System of Particles

Center of Mass

→ Center of mass of a body is defined as a point where total mass of body is supposed to be concentrated.

→ If force is applied ~~ofer~~ on center of mass, it performs translatory motion.

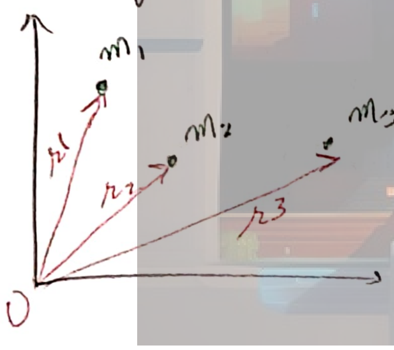
No change in orientation

→ In astronomy, barycenter is the center of mass of two or more bodies that orbit one another and is the point about which the bodies orbit.

→ In Solar System, The planets do not orbit around the sun but the barycentre.

center of mass of all planets & sun

Center of mass of n discrete particles:-



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 \ldots}{m_1 + m_2 + m_3 \ldots}$$

$$\boxed{\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i} \quad (1)$$

M: total mass of the system

$m_i\vec{r}_i$: Mass moment of particle w.r.t origin

x coordinate of center of mass

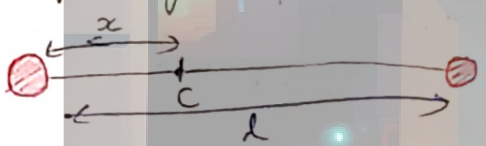
$$\boxed{x_{cm} = \frac{m_1x_1 + m_2x_2 \ldots}{m_1 + m_2 \ldots}}$$

- For a given body the center of mass is the average of position of all the masses that make up the obj.
- From this, It is easy to prove that position of COM lies somewhere ~~D~~ btw greatest & smallest value of r & therefore lies btw the ~~even~~ envelope of entire ~~dso~~ body.
- ~~D~~ COM does not have to be necessarily inside the material of the body as in the case of a circular ~~loop~~ ~~loo~~ loop, where COM is at center of loop & not in loop itself.

Center of Gravity:- It is a point where whole weight of body is supposed to be concentrated.

→ It generally coincides with COM of body for body lies in uniform gravitational field.

Position of COM of two particles:-



$$x_{cm} = \frac{m_1(0) + m_2(l)}{m_1 + m_2}$$

$$\boxed{x_{cm} = \frac{m_2 l}{m_1 + m_2}}$$

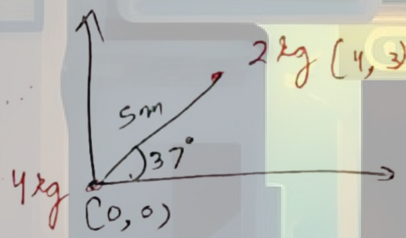
Q1. Two particles $m_1 = 1kg$ & $m_2 = 2kg$ at $x = 0$ & $x = 3m$

find COM

$$x_{cm} = \frac{2 \times 3}{1+2} = \frac{6}{3} = 2$$

$$\boxed{x_{cm} = 2m}$$

Q2. $m_1 = 4kg$ & $m_2 = 2kg$. find COM position.

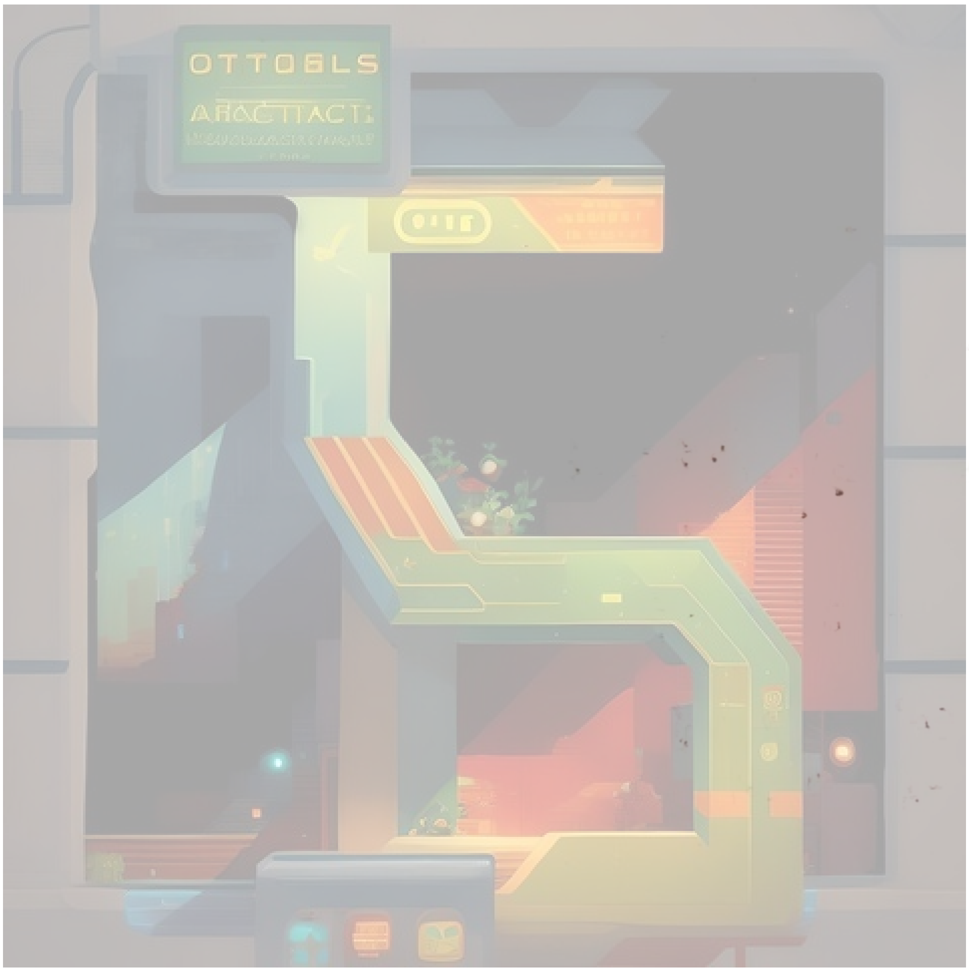


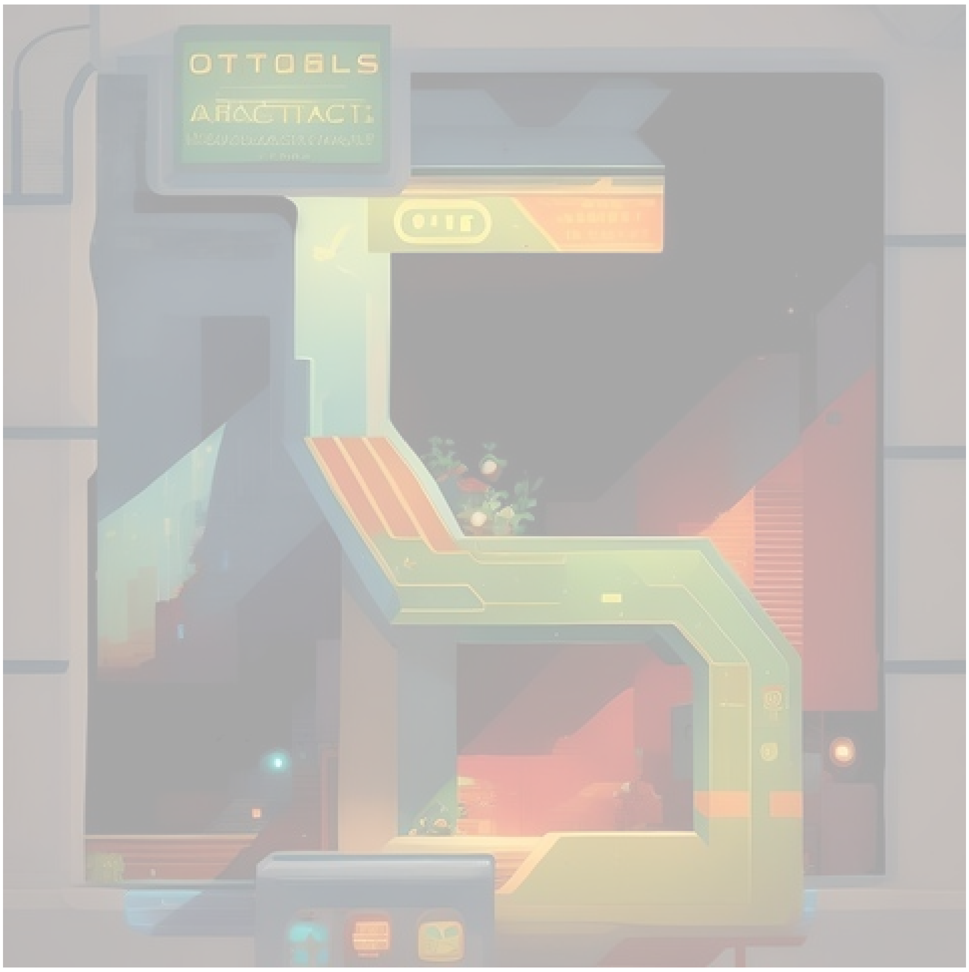
$$\vec{CM} = \frac{2kg \times (4\hat{i} + 3\hat{j})}{2+4}$$

$$= \frac{8\hat{i} + 6\hat{j}}{6}$$

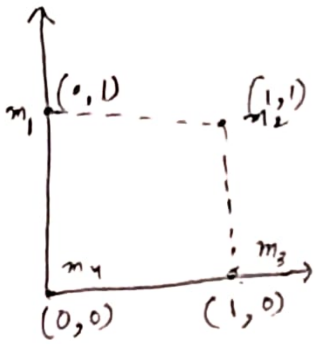
$$\vec{CM} = \frac{4}{3}\hat{i} + \hat{j}$$

$$\boxed{CM = \left(\frac{4}{3}, 1\right)}$$





Q3. find cordinates of center of mass of system of particles as shown.



$m_1 = 1kg$
$m_2 = 2kg$
$m_3 = 3kg$
$m_4 = 4kg$

$$CM = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4}{m_1 + m_2 \dots}$$

$$= \frac{1 \times \hat{j} + 2\hat{i} + 2\hat{j} + 3\hat{i} + 0}{10}$$

$$\cancel{= 4\hat{i}}$$

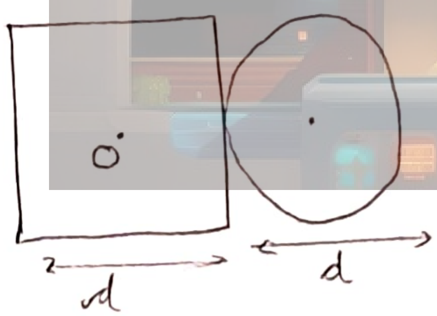
$$= \frac{5\hat{i} + 3\hat{j}}{10}$$

$$= 0.5\hat{i} + 0.3\hat{j}$$

$$\boxed{CM = (0.5, 0.3)}$$

Q4.

The two plates have same mass per unit area. Find position of COM from O (centre of square).



area :-

d^2, $\frac{\pi d^2}{4}$

mass :-

$m_1 = md^2$, $\frac{\pi m d^2}{4} = m_2$

$$\frac{m_2}{m_1} = \frac{\pi m d^2}{4} \times \frac{1}{md^2}$$

$$\boxed{m_2 = \frac{\pi}{4} m_1}$$

$$CM = \frac{m_1 d}{m_1 + m_2}$$

$$= \frac{\frac{R}{4} m_1 \times d}{\left(\frac{R}{4} + 1\right) m_1}$$

$$= \frac{Rd \times 4}{4(R+4)}$$

$$= \frac{Rd}{R+4}$$

$$\cancel{= d + \frac{Rd}{4}}$$

$$\boxed{CM = \frac{Rd}{R+4}}$$

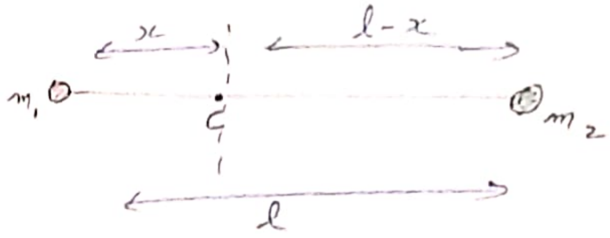
Mass Moment :-

→ It is defined as product of mass of particle and distance of particle from the point about which mass moment is taken. It is a vector quantity.

$$\boxed{\vec{r}_{cm} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i}$$

→ If com is taken as origin, $\sum_{i=1}^{n} m_i \vec{r}_i = 0$. Hence, com is the point about which the vector sum of mass moments of the system is zero.

→ The summation of mass moments of all the components of a system about its center of mass is always equal to zero.



$\rightarrow \vec{x}_{CM} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{x}_i$

$0 = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{x}_i \longrightarrow$ Vector sum of mass moments of all particles about CoM = 0

$\rightarrow m_1 x = m_2 (l - x)$

$m_1 x = m_2 l - m_2 x$

$m_1 x + m_2 x = m_2 l$

$\boxed{x = \frac{m_2 l}{m_1 + m_2}}$

Q 5. Two children A & B of same mass M are sitting on a see-saw. Their mass are M each. Initially, beam is Horizontal. At once, child B throws his cap of mass M/25 which falls at Q. find mass m to be put at P to balance the beam again.

$M(2x) + \frac{M}{25}(x) = mx + \left(M - \frac{M}{25}\right)(2x)$

$\frac{50 x M}{25} + \frac{xM}{25} = xm + \frac{48 xM}{25}$

$\frac{51 x M}{25} - \frac{48 x M}{25} = xm$

$\frac{3M}{25} = m$

$\boxed{m = \frac{3M}{25}}$

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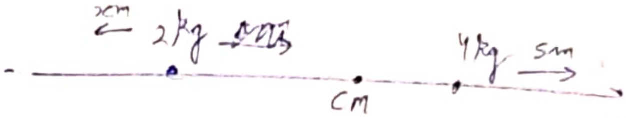
Q6. Two particles 2kg, 4kg lie on same line, If 4kg is displaced right by 5 m by what distance 2kg should move for center of mass remain the same position.

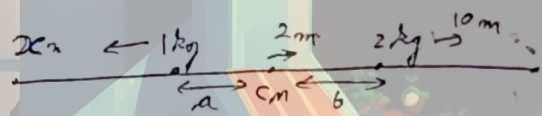


~~$CM_x = 4x$~~

$5 \times \cancel{4}^{2} = \cancel{2} \times x$

$\boxed{x = 10}$

Q7. Two particles of mass 1kg & 2kg lie on the same line. If 2kg is displaced 10m right then what distance 1kg should be displaced so center of mass is displaced 2m right.

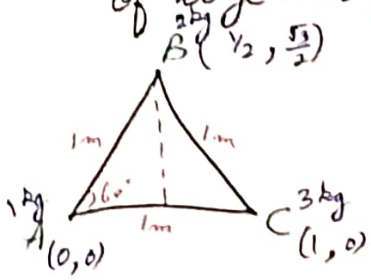


$(10-2)(2) = (1)(x+2)$

$16 = x + 2$

$\boxed{x = 14\,m}$ towards left

Q8. we have 3 particles of mass 1kg, 2kg & 3kg are placed at corners A, B, C respectively of an equilateral $\triangle ABC$ of edge 1m. find distance of COM from A.



$CM_x = \dfrac{(1)(3) + (1/2)(2)}{2+3+1}$

$CM_x = 4/6$

$CM_y = \dfrac{(0)(3) + (\sqrt{3}/2)(2)}{2+3+1}$

$CM_y = \dfrac{\sqrt{3}}{6}$

$Dis = \sqrt{\dfrac{16+3}{36}} = \boxed{\dfrac{\sqrt{19}}{6}\,m}$

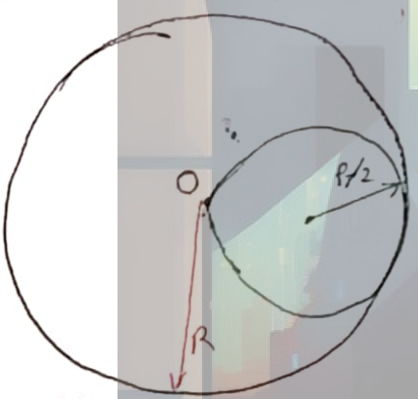
Cut out

→ If a part of a body is cut, to find find center of mass.

$$\boxed{x_{cm} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}}$$ (2)

where, m_1 = mass of original body
m_2 = mass of cut out portion
x_1 = center of mass of original body
x_2 = center of mass of cut-out portion.

Q9. From a disc of uniform mass distribution of radius R, a disc of radius $\frac{R}{2}$ is cut out as shown in figure. Find COM of remaining part of disc from point O?



Let mass of original plate $= M$.
mass of cut out $= m$.

$$\frac{M}{\pi R^2} \times \frac{\pi R^2}{4} = m$$

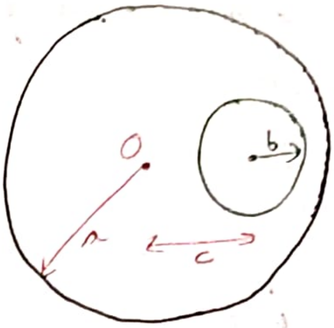
$$m = \frac{M}{4}$$

$$x_{cm} = \frac{M \times 0 - \frac{M}{4} \times \frac{R}{2}}{M - \frac{M}{4}}$$

Consider O as origin.

$$\boxed{x_{cm} = \frac{-R}{6}}$$

Q10. from a disc of radius a, a circular disk of radius b is removed. find COM for remaining.



$$m = \frac{M}{\pi a^2} \times \pi b^2$$

$$m = \frac{Mb^2}{a^2}$$

$$x_{cm} = \frac{-\frac{Mb^2}{a^2} \times c}{\frac{M(a^2-b^2)}{a^2}}$$

$$\boxed{x_{cm} = \frac{-b^2 c}{a^2 - b^2}}$$

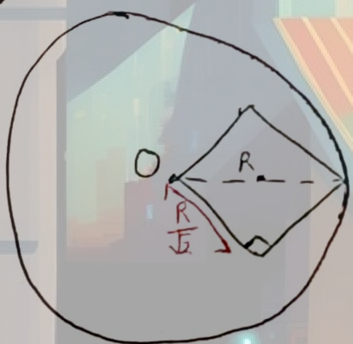
Q11. from a uniform circular disc of radius R, a square is cut out of radius R as diagonal. find new COM.

Let mass of original shape = M

let mass of cut area = m

$$m = \frac{M}{\pi R^2} \times \frac{R^2}{2}$$

$$m = \frac{M}{2\pi}$$

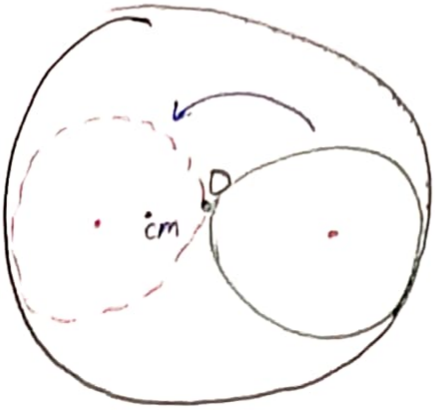


$$x_{cm} = \frac{-\frac{M}{2\pi} \times \frac{R}{2}}{\frac{M(2\pi-1)}{2\pi}}$$

$$x_{cm} = \frac{-R}{4\pi - 2}$$

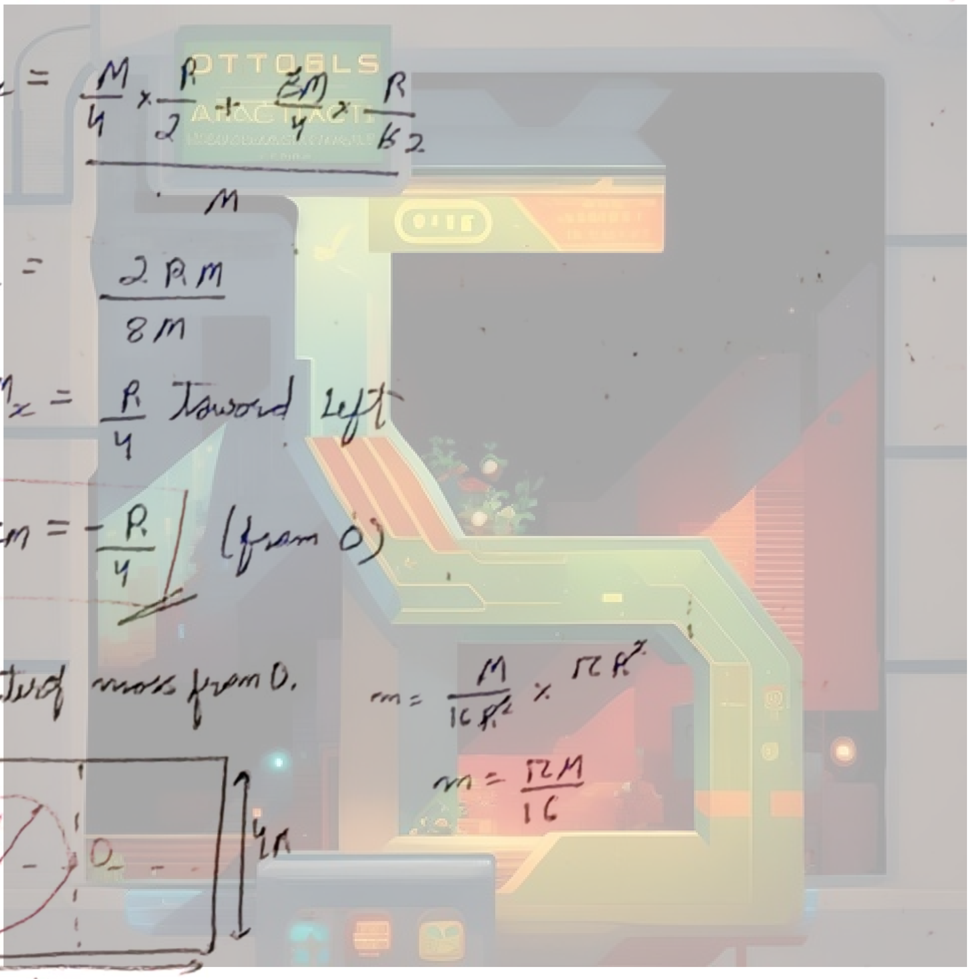
$$\boxed{x_{cm} = \frac{R}{2 - 4\pi}}$$

Q12. from a disc of uniform mass distribution of Radius R, a disc of radius $\frac{R}{2}$ is cut & placed on ther side. find COM.



$m = \frac{M}{4}$

$x_{cm} = -\frac{R}{6}$ $\left(mass = \frac{3M}{4}\right)$



$CM_x = \dfrac{\frac{M}{4} \times \frac{R}{2} + \frac{3M}{4} \times \frac{R}{6}}{M}$

$CM_x = \dfrac{2RM}{8M}$

$CM_x = \frac{R}{4}$ Toward left

$\boxed{x_{cm} = -\frac{R}{4}}$ (from O)

Q13. find center of mass from O.

$m = \frac{M}{16R^2} \times \pi R^2$

$m = \frac{\pi M}{16}$

$4R$

$4R$

$x_{cm} = \dfrac{+\frac{\pi M}{16} \times R}{M - \frac{M\pi}{16}}$

$x_{cm} = \dfrac{M\pi R}{-M(\pi - 16)}$

$\boxed{x_{cm} = \dfrac{\pi R}{-\pi + 16}}$

(62)

Continuous object system

$$R_{cm} = \frac{\int dm\, r}{\int dm} = \boxed{\frac{1}{M}\int dm\, r}$$

→ mass of element

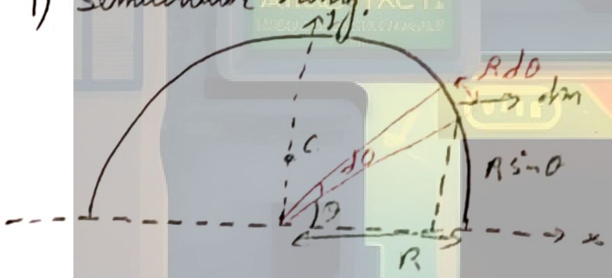
→ com of element

(3)

↓ mass of object

$$x_{cm} = \frac{1}{M}\int dm\, x \qquad y_{cm} = \frac{1}{M}\int dm\, y \qquad z_{cm} = \frac{1}{M}\int dm\, z$$

1) Semicircular ring.



