Frequency Response Representation

- *Graphical Representation Strategy*
- *Bode' Plots*
- *Nyquist Plots*

Graphical Representation Tools

Among the **many possible** ways of **graphically** representing the frequency response, **Bode'** and **Nyquist** plots are the most **commonly** employed forms**.**

An important added **advantage** of these **representations** is the possibility of visualizing **the closed loop** behaviour and applicable **control actions**, in a simple & intuitive manner.

Therefore, the **above plots** are also useful **tools** for the **design** of closed loop **control systems.**

Representation with Bode' Plots

Bode' Diagrams

Bode' diagrams show variation of $|G(j\omega)|$ and $\angle G(j\omega)$ on **two separate plots** as ω varies from 0 to ∞. In these plots, while **dB** is used for **magnitude, degree** is used for **phase**.

Both **$|G(j\omega)|$ and $\angle G(j\omega)$** plots use a **log scale** (either octave or decade) **for ω**, which permits a **large frequency scale** to be shown together.

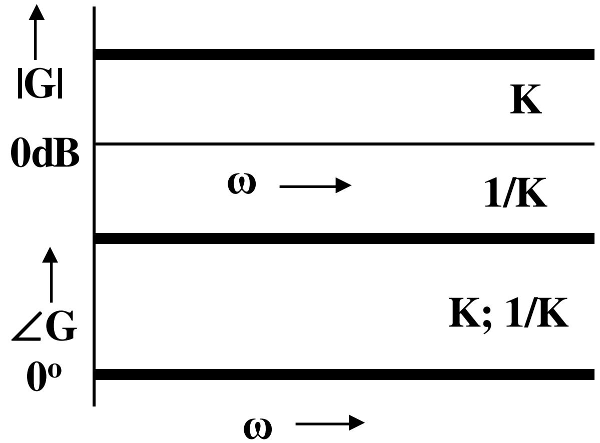
Bode' Plot Creation Process

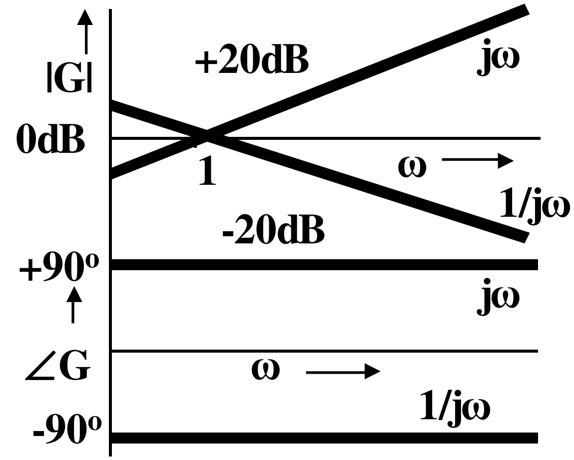
We find that **dB scale ($20\log_{10}$)** converts **multiplication/** division into **addition /** subtraction of **Log magnitudes** of the numerator and denominator **factors** of G(s).

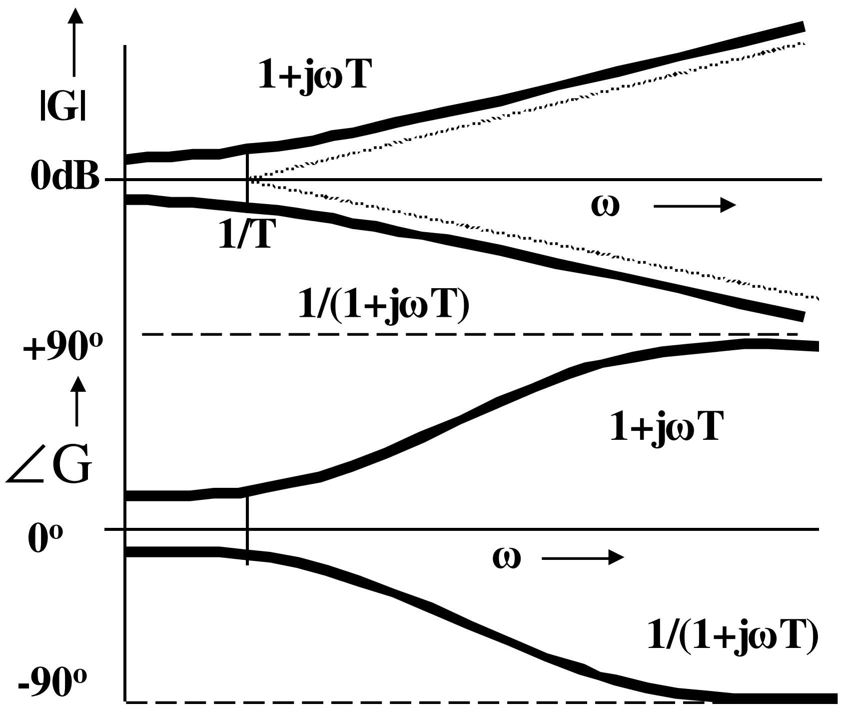
Similarly, **total phase** of the frequency response can also be expressed as **an addition / subtraction** of phase angles of the numerator/ denominator **factors**.

