Quadratic Equation

$x^2 - 2x + 1 = 0$

$x = 1, 1$

Solution - 1 solution [repeated roots are only counted once as solution]

Roots - 2 roots

Quadratic Equation - 2 degree polynomial (2 roots)

- $ax^2 + bx + c = 0$ ($a \to$ leading coefficient)
- $a \neq 0$, If $a = 0$ then Quadratic will be ~~zero~~ linear.
- no. of roots = degree of polynomial

Methods to find roots

1. Shri - Dharacharya $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. factorisation (middle Term Splitting)

3. Perfect Square (leading coefficient = 1)

$\rightarrow \left(\dfrac{\text{coefficient of } x}{2}\right)^2 <^{+}_{-}$

eg 1) $\quad x^2 - 5x + 6 = 0$

$\left(-\dfrac{5}{2}\right)^2 = \dfrac{25}{4} <^{+}_{-}$

$x^2 - 5x + \dfrac{25}{4} - \dfrac{25}{4} + 6 = 0$

$\left(x - \dfrac{5}{2}\right)^2 - \dfrac{1}{4} = 0$

$x - \dfrac{5}{2} = \dfrac{1}{4}$

$x - \dfrac{5}{2} = \pm\dfrac{1}{2}$

$x = \dfrac{1}{2} + \dfrac{5}{2}$

$\boxed{x = 3}$

$x = \dfrac{1}{2} - \dfrac{5}{2}$

$\boxed{x = -2}$

Relation in roots and coefficients/constants

$$ax^2+bx+c=0$$

roots:- α, β

$$\alpha+\beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

→ Difference of Roots - $\boxed{|\alpha-\beta| = \frac{\sqrt{D}}{|a|}}$

Proof:- $(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$

$$\left(\frac{-b}{a}\right)^2 - (\alpha-\beta)^2 = 4\frac{c}{a}$$

$$(\alpha-\beta)^2 = \frac{b^2}{a^2} - \frac{4c}{a}$$

$$(\alpha-\beta)^2 = \frac{b^2-4ac}{a^2}$$

$$|\alpha-\beta| = \frac{\sqrt{D}}{|a|}$$

→ $ax^2+bx+c=0$, α & β are roots so they satisfy Q.E.

$\boxed{a\alpha^2+b\alpha+c=0}$ & $\boxed{a\beta^2+b\beta+c=0}$

Find some values using SoR & PoR (sum of roots & Product of roots)

1. $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

   $\alpha^2+\beta^2 = (\alpha-\beta)^2 + 2\alpha\beta$

2. $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

3. $\alpha^3-\beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

Q $x^2-4x+2=0$ (roots are α & β then find)

① $\alpha+\beta$

$\alpha+\beta = \frac{-b}{a}$

$\boxed{=4}$ ✓

② $\alpha^2+\beta^2$

$\alpha\beta = \boxed{2}$

$\alpha-\beta = \sqrt{\frac{16-8}{1}}$

$= 2\sqrt{2}$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 16 - 4$

$\boxed{=12}$ ✓

③ $\alpha^3+\beta^3$

$\alpha^3+\beta^3 = (4)^3 - 3(2)(4)$

$= 64 - 24$

$\cancel{28}$

$\boxed{=40}$ ✓

④ $\frac{\alpha\beta}{\alpha+\beta}$

$\frac{2}{4} = \boxed{\frac{1}{2}}$ ✓

⑤ $\alpha-\beta = \sqrt{\frac{16-8}{1}}$

$\boxed{=2\sqrt{2}}$ ✓

⑥ $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$\frac{\beta^2+\alpha^2}{(\alpha\beta)^2}$

$\frac{12}{4} = \boxed{3}$ ✓

⑦ $\alpha^2-\beta^2$

$(\alpha+\beta)(\alpha-\beta)$

$(4)(2\sqrt{2})$

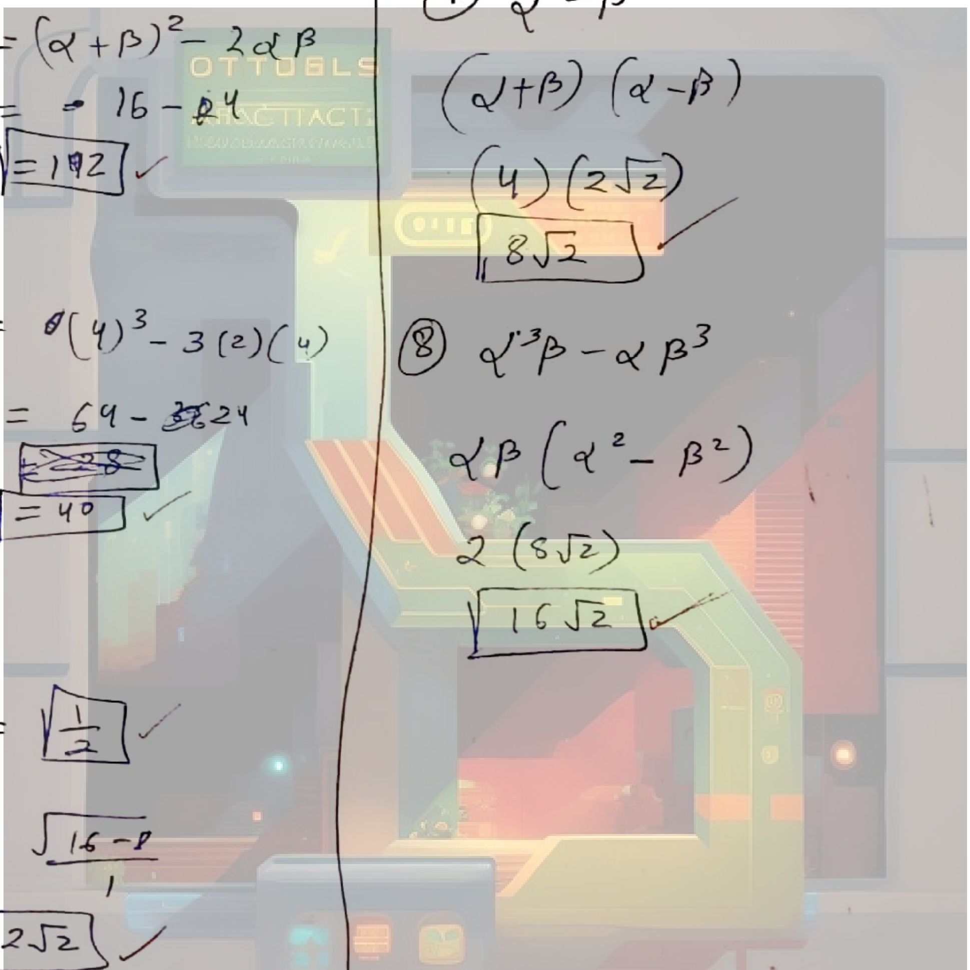
$\boxed{8\sqrt{2}}$ ✓

⑧ $\alpha^3\beta-\alpha\beta^3$

$\alpha\beta(\alpha^2-\beta^2)$

$2(8\sqrt{2})$

$\boxed{16\sqrt{2}}$ ✓



222

Q $3x^2+7x+3=0$

① $\frac{\beta}{\alpha}+\frac{\alpha}{\beta}$

$\frac{\beta^2+\alpha^2}{\alpha\beta}$

$\frac{\frac{49}{9}-2}{1}$

$\frac{49-18}{9}$

$\boxed{=\frac{31}{9}}$

② $\alpha^2\beta+\alpha\beta^2$

$\alpha\beta(\alpha+\beta)$

$1\left(-\frac{7}{3}\right)$

$\cancel{\left(-\frac{7}{3}\right)}\ \boxed{-\frac{7}{3}}$

③ $\alpha^4\beta^7+\alpha^7\beta^4$

$\alpha^4\beta^4(\alpha^3+\beta^3)$

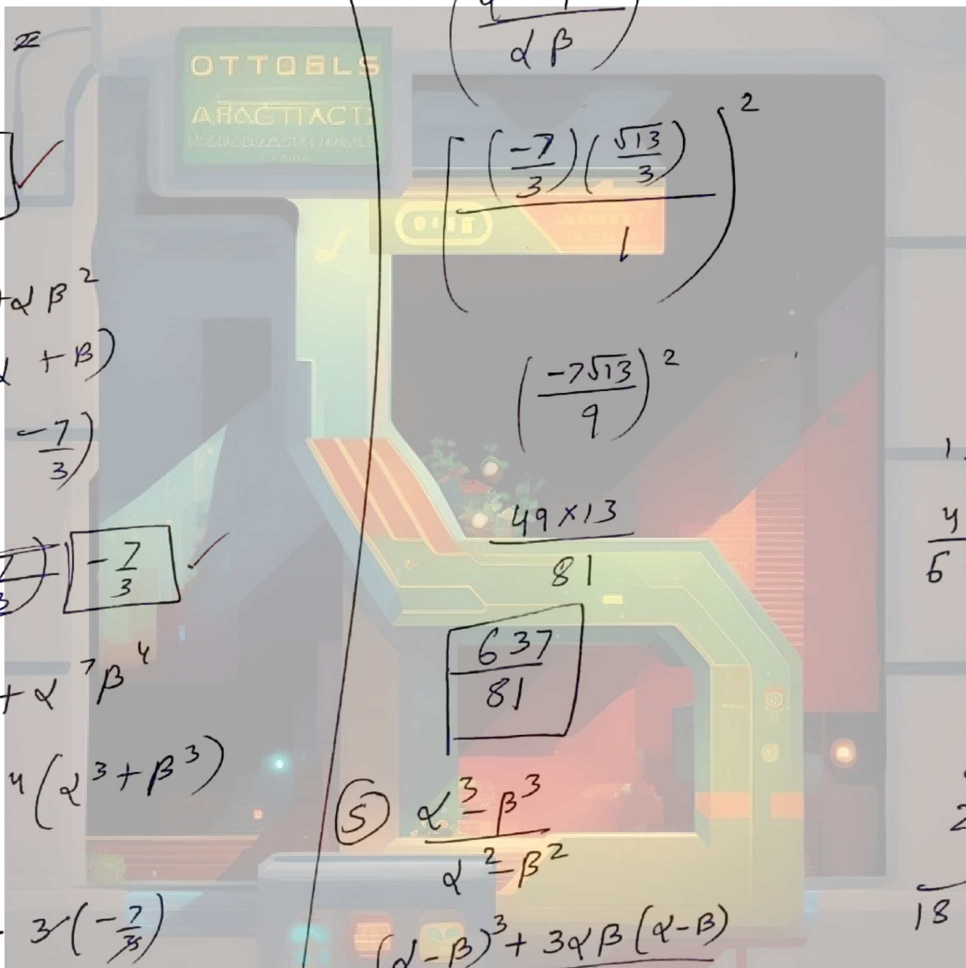
$\left(-\frac{7}{3}\right)^3-3\left(-\frac{7}{3}\right)$

$\frac{-343}{27}+7$

$\cancel{-343+21}$

$\frac{-343+189}{27}$

$\boxed{\frac{-154}{27}}$



$\alpha+\beta=-\frac{7}{3}$

$\alpha\beta=\frac{3}{3}=1$

$\alpha-\beta=\frac{\sqrt{13}}{3}$

④ $\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)^2$

$\left(\frac{\alpha^2-\beta^2}{\alpha\beta}\right)^2$

$\left[\frac{\left(\frac{-7}{3}\right)\left(\frac{\sqrt{13}}{3}\right)}{1}\right]^2$

$\left(\frac{-7\sqrt{13}}{9}\right)^2$

$\frac{49\times 13}{81}$

$\boxed{\frac{637}{81}}$

$27\times 7=189$

$49\times 13=637$

$49\times 7=343$

$27\times 7=189$

⑤ $\frac{\alpha^3-\beta^3}{\alpha^2-\beta^2}$

$\frac{(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)}{-7\sqrt{13}}$

$\frac{(\sqrt{13})^3}{27}+3\times\frac{\sqrt{13}}{3}$

$\frac{(\sqrt{13})^3+27\sqrt{13}}{27(-7\sqrt{13})}$

$\frac{(\sqrt{13})^2+27\sqrt{13}}{-189\sqrt{13}}$

$\frac{13+27}{-\cancel{189}\,21}$

$\boxed{\frac{-40}{\cancel{189}\,21}}$

Q If x^2-x+1 has roots α, β find

① $\alpha^2-\alpha$

$\alpha(\alpha-1)$

$\alpha+\beta = 1$

$\alpha\beta = 1$

~~$\alpha - \beta =$~~

$\alpha(-\beta)$

$-\alpha\beta$

$\boxed{-1}$

Or

α is a root

$\alpha^2-\alpha+1=0$

$\alpha^2-\alpha=-1$

② $\alpha^{15}+\beta^{15}$

~~$(1-\beta)^{15}+\beta^{15}$~~

$\alpha^2-\alpha+1=0$

$\alpha^2-\alpha=-1$

$\alpha^3 - \alpha^2 = -\alpha$

$\alpha^3 = \alpha^2-\alpha$

$\alpha^3 = -1$

$(\alpha^3)^5 = \alpha^{15}$

$\alpha^{15} = (-1)^5$

$\alpha^{15} = -1$

$\beta^{15} = -1$

$-1+(-1)$

$\boxed{-2}$

③ $\alpha^{2025}+\beta^{2025}$

$\alpha^3=-1$

$\alpha^{2025}=-1$

$\beta^{2025}=-1$

$-1+(-1)$

$\boxed{-2}$

Q find qud whose roots are $\frac{1}{5+2\sqrt{6}}$ & $\frac{1}{5-2\sqrt{6}}$

$\alpha+\beta = \frac{5-2\sqrt{6}+5+2\sqrt{6}}{25-24}$

$= 10$

$\alpha\beta = 1$

$\boxed{x^2-10x+1=0}$

H.W. 16-05-2024

DYS-1 {01, 2, 3, 5, 6, 7, 8, 9, 10, 13, 12, 11, 14, 4}

[1, 14]

~~Q find roots $a^2 + a - 444444222222 = 0$~~

$\cancel{a = 0}$ $a(a+1) = 4444444 \mid 2222222$

$\boxed{a = 4444444 2222222}$

$a(a+1) = 4$

$\frac{a(a+1)}{y} = 1$

$\frac{a(a+1)}{y} - 1 = 0$

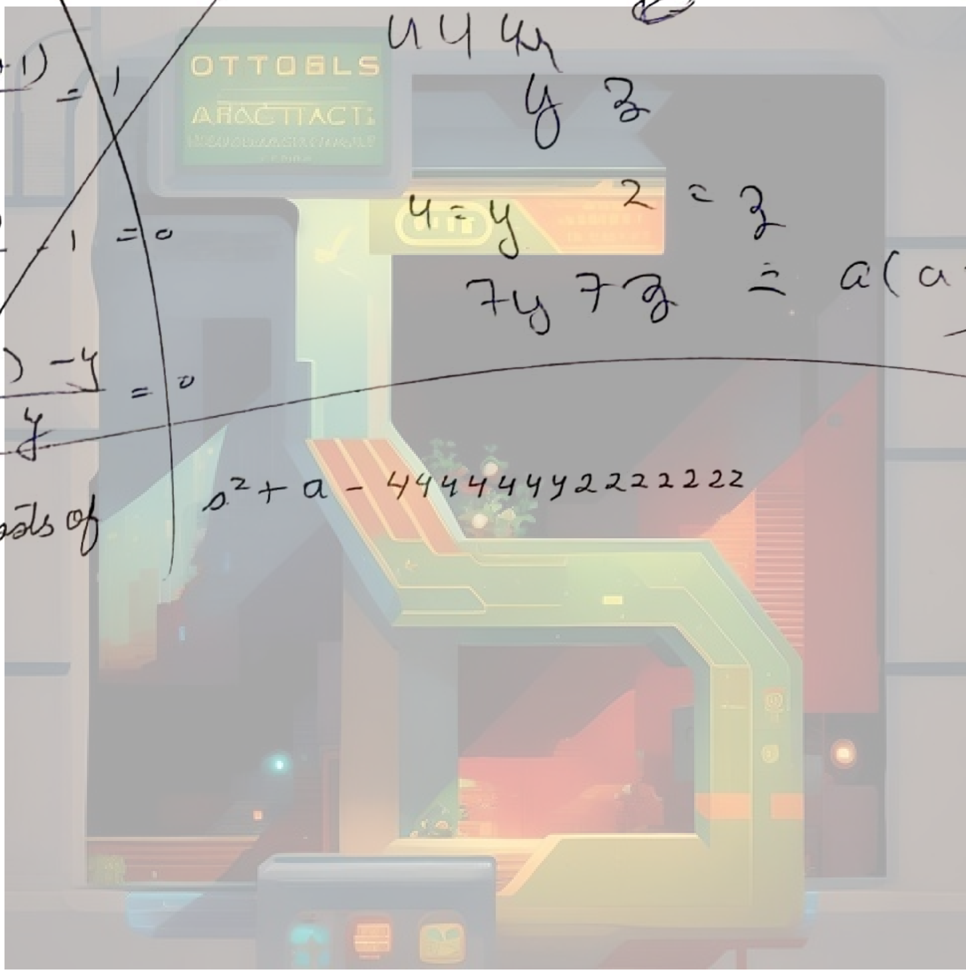
$\frac{a(a+1) - y}{y} = 0$

$\alpha\beta = -1$

$\alpha\beta = -444\ldots222$

Sol

Not defined



444

y z

$4 = y^2 = z$

$7y\ 7z = a(a+1)$

Q find roots of $a^2 + a - 4444444 2222222$

Nature of roots

$ax^2+bx+c \ (a,b,c \in R)$

$D = b^2 - 4ac$

$D > 0$
Distinct real roots

$D = 0$
Equal real roots

$D < 0$
Imaginary roots in pairs.

$D = b^2 - 4ac \ (a,b,c \in Q)$
↳ rational

Rational no.
Perfect Square

Perfect square
Not a perfect square
Roots are irrational in pair
$a+\sqrt{b}$ & $a-\sqrt{b}$

Q Find nature of roots

① $x^2+x+1=0$

$D = (1)^2 - (4)(1)(1)$
$= 1-4$
$= -3$

$\boxed{\text{Imaginary}}\ \boxed{\text{\& Paired}}$

② $2x^2-6x+3=0$

$D = 36-24$
$= 12$

$\boxed{\text{Real, Irrational}}$ in Pair

③ $3x^2-4\sqrt{3}x+4=0$

~~Irrat~~

$D = 48-48$
$= 0$

$\frac{4\sqrt{3}}{6}$

$\boxed{\text{real \&}}\ \boxed{\text{equal, irrational}}$

Q find 'm' if equation $x^2 - x + m^2 = 0$ have no real roots.

$D < 0$

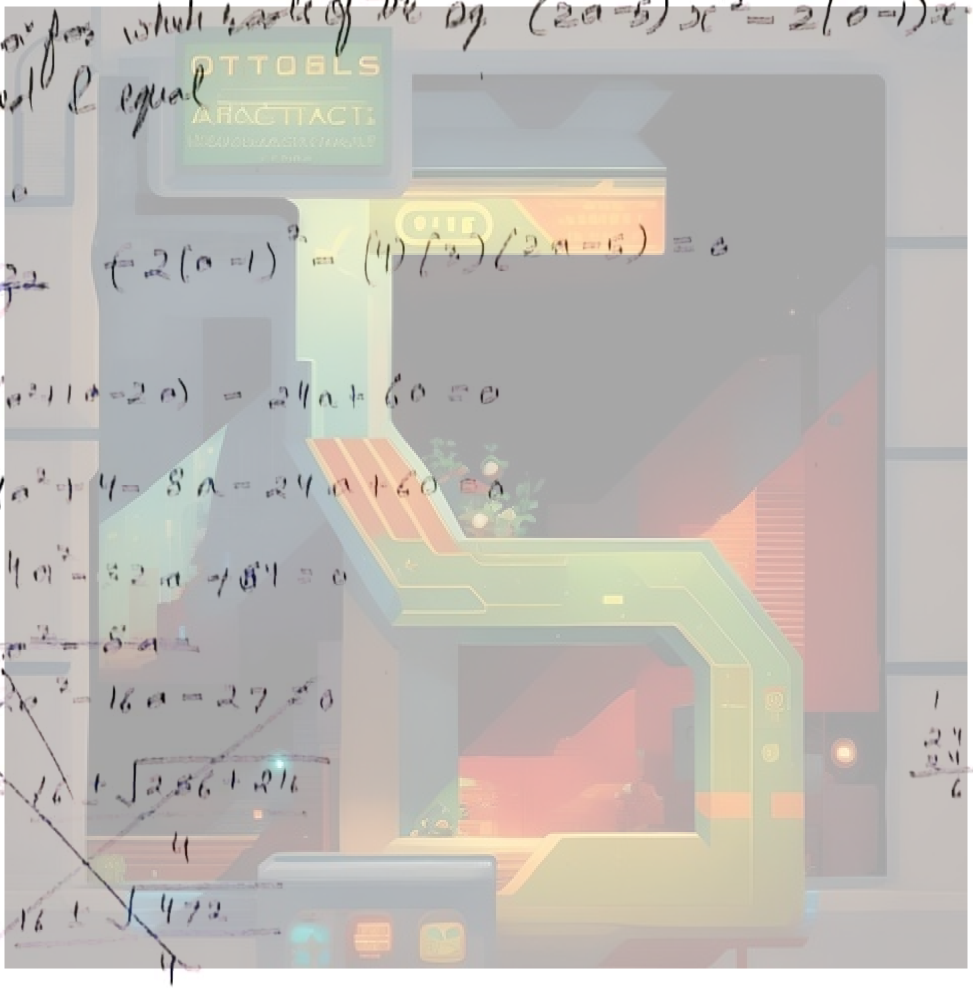
$1 - 4m^2 < 0$

$-\frac{1}{2} \quad \frac{1}{2}$

$\boxed{m \in (-\infty, -\tfrac{1}{2}) \cup (\tfrac{1}{2}, \infty)}$

$1 - 4m^2 = 0$
$4m^2 = 1$
$m^2 = \frac{1}{4}$

Q Value of 'a' for which roots of the eq. $(2a-5)x^2 - 2(a-1)x + 3 = 0$ are real & equal



$D = 0$

$(-2(a-1))^2 - (4)(3)(2a-5) = 0$

$4(a^2 + 1 - 2a) - 24a + 60 = 0$

$4a^2 + 4 - 8a - 24a + 60 = 0$

$4a^2 - 32a + 64 = 0$

~~$a^2 - 8a$~~

~~$2a^2 - 16a - 27 = 0$~~

~~$a = \frac{16 \pm \sqrt{256 + 216}}{4}$~~

~~$a = \frac{16 \pm \sqrt{472}}{4}$~~

$a =$

$a^2 - 8a + 16 = 0$

$a = \frac{8 \pm \sqrt{64 - 64}}{2}$

$\boxed{a = 4}$

Q $x^2 + (2a\sqrt{a^2-3})x + 4 = 0$

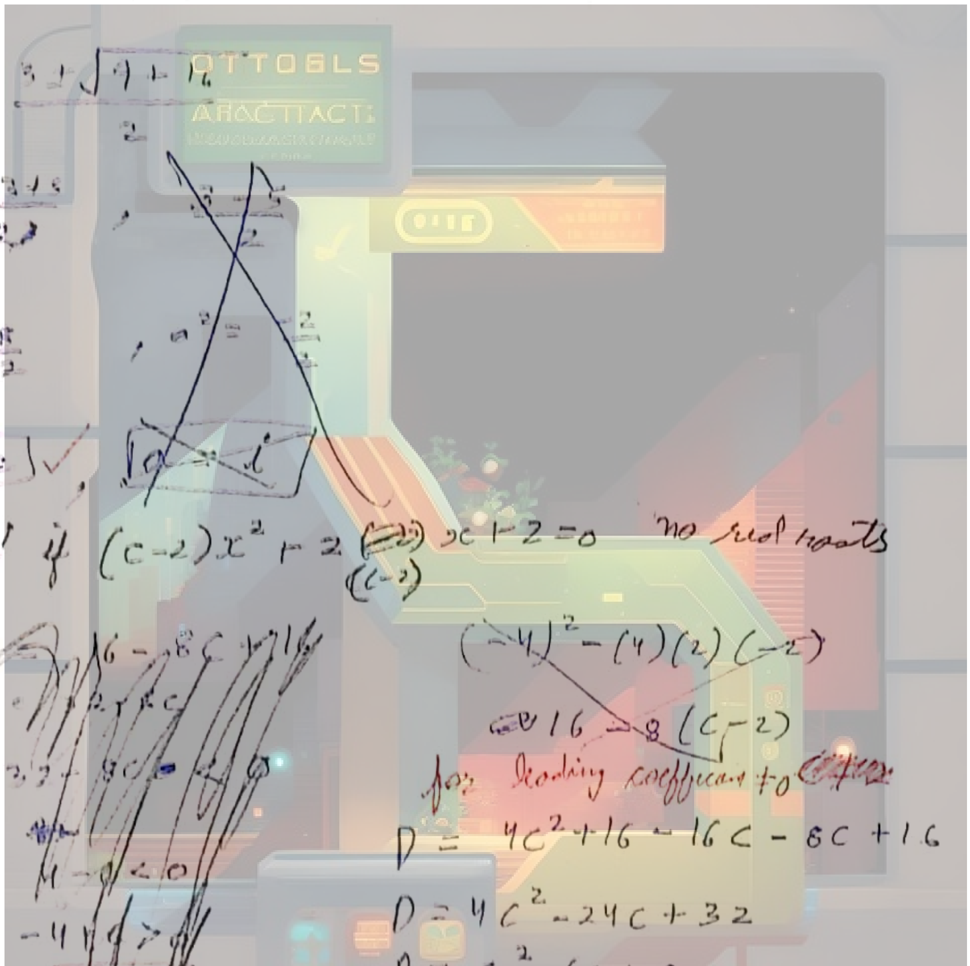
$(2a\sqrt{a^2-3})^2 - (4)(4)(1) = 0$

$4a^2(a^2-3) - 16 = 0$

$4a^4 - 12a^2 - 16 = 0$

$2a^4 - 6a^2 - 8 = 0$

$a^4 - 3a^2 - 4 = 0$



$a^2 = \frac{3 \pm \sqrt{9+16}}{2}$

$a^2 = \frac{3+5}{2}$, $\cancel{\frac{3-5}{2}}$

$a^2 = \frac{8}{2}$, $\cancel{a^2 = \frac{-2}{2}}$

$\boxed{a = \pm 2}$ ✓, $\cancel{\boxed{a = i}}$

Q find 'c' if $(c-2)x^2 + 2(c-2)x + 2 = 0$ no real roots

$\cancel{D = 16 - 8c + 16}$

$\cancel{32 - 8c}$

$\cancel{4 - c < 0}$

$\cancel{-4 + c > 0}$

$\cancel{\boxed{c > 4}}$

$\cancel{(-4)^2 - (4)(2)(-2)}$

$16 - 8(c-2)$

for leading coefficient to ...

$D = 4c^2 + 16 - 16c - 8c + 16$

$D = 4c^2 - 24c + 32$

$D = c^2 - 6c + 8$

$c^2 - 4c - 2c + 8$

$c(c-4) - 2(c-4)$

$(c-2)(c-4)$

$(c-2)(c-4) < 0$

$+ \quad 2 \quad - \quad 4 \quad +$

$\boxed{(2, 4)}$

(228)

Case 2:- leading coefficient $=0$

$C = 2$

$(2-2)x^2 + 2(2-2)x + 2 = 0$

$0x^2 + 0x + 2 = 0$

$2 = 0$

not a real value

so Roots are imaginary

$\boxed{C \in [2, 4)}$

~~Does~~ This is because question not mentioned that given is Quadratic equation. So leading coefficient can be zero making the Equation linear.

If question mensions quadratic equation, take leading coefficient $\neq 0$.

Q DYS-2

Q11. $(K-12)x^2 + 2(K-12)x + 2 = 0$

find integral values of K for which quadratic equation posses no real roots.

$\cancel{(K-12)^2 - (4)(2K-24)}$

$D = 4(K^2 + 144 - 24K) - 8K + 96$

$D = 4K^2 + 576 - 96K - 8K + 96$

$D = 4K^2 - 104K + 672$

$D = K^2 - 26K + 168$

$(K-12)(K-14) < 0$

$+ \quad 12 \quad - \quad 14 \quad +$

$(12, 14)$

so $\boxed{K = 13}$

Q If $x = 1 + 2i$

find $x^3 + x^2 - x + 22$

$x - 1 = 2i$

$(x-1)^2 = -2$

$x^2 + 1 - 2x = -2$

$x^2 - 2x + 1 = -2$

$3x^2 - 6x + 03 = -6$

$x(x^2 + x - 1) + 22$

~~$x(x^2 + x - 1$~~

$x(x^2 - 2x + 1 + 3x - 2) + 22$

$x(3x - 4) + 22$

$3x^2 - 4x + 22$

$3x^2 - 6x + 3 + 17 + 2x$

$-6 + 17 + 2x$

$13 + 2x$

$x^2 - 2x + 3 = 0$

$x = 2 \pm \sqrt{4 -}$

$x = 1 + 2i$

$x - 1 = 2i$

$x^2 + 1 - 2x = -4$

$x^2 - 2x + 5 = 0$

~~$x = 2 \pm \sqrt{4}$~~

$3x^2 - 6x + 15 = 0$

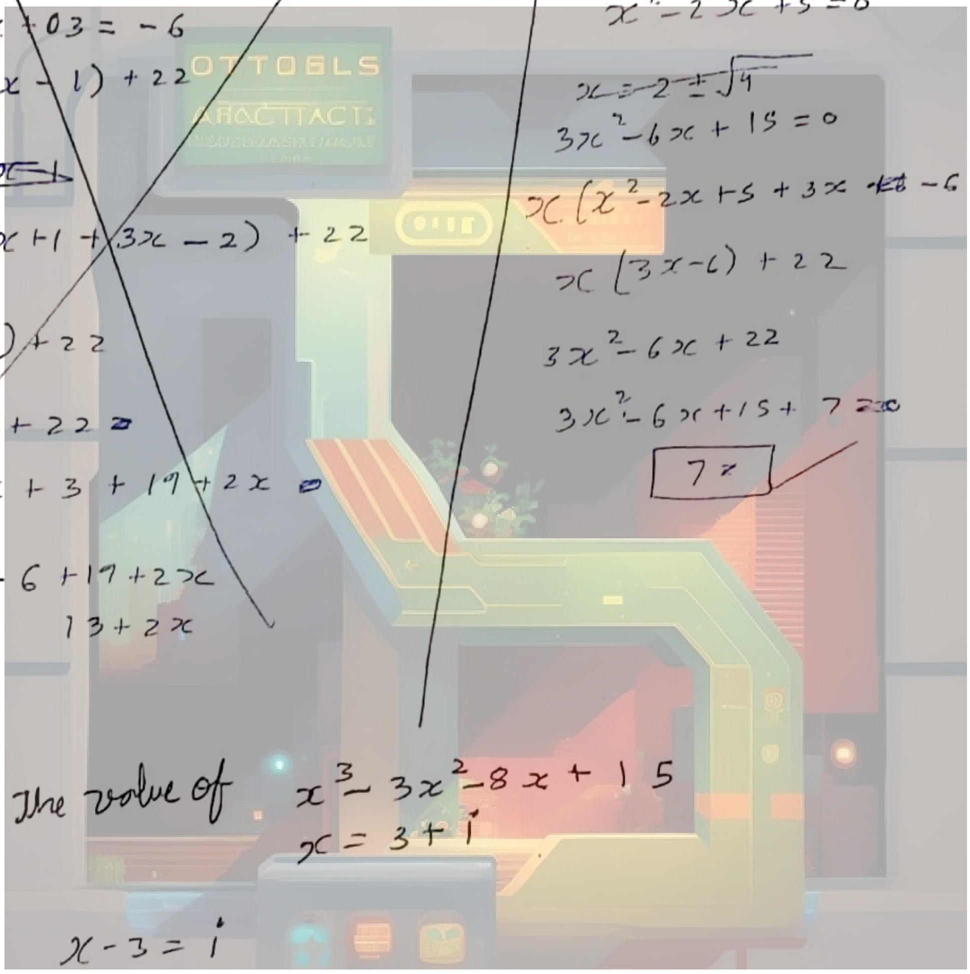
$x(x^2 - 2x + 5 + 3x - 6) + 22$

$x(3x - 6) + 22$

$3x^2 - 6x + 22$

$3x^2 - 6x + 15 + 7 = 0$

$\boxed{7}$



DYQS-2

Q16. find The value of $x^3 - 3x^2 - 8x + 15$

$x = 3 + i$

$x - 3 = i$

$x^2 + 9 - 6x = -1$

$x^2 + 10 - 6x = 0$

$3x^2 + 30 - 18x = 0$

$x(x^2 - 3x - 8) + 15$

$x(x^2 - 6x + 10 + 3x - 18) + 15$

$x(3x - 18) + 15$

$3x^2 - 18x + 15$

$\boxed{-15}$ $\boxed{-15}$

(230)

Scanned with CamScanner

H.W.

