

# Physics - I

## Ch1. Units, Dimensions & Vectors

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## Units & Dimensions

Physical Quantity - Can be measured e.g. time, distance

### Fundamental Quantity (7)

- 1 → Mass → kg
- 2 → Length → m
- 3 → Time - second
- 4 → Temperature - K
- 5 → Electric Current - Ampere
- 6 → Luminous Intensity - Candela (cd)
- 7 → Amount of Substance - mole

### Derived Quantity

e.g. → speed =  $\frac{\text{distance} (\text{m/s})}{\text{time}}$

- velocity
- momentum
- density ( $\text{kg/m}^3$ )

$$1 \text{ g/cc} = 10^3 \text{ kg/m}^3$$

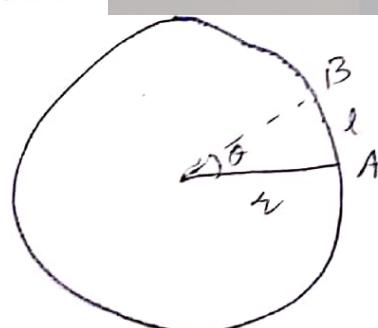
= water density ( $4^\circ\text{C}$ )

water  
 $0^\circ\text{C} \xrightarrow{\text{contract}} 4^\circ\text{C} \xrightarrow{\text{expand}}$

### Supplementary units

1. Plane Angle

SI → radian

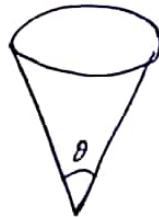


$$\theta = \frac{l}{r}$$

(1)

## 2. Solid Angle

SI  $\rightarrow$  steradian

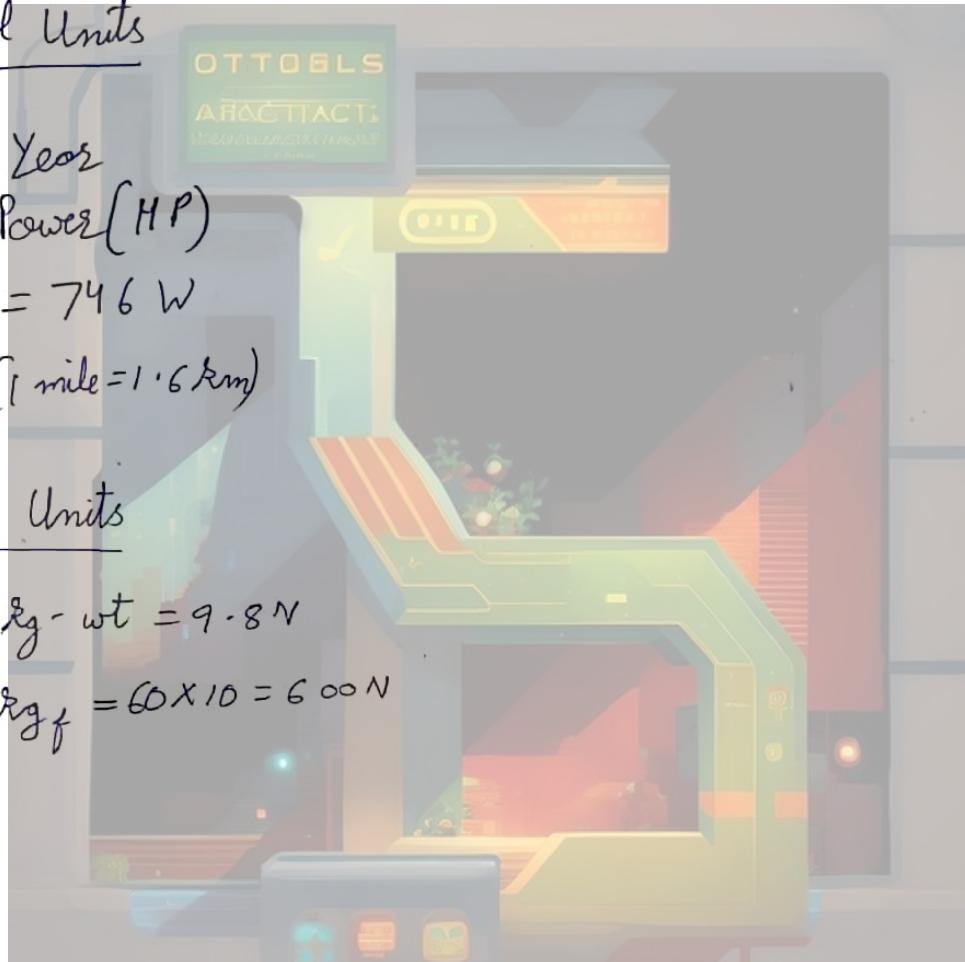


### Practical Units

1. Light Year
2. Horse Power (HP)
3. mile ( $1 \text{ mile} = 1.6 \text{ km}$ )

### Imperial Units

$$1. \text{ } Rg_f = 1 \text{ } Rg - \text{wt} = 9.8 \text{ N}$$
$$\text{eg. } 60 \text{ } Rg_f = 60 \times 10 = 600 \text{ N}$$



(2)

Dimensions → Dimensions of a physical quantity are the powers to which fundamental quantities must be raised to represent a given physical quantity.

e.g. Dimensions of Mass - [M]

Length - [L]

Time - [T]

Temperature - [θ]

Current - [A]

$$\text{Dimensions of speed} = \frac{\text{dist}}{\text{time}} = \frac{[L]}{[T]} = [L T^{-1}]$$

Density energy =

$$\text{work} \hat{=} F \times S = [MLT^{-2}] [L] \\ = [ML^2 T^{-2}]$$

$$\text{Pressure} = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1} T^{-2}]$$

$$\text{Force} = m a = M \times L T^{-2} = [MLT^{-2}]$$

### Applications of Dimensional Analysis

① To find dimensions of physical constants

$$F = \frac{G m_1 m_2}{d^2}$$

$$G = [m^{-1} L^3 T^{-2}]$$

$$N = \frac{G m^2}{L^2}$$

$$G = N \cdot m^2 \cdot kg^{-2}$$

(3)

$$E = h \nu \rightarrow \text{frequency}$$

↓

Planck's constant

$$\nu = \frac{1}{f \tau t}$$

$$ML^2T^{-2} = h T^{-1}$$

$$h = [ML^2T^{-1}]$$

2. To check if a equation is dimensionally correct

e.g.  $F = ma^2$

$$MLT^{-2} = M [LT^{-2}]^2$$

$$MLT^{-2} \neq ML^2T^{-4} \quad (F = ma^2 \text{ is wrong as dimensions are diff})$$

$$F = \underline{m} \underline{v}^2$$

$$= \underline{M} \underline{[LT^{-1}]}^2$$

$$= \underline{ML^2T^{-2}}$$

$$\underline{MLT^{-2}} = \underline{MLT^{-2}} \quad (\text{correct eqn})$$

$$S = ut + \frac{1}{2} at^2 \quad (\text{numbers are dimension less})$$

$$(\underline{[L]} + \underline{[L]} = \underline{[L]})$$

$$L = LT^{-1} \times T + \frac{1}{2} [LT^{-2}] [T^2]$$

$$= L + \cancel{\frac{1}{2} L}$$

$$[L] = [L] \quad \text{correct dimensionally correct}$$

### 3. Derive new equations

$$E = mc^2$$

$$E = f(m, c)$$

$$E = f \propto K m^x c^y \quad (K \text{ is constant})$$

$$MLT^{-2} = K M^x [LT^{-1}]^y$$

$$MLT^{-2} = K M^x L^y T^{-y}$$

$$\begin{aligned} x &= 1 \\ y &= 2 \\ E &= K m c^2 \\ K &= 1 \quad (\text{from experiments}) \\ E &= m c^2 \end{aligned}$$

Q if velocity, force & time are taken as funda-qns. find dimensions of mass & energy

$$vel = \frac{\text{dis}}{\text{time}}$$

$$\text{force} = \text{mass} \times \frac{\text{dis}}{\text{time}^2} \times \frac{\text{vel}}{\text{time}}$$

$$\frac{MLT^{-2}}{LT^{-1}}$$

$$m = f(v, f, t)$$

$$m = K v^x F^y t^z$$

$$m = K [LT^{-1}]^x [MLT^{-2}]^y [F]^z$$

$$m = K L^x M^y L^{x+y} T^{z-y}$$

$$\begin{aligned} y &= 1 \\ x+y &= 0 \end{aligned}$$

(5)

$$E = f(v, f, t)$$

$$ML^2T^{-2} = M^y L^{x+y} T^{-x-2y+2}$$

$$\boxed{y=1}$$

$$\boxed{x=1}$$

$$-x - 2y + z = -2$$

$$-1/2 + 2 = -1/2$$

$$\boxed{z=1}$$

$$\boxed{E = K \cancel{v} v^y F^x T^z}$$

Q if momentum, area & time are taken

$$E = ?$$

$$ML^2T^{-2} = K P^x A^y T^z$$

$$ML^2T^{-2} = K [MLT^{-1}]^x [L^2]^y [T^z]^z$$

$$= K M^x L^{x+2y} T^{-x+2z}$$

$$\text{area} = L^2$$

$$\text{Time} \cancel{T} = T^1$$

$$\text{momentum} = MLT^{-1}$$

$$\boxed{x=1}$$

$$x+2y=2$$

$$2y = 1$$

$$\boxed{y = 1/2}$$

$$\cancel{z=1}$$

$$\boxed{z = 1 + -2}$$

$$\boxed{z = -1}$$

$$\boxed{E = K \cancel{P} P A^{1/2} T^{-1}}$$

⑥

$$\rho = \frac{MLT^{-2}}{L^2} = M L^{-1} T^{-2}$$

$$D = \frac{M}{L^3}$$

$$T' = K \left( ML^{-1} T^{-2} \right)^x \left[ ML^{-3} \right]^y \left[ ML^2 T^{-2} \right]^z$$

$$T' = K M^{x+y+2} L^{-x-3y+2z} T^{-2x-2z}$$

$$= ML^{-3}$$

$$E = ML^2 T^{-2}$$

$$-2x - 2z = 1$$

$$-2z = 2x$$

$$\boxed{z = -x}$$

$$z = \frac{1+2x}{2}$$

$$-x - 3y + 2x = 0$$

$$-3y = x$$

$$x = -y$$

$$-y + y + 2 = 0$$

$$\boxed{z = 0}$$

$$\boxed{x = 0}$$

$$\boxed{y = 0}$$

$$x = -1 - 3 \left( \frac{-3}{10} \right)$$

$$= -1 + \frac{9}{10}$$

$$= \frac{-10 + 9}{10}$$

$$\boxed{x = \frac{-1}{10}}$$

$$z = 1 - \frac{2}{10}$$

$$z = \frac{8}{10}$$

$$\boxed{z = \frac{4}{5}}$$

$$-x - 3y + 2 \left( \frac{1+2x}{2} \right)$$

$$-x + 3y + 1 + 2x$$

$$x + 3y + 1 = 0$$

$$\boxed{x = -1 - 3y}$$

$$-1 - 3y + y + 1 + 2(-1 - 3y)$$

$$-1 - 3y + y + \frac{1 + 2(-1 - 3y)}{2}$$

$$\frac{-2 - 24y + 1 - 2 - 6y}{2}$$

$$\frac{-1 - 3 - 10y}{2} = 0$$

$$y = \frac{3}{10}$$

$$\boxed{y = \frac{-3}{10}}$$

(7)

$$T = K P^x d^y E^z$$

$$M^o L^o T^l = [ML^{-1} T^{-2}]^x [ML^{-3}]^y [ML^2 T^{-2}]^z$$

$$M^o L^o T^l = M^{x+y+z} L^{-x-3y+2z} T^{-2x-2z}$$

$$x + y + z = 0$$

$$-x - 3y + 2z = 0$$

$$-2x - 2z = 1$$

$$-x = \frac{1+2z}{2}$$

$$-x - 3y + 2z = 0$$

$$\frac{1+2z}{2} - 3y + 2z$$

$$1+2z - 6y + 4z = 0$$

$$1-6y + 6z = 0$$

$$y = \frac{6z+1}{6}$$

$$\frac{6z+1}{6} + \frac{-1-2z}{2} + z = 0$$

$$\frac{6z+1 - 3 + 6z + 6z}{6} = 0$$

$$\frac{6z-2}{6} = 0$$

$$6z = 2$$

$$z = \frac{1}{3}$$

$$x = -1 - \frac{2}{3}$$

$$x = \cancel{-1} - \frac{-3-2}{6}$$

$$x = \frac{-5}{6}$$

$$y = \frac{2+1}{6}$$

$$y = \frac{1}{3}$$

$$t = K P^{\frac{-5}{6}} d^{\frac{1}{3}} E^{\frac{-1}{3}}$$

(8)

## System of units

FPS (Foot, Pound, Second) (British system)

CGS (Gaussian system) (centimeter, gram, second)

MKS (Metric system) (meter, kg, second)

$$1 \text{ N} = 10^5 \text{ dyne (c.g.s)}$$

$$1 \text{ J} = 10^7 \text{ erg (G.G.s)}$$

### Formulas

$$1. \sin^2 \theta + \cos^2 \theta = 1$$

$$2. 1 + \tan^2 \theta = \sec^2 \theta$$

$$3. 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$4. \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$5. \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$6. \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$7. \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$8. \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$9. \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$10. \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\begin{aligned} 11. \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \end{aligned}$$

$$12. 1 + \cos \theta = 2 \cos^2 \left( \frac{\theta}{2} \right)$$

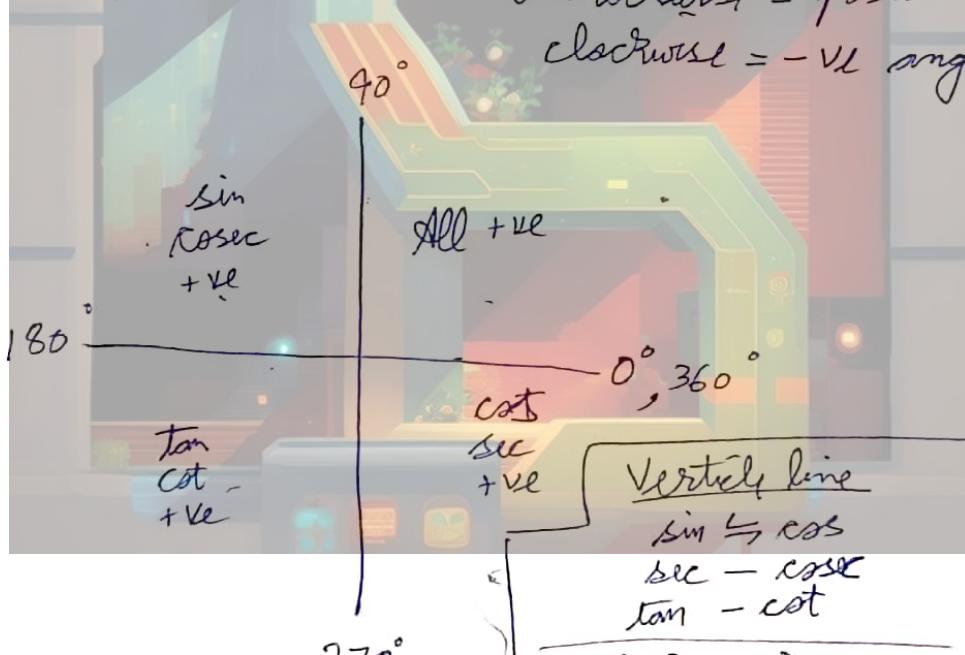
$$13. 1 - \cos \theta = 2 \sin^2 \left( \frac{\theta}{2} \right)$$

Ex.  $\sin 150^\circ = \sin(90 + 60)$

$$\begin{aligned} \sin(A+B) &= \sin 90 \cos 60 + \sin 60 \cos 90 \\ &= 1 \times \frac{1}{2} + 0 \\ &= \frac{1}{2} \end{aligned}$$

~~ATSC~~ rule:-  
ASTC

anticlockwise = positive angle  
clockwise = -ve angle



Horizontal Line (some signs & ratio)

$$\begin{aligned} \sin(180 - \theta) &= + \sin \theta \\ \sin(180 + \theta) &= - \sin \theta \\ \cos(180 - \theta) &= - \cos \theta \\ \tan(180 + \theta) &= \tan \theta \end{aligned}$$

$$\begin{aligned} \sin(90 - \theta) &= + \cos \theta \\ \sin(90 + \theta) &= + \cos \theta \\ \cos(90 + \theta) &= - \sin \theta \\ \cos(90 - \theta) &= + \sin \theta \\ \tan(90 - \theta) &= \cot \theta \\ \tan(90 + \theta) &= - \cot \theta \end{aligned}$$

①

$$\text{eg. } \sin 150^\circ = \sin(90^\circ + 60^\circ) \\ = \cos 60^\circ \\ = \frac{1}{2}$$

$$\tan(270^\circ + \theta) = -\cot \theta$$

$$\text{eg. } \cos(120^\circ) = \cos(90^\circ + 30^\circ) \\ = -\sin 30^\circ$$

$$= -\frac{1}{\sqrt{2}}$$

## # Angles

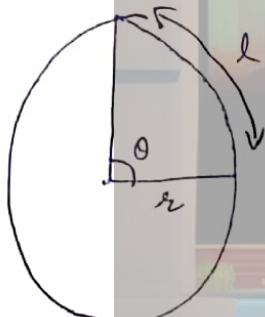
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① Sexagesimal system :- angle is measured in degrees ( $\theta^\circ$ )

$$1^\circ = 60' \text{ (arc minutes)}$$

$$1' = 60'' \text{ (arc seconds)}$$

② Circular system :- angle in radians (rad) ( $\theta^c$ ) ( $\theta^c$ -no. superscript)



$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi^c = 360^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$180^\circ = \pi^c$$

$$90^\circ = \frac{\pi}{2}^c$$

$$45^\circ = \frac{\pi}{4}^c$$

$$30^\circ = \frac{\pi}{6}^c$$

$$60^\circ = \frac{\pi}{3}^c$$

\* If  $\theta$  is very small ( $\theta < 5^\circ$ )

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 \quad \text{or} \quad 1 - \frac{\theta^2}{2}$$

$$\tan \theta \approx \theta$$

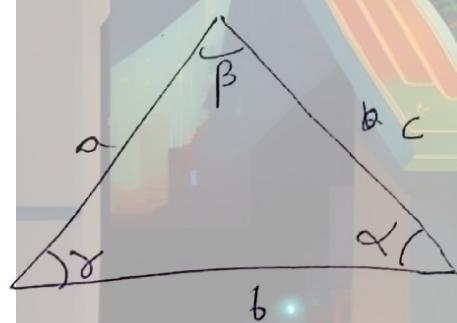
( $\theta$  must in radian)

$$\sin 37^\circ = \frac{3}{5} \quad \sin 53^\circ = \frac{4}{5}$$

$$\cos 37^\circ = \frac{4}{5} \quad \cos 53^\circ = \frac{3}{5}$$

$$\tan 37^\circ = \frac{3}{4} \quad \tan 53^\circ = \frac{4}{3}$$

# Sine Rule



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

# Cosine Rule:-

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

(D)

## # Binomial Theorem (Expression with 2 terms)

Factorial (!)

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120$$

$$4! = 4 \times 3 \times 2 \times 1 = \cancel{2} 24$$

$$3! = 3 \times 2 \times 1 = 6$$

$$2! = 2 \times 1 = 2$$

$$1! = 1 = 1$$

$$0! = 1$$

$n!$  = Product of  $n$  natural numbers

$$= n \times (n-1) \times (n-2) \dots \times 3 \times 2 \times 1$$

$(x+y)$ ,  $(4x-3y)$ ,  $(a+b)^2$   
Binomial Expression :-

# Binomial Formula

$$(x+a)^n = {}^n C_0 x^n a^0 + {}^n C_1 x^{n-1} a^1 + {}^n C_2 x^{n-2} a^2 + \dots + {}^n C_n x^0 a^n$$

$${}^n C_r = \frac{n!}{r! (n-r)!}$$
 ] combination formula

$$\text{Lg. } (a+b)^2 = {}^2C_0 a^2 b^0 + {}^2C_1 a^1 b^1 + {}^2C_2 a^0 b^2$$

$$= \frac{2!}{0! 2!} a^2 + \frac{2!}{1! 1!} ab + \frac{2!}{2! 0!} b^2$$

$$= \frac{a^2}{1} + \frac{2}{1 \times 1} ab + \frac{b^2}{1}$$

$$= a^2 + 2ab + b^2$$

$$\text{Lg. 2. } (a+b)^3 = {}^3C_0 a^3 b^0 + {}^3C_1 a^2 b^1 + {}^3C_2 a^1 b^2 + {}^3C_3 a^0 b^3$$

$$= \frac{3!}{0! 3!} a^3 + \frac{3!}{1! 2!} a^2 b + \frac{3!}{2! 1!} a b^2 + \frac{3!}{3! 0!} b^3$$

$$= \frac{a^3}{1} + \frac{6}{2} a^2 b + \frac{6}{2} a b^2 + \frac{b^3}{1}$$

$$= a^3 + 3a^2 b + 3a b^2 + b^3$$

$$\text{Lg. 3. } (1+x)^6 = {}^6C_0 1^6 x^0 + {}^6C_1 1^5 x^1 + {}^6C_2 1^4 x^2 + {}^6C_3 1^3 x^3 +$$

$$+ {}^6C_4 1^2 x^4 + {}^6C_5 1^1 x^5 + {}^6C_6 1^0 x^6$$

$$= \frac{6!}{0! 6!} x^0 + \frac{6!}{1! 5!} x^1 + \frac{6!}{2! 4!} x^2 + \frac{6!}{3! 3!} x^3 + \frac{6!}{4! 2!} x^4 + \frac{6!}{5! 1!} x^5 + \frac{6!}{6! 0!} x^6$$

$$= 1 + \frac{720}{120} x^1 + \frac{\frac{360}{3240} \cdot 180 \cdot 90 \cdot 45 \cdot 15}{240} x^2 + \frac{\frac{720}{6 \times 6} \cdot 3240}{3 \cdot 3} x^3 + \frac{720}{240} x^4 + \frac{720}{120} x^5 + x^6$$

$$= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$$

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Binomial approximation :-

$$(1+x)^n$$

$x \ll 1$  ( $\ll$  means very small)

$$(1+x)^n \approx 1 + nx \quad x \ll 1$$

$$\text{eg. } (1+x)^2 = 1 + x^2 + 2x \approx 1 + 2x$$

$$(1+x)^3 = 1 + x^3 + 3x^2 + 3x \approx 1 + 3x$$

$x \ll 1$   
 $x^2, x^3 \rightarrow$  higher orders of  $x \rightarrow$  even smaller

$$\text{eg. } g_n = g \left[ \frac{R}{(R+h)} \right]^2$$

$$R = 6400 \text{ km}$$

$$g = 9.8$$

~~$h = \text{height above}$~~

if  $h \ll R$

$$R g_{nh} = g \left[ \frac{R}{R(1+\frac{h}{R})} \right]^2$$

$$= g \left[ \frac{1}{1+\frac{h}{R}} \right]^2$$

$$= g \left( 1 + \frac{h}{R} \right)^{-2}$$

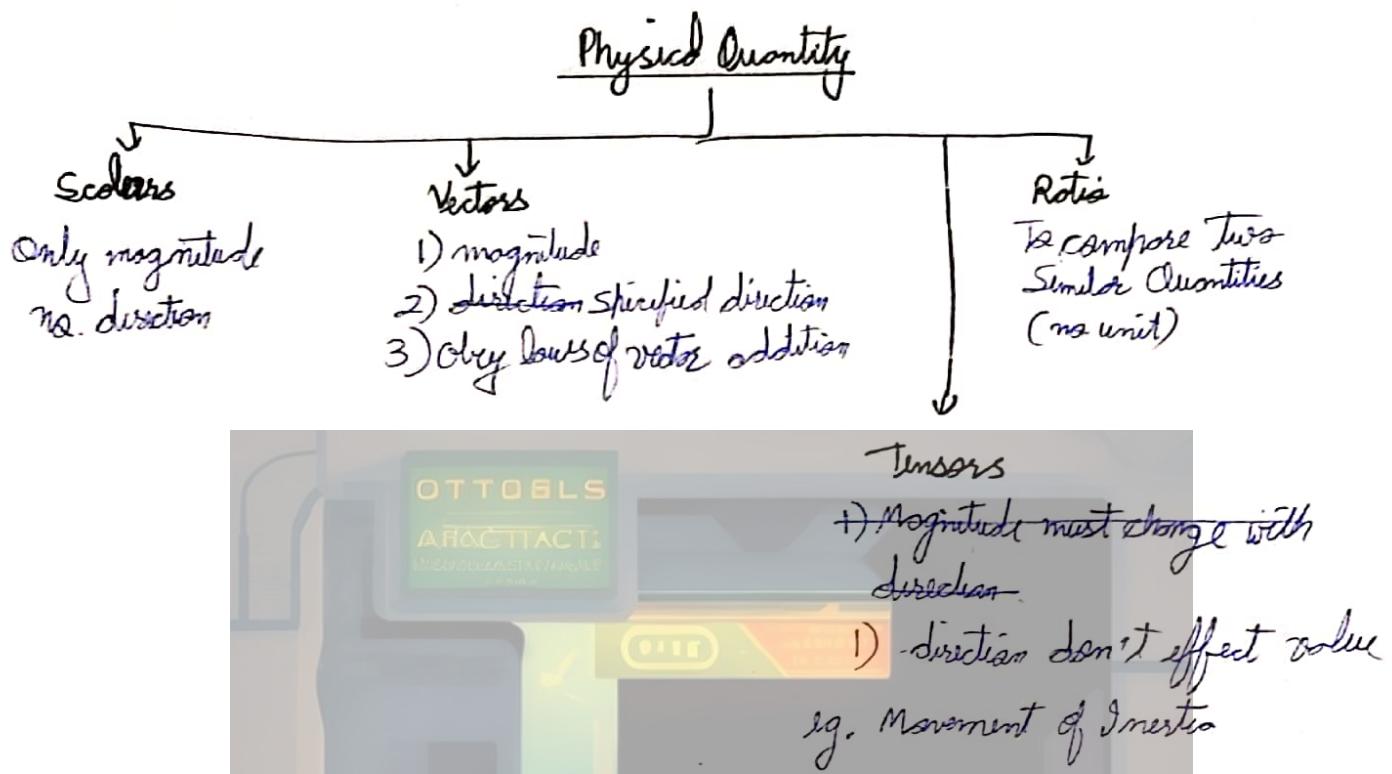
~~$g(1 -$~~

$$h \ll R$$

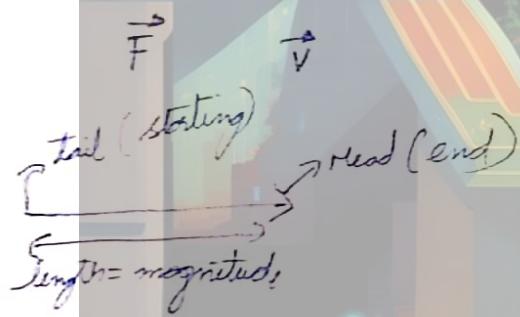
$$\begin{aligned} & \left| \frac{h}{R} \ll 1 \right. \\ & \left. \frac{h}{R} \right. \end{aligned}$$

$$g_h = g \left( 1 - \frac{2h}{R} \right)$$

# Vectors



# Representation of a vector :-

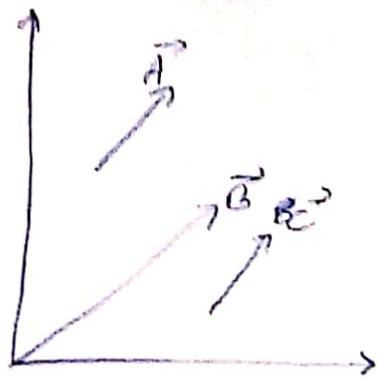


~~$\vec{F}$~~  (taking

→ For two vectors to be same, their magnitude & direction both should be equal.

$|\vec{A}|$  (taking only magnitude of the vector)

→ we can displace a vector parallel to itself without affecting it.

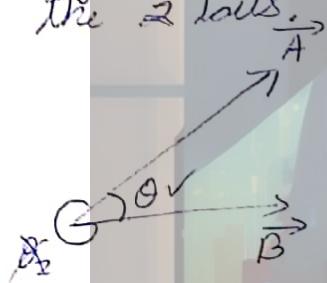


$$\vec{A} = \vec{C}$$

### # Angle between 2 vectors.

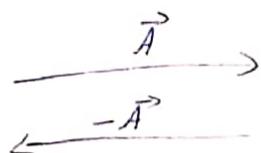
→ Join the 2 vectors by tail.

Angle between 2 vectors is the smaller angle between the 2 tails.



### # Negative of a vector

→ A vector of same magnitude but opposite in direction.



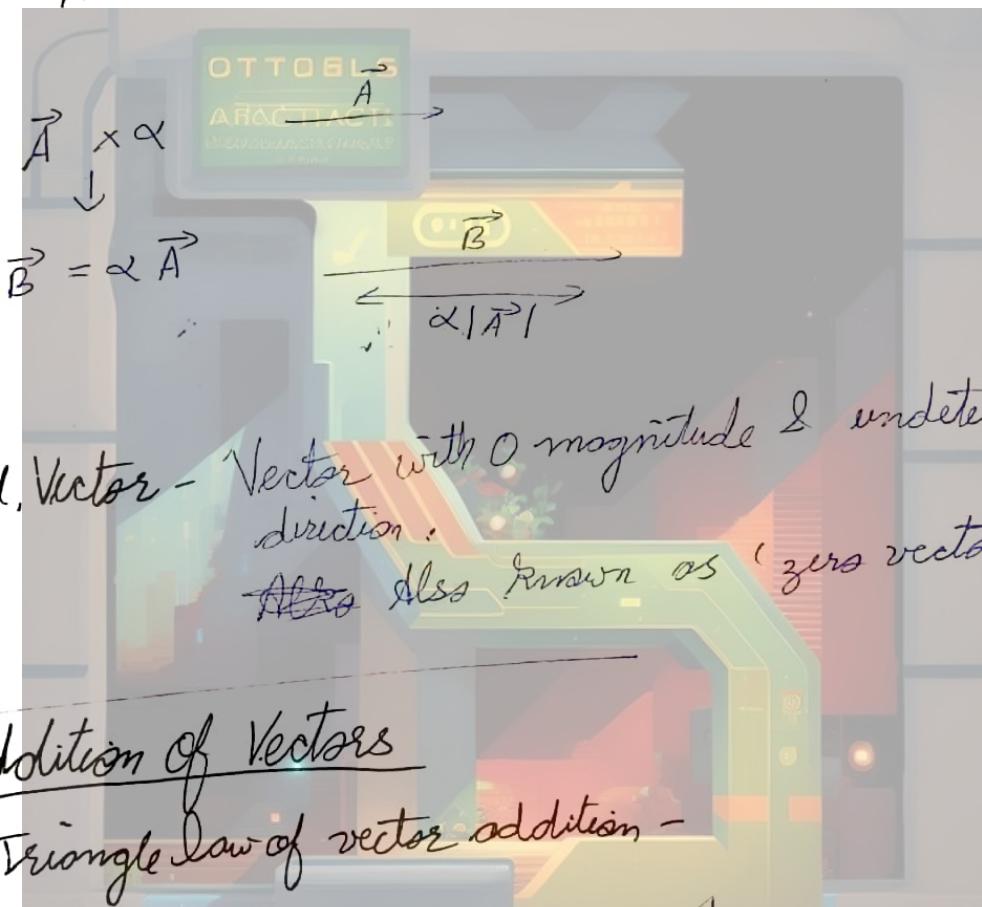
## # Multiply

### ① with Positive number

Multiplying vector  $\vec{A}$  with  $\alpha$  will give new vector  $\vec{B}$  with magnitude  $\alpha$  times & same direction.

### ② with negative number

Opp direction & magnitude gets multiplied.



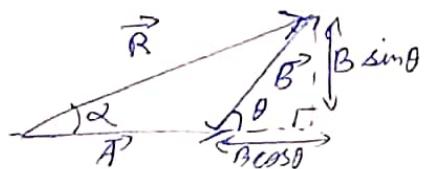
# Null Vector - Vector with 0 magnitude & undetermined direction.

~~Also~~ also known as "zero vector"

## # Addition of Vectors

### ① Triangle Law of vector addition -

$$\vec{A} + \vec{B} = \vec{R}$$



$$\sin \theta = \frac{P}{x}$$

$$p = x \sin \theta$$

$$\cos \theta = \frac{b}{x}$$

$$b = x \cos \theta$$

PGT in

$$R^2 = (B \sin \theta)^2 + (A + B \cos \theta)^2$$

$$\begin{aligned} R^2 &= B^2 \sin^2 \theta + A^2 + B^2 \cos^2 \theta + 2AB \cos \theta \\ &= B^2 (\sin^2 \theta + \cos^2 \theta) + A^2 + 2AB \cos \theta \end{aligned}$$

$$R = \sqrt{B^2 + A^2 + 2AB \cos \theta}$$

Magnitude of resultant vector

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

for  $\theta = 0^\circ$

$$\begin{cases} \theta = 0^\circ \\ R = A + B \end{cases}$$

OTTO BLS  
ARCTIC  
MATERIALS

$$\begin{cases} \theta = 180^\circ \\ R = A - B \end{cases}$$

$$\begin{cases} \theta = 90^\circ \\ R^2 = \sqrt{A^2 + B^2} \end{cases}$$

$$\tan \alpha = \frac{B}{A} \quad |A - B| \leq R \leq |A + B|$$

e.g. Only the vectors of the some type can be added & subtracted & result is also same time.

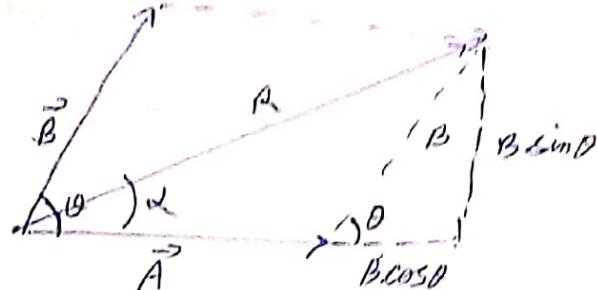
Vector addition is commutative i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Vector addition is associative i.e.

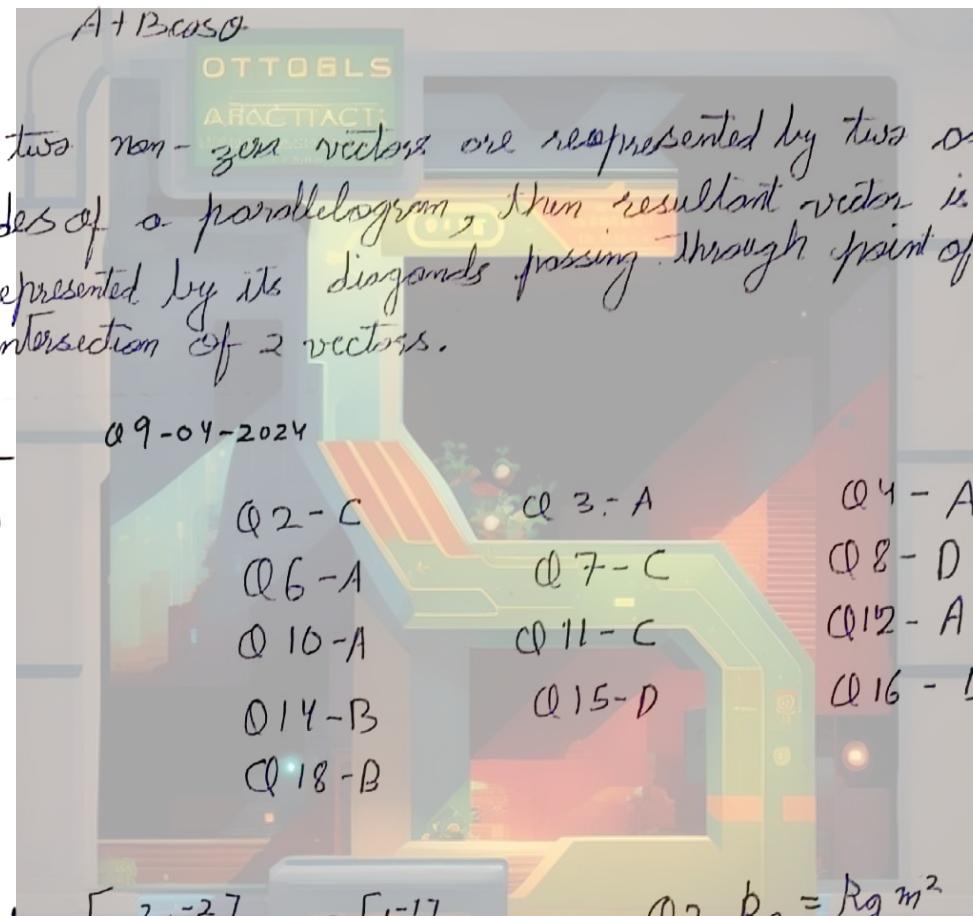
$$(\vec{P} + \vec{Q}) + \vec{R} = \vec{P} + (\vec{Q} + \vec{R}) = \vec{Q} + (\vec{P} + \vec{R})$$

## II Parallelogram Law of Vector addition -



$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\tan \theta = \frac{B \sin \theta}{B \cos \theta}$$



Q-1

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Q-4 - D

Q-5 - B

Q-9 - D

Q-13 - D

Q-17 - C

Q-2 - C

Q-6 - A

Q-10 - A

Q-14 - B

Q-18 - B

Q-3 - A

Q-7 - C

Q-11 - C

Q-15 - D

Q-4 - A

Q-8 - D

Q-12 - A

Q-16 - B

S-4

$$Q-1 - R = [ML^2 F^{-2}] \quad a = [L^{-1}]$$

$$Q-2 - R_B = \frac{kg m^2}{K s^2}$$

$$Q-3 - t = a \sqrt{\frac{m}{K}}$$

$$Q-4 - w = K \sqrt{\frac{m}{s^2}}$$

$$Q-5 - R = 6$$

$$Q-6 - [MLT^{-1}]$$

$$Q-7 i) 24.25$$

$$ii) -\frac{7}{2} s.$$

$$iii) \frac{\sqrt{3}}{2} s$$

$$iv) \frac{1-\sqrt{3}}{2} s$$

$$Q-8 - l = \frac{2\pi}{3} \times 6.4 \times 10^6 m$$

$$Q-9 - a = 9.95$$

$$Q-10 - \frac{\pi}{100} m$$

$$Q-11.$$

Q1-3) Q2-4) Q3-~~4~~) Q4-2) Q5-2) ~~Q6-2~~  
 Q10-3) Q11-4) Q16-4) Q21-C) Q22-C) Q23-B)

Q7  
i)  $\sin 74^\circ$

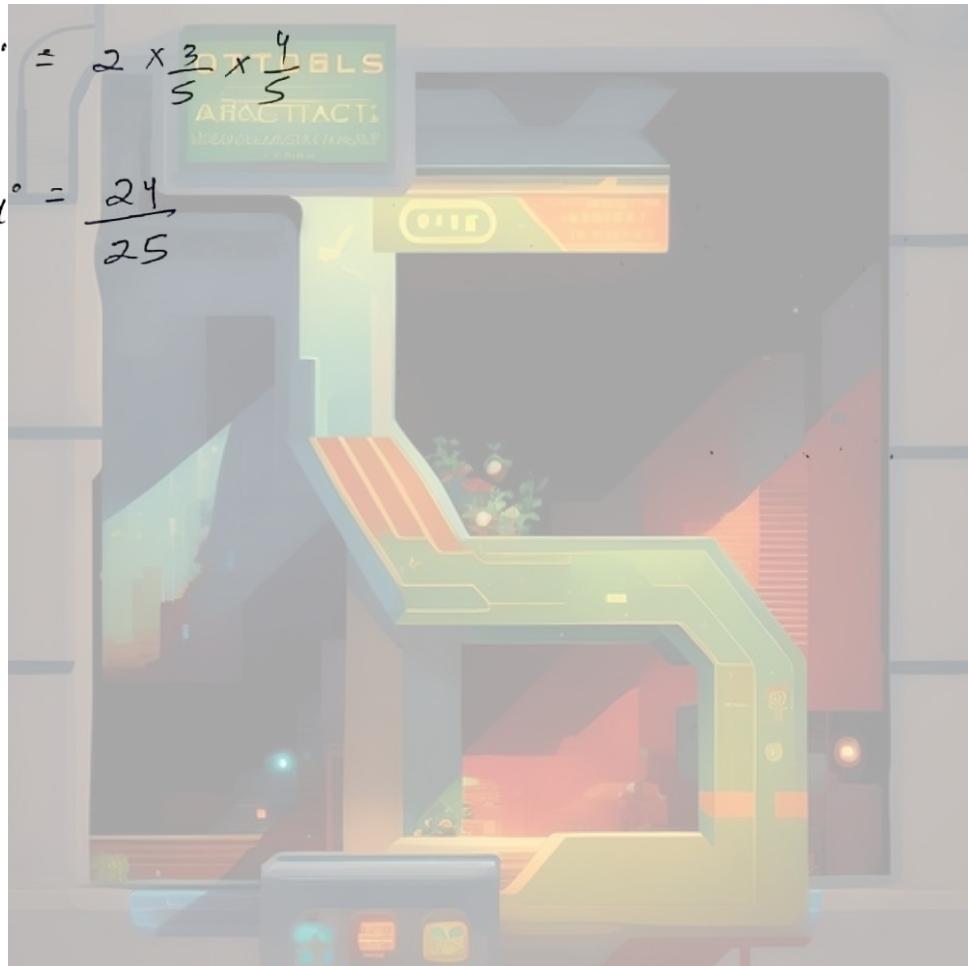
$\sin 2(37^\circ)$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 2(37^\circ) = 2 \sin 37^\circ \cos 37^\circ$$

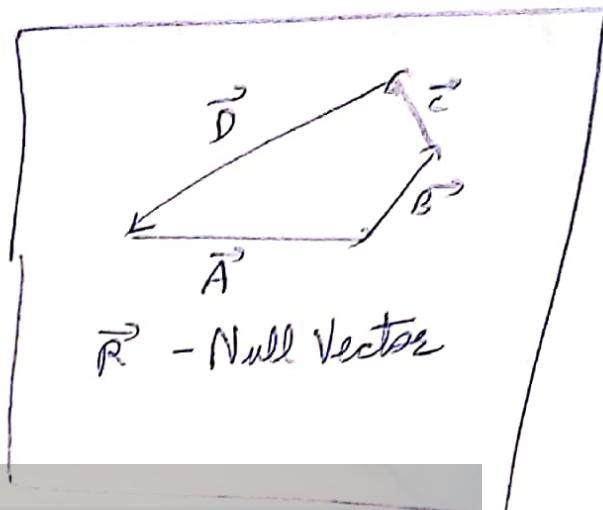
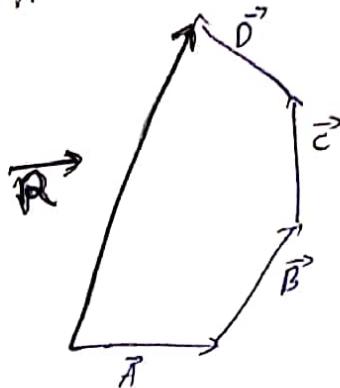
$$\sin 74^\circ = 2 \times \frac{3}{5} \times \frac{4}{5}$$

$$\sin 74^\circ = \frac{24}{25}$$

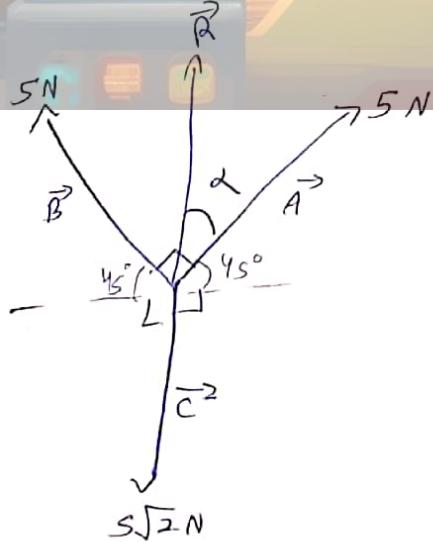
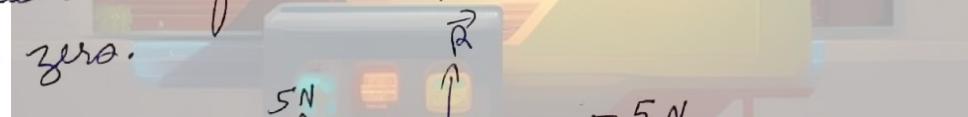


## Polygon law of vector addition:

$$\vec{A} + \vec{B} + \vec{C} + \vec{D}$$



- ★ If all the vectors are arranged to form sides of a polygon in one order then their resultant is represented by closing side of polygon in opposite order.
- ★ If  $n$  vectors are represented by  $n$  sides of a polygon in cyclic manner, the resultant of these vectors is a null vector.
- ★ Resultant of two unequal vectors cannot be zero.
- ★ Resultant of 3-co-planer vectors may or may not be zero.



$$\vec{A} + \vec{B} + \vec{C} = (\vec{A} + \vec{B}) + \vec{C}$$

$$5\sqrt{2} + 5\sqrt{2} = 0$$

$$\alpha = 45^\circ$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$\vec{R} = \sqrt{25 + 25}$$

$$\vec{R} = 5\sqrt{2}.$$

$$\tan \alpha = \frac{B}{A}$$

$$\tan \alpha = 1$$

$$\tan 45^\circ = 1$$

$$45^\circ = 45^\circ$$

$$\vec{R} + \vec{C} = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2}$$

$$= 0$$

Eg 2.



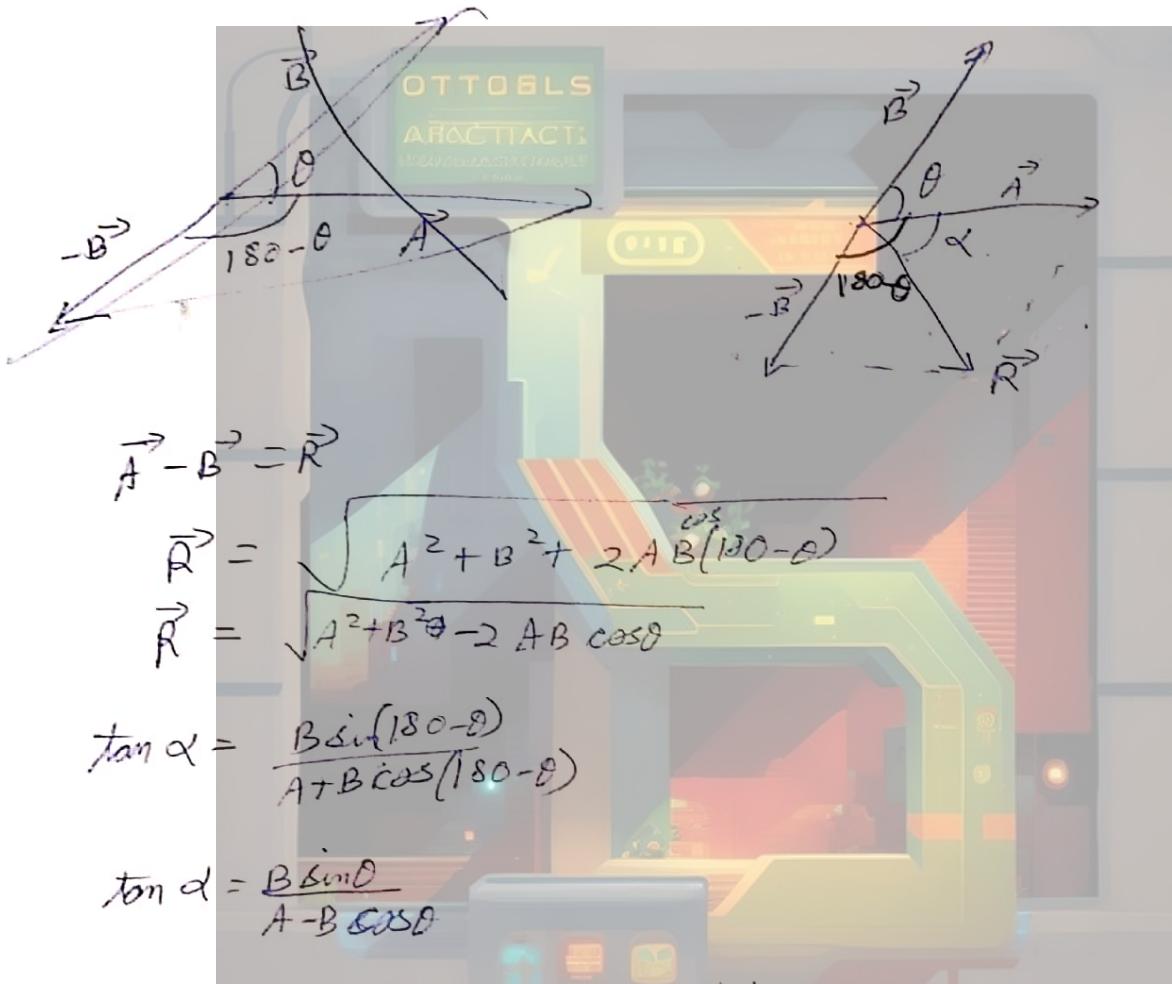
$$\vec{A} + \vec{B} + \vec{C} \neq 0$$

\* The result of non-co-planar vectors cannot be zero

## # Subtraction of vectors

$$\vec{A} - \vec{B}$$

$$\vec{A} + (-\vec{B})$$



$$\vec{A} - \vec{B} = \vec{R}$$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB \cos(180^\circ - \theta)}$$

$$|\vec{R}| = \sqrt{A^2 + B^2 - 2AB \cos\theta}$$

$$\tan \alpha = \frac{B \sin(180^\circ - \theta)}{A + B \cos(180^\circ - \theta)}$$

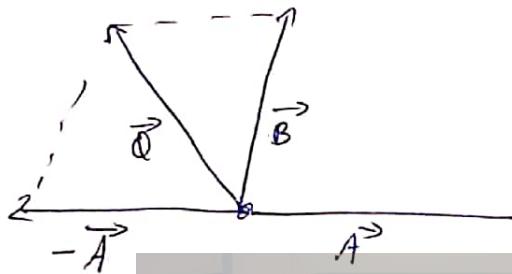
$$\tan \alpha = \frac{B \sin \theta}{A - B \cos \theta}$$

$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

\* Non-commutative

$$\vec{B} - \vec{A} = \vec{0}$$

$$\vec{B} + (-\vec{A})$$



$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

Q Given that  $\vec{C} = \vec{A} + \vec{B}$ .

