! Ch-5 !

Work, Power & Energy

Work

→ Work is said to be done if and when ~~for~~ Applied force produces some Displacement.

→ Whenever Force is Applied & body is displaced, always atleast two ~~forces are~~ bodies are involved,

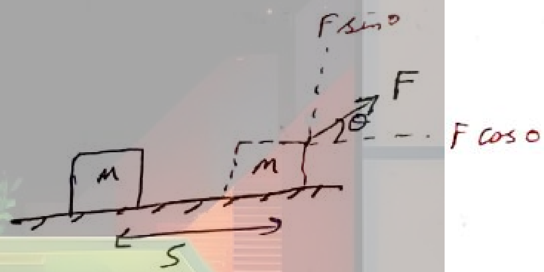
1. which is doing work (whose energy is decreasing)
2. On which the work is being done (whose energy is increasing)

→ work done by a constant force is the product of force in the direction of motion and magnitude of displacement.

$W = F\cos\theta \times S$

$W = FS\cos\theta$

$W = \vec{F} \cdot \vec{S}$



→ It is a scalar quantity

→ SI unit :- Joule (J) - Nm

CGS Unit :- erg = 1 dyne cm

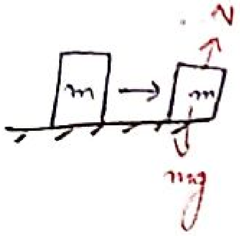
1 erg = 10^{-7} J

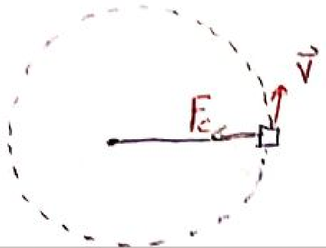
Types of work :-

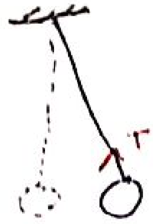
1. Zero Force :-

⇒ Work done by a force is zero if the body is displaced perpendicular to the direction of the force.

→ Work done by a force is zero if body suffers no displacement on application of force.







$W_{mg} = W_N = 0$ $\quad W_{centripetal} = 0$ Always $\quad W_T = 0$

2. Positive Work :-

→ Work done is said to be ⊕ve if Applied force has a component in the direction of displacement.

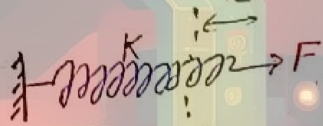
$W = FS\cos\theta$

$0^\circ \leq \theta \leq 90^\circ$ (Acute Angles)

$\cos\theta \rightarrow$ ⊕ve

$W \rightarrow$ ⊕ve

$W_F =$ ⊕ve



$W_F =$ ⊕ve

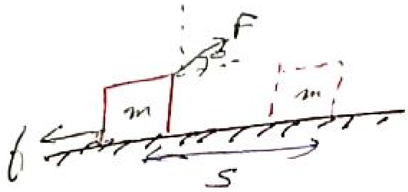
3. Negative Work :-

→ Work done is said to be negative if the applied force has a component in a direction opposite to that of displacement.

$90 < \theta \leq 180$

$\cos\theta =$ ⊖ve

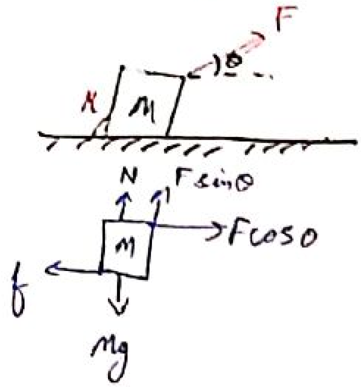
$W =$ ⊖ve



$W_f =$ ⊖ve

Q 1.

Q Travels with uniform velocity, find work done by applied force(F) during displacement d.



$N = Mg - F\sin\theta$

$f_{max} = \mu(Mg - F\sin\theta)$

$F\cos\theta = f_{max} = \mu(Mg - F\sin\theta)$

~~Work = F × s~~
~~$= \mu d(Mg - F\sin\theta)$~~

$F\cos\theta = \mu Mg - \mu F\sin\theta$

$F(\cos\theta + \mu\sin\theta) = \mu Mg$

$$F = \frac{\mu Mg}{\cos\theta + \mu\sin\theta}$$

$W_F = F\cos\theta \times s$

$$\boxed{W_F = \frac{\mu Mg\, d\cos\theta}{\cos\theta + \mu\sin\theta}}$$

Q2. constant $\vec{F} = (3\hat{i} + 2\hat{j} + 2\hat{k})\,N$ acts on a particle. displacement $\vec{r}_1 = (-\hat{i} + \hat{j} - 2\hat{k})\,m$ to new $\vec{r}_2 = (\hat{i} - \hat{j} + 3\hat{k})\,m$. find work.

Displacement, $\vec{s} = \vec{r}_2 - \vec{r}_1$

$= 2\hat{i} - 2\hat{j} + 5\hat{k}$

work $= \vec{F} \cdot \vec{s}$

$= 6 - 4 + 10$

$\boxed{W = 12\,J}$

Q3. Three constant forces act on a body $\vec{F_1} = (2\hat{i} - 3\hat{j} + 2\hat{k})N$, $F_2 = (\hat{i} + \hat{j} - \hat{k})$, $F_3 = (3\hat{i} + \hat{j} - 2\hat{k})$ displaces from. $(1,-1,2)$ to $(-1,-1,3)$ then to $(2,2,0)$ in metres. find Total work done by Forces.

$$S = \hat{i} + 3\hat{j} - 2\hat{k}$$

$$F = 6\hat{i} - \hat{j} - \hat{k}$$

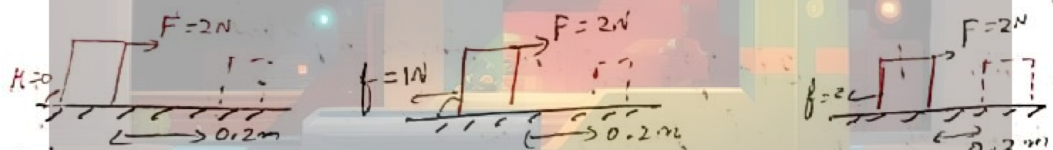
$$\text{Work} = 6 - 3 + 2$$

$$\boxed{= 5J}$$

Important Points on Work.

1. Work is said to be done by a force when its point of application moves by some distance.
2. Work is defined for an interval or displacement. There is no Term like instantaneous work.
3. For a particular displacement, work done by a force is independent of the type of motion i.e. whether it moves with constant velocity, acceleration or retardation etc.

eg.



$W_F = 2 \times 0.2$	$W_F = 0.2 \times 2$	$W_F = 0.2 \times 2$
$= 0.4\,J$	$= 0.4\,J$	$= 0.4\,J$
	$W_f = -0.2J$	$W_f = -0.4\,J$
	$W_{Total} = 0.2\,J$	$W_{Total} = 0\,J$

4. If a body is in dynamic equilibrium (moving with constant velocity) under the action of certain forces, then total work done on the body is zero but work done by individual forces may not be zero.

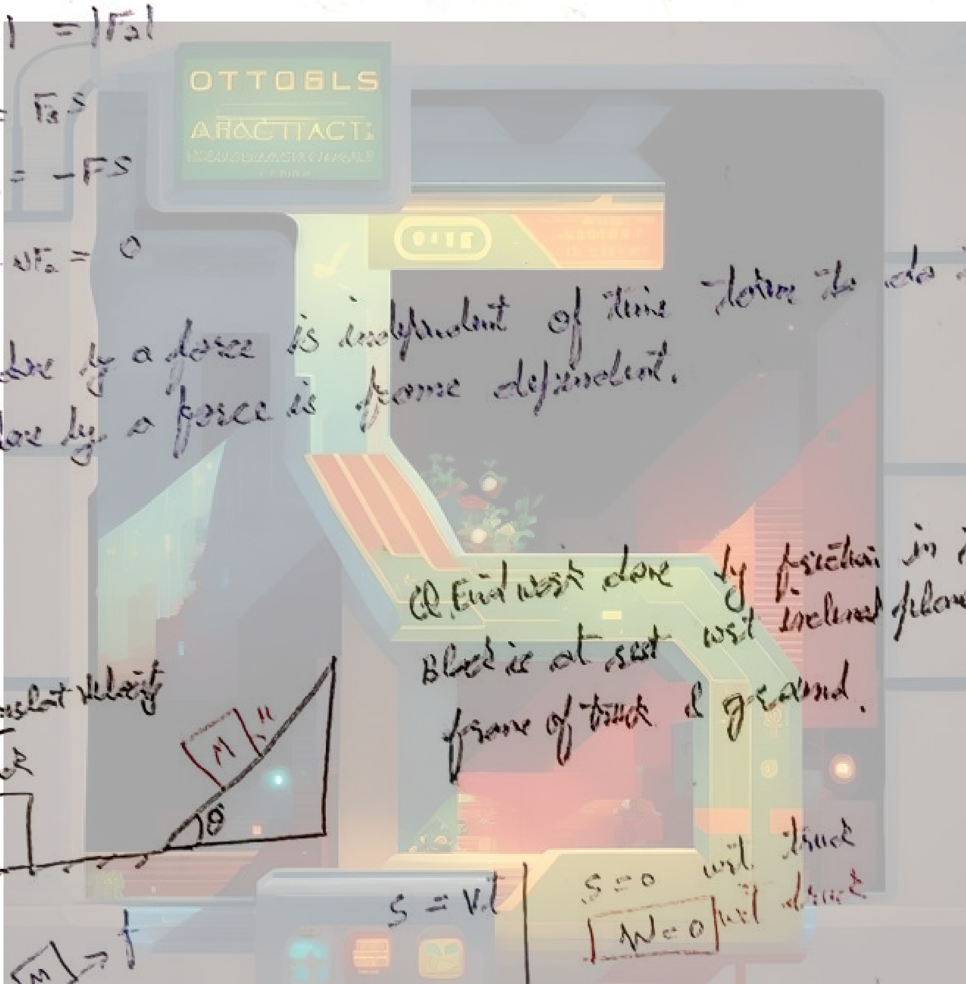
S

$|F_1| = |F_2|$

$W_{F_1} = F_1 S$

$W_{F_2} = -FS$

$W_{F_1} + W_{F_2} = 0$



5. Work done by a force is independent of time taken to do that work.

6. Work done by a force is frame dependent.

Q 4.

Q. Find work done by friction in time t. Block is at rest wrt inclined plane. wrt frame of truck & ground.

Constant velocity

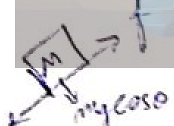
Truck

M μ θ

$S = Vt$

$S = 0$ wrt truck

$\boxed{W = 0}$ wrt truck



$f = Mg\sin\theta$

$W = f\,s\cos(180-\theta)$

$= -vt\,Mg\sin\theta\cos\theta$

$\boxed{\cancel{= Vt\,Mg\sin\theta}}$ ~~wrt ground~~

$\boxed{W = -vt\,Mg\sin\theta\cos\theta}$

wrt ground

Work done by variable force.

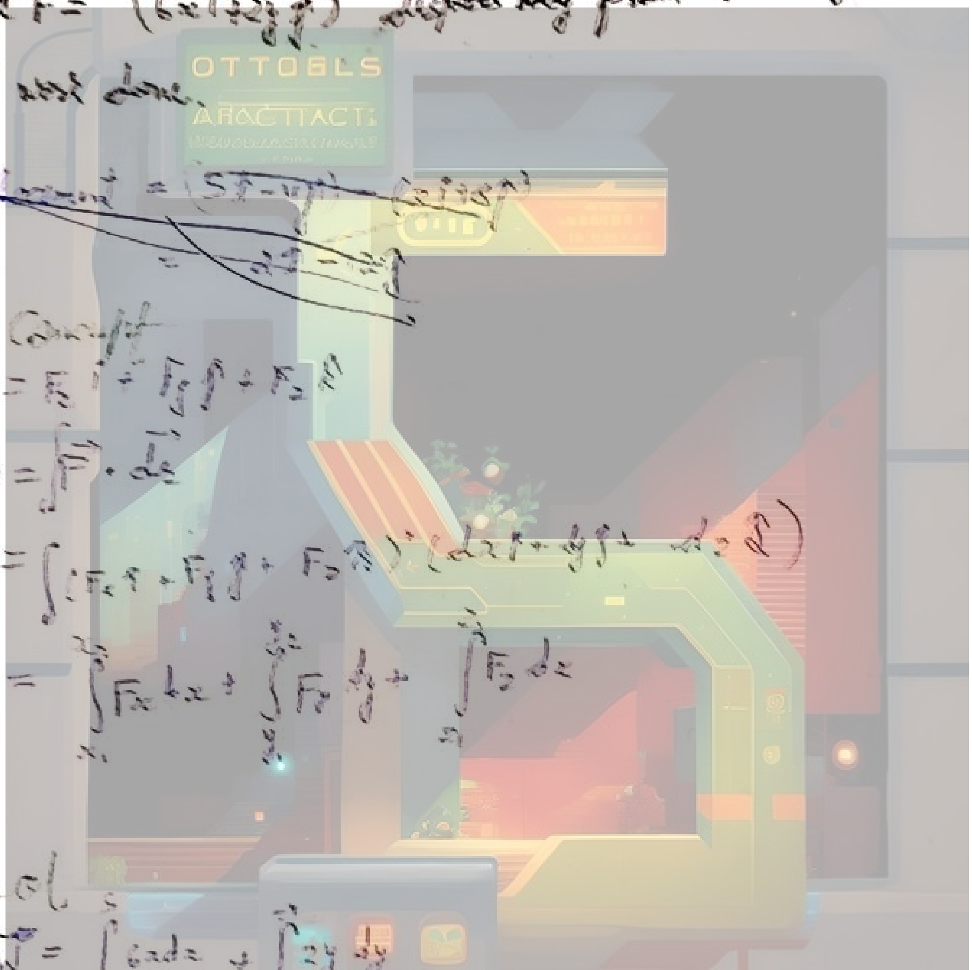
$F = 0$ $\qquad dw = F\,dx$

$F = kx$ $\qquad dw = kx\,dx$

$F = kx$ $\qquad \int dw = \int kx\,dx$

$$\boxed{W = \frac{1}{2}kx^2}$$

Q5. force $F = (6x\hat{i} + 2y\hat{j})$ displaces body from $(3\hat{i}+8\hat{j})$ to $(5\hat{i}-4\hat{j})$ find work done.



$\cancel{\text{Displacement} = (5\hat{i}-4\hat{j}) - (3\hat{i}+8\hat{j})}$

$\cancel{= 2\hat{i} - 12\hat{j}}$

Concept

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$w = \int \vec{F}\cdot d\vec{x}$

$= \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$= \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$

Sol.

$W = \int_3^5 6x\,dx + \int_8^{-4} 2y\,dy$

$W = 3(5)^2 - 3(3)^2 + (-4)^2 - (8)^2$

$W = 48 - 48$

$\boxed{W = 0}$

Q 6. $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$ $\vec{r_1} = (2\hat{i} + 3\hat{j})$ $\vec{r_2} = (4\hat{i} + 6\hat{j})$

$$W = \int_2^4 3x^2\,dx + \int_3^6 2y\,dy$$

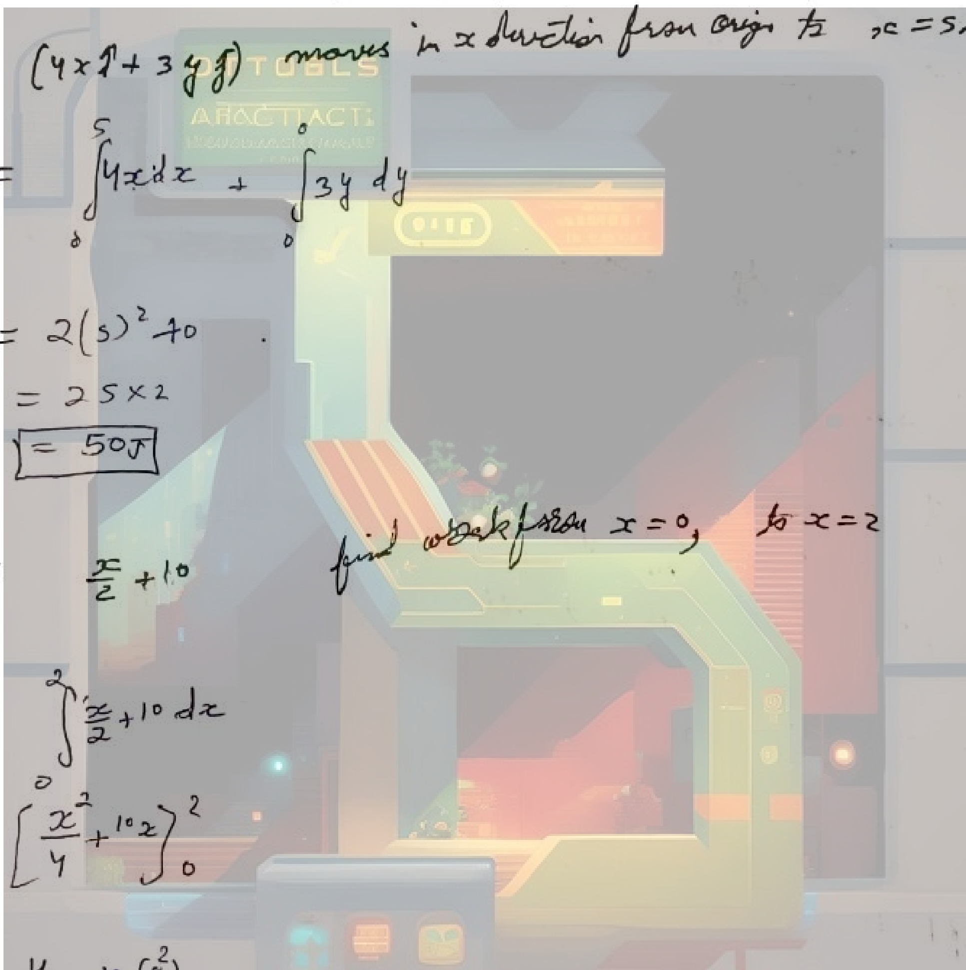
$$= \cancel{(2)^3 - (4)^3} + (6)^2 - (3)^2$$

$$(4)^3 - (2)^3$$

$$= (64 - 8) + (36 - 9)$$

$$= 56 + 27$$

$$\boxed{= 83\ \text{J}}$$



Q 7. $\vec{F} = (4x\hat{i} + 3y\hat{j})$ moves in x direction from origin to $x = 5$ m.