DVS-2 $[1,16] - [\{11,16\} \cup \{5\}]$

$a^2+a = 44444442222222$

$\cancel{=4(1111111)}$

$= 44444440000000 + 2222222$

$= 4444444 \times 10000000 + 2222222$

$= 4444444(9999999+1) + 2222222$

$= 4(1111111)[9(1111111)+1] + 2(1111111)$

$\cancel{=4P(9P+1)+2P}$

$= 4y(9y+1)+2y$

$= 36y^2+4y+2y$

$a^2+a = 36y^2+6y$

a^2-36y^2+a-6y

$(a+6y)(a-6y)-(a+6y)$

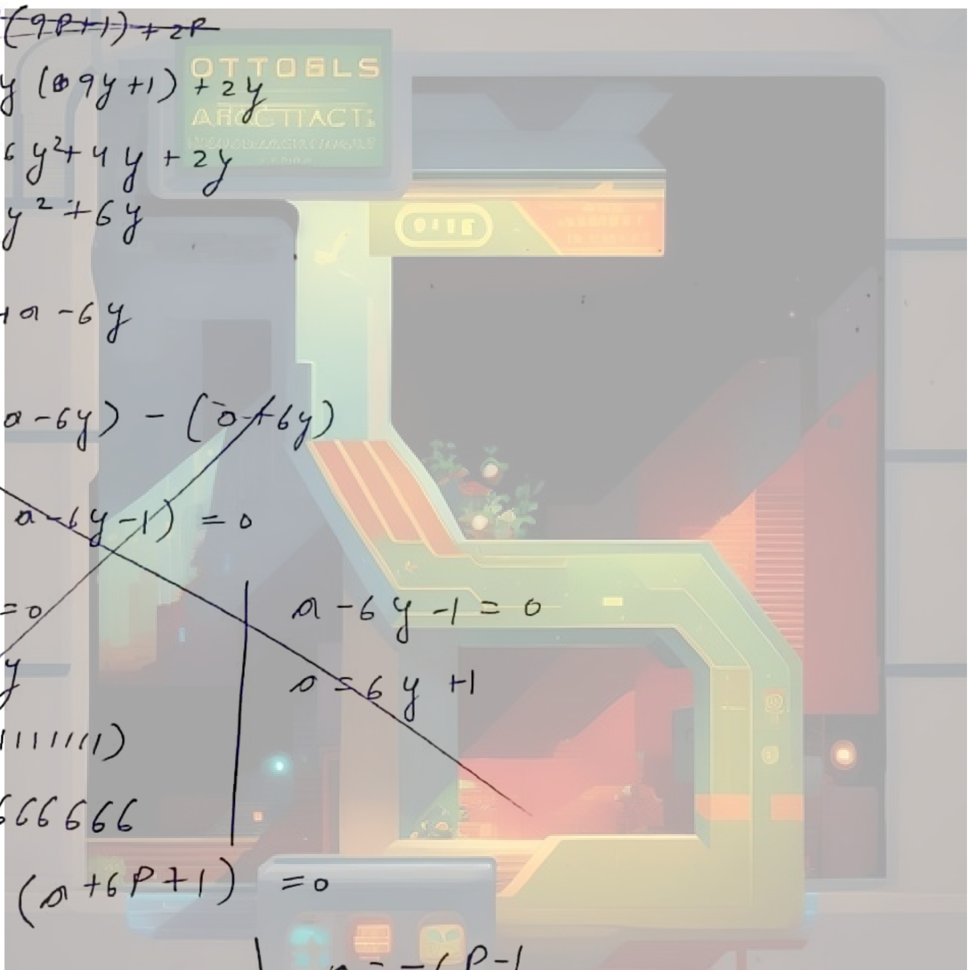
$a+6y\ (a-6y-1)=0$

$a+6y=0$ | $a-6y-1=0$

$a=-6y$ | $a=6y+1$

$a=-6(1111111)$

$a=-6666666$



$(a-6y)(a+6P+1)=0$

$a=6y$ | $a=-6P-1$

$\boxed{a=6666666}$ | $a=-6666666-1$

$\boxed{a=-6666667}$

Q find the quadratic if 1 root is $5+2\sqrt{6}$ & coefficients of quadratic are rational.

x: ~~$\alpha+1$~~ - $\alpha = 5+2\sqrt{6}$

$\beta = 5-2\sqrt{6}$

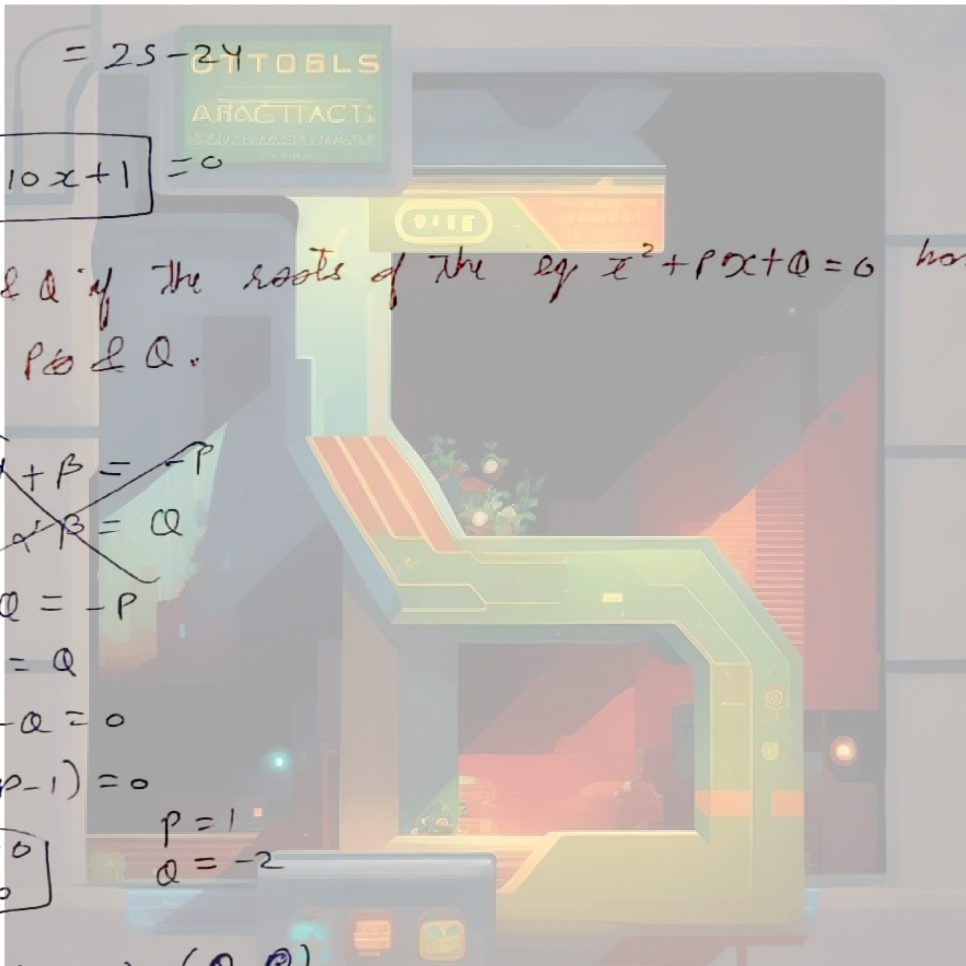
$\alpha+\beta = 5+2\sqrt{6}+5-2\sqrt{6}$

$= 10$

$\alpha\beta = (5+2\sqrt{6})(5-2\sqrt{6})$

$= 25-4\times 6$

$= 25-24$



$\boxed{x^2-10x+1}=0$

Q find P & Q if the roots of the eq $x^2+Px+Q=0$ have ~~P & Q~~ roots P & Q.

~~$\alpha+\beta=-P$~~

~~$\alpha\beta=Q$~~

$P+Q=-P$

$PQ=Q$

$PQ-Q=0$

$Q(P-1)=0$

$\boxed{\begin{array}{l}Q=0\\P=0\end{array}}$ $\quad$ $P=1$, $Q=-2$

$P,Q = (1,-2),(0,0)$

Q $x^2+mx+1=0$ find m if

a) One root is thrice of other

b) ratio of roots is $1/3$

c) Sum of roots is equal to POR.

a) $\alpha = 3\beta$

$\alpha+\beta = -m$

$4\beta = -m$

$\boxed{-\frac{4}{3}\sqrt{3}}$

$m = \boxed{\pm\frac{4}{\sqrt{3}}}$

$\alpha\beta = 1$

$3\beta(\beta) = 1$

$3\beta^2 = 1$

$\beta^2 = \frac{1}{3}$

$\beta = \frac{\sqrt{3}}{3}$

b) $\frac{\alpha}{\beta} = \frac{1}{3}$

$3\alpha = \beta$

$\alpha\beta = 1$

$3\alpha^2 = 1$

$\alpha = \frac{\sqrt{3}}{3}$

$\alpha+\beta = -m$

$\cancel{\frac{\sqrt{3}}{3}}+\beta = -m$

$4\alpha = -m$

$\boxed{\cancel{-\frac{4}{3}\sqrt{3} = m}}$

$\boxed{m = \pm\frac{4}{\sqrt{3}}}$

c) $\alpha+\beta = \alpha\beta$

$-m = 1$

$\boxed{m = -1}$

Symmetric Expression of α and β

If $(\alpha, \beta) = f(\beta, \alpha)$

Eg ① $f(\alpha, \beta) = \alpha^2 + \beta^2$

$f(\beta, \alpha) = \beta^2 + \alpha^2$
$= \alpha^2 + \beta^2$

So, symmetric

Eg ② $f(\alpha, \beta) = \alpha^2 - \beta^2$

$f(\beta, \alpha) = \beta^2 - \alpha^2 \neq \alpha^2 + \beta^2$

So, not symmetric

Q find which are symmetric

① $f(\alpha, \beta) = \alpha^4 - \beta^4$

$f(\beta, \alpha) = \beta^4 - \alpha^4 \neq \alpha^4 - \beta^4$

Not Symmetric

② $f(\alpha, \beta) = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

$f(\beta, \alpha) = \dfrac{\beta}{\alpha} + \dfrac{\alpha}{\beta}$

$= \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

Symmetric

③ $f(\alpha, \beta) = \dfrac{\alpha + \beta}{\alpha\beta}$

$f(\beta, \alpha) = \dfrac{\beta + \alpha}{\beta\alpha}$ Symmetric

$= \dfrac{\alpha + \beta}{\alpha\beta}$

Transformation of roots (valid for symmetric changes)

Eg ①. $x^2 - 4x + 5 = 0$

find quadratic whose roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

$\frac{2}{\alpha}, \frac{2}{\beta} = \frac{2}{x} \to t$

$x = \frac{2}{t}$

(put in Quad)

$\left(\frac{2}{t}\right)^2 - 4\left(\frac{2}{t}\right) + 5 = 0$

$\frac{4}{t^2} - \frac{8}{t} + 5 = 0$

$\boxed{5t^2 - 8t + 4 = 0}$

Q 2. $x^2 - 3x + 2 = 0$

find Quad of roots $\alpha + 1$, $\beta + 1$

$\alpha + 1, \beta + 1 \Rightarrow x + 1 = t$

$x = t - 1$

$(t-1)^2 - 3(t-1) + 2 = 0$

$t^2 + 1 - 2t - 3t + 3 + 2 = 0$

$\cancel{\boxed{t^2 + 5t = 0}}$

$\boxed{t^2 - 5t + 6 = 0}$

Method II

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$x = 1, 2$

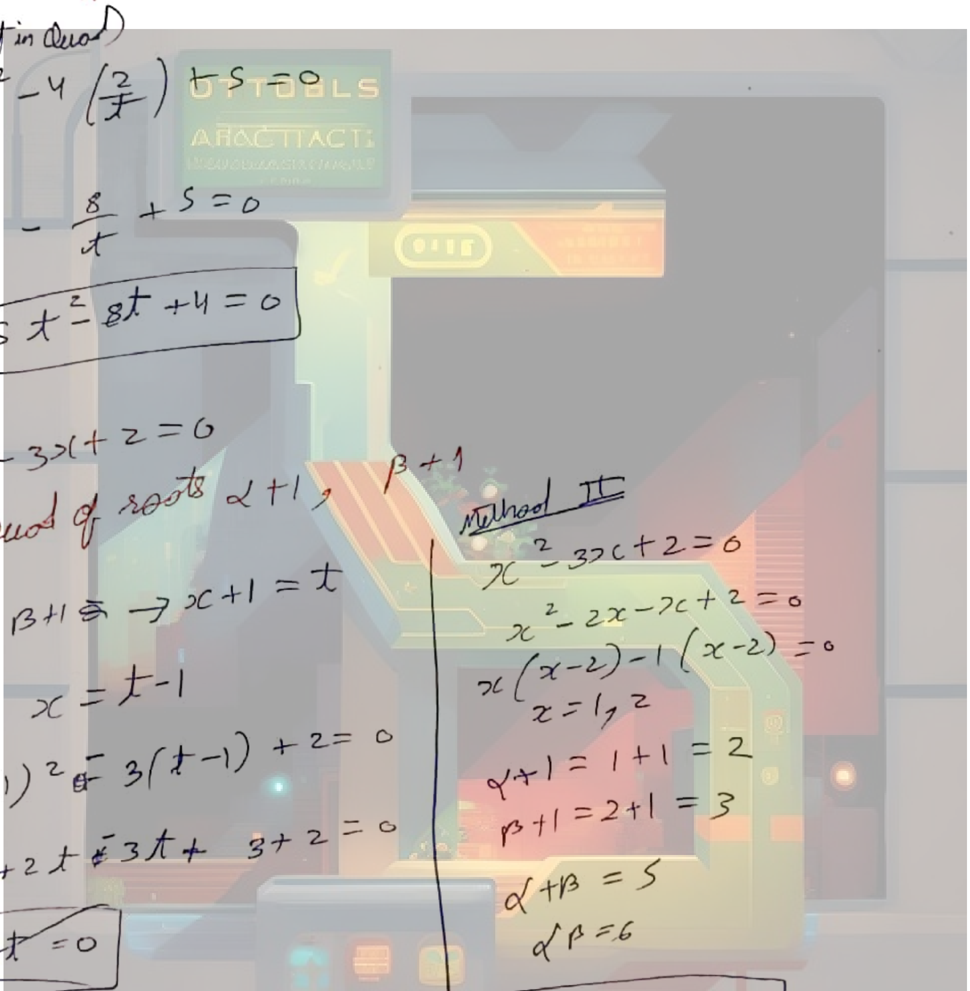
$\alpha + 1 = 1 + 1 = 2$

$\beta + 1 = 2 + 1 = 3$

$\alpha + \beta = 5$

$\alpha\beta = 6$

$\boxed{x^2 - 5x + 6 = 0}$



Q $x^2 - x + 1 = 0$ have roots α and β, find quadratic whose roots are -

(1) $3+\alpha,\ 3+\beta$

$x + 3 = y$

$x = y - 3$

$(y-3)^2 - (y-3) + 1 = 0$

$y^2 + 9 - 6y - y + 3 + 1 = 0$

$\boxed{y^2 - 7y + 13 = 0}$

(2) $1 - \frac{1}{\alpha},\ 1 - \frac{1}{\beta}$

$1 - \frac{1}{x} = y$

$1 - y = \frac{1}{x}$

$x = \frac{1}{1-y}$

$\left(\frac{1}{1-y}\right)^2 - \left(\frac{1}{1-y}\right) + 1 = 0$

$\frac{1}{\cancel{1+y^2-2y}\ (1-y)^2} - \frac{1}{1-y} + 1 = 0$

$(1-y)^2 - (1-y) + 1 = 0$

$y^2 + 1 - 2y - 1 - y + 1 = 0$

$\boxed{y^2 - 3y + 1 = 0}$

(3) $\frac{2}{1+\alpha},\ \frac{2}{1+\beta}$

$\frac{2}{1+x} = y$

$\frac{2}{y} = 1 + x$

$x = \frac{2-y}{y}$

$\left(\frac{2-y}{y}\right)^2 - \left(\frac{2-y}{y}\right) + 1$

$\frac{y^2+4-4y}{y^2} - \frac{(2-y)}{y} + 1$

$y^2 + 4 - 4y - 2y + y^2 + y^2$

$\boxed{3y^2 - 6y + 4 = 0}$

(4) $-\alpha,\ -\beta$

$-x = y$

$x = -y$

$(-y)^2 - (-y) + 1 = 0$

$\boxed{y^2 + y + 1 = 0}$

Q $\cancel{x^2+1}\ x^2 - x + 1 = 0$ have roots α & β

find Quadratic whose roots are.

(1) $\cancel{\mathcal{D}}\ \alpha + 3\beta,\ 3\alpha + \beta$

(2) $(\alpha - \beta)^2,\ (\alpha + \beta)^2$

(1)

$\alpha + \beta = 1$

$\alpha\beta = 1$

$\alpha - \beta = \sqrt{-3}$

$R_1 \cancel{\mathcal{D}} = \alpha + 3\beta$

$R_2 = 3\alpha + \beta$

$R_1 + R_2 = \alpha + 3\beta + 3\alpha + \beta$

$= 4\alpha + \cancel{\beta}\,4\beta$

$= 4\,\cancel{\beta}(\alpha + \beta)$

$= 4(1)$

$= 4$

$R_1 \times R_2 = (\alpha + 3\beta)(3\alpha + \beta)$

$= 3\alpha^2 + 3\beta^2 + \alpha\beta + 9\alpha\beta$

$= 3((\alpha + \beta)^2 - 2\alpha\beta) + 10\alpha\beta$

$= 3(1 - 2) + 10$

$= -3 + 10$

$= 7$

$\cancel{\mathcal{D}}\ x^2 \cancel{0} - (4) + 7$

$\boxed{x^2 - 4x + 7}$

(2)

$\alpha + \beta = 1$

$\alpha\beta = 1$

$\cancel{R}\ R_1 + R_2 = (\alpha - \beta)^2 + (\alpha + \beta)^2$

$= \alpha^2 + \beta^2 - \cancel{2\alpha\beta} + \alpha^2 + \beta^2 + \cancel{2\alpha\beta}$

$= 2(\alpha^2 + \beta^2)$

$= 2(-1)$

$\boxed{= -2}$

$R_1 \times R_2 = (\alpha - \beta)^2(\alpha + \beta)^2$

$= \cancel{(\alpha^2 + \beta^2 - 2\alpha\beta)(\alpha^2 + \beta^2)}$

$= (\sqrt{-3})^2(1)^2$

$= -3 \times 1$

$\boxed{= -3}$

$\boxed{x^2 + 2x - 3}$

Equation (vs) Identity

→ Equation which is true for every value of the variable is a identity

eg. $\sin^2\theta + \cos^2\theta = 1$ $(\theta \in R)$

$(a+b)^2 = a^2 + 2ab + b^2$ $(a, b \in R)$

→ In quadratic if it has more than 2 roots then it will be an identity

Q $(p-1)x^2 + (p^2-3p+2)x + (p^2-4p+3) = 0$ is an identity in x.

find value of p it is

$(p-1)x^2 + (p^2-3p+2)x + (p^2-4p+3) = 0$

$p-1=0$
$p=1$

$p^2-3p+2=0$
$p=2,1$

$p=3,1$

$\boxed{p=1}$

for $p=1$

$\cancel{0\,(1-1)x^2+}$

$0x^2 + 0x + 0 = 0$

Q. find x if Quad in λ have more than 2 roots.

$x^2(\lambda^2-5\lambda-16) + x(\lambda^2+3\lambda+2) + \lambda^2 - 4 = 6$

$\cancel{x^2\lambda^2 - 5x^2\lambda + 6x^2 +}$

$x^2\lambda^2 - 5x^2\lambda + 6x^2 + x\lambda^2 + 3x\lambda + 2x + \lambda^2 - 4 = 0$

$\lambda^2(x^2+x+1) + \lambda(-5x^2+3x) + (6x^2+2x-4)$

$x \in \phi$

no common

$\boxed{x \in \phi}$

x^2+x+1

$x = -1 \pm \sqrt{}$

(238)

Q $\frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-c)(x-a)}{(b-c)(b-a)} + c\frac{(x-a)(x-b)}{(c-a)(c-b)} = x$

$\cancel{ax^2(1-b)}$

let $x = a$

$a + 0 + 0 = a$ $\Big\} \rightarrow \frac{a\cancel{(a-b)}\cancel{(a-c)}}{\cancel{(a-b)}\cancel{(a-c)}} + 0 + 0 = a \Rightarrow a = a$

$x = b$

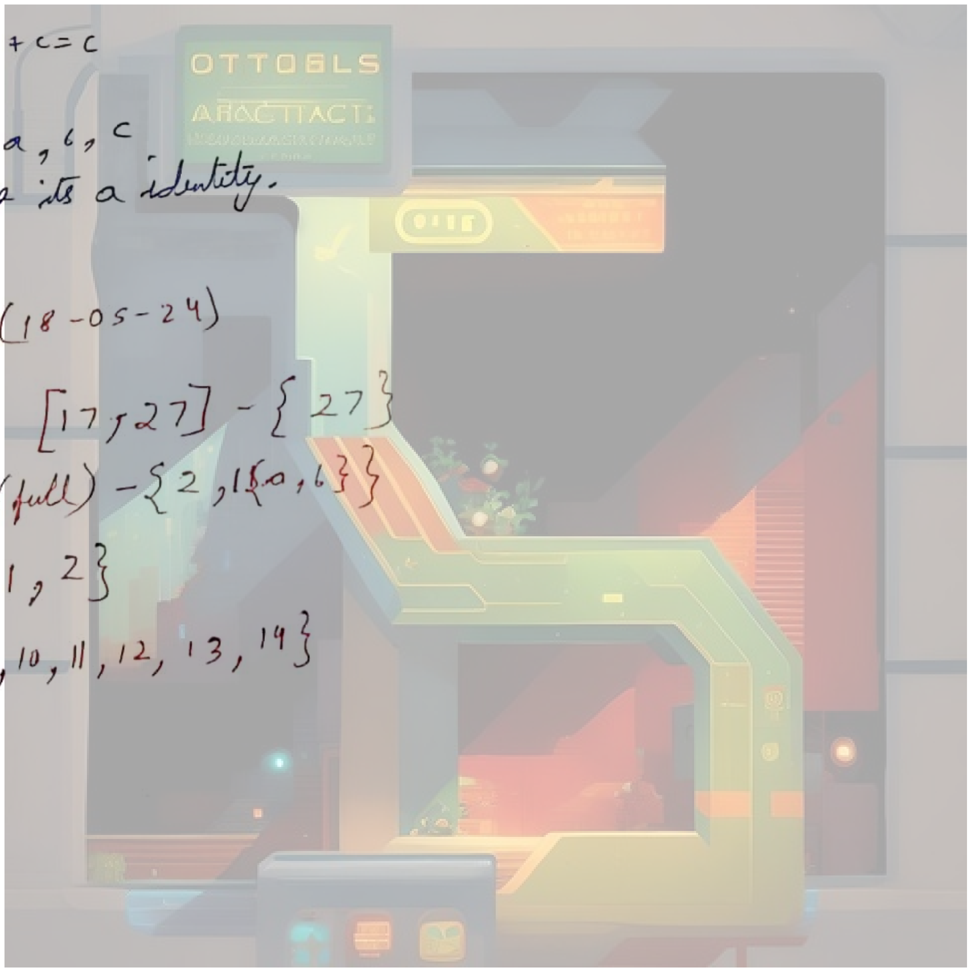
$0 + b + 0 = b$

$x = c$

$0 + 0 + c = c$

So $x = a, b, c$

so its a identity.



H.W. (18-05-24)

DYS-2 [17, 27] - {27}

DYS-3 (full) - {2, 15(a, b)}

DYS-4 {1, 2}

Race - {9, 10, 11, 12, 13, 14}

US-2 (ch-3)

Q24. $x_1 = 9x_2$

$x_1 + x_2 = 3a+2$

$10x_2 = 3a+2$

$x_2 = \frac{3a+2}{10}$

$x_1 x_2 = a^2$

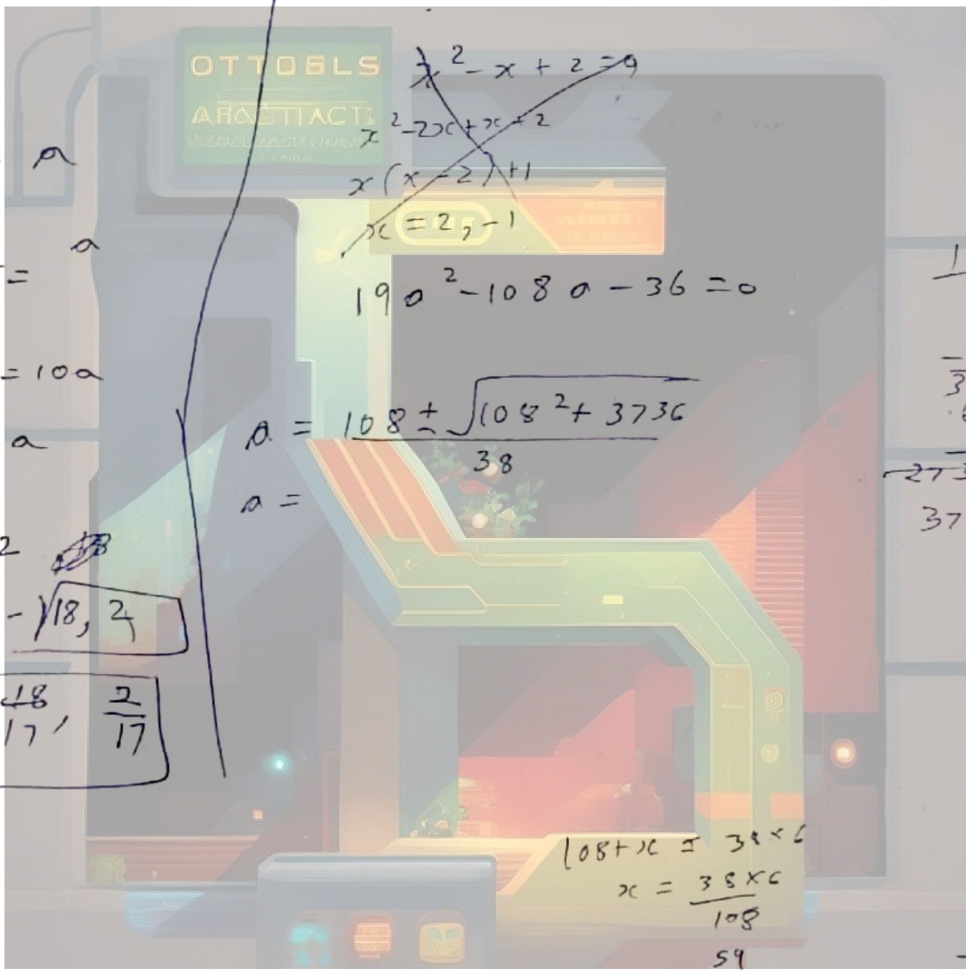
$\cancel{(3x_2)^2 = a^2}$

$\cancel{3x_2 = a}$

$\cancel{9 \times \frac{9a^2+4+12a}{100} = a^2}$

$\cancel{81a^2+36+108a = 100a^2}$

$\cancel{19a^2-108a-36=0}$



$\cancel{x^2 - x + 2 = 0}$

$\cancel{x^2-2x+x+2}$

$\cancel{x(x-2)+1}$

$\cancel{x = 2, -1}$

$19a^2 - 108a - 36 = 0$

$a = \frac{108 \pm \sqrt{108^2 + 3736}}{38}$

$a =$

$3x_2 = a$

$\frac{9a+6}{10} = a$

$9a+6 = 10a$

$6 = a$

$x_2 = 2$

roots - $\boxed{18, 2}$

$\boxed{-\frac{18}{17}, \frac{2}{17}}$

108
108
864
10800
36
720
76
684
11664
2736
3736

11664

34.

$108 + x = 38 \times 6$

$x = \frac{38 \times 6}{108}$

54

65
5
325

63
3
189

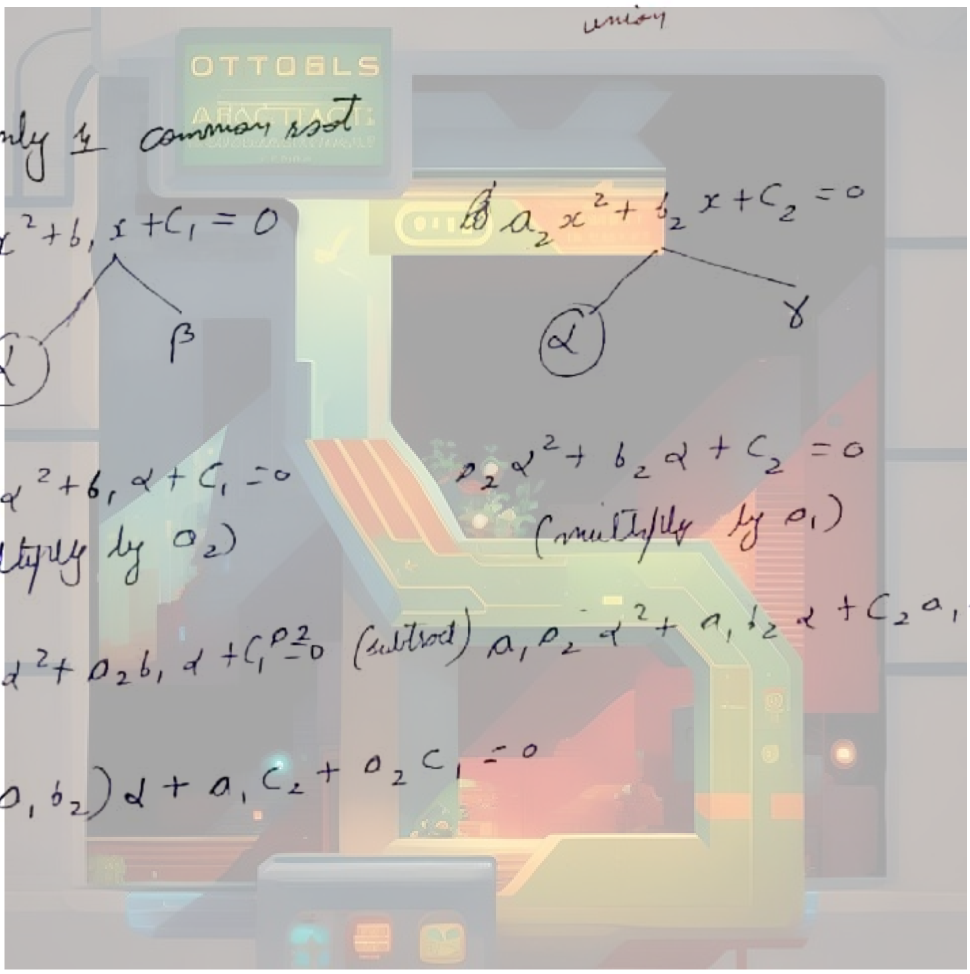
62 64
256

104

Condition for common roots

Types

- Case 1 → Only 1 common root
- Case 2 → Both common roots
- Case 3 → At least one root common
  - Case 1
  - Case 2
  - union
- Case 4 → Quarter question: one root common between a cubic & quadratic



Case 1 → Only 1 common root

$a_1x^2+b_1x+c_1=0$ with roots α, β

$a_2x^2+b_2x+c_2=0$ with roots α, γ

$\therefore\ a_1\alpha^2+b_1\alpha+c_1=0$ (multiply by a_2)

$a_2\alpha^2+b_2\alpha+c_2=0$ (multiply by a_1)

$a_1a_2\alpha^2+a_2b_1\alpha+c_1a_2=0$ (subtract) $a_1a_2\alpha^2+a_1b_2\alpha+c_2a_1=0$

$(a_2b_1-a_1b_2)\alpha+a_1c_2+a_2c_1=0$

Q find λ if $x^2 - \lambda x - 21 = 0$ & $x^2 - 3\lambda x + 35 = 0$ have one root common.

