Test Signals / Building Blocks

– *Test Signals Concepts*

– *Response Building Blocks*

Standard Test Signals as Inputs

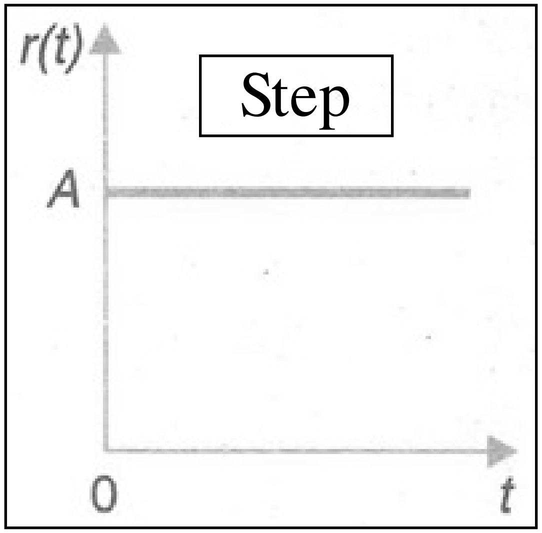
In general, **inputs**, u(t), are **not fully known** ahead of time, and are also **random in nature**. It is, therefore, **difficult to express** the actual input as an expression.

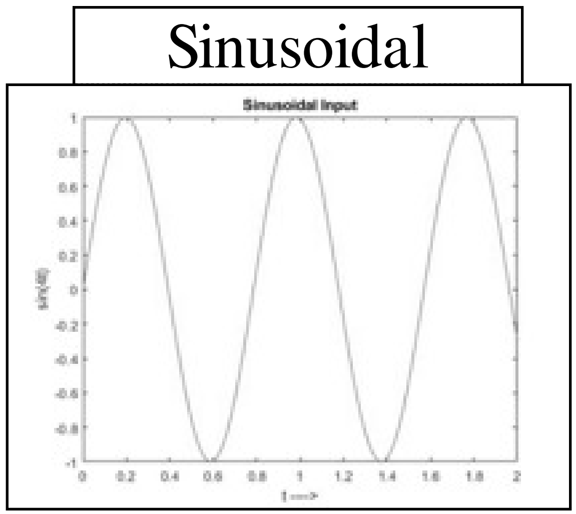
This has given rise to **test sig**nals, which provide a way of **characterizing** the behaviour during **design,** which are **simplified** forms of the **realistic inputs.**

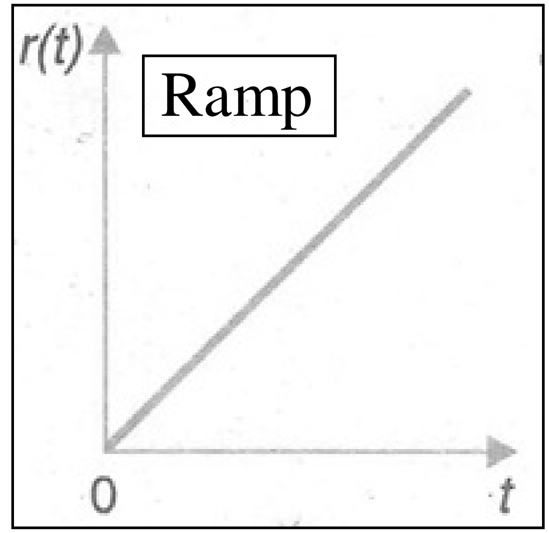
In **control analysis** and design, impulse, step, ramp, parabolic & sinusoidal **inputs** are treated as **test** signals, as these are able to **excit**e the relevant dynamical **features**.