$$W = \int_0^5 4x\,dx + \int_0^0 3y\,dy$$

$$= 2(5)^2 + 0$$

$$= 25 \times 2$$

$$\boxed{= 50\ \text{J}}$$

Q 8. $F = \frac{x}{2} + 10$ find work from $x = 0$, to $x = 2$

$$W = \int_0^2 \frac{x}{2} + 10\,dx$$

$$= \left[\frac{x^2}{4} + 10x\right]_0^2$$

$$= \frac{4}{4} + 10(2)$$

$$= 20 + 1$$

$$\boxed{= 21\ \text{J}}$$

Q9. $v = f\sqrt{x}$ (f = constant)

find work done for displacement $x=0$ to $x=d$

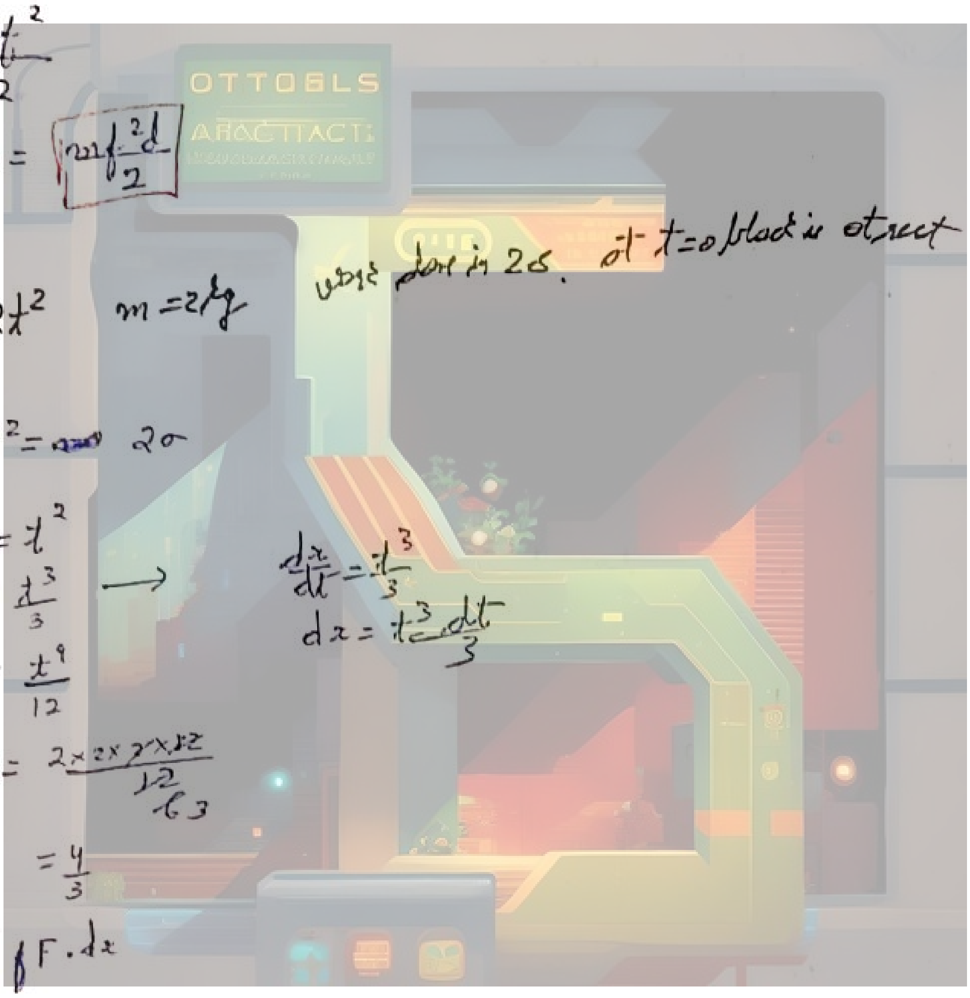
$a = v\frac{dv}{dx}$

$a = f\sqrt{x} \times \frac{f}{2\sqrt{x}}$

$a = \frac{f^2}{2}$

$F = \frac{mf^2}{2}$

Work done = $\boxed{\frac{mf^2 d}{2}}$



Q10. $F = 2t^2$ $m = 2kg$ work done in 2s. at $t=0$ block is at rest

$2t^2 = 2a$

$a = t^2$

$v = \frac{t^3}{3}$ $\longrightarrow$ $\frac{dx}{dt} = \frac{t^3}{3}$

$dx = \frac{t^3 dt}{3}$

$x = \frac{t^4}{12}$

$x = \frac{2\times2\times2\times2}{\cancel{12}\ \cancel{6}\ 3}$

$= \frac{4}{3}$

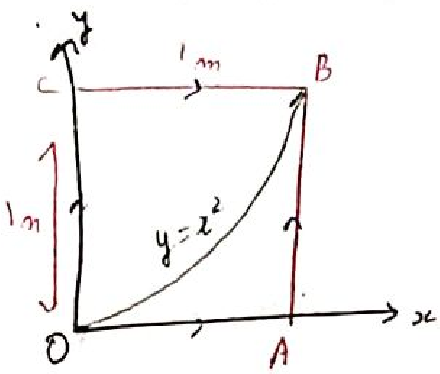
$W = \int_0^2 F\cdot dx$

$W = \int_0^2 F\times\frac{t^3}{3}dt$

$W = \int_0^2 \frac{2}{3}t^5 dt$

$W = \frac{t^6}{\cancel{2}\ 3\times\cancel{6}_3}\Big|_0^2 = \left[\frac{t^6}{9}\right]_0^2 = \boxed{\frac{64}{9}\ J}$

Q11. $F = (xy\,i + x^2y^2\,j)$, work done along 3 paths differently.



① $W_{OAB} = W_{OA} + W_{AB}$

$O \to A \Rightarrow y = 0,\ dy = 0$

$W_{OA} = \int F_x\,dx + \int F_y\,dy$

$= \int xy\,dx + \int x^2y^2\,dy$

$= 0$

$A \to B$

$x = 1,\ dx = 0,\ y\ 0 \to 1$

$W_{AB} = \int 0 + \int_0^1 (1)y^2\,dy$

$= \frac{1}{3}\,J$

$W_{OAB} = 0 + \frac{1}{3} \boxed{= \frac{1}{3}\,J}$

② $W_{OCB} = W_{OC} + W_{OB}$

$O \to C$

$x = 0,\ dx = 0$

$W_{OC} = \int 0 + \int 0 = 0$

$C \to B$

$y = 1\ \ dy = 0 \quad x\ 0 \to 1$

$W_{OB} = \int_0^1 x\,(x(1))\,dx + \int 0$

$= \frac{1}{2}\,J$

$\boxed{W_{OCB} = \frac{1}{2}\,J}$

③ W_{OB}

$O \to B$

$W_{OB} = \int xy\,dx + \int x^2y^2\,dy$

$= \int_0^1 x(x^2)\,dx + \int_0^1 y(y^2)\,dy$

$= \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}\,J}$

Q12. A particle of mass 0.5 kg travels in a straight line with $v = ax^{3/2}$, $a = 5\ m^{-1/2}s^{-1}$ what is work done from $x=0$ to $x=2$.

$$W = \int_0^2 F \cdot dx$$

$$F = ma$$

$$F = \frac{a}{2}$$

$$F = \cancel{\frac{dv}{dt}}\ v\frac{dv}{dx} \times \frac{1}{2}$$

$$F = \frac{1}{2} \times ax^{3/2} \times \frac{d}{dx}(ax^{3/2})$$

$$F = \frac{5}{2}x^{3/2} \times 5 \times \frac{x^{1/2}}{2} \times 3$$

$$F = \frac{75}{4}x^2$$

$$W = \int_0^2 \frac{75}{4}x^2 \cdot dx$$

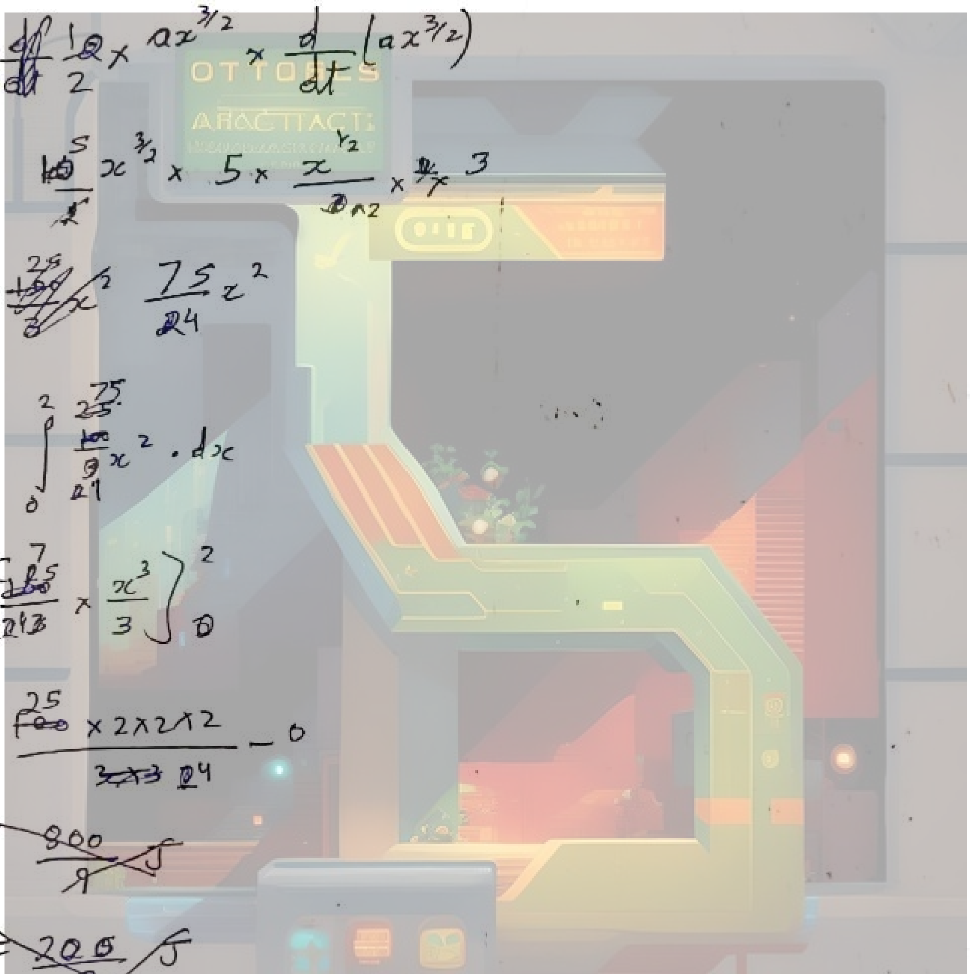
$$= \left[\frac{75}{4} \times \frac{x^3}{3}\right]_0^2$$

$$= \frac{25 \times 2 \times 2 \times 2}{4} - 0$$

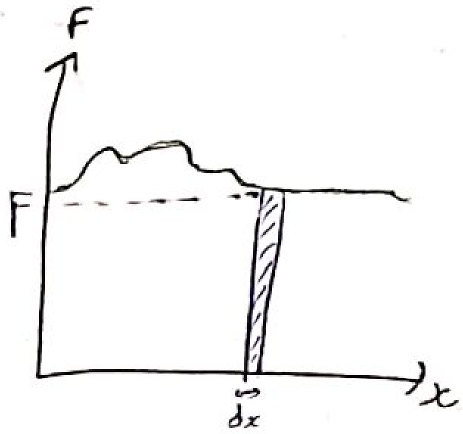
$$= \cancel{\frac{800}{9}\ J}$$

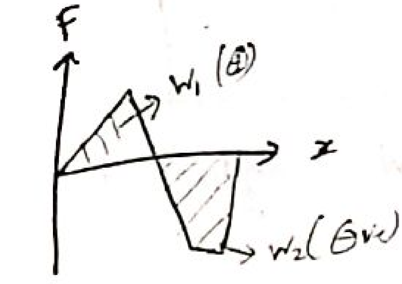
$$= \cancel{\frac{200}{9}\ J}$$

$$\boxed{= 50\ J}$$



Area under Force-Displacement graph.



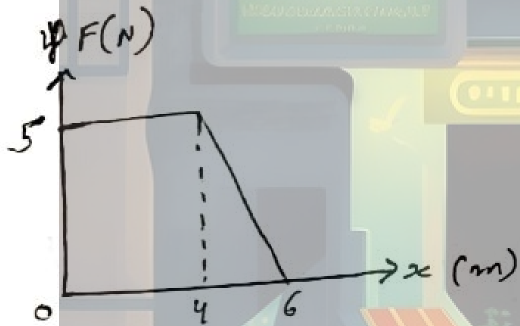


$F = W_1 - W_2$

$W = \int F\,dx$

Area of F-x graph = work done

Q13. calculate work done.

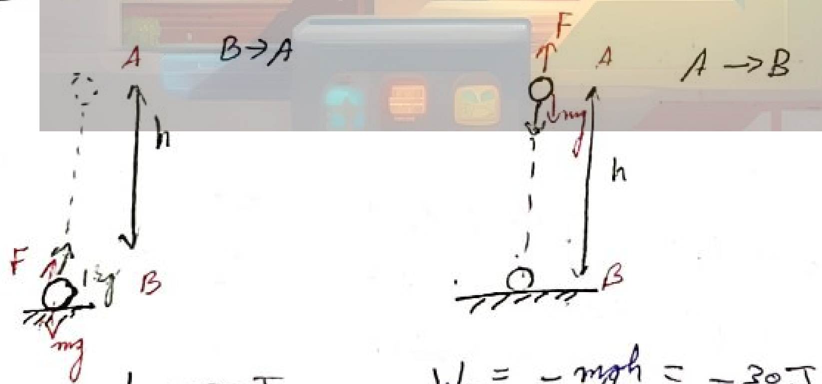


$W = \text{area} = 5\times4 + \cancel{\tfrac{1}{2}\times4\times12}\ \tfrac{1}{2}\times5\times2$

$= 20 + 5$

$= 25\ J$

Work Done By Gravity :-



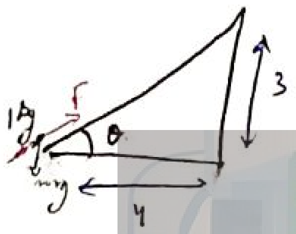
$W_F = mgh = 30\ J$

$W_{mg} = -mgh$

$W_F = -mgh = -30\ J$

$W_{mg} = mgh$

$W_F = mgh = 30\ J$



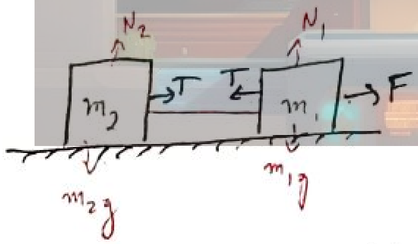
$W = mgh = 30\,J$

$30 = 5 \times F$

$F = 6\,N$

Note :- Work done against gravity depends on initial & final positions (Vertical height) & not depend on path.

Work done by a pair of interacting forces.



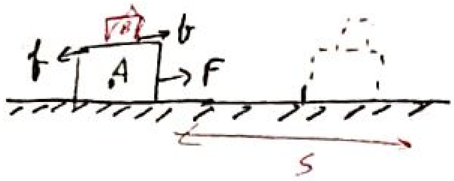
$W_{T(m_2)} = +TS$

$W_{T(m_1)} = -TS$

$W_{T(net)} = 0$

Note :- If There is no relative motion between 2 bodies, then work done by interacting forces is zero as a whole.

Static Friction

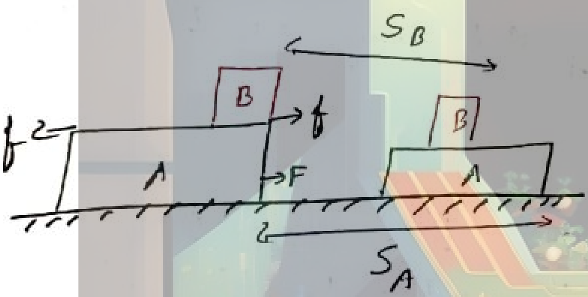


Both move together

$(W_f)_B = +fs$
$(W_f)_A = -fs$
$W_f = 0$

work done by static friction on individual bodies can be ⊕ve & ⊖ve but Total work done by friction is 0.

Kinetic friction.



$W_{f(A)} = -f S_A$
$W_{f(B)} = f S_B$
$W_f = f S_B - f S_A$
$\because S_A > S_B$
$W_f = -ve$

Kinetic Friction does +ve work on one body & more −ve work on another body. So, total work done by kinetic friction is always −ve.

Note:- Total work done by friction Do Not depend on choice of reference frame.

Energy → Ability to Do work.

Kinetic Energy :- (K) –

Energy

↓

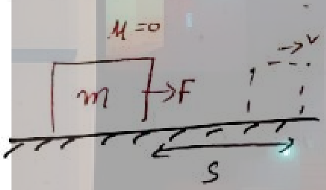
Ability to Do work.

↓

Mechanical Energy

- Kinetic Energy (K)
- Potential Energy (U)
  - Gravitational
  - Elastic

→ The energy possesed by the body due to its motion is called kinetic energy.



$W = FS$

$\because F = ma$

$W = maS$

$\because v^2 = 2aS + 0$

$2aS = \frac{v^2}{2}$

$\boxed{W = \frac{1}{2} m v^2}$

→ It is a scalar Quantity

→ SI unit :- Joule (J)

CGS :- erg

→ It depends on mass & speed of the body

→ KE is always ⊕ve for a moving body.

~~Relation btw K.E. & Momentum~~

Relation btw KE & Momentum.

$K = \frac{1}{2} m v^2 \quad ; \quad p = mv$

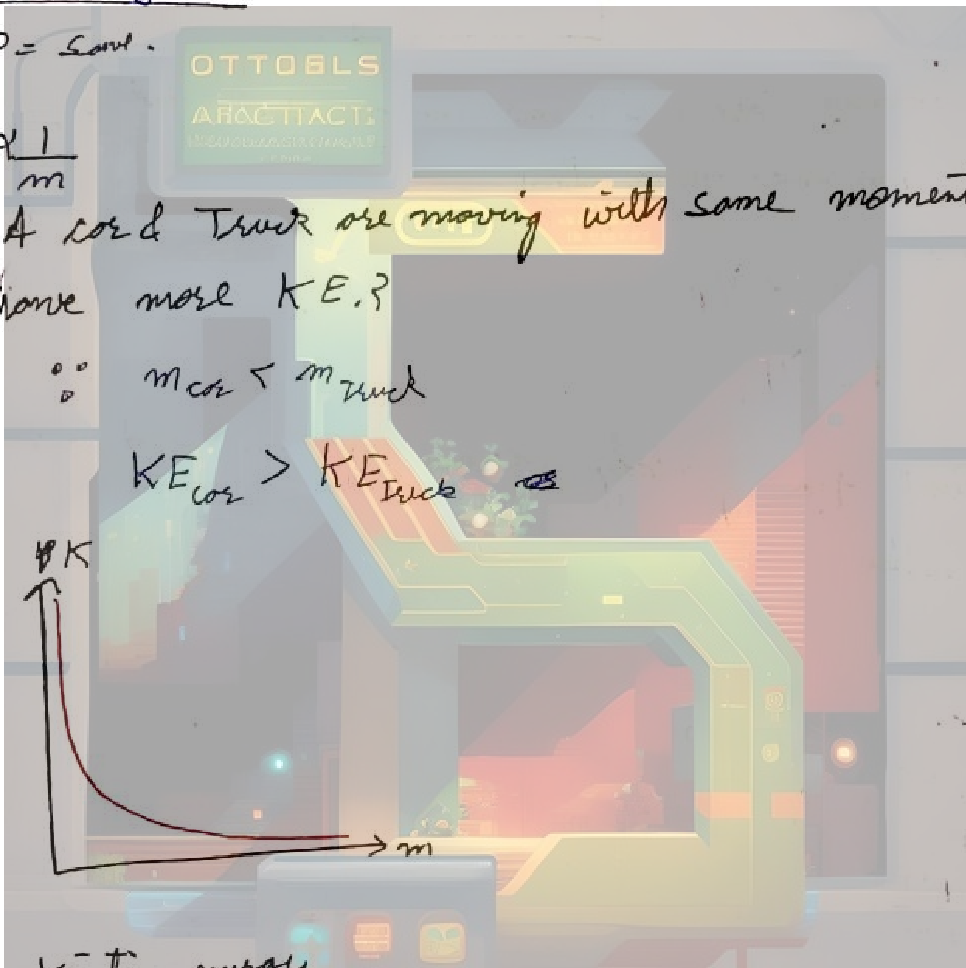
$K = \frac{1}{2m} (mv)^2$

$$\boxed{K = \frac{p^2}{2m}}$$

① Same Momentum

$P = \text{Same}.$

$K \propto \frac{1}{m}$



eg. A car & Truck are moving with same momentum which have more KE.?

