! Determinants !

- Determinant is a scalar value which can be a real no. or complex no.
- It is represented by two vertical lines & b/w these two lines we take the grid of nos. in form of rows & columns.
- Modulus & Determinants are diff.
- Determinants are only possible for square orders.

  eg. 1×1, 2×2, 3×3 ..... $n\times n$ etc.

  where 1^{st} no. denotes no. of rows

  2^{nd} no. denotes no. of columns

- It is represented by $|A|$ or $\det(A)$
- 1×1 order — $|3| = 3$

  $|-2| = 2$

  $\det(-3) = -3$

2×2 order —

$$\begin{array}{cc} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} & \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array} \\ C_1 \quad C_2 & \end{array}$$

$$\begin{array}{cc} \begin{vmatrix} a & b \\ c & d \end{vmatrix} & \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \end{array} \\ C_1 \quad C_2 & \end{array}$$

3×3 order -

$$\begin{array}{cc} \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} & \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \\ C_1 \quad C_2 \quad C_3 & \end{array}$$

Elements of a determinant

a_{ij} — i - no. of rows
— J - no. of columns

1×1 : $A = |a_{11}|$

2×2 : $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

3×3 : $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Q1. make a 2×2 determinant whose elements follow.

i) $a_{ij} = (i+j)^2$

$$A = \begin{vmatrix} 4 & 9 \\ 9 & 16 \end{vmatrix}$$

ii) $a_{ij} = \left(\frac{i}{j}\right)$

$$A = \begin{vmatrix} 1 & 1/2 \\ 2 & 1 \end{vmatrix}$$

Evaluation of determinants.

① 1×1 $|3| = 3$

$|-2| = -2$

② 2×2

i) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1\times4) - (2)(3)$
$= 4-6$
$= -2$

ii) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $ad - bc$

iii) $\begin{vmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{vmatrix}$ $-\sin^2\theta - \cos^2\theta = -1$

③ 3×3 — i)

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix}$$

$= 1(2-0) - 2(-4-0) + 3(-2-0)$
$= 2+8-6$
$= 4$

ii)

$$\begin{vmatrix} 1 & 3 & 6 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} + 6\begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix}$$

$= 1(4-0) - 3(-2-0) + 6(0-2)$
$= 4 + 6 - 12$
$= -2$

Q2. find the values of

① $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \\ 2 & 1 & 3 \end{vmatrix} = 1(9-2) - 2(0-4) - 1(0-6)$
$= 7 + 8 + 6$
$\boxed{= 21}$

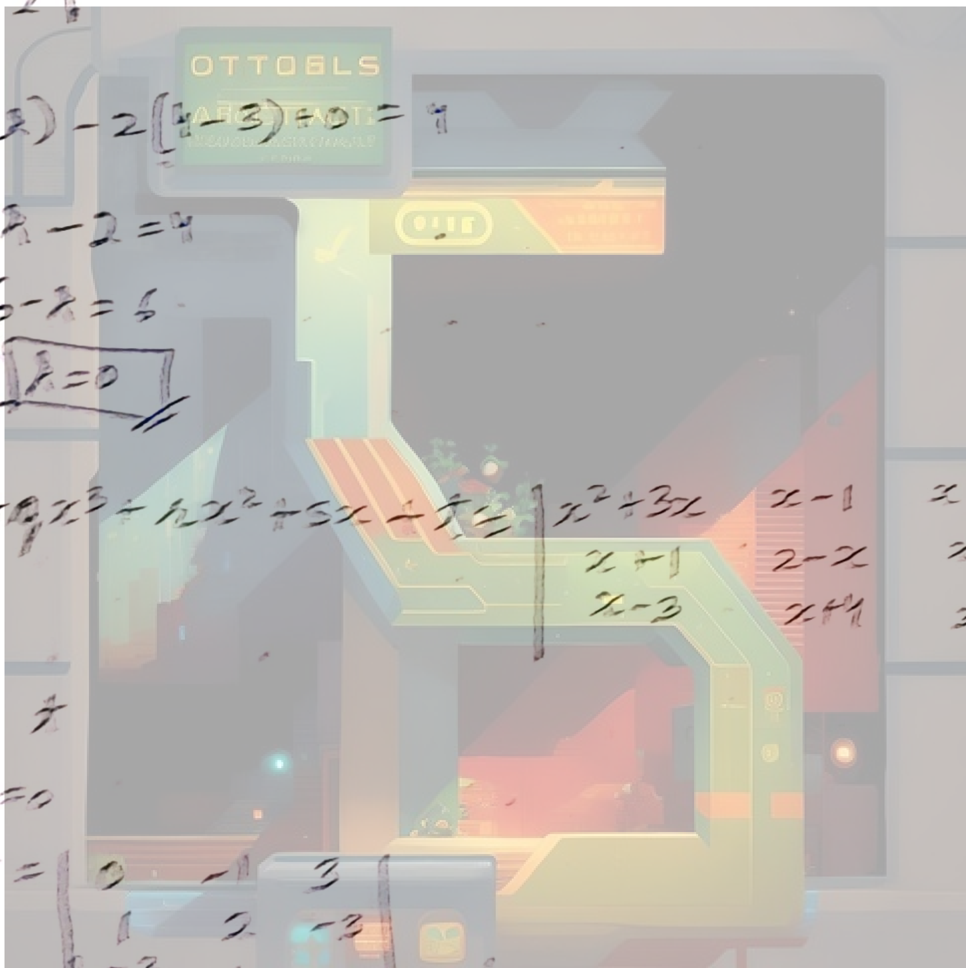
② $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3(1+6) + 4(1+4) + 5(3-2)$

$= 21 + 20 + 5$

$\boxed{= 46}$

Q find λ if :-

$\begin{vmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & \lambda & 2 \end{vmatrix} = 4$



$1(6-\lambda) - 2(4-3) + 0 = 4$

$6 - \lambda - 2 = 4$

$6 - \lambda = 6$

$\boxed{\lambda = 0}$

Q If $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$

find t

at $x = 0$

$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix}$

$t = 1(-9) + 3(4+6)$

$= 30 - 9$

$\boxed{= 21}$

Q find value(s) of x if:

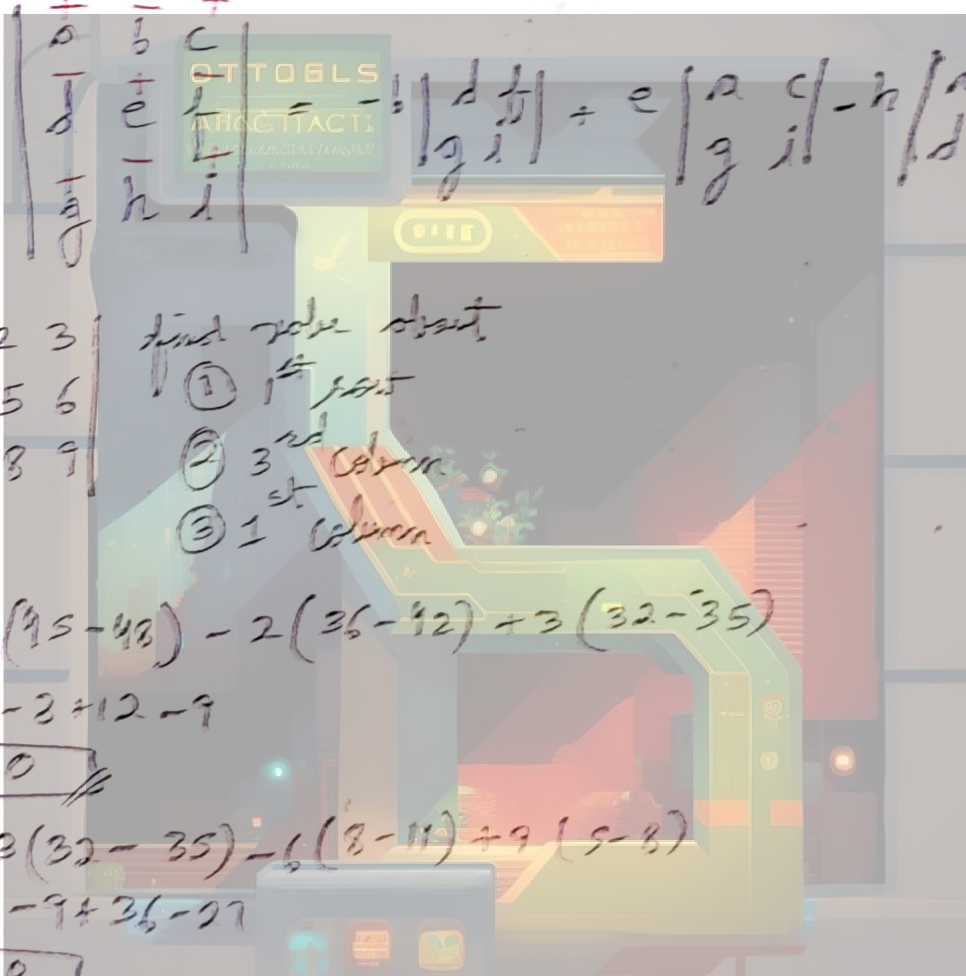
$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$x^2 - 36 = 36 - 36$$

$$x^2 = 36$$

$$\boxed{x = \pm 6}$$

Note -



$$\begin{vmatrix} \overset{+}{a} & \overset{-}{b} & \overset{+}{c} \\ \overset{-}{d} & \overset{+}{e} & \overset{-}{f} \\ \overset{+}{g} & \overset{-}{h} & \overset{+}{i} \end{vmatrix} = -b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + e\begin{vmatrix} a & c \\ g & i \end{vmatrix} - h\begin{vmatrix} a & c \\ d & f \end{vmatrix}$$

Q $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ find value about
① 1^{st} row
② 3^{rd} column
③ 1^{st} column

①
$$= 1(45-48) - 2(36-42) + 3(32-35)$$
$$= -3 + 12 - 9$$
$$\boxed{= 0}$$

② $= 3(32-35) - 6(8-14) + 9(5-8)$
$$= -9 + 36 - 27$$
$$\boxed{= 0}$$

~~③ 1(18-24) - 4(~~

③ $1(45-48) - 4(18-24) + 7(12-15)$

$$-3 + 24 - 21$$

$$\boxed{= 0}$$

Minor & Cofactor

→ Minor - Minor of an element is defined by deleting the elements of row/column in which it lies.

→ It is denoted by M_{ij}.

→ Co-factor - ~~It is denoted by~~ It is related to minor

$$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$$

Q find minor & cofactor of each element in

$$\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$$

$a_{11} = 1$	$a_{12} = 2$	$a_{21} = 3$	$a_{22} = 7$
$M_{11} = 7$	$M_{12} = 3$	$M_{21} = 2$	$M_{22} = 1$
$C_{11} = (-1)^{1+1} \times 7 = 7$	$C_{12} = -3$	$C_{21} = -2$	$C_{22} = 1$

Q find minor & co-factor of each element in

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix}$$

element	Minor	Co-factor
$a_{11} = 1$	$\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$	1
$a_{12} = 2$	$\begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} = +2$	-2
$a_{13} = 3$	$\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = +1$	1
$a_{21} = 0$	$\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 0$	0
$a_{22} = 1$	~~$\begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} = +2$~~ $\begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix} = 6$	~~2~~ 6
$a_{23} = 2$	~~$\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = 1$~~ $\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 4$	~~1~~ -4

$a_{31} = -1$	$\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$	1
$a_{32} = 2$	$\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = 2$	-2
$a_{33} = 3$	$\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$	1

Note:- $C_{ij} = (-1)^{i+j} M_{ij}$

$i+j \in$ odd: $C_{ij} = -M_{ij}$

$i+j \in$ even: $C_{ij} = M_{ij}$

Q find minor & co-factor of 2nd raw in

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$a_{21} = 4$ $M_{21} = \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = 18 - 24 = -6$ $C_{21} = 6$

$a_{22} = 5$ $M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 9 - 21 = -12$ $C_{22} = -12$

$a_{23} = 6$ $M_{22} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$ $C_{23} = 6$

Note:- Value of Determinant can be written as.

$$\boxed{\begin{aligned} \Delta &= a_{11} M_{11} - a_{12} M_{12} + a_{13} M_{13} \\ \Delta &= a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} \end{aligned}}$$

eg. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$

$= -3 + 12 - 9$

$\boxed{= 0}$

Property of Minor & Co-factor

① $\Delta = \Sigma$ (Elements of any row) × (corresponding co-factors of that row)

$\Delta = \Sigma$ (Elements of any column) × (corresponding co-factors of that column)

② Σ (elements of any row) × (corresponding co-factors of any other row) $= 0$

Σ (Elements of any column) × (corresponding co-factors of any other column) $= 0$

Q find values of following in $\Delta = \begin{vmatrix} p^{+} & q^{-} & r^{+} \\ x^{-} & y^{+} & z^{-} \\ l^{+} & m^{-} & n^{+} \end{vmatrix}$

① $pC_{21} + qC_{22} + rC_{23} = 0$

② $xC_{21} + yC_{22} + xC_{23} = \Delta$

③ $-qM_{12} + yM_{22} - mM_{32} = \Delta$

④ $qM_{12} - yM_{22} + mM_{32} = -\Delta$

⑤ $-xM_{21} + yM_{22} - zM_{23} = \Delta$

⑥ $xM_{21} - yM_{22} + zM_{23} = -\Delta$

Q $\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \delta & \theta & \pi \\ e & g & i \end{vmatrix}$

① $\alpha C_{21} + \beta C_{22} + \gamma C_{23} = 0$

② $e M_{11} - g M_{12} + i M_{13} = 0$

③ $\gamma C_{11} - \pi C_{12} + i C_{13} =$ ek colum, ek raw islije, potonchi

④ $-\delta M_{21} + \theta M_{22} - \pi M_{23} = \Delta$

Properties of Determinants:

① Value of det remains same if rows & colums are interchanged

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

② If any two rows/colums are interchanged, only the sign of the determinant changes.