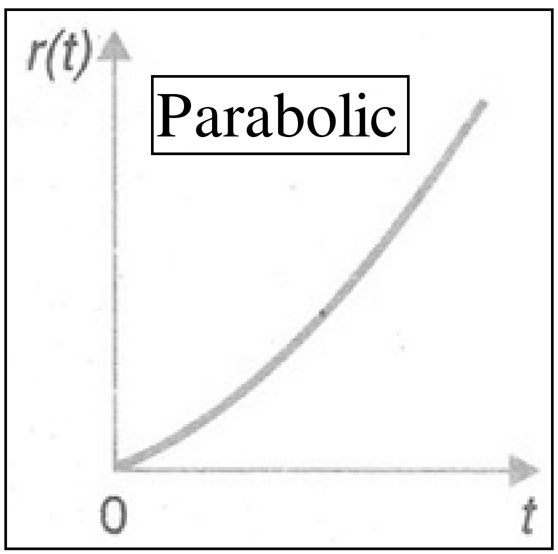
Standard Test Signals

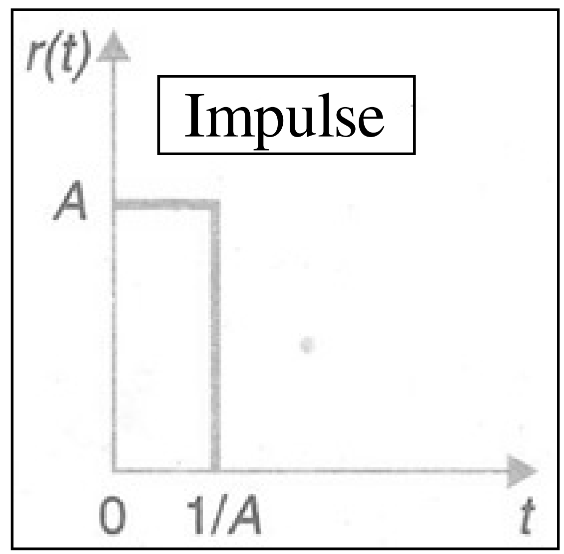
Given below are the **time variations** of these test signals.











Standard Test Signals

Mathematically, these **test** signals can be **expressed as follows.**

Impulse: $\delta(t) = 0, t \neq 0; \quad \int_{-\varepsilon}^{+\varepsilon} \delta(t)dt = 1$

Step: $r(t) = Au(t), u(t) = 1, t > 0; \quad u(t) = 0, t < 0$

Ramp: $r(t) = At, t > 0; \quad r(t) = 0, t < 0$

Parabolic: $r(t) = \dfrac{At^2}{2}, t > 0; \quad r(t) = 0, t < 0$

Sinusoidal: $r(t) = A \sin \omega t$

Summary

Test signals provide a **simple technique** to obtain the system **behaviour** during the **design process.**

Response Building Blocks

Time Response Generation Method

In dealing with **LTI** systems, principle of **superposition** is invoked in order to simplify the **solution procedure**. Thus, total response is a **sum** of natural and forced.

This philosophy is further extended by **decomposing** the general **n^{th} order system** into a number of **1^{st} & 2^{nd}** order systems, whose responses are **added** to get full response.

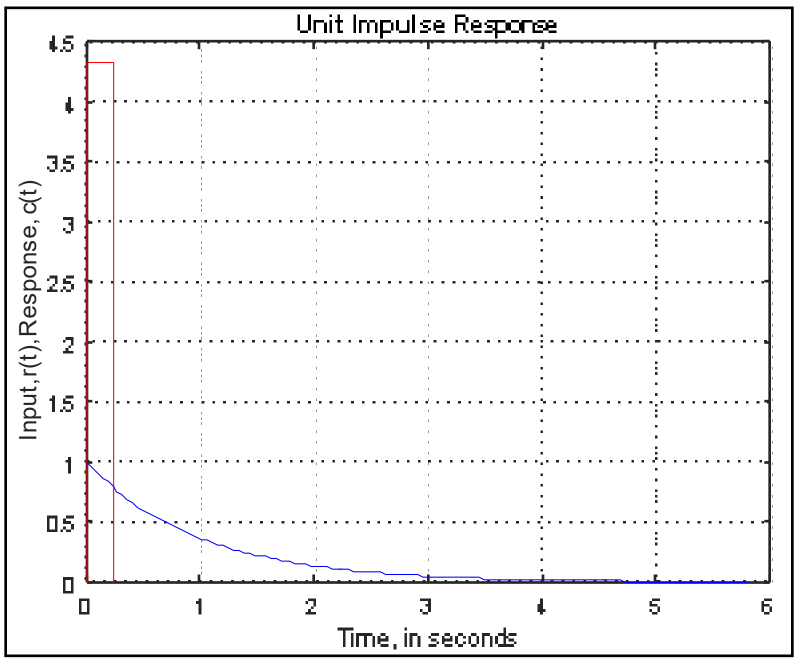
As a consequence, **solution methodologies** give a lot of importance to **1^{st} and 2^{nd} order** system responses, which are also **part** of responses of even **higher order** systems.