$\alpha^2 - \cancel{\alpha}\lambda\alpha - 21 = 0$ --(1) $\qquad \alpha^2 - 3\lambda\alpha + 35 = 0$ --(2)

$\cancel{2\alpha^2}$: (2) - (1)

$-2\lambda\alpha + 56 = 0$

$2\lambda\alpha = 56$

$\boxed{\lambda\alpha = \frac{56}{2} = \frac{28}{\cancel{2}}}$

$\alpha = \frac{28}{\lambda}$

put in (1)

$\left(\frac{28}{\lambda}\right)^2 - \lambda \times \frac{28}{\lambda} - 21 = 0$

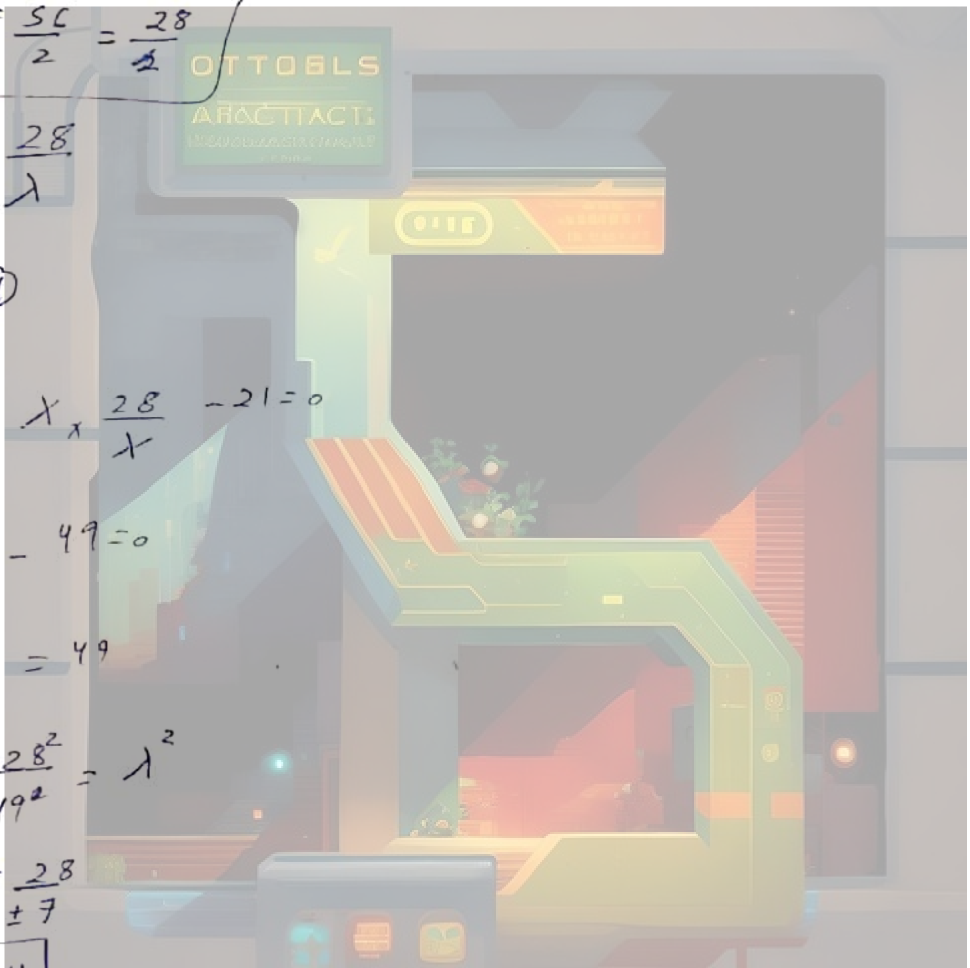
$\frac{(28)^2}{\lambda^2} - 49 = 0$

$\frac{28}{\lambda^2} = 49$

$\frac{\cancel{28}\ 28^2}{49^2} = \lambda^2$

$\lambda = \frac{28}{\pm 7}$

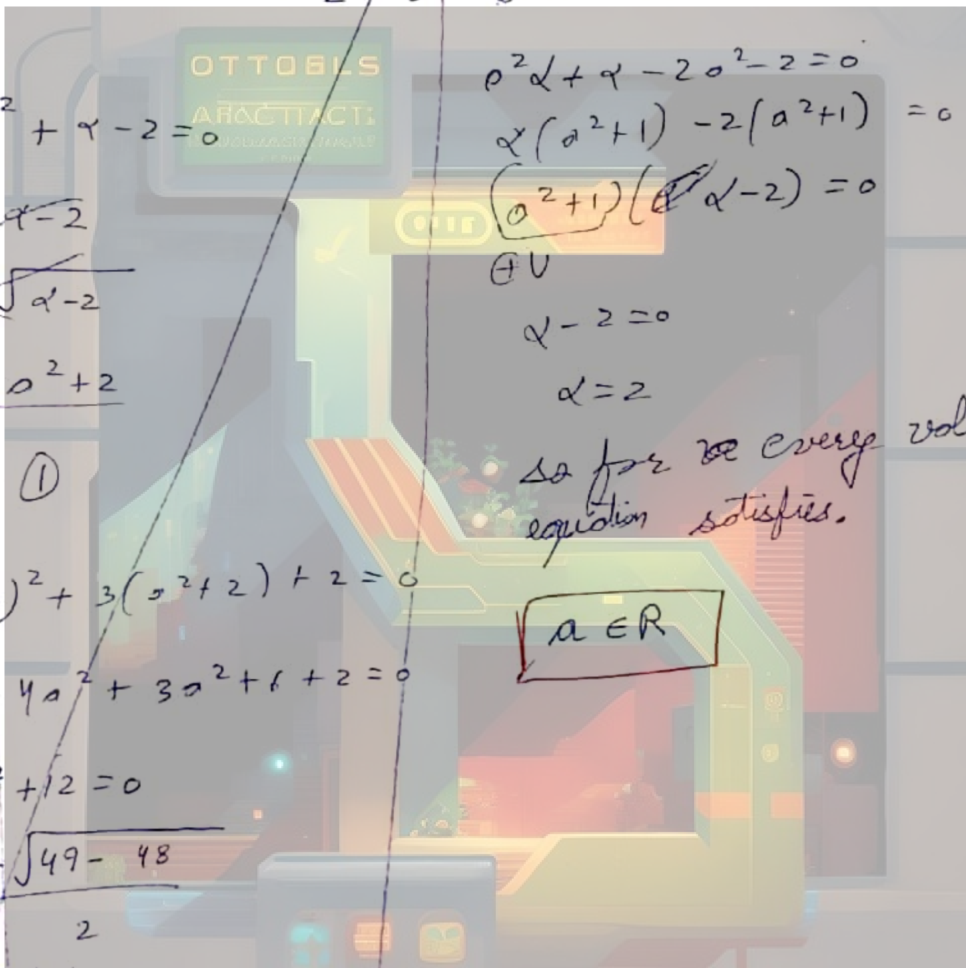
$\boxed{\lambda = \pm 4}$



Q find a if $x^2+(a^2-2)x-2a^2=0$ & x^2-3x+2 have only one root common.

$\alpha^2+(a^2-2)\alpha-2a^2=0$ — ② $\alpha^2-3\alpha+2=0$ — ①

② − ①



$a^2-2\alpha+3\alpha-2a^2-2\neq 0$

$-a^2+\alpha-2=0$

$a^2=\alpha-2$

$a=\sqrt{\alpha-2}$

$\alpha=a^2+2$

put in ①

$(a^2+2)^2+3(a^2+2)+2=0$

$a^4+4+4a^2+3a^2+6+2=0$

$a^4+7a^2+12=0$

$a^2=\frac{-7\pm\sqrt{49-48}}{2}$

$a^2=\frac{-7\pm1}{2}$

$a^2=-4,-3$

✗

$a^2\alpha-2\alpha+3\alpha-2a^2-2=0$

$a^2\alpha+\alpha-2a^2-2=0$

$\alpha(a^2+1)-2(a^2+1)=0$

$(a^2+1)(\alpha-2)=0$

+ve

$\alpha-2=0$

$\alpha=2$

so for every value of a, equation satisfies.

$\boxed{a\in R}$

Q If $x^2 + Px + Q = 0$ & $x^2 + Qx + P = 0$

$P \neq Q$ and one common root.

$\alpha^2 + P\alpha + Q = 0$ $\qquad$ $\alpha^2 + Q\alpha + P = 0$

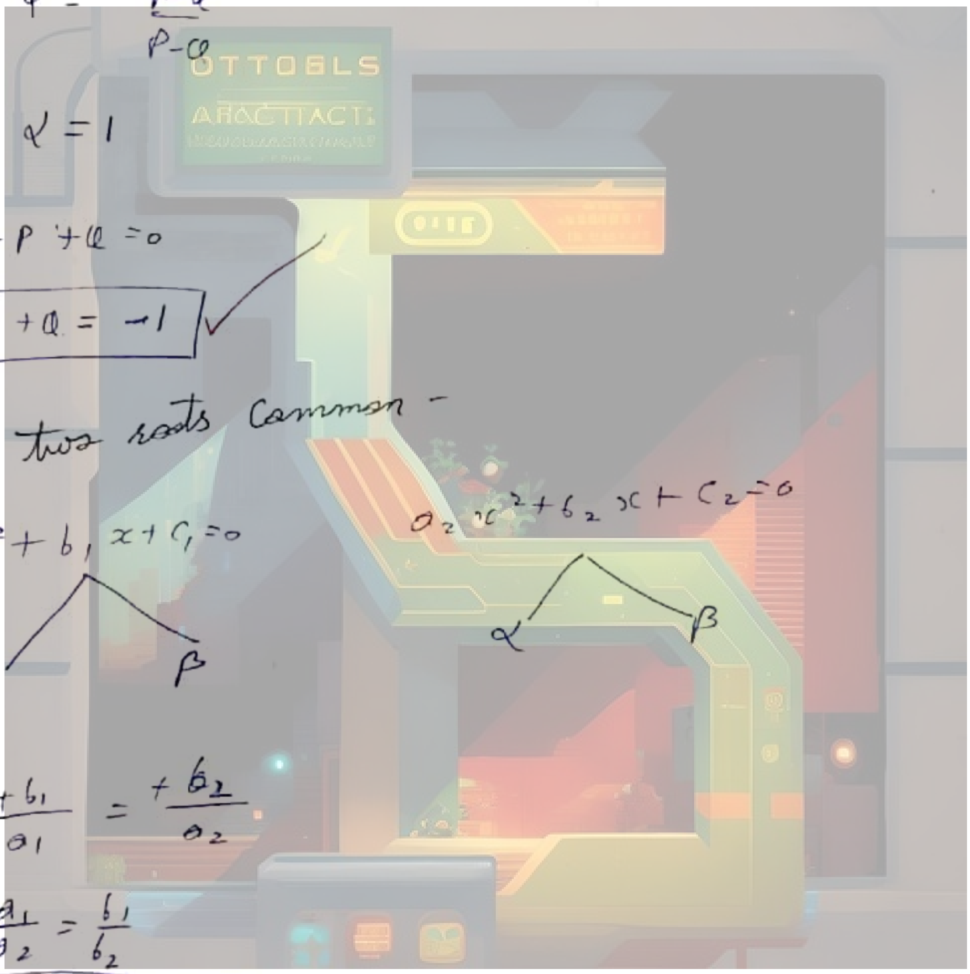
(2) − (1)

$P\alpha - Q\alpha + Q - P = 0$

$(P - Q)\alpha + Q - P = 0$

$P - Q\,\alpha = P - Q$

$\alpha = \frac{P-Q}{P-Q}$



$\alpha = 1$

$(1)^2 + P + Q = 0$

$\boxed{P + Q = -1}$

Case - 2 $\quad$ two roots common -

$a_1x^2 + b_1x + c_1 = 0$ $\qquad$ $a_2x^2 + b_2x + c_2 = 0$

$\alpha \quad \beta$ $\qquad$ $\alpha \quad \beta$

$\alpha + \beta = \frac{+b_1}{a_1} = \frac{+b_2}{a_2}$

$= \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\alpha\beta = \boxed{\frac{c_1}{c_2} = \frac{a_1}{a_2} = \frac{b_1}{b_2}}$

Q If $a, b, c \in R$ & eq $ax^2+bx+c=0$ & $x^2+2x+9=0$ have both roots as common, then find $a:b:c$

~~$\frac{a}{1} = \frac{b}{2} = \frac{c}{9}$~~

~~$a:b:c$~~

~~$\frac{18a}{9}$~~

~~$\frac{18a}{18} = \frac{9b}{18} = \frac{2c}{18}$~~

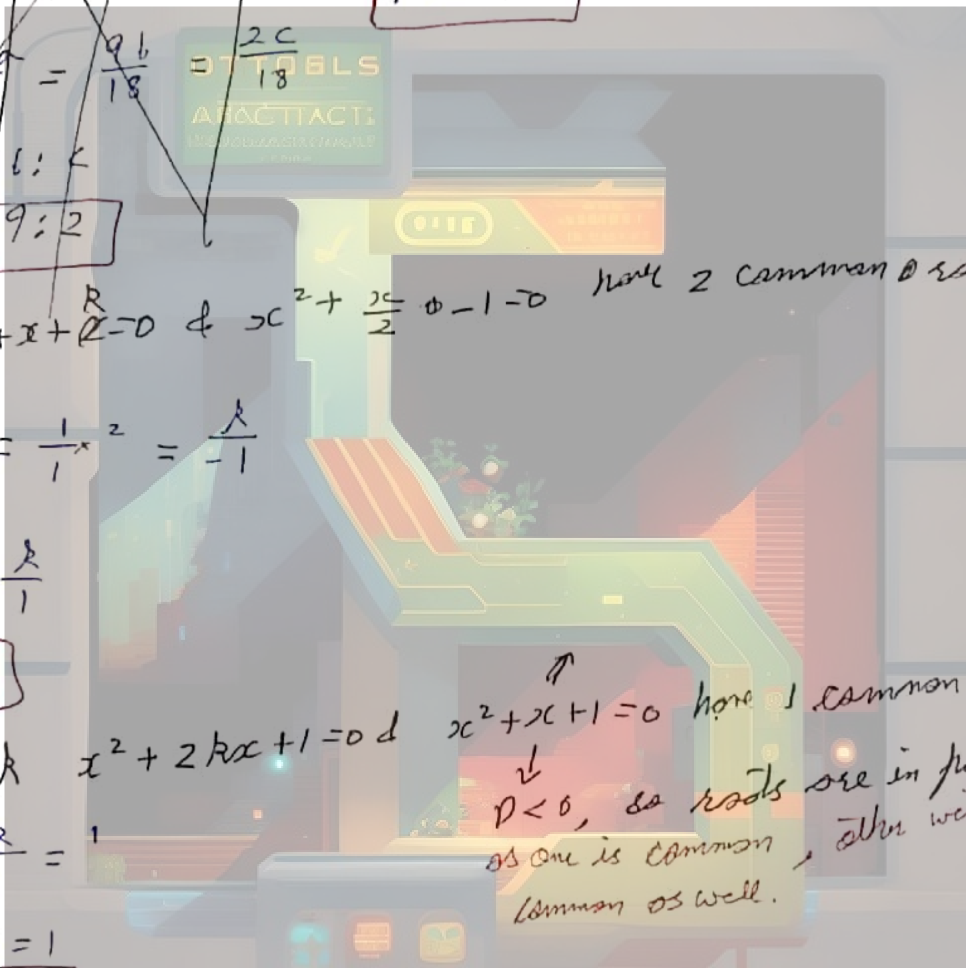
~~$a:b:c$~~

~~$\boxed{18:9:2}$~~

$\frac{1}{a} = \frac{2}{b} = \frac{9}{c}$

$a:b:9$

$\boxed{1:2:9}$



Q. $2x^2+x+k=0$ & $x^2+\frac{x}{2}-1=0$ have 2 common roots find k

$\frac{2}{1} = \frac{1}{1/2} = \frac{k}{-1}$

$\frac{2}{1} = -\frac{k}{1}$

$\boxed{-2 = k}$

Q. find k $x^2+2kx+1=0$ & $x^2+x+1=0$ have 1 common root

$D<0$, so roots are in pair as one is common, other will be common as well.

$\frac{2k}{1} = 1$

$2k = 1$

$\boxed{k = \frac{1}{2}}$

Case 3 At least one root common.

Q Possible values of 'a' for which $x^2+ax+1=0$ & $x^2+x+a=0$ have at least one common root

Case 1 $x^2-x^2+ax-x+1-a=0$

$x(a-1)-1(a-1)=0$

$(a-1)(x-1)=0$

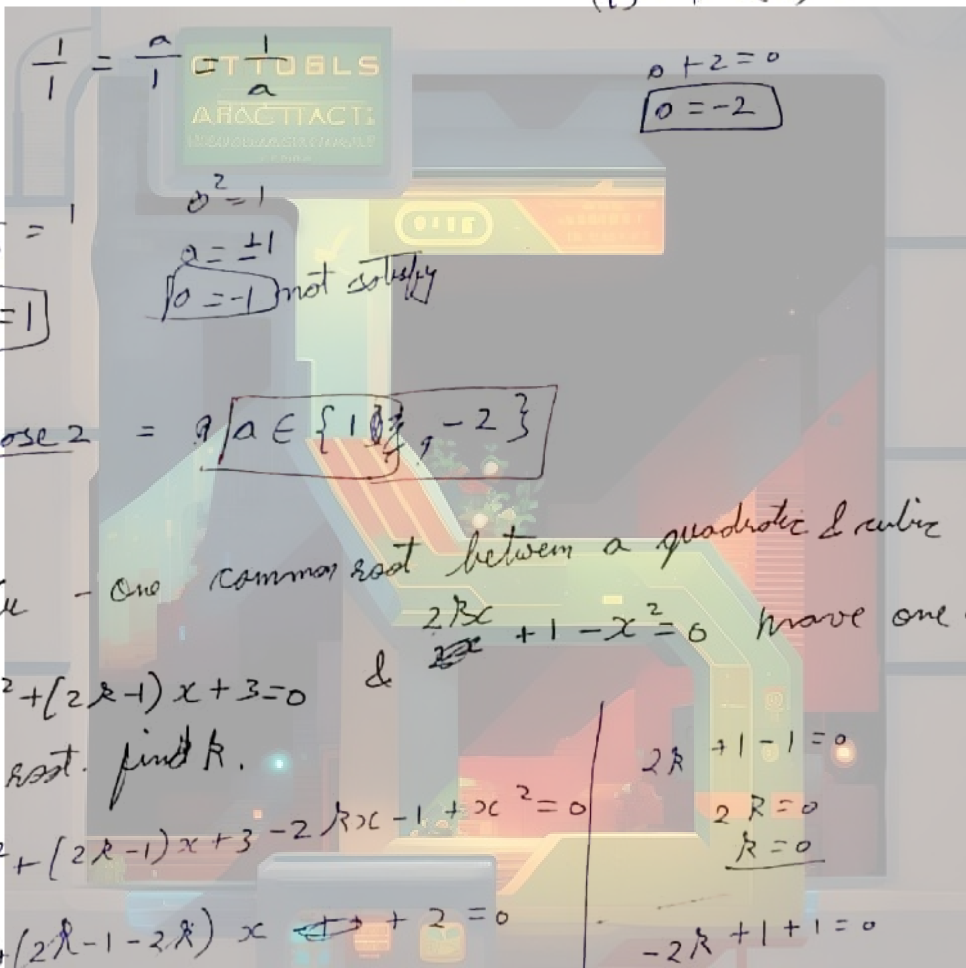
$\boxed{a=1}$ → $x-1=0$

$x=1$

$(1)^2+a(1)+1=0$

$a+2=0$

$\boxed{a=-2}$



Case 2 $\frac{1}{1}=\frac{a}{1}=\frac{1}{a}$

$a^2=1$

$a=\pm1$

$\boxed{a=-1}$ not satisfy

$\frac{1}{a}=1$

$\boxed{a=1}$

Case 1 ∪ case 2 = $\boxed{a\in\{1,-2\}}$

Case 4 Cu Cu - One common root between a quadratic & cubic

Q $x^3-3x^2+(2k-1)x+3=0$ & $2kx+1-x^2=0$ have one common root. find k.

$x^3-3x^2+(2k-1)x+3-2kx-1+x^2=0$

$x^3-2x^2+(2k-1-2k)x+2=0$

$x^3-2x^2-x+2=0$

$\boxed{x=1}$

$x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$\boxed{x=2,-1,1}$

$2k+1-1=0$

$2k=0$

$k=0$

$-2k+1+1=0$

$-2k=-2$

$k=1$

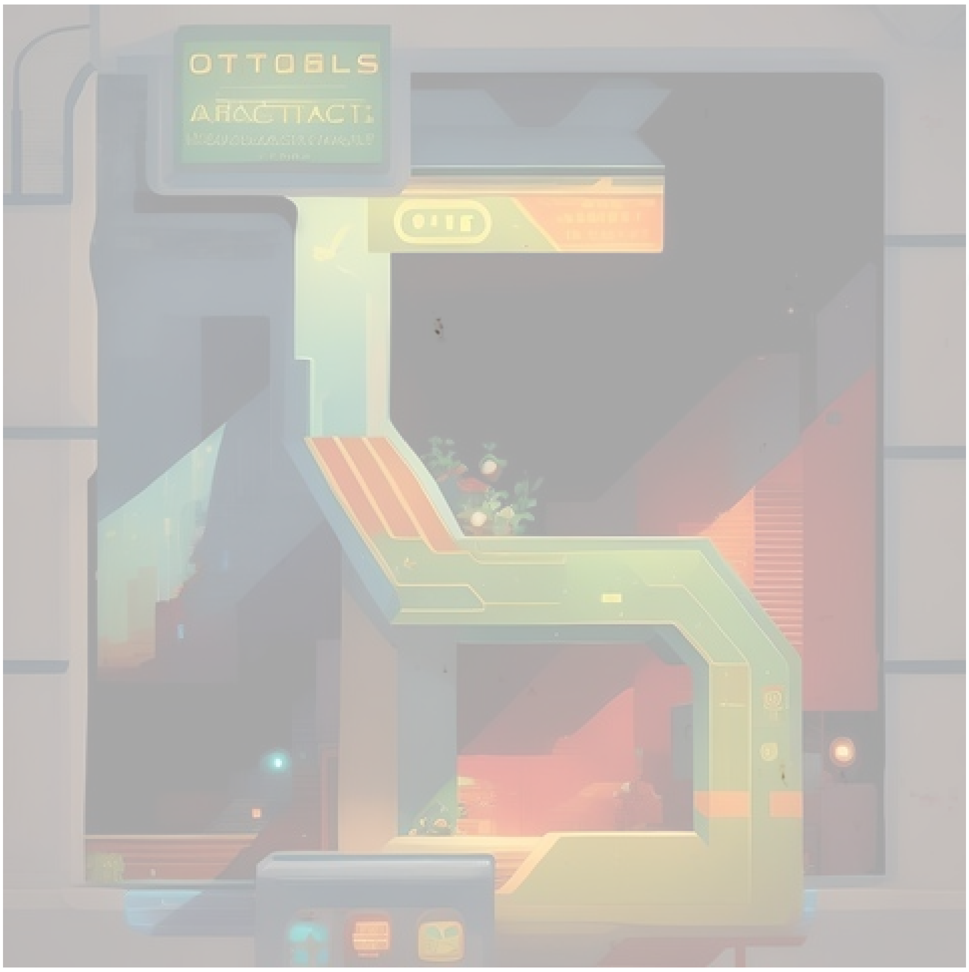
$4k+1-4=0$

$k=\frac{3}{4}$

① H.W. 20-05-2024

DYS-4 {3, 4, 5, 6, 7}

O-1 {{Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}



Q If the eq $ax^2+bx+c=0$ & $x^3+3x^2+3x+2=0$ have 2 common roots. then –

a) $a=b\neq c$

B) $a=-b=c$

c) $a\neq b\neq c$

✓D) $a+b+c=3$

$-1+3-3+2$

$-8+12-6+2$

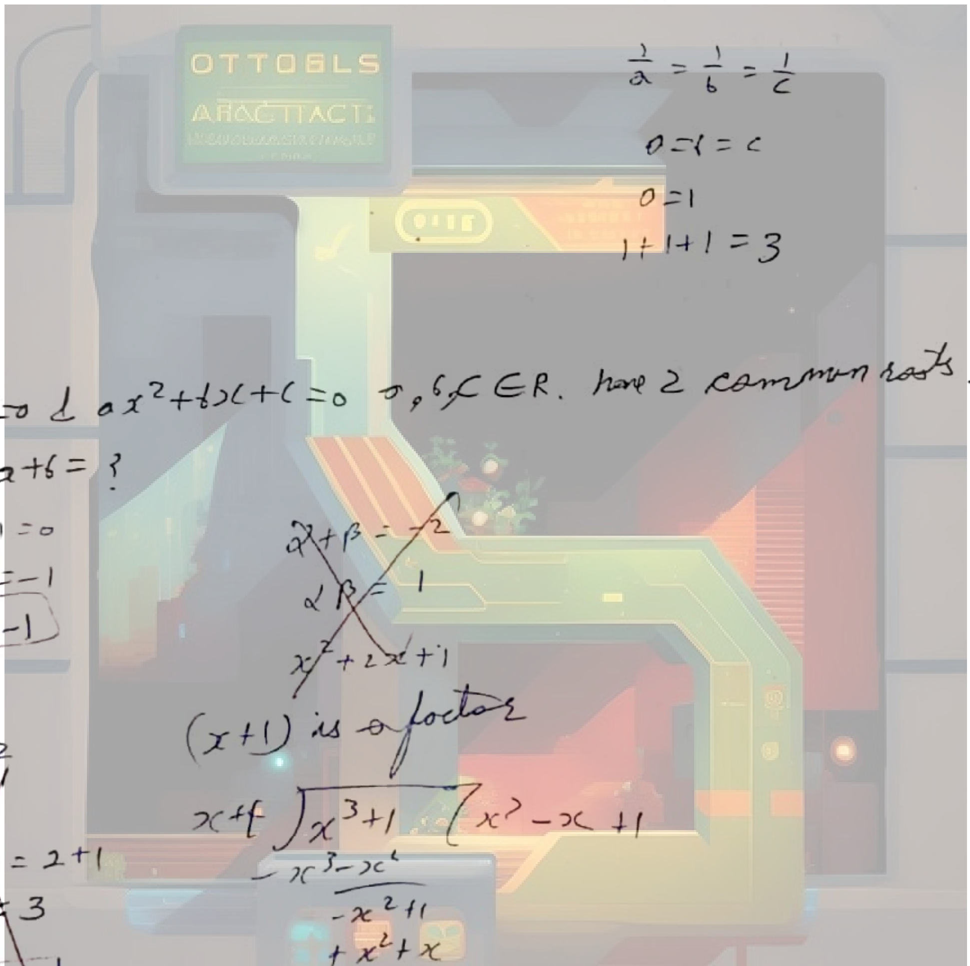
$(x+2)=0$

$\cancel{x^2+x+1}$

$\cancel{x=-1\pm\sqrt{1-4}}$

x^2+x+1

ax^2+bx+c



$\frac{1}{a}=\frac{1}{b}=\frac{1}{c}$

$a=1=c$

$a=1$

$1+1+1=3$

Q If $x^3+1=0$ & $ax^2+bx+c=0$ $a,b,c\in R$. have 2 common roots. then $a+b=$?

$x^3+1=0$

$x^3=-1$

$\boxed{x=-1}$

$\cancel{\alpha+\beta=-2}$

$\cancel{\alpha\beta=1}$

$\cancel{x^2+2x+1}$

$(x+1)$ is a factor

$x+1\,)\,x^3+1\,(\,x^2-x+1$

$\quad -x^3-x^2$

$\quad -x^2+1$

$\quad +x^2+x$

$\quad x+1$

$\cancel{b=2}$

$\cancel{c=1}$

$\cancel{a=1}$

$\cancel{a+b=2+1}$

$\cancel{=3}$

$\cancel{\boxed{D}}$

x^2-x+1

$\cancel{x=}\ D<0$

$x^2-x+1=ax^2+bx+c$

$a=1$

$b=-1$

$\boxed{a+b=0}$

$\boxed{B}$

Quadratic expression and its graphs.

$ax^2+bx+c \quad (a, b, c \in R),\ a \neq 0$

(1) $a<0$ concave ~~up~~ down

$a>0$ concave up

(2) $D>0$ Roots real & unequal $\rightarrow$ Graph cuts X axis at 2 diff points

$D=0$ Roots are equal $\rightarrow$ Graph touch x-axis

$D<0$ Graph does not cut x-axis.

(3) Vertex : $(x, y) = \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

Proof :- $y = ax^2+bx+c$

$$y = a\left[x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$y = \left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2+4ac}{4a^2}\right]$$

$$y = a\left[\left(x+\frac{b}{2a}\right)^2+\left(\frac{-D}{4a^2}\right)\right]$$

$$y = a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a}$$

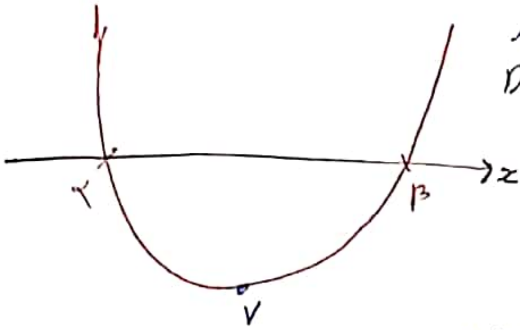
$$\left(y+\frac{D}{4a}\right) = a\left(x+\frac{b}{2a}\right)^2$$

$$\boxed{x = \frac{-b}{2a}}$$

$$\boxed{y = \frac{-D}{4a}}$$

(4) Graphs

(1)



$a>0$
$D>0$

$y = \oplus, \ominus, 0$

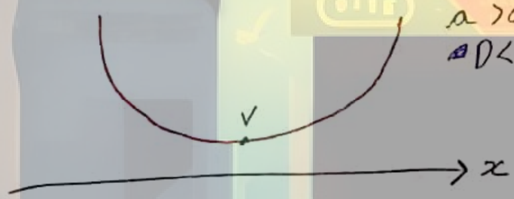
(2)

$a>0$
$D=0$

$y = \oplus, 0$

(3)

$y = \oplus$



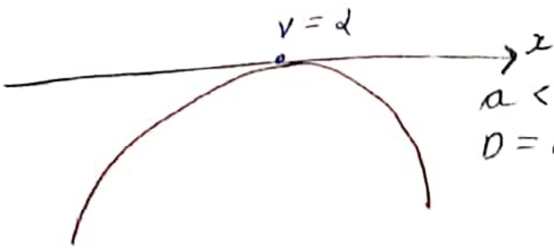
$a>0$
$D<0$

(4)

$a<0$
$D>0$

$y = \oplus, \ominus, 0$

(5)

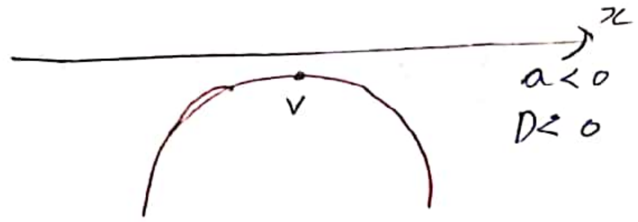


$y = \ominus, 0$

$a<0$
$D=0$

(2)

(6)



$a<0$
$D<0$

$y = \ominus$

$a, b, c \in R$

(1) $ax^2+bx+c>0$ $(a>0 \,\&\, D<0)$ Quadratic always $\oplus$ve

(2) $ax^2+bx+c<0$ $(a<0 \,\&\, D<0)$ Quadratic is always $\ominus$

(3) $ax^2+bx+c \geq 0$ $(a>0 \,\&\, D \leq 0)$ Quadratic is $\oplus$ or 0

Q Find 'a' for which $ax^2+3x+4>0$ $\quad x \in R$

$a>0, \; D<0$

$9-\cancel{4a}\,16a<0$

$16a>9$

$a>9/16 \quad \cup \; a>0$

$\boxed{a>\frac{9}{16}}$ $\quad a \in \left(\frac{9}{16}, \infty\right)$

Q. $ax^2+2ax+\frac{1}{2}<0$

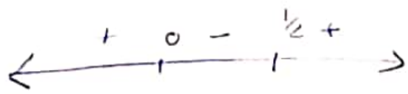
$a<0, \; D<0$

$b^2-4ac<0$

$4a^2-2a<0$

$2a(2a-1)<0$

$a \in (0, 1/2) \cup a<0$



$\boxed{a \in \phi}$

(12)

