Maths -1

Scanned with CamScanner

Sets, Number & Interval

~~Sets Groups of well defined things~~

100 soldiers

Onehand + One leg + One eye = ~~os~~

60 + 50 + 40 = 150

100 ⇒ $(L+E)+(H+E)+(L+H)+(L+H+E) \pm \cancel{L+H+E}$

⇒ ~~L+E+H~~

$\boxed{\text{Sets - Set is a collection of well defined objects}}$

eg. 1. Collection of all natural numbers
2. Collection of all rivers/lakes
3. Collection of all days in a week

Q verify weather the following are sets or not

1. Collection of all boys in a class - ~~yes~~ yes
2. collection of all handsome boys in a class - no
3. collection of most dangerous animals of world no
4. collection of all even integers yes

~~5. A, B, C ... Z~~

~~6. ab,~~

★ A, B, C ... Z ⇒ set denotion

a, b, c ..., 1, 2 ... ⇒ Elements

★ Element a belongs to set A

$a \in A$

belongs to

★ Element a not belongs to A

$a \notin A$

Q Set A is a collection of natural nos less than 7.

① $5 \in A$ ✓

② $8 \in A$ ✗

③ $0 \notin A$ ✓

④ $2 \in A$ ✓

⑤ $10 \notin A$ ✓

Methods to write a set

Roster or Tabular method

1. listing elements seperated by commas (,) and enclosing them in curly brackets.

   eg. $A = \{a, b, c \dots z\}$

2. Order does not matter here

   eg. $A = \{a, b, c\}$ and $A = \{b, a, c\}$ are same

Property or set builder form

1. Write down the property or rule which gives us elements of a set

   eg. $A = \{x : P(x)\}$

   Elements — Such that — Property

$A = \{x :$ All Alphabets in english vocablory $\}$

Roster / Tabular

3. Elements are not generally repeated here.

eg. $A = \{S, C, H, O, O, L\}$ and

$A = \{S, C, H, O, L\}$ are same, duplicates are not counted

Property / builder form

$B = \{x : x$ is prime no. less than $10\}$

$C = \{x : x$ is a odd no. and $1 \leq x \leq 10\}$

Q convert in set builder form

1. $A = \{3, 6, 9, 12\}$

$A = \{x : x$ is ~~divisible~~ multiple of 3 and $3 \geq x \leq 12\}$ or

$A = \{x : x \in 3n$ and $n \in N, 1 \leq n \leq 4\}$

2. $B = \{2, 4, 8, 16, 32\}$

~~$B = \{x : x$ is a natural no. less than $6\}$~~

~~$B = \{2^x : x$ is a natural no. less than $6\}$~~

$B = \{x : x$ is 2^n and n is a natural no. less than $6\}$

3. $C = \{2, 4, 6 \ldots\}$

$C = \{x : x$ is even no. and $x \geq 2\}$

Q convert in roster / tabular form

1. $A = \{x : x$ is an integer and $-\frac{1}{2} < x < \frac{9}{2}\}$

$A = \{0, 1, 2, 3, 4\}$

2. $B = \{x : x$ is a month of a year & not having 31 day$\}$

$B = \{$February, April, June, September, November$\}$

3. $C = \{x : x$ is a consonent in english which proceeds $k\}$

~~$C = \{l, m, n, p, q, r, s, t, v, w, x, y, z\}$~~

$C = \{b, c, d, f, g, h, j\}$

Q let A be the set of Natural nos. and x, y be any two elements of A.

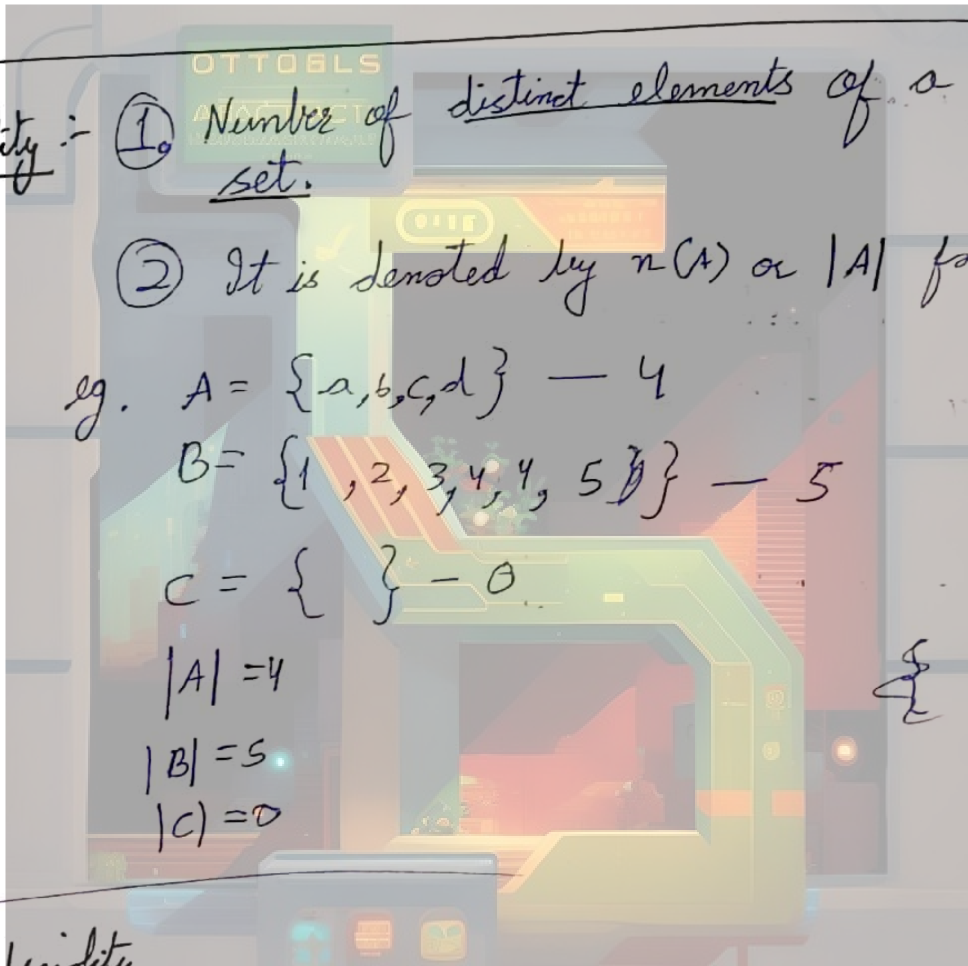
Then:-

a) $x-y \in A$ ✗

b) $x+y \in A$ ✓

c) $xy \in A$ ✓

d) $\frac{x}{y} \in A$ ✗



Cardinality :- ① Number of distinct elements of a ~~set~~ finite set.

② It is denoted by $n(A)$ or $|A|$ for set A

eg. $A = \{a, b, c, d\}$ — 4

$B = \{1, 2, 3, 4, 4, 5\}$ — 5

$C = \{\ \}$ — 0

$|A| = 4$

$|B| = 5$

$|C| = 0$

Q Find Cardinality

1. $P = \{a, d, e, l, k\}$ $|5|$
2. $Q = \{a, a, b, d, d\}$ $|3|$
3. $C = \{a, \{c, d\}, k\}$ $|3|$
4. $D = \{a, \{\ \}, k\}$ $|3|$
5. $E = \{r, \{q, \{r, s\}\}, t\}$ $|3|$ $n(E) = 3$

(4)

Practice Set

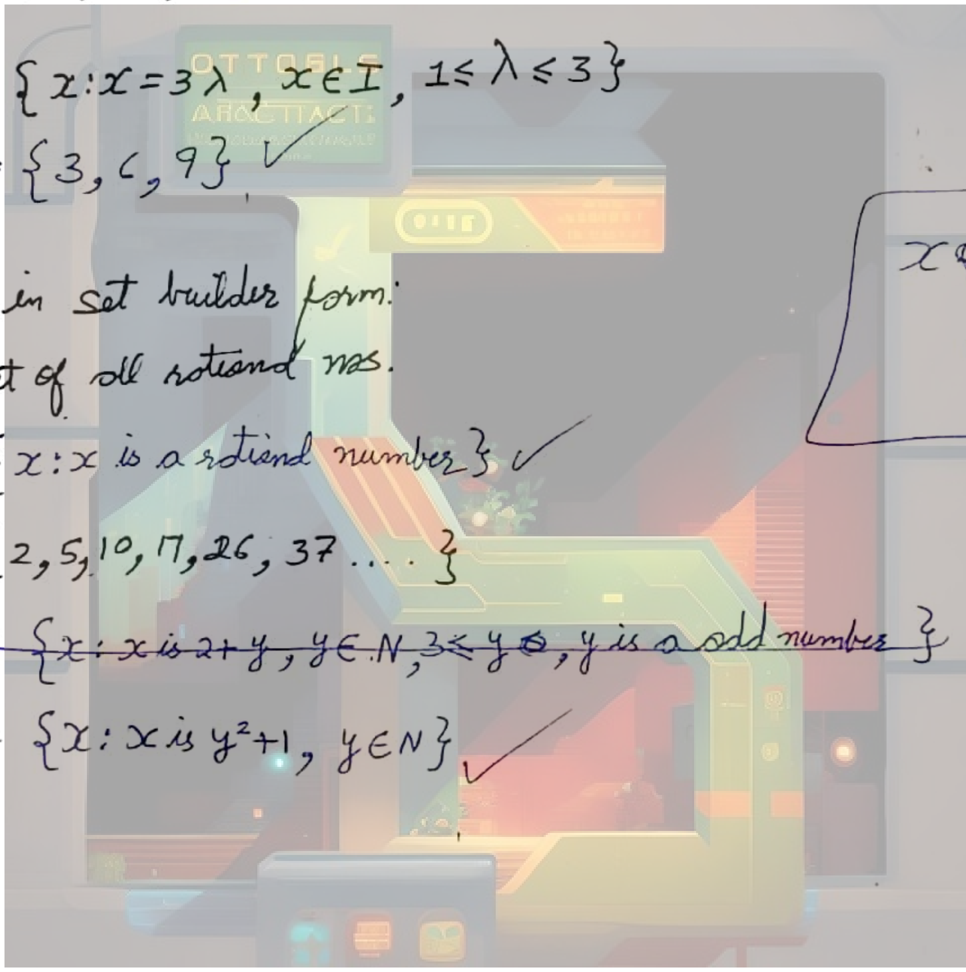
Q1. set or not?

1.	collection of natural nos. between 2 & 20.	Yes ✓
2.	collection of ~~real~~ nos. which satisfy $x^2 - 5x + 6 = 0$	Yes ✓
3.	" prime nos. between 2 & 100	Yes ✓
4.	" all intelligent women in Udaipur.	No ✓

Q2. Write in Tabular form:

1. $A = \{x : x \text{ is a +ve prime} < 10\}$

A $A = \{2, 3, 5, 7\}$ ✓



2. $B = \{x : x = 3\lambda,\ x \in I,\ 1 \le \lambda \le 3\}$

$B = \{3, 6, 9\}$ ✓

Q3. write in set builder form:

A 1. set of all rational nos.

A $A = \{x : x \text{ is a rational number}\}$ ✓

$x \in Q$
Rational

2. $\{2, 5, 10, 17, 26, 37 \ldots\}$

~~$B = \{x : x \text{ is } 2+y,\ y \in N,\ 3 \le y \le \ldots,\ y \text{ is a odd number}\}$~~

$B = \{x : x \text{ is } y^2+1,\ y \in N\}$ ✓

Types of Sets

	Set name	Definition	Example
1	Null/ Void/Empty set	a) set having no elements. ~~b) card~~ b) $A = \{\ \}$ or $A = \phi$ (phi) c) cardinality $= 0$ d) $\phi \neq \{\phi\}$	$A = \{x : x \in N, x < 1\}$ $A = \{x : x \in W, x < 0\}$ $B = \{x : x \neq x\}$ C = set of all months having 32 days.
2	Singleton Set	a) set having only one element. b) cardinality $= 1$	A = set of all months having only 28 days. $B = \{1, 1, 1, 1\}$ $C = \{x : x \in W, x < 1\}$ $D = \{\phi\}$
3.	Finite set	a) set having a ~~fixed~~ finite no. of elements b) All Null & Singleton sets lie in finite set	A = set of having days of a week $B = \{5, 7, 9\}$ C = set of all lakes in udaipur
4.	Infinite Set	a) set having infinite number of elements.	$A = \{1, 2, 3 \ldots\}$ $B = \{x : x \text{ is prime no.}\}$

Q Match the following :-

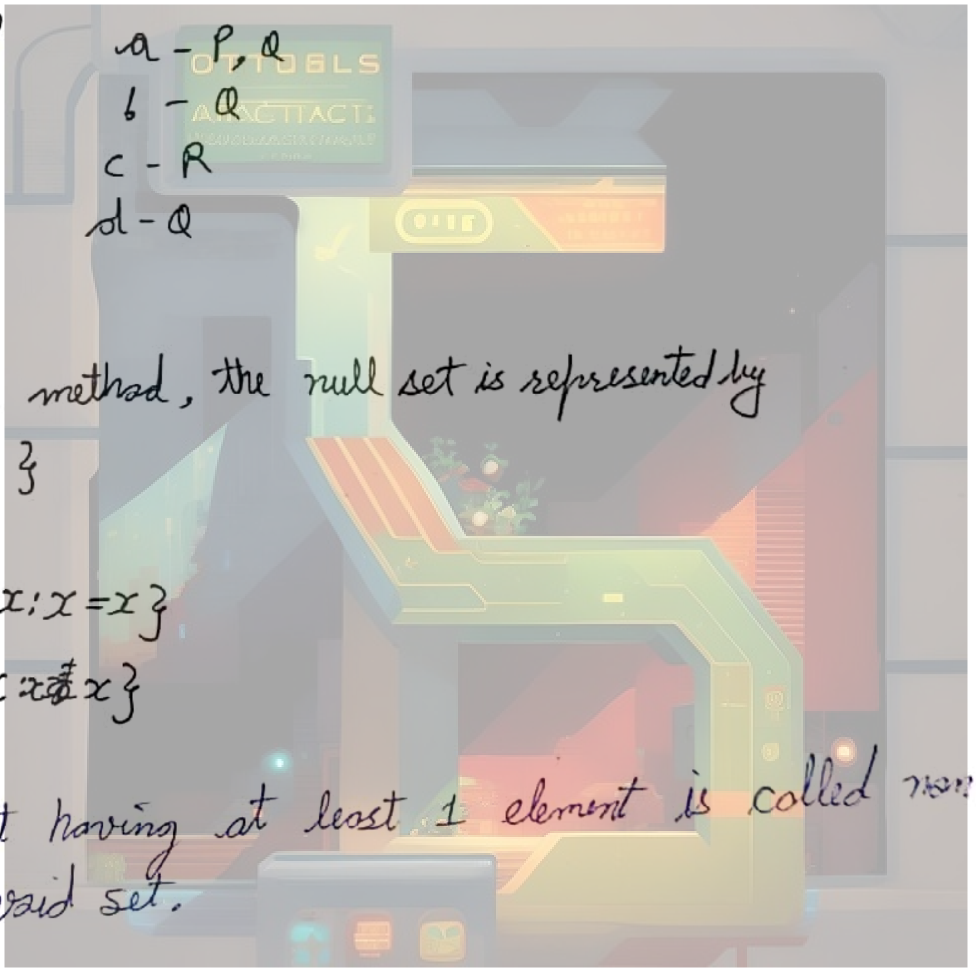
a ~~1~~. $A = \{x : x^2 = 4, x \text{ is odd}\}$ — P) Null set

b) ~~2~~. $B = \{$set of all Men living presently in Udaipur$\}$ — Q) Finite set

c) ~~3~~. $C =$ set of all points on a line — R) Infinite set

d ~~4~~. Set of solutions of $x^2 - 16 = 0$ — S) singleton

~~a - P, Q~~
~~b - Q~~
~~c - R~~
~~d - S~~
~~d - Q~~



a - P, Q
b - Q
c - R
d - Q

Q. In rule method, the null set is represented by

a) $\{ \ \}$

b) ϕ

c) $\{x : x = x\}$

✓ d $\{x : x \neq x\}$

Q A set having at least 1 element is called non-empty or non-void set.

Equivalent Sets

1. Only Cardinal numbers are same.
(Same Cardinality)

2. Elements can be same or different.

3. denoted by '≡' or '~'

4. eg. $A = \{1, 2, 3\}$
$B = \{a, b, c\}$
$C = \{m, n, o, p\}$
$D = \{7, 8, 9\}$
$A \equiv B \equiv D$

Equal Sets/Identical sets

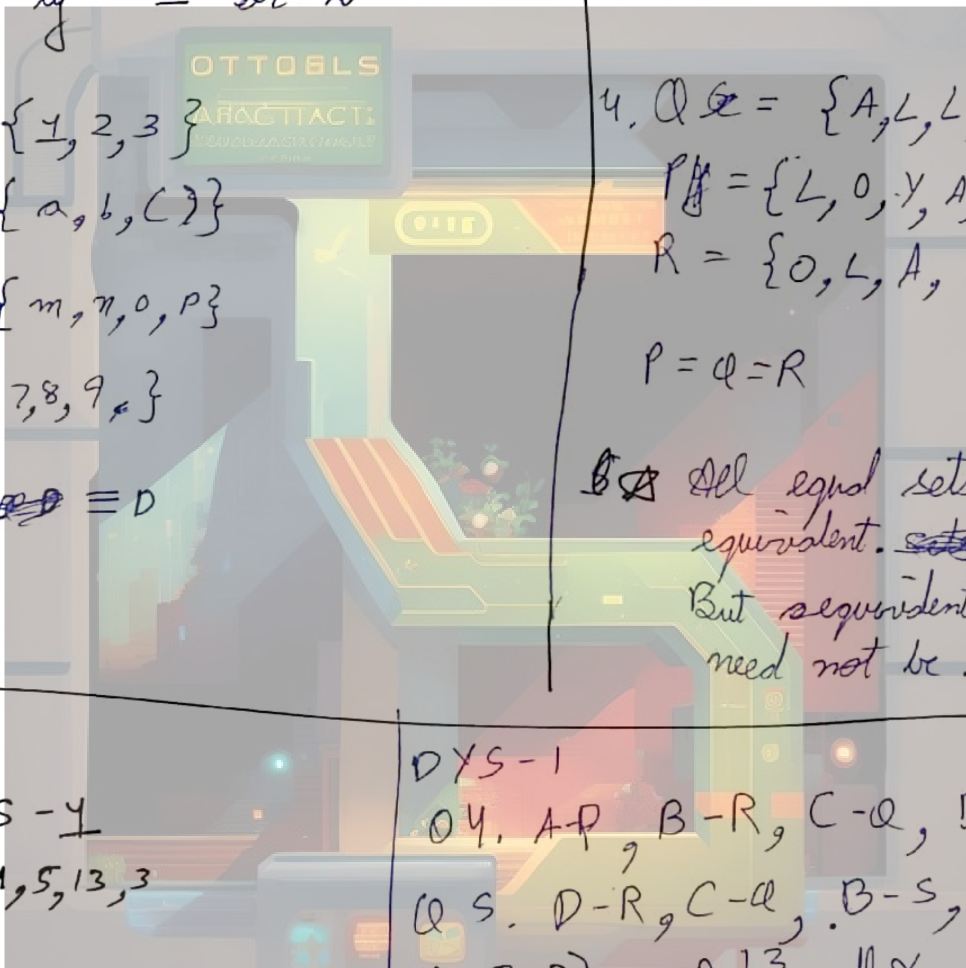
1. Cardinal number and ~~elemen~~ elements in sets are same.

2. Order of elements does not matter

3. denoted by '='

4. $Q = \{A, L, L, O, Y\}$
$P = \{L, O, Y, A, L\}$
$R = \{O, L, A, Y\}$
$P = Q = R$

All equal sets are always equivalent. ~~sets~~
But equivalent sets may need not be equal sets.



H.W.

1. D.Y.S - 1
Q 4, 5, 13, 3

2. O-1
Q 1, 2, 13, 14, 15, 16, 18

3. O-2
Q 10, 11

4. O-3
Q 10

8

DYS - 1

Q 4. A-P, B-R, C-Q, D-S

Q 5. D-R, C-Q, B-S, A-P

Q 3. D)   Q 13. all

O-1

Q 1. B)   Q 2. A)   Q 13. C)

Q 14 C)   Q 15 D)   Q 16. D)

Q 18. A)

O-2

Q 10. B, C, D   Q 11. A, C, D

O-3

Q 10. A-S, B-P, C-Q, D-R

Subset

1. If every element of set-A is an element of set-B then, set-A is called <u>subset</u> of set-B and set-B is called <u>superset</u> of set-A.

eg. J = {R, L, C, GP}

A = {GP}

B = {R, G}

C = {R, L, G}

D = {R, L, G, C}

Sets A, B, C, D are subsets of set J & set J is a superset of sets A, B, C, D.

eg 2. Udaipur is a subset of Rajasthan

→ Subsets are denoted by '$\subseteq$'

eg. $A \subseteq B$
A is subset of B
B is superset of A

eg. A = {a}
subsets = { }, {a}
$= 2^1 = 2$

B = {a, b}
B subsets = { }, {a}, {b}, {a, b}
$= 2^2 = 4$

C = {a, b, c}
C = { }, {a}, {b}, {c}, {a,b}, {a, c}, {b, c}, {a, b, c}
$= 2^3 = 8$

Formula $= 2^{\text{cardinality}}$

* Equal sets are always subsets of each other

$L = \{a, b, b, c, c, c, a\}$

$M = \{a, b, c\}$

$M \subseteq L \qquad L \subseteq M$

Proper Subset

→ If set A is a sub set of set B and $A \neq B$ Then, A is a proper subset of B.

→ It is denoted by '$\subset$'

eg. $B = \{a, b\}$

$$\underbrace{\underbrace{\{\ \}, \{a\}, \{b\}}_{\text{Proper subsets of } B}, \{a, b\}}_{\text{subsets of } B}$$

eg 2. T = {Munni, Chmeli, Sheela}

Proper Subsets = { }, {Munni}, {Chameli}, {Sheela}, {Munni, Chameli}, {Munni, Sheela}, {Chameli, Sheela}

Subsets = P. Subsets and {Munni, Chameli, Sheela}

* Every set is a subset of itself
* Empty set is a subset of every set
* If $\underbrace{n(A)}_{\text{no. of elements in } A} = m$,

then Total no. of subsets of $A = 2^m$

no. of proper subsets of $A = 2^m - 1$

(10)

Q1. Find total no. of subsets & Proper subsets

① $A = \{l, m\}$

$|A| = 2$

no. of subsets $= 2^{|A|}$

$= 2^2$

$\boxed{= 4}$

no. of proper subsets $= 2^{|A|} - 1$

$= 4 - 1$

$\boxed{= 3}$

② $B = \{P, Q, R, S, T\}$

$|B| = 5$

no. of subsets $= 2^{|B|}$

$= 2^5$

$\boxed{= 32}$

no. of Proper subsets $= 32 - 1$

$\boxed{= 31}$

Q2. two finite sets have m & n elements. The total no. of subsets of the first set is 48 more than the total no. of subsets of the second set. The value of m & n are?

a) 7, 6 b) 6, 3 ✓c) 6, 4 d) 7, 4

$2^m + 48 = 2^n$

$\cancel{2^m - 2^n = -48}$

$\cancel{48 = 2^n - 2^m}$

$48 = 2^m - 2^n$

$2^n(2^{m-n} - 1) = 2^4 \times 3$

2^n

$2^n = 2^4$

$n = 4$

$2^{m-n} - 1 = 3$

$2^{m-4} = 4$

$2^{m-4} = 2^2$

$m - 4 = 2$

$\boxed{m = 6}$

$2^n = 3$ $\qquad\qquad$ $2^{m-n} - 1 = 2^4$

$2^{m-n} = 2^4 + 1$

$2^n = 3$ $\qquad\qquad$ $2^{m-n} = 17$

(Reject)

Power Set

→ It is a set containing all the subsets of set A

→ It is denoted by $P(A)$

eg. $A = \{a, b\}$

Subsets $= \{ \{\}, \{a\}, \{b\}, \{a, b\}$

$P(A) = \{\{\}, \{a\}, \{b\}, \{a, b\}\}$

eg2. $X = \{3, 7, 11\}$

$P(X) = \{\{\}, \{3\}, \{7\}, \{11\}, \{3, 7\}, \{3, 11\}, \{7, 11\},$
$\{7, 3, 11\}\}$

① $3 \in X$ ✓

② $\{3\} \in P(X)$ ✓

③ $\{3\} \subset X$ ✓

④ $7 \in X$ ✓

⑤ $\{7\} \in P(X)$ ✓

⑥ $\{7\} \subset X$ ✓

⑦ $\{3, 7\} \subset X$ ✓

⑧ $\{3, 7\} \in P(X)$ ✓

Q $W = \{1, 2\}$

$P(W) = \{\{\ \}, \{1\}, \{2\}, \{1,2\}\}$

(A) $\{1\} \subset W$ ✓

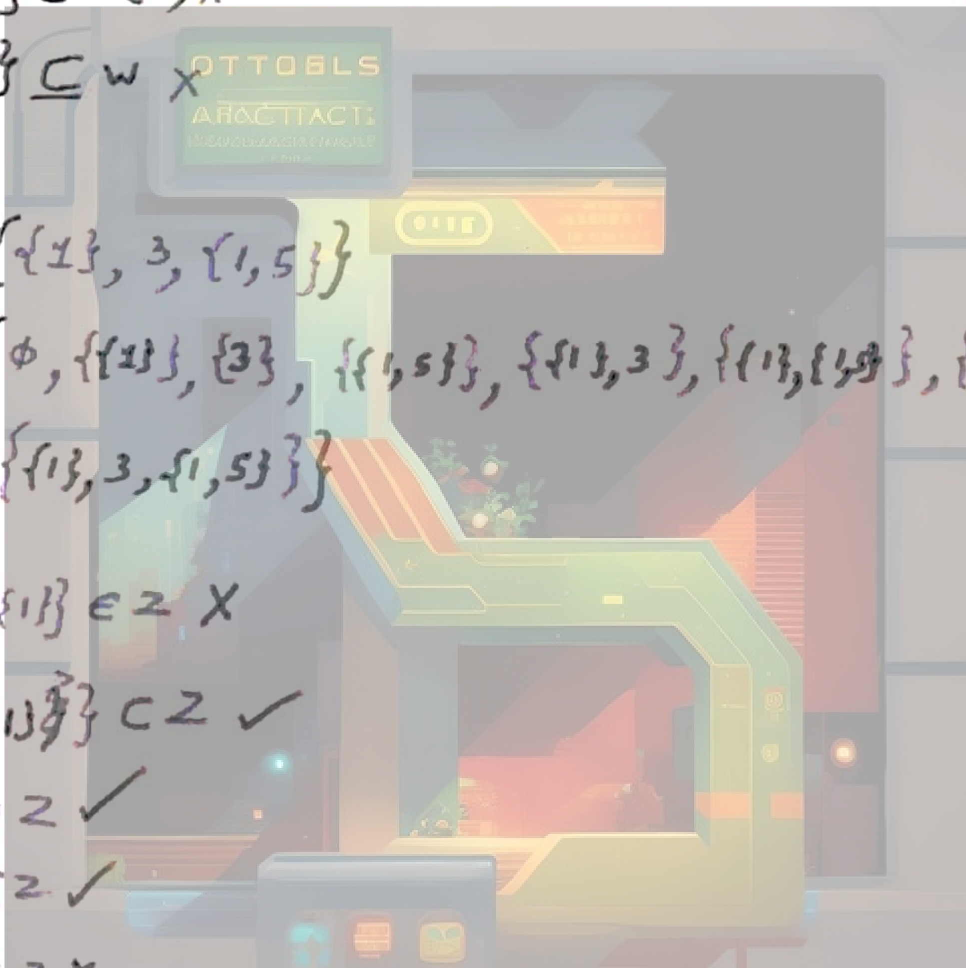
(B) $\{1\} \subset P(W)$ ✗

(C) $\{1\} \in P(W)$ ✓

(D) $\{1,2\} \in P(W)$ ✓

(E) $\{1,2\} \subset P(W)$ ✗

(F) $\{1,2\} \subseteq W$ ✗



Q $Z = \{\{1\}, 3, \{1,5\}\}$

$P(Z) = \{\phi, \{\{1\}\}, \{3\}, \{\{1,5\}\}, \{\{1\}, 3\}, \{\{1\}, \{1,5\}\}, \{3, \{1,5\}\}, \{\{1\}, 3, \{1,5\}\}\}$

(A) $= \{\{1\}\} \in Z$ ✗

(B) $= \{\{1\}\} \subset Z$ ✓

(C) $3 \in Z$ ✓

(D) $\{1\} \in Z$ ✓

(E) $\{3\} \in Z$ ✗

(F) $\{3\} \subset Z$ ✓

M.W. ~~04-04-~~
09-04-2024

DYS-1 (Q6, Q7, 8, 9, 11, 12)

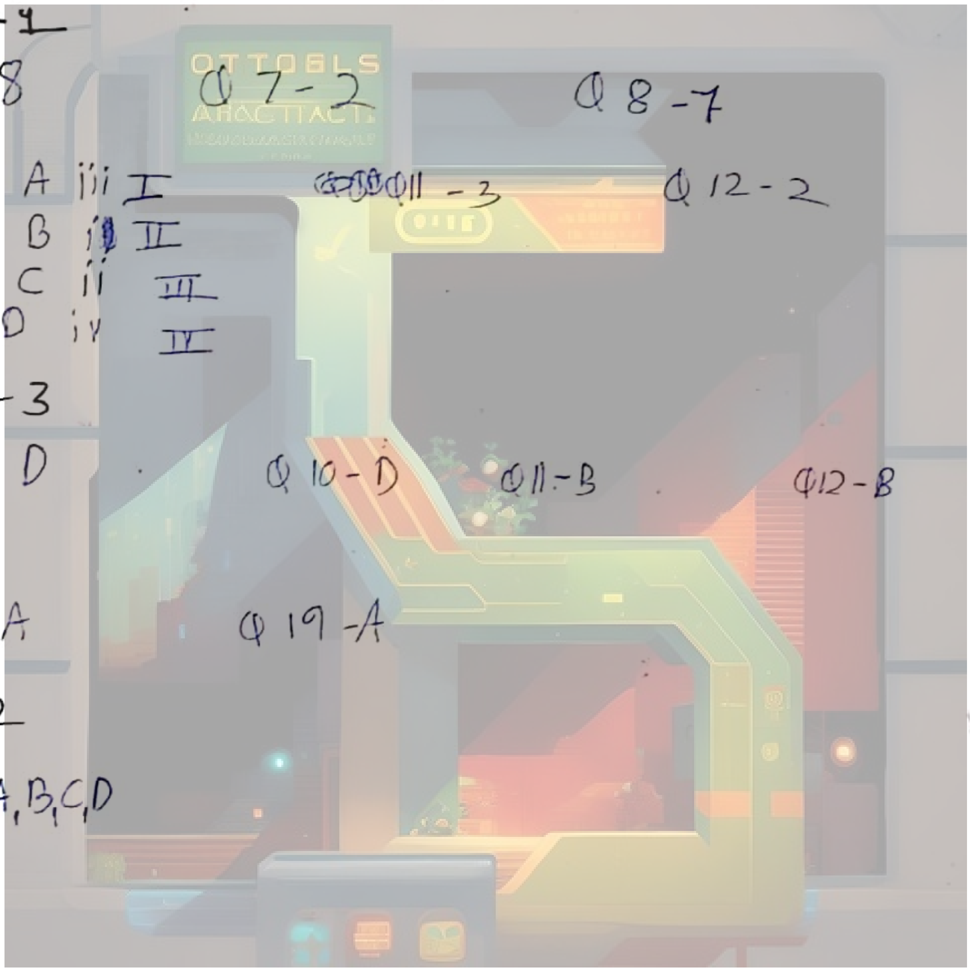
DYS-2 (Q3)

DYS-3 (Q8, 10, 11; 12)

O-1 ~~(Q1, 2,~~ (Q17, 19)

O-2 (Q12)

Answers



DYS-1

Q 6 - 8 Q 7 - 2 Q 8 - 7

Q 9 - A iii I
B i II
C ii III
D iv IV

Q11 - 3 Q 12 - 2

DYS-3

Q 8 - D Q 10 - D Q11 - B Q12 - B

O -1

Q17 - A Q 19 - A

O -2

Q12 - A, B, C, D

★ If set A is a empty set, then

$P(A)$ has 1 element

eg $A = \{\ \}$ or ϕ

$P(A) = \{\{\ \}\}$ or $\{\phi\}$

1. Power set is always non-empty and its minimum value is 1.

Q Determine the power set of the following :-

① $A = \{a\}$

$P(A) = \{\phi, \{a\}\}$

② $B = \{\phi, \{\phi\}\}$

$P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

Q2. Which of the following cannot be the number of elements in the power set of any finite set.

✓a) 26   b) 32   c) 8   D) 16

Q 3. write power set of $\{0\}$.

$P(\{0\}) = \{\phi, \{0\}\}$

Universal Set → A set contains all the elements occur in the discussion.

It is denoted by 'U'

eg. $A = \{1, 2, 3\}$

$B = \{1, 3, 5, 7\}$

$C = \{2, 4, 5, 6\}$

$U = \{1, 2, 3, 4, 5, 6, 7\}$

set A, B, C are subsets of Universal set.

Q $U = \{1, 2, 3, 7, 6, 9, 10, 11\}$

$A = \{1, 2, 3, 7\}$

elements cannot repeat.

$B = \{6, 9, 10, 11\}$

Q find $P(\underset{2}{P(\phi)})$ & $P(P(\underset{4}{P(P(\phi))}))$ 16 256

if $P(\phi) = \{\phi\}$

$\phi = \{\phi\}$

$P\,P(\phi) = \{\phi\}$

$P(\{\phi\}) = \{\phi, \{\phi\}\}$

$P(P(P(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

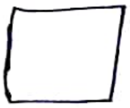
* In any particular discussion, no element can exist out of universal set. It should be noted that universal set is not unique, it may differ in problem to problem.

1 2 4 16 256

0 1 2 4 16

Operation on sets

1. Venn Diagram :- It is a virtual representation of different relations between sets.

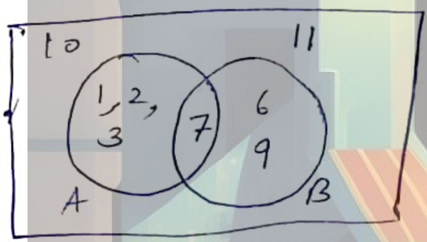
 → shows universal set

 → ~~universal~~ set

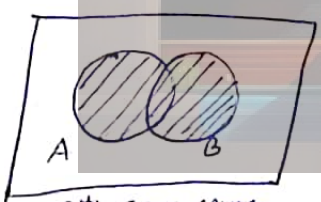
e.g. U = {1, 2, 3, 7, 6, 5, 10, 11}

A = {1, 2, 3, 7}

B = {6, 7, 9}



2. Union (∪) - It contains all the elements which are either in set A or set B or in both.
- It is denoted by A ∪ B

→ 

with some common element

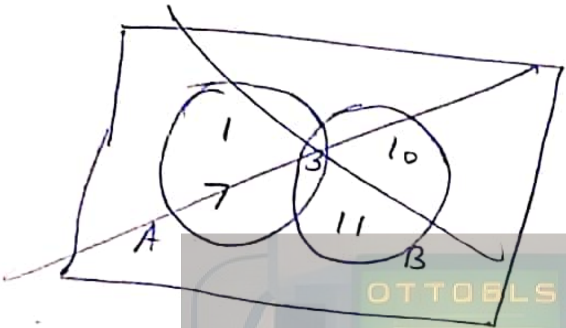
with no common

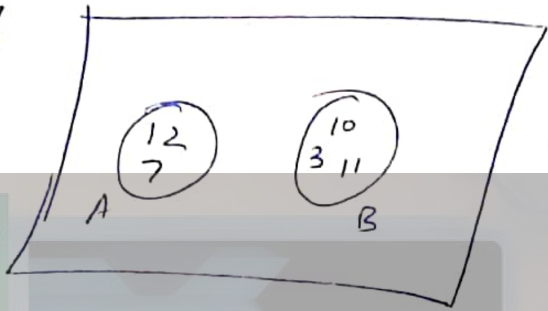
shaded area shows A ∪ B

Q $A = \{1, 2, 3, 7\}$

$B = \{3, 10, 11\}$

$A \cup B = \{1, 2, 3, 7, 10, 11\}$



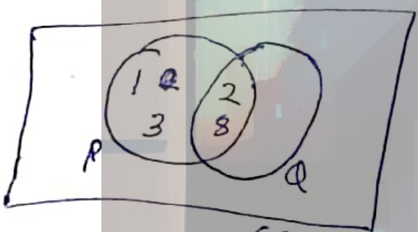


Or

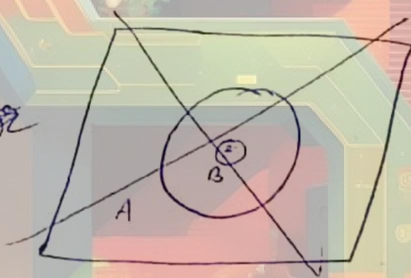
Q2. $P = \{1, 2, 3, 8\}$

$Q = \{2, 8\}$

$P \cup Q = \{1, 2, 3, 8\}$



Or



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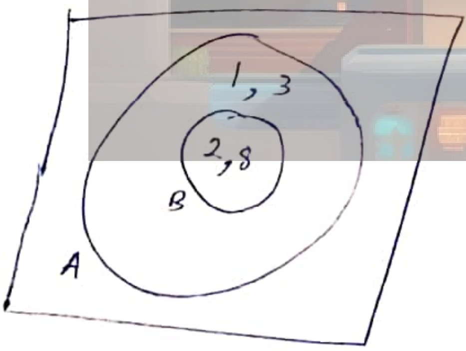


H.W. 12-04-23

~~Q-1~~

DYS-1 (Q 2,)

~~DYS~~

O-2 (9, 13)

JA (Q2, 4)

DYS-1

Q2. c)

O-2

Q9) ABD

Q 10 BCD

J-A

Q2 3)

Q4 2)

Q $A = \{\{a, b\}, c\}$

elements in c

$2^2 = 4$ in $P(c)$

$P(c) = \{\{\}, \{c\}, \{\{a, b\}\}, \{\{a, b\}, c\}\}$

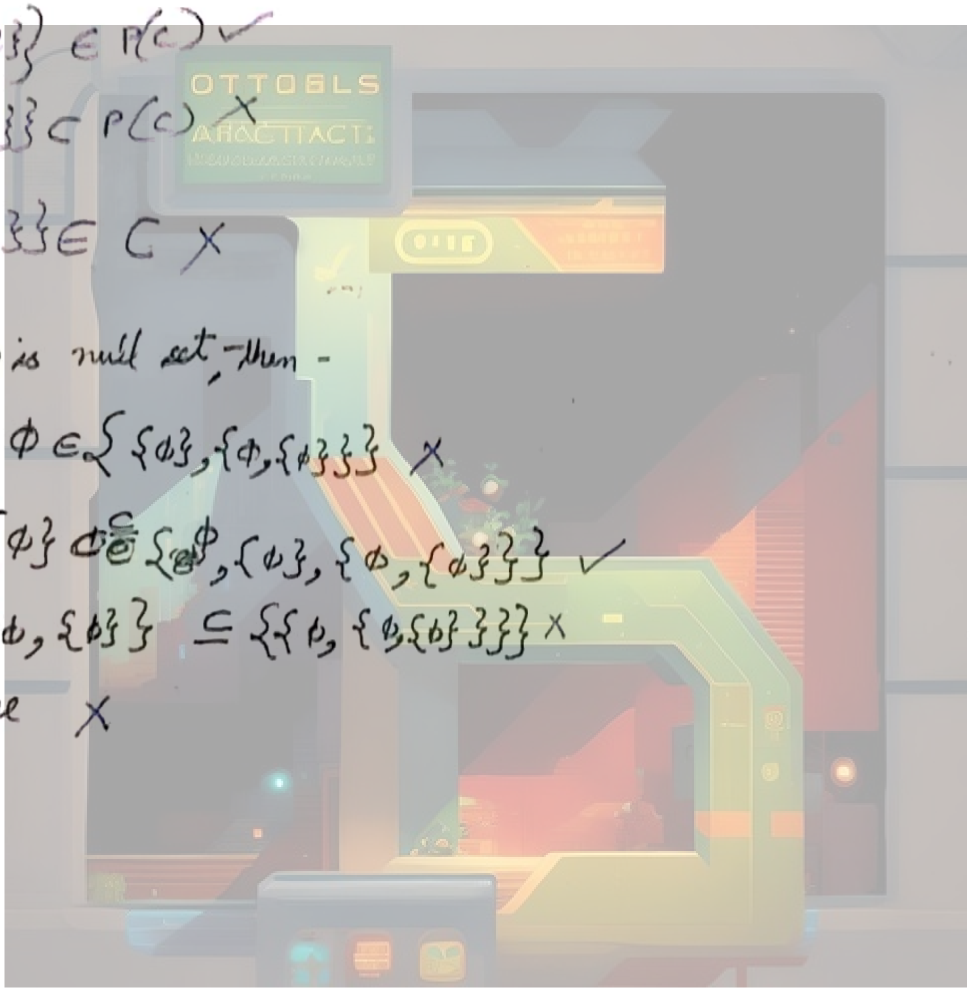
Q $C = \{1, \{2\}\}$

$P(C) = \{\{\}, \{1\}, \{\{2\}\}, \{\{2\}, 1\}\}$

$\{\{2\}\} \in P(C)$ ✓

$\{\{2\}\} \subset P(C)$ ✗

$\{\{2\}\} \in C$ ✗



Q If ϕ is null set, then -

(a) $\phi \in \{\{\phi\}, \{\phi, \{\phi\}\}\}$ ✗

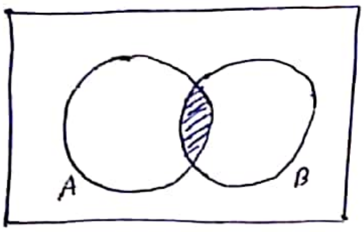
(b) $\{\phi\} \subseteq \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$ ✓

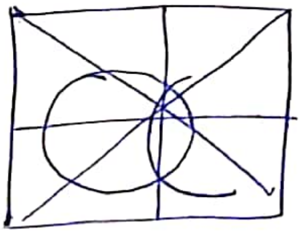
c) $\{\phi, \{\phi\}\} \subseteq \{\{\phi, \{\phi, \{\phi\}\}\}\}$ ✗

d) none ✗

3. <u>Intersection</u> - It contains all elements which are present in set-A and set-B both.

→ It is denoted by $A \cap B$

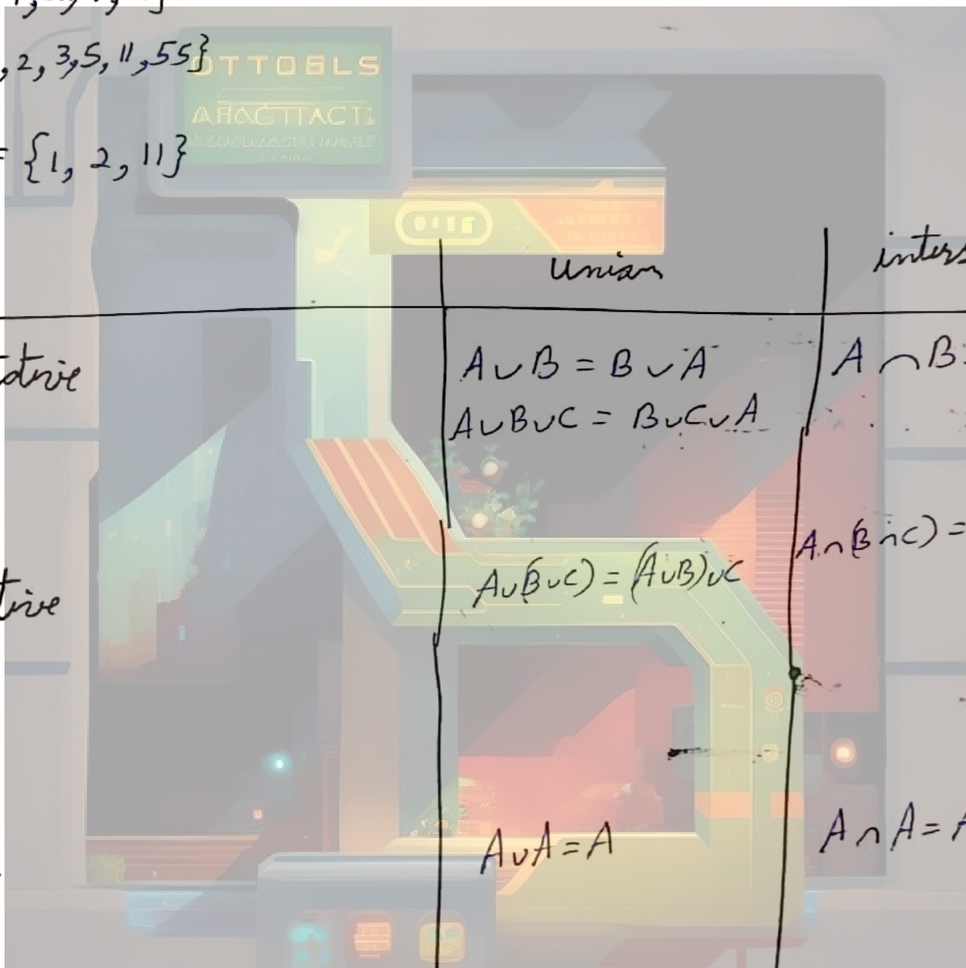




eg. $A = \{1, 2, 7, 11\}$

$B = \{1, 2, 3, 5, 11, 55\}$

$A \cap B = \{1, 2, 11\}$



Property	Union	intersection
① Commutative	$A \cup B = B \cup A$ $A \cup B \cup C = B \cup C \cup A$	$A \cap B = B \cap A$
② Associative	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
③ Idempotent	$A \cup A = A$	$A \cap A = A$
④ Law of identity - compare with ϕ and U	$A \cup U = U$ $A \cup \phi = A$	$A \cap U = A$ $A \cap \phi = \phi$

Note - For two sets A & $B \rightarrow A \cup B \rightarrow A \cap B$

3 sets A, B & $C \rightarrow A \cup B \cup C \rightarrow A \cap B \cap C$

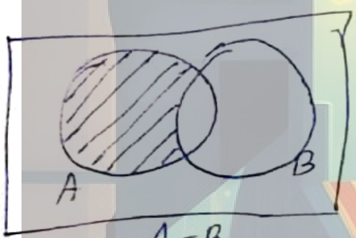
~~n sets A, B, C...Z → A∪B∪C.....Z~~

n sets $A_1, A_2, A_3 \ldots A_n \rightarrow \bigcup_{i=1}^{n} A_i$

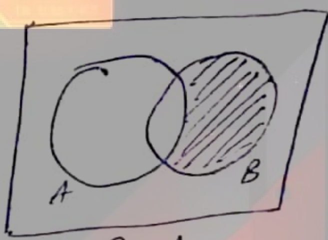
$A \qquad\qquad \rightarrow \bigcap_{i=1}^{n} A_i$

4. **Difference of sets** - It contains all the elements which are in set-A and not present in set-B.

→ It is denoted by $A-B$



$A-B$



$B-A$

eg. * $A-B = A-(A \cap B)$

* $A-B \neq B-A$ (in some cases ~~AB~~ $A-B = B-A$)

~~eg.~~

Q. find difference $A-B$ & $B-A$

$A = \{1, 2, 7, 9\}$

$B = \{-1, 1, 2, 3, 4, 11\}$

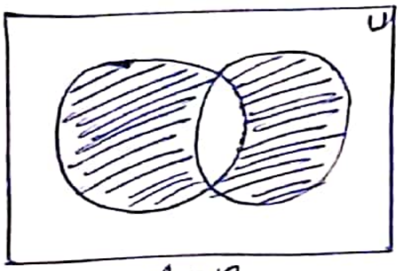
$A-B = \{7, 9\}$

$B-A = \{-1, 3, 4, 11\}$

5. Symmetric Difference of 2 sets :-

→ It is denoted by $A \Delta B$ or $A \oplus B$

→ For two sets A and B it is the part of the sets A & B except $\underline{A \cap B}$.



$A \oplus B$

* $A \Delta B = B \Delta A$ * $A \Delta A = \phi$

Q $A = \{1, 2, 7, 3\}$

$B = \{-1, 1, 3, 7, 5\}$

$A \Delta B = B \Delta A = \{-1, 5, 2\}$

6. Complimentry of set - A set containing all the elements of Universal set which are not present in set-A

$A', \bar{A}, A^c$

→ It is denoted by $\bar{A}, A', A^c$

→ $\bar{A} = U - A$

Q $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 2, 3\}$

$\bar{A} = \{4, 5, 6, 7\}$

Note:-

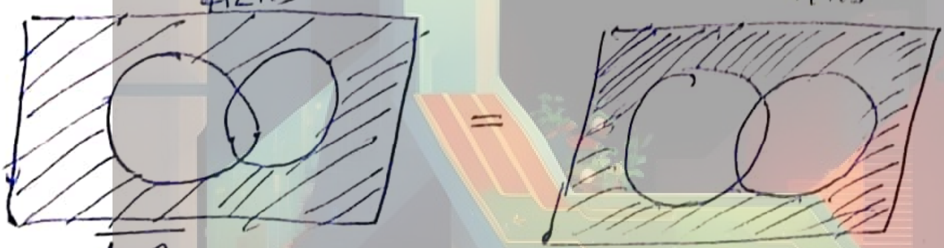
1. $\phi' = U$
2. $U' = \phi$
3. $A \cup A' = U$
4. $A \cap A' = \phi$
5. $\bar{\bar{A}} = A$ (Even times complimentary $= A$)
6. $\bar{\bar{\bar{A}}} = \bar{A}$ (odd times $= \bar{A}$)

Demorgan's principle → For 2 sets A & B (valid for more sets)

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

LHS RHS

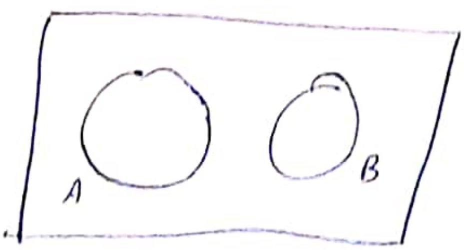


$\overline{A \cup B}$ $\bar{A} \cap \bar{B}$

$$\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} A_i$$

7. **Disjoint Sets** - For any two sets A and B if $A \cap B = \phi$ then set A and set B are called disjoint sets.

eg. $A = \{1, 2, 3, 4\}$
$B = \{P, Q, R\}$
$A \cap B = \phi$
A & B are disjoint sets.



H.W. 13-04-2024

DYS-1 (Q1, 10, all except Q13)

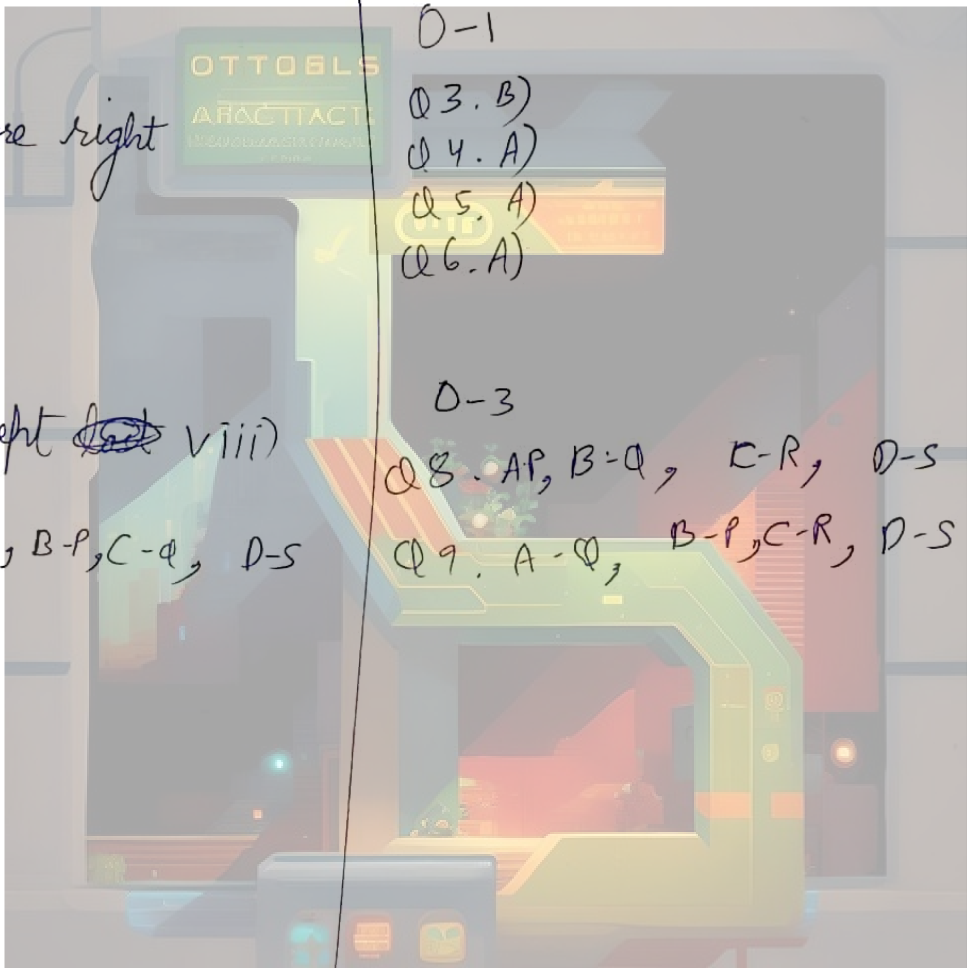
~~DYS-2 (Q1, 2, 3, 4, 5, 5), 6~~

DYS-2 (Q1(1-5), 2, 3, 6, 7)

DYS-3 (Q2, 3, 4, 5, 6, 7)

O-1 (Q3, 4, 5, 6)

O-3 (Q8, 9)



DYS-1

Q1. All are right

Q10. C)

DYS-2

Q1. All except ~~last~~ viii)

Q2. A-R, B-P, C-Q, D-S

Q3. 2

Q6. 2

Q7. 4

DYS-3

Q2. C)

Q3. B)

Q4. B)

Q5. B)

Q6. C)

Q7. C)

O-1

Q3. B)

Q4. A)

Q5. A)

Q6. A)

O-3

Q8. A-P, B-Q, C-R, D-S

Q9. A-Q, B-P, C-R, D-S

Q) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 5\}$

$B = \{2, 4, 6, 7\}$

$C = \{2, 3, 4, 8\}$

① $\bar{A} = \{4, 6, 7, 8, 9, 10\}$

~~② $\overline{(A-B)} = \{2, 4, 6, 7, 8\}$~~

② $\overline{(A-B)} = \{4, 2, 6, 7, 8, 9, 10\}$

③ $\bar{\bar{A}} = \{1, 2, 3, 5\}$

④ $\overline{B \cup C} = \bar{B} \cap \bar{C} = \{1, 9, 10, 5\}$

⑤ $\overline{C \cap A} = \{1, 4, 5, 6, 7, 8, 9, 10\}$

Q2. $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 3, 5, 7, 9\}$

$C = \{2, 4, 8, 10\}$

① $A \cap (B \cup C) = A \cap \{1, 2, 3, 4, 5, 7, 8, 9, 10\} = \{1, 2, 3, 4, 5\}$

② $(A \cap B) \cup (A \cap C) = \{1, 3, 5\} \cup \{2, 4\} = \{1, 2, 3, 4, 5\}$

③ $(A \cup B \cup C)' = \{6\}$

Q 3. If $aN = \{ax : x \in N\}$, then set $3N \cap 7N$.

~~$21, 42 = \boxed{21N}$~~

$3N = \{3x : x \in N\}$

$3N = \{3, 6, 9, 12 \dots\}$

$7N = \{7x : x \in N\}$

$7N = \{7, 14, 21, 28, 35, 42, \dots\}$

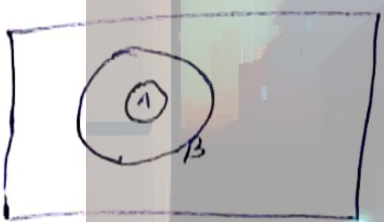
$3N \cap 7N = \{21, 42, 63, \dots\}$

~~$= 7N$~~

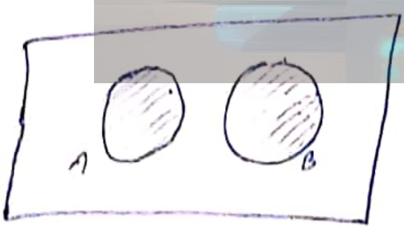
$\text{LCM}(3N, 7N) = \boxed{21N}$ $\text{LCM}(3, 7)N$

Q 4. draw venn diagram

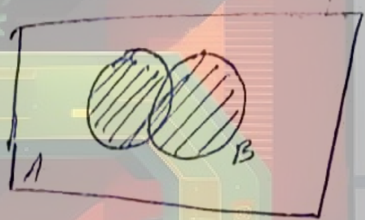
(1) A is a subset of B



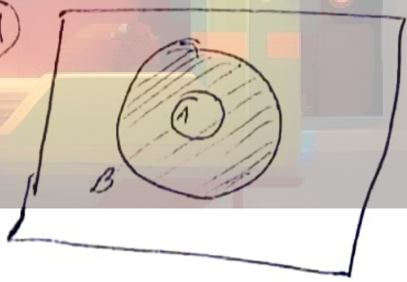
(2) $A \cup B$, when $A \cap B = \phi$



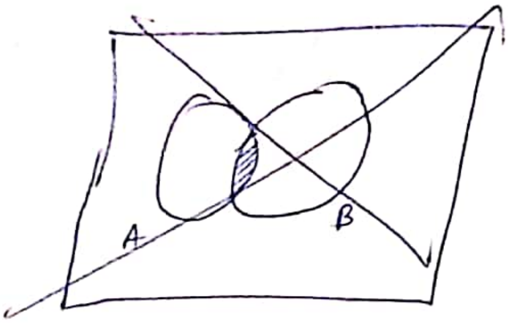
(3) $A \cup B$ when $A \cap B \neq \phi$

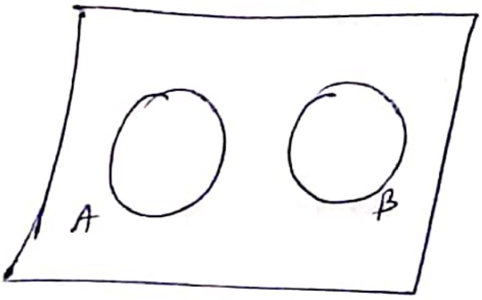


(4) $A \cup B$ when $A \subseteq B$



(5) $A \cap B$ when $A \cap B = \phi$





Q Match The column

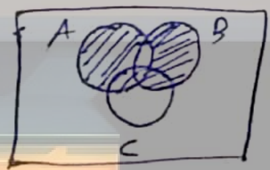
A) $C - (A \Delta B)$

B) $(A \Delta B) - C$

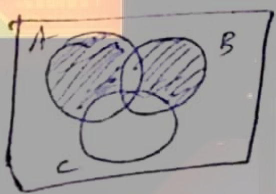
C) $A \Delta B$

D) $(A \Delta B) \Delta C$

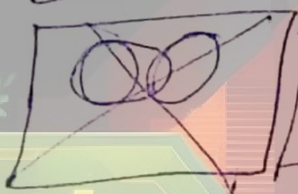
P)

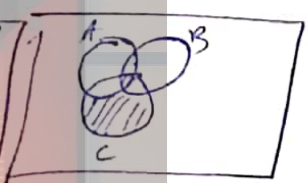


Q)

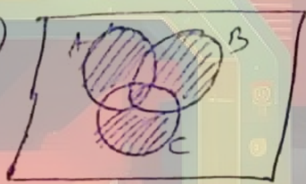


R)





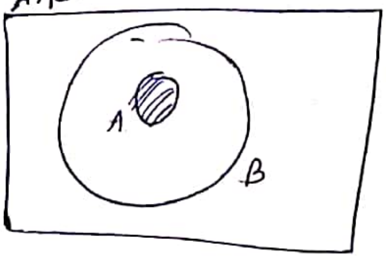
S)



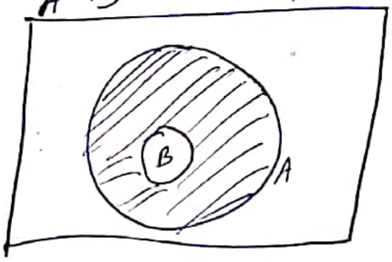
A - R)
B - Q)
C - P)
D - S)

Q Venn diagram

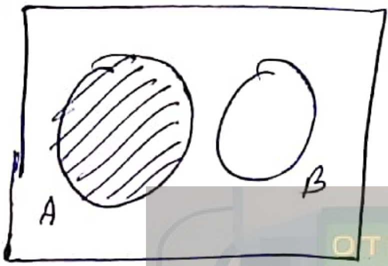
(1) $A \cap B$ & $A \subseteq B$



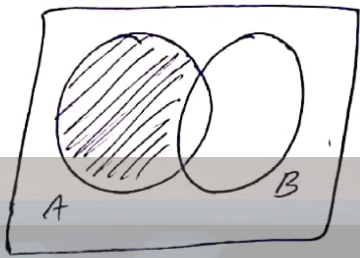
(2) $A - B$ & $B \subseteq A$



(3) $A - B$ & $A \cap B = \phi$

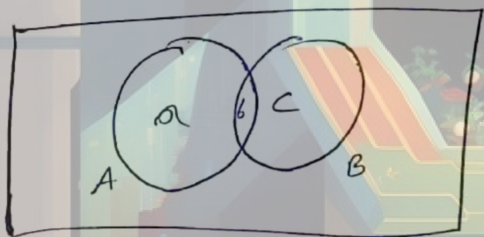


(4) $A - B$ & $A \not\subset B$ & $B \not\subset A$



Q ~~Same~~ Sum results for problem solving

1. For any two finite sets A & B



$n(A) = a + b$

$n(B) = b + c$

$n(A \cup B) = a + b + c$

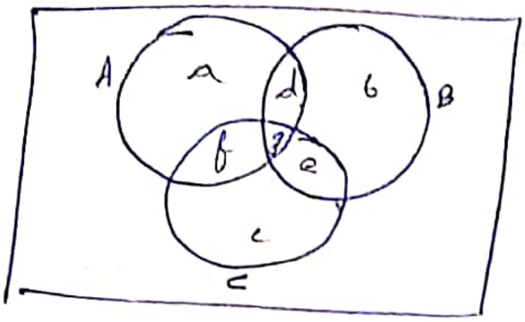
$n(A \cap B) = b$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

2. elements only in set A = a

" " " set B = c

3. for any 3 finite sets,



$n(A) = a+d+f+g$
$n(B) = b+d+e+g$
$n(C) = c+e+f+g$

$\rightarrow n(A)+n(B)+n(C) = a+b+c+2d+2e+2f+3g$

$n(A\cap B) = d+g$
$n(B\cap C) = e+g$
$n(A\cap C) = f+g$

$n(A\cap B)+n(A\cap C)+n(B\cap C) = d+e+f+3g$

$n(A\cap B\cap C) = g$

$n(A\cup B\cup C) = a+b+c+d+e+f+g$

$$\boxed{n(A\cup B\cup C) = n(A)+n(C)+n(B) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)}$$

Only in set A = a
set B = b
set C = c

only in set A & B = d
A & C = f
B & C = e

H.W. 15-04-2024

O-1 (Q7)

Q-2 (Q1, 2, 4, 7, 8, 14, 15, 16)

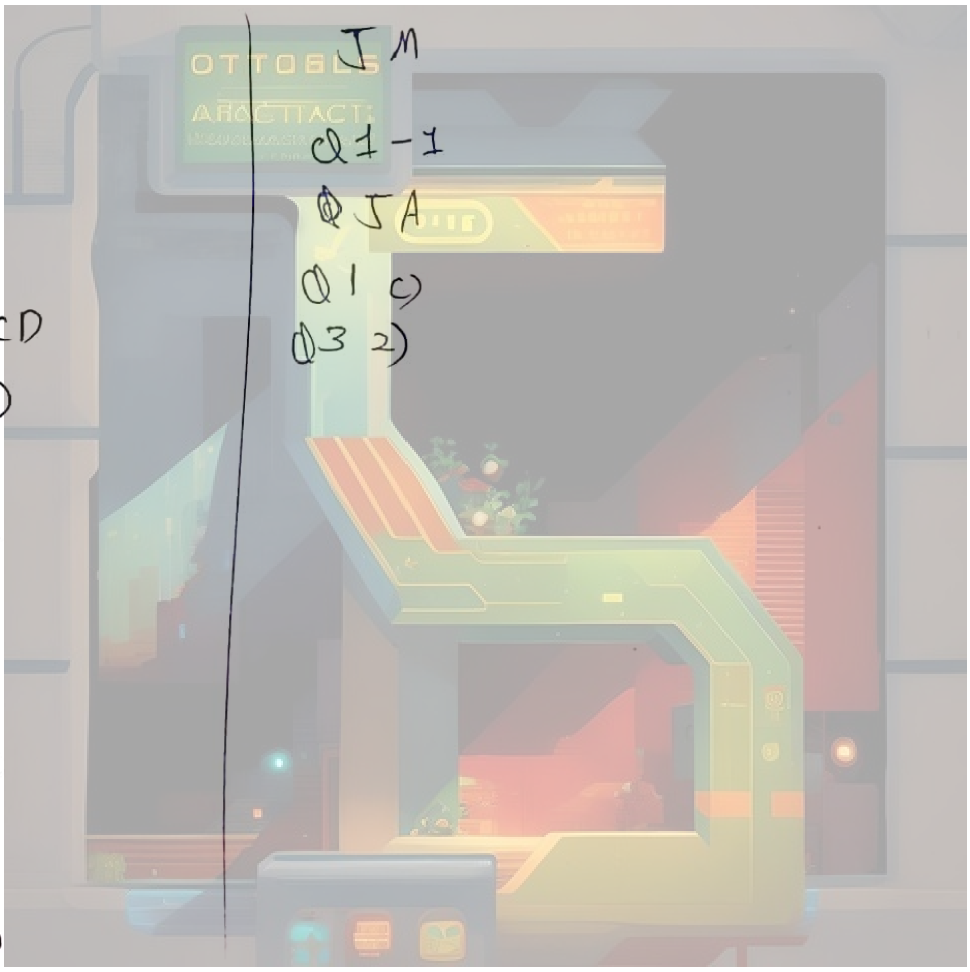
O-3 (Q1, 2, 3)

JM

(Q1)

JA

(Q1, 3)



O-1

Q7 B)

O-2

Q1- ACD

Q2-ABCD

Q4- BC

Q7-ABC

Q8- ABCD

Q14 - AB

Q15 - AD

Q16 - ABCD

O-3

Q1-ABD

Q2-ABCD

Q3- ABCD

JM

Q1 - 1

Q JA

Q1 c)

Q3 2)

4. Number of ~~Elements~~ Elements belonging to exactly two of the sets. ~~A & C B~~

$$\boxed{n(A\cap B) + n(B\cap C) + n(A\cap C) - 3n(A\cap B\cap C) = \text{no. of elements in exactly two sets.}}$$

5. Number of elements belonging to exactly one of the sets A, B and C is

$$\boxed{n(A) + n(B) + n(C) - 2n(A\cap B) - 2n(A\cap C) - 2n(B\cap C) + 3n(A\cap B\cap C)}$$

$$\boxed{\Sigma n(A) - 2\Sigma n(A\cap B) + 3n(A\cap B\cap C)}$$

6. no. of elements in

1. $n(\bar{A}\cup\bar{B}) = n(\overline{A\cap B}) = n(U) - n(A\cap B)$

$n(\bar{A}\cap\bar{B}) = n(\overline{A\cup B}) = n(U) - n(A\cup B)$

$n(A\cap\bar{B}) = n(\overline{\bar{A}\cup B}) = n(A-B) = n(A) - n(A\cap B)$

$n(\bar{A}\cap B) = n(\overline{A\cup\bar{B}}) = n(B-A) = n(B) - n(A\cap B)$

7. For two sets A & B

$A\cap B$ will be maximum when :- $A = B$

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

Q If $n(U) = 700$,
$n(A) = 200$,
$n(B) = 240$
$n(A \cap B) = 100$
$n(\bar{A} \cap \bar{B}) = ?$

$\cancel{n(U) - n(\bar{A} \cup \bar{B}) = 700 - n(A) - n(B) + n(A \cap B)}$
$\cancel{= 700 - 200 - 240 + 100}$
$\cancel{= 800 - 440}$
$\cancel{= 360}$

$n(U) - n(\bar{A} \cap \bar{B}) = 700 - n(A \cap B)$
$= 700 - 100$
$\boxed{= 600}$

Q 2. An investigator interviewed 100 students for the preferences of 3 drinks:- milk (M), coffee (C) & Tea (T)

$n(M \cap C \cap T) = 10$ $\quad$ $n(\text{only } m) = 12$
$n(M \cap C) = 20$ $\quad$ ~~n (C)~~
$n(C \cap T) = 30$ $\quad$ $n(\text{only } c) = 5$
$n(M \cap T) = 25$ $\quad$ $n(\text{only } T) = 8$

$n(M) = 12 + 25 + 20 - 10 - 10$
$= 57 - 20$
$\boxed{= 37} + 10 = 47$

$n(T) = 8 + 30 + 25 - 20$
$\boxed{= 43} + 10 = 53$

$n(C) = 5 + 20 + 30 - 20$
$\boxed{= 35} + 10 = 45$

$n(\overline{M \cup C \cup T}) =$
$25 - (35 + 37 + 43 - 20 - 30 - 25 + 10)$
$25 - (115 - 75 + 10)$
$25 - 115 + 75 - 10$
$100 - 125$

~~25~~

~~MACAT~~

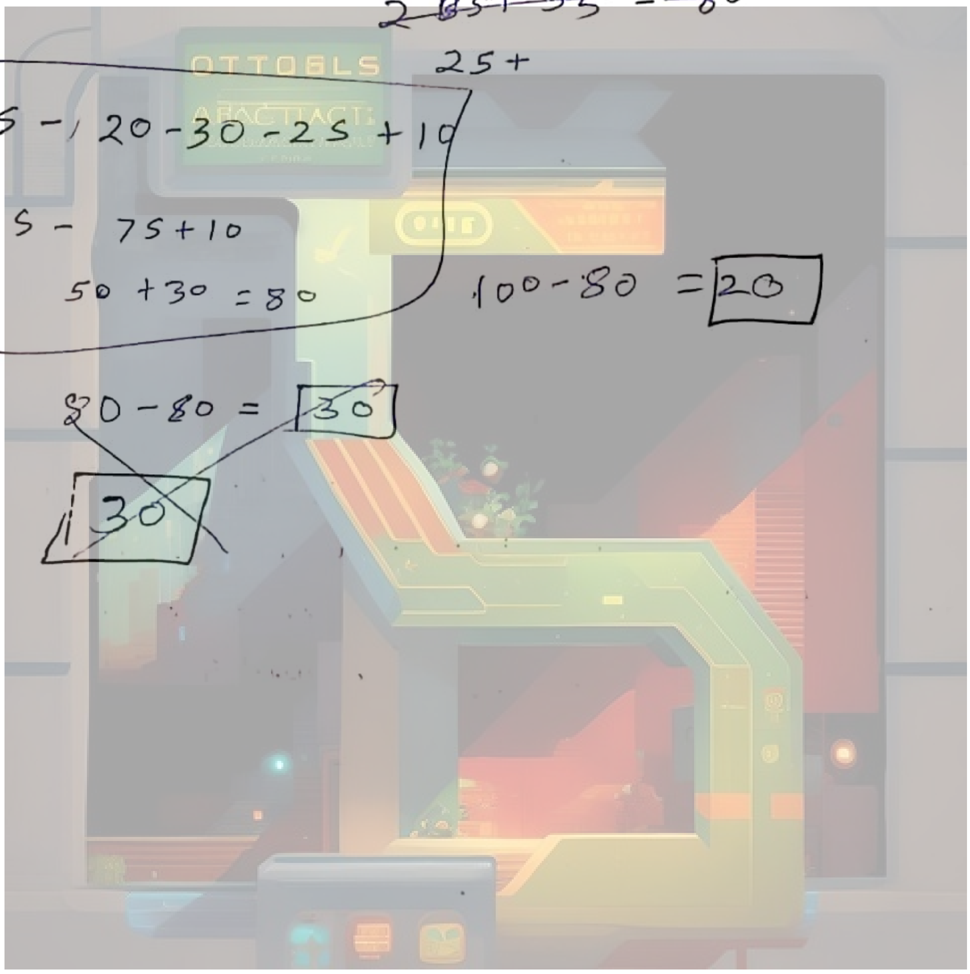
~~115 - 40 - 60 - 25 + 30~~

~~115 - 125 + 30~~

~~145 - 125 = 20~~

~~2 85 + 55 = 80~~

25 +



115 - 20 - 30 - 25 + 10

115 - 75 + 10

50 + 30 = 80

100 - 80 = $\boxed{20}$

80 - 80 = $\boxed{30}$

~~$\boxed{130}$~~

Q In a class of 55 students, the no. of students studying diff subjects are

$n(M) = 23$, $n(P) = 24$, $n(C) = 19$, ~~n(...)~~.

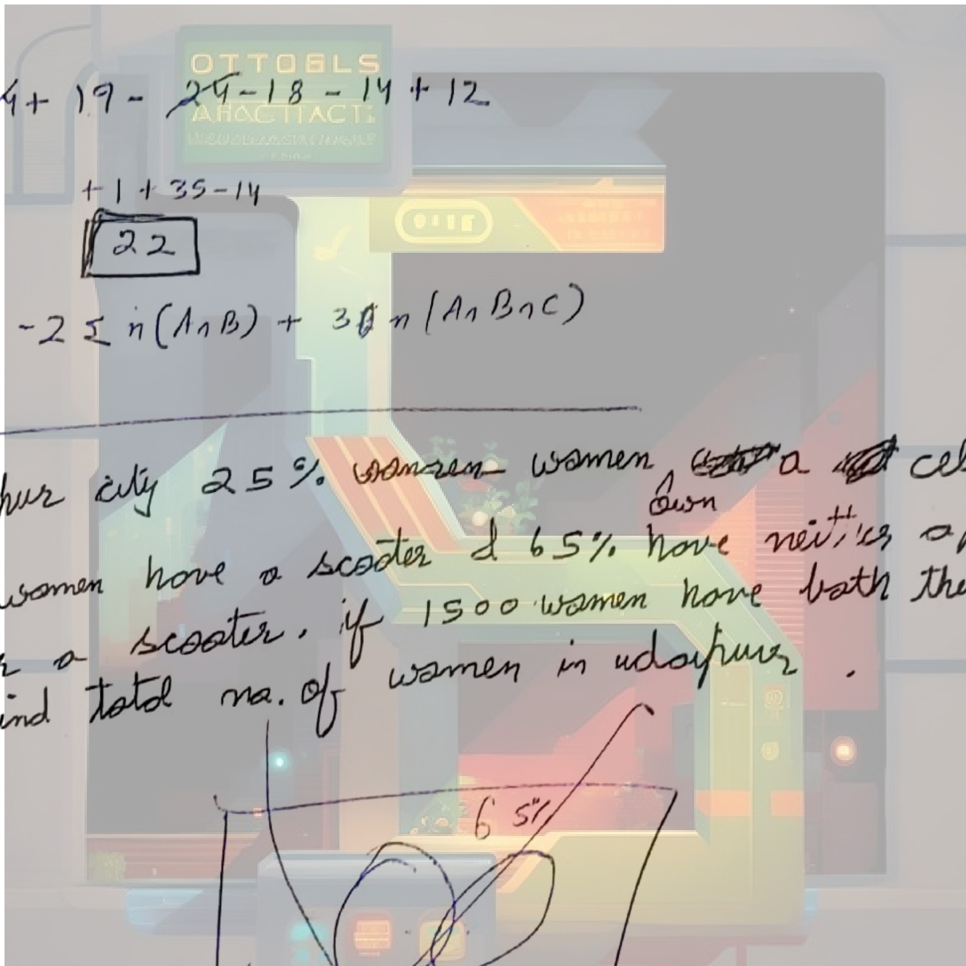
$n(M \cap P) = 12$

$n(M \cap C) = 9$

$n(P \cap C) = 7$

$n(M \cap C \cap P) = 4$

n of exact one sub



$23 + \cancel{24} + 19 - \cancel{24} - 18 - 14 + 12$

$+ 1 + 35 - 14$

$\boxed{22}$

$\Sigma n(A) - 2\Sigma n(A \cap B) + 3 n(A \cap B \cap C)$

Q In udaipur city 25% ~~women~~ women own a cell phone, 15% women have a scooter & 65% have neither a phone nor a scooter. if 1500 women have both them find total no. of women in udaipur.

65%

$n(P) = 25$

$n(S) = 15$

$n(\overline{P \cup S}) = 65$

$P \cup S = 35$

$n(\bar{P} \cap S) = 65$

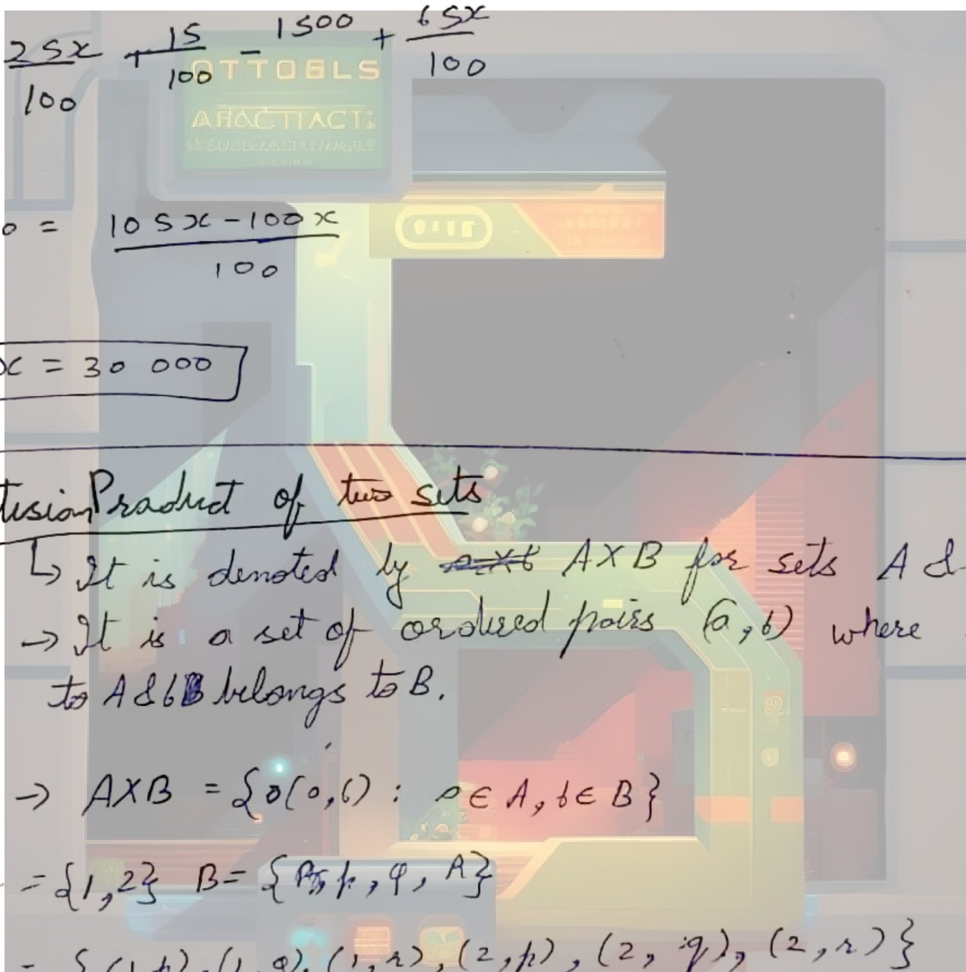
75n

Total $= x$

$n(c) = \frac{25x}{100}$

$n(s) = \frac{15x}{100}$

nothing $= \frac{65x}{100}$



$x = \frac{25x}{100} + \frac{15}{100} - 1500 + \frac{65x}{100}$

$1500 = \frac{105x - 100x}{100}$

$\boxed{x = 30\,000}$

Cartesian Product of two sets

↳ It is denoted by ~~a×b~~ $A \times B$ for sets A & B

→ It is a set of ordered pairs (a, b) where a belongs to A & b belongs to B.

→ $A \times B = \{(a, b) : a \in A, b \in B\}$

eg. $A = \{1, 2\}$ $B = \{p, q, r\}$

① $A \times B = \{(1,p), (1,q), (1,r), (2,p), (2,q), (2,r)\}$

② $B \times A = \{(p,1), (p,2), (q,1), (q,2), (r,1), (r,2)\}$

③ $A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

④ $B \times B = \{(p,p), (p,q), (p,r), (q,p), (q,q), (q,r), (r,p), (r,q), (r,r)\}$

$n(A) \times n(B) = n(A \times B)$

~~H.W. 14-04-2024~~

Q

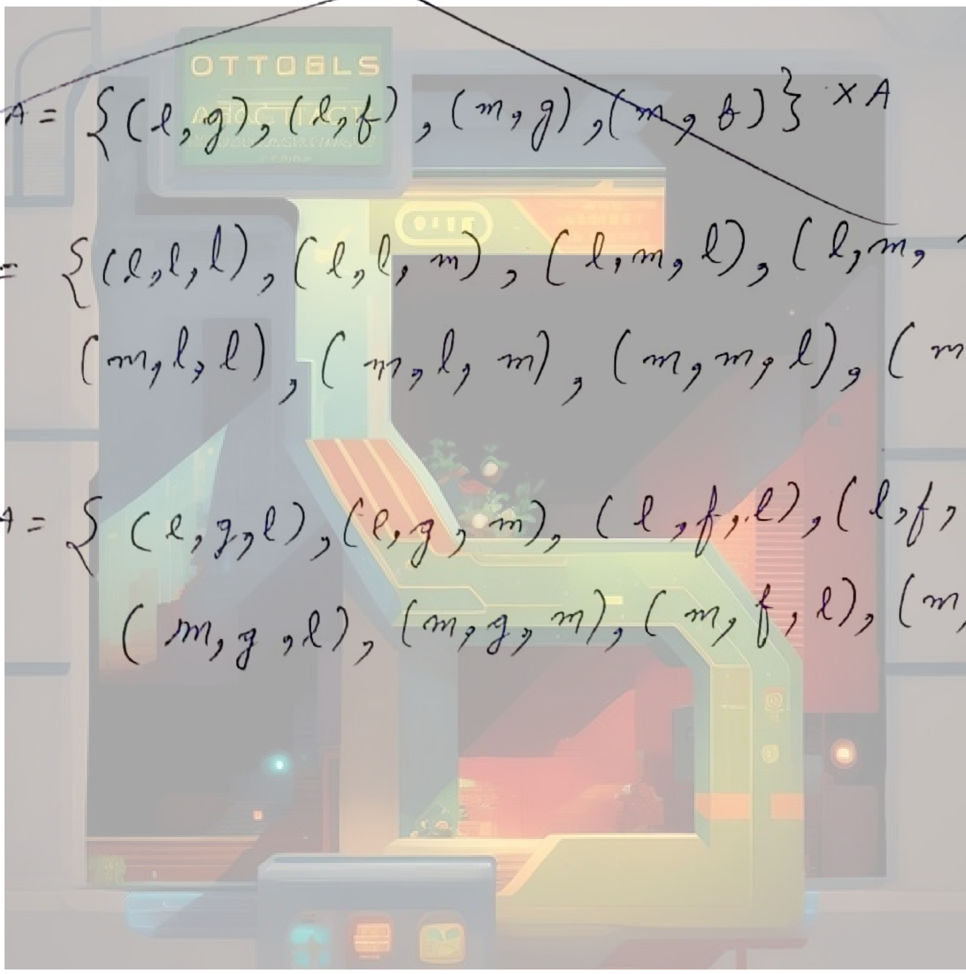
$A = \{l, m)$

$B = \{g, f\}$

$A \times A \times A = \{(l,l), (l,m), (m,l), (m,m\} \times \{l, m\}$

~~$= \{((l,l),l\}, ((l,l),m), (l,m,l), ((l,m),m),$~~

~~$((l,m),l), ((m,l),m), ((m,m),l), ((m,m),m)\}$~~



~~$(A \times B) \times A = \{(l,g), (l,f), (m,g), (m,f)\} \times A$~~

$= \{(l,l,l), (l,l,m), (l,m,l), (l,m,m),$

$(m,l,l), (m,l,m), (m,m,l), (m,m,m)\}$

$(A \times B) \times A = \{(l,g,l), (l,g,m), (l,f,l), (l,f,m),$

$(m,g,l), (m,g,m), (m,f,l), (m,f,m)\}$

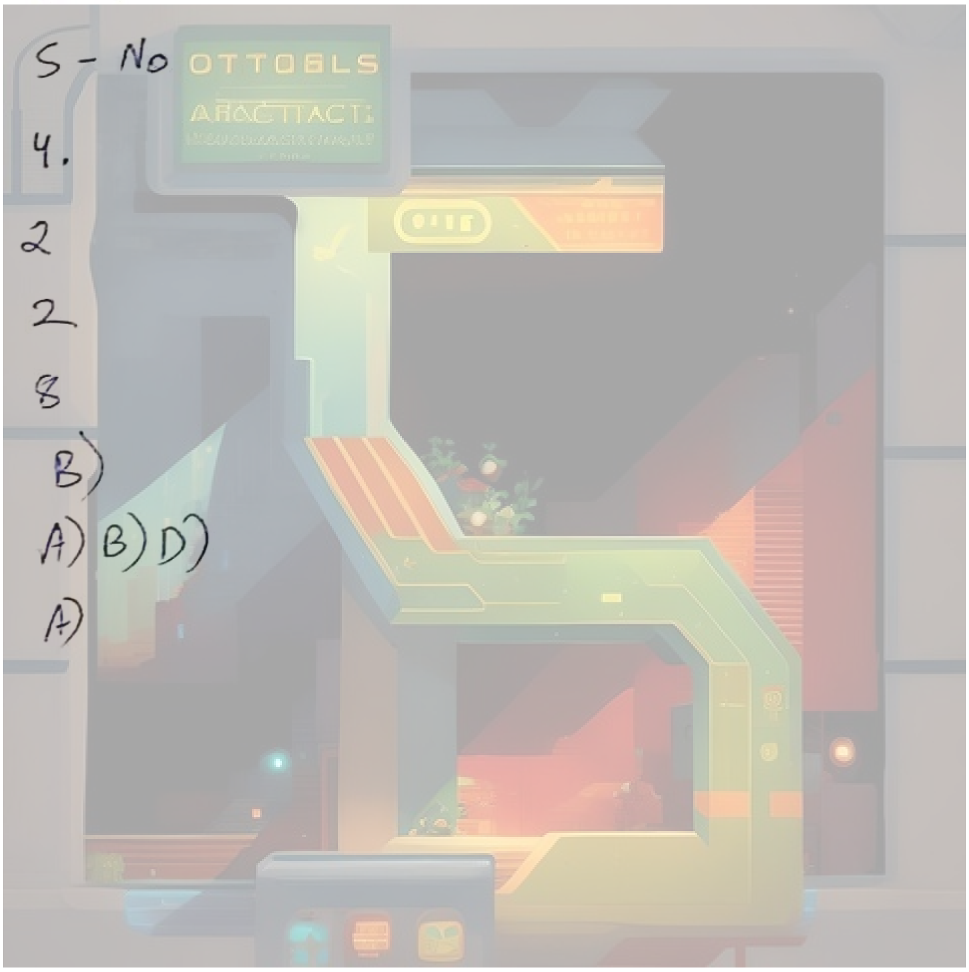
H.W. 14-04-2024

DYS-3 (Q1, 9, 13, 14, 15, 16, 17, 18, 19, 20)

Q1 -

Q 9 - B

Q 13 - P - 30
Q - 19
R - 60



S - No

Q 14 4.

Q 15. 2

Q 16. 2

Q 17. 8

Q 18. B)

Q 19. A) B) D)

Q 20. A)

Note:- ① if $(a,b) = (x,y)$

$$a = b$$
$$x = y$$

② Generally $A \times B \neq B \times A$

③ $A \times B = \phi$, Then either $A = \{\}$ or $B = \{\}$ or both are empty

④ $(A, B) \neq (B, A)$

⑤ If set A and set B have m elements common then number of elements common in $A \times B$ is m^2.

$n(A \cap B) = m$

$n(A \times B \cap B \times A) = m^2$

eg. $A = \{1, 2\}$

$B = \{1, 2, 3\}$

$A \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

$B \times A = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

Q 1. $A = \{\}$
$B = \{1, 2\}$
find $A \times B$
$A \times B = \phi$ or $\{\}$

Q 2. $(2x - y, 25) = (15, 2x + y)$

$2x - y = 15$
$y = 2x - 15$

$2x + y = 25$
$2x + 2x - 15 = 25$
$4x = 40$
$\boxed{x = 10}$

$y = 20 - 15$
$\boxed{y = 5}$

Q3. $a \in A$ & $b \in B$

find (a,b) such that $a^2 < b$.

