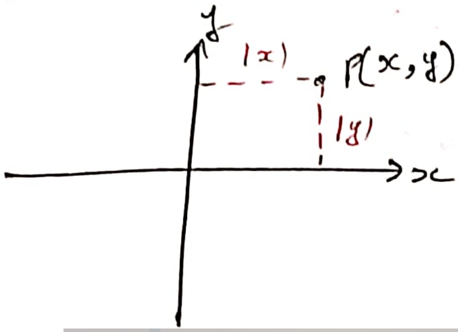
! Co-ordinate Geometry !

→ Algebra + Geometry = Co-ordinate Geometry



Dis. P from x-axis $= |y|$
Dis. P from y-axis $= |x|$

~~Dis P(x, y) & Q(x~~

Dis b/w $P(x_1, y_1)$ & $Q(x_2, y_2)$ $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Q find x if Dis b/w $(x, 0)$ & $(7, 0) = 5$

$$(49 + x^2 - 14x) = 5^2$$

$$\cancel{x^2 - 14x + 44 = 0}$$
$$\cancel{x = 7 \pm \sqrt{49 -}}$$
$$\cancel{x = \frac{14 \pm \sqrt{196 - 176}}{2}}$$
$$\cancel{x = \frac{14 \pm 2\sqrt{5}}{2}}$$
$$\cancel{\boxed{x = 7 \pm \sqrt{5}}}$$
$$\cancel{\boxed{x = 2, 12}}$$

$$x^2 - 14x + 22 = 0$$
$$x = \frac{14 \pm \sqrt{196 - 88}}{2}$$
$$x = \frac{14 \pm 10}{2}$$
$$\boxed{x = 2, 12}$$

Q find the point where x & y are equal & which is equidistant from $(1,0)$ & $(0,3)$

(x, x)

$$(x-1)^2 + (x-0)^2 = (x-0)^2 + (x-3)^2$$

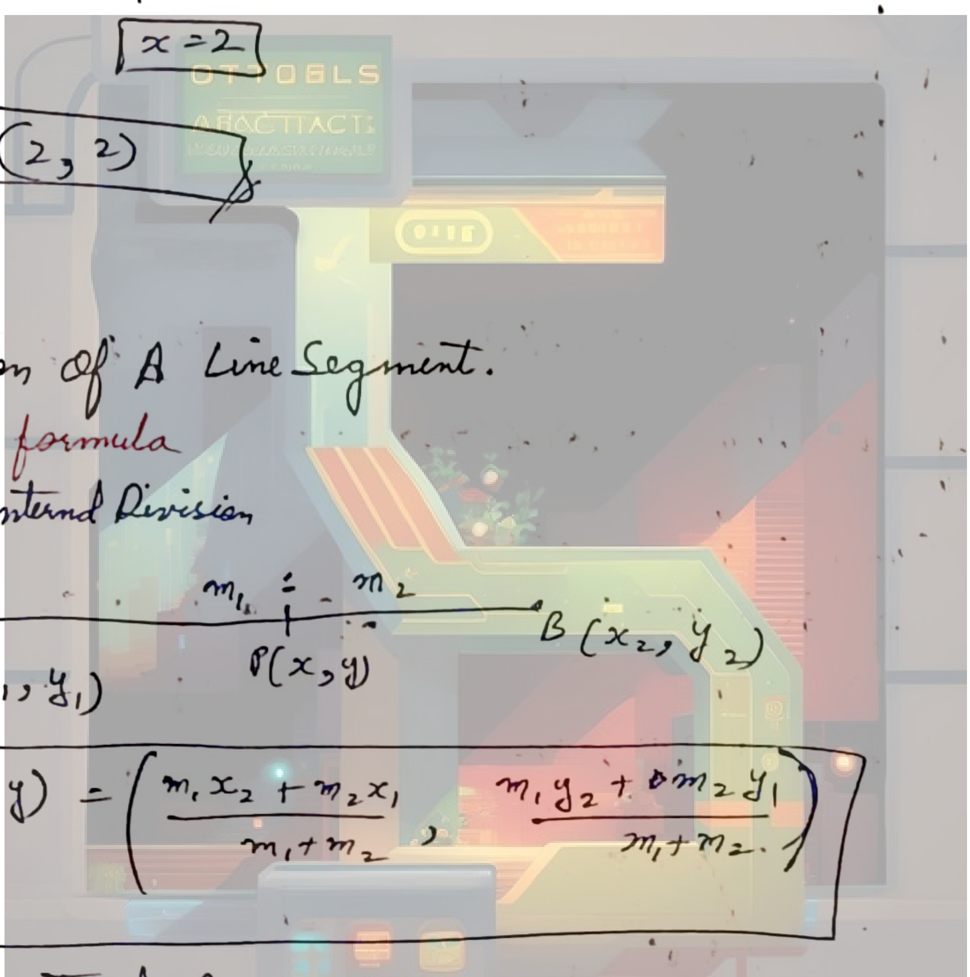
$$x^2 + 1 - 2x + x^2 = x^2 + x^2 + 9 - 6x$$

$$1 - 2x = 9 - 6x$$

$$4x = 8$$

$$\boxed{x = 2}$$

$$\boxed{(2, 2)}$$



Division of A Line Segment.

① Section formula

A) Internal Division

$m_1 : m_2$

$A(x_1, y_1)$ $P(x, y)$ $B(x_2, y_2)$

$$\boxed{P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)}$$

B) External Division

$m_1 : m_2$

$A(x_1, y_1)$ $B(x_2, y_2)$ $(P)\ (x, y)$

$$\boxed{P(x, y) = \left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}\right)}$$

NB. If ratio is asked, we use $\lambda:1$

If λ is ⊕ve then internal division

If λ is ⊖ve then external division

Q find ratio in which x axis divides line joining $(2,-3)$ & $(5,6)$

$\lambda:1$

~~$y=0=\frac{5\lambda+2}{\lambda+1}$~~

~~$5\lambda+2=0$~~

~~$5\lambda=$~~

$y=0=\frac{6\lambda-3}{\lambda+1}$

$6\lambda-3=0$

$\lambda=\frac{1}{2}$

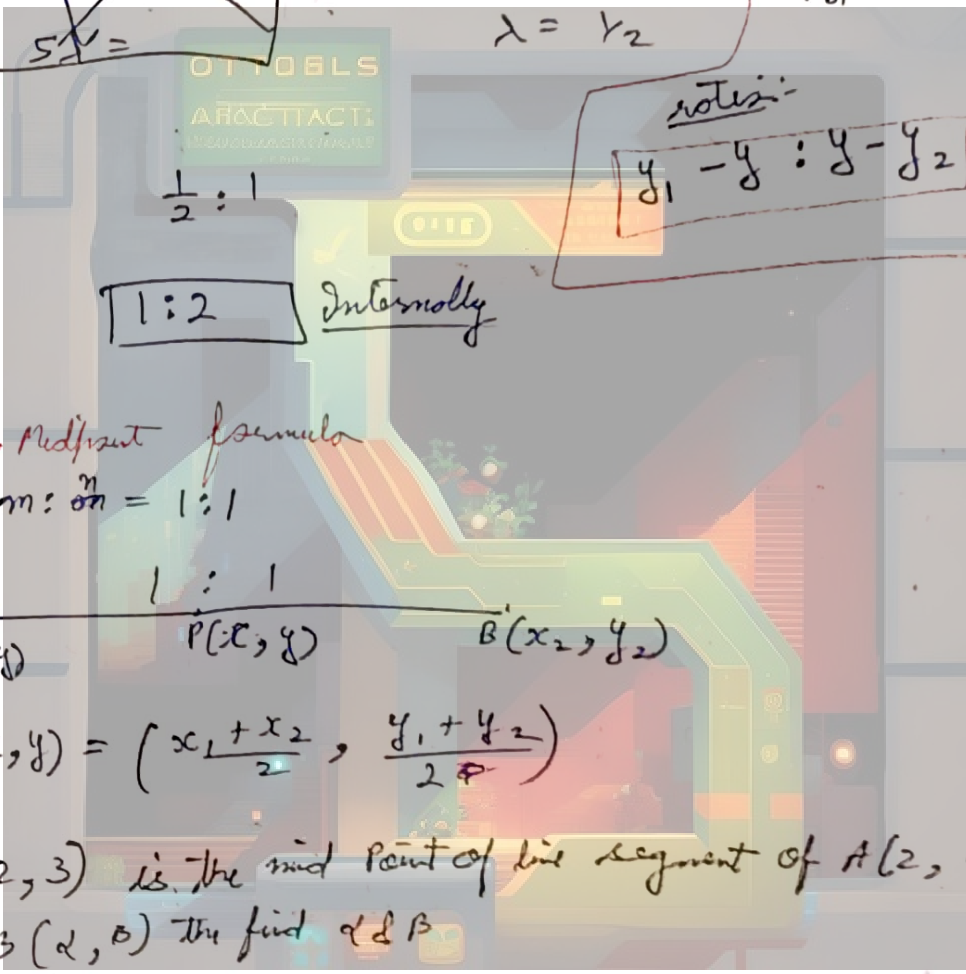
Trick-2 ☆☆

$(2,-3)$ $(x,0)$ $(5,6)$

x_1, y_1 x, y x_2, y_2

ratio:-

$\boxed{y_1-y : y-y_2}$



$\frac{1}{2}:1$

$\boxed{1:2}$ Internally

Q: ② Sec. Midpoint formula

$m:n = 1:1$

$1 : 1$

$A(x_1,y_1)$ $P(x,y)$ $B(x_2,y_2)$

$P(x,y)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

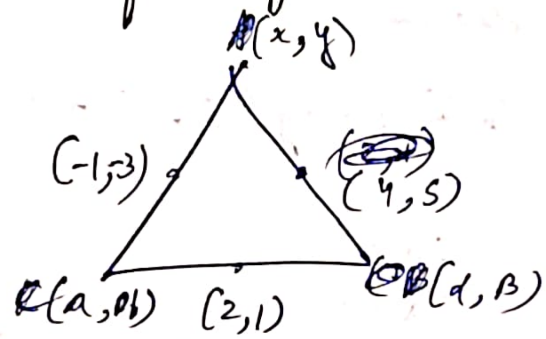
Q if $(2,3)$ is the mid point of line segment of $A(2,9)$ & $B(\alpha,\beta)$ then find α & β

$\frac{\alpha+2}{2}=2$ $\frac{\beta+9}{2}=3$

$\alpha+2=4$ $\beta+9=6$

$\boxed{\alpha=2}$ $\boxed{\beta=-3}$

Q The midpoints of sides of Triangle are $(2,1)$ $(-1,-3)$ & $(4,5)$. find vertices of triangle.



$\frac{a+\alpha}{2} = 2$ $\quad$ $\frac{x+a}{2} = -1$ $\quad$ $\frac{\alpha+x}{2} = 4$

$a+\alpha = 4$ $\quad$ $a+x = -2$ $\quad$ $\alpha + x = 8$

$a - \alpha = -10$

$2a = -6$

$\boxed{a=-3}$

$\boxed{\alpha = 7}$

$\boxed{x = 1}$

$b+\beta = 2$ $\quad$ $b+y = -6$ $\quad$ $\beta + y = 10$

$b - \beta = -16$

$2b = -14$

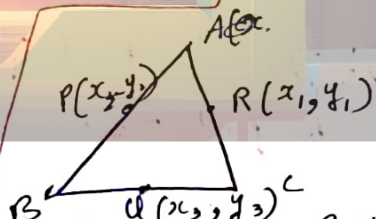
$\boxed{b = -7}$

$\boxed{\beta = 9}$

$\boxed{y = 1}$

$\boxed{{}^{B}(-3,-7),\ {}^{C}(1,1)\ \&\ {}^{A}(7,9)}$

Trick - 4



$A_x = P_x + R_x - Q_x$

$B_x = P_x + Q_x - R_x$

$C_x = R_x + Q_x - P_x$

Harmonic Conjugate

→ If P & Q are two points which divides the line segment AB internally & Externally in the same ratio $(m:n)$ then P & Q are Harmonic conjugate of each other with respect to A & B.

$A(x_1, y_1)$ — $P(a, b)$ — $B(x_2, y_2)$ — $Q(l, m)$

$$\frac{AP}{PB} = \frac{m}{n} \qquad \frac{AQ}{BQ} = \frac{m}{n}$$

P & Q are Harmonic conjugate w.r.t A & B.

→ AP, AQ & AB are in Harmonic series.

Proof:-

$$\frac{AP}{PB} = \frac{AQ}{BQ}$$

$$\frac{PB}{AP} = \frac{BQ}{AQ}$$

$$\frac{AB-AP}{AP} = \frac{AQ-AB}{AQ}$$

$$\frac{AB}{AP} - 1 = 1 - \frac{AB}{AQ}$$

$$\frac{AB}{AP} + \frac{AB}{AQ} = 2$$

$$\frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB}$$

Q3. Determine the ratio in which the point $P(3,5)$ divide the join of $A(1,3)$ & $B(7,9)$ find harmonic conjugate of P wrt AB.

~~$P(x,y)$~~ $\frac{AP}{BP} = \frac{\lambda}{1}$ (let division is internal)

$$\frac{7\lambda+1}{\lambda+1} = 3$$

$$7\lambda + 1 = 3\lambda + 3$$

$$4\lambda = 2$$

$\lambda = \frac{1}{2}$ (Hence, internal division is True)

$$\frac{AP}{BP} = \frac{1}{2}$$

$$Q = \left(\frac{7-2}{1-2}, \frac{\cancel{6-9}\ 9-6}{1-2}\right)$$

$$\boxed{Q(-5,-3)}$$

Q Find Harmonic conjugate of $R(2,4)$ wrt $P(2,2)$ & $Q(2,5)$

$\lambda : 1$

$$\frac{5\lambda+2}{\lambda+1} = 4$$

$$5\lambda + 2 = 4\lambda + 4$$

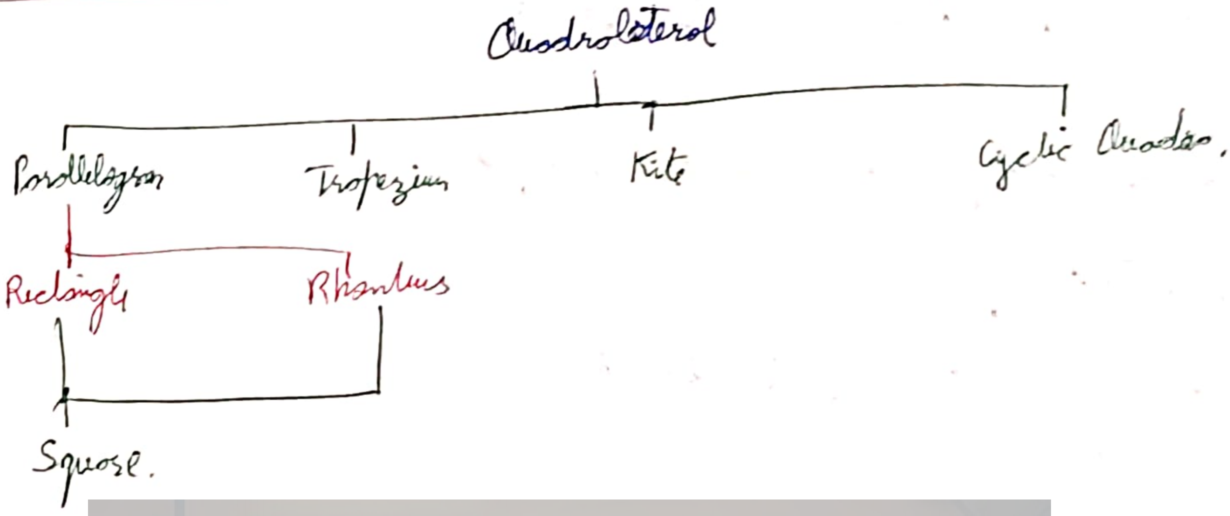
$$\lambda = 2$$

$2:1$

$$S = \left(\frac{2-4}{2-1}, \frac{2-10}{2-1}\right)$$

$$\boxed{S(-2,-8)}$$

Quadrilaterals :-



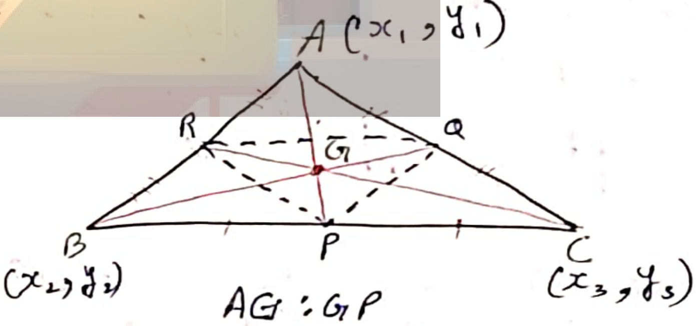
→ Square is a special case of Rectangle with all sides equal.

Diagnols & side based property :-

Parallelogram	Opp sides equal	Diagonals Bisect
Rectangle	opp sides equal	Diagonals bisect & equal in length
Square	all sides equal	Diagonals bisect ~~at~~ at ⊥ & equal length.
Rhombus	all sides equal	Diagonals ⊥.

~~Points of a Triangle & |~~

Triangle (Points & Co-ordinates)



AG : GP
2 : 1

① Centroid (G) -

→ AP, BQ & CR are medians.

→ $\frac{AG}{GP} = \frac{BG}{GQ} = \frac{CG}{GR} = \frac{2}{1}$

→ Area(BGC) = area(BGA) = area(Δ CGA) = $\frac{1}{3}$ area(ΔABC)

→ Area(PQR) = $\frac{\text{Area}(\Delta ABC)}{4}$

→ Ar(ΔABP) = Ar(ΔACP)

→ $AP^2 + CR^2 + BQ^2 = \frac{3}{4}(a^2+b^2+c^2)$

→ Centroid always lies inside triangle.

<u>Cordinates of Centroid</u>

$$\boxed{G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)}$$

Proof:-

$1:1$

$B(x_2, y_2)$ —— P —— $C(x_3, y_3)$

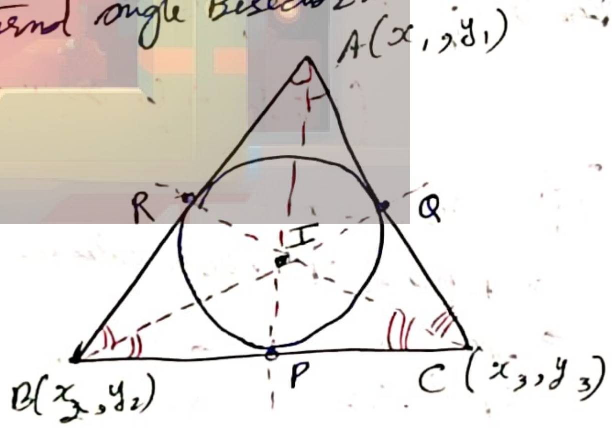
$$P \equiv \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

$2:1$

$A(x_1, y_1)$ —— G —— $P\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) \quad \text{H.P}$$

(2) Incenter (I) → Intersection of Internal angle Bisector.



→ AP, BQ & CR are angle Bisector.

→ $\frac{BP}{PC} = \frac{c}{b}$, $\frac{AR}{RB} = \frac{b}{a}$, $\frac{CQ}{QA} = \frac{a}{c}$

→ $\frac{AI}{IP} = \frac{b+c}{a}$, $\frac{BI}{IQ} = \frac{c+a}{b}$

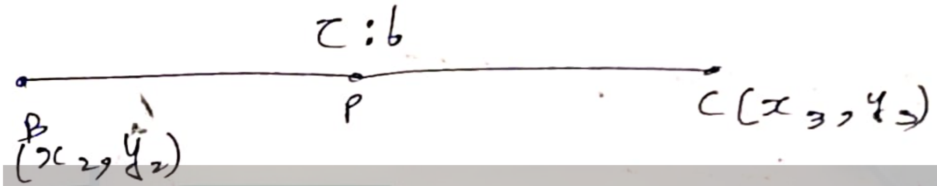
$\frac{CI}{IR} = \frac{a+b}{c}$

→ Incenter always lies inside a triangle.

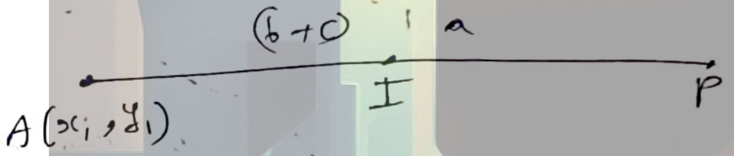
Cordinates of Incenter

$$\boxed{I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)}$$

Proof:-



$$P = \left(\frac{cx_3+bx_2}{c+b}, \frac{cy_3+by_2}{c+b}\right)$$

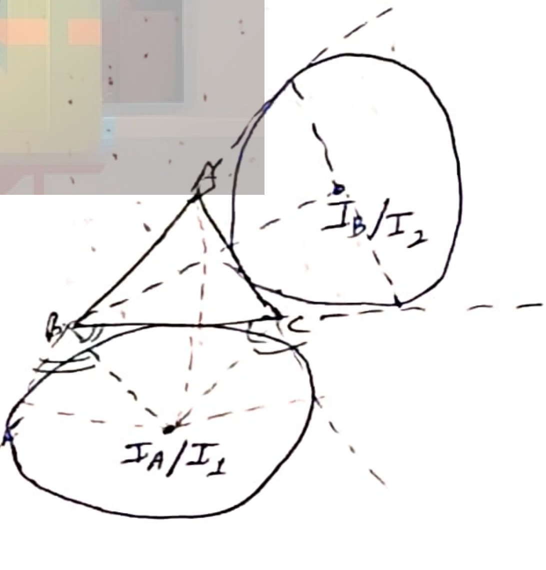


$$I = \left(\frac{(b+c)\frac{(cx_3+bx_2)}{b+c} + ax_1}{a+(b+c)}, \frac{(b+c)\frac{(cy_3+by_3)}{(b+c)} + ay_1}{a+(b+c)}\right)$$

$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

HP

Note:- Ex-center is related to Incenter



$$\boxed{I_A = \left(\frac{bx_2+cx_3-ax_1}{b+c-a}, \frac{by_2+cy_3-ay_1}{b+c-a}\right)}$$

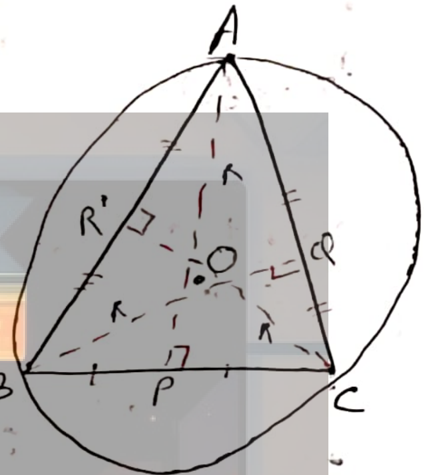
→ One Internal angle Bisector & 2 External angle bisector.

→ always Outside the triangle.

$$I_B \equiv \left(\frac{ax_1+cx_3-bx_2}{a+c-b}, \frac{ay_1+cy_3-by_2}{a+c-b}\right)$$

$$I_C \equiv \left(\frac{ax_1+bx_2-cx_3}{a+b-c}, \frac{ay_1+by_2-cy_3}{a+b-c}\right)$$

③ Circumcenter (O) - Intersection of ⊥ bisectors



→ ~~AP, BQ & CR are circumcenters~~

→ AP, BQ & CR' are ⊥ side bisectors.

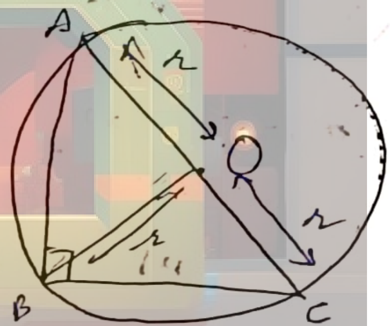
→ OA, OB & OC are circumRadius (R)

→ May or not go through the vertex.

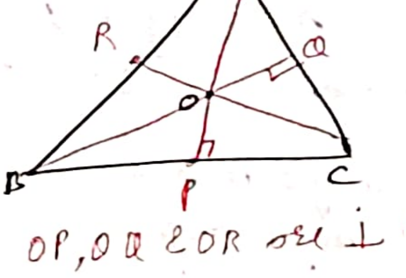
→ $R = \dfrac{abc}{4\Delta}$

→ For finding the circumcenter co-ordinates, we will solve any 2 equations of ⊥ side Bisectors.

Note:- for Right Angle Triangle, circumcenter is the mid point of Hypotenuse.



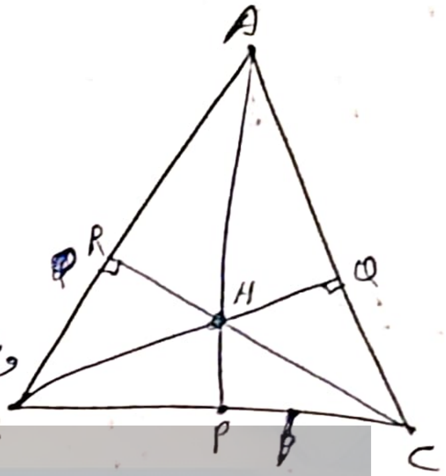
→ In Acute Angle Triangle it lies inside, For Right Angle Δ, It lies on Triangle and for Obtuse angle Triangle, It lies outside the triangle.



OP, OQ & OR are ⊥ bisector

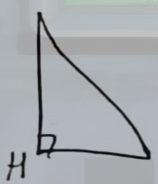
(4) Orthocenter (H) - Intersection of Altitude (⊥)

→ AP, BQ & CR are Altitudes



→ For finding the orthocenter, we will solve equation of any two altitude.

Note:- For A Right Angle Triangle, Orthocenter is the vertex where 90° angle lies.



→ For any acute Angle, orthocenter lies inside the triangle.
any Right Angle Triangle, orthocenter lies on vertex (90°)
any obtuse Angle Triangle, orthocenter lies outside the triangle.

Q. Two vertices of a Δ are (−1, 4) & (5, 2) If its Centroid is (0, −3) find 3rd vertex.

$$\frac{x_1+x_2+x_3}{3} = 0$$

$$\frac{-1+5+x_3}{3} = 0$$

$$x_3 = -4$$

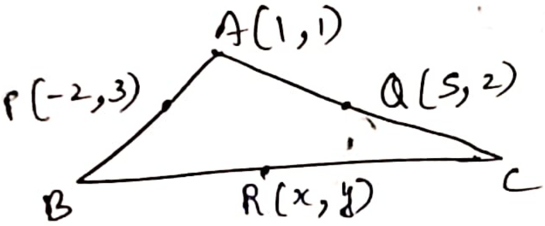
$$\frac{4+2+y_3}{3} = -3$$

$$4+2+y_3 = -9$$

$$y_3 = -15$$

$$\boxed{(-4, -15)}$$

Q



If P & Q are mid points find.