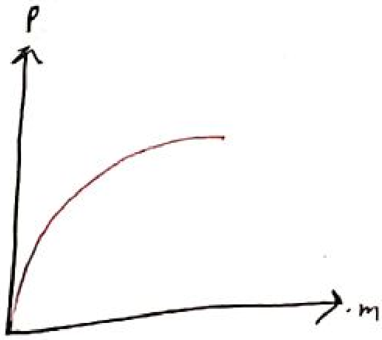
$\because m_{car} < m_{Truck}$

$KE_{car} > KE_{Truck}$

② Same Kinetic energy

$K = \text{Same}.$

$P \propto \sqrt{m}$



eg. A car & Truck is moving with same KE, which has more momentum

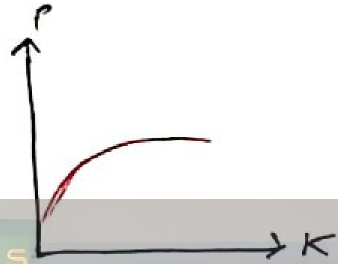
$\because m_{Truck} > m_{car}$

$p_{Truck} > p_{car}$

(3) $m = \text{Same}$

$p^2 \propto K$

$p \propto \sqrt{K}$



eg Two Similar Cars A & B are moving such that $K_A > K_B$. Which has more momentum

$\because K_A > K_B$

$p_A > p_B$

Q14. If Kinetic Energy of a body increases by 100%, find % change in its momentum.

$K = \frac{p^2}{2m}$

$K' = \frac{K}{\cancel{100}} \times 2$

$= 2K$

$K' = \frac{(p')^2}{2m}$

$2K = \frac{(p')^2}{2m}$

$p' = \sqrt{2Km}$

$p = \sqrt{Km}$

$p' - p = \sqrt{2Km} - \sqrt{Km}$

$= (\sqrt{2}-1)\sqrt{Km}$

$\%\,\text{change} = \frac{(\sqrt{2}-1)\sqrt{Km}}{\sqrt{Km}} \times 100$

$= (\sqrt{2}-1) \times 100$

$\boxed{= 41.4\%}$

Q15. If Momentum of a body increases by 50%. Keeping mass Constant, find % change in KE.

$$KE = \frac{p^2}{2m}$$

$$KE' = \frac{\left(\frac{3p}{2}\right)^2}{2m}$$

$$KE' = \frac{9p^2}{8m}$$

$$\%\ \text{change} = \frac{\frac{9p^2}{8m} - \frac{4p^2}{8m}}{\frac{4p^2}{8m}} \times 100$$

$$= \frac{5}{4} \times 100$$

$$\boxed{= 125\%}$$

Work Kinetic Energy Theorem

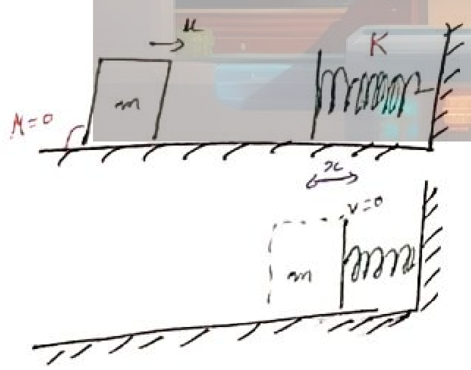
$W = \Delta K$ [change in Kinetic energy]

$W = K_f - K_i$

$$\boxed{K_i \pm W = K_f}$$

eg.

Find maximum compression of spring.

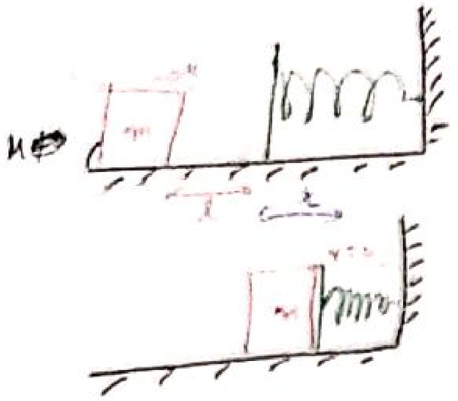


$$\frac{1}{2}mu^2 - \frac{1}{2}Kx^2 = \frac{1}{2}m(0)^2$$

$$mu^2 = Kx^2$$

$$\boxed{x = u\sqrt{\frac{m}{K}}}$$

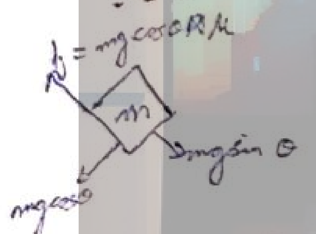
Eg 2.



$$\frac{1}{2}mu^2 - \mu mg(l+x) - \frac{1}{2}Kx^2 = \frac{1}{2}m(0)^2$$

$$\boxed{\frac{1}{2}mu^2 - \mu mg(l+x) - \frac{1}{2}Kx^2 = 0}$$

Q10. write equation for max compression in the spring & distance by which block rebound up the inclined plane.



$W_f = -\mu mg\cos\theta(l+x)$

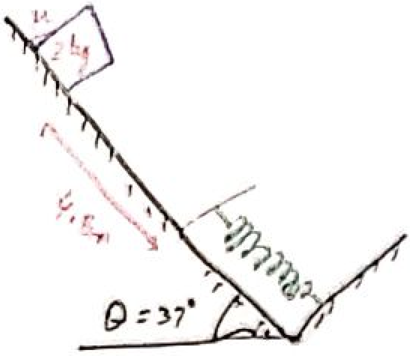
$W_g = +mg\sin\theta(l+x)$

$$\underbrace{0}_{K_i} + \underbrace{mg\sin\theta(l+x)}_{W_g} - \underbrace{\mu mg\cos\theta(l+x)}_{W_f} - \underbrace{\frac{1}{2}Kx^2}_{W_{spring}} = \underbrace{0}_{K_f}$$

$\cancel{0 - mgx}$

$$0 - mg\sin\theta\, d - \mu mg\cos\theta\, d + \frac{1}{2}kx^2 = 0$$

Q17.



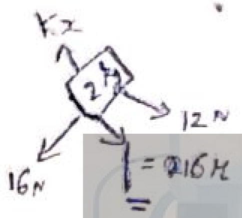
$x = 20\,cm = 0.2\,m = \frac{1}{5}\,m$

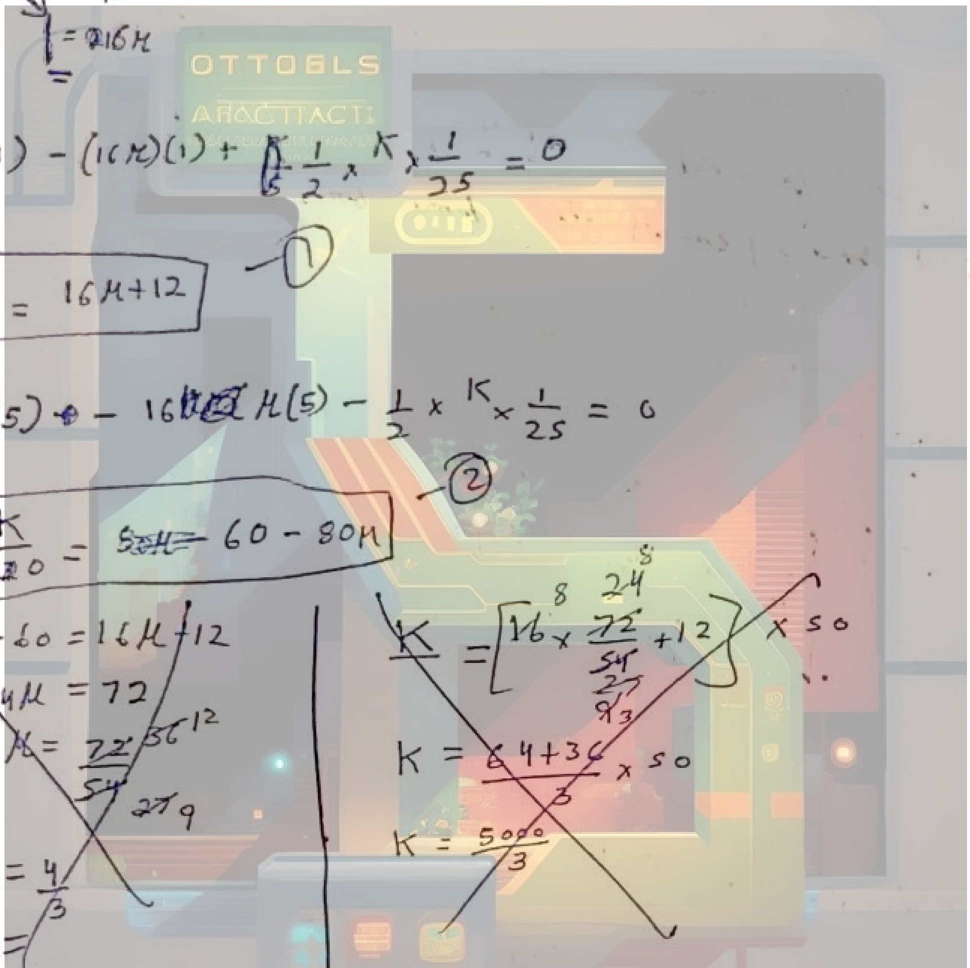
released $= 1\,m$

a) find coefficient of friction btw block & plane

b) find spring constant.

$g = 10\,m/s^2$





$0 - 12(1) - (16\mu)(1) + \frac{1}{2} \times K \times \frac{1}{25} = 0$

$\boxed{\frac{K}{50} = 16\mu + 12}$ — (1)

$0 + 12(5) - 16\mu(5) - \frac{1}{2} \times K \times \frac{1}{25} = 0$

$\boxed{\frac{K}{50} = 60 - 80\mu}$ — (2)

$\cancel{80\mu - 60 = 16\mu + 12}$

$\cancel{54\mu = 72}$

$\cancel{\mu = \frac{72}{54}}$

$\cancel{\mu = \frac{4}{3}}$

$\cancel{K = \left[16 \times \frac{72}{54} + 12\right] \times 50}$

$\cancel{K = \frac{64 + 36}{3} \times 50}$

$\cancel{K = \frac{5000}{3}}$

$60 - 80\mu = 16\mu + 12$

$48 = 96\mu$

$\mu = \frac{1}{2}$

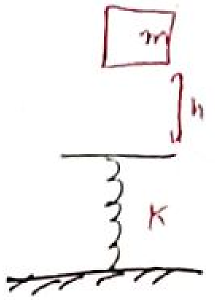
$\boxed{\mu = 0.5}$

$K = (8 + 12)\,50$

$= 20 \times 50$

$\boxed{= 1000}$ N/m

Q. find max compression. (write eqn)



$$0 + mg(h+x) - \frac{1}{2}kx^2 = 0$$

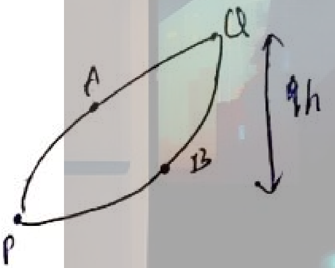
$$\boxed{\frac{1}{2}kx^2 = mg(h+x)}$$

Conservative & non conservative Forces

Conservative → The work done by conservative force on a particle moving between any two points is independent of the path taken by particle.

→ The work done by conservative force on a particle moving through any closed loop is zero.

eg. spring ~~force~~ force, gravitational force etc.

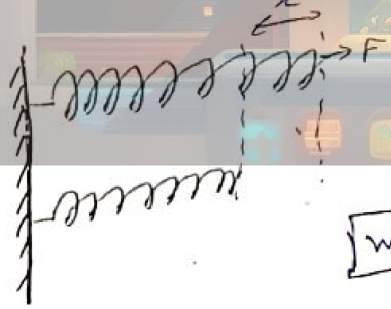


$W_{PAQ} = -mgh$

$W_{QBP} = mgh$

$\boxed{W_T = 0}$

eg2.



$W_{S_1} = -ve$

$W_{SP} = +ve$

$\boxed{W_T = 0}$

Non Conservative - The work done by non-conservative force is not only depends on initial and final positions but also on the path followed.

→ The work done by non conservative force on a particle moving in a closed loop is not zero.

eg. friction force

$W_{f\,A\to B} = -f l_1$

$W_{f\,B\to A} = -f l_2$

$W_{f\,total} = -f(l_1 + l_2)$

Potential Energy

→ When a conservative force acts on a system, It changes energy of system.

→ The energy possessed by a particles of a system due to their position, shape or config of the system is called potential energy.

Work - Potential Energy Theorem

→ The conservative force always does positive work at the expense of its potential energy stored in its field.

eg. Work done by spring is equal to the loss in P.E

$W_{conservative\ force} = -\Delta U \qquad \Delta K + \Delta U = 0$

Gravitational gravitational energy

1. Uniform gravity :-

for $h << R$ (radius of Earth)

$W_F = + mgh$

$W_g = - mgh$

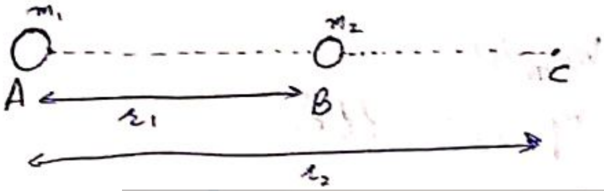
$-mgh = -\Delta U_g$

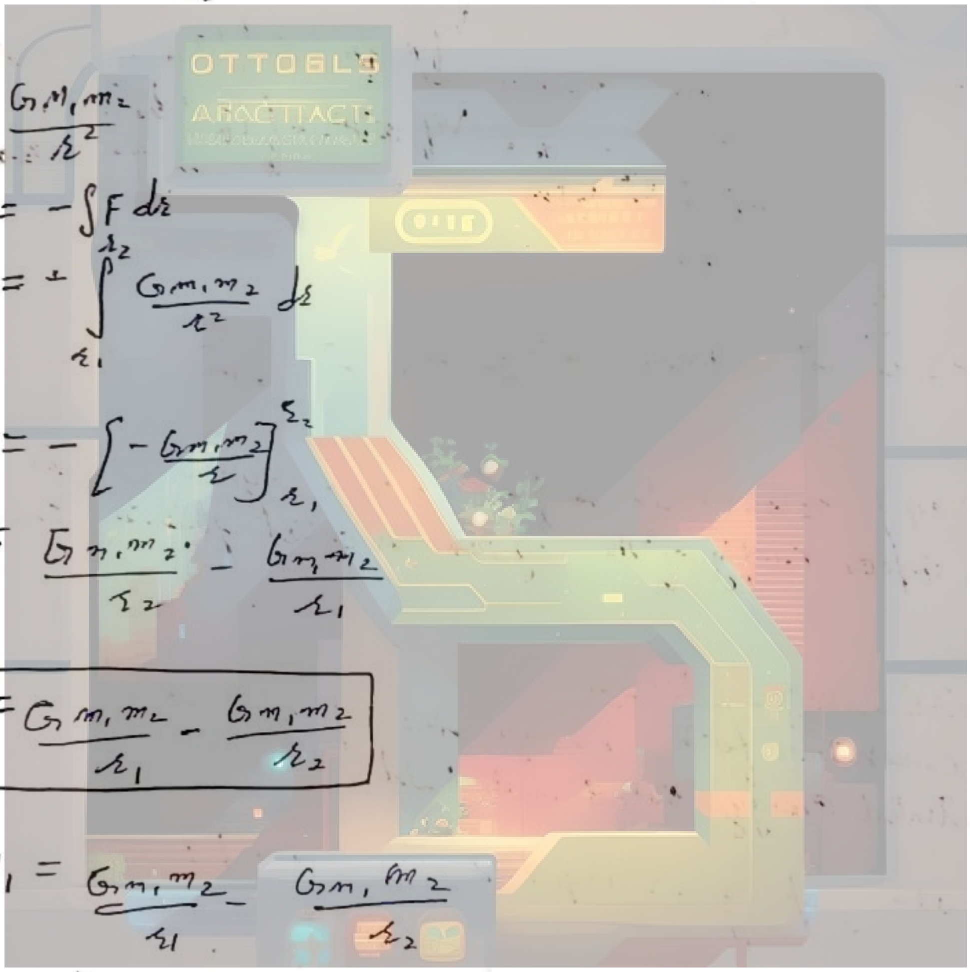
$\Delta U_g = mgh$

$\Delta U_g = mgh$

→ Can be $\oplus$ve, $\ominus$ve or zero
→ Depends on the choice of reference ~~point~~ line.
→ Defined for only conservative forces.

2. Non-Uniform Gravity:-





$$F = \frac{G m_1 m_2}{r^2}$$

$$W_g = -\int F\,dr$$

$$= -\int_{r_1}^{r_2} \frac{G m_1 m_2}{r^2}\,dr$$

$$= -\left[-\frac{G m_1 m_2}{r}\right]_{r_1}^{r_2}$$

$$W_g = \frac{G m_1 m_2}{r_2} - \frac{G m_1 m_2}{r_1}$$

$$\boxed{\Delta U_g = \frac{G m_1 m_2}{r_1} - \frac{G m_1 m_2}{r_2}}$$

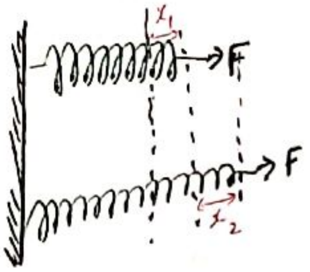
$$U_2 - U_1 = \frac{G m_1 m_2}{r_1} - \frac{G m_1 m_2}{r_2}$$

$$r_2 \to \infty, U_2 = 0$$

$$-U_1 = \frac{G m_1 m_2}{r_1} - 0$$

$$\boxed{U_1 = -\frac{G m_1 m_2}{r_1}}$$

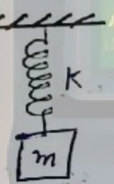
Elastic Potential Energy



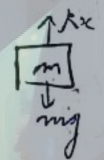
$$\Delta U_{sp} = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2$$

$$W_F = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2 \quad (\text{from } x_1 \to x_2)$$

Q19. Find Maximum ~~length~~ elongation in spring & elongation in equilibrium position. Initially spring is in natural length when block is released.

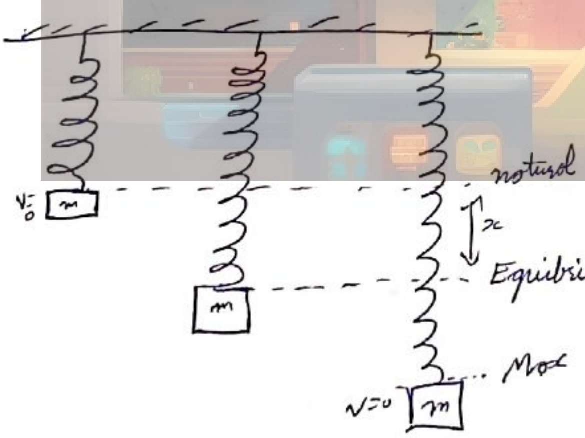


at equilibrium position



$$kx = mg$$

$$\boxed{x = \frac{mg}{k}} \text{ equilibrium position}$$



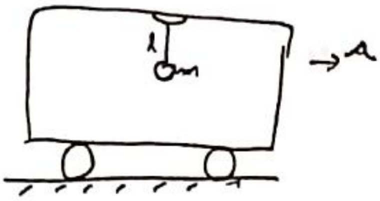
$$K_i + W = K_f$$

$$0 + mg x_{max} - \frac{1}{2} k x_{max}^2 = 0$$

$$mg = \frac{1}{2} k x_{max}$$

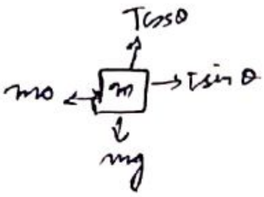
$$\boxed{x_{max} = \frac{2mg}{k}}$$

Q 20.



Find θ deflection θ from vertical
i) in equlibrium. ii) max deflection

i)



$T \sin\theta = ma$

$T = \frac{ma}{\sin\theta}$

$T\cos\theta = mg$

$\frac{ma}{\sin\theta}\cos\theta = mg$

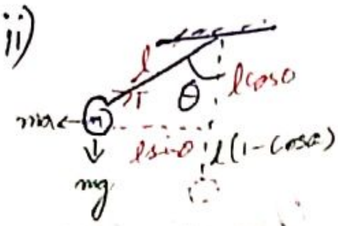
$\cot\theta = \cancel{\frac{mg}{}}\ \frac{g}{a}$

$$\boxed{\theta = \cot^{-1}\left(\frac{g}{a}\right)}$$

Equlibrium.

