! Fundamentals of Geometry !

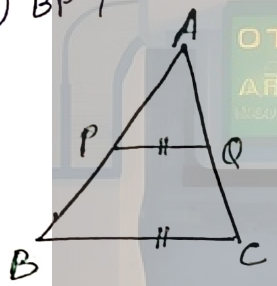
① **Similar & Congruent Triangle** :-

↓ Zooming Effect
↓ Sides Proportional

↳ Exactly Same shape & size (Can overlap)

② **Similar Polygons** – BPT & Converse BPT

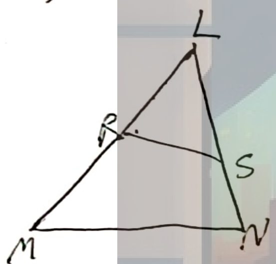
i) BPT



$PQ \parallel BC$

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

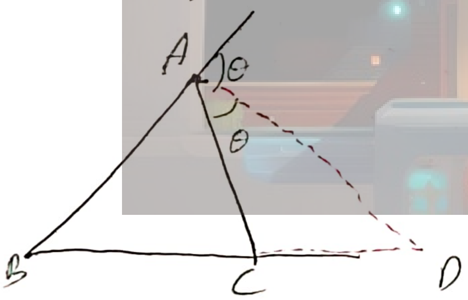
ii) Converse BPT



$$\frac{LR}{RM} = \frac{LS}{SN}$$

Then, $RS \parallel MN$

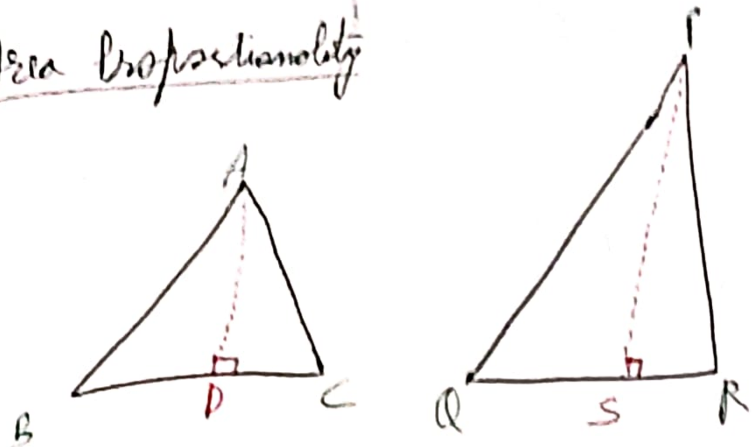
③ External Angle Bisector Theorem



If AD is angle bisector then.

$$\frac{AB}{AC} = \frac{BD}{CD}$$

(4) Area Proportionality

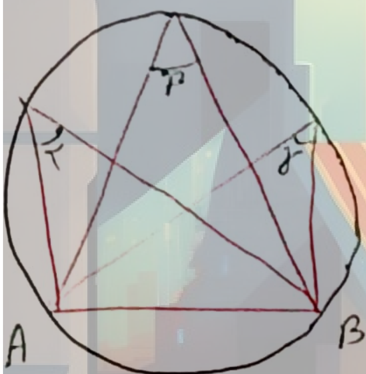


If $\triangle ABC$ & $\triangle PQR$ are similar then—

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

(5) Circle Theorem:

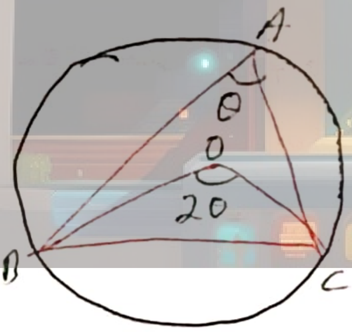
A)



for same chord AB (same segments angles)

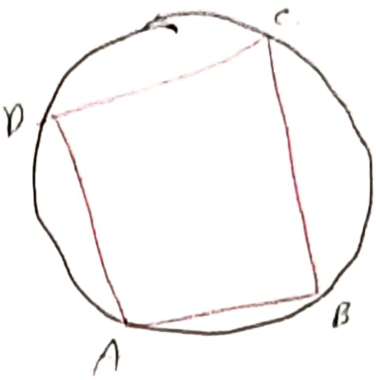
$\angle\alpha = \angle\beta = \angle\gamma$

b)



Angle at center is double of that at circumference for same chord.

⑥ Cyclic Quadrilateral



$\angle A + \angle C = 180$
$\angle B + \angle D = 180°$

☆ Point of intersection of diagonals can or cannot be center of circle.

⑦ Special Points in Δ:

a) Median — Centroid

b) Angle Bisector — Incenter

c) ⊥ bisector — circumcenter

d) Altitude — Orthocentre

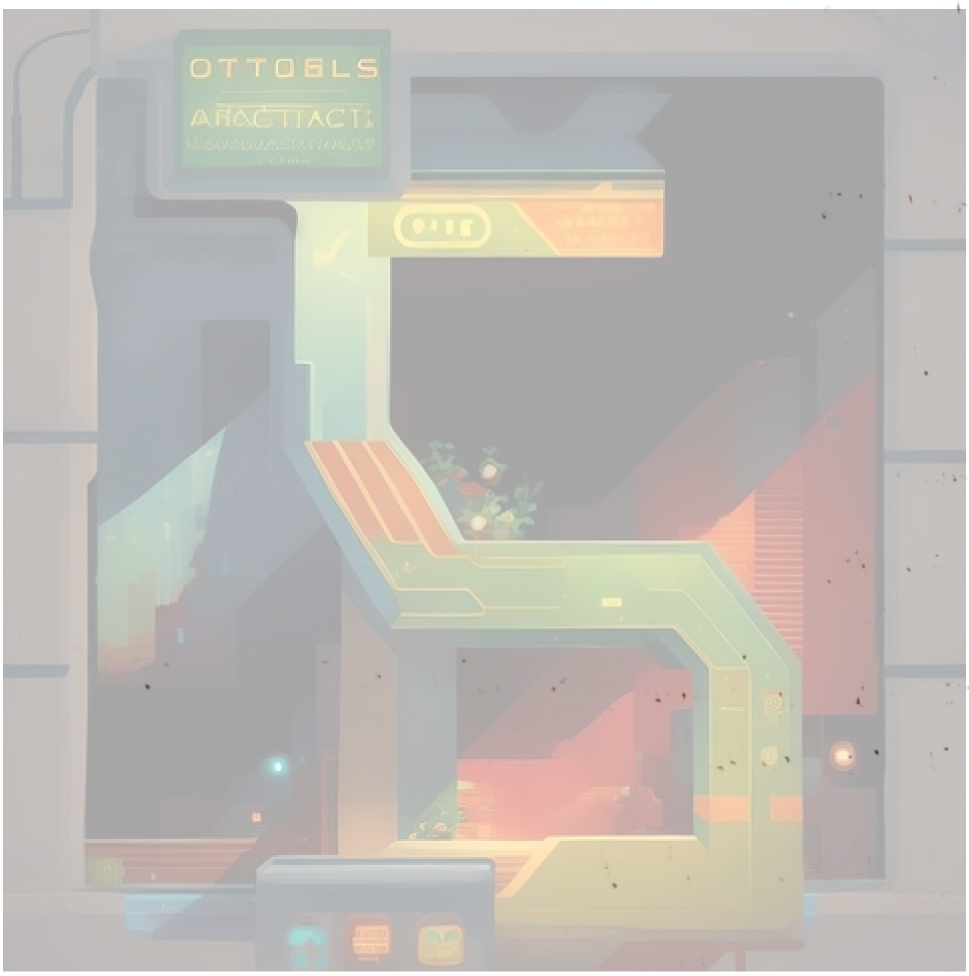
e) Exterior Angle Bisector — Excentre

f) orthocentre, circumcentre, centroid lies on a straight line with 2:1 ratio. ∵ ortho 2 centroid 1 circumcenter

☆ In Equilateral Δ ⇒ All points are same

☆ In Isosceles Δ ⇒ All points are collinear.

H.W. 10-8-24
Compound Angle (JM)
TE (0-1)

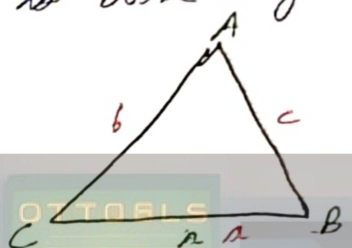


! Solution of Triangle !

→ Process of calculating the sides or angles of a triangle.

→ Here Angles are denoted by capital letters (A, B..., Z) & sides are denoted by small letters (a, b, ... z).

→ Length opposite to above angles are are taken ~~respecto~~ respectively.

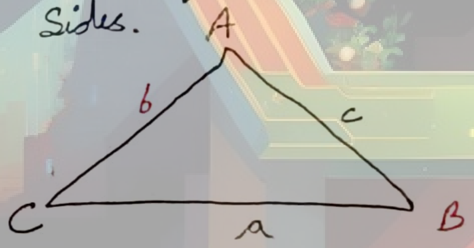


→ All the above 6 elements (3 angles + 3 sides) are not independent they are connected to each other somehow

→ Questions are based of giving 1-2 elements & finding others

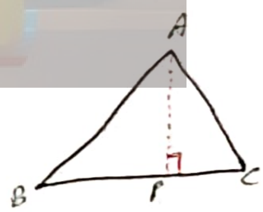
__Basic Laws__

1. __Sine Law__:- Sines of angles are proportional to the opposite sides.



$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

__Proof__:- $\sin B = \frac{AP}{c}$, $\sin C = \frac{AP}{b}$



$c \sin B = b \sin C$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

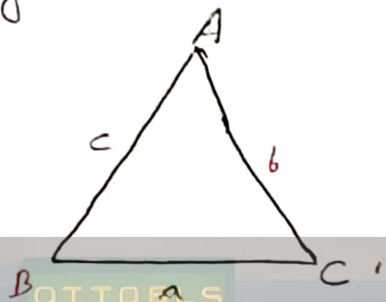
H.P.

Note:- If $a+b > c$

$\sin a + \sin b > \sin c$

2. Cosine Rule:- when all three sides of a triangle are known.

→ Any two sides & Included angle of the triangle are known

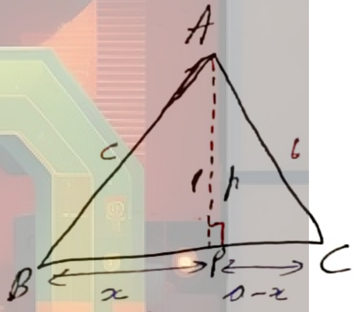


$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\boxed{\cos B = \frac{a^2+c^2-b^2}{2ac}}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Proof:-



in ΔABP,

$p^2 + x^2 = c^2$

$p^2 = c^2 - x^2$ —①

in ΔACP,

$p^2 + (a-x)^2 = b^2$

$p^2 = b^2 - (a-x)^2$ —②

in ΔABP

$\cos B = \frac{x}{c}$

$x = c\cos B$ —③

① = ②

$c^2 - x^2 = b^2 - (a-x)^2$

$c^2 - \cancel{x^2} = b^2 - a^2 - \cancel{x^2} + 2ax$

$a^2 + c^2 - b^2 = \cancel{2(ax-x^2)}\ 2ax$

$a^2 + c^2 - b^2 = 2ac\cos B$

$\frac{a^2+c^2-b^2}{2ac} = \cos B$

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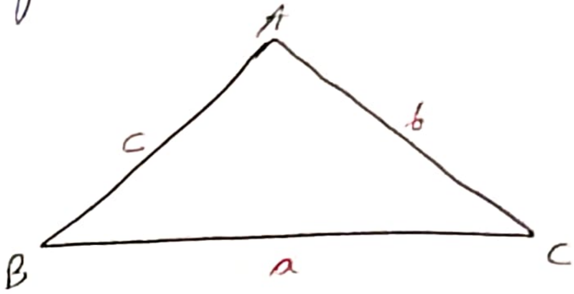
(104)

3. Projection Law:- Any side of a triangle is equal to sum of projections of other two sides on it

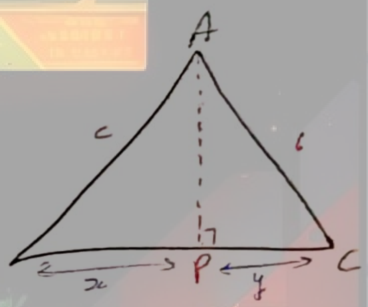


$$\boxed{a = b\cos C + c\cos B}$$

$$b = a\cos C + c\cos A$$

$$c = a\cos B + b\cos A$$

Proof:-



$a = x + y$

in ΔABP,

$$\cos B = \frac{x}{c}$$

$$c\cos B = x \quad -(1)$$

in ΔAPC,

$$\cos C = \frac{y}{b}$$

$$b\cos C = y \quad -(2)$$

(1) + (2)

$$x + y = \cancel{a\cos} \; b\cos C + c\cos B$$

$$a = b\cos C + c\cos B$$

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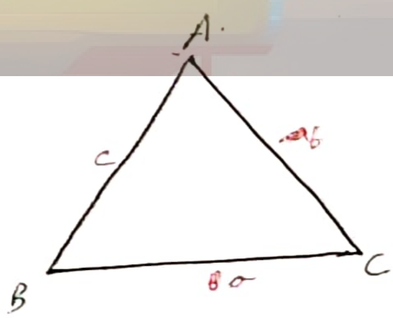
4. Tangent Law (Napier's analogy):-



$$\boxed{\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)}$$

$$\tan\left(\frac{A-C}{2}\right) = \frac{a-c}{a+c}\cot\left(\frac{B}{2}\right)$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

Proof:- $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = \lambda$

$b = \sin B\,\lambda$ —(1)

$c = \lambda \sin C$ —(2)

by (1) & (2)

$\therefore$ $$\frac{b-c}{b+c} = \frac{\lambda \sin B - \lambda \sin C}{\lambda \sin B + \lambda \sin C}$$

$$\frac{b-c}{b+c} = \frac{\cancel{\lambda}\left[2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)\right]}{\cancel{\lambda}\left[2\cos\left(\frac{B-C}{2}\right)\sin\left(\frac{B+C}{2}\right)\right]}$$

$$\frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)\cot\left(\frac{B+C}{2}\right)$$

$$\frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)\cot\left(\frac{180-A}{2}\right) \quad \{A+B+C=180\}$$

$$\frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)\cancel{\cot}\tan\left(\frac{A}{2}\right)$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$$

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Simplify

Q (1) $a \sin(B-C) + b\sin(C-A) + c\sin(A-B)$

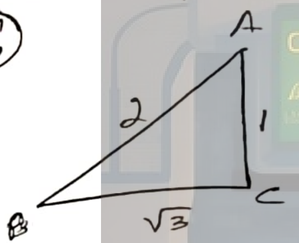
Q (2) Prove:- $a\cos\left(\frac{B-C}{2}\right) = (b+c)\sin\frac{A}{2}$

Q (3) Prove:- $\sin(B-C) = \frac{b^2-c^2}{a^2}\sin A$

Q (4) In ΔABC, If $a\cos A = b\cos B$ then Prove, $a=b$

Q (5) Angles of a Δ are in ratio 4:1:1. find ratio of its greatest side & perimeter.

