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# TUTORIAL-2

$\nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J}$

1. If $\bar{A} = 10\rho^{1.5}\widehat{a_z}$ Wb/m in free space. Find $\bar{J}$
2. We locate a slab of Teflon in the region $0 \leq x \leq a$ and assume free space where $x < 0$ and $x > a$, outside the Teflon there is a uniform field $\overline{E_{out}} = E_0\widehat{a_x}\,V/m$. We seek values for $\bar{D}, \bar{E}$ everywhere.
3. Let the region $z < 0$ be composed of a uniform dielectric material for which $\varepsilon_r = 3.2$. while the region $z > 0$ is characterized by $\varepsilon_r = 2$. Let $\overline{D_1} = -30\widehat{a_x} + 50\widehat{a_y} + 70\widehat{a_z}nC/m^2$ and find $a)\ D_{N1}\ b)\ \overline{D_{t1}}\ c)\ D_{t1}\ d)\ D_1\ e)\ \theta_1\ f)\ \overline{D_{N2}}\ g)\overline{D_{t2}}\ h)\ \overline{D_2}\ i)\ \theta_2$
4. Let the permittivity be $5\ \mu H/m$ in region A where $x < 0$, and $20\ \mu H/m$ in region B where $x > 0$. If there is a surface current density $\bar{K} = 150\widehat{a_y} - 200\widehat{a_z}\ A/m$ at x=0 and if $\overline{H_A} = 300\widehat{a_x} - 400\widehat{a_y} + 500\widehat{a_z}\ A/m$. Find $a)\ |\overline{H_{TA}}|\ b)\ |\overline{H_{NA}}|\ c)\ |\overline{H_{TB}}|\ d)\ |\overline{H_{NB}}|$
5. What values of A and β are required if the two fields $\bar{E} = 120\pi\cos(10^6\pi t - \beta x)\hat{y}\,V/m$, $\bar{H} = A\pi\cos(10^6\pi t - \beta x)\hat{z}\,A/m$
6. A time dependent electric field intensity is given as $\bar{E} = 10\pi\cos(10^6 t - 50z)\,\hat{x}\,V/m$. The field exists in a material with properties $\varepsilon_r = 4$ and $\mu_r = 1$. Given that J=0 and $\rho = 0$. Calculate the magnetic field intensity and magnetic flux density in the material.
7. Let $\mu = \frac{3*10^{-5}H}{m}$, $\varepsilon = \frac{1.2*10^{-10}F}{m}$, $\sigma = 0$ *everywhere*. If $\bar{H} = 2\cos(10^{10}t - \beta x)\hat{z}\,A/m$. Use Maxwell's equations to obtain expressions for **B, D, E,** $\beta$
8. For a current distribution in free space $\bar{A} = (2x^2y + yz)\widehat{a_x} + (xy^2 - xz^3)\widehat{a_y} - (6xyz - 2x^2y^2)\widehat{a_z}Wb/m$. A) calculate $\bar{B}$ B) find the magnetic flux through a loop described by x=1, 0 < y < 2, 0 < z < 2. C) show that $\nabla.\bar{A} = 0$ and $\nabla.\bar{B} = 0$ $\quad \int \vec{B}\cdot d\vec{S} = \phi$
9. Find magnetic field about a long straight wire carrying current I using the vector potential.
10. If $V = 10\sin\omega t$, $\mu_r$=1, $\varepsilon_r$=10. Find $\nabla.\bar{A}$ a) f=50Hz b) f=100THz

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# EE301

## Tutorial 2

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Elec (BTech)

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1. $\mu\vec{H} = \vec{B} = \nabla\times\vec{A}$

$\Rightarrow \nabla\times(\nabla\times\vec{A}) = \mu\vec{J}_c + \mu\frac{\partial\vec{D}}{\partial t} = \mu_0\vec{J}_{tot}$ (free space $\Rightarrow \mu=\mu_0$)