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

③ If a det has two rows/columns identical, then its value becomes 0.

eg. $\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$

$$\begin{vmatrix} a & b & c \\ P & Q & R \\ a & b & c \end{vmatrix} = 0$$

(1) If all the elements of a row or colum are multiplied by a same no., then the value of det is also multiplied by that no.

eg.

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2 \quad \times 3$$

$$\begin{vmatrix} 3 & 6 \\ 3 & 4 \end{vmatrix} = -6$$

Q find value of $\Delta = \begin{vmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 3 \end{vmatrix} \times 3 \times 10 \times 10 \times 10$$

Q If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$ Then value of, $\begin{vmatrix} 2a & 2b & 2c \\ 3d & 3e & 3f \\ 5g & 5h & 5i \end{vmatrix}$ $\times\frac{1}{2}$, $\times\frac{1}{3}$, $\times\frac{1}{5}$

$\cancel{\Delta = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{5} \times \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$

$$\Delta = 2 \times 3 \times 5 \times 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\boxed{\Delta = 60}$$

$\cancel{\Delta = \frac{1}{15}}$

Q $\begin{vmatrix} x & 1 & x^{2013} \\ x^4 & 2 & x^{2016} \\ x^{10} & 3 & x^{2022} \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ x^3 & 2 & x^3 \\ x^9 & 3 & x^9 \end{vmatrix} \times x \times x^{2013}$

$= 0$

$$\boxed{= 0}$$

$$\begin{array}{c} \\ \end{array}\begin{vmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{vmatrix} \rightarrow \begin{vmatrix} a & a & a \\ b & b & b \\ c & c & c \end{vmatrix} \times a \times b \times c$$

$a \quad b \quad c$

$$\boxed{=0}$$

(5) If all the elements of a row or column are zero, the value of the determinant will be zero.

eg.
$$\begin{vmatrix} \alpha & \delta & 0 \\ \beta & \theta & 0 \\ \gamma & \phi & 0 \end{vmatrix} = 0$$

(6) If any row/column is written in terms of addition & subtraction of any 2 elements then we can break the det. accordingly to that row or column. order is important

eg.
$$\begin{vmatrix} 1+2 & 3+4 \\ 5 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 5 & 6 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix}$$

$$\begin{vmatrix} \alpha+1 & 3 \\ \beta+1 & 4 \end{vmatrix} = \begin{vmatrix} \alpha & 3 \\ \beta & 4 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & a+b & p+q \\ 1 & c+d & r+s \\ 1 & e+f & t+u \end{vmatrix} = \begin{vmatrix} 1 & a & p+q \\ 1 & c & r+s \\ 1 & e & t+u \end{vmatrix} + \begin{vmatrix} 1 & b & p+q \\ 1 & d & r+s \\ 1 & f & t+u \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & p \\ 1 & c & r \\ 1 & e & t \end{vmatrix} + \begin{vmatrix} 1 & a & q \\ 1 & c & s \\ 1 & e & u \end{vmatrix} + \begin{vmatrix} 1 & b & p \\ 1 & d & r \\ 1 & f & t \end{vmatrix} + \begin{vmatrix} 1 & b & q \\ 1 & d & s \\ 1 & f & u \end{vmatrix}$$

Q $$\begin{vmatrix} \sqrt{13}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{26}+\sqrt{15} & 5 & \sqrt{10} \\ \sqrt{65}+3 & \sqrt{15} & 5 \end{vmatrix} = \begin{vmatrix} \sqrt{13} & \sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$$

$$\cancel{= \sqrt{13}\times\sqrt{5}\times\sqrt{5} \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix} + \sqrt{3}\times\sqrt{5}\times\sqrt{5}}$$

$$= \sqrt{13}\times\sqrt{5}\times\sqrt{5}\times \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix} + \sqrt{3}\times\sqrt{5}\times\sqrt{5}\times \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$\boxed{=0}$$

(7) In the Ques with Summation Operator (Σ) with det, we can apply it in any one of the row/colum.

eg. $$D = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\sum_{r=1}^{n} D = \begin{vmatrix} \Sigma r & x & \frac{n(n+1)}{2} \\ 2\Sigma r - \Sigma 1 & y & n^2 \\ 3\Sigma r - \Sigma 2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & x & \frac{n(n+1)}{2} \\ n^2 & y & n^2 \\ \frac{n(3n-1)}{2} & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\boxed{=0}$$

(8) If the value of det. of order $n \times n$ is Δ, then the value of det. made by using the co-factors of given data is Δ^{n-1}.

eg. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$ $\quad (\underset{n}{2} \times 2)$

co-factors of a_{12}

$\begin{vmatrix} 4 & -3 \\ -2 & 1 \end{vmatrix} = -2$

$\therefore (-2)^{2-1} = (-2)^1 = -2$

Q If the value of a 3×3 det is -3, value of det. made by its co-factors is ?

$(-3)^{3-1} = (-3)^2 = 9$

Q $\begin{vmatrix} p & q & r \\ a & b & c \\ l & m & n \end{vmatrix} = 5$ find $\begin{vmatrix} bn-mc & lc-an & am-bl \\ mr-qn & pn-lr & lq-pm \\ qc-br & ar-pc & pb-aq \end{vmatrix}$

$(5)^{3-1} = 5^2 = 25$

Q $\underbrace{\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc-a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix}}_{\alpha^2} \times \underbrace{\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}}_{\alpha}$

$\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}^3$

(9) The value of a det. is not altered by adding elements of any row (or column). The same ~~multiple~~ multiples of corresponding elements of any other row / column.
[~~first~~ 1 row must be left on which we do not perform any action]

eg.

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$$

$R_1 \to R_1 + R_2$

$$\begin{vmatrix} 4 & 6 \\ 3 & 4 \end{vmatrix} = -2$$

Note :- ① for 2×2,

$R_1 \to R_1 + \alpha R_2 \quad \alpha \in R$

$C_1 \to C_1 + \alpha C_2 \quad \alpha \in R$

for 3×3,

$R_1 \to R_1 + \alpha R_2 + \cancel{\alpha R_3}\ \beta R_3 \quad \alpha, \beta \in R$

$C_1 \to C_1 + \alpha C_2 + \beta C_3 \quad \alpha, \beta \in R$

ii) 2×2, $R_1 \to x R_1 + \alpha R_2 \quad x, \alpha \in R$

↓

value of Det becomes x times

iii) while applying the other property, atleast one row/column must be unchanged.

$$\begin{vmatrix} 5 & 8 & 10 \\ 77 & 71 & 46 \\ 8 & 7 & 4 \end{vmatrix} = 2 \times \begin{vmatrix} 5 & 8 & 5 \\ 77 & 71 & 23 \\ 8 & 7 & 2 \end{vmatrix}$$

$R_1 \to R_1 + 9R_3$

$$= 2 \times \begin{vmatrix} 77 & 11 & 23 \\ 77 & 11 & 23 \\ 8 & 7 & 2 \end{vmatrix}$$

$\boxed{= 0}$

Q $\begin{vmatrix} a+b+c & p+q+r & l+m+n \\ a+b & p+q & l+m \\ a & p & l \end{vmatrix} = ?$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$\begin{vmatrix} c & r & n \\ b & q & m \\ a & p & l \end{vmatrix} =$

Q $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

~~$C_1 \to C_1 + C_2 + C_3$~~

$R_1 \to R_1 + R_2 + R_3$

$\begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} \boxed{= 0}$

Q $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ if a, b, c are in AP, then its value is

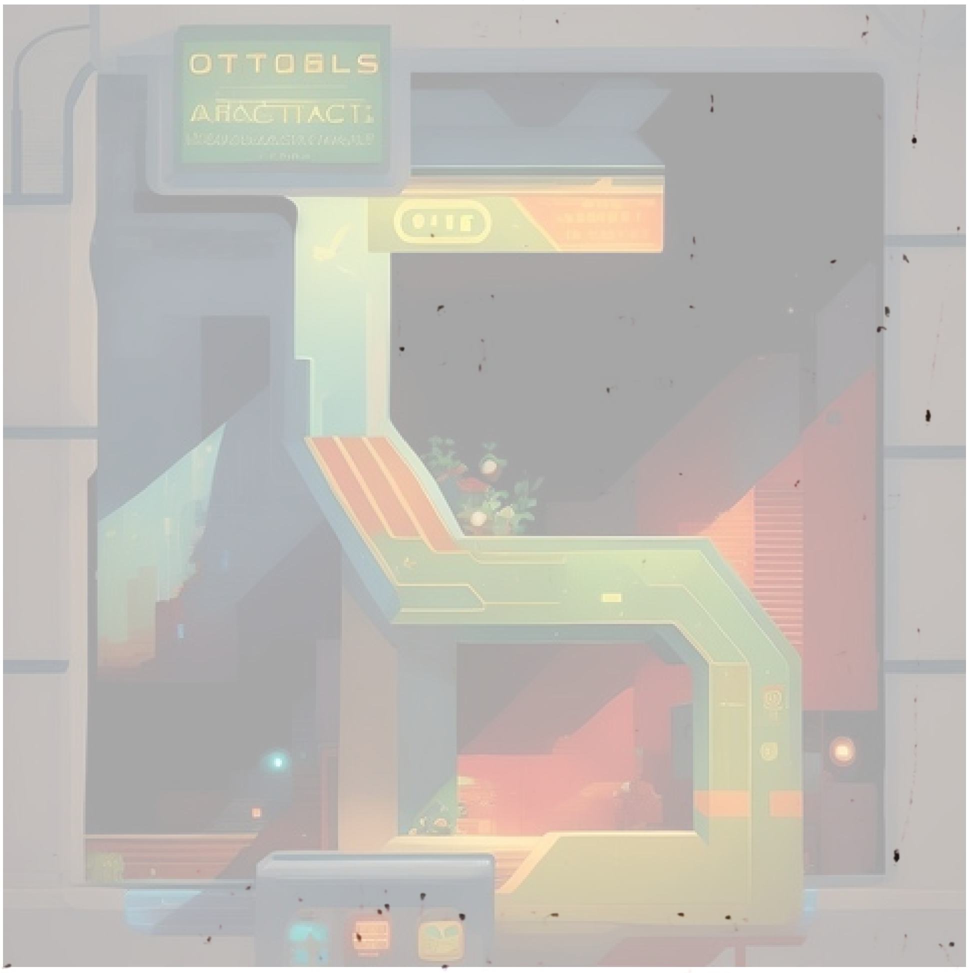
(A) $a+b+c$ (C) 0

(B) $x+a+b+c$ (D) None.

