Ch-3

!! Projectile Motion !!

Projectile motion - एक गेंद object को फेंका जाता and it lands somewhere else.

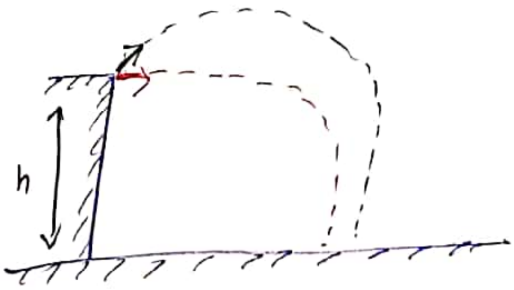
→ the object always moves in a single plane
→ The object moves in parabola path.
→ The object moves only under gravity.
(ignore earth's curvature, air resistance & etc)

→ A projectile is any body that is given an initial velocity and then follows a path determined ~~only~~ entirely by effects of acceleration due to gravity.

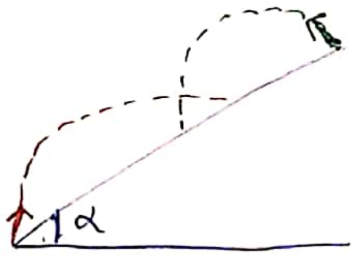
Types of projectial Motion -

① Ground to Ground (oblique) :-

② Projection from a height.



③ Projection on a plane :-



Projectile motion -

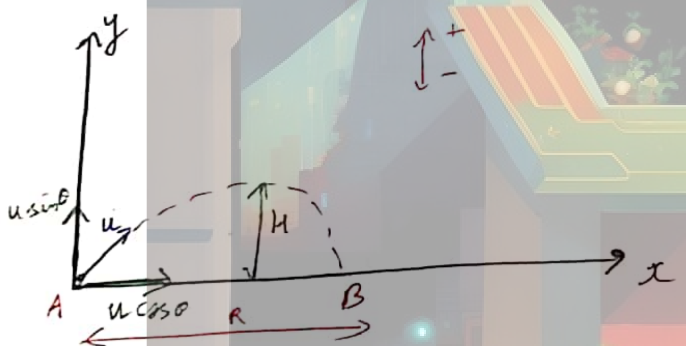
↓

2 dimentional motion (2 d)

(motion in a plane)

Horizontal (uniform motion) ← independent of each other → Verticle (uniformly accelerated motion)

Oblique projection :- (ground to ground projection)



Horizontal (uniform)	Verticle (uniformly accelerated)
$v_x = u\cos\theta$	$v_y = u\sin\theta$
$a_x = 0$	$a_y = -g$

Time of flight (T)

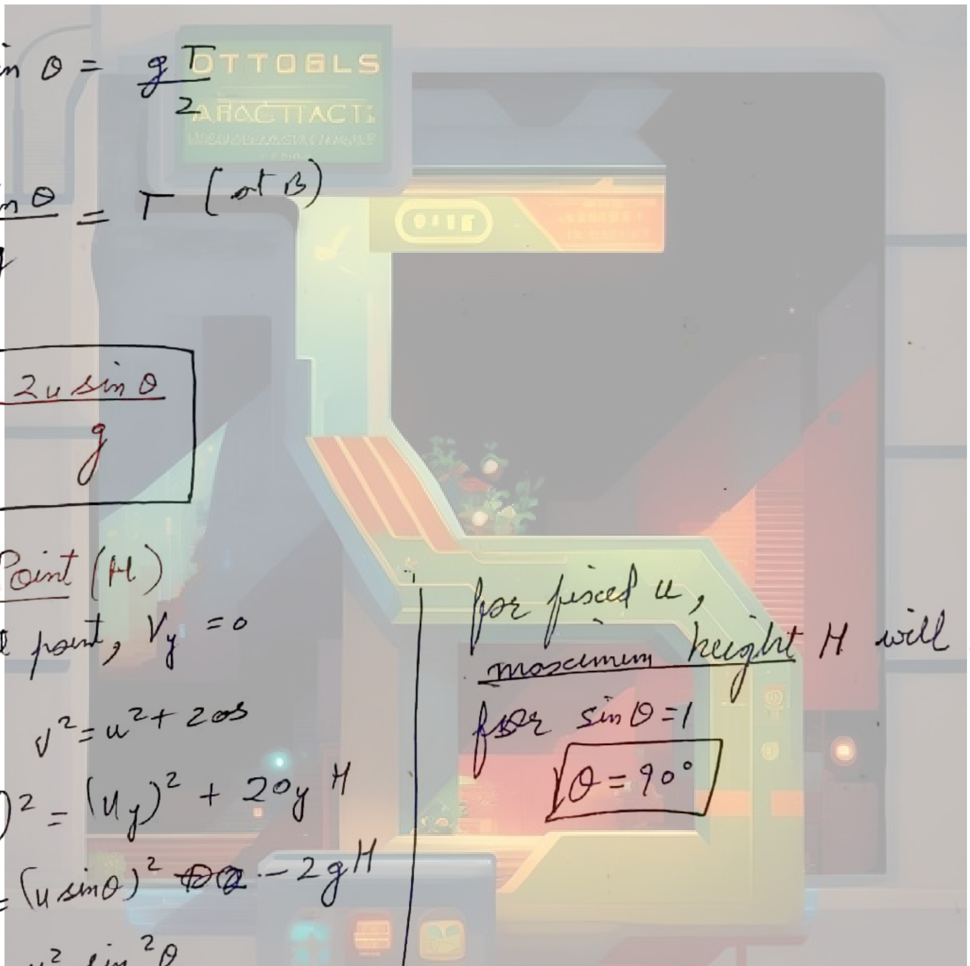
$S_y = 0$

Using $s = ut + \frac{1}{2} at^2$

$0 = u\sin\theta\,(T) + \frac{1}{2}(-g)T^2$

$0 = T\left(u\sin\theta - \frac{gT}{2}\right)$

$T = 0$ (at point of projection)



$u\sin\theta = \frac{gT}{2}$

$\frac{2u\sin\theta}{g} = T$ (at B)

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

Highest Point (H)

at highest point, $V_y = 0$

Using $v^2 = u^2 + 2as$

$(V_y)^2 = (u_y)^2 + 2a_y H$

$0 = (u\sin\theta)^2 - 2gH$

$H = \frac{u^2\sin^2\theta}{2g}$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

for fixed u, maximum height H will be for $\sin\theta = 1$

$\boxed{\theta = 90^\circ}$

Range(R)

$$R = V_x T$$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

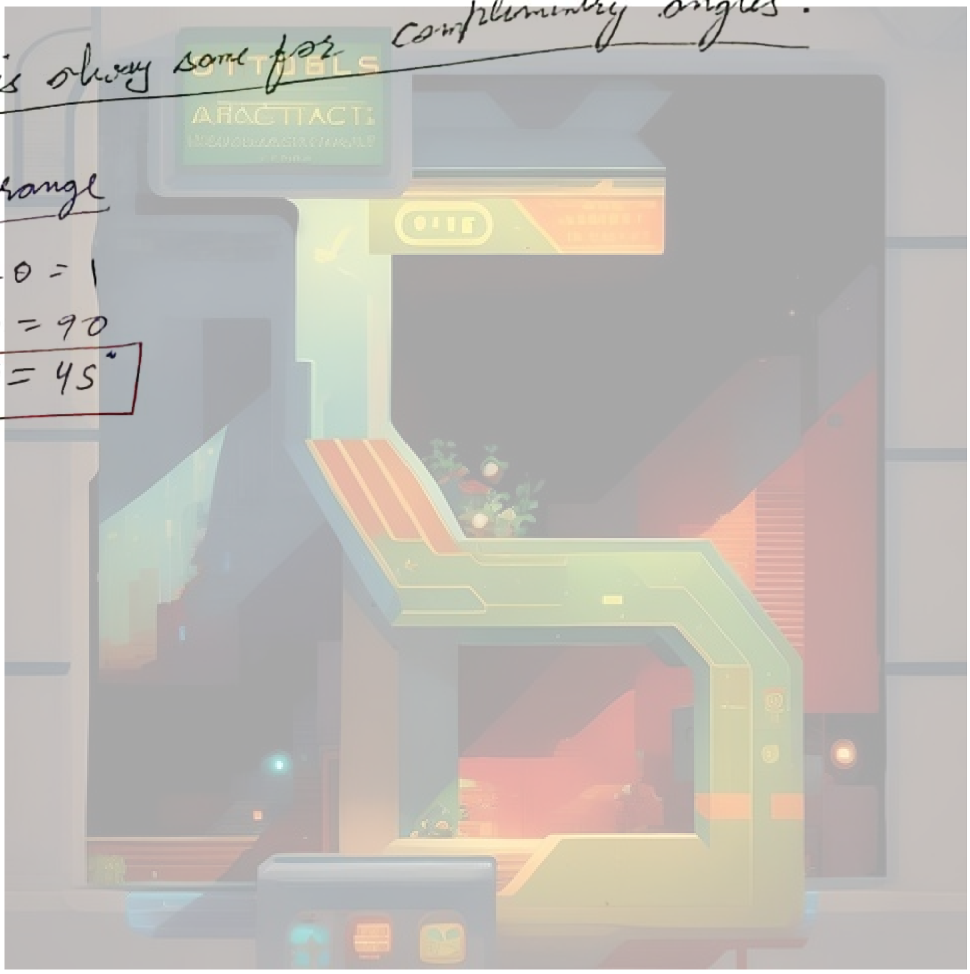
For complimentry angle of projection

$$\theta \rightarrow (90-\theta)$$

$$R' = \frac{u^2 \sin[2(90-\theta)]}{g}$$

$$R' = \frac{u^2 \sin(180-2\theta)}{2}$$

$$\boxed{R' = \frac{u^2 \sin 2\theta}{g}}$$



Range is always same for complimentry angles.

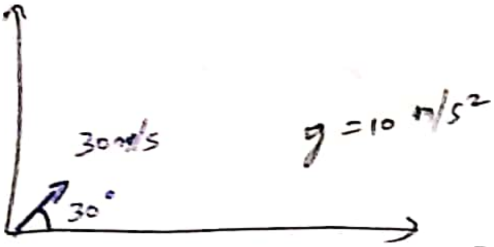
For max range

$$\sin 2\theta = 1$$

$$2\theta = 90$$

$$\boxed{\theta = 45^\circ}$$

Q

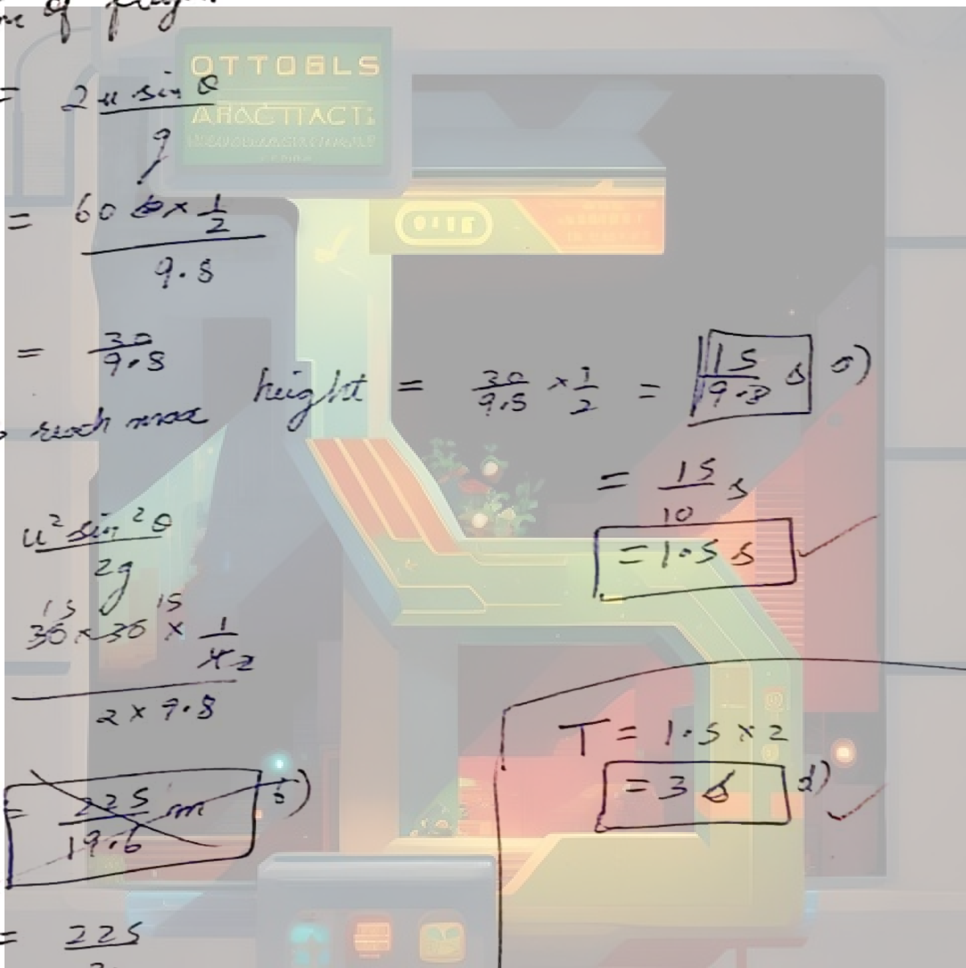


a) time at which ball reaches highest point

b) maximum height reached

c) horizontal range

d) Time of flight



$T = \frac{2u\sin\theta}{g}$

$= \frac{60 \times \frac{1}{2}}{9.8}$

$= \frac{30}{9.8}$

time to reach max height $= \frac{30}{9.8} \times \frac{1}{2} = \boxed{\frac{15}{9.8}\ s}$ a)

$= \frac{15}{10}\ s$

$\boxed{= 1.5\ s}$

$H = \frac{u^2\sin^2\theta}{2g}$

$= \frac{30 \times 30 \times \frac{1}{4}}{2 \times 9.8}$

$\cancel{\boxed{= \frac{225}{19.6}\ m}}$ b)

$T = 1.5 \times 2$

$\boxed{= 3\ s}$ d)

$= \frac{225}{20}$

$= \frac{112.5}{10}$

$\boxed{= 11.25\ m}$ b)

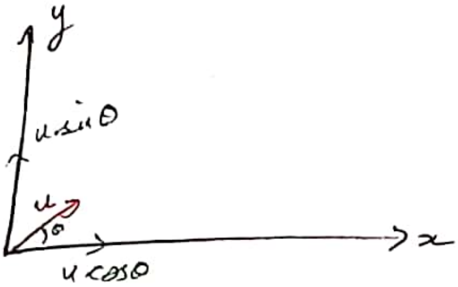
c) $R = \frac{u^2\sin 2\theta}{g}$

$= \frac{30 \times 30 \times \frac{\sqrt{3}}{2}}{}$

$\boxed{= 45\sqrt{3}\ m}$ c)

(204)

Velocity at ~~time~~ time 't' —



$u_x = u\cos\theta$

$V_x = u\cos\theta$

$u_y = u\sin\theta$

$V_y = u_y + a_y t$

$\underline{V_y = u\sin\theta - gt}$

$$\boxed{\vec{V} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}}$$

Q A ball is projected with velocity u at angle of projection θ. After what time ball is moving at right angles to initial direction.

initial direction $= \hat{u} = \frac{\vec{u}}{u} = u\sin\theta\,\hat{j} + u\cos\theta\,\hat{i}$

$\vec{V} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$$= \frac{u^2\cos^2\theta + u\sin\theta(u\sin\theta - gt)}{u\sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2u\sin\theta gt}}$$

$= u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta gt$

$$= \frac{u^2 - u\sin\theta gt}{u\sqrt{u^2 + g^2t^2 - 2u\sin\theta gt}}$$

$= u - \sin\theta g$

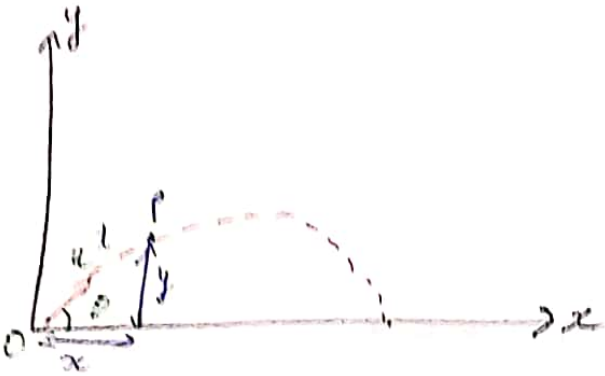
$\vec{u}\cdot\vec{V} = 0$

$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta gt = 0$

$u(u - \sin\theta\, gt) = 0$

$$\boxed{t = \frac{u}{\sin\theta\, g}}$$

Equation of trajectory



$x = u_x t$

$x = u\cos\theta\, t$

$t = \dfrac{x}{u\cos\theta}$ (1)

$s_y = u_y t + \frac{1}{2} a_y t^2$

$y = u\sin\theta\,(t) - \frac{1}{2} g t^2$ ---- (2)

$y = \dfrac{u\sin\theta\,(x)}{u\cos\theta} - \dfrac{1}{2} g \dfrac{x^2}{u^2\cos^2\theta}$

$y = x\tan\theta - \dfrac{g x^2}{2u^2\cos^2\theta}$

$$\boxed{y = x\tan\theta - \dfrac{g x^2}{2u^2\cos^2\theta}}$$

$y = x\tan\theta - \dfrac{g x^2}{2u^2\cos^2\theta}\cdot\dfrac{\sin\theta}{\sin\theta}$

$y = x\tan\theta - \dfrac{x^2 \cancel{\sin\theta}\tan\theta}{\underbrace{\dfrac{2u^2\sin\theta\cos\theta}{g}}_{\text{Range formula}}}$

$y = x\tan\theta - \dfrac{x^2\tan\theta}{R}$

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Q A grasshoper can ~~kep~~ jump into height 'h', find the maximum distance through which it can jump along horizontal ground.

$$h = \frac{u^2 \sin^2\theta}{2g} \qquad h = \frac{u^2}{2g}$$

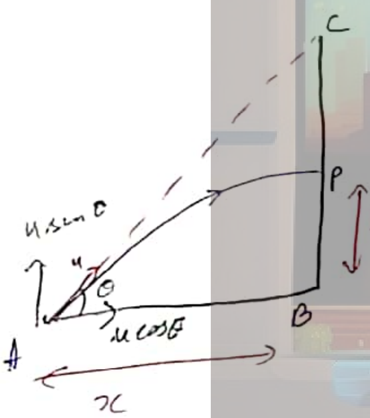
$$\sqrt{2gh} = u$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{2gh \times}{g}$$

$$\boxed{R_{max} = 2h}$$

Q A hunter aims his gun and fires a bullet directly on a monkey on a tree. At the instant bullet is fired, the monkey drops. will the bullet hit the monkey?



$$\tan\theta = \frac{y}{x}$$

for bullet,

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$h = u \sin\theta \, t - \frac{1}{2} g t^2$$

$$x = u \cos\theta \, t \Rightarrow t = \frac{x}{u\cos\theta}$$

$$h = u \sin\theta \left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g t^2$$

$$h = x \tan\theta - \frac{1}{2} g t^2$$

$$h = y - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 = y - h$$

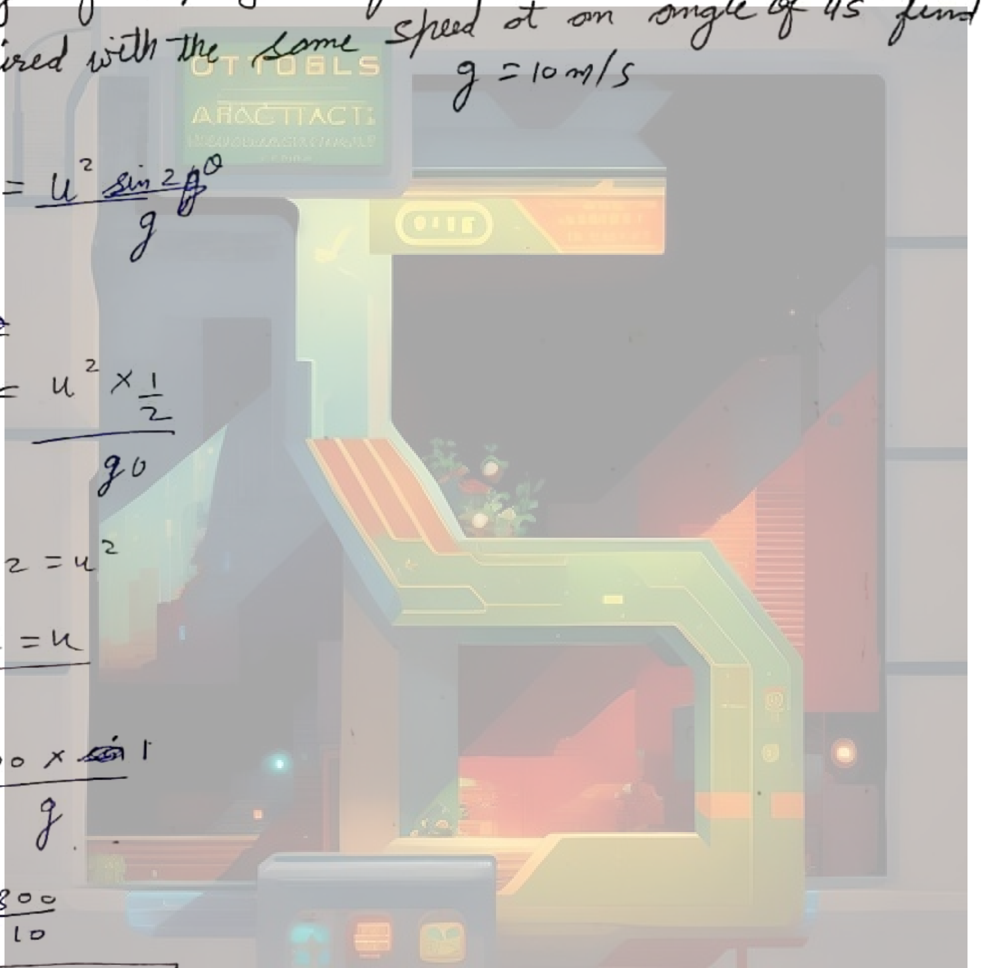
For Monkey

$z = 0 + \frac{1}{2} g t^2$

$z = y - h$

As, the height is same for monkey & bullet at same time,
Yes the bullet will hit monkey.

Q The range of a projectile fired at an angle 15° is 40 m. if it is fired with the same speed at an angle of 45° find its range.
$g = 10 m/s$



$R = \frac{u^2 \sin 2\theta}{g}$

$40 = \frac{u^2 \times \frac{1}{2}}{g 0}$

$400 \times 2 = u^2$

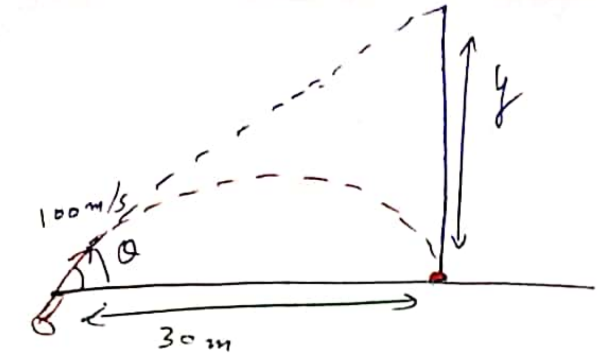
$20\sqrt{2} = u$

$R' = \frac{800 \times 1}{g}$

$R' = \frac{800}{10}$

$\boxed{R' = 80 m}$

Q



How high above the target the gun must be aimed to hit the target on ground at 30 m from the gun?

find y

$$R = \frac{100 \times 10\cancel{0}\ \sin 2\theta}{1\cancel{0}}$$

$$R = 1000 \sin 2\theta$$

$$\frac{3\cancel{0}}{100\cancel{0}} = \sin 2\theta$$

$$\cancel{\frac{3}{100} = 2\sin\theta\cos\theta}$$

$$\cancel{\frac{3}{100} = \sin\theta\cos\theta}$$

$$\sin 2\theta = 0.03 \rightarrow \text{(approximation)}$$

$$2\theta = 0.03$$

$$\theta = 0.015$$

$$\tan\theta = \frac{y}{30}$$

$$30\ \frac{\sin\theta}{\cos\theta} = y$$

$$\tan 0.015 = \frac{y}{30}$$

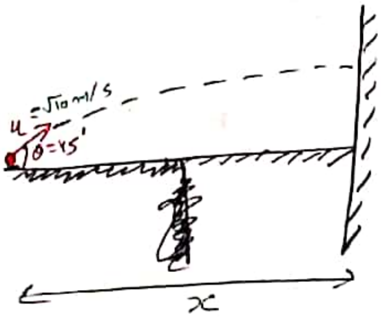
$$30 \times 0.015 = y$$

$$0.15 \times 3 = y$$

$$0.45 = y$$

$$\boxed{y = 0.45\ m}$$

Q



$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{10 \times 1}{10}$$

$$R = 1\,m$$

a) $x = \frac{1}{2}$ m ✓ will water hit wall

b) $x = 2m$ ✗

c) where does it hit wall

d) max height

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$H = \frac{10 \times \frac{1}{2}}{2 \times 10}$$

$$\boxed{H = \frac{1}{4}\,m}$$

$$y = x \tan\theta \left[1 - \frac{x}{R}\right]$$

$$y = \frac{1}{2} \times 1 \left[1 - \frac{\frac{1}{2}}{1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}\right]$$

$$= \frac{1}{4}\,m$$

$$\boxed{y = \frac{1}{4}\,m}$$

Or

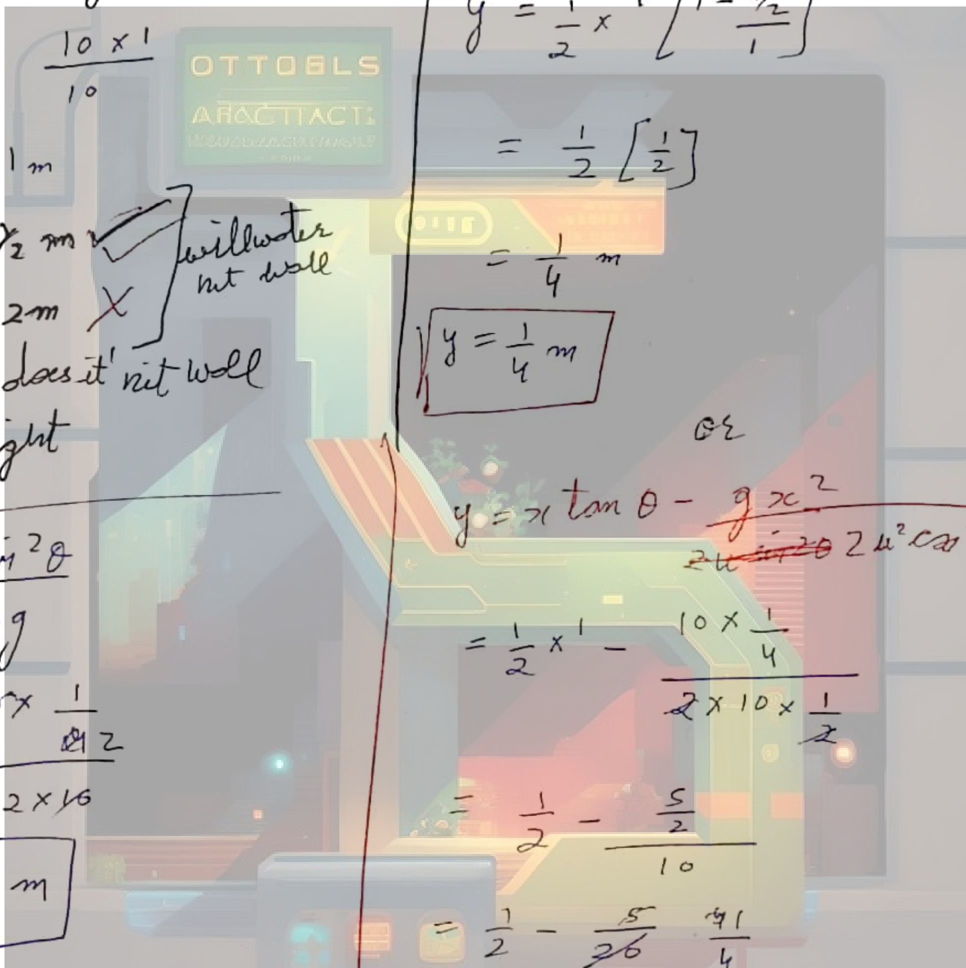
$$y = x\tan\theta - \frac{g x^2}{\cancel{2u^2\sin 2\theta}\; 2u^2\cos^2\theta}$$

$$= \frac{1}{2} \times 1 - \frac{10 \times \frac{1}{4}}{2 \times 10 \times \frac{1}{2}}$$

$$= \frac{1}{2} - \frac{\frac{5}{2}}{10}$$

$$= \frac{1}{2} - \frac{5}{20}$$

$$\boxed{= \frac{1}{4}\,m}$$



Q we have a hose pipe which disposes water at speed of 10 m/s. The safe distance from a building on fire, on ground is 5m. How high can the water go. ($g = 10 m/s^2$)

