Practice problems

8 It is desired to find an algorithm for a computer (or microprocessor) so that the digital system approximates the transfer function C(s) of controller (control algorithm like PD, PID) or H(s) of digital filter. The approximation can be done in various different ways: forward difference, backward difference (see problem 2.a), bilinear transformation. Digital implementation includes a data reconstruction which can also be done in different ways: zero or first-order hold. For implementation of controller C(s) and digital filter H(s) which approximations you would use and why?

8 You have learnt about the bilinear transformation for going from analog domain to digital domain. However one can think of another easier way to implement: what is called at backward difference approximation. In this approximation we assume that the

$$\frac{dy}{dt} = \frac{Y[n] - Y[n-1]}{T}$$

Using the first order differential equation

$$\frac{dy}{dt} + ay = x$$

a) Show that the transformation between s and Z plane with backward difference approximation is given by s $s = \frac{1 - z^{-1}}{T}$

b) Now find and Plot the Locus of the negative s-plane in z domain (stable system mapping.

c) From the plot argue whether a system in s-domain remains stable when transformed to z-domain using this backward difference approximation. What transformation would you prefer: bilinear, backward difference, or based on shift operator?? Why???

8 A pure derivative can but should not be implemented because it will give very large amplification of measurement noise. The gain of derivative must thus be limited. This can be done by approximating transfer function $K_d s$ as $K_d s \approx \frac{K_d s}{1 + K_d s / N}$ where N is typically 3-20. Determine frequency response of this approximation and hence significance of N. Determine corresponding digital domain/microprocessor code for implementation of this smart derivative action.

8 Considering a 'simple pendulum with friction' system input as torque, and output as displacement θ, develop computed torque control strategy to track a trajectory given by $\theta_d(t)$.

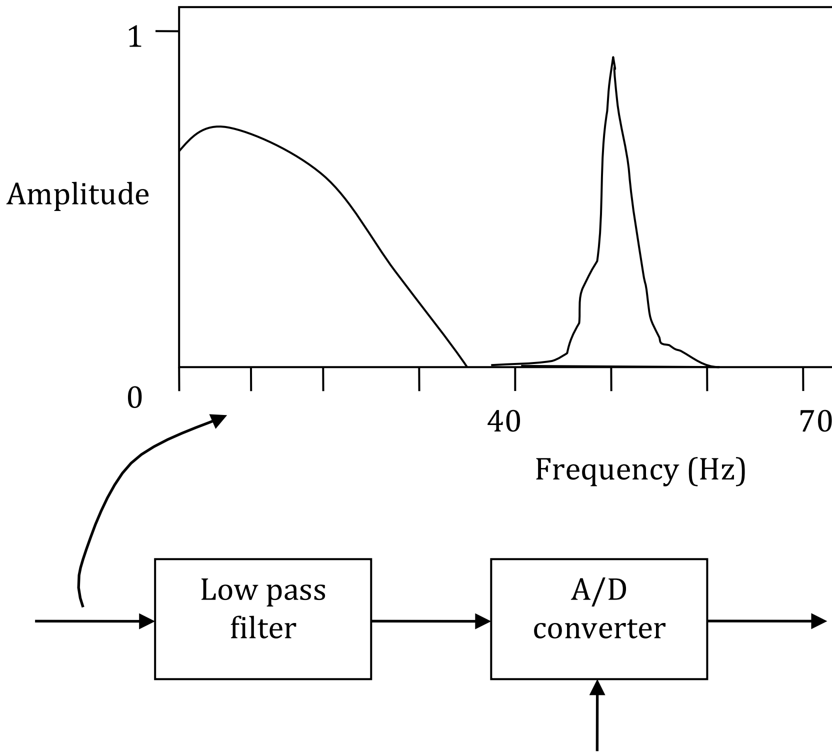
8 Derive discrete-time system corresponding to the following continuous time system representing linearized model of overhead crane with acceleration of the cart as input

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

6. A low pass RC filter has been designed with R=100 KΩ and C= 4μF .What is the transfer function and corner frequency of this filter. Sketch the bode plot of the filter and suggest a situation where you would use this filter.