$C_1 \to C_1 - C_2$

$C_2 \to C_2 - C_1$

$= -1 \times \begin{vmatrix} 1 & 1 & x+a \\ 1 & 1 & x+b \\ 1 & 1 & x+c \end{vmatrix} \boxed{= 0}$

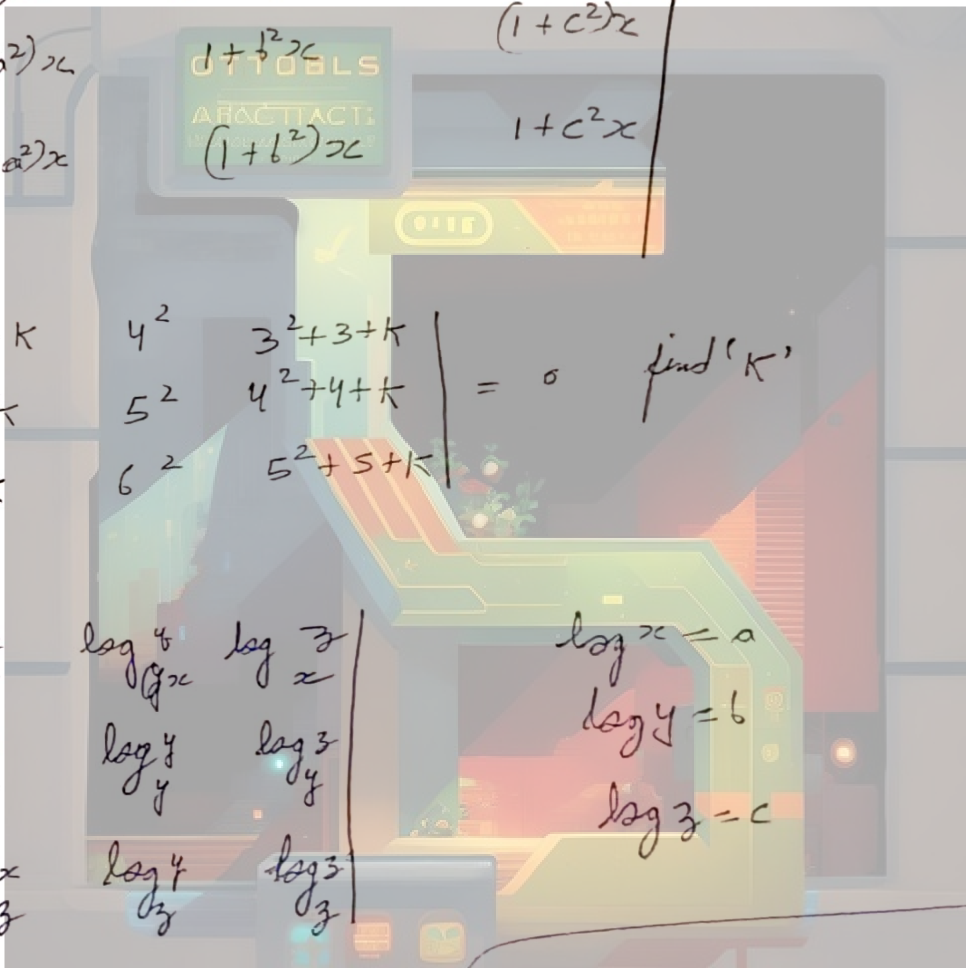


Q 1) If x, y & z are $\oplus$ve integers then, find.

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

Q2. $a^2+b^2+c^2=-2$ find degree of

$$\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$



Q3. $$\begin{vmatrix} 3^2+K & 4^2 & 3^2+3+K \\ 4^2+K & 5^2 & 4^2+4+K \\ 5^2+K & 6^2 & 5^2+5+K \end{vmatrix} = 0$$ find 'K'

Q1. $$\begin{vmatrix} \log_x x & \log_x y & \log_x z \\ \log_y x & \log_y y & \log_y z \\ \log_z x & \log_z y & \log_z z \end{vmatrix}$$

$\log x = a$

$\log y = b$

$\log z = c$

$$\begin{vmatrix} \frac{a}{a} & \frac{b}{a} & \frac{c}{a} \\ \frac{a}{b} & \frac{b}{b} & \frac{c}{b} \\ \frac{a}{c} & \frac{b}{c} & \frac{c}{c} \end{vmatrix} \begin{matrix} \to \frac{1}{a} \\ \to \frac{1}{b} \\ \to \frac{1}{c} \end{matrix}$$

$$\begin{vmatrix} a & b & c \\ a & b & c \\ a & b & c \end{vmatrix} \times \frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}$$

$0 \times \frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}$

$\boxed{=0}$

Q2 $a^2+b^2+c^2+2=0$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+2x+x(a^2+b^2+c^2) & (1+b^2)x & (1+c^2)x \\ 1+2x+x(a^2+b^2+c^2) & 1+b^2x & (1+c^2)x \\ 1+2x+x\underbrace{(a^2+b^2+c^2)}_{-2} & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$ $\quad$ $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 0 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$= 0() - 0() + 1[(x-1)^2 - 0]$

$= (x-1)^2$

$\boxed{\text{2 Degree}}$

Q3.

$$\cancel{3^2+K \quad 4^2}$$

Q3.

$$= \begin{vmatrix} 3^2+K & 4^2 & (3^2+K)+3 \\ 4^2+K & 5^2 & (4^2+K)+4 \\ 5^2+K & 6^2 & (5^2+K)+5 \end{vmatrix} = 0$$

$$= \underbrace{\begin{vmatrix} 3^2+K & 4^2 & 3^2+K \\ 4^2+K & 5^2 & 4^2+K \\ 5^2+K & 6^2 & 5^2+K \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} 3^2+K & 4^2 & 3 \\ 4^2+K & 5^2 & 4 \\ 5^2+K & 6^2 & 5 \end{vmatrix}}_{\substack{R_3 \to R_3 - R_2 \\ R_2 \to R_2 - R_1}} = 0$$

$$\begin{vmatrix} 9+K & 16 & 3 \\ 7 & 9 & 1 \\ 9 & 11 & 1 \end{vmatrix} = 0$$

$$+3(77-81) - (99+11K-144) + (9K+81-112) = 0$$

$$\cancel{45+11K}$$

$$\cancel{(-45+11K) = 12+31-9K}$$

$$\cancel{11K-45 = 43-9K}$$

$$\cancel{20K = 88}$$

$$\cancel{K}$$

$$-12+45-11K-31+9K = 0$$

$$2K = 2$$

$$\boxed{K=1}$$

H.W. Ques

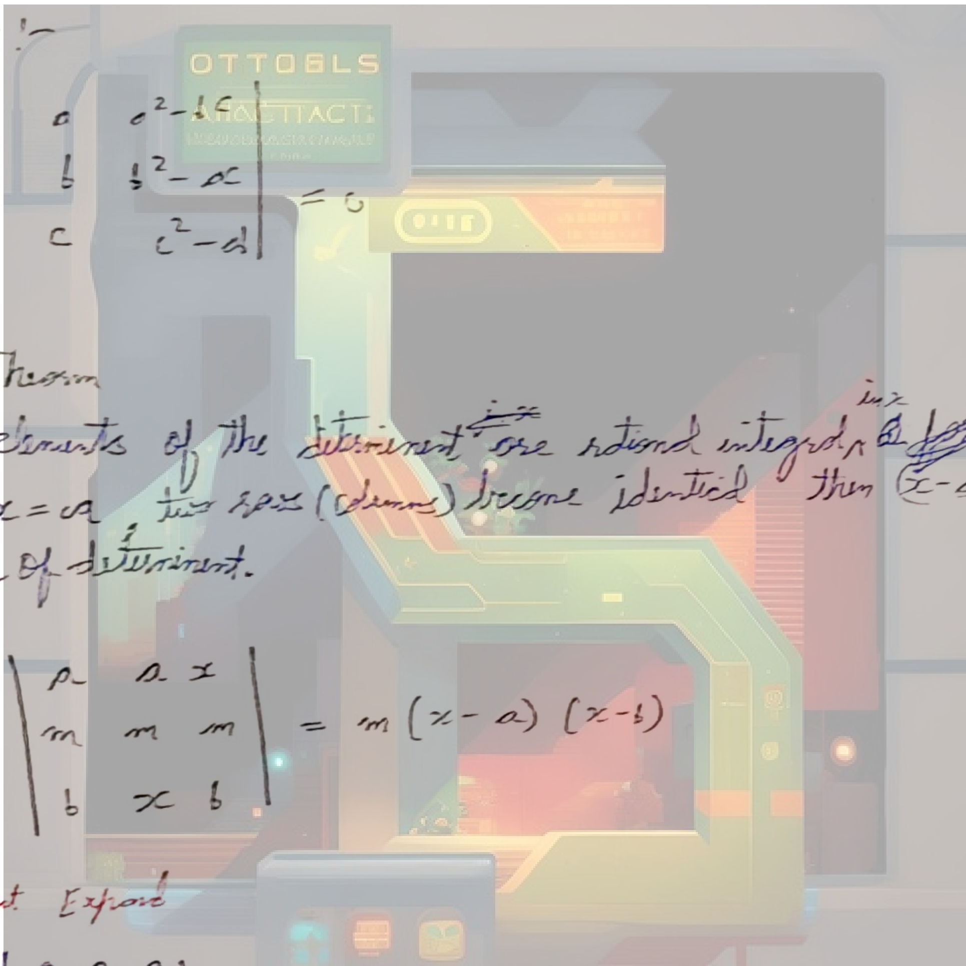
Q1. $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 2\sin 4\phi \\ \sin^2\theta & 1+\cos^2\theta & 2\sin 4\phi \\ \sin^2\theta & \cos^2\theta & 1+2\sin 4\phi \end{vmatrix} = 0$ $\quad R_1 \to R_1 + R_2$, $R_2 \to R_2 + R_1$

Then, (A) $\phi = \frac{3\pi}{4}$ (C) $\phi = \frac{3\pi}{2}$

(B) $\phi = \frac{3\pi}{8}$ ✓ (D) $\phi \in R$

Q2. Prove :-

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = 0$$



※ Factor Theorem

→ If the elements of the determinant are rational integral in x & for $x = a$ two rows (columns) become identical then $(x-a)$ is a factor of determinant.

Q1. Prove. $\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = m(x-a)(x-b)$

M I Direct Expand

M II

$x=a: \begin{vmatrix} a & a & a \\ m & m & m \\ b & a & b \end{vmatrix} = a \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ b & a & b \end{vmatrix} \times m \times a = 0$

$x=b: \begin{vmatrix} a & a & b \\ m & m & m \\ b & b & b \end{vmatrix} = 0$

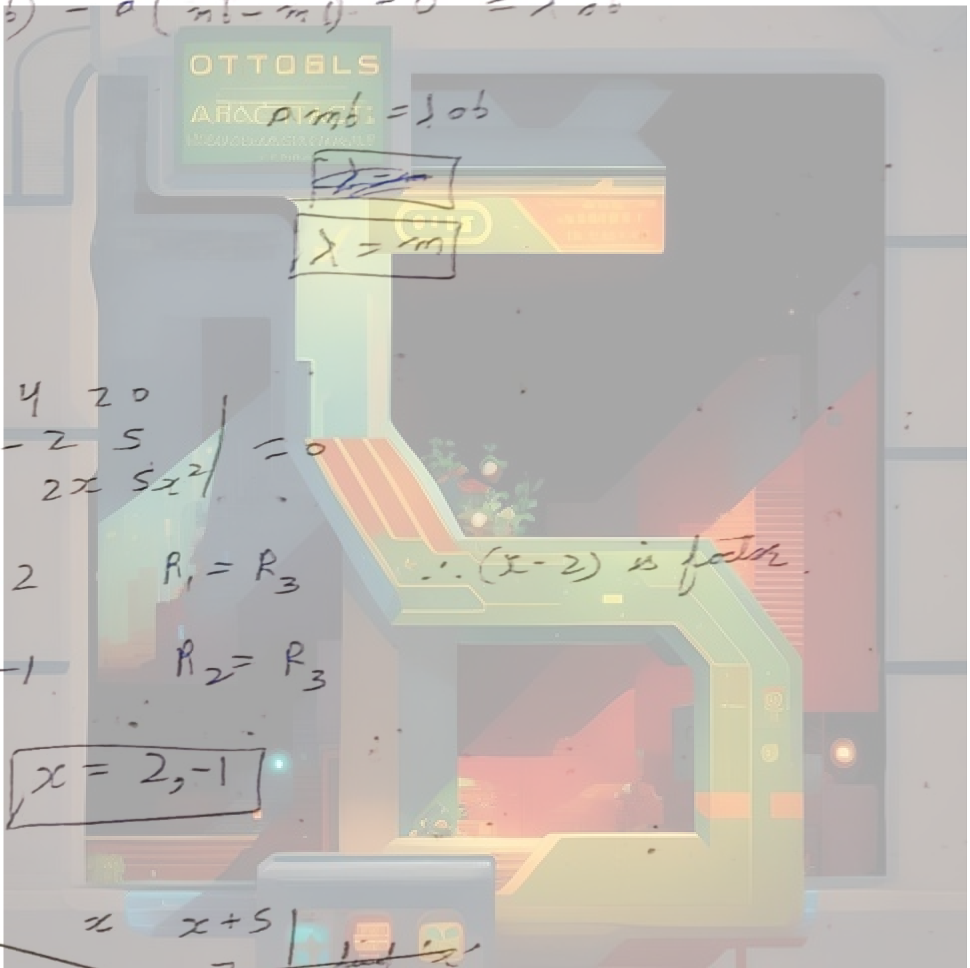
$(x-a)$ & $(x-b)$ are factor

$$\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = \lambda (x-a)(x-b)$$

Put $x=0$

$$\begin{vmatrix} a & a & 0 \\ m & m & m \\ b & 0 & b \end{vmatrix} = \lambda ab$$

$a(mb) - a(mb - mb) + 0 = \lambda ab$



$amb = \lambda ab$

$\cancel{\boxed{\lambda = m}}$

$\boxed{\lambda = m}$

Q. $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$x = 2$ $\quad R_1 = R_3$ $\quad \therefore (x-2)$ is factor.

$x = -1$ $\quad R_2 = R_3$

$\boxed{x = 2, -1}$

H W

~~Q3. $\begin{vmatrix} x^2 & x & x+5 \\ 4 & 2 & 7 \\ 9 & 3 & 8 \end{vmatrix}$ find x~~

