! Circular Motion !

→ If object moves such that ~~its~~ its distance from a fixed point is constant, its motion is called circular motion.

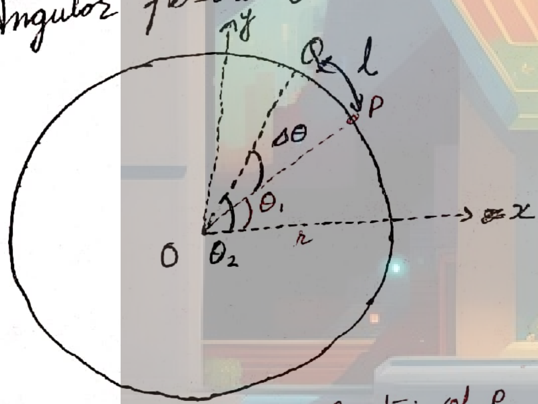
Circular motion

Uniform circular motion
↓
Speed - ~~Changes~~ Constant.
→ Direction changes continuously
→ It is an accelerated motion

non-uniform circular motion
↓
Speed Changing

→ Note :- Both are under Non-Uniform Motion.

Angular position (θ) & Angular Displacement ($\Delta\theta$)



To Decide Angular position, we need to specify two things

i) Origin

ii) reference line

→ The Angle made by position vector w.r.t origin with reference line is called as angular Position (θ)

θ_1 → Angular Position of P

θ_2 → Angular Position of Q

$\Delta\theta$ → Angular Displacement

→ Angle through the position vector of moving particle rotates in a given time interval is called angular displacement of particle ($\Delta\theta$). It depends on origin but not on choice of reference line.

$$\boxed{\Delta\theta = \frac{l}{r}}$$

→ It is Dimensionless

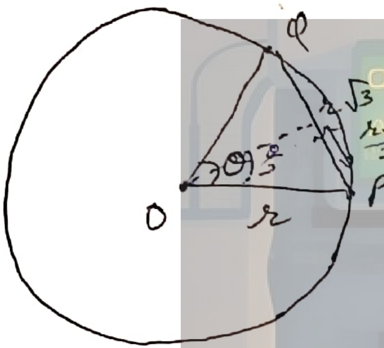
→ S.I. unit is radian.

→ Infinitesimal angular displacements (what chota) is a vector and finite angular displacement is scalar.

→ The direction of small angular displacements is given by right hand thumb rule.

Curl your fingers in the direction of revolution of object and thumb will give direction of angular displacement.

Q1.



A Particle moves from P to Q,

Displacement = $R\sqrt{3}$

find Angular displacement.

$$\sin\frac{\theta}{2} = \frac{r\sqrt{3}}{2r} = \frac{\sqrt{3}}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{3}$$

$$\boxed{\theta = \frac{2\pi}{3}}$$

Average Angular Velocity (ω_{av}):-

$$\boxed{\omega_{av} = \frac{\text{Angular Displacement}}{\text{Time}}}$$

SI unit:- Rad/s

→ It is a scalar Quantity.

Angular Velocity

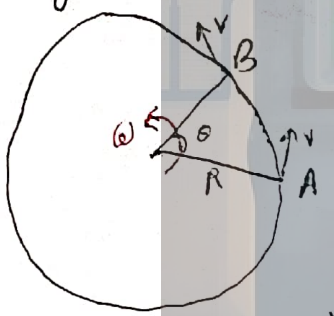
→ Rate of Change of Angular Displacement.

$$\boxed{\vec{\omega} = \frac{d\vec{\theta}}{dt}}$$

→ Since infinitesimal Angular displacement is a vector, angular velocity is also a vector.

- Unit :- rad/s
- Dimensions :- T^{-1}

→ Magnitude of Angular Velocity is called Angular speed.



Angular acceleration (α) :-

$$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}}$$

→ when $\vec{\alpha}$ is in same direction as $\vec{\omega}$, then ω increases.

→ when $\vec{\alpha}$ is opposite to $\vec{\omega}$, ω decreases.

→ For Uniform Circular Motion, $V \to$ constant
$\omega \to$ constant
$\alpha = 0$

Equations for circular with uniform angular acceleration (α) :-

1. $\omega = \omega_0 + \alpha t$
2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega^2 = \omega_0^2 + 2\alpha\theta$

$\omega_0 \to$ initial Angular velocity
$\omega \to$ final Angular Velocity

α = constant
Non-uniform circular motion

Acceleration

Tangential Acceleration (a_T)

→ If the speed of particle changes in a curve, The rate at which speed changes is called tangential acceleration.

$$\boxed{a_T = \frac{dv}{dt} = \frac{d}{dt}|\vec{v}|}$$

Centripetal Acceleration (a_c)

Center seeking

→ It the direction of velocity of a body.

→ It always acts along the center of the circle / ⊥ to the direction of instantaneous velocity.

→ It is also ~~called~~ called as radial / Normal Acceleration.

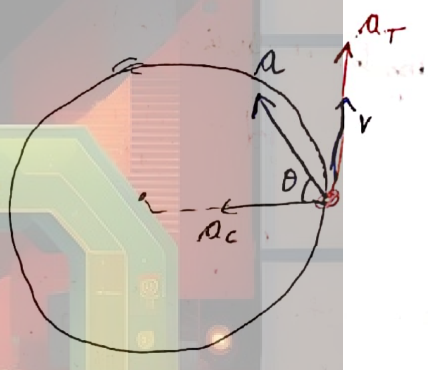
$$\boxed{a_c = \frac{v^2}{r}}$$

→ Net Acceleration (a) → $\vec{a} = \vec{a_c} + \vec{a_T}$

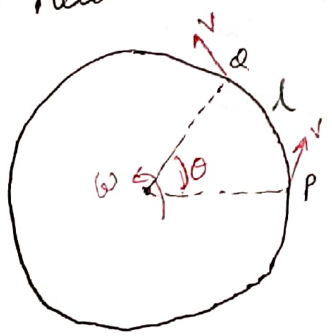
$$\boxed{\vec{a} = \sqrt{a_c^2 + a_T^2}}$$

~~$\tan\theta = \frac{dv}{dt}$~~

$$\boxed{\tan\theta = \frac{a_T}{a_c}}$$



Relation between Linear & Angular Velocities.



$\theta = \frac{l}{r} \Rightarrow l = r\theta$

$v = \frac{l}{t}$

$\frac{dl}{dt} = r\frac{d\theta}{dt}$

$$\boxed{v = r\omega}$$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

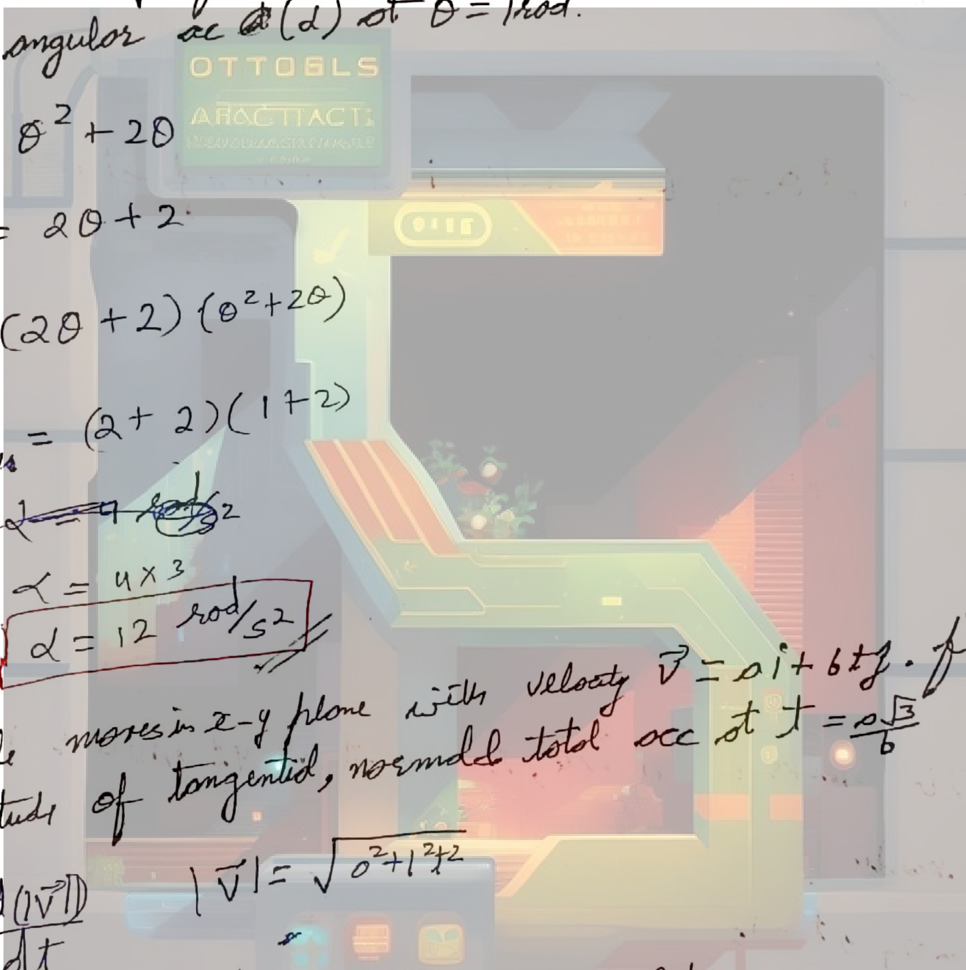
$$\frac{dv}{dt} = \cancel{\ } r\frac{d\omega}{dt}$$

$$\boxed{a_T = r\alpha}$$

$$A_c = \frac{v^2}{r} = \frac{\omega^2 r^2}{r}$$

$$\boxed{A_c = \omega^2 r}$$

Q2 Angular Velocity of a disc depends on angle rotated θ as $\omega = \theta^2 + 2\theta$ find angular acc (α) at $\theta = 1$ rad.



$$\omega = \theta^2 + 2\theta$$

$$\frac{d\omega}{d\theta} = 2\theta + 2$$

$$\alpha = (2\theta + 2)(\theta^2 + 2\theta)$$

$$\alpha\Big|_{\theta = 1} = (2+2)(1+2)$$

$$\cancel{\alpha = 4\ rad/s^2}$$

$$\alpha = 4 \times 3$$

$$\boxed{\alpha = 12\ rad/s^2}$$

Q3. particle moves in x-y plane with velocity $\vec{v} = a\hat{i} + bt\hat{j}$. find magnitude of tangential, normal & total acc at $t = \frac{a\sqrt{3}}{b}$

$$a_T = \frac{d(|\vec{v}|)}{dt} \qquad |\vec{v}| = \sqrt{a^2 + b^2t^2}$$

$$a_T = \frac{d}{dt}(a^2 + b^2t^2)^{1/2} = \frac{1}{2(a^2+b^2t^2)^{1/2}} \times 2b^2t$$

$$a_T = \frac{b^2t}{\sqrt{a^2+b^2t^2}}$$

$$a_T\Big|_{t = \frac{a\sqrt{3}}{b}} = \frac{b^2 a\sqrt{3}}{b\sqrt{a^2 + \frac{b^2a^2(3)}{b^2}}} = \frac{ab\sqrt{3}}{2a} = \boxed{\frac{b\sqrt{3}}{2}}$$

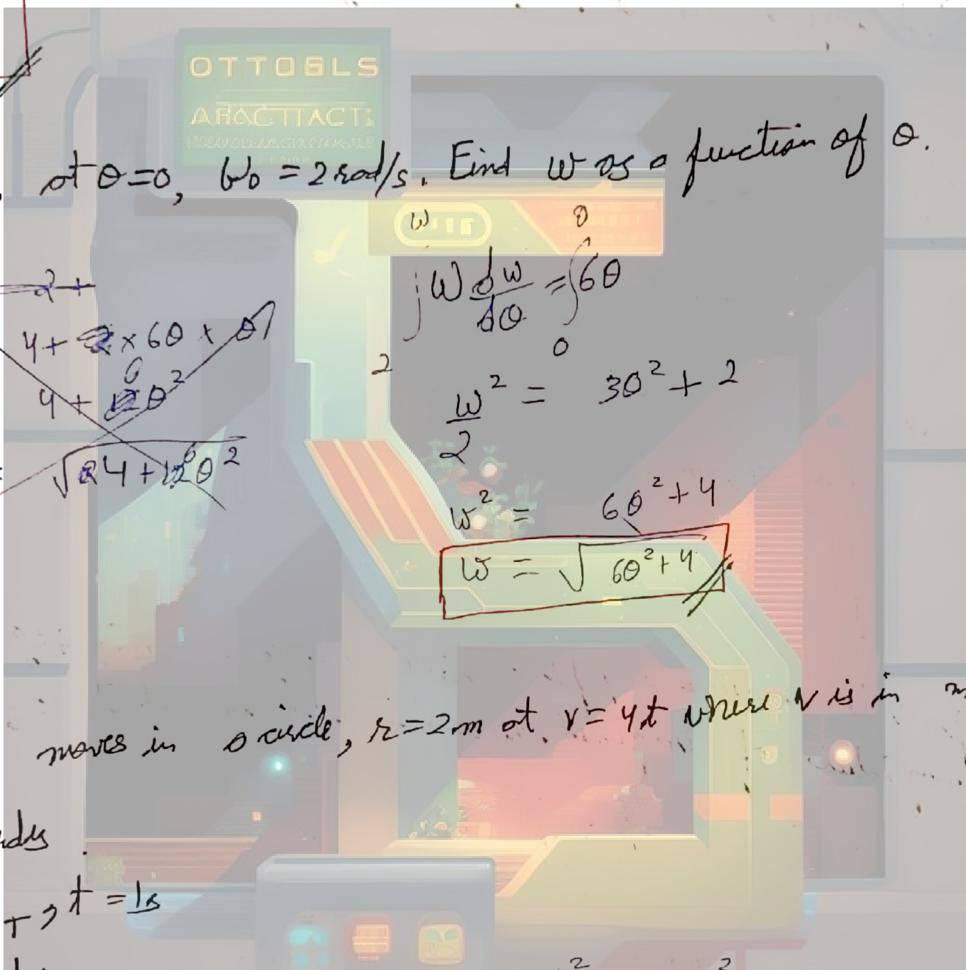
$a_{net} = \frac{d\vec{v}}{dt} = \boxed{6\hat{j}} \Rightarrow |a_{net}| = 6$

$(a_{net})^2 = (a_T)^2 + (a_c)^2$

$(a_c)^2 = 6^2 - \frac{3 \cdot 6^2}{4}$

$(a_c)^2 = \frac{6^2}{4}$

$\boxed{a_c = \frac{6}{2}}$



Q4. $\alpha = 6\theta$, at $\theta = 0$, $\omega_0 = 2$ rad/s. Find ω as a function of θ.

