Quiz 2

Communication Systems (EE 308), Autumn'19

Oct. 24, 2019; Total: 10 marks; Time: 55 minutes

Note:

- *You are allowed to use ONE A4 sheet with handwritten notes on ONE side.*
- *You are allowed to use any result discussed in class without proof. For all other results, a proof needs to be provided.*

QUESTION 1 (2.5 + 0.5 = 3 MARKS)

Let $X(t)$ be a periodic square wave with period T as shown in Fig. 1. The width and height of each pulse are $T/2$ and A respectively. The starting time of the first complete pulse to the right of $t = 0$ is a random variable Θ that is uniform in $[0, T]$.

(a) Find the mean function $\eta_X(t)$ and autocorrelation function $R_X(\tau)$ of the process $X(t)$. (You can assume that $X(t)$ is a WSS process.)

(b) Suppose $X(t)$ is input to a cascade (series) connection of two linear time-invariant systems with impulse responses $h_1(t)$ and $h_2(t)$ respectively. Find the mean function $\eta_Y(t)$ and autocorrelation function $R_Y(\tau)$ of the final output, $Y(t)$, of the cascade system.

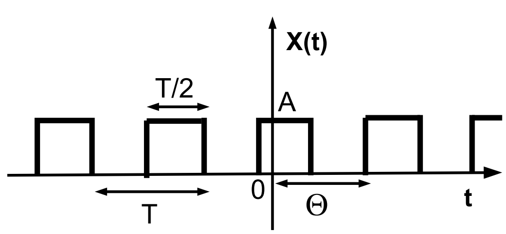

Fig. 1. The process $X(t)$ in Question 1.

QUESTION 2 (0.5 + 1 + 1 + 0.5 = 3 MARKS)

Consider a pulse $s(t) = \text{sinc}(at)\text{sinc}(bt)$, where $a \geq b$.

(a) Sketch the frequency-domain response $S(f)$ of the pulse.

(b) Suppose that the pulse is to be used over an ideal real-baseband channel with one-sided bandwidth 400 Hz. Choose a and b so that the pulse is Nyquist for 4PAM signaling at 1200 bits/ s and exactly fills the channel bandwidth.

(c) Now, suppose that the pulse is to be used over a passband channel spanning the frequency range $2.4 - 2.42$ GHz. Assuming that we use 64QAM signaling at 60 Mbits/ s, choose a and b so that the pulse is Nyquist and exactly fills the channel bandwidth.

(d) Sketch an argument showing that the magnitude of the transmitted waveform in the preceding settings is always finite.

QUESTION 3 (1 + 1 = 2 MARKS)

A zero-mean white Gaussian noise process, $n_w(t)$, with power-spectral density $\frac{N_0}{2}$, is passed through an ideal filter whose passband is from $3 - 11$ kHz. The output process is denoted by $n(t)$.

(a) If $f_c = 7$ kHz, find $S_{n_I}(f)$, $S_{n_Q}(f)$ and $S_{n_I,n_Q}(f)$, where $n_I(t)$ and $n_Q(t)$ are the in-phase and quadrature components of $n(t)$.

(b) Do part (a) with the change that $f_c = 6$ kHz.

QUESTION 4 (1 + 1 = 2 MARKS)

(a) Suppose $X(t)$ is a SSS process and Z is a random variable independent of $X(t)$. Let $Y(t) = X(t-Z)$. Consider the following statement: "The process $Y(t)$ is SSS". State whether this statement is true or false and provide a proof (if true) or counterexample (if false).

(b) Do part (a) with the change that Z is not independent of $X(t)$.