Calculus (Differentiation & Integration)

Differentiation – Differential calculus is used to study the nature (increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) changes independently.

$\frac{dy}{dx}$ (y किस तरह change ho raha hai with respect o to x)

↳ is the instantaneous rate of change of function y with respect to variable x.

$a = \frac{dv}{dt} = \frac{d}{dt}(v)$ $\qquad \frac{dy}{dx} = y'$

$v = \frac{ds}{dt} = \frac{d}{dt}(s)$

~~formulas~~ derivatives of some imp functions.

① $\frac{d}{dx}(x^n) = nx^{n-1}$

② $\frac{d}{dx}(e^x) = e^x$

③ $\frac{d}{dx}(\text{constant}) = 0$ (does not change)

④ $\frac{d}{dx}(\sin x) = \cos x$

⑤ $\frac{d}{dx}(\cos x) = -\sin x$

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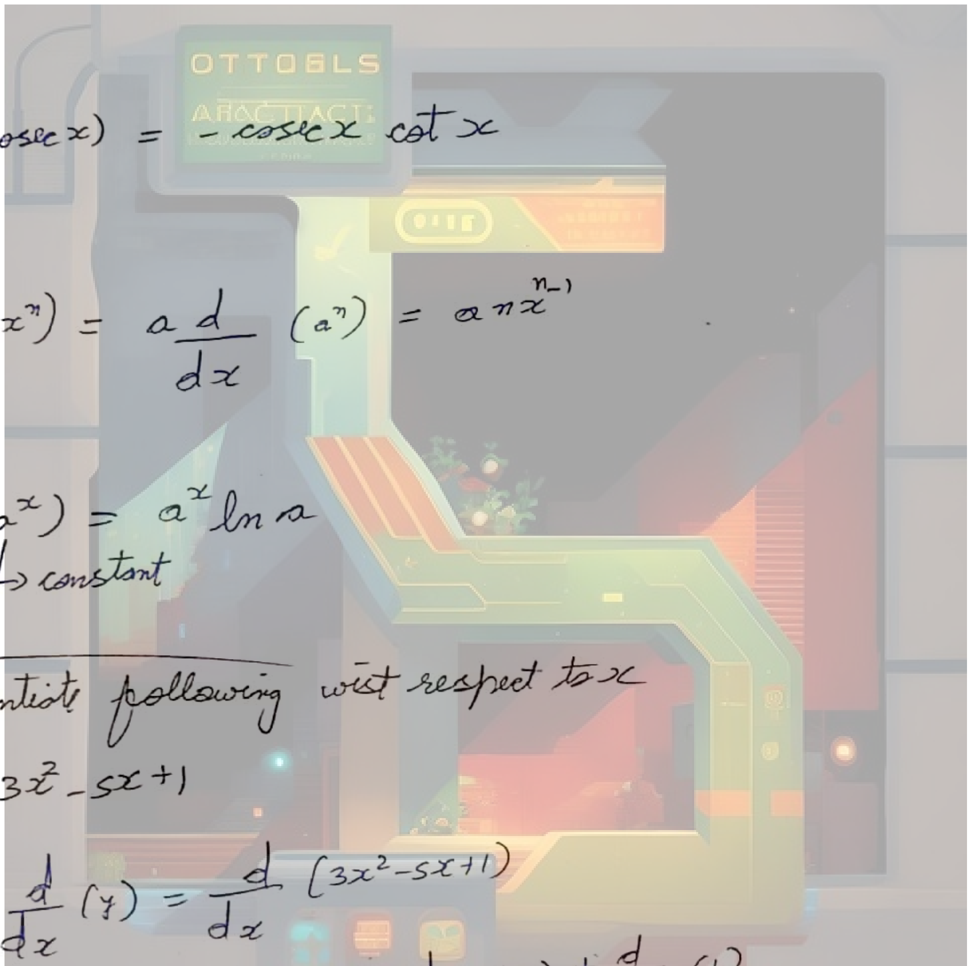
⑥ $\frac{d}{dx}(\tan x) = \sec^2 x$

⑦ $\frac{d}{dx}(\sec x) = \sec x \tan x$

natural log

⑧ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

⑨ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$



⑩ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

⑪ $\frac{d}{dx}(ax^n) = a\frac{d}{dx}(x^n) = anx^{n-1}$

⑫ $\frac{d}{dx}(a^x) = a^x \ln a$

a → constant

Q1. Differentiate following w.r.t respect to x

① $y = 3x^2 - 5x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}(y) = \frac{d}{dx}(3x^2 - 5x + 1)$$

$$= \frac{d}{dx}(3x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

$$= 3\frac{d}{dx}(x^2) - 5\frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$= 3 \times 2 \times x^{2-1} - 5 \times 1 \times x^{1-1} + 0$$

$$\boxed{= 6x - 5}$$

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2. $y = ax^2 + bx + c$

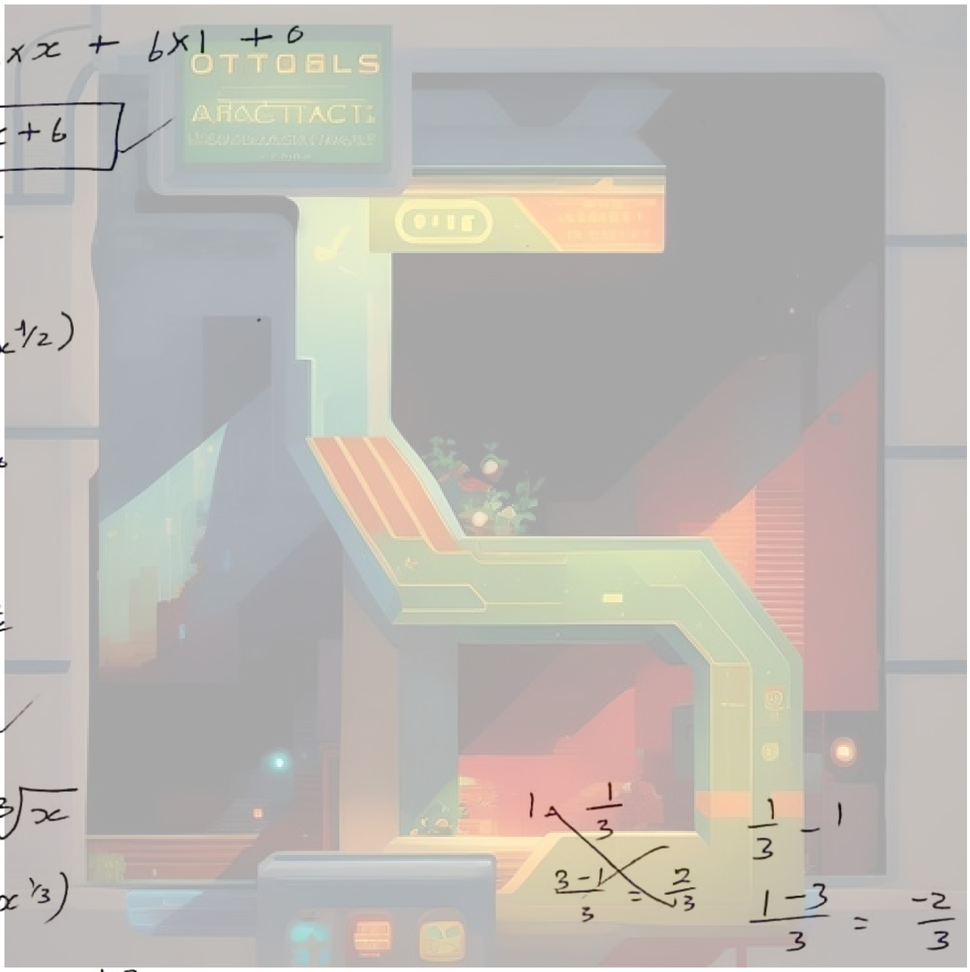
$$\frac{d}{dx}(y)$$

$$= \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)$$

$$= a\frac{d}{dx}(x^2) + b\frac{d}{dx}(x) + 0$$

$$= a \times 2 \times x + b \times 1 + 0$$

$$\boxed{= 2ax + b}$$



3. $y = {}^{1}/_{2}$

$$\frac{d}{dx}(x^{1/2})$$

$$= {}^{1}/_{2} \times x$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{x}}$$

$$\boxed{= \frac{1}{2\sqrt{x}}}$$

4. $y = \sqrt[3]{x}$

$$\frac{d}{dx}(x^{1/3})$$

$1 - \frac{1}{3}$

$\frac{3-1}{3} = \frac{2}{3}$

$\frac{1}{3} - 1$

$\frac{1-3}{3} = \frac{-2}{3}$

$$\frac{1}{3} \times x^{-2/3}$$

$$\boxed{= \frac{1}{3\sqrt[3]{x^2}}}$$

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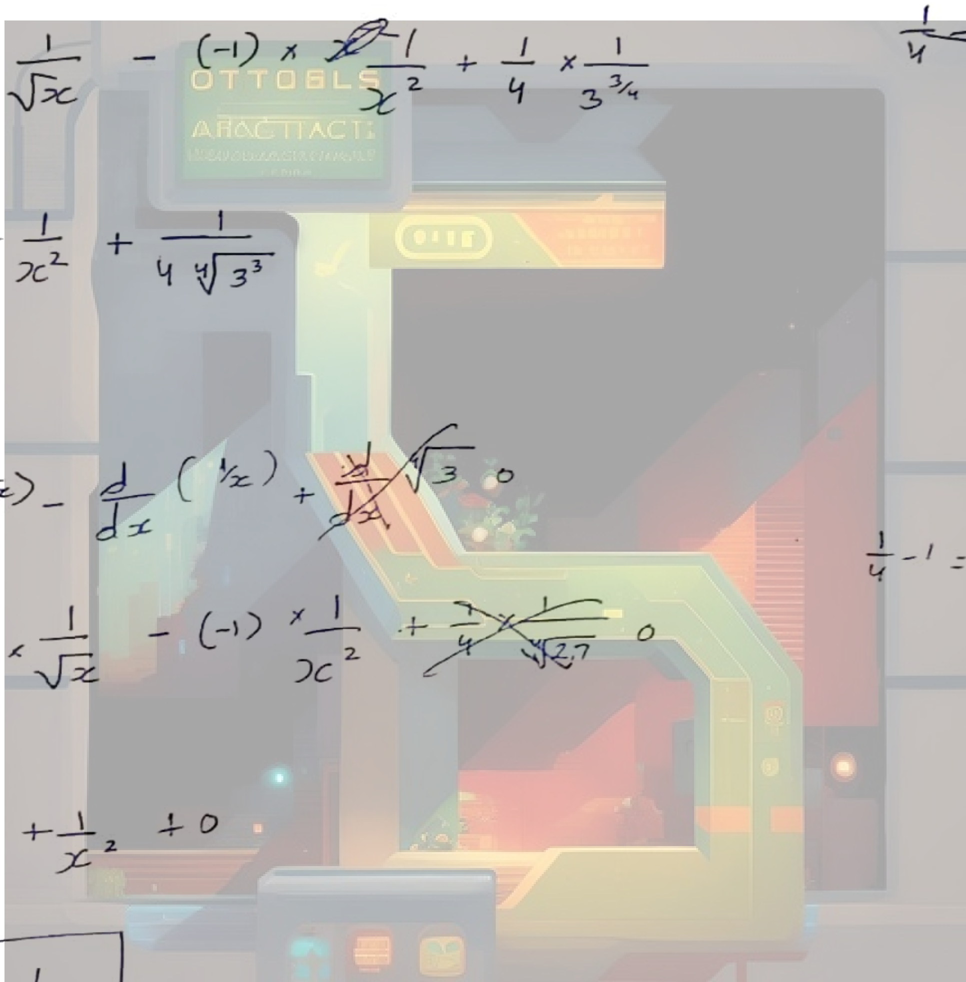
(5) $y = 2\sqrt{x} - \frac{1}{x} + \sqrt[4]{3}$

$\frac{d}{dx}(y) =$

$$= \frac{d}{dx}(2\sqrt{x}) - \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(\sqrt[4]{3})$$

$$= 2\frac{d}{dx}(x^{+1/2}) - \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(3^{1/4})$$



$$= \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{x}} - (-1) \times \frac{1}{x^2} + \frac{1}{4} \times \frac{1}{3^{3/4}}$$

$\frac{1}{4} - \frac{4}{4} = -\frac{3}{4}$

$$= \frac{1}{\sqrt{x}} + \frac{1}{x^2} + \frac{1}{4\sqrt[4]{3^3}}$$

$$= \frac{d}{dx}(2\sqrt{x}) - \frac{d}{dx}(1/x) + \cancel{\frac{d}{dx}}\sqrt{3}\ 0$$

$\frac{1}{4} - 1 = -\frac{3}{4}$

$$= \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{x}} - (-1) \times \frac{1}{x^2} + \cancel{\frac{1}{4} \times \frac{1}{\sqrt[4]{27}}}\ 0$$

$$= \frac{1}{\sqrt{x}} + \frac{1}{x^2} + 0$$

$$\boxed{= \frac{1}{\sqrt{x}} + \frac{1}{x^2}}$$

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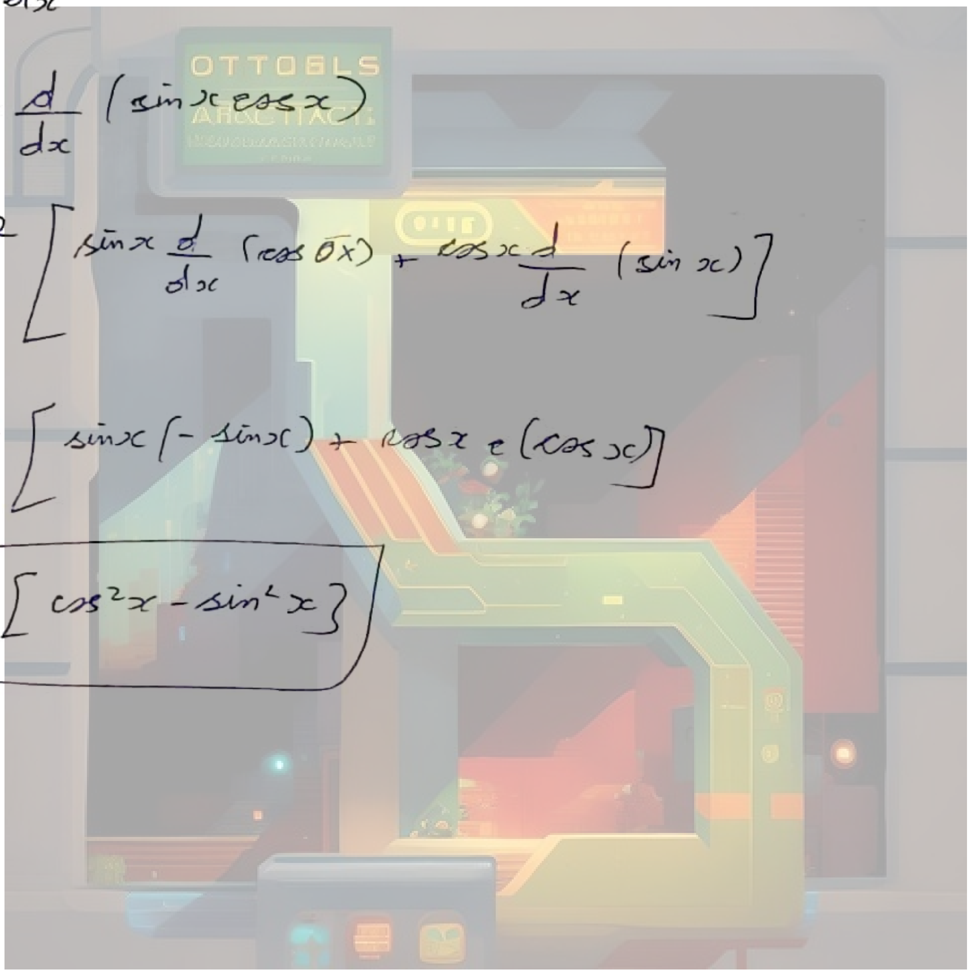
Product Rule

$$y = A \cdot B$$
(A & B are functions of x)

$$\frac{dy}{dx} = A\frac{d}{dx}(B) + B\frac{d}{dx}(A)$$

eg. $y = 2\sin x \cos x$

$$\frac{d}{dx}(2\sin x \cos x$$



$$= 2\frac{d}{dx}(\sin x \cos x)$$

$$= 2\left[\sin x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(\sin x)\right]$$

$$= 2\left[\sin x(-\sin x) + \cos x(\cos x)\right]$$

$$\boxed{= 2\left[\cos^2 x - \sin^2 x\right]}$$

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Quotient Rule

$y = \frac{A}{B}$ (A & B are functions of x)

$$\frac{dy}{dx} = \frac{B \cdot \frac{d}{dx}(A) - A \cdot \frac{d}{dx}(B)}{(B)^2}$$

eg. $y = \frac{\sin x}{\cos x}$

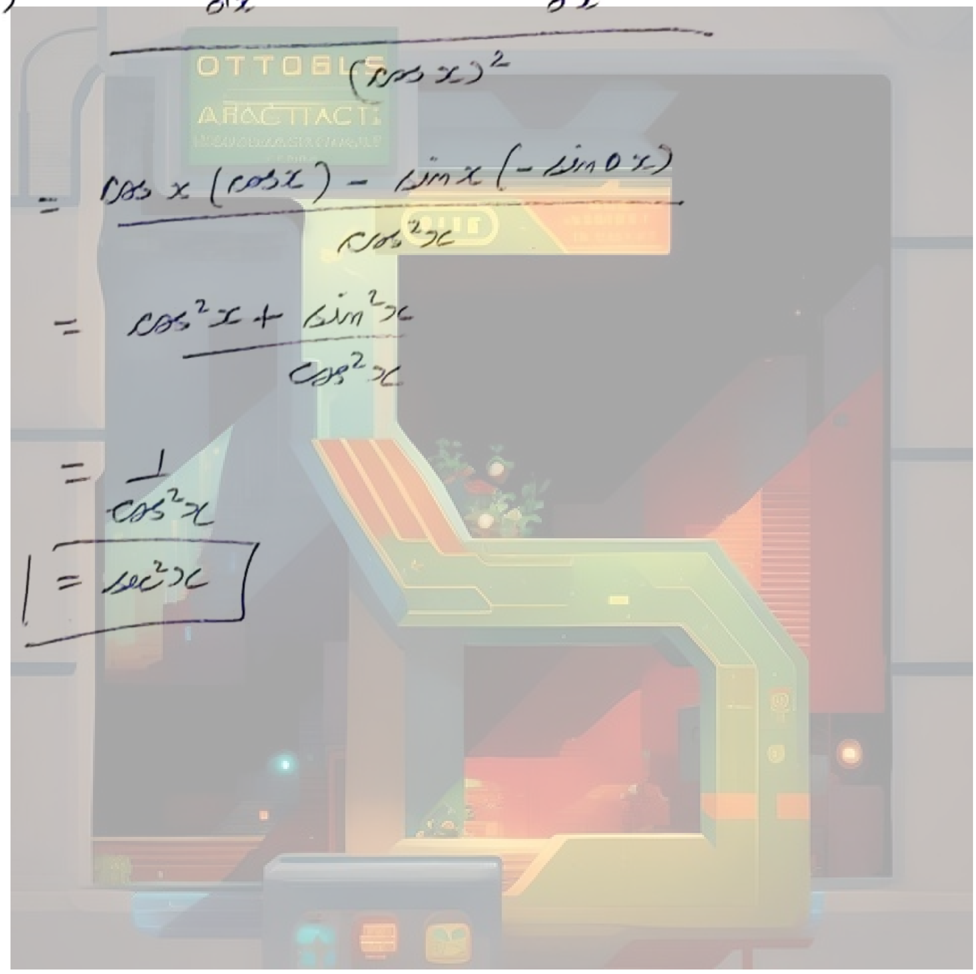
$$\frac{d}{dx}(y) = \frac{\cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{(\cos x)^2}$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\boxed{= \sec^2 x}$$



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Q6. $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$

$\frac{d}{dx}(y)$

$\frac{d}{dx}\left[(x^2 - 3x + 3)(x^2 + 2x - 1)\right]$

$(x^2 + 2x - 1)\frac{d}{dx}(x^2 - 3x + 3) + (x^2 - 3x + 3)\frac{d}{dx}(x^2 + 2x - 1)$

$(x^2 + 2x - 1)(2x - 3 + 0) + (x^2 - 3x + 3)(2x + 2 + 0)$

$(x^2 - 2x - 1)(2x - 3) + (x^2 - 3x + 3)(2x - 2)$

$(2x^3 - 2x^2 - 2x^2 + 6x - 2x + 3) + (2x^3 - 2x^2 - 6x^2 + 6x + 6x - 6)$

$(2x^3 - 4x^2 + 4x + 3) + (2x^3 - 8x^2 + 12x - 6)$

$\boxed{4x^3 - 12x^2 + 16x - 3}$

Q7. $y = (x^3 - 3x + 2)(x^4 + x^2 - 1)$

$\frac{d}{dx}\left[(x^2 - 3x + 2)\ \cancel{x^4 + 2x}\ (x^4 + x^2 - 1)\right]$

$(x^2 - 3x + 2)\frac{d}{dx}(x^4 + x^2 - 1) + (x^4 + x^2 - 1)\frac{d}{dx}(x^2 - 3x + 2)$

$(x^2 - 3x + 2)(4x^3 +$

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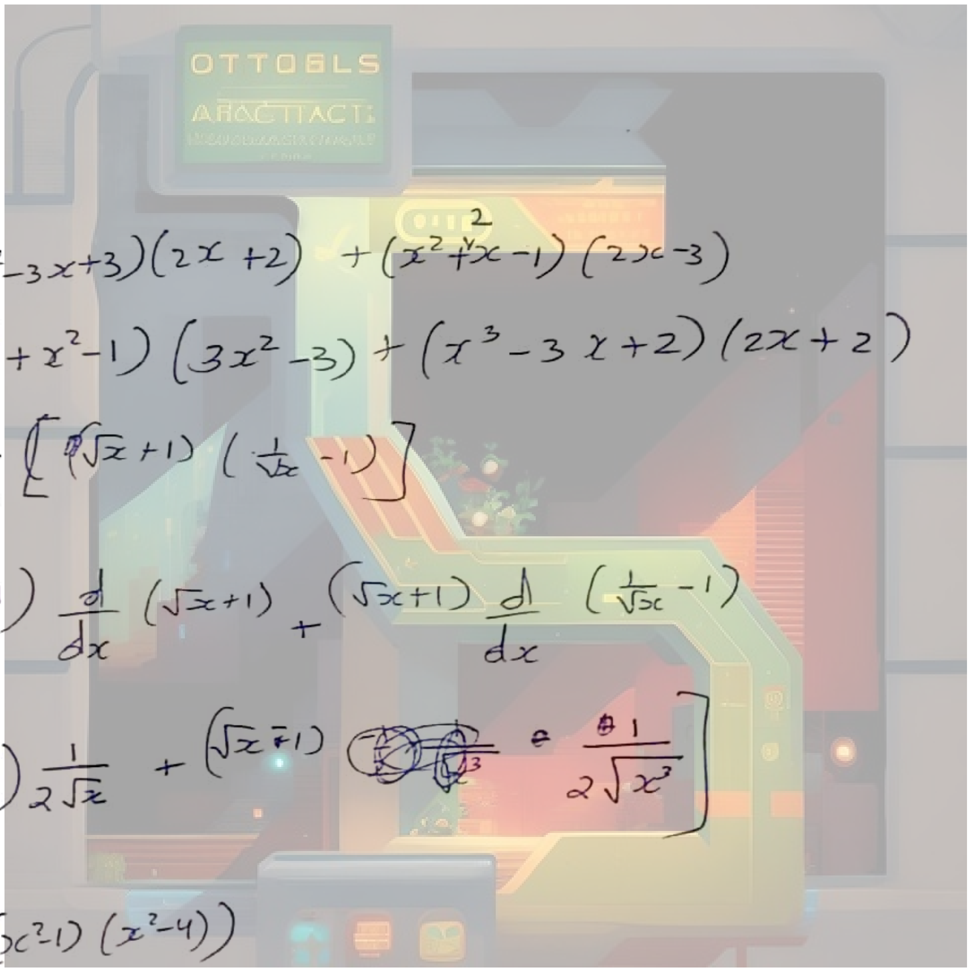
Q6. $y = (x^2-3x+3)(x^2+2x-1)$

Q7. $y = (x^3-3x+2)(x^4+x^2-1)$

Q8. $y = (\sqrt{x}+1)\left[\frac{1}{\sqrt{x}}-1\right]$

Q9. $y = (x^2-1)(x^2-4)$

Q10. $y = \frac{x+1}{x-1}$



Q 6. $(x^2-3x+3)(2x+2) + (x^2+2x-1)(2x-3)$

Q 7. $(x^4+x^2-1)(3x^2-3) + (x^3-3x+2)(2x+2)$

Q 8. $\frac{d}{dx}\left[(\sqrt{x}+1)\left(\frac{1}{\sqrt{x}}-1\right)\right]$

$\left(\frac{1}{\sqrt{x}}-1\right)\frac{d}{dx}(\sqrt{x}+1) + (\sqrt{x}+1)\frac{d}{dx}\left(\frac{1}{\sqrt{x}}-1\right)$

$\left[\left(\frac{1}{\sqrt{x}}+1\right)\frac{1}{2\sqrt{x}} + (\sqrt{x}+1) = \frac{1}{2\sqrt{x^3}}\right]$

Q9. $\frac{d}{dx}\left((x^2-1)(x^2-4)\right)$

$= 2x(2x^2-5)$

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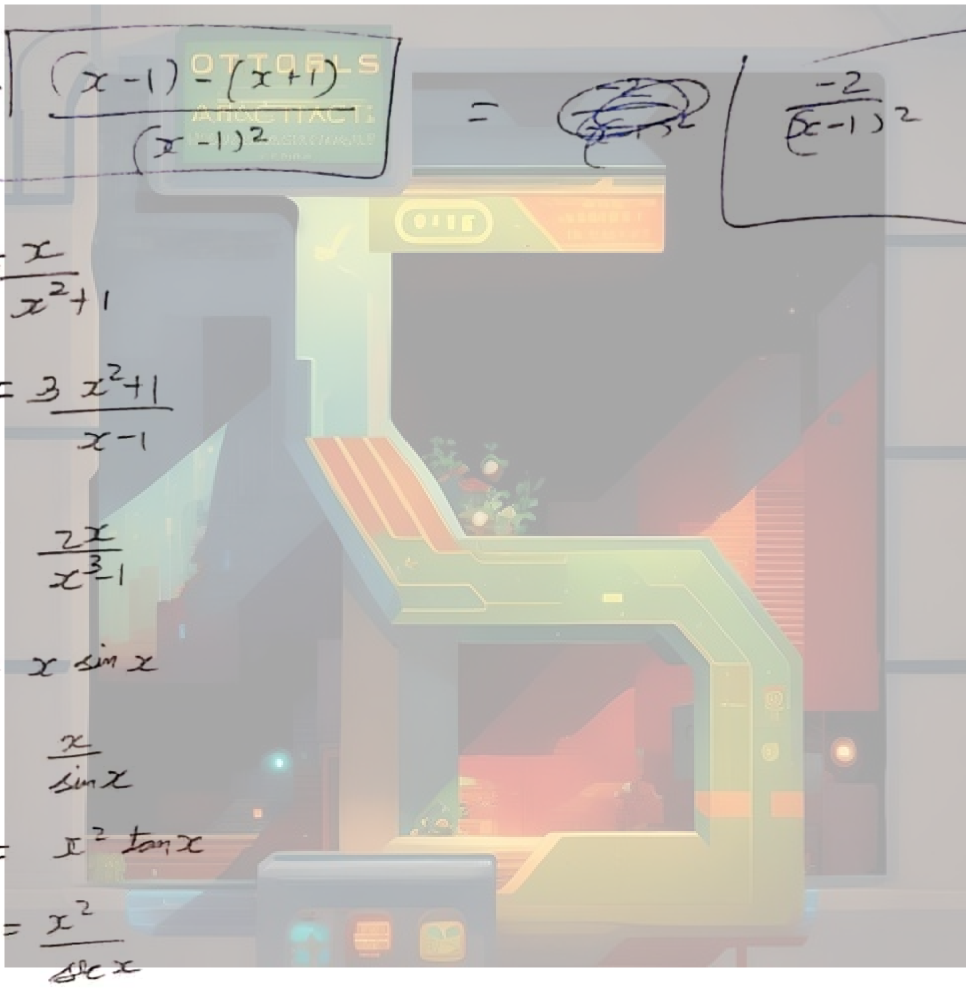
Q10. $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$

$$\frac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$\frac{(x-1)(1) - (x+1)(1)}{x^2+1-2x}$$



$$\boxed{\frac{(x-1)-(x+1)}{(x-1)^2}} = \cancel{\frac{-2}{(x-1)^2}} \boxed{\frac{-2}{(x-1)^2}}$$

Q11. $y = \frac{x}{x^2+1}$

Q12. $y = \frac{3x^2+1}{x-1}$

Q13. $y = \frac{2x}{x^3-1}$

Q14. $y = x \sin x$

Q15. $y = \frac{x}{\sin x}$

Q16. $y = x^2 \tan x$

Q17. $y = \frac{x^2}{\sec x}$

Q18. $y = x \ln x$

Q19. $y = xe^x$

Q20. $y = \frac{x}{e^x}$

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Q11. $\dfrac{dy}{dx} = \dfrac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$

$= \dfrac{x^2+1-2x^2}{\cancel{(x^2+1)^2}(x^2+1)^2}$

$= \boxed{\dfrac{1-x^2}{(x^2+1)^2}}$

Q12. $\dfrac{dy}{dx} = \boxed{\dfrac{(x-1)(6x) - (3x^2+1)(1)}{(x-1)^2}}$

Q13. $\cancel{dy}\ \dfrac{d}{dx}\left(\dfrac{2x}{x^3-1}\right)$

$= \boxed{\dfrac{(x^3-1)(2) - (2x)(3x^2)}{(x^3-1)^2}}$

Q14. $\dfrac{d}{dx}(x \sin x)$

$= \sin x\,(1) + (x)(\cos x)$

$\boxed{= \sin x + x \cos x}$

Q15. $\dfrac{d}{dx}\left(\dfrac{x}{\sin x}\right)$

$= \dfrac{\sin x\,(1) - (x)(\cos x)}{(\sin^2 x)^2}$

$\boxed{= \dfrac{\sin x - x \cos x}{\sin^2 x}}$

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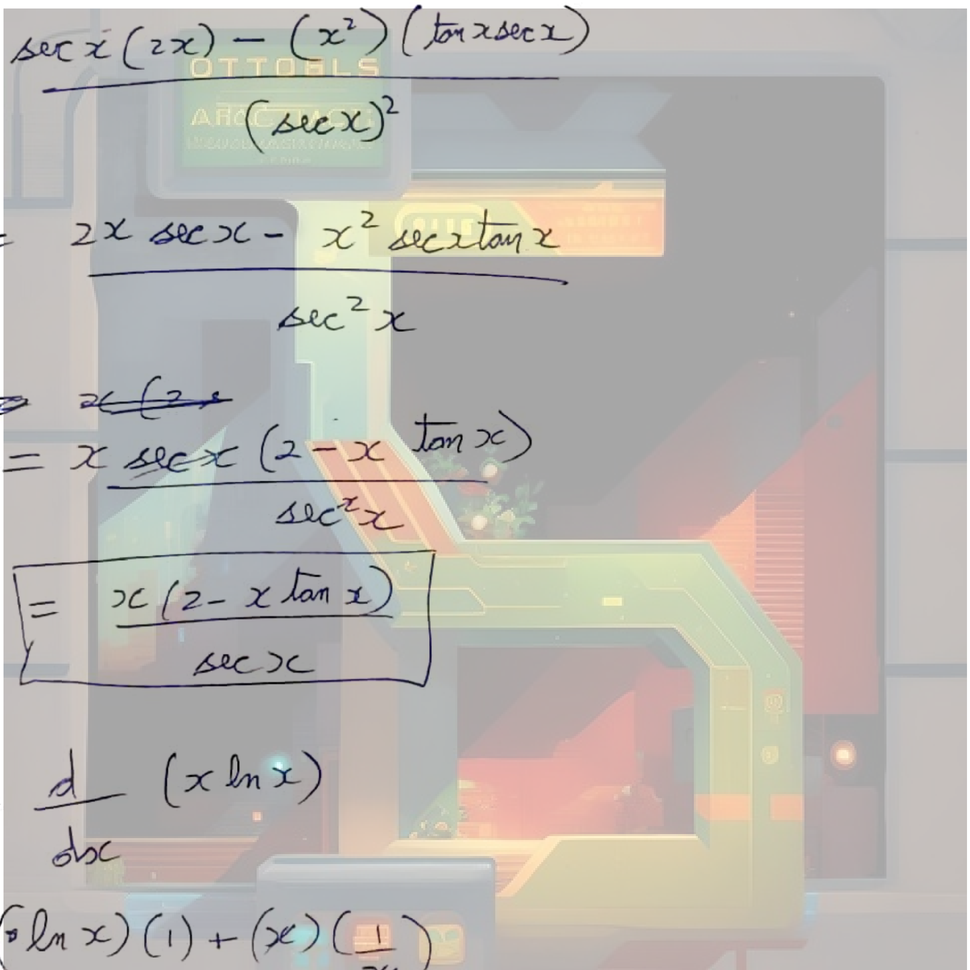
Q16 $\frac{d}{dx}(x^2 \tan x)$

$= \tan x(2x) + (x^2)(\sec^2 x)$

$\boxed{= 2x \tan x + x^2 \sec^2 x}$

Q17. $\frac{d}{dx}\left(\frac{x^2}{\sec x}\right)$



$= \frac{\sec x(2x) - (x^2)(\tan x \sec x)}{(\sec x)^2}$

$= \frac{2x \sec x - x^2 \sec x \tan x}{\sec^2 x}$

$\cancel{= x(2}$

$= \frac{x \sec x(2 - x \tan x)}{\sec^2 x}$

$\boxed{= \frac{x(2 - x \tan x)}{\sec x}}$

Q18. $\frac{d}{dx}(x \ln x)$

$= (\ln x)(1) + (x)\left(\frac{1}{x}\right)$

$\boxed{= \ln x + 1}$

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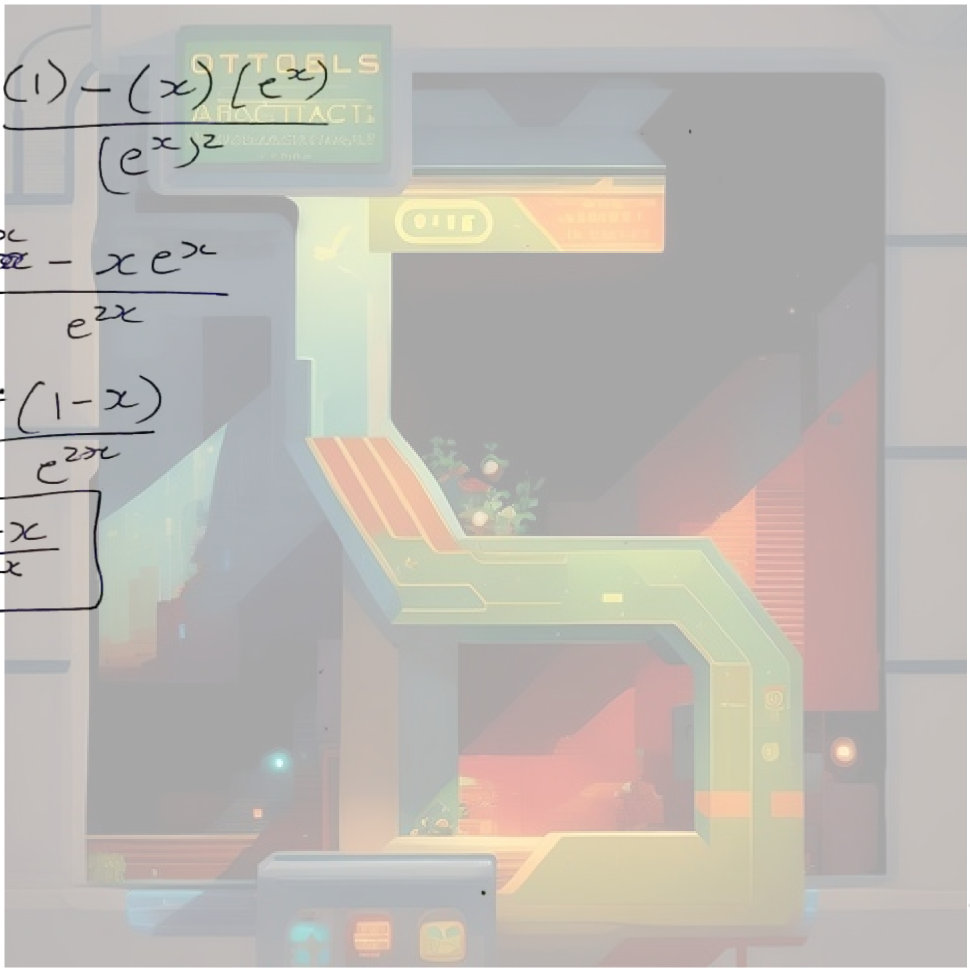
Q19. $\frac{d}{dx}(xe^x)$

$= (e^x)(1) + (x)(e^x)$

$= e^x + xe^x$

$\boxed{= e^x(1+x)}$

Q20. $\frac{d}{dx}\left(\frac{x}{e^x}\right)$



$= \frac{e^x(1) - (x)(e^x)}{(e^x)^2}$

$= \frac{e^x - xe^x}{e^{2x}}$

$= \frac{\cancel{e^x}(1-x)}{e^{2x}}$

$\boxed{= \frac{1-x}{e^x}}$

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Chain rule

eg 1. $y = \sin(x^2 - 4)$

let $t = x^2 - 4$

let $y = \sin t$

$$\frac{d\cancel{x}y}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dt} = \frac{d}{dt}(y) = \frac{d}{dt}(\sin t) = \cancel{\cot}\cos t$$

$$\frac{dt}{dx} = \frac{d}{dx}(t) = \frac{d}{dx}(x^2 - 4) = 2x$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \cancel{\frac{dt}{dt}}\frac{dt}{dx} = 2x\,\cancel{\cot}\cos t$$

$$\boxed{= 2x \cos[x^2 - 4]}$$

eg 2. $y = \sin^2 x = (\sin x)^2$

$t = \sin x$

$y = t^2$

$$\frac{dy}{dt} = 2t$$

$$\frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \cos x \times 2t$$

$$= \cos x \times 2 \sin x$$

$$\boxed{= 2 \sin x \cos x}$$

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eg3. $y = \sin 2x$

$t = 2x$

$y = \sin t$

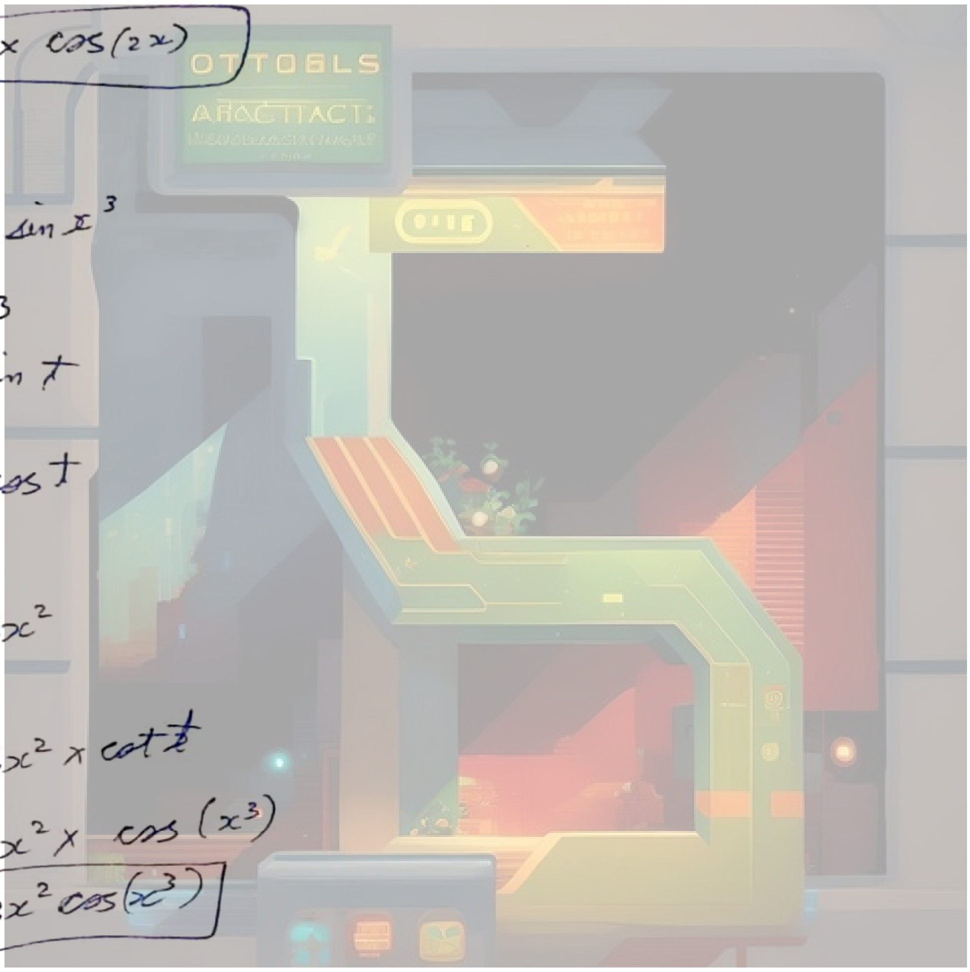
$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 2$$

$$\frac{dy}{dx} = 2\cos t$$

$$\boxed{= 2 \times \cos(2x)}$$

$=$



eg4. $y = \sin x^3$

$t = x^3$

$y = \sin t$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 3x^2$$

$$\frac{dy}{dx} = 3x^2 \times \cos t$$

$$= 3x^2 \times \cos(x^3)$$

$$\boxed{= 3x^2 \cos(x^3)}$$

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Q ~~Q~~1. $y = \sin 3x$

Q 2. $y = 3\sin(3x+5)$

Q 3. $y = \sin(\sin x)$

Q 4. $y = \ln^2 x = [\ln x]^2$

Q 5. $y = \frac{1}{3^x}$

Pg - 41, 42

Q1 - Q ~~29~~ 39

Q 1.

$y = \sin 3x$

let $t = 3x$

$y = \cancel{t} \sin t$

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t) = \cos t$$

$$\frac{dt}{dx} = \frac{d}{dx}(3x) = 3$$

$$\frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dx} = \cancel{3\cos t} \boxed{3\cos t} = 3\cos(3x)$$

Q 2.

$y = 3\sin(3x+5)$

let $t = 3x+5$

$y = 3\sin t$

$$\frac{dy}{dt} = 3\cos t$$

$$\frac{dt}{dx} = 3$$

$$\frac{dy}{dx} = 3 \times 3\cos t$$

$$\boxed{= 9\cos t}$$

$$= 9\cos(3x+5)$$

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Q 3. $y = \sin(\sin x)$

$t = \sin x$

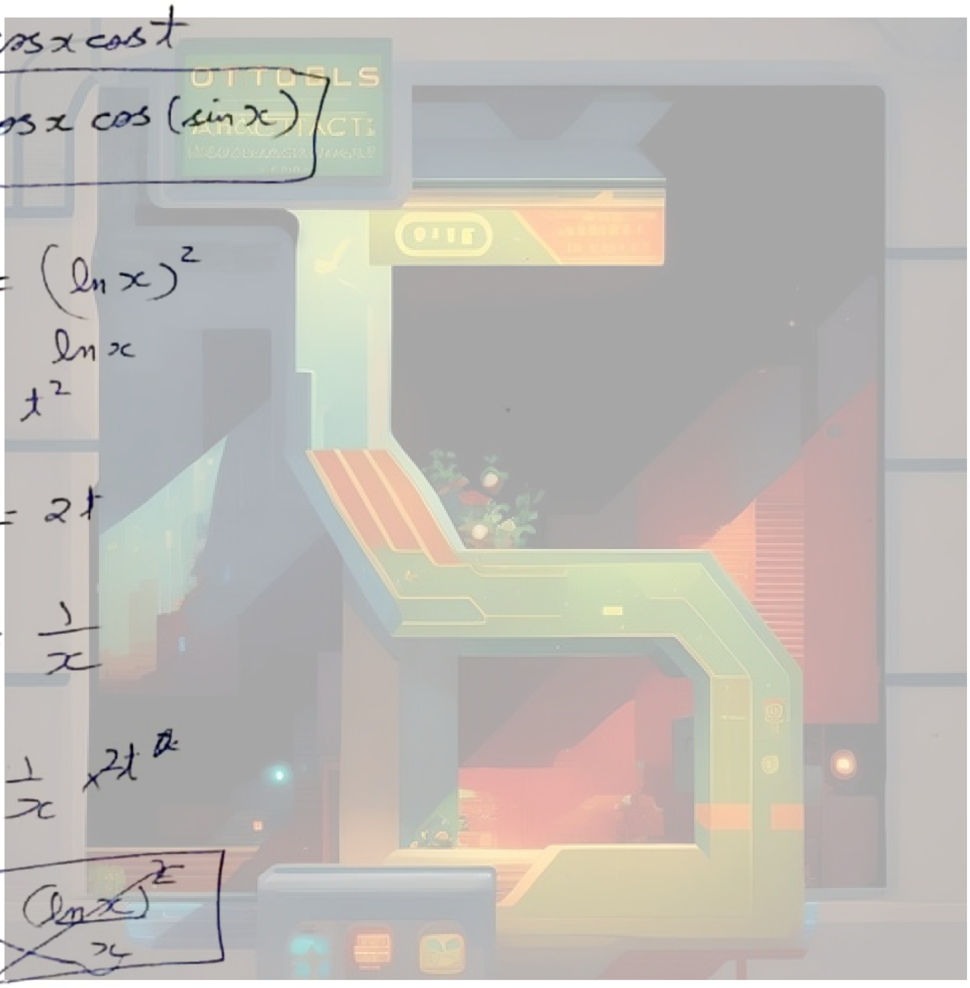
$y = \sin t$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \cos x \cos t$$

$$\boxed{= \cos x \cos(\sin x)}$$



Q 4. $y = (\ln x)^2$

$t = \ln x$

$y = t^2$

$$\frac{dy}{dt} = 2t$$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} \times 2t$$

$$\cancel{\boxed{= \frac{(\ln x)^2}{x}}}$$

$$= \boxed{\frac{2 \ln x}{x}}$$

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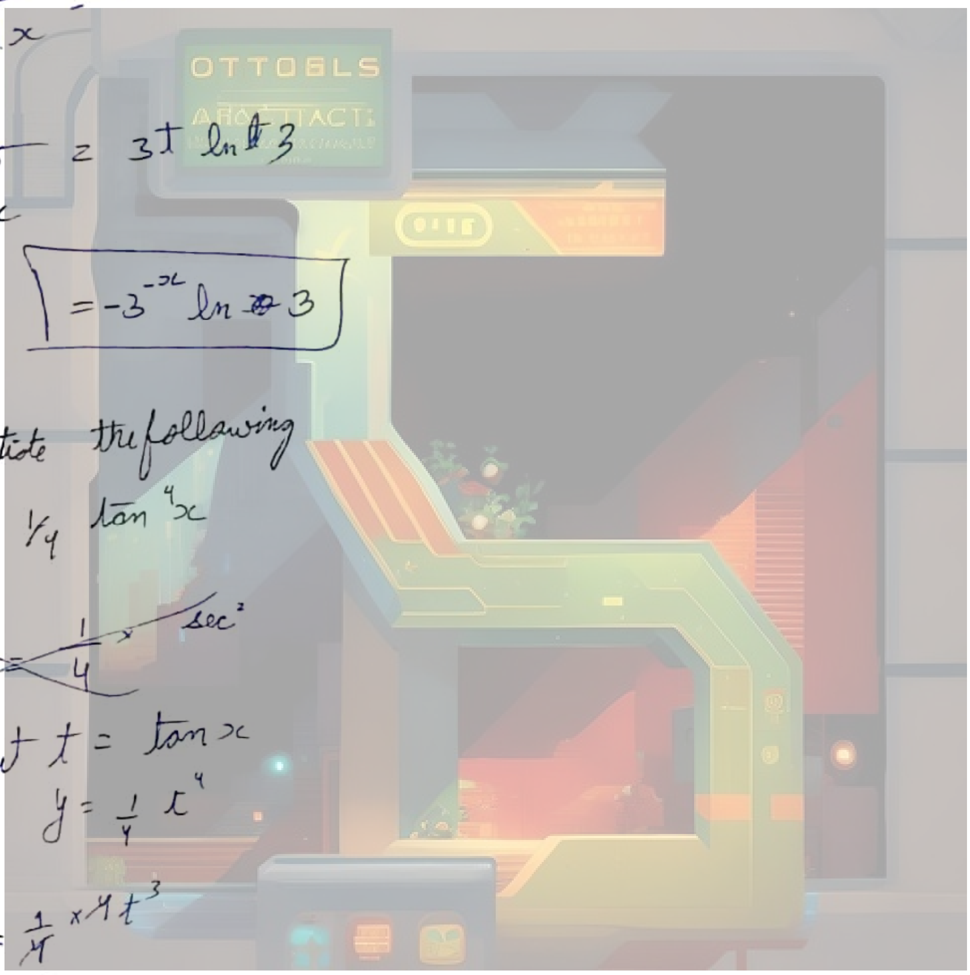
Q 5 $y = \frac{1}{3^x} = 3^{-x}$

let $t = -x$

$y = 3^t$

$$\frac{dy}{dt} = 3^t \ln 3$$

$$\frac{dt}{dx} = -1$$



$$\frac{dy}{dx} = 3^t \ln 3$$

$$\boxed{= -3^{-x} \ln 3}$$

Q Differentiate the following

(1) $y = \frac{1}{4} \tan^4 x$

$\cancel{\frac{dy}{dx} = \frac{1}{4} \times \sec^2}$

let $t = \tan x$

$y = \frac{1}{4} t^4$

$$\frac{dy}{dt} = \frac{1}{4} \times 4 t^3$$
$$= t^3$$

$$\frac{dt}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = t^3 \sec^2 x$$
$$= \boxed{\tan^3 x \sec^2 x}$$

