Closed Loop Relative Stability

– *Frequency Domain Relative Stability*

– *Gain and Phase Margins Concept*

– *Margins from Bode' and Nyquist Plots*

Relative Stability from Bode/Nyquist

Nyquist stability criterion provides the **absolute** stability of **closed** loop system, based on plant **frequency** response.

In a **similar** manner, we can make use of **plant** frequency response to **assess** relative stability of the **corresponding** unity feedback **closed loop** system.

Frequency Response Attributes

As **G(jω)** computation is relatively **simpler**, it would be **convenient** if it can also be **used** to extract the **dominant** closed loop **behaviour** (i.e. poles closest to the '**jω**' axis).

In this context, we can **utilize** the fact that for **closed loop** systems having **poles** on 'jω' axis, corresponding **G(jω) =** '-1+j0', (or **'1 + G(jω)'** = 0) at some **frequency.**

The **above result** can then be quantitatively **extended** to the cases where **dominant** closed loop poles are in **LH s-plane**, though reasonably close to the '**jω**' axis.

Closed Loop Relative Stability

This **quantitative extension** can be done as follows.

Let us **assume** that the plant **G(s)** is such that at least **some roots** of $1 + G(s) = 0$ lie on the **imaginary axis.**

Then, **we know** that there will be **some** frequency at which **G(jω) =** -1+j0.

Next, if we **modify** the plant such that closed loop **poles** are slightly to the **left** of imaginary **axis**, we know that **G(jω)** ≠ -1+j0.

Closed Loop Relative Stability

Therefore, this **change** in poles **must** be reflected in the way modified **G(jω)** behaves in relation to **'-1+j0'.**

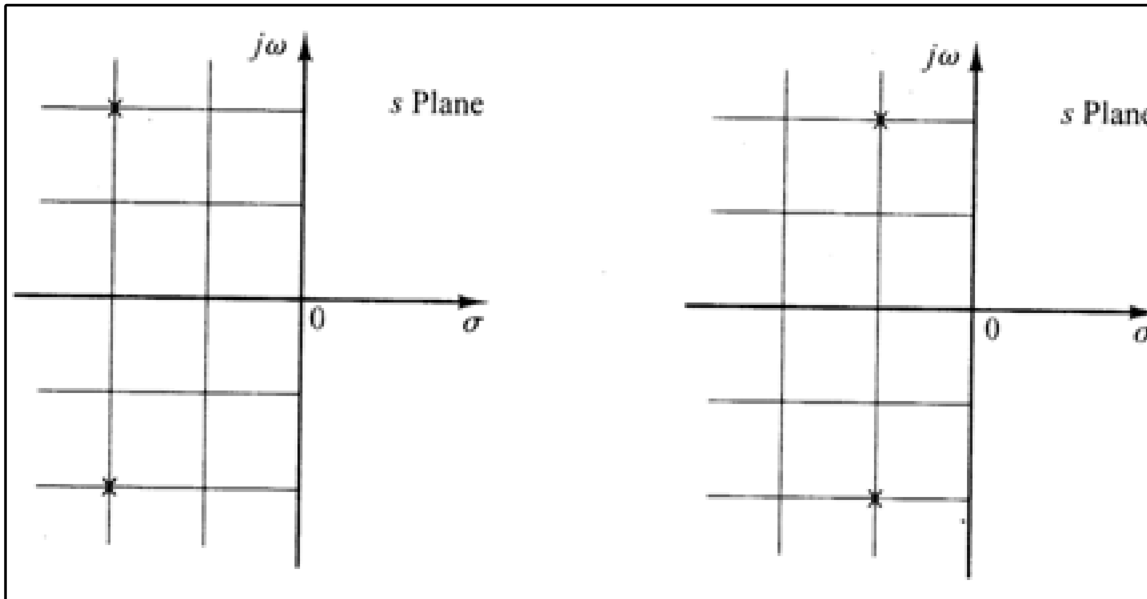
Based on this, we can **conclude** that location of **G(jω)** with respect to **'-1+j0'** can give us the indication of the **location** of dominant **closed loop** poles, and hence, the **margin.**

This **behaviour** can be **better** understood by employing the **conformal** mapping theorem, discussed **earlier.**

s-Plane – G(jω) Plane Mapping

Consider two **closed loop** systems with **poles** as shown **along side.**

We see that **system** in (**b**) is less stable in **relation** to system in (**a**).