$\int_2^{\omega} \omega \frac{d\omega}{d\theta} = \int_0^{\theta} 6\theta$

$\frac{\omega^2}{2} = 3\theta^2 + 2$

$\omega^2 = 6\theta^2 + 4$

$\boxed{\omega = \sqrt{6\theta^2 + 4}}$

Q5. A particle moves in a circle, $r = 2m$ at $v = 4t$ where v is in m/s & t is in seconds

a) a_T, $t = 1s$

b) a, $t = 1s$

a) $a_T = \frac{d|\vec{v}|}{dt} = \frac{d(4t)}{dt}$

$\boxed{a_T = 4 \text{ m/s}^2}$ a)

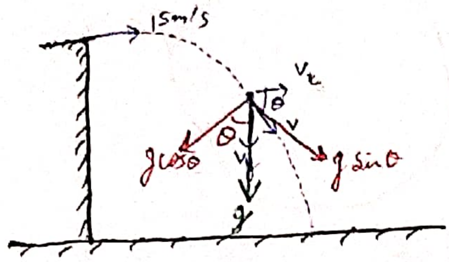
b) $a_c = \frac{16t^2}{2} = 8t^2$

$a_c \big|_{t=1s} = 8 \text{ m/s}^2$

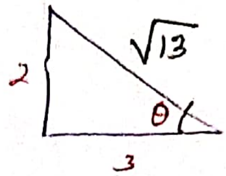
$a_{net} = \sqrt{64 + 16} = \sqrt{80}$

$\boxed{= 4\sqrt{5} \text{ m/s}^2}$ b)

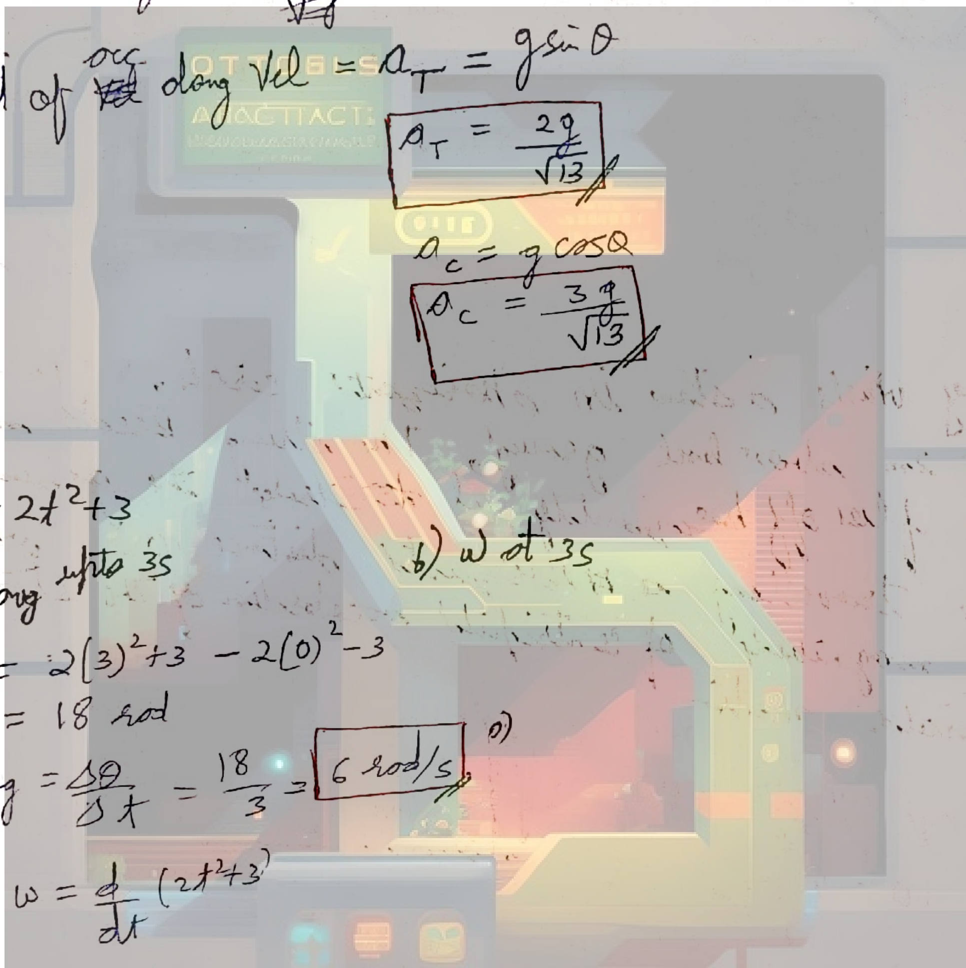
Q6. A stone thrown horizontally from a height with velocity 15 m/s. Determine normal & tangential acc at $t = 1s$.



$\tan\theta = \frac{V_y}{V_x} = \frac{10}{15} = \frac{2}{3}$



~~Radius = Range = $u\sqrt{\frac{2h}{g}}$~~



Component of ~~vel~~ acc along Vel $= a_T = g\sin\theta$

$\boxed{a_T = \frac{2g}{\sqrt{13}}}$

$a_c = g\cos\theta$

$\boxed{a_c = \frac{3g}{\sqrt{13}}}$

Q7. $\theta = 2t^2 + 3$

a) ω_{avg} upto 3s

b) ω at 3s

a) $\Delta\theta = 2(3)^2 + 3 - 2(0)^2 - 3$

$= 18$ rad

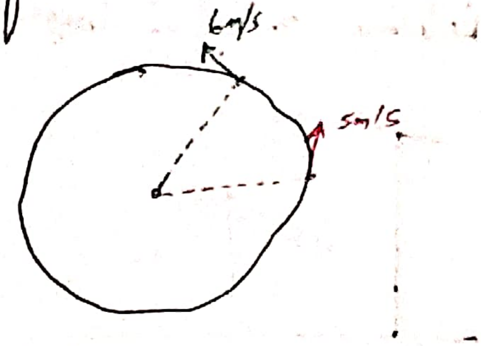
$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{18}{3} = \boxed{6 \text{ rad/s}}$ a)

b) $\omega = \frac{d}{dt}(2t^2+3)$

$= 4t$

$\omega|_{t=3} = \boxed{12 \text{ rad/s}}$ b)

Q 8. $r = 20\,cm$ $v = 5\,m/s$, $v = 6\,m/s$ in $2s$ find α.



M I

$$\omega = \frac{v}{r}$$

$$\omega_0 = \frac{5}{0.2} = \frac{50}{2} = 25$$

$$\omega = \frac{6}{0.2} = \frac{60}{2} = 30$$

$$\omega = \omega_0 + \alpha t$$

$$\frac{30 - 25}{2} = \alpha$$

$$\boxed{\alpha = 5/2\ rad/s^2}$$

M II

$$a_T = \frac{6-5}{2} = \frac{1}{2}\,m/s^2$$

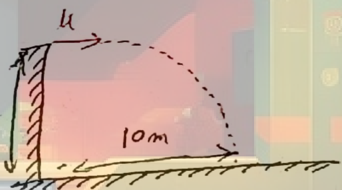
$$a_T = \alpha r$$

$$\alpha = \frac{a_T}{r}$$

$$\alpha = \frac{1}{2} \times \frac{1}{0.2}$$

$$\alpha = \frac{10}{4} = \frac{5}{2} = \boxed{2.5\ rad/s^2}$$

Q9. A boy whirls a stone in a horizontal circle, $r = 1.5\,m$ & height 2m above level ground. The string Breaks and the stone flies off horizontally & ~~to~~ strikes the ground after travelling a ~~h~~ horizontal distance 10m. what is the magnitude of centripetal acceleration of the stone while in circular motion.



$$\text{Range} = u\sqrt{\frac{2h}{g}} = R$$

$$10 = u\sqrt{\frac{2 \times 2}{10}}$$

$$10 = \frac{2u}{\sqrt{10}}$$

$$\boxed{5\sqrt{10} = u}$$

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{250}{1.5}$$

$$\boxed{a_c = \frac{500}{3}\ m/s^2}$$

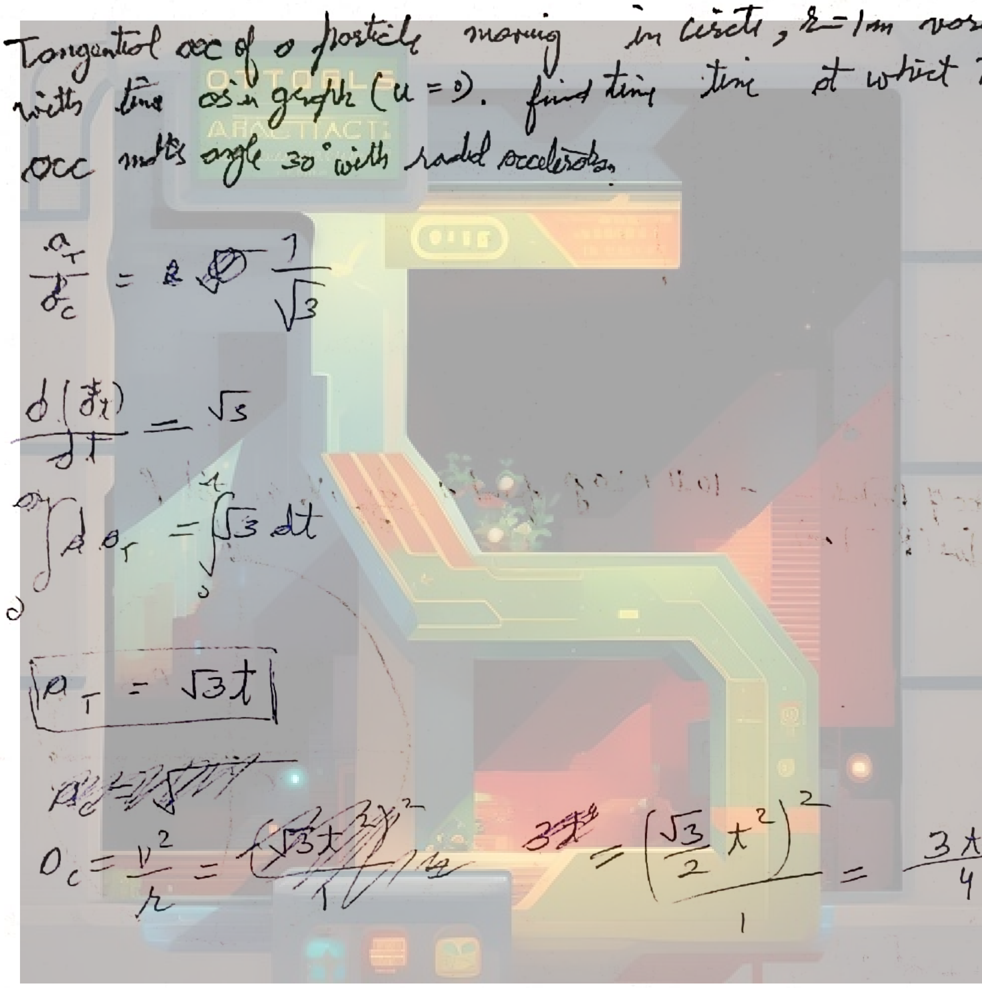
Q10. find magnitude of acceleration of particle moving in $r = 10\,cm$, uniform speed completing circle in 4 s.

$$\omega = \frac{2\pi}{4} = \frac{\pi}{2}\ rad/s$$

$$a_c = \omega^2 r$$

$$= \frac{\pi^2}{4} \times 0.1$$

$$\boxed{= \frac{\pi^2}{40}}$$



Q11. Tangential acc of a particle moving in circle, $r = 1\,m$ varies with time as in graph ($u = 0$). find time at which Total acc makes angle 30° with radial acceleration.

$$\frac{a_T}{a_c} = \cancel{\tan\theta}\ \frac{1}{\sqrt{3}}$$

$$\frac{d(a_T)}{dt} = \sqrt{3}$$

$$\int_0 d a_T = \int_0^t \sqrt{3}\, dt$$

$$\boxed{a_T = \sqrt{3}t}$$

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{\sqrt{3}}{2}t^2\right)^2}{1} = \frac{3t^4}{4}$$

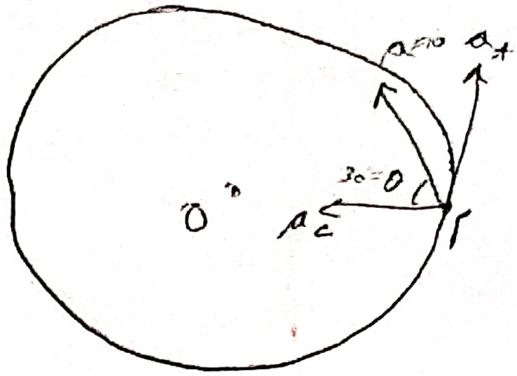
$$\frac{a_T}{a_c} = \frac{\sqrt{3}t}{3t^4} \times 4 = \frac{1}{\sqrt{3}}$$

$$\frac{4}{\sqrt{3}t^3} = \frac{1}{\sqrt{3}}$$

$$t^3 = 4$$

$$\boxed{t = 4^{1/3}}$$

Q12. Net acc^n, $a = 10\,m/s^2$ makes angle $30°$ with normal at a position circular path of radius 2 m. Find a_T, a_c, V & ω.



$\tan 30 = \frac{1}{\sqrt{3}} = \frac{a_T}{a_c}$

$a_c = \sqrt{3}\,a_t$

$\cancel{\frac{V^2}{2} = \sqrt{3} \times \frac{dV}{dt}}$

$a = \sqrt{a_c^2 + a_t^2}$

$a = \sqrt{a_t^2 + 3a_t^2}$

$a = 2a_T$

$10 = 2a_T$

$\boxed{a_T = 5\,m/s^2}$

$a_c = \sqrt{3}\,5$

$\boxed{a_c = 5\sqrt{3}\,m/s^2}$

$5\sqrt{3} = \frac{V^2}{2}$

$\boxed{V = \sqrt{10\sqrt{3}}\,m/s}$

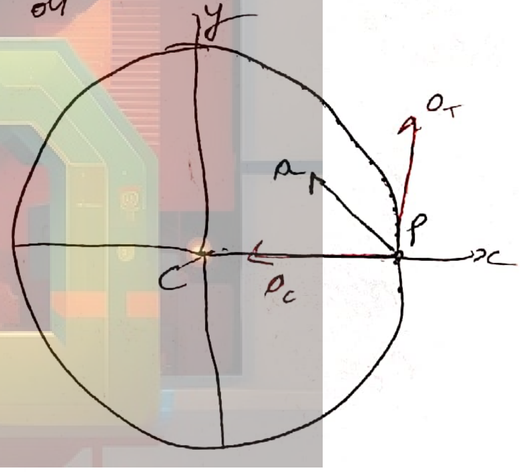
$\cancel{a_T = \omega r}$

$V = \omega r$

$\sqrt{10\sqrt{3}} = 2\omega$

$\boxed{\omega = \sqrt{\frac{5\sqrt{3}}{2}}}$ rad/s

Q13. At P, acc of Particle $= -10\hat{i} + 20\hat{j}$ find α, $\frac{dv}{dt}$, V, ω at P. Radius ($R = 1\,m$)



$a_T = 20\,m/s^2$

$a_c = 10\,m/s^2$

$a_T = \alpha r$

$20 = \alpha$

$\boxed{\alpha = 20\,rad/s^2}$

$\cancel{\frac{dv}{dt} = a_T = \sqrt{100 + 400} = \sqrt{500}}$

$\cancel{\frac{dv}{dt} = 10\sqrt{5}\,m/s^2}$

$\boxed{\frac{dv}{dt} = a_T = 20\,m/s^2}$

$a_c = \frac{V^2}{r}$

$10 = V^2$

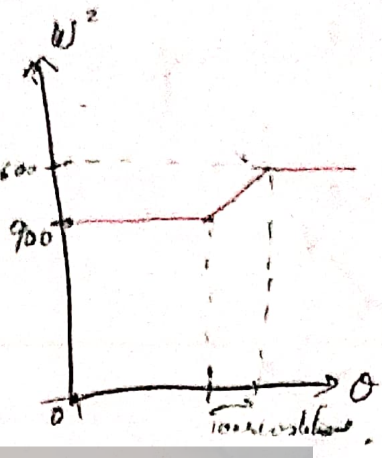
$\boxed{V = \sqrt{10}\,m/s}$

$v = \omega r$

$\sqrt{10} = \omega$

$\boxed{\omega = \sqrt{10}\,rad/s}$

Q14. The square of angular velocity ω of a certain wheel increases linearly with Δθ during 100 revolutions shown. Compute required time t for 100 revolutions.