$A = \{2, 3\}$

$B = \{4, 16, 23\}$

$A \times B = \{(2,4), (2,16), (2,23), (3,4), (3,16), (3,23)\}$

$(a,b) = \cancel{(2,4)}$

$$\boxed{\begin{aligned} &= (2,16) \\ &(2,23) \\ &(3,16) \\ &(3,23) \end{aligned}}$$

Q4. $A \times B = \{(2,3), (2,4), (5,3), (5,4)\}$

find set A & B

$A = \{2, 5\}$

$B = \{3, 4\}$

Q5. Given $A \times B$ has 15 ordered pairs & set A has 5 elements find no. of elements in set B

$n(A) \times n(B) = 15$

$n(B) = \frac{15}{5}$

$\boxed{n(B) = 3}$

Q6. If 2 sets A & B have 44 elements in common, then no. of elements common in $A \times B$ & $B \times A$ are

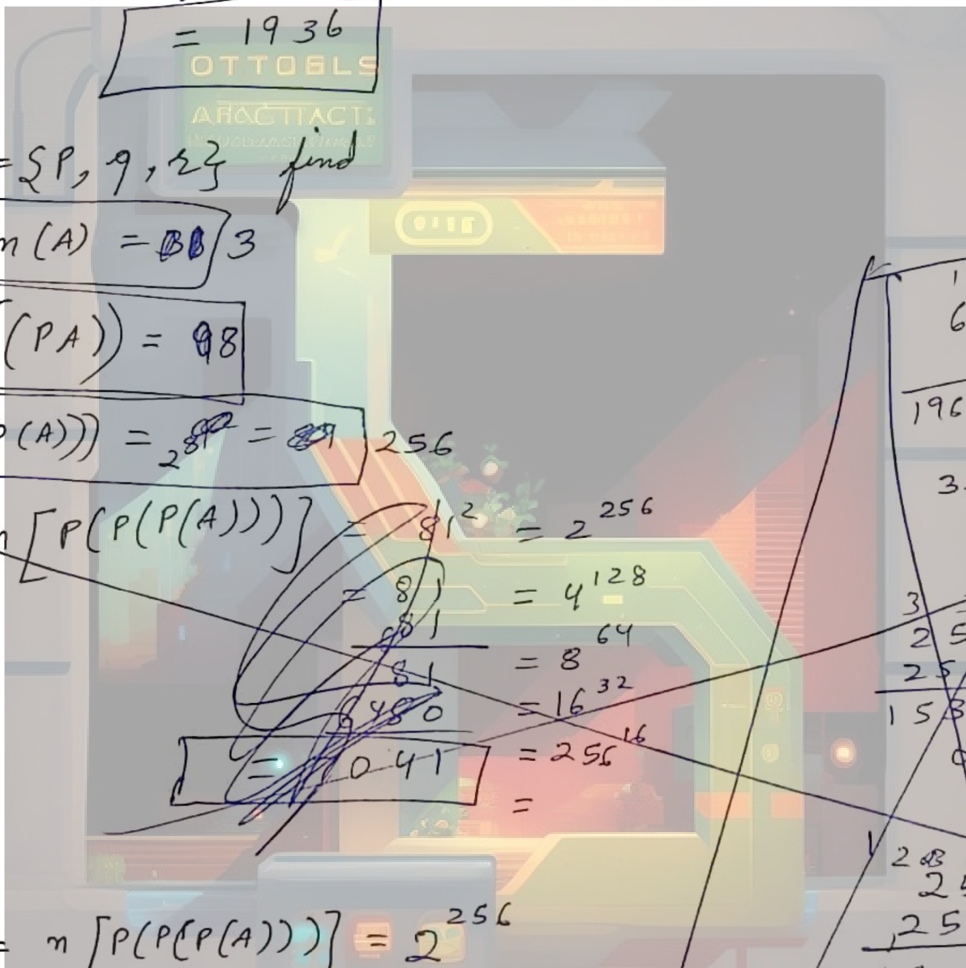
$|A \cap B| = 44$

$|A \times B \cap B \times A| = 44^2$

$= 44 \times 44$

$\begin{array}{r} 44 \\ \times 44 \\ \hline 176 \\ 1760 \\ \hline \end{array}$

$\boxed{= 1936}$



Q7. $A = \{p, q, r\}$ find

a) $\boxed{n(A) = 3}$

b) $\boxed{n(P(A)) = 8}$

c) $\boxed{n(P(P(A))) = 2^8 = 256}$

~~d) $n[P(P(P(A)))] = 81^2 = 2^{256}$~~

$= 4^{128}$

$= 8^{64}$

$= 16^{32}$

$= 256^{16}$

$d = n[P(P(P(A)))] = 2^{256}$

$= 256^{16}$

$= 65536^{8}$

$\begin{array}{r} 256 \\ \times 256 \\ \hline 1536 \\ 12800 \\ 51200 \\ \hline 65536 \end{array}$

$\begin{array}{r} 65536 \\ \times 65536 \\ \hline 393216 \\ 1966080 \\ 32768000 \\ 327680000 \\ 3932160000 \\ \hline 4294967296 \end{array}$

Theory of Numbers

→ It helps us to study the relationship between different type of numbers such as prime nos., edional nos.

Type	Example	Special Point
1. Natural Numbers (N)	$N = \{1, 2, 3, 4 \dots\}$	
2. Whole Number (W)	$W = \{0, 1, 2, 3 \dots\}$	
3. Integers (Z/I)	$Z = \{\dots -2, -1, 0, 1, 2 \dots\}$ $I^+ = \{1, 2, 3 \dots\}$ $I^- = \{-3, -2, -1 \dots\}$	☆ Non negative integers - $\{0, 1, 2, 3\}$ ☆ Non positive integers - $\{\dots -3, -2, -1, 0\}$ ☆ Even Integers - $\{\dots -4, -2, 0, 2, 4 \dots\}$ ☆ odd Integers - $\{\dots -3, -1, 1, 3 \dots\}$
4. Prime Numbers	$\{2, 3, 5, 7, 11, 13, 17\}$	☆ Natural numbers except 1 who have only two factors i.e 1 & number itself ☆ 2 is the only even prime number
5. composite Number	$\{4, 6, 8, 9, 10, 12, 14\}$	☆ Natural numbers except 1 who have more than 2 factors. ☆ 4 is the smallest composite no

⑥ co-prime numbers (Relatively Prime)	$(2,3)$, $(7,8)$ $(10,11)$, $(12,13)$ etc. $(11,13)$, $(17,19)$	☆ HCF of two numbers $=1$ ☆ the numbers are not necessarily prime numbers ☆ any two consecutive nos. are co-prime. ☆ All odd nos. consecutive are co-prime) ☆ any two prime nos. are co-prime
⑦ twin primes	$(3,5)$, $(5,7)$, $(17,19)$ $(11,13)$	☆ two prime numbers having difference of two. ☆ both must be prime.
⑧ rational numbers ($\mathbb{Q}$)	$\frac{1}{2}, \frac{3}{5}, \frac{-9}{8}, \frac{7}{21}$	☆ numbers in form $\frac{P}{Q}$ $(Q \neq 0)$ ☆ P & Q are co-prime and don't have common factor
⑨ Irrational nos. ($\overline{\mathbb{Q}}$ or $\mathbb{Q}^c$)	$\sqrt{2}, \sqrt{3}, \pi, e \ldots$	☆ the numbers which are not rational.
⑩		

10 - <u>Complex Numbers</u> $(z) \Rightarrow$ Numbers which are written in the form of

$a + ib$ (i stands for iota)

$i = \sqrt{-1}$

a, b are real numbers

eg. $3+2i$, $-5+\frac{2}{3}i$, $3(3+i0)$, $-7+2i$

→ two complex numbers $a+ib$ and $p+iq$ are equal. ~~then~~ then $a=p$, $b=q$

→ Conjugate of a complex number

$(2+\sqrt{3} \to 2-\sqrt{3})$

$z = a+ib \;\to\; \bar{z} = a-ib$

Q. find conjugate

a) $3+i2 \rightarrow 3-i2$

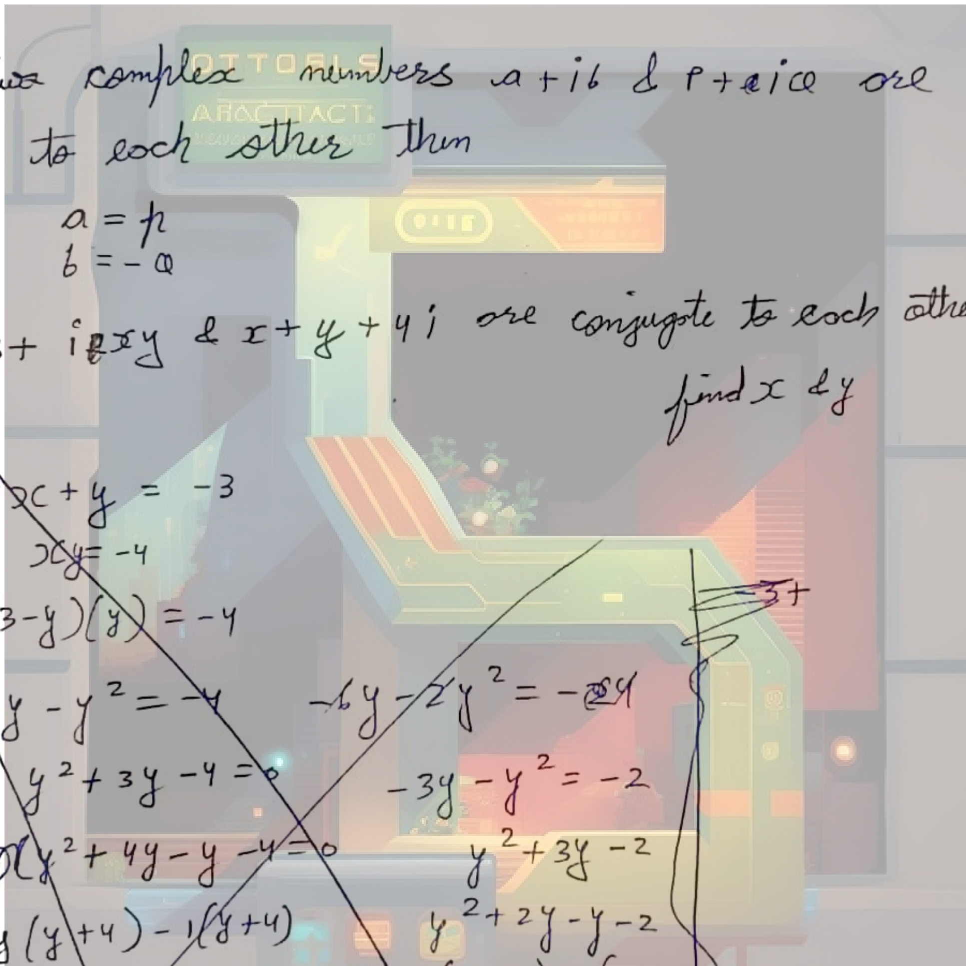
b) $\sqrt{2}+i3 \rightarrow \sqrt{2}-i3$

c) $i2-7 \rightarrow -7-i2$

$-3-i5 \rightarrow -3+i5$

$(8+\sqrt{3})+2i \rightarrow (8+\sqrt{3})-2i$

$6 \rightarrow 6$



→ Two complex numbers $a+ib$ & $p+iq$ are conjugate to each other then

$a = p$
$b = -q$

Q $-3+ixy$ & $x+y+4i$ are conjugate to each other, then find x & y

$x+y = -3$
$xy = -4$

$(-3-y)(y) = -4$

$-3y - y^2 = -4$ $\quad$ $-6y-2y^2 = -4$

$y^2+3y-4=0$ $\quad$ $-3y-y^2=-2$

$y^2+4y-y-4=0$ $\quad$ y^2+3y-2

$y(y+4)-1(y+4)$ $\quad$ $y^2+2y-y-2$

$(y-1)(y+4)$ $\quad$ $y(y+2)-1(y+2$

$y=1$, $y=-4$ | $x=-4$, $y=1$ $\quad$ $\boxed{y=1,\ y=-2}$

$\boxed{x=1,\ x=-2}$

$\boxed{-4,1}$

$\boxed{-2,1}$

(11) ~~Perfect nos.~~ =

$$x + y = -3$$
$$xy = -4$$
$$x = -3 - y$$
$$(-3 - y)(y) = -4$$
$$-3y - y^2 = -4$$
$$y^2 + 3y - 4 = 0$$
$$y^2 + 4y - y - 4 = 0$$
$$y(y+4) - 1(y+4)$$
$$(y-1)(y+4)$$

$$\boxed{\begin{aligned} y &= 1, \quad y = -4 \\ x &= -4, \quad x = 1 \end{aligned}}$$

Note:-

$$\boxed{Z \supset R \supset I \supset W \supset N}$$

11. Perfect number – Sum of ~~an~~ proper ~~p~~ divisors of a number is the number itself then it is called a perfect number.

eg. $6 \begin{cases} 1 \\ 2 \\ 3 \end{cases}$ $1+2+3=6$

$6 \rightarrow$ perfect no.

$28 \begin{cases} 1 \\ 2 \\ 4 \\ 7 \\ 14 \end{cases}$ $1+2+4+7+14 = 28$

$28 \rightarrow$ perfect no.

H.W. 18-04-2024

DYS-4 (Q1-Q6)

J-M (Q2,3)

DYS-4

Q1. $a=3, b=4$

Q2. i) $x = 2/11$

ii) $x = 16/99$

iii) $x = \frac{419}{990}$

Q3. Proved

Q4. i) all rational ii) may or may not iii) all irrational

Q6. irrational

JEE-M

Q2. 4)

Q3. 3)

Note -

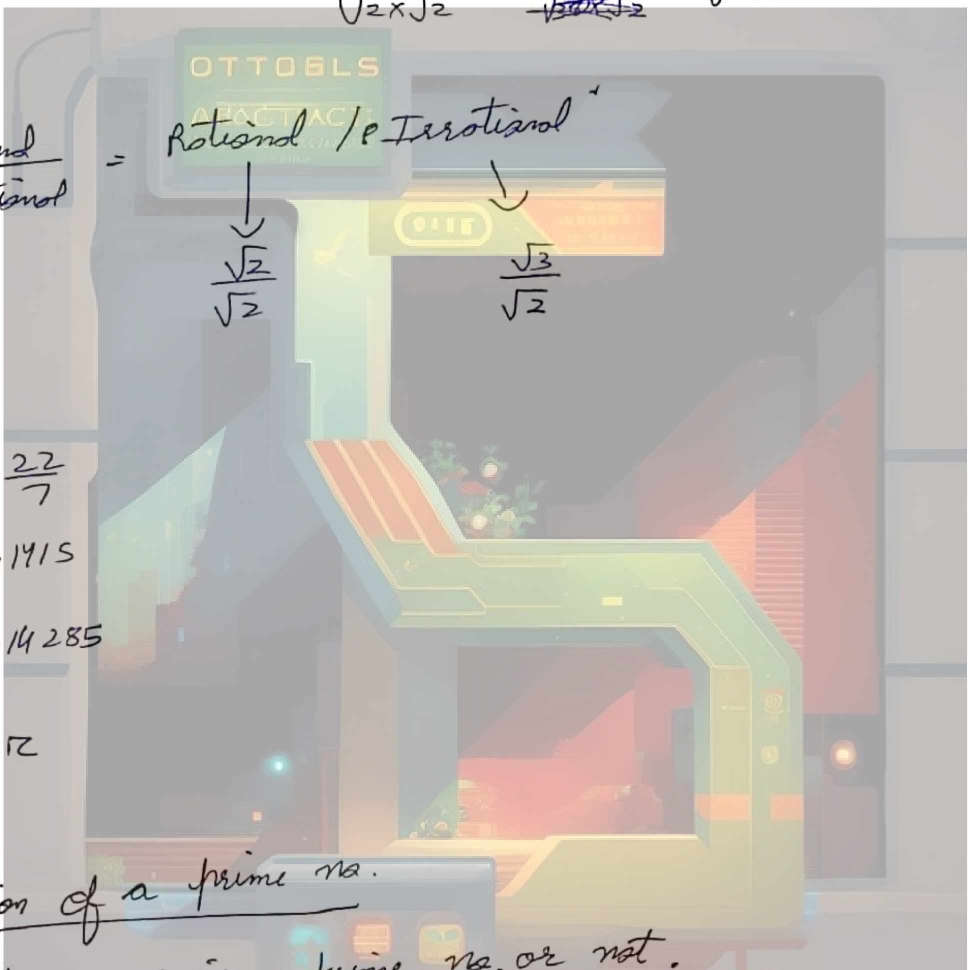
1. Rational $\pm$ Irrational = Irrational
2. Rational $\times$ Irrational = Irrational (for rational $\neq 0$)
3. Irrational $\pm$ Irrational = Rational / Irrational

eg $(2+\sqrt{3}) + (2-\sqrt{3}) \rightarrow$ rational

$(2+\sqrt{3}) + (2-\sqrt{5}) \rightarrow$ irrational

4. Irrational $\times$ Irrational = Rational / Irrational

$\sqrt{2} \times \sqrt{2}$ $\qquad$ $\sqrt{3} \times \sqrt{2}$



5. $\dfrac{\text{Irrational}}{\text{Irrational}}$ = Rational / Irrational

$\dfrac{\sqrt{2}}{\sqrt{2}}$ $\qquad$ $\dfrac{\sqrt{3}}{\sqrt{2}}$

$\pi \approx \dfrac{22}{7}$

$\pi = 3.1415$

$\dfrac{22}{7} = 3.14285$

$\dfrac{22}{7} > \pi$

Investigation of a prime no.

Q. Investigate 17 is a prime no. or not.

$\sqrt{17} = 4.$ something

2 & 3

$\dfrac{4.\text{som}}{2}$ ✗ $\qquad$ $\dfrac{4.\text{som}}{3}$ ✗

so 17 is a prime.

eg 2. 18

$\sqrt{18} = 4.\text{some}$

$= 3\sqrt{2}$

divisible = 3

not prime

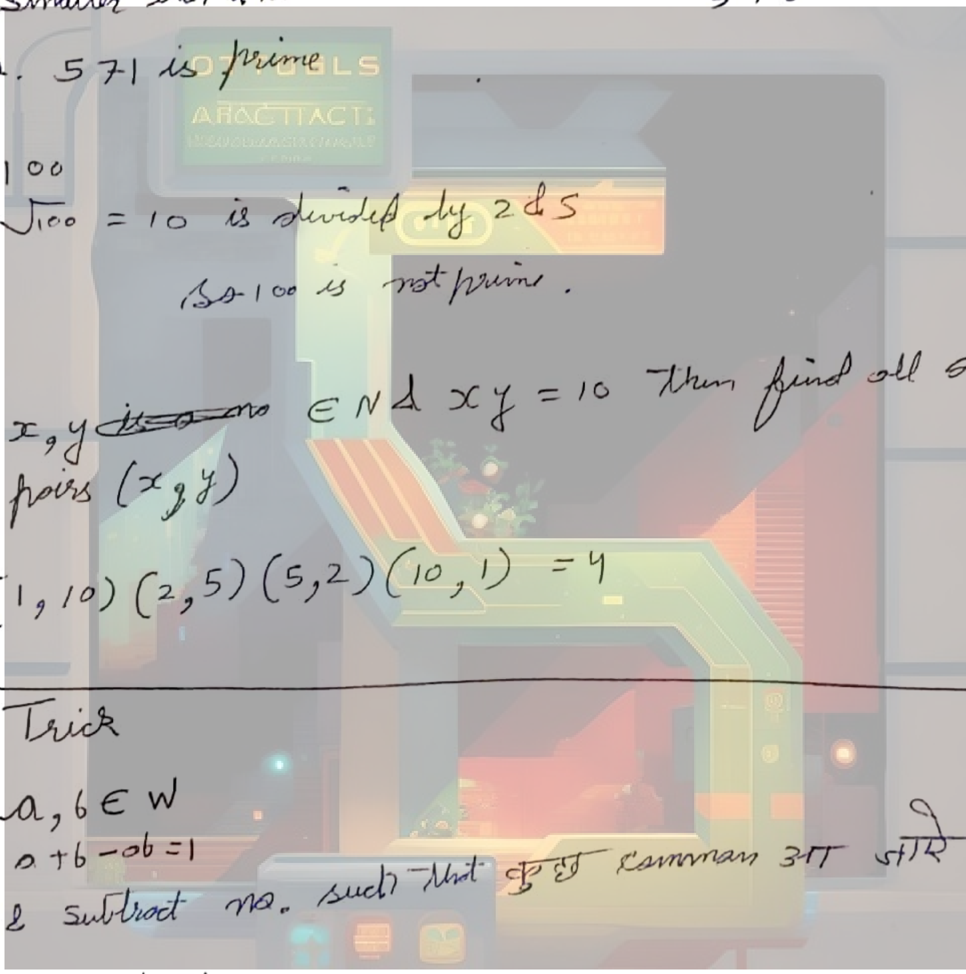
eg 3. 571

$\sqrt{571} = 23.\text{some}$

~~23~~ 23 is not divided by any prime smaller than it

So. 571 is prime.

$$\begin{array}{r} 1 \\ 24 \\ \underline{24} \\ 96 \\ \underline{480} \\ 576 \end{array}$$



eg 4. 100

$\sqrt{100} = 10$ is divided by 2 & 5

So 100 is not prime.

Q If x, y ~~is a no~~ $\in N$ & $xy = 10$ then find all ordered pairs (x, y)

$(1,10)\ (2,5)\ (5,2)\ (10,1) = 4$

★★ Trick

$a, b \in W$

$a + b - ab = 1$

Add & subtract no. such that कुछ तो common आ जाए

eg $a + b - ab = 1$

$a + b - ab - 1 + 1 = 1$

$a - 1 + b - ab + 1 = 1$

$(a-1) + b(1-a) = 0$

$b(1-a) - \cancel{(a-1)}(1-a) = 0$

$(1-a)(b-1) = 0$

$\boxed{\begin{array}{l} a = 1 \\ b = 1 \end{array}}$

$$\text{Q. } 7x + 7y - xy = 49 \quad \text{factorise}$$

$$7x + y(7-x) = 49$$

$$7x + y(7-x) + 49 - 49 = 49$$

$$(7x - 49) + y(7-x) = 0$$

$$7(x-7) + y(7-x) = 0$$

$$y(7-x) - 7(7-x) = 0$$

$$(7-x)(y-7) = 0$$

$$\boxed{\begin{aligned} y &= 7 \\ x &= 7 \end{aligned}}$$

Q find all (x, y) $x, y \in N$ such $xy + 5x = 4y + 38$

$$5x - 4y + xy = 38$$

$$5x + y(-4 + x) = 38$$

$$5x + y(x-4) + 20 - 20 = 38$$

$$y(x-4)\ 5(x-4) = 38 - 20$$

$$(y+5)(x-4) = 18 \longrightarrow$$

$(1, 18)$
$(2, 9)$
$(3, 6)$
$(6, 3)$
$(9, 2)$
$(18, 1)$

$$\boxed{\begin{aligned} &(22, -4)\ (13, -3)\ (10, -2) \\ &(7, 1)\ (6, 4)\ (5, 13) \end{aligned}}$$

$(22, -4)\ (13, -3)\ (10, -2)$ rejected as negative

$(7, 1)$ ✓ $(6, 4)$ ✓ $(5, 13)$ ✓

Note - No. of integers btw a & b $= \boxed{b-a-1}$

eg. Btw 2 & 7 $= 7-2-1$

$= 7-3$

$\boxed{=4}$

Btw 10 & 20 $= 20-10-1$

$= 20-11$

$\boxed{=9}$

btw 11 & 501 $= 501-11-1$

$\boxed{=489}$

Note :- ~~no.~~ no. of integers btw a & b including a & b $= \boxed{b-a+1}$

eg. btw 5 & 10 $= 10-5+1$

$\boxed{=6}$

btw 11 & 40 $= 40-11+1$

$= 40-10$

$\boxed{=30}$

btw 207 & 509 $= 509-207+1$

$= 510-207$

$\boxed{=303}$

Q How many integers in between −200 to 500 (Both inclusive)

a) are multiple of 3 or 5

$500+200+1 = 701$

$= \frac{699}{3}$ $\quad \frac{700}{5}$

$= 233 + 140 + 1$ (zero)

$= \boxed{378} - 47$

$\boxed{= 327}$

b) are multiple of 3 or 5 but not 15.

$373 - \frac{700}{15} + 1(\text{zero})$

$373 - 47$

$373 - 47$

$\boxed{327} - 47$

$\boxed{280}$

~~15)700(~~

H.W. 19-04-2024

DYS-5 (Q1, 2, 3, 4, 5)

Q-1
(Q24)
Q-4
(Q3)

DYS-5

Q1 6

Q2. i) (2, 2)

ii) 3

Q3. 3

Q4. 651

Q5. 293

Q-1

Q24. c)

Q-4

Q3. 7

~~166 166 + 1~~

~~2 3~~ 3

Q How many integers in between -200 to 500 (both inclusive)

a) divisible by 3 or 5

b) divisible by 3 or 5 but not 15.

$$\frac{-200}{3} \leq x \leq \frac{500}{3}$$

$$-66.66 \leqslant x \leqslant 166.6$$

$$-66 \leqslant x \leqslant 166$$

$$166-(-66)+1 = 233 \text{ nos}$$

$$\frac{-200}{5} \leq x \leq \frac{500}{5}$$

$$-40 \leq x \leq 100$$

$$100+40+1 = 141$$

$$\frac{-200}{15} \leq x \leq \frac{500}{15}$$

$$-13 \leq x \leq 33$$

$$33+13+1 = 47$$

a) 3 or 5

$$3 \cup 5 = n(3)+n(5)-n(3 \cap 5)$$

$$= 233+141-47$$

$$= 374-47$$

$$\boxed{=327}$$

b) $$3 \Delta 5 = n(3)+n(5)-2(n\, 3 \cap 5)$$

$$= 327-47$$

$$\boxed{=280}$$

Q which is greater?

① $\frac{8}{9}$ or $\frac{7}{8}$

$\frac{8}{9} > \frac{7}{8}$

② $\sqrt{13} - \sqrt{12}$ or $\sqrt{14} - \sqrt{13}$

$13 + 12 - 2\sqrt{13} \times \sqrt{12}$ $\quad$ $14 + 13 - 2\sqrt{14} \times \sqrt{13}$

$25 - 2\sqrt{13 \times 12}$ $\quad$ $27 - 2\sqrt{14} \times \sqrt{13}$

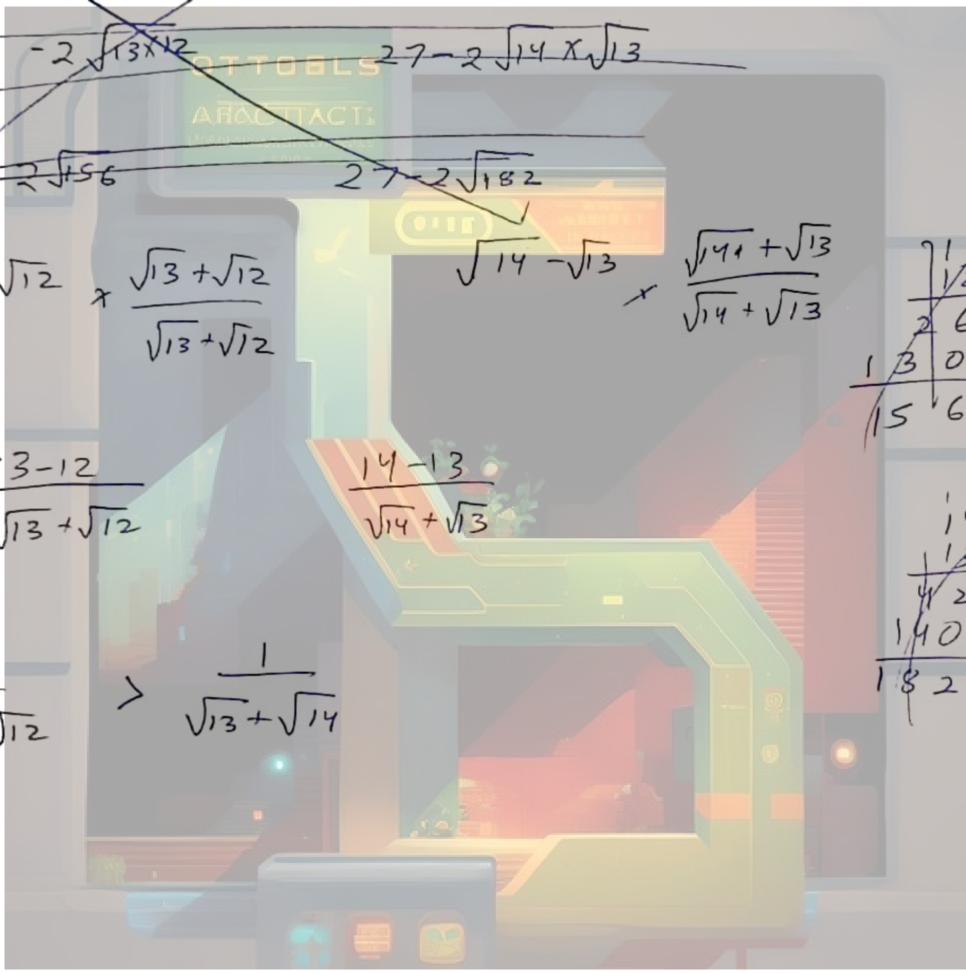
$25 - 2\sqrt{156}$ $\quad$ $27 - 2\sqrt{182}$

$\sqrt{13} - \sqrt{12} \times \frac{\sqrt{13} + \sqrt{12}}{\sqrt{13} + \sqrt{12}}$ $\quad$ $\sqrt{14} - \sqrt{13} \times \frac{\sqrt{14} + \sqrt{13}}{\sqrt{14} + \sqrt{13}}$

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ 130 \\ \hline 156 \end{array} \qquad \begin{array}{r} 14 \\ \times 13 \\ \hline 42 \\ 140 \\ \hline 182 \end{array}$$

$\frac{13 - 12}{\sqrt{13} + \sqrt{12}}$ $\quad$ $\frac{14 - 13}{\sqrt{14} + \sqrt{13}}$

$\frac{1}{\sqrt{13} + \sqrt{12}} > \frac{1}{\sqrt{13} + \sqrt{14}}$



Divisiblity

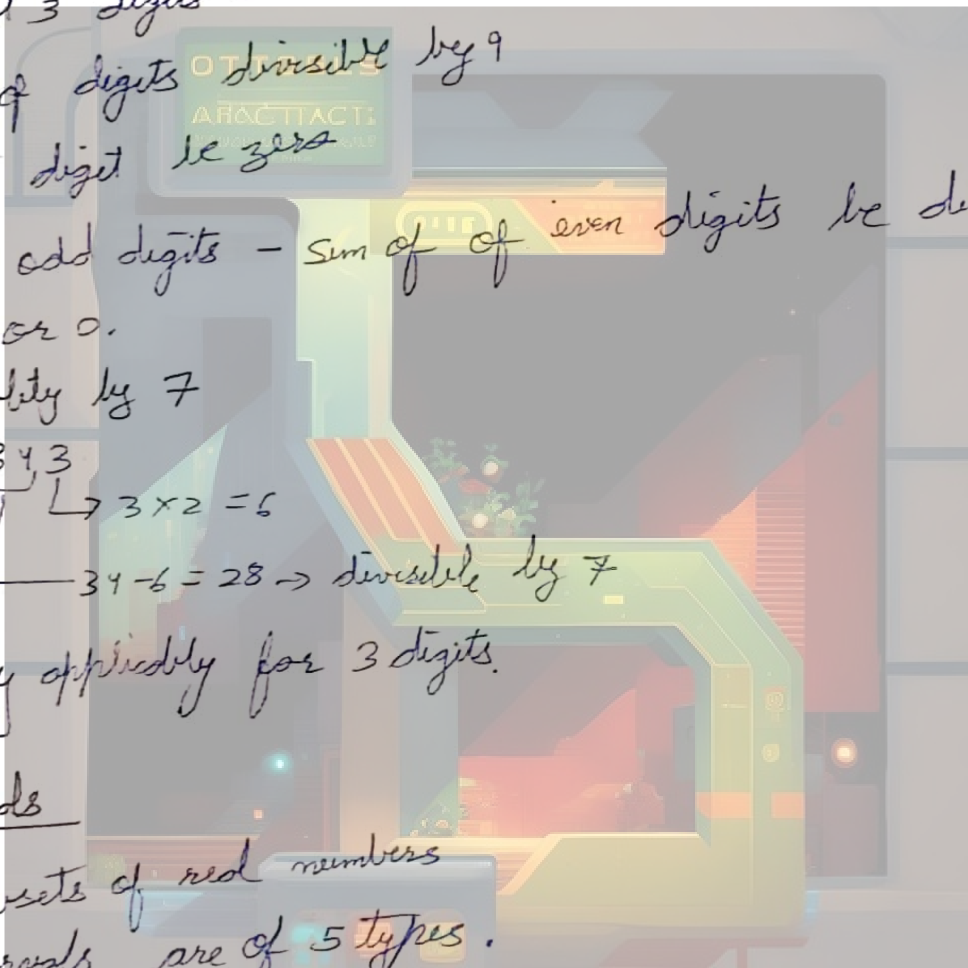
2 → Last digit is even no.

3 → Sum of digits is divisible by 3

4 → last 2 digits divisible by 4

5 → last digit be 0 or 5

6 → divisible by 2 & 3



8 → last 3 digits should be divisible by 8

9 → Sum of digits divisible by 9

10 → last digit be zero

11 → Sum of odd digits − Sum of of even digits be divisible by 11 or 0.

Divisiblity by 7

7 → 343

$$34\,\underline{3} \to 3 \times 2 = 6$$

$$34 - 6 = 28 \to \text{divisible by } 7$$

Only applicable for 3 digits.

Intervals

→ Subsets of real numbers

→ Intervals are of 5 types.

Interval	Notation $(B > A)$	Definition
1. Closed Interval	$[a, b]$ or $a \le x \le b$	All real ~~nount~~ numbers between a & b including a and b.
2. Open Interval	(a, b) or $a < x < b$	All real numbers between a & b excluding a and b.
3. Open-closed Interval	$(a, b]$ or $a < x \le b$	All real numbers between a & b including b but not a.
4. Closed-open Interval	$[a, b)$ or $a \le x < b$	All real numbers between a & b including a but not b.
5. Curly interval	$\{a, b\}$ or	Only a and b.

Q1. Match the column

(A) $[-3, 6]$	(P) 7.1
(B) $(5, 7)$	(Q) 0.9
(C) $(-9, -3)$	(R) -5
(D) $[0, 8]$	(S) 5
(E) $[6, 9]$	(T) 6.1
	(U) 4.3

(A) – Q S U
(B) – ~~Q~~ T
(C) – R
(D) – P Q S T U
(E) – P T

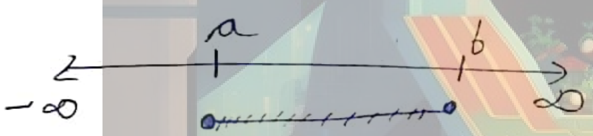
Infinite Intervals - It is of 4 types

① $[a, \infty)$	All real nos. from a to infinite
② (a, ∞)	All real nos. from a to ∞ without a
③ $[-\infty, b]$	All real nos. from $-\infty$ to b including b
④ $(-\infty, b)$	All real nos. from $-\infty$ to b excluding b

Representation of Intervals in graphical form.

① Closed $(b > a)$

$[a, b]$

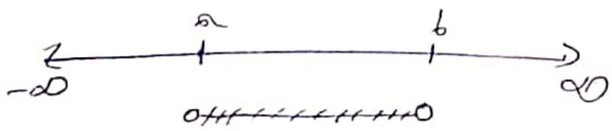


• - Include

o - Exclude

② Open Interval $(b > a)$

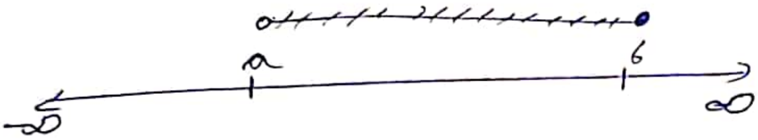
(a, b)



Scanned with CamScanner

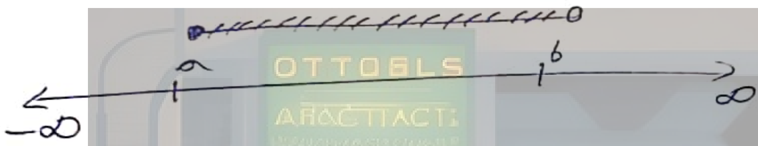
③ Open-closed Interval

(a, b]



④ Closed-open Interval

[a, b)

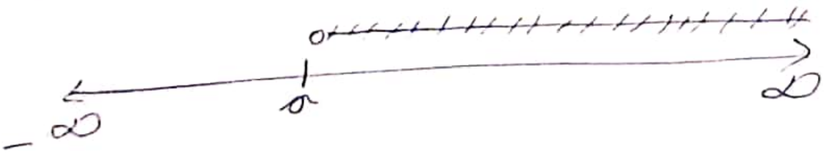


⑤ Curly Interval

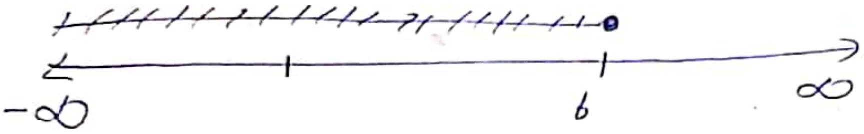
{a, b}

⑥ [a, ∞)

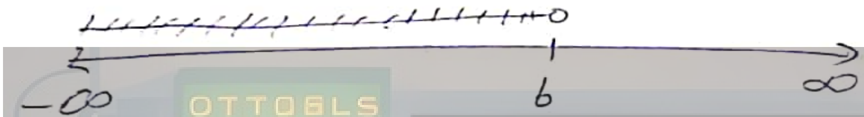
⑦ (a, ∞)



(8) $[-\infty, 6]$

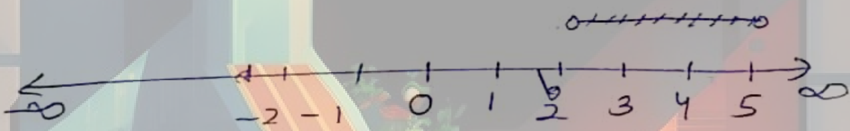


(9) $(-\infty, 6)$

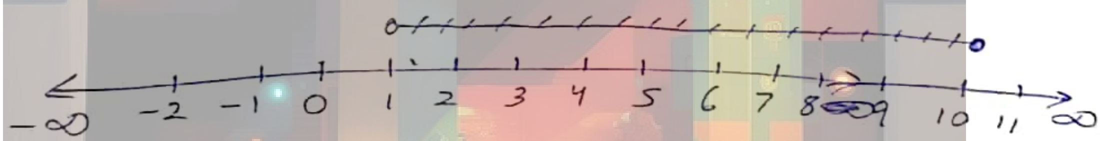


Q1. Represent the ~~sets~~ intervals on number line:

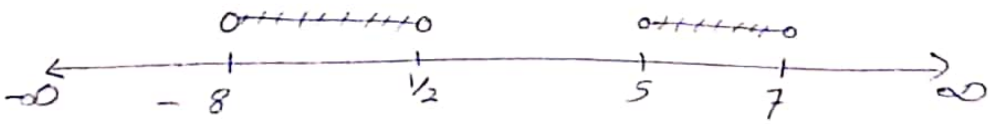
(1) $(2, 5)$



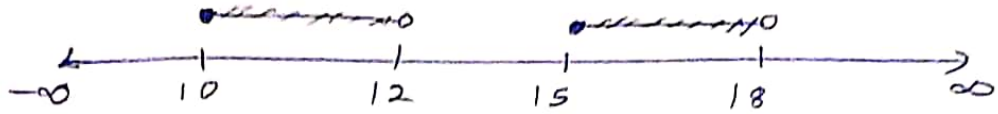
(2) $[1, 10]$



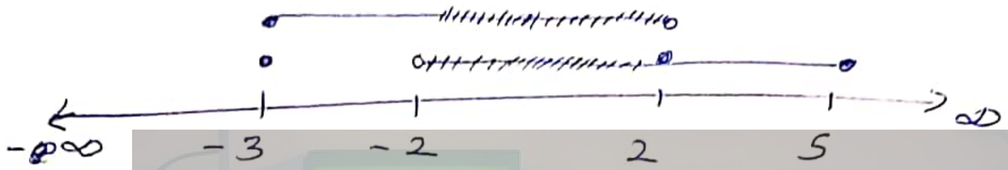
(3) $(-8, \frac{1}{2}) \cup (5, 7)$



④ $[10, 12) \cup [15, 18)$

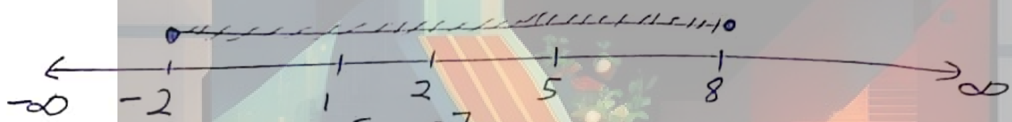


⑤ $(-2, 5] \cap [-3, 2)$



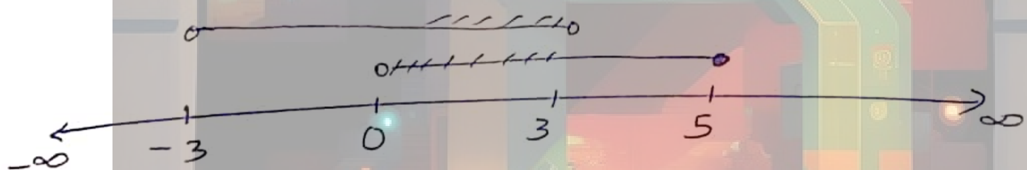
$(-2, 2)$

⑥ $[-2, 5] \cup (1, 2] \cup [5, 8]$



$[-2, 8]$

⑦ $(0, 5] \cap (-3, 3)$



$(0, 3)$

⑧ $\left((-\infty, 5) \cup [4, 10]\right) - (-10, 10)$

$(-\infty, 10] - (-10, 10)$

$(-\infty, -10] \cup \{10\}$

H.W. 20-4-2024

Q1. Represent [1, 4] or [-4, 1) on number line.

Q2. (-8, -1] ∪ [-3, 10] find.

1. Total integers
2. Total (+)ve integers divisible by 3
3. Total (−)ve integers
4. Total real nos.
5. Total prime nos.
6. Total Irrational nos.

Q3. If $x \in [1, 4)$, which of the following lies in it.

(A) 2
(B) $4.1 - 0.\bar{9}$
(C) $4.\bar{1} - 0.0\bar{9}$
(D) $\sqrt{4 - 4\pi + \pi^2}$

DYS-1
Q13 - 3
DYS-2
Q4 4
Q5. A-S, B-P, C-Q, D-P

DYS-1 (Q13)
DYS-2 (Q4, Q5)
DYS-5 (Q6-Q8)
DYS-6 (Q1-10)
O-1
(Q8, 9, 10, ~~3~~1, 23)
O-2
(Q3, 5, 6)
O-3
(Q7)

DYS-5
Q6. - 4
Q7 - 30
Q8 - 3

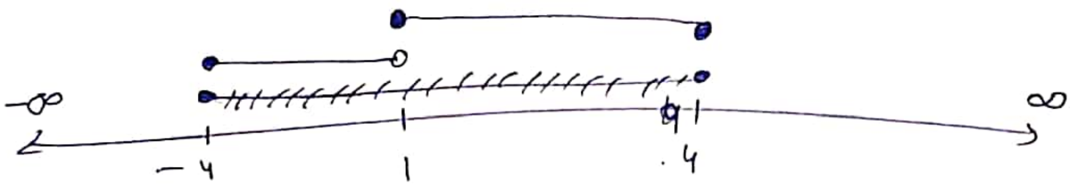
~~DYS~~ O-1
Q8 - C
Q9 - B
Q10 - A
Q23 B

O-2
Q3 AD
Q5 CD
Q6. AB

~~DYS-6~~
O-3
Q7 - A-Q
B-R
C-S
D-P

60

Q1. $[1, 4] \cup [-4, 1)$



Q2. $(-8, -1] \cup [-3, 10]$

~~$[-7$~~

$(-8, 10]$

1. Total integers $= 10 - (-8)$
$\boxed{= 18}$ ✓

2. Total Integers divisible by 3
$-\frac{8}{3} \leq x \leq \frac{10}{3}$
$3 + 3 = \boxed{6}$ ✓

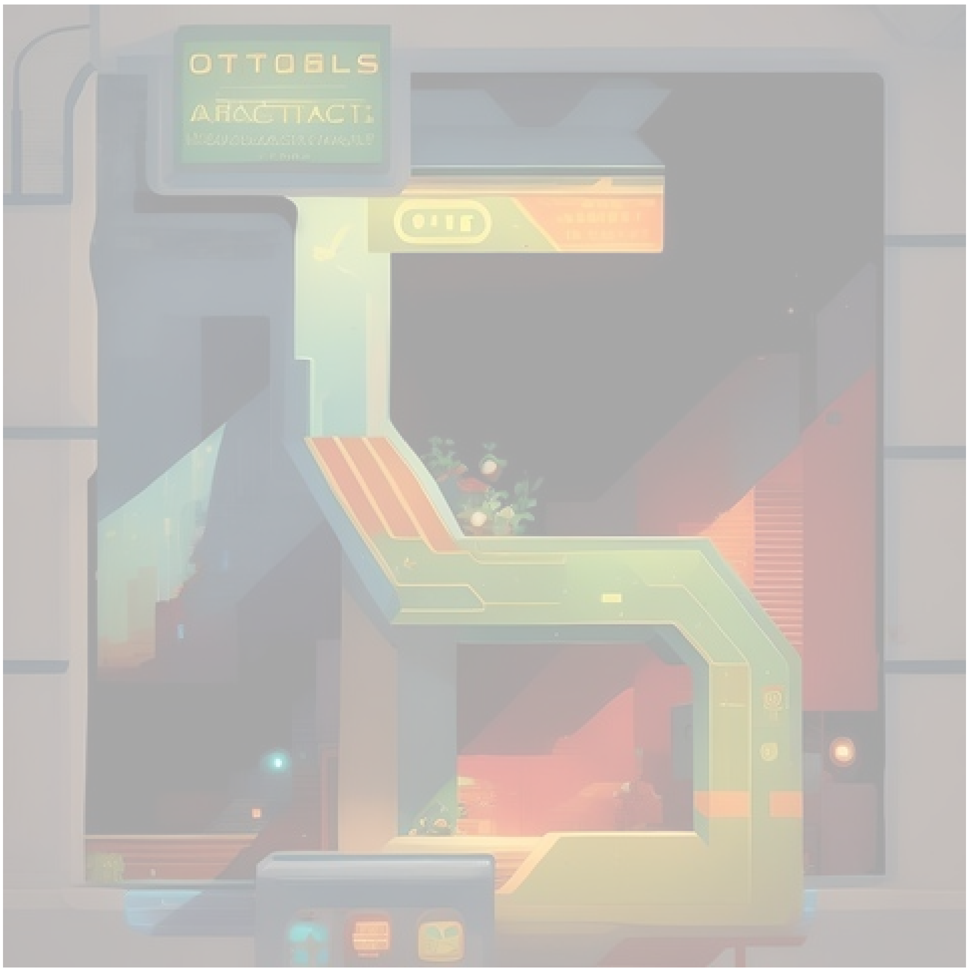
3. $0 - (-8) - 1$
$= 8 - 1$
$\boxed{= 7}$

4. Infinite ✓
5. 4 ✓
6. Infinite ✓

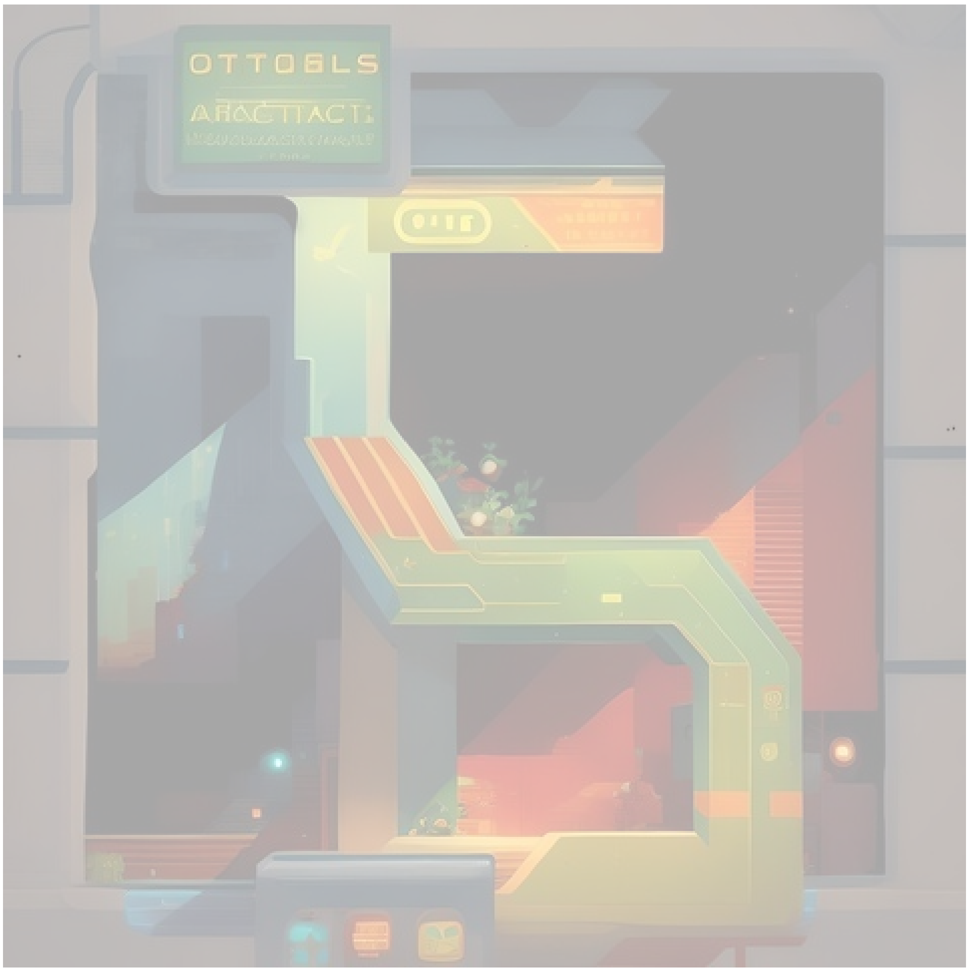
Q3. A B B

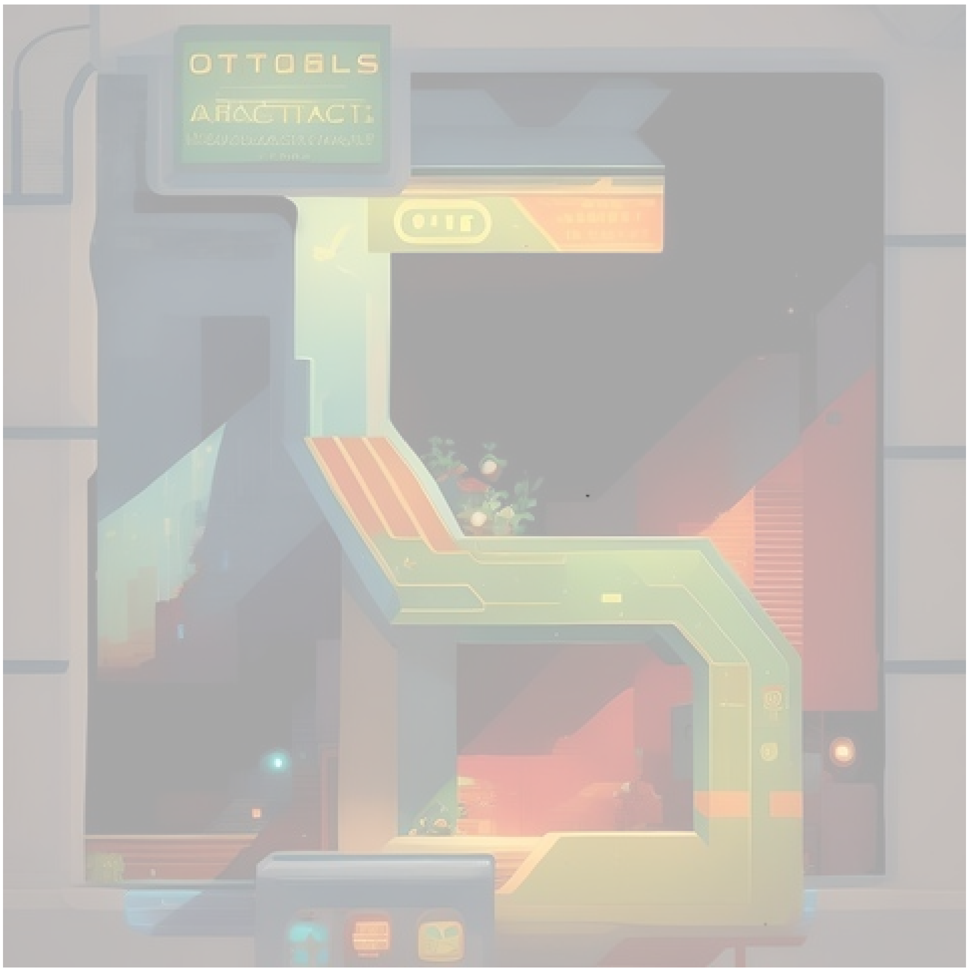
IYS-24

~~Q13~~



(62)

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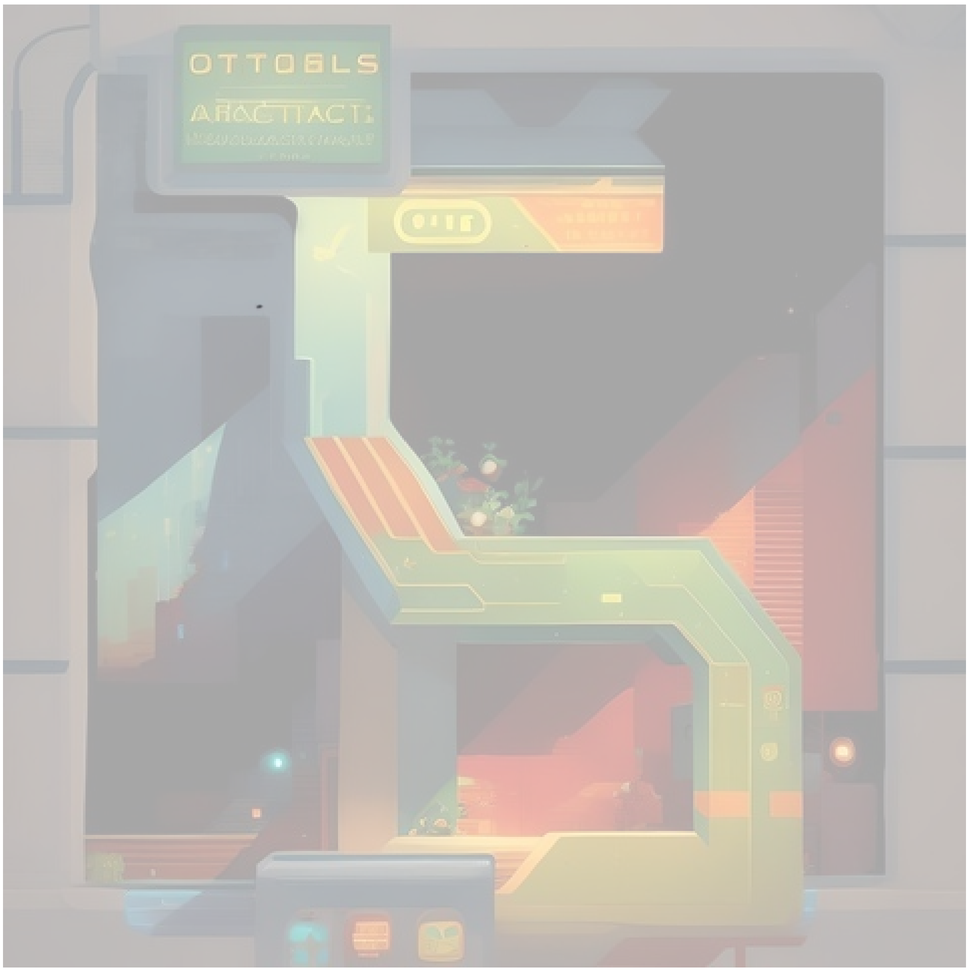
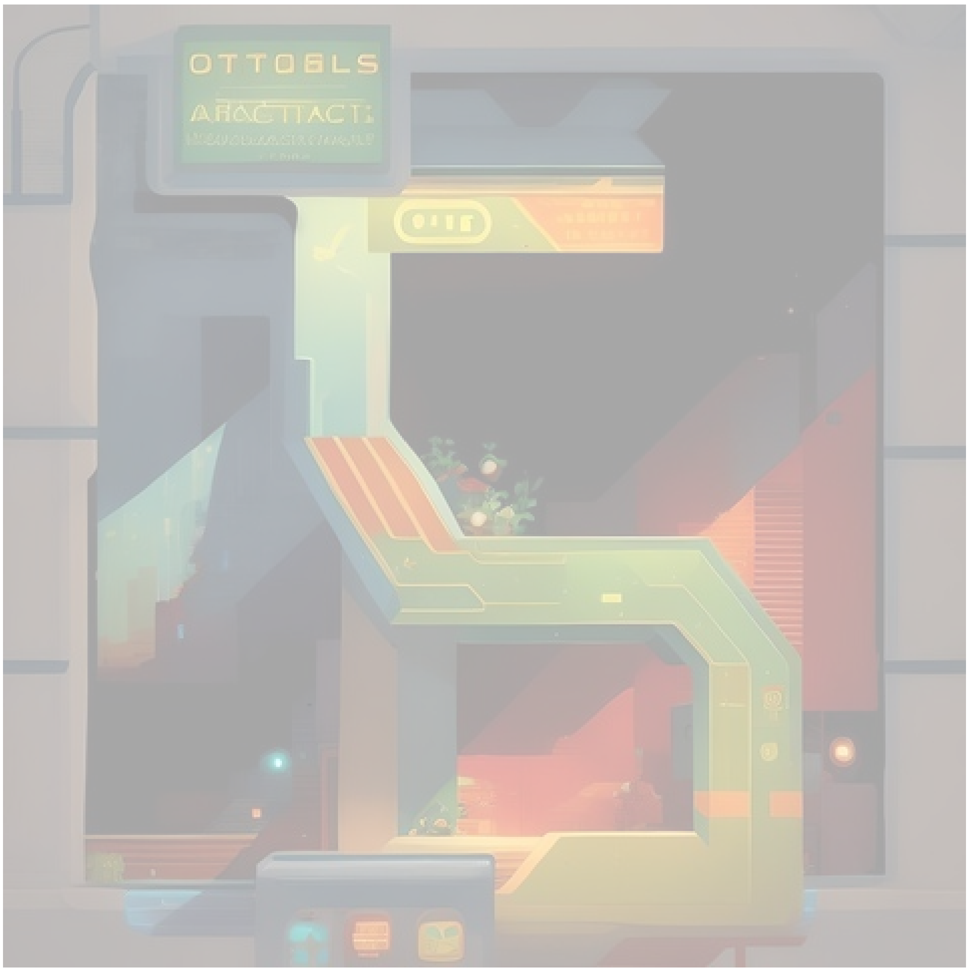




Fundamentals of Algebra

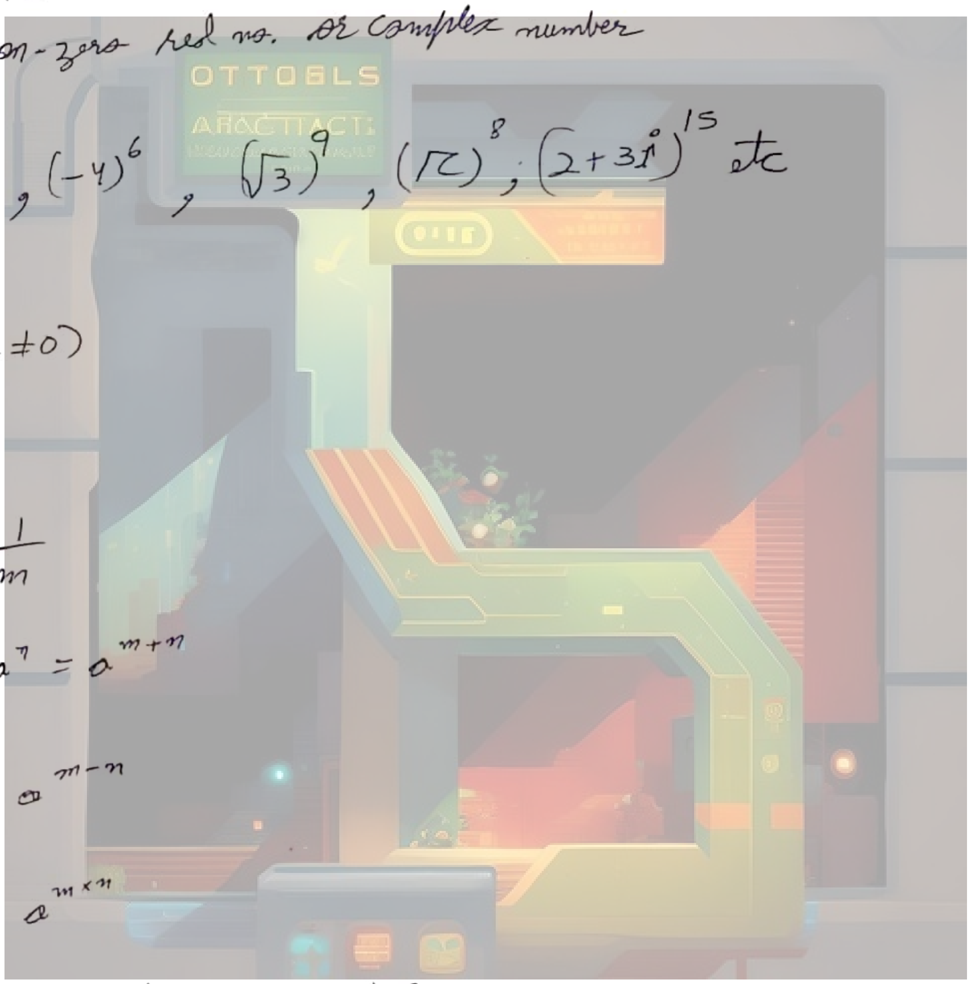
~~Indices/Powers~~

Indices & Surds (Powers & Exponents).

a^m — m → Exponent/Power; a → Base

m → ~~Real no.~~ Integers

a → non-zero real no. or complex number



eg. $2^4, (-4)^6, (\sqrt{3})^9, (\pi)^8, (2+3i)^{15}$ etc

Laws:- $(a \neq 0)$

(1) $a^0 = 1$

(2) $a^{-m} = \frac{1}{m}$

(3) $a^m \times a^n = a^{m+n}$

(4) $\frac{a^m}{a^n} = a^{m-n}$

(5) $(a^m)^n = a^{m \times n}$

(6) $\sqrt[c]{a^b} = a^{b/c}, c \in N, c \geq 2$

Q1. find values

① $\left(\left(256^{-1/2}\right)^{-1/4}\right)^{2}$

$\frac{-1}{\cancel{2}} \times -\frac{1}{4} \times \cancel{2} = +\frac{1}{4}$

$\sqrt{\sqrt{256}} = \sqrt{16} = \boxed{4}$

② $\left(125^{1/3} + 64^{1/3}\right)^{3}$

$\left(\sqrt[3]{125} + \sqrt[3]{64}\right)^{3}$

$(5+4)^{3} = 9^{3} = \cancel{512} = \boxed{729}$

81
$\times 9$
729

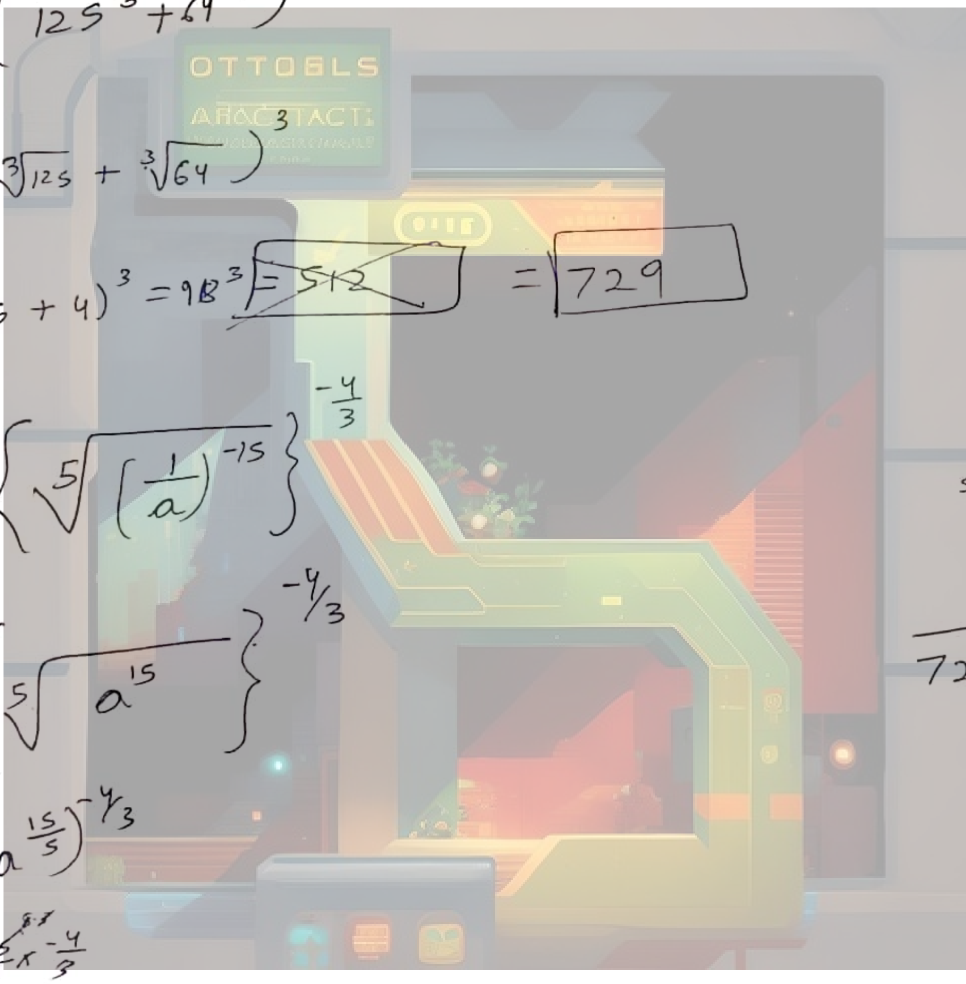
③ $\left\{\sqrt[5]{\left(\frac{1}{a}\right)^{-15}}\right\}^{-4/3}$

$\left\{\sqrt[5]{a^{15}}\right\}^{-4/3}$

$\left(a^{\frac{15}{5}}\right)^{-4/3}$

$a^{\frac{15}{5} \times -\frac{4}{3}}$

$\boxed{a^{-4}}$



4) $$\frac{\sqrt{x^3}\times\sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}}$$

$$\cancel{\frac{x^{\frac{1}{3}}\times x^{\frac{5}{3}}}{x^{\frac{3}{5}}} \times x^{\frac{77}{30}}}$$

$$\cancel{x^{\frac{1}{3}+\frac{5}{3}+\frac{77}{30}-\frac{3}{5}}}$$

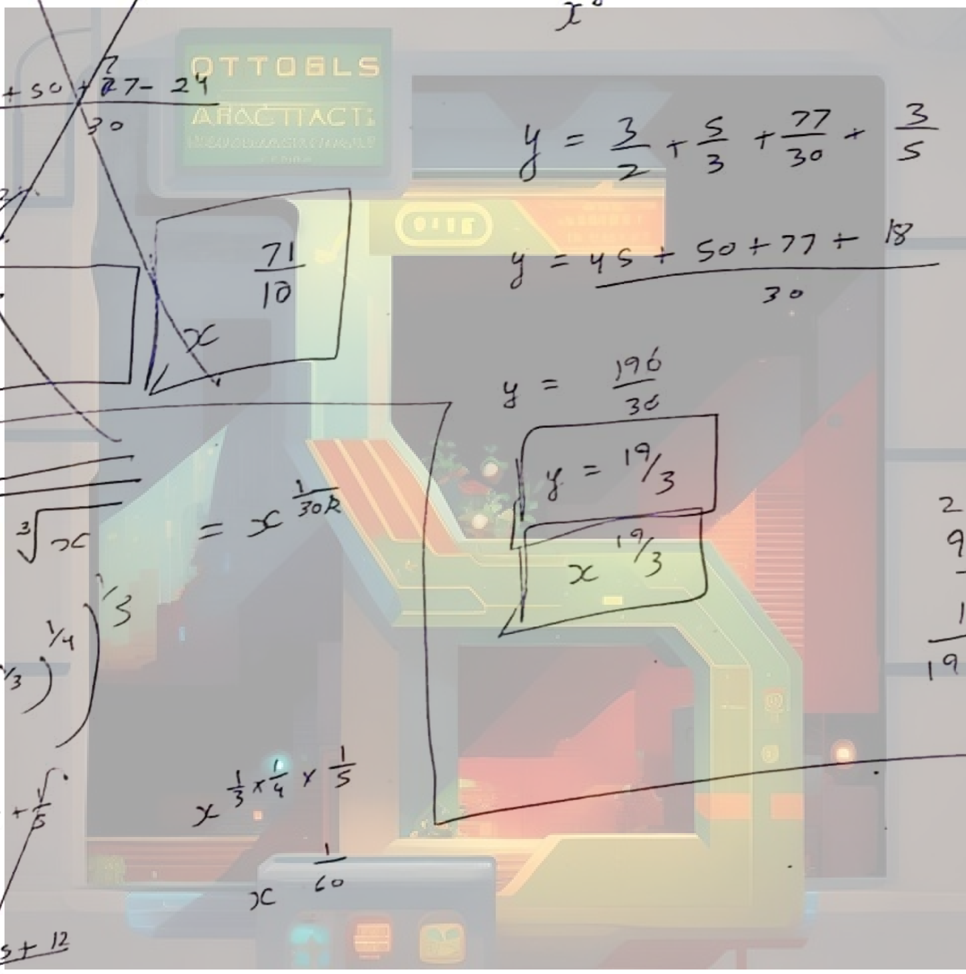
$$\cancel{x^{\frac{10+50+77-24}{30}}}$$

$$\cancel{\sqrt[10]{x^{21}}} \qquad \cancel{\frac{71}{10}}$$

$$\frac{x^{3/2}\times x^{5/3}}{x^{3/5}} \times x^{\frac{77}{30}}$$

$$x^{\frac{3}{2}+\frac{5}{3}+\frac{77}{30}+\frac{3}{5}}$$

$$x^y$$



$$y = \frac{3}{2}+\frac{5}{3}+\frac{77}{30}+\frac{3}{5}$$

$$y = \frac{45+50+77+18}{30}$$

$$y = \frac{190}{30}$$

$$\boxed{y = 19/3}$$

$$\boxed{x^{19/3}}$$

2
95
77
18
190

5) $$\sqrt[5]{\sqrt[4]{\sqrt[3]{x}}} = x^{\frac{1}{30k}}$$

$$\left(\left(x^{1/3}\right)^{1/4}\right)^{1/5}$$

$$\cancel{x^{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}} \qquad x^{\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}}$$

$$x^{\frac{1}{60}}$$

$$\cancel{x^{\frac{20+15+12}{60}}}$$

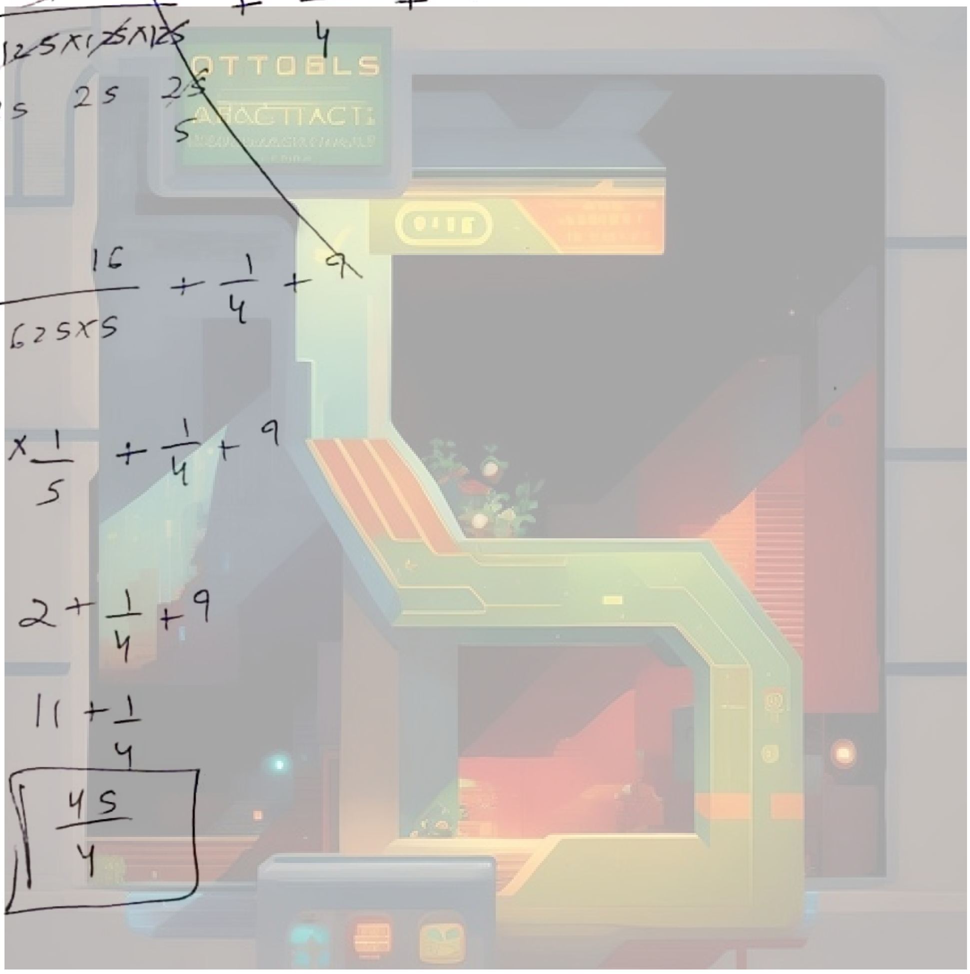
$$\cancel{x^{\frac{47}{60}} = x^{\frac{1}{30k}}} \qquad \frac{1}{60} = \frac{1}{30k}$$

$$\cancel{\frac{47}{60} = \frac{1}{30k}} \qquad \boxed{k = 2}$$

$$\cancel{\boxed{k = \frac{47}{2}}}$$

Q2. $\left(\frac{1}{100}\right)^{1/2} \cdot (125)^{1/3} + (64)^{4/3} \cdot (4)^{-2} + \left((729)^{-\frac{1}{9}}\right)^{-3}$

$$10000 \times \frac{1}{125} \times \frac{1}{125} \times \frac{1}{125} + 4 \times \frac{1}{16} + (729)^{1/3}$$



$$\frac{\cancel{10000}^{\cancel{2000}^{\cancel{400}^{\cancel{80}^{\cancel{40}\ 16}}}}}{\cancel{125}_{25} \times \cancel{125}_{25} \times \cancel{125}_{\cancel{25}\ 5}} + \frac{1}{4} + 9$$

$$\frac{16}{625 \times 5} + \frac{1}{4} + 9$$

$$10 \times \frac{1}{5} + \frac{1}{4} + 9$$

$$2 + \frac{1}{4} + 9$$

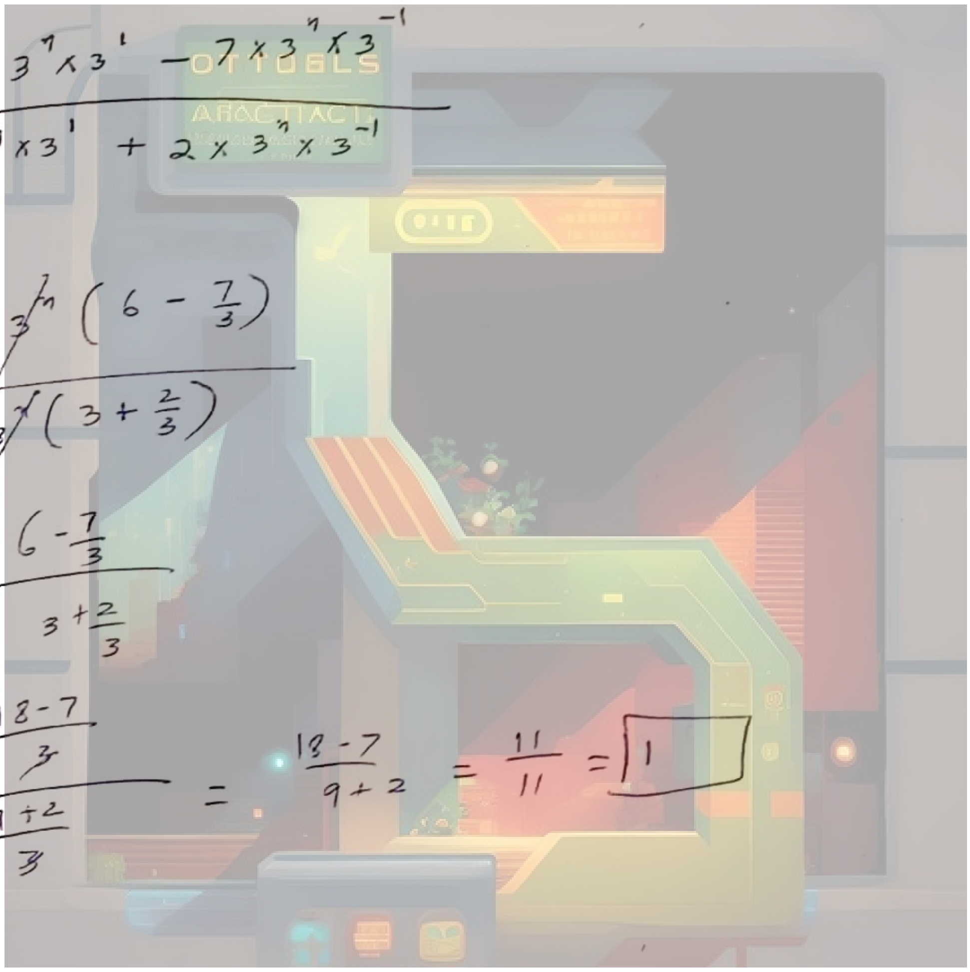
$$11 + \frac{1}{4}$$

$$\boxed{\frac{45}{4}}$$

(70)

Q3. $\cancel{\dfrac{(2\cdot3)^{n+1} - (7\cdot3)^{n-1}}{3^{n+1} + 2\left(\frac{1}{3}\right)^{1-n}}}$

$$\frac{2\times(3)^{(n+1)} - (7\times3)^{(n-1)}}{3^{n+1} + 2\times 3^{(n-1)}}$$



$$\frac{2\times 3^{n}\times 3^{1} - 7\times 3^{n}\times 3^{-1}}{3^{n}\times 3^{1} + 2\times 3^{n}\times 3^{-1}}$$

$$\frac{\cancel{3^{n}}\left(6-\frac{7}{3}\right)}{\cancel{3^{n}}\left(3+\frac{2}{3}\right)}$$

$$\frac{6-\frac{7}{3}}{3+\frac{2}{3}}$$

$$\frac{\frac{18-7}{\cancel{3}}}{\frac{9+2}{\cancel{3}}} = \frac{18-7}{9+2} = \frac{11}{11} = \boxed{1}$$

Q4. $a^x = b$

$b^y = c$

$c^z = a$

prove $xyz = 1$

$a^{xy} = c^1$

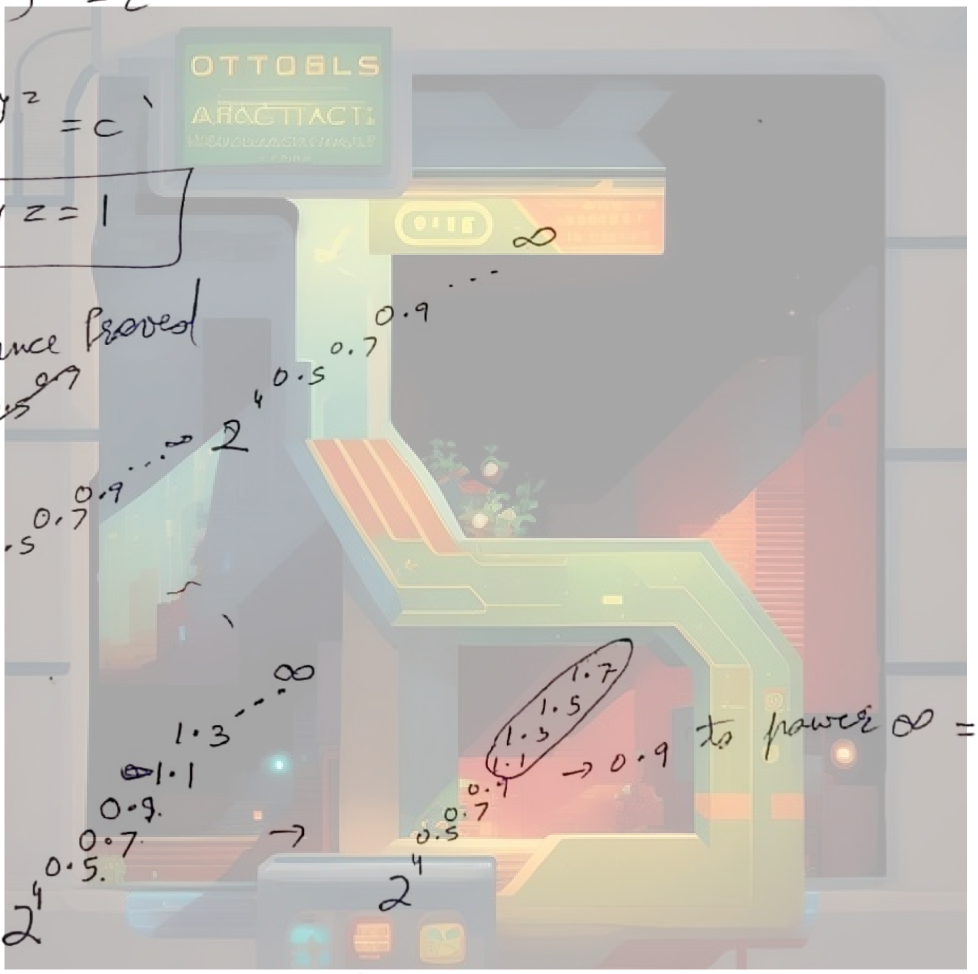
$a = c^z$

$(c^z)^{xy} = c^1$

$c^{xyz} = c^1$

$\boxed{xyz = 1}$

Hence Proved



Q5. $\cancel{2^{4^{0.5^{0.7}}}} \; 2^{4^{0.5^{0.7^{0.9^{\cdots\infty}}}}}$

$16^{0.5^{0.7^{0.9^{\cdots}}}}$

$2^{4^{0.5^{0.7^{0.9^{1.1^{1.3^{\cdots\infty}}}}}}} \to 2^{4^{0.5^{0.7^{0.9^{1.1^{1.3^{1.5^{1.7}}}}}}}} \to 0.9$ to power $\infty = 0$

$2^{4^{0.5^{0.7^{0}}}} \to 2^{4^{0.5^{1}}} \to 16^{1/2} \; \boxed{\to \underline{4}}$

$$(1)^{\infty} = 1$$

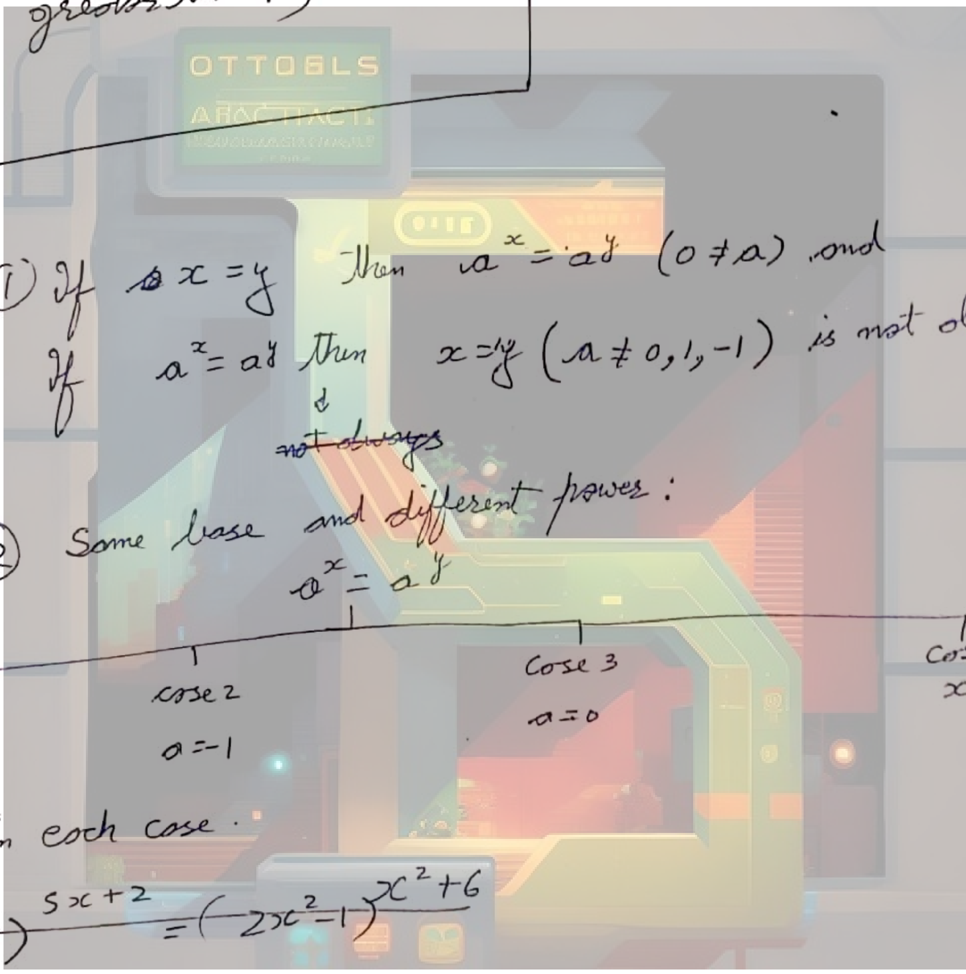
$$1^{0} = 1$$

$$(\text{value btw } 0 \,\&\, 1)^{\cancel{\infty}\,0} = 1$$

$$(\text{value btw } 0 \,\&\, 1)^{\infty} = 0$$

$$(\text{value greater than } 1)^{0} = 1$$

$$(\text{value greater than } 1)^{\infty} = \infty$$



Note:- (1) If $x = y$ then $a^x = a^y$ $(0 \neq a)$ and

If $a^x = a^y$ then $x = y$ $(a \neq 0, 1, -1)$ is not always true.