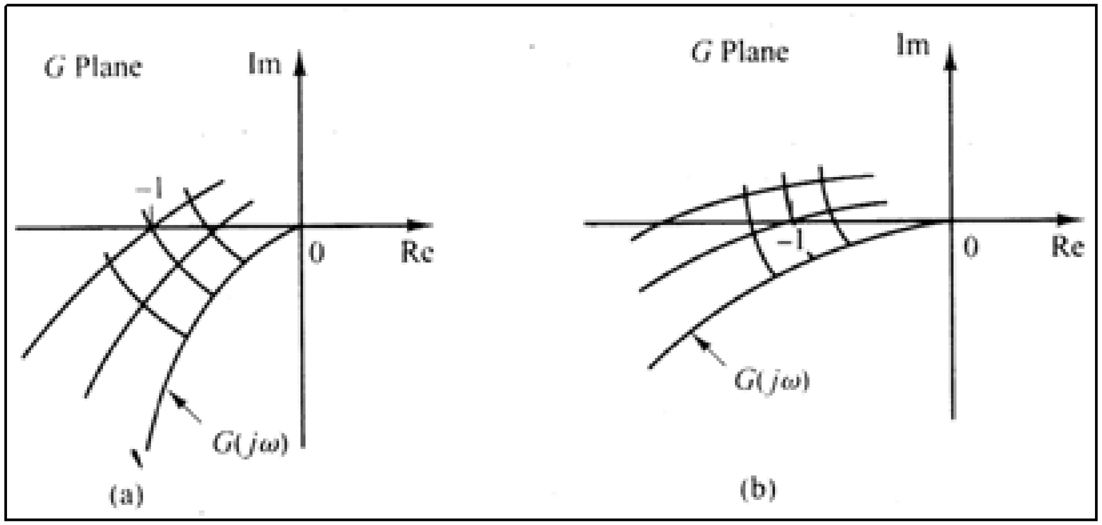
(a) (b)

s-Plane – G(jω)-Plane Mapping

Corresponding **Nyquist plots** are shown **along side.**

We see that **Nyquist** plot in (b) is **closer** to -1+j0 than the plot **in (a)**, indicating a reduction in **stability level.**

Hence, we can **conclude** that **stand-off** distance of G(jω) with **-1+j0** is a measure of stability margin.