$\theta = \tan^{-1}\left(\frac{a}{g}\right)$

ii)



$K_i \pm W = K_f$

$0 \underbrace{- mg\,l(1-\cos\theta)}_{W_g} + \underbrace{ma\,l\sin\theta}_{W_{ma}} + \underbrace{0}_{W_T} = 0$

$a\,l\sin\theta = g\,l(1-\cos\theta)$

$a\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = g\left(2\sin^2\frac{\theta}{2}\right)$

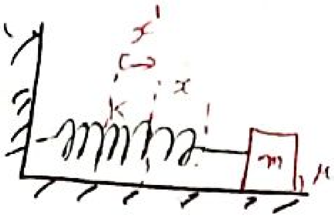
$\frac{a}{g} = \tan\left(\frac{\theta}{2}\right)$

$\frac{\theta}{2} = \tan^{-1}\left(\frac{a}{g}\right)$

$\theta = 2\tan^{-1}\left(\frac{a}{g}\right)$

Q 21. Initial elongation $x = \frac{3Mmg}{k}$, Block is released from rest.

i) initial acc
ii) max compression
iii) max speed.



i) $Kx \leftarrow \boxed{m} \rightarrow f$, N, mg

$f_k = \mu mg$

$Kx - f = ma$

$\cancel{K} \times \frac{3\mu mg}{\cancel{K}} - \mu mg = ma$

$\boxed{2\mu g = a}$ ✓

ii) $K_i \pm W = K_f$

$0 + \frac{1}{2}Kx^2 - \cancel{\mu mg}\ \frac{1}{2}K(x')^2 - \mu mg\,(x + x') = 0$

$\frac{1}{2}Kx^2 = \frac{1}{2}K(x')^2 + \mu mg(x + x')$

$\frac{1}{2}K(x^2 - (x')^2) = \mu mg\,(x + x')$

$\frac{1}{2}K(\cancel{x + x'})(x - x') = \mu mg\,(\cancel{x + x'})$

$\frac{1}{2}K(x - x') = \mu mg$

$x - x' = \frac{2\mu mg}{K}$

$x' = x - \frac{2\mu mg}{K}$

$\boxed{x' = \frac{\mu mg}{K}}$

$\cancel{x + x' = \frac{4\mu mg}{2K}}$

iii) $\cancel{0 + \frac{1}{2}Kx^2 - \mu mg x = \frac{1}{2}mv^2}$

$\cancel{\frac{1}{2}Kx\frac{3\mu mg}{K} - \mu mg \times \frac{3\mu mg}{K} = \frac{1}{2}mv^2}$

$\cancel{\frac{3\mu mg}{2K} - \mu mg}$

$\cancel{\frac{1}{2}mv^2 = \frac{1}{2}Kx^2 - \frac{\mu mg\, x^2}{2}}$

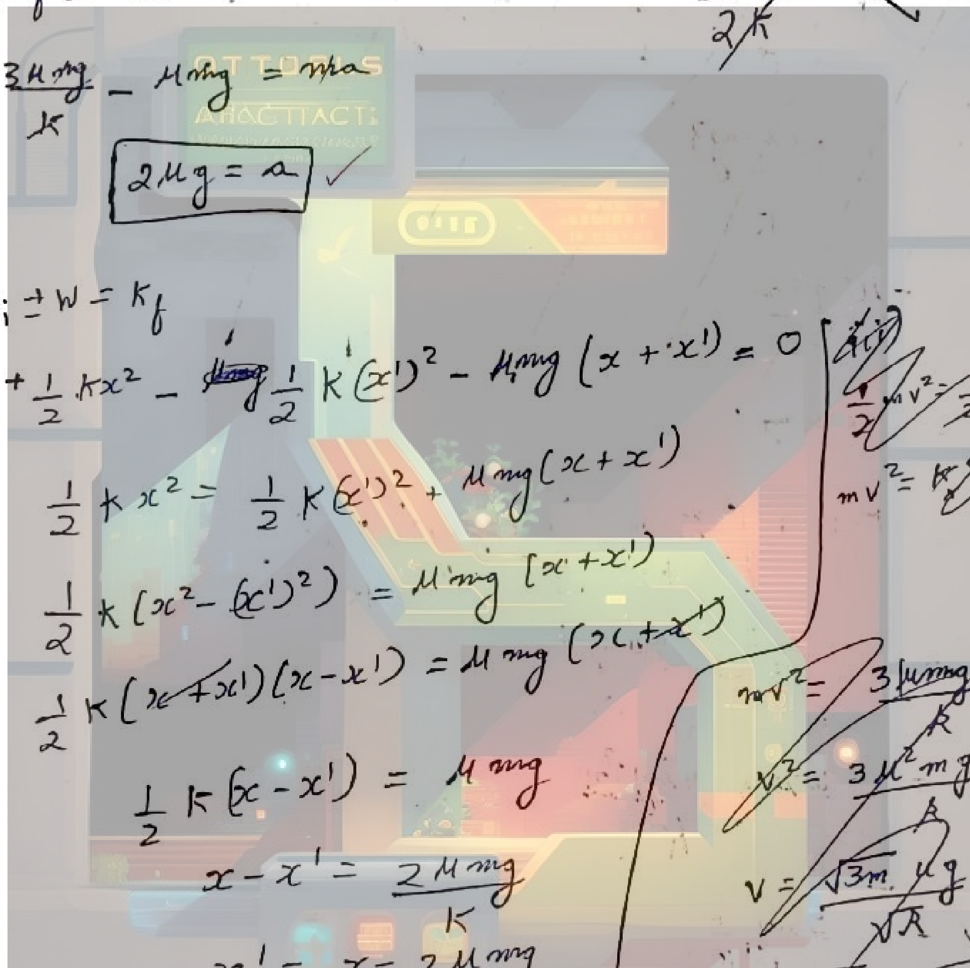
$\cancel{mv^2 = K\frac{9\mu^2m^2g^2}{K^2} - \frac{6(\mu mg)^2}{K}}$

$\cancel{mv^2 = \frac{3(\mu mg)^2}{K}}$

$\cancel{v^2 = \frac{3\mu^2 m g^2}{K}}$

$\cancel{v = \frac{\sqrt{3m}\,\mu g}{\sqrt{K}}}$

$v = \sqrt{\frac{3m}{K}}\,\mu g$



iii) Fim equation bcdan.

$\cancel{Kx = \mu mg}$

$\cancel{K \times \frac{3\mu mg}{K} = \mu mg}$

$\cancel{3 = 1}$

$Ky = \mu mg$

$y = \frac{\mu mg}{K}$

~~from Block → x − y~~

$\cancel{\frac{1}{2} m v^2 = -\frac{1}{2} K y^2 - \mu mg\, y + \frac{1}{2} K x^2}$

$\cancel{m v^2 = -K \times \frac{\mu mg}{K} \times \frac{\mu mg}{K} - 2\mu mg \frac{\mu mg}{K}}$

$\cancel{m v^2 = \frac{-(\mu mg)^2}{K}}$

$\cancel{v^2 = \frac{-\mu^2 g^2 m}{K}}$

$\cancel{m v^2 = 3\mu mg}$

$$\frac{m v^2}{2} = \frac{1}{2} K \left[\frac{8(\mu mg)^2}{K^2}\right] - \frac{(\mu mg)^2 \times 2 \times 2}{2K}$$

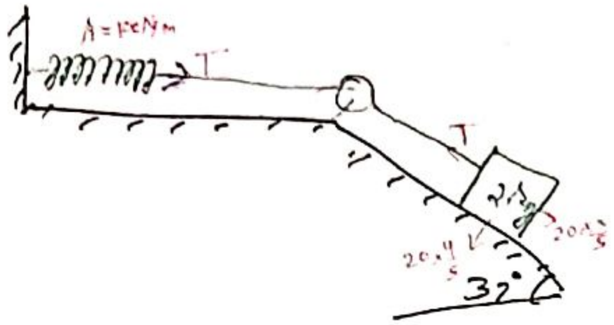
$$m v^2 = \frac{8(\mu mg)^2}{K} - \frac{4(\mu mg)^2}{K}$$

$$v^2 = \frac{4(\mu g)^2 m}{K}$$

$$\boxed{V = 2\mu g\sqrt{\frac{m}{K}}}$$

Q. block is released from rest when spring in unstrength.

a) How far does the block move down before coming momentarily to rest. what is its ac at lowest point.



$\cancel{T = kx}$

$\cancel{T = 100x}$

$\cancel{12 - 100x = 2a}$

$\cancel{6 - 50x = a}$

$K_i + W = K_f$

$0 + 12x - \frac{1}{2} \times 100 \times x^2 = 0$

$12 - 50x = 0$

So $12 = 50x$

$x = \frac{12}{50}$

$x = \frac{6}{25}$

$\boxed{x = 0.24}$

at equilibrium

$kx = 12$

$100x = 12$

$x = \frac{12}{100}$

$x = 0.12$

at lowest point

$kx - 12 = 2a$

$24 - 12 = 2a$

$12 = 2a$

$\boxed{a = 6 \text{ m/s}^2}$

b) If surface is rough and block moves 0.2 m calculate cofficient of friction

$$0 + 12(0.2) - \frac{1}{2}(100)(0.04) - \mu(16)(0.2) = 0$$

$$2.4 - 2 - 3.2\mu = 0$$

$$3.2\mu = 2.4 - 2$$

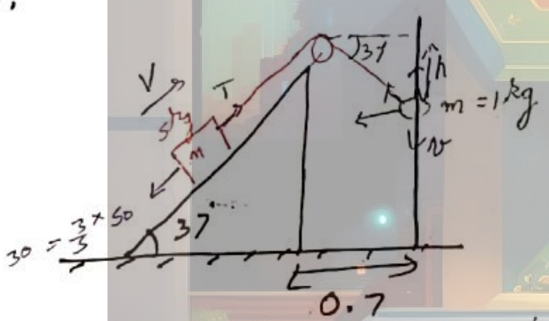
$$\mu = \frac{24-20}{32}$$

$$\mu = \frac{4}{32}$$

$$\mu = \frac{1}{8}$$

$$\boxed{\mu = 0.125}$$

Q 23.



find velocity of ring.

$$V = \frac{5V}{3}$$

$$v^2 = \frac{25V^2}{9}$$

$$\cos 37 = \frac{4}{5} = \frac{0.7}{L_2}$$

$$L_2 = \frac{7}{8}$$

$$L_2 - L_1 = \frac{7}{8} - \frac{7}{10}$$

$$= \frac{70-56}{80}$$

$$= \frac{14}{80}$$

$$\sin 37 = \frac{3}{5} = \frac{h}{7} \times 8$$

$$h = \frac{3 \times 7}{8 \times 5}$$

$$h = \frac{21}{40}$$

$$K_i + W = K_f$$

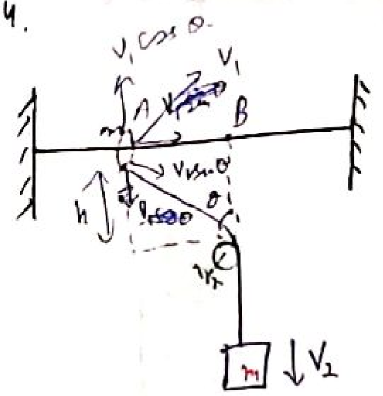
$$\frac{21mg}{40} - \frac{7}{40}\sin 37^\circ Mg = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

$$\frac{21}{40} \times 1 \times 10 - \frac{7}{40} \times \frac{3}{5} \times 5 \times 10 = \frac{1}{2} \times 1 \times \frac{25V^2}{9} + \frac{5}{2} \times V^2$$

$$\frac{21}{4} - \frac{21}{4} = \frac{25V^2}{18} + \frac{45V^2}{18}$$

$$\boxed{\begin{array}{l} V = 0 \\ v = 0 \end{array}}$$

Q24.



$\theta = 60^\circ$

$$\cos\theta = \frac{h}{p}$$

$$p = \frac{h}{\cos\theta}$$

$$p - h = \frac{h}{\cos\theta} - h$$

$$= \frac{h - h\cos\theta}{\cos\theta}$$

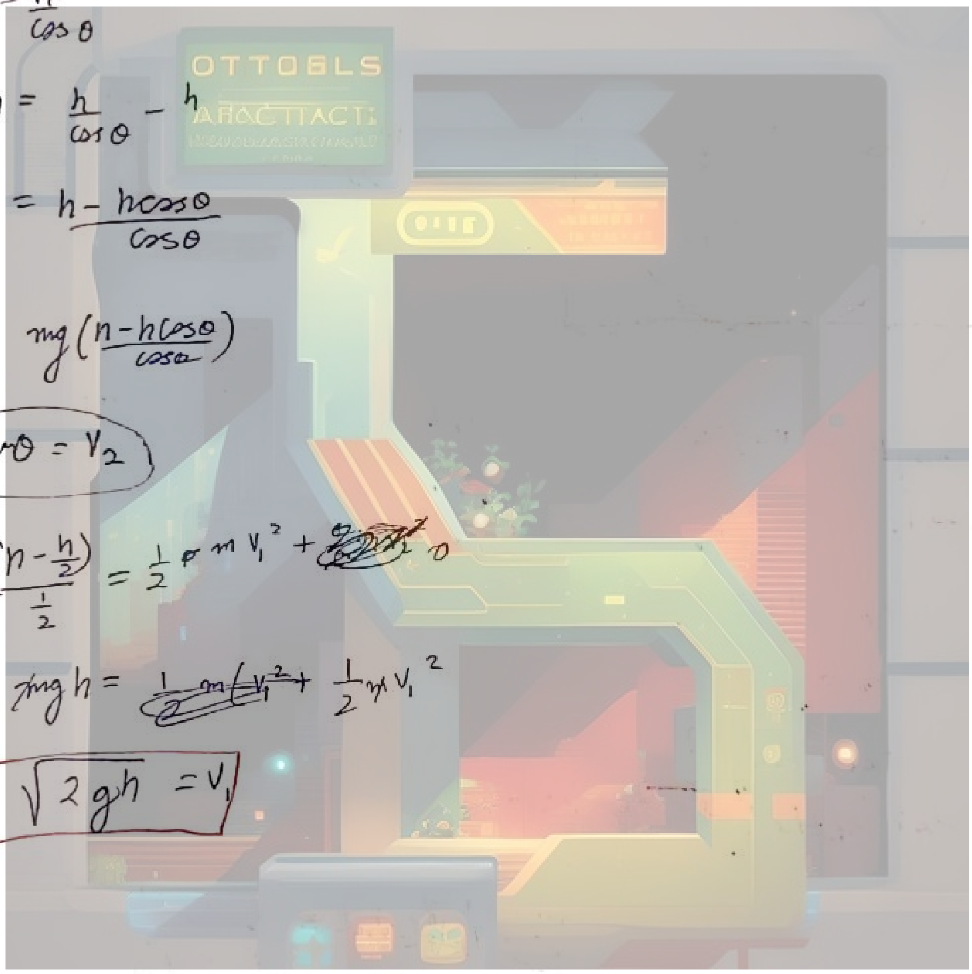
$$W_{mg} = mg\left(\frac{h - h\cos\theta}{\cos\theta}\right)$$

$$V_1 \sin\theta = V_2$$

$$mg\left(\frac{h - \frac{h}{2}}{\frac{1}{2}}\right) = \frac{1}{2} m V_1^2 + \cancel{\;} 0$$

$$mgh = \cancel{\frac{1}{2} m V_1^2} + \frac{1}{2} m V_1^2$$

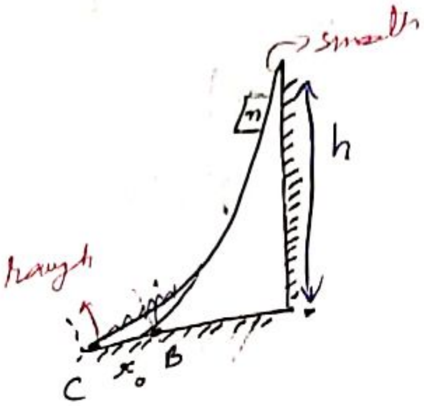
$$\boxed{\sqrt{2gh} = V_1}$$



Q'25. find coefficient of friction if block stops btw B → C.

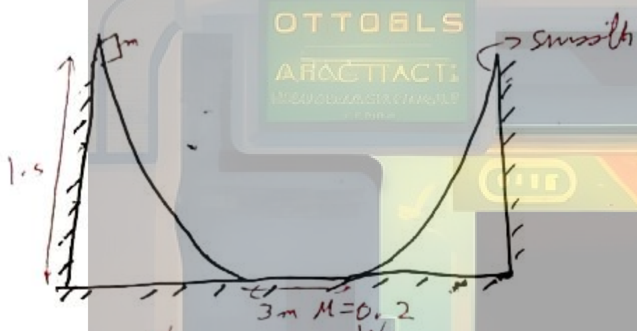
$$\overbrace{mgh}^{\text{gravity}} = \overbrace{\mu mg\,(x_0)}^{\text{friction}}$$

$$\boxed{\mu = \frac{h}{x_0}}$$



→ work done by gravity depends only on height.

Q 26.



When does the block come to rest.

$$\overbrace{mgh}^{W_g} = \overbrace{\mu mg(x_0)}^{W_f}$$

x — Total distance travelled by block on rough surface before comming to rest.

$$1.5 = 0.2\,(x_0)$$

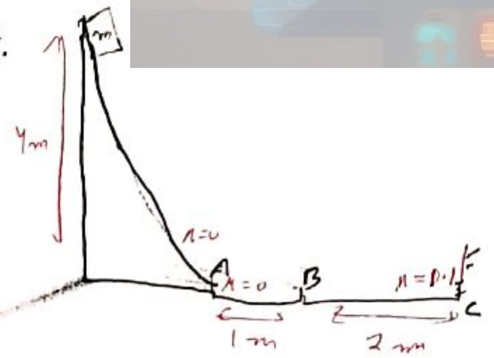
$$x_0 = \frac{1.5}{0.2}$$

$$x_0 = \frac{15}{2}$$

$$x_0 = 7.5$$

so block stops $\boxed{\text{at } 1.5\text{ m from B}}$

Q 27.



m = 0.1 kg
find total distance covered by block on horizontal surface before coming to rest.

$g = 10\ m/s^2$

$m g h = \mu m g x$

$\frac{4}{0.1} = x$

$x = 40$

$2n = 40$

$n = 20$ (n = no. of time move through horizontal)

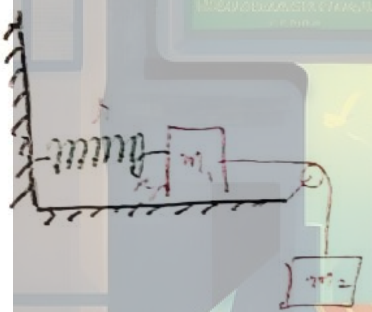
distance $= 60$

stops at B & not A

$\therefore$ distance travelled $= 60 - 1$

$\boxed{= 59\,m}$

Q28.



find μ_k if m_2 falls 'h' before coming to rest.

