

## !! Logarithm !!

→ Every  $\oplus$ ve real number  $N$  can be expressed in exponential form as.

$$a^x = N$$

$a \rightarrow \oplus$ ve real numbers  $> 0$  but  $\neq 1$

$x \rightarrow$  Exponent

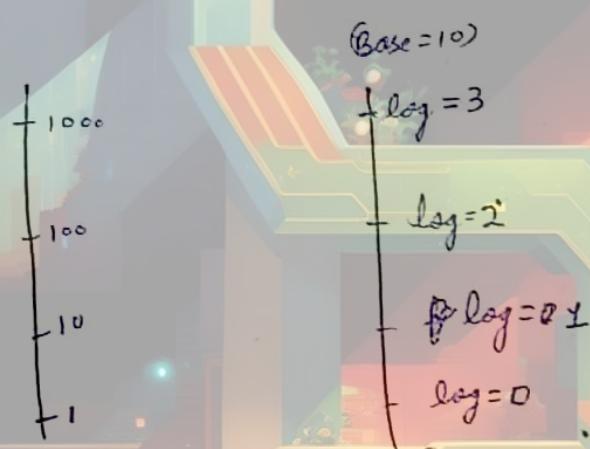
e.g.  $2^2 = 4$ ,  $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$  etc

Reason I

$$2^4 = 4, \left(\frac{1}{2}\right)^4 = \frac{1}{16} \quad (8 \cdot 2)^4 = 16 \cdot 32$$

To find.

Reason II



For  $\text{N} \in$  above reasons we introduced log. It is expressed as

$$\log_a N = x$$

$a \rightarrow$  Base  
 $x \rightarrow$  exponent

$\left[ \begin{array}{l} \text{if Power is } \frac{\text{odd}}{\text{even}} \text{ then } \\ \text{if } N \text{ is } \text{fuge.} \end{array} \right]$

$$\text{eg. } \log_2 8 = 3 \quad [\text{2 का घात Power का फॉर्म रखने के साथ}]$$

$$\log_6 216 = 3$$

$$\log_{\frac{1}{2}} = -4$$

$$\log_{0.6} \left( \frac{25}{9} \right) = \underline{\underline{}}$$

$$0.6 = \underline{\hspace{2cm}}$$

$$= \frac{3}{5}$$

$$= \left(\frac{3}{5}\right)^{-2}$$

$$= \left(\frac{5}{3}\right)^2$$

$$= \frac{25}{9}$$

$$\log_{0.6} \left( \frac{25}{9} \right) = -2$$

$$\log_{\frac{1}{2}}(1) = 0$$

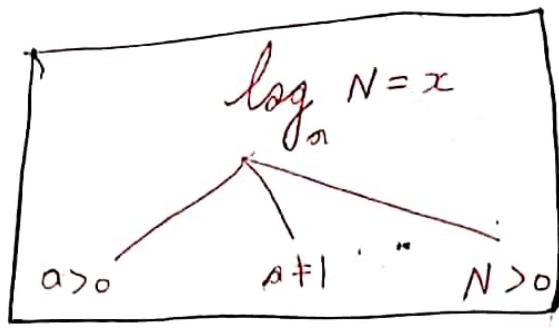
$$\lg \frac{1}{3} = -1$$

$$\log\left(\frac{1}{2}\right) = \text{Not Defined}$$

$$\log_{\frac{1}{2}}(16) = 4 \quad (\text{Wrong})$$

$$\cancel{y^4} = \sqrt{4} = 1\cancel{0}21 = 2 \neq -2$$

Thus Base is  $\oplus \vee$   
 $\neq \perp$



Q Conversion of Logarithm form in exponential form.

$$\textcircled{1} \quad \log_2 3^2 = 5 \rightarrow 2^5 = 3^2$$

$$\textcircled{2} \quad \log_{36} 6 = \frac{1}{2} \rightarrow 36^{\frac{1}{2}} = 6$$

$$\textcircled{3} \quad \log_8 1 = 0 \rightarrow 8^0 = 1$$

$$\textcircled{4} \quad \log_{10} (0.001) = -3 \rightarrow 10^{-3} = 0.001$$

$$\textcircled{5} \quad \text{find the value of } x \text{ if } \log_5 125 = x$$

$$5^x = 125$$

$$5^x = (5)^3$$

$$\boxed{x = 3}$$

$$\textcircled{6} \quad \log_2 m = 1.5$$

$$2^{1.5} = m$$

$$2^{\frac{3}{2}} = m$$

$$\sqrt{2^3} = m$$

$$\sqrt{8} = m$$

$$\boxed{m = 2\sqrt{2}}$$

Note - For some numbers different bases gives different answers.

Q Find  $\log$

①  $3^2$  (base  $2^{-x}$ )

$$\log_{2^{-x}} 3^2 = x$$

$$\left(\frac{1}{2}\right)^x = 3^2$$

$$\frac{1}{2^x} = 3^2$$

$$\frac{1}{2^x} = 2^5$$

$$2^{-x} = 2^5$$

$$-x = 5$$

$$\boxed{x = -5}$$

②  $3^2$  (base 2)

$$\log_2 3^2 = x$$

$$2^x = 3^2$$

$$2^x = 2^5$$

$$\boxed{x = 5}$$

③  $3\sqrt{3}$  (base 3)

$$\log_3 3\sqrt{3} = x$$

$$3^x = 3\sqrt{3}$$

$$3^x = \sqrt{27}$$

$$3^x = 27 \cdot (3)^3$$

$$3^x = 3^{\frac{3}{2}}$$

$$\boxed{x = \frac{3}{2}}$$

④  $3\sqrt{3}$  (base  $2^{-x}$ )

$$\log_{2^{-x}} 3\sqrt{3} = x$$

$$\left(\frac{1}{2}\right)^x = 3\sqrt{3}$$

$$3^{-x} = 3^{\frac{3}{2}}$$

$$-x = \frac{3}{2}$$

$$\boxed{x = -\frac{3}{2}}$$

Note :-

①  $\log_a 1 = 0 \quad (a > 0, a \neq 1)$

②  $\log_N N = 1$

③  $\log_{\frac{1}{N}} N = -1 \quad \text{or} \quad \log_N \frac{1}{N} = -1 \quad (N \geq 0, N \neq 1)$

Q find value of

①  $\log_{2+\sqrt{3}} (2-\sqrt{3}) = x$

$$(2+\sqrt{3})^x = 2-\sqrt{3}$$

$$(2+\sqrt{3})^x = \frac{(2-\sqrt{3})(2+\sqrt{3})}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = \frac{(2-\sqrt{3})^2}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = \frac{2-\cancel{\sqrt{3}}^2}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = (2+\sqrt{3})^{-1}$$

$$\boxed{x = -1}$$

②  $\log_{(1+\sqrt{2})} (\sqrt{2}+1) = x$

$$(1+\sqrt{2})^x = (\sqrt{2}+1)^1$$

$$\boxed{x=1}$$

DYS-1 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Principal Properties of log { $a > 0, a \neq 1$ ;  $m, n > 0$ }

$$\textcircled{1} \quad \log_a(mn) = \log_a m + \log_a n \quad (\text{can put more than two terms})$$

$$\textcircled{2} \quad \log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$$\textcircled{3} \quad \log_a(m^n) = n \log_a m$$

Proof:-

$$\textcircled{1} \quad \log_a(mn) = \log_a m + \log_a n$$

$$\text{let } \log_a m = x, \log_a n = y$$

$$a^x = m$$

$$a^y = n$$

$$\therefore mn = a^x \times a^y$$

$$= a^{x+y}$$

$$\log_a(mn) = x + y$$

$$= \log_a m + \log_a n$$

$$\textcircled{2} \quad \log_a(m^n) = n \log_a m$$

$$\text{LHS} \quad \log_a(m \times m \times m \dots)$$

$$= \log_a m + \log_a m + \dots \text{, } n \text{ times}$$

$$= n \log_a m = \text{RHS}$$

Thus Proved

## Note -

$$\textcircled{1} \log_{1/2} \log_4 2$$

$$\log_{1/2}^2 = \frac{1}{2}$$

$$\log_{1/2}(\frac{1}{2}) = 1$$

$$\boxed{1} x = 1$$

$$\textcircled{2} \log_a N \cdot \log_b M$$

$$\boxed{\log_a N \times \log_b M}$$

$$\textcircled{3} \log_a N \cdot 3$$

$$\log_a N \times 3$$

$$3 \log_a N$$

$$\boxed{\log_a N^3}$$

~~$\textcircled{4} \log 3 + \log (\text{Base } 1)$~~

Q Find value (Base 10)