Let mass of ring = M

$$dm = \frac{M}{\pi R} \times R\,d\theta$$

$$dm = \frac{M\cancel{R}}{\pi}\, d\theta$$

$$y_c = \frac{1}{M}\int dm\, y$$

$$y_c = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi}\, d\theta\, (R\sin\theta)$$

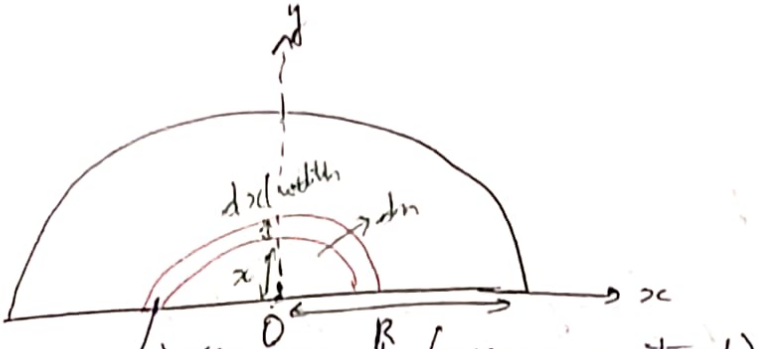
$$y_c = \frac{1}{\cancel{M}}\,\cancel{M}\,\frac{R}{\pi}\int_0^{\pi} R\sin\theta\, d\theta$$

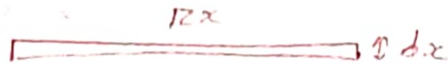
$$y_c = -\frac{R}{\pi}\left[\cos\theta \; \cancel{-\sin\theta}\right]_0^{\pi}$$

$$y_c = -\frac{R}{\pi}\left[\cos\pi - \cos 0\right]$$

$$\boxed{y_c = \frac{2R}{\pi}}$$ (4)

2) Semi-circular disc :-



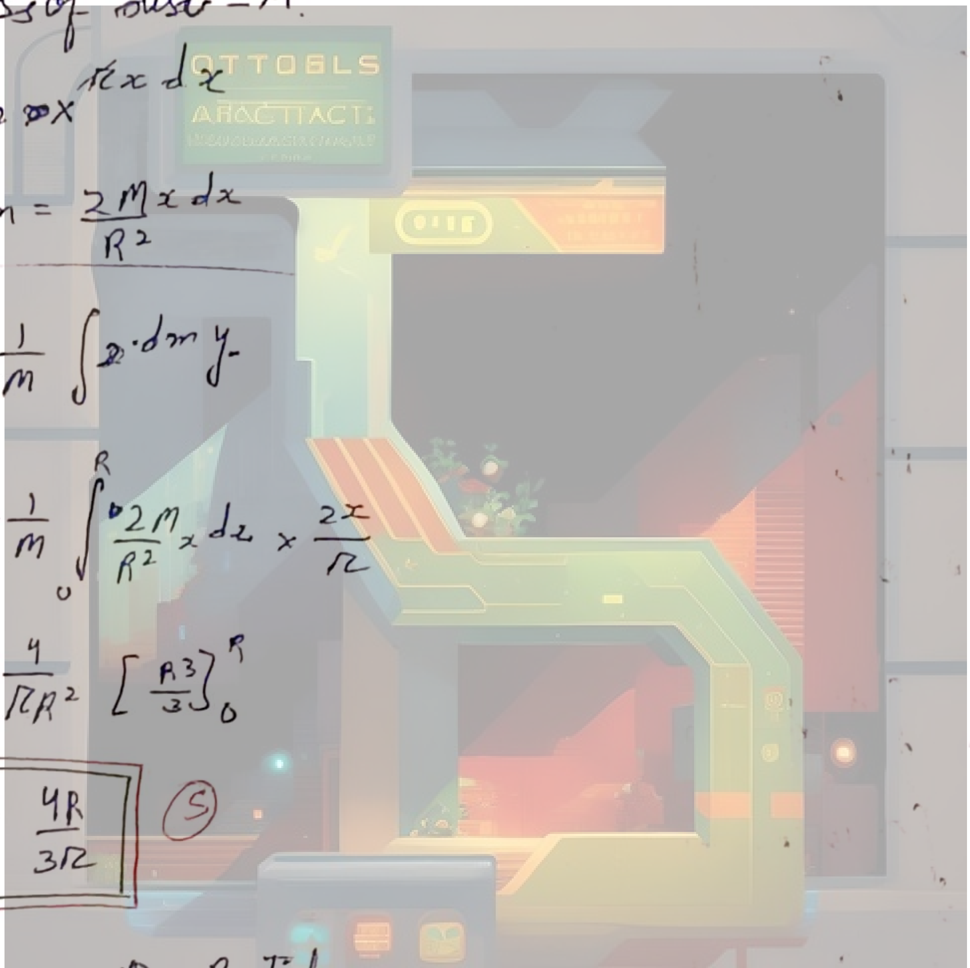


↳ area = $\pi x \, dx$ (assume rectangle)

let mass of disc = M.

$$dm = \frac{2M}{\pi R^2} \times \pi x \, dx$$

$$dm = \frac{2M x \, dx}{R^2}$$



$$y_{cm} = \frac{1}{M} \int dm \, y$$

$$y_{cm} = \frac{1}{M} \int_0^R \frac{2M}{R^2} x \, dx \times \frac{2x}{\pi}$$

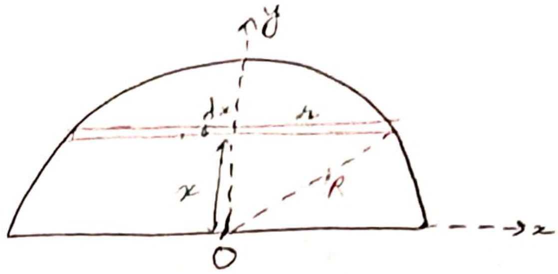
$$y_{cm} = \frac{4}{\pi R^2} \left[\frac{R^3}{3} \right]_0^R$$

$$\boxed{y_{cm} = \frac{4R}{3\pi}}$$ (S)

Note:-

Particle
↓
Ring → Disc → solid sphere.
↓
Hollow sphere

3) Solid Hemisphere :-



$$\frac{M}{\frac{2}{3}\pi R^3} \times \pi r^2 dx = dm$$

$$r^2 + x^2 = R^2$$

$$r^2 = R^2 - x^2$$

$$dm = \frac{3Mr^2 dx}{2R^3}$$

$$y_{cm} = \frac{1}{M}\int dm\, y$$

$$y_{cm} = \frac{1}{M}\int \frac{3Mr^2 dx}{2R^3} \times x$$

$$y_{cm} = \frac{3M}{2MR^3}\int R^2x - x^3\, dx$$

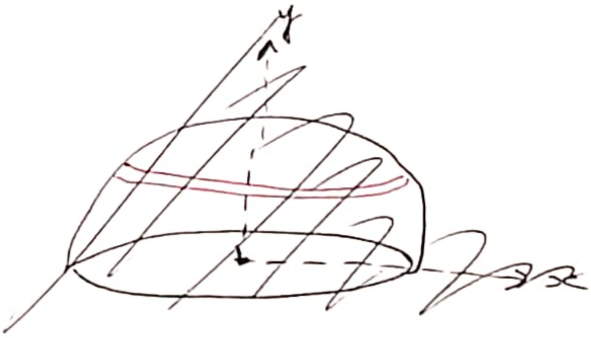
$$y_{cm} = \frac{3}{2R^3}\left[\frac{R^2x^2}{2} - \frac{x^4}{4}\right]_0^R$$

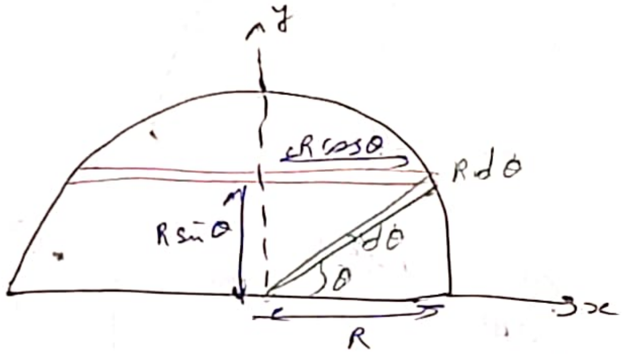
$$y_{cm} = \frac{3}{2R^3}\left[\frac{2R^4}{4} - \frac{R^4}{4}\right]$$

$$y_{cm} = \frac{3R^4}{8R^3}$$

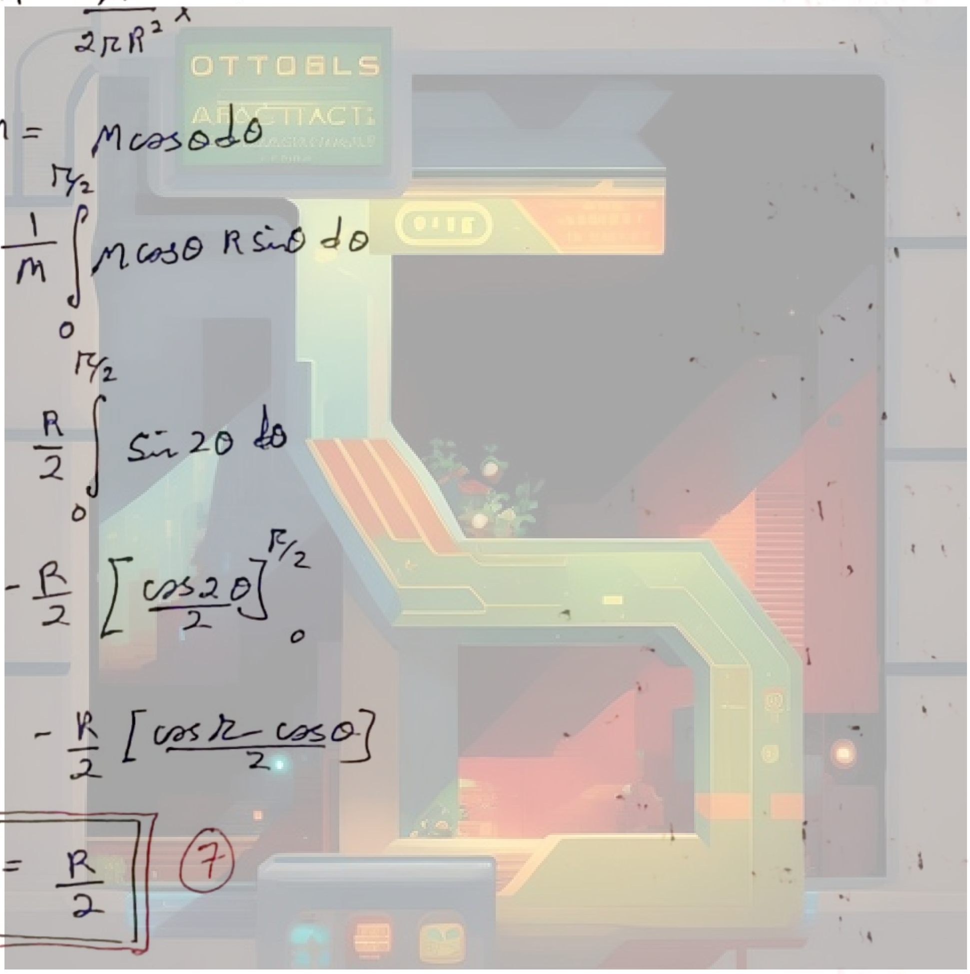
$$\boxed{y_{cm} = \frac{3R}{8}}$$ (6)

(4) Hollow hemisphere.





$$dm = \frac{M}{2\pi R^2} \times 2\pi R \, R\cos\theta \, R\, d\theta$$



$$dm = M\cos\theta \, d\theta$$

$$y_c = \frac{1}{M}\int_0^{\pi/2} M\cos\theta \, R\sin\theta \, d\theta$$

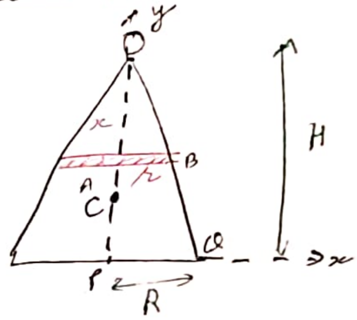
$$y_c = \frac{R}{2}\int_0^{\pi/2} \sin 2\theta \, d\theta$$

$$y_c = -\frac{R}{2}\left[\frac{\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$y_c = -\frac{R}{2}\left[\frac{\cos\pi - \cos 0}{2}\right]$$

$$\boxed{y_c = \frac{R}{2}}$$ (7)

(5) Solid Cone



$dm = \frac{3M}{\pi \cdot R^2 H} \times \pi r^2 dx$

$\frac{x}{H} = \frac{r}{R}$ (Δ AOB & Δ PDQ are similar)

$r = \frac{xR}{H}$

$dm = \frac{3M r^2 dx}{R^2 H}$

$y_c = \frac{1}{M}\int_0^H \frac{3M}{R^2 H} \times \frac{x^2 R^2}{H^2} \times x\, dx$

$y_c = \frac{3}{H^3}\int_0^H x^3\, dx$

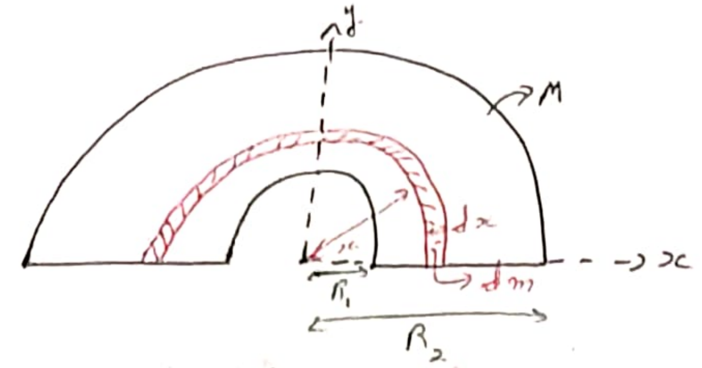
$y_c = \frac{3}{H^3} \times \frac{H^4}{4}$

$\boxed{y_c = \frac{3}{4} H}$ (8) (from apex of cone)

(6) Hollow cone

$\boxed{y_c = \frac{2}{3} H}$ (9) (from apex)

(7) Semi Circular Annular Disc :- (Rolo Ki shape)



$$dm = \frac{2M}{\pi(R_2^2 - \cancel{\pi} R_1^2)} \times \pi x \, dx$$

$$dm = \frac{2Mx \, dx}{R_2^2 - R_1^2}$$

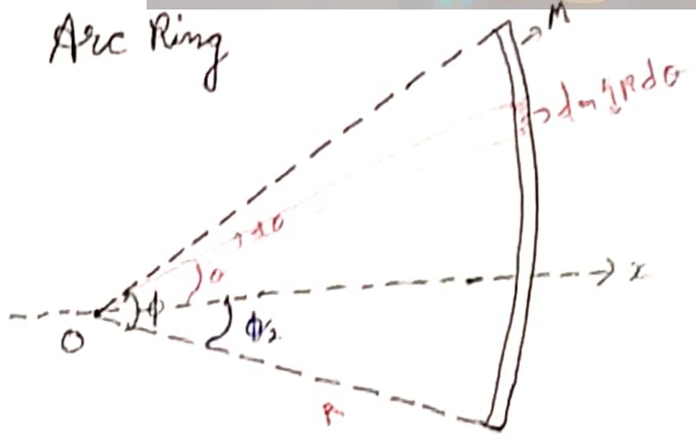
$$y_{cm} = \frac{1}{M}\int_{R_1}^{R_2} \frac{2Mx\,dx}{R_2^2 - R_1^2} \times \frac{2x}{\pi}$$

$$y_{cm} = \frac{4}{\pi(R_2^2 - R_1^2)}\int_{R_1}^{R_2} x^2 \, dx$$

$$y_{cm} = \frac{4}{\pi(R_2^2 - R_1^2)} \times \frac{R_2^3 - R_1^3}{3}$$

$$\boxed{y_{cm} = \frac{4(R_2^3 - R_1^3)}{3\pi(R_2^2 - R_1^2)}} \quad (10)$$

(8) Arc Ring



$$dm = \frac{M}{R\phi} \times R\,d\theta$$

$$dm = \frac{M\,d\theta}{\phi}$$

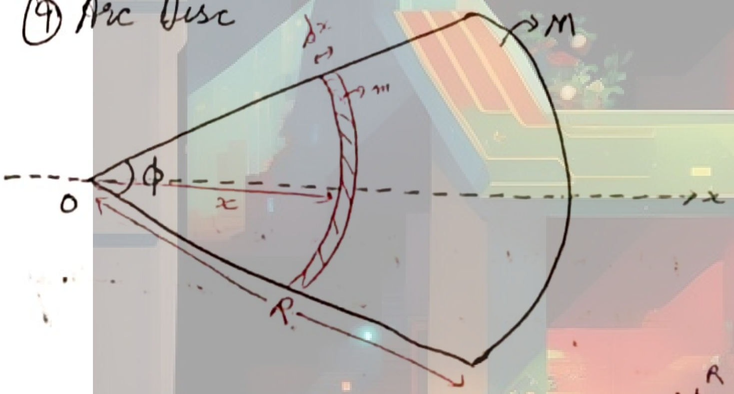
$$x_{cm} = \frac{1}{M}\int_{-\phi/2}^{\phi/2} \frac{M\,d\theta}{\phi} \times R\cos\theta$$

$$x_{cm} = \frac{R}{\phi}\int_{-\phi/2}^{\phi/2} \cos\theta\,d\theta$$

$$x_{cm} = \frac{R}{\phi}\left[\sin\frac{\phi}{2} - \sin\left(-\frac{\phi}{2}\right)\right]$$

$$\boxed{x_{cm} = \frac{2R}{\phi}\sin\left(\frac{\phi}{2}\right)}$$ (11)

(9) Arc Disc



$$dm = \frac{2M}{R^2\phi} \times x\phi\,dx$$

$$dm = \frac{2Mx\,dx}{R^2}$$

$$x_{cm} = \frac{1}{M}\int_0^R \frac{2Mx\,dx}{R^2} \times \frac{2x}{\phi}\sin\frac{\phi}{2}$$

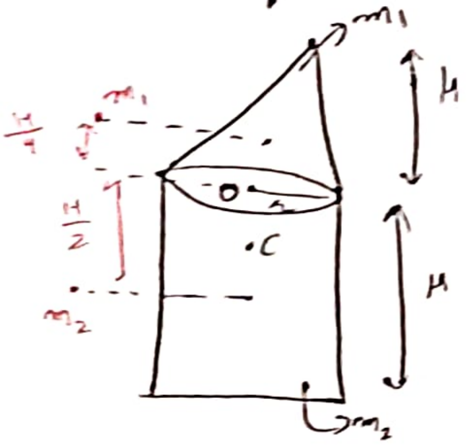
$$x_{cm} = \frac{4}{R^2\phi}\sin\frac{\phi}{2}\left[\frac{R^3-0}{3}\right]$$

$$\cancel{x_{cm} = \frac{4}{3}\frac{R}{\phi}}$$

$$\boxed{x_{cm} = \frac{4R}{3\phi}\sin\left(\frac{\phi}{2}\right)}$$ (12)

... solid cylinder

made of same material from O.



ρ = same $= \frac{m}{v}$

$$\frac{m_1}{\frac{1}{3}\pi r^2 h} = \frac{m_2}{\pi r^2 h}$$

$$\boxed{m_2 = 3m_1}$$

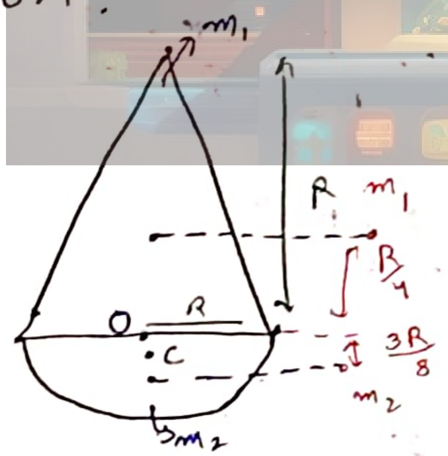
$$y_c = \frac{m_1\left(\frac{H}{4}\right) + m_2\left(-\frac{H}{2}\right)}{m_1 + m_2}$$

$$y_c = \frac{m_1\left(\frac{H}{4}\right) - 3m_1\left(\frac{H}{2}\right)}{3m_1 + m_1}$$

$$y_c = \frac{H - 6H}{4 \times 4}$$

$$\boxed{y_c = \frac{-5H}{16}}$$

Q15. A solid cone ~~of~~ Height $= R$ & radius R is joined with solid hemisphere of radius R. made of same material find coordinates of COM.



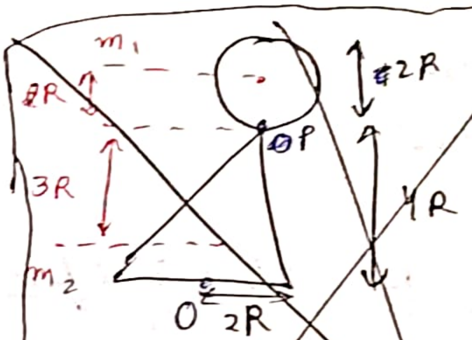
$$\frac{m_1}{\frac{1}{3}\pi R^2 \times R} = \frac{m_2}{\frac{2}{3}\pi R^3}$$

$$m_2 = 2m_1$$

$$y_{cm} = \frac{\frac{2m_1 R}{8} - \frac{2m_1 \times 3R}{8}}{3m_1}$$

$$\boxed{y_{cm} = \frac{-4}{24 \cdot 3} R}$$

Q16. Solid sphere on solid cone. The density of sphere is 12 Times of cone.



$$\frac{m_1}{\frac{4}{3}\pi \times 8R^3} = \frac{12\, m_2}{\frac{1}{3}\pi \times 2R \times 2R \times 4R}$$

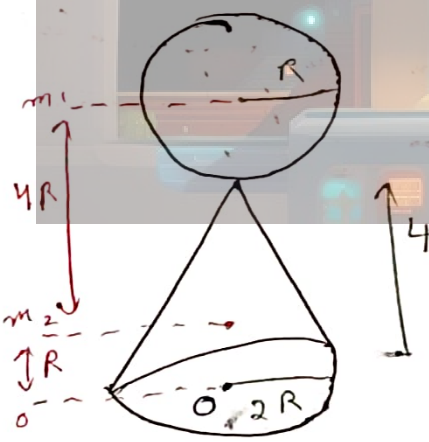
$\boxed{\cancel{m_2 = 6m_1}}$

$\boxed{m_1 = 24\, m_2}$

$$y_{cm} = \frac{24\, m_2 \times 2R + m_2 \times 3R}{25\, m_2}$$

$$y_{cm} = \frac{51R}{25} \text{ (from P)}$$

$$y_{cm} = \frac{51R}{25} + 4R = \frac{151R}{25} \text{ (from O)}$$



$$\frac{m_1}{\frac{4}{3}\pi R^3} = \frac{12\, m_2}{\frac{1}{3}\pi \times 4 \times R^2 \times 4R}$$

$\boxed{m_1 = 3\, m_2}$

$$y_{cm} = \frac{3m_2 \times 5R + m_2 (R)}{4m_2}$$

$$y_{cm} = \frac{16}{4} R$$

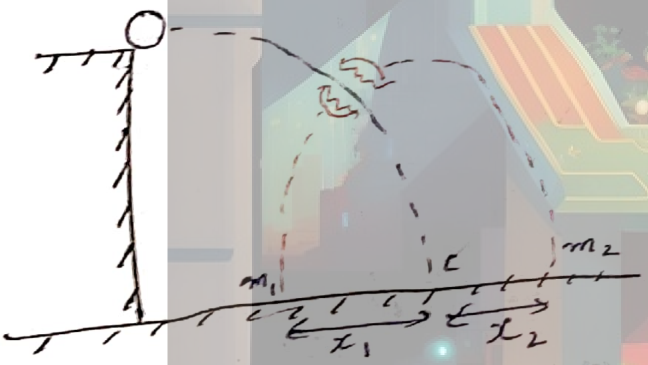
$\boxed{y_{cm} = 4R}$

Motion Of Center Of Mass:-

$$\boxed{\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 \ldots\ldots m_n\vec{v}_n}{m_1 + m_2 \ldots\ldots + m_n} = \frac{\vec{P}_{Total}}{M_{Total}}} \quad (13)$$

$$\boxed{\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 \ldots\ldots}{m_1 + m_2 + m_3 \ldots\ldots} = \frac{\vec{F}_{ext}}{M}} = \frac{\text{Total External force}}{M} \quad (14)$$

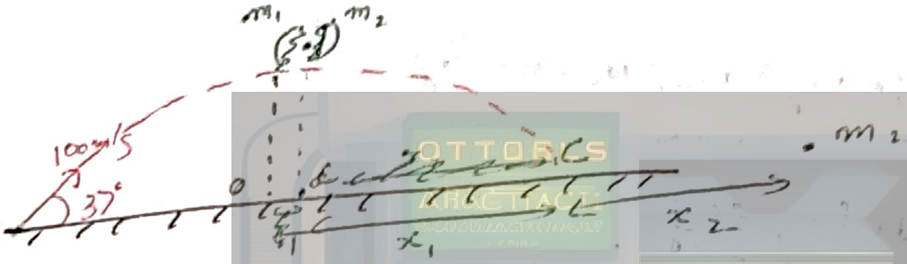
- If $\vec{F}_{ext} = 0$ and $\vec{V}_{cm} = 0$, then COM remains at rest. Individual components of a system may move and have a non-zero momentum ~~but A due to~~ due to internal forces, but net momentum of system remains zero.

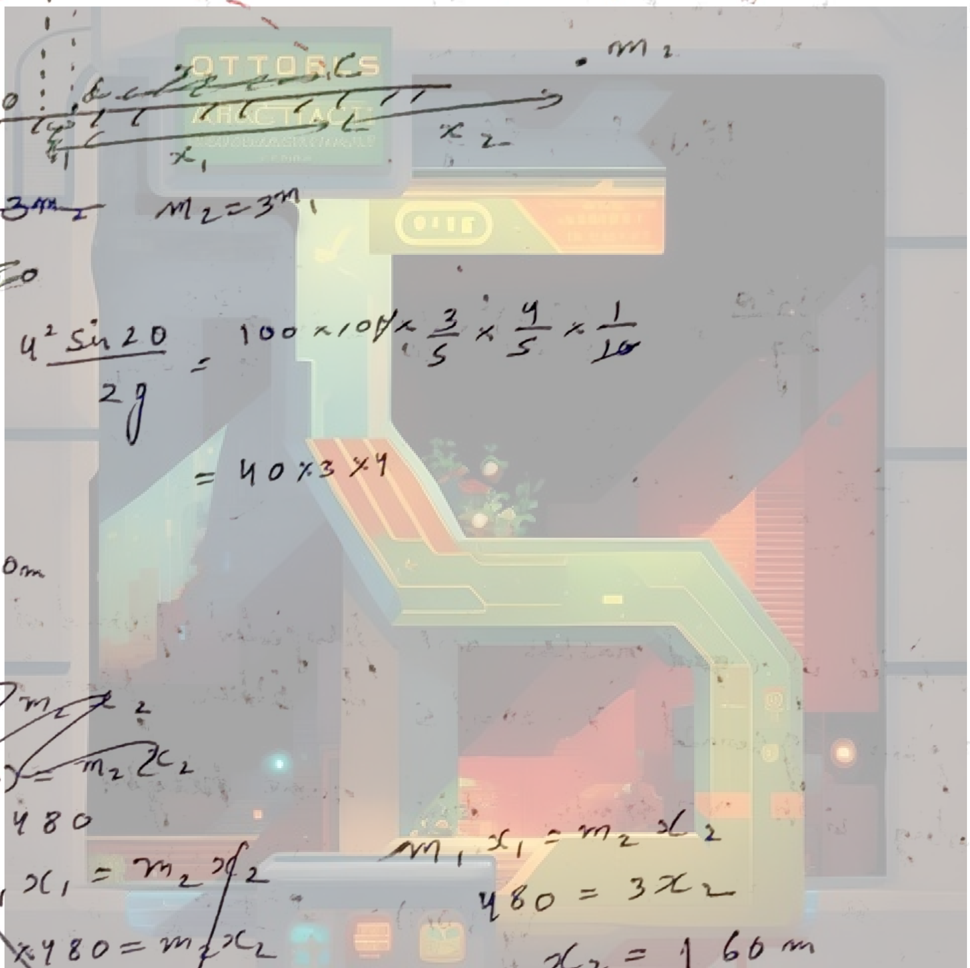


$m_1 x_1 = m_2 x_2$

→ If ~~a~~ a bomb explodes mid-air, its fragments will follow different trajectories but ~~COM~~ COM follows original path.