Scanned with CamScanner

Q $kx^2+x+k>0$

$a>0$

$D<0$

$1-4k^2<0$

$\cancel{4k^2>1}$

$\cancel{k^2>\frac{1}{4}}$

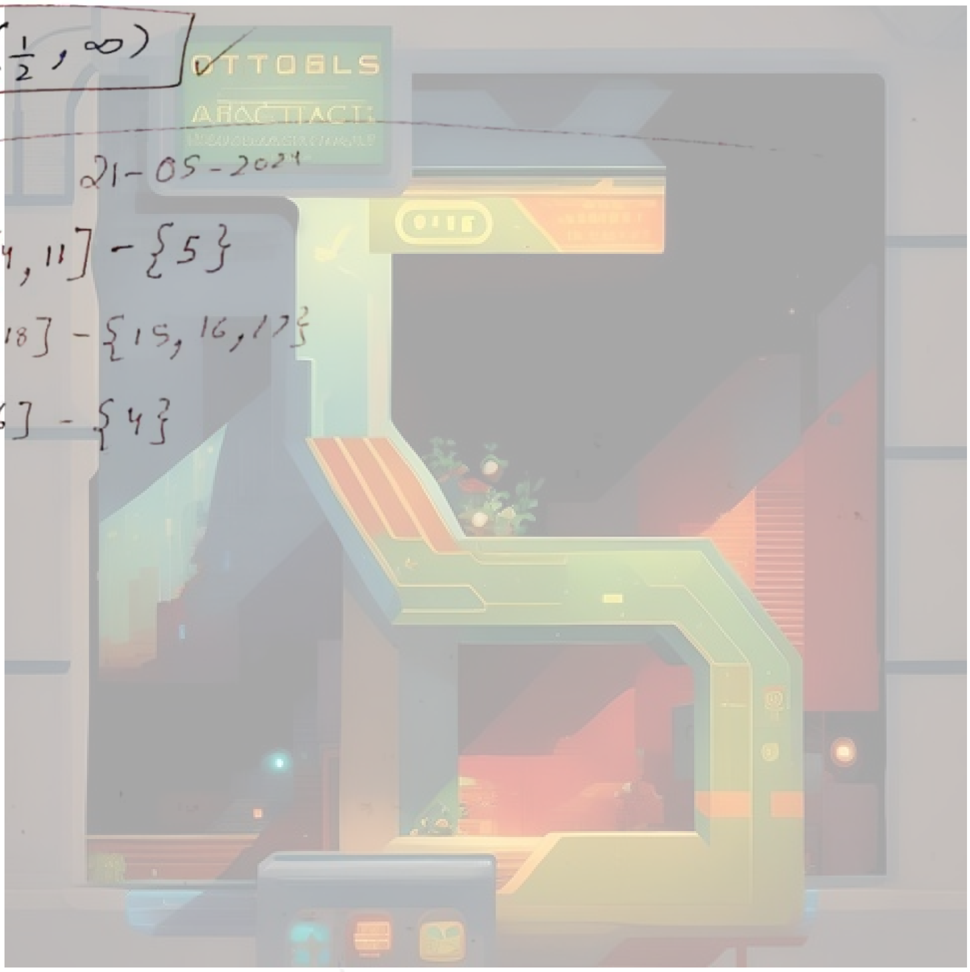
$\cancel{k>\frac{1}{2}}$

$4k^2-1>0$

$+ \quad -\frac{1}{2} \quad - \quad \frac{1}{2} \quad +$

$k\in\left[\left(-\infty,-\tfrac{1}{2}\right)\cup\left(\tfrac{1}{2},\infty\right)\right]\cap(0,\infty)$

$\boxed{k\in\left(\tfrac{1}{2},\infty\right)}$



H.W. 21-05-2024

DYS-8 [4, 11] − {5}

O-1 [11, 18] − {15, 16, 17}

O-2 [1, 6] − {4}

Range of a Quadratic (values of y)

Range $\in [y_{min}, y_{max}]$

Type 1:- $x \in R$

$a > 0$

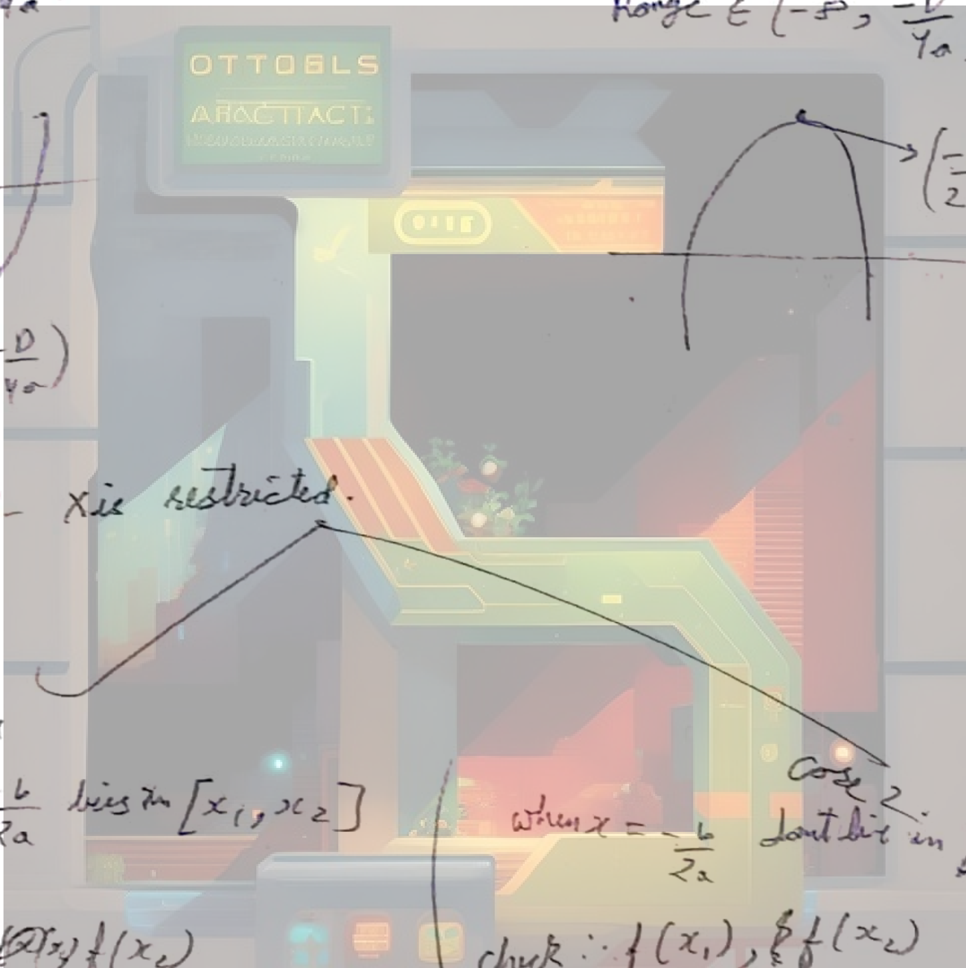
Range $\in \left[\frac{-D}{4a}, \infty\right)$

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$a < 0$

Range $\in \left(-\infty, \frac{-D}{4a}\right]$

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$



Type-2:- x is restricted.

Case 1

when $x = \frac{-b}{2a}$ lies in $[x_1, x_2]$

$\Downarrow$

$f\left(\frac{-b}{2a}\right), f(x_1), f(x_2)$

Case 2

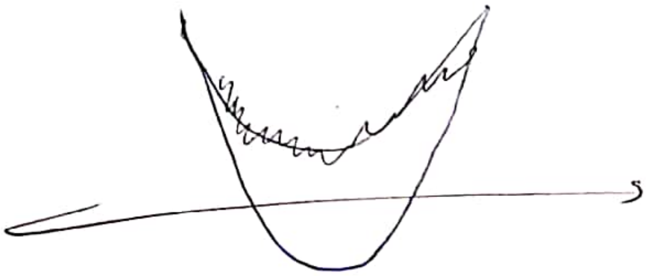
when $x = \frac{-b}{2a}$ don't lie in $[x_1, x_2]$

check :- $f(x_1)$, & $f(x_2)$

Q Draw the graph of $x^2 - 5x + 6 = 0$

① find Minimum value & point where min value occurs.

② Range of quadratic.



① min value $= \frac{-D}{4a}$

$= \frac{-1}{4}$

$\boxed{= -1/4}$

$x_{min} = \frac{5}{2}$

$\boxed{x_{min} = \frac{5}{2}}$

② Range $\in [y_{min}, y_{max}]$

$y_{min} = -1/4$

$y_{max} = \infty$

$\boxed{\text{Range } [-1/4, \infty)}$

Q Draw graph of $x^2 + x + 1 = 0$

① find min value & Point

② Range

① $y_{min} = \frac{-D}{4a}$

$= \frac{4-1}{4}$

$\boxed{= \frac{3}{4}}$

$x_{min} = \boxed{\frac{-1}{2}}$

$x_{min} = \frac{-b}{2a}$

② $y_{max} = \infty$

Range $\boxed{[\frac{3}{4}, \infty)}$

Q find the range of $-x^2+2x-1$

$y_{max} = -\frac{(4-4)}{2}$

$= 0$

$\boxed{(-\infty, 0]}$

Q $y = x^2 - 2x - 3$

① $x \in R$

$y_{min} = -\frac{(4+12)}{4}$

$= -4$

$\boxed{[-4, \infty)}$

② $x \in [0, 3]$

$x \in [-\infty, \infty] \cap [0, 3]$

$\boxed{x \in [0, 3]}$

③ $x \in [-2, 0]$

$\boxed{x \in [-2, 0]}$

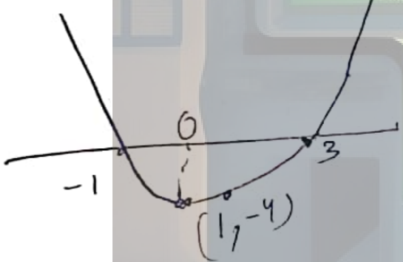
② $x \in [0,3]$

$y|_{x=0} = 0-0-3$
$= -3$

$y|_{x=3} = 9-6-3$
$= 0$

$\boxed{[-4, 0]}$

$y|_{x=1} = 1-2-3$
$= -4$

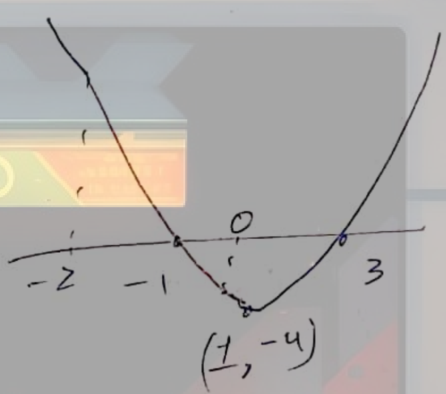


③ $x \in [-2,0]$

$f(-2) = 4+4-3$
$= 8-3$
$= 5$

$f(0) = -3$

$\boxed{[-3, 5]}$



Q $y = f(x) = x^2 - 5x + c$

$y_{min} = \frac{-(25-24)}{4}$

$\boxed{= \frac{-1}{4}} = -0.25$

$x_{min} = \boxed{\frac{5}{2}} = 2.5$

① $[-3, 0]$ $x \in x$

$f(-3) = 9 + 15 + 6$
$= 30$

$f(0) = 6$

$\boxed{y \in [6, 30]}$

② $x \in [1, 5]$

$f(1) = 2$
$f(5) = 6$
$f(2.5) = -\frac{1}{4}$

$\boxed{y \in [-\frac{1}{4}, 6]}$

③ $x \in [3, 4]$

$f(3) = 0$
$f(4) = 16 - 20 + 6$
$= 2$

$\boxed{y \in [0, 2]}$

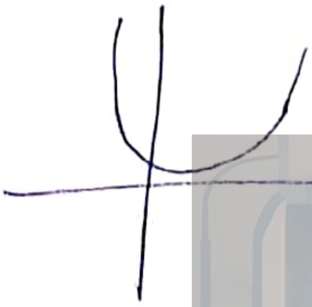
Commenting of Sign of a, b, c

① $y = ax^2 + bx + c$

$a > 0$ concave up

$a < 0$ concave down

② $c \to$ cut on y-axis



c ⊕ve

c ⊖ve

③ $b \to$ no fixed rule

Q Comment on the signs of a, b, c

$a = \oplus ve \to$ concave up

$c = \ominus ve \to$ cut y-axis below x-axis.

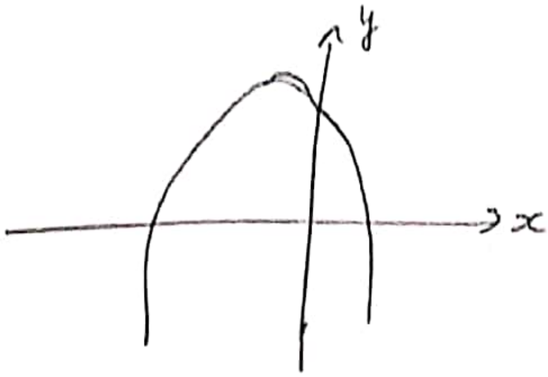
$\frac{-b}{2a} < 0$

⊕v so $-b = \ominus ve$

$\boxed{b = \oplus ve}$

Q Comment on a, b, c signs

(1)



$\boxed{a = \ominus \text{ve}}$

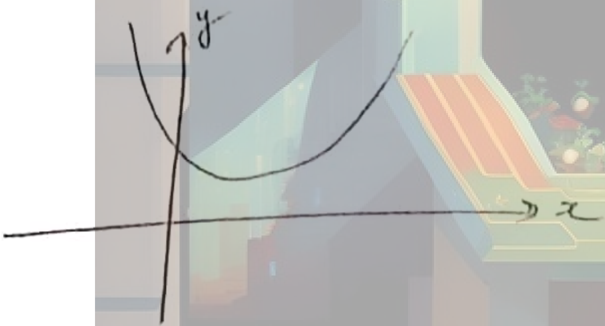
$\boxed{c = \oplus \text{ve}}$

$-\dfrac{b}{2a} = \ominus \text{ve}$

$2a \rightarrow a$ is $\ominus$ve

$\boxed{b = \ominus \text{ve}}$ $\boxed{b = \ominus \text{ve}}$

(2)

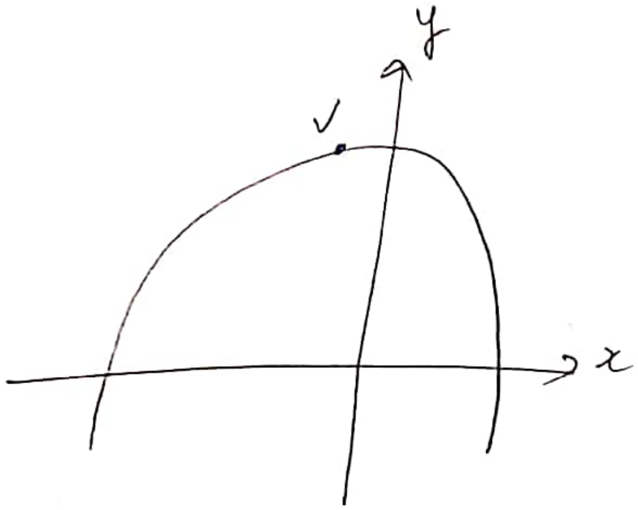


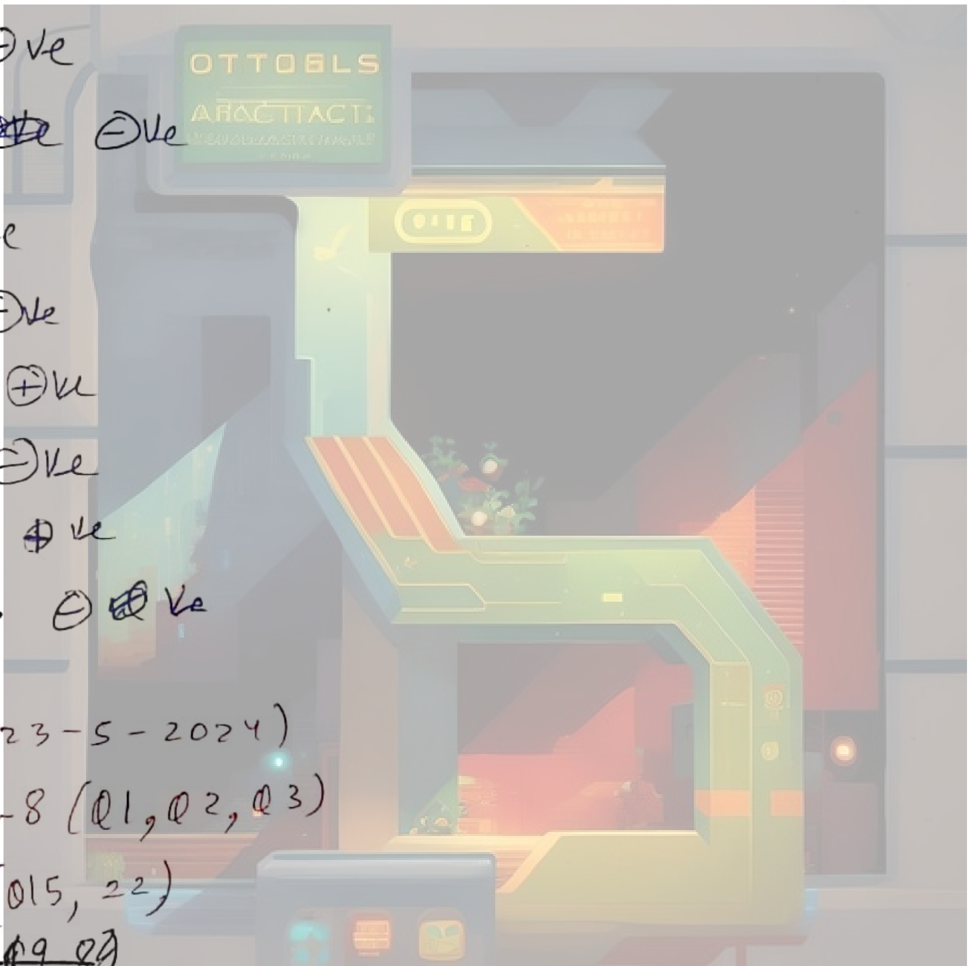
$a = \oplus$ ve

$c = \oplus$ ve

$b = \ominus$ ve

Q





i) a ⊖ve

ii) b ~~⊕ve~~ ⊖ve

iii) c ⊕ve

iv) bc ⊖ve

v) $c-a$ ⊕ve

vi) ab^2 ⊖ve

vii) abc ⊕ve

viii) $\frac{a+b}{c}$ ⊖ ~~⊕~~ ve

H.W. (23-5-2024)

~~O-1~~ DYS-8 (Q1, Q2, Q3)

O-1 (Q15, 22)

~~O-2 (Q9, 8)~~,

O-2 (Q8, 9, 11, 16, 17, 18, 19,)

J-M (Q2, 3, 4, 5, 6, 7, 14)

Q $y = ax^2 + bx + c$, & $c < 0$ does not have any real roots then comment on the signs of - $a < 0, bc < 0$

(A) $c(a+b+c)$

(B) $c(a - b + c)$

(C) $4a + 2b + c$

$\frac{-D}{4a} < 0$

(A) For $x = 1$

$y = a + b + c$

when $y < 0$

$a + b + c < 0$

$c < 0$

$c(a + b + c) = \boxed{\oplus ve}$

$\frac{-b}{a} > 0$

(B) for $x = -1$

$y = \ominus ve$

$c(\ominus ve)$

$\ominus(\ominus ve)$

$\boxed{\oplus ve}$

(C) $x = 2$

$\boxed{y = 4 \ominus ve}$

Q If $c < 0$ & $y = ax^2 + bx + c$ has no real roots find signs.

$c < 0, a < 0$

(A) $9a + 3b + c = y$

for $x = 3$

$\boxed{y = \ominus ve}$

(B) $a + 2b + 4c$

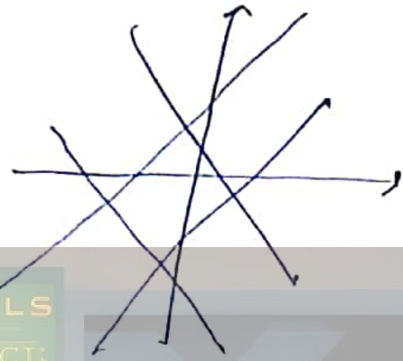
$\ominus ve$ for $x = 1/2$

Range of Linear, $\frac{\text{Linear}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Linear}}$ & $\frac{\text{Quad}}{\text{Quad}}$

① Linear –

$y = ax+b$

$y \in R$ always



eg. $y = 2x+3$

Range – $(-\infty, \infty)$

$y = \sqrt{2}x - \frac{7}{2}$

Range $= (-\infty, \infty)$

② $\frac{\text{Linear}}{\text{Linear}}$ – $y = \frac{ax+b}{cx+d}$

Range $\rightarrow y \in R - \left\{\frac{a}{c}\right\}$

eg – $y = \frac{2x-3}{x+2}$

$y \in R - \left\{\frac{2}{1}\right\}$

$y \in R - \{2\}$

$y \in (-\infty, 2) \cup (2, \infty)$

Q find ranges

① $y = \frac{3x+1}{2x-1}$

$y \in R - \left\{\frac{3}{2}\right\}$

② $y = \frac{x}{2x-1}$ $\cancel{-\left\{\frac{1}{2}\right\}}$

$y \in R - \left\{\frac{1}{2}\right\}$

③ $y = \frac{5x-2}{7-4x}$

$y \in R - \left\{\frac{5}{-4}\right\}$

④ $y = \frac{1}{3x+4}$

$y \in R - \{0\}$

③ $\frac{\text{Quad}}{\text{Linear}}$, $\frac{1}{\text{Quad}}$, $\frac{\text{Quad}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Quad}}$

Process:- Do cross Multiply

Case 1:- when leading cofficient $\neq 0$ then apply $D \geq 0$ & solve inequality in y.

Case 2: when leading coffecient $= 0$, If any value of x is comming then no problem. Otherwise exclude.

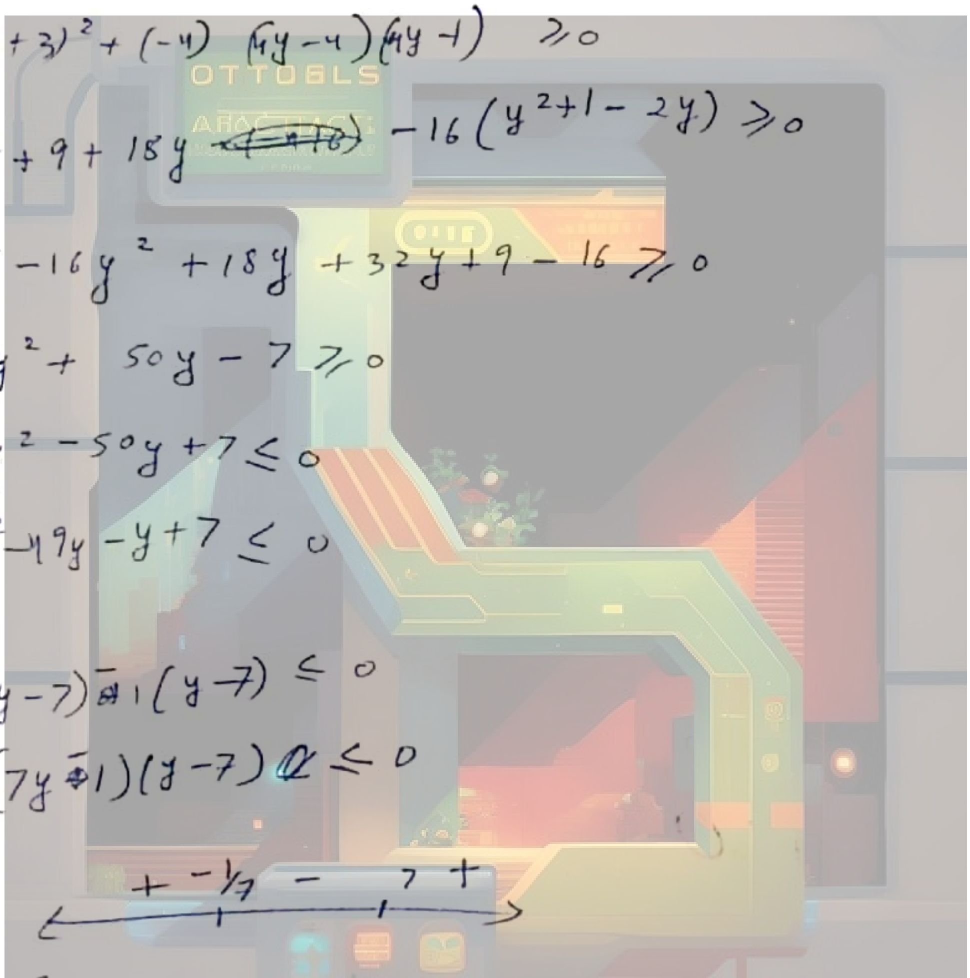
Q find range of $y = \dfrac{x^2-3x+4}{x^2+3x+4}$ $\quad x \in R$

$yx^2+3yx+4y = x^2-3x+4$

$(y-1)x^2 + (3y+3)x + (4y-4)$

Case 1 $\quad y-1 \neq 0$

$D \geq 0$



$(3y+3)^2 + (-4)(4y-4)(4y-1) \geq 0$

$9y^2+9+18y - 16(y^2+1-2y) \geq 0$

$9y^2-16y^2+18y+32y+9-16 \geq 0$

$-7y^2+50y-7 \geq 0$

$7y^2-50y+7 \leq 0$

$7y^2-49y-y+7 \leq 0$

$7y(y-7)-1(y-7) \leq 0$

$(7y-1)(y-7) \leq 0$

$+ \quad -\frac{1}{7} \quad - \quad 7 \quad +$

$y \in \left[\frac{1}{7}, 7\right]$

Case 2:- $y-1=0$

$y=1$

Put in Question

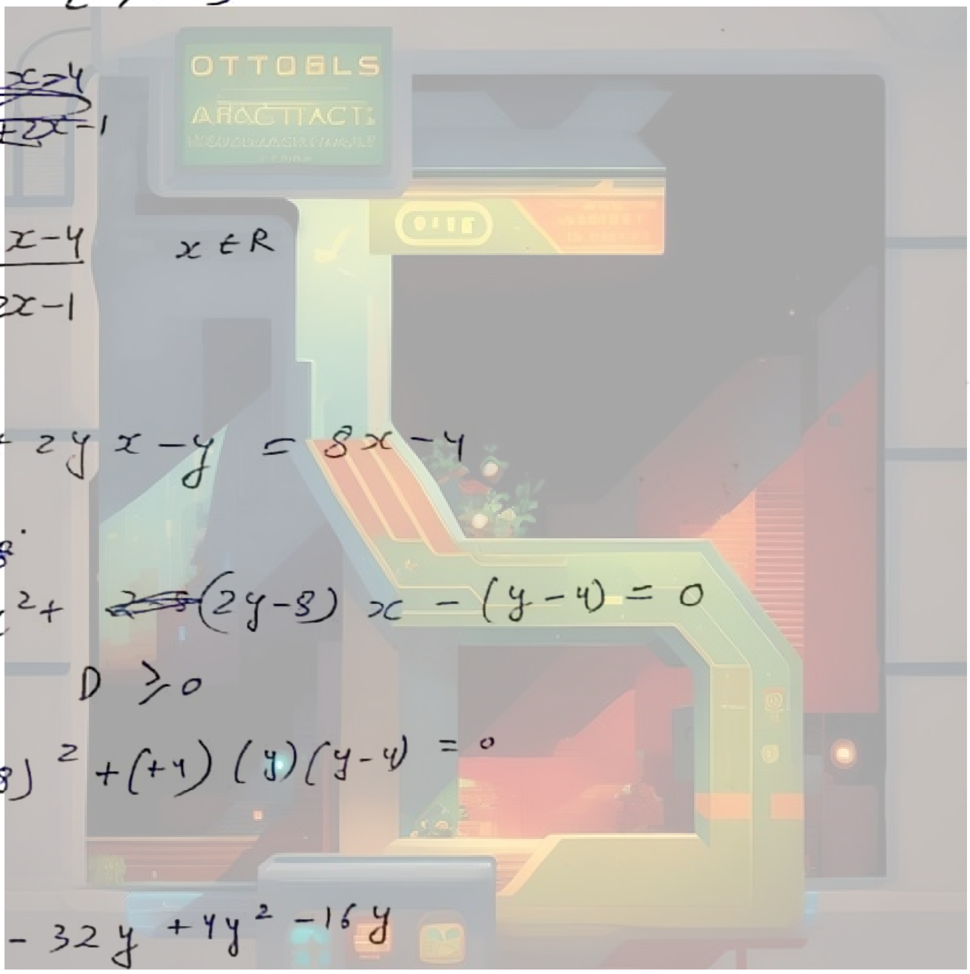
$\cancel{x^2}+3x+4-\cancel{x^2}+3x-4=0$

$6x=0$

$x=0$

$\therefore$ Value of x coming in Case 2 so need to exclude $y=1$

Hence, $y\in\left[\frac{1}{7},7\right]$



~~Q. $y=\frac{8x-4}{x^2+2x-1}$~~

Q) $y=\frac{8x-4}{x^2+2x-1}$ $\quad x\in R$

Case 1:-

$yx^2+2yx-y=8x-4$

$yx^2+(2y-8)x-(y-4)=0$

$D\geq0$

$(2y-8)^2+(+4)(y)(y-4)=0$

$4y^2+64-32y+4y^2-16y$

$8y^2-48y+64\geq0$

$y^2-6y+8\geq0$

$y^2-4y-2y+8\geq0$

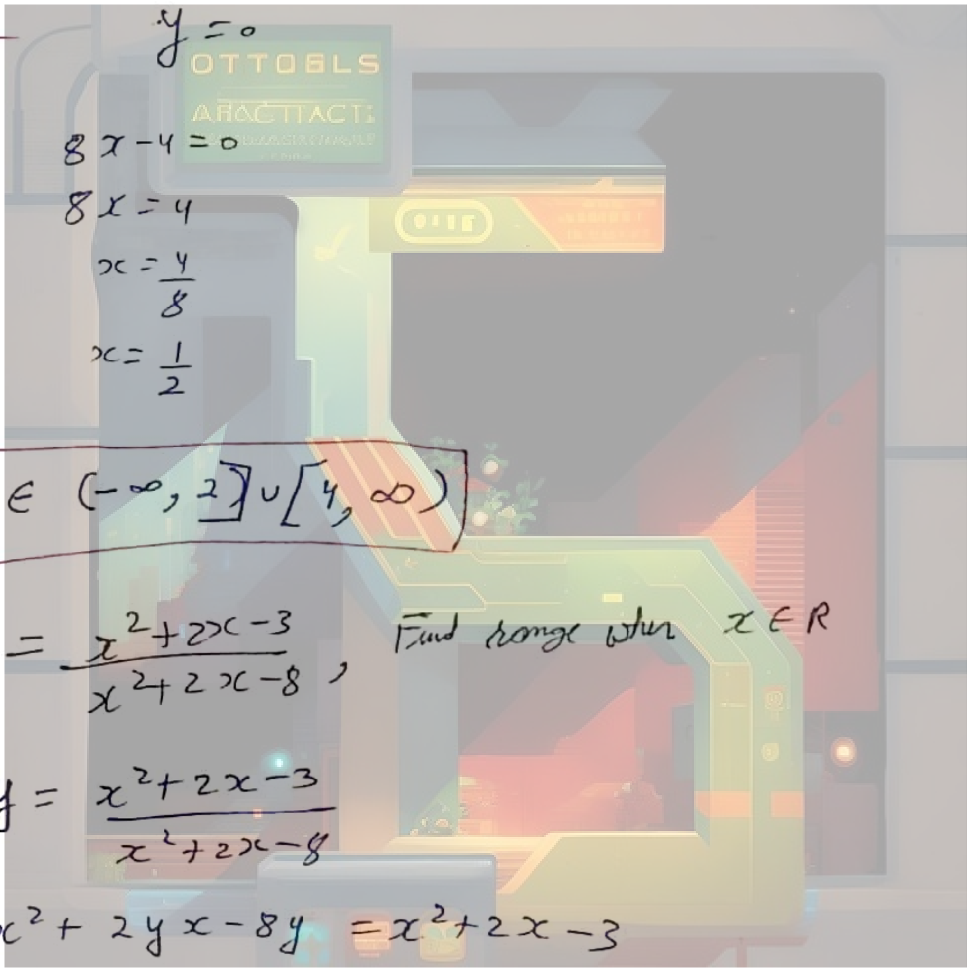
$y(y-4)-2(y-4) \geq 0$

$(y-2)(y-4) \geq 0$

$+ \quad 2 \quad - \quad 4 \quad +$

$y \in (-\infty, 2] \cup [4, \infty)$

Case 2



$y=0$

$8x-4=0$

$8x=4$

$x=\frac{4}{8}$

$x=\frac{1}{2}$

$$\boxed{y \in (-\infty, 2] \cup [4, \infty)}$$

Q $y=\frac{x^2+2x-3}{x^2+2x-8}$, Find range when $x \in R$

$y=\frac{x^2+2x-3}{x^2+2x-8}$

$yx^2+2yx-8y=x^2+2x-3$

$yx^2-x^2+2yx-2x-8y+3=0$

$(y-1)x^2+(2y-2)x-(8y-3)=0$

Case 1

$D \geq 0$

$b^2 - 4ac \geq 0$

$(2y-2)^2 + (+4)(y-1)(8y-3)$

$4y^2 + 4 - 8y + 4(8y^2 - 3y - 8y + 3)$

$4y^2 + 4 - 8y + 32y^2 - 44y + 12$

$36y^2 - 52y + 16 = 0$

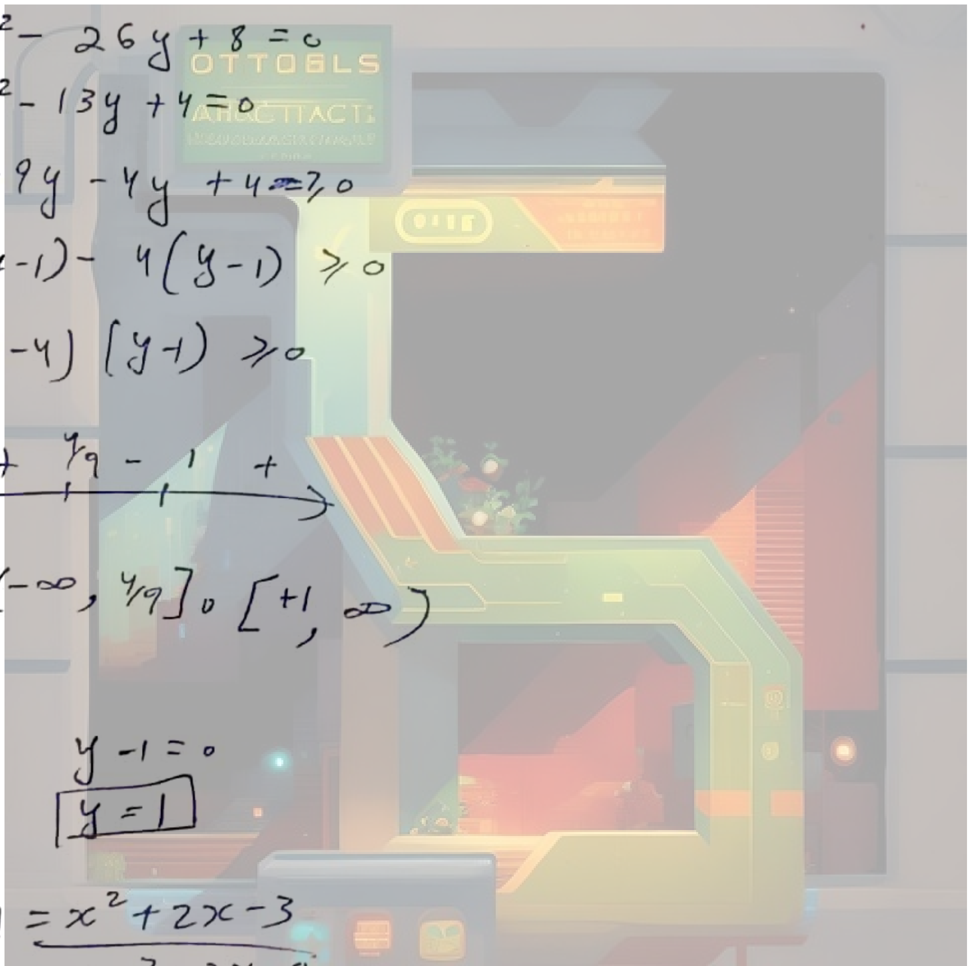
$18y^2 - 26y + 8 = 0$

$9y^2 - 13y + 4 = 0$

$9y^2 - 9y - 4y + 4 \geq 0$

$9y(y-1) - 4(y-1) \geq 0$

$(9y-4)(y-1) \geq 0$



$+ \quad 4/9 \quad - \quad 1 \quad +$

$y \in (-\infty, 4/9] \cup [+1, \infty)$

Case 2

$y - 1 = 0$

$\boxed{y = 1}$

$1 = \dfrac{x^2 + 2x - 3}{x^2 + 2x - 8}$

$\cancel{x^2} + \cancel{2x} - 8 = \cancel{x^2} + \cancel{2x} - 3$

$-8 = -3$

so exclude $\boxed{y = 1}$

$$\boxed{y \in (-\infty, 4/9] \cup (1, \infty)}$$

H.W. 24-05-2024

DYS-9 [All] ✓

O-1 {23}

~~O-2 {~~

J-M {1, 13}

J-A {3, 4}

O-4 {1} ✓

O-3 {7, 8, 1, 2, 3}

~~O-2 {~~

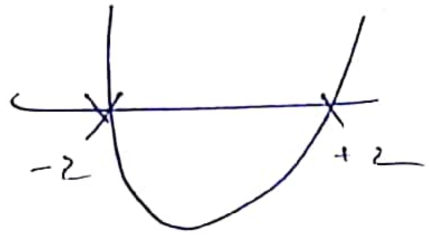
O-2 {7} ✓

Location of roots (Real nos.) (Type 1)

① Roots are equal in magnitude & opposite in sign.