Q (6)



Find $\angle C$

Q (7) In ΔABC, $(a^2-b^2-c^2)\tan A + (a^2-b^2+c^2)\tan B =$

(A) $(a^2+b^2-c^2)\tan C$

(B) $(b^2+c^2-a^2)\tan C$

(C) $(a^2+b^2+c^2)\tan C$

(D) None

Q1. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda$

$a = \sin A\,\lambda$

$\lambda \sin A \sin(B-C) + \lambda \sin B \sin(C-A) + \lambda \sin C \sin(A-B)$

$\frac{\lambda}{2}\Big[\cos(A-B+C) - \cos(A+B-C) + \cos(B-A+B-C) - \cos(-A+B+C)$

$+\cos(A-A+B+C) - \cos(A-B+C)\Big]$

$\boxed{=0}$

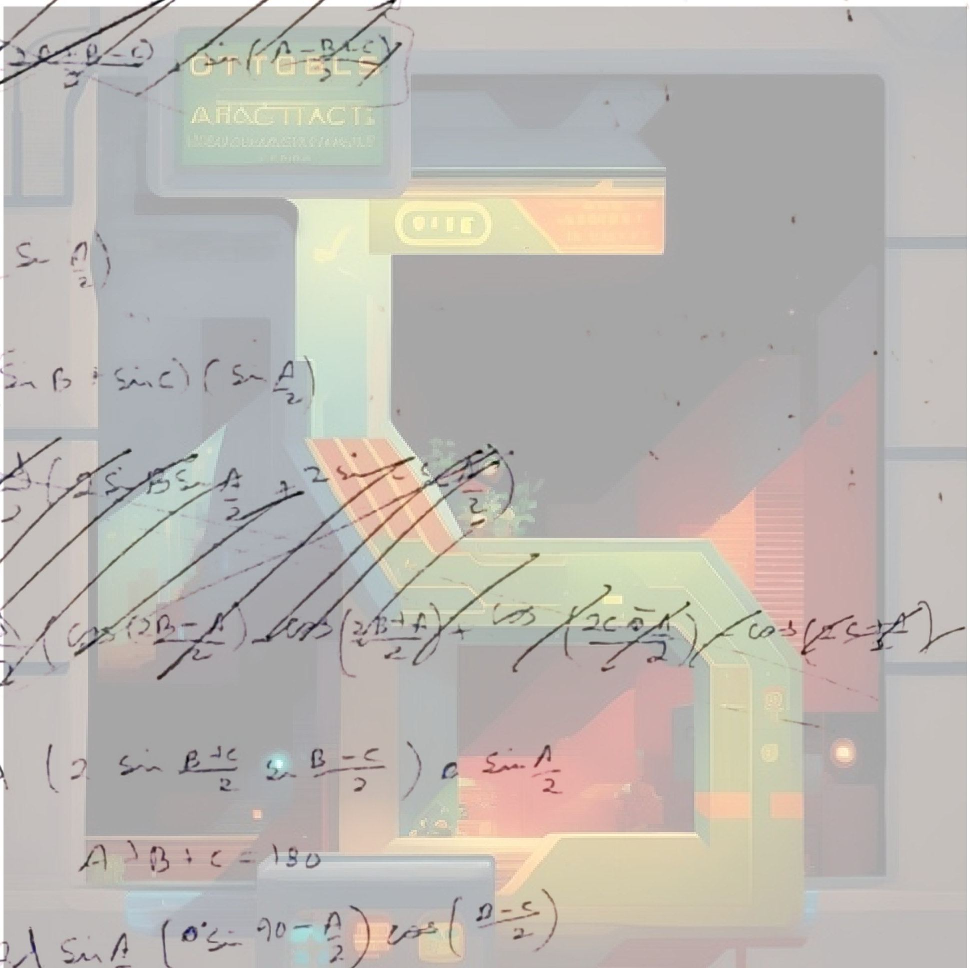
(2) LHS =

$= a\cos\left(\frac{B-C}{2}\right)$

$= \lambda \sin A \cos\left(\frac{B-C}{2}\right)$

$\cancel{= \frac{\lambda}{2}\left[\sin\left(A+\frac{B-C}{2}\right)+\sin\left(A-\frac{B-C}{2}\right)\right]}$

$\cancel{= \frac{\lambda}{2}\left[\sin\left(\frac{2A+B-C}{2}\right)+\sin\left(\frac{A-B+C}{2}\right)\right]}$



RHS =

$= (b+c)\left(\sin\frac{A}{2}\right)$

$= \lambda(\sin B+\sin C)\left(\sin\frac{A}{2}\right)$

$\cancel{= \frac{\lambda}{2}\left(2\sin B\sin\frac{A}{2}+2\sin C\sin\frac{A}{2}\right)}$

$\cancel{= \frac{\lambda}{2}\left(\cos\left(\frac{2B-A}{2}\right)-\cos\left(\frac{2B+A}{2}\right)+\cos\left(\frac{2C-A}{2}\right)-\cos\left(\frac{2C+A}{2}\right)\right)}$

$= \lambda\left(2\sin\frac{B+C}{2}\cos\frac{B-C}{2}\right)\sin\frac{A}{2}$

$A+B+C=180$

$= 2\lambda\sin\frac{A}{2}\left(\sin\left(90-\frac{A}{2}\right)\right)\cos\left(\frac{B-C}{2}\right)$

$= \lambda\, 2\sin\frac{A}{2}\cos\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$= \lambda\sin A\cos\left(\frac{B-C}{2}\right)$

$\boxed{LHS = RHS}$

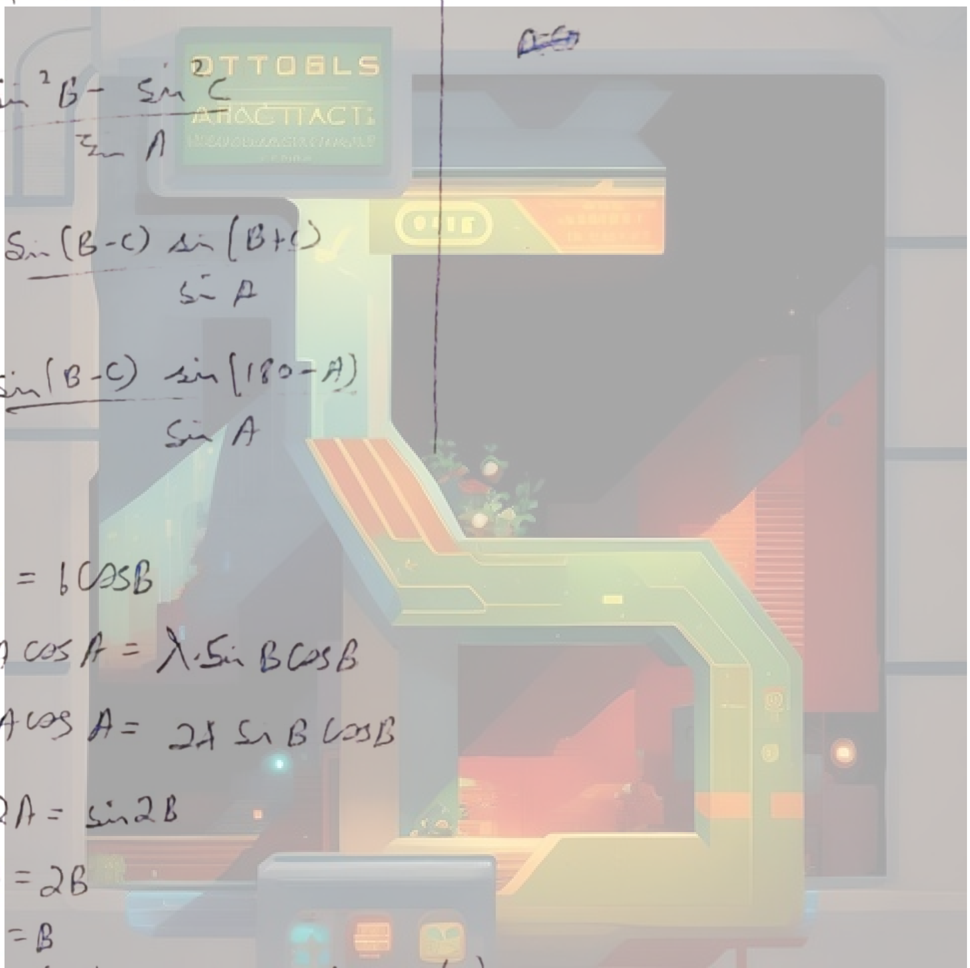
H.W

J-M, Q2 (TE)

Q3 RHS -

$$= \frac{b^2 - c^2}{a^2} \sin A$$

$$= \frac{\lambda^2 \sin^2 B - \lambda^2 \sin^2 C}{\lambda^2 \sin^2 A} \times \sin A$$



$$= \frac{\sin^2 B - \sin^2 C}{\sin A}$$

$$= \frac{\sin(B-C)\sin(B+C)}{\sin A}$$

$$= \frac{\sin(B-C)\sin(180-A)}{\sin A}$$

$$= \frac{\sin(B-C)\sin A}{\sin A}$$

$$= \sin(B-C) = LHS$$

LHS = RHS

Q4. $a\cos A = b\cos B$

$\lambda \sin A \cos A = \lambda \sin B \cos B$

$2\lambda \sin A \cos A = 2\lambda \sin B \cos B$

$\sin 2A = \sin 2B$

$2A = 2B$

$A = B$

$a = b$ (sides opp. to equal angles)

H.S.

Q5 Let angles be $4x, x, x$

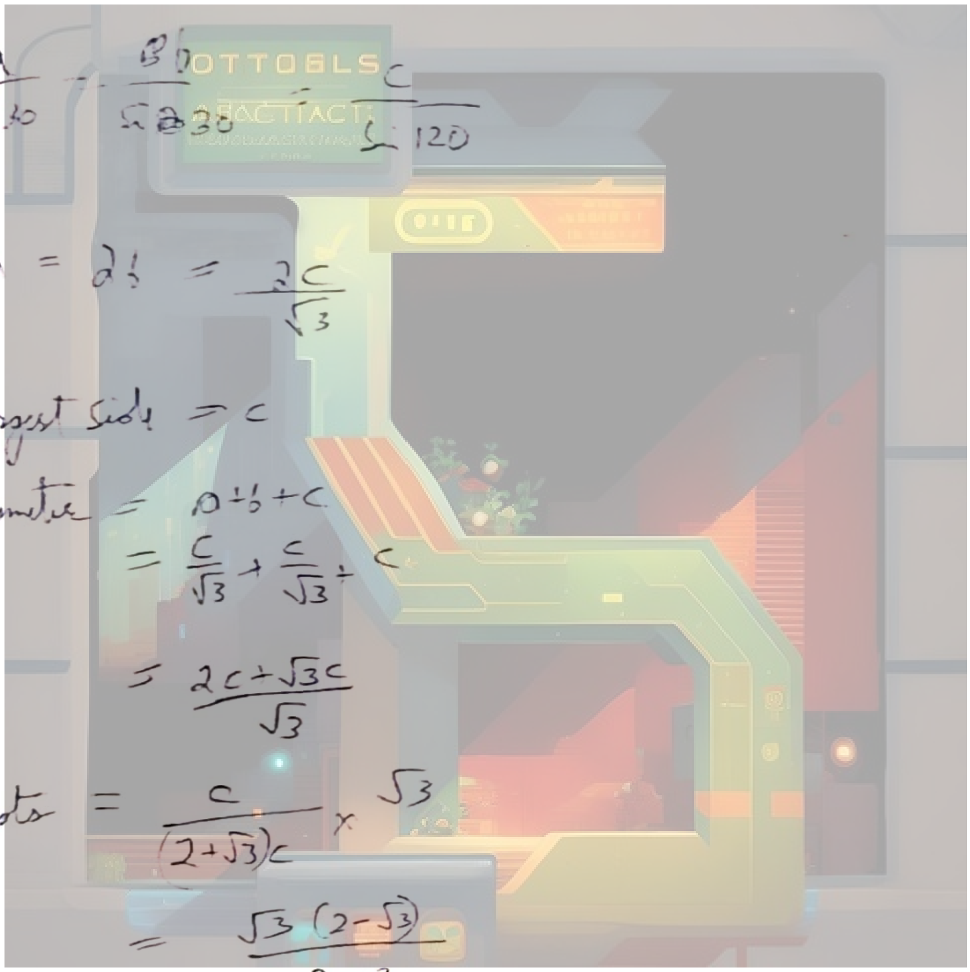
$$4x + x + x = 180$$

$$6x = 180$$

$$x = \frac{180}{6}$$

$$x = 30^\circ$$

~~largest side, s = s=~~



$$\frac{a}{\sin 30} = \frac{b}{\sin 30} = \frac{c}{\sin 120}$$

$$2a = 2b = \frac{2c}{\sqrt{3}}$$

largest side $= c$

perimeter $= a + b + c$

$$= \frac{c}{\sqrt{3}} + \frac{c}{\sqrt{3}} + c$$

$$= \frac{2c + \sqrt{3}c}{\sqrt{3}}$$

ratio $= \dfrac{c}{(2+\sqrt{3})c} \times \sqrt{3}$

$$= \frac{\sqrt{3}(2-\sqrt{3})}{2-3}$$

$$= (\sqrt{3}-2)\sqrt{3}$$

$$\boxed{\cancel{= 3 - 2\sqrt{3}}}$$

$$\boxed{3 - 2\sqrt{3} : 1}$$

Q6

$$\cos C = \frac{a^2+b^2-c^2}{2ba}$$

$$\cos C = \frac{3+1-4}{2\sqrt{3}}$$

$$\cos C = 0$$

$$\boxed{C = 90^\circ}$$

Q7 $-\frac{b\sin A}{\cos A}(b^2+c^2-a^2) + (a^2+c^2-b^2)\frac{\sin B}{\cos B}$

$$= -\frac{\sin A \, 2bc\cos A}{\cos A} + \frac{\sin B \, 2ac\cos B}{\cos B}$$

$$= 2ac\sin B - 2bc\sin A$$

$$\cancel{= 2c(a\sin B - b\sin A)}$$

$$= 2\times\sin B\times a\times\frac{\sin C\times b}{\sin B} - 2\times\sin A\times b\times\frac{\sin C\times a}{\sin A}$$

$$= 2ab\sin C - 2ab\sin C$$

$$\boxed{=0}$$

Q8. In $\triangle ABC$, $B = 30^\circ$ & $c = \sqrt{3}b$ find $\angle A$

$$\frac{\sin C}{c} = \frac{1}{2b}$$

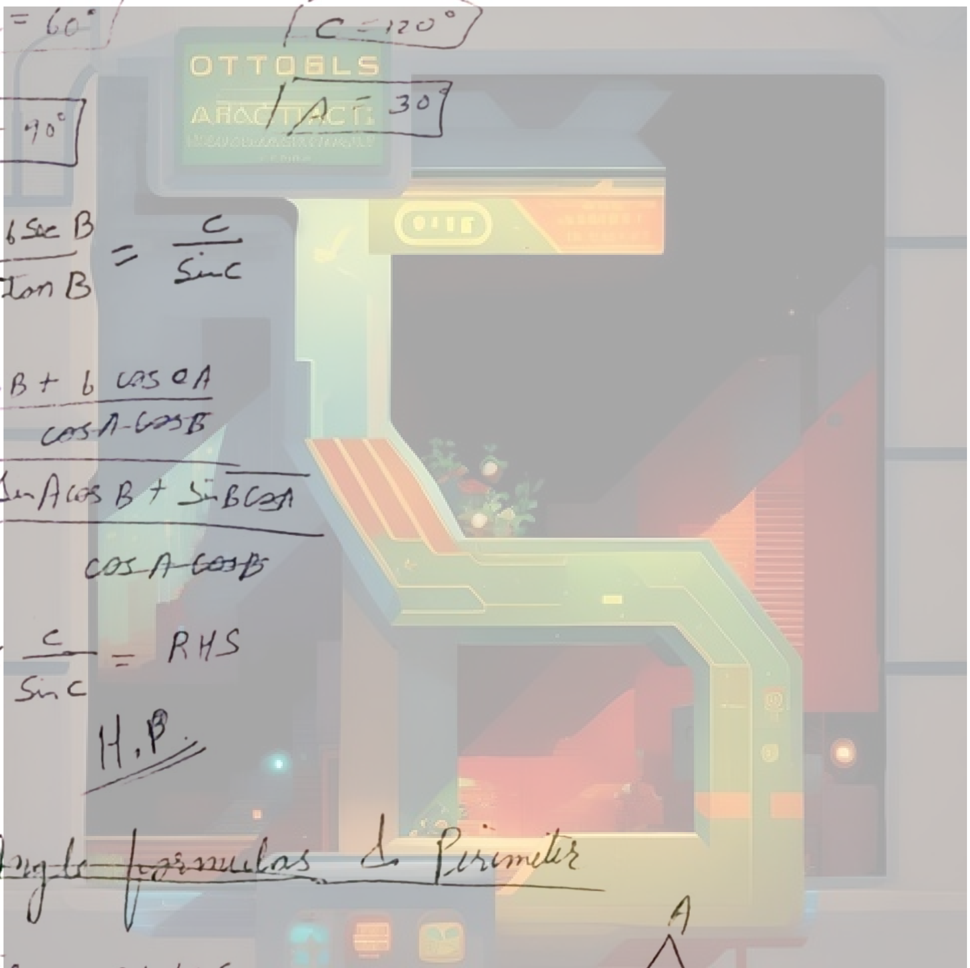
$$2b \sin C = c$$

$$2b \sin C = \sqrt{3}b$$

$$\sin C = \frac{\sqrt{3}}{2}$$

$\boxed{C = 60^\circ}$ $\boxed{C = 120^\circ}$

$\boxed{A = 90^\circ}$ $\boxed{A = 30^\circ}$



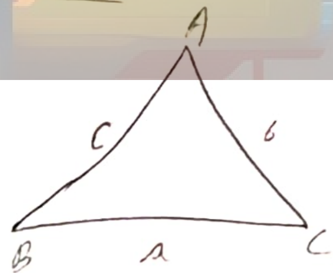
Q9. $\dfrac{a \sec A + b \sec B}{\tan A + \tan B} = \dfrac{c}{\sin C}$

$$= \frac{\dfrac{a \cos B + b \cos A}{\cos A \cos B}}{\dfrac{\sin A \cos B + \sin B \cos A}{\cos A \cos B}}$$

$$= \frac{c}{\sin C} = RHS$$

H.P.

