solution

Tutorial-5

1. $\epsilon_1 = \epsilon_0$, $\epsilon_2 = 1.44\epsilon_0$.

$\mu_1 = \mu_2 = \mu_0$.

(a) $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = \frac{1 - \sqrt{1.44}}{1 + \sqrt{1.44}} = -0.0909$

(b) $T = \frac{2\eta_2}{\eta_1 + \eta_2}$ or. $1 + \Gamma = 0.909$

(c) Incident power $P_{in} = \frac{|E_0|^2}{2\eta} = \frac{(10^{-3})^2}{2 \times 377}$

$= 1.327$ nW/m²

(d) transmitted power: $P_t = \frac{T^2 |E_0|^2}{2\eta_2}$

or. $(1 - |\Gamma|^2) \frac{|E_0|^2}{2\eta_1}$

$= 1.315$ nW/m².

(e) E field in medium 1.

$E_i + E_r = E_0 e^{-j\beta z} + \Gamma E_0 e^{j\beta z}$

$= E_0 e^{-j\beta z} + \Gamma E_0 e^{j\beta z} - \Gamma E_0 e^{-j\beta z} + \Gamma E_0 e^{-j\beta z}$

$= E_0 e^{-j\beta z}(1 + \Gamma) + E_0 \Gamma\, 2j \cdot \sin(\beta z)$

$= E_0 e^{-j\beta z}\left(1 + |\Gamma| e^{j\phi}\right) + E_0 |\Gamma|\, 2j e^{j\phi} \cdot \sin(\beta z)$

$= E_0 e^{-j\beta z}(1 - 0.0909) + -2j E_0 \sin(\beta z)$

$= \underbrace{0.9091 E_0 e^{-j\beta z}}_{T.W} + \underbrace{(-2j) E_0 \sin(\beta z)}_{S.W}$

2.

$z<0$ | $z=0$ | $z>0$

$\mu_1 = \mu_0$, $\epsilon_1 = \epsilon_0$, $\sigma_1 = 0$, $\lambda_1 = 5$ cm

$\mu_2 = \mu_0$, ϵ_2, $\sigma_2 = 0$, $\lambda_2 = 3$ cm.

$$\beta_1 = \omega\sqrt{\mu_1\epsilon_1} \qquad \beta_2 = \omega\sqrt{\mu_2\epsilon_2}$$

$$\Rightarrow \frac{2\pi}{\lambda_1} = \frac{\omega\sqrt{\epsilon_{r1}}}{c} \qquad \Rightarrow \frac{2\pi}{\lambda_2} = \frac{\omega\sqrt{\epsilon_{r2}}}{c}$$

$$\Rightarrow \epsilon_1 = \left(\frac{2\pi}{5}\right)^2 \qquad \Rightarrow \epsilon_2 = \left(\frac{2\pi}{3}\right)^2$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} = -0.25$$

(a) $|\Gamma|^2 = 0.0625$

(b) $1 - |\Gamma|^2 = 0.9375$

*(c) $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.67$

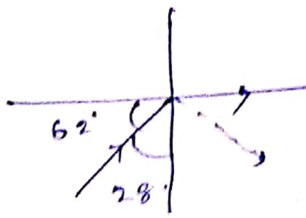
3. Brewster's angle.

$$\sin\theta_{B_{||}} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}} \Rightarrow \tan\theta_{B_{||}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

(a) $\theta_B = \tan^{-1}\sqrt{\frac{1}{81}} = 6.34°$

(b) $\theta_B = \tan^{-1}\sqrt{81} = 83.65°$

5.



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\Rightarrow \sqrt{\epsilon_{r1}}\,\sin\theta_1 = \sin\pi/2$$

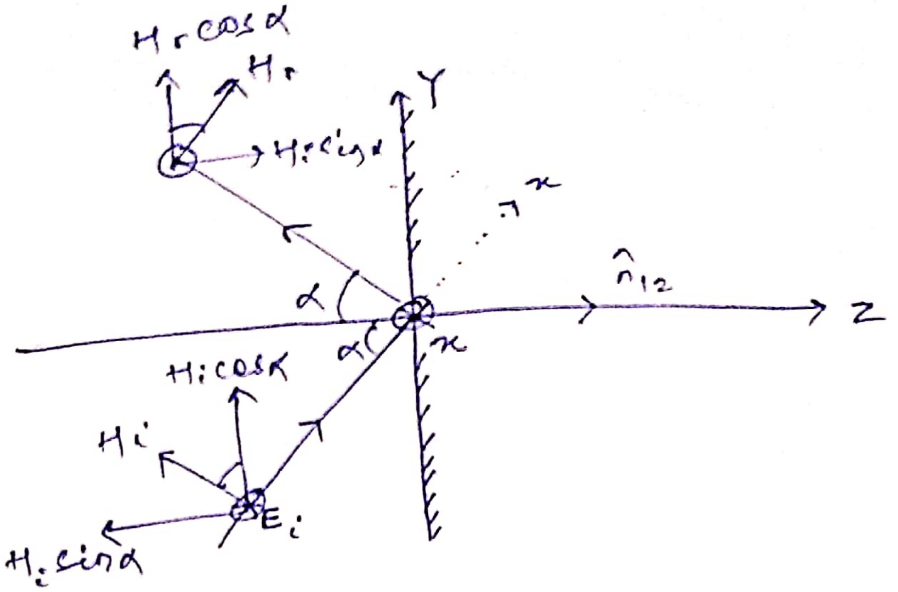
$$\Rightarrow \epsilon_{r1} = \frac{1}{(\sin\theta_1)^2} = 4.53$$

cond^n: $\theta_1 > 28°$

$\epsilon_{r2} < \epsilon_{r1}$ → for non-magnetic material.