$m_2 g h - \frac{1}{2} k x^2 - \mu m_1 g (x) = 0 \quad (x = h)$

$m_2 g h = \frac{1}{2} k h^2 + \mu m_1 g \times h$

$\mu = \dfrac{m_2 g h - \frac{1}{2} k h^2}{m_1 g h}$

$\mu = \dfrac{10 m_2 - \frac{k h}{2}}{10 m_1}$

$\mu = \dfrac{20 m_2 - k h}{20 m_1}$

$\boxed{\mu = \dfrac{2 g m_2 - k h}{2 g m_1}}$

Q spring initial stretched 5 cm, what work to stretch more by 5 cm

$k = 5\times10^3\ N/m$

$W = E_f - E_i$

$W = \frac{1}{2}k(0.1)^2 - \frac{1}{2}k(0.05)^2$

$W = \frac{1}{2}k(0.01 - 0.0025)$

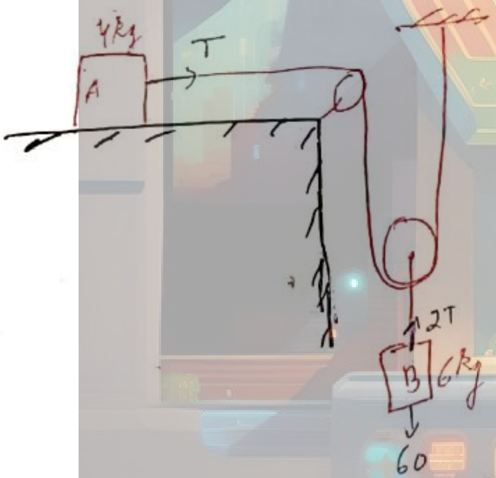
$W = \frac{1}{2}k \times 0.0075$

$W = \frac{1}{2}\times 7.5\times10^{-3}\times 5\times10^{3}$

$W = \frac{37.5}{2}$

$\boxed{W = 18.75\ J}$

Q



Block has speed 2 m/s after decending 1 m. find μ btw table & A.

$v^2 - u^2 = 2as$

$\frac{0.09}{2} = a$

$a = \frac{9}{200}$

$\overbrace{60(1)}^{W_g} - \overbrace{\mu(40)(2)}^{W_f} = \frac{1}{2}\times 6 \times 4^2$

$60 - 80\mu = 12$

$\frac{48}{80} = \mu$

$\mu = \frac{3}{5}$

$\mu = 0.6$

$K_i \pm W_{\phi} = K_f$

$$0 + \overbrace{60(1)}^{W_g} - \overbrace{\mu(40)(2)}^{W_f} = \overbrace{\frac{1}{2} \times 6 \times 4^2}^{KE_A} + \overbrace{\frac{1}{2} \times 4^2 \times 16}^{KE_B}$$

$$60 - 80\mu = 12 + 32$$

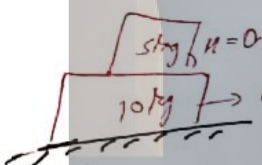
$$80\mu = 16$$

$$\mu = \frac{16}{80}$$

$$\mu = \frac{1}{5}$$

$$\Rightarrow \boxed{\mu = 0.2}$$

Q 31. for Distance of 2m.



$$\overbrace{0}^{K_i} + \overbrace{15(2)}^{W_F} = \overbrace{\frac{1}{2} \times 15 \times v^2}^{K_f}$$

$$\frac{60}{15} = v^2$$

$$v^2 = 4$$

$$\boxed{v = 2\ m/s}$$

$N = 50N$

$f_e = \mu N$

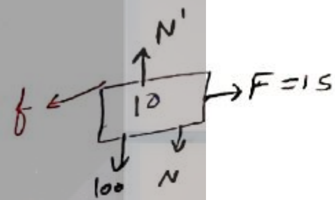
$= 0.5 \times 50$

$= 25N$

If both move together,

$15 = (5+10)a$

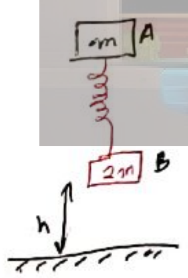
$\boxed{a = 1\ m/s^2}$



Validate for 5kg.

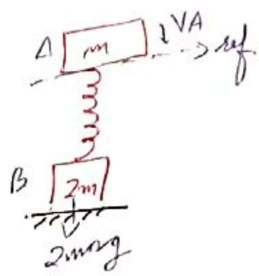
$f = 5 \times 1 = 5N < f_e$

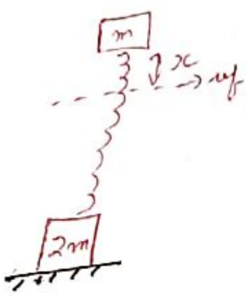
Q32. From what min h must the system be released that after a perfectly inelastic collision, B may be lifted off.



$v^2 = 0 + 2gh$

$v_A = \sqrt{2gh}$





To lift $2m$; $Kx = 2mg$

$$x = \frac{2mg}{k} \text{ (elongation)}$$

$$mgx + \frac{1}{2}Kx^2 + K_A = \frac{1}{2}mv_A^2$$

$$mg\left(\frac{2mg}{K}\right) + \frac{1}{2}K \times \frac{\cancel{2}m^2g^2}{K^{\cancel{2}}} \times 2 + K_A = \frac{1}{2}m(2gh)$$

$$\frac{4m^2g^2}{K} + K_A = mgh$$

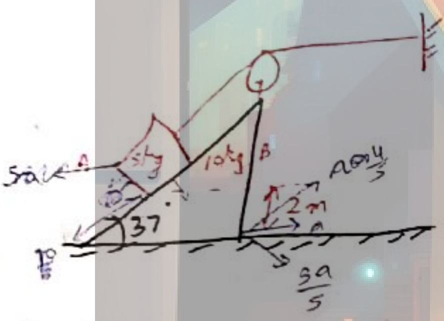
$K_A \geq 0$

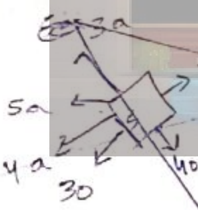
So,

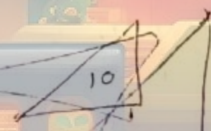
$$\frac{4m^2g^2}{K} \leq mgh$$

$$\boxed{h \geq \frac{4mg}{K}}$$

Q33. find vel of A & B when A is about to touch ground.







$V_A = V_B = V$

$K_i \pm W = K_f$

~~$5 \times 10 \times 2 = \frac{1}{2} \times 5 \times v^2 + \frac{1}{2} \times 10 \times v^2$~~

~~$200 = 15v^2$~~

~~$v^2 = \frac{200}{15}$~~

$V_A = V_B$

$$V_A \text{ (wrt ground)} = \sqrt{V^2 + V^2 + 2V^2(-\cos 37^\circ)}$$

$$= \sqrt{2V^2 - 2V^2 \times \frac{4}{5}}$$

$$= V\sqrt{\frac{2}{5}}$$

$$\underbrace{5 \times 10 \times 2}_{W_g} = \underbrace{\frac{1}{2} \times 10 \times V^2}_{KE_B} + \underbrace{\frac{1}{2} \times 5 \times \frac{2V^2}{5}}_{KE_A}$$

$$100 = 5V^2 + V^2$$

$$V^2 = \frac{100}{6}$$

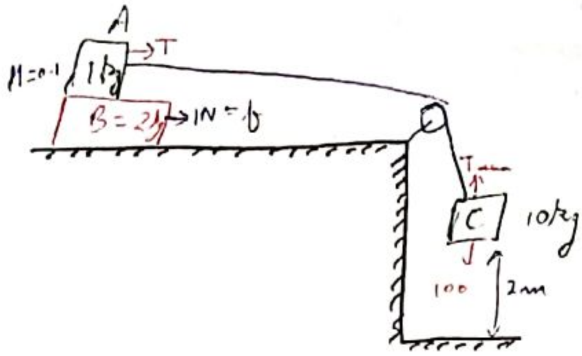
$$\boxed{V = \sqrt{\frac{50}{3}}}$$

$V_B = 5\sqrt{\frac{2}{3}}$ m/s

$V_A = 2\sqrt{\frac{5}{3}}$ m/s

Q24.

find velocity of 3 blocks when C has descended 2m



$f_l = 1N$

$100 - T = 10a$

$T - 1 = a$

$99 = 11a$

$\boxed{A_{A,C} = 9\ m/s}$

$T - 1 = 9$

$T = 10$

$2 = \frac{1}{2} \times 9 \times t^2$

$t^2 = \frac{4}{9} \Rightarrow t = \frac{2}{3}$

$x = \frac{1}{2} \times \frac{1}{2} \times \frac{4}{9}$

$x = \frac{1}{9}$

$1 = 2a$

$\boxed{a_B = 1/2}$

$\cancel{100(2) - 10(2) + 10(2)} - 1\left(\frac{1}{9}\right) = \frac{1}{2} 10 (V_C)^2 + \frac{1}{2} \times 1 (V_A)^2 + \frac{1}{2} \times 2 \times (V_B)^2$

$\cancel{\frac{1799}{9} = 5v^2 + \frac{1}{2}v^2 + V_B^2}$

$\cancel{\frac{1799}{9} = \frac{11}{2} \times 36 + V_B^2}$

$\cancel{\frac{1799}{99 \times 18} = V_B^2}$

$v^2 = 0 + 2 \times 9 \times 2$

$v^2 = 36$

$v = 6$

$\boxed{V_{A,C} = 6}\ m/s$

$V_B = \sqrt{2(\frac{1}{2})(\frac{1}{9})}$

$= \sqrt{\frac{1}{9}}$

$\boxed{V_B = \frac{1}{3}\ m/s}$

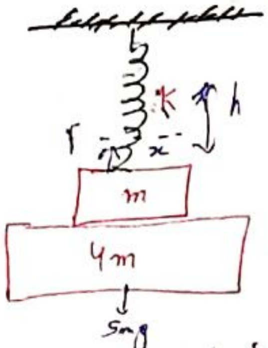
$\cancel{200 - \frac{1}{9} = 180 + 18 + (V_B)^2}$

$\cancel{2 - \frac{1}{9} = (V_B)^2}$

$\cancel{\frac{17}{9} = (V_B)^2}$

Q 35

initially at equilibrium. If 4m falls, how much will 4 m rise,



$5mg\,x = $ Spring Energy

$F = kx = 5mg$

$x = \frac{5mg}{k}$

$\underbrace{-mgh}_{W_g} + \underbrace{\frac{1}{2}\times k \times x^2}_{W_s} - \underbrace{\frac{1}{2}\times k \times (h-x)^2}_{W_s} = 0$

$\cancel{\frac{1}{2}\times k \times \frac{5mg}{k}\times\frac{5mg}{k} = mgh}$

$\cancel{\frac{5mg}{2k} = h}$

(h^2+x^2-2hx)

$\frac{1}{2}\times k \times \frac{5mg}{k}\times \frac{5mg}{k} = \frac{1}{2}\times k \times (h-x)^2 + mgh$

$\frac{25m^2g^2}{2k} = \frac{k}{2}h^2 + \frac{k}{2}x^2 - \frac{2hxk}{2} + mgh$

$\frac{25m^2g^2}{2k} = \frac{kh^2}{2} + \frac{25m^2g^2}{2k} - h(5mg - mg)$

$h(4mg) = \frac{kh^2}{2}$

$\boxed{\frac{8mg}{k} = h}$

M II elongation from equilibrium position = max height from equilibrium position

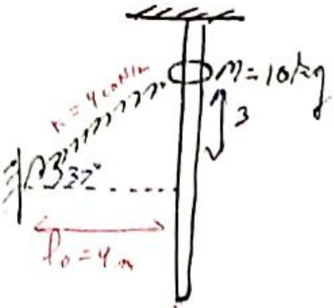
difference in equilibrium positions $= \frac{5mg}{k} - \frac{mg}{k} = \frac{4mg}{k}$

max height from equilibrium $= \frac{4mg}{k}$

initial distance from equilibrium $= \frac{4mg}{k}$

$\boxed{\max h = \frac{8mg}{k}}$

Q 36.



→ natural length = l_0

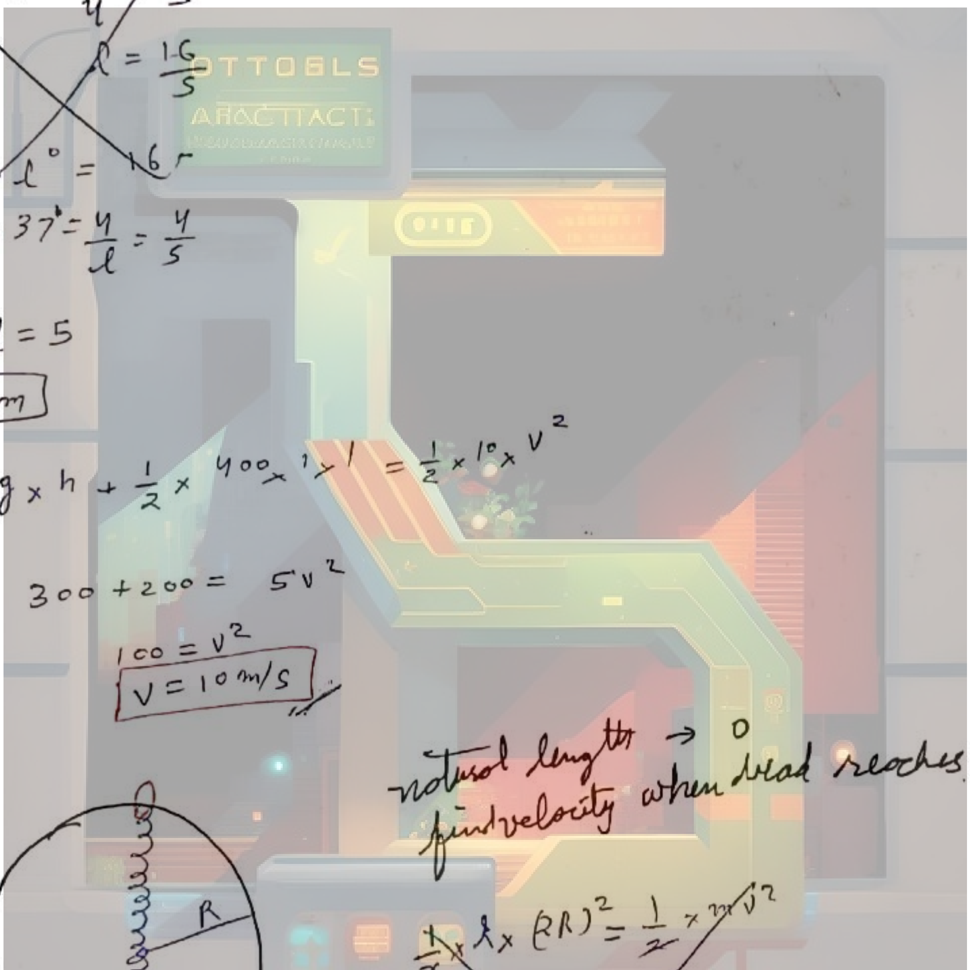
velocity of ring when spring horizontally.

~~elongation = cos 37~~

$\cancel{\cos 37 = \frac{l}{4} = \frac{4}{5}}$

$\cancel{l = \frac{16}{5}}$

$\cancel{l - l^0 = 16}$



$\cos 37° = \frac{4}{l} = \frac{4}{5}$

$l = 5$

$\boxed{x = 1 m}$

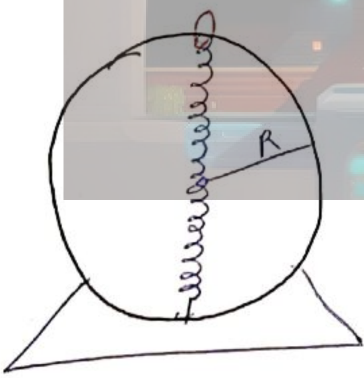
$10 \times g \times h + \frac{1}{2} \times 400 \times 1 \times 1 = \frac{1}{2} \times 10 \times v^2$

$300 + 200 = 5v^2$

$100 = v^2$

$\boxed{v = 10 m/s}$

Q 37.



natural length → 0

find velocity when bead reaches lowest position

$\cancel{\frac{1}{2} \times k \times (2R)^2 = \frac{1}{2} \times m v^2}$

$\cancel{\frac{k(2R)^2}{m} = v^2}$

$\cancel{v = \sqrt{\frac{k}{m}}\, 2R}$

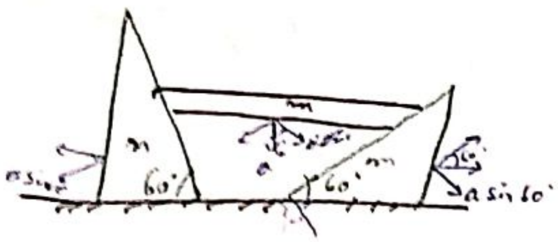
$0 + \frac{1}{2} m v^2 = \frac{1}{2} \times k \times (2R)^2 + \frac{mg\, 2R \times 2}{2}$

$v^2 = \frac{4R^2 k + 4 mg R}{m}$

$\boxed{v = 2\sqrt{\frac{R^2 k + mgR}{m}}}$

Q 38.

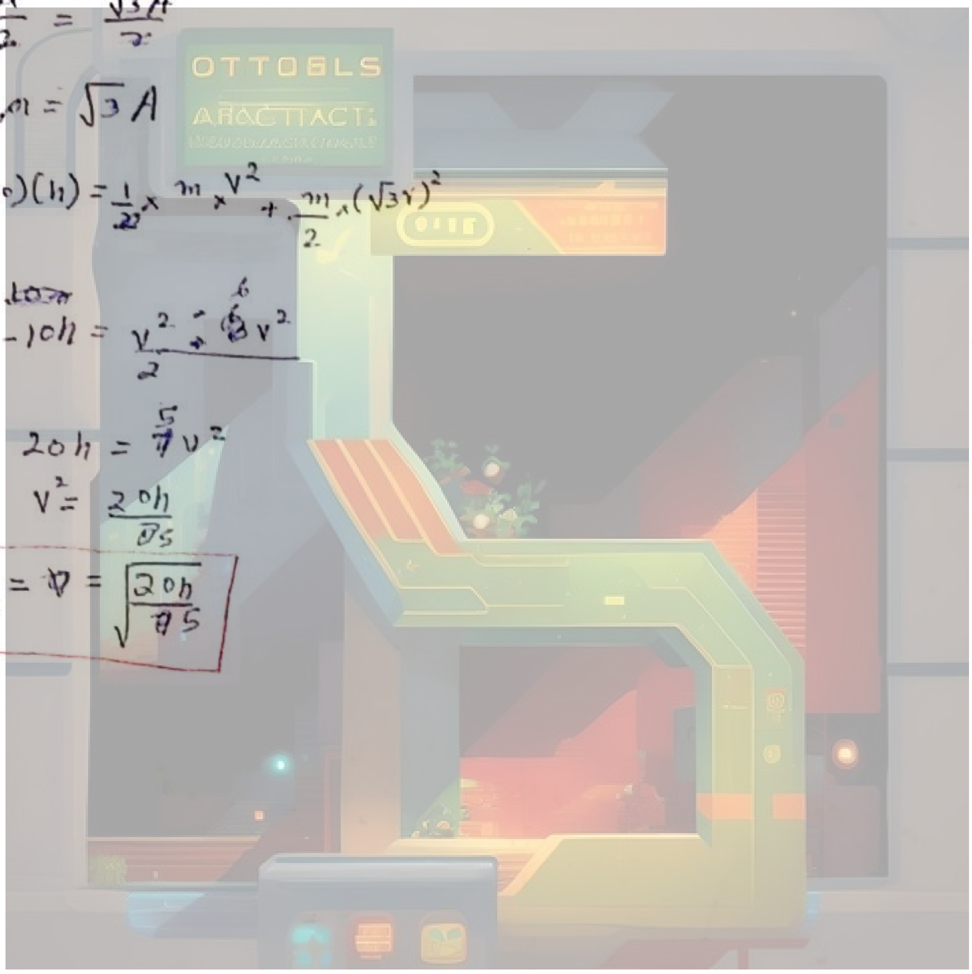
find velocities when m descends h.



