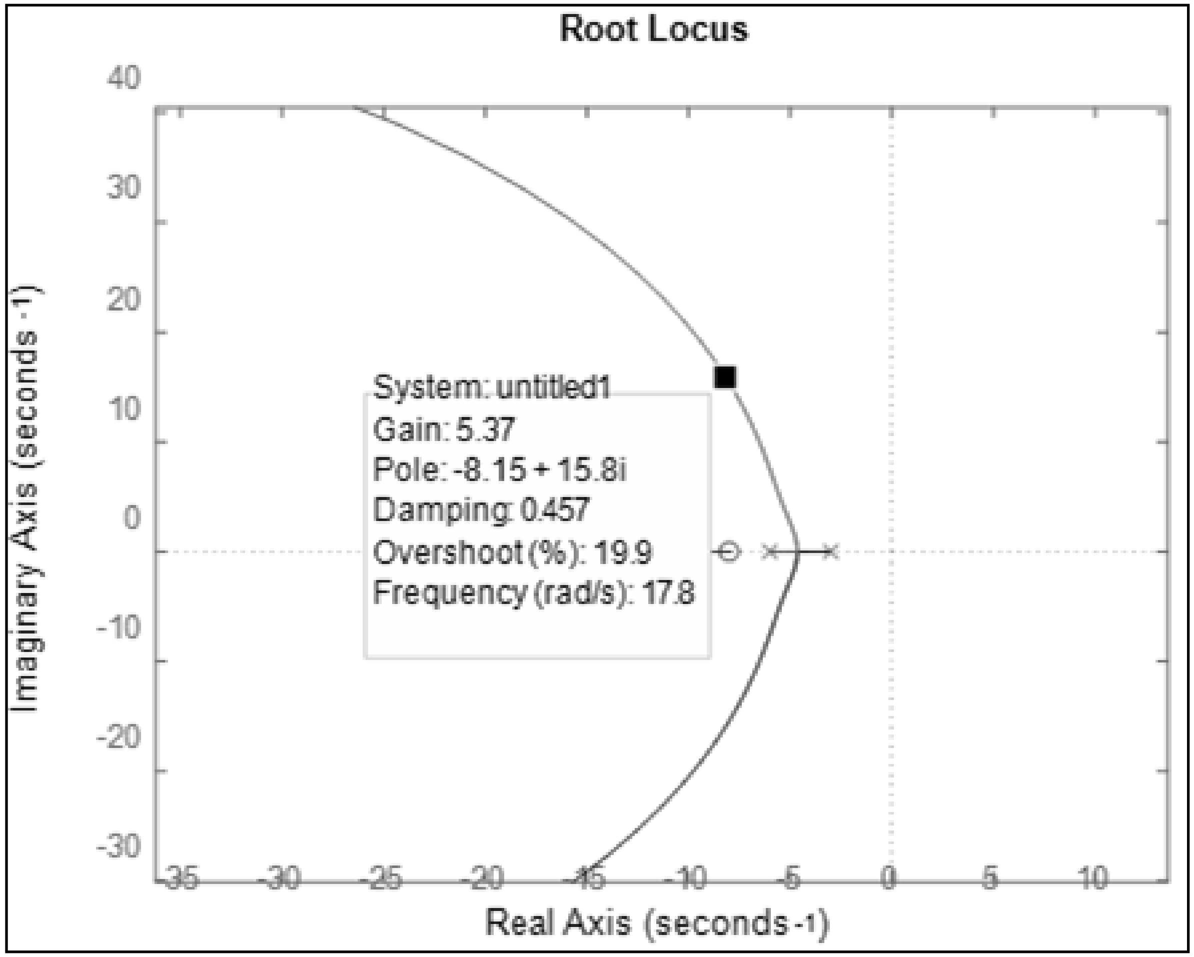
$$z = -8.16 - \frac{15.92}{\tan 18.6} = -55.5; \quad G_{PD}(s) = (s + 55.5)$$

We can now **determine** the new gain, **K** for which the desired **poles** will be on the modified **root locus**.

PD Compensated Solution

Effect of **PD** on the root **locus** is shown alongside.

We see that the **desired** pole is on the root locus, but the gain is **5.37**, so that **additional** DC gain is **298**, instead of **122**.



Design of PI Part of PID

It is seen from the **modified** root locus that dominant **pole** is far to the left and hence any **PI** controller, with zero close to **origin**, should be adequate, for **increasing** type.