$\boxed{b=0 \ \ \& \ D>0}$

eg.



$(x-2)(x+2)=0$

$x^2-4=0$

↳ Hence $b=0$

② Only 1 root is always zero

$\boxed{c=0}$

let ~~α~~ roots $\alpha, 0$

~~③ Roots are is~~

$(x-\alpha)(x-0)=0$

$x^2-\alpha x=0$

↳ Hence $c=0$

③ Roots are reciprocal to each other.

$\boxed{a=c \ \ \& \ D\geq 0}$

$(x-\alpha)\left(x-\frac{1}{\alpha}\right)=0$

$x^2-x\left(\alpha+\frac{1}{\alpha}\right)+1=0$

$\boxed{a=c}$

④ Roots are of opposite sign

$\boxed{POR < 0 \text{ \& } D > 0}$

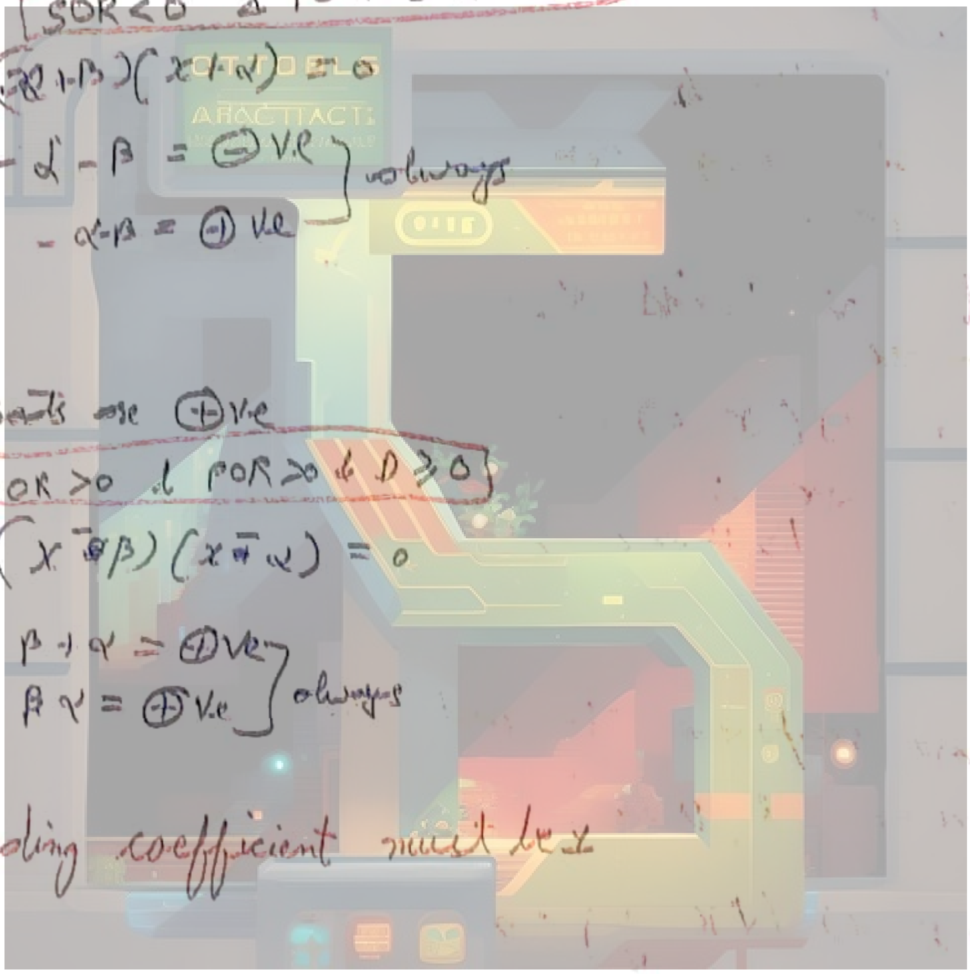
$(x-\alpha)(x+\beta)=0$

$SOR \Rightarrow \alpha - \beta$ (any can be bigger)

$POR \Rightarrow \alpha(-\beta) = \underbrace{-\alpha\beta}_{\text{always } \ominus ve}$

⑤ Both roots are $\ominus$ve

$\boxed{SOR < 0 \text{ \& } POR > 0 \text{ \& } D \geq 0}$



$(x+\beta)(x+\alpha)=0$

$\left.\begin{array}{l} -\alpha-\beta = \ominus ve \\ -\alpha \cdot -\beta = \oplus ve \end{array}\right\}$ always

⑥ Both roots are $\oplus$ve

$\boxed{SOR > 0 \text{ \& } POR > 0 \text{ \& } D \geq 0}$

$(x-\beta)(x-\alpha)=0$

$\left.\begin{array}{l} \beta+\alpha = \oplus ve \\ \beta\alpha = \oplus ve \end{array}\right\}$ always

* leading coefficient must be 1

Q $f(x) = x^2 + 2(a-1)x + (a+5)$ find 'a'.

a) Roots are of opposite sign

$a + 5 < 0$

$\boxed{a < -5}$

$a \in (-\infty, -5)$

$4(a-1)^2 - 4(a+5) > 0$

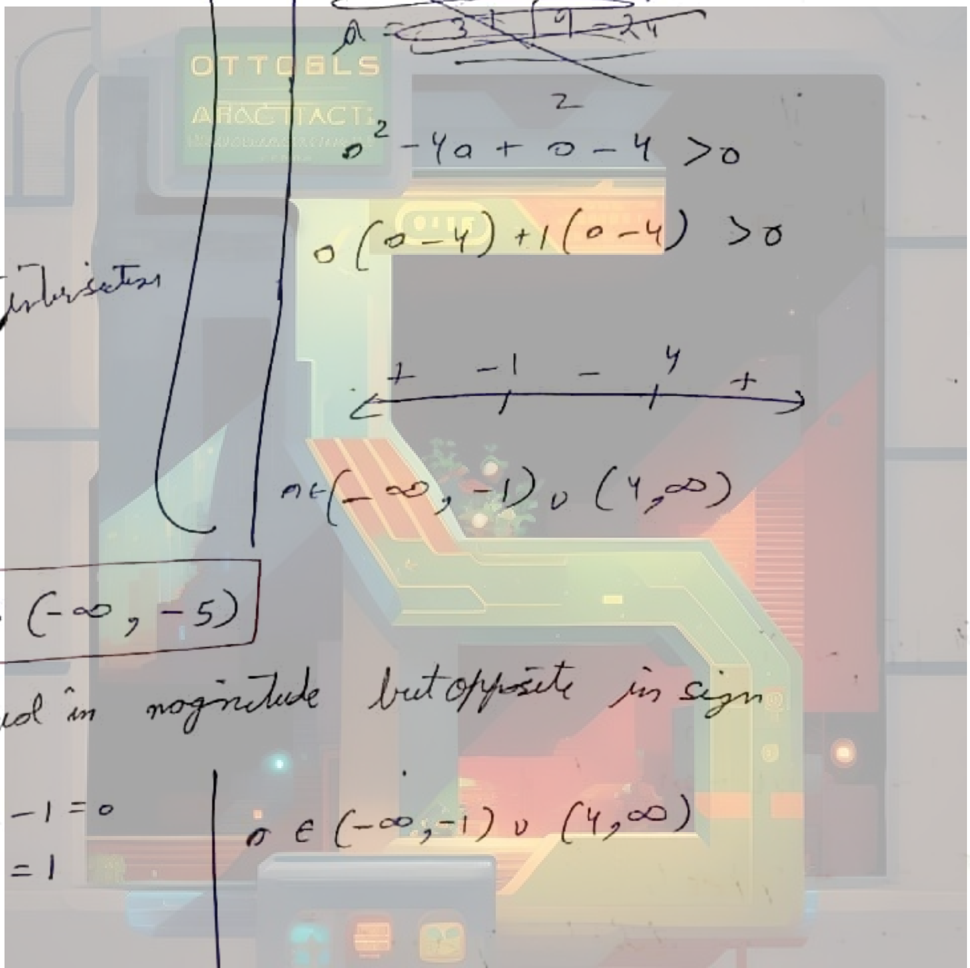
$a^2 + 1 - 2a - a - 5 > 0$

$a^2 - 3a \overset{-4}{\cancel{+6}} > 0$

~~$a^2 - 2a - a + 6 > 0$~~

~~$a(a-2) + (a$~~

~~$a = \frac{3 \pm \sqrt{9 - 24}}{2}$~~



$a^2 - 4a + a - 4 > 0$

$a(a-4) + 1(a-4) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$a \in (-\infty, -1) \cup (4, \infty)$

Intersection

$\boxed{a \in (-\infty, -5)}$

b) Roots equal in magnitude but opposite in sign

$a - 1 = 0$

$a = 1$

$a \in (-\infty, -1) \cup (4, \infty)$

$\boxed{a \in \phi}$

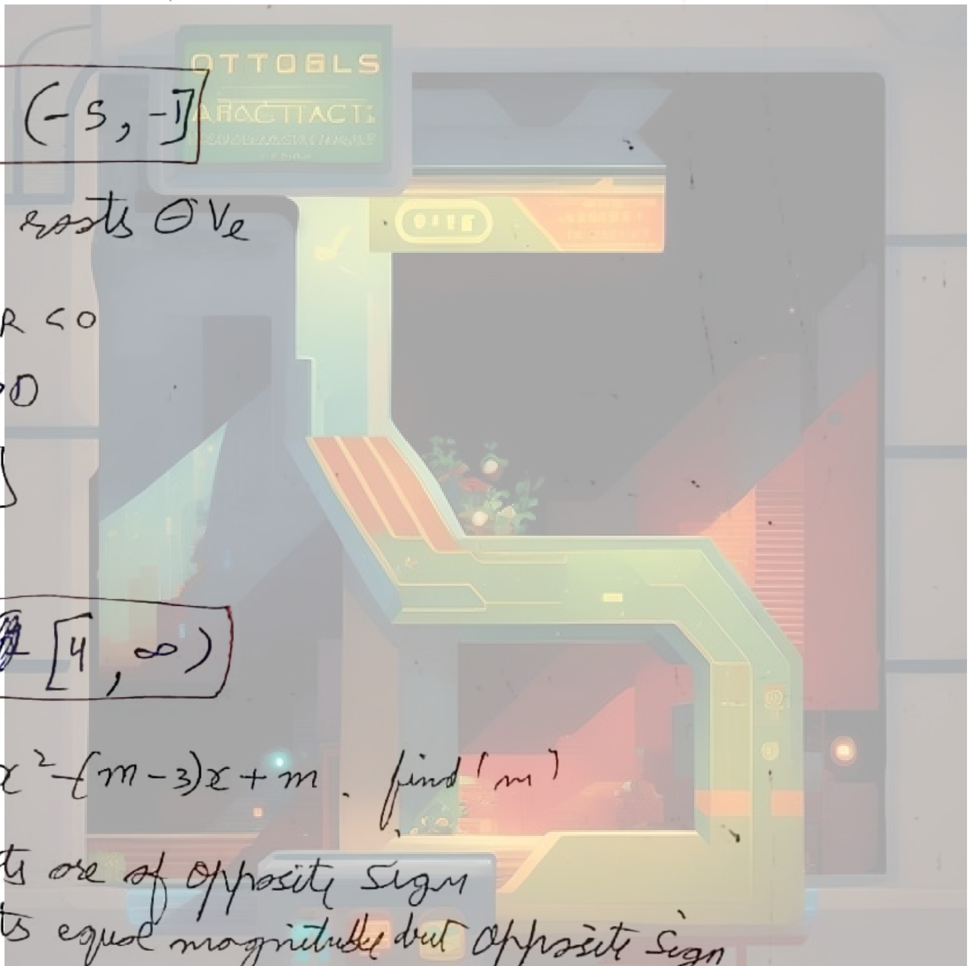
c) Both roots $\oplus$ ve

$SOR > 0$

$POR > 0$

$D \geq 0$

SOR $2(1-a) > 0$	$a + 5 > 0$
$1 - a > 0$	$\boxed{a > -5}$
$\boxed{a < 1}$	$a \in (-\infty, -1] \cup [4, \infty)$



$\boxed{a \in (-5, -1]}$

d) Both roots $\ominus$ ve

$SOR < 0$

$a - 1 > 0$

$\boxed{a > 1}$

$\boxed{a \in [4, \infty)}$

Q $f(x) = x^2 - (m-3)x + m$ find 'm'

a) Roots are of opposite sign

b) roots equal magnitude but opposite sign

c) Both roots are $\oplus$ ve

d) Both roots are $\ominus$ ve

$x^2 - (m-3)x + m$

$(m-3)^2 - 4m = D$

a) $POR < 0$

$D > 0$

$m^2 + 9 - 6m - 4m > 0$

$m^2 + 9 - 10m > 0$

$m^2 - 9m - m + 9 > 0$

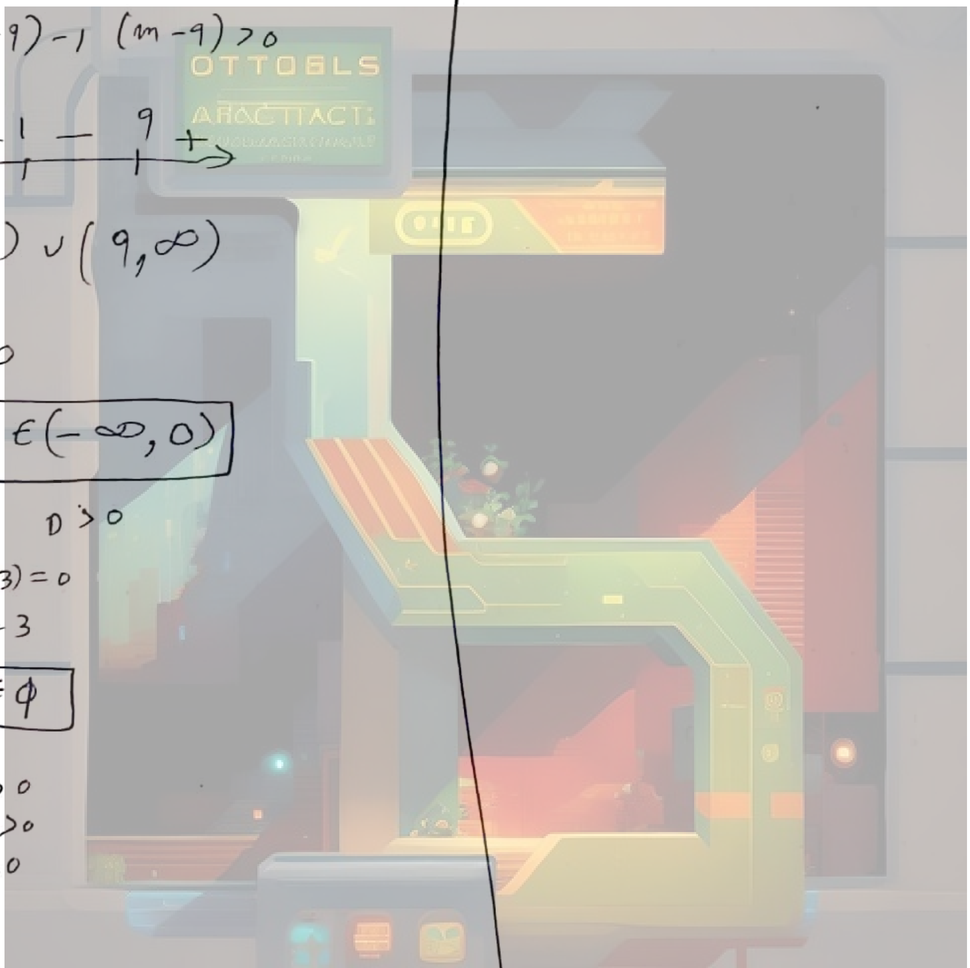
$m(m-9) - 1(m-9) > 0$

$+ \quad 1 \quad - \quad 9 \quad +$

$(-\infty, 1) \cup (9, \infty)$

$m < 0$

$\boxed{m \in (-\infty, 0)}$



b) $b = 0 \quad D > 0$

$-(m-3) = 0$

$m = 3$

$\boxed{m \in \phi}$

c) $SOR > 0$

$POR > 0$

$D \geq 0$

$\underline{m > 0}$

$m - 3 > 0$

$\underline{m > 3}$

$\boxed{m \in [9, \infty)}$

d) $m > 0$

$m - 3 < 0$

$m < 3$

$\boxed{m \in (0, 1]}$

Q. If $f(x) = 3x^2 - 5x + p$ & $f(0)$ & $f(1)$ are of opposite signs find P.

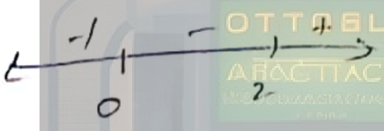
p $\quad \cancel{3-5+p}$

$\quad \cancel{-2+p}$

$f(0) \cdot f(1) < 0$

$p(3-5+p) < 0$

$p(p-2) < 0$



$P \in (0, 2)$

Location of Roots Type-2

$f(x) = ax^2 + bx + c$

$\Rightarrow$ Leading coeff must be 1.

(1) Both ~~the~~ roots of a quad are greater than a number 'd'.

$$\boxed{\begin{array}{l} D \geq 0 \\ f(d) > 0 \\ d < \frac{-b}{2a} \end{array}}$$

Q) find "λ" for both roots of quadratic $x^2-6\lambda x+9\lambda^2-2\lambda+2=0$ are greater than 3

$36\lambda^2 - \cancel{36\lambda^2} + 8\lambda - 8 \geq 0$

$8(\lambda-1) \geq 0$

$\lambda - 1 \geq 0$

$\boxed{\lambda \geq 1}$

$9 - 18\lambda + 9\lambda^2 - 2\lambda + 2 > 0$

$9\lambda^2 - 20\lambda + 11 > 0$

$9\lambda^2 - 11\lambda - 9\lambda + 11 > 0$

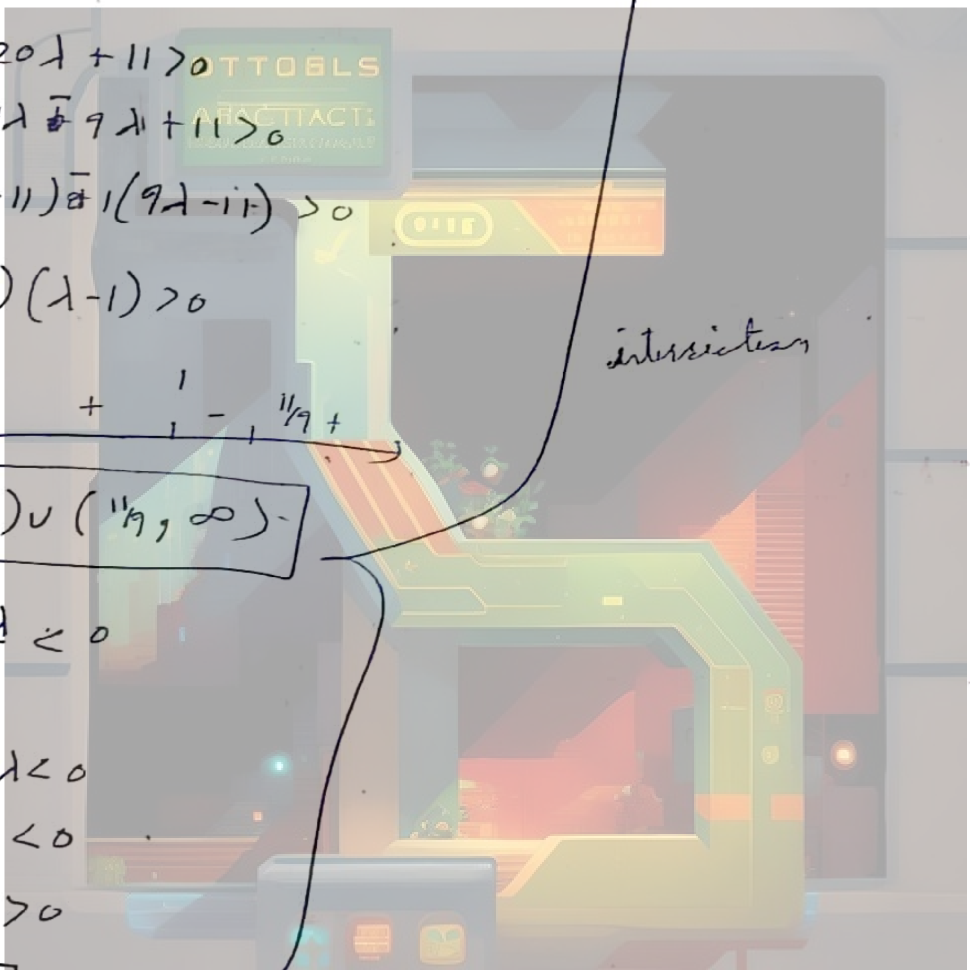
$\lambda(9\lambda - 11) - 1(9\lambda - 11) > 0$

$(9\lambda - 11)(\lambda - 1) > 0$

intersection

$+ \quad 1 \quad - \quad 11/9 \quad +$

$\boxed{\lambda \in (-\infty, 1) \cup (11/9, \infty)}$



$3 + \frac{-6\lambda}{2} < 0$

$6 - 6\lambda < 0$

$(1 - \lambda) < 0$

$\lambda - 1 > 0$

$\boxed{\lambda > 1}$

$\boxed{\lambda \in (11/9, \infty)}$

Q find 'K' for both the roots of the quadratic

$(k+1)x^2 - 3kx + 4k = 0$ are greater than 1

$x^2 - \frac{3k}{k+1}x + \frac{4k}{k+1} = 0$

$\boxed{k \neq -1}$

$D \geq 0$

$\cancel{\frac{9k^2}{k^2+1+2k}}$

$\left(\frac{3k}{k+1}\right)^2 - 4\left(\frac{4k}{k+1}\right) \geq 0$

$\frac{\cancel{9k}\ 9k^2 - 16k^2 - 16k}{k^2+1+2k} \geq 0$

$\frac{7k^2+16k}{k^2+2k+1} \leq 0$

$\frac{k(7k+16)}{(k+1)^2} \leq 0$

$-\frac{16}{7}$, -1, 0 ; $+$ $-$ $-$ $+$

$\boxed{\left[-\frac{16}{7}, -1\right) \cup (-1, 0]}$

$(k+1) - 3k + 4 = 0$

$k+1-3k+4=0$

$\cancel{-2k+5=0}$ $\quad 2k < 1$

$\boxed{k < \frac{1}{2}}$

$\cancel{2k-5<0}$

$\cancel{\boxed{k < \frac{5}{2}}}$

$\cancel{1 + \left(\frac{-3k}{k+1}\right) \times \frac{1}{2} \times \frac{k+1}{4k} \neq 0}$

$\cancel{1 + \frac{(-3k^2-3k)}{8k^2+8} < 0}$

$\cancel{\frac{8k^2+8-3k^2-3k}{8(k^2+1)} < 0}$

$\cancel{\frac{5k^2-3k+8}{k^2+1} < 0}$

$\cancel{5k^2-8k-}$

(36)

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$$\frac{-\left(\frac{-3k}{k+1}\right)}{2} > 1$$

$$\frac{3k}{2(k+1)} - 1 > 0$$

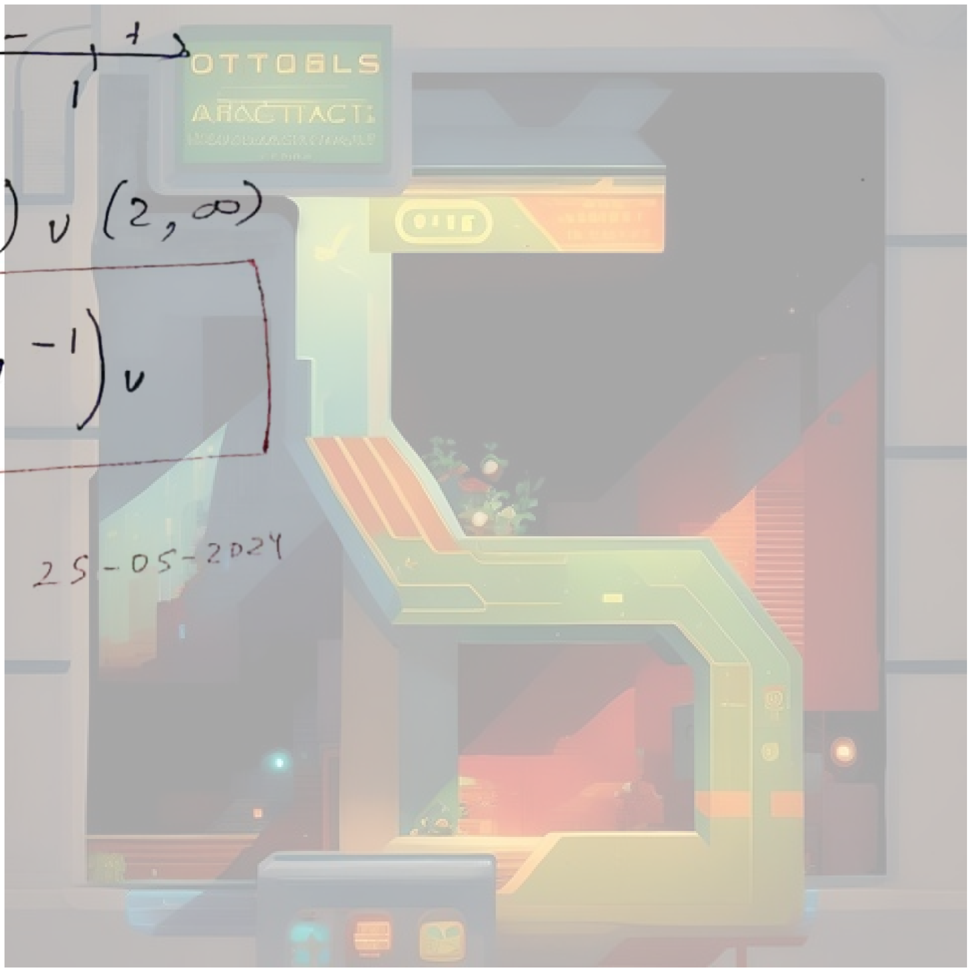
$$\frac{k-1}{k+1} > 0$$

$+ \quad -1 \quad - \quad 1 \quad +$

$$(-\infty,-1) \cup (2,\infty)$$

$$\boxed{\left[\frac{16}{7}, -1\right) \cup}$$

H.W. 25-05-2024

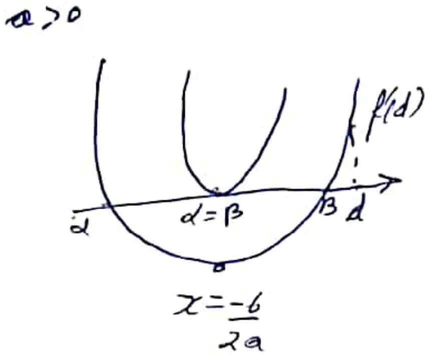


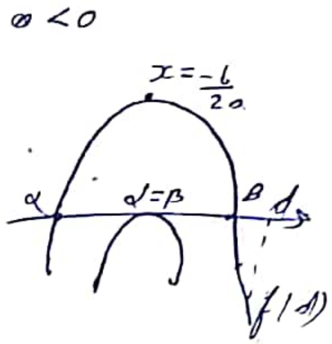
Type (2)

(2) Both roots are less than any specific number 'd'.

intersection

$$\boxed{\begin{array}{l} D \geq 0 \\ \frac{-b}{2a} < d \\ af(d) > 0 \end{array}}$$

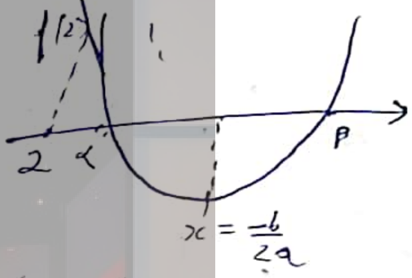




Q let $x^2-(m-3)x+m=0$ $(m \in R)$ be a quadratic equation find the value of m for which.

(1) both roots are greater than 2.

(2) both the roots are smaller than 2.