~~not always~~

(2) Same base and different power:

$$a^x = a^y$$

Case 1	Case 2	Case 3	Case 4
$a = 1$	$a = -1$	$a = 0$	$x = y$

$\Rightarrow$ verify in each case.

$$\cancel{(2x^2-1)^{5x+2}} = (2x^2-1)^{x^2+6}$$

$$\cancel{x^2 + 6 = 5x + 2}$$

Q $\quad (2x^2-1)^{5x+2} = (2x^2-1)^{x^2+6}$

Sol $\quad$ case-1 $\rightarrow a = 1$

$2x^2 - 1 = 1$

$2x^2 = 2$

$x^2 = 1$

$x = \pm 1$

verify $\rightarrow x = +1 \rightarrow (2-1)^{5(1)+2} = (2-1)^{(1)^2+6}$

$1^7 = 1^7$ ✓

$x = -1 \rightarrow (2-1)^{-3} = (2-1)^7$

$1^{-3} = 1^7$

$\frac{1}{1^3} = 1^7$

$1 = 1$ ✓

case 2 $\rightarrow a = 0$

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

verify $\rightarrow x = +\frac{1}{\sqrt{2}} \rightarrow (2 \times \frac{1}{2} - 1)^{\frac{5}{\sqrt{2}}+2} = (2 \times \frac{1}{2} - 1)^{\frac{1}{2}+6}$

$0^{+ve} = 0^{+ve}$ ✓

$x = -\frac{1}{\sqrt{2}} \rightarrow (2 \times \frac{1}{2} - 1)^{-\frac{5}{\sqrt{2}}+2} = (2 \times \frac{1}{2} - 1)^{\frac{1}{2}+6}$

$0^{-ve} = 0^{+ve}$ ✗ (0^{-ve} is not defined)

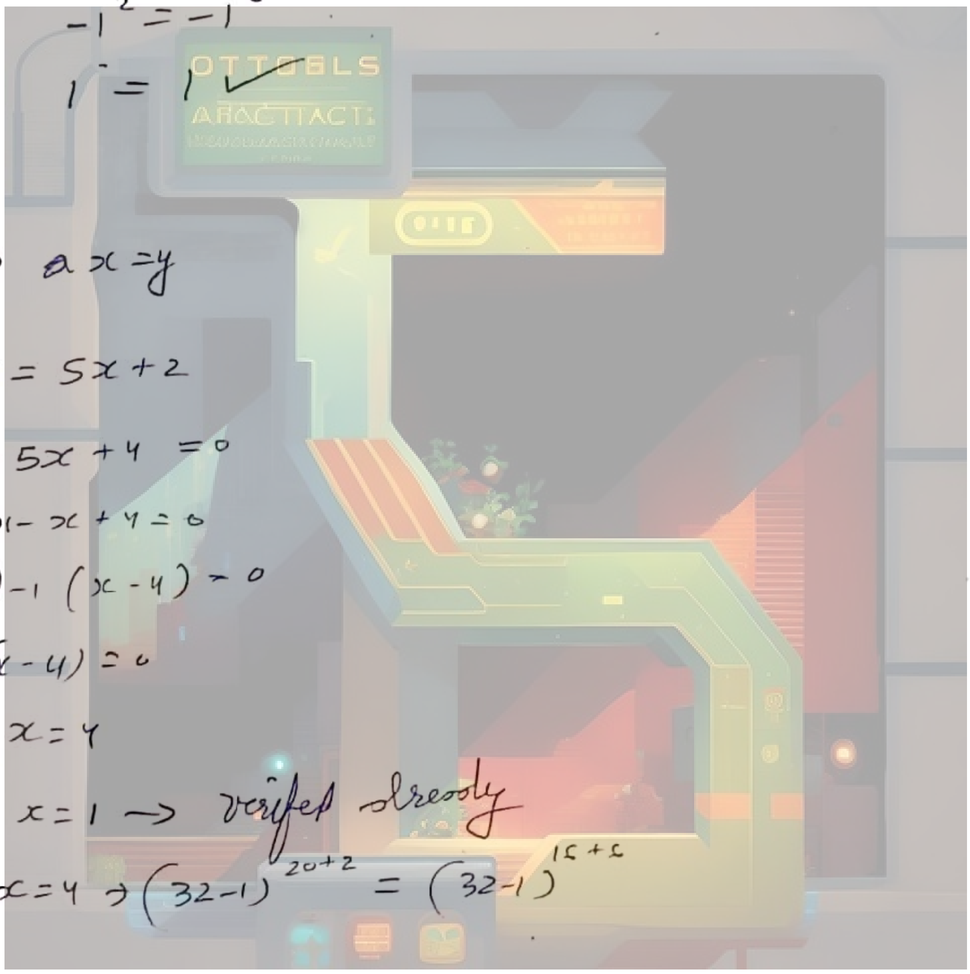
Case-3 → $a = -1$

$$2x^2 \cancel{-1} = \cancel{-1}$$

$$2x^2 = 0$$
$$x^2 = 0$$
$$x = 0$$

verify:- $(0-1)^{0+2} = (0-1)^{0+6}$

$$-1^2 = -1^6$$
$$1 = 1 \checkmark$$



Case 4 → $\cancel{a}\, x = y$

$$x^2+6 = 5x+2$$
$$x^2 - 5x + 4 = 0$$
$$x^2 - 4x - x + 4 = 0$$
$$x(x-4) - 1(x-4) = 0$$
$$(x-1)(x-4) = 0$$
$$x = 1, \quad x = 4$$

verify :- $x = 1$ → verified already

$x = 4 \rightarrow (32-1)^{20+2} = (32-1)^{16+6}$

$$\rightarrow 31^{22} = 31^{22} \checkmark$$

Final answer — $\boxed{x = 1, -1, 0, \frac{1}{\sqrt{2}}, 4 \text{ Answer.}}$

Note:-

③ Same Power and Different Base

$a^x = b^x$

Case-1

a & $b = 1$

or

$a = b$

Case 2

~~a & b = 0~~

Power $= 0$

Case 3

a & $b = -1$

or

$a = -b$

Q find x, $(x+2)^{(x-3)} = (2x-5)^{(x-3)}$

case 1 → $a = b$

$x + 2 = 2x - 5$

$7 = x$

verify

$(7+2)^{(7-3)} = (14-5)^{(7-3)}$

$9^4 = 9^4$ ✓

case 2 → $0(x-3) = 0$

$x - 3 = 0$

$x = 3$

verify:- $(3+5)^{(3-3)} = (6-5)^{(3-3)}$

$8^0 = 1^0$

$1 = 1$ ✓

Case 3 → $a = -6$

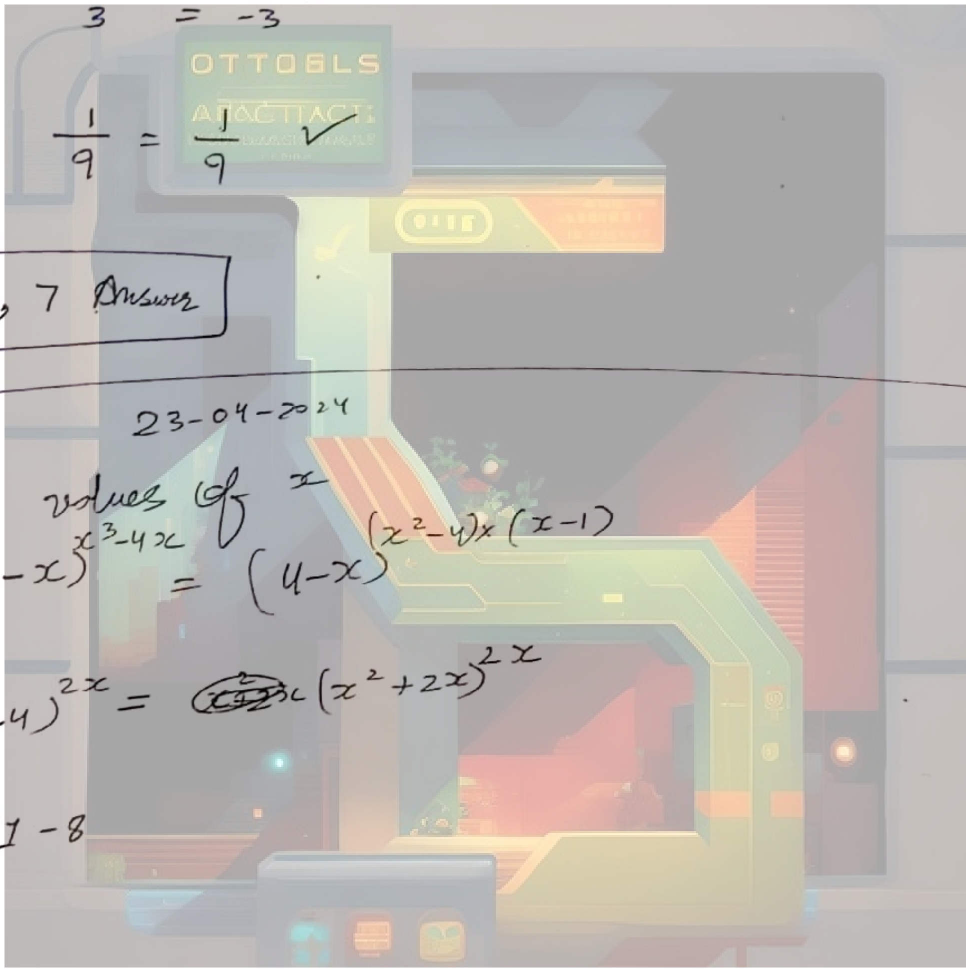
$x + 2 = 5 - 2x$

$3x = 3$

$\underline{x = 1}$

verify :-

$(1+2)^{(1-3)} = (2-5)^{(1-3)}$



$3^{-2} = -3^{-2}$

$\frac{1}{9} = \frac{1}{9}$ ✓

$\boxed{x = 1, 3, 7 \text{ Answer}}$

H.W. 23-04-2024

Q1. Find the values of x

(1) $(4-x)^{x^3-4x} = (4-x)^{(x^2-4)x(x-1)}$

(2) $(x^2-4)^{2x} = (x^2+2x)^{2x}$

Race 1-8

9

Surds

→ Any root of a rational number which cannot be exactly found as a whole or rational number.

$\sqrt[n]{a}$ (a is a rational number)

$n \rightarrow$ degree of surd

eg $\sqrt{2}$, $7^{3/7}$, $(+3)^{4/15}$ → 15 degree

$\sqrt{2}$ ↓ 2 degree, $7^{3/7}$ → 7 degree

→ If a is not rational then $\sqrt[n]{a}$ is not a surd

eg $\sqrt{2+\sqrt{3}}$ is not a surd

→ 1 term → simple surd
2 term → Binomial surd
4 term → Bi Quadratic surd.

→ Two surds which differ ~~differ~~ ~~of~~ only in sign which connects their terms are conjugate or complimentary to each other.

eg. $2\sqrt{7}+5\sqrt{3}$
↓
conjugate
↓
$-2\sqrt{7}+5\sqrt{3}$ Or $2\sqrt{7}-5\sqrt{3}$

→ Product of a surd and its conjugate is rational or irrational.

eg-1. $\sqrt{2}+\sqrt{3} \longrightarrow \sqrt{2}-\sqrt{3}$

$(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$

$(\sqrt{2})^2-(\sqrt{3})^2$

$2-3=1$ (rational)   (if degree = 2, rational)

eg 2. $\sqrt[3]{2}+\sqrt[3]{3} \rightarrow \sqrt[3]{2}-\sqrt[3]{3}$

$(\sqrt[3]{2}+\sqrt[3]{3})(\sqrt[3]{2}-\sqrt[3]{3})$

$(\sqrt[3]{2})^2-(\sqrt[3]{3})^2$

$2^{2/3}-3^{2/3} \rightarrow$ (irrational)

Q. Arrange the following in ascending order

$\sqrt[3]{9}, \sqrt[4]{11}, \sqrt[6]{17}$

$9^{1/3}, 11^{1/4}, 17^{1/6}$ (LCM of 3, 4 & 6 → 24)

$9^8, 11^6, 17^4$

$81^4, 11^6, 17^4$

$81^{4/24}, \cancel{12221}^{46/24}, 17^{4/24}$

1331

$\cancel{\sqrt[4]{11} > \sqrt[3]{9} > \sqrt[6]{17}}$

$\sqrt[3]{9} > \sqrt[4]{11} > \sqrt[6]{17}$

$11 \times 11 = 121$

121×11

121

$\cancel{121}\ 1221$

12221

1331

Square Root

→ $\sqrt{x^2} = \cancel{\pm}|x|$ ($-x$ is not right) (x be $\oplus$ve)

$(\sqrt{x})^2 = x$

→ It gives us only non-negative values
→ All quantities in underroot must be positive
→ Square Root of $a+\sqrt{b}$ ($a \geq 0, b \geq 0$)

$$\sqrt{a+\sqrt{b}} = \sqrt{x}+\sqrt{y} \quad (x, y > 0)$$

Squaring

$$a+\sqrt{b} = x+y+2\sqrt{xy}$$

$$a+\sqrt{b} = c+\sqrt{d} \rightarrow a = c, \; b = d$$

$$\boxed{a = x+y} \rightarrow y = a-x$$

$$\sqrt{b} = 2\sqrt{xy}$$

$$\boxed{b = 4xy}$$

$$b = 4x(a-x)$$

$$b = 4ax - 4x^2$$

$$4x^2 - 4ax + b = 0$$

$$x = \frac{-(-4a) \pm \sqrt{16a^2 - 4(4)(b)}}{2(4)}$$

$$x = \frac{4a \pm \sqrt{16a^2 - 16b}}{8} = \frac{a \pm \sqrt{16a^2-16b}}{2}$$ ($\oplus$ve because x is positive)

$$y = \frac{a - \sqrt{16a^2-16b}}{2}$$

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} \pm \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

H.W. 23-04-2024

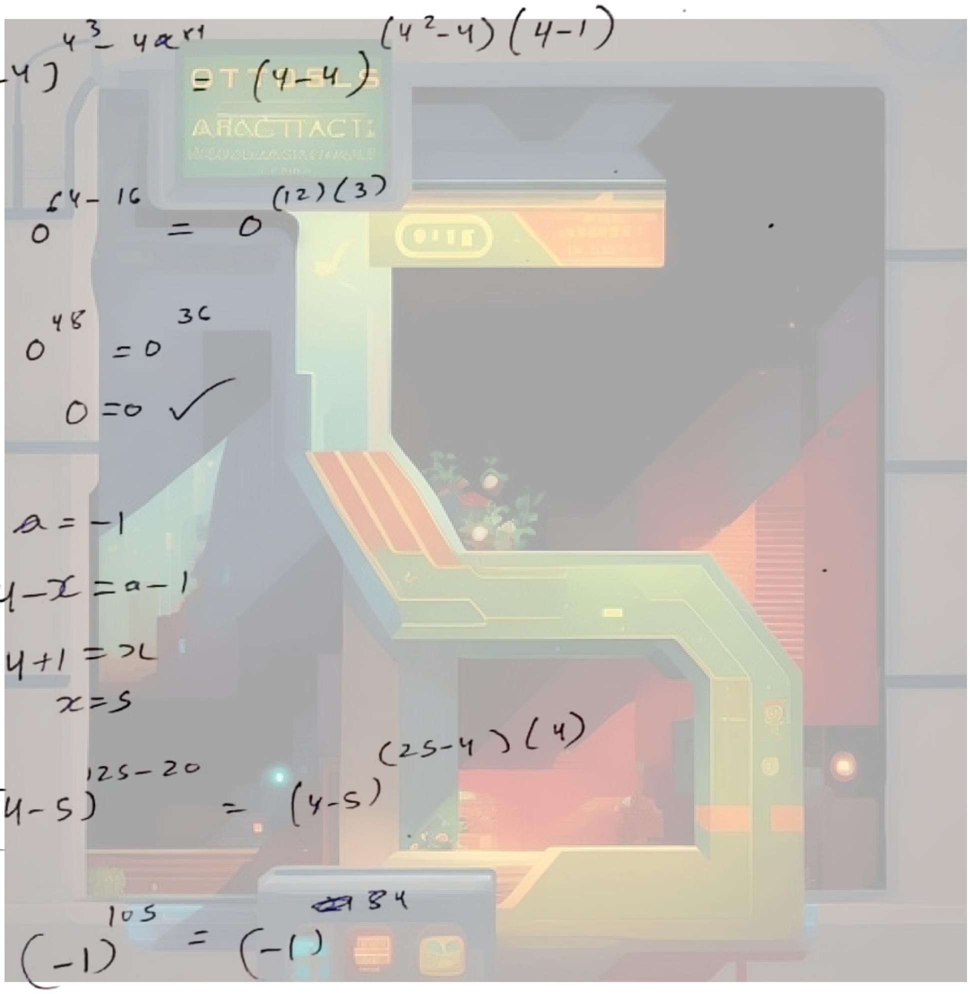
Q1.
(1) $(4-x)^{x^3-4x} = (4-x)^{(x^2-4)(x-1)}$

Case 1:- $a = 0$

$4-x=0$

$x=4$

verify



$(4-4)^{4^3-4\times4} = (4-4)^{(4^2-4)(4-1)}$

$0^{64-16} = 0^{(12)(3)}$

$0^{48} = 0^{36}$

$0=0$ ✓

case 2:- $a = -1$

$4-x = -1$

$4+1 = x$

$x=5$

verify:- $(4-5)^{125-20} = (4-5)^{(25-4)(4)}$

$(-1)^{105} = (-1)^{84}$

$-1 = 1$ ✗

Case 3 :- $a = \cancel{0}1$

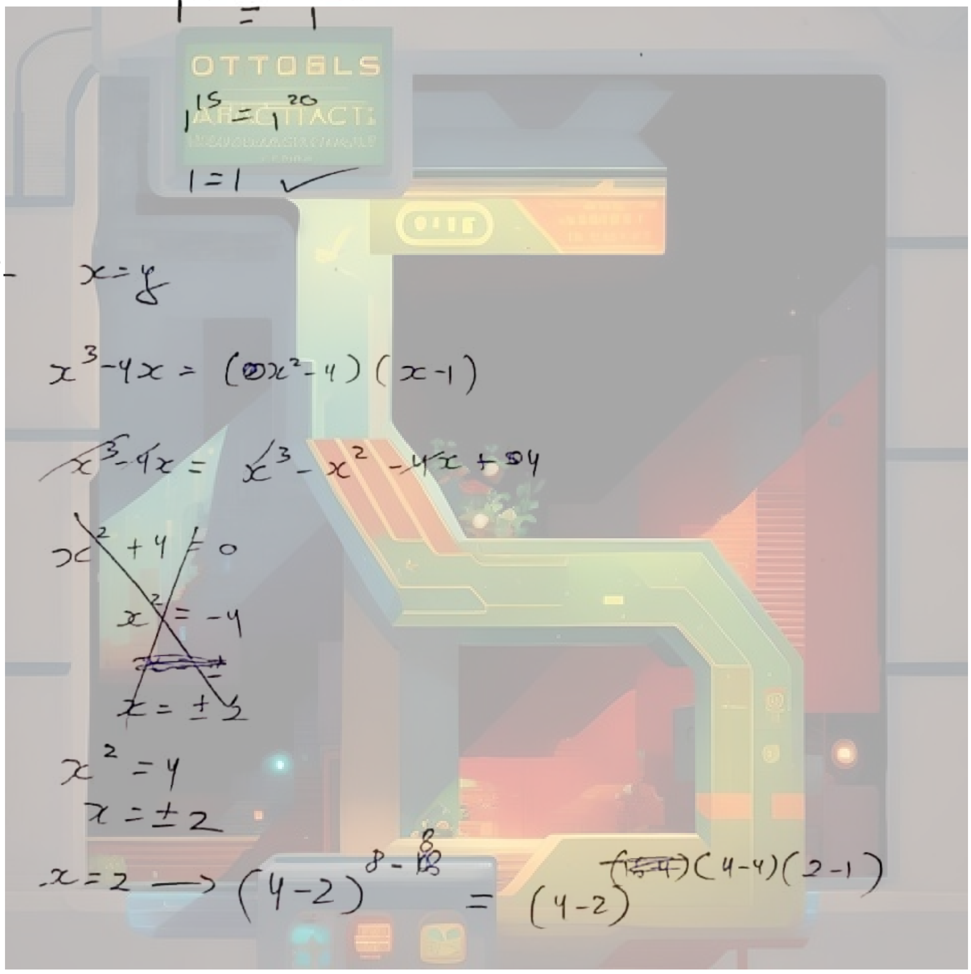
$$4 - x = 1$$

$$x = 3$$

verify :-

$$(4-3)^{27-12} = (4-3)^{(7-4)(3-1)}$$

$$1^{15} = 1^{5 \times 2}$$



$$1^{15} = 1^{20}$$

$$1 = 1 \checkmark$$

Case 4 :- $x = 4$

$$x^3 - 4x = (x^2 - 4)(x - 1)$$

$$\cancel{x^3} - 4x = x^3 - x^2 - 4x + 4$$

$$\cancel{x^2 + 4 = 0}$$

$$\cancel{x^2 = -4}$$

$$\cancel{x = \pm 2}$$

$$x^2 = 4$$

$$x = \pm 2$$

verify $x = 2 \rightarrow (4-2)^{8-8} = (4-2)^{(4-4)(2-1)}$

$$2^0 = 2^0 \checkmark$$

$x = -2 \rightarrow (4+2)^{-8+8} = (4+2)^{(4-4)(-3)}$

$$6^0 = 6^0 \checkmark$$

$$\boxed{x = 2, -2, 3, 4,}$$

② $(x^2-4)^{2x} = (x^2+2x)^{2x}$

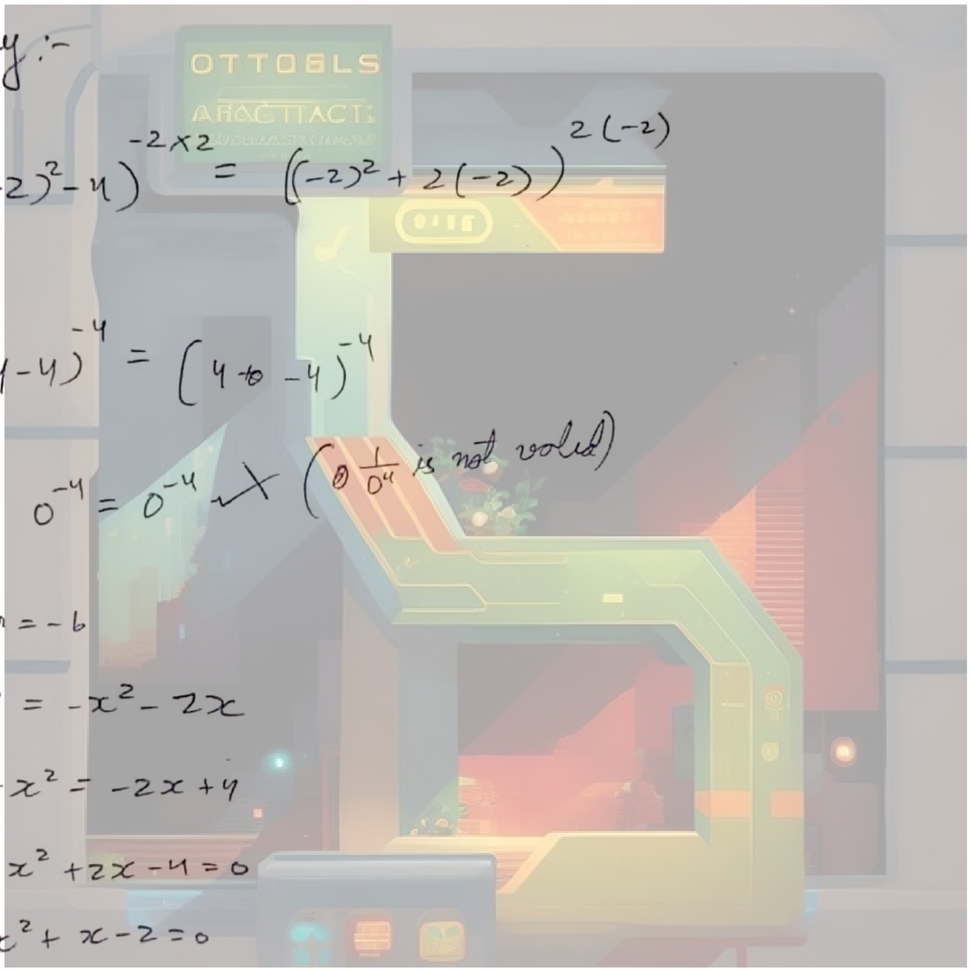
case 1 :- $a = b$

$\cancel{x^2} - 4 = \cancel{x^2} + 2x$

$x = \frac{-4}{2}$

$\underline{x = -2}$

verify :-



$((-2)^2 - 4)^{-2\times 2} = ((-2)^2 + 2(-2))^{2(-2)}$

$(4-4)^{-4} = (4 - 4)^{-4}$

$0^{-4} = 0^{-4}$ ✗ (0 $\frac{1}{0^4}$ is not valid)

Case 2 :- $a = -b$

$x^2 - 4 = -x^2 - 2x$

$x^2 + x^2 = -2x + 4$

$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$(x+2)(x-1)$

$x = -2$ (verified)

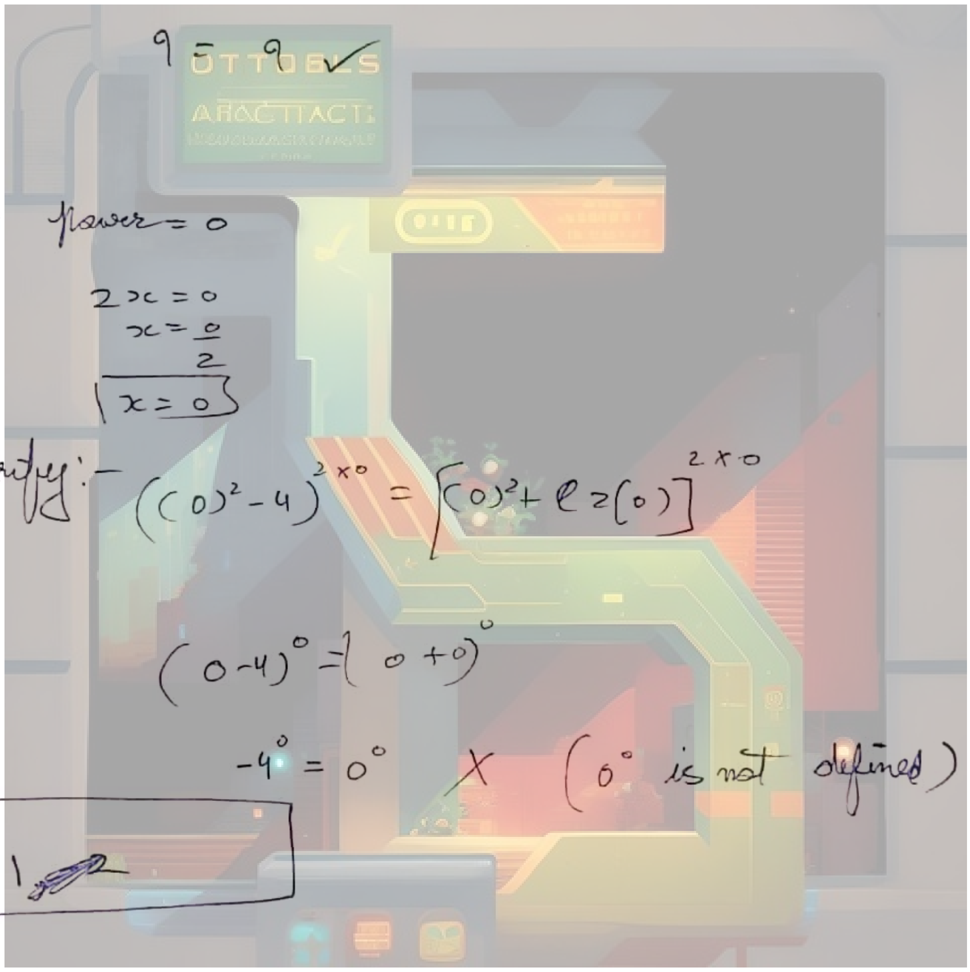
$x = 1$

verify :-

$$((1)^2-4)^{2\times 1} = ((1)^2+2(1))^{2\times 1}$$

$$(1-4)^2 = (1+2)^2$$

$$(-3)^2 = (3)^2$$



$$9 = 9 \checkmark$$

Case 3 :- power = 0

$$2x = 0$$

$$x = \frac{0}{2}$$

$$\boxed{x = 0}$$

Verify :- $((0)^2-4)^{2\times 0} = [(0)^2+2(0)]^{2\times 0}$

$$(0-4)^0 = (0+0)^0$$

$$-4^0 = 0^0 \quad \times \quad (0^0 \text{ is not defined})$$

$$\boxed{x = 1}$$

① find: $\sqrt{8+2\sqrt{15}}$

② $\sqrt{3+\sqrt{5}}$

① $\sqrt{8+2\sqrt{15}} =$

$\sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$

$\sqrt{8+\sqrt{64-}}$

① Method 1

$(a+b)^2 = a^2+b^2+2ab$

$\sqrt{8+2\sqrt{15}}$

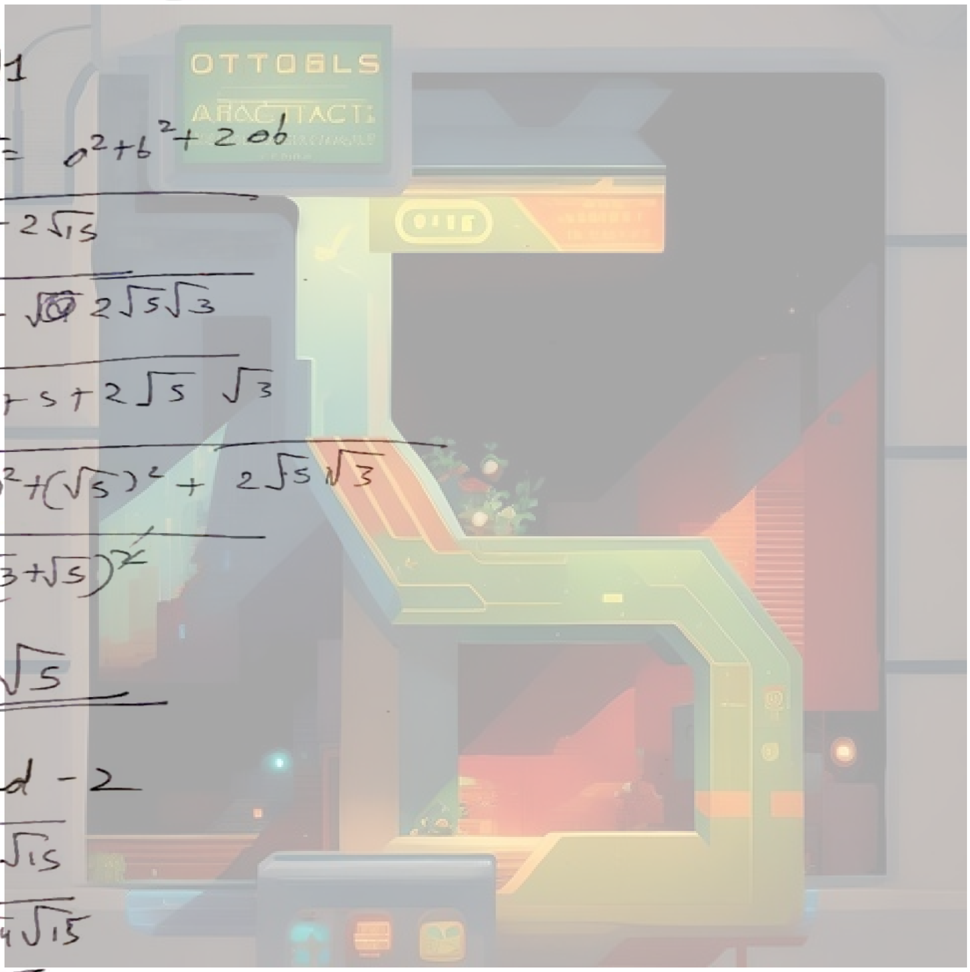
$\sqrt{8+2\sqrt{5}\sqrt{3}}$

$\sqrt{3+5+2\sqrt{5}\sqrt{3}}$

$\sqrt{(\sqrt{3})^2+(\sqrt{5})^2+2\sqrt{5}\sqrt{3}}$

$\sqrt{(\sqrt{3}+\sqrt{5})^2}$

$\sqrt{3}+\sqrt{5}$



Method - 2

$\sqrt{8+2\sqrt{15}}$

$\sqrt{8+\sqrt{4}\sqrt{15}}$

$\sqrt{8+\sqrt{60}}$

$\sqrt{\frac{8+\sqrt{64-60}}{2}} + \sqrt{\frac{8-\sqrt{64-60}}{2}}$

$\sqrt{\frac{8+\sqrt{4}}{2}} + \sqrt{\frac{8-\sqrt{4}}{2}}$

$\sqrt{\frac{8+2}{2}} + \sqrt{\frac{8-2}{2}}$

$\sqrt{5}+\sqrt{3}$

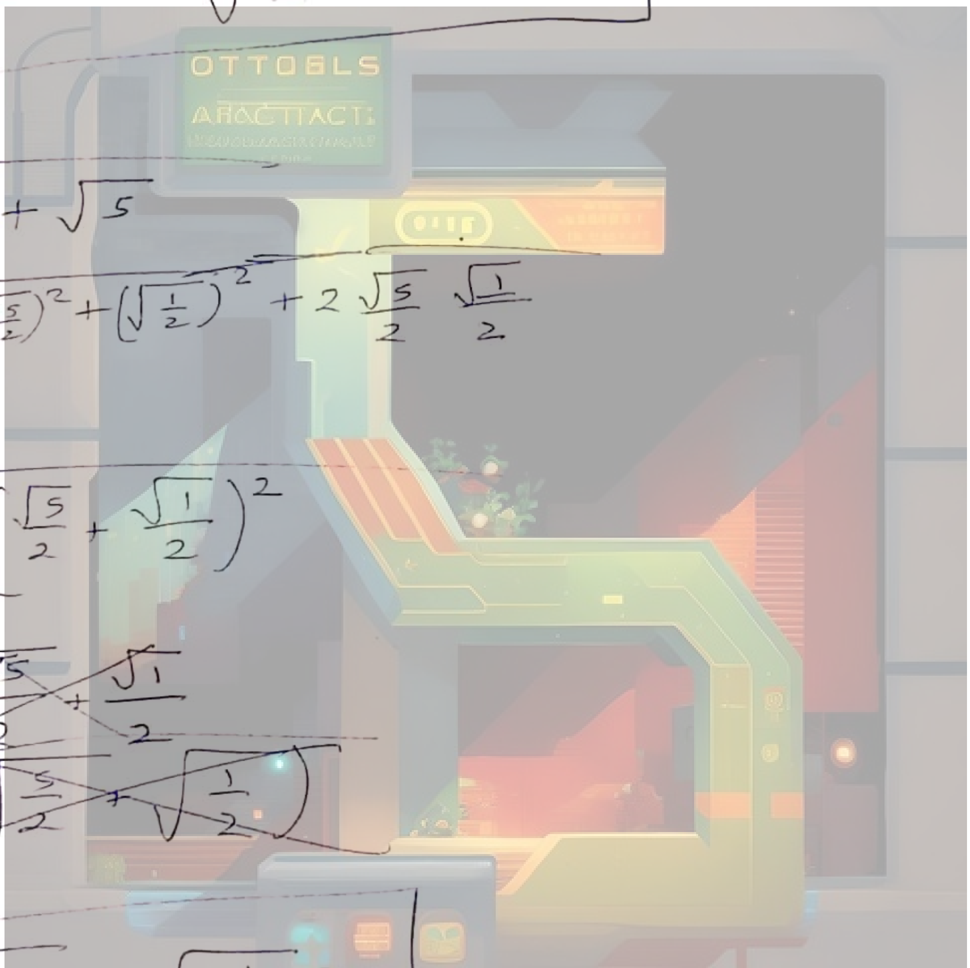
$\cancel{3\sqrt{\ }}$

② $\sqrt{3+\sqrt{5}}$

$$\sqrt{\frac{3+\sqrt{9-5}}{2}} + \sqrt{\frac{3-\sqrt{9-4}}{2}}$$

$$\sqrt{\frac{3+\sqrt{4}}{2}} + \sqrt{\frac{3-\sqrt{4}}{2}}$$

$$\boxed{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \quad ②}$$



$$\sqrt{3+\sqrt{5}}$$

$$\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2+\left(\sqrt{\frac{1}{2}}\right)^2+2\sqrt{\frac{5}{2}}\sqrt{\frac{1}{2}}}$$

$$\sqrt{\left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right)^2}$$

$$\cancel{\frac{\sqrt{5}}{2}+\frac{\sqrt{1}}{2}}$$

$$\cancel{\sqrt{\left(\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}\right)}}$$

$$\boxed{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \quad ②}$$

$$\frac{\sqrt{5}}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$\boxed{\frac{\sqrt{5}+1}{\sqrt{2}}}$$

86

(3) $\sqrt{7-3\sqrt{5}}$

$\cancel{\frac{1}{\sqrt{2}}\sqrt{7-2\times 3\sqrt{5}\sqrt{9}}}$

$\sqrt{7-\sqrt{45}}$

$\sqrt{7-\frac{2\sqrt{45}}{2}}$

$\frac{1}{\sqrt{2}}\sqrt{14-2\sqrt{45}}$

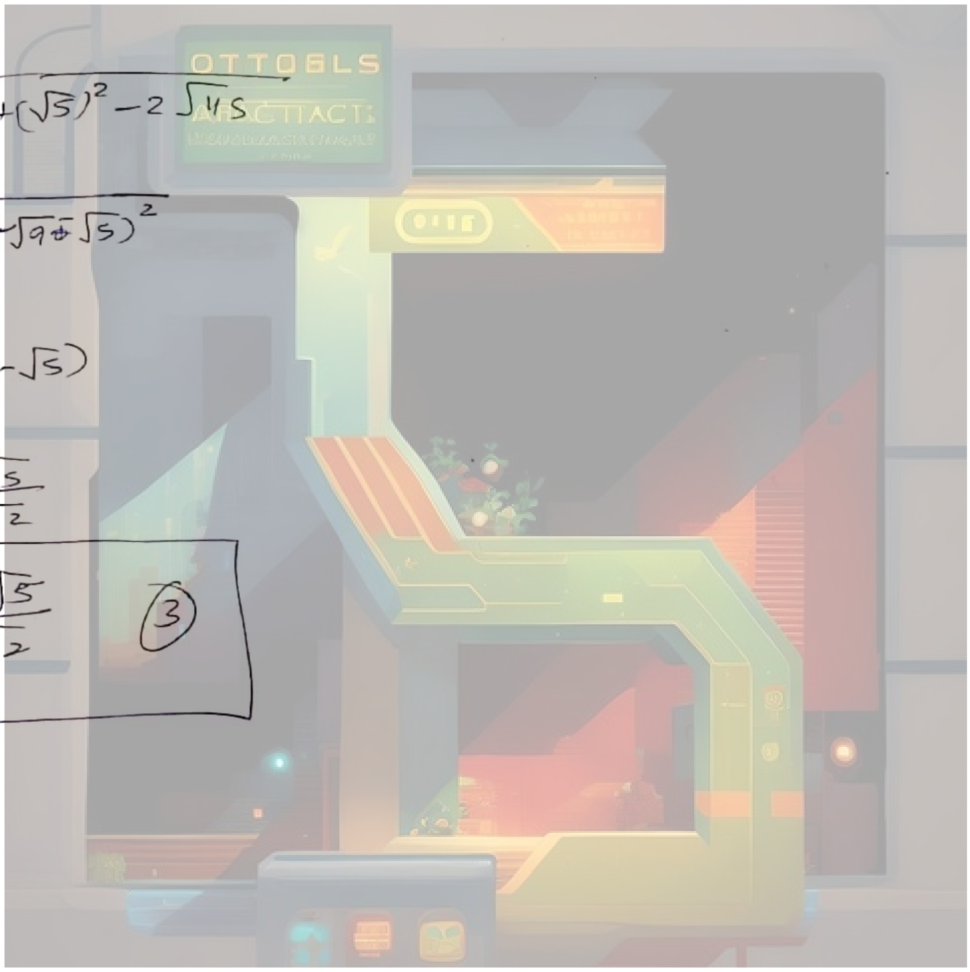
$\frac{1}{\sqrt{2}}\sqrt{(\sqrt{9})^2+(\sqrt{5})^2-2\sqrt{45}}$

$\frac{1}{\sqrt{2}}\sqrt{(\sqrt{9}-\sqrt{5})^2}$

$\frac{1}{\sqrt{2}}(\sqrt{9}-\sqrt{5})$

$\frac{\sqrt{9}}{\sqrt{2}}-\frac{\sqrt{5}}{\sqrt{2}}$

$\boxed{\frac{\sqrt{9}-\sqrt{5}}{\sqrt{2}} \quad (3)}$



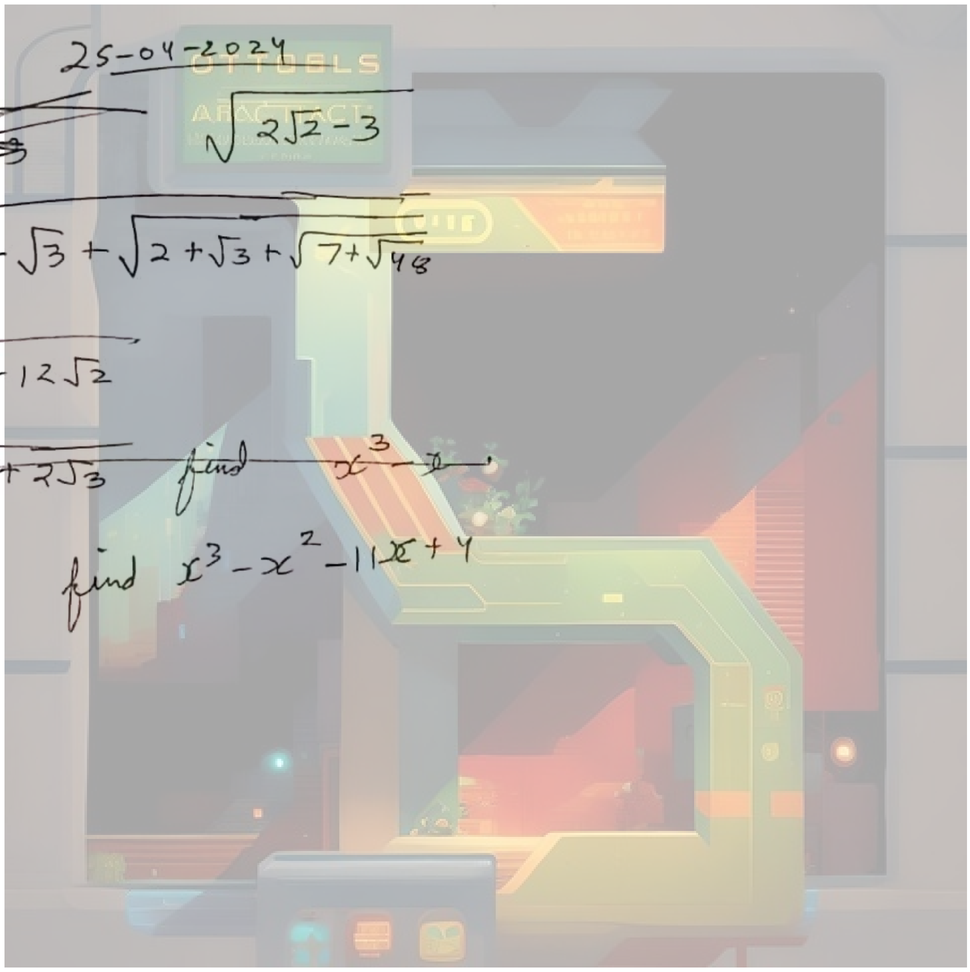
(4) $\sqrt{2\sqrt{3}-3}$

$\cancel{\sqrt{-(3-2\sqrt{3})}}$

$\cancel{\sqrt{-(\ }}$

$\cancel{\sqrt{\sqrt{2\sqrt{3}-\sqrt{9}}}}$

H.W. 25-04-2024



(1) $\sqrt{2\sqrt{2}-3}$

(2) $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}}$

(3) $\sqrt[4]{17+12\sqrt{2}}$

~~(4) $x=\sqrt{3+2\sqrt{3}}$ find x^3-x~~

(4) $x=\frac{1}{2+\sqrt{3}}$ find $x^3-x^2-11x+4$

Q $x = \sqrt{3+2\sqrt{2}}$

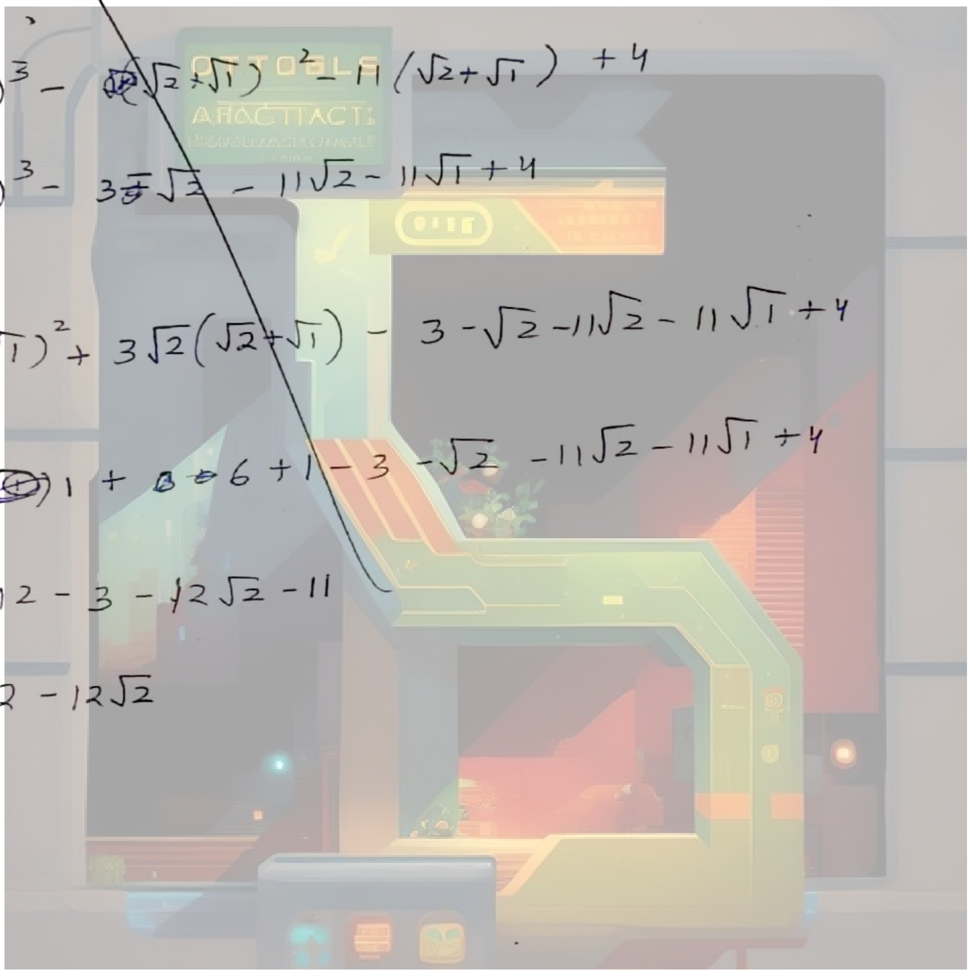
$x^3 - x^2 - 11x + 4$

$x = \sqrt{3+\sqrt{9-12}}$

$x = \frac{1}{\sqrt{2}}\sqrt{4+2\sqrt{3}+2\sqrt{9}}$

$x = \sqrt{(\sqrt{2})^2+(\sqrt{1})^2+2\sqrt{2}\sqrt{1}}$

$= (\sqrt{2}+\sqrt{1})$



$(\sqrt{2}+\sqrt{1})^3 - (\sqrt{2}+\sqrt{1})^2 - 11(\sqrt{2}+\sqrt{1}) + 4$

$(\sqrt{2}+\sqrt{1})^3 - 3\sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$

$(\sqrt{2})^3 + (\sqrt{1})^2 + 3\sqrt{2}(\sqrt{2}+\sqrt{1}) - 3 - \sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$

$(\sqrt{2})^3 + 1 + 6 + 1 - 3 - \sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$

$(\sqrt{2})^3 + 12 - 3 - 12\sqrt{2} - 11$

$(\sqrt{2})^3 - 2 - 12\sqrt{2}$

Q $\sqrt{3+2\sqrt{2}}$ find ① $x^3-x^2-11x+4$

② $2x^3-x^2-8x+120$

①② $\sqrt{3+2\sqrt{2}}$

$\sqrt{(\sqrt{2})^2+(\sqrt{1})^2+2\sqrt{2}\sqrt{1}}$

$x=\sqrt{2}+1$

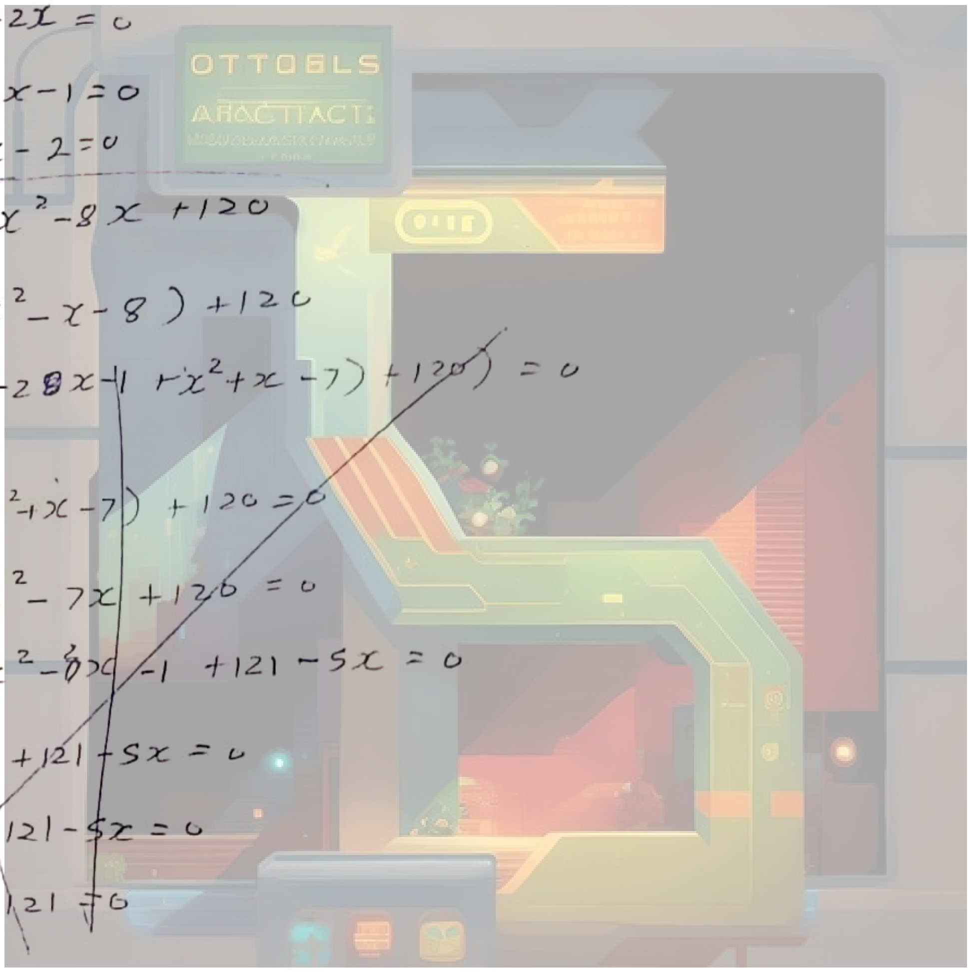
$(x-1)=\sqrt{2}$

$2=x^2+1-2x$

$x^2-1-2x=0$

$x^2-2x-1=0$

$2x^2-4x-2=0$



$2x^3-x^2-8x+120$

$x(2x^2-x-8)+120$

$x((x^2-2\cancel{8}x-1+x^2+x-7)+120)=0$

$x(\cancel{0}x^2+x-7)+120=0$

$x^3+x^2-7x+120=0$

$x^3+x^2-2x-1+121-5x=0$

$x^3+0+121-5x=0$

$x^3+121-5x=0$

$x^3-5x+121=0$

$x(2x^2-4x-2-6+3x)+120=0$

$x(0-6+3x)+120=0$

$3x^2-6x+120=0$

$3(x^2-2x+40)=0$

$3(x^2-2x-1+41)=0$

$3(41)$

90 $\boxed{123}$

① $x^3 - x^2 - 11x + 4$

$x(x^2 - x - 11) + 4$

$x(x^2 - 2x - 1 - 10 + x) + 4$

$x(x - 10) + 4$

$x^2 - 10x + 4$

$x^2 - 2x - 1 + 5 - 8x$

$0 + 5 - 8x$

$5 - 8x$

$5 - 8(\sqrt{2} + 1)$

$5 - 8\sqrt{2} - 8$

$\boxed{-8\sqrt{2} - 3}$

H.W. 25-04-2024

① $\sqrt{2\sqrt{2}-3}$

$\sqrt{-(3-2\sqrt{2})}$

$\sqrt{-((\sqrt{2})^2+(\sqrt{1})^2-2\sqrt{2}\sqrt{1})}$

$\sqrt{-(\sqrt{2}-\sqrt{1})^2}$

~~$\sqrt{2\sqrt{2}-3}$~~

② $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}}$

$\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{3+4+2\sqrt{4}\sqrt{3}}}}$

$\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{3}+\sqrt{4}}}$

$\sqrt{3+\sqrt{3}+\sqrt{4+2\sqrt{3}\sqrt{1}}}$

$\sqrt{3+\sqrt{3}+\sqrt{3+1\,2\sqrt{3}\sqrt{1}}}$

$\sqrt{3+\sqrt{3}+\sqrt{(\sqrt{3})^2+1^2+2\sqrt{3}\sqrt{1}}}$

$\sqrt{3+\sqrt{3}+\sqrt{(\sqrt{3}+1)^2}}$

$\sqrt{3+\sqrt{3}+\sqrt{3}+1}$

$\sqrt{4+2\sqrt{3}}$

$\sqrt{(\sqrt{3}+1)^2}$

92 $\boxed{\sqrt{3}+1 \quad Q2}$

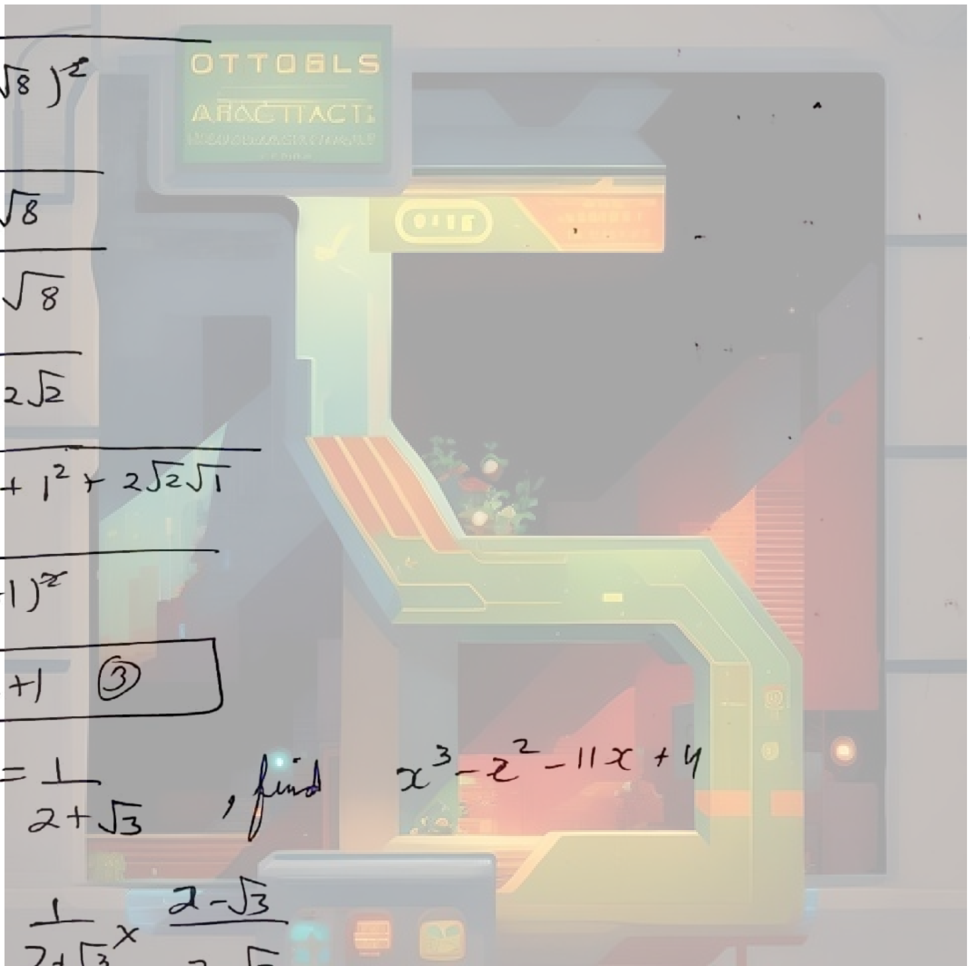
③ $\sqrt[4]{17+12\sqrt{2}}$

$\sqrt[4]{17+2\sqrt{2}\sqrt{36}}$

$\sqrt[4]{17+2\sqrt{72}}$

$\sqrt[4]{9+8+2\sqrt{9}\sqrt{8}}$

$\sqrt[4]{(\sqrt{9})^2+(\sqrt{8})^2+2\sqrt{9}\sqrt{8}}$



$\sqrt[4]{(\sqrt{9}+\sqrt{8})^{\cancel{2}}}$

$\sqrt{\sqrt{9}+\sqrt{8}}$

$\sqrt{3+\sqrt{8}}$

$\sqrt{3+2\sqrt{2}}$

$\sqrt{(\sqrt{2})^2+1^2+2\sqrt{2}\sqrt{1}}$

$\sqrt{(\sqrt{2}+1)^{\cancel{2}}}$

$\boxed{\sqrt{2}+1 \quad ③}$

④ $x=\dfrac{1}{2+\sqrt{3}}$, find $x^3-x^2-11x+4$

$x=\dfrac{1}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$=\dfrac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$=\dfrac{2-\sqrt{3}}{4-3}$

$x=2-\sqrt{3}$

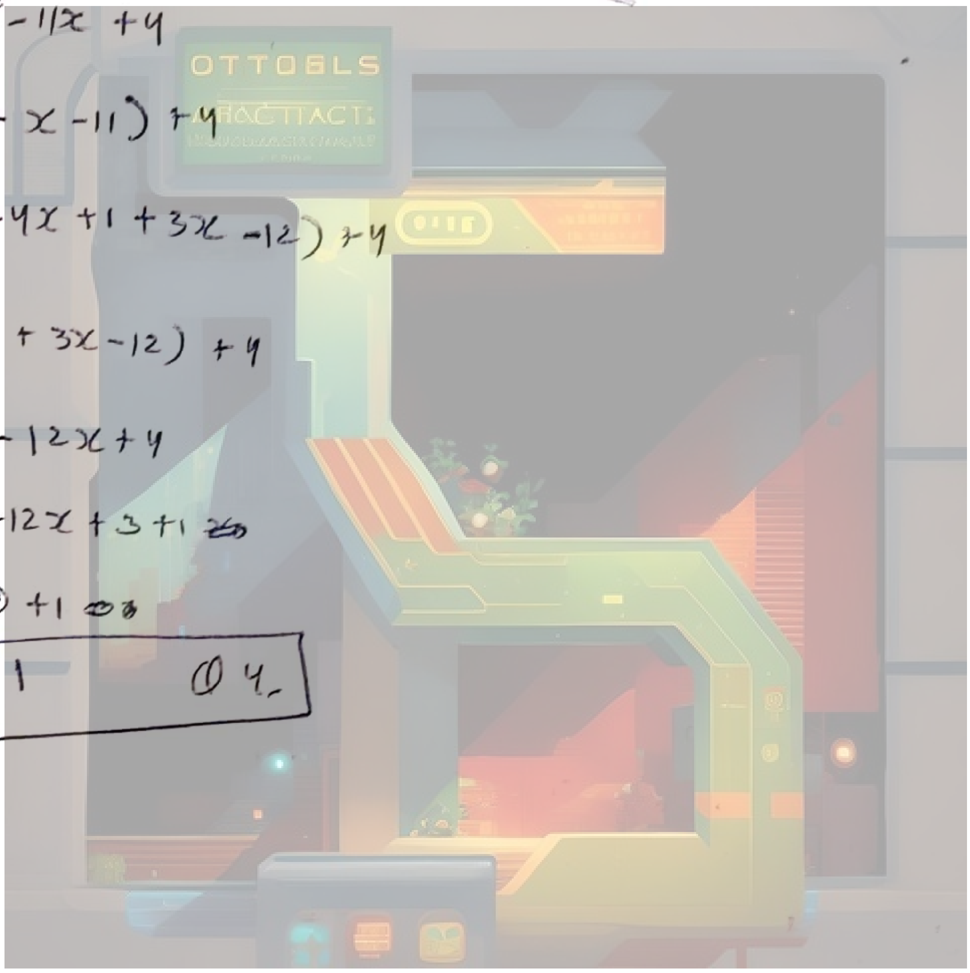
$x = 2 - \sqrt{3}$

$\sqrt{3} = 2 - x$

$(\sqrt{3})^2 = (2 - x)^2$

$3 = 4 + x^2 - 4x$

$x^2 - 4x + 1 = 0$ $\qquad$ $3x^2 - 12x + 3 = 0$



$x^3 - x^2 - 11x + 4$

$x(x^2 - x - 11) + 4$

$x(x^2 - 4x + 1 + 3x - 12) + 4$

$x(0 + 3x - 12) + 4$

$3x^2 - 12x + 4$

$3x^2 - 12x + 3 + 1$

$0 + 1$

$\boxed{= 1 \qquad (1)\ 4.}$

0-3

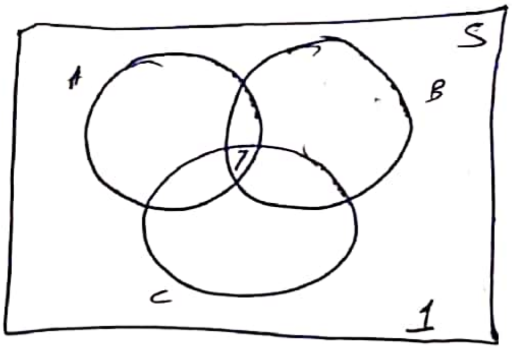
Q4. ✓A) $A = B = C$

✓B) $A = B = C$ (on solving)

✓C) $A - B = (A \cup B \cup C) - (A \cap B \cap C)$

$\phi = \phi$ for $(A \cap B \cap C)$ be max

✗D)

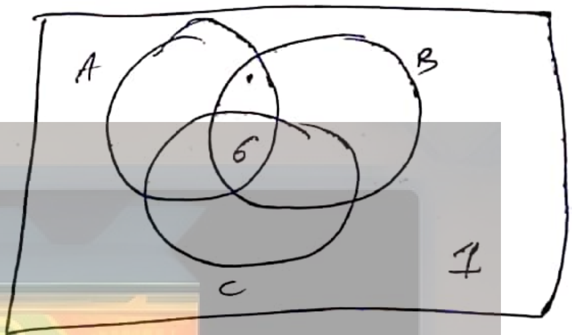


Q5. ✗A)

✓B)

✓C)

✓D)

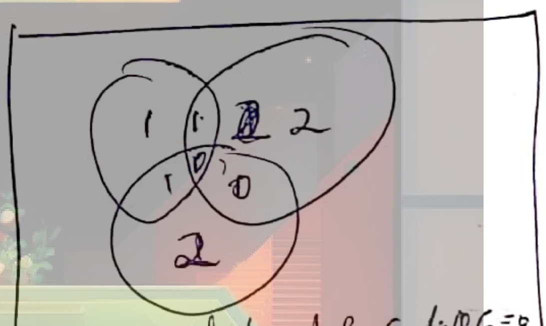


Q6. ✓A)

✓B)

C)

✓D)



min val of $A \cap B \cap C$ for Q6 = 0

7 cases

Q $x=\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{\ldots\infty}}}}}}$

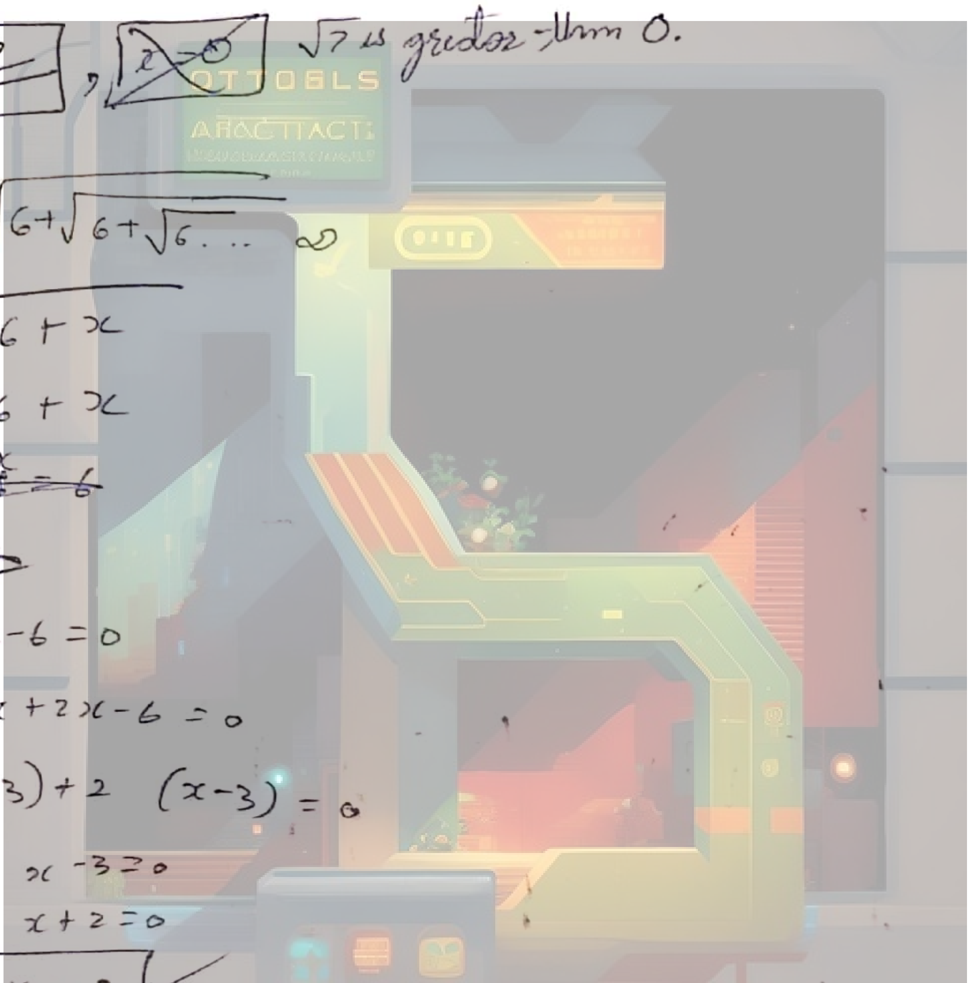
$\cancel{x = 7^{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}}}$

$x = \sqrt{7x}$

$x^2 = 7x$

$\frac{x^2}{x} = 7$

$\boxed{x = 7}$, $\boxed{\cancel{x=0}}$ $\sqrt{7}$ is greater than 0.



Q $x = \sqrt{6+\sqrt{6+\sqrt{6\ldots\infty}}}$

$x = \sqrt{6+x}$

$x^2 = 6 + x$

$\cancel{x^2 - x = 6}$

$\cancel{x(x)}$

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$x - 3 = 0$

$x + 2 = 0$

$\boxed{x = 3}$ ✓

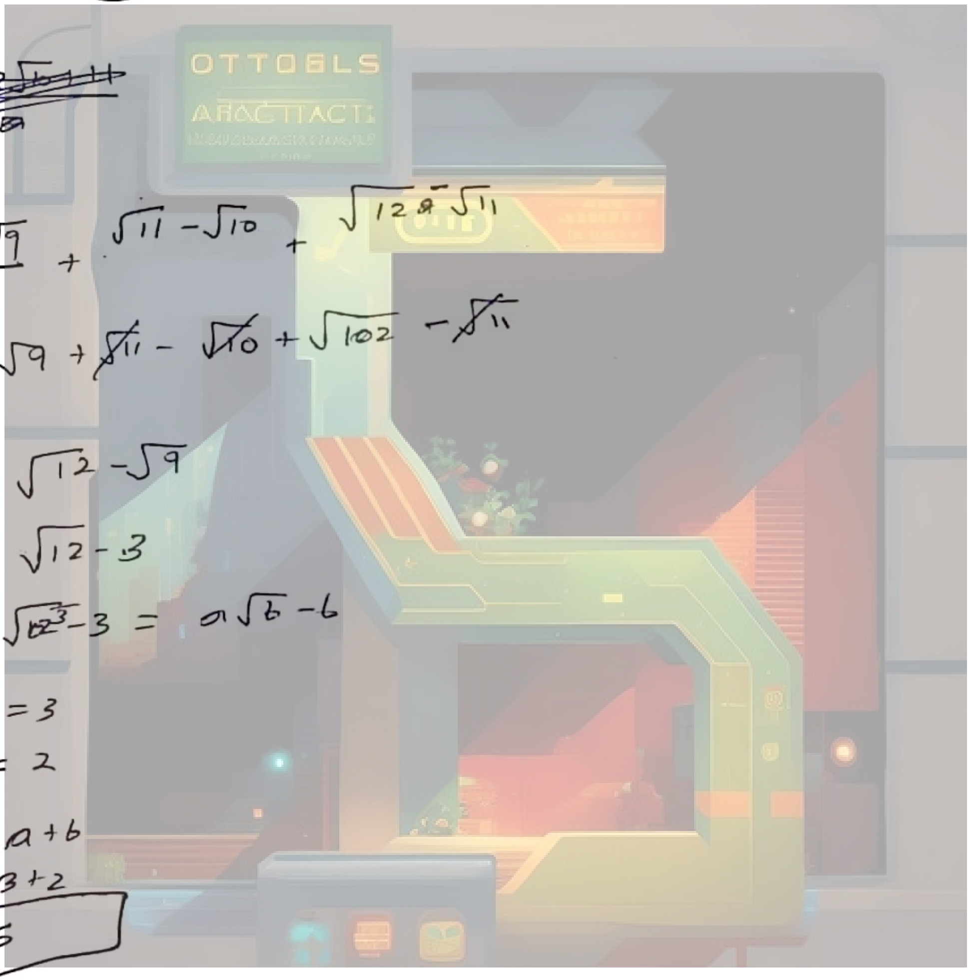
$\boxed{\cancel{x = -2}}$ ✓

Q $\sqrt{\frac{1}{\sqrt{10}+\sqrt{9}}+\frac{1}{\sqrt{11}+\sqrt{10}}+\frac{1}{\sqrt{12}+\sqrt{11}}} = \sqrt{a\sqrt{b}-b}$

$\cancel{\frac{\sqrt{11}+\sqrt{10}+\sqrt{10}+\sqrt{9}}{\sqrt{10}\sqrt{11}+10+\sqrt{9}\sqrt{11}+\sqrt{9}\sqrt{10}}}$

$\cancel{\frac{\sqrt{9}+2\sqrt{10}+\sqrt{11}}{10+\sqrt{110}+\sqrt{99}+\sqrt{90}}}$

$\cancel{\frac{3+2\sqrt{10}+11}{10+\sqrt{a}}}$



$\frac{\sqrt{10}-\sqrt{9}}{1} + \sqrt{11}-\sqrt{10} + \sqrt{12}-\sqrt{11}$

$\cancel{\sqrt{10}}-\sqrt{9}+\cancel{\sqrt{11}}-\cancel{\sqrt{10}}+\sqrt{12}-\cancel{\sqrt{11}}$

$\sqrt{12}-\sqrt{9}$

$\sqrt{12}-3$

$2\sqrt{3}-3 = a\sqrt{b}-b$

$b=3$

$a=2$

$a+b$

$=3+2$

$\boxed{=5}$

H.W. 27-04-2024

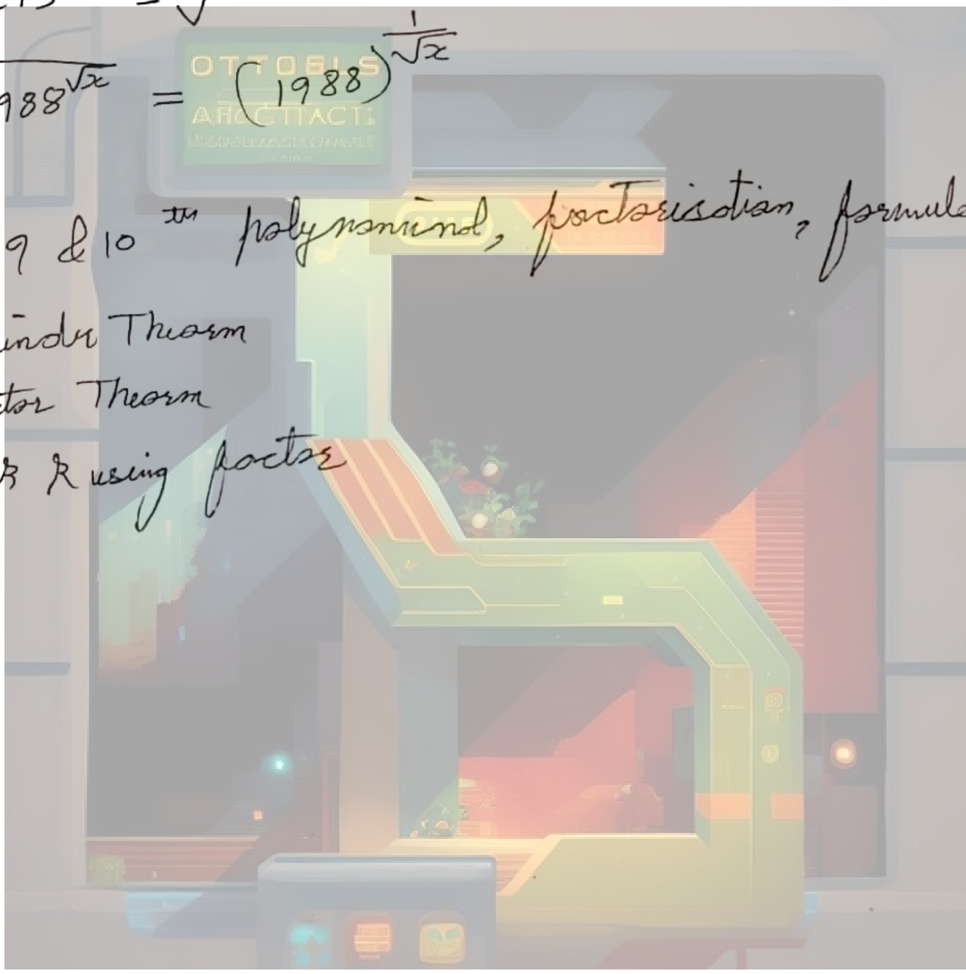
① $\sqrt{x}+\frac{1}{\sqrt{x}}=3$

$x^3+\frac{1}{x^3}=?$

② $\sqrt{8+2\sqrt{15}}+\sqrt{8-2\sqrt{15}}-\sqrt{20}$

③ $\sqrt{2x+3}-\sqrt{3x-5}=1$

④ $\sqrt{1988^{\sqrt{x}}}=(1988)^{\frac{1}{\sqrt{x}}}$



Class 9 & 10th polynomial, factorisation, formula (square cube)

reminder Theorm

factor Theorm

find k using factor

SOS (Sum of Squares)

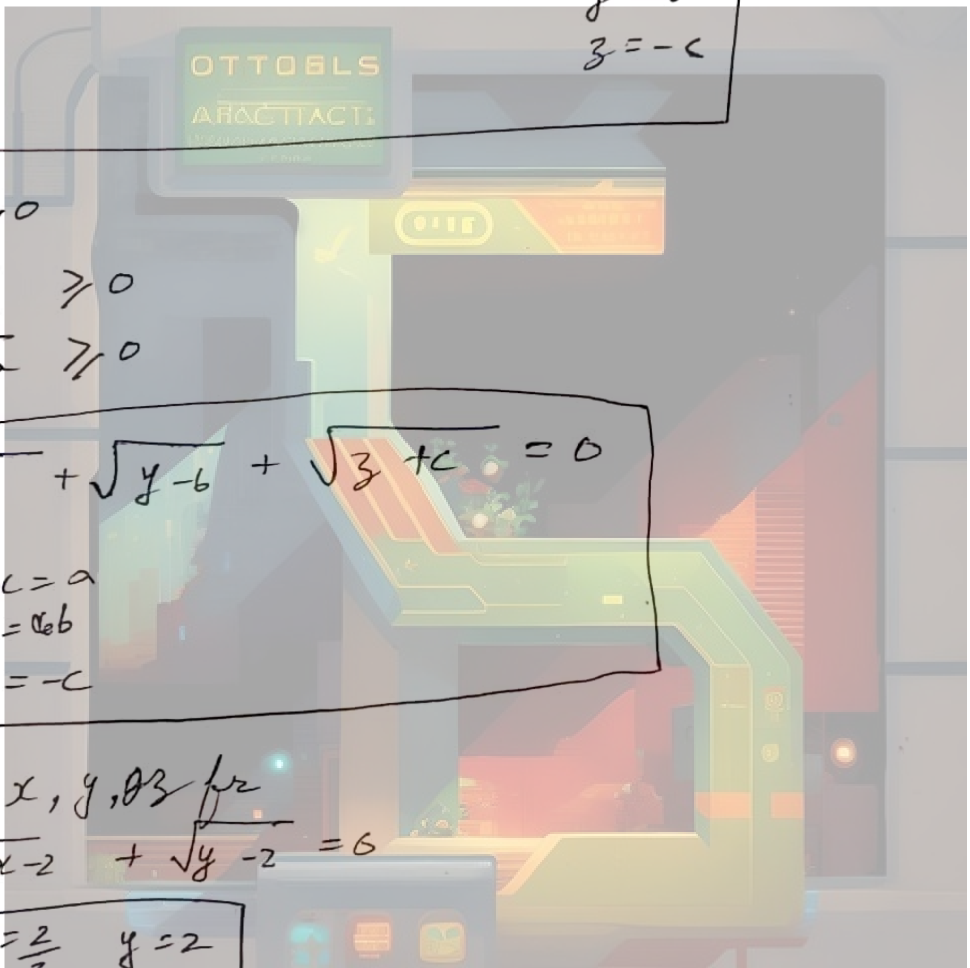
$x^2 \geq 0$

$(x-a)^2 \geq 0$

$(x+a)^2 \geq 0$

$(x-a)^2+(x+b)^2+(x-c)^2 \geq 0$

$$\boxed{(x-a)^2+(y-b)^2+(z+c)^2=0 \text{ if } x=a,\ y=b,\ z=-c}$$



$\sqrt{x} \geq 0$

$\sqrt{x-a} \geq 0$

$\sqrt{x+a} \geq 0$

$$\boxed{\sqrt{x-a}+\sqrt{y-b}+\sqrt{z+c}=0 \text{ if } x=a,\ y=b,\ z=-c}$$

Q1. find x, y, z for

① $\sqrt{3x-2}+\sqrt{y-2}=0$

$\boxed{x=\frac{2}{3} \quad y=2}$

② $\sqrt{x+2}+(y-3)^2+(z+4)^2=0$

$\boxed{x=-2,\ y=3,\ z=-4}$

③ $(p-2)^2+(q-100)^2+(r-3)^2=0$

$$\boxed{\begin{aligned} P&=2\\ Q&=100\\ r&=3\end{aligned}}$$

④ $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}\ ,\ x\in R$

$(x+1)+(x-1)-\cancel{2(x+1)}\sqrt{x+1}\sqrt{x-1}$

$x+1+x-1-2\sqrt{(x+1)(x-1)}$

$2x-2\sqrt{x^2-1}=4x-1$

$2x-4x+1=2\sqrt{x^2-1}$

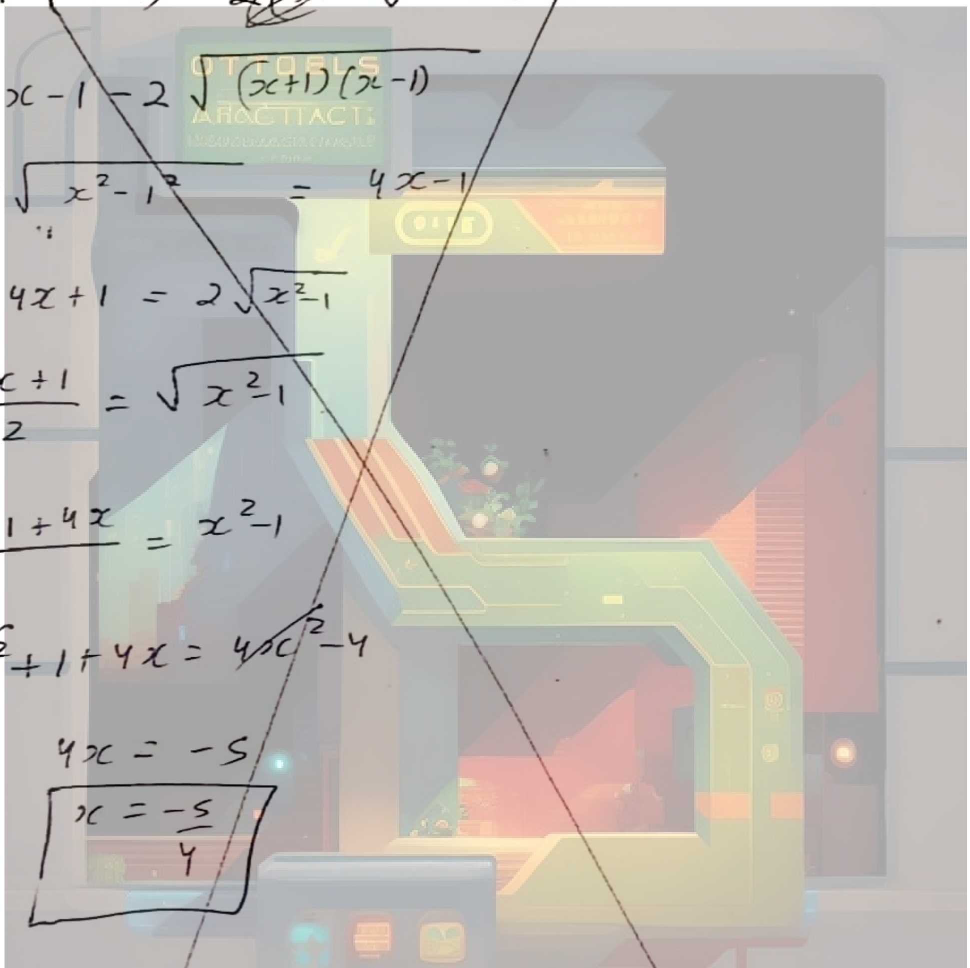
$\frac{-2x+1}{2}=\sqrt{x^2-1}$

$\frac{4x^2+1+4x}{4}=x^2-1$

$\cancel{4x^2}+1+4x=\cancel{4x^2}-4$

$4x=-5$

$$\boxed{x=\frac{-5}{4}}$$



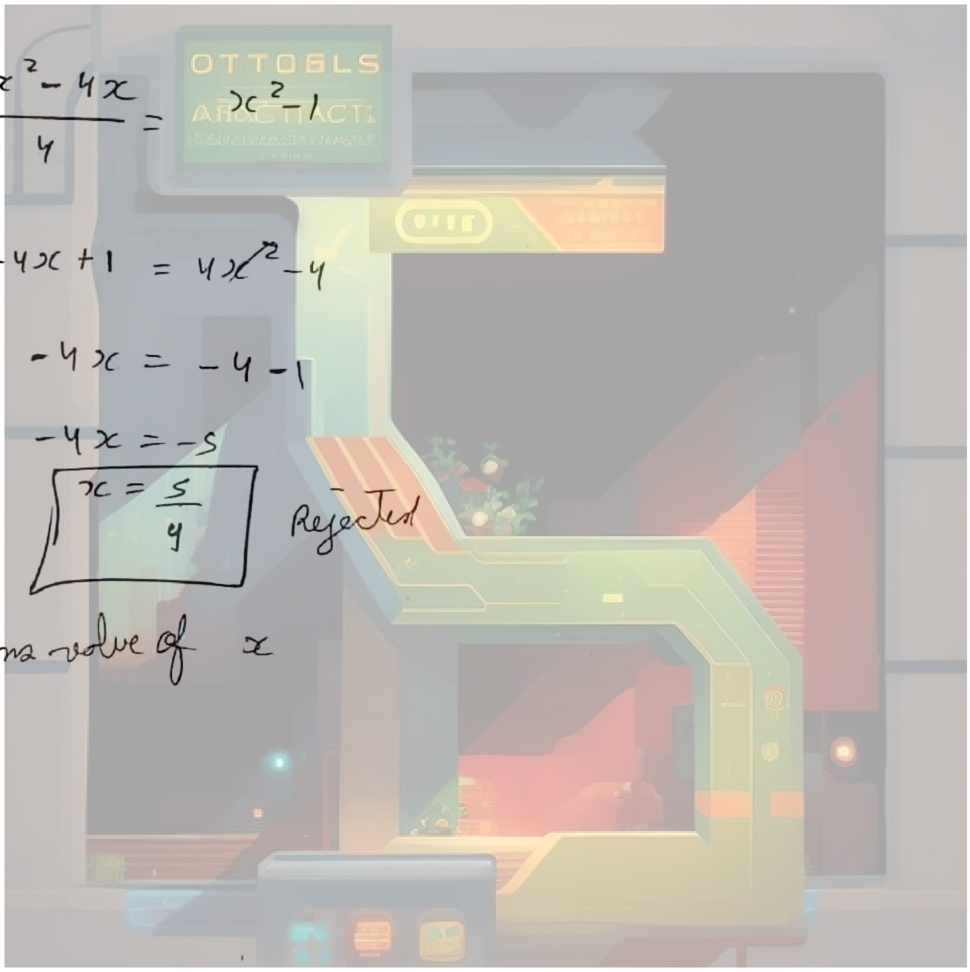
100

$$(x+1)+(x-1) - 2\sqrt{x^2-1} = 4x-1$$

$$x + x \cancel{+1-1} - 2\sqrt{x^2-1} = 4x-1$$

$$\frac{-2x+1}{2} = \sqrt{x^2-1}$$

$$\frac{1-2x}{2} = \sqrt{x^2-1}$$



$$\frac{1+4x^2-4x}{4} = x^2-1$$

$$\cancel{4x^2}-4x+1 = \cancel{4x^2}-4$$

$$-4x = -4-1$$

$$-4x = -5$$

$$\boxed{x = \frac{5}{4}}$$ Rejected

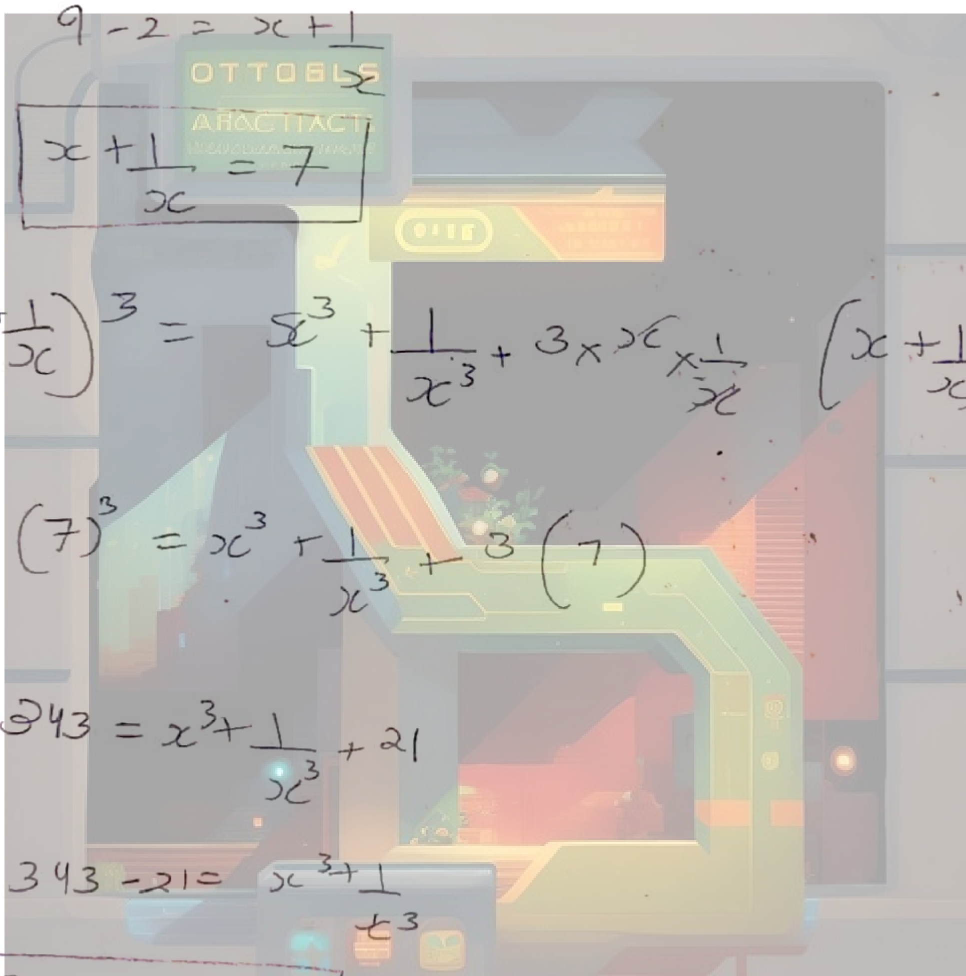
no value of x

H.W. 270-04-2024

Q1. $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2} + 2\times\sqrt{x}\times\frac{1}{\sqrt{x}}$

$(3)^2 = x+\frac{1}{x}+2$

$(3)^2 = x+\frac{1}{x}+2$



$9-2 = x+\frac{1}{x}$

$\boxed{x+\frac{1}{x}=7}$

$\left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$

$(7)^3 = x^3+\frac{1}{x^3}+3(7)$

$343 = x^3+\frac{1}{x^3}+21$

$343-21 = x^3+\frac{1}{x^3}$

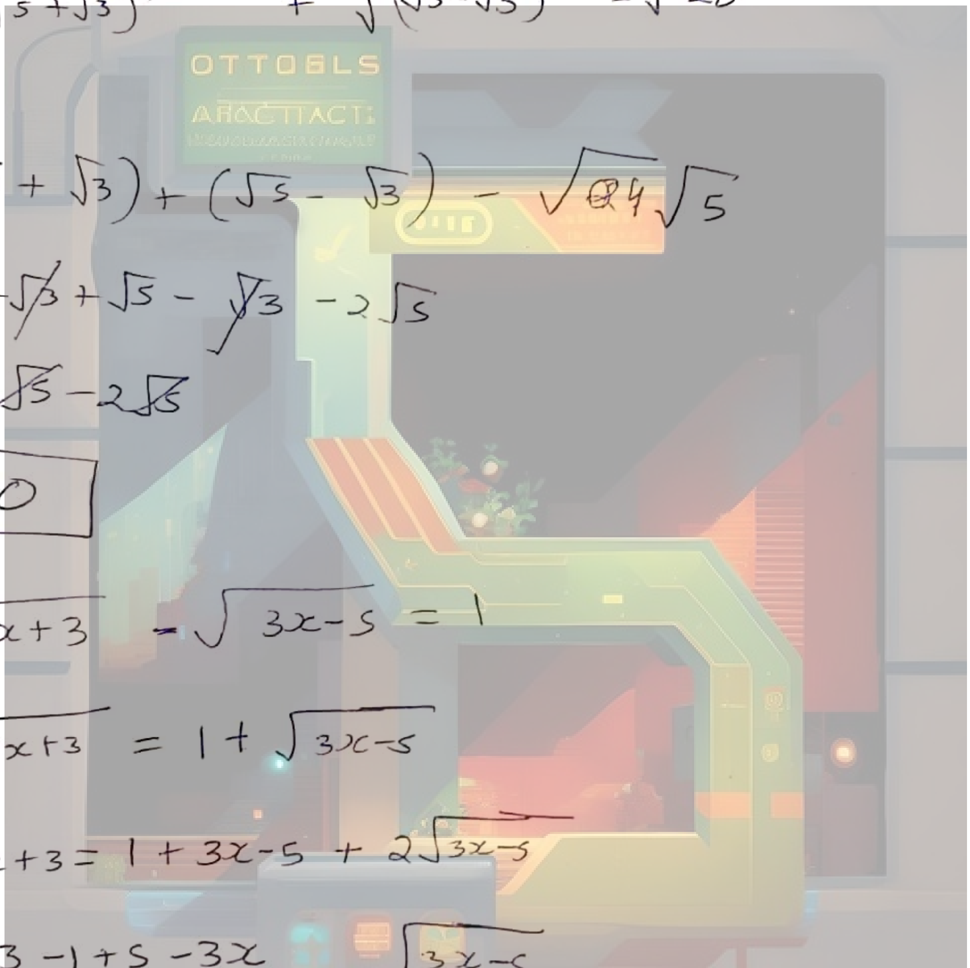
$\boxed{x^3+\frac{1}{x^3}=322}$

② $\sqrt{8+2\sqrt{15}} + \sqrt{\cancel{3x-5}\,8-2\sqrt{15}} - \sqrt{20}$

$\sqrt{\cancel{8}\,5+3+2\sqrt{15}} + \sqrt{\cancel{8}\,5+3-2\sqrt{15}} - \sqrt{20}$

$\sqrt{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\sqrt{3}} + \sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2\sqrt{5}\sqrt{3}} - \sqrt{20}$

$\sqrt{(\sqrt{5}+\sqrt{3})^2} + \sqrt{(\sqrt{5}-\sqrt{3})^2} - \sqrt{20}$



$(\sqrt{5}+\sqrt{3}) + (\sqrt{5}-\sqrt{3}) - \sqrt{4}\sqrt{5}$

$\sqrt{5}+\cancel{\sqrt{3}}+\sqrt{5}-\cancel{\sqrt{3}}-2\sqrt{5}$

$2\sqrt{5}-2\sqrt{5}$

$\boxed{=0}$

Q3. $\sqrt{2x+3} - \sqrt{3x-5} = 1$

$\sqrt{2x+3} = 1+\sqrt{3x-5}$

$2x+3 = 1+3x-5+2\sqrt{3x-5}$

$\dfrac{2x+3-1+5-3x}{2} = \sqrt{3x-5}$

$\dfrac{7-3x}{2} = \sqrt{3x-5}$

$$\frac{49 + 9x^2 - 42x}{4} = 3x - 5$$

$$49 + 9x^2 - 42x = 12x - 20$$

$$9x^2 - 42x - 12x + 49 + 20 = 0$$

$$9x^2 - 54x + 69 = 0$$

$$3x^2 - 18x + 23 = 0$$

$$\cancel{x = +324 \pm \sqrt{}}$$

$$x = +b \pm \sqrt{b^2 - 4ac}$$

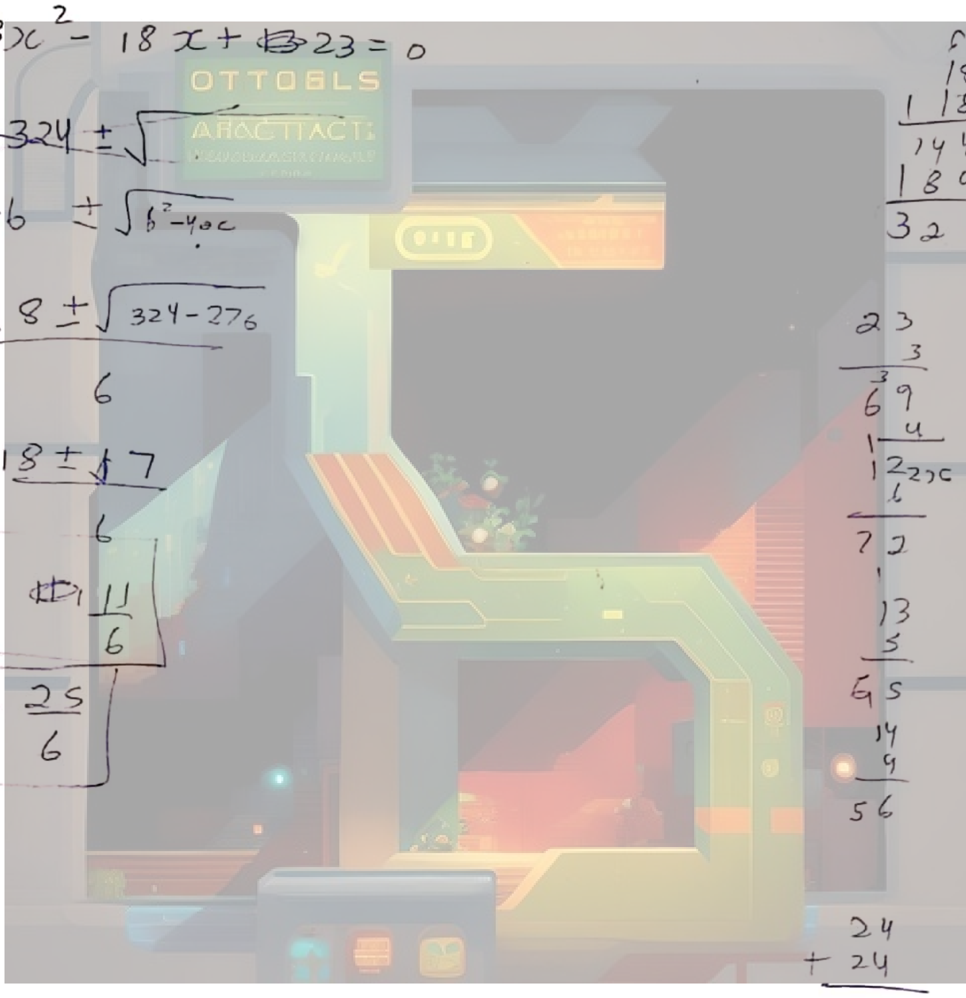
$$x = \frac{18 \pm \sqrt{324 - 276}}{6}$$

$$= \frac{18 \pm 7}{6}$$

$$\boxed{x = \frac{11}{6}}$$

$$\boxed{x = \frac{25}{6}}$$

$$\begin{array}{r} 18 \\ \times 18 \\ \hline 144 \\ 180 \\ \hline 324 \end{array} \qquad \begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array} \qquad \begin{array}{r} 12 \\ \times 6 \\ \hline 72 \end{array} \qquad \begin{array}{r} 13 \\ \times 5 \\ \hline 65 \end{array} \qquad \begin{array}{r} 14 \\ \times 4 \\ \hline 56 \end{array} \qquad \begin{array}{r} 24 \\ + 24 \\ \hline 48 \end{array}$$



Q4.

$$\sqrt{1988^{\sqrt{x}}} = 1988^{\frac{1}{\sqrt{x}}}$$

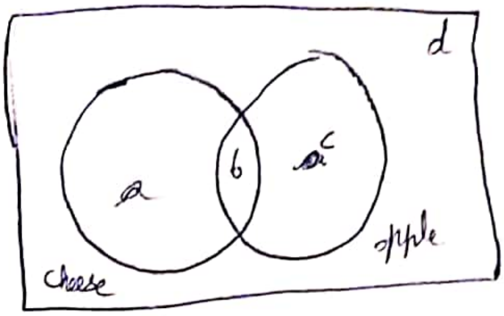
$$1988^{\frac{\sqrt{x}}{2}} = 1988^{\frac{1}{\sqrt{x}}}$$

$$\frac{\sqrt{x}}{2} = \frac{1}{\sqrt{x}}$$

$$\boxed{x = 2}$$

0–4

Q2.



