! Trigonometric Equation !

→ An equation involving 1 or more than 1 TR of unknown angles

eg. $\cos\theta = \frac{1}{2}$

$\tan^2\theta - 4\tan\theta + 3 = 0$

$\sin^2\theta + \cos\theta - 1 = 0$

→ Types of solutions :-

Principal
→ Solutions of TE lying in the interval $[0, 2\pi)$

General Sol
→ All such possible values of θ for which the given TE is satisfied.
→ It is given by a general formula

Particular Sol
→ Solutions which are lying in the Interval given in the question

→ General Sol to be remembered.

(1)	$\sin\theta = 0$	$\theta = n\pi$	$n \in I$
(2)	$\cos\theta = 0$	$\theta = (2n-1)\frac{\pi}{2}$	$n \in I$
		$= (2n+1)\frac{\pi}{2}$	$n \in I$
(3)	$\tan\theta = 0$	$\theta = n\pi$	$n \in I$
(4)	$\cot\theta = 0$	$\theta = (2n-1)\frac{\pi}{2}$ or $(2n+1)\frac{\pi}{2}$	$n \in I$

(5) $\sec\theta = 0$ $\quad \theta \in \phi$

(6) $\operatorname{cosec}\theta = 0$ $\quad \theta \notin \phi$

(7) $\sin(k\theta) = 0$ $\quad k\theta = n\pi \quad n \in I$

$k \in$ constant $\quad \theta = \frac{n\pi}{k}$

(8) $\sin(a\theta + b) = 0$ $\quad \theta = \frac{n\pi - b}{a} \quad n \in I$

$a, b \in$ constant

01. find general sol of

(1) $\sin 3\theta = 0$

$3\theta = n\pi$

$\theta = \frac{n\pi}{3}$

$\boxed{\theta = \frac{n}{3}\pi} \quad n \in I$

(2) $\tan\frac{\theta}{2} = 0$

$\frac{\theta}{2} = n\pi$

$\boxed{\theta = 2n\pi} \quad n \in I$

General Sol of all Trignometric angles :-

(1) $\sin\theta = \sin\alpha \quad \theta = ?$

Proof :-

$\sin\theta - \sin\alpha = 0$

$2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\cos\frac{\theta+\alpha}{2} = 0 \qquad \sin\frac{\theta-\alpha}{2} = 0$

$\theta = 2n\pi + \pi - \alpha$ Or $\theta = 2n\pi + \alpha$

$n=0$ $\theta = \pi - \alpha$ $\theta = \pi$

$n=1$ $\theta = 3\pi - \alpha$ $\theta = 2\pi + \alpha$

$n=2$ $\theta = 5\pi - \alpha$ $\theta = 4\pi + \alpha$

$$\boxed{\theta = n\pi + (-1)^n \alpha}$$

(1) $\sin\theta = \sin\alpha$ $\theta = n\pi + (-1)^n \alpha$. $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

or $\theta = 2n\pi + \alpha$

(2) $\cos\theta = \cos\alpha$ $\theta = 2n\pi \pm \alpha$ $\alpha \in [0, \pi]$

(3) $\tan\theta = \tan\alpha$ $\theta = n\pi + \alpha$ $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$

(4) $\sin^2\theta = \sin^2\alpha$

$\cos^2\theta = \cos^2\alpha$ $\theta = n\pi \pm \alpha$

$\operatorname{cosec}^2\theta = \operatorname{cosec}^2\alpha$

Q2. $2\sin x = 1$ find general sol., Principal sol., particular sol in $(2\pi, 4\pi)$

$\sin x = \frac{1}{2}$

$\sin x = \sin\frac{\pi}{6}$

(1) $\boxed{x = n\pi + (-1)^n \frac{\pi}{6}}$ General

$n=0$ $x = \frac{\pi}{6}$ ✓

$n=1$ $x = \frac{5\pi}{6}$ ✓

$n=$ $x = \frac{17\pi}{6}$ ✗

$n=-1$ $x = -\frac{7\pi}{6}$ ✗

(2) $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ Principal

$n=3 \quad x = \frac{17\pi}{6}$ ✓ $\quad n=2 \quad x = \frac{13\pi}{6}$ ✓

$n=4 \quad x = \frac{25\pi}{6}$ ✗

③ $\boxed{x = \frac{17\pi}{6}, \frac{13\pi}{6}}$ Particular

Q3. a) $\cos x = \frac{\sqrt{3}}{2}$ fin GS., ~~P.S~~ $P_1 \cdot S$, $P_2 \cdot S\left(\frac{\pi}{2}, \frac{5\pi}{2}\right)$

$\cos x = \cos \frac{\pi}{6}$

$\boxed{x = 2n\pi \pm \alpha}$ GS.

$n=0 \quad x = \pm \frac{\pi}{6}$

$n=1 \quad x = \frac{13\pi}{6}, \frac{11\pi}{6}$

$n=2 \quad x = \frac{23\pi}{6}, \frac{25\pi}{6}$

$\boxed{P_1 \cdot S = x = \frac{\pi}{6}, \frac{11\pi}{6}}$

$\boxed{P_2 \cdot S \quad x = \frac{11\pi}{6}, \frac{13\pi}{6}}$

Q 4. $\tan x = -\sqrt{3}$ G.S, P.S

$\tan x = -\tan 60$

$\alpha = -60^\circ$

$\alpha = -\frac{\pi}{3}$

$x = n\pi + \alpha$

$x = n\pi - \frac{\pi}{3}$

$\boxed{x = \frac{(3n-1)\pi}{3}}$ G.S.

$n = 0 \quad -\frac{\pi}{3}$

$n = 1 \quad \frac{2\pi}{3}$

$n = 2 \quad \frac{5\pi}{3}$

P.S $= x = \boxed{\frac{2\pi}{3}, \frac{5\pi}{3}}$ P.S

Q 5. $\tan 3x = 1$

find G.S.

$\tan 3x = \tan \frac{\pi}{4}$

$3x = n\pi + \frac{\pi}{4}$

$3x = \frac{(4n-1)\pi}{4}$

$\boxed{x = \frac{(4n-1)\pi}{12}}$

Q6. $4\tan^2\theta = 3\sec^2\theta$ G.S.