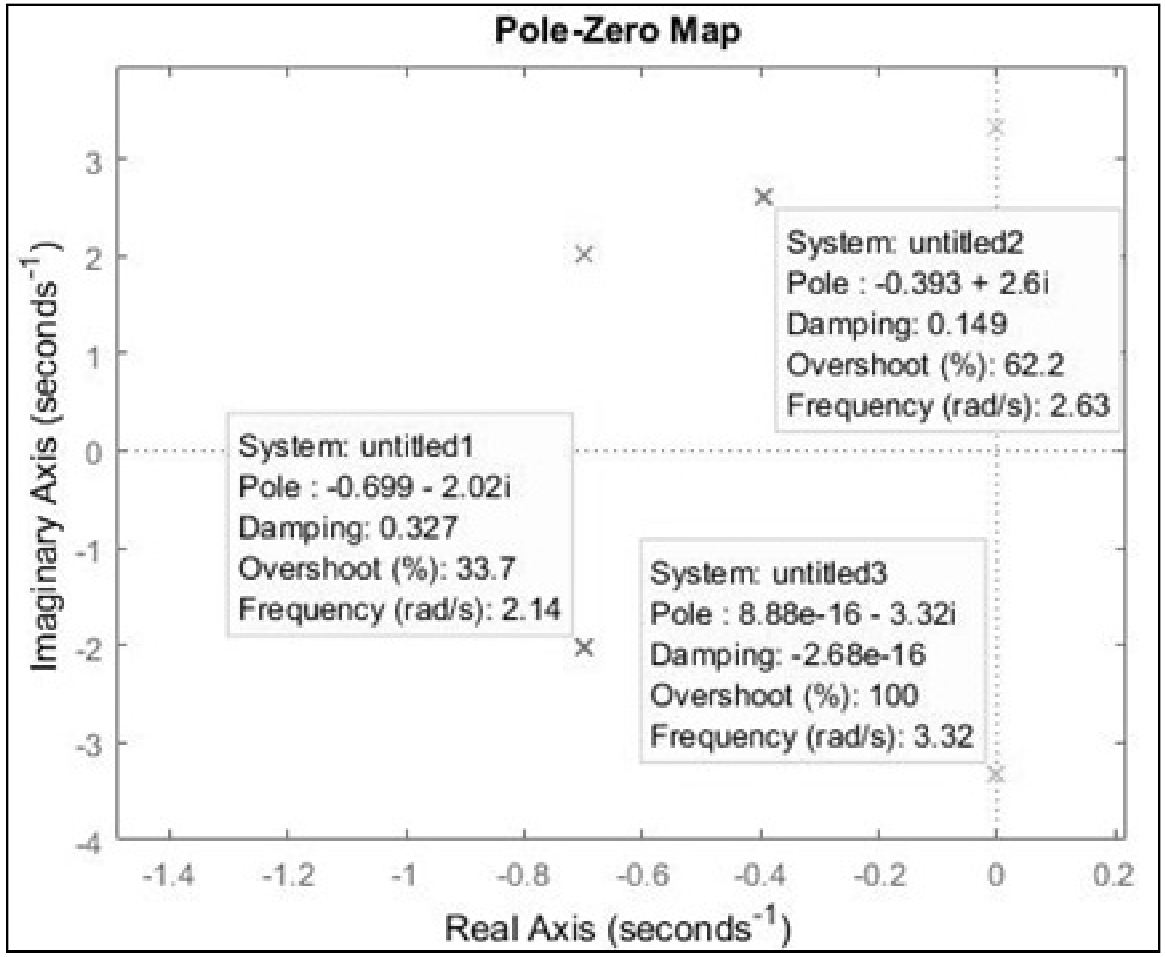


s-Plane – Dominant Pole Location

Consider **closed loop** created from following **plant** for K = 15, 30, 60.