$a+b=63$

$b+c=76$

$a+2b+c=139$

$a+b+c+d=100$

~~a+d=~~

$b=39+d$

$b=39$ (min)

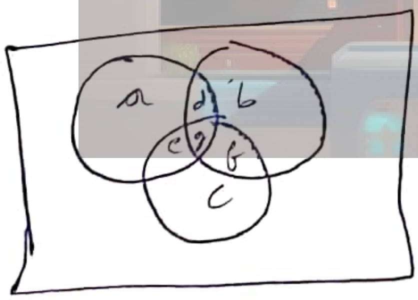
$b=63$ (max)

no. of values = ~~[63–39]~~ $[39,64]$

$=64-39+1$

$\boxed{=25}$

Q3.



$d+e+f=?$

$e+f+c=90$

$b+d+f=120$

$a+d+e=170$

(1) + (2) + (3)

$a+b+c+2(d+e+f)=380$ – (4)

$a+b+c+d+e+f+g=300$ – (5)

4 – 5

$\boxed{d+e+f=80}+30=110$

106

Scanned with CamScanner

Q9. [1, 2000]

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 (6)

14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26 (6)

$\frac{2000}{13} = 153 \times 6$

$= 918$

$\frac{2000}{13}$ 5 x

16

$13 \overline{)1989}$

-13

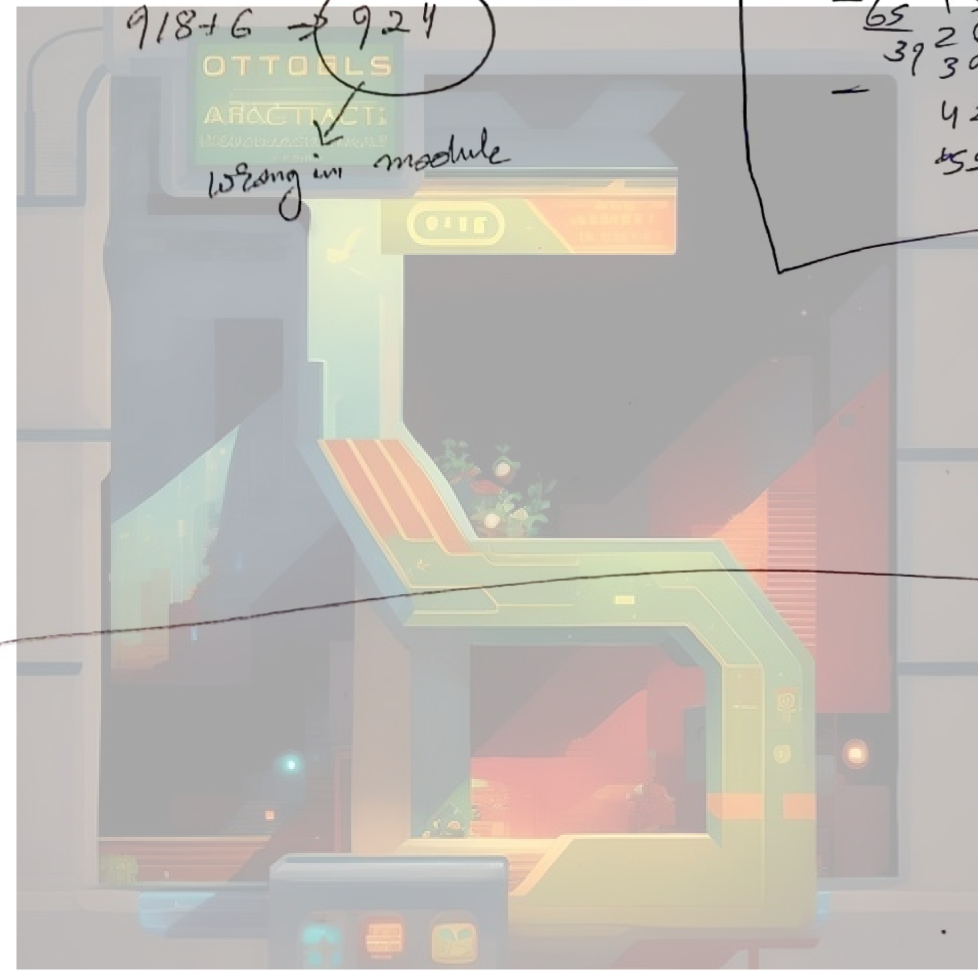
68

-65

39

$153 \times 5 = 765$

13
26
39
42
55



918+6 → 924

wrong in module

1990
1991
1992
1993
1994
1995
1996
1997
1998 (6)
1999
2000

factorisation, factor theorem & cyclic expression

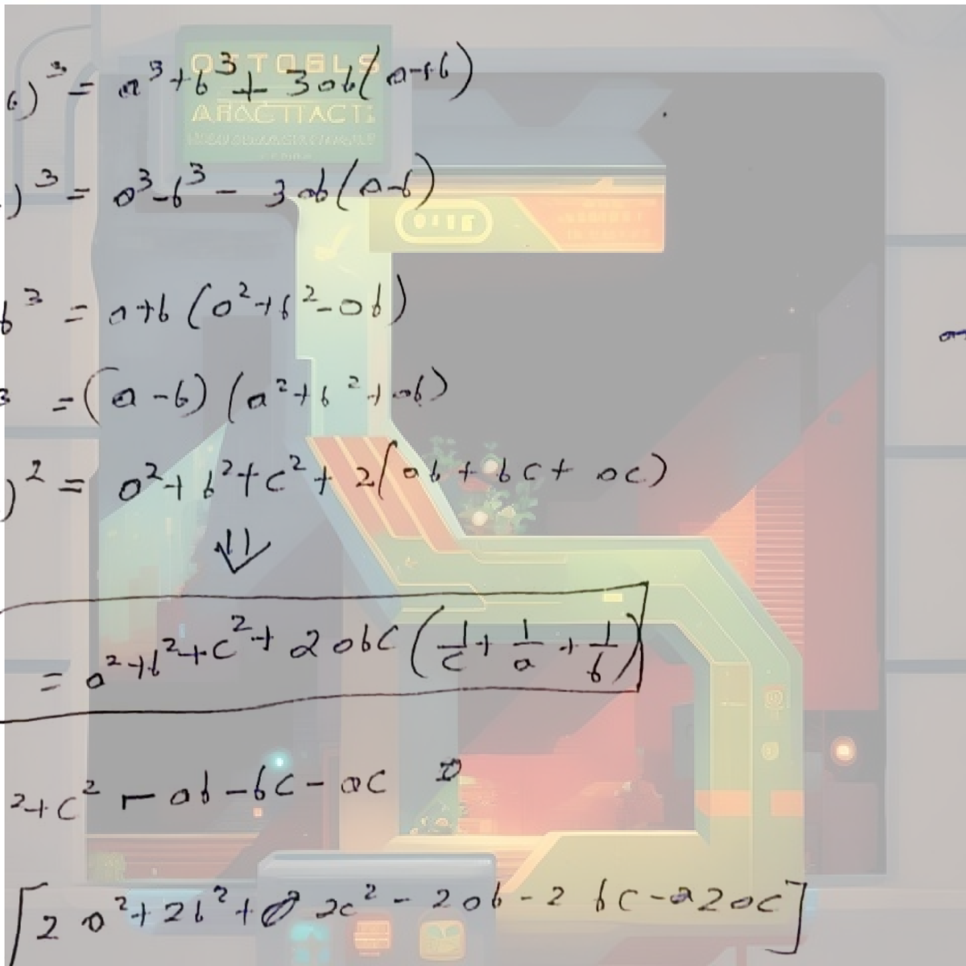
① $a^2-b^2 = (a+b)(a-b)$

② $(a+b)^2 = a^2+b^2+2ab$

③ $(a-b)^2 = a^2+b^2-2ab$

④ ② - ③

$$\boxed{(a+b)^2-(a-b)^2 = 2ab+2ab}$$



⑤ $(a+b)^3 = a^3+b^3+3ab(a+b)$

⑥ $(a-b)^3 = a^3-b^3-3ab(a-b)$

⑦ $a^3+b^3 = a+b(a^2+b^2-ab)$ ~~a+b~~

⑧ $a^3-b^3 = (a-b)(a^2+b^2+ab)$

⑨ $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ac)$

$$\Downarrow$$

$$\boxed{= a^2+b^2+c^2+2abc\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)}$$

⑩ $a^2+b^2+c^2-ab-bc-ac$

$= \frac{1}{2}\left[2a^2+2b^2+2c^2-2ab-2bc-2ac\right]$

$= \frac{1}{2}\left[a^2+a^2+b^2+b^2+c^2+c^2-2ab-2bc-2ac\right]$

$= \frac{1}{2}\left[a^2+b^2-2ab+a^2+c^2-2ac+b^2+c^2-2bc\right]$

$$\boxed{= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (a-c)^2\right]}$$

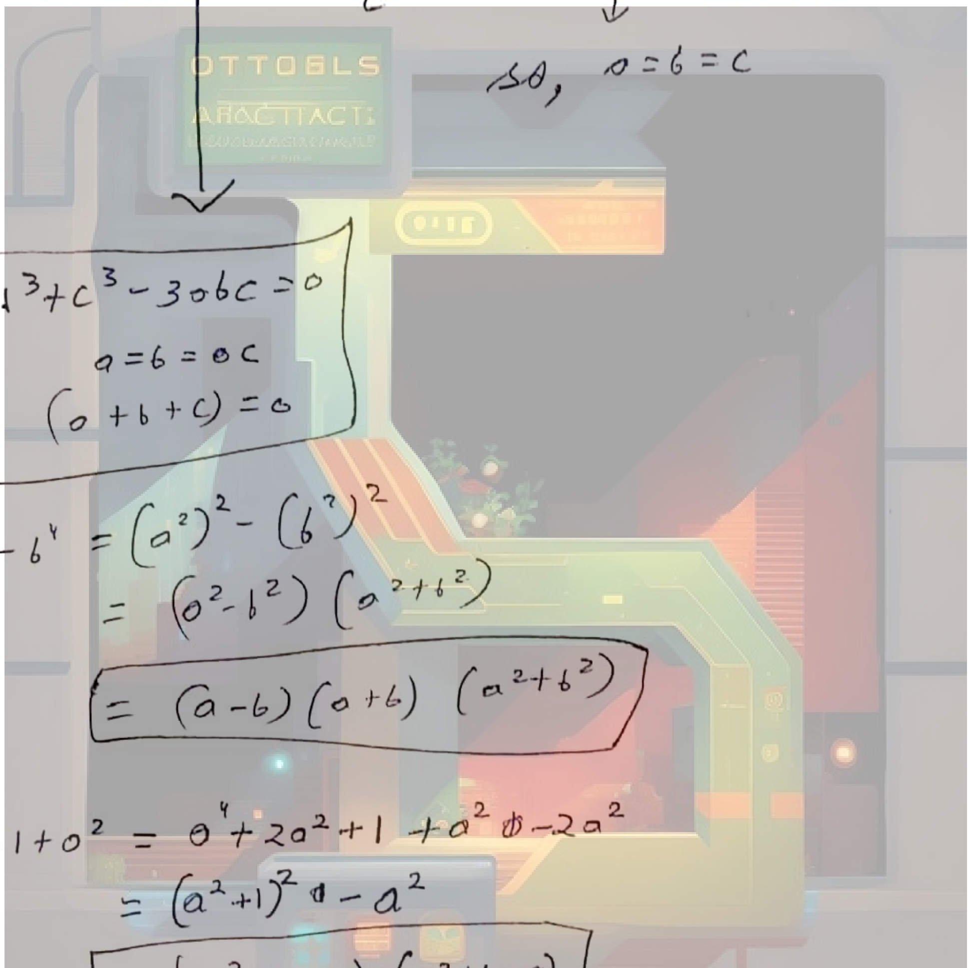
Always greater or equal to zero.

(11) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

(12) $a^3+b^3+c^3-3abc = 0$ if $(a+b+c) = 0$

Or $\left[\frac{1}{2}\left[(a-b)^2+(b-c)^2+(a-c)^2\right]\right] = 0$

so, $a = b = c$



$$\boxed{\begin{gathered} a^3+b^3+c^3-3abc = 0 \\ a = b = c \\ (a+b+c) = 0 \end{gathered}}$$

(13) $a^4-b^4 = (a^2)^2-(b^2)^2$

$= (a^2-b^2)(a^2+b^2)$

$$\boxed{= (a-b)(a+b)(a^2+b^2)}$$

(14) $a^4+1+a^2 = a^4+2a^2+1+a^2-2a^2$

$= (a^2+1)^2 - a^2$

$$\boxed{= (a^2+1+a)(a^2+1-a)}$$

H.W. 29-04-2024

Q find value of the following:-

~~(1) $(x-y)^2$.~~

(1) $$\frac{(x-y)^3+(y-z)^3+(z-x)^3}{3[(x-y)\cancel{(x-z)}(y-z)(z-x)}$$

(2) $$\frac{(2.75)^3-(2.25)^3}{(2.75)^2+2.75\times 2.25+(2.25)^2}$$

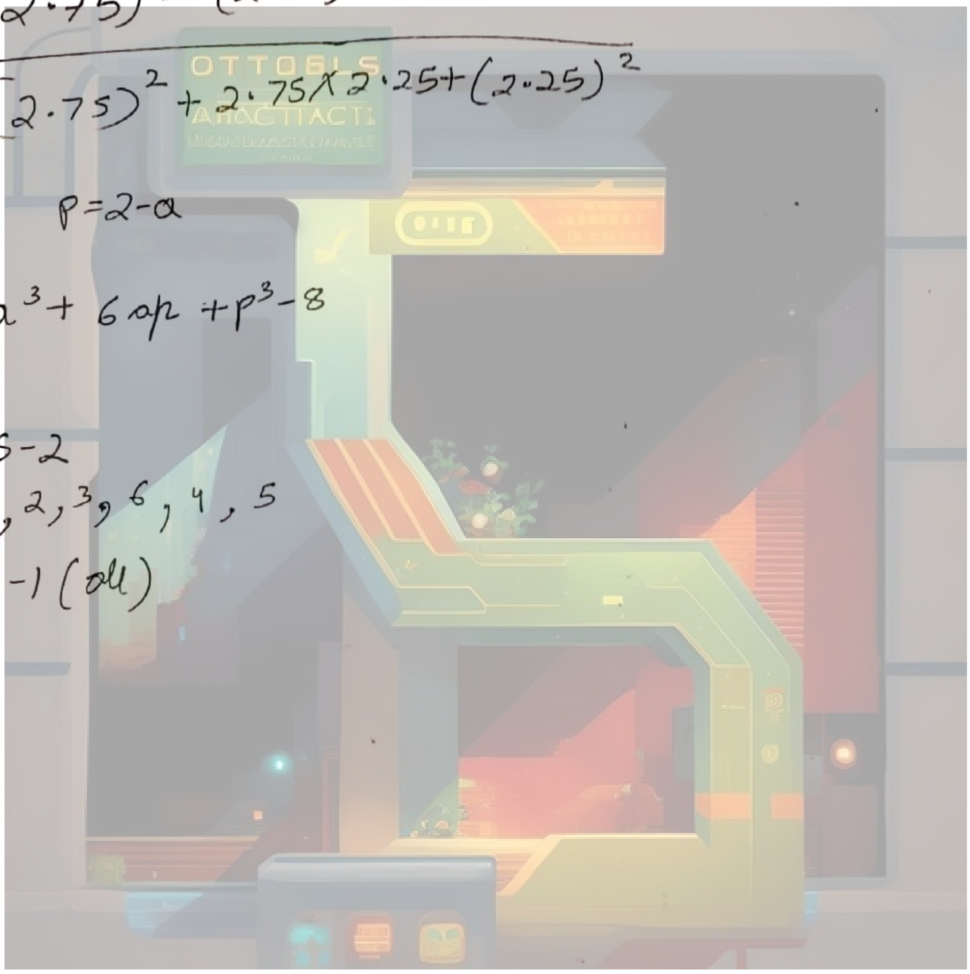
(3) if $p=2-a$

$a^3+6ap+p^3-8$

(4) DYS-2

Q 1, 2, 3, 6, 4, 5

DYS-1 (all)



① $A = x - y$

$B = y - z$

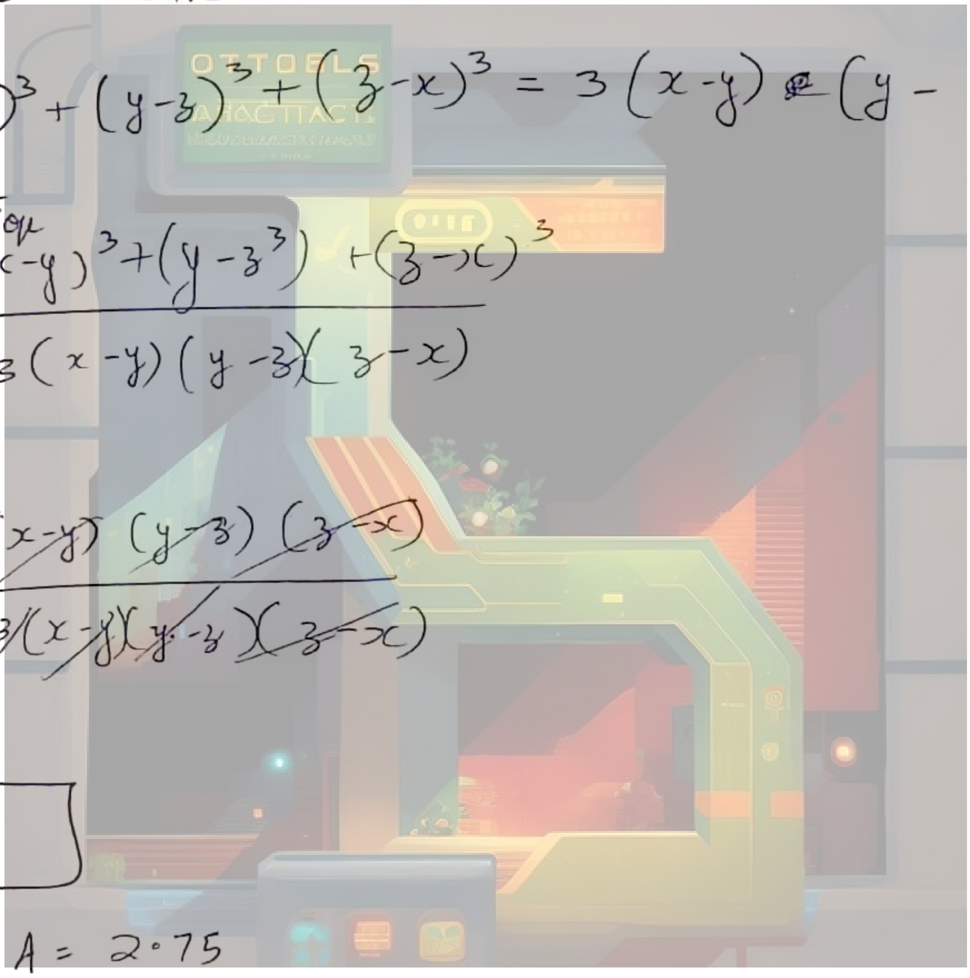
$C = z - x$

$A + B + C = (x-y) + (y-z) + (z-x)$

$= \cancel{x} - \cancel{y} + \cancel{y} - \cancel{z} + \cancel{z} - \cancel{x}$

$= 0$

$A^3 + b^3 + c^3 = 3ABC$



$(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$

Given Eqn

$$\frac{(x-y)^3 + (y-z^3) + (z-x)^3}{3(x-y)(y-z)(z-x)}$$

$$\frac{\cancel{3(x-y)(y-z)(z-x)}}{\cancel{3(x-y)(y-z)(z-x)}}$$

$\boxed{= 1}$

② let $A = 2.75$

$B = 2.25$

Given :-

$$\frac{A^3 - B^3}{A^2 + AB + B^2}$$

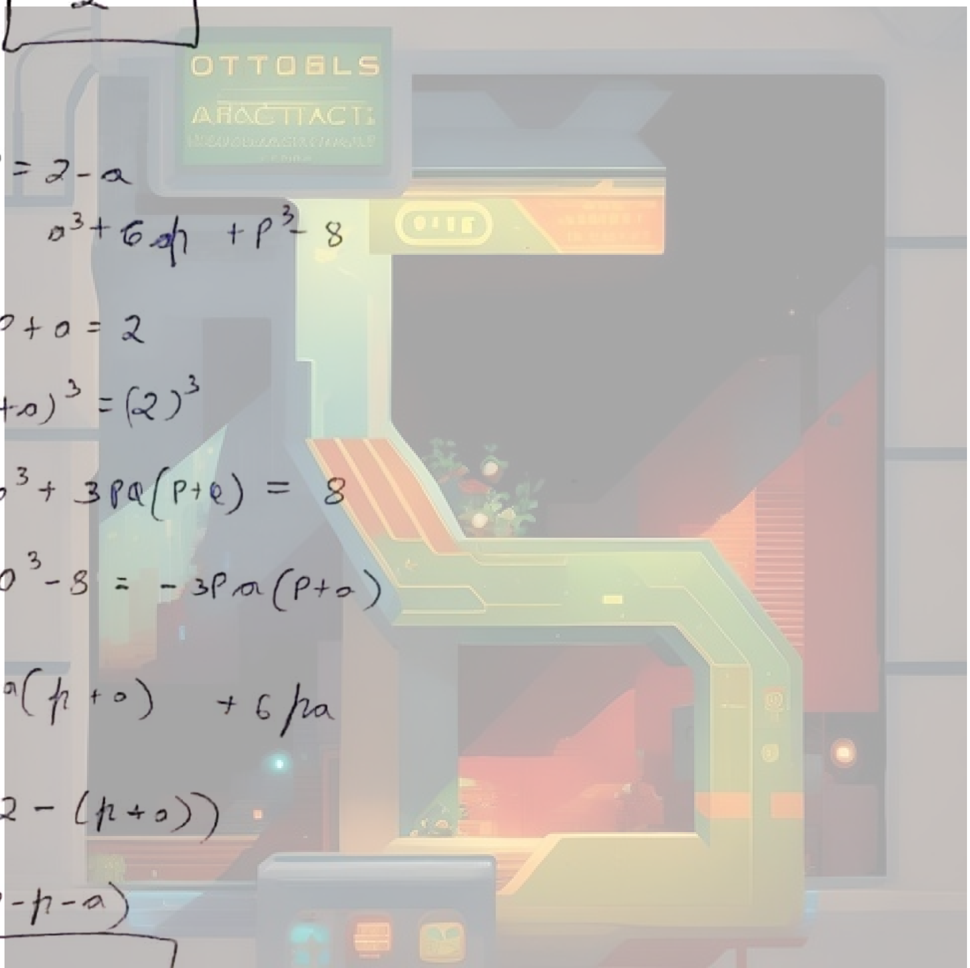
$$= \frac{(A-B)\cancel{(A^2+B^2+AB)}}{\cancel{A^2+B^2+AB}}$$

$$= A - B$$

$$= 2.75 - 2.25$$

$$= 0.50$$

$$\boxed{= \frac{1}{2}}$$



Q3. $p = 2 - a$

find $a^3 + 6ap + p^3 - 8$

$$p + a = 2$$

$$(p+a)^3 = (2)^3$$

$$p^3 + a^3 + 3pa(p+a) = 8$$

$$p^3 + a^3 - 8 = -3pa(p+a)$$

$$-3pa(p+a) + 6pa$$

$$+3pa(2 - (p+a))$$

$$+3(2 - p - a)$$

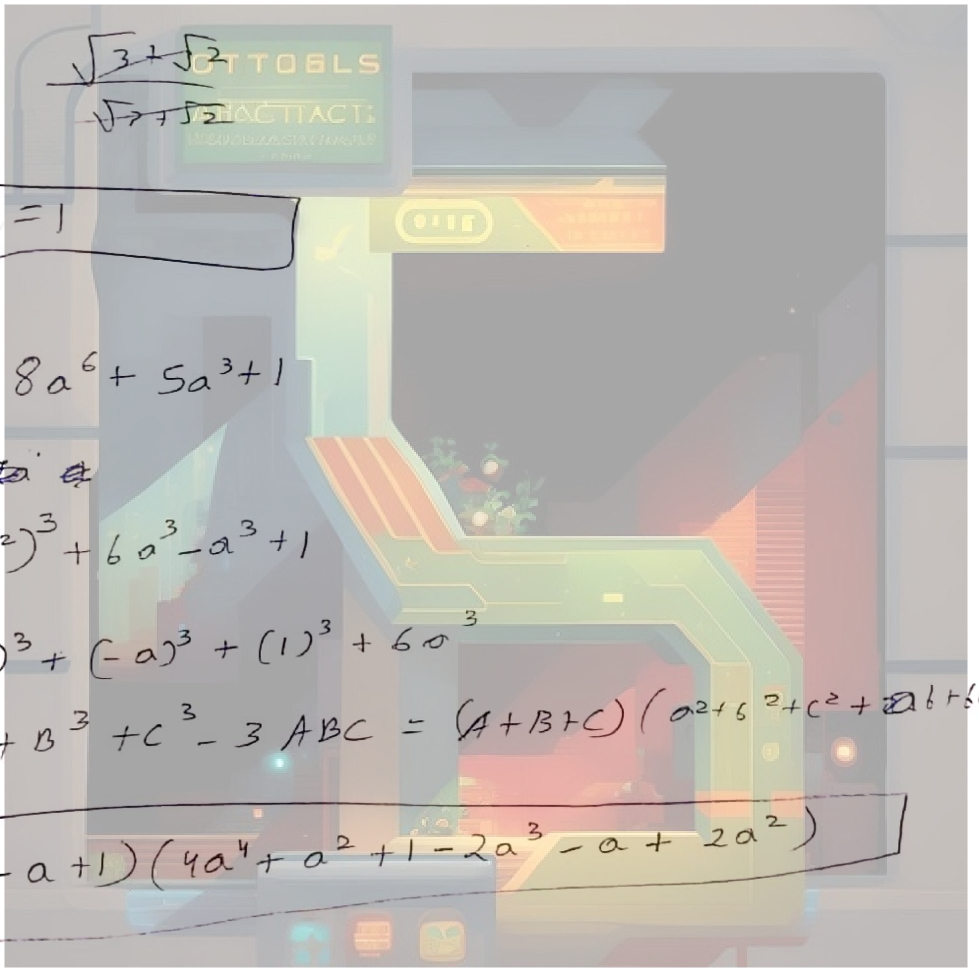
$$\boxed{6 - 3p - 3a}$$

① DYS-1

Q6, iii) $\dfrac{\sqrt{6+2\sqrt{3}+2\sqrt{2}+2\sqrt{6}}-1}{\sqrt{5+2\sqrt{6}}}$

$\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+(\sqrt{1})^2+2\sqrt{2}\sqrt{1}+2\sqrt{3}\sqrt{1}+2\sqrt{3}\sqrt{2}}$

$\dfrac{\cancel{\sqrt{(\sqrt{3}+\sqrt{2}+\sqrt{1})^2}}-1}{\sqrt{3}+\sqrt{2}}$



$\cancel{\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}}$

$\boxed{=1}$

DYS-2

Q6 i) $8a^6+5a^3+1$

$(2a^2)^3+6a^3-a^3+1$

$(2a^2)^3+(-a)^3+(1)^3+6a^3$

$A^3+B^3+C^3-3ABC=(A+B+C)(a^2+b^2+c^2+ab+bc+ac)$

$\boxed{(2a^2-a+1)(4a^4+a^2+1-2a^3-a+2a^2)}$

Q If $4x^2+3y^2+16z^2-4x+12y-24z+14=0$

$x, y, z = ?$

$\cancel{(2x)^2+(\sqrt{3}y)^2+(4z)^2-(2)(x)+}$

$\cancel{(2x)^2+(3y)^2+(4z)^2-4x+12y-24z+14=0}$

$\cancel{(2x)^2+(3y)^2+(4z)^2-4(x-3y+6y)+14=0}$

$4x^2-4x+9y^2+12y+16z^2-24z+14=0$

$(2x)^2-(2)(2x)+(1)^2+(3y)^2+(2)(3y)(2)+(2)^2+(4z)^2-(2)(4z)(3)+(3$

$(2x-1)^2+(3y+2)^2+(4z-3)^2=0$

$2x-1=0$	$3y+2=0$	$4z-3=0$
$2x=1$	$3y=-2$	$4z=3$
$\boxed{x=1/2}$	$\boxed{y=-\frac{2}{3}}$	$\boxed{z=3/4}$

Factorisation:

Method-1 :- Conversion into perfect square.

Q- $x^4+3x^2y^2+4y^4$

$(x^2)^2+(2y)^2+4x^2y^2-x^2y^2$

$(x^2+2y^2)^2-x^2y^2$

$(x^2+2y^2+xy)(x^2+2y^2-xy)$

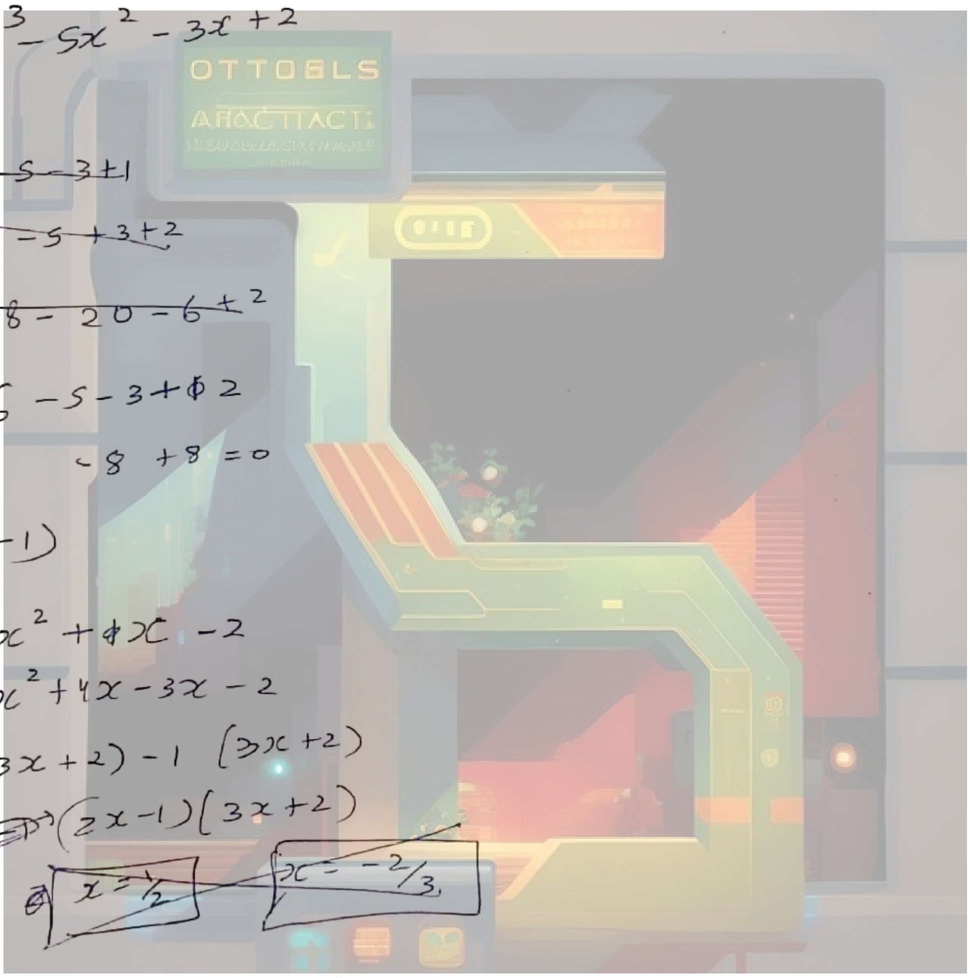
Q $4x^4+81$

$(2x^2)^2 + (9)^2 + 36x^2 - 36x^2$

$(2x^2+9)^2 - (6x)^2$

$\boxed{(2x^2+9-6x)(2x^2+9+6x)}$

Method-2 - Factor Theorm



$6x^3-5x^2-3x+2$

$\cancel{x=1}$

$\cancel{6-5-3+1}$

$\cancel{-6-5+3+2}$

$\cancel{48-20-6+2}$

$6-5-3+2$

$-8+8=0$

$(x-1)$

$6x^2+x-2$

$6x^2+4x-3x-2$

$2(3x+2)-1(3x+2)$

$(2x-1)(3x+2)$

$\cancel{\boxed{x=\frac{1}{2}}}$ $\cancel{\boxed{x=-\frac{2}{3}}}$

$(2x-1)(3x+2)(x-1)$

Method-3

$$6x^3-5x^2-3x+2 = (x-1)(6x^2+bx-2)$$

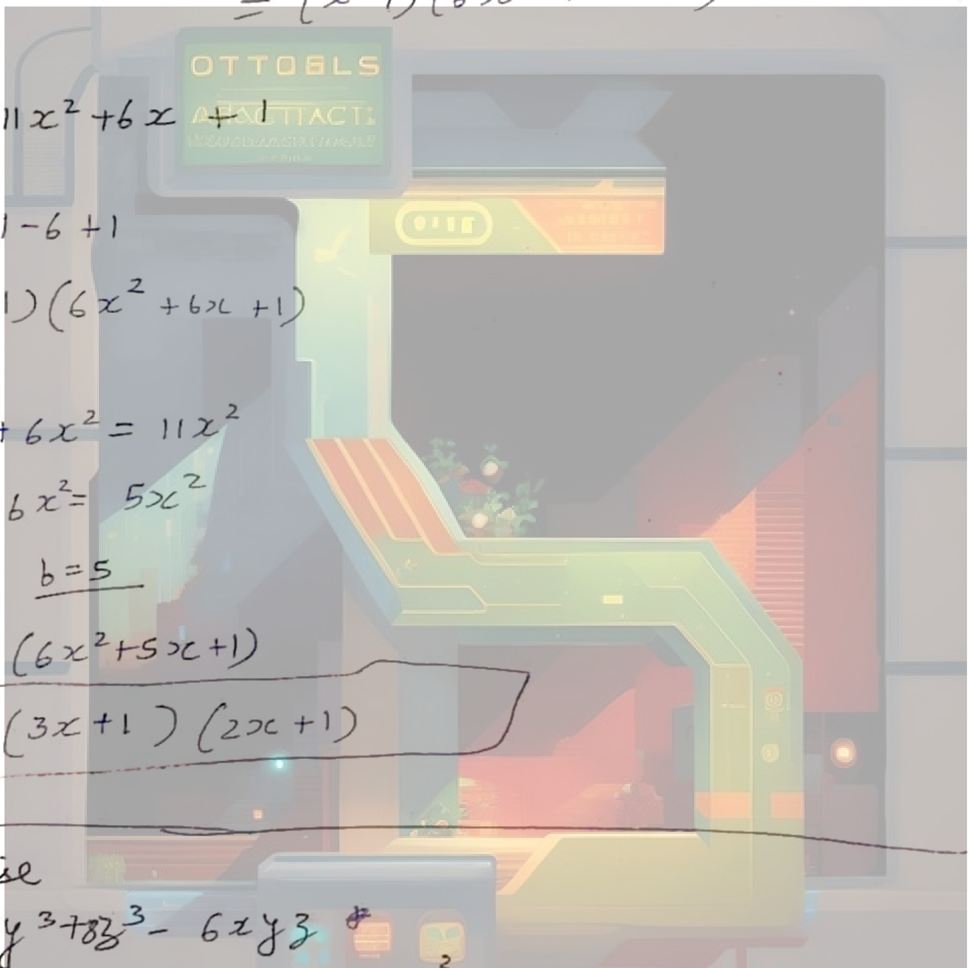
$$\cancel{6x^2}\cdot bx^2-6x^2=-5x^2$$

$$bx^2=\cancel{x}\,6x^2-5x^2$$

$$bx^2=x^2$$

$$\underline{b=1}$$

$$=(x-1)(6x^2+x-2)$$



Q $6x^3+11x^2+6x+1$

$-6+11-6+1$

$(x+1)(6x^2+bx+1)$

$bx+6x^2=11x^2$

$bx^2=5x^2$

$\underline{b=5}$

$(x+1)(6x^2+5x+1)$

$$\boxed{(x+1)(3x+1)(2x+1)}$$

Q Factorise

① $x^3+y^3+8z^3-6xyz$

② $(x-y)^3+(y-z)^3+(z-x)^3$

② $\boxed{(3)(x-y)(y-z)(z-x)}$

① $\cancel{(x+y+z)(x^2+y^2+z^2+xy+yz+xz)}$

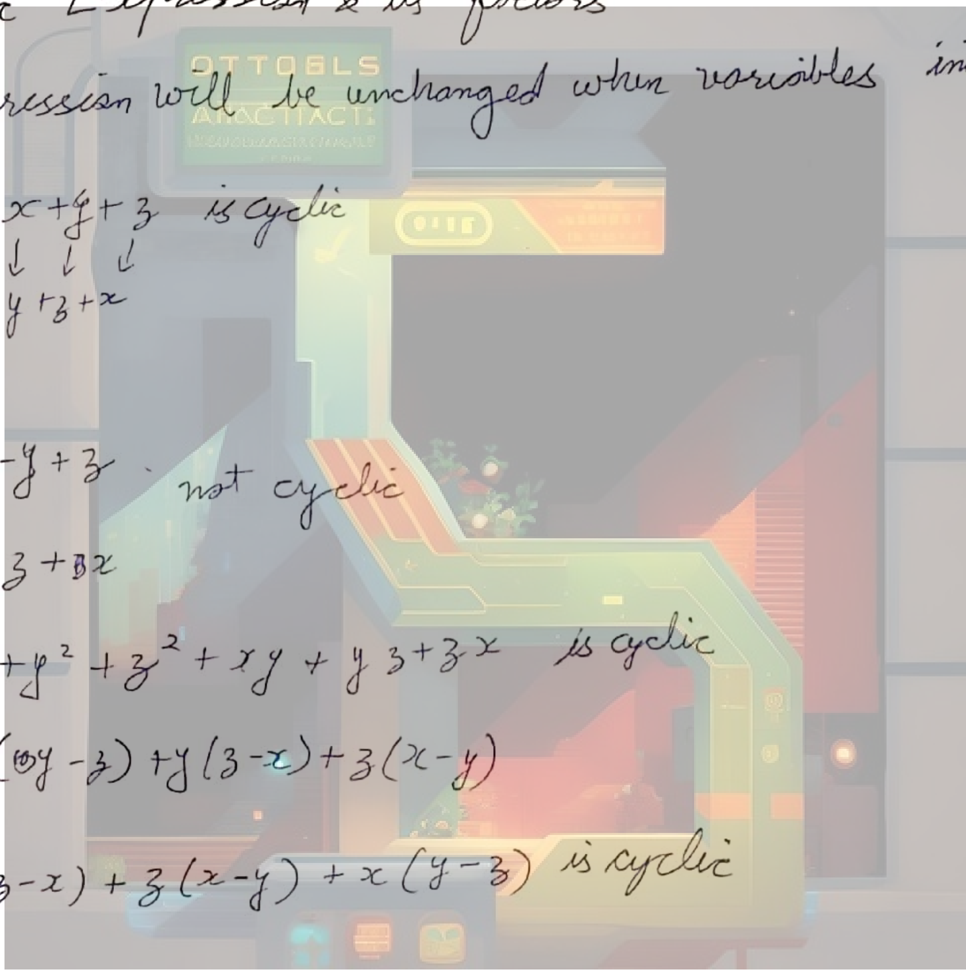
$$(x)^3+(y)^3+(2z)^3+3(2z\times x\times y)$$

$$A^3+B^3+C^3-3ABC=(A+B+C)(A^2+B^2+C^2-AB-BC-AC)$$

$$\boxed{(x+y+2z)(x^2+y^2+4z^2-xy-2xz-2yz)}$$

③ Cyclic Expression & its factors

→ Expression will be unchanged when variables interchanged



eg. $x+y+z$ is cyclic

$\downarrow\ \downarrow\ \downarrow$

$y+z+x$

eg2 $x-y+z$ not cyclic

$\downarrow$

$y-z+x$

eg 3. $x^2+y^2+z^2+xy+yz+zx$ is cyclic

eg 4. $x(y-z)+y(z-x)+z(x-y)$

$y(z-x)+z(x-y)+x(y-z)$ is cyclic

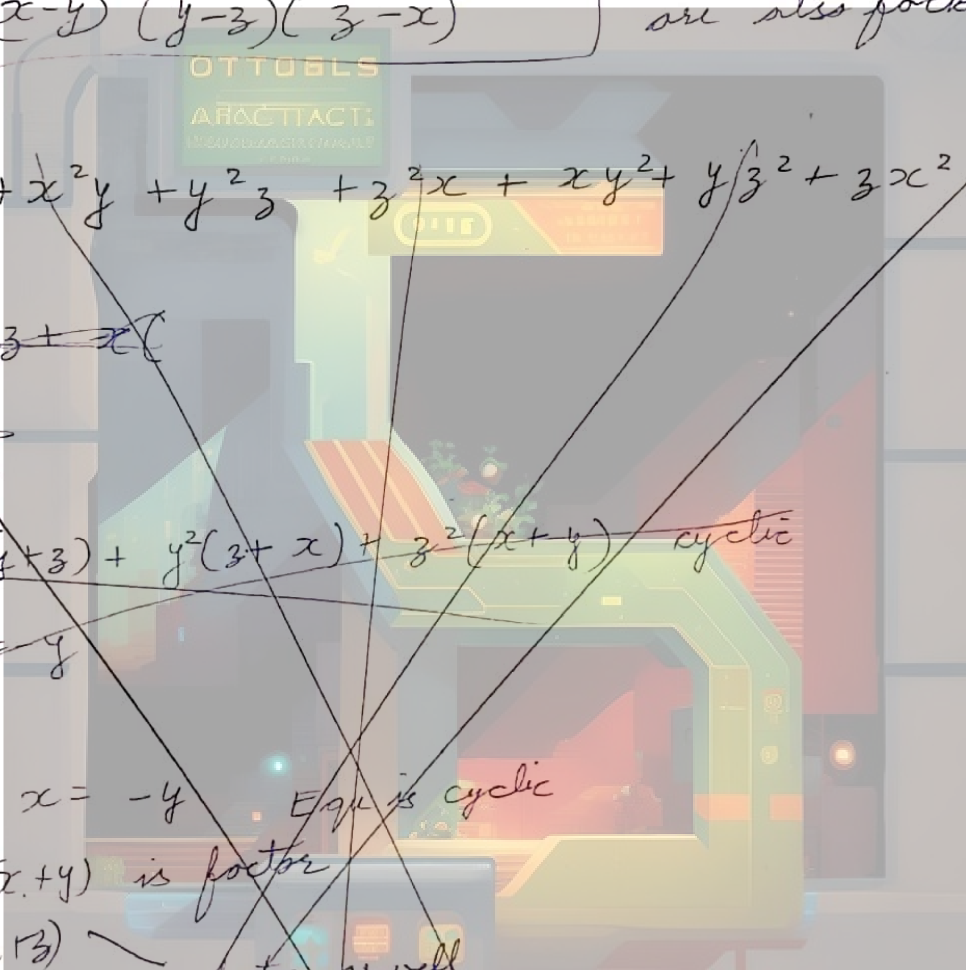
Note- In cyclic Expression if $(x-y)$ is a factor then $(y-z)$ and $(z-x)$ are also factors.

Q $x^2(y-z)+y^2(z-x)+z^2(x-y)$

$x^2(x-z)+x^2(z-x)+\cancel{z^2(x-x)}$

$x^2(x-z+z-x)=0$ so $(x-y)$ is a factor) and it is

cyclic so. $\boxed{(x-y)(y-z)(z-x)}$ are also factors.



Q $2xyz+x^2y+y^2z+z^2x+xy^2+yz^2+zx^2$

$\cancel{2xyz+x(}$

$\cancel{x(y}$

$\cancel{2xyz}+x^2(y+z)+y^2(z+x)+z^2(x+y)$ cyclic

$\cancel{x=y}$

$x=-y$ Eqn is cyclic

$(x+y)$ is factor

$(y+z)$

$(z+x)$ is a factor as well

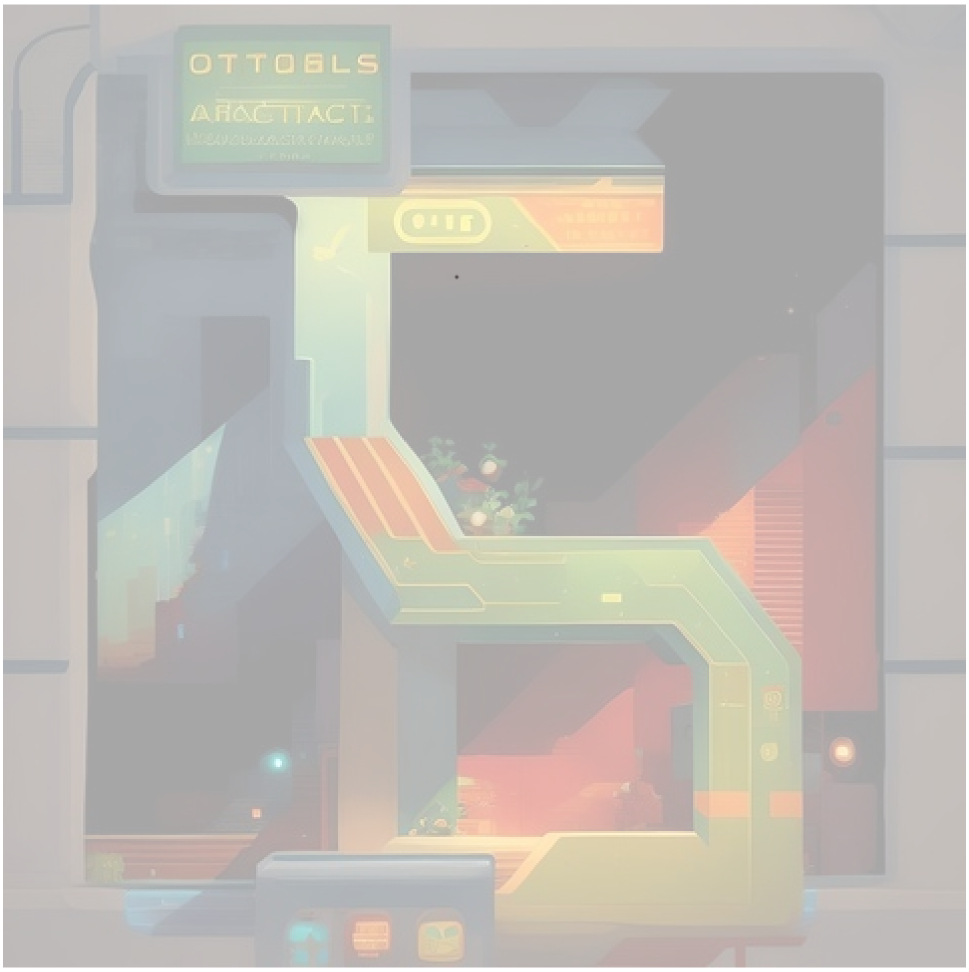
~~DYS-2~~ H.W. 30-04-2024

DYS-2

Q8 - i), ii), iii), iv), v), vi), vii), viii)

DYS-3

Q1, 2, 3, 4, 5, 7



Q $2xyz + x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2$

It is cyclic

$x = -y$, value $= 0$

$(x+y)$ is a factor

so $(y+z)$ are also factors
$(z+x)$

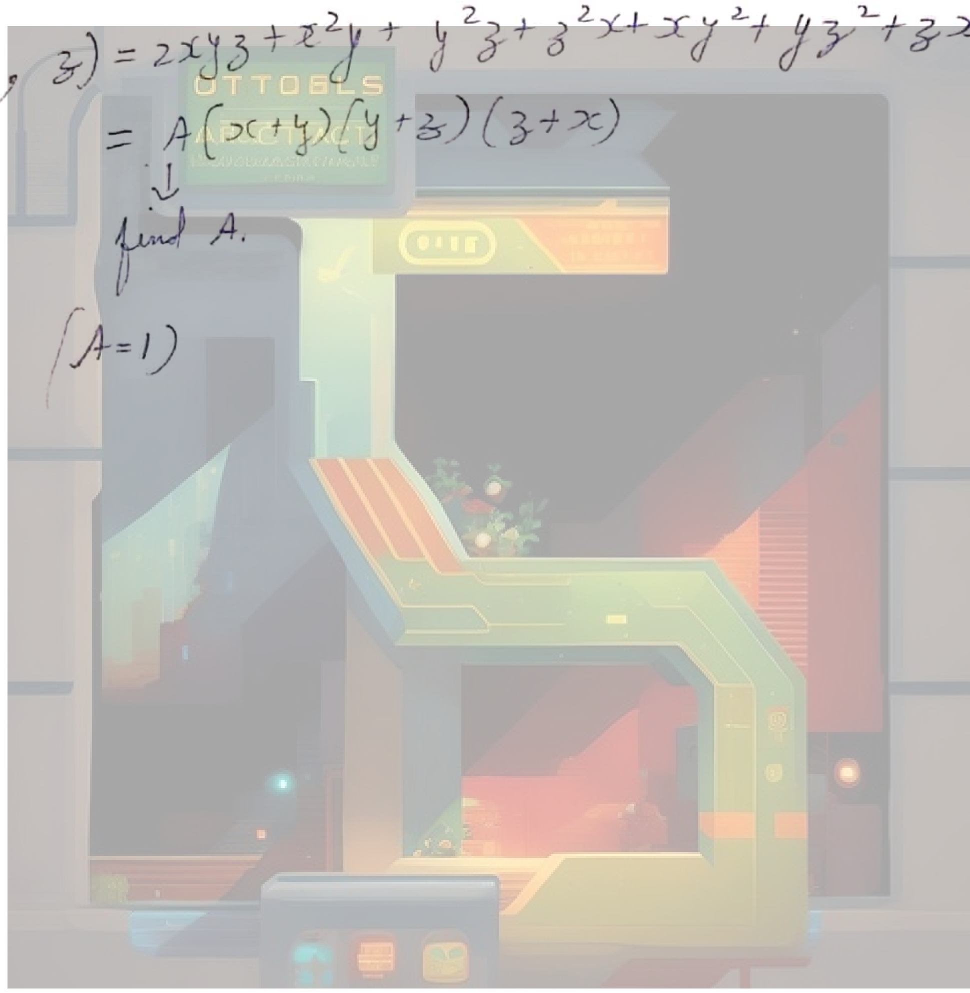
$E(x,y,z) = 2xyz + x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2$

$= A(x+y)(y+z)(z+x)$

$\downarrow$

find A.

$(A=1)$



DYS-3

Q6. $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$

$a^3 = 2+\sqrt{5}$ $\quad b^3 = 2-\sqrt{5}$

$(a^3)^{1/3} + (b^3)^{1/3} = a+b = t$

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$t^3 = 4 + 3(4-5)(t)$

$t^3 = 4 - 3t$

$t^3 + 3t - 4 = 0$

$(t-1)$

$\cancel{t^2+t+4}$ $\quad -t^2+3t-4$

$\cancel{t^2+2}$ $\quad -t$

$2t-4$

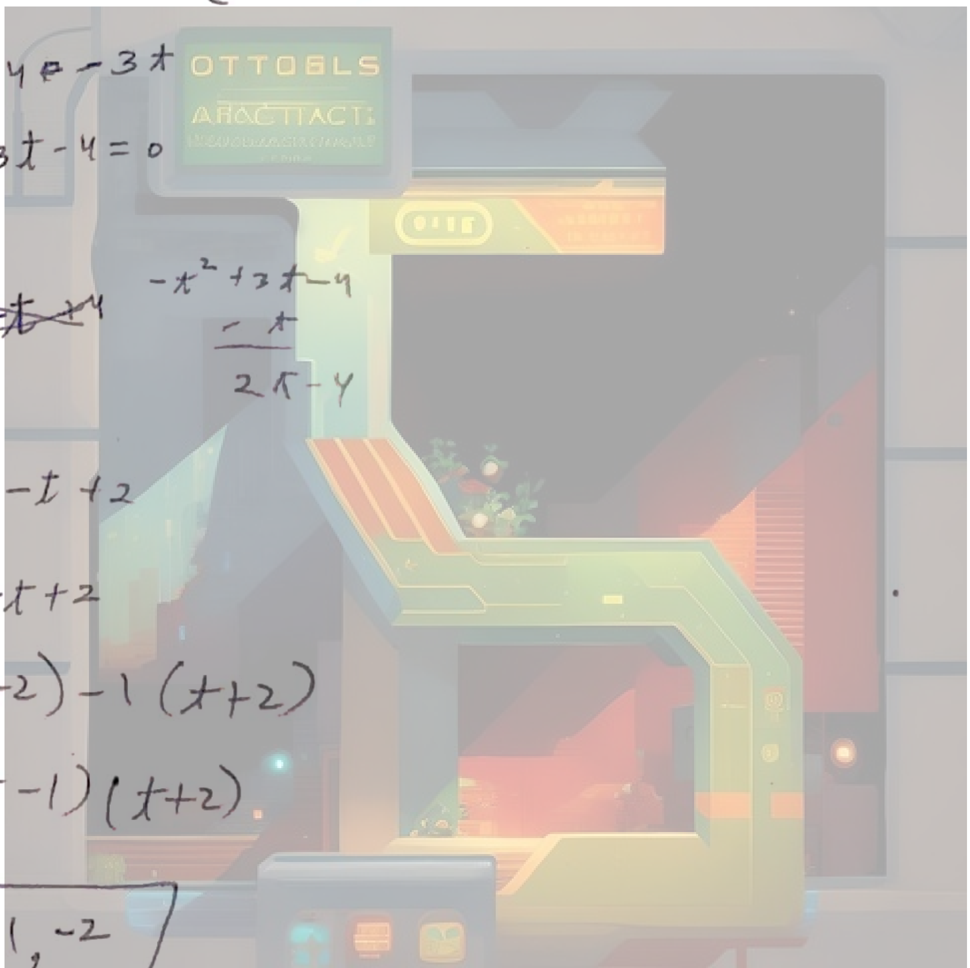
$t^2 - t + 2$

$t^2+2t-t+2$

$t(t+2)-1(t+2)$

$(t-1)(t+2)$

$\boxed{t = 1, -2}$

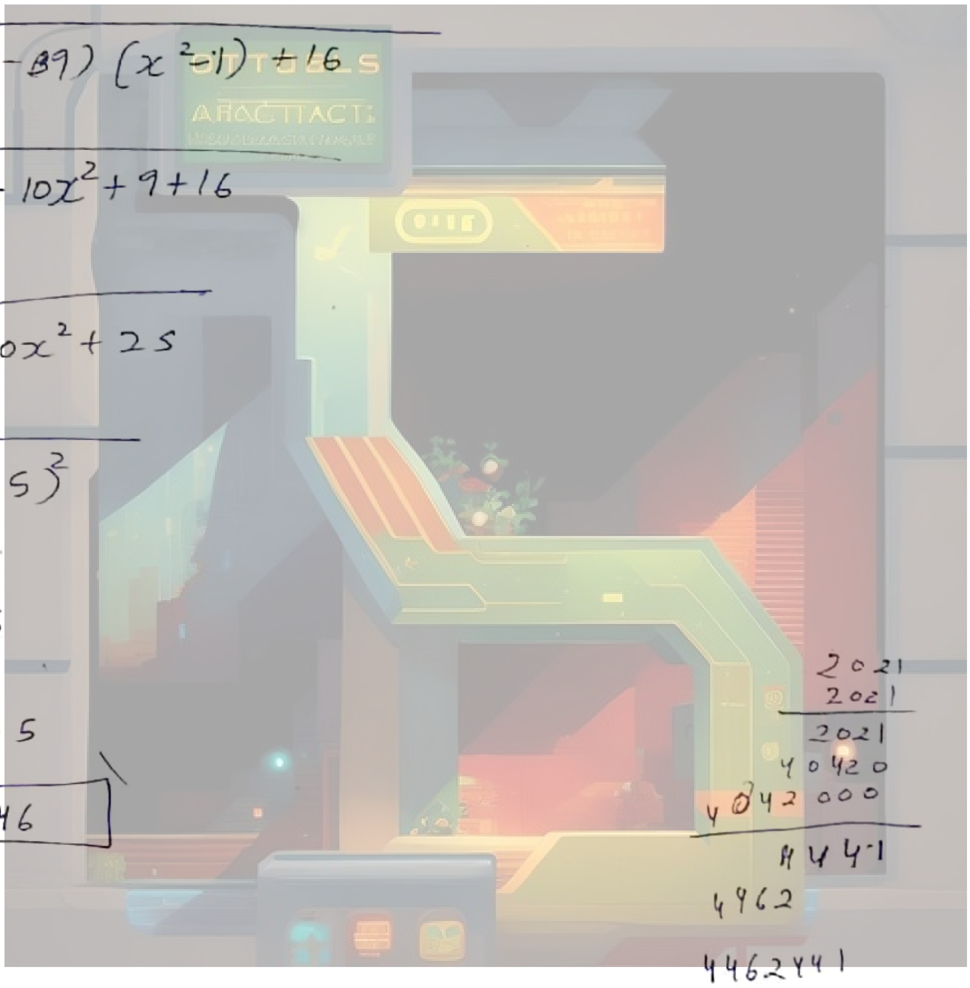


Q. $\sqrt{2024\times 2022\times 2020\times 2018+16}$

$\cancel{4\sqrt{253\times 1011\times 2020\times 2018+1}}$

let $x = 2021$

$\sqrt{(x+3)(x+1)(x-1)(x-3)+16}$



$\sqrt{(x^2-9)(x^2-1)+16}$

$\sqrt{x^4-10x^2+9+16}$

$\sqrt{x^4-10x^2+25}$

$\sqrt{(x^2-5)^2}$

x^2-5

2021^2-5

$\boxed{4462446}$

$$\begin{array}{r} 2021 \\ \times\ 2021 \\ \hline 2021 \\ 40420 \\ 4042000 \\ \hline 4441 \\ 4462 \\ 4462441 \end{array}$$

Q2. $(2+1)(2^2+1)(2^4+1)(2^8+1) - 2^{16}$

$\cancel{(3)(5)(17)(257)} - 2^{16}$

$1 \times (2-1) =$

$$\frac{(2-1)(2+1)(2^2+1)(2^4+1)(2^8+1) - 2^{16}}{\cancel{(2+1)}}$$

$(2^2-1)(2^2+1)$

$(2^4-1)(2^4+1)$

$(2^8-1)(2^8+1)$

$(\cancel{2^{16}}-1) - \cancel{2^{16}}$

$\boxed{-1}$

Polynomial

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots a_2 x^2 + a_1 x + a_0$

$a_n \neq 0$ ($a_n \rightarrow$ leading coefficient)

$n \in$ whole no.

$a_{n-1}, a_{n-2} \dots a_0$

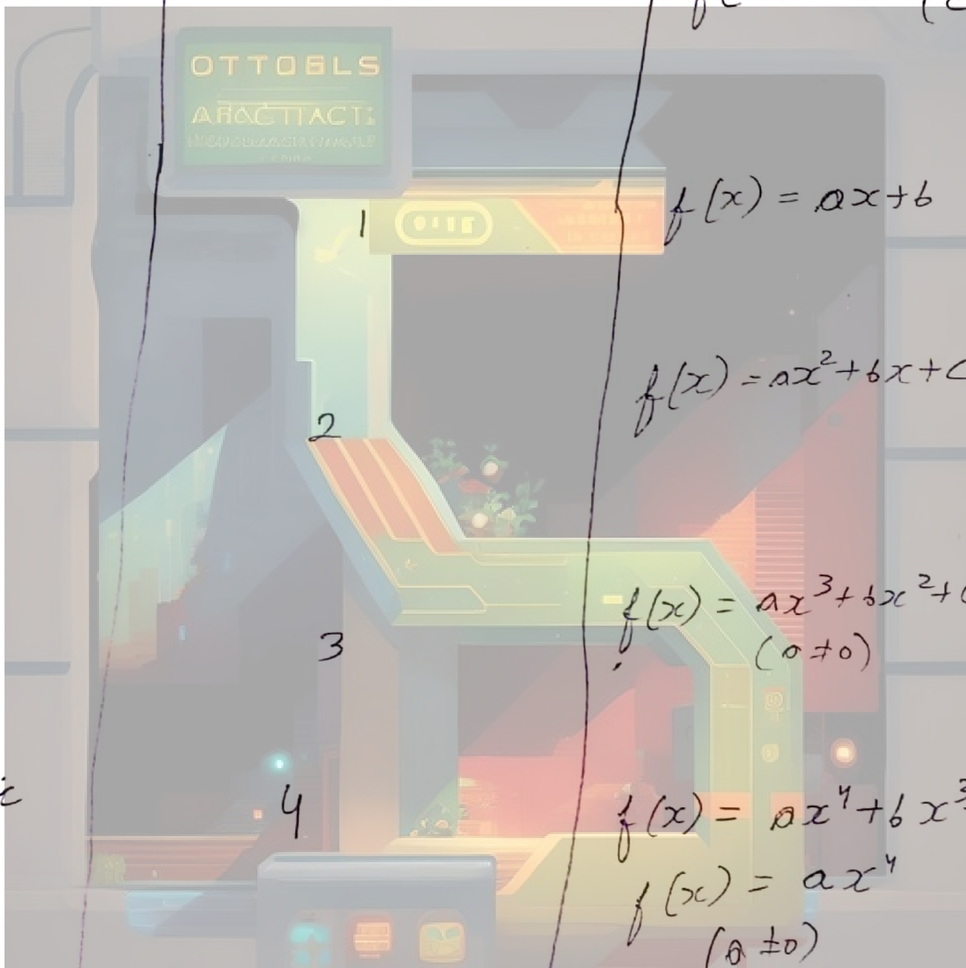
$a_n = 1$ (monic polynomial)

monic polynomial $\rightarrow$ coefficient is 1

monomial $\rightarrow$ one term.

Types of Polynomial

Name	Degree	Format
(1). zero polynomial	Not-Defined	$f(x) = 0$ $f(x) = 0x^{3}$ $= 0x^2$ $= 0x^3$ $\Big\}$ 0
(2). Non-zero polynomial	0	$f(x) = c \quad (c \neq 0)$
(3). Linear	1	$f(x) = ax + b \quad (a \neq 0)$
(4) Quadratic	2	$f(x) = ax^2 + bx + c \quad (a \neq 0)$
(5) Cubic	3	$f(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$
(6) Biquadratic	4	$f(x) = ax^4 + bx^3 + cx^2 + dx + e$ $f(x) = ax^4$ $(a \neq 0)$

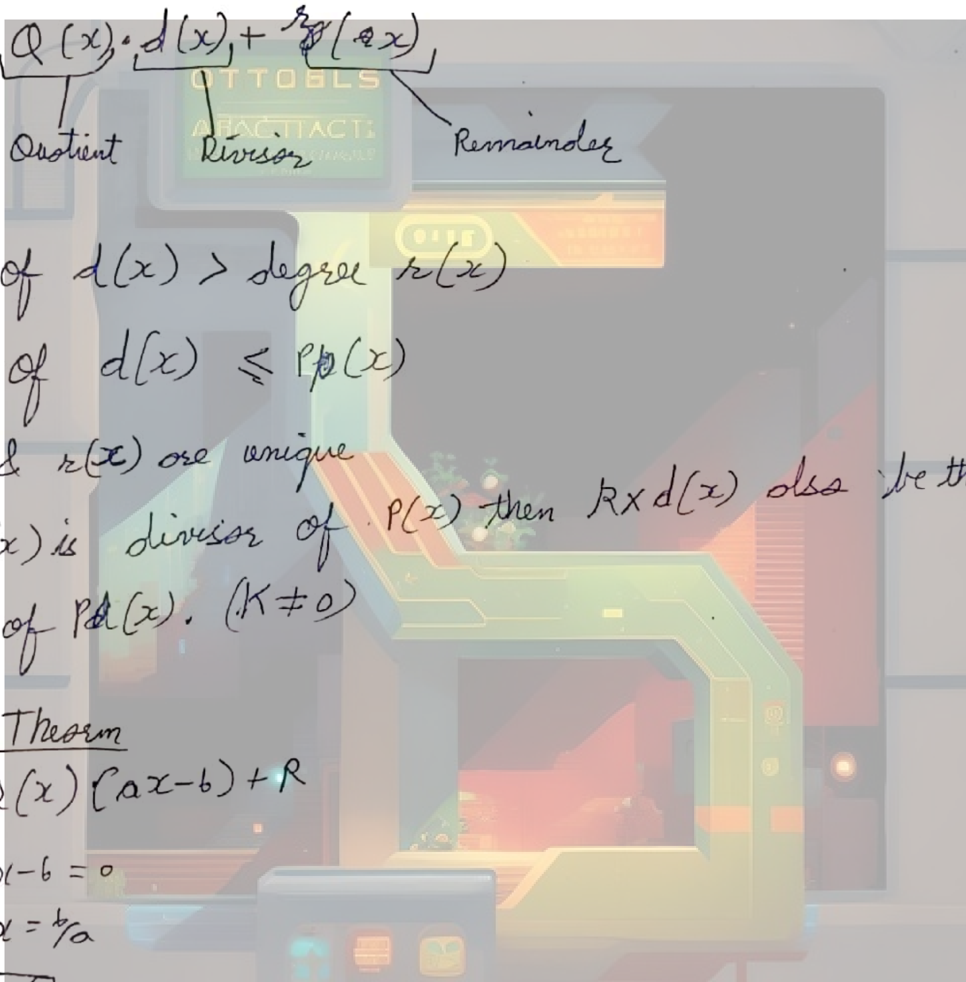


✱ Adding two polynomials may or not result in change in degree

$(x^8+x^7) + (x^8-x^6) = 2x^8+x^7-x^6$

$(x^8+x^7) - (x^8-x^6) = x^{\cancel{8}7}+x^6$ (change)

✱ ~~Mult~~



Division in polynomial

$$\underbrace{P(x)}_{\text{Divident}} = \underbrace{Q(x)}_{\text{Quotient}} \cdot \underbrace{d(x)}_{\text{Divisor}} + \underbrace{r(x)}_{\text{Remainder}}$$

① Degree of $d(x) >$ degree $r(x)$

② Degree of $d(x) \leq P(x)$

③ $Q(x)$ & $r(x)$ are unique

④ If $d(x)$ is divisor of $P(x)$ then $k \times d(x)$ also be the divisor of $P(x)$. $(K \neq 0)$

Remainder Theorm

$P(x) = Q(x)(ax-b)+R$

$ax-b=0$

$x = b/a$

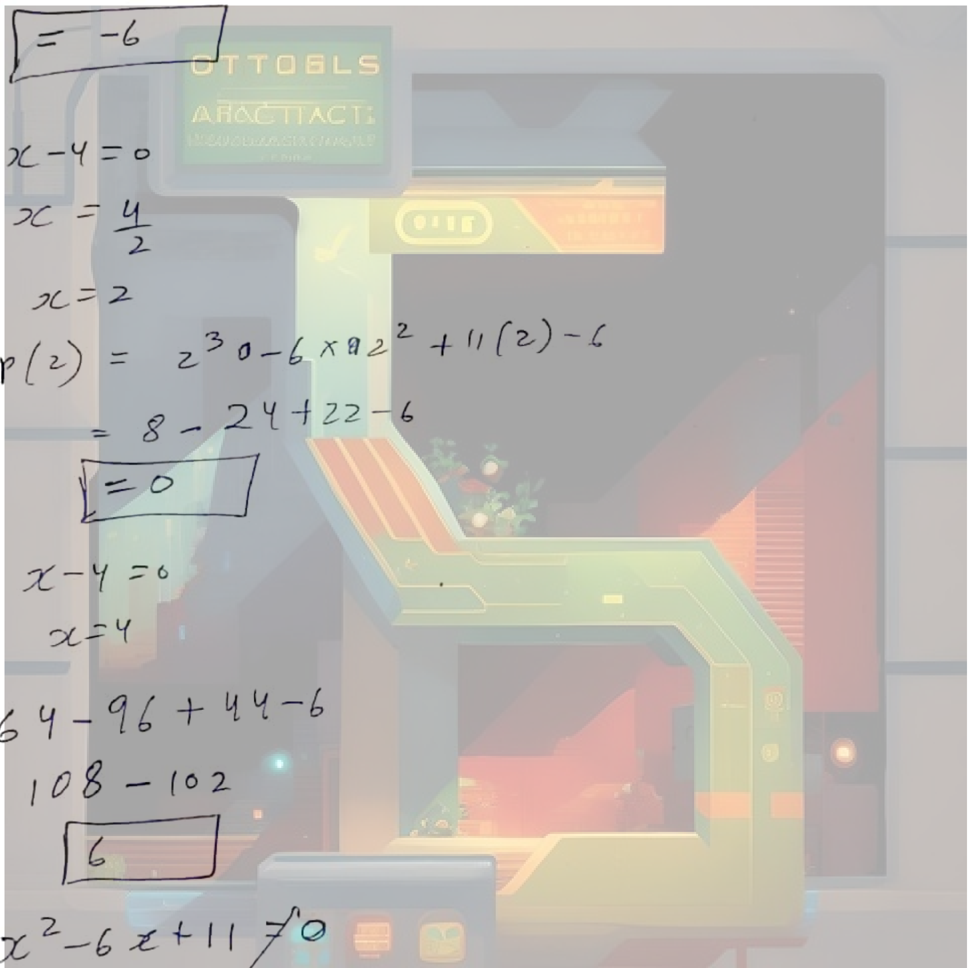
$\cancel{\boxed{P(b/a) = R}}$

$P(b/a) = Q(b/a)\left(a \times \frac{b}{a} - b\right) + R$

$\boxed{P(b/a) = 0+R}$

Q1. find remainder if $P(x) = x^3 - 6x^2 + 11x - 6$ is divided by

1. x
2. $2x - 4$
3. $x - 4$
4. $x^2 - 6x + 11$



(1) $x = 0$

$P(0) = 0 - 0 + 0 - 6$

$\boxed{= -6}$

(2) $2x - 4 = 0$

$x = \frac{4}{2}$

$x = 2$

$P(2) = 2^3 - 6 \times 2^2 + 11(2) - 6$

$= 8 - 24 + 22 - 6$

$\boxed{= 0}$

(3) $x - 4 = 0$

$x = 4$

$64 - 96 + 44 - 6$

$108 - 102$

$\boxed{6}$

(4) $x^2 - 6x + 11 = 0$

$\cancel{6 \pm \sqrt{36 - 44}}$

$\cancel{12x^2 - 11x}$

$\cancel{x + 12}$ $+72x$

$61x$

126

(4) $x^2-6x+11 \overline{\big)\, x^3-6x^2+11x-6\,}\big(\, x$

$-x^3+6x^2-11x$

$\boxed{-6}$

OR

$x^2-6x+11=0$

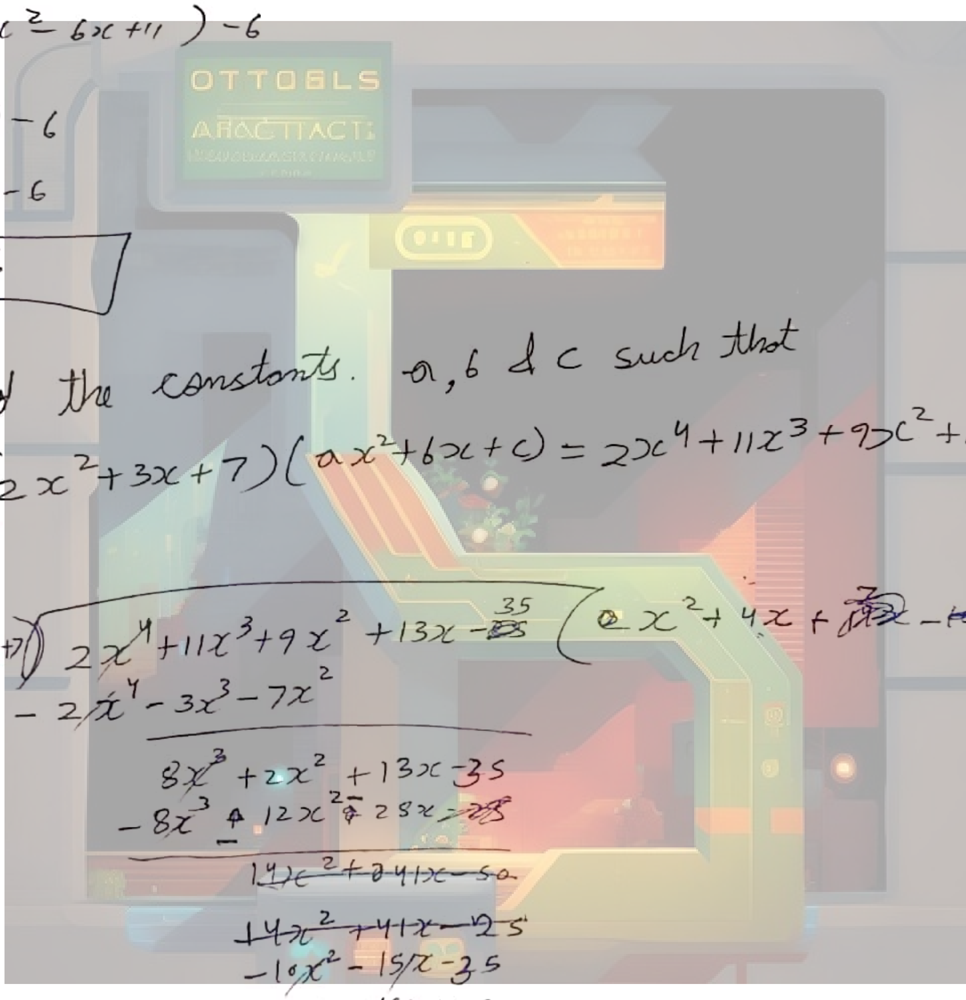
$x^3-6x^2+11x-6$

$x(x^2-6x+11)-6$

$x(0)-6$

$0-6$

$\boxed{-6}$



Q2. find the constants. a, b & c such that
$(2x^2+3x+7)(ax^2+bx+c) = 2x^4+11x^3+9x^2+13x-35$

$2x^2+3x+7 \overline{\big)\, 2x^4+11x^3+9x^2+13x-35\,}\big(\, x^2+4x-5$

$-2x^4-3x^3-7x^2$

$8x^3+2x^2+13x-35$

$-8x^3-12x^2-28x$

~~$14x^2+241x-50$~~

~~$+4x^2+41x-25$~~

$-10x^2-15x-35$

$+10x^2+15x+35$

0

$a = 1$

$b = +4$

$c = -5$

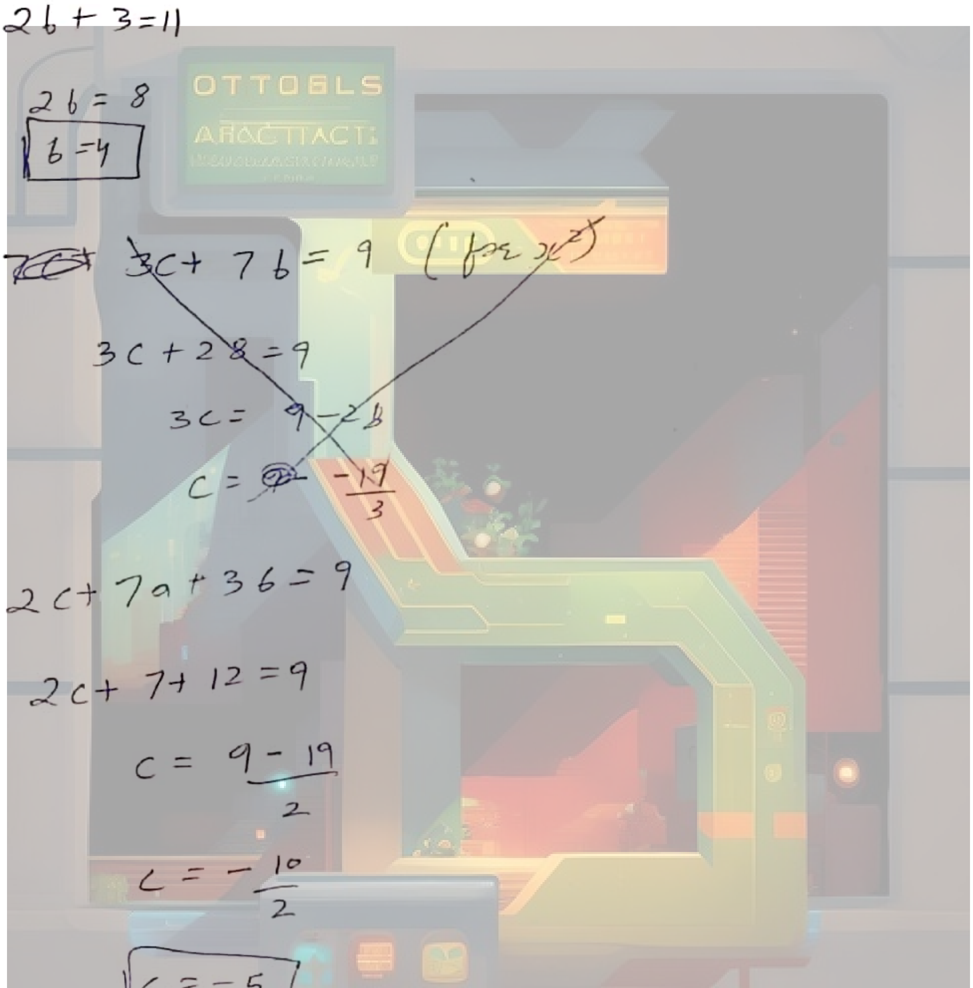
$2x^4 = 2x^2 \times ax^2$

$2x^4 = 2ax^4$

$\therefore \boxed{a=1}$

$2b + 3a = 11$ (for x^3)

$2b + 3 = 11$



$2b = 8$

$\boxed{b=4}$

$3c + 7b = 9$ (for x^2)

$3c + 28 = 9$

$3c = 9 - 28$

$c = -\frac{19}{3}$

$2c + 7a + 3b = 9$

$2c + 7 + 12 = 9$

$c = \frac{9 - 19}{2}$

$c = -\frac{10}{2}$

$\boxed{c=-5}$

Q3. find remainder when $p(x) = x^5-3x^3+2x^2+3x+1$ is divided by x^2-1

$-2x$

$x+3$

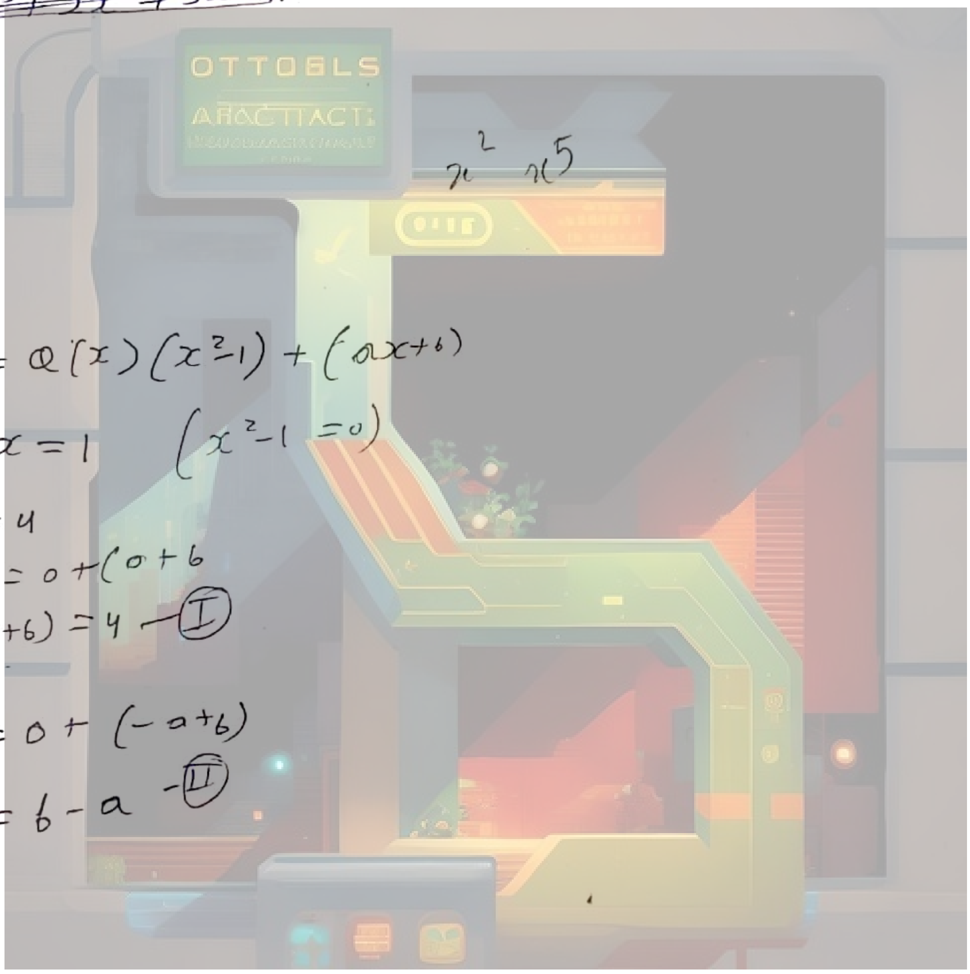
x^3-2x+2

$\boxed{R = x+3}$

~~M-II~~

$\cancel{x^5-3x^3+2x^2+3x+1}$

$\cancel{x^5-x}$



$x^2 \quad x^5$

$P(x) = Q(x)(x^2-1)+(ax+b)$

for $x=1$ $\quad (x^2-1=0)$

$P(1)=4$

$P(1)=0+(a+b$

$(a+b)=4$ — I

$P(-1)=0+(-a+b)$

$2=b-a$ — II

I + II

$2b=6$

$b=3$

$a=4-b$

$a=4-3$

$a=1$

$ax+b$

$\boxed{x+3}$

Q4. $(x-2)$ is factor of x^5-4x^3+x+k

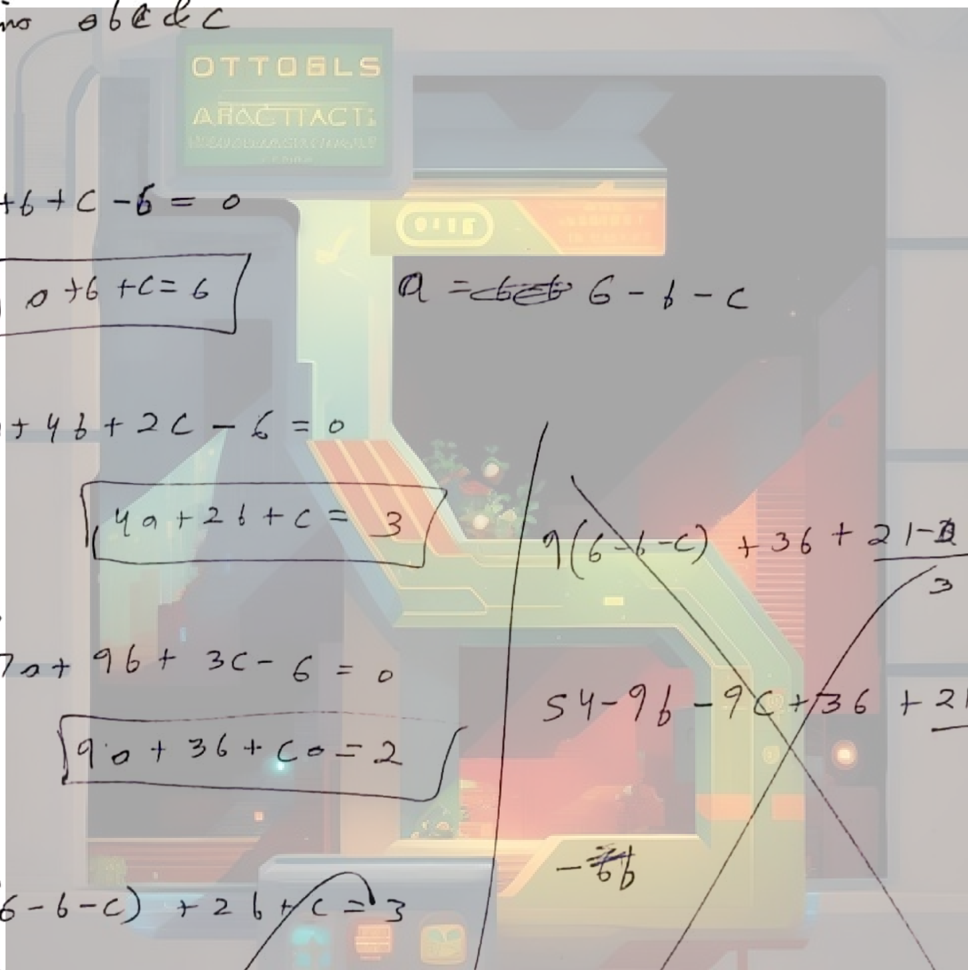
$x-2=0$

$x=2$

$32-32+2+k=0$

$\boxed{k=-2}$

Q5. ax^3+bx^2+cx-6 has $(x-1),(x-2)$ & $(x-3)$ as factors. find a b & c



$x=1$

$a+b+c-6=0$

$\boxed{a+b+c=6}$ $\quad a = 6-b-c$

$x=2$

$8a+4b+2c-6=0$

$\boxed{4a+2b+c=3}$

$9(6-b-c)+3b+\frac{21-2b}{3}=c$

$x=3$

$27a+9b+3c-6=0$

$\boxed{9a+3b+c=2}$

$54-9b-9c+3b+\frac{21-2b}{3}=3$

$-6b$

$4(6-b-c)+2b+c=3$

$24-4b-4c+2b+c=3$

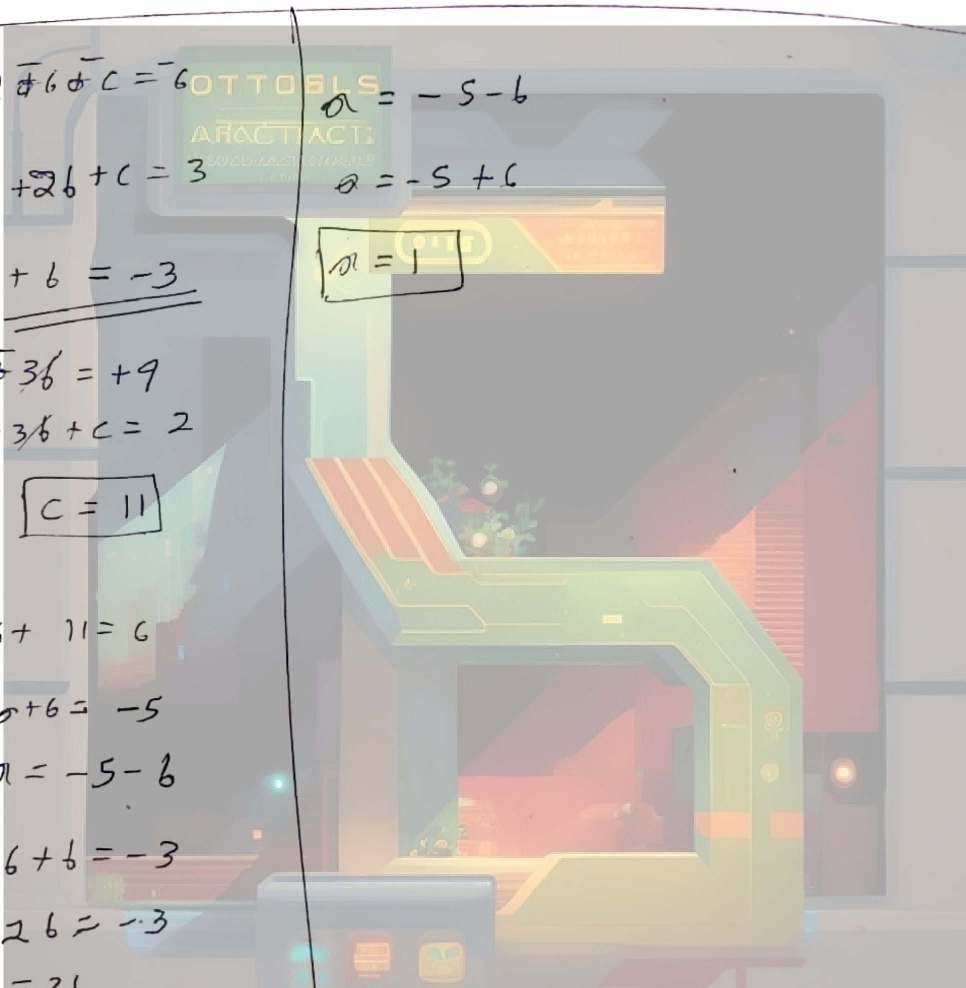
$21=+2b+3c$

$\frac{21-2b}{3}=c$

M.W.

