Homework 2 Solution Outlines

Problem 2

$$R_{12}(\tau) = \int_{-\infty}^{\infty} g_1(t)g_2(t-\tau)dt \quad (1)$$

1. Let $g_3(t) = g_2(-t)$. Then, we get

$$R_{12}(\tau) = \int_{-\infty}^{\infty} g_1(t)g_3(\tau - t)dt \quad (2)$$

   which is convolution of $g_1(t)$ and $g_3(t)$. Using $G_3(f) = G_2^*(f)$, we get the answer.

2. For two real valued signals $g_1(t)$ and $g_2(t)$ to be orthogonal, $R_{12}(0) = 0$. Take $g_1(t) = g(t)$ and $g_2(t) = g_h(t)$ and using the fact that $|G(f)| = |G(-f)|$ for real valued $g(t)$

$$R_{12}(0) = \int_{-\infty}^{\infty} G(f)G_h^*(f)df = \int_{-\infty}^{\infty} jsgn(f)G(f)G^*(f)df = 0 \quad (3)$$

Problem 3

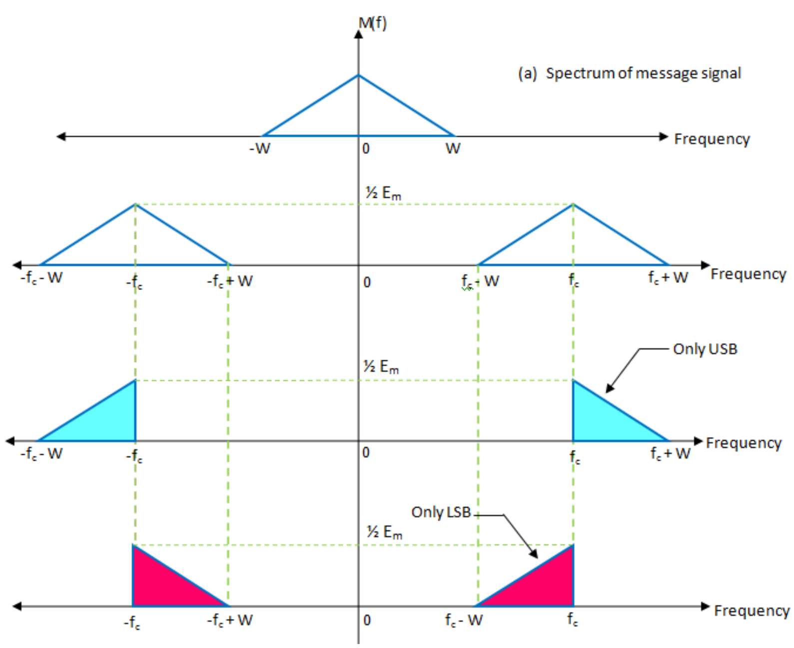

Figure 1: LSB and USB of modulated signal

$$M(f) = M^+(f) + M^-(f) \quad (4)$$

where, M(f) is the Fourier transform of the modulation signal

$$M^{+}(f) = M(f)U(f) \tag{5}$$

where, U(f) is the unit step function

$$\implies M^{+}(f) = M(f)(\frac{1 + sgn(f)}{2}) \tag{6}$$

$$\implies M^{+}(f) = \frac{1}{2}(M(f) + jM_h(f)) \tag{7}$$

where M_h is the Hilbert transform of M(f)
similarly,

$$M^{-}(f) = M(f)U(-f) \tag{8}$$

$$\implies M^{-}(f) = M(f)(\frac{1 - sgn(f)}{2}) \tag{9}$$

$$\implies M^{-}(f) = \frac{1}{2}(M(f) - jM_h(f)) \tag{10}$$

Given in the figure it is evident that:

$$S_{USB}(f) = \frac{1}{2}(M^{+}(f - f_c) + M^{-}(f + f_c)) \tag{11}$$

where, f_c is the carrier frequency

$$\implies S_{USB}(f) = \frac{1}{4}(M(f - f_c) + jM_h(f - f_c) + M(f + f_c) - jM_h(f + f_c)) \tag{12}$$

Upon taking Fourier inverse, we obtain:

$$s_{USB}(t) = \frac{1}{2}m(t)cos(2\pi f_c t) - \frac{1}{2}m_h(t)sin(2\pi f_c t) \tag{13}$$

Similarly working on the for the lower side band, we obtain:

$$S_{LSB}(f) = \frac{1}{2}(M^{-}(f - f_c) + M^{+}(f + f_c)) \tag{14}$$

where, f_c is the carrier frequency

$$\implies S_{LSB}(f) = \frac{1}{4}(M(f - f_c) - jM_h(f - f_c) + M(f + f_c) + jM_h(f + f_c)) \tag{15}$$

Upon taking Fourier inverse, we obtain:

$$s_{LSB}(t) = \frac{1}{2}m(t)cos(2\pi f_c t) + \frac{1}{2}m_h(t)sin(2\pi f_c t) \tag{16}$$

Given

$$m(t) = cos(2\pi f_m t) \tag{17}$$

It's Hilbert transform

$$m_h(t) = sin(2\pi f_m t) \tag{18}$$

Putting the values of $m(t)$ and $m_h(t)$ in 13 and 16, we obtain $s_{USB}(t)$ and $s_{LSB}(t)$