EE 301 Tutorial 3 21/8/19

1. Given electrical field intensity in free space $\hat{E} = 50\hat{x}\cos(10^5 t - 10^5\sqrt{\mu_o\varepsilon_o}\,z)$ (V/m) Compute speed of propagation, frequency, amplitude and direction of propagation of wave

2. $\nabla^2 A = \mu\varepsilon\frac{\partial^2 A}{\partial t^2}$ is wave equation in terms of magnetic vector potential, derive ~~express~~ it in terms of electric flux density D

3. A electric field in a lossless medium with $\varepsilon_r = 9$ and $\mu_r = 1$, and wave travels in x-y plane at an angle 30° to the x-axis and frequency = 50MHz and E_o = 10V/m. Find phasor equation of the wave

4. A AM radio station transmits at 2MHz with amplitude of electric field intensity as 10V/m. Find-
   a. Magnetic field intensity
   b. Electric and magnetic field intensity in time domain (Mention any assumption you made for the same)
   c. ε_r changes from 1.0 to 2.0, calculate change in phase velocity, intrinsic impedance and magnetic field intensity assuming amplitude of electric field does not change

5. The electric field density of a plane EM wave is $\hat{E}(z) = \hat{x}8\cos(10^6\pi t)$ V/m at the point x=0, y=0, z=0. The magnetic field intensity is oscillating in the positive y direction and the wave propagates in a material with properties $\varepsilon = \varepsilon_o$ [F/m], $\mu = \mu_o$ [H/m], and $\sigma = 1.5*10^{-5}$ S/m. Find the magnetic field intensity at a distance 1km from origin in direction of propagation

6. A satellite at the height of 30,000km above the Earth's surface communicates at 30GHz and assume that the atmosphere is 15km thick (assume free space above it). Properties of the atmosphere are $\varepsilon = 1.5\varepsilon_o$ [F/m], $\mu = \mu_o$ [F/m], and $\sigma = 10^{-6}$ S/m:
   a. Calculate phase velocity, propagation constant, intrinsic impedance in atmosphere and free space
   b. If minimum electric field intensity required for reception is 10mV/m then what is the minimum amplitude of transmitter on Earths' surface, assuming satellite completely reflects the signal

7. A room of aluminum walls aims at attenuating electric field of minimum frequency 1Mhz by a factor of 10^6 given its conductivity is 3.7×10^7 S/m what should be the wall thickness. If instead we use iron walls with conductivity 10^7 S/m and relative permeability 100 then what will be the wall thickness

8. A copper conductor of diameter 4mm carries a total current of 100A, given conductivity of copper is 5.7×10^7 S/m. Find:
   a. DC resistance per meter length and current density
   b. AC resistance per meter length and maximum current in the wire. Use frequency of 60Hz and assume maximum allowable current density as that of DC current density

9. The electric field for a linearly polarized electromagnetic wave propagating in free space is given by $\hat{E} = \hat{n}E_0\exp[i(\omega t-2x+4y-4z)]$ where x,y,z are in meters and t in sec, and $\hat{n}=\frac{1}{\sqrt{18}}[4\hat{x}+\hat{y}-\hat{z}]$ represents unit vector along $\hat{E}$

a. What is wavelength and frequency of the wave?

b. Obtain unit vector along direction of propagation of wave

c. Is the wave transverse?

10. Suppose a submarine communicates with another submarine in sea water, and ratio between amplitude at receiver and transmitter must be greater than 10^{-12} then what is maximum range of communication (given relative permittivity is 72 and conductivity is 4S/m) at:

a. 10MHz

b. 100Hz

Review Questions:

1. Does wavelength or frequency change for a propagating wave if medium changes? Why?
2. Is polarization possible in longitudinal waves?

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EE 301

T3

1. $$\vec{E} = 50\cos(10^5 t - 10^5\sqrt{\mu_0\varepsilon_0}\, z)\,\hat{x} \ \ Vm^{-1}$$

$\omega = 10^5 \text{ rad } s^{-1}$, $\beta = 10^5\sqrt{\mu_0\varepsilon_0} \text{ rad } m^{-1}$

(a) Speed of propagation $= \dfrac{\omega}{\beta} = \dfrac{10^5}{10^5\sqrt{\mu_0\varepsilon_0}} = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} = c = \underline{299792458 \ ms^{-1}}$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{10^5}{2\pi} = \underline{1.59\times 10^4 \text{ Hz}}$

(c) Amplitude $= 50 \ Vm^{-1}$

(d) Direction of prop. of wave $= \underline{+z}$

2. $$\nabla^2 A = \mu\varepsilon \frac{\partial^2 A}{\partial t^2}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