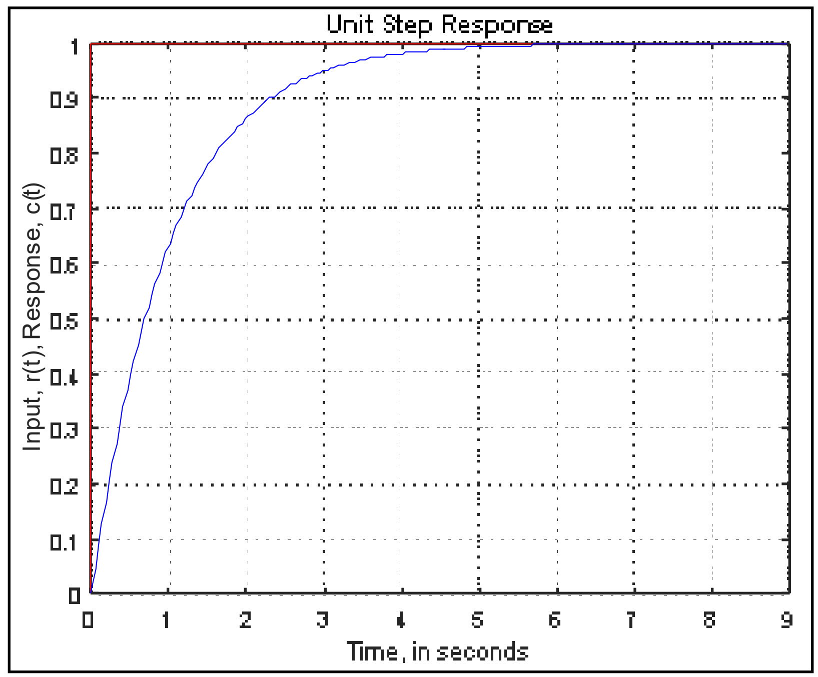
Responses of 1st Order Systems

A typical **1st order system** can be represented through the following **differential equation.**