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

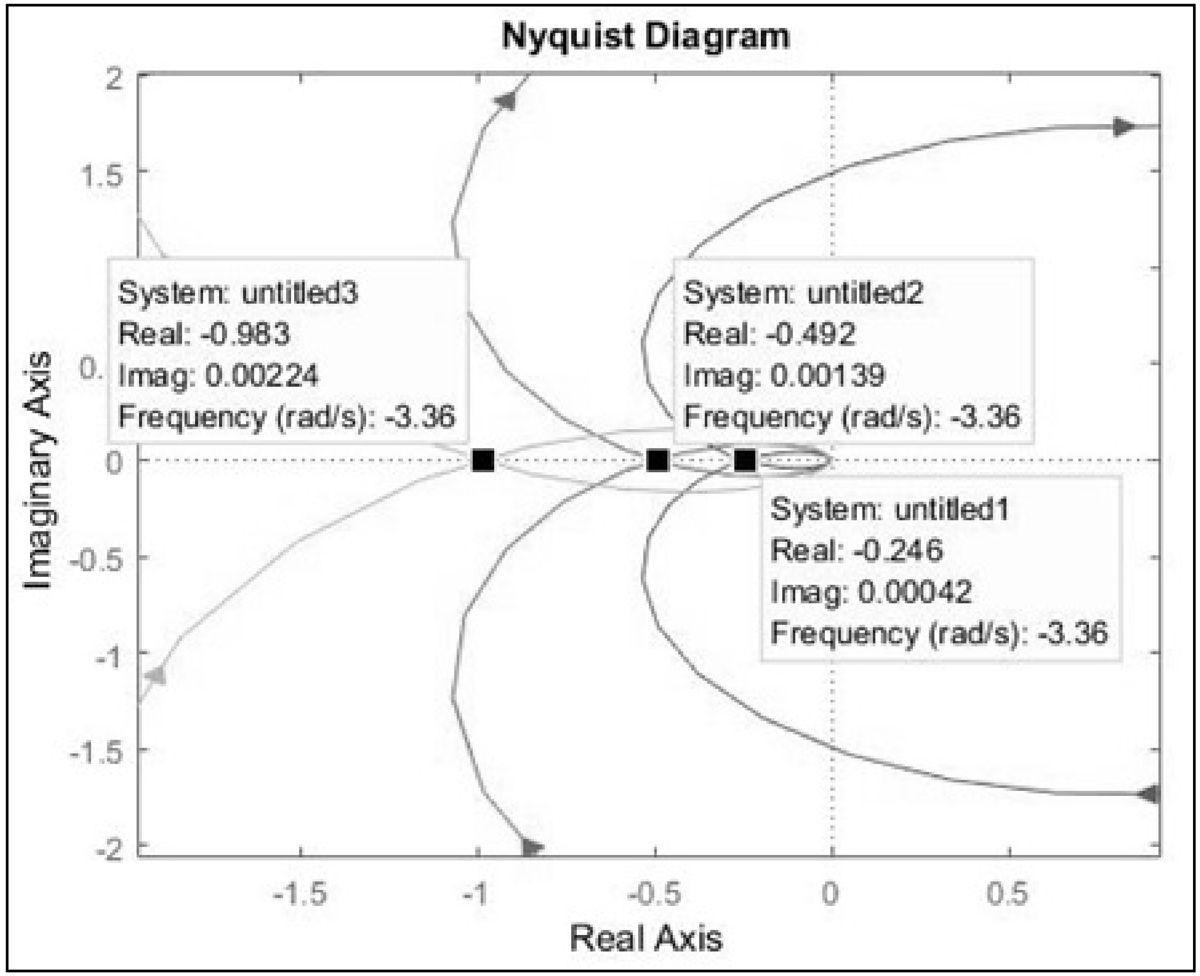
We see that **dominant poles** have 'σ' of 0.7, 0.39 & 0.0 **respectively.**



$G(j\omega)$-Plane – Real Axis Intersection

Given alongside are the corresponding **Nyquist plots** for the three plants.

As **poles move** from left to right, **gap** between **intersections** & -1+j0 are; 0.754, 0.508 & 0.0 respectively.