$\theta \neq (2n-1)\frac{\pi}{2}$

$\frac{4\sin^2\theta}{\cos^2\theta} = \frac{3}{\cos^2\theta}$

$\sin^2\theta = \frac{3}{4}$

$\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$

$\alpha = \frac{\pi}{3}$

$x = n\pi \pm \alpha$

$x = n\pi \pm \frac{\pi}{3}$

$x = \frac{(3n \pm 1)\pi}{3}$

But for $x = \frac{(2n-1)\pi}{2} = \frac{(3n \pm 1)\pi}{3}$

$6n - 3 = (3n \pm 2)$

$3n = \pm 2 + 3$

$n = \frac{3 \pm 2}{4}$

$x = \frac{(3n \pm 1)\pi}{3}$ $n \in R - \left(\frac{3 \pm 2}{4}\right)$

$n \notin \left\{\frac{5}{4}, \frac{1}{4}\right\}$

$4\tan^2\theta\theta = 3 + \tan^2\theta$

$\boxed{\theta = n\pi \pm \frac{\pi}{3}} \quad n \in I$

Q7. Gr 5. $\sin^2\theta - \cos\theta = \frac{1}{4}$

$1 - \cos^2\theta - \cos\theta = \frac{1}{4}$

$\cos^2\theta + \cos\theta - \frac{3}{4} = 0$

$\cos\theta = \frac{-1 \pm \sqrt{1+3}}{2}$

$\cos\theta = \frac{-1 \pm \sqrt{4}}{2}$

$\cos\theta = \frac{-1 \pm 2}{2}$

$\cos\theta = -\frac{3}{2}, \frac{1}{2}$

X

$\cos\theta = \frac{1}{2}$

$\cos\theta = \cos\frac{\pi}{3}$

$\theta = \frac{\pi}{3}$

$\theta = 2n\pi \pm \frac{\pi}{3}$

Q Pg. 5 Q4 4.16 $\frac{\sin^2 x}{\cos x} = 2$

$\sin^2 x = \sin x$

$\sin x = 0 \quad \sin x = 1$

$\sin x = \sin 0° \quad \sin x = \sin \frac{\pi}{2}$

$2^{4\sin^2 x + 2} = 2^{6\sin x}$

$4\sin^2 x - 6\sin x + 2 = 0$

$\sin x = 1$

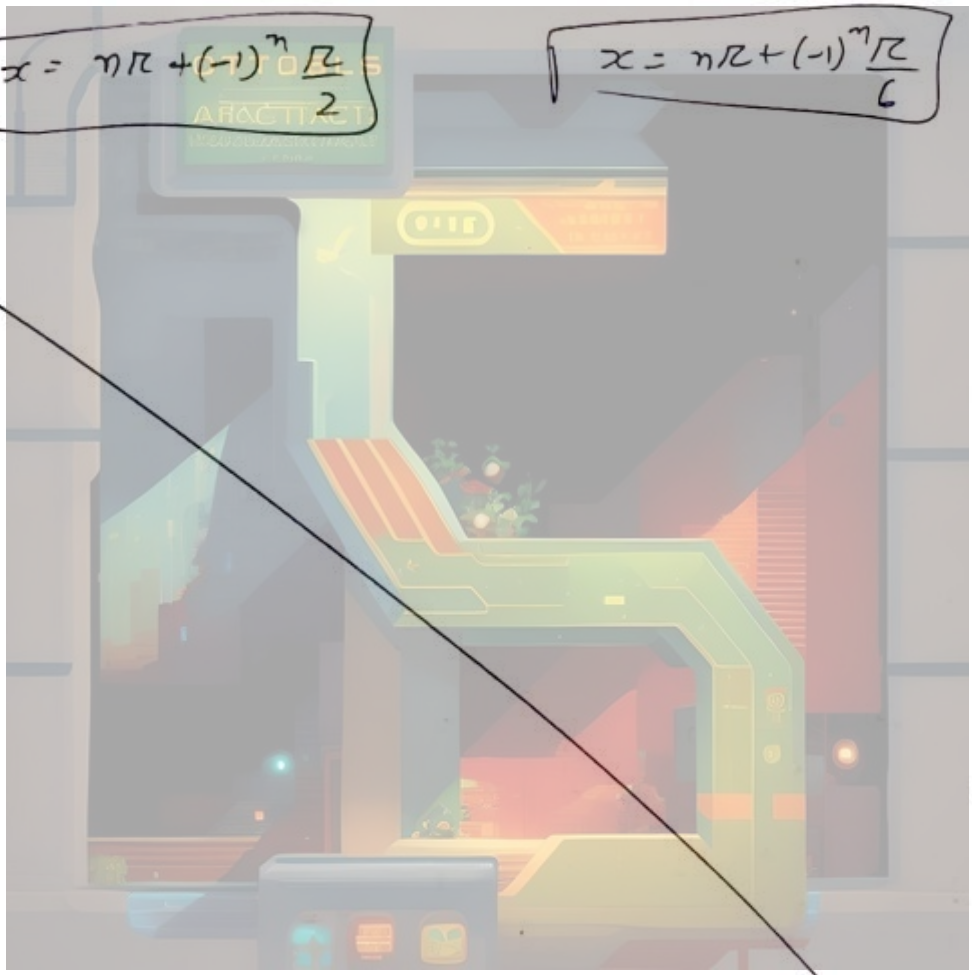
$\sin x = \sin \frac{\pi}{2}$

$\sin x = \frac{1}{2}$

$\sin x = \sin \frac{\pi}{6}$

$x = n\pi + (-1)^n \frac{\pi}{2}$

$x = n\pi + (-1)^n \frac{\pi}{6}$



Q Pr. Sol of $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$

$\log_{1/2}|\sin x| = \log_{1/2}\frac{4}{|\cos x|}$

$|\sin x| = \frac{4}{|\cos x|}$

$\sin x = \frac{4}{\cos x}$ $\quad$ $\sin x = \frac{-4}{\cos x}$

$|\sin x \cos x| = \frac{1}{4}$

$|\sin 2x| = \frac{1}{2}$

$\sin 2x = \frac{1}{2}$ $\quad$ $\sin 2x = -\frac{1}{2}$

$\sin 2x = \sin 30$ $\quad$ $\sin 2x = \sin(-30)$

$2x = n\pi + (-1)^n \frac{\pi}{6}$ $\quad$ $2x = n\pi - (-1)^n \frac{\pi}{6}$

$(\sin 2x \geq 0)$ $\quad$ $(\sin 2x \leq 0)$

Q Find most general values of θ

① $\sin\theta = \frac{-1}{2}$ & $\tan\theta = \frac{1}{\sqrt{3}}$

$\sin\theta = \sin(-30)$ $\quad$ $\tan\theta = \tan(30)$

$\alpha = -30$ $\quad$ $\alpha = 30$

$\theta = n\pi - (-1)^n \frac{\pi}{6}$ $\quad$ $\theta = n\pi + \frac{\pi}{6}$

$n=0$ $\quad -\frac{\pi}{6}$ $\quad$ $\frac{\pi}{6}$

$n=1$ $\quad \frac{7\pi}{6}$ $\quad$ $\frac{7\pi}{6}$

$n=2$ $\quad \frac{11\pi}{6}$ $\quad$ $\frac{13\pi}{6}$

$n=3$ $\quad \frac{19\pi}{6}$ $\quad$ $\frac{19\pi}{6}$

~~$\theta = 2n\pi + \alpha$~~

~~$\theta = n\pi + (-1)^n \alpha$~~

$\theta = (2n-1)\pi + \alpha$ $\quad n \in I$

(2) $\tan^2\alpha + 2\sqrt{3}\tan\alpha - 1 = 0$

$$\tan\alpha = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$$

$$= \frac{-2\sqrt{3} \pm 4}{2}$$

$$= -\sqrt{3} \pm 2$$

$\tan\alpha = -\sqrt{3} - 2$

$\tan\alpha = -(\sqrt{3}+2)$

$\tan\alpha = \frac{-1}{\tan 15^\circ}$

$\tan\alpha = -\tan(-75)$

$\alpha = n\pi - 75^\circ$

$n=0$ $\quad -75$

$n=1$ $\quad \alpha = 180n - 75$

$n=1$ $\quad \alpha = 105$

$n=2$ $\quad \alpha = 285$

$\tan\alpha = 2 - \sqrt{3}$

$\tan\alpha = \tan 15^\circ$

$\alpha = n\pi + 15^\circ$

75

$\alpha = 180n + 15$

$\alpha = 195$

$\alpha = 375$

$\alpha = 180n + (-30 \pm 45)$

$\alpha = 180n\pi + (-30 \pm 45)$

Q $1+\cos 3x - 2\cos 2x = 0$

~~$1+\cos 3x$~~

$1 + 4\cos^3 x - 3\cos x - 2(2\cos^2 x - 1) = 0$

$4\cos^3 x + 1 - 3\cos x - 4\cos^2 x + 2 = 0$

$4\cos^3 x - 4\cos^2 x - 3\cos x + 3 = 0$

~~$\cos x - 1$~~

$4\cos^2 x\,(c-1) - 3(c-1) = 0$

$(4c^2 - 3) = 0$ $\qquad c - 1 = 0$

$c = 1$

$c^2 = \frac{3}{4}$

~~$\cos x = \frac{\sqrt{3}}{2}$~~

$\cos x = \cos 0$

$\cos^2 x = \cos^2 30$

$x = 2n\pi \pm 0$

$x = n\,d \pm 30°$

imp points for solving TE

① Cancellation of terms which are in product is not allowed

eg. $\sin\theta\cos\theta = \sin\theta$

$\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(\cos\theta - 1) = 0$

$\sin\theta = 0$ $\qquad \cos\theta = 1$

② Answer should not contain such values of angle which make any of the terms undefined in the given question

$\tan\theta, \sec\theta$: $\theta \neq \{(2n+1)\frac{\pi}{2}\}$

$\cot\theta, \operatorname{cosec}\theta$: $\theta \neq \{n\pi\}$

Eg. $\frac{\sin 2\theta}{\cos\theta} = 2$

$\Rightarrow \frac{2\sin\theta\cos\theta}{\cos\theta} = 2$

$\Rightarrow 2\sin\theta = 1 \quad \{\cos\theta \neq 0\}$

$\sin\theta = \sin\frac{\pi}{2}$

$\theta = n\pi \pm (-1)^n \frac{\pi}{2}$

but at $\theta = n\pi \pm (-1)^n \frac{\pi}{2}$, $\cos\theta = 0$

so, $\theta \in \phi$

③ Direct Squaring is not allowed as it gives extra undesired solutions

Different Strategies

① factorisation:- Wherever factorisation possible.