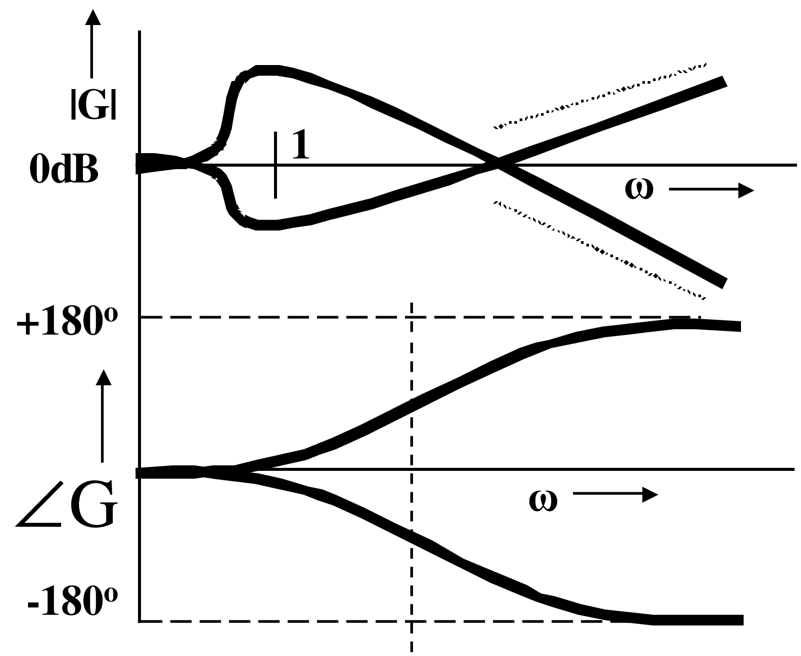
Thus, it is **possible to create** a complex bode' plot through a **building block approach** by synthesizing it using **simple factors**, e.g. pure gain, 1^{st} order and 2^{nd} order.

Bode' Plots of Basic Factors









Basic Features of Plots

We see from the **plots** that all show a specific **pattern** for the **two limiting** frequency points of **'0' and '∞'.**

This is typically in the form of **low** and **high frequency asymptotes**, which provide DC gain (**G(0)**) and relative degree (i.e. **n – m**) of G(s).

We also find that **changes** in the **asymptote angles** occur around **frequencies** that correspond to **poles and zeros.**

Therefore, we can **relate these** features to the plant **transfer function, G(s).**

Transfer Function – Bode' Plot

Poles and zeros are seen as points where **slope** of magnitude plot **changes** & are termed **corner frequencies.**

G(0) & **'k'** are seen as **intercept** which is **20 $\log_{10}$K** & **slope**, which is -**20k dB/decade** for $\omega = 0$. ($G(s) \approx K/s^k$).