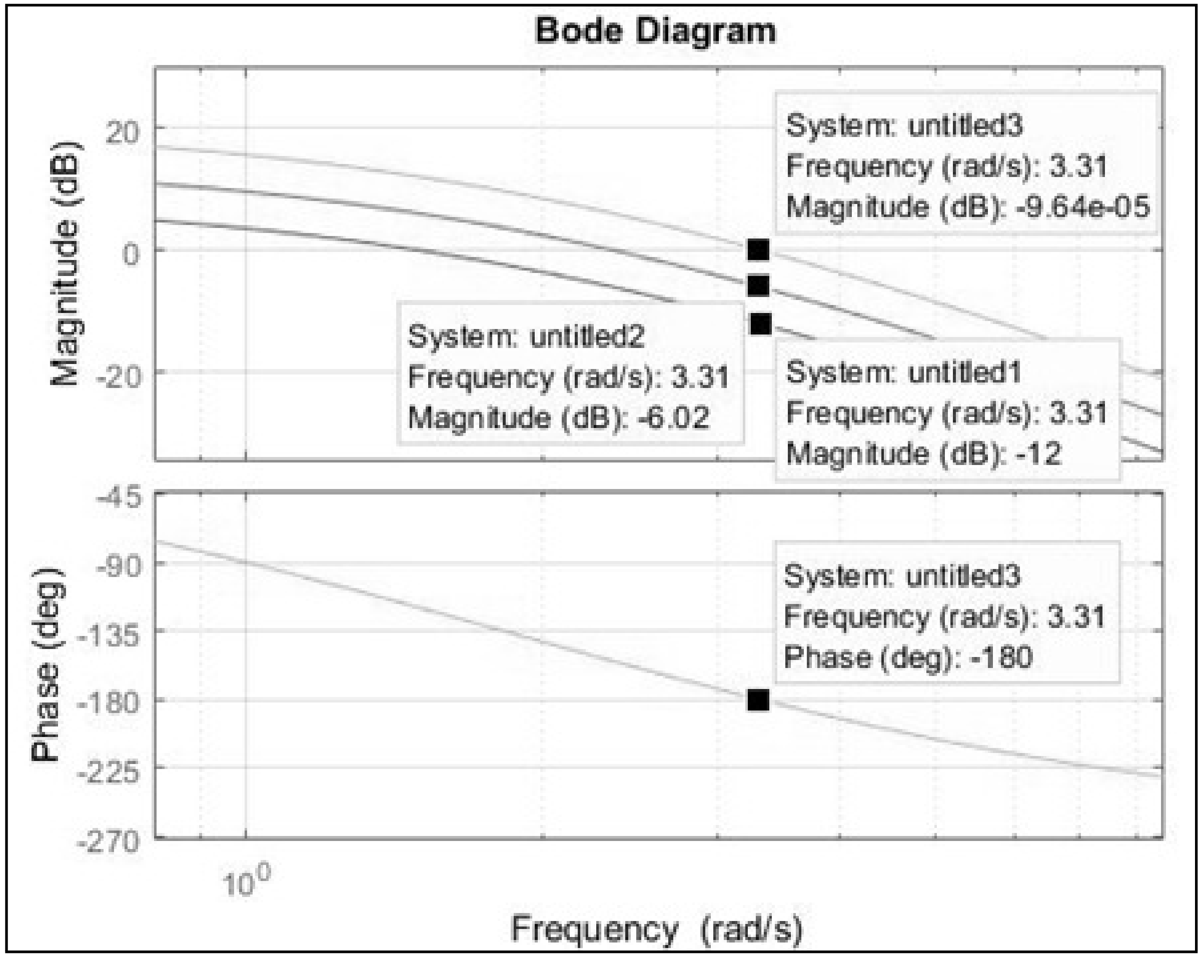


Bode Plot – 180^o Intersection

Given alongside are the bode p**lots** for these **plants.**

Similarly, we see that **bode** magnitudes at **180^o intersectio**n are; -12 dB, -6 dB & 0 dB respectively.

Thus, this **behaviour** is used to **arrive** at the concept of crossovers and **margins**.



Crossover Concept

Crossover Concept

We know that **movement** of closed **loop poles** towards '**jω**' axis, results in **movement** of Nyquist plot towards -**1+j0.**

Therefore, if this continues, **closed loop** system **crosses** from **stable** to **unstable** zone in both **s-** & **G-plane.**

Further, we know that at this **crossover** point, $|G(j\omega)| = 1$ & $\angle G(j\omega) = \pm 180^o$ at **one** value of **frequency.**

However, if **poles** are to the **left/right** of 'jω' axis, then these **conditions** are satisfied at **two** separate **values** of 'ω'.

Gain and Phase Crossover Concepts

The two **frequencies** are called **Gain and phase** crossover frequencies and are **defined** as follows.

Gain crossover (GCO) is defined as **'ω'** at which the **gain** is unity i.e. ω_{GCO} at $\mathbf{|G(j\omega)|=1}$ or $20 \log_{10} |G(j\omega)| = 0$.

Similarly, **phase crossover** (PCO) is defined as **'ω'** at which **phase angle** is $\pm 180^o$, i.e. ω_{PCO} at $\mathbf{\angle G(j\omega)=\pm 180^o}$.