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(2) $y = \cot\sqrt{1+x^2}$

$t = \sqrt{1+x^2}$

$= (1+x^2)^{1/2}$

$\cancel{s = 1+x^2}$ $\cancel{t = 1 + x + 2x^2}$

$S = 1 + x^2$

$t = s^2$

$y = \cot t$

$\frac{dy}{dt} = -\operatorname{cosec} t \cot t$

$\cancel{\frac{ds}{dx}}$

(3) $y = \sin^2 3x$

$t = 3x$

$y = \sin^2 t$

$\frac{dy}{dt} = \cancel{B} \sin t \cos t + \sin t \cos t$

$= 2 \sin t \cos t$

$\frac{dt}{dx} = 3$

$\frac{dy}{dx} = 6 \sin t \cancel{\cot} \cos t$

$\boxed{= 6 \sin 3x \cos 3x}$

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Q(2) $y = \cot\sqrt{1+x^2}$

$y = \cot[1+x^2]^{1/2}$

$t = [1+x^2]^{1/2}$

$y = \cot t$

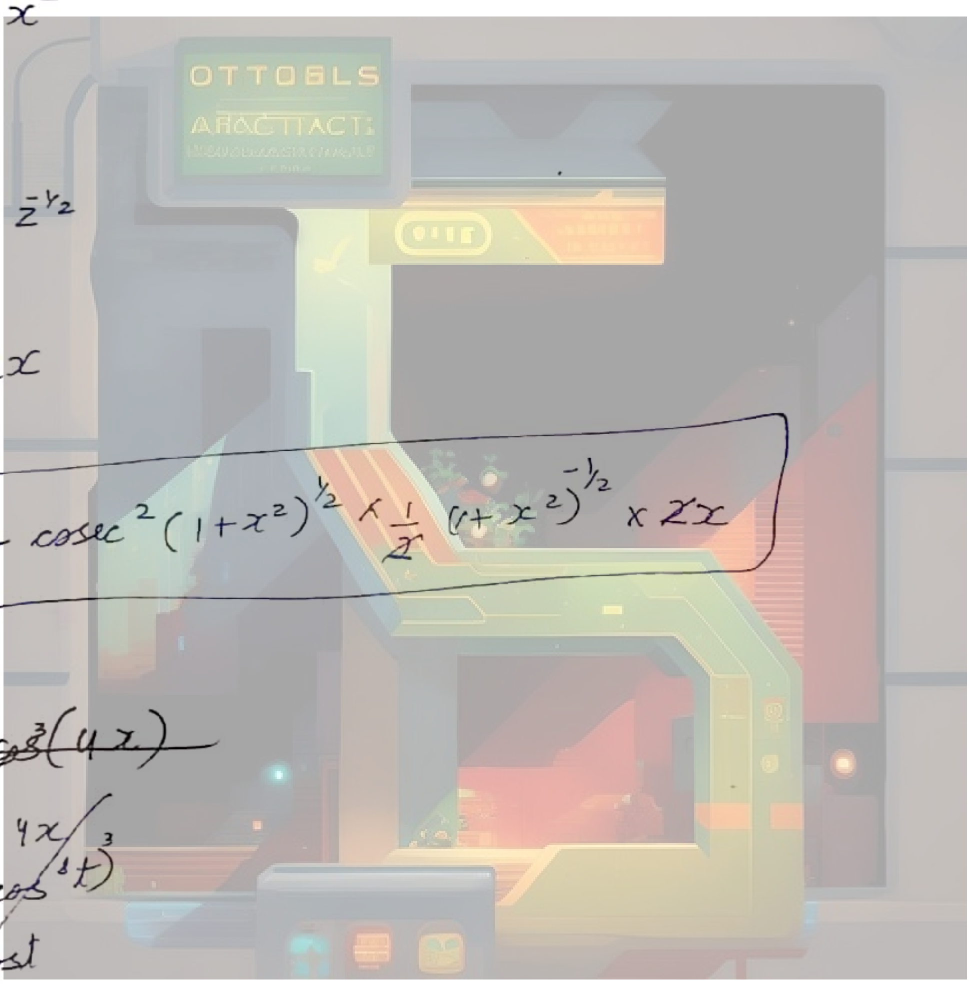
$\frac{dy}{dt} = -\operatorname{cosec}^2 t$

$z = 1+x^2$

$t = z^{1/2}$

$\frac{dt}{dz} = \frac{1}{2} z^{-1/2}$

$\frac{dz}{dx} = 2x$



$\frac{dy}{dx} = \boxed{-\operatorname{cosec}^2(1+x^2)^{1/2} \times \frac{1}{2}(1+x^2)^{-1/2} \times 2x}$

$=$

(4) $\cancel{y = \cos^3(4x)}$

$\cancel{t = 4x}$

$\cancel{y = (\cos^3 t)}$

$\cancel{z = \cos t}$

$\cancel{y = z^3}$

$\frac{dy}{dz}$

④ $y = \cos^3(4x)$

$$\frac{dy}{dx} = 3\,(\cos 4x)^2 \times -\sin 4x \times 4$$

$$\cancel{= 3\cos^2 4x - \sin 4x + 4}$$

$$\boxed{= -12\cos^2 4x \sin 4x}$$

⑤ $y = (1+\sin^2 x)^4$

$$= 4(1+\sin^2 x)^3 \times (2\sin x) \times (\cos x)$$

$$\boxed{= 8\,(1+\sin^2 x)^3 \sin x \cos x}$$

⑥ $y = (3x + \tan^2 x)^2$

$$\frac{dy}{dx} = 2\,(3x + \tan^2 x) \times (3 + 2\tan x \times \sec^2 x)$$

$$= 2\,(3x+\tan^2 x) \times (3 + 2\tan x \sec^2 x)$$

$$\frac{dy}{dx} = 2(3x + \tan^2 x) \times \frac{d}{dx}(3x + \tan^2 x$$

$$= 2(3x + \tan^2 x) \times \left(3 + \frac{d}{dx}(\tan^2 x)\right)$$

$$= 2(3x + \tan^2 x) \times \left(3 + 2\tan x \times \frac{d}{dx}(\tan x)\right)$$

$$= 2(3x + \tan^2 x) \times (3 + 2\tan x \sec^2 x)$$

$$\boxed{= 2(3x + \tan^2 x) \times (3 + 2\tan x \sec^2 x)}$$

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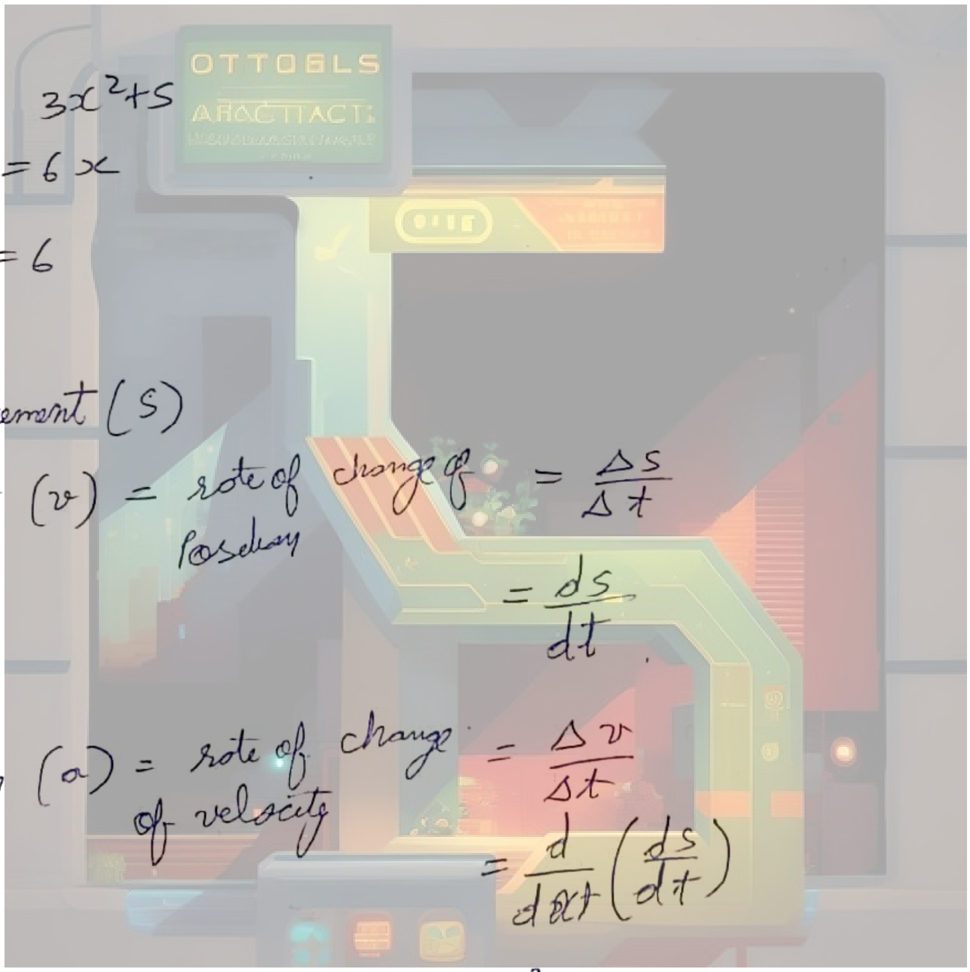
Double differentiation

$$y = f(x)$$

$\frac{dy}{dx} = y'$ (first derivative)

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = y''$ (double differentiation)
↳ (Leibniz notation)



eg $y = 3x^2+5$

$y' = 6x$

$y'' = 6$

Displacement (s)

velocity (v) = rate of change of position $= \frac{\Delta s}{\Delta t}$

$= \frac{ds}{dt}$

acceleration (a) = rate of change of velocity $= \frac{\Delta v}{\Delta t}$

$= \frac{d}{d\cancel{x}t}\left(\frac{ds}{dt}\right)$

$= \frac{d^2 s}{\cancel{dt}dt^2}$

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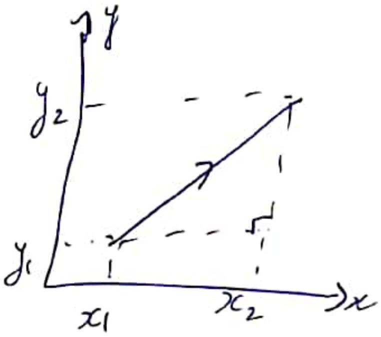
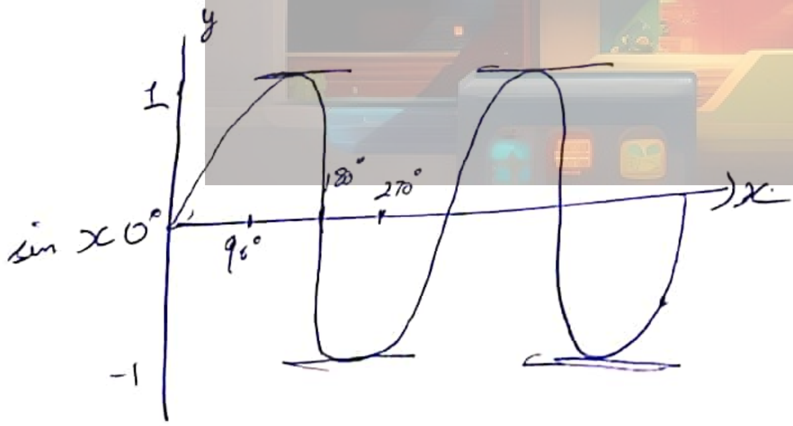
Slope



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{dy}{dx}$$



$y = \sin x$

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at topmost (maxima) and bottommost (minima), slope = 0

$$\boxed{\frac{dy}{dx} = 0}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x$$

$$\cos x = 0$$
$$x = 90^\circ, 270^\circ$$

Curvature → Double Differentiation

maxima → slope decrease → $\frac{d^2y}{dx^2} < 0$

minima → slope increase → $\frac{d^2y}{dx^2} > 0$

For maxima, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} < 0$

For minima, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} > 0$

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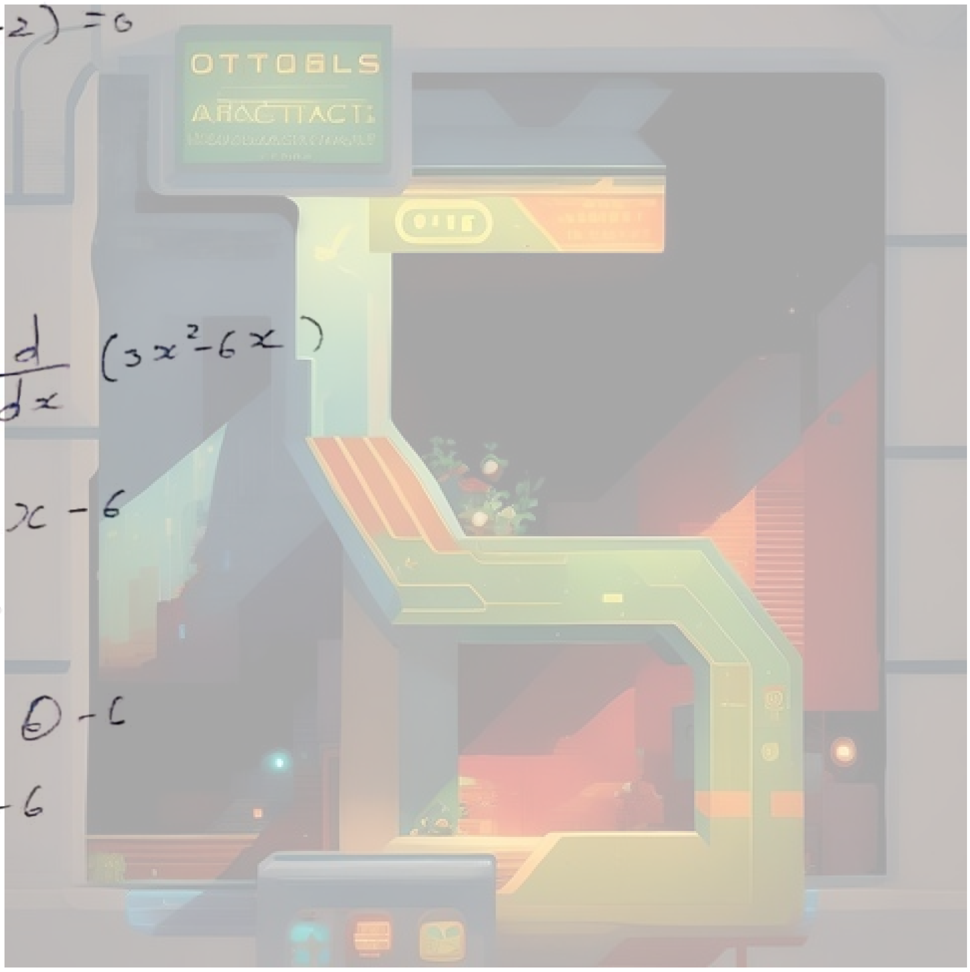
Q $y = x^3 - 3x^2 + 6$

$$\frac{dy}{dx} = 3x^2 - \cancel{3x}\, 6x$$

$$\frac{dy}{dx} = 0$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$



$$\boxed{\begin{array}{l} x = 0 \\ x = 2 \end{array}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 6x)$$

$$= 6x - 6$$

at $x = 0$

$$\frac{d^2y}{dx^2} = 0 - 6$$

$$= -6$$

at $x = 2$

$$\frac{d^2y}{dx^2} = 12 - 6$$

$$= 6$$

Maxima $= (x=0) = 6$

Minim $= (x=2) = 8 - 12 + 6 = 2$

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② $y = 3x^4 + 4x^3 - 12x^2 + 12$

find the points of maxima & minima.

$\frac{d}{dx} = 12x^3 + 12x^2 - 24x$

$\cancel{0=}$

$24x = 12x^3 + 12x^2$

$2 = x^2 + x$

$\cancel{2 = x(x+1)}$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2$

$x(x+2) - 1(x+2)$

$(x-1)(x+2)$

$x = 1 \qquad x = 0$

$x = -2$

$\frac{d^2}{dx^2} = 36x^2 + 24x - 24$

$= 36 + 24 - 24$

$= 36$ (minima, $x = 1$)

$= 36(-2)^2 + 24(-2) - 24$

$= 36 \times 4 - 48 - 24$

$= 144 - 48 - 24$

$= 72$ (minima)

$= 0 + 0 - 24$

$= 0 - 24$ (maxima)

$y_{max} = 12$

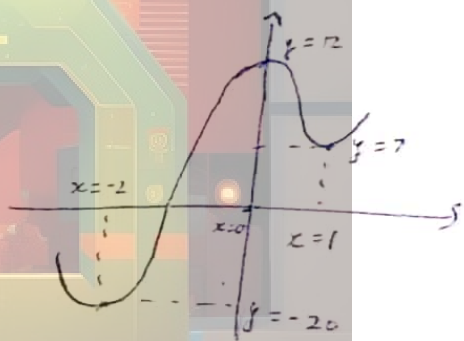
$y_{min} = 3 + 4 - 12 + 12$

$= 7$

$y_{min} = 16 \times 3 - 4 \times 8$

$= -20$

$\boxed{\begin{array}{l} \text{minimum} = -20 \\ \text{maximum} = 12 \end{array}}$



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(3) find minimum value of $5x^2-2x+1$

$$\frac{dy}{dx} = 10x-2 = 0$$

$$\cancel{0 = 10x}$$

$$2 = 10x$$

$$x = \frac{1}{5}$$

$$\frac{d^2y}{dx^2} = 10 \text{ (minima)}$$

$$\text{value} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right)+1$$

$$= \frac{\cancel{5}}{\cancel{25}\,5} - \frac{2}{5}+1$$

$$= \frac{1-2+5}{5}$$

$$\boxed{= \frac{4}{5}} \quad (3) \quad = 0.8$$

(4) find turning points of function $4x^3+12x^2+12x+10$

$$\frac{dy}{dx} = 12x^2+24x+12$$

$$= 12x^2+24x+12$$

$$= x^2+2x+1$$

$$0 = x^2+x+x+1$$

$$= x(x+1)+1(x+1)$$

$$\boxed{= x = -1}$$

$$y = 12 - 24 + 12$$

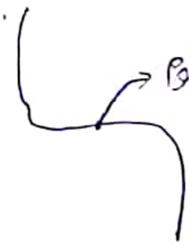
$$= 0$$

$(-1, 0)$

$$\frac{d^2y}{dx^2} = 24x+24$$

$$= -24+24$$

$= 0 \to$ Point of Inflection, no minima or maxima



Point of Inflection

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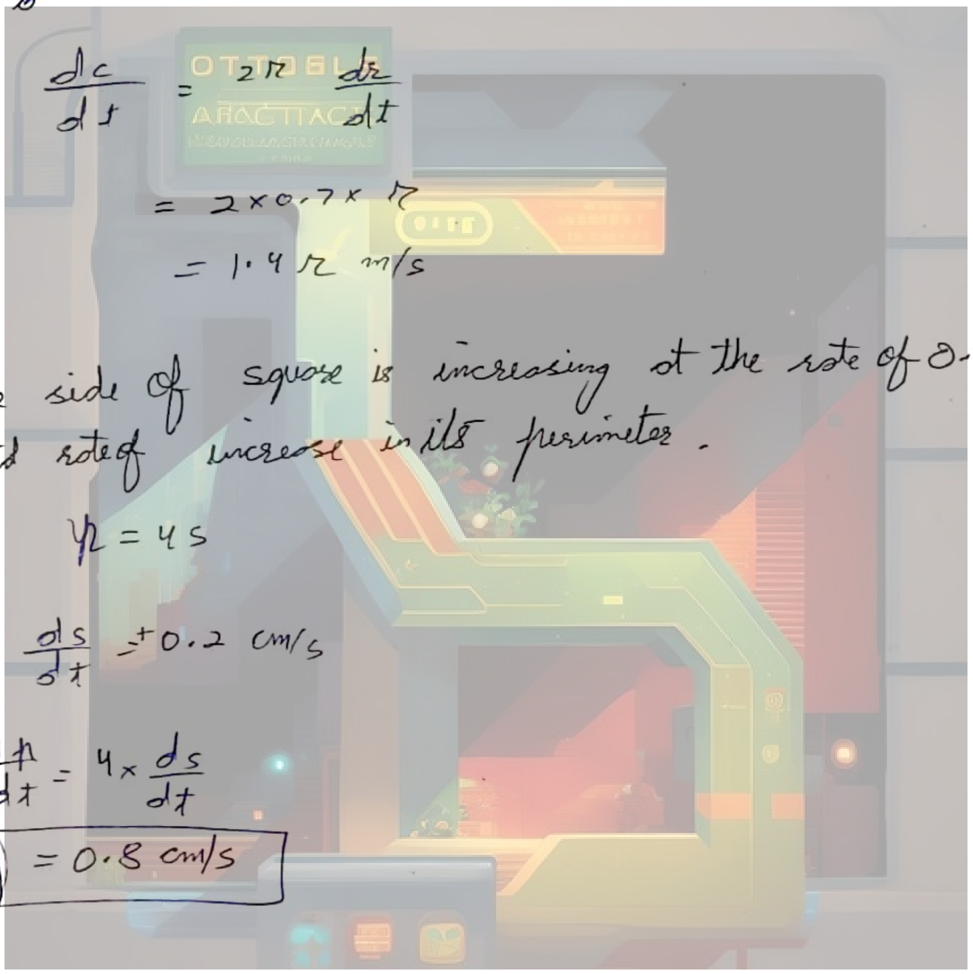
Application of ~~Deravib~~ Derivatives

Q The radius of circle is increasing at the rate of 0.7 cm per second. What is the rate of increase in its circumference.

$$\frac{dr}{dt} = 0.7 \text{ cm/s}$$

$$C = 2\pi r$$



$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$= 2 \times 0.7 \times \pi$$

$$= 1.4\pi \text{ m/s}$$

Q②. The side of square is increasing at the rate of 0.2 cm/s find rate of increase in its perimeter.

$$P = 4s$$

$$\frac{ds}{dt} = 0.2 \text{ cm/s}$$

$$\frac{dP}{dt} = 4 \times \frac{ds}{dt}$$

$$\boxed{= 0.8 \text{ cm/s}}$$

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Q③. The radius of a spherical balloon is decreasing at the rate of 10 cm/s. At what rate is the surface area of the balloon decreasing when its radius is 15 cm.

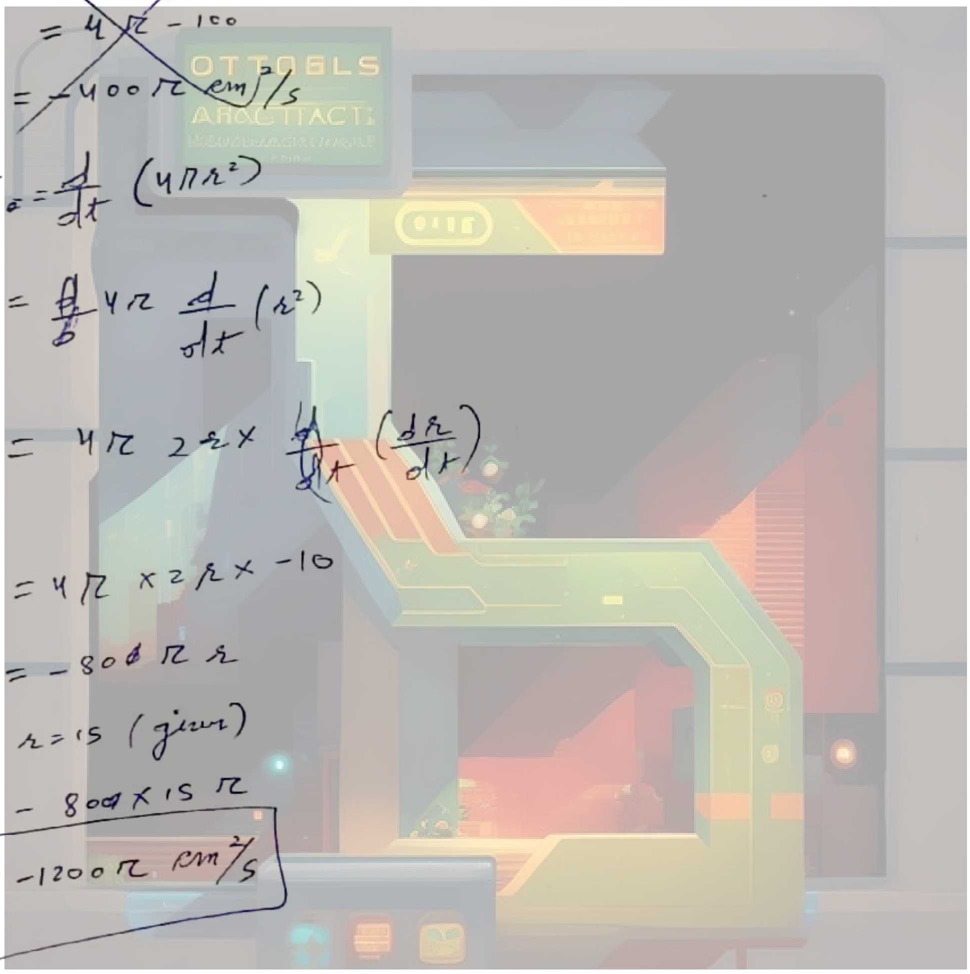
$$\frac{dr}{dt} = -10 \text{ cm/s}$$

$$\text{area} = 4\pi r^2$$

$$\cancel{\frac{da}{dt} = 4\pi(-10)^2}$$

$$\cancel{= 4\pi \; -100}$$

$$\cancel{= -400\pi \text{ cm}^2/s}$$



$$\frac{dA}{dt} = \frac{d}{dt}(4\pi r^2)$$

$$= 4\pi \frac{d}{dt}(r^2)$$

$$= 4\pi \; 2r \times \left(\frac{dr}{dt}\right)$$

$$= 4\pi \times 2r \times -10$$

$$= -80\pi r$$

$r = 15$ (given)

$$= -80 \times 15\pi$$

$$\boxed{= -1200\pi \text{ cm}^2/s}$$

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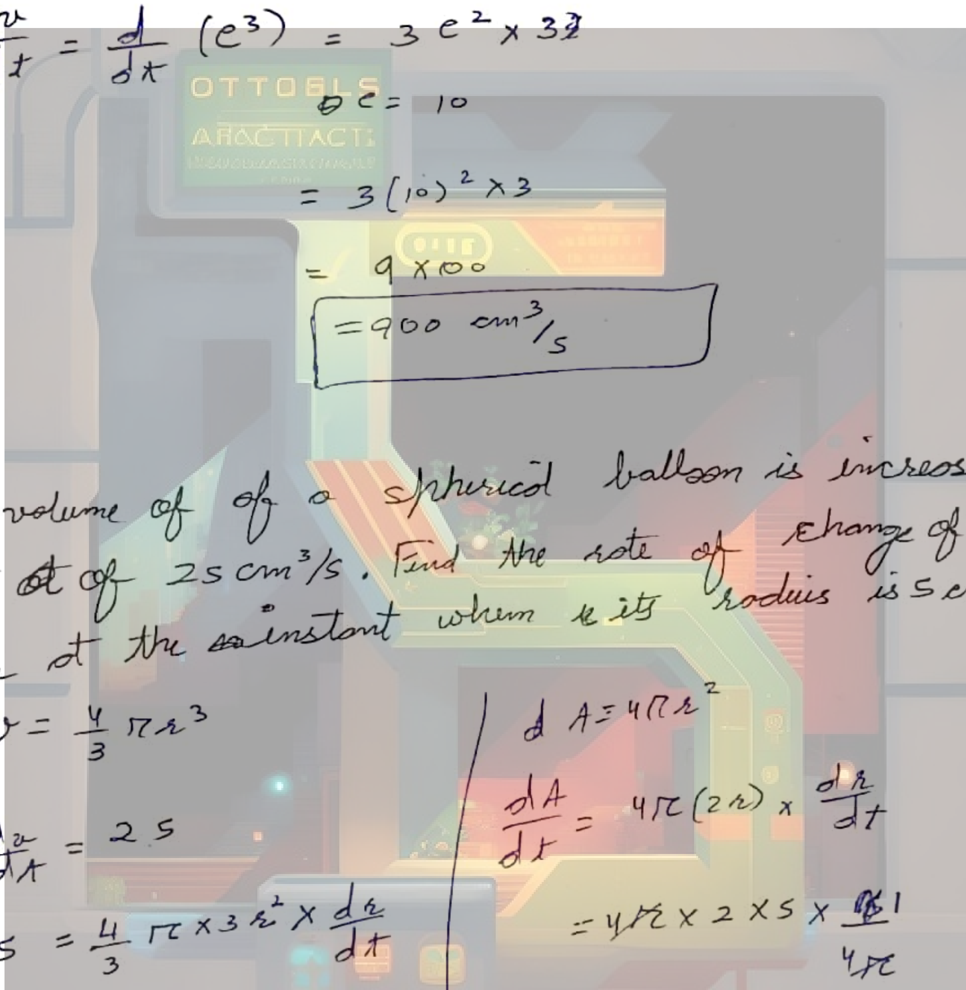
Q4. An Edge of a variable cube ~~Of~~ ~~a va~~ is increasing at a ~~a~~ rate of 3 cm/s. How fast is its volume increasing when the edge is 10 cm long.

$$\frac{de}{dt} = 3\, cm/s$$

$$v = e^3$$



$$\frac{dv}{dt} = \frac{d}{dt}(e^3) = 3e^2 \times 3$$

$$e = 10$$

$$= 3(10)^2 \times 3$$

$$= 9 \times 100$$

$$\boxed{= 900\ cm^3/s}$$

Q5. The volume of ~~of~~ a spherical balloon is increasing at the rate ~~at~~ of 25 cm³/s. Find the rate of change of its surface area at the ~~a~~ instant when ~~r~~ its radius is 5 cm.

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 25$$

$$25 = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$$

$$\frac{\cancel{25}^{5} \times \cancel{3}}{4 \times \pi \times \cancel{3} \times \cancel{5} \times 5}$$

$$\frac{1}{4\pi} = \frac{dr}{dt}$$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 4\pi(2r) \times \frac{dr}{dt}$$

$$= 4\pi \times 2 \times 5 \times \frac{1}{4\pi}$$

$$\boxed{= 10\ cm^2/s}$$

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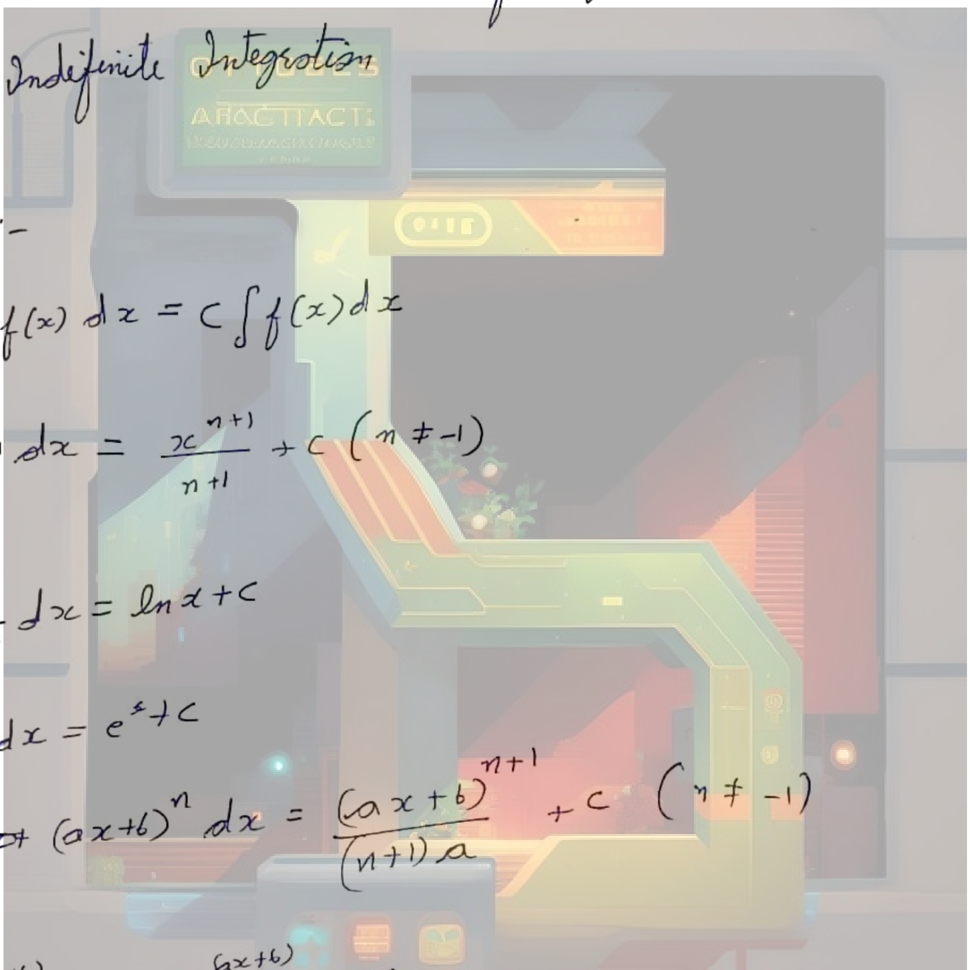
Integration

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^2+1) = 2x$$

$$\frac{d}{dx}(x^3+3) = 2$$

$$\int (2x)\,dx = x^2 + C \rightarrow \text{constant of Integration}$$



Indefinite Integration

formulas:-

(1) $\int c\, f(x)\,dx = c \int f(x)\,dx$

(2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

(3) $\int \frac{1}{x}\,dx = \ln x + c$

(4) $\int e^x\,dx = e^x + c$

(5) $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c \quad (n \neq -1)$

(6) $\int e^{(ax+b)}\,dx = \frac{e^{(ax+b)}}{a} + c$

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(7) $\int \sin x \, dx = -\cos x + c$

(8) $\int \cos x \, dx = \sin x + c$

(9) $\int \sin(ax+b) \, dx = \dfrac{-\cos(ax+b)}{a} + c$

(10) $\int \cos(ax+b) \, dx = \dfrac{\sin(ax+b)}{a} + c$

(11) $\int \sec^2 x \, dx = \tan x + c$

(12) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

(13) $\int \sec x \tan x \, dx = \sec x + c$

(1) $\int \sqrt{x} \, dx$

$\int x^{1/2} \, dx$

$= \dfrac{x^{3/2}}{\frac{3}{2}} + c$

$\boxed{= \dfrac{2\sqrt{x^3}}{3}}$

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(2) $\int \frac{dx}{x^2}$

$\int x^{-2}\,dx$

$\frac{x^{-1}}{-1} + c$

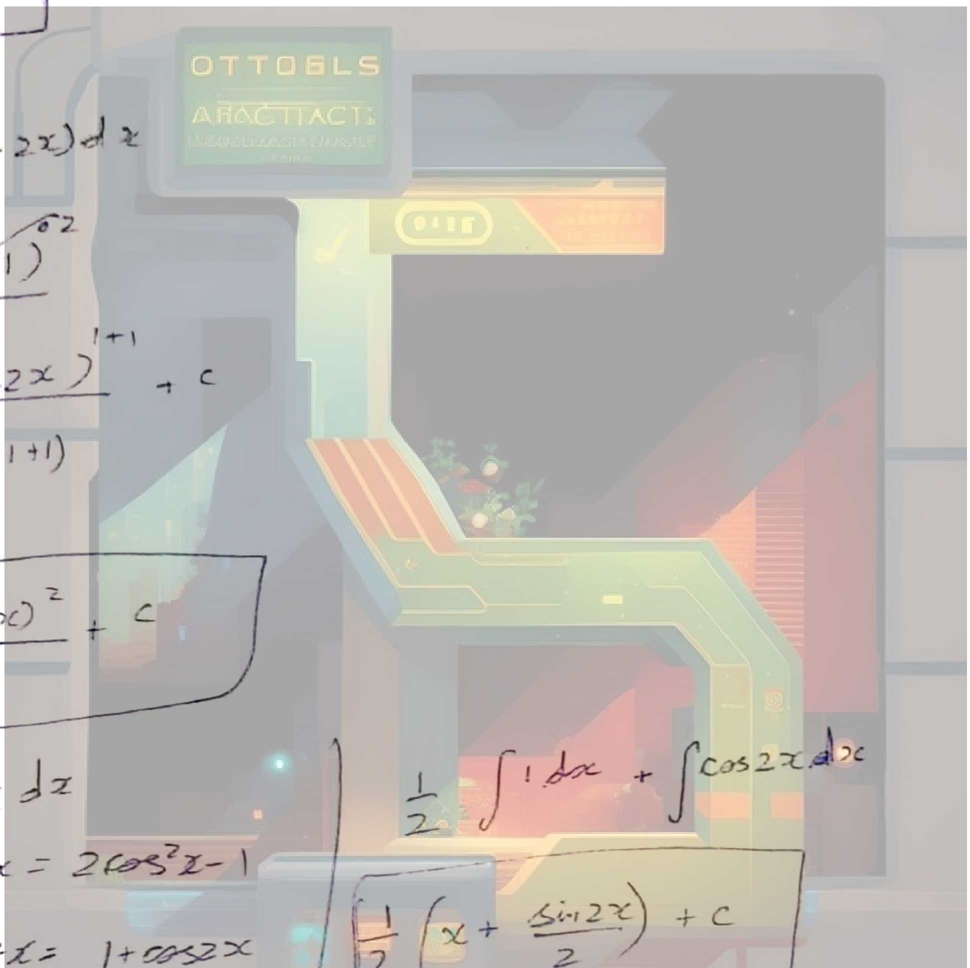
$\boxed{\frac{1}{-x} + c}$

(3) $\int (1-2x)\,dx$

$\cancel{(-2x+1)^{2}}$

$\frac{(1-2x)^{1+1}}{(-2)(1+1)} + c$

$\boxed{\frac{(1-2x)^2}{-4} + c}$



(4) $\int \cos^2 x\,dx$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{1}{2} + \frac{\cos^2 x}{2}\,dx$

$\frac{1}{2}\int 1 + \cos^2 x\,dx$

$\frac{1}{2}\int 1\,dx + \int \cos 2x\,dx$

$\boxed{\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c}$

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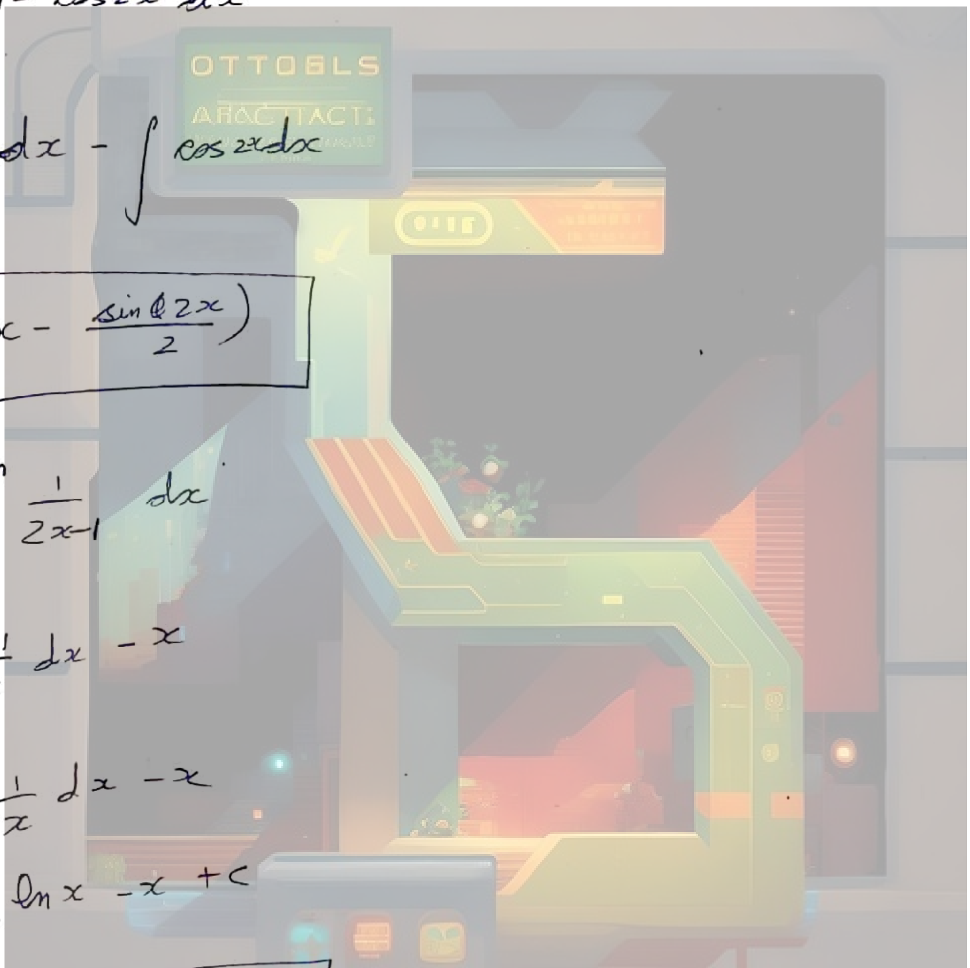
(5) $\int \sin^2 x \, dx$

(6) $\int \frac{1}{2x-1} \, dx$

(5) $\int \frac{1-\cos 2x}{2} \, dx$

$\frac{1}{2} \int 1 - \cos 2x \, dx$



$\frac{1}{2} \int 1 \, dx - \int \cos 2x \, dx$

$\boxed{\frac{1}{2} \left(x - \frac{\sin 2x}{2} \right)}$

(6) $\int \frac{1}{2x-1} \, dx$

$\int \frac{1}{2x} \, dx - x$

$\frac{1}{2} \int \frac{1}{x} \, dx - x$

$\frac{1}{2} \times \ln x - x + c$

$\boxed{\frac{\ln x}{2} - x + c}$

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(7) $\int \left(x - \frac{1}{x}\right) dx$

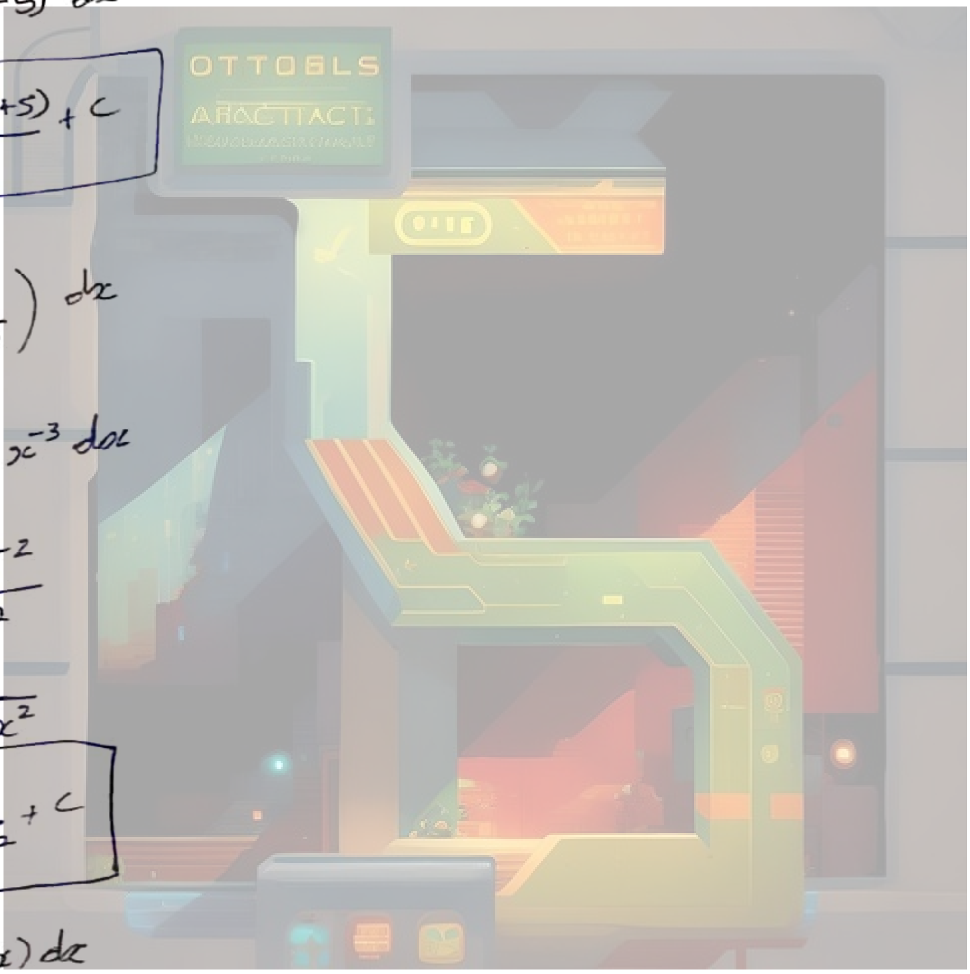
$\int x \, dx - \int \frac{1}{x} \, dx$

$\frac{x^2}{2} - \ln$

$\boxed{\frac{x^2 - 2\ln}{2} + c}$

(8) $\int \cos(2x+5) \, dx$

$\boxed{\frac{\sin(2x+5)}{2} + c}$



(9) $\int \left(e^{2x} + \frac{1}{x^3}\right) dx$

$e^{2x} + \int x^{-3} \, dx$

$e^{2x} + \frac{x^{-2}}{-2}$

$e^{2x} - \frac{1}{2x^2}$

$\boxed{\frac{e^{2x}}{2} - \frac{1}{2x^2} + c}$

(10) $\int \cos(1-2x) \, dx$

$\boxed{\frac{\sin(1-2x)}{-2} + c}$

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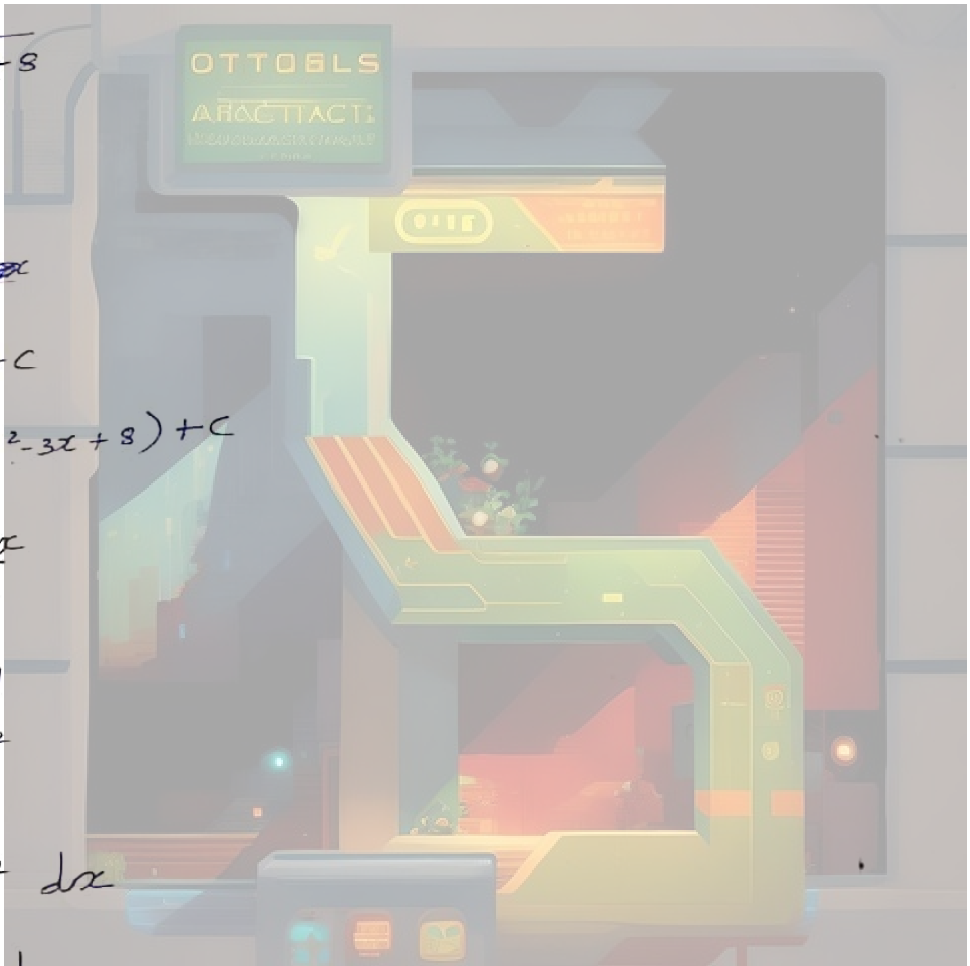
Substitution method