$\therefore \vec{J}_{tot} = \frac{1}{\mu_0}\left(\nabla\times(\nabla\times\vec{A})\right)$

$= \frac{1}{\mu_0}\left(\nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}\right)$

$\vec{A} = 10\rho^{1.5}\,\hat{a}_z$

$\nabla\cdot\vec{A} = \frac{\partial}{\partial z}(10\rho^{1.5})\,\hat{a}_z = 0 \Rightarrow \nabla(\nabla\cdot\vec{A}) = 0$

$-\nabla^2\vec{A} = -\frac{\partial^2}{\partial\rho^2}(10\rho^{1.5})\,\hat{a}_z = -(1.5\times0.5\times10)\rho^{-0.5}\,\hat{a}_z = -\frac{7.5}{\sqrt{\rho}}\,\hat{a}_z$

$\therefore \vec{J}_{tot} = \frac{1}{4\pi\times10^{-7}}\times\left(-\frac{7.5}{\sqrt{\rho}}\right)\hat{a}_z = -\left(\frac{5.968\times10^6}{\sqrt{\rho}}\right)\hat{a}_z\ A\,m^{-2}$

2.

$z>0$ $\quad \varepsilon_r = 2$ $\qquad \vec{D}_1 = (-30\,\hat{a}_x + 50\,\hat{a}_y + 70\,\hat{a}_z)\ nC\,m^{-2}$

$z<0$ $\quad \varepsilon_r = 3.2$

(a) $D_{N_1} = \vec{D}_1\cdot\hat{a}_z = 70\,nC\,m^{-2}$

(b) $\vec{D}_{T_1} = \vec{D}_1 - \vec{D}_{N_1} = (-30\,\hat{a}_x + 50\,\hat{a}_y)\ nC\,m^{-2}$

(c) $D_{T_1} = \sqrt{30^2+50^2} = \sqrt{3400} = 58.31\ nC\,m^{-2}$

(d) $D_1 = \sqrt{D_{T_1}^2 + D_{N_1}^2} = \sqrt{3400+4900} = \sqrt{8300} = 91.10\ nC\,m^{-2}$

(e) $\theta_1 = \cos^{-1}\left(\frac{D_{N_1}}{D_1}\right) = \cos^{-1}\left(\frac{70}{91.10}\right) = 0.22\pi\ \text{rad} = 39.79^\circ$

(f) $\vec{D}_{N_2} = \vec{D}_{N_1} = 70\,\hat{a}_z\ nC\,m^{-2}$

(g) $\vec{D}_{T_2} = \varepsilon_2\vec{E}_{T_2} = \varepsilon_2\vec{E}_{T_1} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)\vec{D}_{T_1} = \frac{2}{3.2}(-30\,\hat{a}_x + 50\,\hat{a}_y)$

$= (-18.75\,\hat{a}_x + 31.25\,\hat{a}_y)\ nC\,m^{-2}$

(h) $\vec{D_2} = \vec{D_{N_1}} + \vec{D_{T_1}}$

$= \left(-18.75\,\hat{a}_x + 31.25\,\hat{a}_y + 70\,\hat{a}_z\right)\ nC\,m^{-2}$

(i) $\theta_2 = \cos^{-1}\left(\frac{70}{78.92}\right) = 27.50^\circ = 0.153\pi \text{ rad}$

4.

| $x<0$ | $x>0$ |
|---|---|
| $5\mu H\,m^{-1}$ | $20\,\mu H\,m^{-1}$ |

$\rightarrow \hat{a}_{N_{AB}}$

$\vec{H_{TA}} = \left(-400\,\hat{a}_y + 500\,a_z\right)\ A\,m^{-1}$

$\vec{H_{NA}} = \left(300\,\hat{a}_x\right)\ A\,m^{-1}$

$\hat{a}_{NAB} = \hat{a}_x$

$\left(\vec{H_{TA}} - \vec{H_{TB}}\right) \times \hat{a}_{NAB} = \vec{K}$

$\Rightarrow \vec{H_{TA}} - \vec{H_{TB}} = \hat{a}_{NAB} \times \vec{K}$

$\Rightarrow \vec{H_{TB}} = \vec{H_{TA}} - \left(\hat{a}_{NAB} \times \vec{K}\right)$ —— (1)