$a \cos 60^\circ = A \sin 60$

$\frac{a}{2} = \frac{\sqrt{3}A}{2}$

$a = \sqrt{3}A$



$m(10)(h) = \frac{1}{2} \times m \times v^2 + \frac{m}{2} \times (\sqrt{3}v)^2$

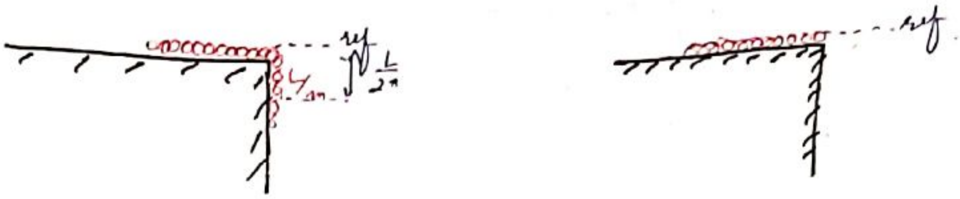
$10h = \frac{v^2 + 3v^2}{2}$

$20h = 5v^2$

$v^2 = \frac{20h}{5}$

$\boxed{V_{wedge} = v = \sqrt{\frac{20h}{5}}}$

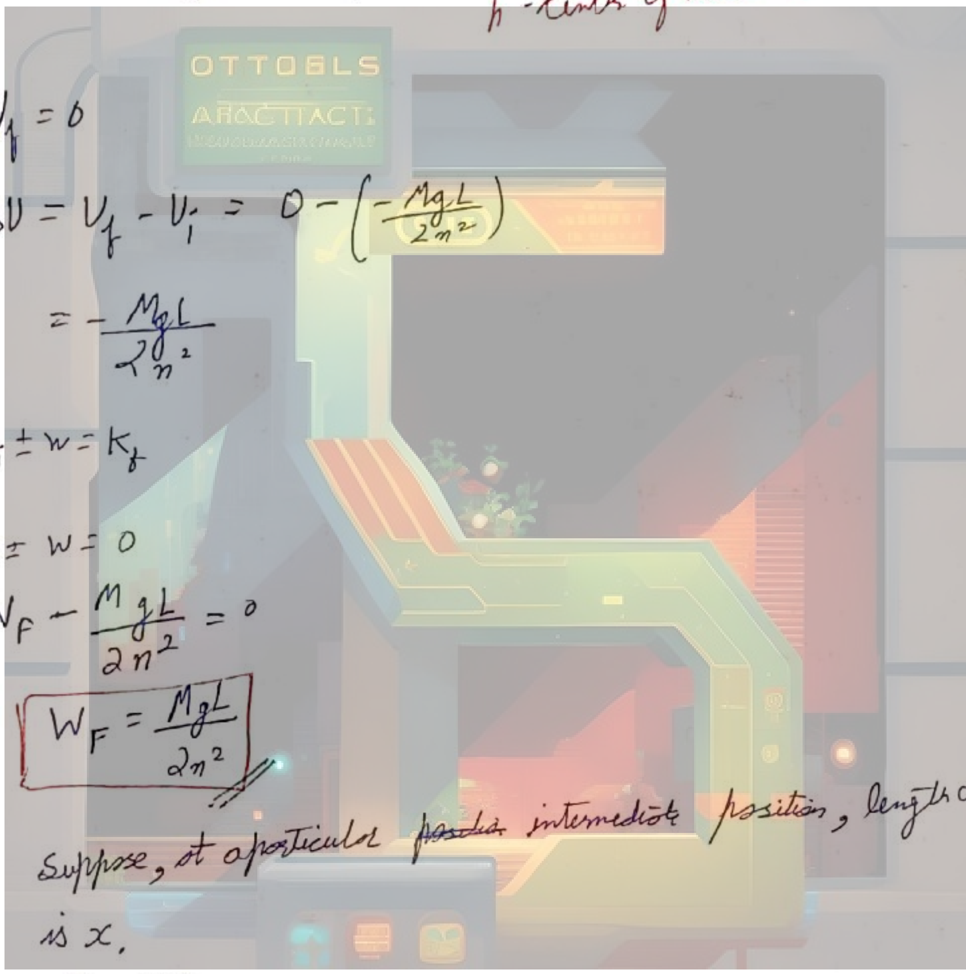
Q 39 A chain of mass M & length L is held on a smooth surface with $(1/n)^{th}$ of length hanging. Calculate ~~work~~ work done in pulling the chain on the Table slowly.



M I

$$U_i = -\frac{M}{n} g \frac{L}{2n} = \frac{-MgL}{2n^2} \qquad (\because U_i = -mgh)$$

h - centre of mass ke distance



$$U_f = 0$$

$$\Delta U = U_f - U_i = 0 - \left(-\frac{MgL}{2n^2}\right)$$

$$= -\frac{MgL}{2n^2}$$

$$K_i \pm W = K_f$$

$$0 \pm W = 0$$

$$W_F - \frac{MgL}{2n^2} = 0$$

$$\boxed{W_F = \frac{MgL}{2n^2}}$$

M II

Suppose, at a particular ~~positio~~ intermediate position, length of hanging part is x.

$$F = mg$$

$$F = \frac{Mx}{L} g$$

$$W = \int F\,dx$$

$$W = \int_0^{L/n} \frac{Mx}{L} g\,dx$$

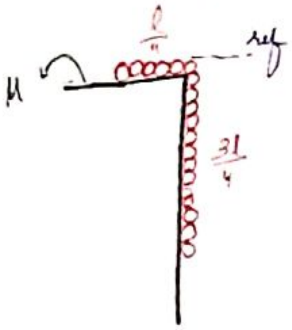
$$W = \left[\frac{Mg x^2}{2L}\right]_0^{L/n}$$

$$W = \frac{Mg}{2\cancel{L}} \times \frac{L}{n} \times \frac{\cancel{L}}{n} \qquad \boxed{W = \frac{MgL}{2n^2}}$$

$$\cancel{W = \frac{Mg}{2n^2}}$$

~~Work Potential Energy theory~~

Q40. chain is on verge of slipping. find velocity when it has slipped.



let mass be M.

$f_1 = \mu \frac{M}{4} g = \frac{3Mg}{4}$

$\boxed{\mu = 3}$

$U_i = -\frac{3M}{4} \times g \times \frac{3l}{8}$

$= -\frac{9Mgl}{32}$

$U_f = -\frac{Mgl}{2}$

$-\Delta U = W_{g} = -(U_f - U_i) = U_i - U_f$

$= -\frac{9Mgl}{32} + \frac{Mgl}{2}$

$= \frac{16Mgl - 9Mgl}{32}$

$\boxed{W_g = \frac{7Mgl}{32}}$

for friction

$f = \mu\left(\frac{Mx}{l}\right)g$

$W_f = \int_{l/4}^{0} \mu\left(\frac{Mx}{l}\right) g \, dx$

$= \left[\mu \frac{Mx^2}{2l} g\right]_{l/4}^{0}$

$= -3 \times \frac{Mg}{2l} \times \frac{l}{4} \times \frac{l}{4}$

$\boxed{W_f = \frac{-3Mgl}{32}}$

$K_i + W = K_f$

$0 + W_g + W_f = \frac{1}{2} MV^2$

$\frac{7Mgl}{32} - \frac{3Mgl}{32} = \frac{1}{2} MV^2$

$\frac{7Mgl - 3Mgl}{32} = \frac{1}{2} MV^2$

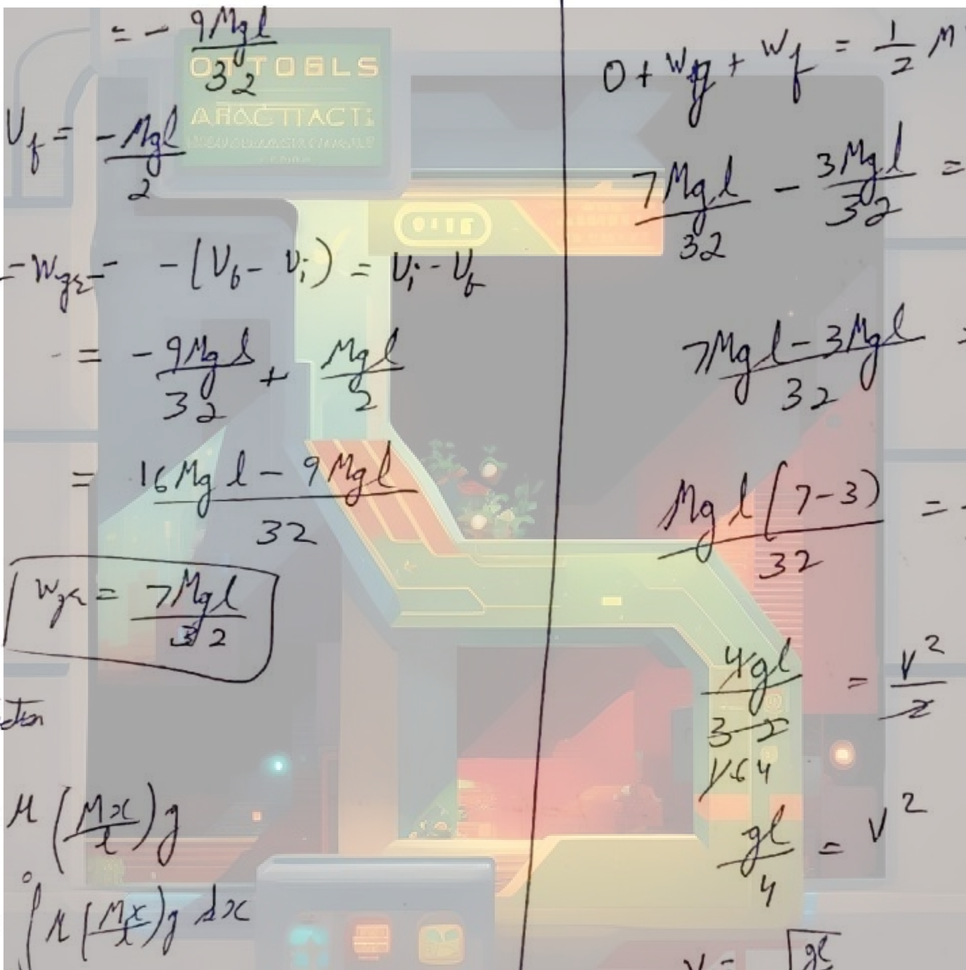
$\frac{Mgl(7-3)}{32} = \frac{1}{2} MV^2$

$\frac{4gl}{\cancel{32}\ 16\ 4} = \frac{V^2}{\cancel{2}}$

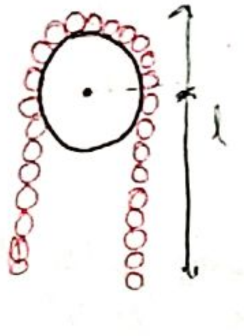
$\frac{gl}{4} = V^2$

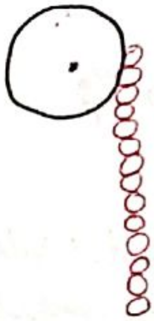
$V = \sqrt{\frac{gl}{4}}$

$\boxed{V = \frac{1}{2}\sqrt{gl}}$



Q41.





Mass - M
find KE when chain becomes straight.

$$F_g = \frac{Mxg}{2l}$$

$$W_g = \int_l^0 \frac{Mx}{2l} g\, dx$$

$$= -\frac{Mg}{2l} \times \frac{1}{2} \times l \times l$$

$$\boxed{W_{g_1} = -\frac{Mgl}{4}}$$

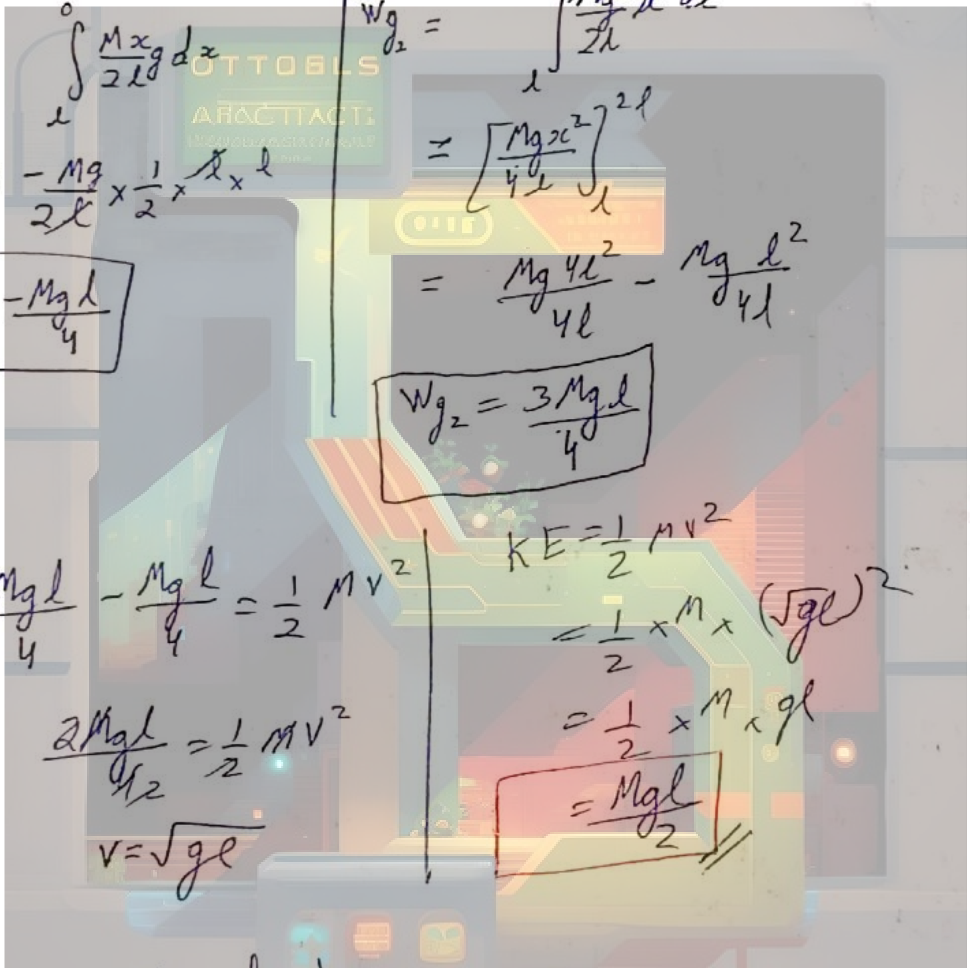
$$F_g = \frac{Mg}{2l} x$$

$$W_{g_2} = \int_l^{2l} \frac{Mg}{2l} x\, dx$$

$$= \left[\frac{Mg x^2}{4l}\right]_l^{2l}$$

$$= \frac{Mg\, 4l^2}{4l} - \frac{Mg\, l^2}{4l}$$

$$\boxed{W_{g_2} = \frac{3Mgl}{4}}$$



$$\frac{3Mgl}{4} - \frac{Mgl}{4} = \frac{1}{2} M v^2$$

$$\frac{2Mgl}{4} = \frac{1}{2} M v^2$$

$$v = \sqrt{gl}$$

$$KE = \frac{1}{2} M v^2$$

$$= \frac{1}{2} \times M \times (\sqrt{gl})^2$$

$$= \frac{1}{2} \times M \times gl$$

$$\boxed{= \frac{Mgl}{2}}$$

M II

$$U_i = -2\frac{M}{2} g \times \frac{l}{2}$$

$$= -\frac{Mgl}{2}$$

$$U_f = -Mgl$$

$$-\Delta U = U_i - U_f$$

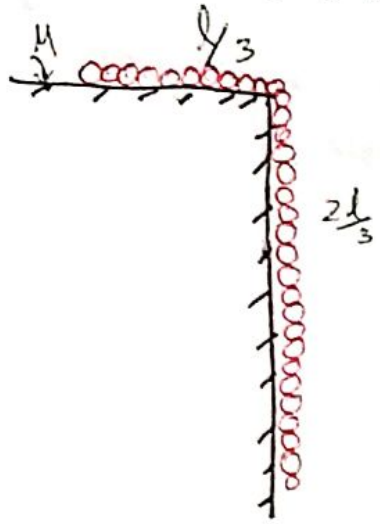
$$W_g = Mgl - \frac{Mgl}{2}$$

$$W_g = \frac{Mgl}{2}$$

$$0 + W_g = KE$$

$$\boxed{KE = \frac{Mgl}{2}}$$

Q Chain is on the verge of slipping.



a) $\mu = ?$

b) $= W_f, W_g$

c) $V = ?$

d) $W_{external}$ at end A of on slowly pulling the chain completely onto the table.

a) let mass $= M$

$f_e =$ gravity

$$\mu\left(\frac{M}{l}\times\frac{l}{3}\times g\right) = \frac{M}{l}\times\frac{2l}{3}\times g$$

$$\mu\frac{Mg}{3} = \frac{2Mg}{3}$$

$$\boxed{\mu = 2}$$ a)

b) $$W_g = \int_{2l/3}^{l/3} \frac{Mg\,x}{l}\,dx$$

$$= \left[\frac{Mg\,x^2}{2l}\right]_{2l/3}^{l/3}$$

$$= \frac{Mg\,l^2}{2l} - \frac{Mg\,4l^2}{l\;9\times 2}$$

$$= \frac{Mg\,l}{2} - \frac{4Mg\,l}{18}$$

$$\boxed{W_g = \frac{5Mgl}{18}\,J}$$ b)

$$W_f = \int_{l/3}^{0} \mu\frac{Mg\,x}{l}\,dx$$

$$W_f = \left[\frac{2Mgx^2}{2l}\right]_{l/3}^{0}$$

$$= -\frac{2Mg}{l}\times\frac{l}{3}\times\frac{l}{3\times 2}$$

$$\boxed{W_f = \frac{-2Mgl}{18}\,J}$$ b)

(212)

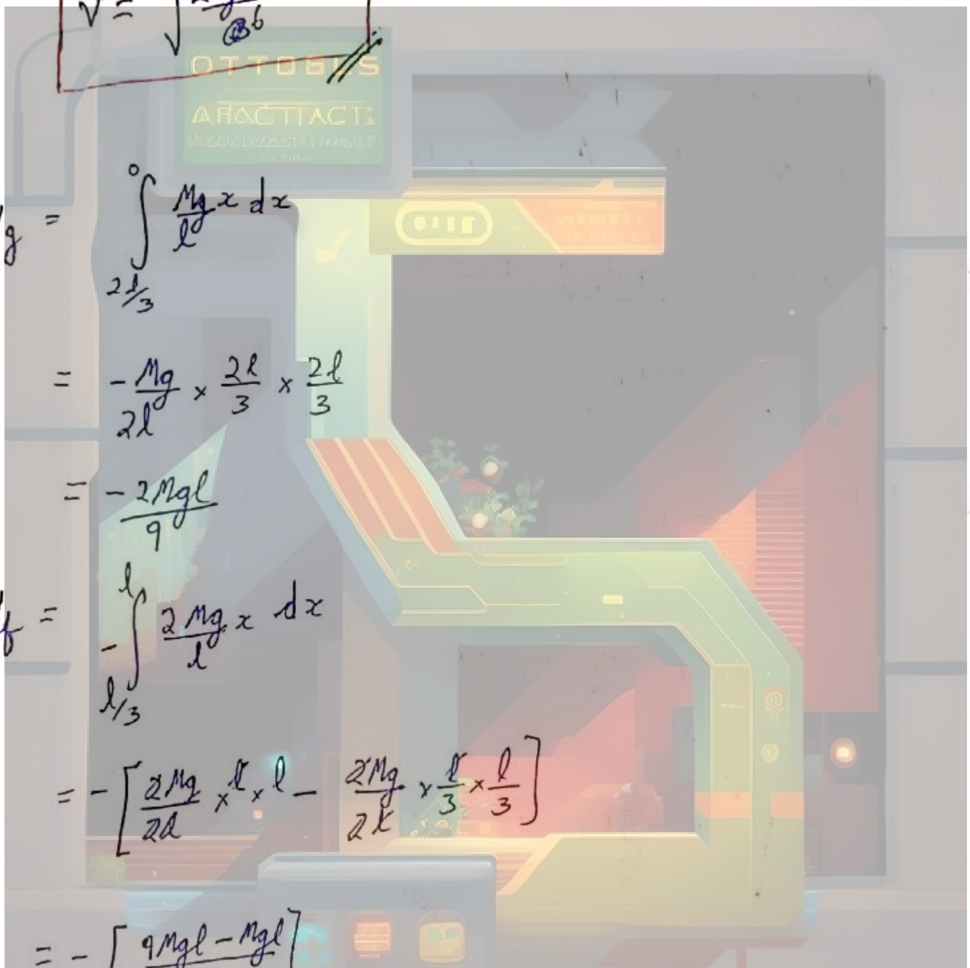
c) $K_i \pm W = K_f$

$$0 + \frac{5Mgl}{\cancel{9}18} - \frac{2Mgl}{18} = \frac{1}{2}Mv^2$$

$$\frac{3Mgl}{\cancel{9}\cancel{3}6} = \frac{1}{2}Mv^2$$

$$\frac{2gl}{\cancel{3}6} = v^2$$

$$\boxed{v = \sqrt{\frac{2gl}{\cancel{3}6}}}$$ c)



d) $$W_g = \int_{2l/3}^{0} \frac{Mg}{l} x\,dx$$

$$= -\frac{Mg}{2l} \times \frac{2l}{3} \times \frac{2l}{3}$$

$$= -\frac{2Mgl}{9}$$

$$W_f = -\int_{l/3}^{l} \frac{2Mg}{l} x\,dx$$

$$= -\left[\frac{2Mg}{2l} \times l \times l - \frac{2Mg}{2l} \times \frac{l}{3} \times \frac{l}{3}\right]$$

$$= -\left[\frac{9Mgl - Mgl}{9}\right]$$

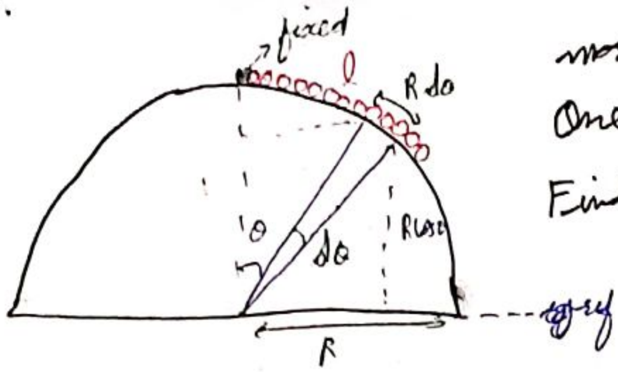
$$= -\frac{8Mgl}{9}$$

$K_i \pm W = K_f$

$$0 + W_{Fexternal} - \frac{2Mgl}{9} - \frac{8Mgl}{9} = 0$$

$$\boxed{W_F = \frac{10Mgl}{9}\ J}$$ d)

Q 43.



mass m & length l.
One end is tied at top of hemisphere.
Find Gravitational P.E.