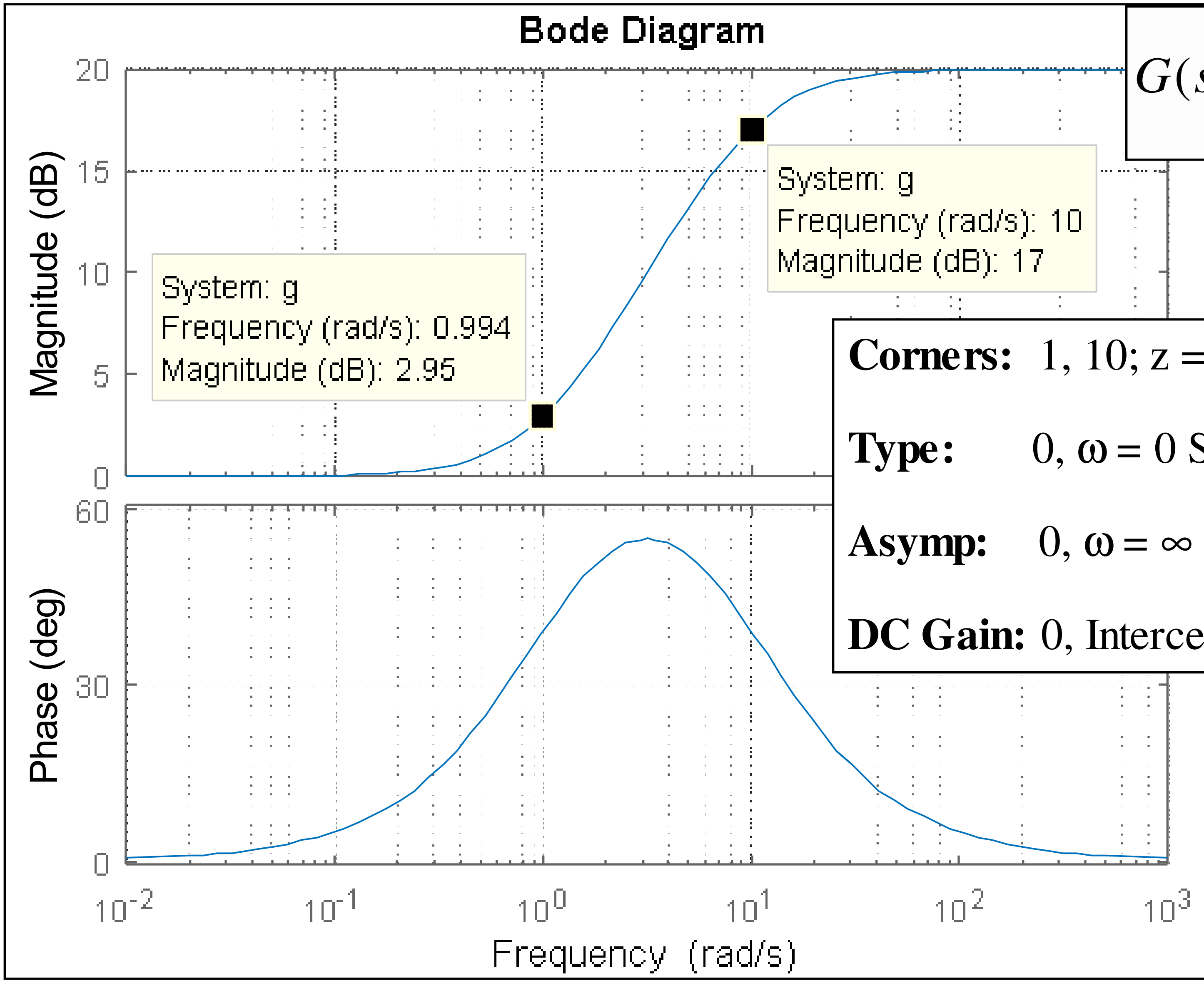
(n-m) is seen as **slope** of curve for $\omega \rightarrow \infty$, which is -**20(n-m) dB/decade**. ($G(s) \approx K/s^{(n-m)}$).

Thus it is seen that we can **reconstruct** the complete **transfer function** from its **magnitude** response.

The **above logic** is used to arrive at **phase plots.**

Bode' Diagram Example

$$G(s) = \frac{10(s+1)}{(s+10)}$$



Corners: 1, 10; z = -1, p = -10

Type: 0, $\omega = 0$ Slope: 0dB/dec

Asymp: 0, $\omega = \infty$ Slope = 0 dB/dec

DC Gain: 0, Intercept = 0 at $\omega = 0$

Summary

Bode' representation, makes the task of **creating and interpreting** the frequency response **simpler.**

Bode' Drawback & Nyquist Concept

Bode' plot **consists of two graphics**, which need to be **interpreted together.**

However, in **some cases**, there is a need to see **complete frequency response** in a single graphic and **Nyquist plot** addresses this need.

