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EE 301(B) Tutorial 1

1. Current density is given in cylindrical coordinates as $\mathbf{J} = -106z^{1.5}\ \mathbf{a}_z$A/m2 in the region $0 \le \rho \le 20\mu$m; for $\rho \ge 20\mu$m, $\mathbf{J} = 0$. (*a*) Find the total current crossing the surface $z = 0.1$ m in the $\mathbf{a}_z$ direction. (*b*) If the charge velocity is 2×10^6 m/s at $z = 0.1$ m, find ρ_v there.

2. A chunk of silicon made in the form of a sphere of radius 100mm is given. The conductivity of silicon is 4 x 10^{-4} S/m, *its relative permittivity is 12 and both are* constant. Suppose that by some means, a uniform volume charge density $\rho_0 = 10^{-6}$ C/m3 is placed in the interior of the sphere at $t = 0$. *Calculate:*
(a) The current produced by the charges as they move to the surface.
(b) The time constant of the charge decay in the silicon.
(c) The divergence of the current density during the transient.

3. A slab of perfect dielectric material ($\varepsilon_r = 2$) is placed in a microwave oven. The oven produces an electric field (as well as a magnetic field). Assume that the electric field intensity is uniform in the slab and sinusoidal in form and that it is perpendicular to the surface of the slab. The microwave oven operates at a frequency of 2.45 GHz and produces an electric field intensity with amplitude 500 V/m inside the dielectric: Calculate the current density in the dielectric.

4. Two charges Qa and Qb are separated by distance of 10m. Qa= 10coswt and Qb= -10coswt where w=10^3 rad/s. Find the magnetic flux density at a point which is at a distance of 10m from both charges.

5. In the medium having dielectric constant 5 and conductivity 10^4 mho/m, the displacement current density is 20 cos(10^7t) A/m2: Find the conduction current density.

6. $\varepsilon = 10^{-11}\ F/m, \mu = 10^{-5}\ H/m, B = 2 \times 10^{-4} cos10^5t.sin10^{-3}yt$
(a) Find E.
(b) Find the total magnetic flux passing through the surface x=0, 0<y<40m, 0<z<2m at t=1 μs.
(c) Find the value of the closed integral of E around the perimeter of the given surface.

7. The circuit given in fig. 1 is situated in a magnetic field

$$B = a_z 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right)\ \mu T$$

Assume R= 15 ohm. Find i.

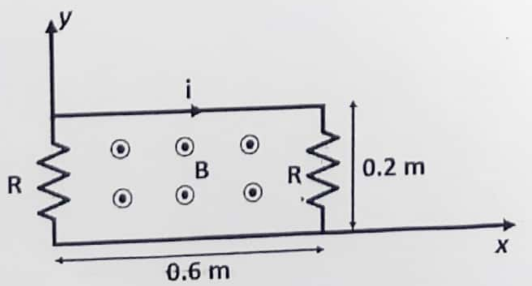

Fig. 1

8. A conducting slide bar oscillates over two parallel conducting rails in a sinusoidally varying magnetic field

$$B = a_z \, 5 \cos \omega t \; mT$$

as shown in Fig. 2. The position of the slider bar is given by x = 0.35 (1- cos wt) m. and the rail terminates in a resistance R = 0.2 ohm. Find i.

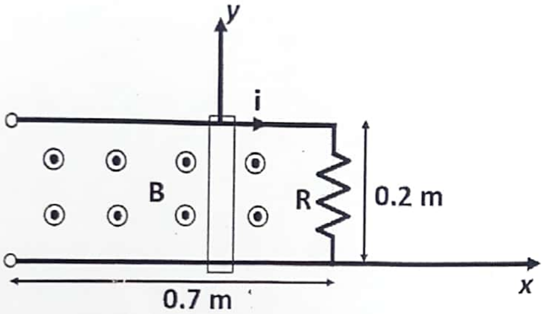

Fig. 2

9. A circular loop of N turns of conducting wire lies in the xy plane with its center at origin of a magnetic field specified by $B = a_z \, B_0 \cos\left(\frac{\pi r}{2b}\right) \sin \omega t$, where b is the radius of the loop and w is the angular frequency. Find the emf induced in the loop.