$$\textcircled{1} \log 3 + \log 5$$

~~$\log_{10}^{(3 \times 5)}$~~

$$\boxed{\log_{10}^{15}}$$

~~10~~

$$\textcircled{2} \log 6 - \log 2$$

$$\log\left(\frac{6}{2}\right)$$

$$\log_{10}^3$$

$$(3) 3 \log^4$$

$$\log(4^3)^8$$

$$\cancel{\log(64)}$$

$$\log_{10} 64$$

$$(4) \log_2 36 - \log 1$$

$$\log_{10} 2^{36}$$

$$(5) 2 \cancel{2} 2 \log 3 - 3 \log_2$$

$$\log 3^2 - 3 \log 2^3$$

$$\log 9 - \log 8$$

$$\log_{10} \left(\frac{1}{8}\right)$$

$$(6) \log_2 + \log 3 + \log 4$$

$$\log(2 \times 3 \times 4)$$

$$-\log(1 \times 2)$$

$$\log_{10} (2^4)$$

$$(7) \log_{10} + 2 \log 3^2 + \log 2$$

$$\log 10^5 + 2 \log 3^2 - \log 2$$

$$\log 10^5 + \log 9 - \log 2$$

$$\log 10^5 + \log \frac{9}{2}$$

$$\log_{10} (4.5 \times 10^5)$$

### Properties

(4) fundamental log. Identity

$$\boxed{a^{\log_a N} = N}$$

$\log_a N$  is powers

Proof:-  $a^{\log_a N} = N$   
Taking  $\log_a$  both sides

$$\log_a (a^{\log_a N}) = \log_a N$$

$$\log_a N \cdot \log_a a = \log_a N$$

$$\log_a N = \log_a N$$

$$N = N$$

Much Proved

### (5) Base Changey Theorem

$$\frac{\log_a m}{\log_a n} = \log_n m$$

Proof:-

$$\log_n m = p \Leftrightarrow n^p = m$$

$$\log_a m = q \Leftrightarrow a^q = m$$

$$\log_a n = r \Leftrightarrow a^r = n$$

$$\begin{aligned} n^p &= a^q \\ (a^r)^p &= a^q \\ a^{qr} &= a^{qp} \end{aligned}$$

$$qr = qp$$

$$\log \frac{q}{r} = p$$

$$\frac{\log_a m}{\log_a n} = \log_n m$$

⑥ DDF

$$\frac{\log_b c}{a} = \cancel{a} \cancel{c} \log_b a$$

Proof

$$\log_b c = x \Leftrightarrow b^x = c$$

$$\text{L.H.S}$$
$$a^x$$

R.H.S

$$\begin{aligned} & c \\ & \log_b c \\ & b^x \log_b a \\ & \circ \log_b a \\ & b \\ & a^x \end{aligned}$$

$$\text{L.H.S} = \text{R.H.S}$$

Hence, Proved

⑦ Base-Power Theorem

$$\log_{b^n} b^m = \frac{m}{n} \log_b a$$

Q find value (Base 10)

①  $2^{\log_2 5}$

$\boxed{5}$

②  $\pi^{\log_{\pi} 60}$

$\boxed{60}$

③  $25^{\log_5 8}$

$5^{2 \log_5 8}$

$5^{\log_5 8^2}$

$64^{x^2}$   
 $\boxed{64}$

④  $(\frac{1}{16})^{\log_2 x^2}$

$2^{\log_2 x^{-4}}$

$\boxed{x^{-4}}$

⑤  $\log_{10} 8 \cdot \log_{10} 8$

$\log_{10} 2^3$

$\frac{3}{6} \log_2^2$

$\frac{3}{6}$

$\boxed{\frac{1}{2}}$

⑥  $\log_3 2 \times \log_4 3 \times \log_5 4$

$\log_4 3^{\log_3 2} \times \log_5 4$

$\log_5 4^{\log_4 3^{\log_3 2}}$

$\log_5 2$

⑦  $4^{\log_3 7} - 7^{\log_3 4}$

⑧  $2^{\log_5 3} + 3^{\log_5 6} - 5^{\log_3 6} - 6^{\log_7 3}$

⑨  $\log_2 [\log_2 \{ \log_3 (\log_3 27^3) \}]$

⑩  $\log(\tan 1^\circ), \log(\tan 2^\circ), \dots, \log(\tan 89^\circ)$

$7^{\log_{10} x^2} + x^2 - 2 = 0$

⑪  $\log(\sin 1^\circ), \log(\sin 2^\circ), \dots, \log(\sin 90^\circ)$

⑫  $\log_{10} \{ (\sqrt[3]{a^2 \cdot b}) (\sqrt[3]{ab^{-3}}) \}$

⑬  $\log_2^3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_n^{(n+1)} = 5$

⑭  $\text{base: } 2^{\frac{\log_2 3}{\log_2 2}} = 3^{\sqrt{\frac{\log 2}{\log 3}}}$

⑮  $\text{base: } 2^{\frac{\log_2 3}{\log_2 2}} = 3^{\sqrt{\frac{\log 2}{\log 3}}}$

$$Q7 \quad 4^{\log_3 7} = 7^{\log_3 4}$$

$$4^{\log_3 7} = 7^{\log_3 4}$$

$$\boxed{10}$$

$$Q8. \quad 2^{\log_3 5} = 5^{\log_3 2}$$

$$3^{\log_7 9} = 6^{\log_7 3}$$

$$\boxed{10}$$

$$Q9. \quad \log_3 27^3 = 9$$

$$\log_3 9 = 2$$

$$\log_2 2 = 1$$

$$\log_2 1 = 0$$

$$\boxed{0}$$

$$Q10. \quad 7^{\log_7 x^2} + x - 2 = 0$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$\boxed{x = 1, -2}$$

$$\textcircled{13} \quad 10^x = (\sqrt{a^{-2} \cdot b})^{(\sqrt[3]{a^2} \cdot b^{-3})}$$

$$10^x = a^{-1} \cdot b^{1/2} \cdot a^{1/3} \cdot b^{-1}$$

$$\cancel{10^x = a^{-2/3} \cdot b^{-1/2}}$$

$$\textcircled{13} \quad 10^x = a^{-2/3} \cdot b^{-1/2}$$

$$\log_{10} (a^{-2/3}) + \log_{10} (b^{-1/2})$$

$$\boxed{-\frac{2}{3} \log a - \frac{1}{2} \log b}$$

\textcircled{10} It includes  $\tan 45^\circ$ ,

$$\log(\tan 45^\circ)$$

$$= \log(1)$$

$$= 0$$

$$\textcircled{12} \quad \log(\sin 90^\circ)$$

$$\log(1)$$

$$= 0$$

$$\textcircled{14} \quad \frac{\log_a^3}{\log_a^2} \times \frac{\log_a^4}{\log_a^3}$$