Also  $|\vec{A}| = 12$ ,  $|\vec{B}| = 5$ ,  $|\vec{C}| = 13$  units

Find angle b/w  $\vec{A}$  &  $\vec{B}$

$$|\vec{A}| = 13$$

$$A = 13 \text{ units}$$

$$B = \frac{13}{12} \text{ units}$$

$$C = \frac{13}{5} \text{ units}$$

$$\frac{13}{5} = \sqrt{(13)^2 + \left(\frac{13}{12}\right)^2 + 2 \times \frac{13}{12} \times 13 \times \cos \theta}$$

$$\frac{169}{25} = 169 + \frac{169}{144} + \frac{169}{6} \times \cos \theta$$

(25)

Ques.

$$C = \sqrt{A^2 + B^2 + 2AB \cos\theta}$$

$$13 = \sqrt{(12)^2 + 5^2 + (12)(5) \cos\theta}$$

$$169 = 144 + 25 + 120 \cos\theta$$

$$\cos\theta = 0$$

$$\theta = 90^\circ = \frac{\pi}{2}$$

A & B are Perpendicular / orthogonal

- Q2. The greatest least resultant of two forces acting at a point is 10 N & 6 N. If each is increased by 3 N, find the resultant of two forces when acting at a point at an angle of  $90^\circ$  with each other.

$$A + B = 10$$

$$A - B = 6$$

$$\underline{A = 6 + B}$$

$$6 + B + B = 10$$

$$2B = 4$$

$$\underline{B = 2}$$

$$\underline{A = 8}$$

$$A = 8 + 3 = 11$$

$$B = 2 + 3 = 5$$

$$\theta = 90^\circ$$

$$\begin{aligned} \vec{R} &= \sqrt{121 + 25} \\ \vec{R} &= \sqrt{146} \text{ N} \end{aligned}$$

$$\tan \alpha = \frac{5}{11}$$

Ans

(26)

Q. The maximum & minimum of two forces are in ratio 7:3  
find force ratio.

$$A+B=7x$$

$$A-B=3x$$

$$A = 3x + B$$

$$3x + 2B = 7x$$

$$\begin{array}{r} A \\ B \end{array}$$

$$2B = 4x$$

$$\underline{\underline{B = 2x}}$$

$$\begin{array}{r} 3x + B \\ B \end{array}$$

$$\begin{array}{r} 3x + B \\ B \\ A = 5x \end{array}$$

$$\frac{A}{B} = \frac{5x}{2x} = \frac{5}{2}$$

$$\boxed{A:B = 5:2} \checkmark$$

$$c = \sqrt{2x^2 + 2x^2 \cos\theta}$$

$$c = 2x \sqrt{2\cos\theta}$$

$$x^2 \cancel{x^2} = 2x^2 + 2x^2 \cos\theta$$

$$-x^2 = 2x^2 \cos\theta$$

$$-\frac{1}{2} = \cos\theta$$

$$\boxed{\theta = 120^\circ} \checkmark$$

$$\boxed{\theta = 120^\circ} \checkmark$$

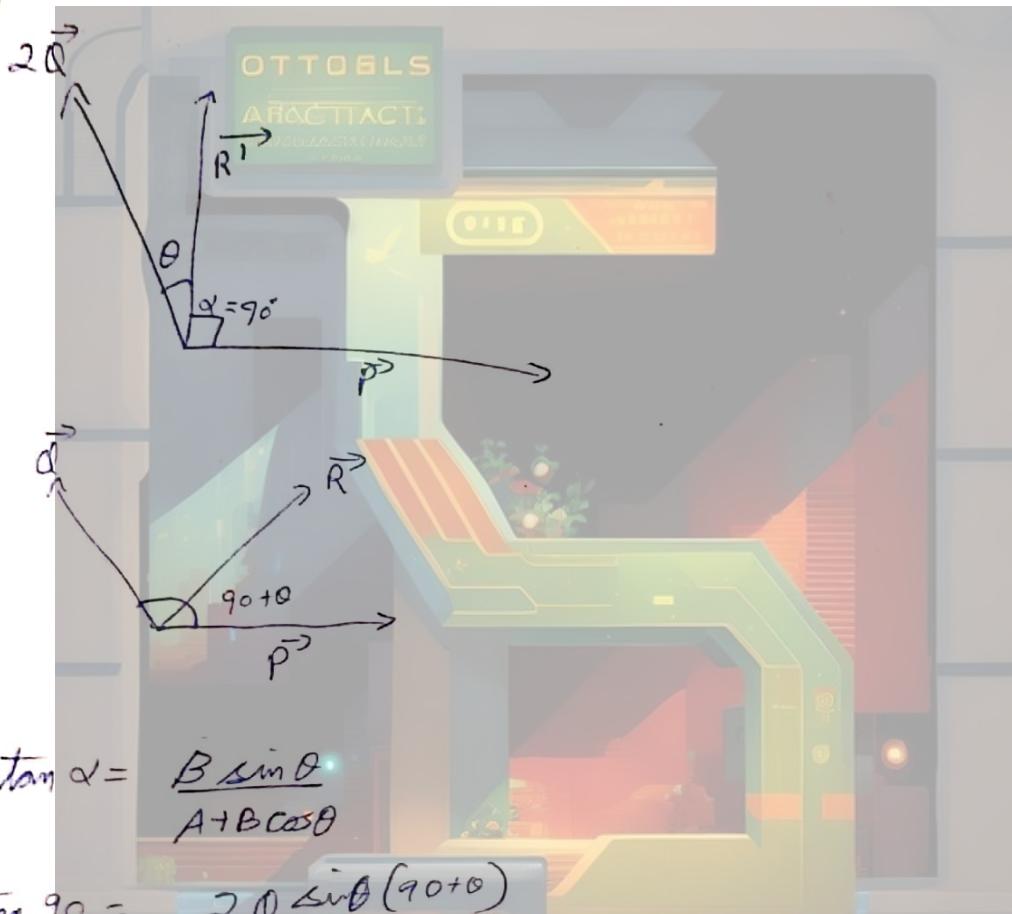
Q Two equal forces have their resultant equal to either. find angle.

(27)

$$\vec{P} + \vec{Q} = \vec{R}$$

$$\tan \theta = \frac{2Q \sin \theta}{P+Q}$$

Q The resultant of two vectors  $\vec{P}$  &  $\vec{Q}$  is  $\vec{R}$ . If the magnitude of  $\vec{Q}$  is doubled, the new vector is  $\perp$  to  $\vec{P}$ . find  $\vec{R}$



$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan 90 = \frac{2Q \sin \theta (90 + \theta)}{P + 2Q \cos (90 + \theta)}$$

$\theta$  (as  $\tan 90$  is undefined)

$$P + 2 \cos \theta (90 + \theta)$$

$$\cos (90 + \theta) = -\frac{P}{2Q}$$

$$R = \sqrt{P^2 + Q^2 + 2 P Q \cos(\theta_0 + \theta)}$$

$$R = \sqrt{P^2 + Q^2 + 2 P Q \cos \frac{\theta}{2}}$$

$$\boxed{R = Q}$$

Q.6 The sum of magnitude of two forces at a point is 18 N and magnitude of resultant is 12 N. If the resultant makes an angle of  $90^\circ$  with the force of smaller magnitude, what are the magnitudes of two forces.

$$A + B = 18 \rightarrow B = 18 - A$$

$$|R| = 12 \text{ N}$$

$$\alpha = 90^\circ$$

$$\tan 90^\circ = \frac{A \sin \theta}{A + B \cos \theta} \Rightarrow 0$$

$$A + B \cos \theta = 0$$

$$\cos \theta = -\frac{A}{B}$$

$$\vec{R} = \vec{A} + \vec{B}$$

$$12 = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$144 = A^2 + B^2 + 2A^2$$

$$144 = B^2 - A^2$$

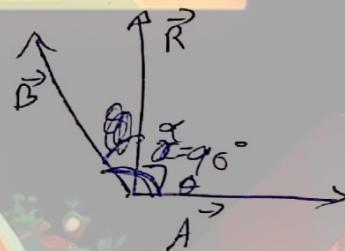
$$144 = (8-A)^2 - A^2$$

$$144 = 324 + A^2 - 36A - A^2$$

$$36A = 324 - 144$$

$$A = \frac{180}{36} = 5$$

$$\begin{array}{r} 118 \\ 144 \\ \hline 180 \\ 324 \end{array}$$



$$\boxed{A = 5 \text{ N}} \\ \boxed{B = 13 \text{ N}} \quad \checkmark$$

A vector  $\vec{B}$  which has a magnitude  $B$ , is added to  $\vec{A}$  which lies along the  $x$ -axis, the resultant lie along  $y$ -axis and has a magnitude that is twice of the magnitude of  $\vec{A}$ . The magnitude of  $\vec{A}$  is \_\_\_\_.

$$A + B \frac{\sin \theta}{\cos \theta}$$

$$\cos \sin \theta = \frac{-A}{B}$$

$$\textcircled{1} \quad 2x(2x)^2 = A^2 + B^2 + 2AB \cos \theta$$

$$4x^2 = x^2 + 64 + 2x4x\cancel{B} \times \frac{-A}{B}$$

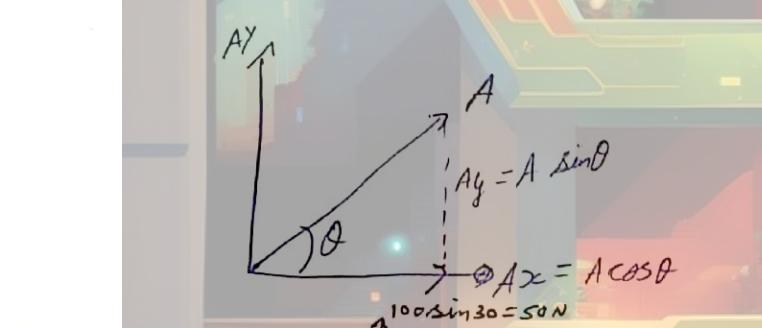
$$4x^2 = 64 - x^2$$

$$5x^2 = 64$$

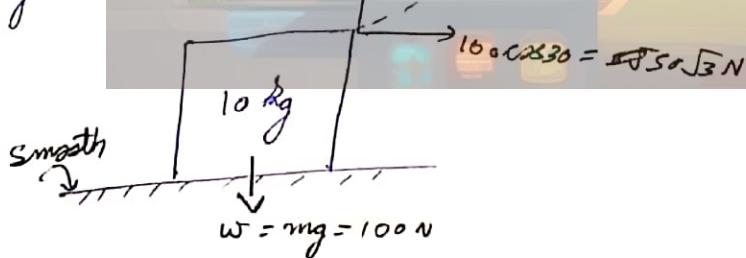
$$x^2 = \frac{64}{5}$$

$$\boxed{x = \frac{8}{\sqrt{5}}}$$

## # Components of Vectors



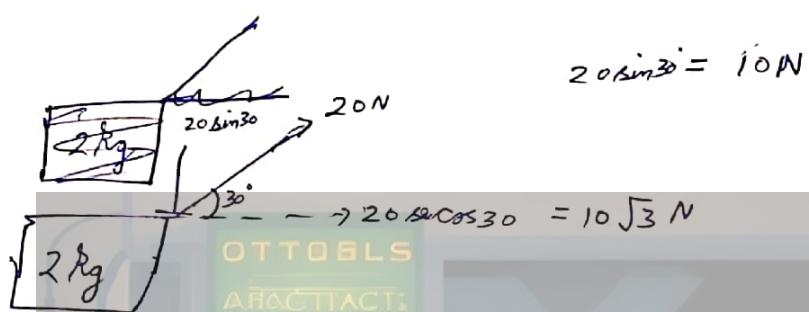
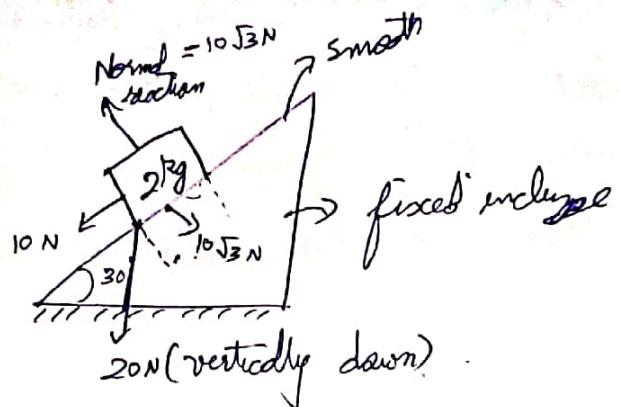
e.g.



$$F = ma$$

$$\frac{50\sqrt{3}}{10} = a = 5\sqrt{3} \text{ m/s}^2$$

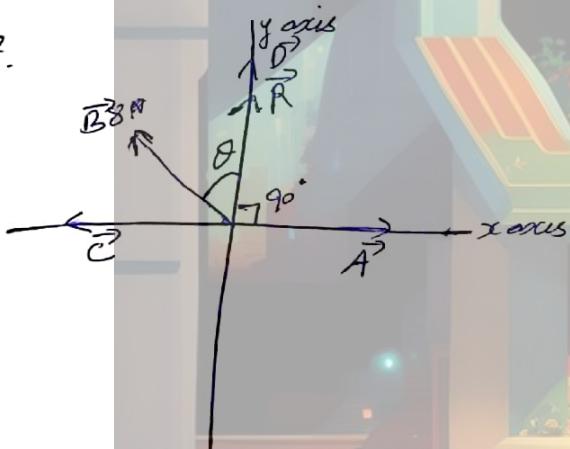
Q2.



$$F = ma$$

$$\frac{10}{2} = a = 5 \text{ m/s}^2$$

Ans 7.



$$|B| = 8$$

$$|R| = 2|A|$$

$$\vec{R} = B \sin \theta$$

$$\vec{R} = AB \cos \theta$$

$$\vec{A} + \vec{C} = 0$$

$$A = C$$

$$A = B \sin \theta$$

$$\vec{R} = R$$

$$B \cos \theta = 2B \sin \theta$$

$$\vec{R} = B \cos \theta$$

$$A^2 + R^2 = B^2 \sin^2 \theta + B^2 \cos^2 \theta$$

$$A^2 + R^2 = B^2$$

$$A^2 + R^2 = 64$$

$$A^2 + 4R^2 = 64$$

$$A^2 = \frac{64}{5}$$

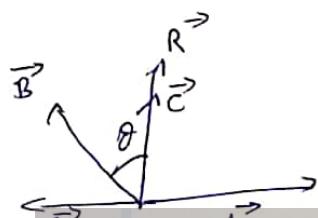
$$A = \frac{8}{\sqrt{5}}$$

(31)

RHS6.

$$|A| + |B| = 18 \quad B = 18 - A$$

$$R = 12$$



$$C = B \cos \theta$$

$$A = B \sin \theta$$

$$R = C = B \cos \theta$$

$$A^2 + R^2 = B^2 \sin^2 \theta + B^2 \cos^2 \theta$$

$$A^2 + R^2 = B^2$$

$$R^2 = B^2 - A^2$$

$$144 = (B-A)(B+A)$$

$$\frac{144}{18} = B-A$$

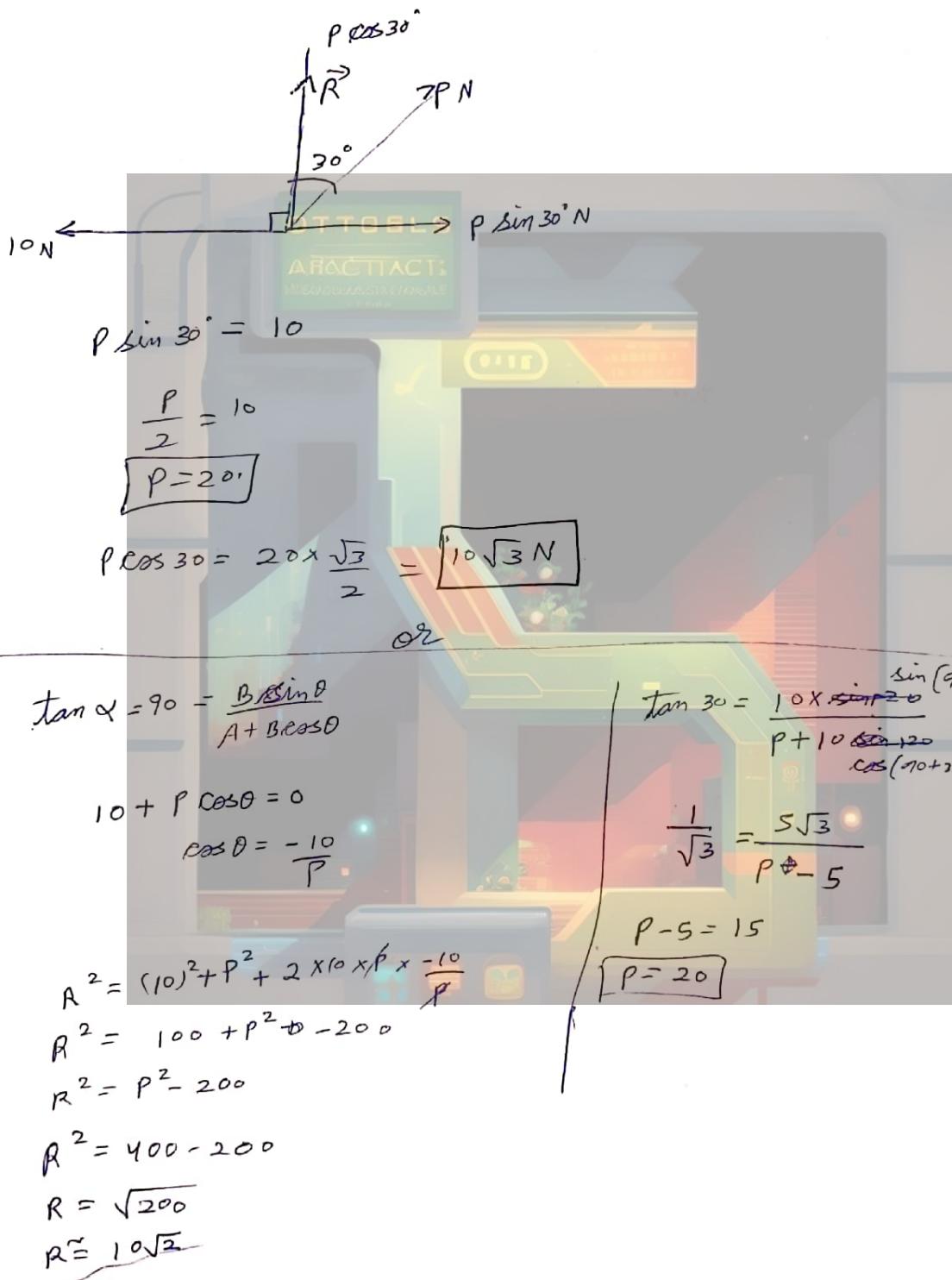
$$B-A = 8$$

$$B+A = 18$$

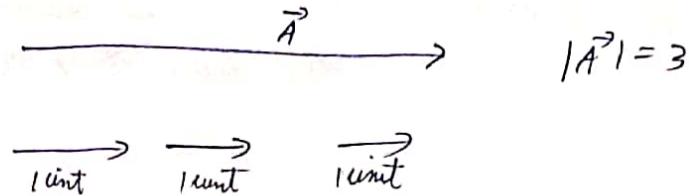
$$2B = 26$$

$$\boxed{\begin{aligned} B &= 13 \\ A &= 5 \end{aligned}}$$

Q8. Two horizontal forces of  $10\text{N}$  &  $P\text{N}$  act on a particle. The force of magnitude  $7\text{N}$  acts west & the force of  $P\text{N}$  acts on a bearing of  $30^\circ$  east of north as shown in figure. The resultant of these two forces acts due north. Find the magnitude of this resultant.



## # Unit Vector

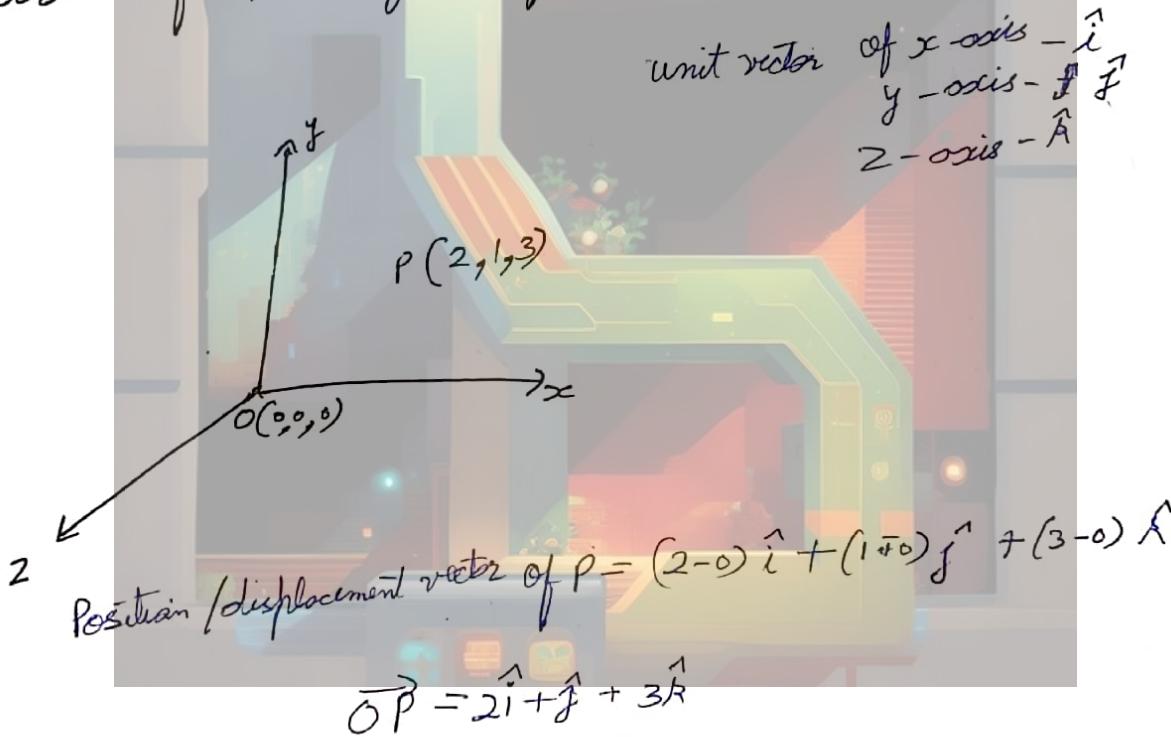


$$\text{unit vector of } \vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

dividing a vector by its magnitude, we get a vector of magnitude 1 & same direction as  $\vec{A}$ .

→ It has no dimensions and no units

→ Cartesian form/rectangular form

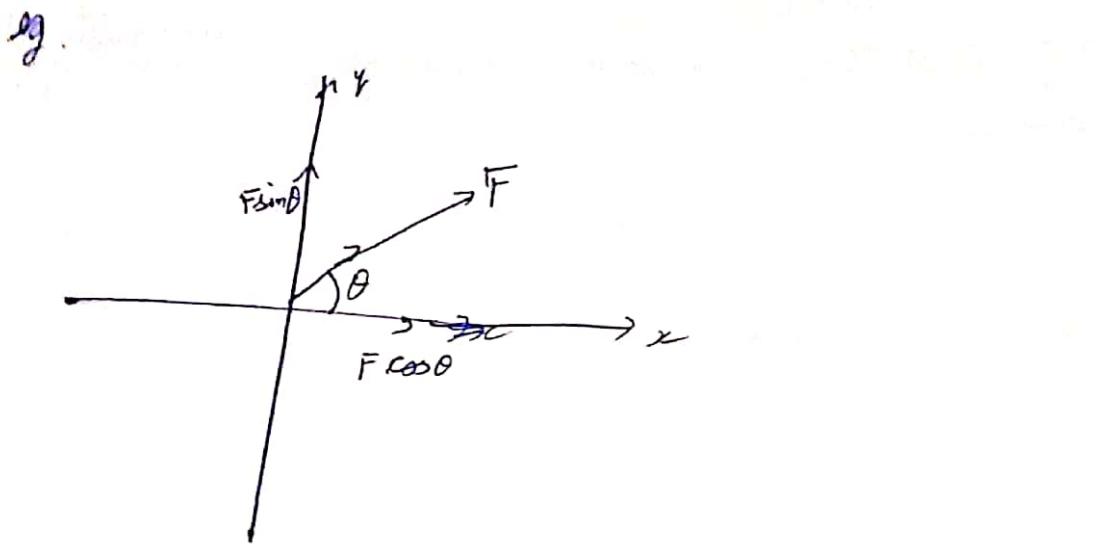


$$\begin{aligned} |\vec{OP}| &= \sqrt{2^2 + 1^2 + 3^2} \\ &= \sqrt{4 + 1 + 9} \\ &= \sqrt{14} \text{ units} \end{aligned}$$

$$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$|\vec{A}| = \sqrt{a^2 + b^2 + c^2}$$

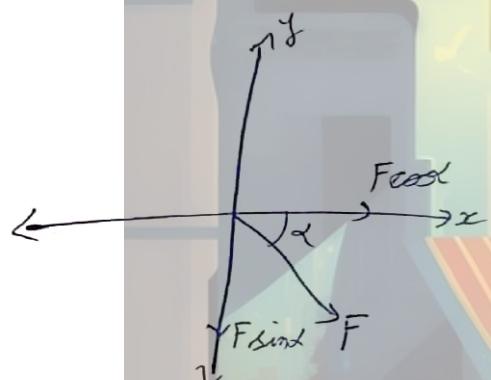
(34)



$$\boxed{\vec{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}}$$

**OTTOBLS**  
ARCTIC  
MICROCOMPUTER

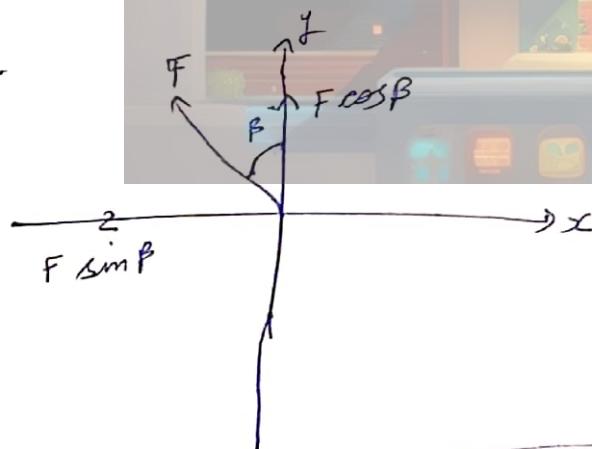
q2



$$\vec{F} = F \cos \alpha \hat{i} + F \sin \alpha \hat{j}$$

$$\boxed{= F \cos \alpha \hat{i} + F \sin \alpha \hat{j}}$$

q3



$$\boxed{\vec{F} = F \cos \beta \hat{i} - F \sin \beta \hat{j}}$$

Q9. find  $c$  if  $0.3\hat{i} + 0.4\hat{j} + c\hat{k}$  is a unit vector.

~~C = 0.3~~

$$c = \sqrt{(0.3)^2 + (0.4)^2 + c^2}$$

$$| \vec{v} | = 0.09 + 0.16 + c^2$$

$$1 - 0.25 = c^2$$

$$0.75 = c^2$$

$$\frac{\sqrt{3}}{\sqrt{100}} = \frac{c}{\sqrt{3}}$$

OTTOSLS  
ATRACTA  
MUSICAL INSTRUMENTS

$$c = \frac{\sqrt{3}}{2}$$

Q10. find unit vector of  $\underline{2\hat{i} + 2\hat{j} - 2\hat{k}}$ .

$$|\vec{A}| = \sqrt{4+1+4}$$

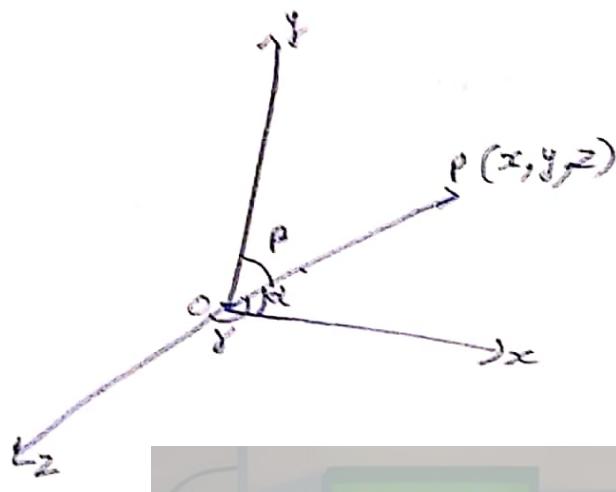
$$|\vec{A}| = \sqrt{9}$$

$$|\vec{A}| = 3$$

$$\hat{\vec{A}} = \frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}$$

$$\hat{\vec{A}} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$$

## Direction cosines :-



angle with  $x$ -axis =  $\alpha$

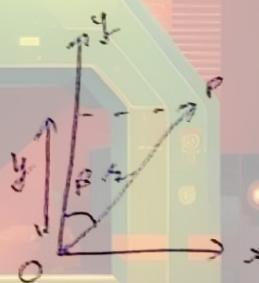
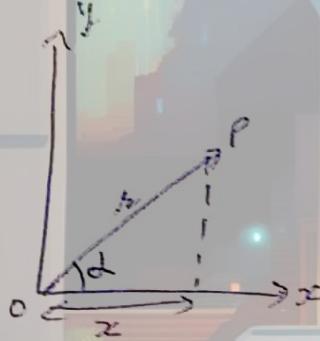
$$\text{OTTOOLS} \\ \text{ANGLE OF ELEVATION} \\ = \beta$$

$$z = r \sin \beta$$

$$\alpha + \beta = 90^\circ$$

$$\overrightarrow{OP} = xi + yj + zk$$

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2} = \Sigma$$



$$\cos \alpha = \frac{x}{\Sigma}$$

$$\cos \beta = \frac{y}{\Sigma}$$

$$\cos \delta = \frac{z}{\Sigma}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$$

$$= \frac{x^2 + y^2 + z^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$$\boxed{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1}$$

The sum of square of direction cosines of vector is always ~~one~~  
equal to one.

Q what are direction cosines of  $\hat{i} + \hat{j} + \hat{k}$ ?

$$\cos \alpha = \frac{x}{r} \quad x=1$$

$$y=1$$

$$z=1$$

$$r = \sqrt{1^2 + 1^2 + 1^2}$$

$$r = \sqrt{3}$$

$$\cos \alpha = \frac{x}{r}$$

$$\boxed{\cos \alpha = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos \beta = \frac{y}{r} = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos \gamma = \frac{z}{r} = \frac{1}{\sqrt{3}}}$$

Q2. if  $\vec{P} = 4\hat{i} - 2\hat{j} + 6\hat{k}$ ,  $\vec{Q} = \hat{i} - 2\hat{j} - 3\hat{k}$   
find angle ( $\vec{P} + \vec{Q}$ ) makes with x axis

$$\vec{P} + \vec{Q} = 5\hat{i} - 4\hat{j} + 3\hat{k}$$

$$r = \sqrt{25 + 16 + 9} \\ = \sqrt{50} \\ r = 5\sqrt{2}$$

$$\cos \alpha = \frac{5}{5\sqrt{2}} \\ \cos \alpha = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\alpha = 45^\circ$$

$$= \frac{45 \times \sqrt{2}}{180} = \frac{\pi}{4}$$

$\sin^{-1}\left(\frac{1}{2}\right)$  is angle re like  
 $\sin \frac{1}{2}$  ago.

Q3. A bird moves with velocity 20 m/s in a direction making an angle of  $60^\circ$  with the eastern line &  $60^\circ$  with vertical upward. Write velocity in rectangular form.

$$\cos 60^\circ = \frac{1}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$2x = r$$

$$2x = 2\sqrt{2}$$

$$x = \sqrt{2}$$

$$2y^2 = r$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{2x^2}$$

$$\vec{v} = 10\hat{i} + 10\hat{k}$$

$$x = 20 \\ y = 10$$

$$|\vec{V}| = 20 \text{ m/s}$$

$$\alpha = 60^\circ$$

$$\beta = 60^\circ$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\frac{1}{4} + \cos^2 \beta + \frac{1}{4} = 1$$

$$\cos^2 \beta = \frac{1}{2}$$

$$\cos \beta = \frac{1}{\sqrt{2}}$$

$$x \cos \alpha = \frac{1}{2} = \frac{x}{20}$$

$$x = 10$$

$$z = 10$$

$$y = 10\sqrt{2}$$

$$\boxed{\vec{V} = 10\hat{i} + 10\sqrt{2}\hat{j} + 10\hat{k}} \text{ m/s}$$

04. find the vector which must be added to the sum of 2 vectors  $\hat{i} + 2\hat{j} + \hat{k}$  and  $\hat{i} - 2\hat{j} + 2\hat{k}$  to get resultant of unit vector along  $z$  axis.

$$\vec{V} = 2\hat{i} + 0\hat{j} + \hat{k}$$

$$\vec{V} - \text{unit vector along } z = -\hat{k}$$

$$-\hat{k} = 2\hat{i} + \hat{k} - \hat{k}$$

$$\boxed{\vec{R} = -2\hat{i}}$$

(40)

Q5. A particle is moving towards east with  $5\text{ m/s}$  in ~~at~~  $10$  seconds, its velocity becomes  $5\text{ m/s}$  towards north. find average acceleration.

$$\vec{u} = 5\hat{i} \text{ m/s}$$

$$\vec{v} = 5\hat{j} \text{ m/s}$$

$$t = 10\text{ s}$$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t}$$

$$\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$$

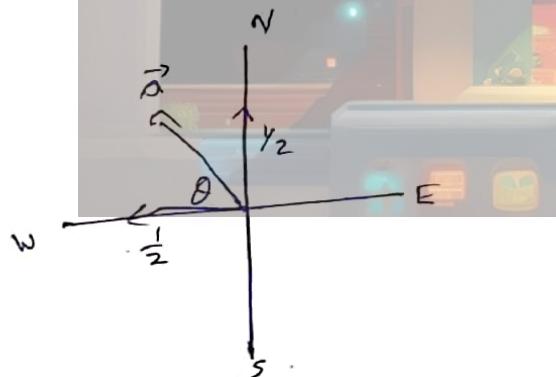
$$\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$$

$$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$r = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

$$|\vec{a}| = \frac{1}{\sqrt{2}} \text{ m/s}^2 \text{ (at an angle } 45^\circ \text{ west of north)}$$

$$\vec{a} = -\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$$



$$\text{or } \cos \theta = \frac{1}{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

Q6. A car is moving towards North with speed of ~~36~~ m/s. In 5s it turns towards left with its speed unchanged. find acc of car.

$$\vec{u} = \cancel{\frac{36}{5}} \hat{j} \text{ m/s}$$

$$\vec{v} = -36 \hat{i} \text{ m/s}$$

$$t = 5 \text{ s}$$

$$\vec{a} = \frac{-36\hat{i} - 36\hat{j}}{5}$$

$$\vec{u} = \frac{36^2 \times \frac{5}{18}}{5} = 10 \hat{j}$$

$$\vec{v} = -10 \hat{i}$$

$$t = 5 \text{ s}$$

$$\vec{a} = - \cancel{(36\hat{i} + 36\hat{j})}$$

$$\vec{a} = -12 \cancel{(\frac{36\hat{i} + 36\hat{j}}{5})}$$

$$|\vec{a}| = \sqrt{36^2 + 36^2}$$

$$(|\vec{a}|) = \sqrt{9+9}$$

$$|\vec{a}| = \frac{\sqrt{18}}{3\sqrt{2} \times 5}$$

$$|\vec{a}| =$$

$$\vec{a} = -\frac{10\hat{i}}{2} - \frac{10\hat{j}}{2}$$

$$\vec{a} = -\left(\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$\vec{a} = -2\hat{i} - 2\hat{j}$$

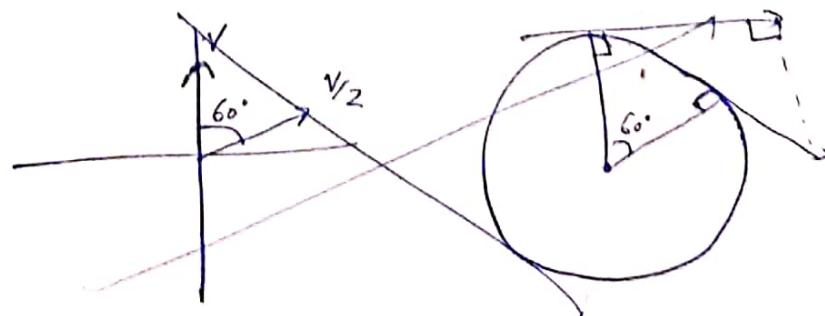
$$|\vec{a}| = \sqrt{4+4}$$

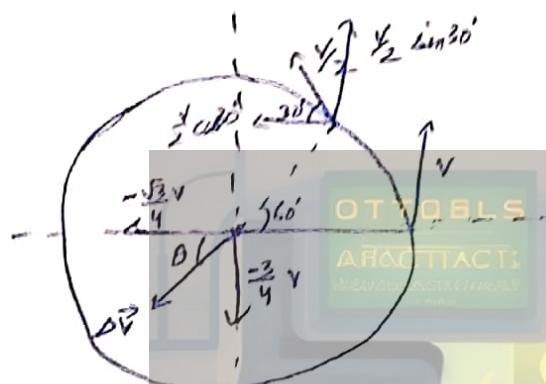
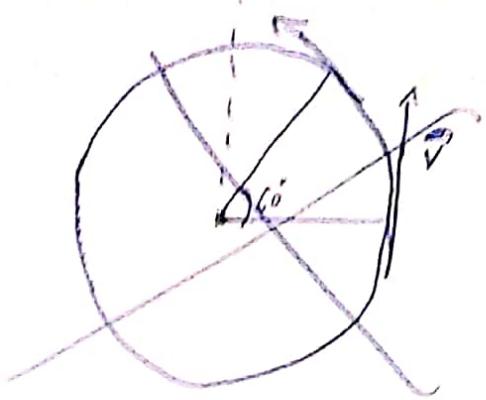
$$= \sqrt{8}$$

$$|\vec{a}| = 2\sqrt{2} \text{ m/s}^2$$

(45° towards south of west)

Q7. A particle is moving on a circular path with speed  $v$  at a particular instant. After some time, when it has described an angle  $60^\circ$  its speed becomes  $\frac{v}{2}$  what is change in velocity.





$$\vec{v} - \vec{u}$$

$$\vec{u} = v \hat{j}$$

$$\vec{v} = \frac{v}{2} \sin 30^\circ (\hat{j}) + \frac{v}{2} \cos 30^\circ (-\hat{x})$$

$$\vec{v} = \frac{v}{4} \hat{j} - \frac{\sqrt{3}}{4} v \hat{x}$$

$$\Delta \vec{v} = \vec{v} - \vec{u}$$

$$= -\frac{\sqrt{3}}{4} v \hat{x} + \frac{v}{4} \hat{j} - v \hat{j}$$

$$\boxed{\Delta \vec{v} = -\frac{\sqrt{3}}{4} v \hat{x} - \frac{3}{4} v \hat{j}}$$

$$|\Delta \vec{v}| = \sqrt{\left(\frac{\sqrt{3}v}{4}\right)^2 + \left(-\frac{3}{4}v\right)^2}$$

$$\boxed{|\Delta \vec{v}| = \frac{\sqrt{3}v}{2}}$$

$$\tan \theta = \frac{\frac{3}{4}v}{\frac{\sqrt{3}v}{4}} = \sqrt{3}$$

$$\boxed{\theta = 60^\circ}$$

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Q 8. A particle whose speed is 50 m/s moves along a line from A(2,1) to B(9,25) find its velocity in rectangular form.

$$u = 2\hat{i} + \hat{j}$$

$$v = 9\hat{i} + 25\hat{j}$$

$$v - u = 7\hat{i} + 24\hat{j}$$

$$\vec{v} = 7\hat{i} + 24\hat{j}$$

Q 8

$$\vec{v} = \frac{7\hat{i} + 24\hat{j}}{\sqrt{(7)^2 + (24)^2}}$$

$$\vec{v} = \frac{7\hat{i} + 24\hat{j}}{25}$$

$\vec{v} \times 50$  = velocity

$$\boxed{\text{Velocity} = 14\hat{i} + 48\hat{j} \text{ m/s}}$$

## # Product of two vectors

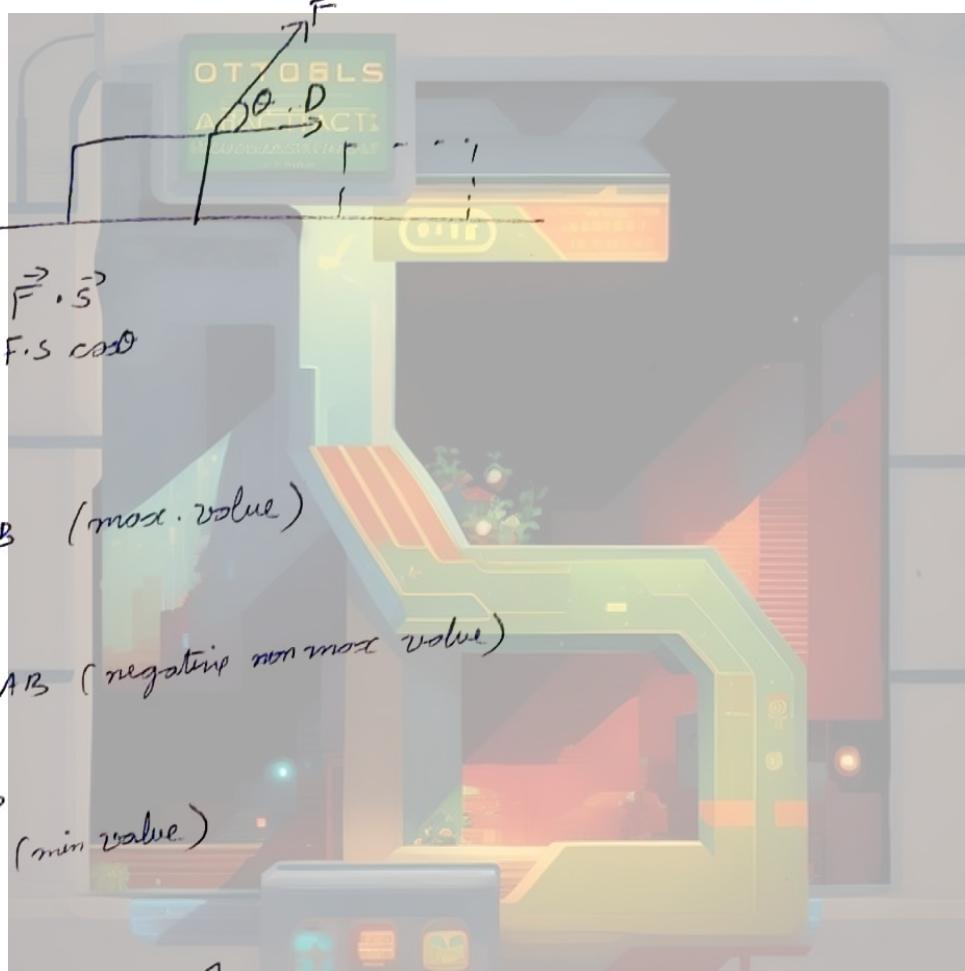
### I) Scalar Product/dot product:-

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos\theta}$$

$$= AB \cos\theta$$


→ The scalar product is a way to multiply two vectors to get a scalar result

e.g.