(2) $x^2-(m-3)x+m=0$

(1) $D = (m-3)^2 - 4m$

$= m^2+9-6m-4m$

$= m^2+9-10m$

~~$= m^2+10-$~~

$= m^2-9m-m+9$

$= m(m-9)-1(m-9)$

$= (m-1)(m-9) \geq 0$

$+ \quad 1 \quad - \quad 9 \quad +$ ...(1)

$\boxed{m \in (-\infty, 1] \cup [9, \infty)}$

$\frac{m-3}{2} < 2$

$m-3 < 4$ ...(2)

$\boxed{m < 7}$

$4-(m-3)\times 2 + m > 0$

$4-2m+6+m > 0$

$10-m > 0$ ...(3)

$\boxed{m < 10}$

(1)∩(2)∩(3)

$\boxed{m \in (-\infty, 1]}$ ✓

(2) $\frac{m-3}{2} > 2$

$m-3 > 4$ ...(1)

$\boxed{m > 7}$

$\boxed{m < 10}$

(1)∩(2)∩(3)

$\boxed{m \in [9, 10)}$ ✓

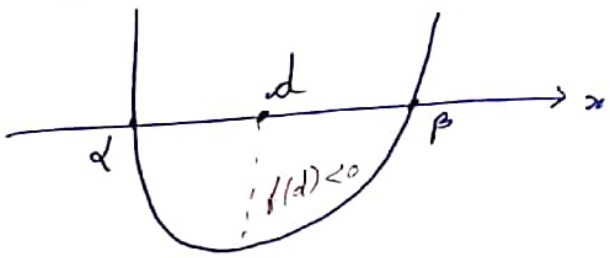
Type ③

(B) Both roots lie on either side of specific no. d

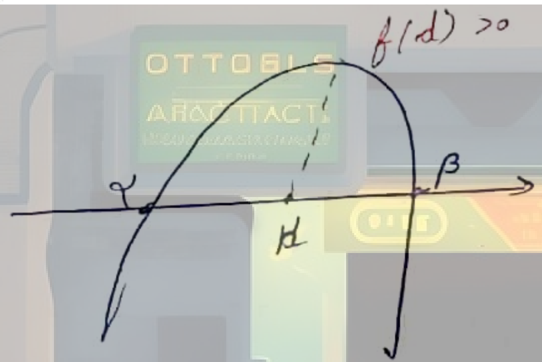
→ One root is greater than D & one is less

→ Specific no. d lies between the roots

$a > 0$



$a < 0$



$\boxed{a \cdot f(d) < 0}$

$\left.\begin{array}{l} a > 0,\ f(d) < 0 \\ a < 0,\ f(d) > 0 \end{array}\right\}$ so $\underline{af(d) < 0}$

Q find all possible values of a for which exactly one root of $x^2 - (a+1)x + 2a = 0$ lie in interval $(0, 3)$.

$f(0) = 2a$

$f(3) = 9 - 3a + 3 + 2a$

$= 6 - a$

$f(0) \cdot f(3) < 0$

$a(6 - a) < 0$

$\xleftarrow{\quad - \quad} 0 \xrightarrow{\quad + \quad} 6 \xrightarrow{\quad - \quad}$

$\underline{(-\infty, 0) \cup (6, \infty)}$

Check at the points of interval

put $a = 0$

$x^2 - x = 0$

$x = 1, 0$ → 1 lie ~~between~~ in interval

So include 0 in answer

put $a = 3$

$D < 0$ so no real roots

$\boxed{a \in (-\infty, 0] \cup (6, \infty)}$

Q find 'K' for which one root of equation $x^2 - (K+1)x + K^2 + K - 8 = 0$
is ~~...~~ greator than 2 & other is less than 2.

$d = 2$

$f(2) = 4 - (K+1)2 + K^2 + K - 8 = 0$

$= 4 - 2K - 2 + K^2 + K - 8$

$= K^2 - K - 6 < 0$

$K^2 - 3K + 2K - 6 < 0$

$K(K-3) + 2(K-3) < 0$

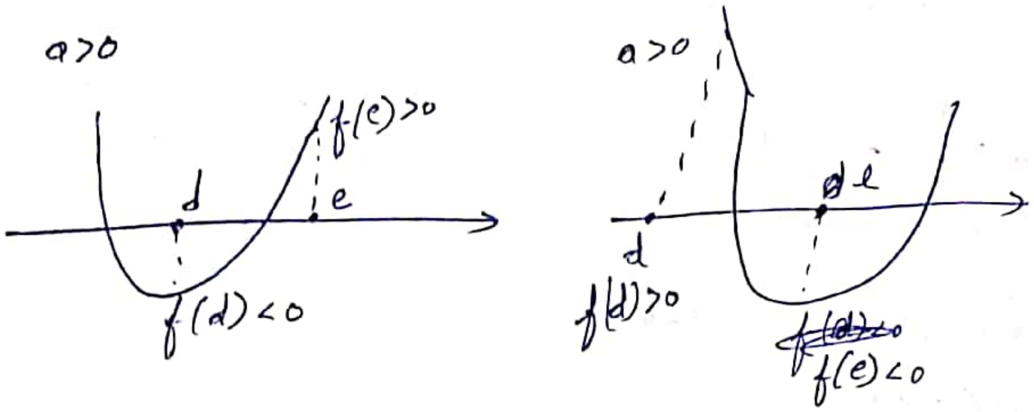
$(K+2)(K-3) < 0$

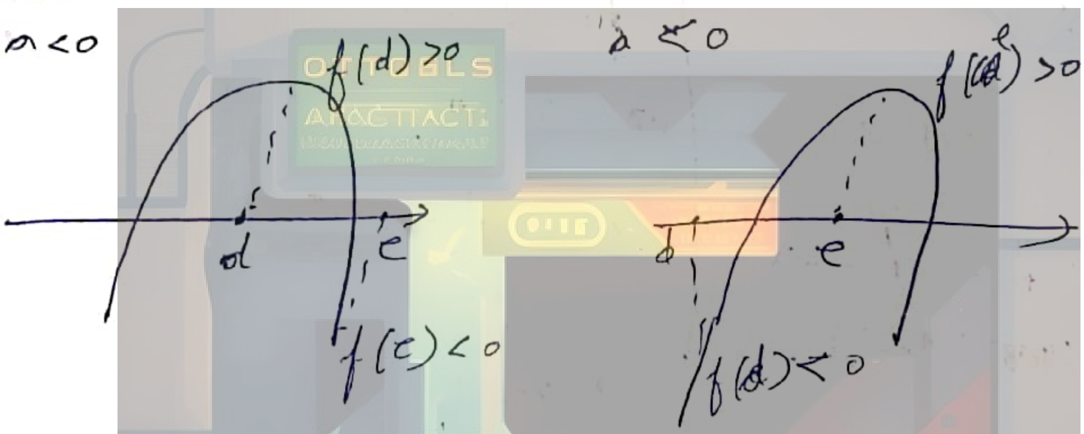
$+ \quad -2 \quad - \quad 3 \quad +$

$\boxed{K \in (-2, 3)}$

② Exactly one root lies in $(d<e)$ the interval (d,e)

Type-4





So $f(d)\cdot f(e) < 0$

$$\boxed{f(d)\cdot f(e) < 0}$$

Note:- Check at the extreme points of Interval.

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Q8. $(c-5)x^2-2cx+(c-4)=0$

$-2x^2-6x-1$

$2x^2+6x+1$

$f(0) = c-4$

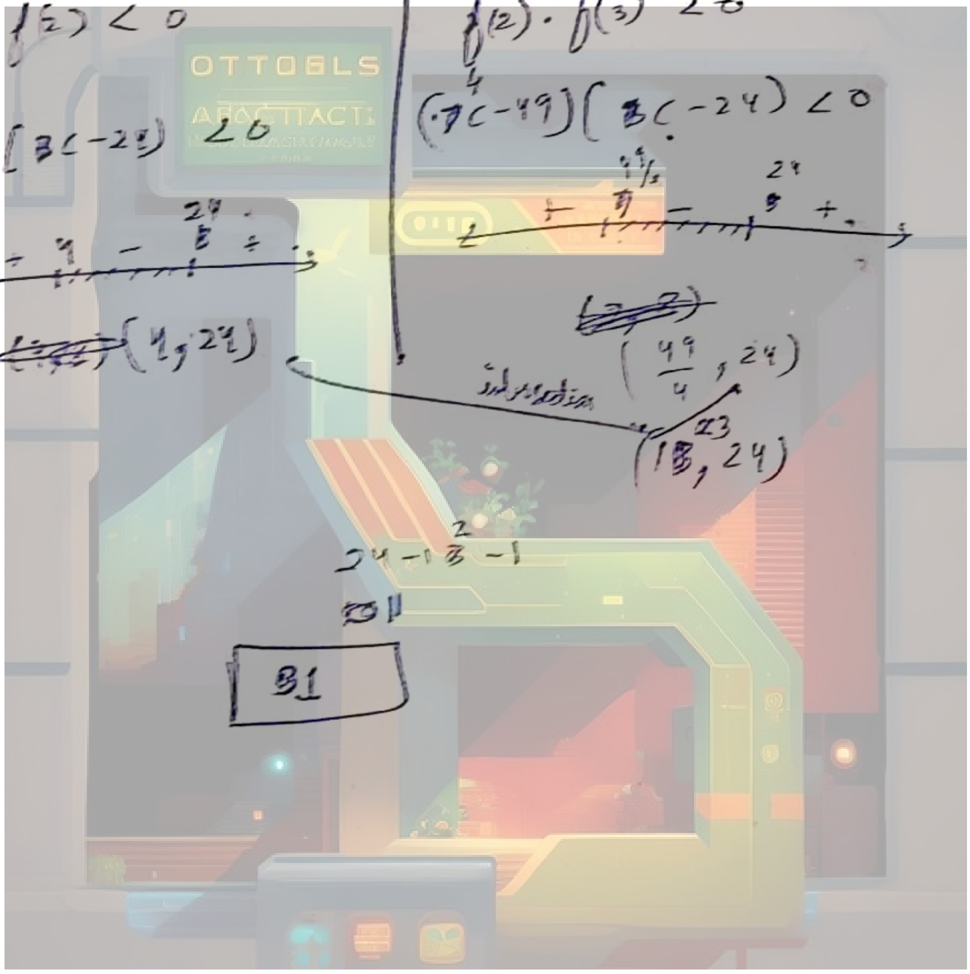
$f(2) = 4(c-5)-2c+c-4=0$

$f(2) = 4c-20-4c+c-4$

$= c-24$

$f(3) = 9c-45-6c+c-4$

$= 4c-49$



$f(0)\cdot f(2) < 0$

$(c-4)(c-24) < 0$

$+ \quad 4 \quad - \quad 24 \quad +$

$(4, 24)$

$f(2)\cdot f(3) < 0$

$(4c-49)(c-24) < 0$

$+ \quad \frac{49}{4} \quad - \quad 24 \quad +$

$\left(\frac{49}{4}, 24\right)$

intersection

$(13, 24)$ 23

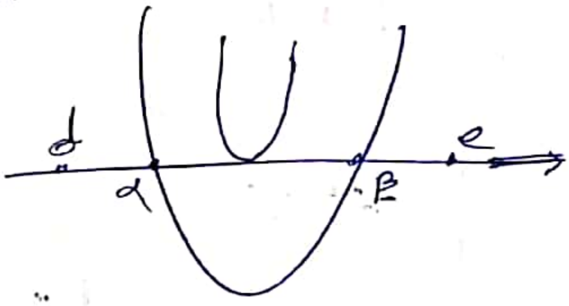
$24-13-1$

$= 11$

$\boxed{11}$

Type - 5 Both the roots lie between number d & e ($d<e$)

$a>0$



$a<0$

Union of ① & ③

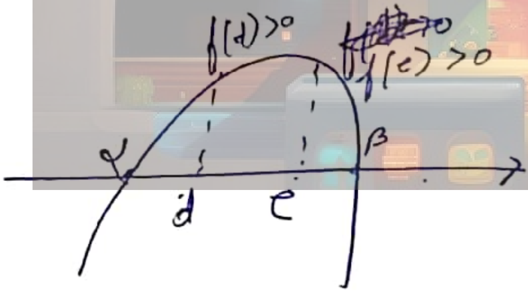
① $D \geq 0$

② $d < \frac{-b}{2a} < e$

③ $a f(d) > 0$

④ $a f(e) > 0$

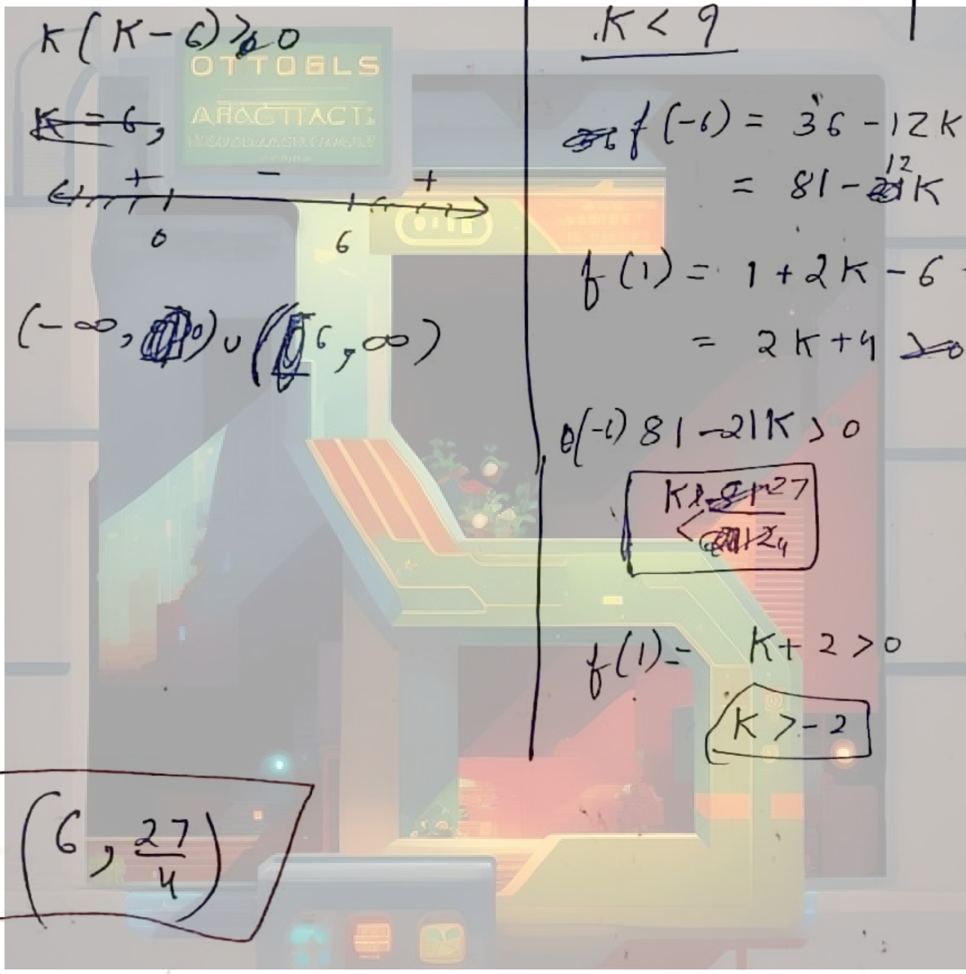
Type - ⑥ one root is greater than e & one is less than d



[$a f(d) < 0$

$a f(e) < 0$]

Q If α, β are the roots of $x^2 + 2(K-3)x + 9 = 0$ if α, β belongs to $(-6, 1)$ find K.



$4(K-3)^2 - 36 \geq 0$

$4K^2 + 36 - 24K - 36 \geq 0$

$K^2 - 6K \geq 0$

$K(K-6) \geq 0$

$\cancel{K = 6},$

$+ \quad - \quad +$ (0, 6)

$K \in (-\infty, 0) \cup (6, \infty)$

$\frac{-2K+6}{2}$

$3 - K$

$-6 < 3 - K$

$K < 9$

$3 - K > 1$

$K > 2$

$f(-6) = 36 - 12K + 36 + 9 = 0$

$= 81 - 12K > 0$

$f(1) = 1 + 2K - 6 + 9$

$= 2K + 4 > 0$

$f(-6)\ 81 - 21K > 0$

$\boxed{K < \frac{27}{4}}$

$f(1) = K + 2 > 0$

$\boxed{K > -2}$

$\boxed{\left(6, \frac{27}{4}\right)}$

Summary (Location of Roots)

① Both roots greater than d. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} > d\right)$

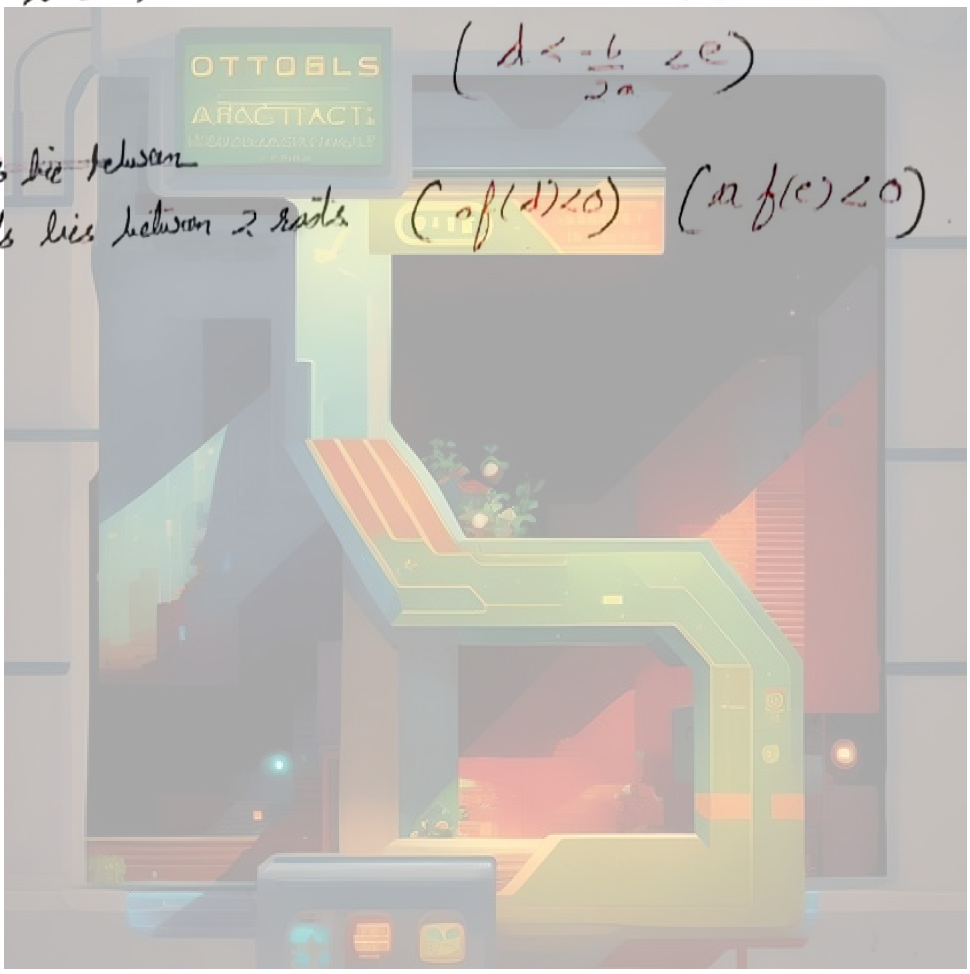
② Both roots less than d. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} < d\right)$

③ 'd' lies between the roots. $(af(d) < 0)$

④ exactly one root lies in (d, e) $(f(d) \cdot f(e) < 0)$

⑤ Both roots lie between d and e $(D \geq 0)$ $(af(d) > 0)$ $(af(e) > 0)$

$\left(d < \frac{-b}{2a} < e\right)$



~~⑥ Both roots lie between~~

⑥ Both points lies between 2 roots $(af(d) < 0)$ $(af(e) < 0)$

Irrational Inequality

① Inequalities having $\sqrt{\ }$ sign.

② Direct Squaring is not allowed without checking

Q $\sqrt{2x-5} < 3$

$2x-5 \geq 0$ [under root quantity is always ⊕)

$2x-5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$x \in \left[\frac{5}{2}, \infty\right)$ ①

$\underbrace{\sqrt{2x-5}}_{\oplus ve} < \underbrace{3}_{\oplus ve}$

Square both sides as we can square

$2x-5 < 9$

$2x < 14$

$x < 7$

$x \in (-\infty, 7)$ ②

① ∩ ②

$\boxed{\left[\frac{5}{2}, 7\right)}$

Q(46)

Q $\sqrt{x+6} < x-6$

$x+6 \geq 0$

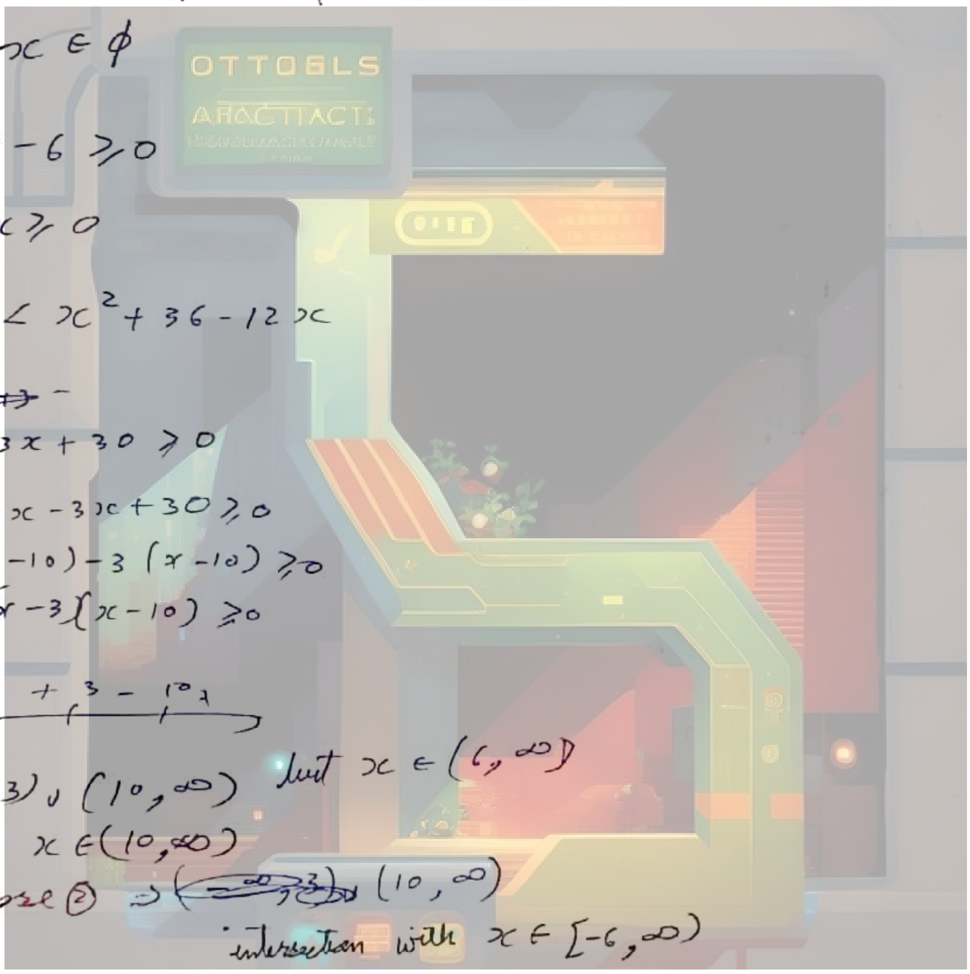
$x \geq -6$

$x \in [-6, \infty)$

Case ① $x-6 < 0$

$x < 6$

$\oplus < \ominus$ not possible



$x \in \phi$

Case ② $x-6 \geq 0$

$x \geq 0$

$x+6 < x^2+36-12x$

$\cancel{x^2+13}$ -

$x^2 - 13x + 30 > 0$

$x^2 - 10x - 3x + 30 \geq 0$

$x(x-10) - 3(x-10) \geq 0$

$(x-3)(x-10) \geq 0$

$+ \quad 3 \quad - \quad 10 \quad +$

$(-\infty, 3) \cup (10, \infty)$ but $x \in (6, \infty)$

$x \in (10, \infty)$

case ① $\cup$ case ② $\Rightarrow \cancel{(-\infty, 3)} \cup (10, \infty)$

intersection with $x \in [-6, \infty)$

$\boxed{x \in (10, \infty)}$

Q $\quad x+\cancel{6}1 \geq \sqrt{5-x}$

$\sqrt{5-x} \leq x+1$

$5-x \geq 0$

$x-5\,\cancel{0} \leq 0$

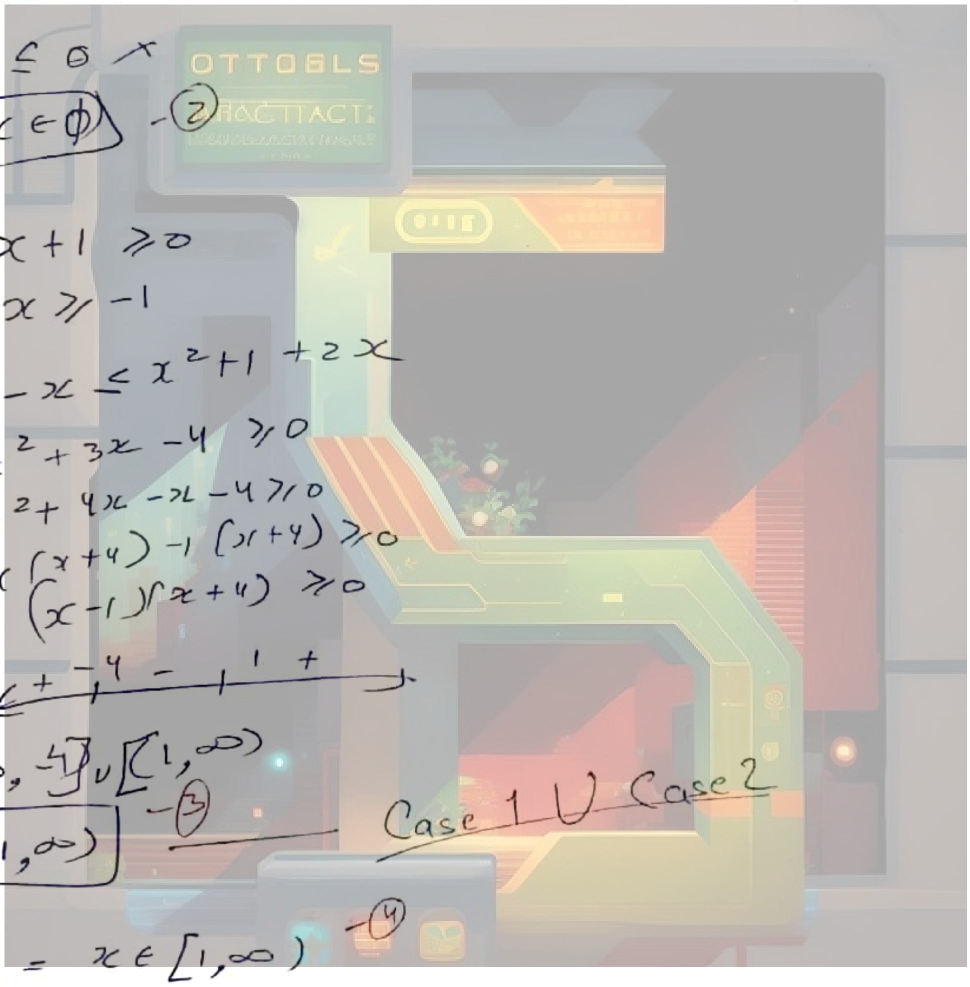
$\boxed{x \leq 5}$ $-①$

Case ① $\quad x+1\,\cancel{0} < 0$

$x < -1$

$⊕ \leq 0$ ✗

$\boxed{x \in \phi}$ $-②$



Case ② $\quad x+1 \geq 0$

$x \geq -1$

$5-x \leq x^2+1+2x$

$x^2+3x-4 \geq 0$

$x^2+4x-x-4 \geq 0$

$x(x+4)-1(x+4) \geq 0$

$(x-1)(x+4) \geq 0$

$+ \quad -4 \quad - \quad 1 \quad +$

$(-\infty,-4] \cup [1,\infty)$

$\boxed{x \in [1,\infty)}$ $-③$ $\qquad$ Case 1 $\cup$ Case 2

$②\cup③ = x \in [1,\infty)$ $-④$

$①\cap④$ $\boxed{[1,5]}$

Q $\sqrt{x+18} < 2-x$

$x+18 \ge 0$

$\boxed{x \ge -18}$

Case (1) $2-x < 0$

$-x < -2$

$x > 2$

(1) ∩ (2)

$x \in \phi$

Case (2) $2-x > 0$

$x < 2$

$x+18 < x^2+4-4x$

$x^2-5x-14 > 0$

$x^2-7x+2x-14 > 0$

$x(x-7)+2(x-7) > 0$

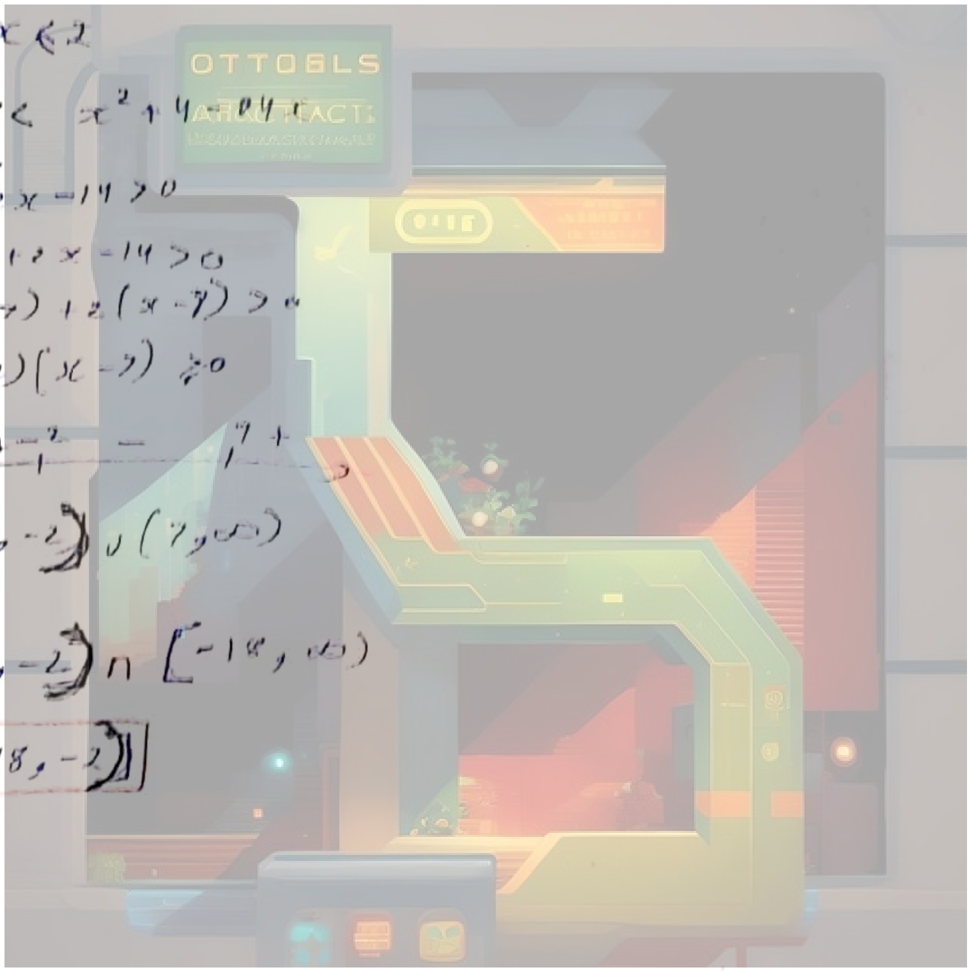
$(x+2)(x-7) \ge 0$

$+ \quad -2 \quad - \quad 7 \quad +$

$(-\infty,-2) \cup (7,\infty)$

$(-\infty,-2) \cap [-18,\infty)$

$\boxed{x \in [-18,-2)}$



d

$$\sqrt{\frac{x-2}{1-2x}} > -1$$

$$\frac{x-2}{1-2x} \geqslant 0$$

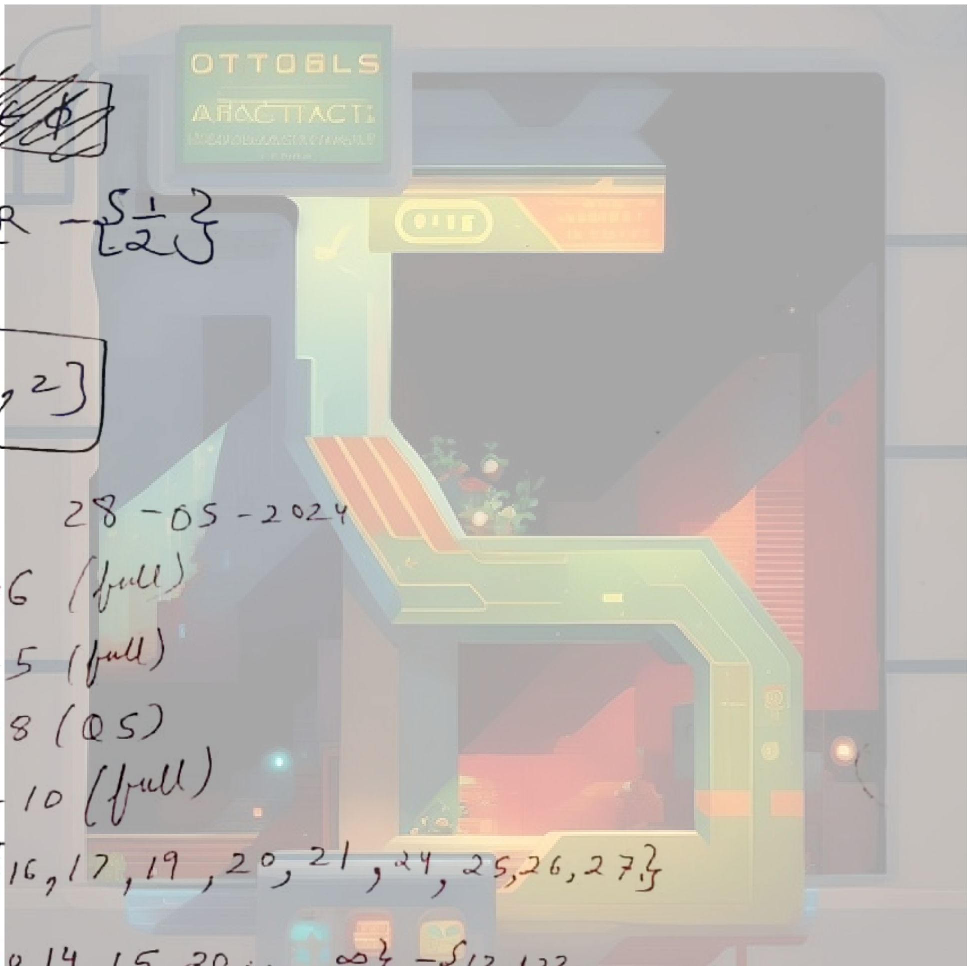
$$- \quad \tfrac{1}{2} \quad + \quad 2 \quad -$$

$$\left(\frac{1}{2}, 2\right]$$

$\cancel{x \in \phi}$

$x \in R - \left\{\frac{1}{2}\right\}$

$$\boxed{\left(\frac{1}{2}, 2\right]}$$



H.W. 28-05-2024

DYS-6 (full)

DYS-5 (full)

DYS-8 (Q5)

DYS-10 (full)

O-1 {16, 17, 19, 20, 21, 24, 25, 26, 27}

O-2 {10, 14, 15, 20 ... ∞} − {12, 13}

~~O-3~~

Modulous Equality.