Half Angle formulas & Perimeter



Perimeter = $a+b+c$

Semi Perimeter $(s) = \dfrac{a+b+c}{2}$

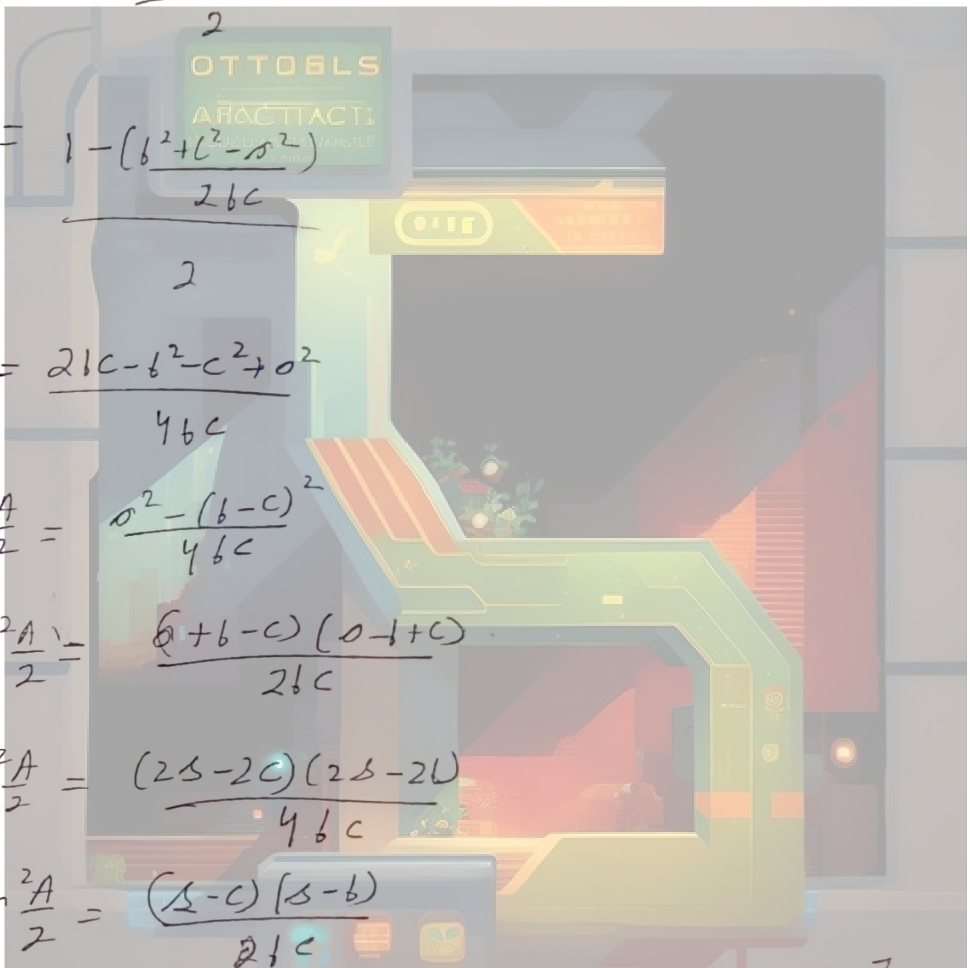
① $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$

$\boxed{\sin\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{ab}}}$ (5)

Proof:-

$\sin^2\frac{A}{2} = \cancel{1-\cos}\ \frac{1-\cos A}{2}$



$\sin^2\frac{A}{2} = \frac{1-\frac{(b^2+c^2-a^2)}{2bc}}{2}$

$\sin^2\frac{A}{2} = \frac{2bc-b^2-c^2+a^2}{4bc}$

$\sin^2\frac{A}{2} = \frac{a^2-(b-c)^2}{4bc}$

$\sin^2\frac{A}{2} = \frac{(a+b-c)(a-b+c)}{2bc}$

$\sin^2\frac{A}{2} = \frac{(2s-2c)(2s-2b)}{4bc}$

$\sin^2\frac{A}{2} = \frac{(s-c)(s-b)}{2bc}$

$\sin\frac{A}{2} = \sqrt{\frac{(s-c)(s-b)}{\cancel{2}bc}}$ [$\ominus$ve reject as its only possible when $A > 180$]

(2) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ (6)

$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$

$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

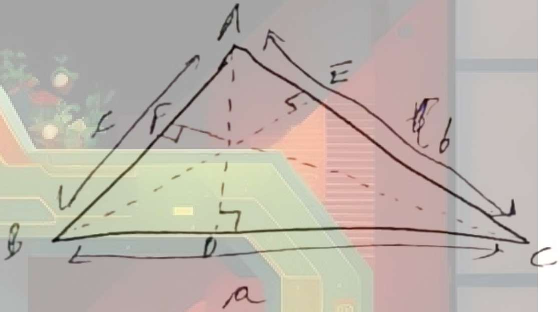
Proof:- $\cos^2\frac{A}{2} = \sqrt{1-\sin^2\frac{A}{2}}$

(3) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ (7)

$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\Delta}{s(s-b)}$; $\Delta \rightarrow$ area

$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\Delta}{s(s-c)}$

(4) Area of Triangle:-



(1) $\frac{1}{2} \times BC \times AD = \text{Area}$

$= \frac{1}{2} \times a \times AD$

$= \frac{1}{2} \times a \times c\sin B$

$= \frac{ac\sin B}{2}$

$$\boxed{\begin{aligned} \text{area} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}ac\sin B \end{aligned}}$$ (8)

(2) $\boxed{\text{area} = ab\sin\frac{C}{2}\cos\frac{C}{2}}$ (9)

$= ac\sin\frac{B}{2}\cos\frac{B}{2}$

$= bc\sin\frac{A}{2}\cos\frac{A}{2}$

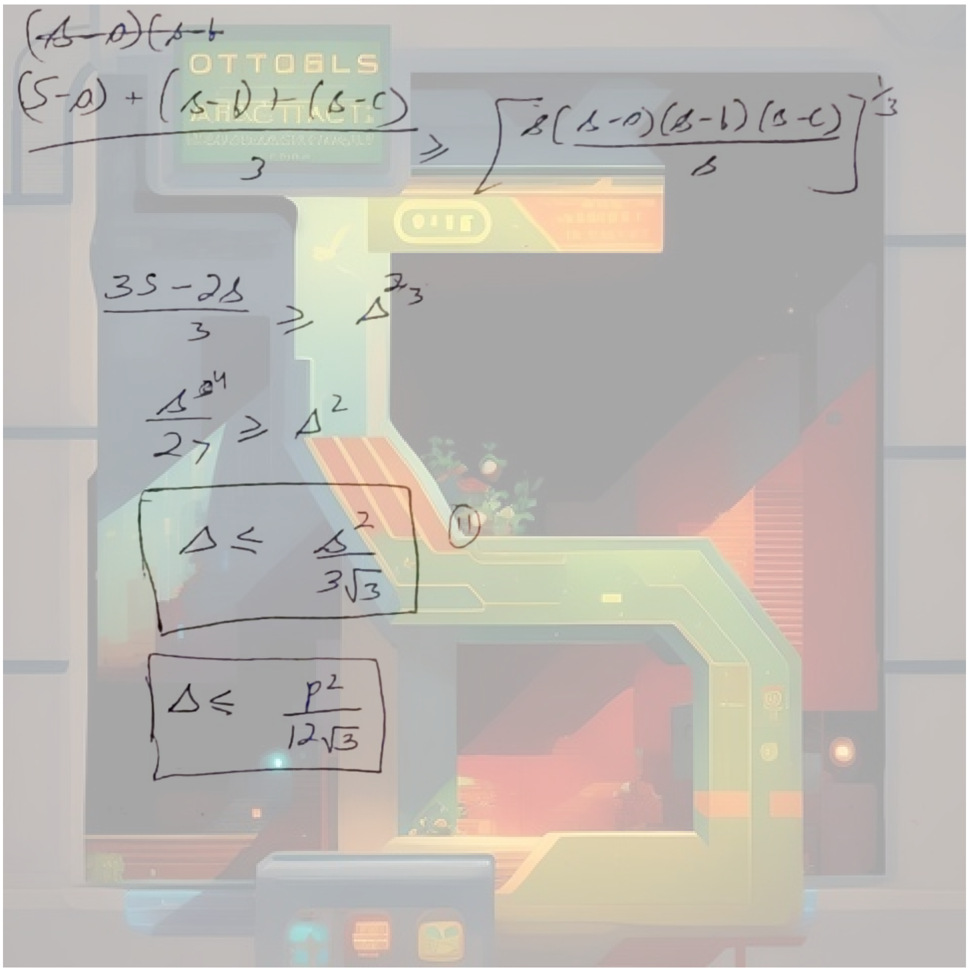
③ $\Delta = bc \sin\frac{A}{2} \cos\frac{A}{2}$

$$= bc\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-a)}{bc}}$$

$$\boxed{= \sqrt{s(s-a)(s-b)(s-c)}} \quad ⑩$$

<u>Note</u>:- Relation in Area of Triangle & Perimeter of Triangle

$AM \geq GM$



$\cancel{(s-a)(s-b}$

$$\frac{(s-a)+(s-b)+(s-c)}{3} \geq \left[\frac{s(s-a)(s-b)(s-c)}{s}\right]^{1/3}$$

$$\frac{3s-2s}{3} \geq \Delta^{2/3}$$

$$\frac{s^4}{27} \geq \Delta^2$$

$$\boxed{\Delta \leq \frac{s^2}{3\sqrt{3}}} \quad ⑪$$

$$\boxed{\Delta \leq \frac{p^2}{12\sqrt{3}}}$$

Q10. If $b+c=3a$ find $\cot\frac{B}{2}\cot\frac{C}{2}$

$$\cot\frac{B}{2} = \sqrt{\frac{ac(s-b)\,\cancel{s}}{(s-a)(s-b)\,\cancel{s}}}$$

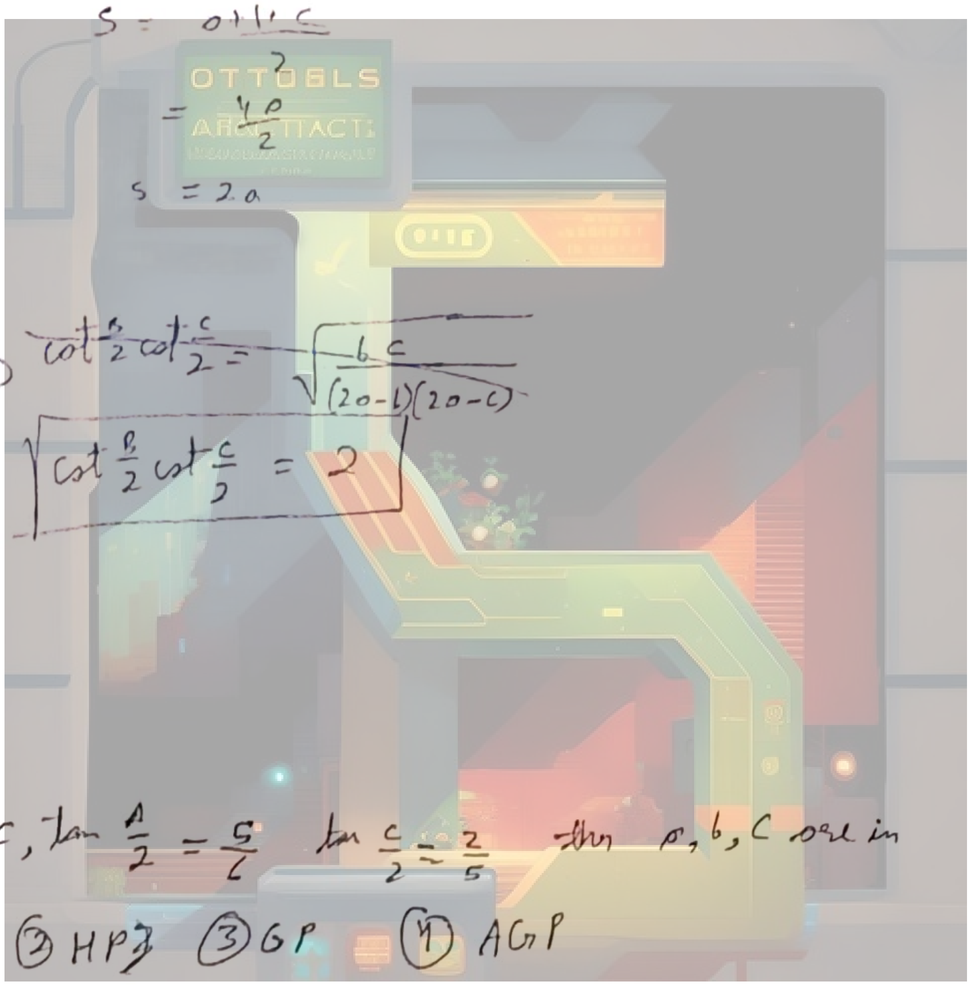
$$\cot\frac{C}{2} = \sqrt{\frac{ab(s-c)\,\cancel{s}}{(s-a)(s-c)\,\cancel{s}}}$$

$$\cot\frac{B}{2}\cot\frac{C}{2} = \sqrt{\frac{ac \times ab \times 4a^2}{(s-a)^2(s-b)(s-c)}}$$

$$s = \frac{a+b+c}{2}$$

$$= \frac{4a}{2}$$

$$s = 2a$$



$$\cancel{\cot\frac{B}{2}\cot\frac{C}{2} = \sqrt{\frac{bc}{(2a-b)(2a-c)}}}$$

$$\boxed{\cot\frac{B}{2}\cot\frac{C}{2} = 2}$$

Q11. In ΔABC, $\tan\frac{A}{2} = \frac{5}{6}$ $\tan\frac{C}{2} = \frac{2}{5}$ then a, b, c are in