DYS-Q 4 (Q 1, 2, 3, 4, 5, 6, 7, 8)

DYS-3 (Q 8)



Q 5. $- \quad a + b + c = 6$

$4a + 2b + c = 3$

$3a + b = -3$

$- \quad 9a + 3b = +9$

$9a + 3b + c = 2$

$\boxed{c = 11}$

$a + b + 11 = 6$

$a + b = -5$

$a = -5 - b$

$-15 - 3b + b = -3$

$-15 - 2b = -3$

$-12 = 2b$

$\boxed{b = -6}$

$a = -5 - b$

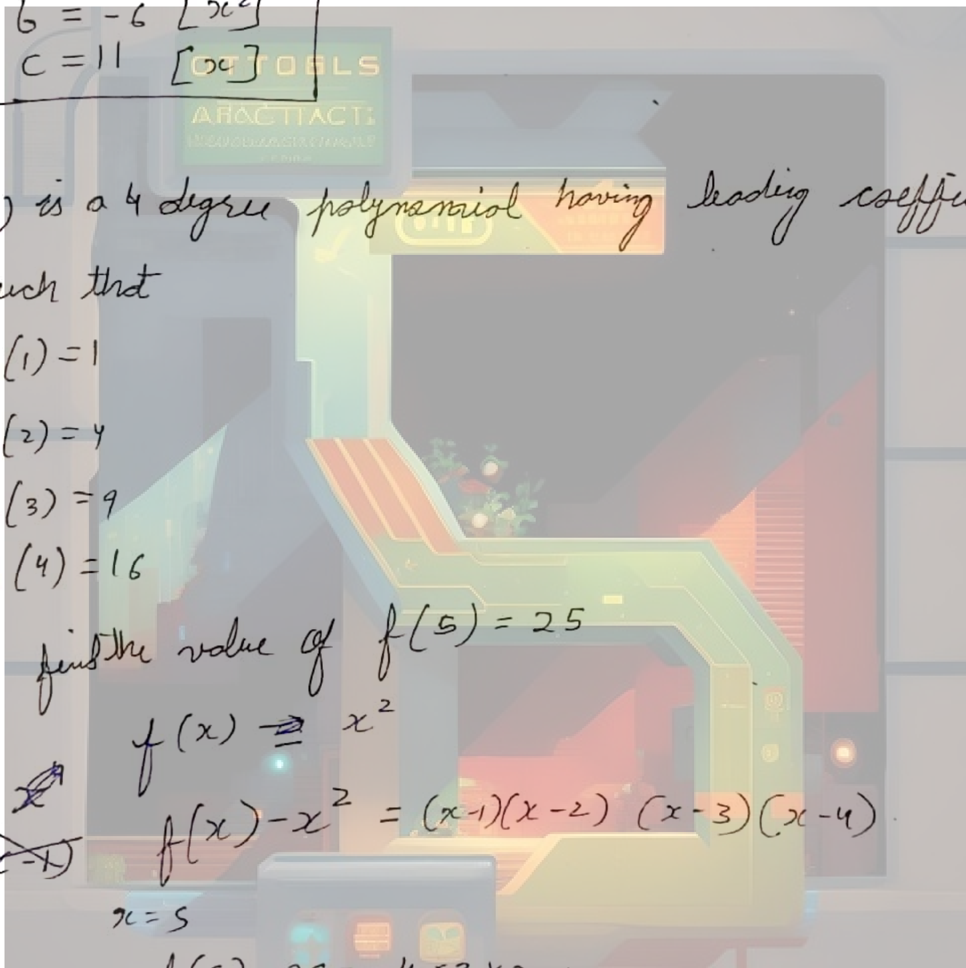
$a = -5 + 6$

$\boxed{a = 1}$

Q5. Method II

$$ax^3+bx^2+cx-6 = (x-1)(x-2)(x-3)$$
$$= (x^2-2x-x+2)(x-3)$$
$$= (x^2-3x+2)(x-3)$$
$$= x^3-3x^2+2x-3x^2+9x-6x$$
$$= x^3-6x^2+11x-6$$

$$\boxed{\begin{array}{ll} a=1 & [x^3] \\ b=-6 & [x^2] \\ c=11 & [x] \end{array}}$$



Q If $f(x)$ is a 4 degree polynomial having leading coefficient 1 such that

$f(1)=1$

$f(2)=4$

$f(3)=9$

$f(4)=16$

then find the value of $f(5) = 25$

$$f(x) = x^2$$

$$f(x)-x^2 = (x-1)(x-2)(x-3)(x-4)$$

$x=5$

$$f(5)-25 = 4\times3\times2\times1$$
$$= 24+25$$
$$f(5) = 49$$

DYS-4

Q10

$f(1) = 1$

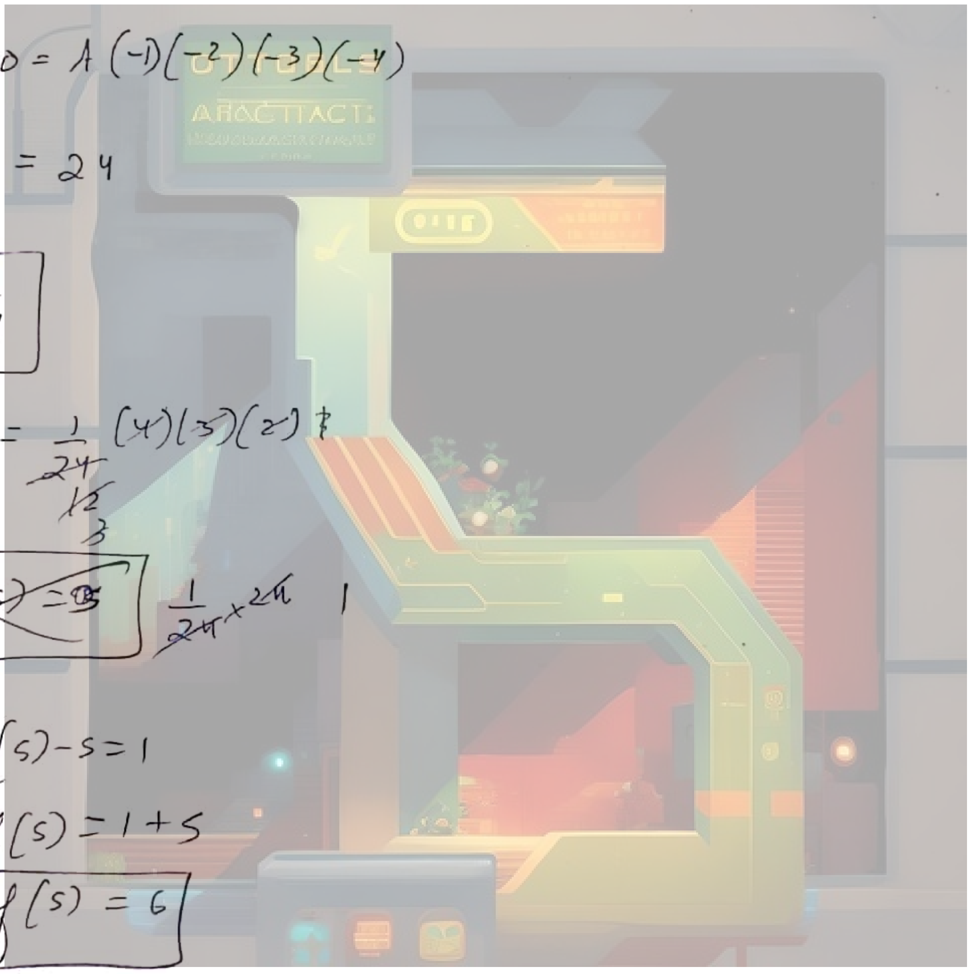
$f(2) = 2$

$f(3) = 3$

$f(4) = 4$

$f(0) = 1$

$f(x) - x = A(x-1)(x-2)(x-3)(x-4)$



$f(0) - 0 = A(-1)(-2)(-3)(-4)$

$\frac{1}{A} = 24$

$\boxed{A = \frac{1}{24}}$

$f(5) - 5 = \frac{1}{\cancel{24}}(4)(3)(2)(1)$

$\cancel{\boxed{f(5) = 25}}$ $\quad \frac{1}{\cancel{24}} \times 24 \quad 1$

$f(5) - 5 = 1$

$f(5) = 1 + 5$

$\boxed{f(5) = 6}$

Q9. $f(x) = (x-1)(x+2)\cancel{(x+1)}$

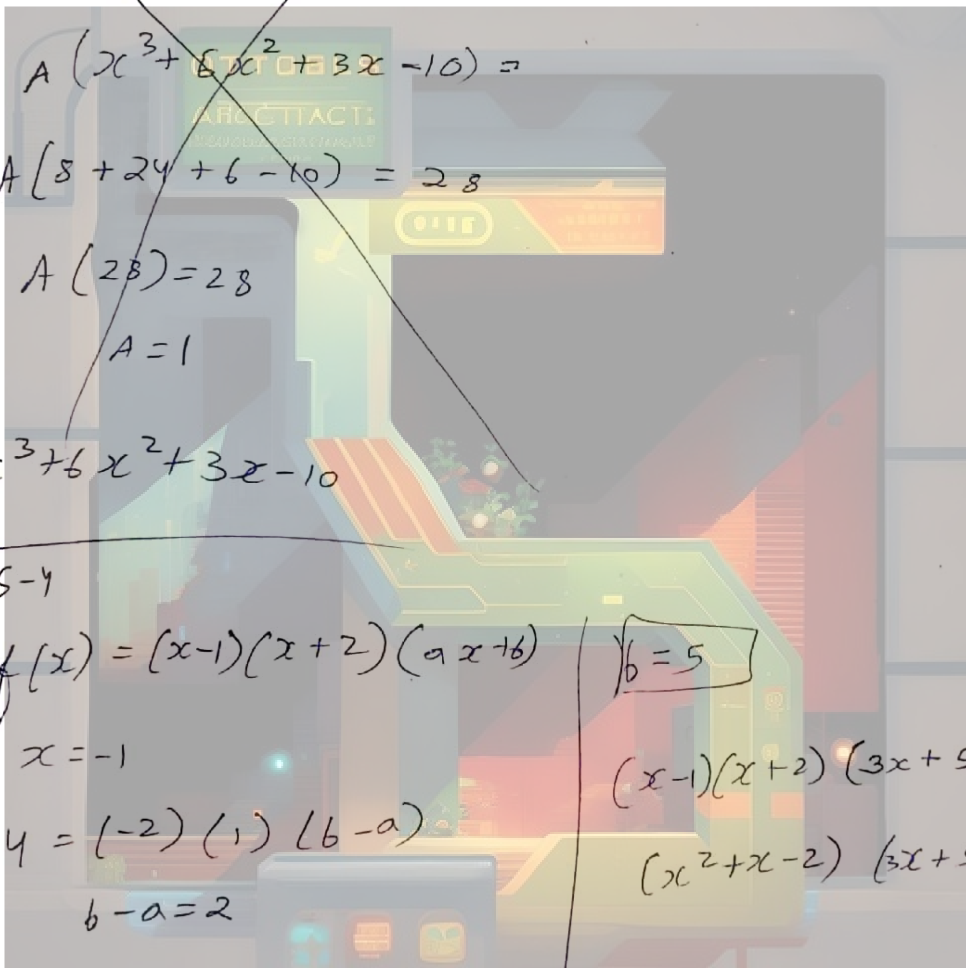
$f(x) \cancel{4} = (x+1) + 4$

$f(x) \cancel{28} = (x-2) + 28$

$(x^2 + 2x - x - 2)(x + \cancel{a}5)$

$(x^2 + x - 2)(x + 5)$

$x^3 + 5x^2 + x^2 + 5x - 2x - 10$



$A(x^3 + 6x^2 + 3x - 10) =$

$A(8 + 24 + 6 - 10) = 28$

$A(28) = 28$

$A = 1$

$x^3 + 6x^2 + 3x - 10$

DYS-4

Q9. $f(x) = (x-1)(x+2)(ax+b)$

$x = -1$

$4 = (-2)(1)(b-a)$

$b - a = 2$

$28 = (1)(4)(2a+b)$

$7 = 2a + b$

$\cancel{2 = b - a}$

$+2 = -b + a$

$9 = 3a$

$\boxed{a = 3}$

$\boxed{b = 5}$

$(x-1)(x+2)(3x+5)$

$(x^2 + x - 2)(3x + 5)$

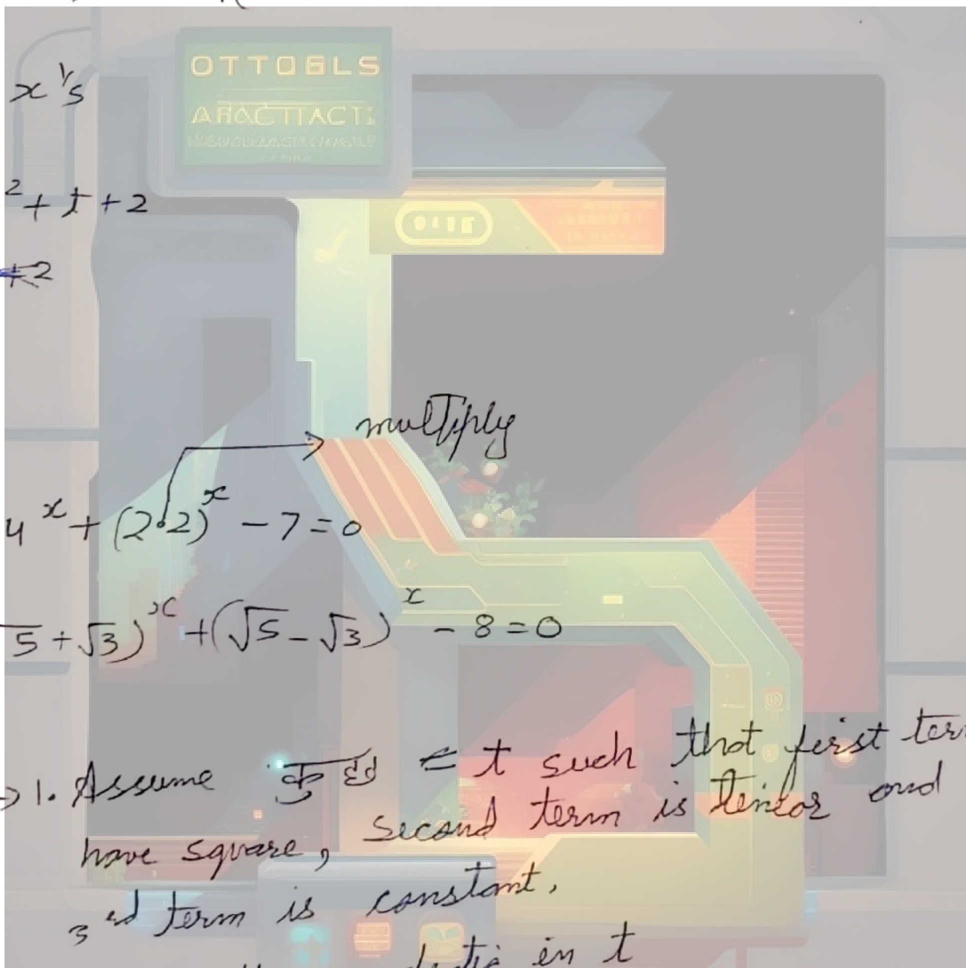
Equations Reducible to quadratic

$a(\text{कुछ})^2 + b(\text{कुछ}) + c = 0$

$\text{कुछ} \to x^{1/5}, x^4, (\sqrt{5}+\sqrt{7})^x, x^2+\frac{1}{x^2}, (x+\frac{1}{x})^2, 2^x$ etc

eg. 1. $x^{2/5} + x^{1/5} + 2 = 0$

$1(x^{1/5})^2 + 1(x)^{1/5} + 2$



$t = x^{1/5}$

$t^2 + t + 2$

$\cancel{t^2+2}$

eg 2. $4^x + (2\cdot 2)^x - 7 = 0$ multiply

eg 3. $(\sqrt{5}+\sqrt{3})^x + (\sqrt{5}-\sqrt{3})^x - 8 = 0$

Method → 1. Assume कुछ = t such that first term have square, second term is linear and 3rd term is constant.

2. Solve the quadratic in t
3. get values of small t & replace t by कुछ

Q1. $5^{2x} - 6 \times 5^{x+1} + 125 = 0$

$(5^{x})^2 - 30(5^{x}) + 125 = 0$

$t = 5^{x}$

$t^2 - 30t + 125$

$t^2 - 25t - 5t + 125$

$t(t-25) - 5(t-25)$

$t = 5$ $\quad\quad$ $t = 25$

$5 = 5^{x}$ $\quad\quad$ $25 = 5^{x}$

$\boxed{x = 1}$ $\quad\quad$ $\boxed{x = 2}$

Q2. $x^{2/3} + x^{1/3} - 2 = 0$

$(\sqrt[3]{x})^2 + 2(\sqrt[3]{x}) - \sqrt[3]{x} - 2 = 0$

$\sqrt[3]{x}(\sqrt[3]{x} + 2) - 1(\sqrt[3]{x} + 2)$

$(\sqrt[3]{x} - 1)$ $\quad$ $(\sqrt[3]{x} + \overset{2}{1})$

$\sqrt[3]{x} = 1$ $\quad\quad$ $\sqrt[3]{x} = -2$

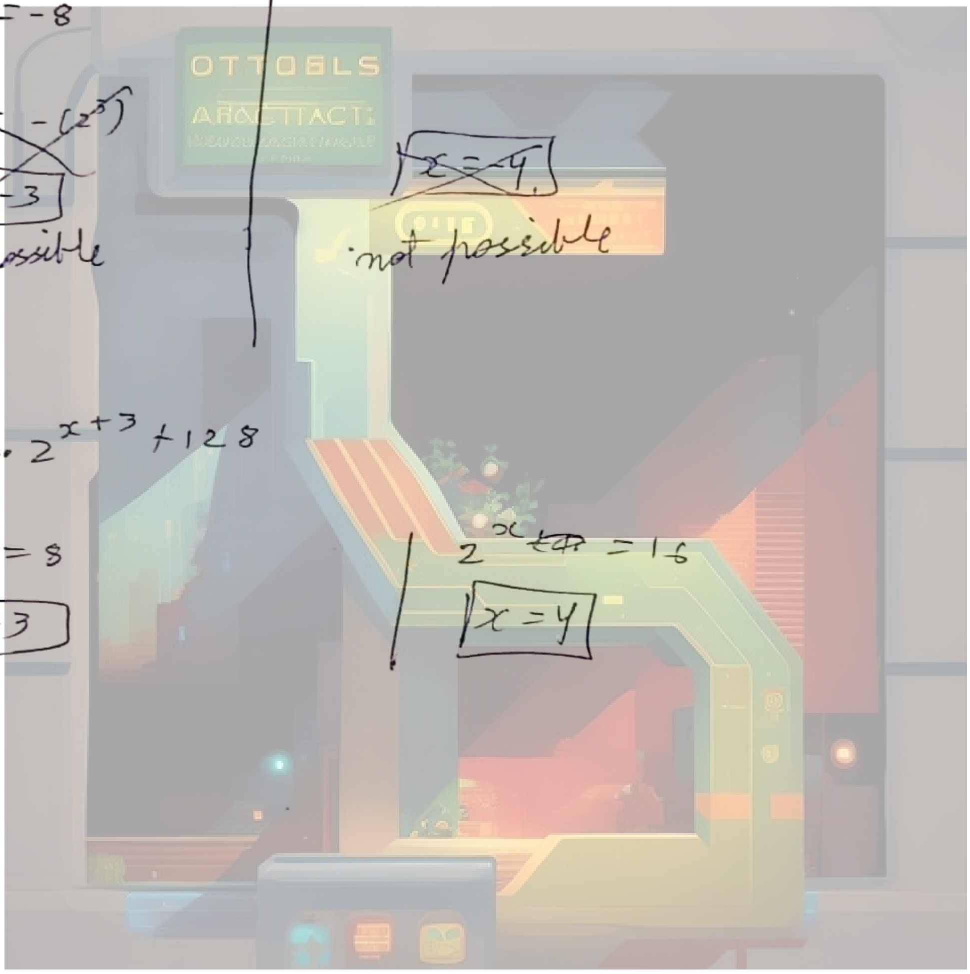
$\boxed{x = 1}$ $\quad\quad$ $\boxed{x = -8}$

Q3. $4^x + 3\cdot 2^{x+3} + 128 = 0$

$(2^x)^2 + 24(2^x) + 128 = 0$

$(2^x)^2 + 16(2^x) + 8(2^x) + 128 = 0$

$2^x(2^x+16) + 8(2^x+16)$



$2^x + 8 = 0$

$2^x = -8$

$\cancel{2^x = -(2^3)}$

$\cancel{\boxed{x = -3}}$

not possible

$2^x + 16 = 0$

$\cancel{\boxed{x = -4}}$

not possible

Q4. $4^x - 3\cdot 2^{x+3} + 128$

$2^x = 8$

$\boxed{x = 3}$

$2^x = 16$

$\boxed{x = 4}$

H.W. 3-5-24

DYS-5

Q 2, 3, Q 5, Q 7, (Q 15-f)

DYS-4

Q 8, 12

Q $3\cdot 4^x + 2\cdot 9^x - 5\cdot 6^x = 0$

$3\cdot 2^{2x} + 2\cdot 3^{2x} - 5\cdot 2^x 3^x$

$\cancel{\dfrac{3\cdot 2^{2x}}{3^x} + \dfrac{2\cdot 3^{2x}}{3^x} - \dfrac{5\cdot 2^x 3^x}{3^x}}$

$\cancel{\dfrac{3\cdot 2^{2x}}{2^x} + \dfrac{2\cdot 3^{2x}}{2^{2x}} - \dfrac{5\cdot 2^x 3^x}{2^x}}$

$\cancel{3\cdot 2^{2x}}$

$\dfrac{3\cdot \cancel{2^{2x}}}{\cancel{2^{2x}}} + \dfrac{2\cdot 3^{2x}}{2^{2x}} - \dfrac{5\cdot 2^x 3^x}{2^{2x}}$

$3 + 2\cdot\left(\frac{3}{2}\right)^{2x} - 5\times\left(\frac{3}{2}\right)^{x}$

$t = \left(\frac{3}{2}\right)^x$

$2t^2 - 5t + 3$

$\dfrac{5 \pm \sqrt{25-24}}{4}$

$\dfrac{5+1}{4}$ $\qquad$ $\dfrac{5-1}{4}$

$\boxed{t = \frac{3}{2}}$ $\qquad$ $\boxed{t = 1}$

$\frac{3}{2} = \frac{3}{2}^{x}$

$\boxed{x = 1}$

$\left(\frac{3}{2}\right)^x = 1$

$\boxed{x = 0}$

Note -

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\because x+\frac{1}{x} = t$$

$$t^2 = x^2+\frac{1}{x^2}+2$$

$$t^2-2 = x^2+\frac{1}{x^2}$$

$$\boxed{\begin{array}{ll} x+\frac{1}{x} = t & x-\frac{1}{x} = t \\ x^2+\frac{1}{x^2} = t^2-2 & x^2+\frac{1}{x^2} = t^2+2 \end{array}}$$

Q1. $3\left(x^2+\frac{1}{x^2}\right) - 16\left(x+\frac{1}{x}\right)+26 = 0$

$$3(t^2-2) - 16(t)+26=0$$

$$3t^2-6-16t+26$$

$$3t^2-16t+20$$

$$3t^2-\cancel{10}6t-10t+20$$

$$3t(t-2)-10(t-2)$$

$$(3t-10)(t-2)$$

$$\boxed{t=2} \qquad \boxed{t=10/3}$$

$$x+\frac{1}{x} = \cancel{2}t$$

$$x^2+1 = 2x$$

$$x^2-2x+1=0$$

$$x+\frac{1}{x} = \frac{10}{3}$$

$$x^2+1 = \frac{10x}{3}$$

$$3x^2-10x+3=0$$

$x^2 - x - x + 1$

$x(x-1) - 1(x-1)$

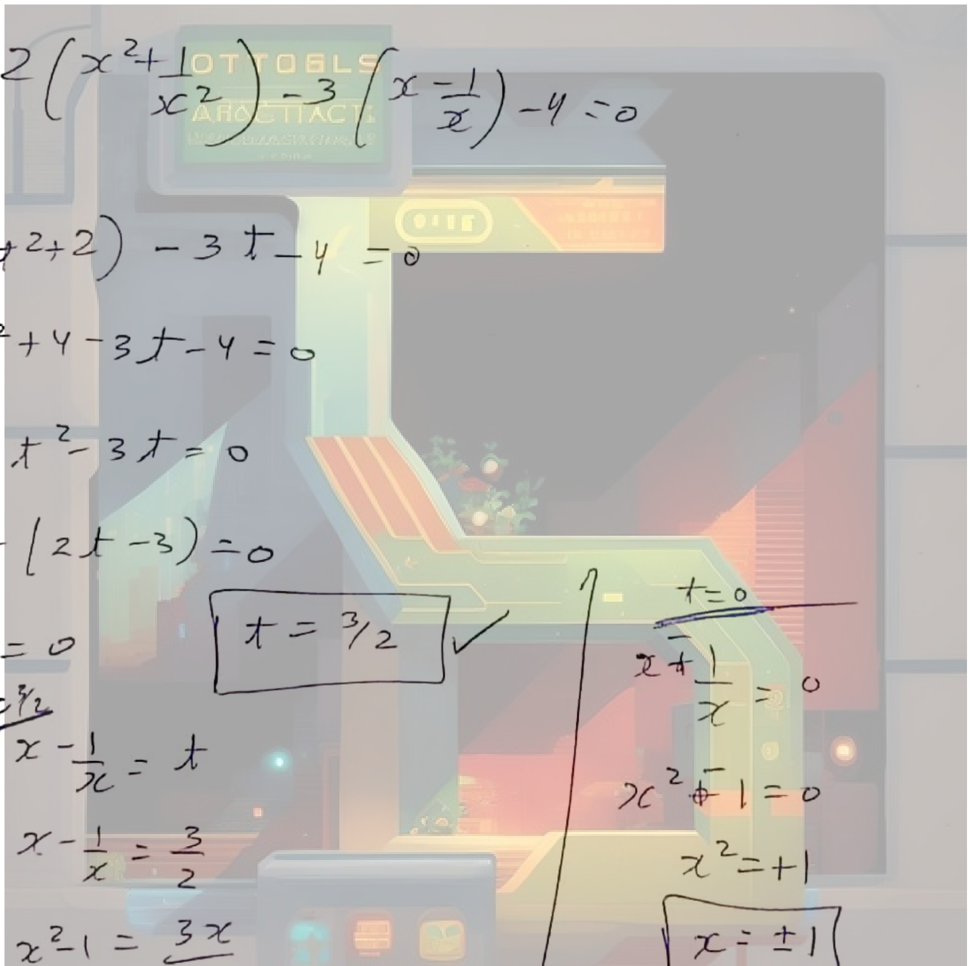
$\boxed{x=1}$

$\frac{10 \pm \cancel{\sqrt{\quad}}\ 8}{6}$

$\frac{2}{6} = \frac{1}{3}$

$\boxed{x=\frac{1}{3}}$ $\boxed{x=3}$

DVS - ds



(6) $2\left(x^2+\frac{1}{x^2}\right) - 3\left(x-\frac{1}{x}\right) - 4 = 0$

$2(t^2+2) - 3t - 4 = 0$

$2t^2+4-3t-4=0$

$2t^2-3t=0$

$t(2t-3)=0$

$t=0$ $\boxed{t=3/2}$ ✓

$t=3/2$

$x - \frac{1}{x} = t$

$x - \frac{1}{x} = \frac{3}{2}$

$x^2 - 1 = \frac{3x}{2}$

$2x^2 - 3x - 2 = 0$

$\frac{3 \pm \sqrt{9+16}}{4}$

$\frac{3+5}{4}$ $\frac{3-5}{4}$

$\boxed{2}$ $\boxed{-\frac{1}{2}}$

$t=0$

$x - \frac{1}{x} = 0$

$x^2 - 1 = 0$

$x^2 = +1$

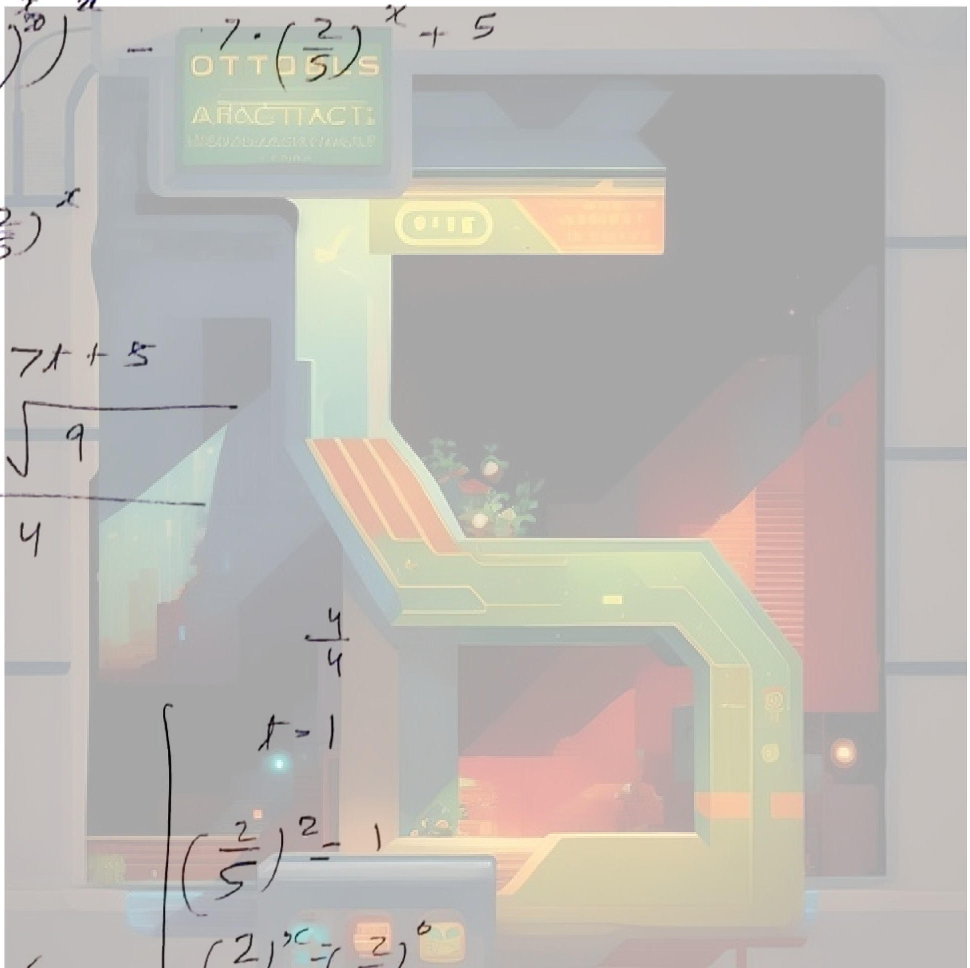
$\boxed{x = \pm 1}$

Q $2^{2x+1} - 7\cdot 10^{x} + 5^{2x+1} = 0$

$2\cdot 2^{2x} - 7\cdot 2^{x}\cdot 5^{x} + 5\cdot 5^{2x}$

$\frac{2\cdot 2^{2x}}{5^{2x}} - \frac{7\cdot 2^{x}\cdot 5^{x}}{5^{2x}} + 5$

$2\cdot\left(\left(\frac{2}{5}\right)^{x}\right)^{2} - 7\cdot\left(\frac{2}{5}\right)^{x} + 5$



$t = \left(\frac{2}{5}\right)^{x}$

$2t^{2} - 7t + 5$

$\frac{7 \pm \sqrt{9}}{4}$

$\frac{10}{4}$ $\qquad$ $\frac{4}{4}$

$t = \frac{5}{2}$ $\qquad$ $t = 1$

$\left(\frac{2}{5}\right)^{x} = \frac{5}{2}$ $\qquad$ $\left(\frac{2}{5}\right)^{x} = 1$

$\boxed{x=-1}$ ✓ $\qquad$ $\left(\frac{2}{5}\right)^{x} = \left(\frac{2}{5}\right)^{0}$

$\boxed{x=0}$ ✓

Q15. F $(5^{2x}-7^{x})-35(5^{2x}-7^{x})=0$

$$(5^{2x}-7^{x})(-34)=0$$

$$5^{2x}=7^{x}$$

$$25^{x}=7^{x}$$

$A \neq B$

$A \neq B$

Power $= \boxed{x=0}$

Q $(5+2\sqrt{6})^{x/2}+(5-2\sqrt{6})^{x/2}=10$

$$5-2\sqrt{6}=5-2\sqrt{6}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}=\frac{25-24}{5+2\sqrt{6}}=\frac{1}{5+2\sqrt{6}}$$

$$(5+2\sqrt{6})^{x/2}+\frac{1}{5+2\sqrt{6}}^{x/2}=10$$

$$t=(5+2\sqrt{6})^{x/2}$$

$$t+\frac{1}{t}=10$$

$$t^2+1=10t$$

$$t^2-10t+1$$

$$t=\frac{10\pm\sqrt{100-4}}{2}$$

$$=5\pm\sqrt{24}$$

$$t=5\pm2\sqrt{6}$$

$t=5+2\sqrt{6}$

$(5+2\sqrt{6})^{x/2}=(5+2\sqrt{6})^{1}$

$\frac{x}{2}=1$

$\boxed{x=2}$

$t=5-2\sqrt{6}$

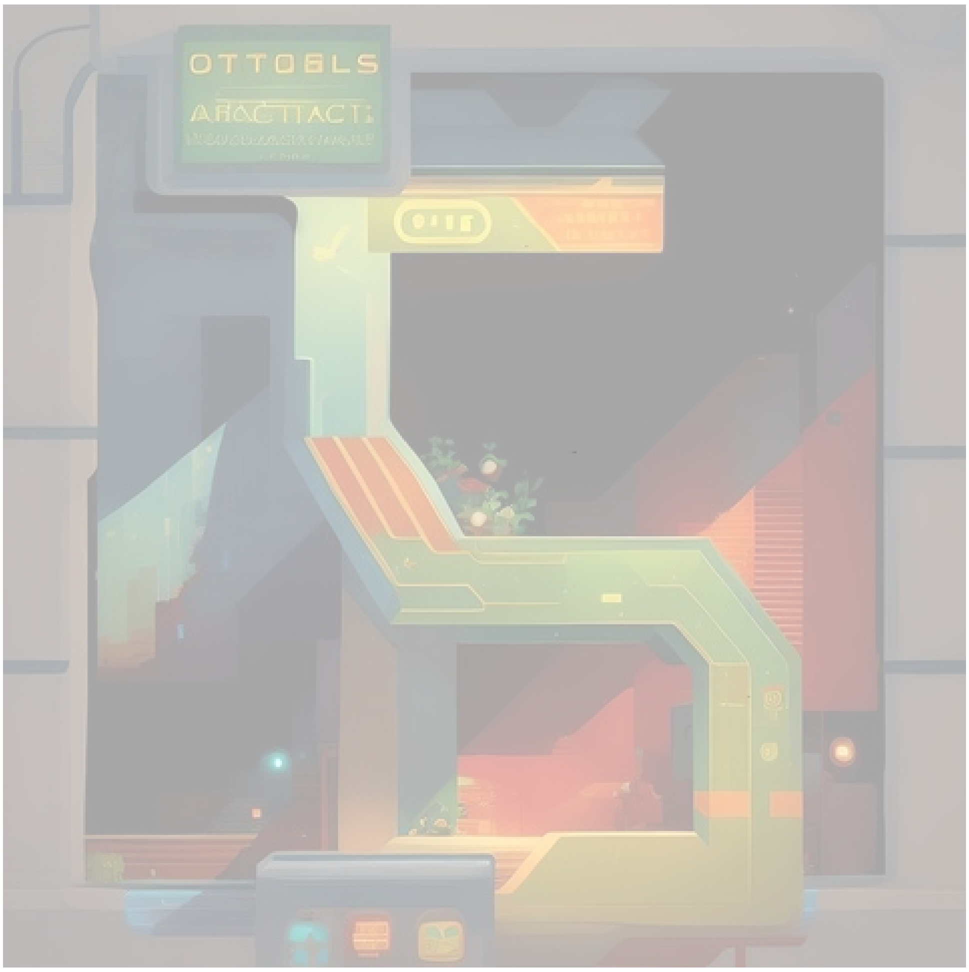
$(5+2\sqrt{6})^{x/2}=\frac{1}{5+2\sqrt{6}}$

$\frac{x}{2}=-1$

$\boxed{x=-2}$

~~Q2. DYS-5~~ H.W
~~Q9~~

DYS-5 (Q10), Q12, Q13, Q8
DYS-2 (Q7)

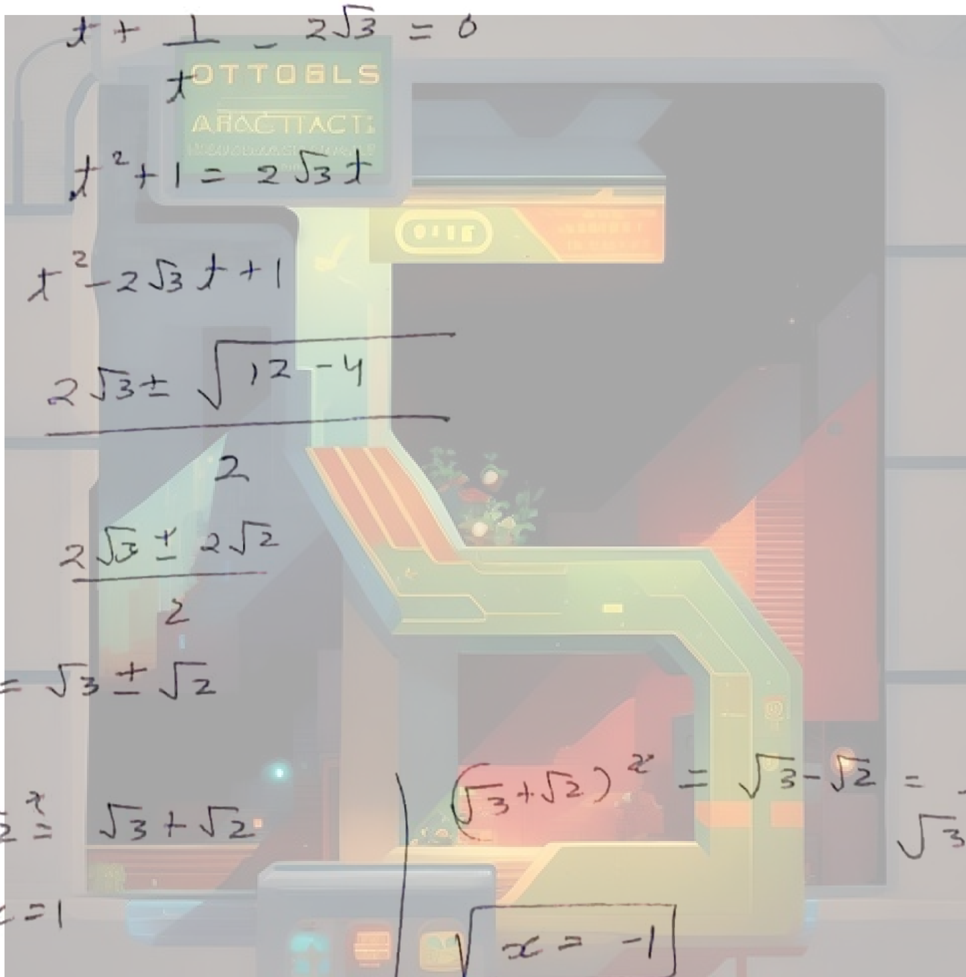


DIS - 5

Q9 $\left(\sqrt{3}+\sqrt{2}\right)^{x}+\left(\sqrt{3}-\sqrt{2}\right)^{x}-2\sqrt{3}=0$

$$\sqrt{3}-\sqrt{2}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{3-2}{\sqrt{3}+\sqrt{2}}=\frac{1}{\sqrt{3}+\sqrt{2}}$$

$$t=\left(\sqrt{3}+\sqrt{2}\right)^{x}$$



$$t+\frac{1}{t}-2\sqrt{3}=0$$

$$t^{2}+1=2\sqrt{3}t$$

$$t^{2}-2\sqrt{3}t+1$$

$$t=\frac{2\sqrt{3}\pm\sqrt{12-4}}{2}$$

$$=\frac{2\sqrt{3}\pm2\sqrt{2}}{2}$$

$$=\sqrt{3}\pm\sqrt{2}$$

$$\sqrt{3}+\sqrt{2}^{\,x}=\sqrt{3}+\sqrt{2}$$

$$x=1$$

$$\left(\sqrt{3}+\sqrt{2}\right)^{x}=\sqrt{3}-\sqrt{2}=\frac{1}{\sqrt{3}+\sqrt{2}}$$

$$\boxed{x=-1}$$

$$\boxed{x=\pm1}$$

Q $\quad x(x+1)(x+2)(x+3)-8=0$

$$x(x+3)(x+1)(x+2)=8$$

$$(x^2+3x)(x^2+3x+2)=8$$

$$x^2+3x=t$$

$$t(t+2)=8$$

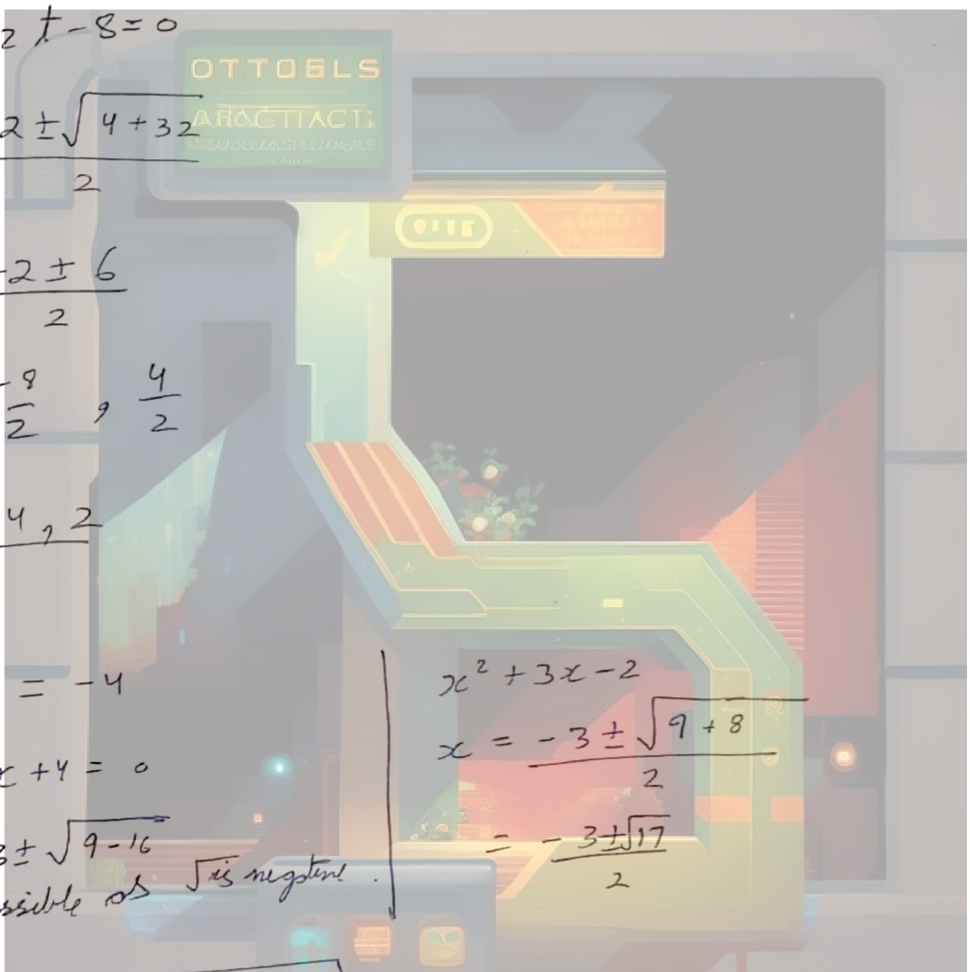
$$t^2+2t-8=0$$

$$D=\frac{-2\pm\sqrt{4+32}}{2}$$

$$=\frac{-2\pm 6}{2}$$

$$=\frac{-8}{2},\ \frac{4}{2}$$

$$t=-4,\ 2$$



$$x^2+3x=-4$$

$$x^2+3x+4=0$$

$$x=-3\pm\sqrt{9-16}$$

Not Possible as $\sqrt{\ }$ is negative.

$$x^2+3x-2$$

$$x=\frac{-3\pm\sqrt{9+8}}{2}$$

$$=\frac{-3\pm\sqrt{17}}{2}$$

$$x=\boxed{\frac{-3\pm\sqrt{17}}{2}}$$

DYS-5

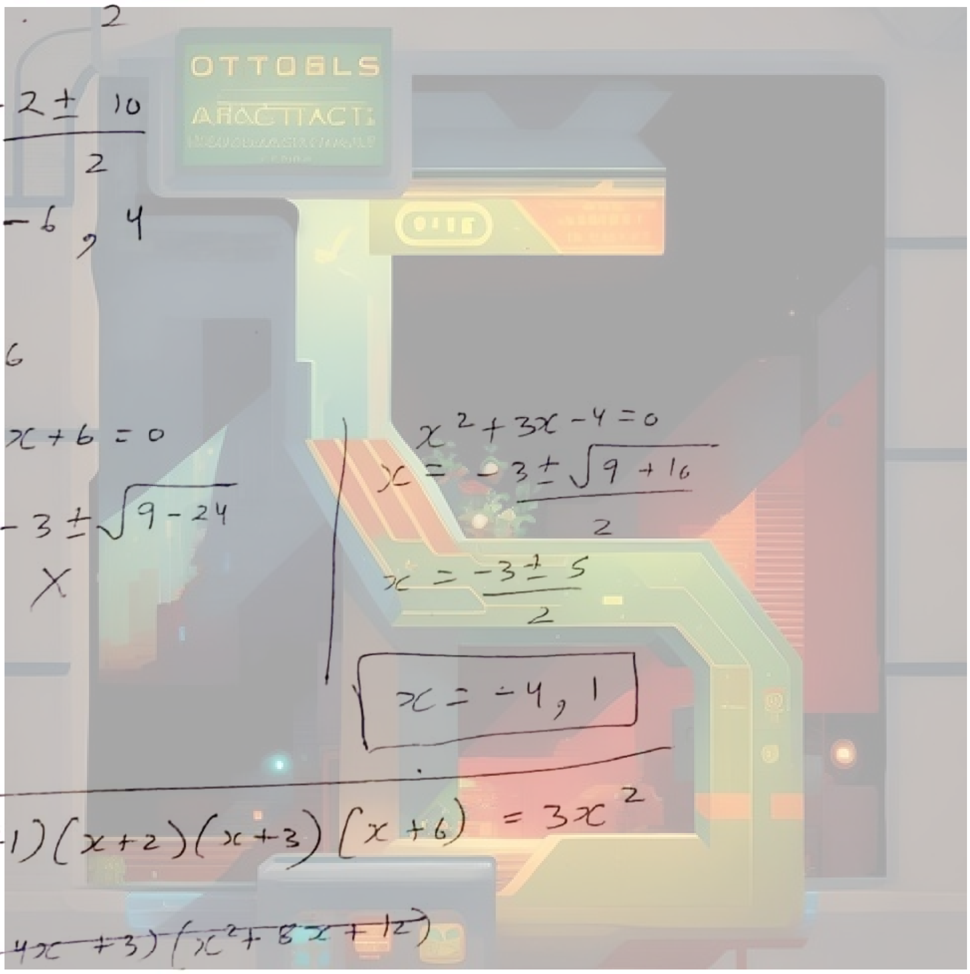
Q 11 $x(x+1)(x+2)(x+3)=24$

$t = x^2+3x$

$t(t+2)=24$

$t^2+2t-24=0$

$t = \dfrac{-2\pm\sqrt{4+96}}{2}$



$= \dfrac{-2\pm 10}{2}$

$= -6, 4$

$t=-6$

$x^2+3x+6=0$

$x = -3\pm\sqrt{9-24}$

X

$x^2+3x-4=0$

$x = \dfrac{-3\pm\sqrt{9+16}}{2}$

$x = \dfrac{-3\pm 5}{2}$

$\boxed{x=-4, 1}$

Q $(x+1)(x+2)(x+3)(x+6)=3x^2$

$\cancel{(x^2+4x+3)(x^2+8x+12)}$

$(x^2+7x+6)(x^2+5x+6)=3x^2$

$\cancel{t = x^2+6}$

$\cancel{(t+7x)(t+5x)=3x^2}$

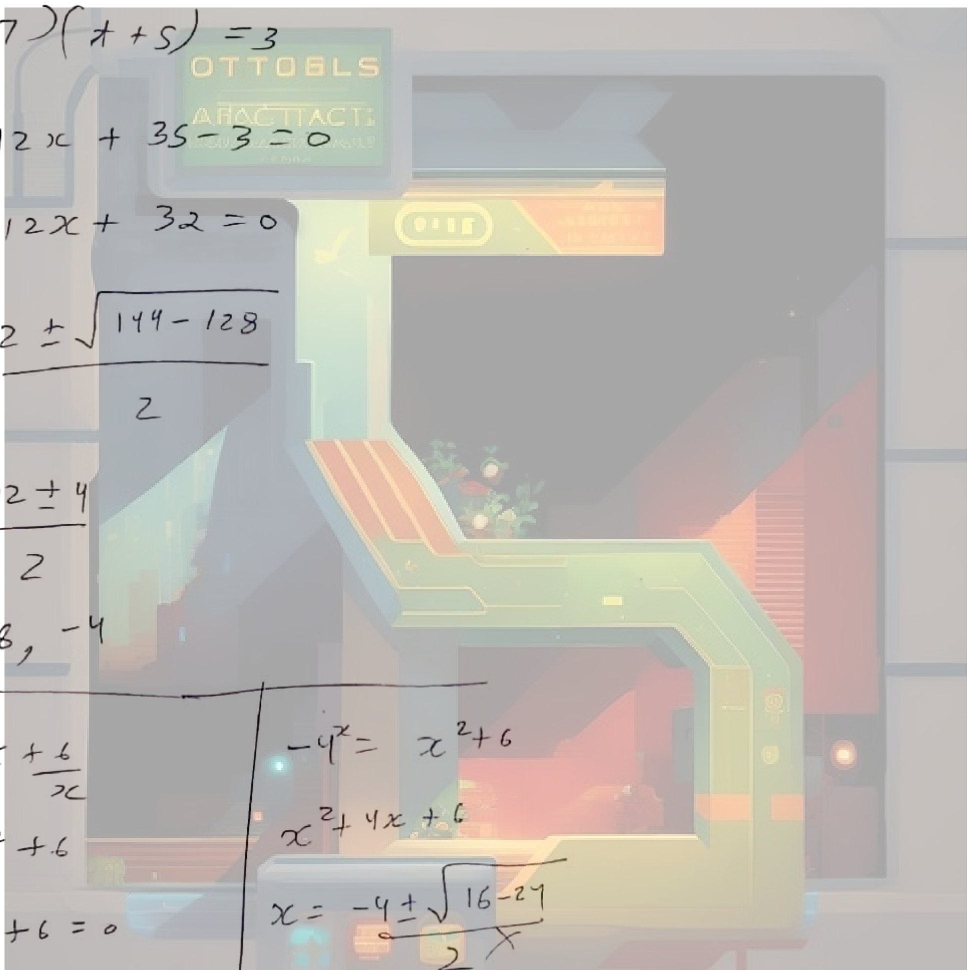
$$\left(\frac{x^2+7x+6}{x}\right)\left(\frac{x^2+5x+6}{x}\right) = 3$$

$$\cancel{x\frac{^2+6}{x} = 7}$$

$$\left(x^{\circ}+7+\frac{6}{x}\right)\left(x+5+\frac{c}{5}\right) = 3$$

$$t = x + \frac{6}{x}$$

$$(t+7)(t+5) = 3$$



$$t^2 + 12x + 35 - 3 = 0$$

$$t^2 + 12x + 32 = 0$$

$$t = \frac{-12 \pm \sqrt{144-128}}{2}$$

$$= \frac{-12 \pm 4}{2}$$

$$= -8, -4$$

$$-8 = x + \frac{6}{x}$$

$$-8x = x^2 + 6$$

$$x^2 + 8x + 6 = 0$$

$$x = \frac{-8 \pm \sqrt{64-24}}{2}$$

$$= -8 \pm \sqrt{40}$$

$$\boxed{x = -4 \pm \sqrt{10}}$$

$$-4^x = x^2 + 6$$

$$x^2 + 4x + 6$$

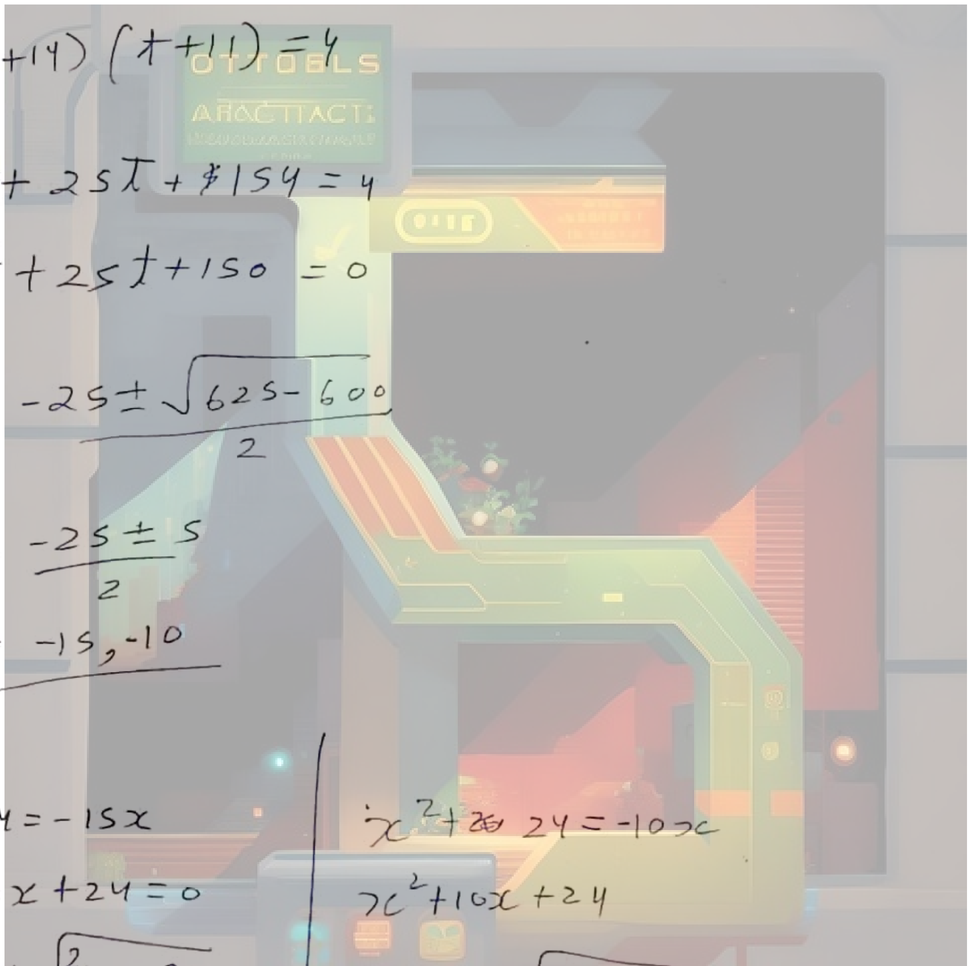
$$x = \frac{-4 \pm \sqrt{16-24}}{2} \times$$

Q $(x+2)(x+3)(x+8)(x+12) = 4x^2$

$$(x^2+14x+24)(x^2+11x+24) = 4x^2$$

$$\left(x+14+\frac{24}{x}\right)\left(x+11+\frac{24}{x}\right)$$

$$t = x+\frac{24}{x}$$



$$(t+14)(t+11) = 4$$

$$t^2+25t+154 = 4$$

$$t^2+25t+150 = 0$$

$$t = \frac{-25 \pm \sqrt{625-600}}{2}$$

$$= \frac{-25 \pm 5}{2}$$

$$= -15, -10$$

$$x^2+24 = -15x$$

$$x^2+15x+24 = 0$$

$$x = \frac{-15 \pm \sqrt{225-96}}{2}$$

$$\boxed{x = \frac{-15 \pm \sqrt{129}}{2}}$$

$$x^2+24 = -10x$$

$$x^2+10x+24$$

$$x = \frac{-10 \pm \sqrt{100-96}}{2}$$

$$= \frac{-10 \pm 2}{2}$$

$$\boxed{= -4, -6}$$

Q $x^4-2x^3+3x^2-2x=0$

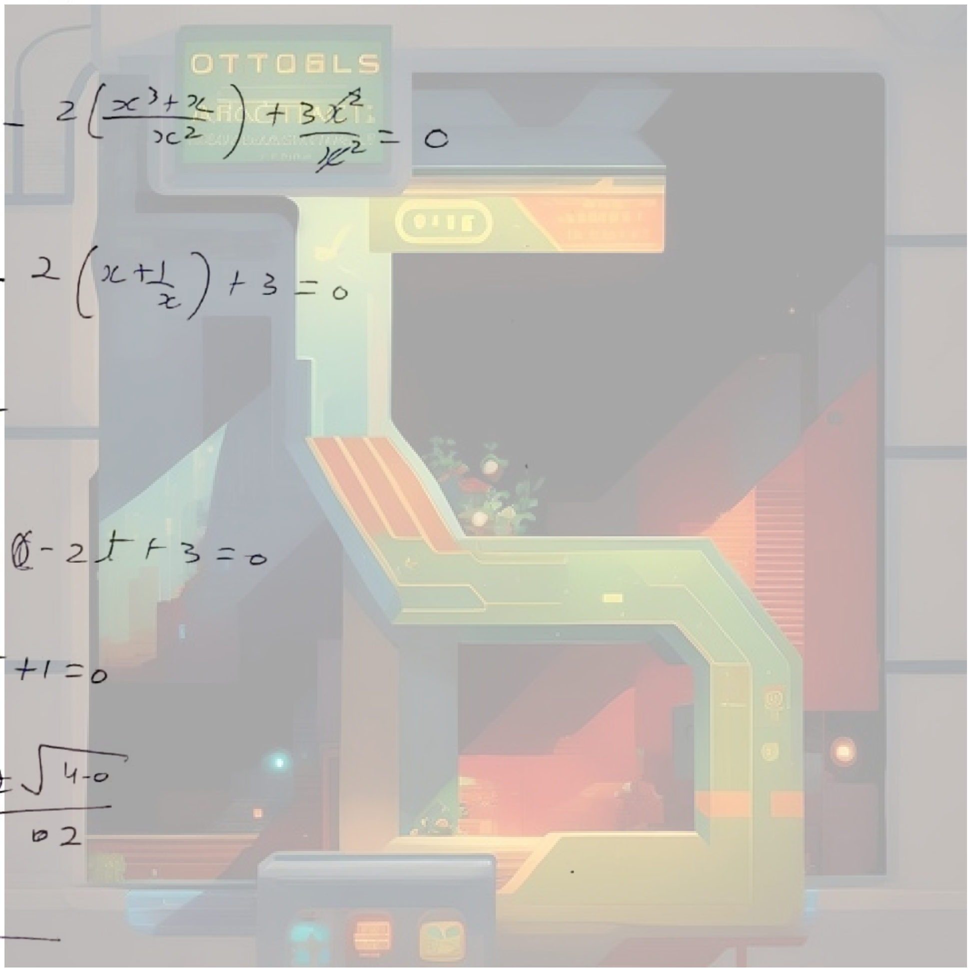
$x(x^3-2x^2+3x-2)=0$

$\boxed{x=0}$

~~Q~~ $\cancel{x^4-2x^3+3x^2-2x+1=0}$

Q $x^4-2x^3+3x^2-2x+1=0$

$x^4+1-2(x^3+x)+3x^2=0$



$\frac{x^4+1}{x^2}-2\left(\frac{x^3+x}{x^2}\right)+\frac{3x^2}{x^2}=0$

$x^2+\frac{1}{x^2}-2\left(x+\frac{1}{x}\right)+3=0$

$x+\frac{1}{x}=t$

$(t^2-2)-2t+3=0$

$t^2-2t+1=0$

$t=\frac{2\pm\sqrt{4-0}}{2}$

$=1$

$x^2+1=x$

$x^2-x+1=0$

$x=\frac{1\pm\sqrt{1-4}}{2}$

Not Possible

DYS-5

Q14 $x^4-2x^3-2x^2+2x+1=0$

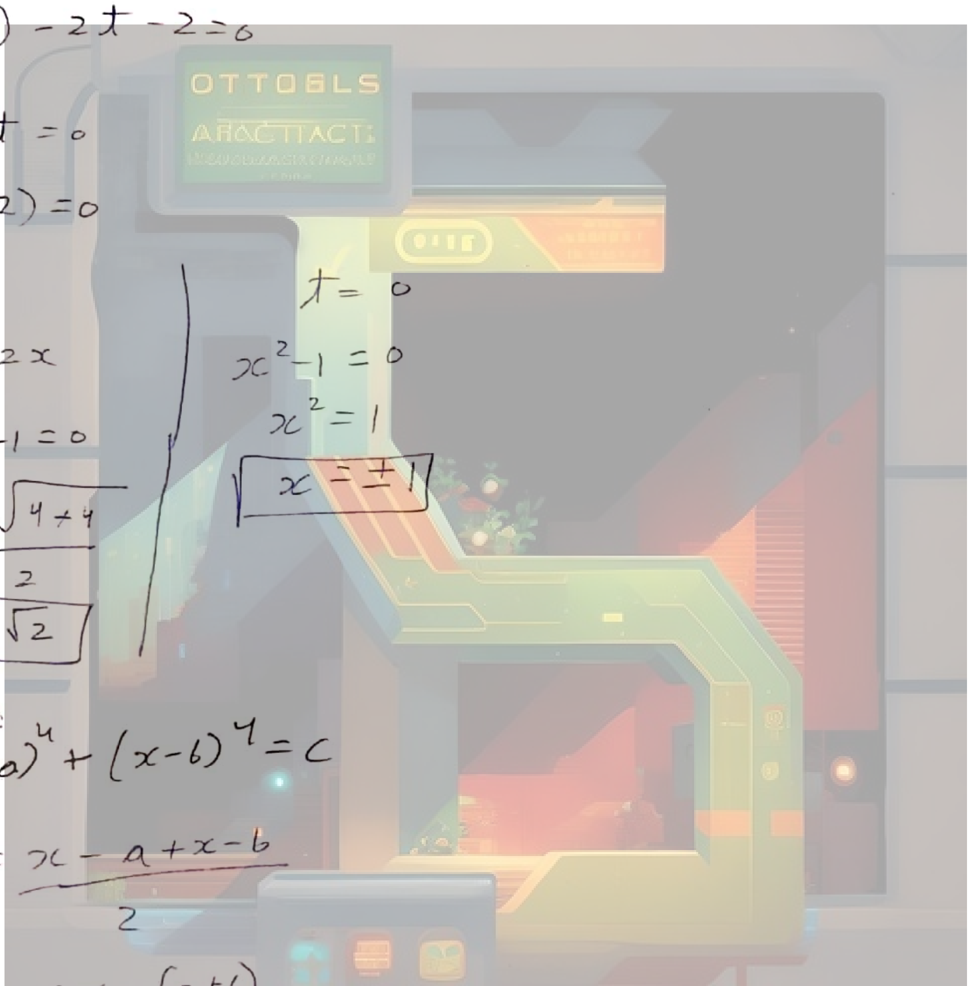
$$\frac{x^4+1}{x^2} - 2\left(\frac{x^3-x}{x^2}\right) - \frac{2x^2}{x^2}=0$$

$$x^2+\frac{1}{x^2} - 2\left(x-\frac{1}{x}\right) - 2 = 0$$

$$(t^2+2) - 2t - 2 = 0$$

$$t^2-2t=0$$

$$t(t-2)=0$$



$t=2$

$x^2-1=2x$

$x^2-2x-1=0$

$$x=\frac{2\pm\sqrt{4+4}}{2}$$

$$\boxed{=1\pm\sqrt{2}}$$

$t=0$

$x^2-1=0$

$x^2=1$

$$\boxed{x=\pm 1}$$

Q $(x-a)^4+(x-b)^4=c$

$$t=\frac{x-a+x-b}{2}$$

$$2t = 2x-(a+b)$$

$$2t+(a+b)=2x$$

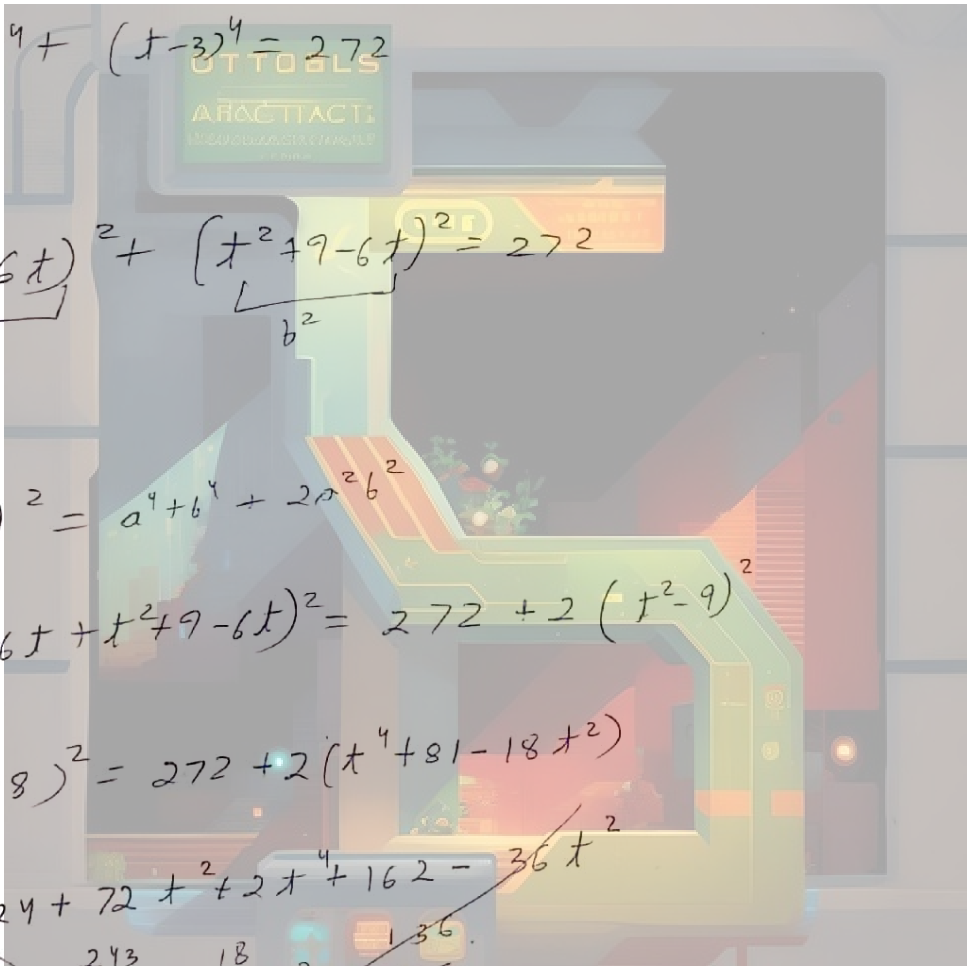
$$\boxed{x = t+\frac{(a+b)}{2}}$$

Q $(x-1)^4+(x-7)^4=272$

$\left(t+\frac{8}{2}-1\right)^4+\left(t+\frac{8}{2}-7\right)^4=272$

$(t+4-1)^4+(t+4-7)^4=272$

$(t+3)^4+(t-3)^4=272$



t^2

$\underbrace{(t^2+9+6t)}_{a^2}{}^2+\underbrace{(t^2+9-6t)}_{b^2}{}^2=272$

$(a^2+b^2)^2=a^4+b^4+2a^2b^2$

$(t^2+9+6t+t^2+9-6t)^2=272+2(t^2-9)^2$

$(2t^2+18)^2=272+2(t^4+81-18t^2)$

$4t^4+324+72t^2+2t^4+162-36t^2$

$243 \quad 18 \quad 136$

$\overset{3}{\cancel{6}}t^4+\overset{243}{\cancel{486}}+\overset{18}{\cancel{36}}t^2=\cancel{272}$

$3t^4+18t^2-136+243=0$

$3t^4+18t^2+107=0$

$t^2=-18\pm\sqrt{324-}$

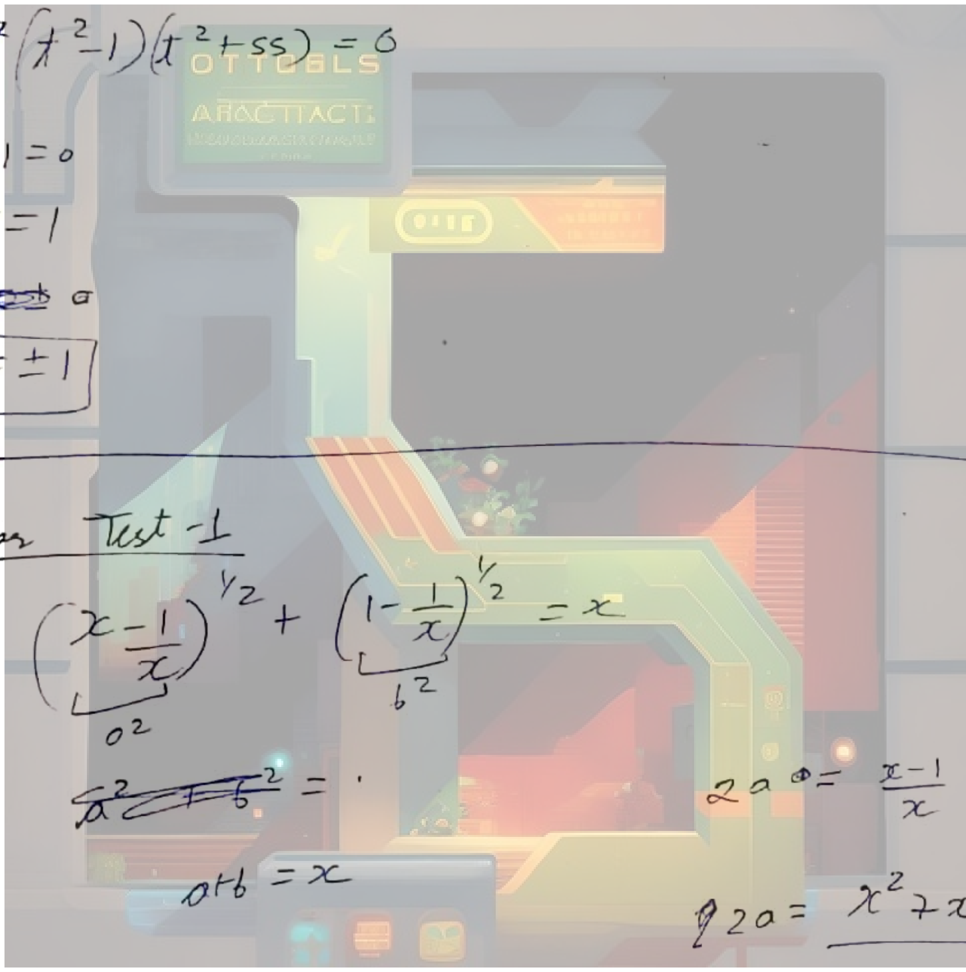
$$4t^4+324+72t^2-2t^4-162+36t^2=272$$

$$2t^4+\cancel{162}^{\,81}+\cancel{108}^{\,54}t^2=\cancel{272}^{\,136}$$

$$t^4+54t^2-55=0$$
$$t^4+55t^2-t^2-55=0$$
$$t^2(t^2+54)-55=0$$

$$\cancel{t^2}(t^2-1)(t^2+55)=0$$

$$t^2-1=0$$
$$t^2=1$$
$$\cancel{t=\pm}\ 0$$
$$\boxed{t=\pm1}$$



P Minor Test -1

Q18. $$\underbrace{\left(\frac{x-1}{x}\right)^{1/2}}_{a^2}+\underbrace{\left(1-\frac{1}{x}\right)^{1/2}}_{b^2}=x$$

$$\cancel{a^2-b^2}=$$

$$a+b=x$$

$$2a=\frac{x-1}{x}+x$$

$$2a=\frac{x^2+x-1}{2x}$$

$$a^2-b^2=\frac{x-1}{x}-\frac{1+1}{x}$$

$D=$ Ove

So irrational

$$(a+b)(a-b)=(x-1)$$

$$(a-b)x=(x-1)$$

$$a-b=\frac{x-1}{x}$$

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Integer type

Q5.

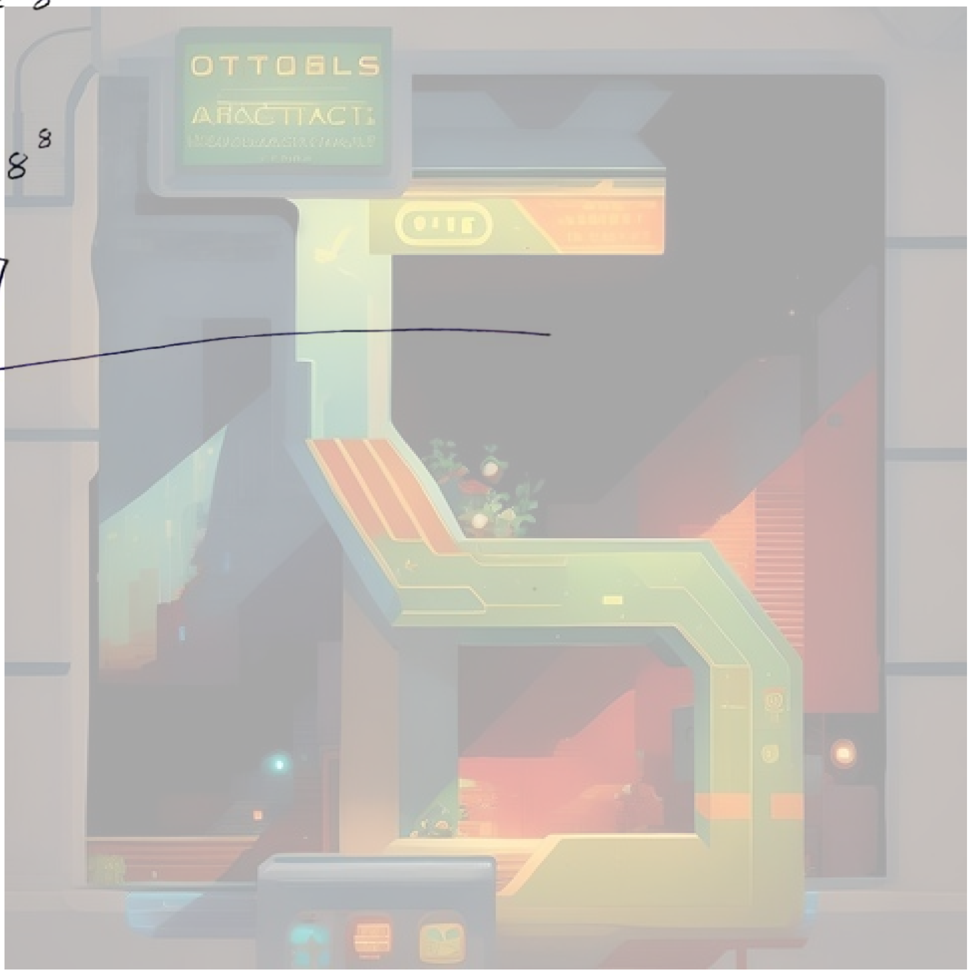
$$x^{x^8} = 8$$

$$x^8 = t$$

$$x^t = 8 \Rightarrow \boxed{x = t^{1/8}}$$

$$(t^{1/8})^t = 8$$

$$t^{t/8} = 8$$



$$t^t = 8^8$$

$$\boxed{t = 8}$$

DYS - 5

Q8. $(x-1)^4+(x-5)^4 = 82$

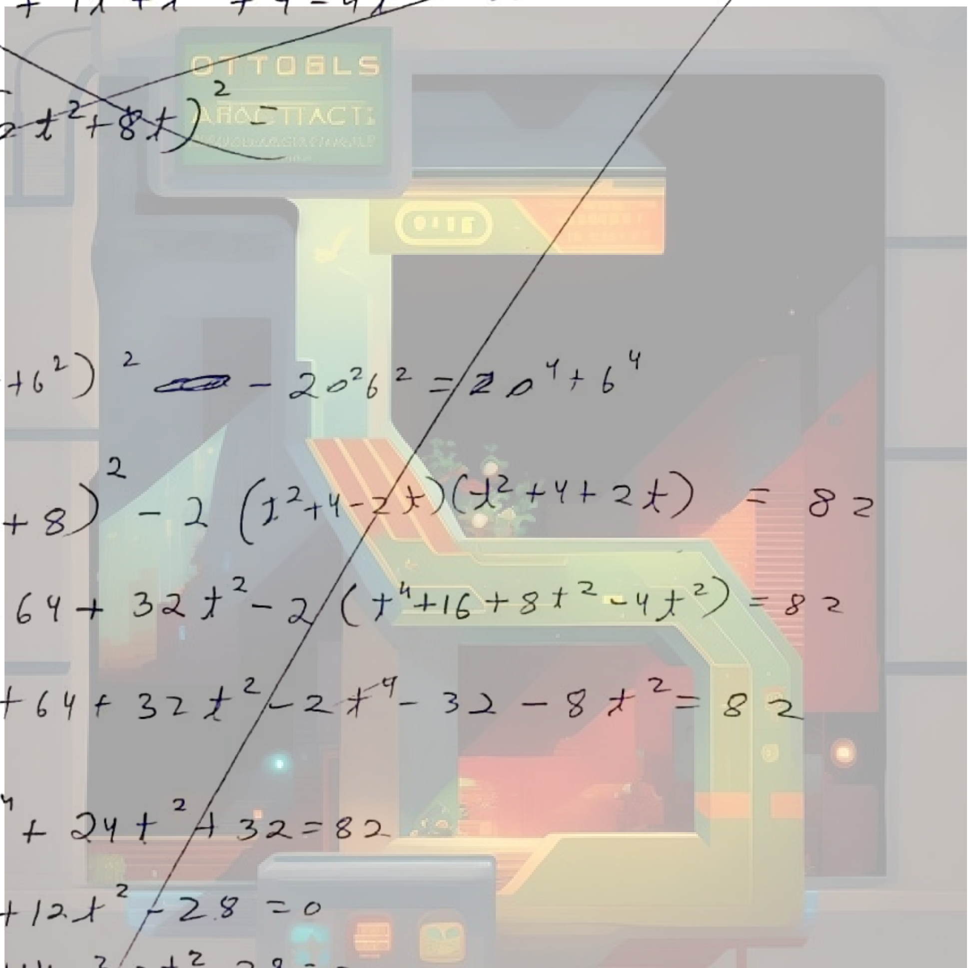
$t = x-3$

$x = t+3$

$(t+2)^4 + (t-2)^4 = 82$

$\cancel{t^2+4+4t+t^2+4-4t = 82}$

$\cancel{(2t^2+8t)^2 =}$



$(a^2+b^2)^2 - 2a^2b^2 = a^4+b^4$

$(2t^2+8)^2 - 2(t^2+4-2t)(t^2+4+2t) = 82$

$4t^4+64+32t^2-2(t^4+16+8t^2-4t^2) = 82$

$4t^4+64+32t^2-2t^4-32-8t^2 = 82$

$2t^4+24t^2+32 = 82$

$t^4+12t^2-28 = 0$

$t^4+14t^2-2t^2-28 = 0$

$t^2(t^2+14)-2(t^2+14) = 0$

$t^2 = 2$

$t = \pm\sqrt{2}$

$t^2 = 14$

$t = \pm 2$

$\boxed{x = 5, 1}$

System of Equation

→ It comprises two or more equations which are satisfied ~~by~~ by the same set of values of variable

Q $x^2 - y^2 = 16$

$x + y = 8$

$x = 8 - y$

$(8-y)^2 - y^2 = 16$

$\cancel{y^2} + 64 - 16y - \cancel{y^2} = 16$

$16y = 64 - 16$

$y = \frac{48}{16}$

$\boxed{y = 3}$

$\boxed{x = 5}$

Q4. $x - y = 1$

$x^3 - y^3 = 7$

$(1+y)^3 - y^3 = 7$

$1 + \cancel{y^3} + 3y(y+1) - \cancel{y^3} = 7$

$3y(y+1) + 1 = 7$

$3y^2 + 3y = 86$

$y^2 + y - 2 = 0$

$y^2 + 2y - y - 2 = 0$

$y(y+2) - 1(y+2)$

$(y-1)(y+2)$

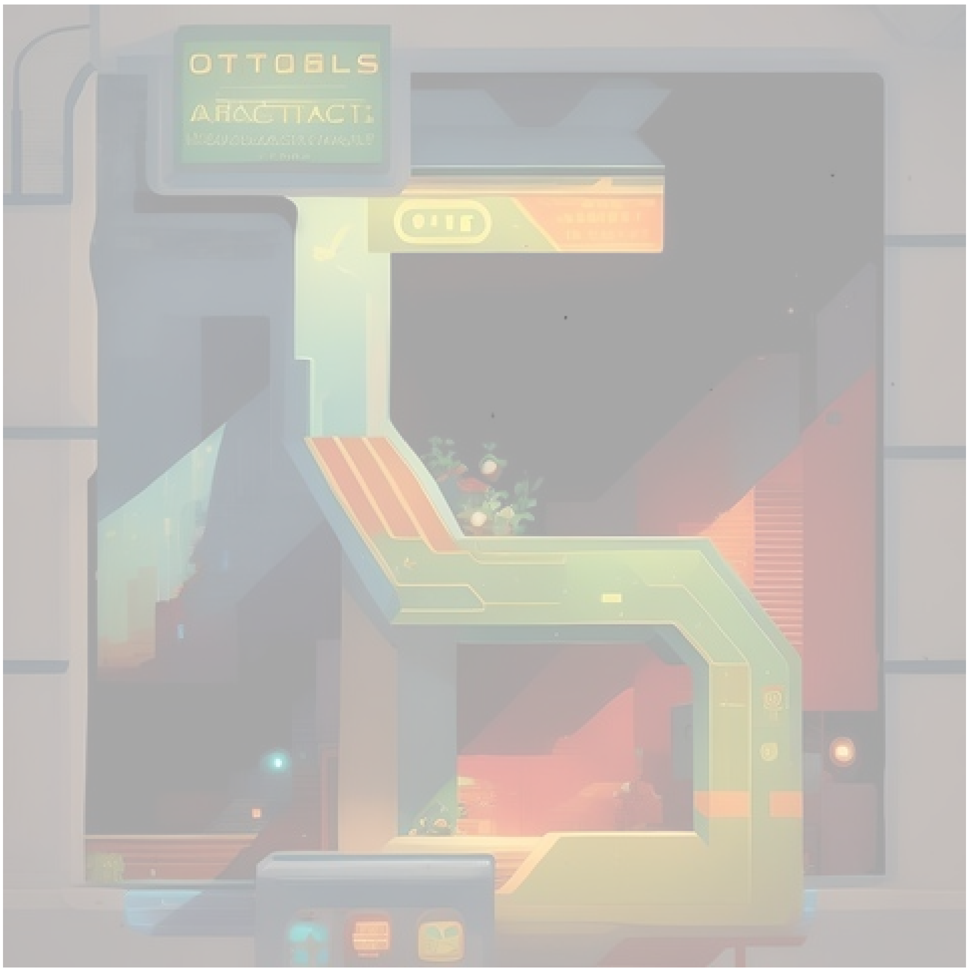
$y = 1$ $x = 2$	$y = -2$ $x = -1$

H.W.