$\Delta\theta = 100 \times 2\pi$

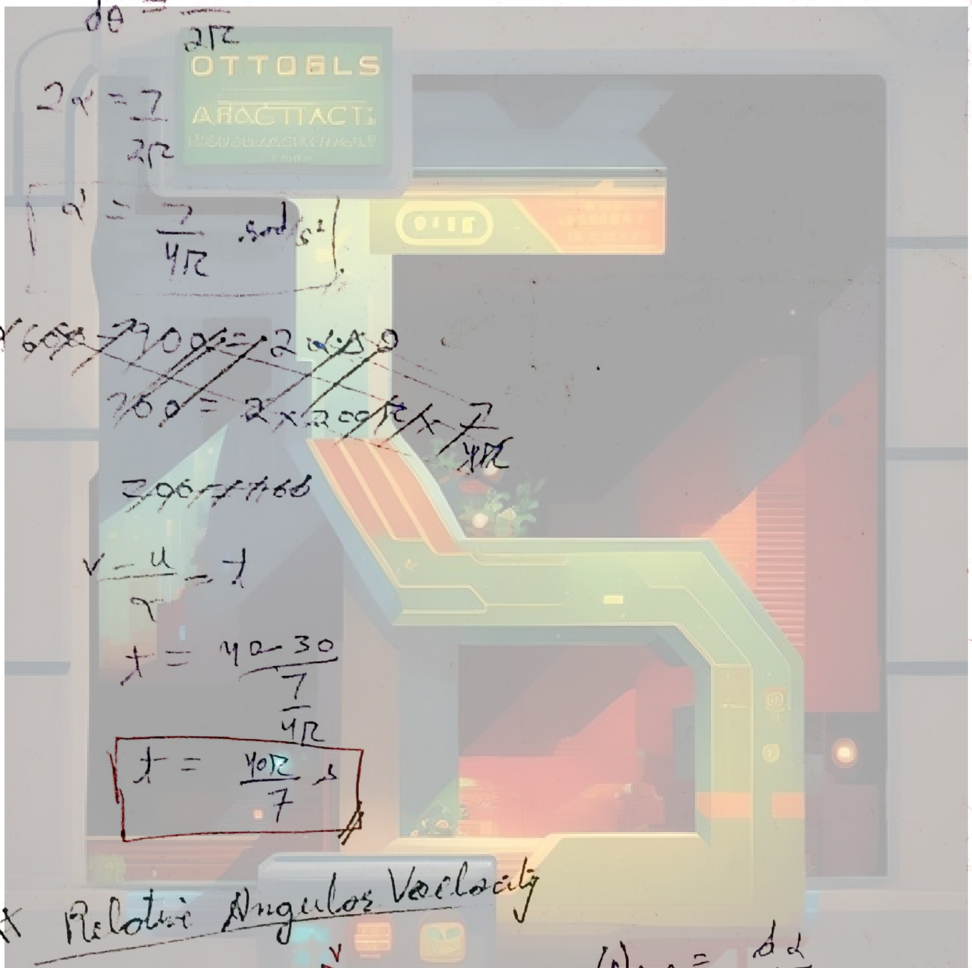
$\boxed{\Delta\theta = 200\pi \text{ rad}}$

$$\frac{d(\omega^2)}{d\theta} = \frac{700}{200\pi} = \frac{7}{2\pi}$$

$$2\omega\frac{d\omega}{d\theta} = \frac{7}{2\pi}$$

$$2\alpha = \frac{7}{2\pi}$$

$$\boxed{\alpha = \frac{7}{4\pi} \text{ rad/s}^2}$$



$\cancel{1600 - 900 = 2\alpha\Delta\theta}$

$\cancel{700 = 2 \times 200\pi \times \frac{7}{4\pi}}$

$\cancel{= 700 \neq 700}$

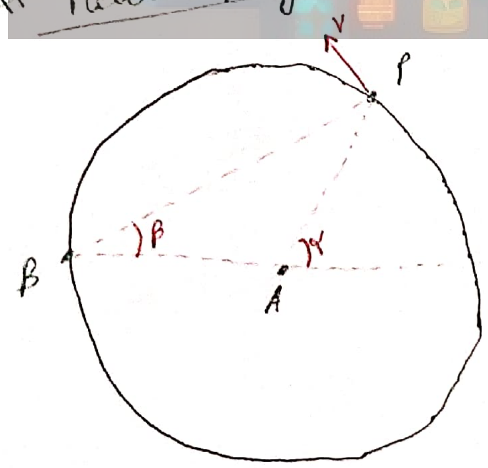
$$\frac{v - u}{\alpha} = t$$

$$t = \frac{40 - 30}{\frac{7}{4\pi}}$$

$$\boxed{t = \frac{40\pi}{7} \text{ s}}$$

Q15. ⇒ Relative Angular Velocity

①

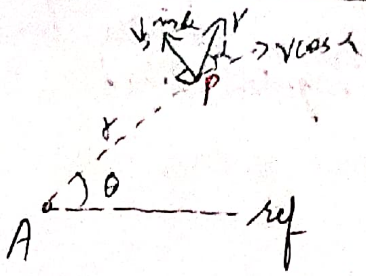


$$\omega_{P,A} = \frac{d\alpha}{dt}$$

$$\omega_{P,B} = \frac{d\beta}{dt}$$

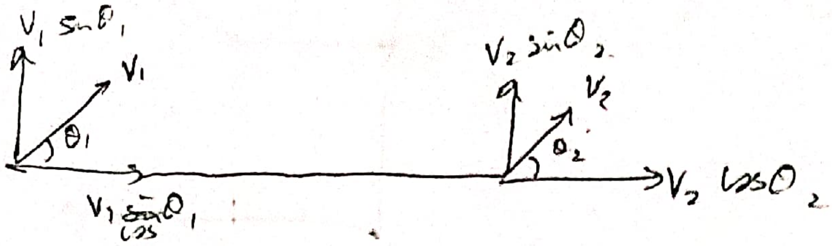
→ Angular velocity of a particle can be different about different points

(2)



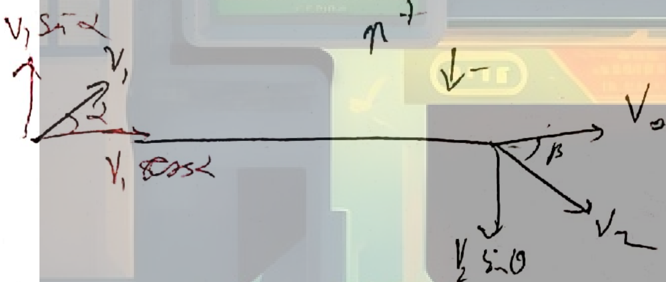
$\omega_{P,A} = \frac{V\sin\alpha}{r}$

(3)

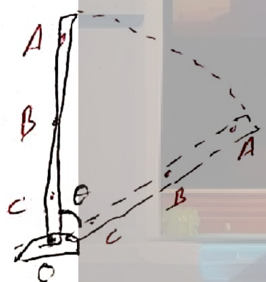


$$\boxed{\omega_{A,B} = \frac{V_1\sin\theta_1 - V_2\sin\theta_2}{r}} = \frac{(V_{rel})_{B,A}\perp}{r}$$

(4)



$$\boxed{\omega_{A,B} = \frac{V_1\sin\alpha + V_2\sin\beta}{r}}$$

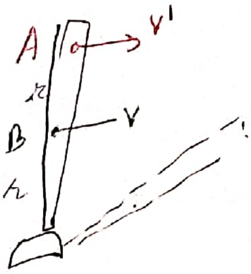


$V_A > V_B > V_C$

$r_A > r_B > r_C$

$\omega_A = \omega_B = \omega_C$

$\theta_A = \theta_B = \theta_C$



$\omega = \frac{V}{r}$

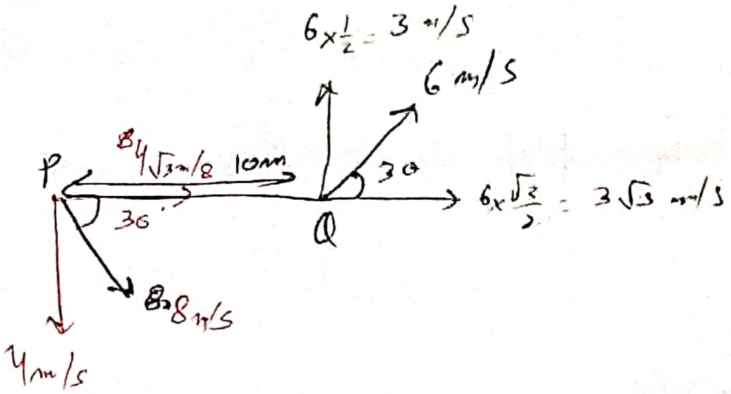
$\omega_A = \omega_B$

$\frac{V'}{2r} = \frac{V}{r}$

$\boxed{V' = 2V}$

(12)

Q15. find Angulos velocity of Q wrt P.

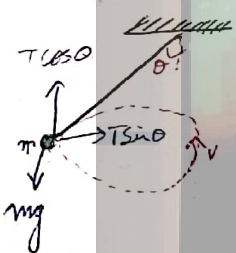


$$\omega_{Q,P} = \frac{3+4}{10} = \frac{7}{10} = \boxed{0.7 \text{ rad/s}}$$

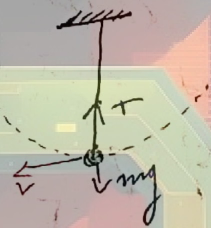
Centripital & Centrifugal Force :-

Centripital → Force responsible for circular motion
→ It does no work ($\perp$) & only changes the direction of motion.

$$\boxed{F_c = \frac{mv^2}{r}}$$

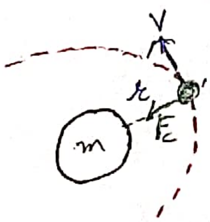


$T\cos\theta = mg$
$T\sin\theta = \frac{mv^2}{r}$

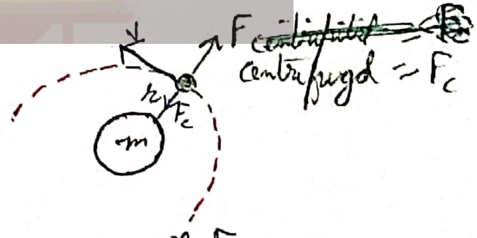


$T - mg = \frac{mv^2}{r}$

Centrifugal force → A pseudo forcing acting ~~away~~ exact opposite & equal to centripital force.

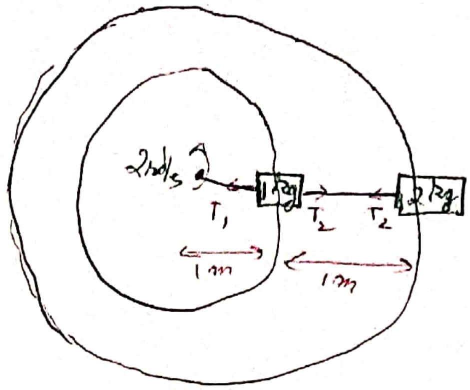


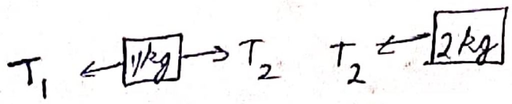
Ground Frame



Non-Inertial Frame.

Q16. find T_1 & T_2





$T_1 \overset{T_2}{=} \frac{mv^2}{r}$

$T_1 - T_2 = \omega^2 r$

$T_1 - T_2 = 4$

$T_1 = 4 + T_2$

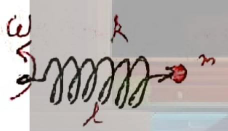
$\boxed{T_1 = 20N}$

$T_2 = m\omega^2 r$

$= 2 \times 2 \times 2 \times 2$

$\boxed{T_2 = 16N}$

Q17. A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The system is rotated about center with an angular velocity ω. Find elongation in spring.



$F_c = m\omega^2(l+x)$

$kx = F_c$

$\cancel{kx = m\omega^2 l}$

$\cancel{x = \frac{m\omega^2 l}{k}}$

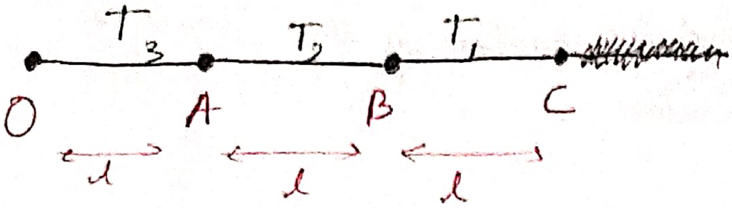
$kx = m\omega^2(l+x)$

$kx = m\omega^2 l + m\omega^2 x$

$x(k - m\omega^2) = m\omega^2 l$

$\boxed{x = \frac{m\omega^2 l}{k - m\omega^2}}$

Q18. all are moving about O. $V_c = V_0$, $T_1 : T_2 : T_3 = ?$ A, B, C are identical



$T_1 = m\omega^2 3l$

$T_2 = m\omega^2 2l + m\omega^2 3l = m\omega^2 5l$

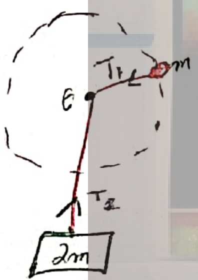
$T_3 = m\omega^2 l + m\omega^2 5l = m\omega^2 6l$

$T_1 : T_2 : T_3$

$\cancel{m\omega^2} 3\cancel{l} : \cancel{m\omega^2} 5l : \cancel{m\omega^2} 6l$

$\boxed{3 : 5 : 6}$

Q19. m rotates freely in circle, r = 1m. & supports mass 2m hanging attached to other end of string falling down.



$T_2 = 2mg$ (2m hanging in equilibrium)

$T = m\omega^2(1)$ (Centripetal force)

$2\cancel{m}g = \cancel{m}\omega^2$

$\omega^2 = 20$

$\boxed{\omega = \cancel{4\sqrt{5}}\ 2\sqrt{5}\ \text{rad/s}}$

Radius Of Curvature.

→ If a particle is moving on a curve, then at any point, If we take a segment of ~~curve~~ curve, then this segment will be part of some circle and the radius of this circle is called as radius of curvature.

→ It is same for all the points in a circular path.

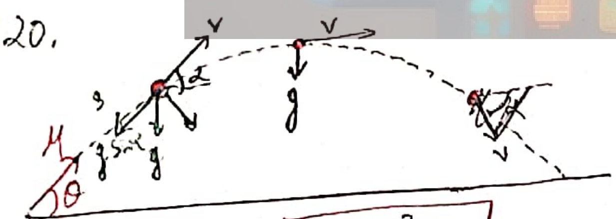
$$R \propto \frac{1}{\text{Curvature}}$$

Case I If velocity & acc given

$$R_c = \frac{(\text{Speed})^2}{(\text{acc} \perp \text{to velocity})}$$

$$R = \frac{v^2}{a_c}$$

Q.20.