Now $\mu\vec{H} = \vec{B}$, $D = \varepsilon\vec{E}$

$$\Rightarrow \frac{\nabla\times\vec{D}}{\varepsilon} = -\mu\frac{\partial H}{\partial t}$$

Take the curl on both sides.

$$\Rightarrow \nabla\times(\nabla\times\vec{D}) = -\mu\varepsilon\frac{\partial}{\partial t}(\nabla\times\vec{H})$$

$$\nabla(\nabla\cdot\vec{D}) - \nabla^2\vec{D} = -\mu\varepsilon\frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$$

$$\nabla^2\vec{D} - \mu\varepsilon\frac{\partial^2\vec{D}}{\partial t^2} = \nabla(\rho_v) + \mu\varepsilon\frac{\partial(\vec{J})}{\partial t} \qquad (\because \nabla\cdot\vec{D} = \rho_v)$$

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In free space, $\rho_v = 0$ and $\vec{J} = \vec{0}$.

$$\Rightarrow \quad \nabla^2 \vec{D} - \mu\varepsilon \frac{\partial^2 \vec{D}}{\partial t^2} = 0$$

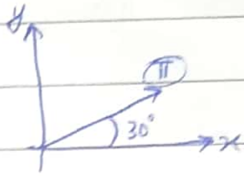
Hence verified

3. $\varepsilon_r = 9, \ \mu_r = 1,$ $f = 50 \text{ MHz}, \ E_0 = 10 \text{ Vm}^{-1}$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c}{3} \approx 1\times10^{8} \text{ ms}^{-1}$$

i.e. $\dfrac{2\pi(50\times10^{6})}{\beta} = 1\times10^{8} \Rightarrow \beta = \dfrac{2\pi\times5\times10^{7}}{10^{8}} = \pi \text{ rad m}^{-1}$

$$\therefore \vec{k} = \pi\left(\frac{\hat{x}\sqrt{3} + \hat{y}}{2}\right)$$



$$\vec{E} = E_0 e^{-j\vec{k}\cdot\vec{r}} \qquad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

i.e. $\underbrace{\vec{E}}_{\text{rep. wave vector}} = 10 \exp\left(-j\left(\pi\left(\dfrac{\sqrt{3}x + y}{2}\right)\right)\right) \ (\text{Vm}^{-1})$

The direction of prop. as mentioned in the qn. is $\left(\dfrac{\hat{x}\sqrt{3} + \hat{y}}{2}\right)$.

5. $\vec{E} = 8\cos(10^{6}\pi t)\,\hat{x} \ (\text{Vm}^{-1})$ at $(x, y, z) = (0, 0, 0)$

$\varepsilon = \varepsilon_0, \ \mu = \mu_0, \ \sigma = 1.5\times10^{-5} \text{ Sm}^{-1}$

Dir. of waves $= \hat{E}\times\hat{H} = \hat{x}\times\hat{y} = \hat{z}$

$\omega = 10^{6}\pi \Rightarrow f = 5\times10^{5} \text{ Hz}$

$$\beta = |k| = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)}$$

$$= (10^{6}\pi)\sqrt{\frac{1}{2c^2}\left(\sqrt{1+\left(\frac{1.5\times10^{-5}}{10^{6}\pi\times8.85\times10^{-12}}\right)^2} + 1\right)}$$

$$= 0.01083 \text{ rad m}^{-1}$$

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$$\alpha = \omega\sqrt{\frac{1}{2c^2}\left(\sqrt{1+\left(\frac{1.5\times10^{-5}}{10^6\pi\times8.85\times10^{-12}}\right)^2}-1\right)}$$

$$= \underline{2.735\times10^{-3}\ \text{Np m}^{-1}}$$

$$E_{1km} = E_0 e^{-\alpha z}, \quad \text{where } z = 1\,\text{km}$$

$$= 8\exp\left(-2.735\times10^{-3}\times10^3\right)$$

$$= \underline{0.519\ \text{Vm}^{-1}}$$

$$\eta = \frac{E}{H} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega(\sigma+j\omega\epsilon)\mu}} = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}}$$

$$= \sqrt{\frac{j(10^6\pi)(4\pi\times10^{-7})}{(1.5\times10^{-5})+j(10^6\pi)(8.85\times10^{-12})}}$$

$$= \underline{342.744 + j\,86.56}$$

$$\Rightarrow H_{1km} = \frac{E}{\eta} = (0.519)\left(2.7427\times10^{-3} - j\,6.9267\times10^{-4}\right)$$

$$= \underline{\left(1.423\times10^{-3} - j\,3.595\times10^{-4}\right)\ \text{Am}^{-1}}$$

$$\vec{H} = H_{1km}\cos(\omega t - \beta z)\,\hat{y}$$

$$= \underline{\left(1.423\times10^{-3} - j\,3.595\times10^{-4}\right)\cos\left(10^6\pi t - 0.0108\right)\hat{y}\ (\text{Am}^{-1})}$$