Q 17. A Projectile is fired at a speed of 100 m/s at angle 37° above the horizontal. At the highest point, the projectile breaks into 2 parts of ratio 1:3. The lighter part comming to rest. Find the distance from the launching point to the point where heavier part lands.





$\cancel{m_1 = 3m_2}$ $\quad m_2 = 3m_1$

$\cancel{x_1 = 0}$

$$\frac{R}{2} = \frac{u^2 \sin 2\theta}{2g} = 100 \times 100 \times \frac{3}{5} \times \frac{4}{5} \times \frac{1}{10}$$

$$= 40 \times 3 \times 4$$

$$\frac{R}{2} = 480\text{m}$$

$\cancel{m_1 x_1 = m_2 x_2}$

$\cancel{3m_2 (0) = m_2 x_2}$

$x_1 = 480$

$\cancel{m_1 x_1 = m_2 x_2}$

$\cancel{3m_2 \times 480 = m_2 x_2}$

$\cancel{x_2 = 480 \times 3}$

$\cancel{x_2 = 1200 + 240}$

$\cancel{x_2 = 1440}$

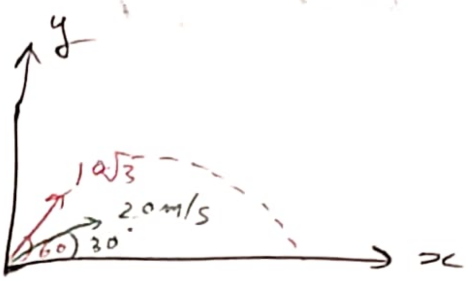
$m_1 x_1 = m_2 x_2$

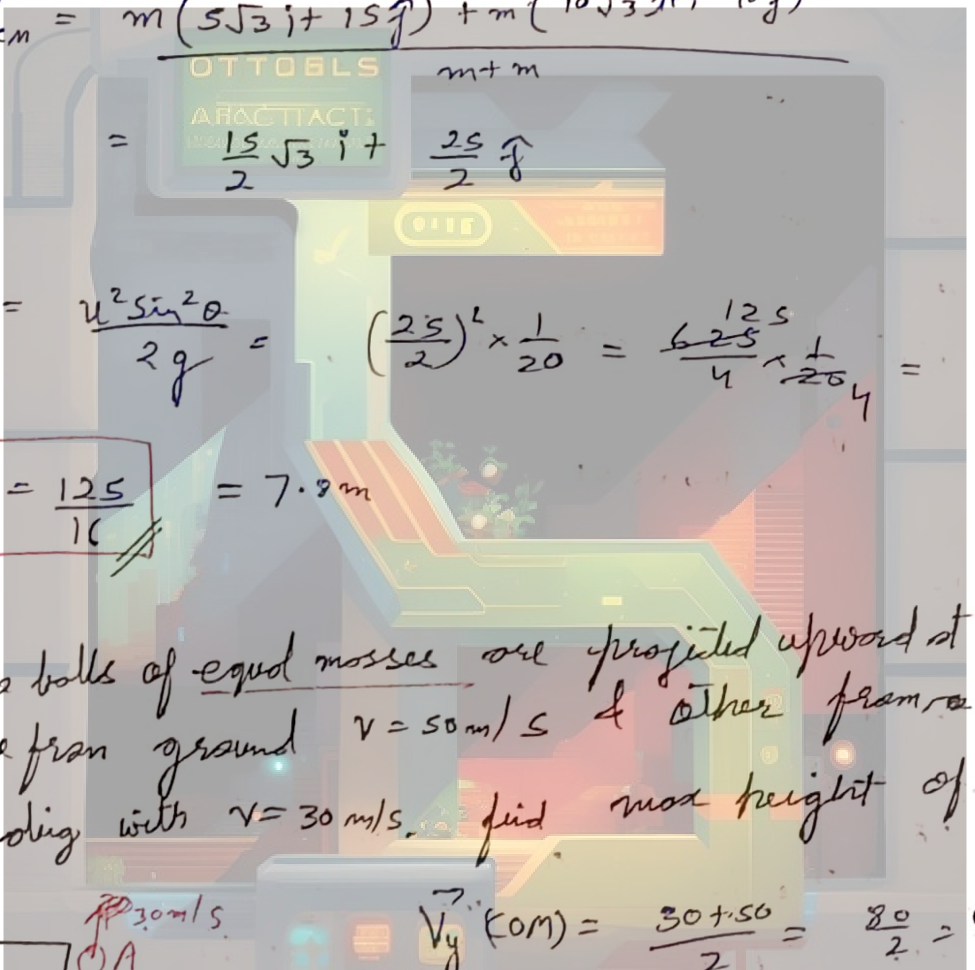
$480 = 3x_2$

$x_2 = 160$ m

$960 + 160 = \boxed{1120 \text{ m}}$

Q16. Two particles of equal mass are projected simultaneously with speeds 20 m/s & $10\sqrt{3}$ m/s as shown find max Height reached by their centre of mass.





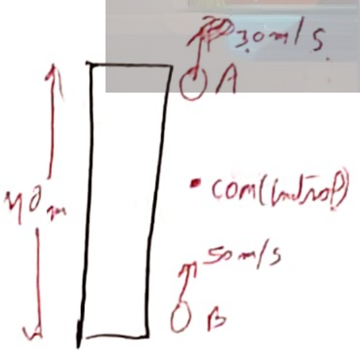
$$\vec{V}_{cm} = \frac{m(5\sqrt{3}\hat{i} + 15\hat{j}) + m(10\sqrt{3}\hat{i} + 10\hat{j})}{m+m}$$

$$= \frac{15}{2}\sqrt{3}\,\hat{i} + \frac{25}{2}\hat{j}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \left(\frac{25}{2}\right)^2 \times \frac{1}{20} = \frac{\overset{125}{\cancel{625}}}{4} \times \frac{1}{\cancel{20}_4} = \cancel{125}$$

$$\boxed{H = \frac{125}{16}} = 7.8\,m$$

Q19. Two balls of equal masses are projected upward at same time. one from ground $v = 50$ m/s & other from a 40m High building with $v = 30$ m/s. find max height of their COM.



$$\vec{V}_y(COM) = \frac{30+50}{2} = \frac{80}{2} = 40\,m/s$$

$$\text{max height (from C)} = \frac{40 \times 4\cancel{0}}{2 \times 1\cancel{0}} = 80\,m$$

$$C_y = \frac{0+40}{2} = 20$$

$$\text{max } H\text{ (from ground)} = 80 + 20$$

$$\boxed{= 100\,m}$$

Q20. A 1 kg particle moving with $\vec{v}(i - j)$ & 2 kg particle with $\vec{v}(2i+3j)$ find velocity of center of mass.

$$V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

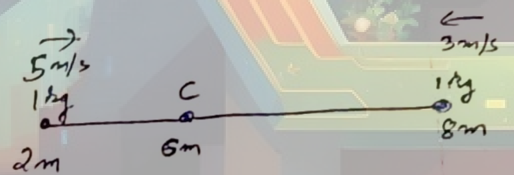
$$= \frac{i - j + 4i + 6j}{3}$$

$$= \frac{5i + 5j}{3}$$

$$\boxed{= \frac{5}{3}(i+j)}$$

Q21. The figure shows positions and velocities of two particles. If the particle move under the mutual attraction of each other, find the position of COM at $t = 1s$.

~~$x_{COM} = \frac{(2)(1) + (8)(1)}{2} = 5m$ (from origin)~~



$$x_{com} = \frac{7+5}{2} = \frac{12}{2} = \boxed{6m}$$

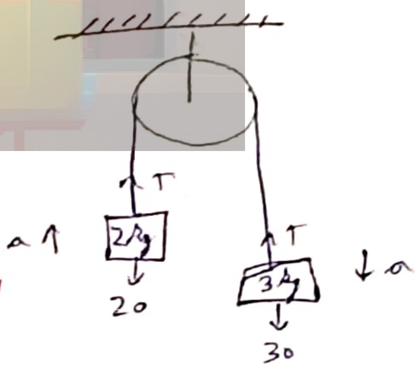
find magnitude of acc of COM.

Q22. $30 - T = 3a$

$T - 20 = 2a$

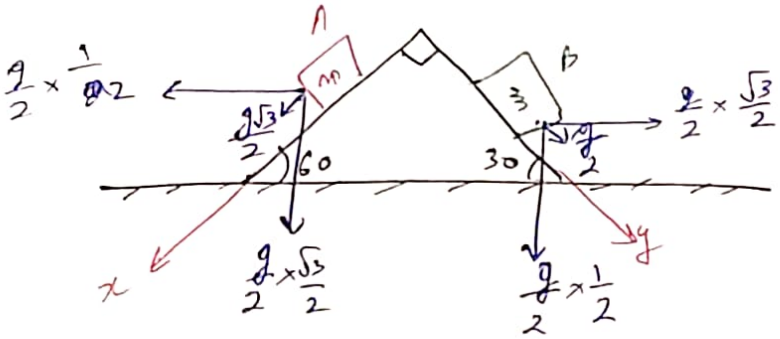
$\frac{10}{5} = a$

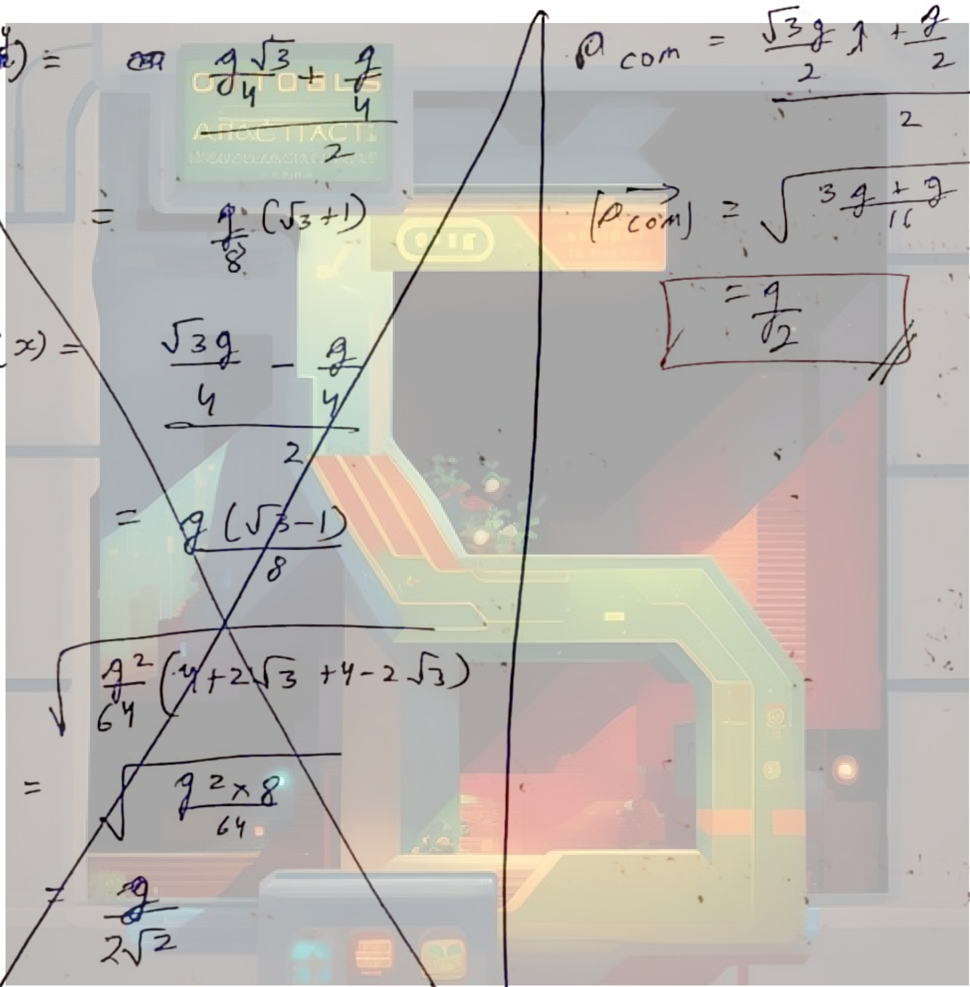
$a = 2\ m/s^2$



$$a_{com} = \frac{(2)(3) - (2)(2)}{5} = \boxed{\frac{2}{5}\ m/s^2}$$

Q 23. find magnitude of acc of com





$$A_{com}(y) = \frac{\frac{g\sqrt{3}}{4} + \frac{g}{4}}{2}$$

$$= \frac{g}{8}(\sqrt{3}+1)$$

$$A_{com}(x) = \frac{\frac{\sqrt{3}g}{4} - \frac{g}{4}}{2}$$

$$= \frac{g(\sqrt{3}-1)}{8}$$

$$|\vec{a}| = \sqrt{\frac{g^2}{64}(4+2\sqrt{3}+4-2\sqrt{3})}$$

$$= \sqrt{\frac{g^2 \times 8}{64}}$$

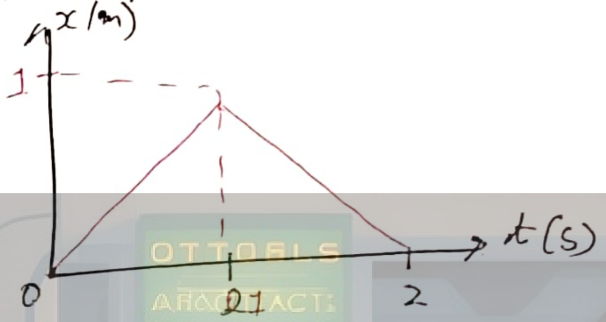
$$= \frac{g}{2\sqrt{2}}$$

$$a_{com} = \frac{\frac{\sqrt{3}g}{2}\hat{i} + \frac{g}{2}\hat{j}}{2}$$

$$|\vec{a}_{com}| = \sqrt{\frac{3g + g}{16}}$$

$$\boxed{= \frac{g}{2}}$$

Q24. Two bodies m_1 & m_2 of mass 1kg & 2kg are moving along x-axis under influence of mutual forces only. The velocity of COM is 2m/s. The x-co-ordinate of m_1 is plotted. Plot x coordinate of x_2 against time. (initially both are at origin)



$t=0 \rightarrow t=1$

$V_{m_1} = 1 m/s$

$2 = \frac{(1)(1) + (2)(V_{m_2})}{3}$

$\frac{5}{2} = V_{m_2}$

$t=1 \rightarrow t=2$

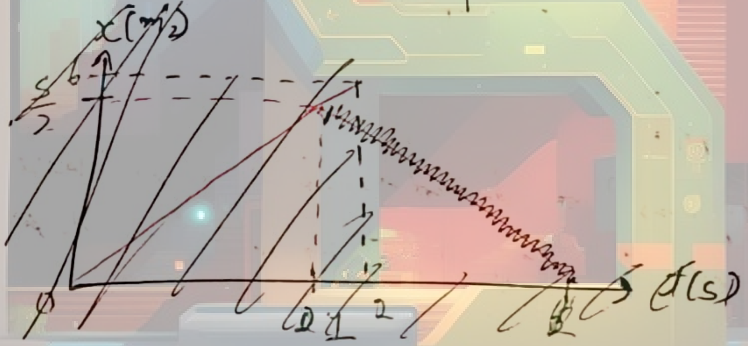
$V_{m_1} = -1 m/s$

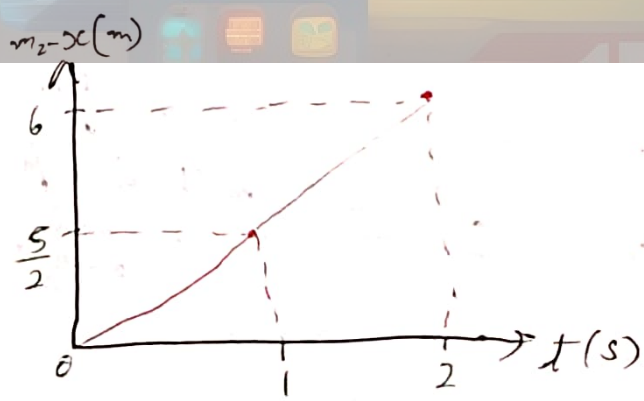
~~$V_{m_1} = -1 m/s$~~

~~$V_{m_2} = \frac{5}{2} m/s$~~

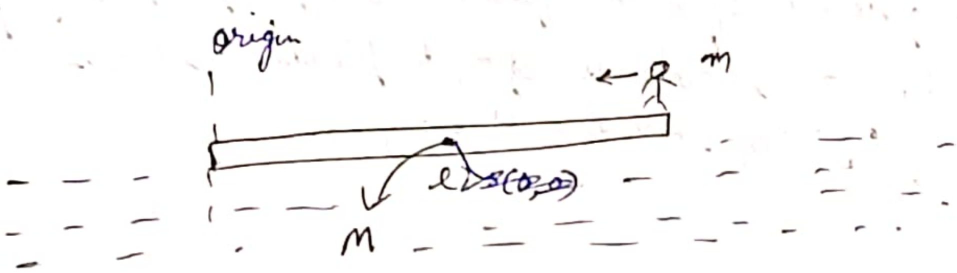
$6 = (1)(-1) + 2(V_2)$

$\frac{7}{2} m/s = V_2$





Q 25. Calculate Displacement of Log : when he reaches other end



COM remains same.

$$\cancel{COM_x = \frac{\frac{l}{2}(m)}{m+M}}$$

$$\cancel{\frac{l}{2}(m) = (x_1)\, m + (x_2)\, M}$$

$$C_i = \frac{\frac{Ml}{2} + ml}{m+M} \qquad C_f = \frac{\frac{Ml}{2} + 0}{M+m}$$

$$\cancel{C_f - C_i =}$$

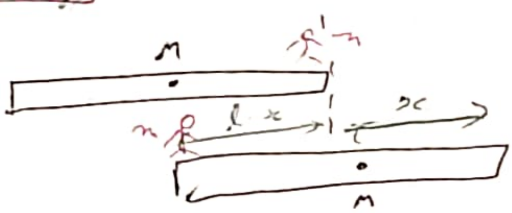
$$\Delta C = C_f - C_i = \frac{\frac{Ml}{2} + 0 - \frac{Ml}{2} - ml}{M+m}$$

$$C_f - C_i = \frac{-ml}{M+m}$$

But center of mass ~~to~~ should remain same as there is no external force so log must ~~has~~ have moved. ~~to~~

$$\boxed{x = \frac{ml}{M+m}}$$

MII

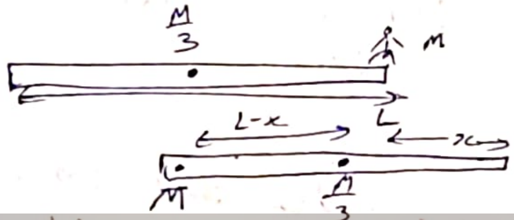


As seen from ground,

$$m(l-x) = Mx$$

$$\boxed{x = \frac{ml}{m+M}}$$

Q26. A man of mass M stands at one end of a plank of length L. which lies at rest on a frictionless surface. the man walks to other end of plank. If mass of plank is $\frac{M}{3}$ then find distance Man moves relative to ground.



$$M(L-x) = \frac{M}{3}(x)$$

$$ML - xMx = \frac{M}{3}x$$

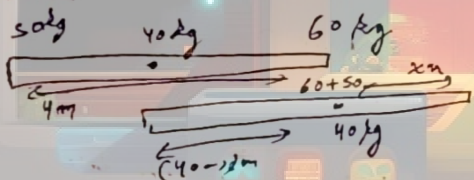
$$ML = \frac{4Mx}{3}$$

$$L = \frac{4x}{3}$$

$$x = \frac{3L}{4}$$

$$L - x = L - \frac{3L}{4} \boxed{= \frac{L}{4}}$$

Q27. Both A & B move at middle of boat. How far does boat move.



$$(2+x)50 + x(40) = (2-x)60$$

$$100 + 90x = 120 - 60x$$

$$20 = 150x$$

$$\boxed{x = \frac{2}{15}\ m}$$

$$\boxed{\cancel{x = 0.67m}}$$

~~0.2 = 0.133~~

Q 28. A man weighing 80 kg is standing on one end of a float boat which is 20 m from shore. He walks 8 m toward shore. Boat weighs 240 kg. How far is he from shore.

$\cancel{(80)(8) = 240x}$

$\cancel{x = \frac{80 \times 8}{240}}$

$\cancel{x = \frac{8}{3} = 2.67\,m}$

$\cancel{12 + 2.67}$

$(80)(8 - x) = 240x$

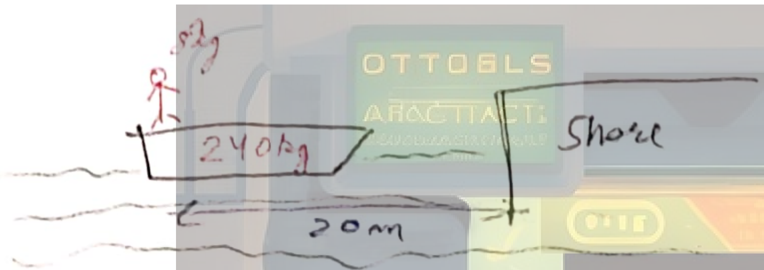
$640 - 80x = 240x$

$\frac{640}{320} = x$

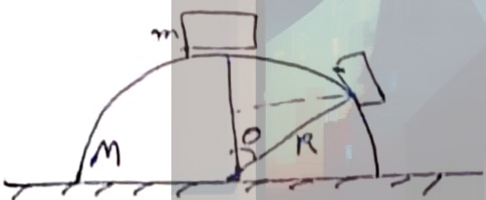
$x = 2$

distance from shore $= (20 - 8) + 2$

$\boxed{= 14\,m}$



Q 29. Calculate Displacement of wedge when block reaches angular position θ.



Since, no external force acts in horizontal, x-co-ordinate of COM remains at same Position.

$\cancel{Mx = mR\cos\theta}$

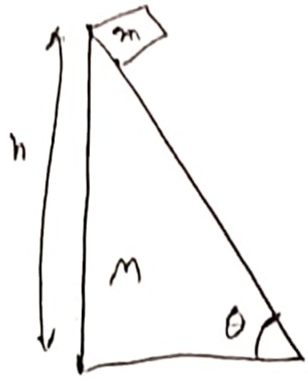
$\cancel{x = \frac{mR\cos\theta}{M}}$

$Mx = m(R\cos\theta - x)$

$Mx + mx = mR\cos\theta$

$\boxed{x = \frac{mR\cos\theta}{(M+m)}}$

Q 30. Calculate Dist of wedge when block reaches bottom



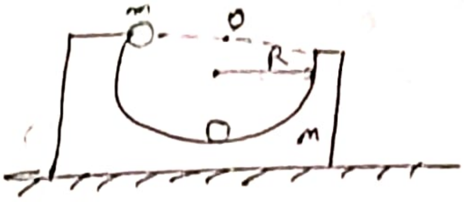
$m(h\cot\theta - x) = Mx$

$mh\cot\theta = x(M + m)$

$\boxed{x = \frac{mh\cot\theta}{(M + m)}}$

x - co-ordinate of COM remains intact.

Q[31] Calculate Displacement of wedge when reaches at bottom of groove. radius of ball (r).



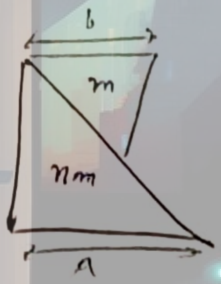
$$Mx = m(R - r - x)$$

$$Mx = mR - mr - mx$$

$$x(M + m) = m(R - r)$$

$$\boxed{x = \frac{m(R - r)}{(M + m)}}$$

Q 32. Two ~~p~~ smooth prisms of similar right triangular section are arranged as shown. lower prism has mass n times upper one. find distance moved by lower prism as upper reaches lower end.



$$nmx = m(a - b - x)$$

$$nmx + mx = m(a - b)$$

$$x = \frac{m(a - b)}{m(n + 1)}$$

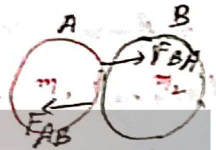
$$\boxed{x = \frac{a - b}{n + 1}}$$

Conservation of Linear Momentum

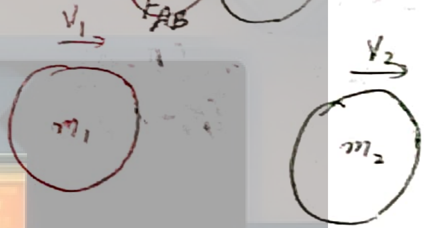
→ "If no external force is acting on the system, the total linear momentum of system always remains conserved."

Total Initial Momentum $= m_1 u_1 + m_2 u_2$

Total Final Momentum $= m_1 V_1 + m_2 V_2$



$$\boxed{m_1 \vec{u_1} + m_2 \vec{u_2} = m_1 \vec{V_1} + m_2 \vec{V_2}}$$ 15



$$m_1 \vec{u}_{1x} + m_2 \vec{u}_{2x} = m_1 \vec{V}_{1x} + m_2 \vec{V}_{2x}$$

$$m_1 \vec{u}_{1y} + m_2 \vec{u}_{2y} = m_1 \vec{V}_{1y} + m_2 \vec{V}_{2y}$$

Proof:- By NLM (III) $|F_{BA}| = |F_{AB}|$

$$F_{AB} = -F_{BA}$$

By NLM (II), $F = m \cdot \left(\frac{V-u}{t}\right)$

$$m_1 \left[\frac{V_1 - u_1}{\Delta t}\right] = -m_2 \left[\frac{V_2 - u_2}{\Delta t}\right]$$

$$m_1 V_1 - m_1 u_1 = m_2 u_2 - m_2 V_2$$

$$m_1 V_1 + m_2 V_2 = m_2 u_2 + m_1 u_1$$

Hence, Proved

Recoil of Gun

By, COLM,

$0 + 0 = mv + MV$

$\boxed{V = \frac{-mv}{M}}$ → Gun recoils in opp direction to bullet

↳ recoil velocity of Gun.

→ when bullet is fired from a gun, it recoils backwards. This is called recoil of gun.

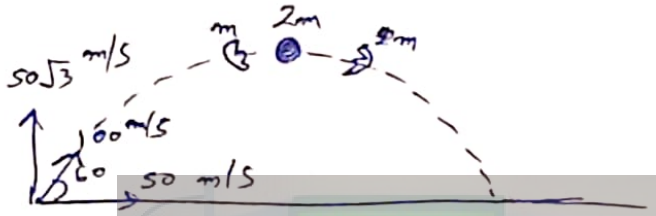
~~Q33. A bullet of mass.~~

Q 33.