D YS-6

QS.

O-1 (Q 01, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18)



Q6. $x^2+y^2+6x+2y=0$

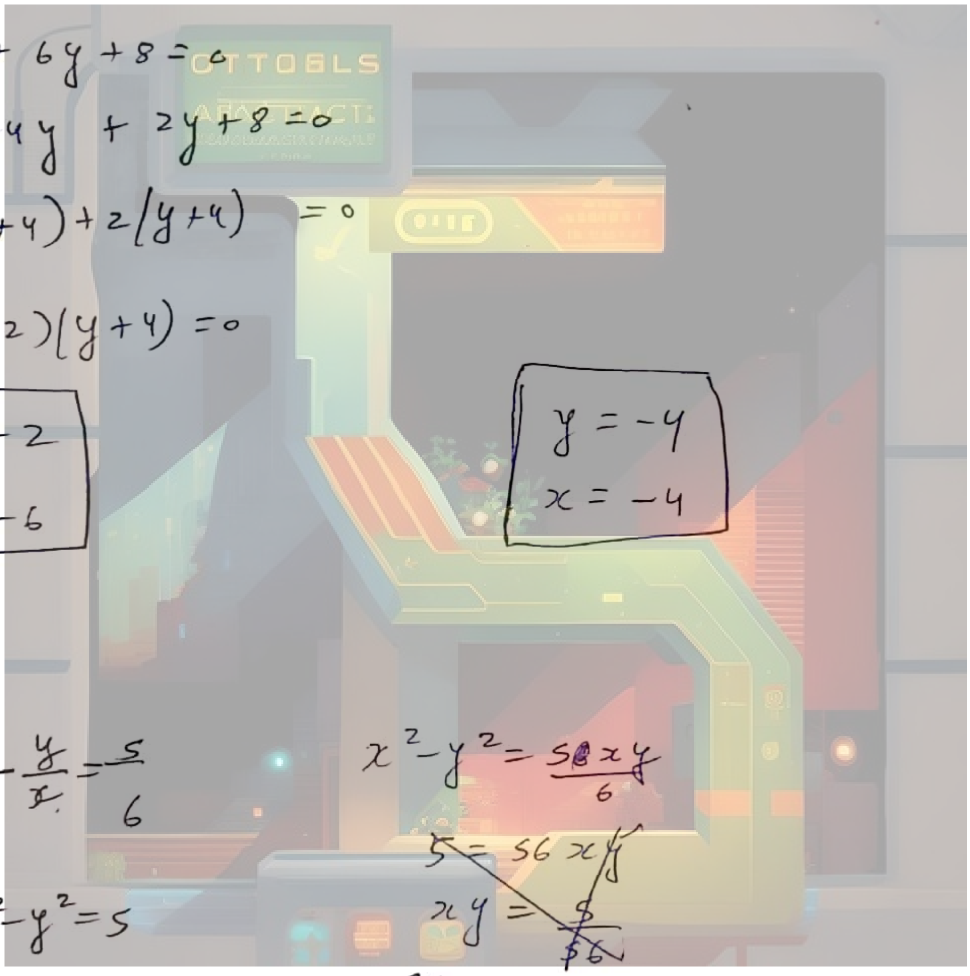
$x+y+8=0$

$x=-(y+8)$

$(y+8)^2+y^2-6y-48+2y=0$

$y^2+64+16y+y^2-6y-48+2y$

$12y+2y^2+16=0$



$y^2+6y+8=0$

$y^2+4y+2y+8=0$

$y(y+4)+2(y+4)=0$

$(y+2)(y+4)=0$

$\boxed{y=-2 \quad x=-6}$ $\boxed{y=-4 \quad x=-4}$

~~Method 2~~

Q7. $\frac{x}{y}-\frac{y}{x}=\frac{5}{6}$

$x^2-y^2=\frac{5xy}{6}$

$\cancel{5}=\cancel{5}6xy$

$xy=\frac{\cancel{5}}{\cancel{56}}$

$x^2-y^2=5$

$\frac{30}{5}=6=xy$

$\frac{6}{y}=x$

$\cancel{(x-y)^2=x^2+y^2-12}$

$\frac{36}{y^2}-y^2=5$

$36-y^4=5y^2$

$$y^4 + 5y^2 - 36 = 0$$

$$y^4 + 9y^2 - 4y^2 - 36$$

$$y^2(y^2+9) - 4(y^2+9)$$

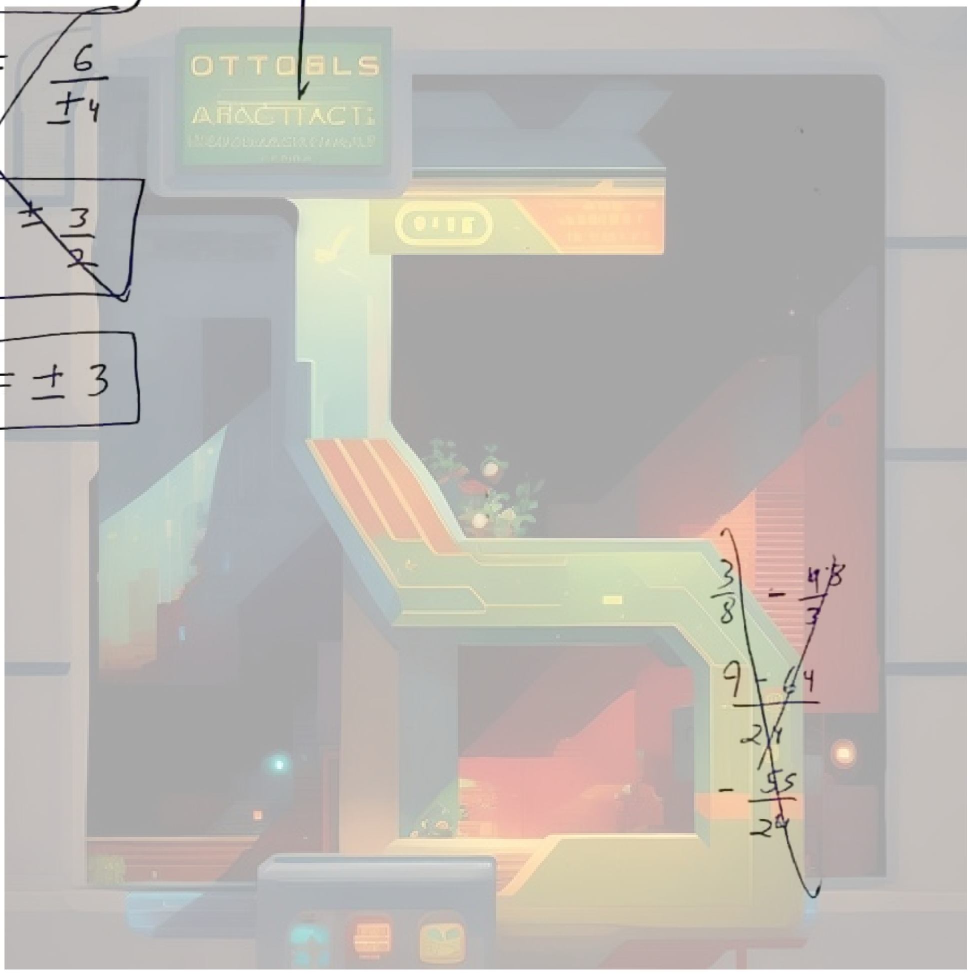
$$(y^2-4) \qquad (y^2+9)$$

$$\boxed{y = \pm 2} \qquad y^2 = -3 \quad \times$$

$$\cancel{x = \frac{6}{\pm 4}}$$

$$\boxed{\cancel{x = \pm \frac{3}{2}}}$$

$$\boxed{x = \pm 3}$$



$$\frac{3}{8} - \frac{4}{3}$$

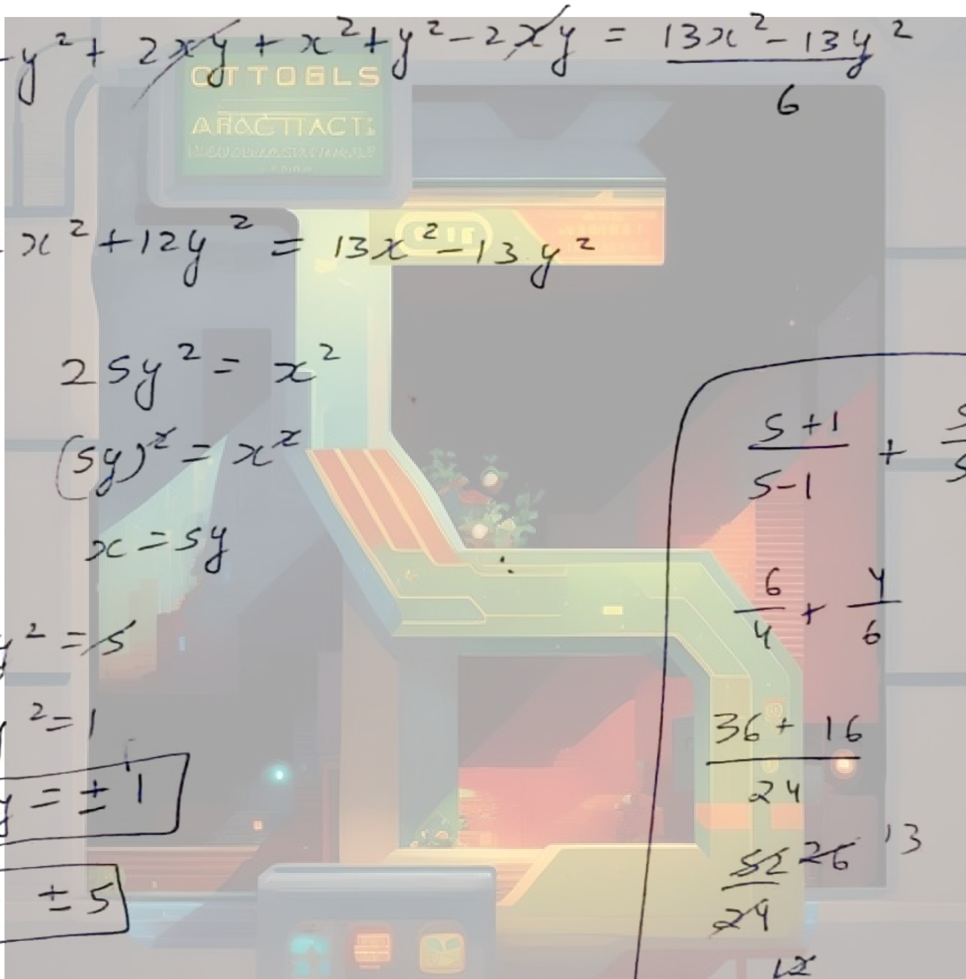
$$\frac{9-64}{24}$$

$$-\frac{55}{24}$$

Q8. $\frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{13}{6}$

$xy = 5$

$$\frac{(x+y)^2 + (x-y)^2}{x^2-y^2} = \frac{13}{6}$$



$$x^2+y^2+\cancel{2xy}+x^2+y^2-\cancel{2xy} = \frac{13x^2-13y^2}{6}$$

$$12x^2+12y^2 = 13x^2-13y^2$$

$$25y^2 = x^2$$

$$(5y)^2 = x^2$$

$$x = 5y$$

$$5y^2 = 5$$

$$y^2 = 1$$

$$\boxed{y = \pm 1}$$

$$\boxed{x = \pm 5}$$

$$\frac{5+1}{5-1} + \frac{5-1}{5+1} =$$

$$\frac{6}{4} + \frac{4}{6}$$

$$\frac{36+16}{24}$$

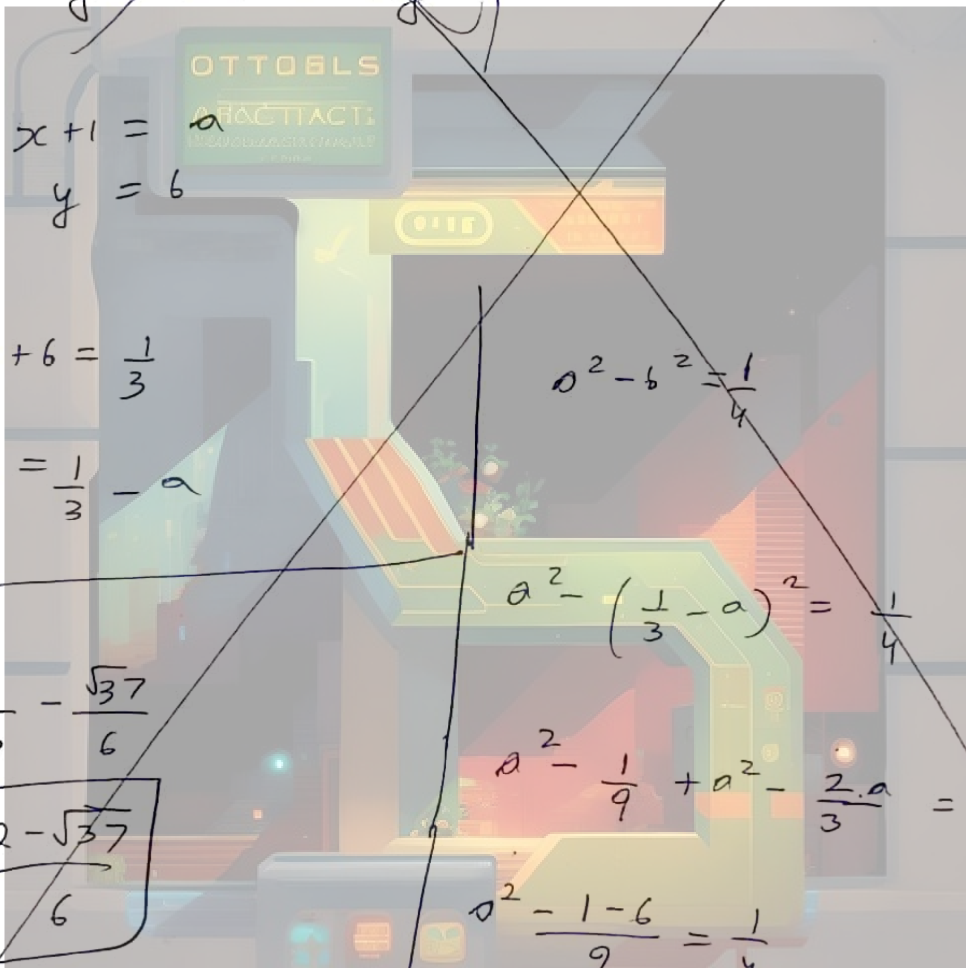
$$\frac{\cancel{52}\ \cancel{26}\ 13}{\cancel{24}\ \cancel{12}\ 6}$$

Q9. $\frac{1}{x+1} + \frac{1}{y} = \frac{1}{3}$ $\qquad$ $\frac{1}{(x+1)^2} - \frac{1}{y^2} = \frac{1}{4}$

$y + x + 1 = \frac{xy + y}{3}$

$3y + 3x + 3 = xy + y$

$2y + 3x + 3 = xy$



$x + 1 = a$

$y = b$

$a + b = \frac{1}{3}$

$b = \frac{1}{3} - a$

$a^2 - b^2 = \frac{1}{4}$

$a^2 - \left(\frac{1}{3} - a\right)^2 = \frac{1}{4}$

$a^2 - \frac{1}{9} + a^2 - \frac{2.a}{3} = \frac{1}{4}$

$a^2 - \frac{1-6}{9} = \frac{1}{4}$

$a^2 - \frac{7}{9} = \frac{1}{4}$

$a^2 = \frac{1}{4} + \frac{7}{9}$

$a^2 = \frac{9+28}{36}$

$a^2 = \frac{37}{36}$

$\boxed{a = \frac{\sqrt{37}}{6}}$

$b = \frac{1}{3} - \frac{\sqrt{37}}{6}$

$\boxed{b = \frac{2-\sqrt{37}}{6}}$

Q11. $\dfrac{1}{y-1} - \dfrac{1}{y+1} = \dfrac{1}{x}$

$y+1-y+1 = \dfrac{y^2-1}{x}$

$2x = y^2-1$

$y^2 = 2x+1$

$y^2 - x - 5 = 0$

$2x+1-x-5=0$

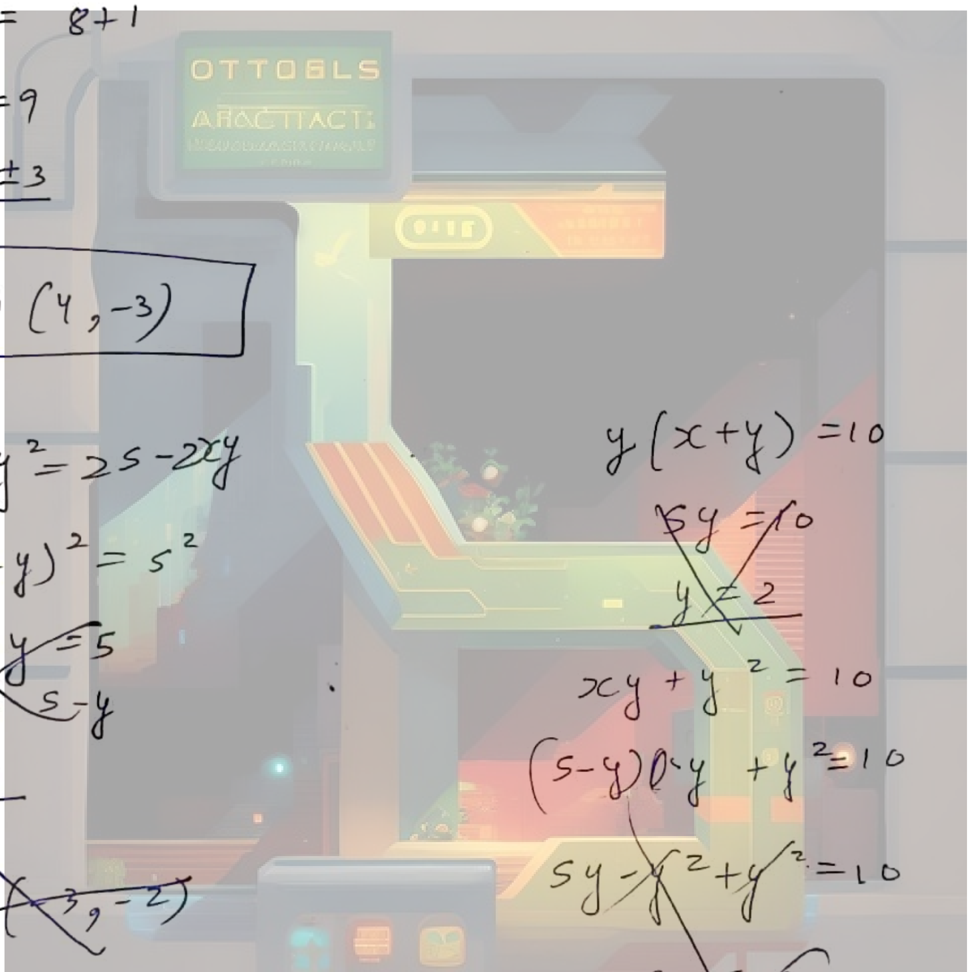
$\underline{x=4}$

$y^2 = 8+1$

$y^2 = 9$

$\underline{y = \pm 3}$

$\boxed{(4, 3)\ (4,-3)}$



Q10. $x^2+y^2 = 25-2xy$

$(x+y)^2 = 5^2$

$\cancel{x+y=5}$

$\cancel{x = 5-y}$

$\cancel{x=3}$

$\cancel{(3,2)\ (-3,-2)}$

$x+y = \pm 5$

$\boxed{(3,2)\ (-3,-2)}$

$y(x+y) = 10$

$\cancel{5y=10}$

$\cancel{y=2}$

$xy + y^2 = 10$

$(5-y)y + y^2 = 10$

$5y - \cancel{y^2} + \cancel{y^2} = 10$

$\cancel{5y = 10}$

$\cancel{y = 2}$

$y(\pm 5) = 10$

$\boxed{y = \pm 2}$

$\boxed{x = \pm 3}$

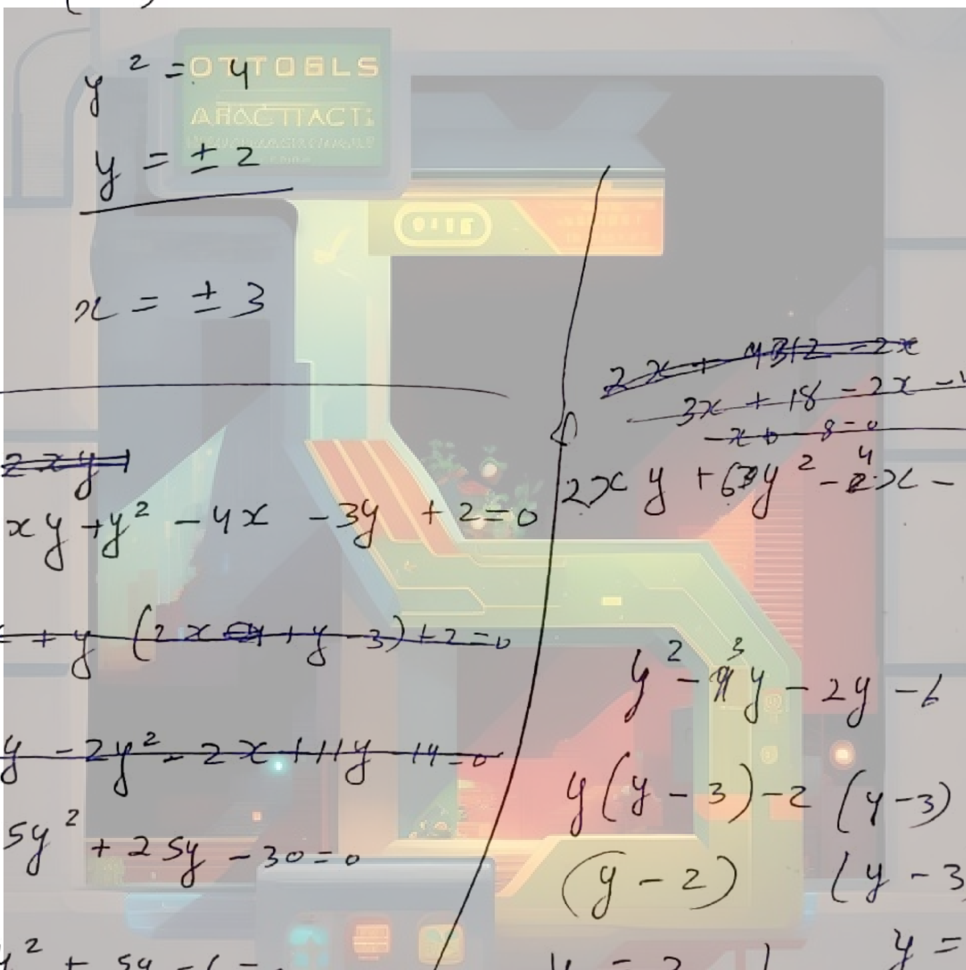
Method - 2

$$y(x+y) = 10$$

$$y^2(x+y)^2 = 100$$

$$(x+y)^2 = 25$$

$$y^2(25) = 100$$



$$y^2 = 4$$

$$y = \pm 2$$

$$x = \pm 3$$

~~Q11. 2xy~~

Q12. $2xy + y^2 - 4x - 3y + 2 = 0$

$$\cancel{-4x + y(2x + y - 3) + 2 = 0}$$

$$\cancel{xy - 2y^2 - 2x + 11y - 14 = 0}$$

$$-5y^2 + 25y - 30 = 0$$

$$-y^2 + 5y - 6 = 0$$

$$y^2 - 5y + 6 = 0$$

$$\cancel{y^2 - 6y + y + 6 = 0}$$

$$\cancel{y(y-6) + 1(y-6)}$$

$$\cancel{2x + 4(3/2) - 2x}$$

$$\cancel{3x + 18 - 2x - 42y + 16}$$

$$\cancel{-x + 8 = 0}$$

$$2xy + \cancel{6}3y^2 - \overset{4}{\cancel{2}}x - \overset{28}{\cancel{21}}y + \overset{32}{\cancel{16}} = 0$$

$$y^2 - \overset{3}{\cancel{4}}y - 2y - 6 = 0$$

$$y(y-3) - 2(y-3)$$

$$(y-2)(y-3)$$

$y = 2$ ✗ $\qquad y = 3$

$$\cancel{x = 8}$$

$$3x + 27 - 2x - 42 + 16$$

$$\boxed{x = -1}$$

H.W.

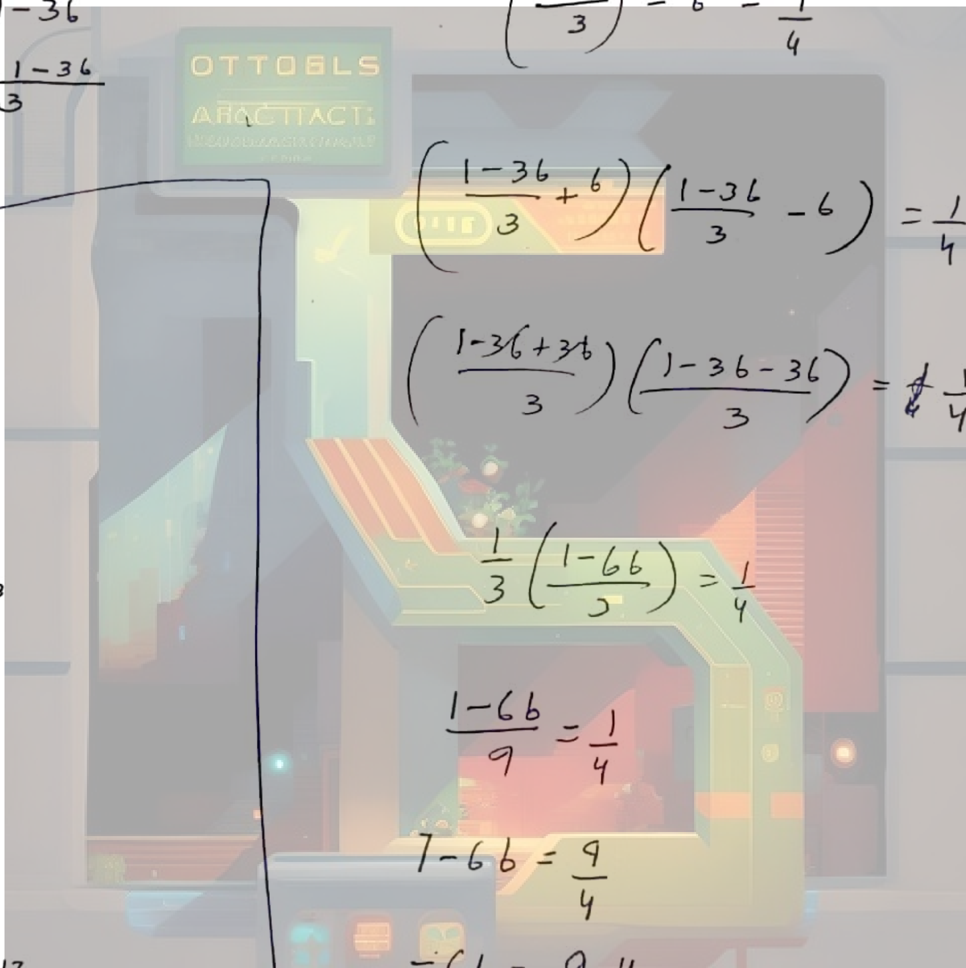
Q1. $\frac{1}{x+1} + \frac{1}{y} = \frac{1}{3}$ $\qquad$ $\frac{1}{(x+1)^2} - \frac{1}{y^2} = \frac{1}{4}$

$a + b = \frac{1}{3}$ $\qquad$ $a^2 - b^2 = \frac{1}{4}$

$3a + 3b = 1$

$3a = 1 - 3b$

$\cancel{3}a = \frac{1-3b}{3}$



$a = \frac{1 + \frac{15}{24}}{3}$

$a = \frac{24+15}{24 \times 3}$

$= \frac{\cancel{39}^{13}}{3 \times 24}$

$= \frac{13}{24}$

$x + 1 = \frac{13}{24}$

$24x + 24 = 13$

$x = \frac{13-24}{24}$

$\boxed{x = \frac{-11}{24}}$

$\boxed{y = \frac{-5}{24}}$

$\left(\frac{1-3b}{3}\right)^2 - b^2 = \frac{1}{4}$

$\left(\frac{1-3b}{3} + b\right)\left(\frac{1-3b}{3} - b\right) = \frac{1}{4}$

$\left(\frac{1-3b+3b}{3}\right)\left(\frac{1-3b-3b}{3}\right) = \cancel{4}\,\frac{1}{4}$

$\frac{1}{3}\left(\frac{1-6b}{3}\right) = \frac{1}{4}$

$\frac{1-6b}{9} = \frac{1}{4}$

$1 - 6b = \frac{9}{4}$

$-6b = \frac{9-4}{4}$

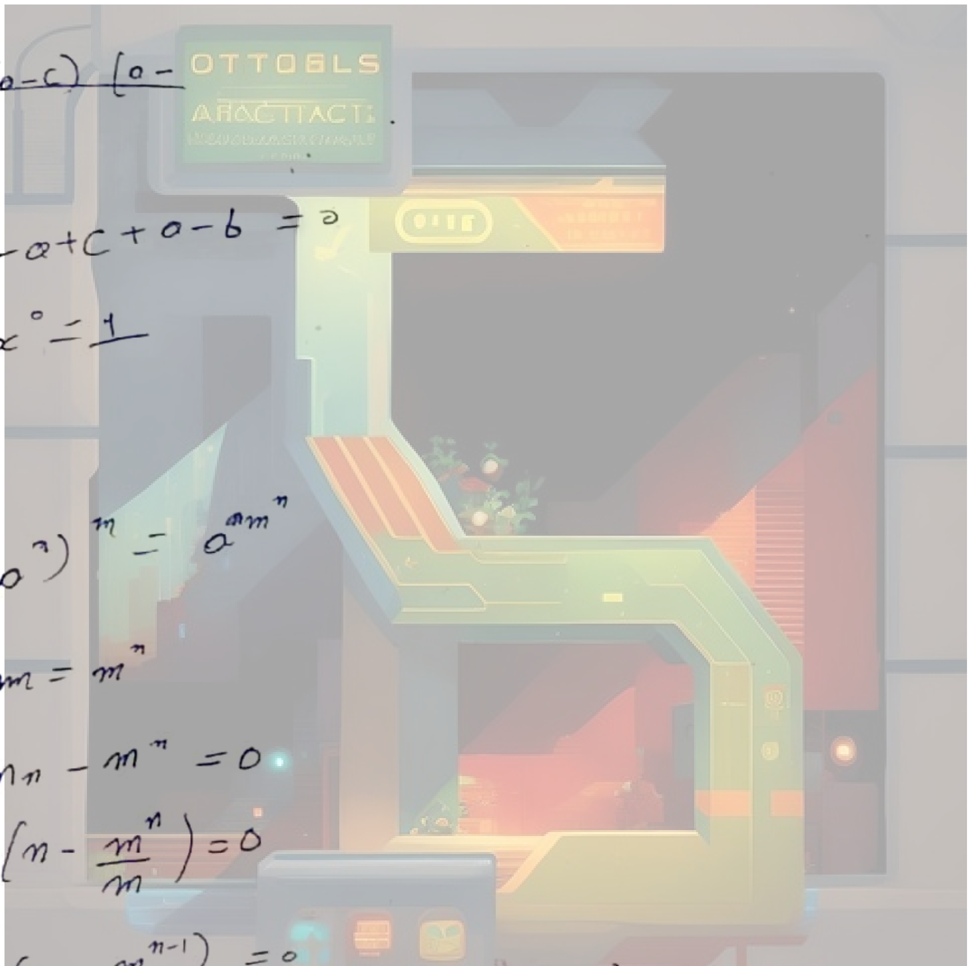
$6b = \frac{-5}{4 \times 6}$

$\cancel{6}\boxed{b = \frac{-5}{24}}$

O-1

QC. $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$

$\frac{1}{(a-b)(a-c)} - \frac{1}{(b-c)(a-b)} + \frac{1}{(a-c)(b-c)}$



$\cancel{(a-b)(a-c)(a-}$

$\cancel{a-b}$

$b-c-a+c+a-b=0$

$x^0 = 1$

Q11.

$(a^n)^m = a^{m^n}$

$nm = m^n$

$mn - m^n = 0$

$m\left(n - \frac{m^n}{m}\right) = 0$

$m(n - m^{n-1}) = 0$

$m = 0$ (rejected $m > 0$ given)

$n - m^{n-1} = 0$

$m^{n-1} = n$

$m^{\frac{n-1}{n-1}} = n^{\frac{1}{n-1}}$

$m = n^{\frac{1}{n-1}}$

$\boxed{A}$

(164)

H.W. 07-05-2024

DYS-6 Q 1 & 3

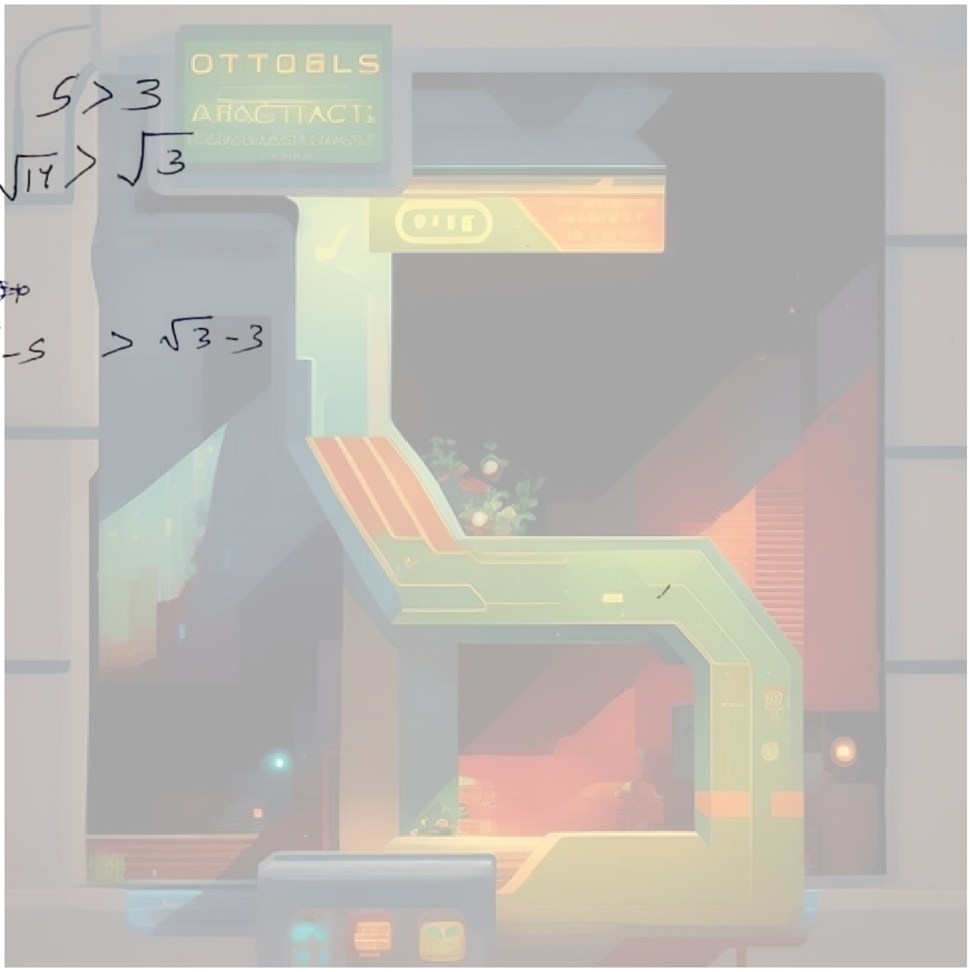
DYS-7 (Q1, 2, 5)

O-1 (Q 22, 23, 24, 25)

O-2 (Q1-5, 6, 8, 9, 10, 11, 12, 13, 14, 15)

DYS-7

Q5.



$5 > 3$

$\sqrt{14} > \sqrt{3}$

$\sqrt{14} - 5 > \sqrt{3} - 3$

DVS-6

Q14.

$x^4 + y^4 = 82$

$xy = 3$

$y = \frac{3}{x}$

$y^4 = \left(\left(\frac{3}{x}\right)^2\right)^2$

$= \left(\frac{9}{x^2}\right)^2$

$= \frac{81}{x^4}$

$x^4 + \frac{81}{x^4} = 82$

$x^8 + 81 = 82x^4$

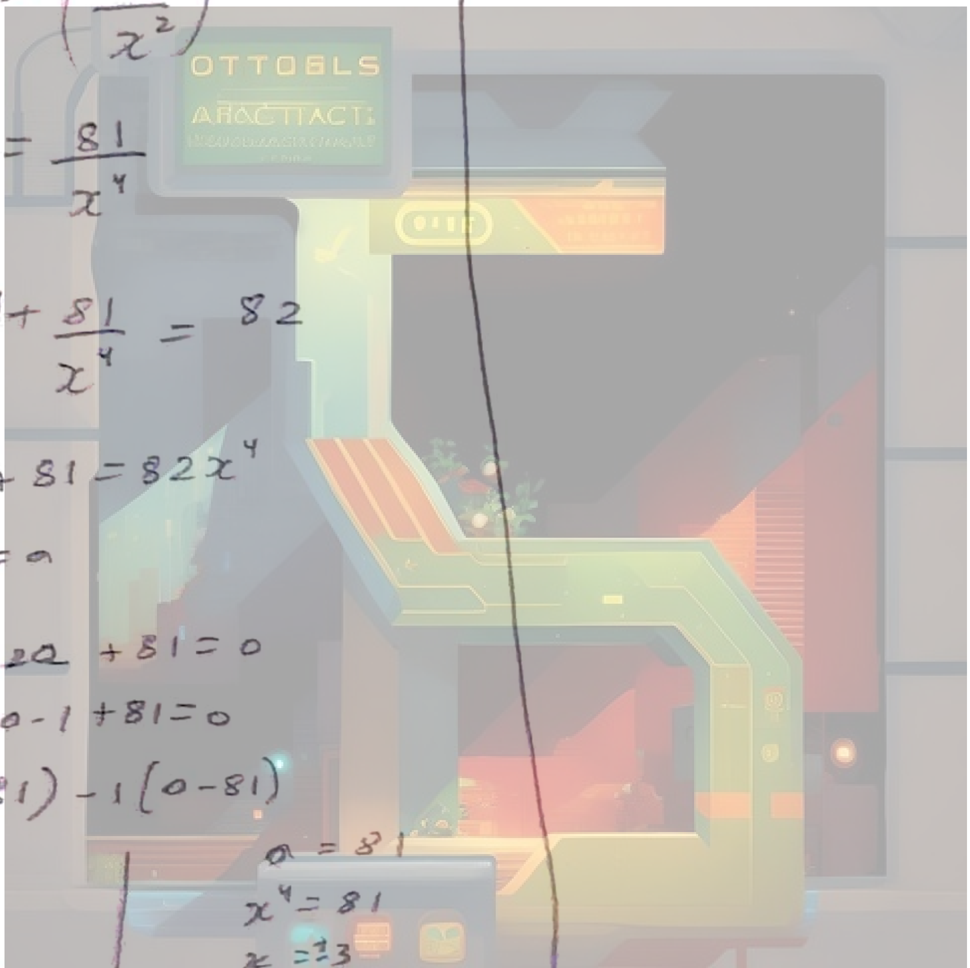
$x^4 = a$

$a^2 - 82a + 81 = 0$

$a^2 - 81a - a + 81 = 0$

$a(a-81) - 1(a-81)$

$a = 1$	$a = 81$
$x^4 = 1$	$x^4 = 81$
$x = \pm 1$	$x = \pm 3$
$y = \pm 3$	$y = \pm 1$
$(1,3)\ (3,1)$	$(-1,-3),\ (-3,-1)$



Method-2

$(x^2+y^2)^2 = x^4+y^4+2x^2y^2$

$(x^2+y^2)^2 = 82 + 2\times(3)^2$

$x^2+y^2 = \pm 10$

put $y = \frac{3}{x}$ & solve

3 Variables

Q 1. $a + b = 10$

$b + c = 15$

$a + c = 25$

Find a, b, c

$b = 10 - a$

$10 - a + c = 15$

$c - a = 5$

$a + c = 25$

$2c = 30$

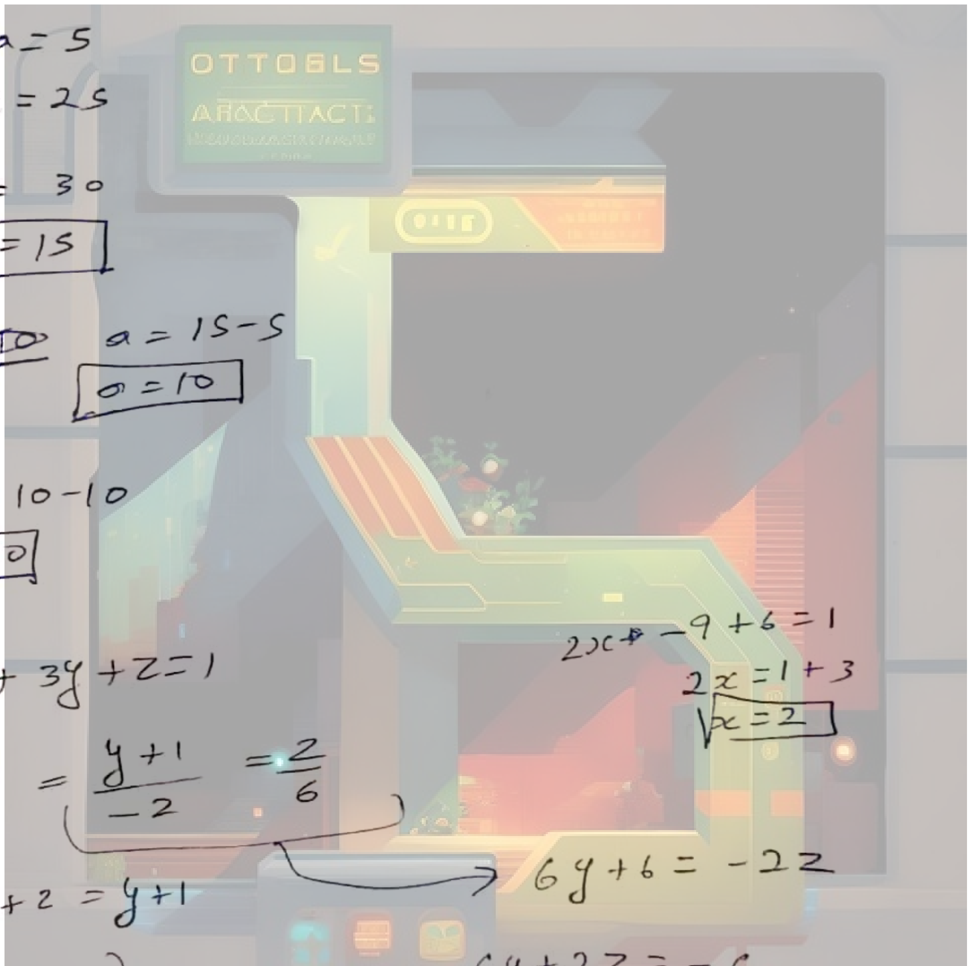
$\boxed{c = 15}$

~~$a = 10$~~ $a = 15 - 5$

$\boxed{a = 10}$

$b = 10 - 10$

$\boxed{b = 0}$



Q 2. $2x + 3y + z = 1$

$\underbrace{\frac{x-1}{1} = \frac{y+1}{-2}} = \frac{z}{6}$

$-2x + 2 = y + 1$

$-(2x + y = 1)$

$2x + 3y + z = 1$

$-(2y + z = 0) \times 2$

$6y + 2z = -6$

$2y = -6$

$\boxed{y = -3}$

$2x - 9 + 6 = 1$

$2x = 1 + 3$

$\boxed{x = 2}$

$6y + 6 = -2z$

$6y + 2z = -6$

$-18 + 2z = -6$

$2z = -6 + 18$

$\boxed{z = 6}$

Method - 2

let $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z}{6} = k$

put k in eq 1

Q13.

$w + x + y = -2$

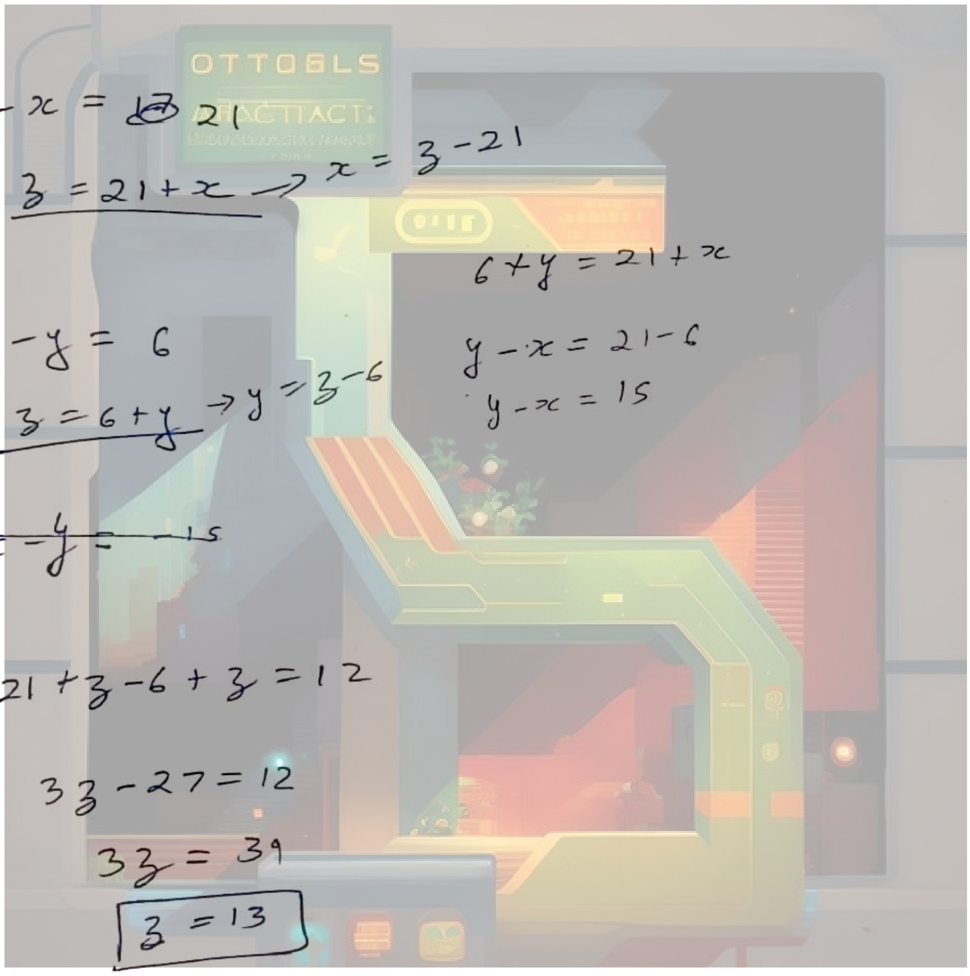
$w + x + z = 4$

$w + y + z = 19$

$x + \cancel{w} + y + z = 12$

$w + y + z = 19$

$-w - x - y = \cancel{8}\,2$



$z - x = \cancel{13}\,21$

$\underline{z = 21 + x} \rightarrow x = z - 21$

$6 + y = 21 + x$

$y - x = 21 - 6$

$y - x = 15$

$z - y = 6$

$\underline{z = 6 + y} \rightarrow y = z - 6$

$\cancel{x - y = -15}$

$z - 21 + z - 6 + z = 12$

$3z - 27 = 12$

$3z = 39$

$\boxed{z = 13}$

$\boxed{x = -8}$

$\boxed{y = 7}$

Q4. $x+y+z=4$

$x^2+y^2+z^2=6$

$x^3+y^3+z^3=8$

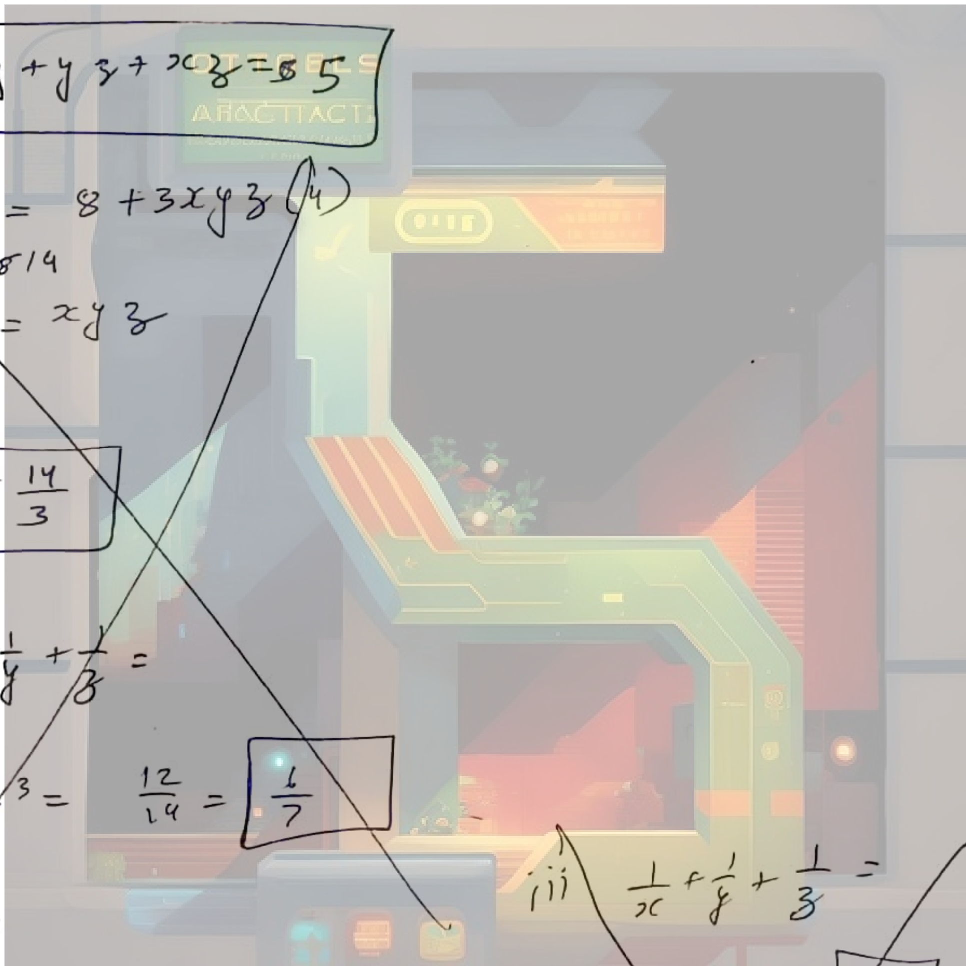
~~i) xyz~~

ii) $xy+yz+xz=?$

$\because (4)^2 = 6 + 2(xy+yz+xz)$

$\frac{16-6}{2} = \frac{10}{2} = 5$

$\boxed{xy+yz+xz=\cancel{8}\,5}$



i) xyz

$64 = 8+3xyz(4)$

$\frac{\cancel{56}^{\cancel{28}\,14}}{\cancel{12}_{\cancel{6}\,3}} = xyz$

$\boxed{xyz=\frac{14}{3}}$

iii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$

$\frac{4\times 3}{14} = \frac{12}{14} = \boxed{\frac{6}{7}}$

i) xyz

$8-3abc=(4)(6-5)$

$8-3abc=4$

$3abc=\cancel{2}4$

$\boxed{abc=\frac{\cancel{2}4}{3}}$

iii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$

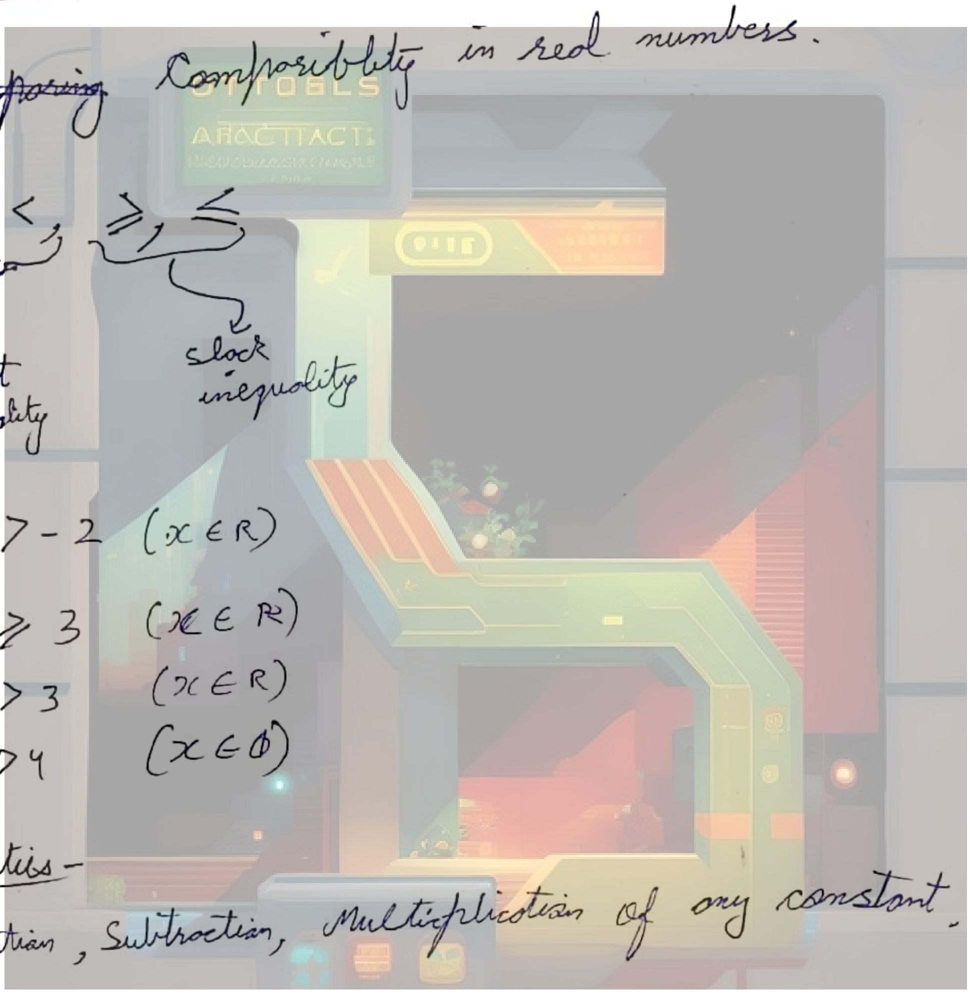
$\frac{4}{\cancel{2}4}\times 3 = \boxed{3}$

$\frac{x+y+z}{xyz} = \frac{4}{\frac{4}{3}} = \boxed{3}$

iii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

$$\frac{xy+yz+xz}{xyz} = \frac{5}{\frac{4}{3}} = \boxed{\frac{15}{4}}$$ ✓

Inequalities



→ ~~Comparing~~ Comparibility in real numbers.

→ $\underbrace{>, <}_{\text{Strict Inequality}}, \underbrace{\geq, \leq}_{\text{slack inequality}}$

eg. $4 > -2 \quad (x \in R)$

$3 \geq 3 \quad (x \in R)$

$4 > 3 \quad (x \in R)$

$3 > 4 \quad (x \in \emptyset)$

→ Properties -

① Addition, Subtraction, Multiplication of any constant.

$5 > 2$

+

$5 > 2$

$5+2 > 2+2$

$7 > 4$

−

$5 > 2$

$5-2 > 2-2$

$3 > 0$

$$5 > 2$$

Multiply ⊕ve

$(3)\ 5 > 2\ (3)$

$15 > 6$

⊖ve

$(-3)\ 5 > 2\ (-3)$

$\boxed{-15 > -2 \text{ (not valid)}}$

Division

$\frac{5}{1} > \frac{2}{1}$

$5 > 1$

⊖ve

$\frac{5}{-1} > \frac{2}{-1}$

$\boxed{-5 > -2 \text{ (not valid)}}$

→ If we multiply ~~or~~ or divide any negative constant, then we have to <u>reverse the sign</u> of inequality.

→ Never multiply or divide any thing whose ~~sig~~ sign we dont know.

eg – $9 > 2$

$9x > 2x$ (not valid)

$x = 3$

$9(3) > 2(3)$

$27 > 6$

$x = -3$

$9(-3) > 2(-3)$

$-27 > -6$

(wrong)

eg ② – $9 > 2$

$9(x^2+2) > 9(x^2+2)$

(we know (x^2+2) is always ⊕ve)

→ Inequalities sign reverse when we take <u>resiprocal both sides</u>

eg $3 > 2$

$\frac{1}{3} < \frac{1}{2}$

→ Cross Multiplication is not valid until unless we dont know the sign.

eg. $\frac{2}{x} > 1$

$2 > x$ (wrong)

$\frac{2}{x} - 1 > 0$

$\frac{2-x}{x} > 0$ (right)

→ If we cancle anything (variables) in inequality then mension it

eg 2. $9 > \frac{\cancel{2-x}}{\cancel{2-x}}$ $\quad x \neq 2$

$9 > 1$ (for $x \in R, x \neq 2$) right

$9 > \frac{\cancel{(2-x)}}{\cancel{(2-x)}}$ wrong as for $x = 2$

$9 > \frac{0}{0}$ (not defined)

R $x \in R$

$9 > 1$

→ we can cancel out anything (constant & variables) which are in addition of subtraction in inequality.

eg. $\cancel{x}+2 > \cancel{x}-1$

$2 > -1$ (right)

$2\cancel{x} > \cancel{x}(-1)$

$2 > -1$ (wrong)

→

$AB > AC \cdot A$

⇓

$B > C$ (wrong)

$AB - AC > 0$

$A(B - C) > 0$ (right)

→ 3. Addition of two inequalities is valid with same sign, but subtraction, multiplication & division is not valid.

eg.

$$\begin{array}{r} 2 > 1 \\ +\ 3 > 2 \\ \hline 5 > 3 \end{array}$$ (right

$$\begin{array}{r} 2 > 1 \\ +\ 3 < 5 \\ \hline 5 \cdot 6 \end{array}$$ (wrong)

eg (2). $\cancel{-3 < 1}$, $\cancel{10 < 11}$

$-3 < 1$ $\quad 10 < 11$

$13 < 10$ (wrong)

Wavy Curvy Method

Q $x - 3 > 0$

Factors → $x - 3 = 0$

$x = 3$

$-\infty \leftarrow$ (−) 3 (+) $\rightarrow +\infty$

two sets $(3, \infty)$ & $(-\infty, 3)$

take random val

$x = 1000$

$1000 - 3 =$ (+)ve

so $(x - 3)$ (+)ve for x in $(3, \infty)$

Scanned with CamScanner

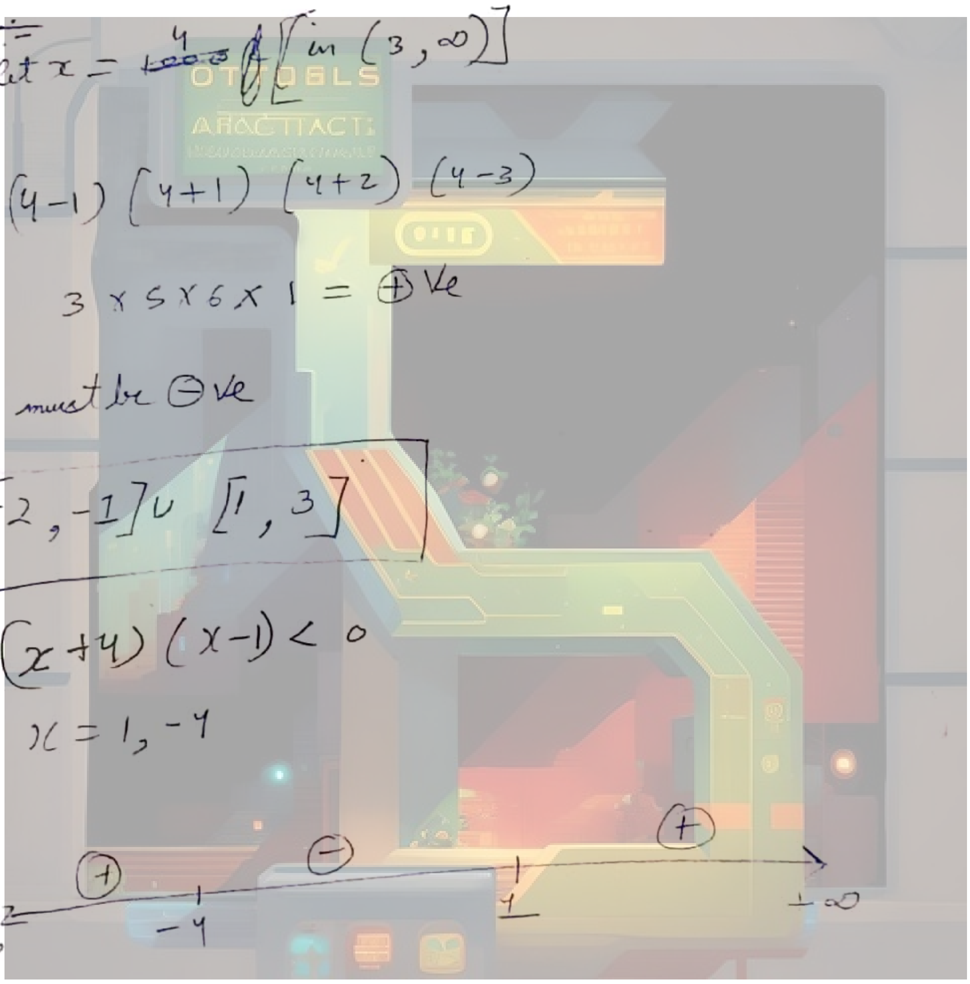
Q2. $(x-1)(x+1)(x+2)(x-3) \le 0$

$x-1=0$

$x = 1, -1, -2, 3$

$$\begin{array}{ccccccccccc} & + & & - & & + & & - & & + & \\ -\infty & & -2 & & -1 & & 1 & & 3 & & +\infty \end{array}$$

~~Intervals:~~



let $x = \cancel{1000}\ 4$ [in $(3, \infty)$]

$(4-1)(4+1)(4+2)(4-3)$

$3 \times 5 \times 6 \times 1 = +ve$

x must be $-ve$

$$\boxed{[-2, -1] \cup [1, 3]}$$

Q3. $(x+4)(x-1) < 0$

$x = 1, -4$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & -4 & & 1 & & +\infty \end{array}$$

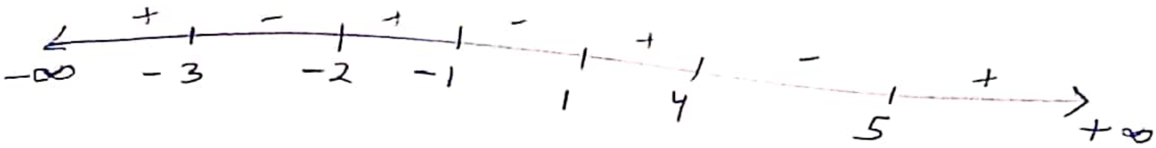
$x = 1000$

$(1004)(999) = +ve$

$$\boxed{(-4, 1)}$$

Q4. $(x-1)(x+3)(x+1)(x+2)(x-4)(x-5) \geq 0$

$x = 1, -3, -1, -2, 4, 5$



let $x=6$

$5 \times 9 \times 7 \times 8 \times 2 \times 1 = \oplus ve$

$$\boxed{x \in [-\infty, -3] \cup [-2, -1] \cup [1, 4] \cup [5, \infty)}$$

Note:-

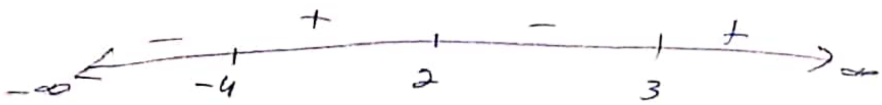
1. $(x-a)(x-b) \leq 0 \rightarrow x \in [a, b]$
2. $(x-a)(x-b) < 0 \rightarrow x \in (a, b)$
3. $(x-a)(x-b) \geq 0 \rightarrow x \in (-\infty, a] \cup [b, \infty)$
4. $(x-a)(x-b) > 0 \rightarrow x \in (-\infty, a) \cup (b, \infty)$

Q5. $(2-x)(4+x)(x-3) < 0$

$-(x-2)(4+x)(x-3) < 0$

(−1 multiply)

$(x-2)(x+4)(x-3) > 0$



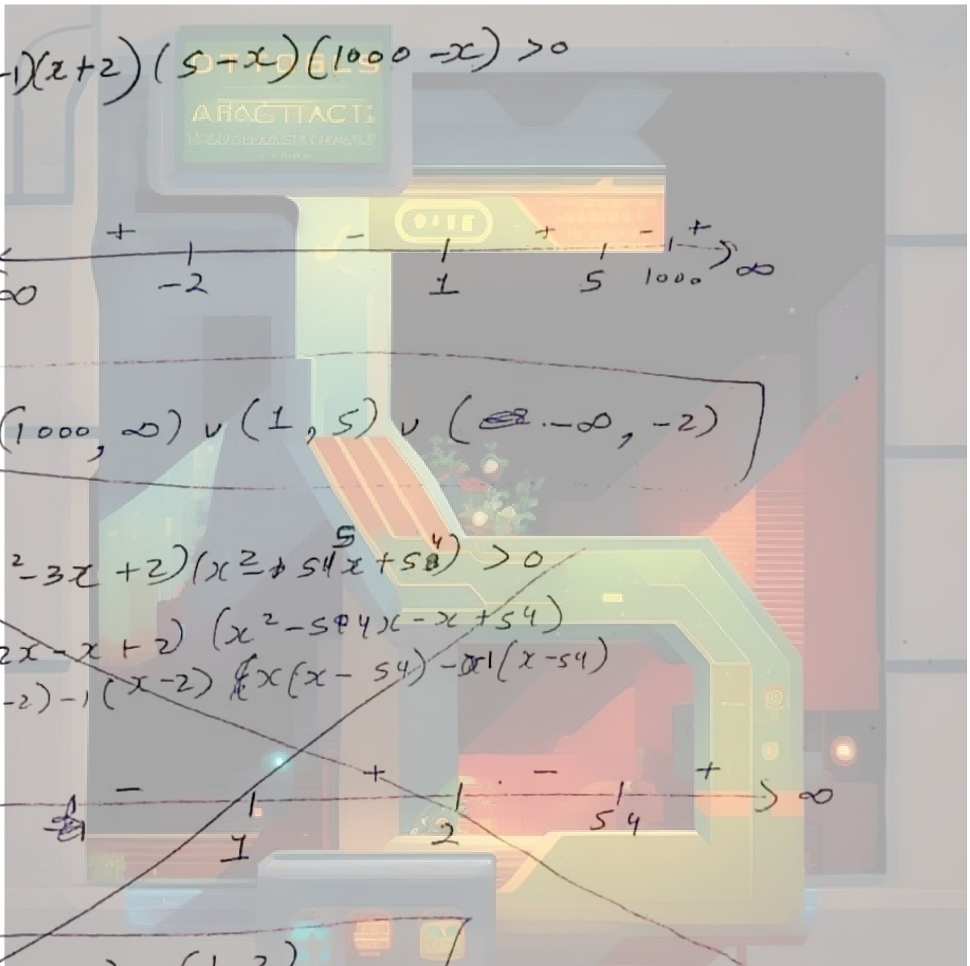
$x \in (-4, 2) \cup (3, \infty)$

Q6. $(4-x)(x+4) \le 0$

$-\infty$ ← (−) −4 (+) 4 (−) → ∞

$\boxed{x \in [4, \infty) \cup (-\infty, -4]}$

Q7. $(x-1)(x+2)(5-x)(1000-x) > 0$



$-\infty$ ← (+) −2 (−) 1 (+) 5 (−) 1000 (+) → ∞

$\boxed{x \in (1000, \infty) \cup (1, 5) \cup (-\infty, -2)}$

Q8. $(x^2-3x+2)(x^2-54x+54) > 0$

$(x^2-2x-x+2)\,(x^2-54x-x+54)$

$x(x-2)-1(x-2) \quad x(x-54)-1(x-54)$

$-\infty$ ← (−) 1 (+) 2 (−) 54 (+) → ∞

$\boxed{x \in (54, \infty) \cup (1, 2)}$

Note :- ① Quantities which have whole even powers does not change sign and for whole odd powers we can ~~chan~~ can change sign

Q8. $(x^2-3x+2)(x^2-55x+54)>0$

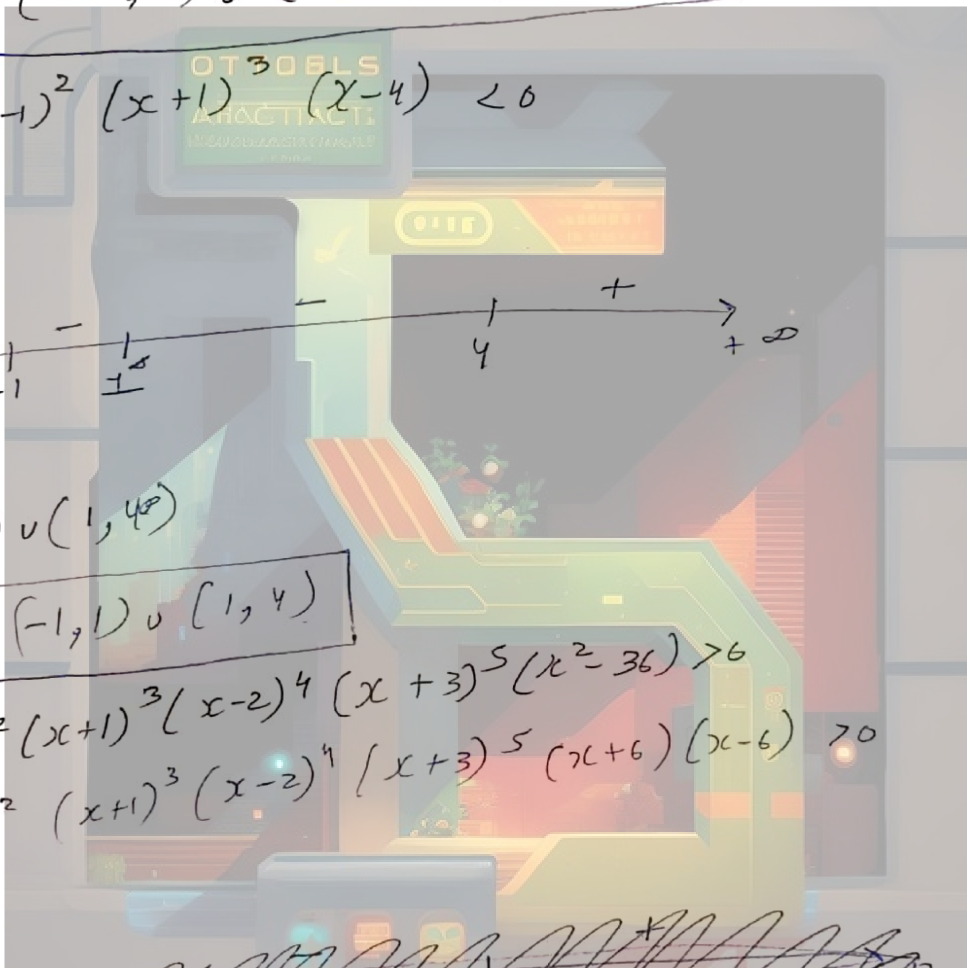
$(x-2)(x-1)\ (x-1)(x-54)>0$

$(x-2)(x-1)^2(x-54)$

$+\quad +\quad -\quad +$

$-\infty\quad 1^*\quad 2\quad 54\quad \infty$

$\boxed{x\in(-\infty,1)\cup(1,2)\cup(54,\infty)}$



Q9. $(x-1)^2(x+1)^3(x-4)<0$

$+\quad -\quad -\quad +$

$-\infty\quad -1\quad 1^*\quad 4\quad +\infty$

$(-1,1)\cup(1,4)$

$\boxed{x\in(-1,1)\cup(1,4)}$

Q10. $(x-1)^2(x+1)^3(x-2)^4(x+3)^5(x^2-36)>0$

$(x-1)^2(x+1)^3(x-2)^4(x+3)^5(x+6)(x-6)>0$

$(x-1)^2(x+1)^3(x-2)^4$

$\boxed{x\in(-\infty,-6)\cup(-3,-1)\cup(6,\infty)}$

$+\quad -\quad +\quad -\quad -\quad -\quad +$

$-\infty\quad -6\quad -3\quad -1\quad 1^*\quad 2^*\quad 6\quad \infty$

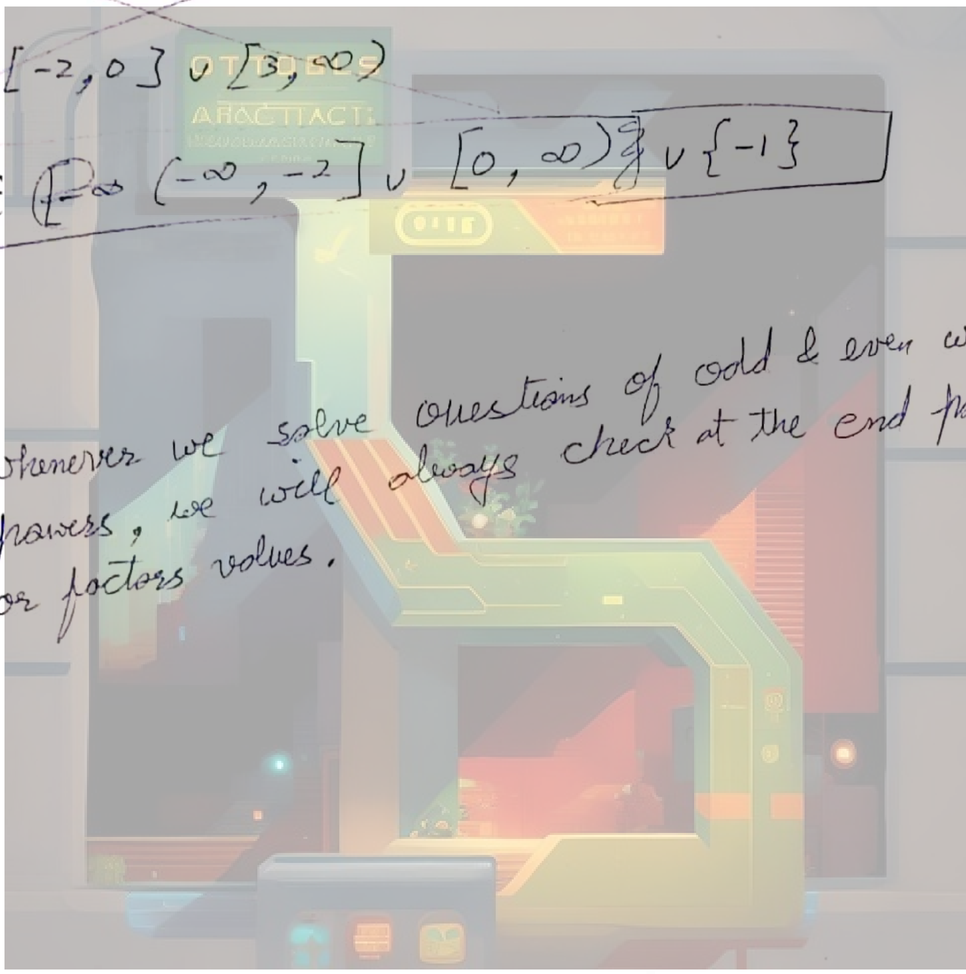
Q11. $(x-1)^2 (x+1)^4 (x) (x-3)^6 (x+2)^7 \geq 0$

$$\underset{-\infty}{} \quad + \quad \underset{-2}{|} \quad - \quad \underset{-1}{|} \quad - \quad \underset{0}{|} \quad + \quad \underset{1}{|} \quad + \quad \underset{3}{|} \quad + \quad \underset{\infty}{}$$

$\cancel{[-2,-1] \cup [-1,0] \cup [3,\infty)}$

$\cancel{x \in [-2,0] \cup [3,\infty)}$

$\boxed{x \in (-\infty,-2] \cup [0,\infty) \cup \{-1\}}$



Note:-

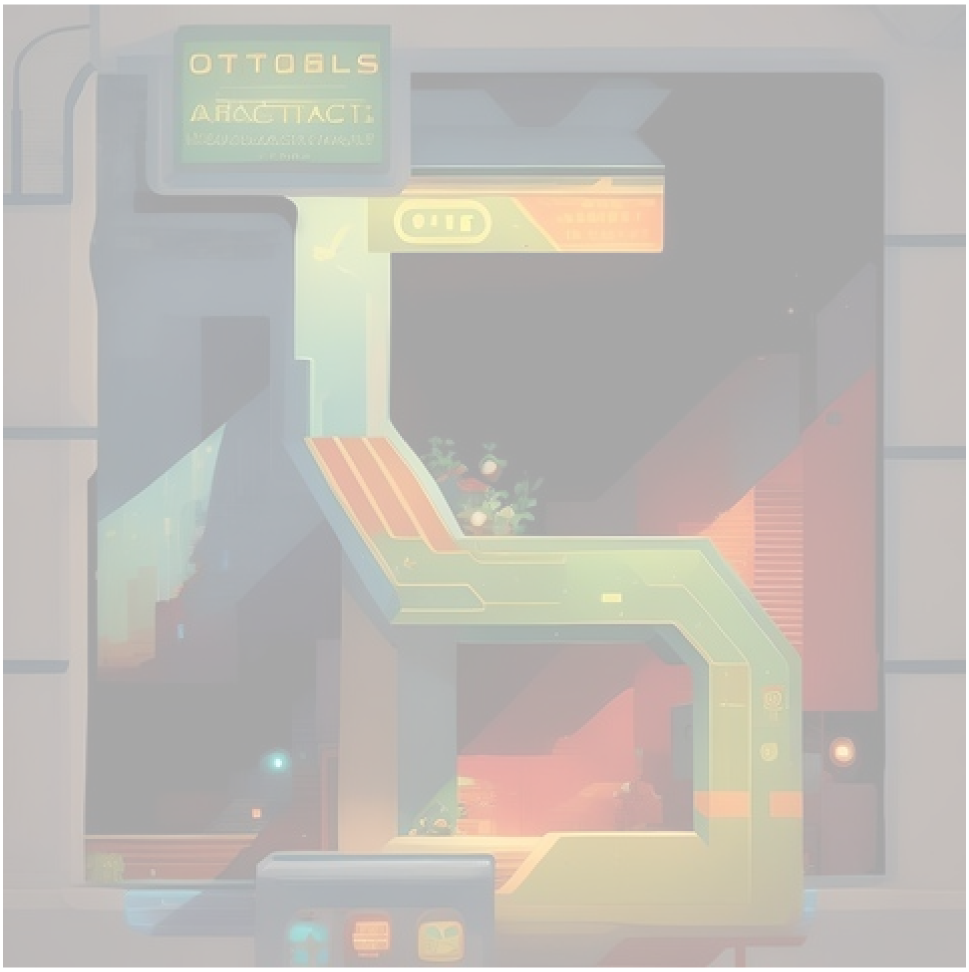
① whenever we solve questions of odd & even whole powers, we will always check at the end points or factors values.

H.W. 09-05-2024

~~Q-3 (Q 1 to 7)~~

Q-3 $[1,7] \cup \{8\}$ (1 to 7)

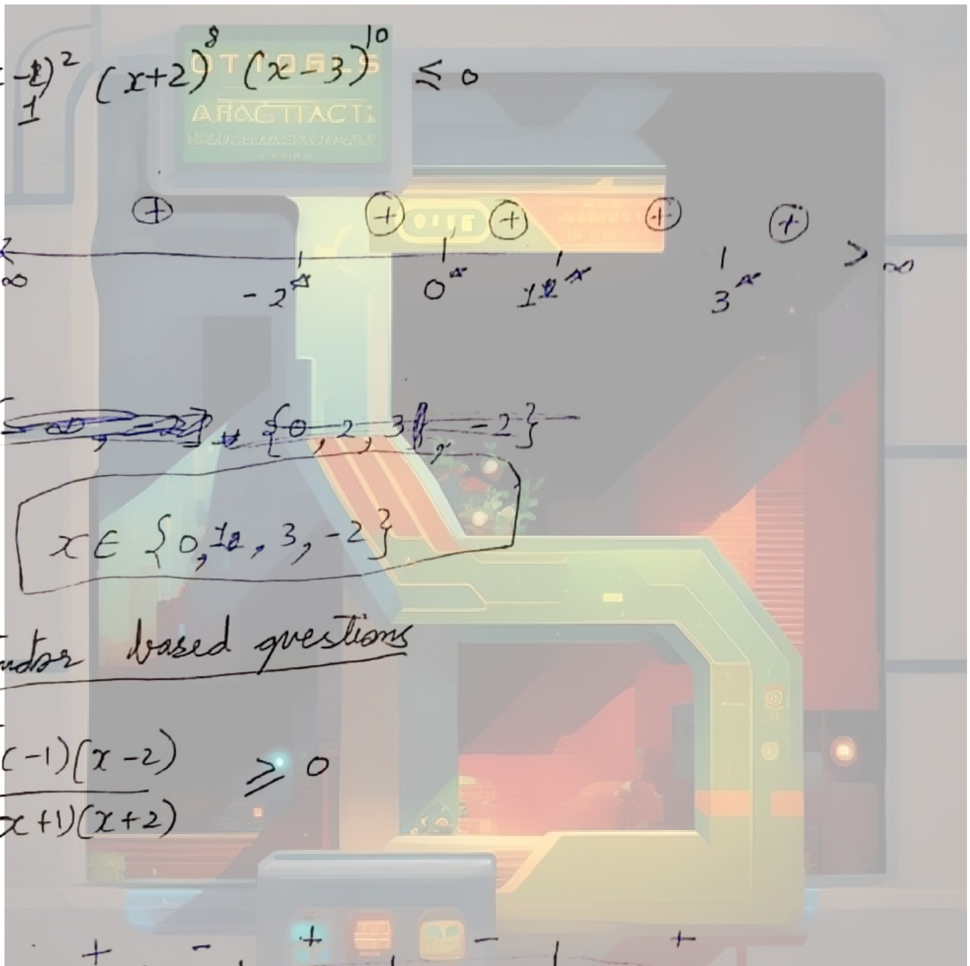
Q-4 $\{1, 2, 3, 4\}$



Q. $(x-1)^{4}(x+1)^{13}(x-4)^{7} < 0$

$$\begin{array}{ccccccccc} & + & & - & & - & & + & \\ -\infty & & -1 & & 1^{*} & & 4 & & \infty \end{array}$$

$\boxed{x \in (-1,1) \cup (1,4)}$



Q $x^{4}(x-1)^{2}(x+2)^{8}(x-3)^{10} \leq 0$

$$\begin{array}{ccccccccccc} & + & & + & & + & & + & & + & \\ -\infty & & -2^{*} & & 0^{*} & & 1^{*} & & 3^{*} & & \infty \end{array}$$

$\cancel{(-\infty, -2]} \cup \{0, 2, 3\} \cancel{, -2\}}$

$\boxed{x \in \{0, 1, 3, -2\}}$

Denominator based questions

Q1. $\dfrac{(x-1)(x-2)}{(x+1)(x+2)} \geq 0$

$$\begin{array}{ccccccccccc} & + & & - & & + & & - & & + & \\ -\infty & & -2 & & -1 & & 1 & & 2 & & \infty \end{array}$$

$x \in (-\infty, -2] \cup [-1, 1] \cup [2, \infty)$

but $x \notin \{-2, -1\}$

$\boxed{x \in (-\infty, -2) \cup (-1, 1] \cup [2, \infty)}$

PYS-10

Q2. $\frac{6x-5}{4x+1} < 0$

$-\infty \quad + \quad -\frac{1}{4} \quad - \quad \frac{5}{6} \quad + \quad +\infty$

$x \notin \{-\frac{1}{4}\}$

$\boxed{x \in (-\frac{1}{4}, \frac{5}{6})}$

Q3. $\frac{x(x-2)(x-5)}{(x+2)(x+1)} > 0$

$-\infty \quad - \quad -2 \quad + \quad -1 \quad - \quad 0 \quad + \quad 2 \quad - \quad 5 \quad + \quad +\infty$

$(-2,-1) \cup \cancel{(2,5)}\ (0,2) \cup (5,\infty)$

$x \notin \{-2,-1\}$

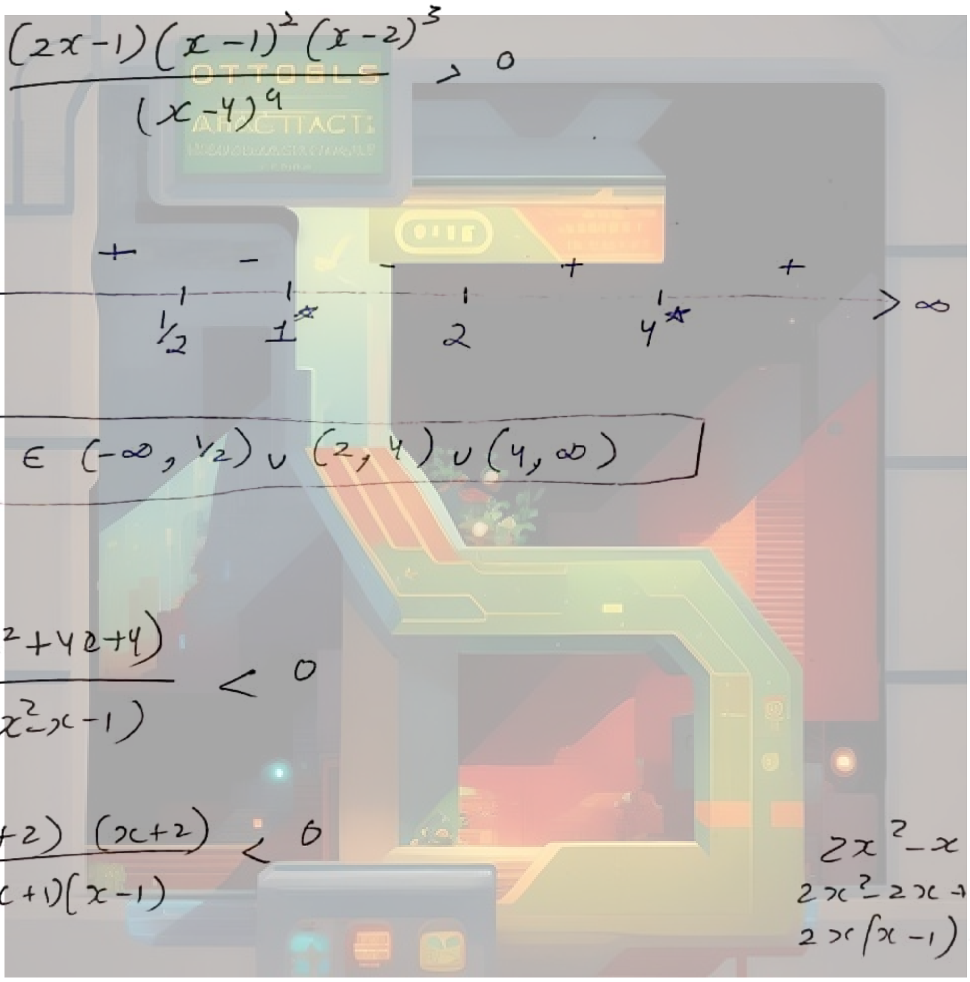
$\boxed{x \in (-2,-1) \cup (0,2) \cup (5,\infty)}$

Q $\dfrac{(x^2-4)(x+3)}{(x-2)} \geq 0$

$-\infty \quad + \quad -3 \quad - \quad -2 \quad + \quad 2 \quad + \quad \infty$

$\boxed{x \in [-3,-2] \cup (2,\infty)}$ $\quad$ $x \in (-\infty,-3] \cup [-2,\infty) - \{2\}$

Q $\dfrac{(2x-1)(x-1)^2(x-2)^3}{(x-4)^4} > 0$



$-\infty \quad + \quad \tfrac{1}{2} \quad - \quad 1 \quad - \quad 2 \quad + \quad 4 \quad + \quad \infty$

$\boxed{x \in (-\infty,\tfrac{1}{2}) \cup (2,4) \cup (4,\infty)}$

Q $\dfrac{(x^2+4x+4)}{(2x^2-x-1)} < 0$

$\dfrac{(x+2)(x+2)}{(2x+1)(x-1)} < 0$

$2x^2-x-1$

$2x^2-2x+x-1$

$2x(x-1)+1(x-1)$

$-\infty \quad + \quad -2 \quad + \quad -\tfrac{1}{2} \quad - \quad 1 \quad + \quad \infty$

$\boxed{x \in (-\tfrac{1}{2},1)}$ $\quad$ $x \notin \{-\tfrac{1}{2},1\}$

Q $$\frac{(x^2-4)(x+3)}{(x-2)} \geq 0$$

$$\frac{(x+2)\cancel{(x-2)}(x+3)}{\cancel{(x-2)}} \geq 0$$

$$(x+2)(x+3) \geq 0$$

$$x \neq 2$$

$+$ $\quad -$ $\quad +$

$-\infty \quad -3 \quad -2 \quad \infty$

$$\boxed{x \in (-\infty,-3] \cup (-2,\infty) - \{2\}}$$

Note - ① we can cancle out factors from numerator & denominator but we have to mention their also that

② ~~we can~~ following quantities are always positive.