Q1. $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

Q2. $2\cos x \cos 2x = \cos x$

Q3. $2\sin^2 2x + 6\cos^2 x = 5$

Q1. $(2S-C)(1+C) = 1-C^2$

$(2S-C)(C+1) = (1+C)(1-C)$

$(2S-C)(1+C) - (1+C)(1-C) = 0$

$(2S-C-1+C)(1+C) = 0$

$(2S-1)(1+C) = 0$

$\cos x = -1$

$\cos x = \cos\pi$

$\therefore x = 2n\pi \pm \pi$ or

$\sin x = 1/2$

$\sin x = \sin\frac{\pi}{6}$

$\therefore x = n\pi + (-1)^n \frac{\pi}{6}$

(2) $2\cos x \cos 2x = \cos x$

$\cos x(2\cos 2x - 1) = 0$

~~$\cos x(-\cos 2x) = 0$~~

~~$\cos 2x = 0$~~ $\cos 2x = 0$

~~$\cos 2x =$~~

$\cos x = 0$

$\cos x = \frac{\pi}{2}$

$x = 2n\pi \pm \frac{\pi}{2}$

$\boxed{x = 2n\pi \pm \frac{\pi}{2}}$

$\cos 2x = \frac{1}{2}$

$\cos 2x = \cos \frac{\pi}{3}$

$2x = 2n\pi \pm \frac{\pi}{3}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

(3) $2\sin^2 2x + 6\sin^2 x = 5$

$2(2\sin x \cos x)^2 + 6\sin^2 x = 5$

$8\sin^2 x \cos^2 x + 6\sin^2 x = 5$

$2\sin^2 x(4\cos^2 x + 3) = 5$

(3) $2(2\sin x \cos x)^2 + 6\sin^2 x = 5$

$8\sin^2 x \cos^2 x + 6\sin^2 x - 5 = 0$

$8\sin^2 x(1 - \sin^2 x) + 6\sin^2 x - 5 = 0$

$8\sin^2 x - 8\sin^4 x + 6\sin^2 x - 5 = 0$

$8\sin^4 x - 14\sin^2 x + 5 = 0$

$\sin^2 x = \frac{14 \pm \sqrt{196 - 160}}{16}$

$\sin^2 x = \frac{14 \pm 6}{16}$

$\sin^2 x = \frac{20}{16}$, $\sin^2 x = \frac{1}{2}$

X

$\sin^2 x = \sin^2(45)$

$x = \frac{\pi}{4}$

$\boxed{x = n\pi \pm \frac{\pi}{4}}$

ALT

$2(1 - \cos^2 2x) + 6\left(\frac{1 - \cos 2x}{2}\right) = 5$

$\cos 2x = t$

$2 - 2t^2 + 3 - 3t - 5 = 0$

$2t^2 + 3t = 0$

~~$\cos 2x = 0$~~

② reducing in Quad

Q1. $\cos 4x + 6 = 7\cos 2x$

$2\cos^2 2x - 1 + 6 = 7\cos 2x$

$2\cos^2 2x - 7\cos 2x + 5 = 0$

$\cos 2x = \frac{7 \pm \sqrt{49 - 40}}{4}$

$= \frac{7 \pm 3}{4}$

$\cos 2x = \frac{10}{4}$ ✗

$\cos 2x = \frac{4}{4}$

$\cos 2x = \cos 0^\circ$

$2x = 2n\pi \pm 0$

$\boxed{x = \frac{n\pi}{2}}$

Q2. $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$

$(\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x \cos^2 2x = \sin 2x \cos 2x$

$1 - 2(1 - \cos^2 2x)\cos^2 2x = 0$

$2x = 0$

$1 - 2\cos^2 0 \sin^2 0 - \sin 0 \cos 0 = 0$

$2\cos^2 0 \sin^2 0 + \sin 0 \cos 0 - 1 = 0$

$\cos 0 \sin 0 = \frac{-1 \pm \sqrt{1+8}}{4}$

$= \frac{-1 \pm 3}{4}$

$\sin 0 \cos 0 = -1, \frac{1}{2}$

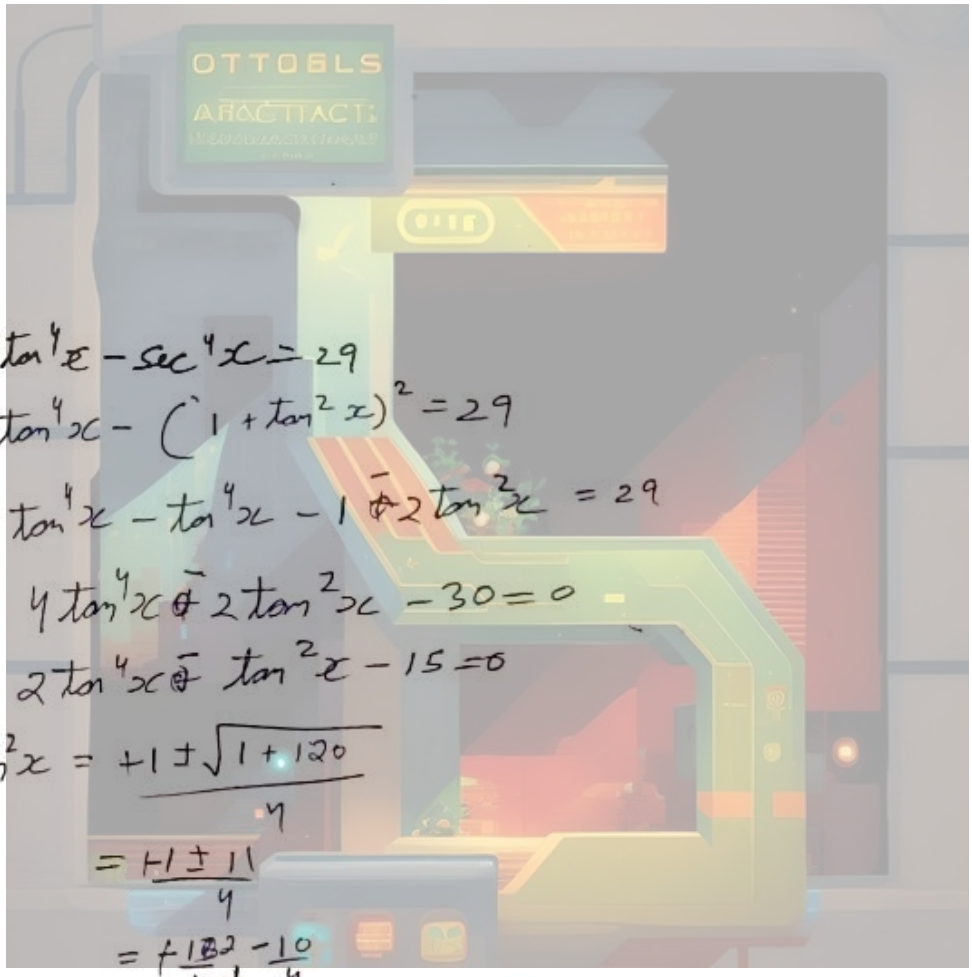
$\sin 4x = -2$ ✗

$\sin 4x = 1$

$\sin 4x = \sin \frac{\pi}{2}$

$4x = n\pi + (-1)^n \frac{\pi}{2}$

$$x = \frac{n\pi}{4} + (-1)^n \frac{\pi}{8}$$



Q3. $5\tan^4 x - \sec^4 x = 29$

$5\tan^4 x - (1 + \tan^2 x)^2 = 29$

$5\tan^4 x - \tan^4 x - 1 - 2\tan^2 x = 29$

$4\tan^4 x - 2\tan^2 x - 30 = 0$

$2\tan^4 x - \tan^2 x - 15 = 0$

$\tan^2 x = \frac{+1 \pm \sqrt{1 + 120}}{4}$

$= \frac{+1 \pm 11}{4}$

$= +\frac{12}{4}, -\frac{10}{4}$

~~$\tan^2 x = \frac{-10}{4}$~~ ✗

$\tan^2 x = -\frac{10}{4}$ ✗

~~$\tan^2 x = -\frac{5}{2}$~~

$\tan^2 x = 3$

$\tan^2 x = \tan^2\left(\frac{\pi}{3}\right)$

$$x = n\pi \pm \frac{\pi}{3}$$

$x \in \{(2n+1)\frac{\pi}{2}\}$

~~③ $a\sin\theta + b\cos\theta = c$~~

③ $a\sin\theta + b\cos\theta = c$ form $\{a, b \in \mathbb{R}\}$

MI multiple & divide LHS by $\sqrt{a^2+b^2}$

MII $\sin\theta = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ & $\cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ of form ques in $\tan\frac{\theta}{2}$