Taking a segment of chain as element from angle θ from vertical & angular width of segment is dθ

$$\text{length of segment} = R\,d\theta$$

$$\text{mass of segment} = \frac{m}{l} \times R\,d\theta$$

$$\text{G.P.E of segment} = \frac{mR\,d\theta\,g}{l} \times R\cos\theta$$

$$= \frac{mg\,R^2\cos\theta\,d\theta}{l}$$

$$\text{G.P.E of chain} = \int_0^{l/R} \frac{mg\,R^2\cos\theta\,d\theta}{l}$$

$$= \frac{mg\,R^2}{l}\left[\sin\theta\right]_0^{l/R}$$

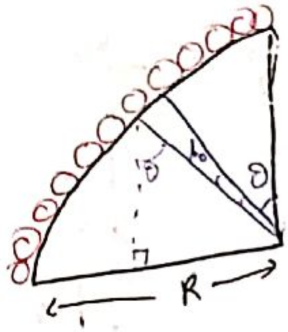
$$\boxed{= \frac{mg\,R^2\sin(l/R)}{l}}$$

Q44.

$l = \pi R/2$

mass per unit length $= \rho$

determine velocity when chain leaves link with groove.



$\cancel{P \cdot E' = \int \rho R\,d\theta}$ per unit length

$\cancel{P \cdot E = \int_0^{\pi/2} \rho R\,d\theta}$

$\cancel{= [\rho R \theta]}$

$\cos\theta = \frac{h}{R}$

$h = R\cos\theta$

P.E per unit length $= \rho g\, R\cos\theta \times R\,d\theta$

$$P \cdot E = \int_0^{\pi/2} \rho g\, R\,d\theta\, R\cos\theta$$

$$= \left[\rho g R^2 \sin\theta\right]_0^{\pi/2}$$

$$\boxed{= \rho g R^2}$$

$$P \cdot E_f = -\frac{\rho \pi R}{2} \times g \times \frac{\pi R}{2 \times 2}$$

$$\boxed{= -\frac{\rho \pi^2 R^2 g}{8}}$$

$$PE_f - PE_i = \Delta U$$

$$\Delta U = -\frac{\rho \pi^2 R^2 g}{8} - \rho g R^2$$

$$-\Delta U = \rho g R^2 \left(\frac{\pi^2}{8} + 1\right)$$

$$\frac{1}{2} M V^2 = -\Delta U$$

$$\frac{1}{2} \times \frac{\rho \pi R}{2} \times V^2 = \rho g R^2 \left(\frac{\pi^2}{8} + 1\right)$$

$$\frac{\pi}{4} V^2 = \frac{g R \pi^2}{8} + gR$$

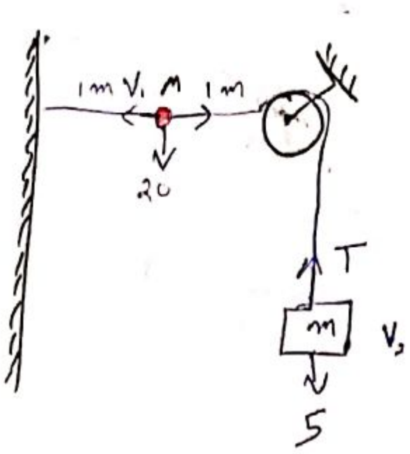
$$V^2 = \frac{g R \pi}{2} + \frac{4 g R}{\pi}$$

$$V^2 = \frac{g R \pi^2 + 8 g R}{2\pi}$$

$$V^2 = \frac{g R (\pi^2 + 8)}{2\pi}$$

$$\boxed{V = \sqrt{\frac{g R (\pi^2 + 8)}{2\pi}}}$$

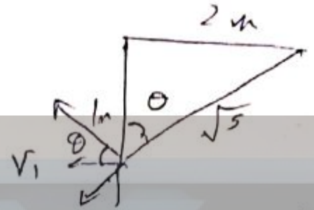
Q 45.



$M = 2kg$

$m = 0.5kg$

find speed with which m strikes the ball.



$$W_g(m) = -(\sqrt{5}-1)\,5$$
$$= -5(\sqrt{5}-1)$$
$$W_g(M) = +20(1)$$
$$= 20$$

$$W_g(m) + W_g(M) = \cancel{\Delta} KE_f$$

$$\cancel{20 - 5\sqrt{\sqrt{5}-1} = \frac{1}{2} \times 2 \times V^2}$$

$$\cancel{V^2 = 20 - 5\sqrt{\sqrt{5}-1}}$$

$$V_1 \sin\theta = V_2$$

$$\frac{2V_1}{\sqrt{5}} = V_2$$

$$KE(m) = \frac{1}{2} \times \frac{1}{2} \times \left(\frac{2V_1}{\sqrt{5}}\right)^2$$
$$= \frac{4V_1}{5 \times 4}$$
$$= \frac{V_1^2}{5}$$

$$\cancel{\tan\theta = \frac{2}{1}}$$

$$\sin\theta = \frac{2}{\sqrt{5}}$$

$$\cancel{20 - 5\sqrt{\sqrt{5}-1}}$$

$$20 - 5(\sqrt{5}-1) = \frac{V_1^2}{5} + V_1^2$$

$$20 - 5(\sqrt{5}-1) = \frac{6V_1^2}{5}$$

$$25 - 5\sqrt{5} = \frac{6V_1^2}{5}$$

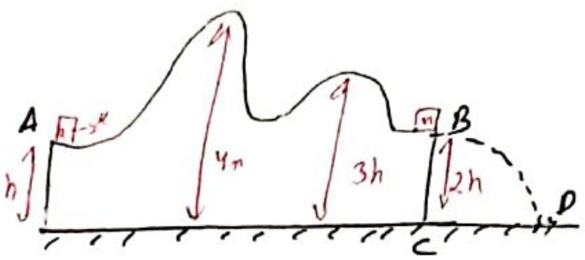
$$V^2 = \frac{5(25 - 5\sqrt{5})}{6}$$

$$V = \sqrt{\frac{25(5-\sqrt{5})}{6}}$$

$$\boxed{V = 5\sqrt{\frac{5-\sqrt{5}}{6}}} \text{ m/s}$$

Q46.

fin min μ for m to reach B.
also find CD.



$\cancel{P.E_i = mgh}$

$\cancel{P.E_f = 2mgh}$

$\cancel{-\Delta U = PE_i - PE_f}$

$\cancel{= mgh - 2mgh}$

W for clim tile 4h by g = $-3mgh$

$\frac{1}{2} \times m v^2 = +3mgh$

$v^2 = 60h$

$\boxed{v = \sqrt{60h}}$

$W_g = 2mgh$

$K_i = 0$

$K_f = ?$

$0 + 2mgh = \frac{1}{2} m v^2$

$4gh = v^2$

$v = \sqrt{4gh}$

$\boxed{v = \sqrt{40h}}$

$2h = \frac{1}{2} \times 10 \times t^2$

$\sqrt{\frac{2h}{5}} = t$

$\cancel{CD = \sqrt{40h}\sqrt{\frac{2h}{5}} - \frac{1}{2} \times}$

$CD = \sqrt{40h}\sqrt{\frac{2h}{5}}$

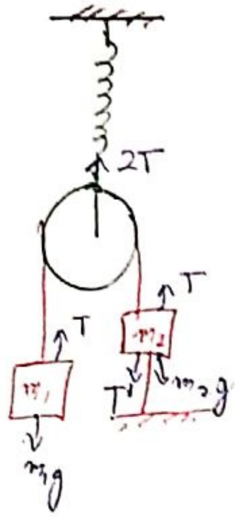
$CD = \sqrt{\frac{40 \times 2}{5} \times h^2}$

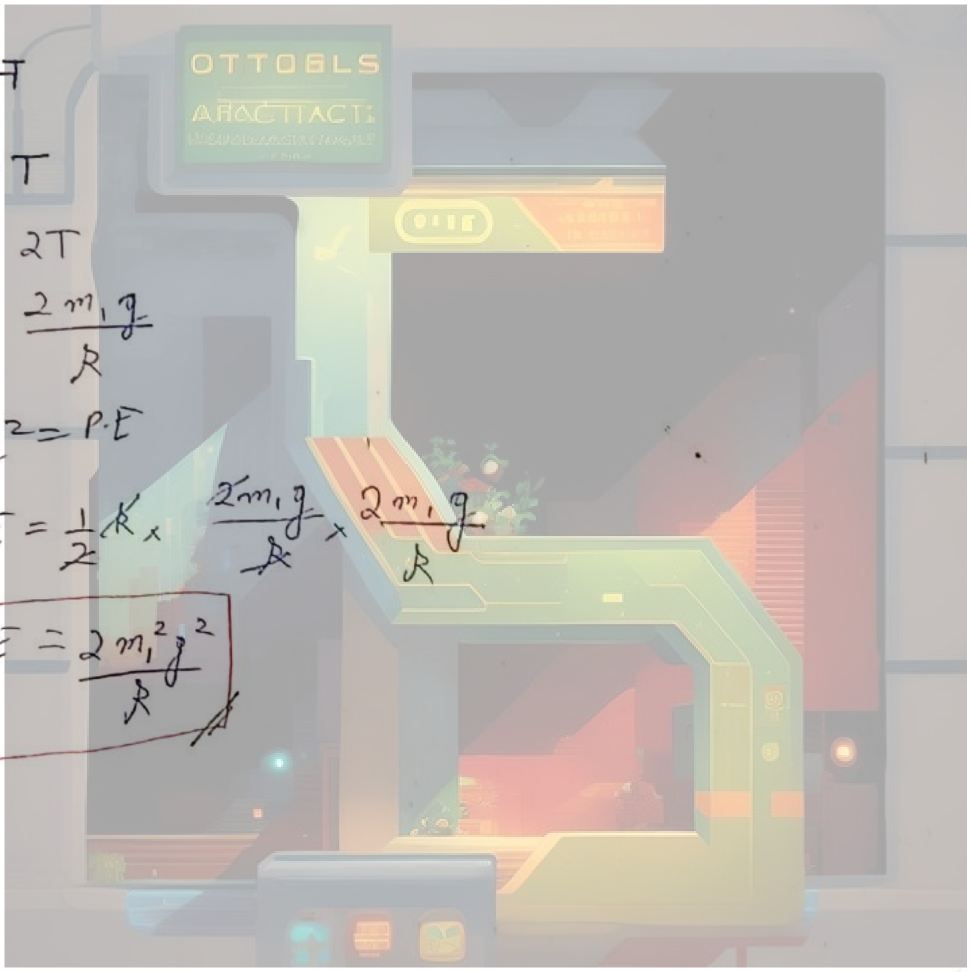
$= h\sqrt{16}$

$\boxed{CD = 4h}$

Q 47.

find P.E spring





$\cancel{m_2 g = T}$

$m_1 g = T$

$kx = 2T$

$x = \frac{2 m_1 g}{k}$

$\frac{1}{2} k x^2 = P.E$

$P.E = \frac{1}{2} \cancel{k} \times \frac{2 m_1 g}{\cancel{k}} \times \frac{2 m_1 g}{k}$

$$\boxed{P.E = \frac{2 m_1^2 g^2}{k}}$$

Power:

→ Work done per unit time is called power.
or
Rate of doing work.

$$\boxed{\text{Power} = \frac{\text{Work}}{\text{Time}}}$$

→ SI unit :- watt

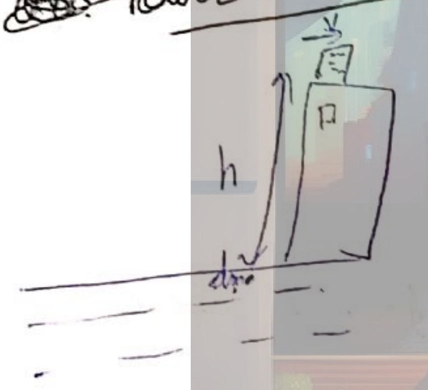
→ 1 W = 1 J/s

→ It is a scalar quantity.

→ Instantaneous Power = Fv
power = Force × velocity

→ 1 HP = 746 W (horsepower)

Power Delivered by Pump



$P = \frac{dw}{dt}$

$P = \frac{1}{dt}\left[(dm) gh + \frac{1}{2}(dm)v^2\right]$

$$\boxed{P = \frac{dm}{dt}\left[gh + \frac{1}{2}v^2\right]}$$

dm → mass of water delivered by pump in dt.

$\frac{dm}{dt}$ → rate of flow of water.

$P = \frac{d(\text{volume})}{dt}\rho\left(gh + \frac{1}{2}v^2\right)$

Q48. A Train has a constant speed of 40 m/s on a level road against resistive force of magnitude $3 \times 10^4 N$. Find power.

$$P = \frac{W}{t}$$

$$P = \frac{F \times s}{t}$$

$$P = V \times F$$

$$P = 40 \times 3 \times 10^4$$

$$P = 120 \times 10^4$$

$$\boxed{P = 1.2 \times 10^6 \text{ watt}}$$

Q49. ball of mass 1 kg dropped from tower. find gravitational of Power at time $t = 2s$.

$$P = F \times V$$

$$V|_{t=2} = 10(2)$$

$$= 20 \, m/s$$

$$P = mg \times v$$

$$P = 1 \times 10 \times 20$$

$$\boxed{P = 200 \text{ watt}}$$

Q50. mass m lying on smooth table, A constant force tangential to the surface is applied.
i) average power over a time interval $t=0$ To $t=t$
ii) instantaneous power as function of t,

ii) $P = FV$

$$P = F \times \frac{F}{m} t$$

$$\boxed{P = \frac{F^2 t}{m}} \text{ ii)}$$

10 i) $P = \frac{\text{work}}{\text{Time}}$ $\qquad S = \frac{1}{2} \times \frac{F}{m} t^2$ $(S = ut + \frac{1}{2} at^2)$

$S = \frac{Ft^2}{2m}$

$P = \frac{FS}{t}$

$P = \frac{F}{t} \times \frac{Ft^2}{2m}$

$\boxed{P = \frac{F^2 t}{2m}}$ i)

Q51. (Q51. $P = 2t$ is applied of mass m.

i) KE & Velocity as function of time

ii) average power over a time interval $t = 0$ to $t = t$.

~~i) KE = Work = Power × time~~

~~$= 2t \times t$~~

~~$= 2t^2$~~

~~$\frac{1}{2} mv^2 = 2t^2$~~

~~$v^2 = \frac{4t^2}{m}$~~

~~$v = \frac{2t}{\sqrt{m}}$~~

i) $P = \frac{dW}{dt}$

$2t = \frac{dW}{dt}$

$\int_0^t 2t\, dt = \int_0^W dW$

$\boxed{W = t^2}$ i)

$\frac{1}{2} mv^2 = t^2$

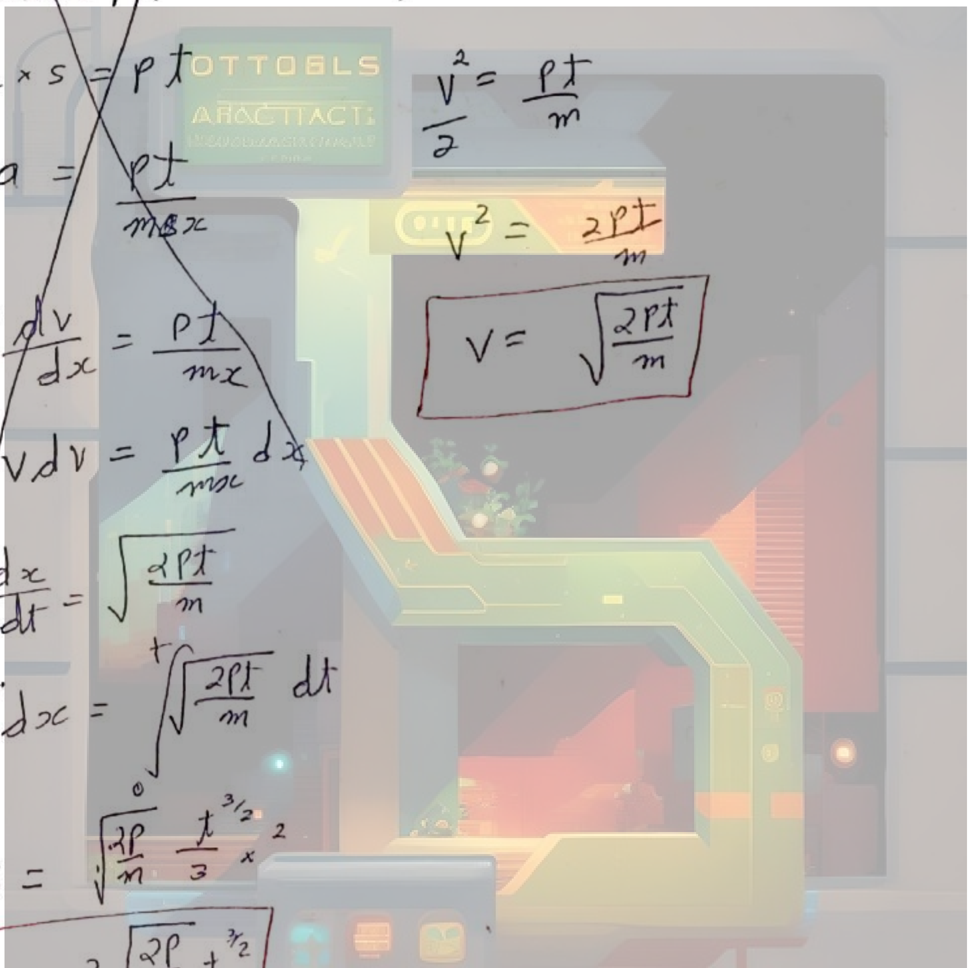
$v^2 = \frac{2t^2}{m}$

$\boxed{v = t\sqrt{\frac{2}{m}}}$ i)

ii) $\frac{\text{Total work}}{\text{Time}} = \frac{t^2}{t}$

$\boxed{= t}$ ii)