Scanned by CamScanner

4.



$\bar{J}_s = \cancel{\hat{n} \times \bar{H}_1}$ or $(H_1 - H_2) \times \hat{n}_{12} = H_1 \times \hat{n}_{12}$

$H_{t1} = \bar{H}_{it} + \bar{H}_{rt}$

$H_i = (\hat{a}_y \cos\alpha - \hat{a}_z \sin\alpha) \dfrac{E_{io}}{\eta} e^{-j\beta(z\cos\alpha + y\sin\alpha)}$ A/m.

$H_r = (\hat{a}_y \cos\alpha + \hat{a}_z \sin\alpha) \dfrac{E_{ro}}{\eta} e^{-j\beta(-z\cos\alpha + y\sin\alpha)}$ A/m

at $z = 0$

$H_{t1} = \hat{a}_y \cos\alpha \dfrac{E_{io}}{\eta} e^{-j\beta y\sin\alpha} + \cancel{E_{ro}}\, \hat{a}_y \cos\alpha . \dfrac{E_{ro}}{\eta} e^{-j\beta y\sin\alpha}$ — (i)

at $z = 0$

$E_t = \hat{a}_x E_{io} e^{-j\beta y\sin\alpha} - \hat{a}_x E_{ro} e^{-j\beta y\sin\alpha} = 0$.

$\Rightarrow \quad E_{io} = E_{ro}$ — (ii)

(ii) on (i)

$H_{t1} = \hat{a}_y \cos\alpha \dfrac{E_{io}}{\eta} \left(e^{-j\beta y\sin\alpha} + e^{-j\beta y\sin\alpha}\right)$

$= 2\hat{a}_y \cos\alpha . \dfrac{E_{io}}{\eta} e^{-j\beta y\sin\alpha}$ A/m

$\bar{J}_s = \cancel{a_x}\ 2\cos\alpha . \dfrac{E_{io}}{\eta} e^{-j\beta y\sin\alpha}\ \hat{a}_y \times \hat{a}_z$

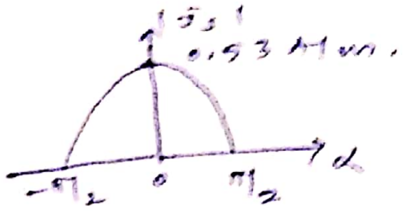
$= \hat{a}_x\, 2\cos\alpha . \dfrac{E_{io}}{\eta} e^{-j\beta y\sin\alpha}$ A/m.

$$\beta = \omega\sqrt{\mu_0\epsilon_0} = \frac{2\pi\times 100\times 10^9}{3\times 10^8} = 2094.4\ \frac{rad}{m}.$$

$$\bar{J}_s = \hat{a}_u\ 0.265\cos\alpha\ e^{-j2094.4\, y\sin\alpha} \times 2$$

$$|\bar{J}_s| = \cancel{0.265}\ 0.53\cos\alpha\ .\ A/m.$$

$$|\bar{J}_s|_{min}\Big|_{\alpha = \pm\pi/2}.$$



6. Incident wave is circularly polarized, the parallel and perpendicular components of the electric field are equal in magnitude but 90° out of phase (in time).

$$E_i = E_{i\parallel} + E_{i\perp} e^{j\phi}$$

$$E_{i\perp} = E_{i\parallel} e^{j\pi/2} = jE_{i\parallel}$$

$$|E_{i\perp}| = |E_{i\parallel}| = A$$

Snell's law of refraction

$$n_1\sin\theta_1 = n_2\sin\theta_2 \Rightarrow \sin 45^\circ = 1.5\sin\theta_2$$

$$\Rightarrow \theta_t = \theta_2 = \sin^{-1}\frac{1}{1.5\sqrt{2}}$$

$$= 28.125$$

$$\cos\theta_2 = 0.88$$

$$\eta_1 = 120\pi\ ,\ \eta_2 = \frac{120\pi}{1.5} = 80\pi\ \left[\because \frac{\eta_2}{\eta_1} = \frac{n_1}{n_2}\right]$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = -0.303\ (\text{real})$$

$$T_\perp = 1 + \Gamma_\perp = 0.6966$$

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = -0.098\ (\text{real})$$

$$T_\parallel = (1 + \Gamma_\parallel)\frac{\cos\theta_t}{\cos\theta_i} = 1 - 0.114 = 0.884$$

$$E_{r\parallel} = A\Gamma_\parallel \ \&\ E_{r\perp} = jA\Gamma_\perp \Rightarrow \frac{E_{r\perp}}{E_{r\parallel}} = \frac{j\Gamma_\perp}{\Gamma_\parallel} = -3.3\angle\pi/2 \neq 1$$

$$E_{t\parallel} = AT_\parallel \ \&\ E_{t\perp} = jAT_\perp \Rightarrow \frac{E_{t\perp}}{E_{t\parallel}} = \frac{jT_\perp}{T_\parallel} = 0.788\angle\pi/2 \neq 1$$

Elliptical polarization