Also, $\frac{\vec{B_{NA}}}{\mu_1} = \frac{\vec{B_{NB}}}{\mu_2}$

$\Rightarrow \vec{H_{NB}} = \frac{\mu_2}{\mu_1}\vec{H_{NA}}$ —— (2)

Now, $\mu_2 = 20\,\mu H\,m^{-1}$, $\mu_1 = 5\,\mu H\,m^{-1}$, $\vec{K} = \left(150\,\hat{a}_y - 200\,\hat{a}_z\right)\ A\,m^{-1}$

(a) $|\vec{H_{TA}}| = 10\sqrt{1600+2500} = \sqrt{410000} = 640.3\ A\,m^{-1}$

(b) $|\vec{H_{NA}}| = 300\ A\,m^{-1}$

(c) $\vec{H_{TB}} = \left(-400\,\hat{a}_y + 500\,\hat{a}_z\right) - \hat{a}_x \times \left(150\,\hat{a}_y - 200\,\hat{a}_z\right)$

$= -400\,\hat{a}_y + 500\,\hat{a}_z - 150\,\hat{a}_z - 200\,\hat{a}_y$

$= \left(-600\,\hat{a}_y + 350\,\hat{a}_z\right)\ A\,m^{-1}$

(d) $\vec{H_{NB}} = \frac{20}{5} \times (300) = \left(1200\,\hat{a}_x\right)\ A\,m^{-1}$

$\therefore \vec{H_B} = \left(1200\,\hat{a}_x - 600\,\hat{a}_y + 350\,\hat{a}_z\right)\ A\,m^{-1}$

5. $\vec{E} = 120\pi \cos(10^6\pi t - \beta x)\,\hat{y}$ $Vm^{-1}$

$\vec{H} = A\pi \cos(10^6\pi t - \beta x)\,\hat{z}$ $Am^{-1}$

$\nabla\times\vec{E} = -\partial\vec{B}/\partial t$ $\qquad\qquad$ $\nabla\times\vec{B} = \vec{J} + \partial\vec{D}/\partial t$

Assume free space, $\vec{J}=0$.

$$\nabla\times\vec{E} = 120\pi\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ 0 & \cos(10^6\pi t-\beta x) & 0\end{vmatrix} = (120\pi)(-\beta)(-\sin(10^6\pi t-\beta x))\,\hat{z}$$

$$= \underline{\underline{120\beta\pi \sin(10^6\pi t - \beta x)\,\hat{z}\ Vm^{-2}}}$$

$$-\frac{\partial\vec{B}}{\partial t} = -\mu\frac{\partial\vec{H}}{\partial t} = \underline{\underline{+\mu A\pi^2(10^6)\sin(10^6\pi t-\beta x)\,\hat{z}\ Vm^{-2}}}$$

$\Rightarrow$ $120\beta\pi = \mu A\pi^2\times 10^6$

$\Rightarrow$ $\beta = \left(\frac{\mu\pi\times 10^5}{12}\right)\cdot A$ ——— (1)

$$\nabla\times\vec{H} = \vec{J}+\frac{\partial\vec{D}}{\partial t} = \frac{\partial\vec{D}}{\partial t} = \varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{H} = A\pi\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ 0 & 0 & \cos(10^6\pi t-\beta x)\end{vmatrix} = \underline{+A\pi(-\beta)(\sin(10^6\pi t-\beta x))\,\hat{y}\ Am^{-2}}$$

$$= \underline{\underline{-A\pi\beta\sin(10^6\pi t-\beta x)\,\hat{y}\ Am^{-2}}}$$

$$\varepsilon\frac{\partial\vec{E}}{\partial t} = \underline{\underline{-\varepsilon(120\pi)(10^6\pi)\sin(10^6\pi t-\beta x)\,\hat{y}}}\ Am^{-2}$$