$\theta = 0^\circ$   
 $\cos\theta = 1$   
 $\vec{A} \cdot \vec{B} = AB$  (max. value)

$\theta = 180^\circ$   
 $\cos\theta = -1$   
 $\vec{A} \cdot \vec{B} = -AB$  (negative non max. value)

$\theta = 90^\circ$   
 $\cos\theta = 0$   
 $\vec{A} \cdot \vec{B} = 0$  (min. value)

$$\begin{aligned} \hat{i} \cdot \hat{i} &= 1 \\ \hat{j} \cdot \hat{j} &= 1 \\ \hat{k} \cdot \hat{k} &= 1 \end{aligned}$$

$$\left| \begin{array}{l} \hat{i} \cdot \hat{j} = 0 \\ \hat{j} \cdot \hat{k} = 0 \\ \hat{k} \cdot \hat{i} = 0 \end{array} \right.$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})(B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

\* To find angle between two vectors

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\boxed{\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}}$$

\*  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$  (Commutative)

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

\* distributive

$$\vec{A}(\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$* \vec{A} \cdot \vec{A} = (A)(A) \cos 0^\circ = A^2$$

$$|\vec{P} + \vec{Q}|^2 = (\vec{P} + \vec{Q})(\vec{P} + \vec{Q})$$

$$= \vec{P} \cdot \vec{P} + \vec{P} \cdot \vec{Q} + \vec{Q} \cdot \vec{P} + \vec{Q} \cdot \vec{Q}$$

$$= P^2 + Q^2 + 2 PQ \cos \theta$$

$$|\vec{P} + \vec{Q}| = \sqrt{P^2 + Q^2 + 2 PQ \cos \theta}$$

$$Q \quad \vec{A} = 2\vec{i} + 3\vec{j} + \vec{k}$$

$$\vec{B} = 5\vec{i} + 2\vec{j} + 7\vec{k}$$

$$\vec{A} \cdot \vec{B} = 2 \times 5 + (-3) \times 2 + (-7) \times 1$$

$$= 10 - 6 - 7$$

$$= 10 - 13$$

$$\boxed{= -3}$$

$$Q2. \quad \vec{A} = \vec{i} + \vec{j} - \vec{k}$$

$$\vec{B} = \vec{i} - \vec{j} + \vec{k}$$

$$\vec{A} \cdot \vec{B} = ?$$

$$\vec{A} \cdot \vec{B} = 1 - 1 - 1$$

$$= -2 + 1$$

$$\boxed{= -1}$$

$$Q3. \quad \vec{A} = 2\vec{i} - 3\vec{j} + \vec{k}$$

$$\vec{B} = 3\vec{i} + 2\vec{j}$$

$$\vec{A} \cdot \vec{B} = 6 - 6 + 0$$

$$\boxed{= 0}$$

Q Find value of  $n$ , such that  $(2\vec{i} - 3\vec{j} + \vec{k})$  is perpendicular to  $\vec{i} + 2\vec{j} + n\vec{k}$ .

$$\cos \theta = \frac{2 - 6 + n}{\sqrt{4+9+1} \times \sqrt{1+4+n^2}}$$

$$\cos \theta = \frac{n-4}{\sqrt{14} \times \sqrt{5+n^2}}$$

$$\cos \theta = \frac{n-4}{\sqrt{70+14n^2}}$$

$$\frac{n-4}{\sqrt{70+14n^2}} = 0$$
$$\boxed{n=4}$$

(47)

Q find angle between  $(\vec{r} + \vec{j})$  &  $(\vec{r} - \vec{j})$

$$\cos \theta = \frac{1-1}{AB}$$

$$\boxed{\theta = 90^\circ} \text{ as } 1-1=0$$

Q find angle b/w  $(\vec{r} + \vec{j})$  &  $(\vec{j} + \vec{k})$

$$\cos \theta = \frac{0+1+0}{AB}$$

$$\cos \theta = \frac{1}{\sqrt{1+1} \times \sqrt{1+1}}$$

$$\cos \theta = \frac{1}{\sqrt{2} \times \sqrt{2}}$$

$$\cos \theta = \frac{1}{2}$$

$$\boxed{\theta = 60^\circ}$$

Q A force  $\vec{F} = (10\vec{i} - 3\vec{j} + 6\vec{k}) N$  acts on a body of mass 100g & displaces it from  $(6\vec{i} + 5\vec{j} + 3\vec{k}) m$  to  $(10\vec{i} - 2\vec{j} + 7\vec{k}) m$  find work done.

$$\text{displacement} = 4\vec{i} - 7\vec{j} + 10\vec{k} m$$

$$\text{work} = \vec{F} \cdot \vec{s}$$

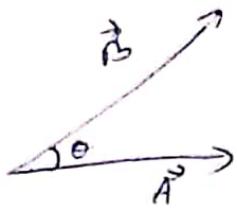
$$= (10\vec{i} - 3\vec{j} + 6\vec{k}) \cdot (4\vec{i} - 7\vec{j} + 10\vec{k})$$

$$= 40 + 21 + 240$$

$$\begin{aligned} &\boxed{= 285 J} \\ &\boxed{= 121 J} \end{aligned}$$

④ 121

## II Cross Product / Vector Product



$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta$$

↑ direction of  $\vec{A} \times \vec{B}$   
(Right hand thumb rule)

- ① out of plane
- ② in plane

✗ not commutative

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB \sin \theta$$

✗ distributive

$$\vec{A}(\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\hat{i} \times \hat{i} = (1)(1) \sin 0^\circ = 0$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = (1)(1) \sin 90^\circ \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{j}$$

$$\hat{i} \times \hat{j} = \hat{j}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

clockwise - (+ve)  
& negative - (-ve)

✗ If cross product is 0, vectors are either equal or parallel

✗ If dot product is 0, vectors are  $\perp$  to each other

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$\vec{A} \times \vec{B} =$

|           |           |           |
|-----------|-----------|-----------|
| $\hat{i}$ | $\hat{j}$ | $\hat{k}$ |
| $A_x$     | $A_y$     | $A_z$     |
| $B_x$     | $B_y$     | $B_z$     |

3 by 3 determinant

$$\vec{A} \times \vec{B} = \hat{i} (A_y - B_z) - \hat{j} (A_x - B_z) + \hat{k} (A_x - B_y)$$

$$\boxed{\vec{A} \times \vec{B} = \hat{i} (A_y B_z - B_y A_z) - \hat{j} (A_x B_z - B_x A_z) + \hat{k} (A_x B_y - A_y B_x)}$$

Q  $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\vec{A} \times \vec{B} =$$

|           |           |           |
|-----------|-----------|-----------|
| $\hat{i}$ | $\hat{j}$ | $\hat{k}$ |
| 3         | 1         | 2         |
| 2         | -2        | 4         |

$$\vec{A} \times \vec{B} = \hat{i} (4 - (-4)) - \hat{j} (12 - 4) + \hat{k} (-6 - 2)$$

$$\boxed{= 8\hat{i} - 8\hat{j} - 8\hat{k}}$$

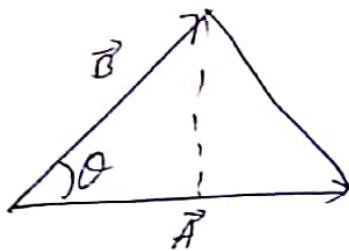
$$|\vec{A} \times \vec{B}| = \sqrt{8^2 + (-8)^2 + (-8)^2}$$

$$\boxed{= 8\sqrt{3}}$$

(30)

## Uses of vector product

1) To calculate area of triangle:-



$$\text{area of triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} (A)(B \sin \theta)$$

$$= \frac{1}{2} |\vec{A} \times \vec{B}|$$

2) To calculate area of || gm:-



Q1. The linear velocity of a rotating body is given by  $\vec{v} = \vec{\omega} \times \vec{r}$

$$\vec{\omega} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & -3 \\ -3 & 4 & 1 \end{pmatrix} \quad \text{find } |\vec{v}|$$

$$\vec{r} = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 2 & 1 \\ 0 & 4 & -3 \end{pmatrix}$$

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & -3 \\ -3 & 4 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ -2 & 2 & 1 \\ 0 & 4 & -3 \end{pmatrix}$$

$$\vec{v} = 1(6-8) - 2(-3-0) + 2(4+2) = -2\vec{i} + 3\vec{j} + 4\vec{k}$$

$$|\vec{v}| = \sqrt{4+9+16}$$

$$= \sqrt{29}$$

~~$$|\vec{v}| = \sqrt{4+9+36}$$~~

~~$$|\vec{v}| = \sqrt{65}$$~~

~~$$= \sqrt{5 \times 13}$$~~

Q2. Calculate area of triangle determined by  $\vec{A} = 3\hat{i} + \hat{j}$   
 $\vec{B} = -3\hat{i} + 7\hat{j}$

$$1/2 \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ -3 & 7 & 0 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(0) - \hat{j}(0) + \hat{k}(21 + 12)$$

$$|\vec{A} \times \vec{B}| = \sqrt{(33)^2} = 33 \hat{k}$$

2  
 Area =  $\frac{33}{2}$   
 $= 16.5 \text{ m}^2$

---

Q3.  $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$   
 $\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

$$\sqrt{p^2 + q^2 + r^2} = \sqrt{25 + 49 + 9} = \sqrt{83}$$

$$p^2 + q^2 + r^2 = 83$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 5 & 7 & 3 \end{vmatrix}$$

$$= \hat{i}(3p - 7q) - \hat{j}(6 - 5q) + \hat{k}(14 - 5p)$$

$$6 - 5Q = 0$$

$$Q = \frac{6}{5}$$

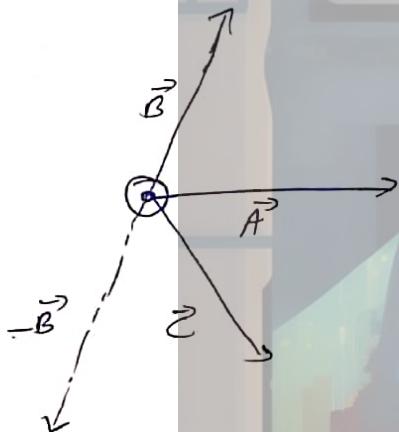
$$14\theta - 5h = 0$$

$$\begin{aligned} h &= 14 \\ h &= \frac{14}{5} \end{aligned}$$

Q3. What is the angle between  $\vec{A} - \vec{B}$  and  $\vec{A} \times \vec{B}$

$$\cos \theta = \frac{(\vec{A} \times \vec{B}) \cdot (\vec{A} - \vec{B})}{|\vec{A} - \vec{B}| |\vec{A} \times \vec{B}|}$$

$$= AB \sin \theta \text{ m.}$$



$$\therefore \theta = 90^\circ$$

$$Q4. \quad |\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$\sqrt{A^2 + B^2 + 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

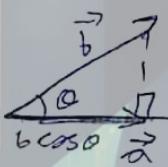
$$4AB \cos \theta = 0$$

$$\theta = 90^\circ$$

# Projection of vector over another:-

$$I) \text{Scalar projection of } \vec{b} \text{ on } \vec{a} = b \cos \frac{\theta}{a} = \frac{b \cos \theta}{a} = \frac{\vec{a} \cdot \vec{b}}{a}$$

जिसपर projection  
होती है।



$$II) \text{Vector projection} = \frac{\vec{a} \cdot \vec{b}}{a} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{a} \frac{\vec{a} \cdot \vec{a}}{a} = \frac{\vec{a}}{a^2} (\vec{a} \cdot \vec{b})$$

Q find scalar vector projection of  $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$  on  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$$\frac{\vec{a} \cdot \vec{b}}{b} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$$

$$\text{vector } \frac{\vec{b}}{b} (\vec{a} \cdot \vec{b}) = \frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{5}{3}\hat{k}$$

① find scalar product of  $\vec{A} = \vec{i} + \vec{j} + \vec{k}$  on  $\vec{B} = \vec{i} + \vec{j}$   
projection and vector projection

$$\frac{\vec{A} \cdot \vec{B}}{6} = \frac{1+1+0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$$

V.P.  $\frac{\vec{B}}{6} \cdot (\vec{a} \cdot \vec{B}) = \frac{\vec{i} + \vec{j}}{2} \times \vec{z} = \boxed{\sqrt{2}}$

②  $\vec{A} + \vec{B} = 2\vec{i}$

$$\vec{A} - \vec{B} = 4\vec{j}$$

$$\begin{aligned} - & \cancel{\vec{A}^2 + \vec{B}^2 + 2AB \cos \theta = A^2 + B^2 + 2AB \cos \theta} \quad 2\vec{i} \\ - & \cancel{\vec{A}^2 + \vec{B}^2 + 2AB \cos \theta = 4\vec{j}} \\ - & 4AB \cos \theta = 2\vec{j} \end{aligned}$$

$$\cos \theta = \frac{-1}{2AB}$$

~~$\cos(90^\circ)$~~

$$\vec{A} + \vec{B} = 2\vec{i}$$

$$\vec{A} - \vec{B} = 4\vec{j}$$

$$\cancel{2\vec{i}}$$

$$\cos \theta = 90^\circ$$

$$\text{Q1. } \vec{A} + \vec{B} = 2\vec{x}$$

$$\vec{A} - \vec{B} = 4\vec{y}$$

$$\theta = ?$$

- A)  $127^\circ$   
 B)  $143^\circ$   
 C)  $53^\circ$   
 D)  $37^\circ$

$$\vec{A} + \vec{B} = 2\vec{x}$$

$$\vec{A} - \vec{B} = 4\vec{y}$$

$$2\vec{A} = 2\vec{x} + 4\vec{y}$$

$$\vec{A} = \vec{x} + 2\vec{y}$$

$$\vec{B} = \vec{x} - 2\vec{y}$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}\vec{B}|}$$

$$= \frac{1 - 4}{\sqrt{5} \times \sqrt{5}}$$

$$\cos\theta = \frac{-3}{5}$$

$$\cos(90 + \theta) = -\sin\theta$$

$$-\sin\theta = \frac{-3}{5}$$

$$\sin 37^\circ = \frac{3}{5}$$

$$90 + 37 = 127^\circ$$

A)  $127^\circ$

Q2. Two forces P & Q are in ratio 1:2 if their resultant is at an angle  $\tan \alpha = \frac{\sqrt{3}}{2}$  to vector P. Then angle between P & Q is.

- A)  $\tan^{-1}(\frac{1}{2})$    B)  $45^\circ$    C)  $30^\circ$    D)  $60^\circ$

$$\tan \alpha = \frac{\sqrt{3}}{2} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\frac{P}{Q} = \frac{1}{2}$$

$$2P = Q \\ Q = 2P$$

$$\sqrt{3}P + \sqrt{3}Q \cos \theta = 2Q \sin \theta$$

$$\sqrt{3}P = 2Q \sin \theta - \sqrt{3}Q \cos \theta$$

$$\sqrt{3}(P + Q \cos \theta) = 2Q \sin \theta$$

$$\sqrt{3}(P + 2P \cos \theta) = 4P \sin \theta$$

$$\sqrt{3}P(1 + 2 \cos \theta) = 4P \sin \theta$$

$$\frac{1 + 2 \cos \theta}{\sin \theta} = \frac{4P}{\sqrt{3}P}$$

$$\frac{1}{\sin \theta} + \frac{2 \cos \theta}{\sin \theta} = \frac{4}{\sqrt{3}}$$

$$\frac{1}{\sin \theta} + 2 \cot \theta = \frac{4}{\sqrt{3}}$$

$$\boxed{\sqrt{3} + 2\sqrt{3} \cos \theta = 4 \sin \theta}$$

$$\sqrt{3} + 2\sqrt{3} \times \frac{1}{2} = 4 \times \frac{1}{2}$$

$$\sqrt{3} + 3 = 2x$$

$$\sqrt{3} + 2\sqrt{3} \times \frac{1}{2} = x^2 \times \frac{\sqrt{3}}{2}$$

$$1+1=2$$

$$LHS = RHS$$

$$\boxed{\int \theta = 60^\circ}$$

$$\boxed{D) 60}$$

$$Q3. \vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$$

find the angle between  $(\vec{A} - \vec{B})$  &  $\vec{C}$

$$A) \theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$B) \cos \theta = \frac{\sqrt{3}}{2}$$

$$C) \theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

D) none.

$$\vec{A} - \vec{B} = -1\hat{i} + 5\hat{j} + \hat{k}$$

$$\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Ans} \frac{-1 + 5 + 2}{\cancel{3\sqrt{3}} \times \cancel{1\sqrt{6}}} = \frac{6}{3\sqrt{18}}$$

$$= \frac{6}{3\sqrt{18}} = \frac{2}{\sqrt{18}}$$

$$\cos \theta = \frac{2}{3\sqrt{2}}$$

$$= \frac{\sqrt{2}}{3}$$

C

On Two forces  $1\hat{i} + \hat{j} + \hat{k} N$  &  $1 + 2\hat{j} + 3\hat{k} N$  acts on a particle and displace it from  $(2, 3, 4)$  to point  $(5, 4, 3)$ .

$$\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{s} = 3\hat{i} + \hat{j} - \hat{k}$$

A) 5 J

B) 4 J

C) 3 J

D) None

$$W = 6 + 3 - 4$$

$$W = 5 J$$

A

Q5. A force  $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$

$$\vec{D} = -4\hat{i} + 2\hat{j} + 2\hat{k}$$

$$W_{23} = 6.5$$

A) 12

B) 0

C) 6

D) 4

$$W_{23} = -12 + 2c + 6$$

$$6 = -6 + 2c$$

$$12 = 2c$$

$$\boxed{c = 6}$$

$$\boxed{C}$$

OTTOBLS  
ARACTAIC

Q6. A force  $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

A)  $6\hat{i} - 8\hat{j} + 10\hat{k}$

B) 100

C)  $10\sqrt{2}$

D) 10

$$6\hat{i} - 8\hat{j} + 10\hat{k} = m \times 1$$

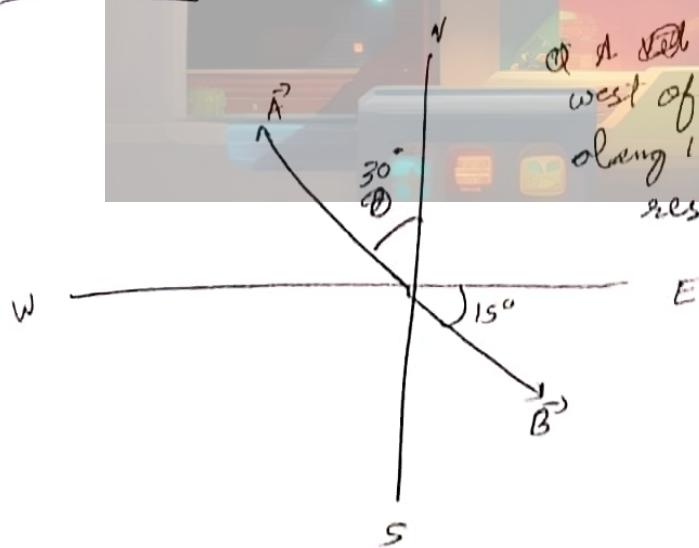
$$m = \sqrt{36 + 64 + 100}$$

$$= \sqrt{200}$$

$$= 10\sqrt{2} \text{ kg}$$

$$\boxed{C}$$

Q7.



a. If vector  $\vec{A}$  is directed  $30^\circ$  west of north & another vector  $\vec{B}$  along  $15^\circ$  south of east. Then resultant cannot be in

$\boxed{\text{South}}$

(57)

Q8. The sum of  $\vec{F}_1 = 100\text{N}$ ,  $\vec{F}_2 = 80\text{N}$  &  $\vec{F}_3 = 60\text{N}$  is 0, angle between  $F_1$  &  $F_2$  is

A) 53  
B) 143

C) 37  
D) 127

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\sqrt{10000 + 6400 + 3600}$$

$$\sqrt{10000 + 6400 + 16000 \cos\theta} = 0$$

$$= \sqrt{32400 \cos\theta}$$

$$= \sqrt{16400 + 16000 \cos\theta}$$

$$= 100 \sqrt{164 + 160 \cos\theta}$$

$$= 20 \sqrt{41 + 40 \cos\theta} = -60$$

$$= \sqrt{41 + 40 \cos\theta} = -3$$

$$900 \times (41 + 40 \cos\theta) + 3600 =$$

$$41 + 40 \cos\theta = 9$$

$$\cos\theta = \frac{9 - 41}{40}$$

$$\cos\theta = \frac{-32}{40} = -\frac{4}{5}$$

$$\cos\theta = -\frac{4}{5}$$

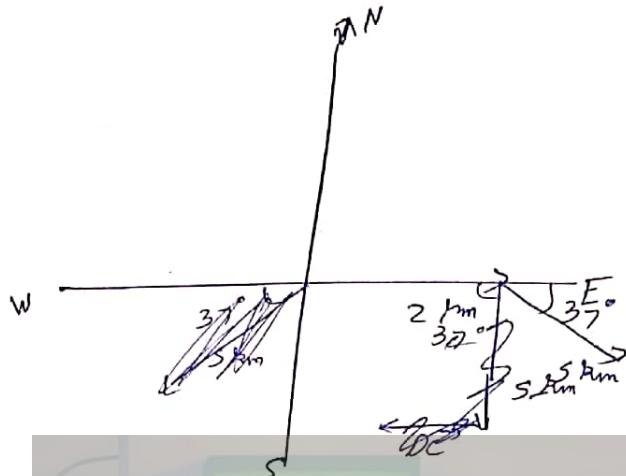
$$\cos(90 + 53^\circ) = -\frac{4}{5}$$

$$\theta = 90 + 53^\circ = 143^\circ$$

B

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Q9. A sail boat sails 2 km east, 5 km  $37^\circ$  south of East and an unknown displacement. The final displacement is 6 km east, find unknown.



$$\vec{A} = 2\hat{i}$$

$$\begin{aligned}\vec{B} &= +5 \times \frac{4}{5} \hat{i} - \frac{3}{5} \times 5 \hat{j} \\ &= -4\hat{i} - 5\hat{j}\end{aligned}$$

$$\vec{C} = ?$$

$$\vec{A} + \vec{B} + \vec{C} = 6\hat{i}$$

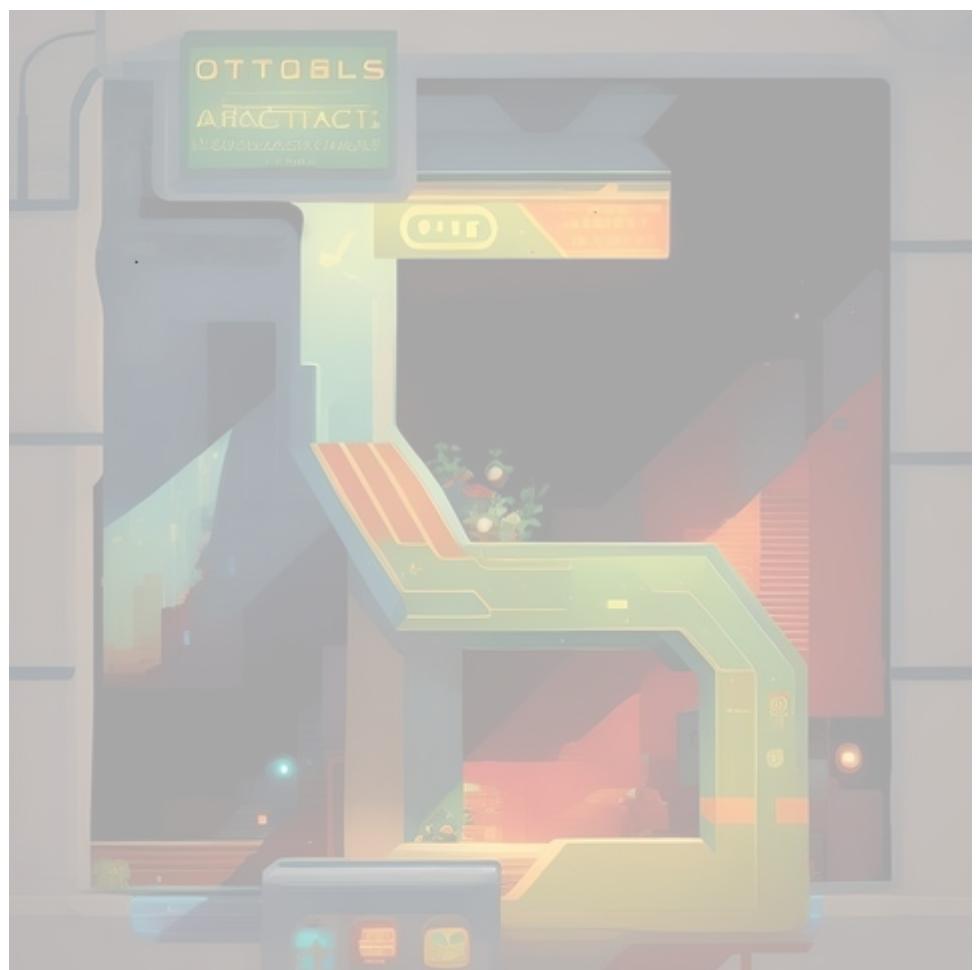
$$\begin{aligned}2\hat{i} + (-4\hat{i}) + \vec{C} - 3\hat{j} &= 6\hat{i} \\ &= 6\hat{i} + 2\hat{i} + 3\hat{j}\end{aligned}$$

$$\boxed{\vec{C} = 3\hat{j}}$$

$$\boxed{= 3\hat{j}} \quad \checkmark$$

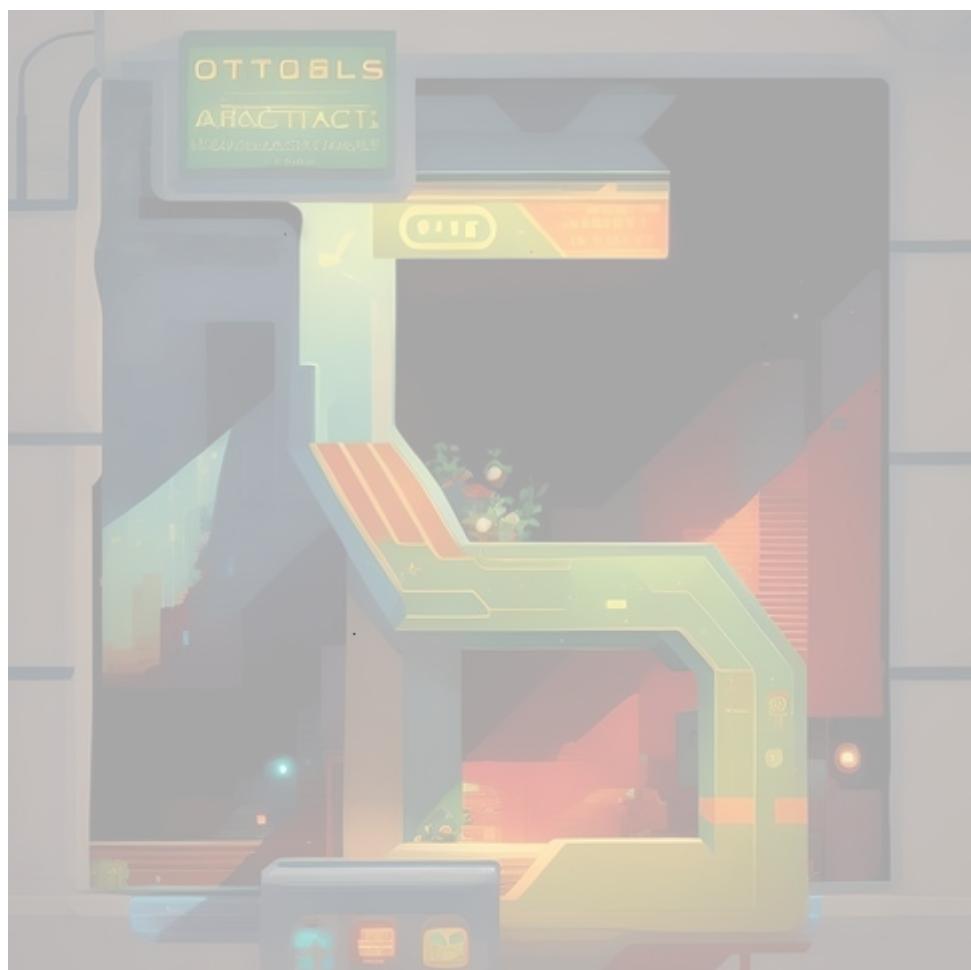


(62)



(63)





(65)

# Calculus (Differentiation & Integration)

Differentiation - Differential calculus is used to study the nature (increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) changes independently.

$\frac{dy}{dx}$  (y first the change is related with respect to x)

is the instantaneous rate of change of function y with respect to variable x.

$$v = \frac{dv}{dt} = \frac{d}{dt}(v)$$

$$\frac{dy}{dx} = y'$$

$$w = \frac{ds}{dt} = \frac{d}{dt}(s)$$

Formulas derivatives of some imp functions.

$$\textcircled{1} \quad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$\textcircled{2} \quad \frac{d}{dx}(e^x) = e^x$$

$$\textcircled{3} \quad \frac{d}{dx}(\text{constant}) = 0 \quad (\text{does not change})$$

$$\textcircled{4} \quad \frac{d}{dx}(\sin x) = \cos x$$

$$\textcircled{5} \quad \frac{d}{dx}(\cos x) = -\sin x$$

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$$\textcircled{6} \quad \frac{d}{dx} (\tan x) = \sec^2 x$$

$$\textcircled{7} \quad \frac{d}{dx} (\sec x) = \sec x \tan x$$

p natural log

$$\textcircled{8} \quad \frac{d}{dx} (\ln x) = \frac{1}{x}$$

$$\textcircled{9} \quad \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$$

$$\textcircled{10} \quad \frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\textcircled{11} \quad \frac{d}{dx} (ax^n) = a \frac{d}{dx} (a^n) = a^n x^{n-1}$$

$$\textcircled{12} \quad \frac{d}{dx} (a^x) = a^x \ln a$$

↳ constant

Q1. Differentiate following w.r.t respect to  $x$

$$\textcircled{1} \quad y = 3x^2 - 5x + 1$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (y) = \frac{d}{dx} (3x^2 - 5x + 1) \\ &= \frac{d}{dx} (3x^2) - \frac{d}{dx} (5x) + \frac{d}{dx} (1)\end{aligned}$$

$$= 3 \frac{d}{dx} (x^2) - 5 \frac{d}{dx} (x) + \frac{d}{dx} (1)$$

$$= 3 \times 2x x^{2-1} - 5 \times 1 \times x^{1-1} + 0$$

$$= 6x - 5$$

$$\textcircled{2}. \quad y = ax^2 + bx + c$$

$$\frac{d}{dx}(y)$$

$$= \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx} \cdot c$$

$$= a \frac{d}{dx}(x^2) + b \frac{d}{dx}(x) + 0$$

$$= a \times 2x \times x + b \times 1 + 0$$

$$= 2ax^2 + b$$

$$\textcircled{3}. \quad y = x^{1/2}$$

$$\frac{d}{dx}(x^{1/2})$$

$$= \frac{1}{2} \times x^{-1/2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$\textcircled{4}. \quad y = \sqrt[3]{x}$$

$$\frac{d}{dx}(x^{1/3})$$

$$\frac{1}{3} \times x^{-2/3}$$

$$1 \times \frac{1}{3}$$

$$\frac{3-1}{3} = \frac{2}{3}$$

$$\frac{1}{3} - 1$$

$$\frac{1-3}{3} = -\frac{2}{3}$$

$$= \frac{1}{3x^{2/3}}$$

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$$⑤ y = 2\sqrt{x} - \frac{1}{x} + \sqrt[4]{3}$$

$$\frac{dy}{dx} (y) =$$

$$= \frac{d}{dx} (2\sqrt{x}) - \frac{d}{dx} (\cancel{2}x^{-1}) + \frac{d}{dx} (\sqrt[4]{3})$$

$$= 2 \frac{d}{dx} (x^{1/2}) - \frac{d}{dx} (x^{-1}) + \frac{d}{dx} (\sqrt[4]{3})$$

$$= 2 \times \frac{1}{2} \times \frac{1}{\sqrt{x}} - (-1) \times \cancel{\frac{1}{x^2}} + \frac{1}{4} \times \frac{1}{3^{3/4}}$$

$$= \frac{1}{\sqrt{x}} + \frac{1}{x^2} + \frac{1}{4\sqrt[4]{3^3}}$$

$$= \frac{d}{dx} (2\sqrt{x}) - \frac{d}{dx} (\frac{1}{x}) + \cancel{\frac{d}{dx} (\sqrt[4]{3})} 0$$

$$\frac{1}{4} - 1 = -\frac{3}{4}$$

$$= 2 \times \frac{1}{2} \times \frac{1}{\sqrt{x}} - (-1) \times \frac{1}{x^2} + \cancel{\frac{d}{dx} (\sqrt[4]{3})} 0$$

$$= \frac{1}{\sqrt{x}} + \frac{1}{x^2} + 0$$

$$= \boxed{\frac{1}{\sqrt{x}} + \frac{1}{x^2}}$$

## Product Rule

$$y = A \cdot B$$

(A & B are functions of  $x$ )

$$\frac{dy}{dx} = A \frac{d}{dx}(B) + B \frac{d}{dx}(A)$$

e.g.  $y = 2 \sin x \cos x$

$$\frac{d}{dx} (2 \sin x \cos x)$$

$$= 2 \frac{d}{dx} (\sin x \cos x)$$

$$= 2 \left[ \sin x \frac{d}{dx} (\cos x) + \cos x \frac{d}{dx} (\sin x) \right]$$

$$= 2 \left[ \sin x (-\sin x) + \cos x (\cos x) \right]$$

$$= 2 \left[ \cos^2 x - \sin^2 x \right]$$

## Quotient Rule

$$y = \frac{A}{B} \quad (A \text{ & } B \text{ are functions of } x)$$

$$\frac{dy}{dx} = \frac{B \cdot \frac{d}{dx}(A) - A \cdot \frac{d}{dx}(B)}{(B)^2}$$

e.g.  $y = \frac{\sin x}{\cos x}$

$$\frac{d}{dx}(y) = \cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)$$

$$\begin{aligned}&= \frac{\cos x (\cos x) - \sin x (-\sin x)}{\cos^2 x} \\&= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\&= \frac{1}{\cos^2 x} \\&= \sec^2 x\end{aligned}$$

$$y = (x^2 - 3x + 3)(x^2 + 2x - 1)$$

$$\frac{dy}{dx} = \frac{d}{dx} (x^2 - 3x + 3)(x^2 + 2x - 1) = (x^2 + 2x - 1) \frac{d}{dx}(x^2 - 3x + 3) + (x^2 - 3x + 3) \frac{d}{dx}(x^2 + 2x - 1)$$

$$(x^2 + 2x - 1)(2x - 3 + 0) + (x^2 - 3x + 3)(2x + 2 + 0)$$

$$(x^2 - 2x - 1)(2x - 3) + (x^2 - 3x + 3)(2x - 2)$$

$$(2x^3 - 2x^2 - 2x^2 + 6x - 2x + 3) + (2x^3 - 2x^2 - 6x^2 + 6x + 6x - 6)$$

$$(2x^3 - 4x^2 + 4x + 3) + (2x^3 - 8x^2 + 12x - 6)$$

$$4x^3 - 12x^2 + 16x - 3$$

$$7. \quad y = (x^3 - 3x + 2)(x^4 + x^2 - 1)$$

$$(x^2 - 3x + 2) \frac{d}{dx}(x^4 + x^2 - 1) + (x^4 + x^2 - 1) \frac{d}{dx}(x^2 - 3x + 2)$$

$$(x^2 - 3x + 2)(4x^3 +$$

$$Q6. \quad y = (x^2 - 3x + 3)(x^2 + 2x - 1)$$

$$Q7. \quad y = (x^3 - 3x + 2)(x^4 + x^2 - 1)$$

$$Q8. \quad y = (\sqrt{x} + 1) \left[ \frac{1}{\sqrt{x}} - 1 \right]$$

$$Q9. \quad y = (x^2 - 1)(x^2 - 4)$$

$$Q10. \quad y = \frac{x+1}{x-1}$$

$$Q6. \quad (x^2 - 3x + 3)(2x + 2) + (x^2 + x - 1)(2x - 3)$$

$$Q7. \quad (x^4 + x^2 - 1)(3x^2 - 3) + (x^3 - 3x + 2)(2x + 2)$$

$$Q8. \quad \frac{d}{dx} \left[ (\sqrt{x} + 1) \left( \frac{1}{\sqrt{x}} - 1 \right) \right]$$

$$\left( \frac{1}{\sqrt{x}} - 1 \right) \frac{d}{dx} (\sqrt{x} + 1) + (\sqrt{x} + 1) \frac{d}{dx} \left( \frac{1}{\sqrt{x}} - 1 \right)$$

$$\left[ \left( \frac{1}{\sqrt{x}} + 1 \right) \frac{1}{2\sqrt{x}} + (\sqrt{x} + 1) \cancel{\left( \frac{1}{2x^3} \right)} = \frac{1}{2\sqrt{x^3}} \right]$$

$$Q9. \quad \frac{d}{dx} ((x^2 - 1)(x^2 - 4))$$

$$= 2x(2x^2 - 5)$$

$$Q10. \frac{d}{dx} \left( \frac{x+1}{x-1} \right)$$

$$\frac{(x-1) \frac{d}{dx}(x+1) - (x+1) \frac{d}{dx}(x-1)}{(x-1)^2}$$

$$\frac{(x-1)(1) - (x+1)(1)}{x^2 + 1 - 2x}$$

$$\left[ \frac{(x-1) - (x+1)}{(x-1)^2} \right] = \left[ \frac{-2}{(x-1)^2} \right]$$

$$Q11. y = \frac{x}{x^2 + 1}$$

$$Q12. y = \frac{3x^2 + 1}{x-1}$$

$$Q13. y = \frac{2x}{x^3 - 1}$$

$$Q14. y = x \sin x$$

$$Q15. y = \frac{x}{\sin x}$$

$$Q16. y = x^2 \tan x$$

$$Q17. y = \frac{x^2}{\sec x}$$

$$Q18. y = x \ln x$$

$$Q19. y = x e^x$$

$$Q20. y = \frac{x}{e^x}$$

$$Q11. \frac{dy}{dx} = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} -$$

$$= \frac{x^2 + 1 - 2x^2}{\cancel{(x^2+1)}} \cdot \cancel{(x^2+1)^2}$$

$$= \boxed{\frac{1-x^2}{(x^2+1)^2}}$$

$$Q12. \frac{dy}{dx} = \boxed{\frac{(x-1)(6x) - (3x^2+1)(1)}{(x-1)^2}}$$

$$Q13. \frac{d}{dx} \left( \frac{2x}{x^3-1} \right)$$

$$= \boxed{\frac{(x^3-1)(2) - (2x)(3x^2)}{(x^3-1)^2}}$$

$$Q14. \frac{d}{dx} (x \sin x)$$

$$= \sin x (1) + (x)(\cos x)$$

$$= \boxed{\sin x + x \cos x}$$

$$Q15. \frac{d}{dx} \left( \frac{x}{\sin x} \right)$$

$$= \frac{\sin x (1) - (x)(\cos x)}{(\sin x)^2}$$

$$= \boxed{\frac{\sin x - x \cos x}{\sin^2 x}}$$

$$Q16 \quad \frac{d}{dx} (x^2 \tan x)$$

$$= \tan x (2x) + (x^2) (\sec^2 x)$$

$$\boxed{= 2x \tan x + x^2 \sec^2 x}$$

$$Q17. \quad \frac{d}{dx} \left( \frac{x^2}{\sec x} \right)$$

$$= \frac{\sec x (2x) - (x^2) (\tan x \sec x)}{(\sec x)^2}$$

$$= \frac{2x \sec x - x^2 \sec x \tan x}{\sec^2 x}$$

$$= \frac{x \sec x (2 - x \tan x)}{\sec^2 x}$$

$$\boxed{= \frac{x (2 - x \tan x)}{\sec x}}$$

$$Q18. \quad \frac{d}{dx} (x \ln x)$$

$$= (\ln x)(1) + (x)\left(\frac{1}{x}\right)$$

$$\boxed{= \ln x + 1}$$

(7c)

$$Q19. \frac{d}{dx} (xe^x)$$

$$= (e^x)(1) + (x)(e^x)$$

$$= e^x + xe^x$$

$$\boxed{= e^x(1+x)}$$

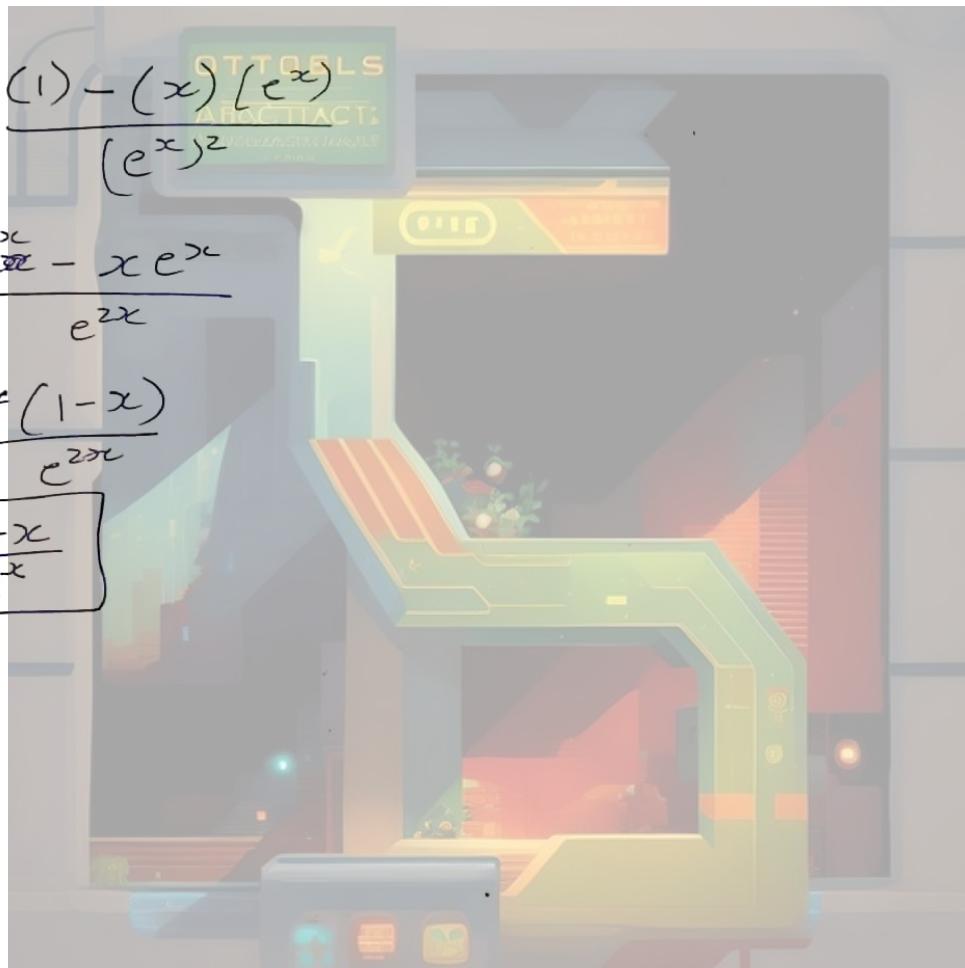
$$Q20. \frac{d}{dx} \left( \frac{x}{e^{2x}} \right)$$

$$= e^{2x} \frac{(1) - (x)(e^x)}{(e^x)^2}$$

$$= \frac{e^{2x} - xe^{2x}}{e^{2x}}$$

$$= \frac{e^{2x}(1-x)}{e^{2x}}$$

$$\boxed{= \frac{1-x}{e^x}}$$



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## Chain rule

$$\text{eg 1. } y = \sin(x^2 - 4)$$

$$\begin{aligned} &\text{let } t = x^2 - 4 \\ &\text{let } y = \sin t \\ &\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \end{aligned}$$

$$\frac{dy}{dt} = \frac{d}{dt}(y) = \frac{d}{dt}(\sin t) = \cos t$$

$$\begin{aligned} \frac{dt}{dx} &= \frac{d}{dx}(t) = \frac{d}{dx}(x^2 - 4) = 2x \\ \frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} = 2x \cos t \\ &= 2x \cos[x^2 - 4] \end{aligned}$$

$$\text{eg 2. } y = \sin^2 x = (\sin x)^2$$

$$t = \sin x$$

$$y = t^2$$

$$\frac{dy}{dt} = 2t$$

$$\frac{dt}{dx} = \cos x$$

$$\begin{aligned} \frac{dy}{dx} &= \cos x \times 2t \\ &= \cos x \times 2 \sin x \end{aligned}$$

$$= 2 \sin x \cos x$$

$$\text{eg 3. } y = \sin 2x$$

$$t = 2x$$

$$y = \sin t$$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 2$$

$$\frac{dy}{dx} = 2 \cos t$$

$$\boxed{= 2 \times \cos(2x)}$$

$$\text{eg 4. } y = \sin x^3$$

$$t = x^3$$

$$y = \sin t$$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 3x^2$$

$$\frac{dy}{dx} = 3x^2 \times \cos t$$

$$= 3x^2 \times \cos(x^3)$$

$$\boxed{= 3x^2 \cos(x^3)}$$

cos

$\Sigma$   
EF = OP

$$Q1. \quad y = \sin 3x$$

$$Q2. \quad y = 3 \sin(3x+5)$$

$$Q3. \quad y = \sin(\sin x)$$

$$Q4. \quad y = \ln^2 x = [\ln x]^2$$

$$Q5. \quad y = \frac{1}{3x}$$

P - 41, 42

Q1 - Q 29 39

$$Q4. \quad y = \sin 3x$$

$$\text{let } t = 3x$$

$$y = t \sin t$$

$$\frac{dy}{dt} = \frac{d}{dt} (\sin t) = \cos t$$

$$\frac{dt}{dx} = \frac{d}{dx} (3x) = 3$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cos t \boxed{3 \cos t} = 3 \cos(3x)$$

$$Q2. \quad y = 3 \sin(3x+5)$$

$$\text{let } t = 3x+5$$

$$y = 3 \sin t$$

$$\frac{dy}{dt} = 3 \cos t$$

$$\frac{dt}{dx} = 3$$

$$\frac{dy}{dx} = 3 \times 3 \cos t$$

$$\boxed{= 9 \cos t}$$

$$= 9 \cos(3x+5)$$

$$Q3. y = \sin(\sin x)$$

$$t = \sin x$$

$$y = \sin t$$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \cos x \cos t$$

$$= \cos x \cos(\sin x)$$

$$Q4. y = (\ln x)^2$$

$$t = \ln x$$

$$y = t^2$$

$$\frac{dy}{dt} = 2t$$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} \cdot x^2 t^2$$

$$= (\ln x)^2$$

$$= \left[ \frac{2 \ln x}{x} \right]$$

$$\text{Q } \textcircled{1} \text{ } \textcircled{1} \text{ } y = \frac{1}{3^x} = 3^{-x}$$

$$\text{Q } t = -x \\ y = 3^{at}$$

$$\frac{dy}{dt} = 3^t \ln 3$$

$$\frac{dt}{dx} = 1$$

$$\frac{dy}{dx} = 3^t \ln 3$$

$$dx$$

$$= -3^{-x} \ln 3$$

Q Differentiate the following

$$\text{Q } \textcircled{1} \text{ } y = \frac{1}{4} \tan^4 x$$

$$\frac{dy}{dx} = \frac{1}{4} x \sec^2 x$$

$$\text{Q } \textcircled{1} \text{ Let } t = \tan x$$

$$y = \frac{1}{4} t^4$$

$$\frac{dy}{dt} = \frac{1}{4} \times 4t^3 \\ = t^3$$

$$\frac{dt}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = \frac{1}{4} \left[ \tan^3 x \sec^2 x \right]$$

$$\textcircled{2} \quad y = \cot \sqrt{1+x^2}$$

$$t = \sqrt{1+x^2}$$

$$= (\sqrt{x^2})^{1/2}$$

$$t = 1 + x^2$$

$$S = 1 + x^2$$

$$t = S^2$$

$$y = \cot t$$

$$\frac{dy}{dt} = -\operatorname{cosec} t \cot t$$

$$\frac{ds}{dt}$$



$$\textcircled{3} \quad y = \sin^2 3x$$

$$t = 3x$$

$$y = \sin^2 t$$

$$\frac{dy}{dx} = \frac{d}{dt} \left( \sin^2 t \right) = 2 \sin t \cos t$$

$$\frac{dt}{dx} = 3$$

$$\frac{dy}{dx} = 6 \sin t \cos t \cdot 3$$

$$= 6 \sin 3x \cos 3x$$

\textcircled{3}

$$Q(2) \quad y = \cot \sqrt{1+x^2}$$

$$y = \cot [1+x^2]^{1/2}$$

$$t = [1+x^2]^{1/2}$$

$$y = \cot t$$

$$\frac{dy}{dt} = -\operatorname{cosec}^2 t$$

$$\# z = 1+x^2$$

$$t = z^{1/2}$$

$$\frac{dt}{dz} = \frac{1}{2} z^{-1/2}$$

$$\frac{dz}{dx} = 2x$$

$$\frac{dy}{dx} = \boxed{-\operatorname{cosec}^2 (1+x^2)^{1/2} \times \frac{1}{2} (1+x^2)^{-1/2} \times 2x}$$

$$Q(3) \quad y = \cos^3(4x)$$

$$t = 4x$$

$$y = (\cos t)^3$$

$$z = \cos t$$

$$y = z^3$$

$$\frac{dy}{dtz}$$

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$$④ y = \cos^3(4x)$$

$$\frac{dy}{dx} = 3(\cos 4x)^2 \times -\sin 4x \times 4$$

$$= 3 \cancel{\cos^2 4x} - \sin 4x \cancel{+ 4}$$

$$= -12 \cos^2 4x \sin 4x$$

$$⑤ y = (1 + \sin^2 x)^4$$

$$= 4(1 + \sin^2 x)^3 \times (2 \sin x) \times (\cos x)$$

$$= 8(1 + \sin^2 x)^3 \sin x \cos x$$

$$⑥ y = (3x + \tan^2 x)^2$$

$$\frac{dy}{dx} = 2(3x + \tan^2 x) \times (3 + 2 \tan x \sec^2 x)$$

$$= 2(3x + \tan^2 x) \times (3 + 2 \tan x \sec^2 x)$$

$$\frac{dy}{dx} = 2(3x + \tan^2 x) \times \frac{d}{dx}(3x + \tan^2 x)$$

$$= 2(3x + \tan^2 x) \times (3 + \frac{d}{dx}(\tan^2 x))$$

$$= 2(3x + \tan^2 x) \times (3 + 2 \tan x \times \frac{d}{dx}(\tan x))$$

$$= 2(3x + \tan^2 x) \times (3 + 2 \tan x \sec^2 x)$$

## # Double differentiation

$$y = f(x)$$

$$\frac{dy}{dx} = y' \text{ (first derivative)}$$

$$\frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d^2 y}{dx^2} = y'' \text{ (double differentiation)} \rightarrow \text{(Leibniz notation)}$$

$$\text{eg } y = 3x^2 + 5$$

$$y' = 6x$$

$$y'' = 6$$

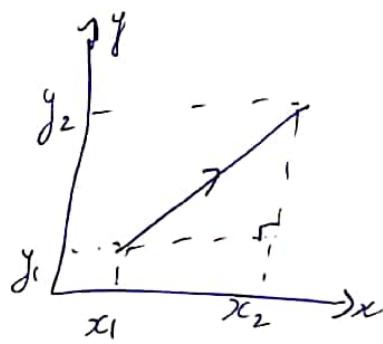
Displacement ( $s$ )

$$\text{velocity } (v) = \text{rate of change of position} = \frac{\Delta s}{\Delta t}$$
$$= \frac{ds}{dt}$$