It is a plot of **Imag[G(jω)]** Vs **Re[G(jω)]** in 2-D complex plane as **ω** varies from **-∞ to +∞**.

Polar plot is the plot from **0 to** ∞ and hence is a **subset of the Nyquist plot**.

Representation with Nyquist Plots

Nyquist Plot Characteristics

In Nyquist plot, **G(jω)** is treated as a **complex** quantity, and represented **as such** in what is called, '**G(jω) – plane**'.

Here, **G(jω)** is a **vector** from **origin** to a point in the **above plane,** whose **length** is **|G(jω|** and **angle** with real axis is **∠G(jω**.

The plot is **locus** of all **terminal points** of these vectors for **ω = 0 to** ∞**.**

In this plot, **frequency** is **a graduation** on the locus.

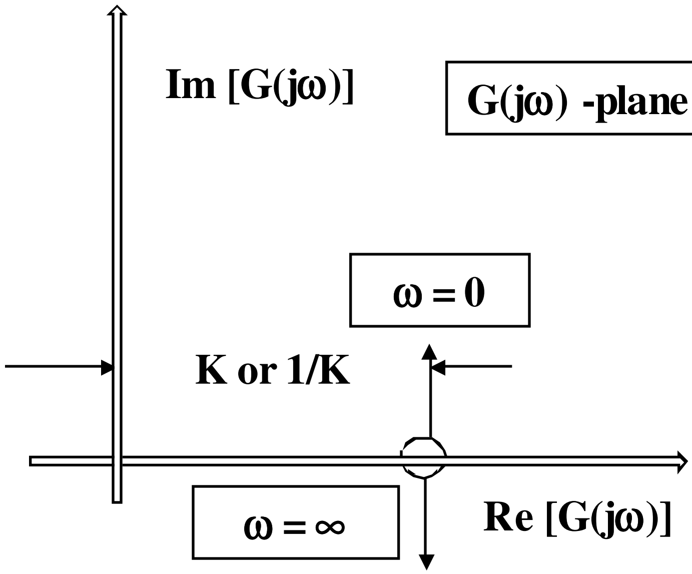
Nyquist Plot Features

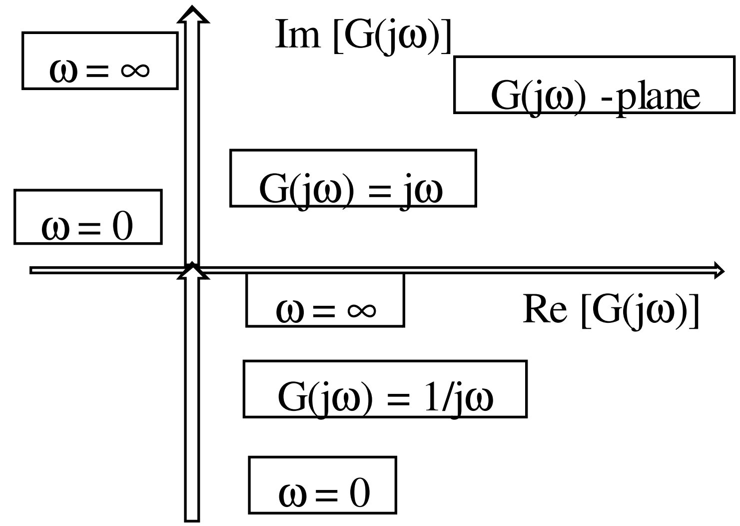
Nyquist plots combine magnitude & phase **into one** and are therefore, **compact**.