① $\int \frac{(2x-3)\,dx}{x^2-3x+8}$

let $t = x^2-3x+8$

$\frac{dt}{dx} = 2x-3$

$\therefore dt = (2x-3)\,dx$



$\int \frac{dt}{x^2-3x+8}$

$\int \frac{dt}{t}$

$\int \frac{1}{t}\,dt \cancel{dx}$

$= \ln t + C$

$= \ln(x^2-3x+8)+C$

② $\int \frac{x^2\,dx}{x^3+1}$

$t = x^3-1$

$\frac{dt}{dx} = 3\cancel{2}x^2$

$dt = 3\cancel{2}x^2\,dx$

$\frac{dt}{3\cancel{2}} = x^2\,dx$

$\int \frac{dt}{\cancel{2}3} \cdot \frac{1}{t}$

$\frac{\ln t}{3\cancel{2}} + c$

$\frac{\ln(x^3+1)}{3\cancel{2}} + c$

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③ $\int \frac{e^{2x}\,dx}{e^{2x}+a^2}$

let $t = e^{2x}+a^2$

$\frac{dt}{dx} = e^{2x} + \cancel{2a}$

$\cancel{dt - 2a\,dx = e^{2x}\,dx}$

$\cancel{dt = e^{2x}\,dx}$

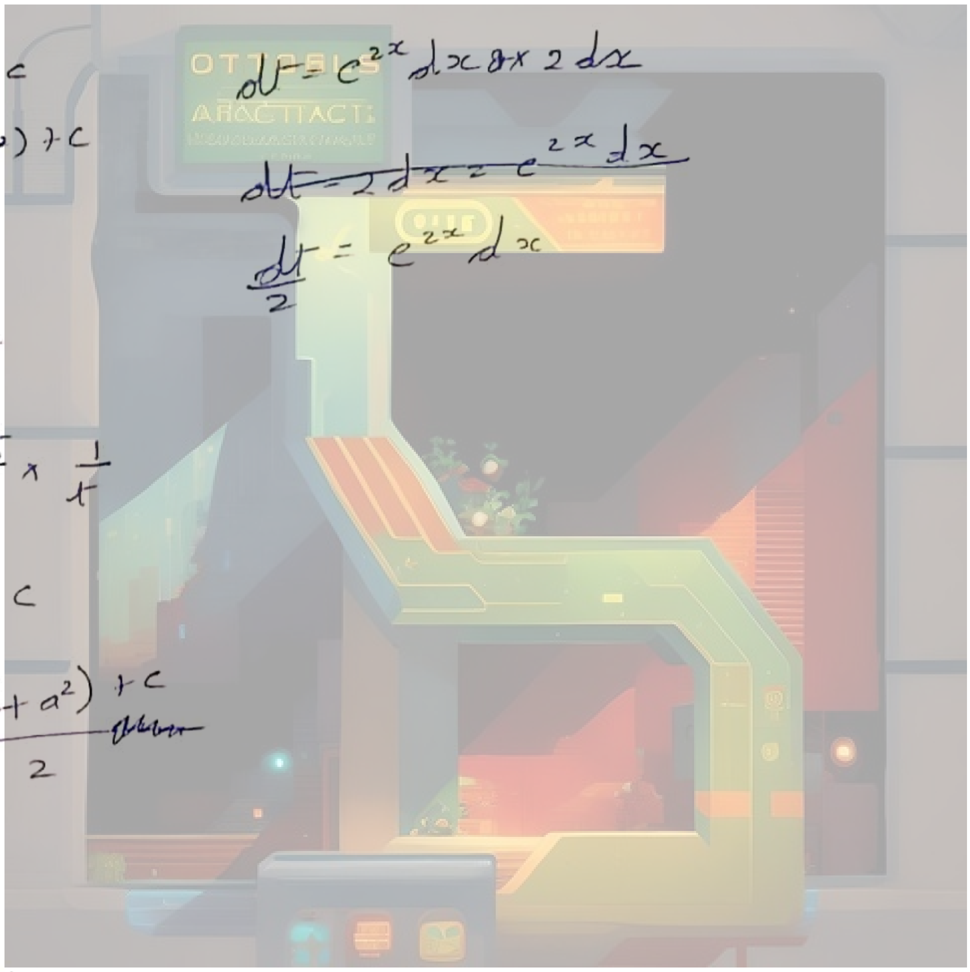
$\cancel{\int \frac{dt}{t}}$

$\cancel{= \ln t + c}$

$\cancel{= \ln(e^{2x}+a^2)+c}$

$\cancel{④ \int \frac{dx}{x \ln x}}$

$\frac{dt}{dx} = e^{2x} \times 2$



$dt = e^{2x}\,dx \times 2\,dx$

$\cancel{dt - 2\,dx = e^{2x}\,dx}$

$\frac{dt}{2} = e^{2x}\,dx$

$\int \frac{dt}{2} \times \frac{1}{t}$

$\frac{\ln t}{2} + c$

$\frac{\ln(e^{2x}+a^2)}{2} + c$

④ $\int \frac{dx}{x \ln x}$

$t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt \times x = dx$

$\int \frac{dt\,x}{x t}$

$\cancel{x} \times \ln t$

$\boxed{\cancel{x} \ln(\ln x)} + C$

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(5) $\int e^x(\sin e^x)\,dx$

let $t = e^x$

$\frac{dt}{dx} = e^x$

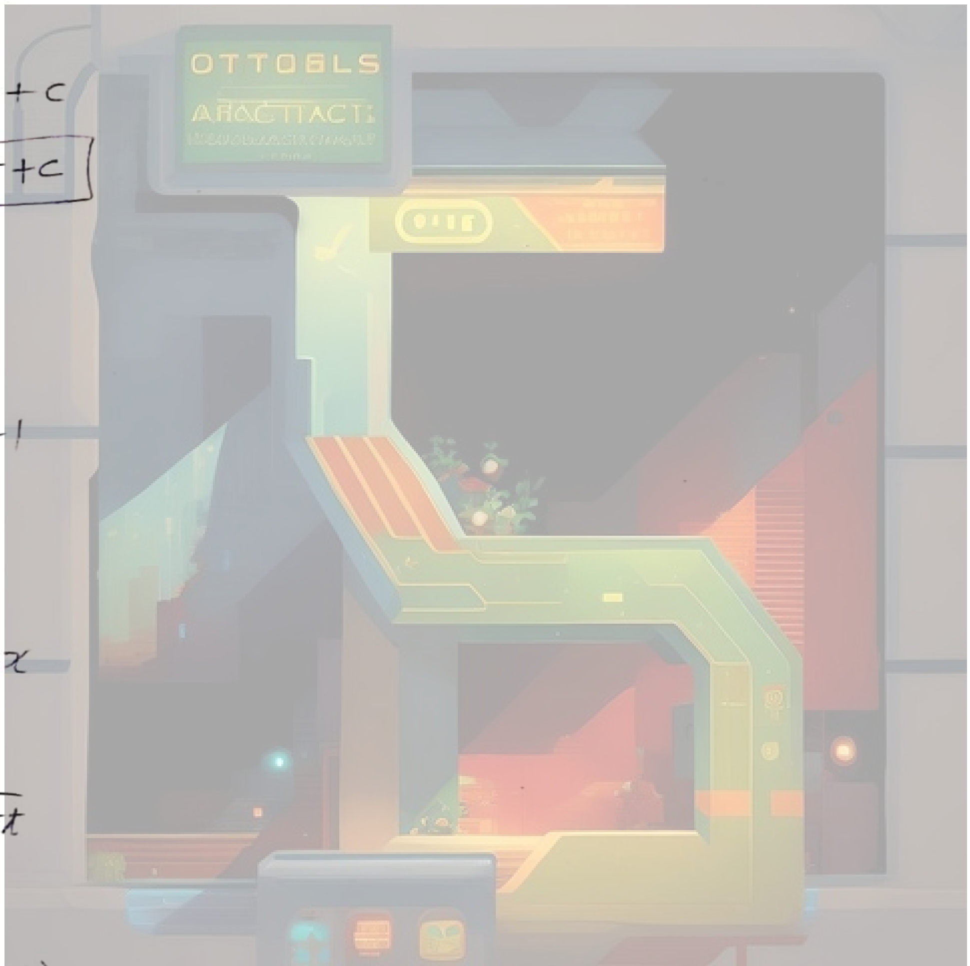
$dt = e^x\,dx$

$\int dt(\sin e^x)$

$\int dt \times \cancel{\cos t}\ \sin t$

$-\cos t + c$

$\boxed{-\cos e^x + c}$



(6) $\int \frac{x\,dx}{x^2+1}$

$t = x^2+1$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x\,dx$

$\int \frac{dt}{2} \times \frac{1}{\cancel{x^2}\,t}$

$\frac{\ln t}{2} + c$

$\boxed{\frac{\ln(x^2+1)}{2} + c}$

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⑦ $\int \frac{(\ln x)^m}{x}\,dx$

$t = \ln x$

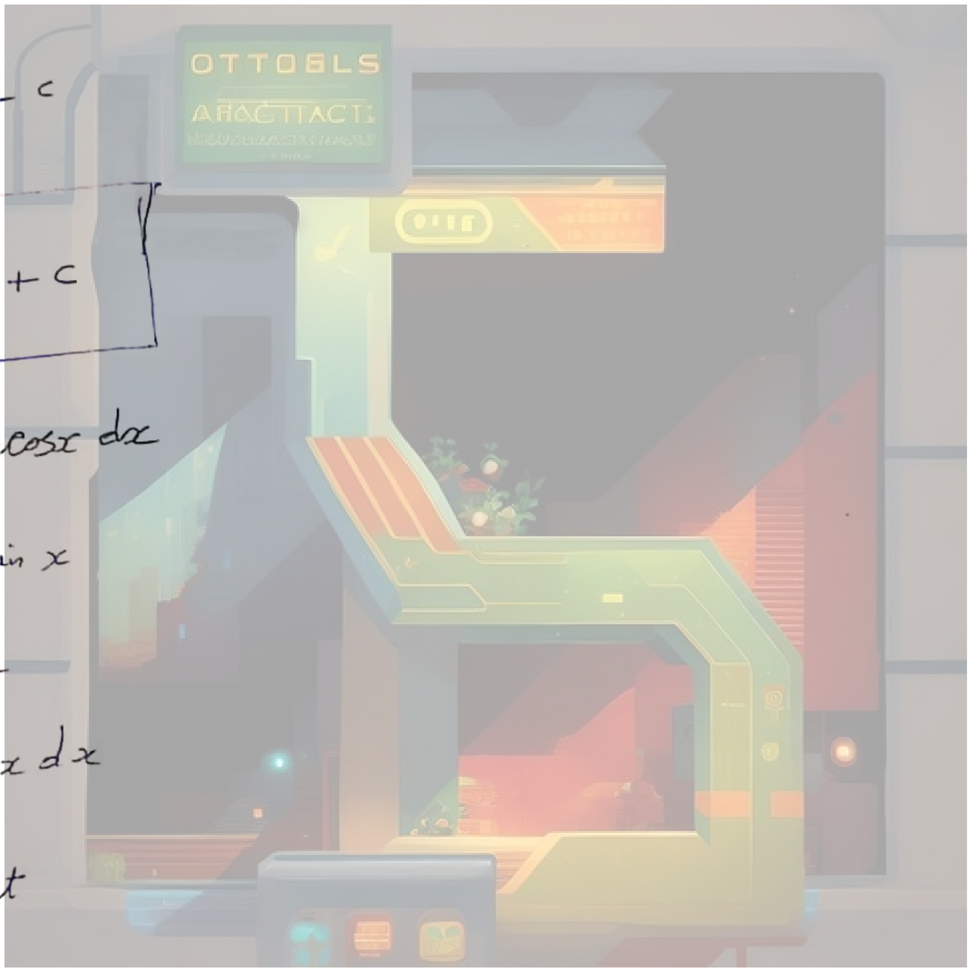
$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{dx}{x}$

$\int \cancel{(\ln x)^m}\, t^m\, dt$

$\frac{t^{m+1}}{m+1} + c$

$\boxed{\frac{(\ln x)^{m+1}}{m+1} + c}$



⑧ $\int e^{\sin x} \cos x\, dx$

let $t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x\, dx$

$\int \cancel{e^{\sin x}}\, e^t\, dt$

e^t

$\boxed{= e^{\sin x} + c}$

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9) $\int e^{x^2} x \, dx$

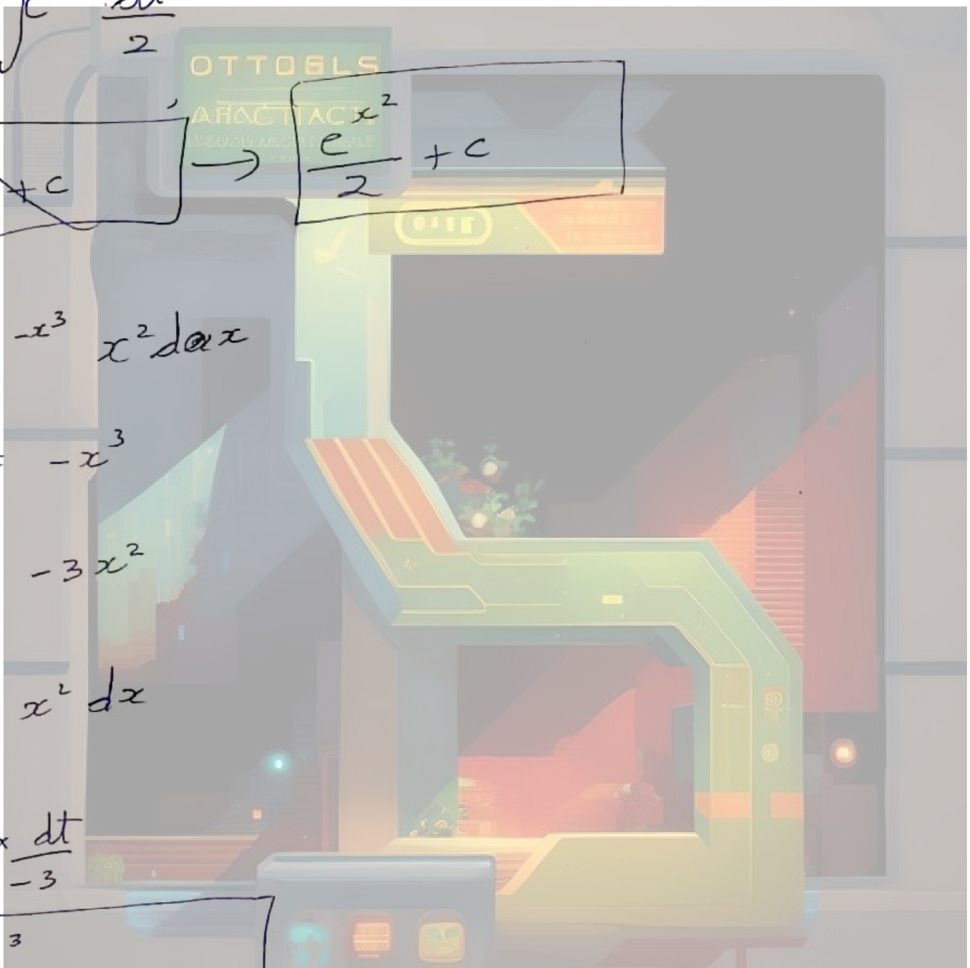
$t = x^2$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x \, dx$

$\int e^t \frac{dt}{2}$

$\cancel{\frac{e^t}{2} + c} \rightarrow \boxed{\frac{e^{x^2}}{2} + c}$



10) $\int e^{-x^3} x^2 \, dx$

$t = -x^3$

$\frac{dt}{dx} = -3x^2$

$\frac{dt}{-3} = x^2 \, dx$

$\int e^t \times \frac{dt}{-3}$

$\boxed{\frac{e^{-x^3}}{-3} + c}$

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(14) $\int \frac{x\,dx}{\sqrt{x^2+1}}$

$t^2 = x^2+1$

$\cancel{2}t\,dt = \cancel{2}x\,dx$

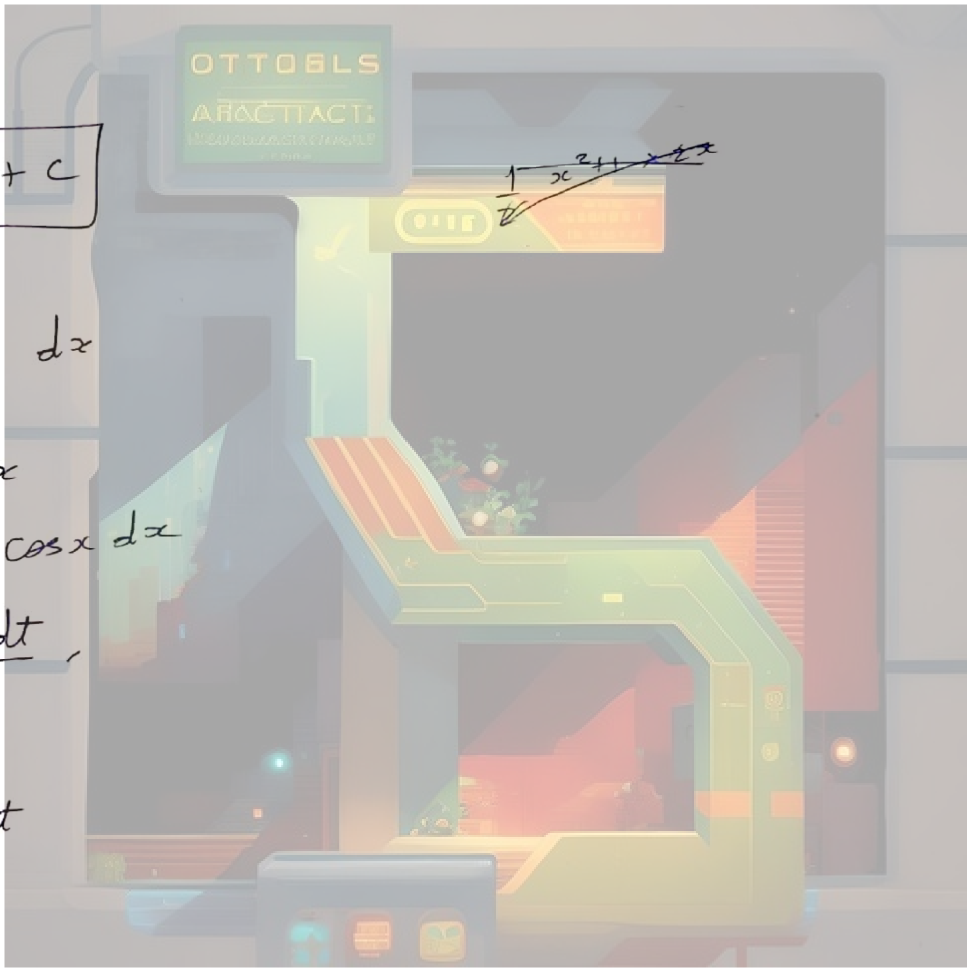
$t\,dt = x\,dx$

$= \int \frac{\cancel{t}\,dt}{\cancel{t}}$

$= \int dt$

$= t$

$\boxed{= \sqrt{x^2+1} + C}$



(15) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

$t^3 = \sin x$

$3t^2\,dt = \cos x\,dx$

$\int \frac{3t^{\cancel{2}}\,dt}{\cancel{t}}$

$3\int t\,dt$

$3 \times \frac{t^2}{2}$

$\boxed{\frac{3}{2}(\sin x)^{2/3} + C}$

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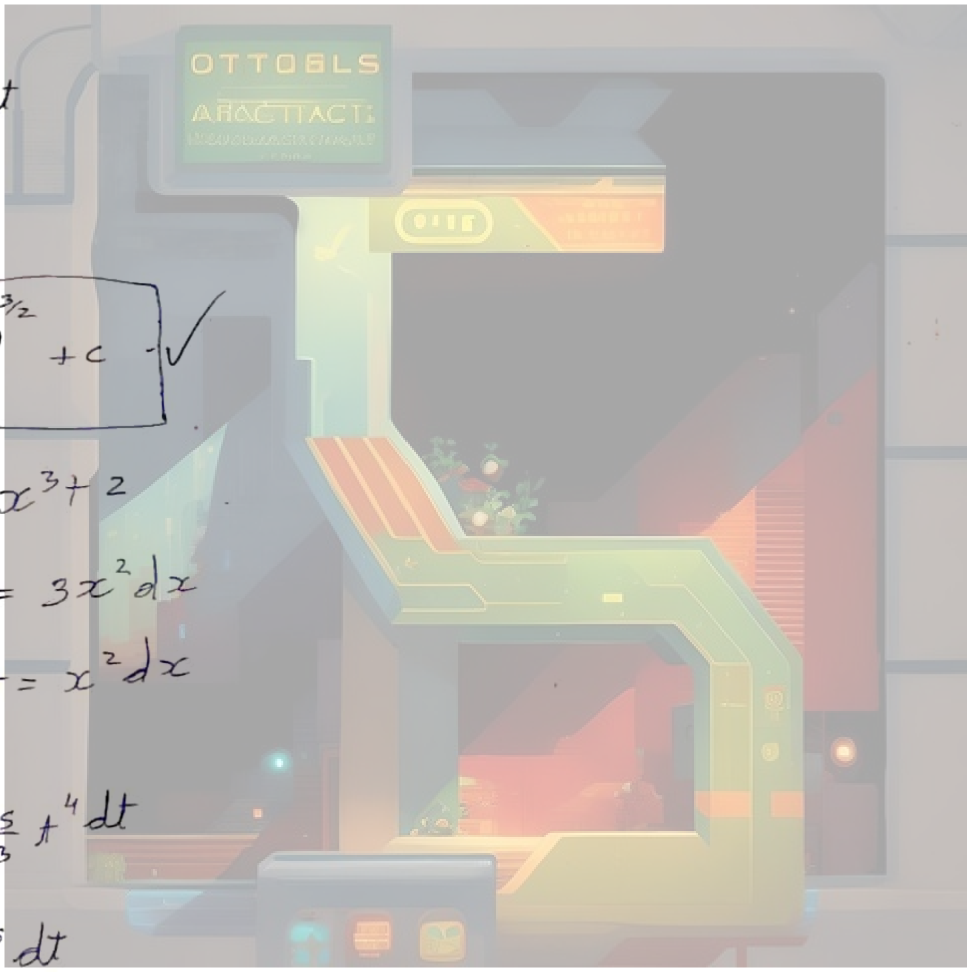
(16) $\int \frac{\sqrt{\ln x}}{x}\, dx$

(17) $\int x^2 \sqrt[5]{x^3+2}\; dx$

(16) $t^2 = \ln x$

$2t\, dt = \frac{1}{x}\, dx$

$\int t \times 2t\, dt$



$2\int t^2\, dt$

$2 \times \frac{t^3}{3}$

$\boxed{\frac{2(\ln x)^{3/2}}{3} + c}$

(17) $t^5 = x^3 + 2$

$5t^4\, dt = 3x^2\, dx$

$\frac{5}{3} t^4\, dt = x^2\, dx$

$\int t \times \frac{5}{3} t^4\, dt$

$\frac{5}{3} \int t^5\, dt$

$\frac{5}{3} \times \frac{t^6}{6}$

$\frac{5}{3} \times \frac{(x^3+2)^{6/5}}{6}$

$\boxed{\frac{5(x^3+2)^{6/5}}{18} + c}$

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(14) $\int \frac{x\,dx}{\sqrt{x^2+1}}$

$t^2 = x^2+1$

$2t\,dt = 2x\,dx$

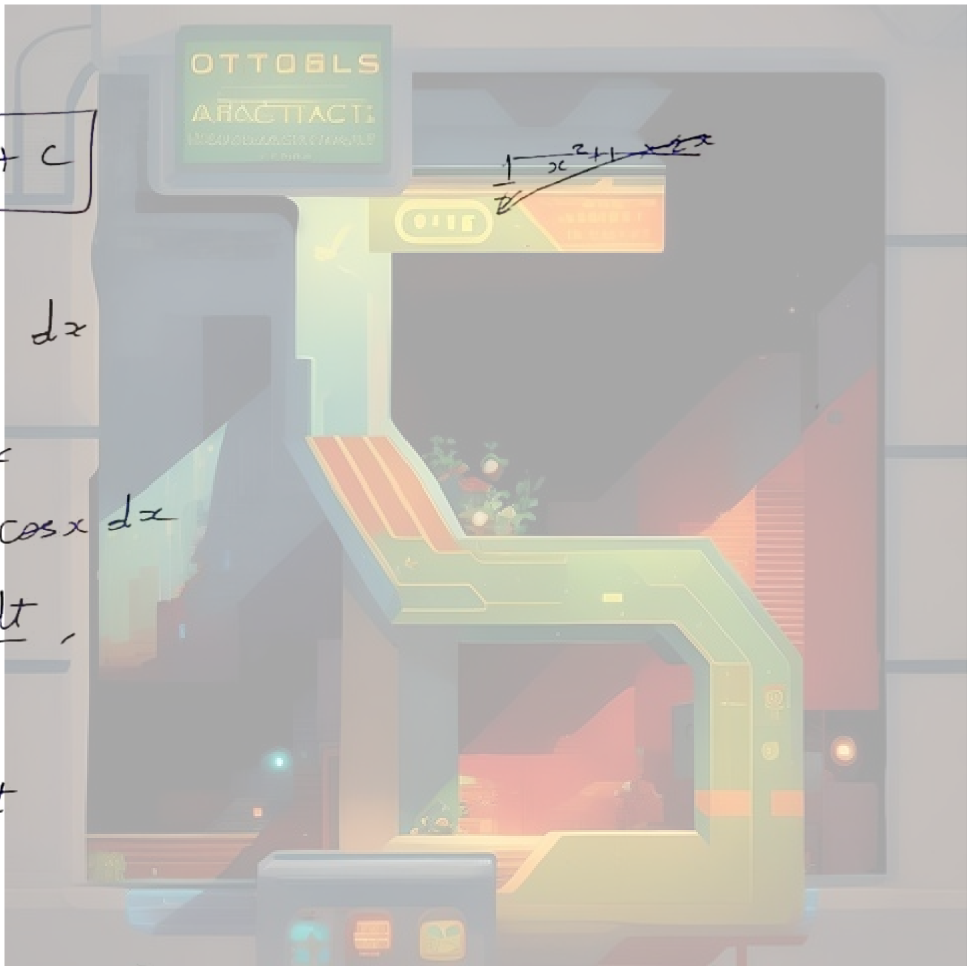
$t\,dt = x\,dx$

$= \int \frac{t\,dt}{t}$

$= \int dt$

$= t$

$\boxed{= \sqrt{x^2+1} + C}$



(15) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

$t^3 = \sin x$

$3t^2\,dt = \cos x\,dx$

$\int \frac{3t^2\,dt}{t}$

$3\int t\,dt$

$3 \times \frac{t^2}{2}$

$\boxed{\frac{3}{2}(\sin x)^{2/3} + C}$

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(16) $\int \frac{\sqrt{\ln x}}{x}\,dx$

(17) $\int x^2 \sqrt[5]{x^3+2}\,dx$

(16) $t^2 = \ln x$

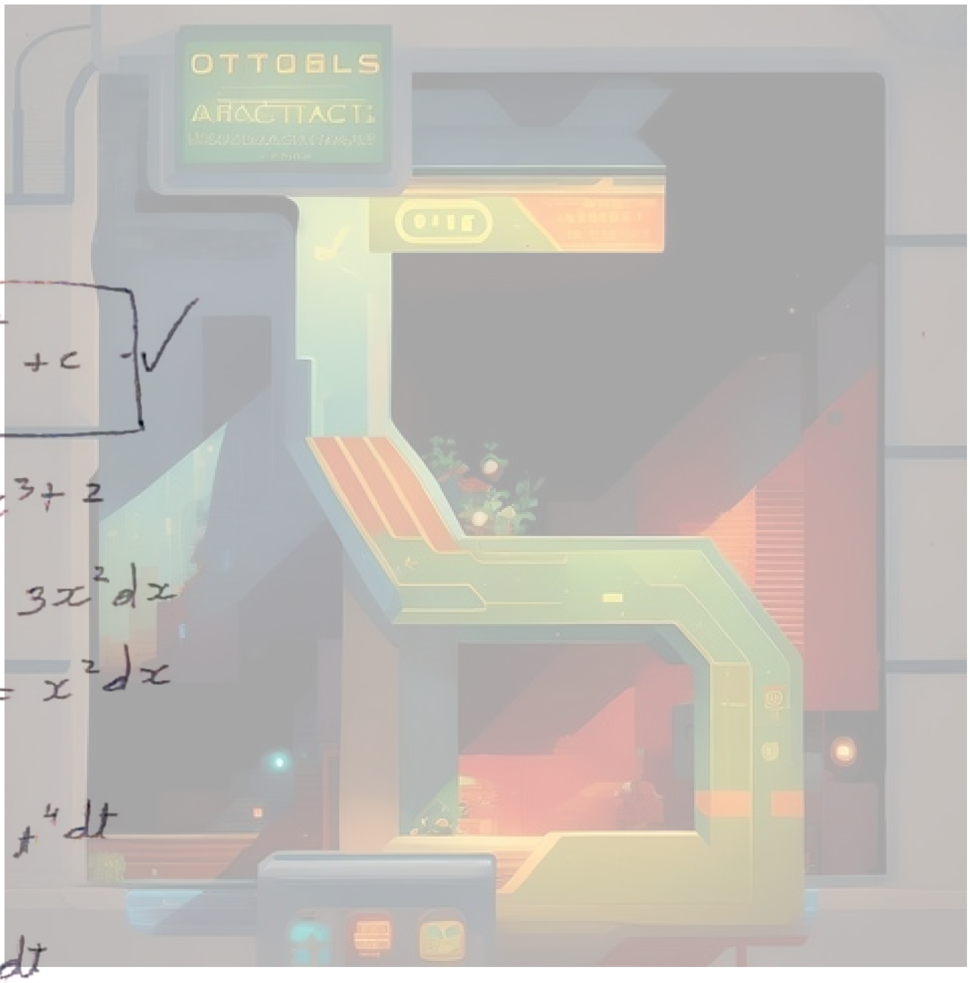
$2t\,dt = \frac{1}{x}\,dx$

$\int t \times 2t\,dt$

$2\int t^2\,dt$

$2 \times \frac{t^3}{3}$

$\boxed{\frac{2(\ln x)^{3/2}}{3} + c}$ ✓



(17) $t^5 = x^3 + 2$

$5t^4\,dt = 3x^2\,dx$

$\frac{5}{3}t^4\,dt = x^2\,dx$

$\int t \times \frac{5}{3}t^4\,dt$

$\frac{5}{3}\int t^5\,dt$

$\frac{5}{3} \times \frac{t^6}{6}$

$\frac{5}{3} \times \frac{(x^3+2)^{6/5}}{6}$

$\boxed{\frac{5(x^3+2)^{6/5}}{18} + c}$ ✓

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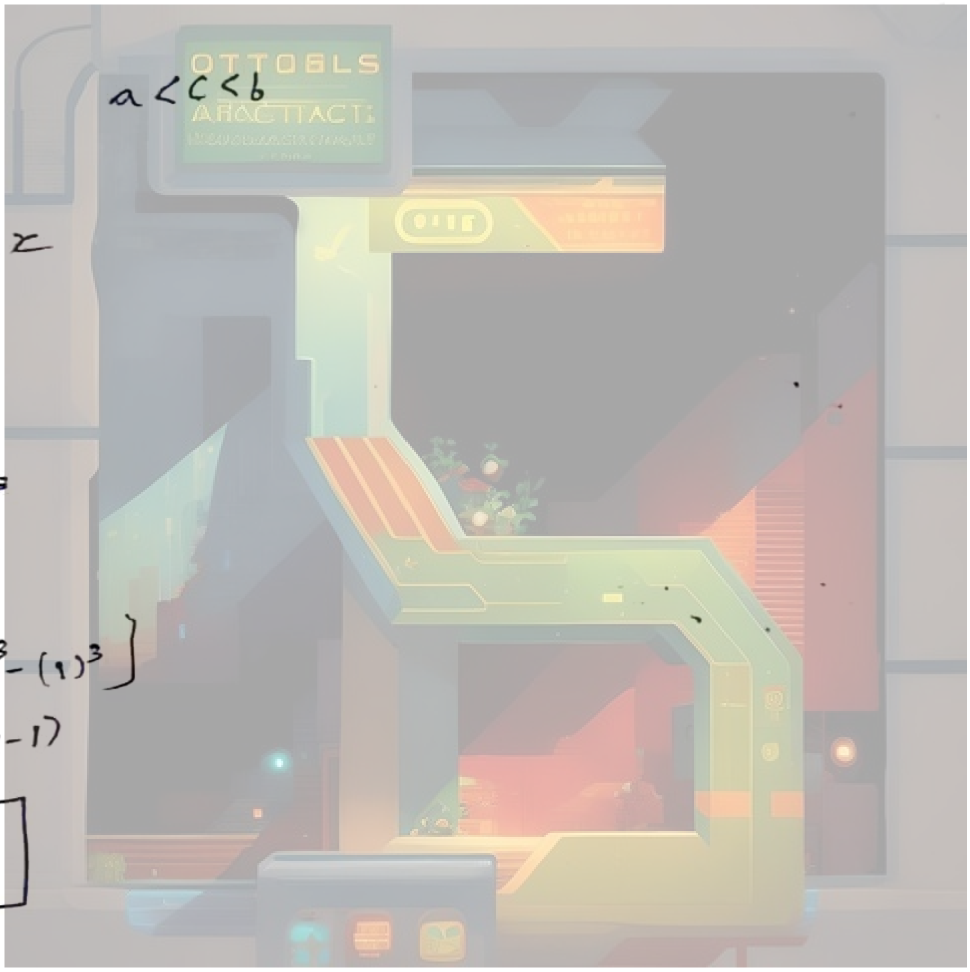
Definite Integral

~~से~~ a से b

$$I = \int_a^b f(x)\,dx = \Big[g(x)\Big]_a^b = g(b) - g(a)$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



$a < c < b$

① $\int_1^3 x^2\,dx$

$\left[\frac{x^3}{3}\right]_1^3$

$\frac{1}{3}\left[x^3\right]_1^3$

$\frac{1}{3}\left[(3)^3 - (1)^3\right]$

$\frac{1}{3}(27-1)$

$\boxed{\frac{26}{3}}$

②

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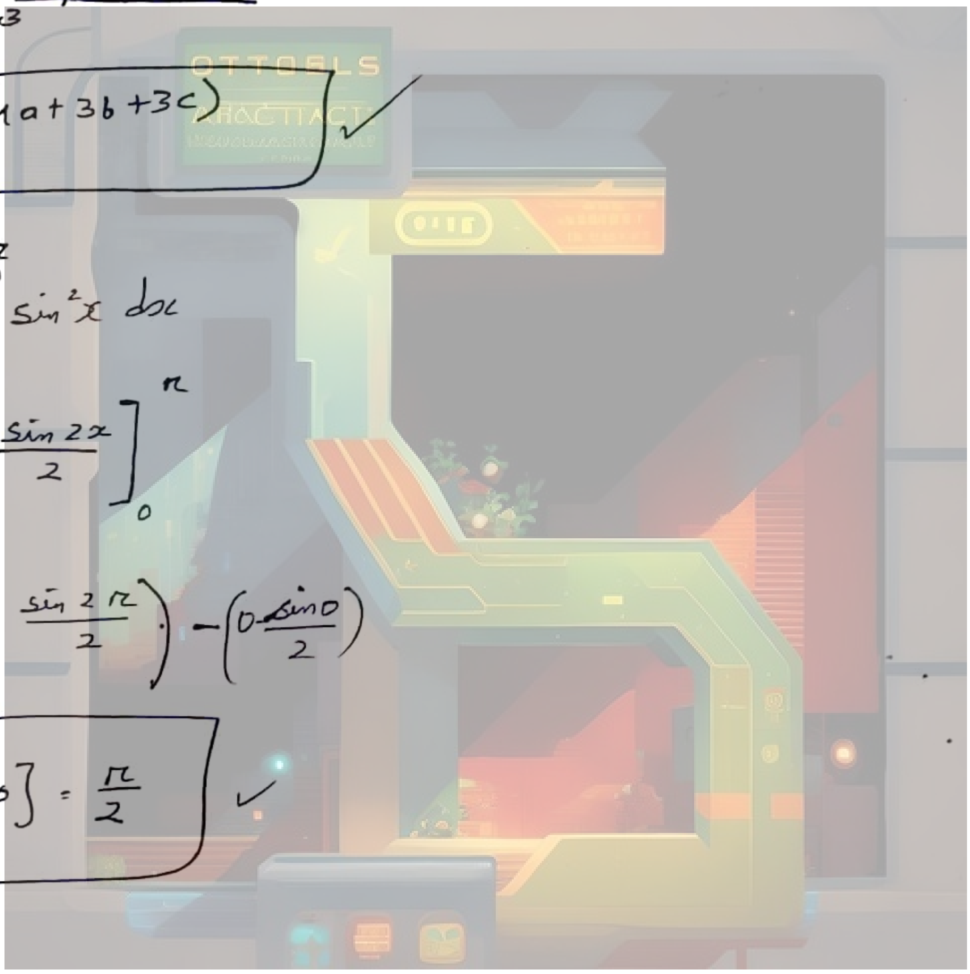
(2) $I = \int_0^2 (ax^2+bx+c)\,dx$

$$\left[\frac{a}{3}x^3+\frac{b}{2}x^2+cx\right]_0^2$$

$$\left(\frac{8a}{3}+\frac{4b}{2}+2c\right)-\left(\cancel{\frac{a}{3}}+\cancel{\frac{b}{2}}\,0+0+0\right)$$

$$\frac{8a+\overset{6}{\cancel{2}}b+\overset{6}{\cancel{2}}c}{3}$$

$$\boxed{\frac{2}{3}(4a+3b+3c)}$$ ✓



(3) $I = \int_0^{\pi} \sin^2 x\,dx$

$$\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\pi}$$

$$\frac{1}{2}\left[\left(\pi-\frac{\sin 2\pi}{2}\right)-\left(0-\frac{\sin 0}{2}\right)\right]$$

$$\boxed{\frac{1}{2}\left[\pi-0\right]=\frac{\pi}{2}}$$ ✓

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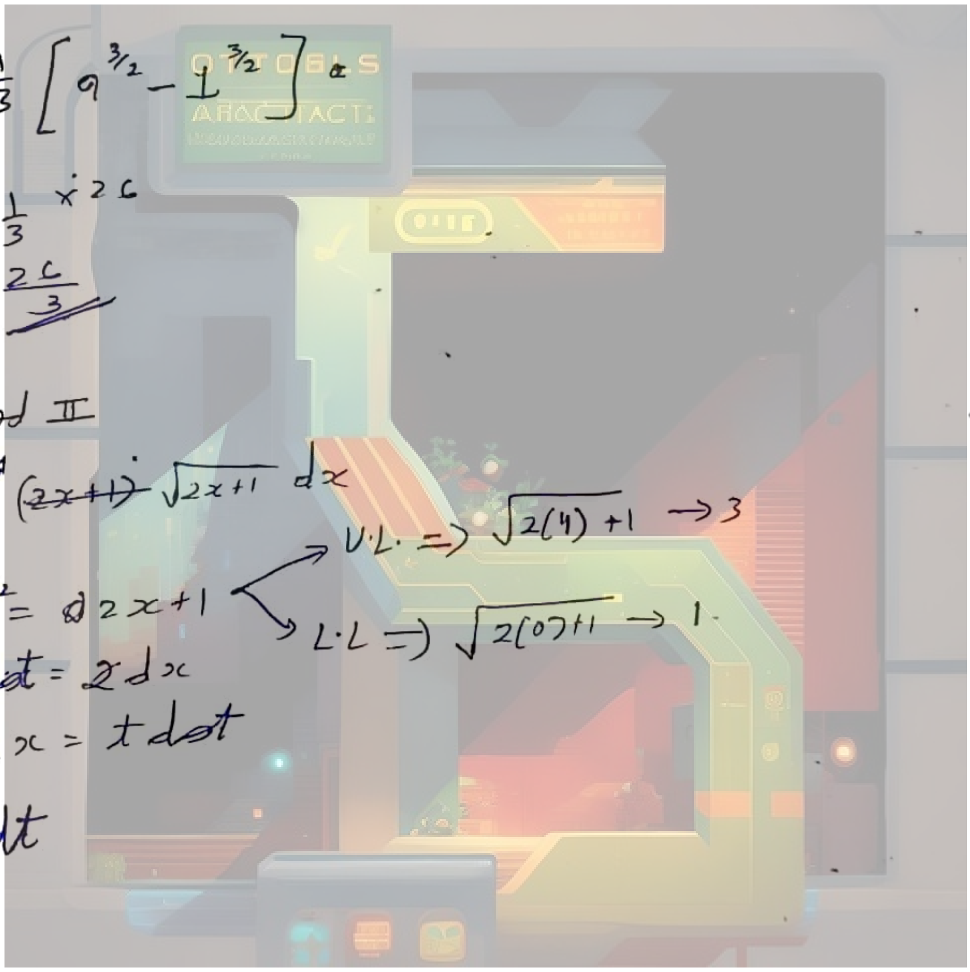
(4) $\int_0^4 \sqrt{2x+1}\, dx$

Method - I

$$I = \int_0^4 (2x+1)^{1/2}\, dx$$

$$= \left[\frac{(2x+1)^{3/2}}{2(\frac{1}{2}+1)}\right]_0^4$$



$$= \frac{1}{3}\left[9^{3/2} - 1^{3/2}\right]$$

$$= \frac{1}{3} \times 26$$

$$= \frac{26}{3}$$

Method II

$$I = \int_0^4 \cancel{(2x+1)} \sqrt{2x+1}\, dx$$

U.L. $\Rightarrow \sqrt{2(4)+1} \to 3$

L.L $\Rightarrow \sqrt{2(0)+1} \to 1$

$$t^2 = 2x+1$$

$$2t\,dt = 2\,dx$$

$$dx = t\,dt$$

$$\int_1^3 t^2\, dt$$

$$\left[\frac{t^3}{3}\right]_1^3$$

$$\frac{1}{3}\left[t^3\right]_1^3$$

$$\frac{1}{3}(27-1)$$

$$\frac{26}{3}$$

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(5) $\int_0^1 x(x^2+1)^3 dx$

$t = x^2+1$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x\,dx$

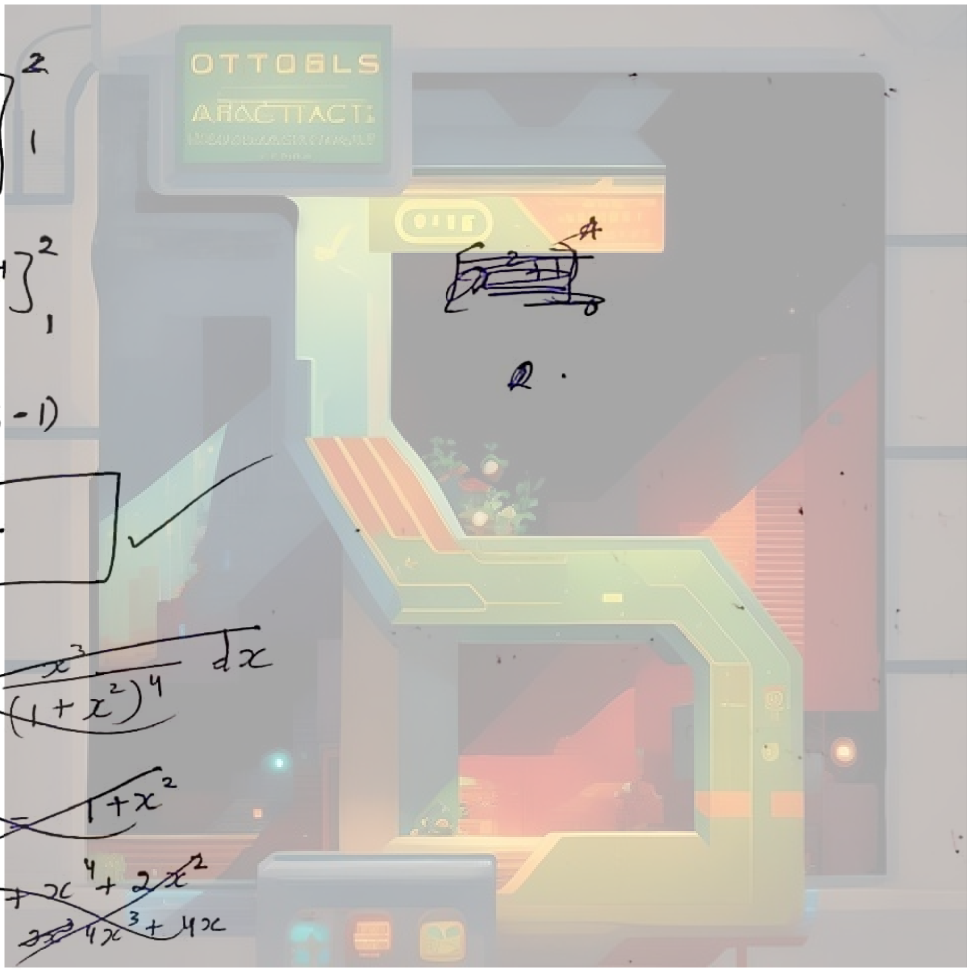
$\int_1^2 \frac{t^3}{2}\, dt$

$\left[\frac{t^4}{2\times 4}\right]_1^2$

$\frac{1}{8}\left[t^4\right]_1^2$

$\frac{1}{8}(16-1)$

$\boxed{\frac{15}{8}}$



(6) $\cancel{\int_0^1 \frac{x^3}{(1+x^2)^4}\,dx}$

$\cancel{t = 1+x^2}$

$T + x^4 + 2x^2$

$\cancel{4x^3 + 4x}$

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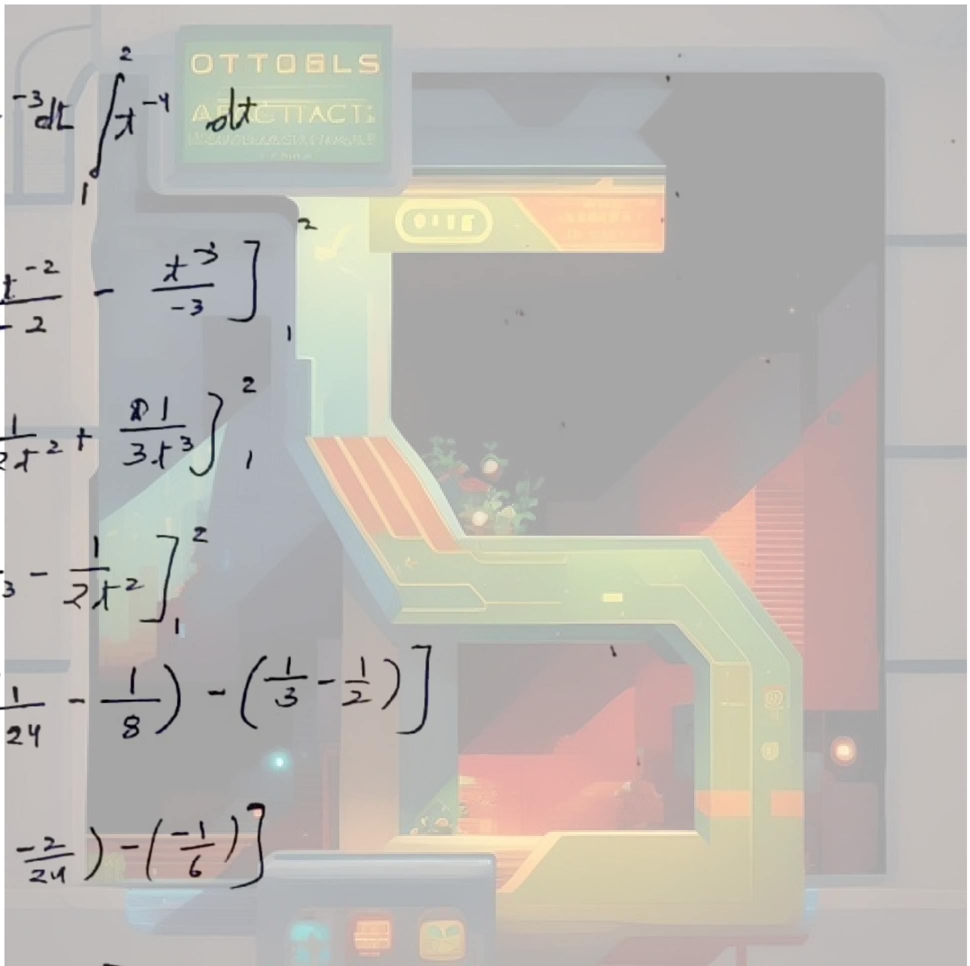
(6) $\displaystyle\int_0^1 \frac{x^3}{(x^2+1)^4}\,dx$

$t = 1 + x^2$

$x^2 = t - 1$

$\frac{dt}{2} = x\,dx$

$$\int_1^2 \frac{(t-1)\frac{dt}{2}}{t^4}$$



$$\frac{1}{2}\int_1^2 t^{-3}\,dt \int_1^2 t^{-4}\,dt$$

$$\frac{1}{2}\left[\frac{t^{-2}}{-2} - \frac{t^{-3}}{-3}\right]_1^2$$

$$\frac{1}{2}\left[-\frac{1}{2t^2} + \frac{1}{3t^3}\right]_1^2$$

$$\frac{1}{2}\left[\frac{1}{3t^3} - \frac{1}{2t^2}\right]_1^2$$

$$\frac{1}{2}\left[\left(\frac{1}{24} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$

$$\frac{1}{2}\left[\left(\frac{-2}{24}\right) - \left(\frac{-1}{6}\right)\right]$$

$$\frac{1}{2}\left[-\frac{1}{12} + \frac{1}{6}\right]$$

$$\frac{1}{2}\left[\frac{2}{12} - \frac{1}{12}\right]$$

$$\frac{1}{2}\left(\frac{1}{12}\right) = \boxed{\frac{1}{24}}$$

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Average value of a continuous function in an Interval:-

$y = f(x)$

→ Average value of y in an interval $a \leq x \leq b$ is given by

$$\langle y \rangle \text{(average value of } y) = \frac{\int_a^b y\, dx}{\int_a^b dx}$$

Q find average value from $x = 0$ to $x = \pi$ for $\sin x$

$$\langle \sin x \rangle = \frac{\int_0^\pi \sin x\, dx}{\int_0^\pi dx}$$

$$= \frac{[-\cos x]_0^\pi}{[x]_0^\pi}$$

$$= \frac{-[\cos \pi - \cos 0]}{\pi - 0}$$

$$= \frac{-(-1-1)}{\pi} = \frac{2}{\pi}$$

Q Determine average value of $y = x+5$ in the interval $0 \leq x \leq 1$

$y = x+5$

$$\langle y \rangle = \frac{\int_0^1 x+5\, dx}{\int_0^1 dx}$$

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$$= \frac{\left[\frac{x^2}{2} + 5x\right]_0^1}{[x]_0^1}$$

$$= \frac{\left(\frac{1}{2} + 5\right) - (0)}{1 - 0}$$

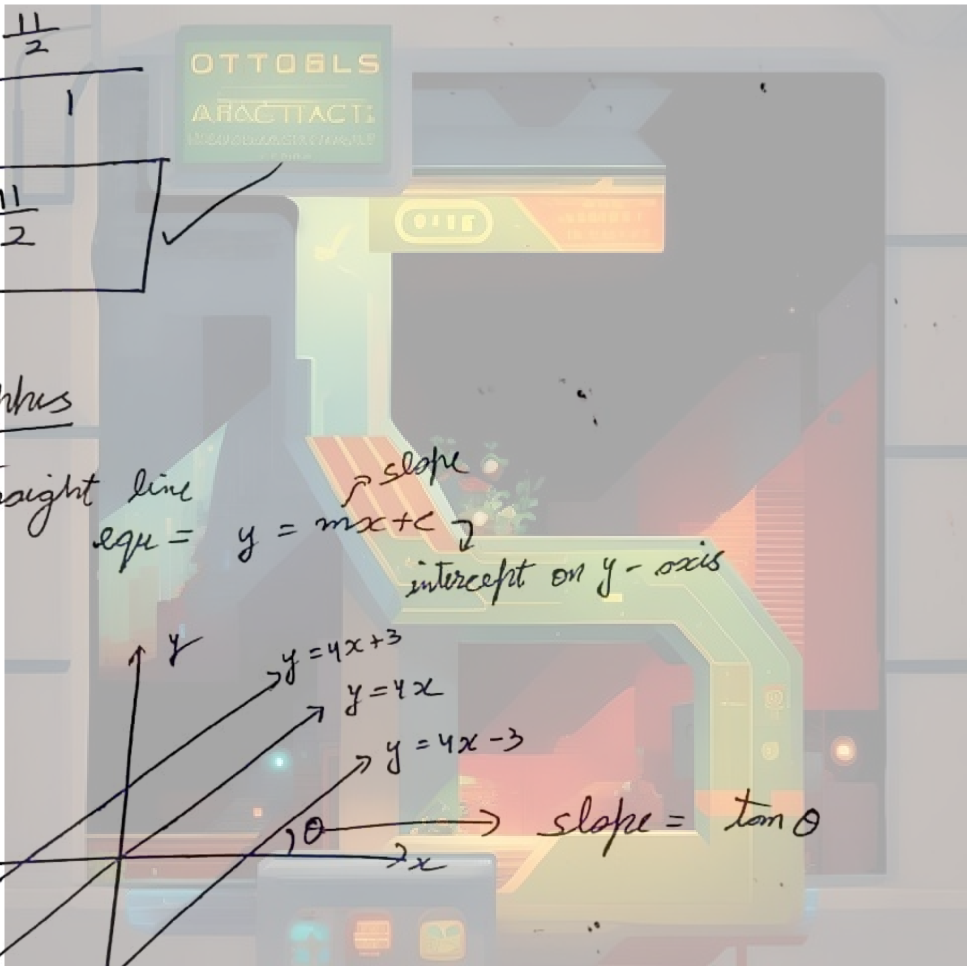
$$= \frac{\frac{11}{2}}{1}$$

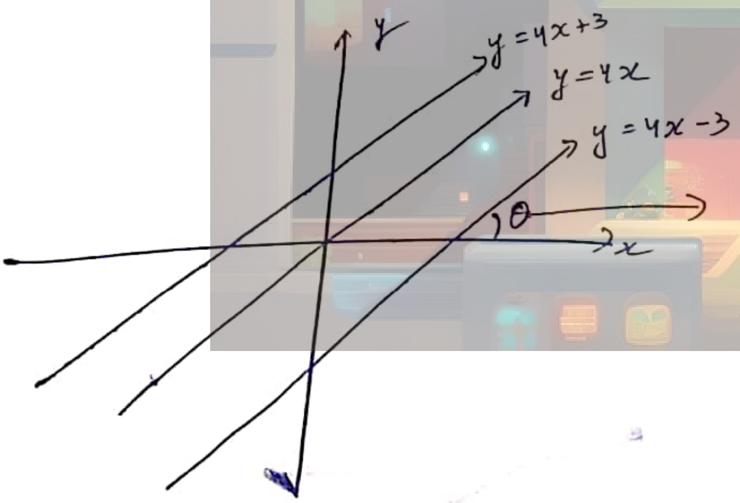
$$\boxed{\langle y \rangle = \frac{11}{2}}$$