$$\boxed{T\dot{c}(t) + c(t) = r(t)}$$

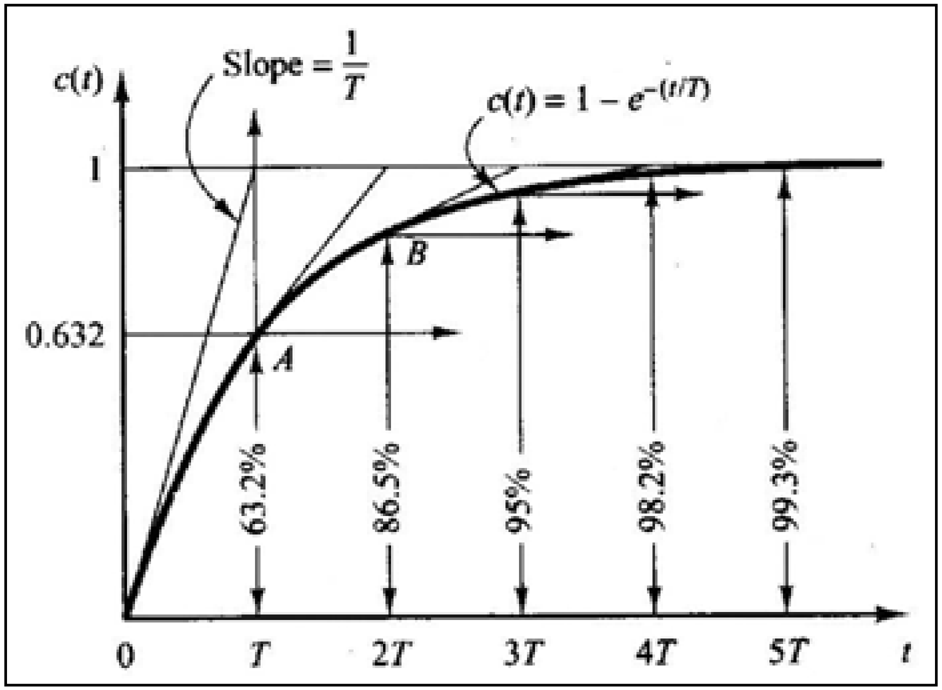
Unit **impulse** and **step** responses are as shown below.





1^{st} Order System Response Features

We see that there is a **specific trend** of the response, which is **related** to the parameter, '**T**', as shown below.



T – Time Constant

95% & 98% instants –
Used as settling time

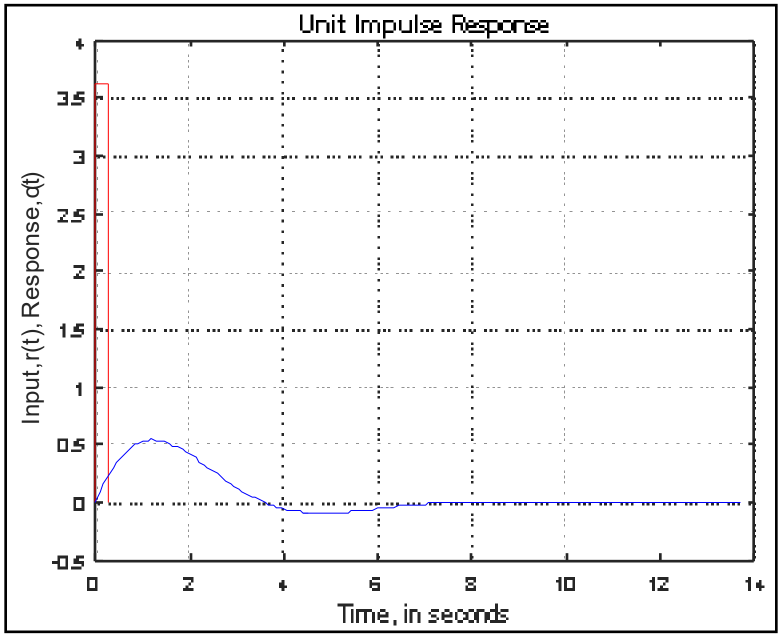
Thus, we can **shape response** by suitably choosing, '**T**'.