(1) AP (2) HP (3) GP (4) AGP

$$\cancel{\frac{A}{B} + \frac{x5}{6} + \frac{2x}{5} + \frac{1}{3} = 1}$$

$$\cancel{25x + 12x = \frac{2}{3} \times 30}$$

$$\cancel{37x = 20}$$

$$\cancel{x = \frac{20}{37}}$$

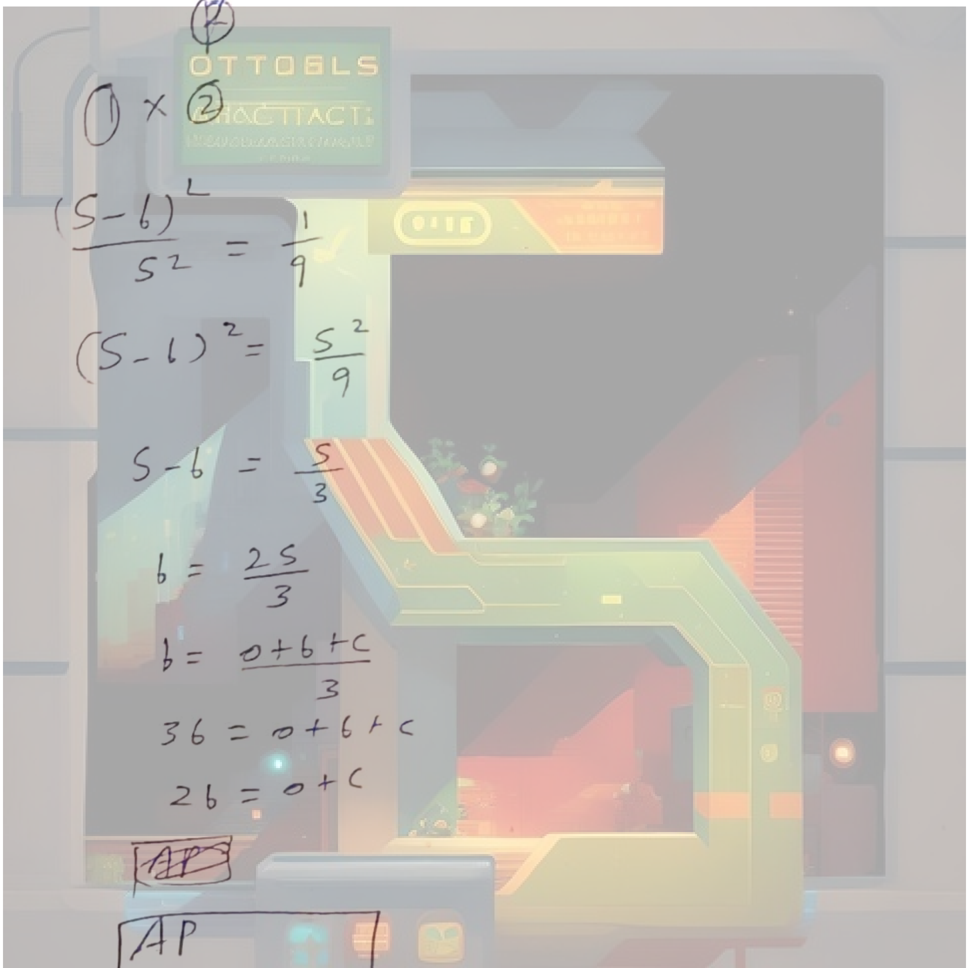
$$\cancel{\tan\frac{B}{2} = \frac{30}{37}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{5}{6}$$

$$\frac{(s-b)(s-c)}{s(s-a)} = \frac{25}{36} \quad - (1)$$

$$\frac{(s-a)(s-b)}{s(s-c)} = \frac{4}{25} \quad - (2)$$

~~So~~ $\cancel{\frac{(1)}{(2)}}$



$(1) \times (2)$

$$\frac{(s-b)^2}{s^2} = \frac{1}{9}$$

$$(s-b)^2 = \frac{s^2}{9}$$

$$s-b = \frac{s}{3}$$

$$b = \frac{2s}{3}$$

$$b = \frac{a+b+c}{3}$$

$$3b = a+b+c$$

$$2b = a+c$$

~~AP~~

$$\boxed{AP}$$

Q 12. ΔABC, if $\tan\frac{A}{2}\tan\frac{B}{2}=\frac{1}{3}$ & $ab=4$ then $\min(c)=?$

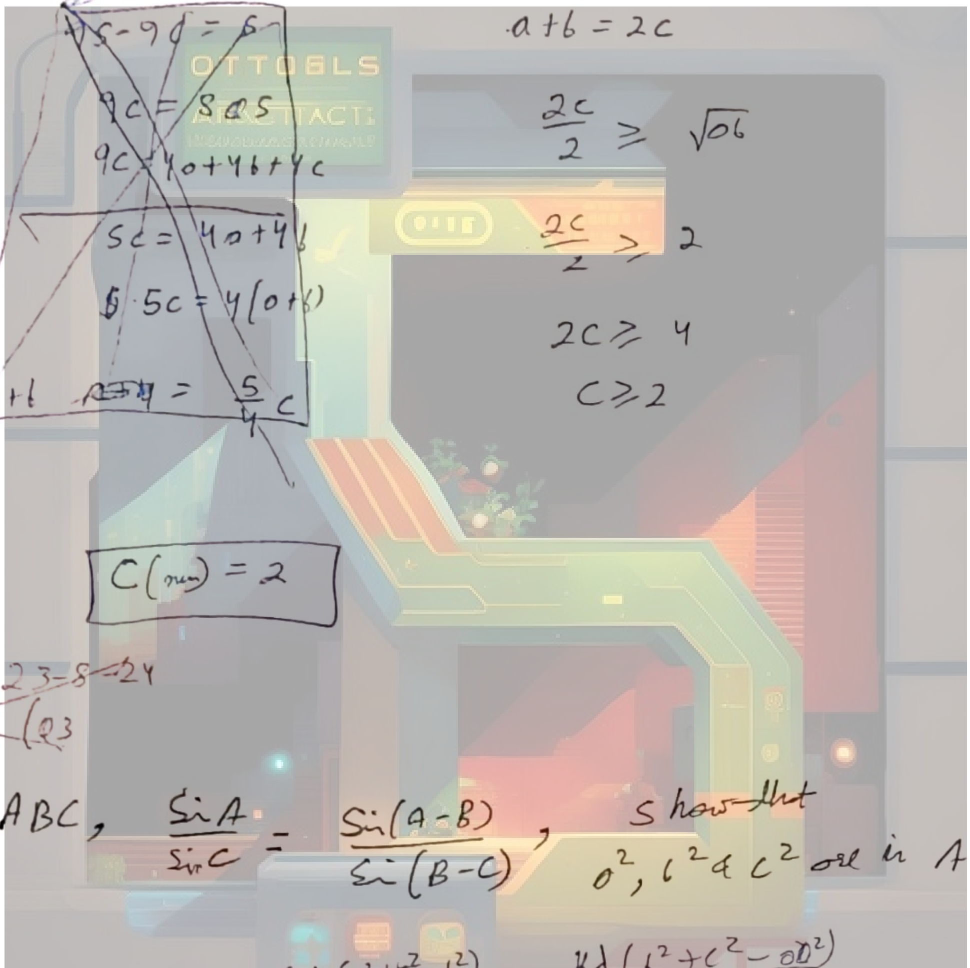
$$\sqrt{\frac{(s-b)(s-c)}{s(s-a)}\times\frac{(s-a)(s-c)}{s(s-b)}}=\frac{1}{3}$$

$$\frac{s-c}{s}=\frac{1}{\cancel{9}\,3}$$

$3s-3c=s$

$2s=3c$

$a+b=2c$



~~$s=$~~

~~$s-9c=s$~~

~~$9c=8as$~~

~~$9c=4a+4b+4c$~~

~~$5c=4a+4b$~~

~~$5c=4(a+b)$~~

~~$a+b=\frac{5}{4}c$~~

$\frac{2c}{2}\geq\sqrt{ab}$

$\frac{2c}{2}\geq 2$

$2c\geq 4$

$c\geq 2$

$\boxed{C(\min)=2}$

H.W. 23-8-24

DYS-X (Q3

Q 13. In ΔABC, $\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$, show that a^2, b^2 & c^2 are in A.P.

$$\frac{a}{c}=\frac{\dfrac{b\lambda(a^2+c^2-b^2)}{ac}-\dfrac{b\lambda(b^2+c^2-a^2)}{bc}}{\dfrac{b\lambda(a^2+b^2-c^2)}{ab}-\dfrac{a\lambda(a^2+c^2-b^2)}{ac}}$$

~~$\frac{a}{c}=\frac{\frac{1}{2c}(2c^2)}{\frac{1}{2a}(2b^2)}$~~

~~$\frac{a}{c}=\frac{2c}{2a}$~~

~~$\boxed{2b^2=c^2}$~~

$$\frac{a}{c}=\frac{a^2-b^2}{c^2}\times\frac{c^2}{b^2-c^2}$$

~~$c(b^2-c^2)=a(a^2-b^2)$~~

$b^2-c^2=a^2-b^2$

$\boxed{2b^2=a^2+c^2}$

(118)

H.W. 24-8-24

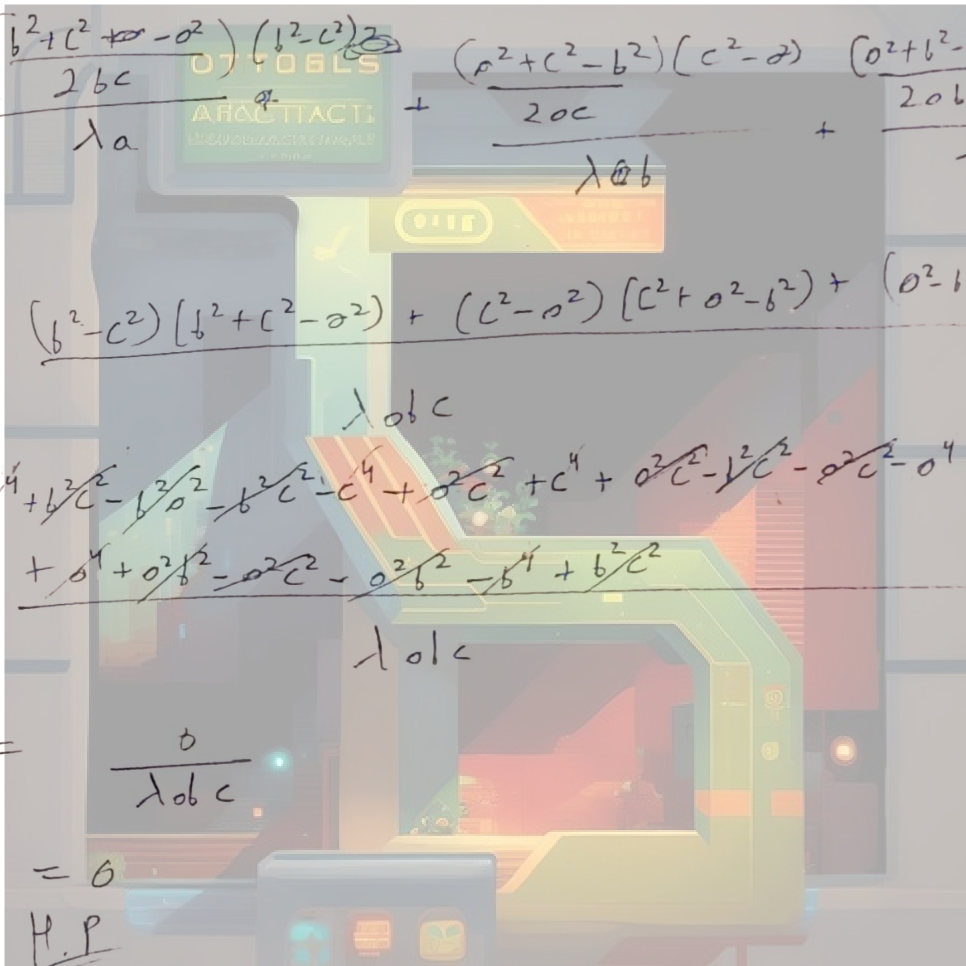
DYS-1 (Q1, 2, 4, 7, 8) & others

DYS-3

DYS-2 – (Q2, 8) mahi karne

DYS-1

Q5 $(b^2-c^2)\cot A + (c^2-a^2)\cot B + (a^2-b^2)\cot C = 0$ (prove)



$$= \frac{\left(\frac{b^2+c^2-a^2}{2bc}\right)(b^2-c^2)}{\lambda a} + \frac{\frac{(a^2+c^2-b^2)(c^2-a^2)}{2ac}}{\lambda ab} + \frac{\frac{(a^2+b^2-c^2)(a^2-b^2)}{2ab}}{\lambda c}$$

$$= \frac{(b^2-c^2)(b^2+c^2-a^2) + (c^2-a^2)(c^2+a^2-b^2) + (a^2-b^2)(a^2+b^2-c^2)}{\lambda abc}$$

$$= \frac{b^4 + \cancel{b^2c^2} - \cancel{b^2a^2} - \cancel{b^2c^2} - \cancel{c^4} + \cancel{a^2c^2} + \cancel{c^4} + \cancel{a^2c^2} - \cancel{b^2c^2} - \cancel{a^2c^2} - a^4 + \cancel{a^2b^2} + \cancel{a^4} + \cancel{a^2b^2} - \cancel{a^2c^2} - \cancel{a^2b^2} - \cancel{b^4} + \cancel{b^2c^2}}{\lambda abc}$$

$$= \frac{0}{\lambda abc}$$

$= 0$

H.P

M II

$b = \lambda \cos B$

$(\lambda^2 \sin^2 B - \lambda^2 \sin^2 C)\cot A$

$\lambda^2 \sin(B+C)\sin(B-C)\cot A$

$\lambda^2 \sin(180-A)\sin(B-C)\frac{\cot A}{\sin A}$

$\lambda^2 \sin(B-C)\cos A + \lambda^2 \sin(C-A)\cos B + \lambda^2 \sin(a-b)\cos C$

$= 0$

Q6. $b^2\cos 2A - a^2\cos 2B = b^2 - a^2$

(M I)

~~$\lambda^2\cos^2 B\cos \times 2\sin A\cos A$~~

~~$\lambda^2\cos^2 B \times 2\sin A$~~

~~$\lambda^2\cos^2 B(\cos^2 A - \sin^2 A) = \lambda^2\cos^2 A\cos^2$~~

$= \lambda^2\sin^2 B\cos 2A - \lambda^2\sin^2 A\cos 2B$

$= \lambda^2\sin^2 B(\cos^2 A - \sin^2 A) - \lambda^2\sin^2 A(\cos^2 B - \sin^2 B)$

$= \lambda^2[\sin^2 B\cos^2 A - \cancel{\sin^2 B\sin^2 A} - \sin^2 A\cos^2 B + \cancel{\sin^2 A\sin^2 B}]$

$= \lambda^2[\sin^2 B\cos^2 A - \sin^2 A\cos^2 B]$

$= b^2 - a^2$

Q9. $\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}$ A, B, C in A.P.

find A, B, C

~~$2A+B = 30°$~~

~~$C-A = 30°$~~

~~$B+2C = -30°$~~

~~$2A-2C = 60$~~

~~$A-C = 30°$~~

~~$A = C$~~

~~$A+B+C = 180$~~

~~$2A+2C+A+C = 180$~~

~~$A+C = 60$~~

~~$A-C = 30$~~

~~$\boxed{A = 15°,\ B = C = 45°,\ B = 30°}$~~ X

$2A+2C+A+C = 180°$

$A+C = 120$

$C-A = 30$

$2C = 150$

$\boxed{C = 75°,\ A = 45°,\ B = 60°}$

PYS - 2

Q 4. A, B, C in AP

$$2\cos\left(\frac{A-C}{2}\right) = \frac{a+c}{\sqrt{a^2+c^2-ac}}$$

M I

LHS

$$= 2\sqrt{\frac{\cos(A-C)+1}{2}}$$

$$= \sqrt{2}\sqrt{\cos A\cos C+\sin A\sin C+1}$$

$$= \sqrt{2}\sqrt{\left(\frac{b^2+c^2-a^2}{2bc}\right)\left(\frac{a^2+b^2-c^2}{2ba}\right)+\lambda^2 ac+1}$$