$u = 10 m/s$

$R = 5 m$

$R = \frac{100 \times \sin 2\theta}{10}$

$\frac{1}{2} = \sin 2\theta$

$2\theta = 30$

$\theta = 15°$

$H = \frac{u^2 \sin^2\theta}{2g}$

$= \frac{100 \times \frac{\sqrt{3}-1}{2\sqrt{2}}}{2 \times 10}$

$= \frac{10\sqrt{3} - 10}{4\sqrt{2}}$

$\boxed{= \frac{5\sqrt{3} - 5}{2\sqrt{2}} m}$

$H = \frac{100 \times \frac{(2-\sqrt{3})}{4}}{20}$

$H = \frac{5(2-\sqrt{3})}{4}$

$\boxed{H = \frac{10 - 5\sqrt{3}}{4}}$

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\frac{\sqrt{3} - 1}{2\sqrt{2}}$

$\frac{1 - \cos 30°}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2}$

$\frac{2 - \sqrt{3}}{4}$

$\frac{1 - \cos\theta}{2} = \sin^2\theta$

$1 - \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\frac{1}{2} - \frac{\sqrt{3}}{4}$

$\frac{2 - \sqrt{3}}{4}$

$$y = x\tan\theta - \frac{g x^2}{2u^2 \cancel{\sec^2}\cos^2\theta}$$

$$y = 5\tan\theta - \frac{10 \times 25}{2 \times 10 \times 10 \times \cos^2\theta}$$

$$y = 5\tan\theta - \frac{5}{4}\sec^2\theta$$

जिसको max करना है

$$\frac{dy}{d\theta} = 0$$

जिससे change kar sakte hai

$$\frac{dy}{d\theta} = 5\sec^2\theta - \frac{5}{4} \times 2\sec\theta \times \sec\theta\tan\theta$$

$$= 5\sec^2\theta - \frac{5}{2}\sec^2\theta\tan\theta$$

$$\frac{5}{2}\cancel{\sec^2}\theta\ \tan = \cancel{5\sec^2\theta}$$

$$\frac{\tan\theta}{2} = 1$$

$$\cancel{\tan\theta = 0}$$

$$\tan\theta = 2$$

$P = 2$

$B = 1$

$H = \sqrt{5}$

$$\sec\theta = \frac{H}{B} = \sqrt{5}$$

$$y_{max} = \cancel{5\times(\sqrt{5})}\ 5 \times 2 - \frac{5}{4} \times 5$$

$$= 10 - \frac{25}{4}$$

$$\boxed{= \frac{15}{4}\, m}$$

Or

$$y = 5\tan\theta - \frac{5}{4}\sec^2\theta$$

$$y = 5\tan\theta - \frac{5}{4}(1 - \tan^2\theta)$$

$$y = 5\tan\theta - \frac{5}{4} + \tan^2\theta$$

$$\tan^2\theta + 5\tan\theta - \frac{5 + 4y}{4}$$

for real roots

$$\cancel{\tan\theta}\ D \geq 0$$

$$\cancel{25 + 5y \geq 0}$$

$$\cancel{5(5+y) \geq 0}$$

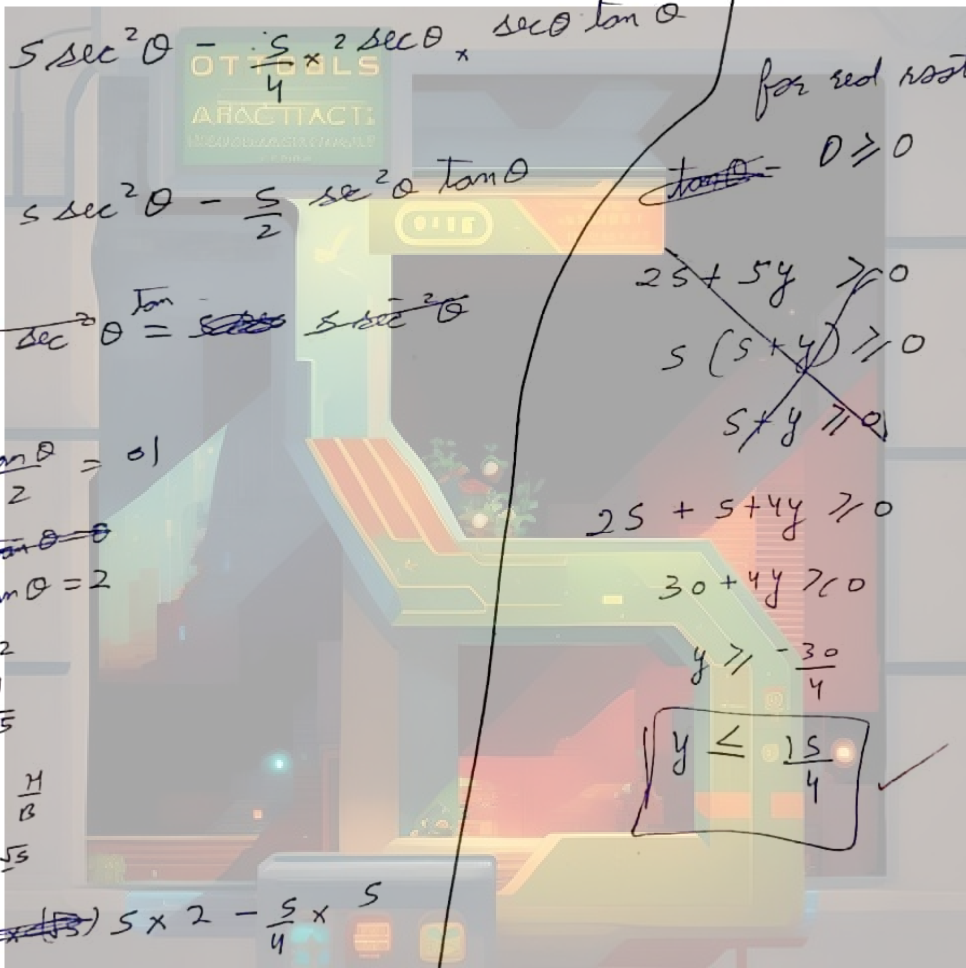
$$\cancel{5+y \geq 0}$$

$$25 + 5 + 4y \geq 0$$

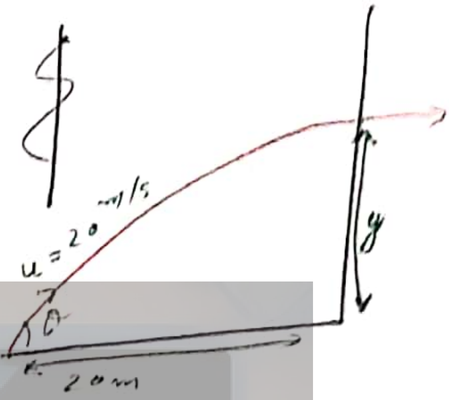
$$30 + 4y \geq 0$$

$$y \geq -\frac{30}{4}$$

$$\boxed{y \leq \frac{15}{4}}$$



Q A shot is fired with a velocity 20 m/s at a vertical wall whose distance from point of projection is 20 m. Find the greatest height above the level of point of projection at which bullet can ~~hit~~ hit the wall.



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = 20\tan\theta - \frac{10\times 20\times 20}{2\times 20\times 20\times\cos^2\theta}$$

$$y = 20\tan\theta - 5\sec^2\theta$$

$$\frac{dy}{d\theta} = 20\sec^2\theta - 10\sec^2\theta\tan\theta$$

$$0 = 2\sec^2\theta - \sec^2\theta\tan\theta$$

$$0 = \sec^2\theta(2-\tan\theta)$$

$$2-\tan\theta = 0$$

$$\tan\theta = 2$$

$P = 2$
$B = 1$
$H = \sqrt{5}$

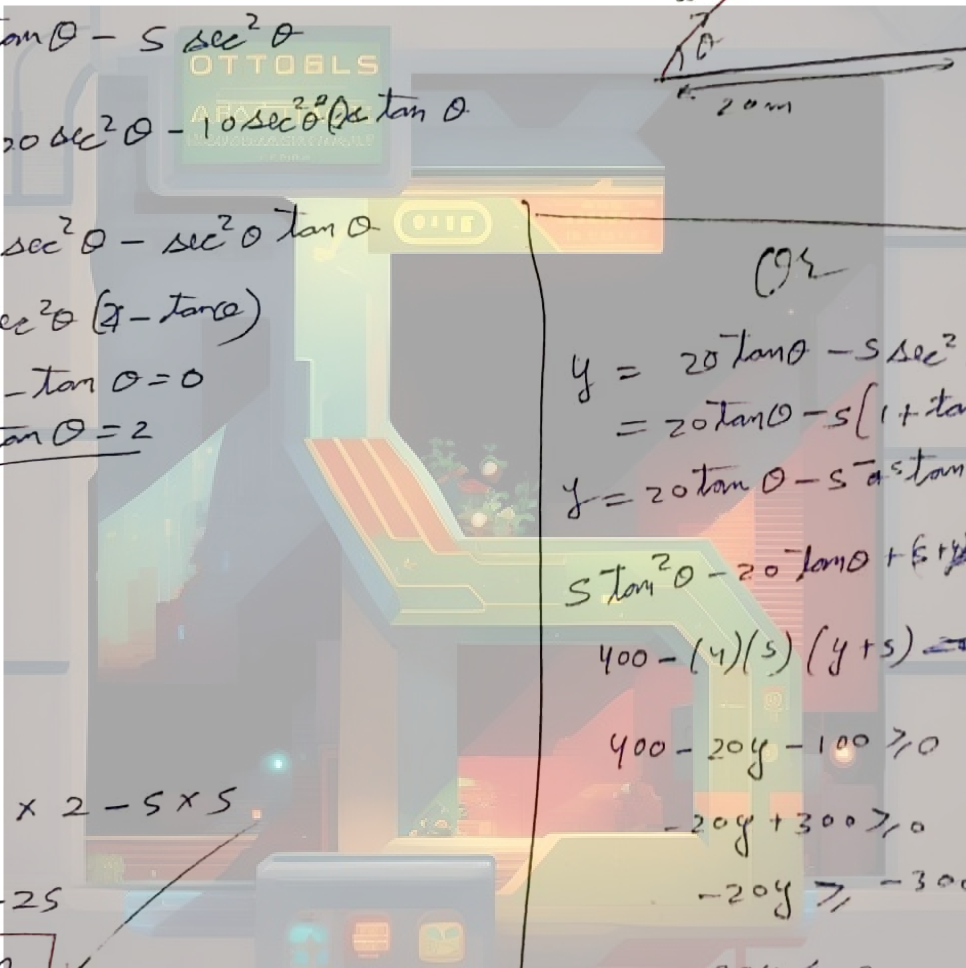
$$\sec\theta = \sqrt{5}$$

$$\sec^2\theta = 5$$

$$y_{max} = 20\times 2 - 5\times 5$$

$$= 40-25$$

$$\boxed{= 15\text{ m}}$$



OR

$$y = 20\tan\theta - 5\sec^2\theta$$

$$= 20\tan\theta - 5(1+\tan^2\theta)$$

$$y = 20\tan\theta - 5 - 5\tan^2\theta$$

$$5\tan^2\theta - 20\tan\theta + (5+y) =$$

$$400 - (4)(5)(y+5) \geq 0$$

$$400 - 20y - 100 \geq 0$$

$$-20y + 300 \geq 0$$

$$-20y \geq -300$$

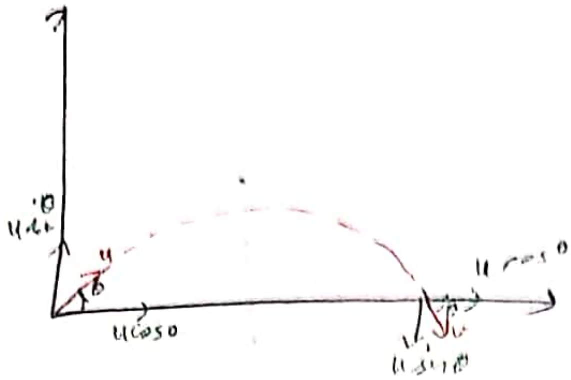
$$20y \leq 300$$

$$y \leq \frac{300}{20}$$

$$\boxed{y \leq 15}$$

Change in momentum between any two positions of projectile

1) Between point of projection & highest point :-



$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v} = u\cos\theta\,\hat{i}$

$\Delta\vec{P} = m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u})$

$\Delta\vec{P} = m(-\sin\theta\,\hat{j})$

$\boxed{\Delta\vec{P} = m(-u\sin\theta\,\hat{j})}$

$\boxed{|\Delta\vec{P}| = mu\sin\theta}$

2) Between initial & final points.

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v} = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

$\Delta\vec{P} = m(\vec{v} - \vec{u})$

$= m(-2\sin\theta\,\hat{j})$

$\boxed{\Delta\vec{P} = -2mu\sin\theta\,\hat{j}}$

$\boxed{|\Delta\vec{P}| = 2mu\sin\theta}$

Q A object of mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 60°. find magnitude of change in momentum after 10s ($g = 9.8\ m/s^2$)

$m = \cancel{10}\ 0.5\ kg$

$u = 98\ m/s$

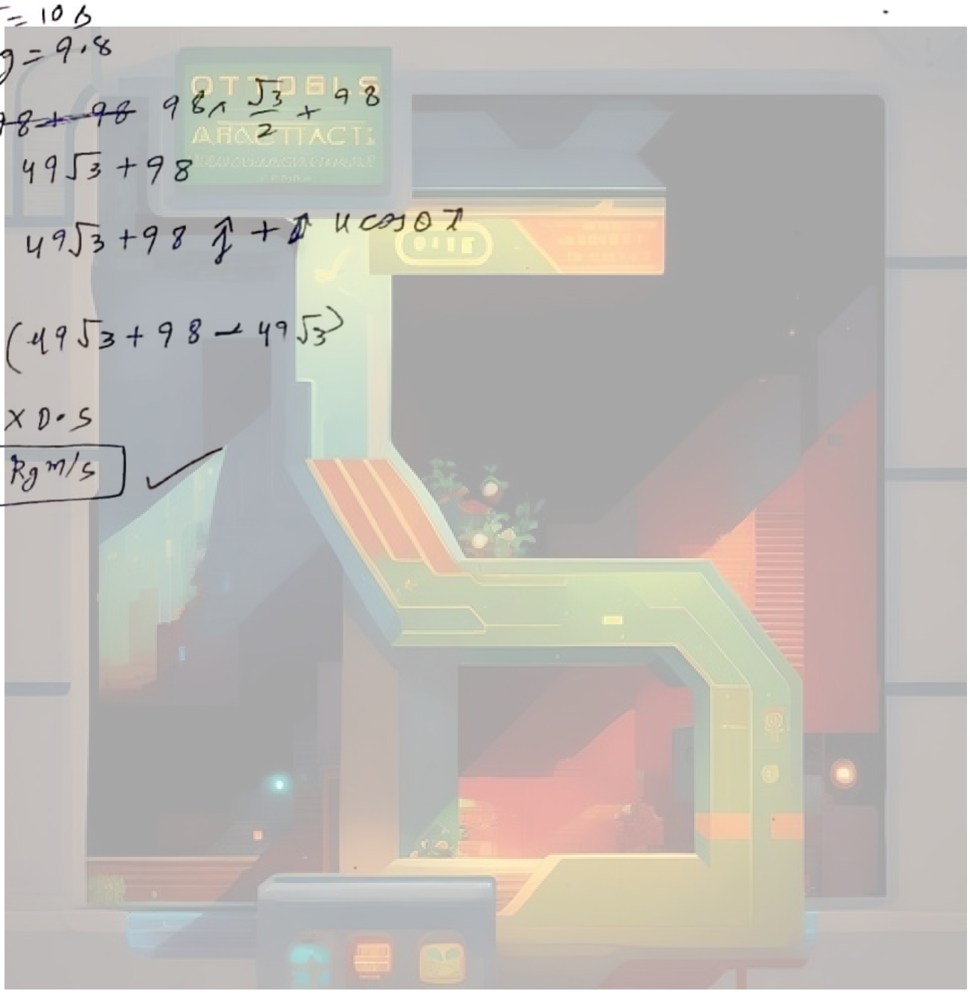
$\theta = 60°$

$\cancel{|\Delta \vec{P}| =}$

$u = 98 \sin\theta\ \hat{j}$

$t = 10\ s$

$g = 9.8$



$v = \cancel{98 + 98}\ 98 \times \frac{\sqrt{3}}{2} + 98$

$v = 49\sqrt{3} + 98$

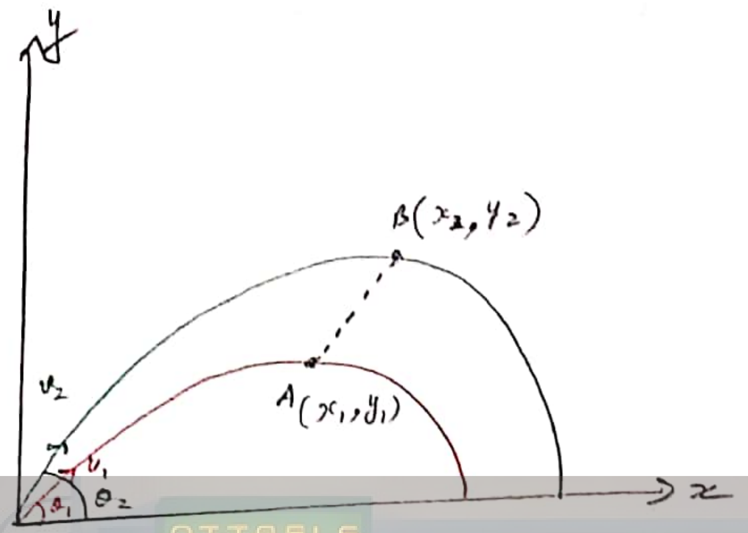
$\vec{v} = 49\sqrt{3} + 98\ \hat{j} + u\cos\theta\ \hat{i}$

$|\Delta \vec{P}| = m(49\sqrt{3} + 98 - 49\sqrt{3})$

$= 98 \times 0.5$

$\boxed{|\Delta \vec{P}| = 49\ kg\ m/s}$

Path of one projectile in reference of another projectile:-



$$\left.\begin{aligned} x_1 &= u_1 \cos_1 t \\ x_2 &= u_2 \cos_2 t \end{aligned}\right] \quad x_2 - x_1 = (u_2 \cos \theta_2 - u_1 \cos \theta_1) t$$

$$\left.\begin{aligned} y_1 &= u_1 \sin \theta_1 t - \frac{1}{2} g t^2 \\ y_2 &= u_2 \sin \theta_2 t - \frac{1}{2} g t^2 \end{aligned}\right] = u_2 \sin \theta_2 - u_1 \sin \theta_1$$

$$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \frac{u_2 \sin \theta_2 - u_1 \sin \theta_1}{u_2 \cos \theta_2 - u_1 \cos \theta_1}}$$

$$y_2 - y_1 = m (x_2 - x_1)$$

m ↳ constant

If $u_1 \cos \theta_1 = u_2 \cos \theta_2$

$$\frac{y_2 - y_1}{x_2 - x_1} = \infty \quad \text{(vertical straight line)}$$

Q.

find after 3s.

a) velocity of ball
b) angle made by ball with horizontal
c) horizontal & vertical displacement

$\theta - 10 \times 3$

~~$u = 60\sqrt{2} \times \frac{1}{\sqrt{2}}$~~

~~$= 60 - 30$~~

~~$\boxed{= 30\hat{j}}$ m/s~~

~~$\vec{v} = 30\hat{j} + 30\hat{i}$~~

~~$= \sqrt{900 + 900}$~~

~~$\boxed{= 30\sqrt{2}}$ m/s~~

$a) \vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$= 60\sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + 60\sqrt{2} \times \frac{1}{\sqrt{2}} - 30\hat{j}$

$= 60\hat{i} + 30\hat{j}$

$|\vec{v}| = \sqrt{60 \times 60 + 30 \times 30}$

$= \sqrt{3600 + 900}$

$= \sqrt{4500}$

$= 10\sqrt{45}$

$\boxed{= 30\sqrt{5} \text{ m/s}}$ ✓

b) ~~$\cos\alpha = \frac{30\sqrt{5}}{60}$~~

~~$= \frac{\sqrt{5}}{2}$~~

~~$\boxed{\alpha = \cos^{-1}\frac{\sqrt{5}}{2}}$~~

$\theta = \tan\theta = \frac{30}{60} = \frac{1}{2}$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

~~$\cos\theta = \frac{\ldots}{\ldots\sqrt{5}}$~~

$B = 1$

$\cos\alpha = \frac{60}{35\sqrt{5}}$

c) horizontal – $\boxed{180 \text{ m}}$

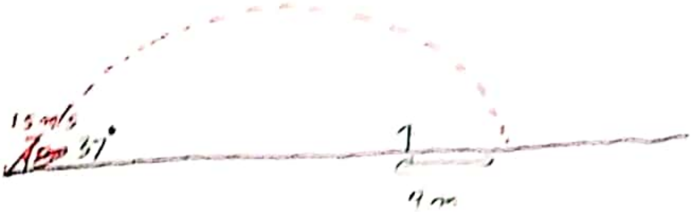
vertical $S = ut + \frac{1}{2}at^2$

$S = 60 \times 3 - \frac{10}{2} \times 9$

$S = 180 - 45$

$\boxed{S = 135 \text{ m}}$

Q with what minimum velocity man should run to catch the ball? $(g = 10\,m/s^2)$



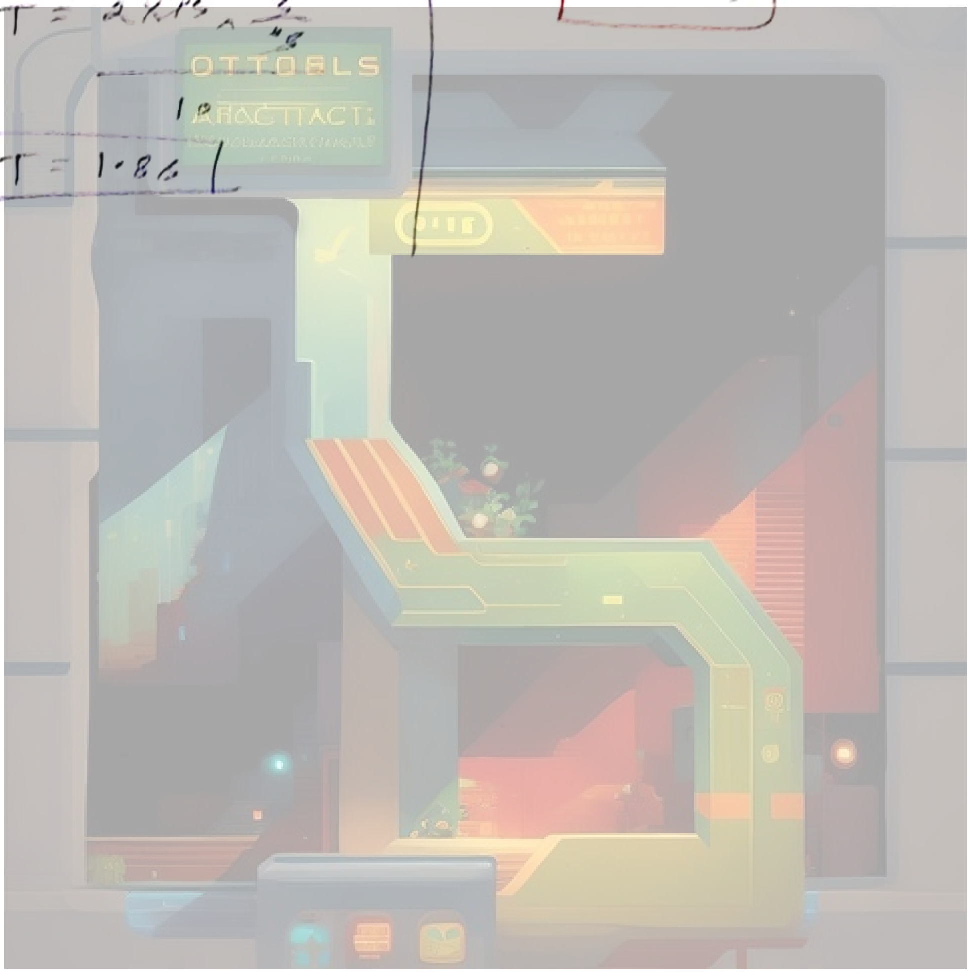
$T = \dfrac{2u\sin\theta}{g}$

$T = \dfrac{2 \times 15 \times \frac{3}{5}}{10}$

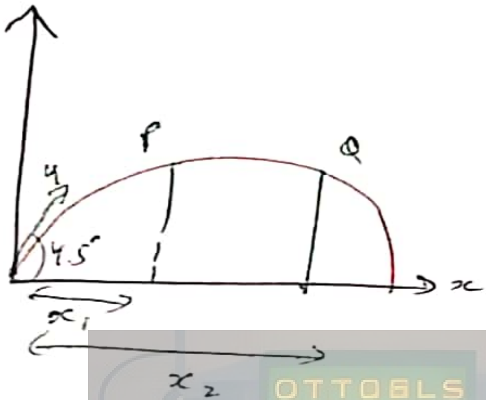
$\boxed{T = 1.8\,s}$

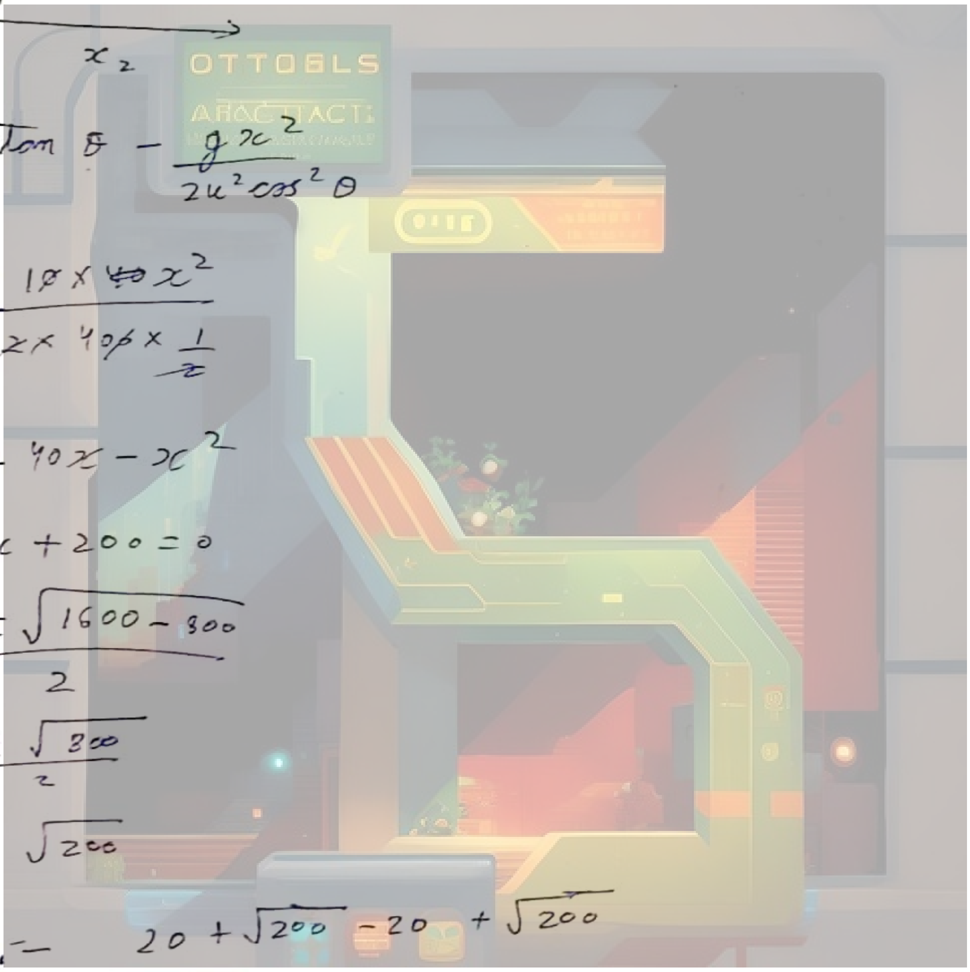
Speed $= \dfrac{9}{1.8}$

$\boxed{= 5\,m/s}$



Q A particle fired with a velocity 20 m/s from a gun adjusted for maximum range. It passes through P & Q whose heights above horizontal are 5 m each. find separation between P & Q?





$$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$$

$$5 = x - \frac{10 \times x^2}{2 \times 400 \times \frac{1}{2}}$$

$$200 = 40x - x^2$$

$$x^2 - 40x + 200 = 0$$

$$x = \frac{40 \pm \sqrt{1600 - 800}}{2}$$

$$x = \frac{40 \pm \sqrt{800}}{2}$$

$$x = 20 \pm \sqrt{200}$$

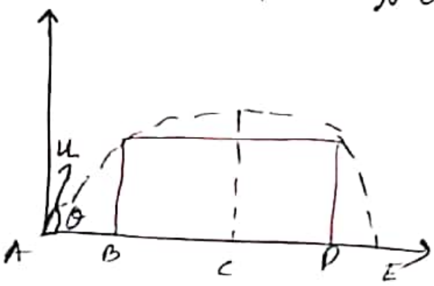
separation $= 20 + \sqrt{200} - 20 + \sqrt{200}$

$$\boxed{= 20\sqrt{2}\ m}$$

Q bird height — h

max height — 2h

find ratio of ~~speed~~ velocity of bird & ~~s~~ horizontal velocity of the stone thrown