Q52. An car of mass m accelerates from rest while engine supplies constant Power P. find its position & instantaneous velocity w.r.t time assuming automobile starts from rest.



$$P = \frac{dw}{dt} = \frac{W}{t}$$

$$W = Pt$$

$$F \times s = Pt$$

$$ma \times s = Pt$$

$$a = \frac{Pt}{mx}$$

$$v\frac{dv}{dx} = \frac{Pt}{mx}$$

$$v\,dv = \frac{Pt}{mx}\,dx$$

$$P = Fv$$

$$P = m\frac{dv}{dt}v$$

$$\int_0^t \frac{P}{m}\,dt = \int_0^v v\,dv$$

$$\frac{v^2}{2} = \frac{Pt}{m}$$

$$v^2 = \frac{2Pt}{m}$$

$$\boxed{v = \sqrt{\frac{2Pt}{m}}}$$

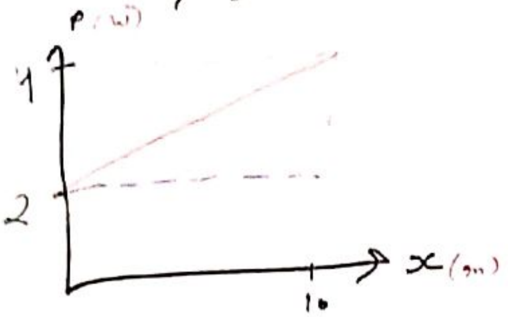
$$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$$

$$\int_0^s dx = \int_0^t \sqrt{\frac{2Pt}{m}}\,dt$$

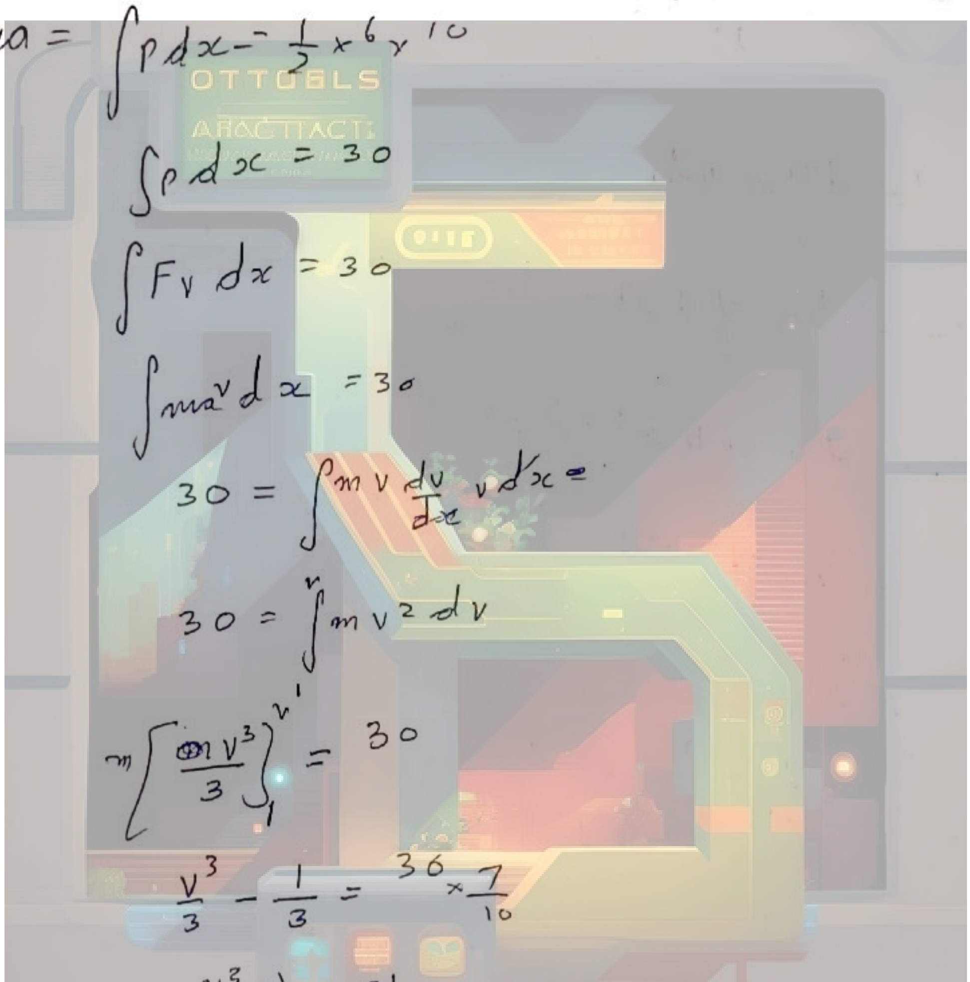
$$s = \sqrt{\frac{2P}{m}}\,\frac{t^{3/2}}{3} \times 2$$

$$\boxed{s = \frac{2}{3}\sqrt{\frac{2P}{m}}\,t^{3/2}}$$

Q53. mass $= \frac{10}{7}$ kg, initial $x = 0$, initial $v = 0$



$\cancel{P = \frac{dW}{dt}}$



area $= \int P\,dx = \frac{1}{2} \times 6 \times 10$

$\int P\,dx = 30$

$\int F v\,dx = 30$

$\int m a v\,dx = 30$

$30 = \int m v \frac{dv}{dx} v\,dx$

$30 = \int_1^v m v^2\,dv$

$m\left[\frac{v^3}{3}\right]_1^v = 30$

$\frac{v^3}{3} - \frac{1}{3} = 30 \times \frac{7}{10}$

$\frac{v^3 - 1}{3} = 21$

$v^3 - 1 = 63$

$v^3 = 64$

$\boxed{v = 4\,m/s}$

Potential Energy Diagrams -

$$W_{conservative} = -\Delta U$$

$$dW = -dU$$

$$F\,dr = -dU$$

$$F = -\frac{dU}{dr}$$

$$\boxed{\vec{F}_{net} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]}$$

$\frac{\partial}{\partial x}$ = partial differentiation

$\frac{\partial U}{\partial x}$: differentiation of U wrt x keeping other variables (y, z) as constant

$\frac{\partial U}{\partial y}$: Differentiation of U wrt y keeping (x, z) as constants

$\frac{\partial U}{\partial z}$: Differentiation of U wrt z keeping (x, y) as constants

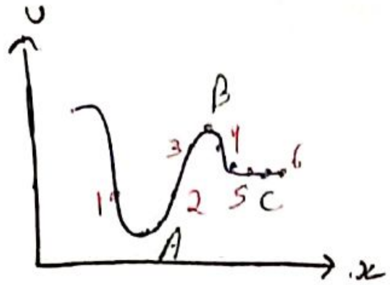
eg: $y = xz^2$

$$\frac{dy}{dx} = x\frac{d}{dx}(z^2) + z^2\frac{d}{dx}(x)$$

$$\frac{\partial y}{\partial x} = z^2\frac{d}{dx}(x) = \boxed{z^2}$$

$$\frac{\partial y}{\partial z} = \frac{d}{dz}(z^2)(x)$$

$$\boxed{= 2xz}$$



A graph plotted between potential energy of a particle and its displacement from the mean position center of the field is called potential energy diagram.

Point 1: slope $\frac{dU}{dx} < 0$, $F > 0$

Point 2: $\frac{dU}{dx} > 0$, $F < 0$

$\Big\}$ Stable Equilibrium $\frac{dU}{dx} = 0 \quad \frac{d^2U}{dx^2} < 0$

Force is always acting toward 'A'

Point 3: $\frac{dU}{dx} > 0$, $F < 0$

Point 4: $\frac{dU}{dx} < 0$, $F > 0$

$\Big\}$ Unstable equilibrium $\frac{dU}{dx} = 0 \quad \frac{d^2U}{dx^2} > 0$

Force is always acting away from 'B'

Point 5: $\frac{dU}{dx} = 0$

Point 6: $\frac{dU}{dx} = 0$

$\Big\}$ neutral equilibrium. 'C'

Q 54. a particle moves along x-axis $U = 2x^3 - 3x^2 - 12x + 1$. Find co-ordinate of equilibrium position & its nature. mass $= 1$ kg

$F = -\frac{dU}{dx}$

$F = -(6x^2 - 6x - 12)$

$F = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x = \frac{1 \pm \sqrt{1+8}}{2}$

$x = \frac{4}{2}, \frac{-2}{2}$

$\boxed{x = 2, -1}$

$\frac{d^2U}{dx^2} = 12x - 6$

$x = 2$

$12(2) - 6 = 24 - 6 = 18$

$\frac{d^2U}{dx^2} > 0$ so, stable equilibrium

$x = -1$

$12(-1) - 6 = -12 - 6 = -18$

$\frac{d^2U}{dx^2} < 0$ so, unstable equilibrium.

Q 55. $V = (2xy + yz)$ Joule. find force acting on particle at $P(x, y, z)$

$$\vec{E}_{net} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\vec{F} = -\left[\frac{\partial(2xy+yz)}{\partial x}\hat{i} + \frac{\partial(2xy+yz)}{\partial y}\hat{j} + \frac{\partial(2xy+yz)}{\partial z}\right]$$

$$\boxed{\vec{F} = -\left[2y\hat{i} + (2x+z)\hat{j} + y\hat{k}\right]}$$

H.W. 6 - 5 - 24

Ex 0-1
Q-1 to Q-45.

Q 56. $V = \frac{a}{r^2} - \frac{b}{r}$, a & b are constant & r is distance from the centre of the field. find value r_0 corresponding to equilibrium & type of equilibrium.

$\cancel{\frac{dV}{dr} = -\frac{2a}{r} + b = 0}$ $\qquad$ $\frac{dV}{dr} = \frac{-2a}{r^3} + \frac{b}{r^2} = 0$

$\frac{2a}{r} = b$ $\qquad$ $\frac{2a}{r^3} = \frac{b}{r^2}$

$\boxed{r_0 = \frac{2a}{b}}$ $\qquad$ $\boxed{r_0 = \frac{2a}{b}}$

$\cancel{\frac{d^2V}{dr^2} =}$

$$\frac{d^2V}{dr^2} = \frac{6a}{r^4} - \frac{2b}{r^3}$$

$$r_0 = \frac{2a}{b}$$

$$\frac{6a}{16a^4} \times b^4 - \frac{2b}{8a^3} \times b^3$$

$$\frac{6b^4 - 4b^4}{16a^3} = 0 \qquad \frac{3b^4}{8a^3} - \frac{b^4}{4a^3} > 0$$

$\boxed{\text{Unstable}}$ $\qquad$ $\boxed{\text{Stable}}$

Q 57. $U = U_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^{6}\right]$ where U_0 & a are constants

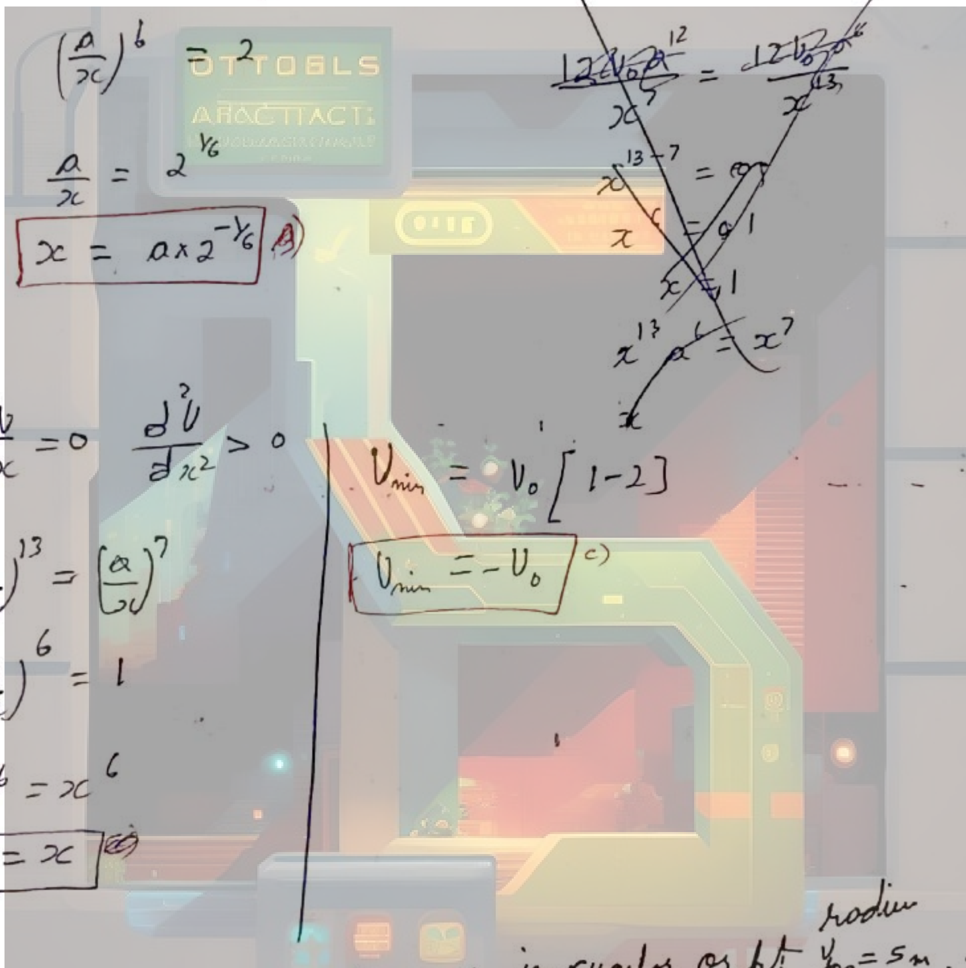
a) x at which $U=0$

b) find F_x

c) x at which PE is min.

a) $U = U_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^{6}\right] = 0$

$\left(\frac{a}{x}\right)^{12} = 2\left(\frac{a}{x}\right)^{6}$



$\left(\frac{a}{x}\right)^{6} = 2$

$\frac{a}{x} = 2^{1/6}$

$\boxed{x = a \times 2^{-1/6}}$ a)

b) $\frac{dU}{dx} = \frac{-12U_0 a^{12}}{x^{13}} + \frac{12U_0 a^6}{x^7} = -F$

$\boxed{F = \frac{12U_0 a^{12}}{x^{13}} - \frac{12U_0 a^6}{x^7}}$ b)

c) ~~PE min, $\frac{dU}{dx} = 0$~~ $\frac{d^2U}{dx^2} > 0$

~~$\frac{12U_0 a^{12}}{x^7} = \frac{12U_0 a^6}{x^{13}}$~~

~~$x^{13-7} = a^6$~~

~~$x^6 = a^6$~~

~~$x = 1$~~

~~$x^{13} a^6 = x^7$~~

c) $\frac{dU}{dx} = 0 \quad \frac{d^2U}{dx^2} > 0$

$\left(\frac{a}{x}\right)^{13} = \left(\frac{a}{x}\right)^{7}$

$\left(\frac{a}{x}\right)^{6} = 1$

$a^6 = x^6$

$\boxed{a = x}$

$U_{min} = U_0[1-2]$

$\boxed{U_{min} = -U_0}$ c)

Q 58. $U = 2r^3$ J. body is moving in circular orbit radius $= 5$ m. find energy

$F_{centripetal} = \frac{mv^2}{r}$, mass $= 2$ kg find energy.

$F = -\frac{dU}{dr} = |-6r^2| = \frac{mv^2}{r}$

$mv^2 = +6r^3$

$E = \frac{1}{2}mv^2 = +3r^3 + U$

$= 3r^3 + 2r^3$

$= 5r^3$

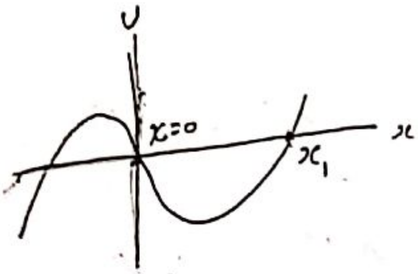
$\boxed{= 625 \text{ J}}$

Q59. move along +x axis under $F(x)$ force. $U(x) = ax^3 - bx^2$

a) find $F(x)$

b) when energy = 0, particle can be between $x=0$ & $x = x_1$. find x_1

c) find Max KE.



a) $F(x) = -\frac{dU}{dx} = -[3ax^2 - b]$

$$\boxed{F(x) = b - 3ax^2}$$ ✓

b) $ma = b - 3ax^2$

$$\frac{v\,dv}{dx} = \frac{b - 3ax^2}{m}$$

$$\int_0^v v\,dv = \int_0^x \frac{b - 3ax^2}{m}\,dx$$

$$\frac{v^2}{2} = \frac{bx}{m} - \frac{3ax^3}{3m}$$

$$\frac{v^2}{2} = \frac{3bx - 3ax^3}{3m}$$

$$v^2 = \frac{6bx - 6ax^3}{3m}$$

$$KE = \frac{1}{2}mv^2 = \frac{3bx - 3ax^3}{3} = bx - ax^3$$

b) $U = 0$

$$ax^3 = bx$$

$$ax^2 = b$$

$$x = \boxed{\sqrt{\frac{b}{a}} = x_1}$$

c) KE = max, V = max, F = 0 ⇓ Equilibrium.

$$F = 0 = b - 3ax^2$$

$$x^2 = \frac{b}{3a}$$

$$x = \sqrt{\frac{b}{3a}}$$

$$KE = \int_0^{\sqrt{\frac{b}{3a}}} F\,dx$$

$$= \left[bx - ax^3\right]_0^{\sqrt{\frac{b}{3a}}}$$

$$= b\sqrt{\frac{b}{3a}} - \sqrt{\frac{b}{3a}} \times \frac{b}{3a} \times a$$

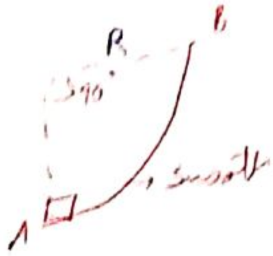
$$\boxed{= \sqrt{\frac{b}{3a}}\left(\frac{2b}{3}\right)}$$

Q60. move m.a. done from A → B

a) F is horizontal

b) ~~~~ tangential to surface

c) towards B.



a) $AP = \frac{R\sqrt{2}}{2}$

$\boxed{W = FR}$ (displacement in horizontal)

b) $W = \int_0^{\pi R/2} F\,ds$

$\boxed{W = \frac{F\pi R}{2}}$

c)

$F/\sqrt{2}$

$F/\sqrt{2}$

$W_x = F/\sqrt{2}\, R$

$W_y = F/\sqrt{2}\, R$

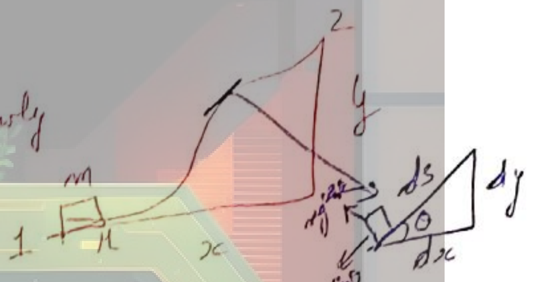
$W_{total} = \frac{2FR}{\sqrt{2}}$

$\boxed{= \sqrt{2}FR}$

Q61.

move slowly

$W_F = ?$



$W_g = -mgy$

$K_i + W = K_f$

$W_F - W_g - W_{fr} = 0$

$dW_{fr} = -\mu mg \frac{dx}{ds} \cdot ds$

$W_{fr} = -\int \mu mg\, dx$

$W_{fr} = -\mu mg x$

$f = \mu mg \cos\theta$

$f = \mu mg \frac{dx}{ds}$

$W_F = mgy + \mu mg x$

$\boxed{W_F = mg(y + \mu x)}$