Q If $\begin{vmatrix} a^n & a^{n+1} & a^{n+2} \\ b^n & b^{n+1} & b^{n+2} \\ c^n & c^{n+1} & c^{n+2} \end{vmatrix} = (a-b)(b-c)(c-a)$. find n. $(a+b+c)$

$$a^n b^n c^n \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$C_2 \to C_2 - C_1$ $\quad$ $C_3 \to C_3 - C_1$

$$a^n b^n c^n \begin{vmatrix} 1 & a-1 & a^2-1 \\ 1 & b-1 & b^2-1 \\ 1 & c-1 & c^2-1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ $\quad$ $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c-1 & c^2-1 \end{vmatrix} \times a^n b^n c^n$$

$$= (a-b)(b-c)(b+c) - (b-c)(a-b)(a+b)$$

$$(a-b)(b-c)(c-a) = (a-b)(b-c)(c-a) \times a^n b^n c^n$$

$$a^n b^n c^n = 1$$

$$\boxed{n=0}$$

Q2. Prove $\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

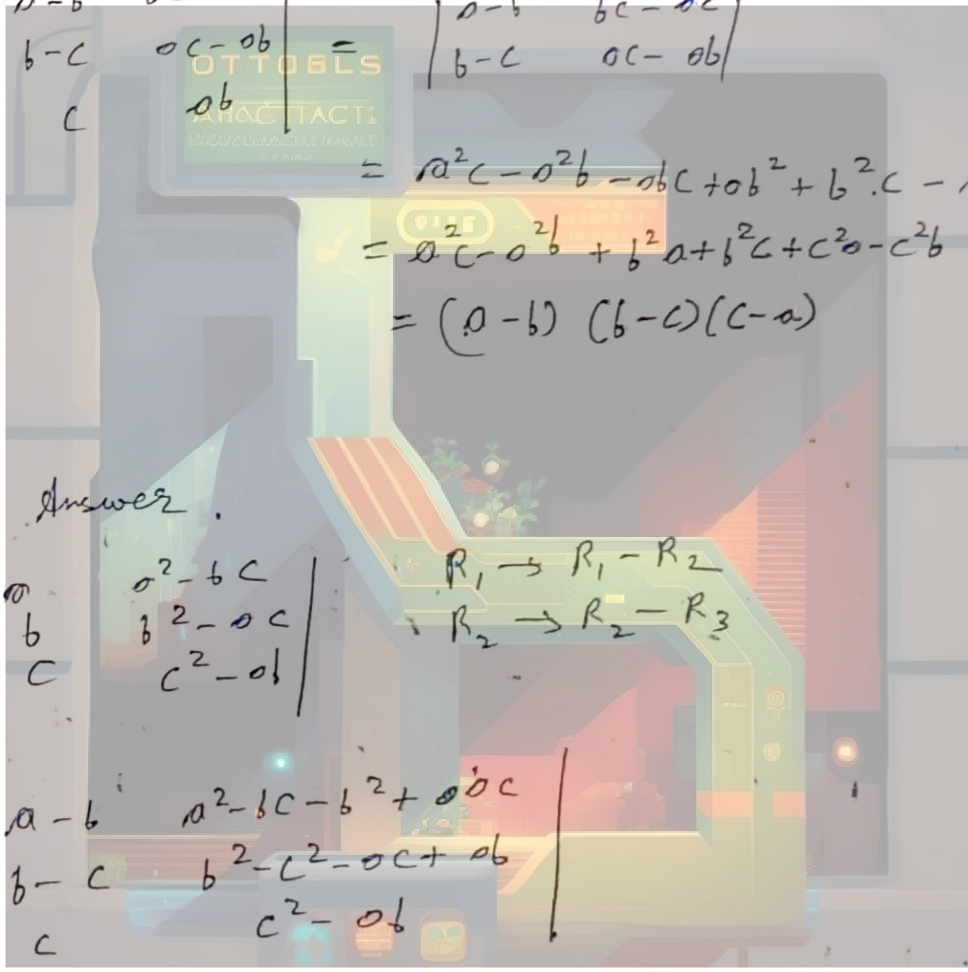
~~at $a=b$, $\det = 0$~~

~~$b=c$, $\Delta = 0$~~

~~$a=c$, $\Delta = 0$~~

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$



$$\begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ac-ab \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} a-b & bc-ac \\ b-c & ac-ab \end{vmatrix}$$

$$= a^2c - a^2b - abc + ab^2 + b^2c - abc - bc^2 + ac^2$$

$$= a^2c - a^2b + b^2a + b^2c + c^2a - c^2b - 2abc$$

$$= (a-b)(b-c)(c-a)$$

H.W Q2 Answer.

$$= \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a-b & a^2-bc-b^2+ac \\ 0 & b-c & b^2-c^2-ac+ab \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= \begin{vmatrix} a-b & a^2-b^2+ac-bc \\ b-c & b^2-c^2+ab-ac \end{vmatrix}$$

$$= (a-b)(b^2-c^2+a(b-c)) - (b-c)[a^2-b^2-c(a-b)]$$

$$= b^2(a-b) - c^2(a-b) - a(a-b)(b-c) - a^2(b-c) + b^2(b-c) + c(b-c)(a-b)$$

$$= (a-b)(b-c)(c-a) \cancel{+b^2(a-c)} - b^2(c-a) - a^2(b-c) - b^2(a-b)$$

$$\boxed{=0}$$

Note :- Special determinant values to remember.

① cyclic det $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$

② $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

③ $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

④ $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

Multiplication of Determinant

2×2

$$\begin{array}{c} \\ R_1 \\ R_2 \end{array} \begin{vmatrix} a & b \\ c & d \end{vmatrix} \times \begin{array}{cc} C_1 & C_2 \\ \hline \end{array}\begin{vmatrix} l & m \\ n & o \end{vmatrix} = \begin{vmatrix} \overset{R_1\to C_1}{al+bn} & \overset{R_1\to C_2}{am+bo} \\ \underset{R_2\to C_1}{cl+dn} & \underset{R_2\to C_2}{cm+do} \end{vmatrix}$$

Q $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \times \begin{vmatrix} -1 & 3 \\ 4 & 2 \end{vmatrix}$

$$= \begin{vmatrix} -1+8 & 3+4 \\ -3+16 & 9+8 \end{vmatrix} = \begin{vmatrix} 7 & 7 \\ 13 & 17 \end{vmatrix} = (17)(7)-(13)(7)$$

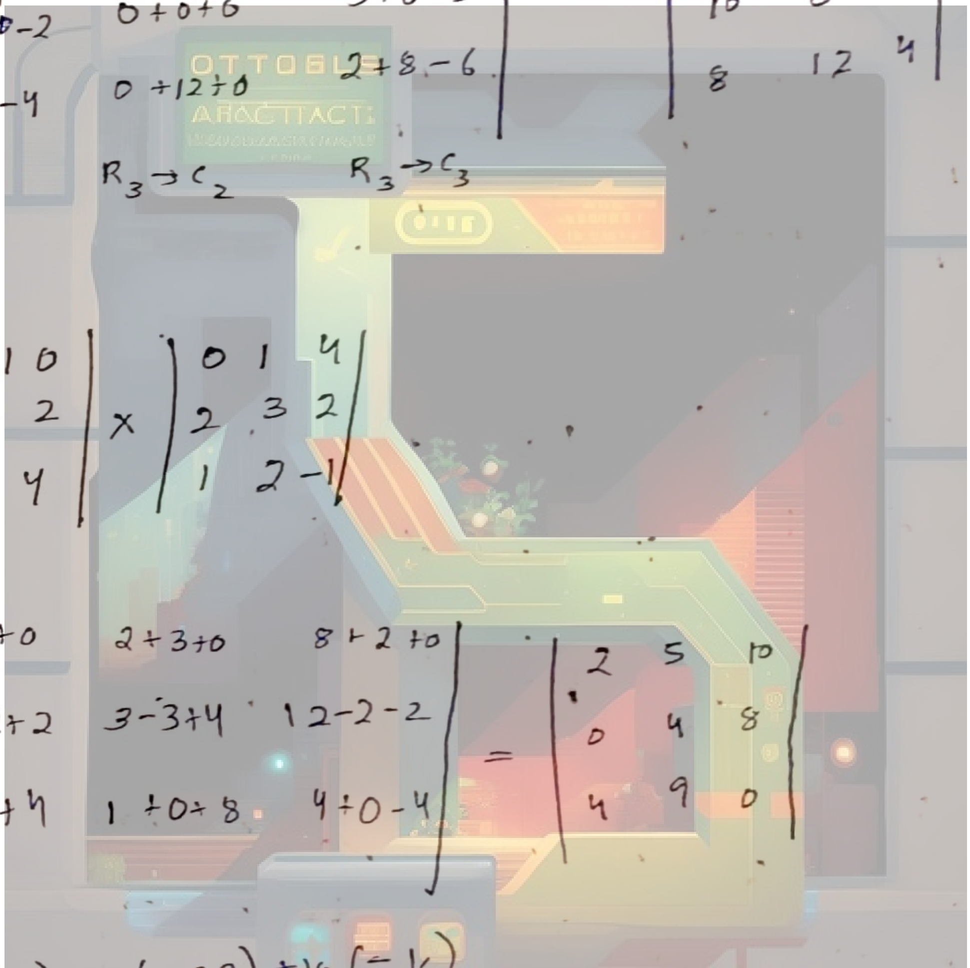
$$= (7)(4)$$

$$\boxed{=28}$$

3x3

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \\ 2 & 4 & -2 \end{vmatrix} \times \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{vmatrix} 4 & 0 & 1 \\ 1 & 3 & 2 \\ 2 & 0 & 3 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} \overset{R_1 \to C_1}{4+2+2} & \overset{R_1 \to C_2}{0+6+0} & \overset{R_1 \to C_3}{1+4+3} \\ \overset{R_2 \to C_1}{12+0\cancel{+0}-2} & \overset{R_2 \to C_2}{0+0+0} & \overset{R_2 \to C_3}{3+0-3} \\ \underset{R_3 \to C_1}{8+4-4} & \underset{R_3 \to C_2}{0+12+0} & \underset{R_3 \to C_3}{2+8-6} \end{vmatrix} = \begin{vmatrix} 8 & 6 & 8 \\ 10 & 0 & 0 \\ 8 & 12 & 4 \end{vmatrix}$$



H.W.

Q $\begin{vmatrix} 2 & 1 & 0 \\ 3 & -1 & 2 \\ 1 & 0 & 4 \end{vmatrix} \times \begin{vmatrix} 0 & 1 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & -1 \end{vmatrix}$

$$= \begin{vmatrix} 0+2+0 & 2+3+0 & 8+2+0 \\ 0+-2+2 & 3-3+4 & 12-2-2 \\ 0+0+4 & 1+0+8 & 4+0-4 \end{vmatrix} = \begin{vmatrix} 2 & 5 & 10 \\ 0 & 4 & 8 \\ 4 & 9 & 0 \end{vmatrix}$$

$= 2(-72) - 5(-32) + 10(-16)$

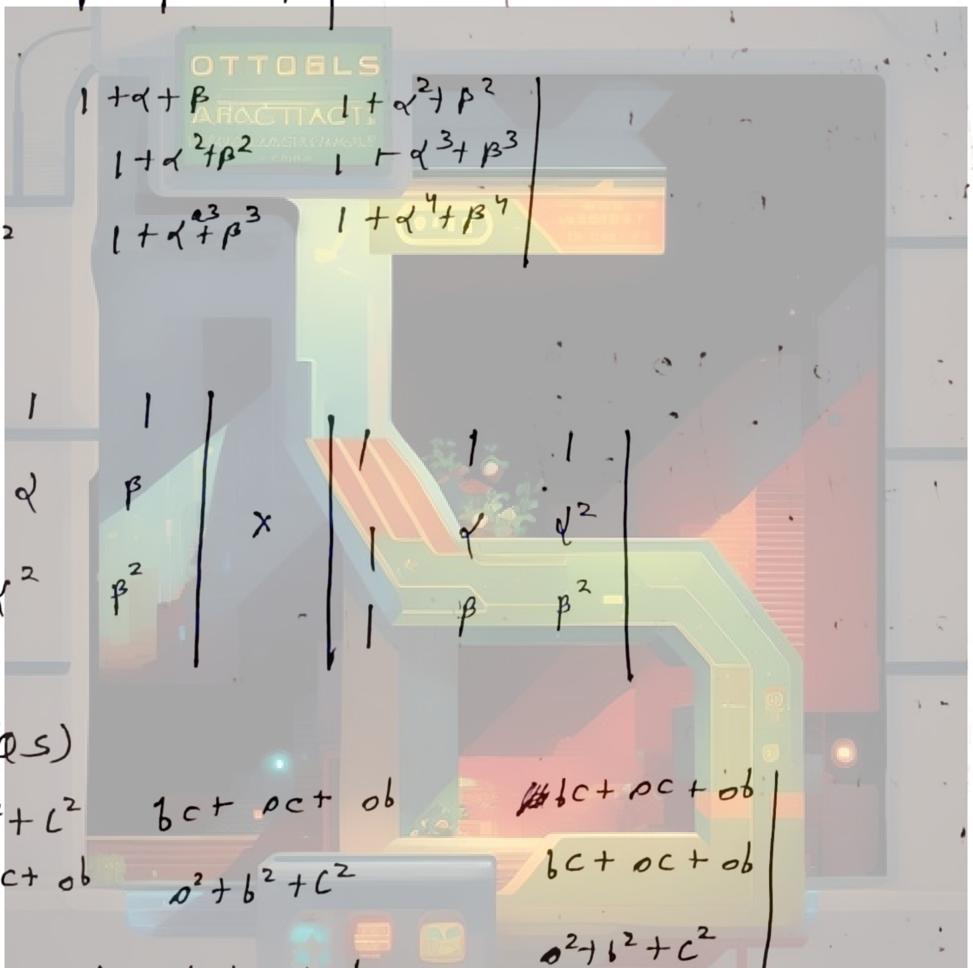
$= -144 + 160 - 160$

$\cancel{\boxed{= -144}}$ $\boxed{= -144}$

Q find both the determinants which gives the determinant by multiplication.