$\Rightarrow$ $-A\pi\beta = -120\pi^2\times 10^6\varepsilon$

$\Rightarrow$ $A\beta = 120\pi\varepsilon\times 10^6$

$\because$ Free space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$.

$$A\beta = A^2\left(\frac{\mu\pi\times 10^5}{12}\right) = 120\pi\varepsilon\times 10^6$$

$$\Rightarrow A = \sqrt{\frac{120\pi\varepsilon\times 10\times 12}{\mu\pi}}$$

$$= 120\sqrt{\varepsilon/\mu}$$

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$$A = 120\sqrt{\varepsilon/\mu} = 120\sqrt{\varepsilon_0/\mu_0} = 120\sqrt{\frac{8.85\times10^{-12}}{4\pi\times10^{-7}}} = \underline{0.318}$$

$$\beta = \left(\frac{\mu_0 \pi \times 10^5}{12}\right) A$$

$$= \frac{4\pi^2\times10^5}{12}\times 0.318\times10^{-7} = \underline{0.0105\ m^{-1}}$$

6. $\vec{E} = 10\pi\cos(10^6 t - 50z)\,\hat{x}\ V m^{-1},\ \varepsilon_r = 4,\ \mu_r = 1$
$\vec{J} = 0,\ \rho = 0$

$$\nabla\times\vec{E} = -\partial B/\partial t; \Rightarrow \nabla\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 10\pi\cos(10^6t-50z) & 0 & 0 \end{vmatrix} = \hat{y}(10\pi)(-50)\sin(\dots) = \underline{-500\pi\sin(\dots)\hat{y}}$$

$$\vec{B} = -\int(\nabla\times\vec{E})\,dt$$
$$= -500\pi\int\sin(10^6t-50z)\,dt\ \hat{y}$$
$$= \frac{+500\pi}{10^6}\left(\cos(10^6t-50z)\right)\hat{y}$$

i.e. $\vec{B} = \underline{5\pi\times10^{-4}\cos(10^6t-50z)\,\hat{y}}\ Wb\,m^{-2}$

$\mu_r = 1 \Rightarrow \mu = \mu_0 \Rightarrow \vec{B} = \mu_0\vec{H}$; i.e. $\vec{H} = \frac{\vec{B}}{\mu_0}$

$$\therefore \vec{H} = \frac{5\pi\times10^{-4}}{4\pi\times10^{-7}}\cos(10^6t-50z)\,\hat{y}$$

$$= \underline{1.25\times10^3\cos(10^6t-50z)\,\hat{y}\ A m^{-1}}$$

7. $\mu = 3\times10^{-5}\ H m^{-1},\quad \varepsilon = 1.2\times10^{-10}\ F m^{-1}$
$\sigma = 0,\quad \vec{H} = 2\cos(10^{10}t - \beta x)\,\hat{z}\ A m^{-1}$

$$\vec{B} = \mu\vec{H} = 3\times10^{-5}\times2\cos(10^{10}t-\beta x) = \underline{6\times10^{-5}\cos(10^{10}t-\beta x)\hat{z}\ Wb\,m^{-2}}$$

$\sigma = 0 \Rightarrow \vec{J} = 0$
$\Rightarrow \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \Rightarrow \vec{D} = \int(\nabla\times\vec{H})\,dt$

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$$\nabla\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & 2\cos(10^{10}t-\beta x) \end{vmatrix} = (-\hat{y})(2)(-\beta)\cos(10^{10}t-\beta x)$$
$$= 2\beta\cos(10^{10}t-\beta x)\,\hat{y}$$

$$\vec{D} = \int(2\beta)\cos(10^{10}t-\beta x)\,dt$$
$$= \frac{2\beta}{10^{10}}\sin(10^{10}t-\beta x) = 2\beta\times10^{-10}\sin(10^{10}t-\beta x)\,\hat{y}\ Cm^{-2}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{2\beta}{1.2}\sin(10^{10}t-\beta x)\,\hat{y}\ Vm^{-1} = \frac{\beta}{0.6}\sin(10^{10}t-\beta x)\hat{y}\ Vm^{-1}$$