10. Find the value of K, so that the fields $E = (Kx - 100t)a_y, H = (x + 20t)a_z$ satisfy the Faraday's law, if μ=0.25 H/m and ε= 0.01 F/m.

Review:

1. **Does the displacement current exist in air? Justify.**
2. **Discuss the physical significance of four Maxwell's equatons. Do you think any of the Maxwell's equation(s) represents radiation? Justify your answer.**
3. **"What happens if a charge is suddenly created at a point-what electromagnetic effects are produced?"- Comment on the validity of the hypothesis written (" "), with equation(s).**

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EE301

Tutorial 1

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1. $\vec{J} = \begin{cases} -10^6 z^{1.5}\ \hat{a}_z\ A\,m^{-2}, & 0 \le \rho \le 20\mu m \\ 0, & \rho \ge 20\mu m \end{cases}$

(a) $\int \vec{J}\cdot\vec{dS} = \int_0^{20\mu m} (-10^6 z^{1.5})(2\pi\rho\,d\rho)(\hat{a}_z\cdot\hat{a}_z)$

$= -(2\pi\times 10^6)(0.1^{1.5})\left[\rho^2/2\right]_0^{20\mu m}$

$= -(2\pi \times 10^6 \times 0.0316 \times 0.5 \times 400 \times 10^{-12})$

$= -4.212\ nA$

(b) $I = \rho A v \Rightarrow \rho_v = \frac{I}{Av}$

$v = 2\times 10^6\ m s^{-1}$ at $z = 0.1 m$

$A = \pi (20\mu)^2\ m^2$

$\rho_v = \frac{-4.212 \times 10^{-9}}{\pi \times 400 \times 10^{-12} \times 2\times 10^6} = 1.676\ \mu C\ m^{-3}$

2. $\nabla\cdot J = -(\partial\rho_v/\partial t) \Rightarrow \sigma(\nabla\cdot E) = -\partial\rho_v/\partial t$

$\Rightarrow \sigma\left(\frac{\rho_v}{\varepsilon}\right) = -\frac{\partial\rho_v}{\partial t}$

$\frac{d\rho_v}{\rho_v} = -\left(\frac{\sigma}{\varepsilon}\right)dt$

$\int_{\rho_0}^{\rho} \frac{d\rho_v}{\rho_v} = -\left(\frac{\sigma}{\varepsilon}\right)\int_0^t dt$

$\rho(t) = \rho_0 e^{-\left(\frac{\sigma t}{\varepsilon}\right)}$

$$\int \nabla\cdot\vec{J}\,dV = -\int\left(\frac{\partial \rho_v}{\partial t}\right)dV$$

$$\oint \vec{J}\cdot\vec{dA} = \int_0^R \left(\frac{\rho_0\sigma}{\varepsilon}\right) e^{-\left(\frac{\sigma t}{\varepsilon}\right)}\left(4\pi r^2 dr\right)$$

$$i = \left(\frac{\rho_0\sigma}{\varepsilon}\right)\left(\frac{4\pi R^3}{3}\right) e^{-\left(\frac{\sigma t}{\varepsilon}\right)}$$

$$= \frac{10^{-6}\times 4\times 10^{-4}}{12\times 8.85\times 10^{-12}} \times \frac{4\pi}{3}(0.1)^3\, e^{-\left(\frac{4\times 10^{-4}\, t}{12\times 8.85\times 10^{-12}}\right)}$$

$$= \underline{\underline{0.01578\,\mathbf{exp}(3.766\times 10^6 t)\ \ A}}$$

(b) $t_{chg.\,decay} = \frac{\varepsilon}{\sigma} = \underline{\underline{2.655\times 10^{-7}\ s}}$