(1) If $a = +ve$ ($a \rightarrow$ constant)

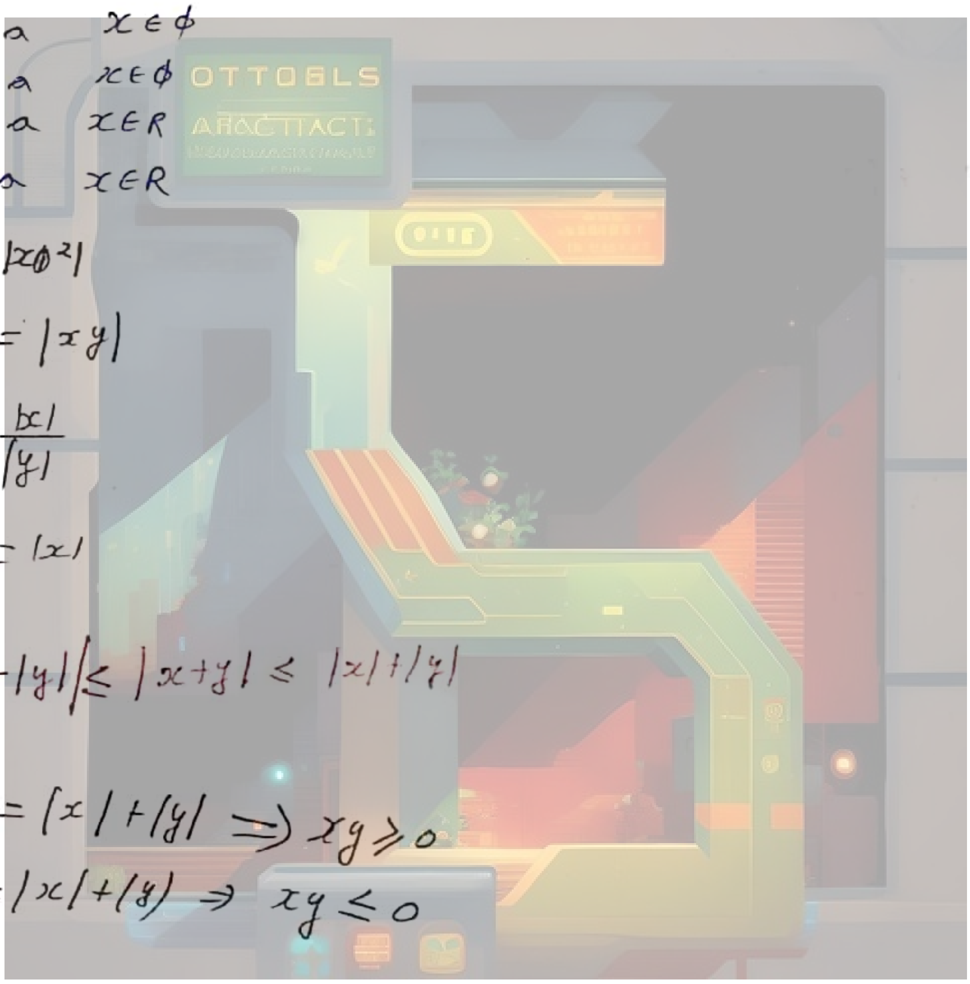
$|x| \leq a \quad x \in [-a, a]$

$|x| < a \quad x \in (-a, a)$

$|x| \geq a \quad x \in (-\infty, -a] \cup [a, \infty)$

$|x| > a \quad x \in (-\infty, -a) \cup (a, \infty)$

(2) If $a = -ve$ ($a \rightarrow$ constant)



$|x| \leq a \quad x \in \phi$

$|x| < a \quad x \in \phi$

$|x| \geq a \quad x \in R$

$|x| > a \quad x \in R$

(3) $|x|^2 = |x^2|$

(4) $|x||y| = |xy|$

(5) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

(6) $\sqrt{x^2} = |x|$

(7) $\big||x| - |y|\big| \leq |x+y| \leq |x| + |y|$

(8) $|x+y| = |x| + |y| \Rightarrow xy \geq 0$

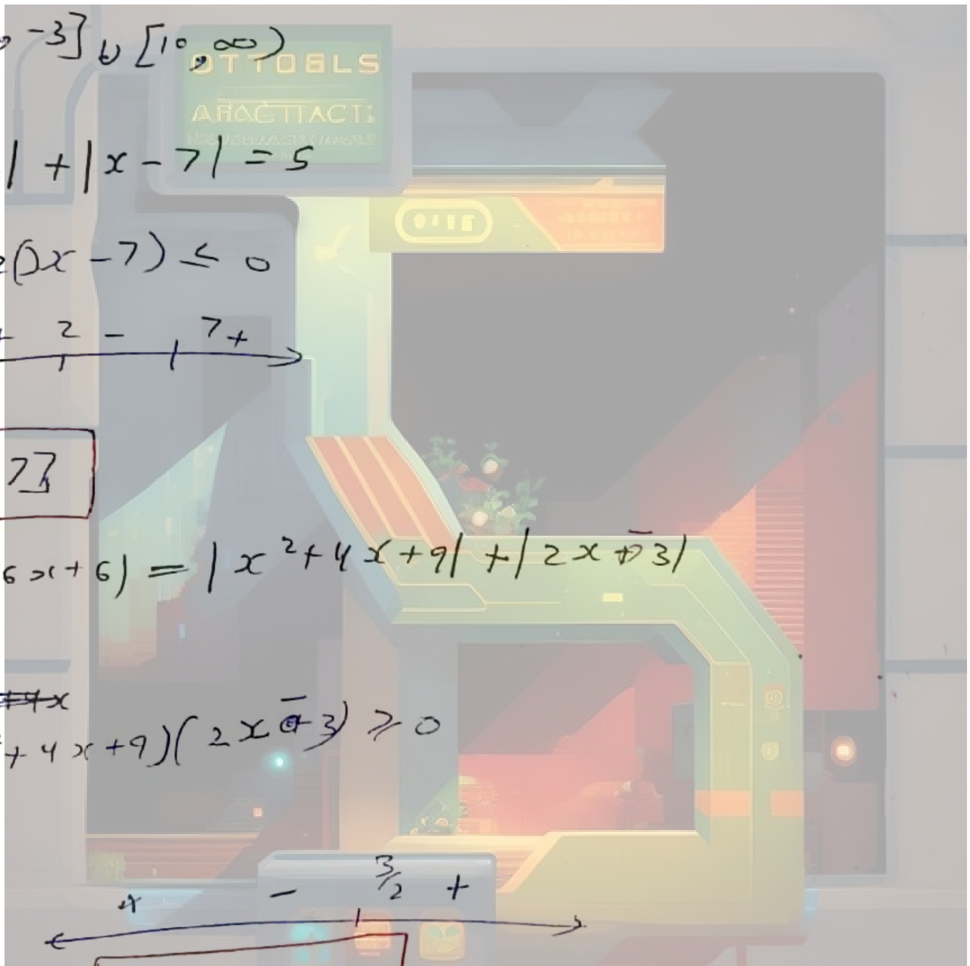
$|x-y| = |x| + |y| \Rightarrow xy \leq 0$

Q $|2x-7| = |x+3| + |x-10|$

$|x+3+x-10| = |x+3| + |x-10|$

$(x+3)(x-10) \geq 0$

$(-\infty, -3] \cup [10, \infty)$



Q $|x-2| + |x-7| = 5$

$(x-2)(x-7) \leq 0$

$\boxed{[2, 7]}$

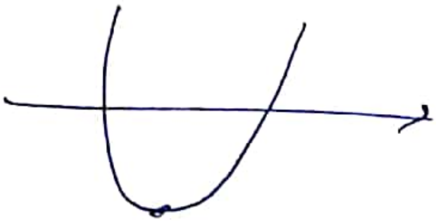
Q $|x^2+6x+6| = |x^2+4x+9| + |2x-3|$

$(x^2+4x+9)(2x-3) \geq 0$

$\boxed{\left(\frac{3}{2}, \infty\right)}$

0-1

Q27.



-1

as it is biggest -ve integer

$$-\frac{p}{4} = -1$$

$$-p = -4$$

$$p = 4$$

$$2x^2 + 4x + 1$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{4}$$

$$= \frac{-4 \pm 2\sqrt{2}}{4}$$

$$= \frac{-2 \pm \sqrt{2}}{2}$$

$$x = \frac{-2 + \sqrt{2}}{2} \quad \times \quad x = \frac{-2 - \sqrt{2}}{2}$$

~~$x = -1$~~

$$x = \frac{(\sqrt{2} - 2)}{2} \quad \times \quad \frac{-(\sqrt{2} + 2)}{2}$$

$$2x^2 + px + 1 = -1$$

$$2x^2 + px + 2 = 0$$

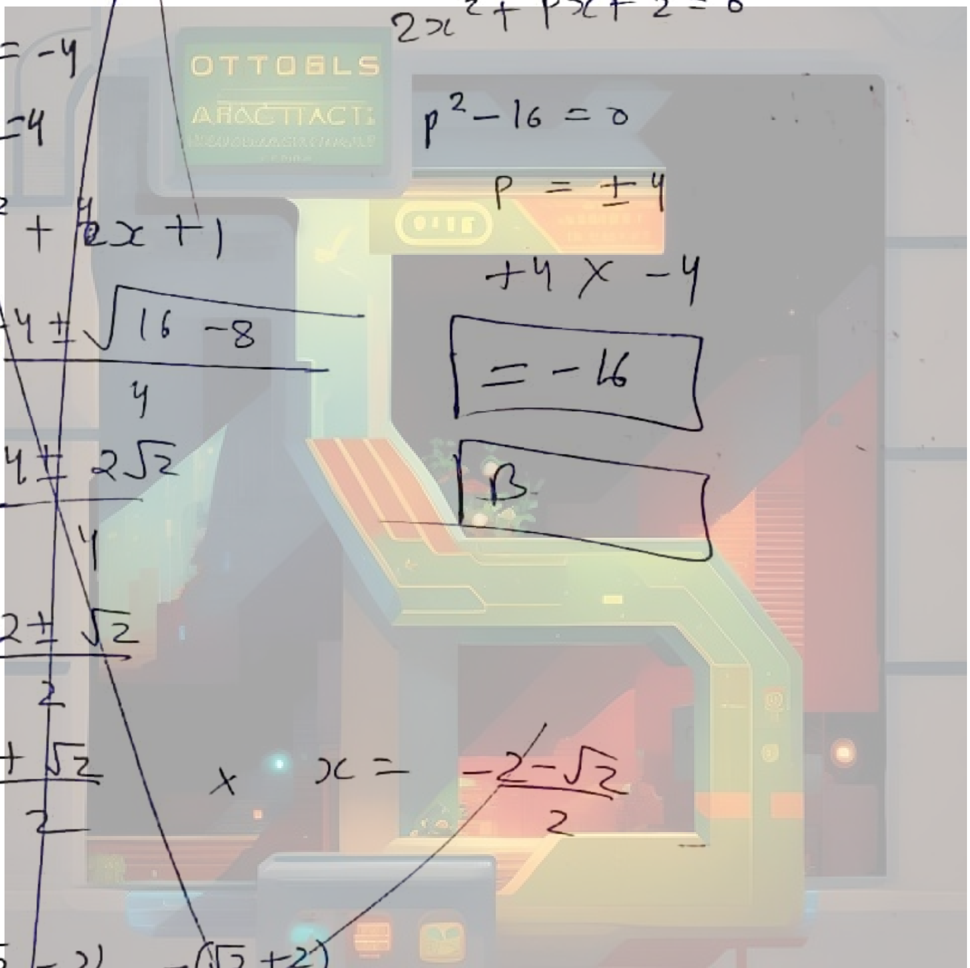
$$p^2 - 16 = 0$$

$$p = \pm 4$$

$$+4 \times -4$$

$$\boxed{= -16}$$

$$\boxed{B}$$



Q find 'x'

$|x| \leq 2$

$|x| \leq a$

$x \in [-a, a]$

$\boxed{x \in [-2, 2]}$

Q $|x-3| \leq 2$

$x - 3 \in [-2, 2]$

$\boxed{x \in [1, 5]}$

Q $|x| \geq 9$

$\boxed{x \in (-\infty, -9] \cup [9, \infty)}$

Q $|x| < \sqrt{3}$

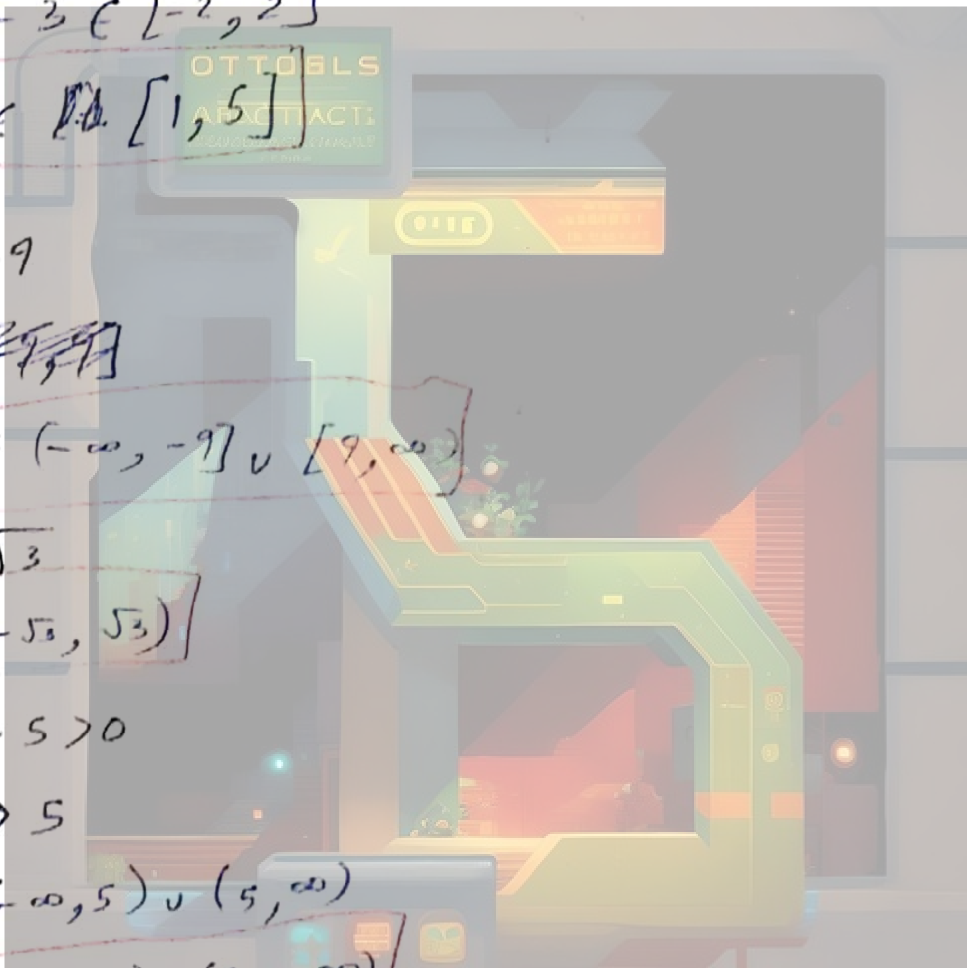
$\boxed{x \in (-\sqrt{3}, \sqrt{3})}$

Q $|2x| - 5 > 0$

$|2x| > 5$

$2x \in (-\infty, 5) \cup (5, \infty)$

$\boxed{x \in (-\infty, 5/2) \cup (5/2, \infty)}$



Q $|2-7x| < 8$

$2-7x \in (-8, 8)$

$-7x \in (-10, 6)$

$\boxed{\cancel{x \in \left(\frac{10}{7}, \frac{-6}{7}\right)}}$ $\boxed{x \in \left(-\frac{6}{7}, \frac{10}{7}\right)}$

Q no. of integral values of x such that $4 \le |x-4| \le 10$

$|x-4| \le 10$

$x-4 \in [-10, 10]$

$x \in [-6, 14]$

$|x-4| \ge 4$

$x-4 \in (-\infty, -4] \cup [4, \infty)$

$x \in (-\infty, 0] \cup [8, \infty)$

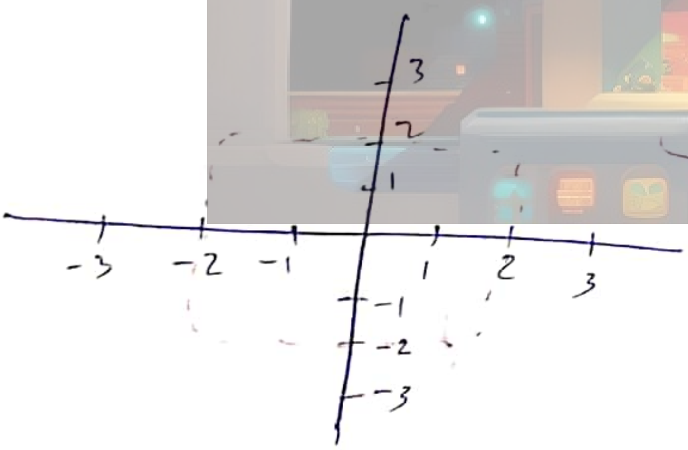
$\boxed{x \in [-6, 0] \cup [8, 14]}$

no. of integral values $= 7+7$

$\boxed{= 14}$

Q $|x| \le 2$ & $|y| \le 2$

$x \in [-2, 2]$ $y \in [-2, 2]$



area $= 4 \times 4$

$\boxed{\text{Area} = 16}$

Q $||x-2|-1| \le 5$

$|x-2|-1 \in [-5, 5]$

$|x-2| \in [-4, 6]$

$-4 \le |x-2| \le 6$

$|x-2| \geq -4$

$\cancel{|x-2| \geq}$

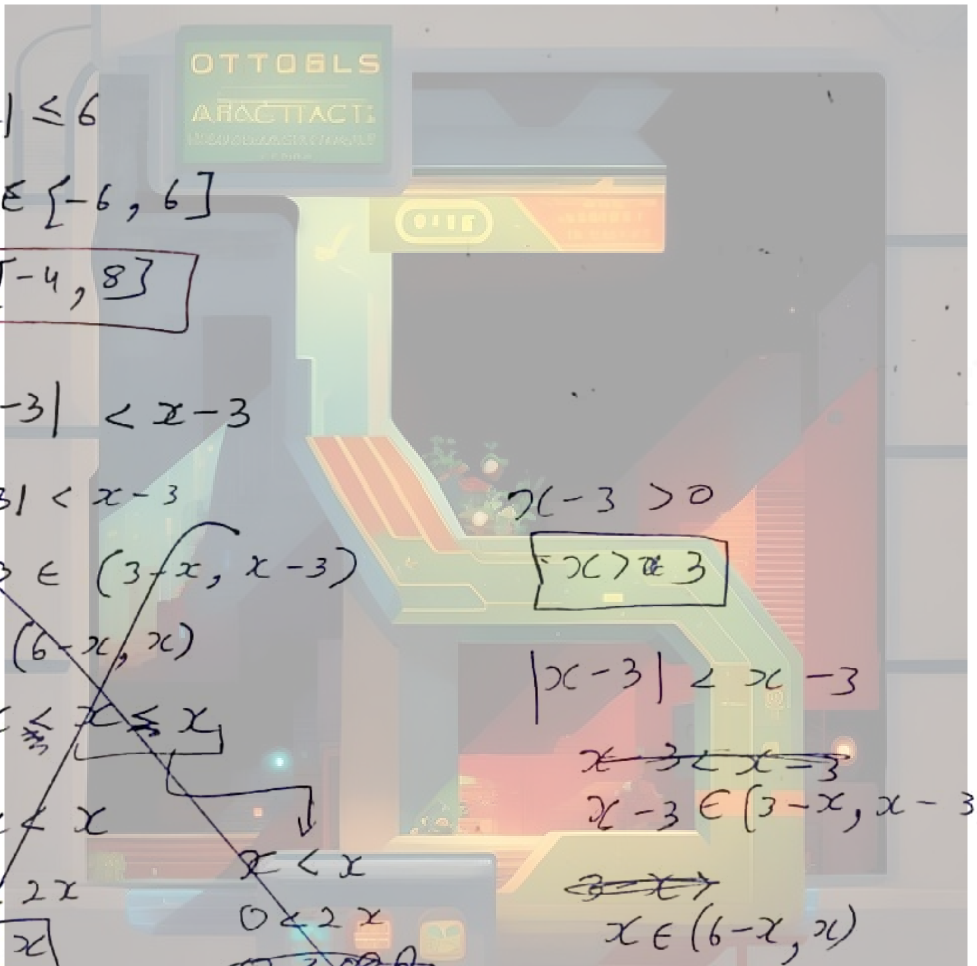
$x \in R$

$|x-2| \le 6$

$x-2 \in [-6, 6]$

$\boxed{x \in [-4, 8]}$

Q $|x-3| < x-3$



$|x-3| < x-3$

$x-3 \in (3-x, x-3)$

$x \in (6-x, x)$

$6-x \le \boxed{x \le x}$

$6-x < x$ $\quad$ $x < x$

$6 < 2x$ $\quad$ $0 < 2x$

$\boxed{3 < x}$ $\quad$ $\cancel{0 < 2x}$

$x \in R$

$x-3 > 0$

$\boxed{x > 3}$

$|x-3| < x-3$

$\cancel{x-3 < x-3}$

$x-3 \in (3-x, x-3)$

$\cancel{3-x}$

$x \in (6-x, x)$

$6-x < \boxed{x < x}$

$\boxed{x \in \phi}$

Theory of Equation

→ Deriving results for polynomials with degree 3 or more.

Let $a \neq 0$

① $ax^2+bx+c=0 \quad \alpha, \beta$

$$\alpha+\beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

② $ax^3+bx^2+cx+d=0 \quad \alpha, \beta, \gamma$

$$\alpha+\beta+\gamma = \frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

③ $ax^4+bx^3+cx^2+dx+e \quad \alpha, \beta, \gamma, \delta$

$$\alpha+\beta+\gamma+\delta = \frac{-b}{a} \quad \text{(sum)}$$

$$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\delta = \frac{c}{a} \quad \text{(2-2 sum)}$$

$$\alpha\beta\gamma+\beta\gamma\delta+\alpha\beta\delta+\alpha\gamma\delta = -\frac{d}{a} \quad \text{(3-3 sum)}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$

general

$$f(x) = \overset{+}{a_0 x^n} + \overset{-}{a_1 x^{n-1}} + \overset{+}{a_2 x^{n-2}} + \overset{-}{a_3 x^{n-3}} \dots + a_n$$

$-$ for odd n $\quad$ $+$ for even n

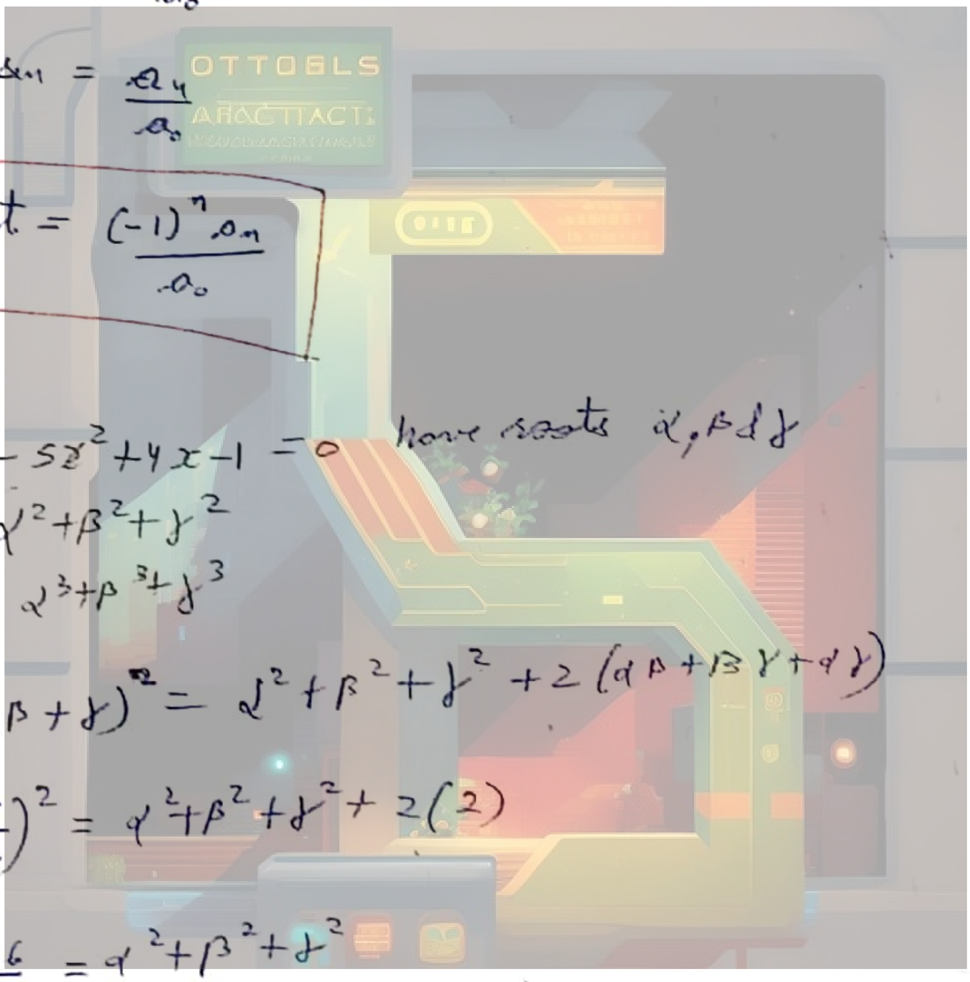
$$\boxed{\text{Sum of Roots} = -\frac{a_1}{a_0}}$$

$$2\text{-}2 \text{ sum} = \frac{a_2}{a_0}$$

$$3\text{-}3 \text{ sum} = \frac{-a_3}{a_0}$$

$$4\text{-}4 \text{ sum} = \frac{a_4}{a_0}$$

$$\boxed{\text{product} = \frac{(-1)^n a_n}{a_0}}$$



Q $2x^3 - 5x^2 + 4x - 1 = 0$ have roots α, β & γ

find (1) $\alpha^2 + \beta^2 + \gamma^2$

(2) $\alpha^3 + \beta^3 + \gamma^3$

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \alpha\gamma)$$

$$\left(\frac{5}{2}\right)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(2)$$

$$\frac{25 - 16}{4} = \alpha^2 + \beta^2 + \gamma^2$$

$$\boxed{\frac{9}{4} = \alpha^2 + \beta^2 + \gamma^2}$$

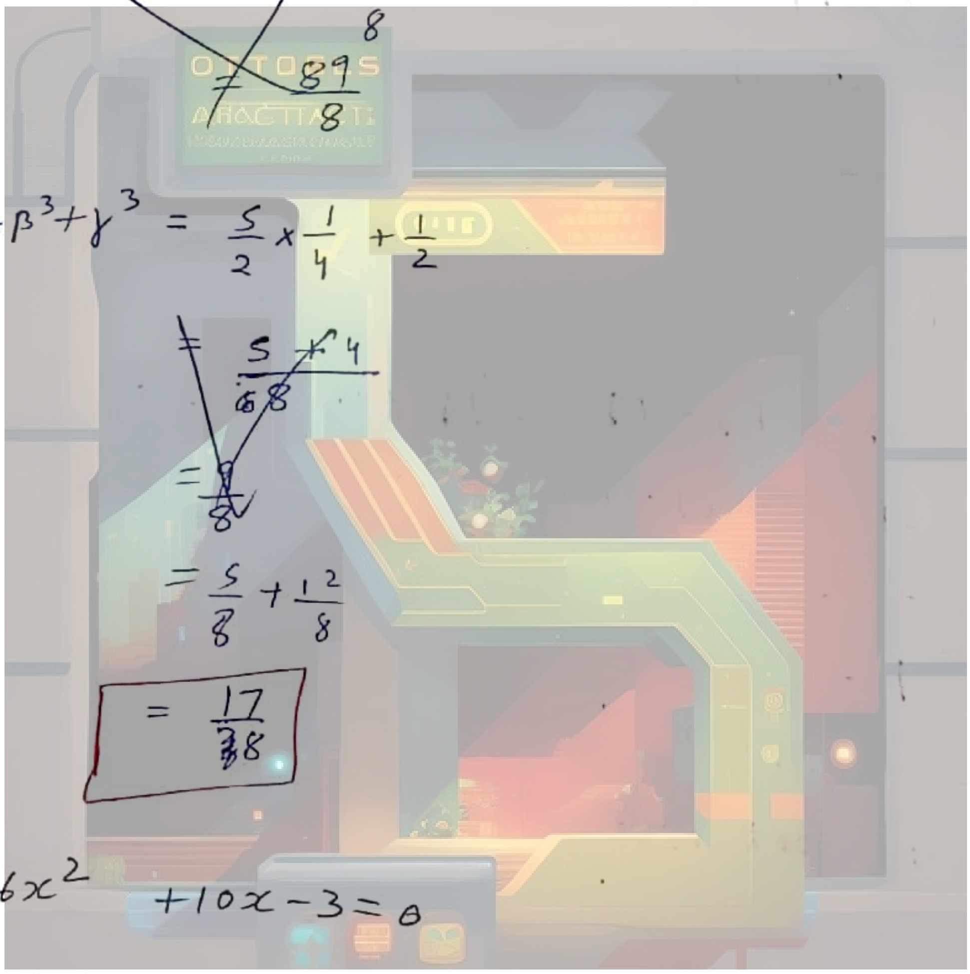
(2) $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\alpha\gamma)$

$\alpha^3+\beta^3+\gamma^3-\frac{1}{2} = \left(\frac{5}{2}\right)\left(\frac{9}{4}+2\right)$

$\cancel{\alpha^3+\beta^3+\gamma^3 = \frac{85}{8}+\frac{1}{2}}$

$\cancel{= \frac{85+4}{8}}$

$\cancel{= \frac{89}{8}}$



$\alpha^3+\beta^3+\gamma^3 = \frac{5}{2}\times\frac{1}{4}+\frac{1}{2}$

$\cancel{= \frac{5+4}{8}}$

$\cancel{= \frac{9}{8}}$

$= \frac{5}{8}+\frac{12}{8}$

$\boxed{= \frac{17}{\cancel{3}8}}$

Q $x^3-6x^2+10x-3=0$

$\left(\alpha-\frac{1}{\beta\gamma}\right)\left(\beta-\frac{1}{\alpha\gamma}\right)\left(\gamma-\frac{1}{\alpha\beta}\right)$