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{u^2\sin^2\theta}{4g} = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{gx^2}{2u^2\cos^2\theta} - x\tan\theta + \frac{u^2\sin^2\theta}{4g} = 0$$

$$x = \frac{\tan\theta \pm \sqrt{\tan^2\theta - \frac{g\,u^2\sin^2\theta}{2u^2\cos^2\theta\, g}}}{\frac{g}{u^2\cos^2\theta}}$$

$$x = \frac{\tan\theta \pm \tan\theta\sqrt{\frac{1}{2}}}{\frac{g}{u^2\cos^2\theta}}$$

$$\frac{\tan\theta + \frac{\tan\theta}{\sqrt{2}}}{\frac{g}{u^2\cos^2\theta}} - \frac{\tan\theta - \frac{\tan\theta}{\sqrt{2}}}{\frac{g}{u^2\cos^2\theta}}$$

$$\frac{\tan\theta\, u^2\cos^2\theta}{\sqrt{2}g} + \frac{\tan\theta\, u^2\cos^2\theta}{\sqrt{2}g}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$h = \frac{u^2\sin^2\theta}{4g}$$

$$u\sin\theta = 2\sqrt{gh}$$

$$\frac{\sqrt{2}\tan\theta\, u^2\cos^2\theta}{g}$$

②

Scanned with CamScanner

$h = ut + \frac{1}{2} g t^2$

$g t^2 + 2ut - 2h = 0$

$g t^2 + 2ut - \frac{u^2 \sin^2\theta}{2g} = 0$

$t = \frac{-2u \pm \sqrt{4u^2 + 2g u^2 \sin^2\theta}}{2g}$

$\frac{-2u + \sqrt{4u^2 + 2g u^2 \sin^2\theta} + 2u + \sqrt{4u^2 + 2g u^2 \sin^2\theta}}{2g}$

$\frac{\sqrt{4u^2 + 2g u^2 \sin^2\theta}}{g}$

$\text{Speed} = \dfrac{\frac{\sqrt{2}\tan\theta \, u^2 \cos^3\theta}{g}}{\frac{\sqrt{4u^2 + 2g u^2 \sin^2\theta}}{g}}$

$= \frac{\sqrt{2}\tan\theta \, u^2 \cos^2\theta}{\sqrt{4u^2 + 2g \times 4gh}}$

$= \frac{\sqrt{2}\tan\theta \, u^2 \cos^2\theta}{\sqrt{4u^2 \sin^2\theta + 8g^2 h}}$ (vertical velocity

$= \frac{\sqrt{2}\tan\theta \, u^2 \cos^2\theta}{\sqrt{8gh + 8g^2 h}}$

$= \frac{\tan\theta \, u^2 \cos^2\theta}{2\sqrt{gh}}$

Speed

$u \cos\theta$

$\boxed{\cancel{\frac{\tan\theta \, u \cos\theta}{2\sqrt{gh}}}}$

$\frac{u \sin\theta}{2\sqrt{gh}}$

$\frac{2\sqrt{gh}}{2\sqrt{gh}} = \boxed{\sqrt{1}\,\sqrt{h}}$

$\boxed{= \frac{2\sqrt{2}}{2+\sqrt{2}}}$

Q2

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$2h = \frac{u^2 \sin^2\theta}{2g}$$

$$u^2 \sin^2\theta = 4gh$$

$$\boxed{u\sin\theta = 2\sqrt{gh}}$$

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$h = u\sin\theta\, t - \frac{1}{2} g t^2$$

$$h = 2\sqrt{gh}\, t - \frac{gt^2}{2}$$

$$gt^2 - 4\sqrt{gh}\, t + 2h = 0$$

$$t = \frac{4\sqrt{gh} \pm \sqrt{16gh - 8gh}}{2g}$$

$$t_1 = \frac{4\sqrt{gh} - 2\sqrt{2gh}}{2g}$$

$$t_2 = \frac{4\sqrt{gh} + 2\sqrt{gh}}{2g}$$

$$AD = AB + BD$$

$$u\cos\theta\, t_2 = u\cos\theta\, t_1 + V_B t_2$$

$$u\cos\theta (t_2 - t_1) = V_B t_2$$

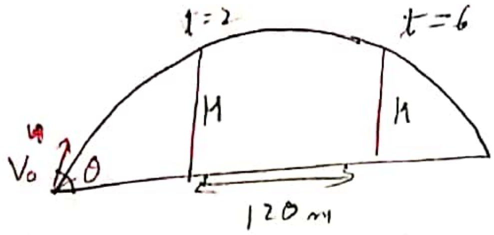
$$\frac{V_B}{u\cos\theta} = \frac{t_2 - t_1}{t_2}$$

$$= \frac{4\sqrt{2gh}}{4\sqrt{gh} + 2\sqrt{2gh}}$$

$$= \frac{4\sqrt{2}}{4 + 2\sqrt{2}}$$

$$\boxed{= \frac{2\sqrt{2}}{2 + \sqrt{2}}}$$

Q



A projectile crosses 2 walls of height H. find

a) time of flight

b) $H = ?$

c) Max height attained $= ?$

d) Range, $R = ?$

e) $\theta = ?$

f) $V_0 = ?$

$g = 10$

$u = V_0$

$u\cos\theta = \frac{120}{4}$ 30

$\boxed{u\cos\theta = 30}$

$S = ut + \frac{1}{2}at^2$

$H = u\sin\theta(t) + 5t^2$

$\cancel{5t^2 + u\sin\theta t - H = 0}$

$\cancel{t = \frac{-u\sin\theta + \sqrt{u^2\sin^2\theta + 20H}}{10}}$

$u\sin(2) + 5(2)^2 = u\sin(6) + 5(6)^2$

$2u\sin\theta - 20 = 6u\sin\theta - 180$

$0 = 4u\sin\theta + 200$

$0 = u\sin\theta + 50$

$\boxed{u\sin\theta = +40}$

$\boxed{u = \frac{40}{\sin\theta}}$

$T = \frac{2u\sin^2\theta}{2 \times g}$

$= \frac{2 \times 40}{10}$

$\cancel{\boxed{T = 40\,s}}$

$\boxed{T = 8\,s}$ a)

$H = \frac{u^2\sin^2\theta}{2g}$

$H = \frac{1600}{2 \times 10}$

$\boxed{H = 80\,m}$ c) max height

$H = 40(2) - 5(4)$

$H = 80 - 20$

$\boxed{H = 60}$ b)

$\frac{40}{\sin\theta} \times \cos\theta = 30$

$\frac{40}{30} = \tan\theta$

$\tan\theta = \frac{4}{3}$

$\boxed{\theta = 60^\circ}$ e)

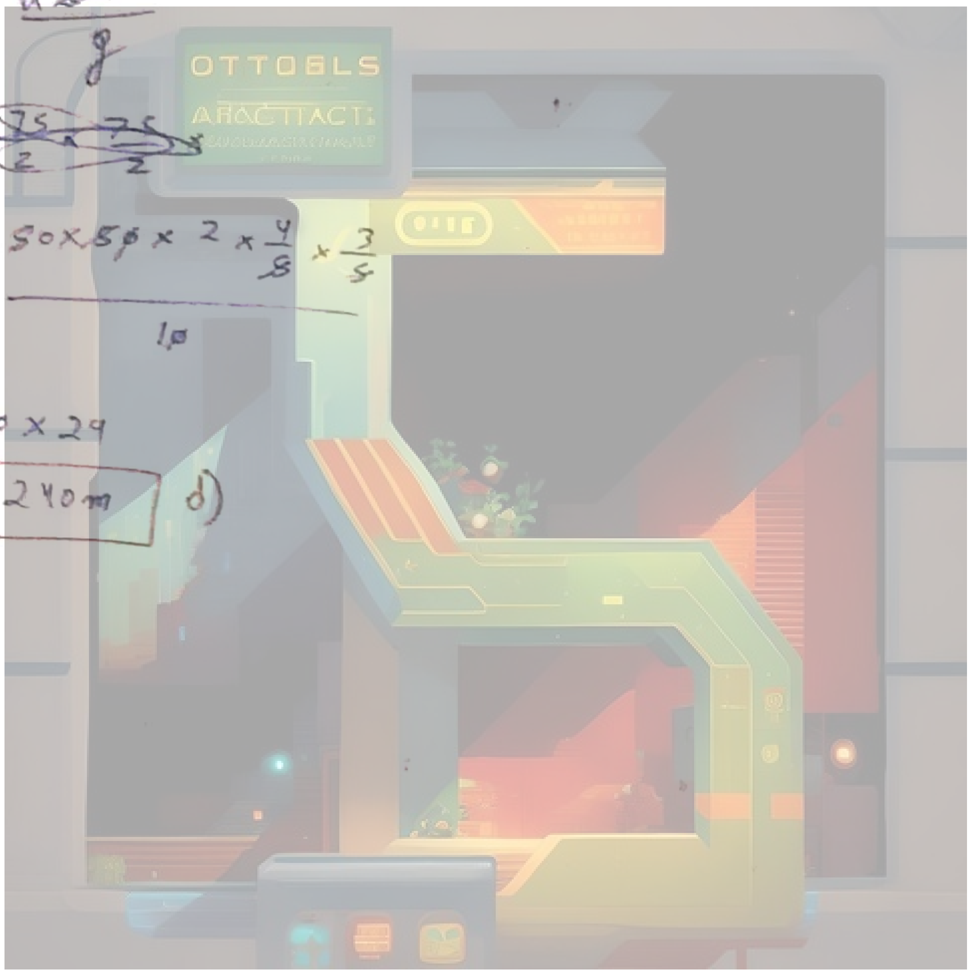
$u \cos\theta = 30$

$u \times \frac{3}{5} = 30$

$u = \frac{30 \times 5}{3}$

$\boxed{u = 50 \text{ m/s}}$ b)

$R = \frac{u^2 \sin 2\theta}{g}$



$R = \frac{50 \times 50 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10}$

$R = 10 \times 24$

$\boxed{R = 240 \text{m}}$ d)

~~Hori~~ Horizantal Projection From A Height

Horizantal

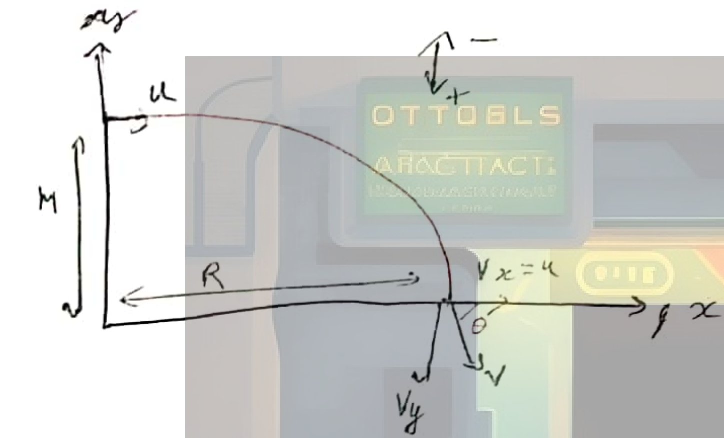
$U_x = u$

$a_x = 0$

Verticle

$U_y = 0$

$a_y = g$



Time of flight (T):-

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$H = \frac{1}{2} g T^2$$

$$\boxed{T = \sqrt{\frac{2H}{g}}}$$

Range $(R) = U_x T$

$$\boxed{R = u\sqrt{\frac{2H}{g}}}$$

$$V_y^2 = u_y^2 + 2 a_y S_y$$

$$V_y^2 = 0 + 2gH$$

$V_y = \sqrt{2gh}$ (Verticle component of velocity on hitting ground)

$$\boxed{V_y = \sqrt{2gh}}$$

~~$\theta = \tan$~~

$$\boxed{\theta = \tan^{-1}\frac{\sqrt{2gH}}{u}}$$

$$\boxed{\vec{V} = u\hat{i} + \sqrt{2gh}\,\hat{j}}$$

Q A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high, Find

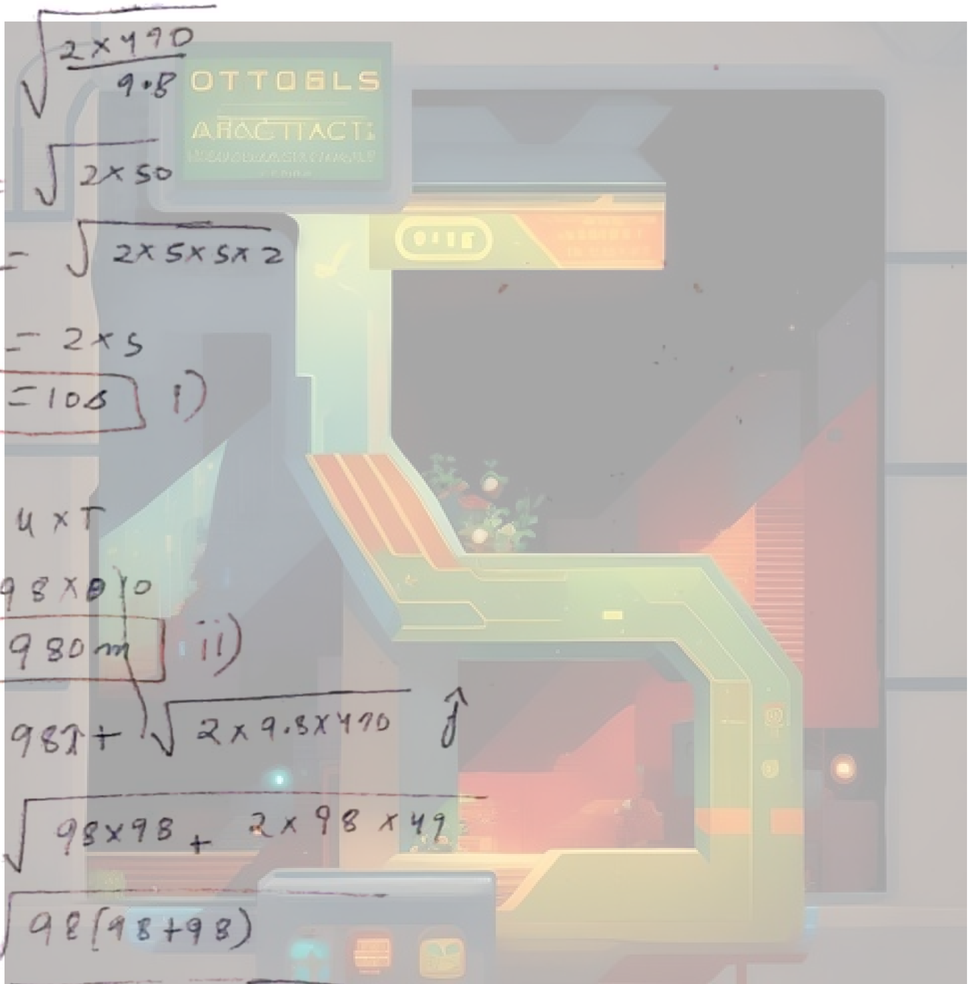
i) time to reach ground

ii) The horizontal distance from foot of hill to ground

iii) The velocity with which it hits the ground. $(g = 9.8)$

$$T = \sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2 \times 490}{9.8}}$$



$$T = \sqrt{2 \times 50}$$

$$T = \sqrt{2 \times 5 \times 5 \times 2}$$

$$T = 2 \times 5$$

$$\boxed{= 10\,s} \quad i)$$

$$R = u \times T$$

$$= 98 \times 10$$

$$\boxed{= 980\,m} \quad ii)$$

$$\vec{v} = 98\hat{i} + \sqrt{2 \times 9.8 \times 490}\ \hat{j}$$

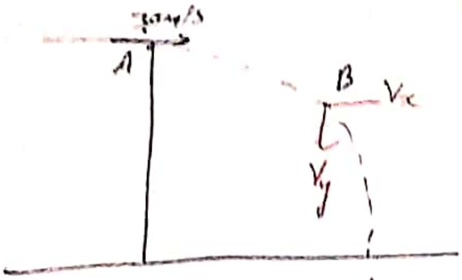
$$v = \sqrt{98 \times 98 + 2 \times 98 \times 49}$$

$$v = \sqrt{98(98 + 98)}$$

$$v = \sqrt{98 \times 98 \times 2}$$

$$\boxed{v = 98\sqrt{2}\ m/s} \quad iii)$$

Q.



$|V_x| = |V_y|$

~~then~~ find time taken to reach point B.

$$T = \sqrt{\frac{2H}{g}}$$

$$V_y = \sqrt{2gH}$$

$$30 = \sqrt{2 \times 10 \times H}$$

$$900 = 20H$$

$$H = \frac{900}{20}$$

$$\boxed{H = 45\,m}$$

$$T = \sqrt{\frac{2 \times 45}{10}}$$

$$T = \sqrt{\frac{45}{5}}$$

$$T = \sqrt{9}$$

$$\boxed{T = 3\,s}$$

Or

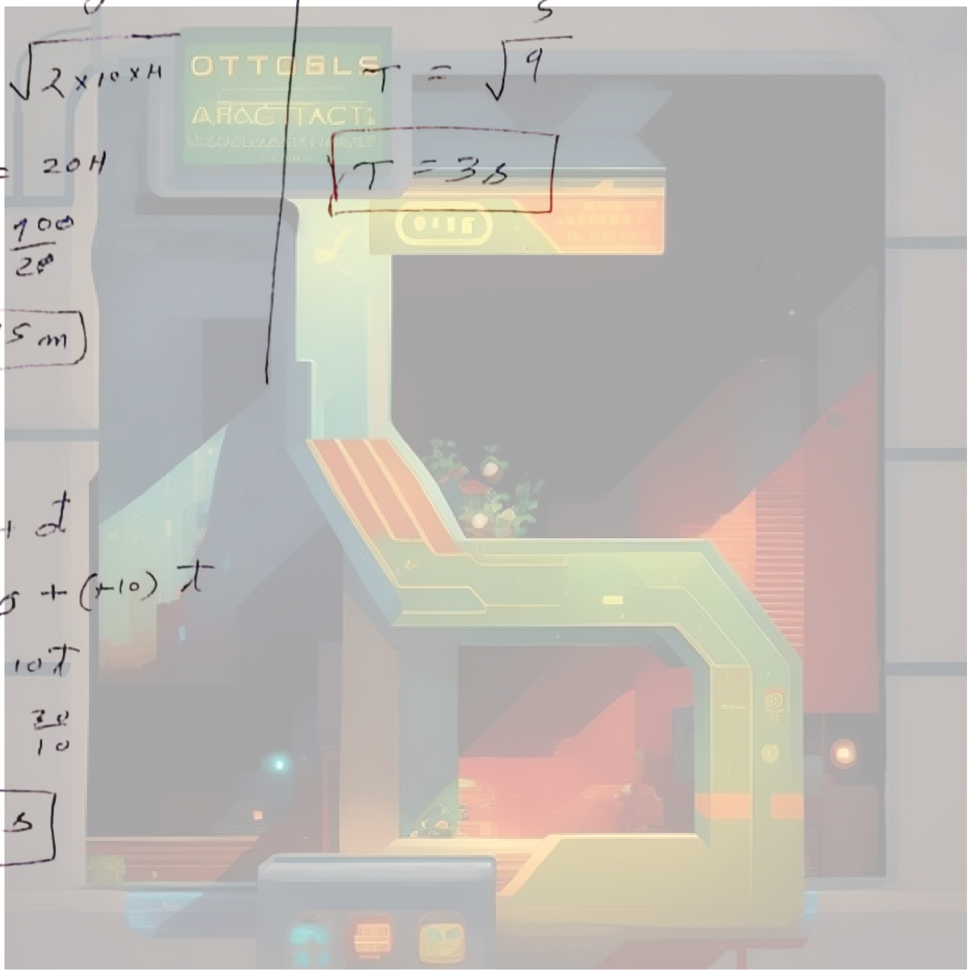
$$v = u + at$$

$$30 = 0 + (+10)t$$

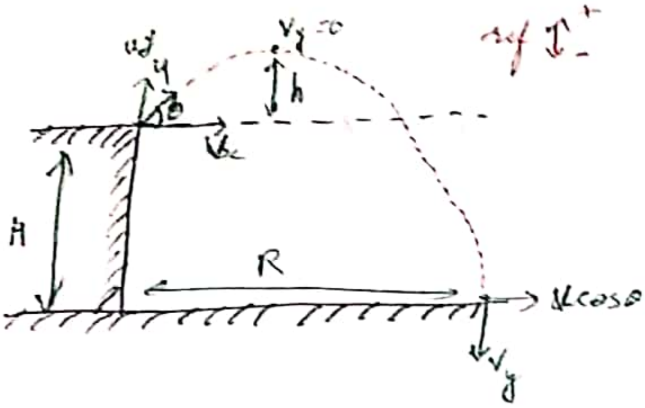
$$30 = 10t$$

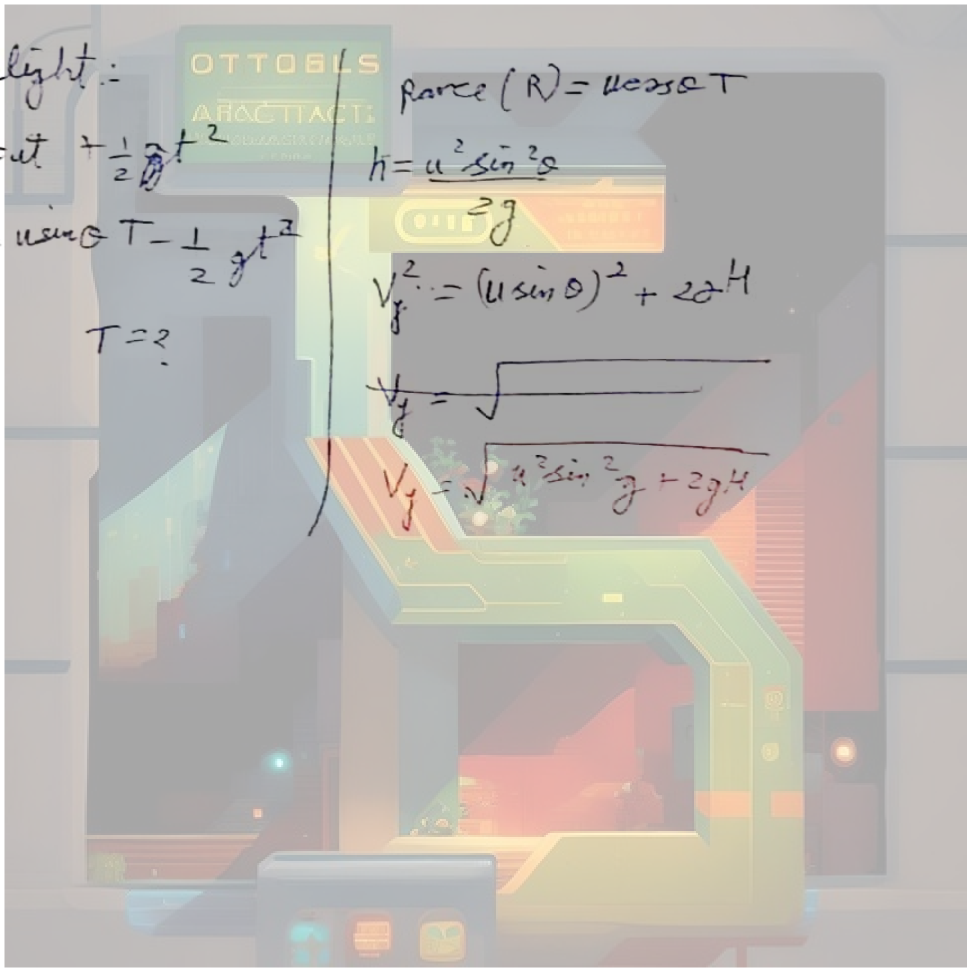
$$t = \frac{30}{10}$$

$$\boxed{t = 3\,s}$$



Projection at an Angle from a height





Time of flight :

$S = ut + \frac{1}{2}gt^2$

$-H = u\sin\theta\, T - \frac{1}{2}gt^2$

$T = ?$

Range $(R) = u\cos\theta\, T$

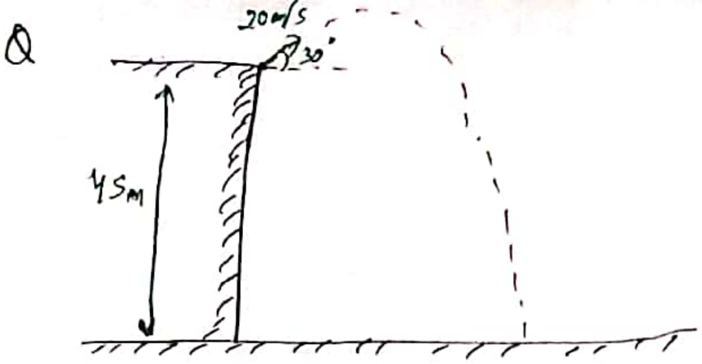
$h = \frac{u^2\sin^2\theta}{2g}$

$v_y^2 = (u\sin\theta)^2 + 2gH$

$v_y = \sqrt{}$

$v_y = \sqrt{u^2\sin^2 g + 2gH}$

Q



find a) T
b) R
c) speed (V)

t

$s = ut + \frac{1}{2} at^2$

$-45 = u\sin\theta \, t - 5t^2$

$5t^2 - 20\times\frac{1}{2} t - 45 = 0$

$t^2 - 2t - 9 = 0$

$t = \frac{2 \pm \sqrt{4+36}}{2}$

$t = \frac{2 \pm \sqrt{40}}{2}$

$t = 1 \pm \sqrt{10}$

$\boxed{t = 1 + \sqrt{10}\, s}$

$R = u\cos\theta \, t$

$R = \cancel{20}^{10} \times \frac{\sqrt{3}}{\cancel{2}} \times (1+\sqrt{10})$

$R = 10\sqrt{3} + 10\sqrt{30}$

$\boxed{R = 10(\sqrt{3}+\sqrt{30})\, m}$

$v = (u^2\sin^2\theta + 2gH)^{1/2}$

$v = 400 \times \frac{1}{4} + 2 \times -10 \times -45$

$v = 100 + 900$

$v = \sqrt{1000}$

$v_y = 10\sqrt{10}$ m/s

$|\vec{v}| = \sqrt{(10\sqrt{10})^2 + (10\sqrt{2})^2}$

$\cancel{= \sqrt{1000 + 200}}$

$\cancel{= \sqrt{1200}}$

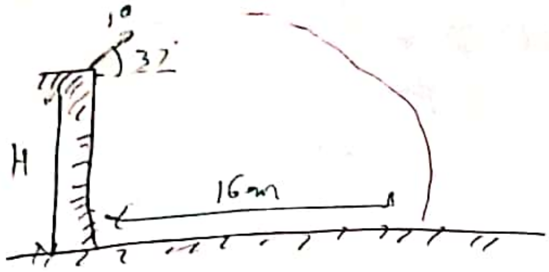
$\cancel{= 10\sqrt{12}}$

$\cancel{\boxed{= 20\sqrt{3} \text{ m/s}}}$

$= \sqrt{1300}$

$\boxed{= 10\sqrt{13} \text{ m/s}}$

Q



$g = 10\ m/s^2$

$-H = 6t - 5t^2$

$5t^2 - 6t - H = 0$

$t = \dfrac{6 \pm \sqrt{36 + 20H}}{2}$

$t = 3 \pm \sqrt{9 + 5H}$

$16 = u\cos\theta\, t$

$16 = 8t$

$t = 2$

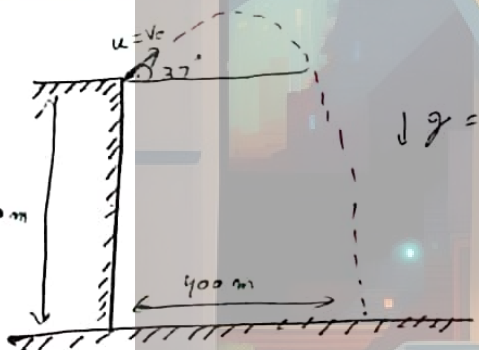
$3 + \sqrt{9+5H} = 2$

$\sqrt{9+5H} =$

$-H = 12 - 20$

$\boxed{H = 8\,m}$

Q



$400 = u \times \frac{4}{5} \times t$

$500 = ut$

$\frac{500}{u} = t$

$-100 = u \times \frac{3}{5} \times \frac{500}{u} - \frac{1}{2} \times 10 \times \frac{500}{u} \times \frac{500}{u}$

$-100 = 300 - \frac{1250000}{u^2}$

$-100u^2 = 300u^2 - 12500$

$12500 = 400u^2$

$\frac{12500}{400} = u^2$

$u = \frac{5\sqrt{5}}{\sqrt{2}}\ m/s$

$u^2 = \frac{6250}{2}$

$u = \frac{25\sqrt{10}}{2}$

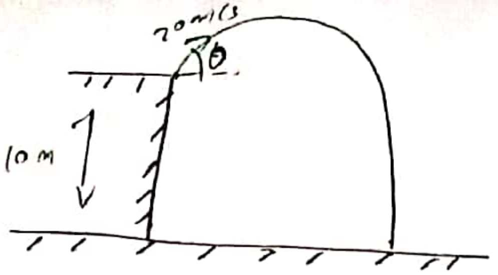
$u^2 = \frac{6250}{2}$

$u^2 = 3125$

$\boxed{u^2 = 25\sqrt{5}}$

(182)