Q1. $\sin x + \cos x = \sqrt{2}$

MI

$$\frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2}} = \sqrt{2}$$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = 1$$

$$\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x = 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos 0^\circ$$

$$x - \frac{\pi}{4} = 2n\pi$$

$$\boxed{x = 2n\pi + \frac{\pi}{4}}$$

MII

$$\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \sqrt{2}$$

$$\frac{2t + 1 - t^2}{t^2+1} = \sqrt{2}$$

$$\sqrt{2}t^2 + \sqrt{2} + t^2 - 2t - 1 = 0$$

$$(\sqrt{2}+1)t^2 - 2t + (\sqrt{2}-1) = 0$$

$$t^2 - \frac{2}{\sqrt{2}+1}t + \frac{\sqrt{2}-1}{\sqrt{2}+1} = 0$$

$$t^2 - 2(\sqrt{2}-1)t + (\sqrt{2}-1)^2 = 0$$

$$t = (\sqrt{2}-1)$$

$$\tan\frac{x}{2} = \tan 22.5$$

$$\frac{x}{2} = n\pi + \frac{\pi}{8}$$

$$\boxed{x = 2n\pi + \frac{\pi}{4}}$$

Q2. $\sqrt{3}\cos x + \sin x = 2$

$$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 1$$

$$\cos\frac{\pi}{6}\cos x + \sin x \sin\frac{\pi}{6} = 1$$

$$\cos\left(x - \frac{\pi}{6}\right) = 1$$

$$\cos\left(x - \frac{\pi}{6}\right) = \cos 0$$

$$x - \frac{\pi}{6} = 2n\pi + 0$$

$$\boxed{x = 2n\pi + \frac{\pi}{6}}$$

Q3. $\sin x + \cos x = \min\{1, a^2 - 4a + 5, 2\}$

$$\min = \frac{-D}{4a}$$

$$= \frac{-(16 - 24)}{4}$$

$$= 2$$

$$\sin x + \cos x = 1$$

$$\sin x \sin\frac{\pi}{4} + \cos x \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\boxed{x = 2n\pi + \frac{\pi}{4} \pm \frac{\pi}{4}}$$

Q4. $1+\sin^3 x+\cos^3 x = \frac{3}{2}\sin 2x$

$(1)^3 + \sin^3 x + \cos^3 x = \frac{3}{2} \times 2\sin x \cos x$

$A^3+B^3+C^3 = 3ABC$

$A+B+C = 0$

$\sin x + \cos x + 1 = -1$

$\sin x \sin\frac{45}{\sqrt{2}} + \cos x \cos\frac{45}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos\left(x - \frac{45}{\sqrt{2}}\right) = \cos(90+45^\circ)$

$x = 2n\pi \pm 135 + \frac{45}{\sqrt{2}}$

Q5. $3\cos x + 4\sin x = 5$

$\frac{3\cos x}{5} + \frac{4\sin x}{5} = 1$

$\cos x \cos 53 + \sin x \sin 53 = 1$

$\cos(x-53) = \cos 90^\circ$

$x - 53 = 2n\pi \pm 90$

$x = 2n\pi \pm 90 + 53$

(4) Sin ⟷ Product

~~A~~ Cos.

Q1. $\cos 3x + \sin 2x - \sin 4x = 0$

Q2. $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Q3. $\sin 5x \cos 3x = \sin 6x \cos 2x$

Q4. $\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

Q5. $8 \cos x \cos 2x \cos 4x = \frac{\sin 6x}{\sin x}$

Q1. $\cos 3x + 2\cos(\theta 3x)\sin(-2x) = 0$

$\cos 3x(1 - 2\sin 2x) = 0$

$\cos 3x = 0$, $1 - 2\sin x = 0$

$\cos 3x = \cos\frac{\pi}{2}$, $\sin x = \frac{1}{2}$

$\sin x = \sin\frac{\pi}{6}$

$3x = 2n\pi \pm \frac{\pi}{2}$

$\boxed{x = \frac{2n\pi}{3} \pm \frac{\pi}{6}}$

$\boxed{x = n\pi + (-1)^n \frac{\pi}{6}}$

Q2. $(\cos\theta + \cos 7\theta) + (\cos 3\theta + \cos 5\theta) = 0$

$(2\cos 4\theta \cos 3\theta) + (2\cos 4\theta \cos\theta) = 0$

$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$

$2\cos 4\theta(2\cos 2\theta \cos\theta) = 0$

$4\cos 4\theta \cos 2\theta \cos\theta = 0$

$\cos\theta = \cos\frac{\pi}{2}$, $\cos 2\theta = \cos\frac{\pi}{2}$, $\cos 4\theta = \cos\frac{\pi}{2}$

$\boxed{\theta = 2n\pi \pm \frac{\pi}{2}}$ or $\boxed{\theta = n\pi \pm \frac{\pi}{4}}$ or $\boxed{\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}}$

Q 3. $2\sin 5x \cos 3x = 2\sin 6x \cos 2x$

$\sin 8x + \sin 2x = \sin 8x + \sin 4x$

$\sin 2x = 2\sin x \cos x$

$2\sin 2x \cos 2x - \sin 2x = 0$

$\sin 2x (2\cos 2x - 1) = 0$

$\sin 2x = 0$

$\sin 2x = \sin 0$

$2x = n\pi + (-1)^n \theta$

$x = \frac{n\pi}{2}$

$2\cos 2x - 1 = 0$

$2\cos 2x = 1$

$\cos 2x = \frac{1}{2}$

$\cos 2x = \cos \frac{\pi}{3}$

$2x = 2n\pi \pm \frac{\pi}{3}$

$x = n\pi \pm \frac{\pi}{6}$

Q4. $\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

$4\cos\theta \cos 2\theta \cos 3\theta = 1$

$2\cos\theta \cos 2\theta \cdot 2\cos 3\theta = 1$

$(\cos 3\theta + \cos\theta) 2\cos 3\theta = 1$

$2\cos 3\theta \cos 3\theta + 2\cos\theta \cos 3\theta = 1$

$\cos 6\theta + 1 + \cos 4\theta + \cos 2\theta = 1$

$\cos 6\theta + \cos 4\theta + \cos 2\theta = 0$

$\frac{\cos 4\theta \sin 6\theta}{\sin 2\theta} = 0$

$\cos 4\theta = 0$

$\cos 4\theta = \cos \frac{\pi}{2}$

$\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$

$\sin 6\theta = 0$

$\sin 6\theta = \sin 0$

$\theta = \frac{n\pi}{6}$

$\left\{\theta \neq \frac{n\pi}{2}\right\}$

05. $8\cos x \cos 2x \cos 4x = \frac{\sin 6x}{\sin x}$ $-\{x \neq n\pi\}$

2

$2 = 4 \times 2 \sin x \cos x \cos 2x \cos 4x = \sin 6x$

$4 \times 2 \sin 2x \cos 2x \cos 4x = \sin 6x$

$2 \sin 4x \cos 4x = \sin 6x$

$\sin 8x = \sin 6x$

$\sin 8x - \sin 6x = 0$

$2 \cos 7x \sin x = 0$

$\cos 7x = \cos \frac{\pi}{2}$ $\quad \sin x = \sin 0$

$7x = 2n\pi \pm \frac{\pi}{2}$ $\quad x = n\pi \pm 0 (-1)^n$

$\boxed{x = n\pi}$ X

$\boxed{x = \frac{2}{7} n\pi \pm \frac{\pi}{14}}$

(5) $(\sin x \pm \cos x)(\sin x \cos x)$ formot

(1) $(\sin x + \cos x)^2 = t$

$\sin x \cos x = \frac{t^2 - 1}{2}$

(2) $(\sin x \mp \cos x)^2 = t$

$\sin x \cos x = \frac{1 - t^2}{2}$

Q1. $\sin x + \cos x = 1 + \sin x \cos x$

$t = \frac{2 + t^2 - 1}{2}$

$t^2 + 1 = 2t$

$t^2 - 2t + 1 = 0$

$t = \frac{2 \pm \sqrt{4 - 4}}{2}$

$t = 1$

MI

$t = 1$

$\sin x \cos x \neq 0$

$\sin x = 0$

$\boxed{x = n\pi}$

$\cos x = 0$

$\boxed{x = 2n\pi \pm \frac{\pi}{2}}$

MII

$\sin x + \cos x = 1$

$\frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2}} = 1$

$\frac{1}{\sqrt{2}} = \sin x \sin 45 + \cos x \cos 45$

$\cos(x - 45) = \frac{1}{\sqrt{2}} \cos 45$

$x - 45 = 2n\pi \pm 45$

$x = 2n\pi + 45 \pm 45$

$x = 2n\pi + \frac{\pi}{2}$ $\qquad$ $x = 2n\pi$

(i) Boundry

→ mostly we use maximum & minimum values of T.R. (y ki values)