Graphs

1. Straight line

eqn = $y = \overbrace{m}^{\text{slope}}x + \underbrace{c}_{\text{intercept on } y\text{-axis}}$





slope = $\tan\theta$

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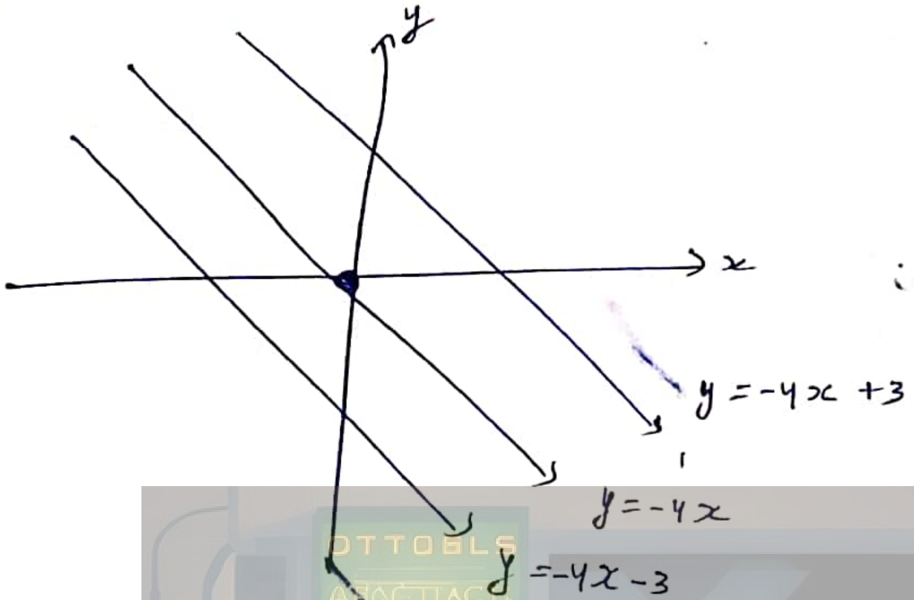
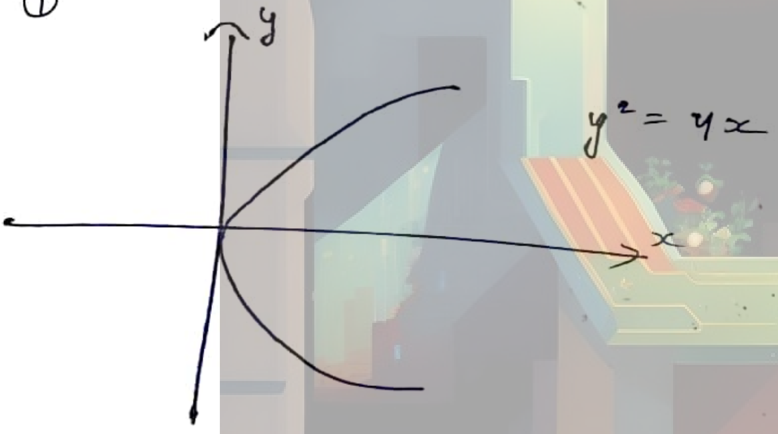


Quadratic ~~Eqn~~ Equation Graph

①



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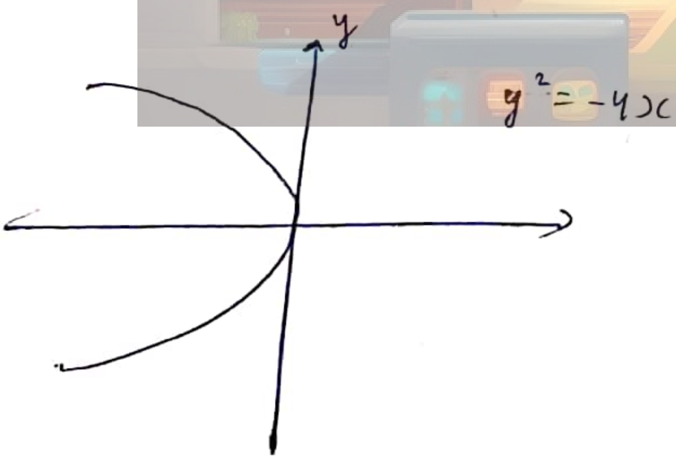

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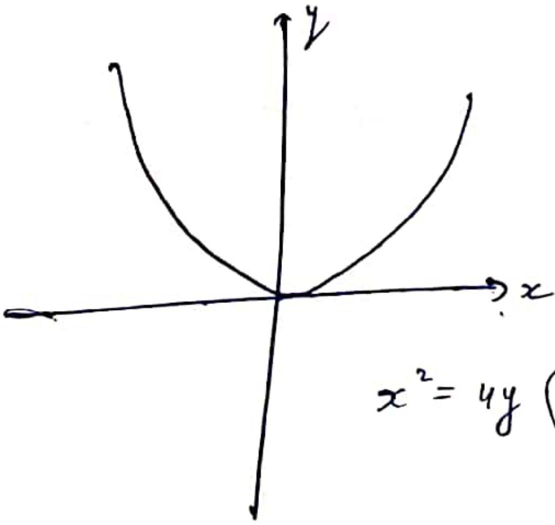
3



$x^2 = 4y$ (mouth of parabola opening upwards

4

$x^2 = -4y$

(mouth of parabola opening downwards)

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Kinematics (Motion in one Dimension)

Kinematics - The branch of theoretical mechanics which deals with study of motion of rigid bodies (shape cannot be changed) without taking into account the cause of motion (i.e. forces acting on them) is called Kinematics.

point object – A particle in its strict sense means an object without dimensions (no size, only a point)

Translatory motion - when their is no change in the orientation of a object while moving. (every point travles same distance)

rectilinear (straight line)

curvilinear (~~curvilinear~~) (curved path)

* Position of a particle in space is determined relative to some fixed point, position depends on position of observer.
  ↳ refrence point.

* The path followed by a point object during its motion is called its trajectory.

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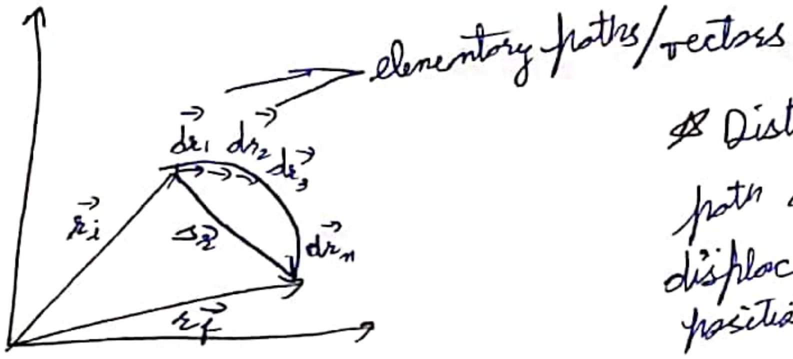
Distance & Displacement



Distance is the length of the path actually traversed ~~which~~ while displacement is the change in position vector.

Displacement; $\Delta\vec{r} = \vec{r_f} - \vec{r_i}$

or

$$\boxed{\Delta\vec{r} = \int \vec{dr}}$$

adding all elementary vectors

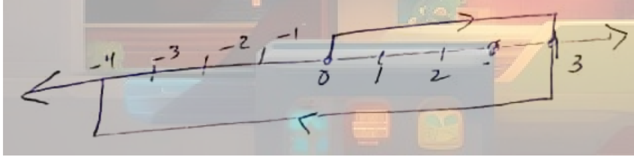
$$\boxed{\text{Distance} = \int |\vec{dr}|}$$

adding of all magnitudes of elementary vectors.

→ Distance is multi valued ~~valued~~ function while displacement is a single valued function.

→ Distance ⩾ Displacement → can be +ve, −ve or zero
↳ always positive

eg.



Distance $= 3+3+4 = 10$

Displacement $\Delta x = x_f - x_i = -4 - 0 = -4$

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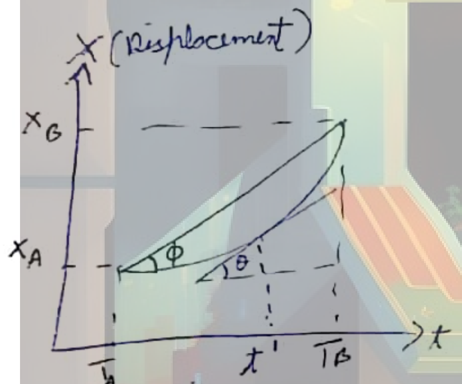
Speed & Velocity -

Average Speed - ~~Total~~ $\dfrac{\text{Total Distance}}{\text{Total Time}}$

Average Velocity $= \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{\Delta \vec{x}}{\Delta t}$

Instantaneous velocity - ~~velocity~~ Rate of change of position at given time instance.

$$\boxed{\text{Velocity} = \frac{dx}{dt}}$$



instantaneous velocity at t_1 ~~= slope~~ slope

$$\boxed{v = \frac{dx}{dt} = \tan\theta}$$

Average velocity = slope of chord

$$\text{Average velocity} = \boxed{\frac{x_B - x_A}{t_B - t_A} = \tan\phi}$$

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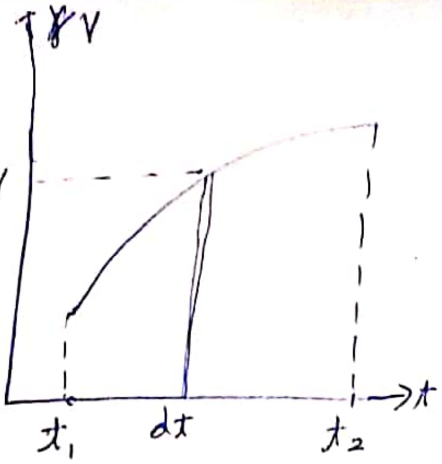

~~Velocity under~~

area under velocity graph = displacement

Vdt = area under small time instance

area under curve is Integration

$$\int_{t_1}^{t_2} v\,dt = \text{area under whole graph (by adding all small time instances)}$$

$$\boxed{\begin{aligned} v &= \frac{dx}{dt} \\ x &= \int dx = \int_{t_1}^{t_2} v\,dt \end{aligned}}$$

x is displacement not distance

Ex.

Distance = I + II + III

Displacement = I − II + III

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~~Q~~ $\cancel{x} = .$

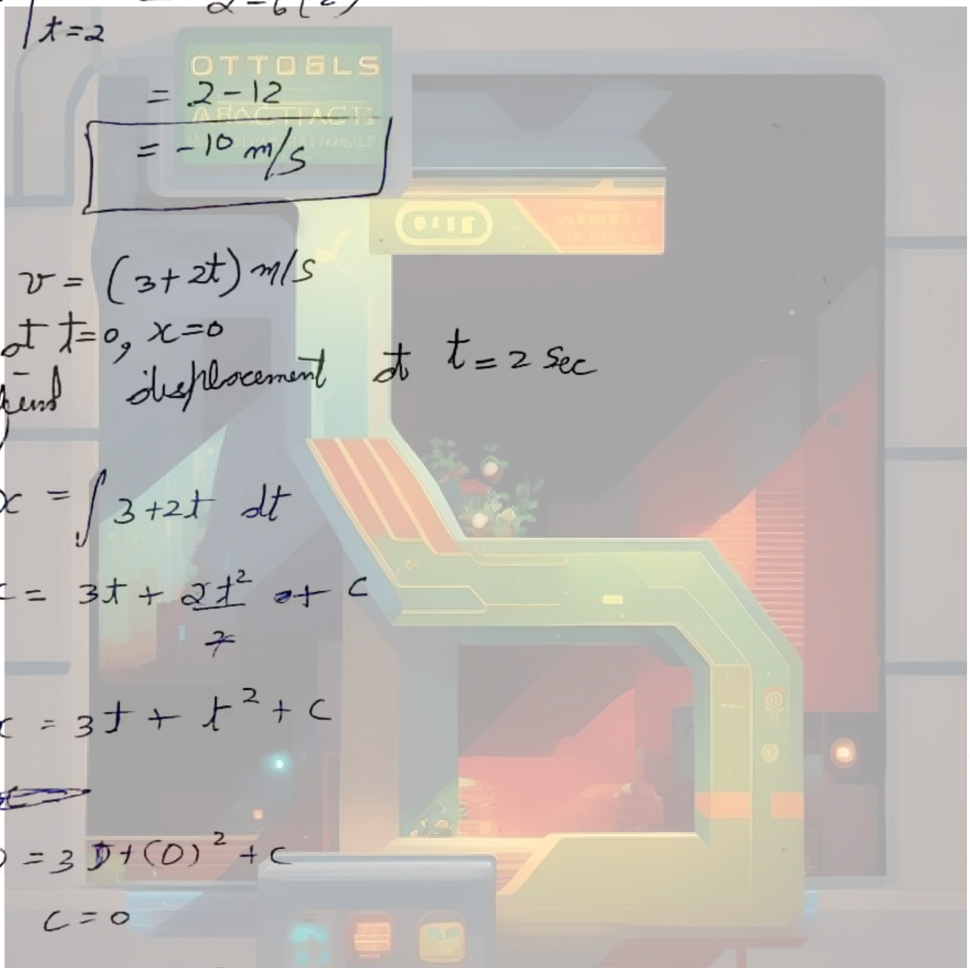
eg 1 $\quad x = 2t - 3t^2$, find velocity at $t = 2s$

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}(2t - 3t^2)$$

$$v = 2 - 6t$$

$$v\Big|_{t=2} = 2 - 6(2)$$



$$= 2 - 12$$

$$\boxed{= -10\ m/s}$$

eg 2. $\quad v = (3 + 2t)\ m/s$

→ at $t = 0$, $x = 0$

find displacement at $t = 2$ sec

$$x = \int 3 + 2t\ dt$$

$$x = 3t + \frac{\cancel{2}t^2}{\cancel{2}} + c$$

$$x = 3t + t^2 + c$$

$\cancel{x}$

$$0 = 3(0) + (0)^2 + c$$

$$c = 0$$

$$x = 3(2) + (2)^2$$

$$x = 6 + 4$$

$$\boxed{x = 10\ m}$$

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Note:-

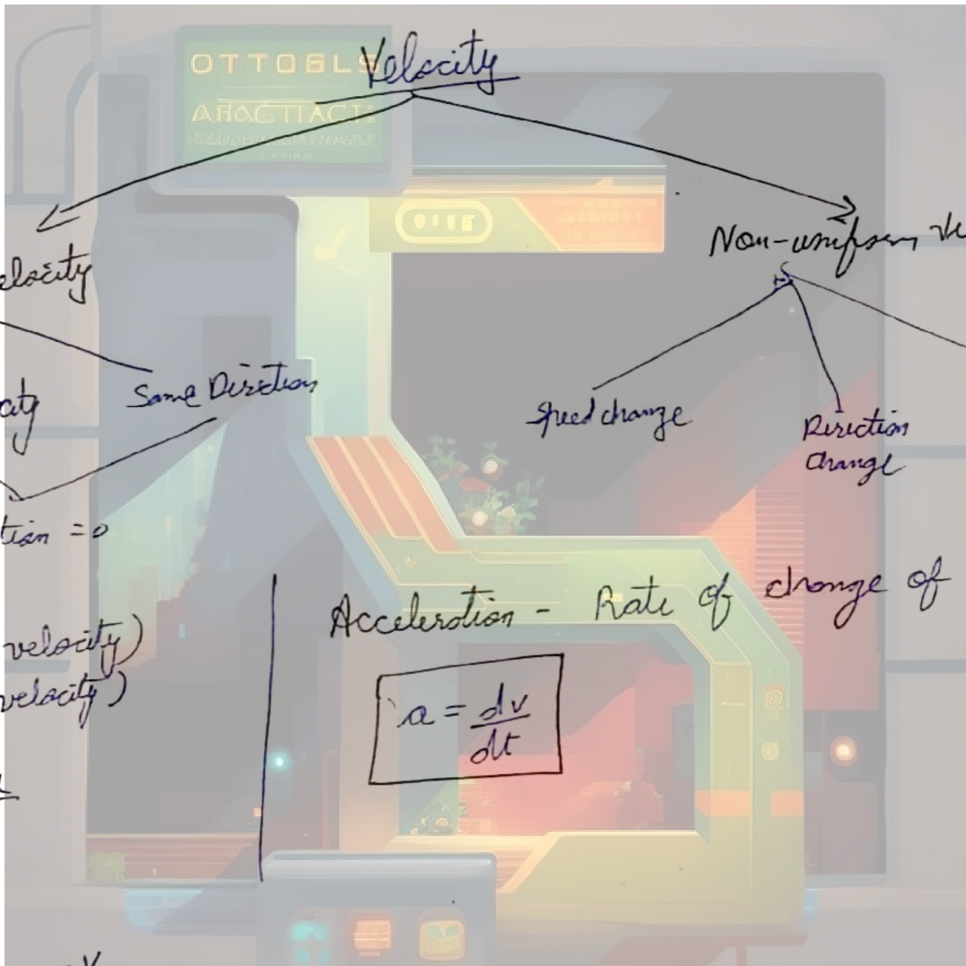
① Slope of Displacement-time graph gives velocity.

$$v = \frac{dx}{dt}$$

② Area under v-t graph with proper sign gives displacement & without sign gives distance

$$x = \int v\,dt$$

Acceleration:-



Velocity

Uniform Velocity

Constant velocity

Same Direction

acceleration = 0

Non-uniform Velocity

Speed change

Direction change

Both change

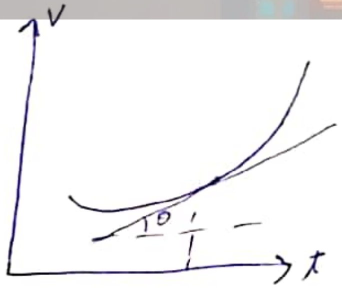
u (initial velocity)

v (final velocity)

$$a = \frac{v-u}{t}$$

Acceleration - Rate of change of velocity

$$\boxed{a = \frac{dv}{dt}}$$



* slope of v-t graph gives acceleration.

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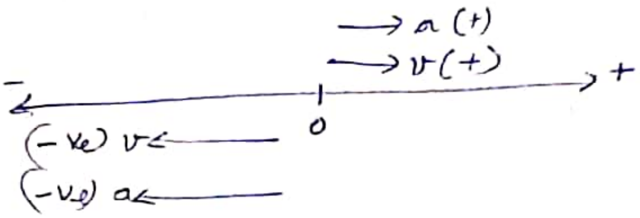
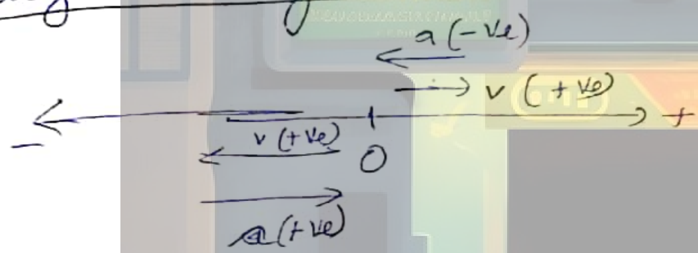
→ Body is speeding up



When velocity & acceleration are of same sign (either ~~of~~ ~~them~~ be ⊕ve or ⊖ve), speed of body will increase.

→ Body is slowing down (Retardation / deceleration)



when $\vec{v}$ & $\vec{a}$ are in opp sign ~~of~~, speed of ~~a~~ body decreases.

→ Velocity ⊥ Acceleration.

speed = constant
direction = change
resulting path — circle

Uniform circular motion

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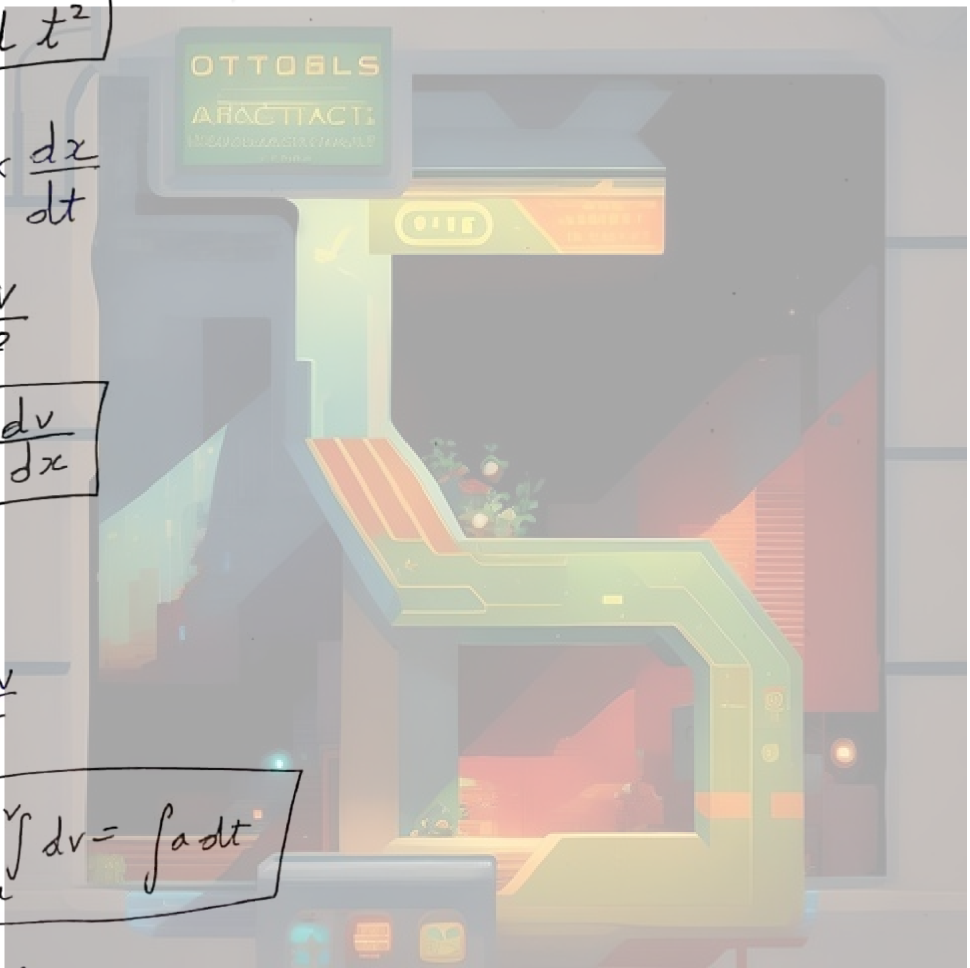
$$\boxed{a = \frac{dv}{dt}}$$

$$\because \; v = \frac{dx}{dt}$$

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$a = \frac{d^2x}{dt^2}$$

$$\boxed{a = \frac{d^2x}{dt^2}}$$



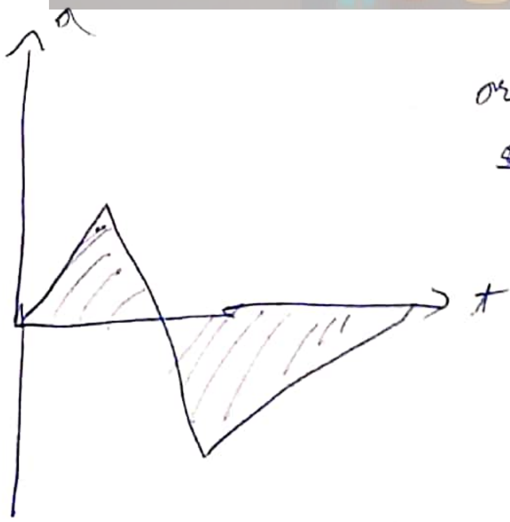
$$a = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$\cancel{a = v\frac{dv}{dx}}$$

$$\boxed{a = v\frac{dv}{dx}}$$

$$a = \frac{dv}{dt}$$

$$\boxed{\Delta v = \int_u^v dv = \int a\,dt}$$



area under a-t graph = velocity

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S-t Graph → Slope → v , $v = \frac{dx}{dt}$

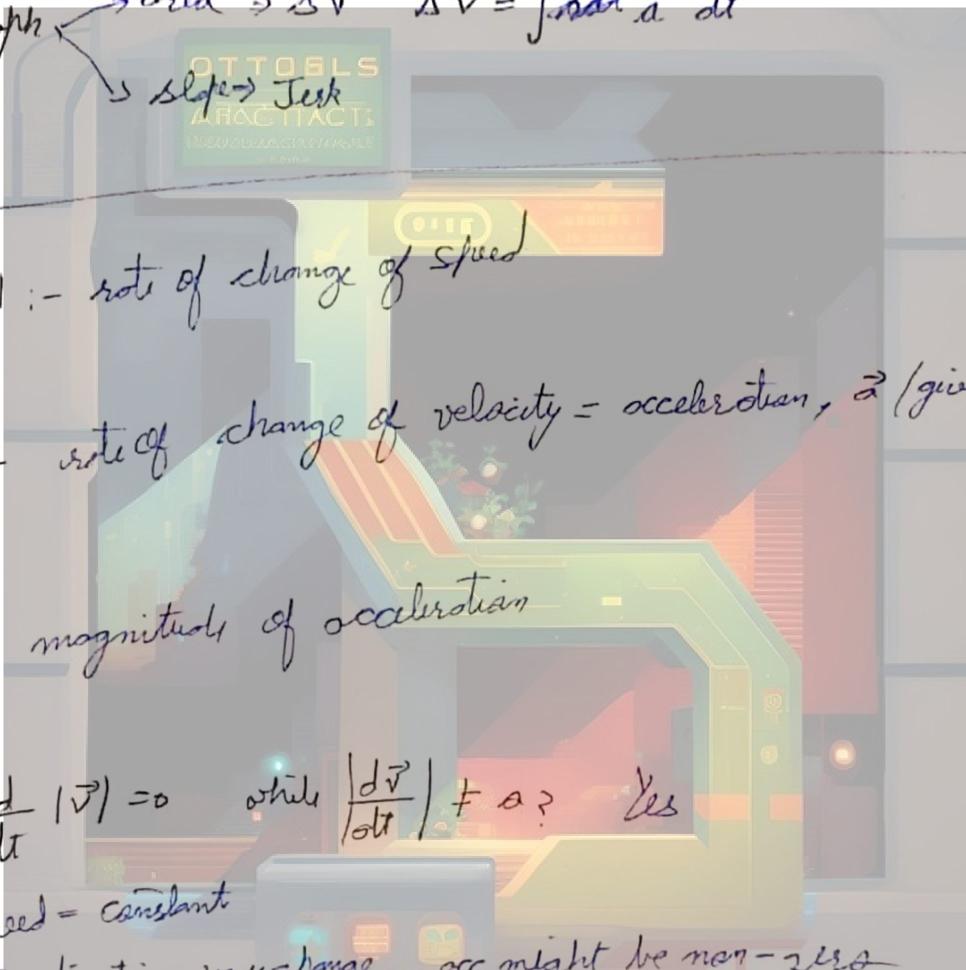
V-t Graph → Slope → a , $a = \frac{dv}{dt}$, $a = v\frac{dv}{dx}$, $a = \frac{d^2x}{dt^2}$

→ area → Displacement , $x = \int v\,dt$

→ Distance

a-t Graph → area → Δv $\Delta v = \int a\,dt$

→ slope → Jerk



$\frac{d}{dt}|\vec{v}|$:- rate of change of speed

$\frac{d\vec{v}}{dt}$:- rate of change of velocity = acceleration, $\vec{a}$ (gives vector)

$\left|\frac{d\vec{v}}{dt}\right|$:- magnitude of acceleration

Q Can $\frac{d}{dt}|\vec{v}| = 0$ while $\left|\frac{d\vec{v}}{dt}\right| \neq 0$? Yes

Speed = constant

Its direction may change, acc might be non-zero

Q Can $\frac{d}{dt}|\vec{v}| \neq 0$ while $\left|\frac{d\vec{v}}{dt}\right| = 0$? No

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126

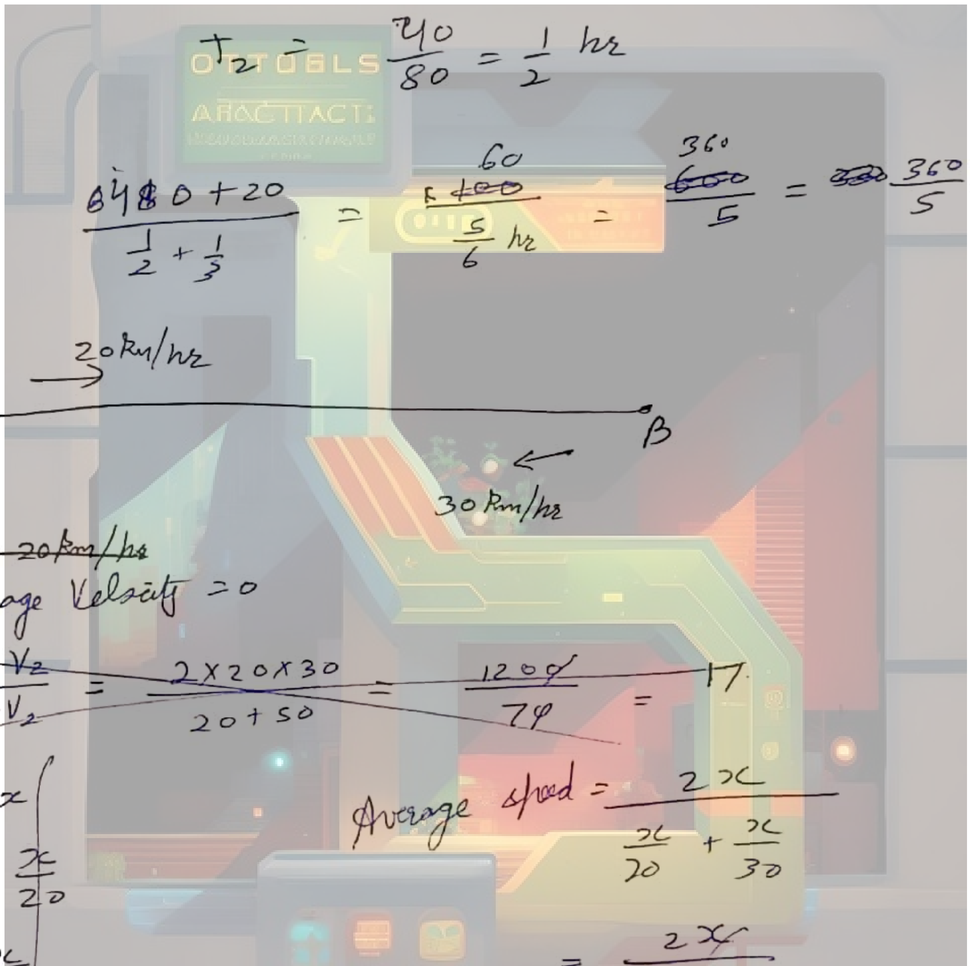
Q1. A train travels a distance of 20 km with a uniform speed of 60 km/hr. It travels another distance of 40 km with a uniform speed of 80 km/hr. Calculate average speed of train.

$$\text{Average speed} = \cancel{80+60}\ \cancel{\frac{V_1+V_2}{2}}$$

$$= \frac{V_1+V_2}{2}$$

$$\cancel{= 70}$$

$$T_1 = \cancel{200}\ \frac{20}{60} = \frac{1}{3}\,hr$$



$$T_2 = \frac{40}{80} = \frac{1}{2}\,hr$$

$$\frac{\cancel{8}4\cancel{8}0+20}{\frac{1}{2}+\frac{1}{3}} = \frac{\overset{60}{\cancel{400}}}{\frac{5}{6}\,hr} = \frac{\overset{360}{\cancel{600}}}{5} = \cancel{300}\ \frac{360}{5} = \boxed{72\ km/hr}$$

Q2.

20 km/hr →

A —————— B

← 30 km/hr

~~V = 20 km/hr~~

Average Velocity = 0

$$\cancel{\frac{2V_1V_2}{V_1+V_2} = \frac{2\times20\times30}{20+50} = \frac{1200}{70} = 17.}$$

dis = x

$$t_1 = \frac{x}{20}$$

$$T_2 = \frac{x}{30}$$

$$A.\ \text{Speed} = \frac{x+x}{\frac{x}{20}+\frac{x}{30}}$$

$$= \frac{2x}{\frac{50x}{600}}$$

$$= \frac{120\cancel{0}\cancel{x}}{5\cancel{0}\cancel{x}}$$

$$\boxed{= 60\ km/hr}$$

$$\text{Average speed} = \frac{2x}{\frac{x}{20}+\frac{x}{30}}$$

$$= \frac{2\cancel{x}}{\cancel{x}\left(\frac{1}{20}+\frac{1}{30}\right)}$$

$$= \frac{2}{\frac{3+2}{60}}$$

$$= \frac{120}{5}$$

$$\boxed{= 24\ km/hr}$$

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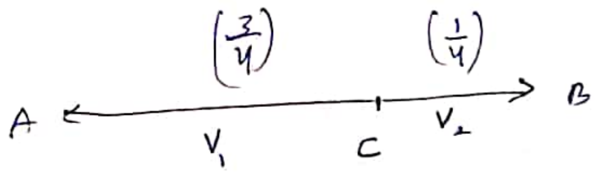
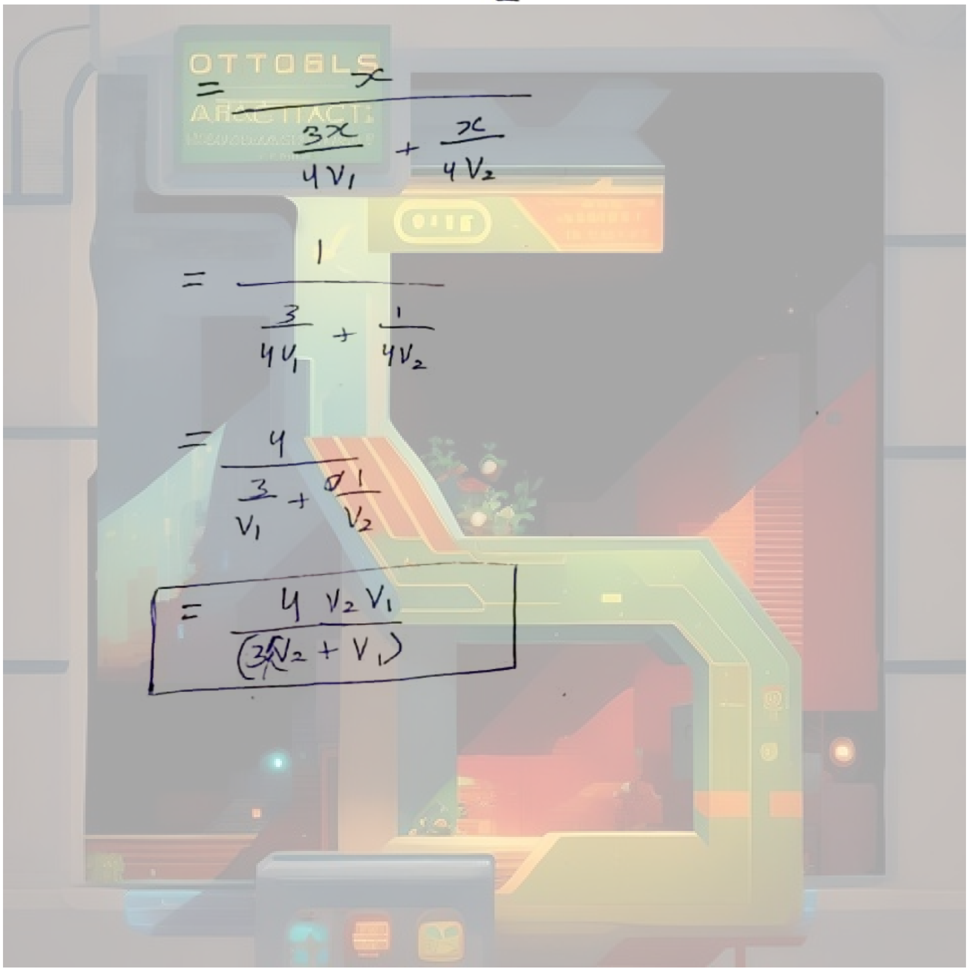
Q3.



$$\text{Average velocity} = \frac{x}{\frac{\frac{3x}{4}}{V_1} + \frac{\frac{x}{4}}{V_2}}$$



$$= \frac{x}{\frac{3x}{4V_1} + \frac{x}{4V_2}}$$

$$= \frac{1}{\frac{3}{4V_1} + \frac{1}{4V_2}}$$

$$= \frac{4}{\frac{3}{V_1} + \frac{1}{V_2}}$$

$$\boxed{= \frac{4\,V_2 V_1}{(3V_2 + V_1)}}$$

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Q4. $x = (5t^2 + 4t + 3)$ m

find velocity and acceleration at $t = 2s$

$$v = \frac{dx}{dt} = \frac{d}{dt}(5t^2 + 4t + 3)$$

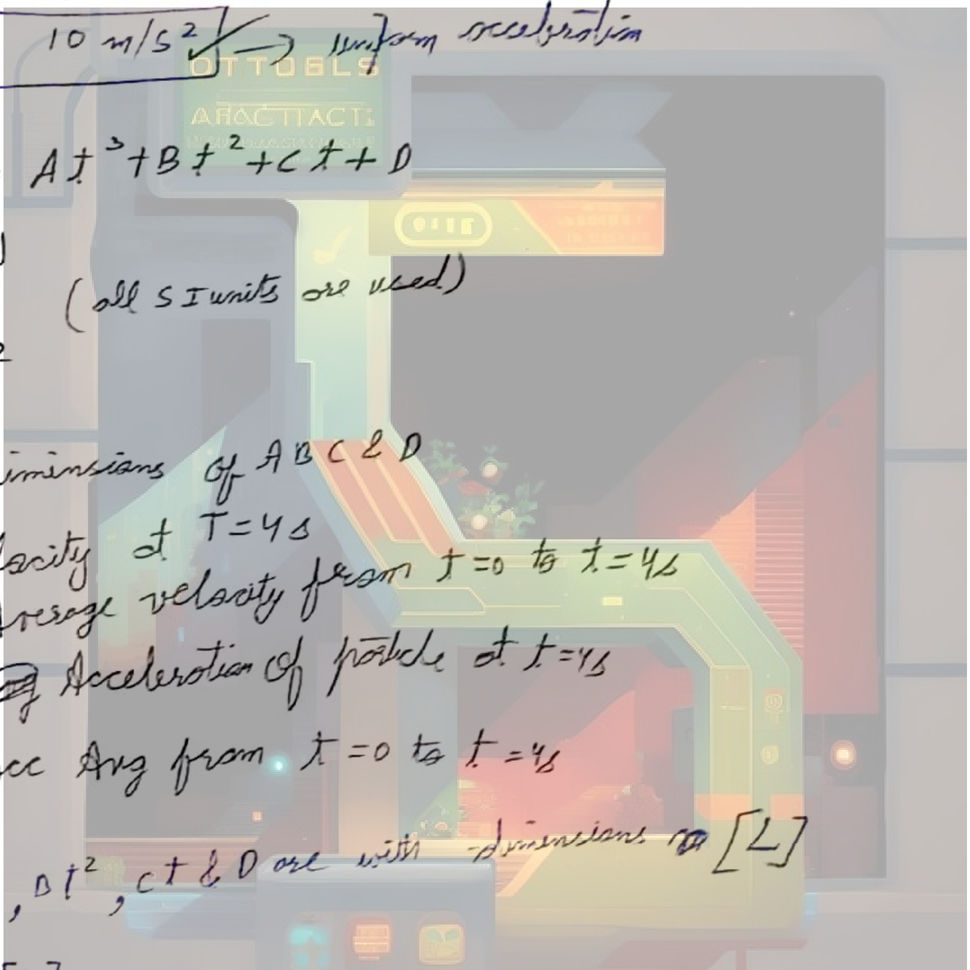
$$= 10t + 4$$

$$= 20 + 4$$

$$\boxed{= 24 \, m/s}$$

$$a = \frac{d}{dt}(10t + 4)$$

$$\boxed{= 10 \, m/s^2} \rightarrow \text{uniform acceleration}$$



Q5. $x = At^3 + Bt^2 + Ct + D$

$A = 1$

$B = 4$ (all SI units are used)

$C = -2$

$D = 5$

a) find dimensions of A B C & D

b) Find velocity at $T = 4s$

c) find Average velocity from $t = 0$ to $t = 4s$

d) find ~~Avg~~ Acceleration of particle at $t = 4s$

e) find acc Avg from $t = 0$ to $t = 4s$

All At^3, Bt^2, Ct & D are with dimensions ~~of~~ $[L]$

$[D] - [L]$

$[C] - \cancel{[T]}$ $[LT^{-1}]$

$[B] - [LT^{-2}]$

$[A] - [LT^{-3}]$

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b) $v = \frac{d}{dt}\left(t^3+4t^20-2t+5\right)$

$\cancel{= (2t^2+4t-2)}$

$\cancel{= 2\times16+4\times4-2}$

$\cancel{= 32+16-2}$

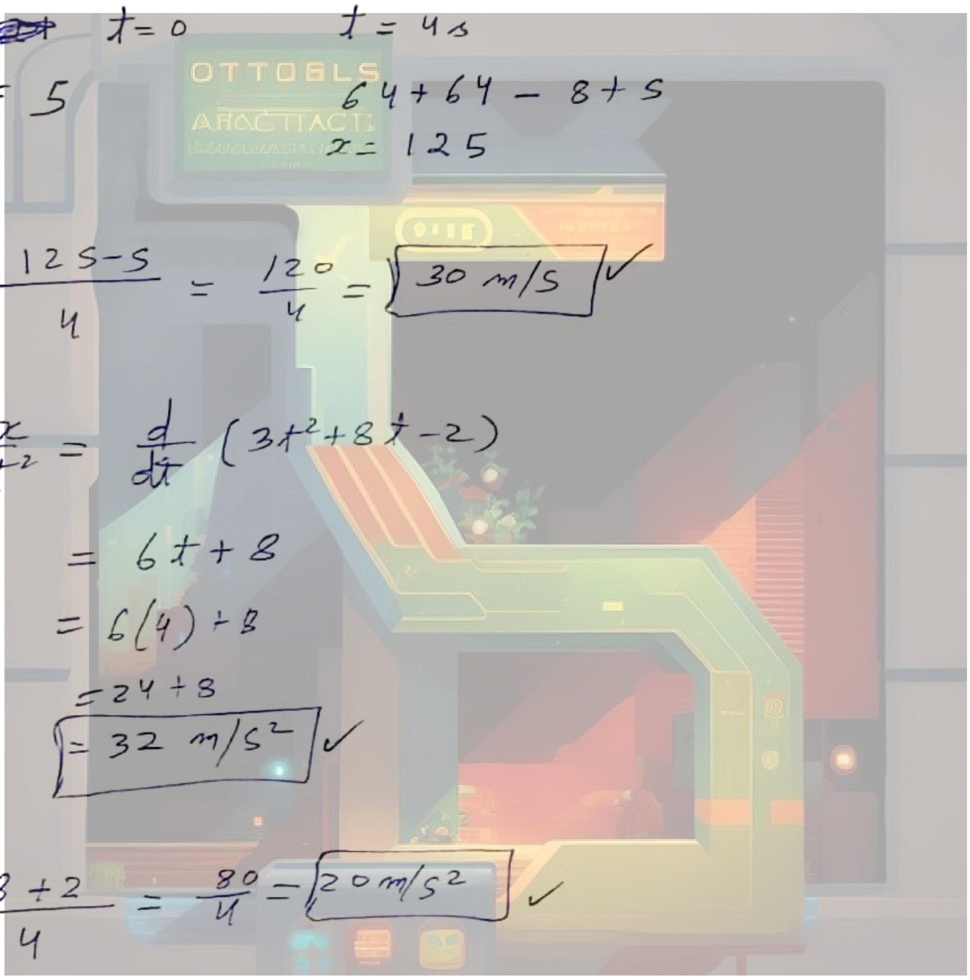
$\cancel{= 48-2}$

$\cancel{\boxed{= 46\ m/s}}$

$= 3t^2+8t-2$

$= 48+32-2$

$\boxed{= 78\ m/s}$ ✓



c) $t=0$ $\qquad$ $t = 4s$

$x = 5$ $\qquad$ $64+64-8+5$

$x = 125$

$\frac{125-5}{4} = \frac{120}{4} = \boxed{30\ m/s}$ ✓

d) $\frac{d^2x}{dt^2} = \frac{d}{dt}(3t^2+8t-2)$

$= 6t+8$

$= 6(4)+8$

$= 24+8$

$\boxed{= 32\ m/s^2}$ ✓

e) $\frac{78+2}{4} = \frac{80}{4} = \boxed{20\ m/s^2}$ ✓

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Q6. $x = t^3 - 6t^2 + 3t + 4$ in meters

find velocity of particle at instant when $a = 0$

$$\frac{dv}{dt} = 0$$

$v =$ constant

$$\frac{dx}{dt} = 3t^2 - \cancel{6t+3}\ 12t + 3$$

$$\cancel{\frac{d^2x}{dt^2} = 6t - 6}$$

$$\cancel{6t - 6 = 0}$$

$$\cancel{6t = 6}$$

$$\cancel{t = 1}$$

$$\cancel{v = 3(1)^3 - 6(1) + 3}$$

$$\cancel{= 3 - 6 + 3}$$

$$\frac{d^2x}{dt^2} = 6t - 12$$

$$6t - 12 = 0$$

$$t = 2$$

$$v = 3(2)^2 - 12(2) + 3$$

$$= 3 \times 4 - 12(2) + 3$$

$$= 12 - 24 + 3$$

$$= -12 + 3$$

$$\boxed{= -9 \text{ m/s}}$$

Q7. $a = 3t^2 + 2t + 2$

$t = 0$, $v = 2$ m/s

find v at $t = 2$ s

$$\Delta v = \int 3t^2 + 2t + 2 \; dt$$

$$2 = \frac{3t^3}{3} + \frac{2t^2}{2} + 2t + C$$

$$\cancel{6 \times 2 = 6 3t^3 + 6t^2 + 12t}$$

$$2 = t^3 + t^2 + 2t + C$$

$$2 = 0 + 0 + 0 + C$$

$$C = 2$$

$$v = 8 + 4 + 4 + 2$$

$$\boxed{v = 18 \text{ m/s}}$$

OR

x जब $t=2, x=?$

$$\int_2^x dv = \int_0^2 3t^2 + 2t + 2$$

जब $t = 0$, $v = 2$

$$[v]_2^x = [t^3 + t^2 + 2t]_0^2$$

$$x - 2 = 8 + 4 + 4$$

$$\boxed{x = 18 \text{ m/s}}$$

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Q7. $a = 6t+6 \ m/s^2$

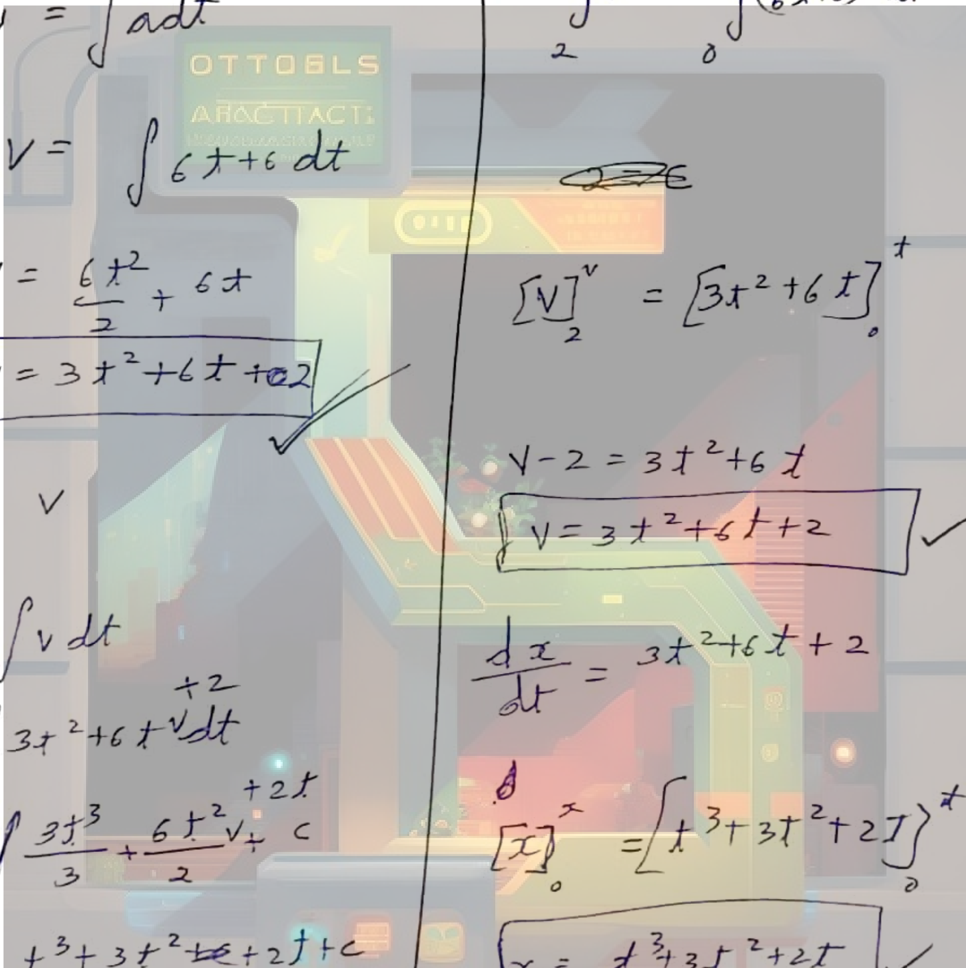
$t=0, x=0, v=2 \ m/s$

velocity & displacement as a function of time.