$5m(v') = mv$

$\boxed{v' = \frac{v}{5}}$

Q 34. A shell is fired with speed 100 m/s at 60° with horizontal. At highest point, shell explodes into two equal fragments. one fragment moves left at 50 m/s find speed of other fragment at the time of explosition.



Speed of 2m at highest point = 50 m/s

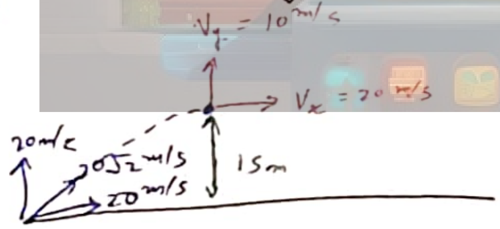
~~Momentum~~

$m(-50) + m(v) = 2m(50)$

$mv = 150m$

$\boxed{v = 150 \text{ m/s}}$

Q 35. A particle of mass 2m is projected 45° with horizontal, $v = 20\sqrt{2}$ m/s After 1s explosition takes placed particle breaks into 2 equal pieces. As a result of explosition, one part comes to rest. find max height attained by other.



at 1s,

$V_y = 20 - 10(1)$

$u_y = 10\hat{j}$

$u_x = 20\hat{i}$

$u = 20\hat{i} + 10\hat{j}$

$s = 20 - 5$

~~s = 15m~~

$s = 15$ m

$2m(20\hat{i} + 10\hat{j}) = 0 + m(v)$

$v = 40\hat{i} + 20\hat{j}$

$0 = 20 - 10(t)$

$t = 2s$

$s = 20(2) - \frac{10}{2}(4)$

$s = 40 - 20$

$s = 20$

Total Height $= 20 + 15$

$\boxed{= 35 \text{ m}}$

Q.36. A man of mass m moves on a plank of mass M with a constant velocity u wrt plank. If plank rests on a smooth surface, find velocity of plank.

$V_m = u + (-v) = u - v$

$V_M = (-v)$

~~$m(u+v) = M(v)$~~

~~$mu + mv = Mv$~~

~~$mu = v(M - m)$~~

~~$v = \frac{mu}{(M-m)}$~~

$m(u - v) = M(+v)$

$mu - mv = +Mv$

$mu = v(m + M)$

$$\boxed{v = \frac{mu}{(m+M)}}$$

Q.37. A man (50kg) standing on (100kg) plank on frictionless floor. initially both rest. If the man starts walking on the plank with speed 6 m/s toward right wrt plank, find amount of muscle energy spent (find energy of system)

$V_m = 6 - v$

$V_{plank} = v$

$50(6 - v) = 100v$

$300 - 50v = 100v$

$\frac{300}{150} = v$

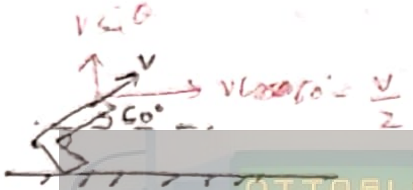
$v = 2$

$E = \frac{1}{2} \times 50 \times (4)^2 + \frac{1}{2} \times 100 \times (2)^2$

$= 400 + 200$

$\boxed{= 600 \text{ J}}$

Q 38. A gun (mass = M) fires a bullet (mass = m) with speed v, relative to barrel of the gun which is inclined at an angle 60° with horizontal. The gun is placed over a smooth horizontal surface. find recoil speed.



Sol recoil speed $= v'$

$\cancel{Mv' = m\left(\frac{v}{2}\right)}$

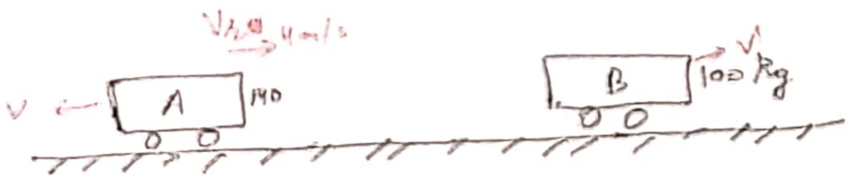
$\cancel{v' = \frac{mv}{2M}}$

$Mv' = \frac{mv}{2} - mv'$

$(M+m)\,v' = \frac{mv}{2}$

$$\boxed{v' = \frac{mv}{2(M+m)}}$$

Q 39. frictionless from rest. A man in std A throws 10 kg bag with horizontal vel 4 m/s wrt himself at trolley B of 100 kg. Mass of trolley A (excluding bag) is 140 kg (with man). Find velocities of trolleys A & B, just after bag lands at B.



$(140)(v) = (4-v)(10)$

$140v \; \cancel{40v} = 40 - 10v$

$\cancel{180v = 40}$ $\quad 150v = 40$

$\cancel{v = \frac{4}{18}}$ $\quad v = \frac{4}{15}$

$\cancel{v = \frac{2}{9}}$

$\cancel{v'(110) = \frac{38}{9}(10)}$

$\cancel{110v' = \frac{380}{9}}$

$\cancel{v' = \frac{380}{990}}$

$\cancel{v' = \frac{38}{99}}$

$\cancel{\frac{v}{v'} = \frac{2}{9} \times \frac{99}{38} = \frac{22}{38} = \frac{11}{19}}$

$\cancel{\boxed{\frac{v}{v'} = \frac{11}{19}}}$

$\cancel{4 - \frac{2}{9} = \frac{36-2}{9}}$

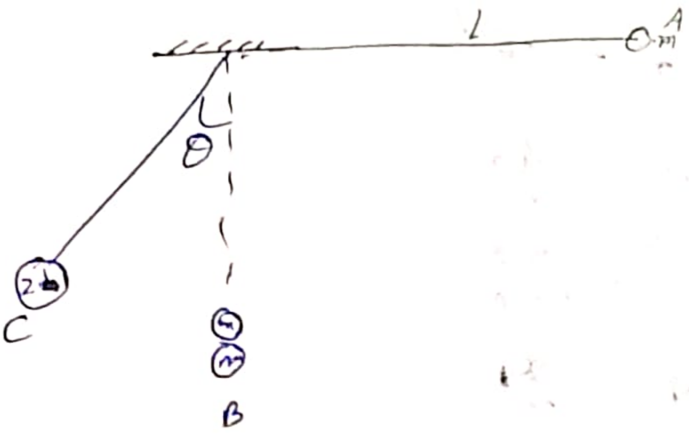
$v'(110) = \frac{56}{15} \times 10$

$v' = \frac{56}{15 \times 11}$

$\frac{v}{v'} = \frac{4}{15} \times \frac{15 \times 11}{56} = \frac{44}{56} = \frac{22}{28} = \frac{11}{14}$

$\boxed{\frac{v}{v'} = \frac{11}{14}}$

Q40. A bob of mass m is attached to string of length l tied on to a point on ceiling released from horizontal. At lowermost point another mass of m attaches to form composite of 2m mass. Find θ from vertical to which it rises



$V_B (\text{of } m) = \sqrt{2gR}$

$\sqrt{2gR}\,(m) = V_{\text{composite}}\,(2m)$

$V = \dfrac{\sqrt{2gR}}{2}$

$m \to 2m$

$R\cancel{l}(1-\cos\theta)\, mg = \frac{1}{2} m \left(\frac{\sqrt{2gR}}{2}\right)^2$

$R\cancel{l}(1-\cos\theta)\cancel{g} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}gRR}{4}$

$(1-\cos\theta) = \dfrac{\cancel{gR}}{4R}$

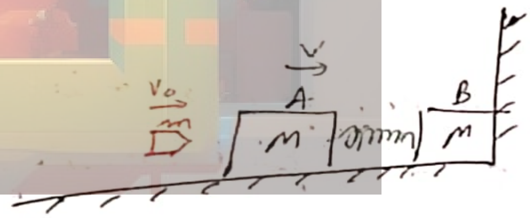
$(1-\cos\theta) = \frac{1}{4}$

$\cos\theta = 1 - \frac{1}{4}$

$\cos\theta = \frac{3}{4}$

$\boxed{\theta = \cos^{-1}\left(\frac{3}{4}\right)}$

Q 31. Two identical mass of 9 kg each on a rough horizontal floor $\mu = 0.1$. B is in contact with fixed wall. Bullet of mass $m = 1$ kg. $V_0 = 10$ m/s hits Block A & gets embeded. Find max compression ($K = 240$, $g = 10$)



$mV_0 = (m+M)\,V$

$V = \dfrac{(1)(10)}{(10)}$

$V = 1$ m/s

$K_i \pm W = K_f$

$\frac{1}{2}(10)(1) - \frac{1}{2}(240)(x^2) - (10)(x) = 0$

$5 = 120x^2 + 10x$

$24x^2 + 2x - 1 = 0$

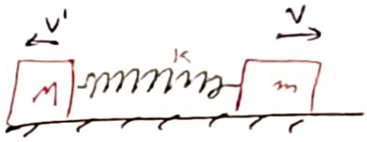
$x = \dfrac{-2 \pm \sqrt{4+96}}{48}$

$x = \dfrac{10-2}{48}$

$x = \dfrac{8}{48}$

$\boxed{x = \frac{1}{6}\text{ m}}$

Q 32. spring initially compressed by distance x. when released blocks aquire velocities in opposite directions. The spring looses contact with block as it aquires natural length find final speed.



$$v' = \frac{mv}{M}$$

$$\frac{1}{2} Kx^2 = \frac{1}{2} \times m \times v^2 + \frac{1}{2} \times M \times \frac{m^2 v^2}{M^2}$$

$$Kx^2 = mv^2 + \frac{m^2 v^2}{M}$$

$$Kx^2 = \left(m + \frac{m^2}{M}\right) v^2$$

$$v^2 = \frac{Kx^2 M}{(Mm + m^2)}$$

$$v = \sqrt{\frac{Kx^2 M}{Mm + m^2}}$$

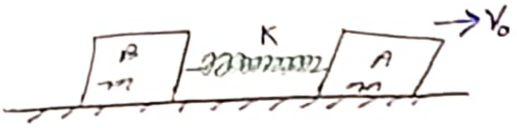
$$\boxed{v = \sqrt{\frac{Kx^2 M}{m(M+m)}}}$$

$$v' = \sqrt{\frac{Kx^2 M}{m(M+m)} \times \frac{m^2}{M^2}}$$

$$\boxed{v' = \sqrt{\frac{Kx^2 m}{M(M+m)}}}$$

Centroidal Frame

M I.



To find max elongation in spring?

velocities of both the blocks same

COLM:-

$$0 + mV_0 = mV + mV$$

$$\boxed{V = \frac{V_0}{2}}$$

COE:-

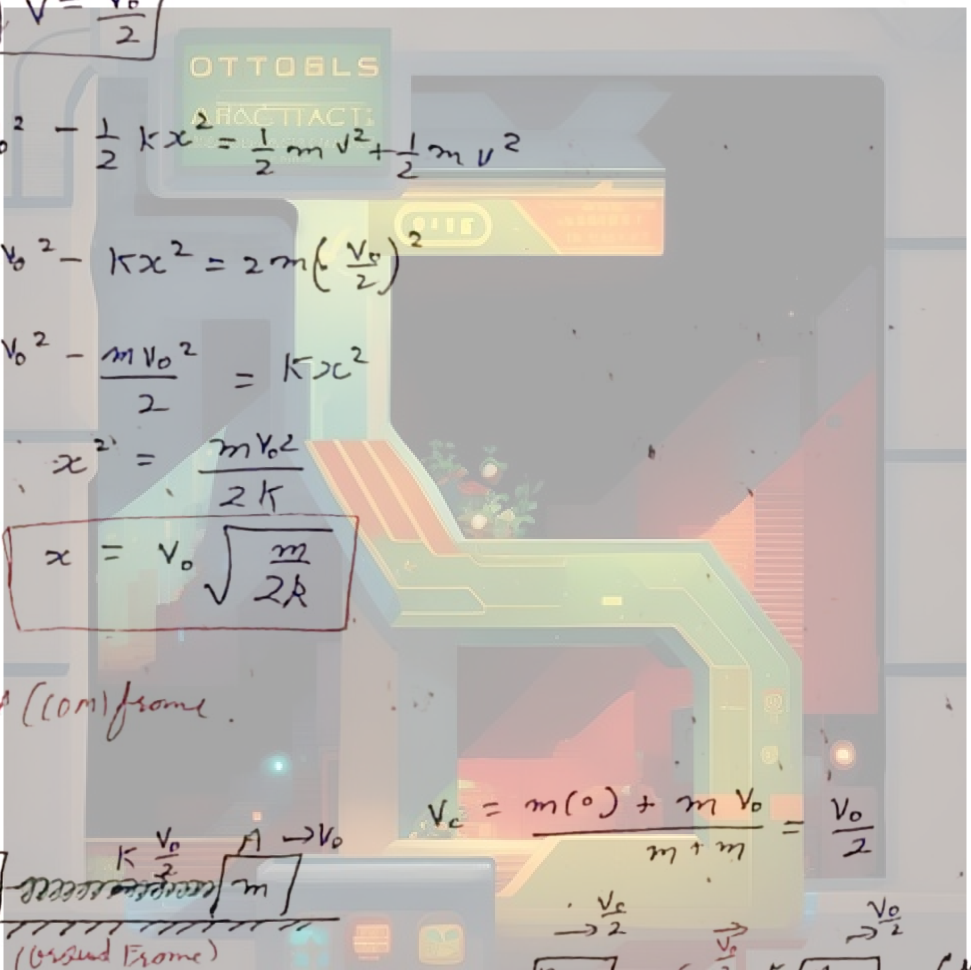
$$\frac{1}{2}mV_0^2 - \frac{1}{2}kx^2 = \frac{1}{2}mV^2 + \frac{1}{2}mV^2$$

$$mV_0^2 - kx^2 = 2m\left(\frac{V_0}{2}\right)^2$$

$$mV_0^2 - \frac{mV_0^2}{2} = kx^2$$

$$x^2 = \frac{mV_0^2}{2k}$$

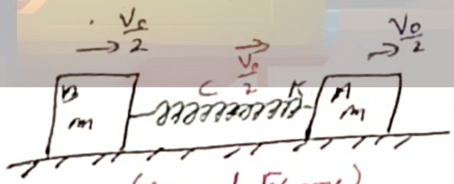
$$\boxed{x = V_0\sqrt{\frac{m}{2k}}}$$



M II Centroidal (COM) frame.

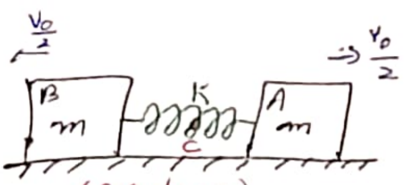
$$V_c = \frac{m(0) + mV_0}{m+m} = \frac{V_0}{2}$$

(Ground Frame)



(Max Elongation)

(Ground Frame)



(COM frame)

$$\boxed{\text{KE of system in COM frame} = \frac{1}{2}\mu (V_{rel})^2}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad \text{(reduced mass)}$$

17

$$K_{i\,cm} \pm W = K_{f\,cm}$$

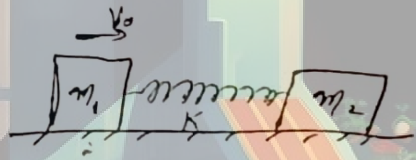
$$\frac{1}{2}\left(\frac{m}{2}\right) v_0^2 - \frac{1}{2} k x^2 = \frac{1}{2}\left(\frac{m}{2}\right)(0)^2$$

$$\frac{m v_0^2}{2} = k x^2$$

$$x^2 = \frac{m v_0^2}{2k}$$

$$\boxed{x = v_0 \sqrt{\frac{m}{2k}}}$$

Q 33. If m_1 is projected with horizontal velocity v_0, find max compression in spring.



MI

COLM:-

$$m_1 v_0 = (m_1 + m_2) v$$

$$v = \frac{m_1 v_0}{(m_1 + m_2)}$$

$$\frac{1}{2} m_1 v_0^2 - \frac{1}{2} k x^2 = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

$$m_1 v_0^2 - k x^2 = (m_1 + m_2)\left(\frac{m_1^2 v_0^2}{(m_1 + m_2)^2}\right)$$

$$m_1 v_0^2 - \frac{m_1^2 v_0^2}{(m_1 + m_2)} = k x^2$$

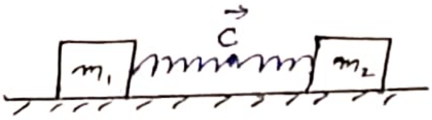
$$\frac{(m_1 + m_2) m_1 v_0^2 - m_1^2 v_0^2}{k(m_1 + m_2)} = x^2$$

$$x^2 = \frac{v_0^2 m_1 [m_1 + m_2 - m_1]}{k(m_1 + m_2)}$$

$$\boxed{x = v_0 \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}}$$

MII

$$V_c = \frac{m_1 V_0}{m_1+m_2}$$



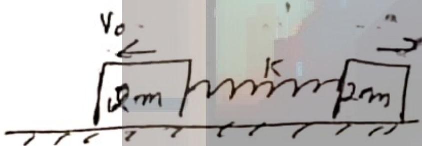
$$\mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\frac{m_1 m_2}{m_1+m_2}\left(V_0 - \frac{m_1 V_0}{m_1+m_2} + \frac{m_1 V_0}{m_1+m_2}\right)^2 = Kx^2$$

$$\frac{m_1 m_2 V_0^2}{(m_1+m_2)K} = x^2$$

$$\boxed{x = V_0\sqrt{\frac{m_1 m_2}{(m_1+m_2)K}}}$$

Q34. find max stretch



MI

$$\frac{2m^2}{3m}\left[3V_0\right]^2 = Kx^2 + \frac{2m^2}{3m}\left[0\right]$$

$$x^2 = \frac{2mV_0^2\times 1}{3K}$$

$$\boxed{x = V_0\sqrt{\frac{\cancel{2}m\times 6}{\cancel{3}K}}}$$

MII

COLM:-

$$mV_0 + 4mV_0 = mV + 2mV$$

$$5mV_0 = 3mV$$

$$V = \frac{5}{3}V_0$$

COE:-

$$\frac{1}{2}mV_0^2 + \cancel{\ldots}4mV_0^2 + \frac{1}{2}Kx^2 = \frac{1}{2}mv^2 + mv^2$$

$$\frac{9mV_0^2}{2} + \frac{1}{2}Kx^2 = \frac{3}{2}m\left(\frac{25V_0^2}{9}\right)$$

$$Kx^2 = \frac{81mV_0^2 - 75mV_0^2}{9}$$

$$Kx^2 = \frac{2mV_0^2}{3}$$

$$x = V_0\sqrt{\frac{2m}{3K}}$$

M.II.

COLM

$$2m(2v_0) + m(-v_0) = (m+2m)v$$

$$3mv_0 = 3mv$$

$$v = v_0$$

COE:-

$$\frac{1}{2}mv_0^2 + \frac{1}{2}(2m)(2v_0)^2 - \frac{1}{2}kx^2 = \frac{1}{2}(m+2m)v^2$$

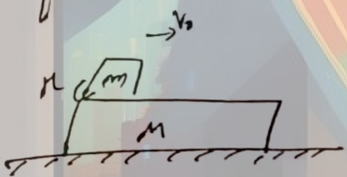
$$9mv_0^2 - 3mv_0^2 = kx^2$$

$$6mv_0^2 = kx^2$$

$$x^2 = \frac{6mv_0^2}{k}$$

$$\boxed{x = v_0\sqrt{\frac{6m}{k}}}$$

Q35. find min distance of relative sliding betw block & plank.

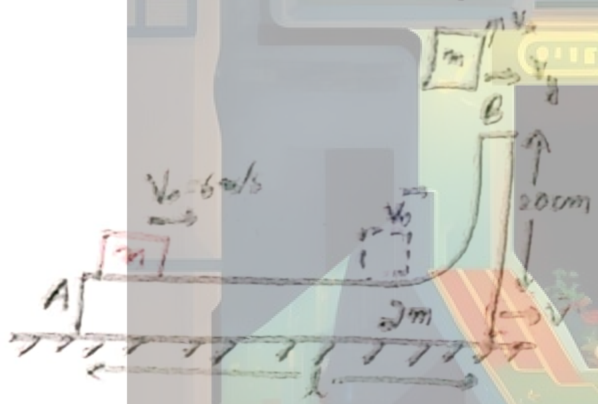


$$\frac{1}{2}\frac{mM}{m+M}[v_0]^2 = \mu mg\, x$$

$$\frac{1}{2}\frac{mMv_0^2}{m+M} = \mu mg\, x$$

$$\boxed{x = \frac{Mv_0^2}{(m+M)2\mu g}}$$

Q (Hard) A wedge of mass (2m) rests on smooth horizontal plane. A small block of mass m rests on it at end A. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity $V_0 = 6\,ms^{-1}$. At highest point of its trajectory, the block collides with a particle of same mass m moving vertically downwards with velocity $v = 2\,ms^{-1}$ and gets stuck with it. If the combined body lands at end point A of body of mass (2m), calculate length l. Neglect friction. ($g = 10\,m\,s^{-2}$)



COLM :-

$0 + m(6) = (2m)(V) + mV$

$V = 2 \quad V = 6/3$

$V = 2\,m/s$

$\mu = \dfrac{m(2m)}{m + 2m} = \dfrac{2m^2}{3m} = \dfrac{2m}{3}$

$K_i + W = K_f$

$\frac{1}{2} \times \frac{2m}{3} \times (6)^2 - (m)(g)(0.2) = \frac{1}{2} \times \frac{2m}{3} \times (2)^2 + E$

$\frac{36}{3} m = 2m + E$

$E = \frac{30m}{3}$

$E = 10m$

$E \rightarrow$ Energy left

$E = \frac{1}{2} m V_y^2$

$10 m = \frac{1}{2} m V_y^2$

$V_y^2 = 20$

$V_y = 4\sqrt{5}$

Max height $H = \frac{u^2}{2g}$

$H = \frac{(4\sqrt{5})^2}{2(10)}$

$H = 1 m$

COLM:

$m(-2) + m(0) = 2m(v)$

$v = \frac{-2}{2}$

$v = -1$ m/s

$v = 1$ m/s (downward)

Time Taken for the block w.r.t particle to come down

$\cancel{(1) = (1)t + \frac{10}{2} t^2}$

$\cancel{5t^2 + t - 1 = 0}$

$\cancel{t = -1 + \sqrt{1}}$

Total Height = 1.2 m

$1.2 = t + 5t^2$

$5t^2 + t - 1.2 = 0$

$t = \frac{-1 \pm \sqrt{1+24}}{10}$

$t = \frac{-1+5}{10}$

$t = 0.4 s$

$V_{rel} = V_{wedge} - V_{block}$ (x direction)

$V_{rel} = 2 - 1$

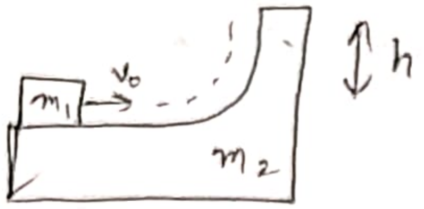
$V_{rel} = 1$ m/s

$l = V_{rel} \times t$

$l = 1 \times 0.4$

$\boxed{l = 0.4 m}$

Q37. find max height attained by block. The block does not leave to wedge. (All surfaces are frictionless)



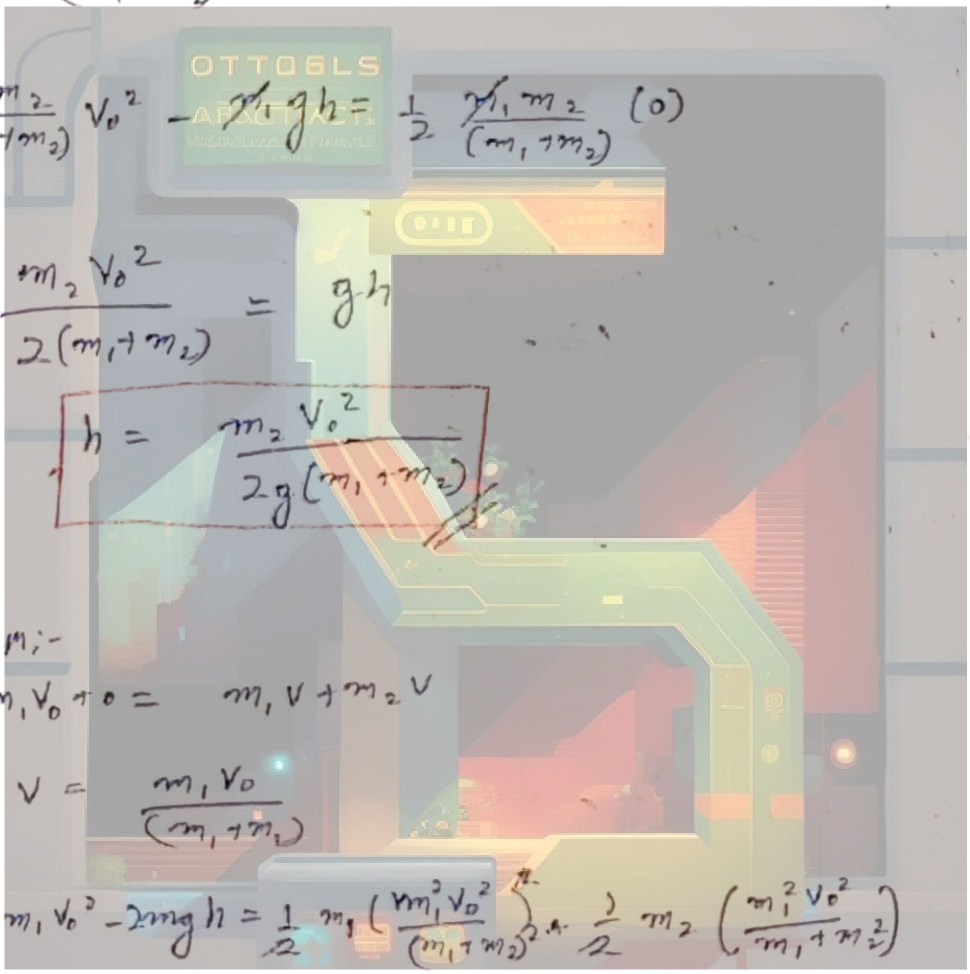
MI

$$\mu = \frac{m_1 m_2}{(m_1+m_2)}$$

$$\frac{1}{2}\frac{\cancel{m_1} m_2}{(m_1+m_2)} v_0^2 - \cancel{m_1} g h = \frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)}(0)$$

$$\frac{m_2 v_0^2}{2(m_1+m_2)} = gh$$

$$\boxed{h = \frac{m_2 v_0^2}{2g(m_1+m_2)}}$$



MII

COLM:-

$$m_1 v_0 + 0 = m_1 v + m_2 v$$

$$v = \frac{m_1 v_0}{(m_1+m_2)}$$

$$\frac{1}{2} m_1 v_0^2 - 2mgh = \frac{1}{2} m_1 \left(\frac{m_1^2 v_0^2}{(m_1+m_2)^2}\right) + \frac{1}{2} m_2 \left(\frac{m_1^2 v_0^2}{(m_1+m_2)^2}\right)$$

$$m_1 v_0^2 - 2mgh = \frac{m_1^2 v_0^2}{(m_1+m_2)}$$

$$\frac{m_1 v_0^2 (m_1+m_2) - m_1^2 v_0^2}{(m_1+m_2)\, 2m_1 g} = h$$

$$\boxed{h = \frac{m_2 v_0^2}{2g(m_1+m_2)}}$$

Q38. two blocks 2 kg & 5 kg lie on frictionless surface. connected by massless string of mass 35 N/m. find max compression.

$$5\,m/s \leftarrow \boxed{2\,kg} \overset{k=35}{\text{-wwww-}} \boxed{5\,kg} \rightarrow 2\,m/s$$

$$\mu = \frac{2 \times 5}{7} = \frac{10}{7}$$

$$\frac{1}{2} \times \frac{10}{\cancel{7}} \times \cancel{49}^{7} - \frac{1}{2} k x^2 = \frac{1}{2} \times \frac{10}{7} \times 0$$

$$\frac{70}{2} = \frac{1}{2} k x^2$$

$$x^2 = \frac{70}{\cancel{35}}$$

$$x^2 = 2$$

$$\boxed{x = \sqrt{2}}$$

Impulse.