Find R at
a) highest point
b) when velocity makes angle α with horizontal.

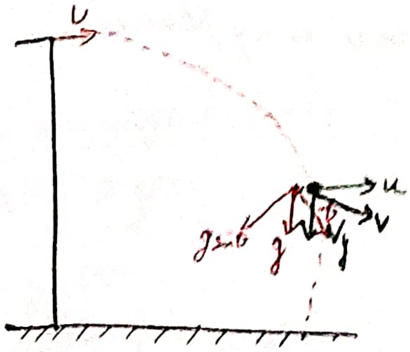
a) $R = \frac{v^2}{g} = \boxed{\frac{(u\cos\theta)^2}{g}}$ a)

b) $R = \frac{v^2}{g\cos\alpha} = \boxed{\frac{u^2\cos^2\theta}{g\cos^3\alpha}}$ b)

$u\cos\theta = v\cos\alpha$

$v = \frac{u\cos\theta}{u\cos\alpha}$

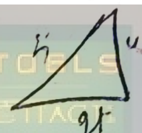
Q21. find R_c at time T



$v_y = gt$

$v = \sqrt{g^2t^2+u^2}$

$\tan\theta = \frac{u}{gt} = \frac{P}{B}$



$h = \sqrt{u^2+g^2t^2}$

$\sin\theta = \frac{u}{\sqrt{u^2+g^2t^2}}$

$\cancel{R = \frac{v^2}{g\sin\theta} = \frac{g^2t^2+u^2}{g\sqrt{u^2+g^2t^2}} = \frac{\sqrt{u^2+g^2t^2}}{g}}$

$R = \frac{v^2}{g\sin\theta} = \frac{\left(\sqrt{g^2t^2+u^2}\right)^2}{g\cdot\frac{u}{(g^2t^2+u^2)^{1/2}}}$

$$\boxed{R = \frac{(g^2t^2+u^2)^{3/2}}{ug}}$$

Case 2:- If equation of trajectory is given.

$y = f(x)$

$$\boxed{R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}}$$

122. The equation of path followed by a particle in xy plane is

$$x^2 = 4y$$

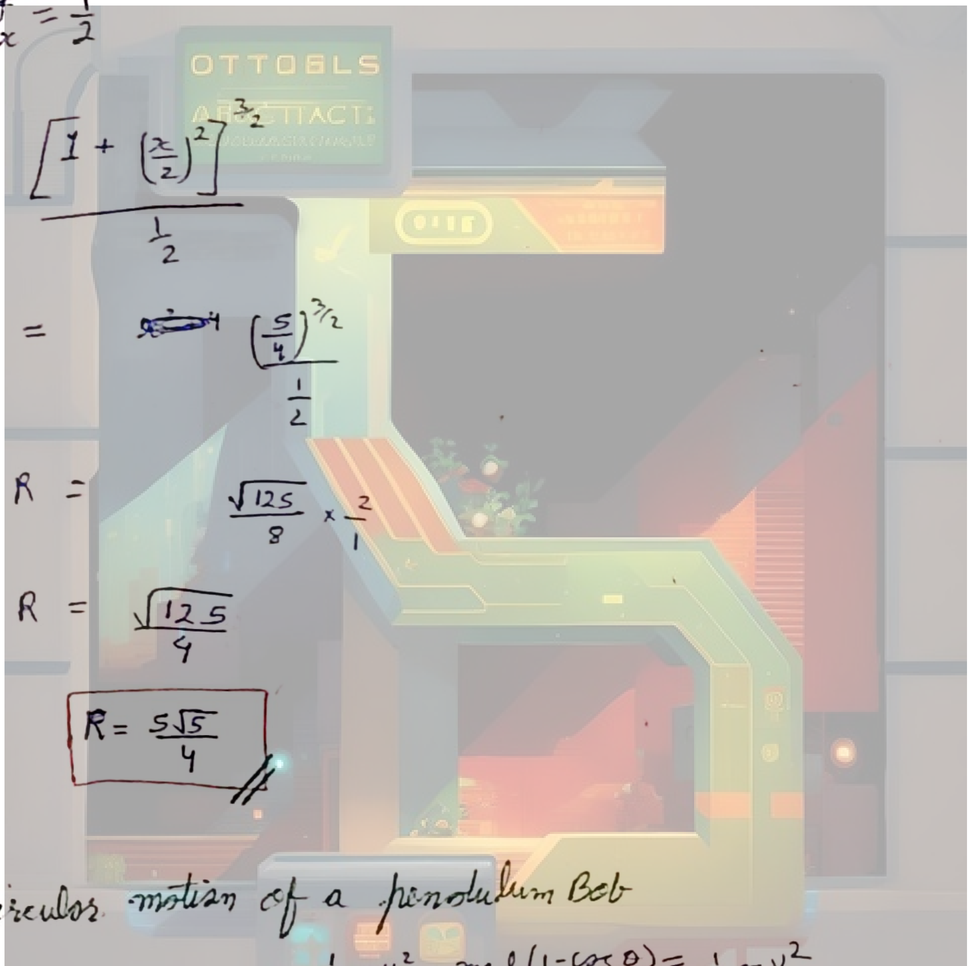
find R at (1,1)

$$x^2 = 4y$$

$$y = \frac{x^2}{4}$$

$$\frac{dy}{dx} = \frac{2}{4}x = \frac{x}{2} = \frac{1}{2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}$$



$$R = \frac{\left[1 + \left(\frac{x}{2}\right)^2\right]^{3/2}}{\frac{1}{2}}$$

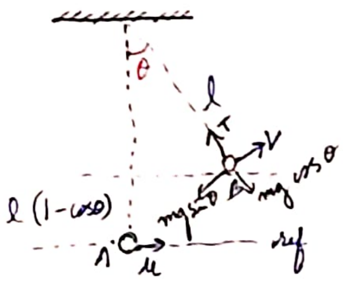
$$R = \frac{\left(\frac{5}{4}\right)^{3/2}}{\frac{1}{2}}$$

$$R = \frac{\sqrt{125}}{8} \times \frac{2}{1}$$

$$R = \frac{\sqrt{125}}{4}$$

$$\boxed{R = \frac{5\sqrt{5}}{4}}$$

Vertical circular motion of a pendulum Bob



$$\frac{1}{2}mu^2 - mgl(1-\cos\theta) = \frac{1}{2}mv^2$$

$$\boxed{v = \sqrt{u^2 - 2gl(1-\cos\theta)}}$$

for $v=0, \quad \theta=\alpha$.

$$u^2 = 2gl(1-\cos\alpha)$$

$$u^2 = 2gl - 2gl\cos\alpha$$

$$2gl\cos\alpha = 2gl - u^2$$

$$\boxed{\cos\alpha = \frac{2gl-u^2}{2gl}}$$

Tension.

To calculate Tension at B,

$$T - mg\cos\theta = \frac{mv^2}{l}$$

$$T = mg\cos\theta + \frac{m}{l}(u^2 - 2gl + 2gl\cos\theta)$$

$$T = mg\cos\theta + \frac{mu^2}{l} - 2mg + 2mg\cos\theta$$

$$\boxed{T = 3mg\cos\theta + \frac{mu^2}{l} - 2mg}$$

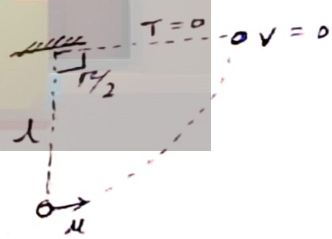
For $T=0, \quad \theta=\phi$.

$$3gl\cos\phi + u^2 = 2gl$$

$$\boxed{\cos\phi = \frac{2gl-u^2}{3gl}}$$

I $\quad u = \sqrt{2gl}$

for $v=0$, $\quad \cos\alpha = \frac{2gl-u^2}{2gl} = \frac{2gl-2gl}{2gl} = 0$



$$\alpha = \frac{\pi}{2}$$

for $T=0$, $\cos\phi = \frac{2gl-2gl}{3gl} = 0$

$$\phi = \frac{\pi}{2}$$

If $u < \sqrt{2gl}$
$\cos\phi < \cos\theta$
$\phi > \theta$

Bob will oscillate in lower semicircle plane & its Tension in string will never be zero.

II $u = \sqrt{4gl}$

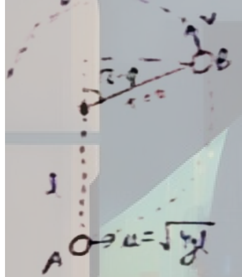
$v = 0 \Rightarrow \cos\alpha = \frac{2gl - 4gl}{2gl} = -1$

$\alpha = \pi$

$T = 0 \Rightarrow \cos\phi = \frac{2gl - 4gl}{3gl} = -\frac{2}{3}$

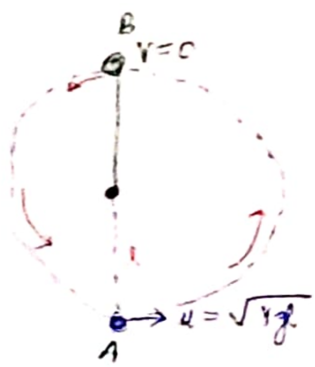
$\frac{\pi}{2} < \phi < \pi$

→ It will follow projectile after Tension is 0 & cannot go till $\alpha = \pi$.



→ Instead of string, If there was a light rod, $T = 0$ is not possible so it can reach $\alpha = \pi$ & so will complete whole circle.
↳ (topmost point)

→ Initial velocity of the bob is sufficient to carry the bob to highest point but tension in thread becomes zero at angle ϕ, & afterward it will no longer be in circular motion. As thread becomes slack, particle is free to move & it follows projectile motion.



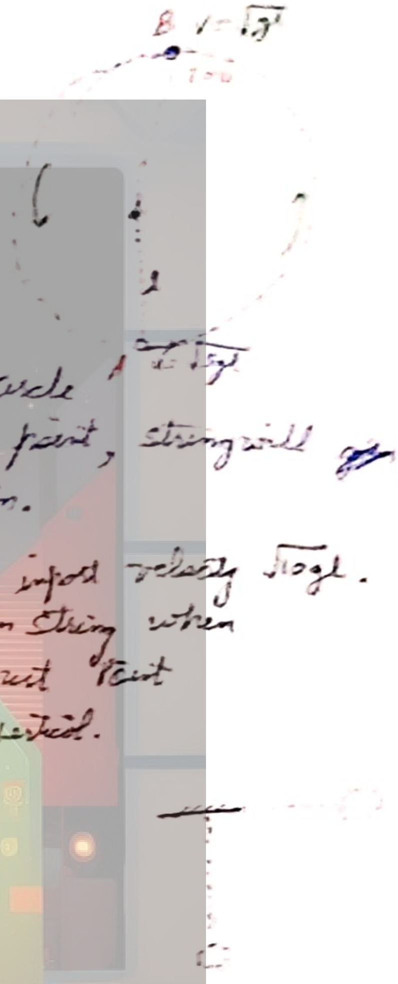
III $u = \sqrt{5gl}$

for $v = 0 \Rightarrow \cos\theta = \frac{2gl - 5gl}{2gl} = -\frac{3}{2}$

So, v is never 0. as $\cos\theta$ is never $-\frac{3}{2}$

for $T = 0 \Rightarrow \cos\phi = \frac{2gl - 5gl}{3gl} = -1$

$\phi = \pi$



V_B,

$\frac{1}{2}m(5gl) - mg(2l) = \frac{1}{2}mv^2$

$\boxed{V_B = \sqrt{gl}}$

$\Rightarrow$ Tension becomes zero at the topmost point of the circle. But as the particle moves ahead of topmost point, string will ~~get~~ gain Tension again & complete the circular motion.

Q23 Bob of simple pendulum given a sharp hit to impart velocity $\sqrt{10gl}$. l is length of the pendulum. Find Tension in string when
i) string is horizontal ii) Bob is at its highest point
iii) string makes angle 60° with upward vertical.

i) $K_i \pm W = K_f$

$\frac{1}{2}m(10gl) - mgl = \frac{1}{2}mv^2$

$v^2 = 8gl$

$v = \sqrt{8gl}$

ii) $\frac{1}{2}m(10gl) - mg(2l) = \frac{1}{2}mv^2$

$5gl - 2gl = \frac{1}{2}v^2$

$v^2 = 6gl$

i) $K_i \pm W = K_f$

$\frac{1}{2}(10gl)m - mgl = \frac{1}{2} m v^2$

$4gl \times 2 = v^2$

$v = \sqrt{8gl}$

$T = \frac{v^2}{l} \times m$

$T = \frac{8gl}{l} \times m$

$\cancel{T = 8g}$ $\boxed{T = 8mg}$ i)

ii) $(5gl - 2gl) \times 2 = v^2$

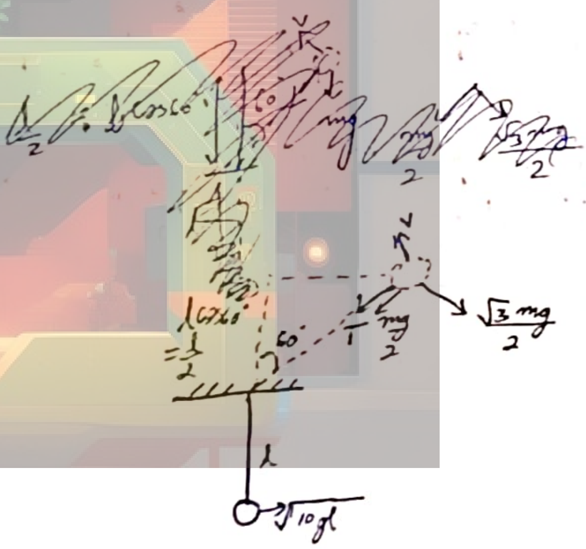
$v = \sqrt{6gl}$

$T + mg = \frac{6gl}{l} \times m$

$\cancel{T = 6g - mg}$

$T = 6mg - mg$

$\boxed{T = 5mg}$ ii)



iii) $\frac{10mgl}{2} - \frac{3mgl}{2} = \frac{1}{2} m v^2$

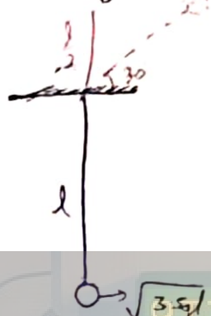
$7gl = v^2$

$T + \frac{mg}{2} = m \times \frac{7gl}{l}$

$T = 7mg - \frac{1}{2}mg$

$\boxed{T = \frac{13mg}{2}}$ iii)

Q24. A block is tied to one end of light string l whose one end is fixed to a rigid support. $u = \sqrt{3.5gl}$ at lowermost point. Find height & speed at which block ~~...~~ leaves circle & max height it finally rises.



for $T = 0$

$$\cos\phi = \frac{2gl - 3.5gl}{3gl} = -\frac{1}{2}$$

$$\phi = \frac{2\pi}{3} = 120^\circ$$

$$\underbrace{\frac{3.5gl}{2}}_{K_i} - \underbrace{\frac{3gl}{2}}_{W} = \underbrace{\frac{1}{2}v^2}_{K_f}$$

$$\sqrt{\frac{gl}{2}} = v$$

$$\boxed{\text{height} = \frac{3l}{2}}$$

$$\boxed{v = \sqrt{\frac{gl}{2}}}$$

$$\text{max height (of projectile)} = \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{\frac{gl}{2} \times \frac{3}{4}}{2g}$$

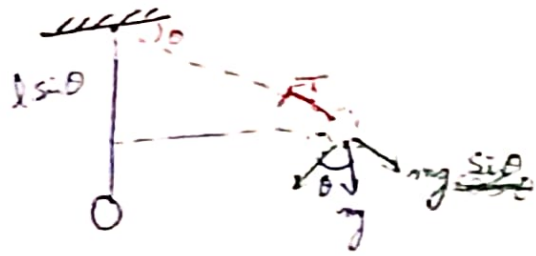
$$= \frac{3l}{16}$$

$$\text{max height} = \frac{3l}{16} + \frac{3l}{2}$$

$$= \frac{3l + 24l}{16}$$

$$\boxed{= \frac{27l}{16}}$$

Q25. In pendulum breaking strength is 2× weight of bob. The bob is released from rest when string is horizontal. String breaks when makes angle θ with horizontal. find θ.