Ans i) Vertices B & C

ii) Centroid of Δ

~~$B \equiv \left(\frac{x+1}{2}, y+\right)$~~

B

$\frac{x_B+1}{2} = -2$

$x_B + 1 = -4$

$x_B = -5$

$\frac{y_B+1}{2} = 3$

$y_B + 1 = 6$

$y_B = 5$

$\boxed{B \equiv (-5, 5)}$

$\frac{x_c+1}{2} = 5$

$x_c = 9$

$\frac{y_c+1}{2} = 2$

$y_c + 1 = 4$

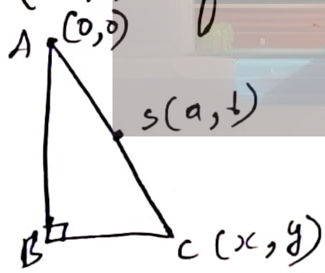
$y_c = 3$

$\boxed{C(9, 3)}$

ii) $G \equiv \left(\frac{-5+1+9}{3}, \frac{1+3+5}{3}\right)$

$\boxed{G \equiv \left(\frac{5}{3}, 3\right)}$

Q The orthocenter of Δ ABC is B & the circumcentre is S(a, b) if A is the origin. Find co-ordinates of C.



A $\frac{x+0}{2} = a$

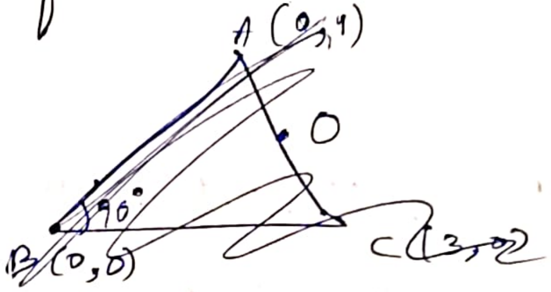
$x = 2a$

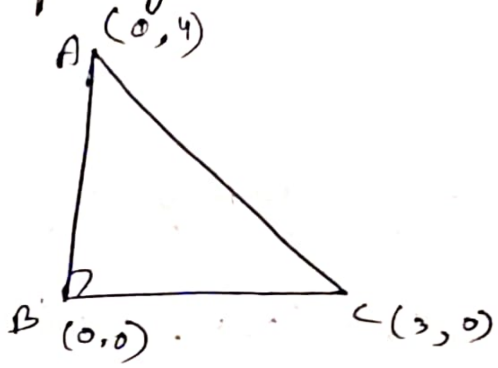
$\frac{y+0}{2} = b$

$y = 2b$

$\boxed{C(2a, 2b)}$

Q triangle with one vertex $(0,0)$ & sides 3, 4 cm $\oplus$ x & y axes. find circumcenter & orthocenter of Triangle

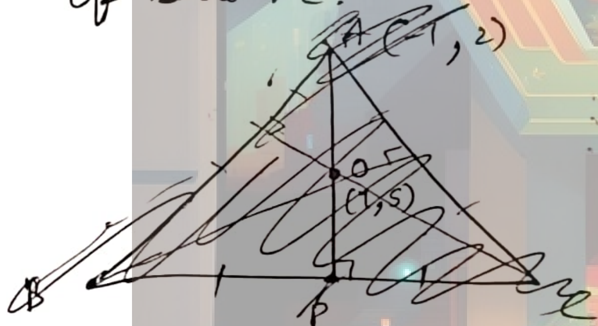


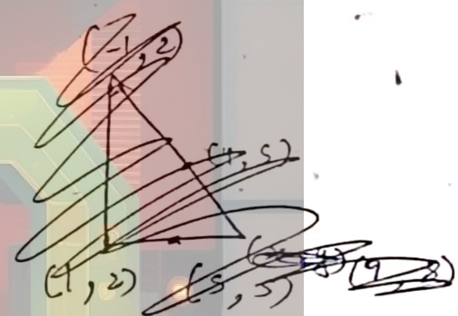


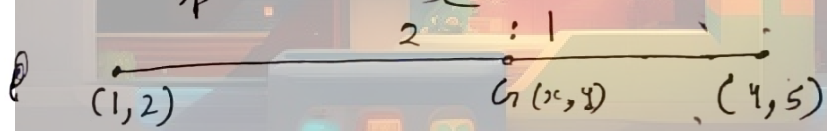
$$\boxed{O = \left(\frac{3}{2}, 2\right)}$$

$$\boxed{H = (0,0)}$$

Q orthocentre & circumcenter of a ΔABC are $(1,2)$ & $(4,5)$. If the co-ordinates of the vertex A are $(-1,2)$ find middle point of side BC.

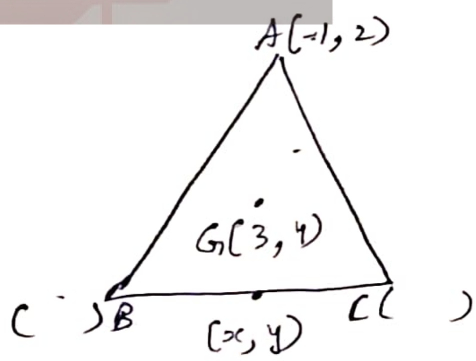






$\cancel{1-x = 2x-8}$ $\cancel{9 = 3x}$ $\cancel{x = 3}$

$\cancel{2-2x = x-4}$ $\cancel{6 = 3x}$ $\cancel{x = 2}$



$1 - x = 2x - 8$

$\boxed{x = 3}$

$2 - y = 2y - 10$

$\boxed{4 = y}$

$$\frac{-1-3}{3-x} = 2$$

$-4 = 6 - 2x$

$\boxed{x = 5}$

$-2 = 8 - 2y$

$\boxed{y = 3}$

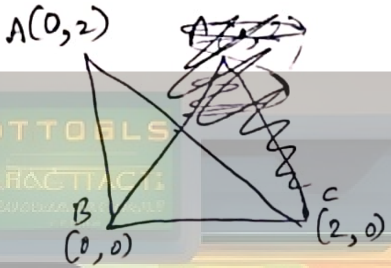
$\boxed{(5, 3)}$

H.W. 13-09-24

0 - 3 [Determinants]
J - A

H.W.
Q1 Find area of Δ formed by origin & POI of line $\frac{x}{6}+\frac{y}{8}=1$ & both axes.

H.W.
Q2. If $(0,0)$ $(0,2)$ $(2,0)$ are vertices of a Δ then, find its incentre.



An2. ~~I =~~ (

$a = 2$

$c = 2$

$b = 2\sqrt{2}$

$I \equiv \left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$

$$\boxed{I \equiv \left(\frac{2}{\sqrt{2}+2}, \frac{2}{\sqrt{2}+2}\right)}$$

A1. at x axis, at y axis $(0,0)$

$(x, 0)$ $(0, 8)$

$(6, 0)$

area $= \frac{1}{2} \times 6 \times 8$

$\boxed{= 24}$

Concurrency of 3 Lines :-

→ when all the given lines passes through a single point then the lines are called concurrent lines.

→ Solve any 2 equations of lines and put values of x & y in 3rd equation.

~~→~~ if it satisfies then the lines are concurrent.

Method -2-

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_3x + b_3y + c_3 = 0$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$ then lines are concurrent.

Q If the lines $y - x = 5$, $3x + 4y = 1$ & $y = mx + 3$ are concurrent find m.

$$\begin{vmatrix} -1 & 1 & -5 \\ 3 & 4 & -1 \\ m & -1 & 3 \end{vmatrix} = 0$$

$-x + y - 5 = 0$

$3x + 4y - 1 = 0$

$mx - y + 3 = 0$

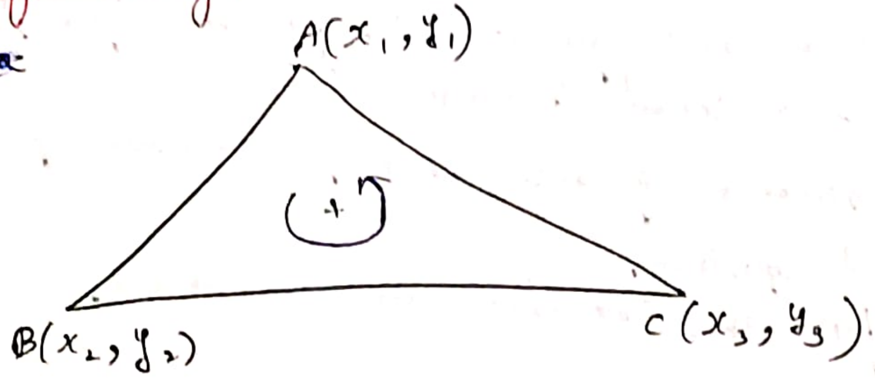
$-11 - 9 - m + 15 + 20m = 0$

$19m = 5$

$$\boxed{m = \frac{5}{19}}$$

Area of Triangle

~~Area~~



$$\boxed{\text{Area} = \left| \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right] \right|}$$

Coordinates :- Anticlockwise :- Area = +ve

Clockwise :- Area = −ve

Q find area of Triangle of vertices. $(1,-1)\ (-1,1)\ (-1,-1)$

$$\text{area} = \frac{1}{2}\left[1(2) + (-1)(0) + (-1)(-2)\right]$$

$$= \frac{1}{2}\left[2+2\right]$$

$$\boxed{=2}$$

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

→ mod

→ Det

Trick-3

$(1,-1), (-1,1), (-1,-1) \quad (1,-1)$

Subtract ~~each term~~ $(1,-1)$ from each

$(0,0) \quad \underset{x_1 \;\; y_1}{(-2,2)} \quad \underset{x_2 \;\; y_2}{(-2,0)}$

$$\boxed{\text{Area} = \frac{1}{2}\left|x_1 y_2 - x_2 y_1\right|}$$

Q find area of △ taken vertices $A(4,4)$ $B(3,-2)$ $C(-3,16)$

$A(0,0)$ $B(-1,-6)$ $C(-7,12)$

$$\text{Area} = \frac{1}{2}\left|(-1)(12) - (-6)(-7)\right|$$

$$\frac{1}{2}\left|-12 - 42\right|$$

$$\boxed{= 27}$$

Note → If Area of △ Triangle ABC $= 0$ then, vertices A, B & C lies on a same line. Hence they are co-linear.

Q prove that the points $A(a, b+c)$ $B(b, c+a)$ & $C(c, a+b)$ are co-linear.

area $= 0$

$$\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} \qquad R_1 \to R_1 + R_2 \qquad = (a+b+c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix}$$

$$= 0$$

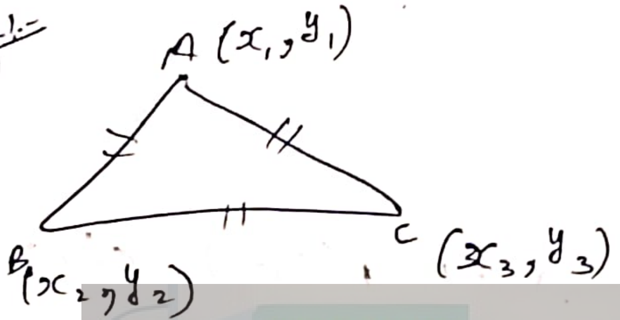
Thus, area $= 0$

HP

Type of Triangle (Trick)

→ (1) In an equilaterd Triangle, co-ordinates of all the vertices can not be rational.

Proof :-



(1) $Ar = \frac{\sqrt{3}a^2}{4} = \underbrace{\frac{\sqrt{3}}{4}}_{\text{Irrational}} \Big(\underbrace{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}_{\text{Rational}}\Big)^2$

Area → Irrational

(2) Area $= \left|\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]\right|$

= rational

So, asumption is wrong.

Q If the points $(4,-4)$, $(-4,4)$ & (x, y) forms a equilateral Δ, find x & y.

$64+64 = (4-x)^2+(y+4)^2$

$128 = 16+x^2-8x+y^2+16-8y$

$96 = x^2+y^2-8x+8y$

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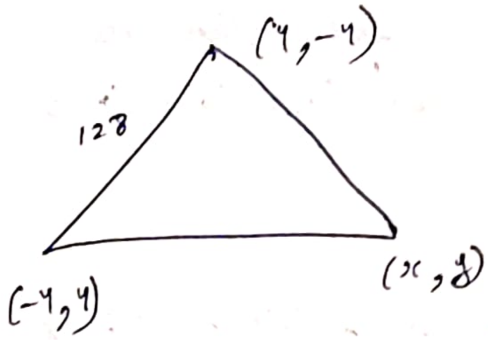


$$128 = (x+4)^2 + (y-4)^2$$

$$128 = 16 + 16 + x^2 + y^2 + 8x - 8y$$

$$x^2 + y^2 + 8x - 8y = x^2 + y^2 - 8x + 8y$$

$$16x = 16y$$

$$x = y$$

$\cancel{128}$

$$96 = x^2 + x^2 + 8x - 8x$$

$$96 = 2x^2$$

$$x^2 = 48$$

$$x = \pm\sqrt{48}$$

$$x = \pm 4\sqrt{3}$$

$$y = \pm 4\sqrt{3}$$

$$(4\sqrt{3}, 4\sqrt{3}) \text{ or } (-4\sqrt{3}, -4\sqrt{3})$$

Trick-4

→ To find 3rd vertex, two given. (equilateral)

$$\boxed{\left(\frac{x_1 + x_2 \pm \sqrt{3}(y_2 - y_1)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_1 - x_2)}{2}\right)}$$

→ (2) No. of Points Having co-ordinates are Integers that lies in the interior of a triangle with vertices $(0,0)$ $(0,n)$ $(n,0)$ are $\boxed{\frac{(n-1)(n-2)}{2}}$

Q. find the no. of integral points which lies inside a triangle with vertices $(0,0)$, $(0,6)$ & $(6,0)$

$\boxed{= 10}$

Q find the area of triangle whose mid Points ~~are~~ of the vertices are. $(0,0)$ $(3,0)$ & $(0,4)$

~~$(6,0)$ & $(0,8)$~~

~~Area $= \frac{1}{2}|48-0|$ = 48 24~~

$$\text{Area} = \frac{1}{2}|3\times4-0| = 6 \text{ (area by mid Point)}$$

$$\text{Area}(\Delta) = 4\times6$$

$$\boxed{=24}$$

Q If area of Δ is 5 & 2 vertices are $(2,1)$ & $(3,-2)$ & the third vertex lies on line $y = x+3$ then find coordinates of 3rd vertex.

$(2-x, 1-y)$ $(3-x, -2-y)$

$10 = +(2-x)(2+y) + (1-y)(3-x)$

$10 = 4+2y-2x-xy+3-3x-3y+xy$

$10 = |7-5x-y|$

$3 = |-5x-y|$

$-5x-y = 3$

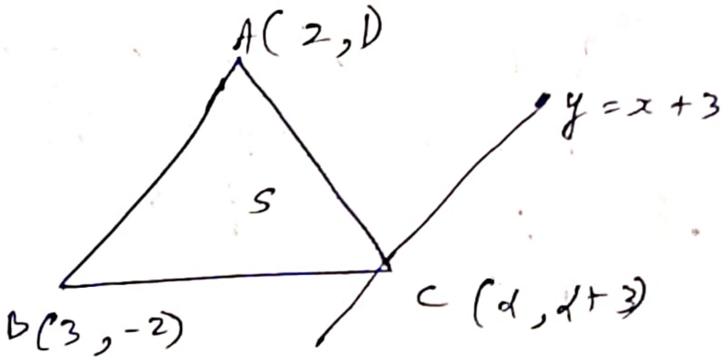
$+y-x = +3$

~~$x = 0$~~

~~$y = 3$~~

$-6x = 6$

$x = -1$



$(0,0)\,(1,-3)\,(\alpha-2,\ \alpha+2)$

$10 = |\alpha+2+3\alpha-6|$

$10 = |4\alpha-4|$

$10 = 4\alpha-4$ | $10 = -4\alpha+4$

$4\alpha = 14$ | $4\alpha = -6$

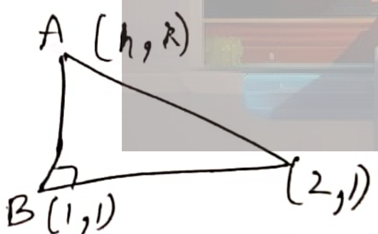
$\alpha = \cancel{3/2}\ 7/2$ | $\alpha = -3/2$

$\cancel{(-3/2,\ 9/2)}$ | $\cancel{(-3/2,\ 9/2)}$

$\cancel{(7/2,\ 9/2)}$ | $(-3/2,\ 3/2)$

$(7/2,\ 13/2)$

Q



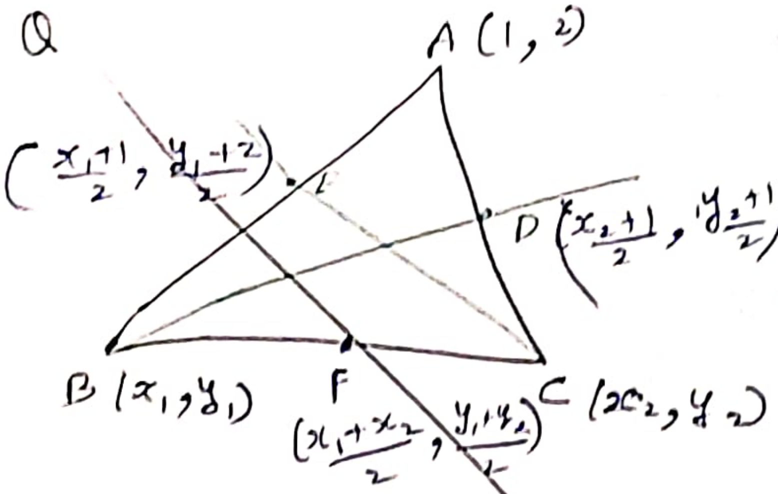
area = 1 unit sq

$\frac{1}{2} \times 1 \times (k-1) = 1$

$k-1 = 2$

$k = 3, -1$

Q



If BD & CE are medians from B & C respectively with eq^{n}s $x+y=5$ & $x=4$ find vertices B & C.

$$\frac{x_2+1}{2} + \frac{y_2+2}{2} = 5$$

$$x_2 + y_2 + 2 = 10$$

$$x_2 + y_2 = 8 \; 7$$

$$\boxed{x_2 = 4}$$

$$\boxed{y_2 = 4}$$

$$\cancel{\frac{x_1+1+y_1+2}{2}} =$$

$$\frac{x_1+1}{2} = 4$$

$$x_1 + 1 = 8$$

$$x_1 = 7$$

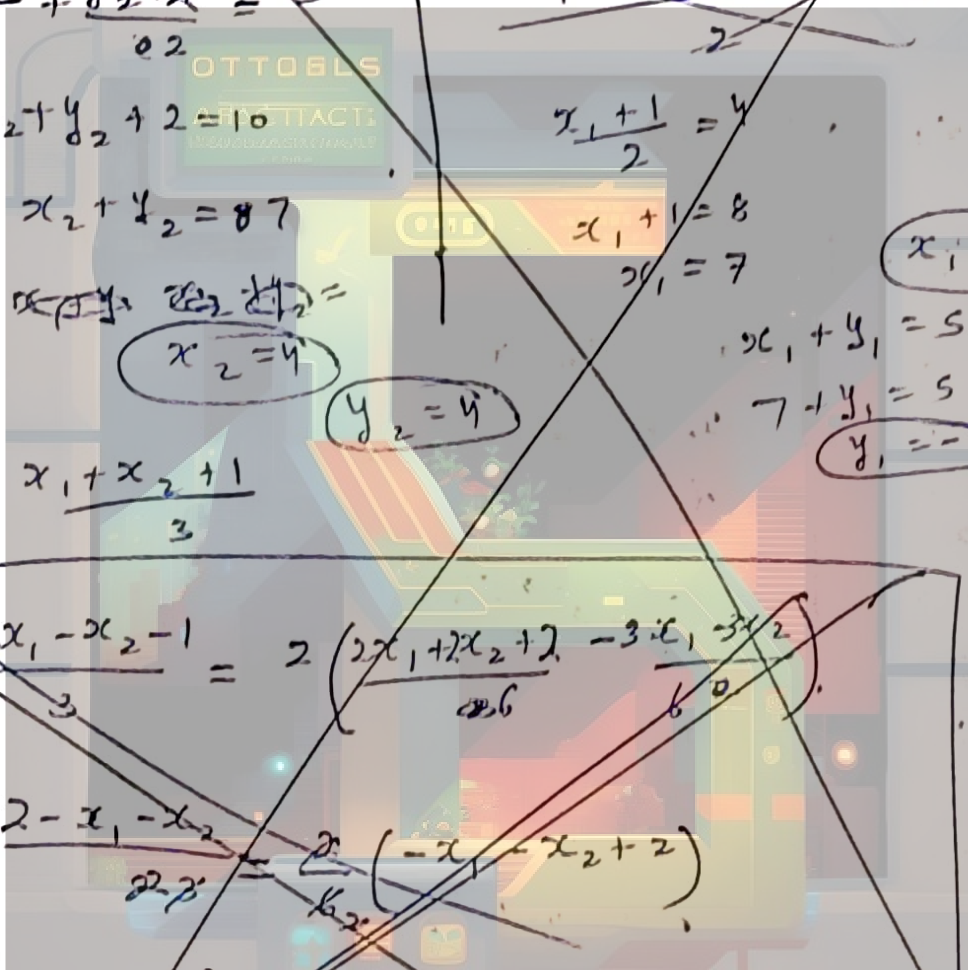
$$\boxed{x_1 = 7}$$

$$x_1 + y_1 = 5$$

$$7 + y_1 = 5$$

$$\boxed{y_1 = -2}$$

$$G_x = \frac{x_1 + x_2 + 1}{3}$$



$$\frac{3 - x_1 - x_2 - 1}{3} = 2\left(\frac{2x_1 + 2x_2 + 2 - 3x_1 - 3x_2}{6}\right)$$

$$\frac{2 - x_1 - x_2}{3} = \frac{2}{6}\left(-x_1 - x_2 + 2\right)$$

$$2 = 2$$

$$\frac{3x_1 - x_1 - x_2 - 1}{3} = 2\left[\frac{2x_1 + 2x_2 + 2 - 3x_2 - 3}{6}\right]$$

$$2x_1 - x_2 - 1 = 2x_1 - x_2 - 1$$

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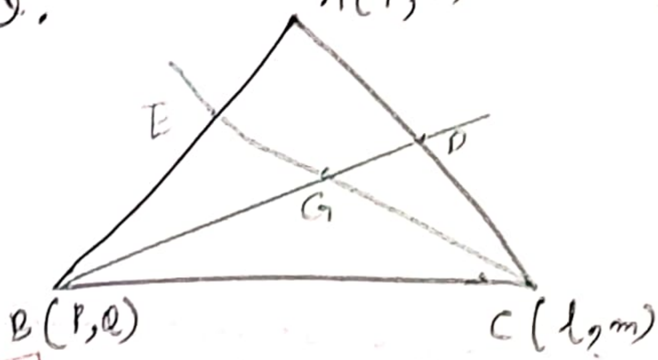
Q. BD & CE are medians satisfying eq.

$x+y=5$

$x=4$

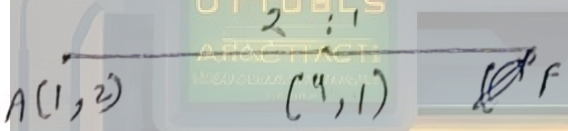
find B & C.



M I

by solving Medians, $G(4,1)$

So, F point is:



$F \equiv \left(\frac{11}{2}, \frac{1}{2}\right)$

$\frac{l+P}{2} = \frac{11}{2}$ | $Q+m=1$

$l+P=11$

~~$P+Q=5$~~

$l=4$ | $P+Q=5$ | ~~$l+m=$~~

$P=7$ | $Q=-2$ | $m=3$

$B=(7,-2)$

$C=(4,3)$

M II

$\frac{P+1}{2}=4$ | $l=4$ | $\frac{l+1}{2}+\frac{m+2}{2}=5$

$P+1=8$ | | $l+m=7$

$P=7$ | | $m=3$

$P+Q=5$

$Q=-2$

H.W.
PYS - 2, 3 (Co-ordinate)
O-1 (Q1, 2, 3, 4, 5)

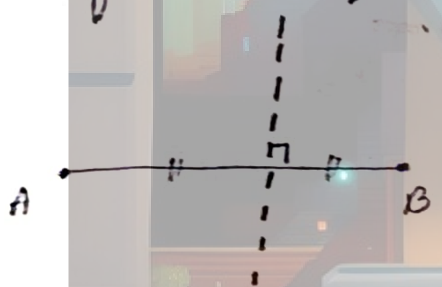
Locus

→ Path traced by a point P(H, K) according to the given condition

eg. locus of a point which moves at a ~~equal~~ same distance from another point will be ~~circle~~. circumference of circle.

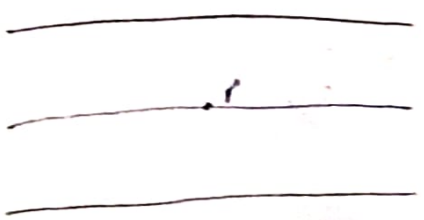
Locus → circumference of a circle

eg2. Locus of a point which moves at a equal distance from two fixed points



Locus:- Perpendicular Bisector of AB.

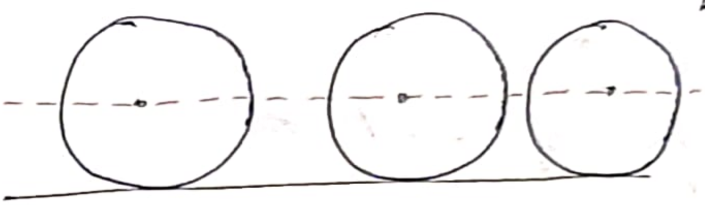
eg3. Locus of a point which moves at a fixed distance from 2 Parallel lines.



Locus :- Parallel lines between given two parallel lines.

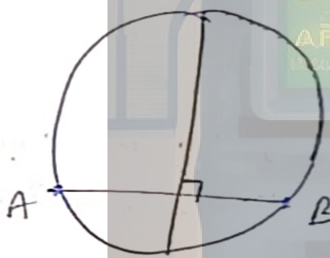
eg4. locus of the center of a circle which moves on a flat surface.

Locus:- A straight line passing through ~~surface~~ centers parallel to flat surface.



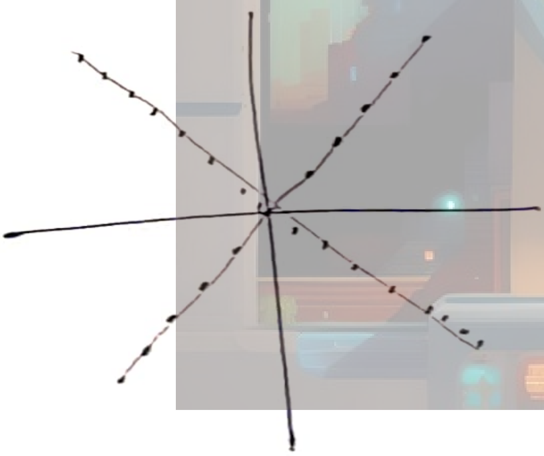
eg5. locus of a point which is inside a circle and equidistant from two fixed points on circumference.