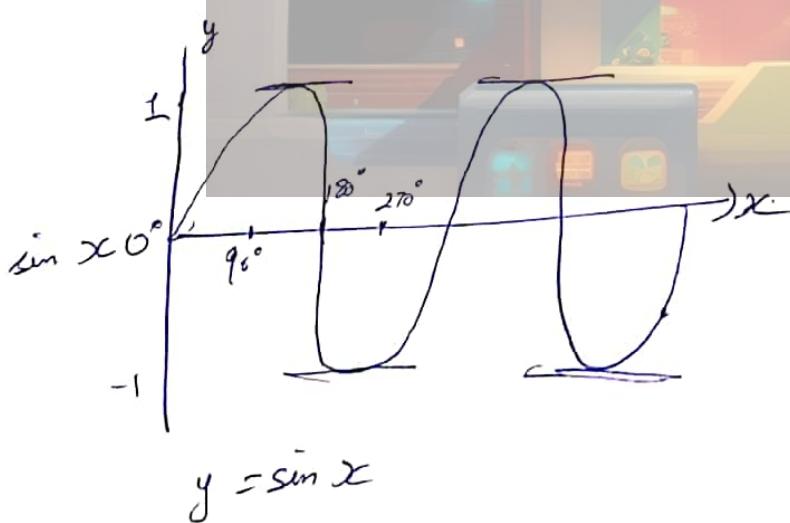
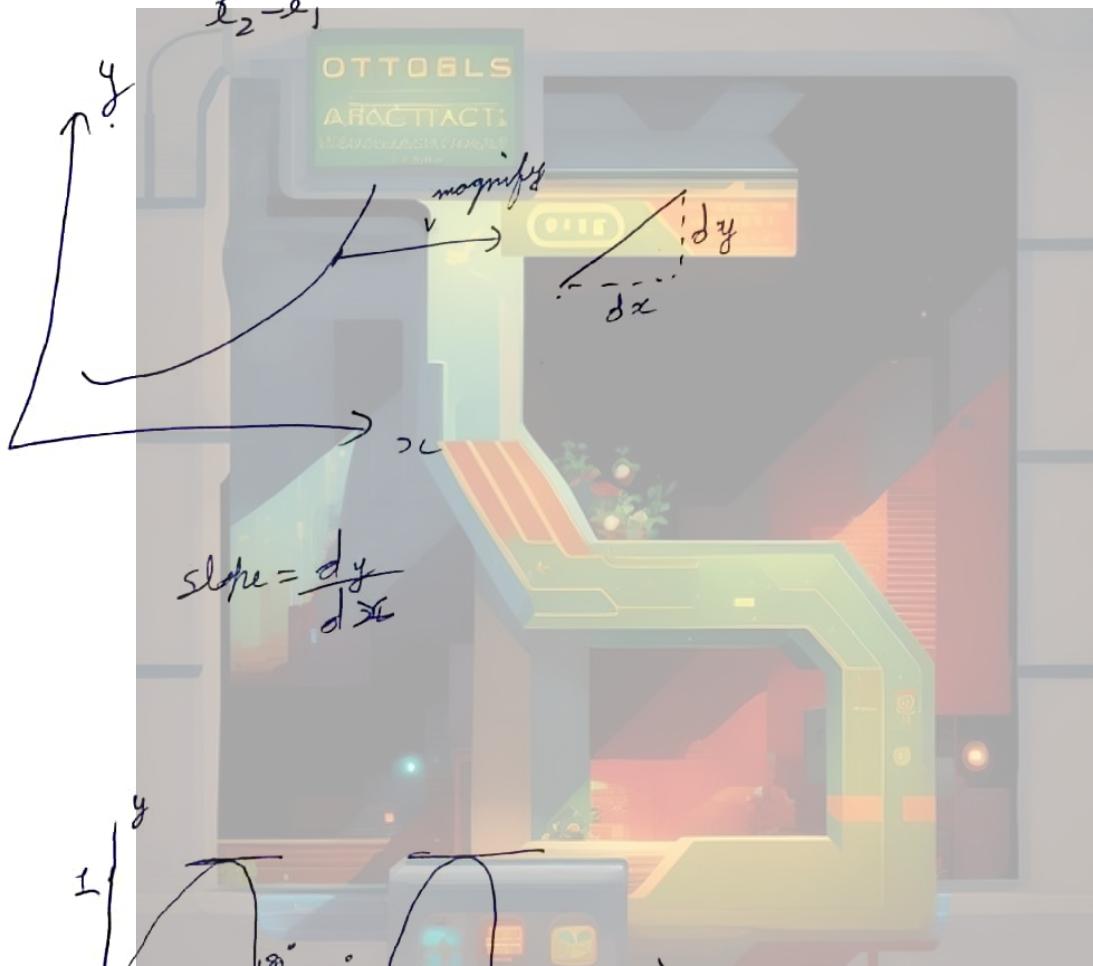
$$\text{acceleration } (a) = \text{rate of change of velocity} = \frac{\Delta v}{\Delta t}$$
$$= \frac{d}{dt} \left( \frac{ds}{dt} \right)$$

$$= \frac{d^2 s}{dt^2}$$

## Slope



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



at topmost (maxima) and bottommost (minima), slope = 0

$$\boxed{\frac{dy}{dx} = 0}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x$$

$$\cos x = 0 \\ x = 90^\circ, 270^\circ$$

Curvature  $\Rightarrow$  Double Differentiation

maxima  $\Rightarrow$  slope decrease  $\Rightarrow \frac{d^2y}{dx^2} < 0$

minima  $\Rightarrow$  slope increase  $\Rightarrow \frac{d^2y}{dx^2} > 0$

For maxima,  $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} < 0$

For minima,  $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} > 0$

$$Q \quad y = x^3 - 3x^2 + 6$$

$$\frac{dy}{dx} = 3x^2 - 6x$$

$$\frac{dy}{dx} = 0$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$\boxed{\begin{array}{l} x=0 \\ x=2 \end{array}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 6x)$$

$$= 6x - 6$$

$$\text{at } x=0$$

$$\frac{d^2y}{dx^2} = 0 - 6$$

$$\text{at } x=2$$

$$\frac{d^2y}{dx^2} = 12 - 6$$

$$= 6$$

$$\boxed{\begin{array}{l} \text{Maxima} = \underset{(x=0)}{0} = 6 \\ \text{Minima} = \underset{(x=2)}{3-12+6} = 2 \end{array}} \checkmark$$

$$\textcircled{2} \quad y = 3x^4 + 4x^3 - 12x^2 + 12$$

find the points of maxima & minima.

$$\frac{dy}{dx} = 12x^3 + 12x^2 - 24x$$

0=

$$24x = 12x^3 + 12x^2$$

$$2 = x^2 + x$$

$$2 = x(x+1)$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$\cancel{x(x+2)}$$

$$(x-1)(x+2) = 0$$

$$x = 1$$

$$x = -2$$

$$\frac{d^2y}{dx^2} = 36x^2 + 24x - 24$$

$$= 36 + 24 - 24$$

$$= 36 \text{ (minima, } x = 1\text{)}$$

$$= 36(-2)^2 + 24(-2) - 24$$

$$= 36 \times 4 + 48 - 24$$

$$= 144 - 48 - 24$$

$$= 72 \text{ (minima)}$$

$$= 0 + 0 - 0.24$$

$$= 0 - 24 \text{ (maxima)}$$

$$y_{\max} = 12$$

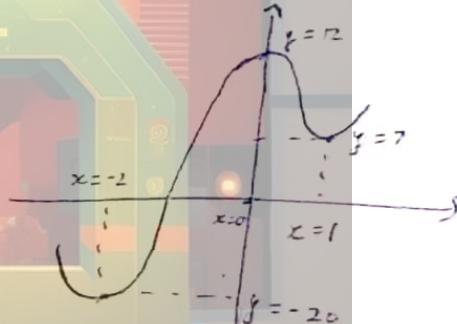
$$y_{\min} = 3 + 4 - 12 + 12 \\ = 7$$

$$y_{\max} = 16 \times 3 + -4 \times 8$$

$$= -20$$

$$\boxed{\text{minimum} = -20}$$

$$\boxed{\text{maximum} = 12}$$



③ find minimum value of  $5x^2 - 2x + 1$

$$\frac{dy}{dx} = 10x - 2 = 0$$

$$0 = 10x$$

$$2 = 10x$$

$$x = \frac{1}{5}$$

$$\frac{d^2y}{dx^2} = 10 \text{ (minimum)}$$

$$\text{value} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1$$

$$= \frac{5x}{25} - \frac{2}{5} + 1$$

$$= \frac{1-2+5}{5}$$

$$\boxed{\frac{4}{5}} \quad \boxed{3} = 0.8$$

④ find turning points of function  $y = x^3 + 12x^2 + 12x + 10$

$$\frac{dy}{dx} = 12x^2 + 24x + 12$$

$$= 12x^2 + 24x + 12$$

$$= x^2 + 2x + 1$$

$$\theta = x^2 + x + x + 1$$

$$= x(x+1) + 1(x+1)$$

$$\boxed{x = -1}$$

$$y = 12\theta - 24 + 12$$

$$= 0$$

$$(-1, 0)$$

$$\frac{d^2y}{dx^2} = 24x + 24$$

$$= -24 + 24$$

$\Rightarrow$  Point of Inflection,  
no minima or maxima

{  
→ Point of Inflection

## Application of Derivative Derivatives

Q The radius of circle is increasing at the rate of 0.7 cm per second. What is the rate of increase in its circumference.

$$\cancel{\frac{dr}{dt}} \quad \frac{dr}{dt} = 0.7 \text{ cm/s}$$

$$C = 2\pi r$$

$$\frac{dc}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2 \times 0.7 \times 17$$
$$= 1.4 \pi \text{ m/s}$$

Q2. The side of square is increasing at the rate of 0.2 cm/s find rate of increase in its perimeter.

$$s = 4s$$

$$\frac{ds}{dt} = 0.2 \text{ cm/s}$$

$$\frac{dp}{dt} = 4 \times \frac{ds}{dt}$$

$$= 0.8 \text{ cm/s}$$

Q③. The radius of a spherical balloon is decreasing at the rate of  $10 \text{ cm/s}$ . At what rate is the surface area of the balloon decreasing when its radius is  $15 \text{ cm}$ .

$$\frac{dr}{dt} = -10 \text{ cm/s}$$

$$\text{area} = 4\pi r^2$$

~~$$\frac{da}{dt} = 4\pi (-10)^2$$~~

~~$$= 4\pi \times 100$$
  

$$= -400\pi \text{ cm}^2/\text{s}$$~~

$$\frac{dA}{dt} = \frac{d}{dt} (4\pi r^2)$$

$$= 8\pi r \frac{d}{dt} (r^2)$$

$$= 4\pi \times 2r \times \frac{d}{dt} (r)$$

$$= 4\pi \times 2r \times -10$$

$$= -80\pi r$$

$$r = 15 \text{ (given)}$$

$$= -80\pi \times 15$$

$$= -1200\pi \text{ cm}^2/\text{s}$$

Q4. An Edge of a variable cube of ~~area~~ is increasing at a rate of 3 cm/s. How fast is its volume increasing when the edge is 10 cm long.

$$\frac{de}{dt} = 3 \text{ cm/s}$$

$$V = e^3$$

$$\frac{dV}{dt} = \frac{d}{dt}(e^3) = 3e^2 \times 3$$

$$e = 10$$

$$= 3(10)^2 \times 3$$

$$= 9 \times 100$$

$$= 900 \text{ cm}^3/\text{s}$$

Q5. The volume of a spherical balloon is increasing at the rate of 25 cm<sup>3</sup>/s. Find the rate of change of its surface area at the instant when its radius is 5 cm.

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 25$$

$$25 = \frac{4}{3} \pi \times 3r^2 \times \frac{dr}{dt}$$

$$\frac{dA}{dt} = 4\pi(2r) \times \frac{dr}{dt}$$

$$= 4\pi \times 2 \times 5 \times \frac{25}{4\pi}$$

$$= 10 \cancel{\pi} \times 5 \times 25 \text{ cm}^2/\text{s}$$

$$\frac{25 \times 3}{4 \times 12 \times 3 \times 8 \times 9}$$

$$\frac{1}{4\pi} = \frac{dr}{dt}$$

## Integration

$$\frac{d}{dx}(x^2) = 2x$$

Q  $\frac{d}{dx}(x^2+1) = 2x$

$$\frac{d}{dx}(x^3+3) = 3x^2$$

$$\int(2x)dx = x^2 + C \rightarrow \text{constant of integration}$$

Indefinite Integration

formulas:-

$$① \int c f(x) dx = c \int f(x) dx$$

$$② \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$③ \int \frac{1}{x} dx = \ln|x| + C$$

$$④ \int e^x dx = e^x + C$$

$$⑤ \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C \quad (n \neq -1)$$

$$⑥ \int e^{(ax+b)} dx = \frac{e^{(ax+b)}}{a} + C$$

$$\textcircled{7} \quad \int \sin x \, dx = -\cos x + C$$

$$\textcircled{8} \quad \int \cos x \, dx = \sin x + C$$

$$\textcircled{9} \quad \int \sin(ax+b) \, dx = \frac{-\cos(ax+b)}{a} + C$$

$$\textcircled{10} \quad \int \cos(ax+b) \, dx = \frac{\sin(ax+b)}{a} + C$$

$$\textcircled{11} \quad \int \sec^2 x \, dx = \tan x + C$$

$$\textcircled{12} \quad \int \csc^2 x \, dx = -\cot x + C$$

$$\textcircled{13} \quad \int \sec x \tan x \, dx = \sec x + C$$

$$\textcircled{14} \quad \int \sqrt{x} \, dx$$

$$\int x^{1/2} \, dx$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{\sqrt{x^3}}{3} \checkmark$$

$$= \frac{2\sqrt{x^3}}{3}$$

$$\textcircled{2} \int \frac{dx}{x}$$

$$\int x^{-2} dx$$

$$\frac{x^{-1}}{-1} + c$$

$$\boxed{\frac{1}{-x} + c}$$

$$\textcircled{3} \int (1-2x) dx$$

$$\cancel{(1-2x)^2}$$

$$\underline{(1-2x)^{1+1}} + c$$

$$(-2)(1+1)$$

$$\boxed{\frac{(1-2x)^2}{-4} + c}$$

$$\textcircled{4} \int \cos^2 x dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \frac{1}{2} + \frac{\cos 2x}{2} dx$$

$$\frac{1}{2} \int 1 + \cos^2 x dx$$

$$\frac{1}{2} \int 1 dx + \int \cos 2x dx$$

$$\boxed{\frac{1}{2} \left( x + \frac{\sin 2x}{2} \right) + c}$$

$$\textcircled{5} \int \sin^2 x dx$$

$$\textcircled{6} \int \frac{1}{2x-1} dx$$

$$\textcircled{5} \int \frac{1 - \cos 2x}{2} dx$$

$$\frac{1}{2} \int 1 - \cos 2x dx$$

$$\frac{1}{2} \int 1 dx - \int \cos 2x dx$$

$$\boxed{\frac{1}{2} \left( x - \frac{\sin 2x}{2} \right)}$$

$$\textcircled{6} \int \frac{1}{2x-1} dx$$

$$\int \frac{1}{2x} dx - x$$

$$\frac{1}{2} \int \frac{1}{x} dx - x$$

$$\frac{1}{2} \ln x - x + C$$

$$\boxed{\frac{1}{2} \ln x - x + C}$$

$$\textcircled{7} \int \left(\frac{x-1}{x}\right) dx$$

$$\int x \ln x dx = - \int \frac{1}{x} dx$$

$$\frac{x^2}{2} - \ln x$$

$$\boxed{\frac{x^2 - 2\ln x + C}{2}}$$

$$\textcircled{8} \int \cos(2x+5) dx$$

$$\boxed{\frac{\sin(2x+5)}{2} + C}$$

$$\textcircled{9} \int \left(e^{2x} + \frac{1}{x^3}\right) dx$$

$$e^{2x} + \int x^{-3} dx$$

$$e^{2x} + \frac{x^{-2}}{-2}$$

$$e^{2x} - \frac{1}{e^{2x} x^2}$$

$$\boxed{\frac{e^{2x}}{2} - \frac{1}{2x^2} + C}$$

$$\textcircled{10} \int \cos(1-2x) dx$$

$$\boxed{\frac{\sin(1-2x)}{-2} + C}$$

## # Substitution method

$$\textcircled{1} \quad \int \frac{(2x-3) dx}{x^2 - 3x + 8}$$

$$\text{let } t = x^2 - 3x + 8$$

$$\frac{dt}{dx} = 2x - 3$$

$$\textcircled{2}, dt = (2x-3) dx$$

$$\begin{aligned} & \int \frac{dt}{x^2 - 3x + 8} \\ & \int \frac{dt}{t} \\ & \int \frac{1}{t} dt \cancel{dx} \\ & \Rightarrow \ln t + C \\ & = \ln(x^2 - 3x + 8) + C \end{aligned}$$

$$\textcircled{2} \quad \int \frac{x^2 dx}{x^3 + 1}$$

$$t = x^3 + 1$$

$$\frac{dt}{dx} = 3x^2$$

$$dt = 3x^2 dx$$

$$\frac{dt}{3x^2} = x^2 dx$$

$$\int \frac{dt}{3x^2} \cdot \frac{1}{t}$$

$$\frac{\ln t}{3} + C$$

$$\frac{\ln(x^3 + 1)}{3} + C$$

$$\textcircled{3} \quad \int \frac{e^{2x}}{e^{2x} + a^2} dx$$

$$\text{let } t = e^{2x} + a^2$$

$$\frac{dt}{dx} = e^{2x} + 2a^2$$

$$dt = 2a^2 dx = e^{2x} dx$$

$$dt = e^{2x} dx$$

$$\begin{aligned} & \int \frac{dt}{t} \\ &= \ln|t| + C \\ &= \ln(e^{2x} + a^2) + C \\ \textcircled{4} \quad & \int \frac{dt}{x \ln t} \end{aligned}$$

$$\frac{dt}{dx} = e^{2x} + 2$$

$$\begin{aligned} dt &= e^{2x} dx \\ dt &= 2 dx = e^{2x} dx \\ \frac{dt}{2} &= e^{2x} dx \end{aligned}$$

$$\int \frac{dt}{2} \times \frac{1}{t}$$

$$\frac{\ln t}{2} + C$$

$$\frac{\ln(e^{2x} + a^2) + C}{2}$$

$$\textcircled{4} \quad \int \frac{dx}{x \ln x}$$

$$t = \ln x$$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$dt \times x = dx$$

$$\int \frac{dt \cdot x}{xt}$$

$$x \times \ln t$$

$$\boxed{x \ln(\ln x)} + C$$

(10)

$$\textcircled{5} \int e^x (\sin e^x) dx$$

$$\text{Let } t = e^x$$

$$\frac{dt}{dx} = e^x$$

$$dt = e^x dx$$

$$\int dt (\sin e^x)$$

$$\int dt \cdot \cancel{e^x} \sin t$$

$$-\cos t + C$$

$$\boxed{-\cos e^x + C}$$

$$\textcircled{6} \int \frac{x dx}{x^2 + 1}$$

$$t = x^2 + 1$$

$$\frac{dt}{dx} = 2x$$

$$\frac{dt}{2} = x dx$$

$$\int \frac{dt}{2} \times \frac{1}{2x^2 t}$$

$$\frac{\ln t}{2} + C$$

$$\boxed{\frac{\ln(x^2+1)}{2} + C}$$

$$\textcircled{7} \int (\ln x)^m dx$$

$$t = \ln x$$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$dt = \frac{dx}{x}$$

$$\int (\ln x)^m t^m dt$$

$$\frac{t^{m+1}}{m+1} + C$$

$$\left[ \frac{(\ln x)^{m+1}}{m+1} + C \right]$$

$$\textcircled{8} \int e^{\sin x} \cos x dx$$

$$\text{let } t = \sin x$$

$$\frac{dt}{dx} = \cos x$$

$$dt = \cos x dx$$

$$\int e^t dt$$

$$e^t =$$

$$= e^{\sin x} + C$$

$$⑨ \int e^{x^2} x \, dx$$

$$t = x^2$$

$$\frac{dt}{dx} = 2x$$

$$\frac{dt}{2} = x \, dx$$

$$\int e^t \frac{dt}{2}$$

$$\left[ \frac{e^t}{2} + c \right] \rightarrow \left[ \frac{e^{x^2}}{2} + c \right]$$

$$⑩ \int e^{-x^3} x^2 \, dx$$

$$t = -x^3$$

$$\frac{dt}{dx} = -3x^2$$

$$\frac{dt}{-3} = x^2 \, dx$$

$$\int e^t \frac{dt}{-3}$$
$$\left[ \frac{e^{-x^3}}{-3} + c \right]$$

$$⑭ \int \frac{x \, dx}{\sqrt{x^2+1}}$$

$$t^2 = x^2 + 1$$

$$2t \, dt = 2x \, dx$$

$$t \, dt = x \, dx$$

$$= \int \frac{1 \, dt}{dt}$$

$$= \int dt$$

$$= t$$

$$\boxed{y = \sqrt{x^2+1} + C}$$

OTTOBLS  
ARACTA  
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TECHNOLOGY

$$\frac{1}{2} \sqrt{x^2+1} \times t^2$$

$$⑮ \int \frac{\cos x}{\sqrt[3]{\sin x}} \, dx$$

$$t^3 = \sin x$$

$$3t^2 \, dt = \cos x \, dx$$

$$\int \frac{3t^2 \, dt}{t}$$

$$3 \int t \, dt$$

$$3 \times \frac{t^2}{2}$$

$$\boxed{\frac{3}{2} (\sin x)^{\frac{2}{3}} + C}$$

$$⑯ \int \frac{\sqrt{\ln x}}{x} dx$$

$$⑰ \int x^2 \cdot \sqrt{x^3 + 2} dx$$

$$⑯ t^2 = \ln x$$

$$2t dt = \frac{1}{x} dx$$

$$\int t \cdot x \cdot 2t dt$$

$$2 \int t^2 dt$$

$$2x \frac{t^3}{3}$$

$$\boxed{\frac{2(\ln x)^{3/2}}{3} + c} \quad \checkmark$$

$$⑰ t^5 = x^3 + 2$$

$$5t^4 dt = 3x^2 dx$$

$$\frac{5}{3} t^4 dt = x^2 dx$$

$$\int t \cdot \frac{5}{3} t^4 dt$$

$$\frac{5}{3} \int t^5 dt$$

$$\frac{5}{3} \times \frac{t^6}{6}$$

$$\frac{5}{3} \times \cancel{\frac{(x^3+2)^{6/5}}{6}}$$
$$\boxed{\frac{5(x^3+2)^{6/5}}{18} + c} \quad \checkmark$$

$$\textcircled{14} \quad \int \frac{x \, dx}{\sqrt{x^2+1}}$$

$$t^2 = x^2 + 1$$

$$2t \, dt = 2x \, dx$$

$$t \, dt = x \, dx$$

$$= \int \frac{t \, dt}{dt}$$

$$= \int dt$$

$$= t$$

$$= \sqrt{x^2+1} + C$$

$$\textcircled{15} \quad \int \frac{\cos x}{\sqrt[3]{\sin x}} \, dx$$

$$t^3 = \sin x$$

$$3t^2 dt = \cos x \, dx$$

$$\int \frac{3t^2 dt}{t}$$

$$3 \int t \, dt$$

$$3 \times \frac{t^2}{2}$$

$$\boxed{\frac{3}{2} (\sin x)^{2/3} + C}$$

$$\textcircled{16} \int \frac{\sqrt{\ln x}}{x} dx$$

$$\textcircled{17} \int x^2 \cdot \sqrt{x^3 + 2} dx$$

$$\textcircled{16} t^2 = \ln x$$

$$2t dt = \frac{1}{x} dx$$

$$\int t \cdot x \cdot 2t dt$$

$$2 \int t^2 dt$$

$$2x \frac{t^3}{3}$$

$$\left[ \frac{2(\ln x)^{3/2}}{3} + C \right] \checkmark$$

$$\textcircled{17} t^5 = x^3 + 2$$

$$5t^4 dt = 3x^2 dx$$

$$\frac{5}{3} t^4 dt = x^2 dx$$

$$\int t \cdot \frac{5}{3} t^4 dt$$

$$\frac{5}{3} \int t^5 dt$$

$$\frac{5}{3} \frac{t^6}{6}$$

$$\frac{5}{3} \cdot \cancel{t^6} \frac{(x^3+2)^{6/5}}{6}$$

$$\left[ \frac{5(x^3+2)^{6/5}}{18} + C \right] \checkmark$$

\textcircled{108}

## # Definite Integral

$$I = \int_a^b f(x) dx = \left[ g(x) \right]_a^b = g(b) - g(a)$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

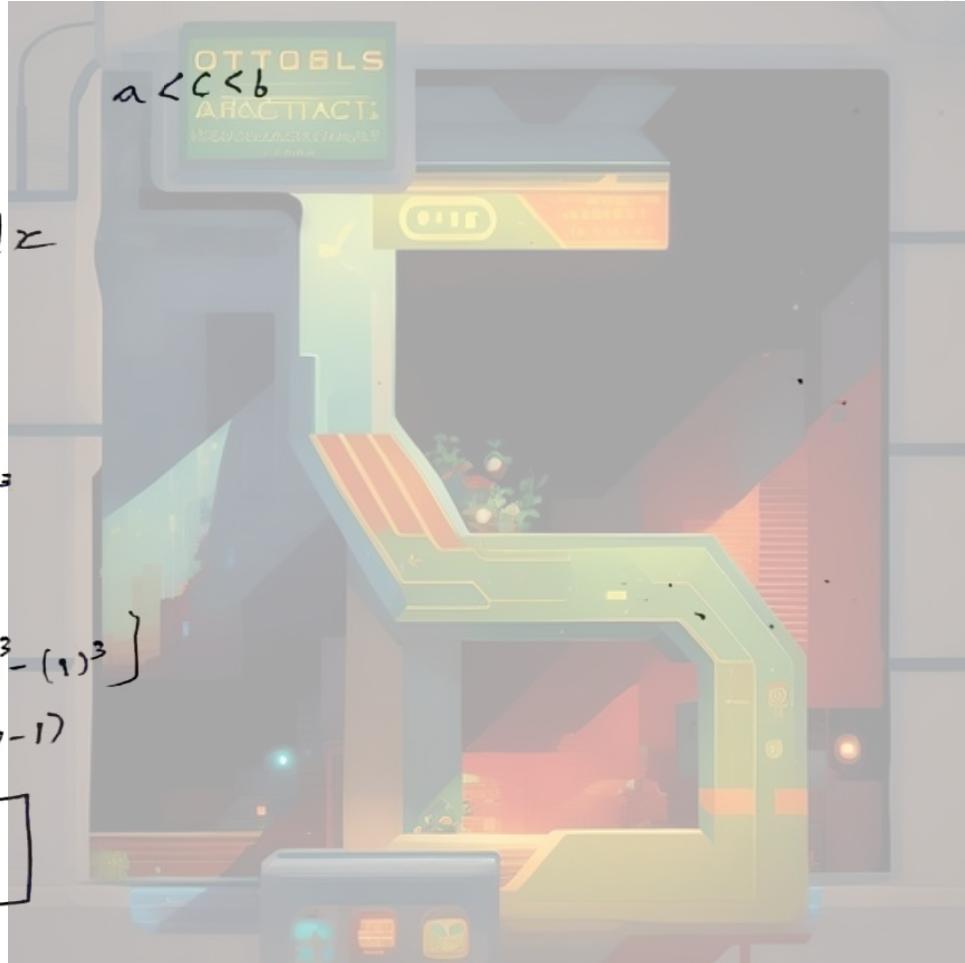
$$\textcircled{1} \quad \int_1^3 x^2 dx$$
$$\left[ \frac{x^3}{3} \right]_1^3$$

$$\frac{1}{3} \left[ x^3 \right]_1^3$$

$$\frac{1}{3} \left[ (3)^3 - (1)^3 \right]$$

$$\frac{1}{3} (27 - 1)$$

$$\boxed{\frac{26}{3}}$$



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$$\textcircled{2} \quad I = \int_0^2 (ax^2 + bx + c) dx$$

$$\left[ \frac{a}{3}x^3 + \frac{b}{2}x^2 + cx \right]_0^2$$

$$\left( \frac{8a}{3} + \frac{4b}{2} + 2c \right) - \left( \cancel{\frac{a}{2}} + \cancel{\frac{b}{2}} 0 + 0 + 0 \right)$$

$$\frac{8a}{3} + 2b + 2c$$

$$\boxed{\frac{2}{3}(4a + 3b + 3c)}$$

$$\textcircled{3} \quad I = \int_0^\pi \sin^2 x dx$$

$$\frac{1}{2} \left[ x - \frac{\sin 2x}{2} \right]_0^\pi$$

$$\frac{1}{2} \left( \pi - \frac{\sin 2\pi}{2} \right) - \left( 0 - \frac{\sin 0}{2} \right)$$

$$\boxed{\frac{1}{2} [\pi - 0] = \frac{\pi}{2}}$$

$$\textcircled{4} \quad \int_0^4 \sqrt{2x+1} dx$$

Method - I

$$I = \int_0^4 (2x+1)^{\frac{3}{2}} dx$$

$$= \left[ \frac{(2x+1)^{\frac{3}{2}}}{2 \cdot \frac{1}{2} + 1} \right]_0^4$$

$$= \frac{1}{3} \left[ 9^{\frac{3}{2}} - 1^{\frac{3}{2}} \right] \text{Ans}$$

$$= \frac{1}{3} \times 2^2$$

$$= \frac{2^2}{3}$$

Method II

$$I = \int_0^4 (2x+1) \sqrt{2x+1} dx$$

$$x = \sqrt{2x+1} \Rightarrow L.L. \Rightarrow \sqrt{2(4)+1} \rightarrow 3$$

$$2x dx = 2 dx \Rightarrow L.L. \Rightarrow \sqrt{2(0)+1} \rightarrow 1$$

$$dx = t dt$$

$$\int_1^3 t^2 dt$$

$$\left[ \frac{t^3}{3} \right]_1^3$$

$$\frac{1}{3} [t^3]^3_1$$

$$\frac{1}{3} (2^3 - 1)$$

$$\frac{26}{3}$$

\textcircled{111}

$$\textcircled{4} \textcircled{5} \int_0^1 x(x^2+1)^3 dx$$

$$t = x^2 + 1$$

$$\frac{dt}{dx} = 2x$$

$$\frac{dt}{2} = x dx$$

$$\int_1^2 \frac{t^3}{2} dt$$
$$\left[ \frac{t^4}{2x^4} \right]_1^2$$

$$\frac{1}{8} [t^4]_1^2$$

$$\frac{1}{8} (16 - 1)$$

$$\boxed{\frac{105}{8}}$$

~~$$\textcircled{4} \textcircled{6} \int_0^1 \frac{x^3}{(1+x^2)^4} dx$$~~

~~$$1+x^2$$~~

~~$$1+x^4 + 2x^2$$~~

~~$$3x^3 + 4x^2 + 4x$$~~

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$$⑥ \int_0^1 \frac{x^3}{(x+1)^4} dx$$

$$t = 1 + x^2$$

$$x^2 = t - 1$$

$$\frac{dt}{2} = x dx$$

$$\int_1^2 \frac{(t-1) \frac{dt}{2}}{t^4}$$

$$\frac{1}{2} \int_1^2 t^{-3} dt \int_1^2 t^{-4} dt$$

$$\frac{1}{2} \left[ \frac{t^{-2}}{-2} - \frac{t^{-3}}{-3} \right]_1^2$$

$$\frac{1}{2} \left[ -\frac{1}{2t^2} + \frac{1}{3t^3} \right]_1^2$$

$$\frac{1}{2} \left[ \frac{1}{3t^3} - \frac{1}{2t^2} \right]_1^2$$

$$\frac{1}{2} \left[ \left( \frac{1}{24} - \frac{1}{8} \right) - \left( \frac{1}{3} - \frac{1}{2} \right) \right]$$

$$\frac{1}{2} \left[ \left( -\frac{2}{24} \right) - \left( -\frac{1}{6} \right) \right]$$

$$\frac{1}{2} \left[ -\frac{1}{12} + \frac{1}{6} \right]$$

$$\frac{1}{2} \left[ \frac{2}{12} - \frac{1}{12} \right]$$

$$\frac{1}{2} \left( \frac{1}{12} \right) = \boxed{\frac{1}{24}}$$

# Average value of a continuous function in an interval:-

$$y = f(x)$$

→ average value of  $y$  in an interval  $a \leq x \leq b$  is given by

$$\langle y \rangle (\text{average value of } y) = \frac{\int_a^b y \, dx}{\int_a^b dx}$$

Q find average value from  $x = 0$  to  $x = \pi$  for  $\sin x$

$$\begin{aligned}\langle \sin x \rangle &= \frac{\int_0^\pi \sin x \, dx}{\int_0^\pi dx} \\ &= \frac{\left[ -\cos x \right]_0^\pi}{[\cos x]_0^\pi} \\ &= \frac{-[\cos \pi - \cos 0]}{\pi - 0} \\ &= \frac{-(-1 - 1)}{\pi} = \frac{2}{\pi}\end{aligned}$$

Q Determine average value of  $y = x + 5$  in the interval  $0 \leq x \leq 1$

$$y = x + 5$$

$$\begin{aligned}\langle y \rangle &= \frac{\int_0^1 x + 5 \, dx}{\int_0^1 dx}\end{aligned}$$

$$= \frac{\left[ \frac{2x^2}{2} + 5x \right]_0^1}{\{x\}_0^1}$$

$$= \frac{\left( \frac{1}{2} + 5 \right) - (0)}{1 - 0}$$

$$= \frac{\frac{11}{2}}{1}$$

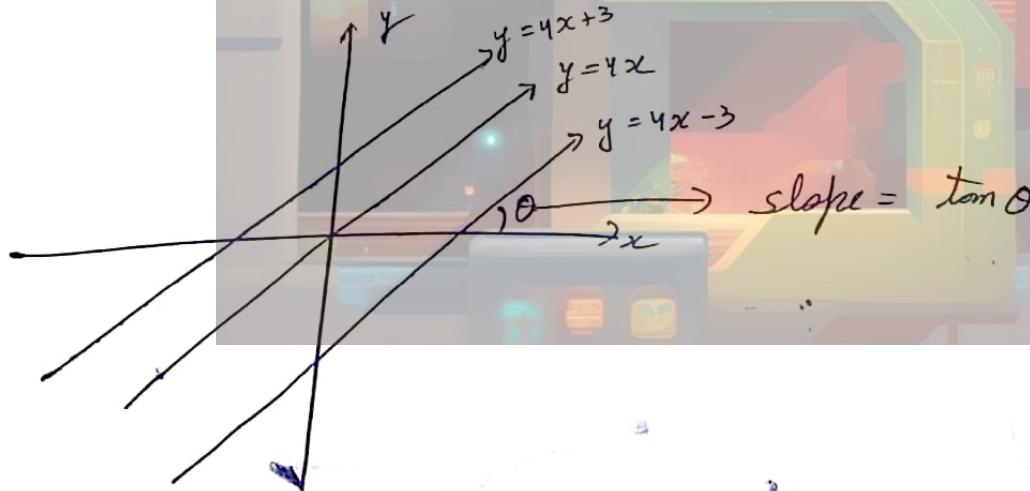
$$\boxed{\langle y \rangle = \frac{11}{2}}$$

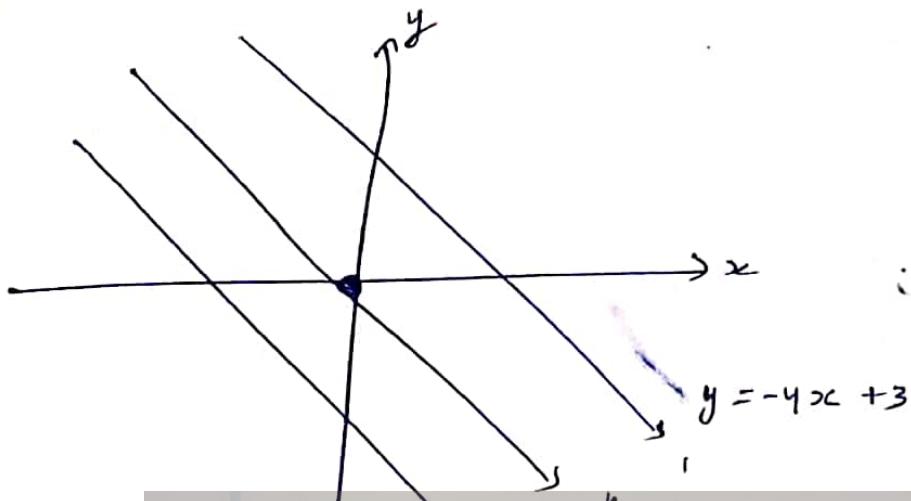
### # Graphs

1. Straight line

$$\text{equ} = y = mx + c$$

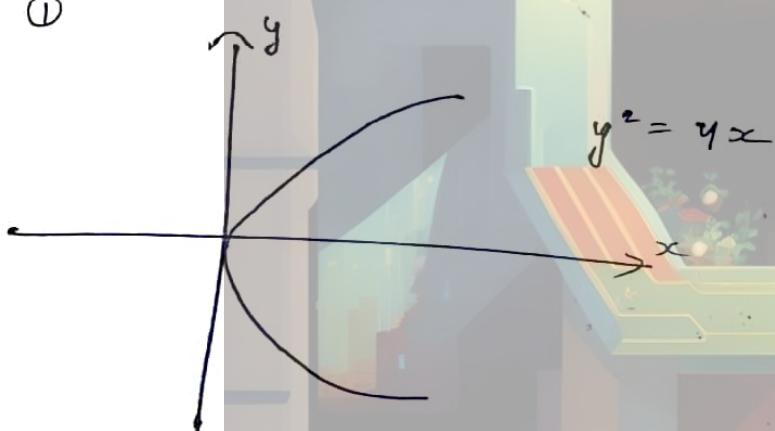
↑ slope  
intercept on y-axis



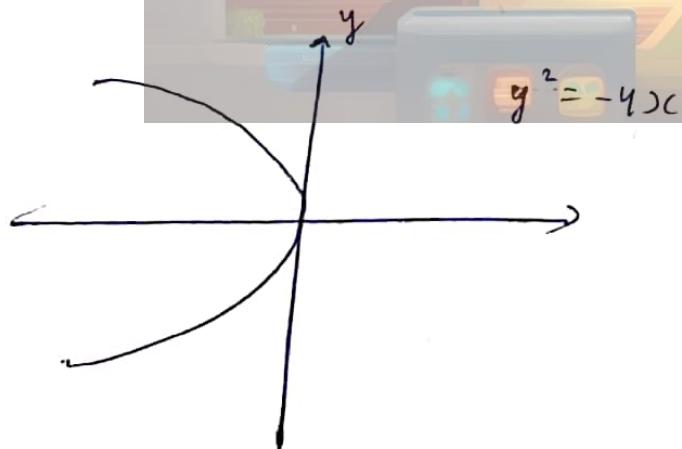


Quadratic Equations Graph

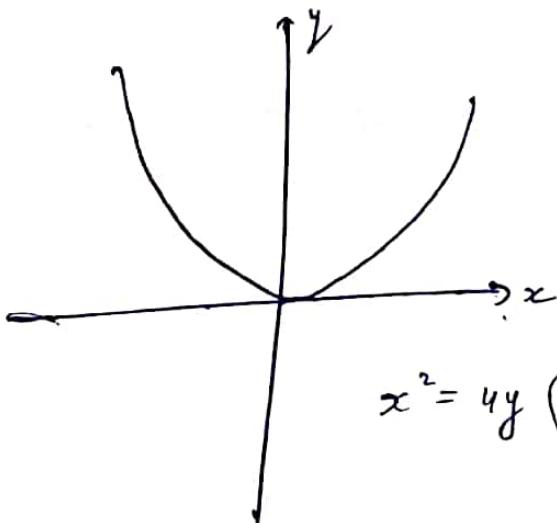
①



②



(ii)



$$x^2 = 4y \text{ (mouth of parabola opening upwards)}$$

(iii)



$$x^2 = -4y$$

(mouth of parabola opening downwards)

(117)

## Kinematics (Motion in one Dimension)

Kinematics - The branch of theoretical mechanics which deals with study of motion of rigid bodies without taking into account (shape cannot be changed).

The cause of motion (i.e. forces acting on them) is called Kinematics.

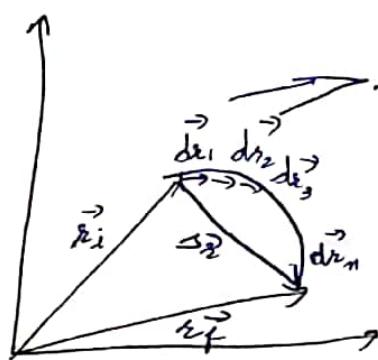
Point object - A particle in its strict sense means an object without dimensions (no size, only a point).

Translatory motion - when there is no change in the orientation of a object while moving. (Every point travels same distance)



- \* Position of a particle in space is determined relative to some fixed point, position depends on position of observer.  
↳ reference points.
- \* The path followed by a point object during its motion is called its trajectory.

## Distance & Displacement



elementary paths/vectors

\* Distance is the length of the path actually traversed ~~which~~ while displacement is the change in position vector.

$$\text{Displacement} ; \Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta \vec{r} = \int \vec{dr}$$

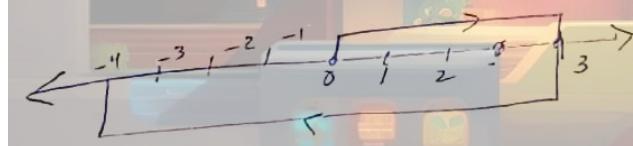
adding all elementary vectors

$$\text{Distance} = \int | \vec{r} |$$

adding of all magnitudes of elementary vectors.

- Distance is multi-valued function while displacement is a single valued function.
- Distance  $\geq$  Displacement  $\rightarrow$  can be +ve, -ve or zero  
↳ always Positive

e.g.



$$\text{Distance} = 3 + 3 + 4 = 10$$

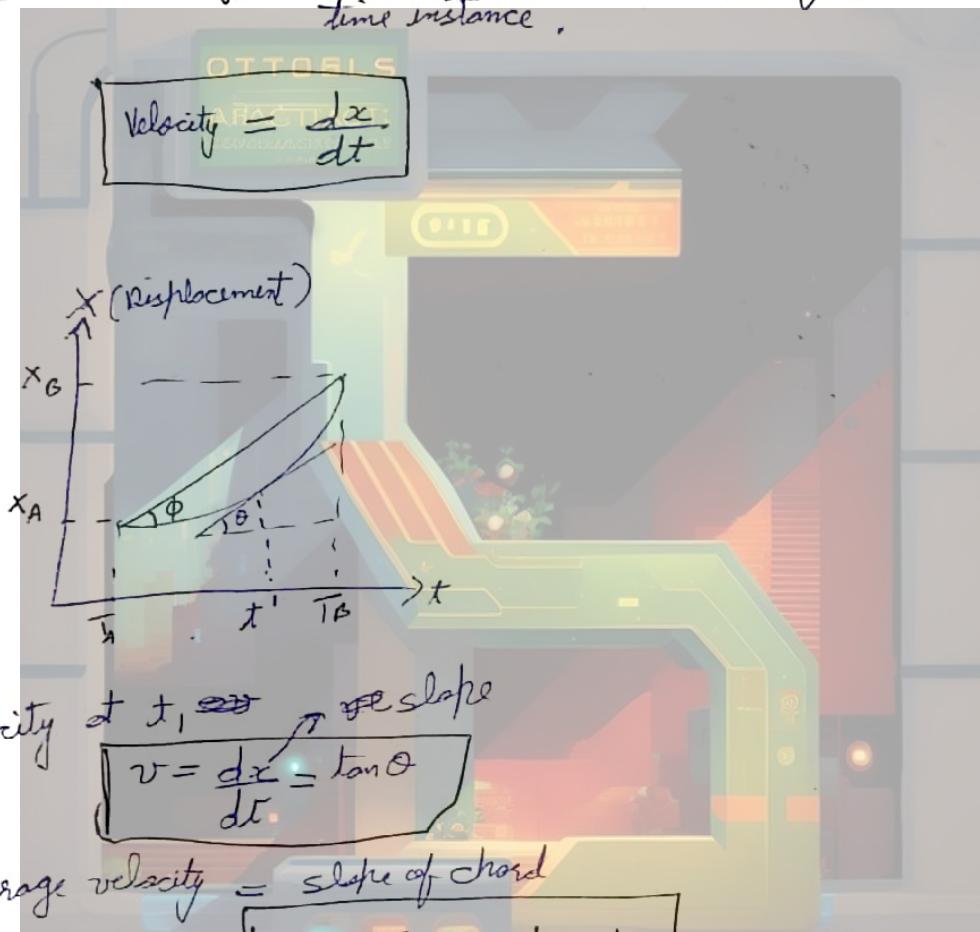
$$\text{Displacement} = x_f - x_i = 3 - (-4) = 7$$

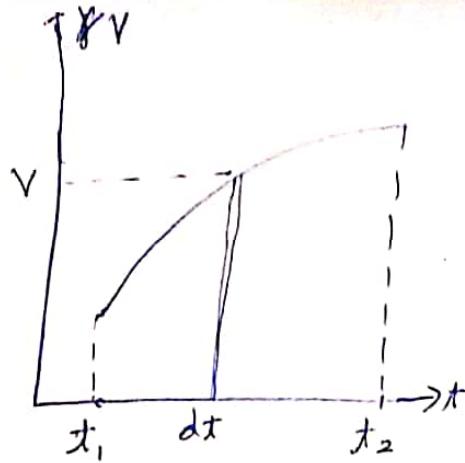
## # Speed & Velocity -

Average Speed -  $\frac{\text{Total Distance}}{\text{Total Time}}$

$$\underline{\text{Average Velocity}} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta \vec{x}}{\Delta t}$$

Instantaneous velocity - ~~velocity~~ Rate of change of position at given time instance.





~~Velocity under~~  
area under velocity graph  
= displacement

✓  $\Delta v = \text{area under small time instance}$

$\int v dt$   $\rightarrow$  area under curve is Integration  
= area under whole graph (by adding all small time instances)

$$v = \frac{dx}{dt}$$

$$x = \int dx = \int v dt$$

$x$  is displacement not distance

Q:



$$\text{Distance} = I + II + III$$

$$\text{Displacement} = I - II + III$$

~~x~~ = .  
eg 1  $x = 2t - 3t^2$ , find velocity at  $t = 2s$

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt} (2t - 3t^2)$$

$$v = 2 - 6t$$

$$v \Big|_{t=2} = 2 - 6(2)$$

$$\begin{aligned} &= 2 - 12 \\ &= -10 \text{ m/s} \end{aligned}$$

eg 2.  $v = (3 + 2t) \text{ m/s}$

$\rightarrow v \Big|_{t=0} = 0, x = 0$   
find displacement at  $t = 2 \text{ sec}$

$$x = \int 3 + 2t \, dt$$

$$x = 3t + 2t^2 + C$$

$$x = 3t + t^2 + C$$

~~x~~

$$0 = 3(0) + (0)^2 + C$$

$$C = 0$$

$$x = 3(2) + (2)^2$$

$$x = 6 + 4$$

$$x = 10 \text{ m}$$

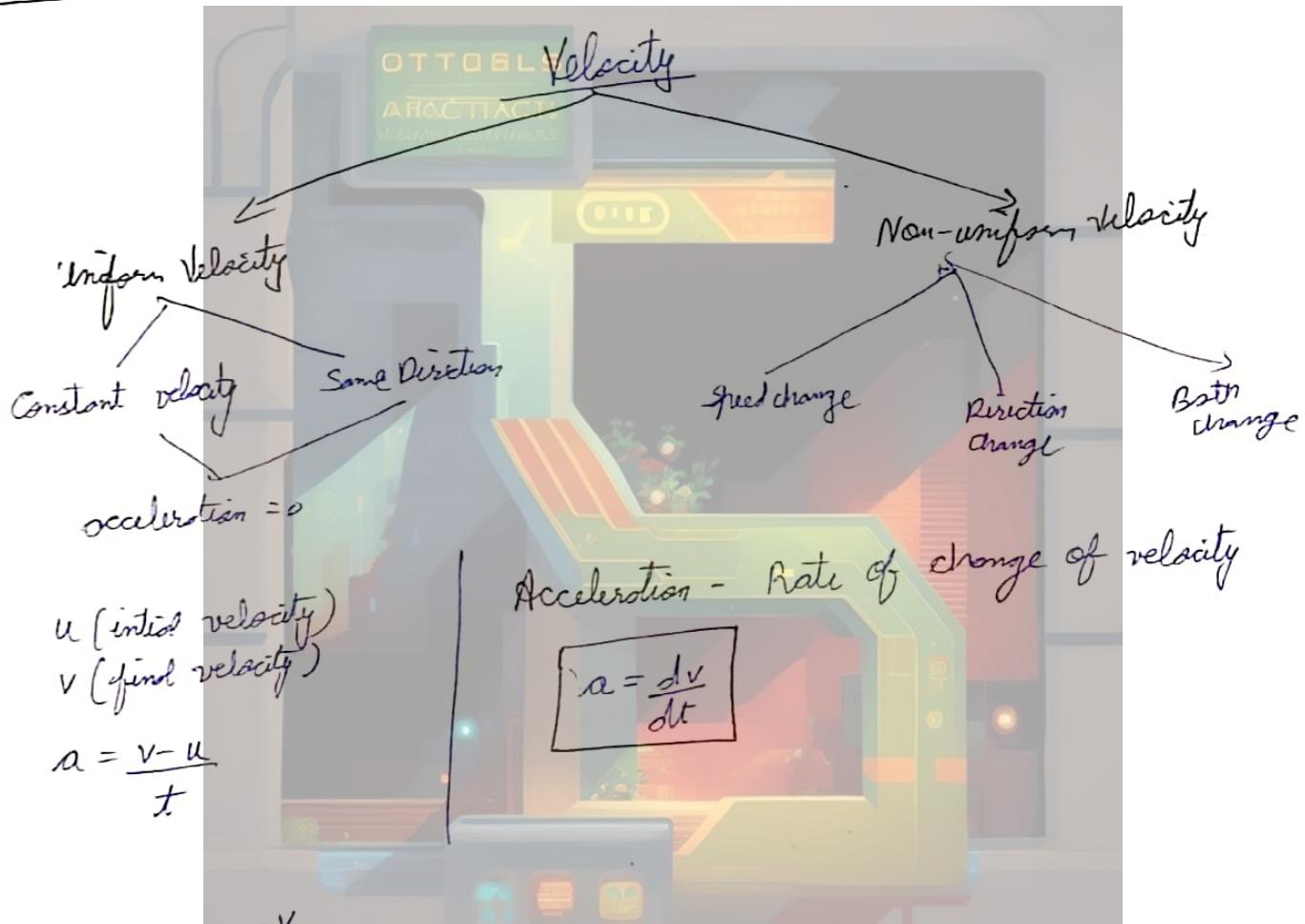
Note:- ① Slope of Displacement-time graph gives velocity.

$$v = \frac{dx}{dt}$$

② Area under v-t graph with proper sign gives displacement & without sign gives distance

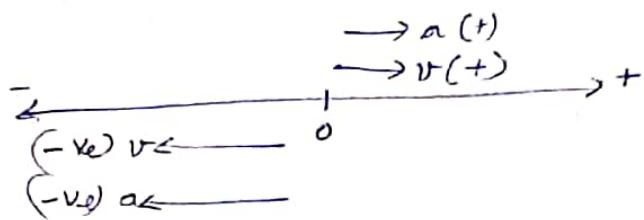
$$x = \int v dt$$

## \*Acceleration:-



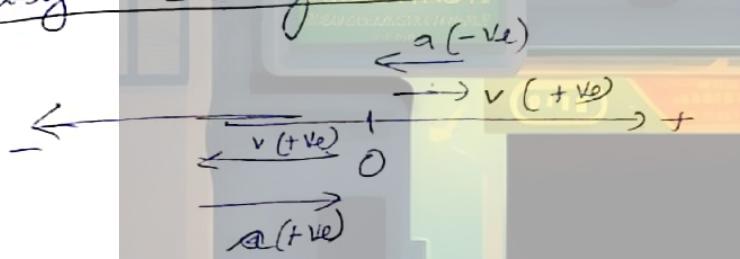
\* Slope of v-t graph gives acceleration.