Q



$$-10 = 20\sin\theta\, t - 5t^2$$

$$-2 = 4\sin\theta\, t - t^2$$

$$t^2 - 4\sin\theta\, t - 2 = 0$$

$$t = \frac{4\sin\theta + \sqrt{16\sin^2\theta + 8}}{2}$$

$$t = 2\sin\theta + \sqrt{4\sin^2\theta + 2}$$

$$0 = 2\cos\theta + \frac{1}{2\sqrt{4\sin^2\theta + 2}}$$

$$0 = 4\cos\theta\sqrt{4\sin^2\theta + 2} + 1$$

$$0 =$$

$$1 = 16\cos^2\theta\,(4\sin^2\theta + 2)$$

$$\sec^2\theta = 64\sin^2\theta + 2$$

~~R = ucosθ t~~

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-10 = x\tan\theta - \frac{10x^2}{2\times 400\times\cos^2\theta}$$

$$-10 = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$-10 = x\tan\theta - \frac{x^2}{80\cos^2\theta}$$

$$-800\cos^2\theta = 80x\sin\theta\cos\theta - x^2$$

$$x^2 - x\sin\theta\cos\theta - 800\cos^2\theta$$

$$\frac{dy}{dx} = 2x -$$

$$-10 = x\tan\theta - \frac{10x^2}{400\times 2\times\cos^2\theta}$$

$$-10 = x\tan\theta - \frac{x^2\sec^2\theta}{80}$$

$$-800 = 80x\tan\theta - x^2\sec^2\theta$$

$$6400x^2 + 4x^2(x^2 - 800) \geq 0$$

$$6400x^2 + 4x^4 - 3200x^2 \geq 0$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$-10 = x\tan\theta - \frac{10x^2\sec^2\theta}{800}$$

$$-10 = x\tan\theta - \frac{x^2(1 - \tan^2\theta)}{80}$$

$$-800 = 80x\tan\theta - x^2 + x^2\tan^2\theta$$

$$x^2\tan^2\theta + 80x\tan\theta - (x^2 - 800) = 0$$

$$b^2 - 4ac \geq 0$$

$$9600 \cancel{3200}x^2 - 4x^4 \geq 0$$

$$2400 \leq R^2$$

$$\boxed{R \leq 20\sqrt{6}}$$

~~2400 tan²θ~~

$$\tan\theta = \frac{1600\sqrt{6} \pm \sqrt{9600x^2 - 4x^4}}{4800}$$

$$\tan\theta = \frac{1600\sqrt{6} + \sqrt{9600\times 2400 + 4\times 2400}}{2400\times 2}$$

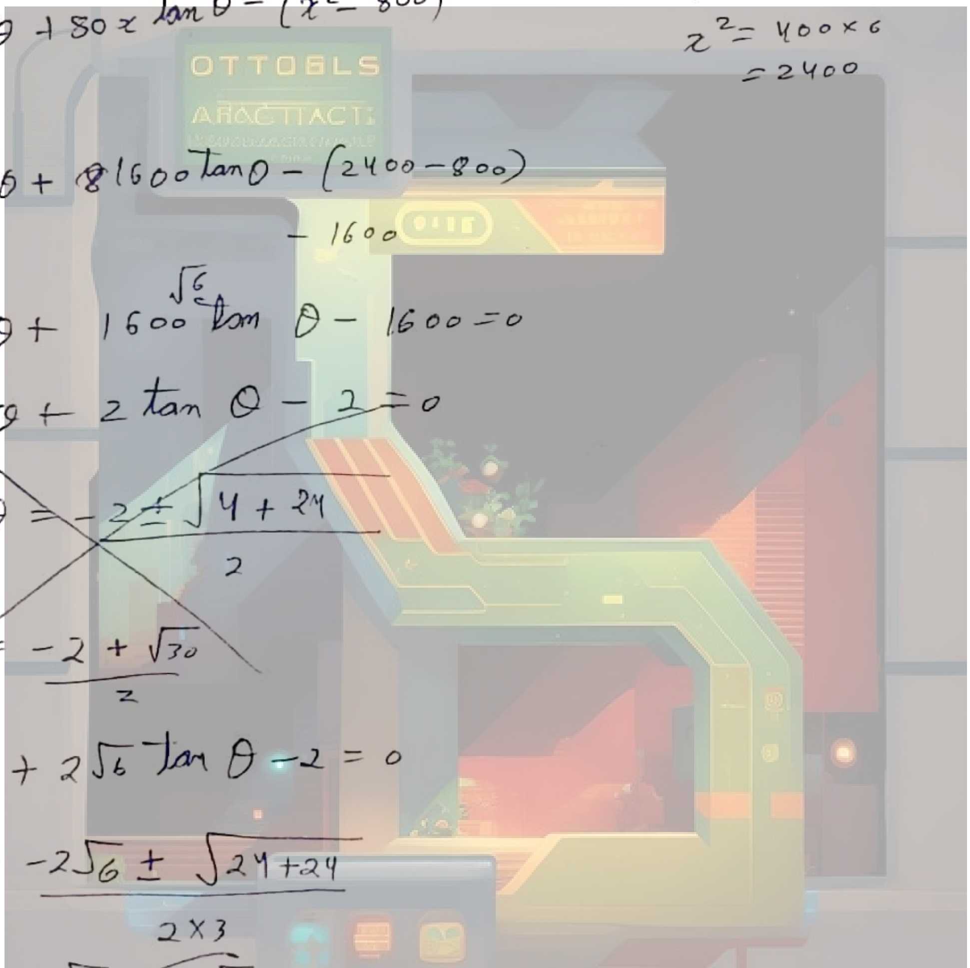
$$\tan\theta = \frac{2/3\sqrt{6} + \sqrt{9600+4}}{2}$$

$$\tan\theta = \frac{1}{3}\sqrt{6} + \sqrt{2401}$$

$$\theta = \tan^{-1}\left[\frac{\sqrt{6}}{3} + \sqrt{2401}\right]$$

$$x = 20\sqrt{6}$$
$$x^2 = 400\times 6 = 2400$$

$$x^2\tan^2\theta + 80x\tan\theta - (x^2 - 800) = 0$$



$$2400\tan^2\theta + \cancel{8}1600\tan\theta - (2400 - 800)$$
$$- 1600$$

$$2400\tan^2\theta + 1600\sqrt{6}\tan\theta - 1600 = 0$$

$$\cancel{3\tan^2\theta + 2\tan\theta - 2 = 0}$$

$$\cancel{\tan\theta = \frac{-2 \pm \sqrt{4+24}}{2}}$$

$$\cancel{\tan\theta = \frac{-2+\sqrt{30}}{2}}$$

$$3\tan^2\theta + 2\sqrt{6}\tan\theta - 2 = 0$$

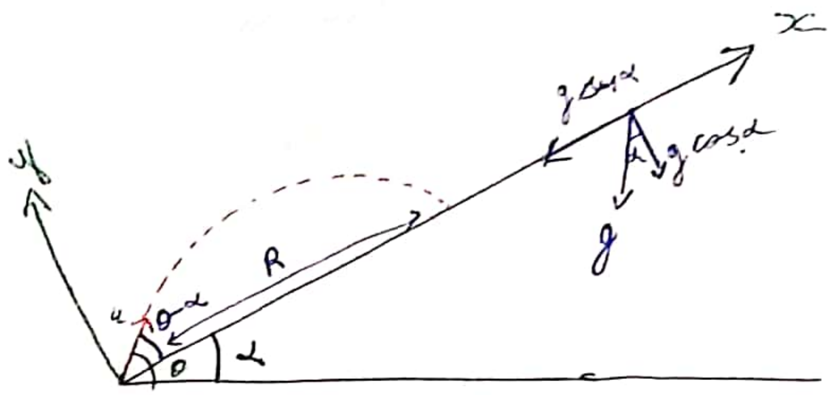
$$\tan\theta = \frac{-2\sqrt{6} \pm \sqrt{24+24}}{2\times 3}$$

$$\cancel{\tan\theta = \frac{-\sqrt{6} \pm 12\sqrt{2}}{3}}$$

$$\tan\theta = \frac{-\sqrt{6} \pm \sqrt{12}}{3}$$

$$\boxed{\tan\theta = \frac{-\sqrt{6} \pm 2\sqrt{3}}{3}}$$

Projection up the inclined plane



Horizontal	Vertical
$u_x = u\cos(\theta-\alpha)$	$u_y = u\sin(\theta-\alpha)$
$a_x = -g\sin\alpha$	$a_y = -g\cos\alpha$

Time of Flight (T)=

$$s = ut + \frac{1}{2}at^2$$

$$0 = u_y t + \frac{1}{2} a_y t^2$$

$$0 = u\sin(\theta-\alpha)\, t + \frac{1}{2}(-g\cos\alpha)\, t^2$$

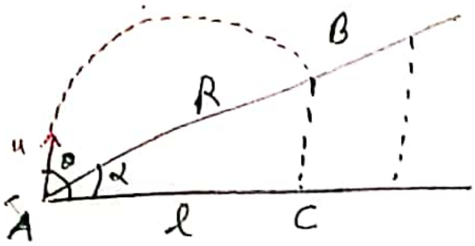
$$\frac{g\cos\alpha}{2} t^{\cancel{2}} = u\sin(\theta-\alpha)\cancel{t}$$

$$t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

$$\boxed{t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}}$$

Range:- $u_x T + \frac{1}{2} a_x T^2$

$$R = u\cos(\theta-\alpha)\, T + \frac{1}{2}(\cancel{-g\cos\alpha})(-g\sin\alpha)\, T^2$$



$l = u\cos\theta\, T$ & $\cos\alpha = \frac{l}{R}$

$$\cos\alpha = \frac{u\cos\theta\, T}{R}$$

$$R = \frac{u\cos\theta\, T}{\cos\alpha}$$

$$R = \frac{u\cos\theta}{\cos\alpha} \times \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

$$\boxed{R = \frac{2u^2\cos\theta\sin(\theta-\alpha)}{g\cos^2\alpha}}$$

$$\boxed{R_{max} = \frac{u^2}{g(1+\sin\alpha)}}$$

JM6 (ArCh-8)

Q8. $x^2 = at^2 + 2bt + c$

$$\frac{d}{dt}(x^2) = \frac{d}{dt}(at^2 + 2bt + c)$$

$$2x \times \frac{dx}{dt} = 2at + 2b$$

$$x\,v = at + b$$

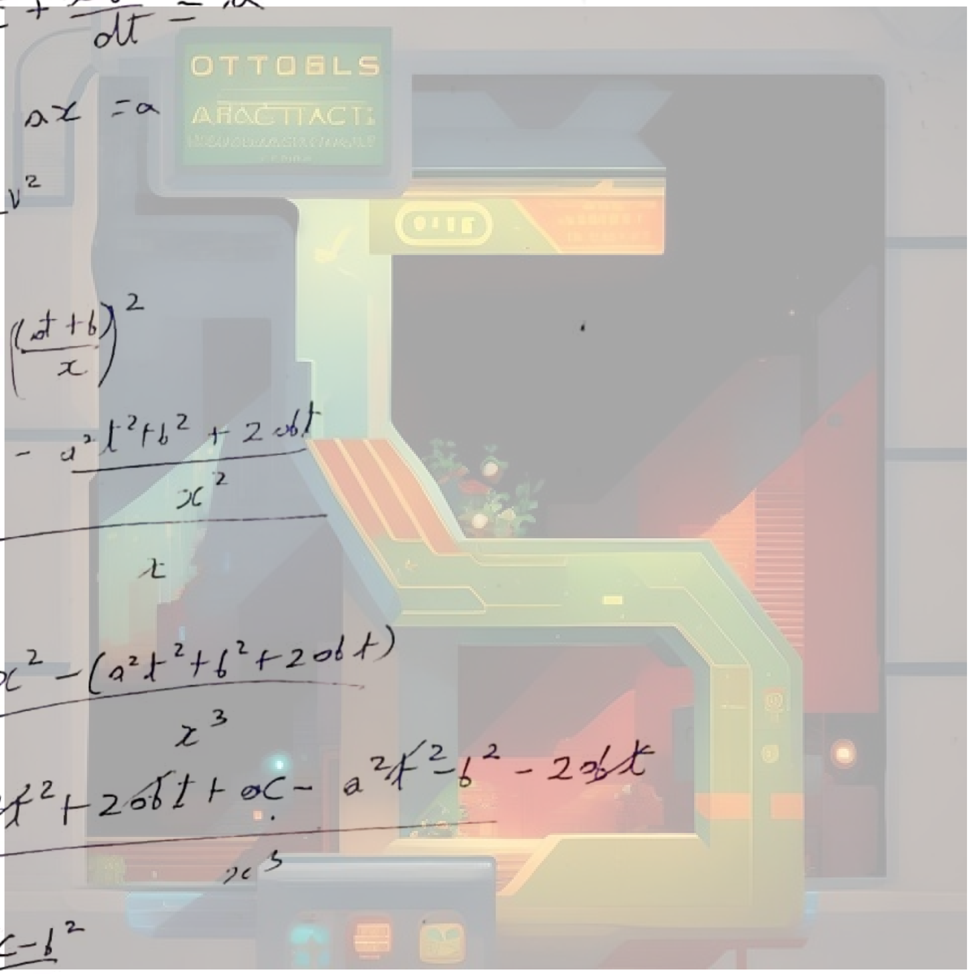
$$v \times \frac{dx}{dt} + x\frac{dv}{dt} = a$$

$$v^2 + Ax = a$$

$$A = \frac{a - v^2}{x}$$

$$A = \frac{a - \left(\frac{at+b}{x}\right)^2}{x}$$

$$A = \frac{a - \frac{a^2t^2 + b^2 + 2abt}{x^2}}{x}$$



$$A = \frac{ax^2 - (a^2t^2 + b^2 + 2abt)}{x^3}$$

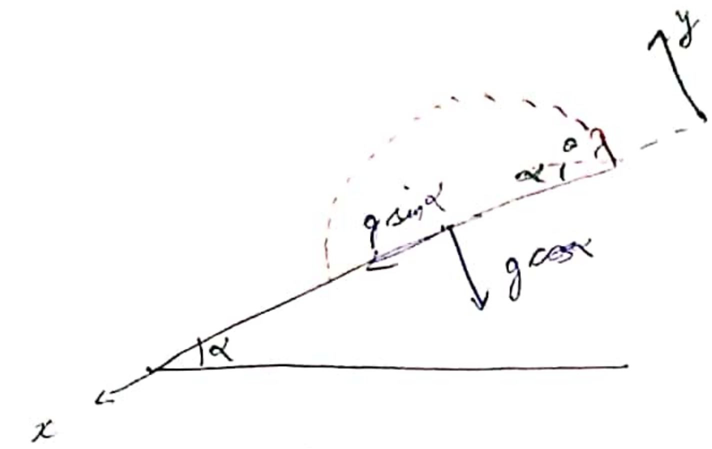
$$A = \frac{a^2t^2 + 2abt + ac - a^2t^2 - b^2 - 2abt}{x^3}$$

$$A = \frac{ac - b^2}{x^3}$$

$$A \propto x^{-3}$$

$$\boxed{n = 3}$$

Projection down the inclined plane



Horizontal	Vertical
$U_x = u\cos(\theta+\alpha)$	$U_y = u\sin(\theta+\alpha)$
$a_x = g\sin\alpha$	$a_y = -g\cos\alpha$

Time of flight (T):–

$$S_y = 0 = U_y T + \frac{1}{2} a_y T^2$$

$$0 = u\sin(\theta-\alpha)T + \frac{1}{2}(-g\cos\alpha)T^2$$

$$\boxed{T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}}$$

$$R = U_x T + \frac{1}{2} a T^2$$

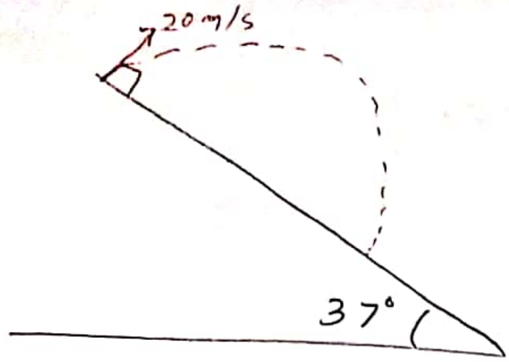
$$R = u\cos(\theta+\alpha)T + \frac{1}{2} g\sin\alpha T^2$$

$$\boxed{R = u\cos(\theta+\alpha)T + \frac{1}{2} g\sin\alpha T^2}$$

$$\boxed{R = \frac{2u^2\sin(\theta+\alpha)\cos\theta}{g\cos^2\alpha}}$$

$$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

Q



$$\cancel{R = \frac{2u^2 \sin(\theta \pm \alpha)\cos\theta}{g\cos^2\alpha}}$$

$$\cancel{R = \frac{2\times 20\times 20 \times \cos 37 \times 1}{10\times(\cos 37)^2}}$$

$$\cancel{R = \frac{80\times\frac{4}{5}}{\frac{16}{25}}}$$

$$\cancel{R = \frac{80\times 4\times 25}{16\times 5}}$$

$$\cancel{R = 100}$$

$$R = \frac{2u^2 \sin(\theta+\alpha)\cos\theta}{g\cos^2\alpha}$$

$$R = \frac{2\times 20\times 20\times 1\times\frac{3}{5}}{10\times\frac{16}{25}}$$

$$R = \frac{80\times 3\times 25}{16\times 5}$$

$$R = 25\times 3$$

$$\boxed{= 75\,m}$$

Or

$$T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$$

$$= \frac{2\times 20\times 1}{2\times 10\times\frac{4}{5}}$$

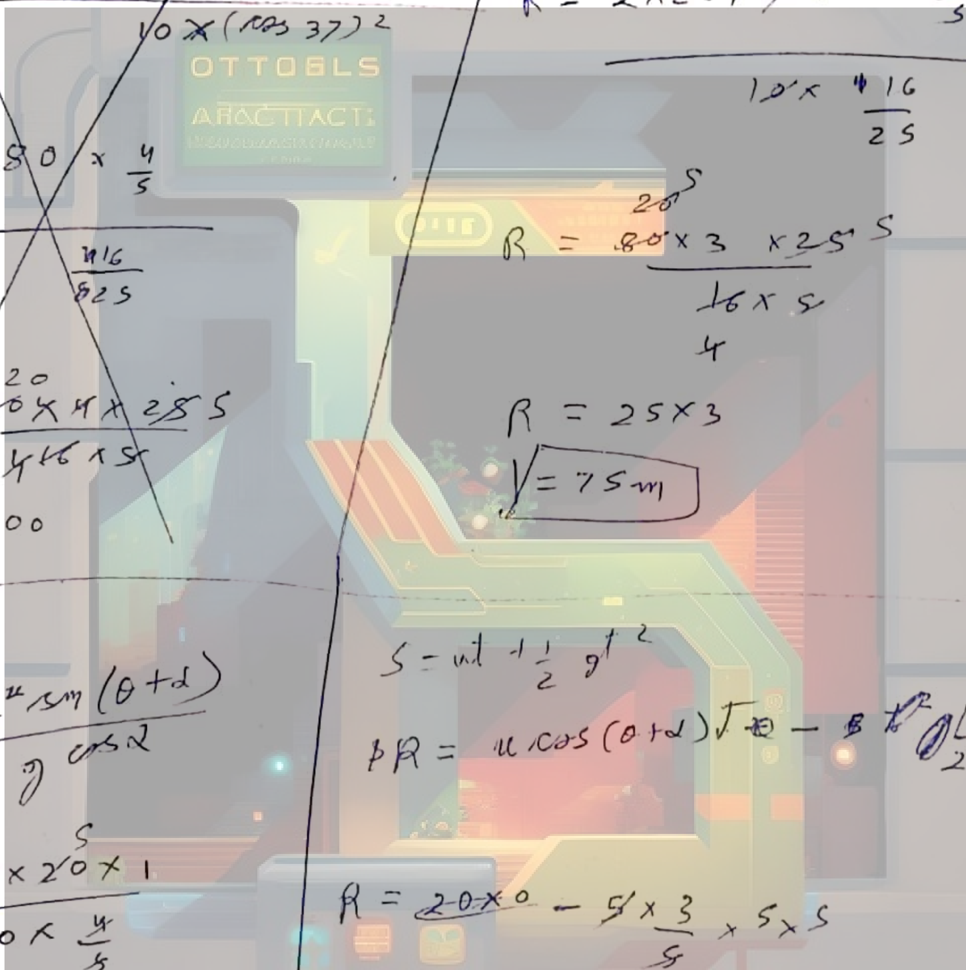
$$\boxed{T = 5s}$$

$$s = ut + \frac{1}{2}gt^2$$

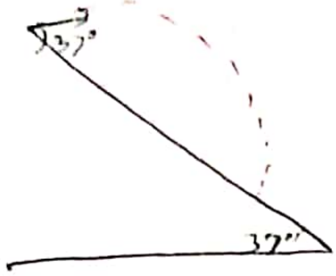
$$R = u\cos(\theta+\alpha)T - \frac{10\times\sin\alpha\times t^2}{2}$$

$$R = 20\times 0 - 5\times\frac{3}{5}\times 5\times 5$$

$$\boxed{R = 75\,m}$$



Q



$T = 5s$

$u = ?$

~~T = 5s find u~~

$$T = \frac{2u \sin(\theta + \alpha)}{g\cos\alpha}$$

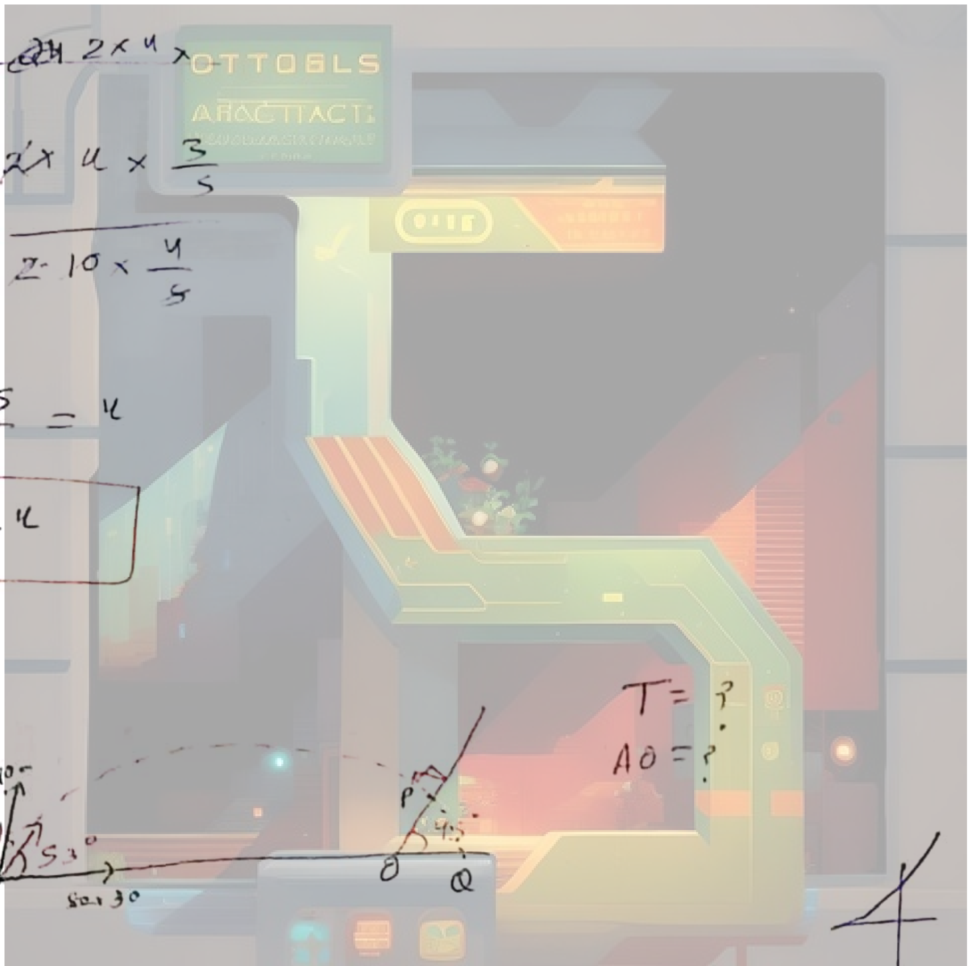
~~T = 2u~~ $2 \times u \times$

$$5 = \frac{2 \times u \times \frac{3}{5}}{2 \cdot 10 \times \frac{4}{5}}$$

$$\frac{20 \times 5}{3} = u$$

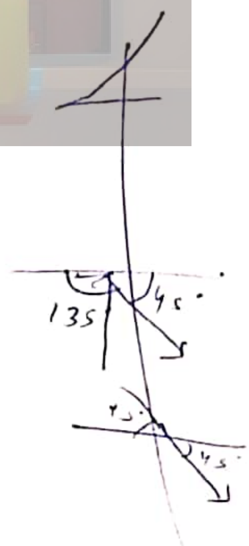
$$\boxed{\frac{100}{3} = u}$$

Q



$T = ?$

$AO = ?$



$\vec{v} = u\cos 45^\circ\,\hat{i} + u\sin 45^\circ\,\hat{j}$

$\vec{v} = u\cos 53^\circ\,\hat{i} + u\sin 53 - gt\,\hat{j}$

$$\frac{50}{\sqrt{2}} = 50 \times \frac{40}{5} - 10t$$

$$10t = 40 - \frac{50}{\sqrt{2}}$$

$$10t = 4 - \frac{5}{\sqrt{2}}$$

$$t = \frac{4\sqrt{2} - 5}{\sqrt{2}}\,s$$

Velocity at P $= \vec{v}$

$\vec{v} = \frac{v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}$

$\vec{u} = 30\hat{i} + 40\hat{j}$

$\frac{v}{\sqrt{2}} = 30$

$v = 30\sqrt{2}$

$v(\text{in } y) = \frac{30\sqrt{2}}{\sqrt{2}}$

$= -30$

Using $v - u = at$

$\frac{-30-40}{-10} = t$

$\frac{-70}{-10} = t$

$\boxed{7 = t}$

$\boxed{t = 7s}$

~~BB~~

$s = ut + \frac{1}{2}at^2$

$PQ = 40(7) + \frac{1}{2}(-10)(7)^2$

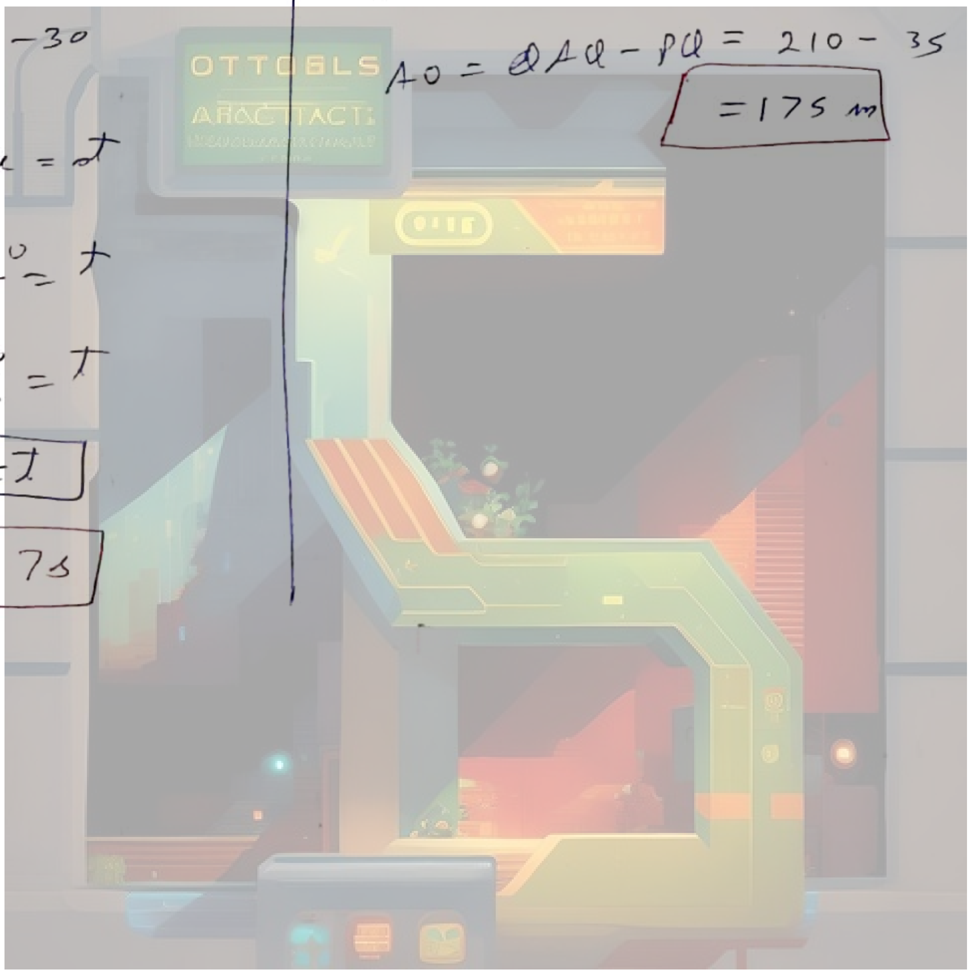
$= 280 - 245$

$= 35 = OQ$

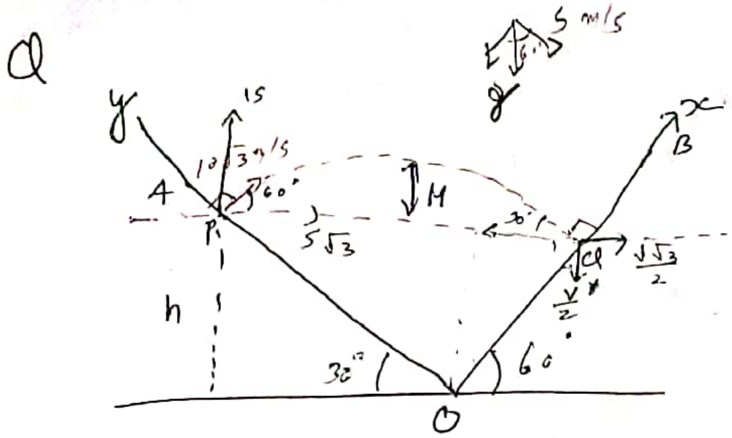
$AQ = 30 \times 7 = 210$

$AO = AQ - PQ = 210 - 35$

$\boxed{= 175 \text{ m}}$



Q



a) Time of flight
b) velocity at OB.
c) $h = ?$
d) Max height attained by particle from O
e) distance PQ.