$\frac{\sin B}{b} = \lambda$

$\frac{\sqrt{3}}{2b} = \lambda$

$b = \frac{\sqrt{3}}{2\lambda}$

$b^2 = \frac{3}{4\lambda^2}$

Note :- If A, B, C in AP, $B = 60^\circ$ in Δ

$$= \sqrt{2}\sqrt{\frac{3 \quad 4c^2\lambda^2 - 4a^2\lambda^2}{4\lambda^2 \times 2\frac{\sqrt{3}}{2\lambda}c} \times \frac{3 \quad 4a^2\lambda^2 - 4c^2\lambda^2}{4\lambda^2 \times 2\frac{\sqrt{3}}{2\lambda}a} + \frac{\lambda^2 ac \quad 4ac \times \frac{3}{4\lambda^2}}{4\lambda^2 \times 2\frac{\sqrt{3}}{2\lambda}ac}\lambda^2 ac+1}$$

$$= \sqrt{2}\sqrt{\quad}$$

M II

$A+B+C = 180^\circ$

$2B = A+C$

$B = 60$

$A+C = 120^\circ$

RHS:

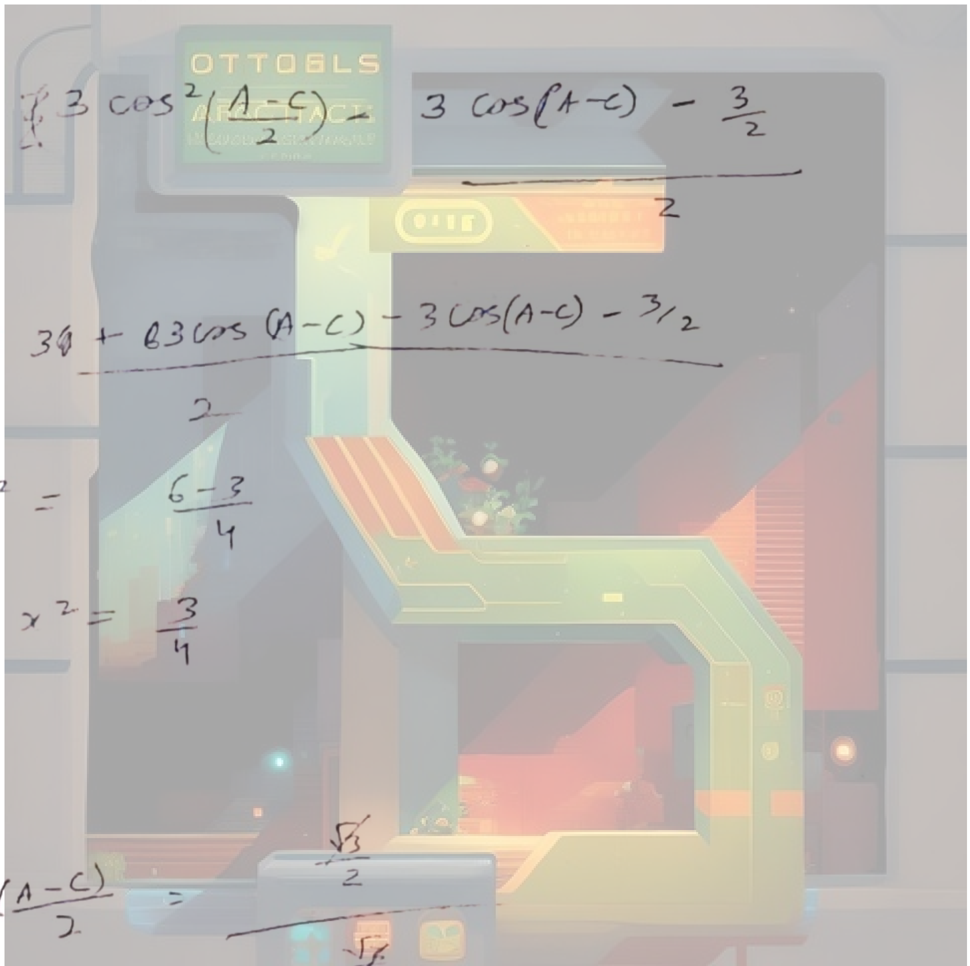
$$= \frac{\sin A+\sin C}{\sqrt{\sin^2 A+\sin^2 C-\sin A\sin C}}$$

$$= \frac{\sqrt{3}\cos\left(\frac{A-C}{2}\right)}{\sqrt{\sin^2 A+\sin^2 C-\sin A\sin C}}$$

$= 2\cos\frac{A-C}{2}$ $\quad \frac{\sqrt{3}}{2}$

$$\sqrt{(\sin A + \sin C)^2 - \frac{6\sin A \sin C}{2}}$$

$$x^2 = \left(2\sin\frac{A+C}{2}\cos\frac{A-C}{2}\right)^2 - \frac{3\cos(A-C) + 3\cos 120}{2}$$



$$x^2 = \frac{3\cos^2\left(\frac{A-C}{2}\right) - 3\cos(A-C) - \frac{3}{2}}{2}$$

$$x^2 = \frac{3 + 3\cos(A-C) - 3\cos(A-C) - 3/2}{2}$$

$$x^2 = \frac{6-3}{4}$$

$$x^2 = \frac{3}{4}$$

LHS

$= 2\cos\left(\frac{A-C}{2}\right) = \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}/2}$

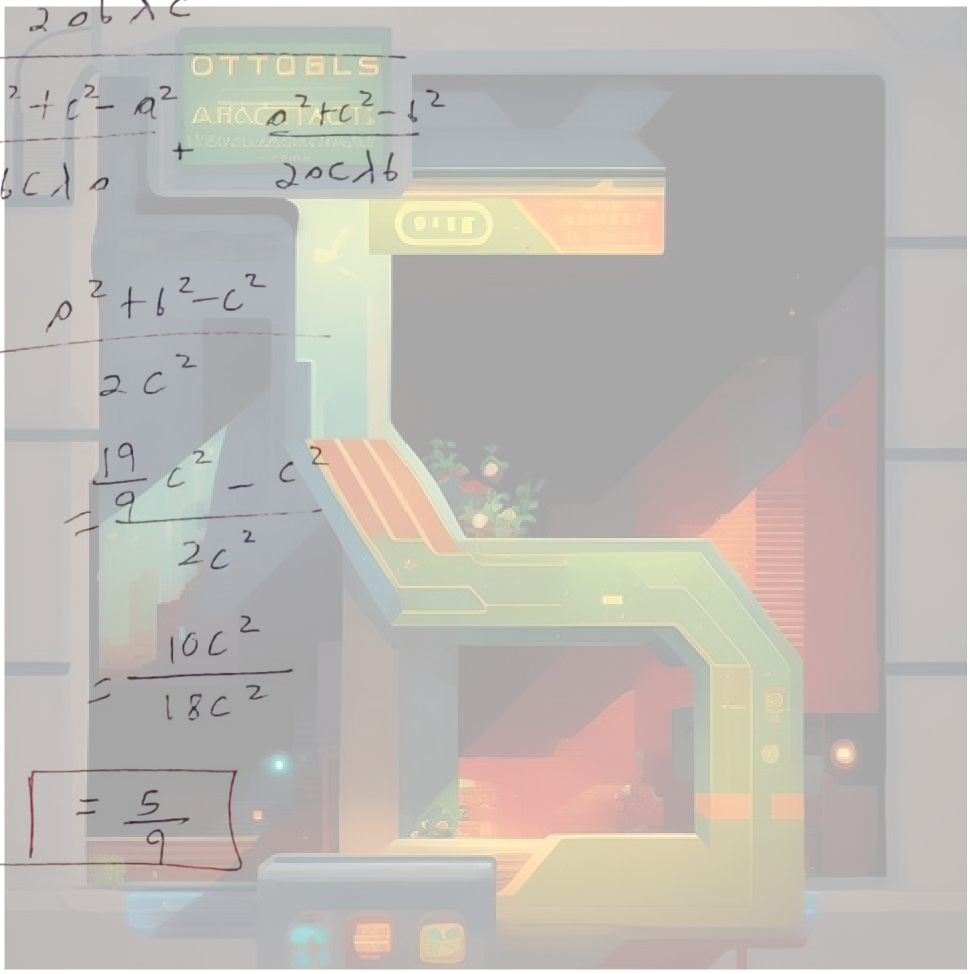
$=$ RHS

अतः सिद्ध हुआ

DYS2

Q8. If $9a^2 + 9b^2 = 19c^2$, find $\dfrac{\cot C}{\cot A + \cot B}$

$$= \frac{\dfrac{\cos C}{\sin C}}{\dfrac{\cos A}{\sin A} + \dfrac{\cos B}{\sin B}}$$



$$= \frac{\dfrac{a^2+b^2-c^2}{2ab\lambda c}}{\dfrac{b^2+c^2-a^2}{2bc\lambda a} + \dfrac{a^2+c^2-b^2}{2ac\lambda b}}$$

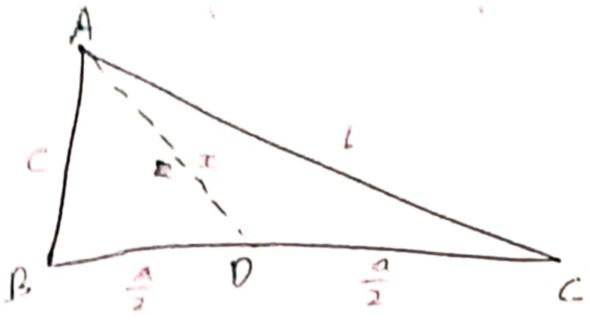
$$= \frac{a^2+b^2-c^2}{2c^2}$$

$$= \frac{\frac{19}{9}c^2 - c^2}{2c^2}$$

$$= \frac{10c^2}{18c^2}$$

$$\boxed{= \frac{5}{9}}$$

11 Length of Median and Angle Bisector



x is median.

in $\triangle ADC$, $\cos C = \dfrac{a^2+4b^2-4x^2}{4ab}$ —(1)

in $\triangle ABC$, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$ —(2)

(1) = (2)

$a^2+4b^2-4x^2 = 2a^2+2b^2-2x^2$

$2b^2 = a^2+2x^2$

$x^2 = \dfrac{2b^2-a^2}{2}$

$x = \sqrt{2b^2}$

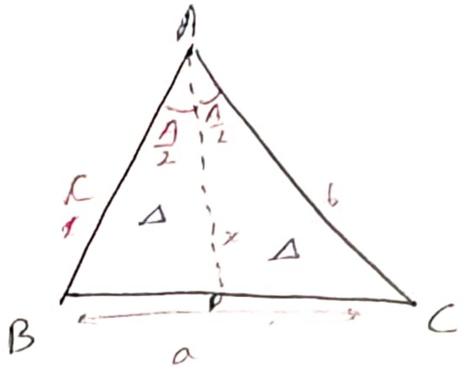
$a^2+4b^2-4x^2 = 2a^2+2b^2-2c^2$

$2b^2-a^2+2c^2 = 4x^2$

$x^2 = \dfrac{2b^2-a^2+2c^2}{4}$

$$\boxed{x = \dfrac{\sqrt{2b^2-a^2+2c^2}}{2}}$$ (12)

Length of Angle Bisector



$$\text{Area}(\Delta ADB) = \frac{1}{2} c x \sin\frac{A}{2} \quad — (1)$$

$$\text{area}(\Delta ADC) = \frac{1}{2} b x \sin\frac{A}{2} \quad — (2)$$

$$\text{area}(\Delta ABC) = \cancel{\frac{1}{2} bc \sin A}$$

$$= \frac{1}{2} bc \sin A \quad — (3)$$

(1) + (2) = (3)

$$\frac{c x \sin\frac{A}{2} + b x \sin\frac{A}{2}}{2} = \frac{1}{2} bc \sin A$$

$$x\left(c \sin\frac{A}{2} + b \sin\frac{A}{2}\right) = 2bc \sin\frac{A}{2}\cos\frac{A}{2}$$

$\cancel{x}$

$\cancel{cx+bx}$

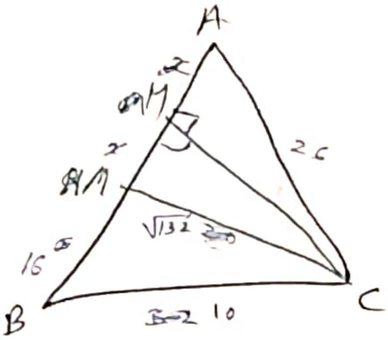
$$x(b+c) = 2bc\cos\frac{A}{2}$$

$$\boxed{x = \frac{2bc\cos\frac{A}{2}}{b+c}} \quad (13)$$

Note :- Formula contains HM of b & c

Q in $\triangle$, CH & CM are lengths of altitude & Median to base AB

If $a=10$, $b=26$ & $c=32$ find HM.



$CM = \frac{\sqrt{2b^2+2c^2-a^2}}{2}$

~~$CM = 30$~~

~~$\cos A = \frac{x}{26}$~~

~~$\cos B = \frac{32-x}{10}$~~

~~$\frac{10}{x} = \frac{26}{32-x}$~~

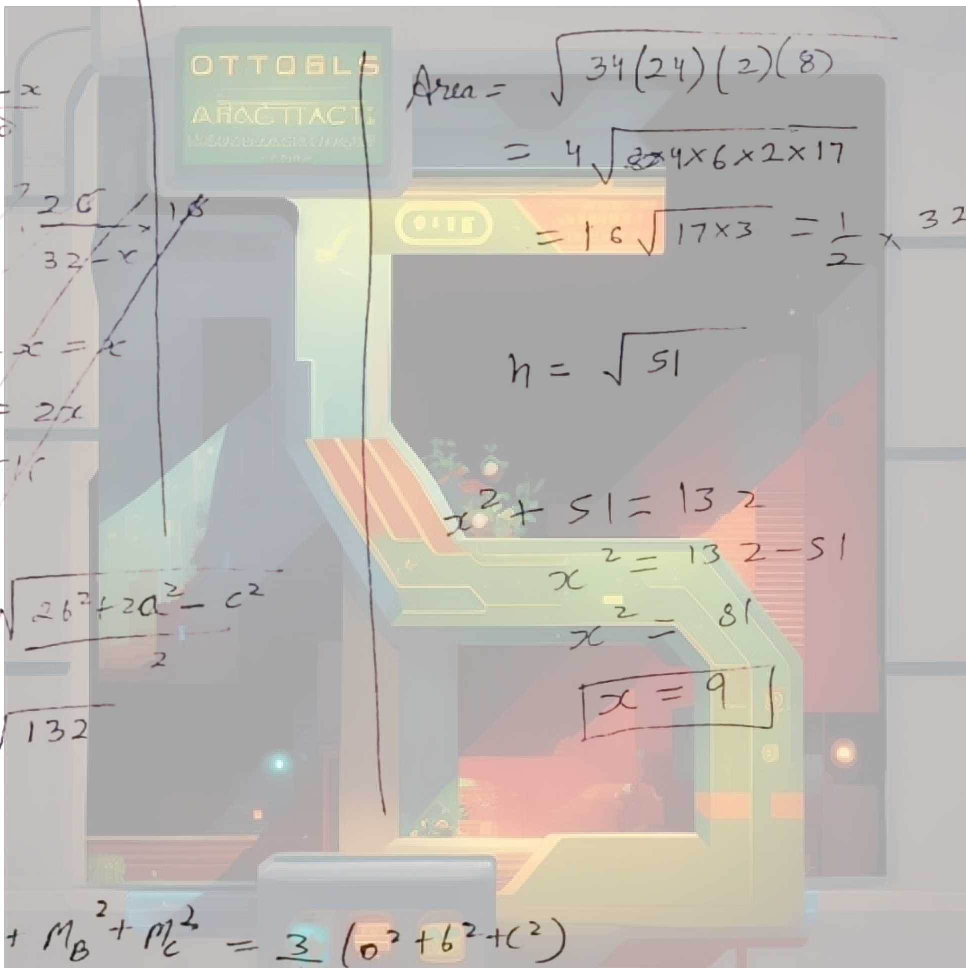
~~$32 - x = x$~~

~~$32 = 2x$~~

~~$x = 16$~~

$CM = \frac{\sqrt{2b^2+2a^2-c^2}}{2}$

$CM = \sqrt{132}$



$\text{Area} = \sqrt{34(24)(2)(8)}$

$= 4\sqrt{4\times6\times2\times17}$

$= 16\sqrt{17\times3} = \frac{1}{2}\times32\times h$

$h = \sqrt{51}$

$x^2 + 51 = 132$

$x^2 = 132 - 51$

$x^2 = 81$

$\boxed{x = 9}$

Note:- $M_A^2 + M_B^2 + M_C^2 = \frac{3}{4}(a^2+b^2+c^2)$

Proof:-

$\frac{2a^2+2b^2-c^2 + 2b^2+2c^2-a^2 + 2a^2+2c^2-b^2}{4}$

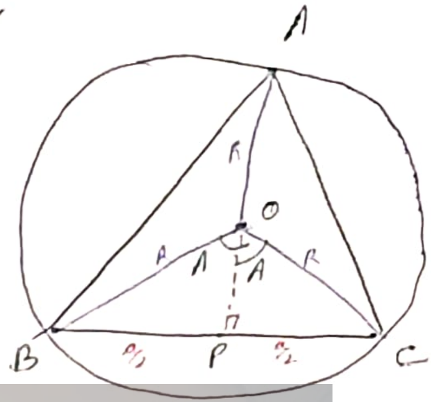
$= \frac{3(a^2+b^2+c^2)}{4}$

H.P.