(c) $\nabla\cdot J = -\frac{\partial\rho_v}{\partial t} = \left(\frac{\rho_0\sigma}{\varepsilon}\right)\exp\left(-\frac{\sigma t}{\varepsilon}\right)$

$$= \underline{\underline{3.766\ \mathbf{exp}(3.766\times 10^6 t)\ \ Am^{-3}}}$$

3. $\nabla\times\vec{H} = \underbrace{\vec{J}}_{J_c} + \underbrace{\left(\frac{\partial\vec{D}}{\partial t}\right)}_{J_d}$

J_d = (current density in dielectric)

$$\vec{J}_d = \frac{\partial\vec{D}}{\partial t} = \varepsilon\frac{\partial E}{\partial t} \qquad \left(E = E_0\sin(2\pi f_0 t)\right)$$

$$= (\varepsilon E_0)(2\pi f_0)\cos(2\pi f_0 t)$$

Assuming $\vec{J}_c = \vec{0}$,

$$\vec{J} = \vec{J}_d = 2\pi\varepsilon_r\varepsilon_0 f_0 E_0\cdot\cos(2\pi f_0 t)$$

$$= (2\pi)(2)(8.85\times 10^{-12})(2.45\times 10^9)(500)\cos(2\pi f_0 t)$$

$$= \underline{\underline{136.235\ \cos(1.539\times 10^{10} t)}}$$

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5. $$J_d = \frac{\partial D}{\partial t} = 20\cos(10^7 t)\ \text{Am}^{-2}$$

$$\Rightarrow \int_{D_0}^{D} dD = \int_{t_0}^{t} 20\cos(10^7 t)\, dt = \frac{20}{10^7}\left(\sin(10^7 t) - \sin(10^7 t_0)\right)$$

$$\Rightarrow D = D_0 + 2\times 10^{-6}\left(\sin(10^7 t) - \sin(10^7 t_0)\right)$$

Assume $D_0 = t_0 = 0$

$$\Rightarrow D = 2\times 10^{-6}\sin(10^7 t)$$

Now $J_c = \sigma E = \frac{\sigma}{\varepsilon} D$

$$= \frac{10^4 \times 2\times 10^{-6}}{5\times 8.85\times 10^{-12}}\sin(10^7 t)$$

$$= \underline{\underline{4.52\times 10^8 \sin(10^7 t)\ \text{Am}^{-2}}}$$

6. $\varepsilon = 10^{-11}\ \text{Fm}^{-1}$, $\mu = 10^{-5}\ \text{Hm}^{-1}$

(a) $$B = 2\times 10^{-4}.\cos(10^5 t)\sin(10^{-3} yt)\,\hat{a}_x$$

$$\nabla\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}\ ; \qquad \nabla\times\vec{B} = \vec{J} + \frac{\partial\vec{D}}{\partial t} = \vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}$$

Assume $\vec{J}_c = 0$

$$\Rightarrow \nabla\times\vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2\times10^{-4}\cos(10^5 t)\sin(10^{-3} yt) & 0 & 0 \end{vmatrix}$$

$$= (\hat{a}_z)(2\times 10^{-4}\cos(10^5 t)\cos(10^{-3} yt)(10^{-3} t)$$

i.e. $$E = \left(\frac{1}{10^{-11}}\right)\left(2\times 10^{-4}\times 10^{-3}\right)\int \cos(10^5 t).\cos(10^{-3} yt)\, t\, dt$$

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(6) $\phi = \int \vec{B} \cdot \vec{dS}$

$$= (2\times10^{-4})\cos(10^5 t)\int_0^2\int_0^{40} \sin(10^{-3}yt)\,dy\,dz$$

$$= -\frac{(2\times2\times10^{-4})}{(10^{-3}t)}\cos(10^5 t)\left[\cos(10^{-3}yt)\right]_0^{40}$$

$$= -0.4\left(\frac{\cos(10^5 t)}{t}\right)\left(\cos(0.04t) - 1\right)$$

7. $B = 3\cos\left(2\pi\left(\frac{5}{2}\times10^7\right)t - 2\pi\left(\frac{1}{3}\right)x\right)\ \mu T\ \hat{a}_z$