Locus:- Diameter ~~[illegible]~~ Perpendicular (⊥) to line joining the given two points.



eg6. Locus of a point which is at equal distance from two intersecting lines

Locus:- Angle Bisector of both given lines.



~~Method to E~~

Method to Find Locus (Locus Equation)

→ Assume the point $P(h, k)$

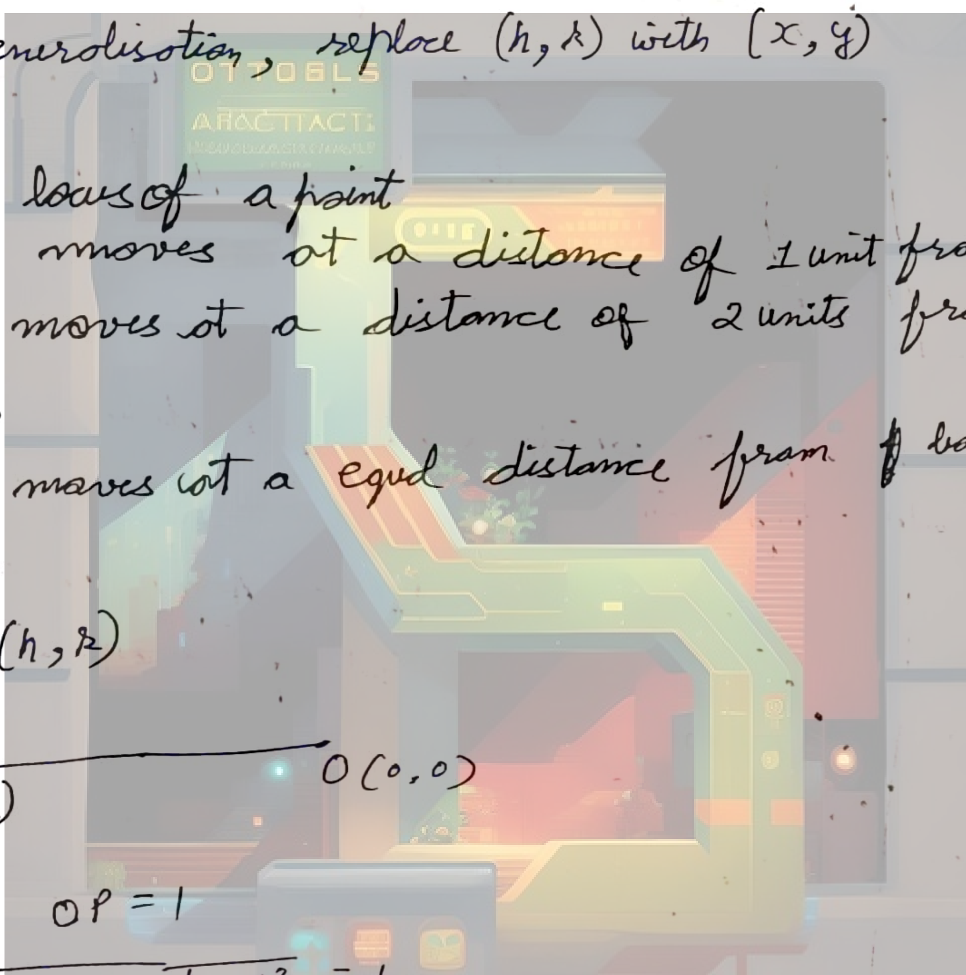
→ Apply given condition of question.

→ Find the equation in the form of h & k

or

find relation in h & k

→ For generalisation, replace (h, k) with (x, y)



Q. Find the locus of a point

① which moves at a distance of 1 unit from origin

② which moves at a distance of 2 units from the y axis.

③ which moves at a equal distance from both the axes.

① Let $P(h, k)$

$P(h,k)$ —— $O(0,0)$

$OP = 1$

$$\sqrt{(h-0)^2 + (k-0)^2} = 1$$

$$h^2 + k^2 = 1$$

$$\boxed{x^2 + y^2 = 1}$$

(2) on y axis, $x = 0$
$\hookrightarrow \cancel{(x,0)} (0, y)$

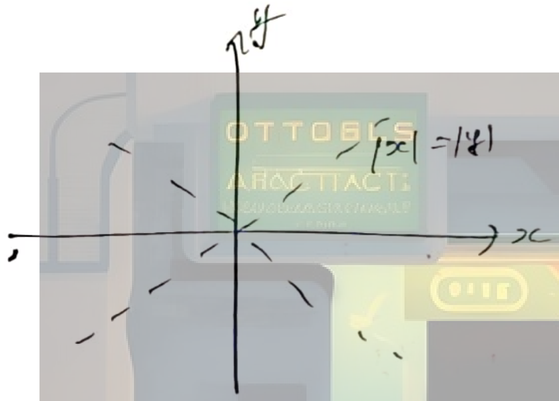
$P(h, k)$

$dis = 2$

$h = |2|$

$\cancel{\boxed{x = 2, x = -2}}$ $\boxed{x = 2,\ x = -2}$

(3)



$P(h, k)$

distance from x-axis = distance from y-axis.

$$|k| = |h|$$

$$\boxed{|x| - |y| = 0}$$

~~(4) which moves such that~~

(4) Distance from the x-axis is 3 times the distance from y axis.

(5) Sum of squares of its distances from the axes $= 3$.

(6) its distance from the point $(3,0)$ is 3 times the distance from the point $(0,2)$.

(4) $3|y| = 3|x|$

$3|x| = |y|$

$|3x| = |y|$

$3x = \pm y$

$\boxed{3x \pm y = 0}$

(5) $|x|^2 + |y|^2 = 3$

$\boxed{x^2 + y^2 = 3}$

(6) $P(x, y)$

$$\sqrt{(x-3)^2 + (y-0)^2} = 3\sqrt{(x-0)^2 + (y-2)^2}$$

$$x^2 + 9 - 6x + y^2 = 9(x^2 + y^2 + 4 - 4y)$$

$$x^2 + y^2 - 6x + 9 = 9x^2 + 9y^2 + 36 - 36y$$

$$\boxed{8x^2 + 8y^2 + 6x - 36y + 27 = 0}$$

~~(7) find the~~

Q find the locus of centroid of a triangle whose vertices are $(a\cos t, a\sin t)$ $(b\sin t, -b\cos t)$ & $(1, 0)$

$$h = \frac{a\cos t + b\sin t + 1}{3} \qquad k = \frac{a\sin t - b\cos t}{3}$$

$$3(h+k) = a\cos t + a\sin t + b\sin t - b\cos t + 1$$

$$h + k = \frac{a(\sin t + \cos t) + b(\sin t - \cos t) + 1}{3}$$

$$x = \frac{a(\sin t + \cos t) + b(\sin t - \cos t) + 1 - 3y}{3}$$

$3h-1 = a\cos t + b\sin t$ $\quad$ $3k = a\sin t - b\cos t$

Square & add

$$9h^2+1-6h+9k^2 = a^2\cos^2 t + b^2\sin^2 t + a^2\sin^2 t + b^2\cos^2 t$$
$$+ 2ab\sin t\cos t - 2ab\sin t\cos t$$

$$\boxed{9h^2+9k^2-6h+1 = a^2+b^2}$$

~~$9h^2 = a^2+b^2+$~~

H.W. 17-9-24

DYS - 4

Slope/Gradient of a line

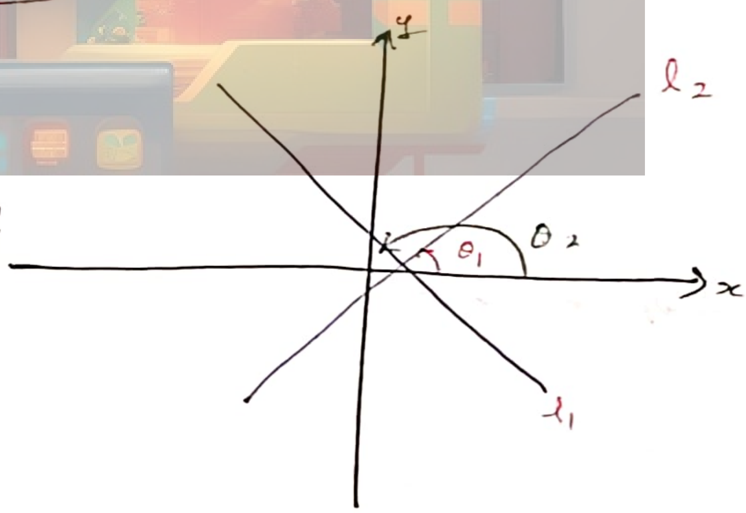
→ Angle made by any line with ⊕ve side of x-axis in anti-clockwise direction is θ, then $\tan\theta$ is called slope.

→ It is denoted by 'm'.

$m(L_1) = \tan\theta_2$

$m(L_2) = \tan\theta_1$

$\theta \in [0, 180°) - \{90°\}$



Q find the slope of the line

① which makes 60° angle from ⊕ve x-axis in anticlockwise direction

② which is ∥ to x-axis

① $\sqrt{3}$

② 0

③ ∥ to y-axis

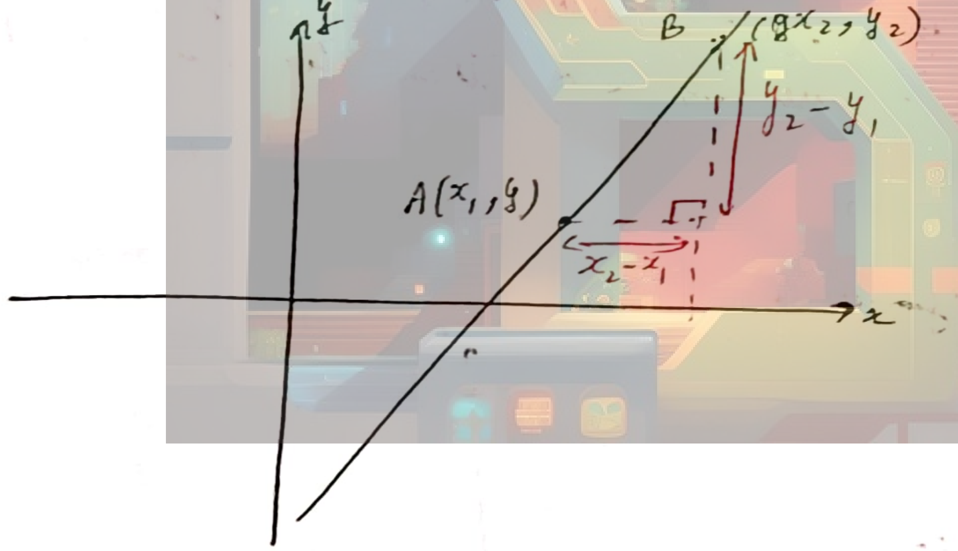
③ not Defined

→ When two lines having slope m_1 & m_2 are given then

parallel → $m_1 = m_2$

Perpendicular → $m_1 m_2 = -1$

Slope of a line when any 2 points on it are given.



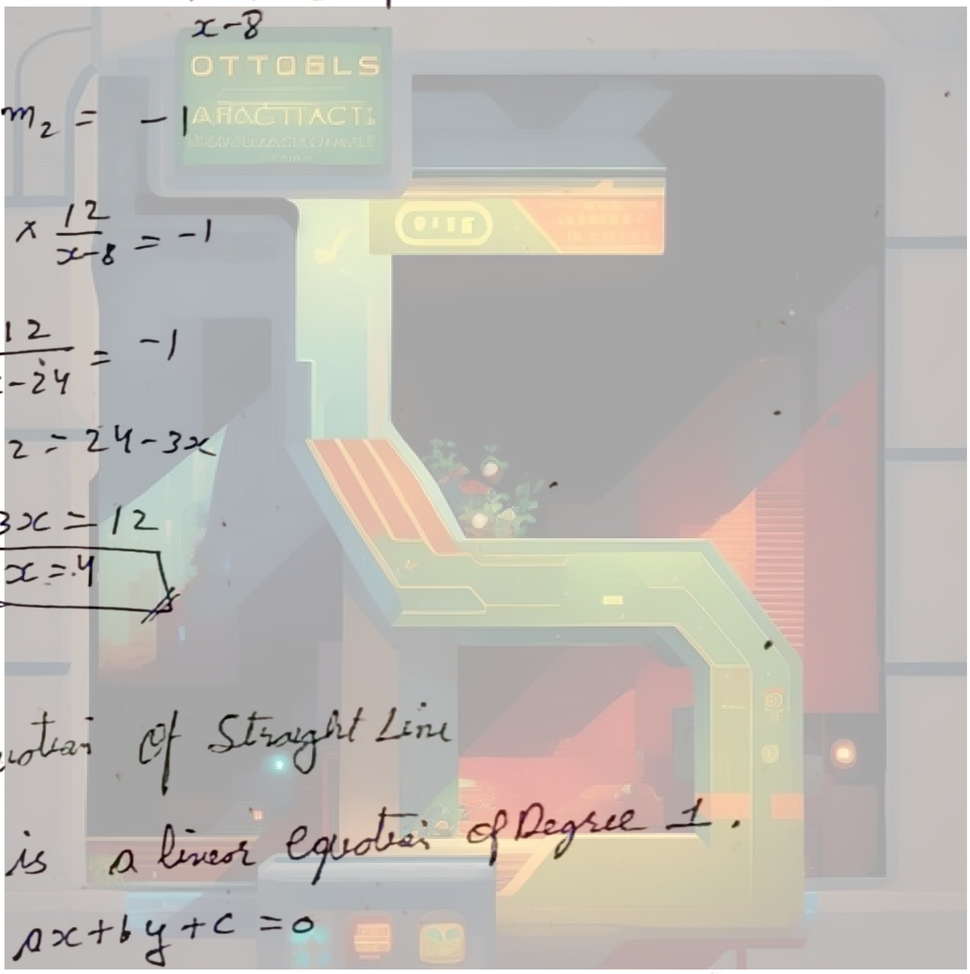
Slope $= m = \tan\theta$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Q. If the line passing through ~~(A-2, 5) & (B,~~ $A(-2,6)$ & $B(4,8)$ is $\perp$ to the line passes through $P(8,12)$ & $Q(x,24)$ find x.

$$m_2(AB) = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3} = m_2$$

$$m_1(PQ) = \frac{12}{x-8} = m_1$$



$$m_1 m_2 = -1$$

$$\frac{1}{3} \times \frac{12}{x-8} = -1$$

$$\frac{12}{3x-24} = -1$$

$$12 = 24 - 3x$$

$$3x = 12$$

$$\boxed{x = 4}$$

Equation of Straight Line

→ It is a linear equation of Degree 1.

$ax + by + c = 0$

eg. $2x - 3y + 7 = 0$

$x + 2y = 0$

$x = 0$; $x = -8$; $y = 3$

→ Equation of y axis ⇒ $x = 0$

Eqn of x-axis ⇒ $y = 0$

Eqn of line ∥ to x-axis at distance 'a' ⇒ $y = \pm a$

Eqn of line ∥ to y-axis at distance 'b' ⇒ $x = \pm b$

Q1. find the equation of line ~~..~~ parallel to x-axis & passes through (1, 2)

Q2. perpendicular to x-axis & passes through (-3, 4)

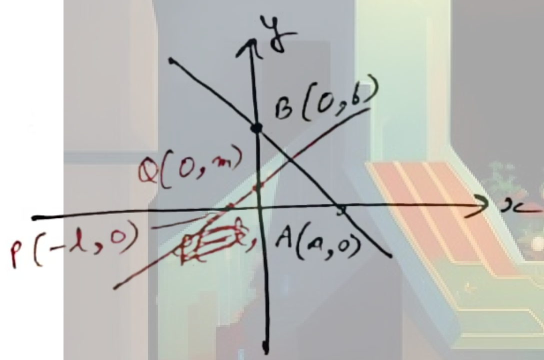
Q1. $y = 2$

Q2. $x = -3$

Intercept

→ It is a point where the given line cuts the x-axis or y-axis.

→ It can be ⊖ve & ⊕ve or 0.



x-intercept = a
y-intercept = b
] AB

x-intercept = -l
y-intercept = m
] PQ

Note:- all lines ∥ to x-axis will have x-intercept = not-Defined
all lines ∥ to y-axis have y-intercept = not-Defined

→ There are ∞ no. of lines with both x & y intercept = 0 (Pass through origin).

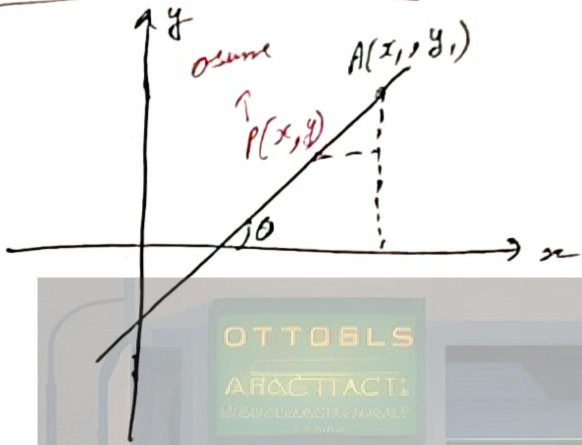
standard forms of straight lines.

→ we need any two things for writing the eqⁿ of a line from slope, point, intercept.

① Point - Slope form

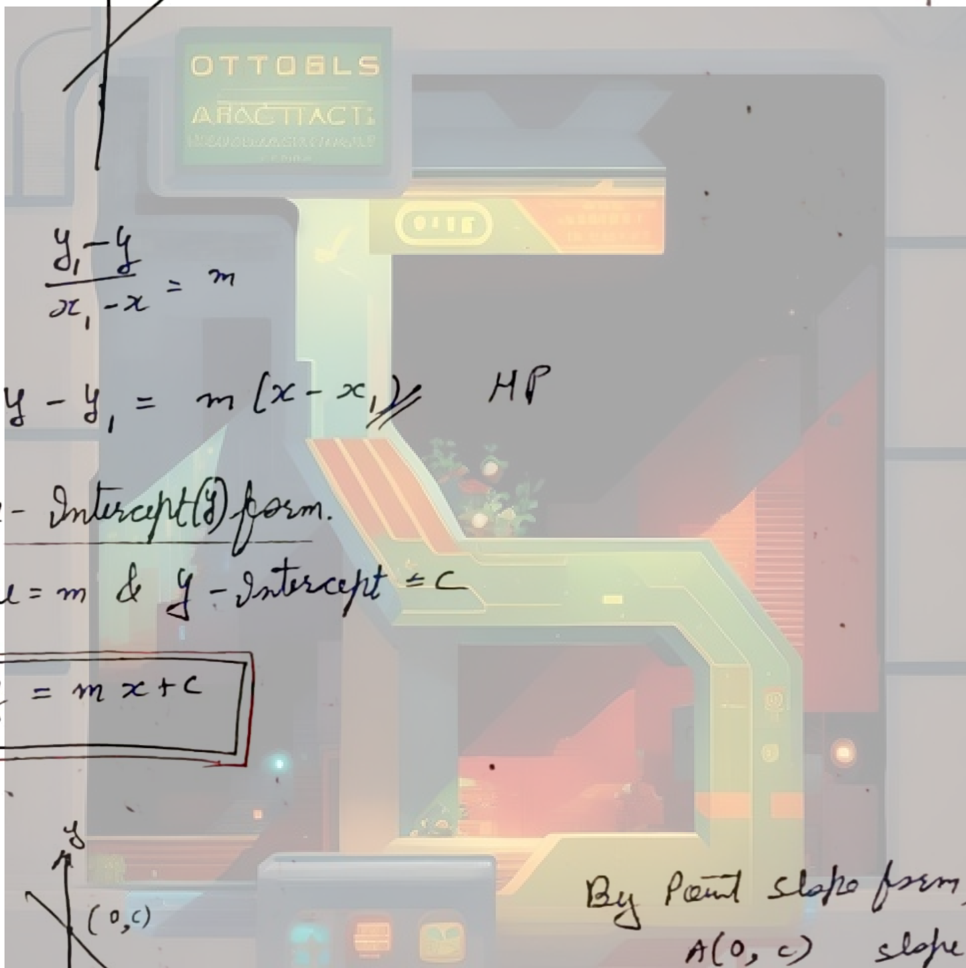
$A(x_1, y_1)$ & Slope $= m$

$$\boxed{y - y_1 = m(x - x_1)}$$



$$\frac{y_1 - y}{x_1 - x} = m$$

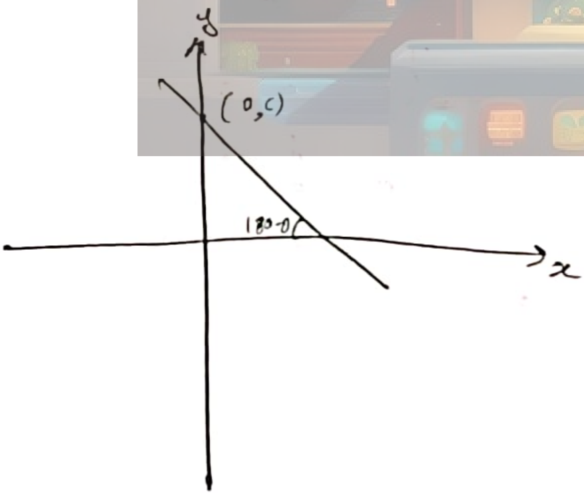
$$y - y_1 = m(x - x_1) \quad \text{MP}$$



② Slope - Intercept(y) form.

Slope $= m$ & y-intercept $= c$

$$\boxed{y = mx + c}$$



By Point slope form,

$A(0, c)$ slope $= m$

$$y - c = m(x)$$

$$y = mx + c \quad \text{MP}$$

(3) 2 Point form

$A(x_1, y_1)$ & $B(x_2, y_2)$

$$\boxed{y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)}$$

↳ slope

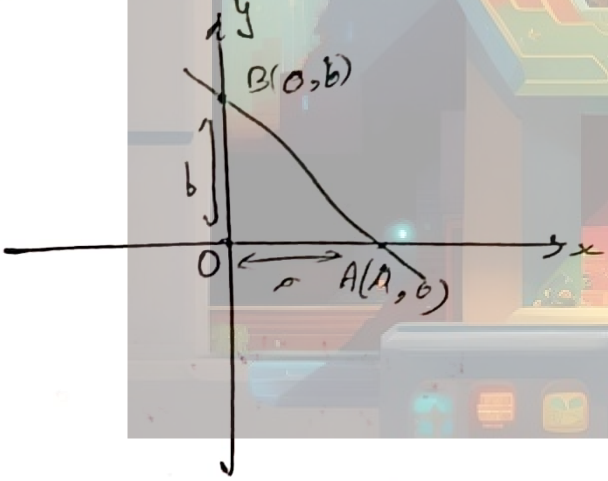
(4) Intercept Form

↳ x & y Intercept given

x - intercept $= a$
y - intercept $= b$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

Proof:-



two point form

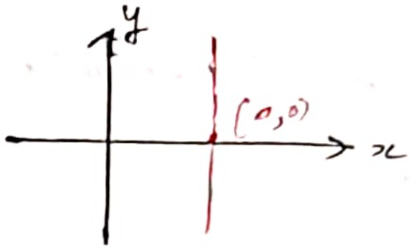
$$y - 0 = \left(\frac{b}{-a}\right)(x - a)$$

$$y = (-b)\left(\frac{x}{a} - 1\right)$$

$$\frac{y}{-b} = \frac{x}{a} - 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Note:- area of $\Delta AOB = \sqrt{a^2 + b^2}$

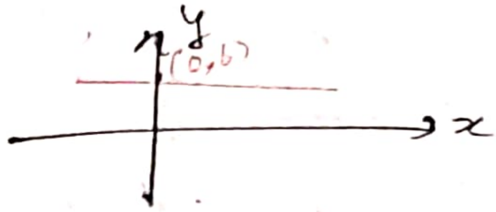


$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{a}+\frac{y}{\infty}=1$$

$$\frac{x}{a}=1$$

$x=a$ HP



$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{\infty}+\frac{y}{b}=1$$

$y=b$ HP

H.W. 19-9-24

DYS-5 [1, 17]

(5) Normal Form (Made by Intercept form)

→ when length of ~~...~~ ⊥ from origin to the line is given & angle made by ⊥ with x axis is given.

$\boxed{x\cos\alpha + y\sin\alpha = p}$ $p \to$ ⊕ve always

Proof: $\frac{x}{OA}+\frac{y}{OB}=1$ (Intercept form)

$OA=\frac{p}{\cos\alpha}$ $OB=\frac{p}{\sin\alpha}$

$$\frac{x\cos\alpha + y\sin\alpha}{p}=1$$

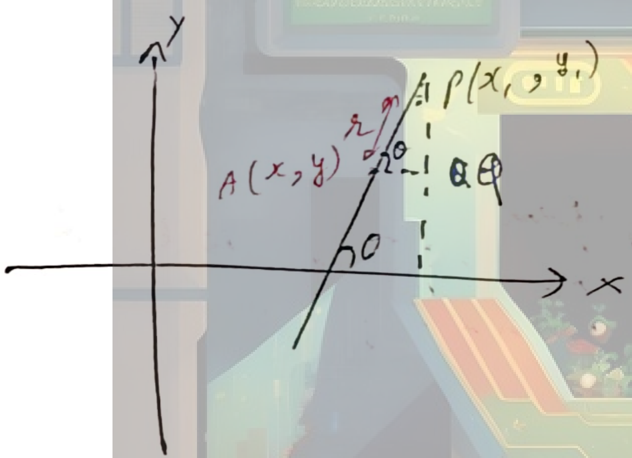
$x\cos\alpha + y\sin\alpha = p$ Hence Proved

⑥ Parametric form (Made by Point Slope)

→ Angle made by line with x-axis (+ve, anticlockwise) & a point $P(x_1, y_1)$ is given.

→ This form is mostly used for finding the co-ordinates of a point which are at some distance from another point.

$$\boxed{\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \pm r}$$



~~Proof~~:- Proof:-

$PQ = y_1 - y$

$AQ = x_1 - x$

$\cos\theta = \frac{x_1 - x}{r}$

$r = \frac{x_1 - x}{\cos\theta}$ H.P.

(7) General Form

$ax + by + c = 0$ $\{a \& b \text{ both} \neq 0\}$

$$\boxed{\text{Slope} = -\frac{a}{b}}$$

Proof:-

$ax + by + c = 0$

$by = -ax - c$

$y = \underbrace{\left(-\frac{a}{b}\right)}_{m} x - \frac{c}{b}$

$m = -\frac{a}{b}$

Q1. find equation of the line.

(1) which passes through origin & makes angle 45° with +ve x axis in anticlock-wise direction.

(2) which makes an angle 60° with x-axis & y intercept of 5 in negative side.

(3) which makes intercept 2 & -3 on X & Y axes respectively

(4) which passes through 2 points A(1,2) & B(3,5)

(5) Passes through (0,0) with slope m.

(6) Intersecting the x-axis at distance of 3 units to the left of the origin with slope = -2.