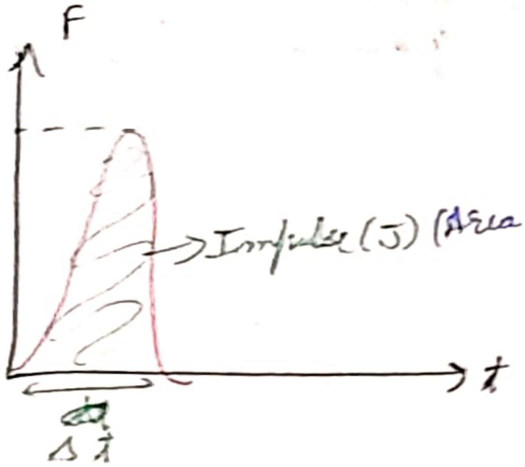
→ A force of relatively higher magnitude & acting for a relatively shorter time is called impulsive force. It is just a relative term. There is no clear boundary between impulsive and non-impulsive.

→ Gravitational & spring forces are always non impulsive.

$$\boxed{\vec{J} = \int F\,dt} \quad (17)$$

$$J = \int m \frac{dv}{dt}\,dt = \int m\,dv = m \int dv$$

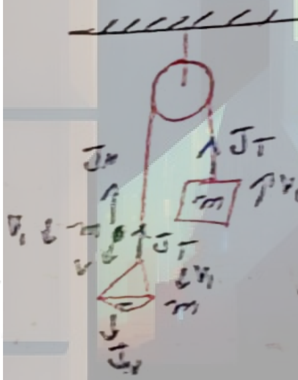
$$\boxed{\vec{J} = \Delta \vec{P}} \quad (18)$$



→ SI unit :- Ns or kg m/s

→ It is a vector quantity.

Q39. find speed just after collision. Also find Impulse of Tension & contact b/w particle & pan.



$-J_N = mv' - mv$ (Particle)

$J_N - J_T = mv' - 0$ (Pan)

$J_T = mv' - 0$ (Block)

add

$0 = 3mv' - mv$

$mv = 3mv'$

$\boxed{v' = v/3}$

$J_T = mv'$

$\boxed{J_T = \frac{mv}{3}}$

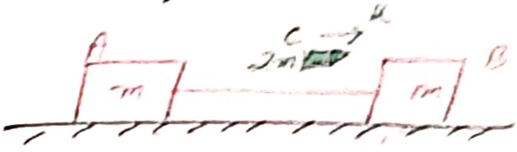
$J_N = mv - mv'$

$J_N = m\left(v - \frac{v}{3}\right)$

$\boxed{J_N = \frac{2mv}{3}}$

Q40. The bullet gets embedded in B, find velocities A, B & C.

M-I



$2mu = 4mv$

$v = \frac{2u}{4}$

$\boxed{v = \frac{u}{2}}$

M-II

~~$J_N = mu$~~

$-J_{AN} = 2mv - 2mu$

$J_N - J_T = mv$

$J_T = mv$

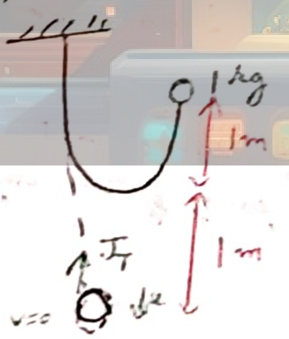
add

$0 = 2mv - 2mu + mv + mv$

$2mu = 4mv$

$\boxed{v = \frac{u}{2}}$

Q41. find impulse by Tension in string immediately after string becomes taut



$v^2 = 0 + 2(10)(2)$

$v^2 = 40$

$v = \sqrt{40}$

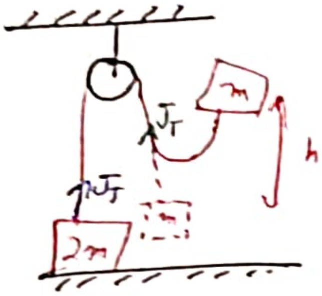
$v = 2\sqrt{10}$

$-J_T = mv - mu$

$-J_T = 0 - (1)(2\sqrt{10})$

$\boxed{J_T = 2\sqrt{10}\ kg\,m/s}$

Q42. Find speed with which 2m begin to rise ?



$v = \sqrt{20h}$

$+J_T = \cancel{\ } \; 2mV - \cancel{2mu}$

$-J_T = + mv - mv$

$J_T = 2mV$

$+J_T = m(\sqrt{20h})$

$\sqrt{20}\ h = 2mV$

$J_T = \sqrt{20}\, h\, m$

$\sqrt{5}\, h = V$

$V = \frac{\sqrt{5}\, h}{\cancel{m}}$

$V = \sqrt{20h}$

$mV = (m + 2m)\, v_1'$

$v' = \frac{v}{3}$

$$\boxed{v' = \frac{\sqrt{2gh}}{3}}$$

Q43. The system is released from rest & it becomes ~~taught~~ taut as B falls 0.5m.

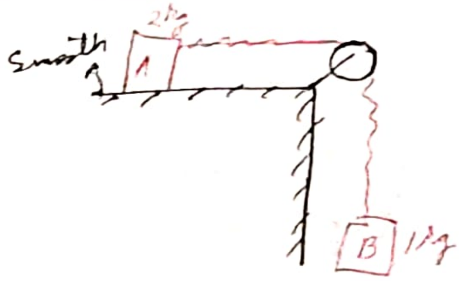
a) find common velocity just after string becomes taut

b) find magnitude of impulse on the pulley ~~of the chain~~

$v^2 = 2 \times 10 \times 0.5$

$v^2 = 10$

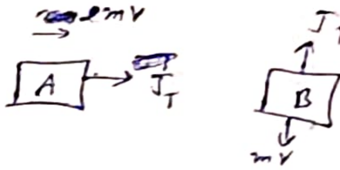
$v = \sqrt{10}$



COLM:-

$(1)\sqrt{10} = 3v'$

$$\boxed{v' = \frac{\sqrt{10}}{3}}$$



~~$-J_T = (1)\sqrt{10}$~~

~~$J_T = -\sqrt{10}$~~

~~$J_T = mv$~~

~~$-\sqrt{10}$~~

$J_T = 2v' - 0$

$J_T = \frac{2\sqrt{10}}{3}$

$-J_T = \frac{\sqrt{10}}{3} - \sqrt{10}$

$-J_T = -\frac{2\sqrt{10}}{3}$

$J_T = \frac{2\sqrt{10}}{3}$

~~$J_T = mv$~~

~~$J_T = 2v$~~

$$J_{net} = \sqrt{\frac{40}{9} + \frac{40}{9}}$$

$$= \frac{\sqrt{80}}{3}$$

$$\boxed{= \frac{4\sqrt{5}}{3}}$$

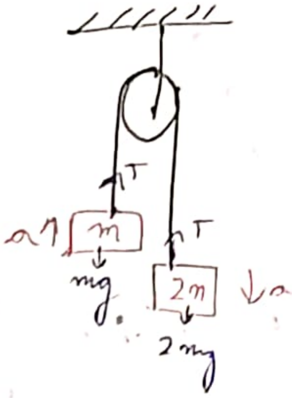
Q44. Two bodies of mass m & 2m. After 4s, a second mass m is suddenly joined to the ascending body.

A) The resulting speed

b) How much kinetic energy is lost by decending body when load of mass m is added.

~~$a = (m_1 + m_2)$~~

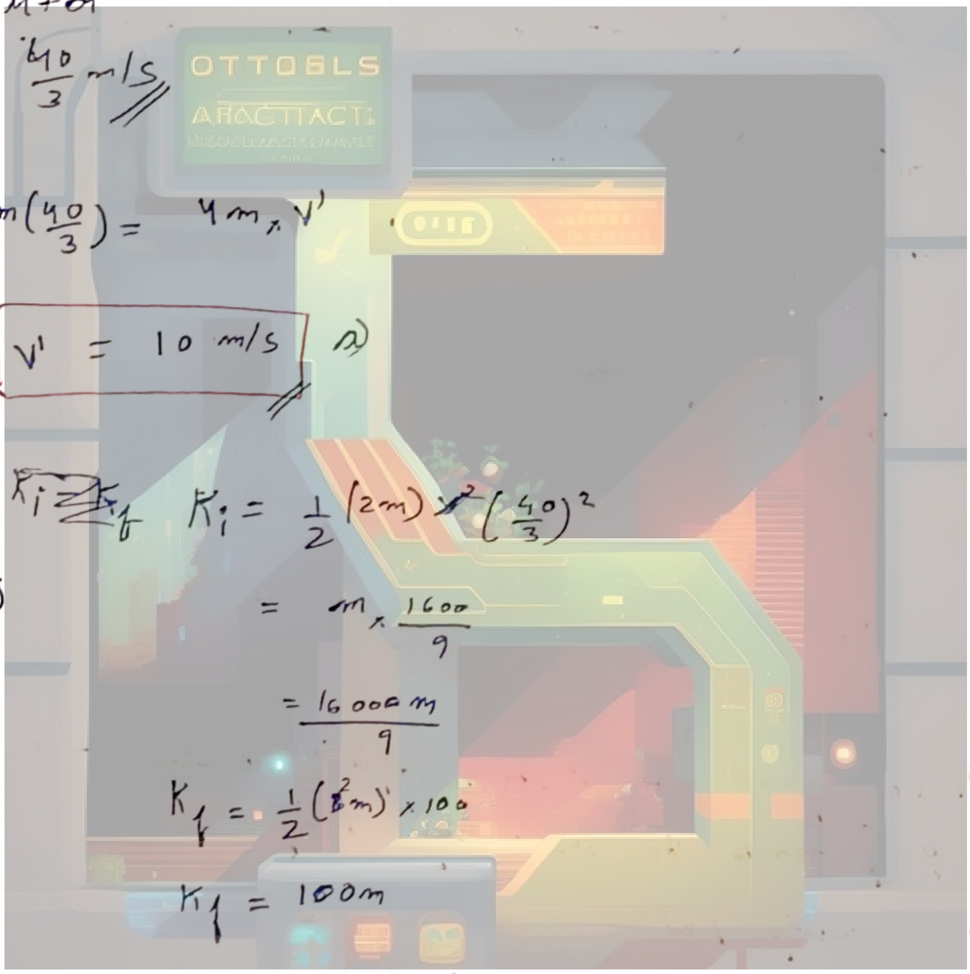
$$2mg - T = 2ma$$
$$T - mg = ma$$



$$mg = 3ma$$
$$\boxed{a = \frac{10}{3}}$$

at 4s,

$$v = u + at$$
$$v = \frac{40}{3} \, m/s$$



$$3m\left(\frac{40}{3}\right) = 4m \times v'$$

$$\boxed{v' = 10 \, m/s} \text{ a)}$$

~~$K_i = K_f$~~ $K_i = \frac{1}{2}(2m)\left(\frac{40}{3}\right)^2$

~~$\frac{1}{2}(2m)$~~

$$= m \times \frac{1600}{9}$$
$$= \frac{16000m}{9}$$
$$K_f = \frac{1}{2}(2m) \times 100$$
$$K_f = 100m$$

~~$K_i - K_f = 1356$~~

~~$K_f - K_i$~~

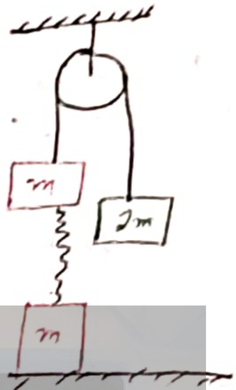
$$K_i - K_f = \frac{1600m - 900m}{9}$$

$$\boxed{= \frac{700}{9} m} \text{ b) Joules}$$

Q45. The atwood machine in figure has a third mass attached by loose string. 2m falls x m as string becomes tight. What is final speed.

$u^2 = \frac{20x}{3}$ $\quad a = \frac{g}{3}$

$u = \cancel{4\sqrt{5x}} \sqrt{\frac{20x}{3}}$



COLM:-

$\cancel{2m(4\sqrt{5x}) = 4m(}$

$\cancel{2m(4\sqrt{5x}) = 4m\,v'}$

$\cancel{v' = 2\sqrt{5x}}$

$\cancel{v' = \sqrt{45x}}$

$\frac{3}{4} \times \sqrt{\frac{20x}{3}} = v'$

$v' = \sqrt{\frac{20x \times 9 \times 3}{3 \times 16}}$

$\boxed{v' = \sqrt{\frac{15x}{4}}}$

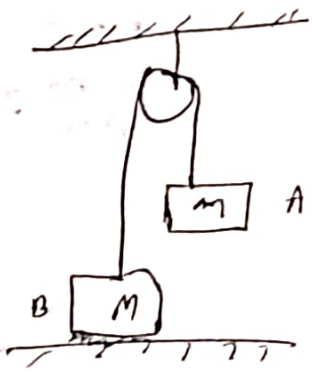
Q46. After falling from rest through a height h, a body of mass m begins to raise a body of mass M, connected to it through a pulley.

a) Determine the time it will take for body of mass M to return to its original position.

b) Find the fraction of kinetic energy lost when M is jerked to motion

$u^2 = 20h$ $\quad A = \frac{(M-m)g}{(M+m)}$

$u = \sqrt{20h}$



$mv = (m + M)v'$

$v' = \frac{m\sqrt{20h}}{m+M}$

$$0 = \frac{m\sqrt{20h}}{M+m}t - \frac{(M-m)10}{(M+m)2}t^2$$

$$\frac{5(M-m)t}{\cancel{(M+m)}} = \frac{m\sqrt{20h}}{\cancel{(M+m)}}$$

$$t = \frac{m\sqrt{20h}}{(M-m)5}$$

$$\boxed{t = \frac{m}{(M+m)}\sqrt{\frac{4h}{5}}\ s}$$

$$K_i = \frac{1}{2}m(20h)$$

$$= 10mh$$

$$K_f = \frac{1}{2}\cancel{a}\left(\frac{m^2\times 20h}{(M+m)^{\cancel{2}}}\right)\cancel{(M+m)}$$

$$K_f = \frac{20h\,m^2}{2(M+m)}$$

$$K_i - K_f = 10mh - \frac{20h\,m^2}{2(M+m)}$$

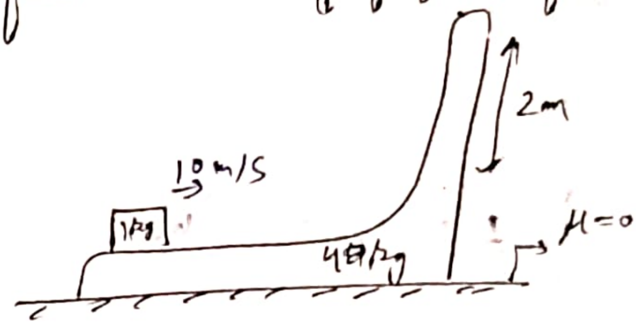
$$= \frac{20(M+m)mh - \cancel{20}\,m^2\times 20h}{2(M+m)}$$

$$= \frac{10mh(M)}{2(M+m)}$$

$$\boxed{= \frac{10mMh}{M+m}\ J}$$

Question Practice :-

Q. find out time of flight for 1kg block when it is in air.



$(1)(10) = (5)(v)$

$v = 2\ m/s$

$\cancel{mgh = \frac{1}{2}\times\frac{4}{5}\times 100}$

$\cancel{100h = 400}$

$\cancel{h = 4m}$

~~FBD~~

$\frac{1}{2}\times\frac{4}{5}\times 100 - (1)(10)(2) = \frac{1}{2}(1)(v^2)$

$40 = v^2$

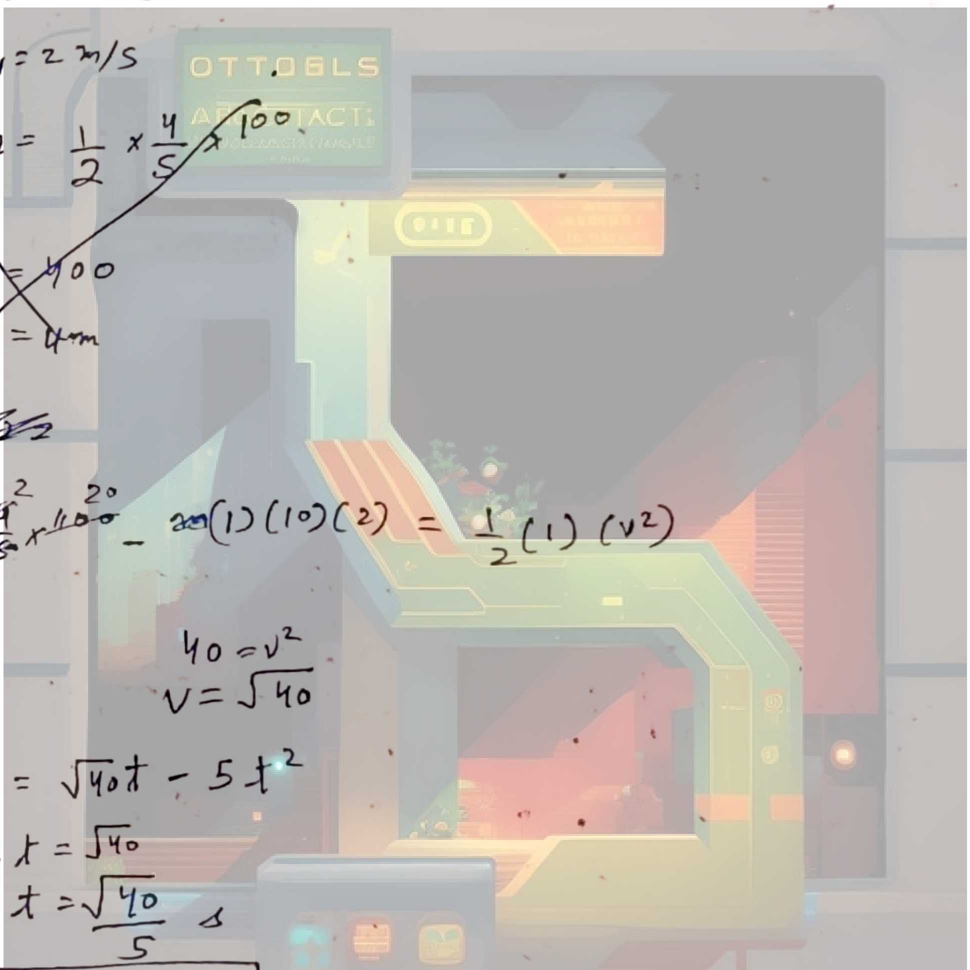
$v = \sqrt{40}$

$0 = \sqrt{40}t - 5t^2$

$5t = \sqrt{40}$

$t = \frac{\sqrt{40}}{5}\ s$

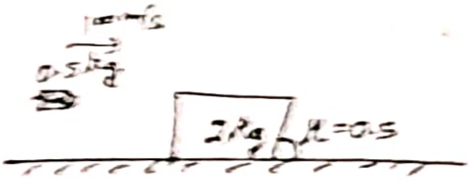
$\boxed{t = 2\sqrt{\frac{2}{5}}}$



Q 2. bullet is embeded in 2 kg block

a) find out work done by friction force

b) find out the distance moved by 2 kg block.



$(0.5)(100) = (2.5)(v)$

$\frac{50}{2.5} = v$

$v = 20\ m/s$

$\mu_k = (0.5)(20) = 10$

$a = \frac{10}{2} = 5$

$0 = 20(400) - (2)(0.5)(25)(x)$

$\frac{400}{10} = x$

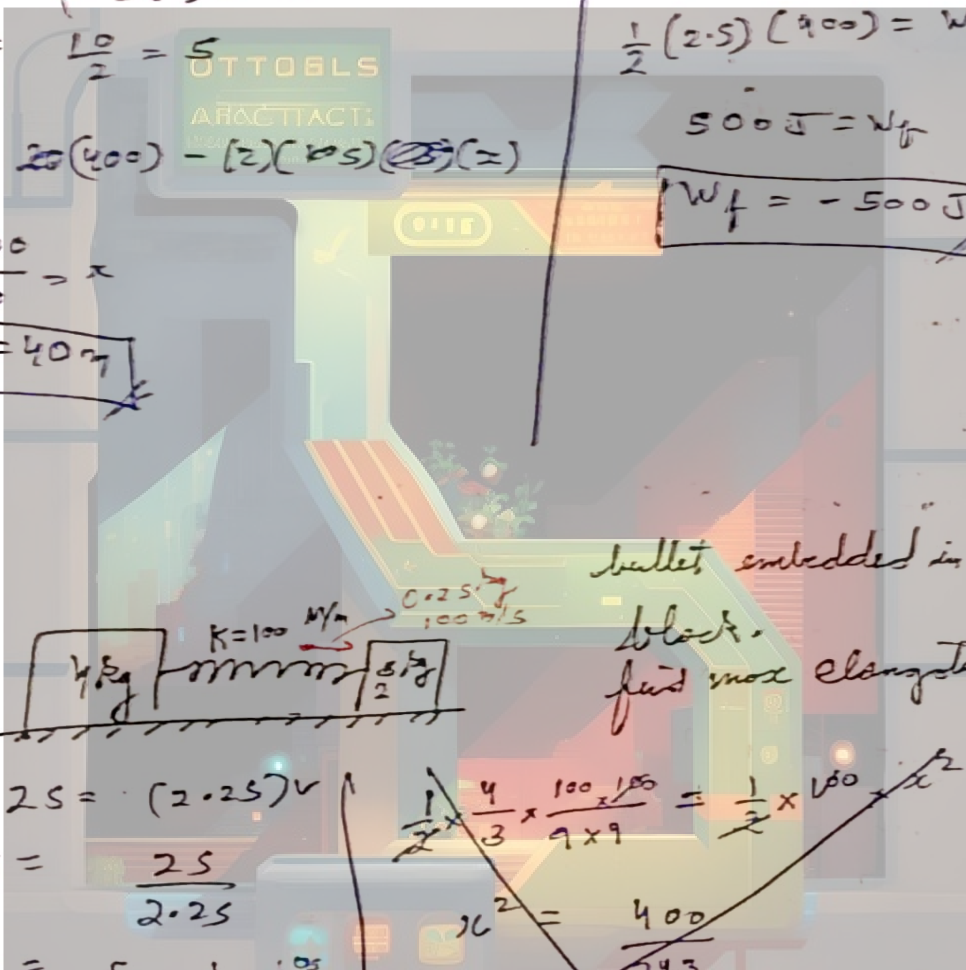
$\boxed{x = 40\ m}$

$\cancel{\frac{1}{2}(0.5) \times (100)^2 = W_f}$

$\frac{1}{2}(2.5)(400) = W_f$

$500\ J = W_f$

$\boxed{W_f = -500\ J}$



Q3. bullet embedded in 2 kg block.

find max elongation

4 kg
K = 100 N/m
0.25 kg
100 m/s
2 kg

$25 = (2.25)v$

$v = \frac{25}{2.25}$

$v = \frac{5}{45} \times \frac{1}{100} \times .05$

$v = \frac{100}{900}\ m/s$

$\cancel{\frac{1}{2} \times \frac{4}{3} \times \frac{100}{9 \times 9} \times 100 = \frac{1}{2} \times 100 \times x^2}$

$\cancel{x^2 = \frac{400}{243}}$

$\cancel{x = \frac{20}{9\sqrt{3}}}$

$\frac{1}{2} \times \frac{4 \times 2.25}{6.25} \times \left(\frac{100}{9}\right)^2 = \frac{1}{2} \times 100 \times x^2$

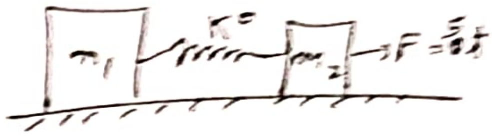
$x = \frac{10}{9}\sqrt{\frac{4 \times 2.25}{6.25}}$

$x = \frac{10}{3}\sqrt{\frac{100}{6.25}}$

$x = \frac{100}{75}$

$x = 4/3$

Q9.