$$\frac{\log_a^4}{\log_a^2}$$

$$\frac{\log_a^{n+1}}{\log_a^2} = \boxed{\frac{\log(n+1)}{2}}$$

\textcircled{15}

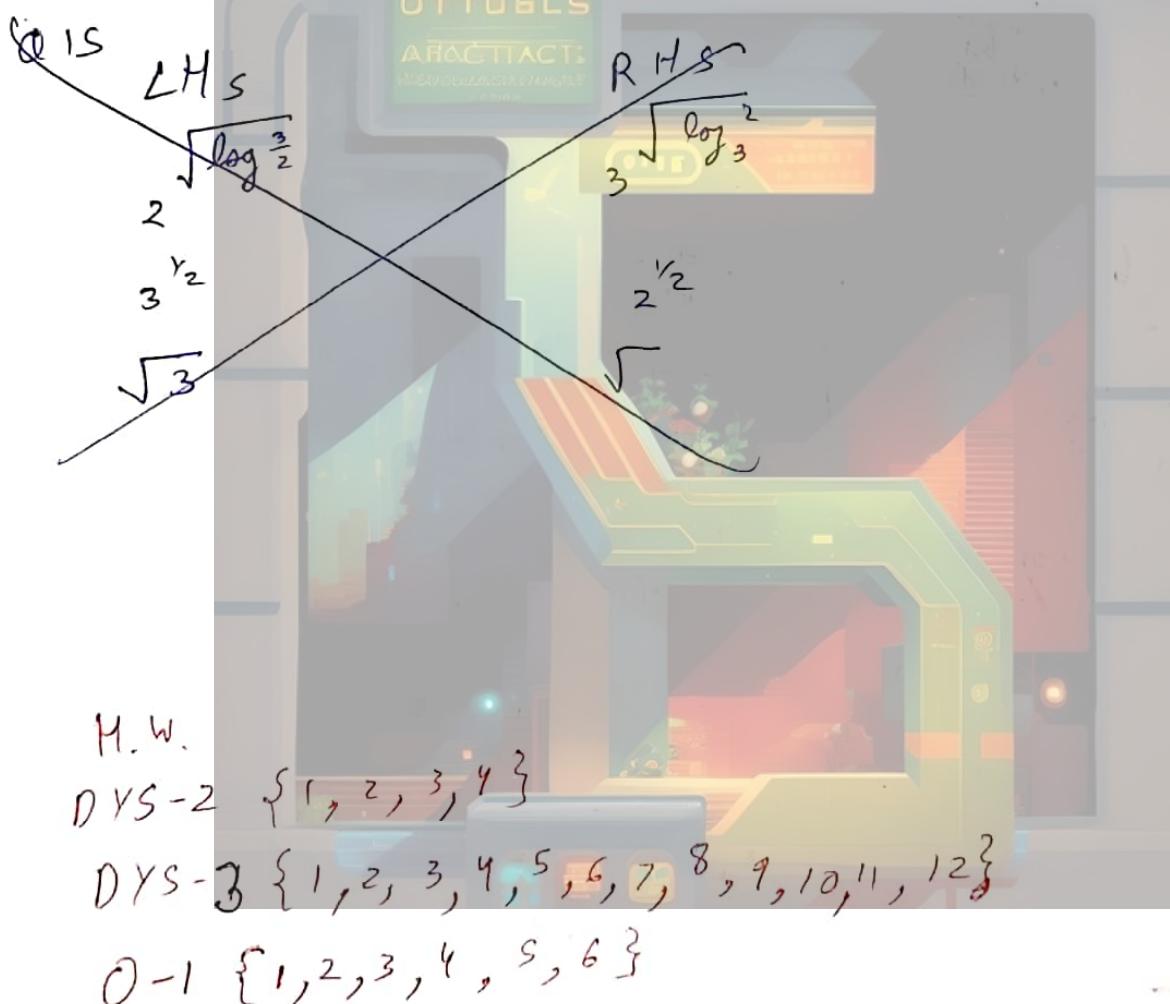
O.S. LHS  
C2  $\sqrt{\log \frac{3}{2}}$

$$\log_2^{(n+1)} = s$$

$$2^s = n + 1$$

$$3^2 = n + 1$$

$$n = 31$$



$$Q15. \quad \log_2 a = \sqrt{\log_2 3}$$

$$a = 2^{\sqrt{\log_2 3}}$$

$$\log_2 2^{\sqrt{\log_2 3}} \times \log_2 2$$

Q15.

$$a = 2^{\sqrt{\log_2 3}}$$

$$b = 3^{\sqrt{\log_2 2}}$$

$$= \log_2 b$$

$$= \log_2 3^{\sqrt{\log_2 2}}$$

$$= \sqrt{\log_2 2} \times \log_2 3$$

$$= \frac{\sqrt{\log_2 2}}{\sqrt{\log_2 3}} = \log_2 3$$

$$\log_3 2 = x$$

$$\frac{1}{\log_3 2} = \frac{1}{x}$$

$$3^x = 2$$

~~$$3^{\cancel{x}} = 2$$~~ 
$$3 = 2^x$$

$$\frac{1}{\log_3 2} = \log_2 3$$

$$\frac{1}{\sqrt{\log_2 3}} \times \log_2 3$$

~~$$3^{\cancel{x}} = 2$$~~

~~$$\frac{1}{\sqrt{\log_2 3}} = 3$$~~

$$\sqrt{\log_2 3} = \log_2 b$$

$$2^{\sqrt{\log_2 3}} = b$$

$$\boxed{a = b}$$

Mence, ~~परिवर्तन~~

Antilog

$$\boxed{\text{Antilog}_a x = a^x}$$

$$\log_a N = x$$

$$\text{Antilog}_a (\log_a N) = \text{Antilog}_a x$$

DYS - 3

Q13 Antilog  $\left(\frac{s}{c}\right)$

$$(64)^{\frac{s}{c}}$$

$$(2)^s$$

$$\boxed{32}$$

~~Eduart~~ Illustration - 8

$$(\log_a \frac{a}{b} \cdot \log_a b - \log_a b) + (\log_b \frac{b}{c} \cdot \log_c b - \log_b c) + (\log_c \frac{c}{a} \cdot \log_a c - \log_c a) = 0$$

$$\frac{\log a}{\log b} \cdot \frac{\log b}{\log c} + \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} + \frac{\log c}{\log a} \cdot \frac{\log a}{\log b} - 3 = 0$$

$$(\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a \log b \log c$$

$$\log a + \log b + \log c = 0$$

$$\log (abc) = 0$$

$$\boxed{abc = 1}$$

## Illustration 10

$$\log_4 18 = x$$

$$4^x = 18$$

$$2^{2x} = 18^2$$

thus irrational.

$$\text{Q1. } 2 \log_2 (\log_2 x) + \log_{\sqrt{2}} \left( \frac{3}{2} + \log_2 x \right) = 1$$

$$\begin{aligned} & 2 \log_2 (\log_2 x) + \log_{\sqrt{2}} \left( \frac{\log_2 x^2}{\log_4 x^2} \right) = 1 \\ & 2 \log_2 x \times \log_2 x + \log_2 x^3 = 2 \\ & \log_2 x^3 = 2 \times \log_2 x \\ & \log_2 x^3 = 3 \\ & 2^3 = x \\ & \boxed{8 = x} \end{aligned}$$

$$\text{Q3. } 6 + \log_{\sqrt{2}} \left( \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \dots}} \right)$$

$$x = \sqrt{4 - \frac{1}{3\sqrt{2}}}$$

$$x^2 = 4 - \frac{x}{3\sqrt{2}}$$

$$3\sqrt{2}x^2 = 12\sqrt{2} - x$$

$$3\sqrt{2}x^2 + x - 12\sqrt{2} = 0$$

$$x = -1 \pm \sqrt{4+288}$$

$$x = \frac{-1 \pm 17}{2\sqrt{2} + 3\sqrt{2}}$$

$$x = \frac{16}{6\sqrt{2}}$$

$$6 + \log_{\sqrt{2}} \frac{1}{4}$$

$$6\sqrt{2}$$

$$\boxed{4}$$

$$6 + \log_{\sqrt{2}} \left( \frac{1}{3\sqrt{2}} \times \frac{16}{3\sqrt{2}} \right)$$

$$6 + \log_{\sqrt{2}} \frac{16}{9} x^2$$

$$\left(\frac{4}{3}\right)^x = \frac{16}{9}$$

$$\left(\frac{2}{3}\right)^{-x} = \frac{8}{9}$$

$$6 - 2 + \log_{\sqrt{2}}^2$$

$$4 + \sqrt{8}$$

$$\boxed{4 + 2\sqrt{2}}$$

$$Q. \quad \log_2 (\log_2 x) + \log_{\frac{1}{2}} \left( \frac{3}{2} + \log_2 x \right) = 1$$

$$\log_2 t^2 + \log_2 \left( \frac{1}{\frac{3}{2} + t} \right) = 1$$

$$\frac{t^2}{\frac{3+2t}{2}} = 1$$

$$\frac{2t^2}{3+2t} = 1$$

$$2t^2 = 3 + 2t$$

$$2t^2 - 2t - 3 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = 3, -1$$

$$\log_2 x = 3$$

$$\log_2 x = -1$$

$$\boxed{x=8}$$

$$\boxed{x=8}$$

H.W

$$J.A - \{5, 6\}$$

$$D \times S - 4 \quad [1, 10]$$

(7)

Q

$$\textcircled{1} \quad 2^{\log_2 x^2} - 3x - 4 = 0$$

$$2^{\log_2 x^2} = 3x + 4$$

$$\log_2 3x + 4 = \log_2 x^2$$

$$3x + 4 = x^2$$

$$x^2 - 3x - 4 = 0$$

$x = 4, -1$

$$\textcircled{2} \quad 2^{\log_2 x} - 3x - 4 = 0$$

$$x = 4, -1$$

-1 is rejected as one not possible in log

$$\textcircled{3} \quad \log_2 (x^2 - 1) = 3$$

$$8 = x^2 - 1$$

$$9 = x^2$$

$$x = \pm 3$$

$$\textcircled{4} \quad \log_2 (x+1) - \log_2 (x-1) = 3$$

$$\log_2 (x+1)(x-1) = 3$$

$$8 = x^2 - 1$$

$$x^2 + 9$$

$$x = \pm 3$$

-3 is rejected

$$\boxed{x = 3}$$

$$\text{Q 5. } x^2 + 7 \log_7 x - 2 = 0$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$x = 1, -2$$