$T = 2mg$

$mgl\sin\theta = \frac{1}{2}mv^2$

$v^2 = 2gl\sin\theta$

$T - mg\sin\theta = \frac{2mgl\sin\theta}{l}$

$T = 2mg\sin\theta + mg\sin\theta$

$2 = 2\sin\theta + \sin\theta$

$\frac{2}{3} = \sin\theta$

$\boxed{\theta = \sin^{-1}\left(\frac{2}{3}\right)}$

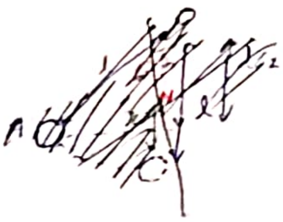
H.W. 22, 23 - 8 - 24

O-1 (Q1-24) Circular Motion

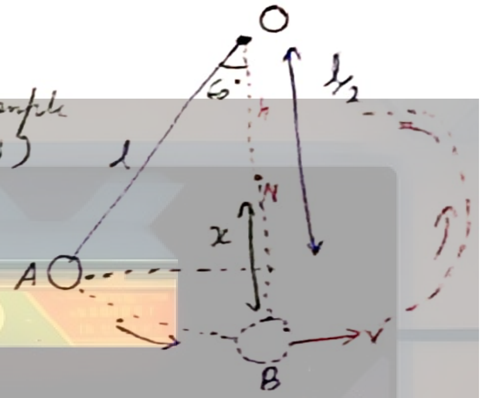
Pg 43. 70, 71, 72, 75

S-1 (Q1-10) circular

Q26. when bob is vertically below O, string makes contact with the nail N placed directly below O at distance and rotates around it. If the particle just complete the vertical circle about N, find h in terms of l.



$$m \times g \times \frac{l}{2} = \frac{1}{2} m v^2$$



$$v = \sqrt{gl}$$

$$v = \sqrt{5gx}$$ (to almost complete circle about N)

$$gl = 5gx$$

$$x = \frac{l}{5}$$

$$h = l - x$$

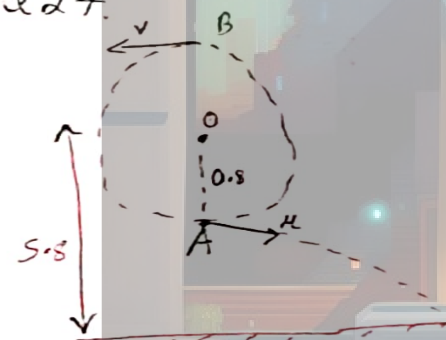
$$h = l - \frac{l}{5}$$

$$\boxed{h = \frac{4l}{5}}$$

Q27. A small sphere tied to the string of length 0.8 m is describing a vertical circle so that maximum & minimum tensions in the string are in ratio 3:1. The fixed end of the string is at a height of 5.8 m above ground. ($g = 10\ m/s^2$)

a) Find the velocity of the sphere at the lowest point.

b) If the string suddenly breaks at the lowest position, when and where will the sphere hit the ground?



$$\underbrace{\frac{1}{2} m u^2}_{K_i} - \underbrace{\frac{2mg(1.6)}{2}}_{W_g} = \underbrace{\frac{1}{2} m v^2}_{K_f}$$

$$v^2 = u^2 - 3.2g$$

$$max\ Tension = \frac{u^2 m}{r} + mg \quad -(1)$$

$$min = \frac{v^2 m}{r} - mg \quad -(2)$$

$$\frac{3v^2 m}{r} - 3mg = \frac{u^2 m}{r} + mg \qquad (1) = 3 \times (2)$$

$$\frac{3u^2 - 9.6g - u^2}{r} = 4mg$$

$$T = \sqrt{\frac{2H}{g}}$$

$$= \sqrt{\frac{2\times5}{10}}$$

$$\boxed{T = 1\,s}\ (b)$$

$$2u^2 - 9.6g = 3.2mg$$

$$2u^2 = 12.8g$$

$$u^2 = 6.4g$$

$$u^2 = 64$$

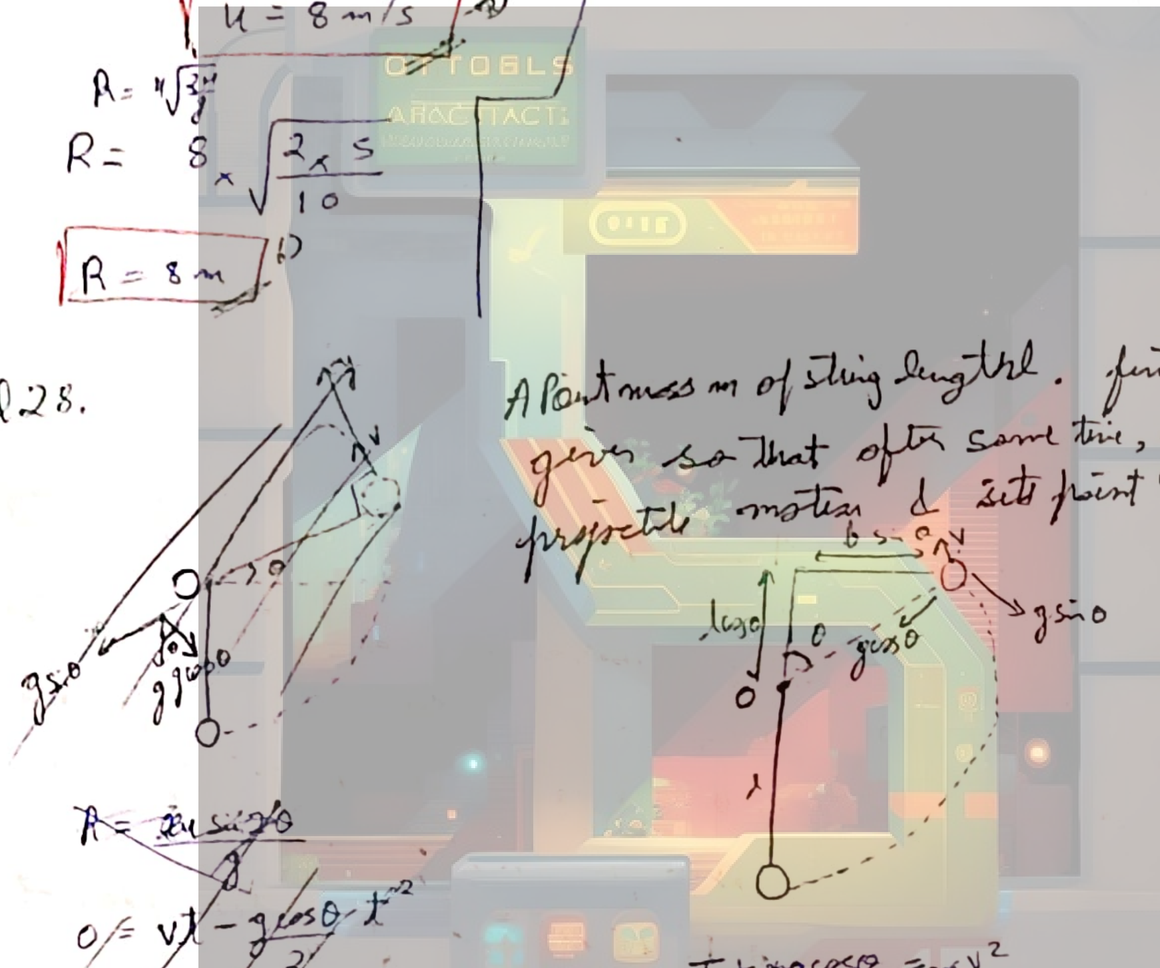
$$\boxed{u = 8\,m/s}\ (a)$$

$$R = u\sqrt{\frac{2H}{g}}$$

$$R = 8\times\sqrt{\frac{2\times5}{10}}$$

$$\boxed{R = 8\,m}\ (b)$$

Q.28. A Point mass m of string length l. find min velocity given so that after some time, It perform projectile motion & hits point 'O'.



$$R = \frac{u^2 \sin 2\theta}{g}$$

$$0 = vt - \frac{g\cos\theta\, t^2}{2}$$

$$t = \frac{2v}{g\cos\theta}$$

$$l = \frac{1}{2}\times g\sin\theta \times \frac{4v^2}{\ }$$

$$T + mg\cos\theta = \frac{mv^2}{l}$$

$T \to 0$

$$v = \sqrt{gl\cos\theta}$$

Let, B be origin

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\cancel{l\sin}$

$$-l\cos\theta = l\sin\theta \times \frac{\sin\theta}{\cos\theta} - g\frac{l^2\sin^2\theta}{2g\cos\theta\cos^2\theta}$$

$$-\cos\theta = \frac{\sin^2\theta}{\cos\theta} - \frac{\sin^2\theta}{2\cos^3\theta}$$

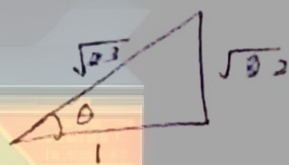
$$-2\cos^4\theta = 2\sin^2\theta\cos^2\theta - \sin^2\theta$$

$$2\cos^4\theta + 2\sin^2\theta\cos^2\theta = \sin^2\theta$$

$$2\cos^2\theta(\sin^2\theta + \cos^2\theta) = \sin^2\theta$$

$$\tan^2\theta = 2$$

$$\tan\theta = \sqrt{2}$$



$$\cos\theta = \frac{1}{\sqrt{3}}$$

$$K_i \pm W = K_f$$

$$\frac{1}{2}mu^2 - mgl(1+\cos\theta) = \frac{1}{2}m\sqrt{gl\cos\theta}$$

$$\frac{u^2}{2} = \frac{gl\cos\theta}{2} + gl(1+\cos\theta)$$

$$\cancel{\frac{u^2}{2} = \frac{gl\sqrt{3}}{1} + 2gl(1+\sqrt{3})}$$

$$u^2 = \frac{gl}{\sqrt{3}} + \frac{2gl(1+\sqrt{3})}{\sqrt{3}}$$

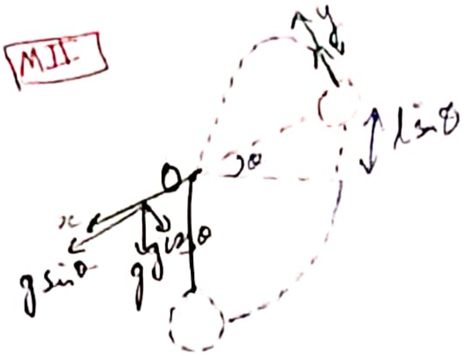
$$u^2 = \frac{3gl + 2\sqrt{3}gl}{\sqrt{3}}$$

$$u^2 = \sqrt{3}gl + 2gl$$

$$u^2 = (2+\sqrt{3})gl$$

$$\boxed{u = \sqrt{(2+\sqrt{3})gl}}$$

MII



$S_y = u_y t + a_y t^2$

$0 = vt - \frac{1}{2} g\cos\theta\, t^2$

$g\cos\theta\, t^2 = 2vt$

$g\cos\theta\, t = 2v$

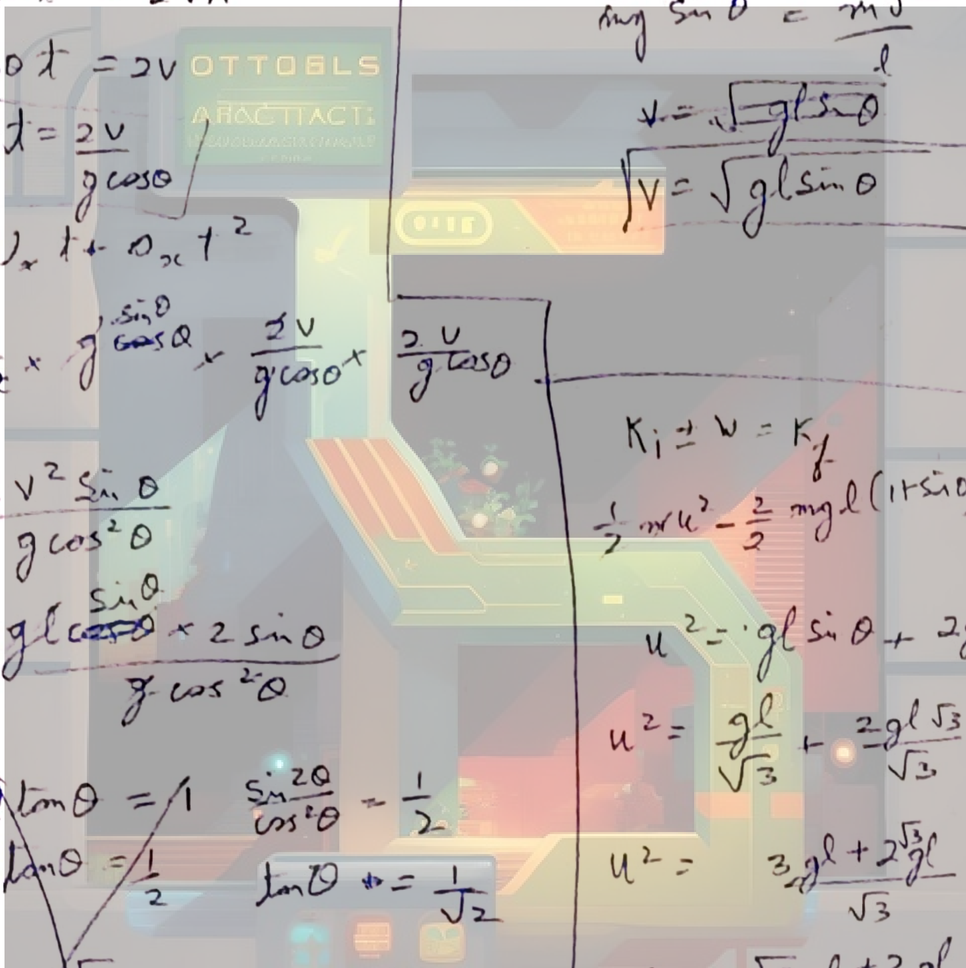
$\boxed{t = \frac{2v}{g\cos\theta}}$

$S_x = u_x t + a_x t^2$

$l = \frac{1}{2} \times g\,\cancel{\cos\theta}\,\sin\theta \times \frac{2v}{g\cos\theta} \times \frac{2v}{g\cos\theta}$

$l = \frac{2v^2 \sin\theta}{g\cos^2\theta}$

$l = \frac{gl\,\cancel{\cos\theta}\,\sin\theta \times 2\sin\theta}{g\cos^2\theta}$