7 (i) : $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right)}$

Now $\left(\frac{\sigma}{\omega\epsilon}\right) \gg 1 \Rightarrow \alpha \approx \omega\sqrt{\frac{\mu\epsilon}{2}\left(\frac{\sigma}{\omega\epsilon}\right)} = \sqrt{\frac{\omega^2\mu\sigma}{2\omega}}$

i.e. $\alpha \approx \sqrt{\pi f \mu\sigma}$, $\sigma = 3.7\times10^7\ \text{Sm}^{-1}$

$$= \sqrt{\pi\times10^6\times4\pi\times10^{-7}\times3.7\times10^7}$$

$$= \underline{1.209\times10^4\ \text{Np m}^{-1}}$$

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$$10^{-6} E_0 = E_0 \exp(-\alpha z) = E_0 \exp(-1.209 \times 10^4 z)$$

i.e. $$z = \frac{\ln(10^6)}{\alpha} = \underline{\underline{1.143 \times 10^{-3}\ m}} = \underline{\underline{1.143\ mm}}$$

(ii) Now consider iron.

$$\alpha \approx \sqrt{\pi f \mu \sigma} = \sqrt{\pi (10^6)(100 \times 4\pi \times 10^{-7})(10^7)} = \underline{\underline{6.28 \times 10^4\ Np\,m^{-1}}}$$

$$10^{-6} E_0 = E_0 \exp(-\alpha z) = E_0 \exp(-6.28 \times 10^4 z)$$

i.e. $$z = \frac{\ln(10^6)}{\alpha} = \underline{\underline{0.2199\ mm}}$$

4. $f = 2\,MHz$, $E_0 = 10\ V m^{-1}$

(a) $$\eta = \frac{E_0}{H_0} = \frac{j\omega\mu}{\gamma} \approx \sqrt{\mu_0/\varepsilon_0} \quad (\because \sigma \approx 0,\ \mu \approx \mu_0,\ \varepsilon \approx \varepsilon_0)$$

$$\Rightarrow \eta = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = \underline{\underline{376.819}}$$

$$H_0 = \frac{E_0}{\eta} = \frac{10}{376.819} = \underline{\underline{0.0265\ A m^{-1}}}$$

(b) Assume dir. of $\vec{E}$ in x axis, that of $\vec{H}$ in y axis $\Rightarrow$ that of the wave in z axis.

$$\beta = \frac{\omega}{v_p} = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi(2 \times 10^6)}{(3 \times 10^8)} = \underline{\underline{0.04189\ rad\,m^{-1}}}$$

$$\omega = 2\pi f = 2\pi(2 \times 10^6) = \underline{\underline{1.257 \times 10^7\ rad\,s^{-1}}}$$

$$\therefore \vec{E} = 10 \cos(1.257 \times 10^7 t - 0.04189 z)\ \hat{x}\ (V m^{-1})$$

$$\vec{H} = 0.0265 \cos(1.257 \times 10^7 t - 0.04189 z)\ \hat{y}\ (A m^{-1})$$

(c) $\varepsilon_r = 2$ ~~and~~ μ supposedly remains the same.

$$\Rightarrow v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{2}} = 211985280\ ms^{-1}$$

$$\eta \approx \sqrt{\mu/\varepsilon} = \sqrt{\mu/2\varepsilon_0} = \frac{376.819}{\sqrt{2}} = 266.45\ VA^{-1} = 266.45\ \Omega$$

$$H_0 = \frac{E_0}{\eta} = \left(\frac{E_0}{\eta}\right)_{org}\sqrt{2} = (H_0)_{org}\sqrt{2} = 0.0375\ Am^{-1}$$