Q1. $\cos x + \cos 2x + \cos 3x = 3$

Q2. $\sin^4 x = 1 + \cos^4 y$

Q3. $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$, $y > 0$ find x

Q4. $2^{\frac{1}{\sin^2 x}} \times \sqrt{y^2 - 2y + 2} = 2$ Find x in $[0, 2\pi]$

Q5. $2\cos^2\frac{x}{2} \sin^2 x = x^2 + \frac{1}{x^2}$ $\qquad$ $x \in [0, \frac{\pi}{2}]$

Q6. $\cos(\pi\sqrt{x-4}) \cdot \cos(\pi\sqrt{x}) = 1$

Q1. $\cos x + \cos 2x + \cos 3x = 3$

$\cos x = 1$

$\cos x = \cos 0$

$x = 2n\pi$

LCM = $2n\pi$

and

$\cos 2x = 1$

$\cos 2x = \cos 0$

$2x = 2n\pi$

$x = n\pi$

LCM = $\frac{2n\pi}{3}(6, 3, 2)$

and

$\cos 3x = 1$

$\cos 3x = \cos 0$

$3x = 2n\pi$

$x = \frac{2n\pi}{3}$

LCM = 6

$2n\pi$

Q2. ~~$\sin^4 x = 1 + \cos^6 y$~~

~~$\sin^4 x - \cos^6 y = 1$~~

Q2. $\sin^4 x = 1 + \cos^6 y$

$\sin^4 x$: max = 1

$1 + \cos^6 y$: min > 1

$\cos^6 y \neq$ ~~$\leq$~~ 0 ✓

$\cos^6 y = 0$

$\cos y = 0$

$y = 2n\pi$

$\sin^4 x = 1$

$\sin x = \sin \frac{\pi}{2}$

$x = 2n\pi \pm \frac{\pi}{2}$

Q3. $\sin x + \cos x \geq \sqrt{y + \frac{1}{y}}$ $y > 0$

$\sin x + \cos x \geq \sqrt{2}$

$\sin x \sin 45 + \cos x \cos 45 \geq 1$

$\cos(x - 45) \geq 1$

~~$\cos x$~~ - but cos max value = 1

$\cos(x - 45) = 1$

$x - 45 = 2n\pi$

$x = 2n\pi + 45$

$(S+0)^2 = y + \frac{1}{y}$

$1 + 2\sin 2x \geq 2$

$\sin 2x = 1$

Q 4.

$y^2 - 2y + 2$

$\min = \frac{-D}{4a} = \frac{4ac-b^2}{4a} = \frac{8-4}{4}$

$= 1$

$\frac{2}{\sin^2 x} \geq 1$

2

$2^1 \geq 2^{\frac{1}{\sin^2 x}}$

$1 \geq \frac{1}{\sin^2 x}$

$\sin^2 x \geq 1$

$\sin^2 x = 1$

$\sin^2 x = \sin^2 \frac{\pi}{2}$

$x = n\pi \pm \frac{\pi}{2}$

$\frac{\pi}{2}, \frac{3\pi}{2}$

Q 5, $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$x^2 + \frac{1}{x^2} + 2 \geq 4$

$x^2 + \frac{1}{x^2} \geq 2$

$\cos^2 \frac{\pi}{2} = 1$

$\cos^2 \frac{\pi}{2} = \cos^2 \frac{\pi}{2}$

$\sin^2 x = 1$

$\sin^2 x = \sin^2 \frac{\pi}{2}$

$x = n\pi \pm \frac{\pi}{2}$ and

$\cos^2 \frac{x}{2} = 1$

$\frac{x}{2} = n\pi$

$x = 2n\pi$

Q6. $\cos(\pi\sqrt{x-4}) = \cos(\pi\sqrt{x})$

either both (1) or (-1)

↓ not possible

$\cos(\pi\sqrt{x-4}) = -1$ | $\cos(\pi\sqrt{x}) = -1$

$\cos(\pi\sqrt{x-4}) = \cos(\pi)$ | $\cos(\pi\sqrt{x}) = \cos\pi$

$\pi\sqrt{x-4} = (2n\pm1)\pi$ | $\pi\sqrt{x} = (2n\pm1)\pi$

$x-4 = (2n\pm1)^2$ | $x = (2n\pm1)^2$

$x = (2n\pm1)^2 + 4$

H.W 8 - 8 - 24

DYS 3, 1, 5

Q6. IM II

$\cos(\pi\sqrt{x-4}) = \sec(\pi\sqrt{x})$

$\cos(\pi\sqrt{x-4}) = 1$ | $\sec(\pi\sqrt{x}) = 1$

$x = 4$
$x = 8$
$x = 20$
⋮

and

$x = 0, 4$
$x = 16$
$x = 36$
⋮

$x = 4$ (common)

$\cos(\pi\sqrt{x-4}) = -1$ and $\sec(\pi\sqrt{x}) = -1$

No common

System of T.E

Q1. $\sin x \sin y = \frac{\sqrt{3}}{4}$ — ① $\quad \cos x \cos y = \frac{\sqrt{3}}{4}$ — ②

② + ①

$$\sin x \sin y + \cos x \cos y = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\cos(x-y) = \cos\frac{\pi}{6}$$

$$x - y = 2n\pi \pm \frac{\pi}{6} \quad \text{— ③}$$

② − ①

$$\cos(x+y) = \cos\frac{\pi}{2}$$

$$x + y = 2k\pi \pm \frac{\pi}{2} \quad \text{— ④}$$

① + ② = $\quad 2x = 2\pi(n+k) \pm \frac{\pi}{6} \pm \frac{\pi}{2}$

$$\boxed{x = 2\pi(n+k) \pm \frac{\pi}{12} \pm \frac{\pi}{4}}$$

Q2. $x + y = \frac{2\pi}{3} \qquad \frac{\sin x}{\sin y} = 2$

~~$\sin(x+y) = \sin\frac{2\pi}{3} \cdot \cos 30$~~ $\qquad \sin x - \sin 30 = \sin y$

~~$\sin(x-y) = \sin x \cos y - \cos x \sin y$~~

M I fix y & put in eq to get A whole eq in x

M II

$$\frac{\sin x + \sin y}{\sin x - \sin y} = \frac{3}{1} = \frac{2\sin\frac{(x+y)}{2}\cos\frac{(x-y)}{2}}{2\sin\frac{(x-y)}{2}\cos\frac{(x+y)}{2}} = 3$$

$$= \frac{\sin 60 \cos\frac{(x-y)}{2}}{\sin\frac{(x-y)}{2}\cos 60} = 3$$

$$\frac{\sqrt{3}}{3} = \tan\left(\frac{x-y}{2}\right)$$

$\tan\left(\frac{x-y}{2}\right) = \frac{1}{\sqrt{3}}$

$\frac{x-y}{2} = n\pi + \frac{\pi}{6}$

$x - y = 2n\pi + 60^\circ$

$2x = 2n\pi + 120^\circ$

$\boxed{x = n\pi + 60^\circ}$

Q $a^2 + 2a + \operatorname{cosec}^2\left(\frac{\pi}{2}(a+x)\right)$

(A) $a = 1$, $\frac{x}{2} \in I$ (C) $a \in R$, $x \in \phi$

(B) $a = -1$ $\frac{x}{2} \in I$ (D) a & x are finite but not possible to find.