$\cancel{2\tan\theta = 1}$ $\quad \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{2}$

$\cancel{\tan\theta = \frac{1}{2}}$ $\quad \tan\theta = \frac{1}{\sqrt{2}}$

$1,\ \sqrt{5},\ 2,\ \theta \qquad 1,\ \sqrt{3},\ \sqrt{2},\ \theta$

~~cosθ~~ $\quad \sin\theta = \frac{1}{\sqrt{3}}$

at B

$\cancel{T + mg\cos\theta = \frac{mv^2}{l}}$

$\cancel{0 + mg\cos\theta = \frac{mv^2}{l}}$

$\cancel{gl\cos\theta = v^2}$

$\cancel{v = \sqrt{gl\cos\theta}}$

at B

$T + mg\sin\theta = \frac{mv^2}{l}$

$T \to 0$

$mg\sin\theta = \frac{mv^2}{l}$

$v = \sqrt{gl\sin\theta}$

$\boxed{v = \sqrt{gl\sin\theta}}$

$K_i + W = K_f$

$\frac{1}{2}mu^2 - \frac{2}{2}mgl(1+\sin\theta) = \frac{1}{2}mv^2$

$u^2 = gl\sin\theta + 2gl + 2gl\sin\theta$

$u^2 = \frac{gl}{\sqrt{3}} + \frac{2gl\sqrt{3}}{\sqrt{3}} + \frac{2gl}{\sqrt{3}}$

$u^2 = \frac{3gl + 2\sqrt{3}gl}{\sqrt{3}}$

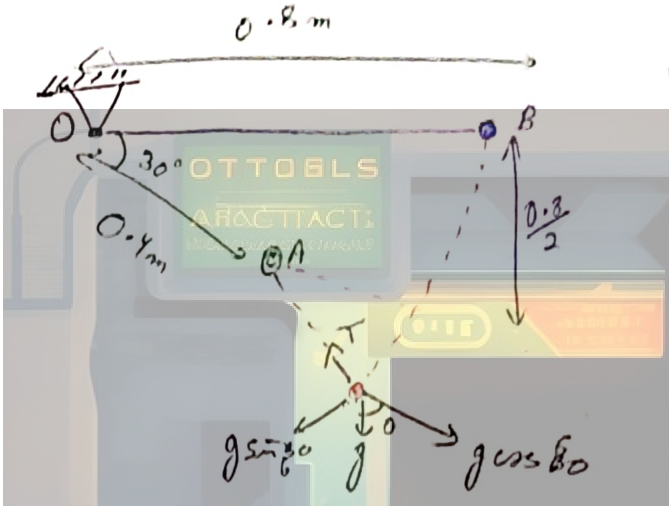
$u^2 = \sqrt{3}gl + 2gl$

$u^2 = (\sqrt{3}+2)gl$

$\boxed{u = \sqrt{(\sqrt{3}+2)gl}}$

Q 29.

A small sphere B of mass m is released from rest as shown and swings freely in a vertical plane, first about O & ~~add~~ and then about the Peg A after ~~cord~~ cord comes in contact with A. find Tension a) just before sphere comes in contact, b) just after it comes in contact.



$\underbrace{mg \cdot 0.4}_{mg} = \underbrace{\frac{1}{2} m v^2}_{K_f}$

$\cancel{v = \sqrt{0.8g}}$

$0.8mg = mv^2$

$0.8 \times g = v^2$

$v^2 = 0.8g$

$V = \sqrt{0.8g}$

$\cancel{T - \frac{\sqrt{3} gm}{2} = \frac{mV^2}{0.8}}$

$\cancel{T = \frac{mV^2}{0.8} + \frac{\sqrt{3} mg}{2}}$

$\cancel{= \frac{0.8 \times m \times g}{0.8} + \frac{\sqrt{3} mg}{2}}$

$\cancel{T = \frac{(\sqrt{3}+2) mg}{2}}$

$T - \frac{mg}{2} = \frac{mv^2}{l}$

$T = \frac{m\,0.8g}{l} + \frac{mg}{2}$

$= mg + \frac{mg}{2}$

$T = \frac{3mg}{2}$

$\boxed{T = 15m}$ a)

$T = \frac{0.8mg}{0.4} + \frac{mg}{2}$

$T = 2mg + \frac{mg}{2}$

$T = \frac{5mg}{2}$

$\boxed{T = 25m}$ b)

Q30. A massless rod length $2l$ carries two equal masses m, can rotate in a vertical plane. What horizontal velocity must be imparted to the mass at A so that it just completes the vertical circle.

M I

$1.5l\,(2m)$ (centre of mass)

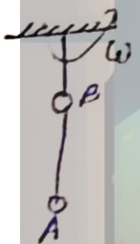
To complete circle, $u = \sqrt{4gx}$

$= \sqrt{4g \times 1.5l}$

$\sqrt{\frac{2}{3}gl}$

$\boxed{= \sqrt{6gl}}$ at C

M II



$V_A = \omega(2l)$

$V_B = \omega l$

$V_A = 2V_B = V$

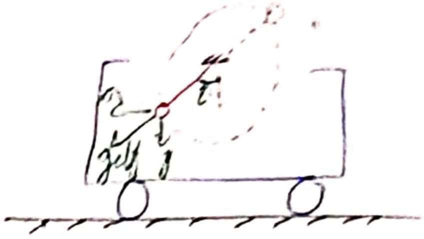
$\frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 - mg(4l) - mg(2l) = 0$

$6gl = \frac{v^2}{2} + \frac{v^2}{8}$

$6gl \times 8 = 5v^2$

$\boxed{\sqrt{\frac{48gl}{5}} = V}$

Vertical circular motion in horizontally accelerated frame.



$\tan\theta = \frac{a}{g}$

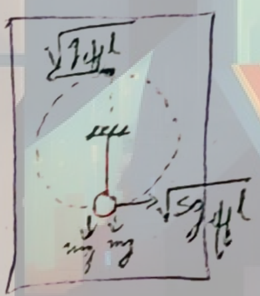
The bob will oscillate about this new eqm position.

$g_{eff} = \sqrt{a^2+g^2}$

min speed $= \sqrt{5\sqrt{a^2+g^2}\, l}$ (To complete vertical circle)

Vertical circular motion in vertical Horizontal Plane.

I



$g_{eff} = g + a$

$\uparrow a$

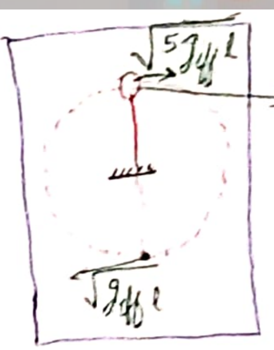
II $\downarrow a\ (<g)$

$g_{eff} = g - a$

III $\downarrow a = g$

Tension $= 0$

IV

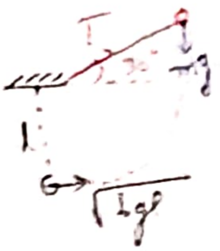


→ Equilibrium Position.

$\downarrow a\ (>g)$

$g_{eff} = a - g$

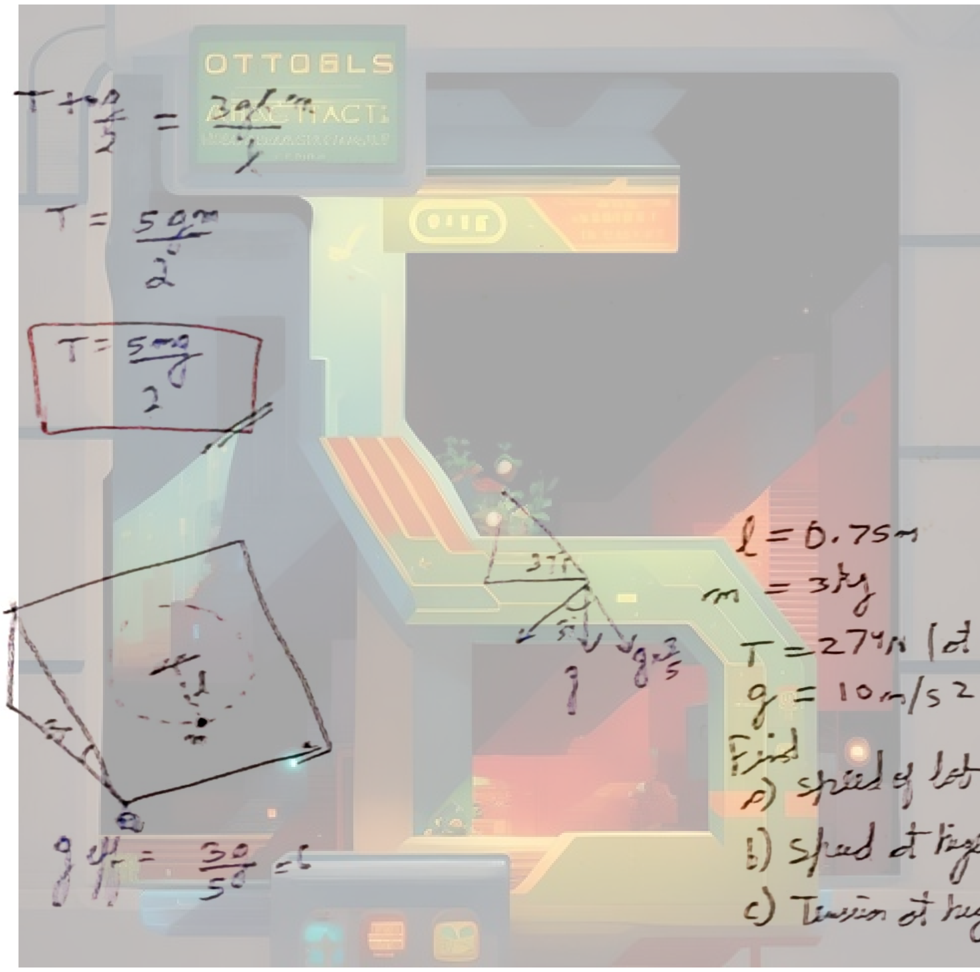
Q.31.



find Tension in the string when deflection is 120° from mean position. mass is m

$$\frac{1}{2}\times m\times 6gl - m\times g\times\frac{3l}{2} = \frac{1}{2}\times m\times v^2$$

$$v=\sqrt{3gl}$$



$$T+\frac{mg}{2} = \frac{3gl\,m}{l}$$

$$T = \frac{5gm}{2}$$

$$\boxed{T=\frac{5mg}{2}}$$

Q.32

$l = 0.75\,m$

$m = 3\,kg$

$T = 274\,N$ (at lowest)

$g = 10\,m/s^2$

Find

a) speed of bob at lowest point

b) speed at highest point

c) Tension at highest point.

$$g_{eff} = \frac{3g}{5} = 6$$

a) $\cancel{T - mg_{eff} = \frac{v^2}{l}}$

$$\cancel{274 - 3\times\frac{3g}{5} = \frac{v^2}{l}}$$

$$\cancel{v^2 = 274\times\frac{3}{4} - 3\times\frac{3g}{5}\times\frac{3}{4}}$$

$$\cancel{v^2 = \frac{822-54}{4}}$$

$$\cancel{v^2 = \frac{768}{4}}$$

$\cancel{u^2 = 192}$

a) ⇒ $T_1 - mg\frac{4}{5} = \frac{mv^2}{r}$

$274 - 18 = \frac{v^2}{\frac{3}{4}} \times 3$

$256 \times \frac{3}{4} = v^2 \times 3$

$3 \times v^2 = 64 \times 3$

$v^2 = 64$

$\boxed{u = 8}$ a)

b) $\frac{1}{2} \times 3 \times 64 - 3 \times 6 \times \frac{2}{4} \times 3 = \frac{1}{2} \times 3 \times v^2$

$64 - 18 = v^2$

$v^2 = 46$

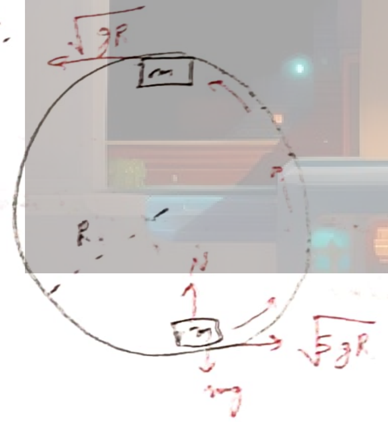
$\boxed{v = \sqrt{46}}$ b)

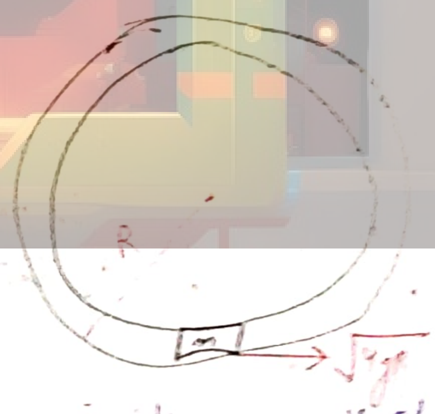
c) $T_2 + 3 \times 6 = \frac{(\sqrt{46})^2}{\frac{3}{4}} \times 3$

$T_2 = 46 \times 4 - 18$

$\boxed{T_2 = 166\,N}$ c)

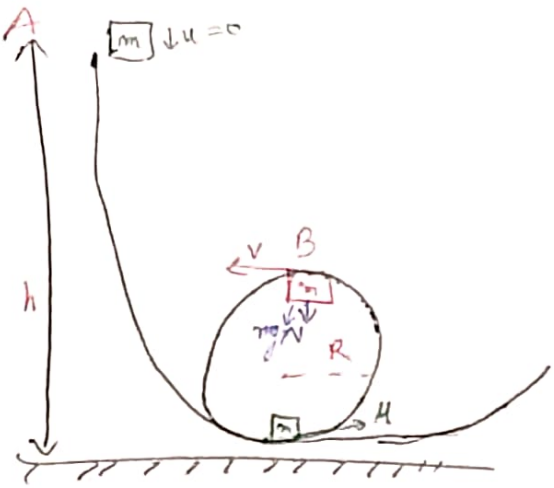
Q33.