- $v_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{10^{10}}{\beta}$

i.e. $\beta = 10^{10}\sqrt{\mu\varepsilon} = 10^{10}\sqrt{3\times10^{-5}\times1.2\times10^{-10}} = 10^{10}\sqrt{36\times10^{-16}}$

$$\beta = 600$$

$\therefore$ $\vec{B} = 6\times10^{-5}\cos(10^{10}t-600x)\,\hat{z}\ Wb\,m^{-2}$

$\vec{D} = 1.2\times10^{-7}\sin(10^{10}t-600x)\,\hat{y}\ C\,m^{-2}$

$\vec{E} = 1000\sin(10^{10}t-600x)\,\hat{y}\ Vm^{-1}$

8. $\vec{A} = (2x^2y+yz)\hat{a}_x + (xy^2-xz^3)\hat{a}_y + (2x^2y^2-6xyz)\hat{a}_z\ Wb\,m^{-1}$

(a) $\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (2x^2y+yz) & (xy^2-xz^3) & (2x^2y^2-6xyz) \end{vmatrix}$

$$= (4x^2y-6xz+3xz^2)\hat{a}_x + (y-4xy^2+6yz)\hat{a}_y + (y^2-z^3-2x^2-z)\hat{a}_z$$

(b) $\phi = \int\vec{B}\cdot d\vec{S} = \int_0^2\int_0^2(\vec{B}\cdot\hat{a}_x)\,dy\,dz = \int_0^2\int_0^2(4x^2y-6xz+3xz^2)\,dy\,dz$

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$$\phi = \int_0^2\int_0^2 (4y - 6z + 3z^2)\, dy\, dz$$

$$= \int_0^2 \left[2y^2 - 6yz + 3yz^2\right]_0^2 dz = \int_0^2 (8 - 12z + 6z^2) dz$$

$$= \left[8z - 6z^2 + 2z^3\right]_0^2 = 16 - 24 + 16 = \underline{\underline{8\ Wb}}$$

(c) $$\nabla\cdot\vec{A} = \frac{\partial}{\partial x}(2x^2y + yz) + \frac{\partial}{\partial y}(xy^2 - xz^3) + \frac{\partial}{\partial z}(2x^2y^2 - 6xyz)$$

$$= 4xy + 2xy - 6xy = \underline{\underline{0}}$$

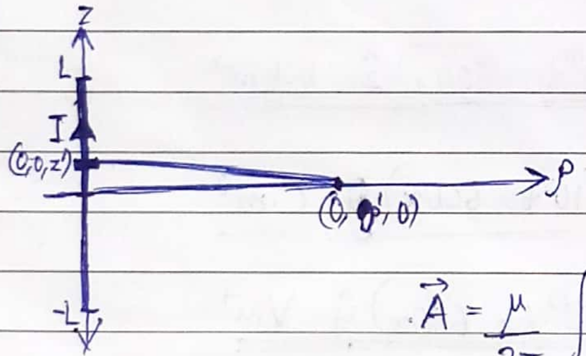
Similarly

$$\nabla\cdot\vec{B} = \frac{\partial}{\partial x}(4x^2y - 6xz + 3xz^2) + \frac{\partial}{\partial y}(y - 4xy^2 + 6yz) + \frac{\partial}{\partial z}(y^2 - z^3 - 2x^2 z)$$

$$= 8xy - 6z + 3z^2 + 1 - 8xy + 6z - 3z^2 - 1 = \underline{\underline{0}}$$

Hence verified

9.