① $\begin{vmatrix} 1+1 & \alpha+\beta \\ \alpha+\beta & \alpha^2+\beta^2 \end{vmatrix}$

$\begin{vmatrix} 1 & 1 \\ \alpha & \beta \end{vmatrix} \times \begin{vmatrix} 1 & \alpha \\ 1 & \beta \end{vmatrix}$.



② $\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$

DYS-5 (Q5)

$\begin{vmatrix} a^2+b^2+c^2 & bc+ac+ab & bc+ac+ab \\ bc+ac+ab & a^2+b^2+c^2 & bc+ac+ab \\ bc+ac+ab & bc+ac+ab & a^2+b^2+c^2 \end{vmatrix}$

$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \times \begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$

* System of linear equation

2 Variable

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

Parallel lines

$$\frac{a_1}{b_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

inconsistent

0 sol^n

Coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Consistent

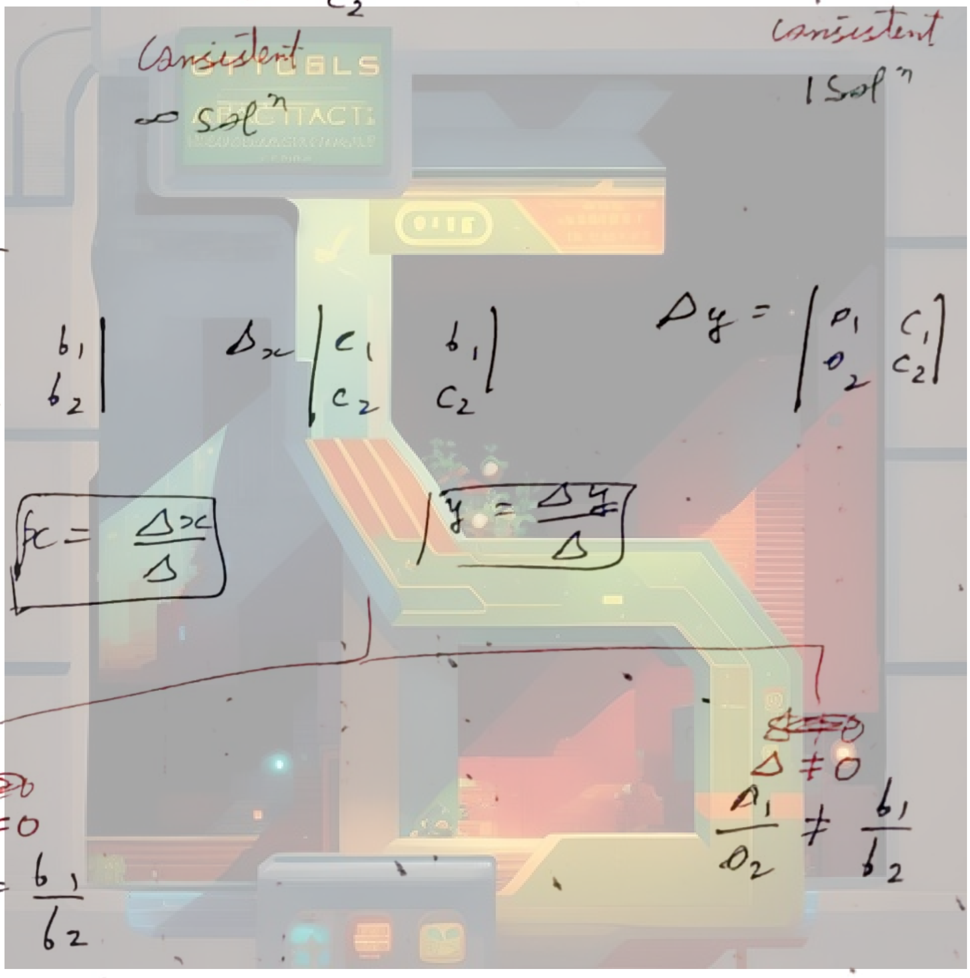
∞ sol^n

Intersect lines

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

consistent

1 Sol^n



Proof:-

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & c_2 \end{vmatrix} \qquad \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$\boxed{x = \frac{\Delta x}{\Delta}} \qquad \boxed{y = \frac{\Delta y}{\Delta}}$$

$\Delta = 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Delta \neq 0$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

eg. $2x - 3y = 4$
$x + y = 3$

$$\Delta = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 5$$

$$\Delta_x = \begin{vmatrix} 4 & -3 \\ 3 & 1 \end{vmatrix} = 13$$

$$\Delta_y = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 2$$

$$x = \frac{\Delta x}{\Delta} = \frac{13}{5}$$

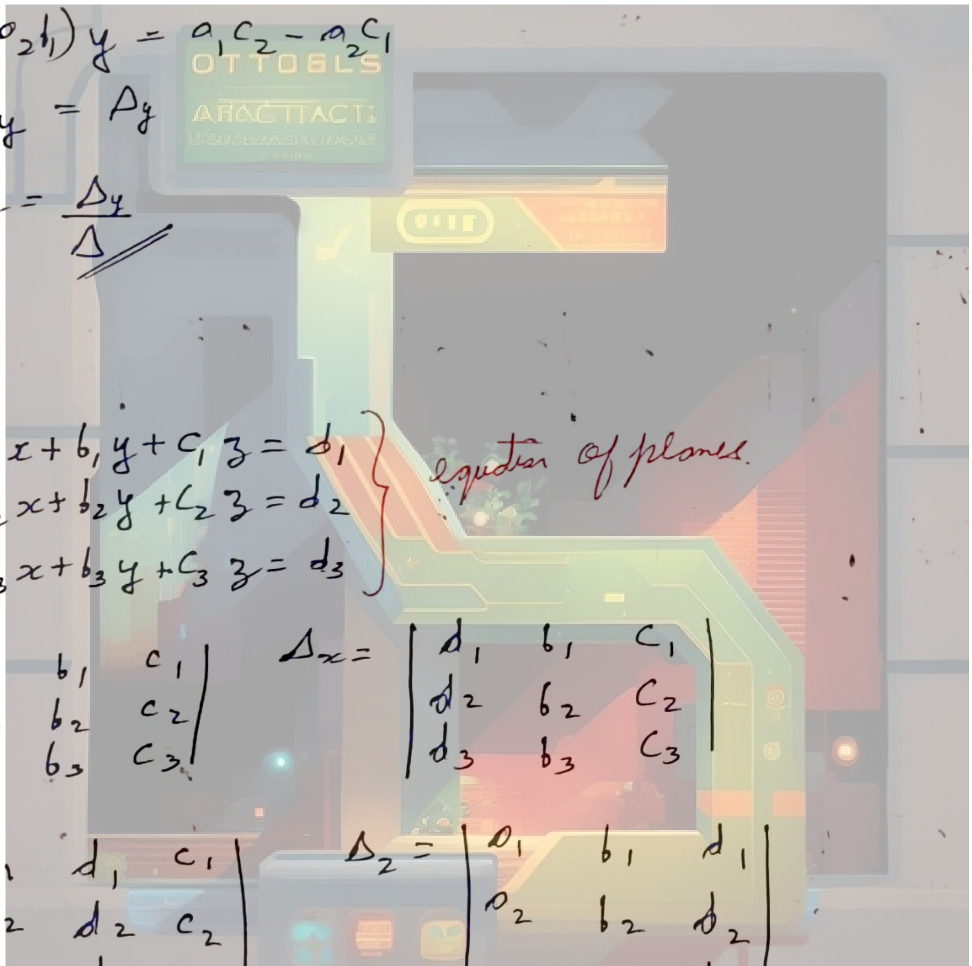
$$y = \frac{\Delta y}{y} = \frac{2}{5}$$

Proof:- (Δy)

$$(a_1x+b_1y=c_1)\times a_2$$
$$(a_2x+b_2y=c_2)\times a_1$$

$$a_1a_2x+a_2b_1y=a_2c_1 \quad -① $$
$$a_1a_2x+a_1b_2y=a_1c_2 \quad -② $$

② − ①



$$(a_1b_2-a_2b_1)y=a_1c_2-a_2c_1$$

$$\Delta y=\Delta_y$$

$$\therefore\ y=\frac{\Delta_y}{\Delta}$$

3 Variables

$$\left.\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3\end{aligned}\right\}\ \text{equation of planes}$$

$$\Delta=\begin{vmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}\qquad \Delta_x=\begin{vmatrix} d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix}$$

$$\Delta_y=\begin{vmatrix} a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix}\qquad \Delta_z=\begin{vmatrix} a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3\end{vmatrix}$$

$$x=\frac{\Delta_x}{\Delta}\qquad y=\frac{\Delta_y}{\Delta}\qquad z=\frac{\Delta_z}{\Delta}$$

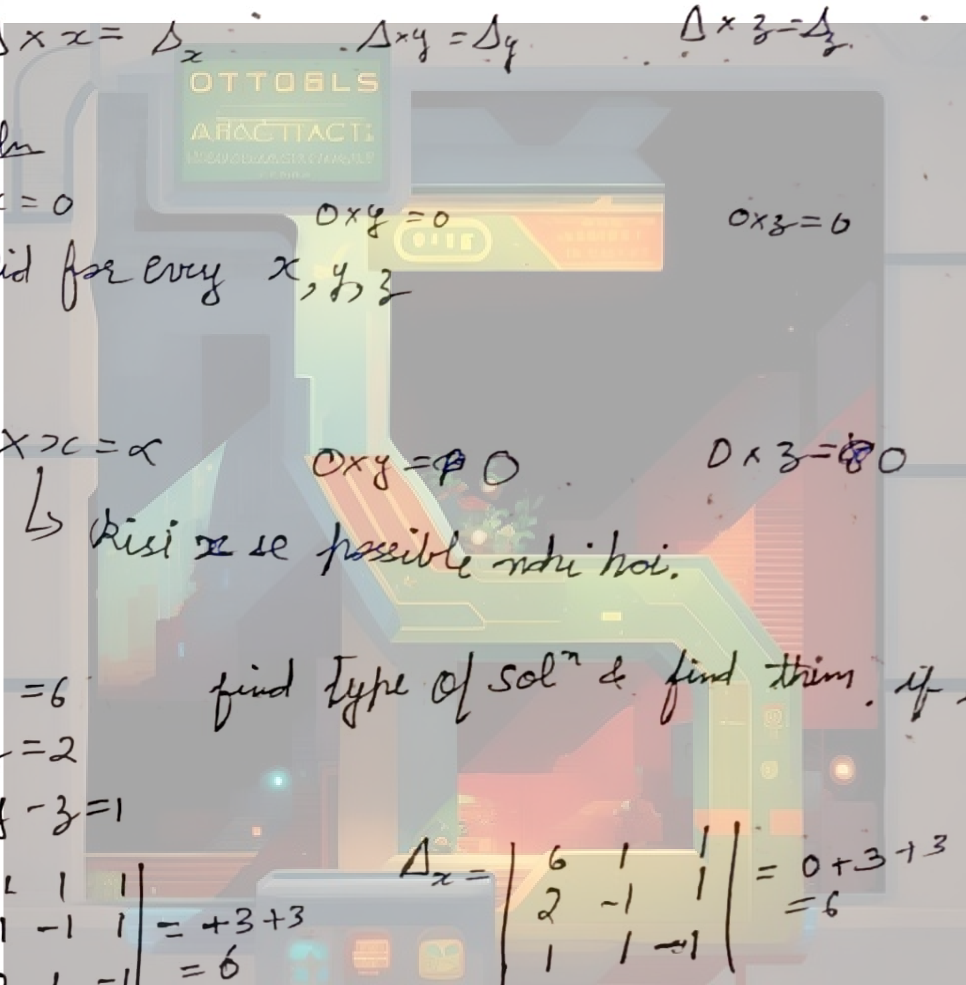
$\Delta \neq 0$ — 1 sol^n, Consistent

~~$\Delta \neq 0$~~ $\Delta = 0$

- $\Delta_x = \Delta_y = \Delta_z = 0$ — ∞ sol^n, Consistent
- At least one of $\Delta_x\ \Delta_y\ \Delta_z \neq 0$ — no sol^n, Inconsistent

Proof:-

$\Delta \times x = \Delta_x \qquad \Delta \times y = \Delta_y \qquad \Delta \times z = \Delta_z$



~~no soln~~ ∞ soln

$0 \times x = 0 \qquad 0 \times y = 0 \qquad 0 \times z = 0$

valid for every x, y, z

no soln

$0 \times x = \alpha \qquad 0 \times y = \cancel{\beta}\ 0 \qquad 0 \times z = \cancel{8}\ 0$

$\hookrightarrow$ kisi x se possible nahi hoi.