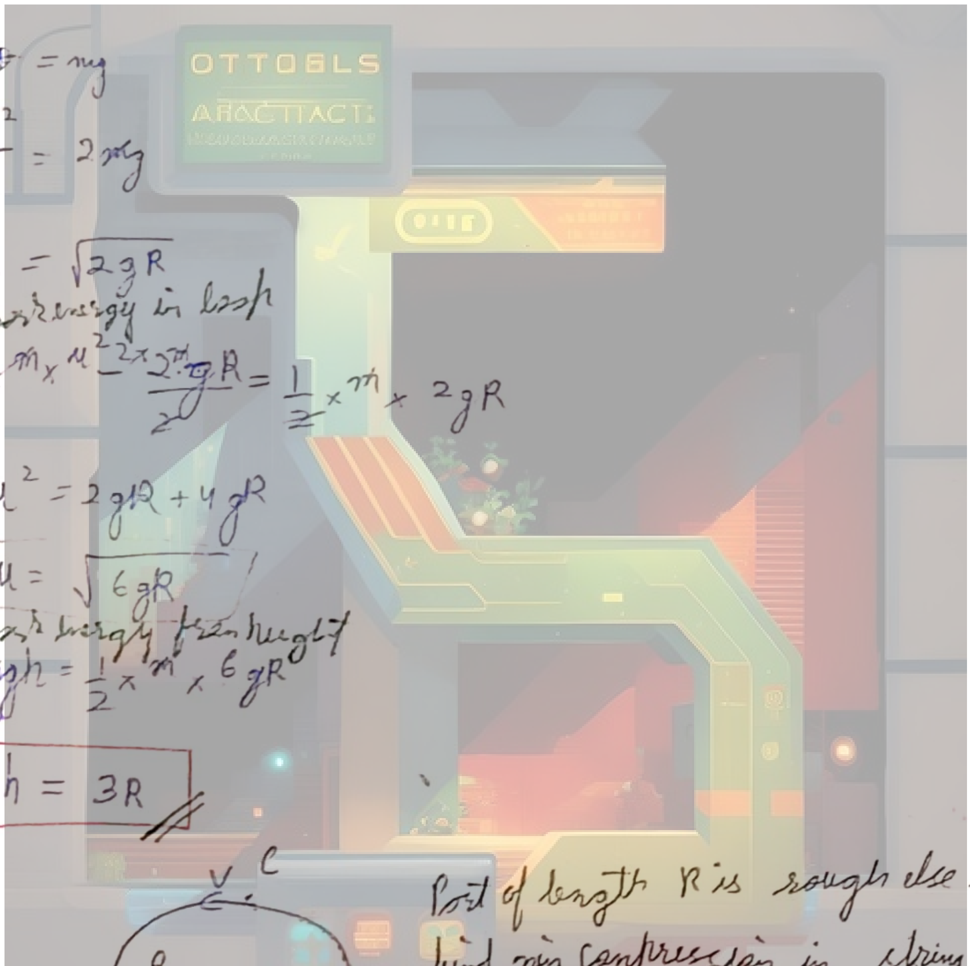
→ If a block is confined to move inside a groove of Radius R, min speed to be given to block to complete circle is $\sqrt{4gR}$

Q 33.



At what height from A it starts so it passes against the ~~B~~ point B with net upward force equal to its weight

$h = ?$



$N = mg$

$\frac{mv^2}{R} = 2mg$

$v = \sqrt{2gR}$

work energy in loop

$\frac{1}{2} \times m \times u^2 - 2 \times \frac{2m}{2} gR = \frac{1}{2} \times m \times 2gR$

$u^2 = 2gR + 4gR$

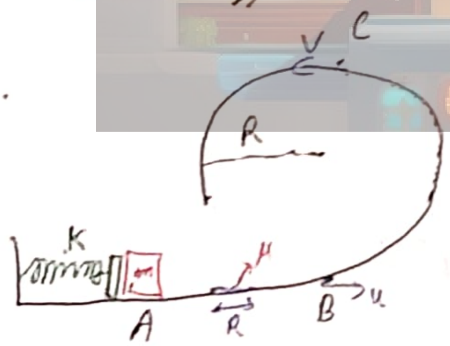
$\boxed{u = \sqrt{6gR}}$

work energy from height

$mgh = \frac{1}{2} \times m \times 6gR$

$\boxed{h = 3R}$

Q 34.



Part of length R is rough else is smooth. find min compression in spring if it don't lose contact with track.

$u = \sqrt{5gR}$ (for not lose contact)

Work Energy Theorem

$$\frac{1}{2} \times K \times x^2 - \cancel{\mu mg}\ \cancel{mg}\ \frac{2\mu mg}{2} R = \frac{1}{2} \times m \times u^2$$

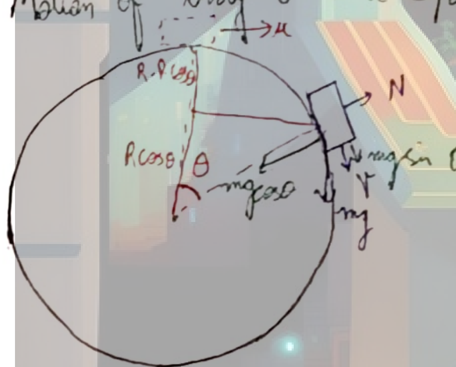
$$Kx^2 - 2\mu mgR = mu^2$$

$$Kx^2 = 2u^2 + 2\mu mgR$$

$$x^2 = \frac{2u(u + mgR)}{K}$$

$$\boxed{x = \sqrt{\frac{2u(u + mgR)}{K}}}$$

* Motion of body outside spherical surface.



$$\theta \quad mg\cos\theta - N = \frac{mv^2}{R}$$

$$\frac{1}{2}mu^2 + mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = u^2 + 2gR(1-\cos\theta)$$

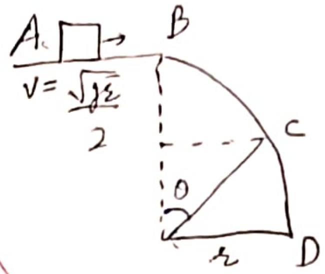
when block losses contact, $N = 0$

$$mg\cos\theta = \frac{mv^2}{R}$$

$$\cos\theta = \frac{v^2}{gR}$$

$$\boxed{\cos\theta = \frac{2gR + u^2}{3gR}}$$

Q.35



$\cancel{m}g\cos\theta = \frac{mv^2}{r}$

$\frac{1}{2}\times \cancel{m}\times v^2 = \frac{1}{2}\times \cancel{m}\times \frac{gr}{4} + \cancel{m}gr(1+\cos\theta)$

$v^2 = \frac{gr}{4} + 2gr(1+\cos\theta)$

$\cancel{m}g\cos\theta = \frac{gr + 8gr + 8gr\cos\theta}{\cancel{r}}$

$\cos\theta = \cancel{1+8+}\ 9 + 8\cos\theta$

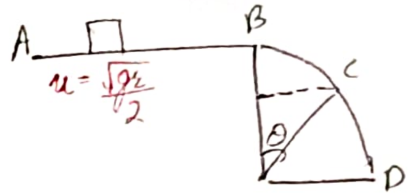
$v^2 = \frac{\cancel{m}\times gr + 8gr + 8gr\cos\theta}{4}$

$4gr\cos\theta = gr + 8gr + 8gr\cos\theta$

$-4\cos\theta = 9$

$\cos\theta = -\frac{9}{4}$

Q 35.



find θ if block leaves surface at C.

$$\cos\theta = \frac{2gr + u^2}{3gr}$$

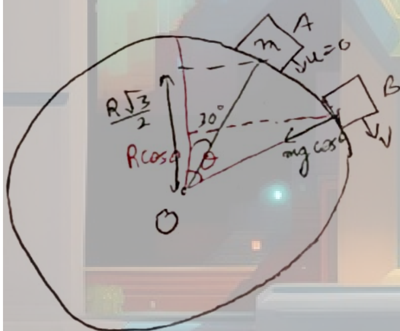
$$\cos\theta = \frac{2gr + \frac{gr}{4}}{3gr}$$

$$\cos\theta = \frac{9gr}{12gr}$$

$$\cos\theta = \frac{3}{4}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{3}{4}\right)}$$

Q 36. mass m ; radius R , block is released when block makes 30° with vertical.



a) what is the force exerted by the sphere on disc just after release

b) At what angle with vertical will it leave the sphere.

a) $mg\cos 30^\circ - N = \frac{mu^2}{r}$

$$\frac{\sqrt{3}}{2} mg = N$$

$$\boxed{N = \frac{\sqrt{3}}{2} mg} \text{ (a)}$$

$$0 + mg(R\cos 30^\circ - R\cos\theta) = \frac{1}{2}mv^2$$

$$\sqrt{3}gR - 2gR\cos\theta = v^2$$

Centripetal

$$mg\cos\theta = \frac{mv^2}{r}$$

$$g\cos\theta = \frac{\sqrt{3}gr - 2gr\cos\theta}{r}$$

$$\cos\theta = \sqrt{3} - 2\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)}$$

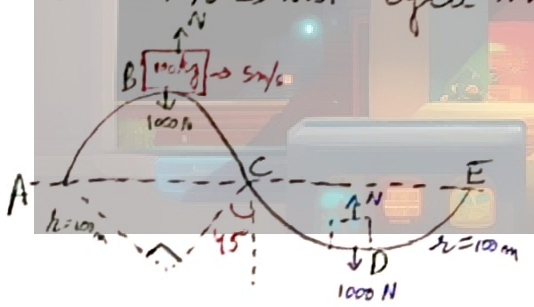
Q37. A Cycle travels at a constant speed. Find.

a) Normal contact force by the road at B & D

b) Force of friction exerted by track on the tyres when the cycle is at B, C & D

c) Normal force btw road & cycle just before & after cycle crosses C

d) Minimum μ so that cycle moves uniform speed.



a) $1000 - N_B = \frac{25}{100} \times 100$

$N_B = 1000 - 25$

$\boxed{N_B = 975}$ a)

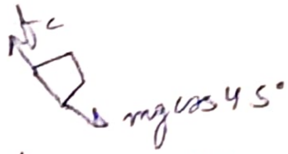
$N - 1000 = 100 \times \frac{5 \times 5}{100}$

$\boxed{N_D = 1025\ N}$ a)

b) $\boxed{F_B = 0}$ b)

$\boxed{F_D = 0}$ b)

at C,



$f_c = mg\cos 45$

$f_c = 100 \times 10 \times \frac{1}{\sqrt{2}}$

$\boxed{f_c = 500\sqrt{2}\,N}$ b)

C) Before C,

$\frac{mg}{\sqrt{2}} - N = m \times \frac{v^2}{r}$

$N = \frac{100 \times 10}{\sqrt{2}} - \frac{100 \times 25}{100}$

$\boxed{N = 500\sqrt{2} - 25}$ c)

After C,

$N - \frac{mg}{\sqrt{2}} = \frac{mv^2}{r}$

$N = \frac{100 \times 25}{100} + \frac{100 \times 10}{\sqrt{2}}$

$\boxed{N = 500\sqrt{2} + 25}$ c)

~~d) $f_c = 500\sqrt{2}$~~

~~$N = 1000$~~

~~$f_c = \mu N$~~

~~$\frac{500\sqrt{2}}{1000} = \mu$~~

~~$\mu = \frac{1}{\sqrt{2}}$~~

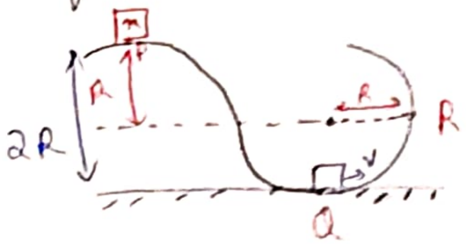
d) $f_c = 500\sqrt{2}$

$N = 500\sqrt{2} - 25$

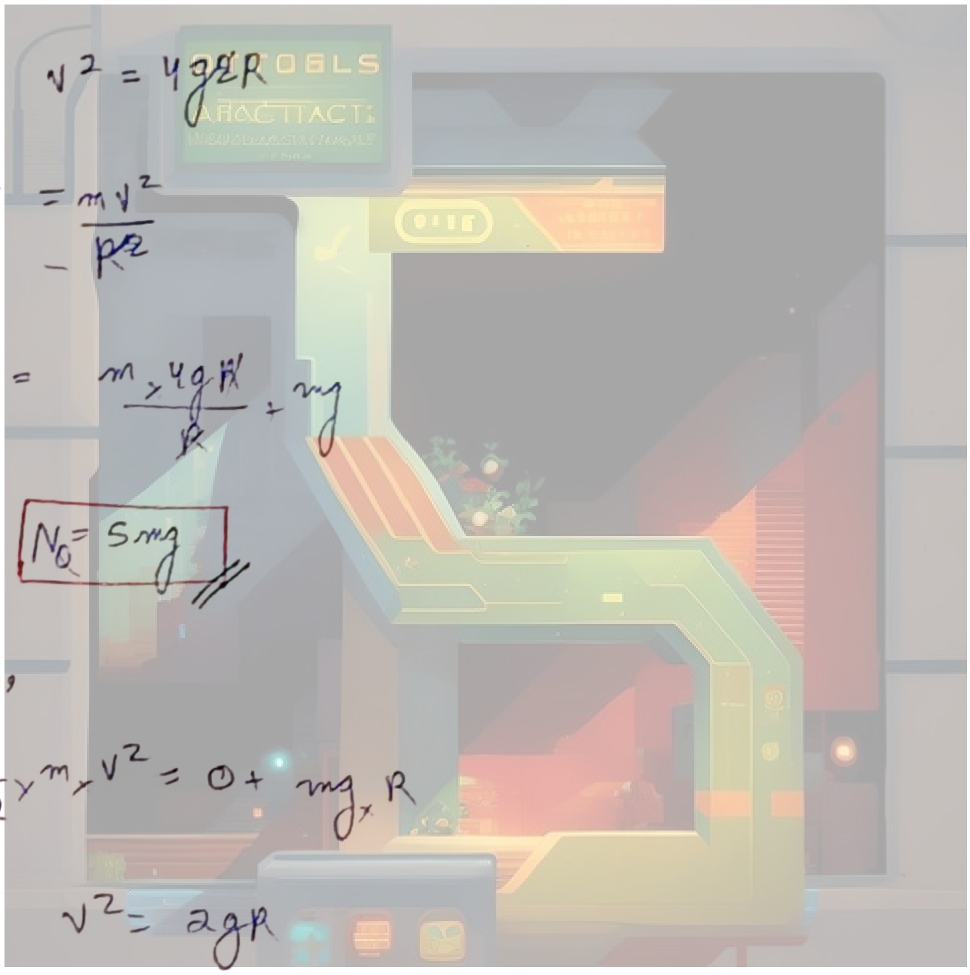
$\mu = \frac{500\sqrt{2}}{500\sqrt{2} - 25}$

$\boxed{\mu = 0.103}$ d)

Q 38. Track is smooth, The block starts sliding from rest. Find normal forces at Q & R.