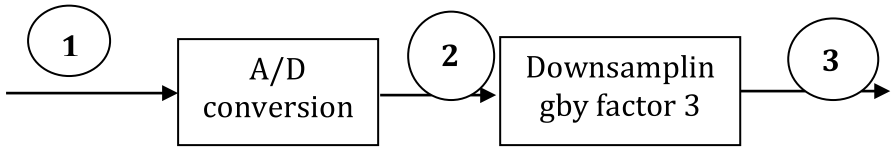
7. A signal coming from a sensor as an input to A/D converter has the following frequency characteristics.



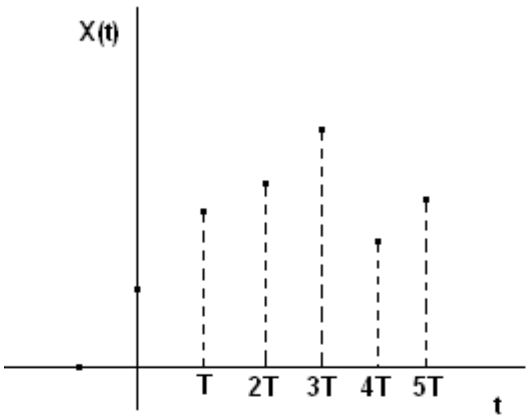
Supposing a low pass filter is used before A/D converter as shown in the figure such that the 50 Hz noise component is reduced to 10% of the original. The frequencies below 40 Hz can be assumed to be unchanged. If a sampling frequency of 80 Hz is used, sketch the frequency spectra of sampled signal between 0 and 40 Hz.

Would you recommend an anti aliasing filter to be used?

8. (4marks) Shown below is a block diagram for signal processing. If the signal of interest is band limited to ($-\omega_0$, ω_0), draw signal as seen in FREQUENCY domain at point 1,2, and 3. Show the quantitative values at important points in your sketch. Downsampling by factor 2 means you take every alternate (2nd) sample from SAMPLED signal and skip others to represent the signal with reduced memory requirement.



9. (6marks) Samples of signal x(t) are shown in the following figure.



Draw reconstruction of this signal using

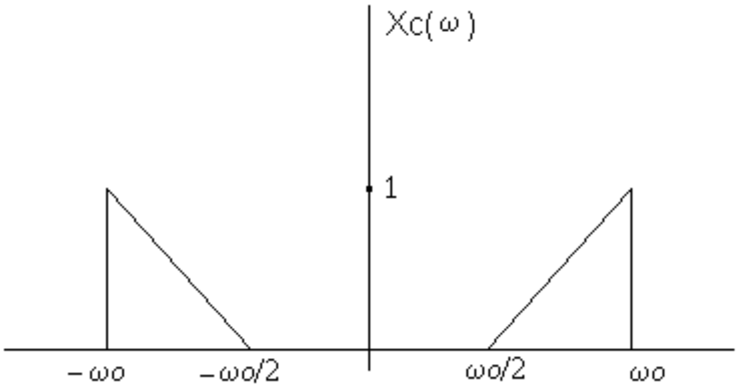
a. Shannon's reconstruction theorem

b. Zero Order Hold

c. causal and non-causal First Order Hold

How will you mathematically represent the signal reconstructed using causal First Order Hold?

10. (4 marks) An analog signal xc(t), with Fourier transform Xc(w) shown in the figure is sampled with sampling period T = 2*pi/wo to form sequence xc(kT).



a. Sketch the Fourier transform of sampled signal

b. In terms of wo, for what range of values of T can xc(t) be recovered?

11. Given a first order filter $G(s) = \dfrac{a}{s+b}$ in analog domain. Convert it in digital domain by using zero order hold (ZOH) with sampling time T, and find the pulse transfer function. Replace q by z to get z domain equivalent (since shift property of q is same as that of z). Explain: how will you implement this z domain filter in the real time microprocessor with all the steps in code mentioned clearly.

12. Derive the expression for pulse response function in discrete domain. (equivalent to impulse response in continuous domain. HINT: convolution involving impulse response function is similar to summation in discrete domain)