~~$x = $ const~~

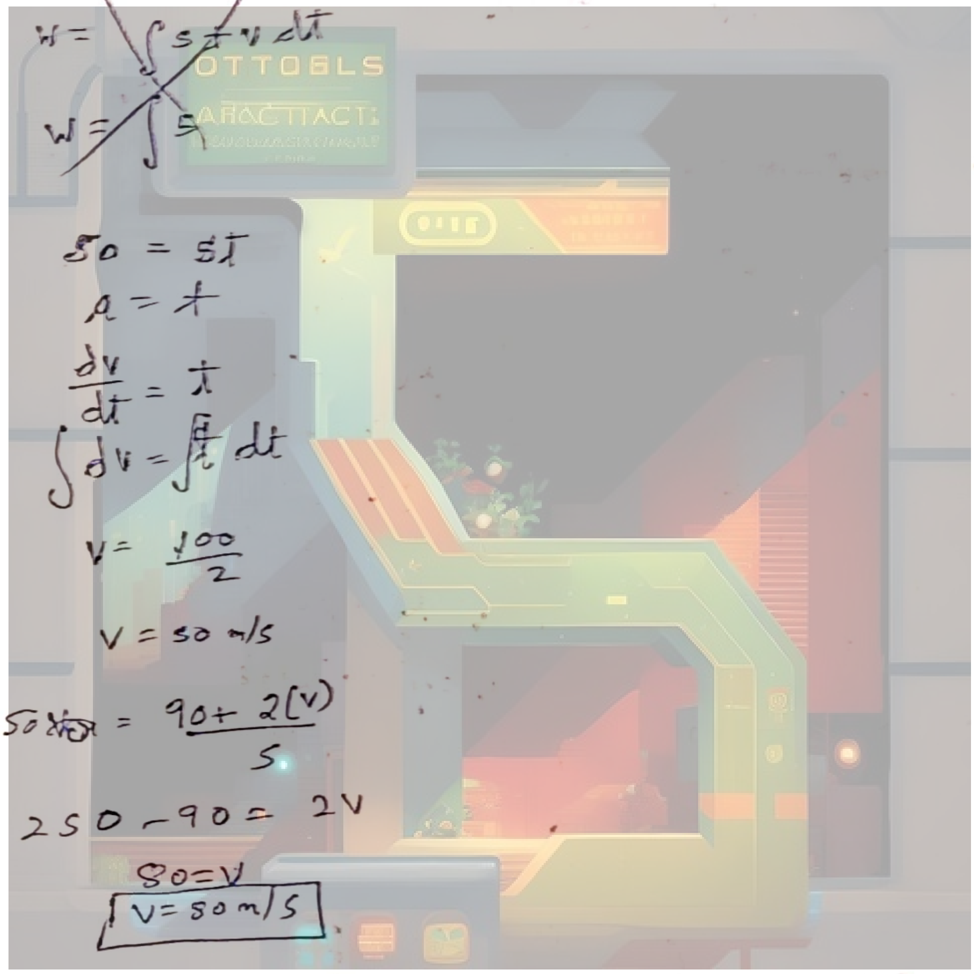
$k = 50$ N/m

$m_1 = 3$ kg

$m_2 = 2$ kg

friction force to be less

After 10s, vel of m_1 is 30 m/s find vel of m_2.



$\cancel{W = \int 5t\, dx}$

$\cancel{W = \int 5t\, v\, dt}$

$\cancel{W = \int 5}$

$50 = 5t$

$a = t$

$\frac{dv}{dt} = t$

$\int dv = \int t\, dt$

$v = \frac{100}{2}$

$v = 50$ m/s

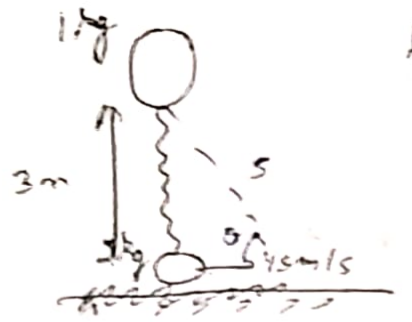
$50 = \frac{90 + 2(v)}{5}$

$250 - 90 = 2v$

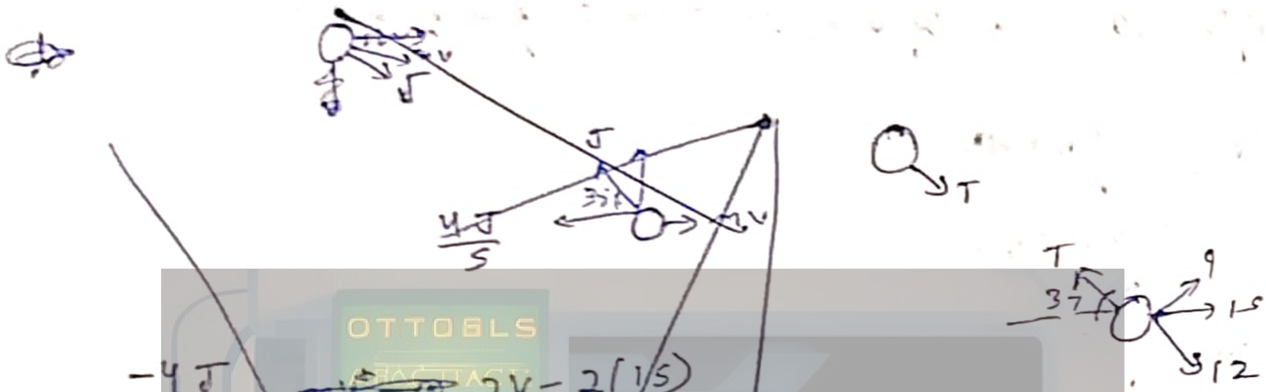
$80 = v$

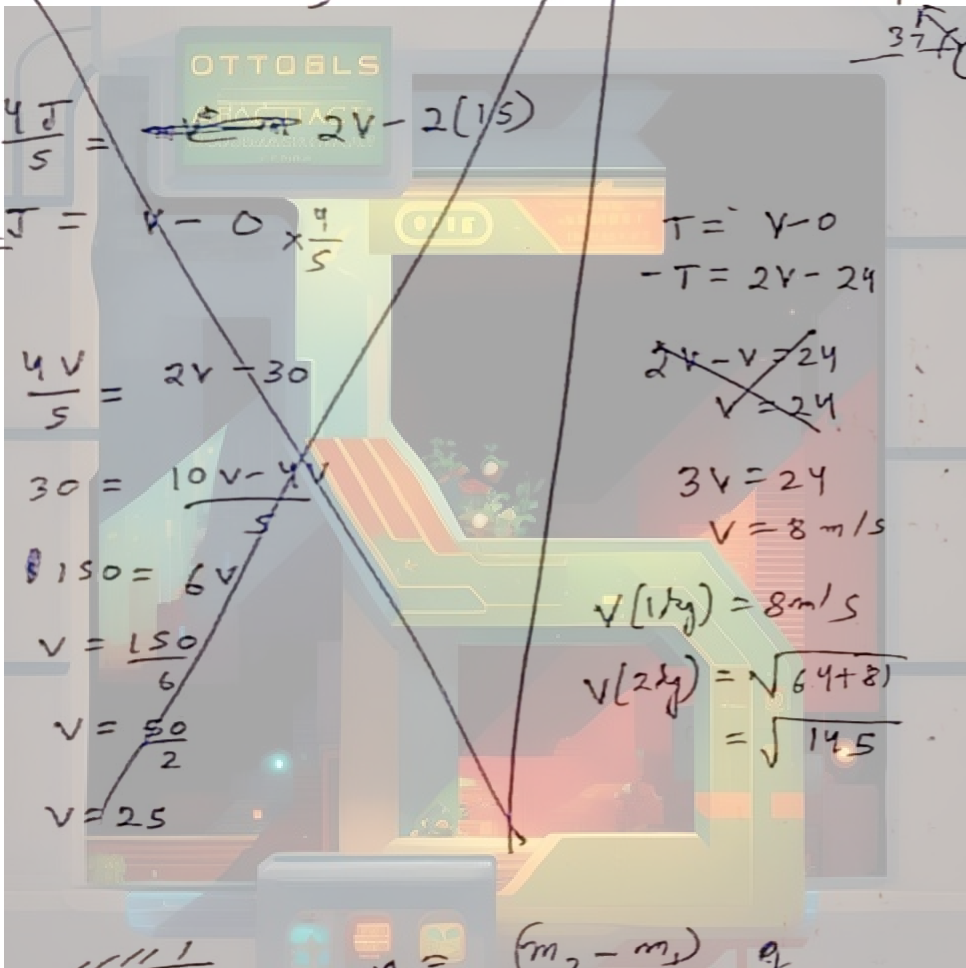
$\boxed{v = 80 \text{ m/s}}$

Q



$l(\text{spring}) = 5m$





$-\frac{4J}{5} = 2v - 2(15)$

$\frac{4 \times J}{5} = v - 0 \times \frac{4}{5}$

$\frac{4v}{5} = 2v - 30$

$30 = \frac{10v - 4v}{5}$

$150 = 6v$

$v = \frac{150}{6}$

$v = \frac{50}{2}$

$v = 25$

$T = v - 0$

$-T = 2v - 24$

$\cancel{2v - v = 24}$

$\cancel{v = 24}$

$3v = 24$

$v = 8\ m/s$

$v(1kg) = 8\ m/s$

$v(2kg) = \sqrt{64+81}$

$= \sqrt{145}$

Q

$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$

$a_{cm} = \frac{m_1 g(m_2 - m_1) + m_2 g(m_2 - m_1)}{(m_1 + m_2)^2}$

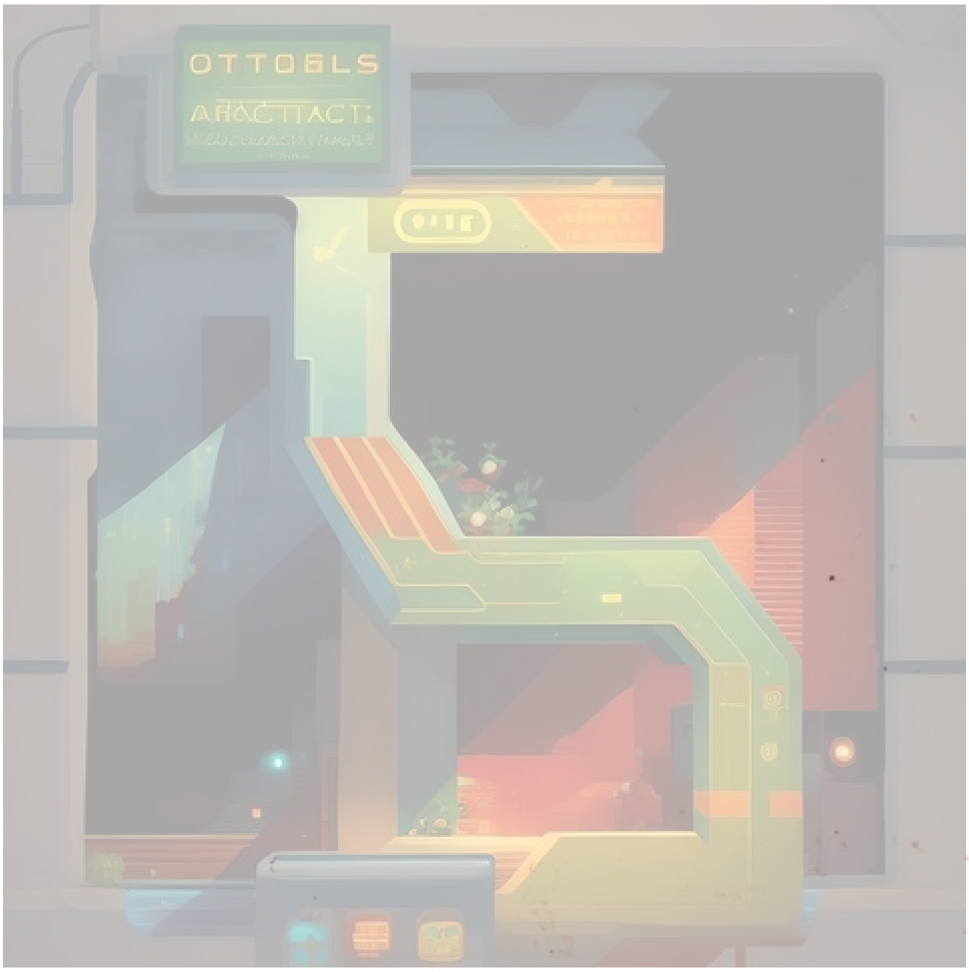
$a_{cm} = \frac{(m_2 - m_1) g (m_1 + m_2)}{(m_1 + m_2)^2}$

$\boxed{a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}}$

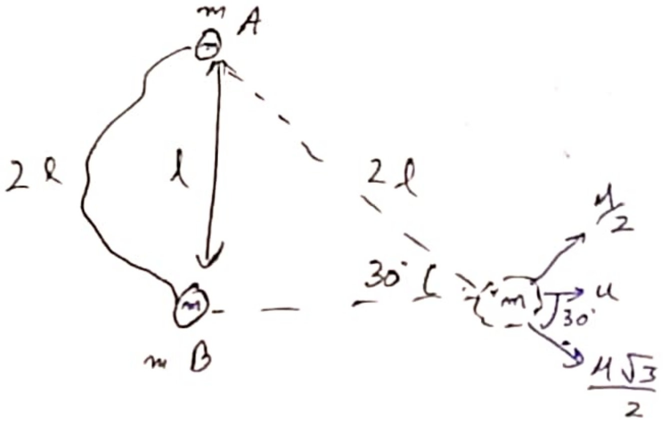
$$A = \frac{(m_1 - m_2)\, g}{(m_1 + m_2)}$$

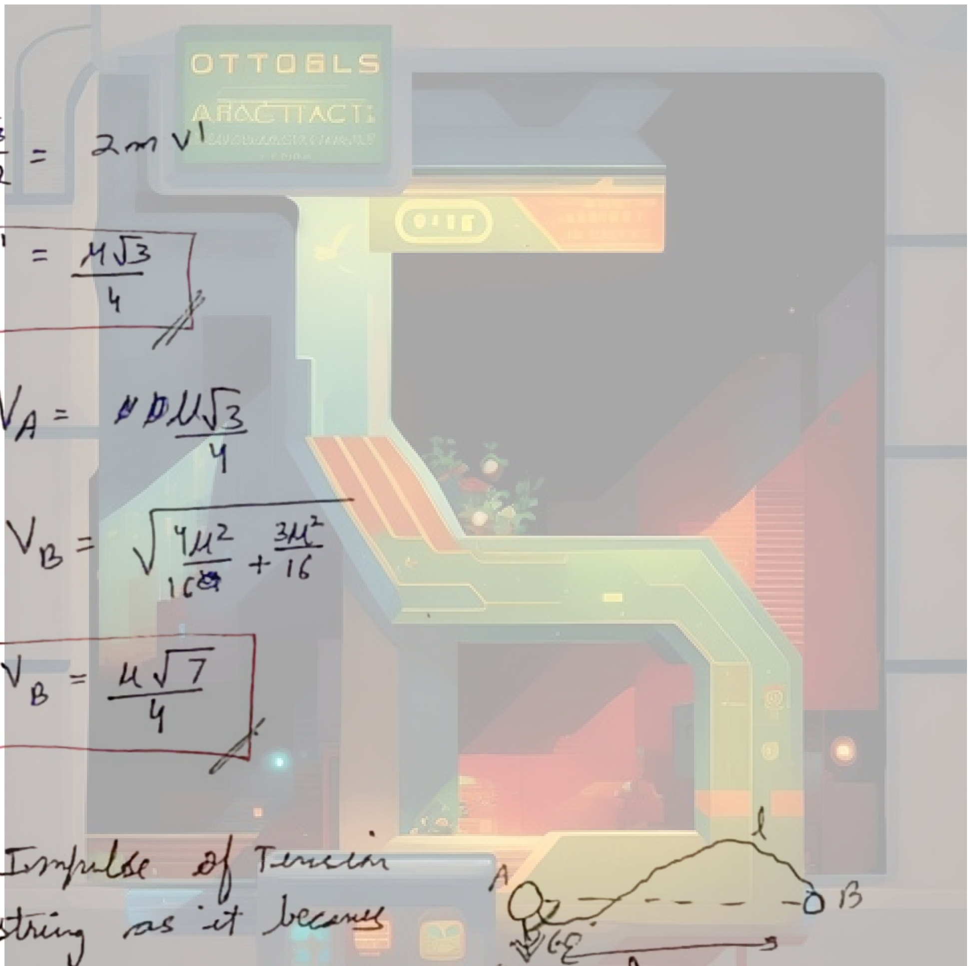
$$a_{cm} = \frac{m_1 g (m_1 - m_2) - m_2 g (m_1 - m_2)}{(m_1 + m_2)^2}$$

$$a_{cm} = \frac{g (m_1 - m_2)^2}{(m_1 + m_2)^2}$$



Q47. Find velocities of A & B after string is taut (Motion on Horizontal plane)





$$m u \frac{\sqrt{3}}{2} = 2 m v'$$

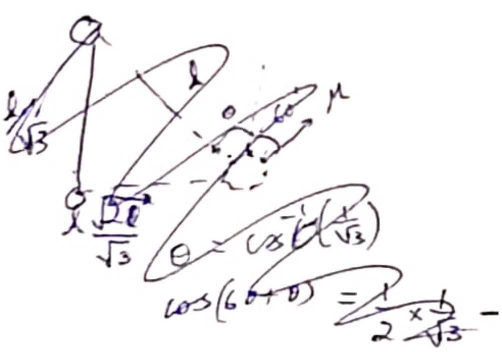
$$\boxed{v' = \frac{u\sqrt{3}}{4}}$$

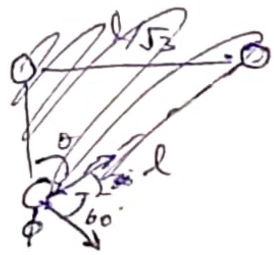
~~Vel A~~ $V_A = \frac{u\sqrt{3}}{4}$

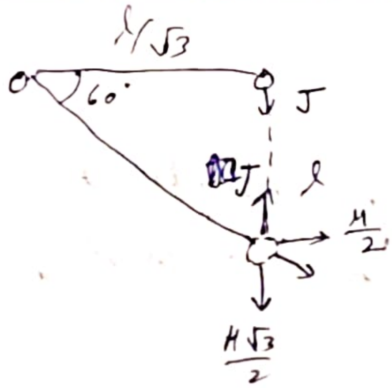
$$V_B = \sqrt{\frac{4u^2}{16} + \frac{3u^2}{16}}$$

$$\boxed{V_B = \frac{u\sqrt{7}}{4}}$$

Q48. find Impulse of Tension in string as it becomes taut.







$J = mv - 0$

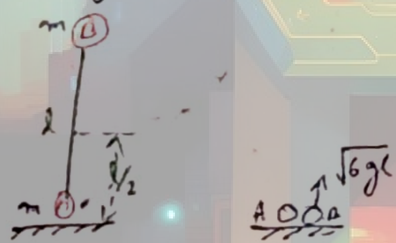
$-J = mv - \frac{mu\sqrt{3}}{2}$

$\frac{mu\sqrt{3}}{2} = 2mv$

$v = \frac{u\sqrt{3}}{4}$

$\boxed{J = \frac{mu\sqrt{3}}{4}}$

Q49. A & B are tied by string of length l. B is projected up with velocity $\sqrt{6gl}$. find maximum height upto which center of mass of the system rise.



V_B at length l.

$V_B^2 = 6gl - 2gl$

$V_B^2 = 4gl$

$V_B = \sqrt{4gl}$

COLM:-

$m\sqrt{4gl} = 2mv_2$

$V = \sqrt{gl}$

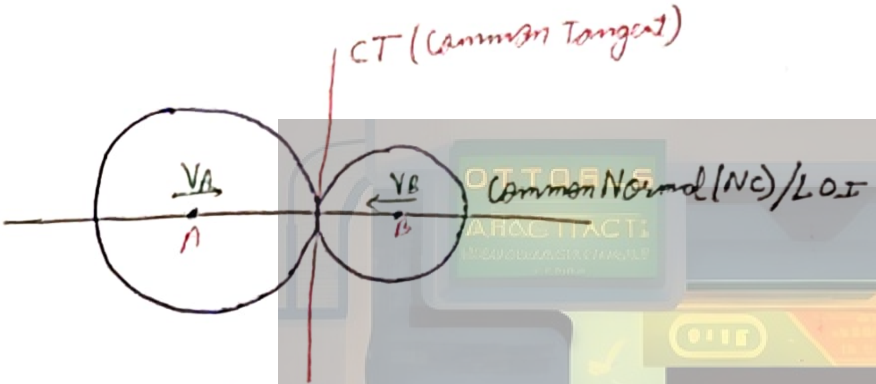
$V_{cm} = \sqrt{gl}$

$H = \frac{gl}{2g} = \frac{l}{2}$

Total Height $= \frac{l}{2} + \frac{l}{2} = \boxed{l}$

* Collision / Impact

Line of Impact (LOI) – The line passing through the common normal to the surface in contact is called line of impact.



Head on collision

Velocities of both bodies are along same ~~side~~ line before & after collision.

Oblique collision

Velocities of both the bodies are Along different lines.

Classification of collisions - (Basis of energy loss)

i) Elastic collision:- The particle regain their speed and size ~~comp~~ Completely after collision.

KE before collision = KE after collision

→ No fraction of mechanical energy remains stored as deformation P.E in bodies.

→ Thus, linear momentum and K.E. both remains conserved.

ii) inElastic collision - The particles do not regain their shape & size completely ~~f~~ after collision.

→ Some fraction of mechanical energy is retained by the colliding particles in form of deformation P.E.

Thus, K.E is no longer conserved.

Linear Momentum remains conserved.

iii) Perfectly Inelastic collision - Both particles stick together after collision and move with some velocity.

Thus, K.E is no longer conserved.

Linear Momentum remains Conserved.

Coefficient of restitution (e) :-

$$\cancel{e = \frac{V_2 - V_1}{u_2 - u_2}}$$

$$e = \frac{\text{Velocity of separation along LOI}}{\text{Velocity of approach along LOI}}$$

$$\boxed{e = \frac{V_2 - V_1}{u_1 - u_2}} \quad (19)$$

m_1 $\quad$ m_2

$\xrightarrow{u_1}$ $\quad$ $\xrightarrow{u_2}$

$u_1 > u_2$

m_1 $\quad$ m_2

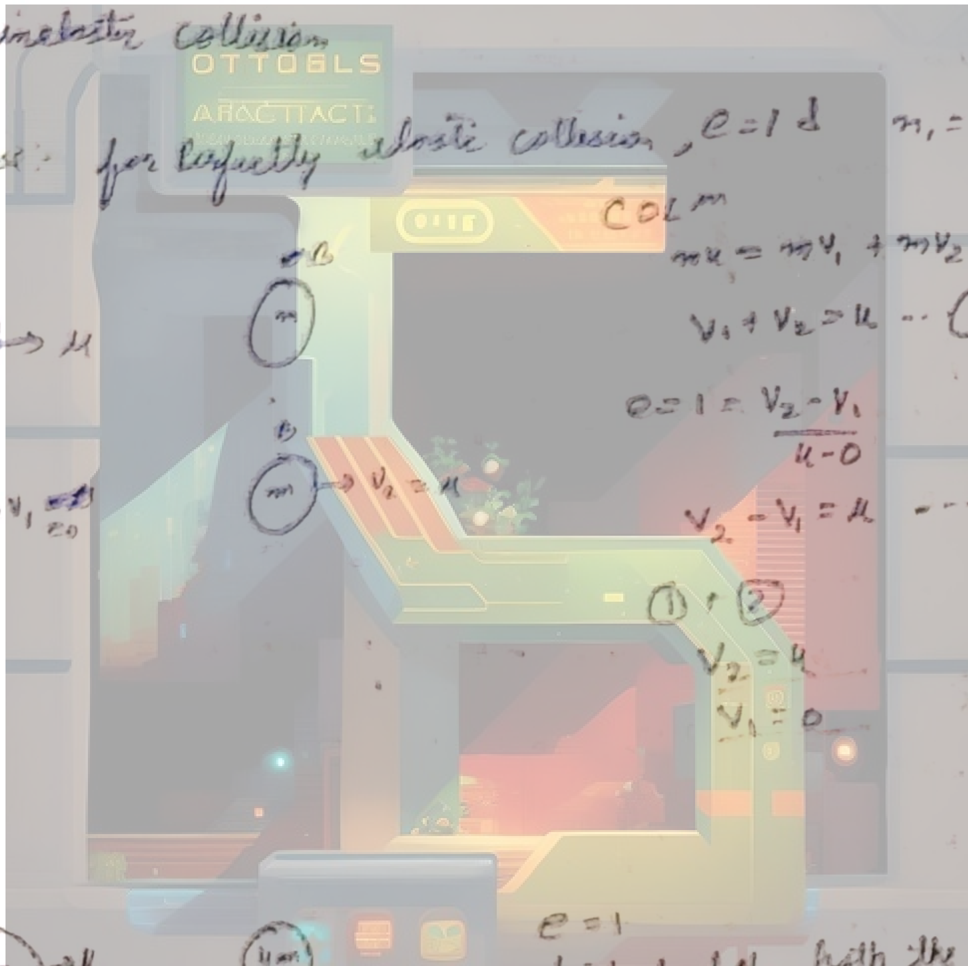
$\xrightarrow{v_1}$ $\quad$ $\xrightarrow{v_2}$

$v_2 > v_1$

$e=0 \Rightarrow$ perfectly inelastic collision

$e=1 \Rightarrow$ elastic collision

$0<e<1 \Rightarrow$ inelastic collision



Special case :- for perfectly elastic collision, $e=1$ & $m_1 = m_2 = m$

COLM

A: $(m) \rightarrow u$ $\quad$ B: (m)

A: $(m) \rightarrow v_1 = 0$ $\quad$ B: $(m) \rightarrow v_2 = u$

$mu = mv_1 + mv_2$

$v_1 + v_2 = u \quad \text{--- (1)}$

$e = 1 = \dfrac{v_2 - v_1}{u - 0}$

$v_2 - v_1 = u \quad \text{--- (2)}$

(1) + (2)

$v_2 = u$

$v_1 = 0$

Q.50. $(m) \rightarrow u$ $\quad$ $(4m)$ (initially at rest) $\quad$ $e=1$

find speed of both the bodies after collision.