Of course, if a $\mathbf{K_V}$ is desired, then zero **location** would be based on it. In the present case, a **PI** of the following **form** is considered to be **adequate**.

$$G_{PI}(s) = \frac{s+0.5}{s}; \quad G_{PID}(s) = \frac{K(s+0.5)(s+55.5)}{s}$$

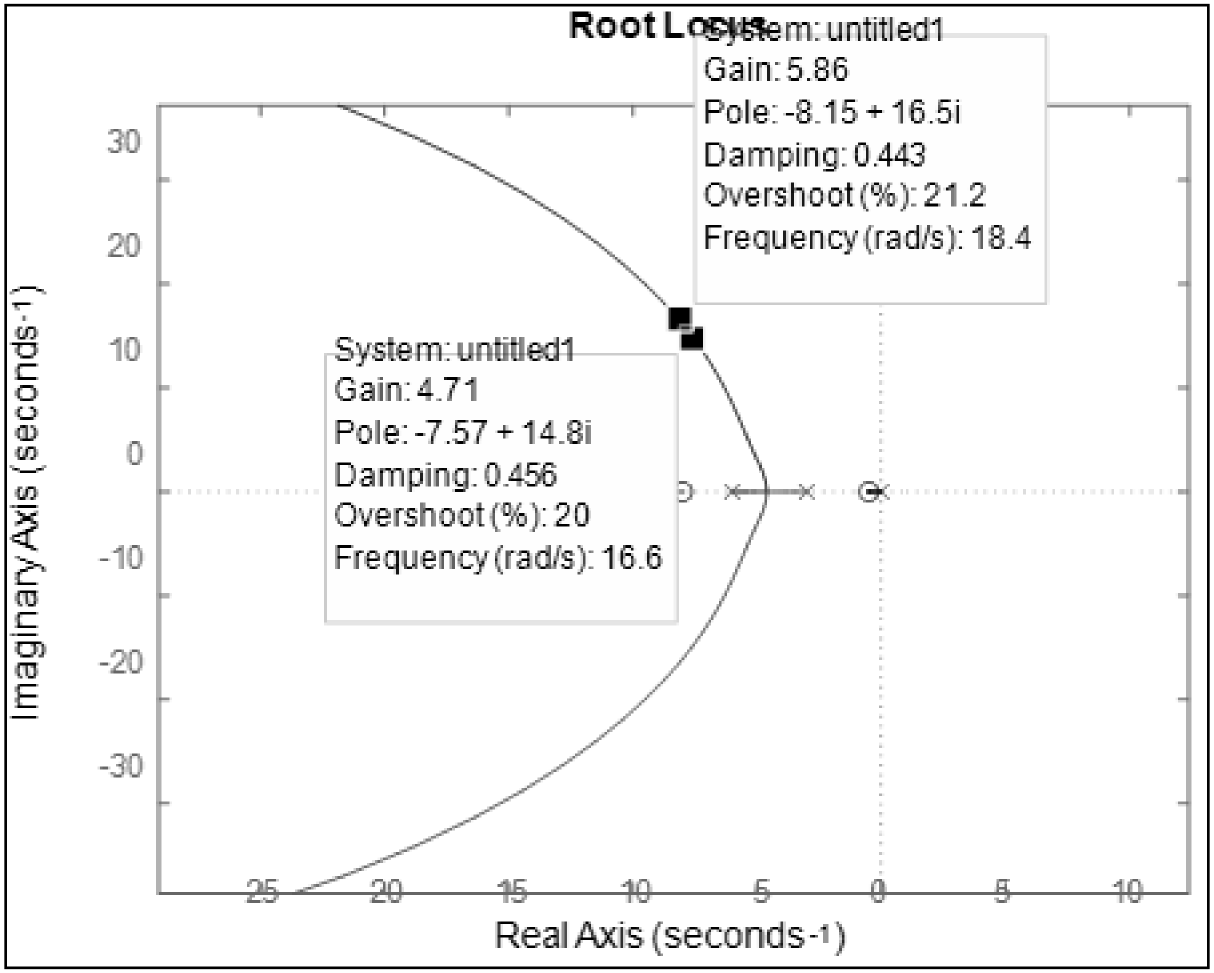
K can be obtained from the **final** root locus.

PD Compensated Solution

The **final** root locus with **PI** and **PD** compensated **system** is as shown alongside.

We see that if we **wish** for desired **pole**, gain is **5.37**, but if we wish for only ‘ζ’, we can use a **lower** gain of **4.71**.

Net **DC** gain **decreases** by a factor of **0.5**.