→ Body is speeding up



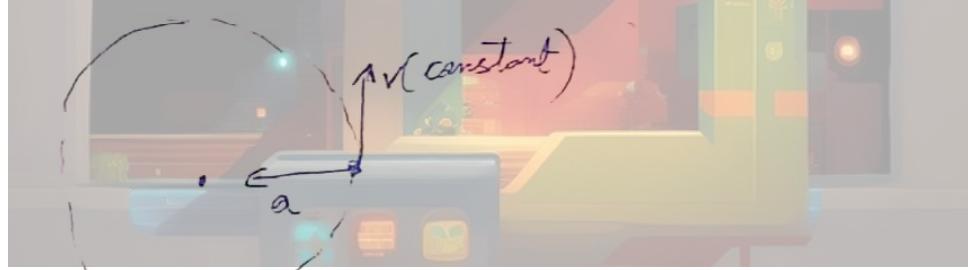
When velocity & acceleration are of same sign. (either both be +ve or -ve), speed of body will increase.

→ Body is slowing down (retardation / deceleration)



When  $\vec{v}$  &  $\vec{a}$  are in opp sign. off, speed of body decreases.

→ Velocity  $\perp$  Acceleration



speed = constant  
direction = change  
resulting path = circle

uniform circular motion

$$a = \frac{dv}{dt}$$

$$\therefore v = \frac{dx}{dt}$$

$$a = \frac{d}{dt} \left( \frac{dx}{dt} \right)$$

$$a = \frac{d^2x}{dt^2}$$

$$a = \frac{d^2x}{dt^2}$$

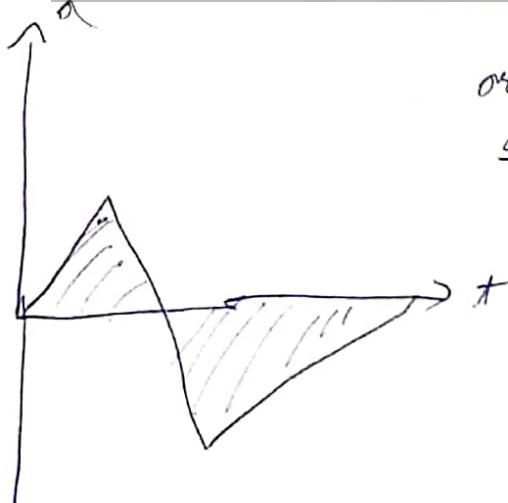
$$a = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$a = v \frac{dv}{dx}$$

$$a = v \frac{dv}{dx}$$

$$a = \frac{dv}{dt}$$

$$\Delta v = \int_u^v dv = \int a dt$$



area under  $a-t$  graph = velocity

$\therefore$

S-t Graph  $\rightarrow$  Slope  $\rightarrow v$ ,  $v = \frac{dx}{dt}$

V-t Graph  $\rightarrow$  Slope  $\rightarrow a$ ,  $a = \frac{dv}{dt}$ ,  $a = v \frac{dv}{dx}$ ,  $a = \frac{d^2x}{dt^2}$

$\rightarrow$  area  $\rightarrow$  Displacement  $\rightarrow$  Distance  $x = \int v dt$

a-t Graph  $\rightarrow$  area  $\rightarrow \Delta v$   $\Delta v = \int a dt$

OTTOEBS  
ARCHITECTS

$\frac{d|\vec{v}|}{dt}$  :- rate of change of speed

$\frac{d\vec{v}}{dt}$  :- rate of change of velocity = acceleration,  $\vec{a}$  (gives vector)

$\left| \frac{d\vec{v}}{dt} \right|$  :- magnitude of acceleration

Q Can  $\frac{d|\vec{v}|}{dt} = 0$  while  $\left| \frac{d\vec{v}}{dt} \right| \neq 0$ ? Yes

Speed = constant

Its direction may change, acc might be non-zero

Q Can  $\frac{d|\vec{v}|}{dt} \neq 0$  while  $\left| \frac{d\vec{v}}{dt} \right| = 0$ ? No

Q1. A train travels a distance of 20 km with a uniform speed of 60 km/hr. It travels another distance of 40 km with a uniform speed of 80 km/hr. Calculate average speed of train.

$$\text{Average speed} = \frac{V_1 + V_2}{2}$$

$$= \frac{60 + 80}{2}$$

$$\text{Time } T_1 = \frac{20}{60} = \frac{1}{3} \text{ hr}$$

$$T_2 = \frac{40}{80} = \frac{1}{2} \text{ hr}$$

$$\frac{60 + 80 + 20}{\frac{1}{2} + \frac{1}{3}} = \frac{160}{\frac{5}{6}} = \frac{960}{5} = \frac{360}{5} = 72 \text{ km/hr}$$

$$\text{Average Velocity} = 0$$

$$\frac{2V_1 V_2}{V_1 + V_2} = \frac{2 \times 20 \times 30}{20 + 50} = \frac{1200}{70} = 17$$

$$\text{dis} = x$$

$$t_1 = \frac{x}{20}$$

$$T_2 = \frac{x}{30}$$

$$\text{Average speed} = \frac{2x}{\frac{x}{20} + \frac{x}{30}}$$

$$= \frac{2x}{x(\frac{1}{20} + \frac{1}{30})}$$

$$\text{A. Speed} = \frac{2x + 2x}{\frac{x}{20} + \frac{x}{30}}$$

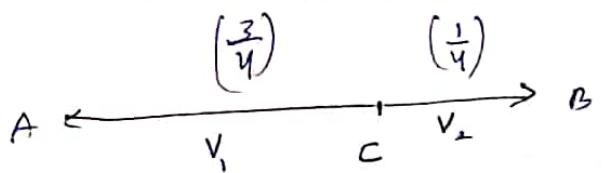
$$= \frac{2x}{\frac{30x}{60}}$$

$$= \frac{120x}{5x}$$

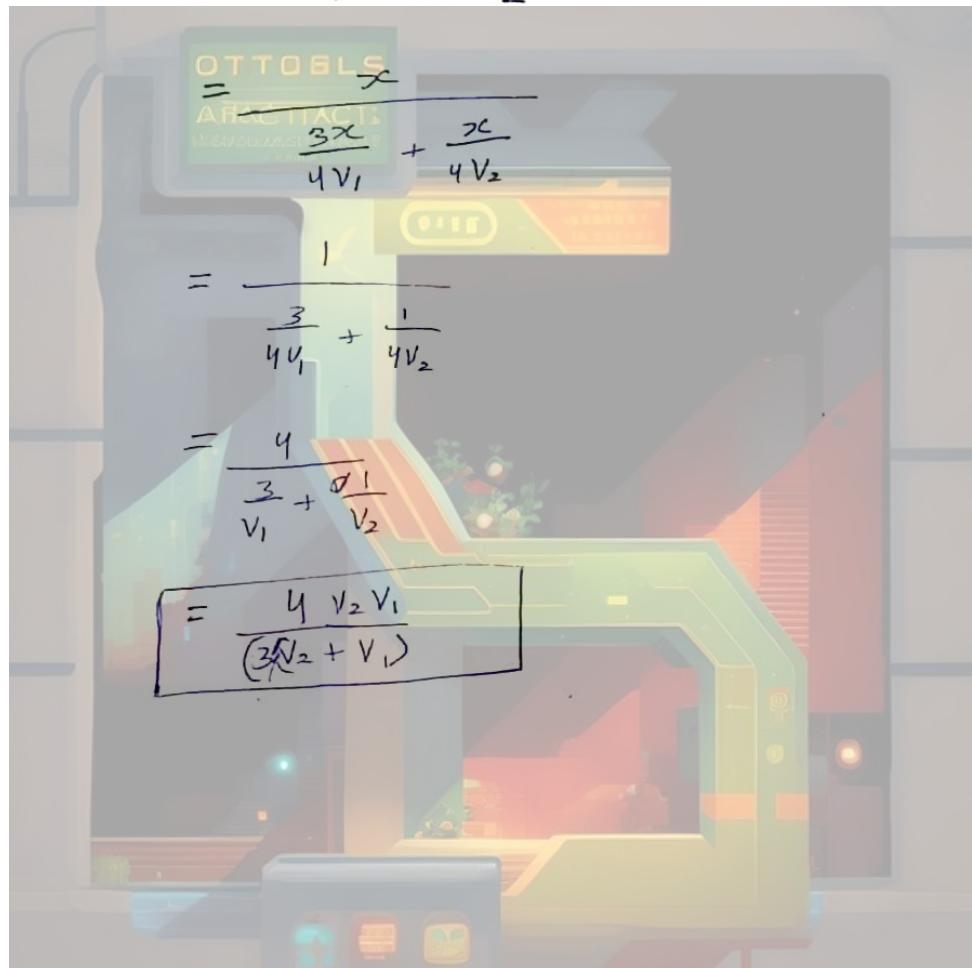
$$= 60 \text{ km/hr}$$

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Q3.



$$\text{Average velocity} = \frac{x}{\frac{\frac{3x}{4}}{v_1} + \frac{\frac{x}{4}}{v_2}}$$



Q4.  $x = (5t^2 + 4t + 3) \text{ m}$   
find velocity and acceleration at  $t = 2\text{s}$

$$v = \frac{dx}{dt} = \frac{d}{dt}(5t^2 + 4t + 3)$$
$$= 10t + 4$$
$$= 20 + 4$$
$$\boxed{= 24 \text{ m/s}}$$

$$a = \frac{d}{dt}(10t + 4)$$

$$\boxed{= 10 \text{ m/s}^2} \rightarrow \text{uniform acceleration}$$

Q5.  $x = At^3 + Bt^2 + Ct + D$

$$A = 1$$

$$B = 4 \quad (\text{all SI units are used})$$

$$C = -2$$

$$D = 5$$

a) find dimensions of A B C & D

b) Find velocity at  $t = 4\text{s}$

c) find Average velocity from  $t = 0$  to  $t = 4\text{s}$

d) find Acceleration of particle at  $t = 1\text{s}$

e) find acc avg from  $t = 0$  to  $t = 4\text{s}$

All  $At^3$ ,  $Bt^2$ ,  $Ct$  &  $D$  are with dimensions  $\boxed{[L]}$

$$[D] = [L]$$

$$[C] = \boxed{[T]} \quad [LT^{-1}]$$

$$[B] = \boxed{[T^{-2}]}$$

$$[A] = \boxed{[LT^{-3}]}$$

$$b) v = \frac{d}{dt} (t^3 + 4t^2 - 2t + 5)$$

$$\begin{aligned} &= (2t^2 + 4t - 2) \\ &= 2 \times 16 + 4 \times 4 - 2 \\ &= 32 + 16 - 2 \\ &= 48 - 2 \\ &\boxed{= 46 \text{ m/s}} \end{aligned}$$
$$\begin{aligned} &= 3t^2 + 8t - 2 \\ &= 3 \times 16 + 8 \times 4 - 2 \\ &= 48 + 32 - 2 \\ &\boxed{= 78 \text{ m/s}} \end{aligned}$$

$$c) \quad t = 0 \quad t = 4 \text{ s}$$

$$x = 5 \quad 64 + 64 - 8 + 5 \\ x = 125$$

$$\frac{125 - 5}{4} = \frac{120}{4} = \boxed{30 \text{ m/s}} \checkmark$$

$$d) \quad \frac{d^2x}{dt^2} = \frac{d}{dt} (3t^2 + 8t - 2)$$

$$\begin{aligned} &= 6t + 8 \\ &= 6(4) + 8 \\ &= 24 + 8 \\ &\boxed{= 32 \text{ m/s}^2} \end{aligned}$$

$$e) \quad \frac{78 - 2}{4} = \frac{80}{4} = \boxed{20 \text{ m/s}^2} \checkmark$$

$$Q6. x = t^3 - 6t^2 + 3t + 4 \text{ in meters}$$

find velocity of particle at instant when  $a=0$

$$\frac{dv}{dt} = 0$$

$\Rightarrow v = \text{constant}$

$$\frac{dx}{dt} = 3t^2 - 6t + 3$$

$$\frac{d^2x}{dt^2} = 6t + 6$$

$$\begin{aligned} 6t + 6 &= 0 \\ 6t &= -6 \\ t &= -1 \end{aligned}$$

$$\begin{aligned} v &= 3(1)^3 - 6(1) + 3 \\ &= 3 - 6 + 3 \end{aligned}$$

$$\frac{d^2x}{dt^2} = 6t - 12$$

$$6t - 12 = 0$$

$$\underline{\underline{t = 2}}$$

$$\begin{aligned} v &= 3(2)^2 - 12(2) + 3 \\ &= 3 \times 4 - 12(2) + 3 \\ &= 12 - 24 + 3 \\ &= -12 + 3 \\ &= -9 \text{ m/s} \quad \checkmark \end{aligned}$$

$$Q7. a = 3t^2 + 2t + 2$$

$$t=0, v=2 \text{ m/s}$$

find  $v$  at  $t=2$

$$\Delta v = \int 3t^2 + 2t + 2 \, dt$$

$$2 = \frac{3t^3}{3} + \frac{2t^2}{2} + 2t + C$$

$$6 \times 2 = 6t^3 + 6t^2 + 12t$$

$$2 = t^3 + t^2 + 2t + C$$

$$2 = 0 + 0 + 0 + C$$

$$\underline{\underline{C=2}}$$

$$v = 8 + 4 + 4 + 2$$

$$\boxed{v = 18 \text{ m/s}}$$

$$\int dv = \int 3t^2 + 2t + 2$$

$$\begin{aligned} &\text{at } t=0 \quad v=2 \\ &\text{at } t=2 \quad v=? \end{aligned}$$

$$[v]_2^\infty = \left[ t^3 + t^2 + 2t \right]_0^2$$

$$v|_{t=2} = 8 + 4 + 4$$

$$\boxed{v = 18 \text{ m/s}}$$

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$$Q7. \quad a = 6t + 6 \text{ m/s}^2$$

$$t=0, x=0, v=2 \text{ m/s}$$

velocity & displacement as a function of time

$$\frac{dv}{dt} = a$$

$$\frac{dv}{dt} = a dt$$

$$\int dv = \int a dt$$

$$\Rightarrow v = \int (6t + 6) dt$$

$$v = \frac{6t^2}{2} + 6t$$

$$\boxed{v = 3t^2 + 6t + 2}$$

$$\frac{dx}{dt} = v$$

$$\Rightarrow x = \int v dt$$

$$x = \int 3t^2 + 6t + 2 dt$$

$$x = \int \frac{3t^3}{3} + \frac{6t^2}{2} + 2t + C$$

$$x = t^3 + 3t^2 + 2t + C$$

$$\boxed{x = t^3 + 3t^2 + 2t} \checkmark$$

Q2

$$\frac{dv}{dt} = 6t + 6$$

$$\Rightarrow v +$$

$$\int dv = \int (6t + 6) dt$$

~~2~~ E

$$\boxed{\left[ v \right]_2^t = \left[ 3t^2 + 6t \right]_0^t}$$

$$v - 2 = 3t^2 + 6t$$

$$\boxed{v = 3t^2 + 6t + 2} \checkmark$$

$$\frac{dx}{dt} = 3t^2 + 6t + 2$$

$$\boxed{\left[ x \right]_0^t = \int t^3 + 3t^2 + 2t dt} \checkmark$$

$$\boxed{x = t^3 + 3t^2 + 2t} \checkmark$$

Q8.  $x = \left( -\frac{2}{3}t^2 + 16t + 2 \right) m$   
at  $t=0, \underline{\underline{x}} = 0$

a)  $v$  (initial velocity)

$$\frac{dx}{dt}$$

$$\cancel{dv = \int -\frac{2}{3}t^2 + 16t + 2 dt}$$

$$\frac{dx}{dt} = -\frac{4}{3}t + 16$$

$$= -\frac{4}{3}(0) + 16$$

$$= 16 \text{ m/s} \quad \checkmark$$

b) how long to came to rest

$$-\frac{4}{3}t + 16 = 0$$

$$t = \frac{-16 \times 3}{-4}$$

$$t = 4 \times 3$$

$$t = 12 \text{ s} \quad \checkmark$$

c)  $-acc, t = 12$

$$a = -\frac{4}{3} \text{ m/s}^2 \quad \checkmark$$

- Q9:- The position of particle is given by  $x = 10 + 20t - 5t^2$  in meter  
 find a) displacement in 1s  
 b) distance in 3s  
 c) initial acceleration  
 d) velocity at 4s

$$\frac{dx}{dt} (10 + 20t - 5t^2)$$

$$= -10t + 20$$

$$x \Big|_{t=1} = 20 - 10(1)$$

$$= 10 \text{ m}$$

$$x = \int v dt$$

$$x \text{ Distance} = 0$$

a)  $\Delta x = x_f - x_i$

$$= (10 + 20 - 5) - 10$$

$$= 15 \text{ m} \checkmark$$

b)  $v = \frac{dx}{dt} = 0 + 20 - 10t$

$$v = 20 - 10t$$

for  $v = 0$

$$20 - 10t = 0$$

$$t = 2 \text{ s}$$

$$t = 0$$

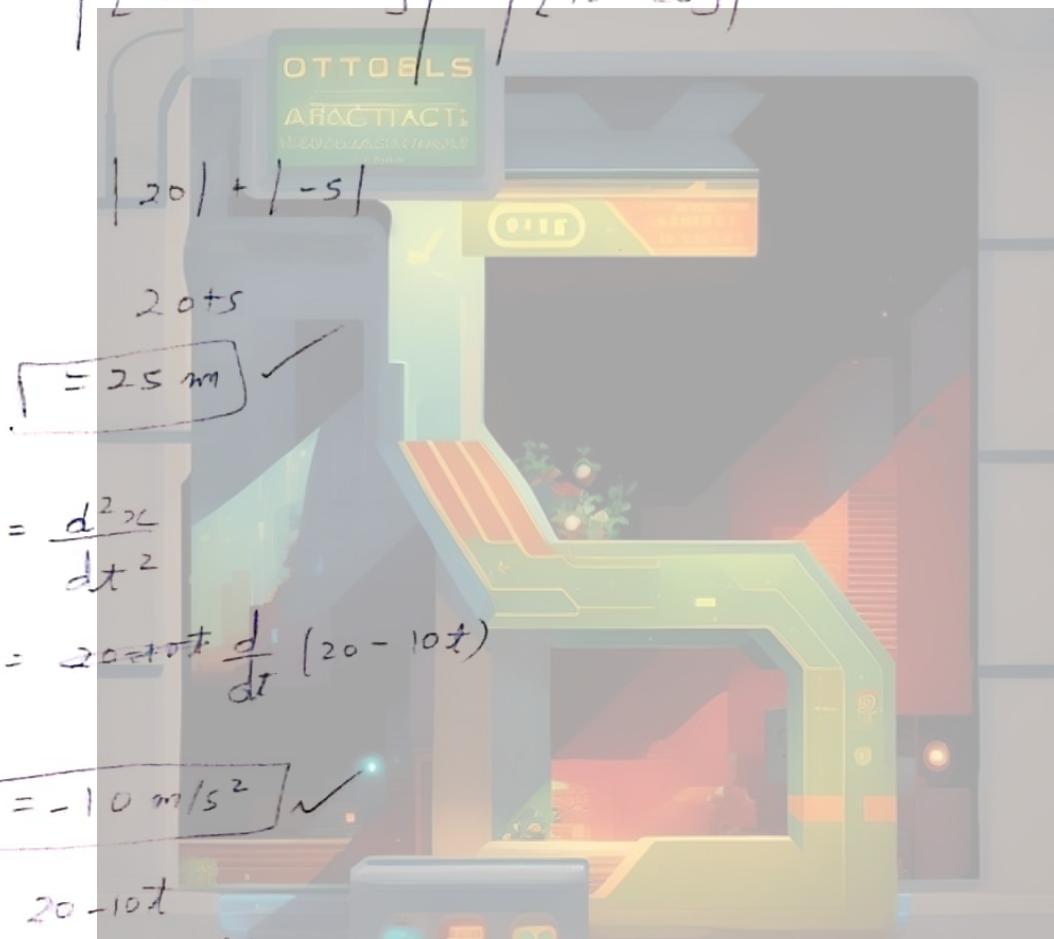
$$t = 2$$

$$\Delta x \Big|_{t=3}$$

$$\text{Distance} = \left| \int_0^2 (20 - 10t) dt \right| + \left| \int_2^3 (20 - 10t) dt \right|$$

$$= \left| \left[ 20t - 5t^2 \right]_0^2 \right| + \left| \left[ 20t - 5t^2 \right]_2^3 \right|$$

$$= |[20 \cancel{t} \rightarrow 20 - 0]| + |[15 - 20]|$$



c)  $a = \frac{d^2 x}{dt^2}$

$$a = 20 - 10t \cdot \frac{d}{dt} (20 - 10t)$$

$$a = -10 \text{ m/s}^2 \checkmark$$

d)  $v = 20 - 10t$   
 $= 20 - 10(1)$

$$= 20 - 10$$

$$= -10 \text{ m/s} \checkmark$$

Q10. The acc of body is given by  $(4-2t)m/s^2$ . at  $t=0$   
find distance at  $12s$ .  $v=5m/s$

$$v = \int a dt$$

$$v = \int 4-2t dt$$

$$v = 4t - t^2 + c$$

$$5 = 0 - 0 + c$$

$$c = 5$$

$$v = 4t - t^2 + 5$$

distance =

~~at~~

$$0 = 4t - t^2 + 5$$

$$t^2 - 4t + 5 = 0$$

~~$t^2 - 5t + t - 5$~~

$$t(t-5) + 1(t-5)$$

$$t = 5$$

$$t = -1 \times$$

$$v=0, t=5$$

$$\text{Displacement} = \int v dt$$

$$= \left[ 2t^2 - \frac{t^3}{3} + 5t \right]_0^{12}$$

$$= 288 - \frac{1728}{3} + 60$$

$$= -228 m \checkmark$$

(36)

$$\text{dis} = \left| \int_0^5 4t - t^2 dt + s \right| + \left| \int_5^{12} 4t - t^2 + s dt \right|$$

$$\text{dis} = \left| \left[ 2t^2 - \frac{t^3}{3} + 5t \right]_0^5 \right| + \left| \left[ 2t^2 - \frac{t^3}{3} + st^2 \right]_5^{12} \right|$$

$$= \left| \left[ 50 - \frac{125}{3} + 25 - 0 \right] \right| + \left| \left[ 288 - \frac{1728}{3} + 60 \right. \right. \\ \left. \left. - 50 + \frac{125}{3} - 25 \right] \right|$$

$$= \left| 75 - \frac{125}{3} \right| + \left| -228 - 75 + \frac{125}{3} \right|$$

$$= \left| 75 - \frac{125}{3} \right| + \left| -303 + \frac{125}{3} \right|$$

$$= \frac{225 - 125}{3} + \frac{178 - 784}{3}$$

$$= \frac{100 + 178}{3}$$

$$= \frac{884}{3} m \checkmark$$

$$\begin{array}{r} 744 \\ 12 \\ \hline 288 \\ 144 \\ \hline 1728 \end{array}$$

$$Q11. \quad a = -\cos t$$

$$t=0; v=0, x=1$$

a) position at  $t = \pi$

b) distance from  $t = 0$  to  $t = 2\pi$

$$a) v = \int -\cos t \, dt$$

$$v = -\sin t + c$$

$$0 = c$$

$$v = -\underline{\sin t}$$

$$x = \int v \, dt$$

$$\Delta x = \int -\sin t \, dt$$

$$x = \cos t + c$$

$$0 + = 90 + c \quad l = l + c$$

$$c = -90$$

$$\Delta x = \cos t \cos t - \underline{-90}$$

$$\Delta x = \cos(180^\circ) - 90$$

$$x = -90 \text{ m}$$

$$b) v = 0; \text{ for } t = 90, 180, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, 2\pi$$

$$x = \int v \, dt$$

$$x = \cos t + c$$

$$10 = \cos 0 + c$$

$$1 = l + c$$

$$c = 0$$

$$\Delta x = \cos t$$

$$x = \cos t \quad (180^\circ)$$

$$x = -1 \text{ m}$$

$$6) \quad \cos t \quad v=0$$

$$-\sin t = 0 \quad \rightarrow \quad t=0$$

$$\sin t = 0 \quad \rightarrow \quad t=\pi$$

$$D = \left| \int v dt \right|$$

$$D = \left| \int_0^\pi -\sin t dt \right| + \left| \int_\pi^{2\pi} -\sin t dt \right|$$

$$D = \left| [\cos t]_0^\pi \right| + \left| [\cos t]_\pi^{2\pi} \right|$$

$$D = | \cos \pi - \cos 0 | + | \cos 2\pi - \cos \pi |$$

$$D = |-1 - 1| + |1 - (-1)|$$

$$= | -2 | + | 2 |$$

$$= 4 \text{ m}$$

$$Q12. \quad v = 6t - 3t^2 \text{ m/s}$$

find average speed and average velocity for first 4 seconds.

Q. 20

$$v=0$$

$$0 = 6t - 3t^2$$

$$3t^2 - 6t = 0$$

~~$$3t(t-2) = 0$$~~

$$t = 2$$

$$t = 0$$

$$x_4 = \left| \int_0^2 (6t - 3t^2) dt \right| + \left| \int_2^4 (6t - 3t^2) dt \right|$$

$$= \left| [3t^2 - t^3]_0^2 \right| + \left| [3t^2 - t^3]_2^4 \right|$$

$$= |4| + |48 - 16 - 4|$$

$$= 4 + 20$$

$$= 24 \text{ m}$$

$$\frac{24}{4} = \boxed{6 \text{ m/s}} \quad \checkmark \quad \text{Average speed}$$

$$x = \int v dt$$

$$x = \int_{t=0}^4 3t^2 - t^3$$

$$\boxed{x = -16 \text{ m}}$$

$$\frac{-16}{4} = \boxed{-4 \text{ m/s}} \quad \underline{\text{Average Velocity}}$$

Q 13.  $v = (t-2) \text{ m/s}$

calculate

distance & displacement from  $t=0$  to  $t=4$

$$x = \int v dt = \int (t-2) dt$$

$$x = \frac{t^2}{2} - 2t$$

$$x \Big|_{t=4} = \frac{16}{2} - 8$$

$$= 0 \text{ m} \quad \checkmark$$

$$\Delta x \text{ distance} = \left| \int_0^2 (t-2) dt \right| + \left| \int_2^4 (t-2) dt \right|$$

$$= \left| \left[ \frac{t^2}{2} - 2t \right]_0^2 \right| + \left| \left[ \frac{t^2}{2} - 2t \right]_2^4 \right|$$

$$= \left| -2 \right| + \left| 2 \right|$$

$$= 2 + 2$$

$$\boxed{= 4 \text{ m}} \quad \checkmark$$

Q14. A particle starts moving along straight line such that

$$v = t^2 - t \text{ m/s}$$

find the interval for which particle retards.

$$a = \int t^2 - t \, dt$$

$$a = \frac{t^3}{3} - \frac{t^2}{2} \text{ m/s}^2$$

$$av < 0$$

$$\frac{t^3}{3} - \frac{t^2}{2} (t^2 - t) < 0$$

$$\frac{2t^3 - 3t^2 (2t^2 - t)}{6} < 0$$

$$24t^5 - 2t^4 - 6t^4 + 3t^3 < 0$$

$$t^3 (4t^2 - 2t - 6t + 3) < 0$$

$$4t^2 - 2t - 6t + 3 < 0$$

$$t(4t-2)$$

$$4t^2 - 8t + 3 < 0$$

$$4t^2 - 6t - 2t + 3 < 0$$

$$2t(2t-3) - 1(2t-3) < 0$$

$$(2t-1)(2t-3) < 0$$

$$t < \frac{1}{2} \quad t < \frac{3}{2} \quad \times$$



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Q14 A particle starts moving along straight line such that

$$v = t^2 - t \text{ m/s}$$

find interval when particle retards

$$v = t^2 - t$$

$$a = \frac{d}{dt}(t^2 - t) = 2t - 1$$

$$av < 0$$

$$(2t-1)(t^2-t) < 0$$

$$t^2(t-1)(2t-1) < 0$$

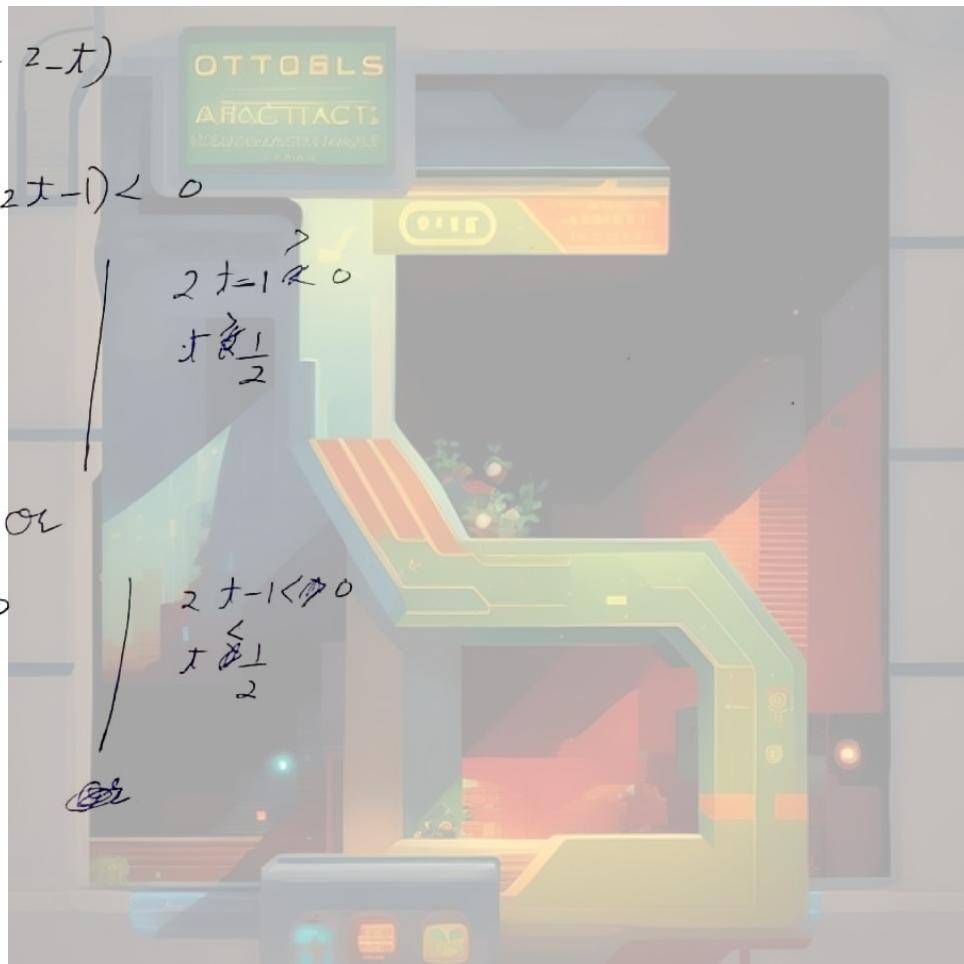
$$t-1 < 0$$

$$t < 1$$

or

$$2t-1 > 0$$

$$t > \frac{1}{2}$$



Q15. Velocity of a particle moving in the +ve x-axis varies as  $v = a\sqrt{x}$  where a is a constant & x is displacement at  $t=0$ ,  $x=0$  find time dependence of velocity.

$$x = \int v dt$$

$$\frac{dx}{dt} = v$$

$$v = \frac{dx}{dt} = a\sqrt{x}$$

$$\int dx = \int a\sqrt{x} dt$$

$$\int \frac{dx}{\sqrt{x}} = \int dt$$

$$\int \frac{1}{\sqrt{x}} dx = t$$

$$\frac{1}{a} \frac{x^{1/2}}{\frac{1}{2}}$$

$$\frac{2\sqrt{x}}{a} = t$$

$$\frac{a}{\sqrt{x}} = \frac{at}{2}$$

$$x = \frac{a^2 t^2}{4}$$

$$\sqrt{x} + c = t$$

$$\boxed{\sqrt{x} + c = t}$$

$$\boxed{x = \frac{a^2 t^2}{4}}$$

$$\frac{dx}{dt} = v$$

$$\frac{d}{dt} \left( \frac{a^2 t^2}{4} \right) = v$$

$$\boxed{\frac{a^2 t}{2} = v}$$

$$\begin{aligned} \frac{dv}{dt} &= \\ &= \frac{d}{dt} \left( \frac{a^2 t}{2} \right) \\ &= \boxed{\frac{a^2}{2}} \end{aligned}$$

$$Q16. \vec{v} = 4t\hat{i} + 3\hat{j}$$

$$\text{at } t=0$$

$$\vec{x} = 2\hat{j}$$

a) find position vector at  $t=2s$

b) find average acc for  $t=0s$  to  $t=2s$ .

$$\vec{v} = 4t\hat{i} + 3\hat{j}$$

~~$$\|\vec{v}\| = \sqrt{16t^2 + 9}$$~~

~~$$\vec{x} = \int \sqrt{16t^2 + 9} dt$$~~

~~$$\vec{x} = \int 4t\hat{i} + 3\hat{j} dt$$~~

~~$$\vec{x} = 4t^2\hat{i} + 3t\hat{j}$$~~

~~$$\vec{x}|_{t=2} = 8\hat{i} + 6\hat{j}$$~~

~~$$\int \vec{x} =$$~~

$$\vec{r} = 4t\hat{i} + 3\hat{j}$$

$$\int_0^x \vec{x} dt = \int_0^t 4\hat{i} + 3\hat{j} dt$$

$$\vec{x} - \vec{x}|_{t=0} = \left[ 2t^2 + 3t \right]_0^t$$

$$\vec{x} - \vec{x}|_{t=0} = 2t^2\hat{i} + 3t\hat{j}$$

$$\vec{x}|_{t=0} = 2(2)^2\hat{i} + 3(2)\hat{j} = 8\hat{i} + 6\hat{j}$$

$$\boxed{\vec{r} = 8\hat{i} + 6\hat{j}}$$

$$\text{avg } \alpha = \frac{1}{2} \int_0^2 4t + 1 + 3\vec{f} dt$$

$$\text{avg } \alpha = \frac{\vec{v} - \vec{u}}{T}$$

$$= \frac{8t + 3\vec{f} - 3\vec{f}}{2}$$

$$\int = 4t \text{ m/s}^2$$

$$(Q17) \quad \vec{a} = 2\vec{i} - \vec{j}$$

$$t=0, u=0, \vec{x} = 3\vec{i} + \vec{j}$$

find  $\vec{x}$  at time t

$$dv = \int_0^t 2\vec{i} - \vec{j} dt$$

$$\int_{3\vec{i} + \vec{j}}^x d\vec{x} = \int_0^t 2\vec{i} - \vec{j}$$

$$x - 3\vec{i} - \vec{j} = t^2\vec{i} - \frac{t^2}{2}\vec{j}$$

$$x = t^2\vec{i} + 3\vec{i} - \frac{t^2}{2}\vec{j} + \vec{j}$$

$$x = (t^2 + 3)\vec{i} - \left(\frac{t^2 - 2}{2}\right)\vec{j}$$

Q18. A particle with velocity  $v$  at  $t=0$  is decelerated at rate  $a = -\alpha \sqrt{v}$   
 ( $\alpha$  is a +ve constant).

- find time after which it comes to rest,
- distance.

$$\cancel{\frac{dv}{dt} = -\alpha \sqrt{v}}$$

$$\cancel{\frac{dv}{dt} = -\alpha \sqrt{v}}$$

$$dv = -\alpha \sqrt{v} dt$$

$$\int dv = -\alpha \sqrt{v} dt$$

$$v =$$

$$\frac{da}{dv} = \alpha \sqrt{v}$$

$$da = \alpha \sqrt{v} dv$$

$$a = \frac{2\alpha \sqrt{v}}{3}$$

$$-\alpha \sqrt{v} = \frac{2\alpha \sqrt{v}}{3}$$

$$v = \frac{4}{9} v^3$$

$$\frac{9}{4} v^2$$

$$v = \frac{3}{2}$$

$$a = -\alpha \sqrt{v}$$

$$\frac{dv}{dt} = -\alpha \sqrt{v}$$

$$\frac{dv}{\sqrt{v}} = -\alpha dt$$

$$\left[ 2\sqrt{v} \right]_{v_0}^v = -\alpha t$$

$$2\sqrt{v} - 2\sqrt{v_0} = -\alpha t$$

$$\boxed{2\sqrt{v} = 2\sqrt{v_0} - \alpha t}$$

$$\textcircled{a} \quad v = 0$$

$$v_0 = 2\sqrt{v_0} - \alpha t$$

$$\cancel{\alpha t}$$

$$\alpha t = 2\sqrt{v_0}$$

$$\boxed{t = \frac{2\sqrt{v_0}}{\alpha}}$$

$$\textcircled{b} \quad \sqrt{v} = \sqrt{v_0} - \frac{\alpha t}{2}$$

$$v = \left( \sqrt{v_0} - \frac{\alpha t}{2} \right)^2$$

$$\frac{dv}{dt} = \left( \sqrt{v_0} - \frac{\alpha t}{2} \right)^2$$

$$\int dx = \int \left( \sqrt{v_0} - \frac{\alpha t}{2} \right)^2 dt$$

$$x = \left[ \frac{\left( \sqrt{v_0} - \frac{\alpha t}{2} \right)^3}{3 \left( -\frac{\alpha}{2} \right)} \right]_0$$

$$x = \frac{-2}{3\alpha} \left[ \sqrt{v_0} - \frac{\alpha + 2\sqrt{v_0}}{2\alpha} \right]^3 - (\sqrt{v_0})^3$$

$$x = \frac{-2}{3\alpha} \left[ \sqrt{v_0} - \sqrt{v_0} \right]^3 - (\sqrt{v_0})^3$$

$$x = \frac{-2}{3\alpha} (\sqrt{v_0})^3$$



## Equations of Motion :-

① Velocity Time relation ( $v = u + at$ )

$$a = \frac{dv}{dt}$$

$$dv = a dt$$

$$\int_u^v dv = \int_0^t a dt$$

$$[v]_u^v = [at]_0^t$$

$$v - u = at$$

$$\boxed{v = u + at}$$

$$v = u + at$$

② Displacement time relation ( $s = ut + \frac{1}{2} at^2$ )

$$v = \frac{dx}{dt}$$

$$dx = v dt$$

$$v = u + at$$

$$dx = v t + at^2 dt$$

$$\int dx = \int_0^t (u + at) dt$$

$$s = \left[ ut + \frac{at^2}{2} \right]_0^t$$

$$\boxed{s = ut + \frac{1}{2} at^2}$$

③ Velocity - Displacement relation ( $v^2 = u^2 + 2as$ )

$$a = v \frac{dv}{dx}$$

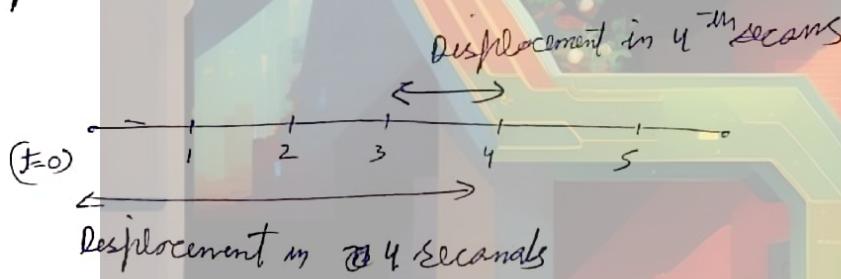
$$\int_u^v v dv = \int_0^s a dx$$

$$\left[ \frac{v^2}{2} \right]_u^v = a [x]_0^s$$

$$\frac{v^2}{2} - \frac{u^2}{2} = as$$

$$v^2 - u^2 = 2as$$

④ Displacement in  $n^{\text{th}}$  second :-



$$S_n^{\text{th}} = S_n - S_{(n-1)}$$

$$= un - \frac{1}{2} \alpha n^2 - \left[ u(n-1) - \frac{1}{2} \alpha (n-1)^2 \right]$$

$$= un - \frac{\alpha n^2}{2} - \left[ un - u - \frac{\alpha n^2}{2} - \frac{\alpha}{2} + \frac{\alpha n}{2} \right]$$

$$= un - un - \frac{\alpha n^2}{2} + \frac{\alpha n^2}{2} + u + \frac{\alpha}{2} - \alpha n$$

$$= u + \alpha n - \frac{\alpha}{2}$$

$$= u + \frac{\alpha}{2} (2n-1)$$

$$S_n^{\text{th}} = u + \frac{\alpha}{2} (2n-1)$$

Q A Train is travelling at speed 90 km/hr brakes are applied so as to produce a uniform retardation of  $0.5 \text{ m/s}^2$ . find how far the train goes before coming to rest.

$$u = 90 \times \frac{5}{18} = 5 \times 5 = 25 \text{ m/s}$$

$$\alpha = 0.5$$

$$\underline{v^2 - u^2 = 2as}$$

$$\frac{+ 625}{2 \times 0.5} = 5$$

$$S = 625 \text{ m}$$

Q2. A particle moving with constant acceleration from A to B in a straight line AB has velocities  $u$  &  $v$  at A & B. If C is the mid of C, find velocity while passing through

$$S = \frac{AB}{2}$$

$$u = u$$

$$V_c = ?$$

$$\alpha = a$$

$$V_c^2 = u^2 + 2as$$

$$V_c^2 = u^2 + 2as$$

$$V_c = \sqrt{u^2 + 2as}$$

$$\begin{aligned} s &= \frac{v-u}{a} \\ V_c^2 &= 2axAB \div v^2 \\ V_c^2 &= v^2 - aAB \\ \sqrt{v^2 - aAB} &= V_c \\ 2V_c^2 &= v^2 + u^2 \\ V_c &= \sqrt{\frac{v^2 + u^2}{2}} \end{aligned}$$

Q3. A particle starts from rest with uniform velocity acceleration  $a$ . Its velocity after  $n$  seconds is  $v$ . Find the displacement of body in last two seconds

$$u = 0$$

$$v = v$$

$$t = n$$

$$S_{(n-2)}^{\text{th}} = u + \frac{a}{2} (2n-1)$$

$$= u + \frac{a}{2} (2(n-2)-1)$$

$$= u + \frac{a}{2} (2n-4-1)$$

$$= u + \frac{a}{2} (2n-5)$$

$$= u + an - \frac{5a}{2}$$

$$S_n + S_{(n-1)} = u + \frac{a}{2} (2n-1) + u + \frac{a}{2} (2n-3)$$

$$= 2u + an - \frac{a}{2} + an - \frac{3a}{2}$$

$$= 2u + 2an - 2a$$

$$= 2(u + a + an)$$

$$= 2[u + a(1+n)]$$

~~or~~

$$u = 0 \quad \text{or} \quad = 2[a(n-1)]$$

$$v = an \dots (1) \dots$$

$$\text{Disp in last 2 sec} = S_n - S_{(n-2)}$$

$$= \frac{1}{2} an^2 - \frac{1}{2} a(n-2)^2$$

$$= \frac{an^2 - a(n^2 - 4n + 4)}{2}$$

$$= \frac{a}{2} (4 - 2n) = \frac{2a - an}{2} = a(n-1)$$

Q4. A body starting from rest moves with constant acceleration. Find the ratio of distance covered during fifth to that covered in five seconds.

$$= \frac{\frac{a}{2} (2n-1)}{\frac{1}{2} a n^2}$$

$$= \frac{a(10-1)}{2 \times 5^2}$$

$$= \frac{9}{25}$$

$$\boxed{9:25}$$

Q5. A particle moves with constant acceleration for 6 sec. after starting from rest. Find the ratio of distance travelled it during the intervals of two seconds each.

$$t = 6$$

$$u = 0$$

$$\frac{s_4 - s_2}{s_2 - s_0}$$

$$\frac{\frac{a}{2} \times 4^2 - \frac{a}{2} \times 2^2}{\frac{a}{2} \times 2^2 - \frac{a}{2} \times 0^2} = \frac{32 - 8}{8 - 0} = \frac{10}{6}$$

$$= \frac{5}{3}$$

$$\frac{8a - 2a}{2a} =$$

$$= \frac{6a}{2a}$$

$$\boxed{3:1}$$

$$\boxed{\cancel{5:3:1}}$$
  

$$\boxed{1:2:3:5}$$

Q6. A driver driving a truck at a constant speed of  $20 \text{ m/s}$  suddenly saw a parked car ahead of him by  $95 \text{ m}$ . He could apply the brake after some time to produce retardation of  $2.5 \text{ m/s}^2$ . If an accident was just avoided, find his reaction time.

$$a = -2.5 \text{ m/s}^2$$

$$u = 20 \text{ m/s}$$

$$v = 0$$

$$s = 95 \text{ m}$$

$$\frac{u + 20}{-2.5} = t$$

$$8 \text{ s} = t$$

$$s = u \cdot 20(8) + \frac{1}{2} (-2.5)(64)$$

$$s = 160 - 80$$

$$s = 80 \text{ m}$$

$$t = \frac{15}{20} \boxed{\frac{3}{4} \text{ sec}}$$

Q 7. A body starts its motion with initial velocity of 9 m/s towards east and its acceleration is 2 m/s<sup>2</sup> west. find distance covered in fifth second of its motion.

$$u = 9 \text{ m/s}$$

$$a = -2 \text{ m/s}^2$$

$$S_s = u + \frac{1}{2} a (t^2 - 2)$$

$$= 9 - 8$$

$$\boxed{= 1 \text{ meter}}$$

$$S = 9 \times 4.5 - \frac{1}{2} \times 2 (4.5)^2$$

$$= 9 \times 4.5 (9 - 4.5)$$

$$\boxed{= 4.5 \text{ meters}}$$

$$S = 9 \times 0.5 + \frac{1}{2} \times 2 (0.5)^2$$

$$= 0.5 (9 - 0.5)$$

$$= 8.5 \times 0.5$$

$$= 4.25 \text{ cm} \times 2$$

$$\boxed{= 8.5 \text{ meter}}$$

$$\begin{array}{r} 2 \\ 45 \\ 45 \\ \hline 22 \\ 22 \\ \hline 18 \\ 18 \\ \hline 2 \end{array}$$

$$S = AC - AB$$

$$= ut + \frac{1}{2} at^2 - ut \cancel{\frac{1}{2}} + \frac{1}{2} at^2$$

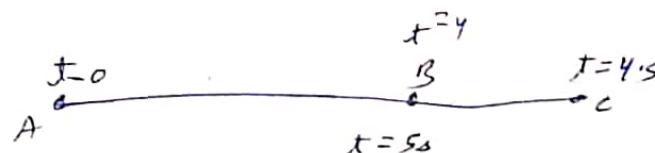
$$= 9 \times 4.5 - \frac{1}{2} \times 2 (4.5)^2 - 9 \times 4 - \frac{1}{2} \times 2 (4)^2$$

$$= 4.5 \times 4.5 - 20$$

$$= 20.25 - 20$$

$$= \frac{1}{4} \times 2$$

$$\boxed{= \frac{1}{2} \text{ meter}}$$



Q 8. A particle starts with velocity  $10 \text{ m/s}$  & deceleration  $5 \text{ m/s}^2$   
find displacement & distance covered in  $6 \text{ s}$ .

$$S = \frac{1}{2} \times S \rightarrow 3818$$

$$\boxed{S = 90 \text{ m}}$$

$$S = 60 - 90$$

$$\boxed{S = -30 \text{ m}}$$

$$S_1 = 20 - \frac{1}{2} \times S(4)^2$$

$$S_1 = 10 \text{ m}$$

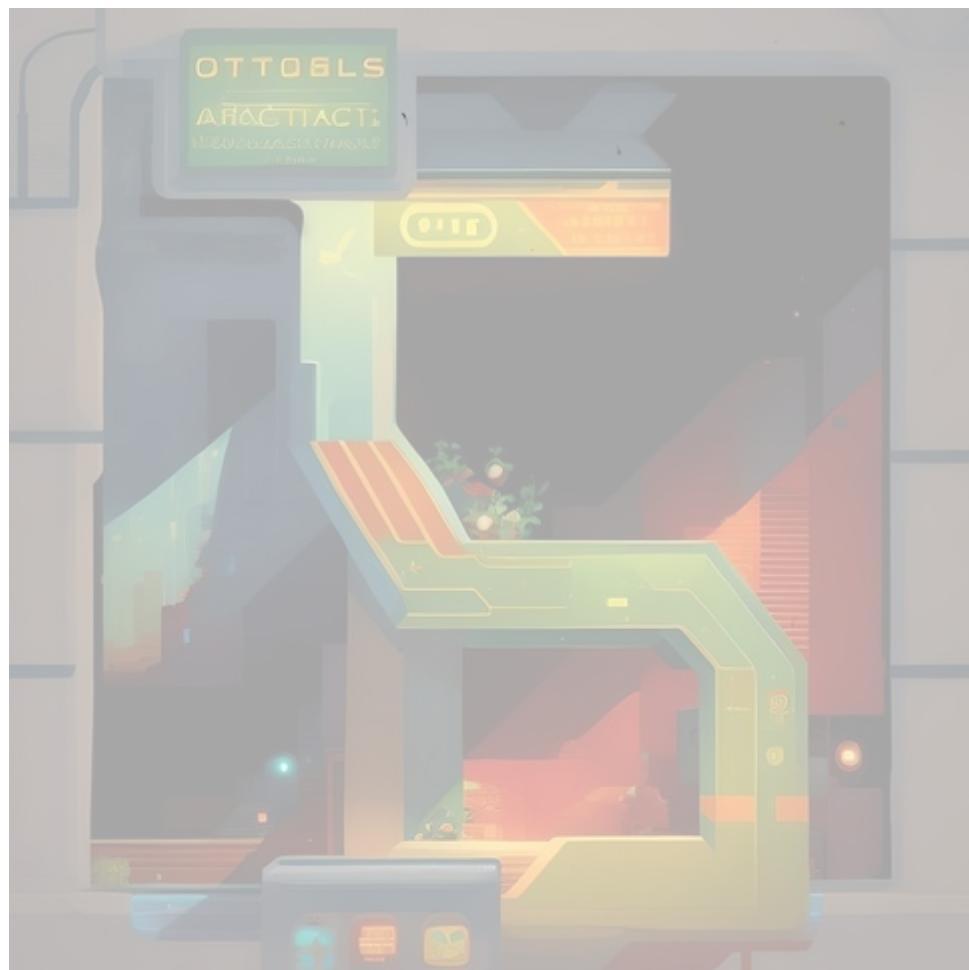
$$S_2 = \frac{1}{2} \times S(4) \times 4^2$$

$$S_2 = 40 \text{ m}$$

$$\boxed{S_1 + S_2 = 50 \text{ m}}$$

H. W. 06-05-2024

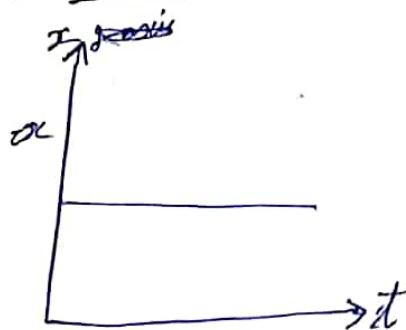
b 47 → d 1 - 25



# Graphs

## 1) Position - Time Graph ( $x-t$ graph)

a) At rest



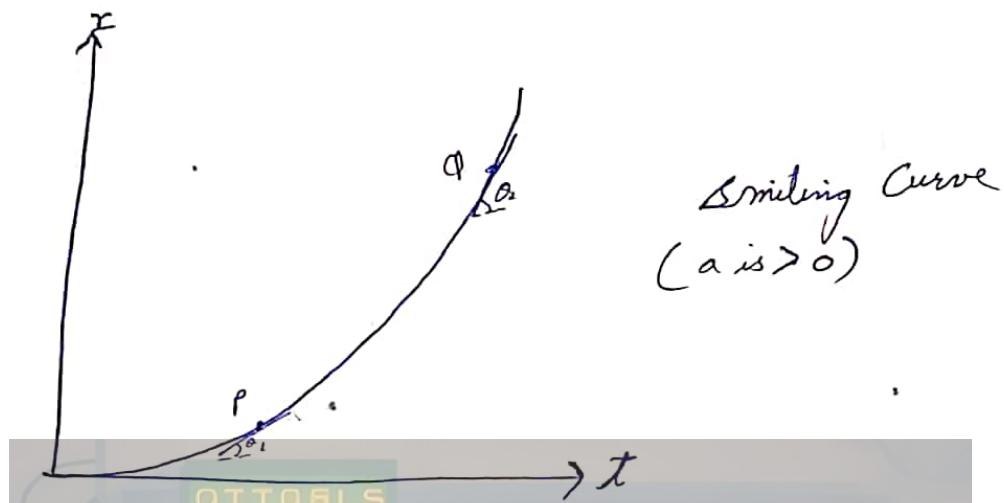
→ straight line parallel to  $x$ -time axis  
→ slope =  $\frac{dx}{dt} = v = 0$   
 $= \tan \theta = \tan 0^\circ = 0$   
↳ angle with time axis.

b) Uniform velocity



$$\begin{aligned}\text{slope} &= \tan \theta = \text{constant} \\ &= \frac{dx}{dt} = v = \text{constant}\end{aligned}$$

iii) Non-Uniform Velocity (increasing with time) :-



$$\theta_1 < \theta_2$$

$$\tan \theta_1 < \tan \theta_2$$

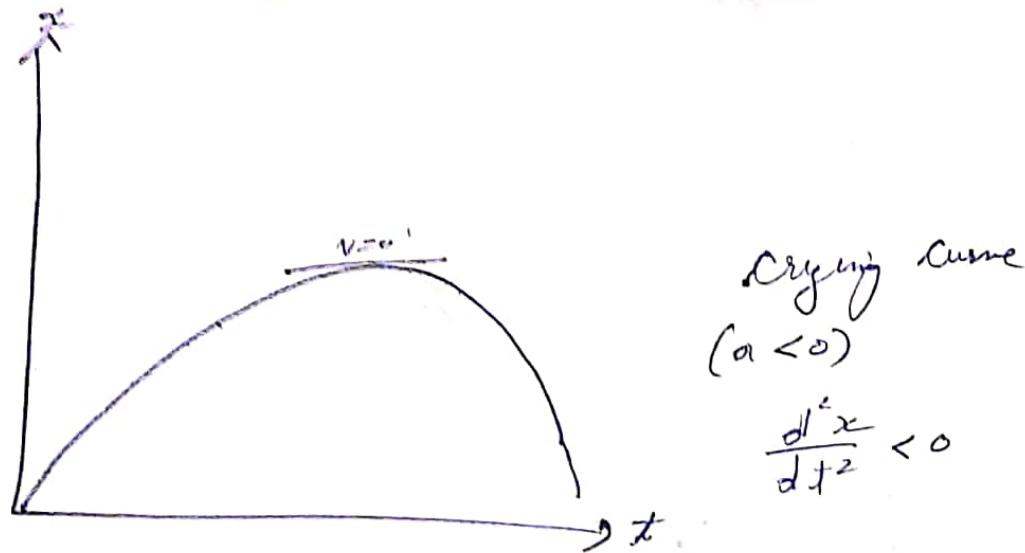
$$v_1 < v_2$$

so velocity is increasing.