MI

$$a = \frac{-2 \pm \sqrt{4 - 4\operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right]}}{2}$$

$$a = -1 \pm \sqrt{1 - \operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right]}$$

$\boxed{a = -1}$

$$\operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right] = 1$$

$$\frac{\pi}{2}(a+x) = n\pi \pm \pi$$

$$a + x = 2n \pm 2$$

$$x = 2n \pm 2 + 1$$

$\boxed{B}$

MII

$$a^2 + 2a + 1 + \operatorname{cosec}^2\left(\frac{\pi}{2}(a+x)\right) = 1$$

$$\underbrace{(a+1)^2}_{0} + \underbrace{\operatorname{cosec}^2\left(\frac{\pi}{2}(a+x)\right)}_{1} = 1$$

Q4. $(\sin x - 1)^2 + \cos^4 y + \left|\tan^2 z - \sqrt{3}\right| = 0$ find x, y, z

$\sin x = 1$ $\cos y = 0$ $\tan^2 z = \sqrt{3}$

$\boxed{x = n\pi + (-1)^n \frac{\pi}{2}}$ and $\boxed{y = 2n\pi}$ and $\boxed{z = n\pi \pm \frac{\pi}{3}}$

$\tan^2 z = 3$

$\boxed{z = n\pi \pm \frac{\pi}{3}}$

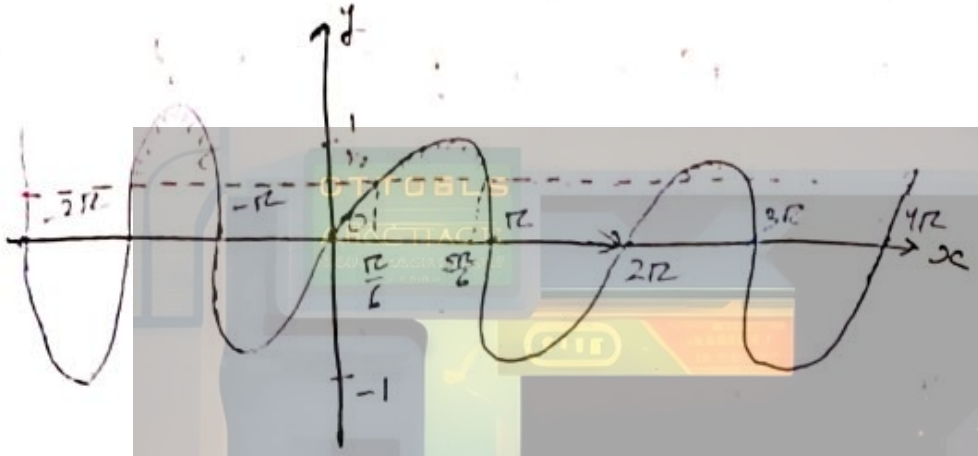
Trigonometric Inequalities

Q find the values of x for $\sin x > \frac{1}{2}$

i) when $x \in [0, 2\pi]$

ii) $x \in [-2\pi, 4\pi]$

iii) $x \in \mathbb{R}$

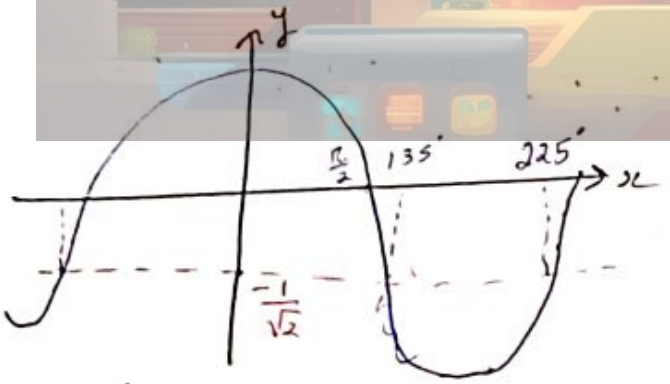


i) $x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

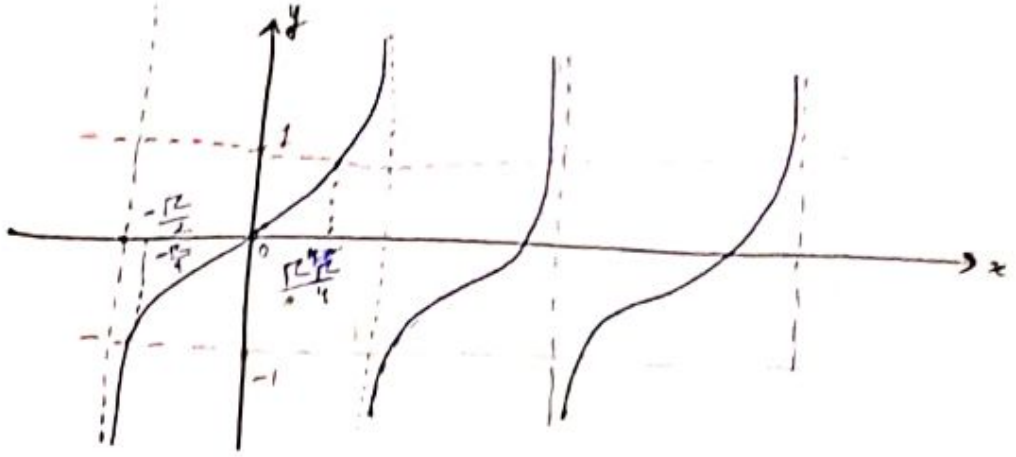
ii) $x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \cup \left(2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}\right) \cup \left(\frac{\pi}{6} - 2\pi, \frac{5\pi}{6} - 2\pi\right)$

iii) $x \in \left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$

Q2. $\cos x > -\frac{1}{\sqrt{2}}$



$x \in (2n\pi + 135^\circ, 2n\pi - 135^\circ)$



$x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]$

Q4. $\tan^2 x - (\sqrt{3}+1)\tan x + \sqrt{3} < 0$

$\tan^2 x - \sqrt{3}\tan x - \tan x + \sqrt{3} < 0$

$\tan x(\tan x - \sqrt{3}) - 1(\tan x - \sqrt{3})$

$\tan x = \sqrt{3} \qquad \tan x = 1$

$\tan x \in [1, \sqrt{3})$

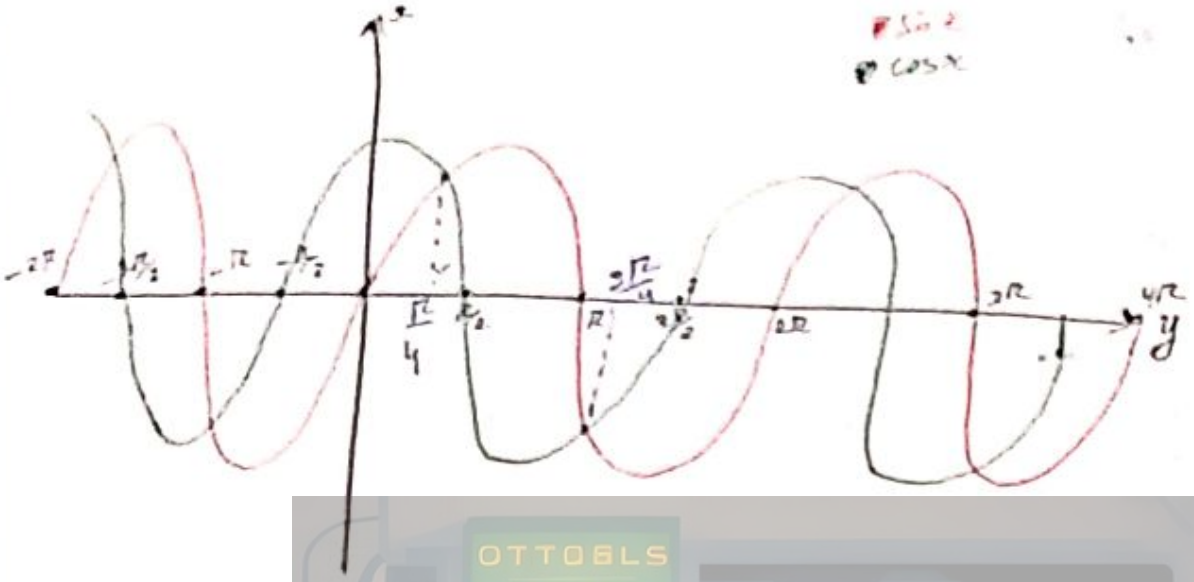
$x \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}\right)$

Q5. $|\tan x| < -1$

Asomthoor

$x \in \phi$

① $\sin x > \cos x \quad x \in [0, 2\pi]$



$$\boxed{x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)}$$ ✓

② $\operatorname{cosec}^2 x - \tan x \geq 0 \qquad x \in [0, 2\pi]$

$$\frac{1}{\sin x \cos x} \geq 0$$

$\sin x > 0$ and $\cos > 0$, $\sin x < 0$ and $\cos x < 0$

$\downarrow$

$(2n\pi, 2n\pi + \pi) \cap \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$