-2 rejected

$$\boxed{x = 1}$$

$$\text{Q 7. } 5^{(\log_5 x)^2} + x^{\log_5 x}$$

$$5^y^2 + x^y = 1250$$

$$\log_5 x = y$$

$$5^y = x$$

$$5^{y^2} + 5^{y^2} = 1250$$

$$5y^2 - 625 = 0$$

$$5y^2 = 5^4 \quad y = \pm 2$$

$$\text{Q 6. } \log_4 (2 \log_3 (1 + \log_2 (1 + 3 \log_3 x))) = \frac{1}{2}$$

$$2^{\frac{1}{2}} = 2 \log_3 (1 + \log_2 (1 + 3 \log_3 x))$$

$$2 = \log_3 (1 + 3 \log_2 x)$$

$$\Leftrightarrow 1 = \log_2 x$$

$$\Leftrightarrow 1 = \log_2 x$$

$$\boxed{x = 2}$$

$$2^{\log_2 (3-x)} = 10^{\log_{10} (3-x)}$$

$$\text{Q 8. } \log_2 (9-2^x) = 10^{\log_{10} (3-x)}$$

$$2^{3-x} = 9-2^x$$

$$2^{3-x} = 9-2^x$$

$$8y = 9-y$$

$$y = 8, 1$$

$$x = 3, 0$$

3 rejected

$$\boxed{x = 0}$$

$$Q9. \log_2 \log_5 (\sqrt{x+s} + \sqrt{x}) = 0$$

$$\sqrt{x+s} + \sqrt{x} = s$$

$$s = \sqrt{x+s} + \sqrt{x}$$

$$2s = x+s+x+2\sqrt{(x+s)x}$$

$$2s = 10 = x + \sqrt{x^2+sx}$$

$$10-x = \sqrt{x^2+sx}$$

$$100 = 25x$$

$$\boxed{x=4}$$

$$Q10. (x+1) \log_{10}(x+1) = 100(x+1)$$

$$x+1 = y$$

$$y \log_{10} y = 100y$$

$$\log_{10} y = \log_y 100y \quad (\text{take } \log_{10} \text{ both sides})$$

$$\log_{10} y = \log_y y + \log_y 100$$

$$\log_{10} y = 1 + 2 \log_y 10$$

$$z = 1 + \frac{2}{z}$$

$$\underline{z = 2, -1}$$

$$\log_{10} y = -1$$

$$y = \frac{1}{10}$$

$$x+1 = \frac{1}{10}$$

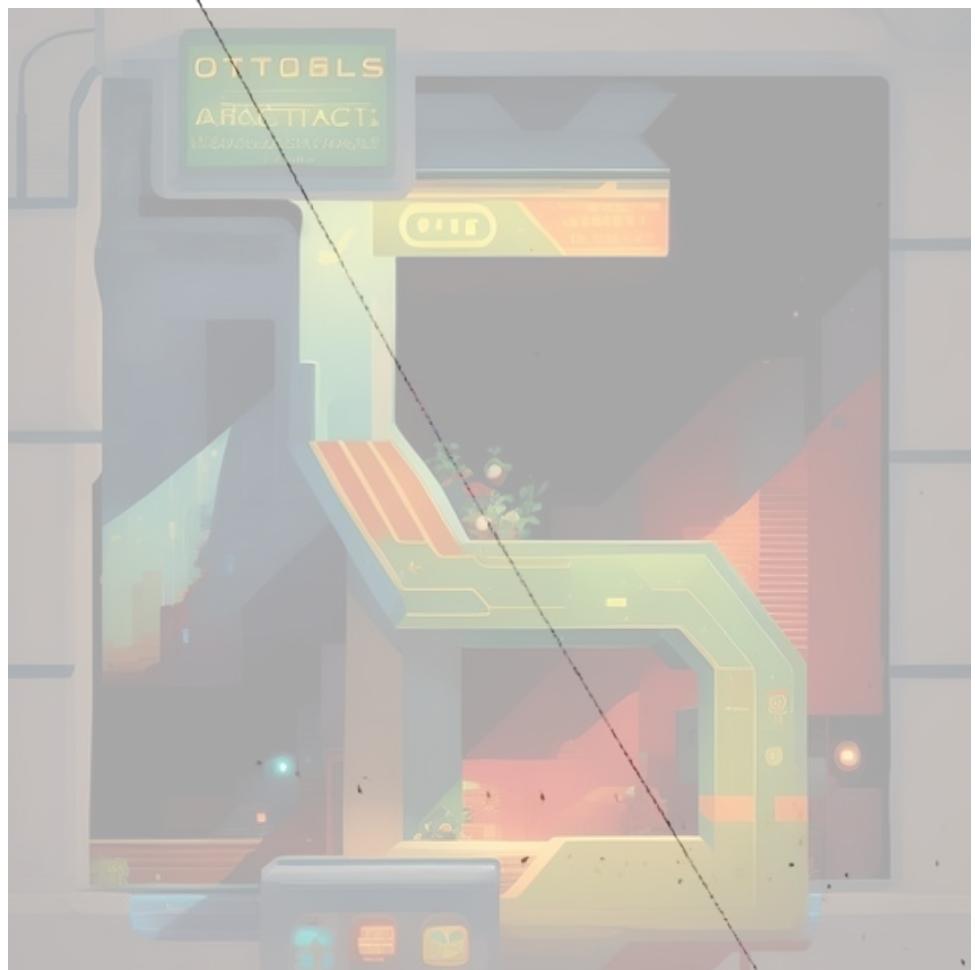
$$\boxed{x = -\frac{9}{10}}$$

$$\log_{10} y = 2$$

$$y = 100$$

$$x+1 = 100$$

$$\boxed{x = 99}$$



Q ⑪

$$\log_{x-1}(4) = 1 + \log_2(x-1)$$

$$\log_y 4 = 1 + \log_2 y$$

~~$y^{1+\log_2 y}$~~

$$2 \log_y 2 = 1 + \log_2 y$$

$$\log_2 y = 2$$

$$\frac{2}{z} = 1 + 2$$

$$2 = z + z^2$$

$$z^2 + z - 2 = 0$$

$$z = -1 \pm \sqrt{\frac{1+8}{2}}$$

$$z = -1 \pm 3$$

$$z = -2, 1$$

$$\log_2 y = -2$$

$$\frac{1}{4} = y$$

$$x-1 = y$$

$$x-1 = \frac{1}{4}$$

$$x = \frac{1}{4} + \frac{1}{4}$$

$$x = \frac{5}{4}$$

$$\log_2 y = 1$$

$$y = 2$$

$$x-1 = 2$$

$$x = 3$$

$$x = 3, \frac{5}{4}$$

(12)

sum of values of  $x$ : A) 1, B) 4 C) 0 D) 3

$$\log_{2x-1}(x^3 + 3x^2 - 13x + 10) = 2$$

$$(2x-1)^2 = x^3 + 3x^2 - 13x + 10$$

$$4x^2 + 1 - 4x = x^3 + 3x^2 - 13x + 10$$

$$x^3 - x^2 - 9x + 9 = 0$$

$$x=1$$

$$(x-1)$$

$$x^2 - 9 = 0$$

$$x = \pm 3$$

$$x = 1, 3, -3$$

~~3 + 3 + 1~~ but, -3 is rejected as  $2x-1$  would be  $\leq 0$

~~A~~ ~~C~~

~~A~~

I reject as  $2x-1$  would be 1

$$x = 3$$

~~D~~

(13)

$$5^{1+\log_4 x} + 5^{(\log_4 x)-1} = \frac{26}{5}$$

$$5^{1+\log_4 x} + 5^{-(\log_4 x+1)} = \frac{26}{5}$$

~~$$5^y + \frac{1}{5^y} = \frac{26}{5}$$~~

~~$$5^{2y} + 1 = 26 \cdot 5^y$$~~

~~$$5^x \cdot 5^{2y} + 1 = 26 \cdot 5^y$$~~

~~$$5^x \cdot 5^{x^2} + 1 = 26$$~~

~~$$5^x \cdot 2^2 + 1 = 26$$~~

~~$$5^x \cdot 4 - 26 + 1 = 0$$~~

~~$$2 = \frac{26 \pm \sqrt{676 - 20}}{2}$$~~

~~$$2 = \frac{26 \pm \sqrt{656}}{2}$$~~

~~$$5^x t + \frac{1}{5^x t} = \frac{26}{5}$$~~

~~$$25t^2 + 1 = \frac{26 \times 5t}{8}$$~~

~~$$25t^2 - 26t + 1$$~~

~~$$25t^2 - 25t^2 - t + 1 \\ 25t(t-1) - 1(t-1)$$~~

~~$$t = 1, \frac{1}{25}$$~~

~~$$t^2 + 1 = \frac{26t}{5}$$~~

~~$$5t^2 - 26t + 1 = 0$$~~

$$5^{\log_4 x} = 1$$

~~$$\log_{10} 1 = \log_{10} x$$~~

~~$$\log_{10} x = 0$$~~

~~$$4^0 = x$$~~

$$\boxed{x=1}$$

$$5^{\log_4 x} = \frac{1}{25}$$

$$\log_4 x = -2$$

$$\frac{1}{16} = x$$

$$\boxed{x = 1, \frac{1}{16}}$$

$$\begin{array}{r}
 3 \\
 26 \\
 26 \\
 \hline
 52 \\
 42 \\
 \hline
 56 \\
 56 \\
 \hline
 0 \\
 31 \\
 26 \\
 \hline
 56 \\
 520 \\
 \hline
 676
 \end{array}$$