(7) Passes through 2 points $(\sin\alpha, \cos\alpha)$ & $(\sin\beta, \cos\beta)$

① $m = \tan 45^\circ = 1$

$(0, 0)$

$(y - 0) = 1(x - 0)$

$y = x$

$\boxed{x = y}$

② $m = \tan 60 = \sqrt{3}$

$(0, -5)$

$y = mx + c$

$y = \sqrt{3}x - 5$

$\boxed{\sqrt{3}x - y - 5 = 0}$

③ $\frac{x}{2} + \frac{y}{-3} = 1$

$\frac{x}{2} = \frac{3 + y}{3}$

$3x = 6 + 2y$

$\boxed{3x - 2y + 6 = 0}$

④ $(y - 2) = (x - 1)\left(\frac{3}{2}\right)$

$2y - 4 = 3x - 3$

$\boxed{3x - 2y + 1 = 0}$

⑤ $m = m$

$(0, 0)$

$(y - 0) = m(x - 0)$

$\boxed{y = mx}$

⑥ $(-3, 0)$

$m = -2$

$(y - 0) = (x + 3)(-2)$

$-2x - 6 = y$

$\cancel{2x + y + 6 = 0}$

$\boxed{2x + y + 6 = 0}$

⑦ $m = \frac{\cos\alpha - \cos\beta}{\sin\alpha - \sin\beta}$

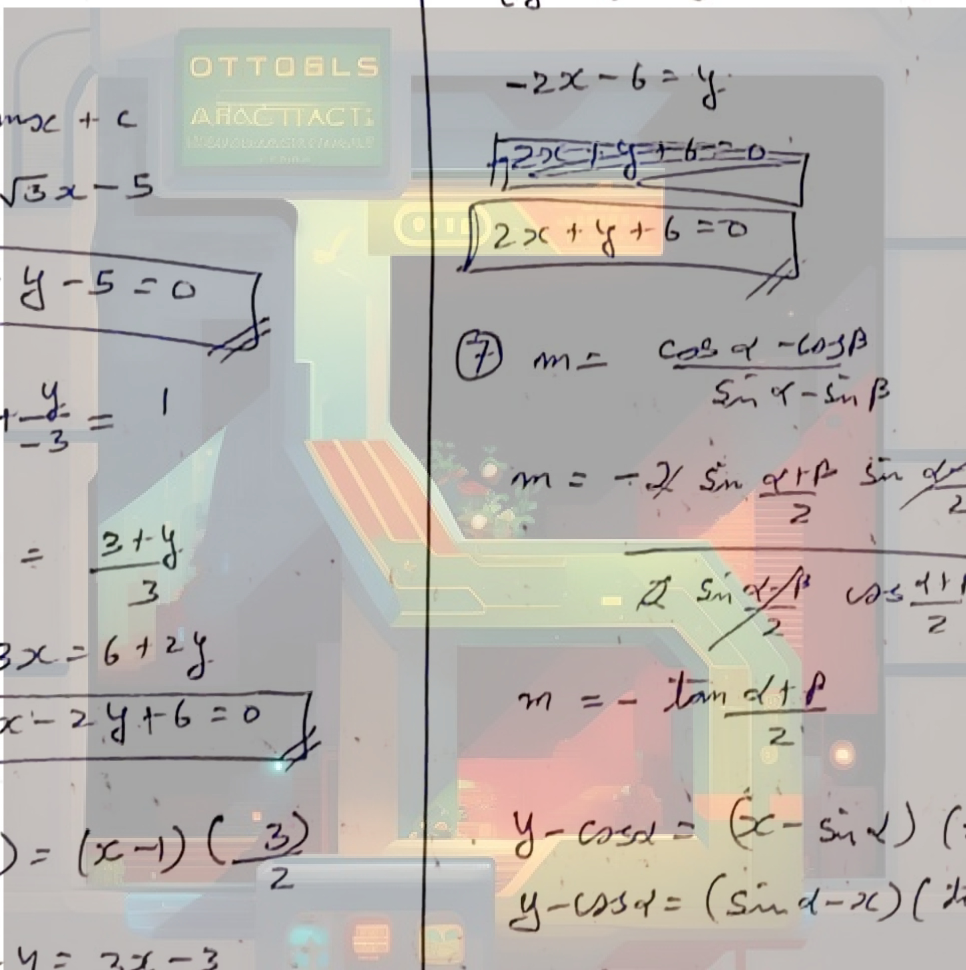
$m = \frac{-\cancel{2} \sin\frac{\alpha+\beta}{2} \cancel{\sin\frac{\alpha-\beta}{2}}}{\cancel{2} \cancel{\sin\frac{\alpha-\beta}{2}} \cos\frac{\alpha+\beta}{2}}$

$m = -\tan\frac{\alpha+\beta}{2}$

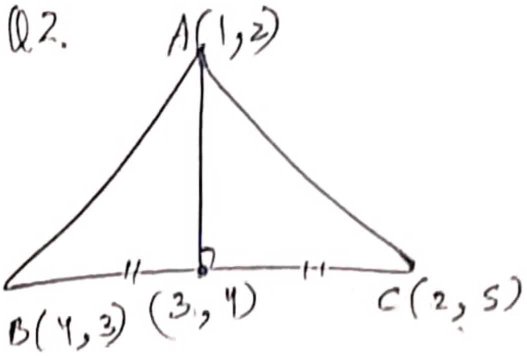
$y - \cos\alpha = (x - \sin\alpha)\left(-\tan\frac{\alpha+\beta}{2}\right)$

$y - \cos\alpha = (\sin\alpha - x)\left(\tan\frac{\alpha+\beta}{2}\right)$

$y + \tan\left(\frac{\alpha+\beta}{2}\right)x - \cos\alpha - \sin\alpha \tan\left(\frac{\alpha+\beta}{2}\right)$



Q2.



find equations.

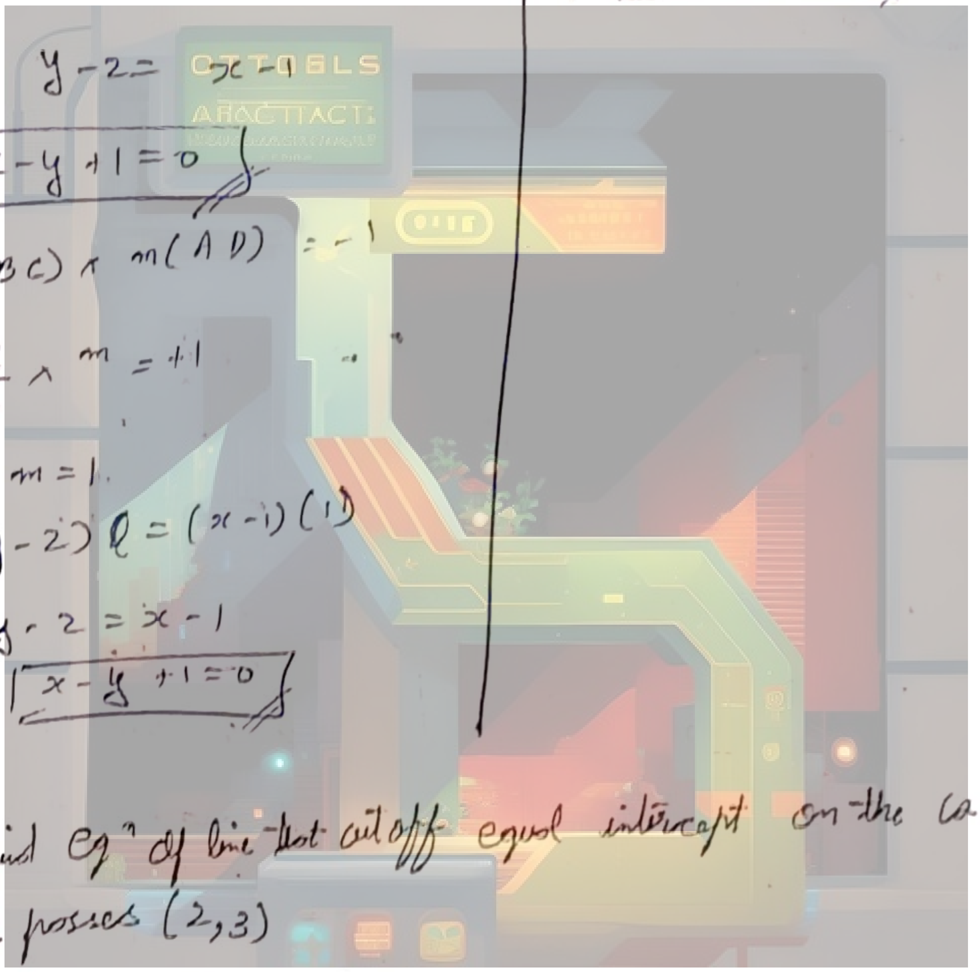
a) Median from A

b) Altitude from A

c) $\perp$ side bisector from A.

A)

$$(y-2) = (x-1)\left(\frac{2}{2}\right)$$



$$y-2 = x-1$$

$$\boxed{x-y+1=0}$$

b) $m(BC) \times m(AD) = -1$

$$\frac{2}{-2} \times m = -1$$

$$m = 1.$$

$$(y-2) = (x-1)(1)$$

$$y-2 = x-1$$

$$\boxed{x-y+1=0}$$

c) $m = 1$ (B/f $\perp$)

$$\boxed{x-y+1=0}$$

Q3. find eq^n of line that cut off equal intercept on the co-ordinate axes & passes (2,3)

$$(y-3) = (x-2)$$

$$y-3 = (x-2)(-1)$$

$$\boxed{\cancel{x-y+1=0}}$$

$$y-3 = 2-x$$

$$\boxed{x+y-5=0}$$

H. W. ~~⊕~~ 20-09-24

DYS-5 [108, ∞)

~~DYS-5~~

Broadcast

Q4. Find the equation of the line which passes through (3, 4) & have intercepts on axes.

1) equal in magnitude but opp in sign

2) such that their sum = 14

1) $m = 1$

$y - 4 = x - 3$

$\boxed{x - y + 1 = 0}$

2) $(0, x)\ (x-14, 0)\ (3, 4)$

$\frac{4-x}{3} = \frac{4}{-x-11}$

$-4x - 44 + x^2 + 11x = 12$

$x^2 + 7x - 56 = 0 \quad (x = h)$

$h = \frac{-7 \pm \sqrt{49+224}}{2}$

$h = \frac{-7 \pm 9\sqrt{3}}{2}$

$m = \frac{4-h}{3}$

$(y-4) = (x-3)\left(\frac{4-h}{3}\right)$

$(y-4) = (x-3)\left(\frac{8+7 \mp 9\sqrt{3}}{2}\right)$

2) $\frac{x}{a} + \frac{y}{b} = 1$

$a + b = 14$

$\frac{x}{a} + \frac{y}{14-a} = 1$

$\frac{3}{a} + \frac{4}{14-a} = 1$

$42 - 3a + 4a = 14a - a^2$

$a^2 - 13a + 42 = 0$

$a = 7, 6$

$\boxed{\frac{x}{6} + \frac{x}{8} = 1}$

(224)

Q 4. find line eq. when length of $\perp$ from origin is 4 & its inclination from x - axis is 30°.

$\underline{x \cos\alpha} + y \sin\alpha = p$

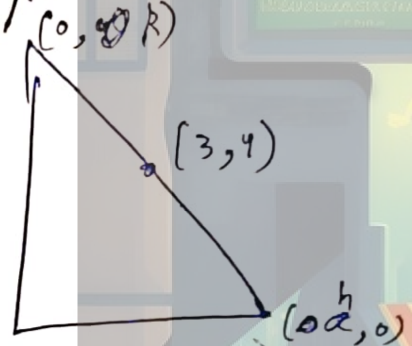
$\cancel{\frac{x}{2} + \frac{y\sqrt{3}}{2} = 4}$

$\cancel{\boxed{x + \sqrt{3}y = 8}}$

$\frac{\sqrt{3}x}{2} + \frac{y}{2} = 4$

$\boxed{\sqrt{3}x + y = 8}$

Q.5. find the equation of the line which passes through A(3, 4) & point A bisects the line segment b/w both axes.



$\cancel{\frac{x+0}{2} = 3}$

$\frac{h+0}{2} = 3$

$h = 6$

$k + 0 = 8$

$k = 8$

$(3, 4)\ (6, 0)\ \&\ (0, 8)$

$m = \frac{4}{-3} = \frac{-4}{3}$

$(y - 4) = (x - 3)\frac{-4}{3}$

$3y - 12 = -4x + 12$

$\boxed{4x + 3y - 24 = 0}$

Conversion of line equation

Q1. Convert the eqⁿ ~~of~~ of the line $x + \sqrt{3}y = 2$ in
① Slope intercept form & find Y intercept
② Intercept form & find X & Y intercept
③ Normal form & find length ⊥ drawn ~~on~~ it from origin.

①
$$x + \sqrt{3}y = 2$$
$$\frac{x}{\sqrt{3}} + y = \frac{2}{\sqrt{3}}$$
$$y = \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}x$$
$$y = \left(-\frac{1}{\sqrt{3}}\right)x + \frac{2}{\sqrt{3}}$$
$$\boxed{y\text{-intercept} = \frac{2}{\sqrt{3}}}$$

②
$$\frac{x}{a} + \frac{y}{b} = 1$$
$$x + \sqrt{3}y = 2$$
$$\frac{x}{2} + \frac{y}{2/\sqrt{3}} = 1$$
$$\boxed{\begin{array}{l} a = 2 \ (\text{x intercept}) \\ b = 2/\sqrt{3} \ (\text{Y intercept}) \end{array}}$$

③
$$\frac{x}{2} + \frac{\sqrt{3}}{2}y = 1 \quad \left[\text{multiply \& Divide by } \sqrt{a^2+b^2}\right.$$
$$\cos 60 = 1/2$$
$$\cancel{\cos 30}\ \sin 60 = \frac{\sqrt{3}}{2}$$
$$x\cos 60 + y\sin 60 = 1$$
$$\boxed{p = 1} \qquad \alpha = 60^\circ$$

Q2. Same as previous in eq $x+y=-\sqrt{2}$.

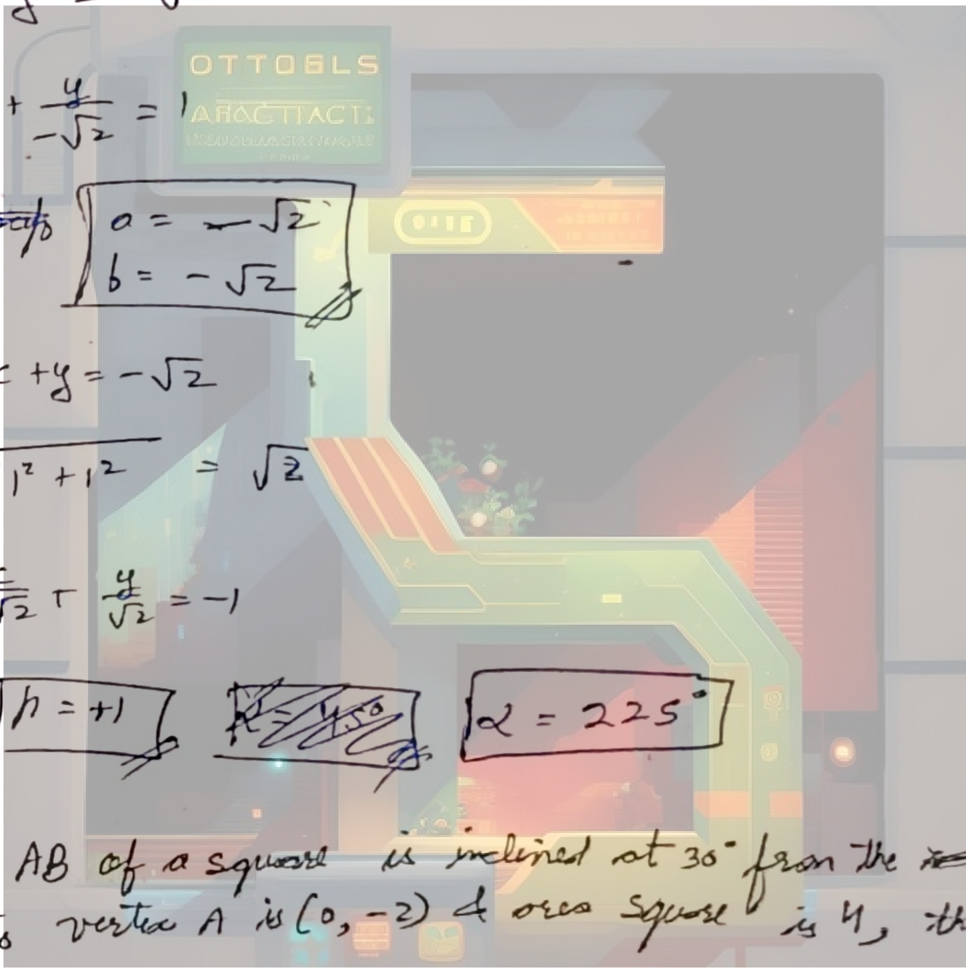
(1) $x+y=-\sqrt{2}$

$y = -x-\sqrt{2}$

$y=(-1)x+(-\sqrt{2})$

$\boxed{y \text{ intercept} = (-\sqrt{2})}$

(2) $x+y=-\sqrt{2}$



$\frac{x}{-\sqrt{2}}+\frac{y}{-\sqrt{2}}=1$

~~A intercepts~~ $\boxed{\begin{array}{l} a=-\sqrt{2} \\ b=-\sqrt{2} \end{array}}$

(3) $x+y=-\sqrt{2}$

$\sqrt{1^2+1^2}=\sqrt{2}$

$\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=-1$

$\boxed{p=+1}$ $\boxed{\cancel{\alpha=45^\circ}}$ $\boxed{\alpha=225^\circ}$

Q3. Side AB of a square is inclined at 30° from the x-axis. If its vertex A is (0, −2) & area square is 4, then find vertex B.

~~$m(AB)=1/\sqrt{3}$~~

eq. $y+2=\frac{x}{\sqrt{3}}$

$x=\sqrt{3}y+2\sqrt{3}$

$\boxed{\cancel{B(2\sqrt{3},0)}}$

B $(\sqrt{3}+2\sqrt{3}, 1)$

$B(\sqrt{3}h+2\sqrt{3}, h)$

$AB=2$

$3h^2+12+h^2+4h=4$

$4h^2+4h+8=0$

$(\sqrt{3}+2)^2h^2+(h+2)^2=4$

$(7+4\sqrt{3})h^2+h^2+4+4h=4$

$(8+4\sqrt{3})h^2+4h=0$

$h=0$, same

$h^2+h-2=0$

$h=\frac{-1\pm\sqrt{1+8}}{2}$

$h=-2, 1$

$\frac{x - x_1}{\cos\theta} = \pm r$ $B(x, y)$

$\frac{2(x-0)}{\sqrt{3}} = \pm 2$

$x = \frac{\pm 2\sqrt{3}}{2}$

$x = \pm\sqrt{3}$

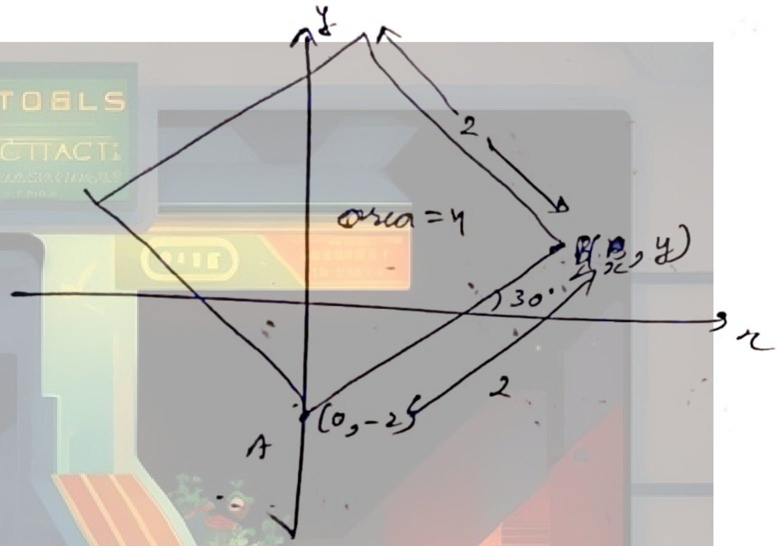
$2(y+2) = \pm 2$

$y + 2 = \pm 1$

$y = \pm 1 - 2$

$B(\sqrt{3}, -1)$

$B(-\sqrt{3}, -3)$



Equation of line ∥ or ⊥ to given line

$ax + by + c = 0$

∥ line

$\boxed{ax + by + c_1 = 0}$

$m_1 = \frac{-a}{b}$

⊥ line

$\boxed{bx - ay + c_2 = 0}$

$m_2 = \frac{b}{a}$

→ will find C_1 & C_2 by other given condition

Q find the $\parallel$ line which passes through $(1, 2)$ to line

$$2x + 3y - 7 = 0.$$

$$2x + 3y + c_1 = 0$$

$$2 + 6 + c_1 = 0$$

$$c_1 = -8$$

$$\boxed{2x + 3y - 8 = 0}$$

Q Find the line $\perp$ to given ~~the~~ line $4x - 3y = 0$ which passes through $(2,3)$

$$-3x - 4y + c_1 = 0$$

$$3x + 4y - c_1 = 0$$

$$6 + 12 - c_1 = 0$$

$$c_1 = 18$$

$$\boxed{3x + 4y - 18 = 0}$$

H.W. 21-9-234

DYS-6 – {7}

DYS-7 • {2, 3, 4, 5}

O-1 ~~{1-5}~~ {6, 7, 8, 9, 10, 11, 13}

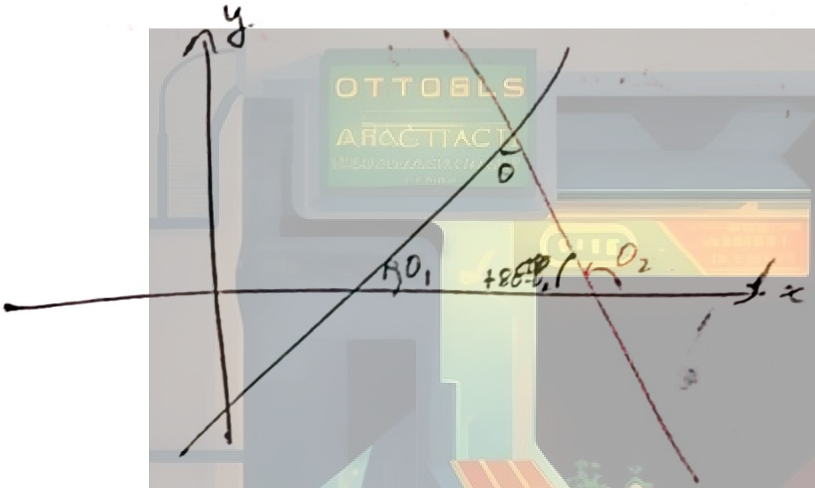
O-2 {1, 2, 4}

Angle btw two lines :

→ Always consider acute Angle

$$\boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|}$$

Proof :-



$\theta + \theta_1 + 180 - \theta_2 = 180$

$\theta = \cancel{\theta_1}\ \theta_2 - \theta_1$

$\tan\theta = \cancel{\tan\theta_2 - \theta\tan\theta_1}\ \tan(\theta_2 - \theta_1)$

$$\tan\theta = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \rightarrow \text{mod so } \cancel{\tan}\ \theta \text{ is } 0° \text{ to } 90°$$

2 lines are ∥

$\theta = 0$

$$0 = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$m_1 = m_2$

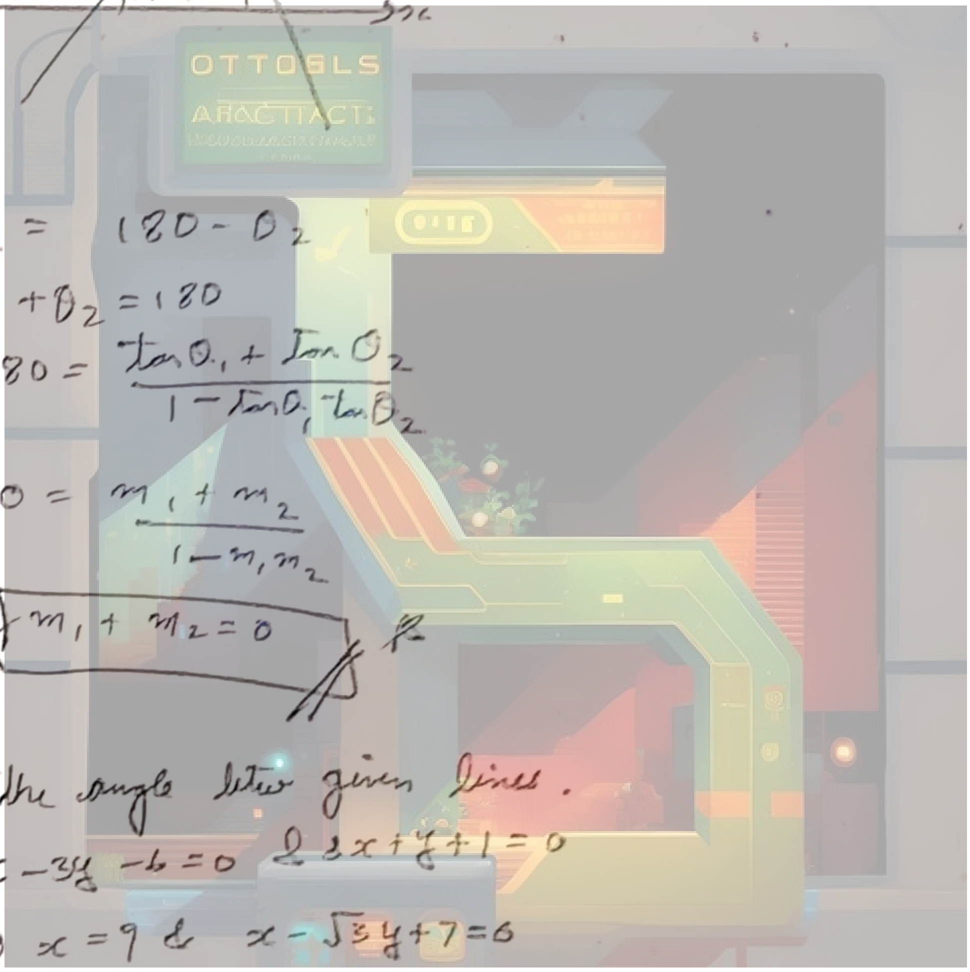
2 lines are ⊥

$\theta = 90$

$$\frac{1}{0} = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$\therefore m_1 m_2 = -1$

Note :- when two lines makes isosceles Δ with x axis & having slopes m_1 & m_2



$\theta_1 = 180 - \theta_2$

$\theta_1 + \theta_2 = 180$

$\tan 180 = \dfrac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$

$0 = \dfrac{m_1 + m_2}{1 - m_1 m_2}$

$\boxed{m_1 + m_2 = 0}$

Q Find the angle between given lines.