(126)

Circumcircle & in circle

Circumcircle :- A circle passes through the vertices of a triangle & radius is called circum-radius (R).



$$\sin A = \frac{a}{2} \times \frac{1}{R}$$

$$\frac{\sin A}{a} = \frac{1}{2R}$$

$$\therefore \boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R} \quad (14)$$

$$\text{Area}(\Delta ABC) = \frac{1}{2} bc \sin A$$

$$\Delta = \frac{abc}{4R}$$

$$\boxed{R = \frac{abc}{4\Delta}} \quad (15)$$

Q In ΔABC, P.T.

$$s = 4R \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$$

$$s = 4R\sqrt{\frac{s(s-a)\, s(s-c)\, s(s-b)}{bc \times ab \times abc}}$$

$$s = \frac{4R}{abc}\, s\sqrt{s(s-a)(s-b)(s-c)}$$

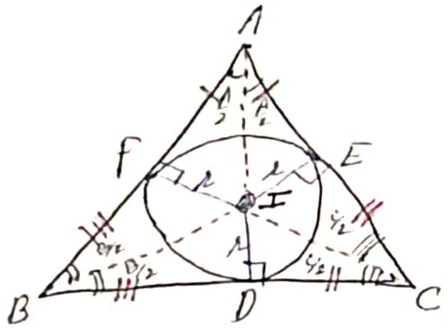
$$s = \frac{4R}{abc}\, s\Delta$$

$$s = 4 \times \frac{abc}{4\Delta} \times \frac{1}{abc} \times \Delta \times s$$

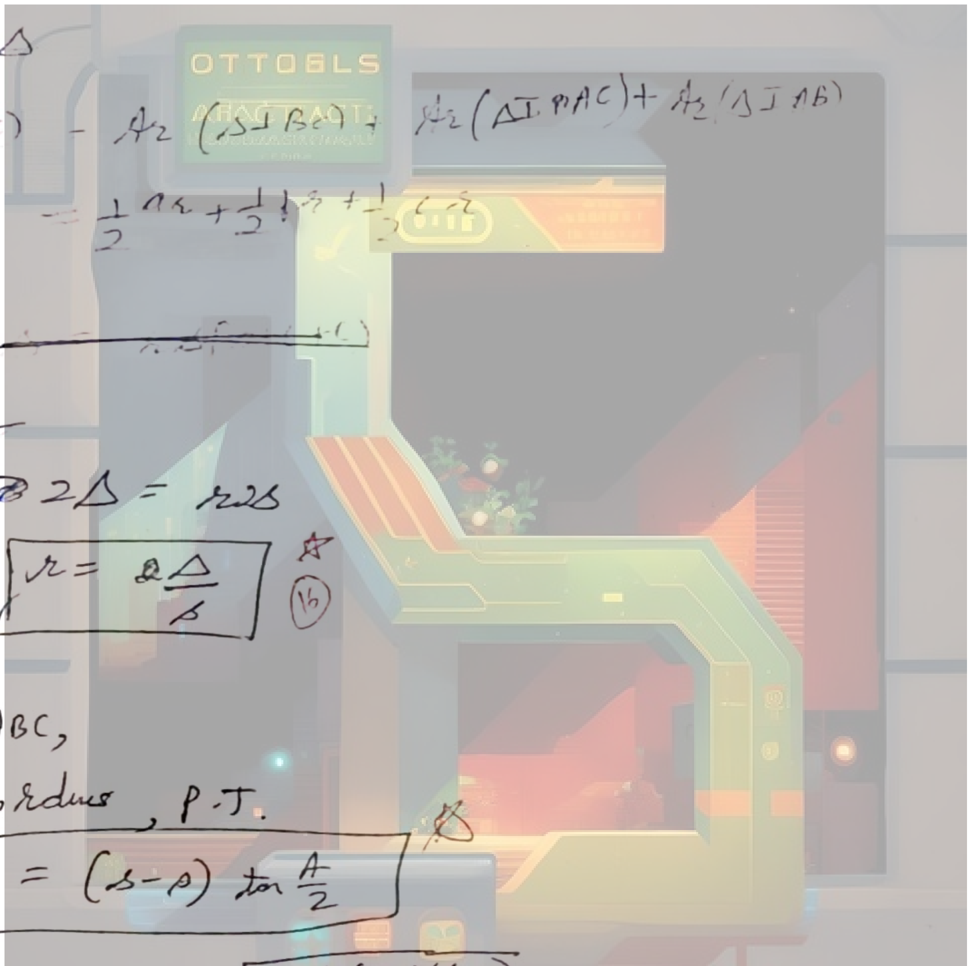
$$s = s$$

H.P

Incircle: The circle touching all the 3 sides of a triangle internally is called incircle and radius of this circle is called in-radius (r).



→ All the 3 sides of the Δ are $\perp$ to the in-radius



$\Delta = \frac{1}{2}\Delta$

$A(\Delta ABC) = A_2(\Delta IBC) + A_2(\Delta IAC) + A_2(\Delta IAB)$

$\Delta = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$

$2\Delta = r2s$

$\boxed{r = \frac{2\Delta}{s}}$ (15)

① In a ΔABC,
r is inradius, P.T.

$\boxed{r = (s-a)\tan\frac{A}{2}}$

$= (s-a)\sqrt{\frac{s(s-b)(s-c)(s-a)}{s^2(s-a)^2}}$

$= \frac{(s-a)}{s(s-a)} \times \Delta$

$= \frac{\Delta}{s}$

$= r$

H.P

Relation in Circumcircle & Inradius

$$\boxed{r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}} \quad (17)$$

Proof:- [M-I]

RHS

$$= 4R \times \sqrt{\frac{\cancel{s^2}\, s(s-b)(s-c)(s-a)(s-c)(s-a)(s-b)}{bc \times ac \times ab}}$$

$$= \frac{4R}{abc} \times s(s-a)(s-b)(s-c)$$

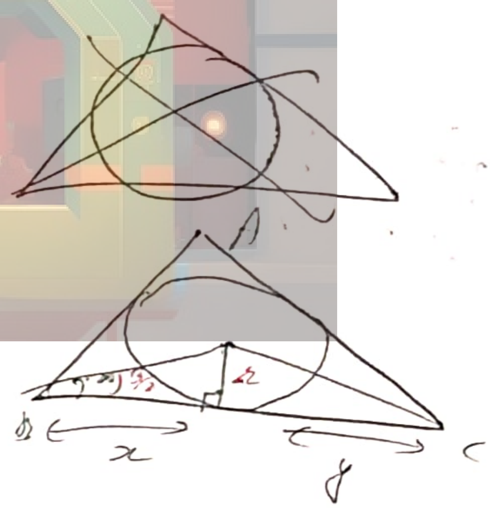
$$= \frac{1}{\Delta}(s-a)(s-b)(s-c) \times \frac{s}{s}$$

$$= \frac{\Delta^2}{\Delta s}$$

$$\boxed{= \frac{\Delta}{s}}$$

H.P

[M-II]



$$a = x + y$$

$$a = r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right)$$

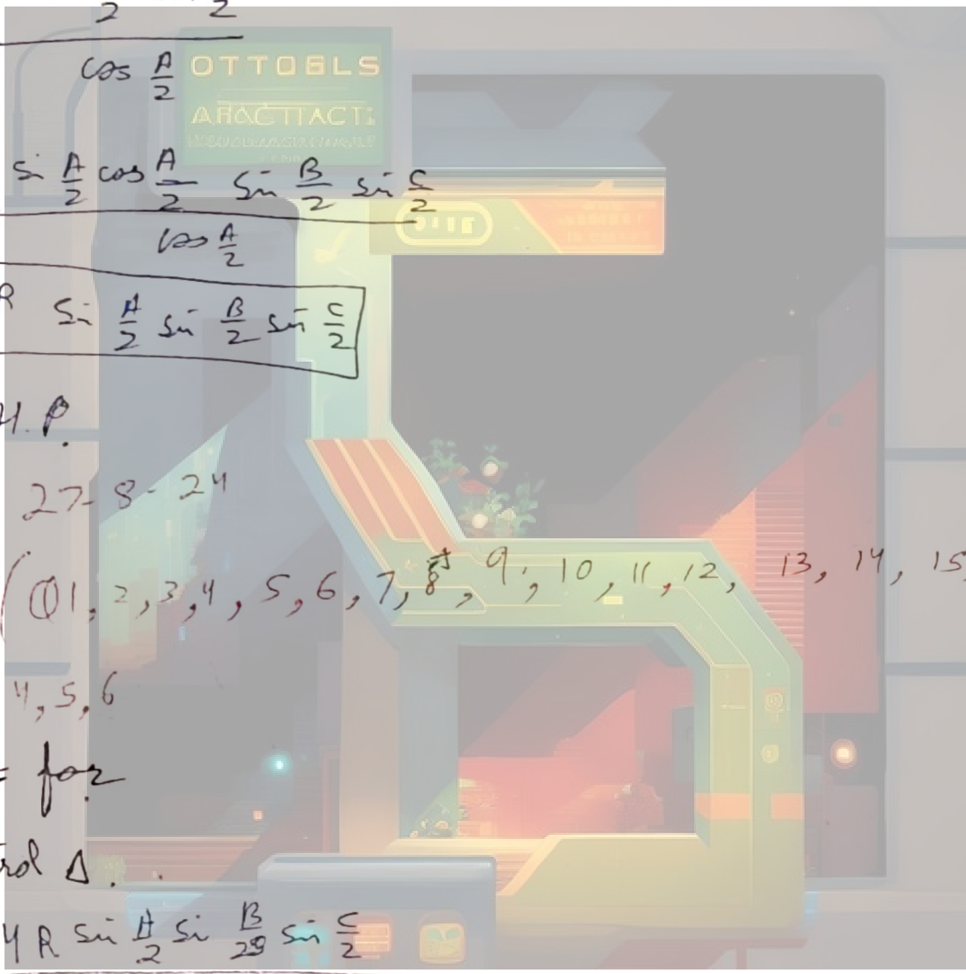
$$a = r\left(\frac{\cos\frac{B}{2}}{\sin\frac{B}{2}} + \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}\right)$$

$$a = \frac{r\left(\cos\frac{B}{2}\sin\frac{C}{2} + \cos\frac{C}{2}\sin\frac{B}{2}\right)}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$a = \frac{r \sin\frac{(B+C)}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$a = \frac{r\cos\frac{A}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$r = \frac{a\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$$



$$r = \frac{4R\sin\frac{A}{2}\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$$

$$\boxed{r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

H.P.

H.W 27-8-24

Q-1 (Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

DYS - 4, 5, 6

Q find $\frac{r}{R}$ for

(1) equilateral Δ.

$$\frac{r}{R} = \frac{4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{R}$$

$$\frac{r}{R} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$A = B = C = 60°$

$$\frac{r}{R} = 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$\boxed{\frac{r}{R} = \frac{1}{2}}$$

(2) isosceles right angled Δ, $C = 90^\circ$, $A = 45^\circ$, $B = 45^\circ$

$$\frac{r}{R} = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{\cancel{2}2}$$

$$\frac{r}{R} = 4 \times \frac{1}{\sqrt{2}} \times \cancel{} \sin^2 22.5$$

$$\frac{r}{R} = \frac{4}{\sqrt{2}} \times \frac{(1-\cos 45)}{2}$$

$$\frac{r}{R} = \sqrt{2} \times \frac{(\sqrt{2}-1)}{\sqrt{2}}$$

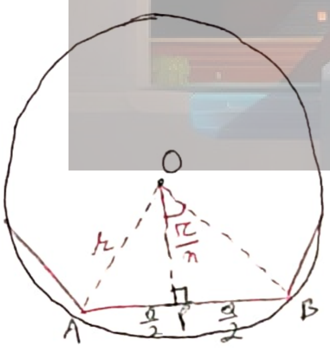
$$\boxed{\frac{r}{R} = \sqrt{2}-1}$$

Regular Polygon

→ All sides and all angles are equal

$$\boxed{\text{Sum of all int. angles} = (n-2)\pi}$$ $n =$ no. of sides (18)

(I) Perimeter & Area of regular polygon ~~inscr~~ inscribed in a circle.



$$\sin\left(\frac{\pi}{n}\right) = \frac{a/2}{r}$$

$$a = 2r\sin\left(\frac{\pi}{n}\right)$$

$n =$ no. of sides

$r =$ radius

$$\boxed{\text{Perimeter} = 2rn\sin\left(\frac{\pi}{n}\right)}$$ (19)

$OP = r\cos\left(\frac{\pi}{n}\right)$

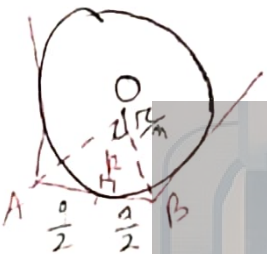
$$\text{area}\,(\Delta\, OAB) = \frac{1}{2} \times a \times OP$$

$$= \frac{1}{2} \times \cancel{a} \; 2r\sin\left(\frac{\pi}{n}\right) \times r\cos\left(\frac{\pi}{n}\right)$$

$$\text{area}(\triangle OAB) = \frac{r^2}{2}\sin\left(\frac{2\pi}{n}\right)$$

$$\therefore \boxed{\text{Area(Polygon)} = \frac{nr^2}{2}\sin\left(\frac{2\pi}{n}\right)} \quad (20)$$

(II) Area & Perimeter of ~~Triang~~ regular Polygon circumscribing a circle.