(97)

$$\textcircled{12} \quad \log^2(x-2) + \log(x-2)^5 - 12 = 10^2 \log(x-2)$$

$$(x-2)^5$$

$$x-2=y$$

$$y(\log y)^2 + \log y^5 - 12 = 10^2 \log y$$

$$\log y =$$

$$(x-2)^2 =$$

$$x-2 = y$$

$$y \log y (\log y)^2 + 5 \log y - 12 = 10^2 \log y$$

$$y \log y (\log y + 5) - 12 = 10^2 \log y^2$$

$$2 \log y^2 \log 10 = \log y (\log y + 5) - 12$$

$$2 \log y \log 10 = \log y + 5 - 12$$

$$2 \log y \log 100 = \frac{(\log y)^2 + 5 \log y - 12}{\log y}$$

95

$$Q 12. \quad (x-2)^{\log_{10}y + 5 \log_{10}y - 12} = 10^{\log_{10}y^2}$$

Note - can assume some base on both sides.

$$y^{\log_{10}y + 5 \log_{10}y - 12} = 10^{\log_{10}y^2}$$

$$y^{\log_{10}y(\log_{10}y + 5) - 12} = y^2 \quad \left| \begin{array}{l} \log_{10}y = 2 \\ \log_{10}y = -7 \end{array} \right.$$

$$\log_{10}y(\log_{10}y + 5) - 12 = 2$$

$$100 = y^{\log_{10}y + 5} - 12$$

$$100 = 5y + y \log_{10}y$$

$$\cancel{100=}$$

$$2 \cancel{100} = (\log_{10}y)^2 + 5 \log_{10}y - 12$$

$$14 = z^2 + 5z$$

$$z^2 + 5z - 14 = 0$$

$$z = \frac{-5 \pm \sqrt{25 + 56}}{2}$$

$$z = \frac{-5 \pm 9}{2}$$

$$z = -7, 2$$

$$y = 100$$

$$x-2 = 100$$

$$\sqrt{x} = 102$$

$$\frac{1}{10^7} = y$$

$$x = \frac{1}{10^7} + 2$$

$$x = \frac{1 + 20000000}{10000000}$$

$$x = \frac{20000001}{10000000}$$

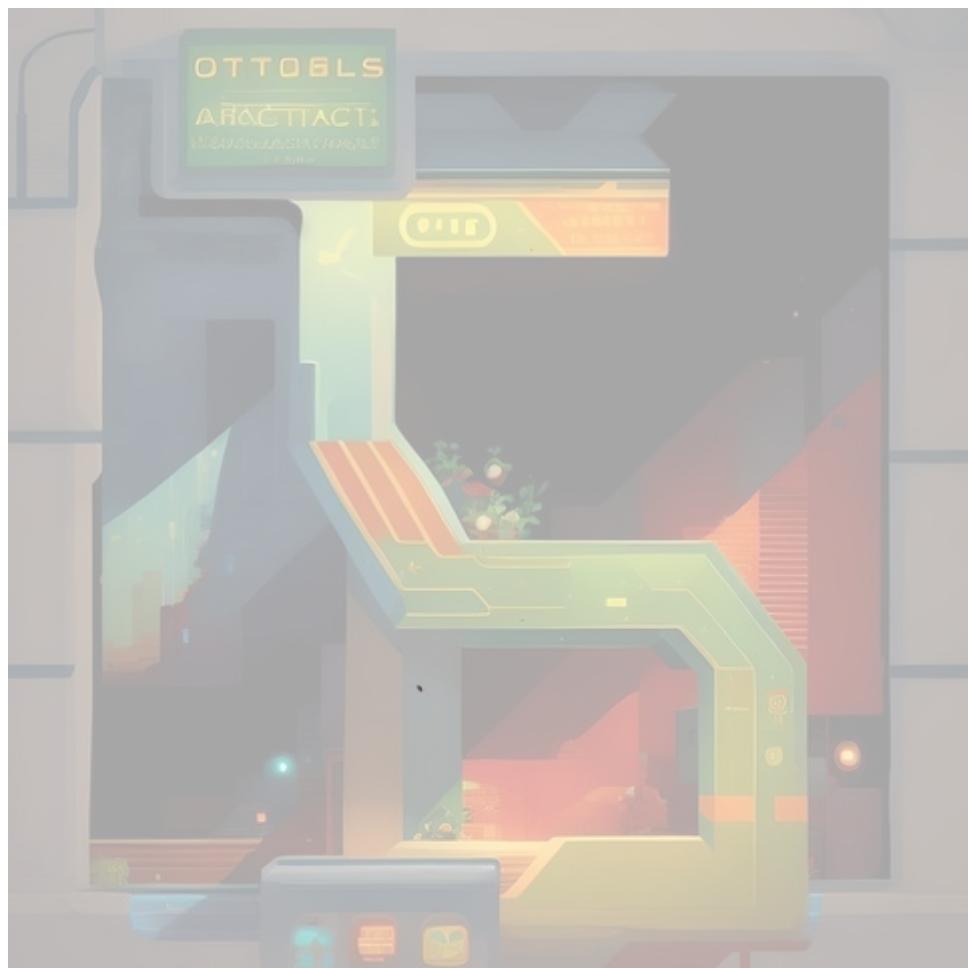
$$x = 2.0000001$$

$$\boxed{x = 102, 2.0000001}$$

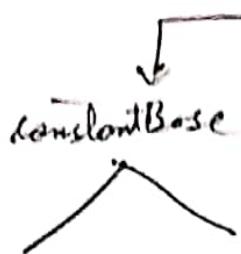
H.W. 07-06-2024

DYS-4 [10:]

Logarithm



# Logarithmic Inequalities



$$\in (0, 1)$$

sign change

$$(1, \infty)$$

no sign change

e.g.  $\log_{\frac{1}{2}} x > \log_{\frac{1}{2}} (2x-1)$       If.  $\log_2 x > \log_2 (2x-1)$

Variable Base

$$\text{case 1} \quad \text{base } (0, 1)$$

$$\text{case 2} \quad \text{base } (1, \infty)$$

union

$$x < 2^{x-1}$$

$$\begin{cases} 1 < x \\ \sqrt{x} > 1 \end{cases}$$

$$\begin{array}{l} x > 2^{x-1} \\ \text{OTTOBLIS} \\ \text{ARACT ACT} \\ \boxed{x < 1} \end{array}$$

Q  $\log_{0.5} (x-3) > \log_{0.5} (2^x)$

$$x-3 < 2^x$$

$$-3 < x$$

$$x > -3$$

$$\therefore x \in (-3, \infty) - \textcircled{1}$$

For 2  $\log_{0.5} (x-3) = x-3 > 0 \quad x > 3 - \textcircled{2}$

$\log_{0.5} (2^x) = 2^x > 0 \quad x > 0 - \textcircled{3}$

∴  $\textcircled{1} \cap \textcircled{2} \cap \textcircled{3}$

$$\boxed{x \in (3, \infty)}$$

$$Q2. \log_7(x^2 - 3x) \geq \log_7(2x - 6)$$

$$x^2 - 3x \geq 2x - 6$$

$$x^2 - 5x + 6 \geq 0$$

$$(x-2)(x-3) \geq 0$$

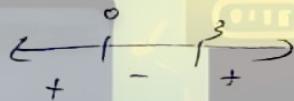


$$x \in (-\infty, 2] \cup [3, \infty) \quad \text{--- } ①$$

$$\log_7(x^2 - 3x) \geq 0$$

$$x^2 - 3x \geq 0$$

$$x(x-3) \geq 0$$



$$x \in (-\infty, 0) \cup (3, \infty) \quad \text{--- } ②$$

$$\log_7(2x - 6) \geq 0$$

$$2x - 6 \geq 0$$

$$2x \geq 6$$

$$x \geq 3$$

$$x \in (3, \infty) \quad \text{--- } ③$$

$$\textcircled{1} \cap \textcircled{2} \cap \textcircled{3}$$

$$\boxed{x \in (3, \infty)}$$

$$Q \log_x(2^x) > 2$$

$$x \in (0, 1)$$

$$\text{Case 1} \quad 2x < x^2$$

$$0 < x^2 - 2x$$

$$x(x-2) > 0$$

$$\Rightarrow x \in \emptyset$$



$$x \in (-\infty, 0) \cup (2, \infty)$$

(102)