$\frac{V}{2} = 5\sqrt{3}$

$\boxed{V = 10\sqrt{3}}$

$\frac{V\sqrt{3}}{2} = \frac{10\sqrt{3} \times \sqrt{3}}{2} = \frac{30}{2} = 15$

$\frac{V\sqrt{3}}{2} = 5\sqrt{3}$

$\boxed{V = 10}$ b)

$\frac{-5-15}{-10} = t$

$\frac{10}{10} = t$

$\boxed{t = 1s}$ a)

$\frac{-20}{-10} = \boxed{2s}$ a)

$T = \sqrt{\frac{2H}{g}}$

$2 = \sqrt{\frac{2 \times PO}{10}}$

$4 = \frac{2PO}{10}$

$\frac{40}{2} = PO$

$\boxed{PO = 20}$

$\tan 30 = \frac{h}{PO}$

$\frac{1}{\sqrt{3}} = \frac{h}{20}$

$\boxed{h = \frac{10}{\sqrt{3}}}$ c)

$\tan 60^\circ = \frac{PO}{PQ}$

$\sqrt{3} = \frac{20}{PQ}$

$\boxed{PQ = \frac{20\sqrt{3}}{3}}$ e)

$H = \frac{u^2 \sin^2\theta}{2g}$

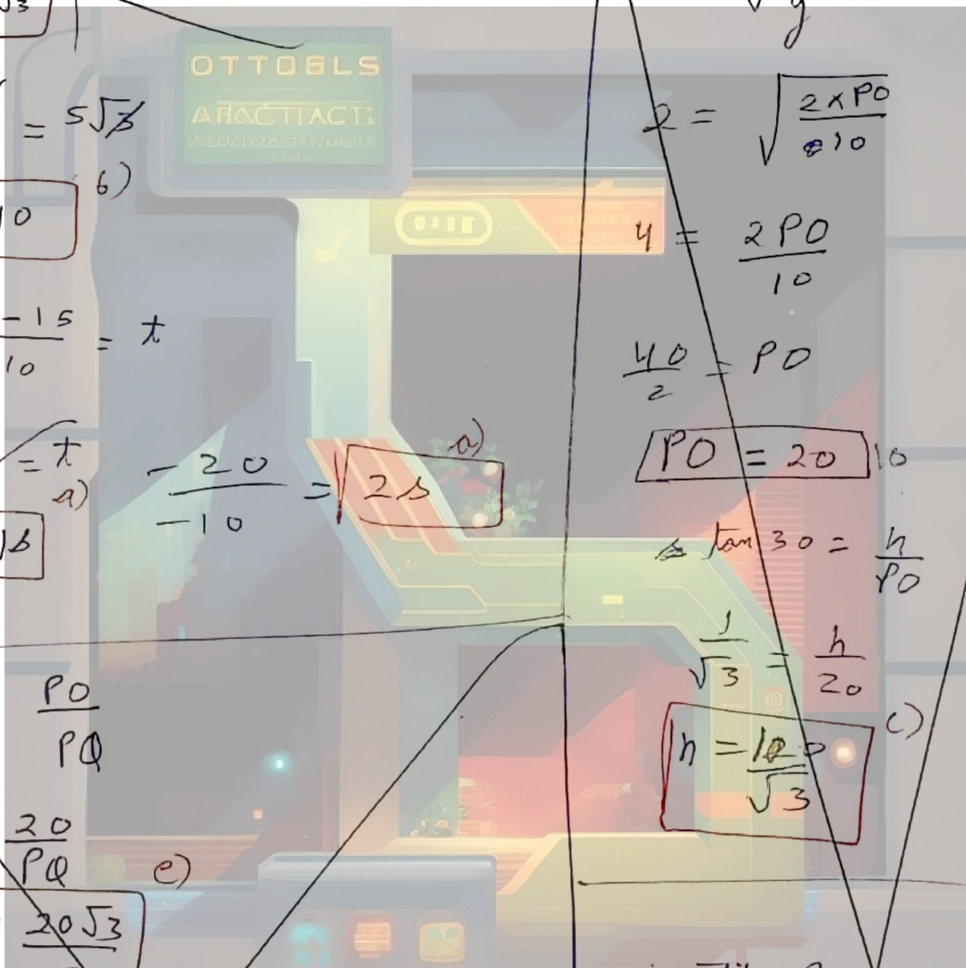
$H = \frac{10\sqrt{3} \times 10\sqrt{3} \times \frac{3}{4}}{10 \times 10}$

$H = \frac{9}{4}$

height $= \frac{9}{4} + \frac{20}{\sqrt{3}}$

$\boxed{= \frac{9\sqrt{3} + 80}{4\sqrt{3}} \text{ m}}$ d)

$\frac{10}{PQ} = \frac{\sqrt{3}}{2}$



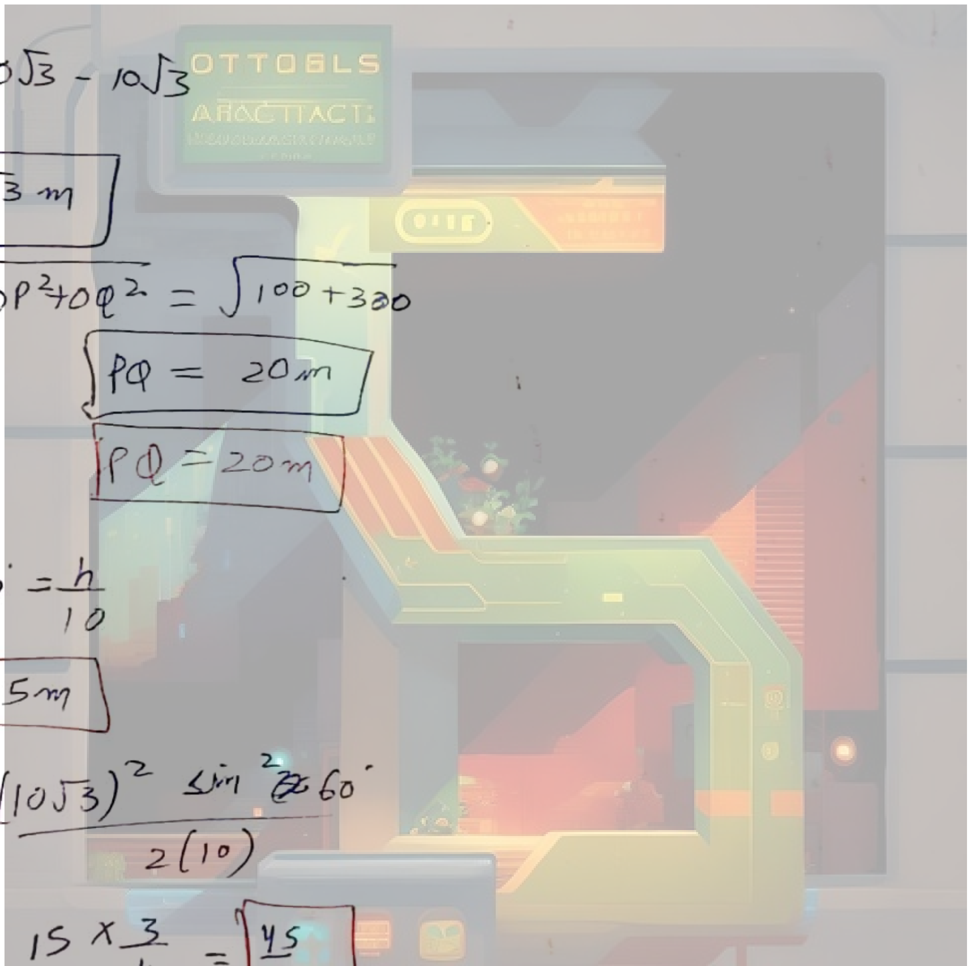
$$-OP = 0 + \frac{1}{2}(-5)(2)^2$$

$$S_y = V_y T + \frac{10}{2} y T^2$$

$$\boxed{OP = 10}$$

$$S_x = V_x T + \frac{1}{2} a_x T^2$$

$$OQ = 10\sqrt{3}(2) + \frac{1}{2}(-5\sqrt{3})(2)^2$$



$$= 20\sqrt{3} - 10\sqrt{3}$$

$$\boxed{OQ = 10\sqrt{3}\ m}$$

$$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{100+300}$$

$$\boxed{PQ = 20m}$$

$$\boxed{PQ = 20m}$$

$$\sin 30^\circ = \frac{h}{10}$$

$$\boxed{= 5m}$$

$$H = \frac{(10\sqrt{3})^2 \sin^2 \cancel{30}\, 60^\circ}{2(10)}$$

$$H = 15 \times \frac{3}{4} = \boxed{\frac{45}{4}}$$

Relative Velocity

$$\boxed{\vec{V}_{obj,\,observer} = \vec{V}_{obj} - \vec{V}_{observer}}$$

→ Velocity of Object with respect to observer.

→ If distance increases, velocity of separation else velocity of approach

eg.

$\xleftarrow{-}\,\xrightarrow{+}$

eg ①

15 m/s →   → 20 m/s

$V_{AB} = V_A - V_B = 15 - 20 = -5\ m/s$

$V_{BA} = V_B - V_A = 20 - 15 = 5\ m/s$

eg ②

$\xleftarrow{-}\,\xrightarrow{+}$

← 15 m/s   → 20 m/s

$V_{AB} = V_A - V_B = -15 - 20 = -35\ m/s$

$V_{BA} = V_B - V_A = 20 - (-15) = 20 + 15 = 35\ m/s$

eg ③.

→ v   → v

A   B

$V_{AB} = V_A - V_B = v - v = 0$

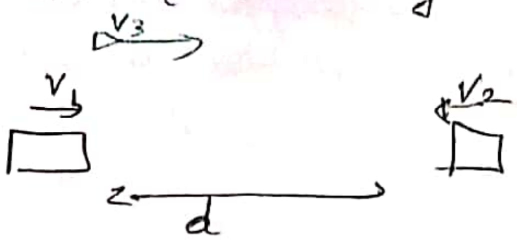
④ -

→ v   ← v

A   B

$V_{AB} = V_A - V_B = v - (-v) = v + v = 2v$

Q find distance travelled by bird till car meets.



$$V_{AB} = V_A - (-V_2) = V_1 + V_2$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

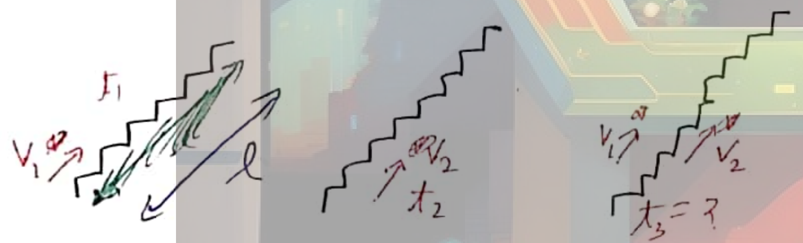
$$\boxed{\text{Time} = \frac{d}{V_1 + V_2}}$$

distance by bird = speed × time

$$= V_3 \times \frac{d}{V_1 + V_2}$$

$$\boxed{= \frac{V_3 d}{V_1 + V_2}}$$

Q



$$V_1 + V_2 = V_3$$

$$V_1 = \frac{l}{t_1} \qquad V_2 = \frac{l}{t_2}$$

$$V_3 = \frac{l}{t_1} + \frac{l}{t_2}$$

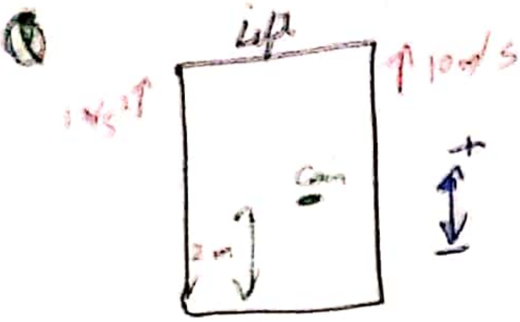
$$\frac{l}{V_3} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$$

$$= \frac{t_2 l + t_1 l}{t_1 t_2}$$

$$\frac{l}{V_3} = t_3 = \frac{l}{\frac{l(t_1 + t_2)}{t_1 t_2}}$$

$$\boxed{= \frac{t_1 t_2}{t_1 + t_2}}$$

Q.



A coin is dropped inside lift. Find time it takes to reach the floor ($g = 10\ m/s^2$)

$$u_{relative} = u_{c,l} = u_c - u_l = 10 - 10 = 0\ m/s$$

$$a_{rel} = a_{c,l} = a_c - a_l = -10 - (+1) = -11\ m/s^2$$

$$s_{rel} = -2\ m$$

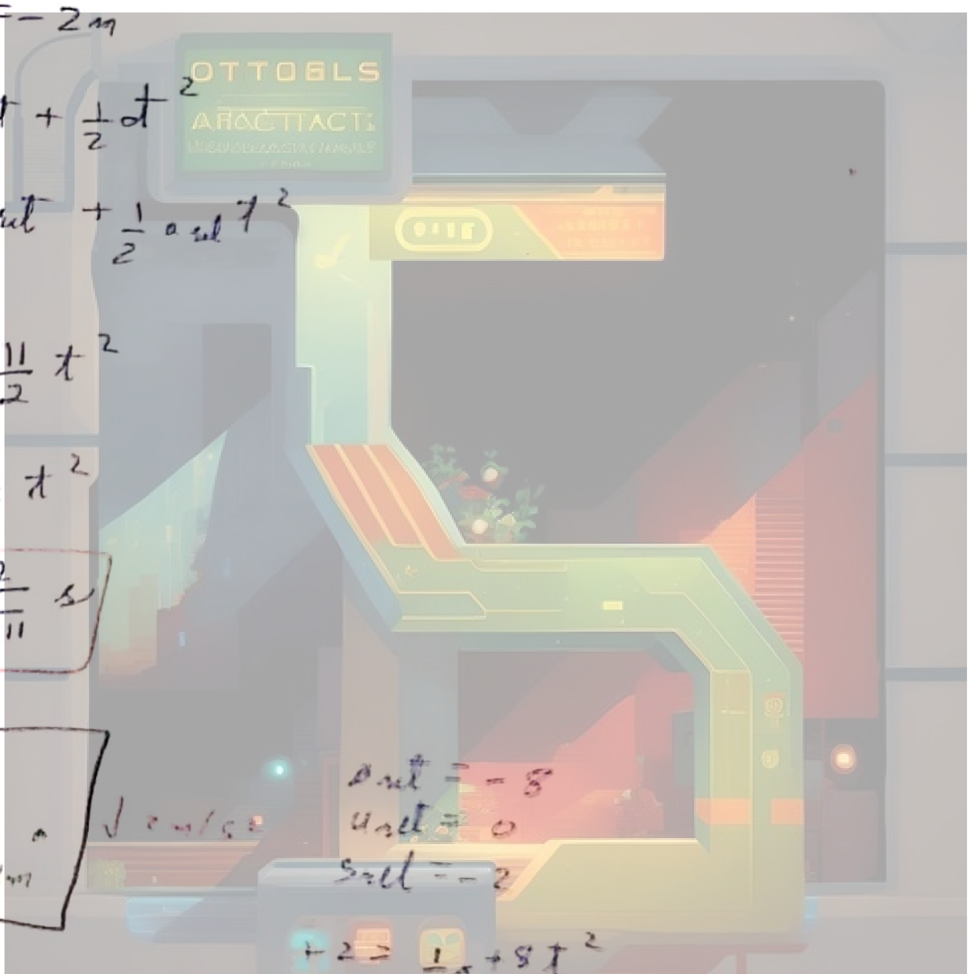
$$s = ut + \frac{1}{2}at^2$$

$$s_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$$

$$-2 = -\frac{11}{2}t^2$$

$$\frac{4}{11} = t^2$$

$$\boxed{t = \frac{2}{\sqrt{11}}\ s}$$



Q.

$$a_{rel} = -8$$

$$u_{rel} = 0$$

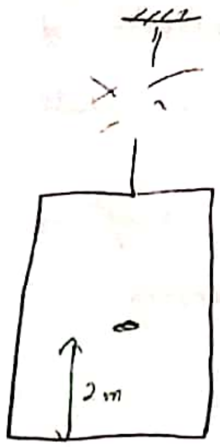
$$s_{rel} = -2$$

$$+2 = \frac{1}{2} \times 8t^2$$

$$\frac{4}{8} = \frac{1}{2} = t^2$$

$$\boxed{t = \frac{1}{\sqrt{2}}\ s}$$

Q



~~Ans~~ = $V_{cl} = 0\ m/s$

$a_{c,l} = -10 - (-10) = -10 + 10 = 0$

$-2 = 0 + \frac{1}{2}(0)t^2$

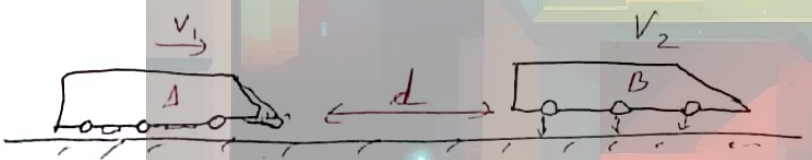
$-2 = 0t^2$

$\frac{-2}{0} = t^2$

$t^2 = \infty$

$\boxed{t = \infty\ s}$ The coin will never reach the floor of lift.

Q



$V_1 > V_2$ (initially)

~~If~~ Applies break for $a = -\alpha$ within distance is d, find min d to avoid collision. Speed of A must be equal to V_2 after relative distance d.

$V_3 = V_1 - V_2 = V_{rel}$

$a = -\alpha - 0 = -\alpha$

$S = d$

~~$d = V_3 t + 0 - \frac{1}{2}\alpha t^2$~~

$0 - V_3^2 = 2 \times (-\alpha) \times d$

$\boxed{\frac{+(V_1 - V_2)^2}{+2\alpha} = d}$

Q A platform is moving upwards with a constant acceleration of $2\,m/s^2$. At $t=0$, a boy standing on platform throws a ball upwards with a relative speed of $8\,m/s$. At this instant, platform was at height of 4 m from the ground and was moving with a speed of $2\,m/s$. Find

a) when & where ball strikes the platform

b) Max height of ball from ground

c) Max height of ball from platform.

$a_{rel} = -12\,m/s^2$

$u_{rel} = 8\,m/s$

$v = 0$

$\frac{-8}{-12} = t$

$t = \frac{2}{3}\,s$ (to go up)

~~$t = 2s$~~ $\boxed{t = \frac{4}{3}}$ a)

~~$S = H + \frac{1}{2} \times 2 \times \ldots$~~

~~$S = 8\,m$~~

~~$= 12\,m$~~

~~$S = 4 + \frac{1}{2} \times$~~

$S = 2 \times \frac{4}{3} + \frac{1}{2} \times 2 \times \frac{4}{3} \times \frac{4}{3}$

$S = \frac{8 \times 3}{9} + \frac{16}{9}$

$S = \frac{40}{9} + 4$

~~$S = 4.66\,m$~~ a)

$\boxed{S = 8.66\,m}$ a)

~~$s = 8 \times 0 + \frac{1}{2}$~~

~~$V + 64 = 2 \times + 12 \times H$ (from const)~~

~~$H = 4\,m$~~

~~$H = \frac{8}{3}\,m$~~ c)

$v + 100 = 2 \times + 10 \times h$ (from ground) $+4$

$\frac{100}{20} + 4 = h$

$h = 5 + 4$

$\boxed{h = 9\,m}$ b)

W.r.t platform, max ball height will be for velocity of ball = velocity of platform.

$U_{BP} = 8\,m/s$

$V_{BP} = 0$

$a_{BP} = -12\,m/s^2$

$S_{BP} = ?$

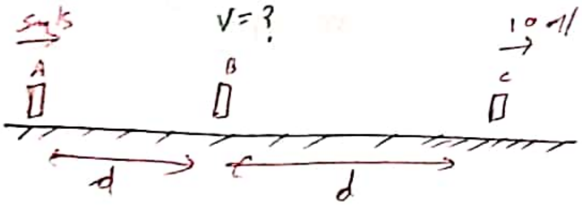
$v^2 - u^2 = 2as$

$0 + 64 = 2 \times 12 \times h$

$\frac{64}{24} = h$

$\boxed{h = \frac{8}{3}\,m}$ c)

Q



~~when A catches B, sep~~
when B catches C, separation between A & C is 3d.
find speed of B.

Time taken by B to catch C, $t_1 = \frac{d - 0}{V - 10}$

A & C

change in seperation

$10 - 5 = \frac{3d - 2d}{T_1}$

$10 - 5 = \frac{d}{\frac{d}{V-10}}$

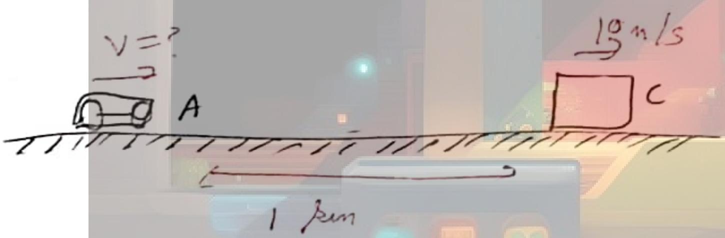
$10 - 5 = V - 10$

$5 + 10 = V$

$\boxed{V = 15 \text{ m/s}}$

Q

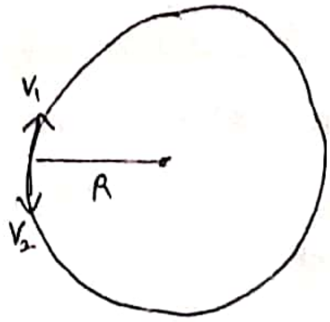
find the speed with which A should chase C to overtake it in 100 s.



$(V - 10) \times 100 = 1000$

$\boxed{V = 20 \text{ m/s}}$

Q



After what time will they meet

$$\frac{2\pi R}{V_1 + V_2}$$

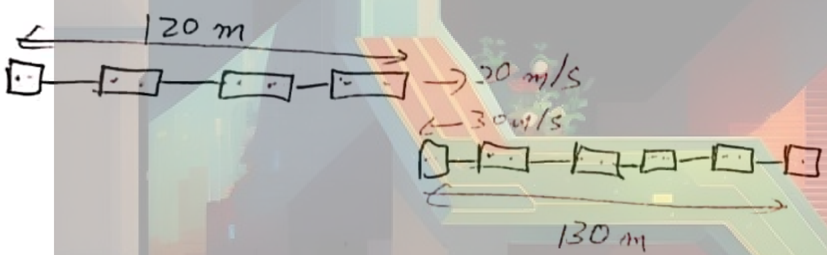
rel Velocity = $\cancel{V_1 - V_2}$ $V_1 - (-V_2) = V_1 + V_2$

distance $= 2\pi R$ (circumference)

Time $= \frac{\text{dis}}{\text{Time}}$

$$\boxed{= \frac{2\pi R}{V_1 + V_2}}$$

Q



$V_1 = 20$

$V_2 = 30$

$V_{rel} = 30 + 20$

$= 50$

distance $= 120 + 130$

$= 250$

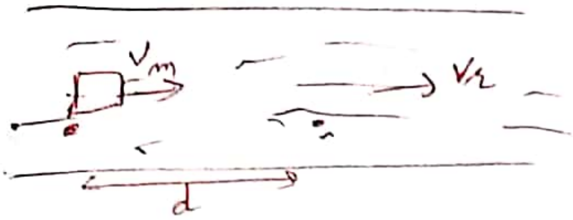
Time $= \frac{\text{distance}}{\cancel{\text{Total}}\ \text{Speed}}$

$= \frac{250}{50}$

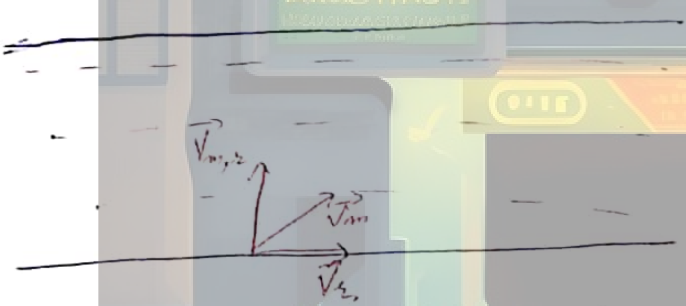
$\boxed{= 5s}$

river man problems.

upstream : against the river flow
downstream : with the river flow



$$t_{down} = \frac{d}{V_m + V_r} \quad (A \to B) \qquad T_{up} = \frac{d}{V_m - V_r} \quad (B \to A)$$

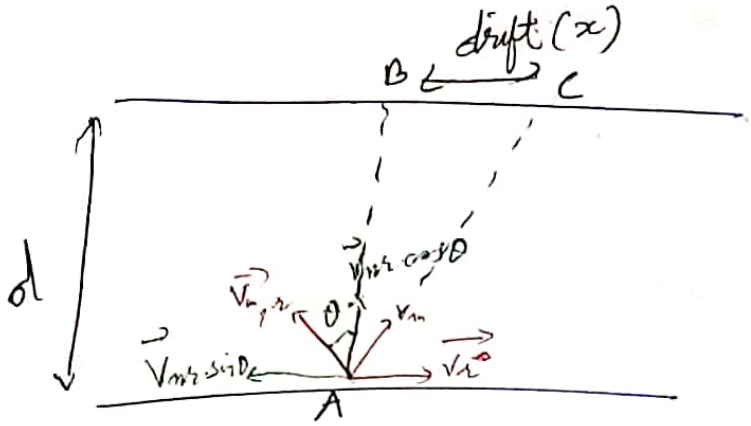


$\vec{V}_{m,r}$ = velocity of man w.r.t river (man generated v in still water)

$\vec{V}_r$ = Velocity of flow of river

$\vec{V}_m$ = Net velocity of man

$$\vec{V}_m = \vec{V}_{m,r} + \vec{V}_r$$



Time Taken $= \dfrac{d}{\;} \quad \boxed{\dfrac{d}{V_{mr}\cos\theta}}$

$$\text{drift}(x) = (V_r - V_{mr}\sin\theta) \times t$$

$$\boxed{x = \frac{(V_r - V_{mr}\sin\theta)\,d}{V_{mr}\cos\theta}}$$

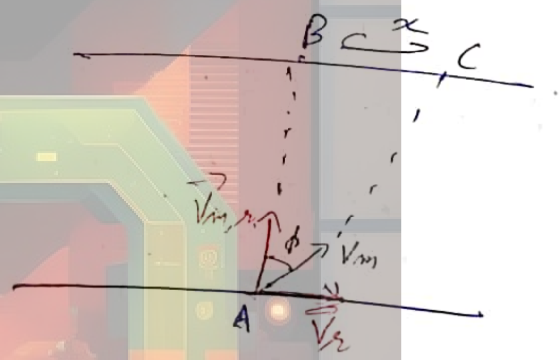
Case I :- For minimum time

$\cos\theta = 1$ (max)

$\theta = 0^\circ$

$$\boxed{V_m = \sqrt{V_{mr}^2 + V_r^2}}$$

man should swim $\perp$ to river flow

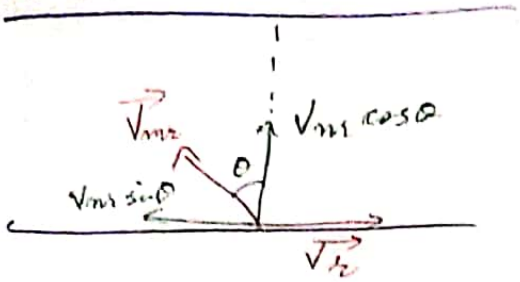


$x = V_r t$

$$\boxed{x = \frac{V_r d}{V_{mr}}}$$

$$\boxed{\tan\phi = \frac{V_r}{V_{mr}}}$$

case 2



II To cross river with zero drift.