$R = 15\,\Omega$

$$\phi = \int_0^{0.6} 3\cos\left(2\pi\left(\tfrac{5}{2}\times10^7\right)t - 2\pi\left(\tfrac{1}{3}\right)x\right)\times 0.2\,dx$$

$$i = \frac{-1}{R}\frac{d\phi}{dt} = \frac{3\times0.2\times5\pi\times10^7}{30}\int_0^{0.6}\sin\left(5\pi\times10^7 t - \frac{2\pi x}{3}\right)dx$$

$$= \frac{2\pi}{2}\times10^6\left(\frac{3}{2\pi}\right)\left[\cos\left(5\pi\times10^7 t - \frac{2\pi}{3}x\right)\right]_0^{0.6}$$

$$= 1.5\times10^6\left(\cos(5\pi\times10^7 t - 0.4\pi) - \cos(5\pi\times10^7 t)\right)\ A$$

8. $B = 5\cos(\omega t)\ mT\ \hat{a}_z$

$$\phi = \int \vec{B}\cdot\vec{dS}$$

$$\vec{dS} = (0.2)dx = 0.2\times0.35\omega\sin(\omega t)\,dt = 0.07\omega\sin(\omega t)dt$$

$$\rightarrow \phi = 5\times0.035\omega\int\sin(2\omega t)\,dt = \frac{0.175\omega}{2\omega}\left[-\cos(2\omega t)\right]$$

$$\phi = -0.0875\cos(2\omega t)$$

$$i = -\frac{1}{R}\frac{d\phi}{dt} = -\frac{1}{0.2}(-0.0875)(-2\omega\sin(2\omega t))$$

$$= -0.875\,\omega\sin(2\omega t)\ A$$

9. $\phi = N\int \vec{B}\cdot\vec{dS}$

$$= NB_0 \sin(\omega t)\int_0^b \cos\left(\frac{\pi r}{2b}\right)\cdot 2\pi r\,dr$$

$$= \frac{8b^2}{\pi}\cdot NB_0 \sin(\omega t)\int_0^{\pi/2} \cos(r)\cdot r\,dr$$

$$= \frac{8b^2 NB_0}{\pi}\sin(\omega t)\Big[r\sin(r) + \cos(r)\Big]_0^{\pi/2}$$

$$= \frac{8b^2 NB_0}{\pi}\sin(\omega t)\left(\frac{\pi}{2} - 1\right)$$

$$\text{EMF} = -\frac{\partial\phi}{\partial t} = -\frac{8b^2 NB_0}{\pi}\left(\frac{\pi}{2}-1\right)\omega\cos(\omega t)$$

10. $E = (Kx - 100t)\,\hat{a}_y$, $H = (x + 20t)\,\hat{a}_z$, $\mu = 0.25\ Hm^{-1}$
$\varepsilon = 0.01\ Fm^{-1}$

$\nabla\times\vec{E} = -\partial B/\partial t = -\mu\,\partial H/\partial t$ ($\because \mu$ is constant)

$$\nabla\times\left((Kx-100t)\hat{a}_y\right) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & (Kx-100t) & 0 \end{vmatrix} = (\hat{a}_z)\,K$$

$$-\mu\,\partial H/\partial t = -0.25\times 20\,\hat{a}_z = -5(\hat{a}_z)$$

$$K\hat{a}_z = -5\hat{a}_z \Rightarrow K = -5$$

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Review

1. Yes, displacement current can exist in air. In such a scenario, air acts as the dielectric between the two plates of an air-core capacitor.

2. Maxwell's equations represent the relationship between the electric domain and the magnetic domain.

$$\nabla \times E = -\partial B/\partial t \quad \text{and} \quad \nabla \times H = J + \frac{\partial D}{\partial t}$$

together represent the effect of electromagnetic radiation.