$$\frac{dv}{dt} = a$$

$$\frac{dv}{dt} = a\,dt$$

$$\int dv = \int a\,dt$$



$$v = \int 6t+6\,dt$$

$$v = \frac{6t^2}{2} + 6t$$

$$\boxed{v = 3t^2+6t+2}$$

$$\frac{dx}{dt} = v$$

$$x = \int v\,dt$$

$$x = \int 3t^2+6t+2\,dt$$

$$x = \int \frac{3t^3}{3} + \frac{6t^2}{2} + 2t + c$$

$$x = t^3+3t^2+2t+c$$

$$\boxed{x = t^3+3t^2+2t}$$

OR

$$\frac{dv}{dt} = 6t+6$$

$$\int_2^v dv = \int_0^t (6t+6)\,dt$$

$$[v]_2^v = [3t^2+6t]_0^t$$

$$v-2 = 3t^2+6t$$

$$\boxed{v = 3t^2+6t+2}$$

$$\frac{dx}{dt} = 3t^2+6t+2$$

$$[x]_0^x = [t^3+3t^2+2t]_0^t$$

$$\boxed{x = t^3+3t^2+2t}$$

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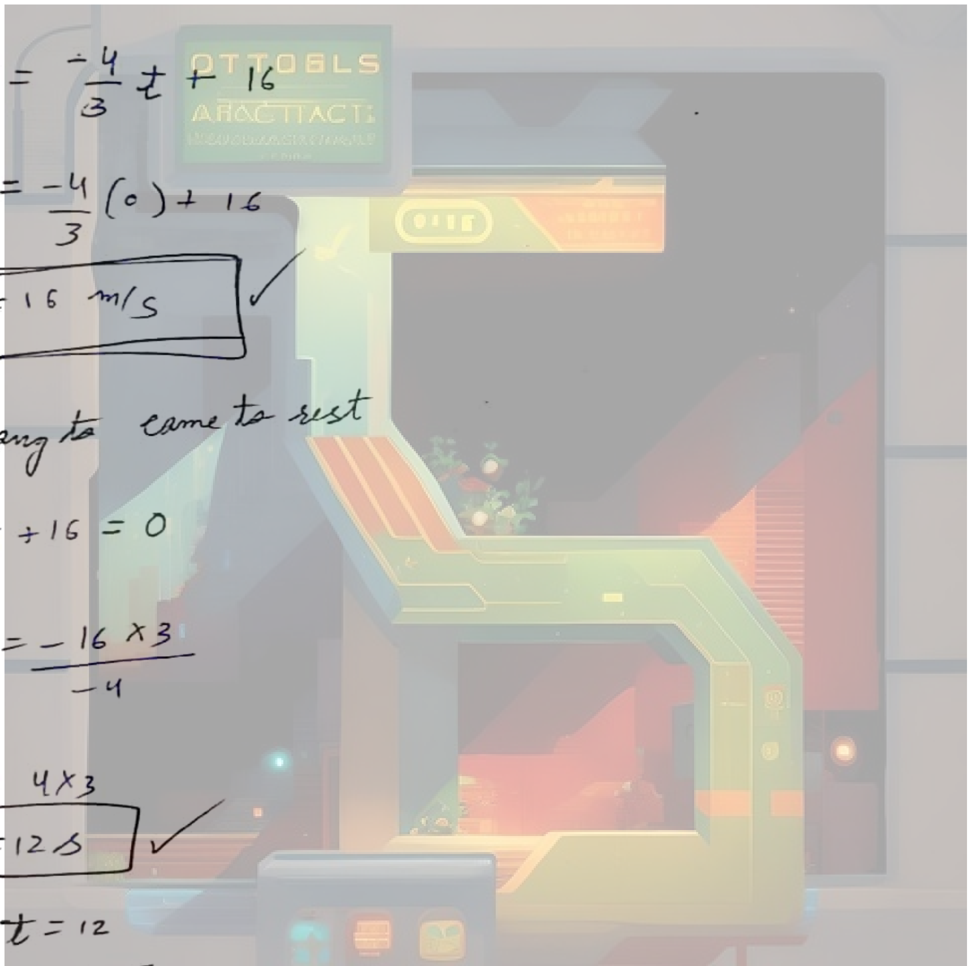
Q8. $x = \left(-\frac{2}{3}t^2 + 16t + 2\right) m$

at $t=0$, $x=0$

a) u (initial velocity

$\cancel{\frac{dv}{dt}}$

$\cancel{\int dv = \int -\frac{2}{3}t^2 + 16t + 2 \, dt}$



$\frac{dx}{dt} = -\frac{4}{3}t + 16$

$= -\frac{4}{3}(0) + 16$

$\boxed{= 16 \ m/s}$

b) how long to came to rest

$-\frac{4}{3}t + 16 = 0$

$t = \frac{-16 \times 3}{-4}$

$t = 4 \times 3$

$\boxed{t = 12 s}$

c) acc, $t = 12$

$\boxed{a = -\frac{4}{3} m/s^2}$

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Q7:- The position of particle is given by $x = 10 + 20t - 5t^2$ in metre

find a) displacement in 1s

b) distance in 3s

c) initial acceleration

d) velocity at 4s

$\frac{d}{dt}(10+20t-5t^2)$

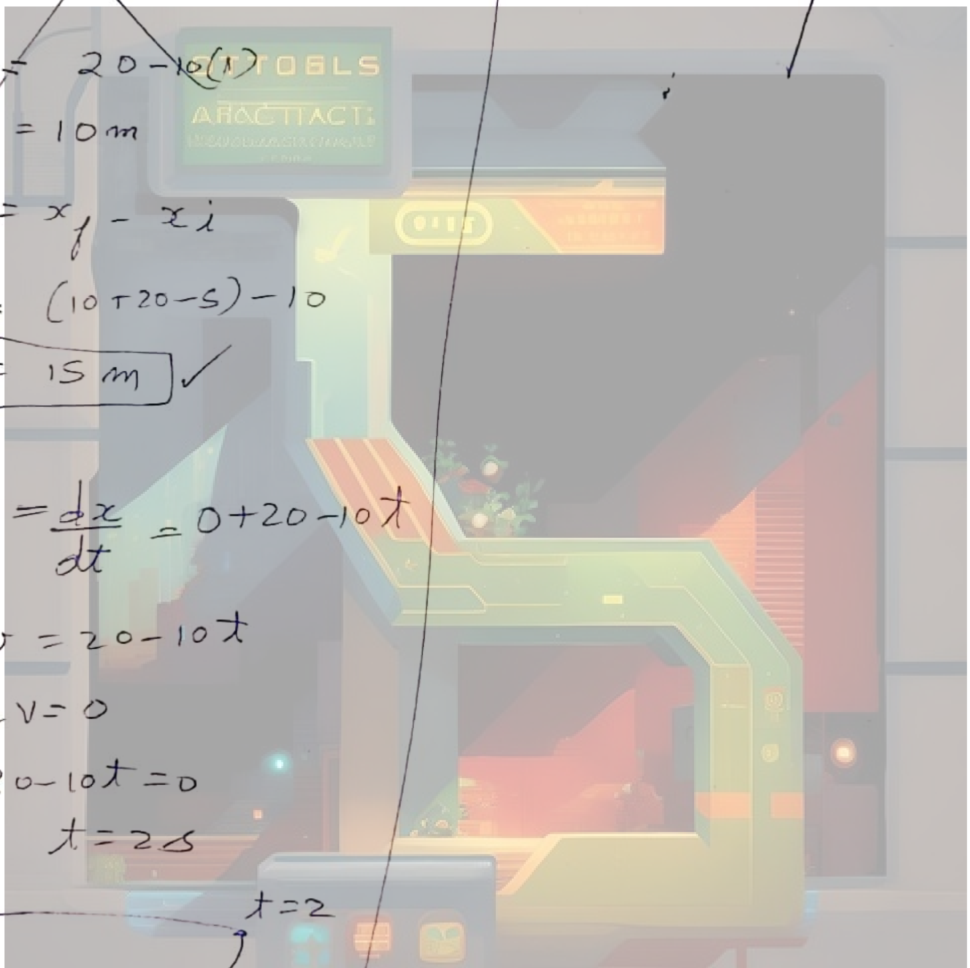
$= -10t + 20$

$x\Big|_{t=1} = 20 - 10(1)$

$= 10\,m$

$x = \int v\,dt$

Distance = 0



a) $\Delta x = x_f - x_i$

$= (10+20-5) - 10$

$\boxed{= 15\,m}$ ✓

b) $v = \frac{dx}{dt} = 0 + 20 - 10t$

$v = 20 - 10t$

for $v = 0$

$20 - 10t = 0$

$t = 2s$

$t = 0$

$t = 2$

Δx $t = 3$

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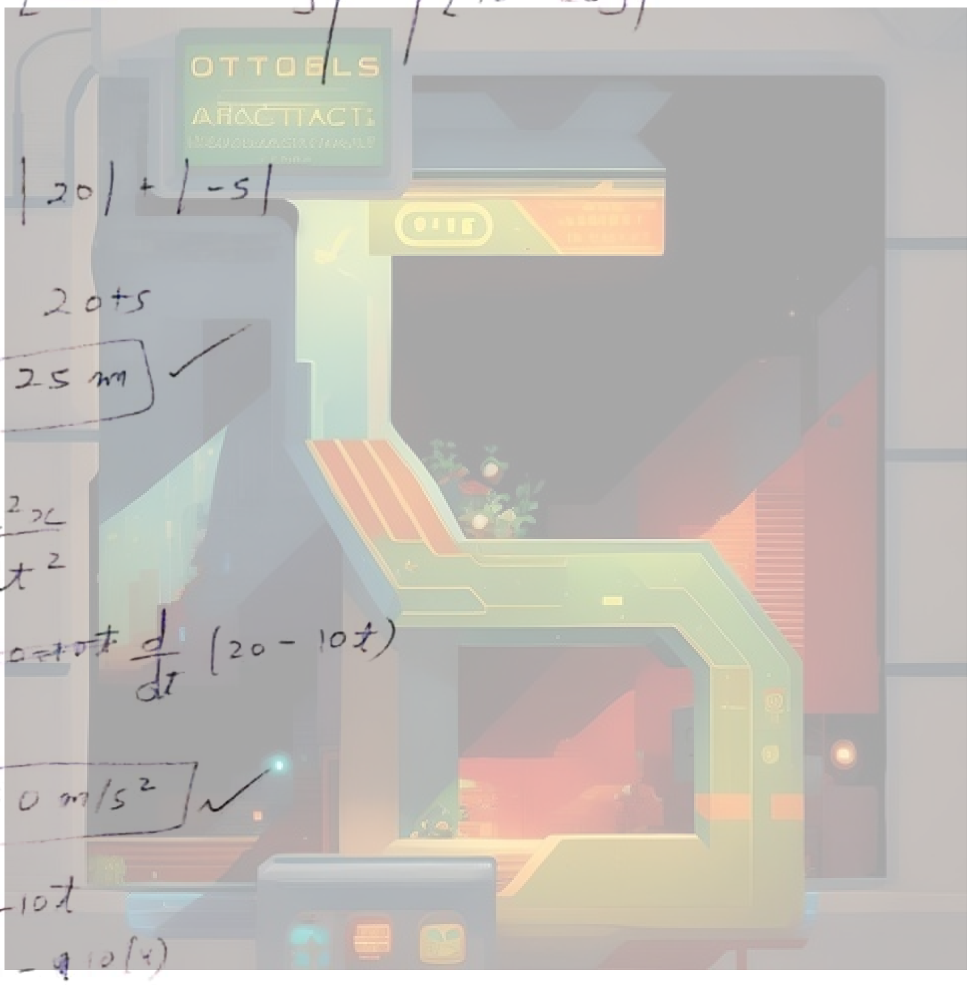
$$\text{Distance} = \left| \int_0^2 (20-10t)\,dt \right| + \left| \int_2^3 (20-10t)\,dt \right|$$

$$\left| \left[20t - 5t^2 \right]_0^2 \right| + \left| \left[20t - 5t^2 \right]_2^3 \right|$$

$$\left| \left[\cancel{20(2)} \; 20 - 0 \right] \right| + \left| \left[15 - 20 \right] \right|$$



$$|20| + |-5|$$

$$20 + 5$$

$$\boxed{= 25\text{ m}}$$

c) $a = \dfrac{d^2x}{dt^2}$

$a = \cancel{20-10t}\; \dfrac{d}{dt}(20 - 10t)$

$$\boxed{a = -10\text{ m/s}^2}$$

d) $v = 20 - 10t$

$= 20 - 10(4)$

$= 20 - 40$

$$\boxed{= -20\text{ m/s}}$$

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Q10. The acc of body is given by $(4-2t)m/s^2$ at $t=0$ find distance at 12s. $v=5m/s$

$$v = \int a\,dt$$

$$v = \int 4-2t\,dt$$

$$v = 4t - t^2 + c$$

$$5 = 0 - 0 + c$$

$$c = 5$$

$$v = 4t - t^2 + 5$$

~~distance =~~

$$0 = 4t - t^2 + 5$$

$$t^2 - 4t - 5 = 0$$

$$t^2 - 5t + t - 5$$

$$t(t-5) + 1(t-5)$$

$$t = 5$$

$$t = -1 \times$$

$$v = 0,\ t = 5$$

$$\text{Displacement} = \int_0^{12} v\,dt$$

$$= \left[2t^2 - \frac{t^3}{3} + 5t\right]_0^{12}$$

$$= 288 - \frac{1728}{3} + 60$$

$$\boxed{= -228\ m}$$

$$dis = \left|\int_0^5 4t - t^2 + 5\,dt\right| + \left|\int_5^{12} 4t - t^2 + 5\right|$$

$$dis = \left|\left[2t^2 - \frac{t^3}{3} + 5t\right]_0^5\right| + \left|\left[2t^2 - \frac{t^3}{3} + 5t\right]_5^{12}\right|$$

$$= \left|\left[50 - \frac{125}{3} + 25 - 0\right]\right| + \left|\left[288 - \frac{1728}{3} + 60 - 50 + \frac{125}{3} - 25\right]\right|$$

$$= \left|75 - \frac{125}{3}\right| + \left|-228 - 75 + \frac{125}{3}\right|$$

$$= \left|75 - \frac{125}{3}\right| + \left|-303 + \frac{125}{3}\right|$$

$$= \frac{225 - 125}{3} + \cancel{\frac{178}{3}} + \frac{784}{3}$$

$$\cancel{= \frac{100 + 178}{3}}$$

$$\cancel{= \frac{278}{3}}$$

$$\boxed{= \frac{884}{3}\ m}$$

$$\begin{array}{r} 144 \\ \times 12 \\ \hline 288 \\ 144 \\ \hline 1728 \end{array}$$

(136)

Q11. $a = -\cos t$

$t = 0;\ v = 0,\ x = 1$

find

a) position at $t = \pi$

b) distance from $t = 0$ to $t = 2\pi$

a) $v = \int -\cos t \, dt$

$v = -\sin t + c$

$0 = c$

$v = -\sin t$

$x = \int v \, dt$

$x = \int -\sin t \, dt$

$x = \cos t + c$ $\quad 1 = 1 + c$

$\cancel{0 + = 90 + c}$

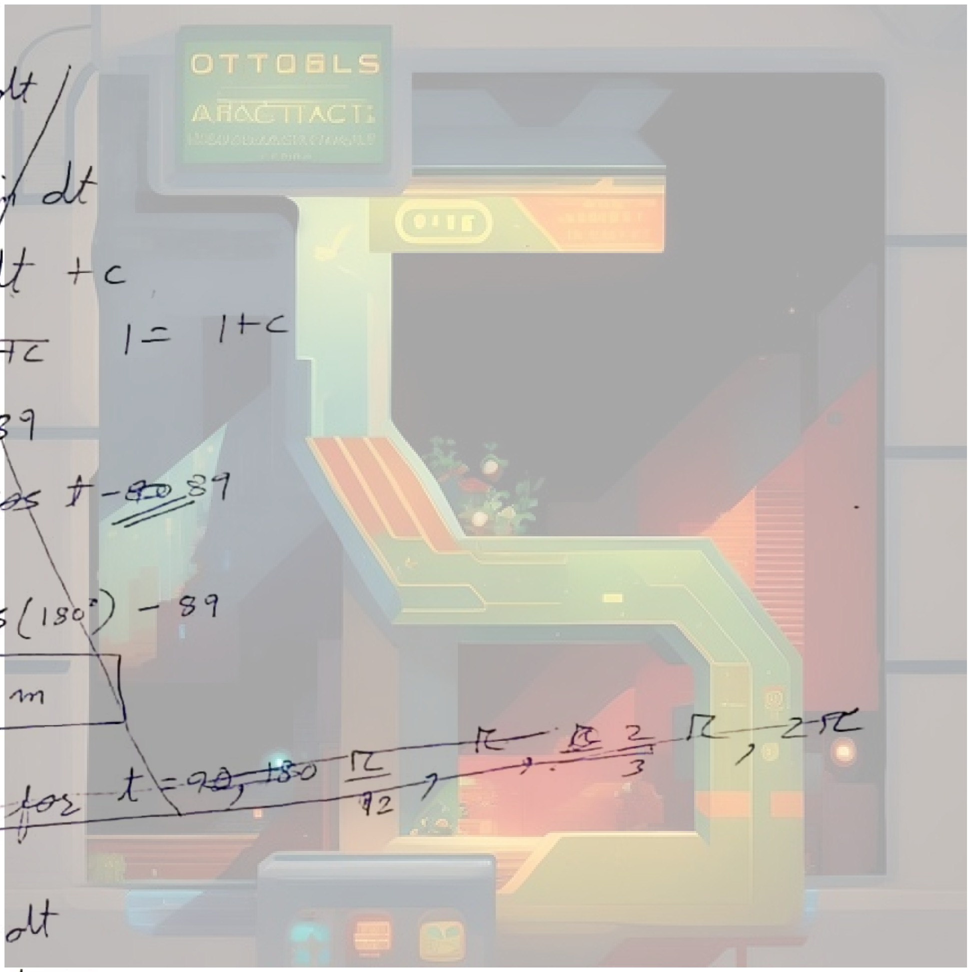
$c = -89$

$x = \cos t - 89$

$x = \cos(180^\circ) - 89$

$\boxed{x = -90 \text{ m}}$

b) $v = 0$; for $t = \cancel{90, 180}\ \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$



$x = \int v \, dt$

$x = \cos t + c$

$1 = \cos 0 + c$

$1 = 1 + c$

$c = 0$

$x = \cos t$

$x = \cos(180^\circ)$

$\boxed{x = -1 \text{ m}}$

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6) ~~cos t~~ $v = 0$

$-\sin t = 0$

$\sin t = 0 \rightarrow t = 0$, $t = \pi$

$$D = \left| \int v \, dt \right|$$

$$D = \left| \int_0^{\pi} -\sin t \, dt \right| + \left| \int_{\pi}^{2\pi} -\sin t \, dt \right|$$

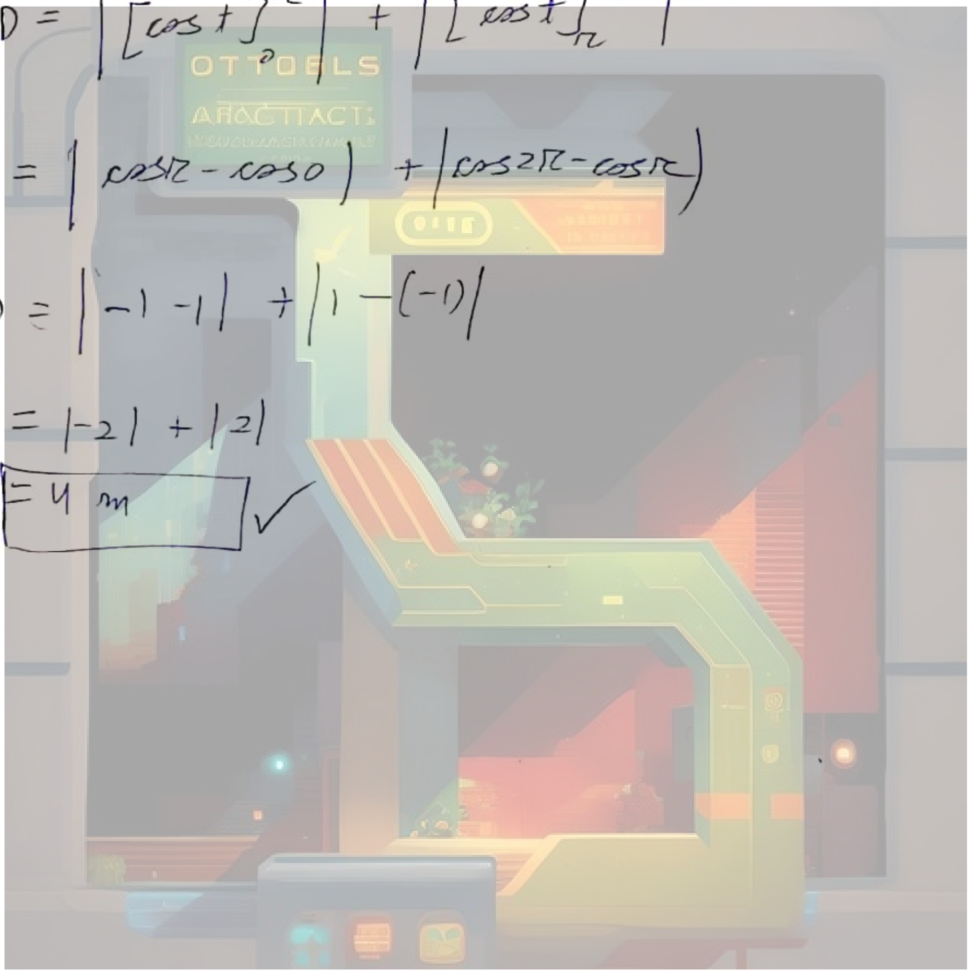
$$D = \left| [\cos t]_0^{\pi} \right| + \left| [\cos t]_{\pi}^{2\pi} \right|$$

$$D = \left| \cos \pi - \cos 0 \right| + \left| \cos 2\pi - \cos \pi \right|$$

$$D = |-1 - 1| + |1 - (-1)|$$

$$= |-2| + |2|$$

$$\boxed{= 4 \text{ m}}$$



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Q12. $V = 6t - 3t^2$ m/s

find average speed and average velocity for first 4 seconds.

$\cancel{V.\ x}$

$V = 0$

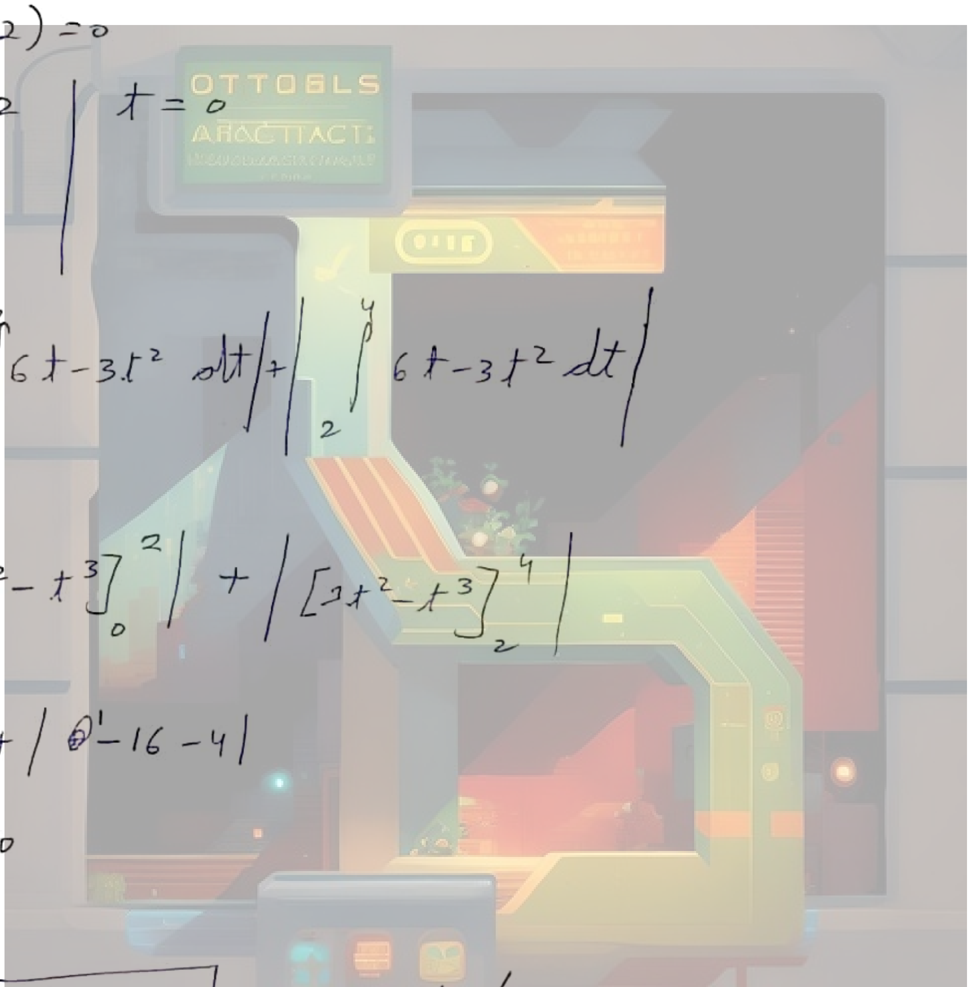
$0 = 6t - 3t^2$

$3t^2 - 6t = 0$

$\cancel{3t(3-t)}$

$3t(t-2) = 0$

$t = 2 \quad | \quad t = 0$



$$x_4 = \left| \int_0^2 6t - 3t^2 \, dt \right| + \left| \int_2^4 6t - 3t^2 \, dt \right|$$

$$= \left| \left[3t^2 - t^3\right]_0^2 \right| + \left| \left[3t^2 - t^3\right]_2^4 \right|$$

$$= |4| + |\cancel{0} - 16 - 4|$$

$$= 4 + 20$$

$$= 24 \text{ m}$$

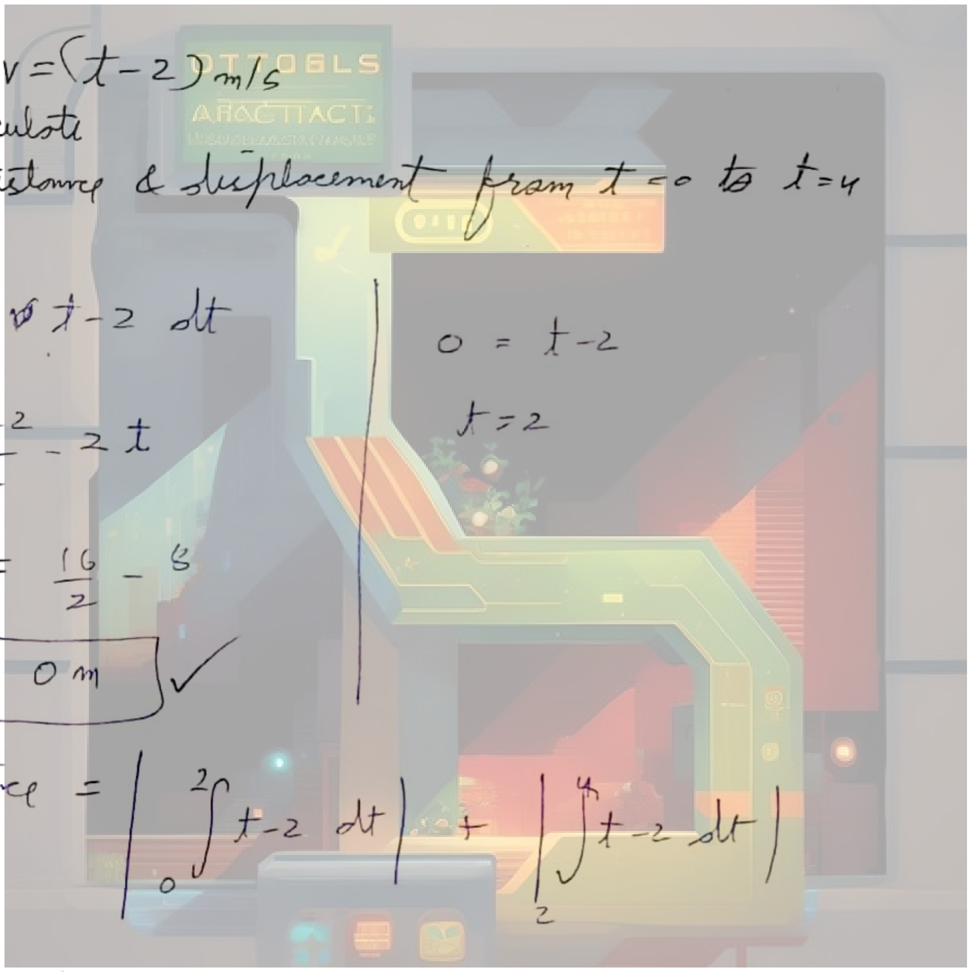
$$\frac{24}{4} = \boxed{6 \text{ m/s}}$$ Average speed

$$x = \int v\,dt$$

$$x\Big|_{t=4} = \int 3t^2 - t^3$$

$$\boxed{x = \cancel{3t}\ -16\ m}$$

$$\frac{-16}{4} = \boxed{-4\ m/s}$$ Average Velocity



Q 13. $v = (t-2)\ m/s$

calculate

distance & displacement from $t=0$ to $t=4$

$$x = \int \cancel{v}\ t-2\ dt$$

$$0 = t-2$$

$$t = 2$$

$$x = \frac{t^2}{2} - 2t$$

$$x\Big|_{t=4} = \frac{16}{2} - 8$$

$$\boxed{= 0\ m}$$

~~Δx~~ distance $= \left|\int_0^2 t-2\ dt\right| + \left|\int_2^4 t-2\ dt\right|$

$$= \left|\left[\frac{t^2}{2} - 2t\right]_0^2\right| + \left|\left[\frac{t^2}{2} - 2t\right]_2^4\right|$$

$$= |-2| + |2|$$

$$= 2 + 2$$

$$\boxed{= 4\ m}$$

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Q14. A Particle starts moving along straight line such that

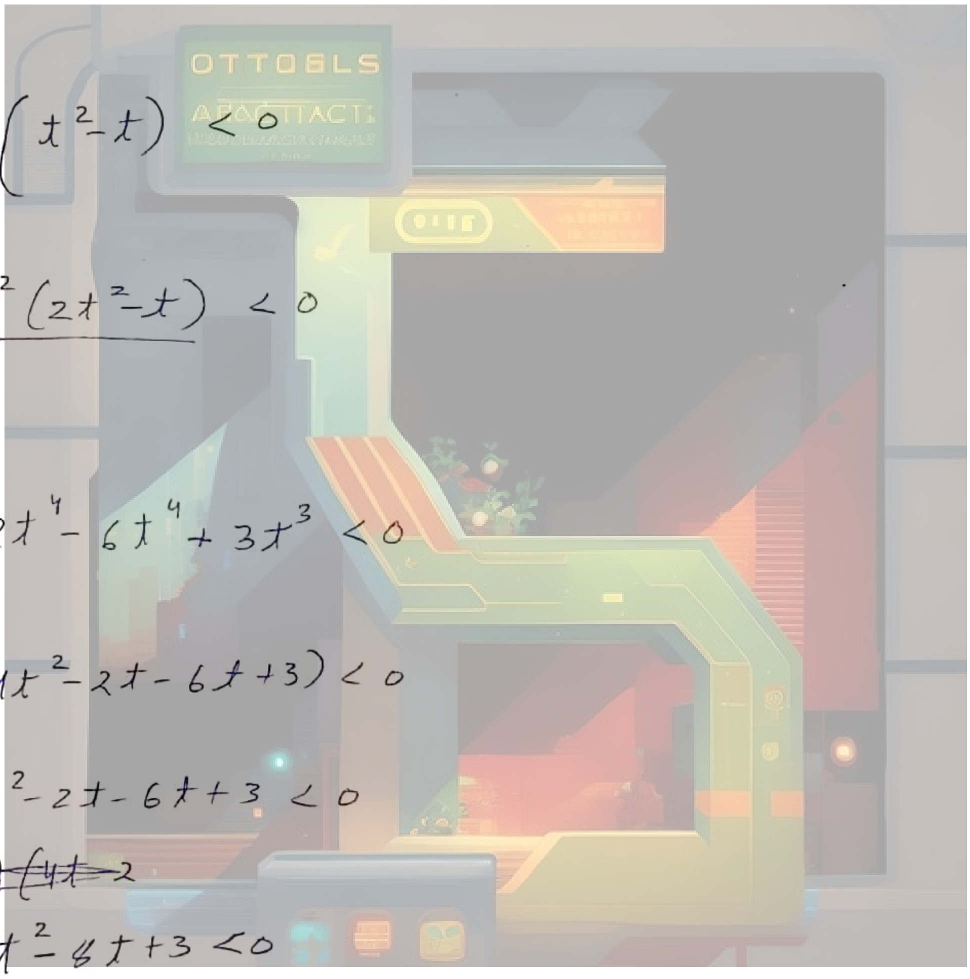
$v = t^2 - t$ m/s

find the interval for which particle retards.

$a = \int t^2 - t \, dt$

$a = \frac{t^3}{3} - \frac{t^2}{2}$ m/s^2

$av < 0$



$\frac{t^3}{3} - \frac{t^2}{2} \left(t^2 - t\right) < 0$

$\frac{2t^3 - 3t^2 (2t^2 - t)}{6} < 0$

$\cancel{2}4t^5 - 2t^4 - 6t^4 + 3t^3 < 0$

$t^3 (4t^2 - 2t - 6t + 3) < 0$

$4t^2 - 2t - 6t + 3 < 0$

$\cancel{t(4t-2}$

$4t^2 - 8t + 3 < 0$

$4t^2 - 6t - 2t + 3 < 0$

$2t(2t-3) - 1(2t-3) < 0$

$(2t-1)(2t-3) < 0$

$t < \frac{1}{2}$ $\quad t < \frac{3}{2}$ ×

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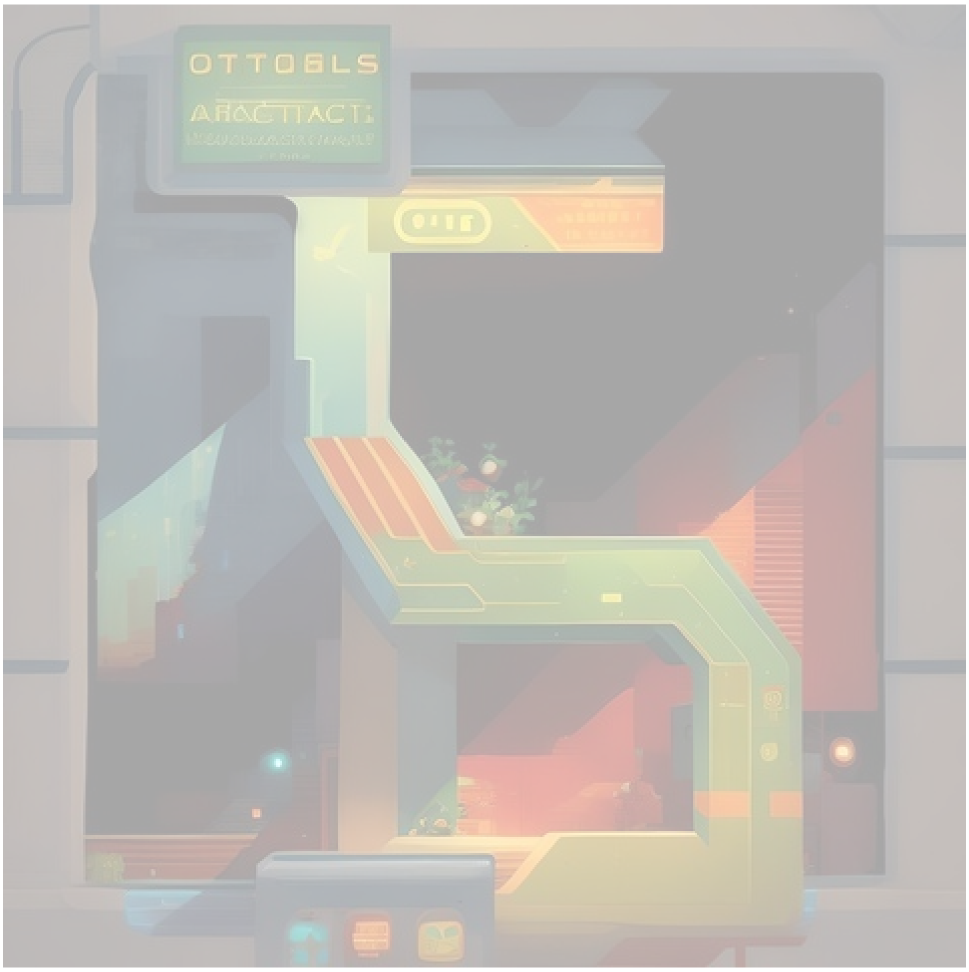

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Q14. A particle starts moving along straight line such that

$v = t^2 - t$ m/s

find interval when particle retards

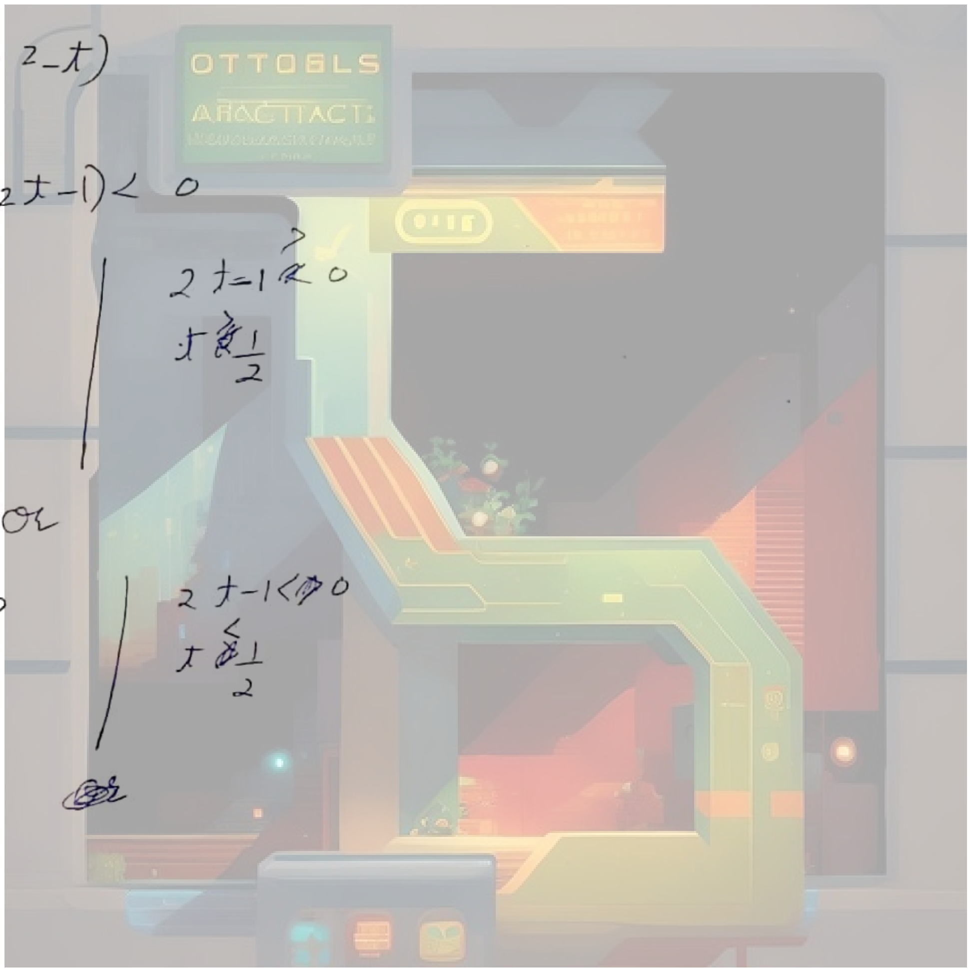
$v = t^2 - t$

$a = \frac{d}{dt}(t^2 - t) = 2t - 1$

$av < 0$

$(2t-1)(t^2-t)$

$t(t-1)(2t-1) < 0$



$t - 1 < 0$ | $2t - 1 > 0$

$t < 1$ | $t > \frac{1}{2}$

or

$t - 1 > 0$ | $2t - 1 < 0$

$t > 1$ | $t < \frac{1}{2}$

✗

or

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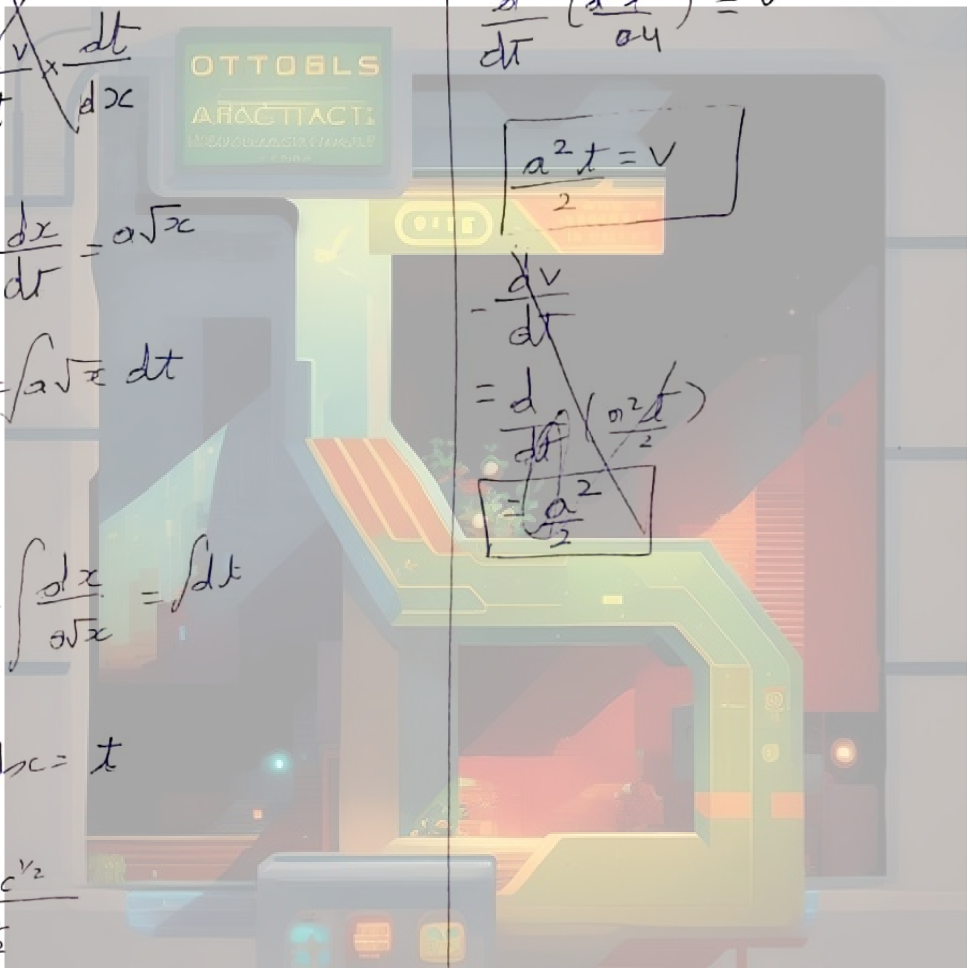
Q15. Velocity of a particle moving in the (+)ve x-axis varies as $v = a\sqrt{x}$ where a is a constant & x is displacement

at $t = 0$, $x = 0$

find time dependence of velocity.



$\cancel{x = \int v\,dt}$

$\cancel{\frac{dv}{dt} \times \frac{dt}{dx}}$

$v = \frac{dx}{dt} = a\sqrt{x}$

$\int dx = \int a\sqrt{x}\,dt$

$\int \frac{dx}{a\sqrt{x}} = \int dt$

$\int \frac{1}{a\sqrt{x}}\,dx = t$

$\frac{1}{a}\frac{x^{1/2}}{\frac{1}{2}}$

$\frac{2\sqrt{x}}{a} = t$

$\sqrt{x} = \frac{at}{2}$

$\cancel{x = \frac{a^2t^2}{2}}$

$\sqrt{x} + c = t$

$c = 0$

$\boxed{x = \frac{a^2t^2}{4}}$

$\frac{dx}{dt} = v$

$\frac{d}{dt}\left(\frac{a^2t^2}{4}\right) = v$

$\boxed{\frac{a^2t}{2} = v}$

$\cancel{\frac{dv}{dt} = \frac{d}{dt}\left(\frac{a^2t}{2}\right) = \boxed{\frac{a^2}{2}}}$

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Q16. $\vec{v} = 4t\hat{i} + 3\hat{j}$

at $t = 0$

$\vec{x} = 2\hat{j}$

a) find position vector at $t = 2s$

b) find average acc for $t = 0s$ और $t = 2s$.

$\cancel{|\vec{v}| = 4t\hat{i} + 3\hat{j}}$

$\cancel{|\vec{v}| = \sqrt{16t^2+9}}$

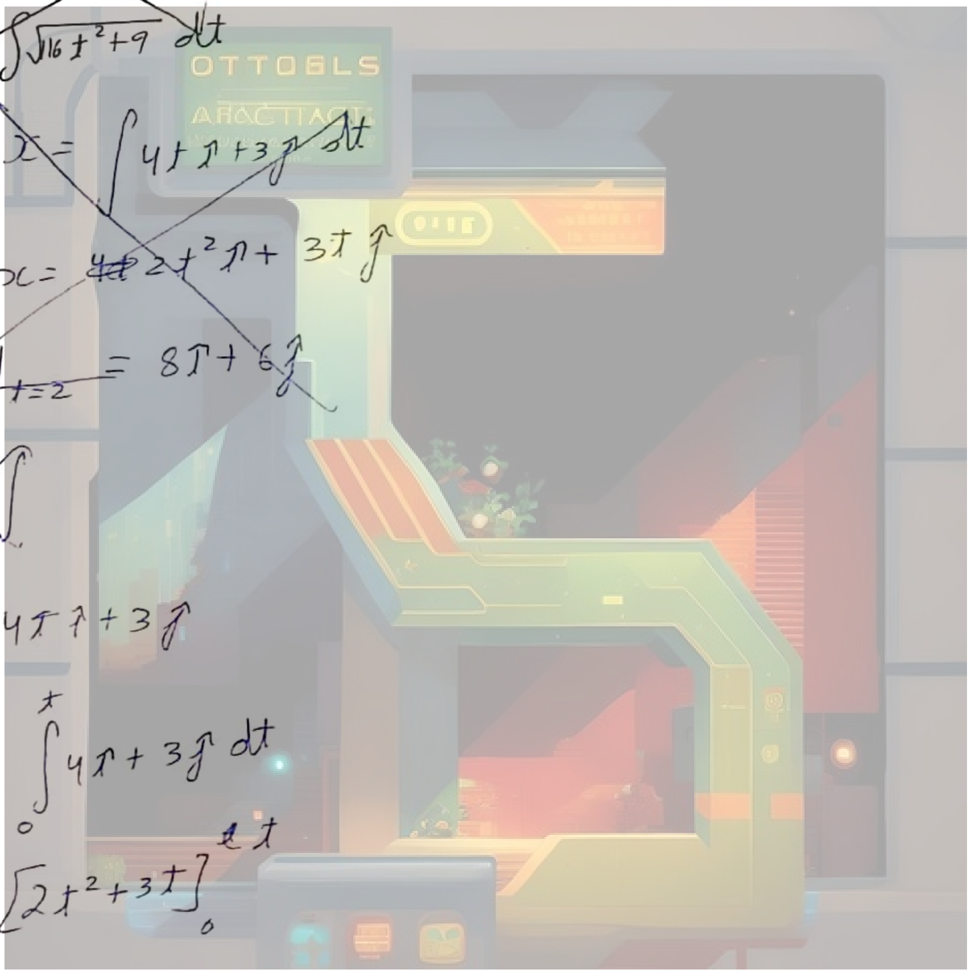
$\cancel{x = \int \sqrt{16t^2+9}\,dt}$

$\cancel{x = \int 4t\hat{i} + 3\hat{j}\,dt}$

$\cancel{x = 2t^2\hat{i} + 3t\hat{j}}$

$\cancel{x|_{t=2} = 8\hat{i} + 6\hat{j}}$

$\cancel{\int_0^t x = \int}$



$\vec{v} = 4t\hat{i} + 3\hat{j}$

$$\int_{2\hat{j}}^{x} dx = \int_0^t 4t\hat{i} + 3\hat{j}\,dt$$

$$x - 2\hat{j} = \left[2t^2 + 3t\right]_0^t$$

$$x - 2\hat{j} = 2t^2\hat{i} + 3t\hat{j}$$

$$x|_{t=0} = 2(2)^2\hat{i} + 3(2)\hat{j} + 2\hat{j}$$

$$\boxed{r = 8\hat{i} + 8\hat{j}}$$

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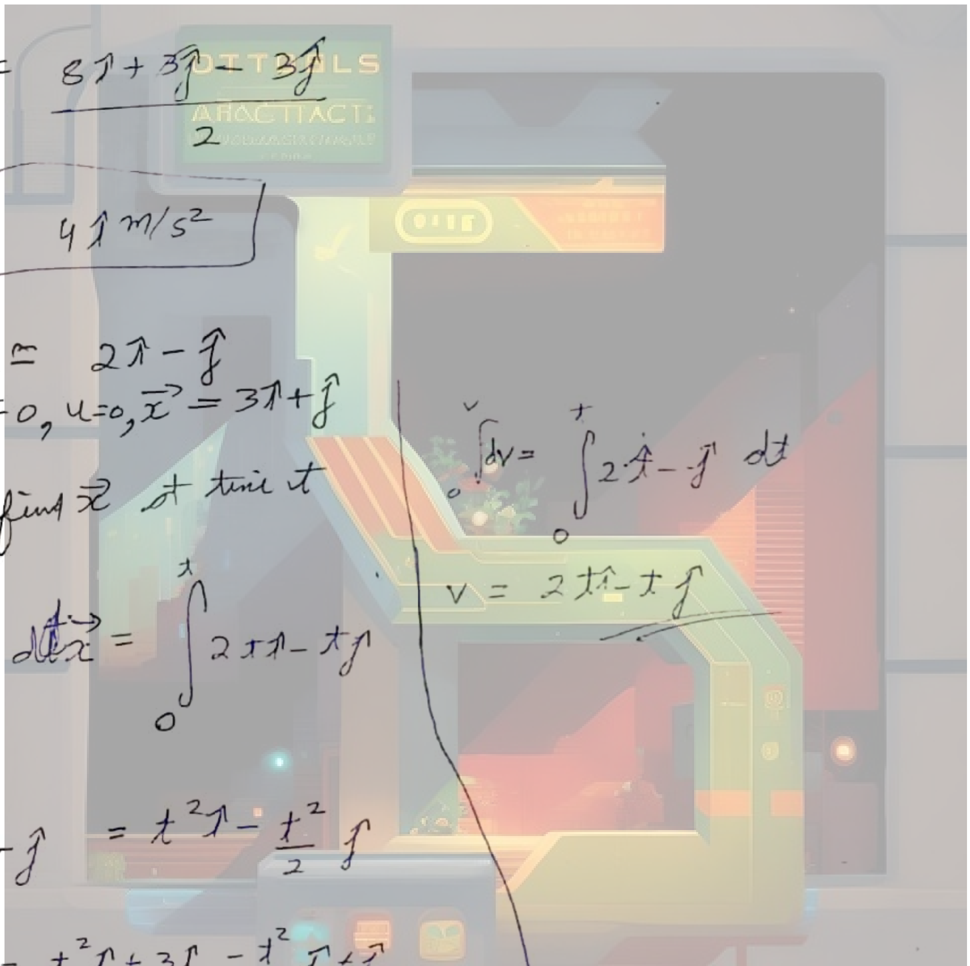
$\cancel{\int a\, dt}$

$$\cancel{avg\ a = \int_0^2 4t\hat{i} + 3\hat{j}\, dt}$$

$\cancel{=}$

$$a_{avg} = \frac{\vec{v} - \vec{u}}{t}$$



$$= \frac{8\hat{i} + 3\hat{j} - 3\hat{j}}{2}$$

$$\boxed{= 4\hat{i}\ m/s^2}$$

Q17. $\vec{a} = 2\hat{i} - \hat{j}$

$t=0,\ u=0,\ \vec{x} = 3\hat{i} + \hat{j}$

find $\vec{x}$ at time t

$$\int_0^v dv = \int_0^t 2\hat{i} - \hat{j}\, dt$$

$$v = 2t\hat{i} - t\hat{j}$$

$$\int_{3\hat{i}+\hat{j}}^{x} d\vec{x} = \int_0^t 2t\hat{i} - t\hat{j}$$

$$x - 3\hat{i} - \hat{j} = t^2\hat{i} - \frac{t^2}{2}\hat{j}$$

$$x = t^2\hat{i} + 3\hat{i} - \frac{t^2}{2}\hat{j} + \hat{j}$$

$$\boxed{x = (t^2+3)\hat{i} - \left(\frac{t^2-2}{2}\right)\hat{j}}$$

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Q18. A particle with velocity v_0 at $t=0$ is decelerated at rate $|a| = \alpha\sqrt{v}$ (α is a +ve constant)

a) find time after which it comes to rest.
b) distance.