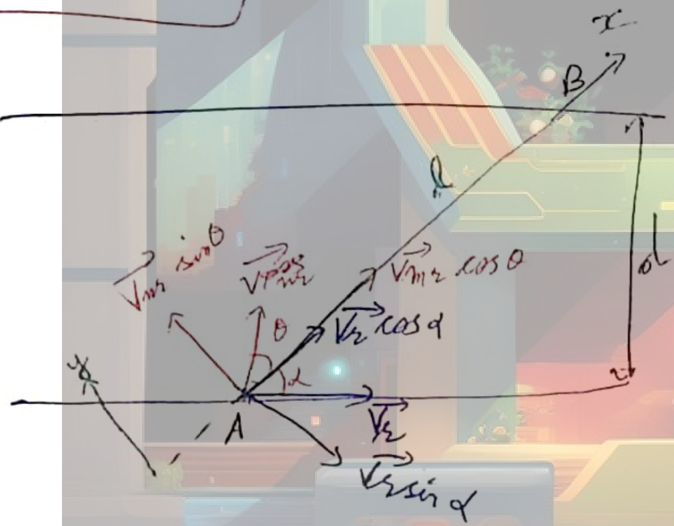
$$V_r = V_{Mr}\sin\theta$$

$$\sin\theta = \frac{V_r}{V_{mr}}$$ Only valid till $V_{mr} > V_r$.

$$\boxed{\theta = \sin^{-1}\left[\frac{V_r}{V_{mr}}\right]}$$

$$\boxed{t = \frac{d}{V_{mr}\cos\theta}}$$

Q



$$V_{mr}\sin\theta = V_r\sin\alpha$$

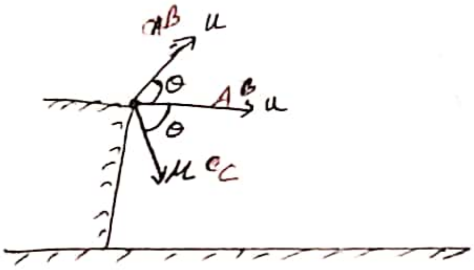
$$\boxed{\sin\theta = \frac{V_r}{V_{mr}}\sin\alpha}$$

↳ Angle with which boat will be steered.

$$\boxed{t = \frac{l}{V_r\cos\alpha + V_{mr}\cos\theta}}$$

Extra Questions (Ch-1,2,3)

Q1.



A) $V_A > V_B > V_C$

B) $V_A = V_B = V_C$

C) $V_C > V_B > V_A$

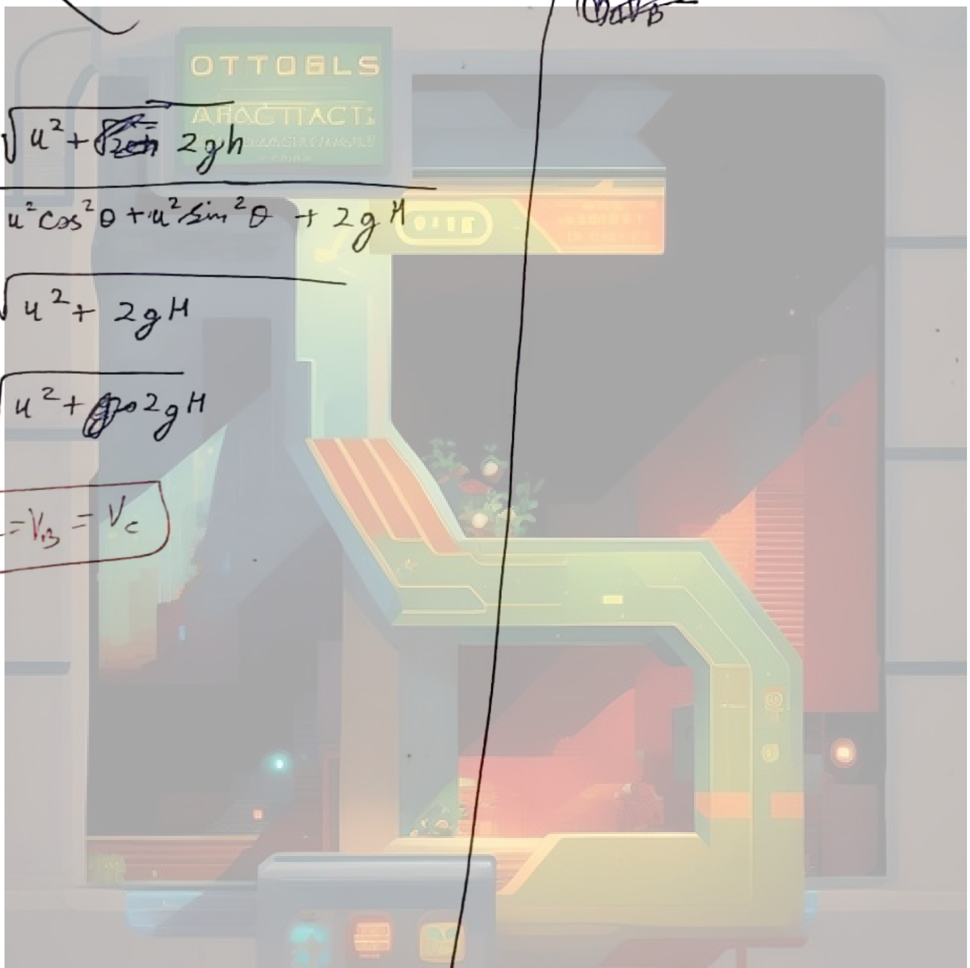
D) cannot find.

$\cancel{V_{B_y} = \sqrt{2gh} = \sqrt{20h}}$

$\cancel{V_{A_y} = u\sin\theta + 10}$

$\cancel{V_C =}$

$\cancel{V_B}$



$\vec{V}_{BA} = \sqrt{u^2 + \cancel{20h}\ 2gh}$

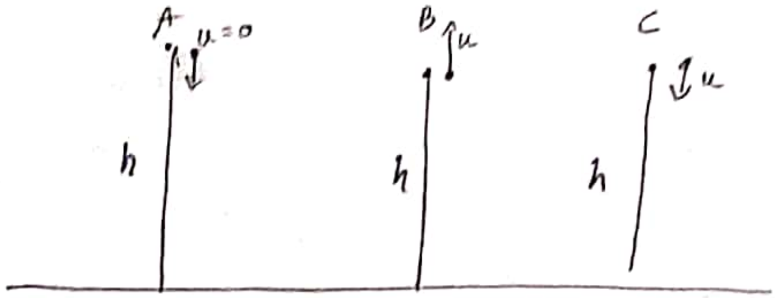
$V_B = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + 2gH}$

$= \sqrt{u^2 + 2gH}$

$V_C = \sqrt{u^2 + \cancel{g}\ 2gH}$

$\boxed{B)\ V_A = V_B = V_C}$

Q2.



a) velocity relation

$V_A^2 = 2gH$

$V_B^2 = 2gH + u^2$

$V_C^2 = 2gH + u^2$

A) $V_C > A_B > V_A$

B) $V_A < V_B < V_C$

C) $V_A < V_B = V_C$

d) Nota

Q3. If initial vel. of a particle projected from ground is $\vec{u} = 6i + 8j$ m/s find the horizontal range

$R = \frac{u^2 \sin 2\theta}{g} = \frac{2 u\sin\theta \, u\cos\theta}{g} = \frac{2 \times 8 \times 6}{10} = \frac{96}{10} = 9.6\ m$

Q4. In the situation shown a player kicks the ball at 45° with 20 m/s vel the distance of a 3m high goal post is 25m from the player. find wether there will be a goal or not

$= 25 - \frac{10 \times 625}{2 \times 20 \times 20 \times \frac{1}{2}}$

$= 25 - \frac{625}{40}$

$= \frac{1000 - 625}{40}$

$= \frac{375}{40}$

$= \frac{93.\text{something}}{10}$

$y = 9.3$ something

at 25m, ball's height will be 9.3 so there will be no goal.

Q 5. A particle is thrown from ground at angle 30° with horizontal at 40 m/s find time for which it will be at the height more than 20 m

$$20 = \overset{20}{\cancel{40}} \times \frac{1}{\cancel{2}} t - 5t^2$$

$$\frac{\overset{20}{\cancel{40}} \times \overset{10}{\cancel{40}} \times \frac{1}{4}}{} = 2 \times 10$$

$$5t^2 - 20t + 20 = 0$$

$$t^2 - 4t + 4 = 0$$

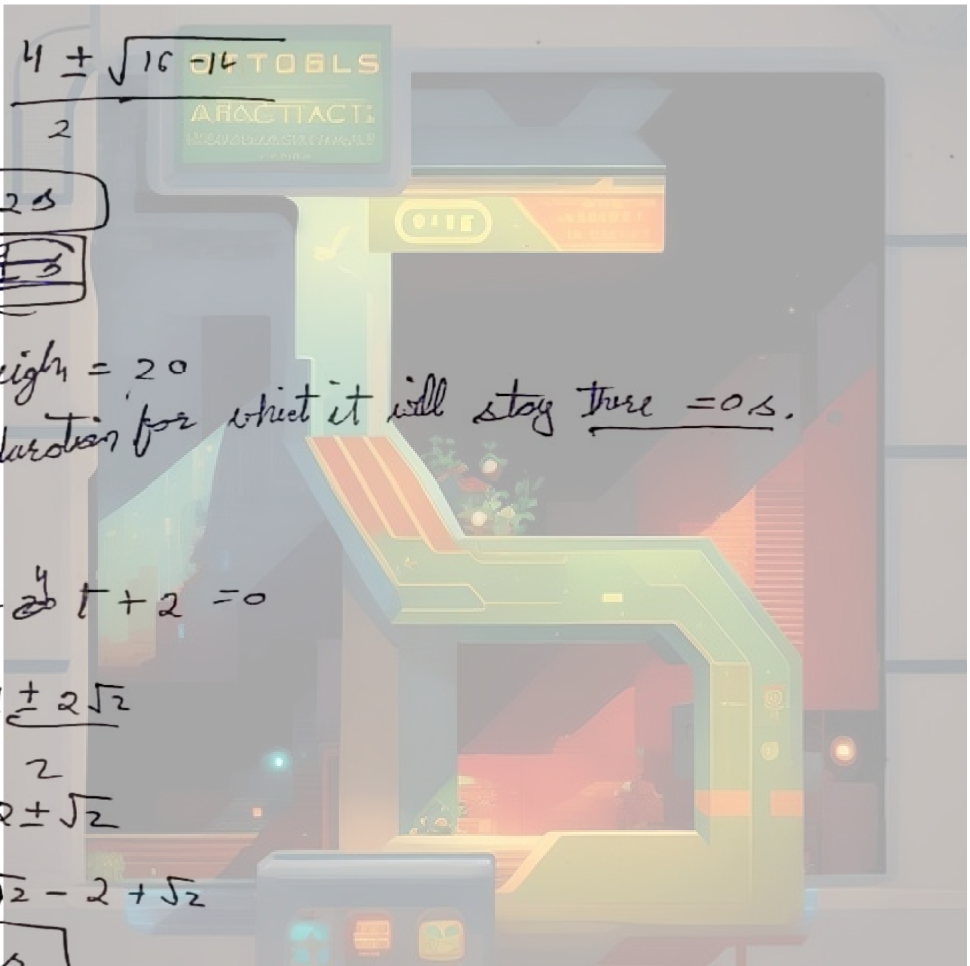
$$t = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$\boxed{t = 2s}$$

$$\boxed{\cancel{t = 2 + s}}$$

Max heigh = 20

time duration for which it will stay there = 0 s.



$$t^2 - 4t + 2 = 0$$

$$t = \frac{4 \pm 2\sqrt{2}}{2}$$

$$t = 2 \pm \sqrt{2}$$

$$2 + \sqrt{2} - 2 + \sqrt{2}$$

$$\boxed{2\sqrt{2}\, s}$$

Q6. चोर is running away on a straight road at 9 m/s. A police man chases him on a motorcycle at 10 m/s if the distance between them is 100 m how long will it take for police to catch the thief.

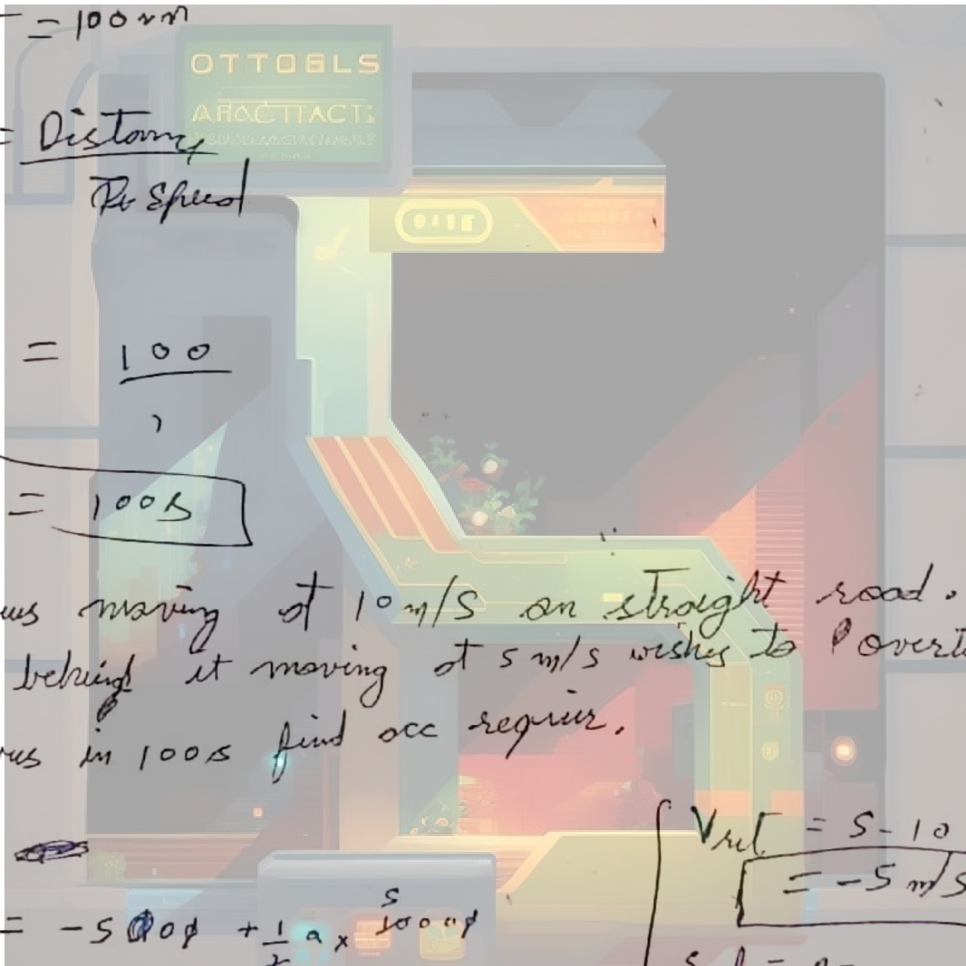
$V_t = 9$

$V_p = 10$

$V_{rel} = 10 - 9$

$= 1\ m/s$

$S_{rel} = 100\ m$



$$Time = \frac{Distance}{Re\ Speed}$$

$$Time = \frac{100}{1}$$

$$\boxed{= 100\ s}$$

Q 7. A bus moving at 10 m/s on straight road, a bikes 80 m behind it moving at 5 m/s wishes to overtake the bus in 100 s find acc requir.

$V_{rel} = 5 - 10$

$\boxed{= -5\ m/s}$

$S_{rel} = 80\ m$

$t = 100\ s$

$8\cancel{0} = -5\cancel{00} + \frac{1}{2} a \times \cancel{1000}\ 5$

$8 + 50 = a \times 500$

$a = \frac{58}{500}$

$a = \frac{58}{5 \times 100}$

$a = 11.6$

$\boxed{a = 0.116\ m/s^2}$

H.W.

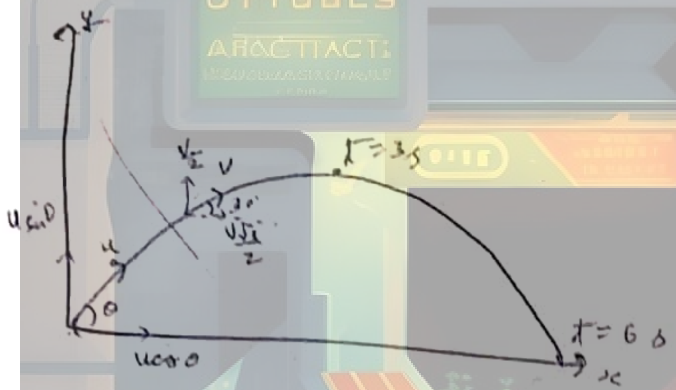
Q. A body is projected with velocity ~~u~~ u at angle θ ~~with~~ with horizontal. The body ~~----~~ makes angle 30° with horizontal at $t = 2s$ & after 1s reaches max height.

a) angle of projection

b) speed of projection.

~~$v = u\sin\theta = 20$~~

~~$u\cos\theta = u\sin\theta\ 20 \times \sqrt{3}$~~



$$T = \frac{2u\sin\theta}{g}$$

$$\frac{6 \times 10}{2} = u\sin\theta$$

$$30 = u\sin\theta$$

$$\boxed{u = \frac{30}{\sin\theta}}$$

$$\frac{v}{2} = u\sin\theta - gt$$

$$= 30 - 20$$

$$\frac{v}{2} = 10$$

$$v = 20$$

$$\frac{20\sqrt{3}}{2} = u\cos\theta$$

$$\cancel{20\sqrt{3}}$$

$$10\sqrt{3} = 30\cot\theta$$

$$\frac{30}{10\sqrt{3}} = \tan\theta$$

$$\sqrt{3} = \tan\theta$$

$$\boxed{\theta = 60^\circ}$$

$$u = \frac{30}{\sin 60}$$

$$= \frac{30}{\sqrt{3}} \times 2$$

$$\boxed{u = 20\sqrt{3}\ m/s}$$

Ch-3

Q1. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min. with a drift 120 m. If he crosses the river taking shortest route he takes 12.5 min, find velocity of boat w.r.t water.

$\theta = 0$

$600 = \dfrac{d}{V_{mr}}$

$120 = 600\, u_r$

$u_r = \dfrac{120}{600}$ m/s

$\boxed{u_r = \dfrac{1}{5} \text{ m/s}}$

$\sin\theta = \dfrac{V_{mr}}{V_r}$

$\sin\theta = V_{mr} \times 5$

$\sin\theta = 5 V_{mr}$

$750 = \dfrac{d}{V_{mr}\cos\theta}$

$\cancel{750 = \dfrac{5d}{V_m}}$

$750 = \dfrac{d}{V_{mr}\cos\theta}$

$\dfrac{600}{750} = \cos\theta$

$\cos\theta = \dfrac{4}{5}$

$\cancel{750 = \dfrac{d}{V_{mr}} \times \dfrac{5}{4}}$

$\cancel{600 = \dfrac{d}{V_m}}$

$\theta = 37^\circ$

$V_{mr} \times \dfrac{3}{5} = \dfrac{1}{5}$

$V_{mr} = \dfrac{1}{3}$ m/s

$= \dfrac{1}{3} \times 60$

$\boxed{= 20 \text{ m/min}}$

Q2. A man can swim at the rate of 5 km/hr in still water. A 1 km wide river flows at 3 km/h. The man wishes to swim across the river directly opposite to starting point.

a) along what direction should the man swim?
b) What should be his resultant velocity?
c) Find time taken to cross the river.

$d = 1\,km$

$V_r = 3\,km/h$

$V_{mr} = 5\,km/hr$

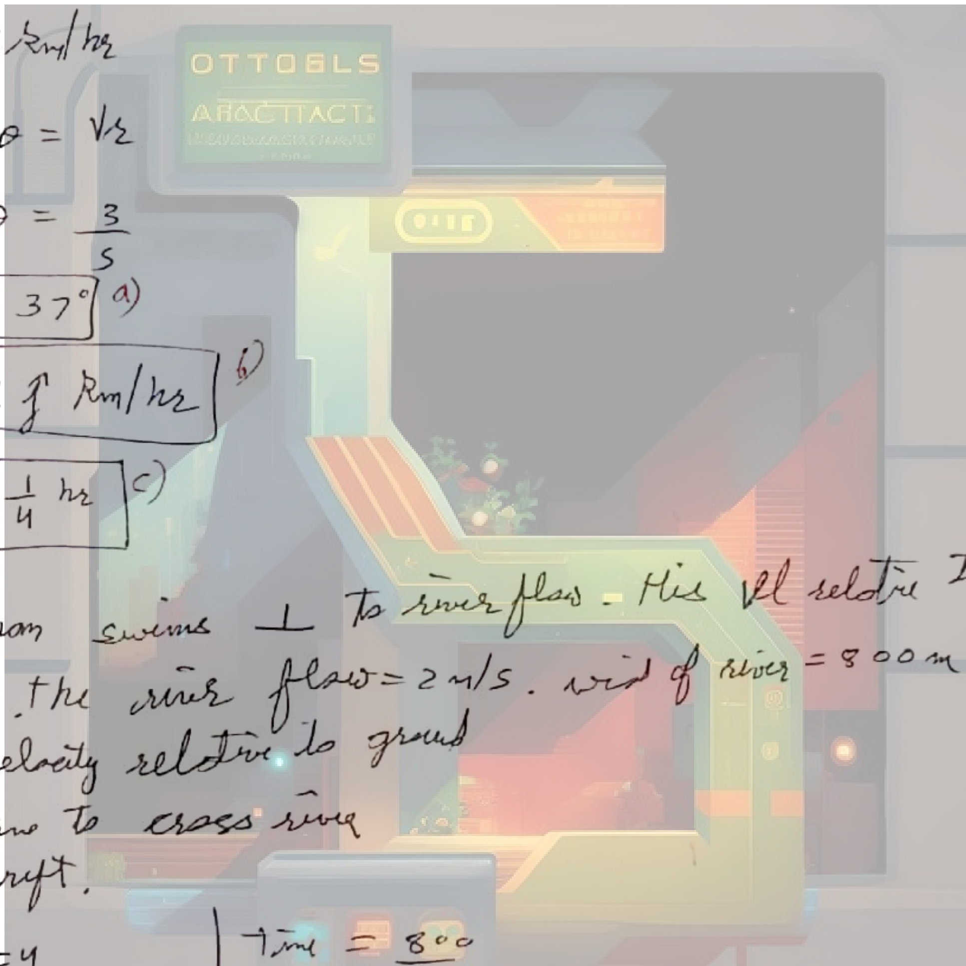
$V_{mr} \sin\theta = V_r$

$\sin\theta = \frac{3}{5}$

$\boxed{\theta = 37°}$ a)

b) $\boxed{\vec{V} = 4\,\hat{j}\ km/hr}$

$\boxed{Time = \frac{1}{4}\,hr}$ c)



Q3. A man swims $\perp$ to river flow. His vel relative to water is 4 m/s the river flow = 2 m/s. wid of river = 800 m

a) velocity relative to ground
b) time to cross river
c) drift.

$V_{mr} = 4$

$V_r = 2$

$d = 800$

$V_m = \sqrt{16 + 4}$

$\boxed{= \sqrt{20}\ m/s}$ a)

$Time = \frac{800}{4}$

$\boxed{= 200\,s}$ b)

$drift = \cancel{400\,m}\ \frac{distance}{\cancel{Time}\ Speed}$

$= \frac{800}{\cancel{200}\ 2}$

$= \frac{800}{2}$

$\boxed{= 400\,m}$ c)

Q4. A swimmer crosses a 200 m wide river & return 10 minutes at a point away from starting point (downstream). Find velocity of man with respect to ground if he heads towards the bank & at right angles all the times.

$d = 200\,m$

$T = 300\,s$

$x = 150\,m$

$\theta = 90$

$V_{mr} = \frac{2\cancel{0}\cancel{0}}{3\cancel{0}\cancel{0}}$

$= \frac{2}{3}\,m/s$

$\cancel{\frac{V_r 150 \times 3}{2} = V_r}$

$\cancel{225}\ \frac{150}{300} = \frac{1}{2} = V_r$

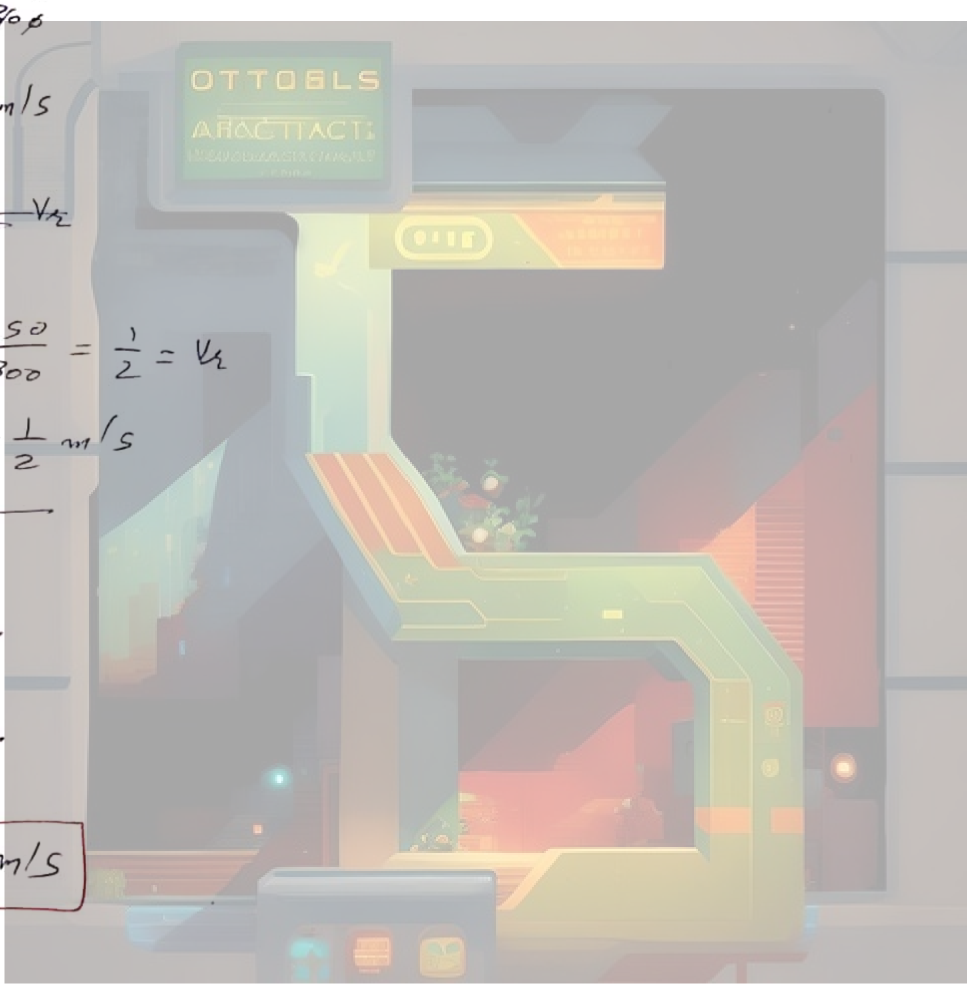
$V_r = \frac{1}{2}\,m/s$

$\sqrt{\frac{1}{4} + \frac{4}{9}}$

$\sqrt{\frac{9+16}{36}}$

$\sqrt{\frac{25}{36}}$

$\boxed{\frac{5}{6}\,m/s}$



Q6.

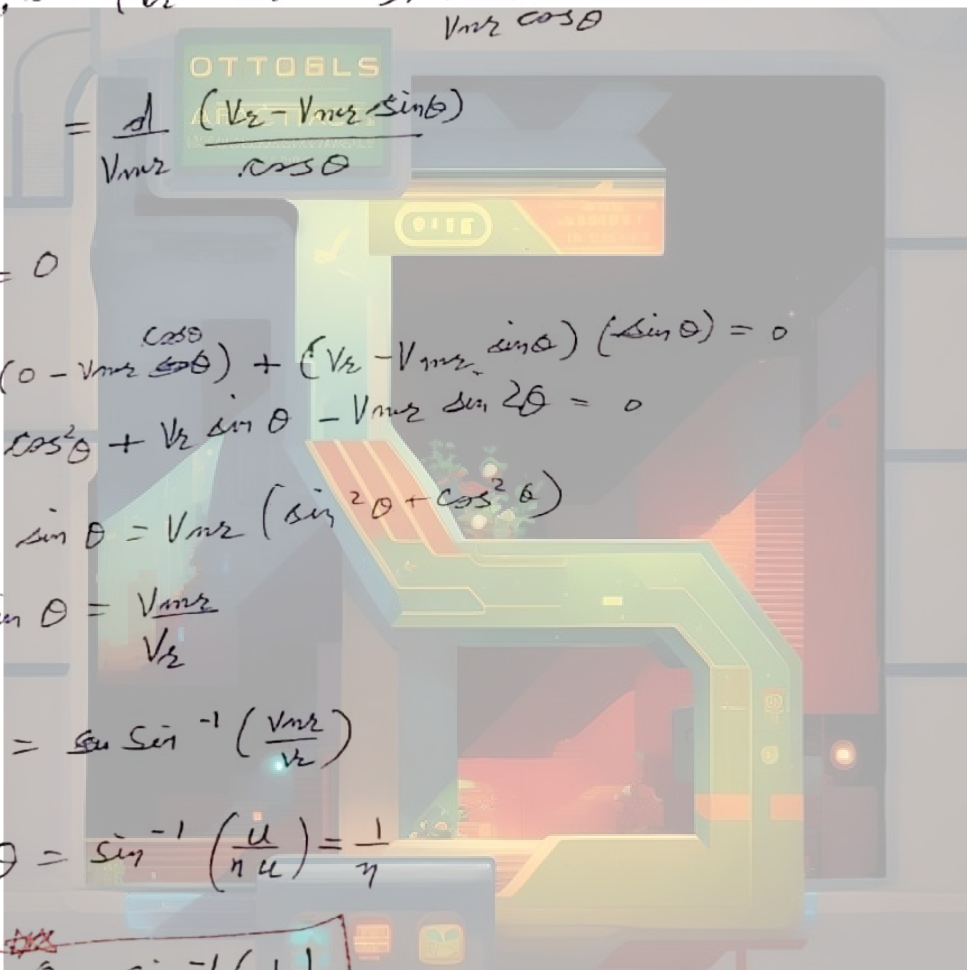
Q6. A boat moves relative to water with velocity v which is η times less than river flow u. At what angle river flow must the boat move to minimise drifting?