(1) $2x - 3y - 6 = 0$ & $3x + y + 1 = 0$

(2) $x = 9$ & $x - \sqrt{3}y + 7 = 0$

(1) $m_1 = \dfrac{-2}{-3} = \dfrac{2}{3}$ $\quad m_2 = \dfrac{-3}{1}$

$\tan\theta = \left| \dfrac{\frac{2}{3} + \frac{9}{3}}{1 - 2} \right|$ $\quad \tan\theta = \dfrac{11}{3}$

$\boxed{\theta = \tan^{-1}\left(\frac{11}{3}\right)}$

(2)

$\tan\theta = \tan(\theta_2 - \theta_1)$

$\tan\theta = \tan(90 - 30)$

$\tan\theta = \tan 60$

$\boxed{\theta = 60^\circ}$

$\tan\theta_1 = \frac{1}{\sqrt{3}}$

$\theta_1 = 30^\circ$

Q2. If $A(-2,1)$, $B(2,3)$ & $C(-2,4)$ are three points, then find angle btw BA & BC.

$m(BA) = \frac{2}{4} = \frac{1}{2}$ $\qquad m(BC) = \frac{1}{-4} = -\frac{1}{4}$

$$\tan\theta = \frac{\frac{1}{2} + \frac{1}{4}}{1 - \frac{1}{8}}$$

$$\tan\theta = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{6}{7}$$

$\boxed{\theta = \tan^{-1}\left(\frac{6}{7}\right)}$

H.W

Q3. Find the angle btw the lines $y = x + 5$ & $y = \sqrt{3}x - 4$.

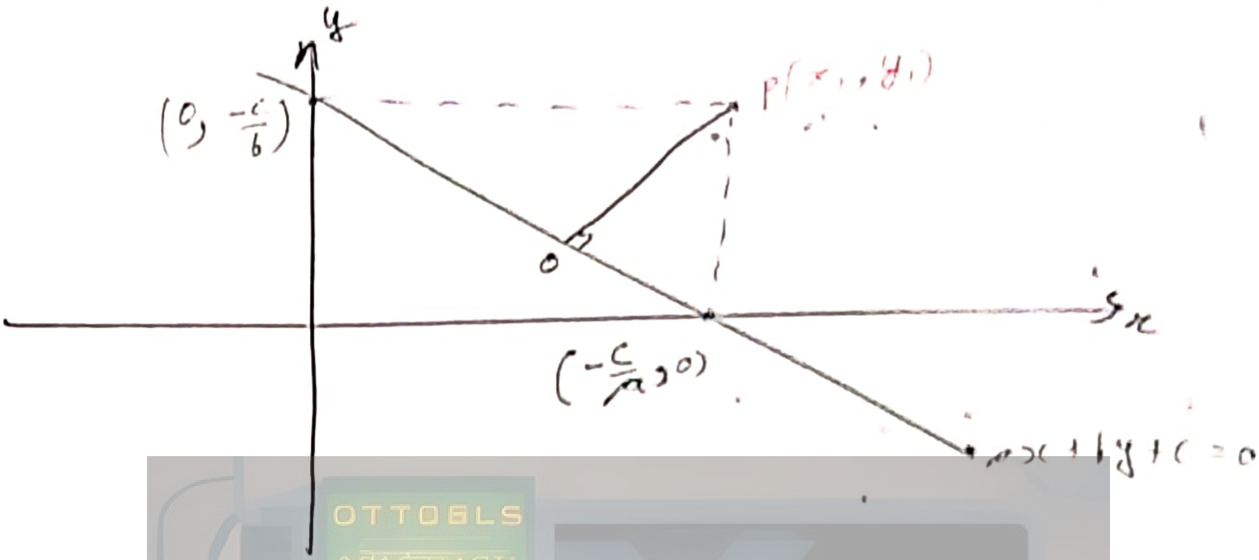
$x - y + 5 = 0 \qquad \sqrt{3}x - y - 4 = 0$

$m_1 = 1 \qquad m_2 = \sqrt{3}$

$$\tan\theta = \left|\frac{1-\sqrt{3}}{1+\sqrt{3}}\right| = \left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| = \tan 15^\circ$$

$\boxed{\theta = 15^\circ}$

$\perp$ length of $P(x_1, y_1)$ from line $ax+by+c=0$



$$\boxed{OP = \left| \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|}$$

Proof:- Area of $\Delta PAB = \frac{1}{2} \times AB \times OP = \frac{1}{2} \times OP \times \sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}}$

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ 0 & -\frac{c}{b} & 1 \\ -\frac{c}{a} & 0 & 1 \end{vmatrix}$$

$$\frac{1}{2} \times c\sqrt{\frac{a^2+b^2}{a^2b^2}} \times OP = \frac{1}{2} \times c \left| \frac{c}{ab} + \frac{x_1}{b} + \frac{y_1}{a} \right|$$

$$\frac{\sqrt{a^2+b^2} \times OP}{ab} = \left| \frac{c + ax_1 + by_1}{ab} \right|$$

$$OP = \left| \frac{ax_1+by_1+c}{\cancel{ab}\sqrt{a^2+b^2}} \right|$$

Q find the length of $\perp$ from point $(3,4)$ on the line

$3x+4y+10=0$

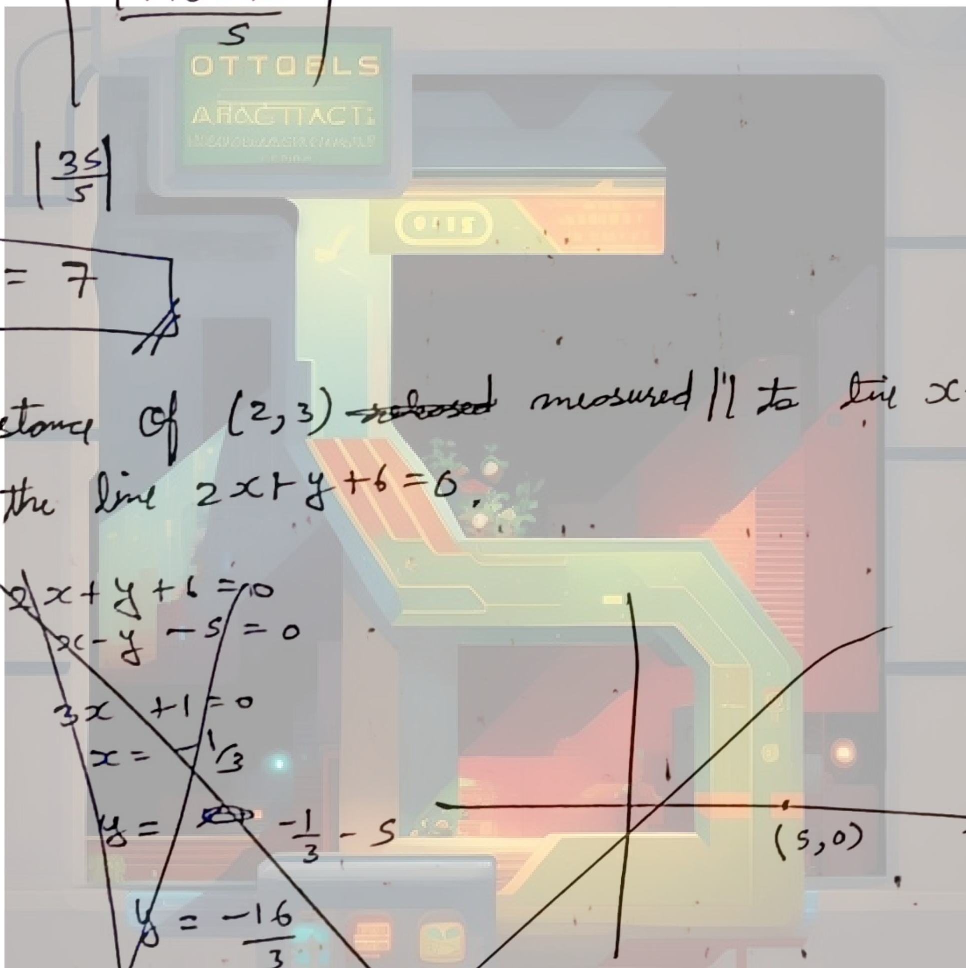
$$\perp = \left| \frac{(3)(3)+(4)(4)+10}{\sqrt{9+16}} \right|$$

$$\perp = \left| \frac{9+16+10}{5} \right|$$

$$\perp = \left| \frac{35}{5} \right|$$

$$\boxed{\perp = 7}$$

Q find distance of $(2,3)$ ~~[illegible]~~ measured $\parallel$ to the line $x-y=5$ from the line $2x+y+6=0$.



$2x+y+6=0$

$x-y-5=0$

$3x+1=0$

$x = -\frac{1}{3}$

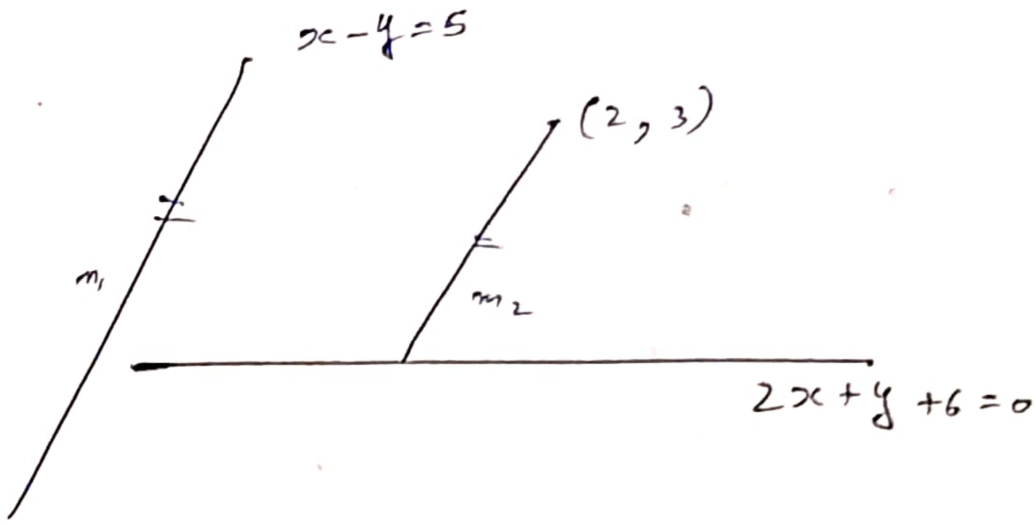
$y = -\frac{1}{3} - 5$

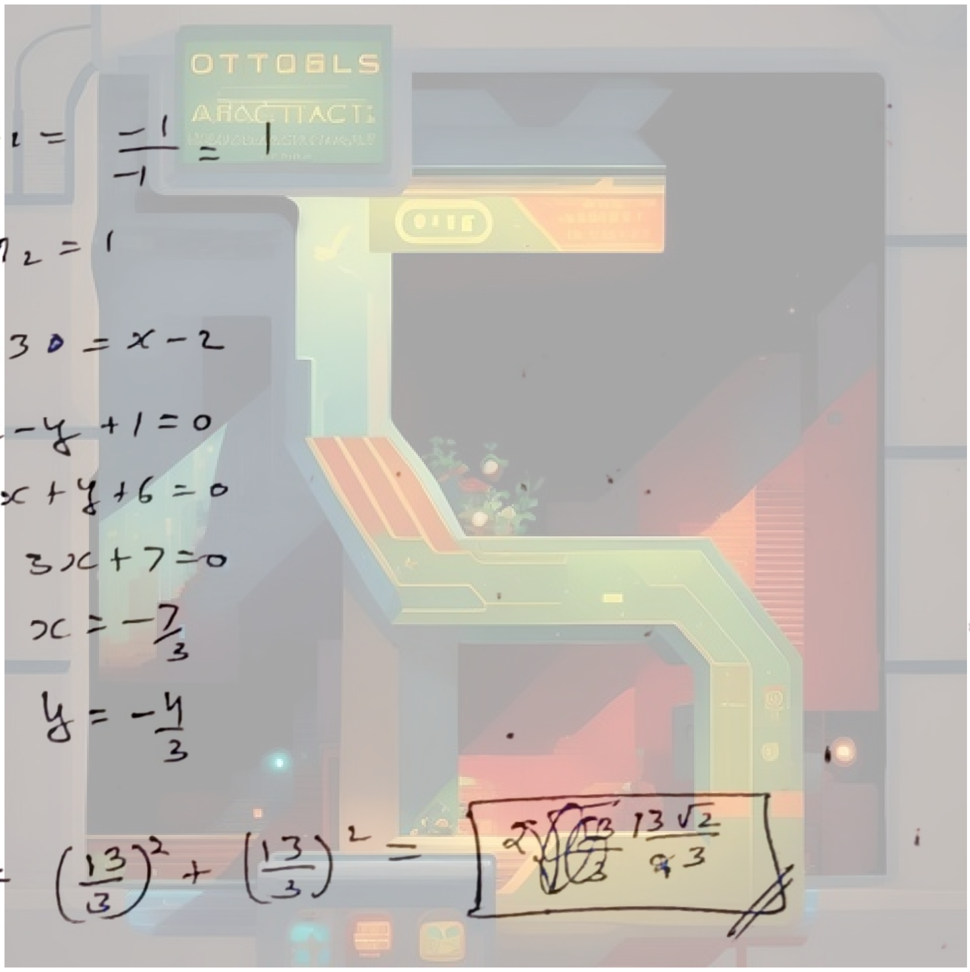
$y = -\frac{16}{3}$

$\left(-\frac{1}{3}, -\frac{16}{3}\right)$

$\frac{49}{9} + \frac{625}{9}$

$(5,0)$





$m_1 = \frac{-1}{-1} = 1$

$m_2 = 1$

$y - 3 = x - 2$

$x - y + 1 = 0$

$2x + y + 6 = 0$

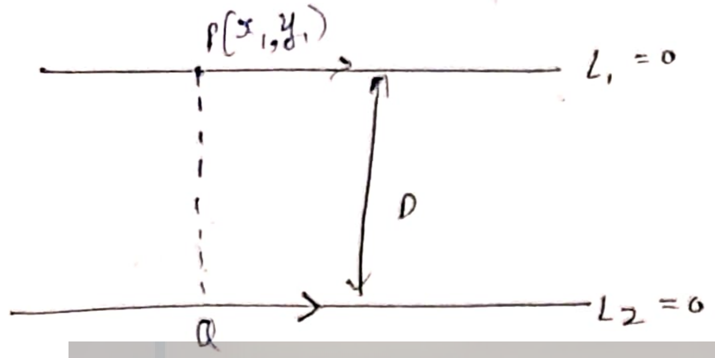
$3x + 7 = 0$

$x = -\frac{7}{3}$

$y = -\frac{4}{3}$

Dis $= \left(\frac{13}{3}\right)^2 + \left(\frac{13}{3}\right)^2 = \boxed{\frac{13\sqrt{2}}{3}}$

Distance btw two parallel lines.



$L_1 : y = mx + c_1$

$L_2 : y = mx + c_2$

$y_1 = mx_1 + c_1$ — (1)

+ Distance of $P(x_1, y_1)$ from $L_2 = 0$

$$PQ = \left| \frac{mx_1 - y_1 + c_2}{\sqrt{m^2+1}} \right|$$

$$\boxed{PQ = \left| \frac{c_2 - c_1}{\sqrt{m^2+1}} \right|}$$

Note :- Before Applying This formula, convert the line equation in slope intercept form.

① Find the Distance b/w given || line -

(1) $3x + 4y = 9$ & $3x + 4y = -1$

$\cancel{m = \frac{-3}{4}}$

$4y = (-3)x + \frac{9}{4}$

$y = \left(\frac{-3}{4}\right)x + \frac{9}{4}$

$$Dis = \left| \frac{-\frac{1}{4} - \frac{9}{4}}{\sqrt{1 + \frac{9}{16}}} \right| = \left| \frac{-10}{5} \right| = |-2| = \boxed{2}$$

$$\cancel{= \left| \frac{\frac{-37}{4}}{\frac{5}{4}} \right|}$$

$$\cancel{= \left| \frac{-37}{5} \right|}$$

$$\cancel{\boxed{\frac{37}{5}}}$$

(2) $2x + 4y = 15$ & $x + 2y = 5$

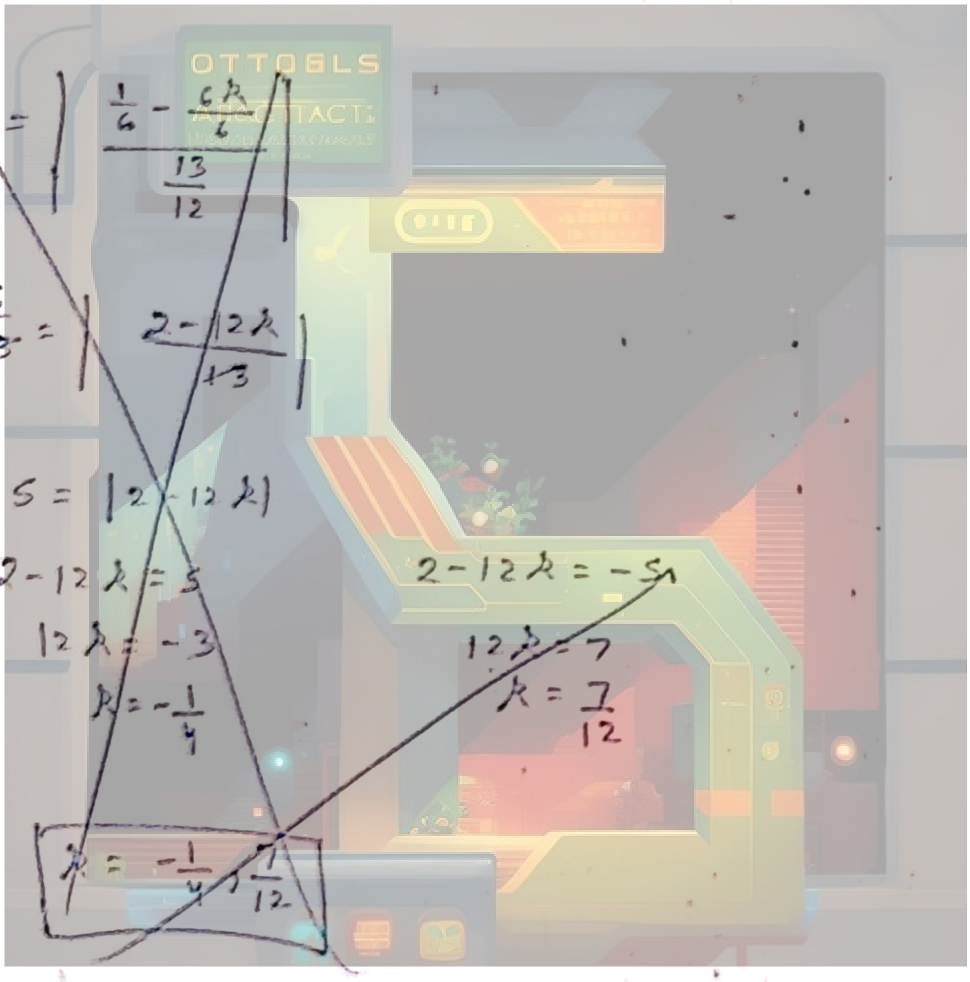
$y = \left(-\frac{1}{2}\right)x + \frac{15}{4}$ $\qquad$ $y = \left(-\frac{1}{2}\right)x + \frac{5}{2}$

$$Dis = \left| \frac{\frac{15}{4} - \frac{10}{4}}{\sqrt{1 + \frac{1}{4}}} \right| = \left| \frac{\frac{5}{4}}{\frac{\sqrt{5}}{2}} \right| = \frac{5}{2\sqrt{5}} = \boxed{\frac{\sqrt{5}}{2}}$$

Q. The Dis btw 2 || lines $5x - 12y + 2 = 0$ & $5x - 12y + k = 0$ is $\frac{5}{13}$ units, find 'K'.

$\left(\frac{5}{12}\right)x + \left(\frac{2}{12}\right) = y$ $\qquad$ $y = \left(\frac{5}{12}\right)x + \frac{k}{12}$

$y = \left(\frac{5}{12}\right)x + \left(\frac{1}{6}\right)$



$\frac{5}{13} = \left| \frac{\frac{1}{6} - \frac{6k}{6}}{\frac{13}{12}} \right|$

$\frac{5}{13} = \left| \frac{2 - 12k}{13} \right|$

$5 = |2 - 12k|$

$2 - 12k = 5$ $\qquad$ $2 - 12k = -5$

$12k = -3$ $\qquad$ $12k = 7$

$k = -\frac{1}{4}$ $\qquad$ $k = \frac{7}{12}$

$\boxed{k = -\frac{1}{4}, \frac{7}{12}}$

$\pm \frac{5}{13} = \frac{K - 2}{13}$

$K - 2 = \pm 5$

$\boxed{K = 7, -3}$

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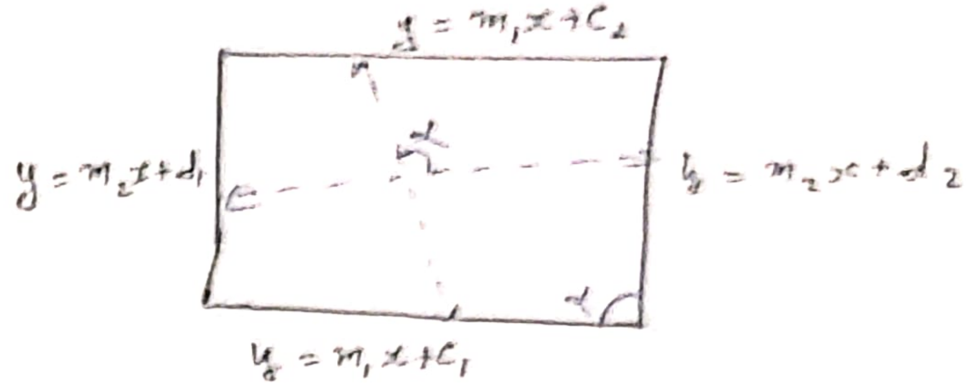
Note:- area of ||gm with all 4 side equations given



$$\boxed{\text{Area} = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|}$$

derived by

$$\text{area} = \frac{p_1\, p_2}{\sin\alpha}$$

Q find area of ||gm with sides

$$3x - 2y + 33 = 0$$
$$x + 3y - 11 = 0$$
$$3x - 2y + 77 = 0$$
$$x + 3y + 44 = 0$$

$$y = \left(\frac{3}{2}\right)x + \frac{33}{2} = 0$$
$$y = \left(\frac{-1}{3}\right)x + \frac{11}{3} = 0$$
$$y = \left(\frac{3}{2}\right)x + \frac{77}{2} = 0$$
$$y = \left(\frac{-1}{3}\right)x + \left(\frac{-44}{3}\right) = 0$$

$\cancel{\text{area} = \left|\left(\frac{33}{2}\right)\left(\frac{55}{3}\right)\right|}$ area $= \left| \dfrac{\frac{44}{2} \times \frac{55}{3}}{\frac{9}{6} + \frac{2}{6}} \right|$ area $= \left| \dfrac{44 \times 5 \times 11}{11} \right| = \boxed{220}$

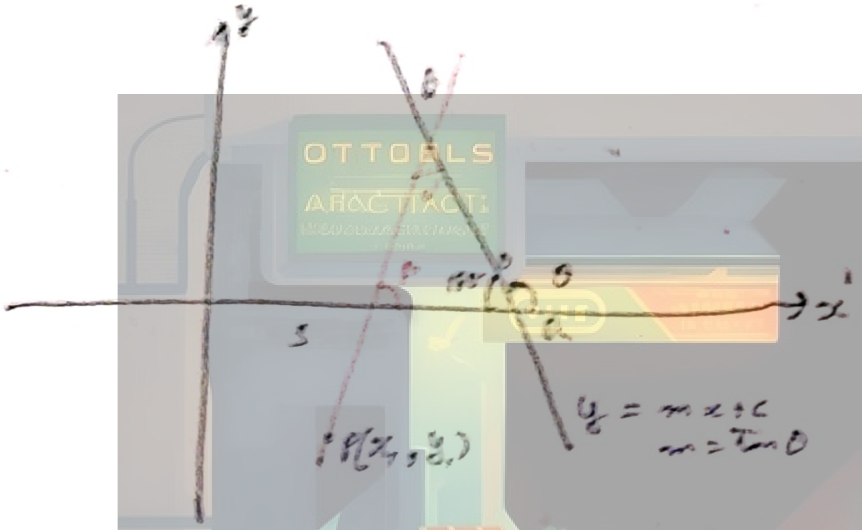
H W 22-9-24

DYS-7 (1, 7, 8, 9, 10)

DYS-8 (Full)

O-1 (OM)

Equation of line which makes angle α with another line $y = mx + c$



$\theta = \theta - \alpha$

Eqn of line

$$y - y_1 = \tan(\theta - \alpha)(x - x_1)$$

$$\boxed{y - y_1 = \tan(\theta \pm \alpha)(x - x_1)}$$

$\alpha \to$ angle b/w lines

$\theta \to$ angle of given line

Q find the eqⁿ of SL passing through $(2,3)$ & inclined at $\frac{\pi}{4}(45^\circ)$ to given line $2x+3y=5$.

$\tan\theta = \frac{-2}{3}$

$\tan\beta = \dfrac{\frac{-2}{3} - 1}{1 + \frac{2}{3}}$

$\tan\beta = \frac{-5}{5}$

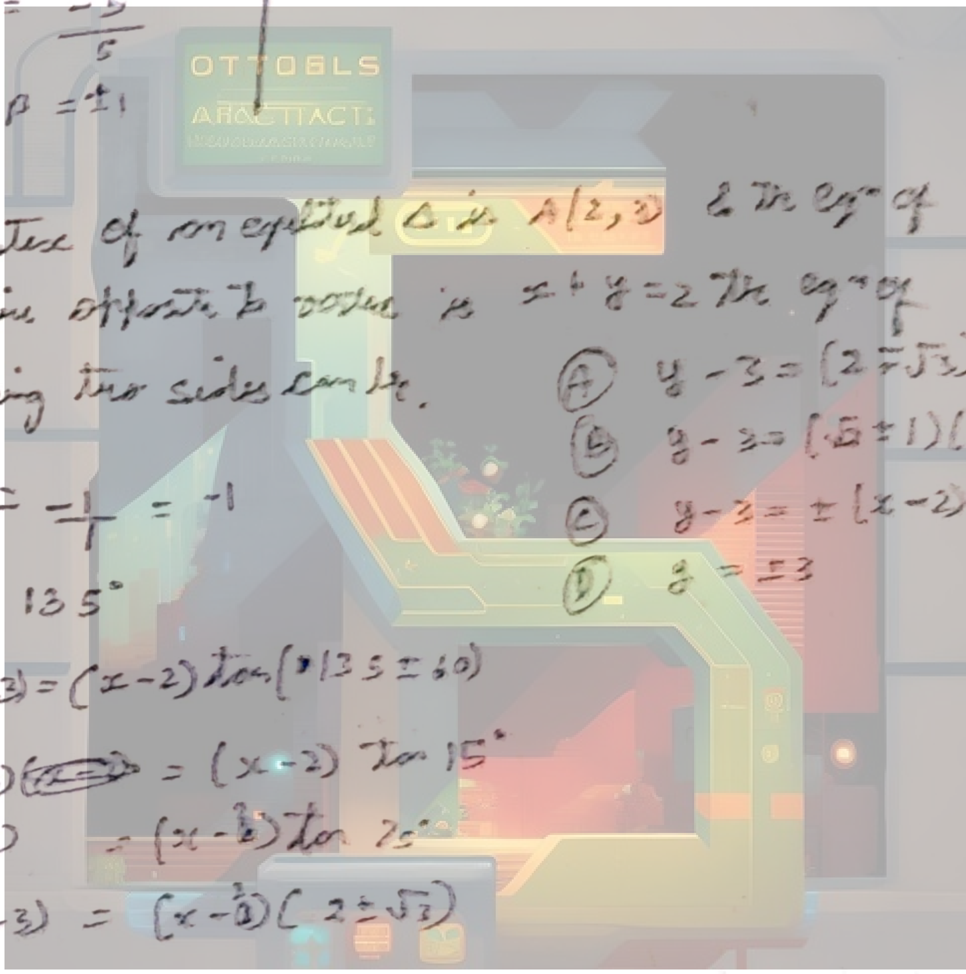
$\tan\beta = \pm 1$

eqⁿ

$y-3 = (x-2)(\pm 1)$

$y-3 = 2-x$ $\qquad$ $y-3 = x-2$

$\boxed{x+y-5=0}$ $\qquad$ $\boxed{x-y+1=0}$



Q One vertex of an equilateral Δ is $A(2,3)$ & the eqⁿ of the line opposite to vertex is $x+y=2$ the eqⁿ of remaining two sides can be.