$$\tan\left(\frac{\pi}{n}\right) = \frac{a/2}{r}$$

$$a = 2r\tan\left(\frac{\pi}{n}\right)$$

$$\boxed{\text{Perimeter} = 2nr\tan\left(\frac{\pi}{n}\right)} \quad (21)$$

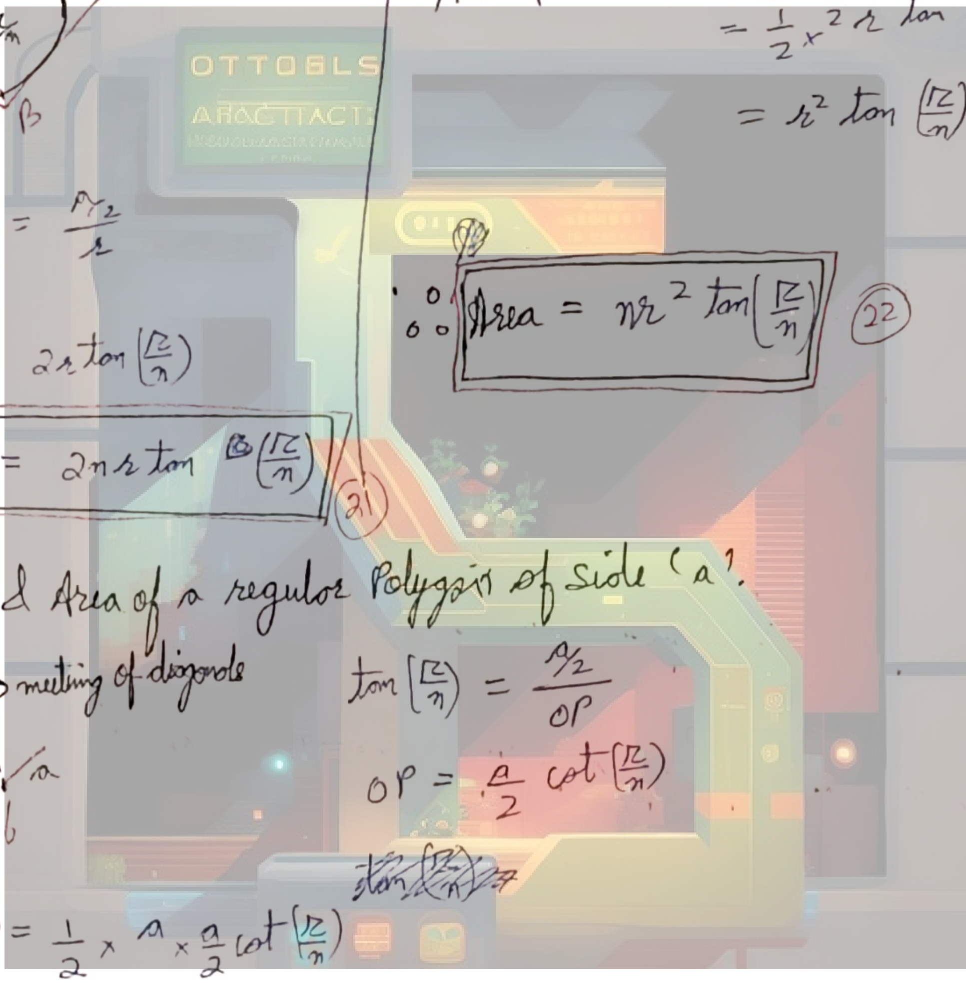
~~Area = nr^2~~.

$$\text{Area}(\triangle OAB) = \frac{1}{2}\times a \times r$$

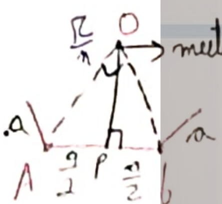
$$= \frac{1}{2}\times 2r\tan\frac{\pi}{n}\times r$$

$$= r^2\tan\left(\frac{\pi}{n}\right)$$

$$\therefore \boxed{\text{Area} = nr^2\tan\left(\frac{\pi}{n}\right)} \quad (22)$$



(III) Perimeter & Area of a regular Polygon of side 'a'.



$$\tan\left(\frac{\pi}{n}\right) = \frac{a/2}{OP}$$

$$OP = \frac{a}{2}\cot\left(\frac{\pi}{n}\right)$$

$$\text{ar}(\triangle OAB) = \frac{1}{2}\times a \times \frac{a}{2}\cot\left(\frac{\pi}{n}\right)$$

$$= \frac{a^2}{4}\cot\left(\frac{\pi}{n}\right).$$

$$\therefore \boxed{\text{Area of Polygon} = \frac{na^2}{4}\cot\left(\frac{\pi}{n}\right)} \quad (23)$$

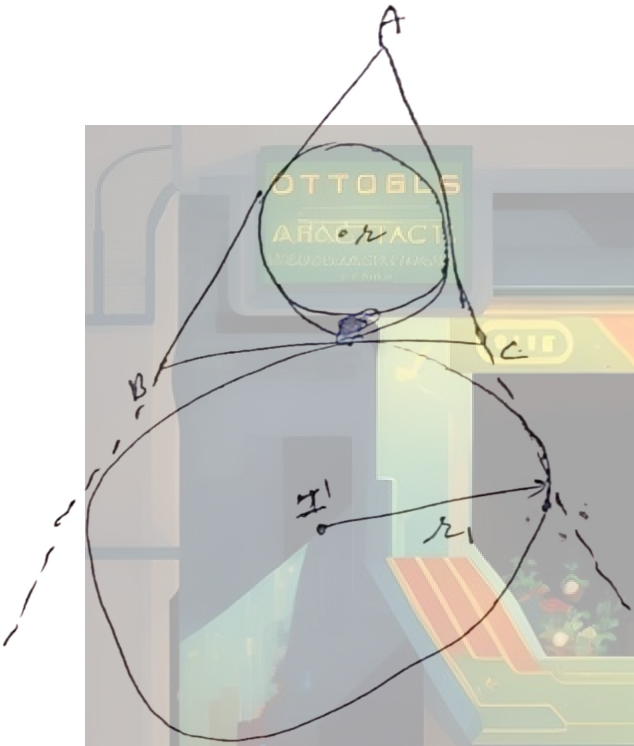
$$\boxed{\therefore \text{Perimeter} = n\times a} \quad (24)$$

If $a^2+b^2+c^2=8R^2 \rightarrow$ Right Angle Triangle

$\downarrow$

~~equilateral Triangle~~

Ex-center :-



$r_1 = \frac{\Delta}{s-a}$ → exradius

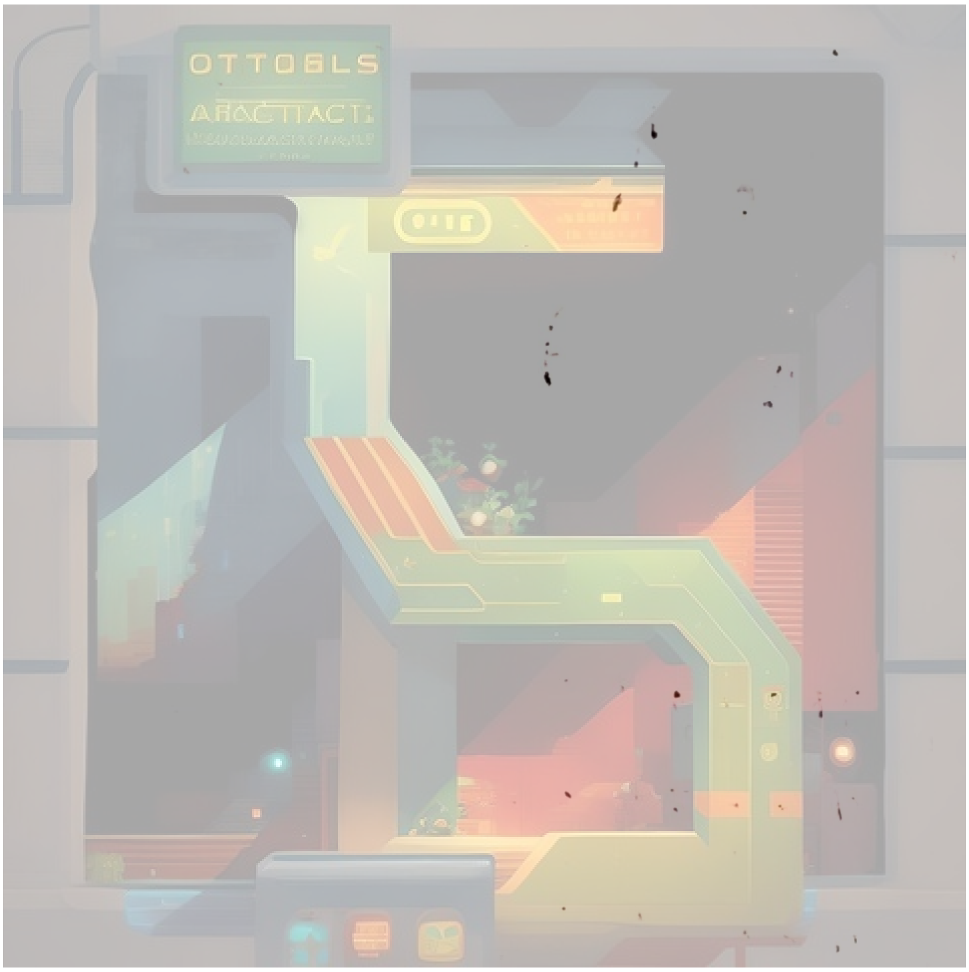
$r_2 = \frac{\Delta}{s-b}$

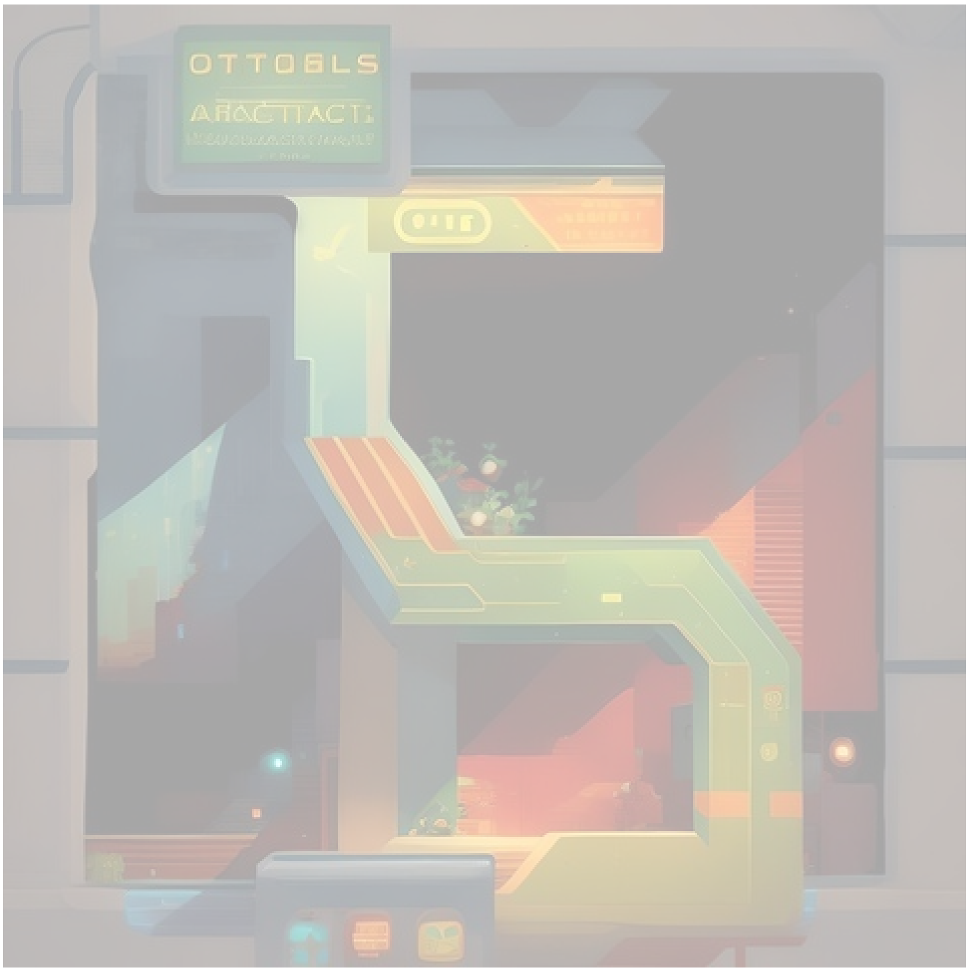
$r_3 = \frac{\Delta}{s-c}$

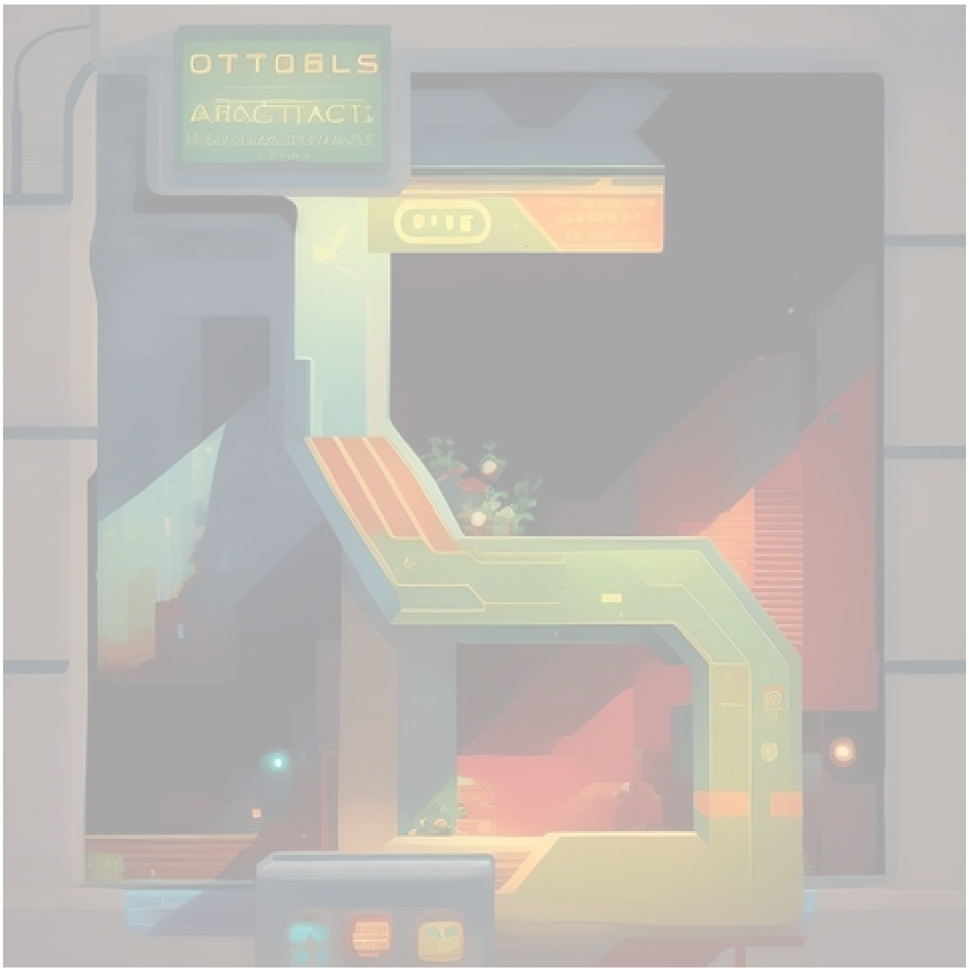
$r_1 = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$

$r_2 = 4R \cos\frac{A}{2} \sin\frac{B}{2} \cancel{\sin} \cos\frac{C}{2}$

$r_3 = 4R \cos\frac{A}{2} \cos\frac{B}{2} \sin\frac{C}{2}$

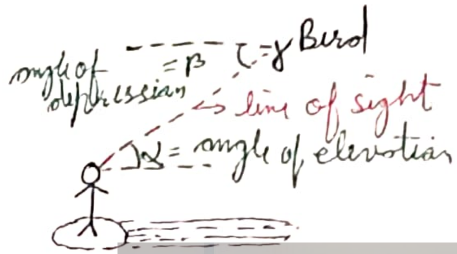


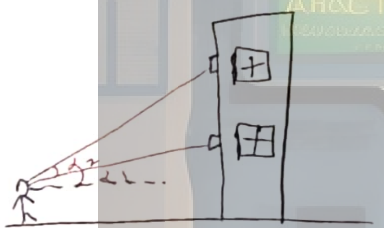




Heights & Distances

i) Line of sight
ii) Angle of Elevation
iii) Angle of Depression.