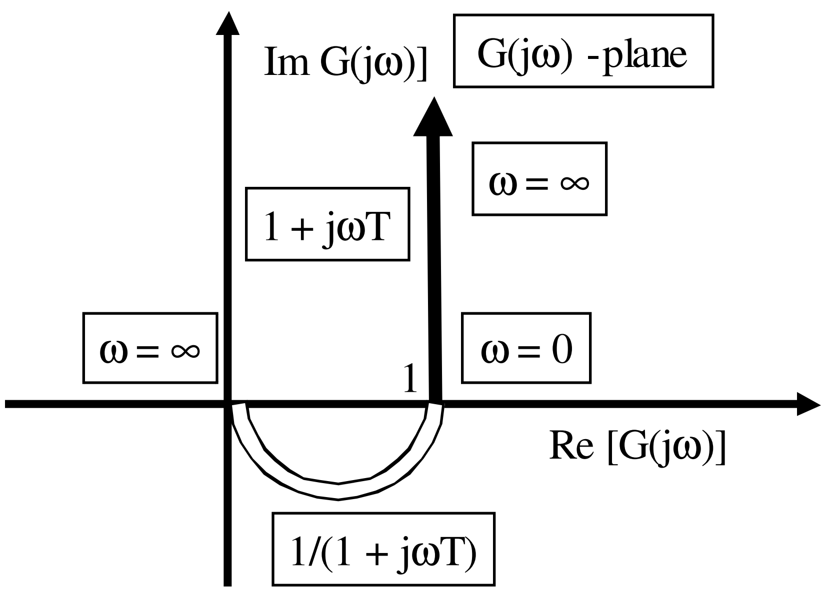
However, **Nyquist plots cannot** clearly indicate **contributions of various factors**, like Bode' plots do.

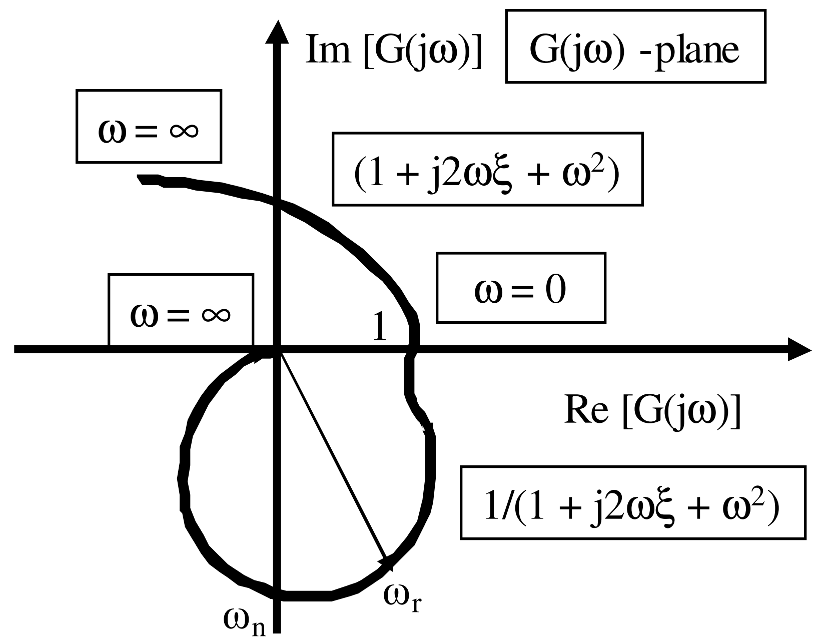
Even then, factors like, $K^{\pm 1}$, $(j\omega)^{\pm 1}$, $(1+j\omega T)^{\pm 1}$, $(1+2\xi\omega j+(j\omega)^2)^{\pm 1}$ have **certain distinguishing features**, as shown next.