(A) $y-3=(2\mp\sqrt{3})(x-2)$

(B) $y-3=(\sqrt{3}\pm 1)(x-2)$

(C) $y-3=\pm(x-2)$

(D) $y=\pm 3$

$\tan\theta = \frac{-1}{1} = -1$

$\theta = 135^\circ$

$(y-3)=(x-2)\tan(135\pm 60)$

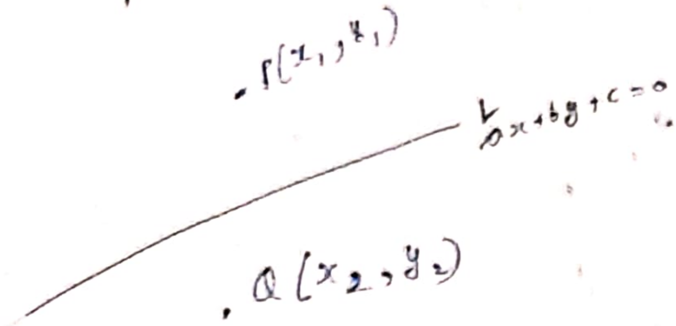
$(y-3) = (x-2)\tan 15^\circ$

$(y-3) = (x-2)\tan 75^\circ$

$(y-3) = (x-2)(2\pm\sqrt{3})$

$\boxed{A}$

Position of line w.r.t line.



⇒ $\dfrac{L(x_1,y_1)}{b} > 0$

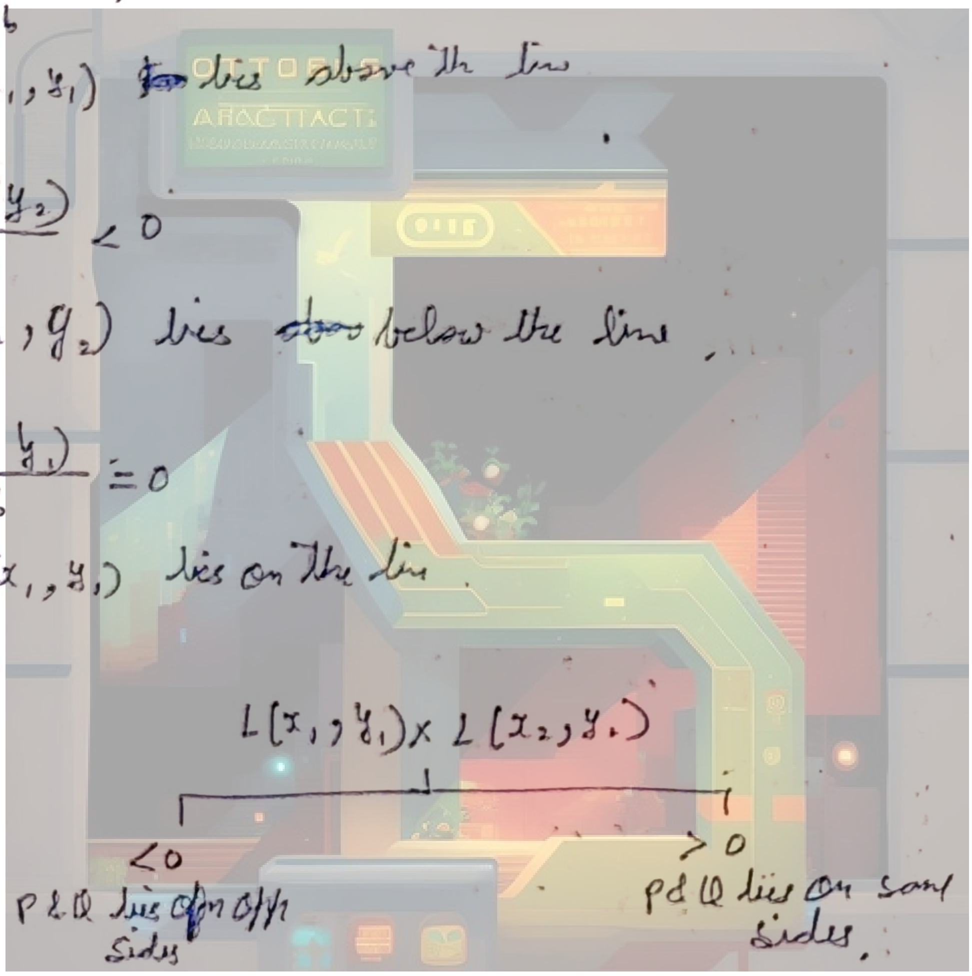
$P(x_1,y_1)$ lies above the line

⇒ $\dfrac{L(x_2,y_2)}{b} < 0$

$Q(x_2,y_2)$ lies ~~abov~~ below the line.

⇒ $\dfrac{L(x_1,y_1)}{b} = 0$

$P(x_1,y_1)$ lies on the line

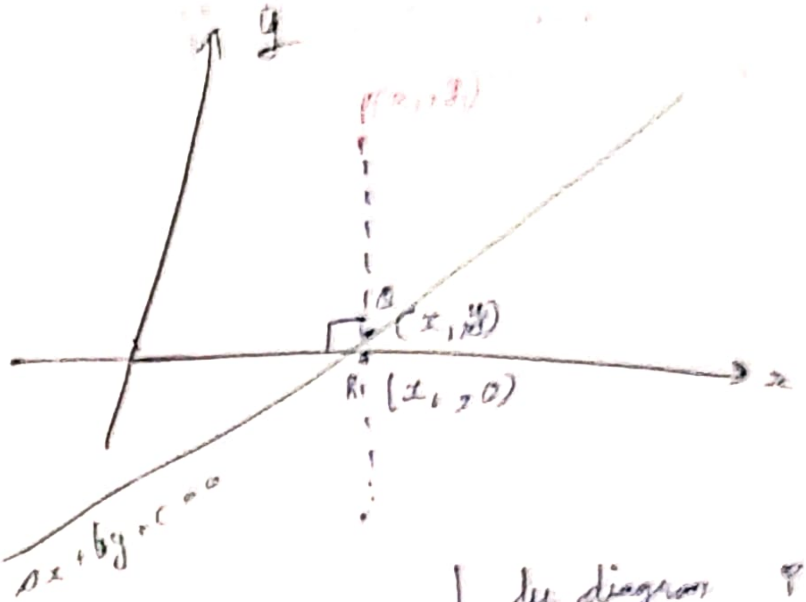


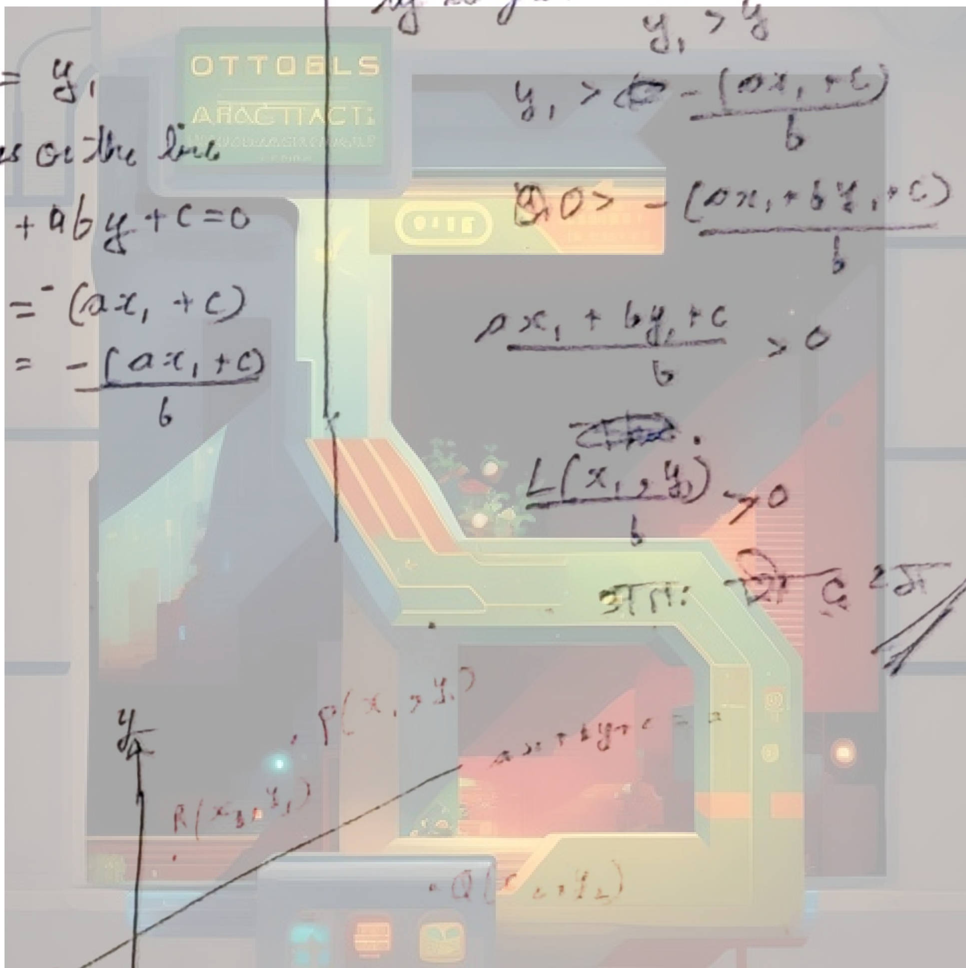
$L(x_1,y_1)\times L(x_2,y_2)$

<0 : P & Q lies on opp sides

>0 : P & Q lies on same sides.

Proof:- Let $P(x_1,y_1)$ lie above the line $ax+by+c=0$





$PR = y_1$

Q lies on the line

$ax_1 + ab y + c = 0$

$by = -(ax_1 + c)$

$y = \frac{-(ax_1+c)}{b}$

by diagram $PR > QR$

$y_1 > y$

$y_1 > \frac{-(ax_1+c)}{b}$

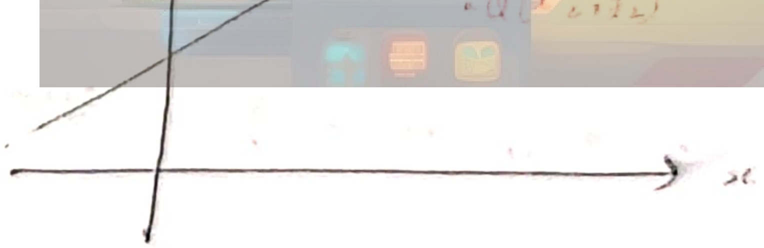
$0 > \frac{-(ax_1+by_1+c)}{b}$

$\frac{ax_1+by_1+c}{b} > 0$

$\frac{L(x_1,y_1)}{b} > 0$

अतः सिद्ध

Proof 2



$\frac{L(x_1,y_1)}{b} > 0 \quad \frac{L(x_2,y_2)}{b} < 0 \quad \frac{L(x_3,y_3)}{b} > 0$

$$\frac{L(x_1,y_1)\,L(x_2,y_2)}{b^2} = \frac{(+)(-)}{(+)} = (-)$$

Q find the position of $(3,-4)$ (1) wrt line $3x-4y+5=0$
(2) wrt origin in above line.

(1) $L(3,-4)$

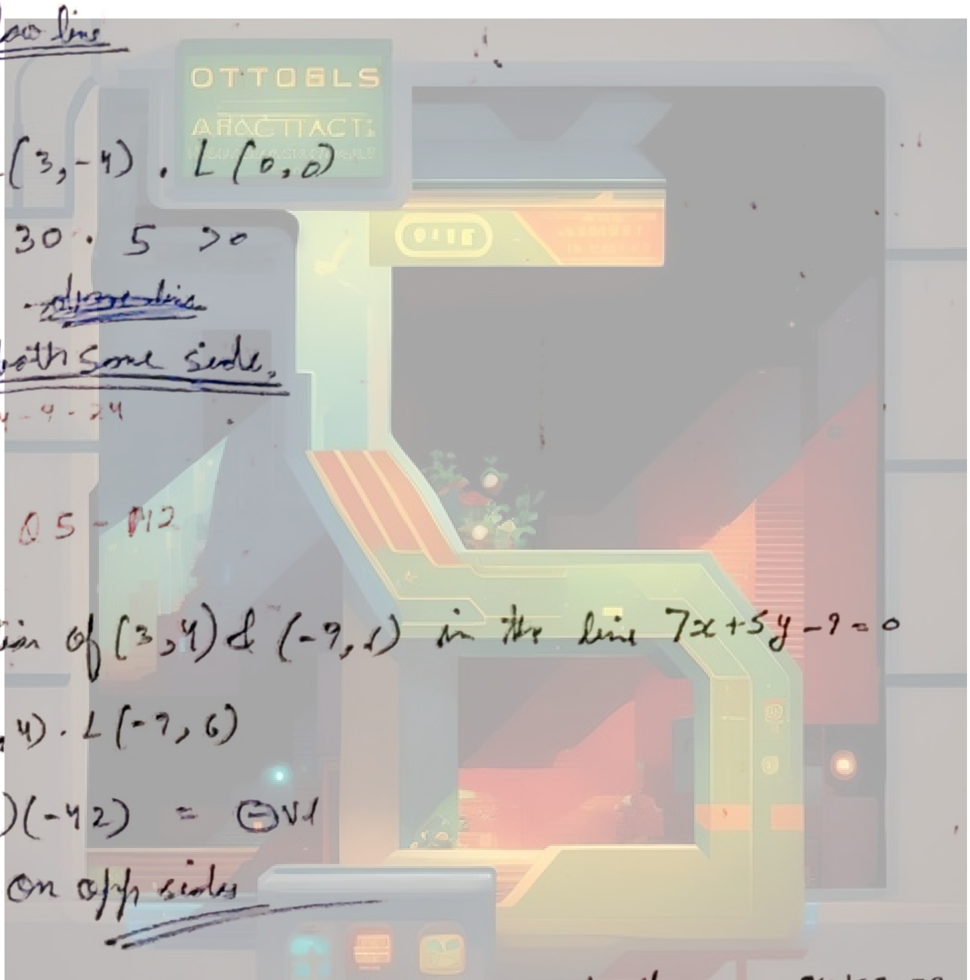
$= 3(3)-4(-4)+5$

$= 9+16+5$

$= 30$

$\frac{30}{-4} < 0$

below line



(2) $L(3,-4) \cdot L(0,0)$

$30 \cdot 5 > 0$

~~opposite line~~

both same side.

H.W.
DYS - 9 $24-4-24$
Ex 0-2 $05-012$

Q find position of $(3,4)$ & $(-7,6)$ in the line $7x+5y-9=0$

$L(3,4) \cdot L(-7,6)$

$(32)(-42) = \ominus ve$

On opp sides

Q If the point $(1,2)$ & $(3,4)$ are on opposite side of $3x-5y+a=0$
find a.

$L(1,2) \cdot L(3,4) < 0$

$(3-10+a)(9-20+a) < 0$

$(a-7)(a-11) < 0$

$\boxed{a \in (7,11)}$

Q If $P(2, \frac{1}{2})$ & ΔABC is $A(2,1)$ $B(0,0)$ & $C(3,0)$ then P lies in.

(A) ① Inside Δ (B) outside Δ (C) above or my side (D) P, 0, B & C are co-linear

area ≠ 0

$AB \Rightarrow y = \frac{1}{2}x$ $BC \Rightarrow y = 0$ $AC \Rightarrow y = (x-2) - 1$

$x = 2y$ $y = 3 - x$

$x - 2y = 0$ $x + y - 3 = 0$

$L(2, \frac{1}{2})$ $L'(2, \frac{1}{2})$ $L_2(2, \frac{1}{2})$

$= 1$ $= -\frac{1}{2}$

above above below

$\boxed{A}$

Reflection

$P(x, y)$

→ Reflection in x axis $Q(x, -y)$

→ Reflection in y axis $R(-x, y)$

→ Reflection in $y = x$ $S(y, x)$

→ Reflection in $y = -x$ $T(-y, -x)$

→ Reflection in origin. $U(-x, -y)$

→ Take reflection in x axis & then in y-axis.

Q find the image of the point /reflection

① (1, 2) in x-axis
② (−3, 4) in y-axis
③ (4, 2) in x = y
④ (−3, 6) in y = −x
⑤ (−2, 4) in origin

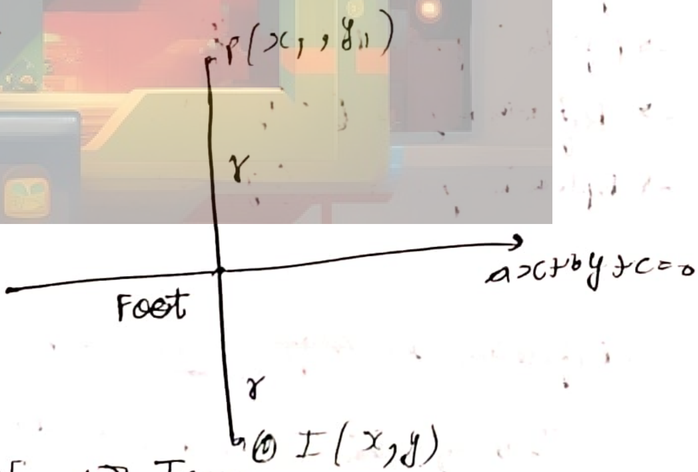
① (1, −2)
② (3, 4)
③ (2, 4)
④ (−6, 3)
⑤ (2, −4)

※ General formula of reflection in any line $ax+by+c=0$

$$\boxed{\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}}$$

Proof:- $m_{FP}=\dfrac{b}{a}$

$\tan\theta=\dfrac{b}{a}$ → $\cos\theta=\dfrac{a}{\sqrt{a^2+b^2}}$, $\sin\theta=\dfrac{b}{\sqrt{a^2+b^2}}$



$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=\pm r$$

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}\ [\text{foot}]$$ Image

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}\ [\text{image}]$$

Q find the foot of & the image of the point $(2,3)$ in the line $3x+4y+7=0$

$I(x,y)$

$\frac{x-2}{3} = \frac{-2(25)}{25}$ | $\frac{y-3}{4} = \frac{-2(25)}{25}$

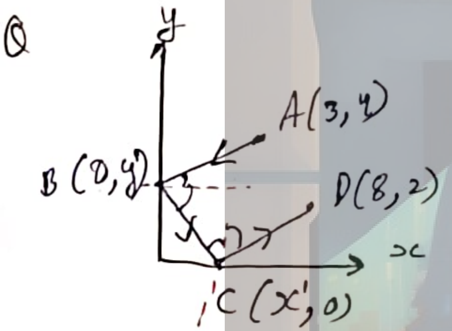
$x-2=-6$ | $y-3=-8$

$x=-4$ | $y=-5$

$\boxed{I(-4,-5)}$

$\boxed{F(-1,-1)}$

Q



find (x,y).

~~BC ⇒~~ $\cancel{y = (x - x')\frac{-y'}{x'}}$

~~BC ⇒~~ $\cancel{y x' = x'y' - x y'}$

$\cancel{x'y + y'x = x'y'}$

$\cancel{\frac{0-3}{y'x'} = \frac{y-4}{x'y'} = \frac{-2(3y'+4x'-x'y')}{(x')^2+(y')^2}}$

$A'(-3,4)$

$D'(8,-2)$

$A'D' \Rightarrow \frac{6}{-11} = -\frac{6}{11}$

$A'B' \Rightarrow y-4 = (x+3)\left(\frac{-6}{11}\right)$

$11y - 44 = -6x - 18$

$6x + 11y - 26 = 0$

$\boxed{y = \frac{26}{11}, \quad \frac{13}{3} = x}$

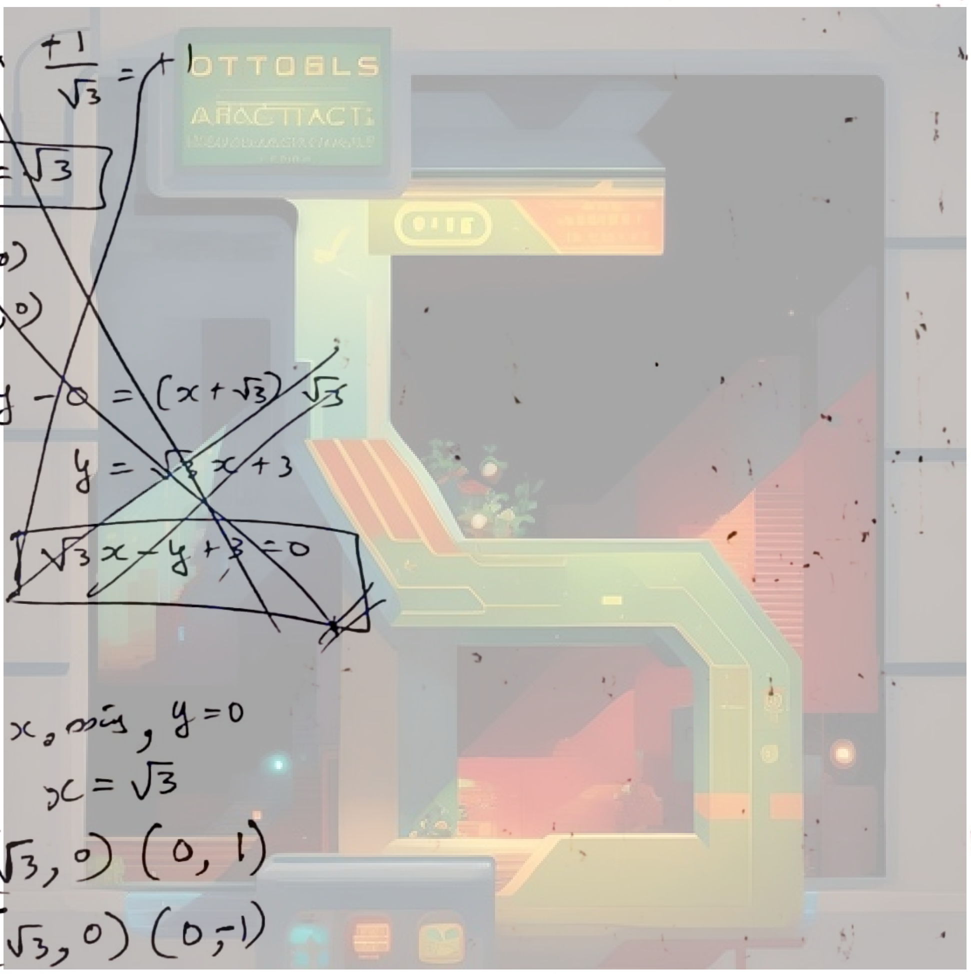
Q A ray of light along $x + \sqrt{3}y = \sqrt{3}$ is incedented on x axis & gets reflected, find the eq^n of reflected ray.

~~$\left(1, \frac{\sqrt{3}-1}{\sqrt{3}}\right)$~~ ~~$(\sqrt{3}, 0)$~~

~~$A' = \left(-1, \frac{\sqrt{3}-1}{\sqrt{3}}\right)$~~ ~~$A' = (-\sqrt{3}, 0)$~~

~~$B' = (0.\sqrt{3}, 0)$~~

~~$B' =$~~



~~$m(1) \times \frac{+1}{\sqrt{3}} = +1$~~

~~$\boxed{m = \sqrt{3}}$~~

~~$(\sqrt{3}, 0)$~~

~~$A'(-\sqrt{3}, 0)$~~

line $\Rightarrow$ ~~$y - 0 = (x+\sqrt{3})\sqrt{3}$~~

~~$y = \sqrt{3}x + 3$~~

~~$\boxed{\sqrt{3}x - y + 3 = 0}$~~

at x, axis, $y = 0$

$x = \sqrt{3}$

$(\sqrt{3}, 0)\ (0, 1)$

$(\sqrt{3}, 0)\ (0, -1)$

$m = \frac{1}{\sqrt{3}}$

$y - 0 = (x - \sqrt{3})\frac{1}{\sqrt{3}}$

$\sqrt{3}y = x - \sqrt{3}$

$\boxed{x - \sqrt{3}y - \sqrt{3} = 0}$

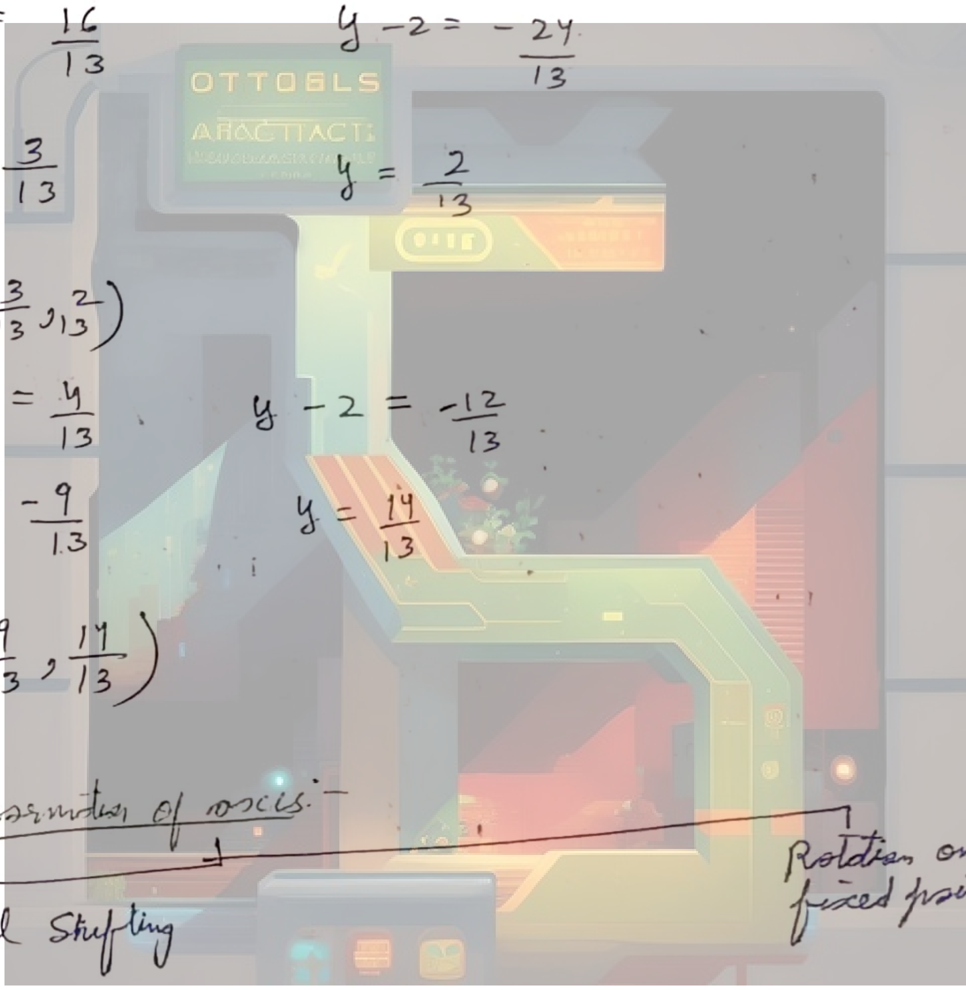
H.W.