6. Atmosphere :

$$v_p = \frac{\omega}{\beta}. \quad \text{Now} \quad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right)}$$

$$= 2\pi(30\times10^9)\sqrt{\frac{1.5}{2c^2}\left(\sqrt{1+\left(\frac{10^{-6}}{30\times10^9\times\varepsilon_0\times2\pi\times1.5}\right)^2}+1\right)}$$

$$\beta \approx \frac{2\pi(30\times10^9)\sqrt{1.5}}{c} = 769.53\ rad\ m^{-1}$$

$$\Rightarrow v_p = \frac{2\pi\times30\times10^9}{769.53} = 244{,}948{,}974.30\ ms^{-1}$$

$$\alpha = \frac{2\pi(30\times10^9)}{c}\sqrt{\frac{1.5}{2}\left(\sqrt{1+\left(\frac{10^{-6}}{30\times10^9\times\varepsilon_0\times2\pi\times1.5}\right)^2}-1\right)}$$

$$= 2.176\times10^{-4}\ Np\ m^{-1}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\varepsilon}} = \sqrt{\frac{j(2\pi\times30\times10^9)(4\pi\times10^{-7})}{10^{-6}+j(2\pi\times30\times10^9)(1.5\times8.85\times10^{-12})}}$$

$$= \sqrt{\frac{236870j}{10^{-6}+j2.502}} = \sqrt{94672+j0.0378}$$

$$= (307.688+j(6.148\times10^{-5}))\ \Omega$$

$$\approx 307.688\ \Omega$$

Free space:

$$\sigma = 0, \quad \mu = \mu_0, \quad \varepsilon = \varepsilon_0$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = \underline{\underline{299792458 \ ms^{-1}}}$$

$$\alpha = \underline{\underline{0 \ Npm^{-1}}} \ \because \ \frac{\sigma}{\omega\varepsilon} = 0 \ \because \ \sigma = 0$$

$$\eta = \sqrt{\mu_0/\varepsilon_0} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = \underline{\underline{376.819 \ \Omega}}$$

(b)

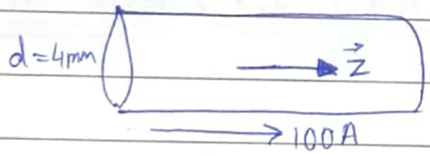
$I(15 \text{ km}) \Rightarrow d = z = (15 \text{ km}) \times 2 = \underline{\underline{30 \text{ km}}}$

$$10 = E_0 \exp(-\alpha(30 \text{ km}))$$

i.e.

$$E_0 = 10 \exp(2.176 \times 10^{-4} \times 30 \times 10^3)$$
$$= 10 \times 684.029$$
$$= \underline{\underline{6840.29 \ Vm^{-1}}}$$

8.



$\sigma = 5.7 \times 10^7 \ Sm^{-1}$

(a) $R = \frac{\rho l}{A} = \frac{l}{\sigma A} =$

$$(R/l) = \frac{1}{5.7 \times 10^7 \times \pi (2 \times 10^{-3})^2} = \underline{\underline{13.96 \times 10^{-4} \ \Omega \ m^{-1}}}$$

$$\vec{J} = \frac{100}{\pi (2 \times 10^{-3})^2} \hat{z} = \underline{\underline{7.956 \times 10^6 \ A m^{-2}}}$$

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9. $\vec{E} = \hat{n} E_0 \exp\left(j\left(\omega t - \gamma(2x + 4y - 4z)\right)\right)$

$\hat{n} = \frac{1}{\sqrt{18}}\left(4\hat{x} + \hat{y} - \hat{z}\right)$

(a) $\vec{k} \cdot \vec{r} = 2x + 4y - 4z$

Here, $\vec{r} = \frac{1}{\sqrt{18}}\left(4\hat{x} + \hat{y} - \hat{z}\right)$

i.e. $\frac{4k_x}{\sqrt{18}} = 2, \quad \frac{k_y}{\sqrt{18}} = 4, \quad \frac{-k_z}{\sqrt{18}} = -4$

$\Rightarrow k_x = \frac{1}{2\sqrt{18}}, \quad k_y = \frac{4}{\sqrt{18}}, \quad k_z = \frac{4}{\sqrt{18}}$

$\vec{k} = \frac{1}{\sqrt{18}}\left(\frac{\hat{x}}{2} + 4\hat{y} + 4\hat{z}\right)$

i.e. $|\beta| = \frac{1}{\sqrt{18}}\left(16 + 16 + 1/4\right)^{1/2} = \underline{\underline{1.3385 \text{ rad m}^{-1}}}$

But $|\beta| = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{1.3385} = \underline{\underline{4.694 \text{ m}}}$

Now $v_p = f\lambda$

$c = f(4.694) \Rightarrow f = \frac{299792458}{4.694} = \underline{\underline{6.387 \times 10^7 \text{ Hz}}}$

(b)

