EE 308

Homework 1 Solution Outlines

August 22, 2019

Problem 2

$$g(t) = cos(2\pi(f_c + f_m)t) \tag{1}$$

where, $f_c >> f_m$

(a) Expanding $g(t)$ in terms of $cos(2\pi f_c t)$ and $sin(2\pi f_c t)$ we get:

$$g(t) = cos(2\pi f_c t)cos(2\pi f_m t) - sin(2\pi f_c t)sin(2\pi f_m t) \tag{2}$$

Now recall the complex baseband representation

$$g(t) = g_I(t)cos(2\pi f_c t) - g_Q(t)sin(2\pi f_c t) \tag{3}$$

In our case, $g_I = cos(2\pi f_m t)$ and $g_Q = sin(2\pi f_m t)$

The complex baseband representation is given by

$$\tilde{g}(t) = g_I(t) + jg_Q(t) \tag{4}$$

$$\tilde{g}(t) = cos(2\pi f_m t) + jsin(2\pi f_m t) = e^{2\pi j f_m t} \tag{5}$$

(b) Fourier transform of $\tilde{g}(t)$ is:

$$\tilde{G}(f) = \delta(f - f_m) \tag{6}$$

Also:

$$G(f) = \frac{1}{2}\left[\tilde{G}(f - f_c) + \tilde{G}^*(-f - f_c)\right]. \tag{7}$$

So:

$$G(f) = \frac{\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))}{2} \tag{8}$$

Problem 3

$$g(t) = e^{-a|t|}cos(2\pi f_c t) \tag{9}$$

where, $a > 0$ and f_c is large enough

$$H(f) = \begin{cases} e^{-j2\pi|f|t_0}, & \text{if } f_c - B \le |f| \le f_c + B \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

where, $t_0, B > 0$. Complex envelope of $g(t)$ is $\tilde{g}(t) = e^{-a|t|}$. The fourier transform $\tilde{H}(f)$ of complex envelope $\tilde{h}(t)$ comes out to be:

$$\tilde{H}(f) = \begin{cases} 2e^{-j2\pi(f+f_c)t_0}, & \text{if } -B \le f \le B \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

We get $\tilde{h}(t) = 4Be^{-j2\pi f_c t_0}sinc(2B(t-t_0))$. Use $\tilde{y}(t) = \frac{1}{2}(\tilde{h}(t)*\tilde{g}(t))$ and $y(t) = Re[\tilde{y}(t)exp(j2\pi f_c t)]$.