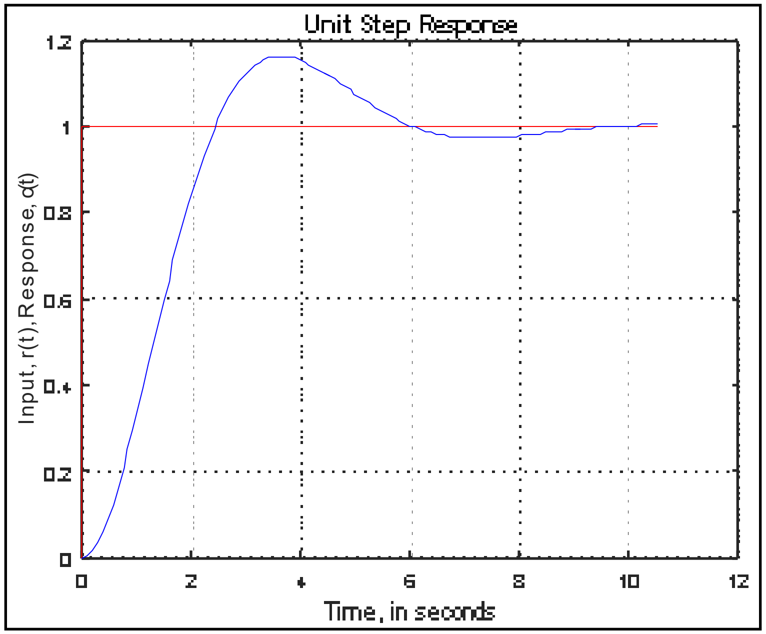
Responses of 2nd Order Systems

A typical **2nd order system** can be represented through the following **differential equation.**

$$\ddot{c}(t) + 2\zeta\omega_n \dot{c}(t) + \omega_n^2 c(t) = \omega_n^2 r(t)$$

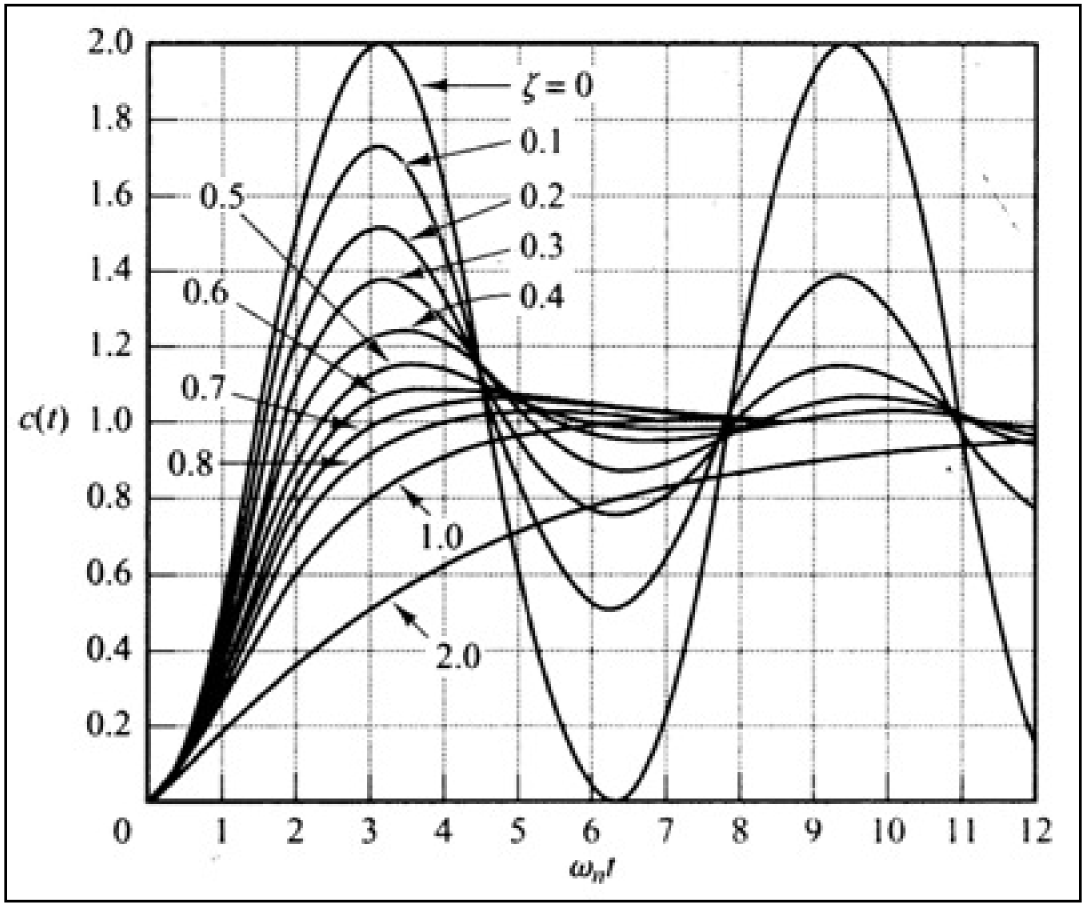
Unit **impulse** and **step** response are as follows.