$\frac{(\alpha\beta\gamma-1)}{\beta\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\beta}$

$\frac{(3-1)^3}{(3)^2} = \boxed{\frac{8}{9}}$

Q $x^3-3x^2+2x+1=0$

(i) $(\alpha-2)(\beta-2)(\gamma-2)$

$\alpha+\beta+\gamma-6$

$\boxed{=-3}$

$\cancel{\alpha\beta+}$

$\alpha\beta-2\alpha-2\beta+4+\beta\gamma-2\beta-2\gamma+4+\alpha\gamma-2\alpha-2\gamma+4$

$\alpha\beta+\alpha\gamma+\beta\gamma+12-4(\alpha+\beta+\gamma)$

$-2+12-4(+3)$

$\cancel{-2+12+12}$

$-2+12+12$

$\boxed{=22}$

$\alpha\beta\gamma-2\alpha\gamma-2\beta\gamma+4\gamma-2\alpha\beta+4\alpha+4\beta-8$

$\alpha\beta\gamma-2(\alpha\beta+\beta\gamma+\alpha\gamma)+4(\alpha+\beta+\gamma)-8$

$-1-4+12-8$

$\boxed{\cancel{-25}}$ $\boxed{-1}$

$\cancel{t^3+3+2t+2}$

$\boxed{t^3+3t^2+2t+\cancel{123}}$

② cubic with roots $\frac{\alpha+1}{2}, \frac{\beta+1}{2}, \frac{\gamma+1}{2}$

$$\frac{x+1}{2} = t$$

$$x+1 = 2t$$

$$x = 2t-1$$

$$(2t-1)^3 - 3(2t-1)^2 + 2(2t-1) + 1 = 0$$

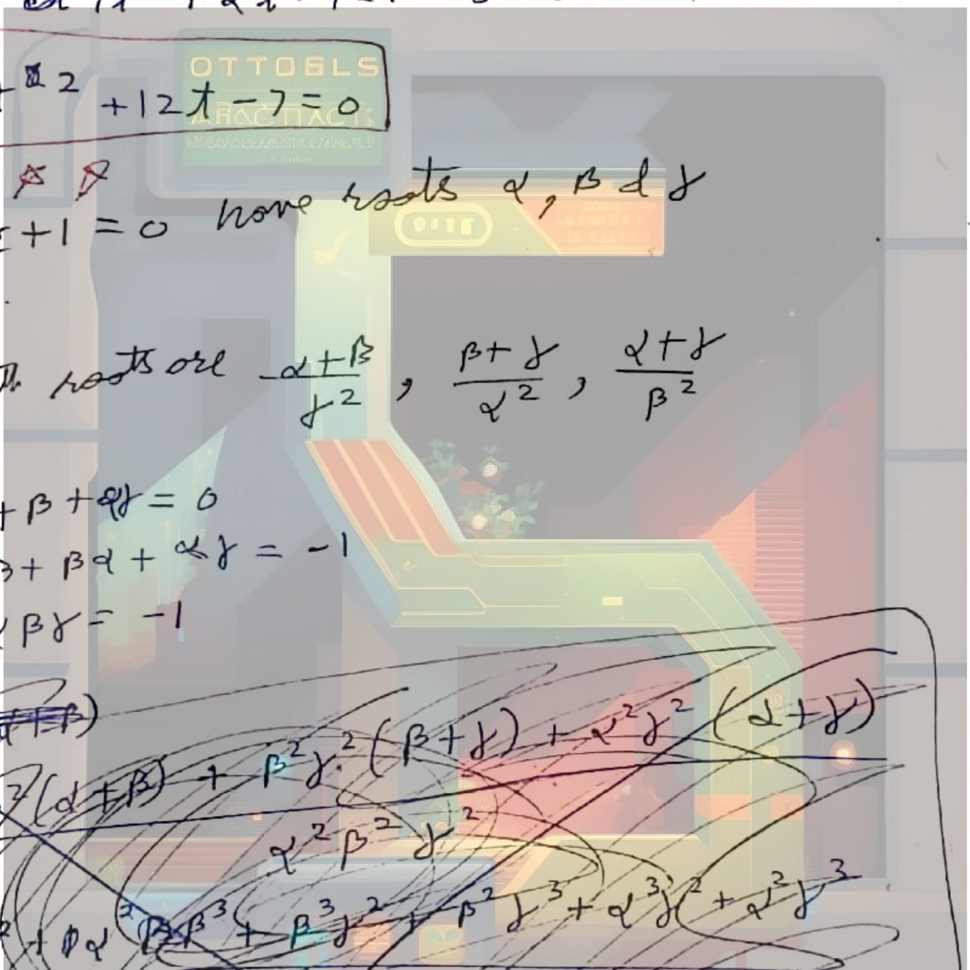
$$8t^3 - 3 \cdot 4t^2 + 2t - 12t^2 - 3 + 6t + 4t - 2 + 1 = 0$$

$$\boxed{8t^3 - 16t^2 + 12t - 7 = 0}$$

Q $x^3 - x + 1 = 0$ have roots α, β & γ.

find

① Eq with roots are $\frac{\alpha+\beta}{\gamma^2}, \frac{\beta+\gamma}{\alpha^2}, \frac{\alpha+\gamma}{\beta^2}$



$$\alpha+\beta+\gamma = 0$$

$$\alpha\beta + \beta\alpha + \alpha\gamma = -1$$

$$\alpha\beta\gamma = -1$$

$$\cancel{\frac{\alpha^2\beta^2(\alpha+\beta) + \beta^2\gamma^2(\beta+\gamma) + \alpha^2\gamma^2(\alpha+\gamma)}{\alpha^2\beta^2\gamma^2}}$$

$$\cancel{\frac{\alpha^3\beta^2 + \alpha^2\beta^3 + \beta^3\gamma^2 + \beta^2\gamma^3 + \alpha^3\gamma^2 + \alpha^2\gamma^3}{(\alpha\beta\gamma)^2}}$$

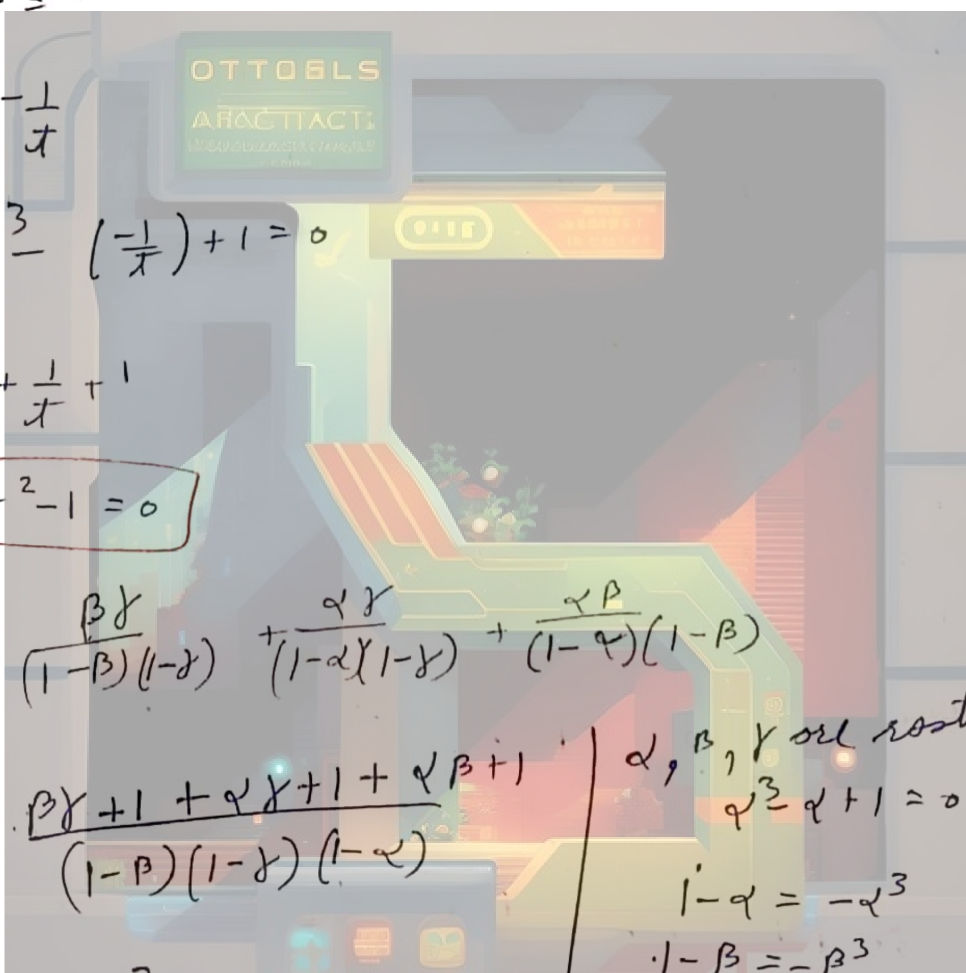
$$\frac{\alpha+\beta}{\gamma^2}$$

$$\alpha+\beta+\gamma=0$$

$$\alpha+\beta=-\gamma$$

$$\frac{-\gamma}{\gamma^2} = -\frac{1}{\gamma}, \frac{-1}{\alpha}, -\frac{1}{\beta} \text{ are roots.}$$

$$-\frac{1}{x} = t$$



$$x = -\frac{1}{t}$$

$$\left(-\frac{1}{t}\right)^3 - \left(\frac{-1}{t}\right) + 1 = 0$$

$$\frac{-1}{t^3} + \frac{1}{t} + 1$$

$$\boxed{t^3 + t^2 - 1 = 0}$$

(2) Value of $\frac{\beta\gamma}{(1-\beta)(1-\gamma)} + \frac{\alpha\gamma}{(1-\alpha)(1-\gamma)} + \frac{\alpha\beta}{(1-\alpha)(1-\beta)}$

α, β, γ are roots

$$\alpha^3 - \alpha + 1 = 0$$

$$1-\alpha = -\alpha^3$$

$$1-\beta = -\beta^3$$

$$1-\gamma = -\gamma^3$$

$$\frac{\beta\gamma+1+\alpha\gamma+1+\alpha\beta+1}{(1-\beta)(1-\gamma)(1-\alpha)}$$

$$= \frac{2}{(1-\beta)(1-\gamma)(1-\alpha)}$$

$$= \frac{2}{-(\alpha\beta\gamma)^3}$$

$$= \frac{2}{1}$$

$$\boxed{=2} \qquad \boxed{=2}$$

(62)

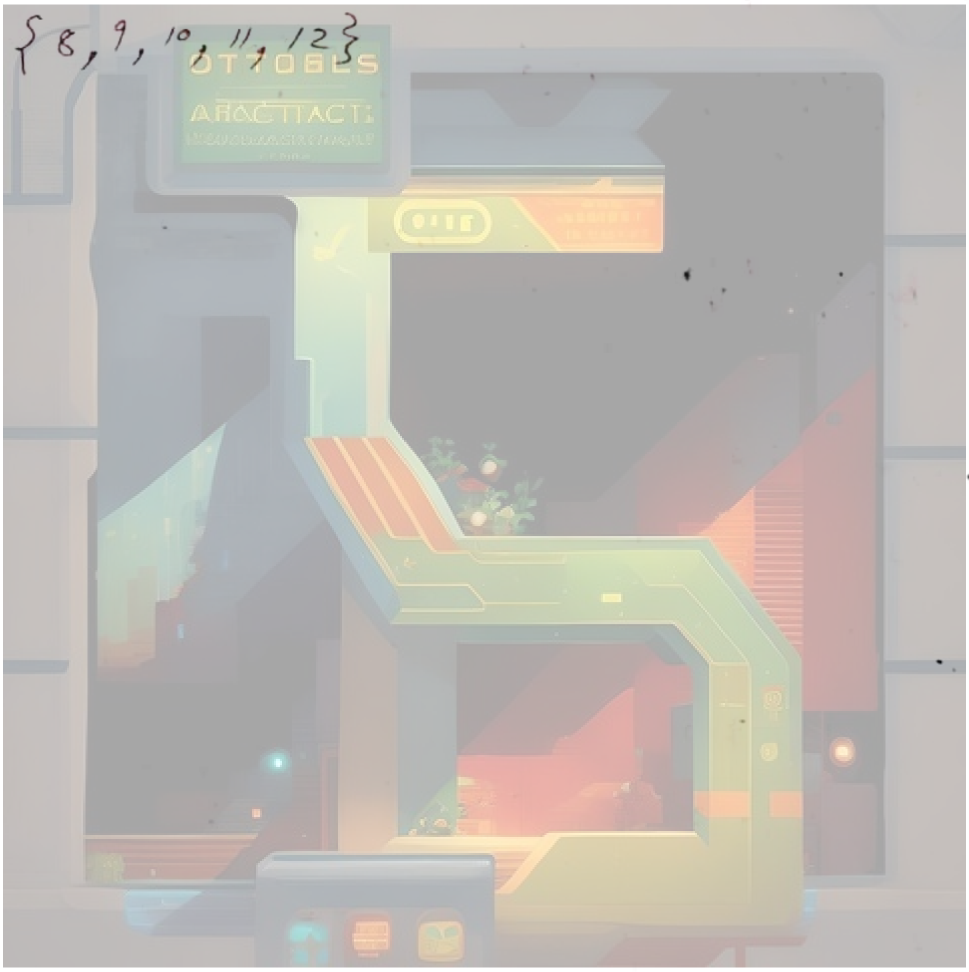
Q $\boxed{\begin{aligned}&\sum\alpha = \alpha+\beta+\gamma\\ &\sum\alpha^2 = \alpha^2+\beta^2+\gamma^2\end{aligned}}$

H.W. 30-5-2024

Q-1 {28, 29, 30} *

Q-3 {~~5-6~~ 4-6, 9, 10}

Q J-M {8, 9, 10, 11, 12}



General quadratic equation in two variable

$$f(x,y) = ax^2 + by^2 + 2hxy + 2gx + 2fy + c$$

↳ two linear factors when

$$\boxed{abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$$

Proof:- consider quadratic in x

$$\underbrace{ax^2}_{x^2} + \underbrace{(2hy + 2g)x}_{x} + \underbrace{by^2 + 2fy + c}_{\text{constant}} = 0$$

$$x = \frac{-(2hy+2g) \pm \sqrt{(2hy+2g)^2 - 4(a)(by^2+2fy+c)}}{2a}$$

$\cancel{x = -2hy - 2g}$

Q find weather $x^2 + 2xy + 2x + 6y - 3 = 0$ have two linear factors or not.

$$2 \times 3 - 9 + 3 = 0$$

So it resolved in 2 linear factors.

Q $x^2 + 2xy + 2x + ky^2 + k = 0$ find k if the above equation has two linear factors

$a = 1$
$b = k$
$c = k$
$h = 1$
$g = 1$
$f = 0$

$k^2 - k - k = 0$
$k^2 - 2k = 0$
$\boxed{k = 0, 2}$

Type-2 – when two homogeneous equation have ~~common~~ common linear factors.

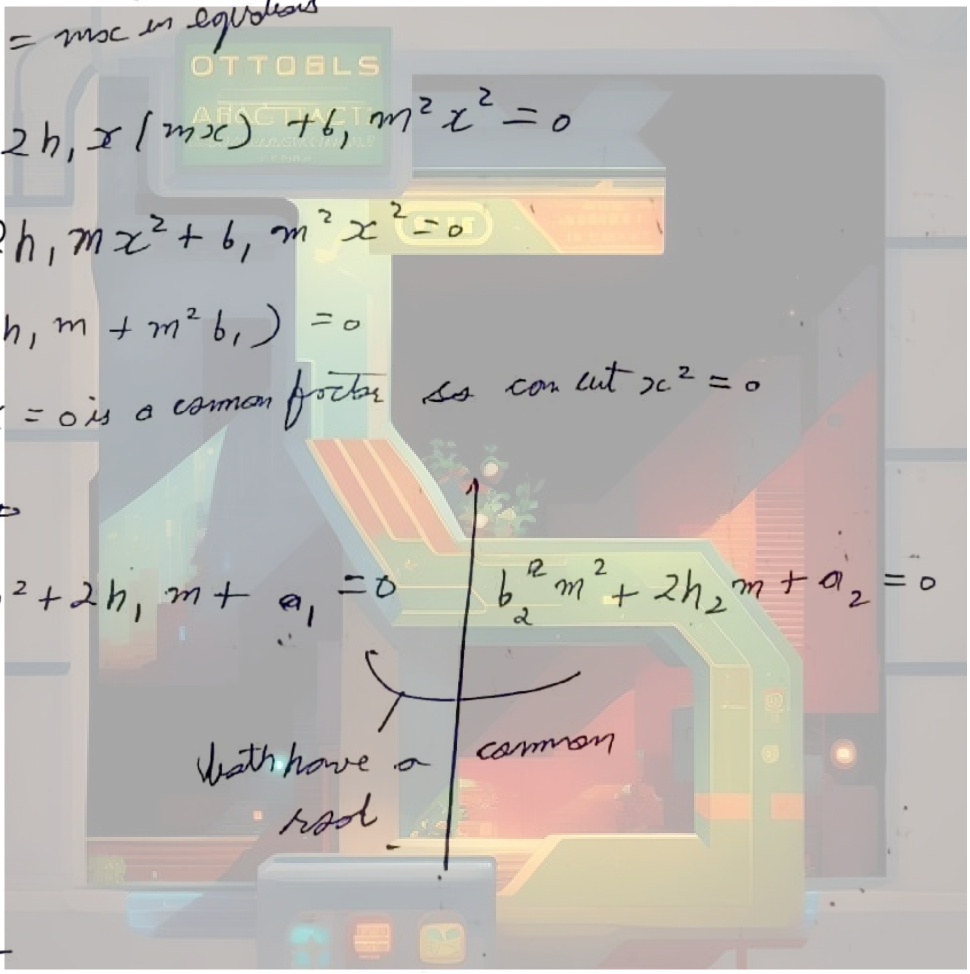
Homogeneity → when degree of all terms is same (xy degree = 2)

$a_1x^2 + 2h_1xy + b_1y^2 = 0$ $\qquad$ $a_2x^2 + 2h_2xy + b_2y^2 = 0$

Assume $y - mx = 0$ is a common factor

$x = 0$ is a factor

put $y = mx$ in equations



$a_1x^2 + 2h_1x(mx) + b_1m^2x^2 = 0$

$a_1x^2 + 2h_1mx^2 + b_1m^2x^2 = 0$

$x^2(a_1 + 2h_1m + m^2b_1) = 0$

we know $x = 0$ is a common factor so can cut $x^2 = 0$

~~$a_1 + 2h_1$~~

or $b_1m^2 + 2h_1m + a_1 = 0$ $\qquad$ $b_2m^2 + 2h_2m + a_2 = 0$

both have a common root

Method 2:-

$m^2b_1 + 2h_1m + a_1 = 0$

$m^2b_2 + 2h_2m + a_2 = 0$

$$\frac{m^2}{\begin{matrix}2h_1 & a_1\\ 2h_2 & a_2\end{matrix}} = \frac{-m}{\begin{matrix}b_1 & a_1\\ b_2 & a_2\end{matrix}} = \frac{1}{\begin{matrix}b_1 & 2h_1\\ b_2 & 2h_2\end{matrix}}$$

Cross multiply

$$\frac{m^2}{2h_1a_2-2h_2a_1} = \frac{-m}{b_1a_2-b_2a_1} = \frac{1}{2b_1h_2-2h_1b_2}$$

$$\cancel{m = \frac{b_1a_2-b_2a_1}{}}$$

$$m = \frac{b_2a_1-b_1a_2}{2b_1h_2-2h_1b_2}$$

Now.

$$\frac{m^2}{2b_1a_2-2b_2a_1} = \frac{1}{2b_1h_2-2h_1b_2}$$

Put value of m

$$\frac{\left(\frac{b_2a_1-ba_2}{2b_1h_2-2h_1b_2}\right)^2}{2h_1a_2-2b_2a_1} = \frac{1}{2b_1b_2-2h_1b_2}$$

$$\frac{(b_2a_1-b_1a_2)^2}{(2b_1h_2-2h_1b_2)^2(2h_1a_2-2h_2a_1)} = \frac{1}{(2b_1h_2-2h_1b_2)}$$

$$\frac{(b_2a_1-b_1a_2)^2}{(2b_1h_2-2h_1b_2)(2h_1a_2-2h_2a_1)} = 1$$

$$(b_2a_1-b_1a_2)^2 = (2b_1h_2-2h_1b_2)(2h_1a_2-2h_2a_1)$$

48 66

DYS-11 - Complete H.W.

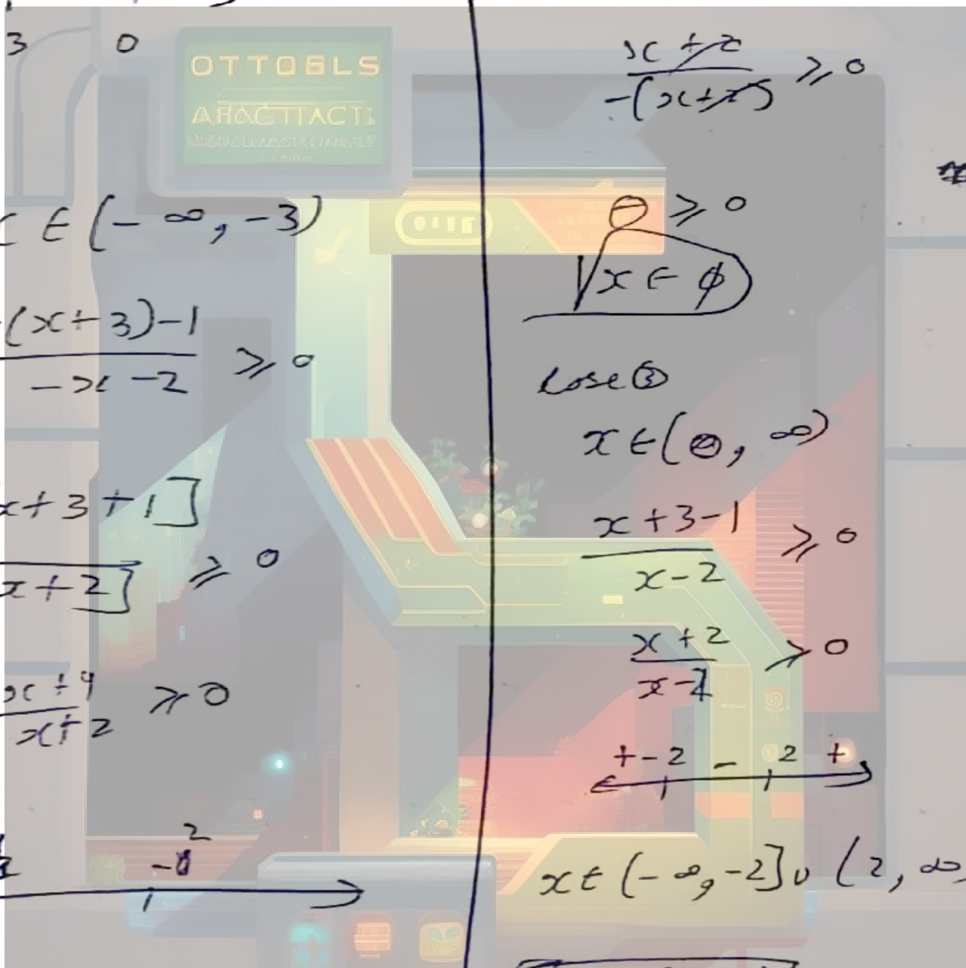
~~DS~~ J-A - Complete

J-M

Q16.

$$\frac{|x+3|-1}{|x|-2} \geq 0$$

$$\xleftarrow{\quad --\quad \underset{-3}{|} \quad +-\quad \underset{0}{|}\quad ++\quad}$$



Case ①

$x \in (-\infty, -3)$

$$\frac{-(x+3)-1}{-x-2} \geq 0$$

$$\frac{[x+3+1]}{[x+2]} \geq 0$$

$$\frac{x+4}{x+2} \geq 0$$

$$\xleftarrow{\quad \underset{-4}{|} \quad\quad \underset{-2}{|} \quad}$$

$x \in (-\infty, -4] \cup (-2, \infty)$

$\boxed{x \in (-\infty, -4)}$

Case ②

$x \in [-3, 0]$

$$\frac{x+3-1}{-(x+2)} \geq 0$$

$$\frac{\cancel{x+2}}{-(\cancel{x+2})} \geq 0$$

$\ominus \geq 0$

$x \in \phi$

Case ③

$x \in (0, \infty)$

$$\frac{x+3-1}{x-2} \geq 0$$

$$\frac{x+2}{x-2} \geq 0$$

$$\xleftarrow{\quad + \quad \underset{-2}{|} \quad - \quad \underset{2}{|} \quad + \quad}$$

$x \in (-\infty, -2] \cup (2, \infty)$

$\boxed{x \in (2, \infty)}$

$x \in [-6, -2) \cup (-2, 2) \cup (2, 3]$

$\boxed{x \in [-6, -4] \cup (2, 3]}$

Part 2 - $x^2-7|x|+9 \le 0$

$x^2 = |x|^2$

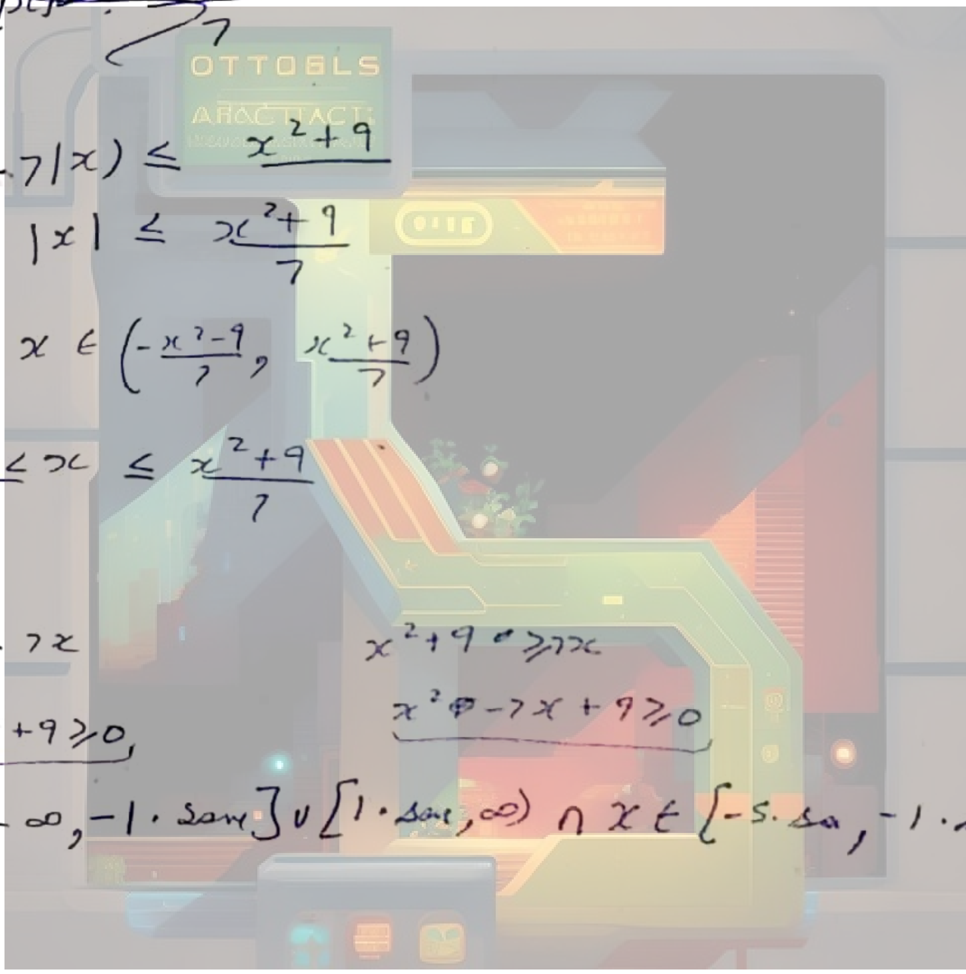
$|x|^2 - 7|x| + 9 \le 0$

$\cancel{t^2 - 7t + 9 \le 0}$

$\cancel{t = 7}$

$\cancel{+7|x| = x^2+9}$

$\cancel{|x| = \frac{x^2+9}{7}}$



$-7|x| \le x^2+9$

$|x| \le \frac{x^2+9}{7}$

$x \in \left(-\frac{x^2-9}{7}, \frac{x^2+9}{7}\right)$

$-\frac{x^2-9}{7} \le x \le \frac{x^2+9}{7}$

$-x^2-9 \le 7x$ $\quad$ $x^2+9 \ge 7x$

$\underline{x^2+7x+9 \ge 0}$ $\quad$ $\underline{x^2-7x+9 \ge 0}$

$x \in (-\infty, -1.\text{sam}] \cup [1.\text{sam}, \infty) \cap x \in [-5.\text{sa}, -1.\text{sam}] \cup [1.\text{sa}, 5.\text{sam}]$

$x \in [-5.\text{same}, -1.\text{sam}] \cup [1.\text{sam}, 5.\text{sam}]$

Q 10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$x+1 = y$

$y^{\log_{10} y} = 100y$

take $\log_y$ both sides

$\log_{10} y = \log_y 100y$

$1 = \log_y (100y + 10)$

$y = 100y + 10$

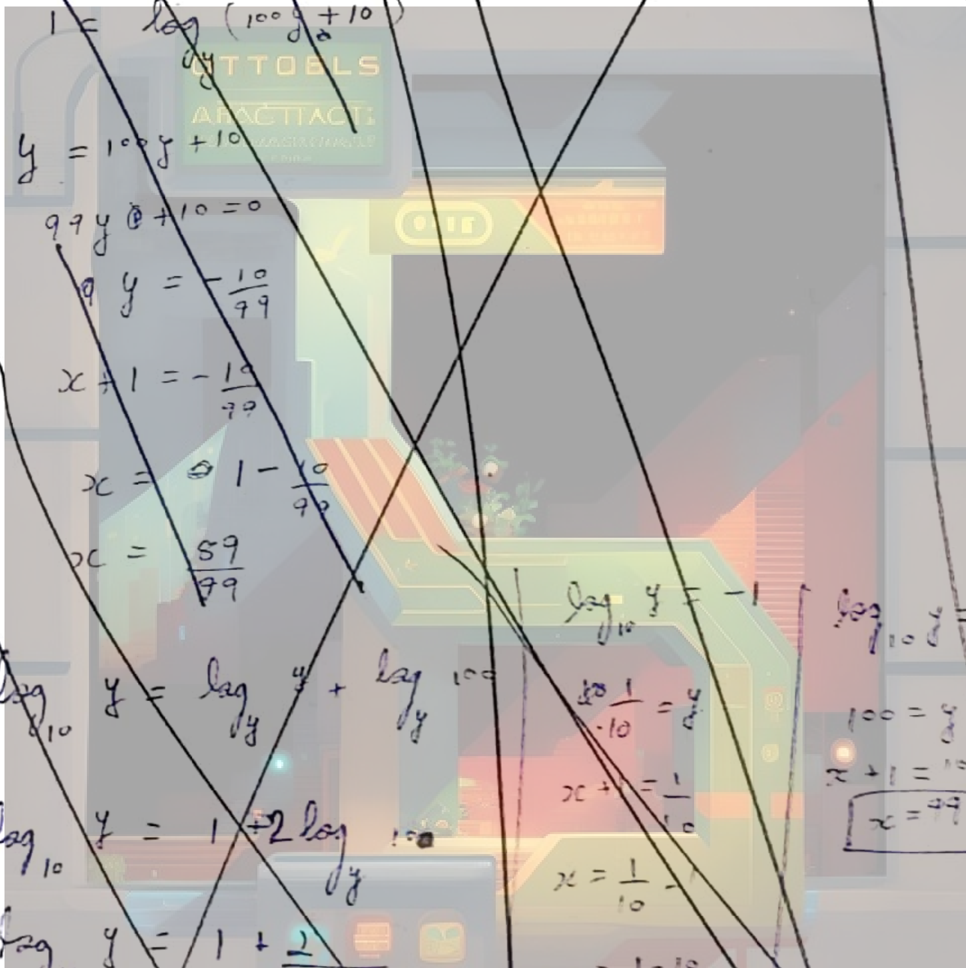
$99y + 10 = 0$

$y = -\frac{10}{99}$

$x+1 = -\frac{10}{99}$

$x = 1 - \frac{10}{99}$

$x = \frac{59}{99}$



$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$\log_{10} y = 1 + \frac{2}{\log_{10} y}$

$z = 1 + \frac{2}{z}$

$z^2 = z + 2$

$z^2 - z - 2 = 0$

$z = 2, -1$

$\log_{10} y = -1$

$\frac{1}{10} = y$

$x + 1 = \frac{1}{10}$

$x = \frac{1}{10} - 1$

$x = \frac{1-10}{10}$

$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$100 = y$

$x + 1 = 100$

$\boxed{x = 99}$