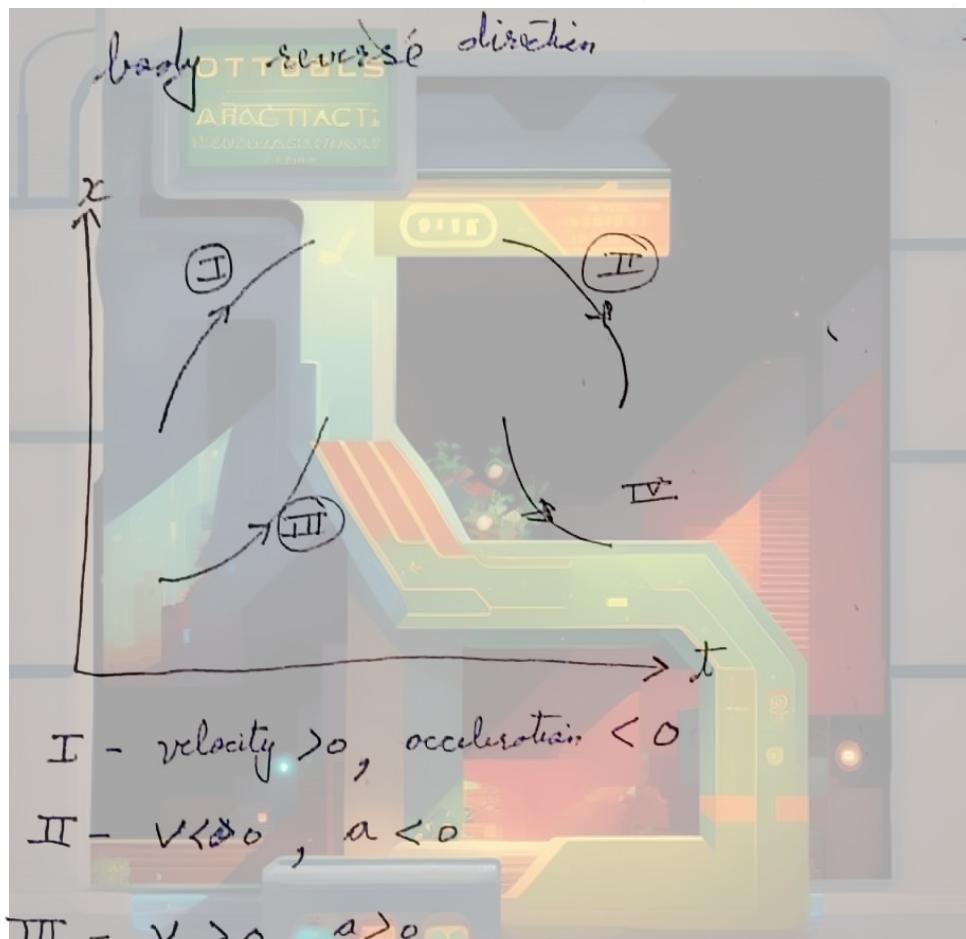
iv) Non-Uniform Velocity (decreasing with time)



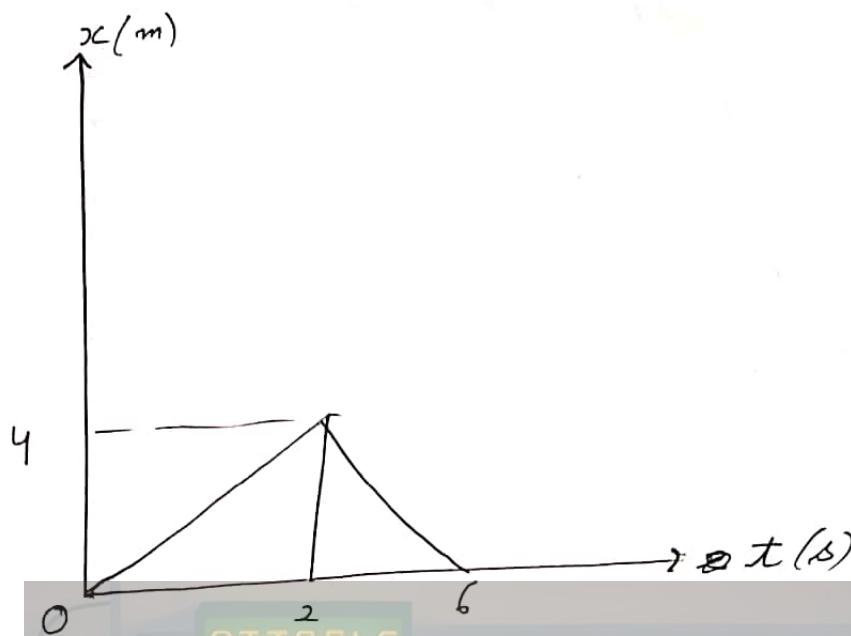
body finally stops



Q1.



Q2.



find velocity

- a) 0 - 2 s
- b) 2 - 6 s

c)  $\frac{dx}{dt} = \text{velocity}$

~~$v = \frac{d}{dt}$~~

$$v = \frac{4}{2}$$

$$\boxed{v = 2 \text{ m/s}}$$

d)  $v = \frac{d}{dt}$

$$= \frac{-4}{4}$$

$$\boxed{v = -1 \text{ m/s}}$$

## 2) Velocity - Time Graph ( $v-t$ graph)

### a) Uniform Velocity -

$$v = \text{constant}$$

$$a = 0$$

$$\frac{dv}{dt} = 0$$

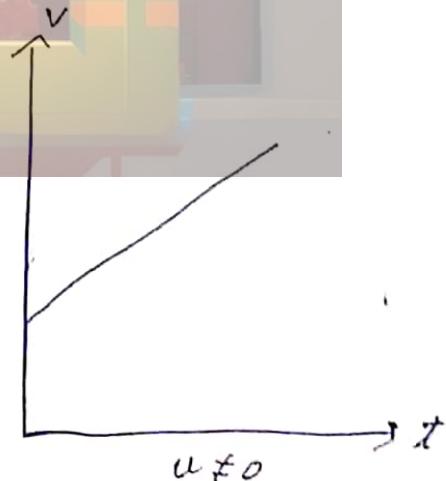
$$\text{slope} = \frac{dv}{dt} = 0$$



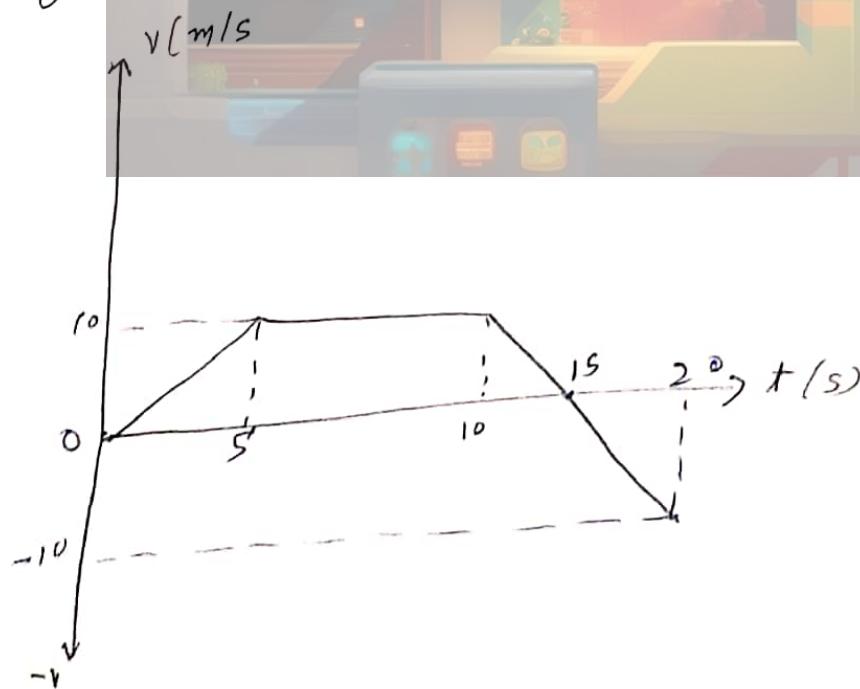
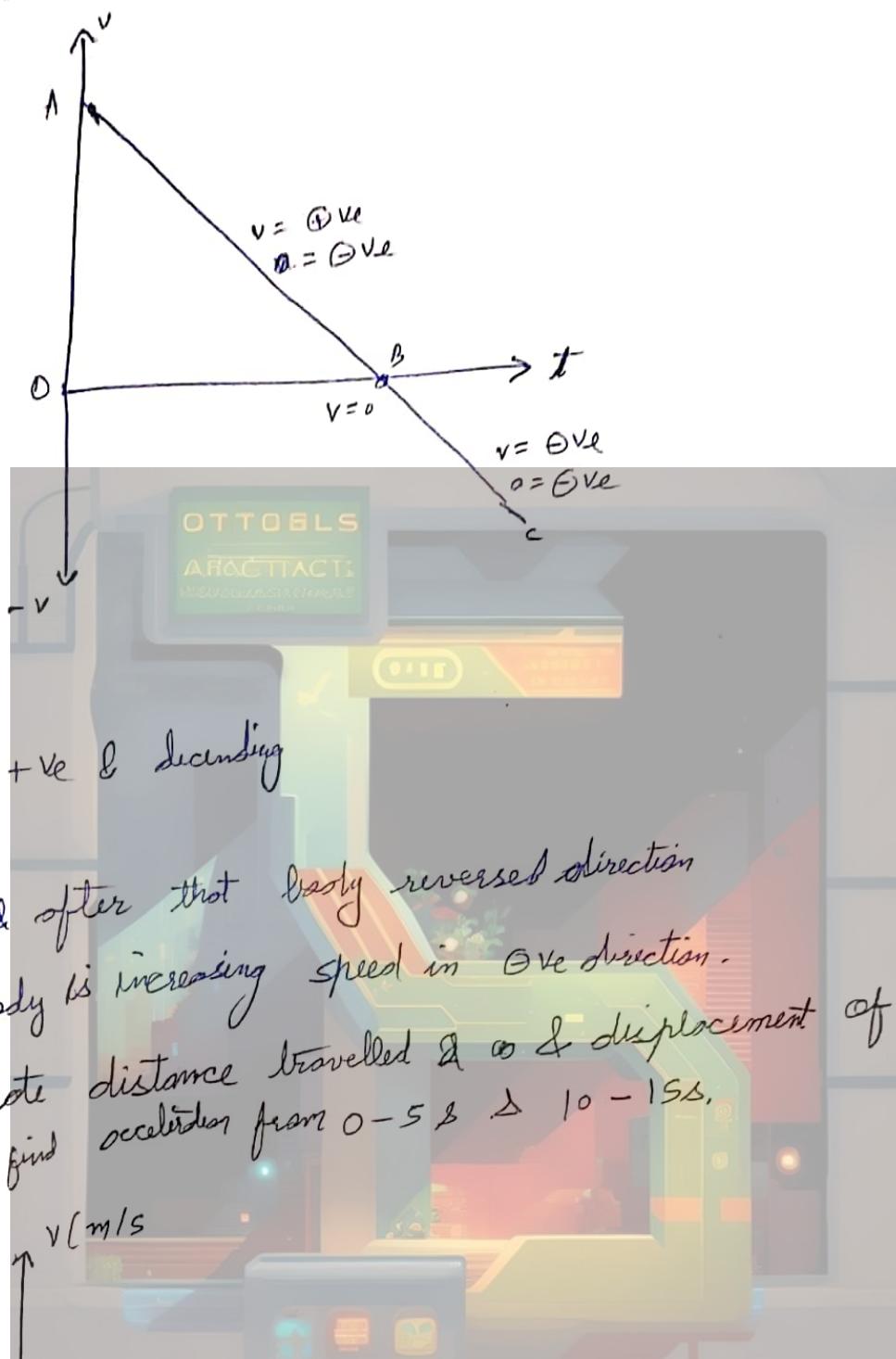
### b) Uniform Velocity

$$a = \text{const}$$

$$\frac{dv}{dt} = \text{constant} = \text{slope}$$



C) Uniform Acceleration. Retardation



~~class~~

$$S(\text{displacement}) = \frac{1}{2} \times \frac{10}{5} \times 20 \times 20 \cancel{\times 2}$$
$$= 200 \times 2$$
$$= 400$$

$$S_1 = \frac{1}{2} \times \frac{10}{5} \times 15 \times 15$$

Distance

$$= \frac{1}{2} \times \frac{10}{5} \times 5 + 15 + 10 \times 5 \times \frac{1}{2}$$
$$= 250 + 25$$

~~class~~

$$\sqrt{ } = 125 \text{ m}$$

Displacement

$$= 100 - 25$$
$$= 75 \text{ m}$$

~~class~~

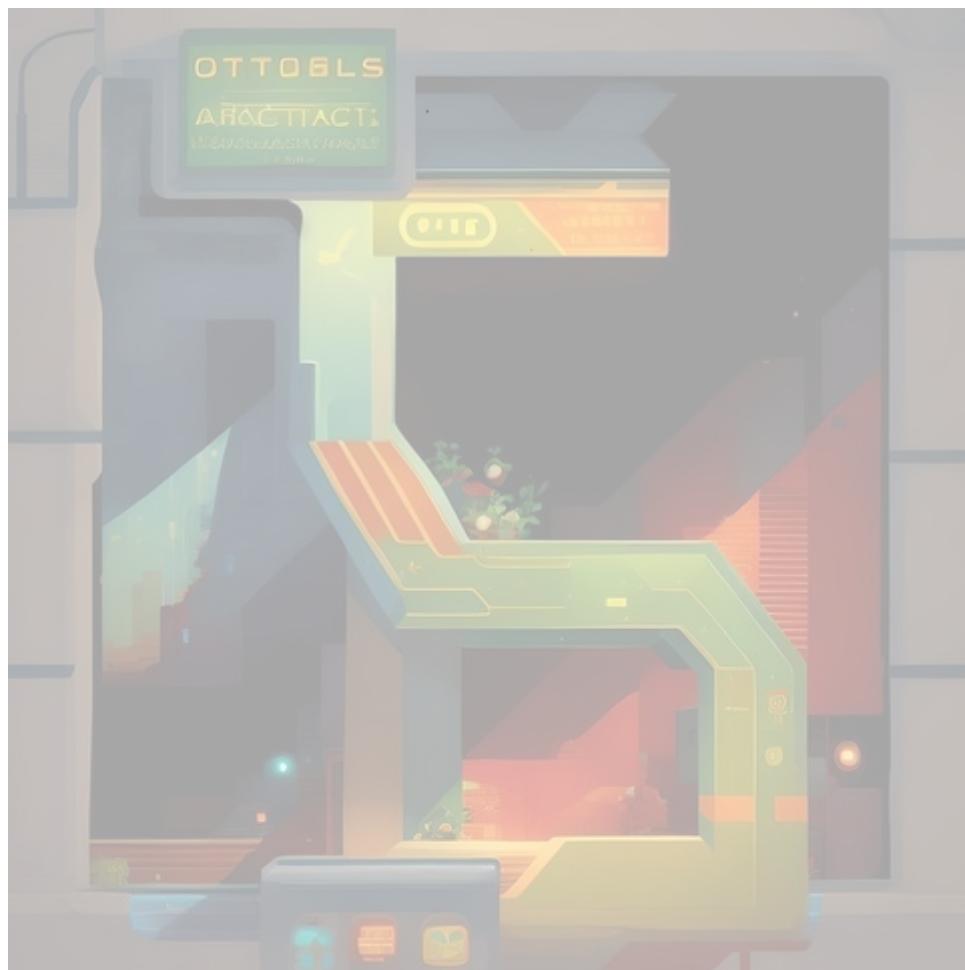
$$a_{0-s} \rightarrow \frac{10}{5}$$
$$\sqrt{ } = 2 \text{ m/s}^2$$

$$a_{10-15} \rightarrow -\frac{10}{5}$$
$$\sqrt{ } = -2 \text{ m/s}^2$$

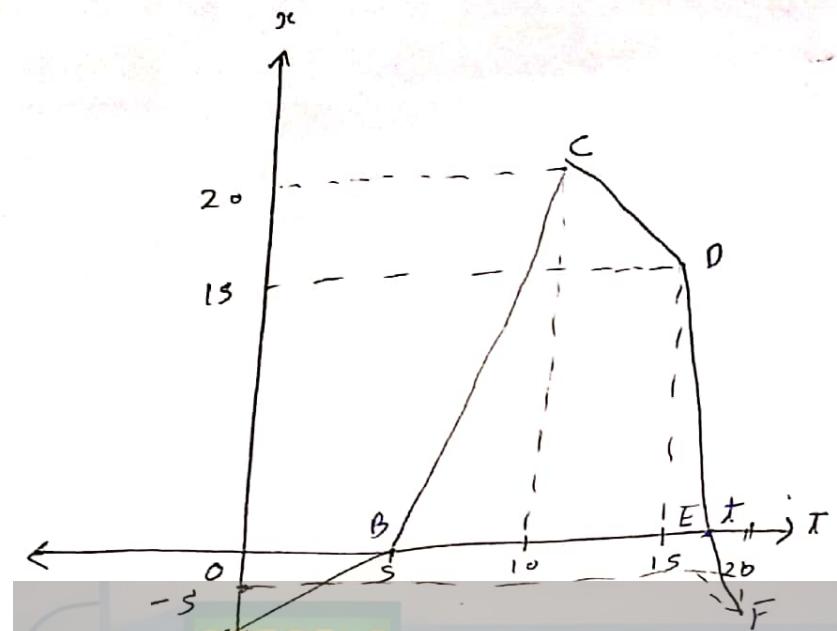
H. W.

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Pg 55 - ① 1-15



Q1.



a) find distance covered & displacement from  $t=0$  to  $20\text{s}$ .

- find distance covered & velocity from  $t=0$  to  $15\text{s}$
- find average speed & velocity from  $t=0$  to  $20\text{s}$
- find velocity of particle in  $t=5$  to  $10\text{s}$
- find average velocity of particle in  $t=10$  to  $15\text{s}$

a) Distance =  $10 + 20 + 5 + 15 + 5$

$$= 55 \text{ m} \checkmark$$

Displacement  $= +5 \text{ m}$  ✓

- e) find  $t$

f) How many times particle has turned during  $t=0$  to  $20\text{s}$

b) average speed =  $\frac{35\text{s}}{120} = \frac{7}{3} \text{ m/s}$   
 $= 2.75 \text{ m/s}$

average velocity =  $\frac{+5}{20} = +0.25 \text{ m/s}$

$$\frac{25}{15} = \frac{5}{3} \text{ m/s}$$

c) velocity

$$\left| \begin{array}{l} t = 5 \text{ to } 10 \text{ s} \\ \text{distance} \\ \text{time} \end{array} \right. = \frac{\text{distance}}{\text{time}}$$

$$= \frac{20}{5}$$

$$\boxed{= 4 \text{ m/s}} \quad \checkmark$$

d) velocity

$$\left| \begin{array}{l} t = 10 - 15 \text{ s} \\ \text{dis} \\ \text{time} \end{array} \right. = \frac{-5}{5}$$

$$\boxed{= -1 \text{ m/s}} \quad \checkmark$$

e) ~~average velocity~~ =  $\frac{20}{5} = 4$  or DEF is a straight line

$\text{dis} = 15$

$t_{\text{time}} = \frac{15}{4} \text{ s}$

$t = 15 + \frac{15}{4} \text{ s}$

$= 15 + 3 \cdot 75$

$= 18.75 \text{ s}$

f)  $1 \text{ time}$  at Point C

$\text{slope}_{DE} = \text{slope}_{EF}$

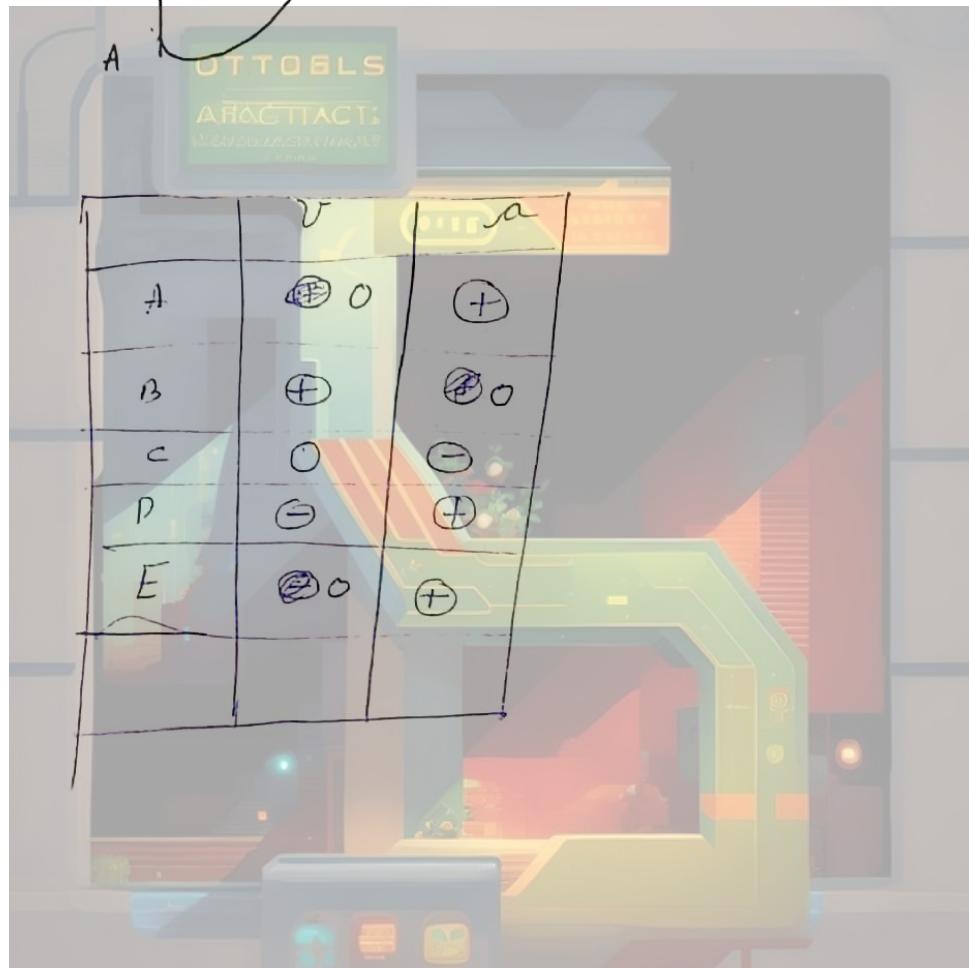
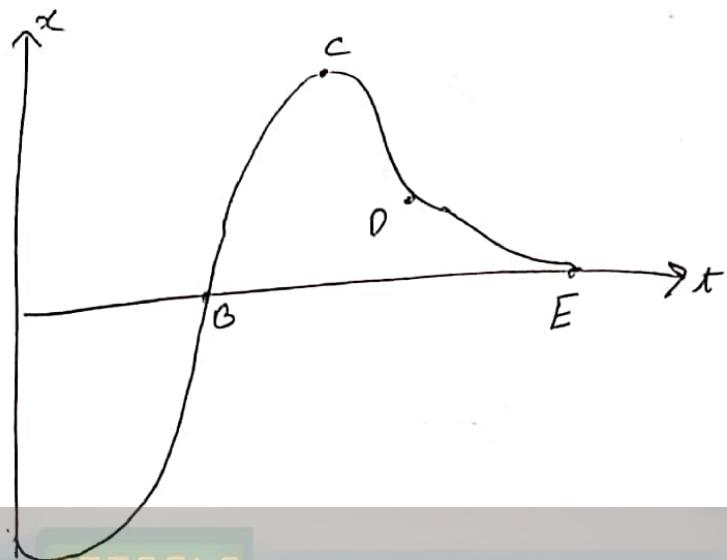
$$\frac{0 - 15}{t - 15} = \frac{-5 - 0}{20 - t}$$

$$-300 + 15t = -5t + 75$$

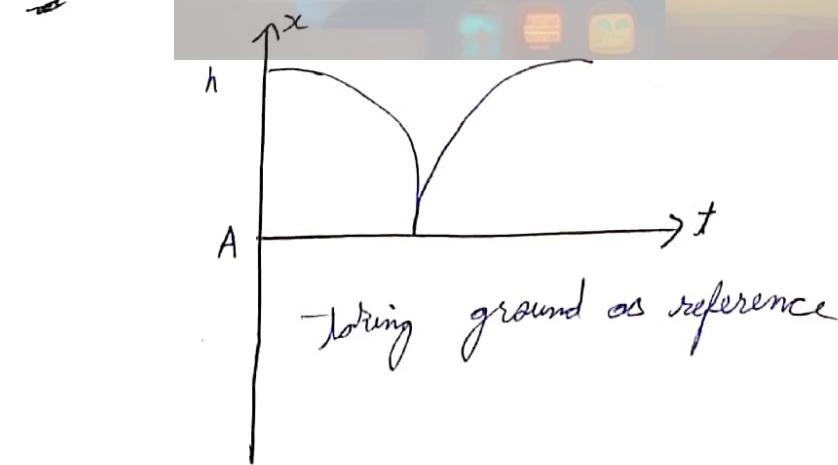
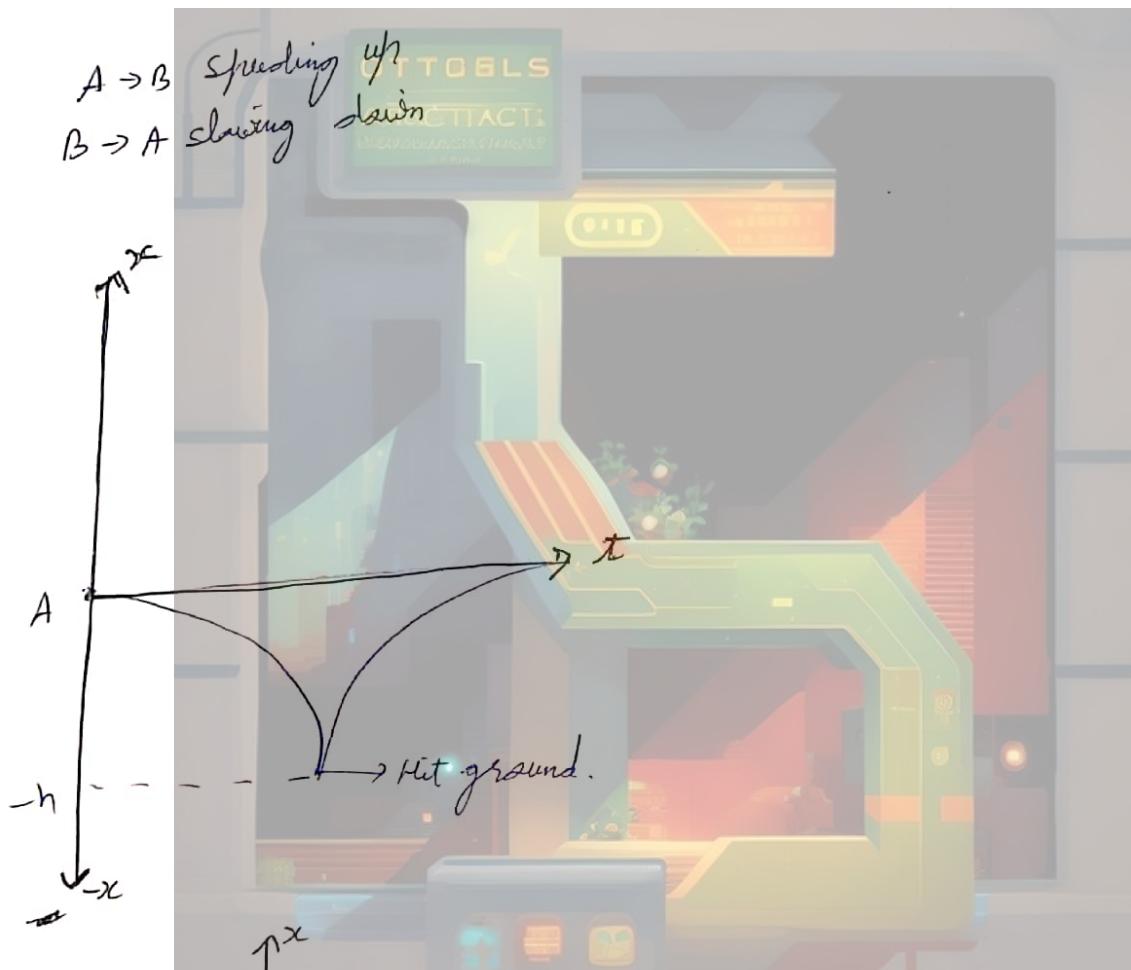
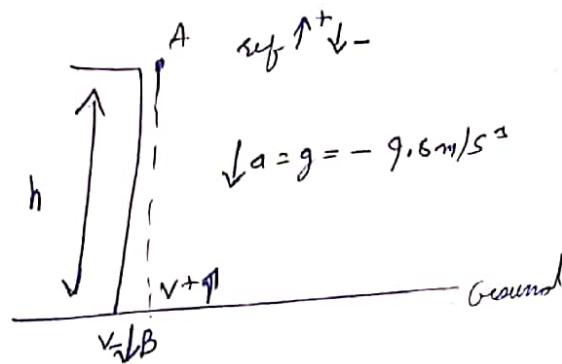
$$20t = 375$$

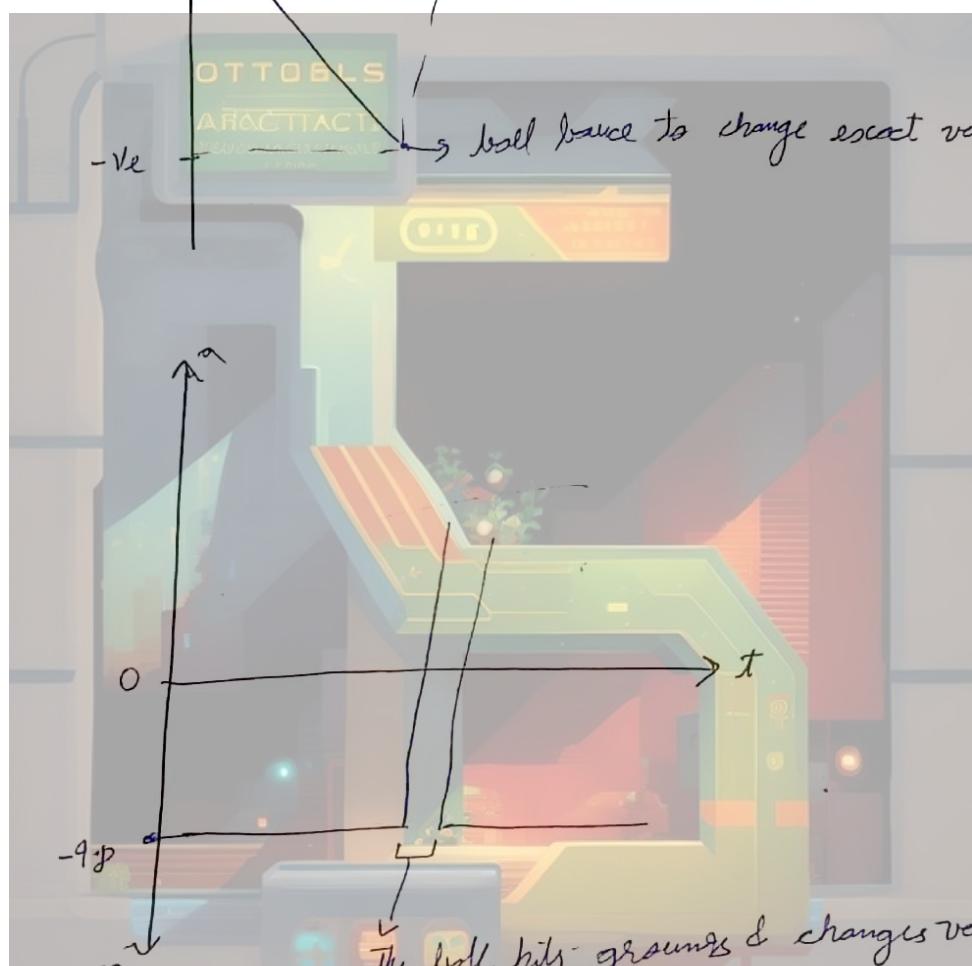
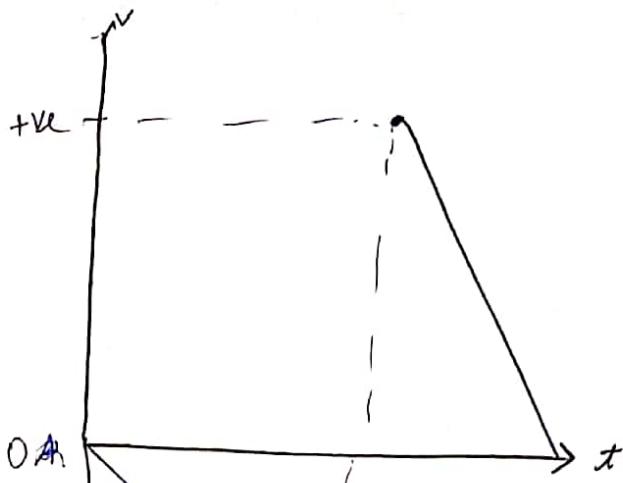
$$\boxed{t = \frac{375}{20} \text{ s}} \quad \checkmark$$

Q2.



Q3. A body is dropped from a height, it strikes the ground elastically. draw all the 3 kinematic graphs taking upward direction as +ve.



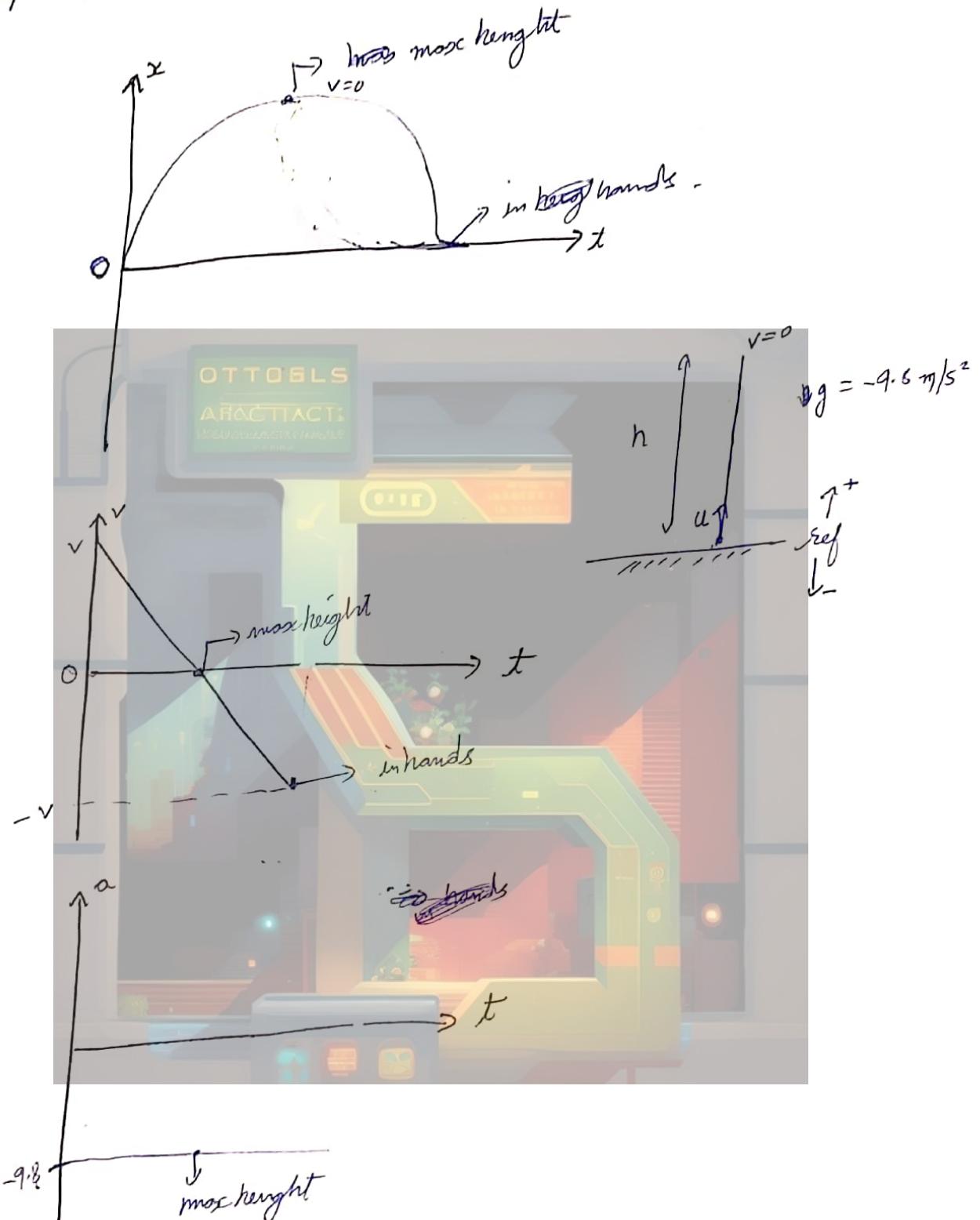


$-v$  to  $+v$

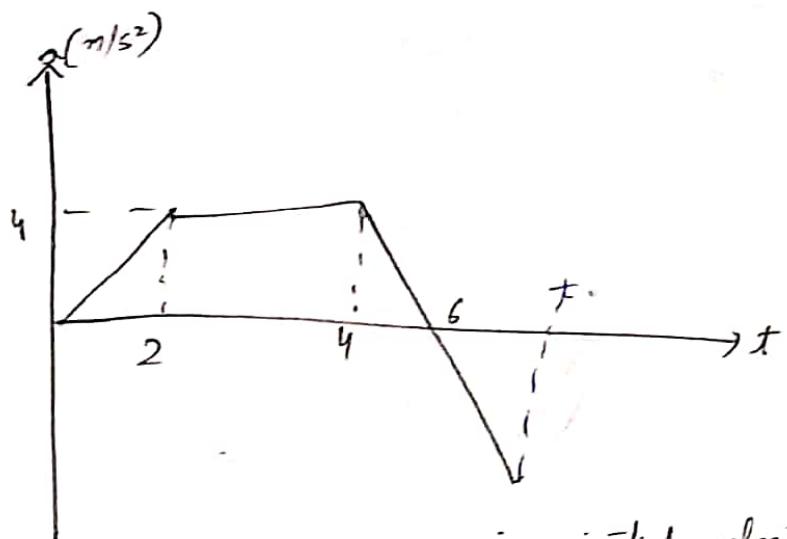
$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

$\therefore a = \text{infinite in +ve direction}$

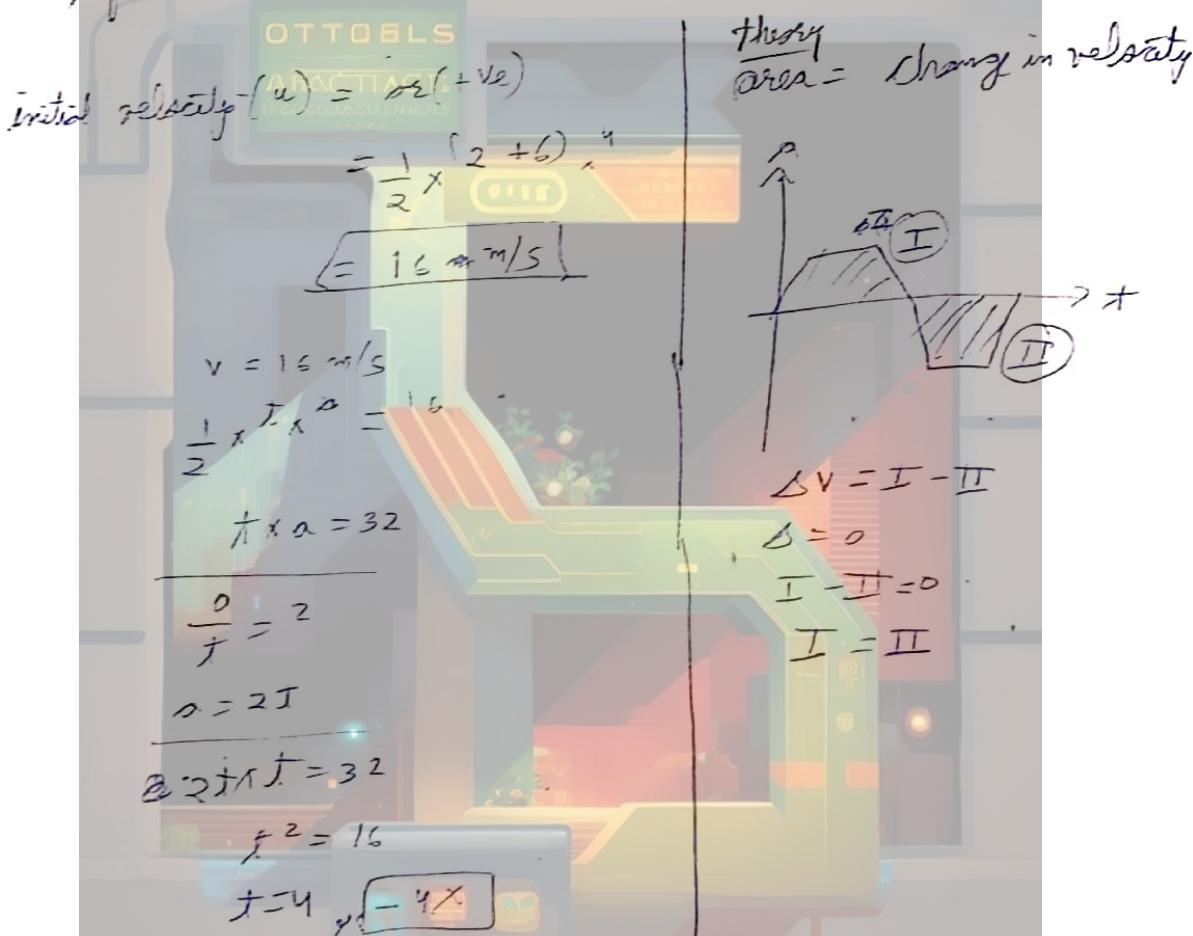
Q:- A body thrown upwards with some velocity. It reaches max height and again reaches hands of thrower. Draw all 3 kinematic graphs (take upward as +ve)



Q5.



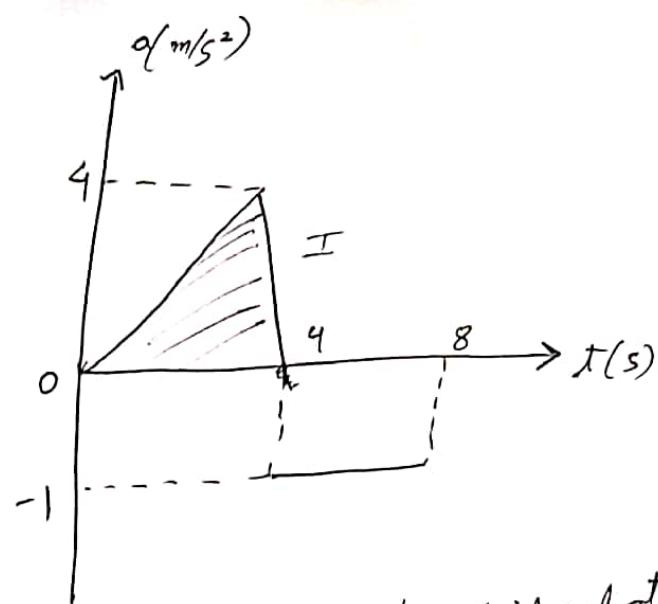
- a) find time at which particle attains initial velocity.



$$4+6=t$$

$$\boxed{t=10 \text{ s}}$$

Q6.