2^{nd} Order System Response Features

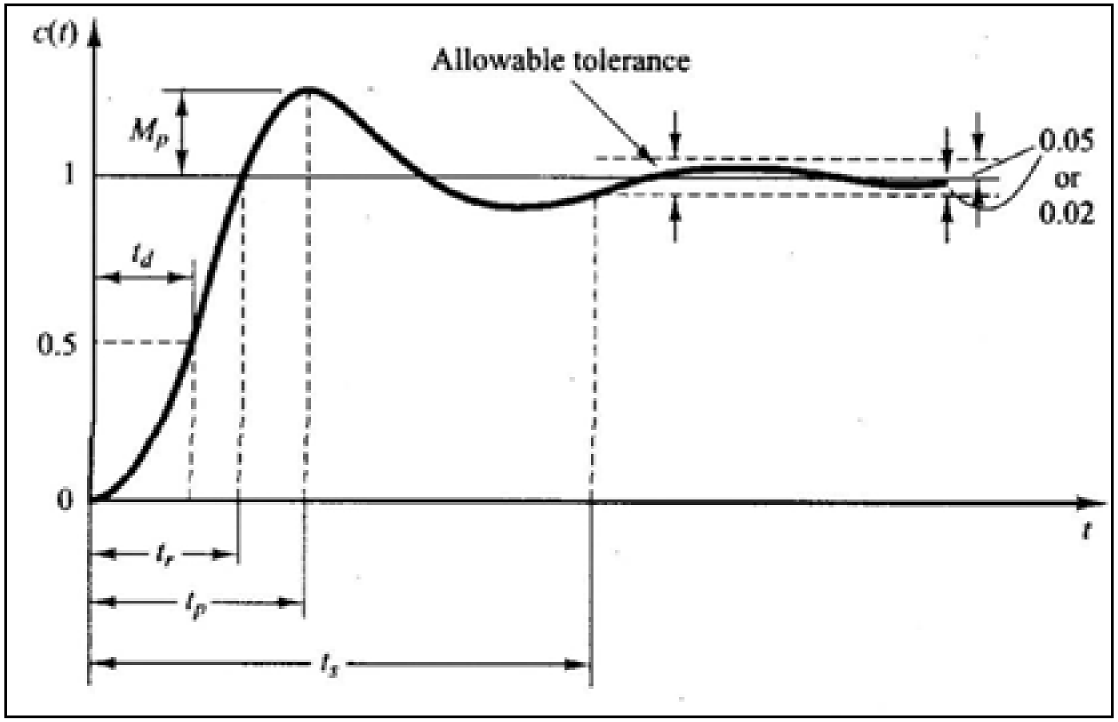
Response is a function of 'ζ' and 'ω_n', as shown below.



Thus, we can choose 'ζ' and 'ω_n', for a desirable **2^{nd} order** response.

2^{nd} Order System Response Features

2^{nd} order system response contains **important features**, as described below.



M_p – Peak Overshoot
t_d – Dead Time
t_r – Rise Time
t_p - Peak Time
t_s – Settling Time

Response to General Inputs

While, we can obtain **1st** and **2nd** order responses through **assumed functions**, as **most systems** are of higher **order** & experience **complex** inputs, we need a **generic procedure.**

As **integrating factor** for a general input, is **not feasible**, an alternative strategy, which makes use of **impulse response** as basic **building block**, is employed.

'**Convolution**' is such a technique, which is **based** on the concept of **assembling** a large number of **impulse** responses to arrive at response to **general** inputs.

Summary

Response of **1st and 2nd order** systems is used as **building block** for obtaining the time responses of the **higher order** systems.