1. quadratic equations with D ⊖ve & a ⊕ve

   $D < 0$ & $a > 0$

2. Quantities with whole even powers.
3. Modulus.

Q $$\frac{(x-1)(x+2)}{(x^2+1)(x^2+x+1)} < 0$$

$D = 1-4(1)(1) < 0$

$a = \oplus ve$

$D<0, a>0$

Positive

$$(x-1)(x+2) < 0\,(x^2+1)(x^2+x+1)$$

$$(x-1)(x+2) < 0$$

$$x \in (-2,1)$$

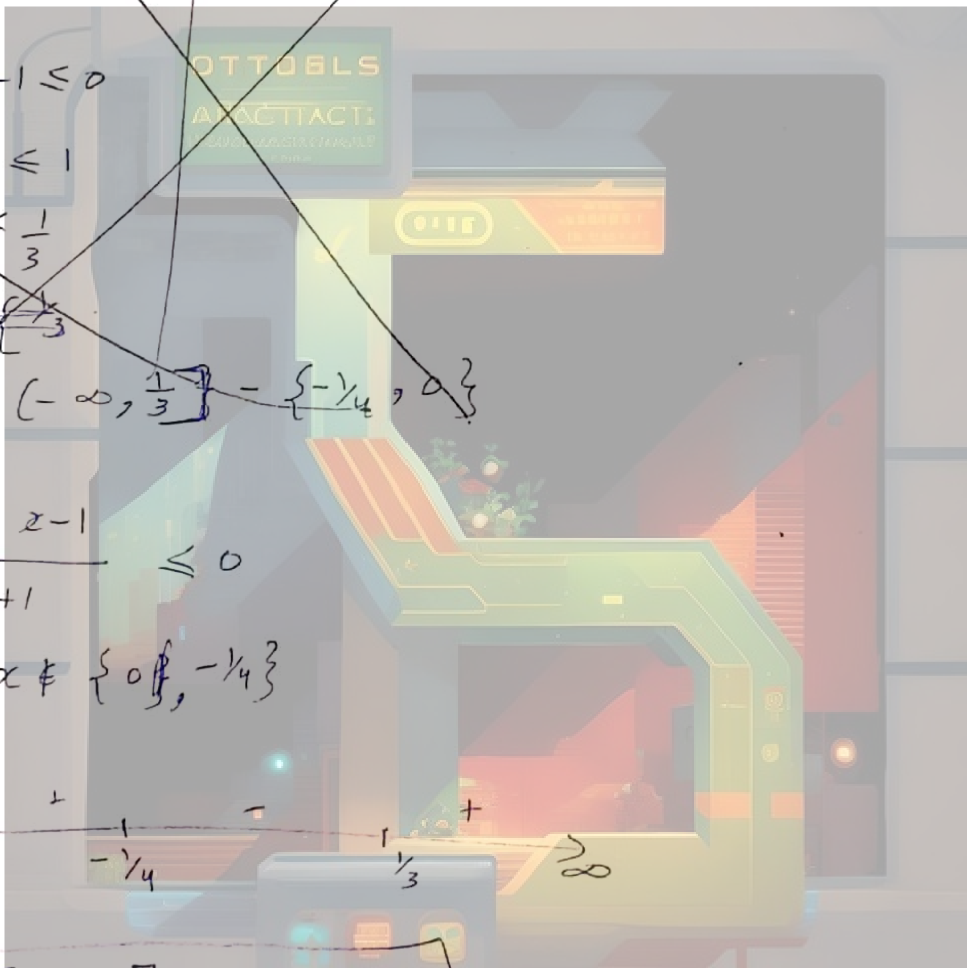
① $\dfrac{3x-1}{(4x+1)(x^2)} \leq 0$

$D = 0 - 4 \times 1 \times 4$

$3x-1 \leq 0$

$x \notin \left\{-\frac{1}{4}, 0\right\}$

$-\infty \qquad \frac{1}{3} \qquad +\infty$



$3x-1 \leq 0$

$3x \leq 1$

$x \leq \frac{1}{3}$

$\cancel{x \in]\tfrac{1}{3}}$

$x \in \left]-\infty, \frac{1}{3}\right] - \left\{-\frac{1}{4}, 0\right\}$

$\dfrac{3x-1}{4x+1} \leq 0$

$x \notin \left\{0, -\frac{1}{4}\right\}$

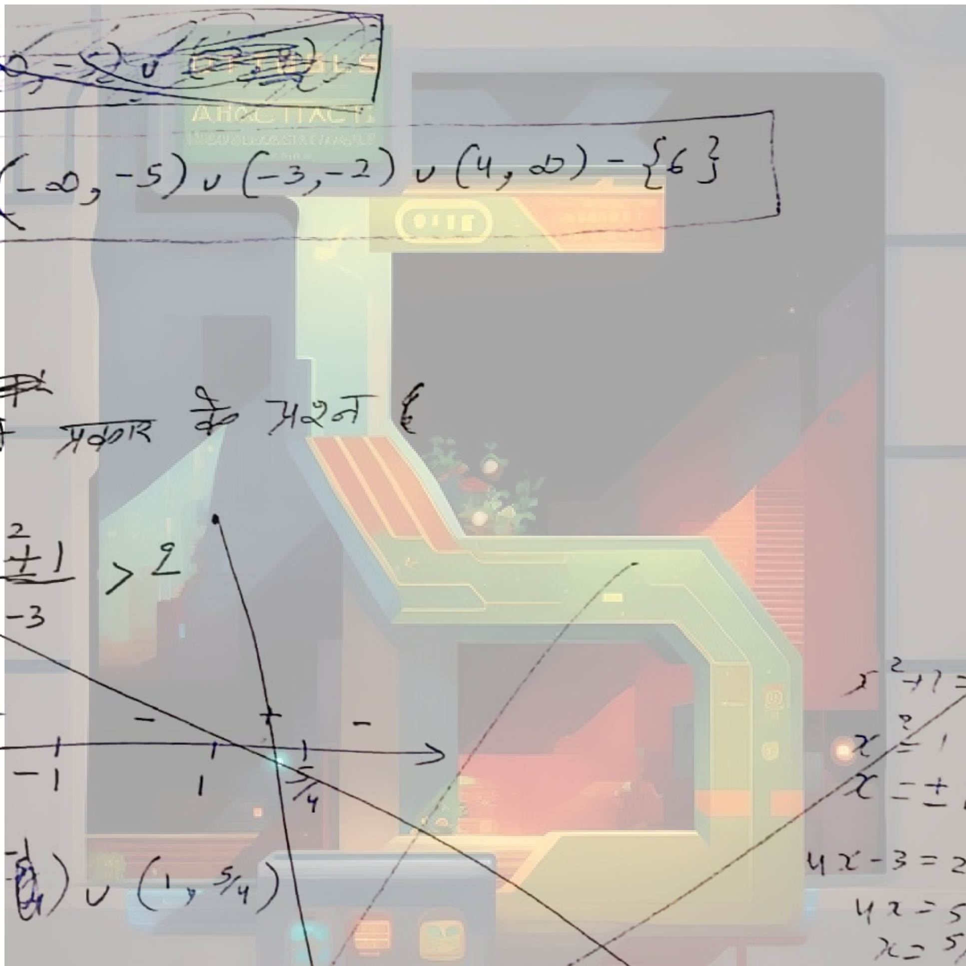
$-\infty \quad + \quad -\frac{1}{4} \quad - \quad \frac{1}{3} \quad + \quad \infty$

$\boxed{x \in \left]-\frac{1}{4}, \frac{1}{3}\right] - \{0\}}$

Q2. $$\frac{(x+2)(x+3)(x+5)}{(x-4)^3\,(x-6)^8} > 0$$

$x \notin \{6, 4\}$

$$\begin{array}{ccccccccccc} & + & & - & & + & & - & & + & \\ -\infty & & -5 & & -3 & & -2 & & 4 & & \infty \end{array}$$



$$\boxed{x \in (-\infty, -5) \cup (-3, -2) \cup (4, \infty) - \{6\}}$$

नये प्रकार के प्रश्न है

Q1. $$\frac{x^2+1}{4x-3} > 2$$

$$\begin{array}{ccccccc} + & & - & & + & & - \\ & -1 & & 1 & & 5/4 & \end{array}$$

$(-\infty, -1) \cup (1, 5/4)$

$x^2+1 = 2$

$x^2 = 1$

$x = \pm 1$

$4x-3 = 2$

$4x = 5$

$x = 5/4$

$\frac{41}{16}$

$\frac{41}{32}$

Q1. $\dfrac{x^2+1}{4x-3} > 2$

$$\frac{x^2+1-2(4x-3)}{4x-3} > 0$$

$$\frac{x^2-8x+7}{4x-3} > 0$$

$$\frac{x^2-7x-x+7}{4x-3} > 0$$

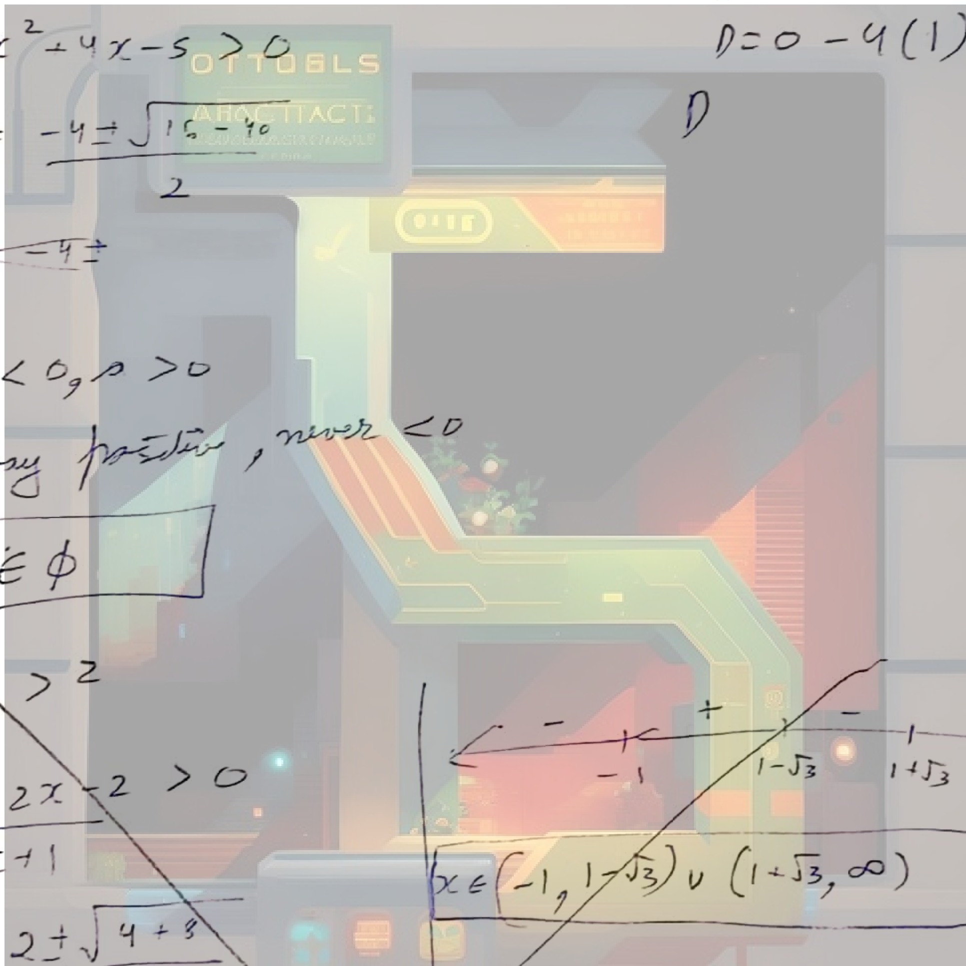
$$\frac{x(x-7)-1(x-7)}{4x-3} > 0$$

$$\frac{(x-1)(x-7)}{4x-3} > 0$$

$-$ $+$ $-$ $+$

$\frac{3}{4}$ 1 7

$$\boxed{x \in \left(\frac{3}{4}, 1\right) \cup (7, \infty)}$$



(2) $$\frac{4x-3}{x^2+1} > 2$$

$$\frac{4x-3-2x^2-2}{x^2+1} > 0$$

$$4x-2x^2-5 > 0$$

$$D = b^2-4ac$$

$$D = 0-4(1)(1)$$

D

$$-2x^2+4x-5 > 0$$

$$x = \frac{-4 \pm \sqrt{16-40}}{2}$$

$\cancel{x = -4 \pm}$

$D < 0, a > 0$

always positive, never < 0

$$\boxed{x \in \phi}$$

(3) $$\frac{x}{x+1} > 2$$

$$\frac{x-2x-2}{x+1} > 0$$

$$x = \frac{2 \pm \sqrt{4+8}}{2}$$

$$x = 1 \pm \sqrt{3}$$

$x = 1+\sqrt{3}$, $1-\sqrt{3}$, -1

$x \neq -1$

$-$ $+$ $-$ $+$

-1 $1-\sqrt{3}$ $1+\sqrt{3}$

$$\boxed{x \in (-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, \infty)}$$

Q4. $\frac{x+1}{(x-1)^2} < 1$

$\frac{x+1}{x^2+1-2x} < 1$

$\frac{x+1-(x-1)^2}{(x-1)^2} < 0$

$x+1-x^2-1+2x$

$\cancel{3x-3x}$

$\frac{3x-x^2}{(x-1)^2} < 0$

$\frac{x(3-x)}{(x-1)^2} < 0$

$-\quad -\quad +\quad -$

$-\infty \quad 0 \quad 1 \quad 3 \quad \infty$

~~$x \in (0,3) - \{1\}$~~

$\boxed{x \in (-\infty,0) \cup (3,\infty)}$

Method -2

$x+1 < (x-1)^2$

$x+1 < x^2-2x+1$

$x < x^2-2x$

$\cancel{0 < x-x}$

$\cancel{x-2 > 0}$

$0 < x^2-3x$

$x^2-3x > 0$

$+\quad -\quad +$

$-\infty \quad 0 \quad 3 \quad \infty$

$\boxed{(-\infty,0) \cup (3,\infty) \in x}$

Q 3. $\dfrac{x}{x+1} > 2$

$$\frac{x}{x+1} - \frac{2}{1} > 0$$

$$\frac{x - 2(x+1)}{x+1} > 0$$

$$\frac{x - 2x - 2}{x+1} > 0$$

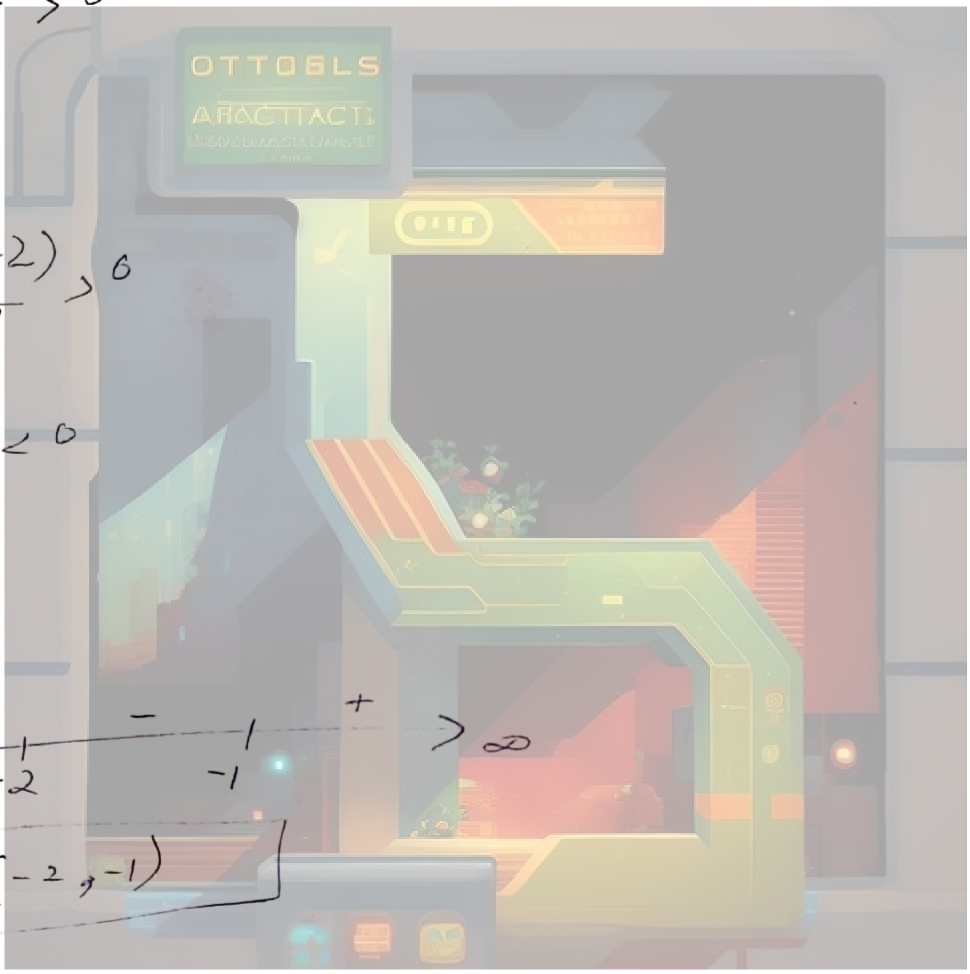
$$\frac{-x - 2}{x+1} > 0$$

$\cancel{x(+}$

$$\frac{-(x+2)}{x+1} > 0$$

$$\frac{x+2}{x+1} < 0$$

$x \neq -1$



$+ \qquad - \qquad +$

$-\infty \quad -2 \quad -1 \quad \infty$

$$\boxed{x \in (-2, -1)}$$

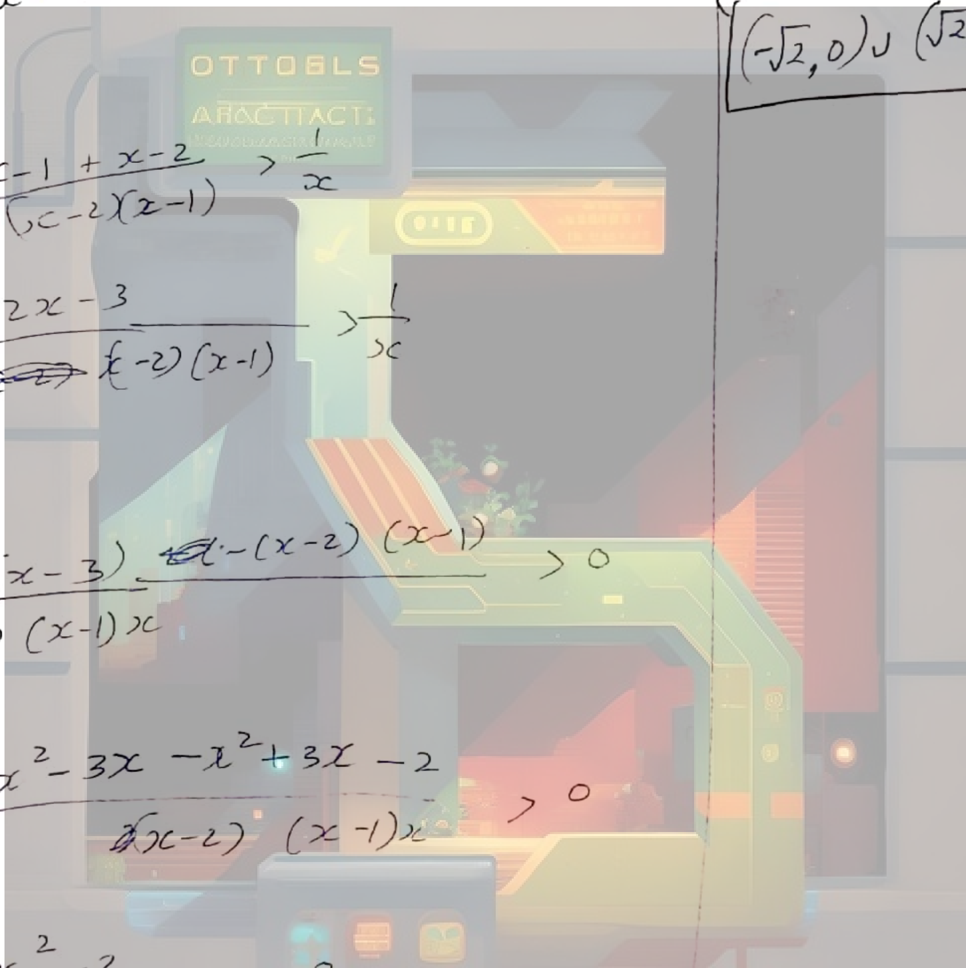
HW. 10-~~04~~5-2024

DYS-10

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16\}$

$[1, 16] - \{13\}$

Q13. $\frac{1}{x^2-2} + \frac{1}{x-1} > \frac{1}{x}$



$\frac{x-1+x-2}{(x-2)(x-1)} > \frac{1}{x}$

$\frac{2x-3}{(x-2)(x-1)} > \frac{1}{x}$

$\frac{x(2x-3) - (x-2)(x-1)}{(x-2)(x-1)x} > 0$

$\frac{2x^2-3x-x^2+3x-2}{(x-2)(x-1)x} > 0$

$\frac{x^2-2}{(x-2)(x-1)x} > 0$

$\frac{(x-\sqrt{2})(x+\sqrt{2})}{(x-2)(x-1)x} > 0$

$-\infty \quad - \quad -\sqrt{2} \quad + \quad 0 \quad - \quad \sqrt{2} \quad + \quad 1 \quad - \quad 2 \quad + \quad \infty$

$(-\sqrt{2}, 0) \cup (\sqrt{2}, 1) \cup$
$(2, \infty)$

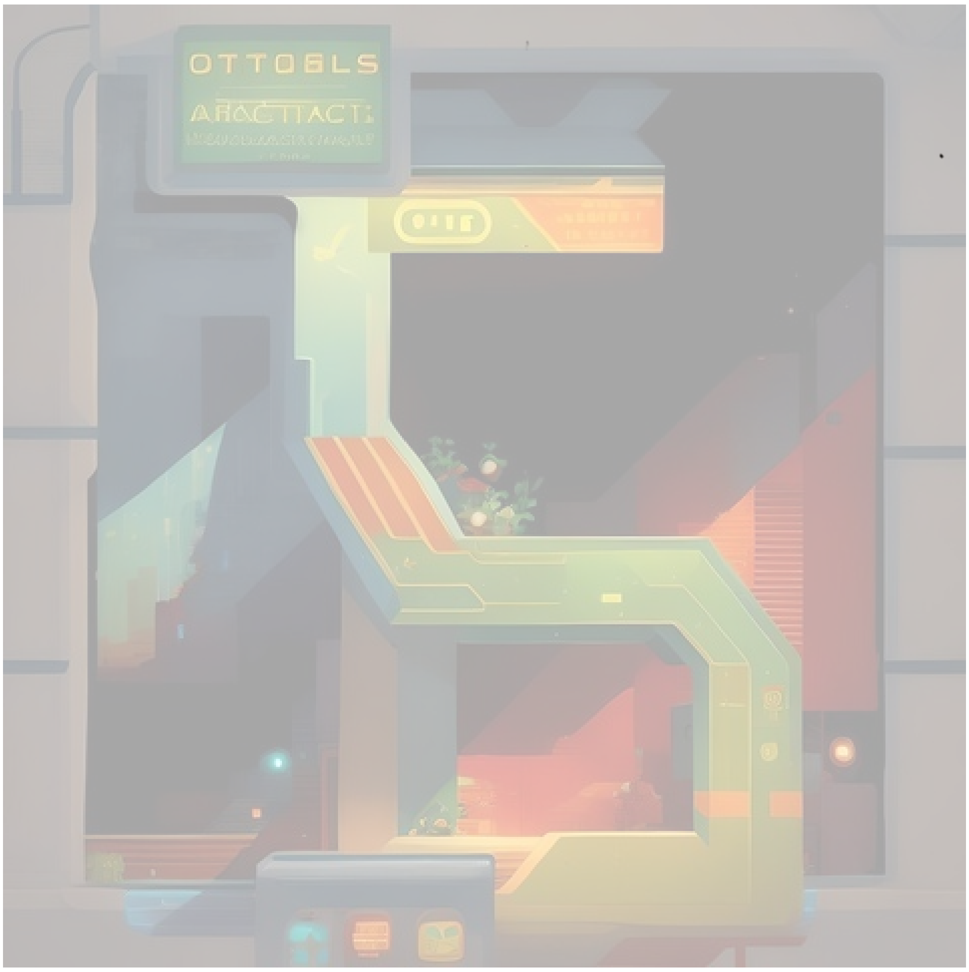
$\boxed{(-\sqrt{2}, 0) \cup (\sqrt{2}, 1) \cup (2, \infty)}$

H.W. 21-05-2024

PYS-10 Q17

O-1 Q19-20-21

O-4 Q5, 6

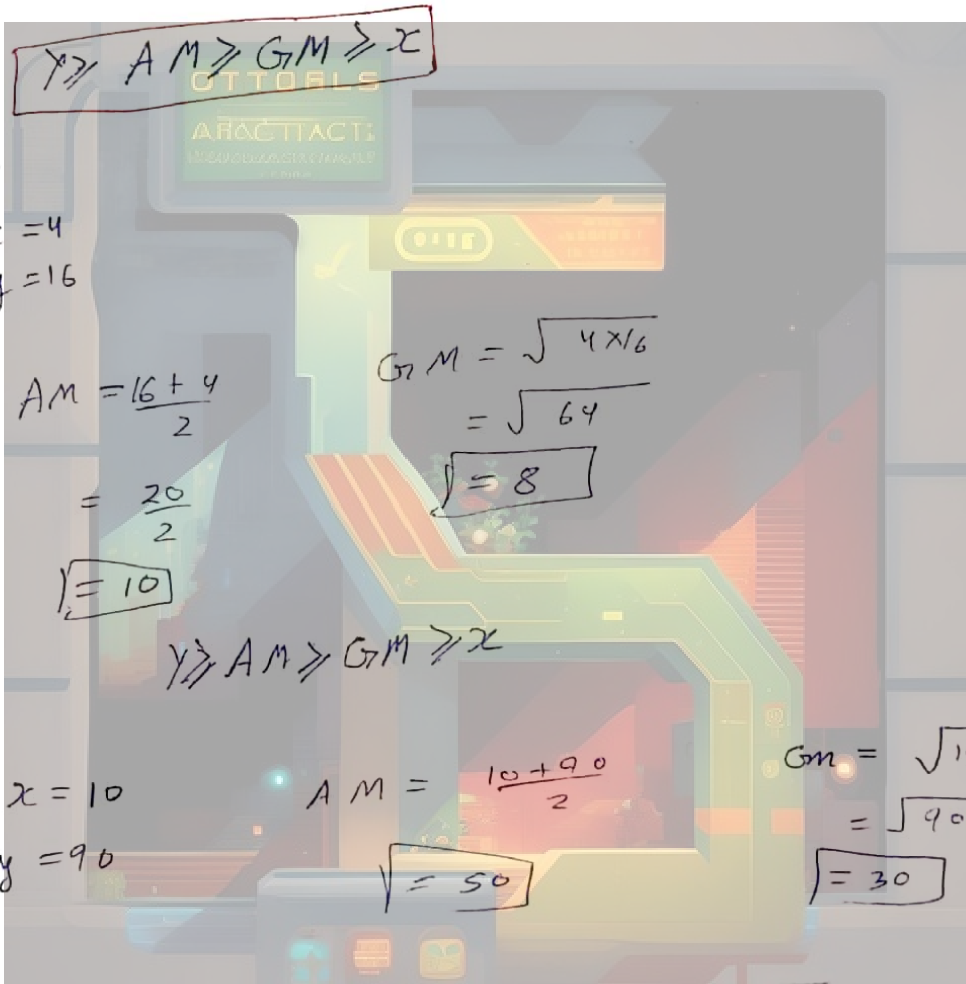


Meon

→ For Any two positive real numbers, x & y $(y \geq x)$

$$AM \text{(Airthmetic mean)} = \frac{x+y}{2}$$

$$\boxed{GM \text{(Geometric mean)} = \sqrt{xy}}$$



$$\boxed{y \geq AM \geq GM \geq x}$$

Ex ①.

$x = 4$
$y = 16$

$$AM = \frac{16+4}{2} = \frac{20}{2} = \boxed{10}$$

$$GM = \sqrt{4 \times 16} = \sqrt{64} = \boxed{8}$$

$$y \geq AM \geq GM \geq x$$

②.

$x = 10$
$y = 90$

$$AM = \frac{10+90}{2} = \boxed{50}$$

$$Gm = \sqrt{10 \times 90} = \sqrt{900} = \boxed{30}$$

③.

$x = 10$
$y = 10$

$$AM = \frac{10+10}{2} = \boxed{10}$$

$$GM = \sqrt{10 \times 10} = \sqrt{100} = \boxed{10}$$

Note:- $AM = GM$ when $x = y$

→ In AM & GM, equality holds when $x = y$

→ For 3 ⊕ve quantities x, y, z

$$AM = \frac{x+y+z}{3}$$

$$\boxed{GM = \sqrt[3]{xyz}}$$

→ For n quantities $a_1, a_2, a_3 \ldots a_n$

$$AM = \frac{a_1 + a_2 + a_3 + a_4 \ldots a_n}{n}$$

$$\boxed{GM = \sqrt[n]{a_1 \times a_2 \times a_3 \ldots a_n}}$$

Hints

→ Maximum / Minimum or largest / smallest is in question

→ Given all quantities are ⊕ve

→ Multiplication of quantities will be a constant

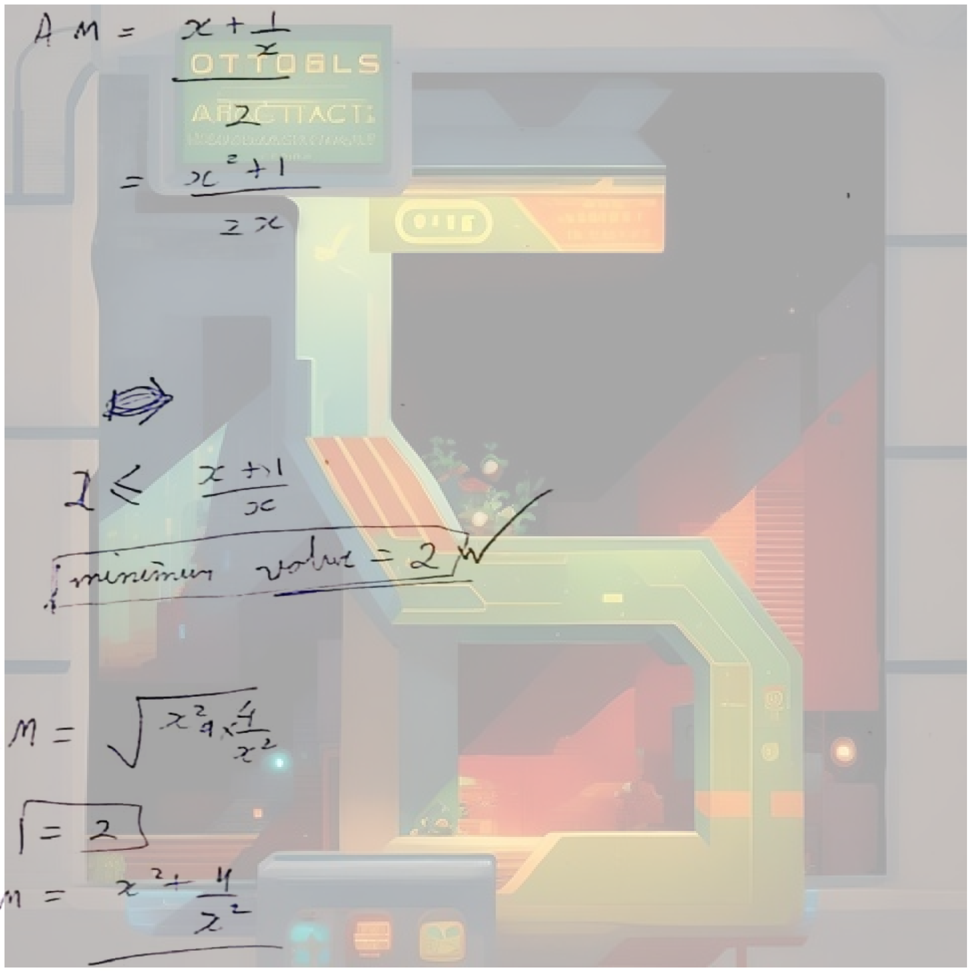
Q Find the ~~maximum~~ minimum value of (1) $x + \frac{1}{x}$ (x is +ve)

(2) $x^2 + \frac{4}{x^2}$

(1) $GM = \sqrt{x \times \frac{1}{x}}$

$= \sqrt{1}$

$\boxed{=1}$



$AM = \frac{x + \frac{1}{x}}{2}$

$= \frac{x^2+1}{2x}$

$\Rightarrow$

$2 \leqslant \frac{x^2+1}{x}$

$\boxed{\text{minimum value} = 2}$

(2) $GM = \sqrt{x^2 \times \frac{4}{x^2}}$

$\boxed{= 2}$

$AM = \frac{x^2 + \frac{4}{x^2}}{2}$

$x^2 + \frac{4}{x^2} \geqslant 4$

$\boxed{\text{minimum} = 4}$

Q 2. find min value. $(x>0)$

① $x + \frac{1}{x} + 3$

$a = x$, $b = \frac{1}{x}$, 3

$AM = \frac{x + \frac{1}{x} + 3}{2}$

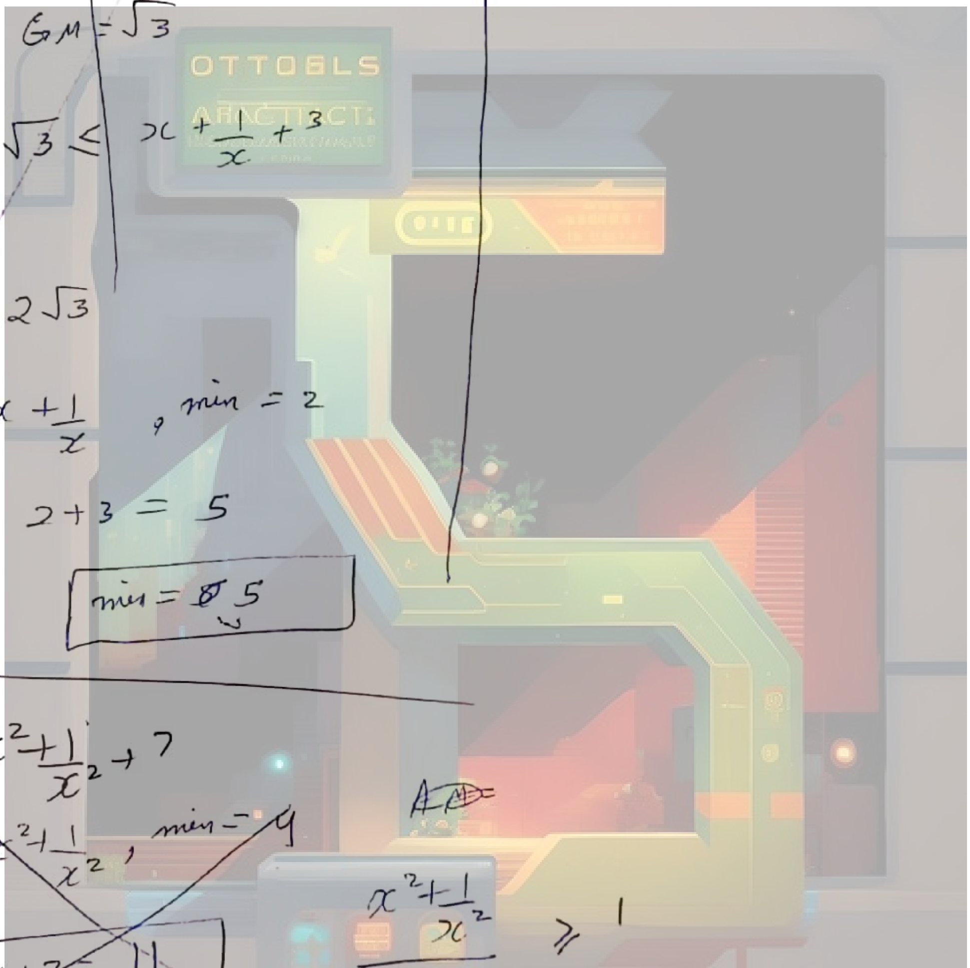
$GM = \sqrt[2]{x\left(\frac{1}{x} + 3\right)}$

$x = \frac{1}{x} = 3$ (not possible for any value of x)

So cannot use AM & GM.

$GM = \sqrt[2]{3}$

$GM = \sqrt{3}$



$2\sqrt{3} \le x + \frac{1}{x} + 3$

min $2\sqrt{3}$

$x + \frac{1}{x}$, min $= 2$

$2 + 3 = 5$

$\boxed{\text{min} = 5}$

---

② $x^2 + \frac{1}{x^2} + 7$

$x^2 + \frac{1}{x^2}$, min $= 4$

$\cancel{\boxed{4 + 7 = 11}}$

$AM = \frac{x^2 + \frac{1}{x^2}}{2} \geq 1$

$x^2 + \frac{1}{x^2} \geq 2$

$\boxed{2 + 7 = 9}$

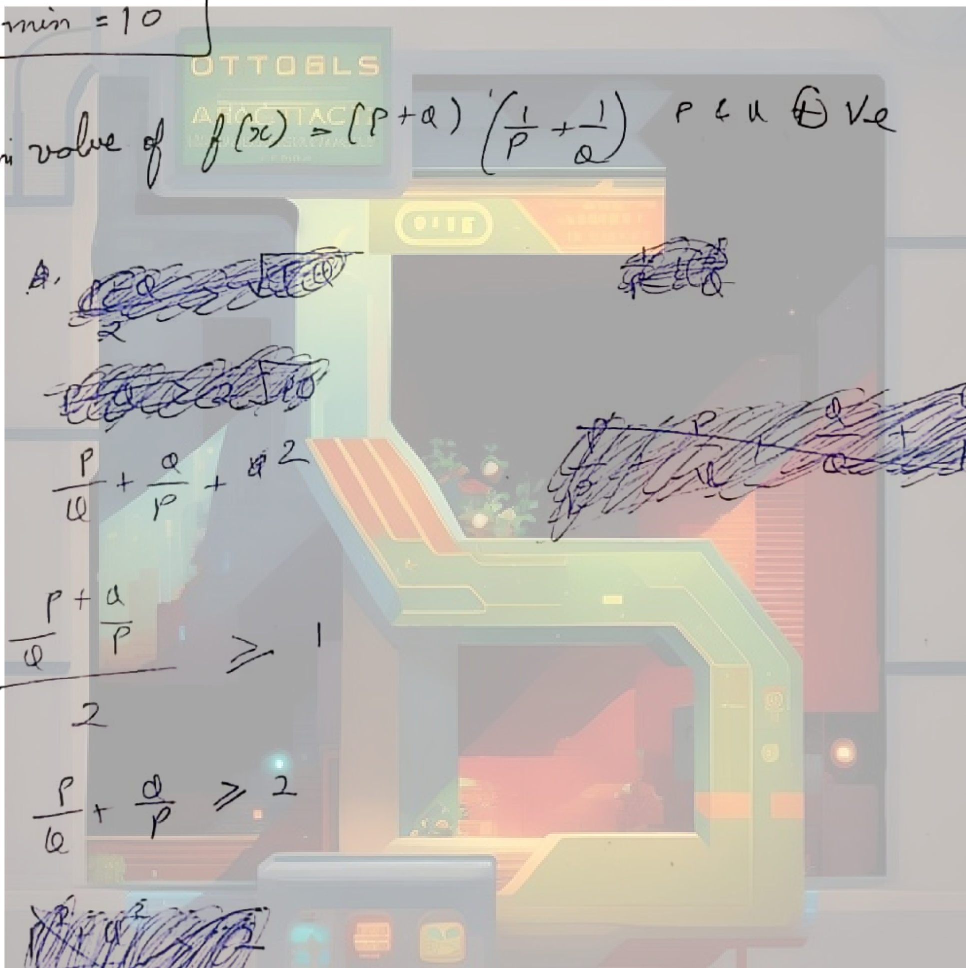
Q3. $f(x) = \dfrac{x^2+1+8x}{x}$

find min value of x is (+)ve

$$\underbrace{x + \frac{1}{x}}_{min = 2} + 8$$

$2 + 8$

$\boxed{min = 10}$



Q4. mini value of $f(x) = (p+q)\left(\dfrac{1}{p}+\dfrac{1}{q}\right)$ p & q (+)ve

$$\frac{p}{q} + \frac{q}{p} + 2$$

$$\frac{\frac{p}{q} + \frac{q}{p}}{2} \geq 1$$

$$\frac{p}{q} + \frac{q}{p} \geq 2$$

$2 + 4$

$\boxed{4 = min}$

Ratio & Proportion

Ratio

→ Comparision of quantities by the method of division.

→ It says how many times one quantity is equal to the another quantity.

→ two numbers in ratio are compared only when they have the same unit.

→ a to b

$a:b$

a/b

$a \rightarrow$ Antecedent

$b \rightarrow$ Consequent

Proportion

→ It is a equation which defines that two given ratios are equivalent to each other.

→ $::$ or $=$

→ 3 Types

1. Direct proportion $(a \propto b)$
2. Inverse proportion $(a \propto \frac{1}{b})$
3. Continued proportion

$\frac{a}{b} = \frac{c}{d}$ or $a:b::c:d$

Properties of ratio

① $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

then :- $\frac{a+c+e}{b+d+f} = \frac{a}{b}$

eg. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

$\frac{1+2+3}{2+4+6} = \frac{6}{12} = \frac{1}{2}$

(2) ratio $a:b$

Duplicate → $a^2 : b^2$

Triplicate → $a^3 : b^3$

Sub Duplicate → $\sqrt{a} : \sqrt{b}$

Sub ~~of~~ triplicate → $\sqrt[3]{a} : \sqrt[3]{b}$

Q1. ~~A~~ are the 2 ratios ~~10:8~~ 8:10 & 7:10 in proportion or not

~~8~~ $\frac{8}{10} \neq \frac{7}{10}$

not in proportion

~~Q~~. Properties of proportion –

(1) Componendo – $\frac{a}{b} = \frac{c}{d}$

componendo is $\frac{a+b}{b} = \frac{c+d}{d}$

(2) dividendo of $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{a-b} = \frac{c}{c-d}$

(3) Componendo – dividendo → $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

(4) Alternendo → $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{a}{c} = \frac{b}{d}$

(5) Invertendo → $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{b}{a} = \frac{d}{c}$

(6) reverse componendo-dividendo — $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

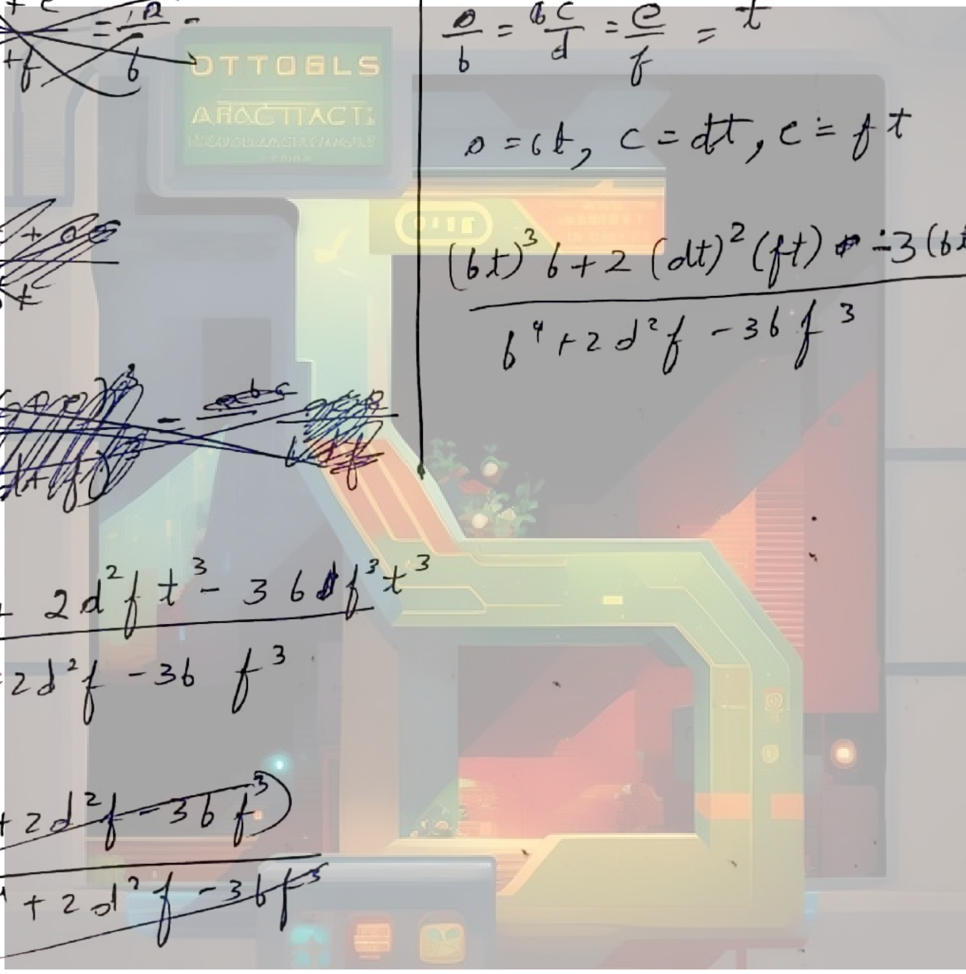
$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Q If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ show that $\frac{a^3b + 2c^2e - 3ae^2f}{b^4 + 2d^2f - 3bf^3} = \frac{ace}{bdf}$

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = t$$

$$a = bt,\ c = dt,\ e = ft$$

$$\frac{(bt)^3 b + 2(dt)^2(ft) - 3(bt)(ft)^2(ft)}{b^4 + 2d^2f - 3bf^3}$$



$$\frac{b^4t^3 + 2d^2ft^3 - 3bf^3t^3}{b^4 + 2d^2f - 3bf^3}$$

$$\frac{t^3\cancel{(b^4 + 2d^2f - 3bf^3)}}{\cancel{b^4 + 2d^2f - 3bf^3}}$$

$$t^3 = \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$$

$$= \frac{a \times c \times e}{b \times d \times f}$$

$$= \frac{ace}{bdf}$$

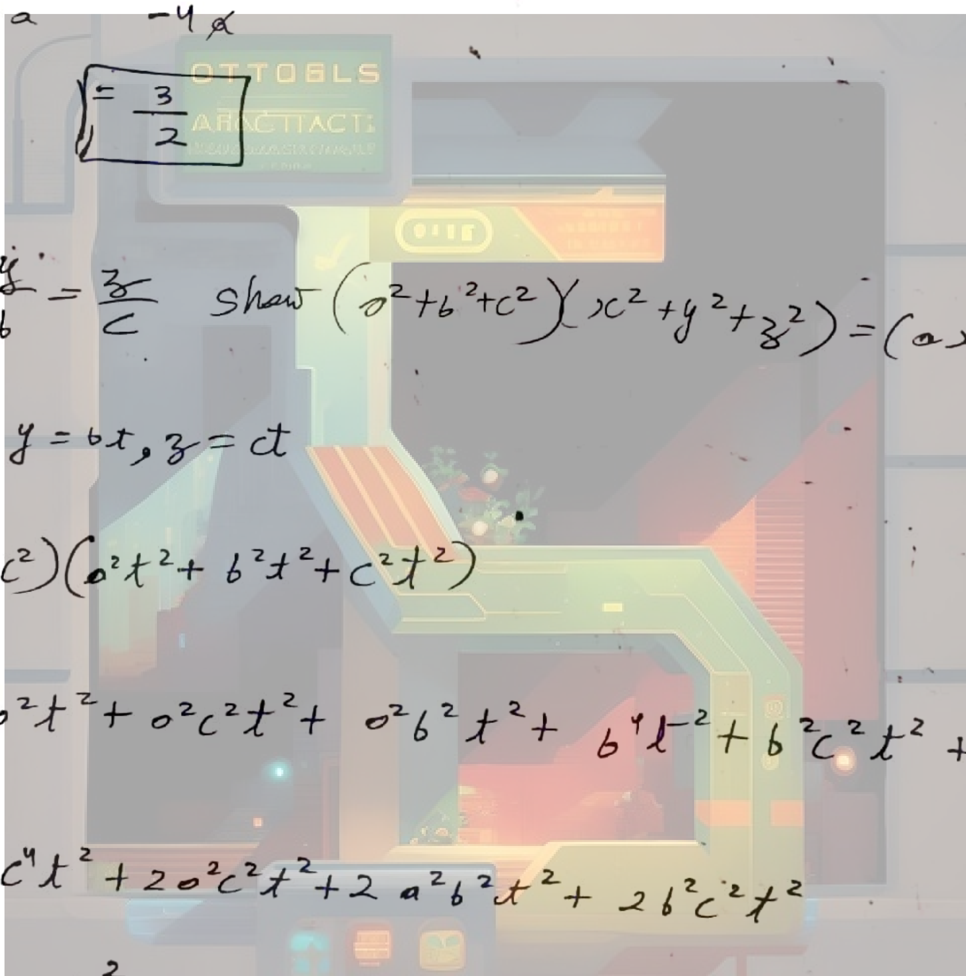
Q if $a:b = 1:2$ $\quad \dfrac{b-8a}{b-6a}$

$$\frac{a}{b} = \frac{1}{2}$$

$$2a = b$$

$$\frac{2a-8a}{2a-6a} = \frac{-6\cancel{a}}{-4\cancel{a}}$$

$$\boxed{= \frac{3}{2}}$$



Q2. $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$ show $(a^2+b^2+c^2)(x^2+y^2+z^2) = (ax+by+cz)^2$

$x = at,\ y = bt,\ z = ct$

$$(a^2+b^2+c^2)(a^2t^2+b^2t^2+c^2t^2)$$

$$a^4t^2+b^2a^2t^2+a^2c^2t^2+a^2b^2t^2+b^4t^2+b^2c^2t^2+a^2c^2t^2+b^2c^2t^2+c^4t^2$$

$$a^4t^2+b^4t^2+c^4t^2+2a^2c^2t^2+2a^2b^2t^2+2b^2c^2t^2$$

$$t^2(a^2+b^2+c^2)^2 = LHS.$$

$$(a(at)+b(bt)+c(ct))^2$$

$$(a^2t+b^2t+c^2t)^2$$

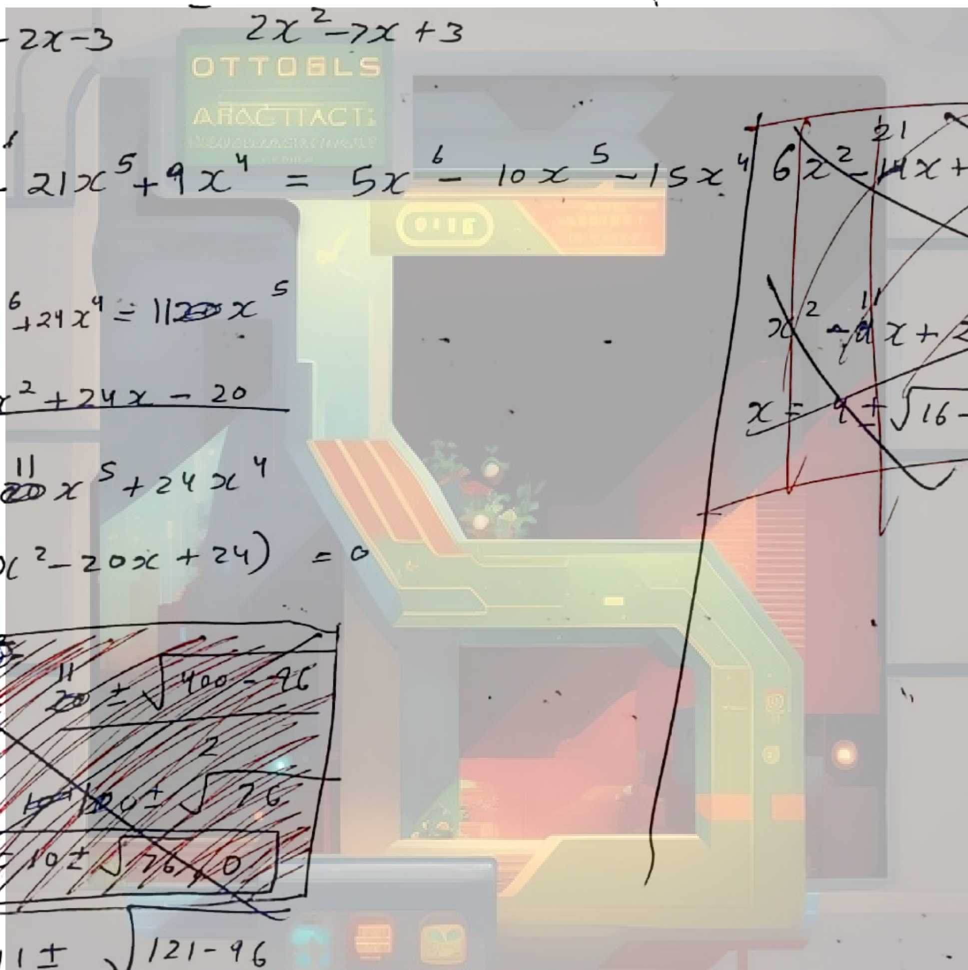
$$t^2(a^2+b^2+c^2)^2 = RHS.$$

$$\underline{LHS = RHS}$$

Q 4. $\dfrac{3x^4+x^2-2x-3}{3x^4-x^2+2x+3} = \dfrac{5x^4+2x^2-7x+3}{5x^4-2x^2+7x-3}$ find x.

$$\frac{3x^4+(x^2-2x-3)}{3x^4-(x^2-2x-3)} = \frac{5x^4+(2x^2-7x+3)}{5x^4-(2x^2-7x+3)}$$

$$\frac{3x^4}{x^2-2x-3} = \frac{5x^4}{2x^2-7x+3}$$



$$6x^6-21x^5+9x^4 = 5x^6-10x^5-15x^4$$

$$\cancel{6x^2-21x+9 = 5x^2-10x-15}$$

$$\cancel{x^2-11x+24=0}$$

$$\cancel{x = 4 \pm \sqrt{16-}}$$

$$x^6+24x^4 = 11x^5$$

$$\cancel{x^4(x^2+24x-20}$$

$$x^6-11x^5+24x^4$$

$$x^4(x^2-20x+24)=0$$

$$\cancel{x = \frac{20 \pm \sqrt{400-96}}{2}}$$

$$\cancel{x = 10 \pm \sqrt{76}}$$

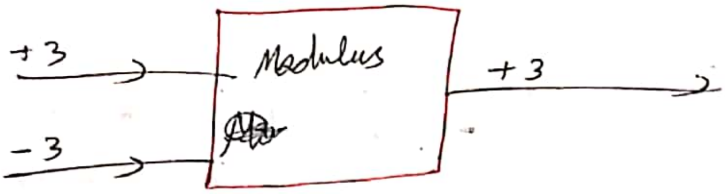
$$\cancel{x = 10 \pm \sqrt{76}, 0}$$

$$x = \frac{11 \pm \sqrt{121-96}}{2}$$

$$x = \frac{11 \pm 5}{2}$$

$$\boxed{x = 8, 3, 0}$$

Modulus :-



→ Also called absolute value
→ $|x|$ (denotion)

$$f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

$$f(x) = |x-2| = \begin{cases} (x-2) & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$$

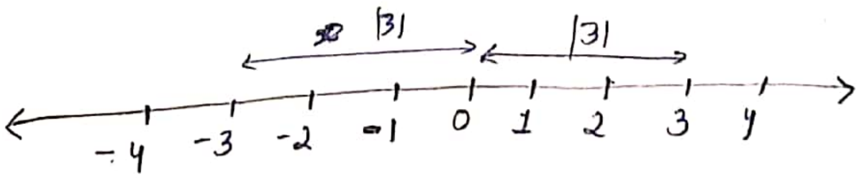
कहाँ पर zero हो?

$$f(x) = |x+3| = \begin{cases} (x+3) & x \geq -3 \\ -(x+3) & x < -3 \end{cases}$$

Geometrical Representation -

$|x| = 3$ → Distance 3 from origin.

$x = 3, -3$



• $|x| = 5 \Rightarrow$ Distance of x from origin is 5

$x = \pm 5$

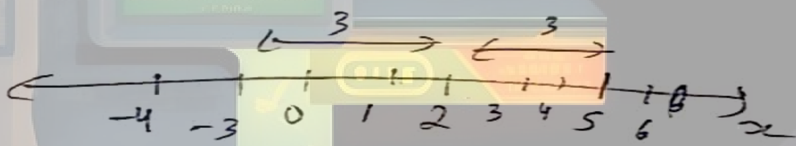
• $|x| = -2 \longrightarrow$ Not Possible

↳ Distance of any point from origin is -2. not possible.

So, x is empty set.

• $|x-2| = 3$

↳ Distance of any point from 2 is 3

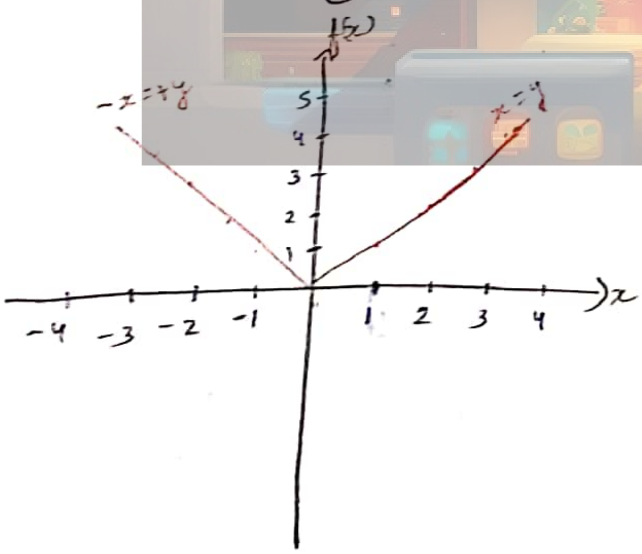


• $|x+5| = 7$

↳ Distance 7 from (-5).

Graphical Representation

$$f(x) = |x| \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



x	0	1	2	3
$f(x)$	0	1	2	3

$-x$	-1	-2	-3
$f(x)$	1	2	3

Note:-

① $|x| = |-x|$

$|x-2| = |2-x|$

$|x+3| = |-x-3|$

$|x| \neq -|x|$

② $\sqrt{x^2} = |x|$

③ value of modulus cannot be negative

Q Find the value of ~~your~~ x.

① $|3x-2| = 2$

$3x-2 = 2$

$3x = 4$

$\boxed{x = 4/3}$

$-3x+2 = 2$

$-3x = 0$

$\boxed{x = 0}$

② $|8x+1| = 7$

③ $|x-7| = 0$

④ $|2x-3| = -3$ - not possible

⑤ $\left|\frac{5x-10}{3}\right| = 4$

② $8x+1 = 7$

$8x = 6$

$\boxed{x = 3/4}$

$-8x-1 = 7$

$-8x = 8$

$\boxed{x = -1}$

③ $x-7 = 0$

$\boxed{x = 7}$

$-x+7 = 0$

$\boxed{x = 7}$

④ $x \in \phi$

⑤ $\frac{5x-10}{3} = 4$

$5x-10 = 12$

$5x = 22$

$\boxed{x = \frac{22}{5}}$

$\frac{5x-10}{3} = -4$

$5x-10 = -12$

$5x = -2$

$\boxed{x = -\frac{2}{5}}$

H.W. 13-05-2023

DYBS-8 (full) = $\{x : x \in \text{Left}\}$

DYS-9 (full)

DYS-10 = ϕ

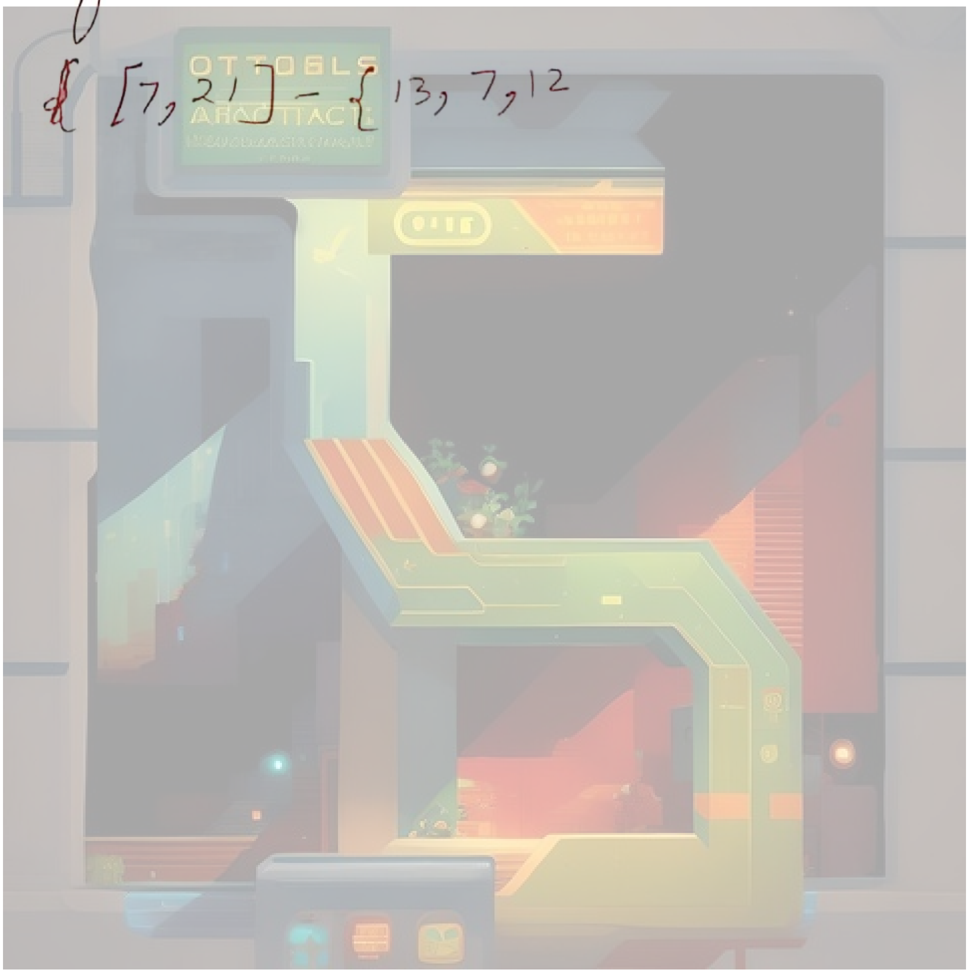
DYS-11 = $\{1, 2, 3, 4, 5, 6\}$

O-4 = $\{7, 8, 9, 10\}$

J-M, J-A full

& others left

DYS-11 & $[7, 21] - \{13, 7, 12$

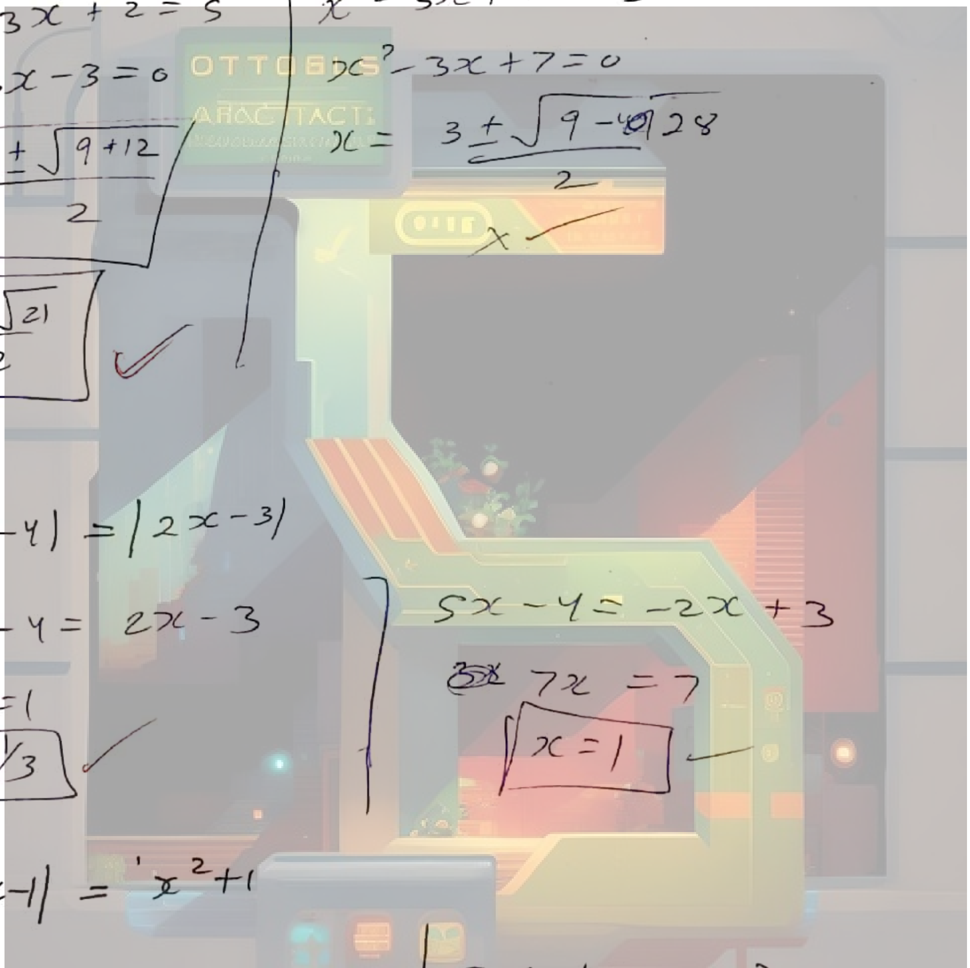


Q

Hint:- with single modulus if constant is given in RHS or some positive quantity then we will solve modulus directly.

Q

(1) $|x^2-3x+2| = 5$



$x^2-3x+2=5$

$x^2-3x-3=0$

$\boxed{x = \frac{3 \pm \sqrt{9+12}}{2}}$

$\boxed{x = \frac{3 \pm \sqrt{21}}{2}}$

$x^2-3x+2=-5$

$x^2-3x+7=0$

$x = \frac{3 \pm \sqrt{9-28}}{2}$

$\cancel{x}$

(2) $|5x-4| = |2x-3|$

$5x-4 = 2x-3$

$3x = 1$

$\boxed{x = 1/3}$

$5x-4 = -2x+3$

$7x = 7$

$\boxed{x = 1}$

(3) $|8x-1| = x^2+1$

$8x-1 = x^2+1$

$x^2-8x+2=0$

$x = \frac{8 \pm \sqrt{64-8}}{2}$

$x = \frac{8 \pm \sqrt{56}}{2}$

$\boxed{x = 4 \pm \sqrt{14}}$

$8x-1 = -x^2-1$

$x^2+8x=0$

$x(x+8)=0$

$\boxed{x=0}$

$\boxed{x=-8}$

Q $2|x+3| = 3|x-4|$

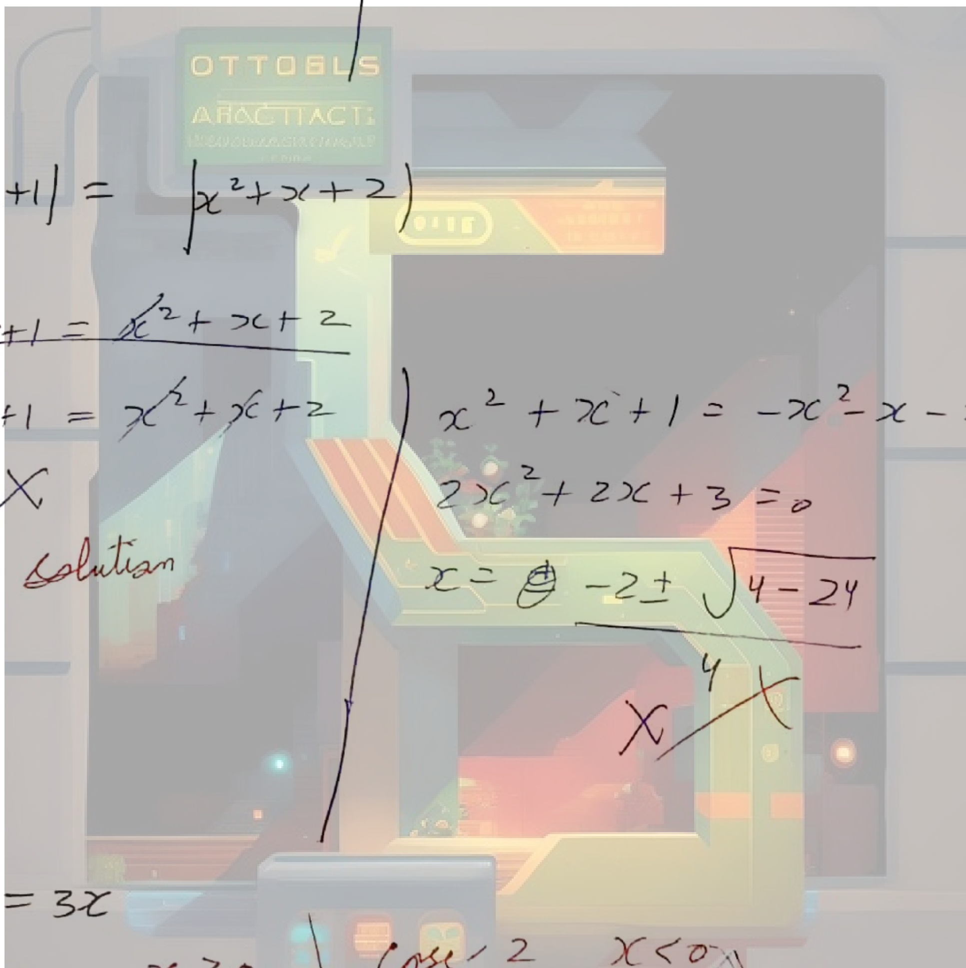
$2x+6 = 3x-12$

$\boxed{18 = x}$

$2x+6 = -3x+12$

$5x = 6$

$\boxed{x = 6/5}$



Q $|x^2+x+1| = |x^2+x+2|$

~~$x^2+x+1 = x^2+x+2$~~

$\cancel{x^2}+\cancel{x}+1 = \cancel{x^2}+\cancel{x}+2$

X

No Solution

$x^2+x+1 = -x^2-x-2$

$2x^2+2x+3 = 0$

$x = \dfrac{-2 \pm \sqrt{4-24}}{4}$

X

Q $|x| = 3x$

Case-1 $x \geq 0$

$x = 3x$

$x - 3x = 0$

$-2x = 0$

✓ $\boxed{x = 0}$

Case-2 $x < 0$

$|x| = -x$

$x = -3x$

$x + 3x = 0$

$4x = 0$

$\boxed{x = 0}$ X

Q ~~$|x^2+3x+2|$~~

Q $|x-1| = x+1$

case 1 $\quad x-1 \geq 0$

$x \geq 1$

$\cancel{x}-1 = \cancel{x}+1$

$-1 = +1$

$x \in \phi$

Case 2 $\quad \cancel{x-1<0}$

$x-1 < 0$

$x < 1$

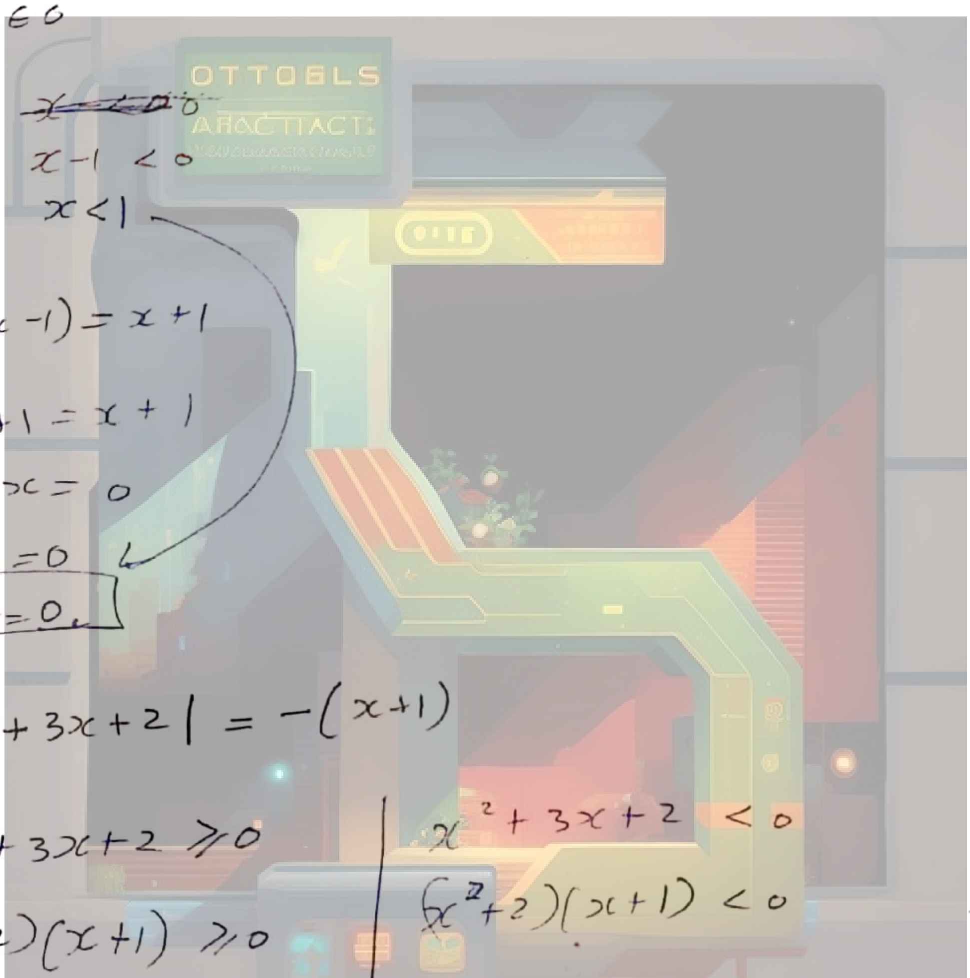
$-(x-1) = x+1$

$-x+1 = x+1$

$2x = 0$

$x = 0$

$\boxed{x=0}$



Q $|x^2+3x+2| = -(x+1)$

$x^2+3x+2 \geq 0$	$x^2+3x+2 < 0$
$(x+2)(x+1) \geq 0$	$(x+2)(x+1) < 0$

$+ \quad -2 \quad - \quad -1 \quad +$

Case 1 $\quad x < -2$

$$|(x+2)(x+1)| = (x+2)(x+1) \quad (\because \text{take } |x| = x, \text{ as } x \oplus)$$

$$(x+2)(x+1) = -(x+1)$$

$$(x+2)(x+1) - (x+1)$$

$$(x+1)(x+2+1)$$

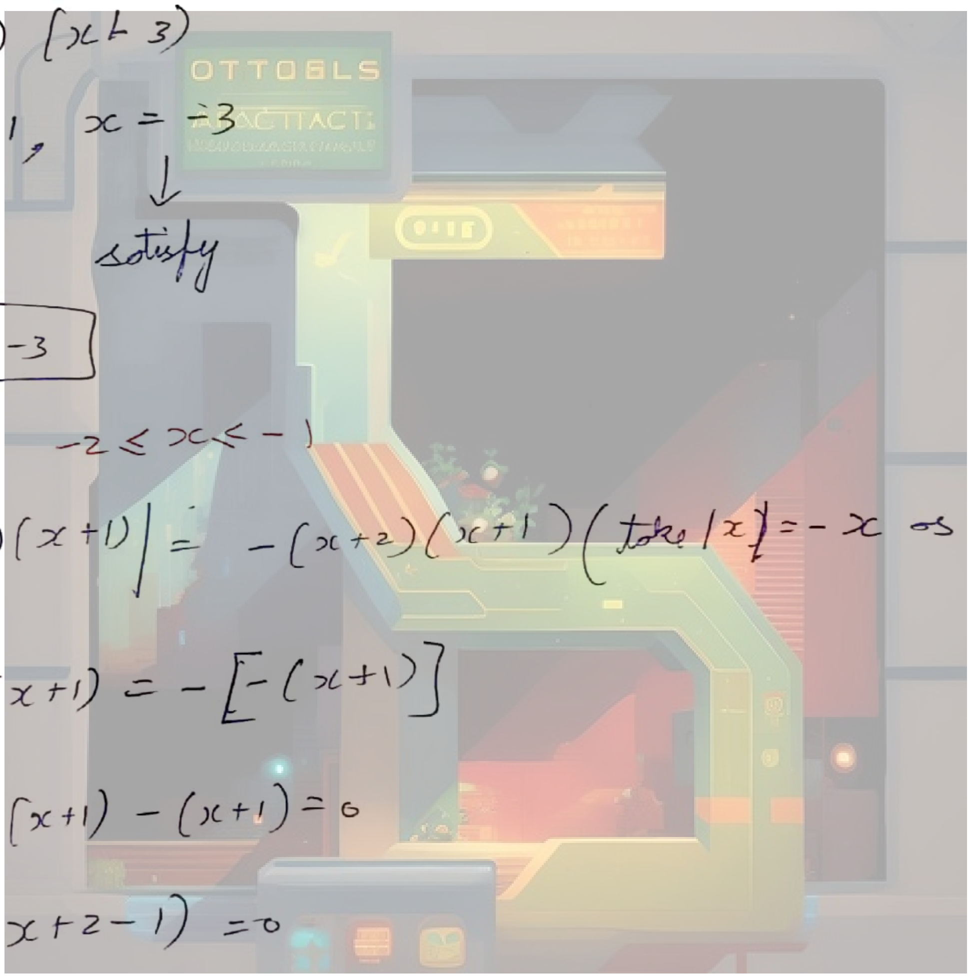
$$(x+1)(x+3)$$

$$x = -1, \quad x = -3$$

$\downarrow$

satisfy

$$\boxed{x = -3}$$



Case 2 - $\quad -2 \leq x \leq -1$

$$|(x+2)(x+1)| = -(x+2)(x+1) \quad (\text{take } |x| = -x \text{ as } x \ominus)$$

$$(x+2)(x+1) = -[-(x+1)]$$

$$(x+2)(x+1) - (x+1) = 0$$

$$(x+1)(x+2-1) = 0$$

$$(x+1)(x+1) = 0$$

$$\boxed{x = -1} \text{ (satisfy)}$$

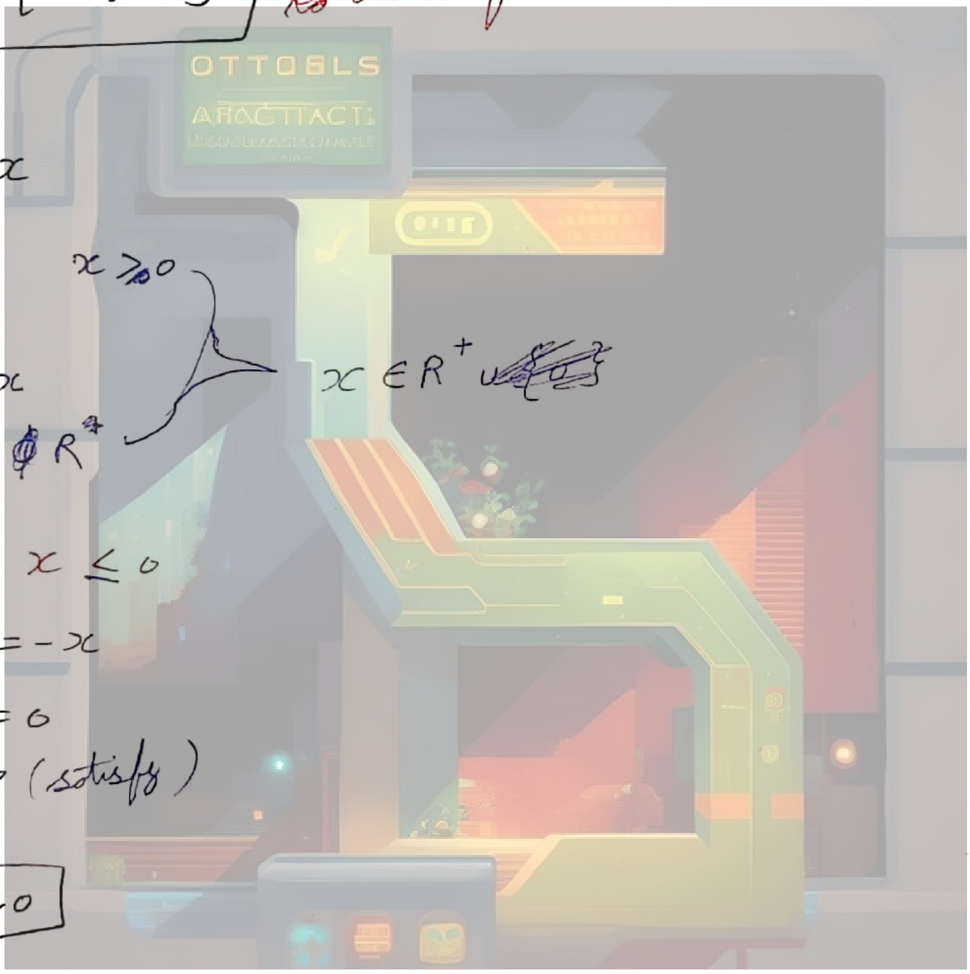
Case-3 $\quad x > -1$

$$(x+2)(x+1) = -(x+1)$$

$x = -1, \; x = -3$

(not satisfy)

$\boxed{x \in \{-3, -1\}}$ Combining answer from diff cases –



Q $\quad |x| = x$

Case 1 $\quad x \geq 0$

$x = x$

$x \in R^+$

$x \in R^+ \cup \cancel{\{0\}}$

Case 2 $\quad x \leq 0$

$x = -x$

$2x = 0$

$x = 0$ (satisfy)

$\boxed{x \geq 0}$

Q $|x+2| = -(x+1)$

Case 1 $x \geq 0$

$x+2 \geq 0$

$x \geq -2$

$x+2 = -(x+1)$

$x+2 = -x-1$

$2x = -3$

$\boxed{x = -3/2}$ (~~not~~ satisfy)

Case 2 $x \leq 0$

$x+2 = x+1$

$x+1 = x$

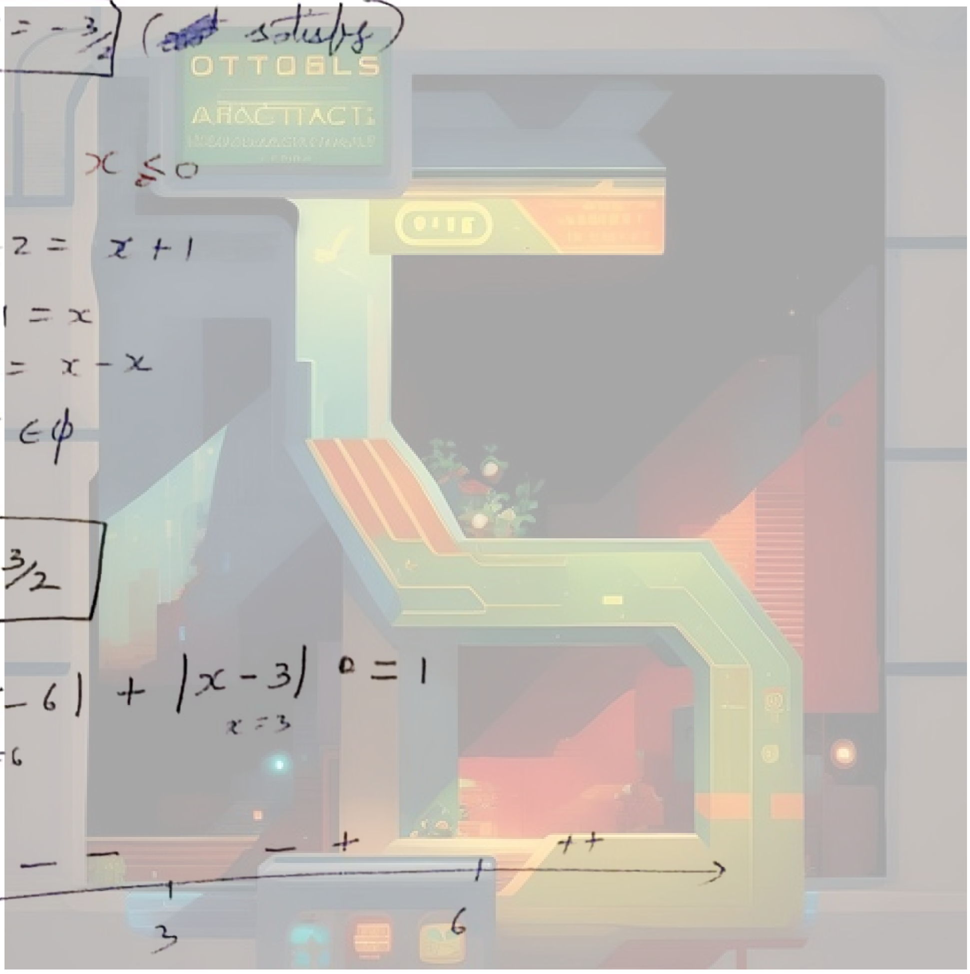
$+1 = x - x$

$x \in \phi$

$\boxed{x = -3/2}$

Q $|x-6| + |x-3| = 1$

$x=6$ $\quad$ $x=3$



— — $\quad$ — + $\quad$ + +

3 $\quad$ 6

Case $x<3$

$-(x-6) - (x-3) = 1$

$-x+6-x+3 = 1$

$-2x+9 = 1$

$9-1 = 2x$

$8 = 2x$

$x = 4$ (not satisfy)