PID Design with Zeigler-Nichols

Design a **PID controller**, for the following system, to achieve, a **peak** overshoot of **less than 25%.**

$$G(s) = \frac{1}{s(s+1)(s+5)}$$

Design Example

The **Routh's analysis** is shown below.

$$G(s) = \frac{1}{s(s+1)(s+5)}; \quad \frac{C}{R} = \frac{K_{cr}}{s^3 + 6s^2 + 5s + K_{cr}}$$

$$K_{cr} = 30; \quad P_{cr} = 2.81$$

The **controller TF** is given below.

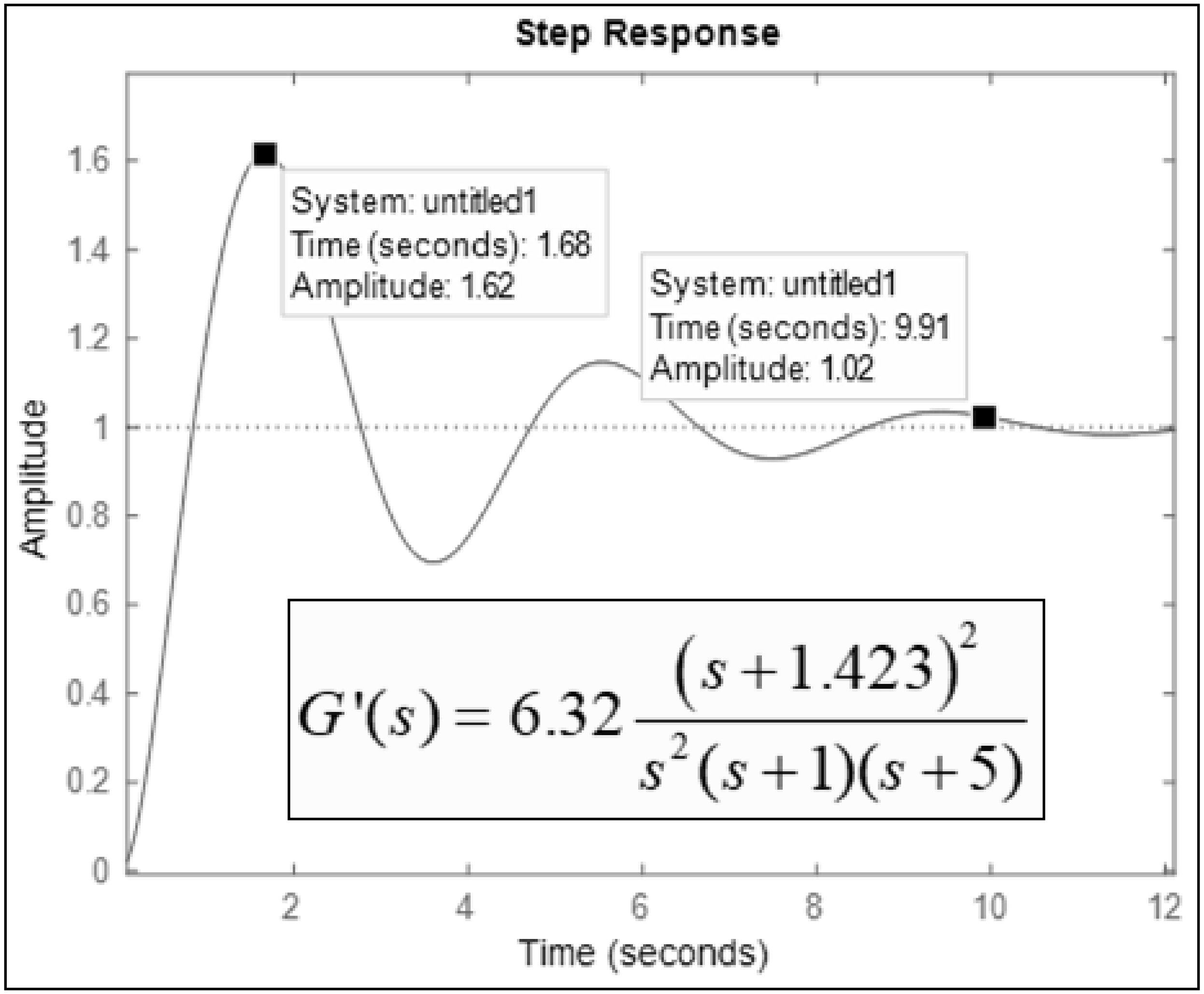
$$G_c(s) = 0.075 K_{cr} P_{cr} \frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s} = 6.32 \frac{(s+1.423)^2}{s}$$

Design Example

The **basic** step response of the **closed loop** system is shown alongside.