Polar Plots of Basic Factors









Transfer Function – Nyquist Plot

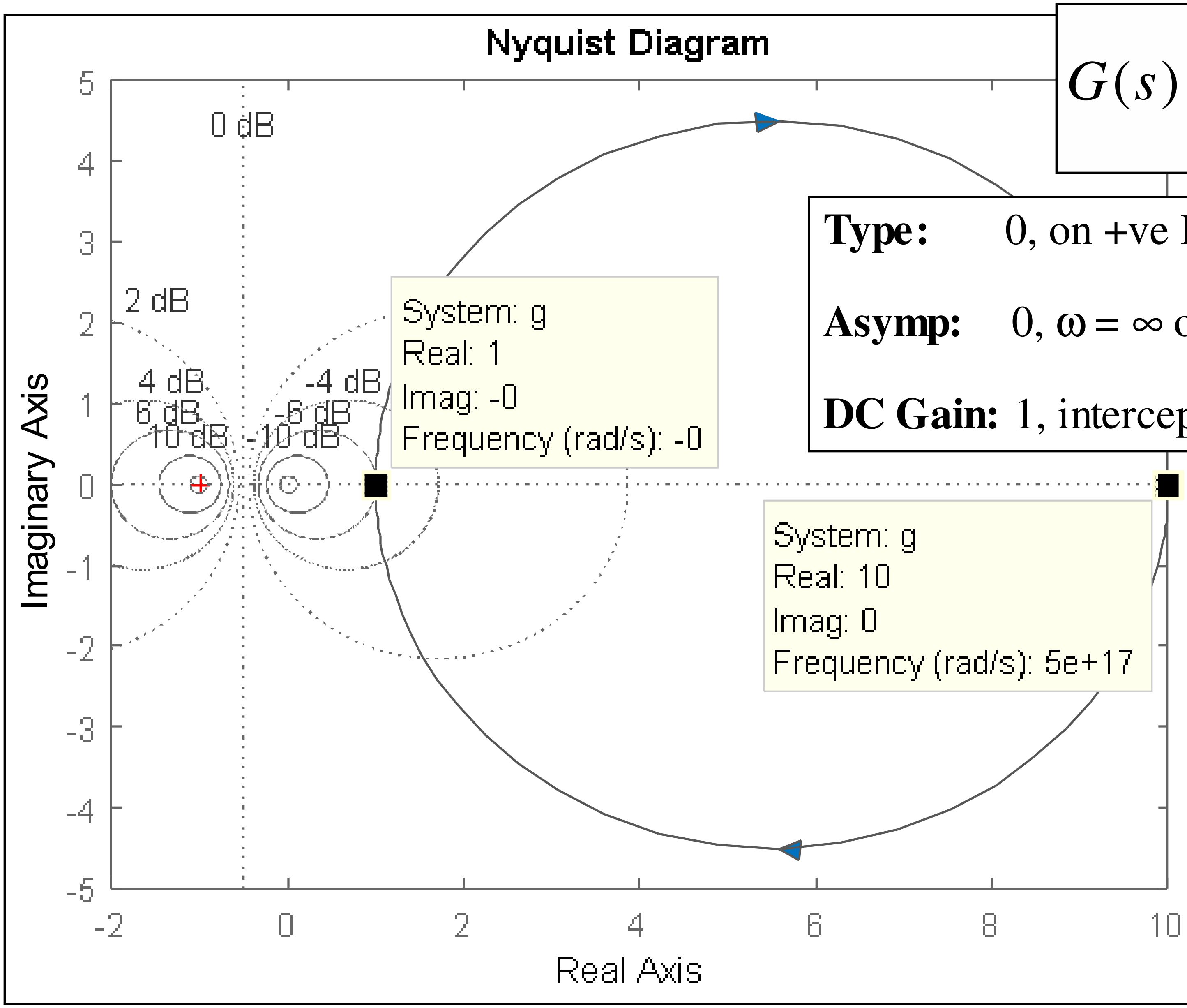
Poles and zeros are not seen in Nyquist plot and for that reason, **no corner frequencies** are possible.

System type is seen in Nyquist plot in terms of the **axis** from which it start at **ω = 0.** E.g., **type '0' systems** start from **+ve real axis**, type '1' systems **start** from **–ve imaginary** axis etc.

(n-m) is seen in terms of the **axis close to ω = ∞**. E.g. closeness to **+ve imaginary axis** indicates n-m = **3**.

DC gain is seen as **intercept** of plot on **real axis** at ω=0.

Nyquist Plot Example



$$G(s) = \frac{10(s+1)}{(s+10)}$$

Type: 0, on +ve Real Axis

Asymp: 0, $\omega = \infty$ on +ve Real Axis

DC Gain: 1, intercept = 1 at $\omega = 0$

Summary

Nyquist plots are more **elegant**, but are not very easy to interpret.

However, both **Bode' & Nyquist** representations can be used for interpreting the **transfer function features** through attributes **captured** in these plots.