$V_{mr} = \frac{u}{n}$

$V_r = u$

Time taken to cross river, $t = \frac{d}{V_{mr}\cos\theta}$

Drift, $x = (V_r - V_{mr}\sin\theta) \times \frac{d}{V_{mr}\cos\theta}$



$= \frac{d}{V_{mr}} \frac{(V_r - V_{mr}\sin\theta)}{\cos\theta}$

$\frac{dx}{d\theta} = 0$

$\cos\theta(0 - V_{mr}\cos\theta) + (V_r - V_{mr}\sin\theta)(\sin\theta) = 0$

$-V_{mr}\cos^2\theta + V_r\sin\theta - V_{mr}\sin^2\theta = 0$

$V_r \sin\theta = V_{mr}(\sin^2\theta + \cos^2\theta)$

$\sin\theta = \frac{V_{mr}}{V_r}$

$\theta = \sin^{-1}\left(\frac{V_{mr}}{V_r}\right)$

$\theta = \sin^{-1}\left(\frac{u}{nu}\right) = \frac{1}{n}$

$\boxed{\theta = \sin^{-1}\left(\frac{1}{n}\right)}$

Angle with river flow $= \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{n}\right)$

Q 7. Two swimmers leave point A on the bank of river to reach B right cross other bank. One of them crosses river along AB and other swims at right angles to river and then walks the drift distance to get to point B. find velocity u of his walking if both arrive at B at same time. ($V_r = 2$ km/hr, $V_{mr} = 2.5$ km/h)

$\sin\theta = \frac{\cancel{2}\,4}{\cancel{2.5}\,5}$

$\underline{\theta = 53^\circ}$

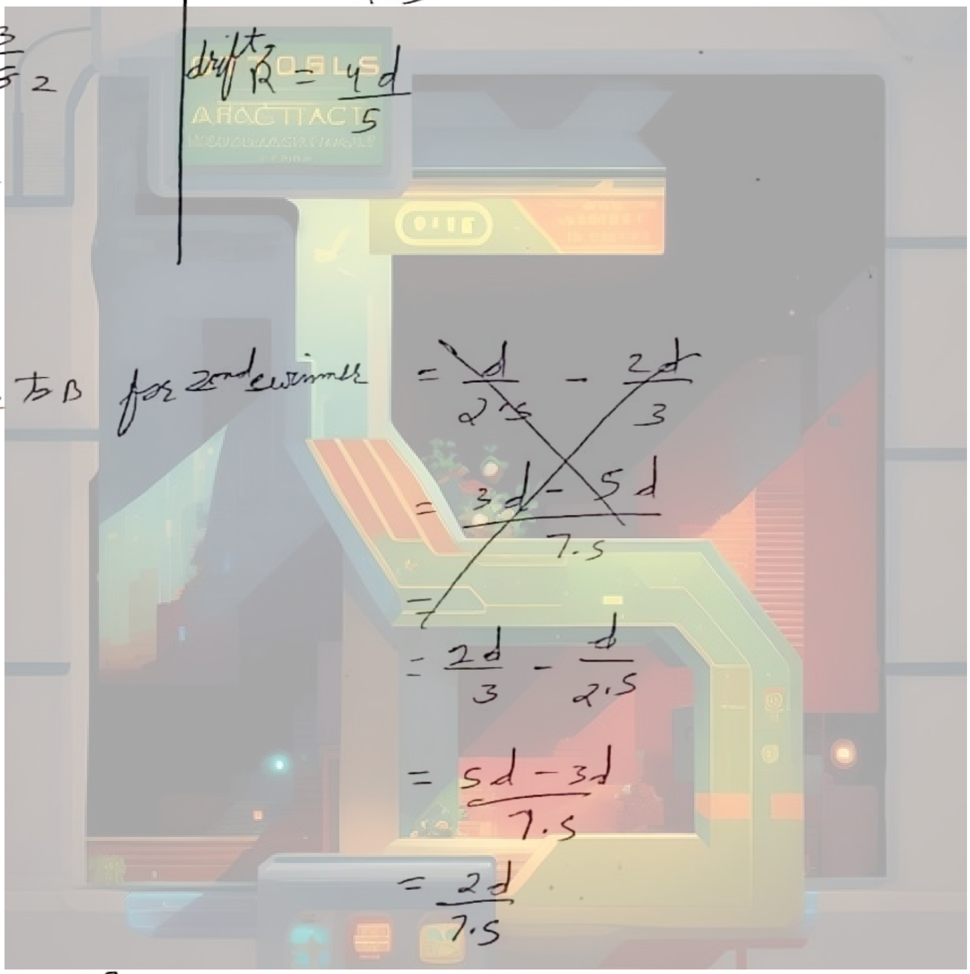
$T = \frac{d}{2.5 \times \frac{3}{5}}$

$T_1 = \frac{2d}{3}$

$T_2 = \frac{d}{2.5}$

$R = 2 \times \frac{d}{2.5}$

drift $R = \frac{4d}{5}$



time to come to B for 2nd swimmer $= \cancel{\frac{d}{2.5} - \frac{2d}{3}}$

$= \cancel{\frac{3d - 5d}{7.5}}$

$= \frac{2d}{3} - \frac{d}{2.5}$

$= \frac{5d - 3d}{7.5}$

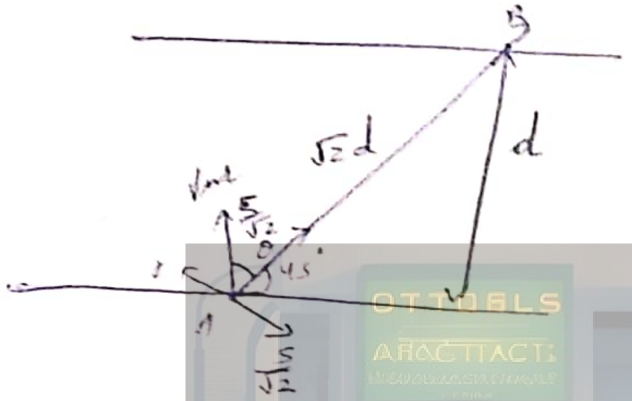
$= \frac{2d}{7.5}$

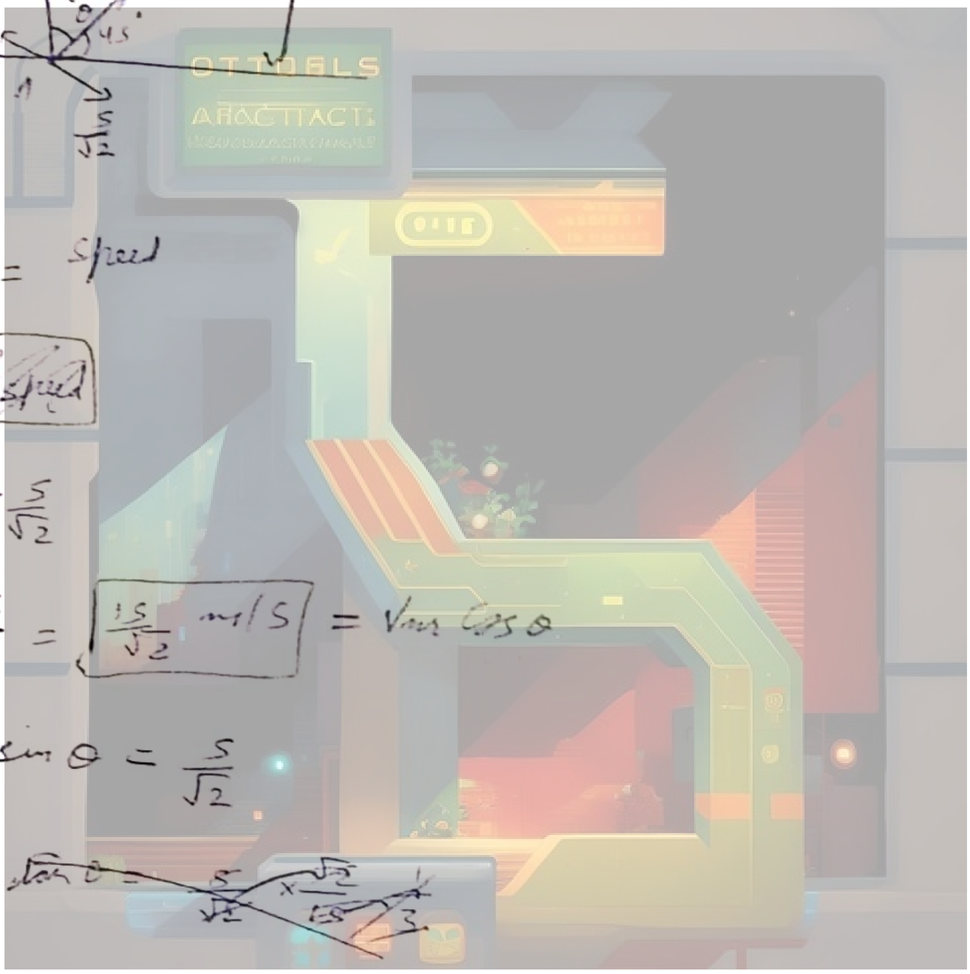
speed $= \frac{4\cancel{d}^{\,2}}{5} \times \frac{7.5}{\cancel{2d}}$

$= \frac{15}{5}$

$\boxed{V = 3\ km/h}$

Q8. A swimmer crosses a river along the line making an angle of 45° with the direction of flow. Velocity of river water is 5 m/s. Swimmer takes 6 s to cross the river of width 60 m. Find velocity of swimmer w.r.t. water.





$$\frac{\sqrt{2}d}{6} = \text{speed}$$

$$\boxed{\cancel{10\sqrt{2} = \text{speed}}}$$

$$10\sqrt{2} - \frac{5}{\sqrt{2}}$$

$$\frac{20-5}{\sqrt{2}} = \boxed{\frac{15}{\sqrt{2}}\ m/s} = V_{mr}\cos\theta$$

$$V_{mr}\sin\theta = \frac{5}{\sqrt{2}}$$

$$\Rightarrow \cancel{\tan\theta = \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{15} = \frac{1}{3}}$$

$$V_{mr}^2 = \frac{25}{2} + \frac{225}{2}$$

$$V_{mr}^2 = \frac{250}{2}$$

$$V_{mr} = \frac{5\sqrt{10}}{\sqrt{2}}$$

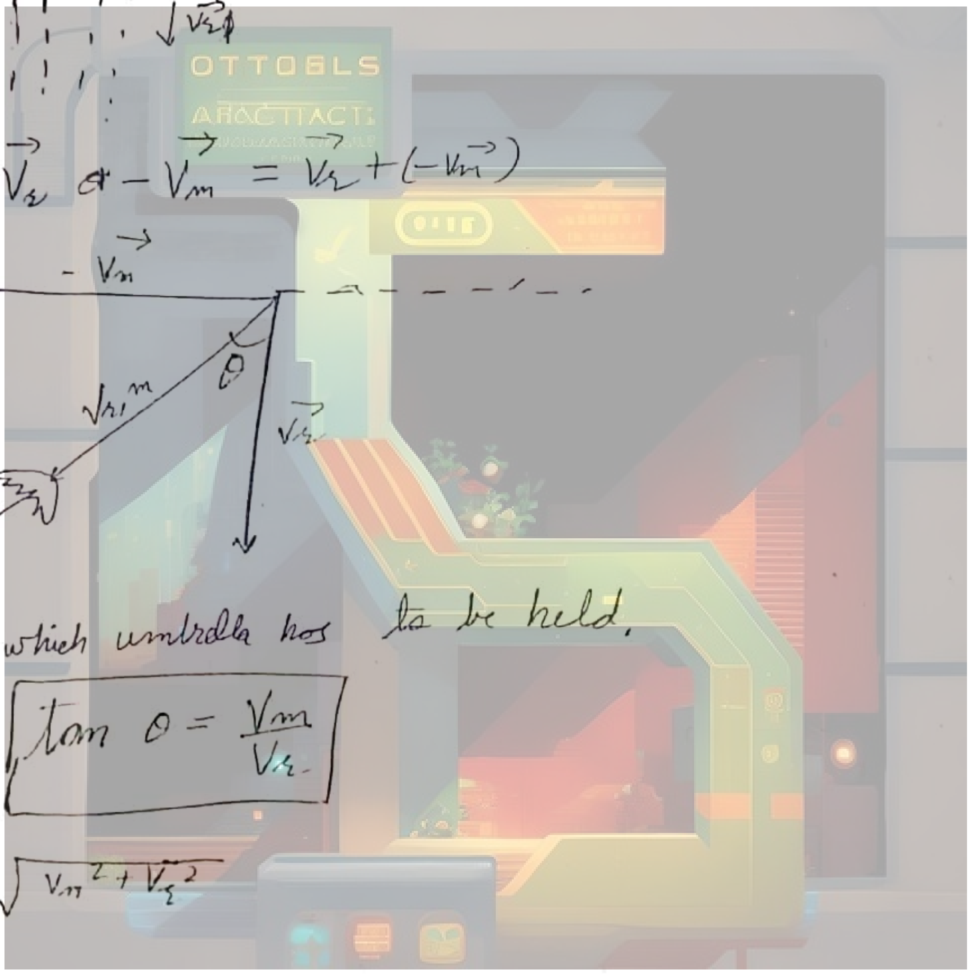
$$\boxed{V_{mr} = 5\sqrt{5}\ m/s}$$

H. W.

Pg 72 Q 29-45

Rain Man Problems

If rain is falling vertically with velocity $\vec{V_r}$ and ~~the~~ observer is moving horizontally with velocity $\vec{V_m}$, then velocity of rain w.r.t. man, $\vec{V_{r,m}} = \vec{V_r} - \vec{V_m}$



$\vec{V_m}$ $\longrightarrow$ $\downarrow \vec{V_r}$

$\vec{V_{r,m}} = \vec{V_r} - \vec{V_m} = \vec{V_r} + (-\vec{V_m})$

$-\vec{V_m}$

θ

$V_{r,m}$

$\vec{V_r}$

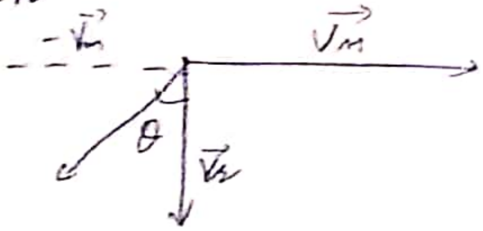
Angle at which umbrella has to be held.

$$\boxed{\tan\theta = \frac{V_m}{V_r}}$$

$V_{r,m} = \sqrt{V_m^2 + V_r^2}$

Q Rain falls vertically at speed of 10 m/s. A man walks with a speed of 6 m/s on a horizontal road. find angle at which man should hold his umbrella to avoid getting wet?

$\cancel{V_r = 10}$



$|\vec{V_r}| = 10\,m/s$

$\vec{V_r} = -10\hat{j}\,m/s$

$|\vec{V_m}| = 6\,m/s$

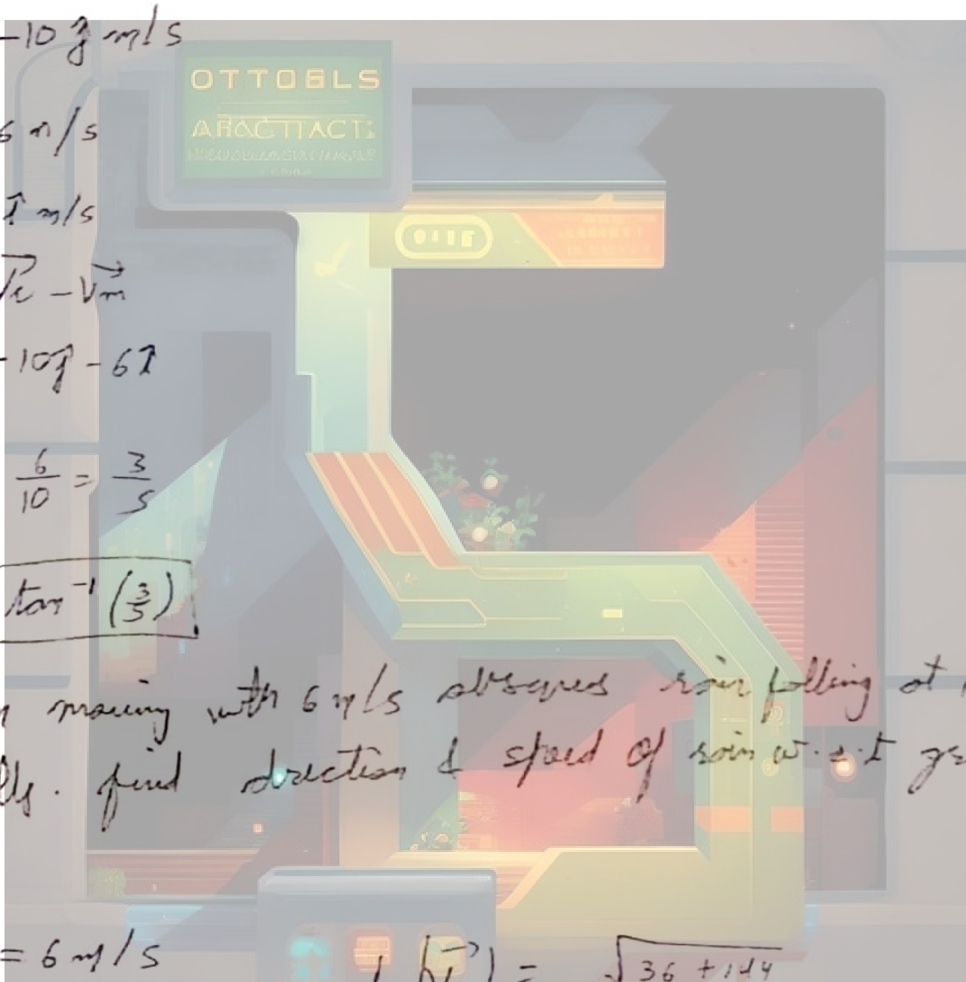
$\vec{V_m} = 6\hat{i}\,m/s$

$\vec{V_{r,m}} = \vec{V_r} - \vec{V_m}$

$= -10\hat{j} - 6\hat{i}$

$\tan\theta = \frac{6}{10} = \frac{3}{5}$

$\boxed{\theta = \tan^{-1}\left(\frac{3}{5}\right)}$



Q A man moving with 6 m/s observes rain falling at 12 m/s vertically. find direction & speed of rain w.r.t ground.

$V_m = 6\,m/s$

$V_{r,m} = 12$

$\vec{V_{r,m}} = -12\hat{j}$

$\vec{V_m} = 6\hat{i}$

$\vec{V_r} = \vec{V_m} + \vec{V_{r,m}}$

$= 6\hat{i} - 12\hat{j}$

$|\vec{V_r}| = \sqrt{36+144}$

$= \sqrt{180}$

$= \sqrt{3\times3\times2\times10}$

$= 3\sqrt{20}$

$\boxed{= 6\sqrt{5}\,m/s}$

$\cos\beta = \frac{+12}{6\sqrt{5}}$

$= \frac{+2}{\sqrt{5}}$

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\tan\theta = \frac{6}{12}$

$\tan\theta = \frac{1}{2}$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

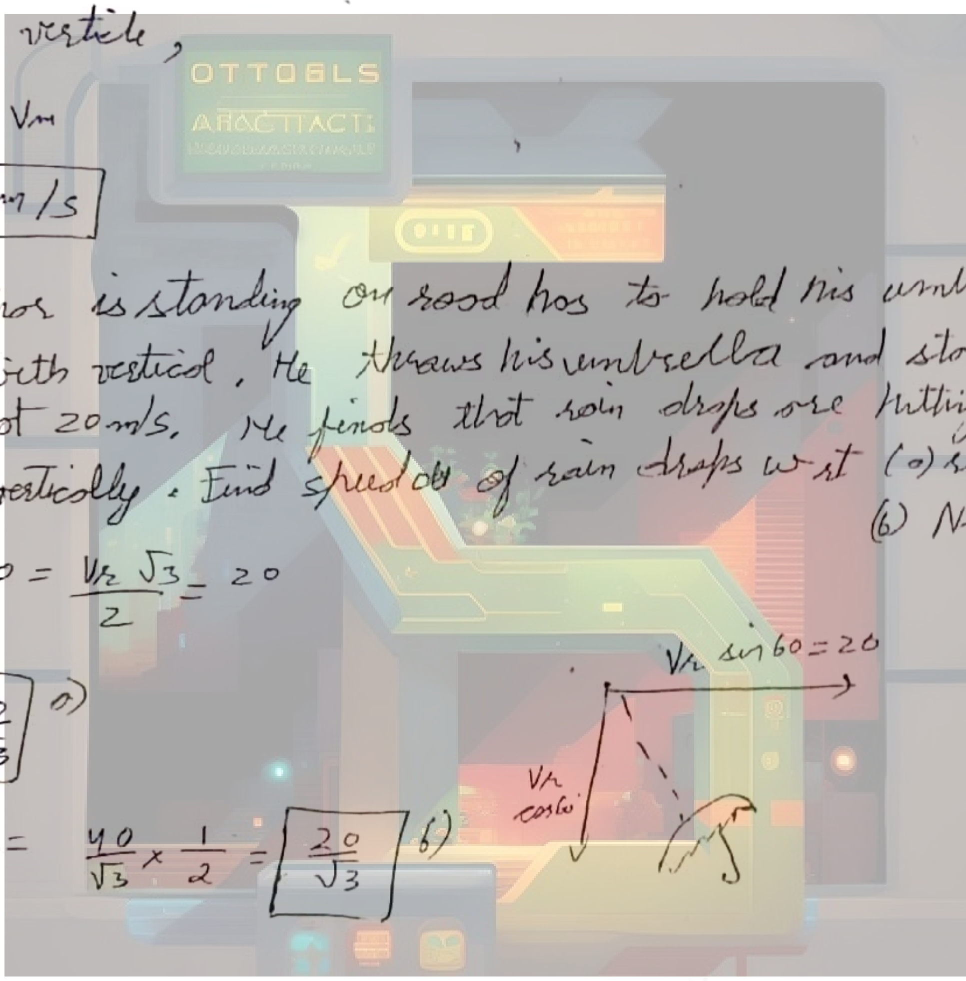
Q3. Rain is falling with velocity of 20 m/s at an angle 30° with vertical. How fast should he move so that rain appears ⊥ to vertical to him?

$|\vec{V_r}| = 20\ m/s$

$\vec{V_r} = 20\sin 30°\ \hat{i} - 20\cos 30°\ \hat{j}$

$= 20\times\frac{1}{2}\ \hat{i} - 20\times\frac{\sqrt{3}}{2}\ \hat{j}$

$= 10\hat{i} - 10\sqrt{3}\ \hat{j}$



If rain is vertical,

$10 = V_m$

$\boxed{V_m = 10\ m/s}$

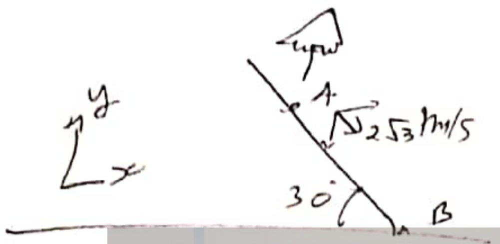
Q4. Nirjhar is standing on road has to hold his umbrella at 60° with vertical. He throws his umbrella and starts moving at 20 m/s. He finds that rain drops are hitting his head vertically. Find speed of rain drops w.r.t (a) road, (b) Nirjhar

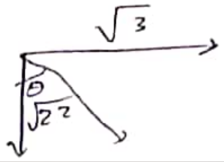
$\vec{V_r}\sin 60 = \frac{V_r\sqrt{3}}{2} = 20$

$\boxed{\vec{V_r} = \frac{40}{\sqrt{3}}}$ a)

$V_r\cos 60 = \frac{40}{\sqrt{3}}\times\frac{1}{2} = \boxed{\frac{20}{\sqrt{3}}}$ b)

Q The man is standing on a ramp with speed $2\sqrt{3}$ m/s and keeps umbrella vertical. Actual speed of rain is 5 m/s. At what angle should he keep his umbrella with vertical when he reaches & stops at B





$\vec{A} = \frac{2\sqrt{3}}{2}\hat{i} - 2\sqrt{3} \times \frac{\sqrt{3}}{2}\hat{j}$

$= \sqrt{3}\hat{i} - 3\hat{j}$

$\vec{R} = \sqrt{3}\hat{i} - \sqrt{22}\hat{j}$

$\frac{\sqrt{22}}{\sqrt{3}} = \tan\theta$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{22}}$

$\vec{A} = 3\hat{i} - \sqrt{3}\hat{j}$

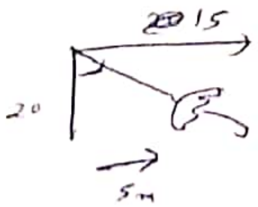
$\vec{R} = 3\hat{i} - 4\hat{j}$

$\tan\theta = \frac{3}{4}$

$\boxed{\cancel{\theta = 30^\circ}}$ $\boxed{\cancel{\theta = 37^\circ}}$

$\boxed{\theta = 37^\circ}$

Q Rain is falling with a speed 30 m/s. A Person running in rain with a velocity of 5 m/s & wind is also blowing with a speed of 15 m/s (both from west) The angle with the vertical at which the person should hold his umbrella so that he may not get drenched is.