Case 2 $\quad 3 \le x \le 6$

$$-(x-6)+(x-3)=1$$

$$-\cancel{x}+6+\cancel{x}-3=1$$

$$3=1$$

$$x \in \phi$$

Case 3 $\quad x > 6$

$$(x-6)+(x-3)=1$$

$$x-6+x-3=1$$

$$2x-9=1$$

$$2x=10$$

$x=5$ (not satisfy)

$$\boxed{x \in \phi}$$

Q $|2x-1|+|2x+3|=6$

$x = 1/2 \qquad x = -3/2$

$- \; - \quad -3/2 \quad - \; - \; + \quad 1/2 \quad + \; +$

Case 1 $\quad x < -3/2$

$$-(2x-1)-(2x+3)=6$$

$$-2x+1-2x-3=6$$

$$-4x-2=6$$

$$-4x=8$$

$$\boxed{x=-2}$$

Case 2 $\quad -\frac{3}{2} \le x \le \frac{1}{2}$

$$-(2x-1) + (2x+3) = 6$$

$$-\cancel{2x}+1 + \cancel{2x}+3 = 6$$

$$4 = 6$$

$$x \in \phi$$

Case 3 $\quad x \geq \frac{1}{2}$

$$(2x-1)+(2x+3) = 6$$

$$4x+2=6$$

$$4x=4$$

$\boxed{x=1}$ (satisfy)

$$\boxed{x \in \{-2, 1\}}$$

Double Inequality

① $-1 \le \underbrace{8x-3 \le 5}$

$-1 \le 8x-3 \qquad 8x-3 \le 5$

$2 \le 8x \qquad 8x \le 8$

$\frac{1}{4} \le x \qquad x \le 1$

$x \in [\frac{1}{4}, \infty) \xleftrightarrow{\cap \text{ intersection}} x \in (-\infty, 1]$

Q $1 \le \frac{x^2-5x-15}{x^2+x+1} \le 2$

$\hookrightarrow 0 \le 0$

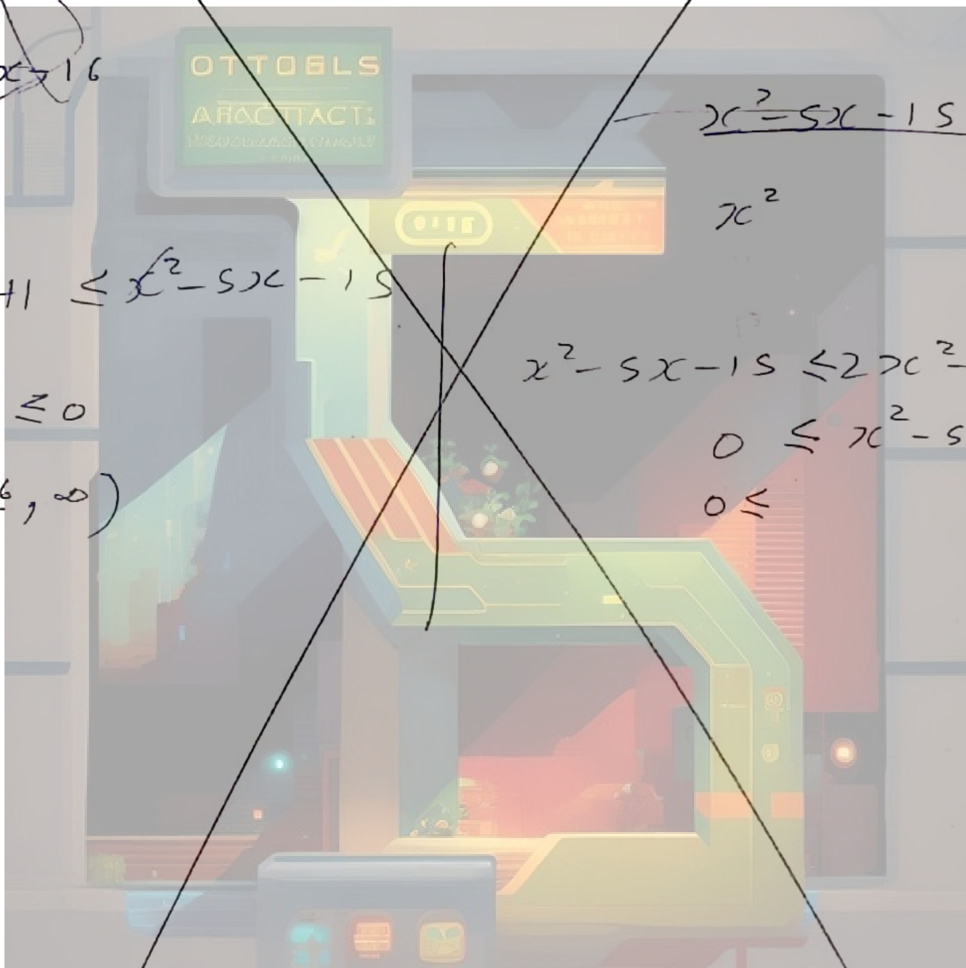
$\cancel{x^2+x+1 \le x^2-5x-15}$

$\cancel{0 \le -6x-16}$

$\cancel{16+6x \le 0}$

$\cancel{0 \le x^2-5x+16}$

$\cancel{x^2-5}$



$x^2+x+1 \le x^2-5x-15$

$6x+16 \le 0$

$x \in \left[\frac{-16}{6}, \infty\right)$

$\frac{x^2-5x-15}{x^2}$

$x^2-5x-15 \le 2x^2-10x-30$

$0 \le x^2-5x-15$

$0 \le$

Q $$1 \le \frac{x^2-5x-15}{x^2+x+1} \le 2$$

x^2+x+1: $a>0,\ D<0$

$\oplus$ ve

$$x^2+x+1 \le x^2-5x-15 \le 2(x^2+x+1)$$

$x^2+x+1 \le x^2-5x-15$

$6x \le -16$

$x \le -\frac{8}{3}$

$x \in \left(-\infty, -\frac{8}{3}\right]$

$x^2-5x-15 \le 2x^2+2x+2$

$0 \le x^2+7x+17$

$x^2+7x+17 \ge 0$

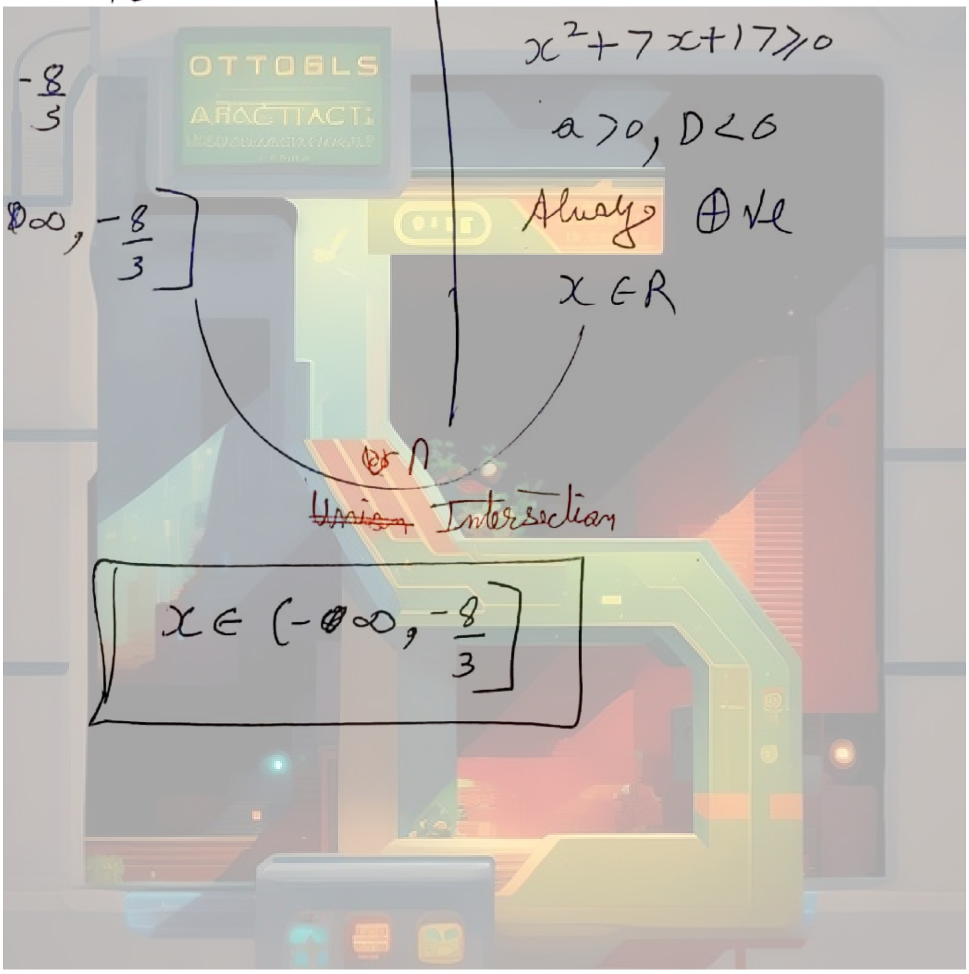
$a>0,\ D<0$

Always $\oplus$ ve

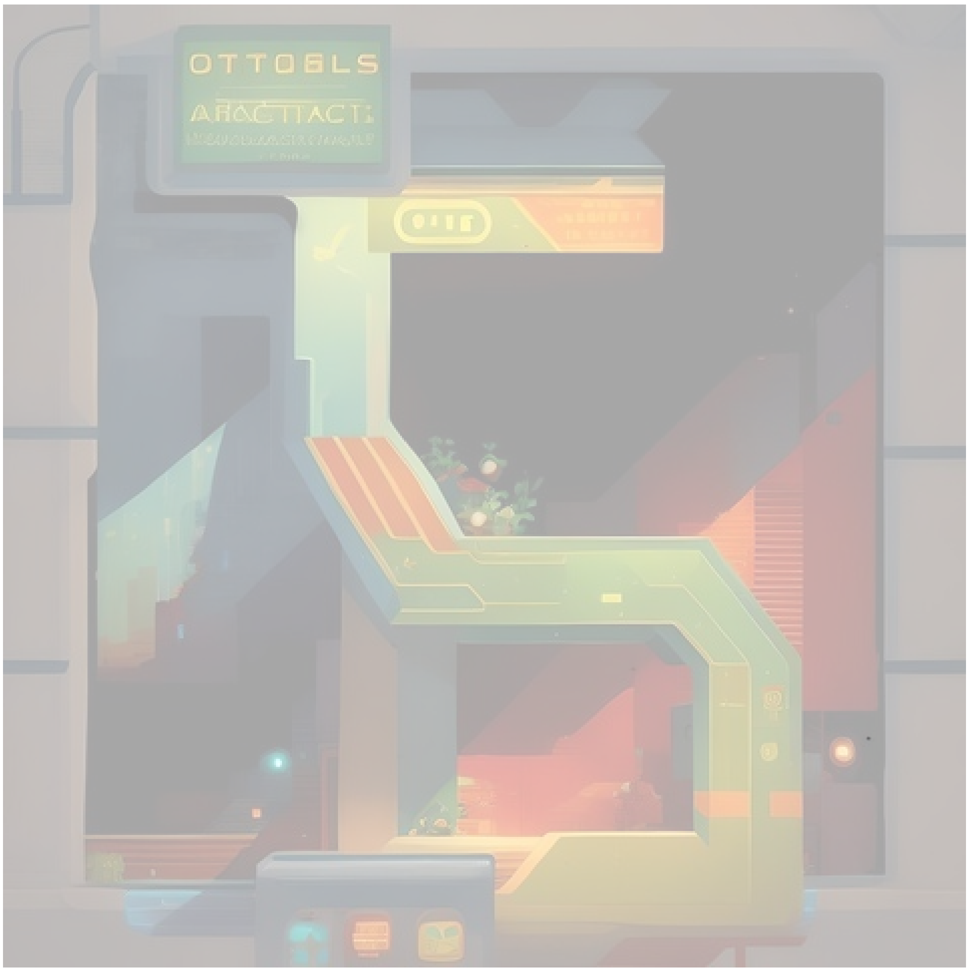
$x \in R$

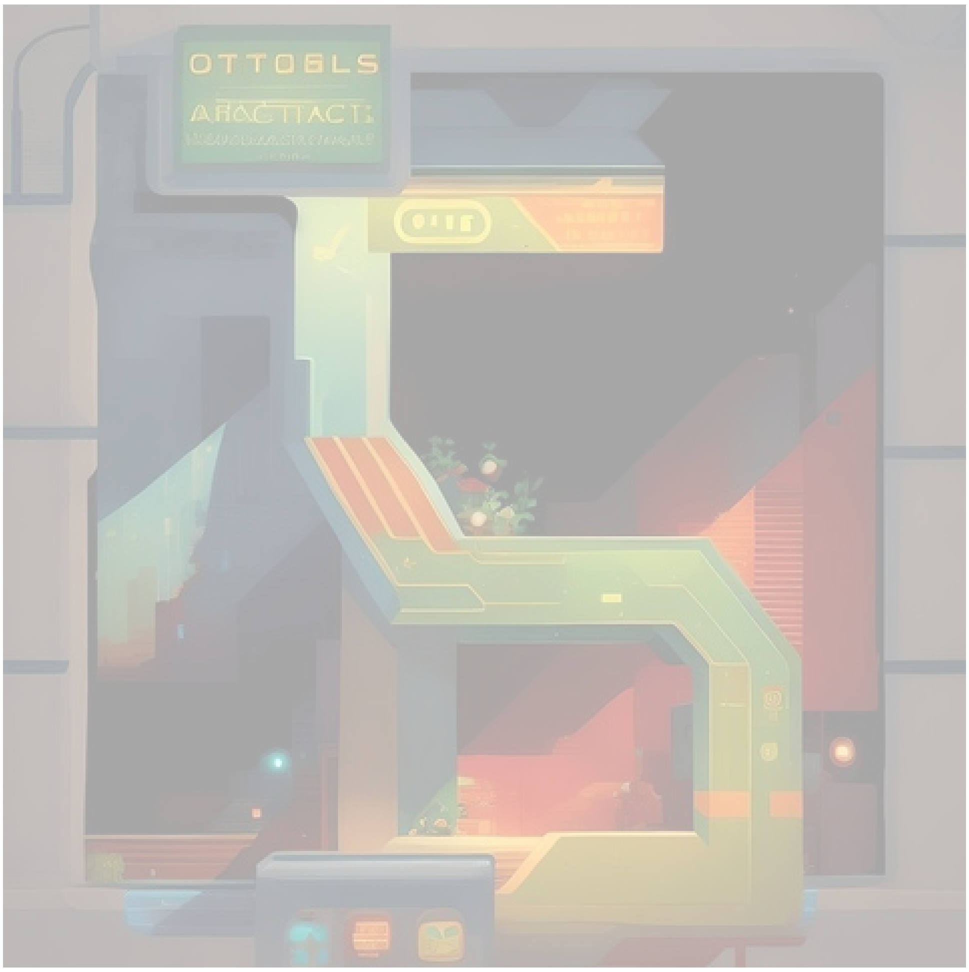
$\cap$

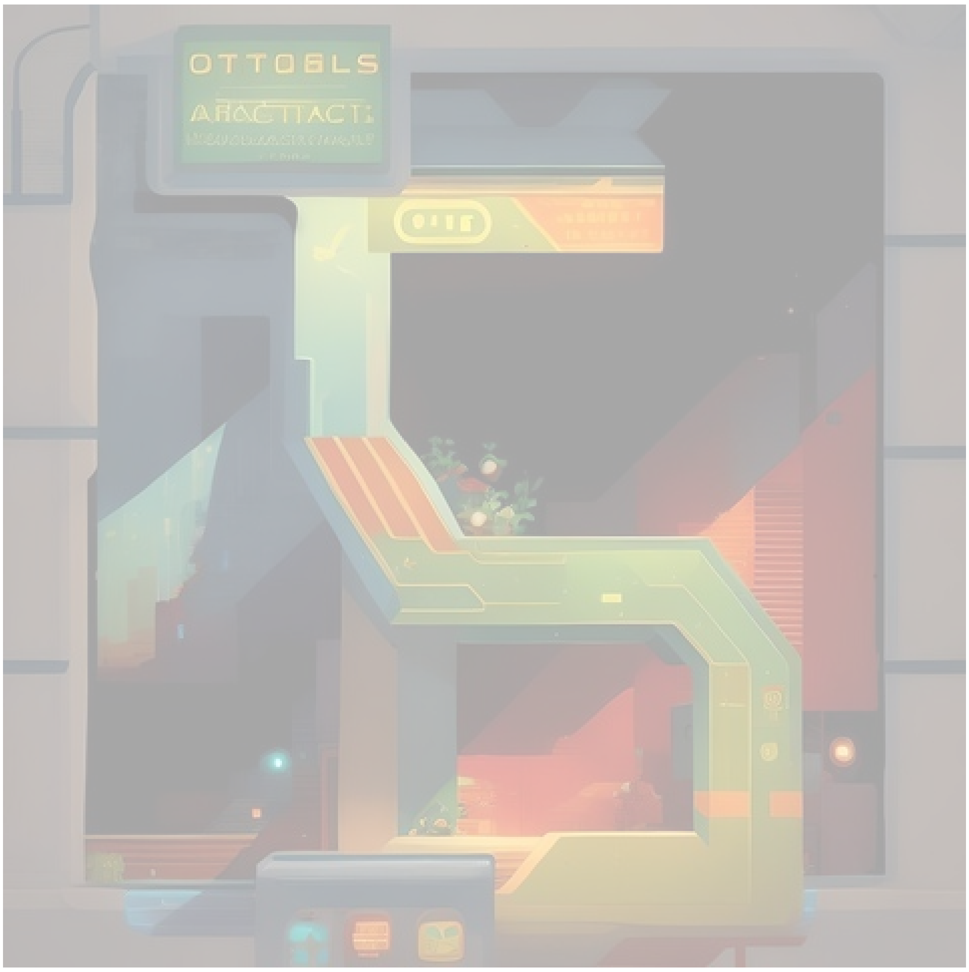
~~Union~~ Intersection

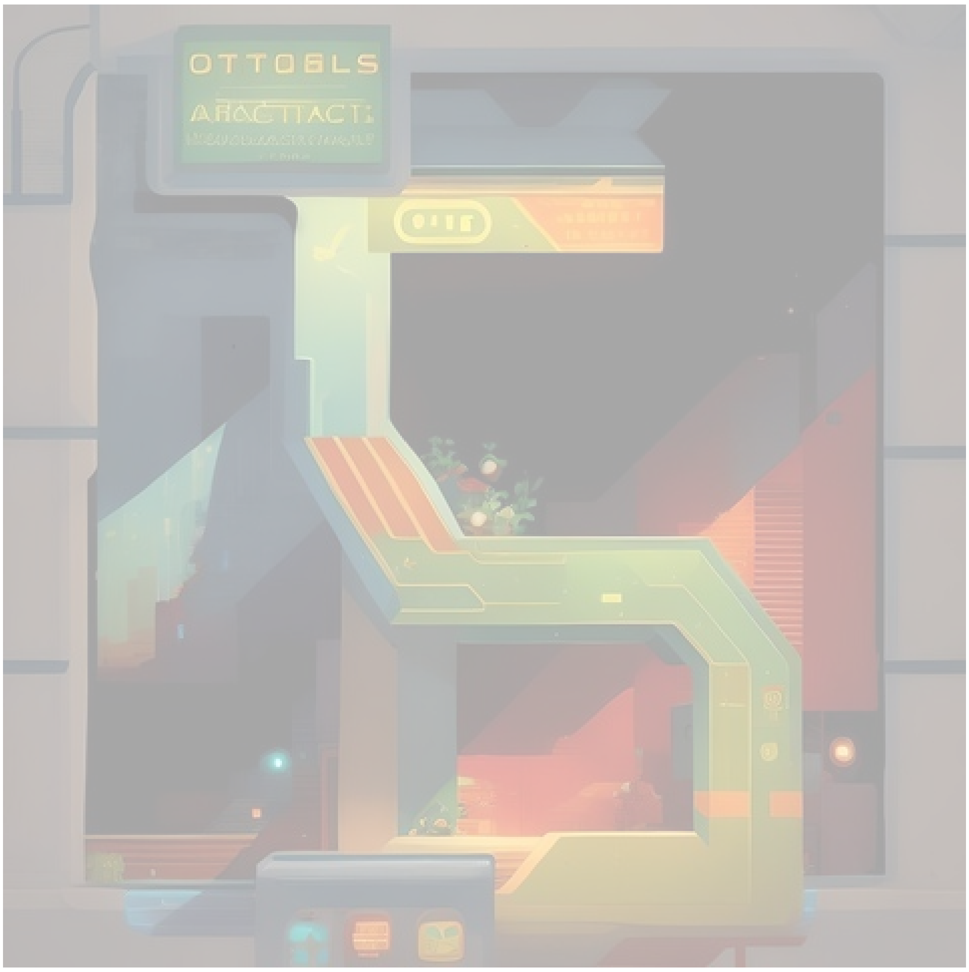


$$\boxed{x \in \left(-\infty, -\frac{8}{3}\right]}$$









Quadratic Equation

$x^2 - 2x + 1 = 0$

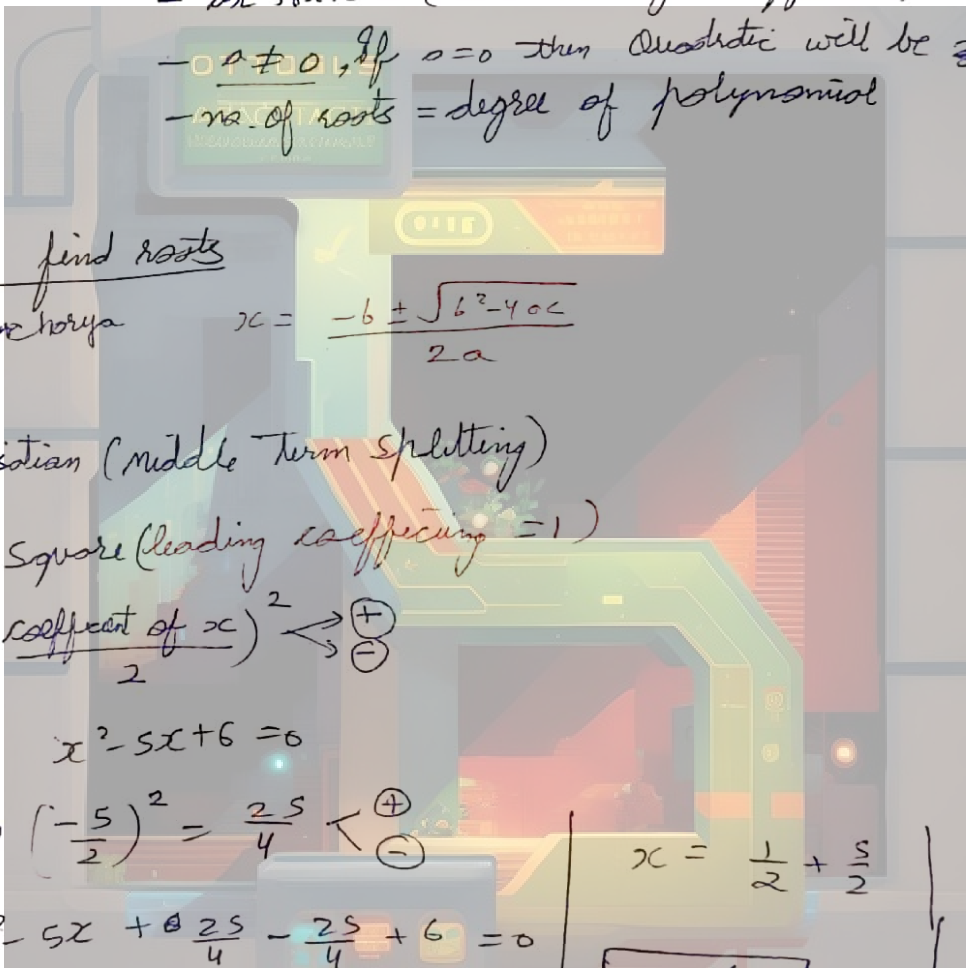
$x = 1, 1$

Solution - 1 solution [repeated roots are only counted once as solution]

Roots - 2 roots

Quadratic Equation - 2 degree polynomial (2 roots)

- $ax^2+bx+c=0$ ($a \to$ leading coefficient)
- $a \neq 0$, if $a=0$ then Quadratic will be ~~zero~~ linear.
- no. of roots = degree of polynomial



Methods to find roots

(1) Shri-Dharacharya $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

(2) factorisation (middle term splitting)

(3) Perfect square (leading coefficient $=1$)

$\rightarrow \left(\dfrac{\text{coefficient of } x}{2}\right)^2 < \begin{matrix} + \\ - \end{matrix}$

eg (1) $x^2 - 5x + 6 = 0$

$\left(-\dfrac{5}{2}\right)^2 = \dfrac{25}{4} < \begin{matrix} + \\ - \end{matrix}$

$x^2 - 5x + \dfrac{25}{4} - \dfrac{25}{4} + 6 = 0$

$\left(x - \dfrac{5}{2}\right)^2 - \dfrac{1}{4} = 0$

$x - \dfrac{5}{2} = \dfrac{1}{4}$

$x - \dfrac{5}{2} = \pm\dfrac{1}{2}$

$x = \dfrac{1}{2} + \dfrac{5}{2}$ | $x = \dfrac{1}{2} - \dfrac{5}{2}$

$\boxed{x = 3}$ | $\boxed{x = -2}$

Relation in roots and coefficients/constants

$$ax^2+bx+c=0$$

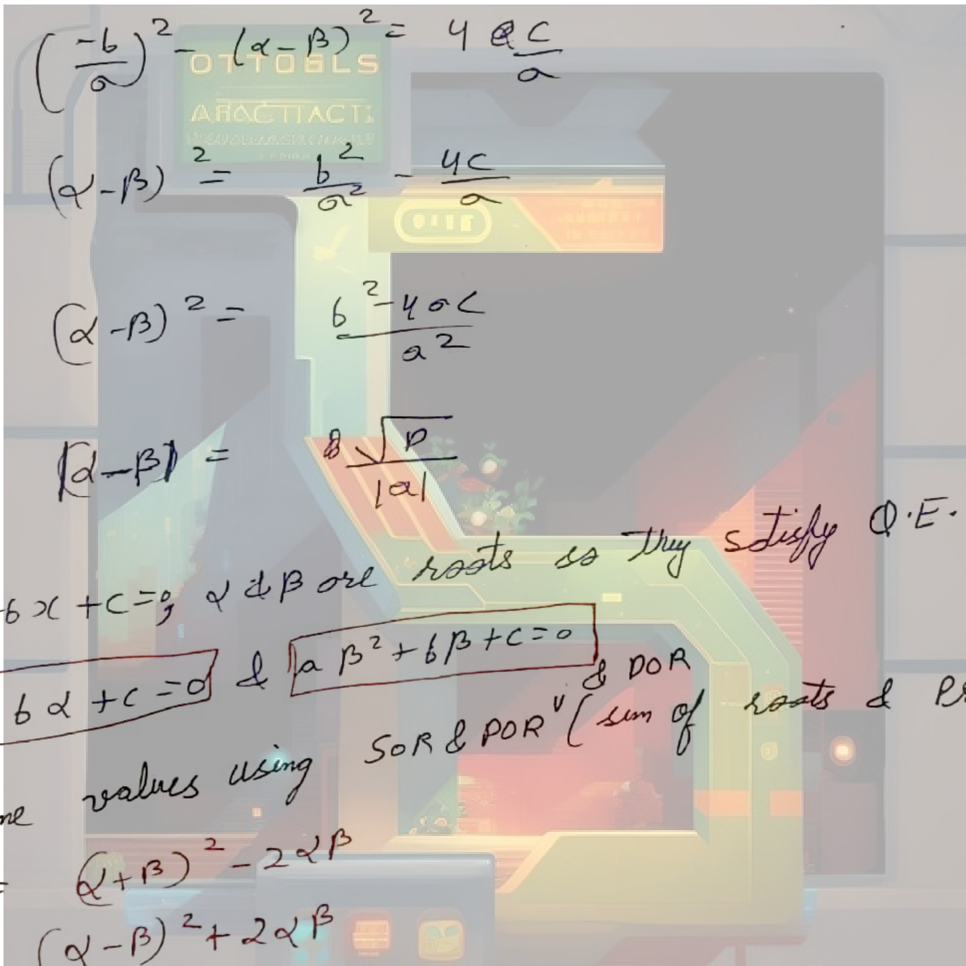
roots:- α, β

$$\alpha+\beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

→ Difference of Roots - $\boxed{|\alpha-\beta| = \frac{\sqrt{D}}{|a|}}$

Proof:- $(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$



$$\left(\frac{-b}{a}\right)^2 - (\alpha-\beta)^2 = 4\frac{c}{a}$$

$$(\alpha-\beta)^2 = \frac{b^2}{a^2} - \frac{4c}{a}$$

$$(\alpha-\beta)^2 = \frac{b^2-4ac}{a^2}$$

$$|\alpha-\beta| = \frac{\sqrt{D}}{|a|}$$

→ $ax^2+bx+c=0$, α & β are roots so they satisfy Q.E.

$\boxed{a\alpha^2+b\alpha+c=0}$ & $\boxed{a\beta^2+b\beta+c=0}$

Find some values using SoR & PoR (sum of roots & Product of roots)

① $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\alpha^2+\beta^2 = (\alpha-\beta)^2 + 2\alpha\beta$

② $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

③ $\alpha^3-\beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

Q $x^2-4x+2=0$ (roots are α & β then find)

① $\alpha+\beta$

$\alpha+\beta = \frac{-b}{a}$

$\boxed{= 4}$

② $\alpha^2+\beta^2$

$\alpha\beta = \boxed{2}$

$\alpha-\beta = \frac{\sqrt{16-8}}{1}$

$= 2\sqrt{2}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 16 - 4$

$\boxed{= 12}$

③ $\alpha^3+\beta^3$

$\alpha^3+\beta^3 = (4)^3 - 3(2)(4)$

$= 64 - 24$

$\cancel{28}$

$\boxed{= 40}$

④ $\frac{\alpha\beta}{\alpha+\beta}$

$\frac{2}{4} = \boxed{\frac{1}{2}}$

⑤ $\alpha-\beta = \frac{\sqrt{16-8}}{1}$

$\boxed{= 2\sqrt{2}}$

⑥ $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$\frac{\beta^2+\alpha^2}{(\alpha\beta)^2}$

$\frac{12}{4} = \boxed{3}$

⑦ $\alpha^2-\beta^2$

$(\alpha+\beta)(\alpha-\beta)$

$(4)(2\sqrt{2})$

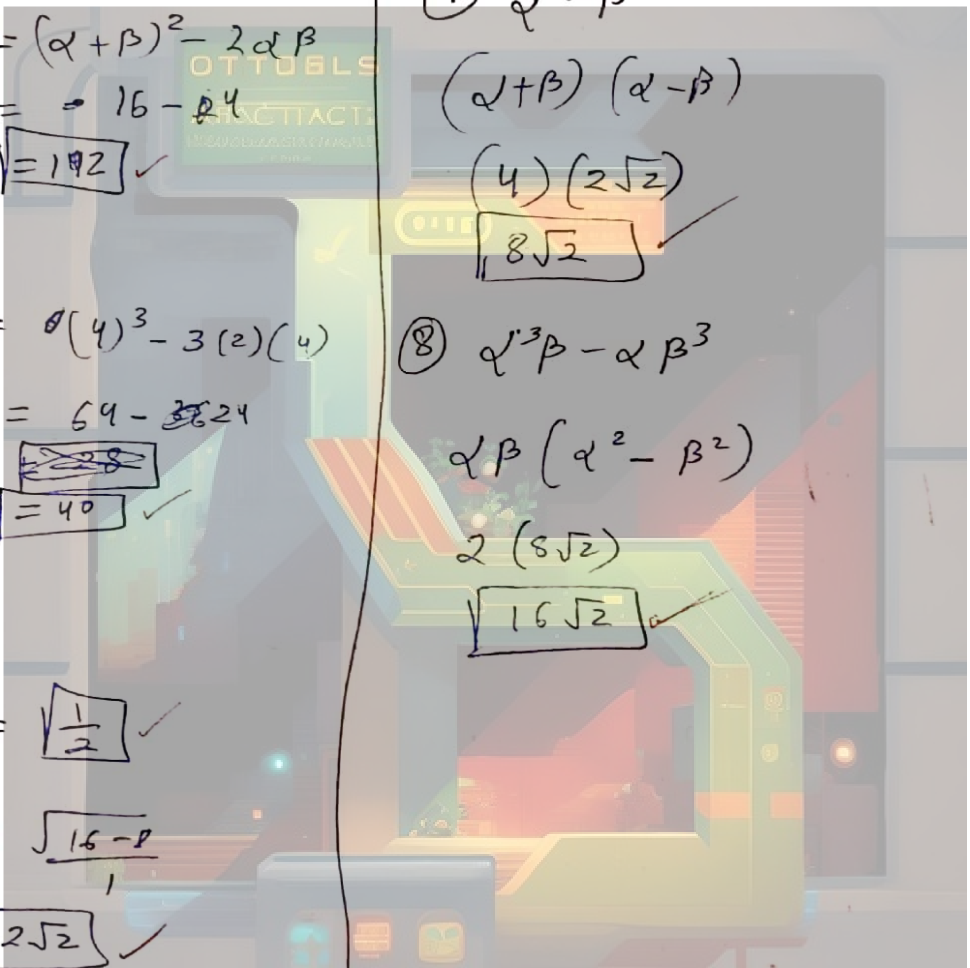
$\boxed{8\sqrt{2}}$

⑧ $\alpha^3\beta - \alpha\beta^3$

$\alpha\beta(\alpha^2-\beta^2)$

$2(8\sqrt{2})$

$\boxed{16\sqrt{2}}$



Q $3x^2+7x+3=0$

(1) $\frac{\beta}{\alpha}+\frac{\alpha}{\beta}$

$\frac{\beta^2+\alpha^2}{\alpha\beta}$

$\frac{\frac{49}{9}-2}{1}$

$\frac{49-18}{9}$

$\boxed{=\frac{31}{9}}$

$\alpha+\beta=\frac{-7}{3}$

$\alpha\beta=\frac{3}{3}=1$

$\alpha-\beta=\frac{\sqrt{13}}{3}$

(2) $\alpha^2\beta+\alpha\beta^2$

$\alpha\beta(\alpha+\beta)$

$1\left(\frac{-7}{3}\right)$

$\boxed{-\frac{7}{3}}$

(3) $\alpha^4\beta^7+\alpha^7\beta^4$

$\alpha^4\beta^4(\alpha^3+\beta^3)$

$\left(-\frac{7}{3}\right)^3-3\left(-\frac{7}{3}\right)$

$\frac{-343}{27}+7$

$\frac{-343+189}{27}$

$\boxed{\frac{-154}{27}}$

(4) $\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)^2$

$\left(\frac{\alpha^2-\beta^2}{\alpha\beta}\right)^2$

$\left[\frac{\left(\frac{-7}{3}\right)\left(\frac{\sqrt{13}}{3}\right)}{1}\right]^2$

$\left(\frac{-7\sqrt{13}}{9}\right)^2$

$\frac{49\times13}{81}$

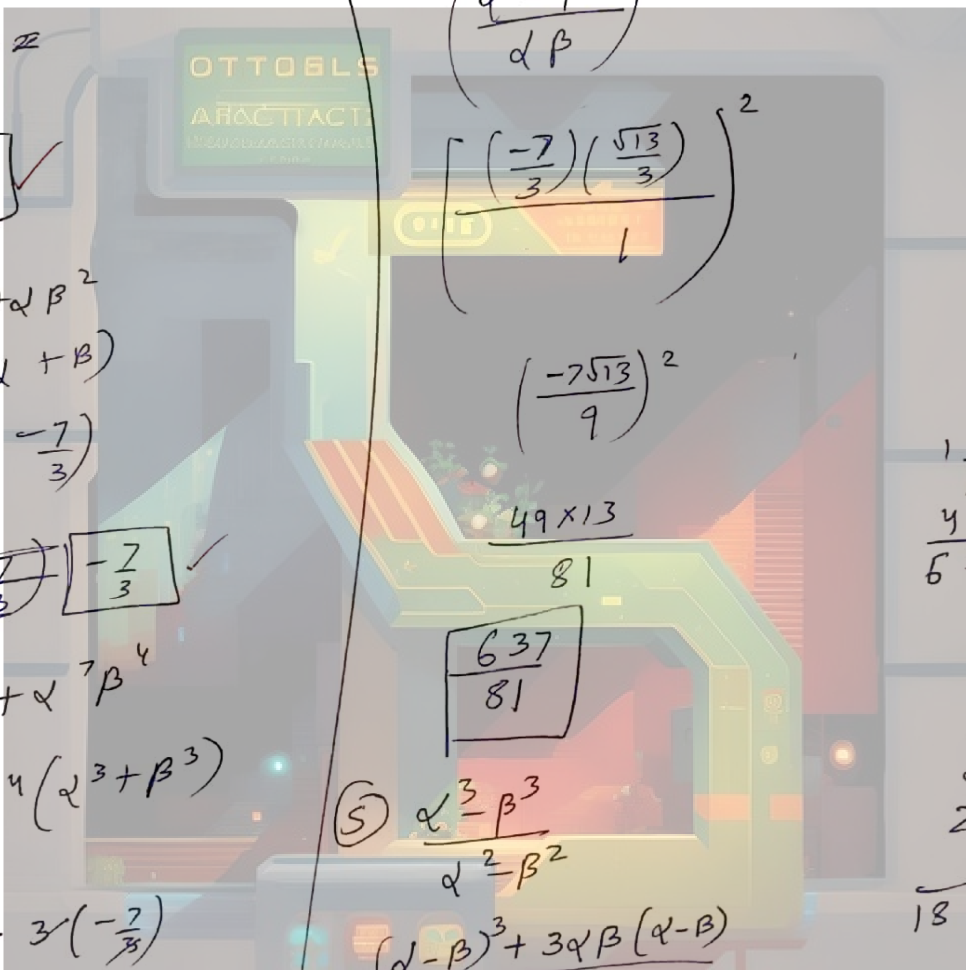
$\boxed{\frac{637}{81}}$

$27 \times 7 = 189$

$49 \times 13 = 147 + 490 = 637$

$49 \times 7 = 343$

$27 \times 7 = 189$



(5) $\frac{\alpha^3-\beta^3}{\alpha^2-\beta^2}$

$\frac{(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)}{-7\sqrt{13}}$

$\frac{(\sqrt{13})^3}{27}+3\times\frac{\sqrt{13}}{3}$

$\frac{(\sqrt{13})^3+27\sqrt{13}}{27(-7\sqrt{13})}$

$\frac{(\sqrt{13})^3+27\sqrt{13}}{-189\sqrt{13}}$

$\frac{13+27}{-21}$

$\boxed{\frac{-40}{21}}$

Q If x^2-x+1 has roots α,β find

① $\alpha^2-\alpha$

$\alpha(\alpha-1)$

$\alpha+\beta=1$

$\alpha\beta=1$

$\cancel{\alpha-\beta=}$

$\alpha(-\beta)$

$-\alpha\beta$

$\boxed{-1}$

OR

α is a root

$\alpha^2-\alpha+1=0$

$\alpha^2-\alpha=-1$

② $\alpha^{15}+\beta^{15}$

$\cancel{(1-\beta)^{15}+\beta^{15}}$

$\alpha^2-\alpha+1=0$

$\alpha^2-\alpha=-1$

$\alpha^3-\alpha^2=-\alpha$

$\alpha^3=\alpha^2-\alpha$

$\alpha^3=-1$

$(\alpha^3)^5=\alpha^{15}$

$\alpha^{15}=(-1)^5$

$\alpha^{15}=-1$

$\beta^{15}=-1$

$-1+(-1)$

$\boxed{-2}$

③ $\alpha^{2025}+\beta^{2025}$

$\alpha^3=-1$

$\alpha^{2025}=-1$

$\beta^{2025}=-1$

$-1+(-1)$

$\boxed{-2}$

Q find qud whose roots are $\frac{1}{5+2\sqrt6}$ & $\frac{1}{5-2\sqrt6}$

$\alpha+\beta=\frac{5-2\sqrt6+5+2\sqrt6}{25-24}$

$=10$

$\alpha\beta=1$

$\boxed{x^2-10x+1=0}$

M.W. 16-05-2024

DYS-1 {01, 2, 3, 5, 6, 7, 8, 9, 10, 13, 12, 11, 14, 4}

[1, 14]

~~Q find roots $a^2 + a - 444444222222 = 0$~~

$a(a+1) = 4444444|2222222$

$\boxed{a = 444444422222222}$

$a(a+1) = 4$

$\frac{a(a+1)}{y} = 1$

$\frac{a(a+1)}{y} - 1 = 0$

$\frac{a(a+1) - y}{y} = 0$

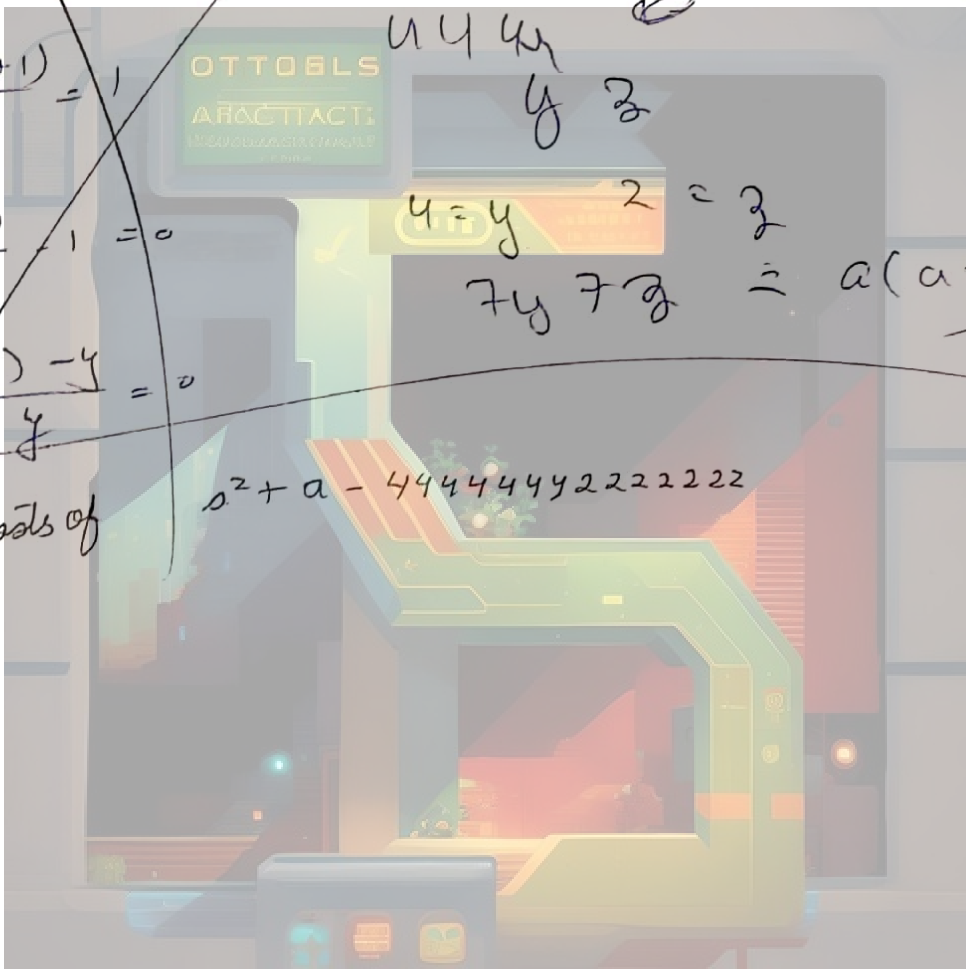
$\alpha\beta = -1$

$\alpha\beta = -444$

Sol

Not defined

0



444

y z

$4 = y^2 = z$

$7y\,7z = a(a+1)$

Q find roots of $a^2 + a - 44444442222222$

Nature of roots

$ax^2+bx+c \quad (a,b,c \in R)$

$D = b^2-4ac$

- $D>0$: Distinct real roots
- $D=0$: Equal real roots
- $D<0$: Imaginary roots in pairs.

$D = b^2-4ac \quad (a,b,c \in Q)$ ↳ rational

- Rational no. Perfect Square
- Perfect square; Not a perfect square: Roots are irrational in pair $a+\sqrt{b}$ & $a-\sqrt{b}$

Q Find nature of roots

① $x^2+x+1=0$

$D = (1)^2-(4)(1)(1)$

$= 1-4$

$= -3$

$\boxed{\text{Imaginary}}\ \boxed{\text{\& Paired}}$ ✓

② $2x^2-6x+3=0$

$D = 36-24$

$= 12$

$\boxed{\text{Real, Irrational}}$ in Pair ✓

③ $3x^2-4\sqrt{3}x+4=0$

~~Irrat~~

$D = 48-48$

$= 0$

$\frac{4\sqrt{3}}{6}$

$\boxed{\text{real \&}}\ \boxed{\text{equal, irrational}}$ ✓

Q find 'm' if equation $x^2 - x + m^2 = 0$ have no real roots.

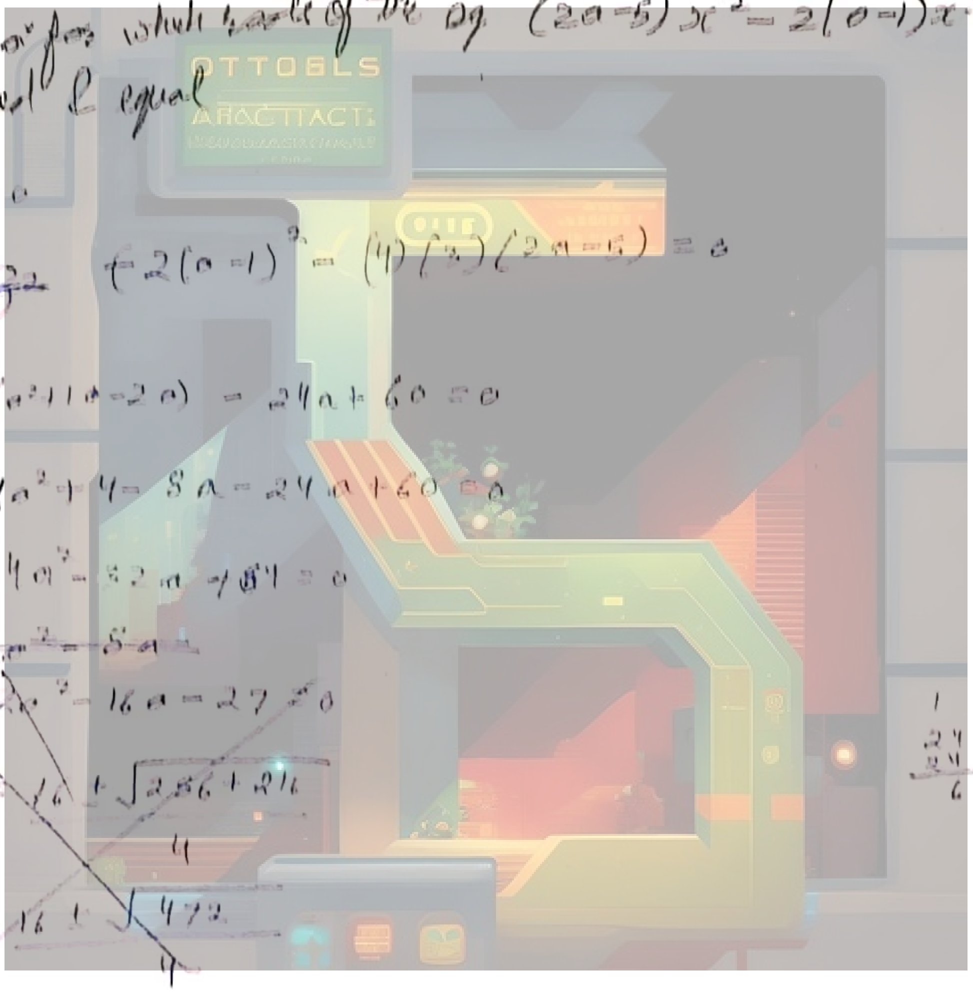
$D < 0$

$1 - 4m^2 < 0$

$-\tfrac{1}{2} \qquad \tfrac{1}{2}$

$\boxed{m \in (-\infty, -\tfrac{1}{2}) \cup (\tfrac{1}{2}, \infty)}$

$1 - 4m^2 = 0$
$4m^2 = 1$
$m^2 = \frac{1}{4}$



Q Value of 'a' for which roots of the eq. $(2a-5)x^2 - 2(a-1)x + 3 = 0$ are real & equal

$D = 0$

$(-2(a-1))^2 - (4)(3)(2a-5) = 0$

$4(a^2 + 1 - 2a) - 24a + 60 = 0$

$4a^2 + 4 - 8a - 24a + 60 = 0$

$4a^2 - 32a + 64 = 0$

$\cancel{2a^2 - 16a - 27 = 0}$

$\cancel{a = \frac{16 \pm \sqrt{256 + 216}}{4}}$

$\cancel{a = \frac{16 \pm \sqrt{472}}{4}}$

$a =$

$a^2 - 8a + 16 = 0$

$a = \frac{8 \pm \sqrt{64 - 64}}{2}$

$\boxed{a = 4}$

Q $x^2+(2a\sqrt{a^2-3})x+4=0$

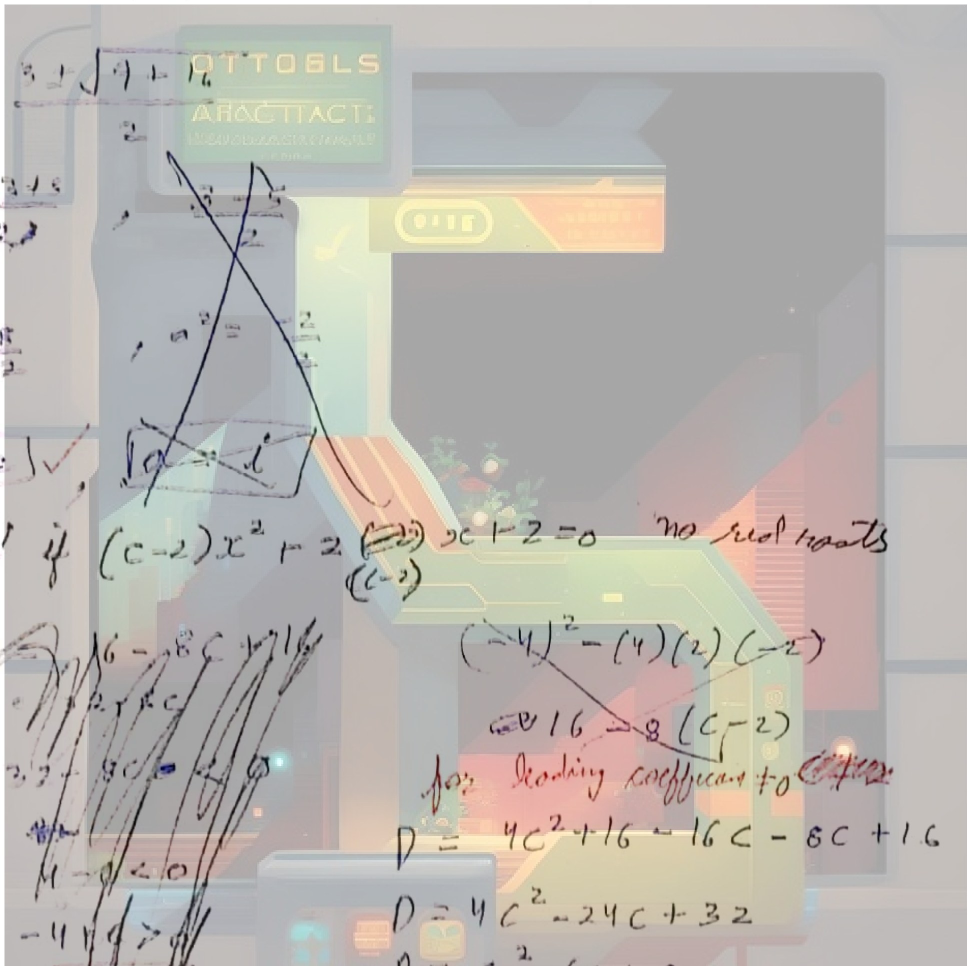
$(2a\sqrt{a^2-3})^2-(4)(4)(1)=0$

$4a^2(a^2-3)-16=0$

$4a^4-12a^2-16=0$

$2a^4-6a^2-8=0$

$a^4-3a^2-4=0$



$a^2=\frac{3\pm\sqrt{9+16}}{2}$

$a^2=\frac{3+5}{2}$, $\cancel{a^2=\frac{3-5}{2}}$

$a^2=\frac{8}{2}$, $\cancel{a^2=\frac{-2}{2}}$

$\boxed{a=\pm 2}$ ✓, $\cancel{\boxed{a=\pm i}}$

Q find 'c' if $(c-2)x^2+2(c-2)x+2=0$ no real roots

$\cancel{(-4)^2-(4)(2)(c-2)}$

$16-8(c-2)$

for leading coefficient to ...

$D=4c^2+16-16c-8c+16$

$D=4c^2-24c+32$

$D=c^2-6c+8$

$c^2-4c-2c+8$

$c(c-4)-2(c-4)$

$(c-2)(c-4)$

$(c-2)(c-4)<0$

$+\quad 2 \quad - \quad 4 \quad +$

$\boxed{(2,4)}$

(228)

Case 2:- leading coefficient $=0$

$C = 2$

$(2-2)x^2 + 2(2-2)x + 2 = 0$

$0x^2 + 0x + 2 = 0$

$2 = 0$

not a real value

so Roots are imaginary

$\boxed{C \in [2, 4)}$

~~Does~~ This is because questions not mentioned that given is Quadratic equation. So leading coefficient can be zero making the Equation linear

If question mensions quadratic equation, take leading coefficient $\neq 0$.

Q DYS-2

Q11. $(K-12)x^2 + 2(K-12)x + 2 = 0$

find integral values of K for which quadratic equation posses no real roots.

$\cancel{(K-12)^2 - (4)(2K-24)}$

$D = 4(K^2 + 144 - 24K) - 8K + 96$

$D = 4K^2 + 576 - 96K - 8K + 96$

$D = 4K^2 - 104K + 672$

$D = K^2 - 26K + 168$

$(K-12)(K-14) < 0$

$+ \quad 12 \quad - \quad 14 \quad +$

$(12, 14)$

so $\boxed{K = 13}$

Q If $x = 1 + 2i$

find $x^3 + x^2 - x + 22$

$x - 1 = 2i$

$(x-1)^2 = -2$

$x^2 + 1 - 2x = -2$

$x^2 - 2x + 1 = -2$

$3x^2 - 6x + 03 = -6$

$x(x^2 + x - 1) + 22$

~~$x(x^2 + x - 1$~~

$x(x^2 - 2x + 1 + 3x - 2) + 22$

$x(3x - 4) + 22$

$3x^2 - 4x + 22$

$3x^2 - 6x + 3 + 17 + 2x$

$-6 + 17 + 2x$

$13 + 2x$

$x^2 - 2x + 3 = 0$

$x = 2 \pm \sqrt{4 -}$

$x = 1 + 2i$

$x - 1 = 2i$

$x^2 + 1 - 2x = -4$

$x^2 - 2x + 5 = 0$

~~$x = 2 \pm \sqrt{4}$~~

$3x^2 - 6x + 15 = 0$

$x(x^2 - 2x + 5 + 3x - 6) + 22$

$x(3x - 6) + 22$

$3x^2 - 6x + 22$

$3x^2 - 6x + 15 + 7 = 0$

$\boxed{7}$

DYQS-2

Q16. find the value of $x^3 - 3x^2 - 8x + 15$

$x = 3 + i$

$x - 3 = i$

$x^2 + 9 - 6x = -1$

$x^2 + 10 - 6x = 0$

$3x^2 + 30 - 18x = 0$

$x(x^2 - 3x - 8) + 15$

$x(x^2 - 6x + 10 + 3x - 18) + 15$

$x(3x - 18) + 15$

$3x^2 - 18x + 15$

$\boxed{-15}$ $\boxed{-15}$

230

H.W.

DVS-2 $[1,16] - [\{11,16\} \cup \{5\}]$

$a^2 + a = 44444442222222$

$\cancel{= 4(1111111)}$

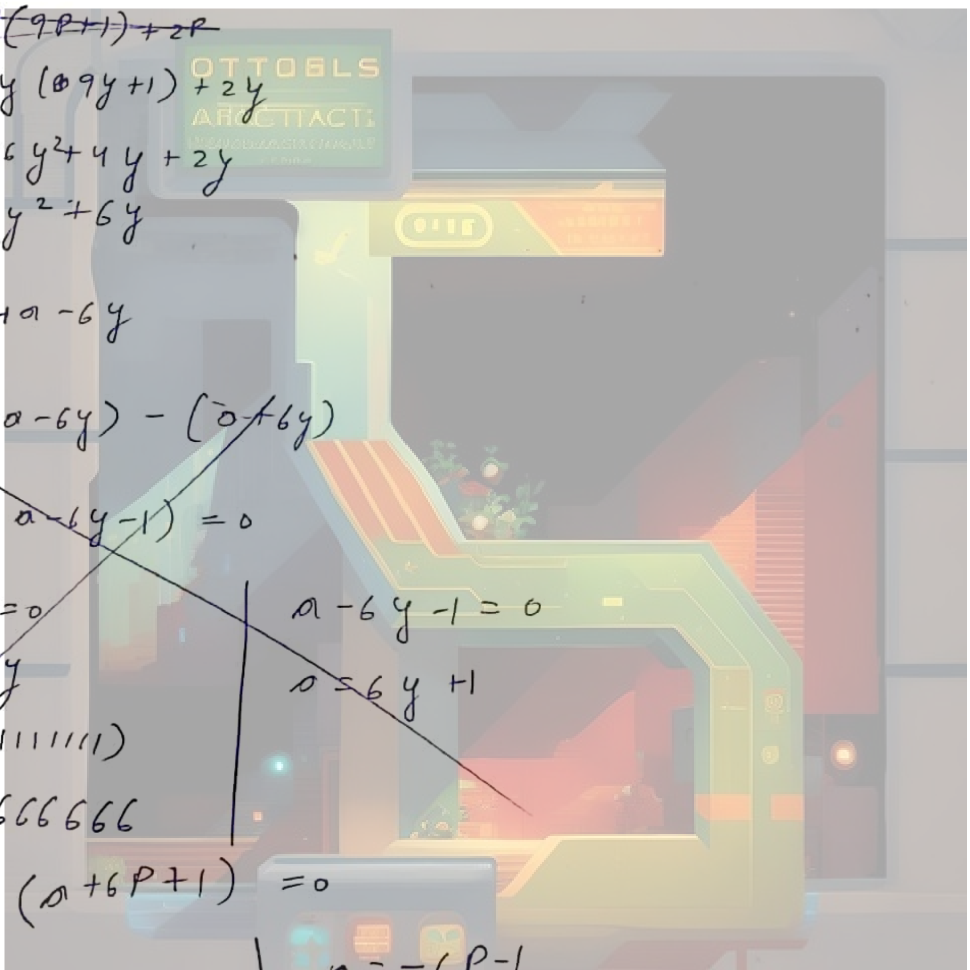
$= 44444440000000 + 2222222$

$= 4444444 \times 10000000 + 2222222$

$= 4444444(9999999 + 1) + 2222222$

$= 4(1111111)[9(1111111) + 1] + 2(1111111)$

$\cancel{= 4P(9P+1) + 2P}$



$= 4y(9y+1) + 2y$

$= 36y^2 + 4y + 2y$

$a^2 + a = 36y^2 + 6y$

$a^2 - 36y^2 + a - 6y$

$(a+6y)(a-6y) - (a+6y)$

$a+6y \; (a-6y-1) = 0$

$a + 6y = 0$	$a - 6y - 1 = 0$
$a = -6y$	$a = 6y + 1$
$a = -6(1111111)$	
$a = -6666666$	

$(a-6y)(a+6P+1) = 0$

$a = 6y$	$a = -6P - 1$
$\boxed{a = 6666666}$	$a = -6666666 - 1$
	$\boxed{a = -6666667}$

Q find the quadratic if 1 root is $5+2\sqrt{6}$ & coefficients of quadratic are rational.

x: ~~$ax+1$~~ $\alpha = 5+2\sqrt{6}$

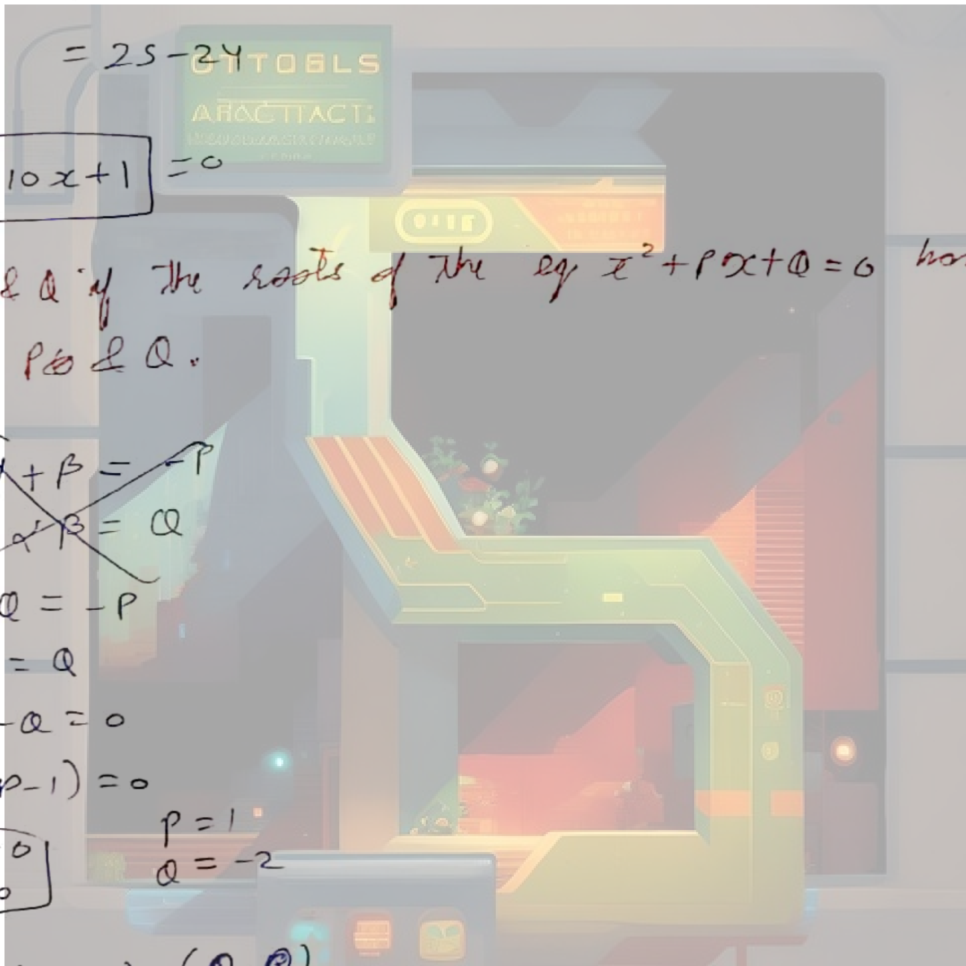
$\beta = 5-2\sqrt{6}$

$\alpha+\beta = 5+2\sqrt{6}+5-2\sqrt{6}$

$= 10$

$\alpha\beta = (5+2\sqrt{6})(5-2\sqrt{6})$

$= 25-4\times 6$



$= 25-24$

$\boxed{x^2-10x+1} = 0$

Q find P & Q if the roots of the eq $x^2+Px+Q=0$ have ~~[illegible]~~ roots P & Q.

~~$\alpha+\beta = -P$~~

~~$\alpha\beta = Q$~~

$P+Q = -P$

$PQ = Q$

$PQ - Q = 0$

$Q(P-1) = 0$

$\boxed{\begin{matrix} Q=0 \\ P=0 \end{matrix}}$ $\quad$ $P=1$, $Q=-2$

$P, Q = (1,-2), (0,0)$

Q $x^2 + mx + 1 = 0$ find m if

a) On root is thrice of other

b) ratio of roots is $1/3$

c) Sum of roots is equal to POR.

a) $\alpha = 3\beta$

$\alpha + \beta = -m$

$4\beta = -m$

$\alpha\beta = 1$

$3\beta(\beta) = 1$

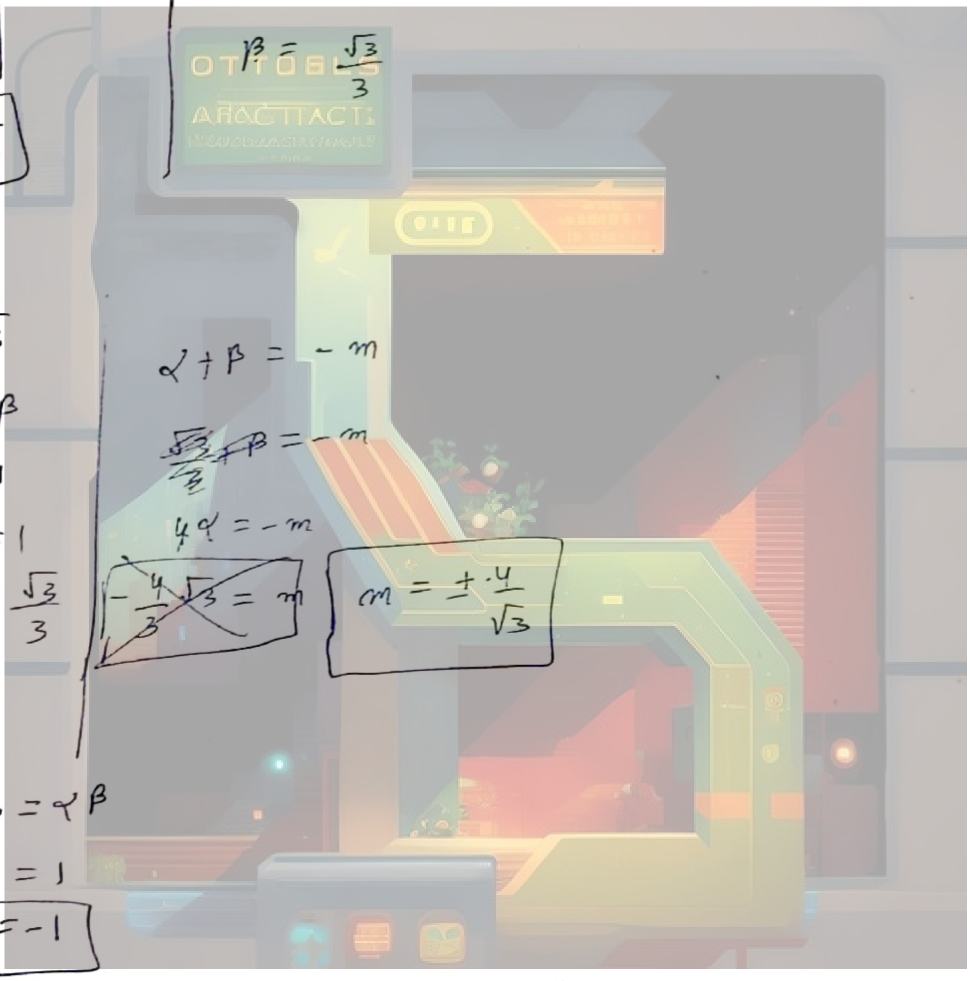
$3\beta^2 = 1$

$\beta^2 = \frac{1}{3}$

$\beta = \frac{\sqrt{3}}{3}$

$\boxed{-\frac{4\sqrt{3}}{3}}$

$m = \boxed{\pm\frac{4}{\sqrt{3}}}$



b) $\frac{\alpha}{\beta} = \frac{1}{3}$

$3\alpha = \beta$

$\alpha\beta = 1$

$3\alpha^2 = 1$

$\alpha = \frac{\sqrt{3}}{3}$

$\alpha + \beta = -m$

$\cancel{\frac{\sqrt{3}}{3} + \beta} = -m$

$4\alpha = -m$

$\cancel{-\frac{4\sqrt{3}}{3} = m}$

$\boxed{m = \pm\frac{4}{\sqrt{3}}}$

c) $\alpha + \beta = \alpha\beta$

$-m = 1$

$\boxed{m = -1}$

Symmetric Expression of α and β

If $f(\alpha, \beta) = f(\beta, \alpha)$

Eg ① $f(\alpha, \beta) = \alpha^2 + \beta^2$

$f(\beta, \alpha) = \beta^2 + \alpha^2$
$= \alpha^2 + \beta^2$

so, symmetric

Eg ② $f(\alpha, \beta) = \alpha^2 - \beta^2$

$f(\beta, \alpha) = \beta^2 - \alpha^2 \neq \alpha^2 + \beta^2$

so, not symmetric

Q find which are symmetric

① $f(\alpha, \beta) = \alpha^4 - \beta^4$

$f(\beta, \alpha) = \beta^4 - \alpha^4 \neq \alpha^4 - \beta^4$

Not Symmetric

② $f(\alpha, \beta) = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$f(\beta, \alpha) = \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Symmetric

③ $f(\alpha, \beta) = \frac{\alpha + \beta}{\alpha\beta}$

$f(\beta, \alpha) = \frac{\beta + \alpha}{\beta\alpha}$ Symmetric

$= \frac{\alpha + \beta}{\alpha\beta}$

Transformation of roots (valid for symmetric changes)

Eg ①. $x^2 - 4x + 5 = 0$

find quadratic whose roots are $\frac{2}{\alpha}$ & $\frac{2}{\beta}$

$\frac{2}{\alpha}, \frac{2}{\beta} = \frac{2}{x} \to t$

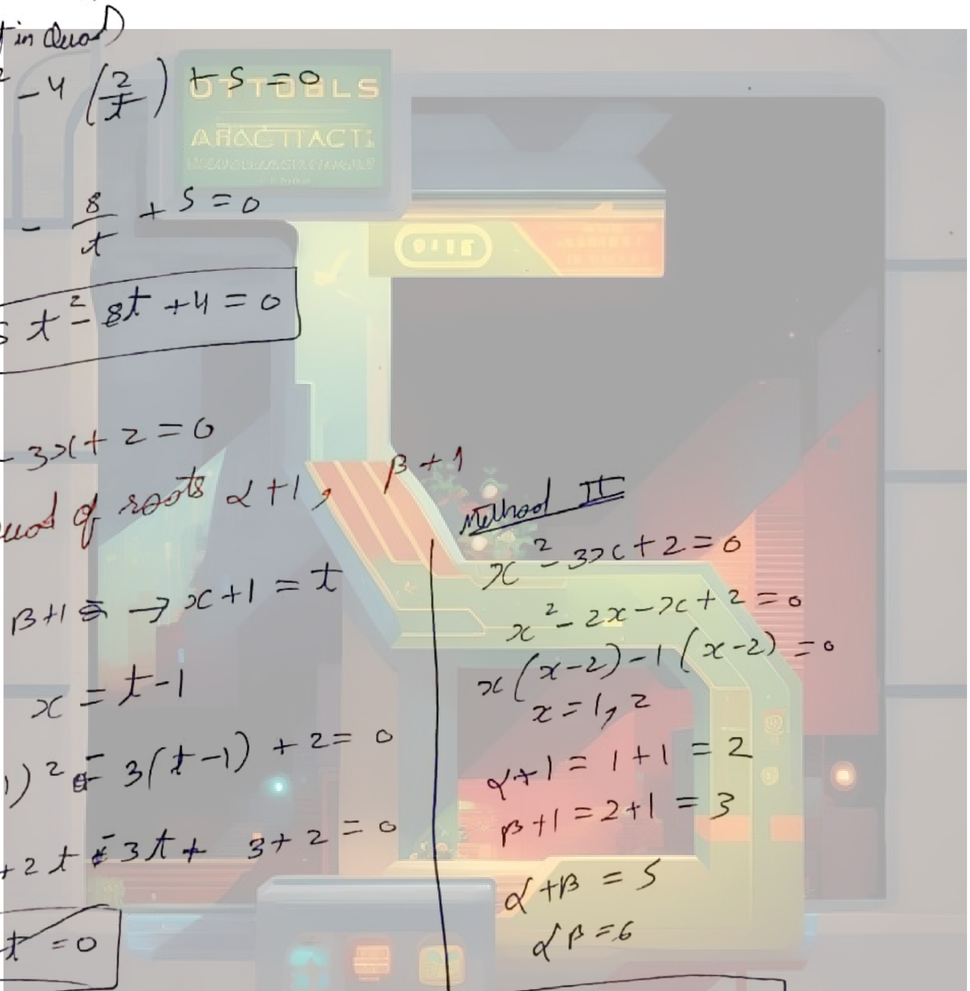
$x = \frac{2}{t}$

(put in Quad)

$\left(\frac{2}{t}\right)^2 - 4\left(\frac{2}{t}\right) + 5 = 0$

$\frac{4}{t^2} - \frac{8}{t} + 5 = 0$

$\boxed{5t^2 - 8t + 4 = 0}$



Q 2. $x^2 - 3x + 2 = 0$

find Quad of roots $\alpha + 1$, $\beta + 1$

$\alpha + 1, \beta + 1 \Rightarrow x + 1 = t$

$x = t - 1$

$(t-1)^2 - 3(t-1) + 2 = 0$

$t^2 + 1 + 2t - 3t + 3 + 2 = 0$

$\boxed{\cancel{t^2 + 5t = 0}}$

$\boxed{t^2 - 5t + 6 = 0}$

Method II

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$x = 1, 2$

$\alpha + 1 = 1 + 1 = 2$

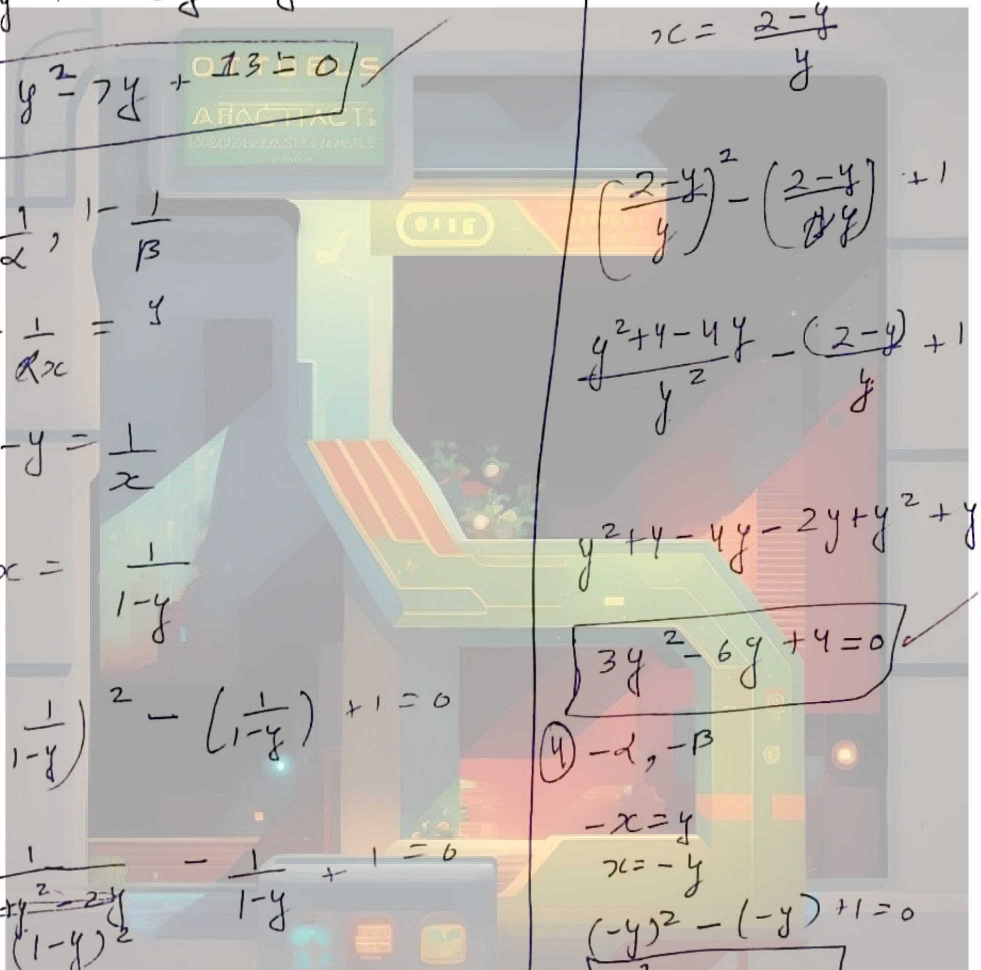
$\beta + 1 = 2 + 1 = 3$

$\alpha + \beta = 5$

$\alpha\beta = 6$

$\boxed{x^2 - 5x + 6 = 0}$

Q $x^2 - x + 1 = 0$ have roots ~~[illegible]~~ α and β, find quadratic whose roots are -



① $3+\alpha,\ 3+\beta$

$x + 3 = y$

$x = y - 3$

$(y-3)^2 - (y-3) + 1 = 0$

$y^2 + 9 - 6y - y + 3 + 1 = 0$

$\boxed{y^2 - 7y + 13 = 0}$

② $1 - \frac{1}{\alpha},\ 1 - \frac{1}{\beta}$

$1 - \frac{1}{x} = y$

$1 - y = \frac{1}{x}$

$x = \frac{1}{1-y}$

$\left(\frac{1}{1-y}\right)^2 - \left(\frac{1}{1-y}\right) + 1 = 0$

$\frac{1}{\cancel{1+y^2-2y}\ (1-y)^2} - \frac{1}{1-y} + 1 = 0$

$(1-y)^2 - (1-y) + 1 = 0$

$y^2 + 1 - 2y - 1 - y + 1 = 0$

$\boxed{y^2 - 3y + 1 = 0}$

③ $\frac{2}{1+\alpha},\ \frac{2}{1+\beta}$

$\frac{2}{1+x} = y$

$\frac{2}{y} = 1 + x$

$x = \frac{2-y}{y}$

$\left(\frac{2-y}{y}\right)^2 - \left(\frac{2-y}{y}\right) + 1$

$\frac{y^2 + 4 - 4y}{y^2} - \frac{(2-y)}{y} + 1$

$y^2 + 4 - 4y - 2y + y^2 + y^2$

$\boxed{3y^2 - 6y + 4 = 0}$

④ $-\alpha,\ -\beta$

$-x = y$

$x = -y$

$(-y)^2 - (-y) + 1 = 0$

$\boxed{y^2 + y + 1 = 0}$

Q $\cancel{x^2+1}$ $x^2 - x + 1 = 0$ have roots α & β

find Quadratic whose roots are.

1) $\alpha + 3\beta,\ 3\alpha + \beta$

2) $(\alpha - \beta)^2,\ (\alpha + \beta)^2$

1)

$\alpha + \beta = 1$

$\alpha\beta = 1$

$\alpha - \beta = \sqrt{-3}$

$R_1 = \alpha + 3\beta$

$R_2 = 3\alpha + \beta$

$R_1 + R_2 = \alpha + 3\beta + 3\alpha + \beta$

$= 4\alpha + 4\beta$

$= 4(\alpha + \beta)$

$= 4(1)$

$= 4$

$R_1 \times R_2 = (\alpha + 3\beta)(3\alpha + \beta)$

$= 3\alpha^2 + 3\beta^2 + \alpha\beta + 9\alpha\beta$

$= 3((\alpha+\beta)^2 - 2\alpha\beta) + 10\alpha\beta$

$= 3(1-2) + 10$

$= -3 + 10$

$= 7$

$x^2 - (4) + 7$

$\boxed{x^2 - 4x + 7}$

2)

$\alpha + \beta = 1$

$\alpha\beta = 1$

$R_1 + R_2 = (\alpha - \beta)^2 + (\alpha + \beta)^2$

$= \alpha^2 + \beta^2 - 2\alpha\beta + \alpha^2 + \beta^2 + 2\alpha\beta$

$= 2(\alpha^2 + \beta^2)$

$= 2(-1)$

$\boxed{= -2}$

$R_1 \times R_2 = (\alpha - \beta)^2 (\alpha + \beta)^2$

$= \cancel{(\alpha^2 + \beta^2 - 2\alpha\beta)(\alpha^2 + \beta^2)}$

$= (\sqrt{-3})^2 (1)^2$

$= -3 \times 1$

$\boxed{= -3}$

$\boxed{x^2 + 2x - 3}$

Equation (vs) Identity

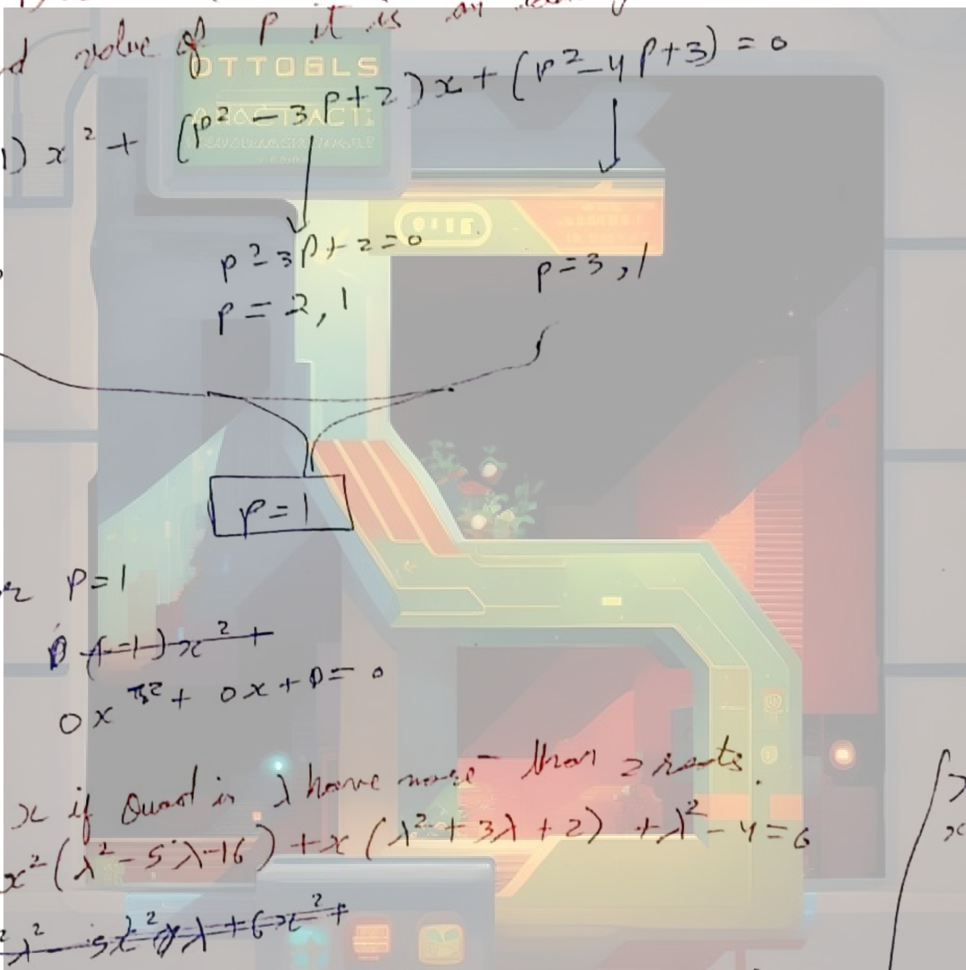
→ Equation which is true for every value of the variable is a identity

eg. $\sin^2\theta + \cos^2\theta = 1$ $(\theta \in R)$

$(a+b)^2 = a^2 + 2ab + b^2$ $(a, b \in R)$

→ In quadratic if it has more than 2 roots then it will be an identity

Q $(p-1)x^2 + (p^2-3p+2)x + (p^2-4p+3) = 0$ is an identity in x.
find value of p it is



$(p-1)x^2 + (p^2-3p+2)x + (p^2-4p+3) = 0$

$p-1=0$
$p=1$

$p^2-3p+2=0$
$p=2,1$

$p=3,1$

$\boxed{p=1}$

for $p=1$

$\cancel{0\,(1-1)x^2+}$

$0x^2 + 0x + 0 = 0$

Q. find x if Quad in λ have more than 2 roots.

$x^2(\lambda^2-5\lambda-16) + x(\lambda^2+3\lambda+2) + \lambda^2 - 4 = 6$

$\cancel{x^2\lambda^2 - 5x^2\lambda + 6x^2 +}$

$x^2y^2 - 5x^2\lambda + 6x^2 + x\lambda^2 + 3x\lambda + 2x + \lambda^2 - 4 = 0$

$\lambda^2(x^2+x+1) + \lambda(-5x^2+3x) + (6x^2+2x-4)$

$x \in \phi$

no common

$\boxed{x \in \phi}$

x^2+x+1
$x = -1 \pm \sqrt{}$

Q $\frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-c)(x-a)}{(b-c)(b-a)} + c\frac{(x-a)(x-b)}{(c-a)(c-b)} = x$

~~$ax^2(1-b)$~~

let $x = a$

$a + 0 + 0 = a$ $\rightarrow$ $\frac{a\cancel{(a-b)}\cancel{(a-c)}}{\cancel{(a-b)}\cancel{(a-c)}} + 0 + 0 = a \Rightarrow a = a$

$x = b$

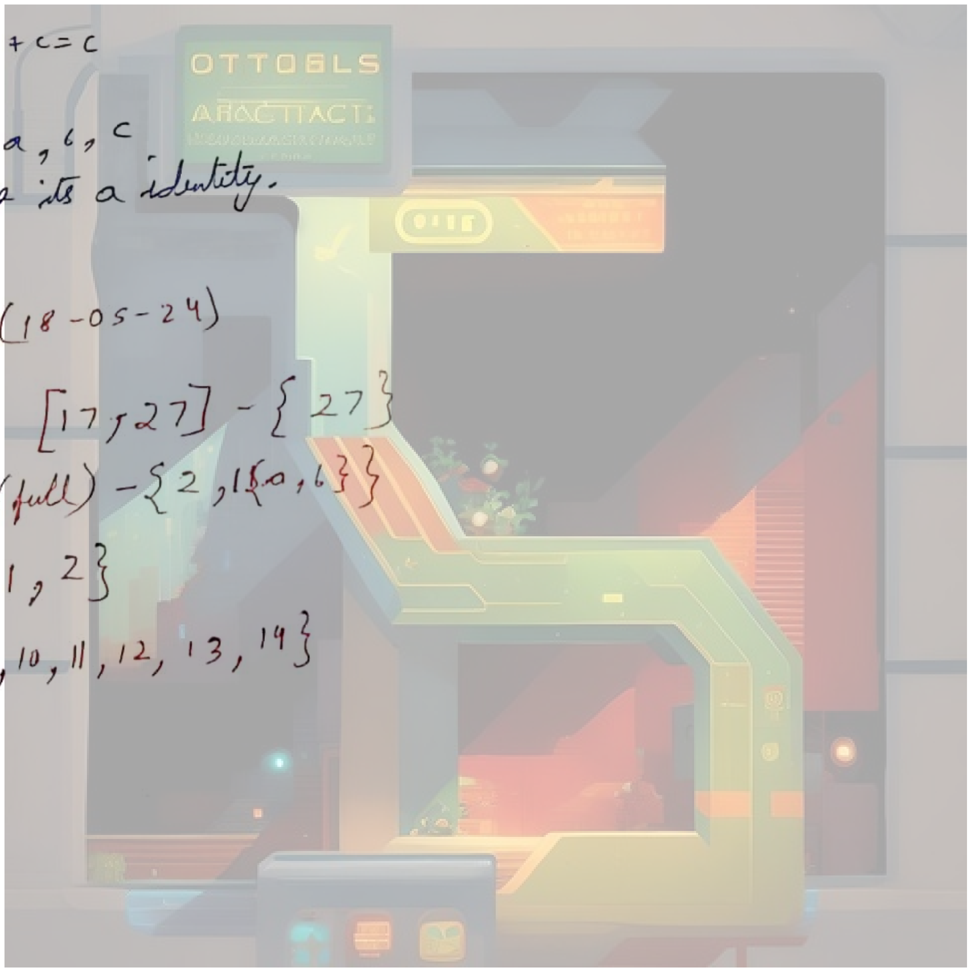
$0 + b + 0 = b$

$x = c$

$0 + 0 + c = c$

So $x = a, b, c$

so its a identity.



H.W. (18-05-24)

DYS-2 [17, 27] - {27}

DYS-3 (full) - {2, 15(a, b)}

DYS-4 {1, 2}

Race - {9, 10, 11, 12, 13, 14}