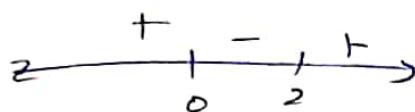
Case 2:-

$$x \in (1, \infty)$$

$$2x > x^2$$

$$0 > x^2 - 2x$$

$$x(x-2) < 0$$



$$x \in (0, 2)$$

$$\cap \rightarrow x \in (1, 2)$$

case 1 v Case 2:  $(x \in \emptyset) \cup (x \in (1, 2))$

$$x \in (1, 2) \rightarrow \textcircled{1}$$

$$\log x \quad 2x > 0 \quad x > 0$$

~~$x > 0, x \neq 1$~~

$$x \in (0, 1) \cup (1, \infty) \rightarrow \textcircled{2}$$

$$\textcircled{1} \cap \textcircled{2}$$

$$(x \in (1, 2)) \cap (x \in (0, 1) \cup (1, \infty))$$

$$\boxed{x \in (1, 2)}$$

(10.3)

$$Q) \textcircled{1} \log_{\frac{1}{3}}\left(\frac{1-2x}{x^3}\right) \leq 0$$

$$\left(\frac{1}{3}\right)^0 \geq \left[\frac{1-2x}{x^3}\right]$$

$$\frac{1-2x}{x^3} \leq 1$$

$$\begin{aligned} 1-2x &\leq 1 \\ -2x &\leq 0 \\ -2x &\leq -2 \end{aligned}$$

$$2x = 2$$

$$x = 1$$

$$1-2x \leq 3$$

$$-2x \leq -2$$

$$2x \geq 2$$

$$x \geq 1$$

$$\log_{\frac{1}{3}}\left(\frac{1-2x}{x^3}\right) \rightarrow \frac{1-2x}{x^3} > 0$$

$$1 > \frac{2x}{3}$$

$$3 > 2x$$

$$x < \frac{3}{2}$$

$$x \in (1, 1.5)$$

$$\frac{1-2x}{x^3} > 1$$

$$\frac{1-2x-x}{x^3} > 0$$

$$\frac{1-3x}{x^3} > 0$$

$$\begin{array}{c} \xleftarrow{-} \xrightarrow{+} \xleftarrow{-} \\ \hline - & + & - \end{array}$$

$$(-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \\ x \in (0, \frac{1}{3}] \quad \textcircled{1}$$

$$\log_{\frac{1}{3}}\left(\frac{1-2x}{x^3}\right) \rightarrow \frac{1-2x}{x^3} > 0$$

$$\begin{array}{c} \xleftarrow{-2x} \xrightarrow{+} \xleftarrow{-} \\ \hline - & + & - \end{array}$$

$$x \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$$

$$x \in (0, \frac{1}{2}) \quad \textcircled{2}$$

$$\textcircled{1} \cap \textcircled{2}$$

$$(0, \frac{1}{3}]$$

$$\textcircled{2} \quad \frac{\log(x^2 - 5x + 6)}{2x} < 1$$

Case 1  $2x \in (0, 1)$

$$x \in (0, \frac{1}{2})$$

$$x^2 - 5x + 6 > 2x$$

$$x^2 - 7x + 6 > 0$$

$$x^2 - 6x - x + 6 > 0$$

$$(x-6)(x-1) > 0$$

$$x \in (-\infty, 1) \cup (6, \infty)$$

$$x \in (0, \frac{1}{2}) - \textcircled{1}$$

Case 2  $2x \in (1, \infty)$

$$x \in (\frac{1}{2}, \infty)$$

$$(x-6)(x-1) < 0$$

$$x \in (1, 6)$$

$$x \in (1, 6) - \textcircled{2}$$

$$\textcircled{1} \cup \textcircled{2}$$

$$x \in (0, \frac{1}{2}) \cup (1, 6) - \textcircled{3}$$

$$x^2 - 5x + 6 > 0$$

$$x^2 - 3x - 2x + 6 > 0$$

$$x(x-3) - 2(x-3) > 0$$

$$(x-2)(x-3) > 0$$

$$x \in (-\infty, 2) \cup (3, \infty)$$

$$2x > 0$$

$$x > 0, \neq \frac{1}{2}$$

$$x \in (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$$

$$x \in (0, \frac{1}{2}) \cup (\frac{1}{2}, 2) \cup (3, \infty) \quad \textcircled{4}$$

$$\textcircled{3} \cap \textcircled{4}$$

$$\boxed{(0, \frac{1}{2}) \cup (1, 2) \cup (3, 6)}$$

DYS-5 [1, 2, 3, 5, 7, 8, 9, 10]

O-1 [7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]

$$\log_{0.5} \left( \log_6 \left( \frac{x^2+x}{x+4} \right) \right) < 0$$

 ~~$\log_{0.5}$~~ 

$$\log_6 \left( \frac{x^2+x}{x+4} \right) > 1$$

$$\frac{x^2+x}{x+4} > 6$$

$$\frac{x^2+x-6x-24}{x+4} > 0$$

$$\frac{x^2-5x-24}{x+4} > 0$$

$$\frac{x^2-8x+3x-24}{x+4} > 0$$

$$\frac{(x-8)(x+3)}{x+4} > 0$$

$$\begin{array}{c} -4 \\ -1 \quad -3 \quad 8 \\ - \quad + \quad - \quad + \end{array}$$

$$x \in (-4, -3) \cup (8, \infty)$$

$$\frac{x^2+x}{x+4} > 0$$

$$\frac{x(x+1)}{x+4} > 0$$

$$\begin{array}{c} -1 \quad 0 \\ -1 \quad - \quad + \\ (-4, -1) \cup (0, \infty) \end{array}$$

$$\frac{x^2+x}{x+4} > 1$$

$$\frac{x^2+x-x-4}{x+4} > 0$$

$$\frac{(x^2+2)(x-2)}{x+4} > 0$$

$$\begin{array}{c} -4 \quad -2 \quad 2 \\ - \quad + \quad - \quad + \\ x \in (-4, -2) \cup (2, \infty) \end{array}$$

$$x \in (-4, -2) \cup (2, \infty)$$

$$x \in (-4, -3) \cup (3, \infty)$$

$$\boxed{x \in (-4, -3) \cup (3, \infty)}$$

$$Q \log_2 \log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$$

$$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 1$$

$$\log_7 \log_{1/3} (2x-3) < \frac{1}{2}$$

$$\log_{1/3} (2x-3) < \sqrt{7}$$

$$\log$$

$$2x-3 > \frac{1}{3^{\sqrt{7}}}$$

$$2x-3 > 0$$

$$\log_{1/3} (2x-3) > 0$$

$$\log_7 \log_{1/3} (2x-3) > 0$$

$$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$$

### Exponentiated Inequalities

→ Move the base some  
→

Base  $\in (0, 1)$   
Sign change

Base  $\in (1, \infty)$   
no sign change

$$①. 2^{x+2} > (\frac{1}{4})^{\frac{1}{x}}$$

$$2^{x+2} > 2^{-\frac{2}{x}}$$

$$x+2 > -\frac{2}{x}$$

$$x+2 + \frac{2}{x} > 0$$

$$\frac{x^2 + 2x + 2}{x} > 0$$

$$-\frac{1}{x} > 0$$

$$x = -1 \rightarrow -3$$

$$\boxed{(-\infty, 0)}$$

$$\textcircled{1} \quad 2 \cdot (1.25)^{-x} < (0.64)^{\frac{2}{(1+5x)}}$$

$$\textcircled{2} \quad \left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \frac{2^x}{4}$$

$$\frac{2}{5} > \left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}}$$

$$1 < \frac{1}{2} \left(\frac{6-5x}{2+5x}\right)$$

$$\frac{6-5x-2-5x}{2+5x} > 0$$

$$\frac{4x-10x}{2+5x} > 0$$

$$\frac{10x-4}{5x+2} < 0$$

PA

$$1 + \frac{12-5x}{2+5x} < 0$$

$$2+5x < 12-5x < 0$$

$$12 < 6+5x$$

$$6+5x < 0$$

$$-6 < 5x$$

$$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{2}{5}\right)^{-2}$$

$$\frac{5x-6}{5x+2} < 2$$

$$\frac{5x-6-10x+1}{5x+2} < 0$$

$$\frac{2+5x}{5x+2} > 0$$

$$\frac{5x+2}{5x+2} > 0$$

$$\begin{array}{c|cc|c} & -\frac{2}{5} & \frac{2}{5} & \\ \hline & + & - & + \\ (-\infty, -\frac{2}{5}) \cup (\frac{2}{5}, \infty) & & & \end{array}$$