Q Find the foot & Image of point $P(-1, 2)$ in the line mirror

$2x - 3y + 4 = 0$

$$\frac{-2(-2-6+4)}{4+9} = \frac{8}{13}$$

$$\frac{x+1}{2} = \frac{8}{13} \qquad \frac{y-2}{-3} = \frac{8}{13}$$



$$x+1 = \frac{16}{13} \qquad y-2 = -\frac{24}{13}$$

$$x = \frac{3}{13} \qquad y = \frac{2}{13}$$

$$I = \left(\frac{3}{13}, \frac{2}{13}\right)$$

$$x+1 = \frac{4}{13} \qquad y-2 = \frac{-12}{13}$$

$$x = \frac{-9}{13} \qquad y = \frac{14}{13}$$

$$F = \left(\frac{-9}{13}, \frac{14}{13}\right)$$

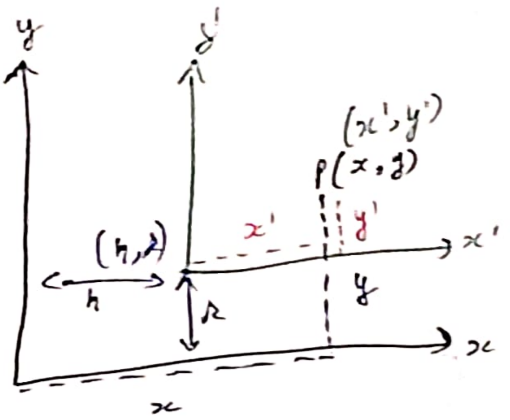
* Transformation of axis:-

Parallel Shifting

Rotation on a fixed point

Parallel Shifting

Parallel Shifting



$\boxed{x = h + x'}$ $\boxed{y = y' + k}$

(h, k) jaha origin ko le gaye.

(x', y') new coordinate

(x, y) old coordinate

Q If axes are transformed from origin to the point (−2, 1), then the new co-ordinates of (4, −5) is –

(A) (6, 4) (B) (−2, 6) (C) (6, −6) (D) (2, −4)

$x' = x - h$ | $y' = y - k$
$x' = 4 - (-2)$ | $y' = -5 - 1$
$x' = 6$ | $y' = -6$

6, −6

$\boxed{C}$

H.W. 25-07-24

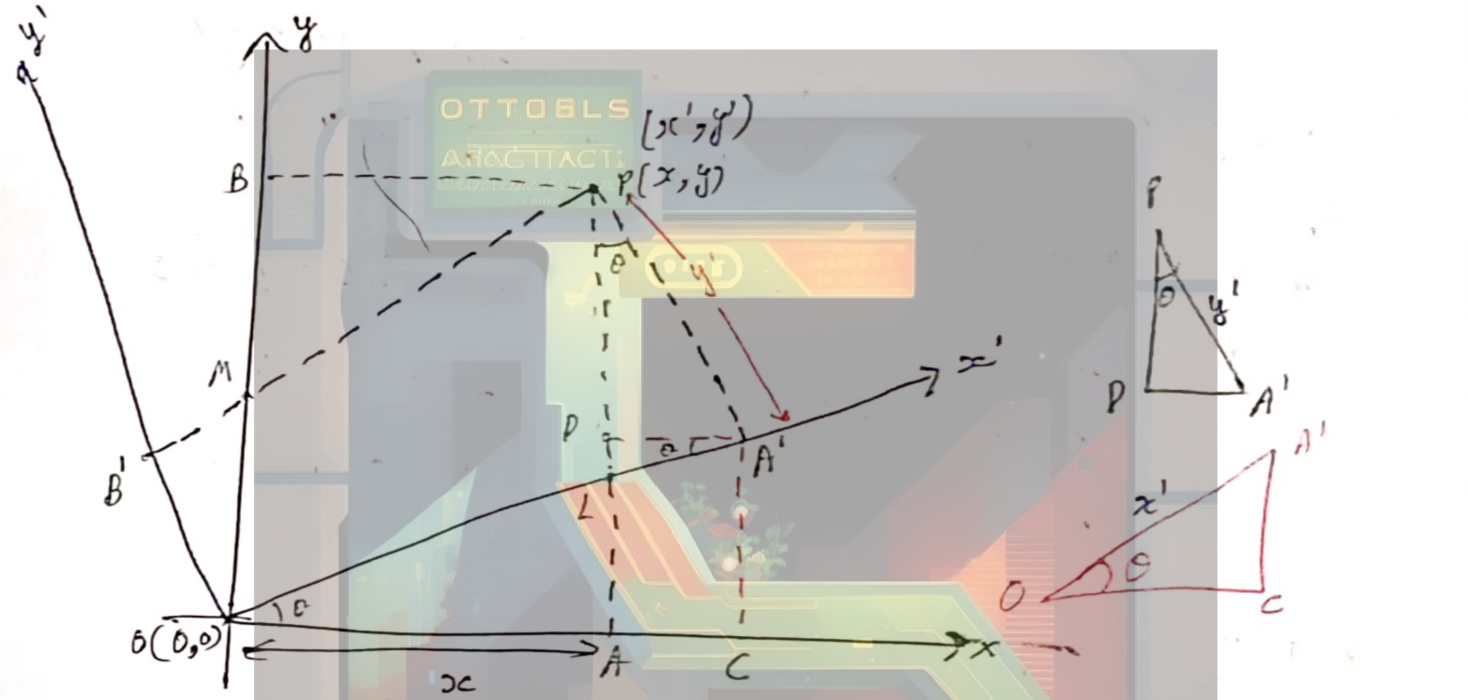
DYS-10 {1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}.

O-1 {15, 16, 20}

O-2 {9, 10, 6-8, 4-5}

O-4 {4, 6, 7, 8, 9, 10} + {2, 3}

(2) Rotation on fixed point. → Here we consider rotation in anticlockwise dir. So in question if axis are rotated clockwise then we use $-\theta$ everywhere.



$OC = OA + AC$

$OC = x + AC$

$x = OC - AC$

$AP = AD + DP$

$y = AD + DP$

in $\Delta A'DP$

$\cos\theta = \frac{DP}{y'}$

$\sin\theta = \frac{AC}{y'}$

in $\Delta OCA'$

$\cos\theta = \frac{OC}{x'}$

$\sin\theta = \frac{AP}{x'}$

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

old / new	x	y
x'	$\cos\theta$	$\sin\theta$
y'	$-\sin\theta$	$\cos\theta$

$$\boxed{\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}}$$

$$\boxed{\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}}$$

Q Keeping the origin constant, axes are rotated at an angle of $30°$ in ACW. Find the co-ordinate of $(2,1)$ wrt to new ~~axes~~ axes.

$x' = x\cos\theta + y\sin\theta$

$x' = 2\cos 30 + \sin 30$

$x' = \sqrt{3} + \frac{1}{2}$

$y' = -x\sin\theta + y\cos\theta$

$y' = -2\sin 30 + \cos 30$

$y' = \frac{\sqrt{3}}{2} - 1$

$$\boxed{\left(\sqrt{3} + \frac{1}{2}, \frac{\sqrt{3}}{2} - 1\right)}$$

Q If the origin is taken to $(1,1)$ & the axes are rotated by $45°$ ACW. Find the co-ordinates of $(4,5)$ wrt find transformation

$x_1 = 4 - 1$ | $y_1 = 5 - 1$

$x_1 = 3$ | $y_1 = 4$

$(3,4)$

$x' = 3\cos 45 + 4\sin 45$

$x' = \frac{7}{\sqrt{2}}$

$y' = -3\sin 45 + 4\cos 45$

$y' = \frac{1}{\sqrt{2}}$

$$\boxed{\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}}$$

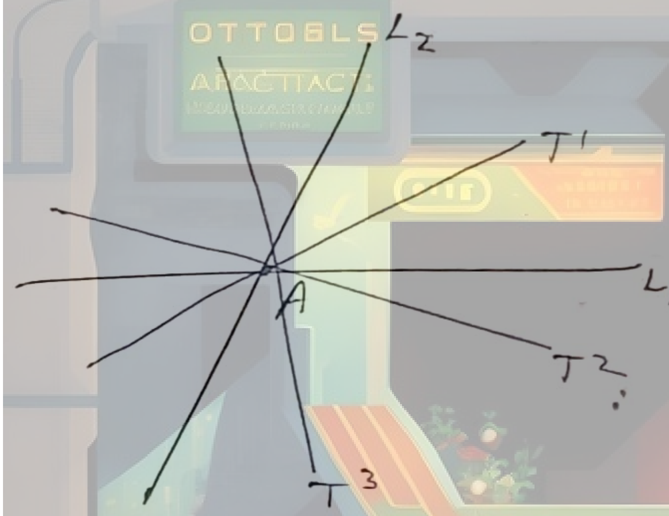
Family of Lines

→ ~~&~~ Group of similar lines having something in common.

eg. 1. all lines passing through $(0,0)$.

2. all lines $\parallel$ to y-axis.

→ by solving L_1 & L_2 we get co-ordinates of A.



L_1, L_2, T_1, T_2, T_3 forms a family of ~~Eq.~~ lines.

→ $L_1 + \lambda L_2 = 0$

$\lambda \in R$

Type 1 finding equations.

Q1. $L_1 \Rightarrow 3x + 4y + 6 = 0$

$L_2 \Rightarrow x + y + 2 = 0$

find eq^n of line ~~w~~ passes through POI of L_1 & L_2 &

i) through $(2, -3)$

ii) $\parallel$ to the line $(x + 2y + 3) = 0$

iii) $\perp$ to the line $2x - 3y + 1 = 0$

iv) at distance 2 unit from origin.

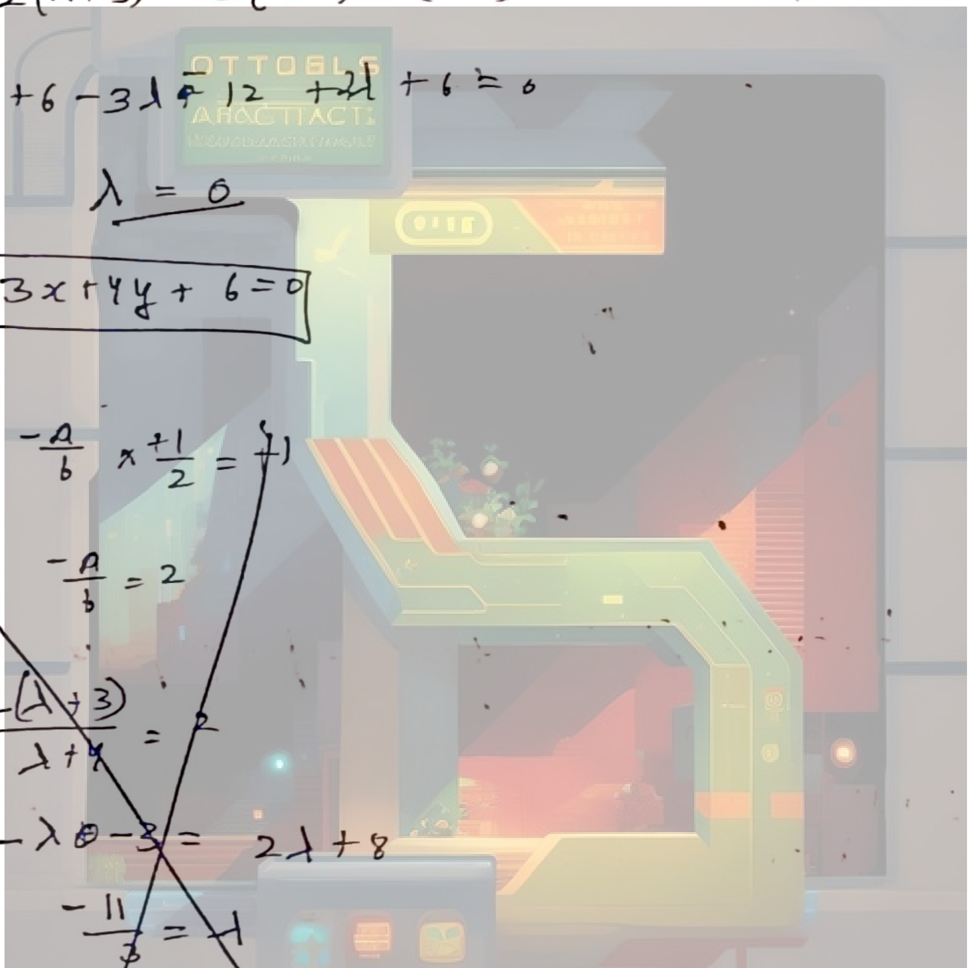
(1) $L_1 + \lambda L_2 = 0$

$(3x+4y+6) + \lambda(x+y+2) = 0$

$(\lambda+3)x + (\lambda+4)y + (2\lambda+6) = 0$

i) $(2, -3)$

$2(\lambda+3) - 3(\lambda+4) + (\lambda+6) = 0$



$2\lambda + 6 - 3\lambda - 12 + 2\lambda + 6 = 0$

$\lambda = 0$

$\boxed{3x + 4y + 6 = 0}$

ii) $-\frac{a}{b}x + \frac{1}{2} = y$

$-\frac{a}{b} = 2$

$\frac{-(\lambda+3)}{\lambda+4} = 2$

$-\lambda - 3 = 2\lambda + 8$

$-\frac{11}{3} = \lambda$

$-\frac{2}{3}x + \frac{y}{3} + -\frac{4}{3} = 0$

$\boxed{2x - y + 4 = 0}$

ii) $m_1 = m_2$

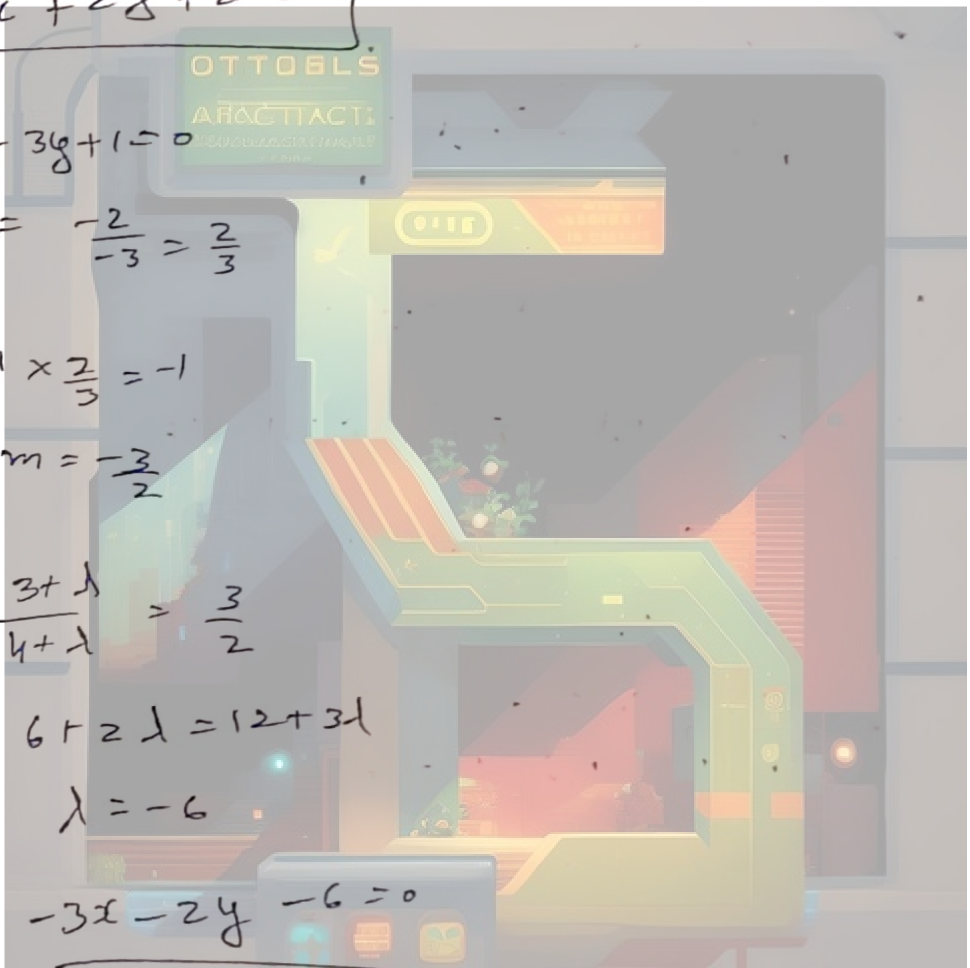
$x + 2y + 3 = 0$

$$\frac{\pm(3+\lambda)}{4+\lambda} = \pm\frac{1}{2}$$

$\cancel{2}\ 6 + 2\lambda\ \cancel{0} = 4 + \lambda$

$\lambda = -2$

$\boxed{x + 2y + 2 = 0}$



iii) $2x - 3y + 1 = 0$

$$m' = \frac{-2}{-3} = \frac{2}{3}$$

$$m \times \frac{2}{3} = -1$$

$$m = -\frac{3}{2}$$

$$\frac{3+\lambda}{4+\lambda} = \frac{3}{2}$$

$6 + 2\lambda = 12 + 3\lambda$

$\lambda = -6$

$-3x - 2y - 6 = 0$

$\boxed{3x + 2y + 6 = 0}$

iv) $x(3+\lambda)^2 + (4+\lambda)^2 = 94$ 0

$\lambda^2 + 9 + 6\lambda + \lambda^2 + 16 + 8\lambda = 9$

$2\lambda^2 + 14\lambda + \cancel{2} + 16 = 0$

$\lambda^2 + 7\lambda + 8 = 0$

$$\lambda = \frac{-7 \pm \sqrt{49 - 32}}{2}$$

iv) $\lambda^2+6\lambda+9+\lambda^2+8\lambda+16=4$

$2\lambda^2+14\lambda+21=0$

$\lambda = \dfrac{-14 \pm \sqrt{196-168}}{4}$

$\lambda = \dfrac{-7 \pm \sqrt{7}}{4}$

iv) $2 = \left| \dfrac{6+2\lambda}{\sqrt{(3+\lambda)^2+(4+\lambda)^2}} \right|$

$6+2\lambda = 2\sqrt{2\lambda^2+14\lambda+25}$

$4\lambda^2+36+24\lambda = 8\lambda^2+56\lambda+100$

$4\lambda^2+\cancel{20\lambda}\,32\lambda+64=0$

$\lambda^2+8\lambda+16=0$

$\lambda^2+4\lambda+4\lambda+16=0$

$\lambda(\lambda+4)+4(\lambda+4)=0$

$\lambda=-4$

$-x-2=0$

$\boxed{x=-2}$

H.W. 26-9-24

DYS-11 {2, 3, 10, 11}

O-1 {17, 20, 18, 23, 26, 27, 29}
imp

Type-2 finding fix point.

Q Find the fixed point from which the given family of lines passes, $\lambda \in R, \theta \in R, a, b \in R$

① $(x-2y+1)+\lambda(x+y)=0$

② $x(\lambda+1)+y(2-\lambda)+5=0$

③ $ax(a+2b)+y(a+3b)-(a+b)=0$

④ $x(\cos\theta+\sin\theta)+y(\cos\theta-\sin\theta)-3(3\cos\theta+\sin\theta)=0$

① $x-2y+1=0$

$-x-y=0$

$-3y=-1$

$y=1/3$

$x=-1/3$

$\boxed{(-1/3, 1/3)}$

② $(x+2y+5)+\lambda(x-y)=0$

$x+2y+5=0$

$2x-2y=0$

$3x=-5$

$x=-\frac{5}{3}$

$y=-\frac{5}{3}$

$\boxed{\left(-\frac{5}{3}, -\frac{5}{3}\right)}$

③ $a[x+y-1]+\frac{b}{a}[2x+3y-1]=0$

$2x+2y-2=0$

$2x+3y-1=0$

$y+1=0$

$y=-1$

$x=2$

$\boxed{(2,-1)}$

Note:- $L_1+\lambda L_2=0$
↳ gives all the lines except L_2

for $L_2=0$, $\lambda=0$ not possible.

$L_2=0+(L_1+\lambda L_2=0)$

(9) $(x+y-9)+\frac{\cos\theta}{\sin\theta}(x-y-3)=0$

$x+y-9=0$

$x-y-3=0$

$2x=12$

$x=6$

$y=9-6$

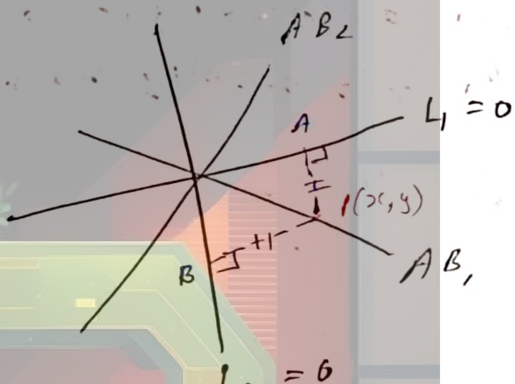
$y=3$

$\boxed{\checkmark(6,3)}$

Eqⁿ of angle bisectors b/w 2 given lines:

$L_1: \; a_1x+b_1y+c_1=0$

$L_2: \; a_2x+b_2y+c_2=0$



$AB_1 \rightarrow$ locus of $P(x,y)$

$PA=PB$

$$\left|\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right|=\left|\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right|$$

$$\boxed{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)=\pm\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)}$$

$\hookrightarrow$ gives both AB_1 & AB_2

$$(a_1x+b_1y+c_1)=\underbrace{\pm\sqrt{\frac{a_1^2+b_1^2}{a_2^2+b_2^2}}}_{\lambda}(a_2x+b_2y+c_2)$$

Note:- Both angle bisectors are mutually $\perp$ to each other.

1) Identification of acute/obtuse $\angle$ bisector

→ Make c_1 & c_2 $\oplus$ve in eq^n of $\angle$ bisector

→ Check the sign of $\boxed{a_1 a_2 + b_1 b_2}$

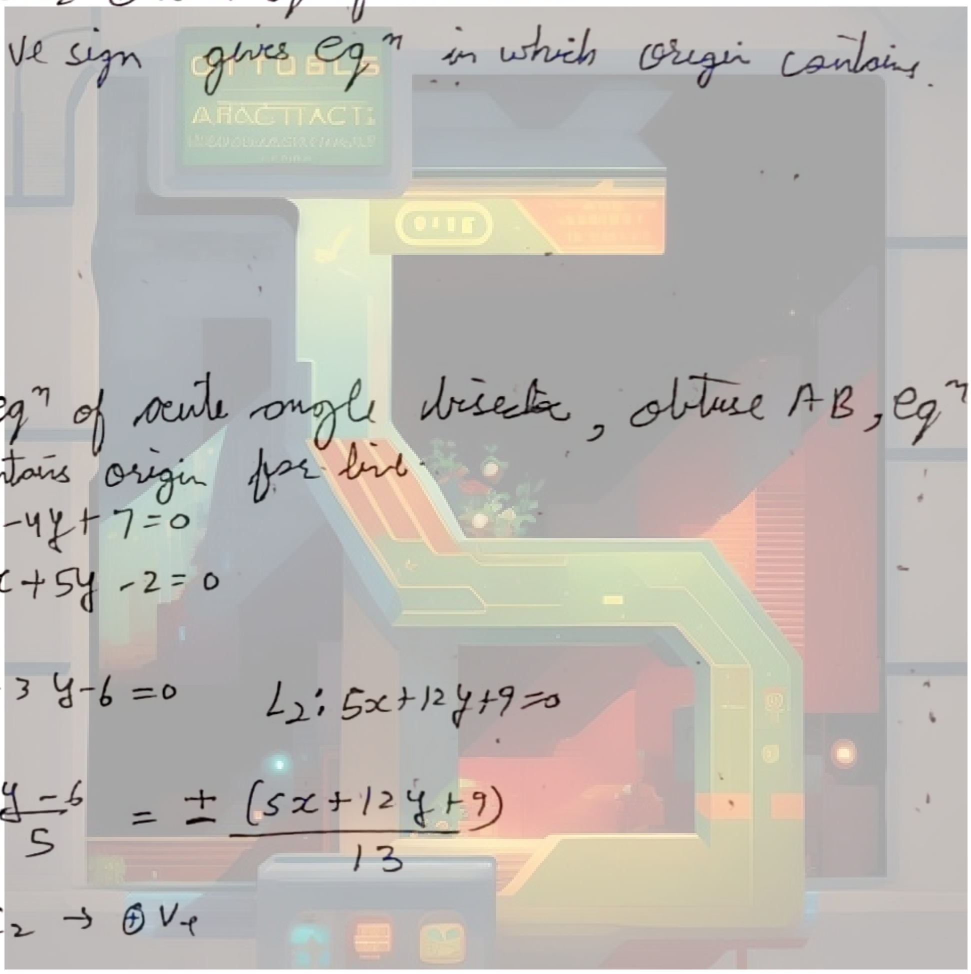
→ If it is $\ominus$ve then $\frac{\oplus\text{ve sign}}{\text{of eq}^n}$ gives acute angle bisector

→ If it is $\oplus$ve then $\ominus$ve sign gives acute bisector.

2) On which region of bisector, origin lie

→ make c_1 & c_2 $\oplus$ve in eq^n of $\angle$ bisector

→ Make $\oplus$ve sign gives eq^n in which origin contains.