$\cancel{a = \alpha\sqrt{v}}$

$\cancel{\frac{dv}{dt} = \alpha\sqrt{v}}$

$\cancel{dv = \alpha\sqrt{v}\,dt}$

$\cancel{\int_0^v dv = \int_0 \alpha\sqrt{v}\,dt}$

$\cancel{v =}$

$\cancel{\int \frac{da}{dv} = \alpha\sqrt{v}}$

$\cancel{da = \alpha\sqrt{v}\,dv}$

$\cancel{a = \frac{2\alpha\sqrt{v^3}}{3}}$

$\cancel{\alpha\sqrt{v} = \frac{2\alpha\sqrt{v^3}}{3}}$

$\cancel{v = \frac{4v^3}{9}}$

$\cancel{\frac{9}{4} = v^2}$

$\cancel{v = \frac{3}{2}}$

$a = -\alpha\sqrt{v}$

$\frac{dv}{dt} = -\alpha\sqrt{v}$

$\frac{dv}{\sqrt{v}} = -\alpha\,dt$

$\left[2\sqrt{v}\right]_{v_0}^{v} = -\alpha t$

$2\sqrt{v} - 2\sqrt{v_0} = -\alpha t$

$\boxed{2\sqrt{v} = 2\sqrt{v_0} - \alpha t}$

a) $v = 0$

$0 = 2\sqrt{v_0} - \alpha t$

$\alpha t = 2\sqrt{v_0}$

$\boxed{t = \frac{2\sqrt{v_0}}{\alpha}}$

b) $\sqrt{v} = \sqrt{v_0} - \frac{\alpha t}{2}$

$v = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$

$\frac{dx}{dt} = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$

$\int_0^x dx = \int_0^{\frac{2\sqrt{v_0}}{\alpha}} \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2 dt$

$x = \left[\frac{\left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^3}{3\left(\frac{-\alpha}{2}\right)}\right]_0^{\frac{2\sqrt{v_0}}{\alpha}}$

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$$x = \frac{-2}{3\alpha}\left[\left(\sqrt{v_0} - \frac{\alpha \times 2\sqrt{v_0}}{2\alpha}\right)^3 - \left(\sqrt{v_0}\right)^3\right]$$

$$x = \frac{-2}{3\alpha}\left[\sqrt{v_0} - \sqrt{v_0}\right]^3 - \left(\sqrt{v_0}\right)^3$$

$$x = \frac{-2}{3\alpha} \times -\left(\sqrt{v_0}\right)^3$$

$$\boxed{x = \frac{2\left(\sqrt{v_0}\right)^3}{3\alpha}}$$

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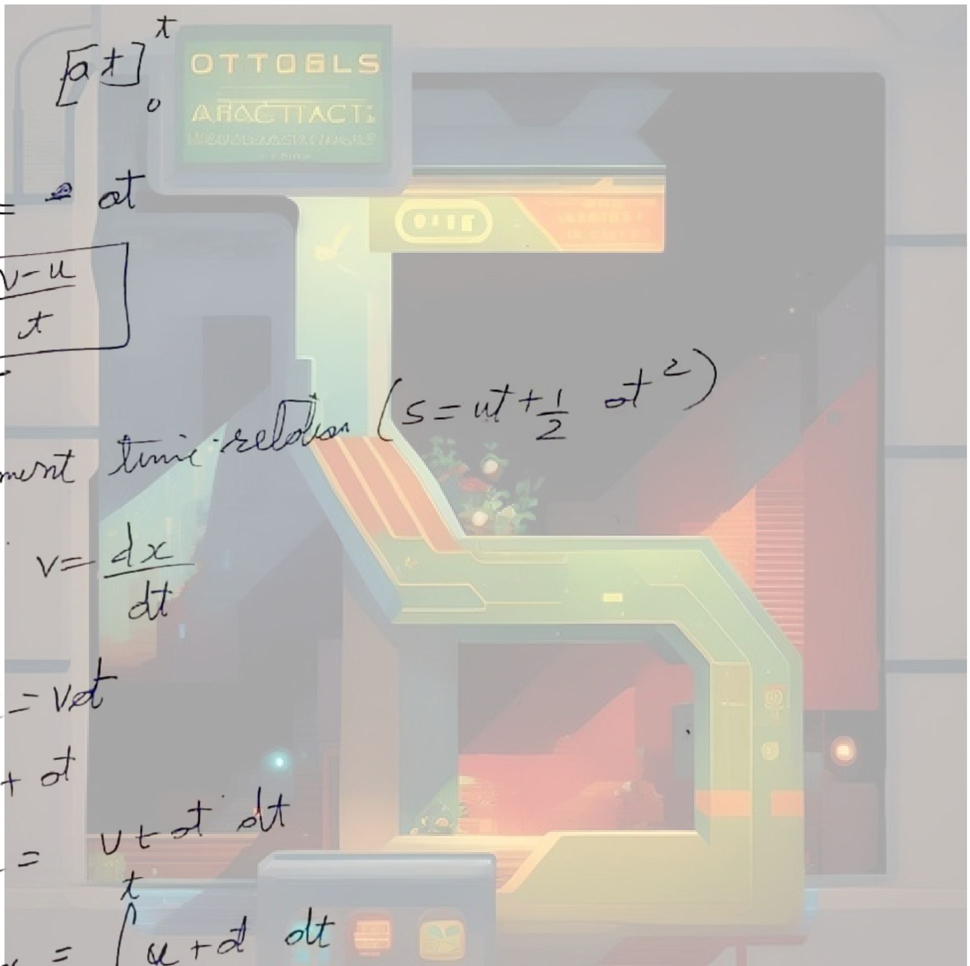
Equations of Motion :-

① Velocity Time relation ($v = u + at$)

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$\int_u^v dv = \int_0^t a\,dt$$



$$[v]_u^v = [at]_0^t$$

$$v - u = at$$

$$\boxed{a = \frac{v-u}{t}}$$

2) Displacement time relation ($s = ut + \frac{1}{2}at^2$)

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$v = u + at$$

$$dx = u + at\,dt$$

$$\int_0^s dx = \int_0^t u + at\,dt$$

$$s = \left[ut + \frac{at^2}{2}\right]_0^t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

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(3) Velocity - Displacement relation ($v^2 = u^2 + 2as$)

$$a = v\frac{dv}{dx}$$

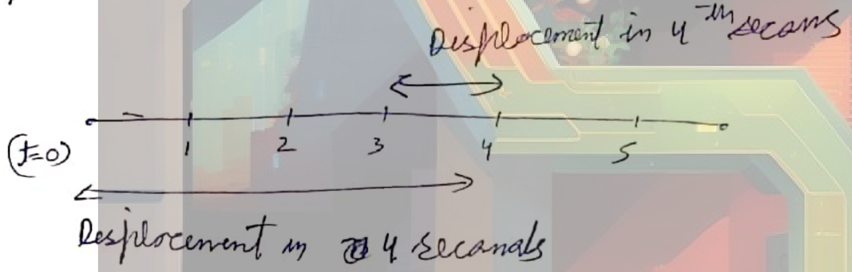
$$\int_u^v v\,dv = \int_0^s a\,dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a\left[x\right]_0^s$$

$$\frac{v^2}{2} - \frac{u^2}{2} = as$$

$$\boxed{v^2 - u^2 = 2as}$$

(4) Displacement in n^{th} second :-



$$S_{n^{th}} = S_n - S_{(n-1)}$$

$$= un - \frac{1}{2}an^2 - \left[u(n-1) - \frac{1}{2}a(n-1)^2\right]$$

$$= un - \frac{an^2}{2} - \left[un - u - \frac{an^2}{2} - \frac{a}{2} + \frac{an}{2}\right]$$

$$= un - un - \frac{an^2}{2} + \frac{an^2}{2} + u + \frac{a}{2} - an$$

$$= u + an - \frac{a}{2}$$

$$\boxed{= u + \frac{a}{2}(2n-1)}$$

$$\boxed{S_n^{th} = u + \frac{a}{2}(2n-1)}$$

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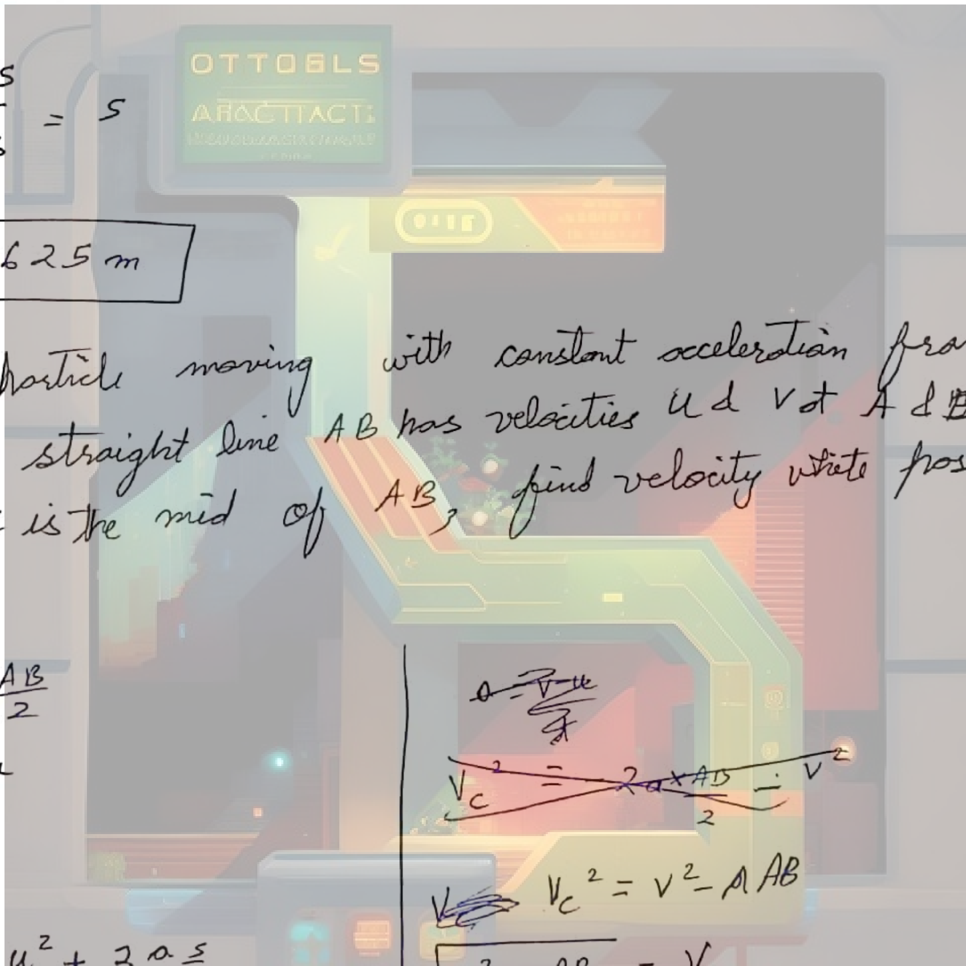
Q A Train is travelling at speed 90 km/hr breaks are applied so as to produce a uniform retardation of $0 \cdot 5 m/s^2$. find how far the train goes before comming to rest.

$$u = 90 \times \frac{5}{18} = 5 \times 5 = 25 \, m/s$$

$$a = 0.5$$

$$\underline{v^2 - u^2 = 2as}$$



$$\frac{+625}{2 \times 0.5} = s$$

$$\boxed{s = 625 \, m}$$

Q2. A particle moving with constant acceleration from A to B in a straight line AB has velocities u & v at A & B. If C is the mid of AB, find velocity while passing through C.

$$s = \frac{AB}{2}$$

$$u = u$$

$$V_c = ?$$

$$a = a$$

$$V_c^2 = u^2 + 2a\frac{s}{2}$$

$$V_c^2 = u^2 + \cancel{2}as$$

$$\boxed{V_c = \sqrt{u^2 + a\cancel{B}AB}}$$

$$\cancel{a = \frac{v-u}{t}}$$

$$\cancel{V_c^2 = 2a \times \frac{AB}{2} - v^2}$$

$$V_c^2 = v^2 - a AB$$

$$\underline{\sqrt{v^2 - aAB} = V_c}$$

$$2V_c^2 = v^2 + v^2$$

$$x = \boxed{\frac{v^2 + u^2}{2}}$$

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Q3. A particle starts from rest with uniform ~~velocity~~ acceleration a. Its velocity after n seconds is v. Find the displacement of body in last two seconds

$u = 0$
$v = v$
$t = n$

$$S_{(n-2)}^{th} = u + \frac{a}{2}(2n-1)$$

$$= u + \frac{a}{2}(2(n-2)-1)$$

$$= u + \frac{a}{2}(2n-4-1)$$

$$= u + \frac{a}{2}(2n-5)$$

$$= u + an - \frac{5a}{2}$$

$$S_n^{th} + S_{(n-1)}^{th} = u + \frac{a}{2}(2n-1) + u + \frac{a}{2}(2n-3)$$

$$= 2u + an - \frac{a}{2} + an - \frac{3a}{2}$$

$$= 2u + 2an - 2a$$

$$= 2(u + a + an)$$

$$= 2[u + a(1+n)]$$

or

$$= 2[a(n-1)]$$

$u = 0$ or
$v = an \ldots (1) \ldots$

Disp in last 2 sec $= S_n - S_{(n-2)}$

$$= \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= \frac{a}{2}(n^2 - n^2 + 4 - 2n)$$

$$= \frac{a}{2}(4 - 2n) = 2a - an$$

$$= 2a(n-1)$$

(152)

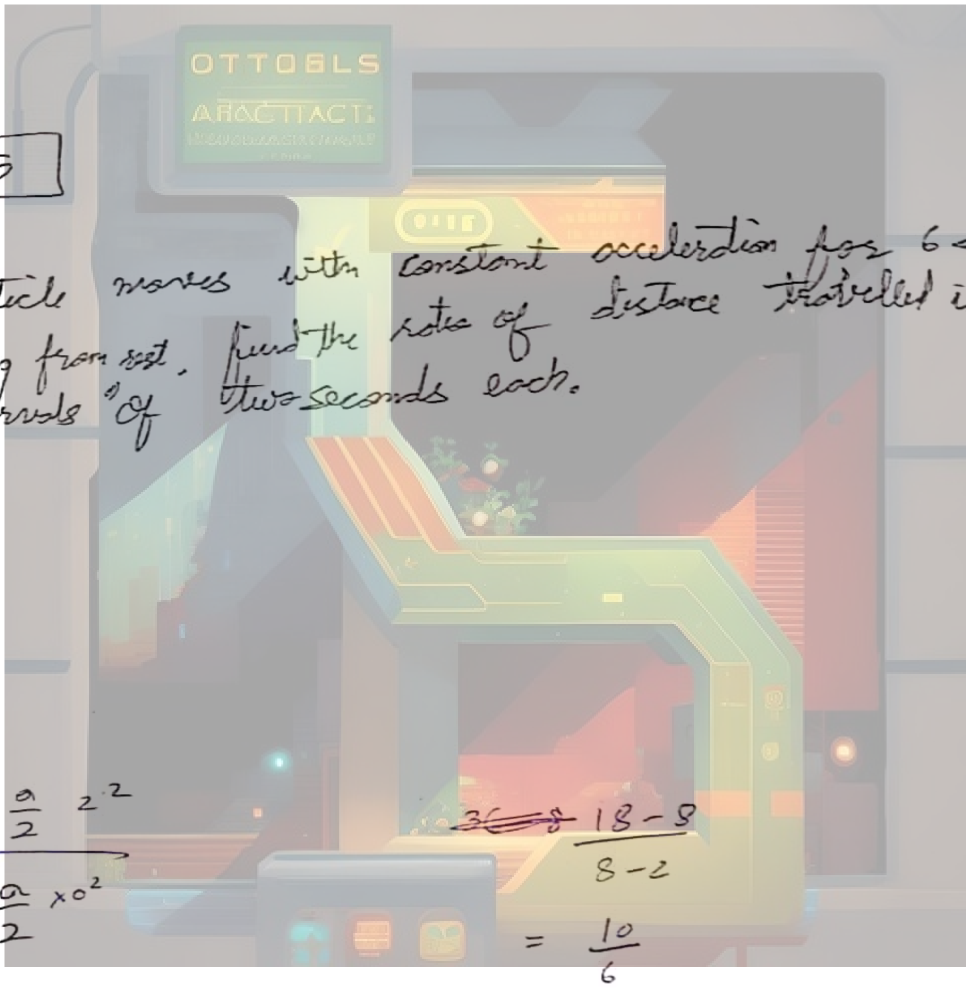
Q4. A body starting from rest moves with constant acceleration. Find the ratio of distance covered during fifth to that covered in five seconds.

$$= \frac{\frac{a}{2}(2n-1)}{\frac{1}{2}an^2}$$

$$= \frac{a(10-1)}{2\times 5^2}$$

$$= \frac{9}{25}$$

$\boxed{9:25}$



Q5. A particle moves with constant acceleration for 6 sec. after starting from rest. Find the ratio of distance travelled it during the intervals of two seconds each.

$a = a$

$t = 6$

$u = 0$

$$\frac{S_4 - S_2}{S_2 - S_0}$$

$$\frac{\frac{a}{2}\times 4^2 - \frac{a}{2}\, 2^2}{\frac{a}{2}\times 2^2 - \frac{a}{2}\times 0^2}$$

$$\frac{8a - 2a}{2a}$$

$$= \frac{6a}{2a}$$

$\boxed{= 3:1}$

$$\cancel{36-8}\ \frac{18-8}{8-2}$$

$$= \frac{10}{6}$$

$$= \frac{5}{3}$$

$\cancel{\boxed{5:3:1}}$

$\boxed{1:3:5}$

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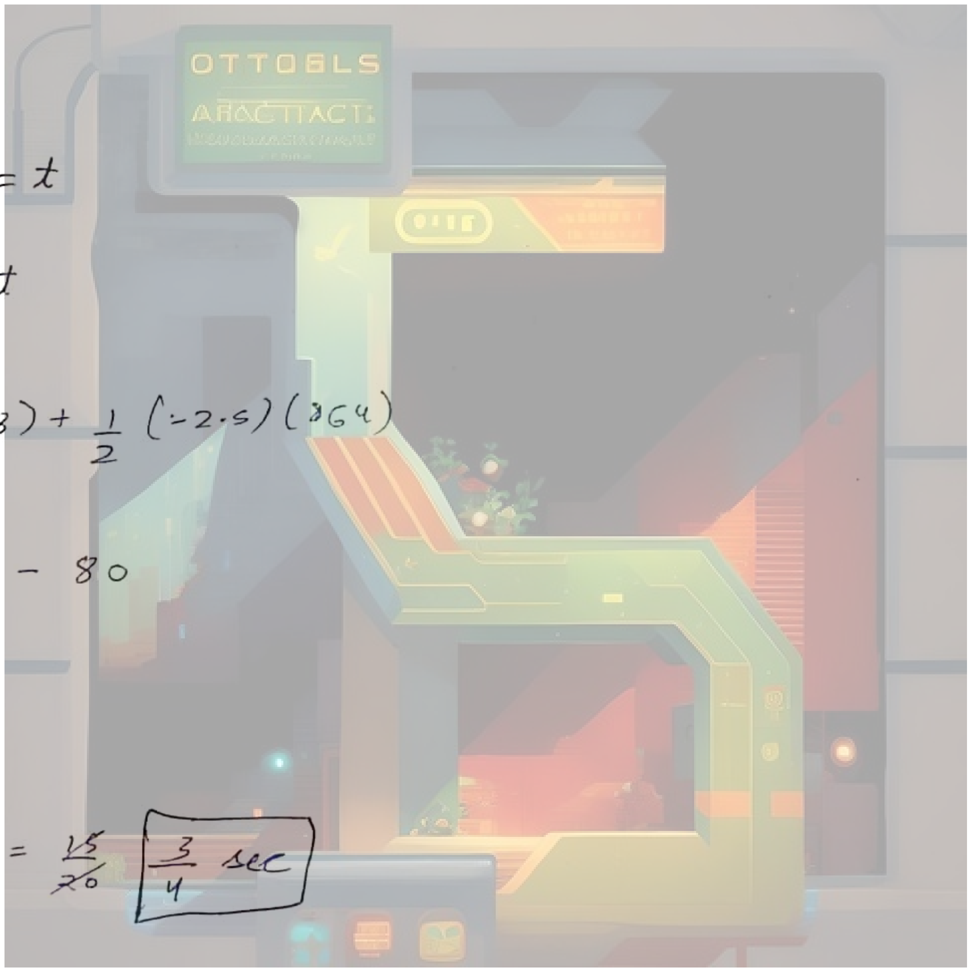
Q6. A driver driving a truck at a constant speed of 20 m/s suddenly saw a parked car ahead of him by 95 m. He could apply the brake after some time to produce retardation of $2.5\ m/s^2$. If an accident was ~~just~~ just avoided, find his reaction time.

$a = -2.5\ m/s^2$

$u = 20\ m/s$

$v = 0$

$S = 95\ m$



$$\frac{0 - 20}{-2.5} = t$$

$8\ s = t$

$S = 20(8) + \frac{1}{2}(-2.5)(64)$

$S = 160 - 80$

$S = 80\ m$

$t = \frac{15}{20}$ $\boxed{\frac{3}{4}\ sec}$

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Q 7. A body starts its motion with initial velocity of 9 m/s towards east and its acceleration is 2 m/s² west. find distance covered in fifth second of its motion.

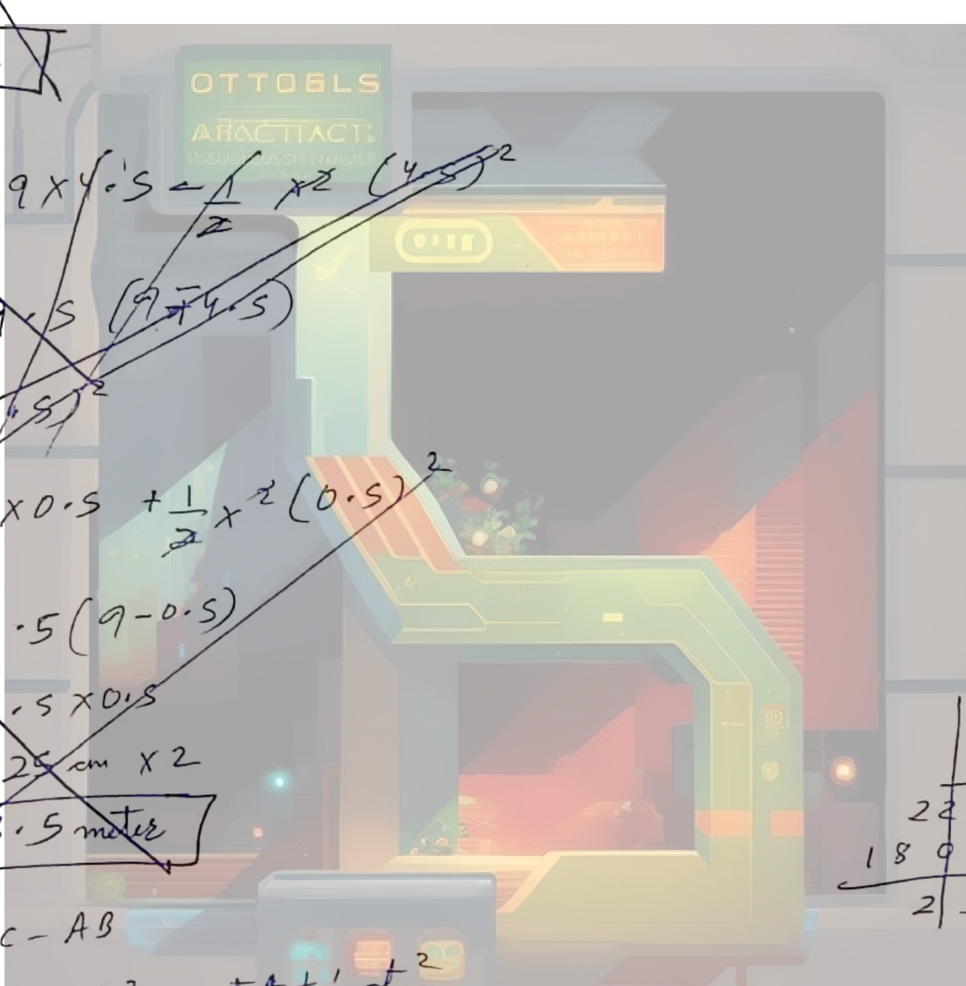
$u = 9\ m/s$

$a = -2\ m/s^2$

$S_{5^{th}} = 9 + \frac{-2}{2}(10-2)$

$= 9 - 8$

$\boxed{= 1\ meter}$



$S = 9 \times 4.5 - \frac{1}{2} \times 2 (4.5)^2$

$= 4.5 (9 - 4.5)$

$= (4.5)^2$

$S = 9 \times 0.5 + \frac{1}{2} \times 2 (0.5)^2$

$= 0.5 (9 - 0.5)$

$= 8.5 \times 0.5$

$= 4.25\ cm \times 2$

$\boxed{= 8.5\ meter}$

4.5×4.5: 225, 1800, 20.25

$S = AC - AB$

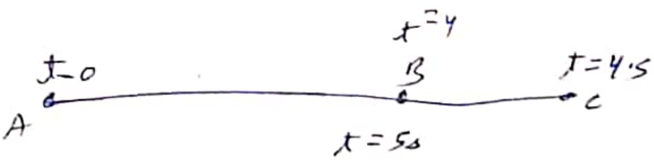
$= ut + \frac{1}{2}at^2 - ut + \frac{1}{2}at^2$

$= 9 \times 4.5 - \frac{1}{2} \times 2 (4.5)^2 - 9 \times 4 - \frac{1}{2} \times 2 (4)^2$

$= 4.5 \times 4.5 - 20$

$= 20.25 - 20$

$= \frac{1}{4} \times 2$



$\boxed{= \frac{1}{2}\ meter}$

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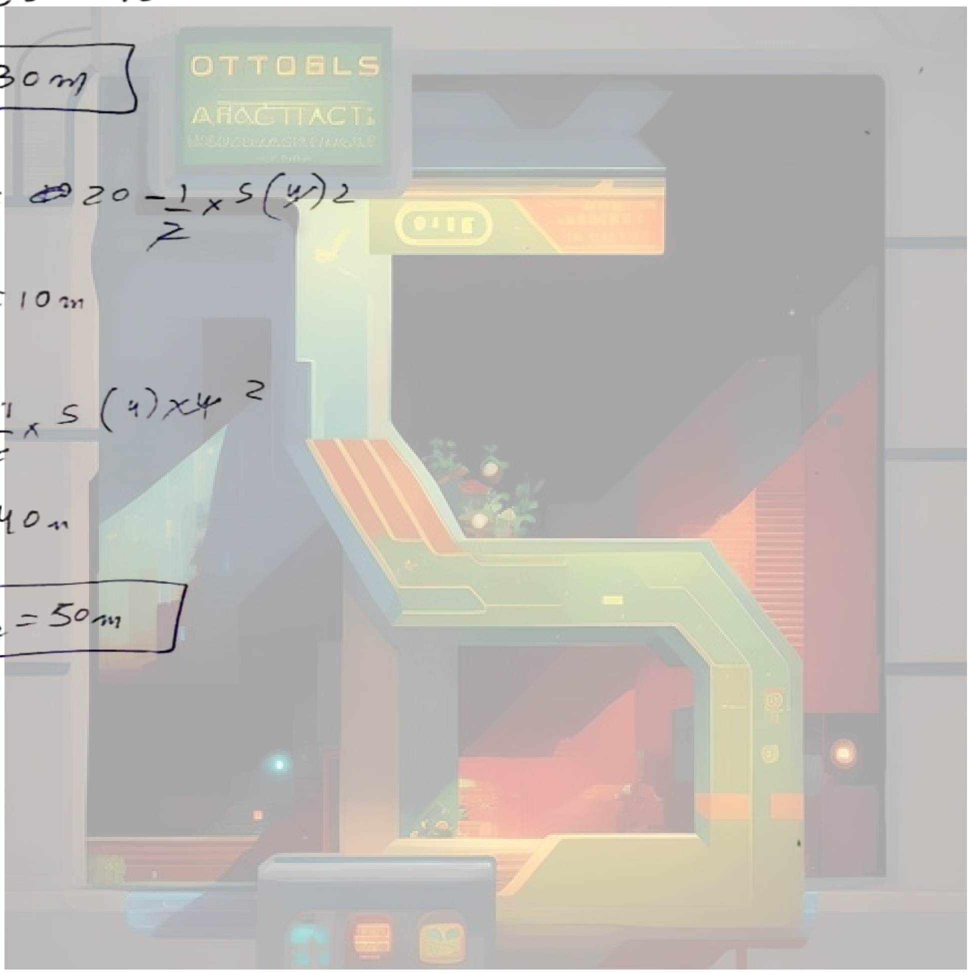
Q 8. A particle starts with velocity 10 m/s & deceleration 5 m/s² find displacement & distance covered in 6 s.

$S = \frac{1}{2} \times 5 \times 36/18$

$\cancel{\boxed{S = 90\ m}}$

$S = 60 - 90$

$\boxed{S = -30\,m}$



$S_1 = 20 - \frac{1}{2} \times 5 (\cancel{4})2$

$S_1 = 10\,m$

$S_2 = \frac{1}{2} \times 5 (4) \times 4^2$

$S_2 = 40\,m$

$\boxed{S_1 + S_2 = 50\,m}$

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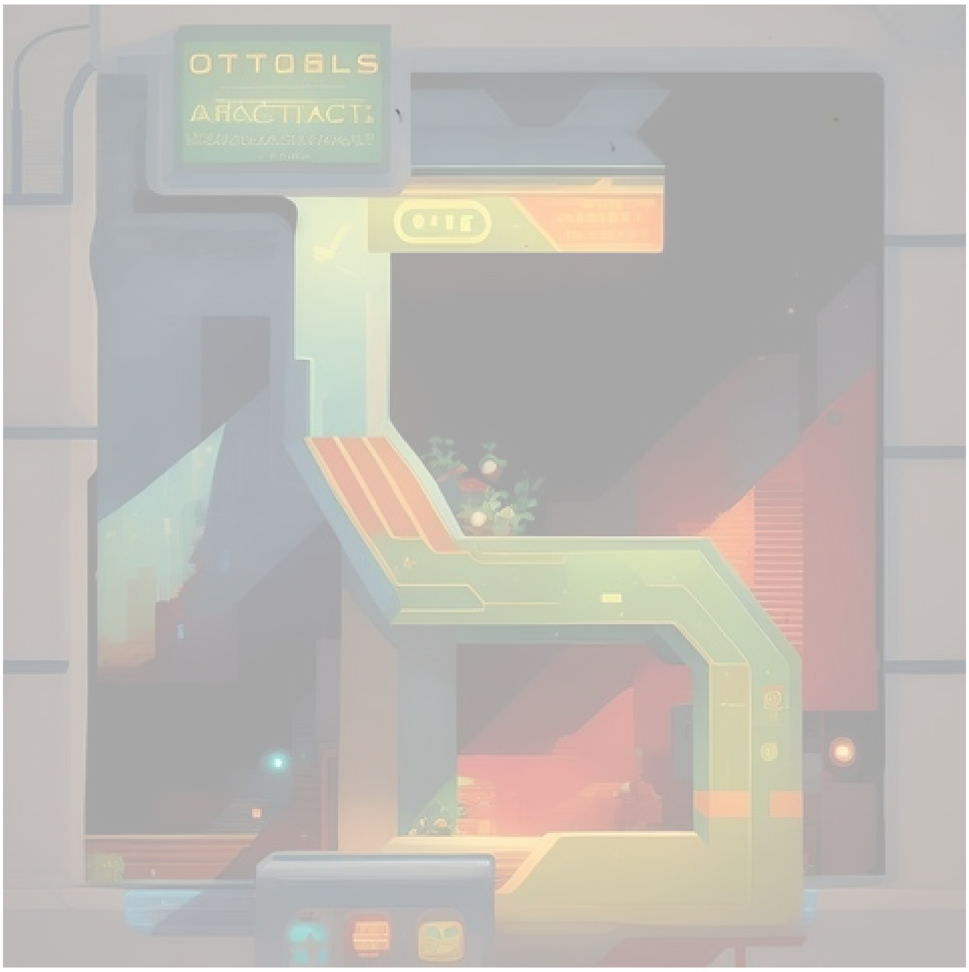
Pg 47 → Q1-25



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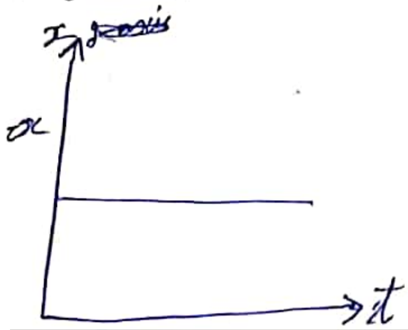
Graphs

1) Position-Time Graph (x-t graph)

a) At rest



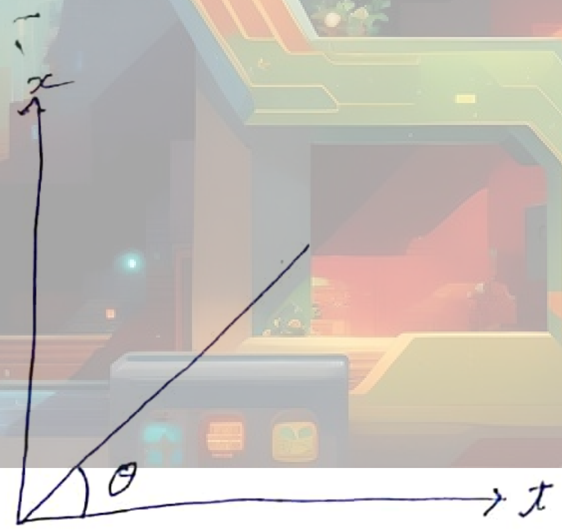
→ straight line parallel to ~~x~~-time axis

→ slope $= \frac{dx}{dt} = v = 0$

$= \tan\theta = \tan 0^\circ = 0$

↳ angle with time axis.

b) Uniform velocity



slope $= \tan\theta =$ constant

$= \frac{dx}{dt} = v =$ constant

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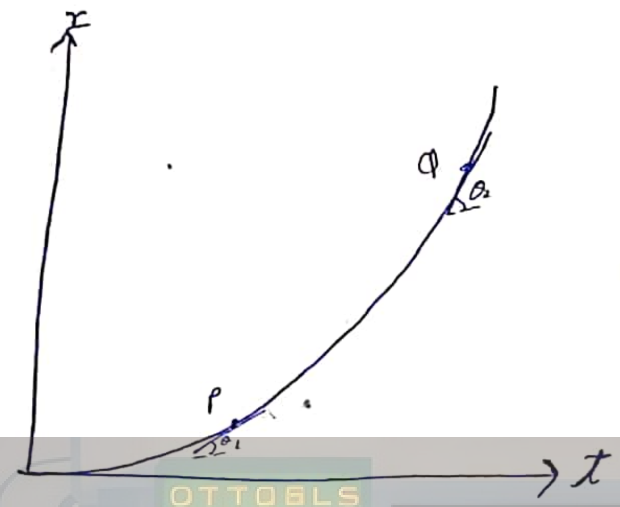
iii) Non-Uniform Velocity (increasing with time) :-



Smiling Curve
(a is > 0)

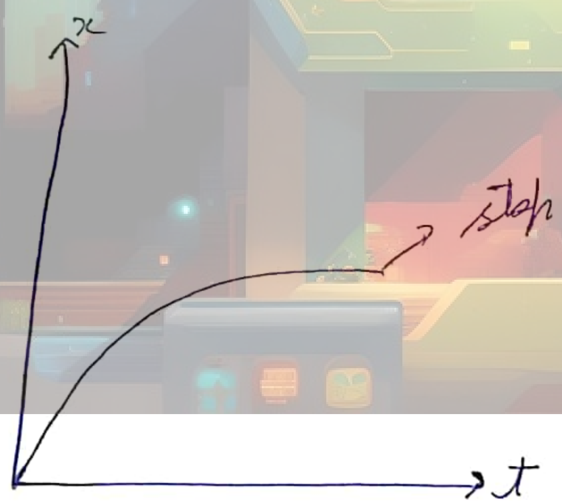
$\theta_1 < \theta_2$

$\tan\theta_1 < \tan\theta_2$

$v_1 < v_2$

so velocity is increasing.

iv) Non-Uniform Velocity (decreasing with time)



body finally stops

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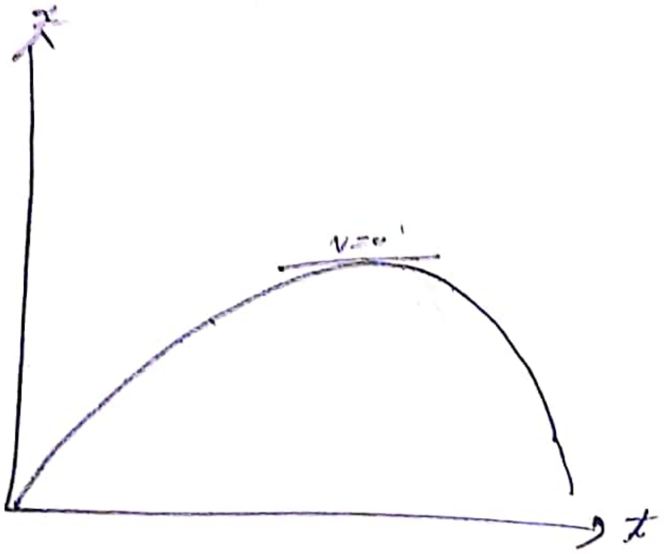

Crying curve
$(a<0)$

$$\frac{d^2x}{dt^2} < 0$$

body reverse direction

Q1.

I - velocity >0, acceleration <0

II - $v<0$, $a<0$

III - $v>0$, $a>0$

IV - $v<0$, $a>0$

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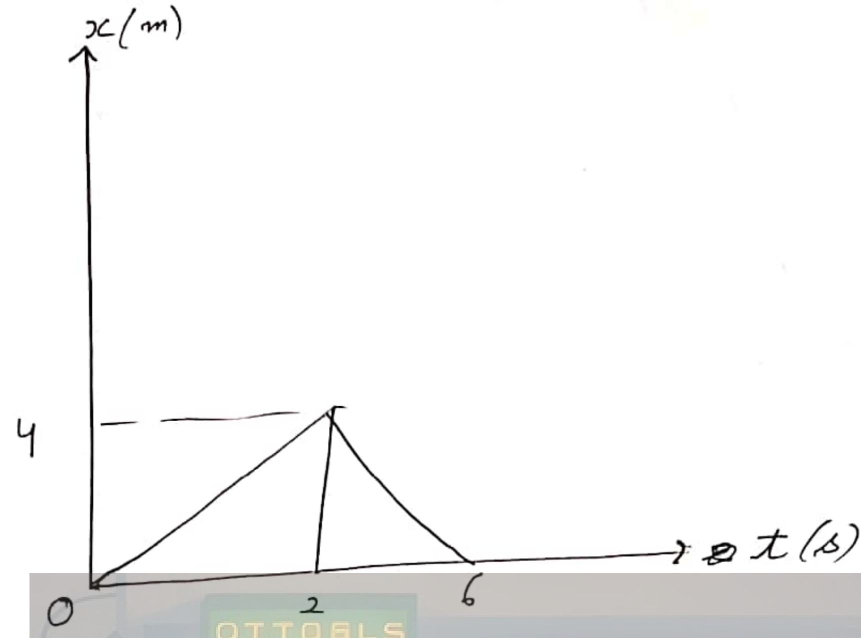
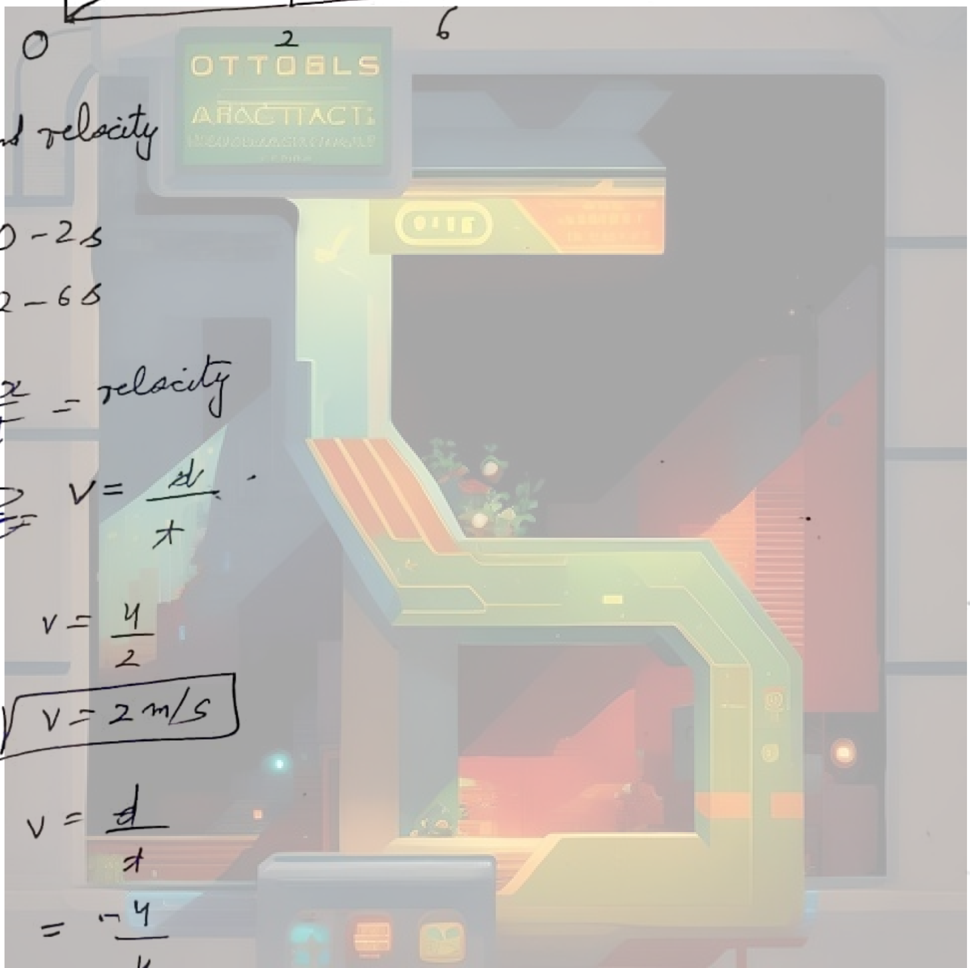
Q2.





find velocity

a) 0-2s

b) 2-6s

a) $\frac{dx}{dt}$ = velocity

~~$v = \frac{d}{dt}$~~ $v = \frac{d}{t}$.

$v = \frac{4}{2}$

$\boxed{v = 2\ m/s}$

b) $v = \frac{d}{t}$

$= \frac{-4}{4}$

$\boxed{v = -1\ m/s}$

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2) Velocity - Time Graph (v-t graph)

a) Uniform Velocity -

$v = \text{constant}$

$a = 0$

$$\frac{dv}{dt} = 0$$

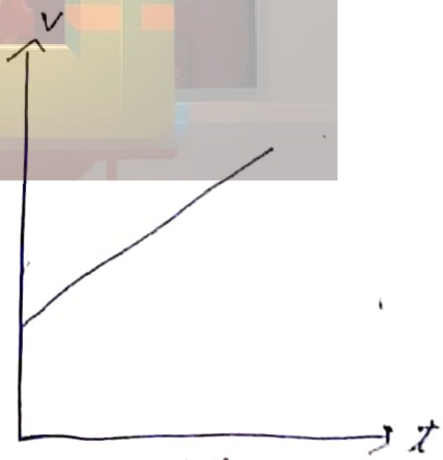
$$\text{slope} = \frac{dv}{dt} = 0$$

6) Uniform Velocity

$a = \text{const}$

$$\frac{dv}{dt} = \text{constant} = \text{slope}$$

$u = 0$



$u \neq 0$

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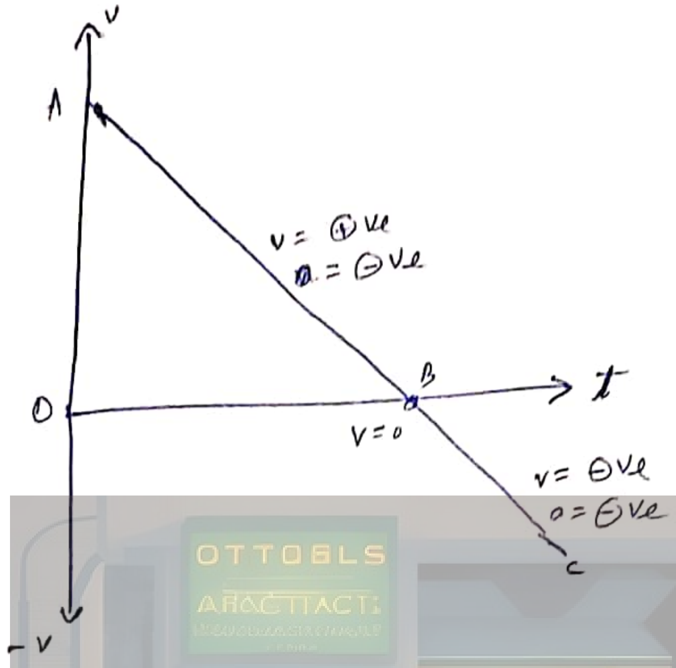
c) Uniform ~~Acceleration~~ Retardation

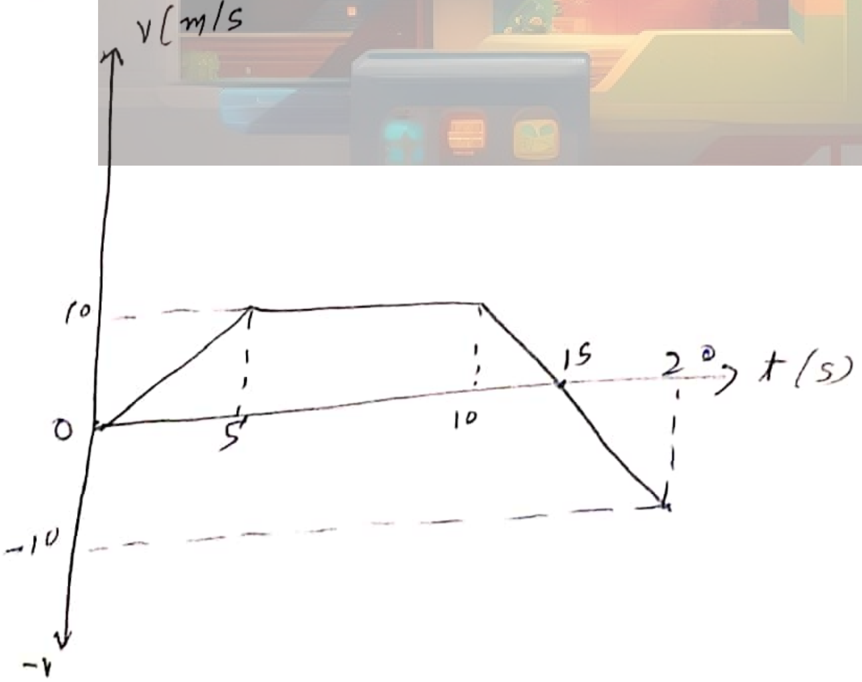


A → B
v = +ve & decreasing

B → v = 0 & after that body reversed direction

B → C body is increasing speed in ⊖ve direction.