$$\frac{7x-10x+12}{25x+2} < 0$$

$$\frac{3x-12}{5x+2} > 0$$

$$\begin{array}{c|cc|c} & -\frac{2}{5} & 4 & \\ \hline & - & + & - \\ (-\infty, -\frac{2}{5}) \cup (4, \infty) & & & \end{array}$$

$$\textcircled{2} \quad (1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$$

$$\left(\frac{1.25}{100}\right)^{1-x} < \left(\frac{64}{100}\right)^{2(1+\sqrt{x})}$$

$$\left(\frac{5}{4}\right)^{1-x} < \left(\frac{8}{4}\right)^{-4(1+\sqrt{x})}$$

$$1-x < -4(1+\sqrt{x})$$

$$4(1+\sqrt{x}) < x-1$$

$$4 + 4\sqrt{x} < x^2$$

$$x^2 - 4\sqrt{x} - 5 > 0$$

$$x - 5\sqrt{x} + 5 > 0$$

$$x(t-s) + 1(t-s) > 0$$

$$(t+1)/(t-s) > 0$$

$$\begin{array}{c} + - + - \\ \swarrow \quad \searrow \end{array}$$

$$t \in (-\infty, -1) \cup (5, \infty)$$

$$x \notin \{-\infty\}$$

$$x \notin (5, \infty)$$

$$\textcircled{3} \quad 2\left(\frac{5}{2}\right)^{\frac{5x-6}{5x+2}} < \left(\frac{5}{2}\right)^2$$

$$\frac{5x-10}{5x+2} < 2$$

$$\frac{5x+2}{5x+2} > 0$$

$$x \in (-\infty, -2) \cup (-2, \infty)$$

DYS-6

$$\textcircled{8} \quad \left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > 1$$

$$\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > \left(\frac{2}{3}\right)^0$$

$$\frac{|x|-1}{|x|+1} < 0$$

$$|x|-1 < 0$$

$$\begin{cases} |x| < 1 \\ x \in (-1, 1) \end{cases}$$

H.W. ~~6~~

DYS-6 - (Full)

$$\begin{array}{l} \text{ACTACT} \\ 0-1(21, 22, 23, 24, 25, 26, 27, 28, 29) \\ 0-2(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15) \end{array}$$

$$c = 207$$

$$\textcircled{4} \quad \text{DYS-9S} \quad (\log_{10} 100x)^2 + (\log_{10} 10x)^2 + \log_{10} x < 14$$

$$\cancel{(\log_{10} 100x)^2} + \cancel{(\log_{10} 10x)^2} + \cancel{\frac{\log x}{\log 10}}$$

$$\cancel{(\log_{10} 100x)^2} + \cancel{(\log_{10} 10x)^2} + \cancel{(\log_{10}^2 + \log^2 x)}$$

$$\cancel{\log_{10} - \log 10}$$

$$\textcircled{2} \quad 9 \left(2 + \log_{10} x\right)^2 + \left(1 + \log_{10} x\right)^2 + \log_{10} x < 14$$

$$4y + y^2 + 4y + 1 + y^2 + 2y + y < 14$$

$$2y^2 + 7y - 9 < 0 \quad 2y^2 + 7y - 2y - 9 = 0$$

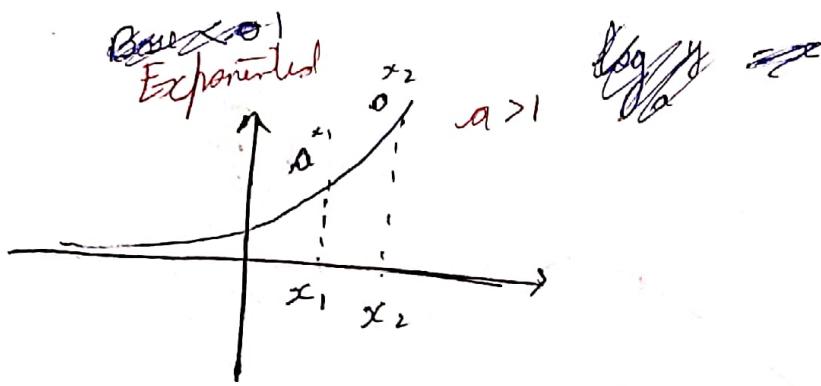
$$\xrightarrow{-9/2} \quad \xrightarrow{1}$$

$$\log_{10} x \in (-\cancel{\infty}, -9/2, 1)$$

$$\boxed{x \in (10^{-9/2}, 10)} \quad \checkmark$$

(110)

# Exponential graphs and Logarithmic graphs

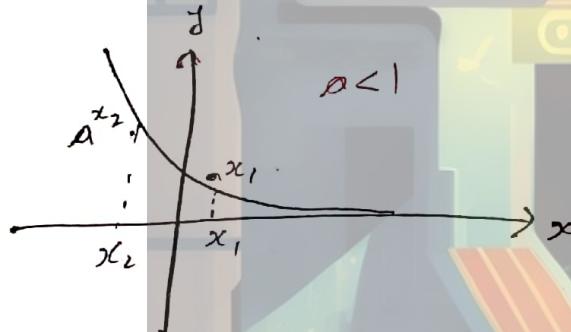


$$x_1 < x_2 \quad a^{x_1} < a^{x_2}$$

$$a^{x_1} < a^{x_2} \quad \left. \begin{array}{l} f(x) \\ \log_a f(x) \end{array} \right\} > b \quad \left. \begin{array}{l} a > 1 \\ 0 < a < 1 \end{array} \right\} \Rightarrow f(x) > \log_a b$$

$$a > 1 \Rightarrow f(x) > \log_a b$$

$$0 < a < 1 \Rightarrow f(x) < \log_a b$$



$$x_2 < x_1 \quad a^{x_2} > a^{x_1}$$

Domain (Range of  $x$ )  $\rightarrow (-\infty, \infty)$

Range  $\rightarrow (0, \infty)$

$$y = \log_a x$$

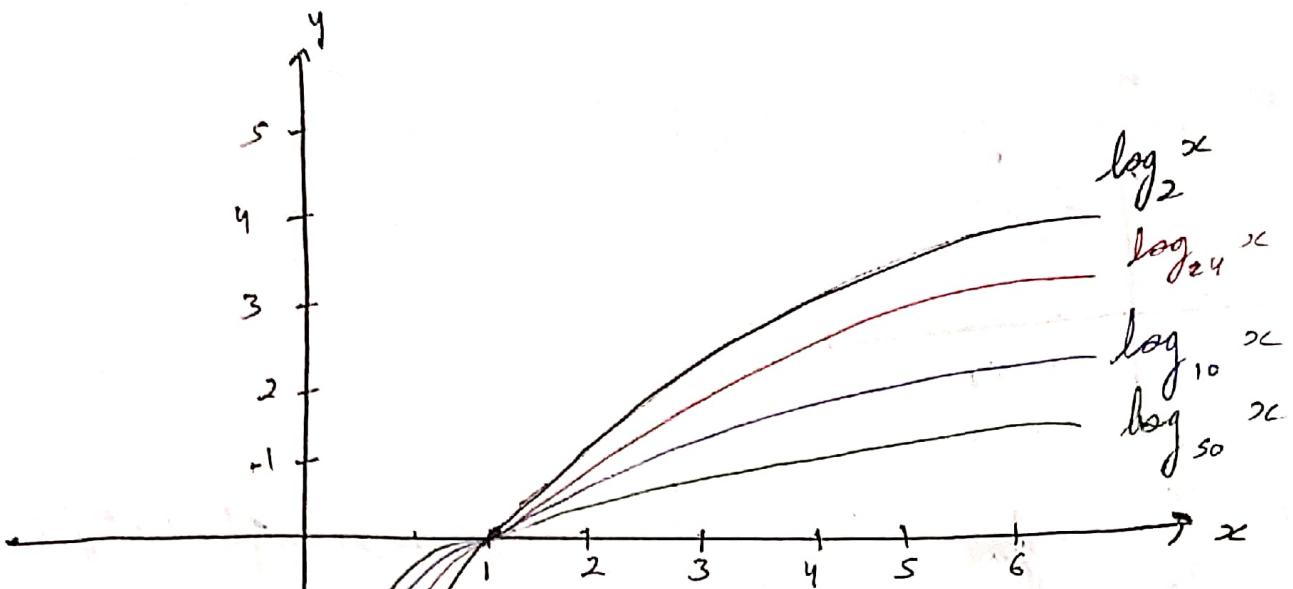
$$a > 0 \quad x > 0$$

$$a=2 \quad x=2, \quad y=1, \quad x=4, \quad y=2, \quad x=8, \quad y=3, \quad x=16, \quad y=4$$

$$a=4 \quad x=4, \quad y=1, \quad x=16, \quad y=2, \quad x=64, \quad y=3, \quad x=256, \quad y=4$$

$$a=10 \quad x=10, \quad y=1, \quad x=100, \quad y=2, \quad x=10^3, \quad y=3, \quad x=10^4, \quad y=4$$