$\left(2n\pi, 2n\pi + \frac{\pi}{2}\right)$

$\left(2n\pi + \pi, 2n\pi + 3\frac{\pi}{2}\right)$

Union → answer

H.W. 9-8-24

DYS - 6

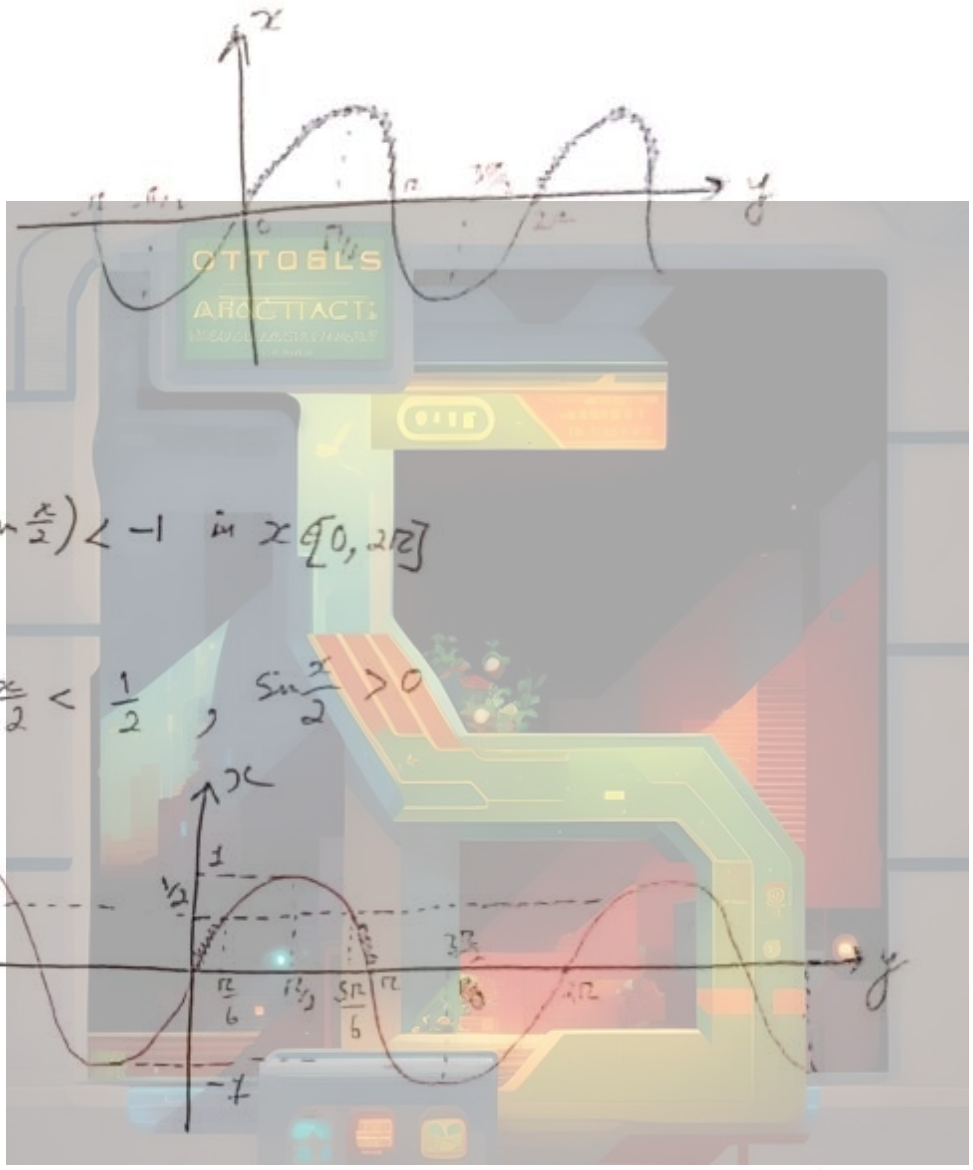
O-1 [1 - 15]

a) $\sqrt{\sin x} \leq 1 \quad \sin x \leq 1$

↳ $\sin x \geq 0$

$\sin x \in [0, 1]$

$$\boxed{x \in [2n\pi, 2n\pi + \pi]}$$

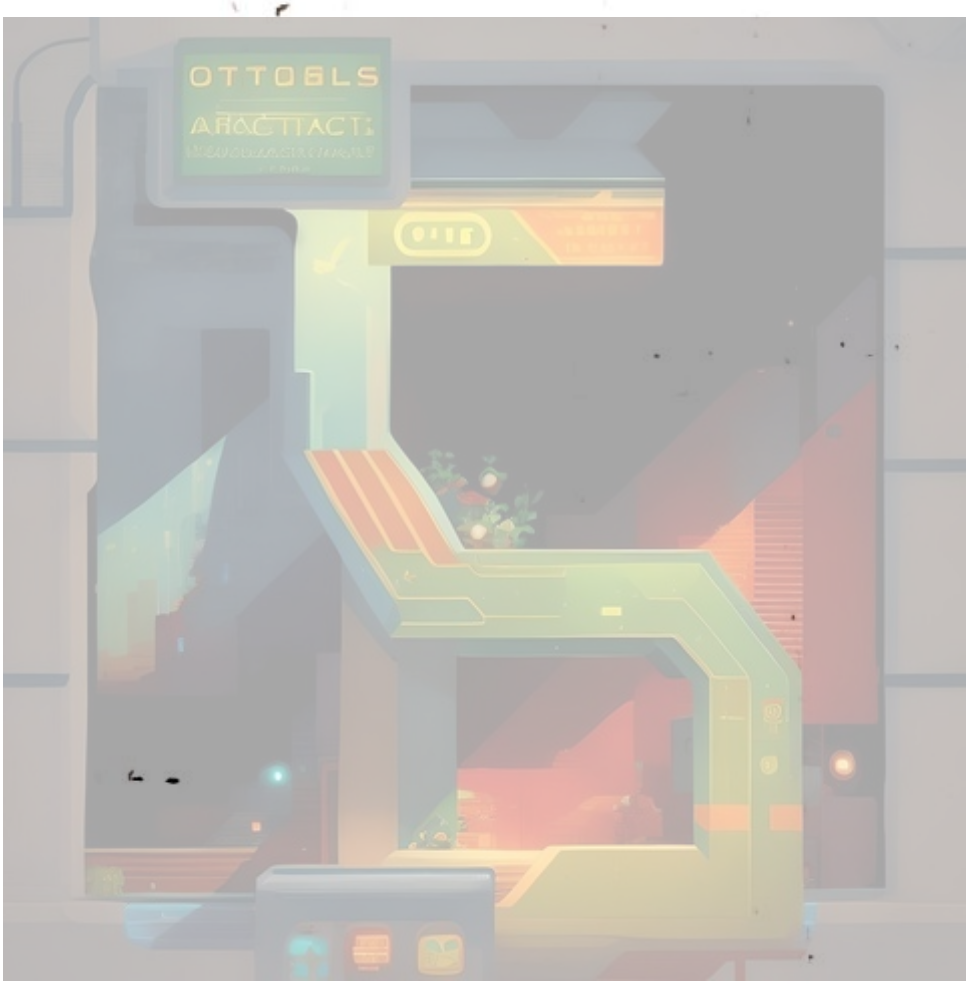


b) $\log_2 (\sin \frac{x}{2}) < -1 \quad \text{în } x \in [0, 2\pi]$

$\sin \frac{x}{2} < \frac{1}{2}$, $\sin \frac{x}{2} > 0$

$\frac{x}{2} \in (2n\pi, 2n\pi + \frac{\pi}{6}) \cup (6 \cdot 2n\pi + \frac{5\pi}{6}, 2n\pi + 2\pi)$