(a) If initial velocity =  $3 \text{ m/s}$  find vel at  $t = 8$

$$I = \frac{1}{2} \times u \times t^2$$
$$= 8$$

$$\Delta v = 8 \text{ m/s}$$

$$u = 3$$
$$v = 11 \text{ m/s at } t = 8$$

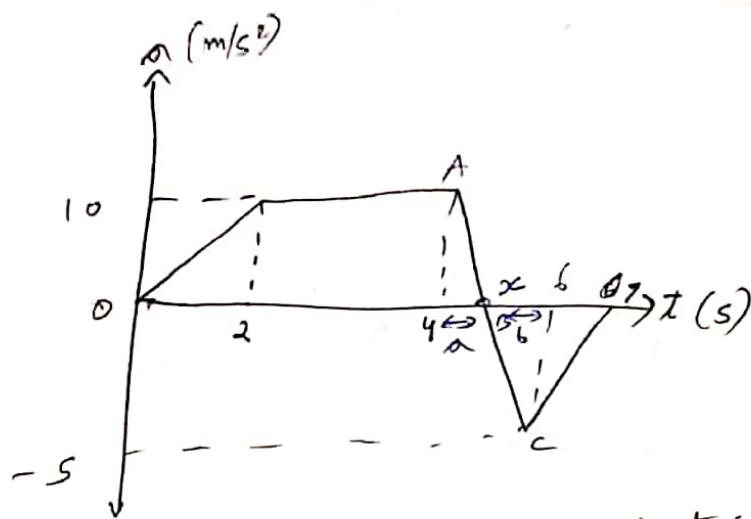
$$a = (-1) \text{ m/s}^2$$

$$\frac{v-u}{t} = a$$
$$(8-4)$$

$$v = (-1)(4) + 11$$

$$v = -4 + 11$$
$$\boxed{v = 7 \text{ m/s}} \checkmark$$

Q7.



If body starts from rest final time at which velocity is maximum.

$$\max v, 0 = 0^2$$

$$v = x$$

$$\text{slope}_{AB} = \text{slope}_{BC}$$

$$\frac{10}{a} = \frac{-5}{0.6}$$

$$10.1 = 5a$$

$$2a = a$$

$$a + 6 = 2$$

$$3a = 2$$

$$a = \frac{2}{3}$$

$$a = 2 - \frac{2}{3}$$

$$a = \frac{4}{3}$$

$$4 + \frac{4}{3}$$

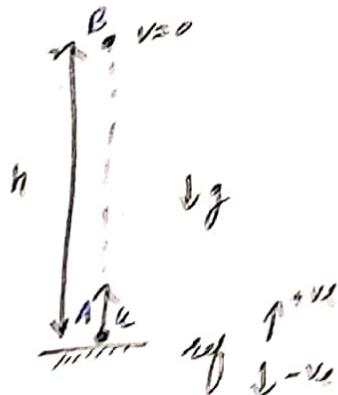
$$\frac{12 + 4}{3}$$

|                       |
|-----------------------|
| $\frac{16}{3} \Delta$ |
| $5.33 \Delta$         |

$$\begin{array}{r} 16 \\ 3 ) \overline{16} \\ - 15 \\ \hline 10 \\ - 9 \\ \hline 10 \end{array}$$

## Motion Under Gravity

### ① Case 1 →



→ Take point of projection as reference.

$$0 - (u)^2 = 2(-g)(+h)$$

$$h = \frac{u^2}{2g} \quad (\text{maximum height})$$

→ Time of flight = time of ascent + time of descent

$$v = u + at$$

$$A \rightarrow B$$

$$0 = u + (-g)t$$

$$t_1 = \frac{u}{g}$$

$$s = ut + \frac{1}{2}at^2$$

$$-h = 0 + \frac{1}{2}(-g)t^2$$

$$\frac{u^2}{2g} = \frac{1}{2}gt^2$$

$$\frac{u^2}{2g} \times \frac{2}{g} = t^2$$

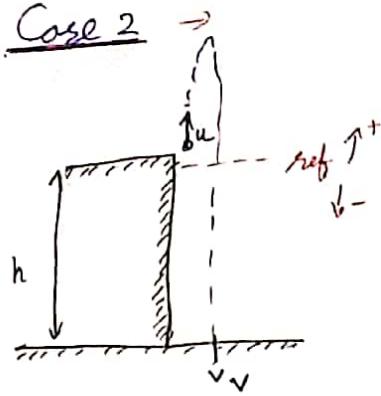
$$\frac{u^2}{g^2} = t^2$$

$$t_2 = \frac{u}{g}$$

Time of ascent = time of descent

$$\text{Time of flight} = \frac{2u}{g}$$

② Case 2



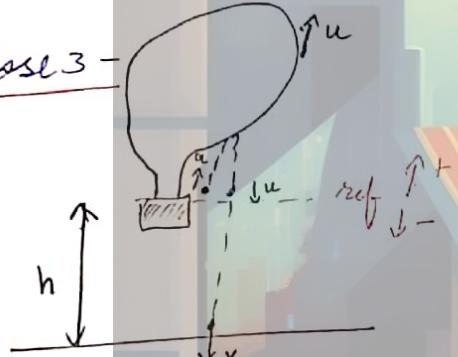
$$\begin{aligned} v^2 &= u^2 + 2 \cdot g \cdot s \\ (-v)^2 &= u^2 + 2(-g)(-h) \\ v^2 &= u^2 + 2gh \\ v &= \sqrt{u^2 + 2gh} \end{aligned}$$

→ To calculate Time of flight

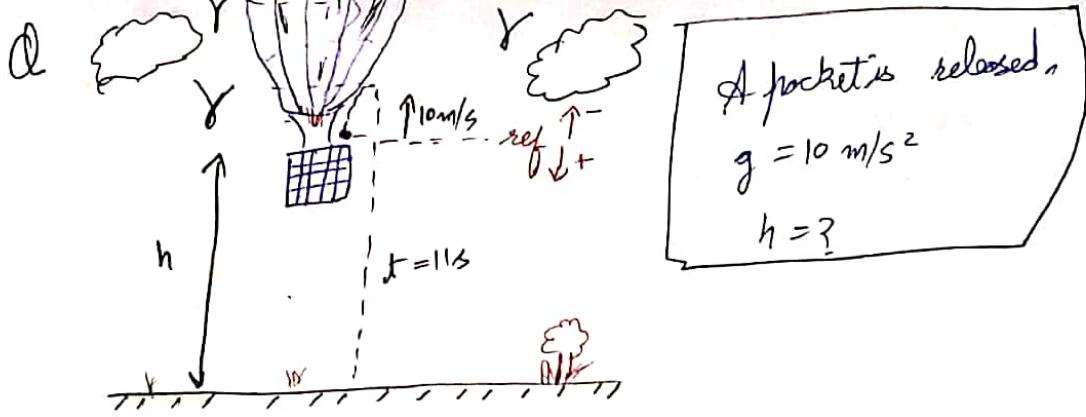
$$s = ut + \frac{1}{2} at^2$$

$$-h = ut + \frac{1}{2} (-g)t^2$$

③ Case 3



$u$  = velocity of balloon



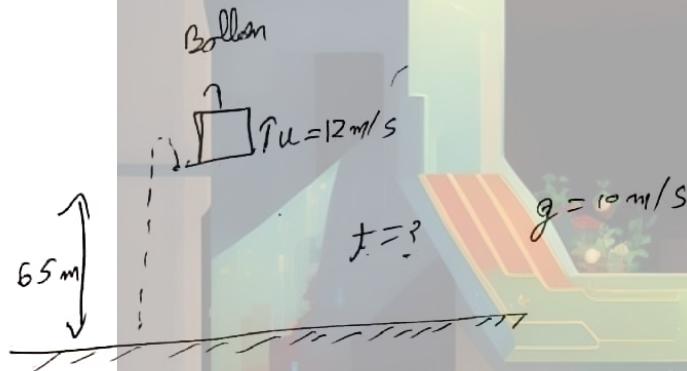
$$h = (-10 \times 11) + \frac{1}{2} \times 10 \times 121$$

$$s = ut + \frac{1}{2} at^2$$

$$h = -110 + 605$$

$\boxed{h = 495 \text{ m}}$

Q2.



$$65 = -12t + \frac{1}{2} \times 10t^2$$

$$65 = -12t + 5t^2$$

$$5t^2 - 12t - 65 = 0$$

$$t = \frac{-12 \pm \sqrt{144 + 1300}}{10}$$

$$t = \frac{-12 \pm \sqrt{1444}}{10}$$

$$t = \frac{-12 + 38}{10}$$

$$t = \frac{+50}{10}$$

$\boxed{t = 5.6}$

$$\begin{array}{r} & 68 \\ 3 & | 1444 \\ & \underline{9} \\ & 544 \end{array}$$

$$\begin{array}{r} 2 \\ 65 \\ \hline 325 \\ 167 \\ \hline 158 \\ 69 \\ \hline 84 \end{array}$$

Q3. A balloon starts ascending from rest at constant  $a = 2 \text{ m/s}^2$  when it was at 100 m height a pocket was released from it. after how much time and with what velocity will it hit ground.

$$u=0$$

$$v^2 - u^2 = 2as$$

$$v^2 = 2 \times 2 \times 100$$

$$v^2 = 400$$

$$v = 20 \text{ m/s}$$

for pocket

$$u = 20 \text{ m/s}$$

$$100 = 20t + \frac{1}{2} \times 10(t^2)$$

$$100 = 20t + 5t^2$$

$$20t = -4t + t^2$$

$$t^2 - 4t - 20 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{4 \pm \sqrt{96}}{2}$$

$$t = 2 \pm \sqrt{24}$$

$$t = 2 \pm 2\sqrt{6} \text{ s}$$

$$t = 2 + 2\sqrt{6} \text{ s}$$

$$v^2 = u^2 + 2as$$

$$= 400 + 2 \times 10 \times 100$$

$$= 400 + 2000$$

$$v^2 = 2400$$

$$v = \sqrt{2400}$$

$$v = 20\sqrt{6} \text{ m/s}$$

$$v = 20\sqrt{6} \text{ m/s}$$

Q4. A parachutist falls out from an aeroplane and after 40m, he ~~sits~~ opens parachute and decelerates at  $2 \text{ m/s}^2$ . If he reaches ground with speed  $2 \text{ m/s}$ , How long was he in air. At what height he jumped off the plane? ( $g = 10 \text{ m/s}^2$ )

$$\cancel{v = 2 \text{ m/s}}$$

$$s = 40 \text{ m}$$

$$u = 0$$

$$g = 10 \text{ m/s}^2$$

$$\therefore v^2 = 2 \times 10 \times 40$$

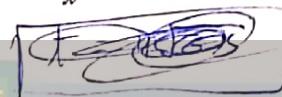
$$v^2 = 800$$

$$\boxed{v = \cancel{20\sqrt{2}}} \quad \checkmark$$

$$40 = \frac{1}{2} \times 10 \times t^2$$

$$80 = t^2$$

$$t^2 = 2\cancel{0}8$$



$$\boxed{t_1 = 2\sqrt{2}} \quad \checkmark$$

After Parachute

$$u = 20\sqrt{2}$$

$$a = -2 \text{ m/s}$$

$$v = 2 \text{ m/s}$$

$$v^2 - u^2 = 2 a s$$

$$4 + 800 = 2x + 2 \times s$$

$$\frac{796}{4} = s$$

$$s = 199 \text{ m} \quad \checkmark$$

$$\phi \quad 2 - 20\sqrt{2} = t$$

$$-2$$

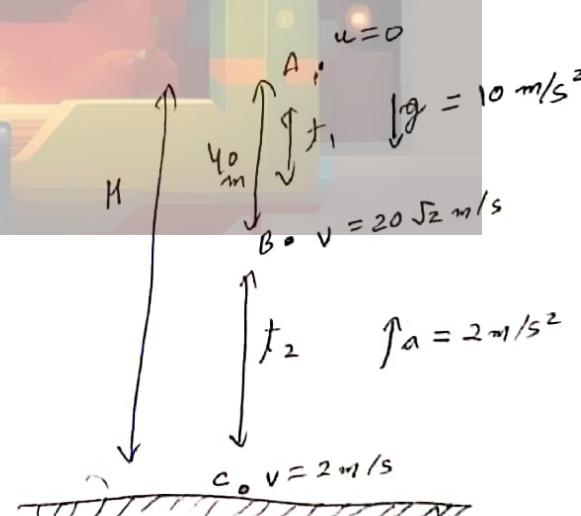
$$-1 + 10\sqrt{2} = t$$

$$\boxed{\sqrt{t} = 10\sqrt{2} - 1} \quad \checkmark$$

$$\text{Total height} = 199 + 40 = \boxed{239 \text{ m}} \quad \checkmark$$

$$\text{Total time} = 10\sqrt{2} - 1 + 2\sqrt{2}$$

$$= \boxed{12\sqrt{2} - 1} \quad \checkmark$$



Q5. A body throw from top of a cliff vertically up with velocity  $v$ ,  
 & when thrown vertically down with same velocity.  
 takes time  $t_2$ . If dropped freely takes time  $t$ , find relation among  
 three times.

$$S = \frac{vt + \frac{1}{2}gt^2}{2}$$

$$S = \frac{1}{2}at^2$$

$$S = -vt + \frac{1}{2}at_2^2$$

$$+2S = -2vt + gt_1^2$$

$$= gt^2$$

$$= -2vt + at_2^2$$

$$= 2vt_2 - at_2^2$$

$$2vt_1 + gt_1^2 = 2vt_2 + -gt_2^2$$

$$-h = vt_1 + \frac{1}{2}(-g)(t_1)^2$$

$$-h = vt_1 + \frac{1}{2}gt_1^2 \quad \text{--- (1)} \times t_2$$

$$-h = -vt_2 - \frac{1}{2}gt_2^2 \quad \text{--- (2)} \times t_1$$

$$2h = +\frac{1}{2}gt^2 \quad \text{--- (3)}$$

$$-ht_2 = vt_1t_2 - \frac{1}{2}gt_1^2t_2$$

$$+ht_1 = vt_1t_2 + \frac{1}{2}gt_2^2t_1$$

$$+h(t_1 + t_2) = +\frac{1}{2}g(t_1 t_2)(t_1 + t_2)$$

$$+h = \frac{t_1 t_2 g}{2} \quad \text{--- (4)}$$

$$\frac{t_1 t_2 g}{2} = \frac{1}{2}gt^2 \quad (4)$$

$$t_1 t_2 = t^2$$

$$t = \sqrt{t_1 t_2}$$

- Q1. A particle is projected vertically up such that it passes a fixed point P after time  $t_1$  and  $t_2$  respectively. Find
- Height at which point is located w.r.t point of projection.
  - speed of projection of ball
  - velocity at point P
  - Max height reached by ball w.r.t point of projection
  - Max height reached by ball w.r.t P.

$$S = ut + \frac{1}{2}gt^2$$

$$h = ut - \frac{gt^2}{2}$$

$$2h = 2ut - gt^2$$

$$gt^2 - 2ut + 2h = 0$$

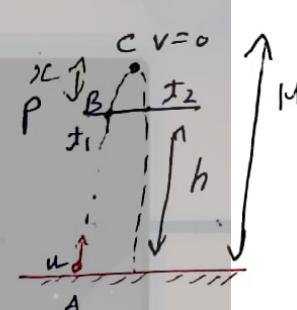
$$t = \frac{2u \pm \sqrt{4u^2 - (4)(2h)(g)}}{2g}$$

$$t = \frac{2u \pm \sqrt{4(u^2 - 2gh)}}{2g}$$

$$t = \frac{u \pm \sqrt{u^2 - 2gh}}{g}$$

$$t_1 = \frac{u - \sqrt{u^2 - 2gh}}{g}$$

$$t_2 = \frac{u + \sqrt{u^2 - 2gh}}{g}$$



$$t_1 + t_2 = \frac{2u}{g}$$

$$u = \frac{g(t_1 + t_2)}{2} \quad b)$$

A - B

$$v = u + at$$

$$v = g \frac{(t_1 + t_2)}{2} - gt_1$$

$$v = \frac{gt_2 + gt_1 - 2gt_1}{2}$$

$$v = \frac{gt_2 - gt_1}{2}$$

$$v = \frac{g}{2}(t_2 - t_1)$$

$$\left. v = \frac{g}{2}(t_2 - t_1) \right] \begin{matrix} c) \\ b) \end{matrix}$$

from A  $\rightarrow$  C

$$v=0$$

$$v^2 = u^2 + 2as$$

$$v^2 = \left( \frac{g}{2} (t_1 + t_2) \right)^2 \Rightarrow 2 \times g \times H$$

$$0 = \frac{g^2}{4} (t_1 + t_2)^2 \Leftrightarrow 2gH$$

$$2gH = \frac{g^2}{4} (t_1 + t_2)^2$$

$$H = \frac{g}{8} (t_1 + t_2)^2$$

$$\boxed{H = \frac{g}{8} (t_1 + t_2)^2 d}$$

from B  $\rightarrow$  C

$$v=0 \\ a_u = \frac{g(t_2 - t_1)}{2}$$

$$v^2 - u^2 = 2as$$

$$\cancel{\frac{g^2}{4} (t_2 - t_1)^2} = + 2 \times g \times x$$

$$\boxed{\frac{g}{8} (t_2 - t_1)^2 = x} \quad c)$$

$\theta H - x = \text{height of P}$

$$\frac{g}{8} (t_1 + t_2)^2 - \frac{g}{8} (t_2 - t_1)^2$$

$$\frac{g}{8} ((t_1)^2 + (t_2)^2 + 2t_1 t_2 - (t_2)^2 - (t_1)^2 + 2t_1 t_2)$$

$$\frac{g}{8} (4t_1 t_2)$$

$$\boxed{\frac{g(t_1 t_2)}{2}}$$

Q2. Drops of water fall from the roof of a building 9 m high at regular intervals of time, the first drop reaching the ground at the same instant fourth drop starts to fall. What are the distances of the second and third drop from the roof ( $g = 10 \text{ m/s}^2$ )

$$u = 0$$

$$h = 9 \text{ m}$$

$$g = 10 \text{ m/s}^2$$

$$9 = \frac{1}{2} \times 10 \times t^2$$

$$\frac{18}{10} = t^2$$

$$t^2 = 1.8$$

$$t = \sqrt{1.8}$$

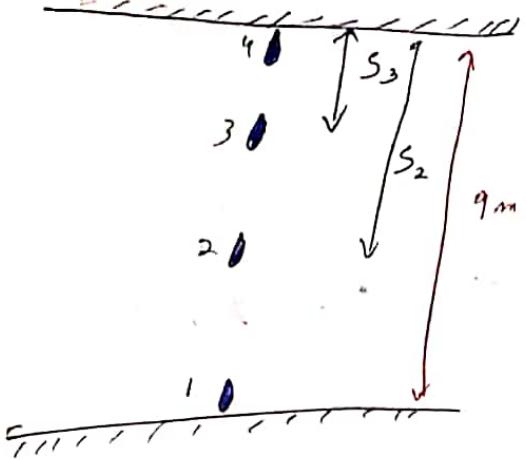
$$\frac{3 \times \sqrt{2}}{\sqrt{10}} \times \frac{1}{4} = 0.75$$

$$S = \frac{1}{2} \times 10 \times \left( \frac{3 \times \sqrt{2}}{\sqrt{10}} \times \frac{1}{4} \right)^2$$

$$S = S \times \left( \frac{9 \times \frac{1}{4}}{10} \times \frac{1}{16} \right)$$

$$S = S \times \frac{9}{160}$$

$$\boxed{S = \frac{9}{16} \text{ m}}$$



$$\frac{9\sqrt{282}}{3\sqrt{10}} - \frac{3\sqrt{2}}{4} = t_2$$

$$S_2 = \frac{1}{2} \times 10 \times \left( \frac{2\sqrt{2}}{3\sqrt{10}} \right)^2$$

$$= \cancel{10} \times \cancel{10} \times \frac{16}{160}$$

$$= S \times \frac{8}{10}$$

$$S_2 = 4 \frac{\cancel{8}}{\cancel{10}} \text{ m}$$

$$S_3 = \frac{1}{2} \times 10 \times \left( \cancel{2\sqrt{2}} \right) \left( \frac{2\sqrt{2}}{10} \right)^2$$

$$= 5 \times \frac{32}{160} = S \times \frac{2}{10}$$

$$= \cancel{X2} \frac{32}{\cancel{32}} \cancel{c}^2 c$$

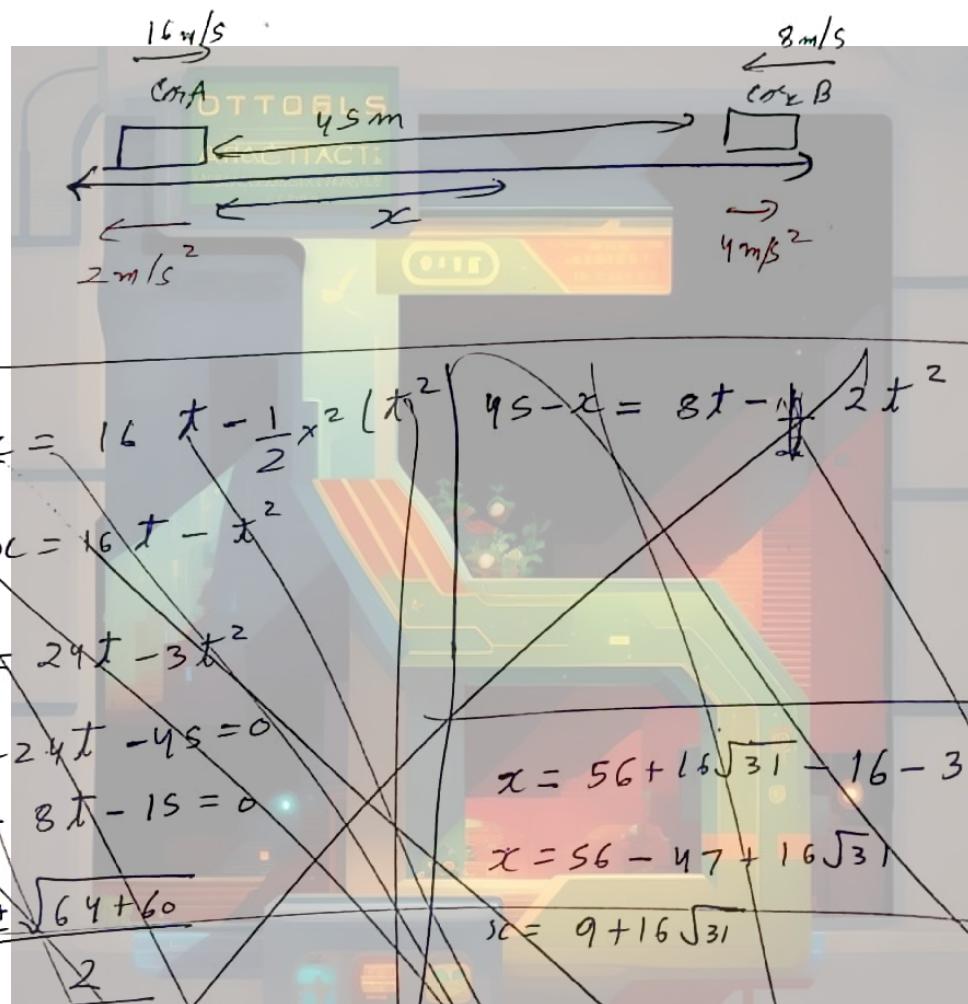
$$\sqrt{=} 1 \text{ m}$$

$$S_3 = 1 \text{ m}$$

$$S_2 \cancel{+} S_3$$

$$\cancel{\frac{81}{16} - \frac{31}{16}} = \frac{45}{16} \text{ meter}$$

Q 3. Two cars approach each other on a straight road. Car A moves at 16 m/s and car B moves at 8 m/s. When they are 45 m apart, both drivers apply their brakes. Car A slows down at  $2 \text{ m/s}^2$ , while car B slows down at  $4 \text{ m/s}^2$ . When and where do they collide?



$$t = \frac{8 \pm \sqrt{129}}{2}$$

$$t = 4 \pm \sqrt{31}$$

$$t = 4 + \sqrt{31}$$

Car - B

$$u = 8$$

$$a = -4$$

$$v = 0$$

$$t = 2\Delta$$

$$S = (8)(2) - \frac{1}{2} \times 4 \cdot (4)$$

$$S = 16 - 8$$

$$\boxed{S = 8 \text{ m}}$$

Collision at 8m from B.

$$4S - 8 = 37$$

$$37 = 16t - t^2$$

$$t^2 - 16t + 37 = 0$$

$$t = \frac{16 \pm \sqrt{256 - 198}}{2}$$

$$t = \frac{16 \pm \sqrt{108}}{2}$$

$$t = 8 \pm \sqrt{27}$$

$$\boxed{t = 8 + \sqrt{27} \Delta}$$

$$\boxed{t = 8 \pm \sqrt{27} \Delta}$$

Car - A

$$u = 16$$

$$a = -2$$

$$v = 0$$

$$t = 8\Delta$$

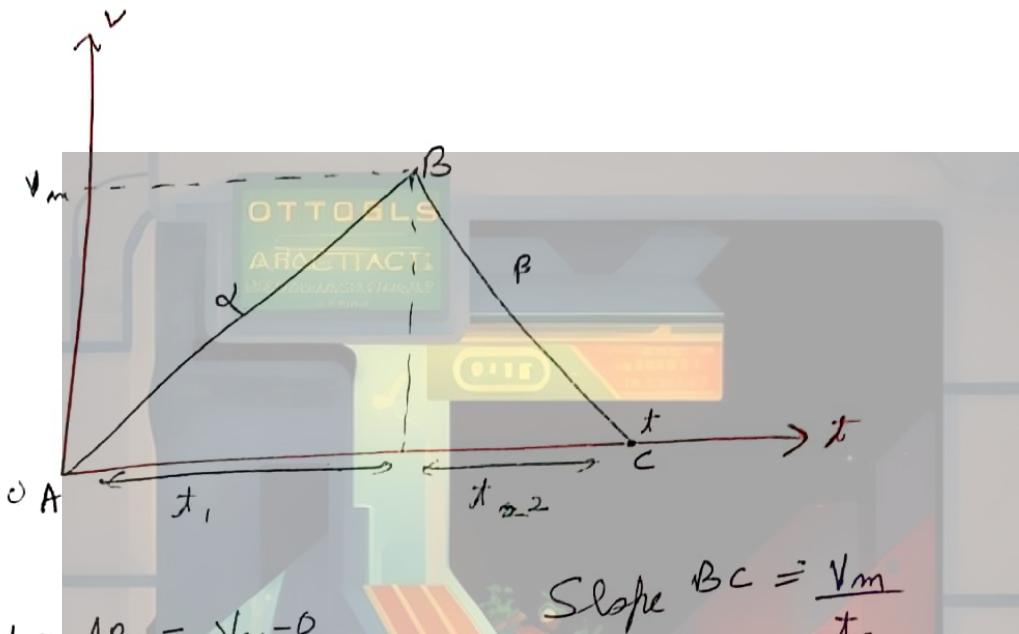
$$S = (16)(8) - \frac{1}{2} \times 2(64)$$

$$S = 128 - 64$$

$$\boxed{S = 64 \text{ m}}$$

Q A car accelerates from rest at a constant rate  $\alpha$  for some time and then decelerates at constant rate  $\beta$  to come to rest. If total time elapsed is  $t$  then, calculate

- Maximum velocity obtained
- Total distance travelled.



$$\text{Slope } AB = \frac{V_m - 0}{t_1}$$

$$\alpha = \frac{V_m}{t_1}$$

$$t_1 = \frac{V_m}{\alpha}$$

$$t_1 + t_2 = t$$

$$\frac{V_m}{\alpha} + \frac{V_m}{\beta} = t$$

$$V_m = \frac{t \alpha \beta}{\alpha + \beta}$$

$$V_m = \frac{t \alpha \beta}{\alpha + \beta}$$

$$\text{Slope } BC = \frac{V_m}{t_2}$$

$$t_2 = \frac{V_m}{\beta}$$

$$\text{Distance} = \frac{1}{2} \times V_m \times t$$

$$= \frac{1}{2} \times t \times \frac{t \alpha \beta}{\alpha + \beta}$$

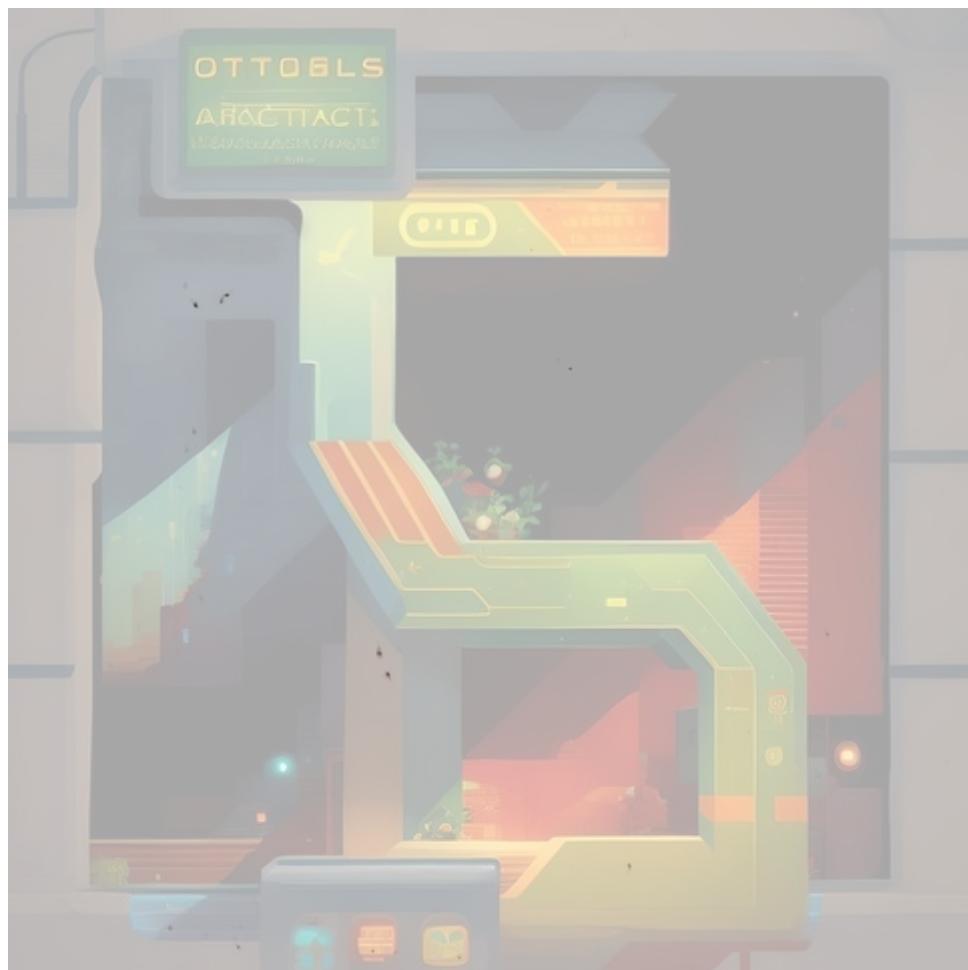
$$= \frac{\alpha \beta t^2}{2(\alpha + \beta)}$$

$$\text{Dis} = \frac{\alpha \beta t^2}{2(\alpha + \beta)}$$

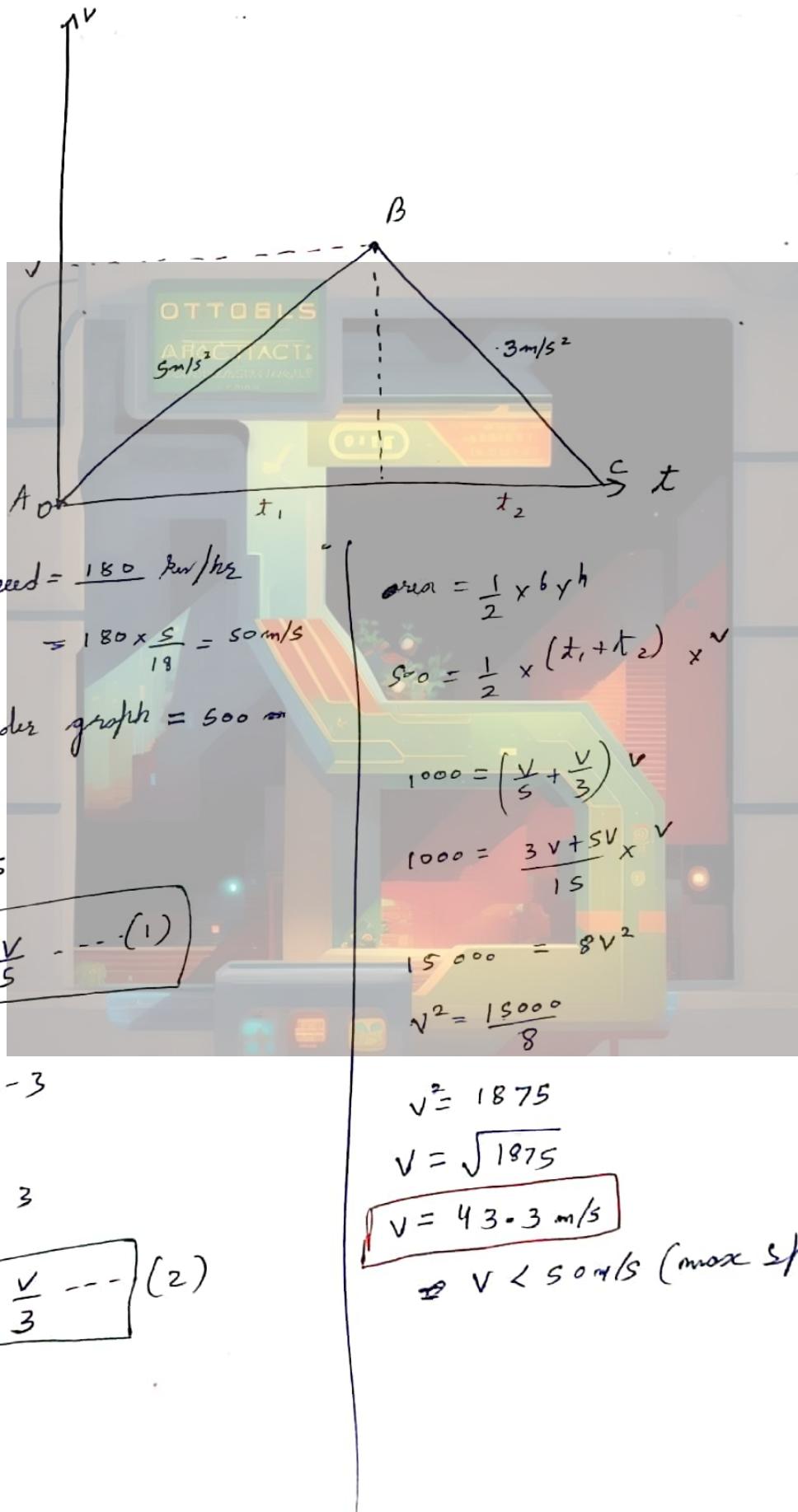
H.W.

11-05-2024

O-I (full)



Q A car can travel at maximum speed of 180 km/hr and can have maximum acceleration  $5 \text{ m/s}^2$  and retardation  $3 \text{ m/s}^2$ . How fast can it start from rest and come to rest in travelling 500 m?



$$t = t_1 + t_2$$

$$= \frac{v}{5} + \frac{v}{3}$$

$$= \frac{3v + 5v}{15}$$

$$= \frac{8v}{15}$$

$$= \frac{8}{15} \times 43.3$$

$$= 0.53 \times 43.3$$

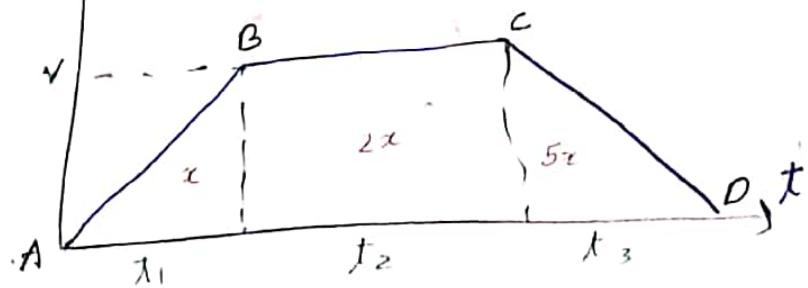
$$= 22.949 \text{ s}$$

$$\boxed{t = 22.949 \text{ s}}$$

$$\begin{array}{r}
 & 1 & 1 \\
 4 & 8 & 3 & \cdot & 3 \\
 & 5 & 3 \\
 \hline
 & 1 & 2 & 9 & 9 \\
 \hline
 & 2 & 1 & 6 & 5 & 0 \\
 \hline
 & 2 & 2 & 9 & 4 & 9
 \end{array}$$

$$\begin{array}{r}
 15 ) 80 ( 0.53 \\
 - 75 \\
 \hline
 50 \\
 - 45 \\
 \hline
 5
 \end{array}$$

Q A particle starts from rest and travels a distance  $x$  with uniform acceleration, then moves uniformly a distance  $2x$  and finally comes to rest after moving further distance  ~~$x$~~  so  $5x$  with uniform deceleration. If the ratio of average speed to maximum speed is  $R/7$ . Then find value of  $R$ .



(190)

$$\text{average speed} = \frac{x + 2x + 5x}{t}$$

$$= \frac{8x}{t}$$

~~max speed~~

$$\frac{1}{2} \times v \times t_1 = x$$

$$\frac{v}{2x} = \frac{1}{t}$$

$$t_1 = \frac{2x}{v}$$

$$t_2 = \underline{4x}$$

$$t_2 \times v = 2x$$

$$t_2 = \frac{2x}{v}$$

$$t_3 \times \frac{1}{2} \times v = 5x$$

$$t_3 = \frac{10x}{v}$$

$$t = \frac{2x + 2x + 10x}{v}$$

$$t = \frac{14x}{v}$$

$$v = \frac{14x}{t}$$

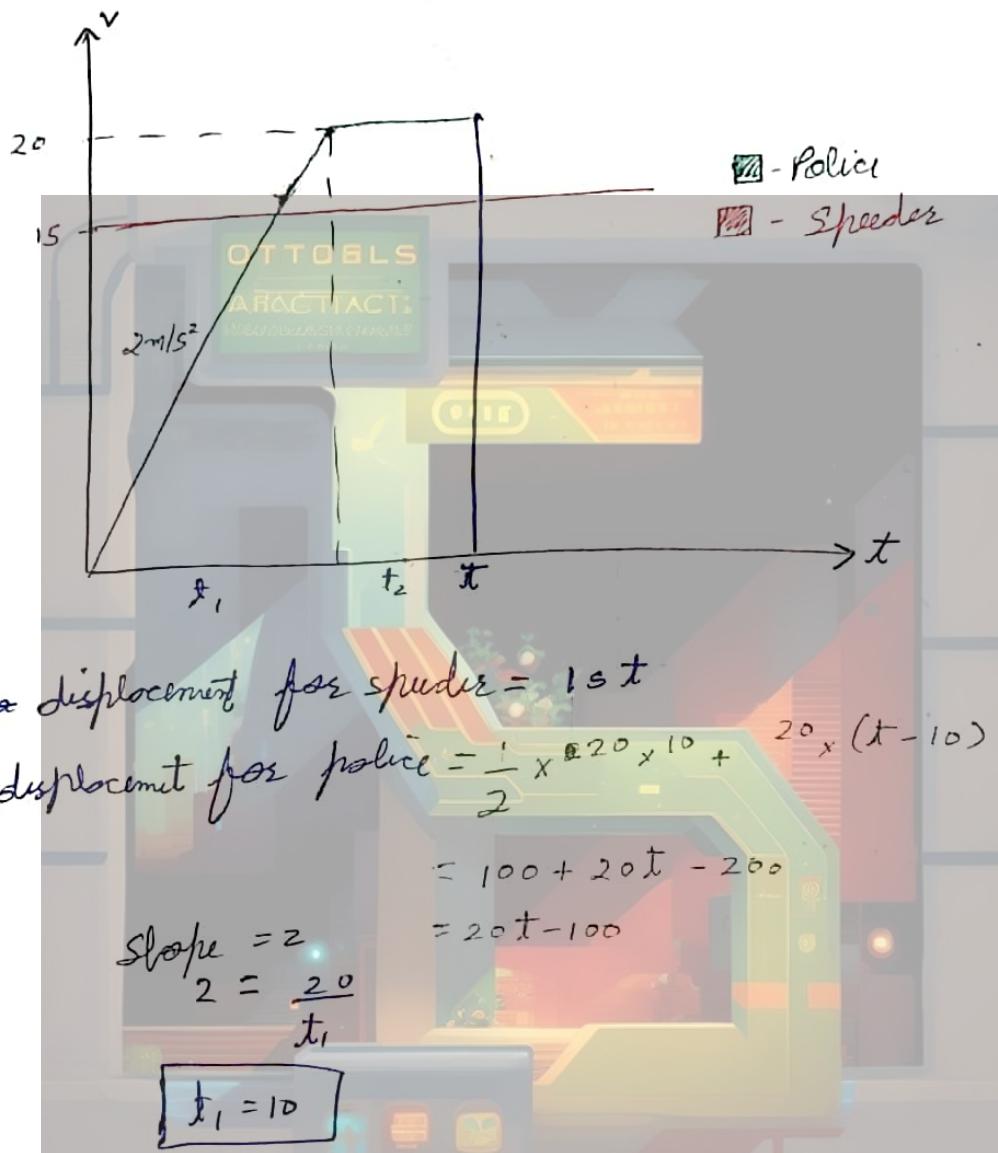
$$\text{max speed} = \frac{14x}{t}$$

$$\frac{\frac{8x}{t}}{\frac{14x}{t}} = \frac{k}{7}$$

$$\frac{8x}{14x} = k$$

$$k = 4$$

Q A speeder moves at a constant speed  $15 \text{ m/s}$  in a school zone. A police car starts from rest just as the speeder passes it. The police car accelerates at  $2 \text{ m/s}^2$  until it reaches its maximum velocity of  $20 \text{ m/s}$ . When and where does the speeder get caught?



$$15t = 20t - 100$$

$$100 = 5t$$

$t = 20 \text{ s}$

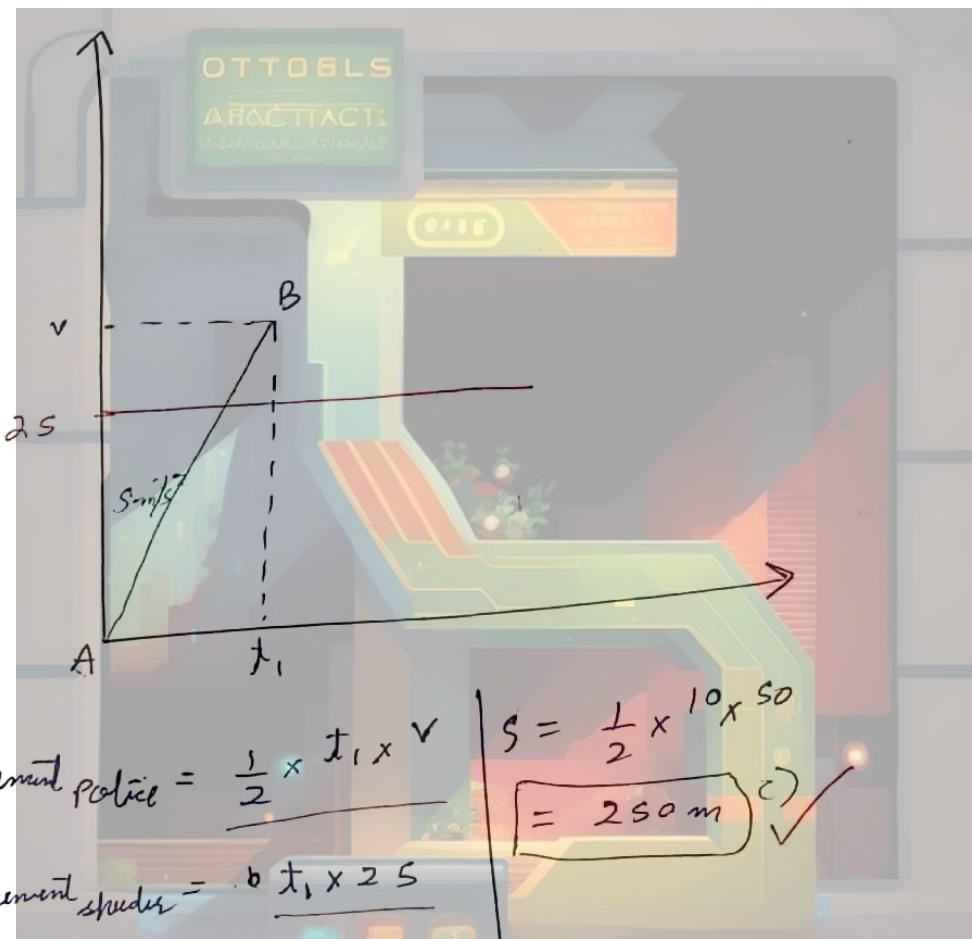
$$S = 15t$$

$$= 15 \times 20$$

$= 300 \text{ m}$

Q A car is speeding at 25 m/s in a low speed zone. A police car starts from rest as the speedster passes and accelerates at a constant rate of  $5 \text{ m/s}^2$ .

- When does the police car catch the speeding car?
- How fast is the police car at the instant when it catches up with the speedster?
- How far have the cars travelled when the police car catches the speedster?