We find that we **need to increase, 'ζ',** to improve M_p.

Thus, we look at the **root locus** of the system to **arrive** at **tuned** controller.



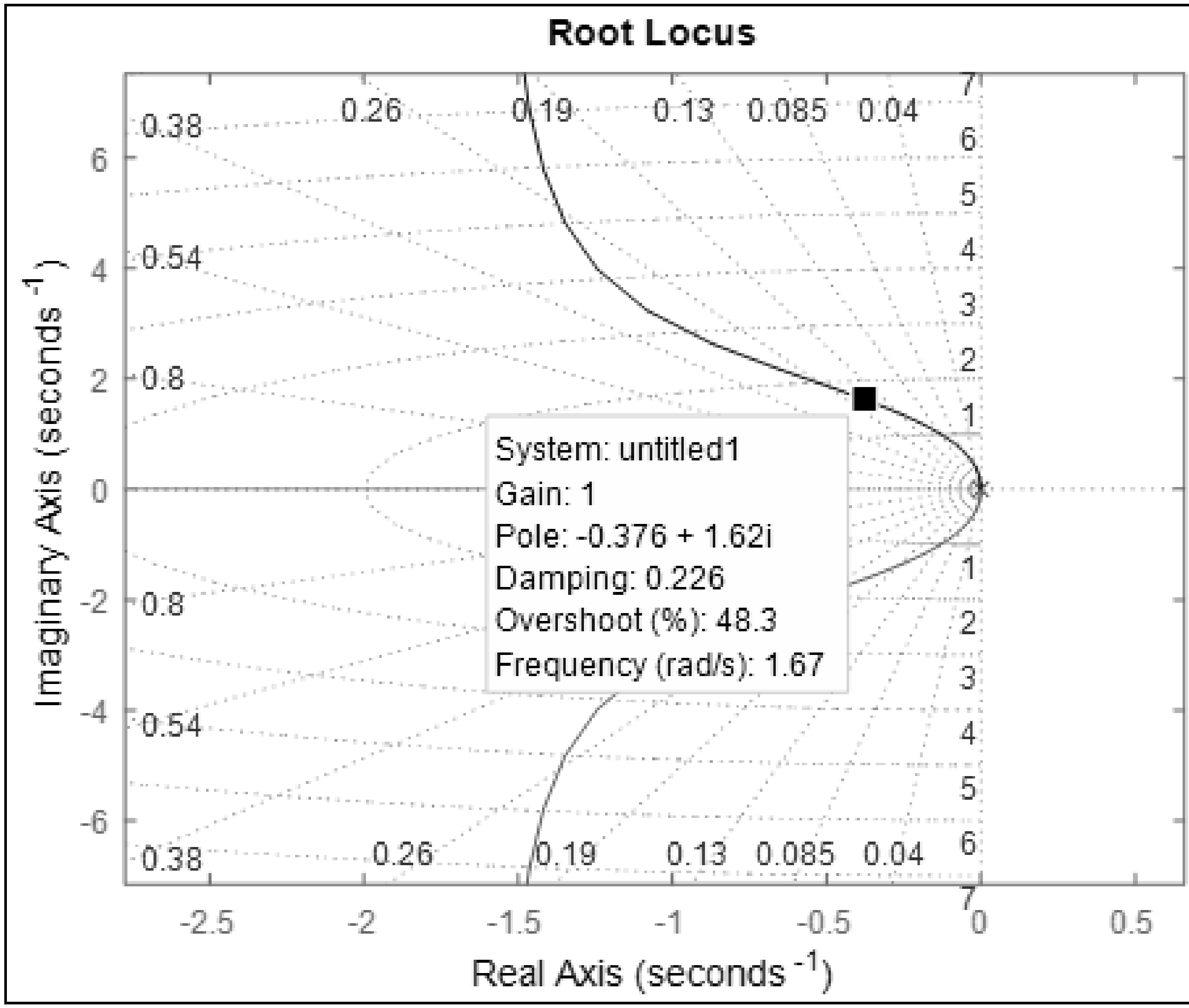
$$G'(s) = 6.32\frac{(s+1.423)^2}{s^2(s+1)(s+5)}$$

Design Example

The **root locus** is shown **alongside.**

We see that **existing** system has **damping** of **0.226,** which needs to be raised to **0.4.**

We also find that **ζ = 0.4 line** does not **intersect** the root locus**,** so that we need to **pull** it to **left** by shifting **zeros** to right.