$(m) \rightarrow v_1$ $\quad$ $(4m) \rightarrow v_2$

$mu = mv_1 + 4mv_2$

$u = v_1 + 4v_2$

$e = 1 = \dfrac{v_2 - v_1}{u - 0} = 1$

$v_2 - v_1 = u$

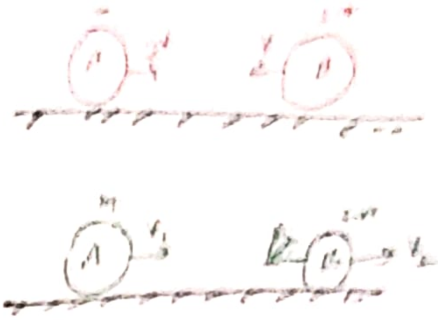
$v_2 = u + v_1$

$u = v_1 + 4u + 4v_1$

$5v_1 = -3u$

$\boxed{v_1 = -\dfrac{3u}{5}}$ $\quad$ $\boxed{v_2 = \dfrac{2u}{5}}$

Q.51. find vel of A & B after collision (e = 1)



$2mV - 2mV = mV_1 + 2mV_2$ | $1 = \frac{V_2 - V_1}{2V + V}$

~~$mV_1 + 2$~~

$V_1 + 2V_2 = 0$ | $3V = V_2 - V_1$

add

$3V_2 = 3V$ | $V_1 = V_2 - 3V$

$\boxed{V_2 = \frac{V}{3}}$ | $V_1 = \frac{V}{3} - 3V$

$\boxed{V_1 = -\frac{2V}{3}}$

Q.52. $e = \frac{1}{3}$, find speed after collision

~~$mV - 2m$~~

$mV - 4mV = mV_1 + 2mV_2$ | $\frac{1}{3} = \frac{V_2 - V_1}{3V}$

$-3V = V_1 + 2V_2$ | $V = V_2 - V_1$

$3V_2 = -2V$

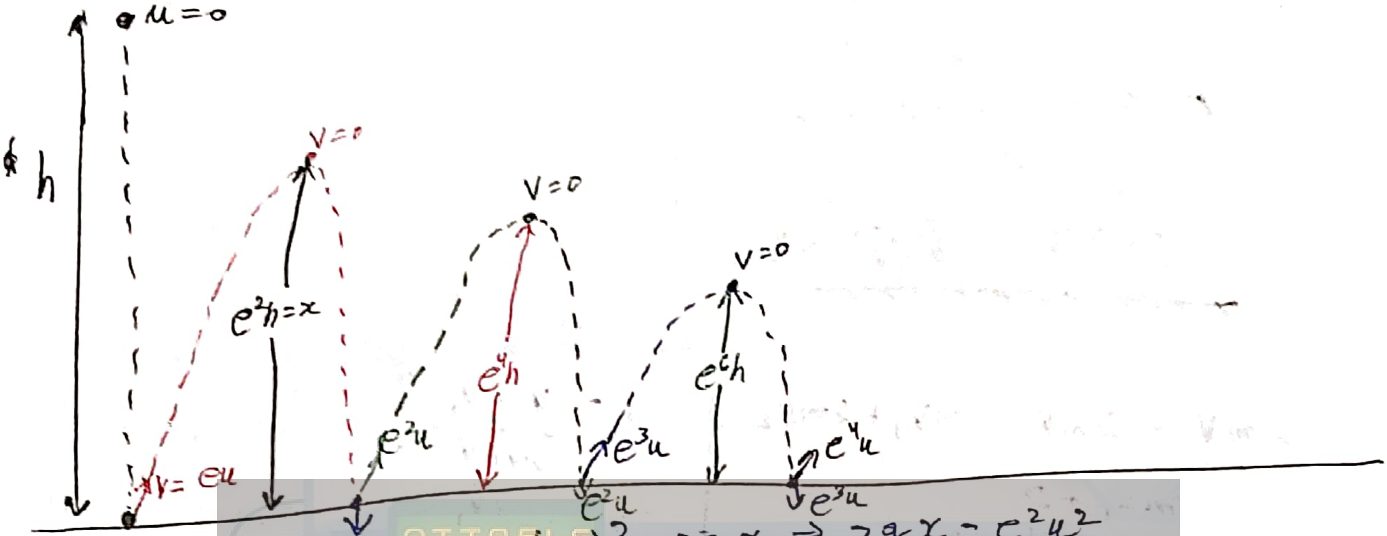
$\boxed{V_2 = -\frac{2V}{3}}$

$V_1 = V_2 - V$

$V_1 = -V - V - \frac{2V}{3} - \frac{2V}{3}$

$\boxed{V_1 = -\frac{5V}{3}}$

Dropping ball from height.



$u=\sqrt{2gh}$

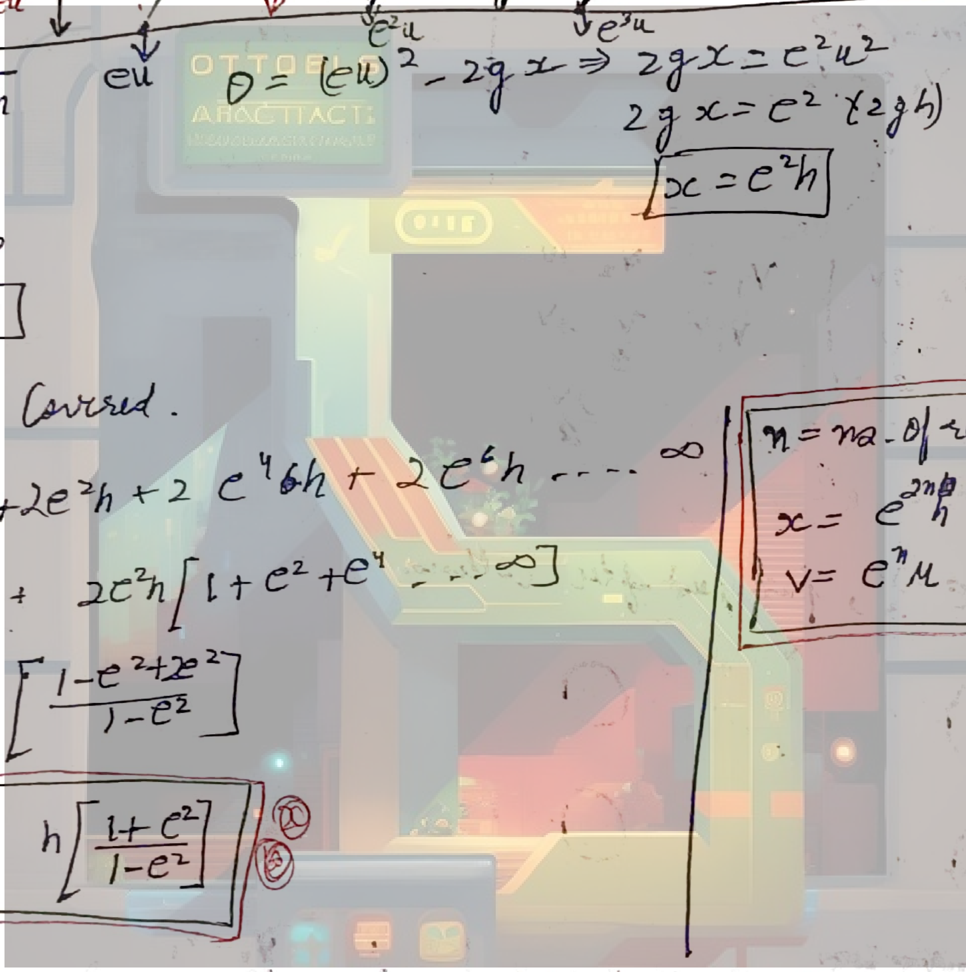
$0=(eu)^2-2gx \Rightarrow 2gx=e^2u^2$

$2gx=e^2(2gh)$

$\boxed{x=e^2h}$

$e=\dfrac{v-0}{u-0}$

$\boxed{v=eu}$



Distance Covered.

$D=h+2e^2h+2e^4h+2e^6h \ldots\ldots \infty$

$D=h+2e^2h\left[1+e^2+e^4 \ldots\ldots \infty\right]$

$D=h\left[\dfrac{1-e^2+2e^2}{1-e^2}\right]$

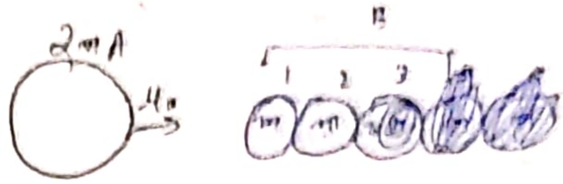
$\boxed{D=h\left[\dfrac{1+e^2}{1-e^2}\right]}$

n = no. of rebounds

$x=e^{2n}h$

$v=e^nu$

Q53. A heavy ball (2m) with velocity u_0. Determine velocity of each ball after collision. ($e=1$)



when collide with ball 1.

① $\cancel{2mu = mV_3}$

$\cancel{2u = V_3}$

$2mu = 2mV_A + mV_B$

$2u = 2V_A + V_B$

$\frac{V_B - V_A}{u} = 1$

$V_B - V_A = u$

$2V_B - 2V_A = 2u$

$3V_B = 4u$

$V_B = \frac{4u}{3}$

$\boxed{V_3 = \frac{4u_0}{3}}$

$V_A = V_B - u$

$V_A = \frac{4u}{3} - u$

$\boxed{V_A = \frac{u_0}{3}}$

when collide with 2

② $\frac{2mu}{3} = 2mV_A + mV_B$

$2u = 6V_A + 3V_B$

$\frac{3(V_B - V_A)}{u} = 1$

$3V_B - 3V_A = u$

$6V_B - 6V_A = 2u$

$9V_B = 4u$

$V_B = \frac{4u}{9}$

$\boxed{V_2 = \frac{4u_0}{9}}$

$V_A = \frac{3V_B - u}{3}$

$\boxed{V_A = \frac{u}{9}}$

when collide $\bar{c}$ 1

③ $\frac{2mu}{9} = 2mV_A + mV_B$

$2u = 18V_A + 9V_B$

$9V_B - 9V_A = u$

$18V_B - 18V_A = 2u$

$27V_B = 4u$

$V_B = \frac{4u}{27}$

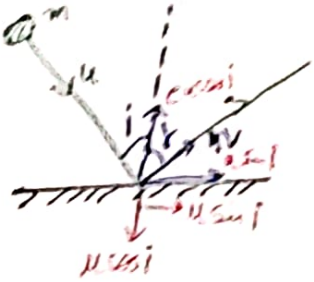
$\boxed{V_1 = \frac{4u_0}{27}}$

$V_A = \frac{9V_B - u}{9}$

$\boxed{V_A = \frac{u_0}{27}}$

Oblique collision

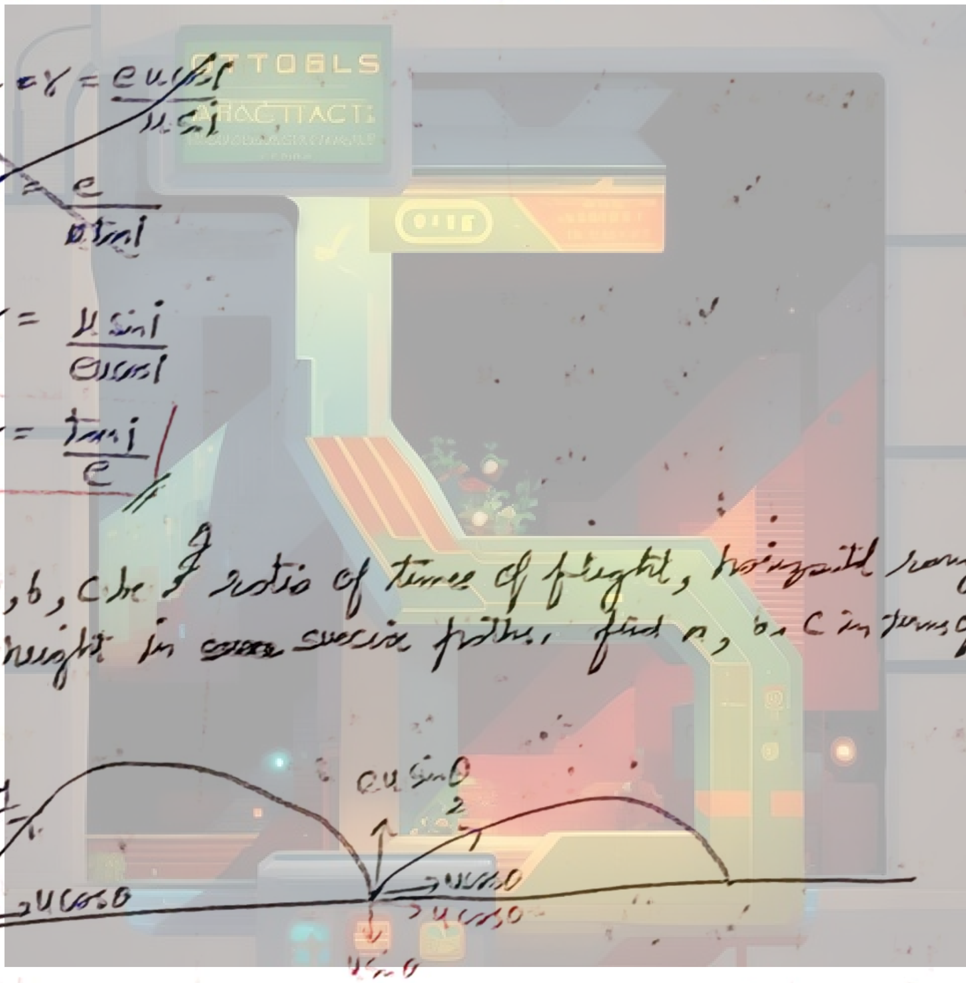
Q 54. what will be the angle of reflection in case of an inelastic collision? (coefficient of restitution = e). Also find velocity after Impact.



$$v = \sqrt{(eu\cos i)^2 + (u\sin i)^2}$$

$$v = \sqrt{e^2u^2\cos^2 i + u^2\sin^2 i}$$

$$\boxed{v = u\sqrt{(e\cos i)^2 + (\sin i)^2}}$$



$$\therefore \tan r = \frac{eu\cos i}{u\sin i}$$

$$\tan r = \frac{e}{\tan i}$$

$$\tan r = \frac{u\sin i}{eu\cos i}$$

$$\boxed{\tan r = \frac{\tan i}{e}}$$

Q.55. let a, b, c be the ratio of times of flight, horizontal range & max height in successive paths. find a, b, c in terms of e.

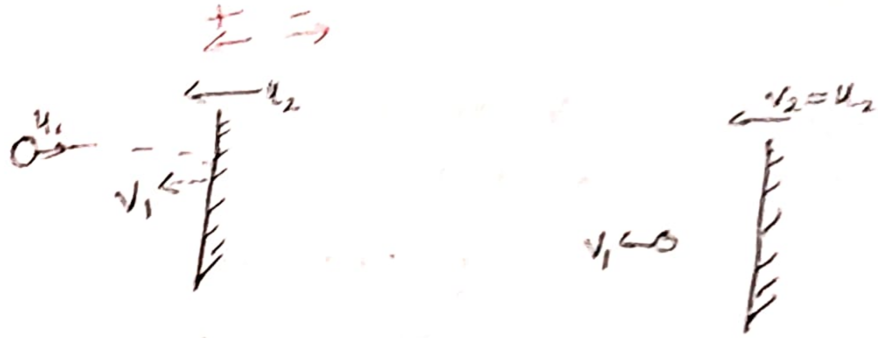
$$a = \frac{T_1}{T_2} = \frac{u\sin\theta}{eu\sin\theta} = \boxed{\frac{1}{e} = a}$$

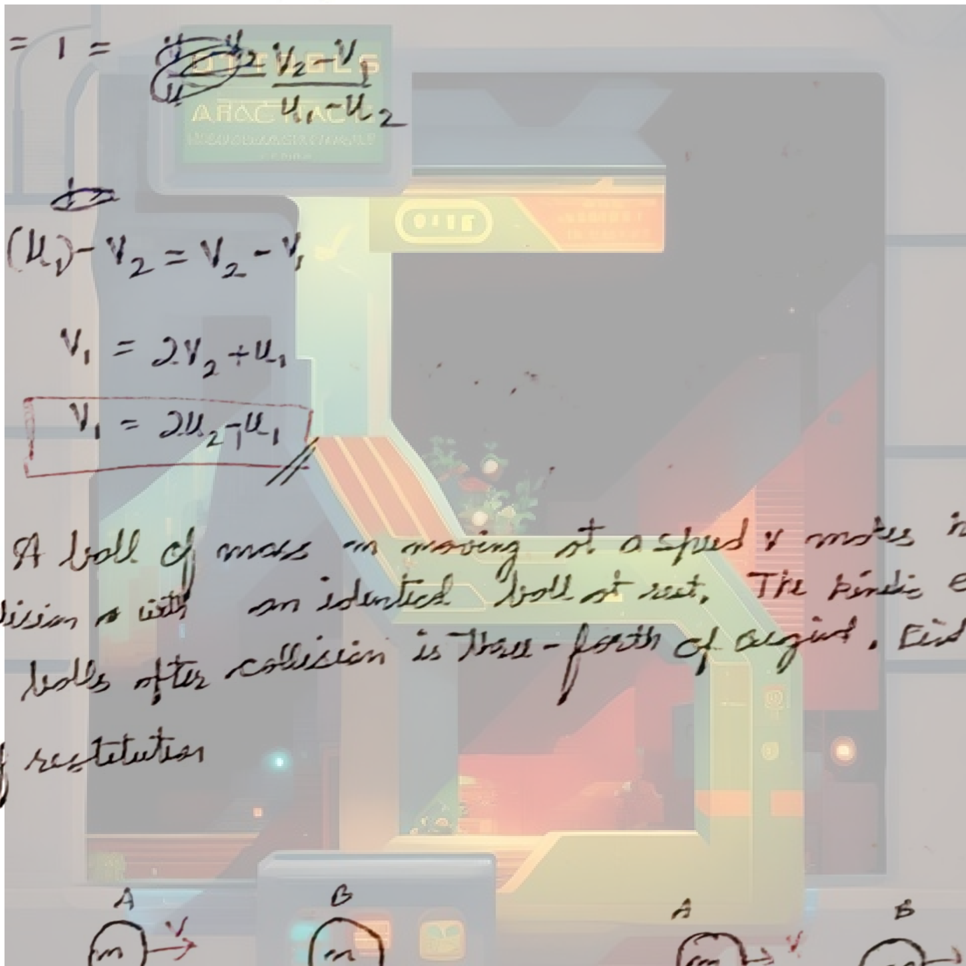
$$b = \frac{R_1}{R_2} = \frac{2\times u\cos\theta\times u\sin\theta}{2\times u\cos\theta\times eu\sin\theta}$$

$$\boxed{b = \frac{1}{e}}$$

$$c = \frac{H_1}{H_2} = \frac{u^2\sin^2\theta}{e^2u^2\sin^2\theta} = \boxed{\frac{1}{e^2} = c}$$

Q.56. find vel after collision. Speed of wall do not change ($e=1$)





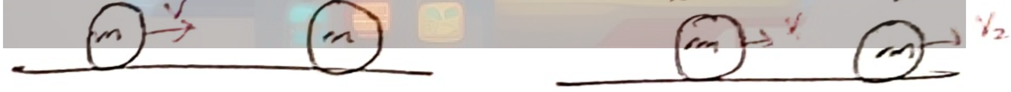
$$e = 1 = \frac{v_2 - v_1}{u_1 - u_2}$$

$$-(u_1) - v_2 = v_2 - v_1$$

$$v_1 = 2v_2 + u_1$$

$$\boxed{v_1 = 2u_2 - u_1}$$

Q.57. A ball of mass m moving at a speed v makes head on collision with an identical ball at rest. The kinetic energy of balls after collision is three-fourth of original. Find coefficient of restitution



$mv = mv_1 + mv_2$

$v_1 + v_2 = v$

$v_2 = v - \frac{v\sqrt{3}}{2}$

$v_2 = \frac{v(2-\sqrt{3})}{2}$

$\frac{3}{4}\cdot\frac{1}{2} m v^2 = \frac{1}{2} m (v_1)^2$

$(v_1)^2 = \frac{3v^2}{4}$

$v_1 = \frac{v\sqrt{3}}{2}$

$e = \frac{v\left(\frac{2-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)}{0 - v}$

$e = \frac{-2+2\sqrt{3}}{2}$

$\cancel{e = -1+\sqrt{3}}$

$e = \sqrt{3} - 1$

$$\frac{3}{4}\times\cancel{\frac{1}{2}}\cancel{m}\,V^2 = \cancel{\frac{1}{2}}\cancel{m}\,(V_1^2+V_2^2)$$

$$V_1^2+V_2^2=\frac{3V^2}{4}$$

$$V_1+V_2=V$$

$$V_1^2+V_2^2+2V_1V_2=V^2$$

$$\frac{3V^2}{4}+2V_1V_2=V^2$$

$$2V_1V_2=\frac{1}{4}V^2$$

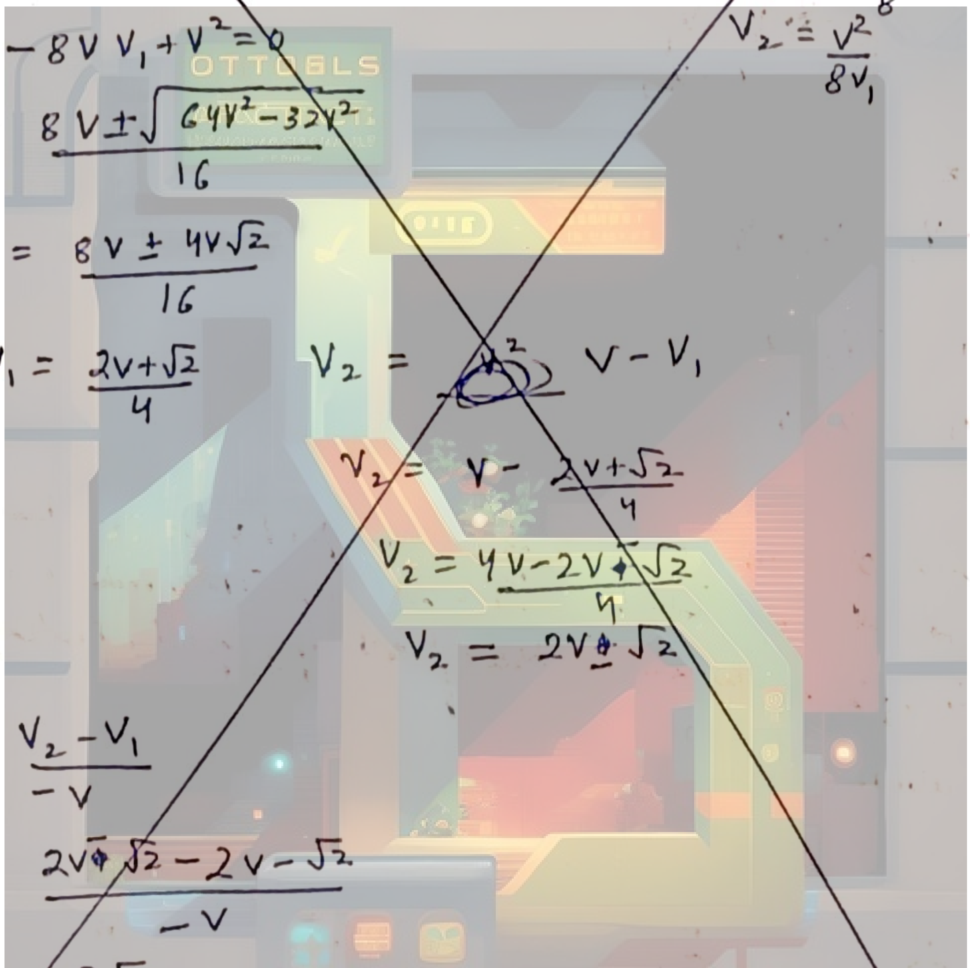
$$V_1V_2=\frac{V^2}{8}$$

$$V_2=\frac{V^2}{8V_1}$$

$$V_1+\frac{V^2}{8V_1}=V$$

$$(8V_1)^2+V^2=8V_1V$$

$\cancel{8V}$



$$8V_1^2-8VV_1+V^2=0$$

$$V_1=\frac{8V\pm\sqrt{64V^2-32V^2}}{16}$$

$$V_1=\frac{8V\pm4V\sqrt{2}}{16}$$

$$V_1=\frac{2V+\sqrt{2}}{4}\qquad V_2=V-V_1$$

$$V_2=V-\frac{2V+\sqrt{2}}{4}$$

$$V_2=\frac{4V-2V\pm\sqrt{2}}{4}$$

$$V_2=2V\pm\sqrt{2}$$

$$e=\frac{V_2-V_1}{-V}$$

$$e=\frac{2V\pm\sqrt{2}-2V-\sqrt{2}}{-V}$$

$$e=-2\sqrt{2}$$

$$v_1 + v_2 = v$$

$$e = \frac{v_2 - v_1}{v}$$

$$v_2 - v_1 = ev$$

$$2v_2 = (e+1)v$$

$$v_2 = \frac{(e+1)v}{2}$$

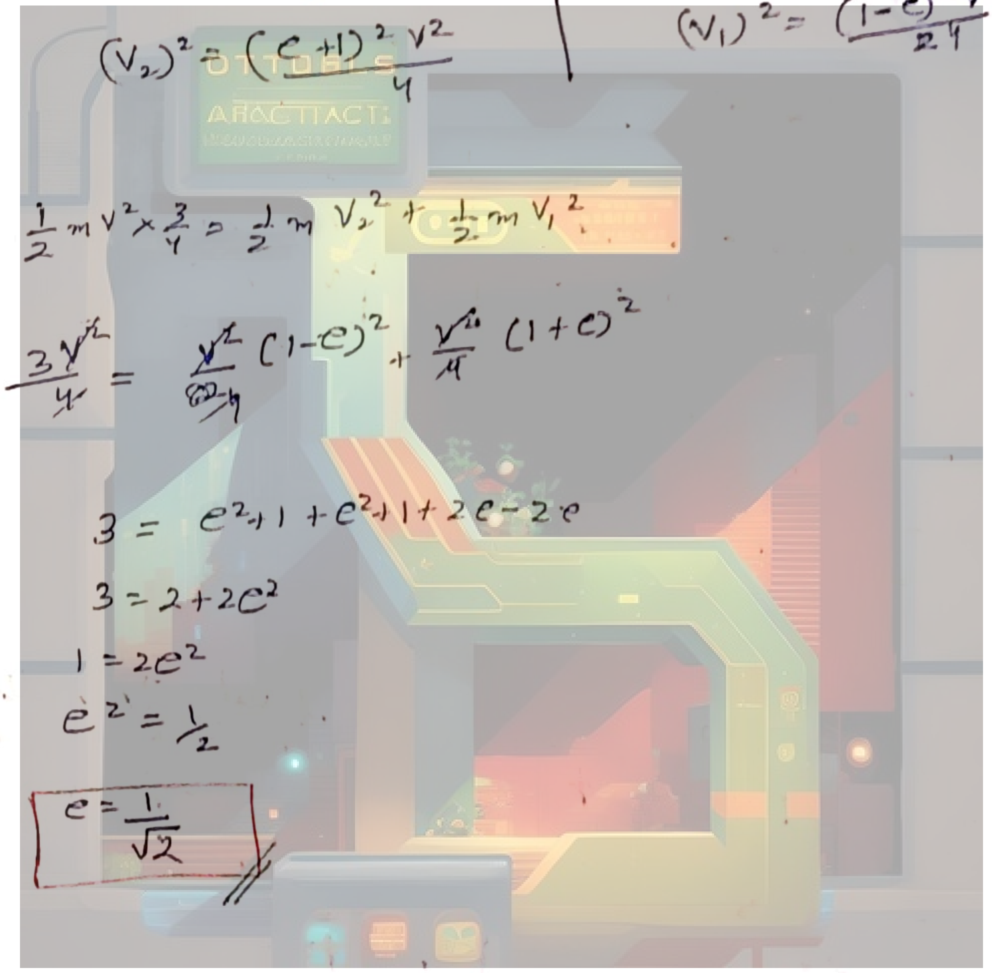
$$v_1 = v - \frac{(e+1)v}{2}$$

$$v_1 = \frac{2v - ev - v}{2}$$

$$v_1 = \frac{v - ev}{2}$$

$$v_1 = \frac{(1-e)v}{2}$$

$$(v_1)^2 = \frac{(1-e)^2 v^2}{4}$$



$$(v_2)^2 = \frac{(e+1)^2 v^2}{4}$$

$$\frac{1}{2} m v^2 \times \frac{3}{4} = \frac{1}{2} m v_2^2 + \frac{1}{2} m v_1^2$$

$$\frac{3v^2}{4} = \frac{v^2}{4}(1-e)^2 + \frac{v^2}{4}(1+e)^2$$

$$3 = e^2 + 1 + e^2 + 1 + 2e - 2e$$

$$3 = 2 + 2e^2$$

$$1 = 2e^2$$

$$e^2 = \frac{1}{2}$$

$$\boxed{e = \frac{1}{\sqrt{2}}}$$

Q58. A sphere of mass m_1 in motion hits ... sticks to it, the total kinetic energy after collision is 2/3 of total KE before. find ratio m_1 & m_2

$$m_1 u = \cancel{(m_1+m_2)v} \quad m_1 V + m_2 V$$

$$m_1 u = (m_1+m_2) V$$

$$V = \frac{m_1 u}{(m_1+m_2)}$$

$$\frac{1}{\cancel{2}} m u^2 \times \frac{2}{3} = \frac{1}{\cancel{2}} m_1 V^2 + \frac{1}{\cancel{2}} m_2 V^2$$

$$m_1 u^2 \times \frac{2}{3} = \cancel{(m_1+m_2)} \frac{m_1^2 u^2}{(m_1+m_2)^{\cancel{2}}}$$

$$2m_1 + 2m_2 = 3m_1$$

$$2m_2 = m_1$$

$$\boxed{\frac{m_1}{m_2} = 2:1}$$

$$e = 0 = \frac{V_2 - V_1}{u - 0}$$

$$(u-0)\,0 = V_2 - V_1$$

$$(u)\cancel{(t)}(0) - 0(0) = V_2 - V_1$$

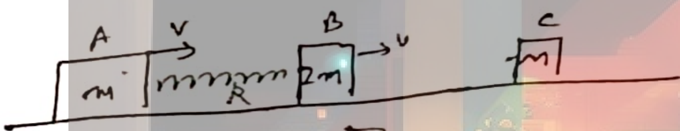
$$0 - 0 = V_2 - V_1$$

$$V_2 - V_1 = 0$$

$$V_2 = -(-V_1)$$

$$V_2 = +V_1$$

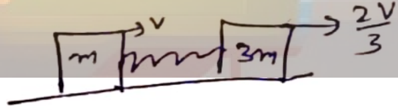
$$\boxed{V_2 = V_1}$$

Q59. 