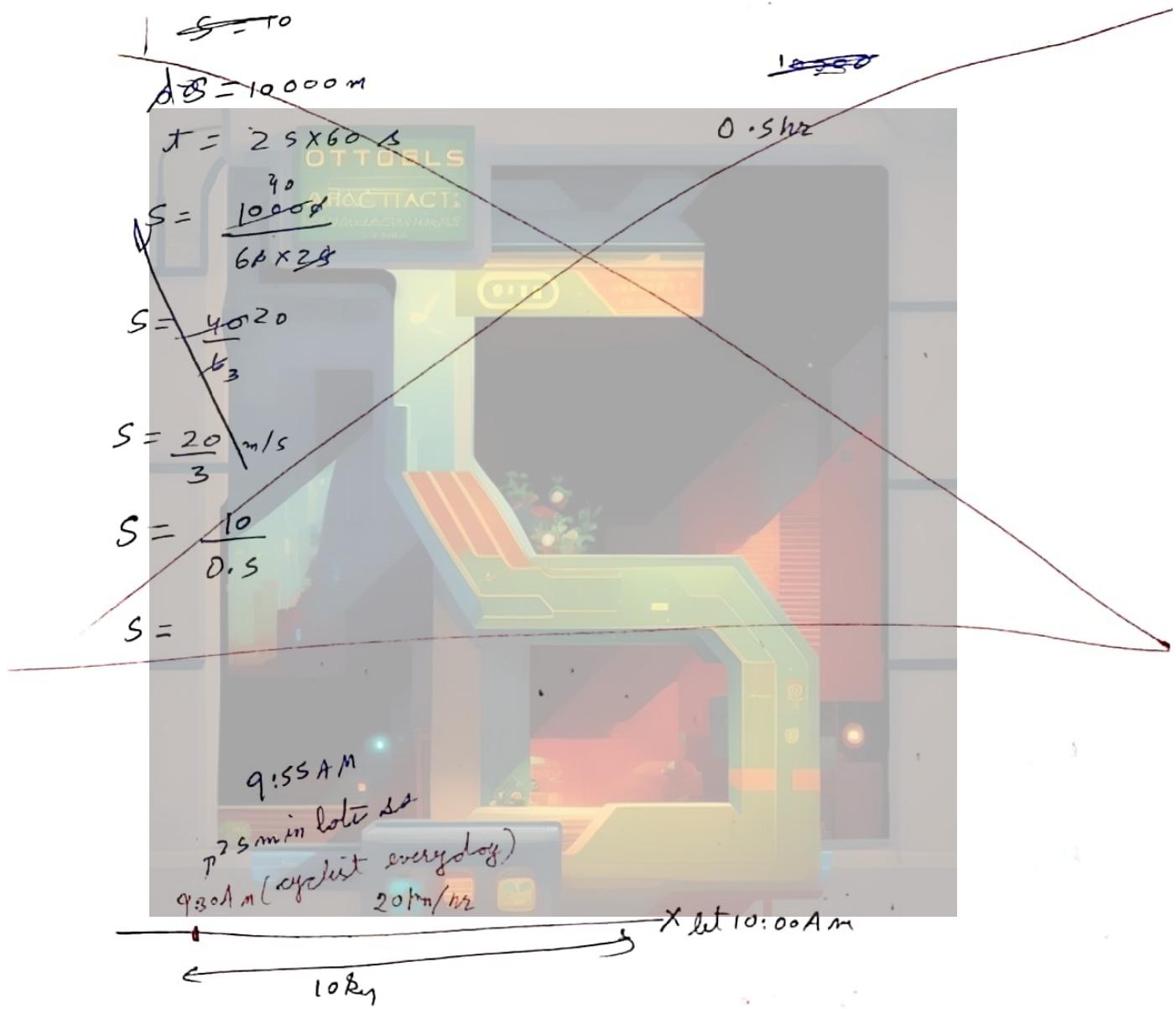


$$\text{slope}_{AB} = \frac{v}{t_1} = s$$

$$50 = st_1$$

$$t_1 = 10 \text{ s}$$

Q A railway track runs parallel to a road until a train  
 brings the road to railway crossing. A cyclist rides  
 along the road everyday at a constant speed of 20 km/hr. He normally  
 meets a train that travels in some direction at the crossing -  
 One day he was late by 25 minutes and met the train 10 km  
 before crossing. find train speed.



$$T_{\text{late}} = \frac{s_{\text{min}}}{s} = \frac{25}{60} \text{ hr}$$

$$\text{speed} = \frac{10^2}{60} \times 60$$

$$= 120 \text{ km/hr}$$

① A body starts with an initial velocity of  $10 \text{ m/s}$  and moves along a straight line with constant acceleration  $a$  ( $a < 0$ ) reversed in direction. Find the velocity of the particle when it reaches the starting point.

$$S = \frac{s_0^2 - v_0^2}{2a}$$

$$S = \frac{2s_0v_0 - v_0^2}{2a}$$

$$S = \frac{2s_0v_0}{2a}$$

$$S = \frac{1200}{a}$$

$$s_0 - 10 = at$$

$$\frac{v_0}{a} = t$$

$$S = \frac{1200}{a}$$

$$u = s_0$$

$$v = ?$$

$$2 \times a \sqrt{\frac{1200}{a}} = v^2 - 2s_0$$

$$2500 + 2400 = v^2$$

$$4900 = v^2$$

$$V = 70 \text{ m/s}$$

$$u = 60$$

$$S = \frac{1200}{a}$$

$$\frac{1200}{a} = s_0 + \frac{1}{2} a t^2$$

$$\frac{-10}{a} = t \quad 0 = \frac{-10}{a}$$

$$\frac{1200}{a} = s_0 + \frac{1}{2} (-\frac{10}{a}) t^2$$

$$\frac{-1200t}{40} = s_0 t + 20t$$

$$-1200t = 2800t$$

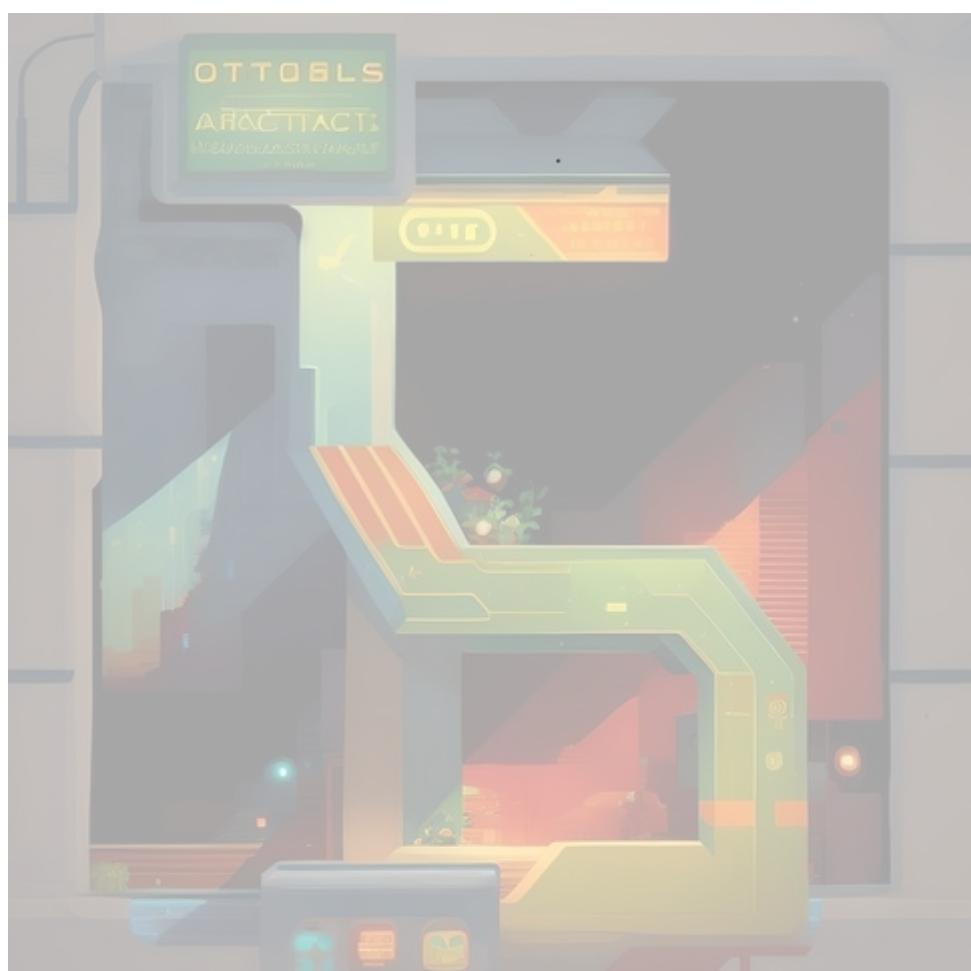
$$2800t + 1200t = 0$$

$$t = 0.8$$

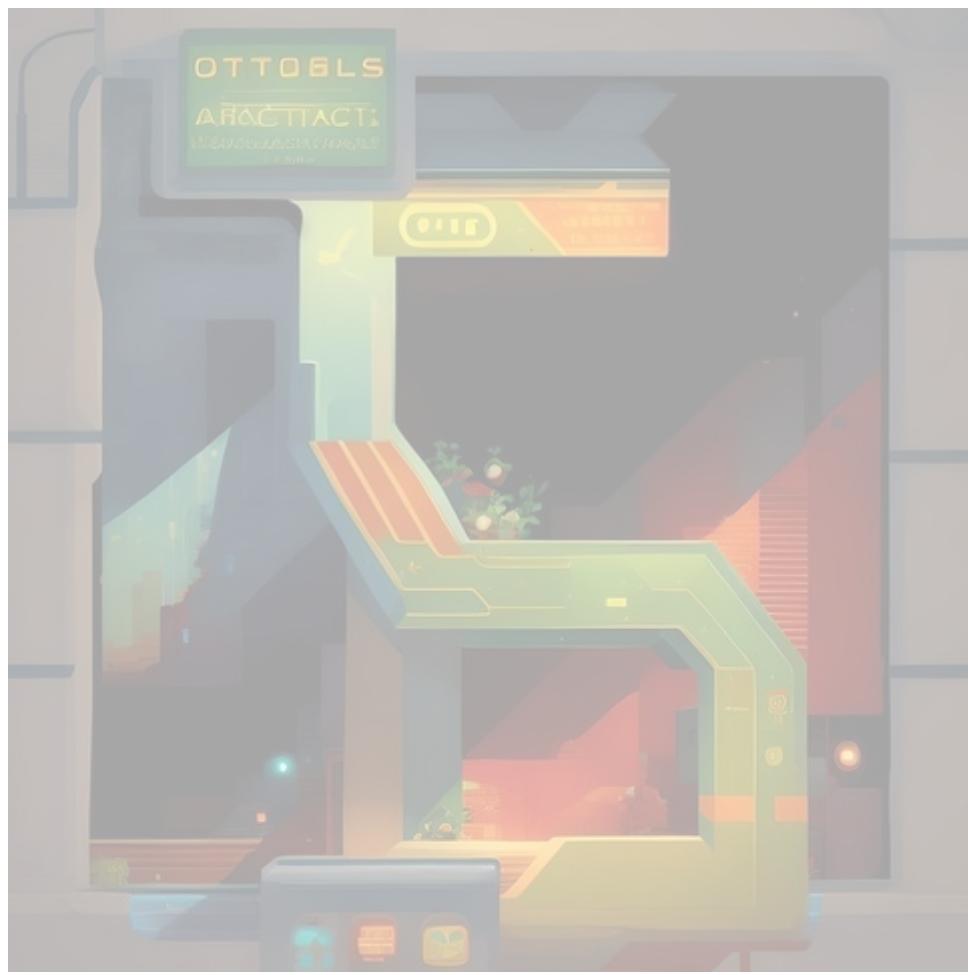




(197)



(198)



# !! Projectile Motion !!

Projectile motion - A ~~for~~ object has ~~that~~ got ~~it~~ and it lands somewhere else.

→ the object always moves in a single plane

→ The object moves in parabola path.

→ The object moves only under gravity.

→ The object moves only under gravity, (ignores earth's curvature, air resistance etc.)

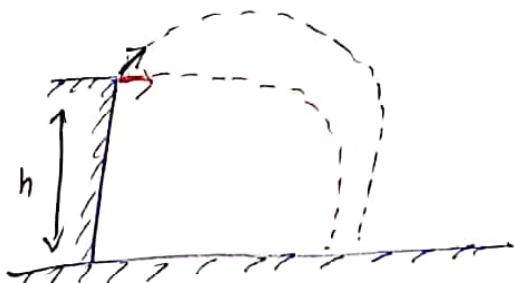
→ A projectile is any body that is given an initial velocity and then follows a path determined entirely by effects of acceleration due to gravity.

Types of projectile Motion -

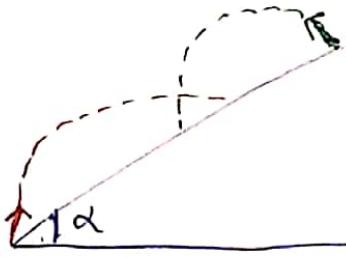
① Ground to ground (oblique) :-



② Projection from a height



### ③ ~~projection~~ on a plane:-

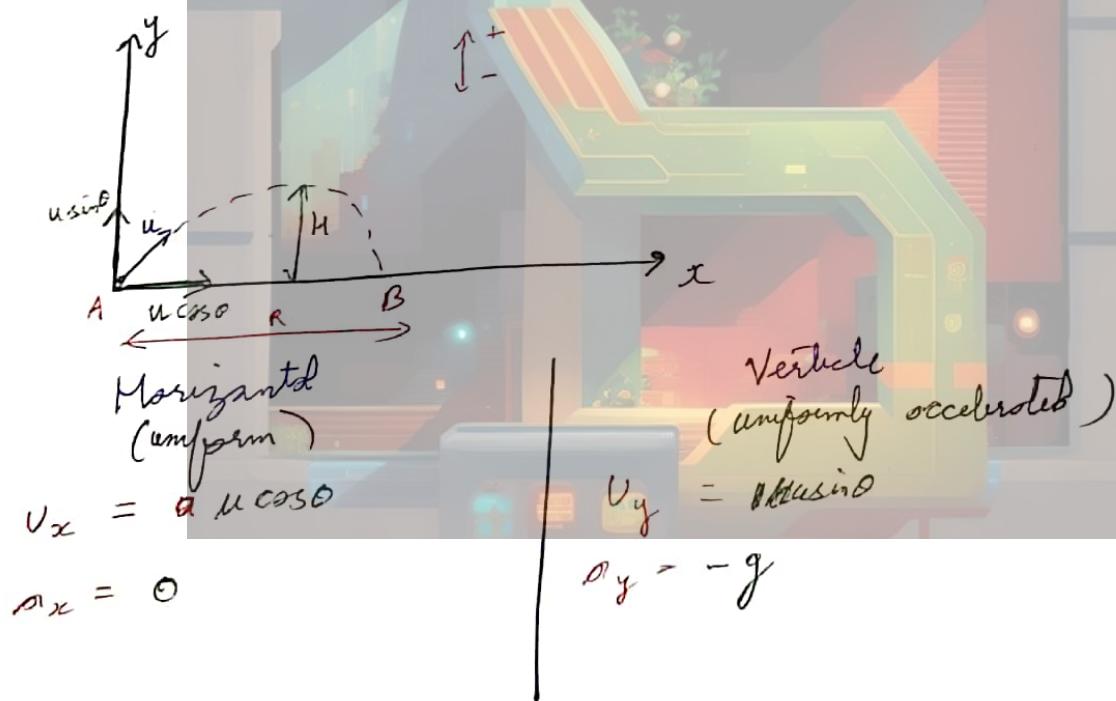


Projectile motion -

↓  
2 dimensional motion (2 d)  
(motion in a plane)

Horizontal (uniform motion) ← independent of each other. → Vertical (uniformly accelerated motion)

Oblique projection :- (ground to ground projection)



## Time of flight ( $T$ )

$$s_y = 0$$

Using  $s = ut + \frac{1}{2} at^2$

$$0 = u \sin \theta (T) + \frac{1}{2} (-g) T^2$$

$$0 = T \left( u \sin \theta - \frac{g T}{2} \right)$$

$T = 0$  (at point of projection)

$$u \sin \theta = \frac{g T}{2}$$

$$\frac{2u \sin \theta}{g} = T \quad (\text{at } B)$$

$$T = \frac{2u \sin \theta}{g}$$

## Highest Point ( $H$ )

At highest point,  $v_y = 0$

Using  $v^2 = u^2 + 2as$

$$(v_y)^2 = (u_y)^2 + 2 \cdot g \cdot H$$

$$0 = (u \sin \theta)^2 + 2 \cdot g \cdot H$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

for fixed  $u$ ,  
maximum height  $H$  will be  
for  $\sin \theta = 1$

$$\theta = 90^\circ$$

## Range (R)

$$R = U_x T$$

$$R = \frac{U \cos \theta \times 2 U \sin \theta}{g}$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

For complimentary angle of projection

$$\theta \rightarrow (90 - \theta)$$

$$R' = \frac{U^2 \sin [2(90 - \theta)]}{g}$$

$$R' = \frac{U^2 \sin (180 - 2\theta)}{2}$$

$$R' = \frac{U^2 \sin 2\theta}{g}$$

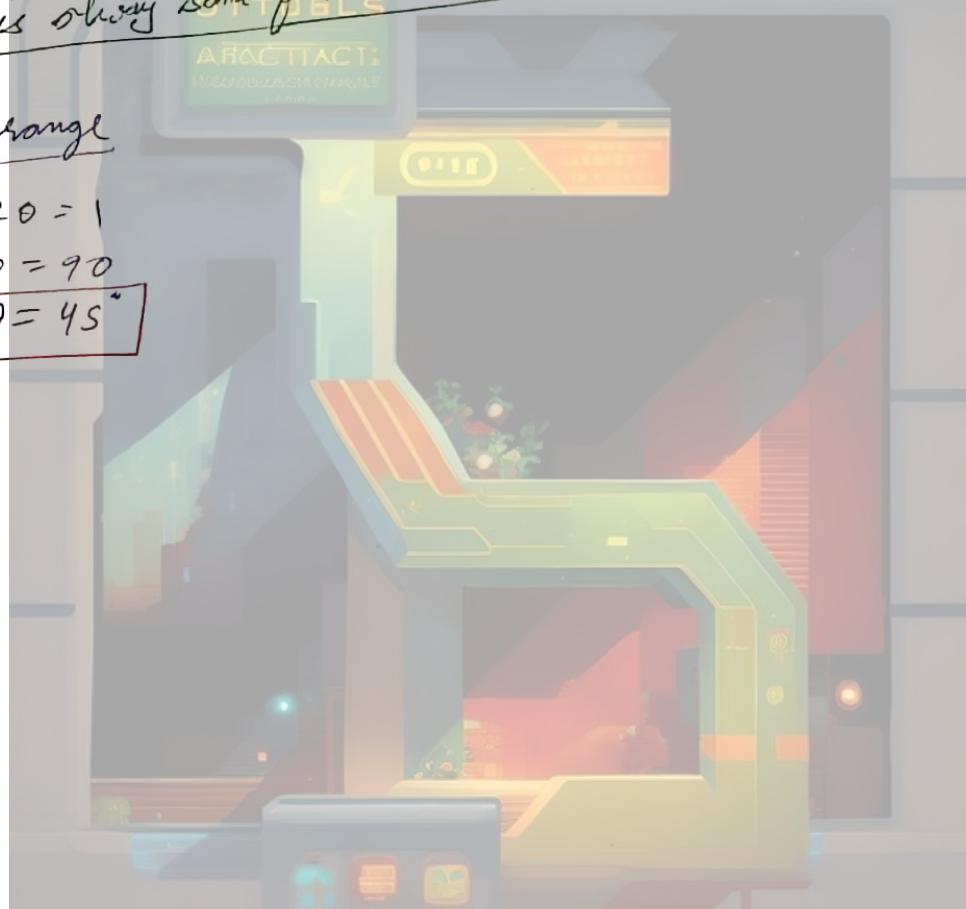
Range is always same for complimentary angles.

For max range

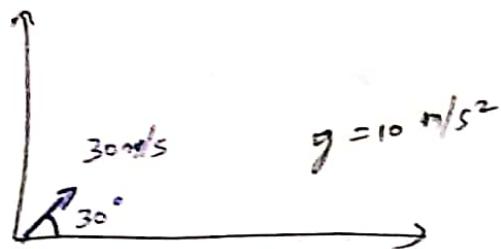
$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$



Q



a) time at which ball reaches highest point

- ~~b)~~) maximum height reached
- c) horizontal range
- d) time of flight

$$T = \frac{2u \sin \theta}{g}$$

$$= \frac{60 \times \frac{1}{2}}{9.8}$$

$$= \frac{30}{9.8}$$

time to reach max height =  $\frac{30}{9.8} \times \frac{1}{2} = \boxed{\frac{15}{9.8} \text{ s}}$  d)

$$= \frac{15}{10}$$

$$= 1.5 \text{ s}$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$= \frac{15 \times 15}{30 \times 30} \times \frac{1}{2}$$

~~$$= \frac{225}{196} \text{ m}$$~~

$$= \frac{225}{20}$$

$$= \frac{112.5}{10}$$

$$\boxed{= 11.25 \text{ m}} \quad \checkmark$$

Q)  $R = \frac{u^2 \sin 2\theta}{g}$

$$= 30 \times 30 \times \frac{\sqrt{3}}{2}$$

~~$$= 45\sqrt{3} \text{ m}$$~~

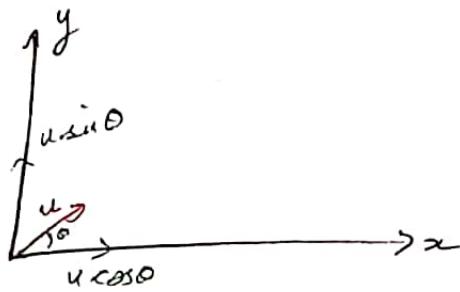
$$\boxed{c)} \quad \checkmark$$

$$T = 1.5 \times 2$$

$$\boxed{= 3 \text{ s}} \quad \checkmark$$

Q 04

Velocity at ~~time~~ time 't' -



$$u_x = u \cos \theta$$

$$v_x = u \cos \theta$$

$$u_y = u \sin \theta$$

$$\bullet v_y = u_y + \alpha_y t$$

$$v_y = u \sin \theta - gt$$

$$\vec{v} = (u \cos \theta) \hat{i} + (u \sin \theta - gt) \hat{j}$$

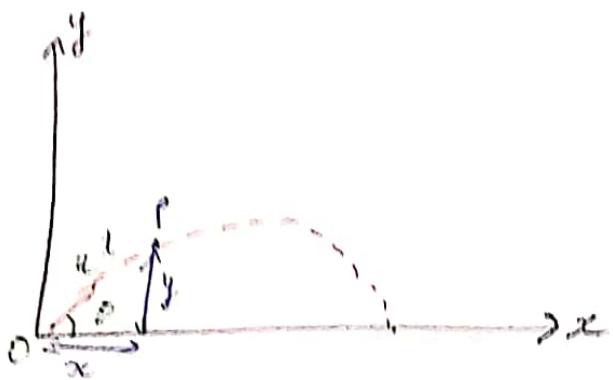
A ball is projected with velocity  $u$  at angle of projection  $\theta$ . After what time ball is moving at right angles to initial direction.  
initial direction =  $\theta = \tan^{-1} \frac{u \sin \theta}{u \cos \theta} = u \sin \theta \hat{i} + u \cos \theta \hat{j}$

$$\vec{v} = (u \cos \theta) \hat{i} + (u \sin \theta - gt) \hat{j}$$
$$\neq \frac{u^2 \cos^2 \theta + u \sin \theta (u \sin \theta - gt)}{\mu \sqrt{u^2 \cos^2 \theta + u^2 \sin^2 \theta + g^2 t^2 - 2 u \sin \theta g t}}$$
$$= u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta g t$$
$$= u^2 - u \sin \theta g t$$
$$\frac{u \sqrt{u^2 + g^2 t^2 - 2 u \sin \theta g t}}{u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta g t}$$
$$= u - \sin \theta g$$
$$\vec{u}, \vec{v} = 0$$
$$u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta g t = 0$$

$$u(u - \sin \theta g t) = 0$$

$$\boxed{t = \frac{u}{\sin \theta g}}$$

# Equation of trajectory



$$x = u_x t \quad \left| \begin{array}{l} s_y = u_y t + \frac{1}{2} a_y t^2 \\ s_y = u \sin \theta (t) - \frac{1}{2} g t^2 \end{array} \right. \quad (1)$$

$$y = \frac{u \sin \theta (x)}{u \cos \theta} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

$$y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta}$$

$$\boxed{y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta}}$$

$$y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta} \quad \frac{\sin \theta}{\sin \theta}$$

$$y = x \tan \theta - \frac{x^2 \cancel{\sin \theta \tan \theta}}{2u^2 \sin \theta \cos \theta} \quad \boxed{\text{Range formula}}$$

$$y = x \tan \theta - \frac{x^2 \tan \theta}{R}$$

$$\boxed{y = x \tan \theta \left[ 1 - \frac{x}{R} \right]}$$

Q A grasshopper can jump its height 'h', find the maximum distance through which it can jump along horizontal ground.

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$h = \frac{u^2}{2g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

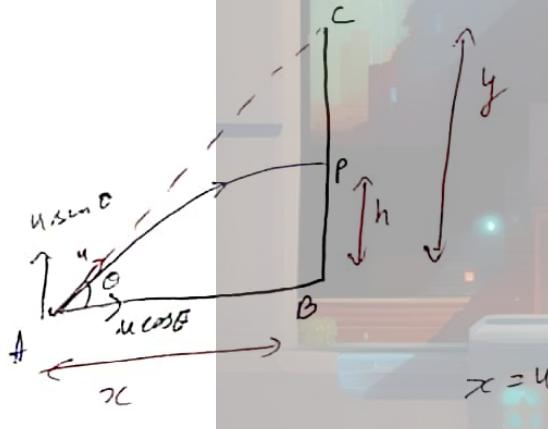
$$\sqrt{2gh} = u$$

$$R = \frac{2gh}{g} \times$$

$$R_{\text{max}} = 2h$$

OTTOSLS  
ARACTAIC  
MUSICAL INSTRUMENTS

Q A hunter aims his gun and fires a bullet directly on a monkey on a tree. At the instant bullet is fired, the monkey drops. Will the bullet hit the monkey?



$$\tan \theta = \frac{y}{x}$$

for bullet,

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$h = u \sin \theta t - \frac{1}{2} g t^2$$

$$x = u \cos \theta t \Rightarrow t = \frac{x}{u \cos \theta}$$

$$h = u \sin \theta \left( \frac{x}{u \cos \theta} \right) - \frac{1}{2} g t^2$$

$$h = x \tan \theta - \frac{1}{2} g t^2$$

$$h = y - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 = y - h$$

For monkey

$$z = 0 + \frac{1}{2} g t^2$$

$$z = y - h$$

As, the height is same for monkey & bullet at some time,  
Yes the bullet will hit monkey.

Q The range of a projectile fired at an angle  $15^\circ$  is 40 m. If it is fired with the same speed at an angle of  $45^\circ$  find its range.

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R \Rightarrow$$

$$u_0 = \frac{u^2 \times \frac{1}{2}}{g_0}$$

$$400 \times 2 = u^2$$

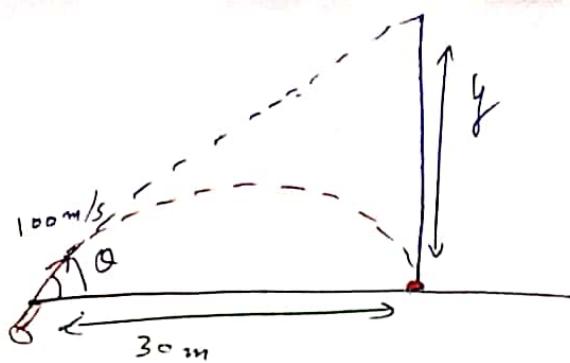
$$20\sqrt{2} = u$$

$$R' = \frac{800 \times \sin 45^\circ}{g}$$

$$R' = \frac{800}{10}$$

$$\boxed{b) R' = 80 \text{ m}}$$

Q



At how high above the target  
the gun must be aimed to hit  
the target on ground at 30 m  
from the gun?

Find  $y$

$$R = \frac{100 \times 10 \phi \sin 2\theta}{g}$$

$$R = 1000 \sin 2\theta$$

$$\tan \theta = \frac{4}{3}$$

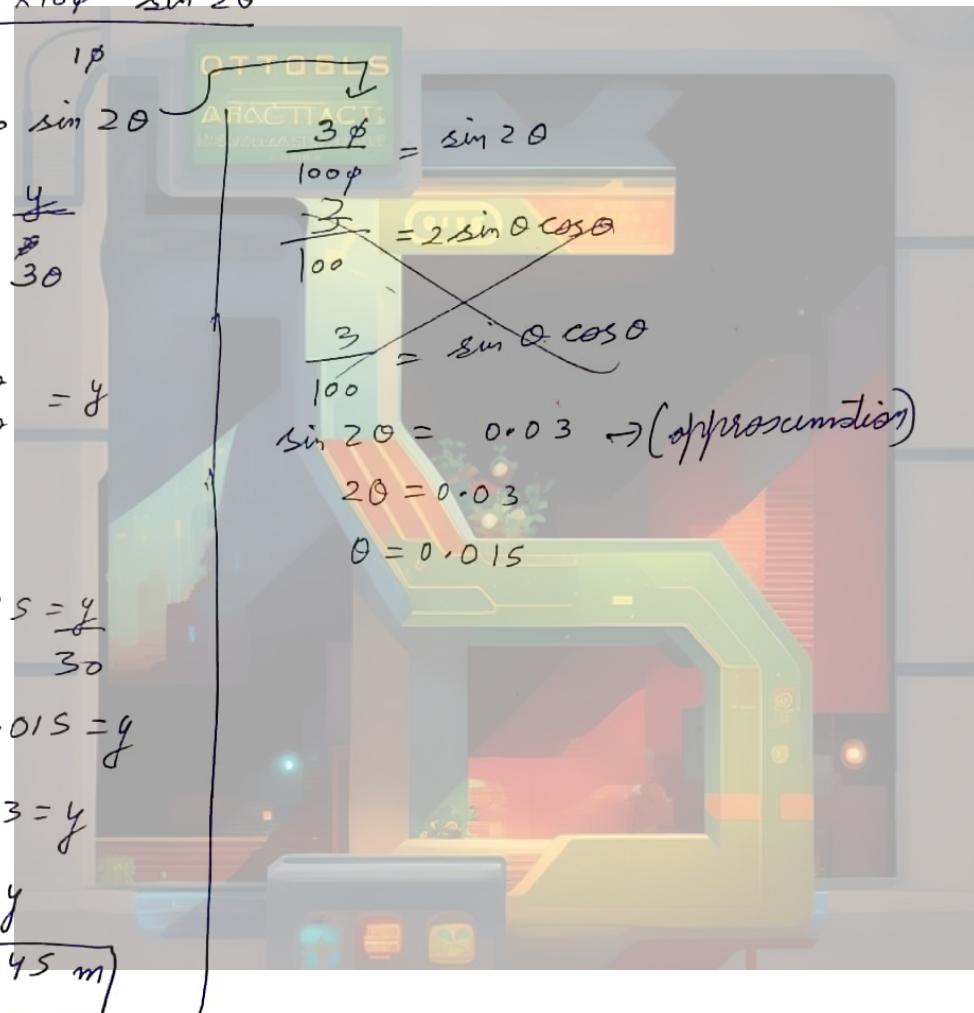
$$30 \frac{\sin \theta}{\cos \theta} = y$$

$$\tan 0.015 = \frac{y}{30}$$

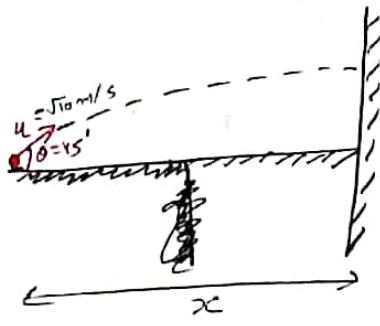
$$30 \times 0.015 = y$$

$$0.45 = y$$

$$\boxed{y = 0.45 \text{ m}}$$



Q



$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{10 \times 1}{10}$$

$$R = 1 \text{ m}$$

- a)  $x = y_2 \text{ m}$  ✓ will water hit wall  
 b)  $x = 2 \text{ m}$  X  
 c) where does it hit wall  
 d) max height

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = 10 \times \frac{1}{2 \times 10}$$

$$H = \frac{1}{4} \text{ m}$$

$$y = x \tan \theta \left[ 1 - \frac{x}{R} \right]$$

$$y = \frac{1}{2} \times 1 \left[ 1 - \frac{1}{1} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{2} \right]$$

$$= \frac{1}{4} \text{ m}$$

$$\boxed{y = \frac{1}{4} \text{ m}}$$

OE

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$= \frac{1}{2} \times 1 - \frac{10 \times \frac{1}{4}}{2 \times 10 \times \frac{1}{2}}$$

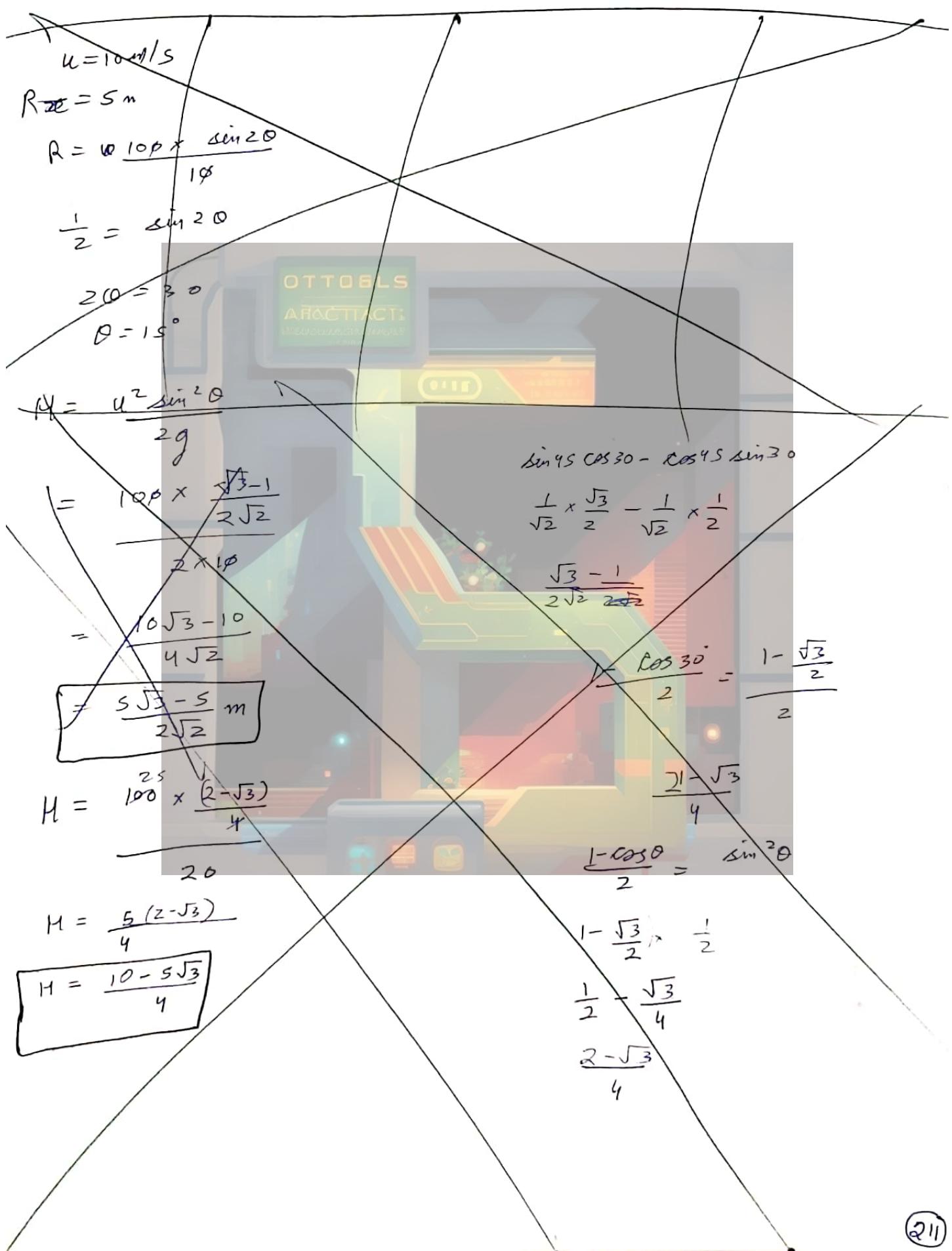
$$= \frac{1}{2} - \frac{\frac{5}{2}}{10}$$

$$= \frac{1}{2} - \frac{5}{20} = \frac{9}{4}$$

$$\boxed{= \frac{1}{4} \text{ m}}$$

210

Q we have a hose pipe which disposes water at speed of  $10 \text{ m/s}$ .  
 The safe distance from a building on fire, on ground, is  $5\text{m}$ . How  
 high can this water go. ( $g = 10 \text{ m/s}^2$ )



$$y = x \tan \theta - \frac{g x^2}{2 u^2 \sec^2 \theta}$$

$$y = s \tan \theta - \frac{10 \times 25^2}{2 \times 10 \times 16 \times \sec^2 \theta}$$

$$y = s \tan \theta - \frac{s \sec^2 \theta}{4}$$

~~first diff~~ max height at  $\theta = \frac{\pi}{4}$

$$\frac{dy}{d\theta} = 0$$

~~first diff~~ change from stationary

$$\frac{dy}{d\theta} = s \sec^2 \theta - \frac{s \sec^2 \theta \tan \theta}{4}$$

$$= s \sec^2 \theta - \frac{s}{2} \sec^2 \theta \tan \theta$$

$$\frac{s}{2} \sec^2 \theta = \cancel{s} \sec^2 \theta$$

$$\frac{\tan \theta}{2} = 0$$

$$\tan \theta = 0$$

$$\tan \theta = 2$$

$$P = 2$$

$$B = 1$$

$$H = \sqrt{5}$$

$$\sec \theta = \frac{H}{B}$$

$$= \sqrt{5}$$

$$y_{\text{max}} = \cancel{(15)} 5 \times 2 - \frac{s}{4} \times 5$$

$$= 10 - \frac{25}{4}$$

$$= \frac{15}{4} \text{ m}$$

Or

$$y = s \tan \theta - \frac{s}{4} \sec^2 \theta$$

$$y = s \tan \theta - \frac{s}{4} (1 - \tan^2 \theta)$$

$$y = s \tan \theta - \frac{s}{4} + \tan^2 \theta$$

$$\tan^2 \theta + s \tan \theta - \frac{5s+4y}{4}$$

for real roots

$$\tan^2 \theta - 0 \geq 0$$

$$25 + 5y \geq 0$$

$$s(s+y) \geq 0$$

$$s+y \geq 0$$

$$25 + s+4y \geq 0$$

$$30 + 4y \geq 0$$

$$y \geq -\frac{30}{4}$$

$$y \leq \frac{15}{4}$$

Q A shot is fired with a velocity 20 m/s at a vertical wall whose distance from point of projection is 20m. Find the greatest height above the level of point of projection at which bullet can ~~not~~ hit the wall.

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$y = 20 \tan \theta - \frac{10 \times 20 \times 20}{2 \times 20 \times 20 \times \cos^2 \theta}$$

$$y = 20 \tan \theta - 5 \sec^2 \theta$$

$$\frac{dy}{d\theta} = 20 \sec^2 \theta - 10 \sec^2 \theta \tan \theta$$

$$0 = 2 \sec^2 \theta - \sec^2 \theta \tan \theta$$

$$\sec^2 \theta (2 - \tan \theta)$$

$$2 - \tan \theta = 0$$

$$\tan \theta = 2$$

$$P = 2$$

$$B = 1$$

$$u = \sqrt{5}$$

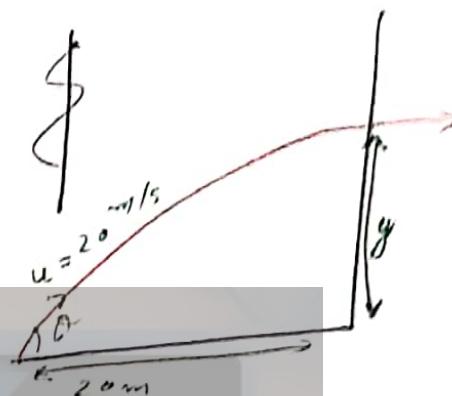
$$\sec \theta = \sqrt{5}$$

$$\sec^2 \theta = 5$$

$$y_{max} = 20 \times 2 - 5 \times 5$$

$$= 40 - 25$$

$$= 15 \text{ m}$$



OR

$$y = 20 \tan \theta - 5 \sec^2 \theta$$

$$= 20 \tan \theta - 5(1 + \tan^2 \theta)$$

$$y = 20 \tan \theta - 5 + 5 \tan^2 \theta$$

$$5 \tan^2 \theta - 20 \tan \theta + 5 + 20 =$$

$$400 - (4)(5)(y + 5) \geq 0$$

$$400 - 20y - 100 \geq 0$$

$$-20y + 300 \geq 0$$

$$-20y \geq -300$$

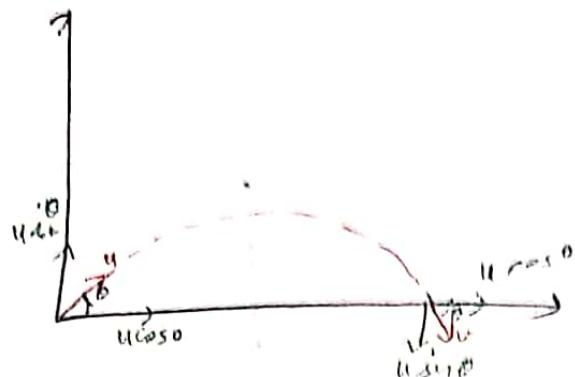
$$20y \leq 300$$

$$y \leq \frac{300}{20}$$

$$y \leq 15$$

Change in momentum between any two positions of projectile

I) Between point of projection & highest point :-



$$\begin{aligned}\vec{u} &= u \cos \theta \hat{i} + u \sin \theta \hat{j} \\ \vec{v} &= u \cos \theta \hat{i} \\ \Delta \vec{p} &= m \vec{v} - m \vec{u} \\ &= m (\vec{v} - \vec{u}) \\ \Delta \vec{p} &= m (-u \sin \theta \hat{j}) \\ \boxed{\Delta \vec{p} = m (-u \sin \theta \hat{j})} \\ |\Delta \vec{p}| &= m u \sin \theta\end{aligned}$$

2) Between initial & final points.

$$\begin{aligned}\vec{u} &= u \cos \theta \hat{i} + u \sin \theta \hat{j} \\ \vec{v} &= u \cos \theta \hat{i} - u \sin \theta \hat{j} \\ \Delta \vec{p} &= m (\vec{v} - \vec{u}) \\ &= m (-2 u \sin \theta \hat{j}) \\ \Delta \boxed{\Delta \vec{p} = -2 m u \sin \theta \hat{j}} \\ |\Delta \vec{p}| &= 2 m u \sin \theta\end{aligned}$$

Q A object of mass 0.5 kg is projected under gravity with a speed  $u$  of 98 m/s at an angle of  $60^\circ$ . find magnitude of change in momentum after 10s ( $g = 9.8 \text{ m/s}^2$ )

$$m = 0.5 \text{ kg}$$

$$u = 98 \text{ m/s}$$

$$\theta = 60^\circ$$

$|\Delta \vec{p}| =$

$$\theta u = 98 \sin 60^\circ \uparrow$$

$$t = 10 \text{ s}$$

$$g = 9.8$$

$$v = 98 + 98 \sin 60^\circ \uparrow$$

$$v = 49\sqrt{3} + 98$$

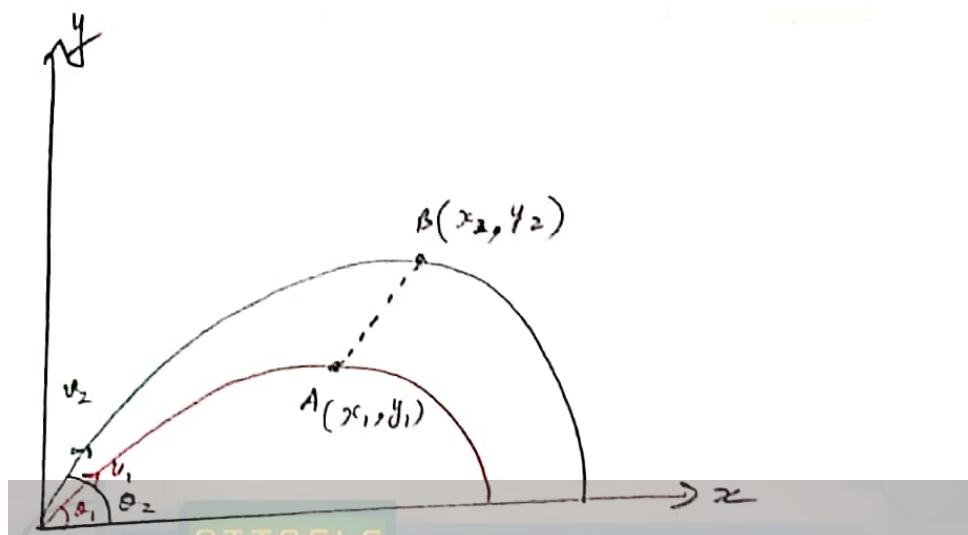
$$\vec{v} = 49\sqrt{3} + 98 \hat{j} + u \cos 60^\circ \hat{i}$$

$$|\Delta \vec{p}| = m (49\sqrt{3} + 98 - 49\sqrt{3})$$

$$= 98 \times 0.5$$

$$|\Delta \vec{p}| = 49 \text{ kg m/s}$$

~~Path of one projectile in reference of another projectile~~



$$\left. \begin{array}{l} x_1 = u_1 \cos \theta_1 t \\ x_2 = u_2 \cos \theta_2 t \end{array} \right\} x_2 - x_1 = (u_2 \cos \theta_2 - u_1 \cos \theta_1) t$$

$$\left. \begin{array}{l} y_1 = u_1 \sin \theta_1 t - \frac{1}{2} g t^2 \\ y_2 = u_2 \sin \theta_2 t - \frac{1}{2} g t^2 \end{array} \right\} = u_2 \sin \theta_2 - u_1 \sin \theta_1$$

$$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \frac{u_2 \sin \theta_2 - u_1 \sin \theta_1}{u_2 \cos \theta_2 - u_1 \cos \theta_1}}$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$\hookrightarrow$  constant

If  $u_1 \cos \theta_1 = u_2 \cos \theta_2$

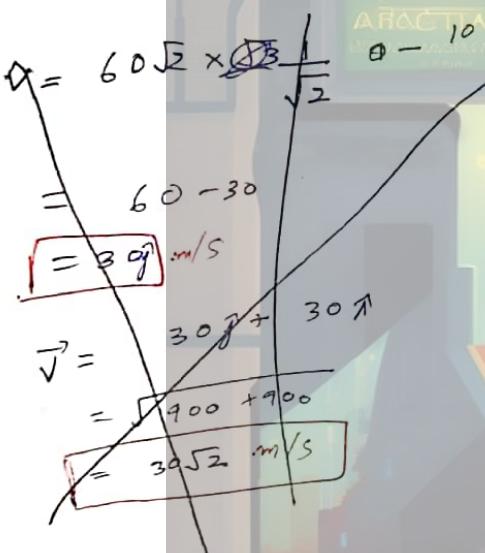
$$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \infty \infty \text{ (vertical straight line)}}$$

Q.

$$\frac{60\sqrt{2}}{10\sqrt{5}}$$

find after 3s.

- velocity of ball
- angle made by ball with horizontal
- horizontal & vertical displacement.



$$\begin{aligned}\vec{v} &= (u \cos \theta) \hat{i} + (u \sin \theta - gt) \hat{j} \\ &= 60\sqrt{2} \times \frac{1}{\sqrt{2}} \hat{i} + 60\sqrt{2} \times \frac{1}{\sqrt{2}} - 30 \hat{j} \\ &= 60 \hat{i} + 30 \hat{j}\end{aligned}$$

$$\begin{aligned}|\vec{v}| &= \sqrt{60^2 + 30^2} \\ &= \sqrt{3600 + 900} \\ &= \sqrt{4500} \\ &= 10\sqrt{45} \\ &= 30\sqrt{5} \text{ m/s}\end{aligned}$$

c) horizontal - ~~180 m~~  $180 \text{ m}$

$$\text{vertical } s = ut + \frac{1}{2}at^2$$

$$s = 60 \times 3 - \frac{1}{2} \times 9$$

$$s = 180 - 45$$

$$s = 135 \text{ m}$$

$$\cos \alpha = \frac{30\sqrt{5}}{60} \quad \theta = \tan^{-1} \frac{30}{60} = \frac{1}{2} \alpha$$

$$\theta = \tan^{-1} \left( \frac{1}{2} \right)$$

$$\alpha = \cos^{-1} \frac{\sqrt{5}}{2}$$

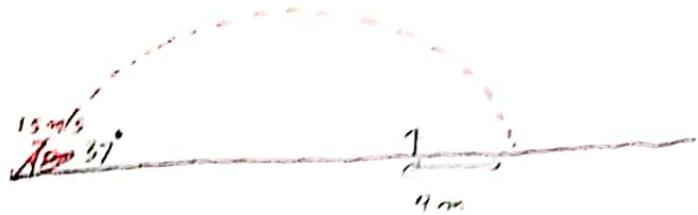
$$\cos \theta = \frac{\sqrt{2}}{\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$B = 1$$

$$\cos \alpha = \frac{6}{30\sqrt{5}}$$

(217)

Q With what minimum velocity man should run to catch the ball?  
( $g = 10 \text{ m/s}^2$ )



$$T = \frac{24 \sin \theta}{g}$$

$$\text{Speed} = \frac{9}{1.8}$$

$$= 5 \text{ m/s}$$

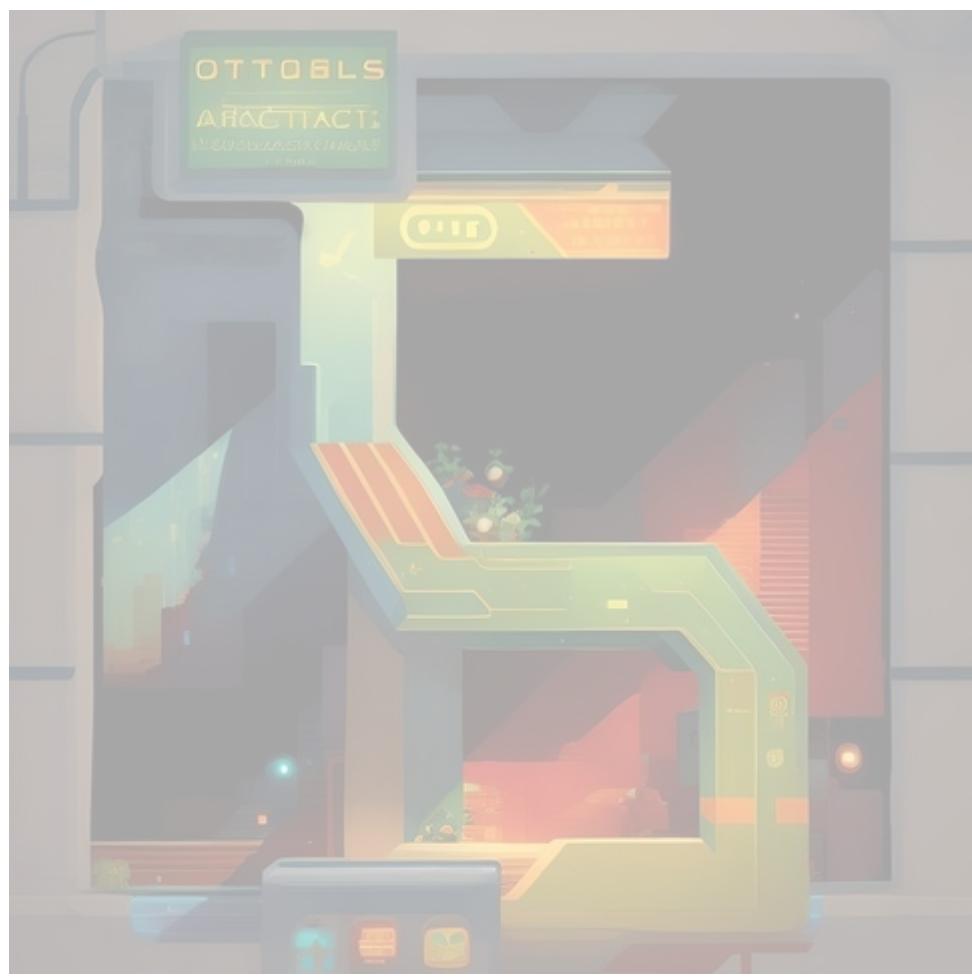
$$T = \frac{24 \sin 37}{10}$$

$$T = 1.86 \text{ s}$$









(22)