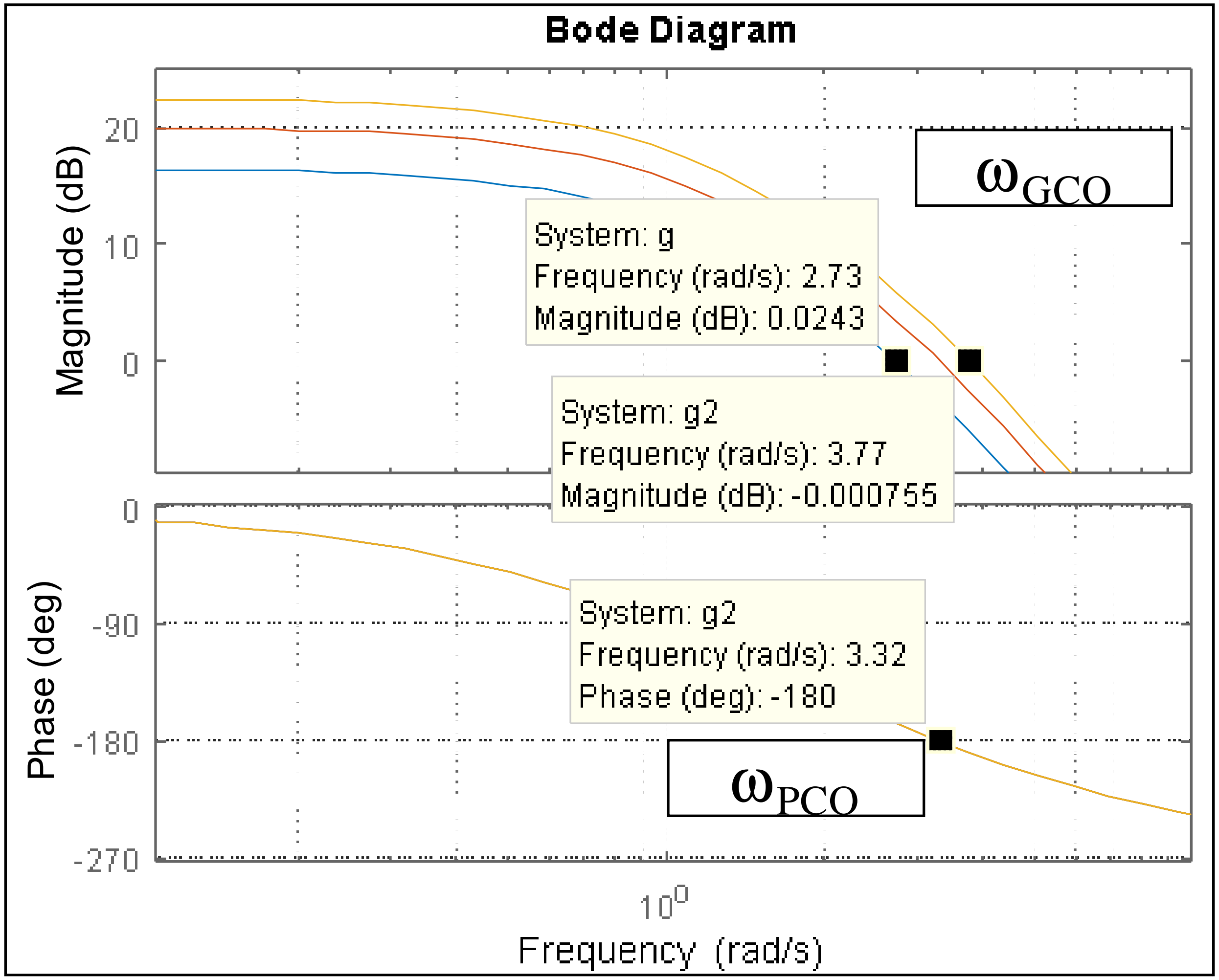
It is **clear** that if these **two** frequencies coincide, the **closed** loop **poles** lie on 'jω' **axis** and the margin is **zero.**

Gain and Phase Crossover – Bode

Consider **bode plot**, given alongside, of the **plant** given below.

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$
$$K = 40, 60, 80$$

We see that **GCO is same** as PCO for **K = 60**, for which we know that **closed loop poles** are on ‘jω’ axis.