Q $x+y+z=6$ find type of sol^n & find them. if exists.

$x-y+z=2$

$2x+y-z=1$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = +3+3 = 6 \qquad \Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 0+3+3 = 6$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} \qquad \Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} =$$

$\boxed{\Delta = 0,\ 1\ \text{sol}^n}$

$\boxed{x = 1}$

$3 + y = 5$

$3 - y = 1$

$\boxed{z = 3} \qquad \boxed{y = 2}$

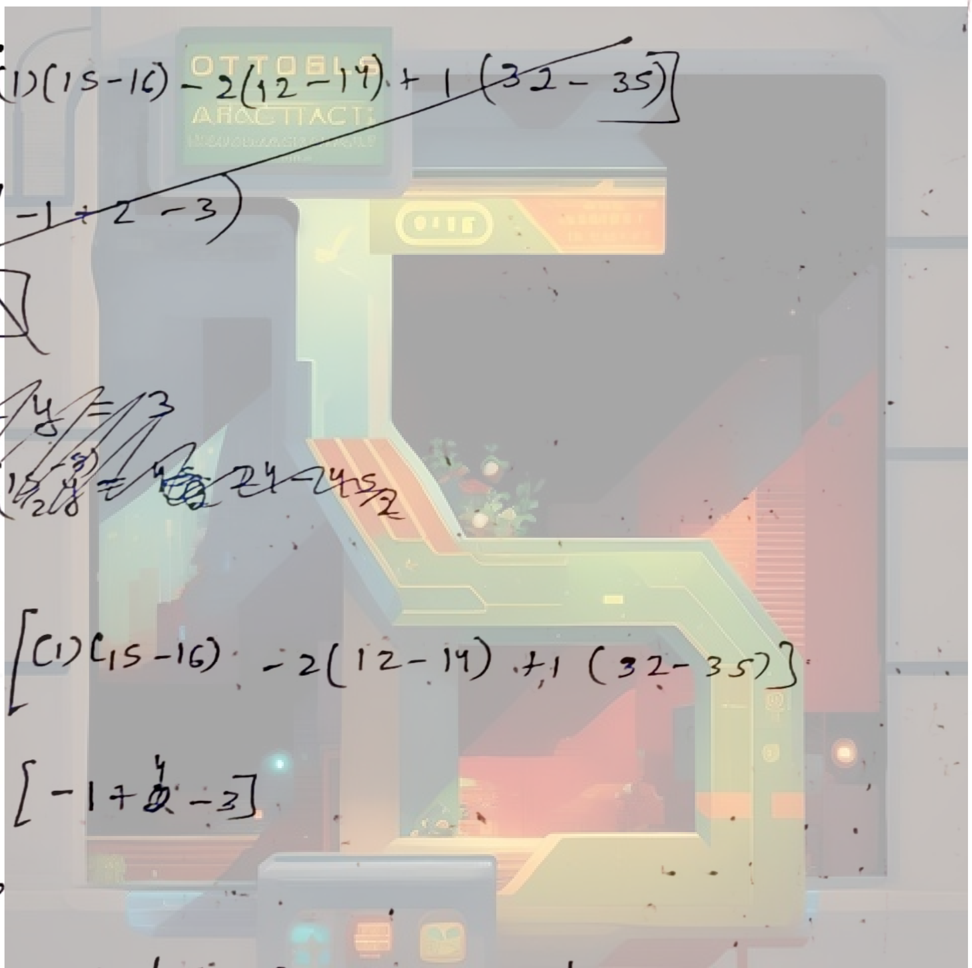
Q $x+2y+3z=6$

$4x+5y+6z=15$

$7x+8y+9z=24$

find type of sol^n & find them if possible.

$$\Delta = \begin{vmatrix}1&2&3\\4&5&6\\7&8&9\end{vmatrix} = 3\begin{vmatrix}1&2&1\\4&5&2\\7&8&3\end{vmatrix}$$



$$\cancel{\Delta = 3\times\left[(1)(15-16)-2(12-14)+1(32-35)\right]}$$

$$\cancel{= 3(-1+2-3)}$$

$$\cancel{\boxed{\Delta=-6}}$$

$$\Delta = 3\left[(1)(15-16)-2(12-14)+1(32-35)\right]$$

$$\Delta = 3\left[-1+4-3\right]$$

$$\Delta = 0$$

$$\Delta_x = \begin{vmatrix}6&2&3\\15&5&6\\24&8&9\end{vmatrix} = 3\begin{vmatrix}2&2&3\\5&5&6\\8&8&9\end{vmatrix} = 0$$

$$\Delta_x = 0$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 3 \\ 4 & 15 & 6 \\ 7 & 24 & 9 \end{vmatrix} = 3\times 3 \begin{vmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 3 \end{vmatrix}$$

$$= 9\left[(1)(15-16) - 2(12-14) + 1(32-35)\right]$$

$$\boxed{=0}$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 6 \\ 4 & 5 & 15 \\ 7 & 8 & 24 \end{vmatrix} = 3 \begin{vmatrix} 1 & 2 & 2 \\ 4 & 5 & 5 \\ 7 & 8 & 8 \end{vmatrix}$$

$$= 0$$

$\Delta_z = 0$

$\Delta_x = \Delta_y = \Delta_z = 0$

$\boxed{\infty \text{ sol}^n}$

Put $x = \lambda$
put in ①
$\lambda + 2y + 3z = 6$
$2y + 3z = 6 - \lambda$
Put in ②
$5y + 6z = 15 - 4\lambda$

$$\boxed{\begin{aligned} y &= 3 - 2\lambda \\ z &= \lambda \\ x &= \lambda \end{aligned}} \quad \lambda \in I$$

Q $x + y + z = 3$
$x + 2y + 3z = 4$
$2x + 3y + 4z = 8$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = -1 + 2 - 1 = 0$$

$$\Delta_x = \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 3 \\ 8 & 3 & 4 \end{vmatrix} = 3(-1) - 1(-4) + 1(12-16)$$
$$= -3 + 4 - 4$$
$$= -3$$

$\boxed{\Delta = 0,\ \Delta_x \neq 0,\ \text{no sol}^n}$

Q $2x + Py + 6z = 8$
$x + 2y + Qz = 5$
$x + y + 3z = 4$

find P & Q if system of eq^n has
① no sol^n
② Unique sol^n
③ ∞ sol^n

②①③ $\Delta = \begin{vmatrix} 2 & P & 6 \\ 1 & 2 & Q \\ 1 & 1 & 3 \end{vmatrix} \neq 0$

$2(6-Q) - P(3-Q) + 6(-1) = 0$

$12 - 2Q - 3P + PQ - 6 = 0$

$2Q + 3P - PQ = 6$ — ①

$\Delta = (P-2)(Q-3)$

$\Delta_y = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & Q \\ 1 & 4 & 3 \end{vmatrix} = 0$

$30 - 8Q - 24 + 8Q - 6 = 0$ — ③

$0 = 0$

$\Delta_y = 0$

at $P = 2$, in ①

$2Q + 6 - 2Q = 6$

$\Delta = 0$ ✓

$\Delta_x = \begin{vmatrix} 8 & P & 6 \\ 5 & 2 & Q \\ 4 & 1 & 3 \end{vmatrix} = 0$

$8(6-Q) - P(15-4Q) + 6(5-8) = 0$

$48 - 8Q - 15P + 4PQ + 30 - 48 = 0$

$8Q + 15P - 4PQ = 30$ — ②

$\Delta_x = (4Q-15)(P-2)$

$\Delta_z = \begin{vmatrix} 2 & P & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix}$

$\Delta_z = 6 + P - 8$

$\Delta_z = P - 2 = 0$ — ④

$P = 2$

$\Delta_z = (P-2)$

in ②

$8Q + 30 - 8Q = 30$ ✓

③ $\boxed{\text{at } P = 2,\ \infty \text{ sol}^n}$ iii)

② $\Delta \neq 0$

$P \neq 2,\ Q \neq 3$

$\boxed{P \in R - \{2\} \text{ and } Q \in R - \{3\}}$ ii)

① $\Delta = 0$

$P = 2$ or $Q = 3$

$\Delta_x, \Delta_y, \Delta_z \neq 0$

at $P = 2$, $\Delta_x = \Delta_y = \Delta_z = 0$

$\boxed{\text{at } Q = 3,\ P \in R - \{2\}\ \ 0 \text{ sol}^n}$ i)

Q $2x - y + 2z = 2$ find λ for no solⁿ

$x - 2y + z = -4$

$x + y + \lambda z = 4$

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$2(-2\lambda - 1) - 1(\overset{\lambda - 1}{\cancel{1+2}}) + 2(1+2) = 0$$

$$-4\lambda - 2 - \lambda + 1 + \cancel{2}6 = 0$$

$$-5\lambda + 5 = 0$$

$$\boxed{\lambda = 1}$$

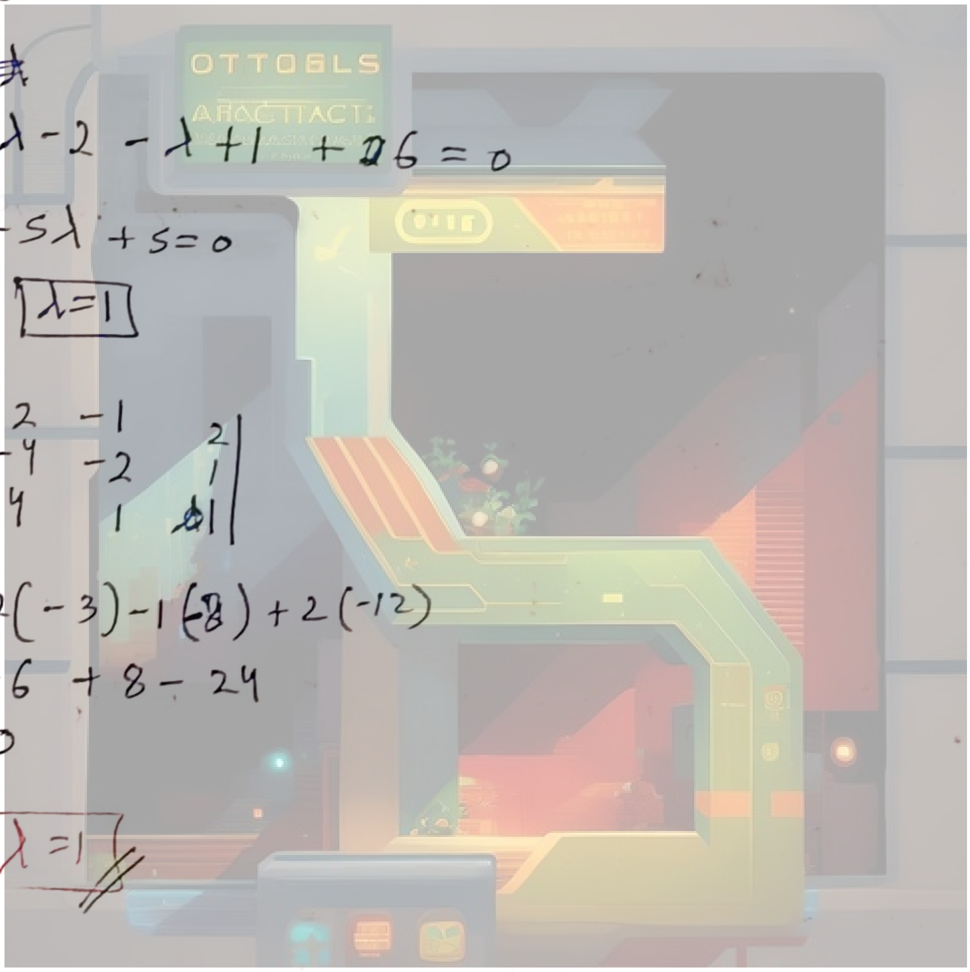
$$\Delta_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

$$= 2(-3) - 1(-8) + 2(-12)$$

$$= -6 + 8 - 24$$

$$\Delta_x \neq 0$$

$$\boxed{\lambda = 1}$$



Homogeneous equ in 3 variable.