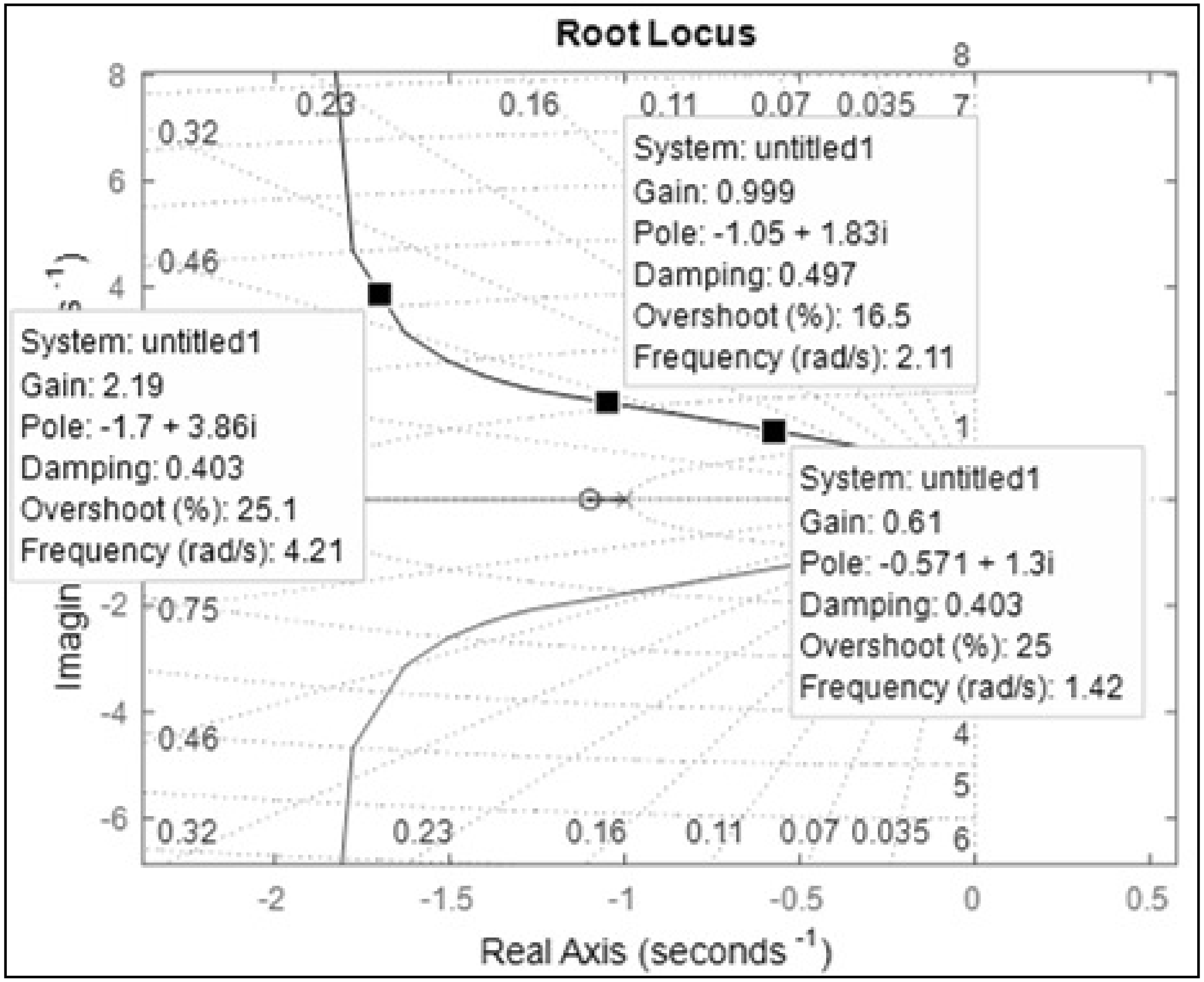
Design Example

While, **this** can be done in **many ways**, let us **keep** the **DC gain** constant.

Let us **use** a trial **controller** as shown below.

$$G'_c(s) = 10.58\frac{(s+1.1)^2}{s}$$

New root locus, **provides** two values of K = **0.61**, **2.19**.



PID Controller Bode Design Example

Consider a **system** as given below.

$$G(s) = \frac{s+0.1}{s^2+1}$$

It is required to **design a PID** controller so that $\mathbf{K_V}$ **is 4**, PM is at least $\mathbf{50^o}$ and GM is more than **10 dB.**

We first add **K = 40** and, 's' in **denominator** to ensure the desired $\mathbf{K_V}$.