C at rest. The collision is ($e = 0$). calculate max compression.

$$2mV = 3mV_0' \quad \text{coln}$$

$$V' = \frac{2V}{3}$$

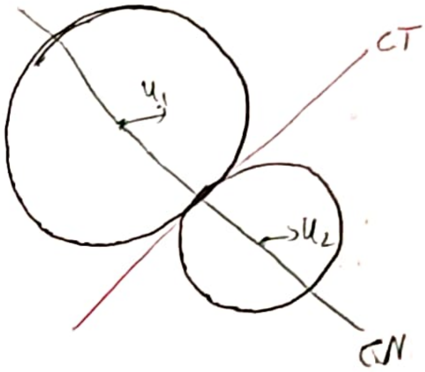


$$\overbrace{\frac{1}{\cancel{2}} \times \frac{3}{4} m \times \frac{V^2}{\cancel{9}_3}}^{ki} = \overbrace{\frac{1}{\cancel{2}} \times k \times x^2}^{W} + 0^{kf}$$

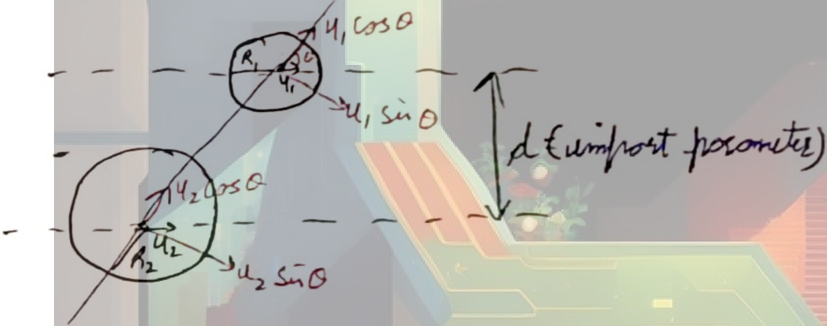
$$x^2 = \frac{V^2 m}{12k}$$

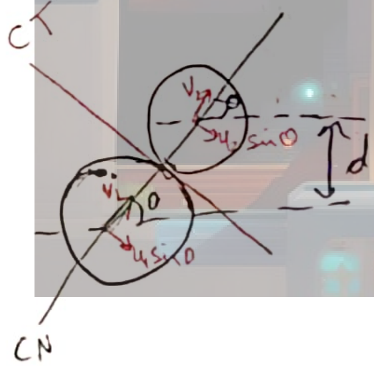
$$\boxed{x = V\sqrt{\frac{m}{12k}}}$$

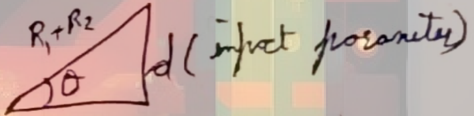
Oblique collision.



→ No component of impulse acts along common tangent direction. Hence linear momentum or linear velocity of individual particles do not change along common tangent direction.







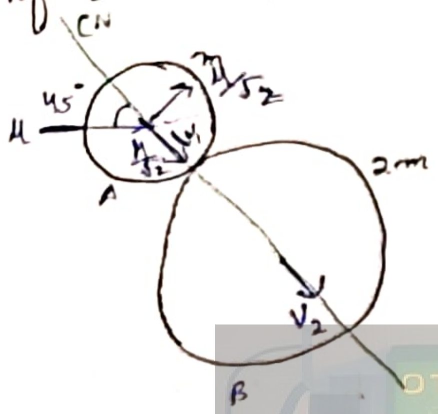
$$\sin\theta = \frac{d}{R_1 + R_2}$$

$$e = \frac{V_2 - V_1}{u_1 \cos\theta - u_2 \cos\theta}$$

COLM:-

$$m_1 u_1 \cos\theta + m_2 u_2 \cos\theta = m_1 V_1 + m_2 V_2$$

Q60. Disc A of mass m collides with stationary disc B of mass 2m as shown. Find e for which two discs move in ~~~~ perpendicular direction after collision.



COLM :-

$$\frac{mu}{\sqrt{2}} = mv_1 + 2mv_2$$

$$v_1 + 2v_2 = \frac{u}{\sqrt{2}}$$

$$v_1 \to 0$$

$$2v_2 = \frac{u}{\sqrt{2}}$$

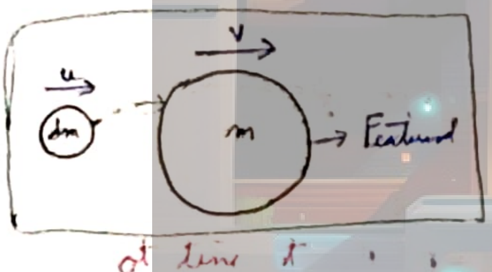
$$v_2 = \frac{u}{2\sqrt{2}}$$

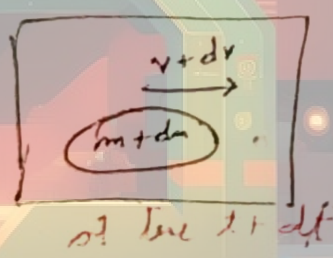
$$e = \frac{\frac{u}{2\sqrt{2}} - 0}{\frac{u}{\sqrt{2}}}$$

$$e = \frac{\sqrt{2}u}{2\sqrt{2}u}$$

$$\boxed{e = 1/2}$$

Q61: Variable mass system





$$J = \Delta P$$

$$F_{ext}\,dt = [(m + dm)(v + dv)] - [u(dm) + mv]$$

$$F_{ext}\,dt = \cancel{mv} + m\,dv + v\,dm + \underbrace{\cancel{dm\,dv}}_{\text{negligible}}^{\to 0} - dm\,u - \cancel{mv}$$

$$F_{ext}\,dt = m\,dv + dm(v - u)$$

$$F_{ext}\,dt + (u - v)dm = m\,dv$$

$F_{ext} + (u - v)\dfrac{dm}{dt} = m\dfrac{dv}{dt}$

$u - v = V_{rel}$ (relative vel of elementary mass wrt initial of mass of system)

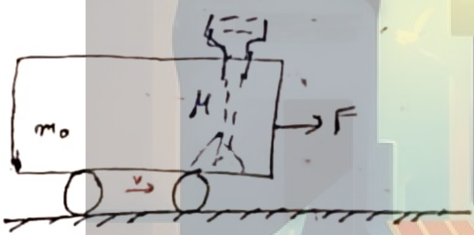
$$\boxed{\vec{F}_{ext} + \vec{V}_{rel}\frac{dm}{dt} = m\frac{d\vec{v}}{dt}} \quad (22)$$

$\frac{d\vec v}{dt}$ → acceleration

m → mass of system at time t

$\frac{dm}{dt}$ → rate of change of mass

Q 61. A flatcar (m_0) ~~moves right~~ starts moving to the right due constant force F. Sand spills on the flat car from a stationary hopper. The rate of loading is μ kg/s. find time dependence of velocity & acceleration of the flat car.



Sol $\vec{F}_{ext} + \vec{V}_{rel}\dfrac{dm}{dt} = m\dfrac{d\vec{v}}{dt}$

$V_{rel} = u - V = 0 - V = -V$

$\dfrac{dm}{dt} = \mu$

$m = m_0 + \mu t$

$F - V\mu = (m_0 + \mu t)\dfrac{dv}{dt}$

$\dfrac{dv}{dt} = \boxed{a = \dfrac{F - \mu V}{m_0 + \mu t}}$

$\dfrac{dv}{dt} = \dfrac{F - \mu V}{m_0 + \mu t}$

$\displaystyle\int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$

$\left[\dfrac{\ln(F - \mu V)}{-\mu}\right]_0^v = \left[\dfrac{\ln(m_0 + \mu t)}{\mu}\right]_0^t$

$-\left[\ln(F - \mu V) - \ln(F)\right] = \left[\ln(m_0 + \mu t) - \ln(m_0)\right]$

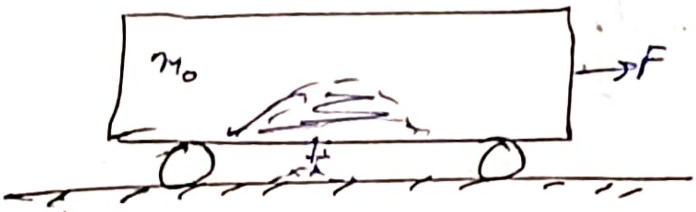
$\ln\left(\dfrac{F}{F - \mu V}\right) = \ln\left(\dfrac{m_0 + \mu t}{m_0}\right)$

$\dfrac{F}{F - \mu V} = \dfrac{m_0 + \mu t}{m_0}$

$V(m_0 + \mu t) = Ft$

$$\boxed{V = \frac{Ft}{m_0 + \mu t}}$$

Q62. sand spills through a hole in bottom $\bar{c}$ constant rate μ kg/s. find velocity & acceleration (cart starts from rest & system initial mass is m_0).



$V_{rel} = \mu - V = V_0 - V = 0$

$\frac{dm}{dt} = -\mu$

$m = m_0 - \mu t$

$F + \cancel{V\mu} = (m_0 - \mu t)\frac{dv}{dt}$

$$\boxed{\frac{dv}{dt} = \frac{F + \cancel{\mu V}}{m_0 - \mu t}}$$

$$\boxed{a = \frac{F}{m_0 - \mu t}}$$

$\cancel{\left[\frac{\ln(F+\mu V)}{\mu}\right]_0^V = -\left[\frac{\ln(m_0 - \mu t)}{+\mu}\right]_0^t}$

$\cancel{\ln F +}$

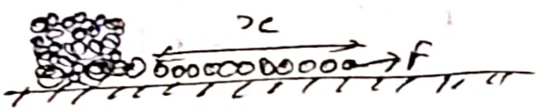
$$\frac{V}{F} = -\left[\frac{\ln(m_0 - \mu t)}{\mu}\right]_0^t$$

$$\frac{V}{F} = -\frac{\ln(m_0 - \mu t)}{\mu}$$

$\cancel{V = -\frac{\ln F(m_0 - \mu t)}{\mu}}$

$$\boxed{V = \frac{F}{\mu}\ln\left(\frac{m_0}{m_0 - \mu t}\right)}$$

Q63. A pile of loose link chain, mass per unit length ρ, lies on a rough surface with coefficient of kinetic friction μ. one end of the chain is being pulled horizontally along the surface by a constant force F. Determine acc of chain in terms of x and $\frac{dx}{dt} = V$.



~~$V_{ret} =$~~

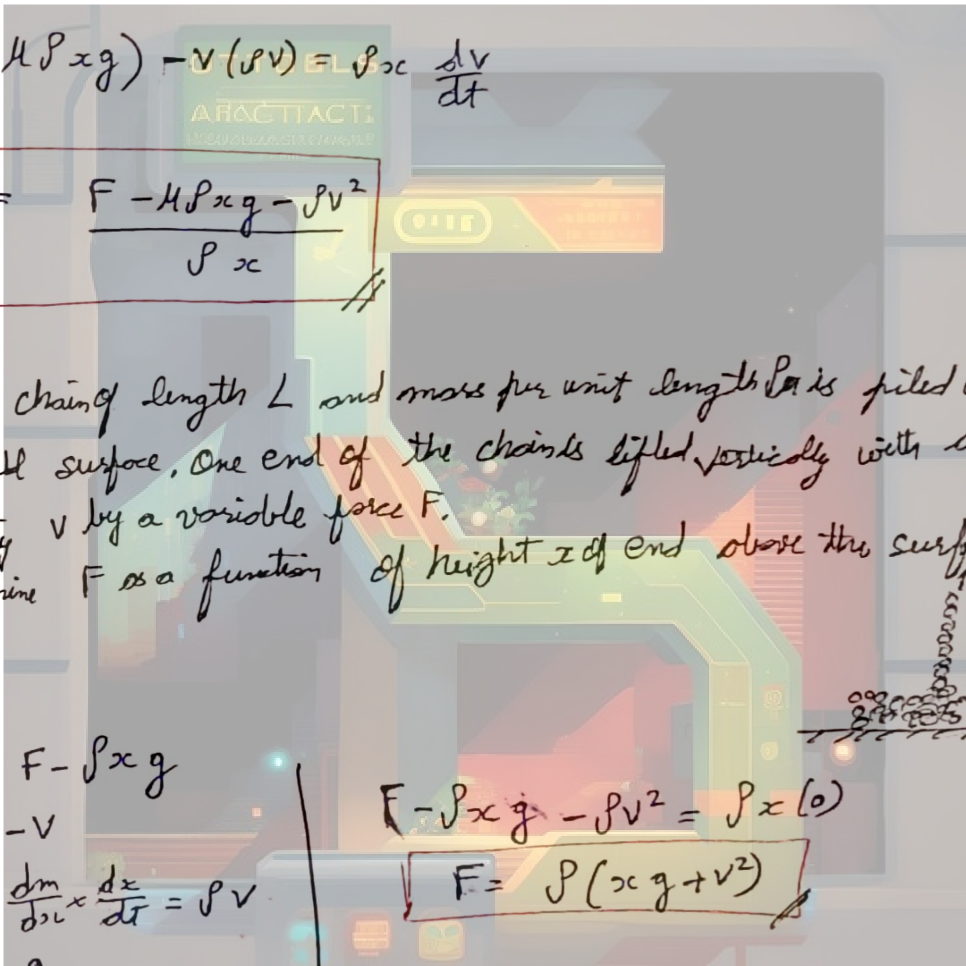
$$F_{ext} + V_{rel}\frac{dm}{dt} = m\frac{dv}{dt}$$

$$F_{ext} = F - \mu \rho x g$$

$$V_{rel} = u - v = -v$$

$$\frac{dm}{dt} = \frac{dm}{dx} \times \frac{dx}{dt} = \rho v$$

$$m = \rho x$$



$$(F - \mu \rho x g) - v(\rho v) = \rho x \frac{dv}{dt}$$

$$\boxed{a = \frac{F - \mu \rho x g - \rho v^2}{\rho x}}$$

Q64. A chain of length L and mass per unit length ρ is piled on a horizontal surface. One end of the chain is lifted vertically with a constant velocity v by a variable force F. Determine F as a function of height x of end above the surface.

$$F_{ext} = F - \rho x g$$

$$V_{rl} = -v$$

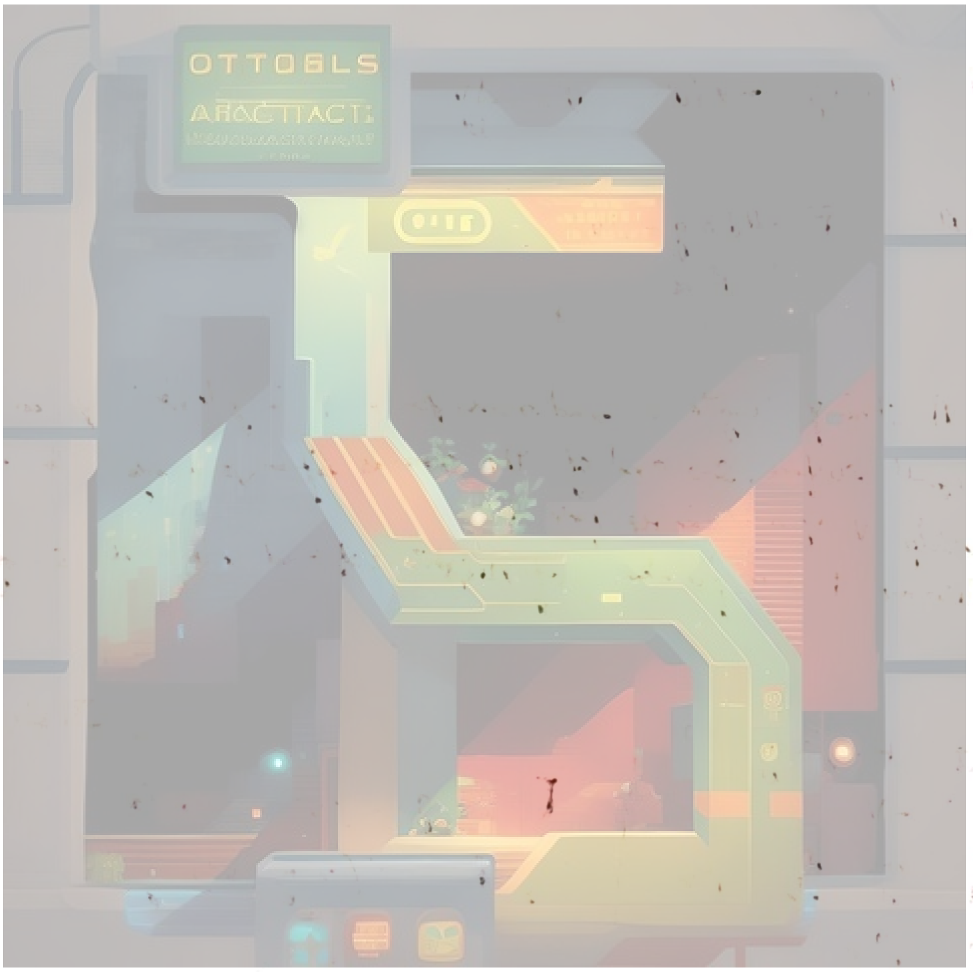
$$\frac{dm}{dt} = \frac{dm}{dx} \times \frac{dx}{dt} = \rho v$$

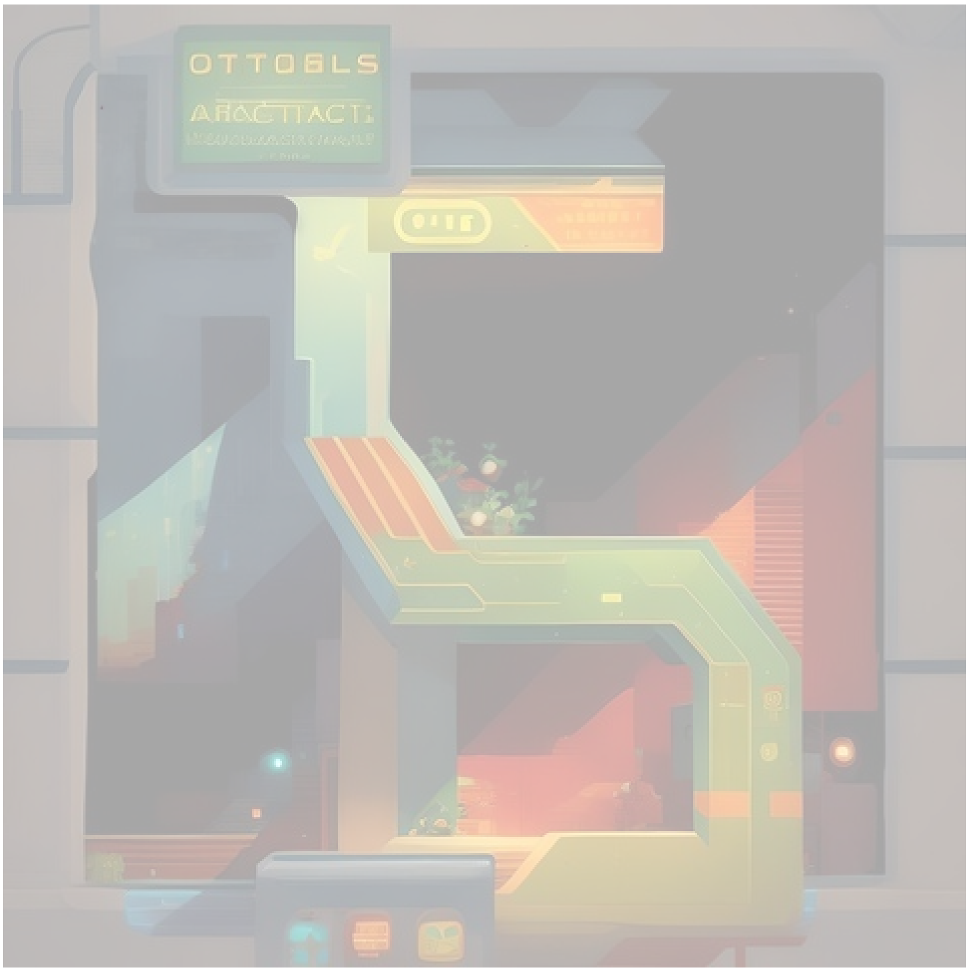
$$m = \rho x$$

$$a = 0 = \frac{dv}{dt}$$

$$F - \rho x g - \rho v^2 = \rho x(0)$$

$$\boxed{F = \rho(xg + v^2)}$$

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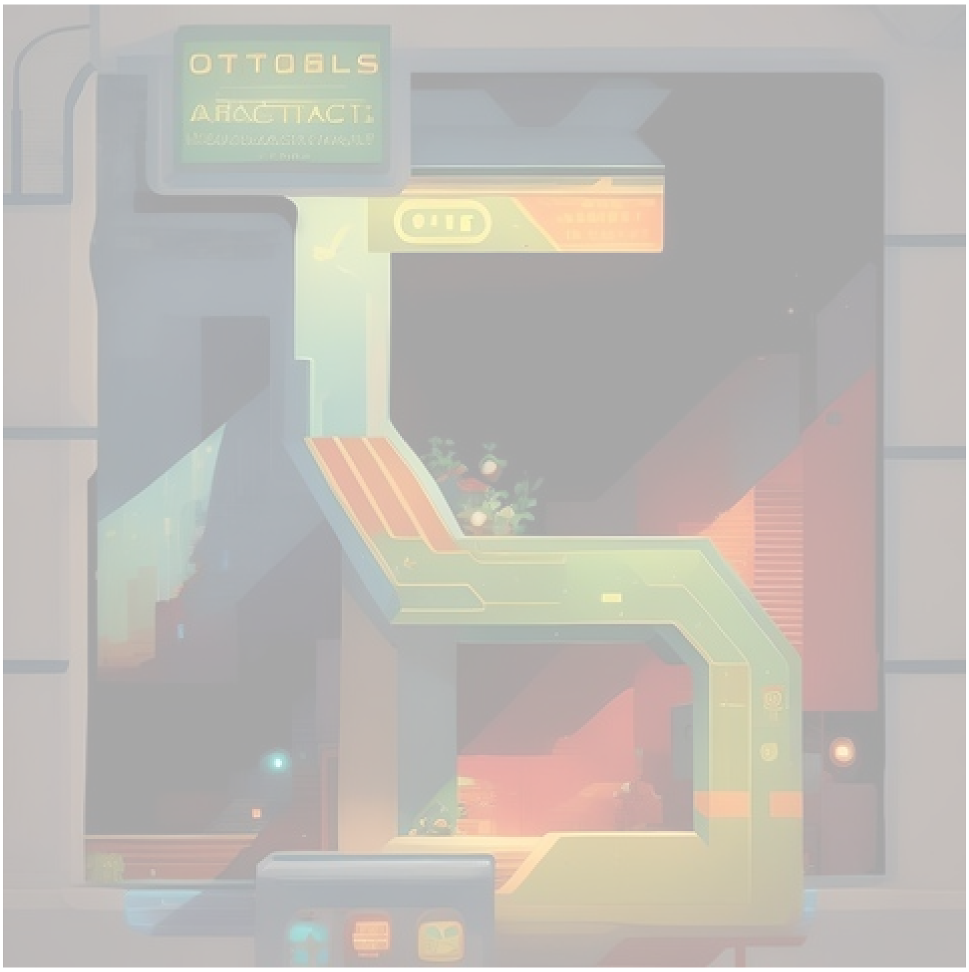
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