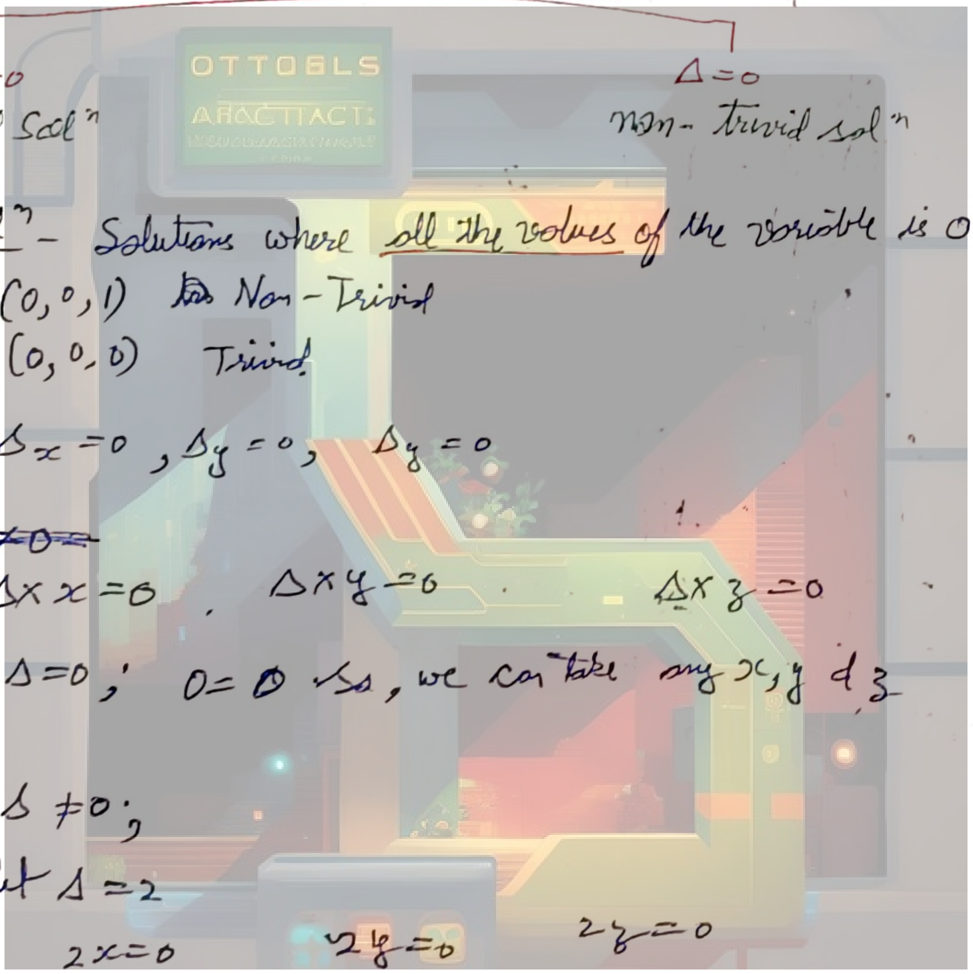
$$\left.\begin{aligned} a_1x + b_1y + c_1z &= 0 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \end{aligned}\right\} \text{3 Homogeneous equ.}$$

$x = 0, y = 0, z = 0$ must be a solⁿ

Δ

$\Delta \neq 0$ — Trivial Solⁿ

$\Delta = 0$ — Non-trivial solⁿ



Trivial Solⁿ – Solutions where all the values of the variable is 0.

eg. $(0, 0, 1)$ ~~is~~ Non-Trivial

$(0, 0, 0)$ Trivial

Proof:- $\Delta_x = 0, \Delta_y = 0, \Delta_z = 0$

~~$\Delta \neq 0$~~

$\Delta \times x = 0 \quad \Delta \times y = 0 \quad \Delta \times z = 0$

If $\Delta = 0$; $0 = 0$ so, we can take any x, y & z

If $\Delta \neq 0$;

let $\Delta = 2$

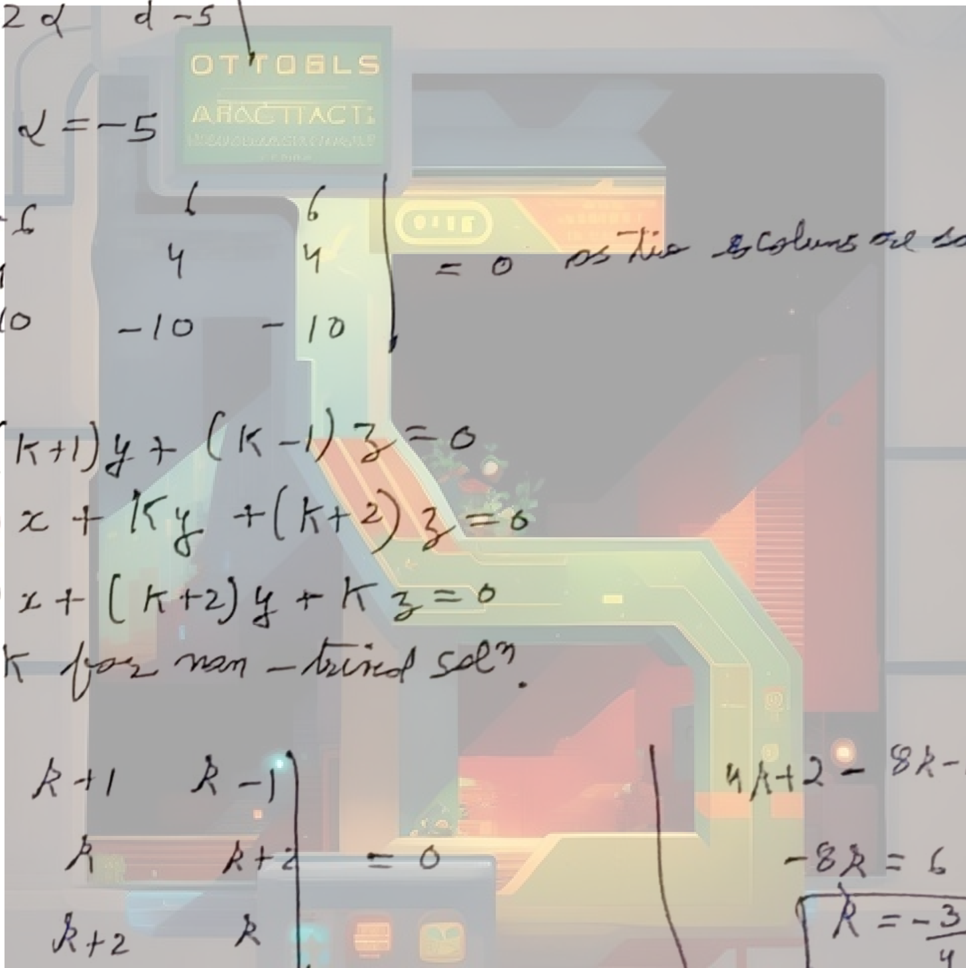
$2x = 0 \quad 2y = 0 \quad 2z = 0$

Trivial Solⁿ

Q $(1-\alpha)x + 6y + 6z = 0$
$4x - (\alpha+1)y + 4z = 0$
$2\alpha x + 2\alpha y + (\alpha-5)z = 0$
has non trivial sol^n find α

$\Delta = 0$

$$\begin{vmatrix} 1-\alpha & 6 & 6 \\ 4 & -(\alpha+1) & 4 \\ 2\alpha & 2\alpha & \alpha-5 \end{vmatrix} = 0$$



at $\alpha = -5$

$$\begin{vmatrix} 6 & 6 & 6 \\ 4 & 4 & 4 \\ -10 & -10 & -10 \end{vmatrix} = 0$$ as the 3 colums are same.

Q $kx + (k+1)y + (k-1)z = 0$
$(k+1)x + ky + (k+2)z = 0$
$(k-1)x + (k+2)y + kz = 0$
find k for non-trivial sol^n.

$$\begin{vmatrix} k & k+1 & k-1 \\ k+1 & k & k+2 \\ k-1 & k+2 & k \end{vmatrix} = 0$$