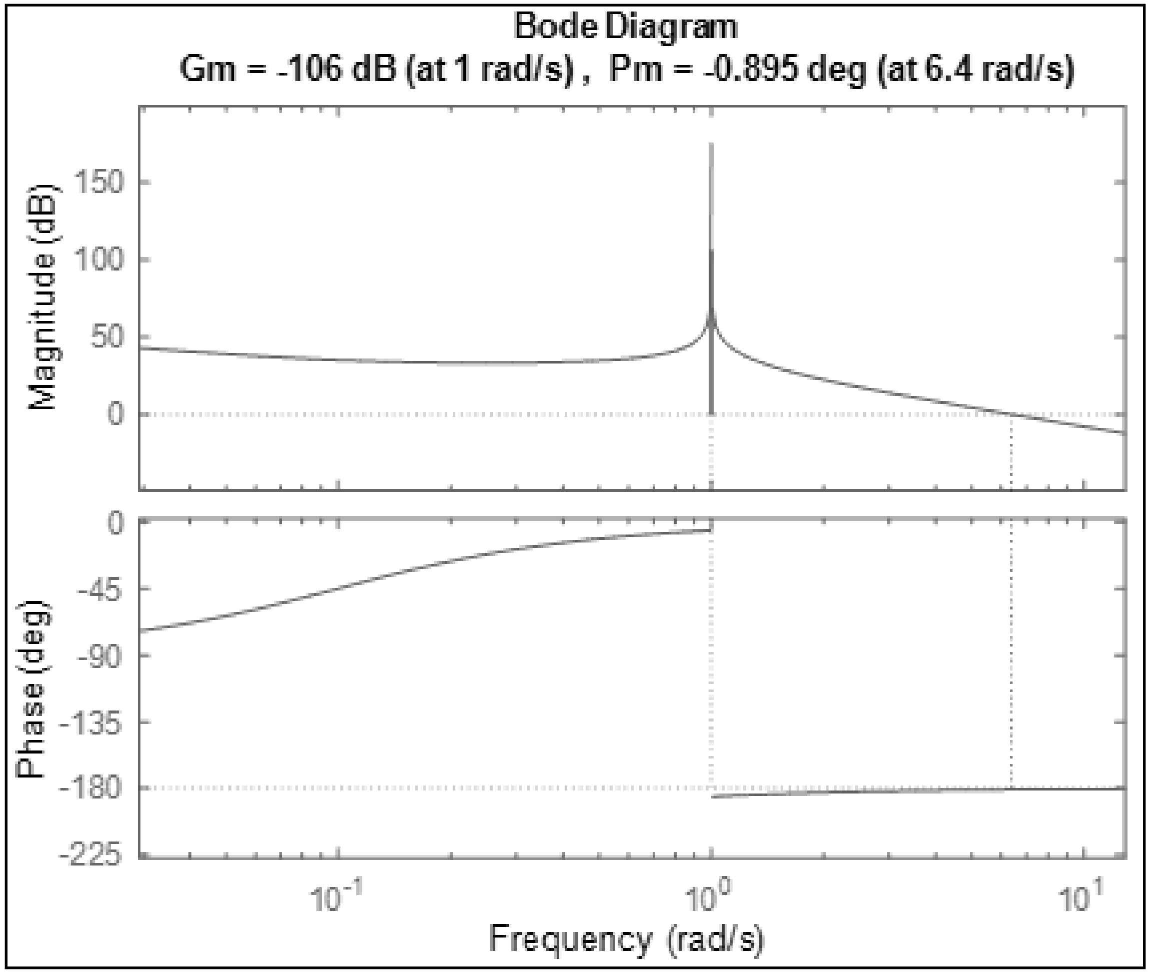
Gain – Integral Adjusted Bode Plot

The **modified** plant is as given below, and **bode plot** is alongside.

$$G'(s) = \frac{40(s+0.1)}{s\left(s^2+1\right)}$$

We see that **GCO** is 6.4 rad/s and PM is ~ **-1°.**

Thus, we need a '**zero**' to add the desired **phase.**



Bode Plot with Double Zero

As **PM** required is 50^o, and as **GCO** is higher, we add a **double zero** so that each adds about **25^o** at the GCO of **6.4.**

The final **TF** and the **bode plot** are given alongside.

We find that **PM** is more than **50^o,** as per the **requirements.**