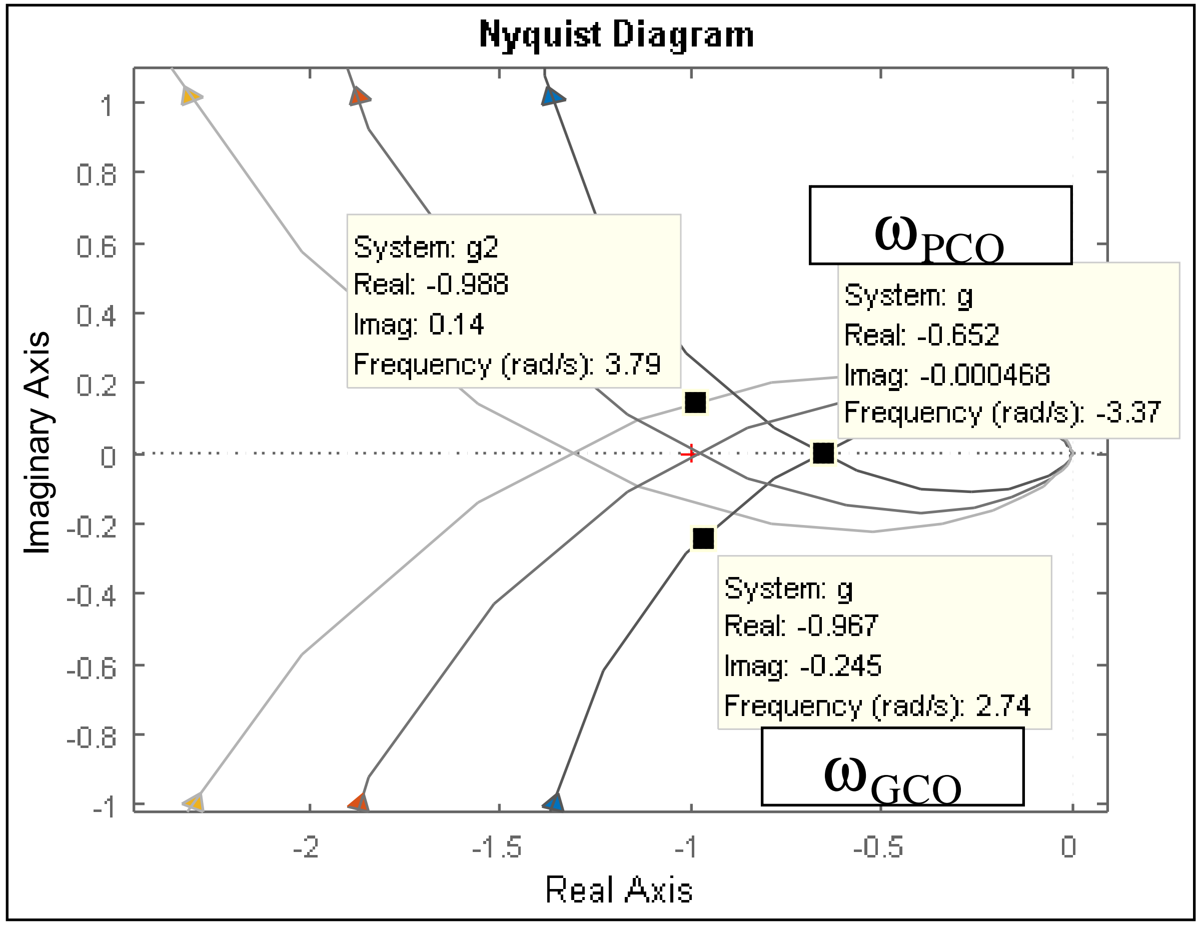


Gain and Phase Crossover – Nyquist

Consider **Nyquist plot**, given alongside, of the **plant** given below.

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$
$$K = 40, 60, 80$$

We see that while, for **K = 40,** plot does not encircle -1+j0, for **K = 80**, the plot **crosses over**, resulting in **unstable** closed loop.



Features of Crossovers

GCO & PCO contain **stability** information inasmuch as that these are **related** to the corresponding **location** of closed loop poles in **s-plane.**

Therefore, **magnitude** and **phase** at PCO and GCO are used as **quantitative measures** of relative stability of the **closed loop** system, also called **margins.**

Summary

Gain and phase crossover frequencies provide an **elegant** mechanism to examine **relative** stability of the **closed** loop system from **plant** frequency response.