$a > 0$	$x < 0$	
$a=2$	$x=y_2$	$y_4$
	$y=-1$	-3
$a=4$	$x=y_4$	$y_{16}$
	$y=-1$	-3
$a=10$	$x=y_{10}$	$y_{100}$
	$y=-1$	-3

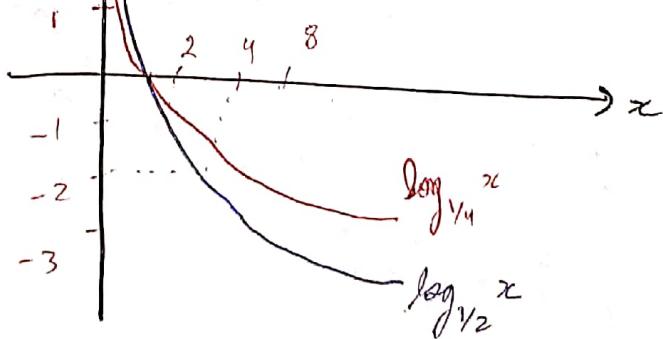


Point of intersection = (1, 0)

I draw graph  $y = \log_{r_2} x$        $y = \log_{r_4} x$

$$\begin{array}{lllll} y = & 1 & 2 & 3 & 4 \\ x = & r_2 & r_4 & r_8 & r_{16} \end{array}$$

$0 < a < 1$



## Conclusions

- Value of log can be +ve or can be -ve or zeros.
- Graph of log lies in I<sup>st</sup> & IV<sup>th</sup> quadrant.
- When base 'a' and number 'x' lies on either side of 1  
(i.e. away from) then value of log will be +ve.  
When base 'a' and number 'x' lies on the same side of one  
then the value of log will be -ve.

$$\begin{cases} \text{Base } a < 1 \\ x \in (0, 1) \end{cases} \text{ +ve}$$

$$x \in (1, \infty) \text{ -ve}$$

$$\begin{cases} \text{Base } a > 1 \\ x \in (1, \infty) \end{cases} \text{ +ve}$$

$$x \in (0, 1) \text{ -ve}$$

for  $\log_a x$

Q which of the following is +ve

①  $(\log_{10} 7)^{10}$       (+) ve

②  $\log_{10} 7 \cdot \log_{10} 2$       (+) ve

③  $\log_2 10$       (-) ve

④  $\log_2 8$       (+) ve

d. find signs

①  $\log_9 (\log_4 5)$       (+) ve

②  $\log_5 (\log_4 2)$       (+) ve

③  $\log_{\log_5 11} \left(\frac{3}{7}\right)$       (+) ve

## Characteristic & Mantissa

↓  
log to logarithmic  
part

→ fractional part ( $0 \sim 1$ )  
~~Common base 10<sup>100</sup>~~

$$1.7 \\ + 0.7$$

$$\sim 4.3 \\ - 4 + -0.3 \quad X$$

$$\sim 4.3 \\ - 5 + 0.7 \quad \checkmark$$

$$\log_{10} 20 = 1.3010$$

$$1 \\ C + 0.3010$$

$$\log_{10} \frac{1}{20} = -1.3010$$

$$2 \\ C + 0.6990$$

$$\log_{10} 1300$$

$$-3 \\ C 0.5229$$

Applications :-  
→ Finding the number of digits using common log.  
Base - 10

No. of ~~zero~~ digits =  $C + 1$

\*  $\log_{10} 2 = 0.3010$

\*  $\log_{10} 3 = 0.4771$

$$\text{Q. } \log_{10} 10^2$$

$$\log_{10} 10^2 = 2$$

$$c = 2 \quad m = 0$$

$$\begin{aligned}\text{no. of digits} &= c+1 \\ &= 2+1 \\ &= 3\end{aligned}$$

$$\text{Q. } \log_{10} 10^3 = 3$$

$$\begin{aligned}c &= 3 \quad m = 0 \\ \text{no. of digits} &= c+1 \\ &= 3+1 \\ &= 4\end{aligned}$$

$$\text{Q. } \log_{10} 10^4 = 4$$

$$\begin{aligned}c &= 4 \quad m = 0 \\ \text{no. of digits} &= c+1 \\ &= 4+1 \\ &= 5\end{aligned}$$

Q. Find the no. of digits in  $6^{50}$

$$\log_{10} 6^{50}$$

$$\text{so } \log_{10} 6$$

$$\text{so } (\log_{10} 2 + \log_{10} 3)$$

$$\text{so } (0.3010 + 0.4771)$$

$$0.7781$$

$$38.905$$

$$c = 38$$

$$m = 0.905$$

$$\begin{aligned}\text{no. of digits} &= 38+1 \\ &= 39\end{aligned}$$

DYS 7

Q 2.  $18^{20}$

$$\log_{10} 18^{20}$$

$$20 \log_{10} 18$$

$$20(\log_{10} 3 + \log_{10} 3 + \log_{10} 2)$$

$$20(0.3010 + 0.4771 + 0.4771)$$

$$20(1.2552)$$

$$C=25 \quad M=?$$

$$\text{no. of digits} \quad \boxed{\begin{array}{l} = 25 + 1 \\ = 26 \end{array}}$$

H.W. ~~0.23~~ 17-6-24

0-3 (full)

→ Finding no. of zeros after the decimal & before the significant figure using common log  
Significant figure - (nos. apart from zero)  
no. of zeros =  $|C| - 1$

e.g.,  $0.01$

$$\log_{10} 0.01 = -2$$
$$C = -2 \quad M = 0$$

$$\text{no. of zeros} = 2 - 1$$
$$= 1$$

$$0.0001$$

$$\log_{10} 10^{-4} = -4$$

$C = -4 \quad M = 0$

$$\text{no. of zeros} = 4 - 1$$

$$= 3$$

$$0.000001$$

$$\log_{10} 10^{-6} = -6$$

$C = -6 \quad M = 0$

$$\text{no. of zeros} = 6 + 1$$

$$= 5$$

- Q Find the no. of zeros after a decimal & before a significant figure in  $2^{-100}$

$$\log_{10} 2^{-100}$$

$$-100 (0.3010)$$

$$-3.010 \times$$

$$-30.10$$

$C = -301 \quad M = 0.9$

$$\text{no. of zeros} = C - 1$$

$$= 301 - 1$$

$\boxed{= 30}$

DYS-7

Q 2.  $\log_{10} 3^{-100}$

$-100 (0.4771)$

$-47.71$

$c = -48$

$m =$

$48 - 1$

$= 47$

$\log_{10} c^{15}$

$15(0.3010 + 0.4771)$

$15 \times (0.7781)$

$11.6715$

$c = 11$

$11 + 1$

$= 12$

Q 3.  $\log_{10} 2^{200}$

$-200 (0.3010 + 0.4771)$

$-200 (0.7781)$

$-1.5562$

$c = -156$

$156 - 1$

$155$

Q Evaluat:  $\log_{10}(0.06)^6$  find characteristic & Mantissa

~~$\log_{10} 6 + \log_{10} 10^{-2}$~~

~~$6[(0.7781) \oplus -2]$~~

~~$4.6686 - 812$~~

~~$2.6686$~~

~~$8.6686$~~

~~$c = 2$~~

~~$M = 0.6686$~~

~~$c = -9$~~

~~$M = 0.3314$~~

$$c \log_{10} \left( \frac{6}{100} \right)$$

$$6(0.7781 - 1)$$

~~so x (→)~~

$$6(-1.2219) = -7.331$$

$$C = -8 \quad M = 0.668$$

DYS-7

Q S.

$$\log_{10} \frac{6}{100}$$

$$(0.7781 - 5)$$

$$-4.2219$$

$$C = -5 \quad M =$$

$$\log_3 750$$

$$3 \log_3 5 + \log_3^2 + \log_3 3$$

$$\frac{3 \log_{10} 10}{\log_3 10} + \frac{\log_{10}^2}{\log_3 10} + \frac{\log_3 3}{\log_3 10}$$

$$\frac{3(1 - 0.3010)}{0.4771} + \frac{0.3010}{0.4771} + 1$$

$$3 - 0.9030 + 0.3010 + 0.4771$$

$$\frac{2.8751}{0.4771}$$

$$= 6 \dots$$

$$C = G$$

$$G \cdot (-s)$$

$$= 1$$

H.W. 18-6-29

Race 9-19

O-7

O-4 (Androste)

T-A