9. $\vec{E} = E_0 \exp\left(j\left(\omega t + \left(2x + 4y - 4z\right)\right)\right) \hat{n}$

$$\hat{n} = \frac{1}{\sqrt{18}}\left(4\hat{x} + \hat{y} - \hat{z}\right)$$

(a) $\vec{\beta} = 2\hat{x} + 4\hat{y} - 4\hat{z}$

$\Rightarrow \beta = \sqrt{4+16+16} = \underline{6 \text{ rad } m^{-1}}$

Wavelength $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \underline{\frac{\pi}{3} m} = \underline{1.0471 \text{ m}}$

$$f = \frac{c}{\lambda} = \frac{299792458}{1.0471} = \underline{286,280,709.60 \text{ Hz}}$$

(b) Dir. of prop. = dir. of $\vec{\beta}$ = $\frac{\vec{\beta}}{\beta} = \underline{-\frac{1}{3}\hat{x} + \frac{2}{3}\hat{y} - \frac{2}{3}\hat{z}}$

(c) Transverse $\Rightarrow$ Dir. of prop. $\perp$ oscillations (of particles)

Now $\hat{\beta} \cdot \hat{n} = \frac{1}{3\sqrt{18}}\left(-4 + 2 + 2\right) = \underline{0}$

$\Rightarrow$ <u>Transverse</u>

10. $\exp(-\alpha z) > 10^{-12}$

$\Rightarrow \exp(\alpha z) < 10^{12}$

$\Rightarrow \alpha z < \ln(10^{12})$

$\Rightarrow \boxed{z < \frac{12 \ln(10)}{\alpha}}$

Now $\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)}$

Now $\left(\frac{\sigma}{\omega\varepsilon}\right) >> 1$ for both given ω.

$\Rightarrow \alpha \approx \sqrt{\pi f \sigma \mu}$

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$$\cancel{\text{i.e. } z_{max} = \frac{12 \ln 10}{2\pi} \left(\frac{1}{f\sqrt{1+\left(\frac{\sigma}{2\pi f \epsilon}\right)^2} - 1} \right)^{1/2}}$$

$$\cancel{= 4.3976 \left(\frac{1}{f\sqrt{1+\left(\frac{4}{2\pi \times 72 \times 8.85 \times 10^{-12}}\right)^2 \frac{1}{f^2}} - 1} \right)^{1/2}}$$

$$\cancel{= 4.3976 \left[\frac{1}{f\sqrt{1+\frac{9.9818 \times 10^{17}}{f^2}} - 1} \right]^{1/2}}$$

$$\cancel{\text{(a) } f = 10 \text{ MHz} = 10^7 \text{ Hz}}$$

$$\cancel{z_{max} = 4.3976 \left(\frac{1}{10^7 \left(\sqrt{1+(9.9818 \times 10^3)} - 1\right)} \right)^{1/2}}$$

$$\text{i.e. } z_{max} = \left(\frac{12 \ln(10)}{\sqrt{\pi \sigma \mu_0}} \right) \frac{1}{\sqrt{f}} = \frac{12 \ln(10)}{\sqrt{\pi \times (4) \times (4\pi \times 10^{-7})}} \left(\frac{1}{\sqrt{f}} \right)$$

$$= \frac{6953.237}{\sqrt{f}} \quad //$$

(a) $f = 10^7$ Hz $\Rightarrow z_{max} = \dfrac{6953.237}{\sqrt{10^7}} = \underline{\underline{2.199 \text{ m}}}$

(b) $f = 100$ Hz $\Rightarrow z_{max} = \dfrac{6953.237}{\sqrt{100}} = \underline{\underline{695.32 \text{ m}}}$

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Review Questions

1. Wavelength changes but frequency does not.

Frequency does not vary because of the following reason:-
At the boundary, $E_{\parallel}(x_-) = E_{\parallel}(x_+)$

$$\Rightarrow E_0 \cos(\omega_1 t - \beta_1 x_-) = E_0 \cos(\omega_2 t - \beta_2 x_+)$$

For diff ω_1 and ω_2, this equality does not hold for all t.

$\Rightarrow \omega_1 = \omega_2$

i.e. Frequency remains the same.

But the speed changes $\because$ change in ε and μ.

$\Rightarrow$ Wavelength changes in the absence of frequency change.

2. Polarization is not possible in a longitudinal waves because all particles move in the dir of propagation of the wave. This implies that the rotation of any vector of propagation is along its own axis, implying the same vector. As a result, a wave which seems to have undergone some sort of polarization is the exact copy as any other wave of the same frequency, amplitude, wavelength and similar non-polarization values.