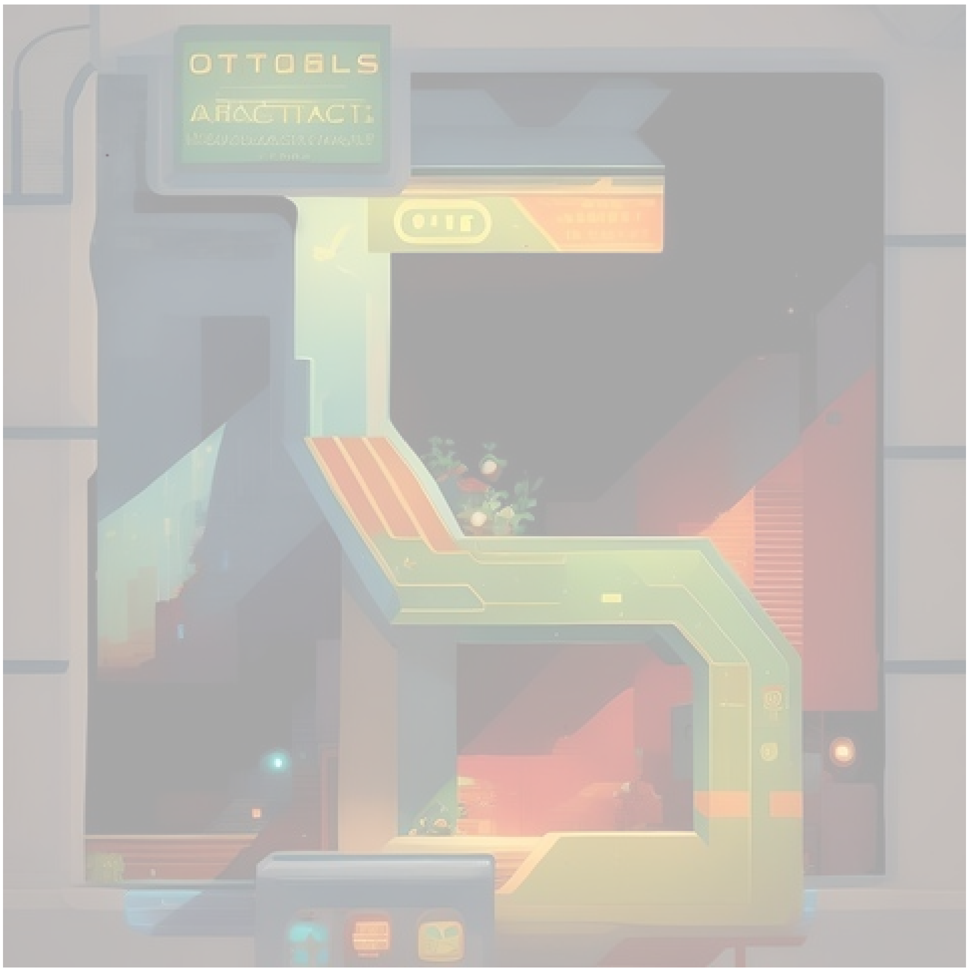
$R_1 \to R_1 - R_2$
$R_2 \to R_2 - R_3$

$$\begin{vmatrix} -1 & 2 & k-1 \\ 1 & -2 & k+2 \\ -3 & 2 & k \end{vmatrix} = 0$$

$4k + 2 - 8k - 12 - 4k + 4 = 0$

$-8k = 6$

$\boxed{k = -\frac{3}{4}}$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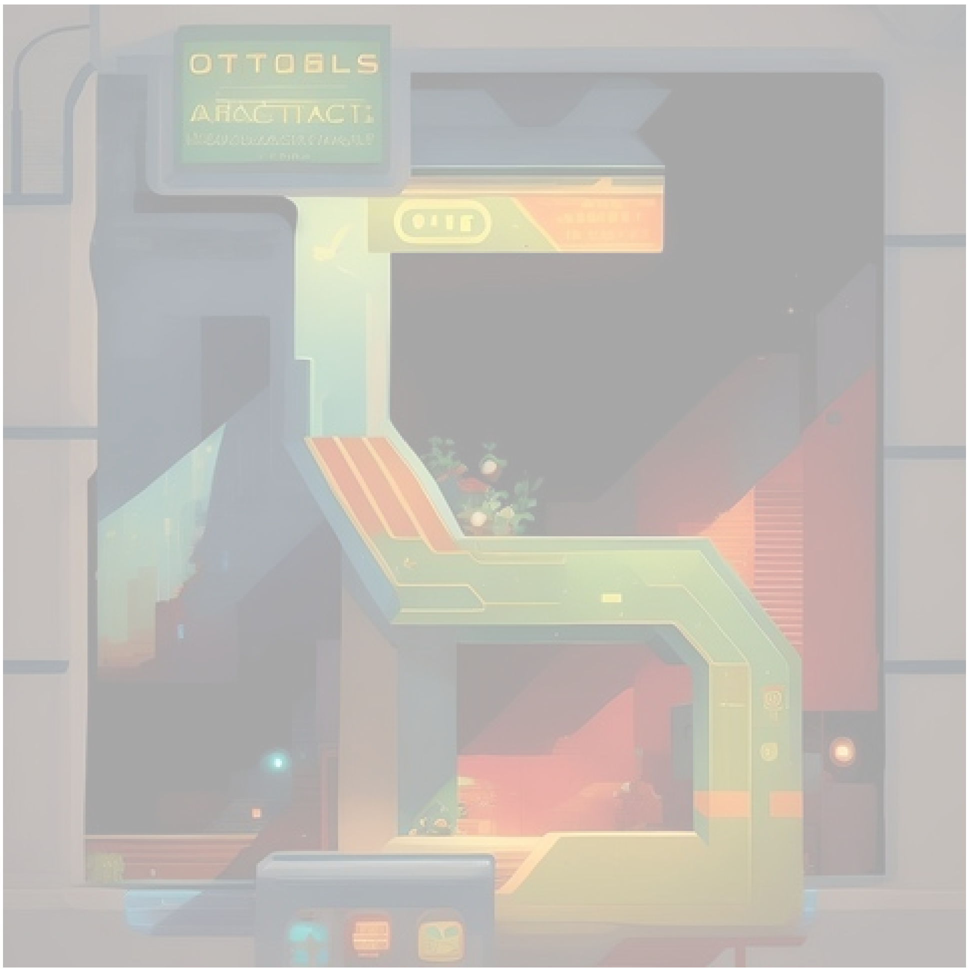
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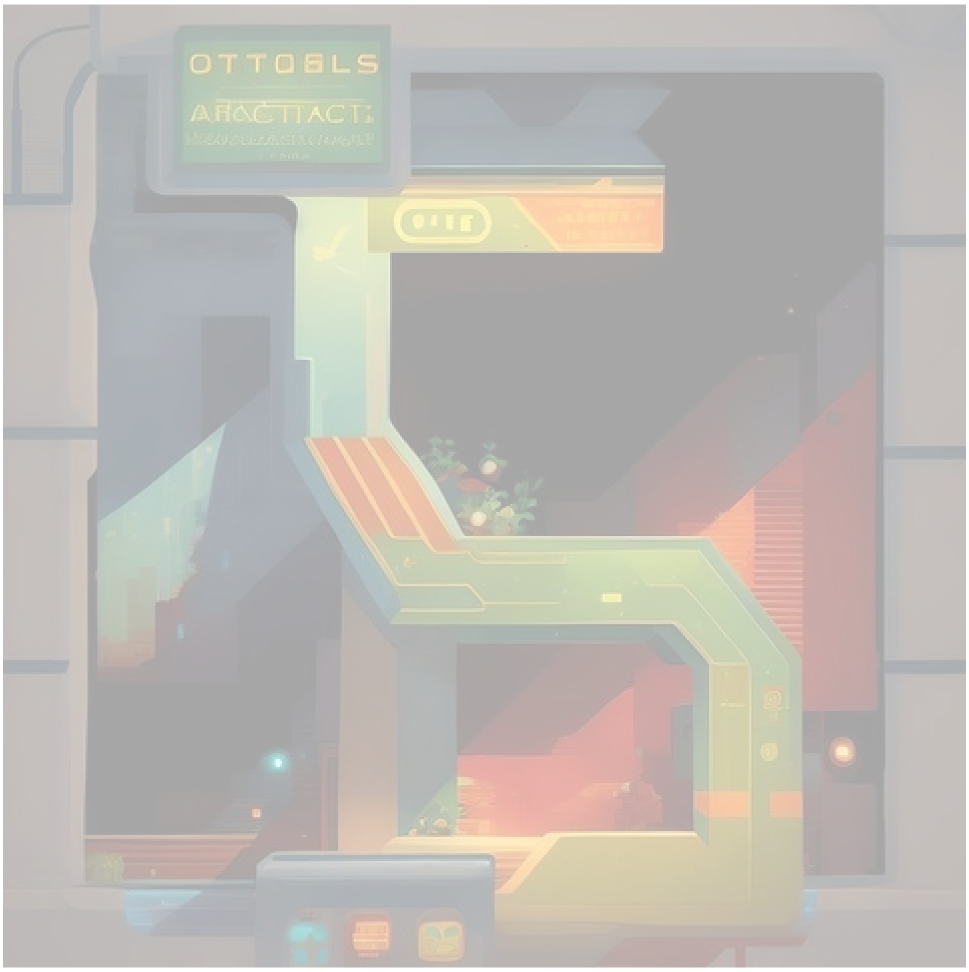

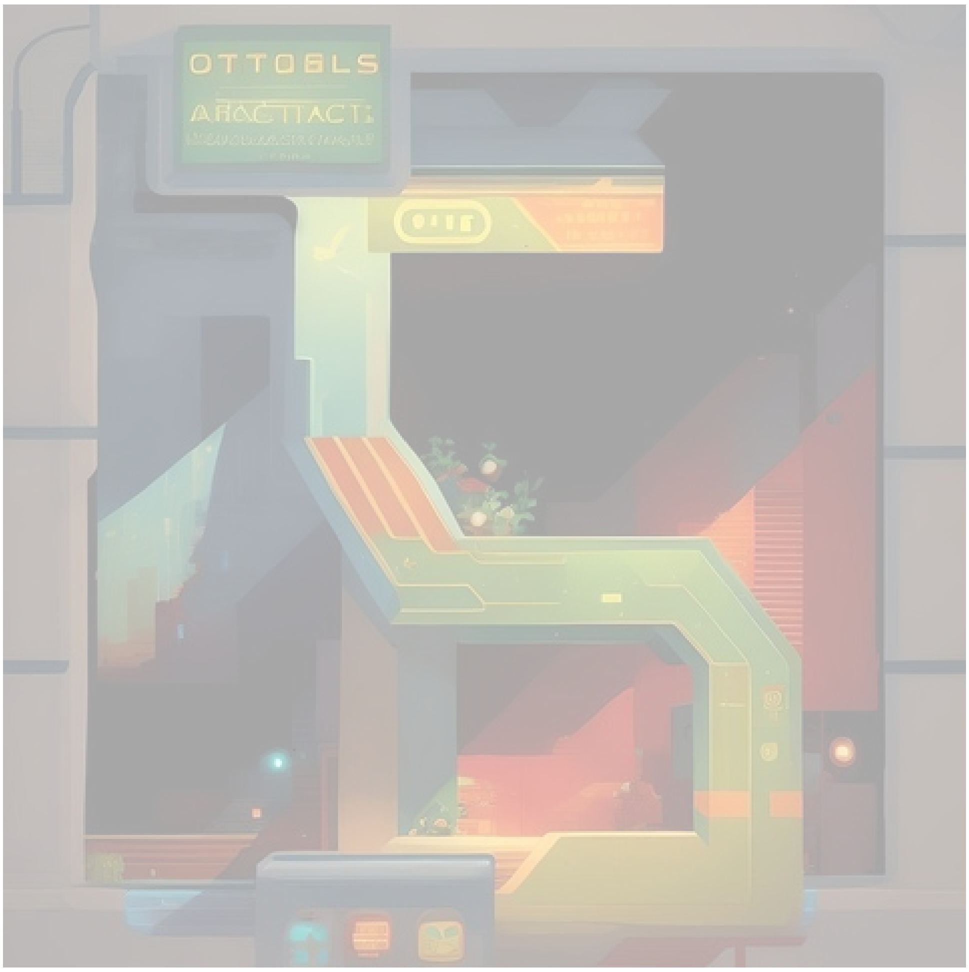
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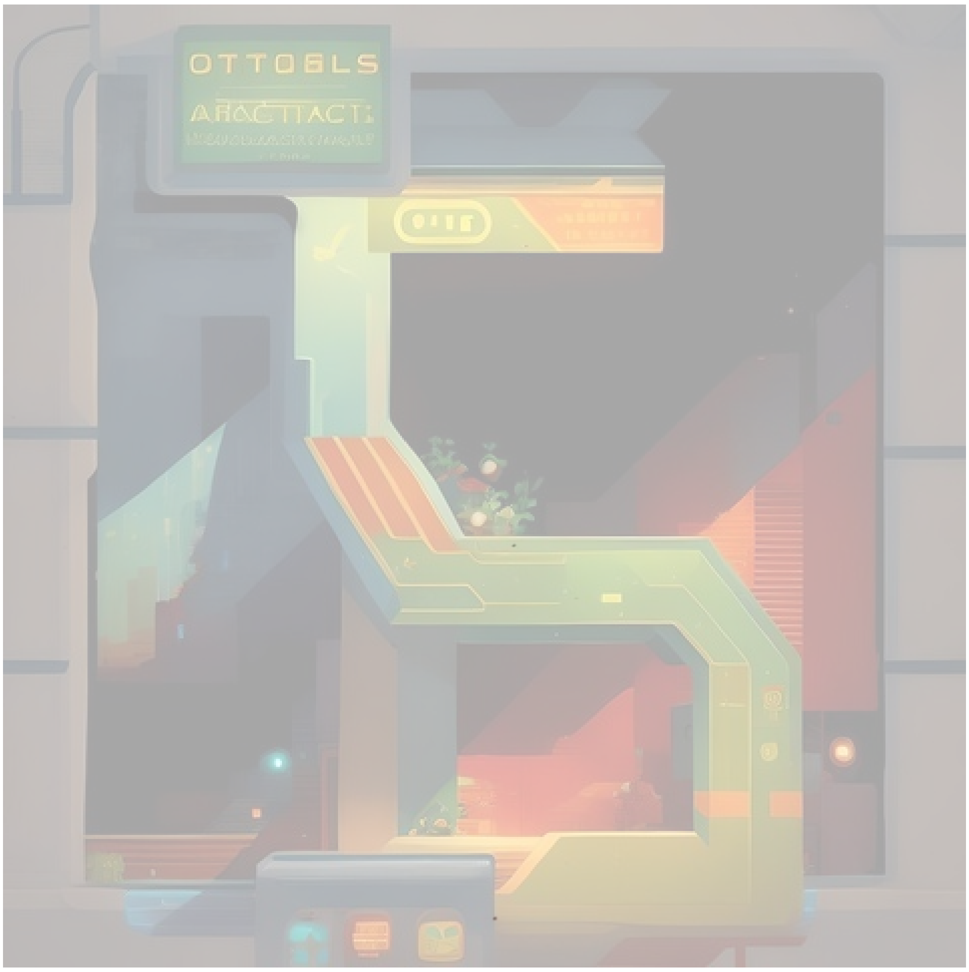
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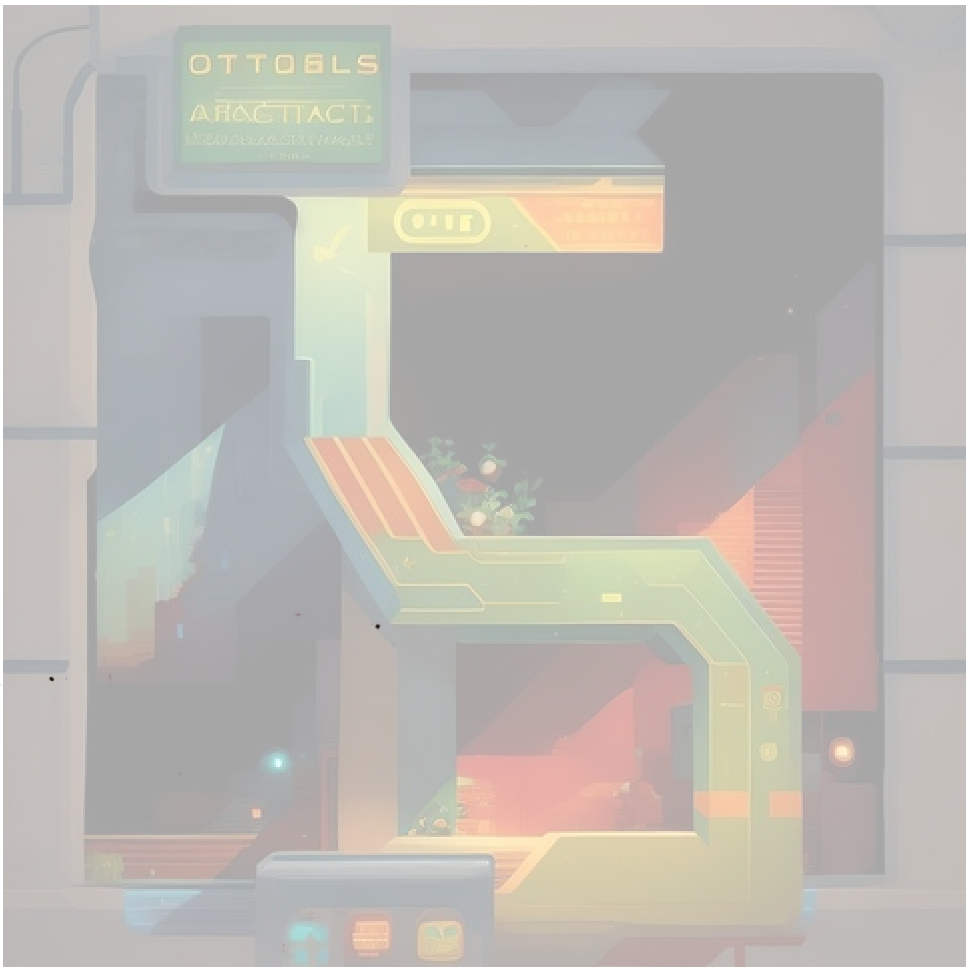
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