Q find the eq^n of acute angle bisector, obtuse AB, eq^n of AB that contains origin for line

① $L_1: 3x - 4y + 7 = 0$
$L_2: 12x + 5y - 2 = 0$

② $L_1: 4x + 3y - 6 = 0$ $\quad$ $L_2: 5x + 12y + 9 = 0$

② $\frac{4x + 3y - 6}{5} = \pm \frac{(5x + 12y + 9)}{13}$

$c_1, c_2 \rightarrow \oplus$ve

$$\frac{-4x - 3y + 6}{5} = \pm \frac{(5x + 12y + 9)}{13} \quad — ①$$

$a_1 a_2 + b_1 b_2 = -20 - 36 < 0 = \ominus$ve

→ Put $\oplus$ve in eq^n ① where c_1 & c_2 are $\oplus$ve

$-52x - 39y + 78 = 25x + 60y + 45$

$77x + 99y - 33 = 0$

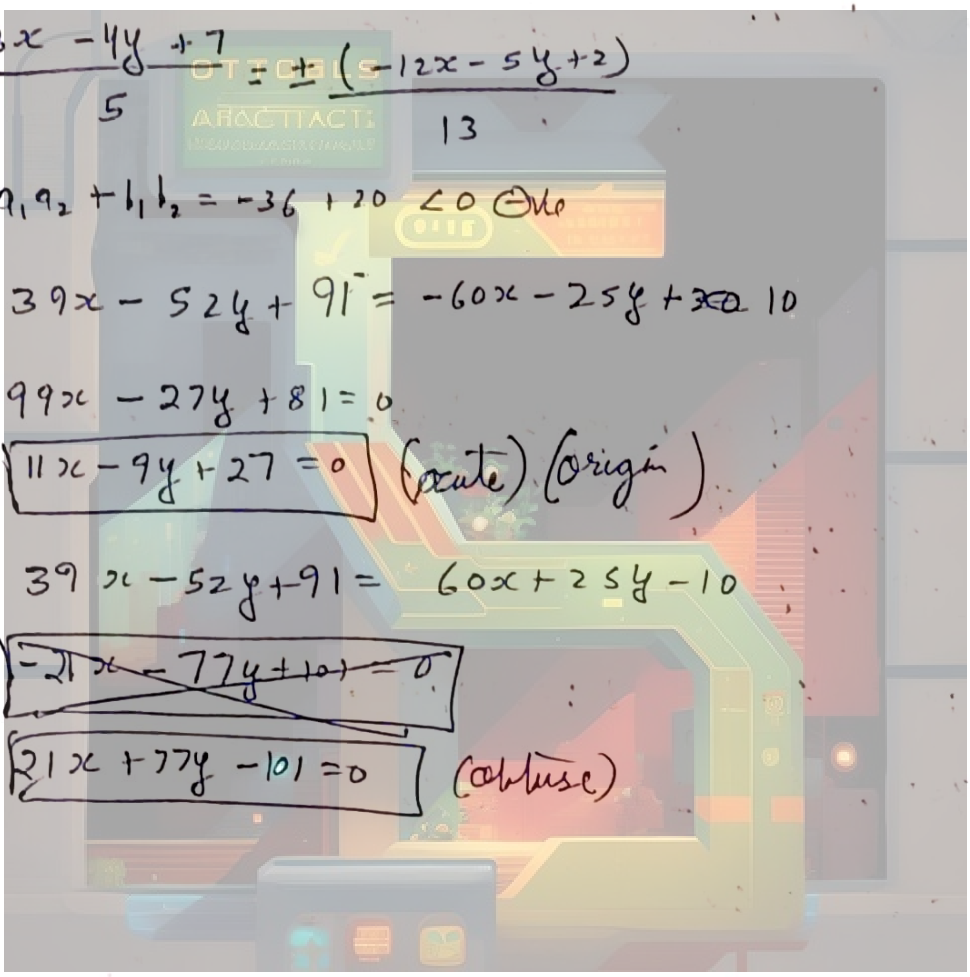
$\boxed{7x + 9y - 3 = 0}$ (acute) (origin)

$$\frac{4x+3y-6}{5} = \frac{5x+12y+9}{13}$$

$$52x + 39y - 78 = 25x + 60y + 45$$

$$27x - 21y - 123 = 0$$

$$\boxed{9x - 7y - 41 = 0}$$ (obtuse)



(1)

$$\frac{3x-4y+7}{5} = \pm\frac{(-12x-5y+2)}{13}$$

$$a_1a_2 + b_1b_2 = -36 + 20 < 0 \ominus ve$$

$$39x - 52y + 91 = -60x - 25y + 10$$

$$99x - 27y + 81 = 0$$

$$\boxed{11x - 9y + 27 = 0}$$ (acute) (origin)

$$39x - 52y + 91 = 60x + 25y - 10$$

$$\cancel{-21x - 77y + 101 = 0}$$

$$\boxed{21x + 77y - 101 = 0}$$ (obtuse)

Q find the eq^n of Acute Angle bisector btw the lines

$\cancel{L_1 : 3x-4y+7=0}$

$\cancel{L_2 : 12x+5y-2=0}$

(1) $L_1 : 3x-4y+10=0 \qquad L_2 : 12x+5y-1=0$

(2) $3x+4y-5=0$ & $12x+5y-7=0$

(1)

$$\frac{3x-4y+10}{5} = \pm\left(\frac{-12x-5y+1}{13}\right)$$

$$39x-52y+130 = -60x-25y+5$$

$$\boxed{99x-27y+125=0} \text{ (acute)}$$

$$\cancel{39x-52y+130 = 60x+25y-5}$$

(2)

$$\frac{-3x-4y+5}{5} = \pm\left(\frac{-12x-5y+7}{13}\right)$$

$$a_1a_2+b_1b_2>0$$

$$-39x-52y+65 = +60x+25y\mp35$$

$$\cancel{21x-27y+30=0}$$

$$\cancel{\boxed{7x-9y+10=0} \text{ (acute)}} \qquad \boxed{99x+77y-100=0} \text{ (acute)}$$

Q find the eq^n of bisector of $\angle ABC$ ~~where~~ if A, B & C are $(-2,7)$, $(4,1)$ & $(-3,0)$

$AB \Rightarrow (y-1)=(x-4)(-1) \qquad BC \Rightarrow y=(x+3)\left(\frac{1}{7}\right)$

$x+y-5=0 \qquad x-7y+3=0$

$$\frac{-x-y+5}{\sqrt{2}} = \pm\frac{(x-7y+3)}{5\sqrt{2}}$$

$$5x+5y-25 = x-7y+3$$

$$4x+12y-28=0$$

$$\boxed{x+3y-7=0}$$

H.W. 27-9-24

Pair of Straight line

① Homogenous Eqⁿ of (2) degree.

→ Always pass through origin (0,0).

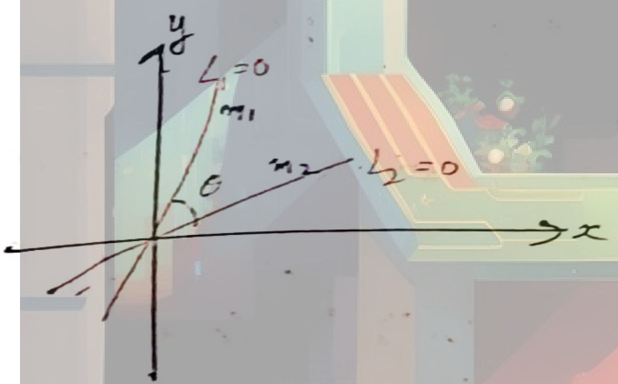
→ Degree of variables is same in each term

$$ax^2+2hxy+by^2=0$$

$$a+2h\left(\frac{y}{x}\right)+b\left(\frac{y}{x}\right)^2=0$$

$$bm^2+2hm+a=0 \quad ① \quad \left(\frac{y}{x}=m\right)$$

$$m_1+m_2=\frac{-2h}{b} \qquad m_1m_2=\frac{a}{b}$$



$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

If $\theta=0$, $\boxed{h^2=ab}$ lines are coincident

If $\theta=90°$, $\boxed{a+b=0}$ lines are ⊥.

$$\boxed{\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}}$$

If $a=b$ → slopes of lines are reciprocals. (m & $\frac{1}{m}$)

If $h=0$ → slopes are opposite sign same magnitude.

from eq ①

$D = 4h^2 - 4ab$

$D = 4(h^2 - ab)$

$h^2 - ab$

>0 real & diff lines

$=0$ real & coincide

<0 lines are imaginary but POI is $(0,0)$.

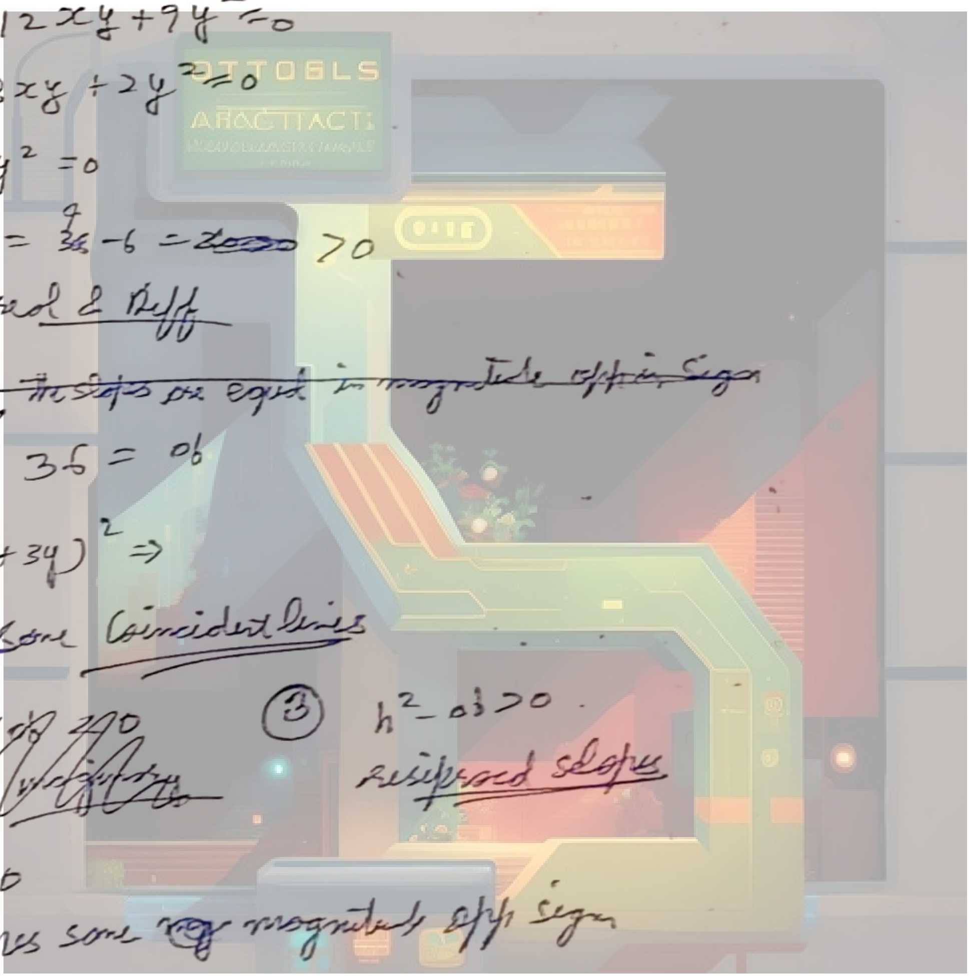
Q ~~Identify~~ Identify the lines.

① $2x^2 + 6xy + 3y^2 = 0$

② $4x^2 + 12xy + 9y^2 = 0$

③ $2x^2 + 8xy + 2y^2 = 0$

④ $4x^2 - 9y^2 = 0$



① $h^2 - ab = \cancel{36}\ 9 - 6 = \cancel{30} > 0$

real & diff

~~② real, the slopes are equal in magnitude opp in sign~~

② $h^2 = 36 = ab$

$(2x + 3y)^2 \Rightarrow$

Same coincident lines

③ $h^2 - ab > 0$

reciprocal slopes

④ $h = 0$

Slopes same ~~mag~~ magnitude opp sign

Q. solve ~~x^2~~ ~~$6y^2$~~ $= 0$ & find eq^n of lines

$x^2 - 5xy + 6y^2$

$bm^2 + 2mh + a \neq 0$

$6m^2$ ~~...~~ $-10m + 1 = 0$

$m = \dfrac{10 \pm \sqrt{100 - 36}}{12}$

$m = \dfrac{10 \pm 8}{12}$

$m = -\frac{1}{6}$ ~~$\frac{1}{3}$~~ $\frac{3}{2}$

$\frac{y}{x}$.

$$x^2 - 2xy - 3xy + 6y^2 = 0$$

$$x(x-2y) - 3y(x-2y) = 0$$

$$\cancel{x(x-3y)}$$

$$(x-3y)(x-2y) = 0$$

$$\boxed{x-3y=0} \qquad \boxed{x-2y=0}$$

// Pair of SOL $\perp$ to $ax^2+2hxy+by^2=0$

$$ax^2+2hxy+by^2=0$$

$m_1\ (0,0)$ $\qquad$ $m_2\ (0,0)$

$y = m_1 x$ $\qquad$ $y = m_2 x$

$\downarrow \perp$ $\qquad$ $\downarrow \perp$

$y = -\frac{1}{m_1} x$ $\qquad$ $y = -\frac{1}{m_2} x$

$$\left(y+\frac{1}{m_1}x\right)\left(y+\frac{1}{m_2}x\right) = 0$$

$$y^2 + xy\left(\frac{m_1+m_2}{m_1 m_2}\right) + \frac{x^2}{m_1 m_2} = 0$$

$$y^2 + xy\left(\frac{-2h}{a}\right) + \frac{bx^2}{a} = 0$$

$$ay^2 - 2hxy + bx^2 = 0$$

$$\boxed{bx^2 - 2hxy + ay^2 = 0}$$

Q find the combined eq^n of $\perp$ lines

(1) $2x^2+6xy+3y^2=0$ (2) $2x^2+3xy-3y^2=0$ (3) $4x^2-9y^2=0$

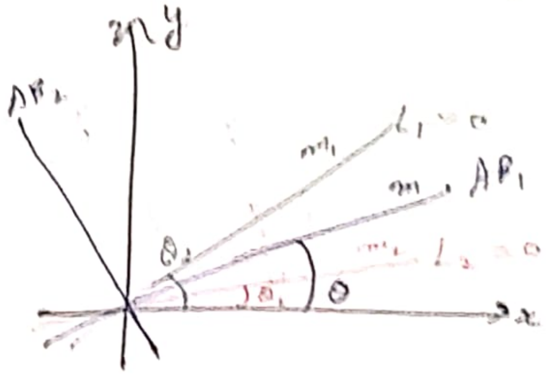
(1) $3x^2-6xy+2y^2=0$

(2) $-3x^2-3xy+2y^2=0$

(3) $-9x^2+4x^2=0$

~~Angle bisectors~~

Combined eqⁿ of angle bisector of a homogeneous eqⁿ



→ Always pass through $(0,0)$ (∠ bisectors).

$$\theta - \theta_1 = \theta_2 - \theta$$

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

$$\frac{2m}{1-m^2} = \frac{m_1 + m_2}{1 - m_1 m_2}$$

$$\cancel{\frac{2m}{1-m^2} = -2h}$$

$$\frac{\frac{2y}{x}}{1 - \frac{y^2}{x^2}} = \frac{-2h}{b-a}$$

$$\frac{2xy}{x^2-y^2} = \frac{2h}{a-b}$$

$$axy - bxy = hx^2 - hy^2$$

$$\boxed{hx^2 - (a-b)xy - hy^2 = 0}$$

$$\boxed{\frac{x^2-y^2}{a-b} = \frac{xy}{h}}$$

Q find the Cambined eq^n of ABs cfor

① $2x^2+3xy-y^2=0$

② $x^2-5xy+6y^2=0$

① $\frac{3}{2}x^2-3xy-\frac{3}{2}y^2=0$

$3x^2-6xy-3y^2=0$

$x^2-2xy-y^2=0$

② $-\frac{5}{2}x^2+\frac{5}{2}y^2+5xy$

$5x^2-10xy-5y^2=0$

$x^2-2xy-y^2=0$

General eq^n of 2nd degree representing POSL:

$ax^2+2hxy+2gx+2fy+by^2+c=0$

↳ condition for 2 lines.

$\Delta = abc+2fgh-ch^2-bg^2-af^2=0$

$$\Delta = \begin{vmatrix} a & f & g \\ -f & b & h \\ -g & -h & c \end{vmatrix} = 0$$

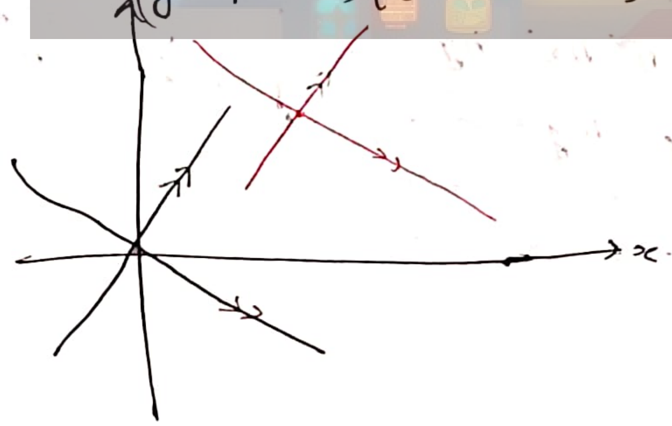
$\underline{ax^2+by^2+2hxy}+2gx+2fy+c=0$

↓

$(y-m_1x)(y-m_2x)$

↓

$(y-m_1x+c_1)(y-m_2x+c_2)=0$



Q $3x^2+8xy-3y^2+2x-4y-1=0$

i) find both lines

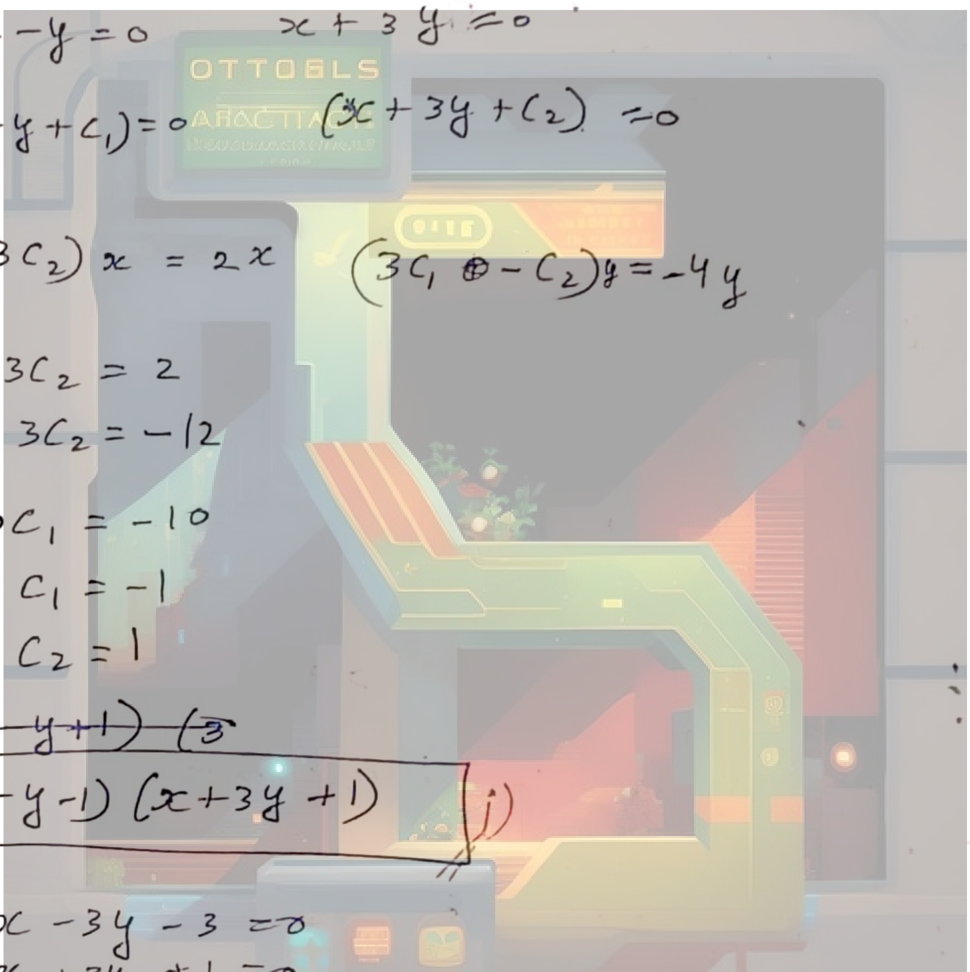
ii) find their POI.

$3x^2+8xy-3y^2$

$3x^2+9xy-xy-3y^2$

$\cancel{x(3x}$

$3x(x+3y)-y(x+3y)=0$



$3x-y=0 \qquad x+3y=0$

$(3x-y+c_1)=0 \qquad (x+3y+c_2)=0$

$(c_1+3c_2)x=2x \qquad (3c_1-c_2)y=-4y$

$c_1+3c_2=2$

$9c_1-3c_2=-12$

$10c_1=-10$

$c_1=-1$

$c_2=1$

$\cancel{(3x-y+1)(3}$

$\boxed{(3x-y-1)(x+3y+1)}$ i)

$9x-3y-3=0$

$x+3y+1=0$

$10x=2$

$x=1/5, \quad y=\frac{-2}{5}$

$\boxed{\left(\frac{1}{5}, \frac{-2}{5}\right)}$ ii)

Trick[5] ★ ★
for POI. Only POI is same not lines

$$\frac{\partial f}{\partial x} = 0$$

$$\frac{\partial f}{\partial y} = 0$$

[M II]

$$\frac{\partial f}{\partial x} = 6x + 8y + 2$$

$$\frac{\partial f}{\partial y} = 8x - 6y - 4 = 0$$

$$4x - 3y - 2 = 0$$

$$3x + 4y + 2 = 0$$

$$\cancel{7x + y = 0}$$

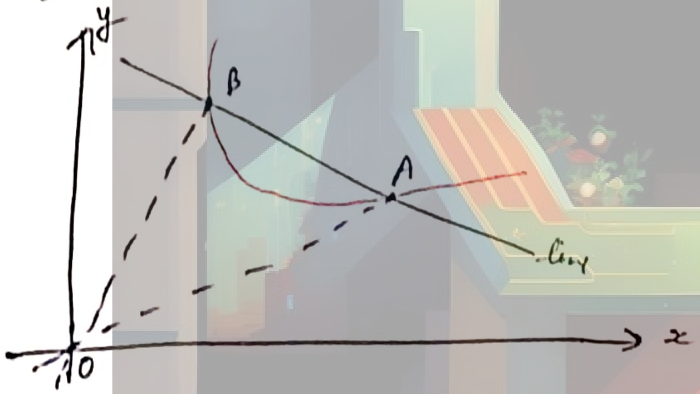
$$\cancel{25x}$$

$$\boxed{\left(\frac{1}{5}, \frac{-2}{5}\right)}$$

Angle btw lines

$$\boxed{\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|}$$

Homogenisation



→ Trying to make degree same in each term.

Q $x^2 - y^2 - xy + 3x - 6y + 18 = 0$

$2x - y = 3$

Homogenise it. / find the eqn of lines passing through $(0,0)$ & POI of line & curve

$2x - y = 3$

$1 = \frac{2x-y}{3}$ — ①

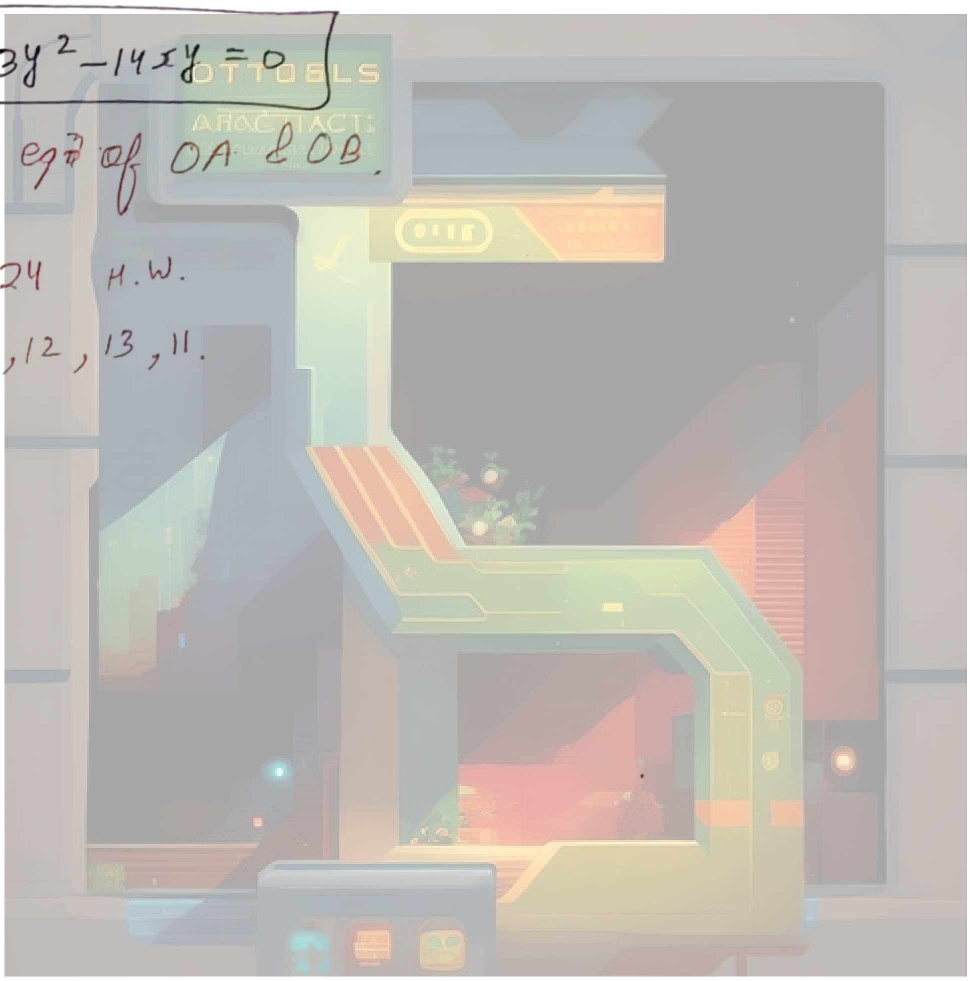
$x^2 - y^2 - xy + 3x(1) - 6y(1) + 18(1)^2 = 0$

$x^2 - y^2 - xy + \cancel{3}x\left(\frac{2x-y}{\cancel{3}}\right) - \overset{2y}{\cancel{6}}\left(\frac{2x-y}{\cancel{3}}\right) + \overset{2}{\cancel{18}} \times \frac{(2x-y)^2}{\cancel{9}}$

$x^2 - y^2 - xy + 2x^2 - xy - 4xy + 2y^2 + 8x^2 + 2y^2 - 8xy =$

$\boxed{11x^2 + 3y^2 - 14xy = 0}$

↳ eqn of OA & OB.



28-9-24 H.W.

DYS ~~14~~, 12, 13, 11.