$$\frac{1}{2}\times m\times v^2 = \cancel{\tfrac{1}{2}}\, 0 + mg\times 2R$$



$$v^2 = 4gR$$

$$N - mg = \frac{mv^2}{R}$$

$$N = \frac{m\times 4g\cancel{R}}{\cancel{R}} + mg$$

$$\boxed{N_Q = 5mg}$$

at R,

$$\frac{1}{2}\times m\times v^2 = 0 + mg\times R$$

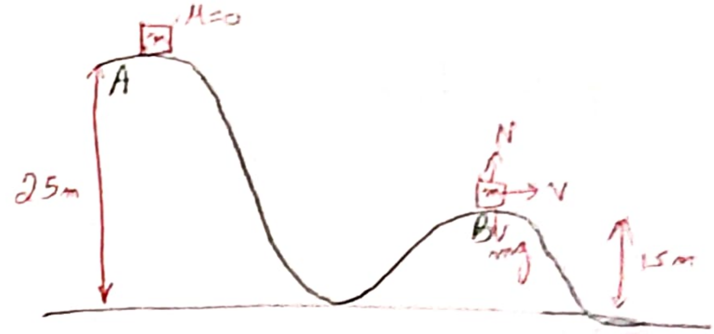
$$v^2 = 2gR$$

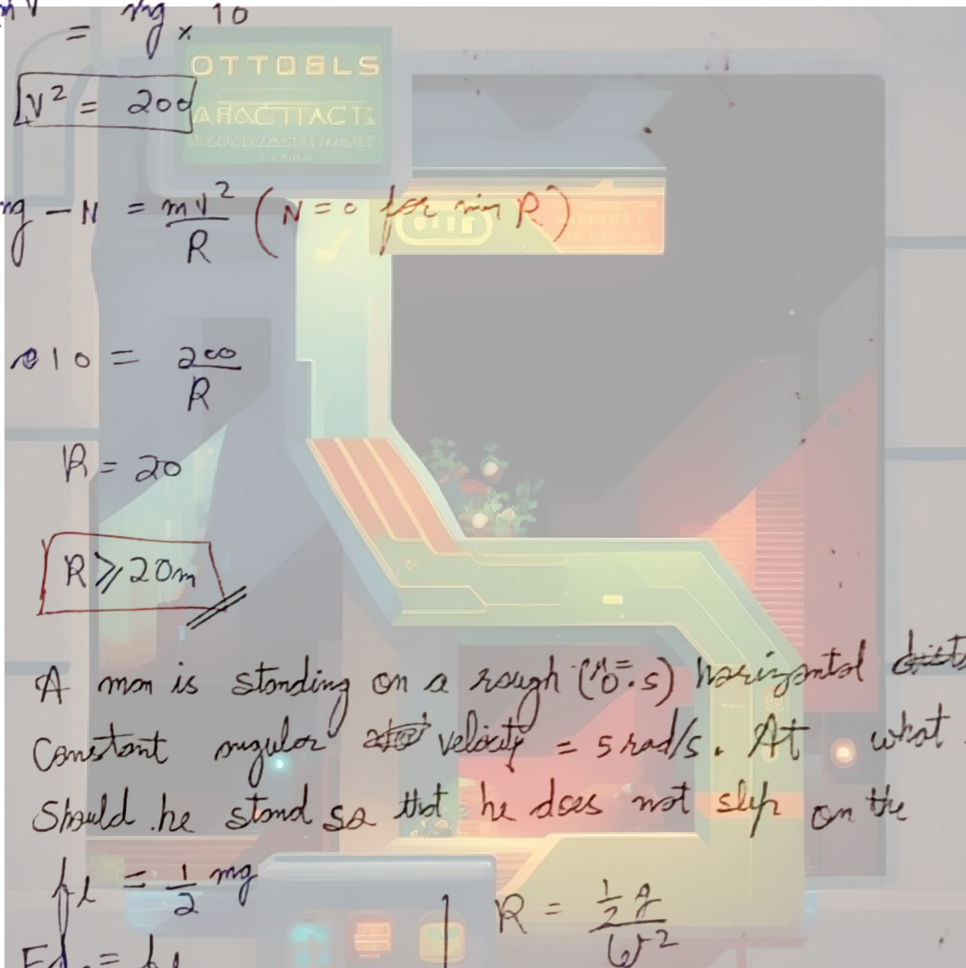
$$N = \frac{mv^2}{R}$$

$$N = \frac{m\times 2gR}{R}$$

$$\boxed{N = 2mg}$$

Q39. Find min safe value of radius of curvature of B, so that car do not leave the track?





$$\frac{1}{2} m v^2 = mg \times 10$$

$$\boxed{v^2 = 200}$$

$$mg - N = \frac{mv^2}{R} \quad (N = 0 \text{ for min } R)$$

$$10 = \frac{200}{R}$$

$$R = 20$$

$$\boxed{R \geq 20\text{m}}$$

Q40. A man is standing on a rough ($\mu = 0.5$) horizontal ~~distance~~, disc, Constant angular velocity $= 5$ rad/s. At what distance should he stand so that he does not slip on the disc.

$$f_l = \frac{1}{2} mg$$

$$F_c = f_l$$

$$\frac{mv^2}{R} = \frac{1}{2} mg$$

$$v^2 = \frac{1}{2} gR$$

$$\omega^2 R^2 = \frac{1}{2} gR$$

$$\frac{1}{2} g = \omega^2 R$$

$$R = \frac{\frac{1}{2} g}{\omega^2}$$

$$R = \frac{5}{25}$$

$$R = \frac{1}{5}$$

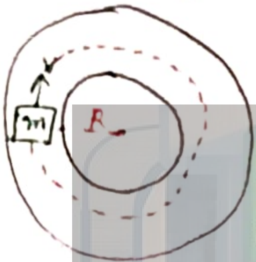
$$\boxed{R \leq 0.2\text{ m}}$$

Circular Turning of Roads

→ when vehicles take a circular turn some force must be present to provide the required centripetal force.

→ It can be provided by 3 ways.

(1) Friction only:-



$$F_c = \frac{mv^2}{R}$$

$$\cancel{f_k} = f_l = \mu mg$$

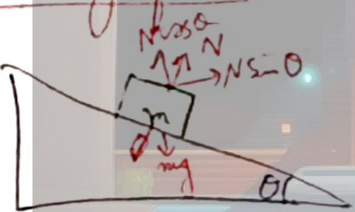
for safe turn

$$F_c \leq f_l$$

$$\frac{mv^2}{R} \leq \mu mg$$

$$\boxed{V \leq \sqrt{\mu R g}}$$

(2) By Banking of Roads -



$$N\cos\theta = mg \quad -(1)$$

$$N\sin\theta = \frac{mv^2}{R} \quad -(2)$$

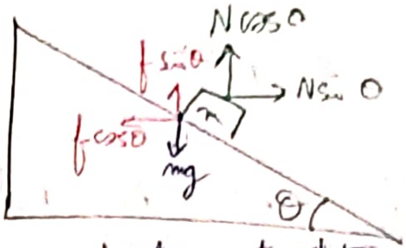
(2) ÷ (1)

$$\boxed{\tan\theta = \frac{V^2}{Rg}}$$

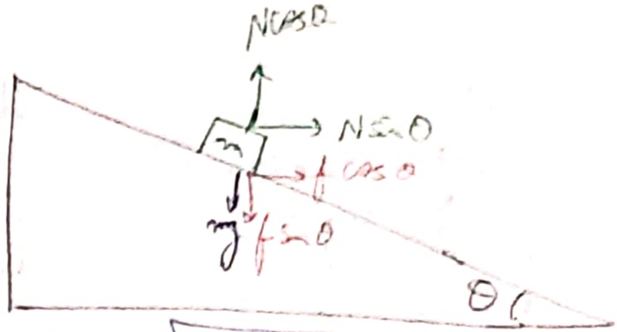
This is also the angle, riders tilt to maintain smooth curve without skidding.

③ Both

i)



ii)



→ $v=0$, friction acts out to balance $mg\sin\theta$

→ $v < \sqrt{gr\tan\theta}$, friction acts outward

→ $v > \sqrt{gr\tan\theta}$, friction acts inward.

→ $v = \sqrt{gr\tan\theta}$, $f = 0$

for min speed,

$$N\cos\theta + f\sin\theta = mg \Rightarrow N\cos\theta + \mu N\sin\theta = mg$$

$$N\sin\theta - f\cos\theta = \frac{mv^2}{R} \Rightarrow N\sin\theta - \mu N\cos\theta = \frac{mv^2}{R}$$

$$\frac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta} = \frac{gR}{v^2}$$

$$\boxed{V_{min} = \sqrt{\frac{gr(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}}}$$

for max speed,

$$N\cos\theta = mg + f\sin\theta \Rightarrow mg = N\cos\theta - \mu N\sin\theta$$

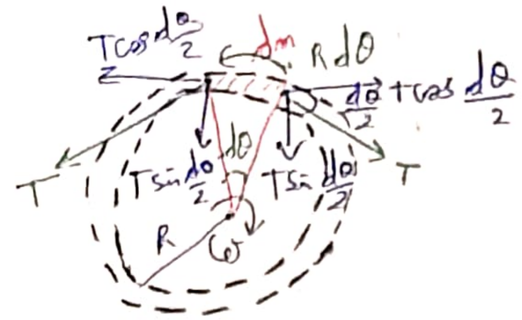
$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \Rightarrow \frac{mV^2}{R} = N\sin\theta + \mu N\cos\theta$$

divide,

$$\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta} = \frac{gR}{V^2}$$

$$\boxed{V_{max} = \sqrt{\frac{gr(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}}}$$

Tension in revolving chain



$$d\theta = \frac{l}{R} \Rightarrow l = R\,d\theta$$

$$\left.\begin{array}{l} 2\pi R \rightarrow M \\ R\,d\theta \rightarrow dm \end{array}\right\} \quad dm = \frac{M}{2\pi \cancel{R}} \cancel{R}\,d\theta$$

$$dm = \frac{M\,d\theta}{2\pi}$$

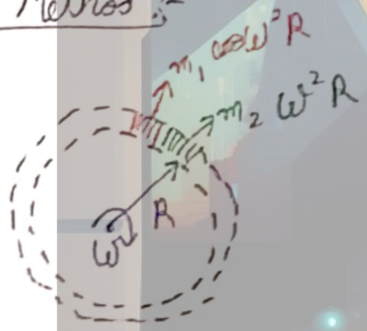
$$dm\,\omega^2 R = 2 \times T \sin\frac{d\theta}{2}$$

θ is very small, $\sin\theta \approx \theta$

$$2T\left(\frac{d\theta}{2}\right) = \frac{M}{2\pi}\,d\theta\,\omega^2 R$$

$$\boxed{T = \frac{M\omega^2 R}{2\pi}}$$

Short Method :-



$$T = \frac{m_1\omega^2 R + m_2\omega^2 R + m \ldots}{2\pi}$$

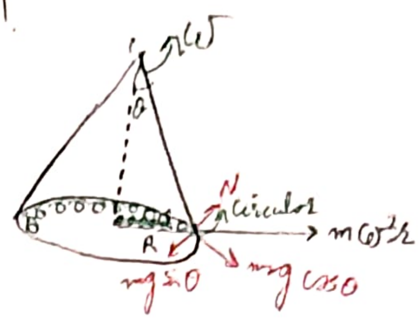
$$T = \frac{\omega^2 R\,(m_1 + m_2 \ldots)}{2\pi}$$

$$T = \frac{M\omega^2 R}{2\pi}$$

→ Net radial force on chain & divide by 2π.

→ we here, assume there is not whole chain but a singular segment of Mass M moving in a circle of R radius & angular vel ω.

Q41.



$$N = mg\sin\theta - m\omega^2 r\cos\theta \quad -(1)$$

$$F = N\cos\theta + m\omega^2 r$$

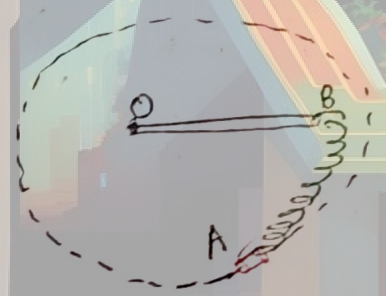
↳ net radial force

$$T = \frac{N \cos\theta + m\omega^2 r}{2\pi r}$$

$$T = \frac{mg\sin\theta\cos\theta + m\omega^2 r(1-\cos^2\theta)}{2\pi r}$$

$$\boxed{T = \frac{\sin\theta}{2\pi r} m\left(g\cos\theta + \omega^2 r\sin\theta\right)}$$

Q42.



length of L shaped rod = l

natural spring length = l

mass (A) = m

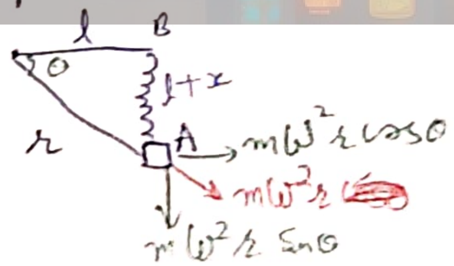
spring stiff = K

angular vel. = ω about O

elongation = x (find)

How is resultant affected by rotation direction

Top.



$$\sin\theta = \frac{l+x}{r}$$

$$r\sin\theta = l+x$$

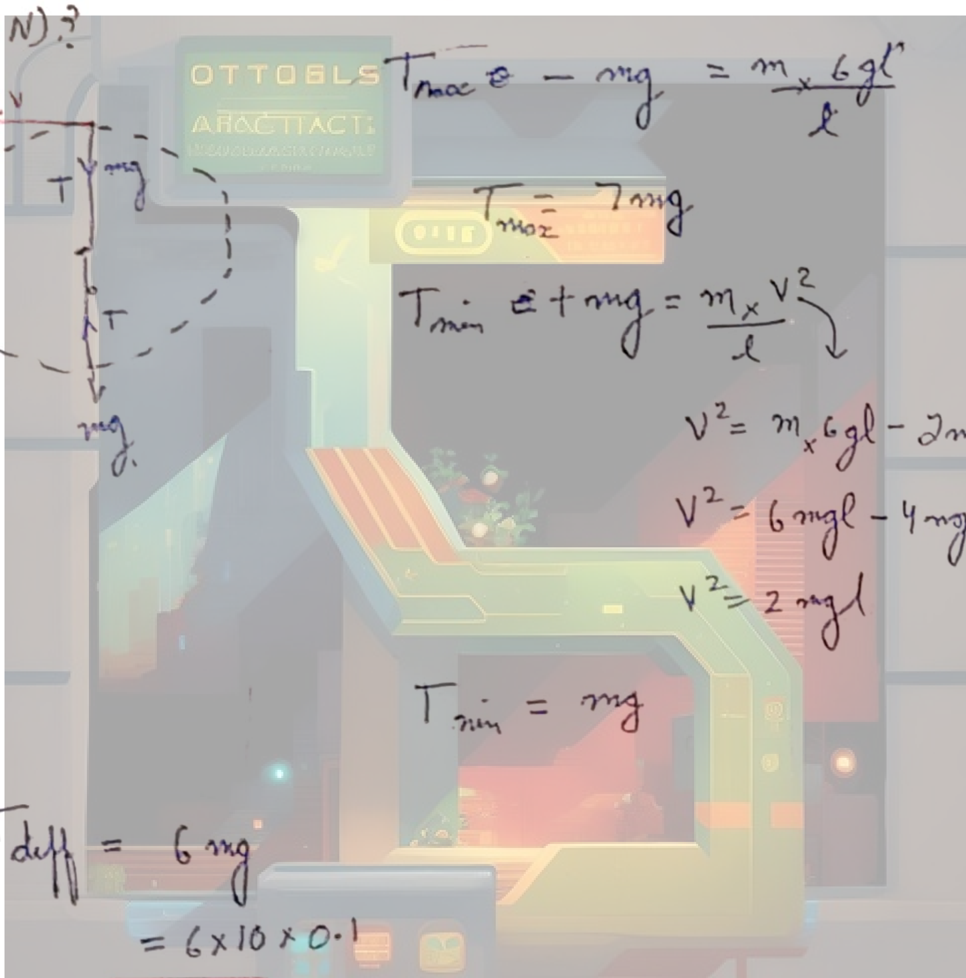
$kx = m\omega^2 r \sin\theta$

$kx = m\omega^2 l + m\omega^2 x$

$x(k - m\omega^2) = m\omega^2 l$

$\boxed{x = \dfrac{m\omega^2 l}{k - m\omega^2}}$ → unaffected by direction of rotation

Q43. A bob of mass 100 g, length l, $u = \sqrt{6gl}$ completes vertical circle; what is the difference btw max tension & min tension (in N)?



$T_{max} - mg = m \times \dfrac{6gl}{l}$

$T_{max} = 7mg$

$T_{min} + mg = m \times \dfrac{v^2}{l}$

$v^2 = m \times 6gl - 2mg(2l)$

$v^2 = 6mgl - 4mgl$

$v^2 = 2mgl$

$T_{min} = mg$

$T_{diff} = 6mg$

$= 6 \times 10 \times 0.1$

$\boxed{= 6N}$