$$\boxed{x \in (0, \frac{\pi}{3}) \cup (\frac{5\pi}{3}, 2\pi)}$$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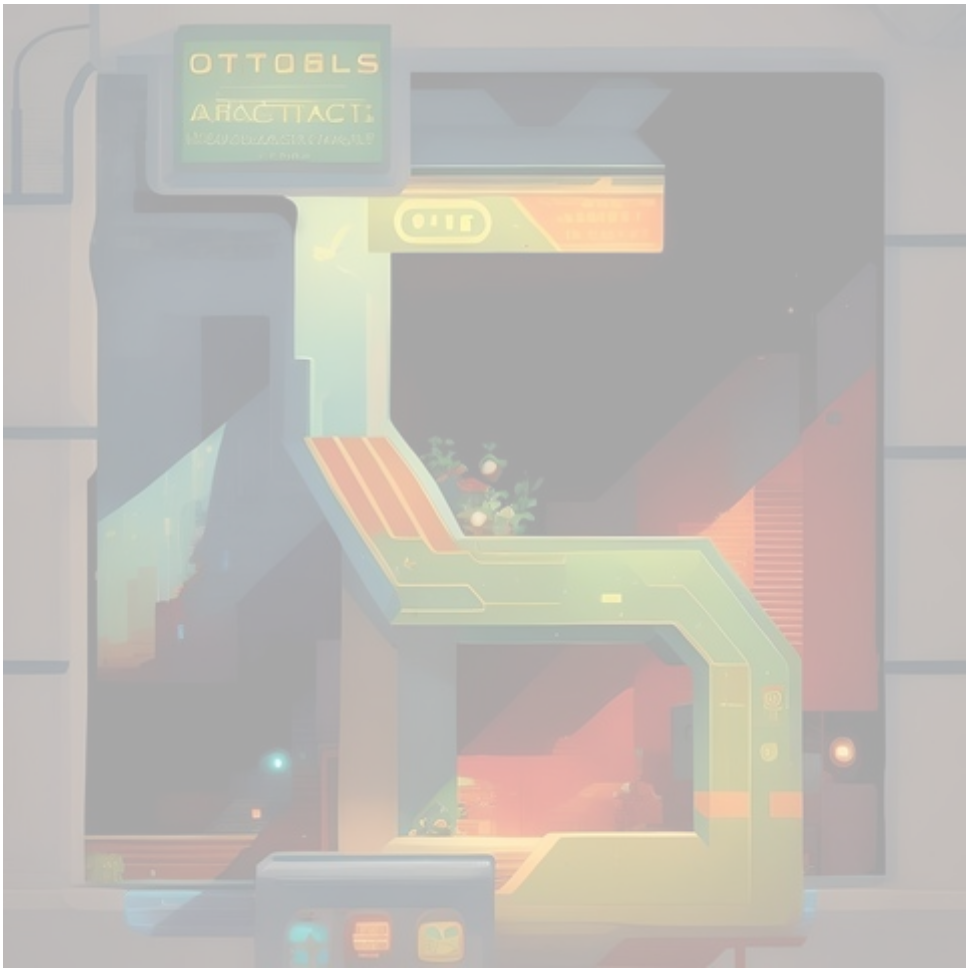
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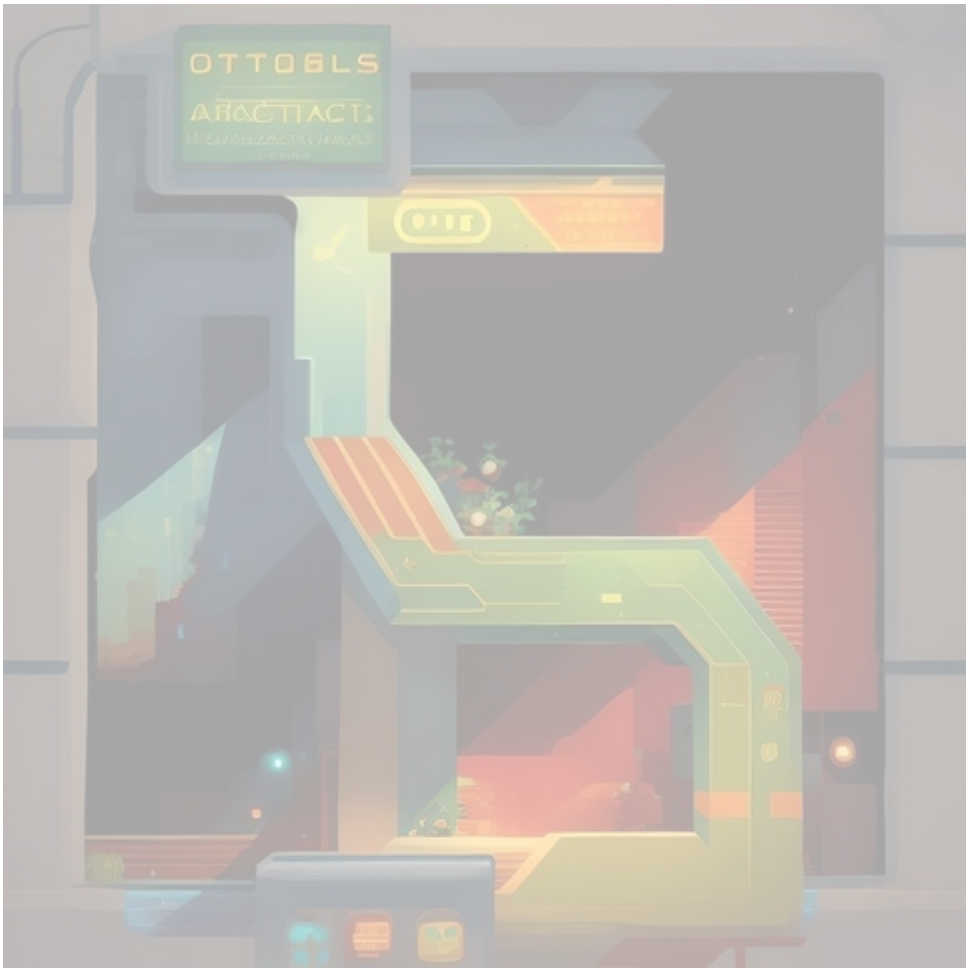

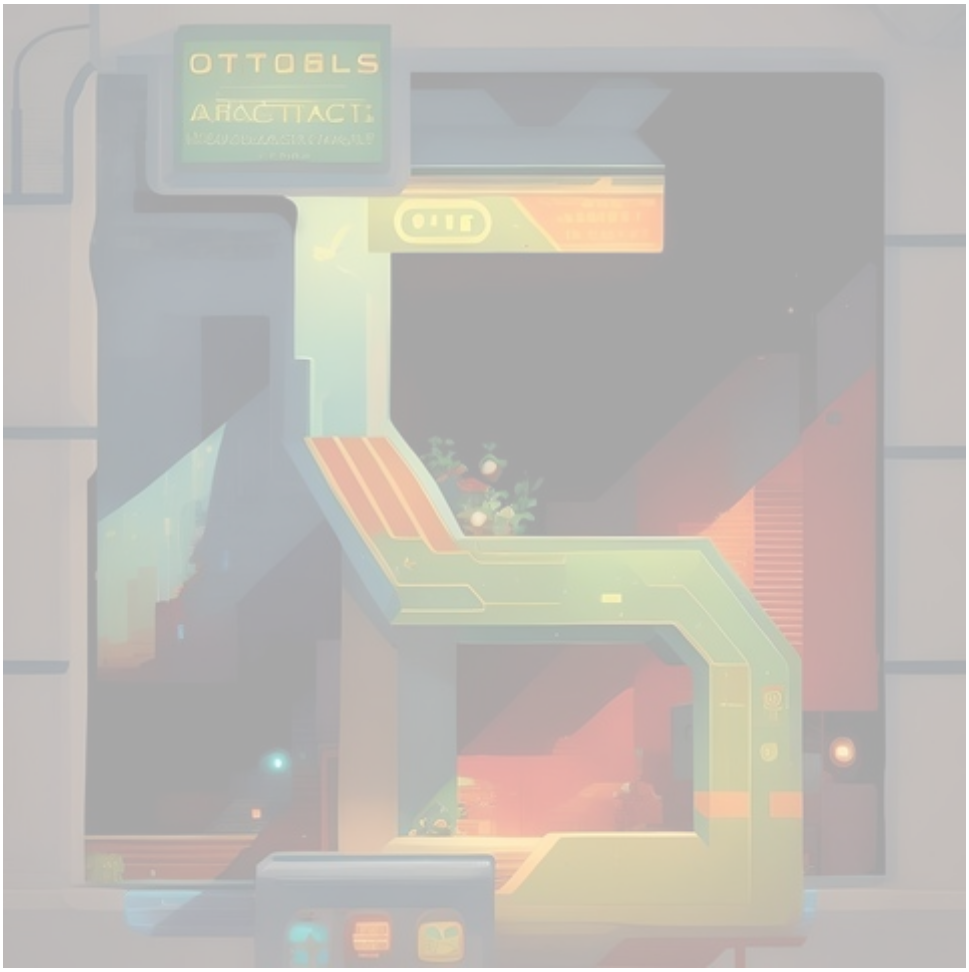
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