$$A = \int \frac{\mu}{4\pi}\frac{I\,dl'}{R} = \int_{-L}^{L} \frac{\mu}{4\pi}\frac{I\,dz'\,\hat{a}_z}{\sqrt{z'^2+\rho'^2}}$$

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{A} = \frac{\mu}{2\pi}\int_0^L \frac{I\,dz'}{\sqrt{z'^2+\rho'^2}}\,\hat{a}_z = \frac{\mu I}{2\pi}\left[\ln\left(z' + \sqrt{z'^2+\rho'^2}\right)\right]_0^L \hat{a}_z$$

$$= \frac{\mu I}{2\pi}\left(\ln\left(L + \sqrt{L^2+\rho'^2}\right) - \ln(\rho')\right)\hat{a}_z$$

If $L >> \rho'$, $A = \frac{\mu I}{2\pi}\left(\ln(2L) - \ln(\rho')\right)\hat{a}_z$

$$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \hat{\rho} & \hat{z} & \rho\hat{\phi} \\ \partial/\partial\rho & \partial/\partial z & \partial/\partial\phi \\ A_\rho & A_z & \rho A_\phi \end{vmatrix}\cdot\frac{1}{\rho'} = \frac{1}{\rho'}\begin{vmatrix} \hat{\rho} & \hat{z} & \rho\hat{\phi} \\ \partial/\partial\rho & \partial/\partial z & \partial/\partial\phi \\ 0 & \frac{\mu I}{2\pi}\left(\ln(2L) - \ln(\rho')\right) & 0 \end{vmatrix}$$

$$= (\rho'\hat{\phi})\left(\frac{\mu I}{2\pi}\right)\left(\frac{-1}{\rho'}\right)\left(\frac{1}{\rho'}\right) = \underline{\underline{-\frac{\mu I}{2\pi\rho}\hat{\phi}}}$$

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10 $V = 10\sin(\omega t)$, $\mu_r = 1$, $\varepsilon_r = 10$

$$\nabla\cdot A = -\mu\varepsilon \frac{\partial V}{\partial t} = -\mu_0\varepsilon_0 \times \mu\varepsilon \times \frac{\partial V}{\partial t}$$

$$= \frac{-1\times 10\times 1}{(299792458)^2} \times 10\,\omega\cos(\omega t)$$

$$= 1.1127\times 10^{-15}\,\omega\cos(\omega t) \Rightarrow \underline{|\nabla\cdot A| = 1.1127\times 10^{-15}\,\omega\times 2\pi}$$

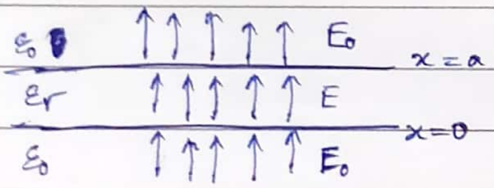
$$= \underline{6.99\times 10^{-15}\,\omega}$$

(a) $\omega = 50\ Hz$

$$\Rightarrow |\nabla\cdot A| = 1.1127\times 10^{-15}\times 50\times 2\pi = 3.495\times 10^{-13} \cong \underline{\underline{0}}$$

(b) $\omega = 100\times 10^{12}\ Hz$

$$\Rightarrow |\nabla\cdot A| = 1.1127\times 10^{-15}\times 100\times 10^{12}\times 2\pi = \underline{\underline{0.699}}$$

2. 


$E_{out} = \underline{\underline{E_0}}$

$D_{out} = \underline{\underline{\varepsilon_0 E_0}}$

$D_{in} = D_{out} = \underline{\underline{\varepsilon_0 E_0}} = \varepsilon_r\varepsilon_0 E$

$\therefore E = \frac{E_0}{\varepsilon_r}$ //

$$\therefore \vec{E} = \begin{cases} E_0, & x<0,\ x>a \\ E_0/\varepsilon_r, & 0\le x\le a \end{cases}$$

$\vec{D} = \varepsilon_0 E_0$ always