Q Calculate distance travelled & ~~co~~ & displacement of body. find acceleration from 0 – 5 s & 10 – 15 s.



~~dos~~

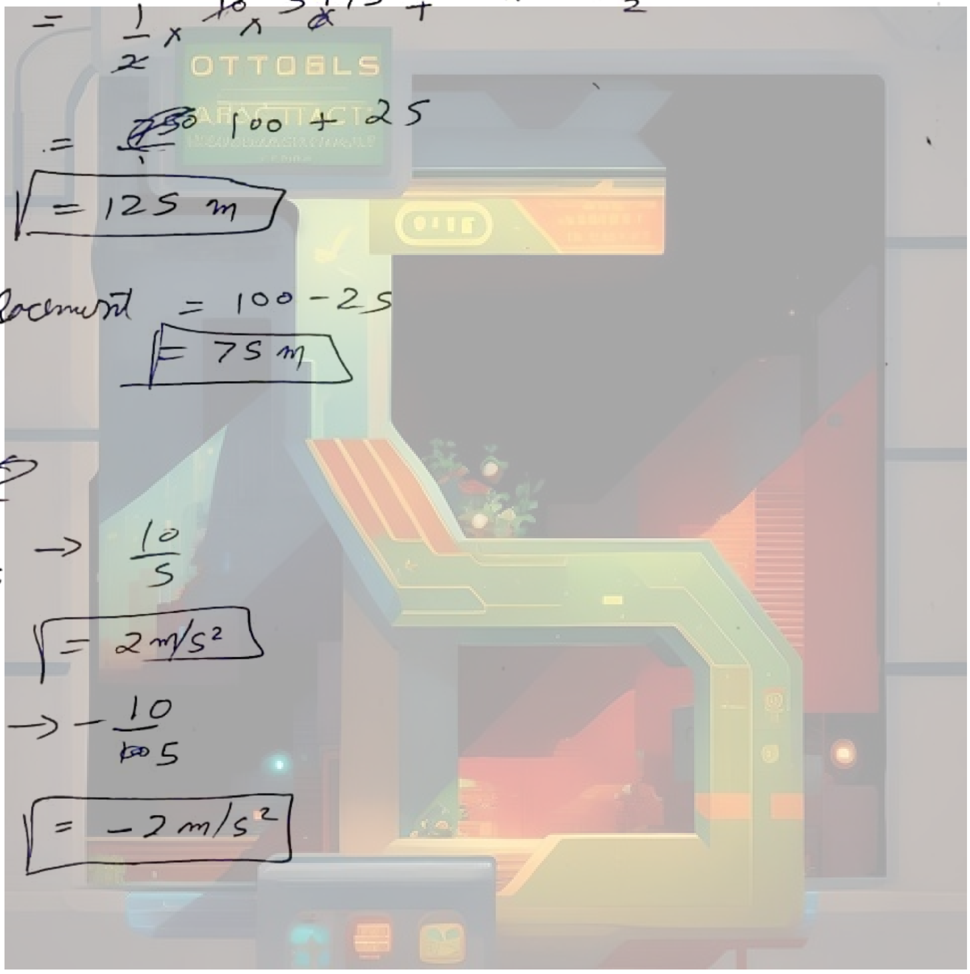
$\cancel{s \text{(displacement)} = \frac{1}{\cancel{2}} \times \frac{10}{\cancel{5}} \times 20 \times \cancel{20}\,\cancel{4}\,2}$

$\cancel{= 200 \times 2}$

$\cancel{= 400}$

$\cancel{S_1 = \frac{1}{2} \times \frac{10}{5} \times 15 \times 15}$

Distance $= \frac{1}{\cancel{2}} \times \cancel{10}^{5} \times \frac{5+15}{\cancel{2}} + 10 \times 5 \times \frac{1}{2}$



$= 100 + 25$

~~dos~~ $\boxed{= 125\ m}$

Displacement $= 100 - 25$

$\boxed{= 75\ m}$

~~a =~~

$a_{0-5} \rightarrow \frac{10}{5}$

$\boxed{= 2\ m/s^2}$

$a_{10-15} \rightarrow -\frac{10}{5}$

$\boxed{= -2\ m/s^2}$

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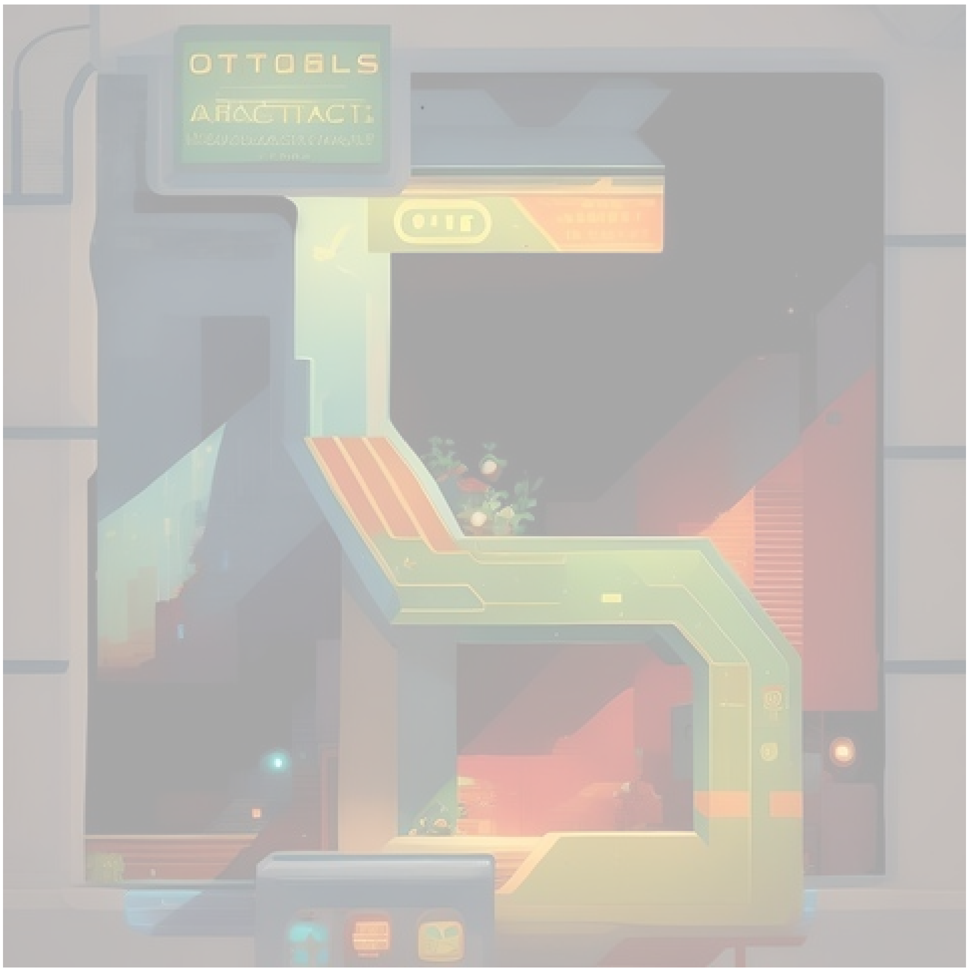
Pg 55 – Q1-15



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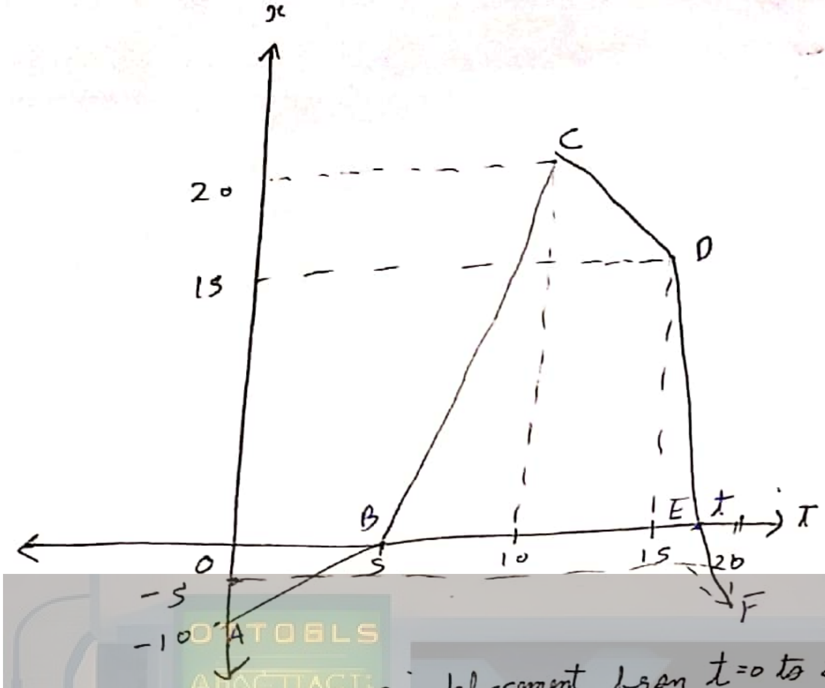
Q 1.



a) find distance covered & Displacement from t=0 to 20s.

b) find average speed & velocity from t=0 to ~~15~~ 15 sec.

c) find velocity of particle in t=5 to 10s

d) find velocity of particle in t=10 to 15s

a) Distance = 10 + 20 + 5 + 15 + 5

$\boxed{= 55 m}$ ✓

Displacement $\boxed{= +5 m}$ ✓

e) find t

f) How many time particle has taken turn during t=0 to 20s

b) average speed = $\frac{35}{\cancel{12}0}$ = $\boxed{\frac{7}{3} m/s}$

~~= 2.75 m/s~~

average velocity = $\cancel{\frac{+5}{20}}$ $\qquad \frac{25}{15} = \boxed{\frac{5}{3} m/s}$

~~= +0.25 m/s~~

(166)

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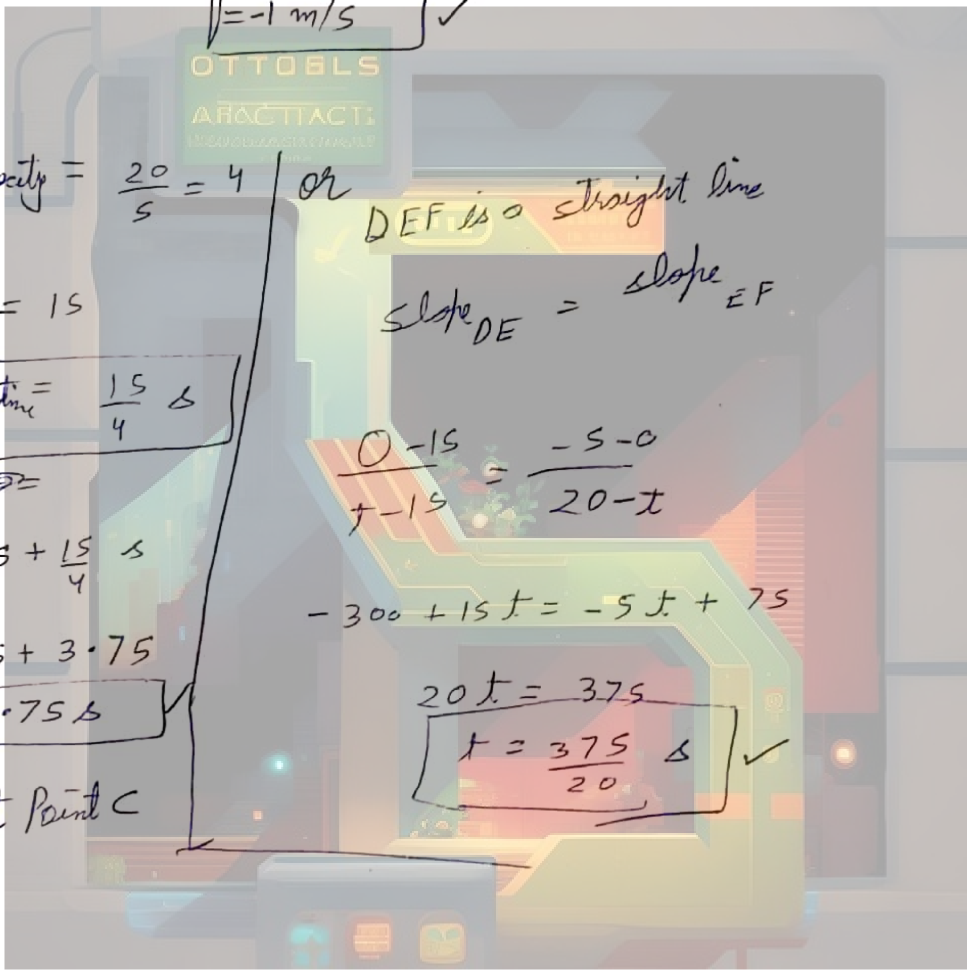
c) $\text{velocity}\Big|_{t=5 \text{ to } 10s} = \dfrac{\text{distance}}{\text{Time}}$

$$= \frac{20}{5}$$

$$\boxed{= 4 \text{ m/s}}$$

d) $\text{velocity}\Big|_{t=10-15s} = \dfrac{-5}{5}$

$$\boxed{= -1 \text{ m/s}}$$



e) ~~speed~~ velocity $= \dfrac{20}{5} = 4$

dis $= 15$

$\boxed{\text{time} = \dfrac{15}{4} \text{ s}}$

$t = 15 + \dfrac{15}{4}$ s

$= 15 + 3.75$

$\boxed{= 18.75 \text{ s}}$

or

DEF is a straight line

$\text{slope}_{DE} = \text{slope}_{EF}$

$$\frac{0-15}{t-15} = \frac{-5-0}{20-t}$$

$$-300 + 15t = -5t + 75$$

$$20t = 375$$

$$\boxed{t = \frac{375}{20} \text{ s}}$$

f) $\boxed{1 \text{ time}}$ at Point C

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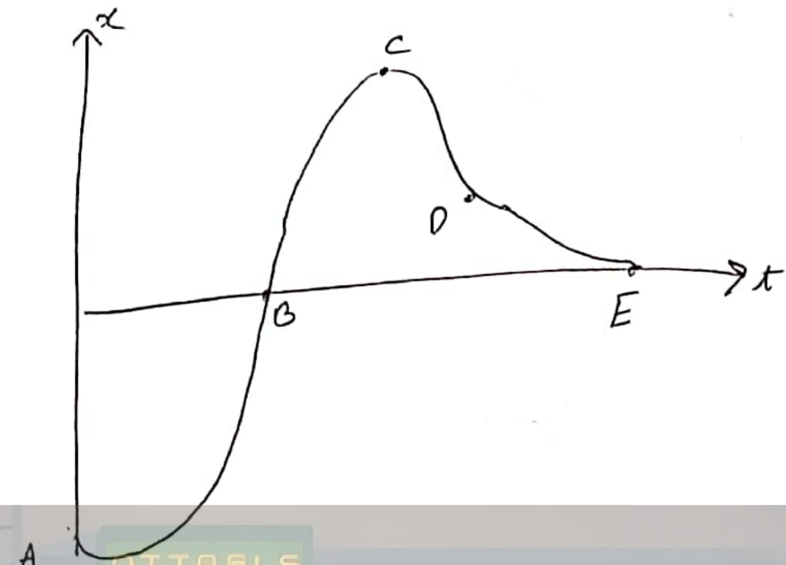
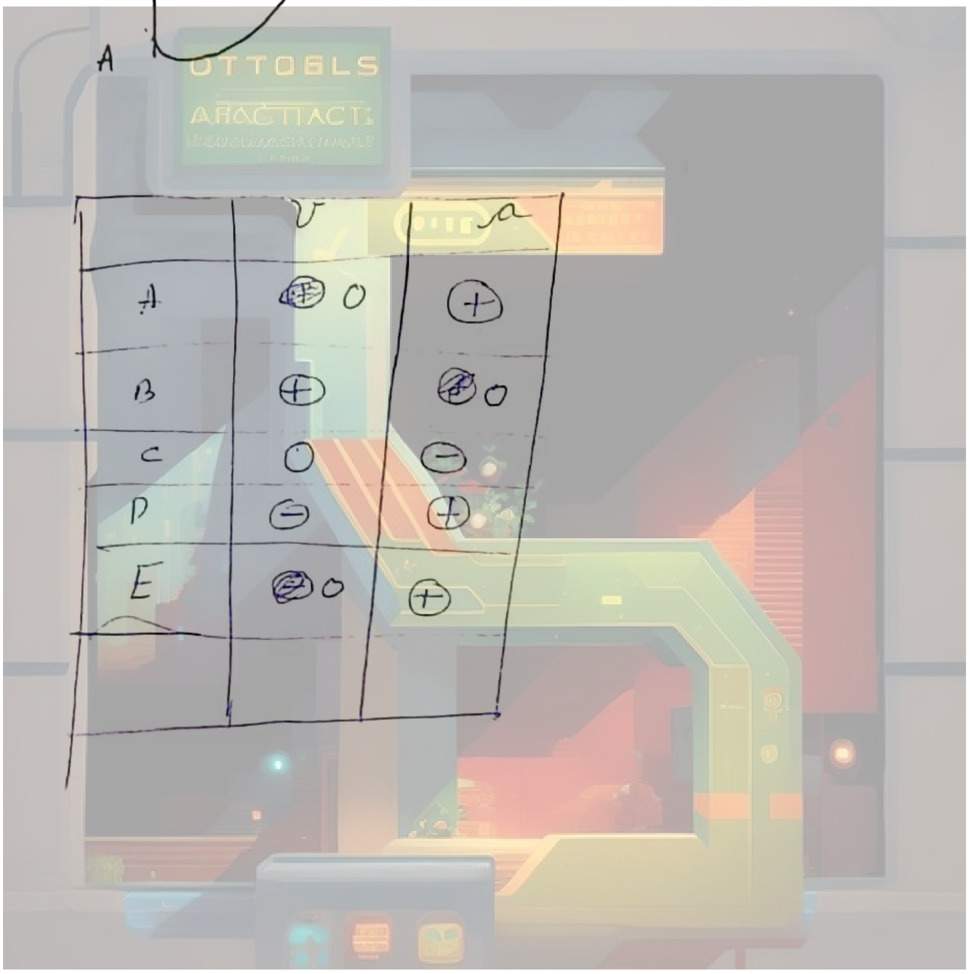
Q2.





	v	a
A	0	+
B	+	0
C	0	−
D	−	+
E	0	+

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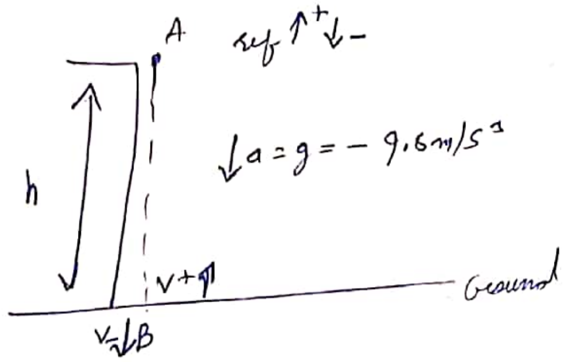
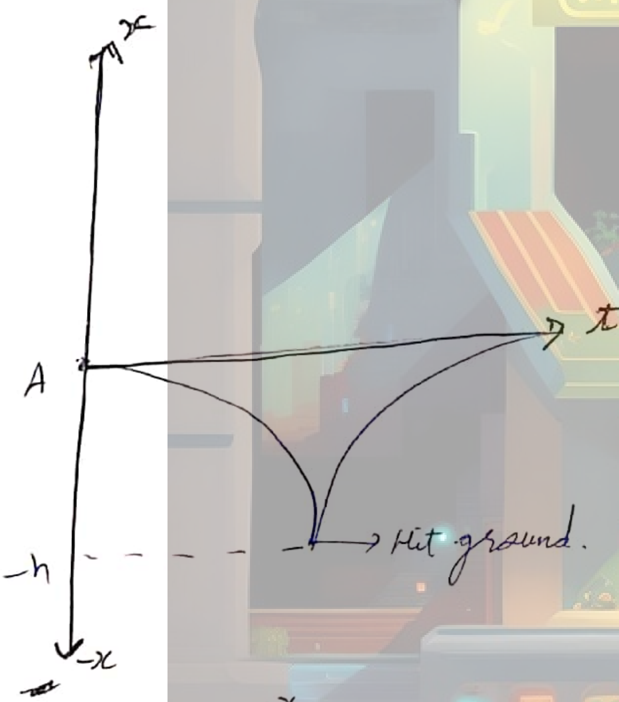
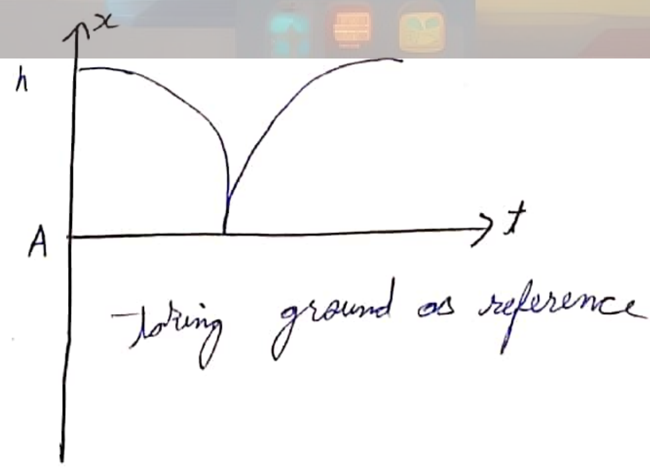
Q3. A body is dropped from a height, It strikes the ground elastically. draw all the 3 kinematic graphs taking upward direction as +ve.



A → B speeding up
B → A slowing down





taking ground as reference

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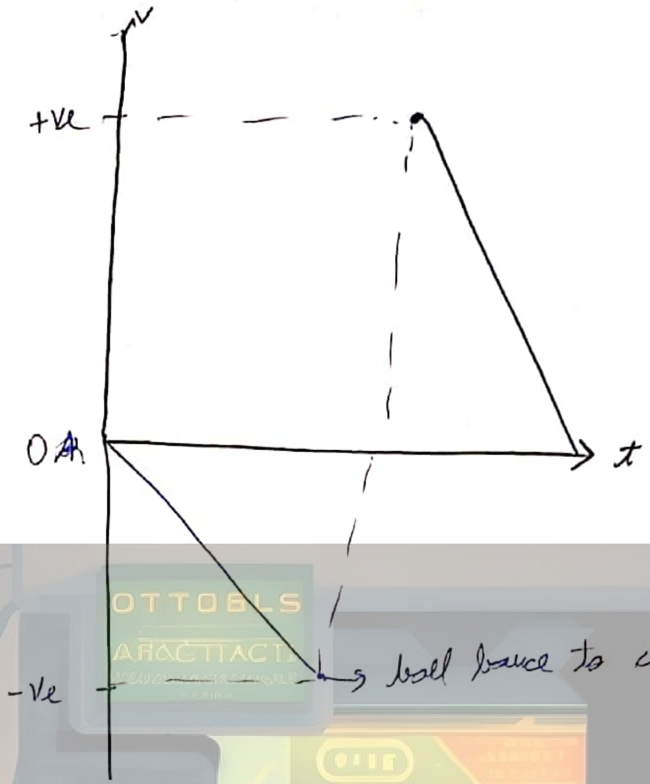
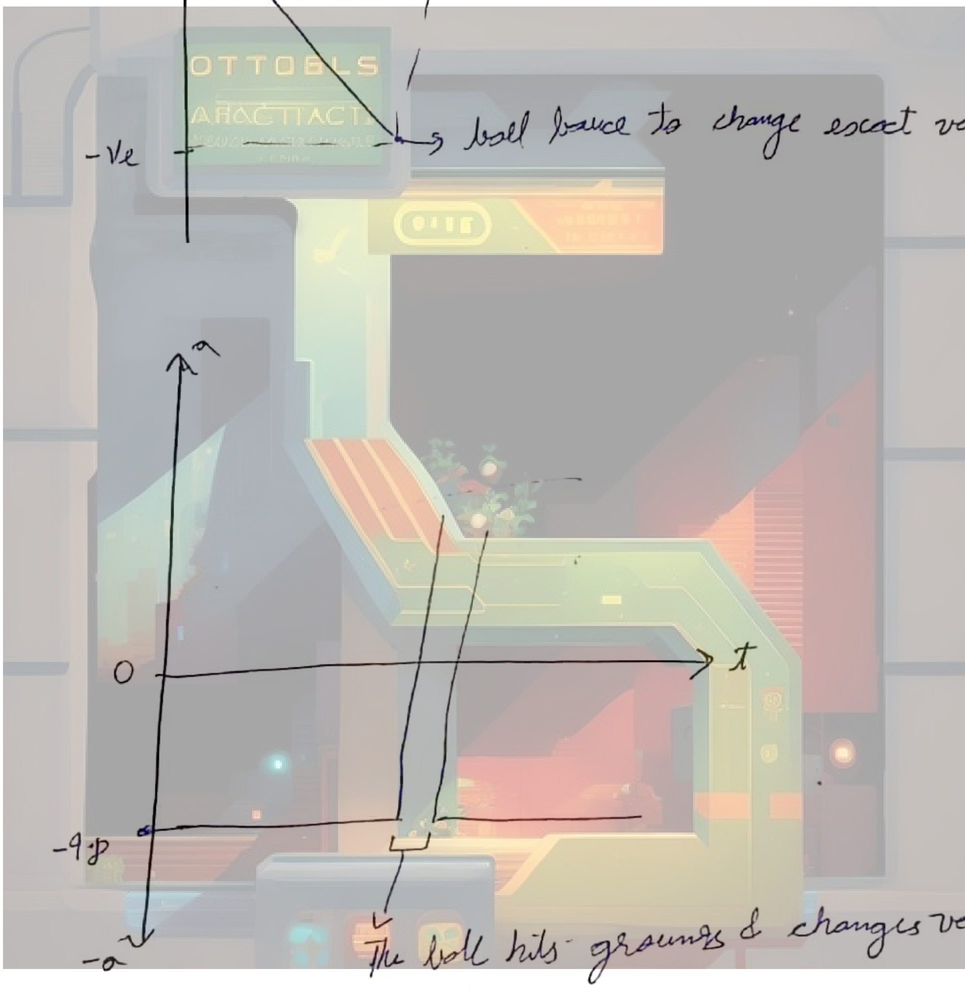


→ ball bounce to change exact velocity to ⊖ve



The ball hits grounds & changes vel from
$-v$ to $+v$

$$a = \frac{\Delta v}{\Delta t \to 0}$$

so a = infinite in +ve direction

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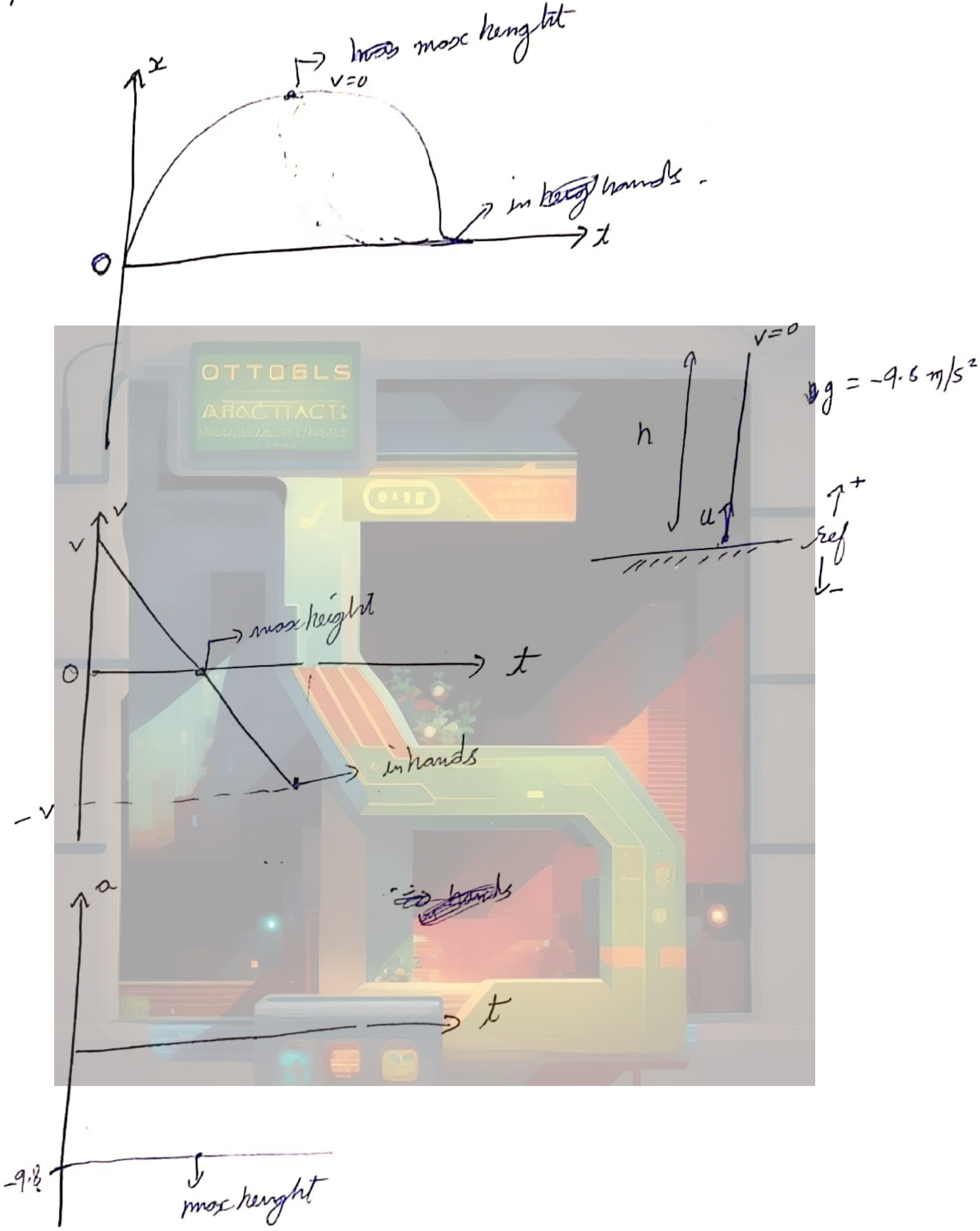
Q:- A body thrown upwords with same velocity. It reaches max height and again reaches hands of thrower. Draw all 3 kinematic graphs (take upward as +ve)



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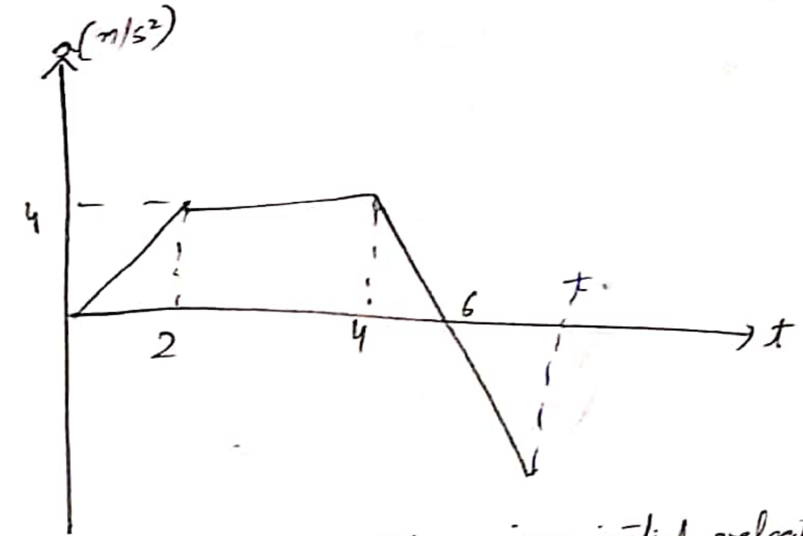
Q 5.



a) find time at which particle aquires initial velocity.

initial velocity (u) = area $(+ve)$

$= \frac{1}{2} \times (2+6) \times 4$

$\boxed{= 16\ m/s}$

$v = 16\ m/s$

$\frac{1}{2} \times t \times a = 16$

$t \times a = 32$

$\frac{a}{t} = 2$

$a = 2t$

$2t \times t = 32$

$t^2 = 16$

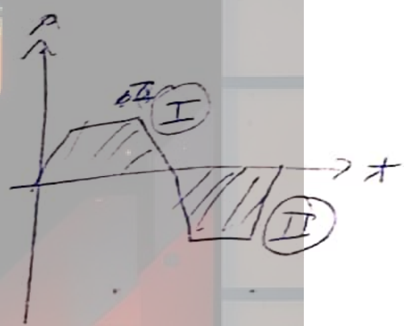
$t = 4$, $\boxed{\cancel{-4}}$

$4 + 6 = t$

~~$10 = t$~~

$\boxed{t = 10\ s}$

theory
area = change in velocity



$\Delta v = I - II$

$\Delta = 0$

$I - II = 0$

$I = II$

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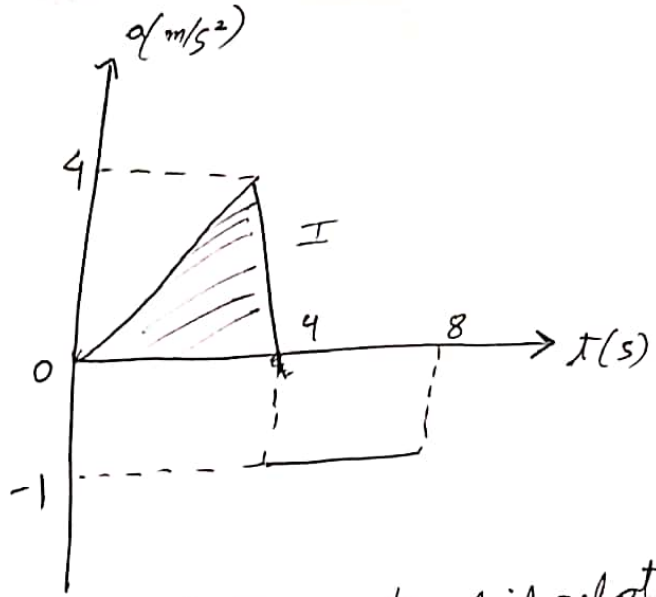
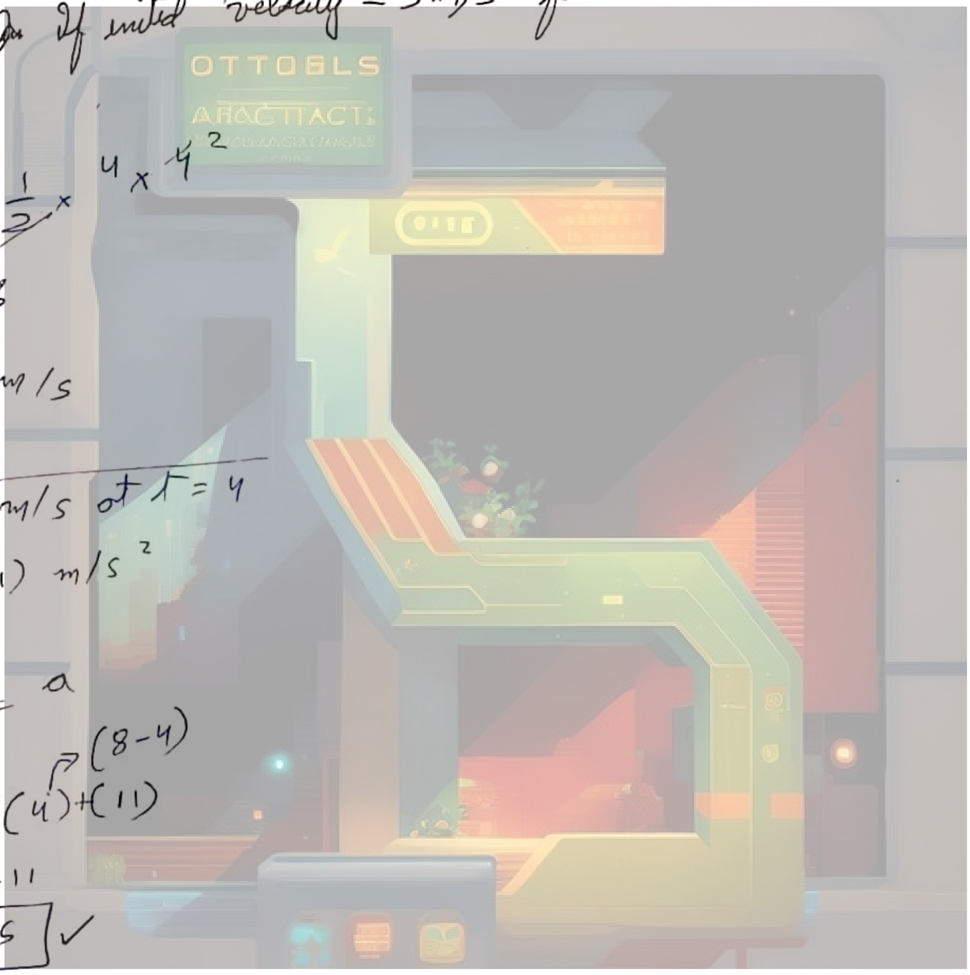
Q6.



b) a) If initial velocity = 3 m/s find vel at t = 8 s



$I = \frac{1}{2} \times 4 \times 4$

$= 8$

$\Delta v = 8 \text{ m/s}$

$u = 3$

$v = 11 \text{ m/s}$ at $t = 4$

$a = (-1) \text{ m/s}^2$

$\frac{v-u}{t} = a$

$v = (-1)(4) + (11)$ ← (8-4)

$v = -4 + 11$

$\boxed{v = 7 \text{ m/s}}$ ✓

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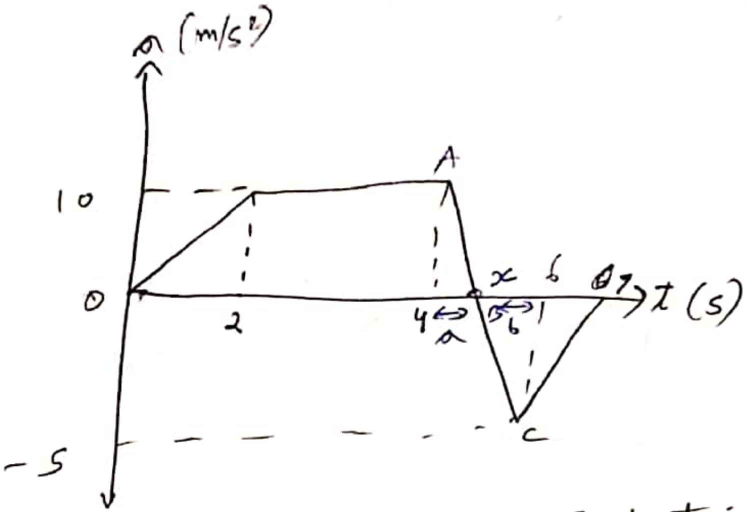
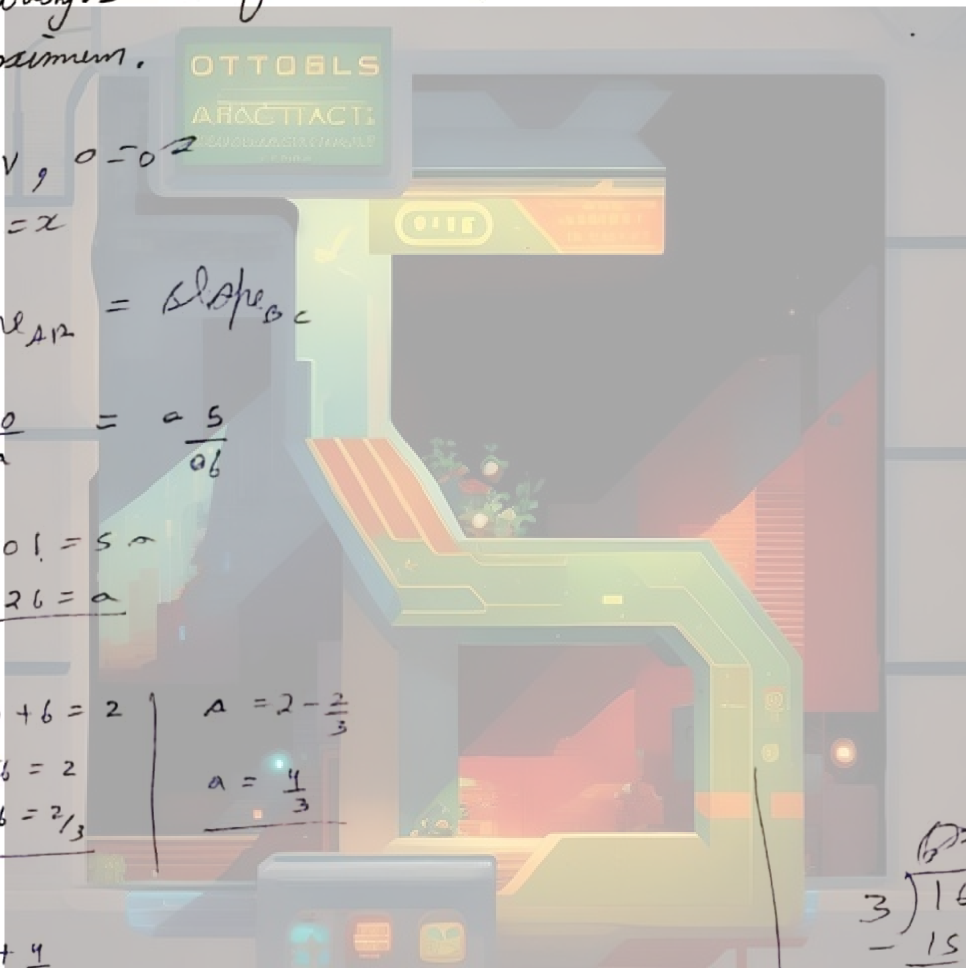
Q 7.



If body starts from rest find time at which velocity is maximum.



max v, $a = 0$

$v = x$

$slope_{AB} = slope_{BC}$

$\frac{10}{a} = \frac{5}{b}$

$10b = 5a$

$2b = a$

$a + b = 2$

$3b = 2$

$b = 2/3$

$a = 2 - \frac{2}{3}$

$a = \frac{4}{3}$

$4 + \frac{4}{3}$

$\frac{12+4}{3}$

$\boxed{\frac{16}{3}\,s}$

$\boxed{5.33\,s}$

$16 \div 3 = 5.3$

-15

10

-9

10

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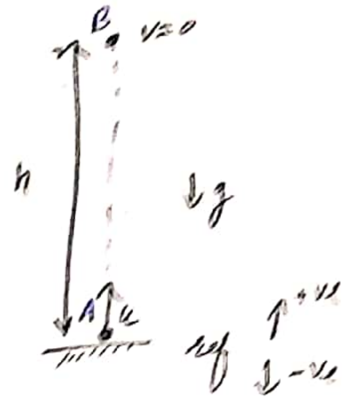
Motion Under Gravity

① Case 1 →



→ Take point of projection as reference.

$$0-(u)^2 = 2(-g)(+h)$$

$$\boxed{h = \frac{u^2}{2g}}$$ (maximum hight)

→ Time of flight = time of ascent + time of descent

$v = u + at$

A → B

$0 = u + (-g)t$

$$\boxed{t_1 = \frac{u}{g}}$$

B → A

$s = ut + \frac{1}{2}at^2$

$-h = 0 + \frac{1}{2}(-g)t^2$

$\frac{u^2}{2g} = \frac{1}{2}gt^2$

$\frac{u^2}{2g} \times \frac{2}{g} = t^2$

$\frac{u^2}{g^2} = t^2$

$$\boxed{t_2 = \frac{u}{g}}$$

time of ascent = time of descent

$$\boxed{\text{time of flight} = \frac{2u}{g}}$$

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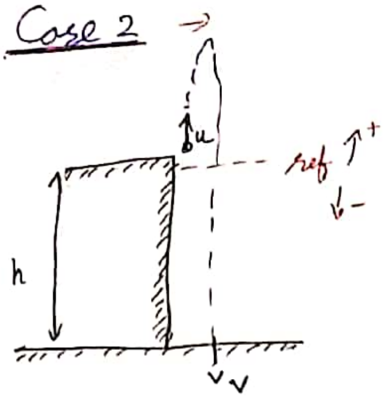
(2) Case 2 →



$$v^2 = u^2 + 2as$$

$$(-v)^2 = u^2 + 2(-g)(-h)$$

$$v^2 = u^2 + 2gh$$

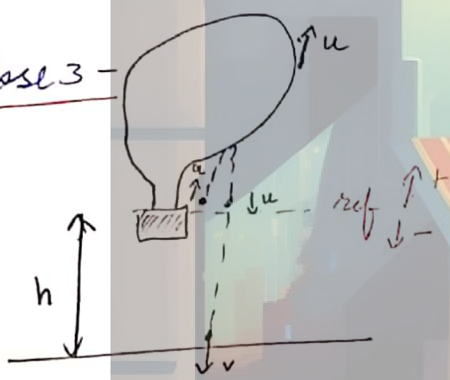
$$\boxed{v = \sqrt{u^2 + 2gh}}$$

→ To calculate time of flight

$$s = ut + \frac{1}{2}at^2$$

$$\boxed{-h = ut + \frac{1}{2}(-g)t^2}$$

(3) Case 3 –



u = velocity of balloon.

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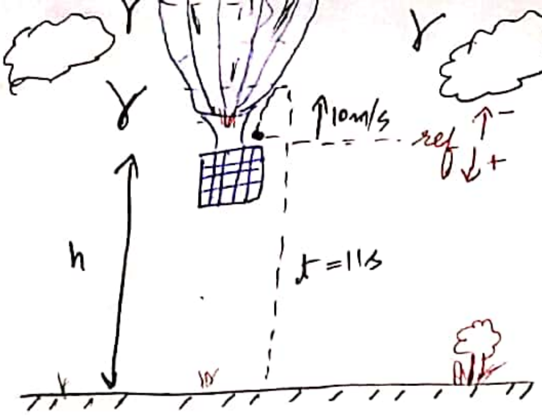
Q



A packet is released,

$g = 10\ m/s^2$

$h = ?$

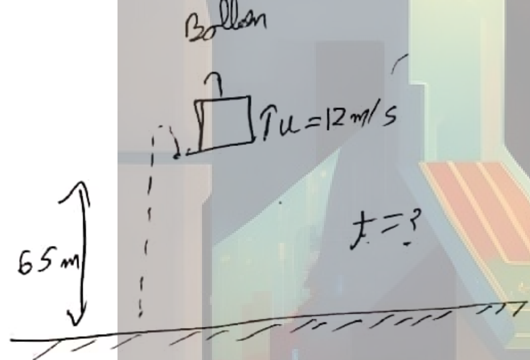
$s = ut + \frac{1}{2} at^2$

$h = (-10)(11) + \frac{1}{2} \times 10 \times 121$

$h = -110 + 605$

$\boxed{h = 495m}$

Q2.



$65 = -12t + \frac{1}{2} \times 10t^2$

$65 = -12t + 5t^2$

$5t^2 - 12t - 65$

$t = \frac{+12 \pm \sqrt{144 + 1300}}{10}$

$t = \frac{+12 \pm \sqrt{1444}}{10}$

$t = \frac{+12 + 38}{10}$

$t = \frac{+50}{10}$

$\boxed{t = 5s}$

268
3 | 1444
9
544

2
65
5
325
4
67
7
469
6
69
9
544

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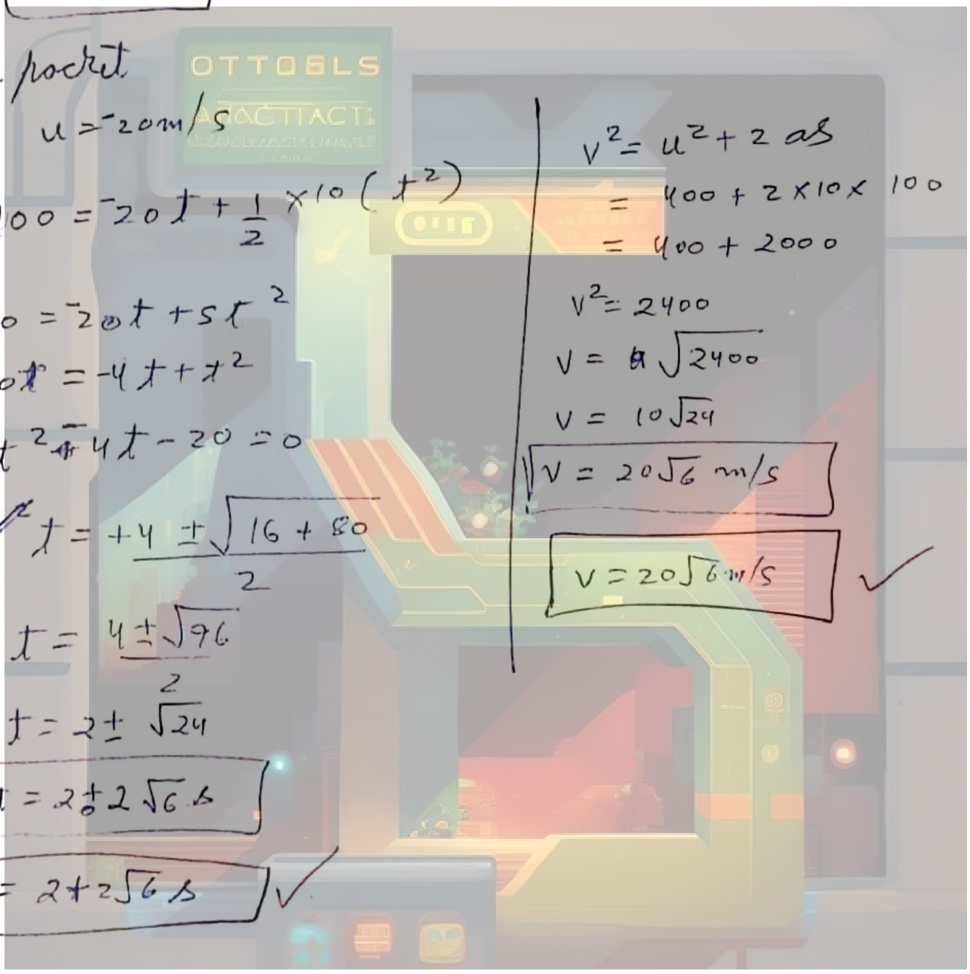
Q3. A balloon starts ascending from rest at constant ~~acc~~ $a = 2\,m/s^2$ when it was at 100 m height a pocket was released from it. after ~~tim~~ how much time and with what velocity will it hit ground.