$\alpha_1, \alpha_2 \rightarrow$ angle of elevation.

I Results to Remember

① In a $\triangle ABC$, if $BD : DC = m : n$,

$\angle BAD = \alpha$

$\angle ADC = \theta$

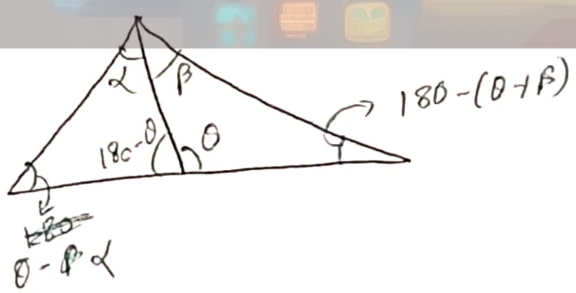
$\angle CAD = \beta$

Then,

$$\boxed{(m+n)\cot\theta = m\cot\alpha - n\cot\beta}$$

(m-n theorm)

Proof:-



Proof:- $\triangle ABD$ -

$$\frac{\sin\alpha}{BD} = \frac{\sin(\theta-\alpha)}{AD} \quad -①$$

$\triangle ADC$ -

$$\frac{\sin\beta}{DC} = \frac{\sin(180-(\theta+\beta))}{AD} \quad -②$$

① ÷ ②

$$\frac{\frac{\sin\alpha}{BD}}{\frac{\sin\beta}{DC}} = \frac{\sin(\theta-\alpha)}{AD} \times \frac{AD}{\sin(\theta+\beta)}$$

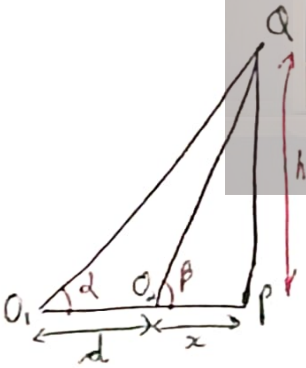
$$\frac{n\sin\alpha}{m\sin\beta} = \frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\sin\theta\cos\beta + \cos\theta\sin\beta} \quad \left\{\frac{BD}{DC} = \frac{m}{n}\right\}$$

cross multiply

② The angles of elevation of two objects are α & β respectively and the distance btw ~~s~~ two objects is d.

Then,

$$\boxed{d = h(\cot\alpha - \cot\beta)}$$



Proof:-

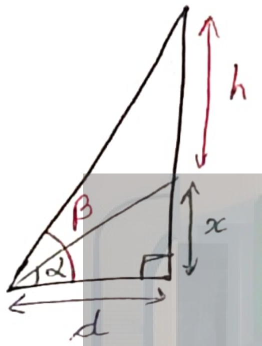
$$\cot\alpha = \frac{d+x}{h} \Rightarrow (x = h\cot\alpha - d) \quad -①$$

$$\cot\beta = \frac{x}{h} \Rightarrow (x = h\cot\beta) \quad -②$$

① = ②

$$h(\cot\alpha - \cot\beta) = d$$

Q A vertical tower stands on a horizontal plane and is ~~surround~~ surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom & the top of the flag staff are α & β respectively. Prove that the height of the tower is $\frac{h\cot\beta}{\cot\alpha - \cot\beta}$.



$$\cot\beta = \frac{d}{h+x} \Rightarrow d = (h+x)\cot\beta \quad \text{—} ①$$

$$\cot\alpha = \frac{d}{x} \Rightarrow d = x\cot\alpha \quad \text{—} ②$$

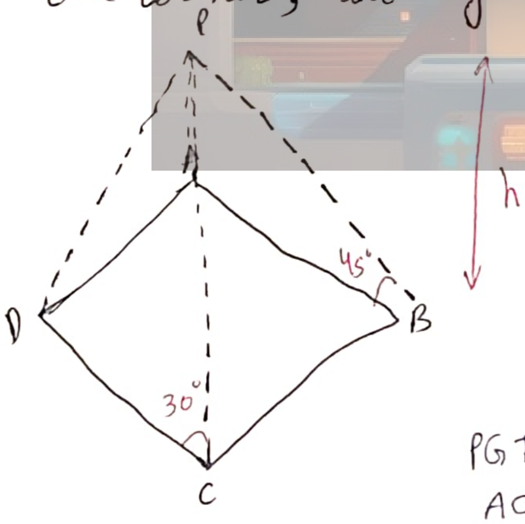
$① = ②$

$$h\cot\beta + x\cot\beta = x\cot\alpha$$

$$\Rightarrow x = \frac{h\cot\beta}{\cot\alpha - \cot\beta}$$

H.P

Q A vertical lamp post of height 9m stands at the corner of a rectangular field. The angle of elevation of its top from the farthest corner of field is 30° while from the ~~oth~~ other one corner, the angle is 45°. Find the area of the field.



ΔBAP –

$$\tan 45^\circ = \frac{9}{AB} \Rightarrow AB = 9\,m$$

ΔCAP –

$$\tan 30^\circ = \frac{9}{AC} \Rightarrow AC = 9\sqrt{3}\,m$$

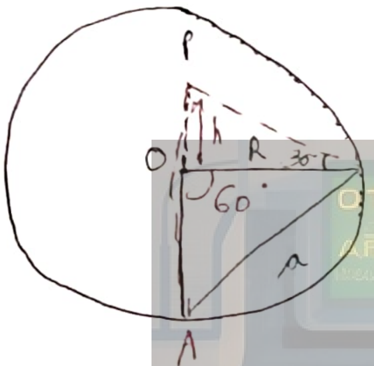
PGT in ΔABC –

$$AC^2 = AB^2 + BC^2$$

$$BC = 9\sqrt{2}\,m$$

$$\therefore \text{Area} = 9 \times 9\sqrt{2} = \boxed{81\sqrt{2}\,m^2}$$

Q A tower stands at the top centre of a circular path. A & B are 2 points on the boundary of the park such that AB = a and it subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30°. find h of tower in terms of a.



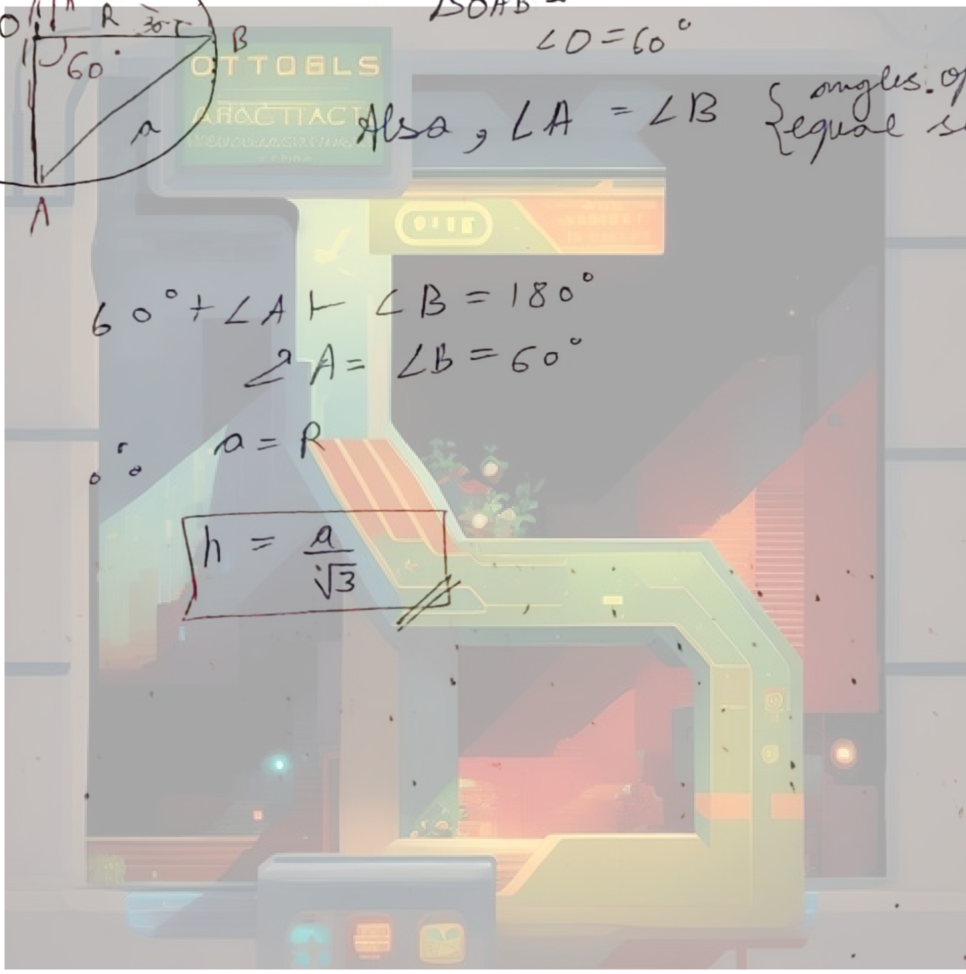
$\triangle BOP$ -

$\tan 30° = \frac{h}{R} \Rightarrow h = \frac{R}{\sqrt{3}}$

$\triangle OAB$ -

$\angle O = 60°$

Also, $\angle A = \angle B$ {angles opp. to equal sides of $\triangle$s}

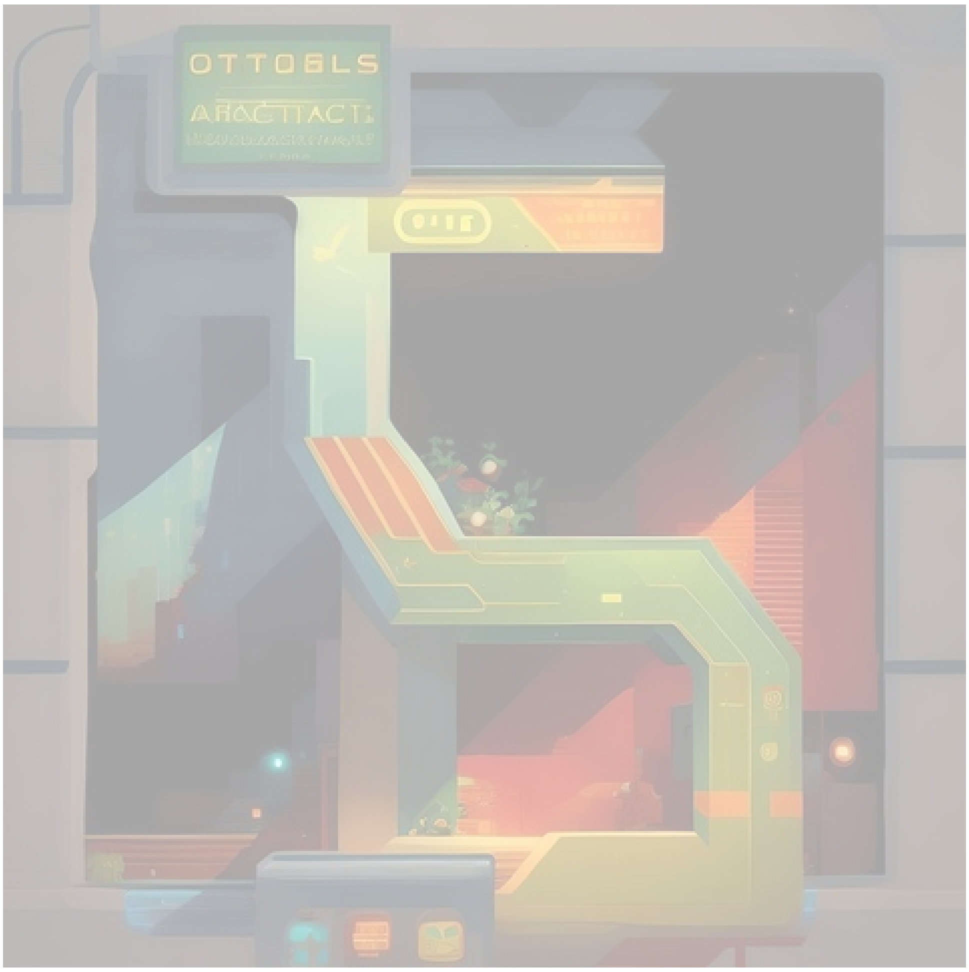


$\therefore$ $60° + \angle A + \angle B = 180°$

$\angle A = \angle B = 60°$

$\therefore$ $a = R$

$$\boxed{h = \frac{a}{\sqrt{3}}}$$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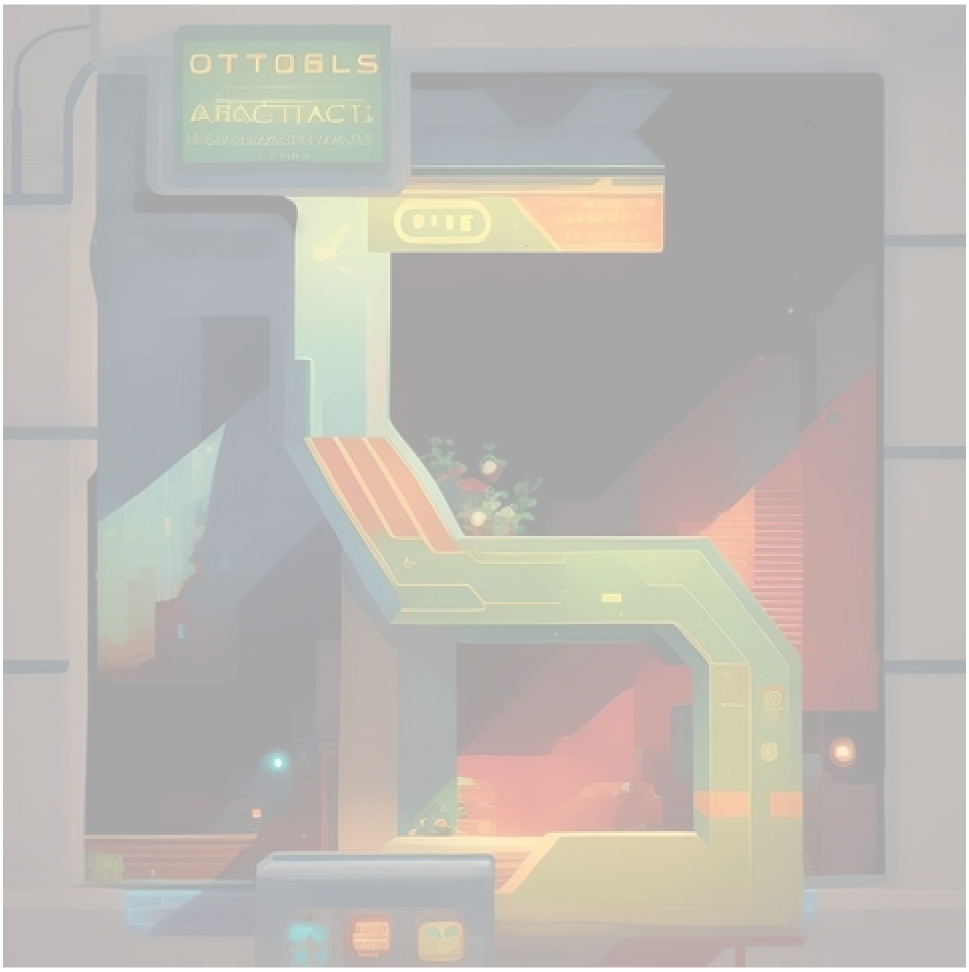
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