$V_{rain,m} = -20\hat{j} + 15 - 5\hat{i}$

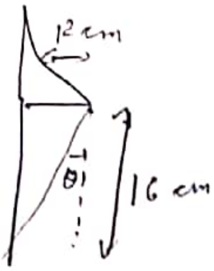
$= -20\hat{j} + 10\hat{i}$

$\tan\theta = \frac{10}{20}$

$= \frac{1}{2}$

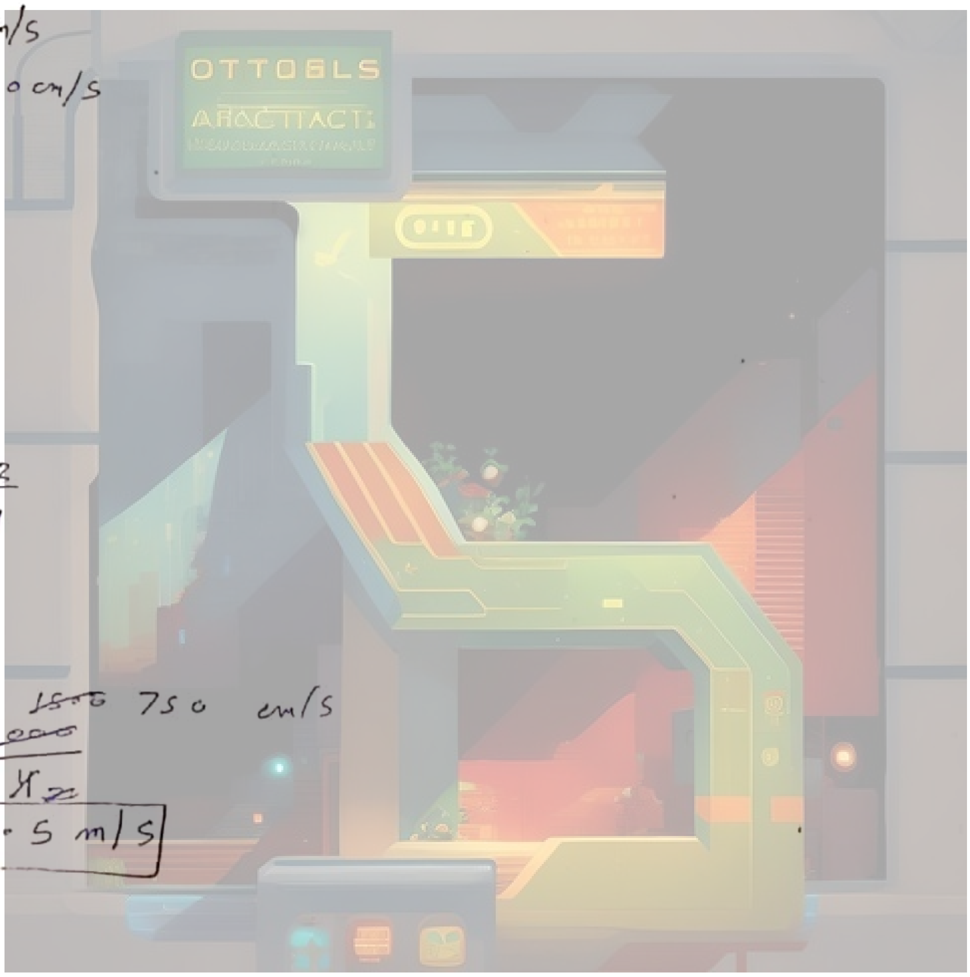
$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

Q A wearing a hat of extended length 12 cm is running in rain falling vertically with 10 m/s. find max speed with which man can run so that raindrops does not fall on his face. (length of face is 16 cm)



V_r (vertical) $= 10$ m/s

$= 1000$ cm/s



$\tan\theta = \frac{12}{16}$

$\tan\theta = \frac{3}{4}$

$\theta = 37^\circ$

$\frac{V_m}{V_r(\text{vertical})} = \frac{3}{4}$

$\frac{V_m}{1000} = \frac{3}{4}$

$V_m = \frac{\cancel{3000}}{\cancel{4}} = \cancel{1500}\ 750$ cm/s

$\boxed{V_m = 7.5 \text{ m/s}}$

Q

a

$\tan\theta = \frac{\text{velocity}}{2}$

$\cancel{\theta = \tan^{-1}}$

$\frac{d\theta}{dt}$

$\frac{dv}{dt} = 2\,m/s^2, \quad v = 2t$

$\cancel{\frac{d\theta}{dv} \times \frac{dv}{dt} = \frac{d\theta}{dt} = 2 \times \tan^{-1}\left(\frac{v}{2}\right)}$

$\theta =$

$\frac{d\theta}{dt} = \frac{d\theta}{dv} \times \frac{dv}{dt}$

$= 2 \times \frac{\sec^{-2}\left(\frac{v}{2}\right)}{2}$

$\boxed{\frac{d\theta}{dt} = \left[\sec^{-1}(t)\right]^2}$

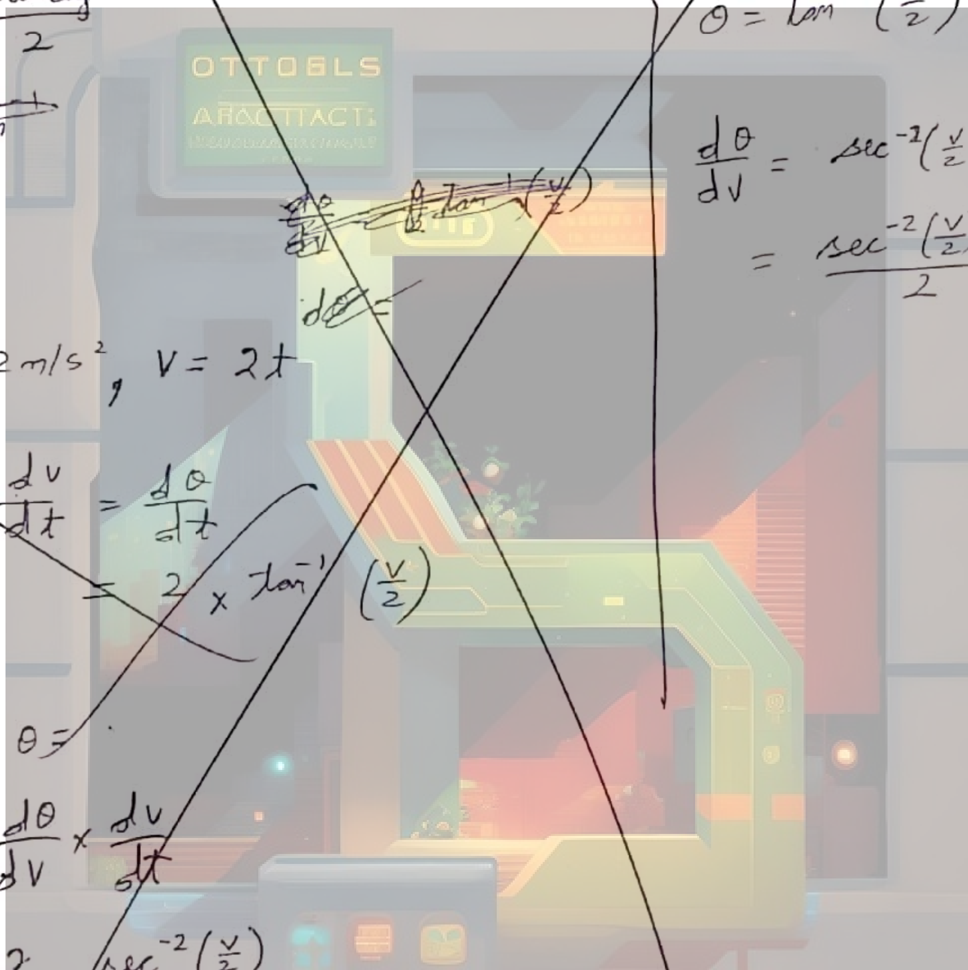
$\theta = \tan^{-1}\left(\frac{v}{2}\right)$

$\cancel{\frac{d\theta}{dv} = \frac{d}{dv}\tan^{-1}\left(\frac{v}{2}\right)}$

$\cancel{d\theta =}$

$\frac{d\theta}{dv} = \sec^{-2}\left(\frac{v}{2}\right) \times \frac{1}{2}$

$= \frac{\sec^{-2}\left(\frac{v}{2}\right)}{2}$



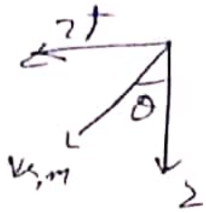
Q rain falling vertical with 2 m/s. A boy at rest accelerates at 2 m/s² along straight road. find rate of change of angle of umbrella ?

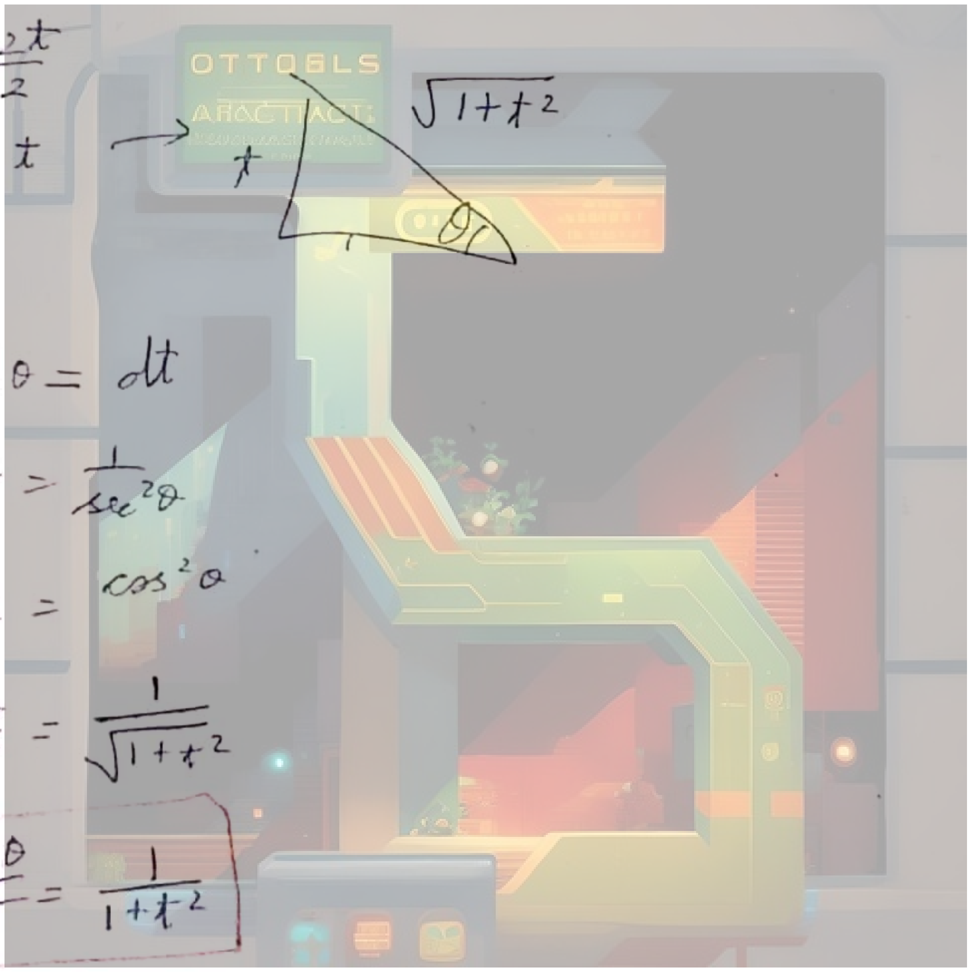
$V_r = -2\hat{j}$

at t $V_m = 2t\hat{i}$

$V_{rm} = V_r - V_m$

$V_{rm} = -2\hat{j} - 2t\hat{i}$





$\tan\theta = \frac{2t}{2}$

$\tan\theta = t$

$\sqrt{1+t^2}$

t

θ

1

$\sec^2\theta \, d\theta = dt$

$\frac{d\theta}{dt} = \frac{1}{\sec^2\theta}$

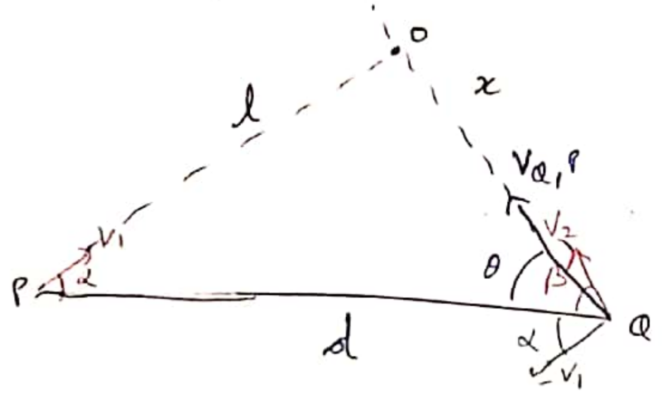
$\frac{d\theta}{dt} = \cos^2\theta$

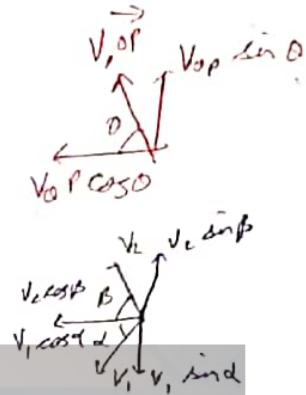
$\frac{d\theta}{dt} = \frac{1}{\sqrt{1+t^2}}$

$\boxed{\frac{d\theta}{dt} = \frac{1}{1+t^2}}$

Shortest Distance Between two moving particles

(Distance of closest approach)





$V_{Q,P} = \vec{V_Q} - \vec{V_P}$

$\vec{V_{QP}} = \vec{V_2} - \vec{V_1} = \vec{V_2} + (-\vec{V_1})$

$V_{QP}\cos\theta = V_1\cos\alpha + V_2\cos\beta$ --- (1)

$V_{QP}\sin\theta = V_2\sin\beta + V_1\sin\alpha$ --- (2)

$(1)^2 + (2)^2$

$(V_{QP})^2 = (V_1\cos\alpha + V_2\cos\beta)^2 + (V_2\sin\beta + V_1\sin\alpha)^2$

$V_{QP} = ?$

$(2) \div (1)$

$\tan\theta = \dfrac{V_2\sin\beta + V_1\sin\alpha}{V_1\cos\alpha + V_2\cos\beta}$

$\theta = ?$

$\sin\theta = \dfrac{l}{d}$

$\boxed{l = d\sin\theta}$ (for particles to collide, $d = 0$)

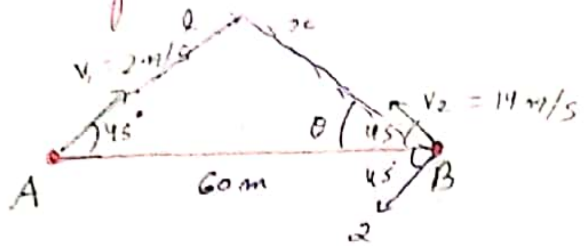
$\cos\theta = \dfrac{x}{d}$

$x = d\cos\theta$

$\text{time} = \dfrac{x}{V_{QP}}$

$\boxed{= \dfrac{d\cos\theta}{V_{QP}}}$

Q1. Motion of A & B takes place in horizontal plane



a) find distance of closest approach

b) At what time is the separation b/t them minimum.

$V\cos\theta = \cos 45 \times 2 + \cos 45 \times 14$

$= \frac{2}{\sqrt{2}} + \frac{14}{\sqrt{2}}$

$= \frac{16}{\sqrt{2}}$

$= 8\sqrt{2}$

$V\sin\theta = \sin 45 \times 2 + \sin 45 \times 14$

$= 6\sqrt{2}$

$\tan\theta = \frac{6\sqrt{2}}{8\sqrt{2}}$

$= \frac{3}{4}$

$\boxed{\cancel{\theta = 45°}}$ $\boxed{\theta = 37°}$

Q ~~$\sin\theta = \frac{l}{60}$~~

~~$\frac{1}{\sqrt{2}} = \frac{l}{60}$~~

~~$\boxed{30\sqrt{2} = l}$~~ a)

$\sin 37 = \frac{l}{60}$

$\frac{3}{5} \times 60 = l$

$\boxed{36 = l}$ a)

~~$\frac{V}{\sqrt{2}} = 8\sqrt{2}$~~

~~$V = 8 \times 2$~~

~~$\boxed{V = 16\,m/s}$~~

~~$\cos 45 = \frac{x}{60}$~~

~~$\boxed{30\sqrt{2} = x}$~~

~~$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$~~

~~$= \frac{30\sqrt{2}}{16}$~~

$\frac{4V}{5} = 8\sqrt{2}$

$V = \frac{5 \times 8\sqrt{2}}{4}$

$\boxed{V = 10\sqrt{2}}$

$\cos 37° = \frac{x}{60}$

$\frac{4 \times 60}{5} = x$

$\boxed{x = 48}$

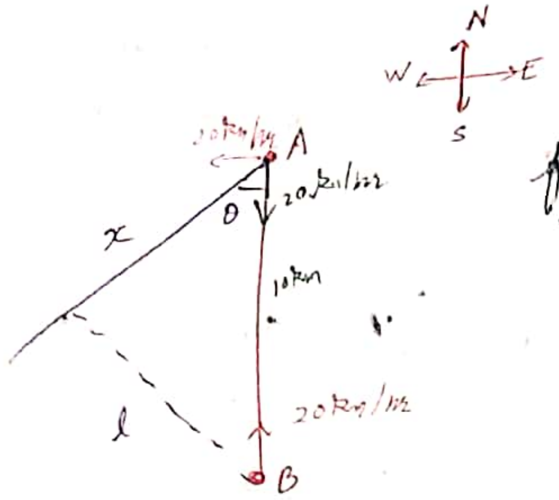
$\text{Time} = \frac{48}{10\sqrt{2}}$

$= \frac{48\sqrt{2}}{20}$

$= \frac{24\sqrt{2}}{10}$

$\boxed{= \frac{12\sqrt{2}}{5}}$ b)

Q 2.



find distance of closest approach & time

$\theta = 45$

$V = \sqrt{400 + 400}$

$= 20\sqrt{2}$

$\sin 45 = \frac{l}{10}$

$\frac{1}{\sqrt{2}} = \frac{l}{10}$

$\boxed{5\sqrt{2} = l}$ a)

$\cos 45 = \frac{l}{10}$

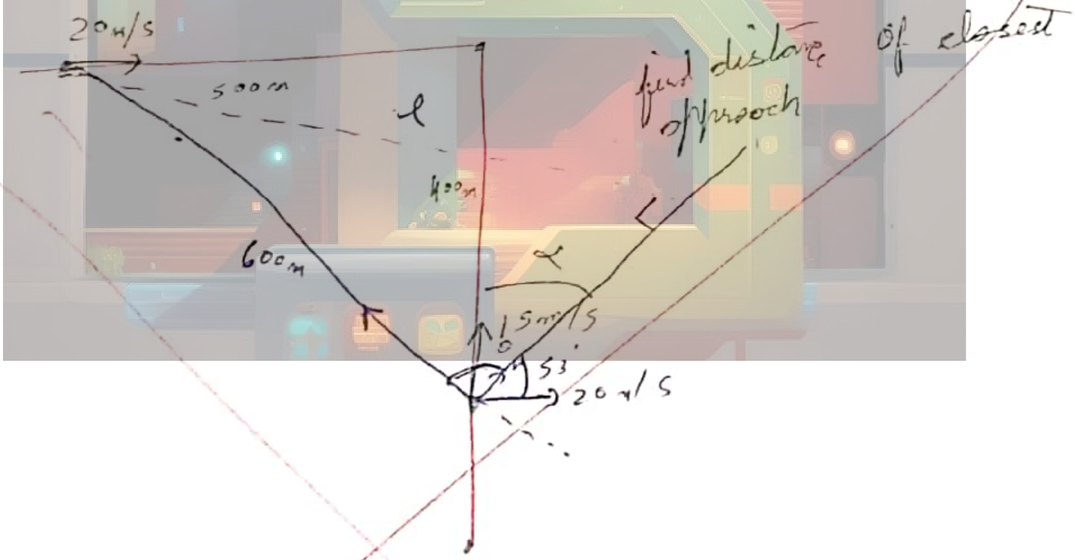
$\boxed{5\sqrt{2} = x}$

$\text{time} = \frac{5\sqrt{2}}{20\sqrt{2}}$

$= \frac{1}{4}\ \text{hr}$

$\boxed{= 15\ \text{min}}$ b)

Q 3.



find distance of closest approach

$V = \sqrt{400 + 225}$

$= \sqrt{625}$

$\boxed{= 25\ m/s}$

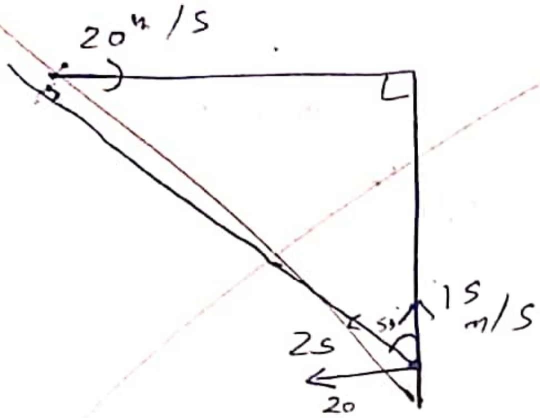
θ

$\alpha = 37^\circ$

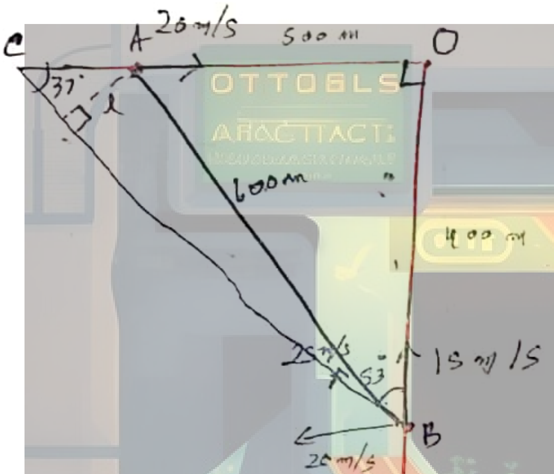
$15 \sin 37 = \sin 53 \times 20$

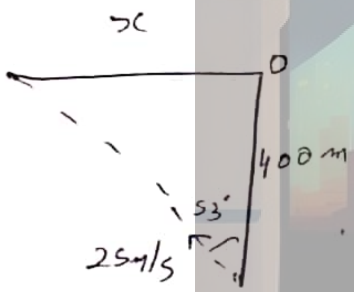
$15 \times \frac{3}{5} = \frac{4}{5} \times 20$

$9 =$



Q3. find distance of closest approach





$CA = \frac{1600-1500}{3}$

$= \frac{100}{3}$

$\sin 37 = l \times \frac{3}{100}$

$\frac{100}{3} \times \frac{3}{5} = \boxed{l = 20m}$

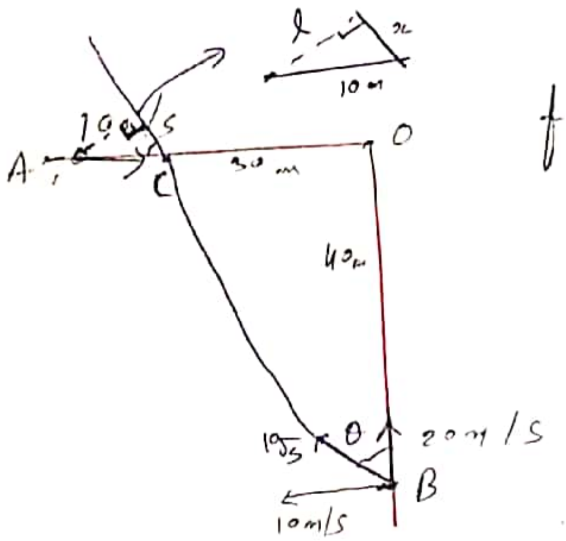
$\tan 53^\circ = \frac{x}{400}$

$\frac{4}{3} = \frac{x}{400}$

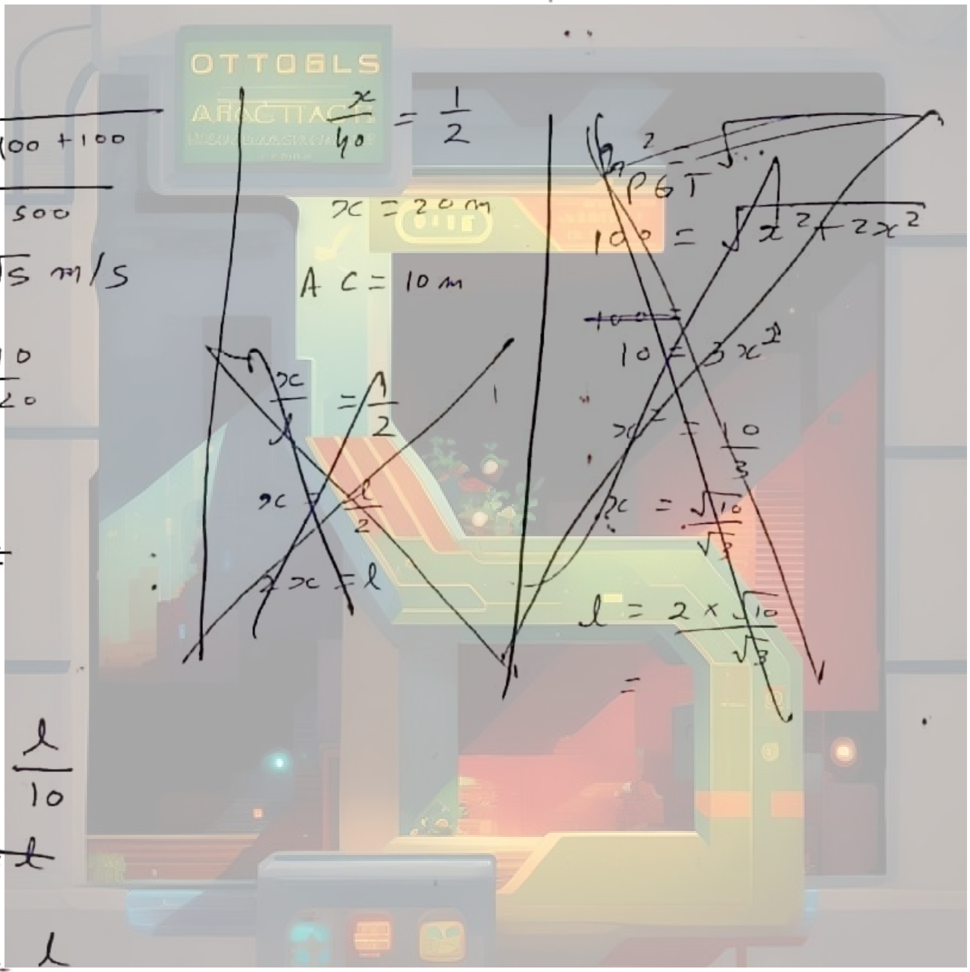
$\boxed{\frac{1600}{3} = x}$

$\frac{x}{400} = \frac{4}{3}$

Q



find distance of closest approach



$V_A = \sqrt{400 + 100}$

$= \sqrt{500}$

$= 10\sqrt{5}$ m/s

$\tan\theta = \frac{10}{20}$

$= \frac{1}{2}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\frac{x}{40} = \frac{1}{2}$

$x = 20$ m

$AC = 10$ m

$\cancel{\frac{x}{l} = \frac{1}{2}}$

$\cancel{x = \frac{l}{2}}$

$\cancel{2x = l}$

$\cancel{PGT}$

$\cancel{100 = \sqrt{x^2 + 2x^2}}$

$\cancel{10 = 3x^2}$

$\cancel{x^2 = \frac{10}{3}}$

$\cancel{x = \frac{\sqrt{10}}{\sqrt{3}}}$

$\cancel{l = 2 \times \frac{\sqrt{10}}{\sqrt{3}}}$

$\frac{2}{\sqrt{5}} = \frac{l}{10}$

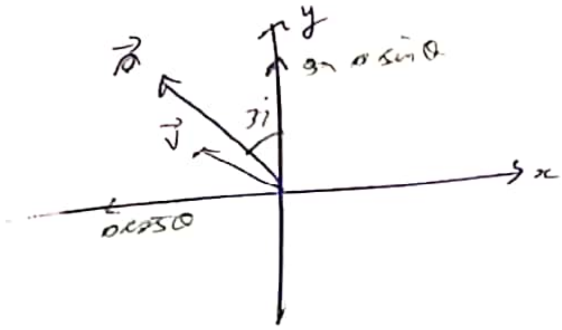
$\cancel{2\sqrt{5} = l}$

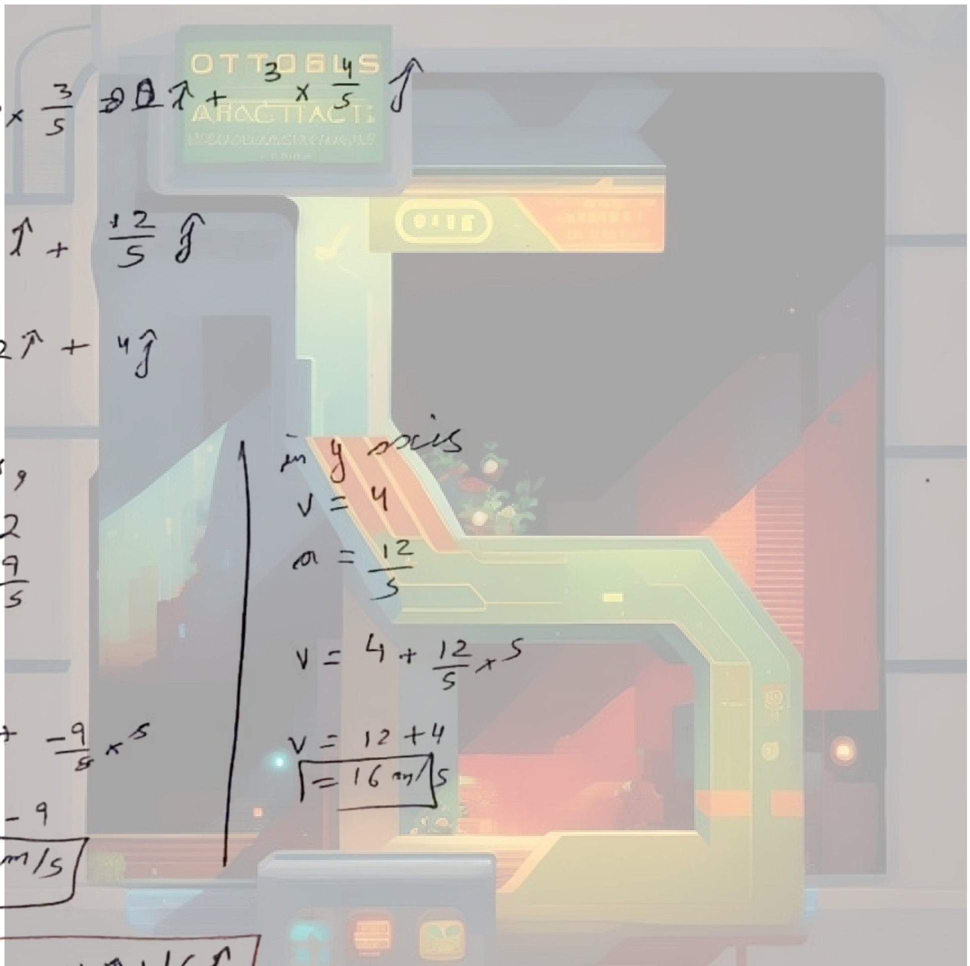
$\frac{10 \times 2}{\sqrt{5}} = l$

$\boxed{l = 4\sqrt{5}}$

General 2-D motion questions

Q. A Particle with initial velocity $\vec{V_0} = (-2\hat{i} + 4\hat{j})$ undergoes constant acceleration of 3 m/s² at $\theta = 127°$ from +ve x-axis, find $\vec{v}$ at $t = 5s$





$$\vec{a} = 3 \times \frac{3}{5} \hat{i} + 3 \times \frac{4}{5} \hat{j}$$

$$= \frac{-9}{5} \hat{i} + \frac{12}{5} \hat{j}$$

$$\vec{V} = -2\hat{i} + 4\hat{j}$$

in x axis,

$v = -2$

$a = -\frac{9}{5}$

$t = 5$

$v = -2 + \frac{-9}{5} \times 5$

$= -2 - 9$

$\boxed{v = -11 \text{ m/s}}$

in y axis

$v = 4$

$a = \frac{12}{5}$

$v = 4 + \frac{12}{5} \times 5$

$v = 12 + 4$

$\boxed{= 16 \text{ m/s}}$

final $\boxed{\vec{v} = -11\hat{i} + 16\hat{j}}$