$\cancel{u=0}$

$v^2 - 0 = 2 \times 2 \times 100$

$v^2 = 400$

$\boxed{v = 20\,m/s}$



for pocket

$u = -20\,m/s$

$100 = -20t + \frac{1}{2} \times 10\,(t^2)$

$100 = -20t + 5t^2$

$20 = -4t + t^2$

$t^2 - 4t - 20 = 0$

$t = \frac{+4 \pm \sqrt{16 + 80}}{2}$

$t = \frac{4 \pm \sqrt{96}}{2}$

$t = 2 \pm \sqrt{24}$

$\boxed{t = 2 \pm 2\sqrt{6}\,s}$

$\boxed{t = 2 + 2\sqrt{6}\,s}$

$v^2 = u^2 + 2as$

$= 400 + 2 \times 10 \times 100$

$= 400 + 2000$

$v^2 = 2400$

$v = \sqrt{2400}$

$v = 10\sqrt{24}$

$\boxed{v = 20\sqrt{6}\,m/s}$

$\boxed{v = 20\sqrt{6}\,m/s}$

8

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Q4. A parachutists bails out from an aeroplane and after 40 m, he ~~S = 40 m~~ opens parachute and decelerates at $2\ m/s^2$. If he reaches ground with speed $2\ m/s$, How long was he in air. At what height he jumped off the plane? $(g = 10\ m/s^2)$

~~$v = 2\ m/s$~~

$S = 40\ m$

$u = 0$

$g = 10\ m/s$

$v^2 = 2 \times 10 \times 40$

$v^2 = 800$

$\boxed{v = 20\sqrt{2}}$

$40 = \frac{1}{2} \times 10 \times t^2$

$80 = t^2$

$t^2 = 8$

$\boxed{t_1 = 2\sqrt{2}}$

After Parachute

$u = 20\sqrt{2}$

$a = -2\ m/s$

$v = 2\ m/s$

$\frac{2 - 20\sqrt{2}}{-2} = t$

$-1 + 10\sqrt{2} = t$

$\boxed{t = 10\sqrt{2} - 1}$

$v^2 - u^2 = 2as$

$4 - 800 = 2 \times -2 \times s$

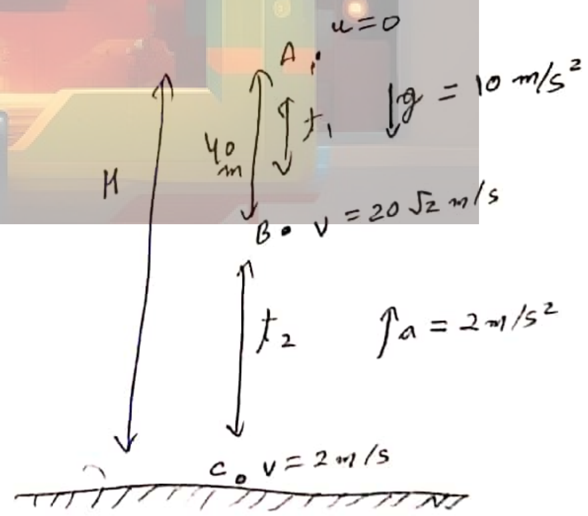
$\frac{796}{4} = s$

$\boxed{S = 199\ m}$

Total height $= 199 + 40 = \boxed{239\ m}$

Total time $= 10\sqrt{2} - 1 + 2\sqrt{2}$

$= \boxed{12\sqrt{2} - 1\ s}$



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Q 5. A body throw from top of a cliff vertically up with velocity v, takes time t_1, the same body when thrown vertically down with same velocity takes time t_2. If dropped freely takes time t, find relation among three times.

$S = \cancel{vt_1 + \frac{1}{2} at_1}$

$S = \frac{1}{2} at^2$

$S = -vt \mp \frac{1}{2} a t_2^2$

$2S = -2vt_1 + \cancel{2}gt_1^2$

$= gt^2$

$= \cancel{-(2vt + at_2^2)}$

$= 2vt_2 - at_2^2$

$\cancel{2vt_1 + gt_1^2 = -2vt_2 - gt_2^2}$

$-h = ut_1 + \frac{1}{2}(-g)(t_1)^2$

$-h = ut_1 - \frac{1}{2} gt_1^2$ ---- ① $\times t_2$

$-h = -ut_2 - \frac{1}{2} g t_2^2$ ---- ② $\times t_1$

$-ht_2 = \cancel{ut_1t_2} - \frac{1}{2} g t_1^2 t_2$

$+ ht_1 = \mp \cancel{ut_1t_2} \mp \frac{1}{2} g t_2^2 t_1$

$+h(t_1 + t_2) = +\frac{1}{2} g\, t_1 t_2 (t_1 + t_2)$

$+h = \frac{t_1 t_2 g}{2}$ ---- ④

$+h = +\frac{1}{2} gt^2$ ---- ③

$\frac{t_1 t_2 g}{\cancel{2}} = \frac{1}{\cancel{2}} gt^2$ (4)

$t_1 t_2 = t^2$

$\boxed{t = \sqrt{t_1 t_2}}$ ✓

(180)

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Q1. A particle is projected vertically up such that it posses a fixed point P after time t_1 and t_2 respectively. Find

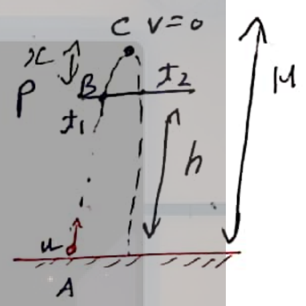
a) Height at which point is located w.r.t point of projection.

b) speed of projection of ball

c) velocity at point P

d) Max height reached by the ball w.r.t point of projection

e) Max height reached by ball w.r.t P.



a) $S = ut_1 + 4.9(t_1)^2$

$h = ut - \frac{gt^2}{2}$

$2h = 2ut - gt^2$

$gt^2 - 2ut + 2h = 0$

$t = \frac{2u \pm \sqrt{4u^2 - (4)(2h)(g)}}{2g}$

$t = \frac{2u \pm \sqrt{4(u^2 - 2gh)}}{2g}$

$t = \frac{u \pm \sqrt{u^2 - 2gh}}{g}$

$t_1 = \frac{u - \sqrt{u^2 - 2gh}}{g}$

$t_2 = \frac{u + \sqrt{u^2 - 2gh}}{g}$

$t_1 + t_2 = \frac{2u}{g}$

$$\boxed{u = \frac{g(t_1 + t_2)}{2}} \quad b)$$

A – B

$v = u + at$

$v = g\frac{(t_1 + t_2)}{2} - gt_1$

$v = \frac{gt_1 + gt_2 - 2gt_1}{2}$

$v = \frac{gt_2 - gt_1}{2}$

$v = \frac{g}{2}(t_2 - t_1)$

$$\boxed{v = \frac{g}{2}(t_2 - t_1)} \quad c)$$

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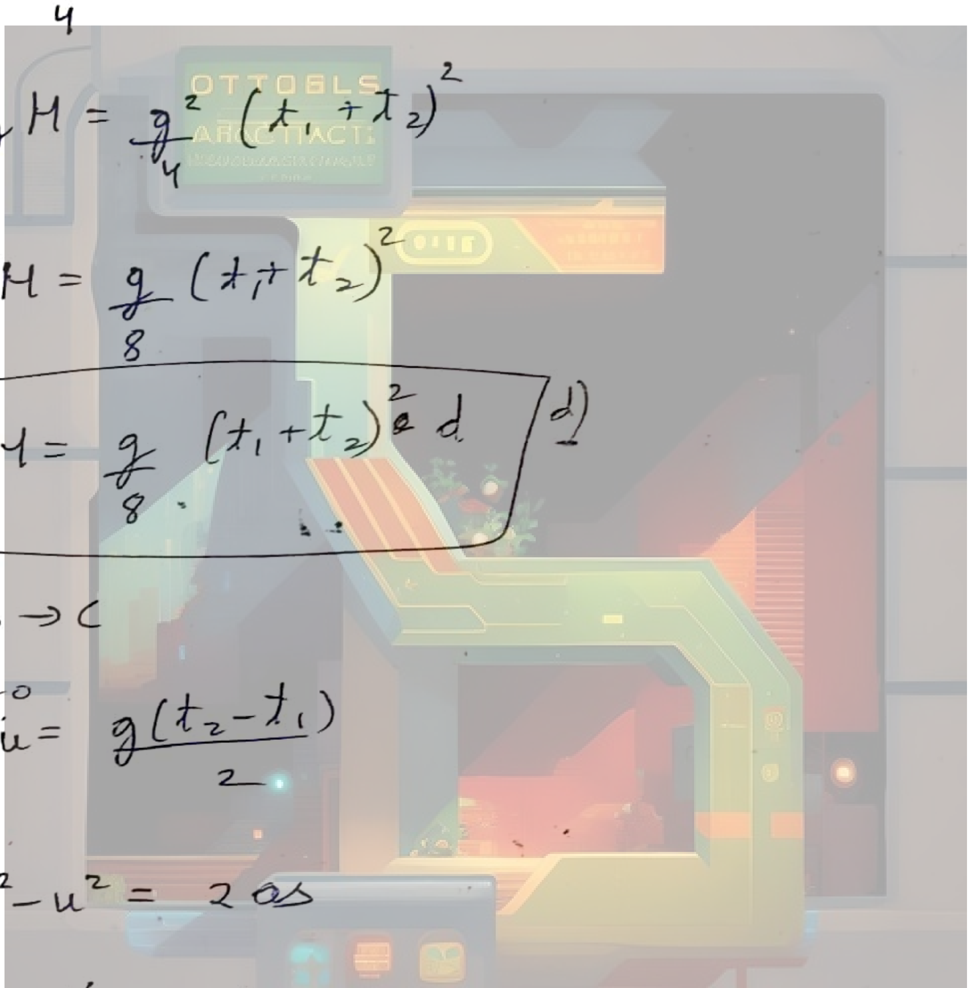
from $A \to C$

$v = 0$

$v^2 = u^2 + 2as$

$v^2 = \left(\frac{g}{2}(t_1 + t_2)\right)^2 - 2 \times g \times H$

$0 = \frac{g^2}{4}(t_1 + t_2)^2 - 2gH$



$2gH = \frac{g^2}{4}(t_1 + t_2)^2$

$H = \frac{g}{8}(t_1 + t_2)^2$

$\boxed{H = \frac{g}{8}(t_1 + t_2)^2 \ d}$ d)

from $B \to C$

$v = 0$

$u = \frac{g(t_2 - t_1)}{2}$

$v^2 - u^2 = 2as$

$-\frac{g^2}{4}(t_2 - t_1) = -2 \times g \times x$

$\boxed{\frac{g}{8}(t_2 - t_1)^2 = x}$ e)

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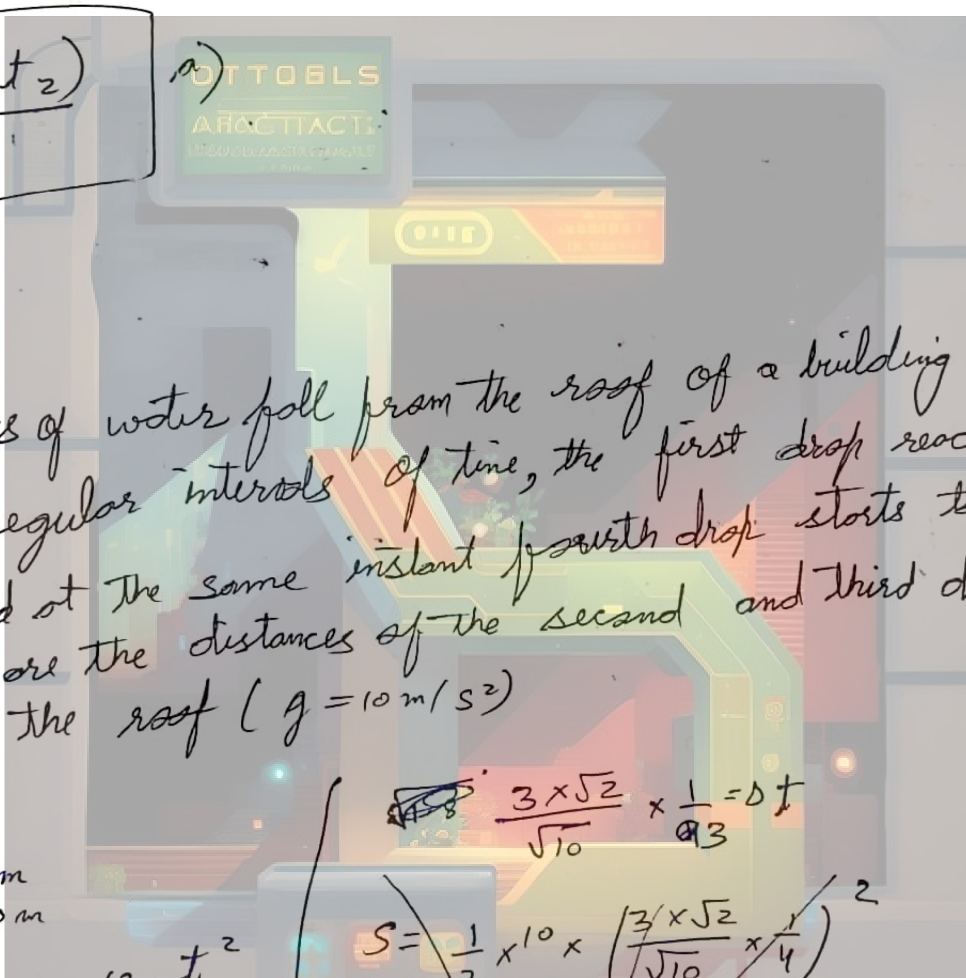
$\therefore H - x =$ hight of P

$$\frac{g}{8}(t_1+t_2)^2 - \frac{g}{8}(t_2-t_1)^2$$

$$\frac{g}{8}\left(\cancel{(t_1)^2}+\cancel{(t_2)^2}+2t_1t_2-\cancel{(t_2)^2}-\cancel{(t_1)^2}+2t_1t_2\right)$$

$$\frac{g}{8}(4t_1t_2)$$

$$\boxed{\frac{g(t_1t_2)}{2}}$$



Q2. Drops of water fall from the roof of a building 9 m high at regular intervals of time, the first drop reaching the ground at the same instant fourth drop starts to fall. What are the distances of the second and third drop from the roof ($g = 10\,m/s^2$)

$u = 0$
$h = 9\,m$
$g = 10\,m$

$9 = \frac{1}{2}\times 10 \times t^2$

$\frac{18}{10} = t^2$

$t^2 = 1.8$

$t = \sqrt{1.8}$

$\frac{3\times\sqrt{2}}{\sqrt{10}} \times \frac{1}{93} = \Delta t$

$S = \frac{1}{2}\times 10 \times \left(\frac{3\times\sqrt{2}}{\sqrt{10}} \times \frac{1}{4}\right)^2$

$S = 5 \times \left(\frac{9\times 2}{10} \times \frac{1}{168}\right.$

$S = 8 \times \frac{9}{80}$ 16

$\boxed{S = \frac{9}{16}\,8\,m}$

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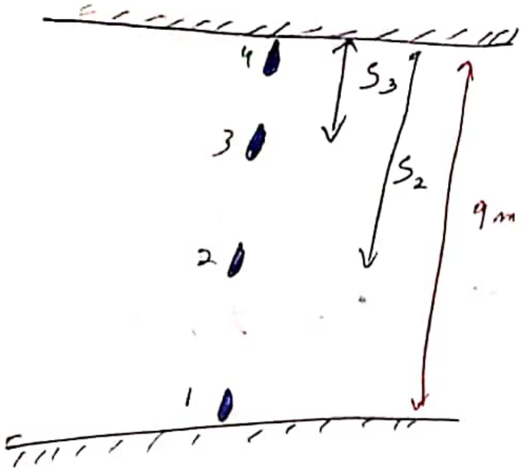

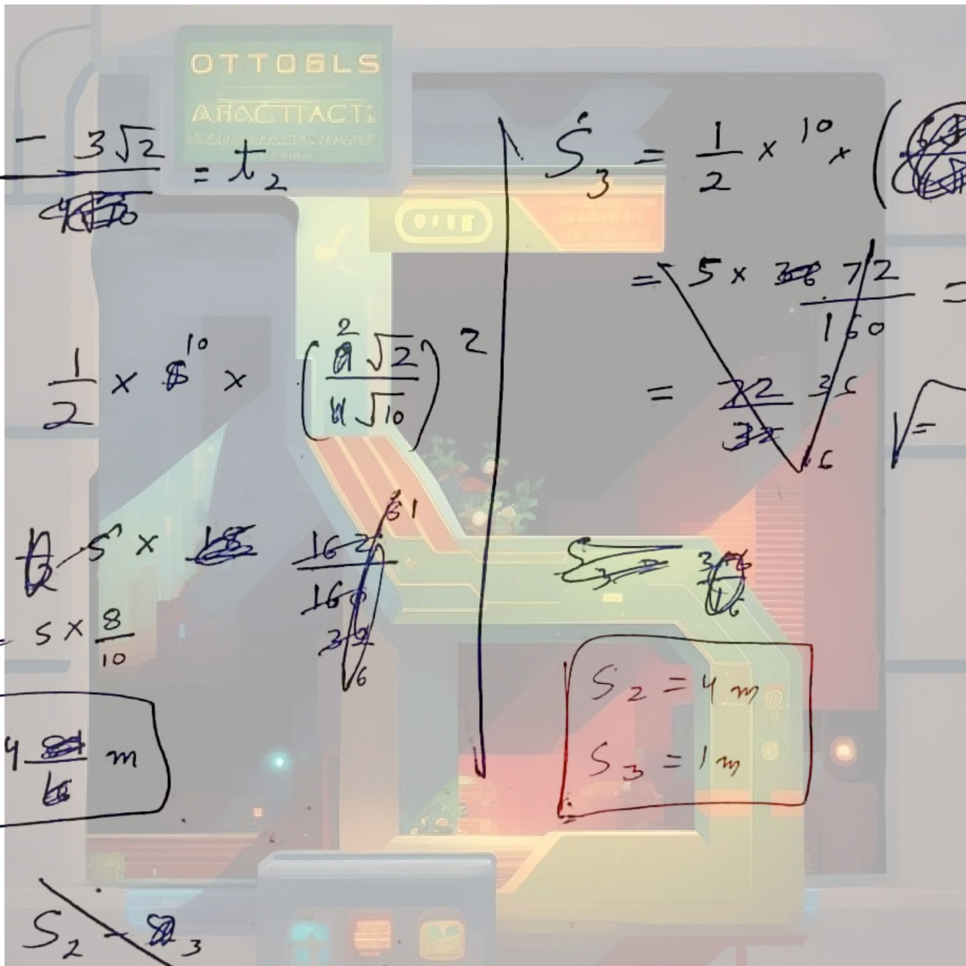

$$\frac{9\sqrt{2}}{3\sqrt{10}} - \frac{3\sqrt{2}}{4\sqrt{10}} = t_2$$

$$S_2 = \frac{1}{2} \times 10 \times \left(\frac{9\sqrt{2}}{4\sqrt{10}}\right)^2$$

$$= 5 \times \frac{162}{160}$$

$$= 5 \times \frac{8}{10}$$

$$\boxed{S_2 = 4\ m}$$

$$S_3 = \frac{1}{2} \times 10 \times \left(\frac{2\sqrt{2}}{10}\right)^2$$

$$= 5 \times \frac{72}{160} = 5 \times \frac{2}{10}$$

$$= \frac{72}{32} \quad \boxed{= 1\ m}$$

$$\boxed{\begin{aligned} S_2 &= 4\ m \\ S_3 &= 1\ m \end{aligned}}$$

~~$S_2 - S_3$~~

~~$\frac{81}{16} - \frac{36}{16} = \boxed{\frac{45}{16}\ \text{meter}}$~~

(184)

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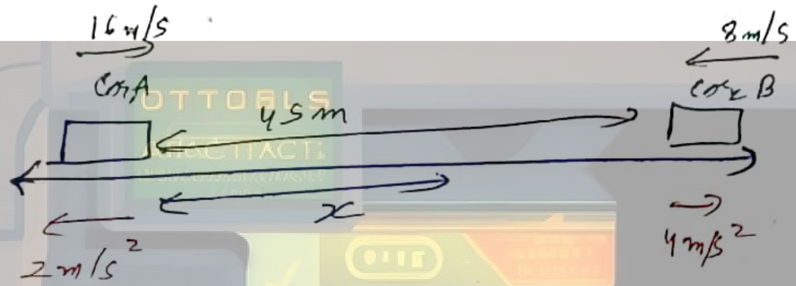
Q 3. Two cars approach each other on a straight road. Car A moves at 16 m/s and car B moves at 8 m/s. When they are 45 m apart, both ~~d~~ drivers apply their brakes. Car A slows down at $2 m/s^2$, while car ~~B~~ B slows down at ~~a~~ $4 m/s^2$. Where and when do they collide?



$x = 16t - \frac{1}{2} \times 2 (t)^2$

$x = 16t - t^2$

$45 - x = 8t - \frac{1}{2} \times 2t^2$

$45 = 24t - 3t^2$

$3t^2 - 24t - 45 = 0$

$t^2 - 8t - 15 = 0$

$t = \frac{8 \pm \sqrt{64 + 60}}{2}$

$t = \frac{8 \pm \sqrt{124}}{2}$

$t = 4 \pm \sqrt{31}$

$t = 4 + \sqrt{31}$

$x = 56 + 16\sqrt{31} - 16 - 31$

$x = 56 - 47 + 16\sqrt{31}$

$x = 9 + 16\sqrt{31}$

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Car - B

$u = 8$

$a = -4$

$v = 0$

$t = 2s$

$s = (8)(2) - \frac{1}{2} \times 4 \cdot (4)$

$s = 16 - 8$

$\boxed{s = 8\ m}$

collision ~~when~~ 8m from B.

$45 - 8 = 37$

$37 = 16t - t^2$

$t^2 - 16t + 37 = 0$

$t = \frac{16 \pm \sqrt{256 - 148}}{2}$

$t = \frac{16 \pm \sqrt{108}}{2}$

$t = 8 \pm \sqrt{27}$

$\boxed{t = 8 + \sqrt{27}\ s}$

$\boxed{t = 8 - \sqrt{27}\ s}$

Car - A

$u = 16$

$a = -2$

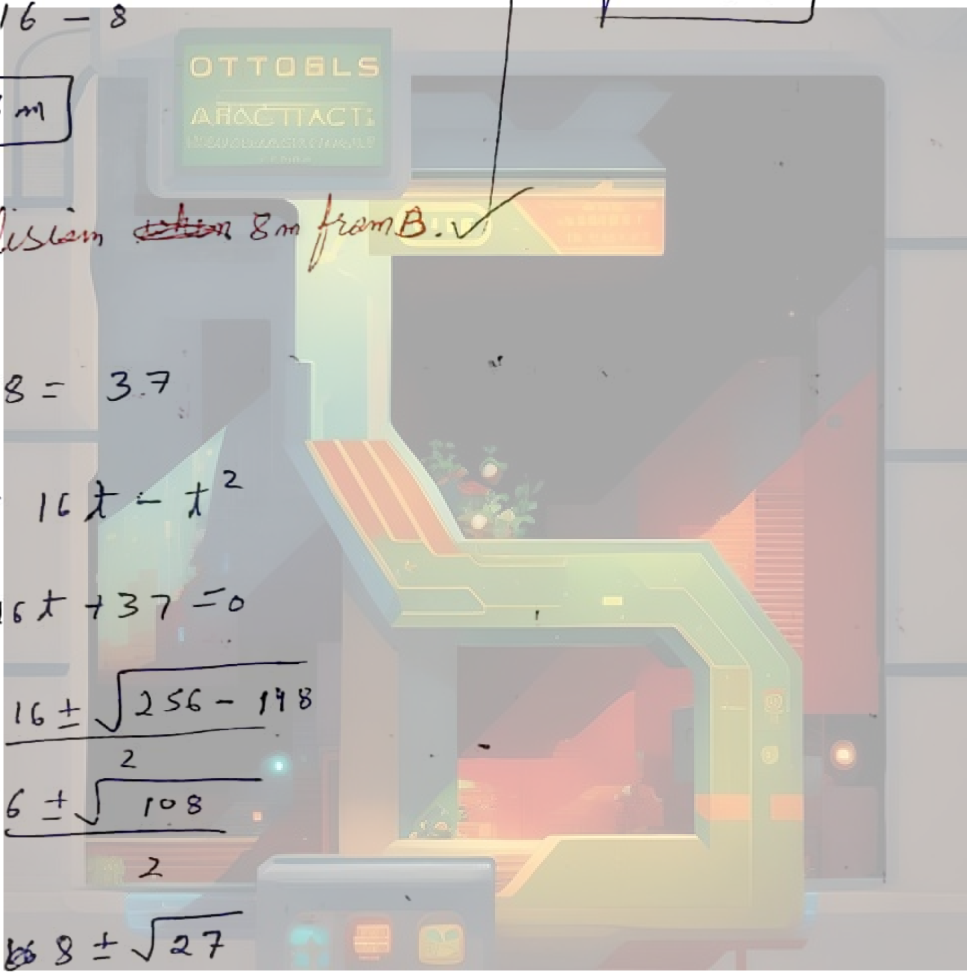
$v = 0$

$t = 8s$

$s = (8)(16) - \frac{1}{2} \times 2(64)$

$s = 128 - 64$

$\boxed{s = 64\ m}$



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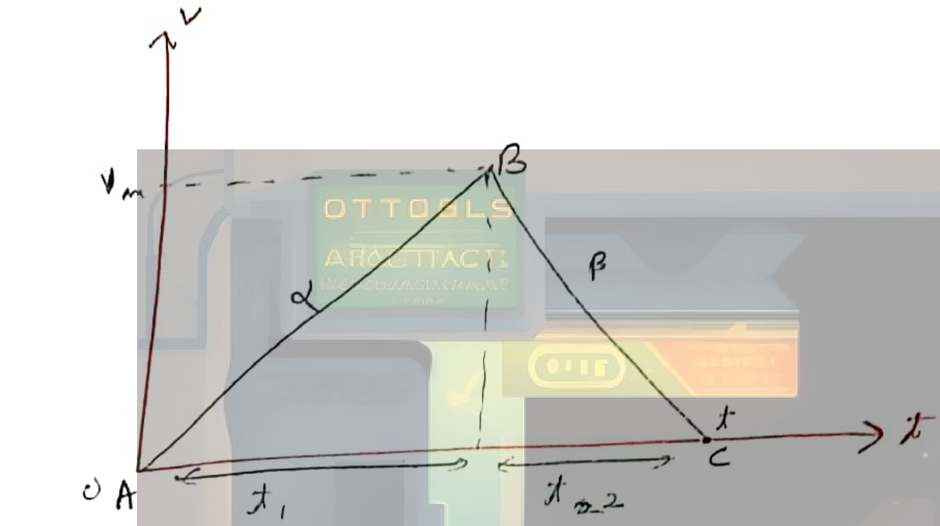
Q A car accelerates from rest at a constant rate α for some time and then decelerates at constant rate β to come to rest, find Total time elapsed is t then, calculate

a) Maximum velocity attained

b) Total distance travelled.



$$\text{Slope } AB = \frac{V_m - 0}{t_1}$$

$$\alpha = \frac{V_m}{t_1}$$

$$\boxed{t_1 = \frac{V_m}{\alpha}}$$

$$\text{Slope } BC = \frac{V_m}{t_2}$$

$$\boxed{t_2 = \frac{V_m}{\beta}}$$

$$t_1 + t_2 = t$$

$$\frac{V_m}{\alpha} + \frac{V_m}{\beta} = t$$

$$\boxed{V_m = \frac{t\alpha\beta}{\alpha+\beta}}$$

$$\boxed{V_m = \frac{t\alpha\beta}{\alpha+\beta}}$$

$$\text{Distance} = \frac{1}{2} \times V_m \times t$$

$$= \frac{1}{2} \times t \times \frac{t\alpha\beta}{\alpha+\beta}$$

$$= \frac{\alpha\beta t^2}{2(\alpha+\beta)}$$

$$\boxed{\text{Dis} = \frac{\alpha\beta t^2}{2(\alpha+\beta)}}$$

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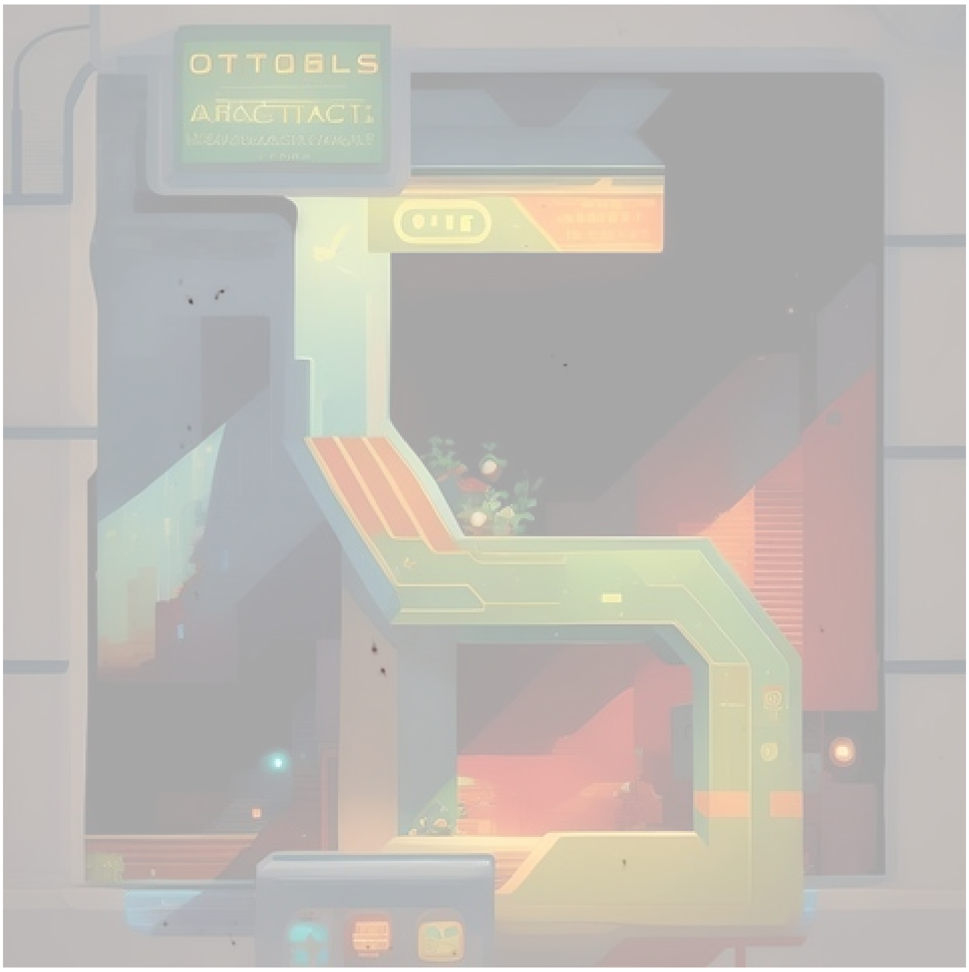
H.W. 11-05-2024

Q-1 (full)



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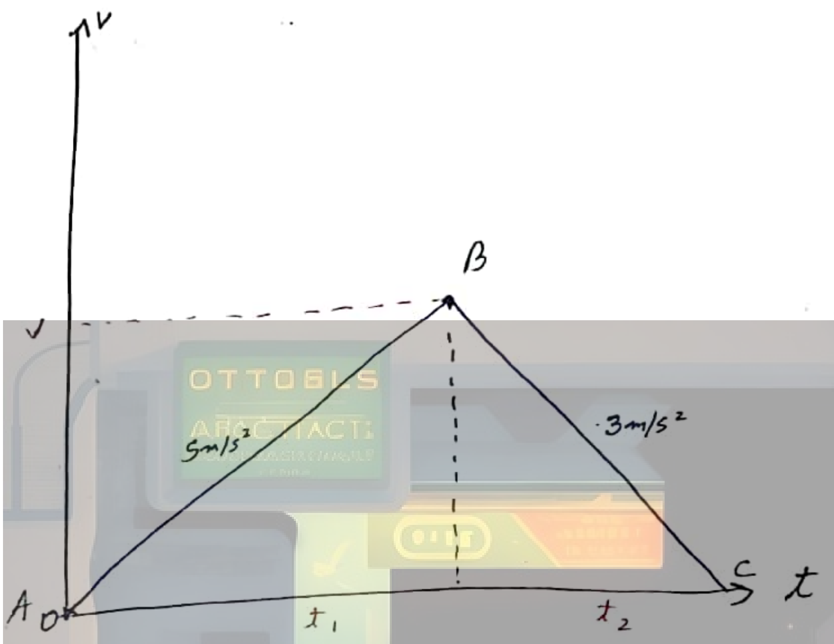
Q A car can travel at maximum speed of 180 km/hr and can have maximum acceleration $5 m/s^2$ and retardation $3 m/s^2$. How fast can it start from rest and come to rest in travelling 500 m?



max speed = 180 km/hr

$= 180 \times \frac{5}{18} = 50 m/s$

area under graph = 500 m

$A \to B$

$\frac{v}{t_1} = 5$

$\boxed{t_1 = \frac{v}{5}}$ --- (1)

$B \to C$

$-\frac{v}{t_2} = -3$

$\frac{v}{t_2} = 3$

$\boxed{t_2 = \frac{v}{3}}$ --- (2)

area $= \frac{1}{2} \times b \times h$

$500 = \frac{1}{2} \times (t_1 + t_2) \times v$

$1000 = \left(\frac{v}{5} + \frac{v}{3}\right) v$

$1000 = \frac{3v + 5v}{15} \times v$

$15000 = 8v^2$

$v^2 = \frac{15000}{8}$

$v^2 = 1875$

$v = \sqrt{1875}$

$\boxed{v = 43.3 m/s}$

$\because v < 50 m/s$ (max speed)

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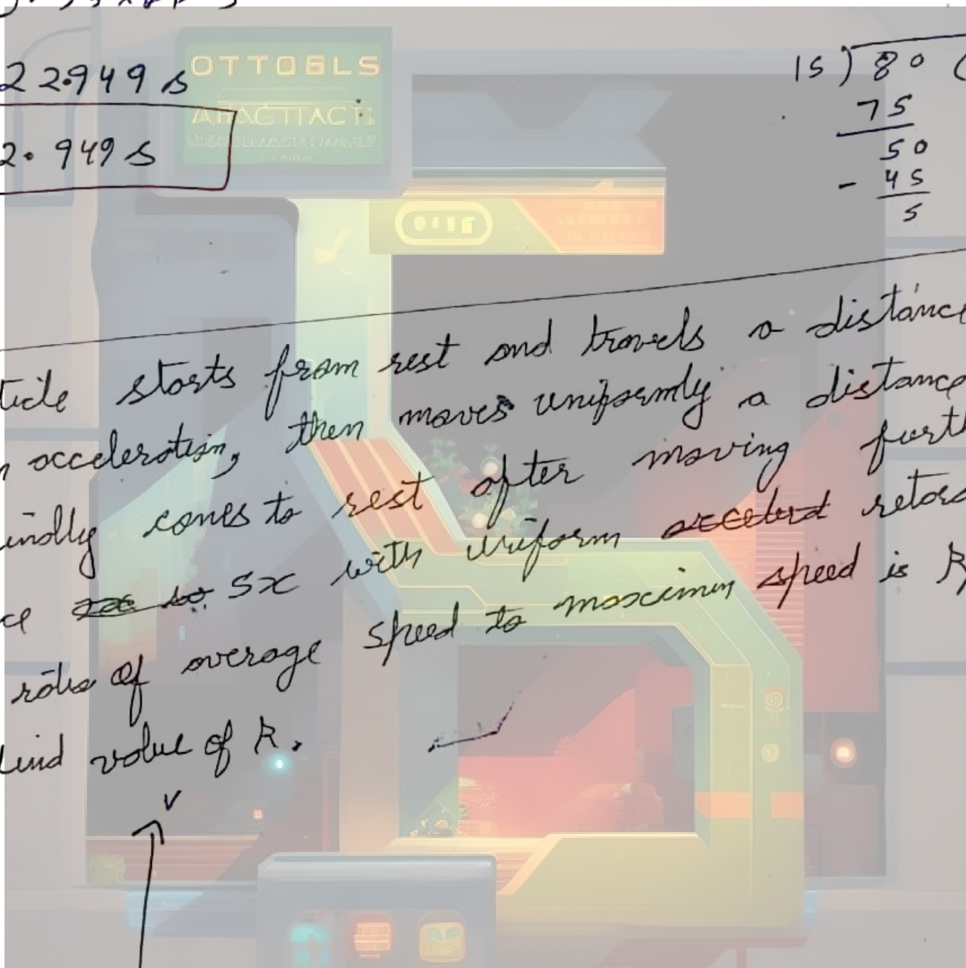
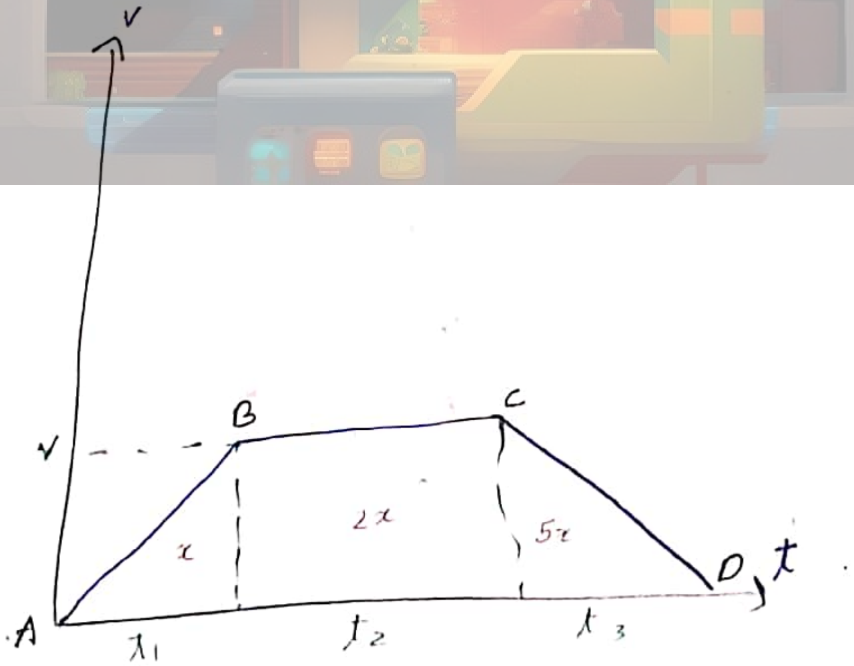
$$t = t_1 + t_2$$
$$= \frac{v}{5} + \frac{v}{3}$$
$$= \frac{3v + 5v}{15}$$
$$= \frac{8v}{15}$$
$$= \frac{8}{15} \times 43.3$$
$$= 0.53 \times 43.3$$
$$= 22.949\,s$$
$$\boxed{t = 22.949\,s}$$

$$\begin{array}{r} 43.3 \\ \times\ 53 \\ \hline 1299 \\ 21650 \\ \hline 22949 \end{array}$$

$$\begin{array}{l} 15\,)\,80\,(\,0.53 \\ \quad 75 \\ \hline \quad\ 50 \\ \quad -45 \\ \hline \quad\ \ 5 \end{array}$$



Q A particle starts from rest and travels a distance x with uniform acceleration, then moves uniformly a distance $2x$ and finally comes to rest after moving further distance ~~2x to~~ $5x$ with uniform ~~accelerat~~ retardation. If the ratio of average speed to maximum speed is $k/7$ then find value of k.



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$$\text{average speed} = \frac{x + 2x + 5x}{t}$$

$$\boxed{= \frac{8x}{t}}$$

~~max speed =~~

$$\frac{1}{2} \times v \times t_1 = x$$

$$\frac{v}{2x} = \frac{1}{t}$$

$$t_1 = \frac{2x}{v}$$

$$t_2 \times v = 2x$$

$$t_2 = \frac{2x}{v}$$

$$t_3 \times \frac{1}{2} \times v = 5x$$

$$t_3 = \frac{10x}{v}$$

$$t = \frac{2x + 2x + 10x}{v}$$

$$t = \frac{14x}{v}$$

$$\boxed{v = \frac{14x}{t}}$$

$$\text{max speed} = \frac{14x}{t}$$

$$\frac{\frac{8x}{t}}{\frac{14x}{t}} = \frac{k}{7}$$

$$\frac{8 \times 7}{14} = k$$

$$\boxed{k = 4}$$

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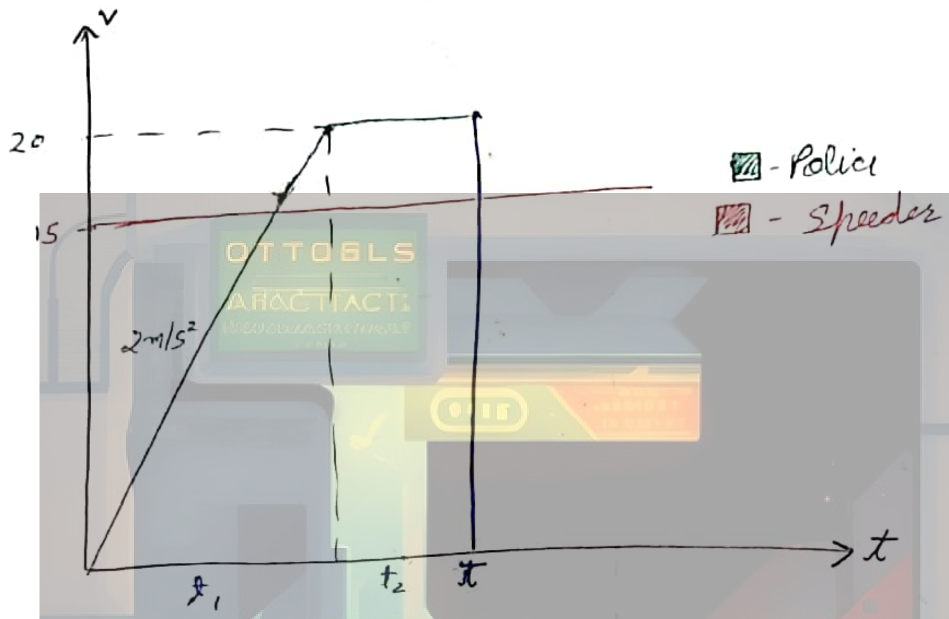
Q A speeder moves at a constant speed 15 m/s in a school zone. A police car starts from rest just as the speeder passes it. The police car accelerates at 2 m/s2 until it reaches its maximum velocity of 20 m/s. when and where does the speeder get caught?



~~area~~ displacement for speeder = $15t$

displacement for police $= \frac{1}{2} \times 20 \times 10 + 20 \times (t-10)$

$= 100 + 20t - 200$

$= 20t - 100$

Slope $= 2$

$2 = \frac{20}{t_1}$

$\boxed{t_1 = 10}$

$15t = 20t - 100$

$100 = 5t$

$\boxed{t = 20\,s}$

$S = 15t$

$= 15 \times 20$

$\boxed{= 300\ m}$

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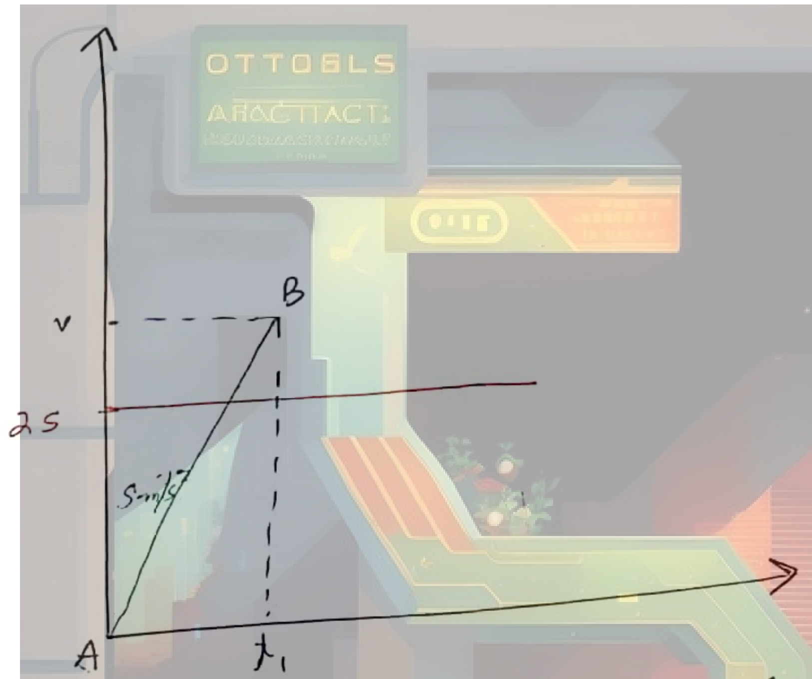
Q. A car is speeding at 25 m/s in a low speed zone. A police car starts from rest as the speedster passes and accelerates at a constant rate of 5 m/s².

a) When does the police car catch the speeding car?

b) How fast is the police car at the instant when it catches up with the speedster?

c) How far have the cars travelled when the police car catches the speedster?



Displacement police $= \frac{1}{2} \times t_1 \times v$

Displacement speeder $= t_1 \times 25$

$\frac{1}{2} t_1 v = 25 t_1$

$\boxed{v = 50 \text{ m/s}}$ b)

slope$_{AB}$ $= \frac{v}{t_1} = 5$

$50 = 5 t_1$

$\boxed{t_1 = 10 s}$ a)

$s = \frac{1}{2} \times 10 \times 50$

$\boxed{= 250 \text{ m}}$ c)

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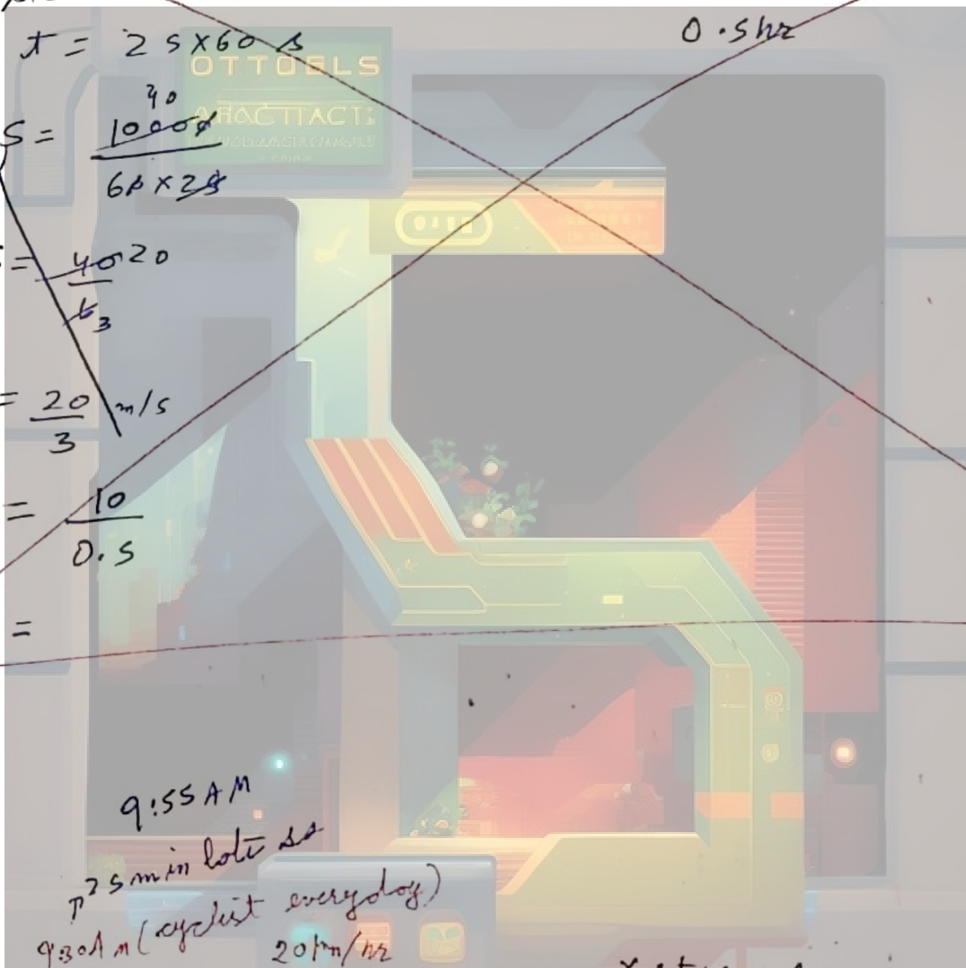
Q A railway track runs parallel to a road until a turn brings the road to railway crossing. A cyclist rides along the road everyday at a constant speed of 20 km/h. He normally meets a Train that Travels in same direction at the crossing. One day he was late by 25 minutes and met the train 10 km before crossing. find train speed.

$\cancel{s} = 10$

$d = 10000\ m$ $\quad \cancel{10000}$

$t = 25 \times 60\ s$ $\quad 0.5\ hr$



$S = \frac{10000}{60 \times 25}$

$S = \frac{40}{6}$

$S = \frac{20}{3}\ m/s$

$S = \frac{10}{0.5}$

$S =$

9:55 AM

25 min late so

9:30 AM (cyclist everyday) 20 km/hr

X let 10:00 AM

10 km

$Time = 5\ min$

$= \frac{5}{60}\ hr$

$speed = \frac{10}{5} \times 60$

$\boxed{= 120\ km/hr}$

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Q A body starts with an initial velocity of 10 m/s and moves along a straight line with constant acceleration is reversed in direction (at v = 0 m/s). Find the velocity of the particle when it reaches the starting point.

$S = \frac{50^2 - 10^2}{2a}$

$S = \frac{2500 - 100}{2a}$

$S = \frac{2400}{2a}$

$S = \frac{1200}{a}$

$50 - 10 = at$

$\frac{40}{a} = t$

$S = \frac{1200}{a}$

$u = 50$

$v = ?$

$2 \times a \times \frac{1200}{a} = v^2 - 2500$

$2500 + 2400 = v^2$

$4900 = v^2$

$\boxed{V = 70\ m/s}$

$u = 50$

$S = \frac{1200}{a}$

$\frac{1200}{a} = 50t - \frac{1}{2} a t^2$

$\frac{-40}{a} = t \qquad 0 = \frac{-40}{t}$

$\frac{1200}{a} = 50 \times \frac{-40}{a} - \frac{1}{2} \times a \times \frac{1600}{a^2}$

$\frac{1200}{a} = \frac{-2000}{a} - \frac{800}{a}$

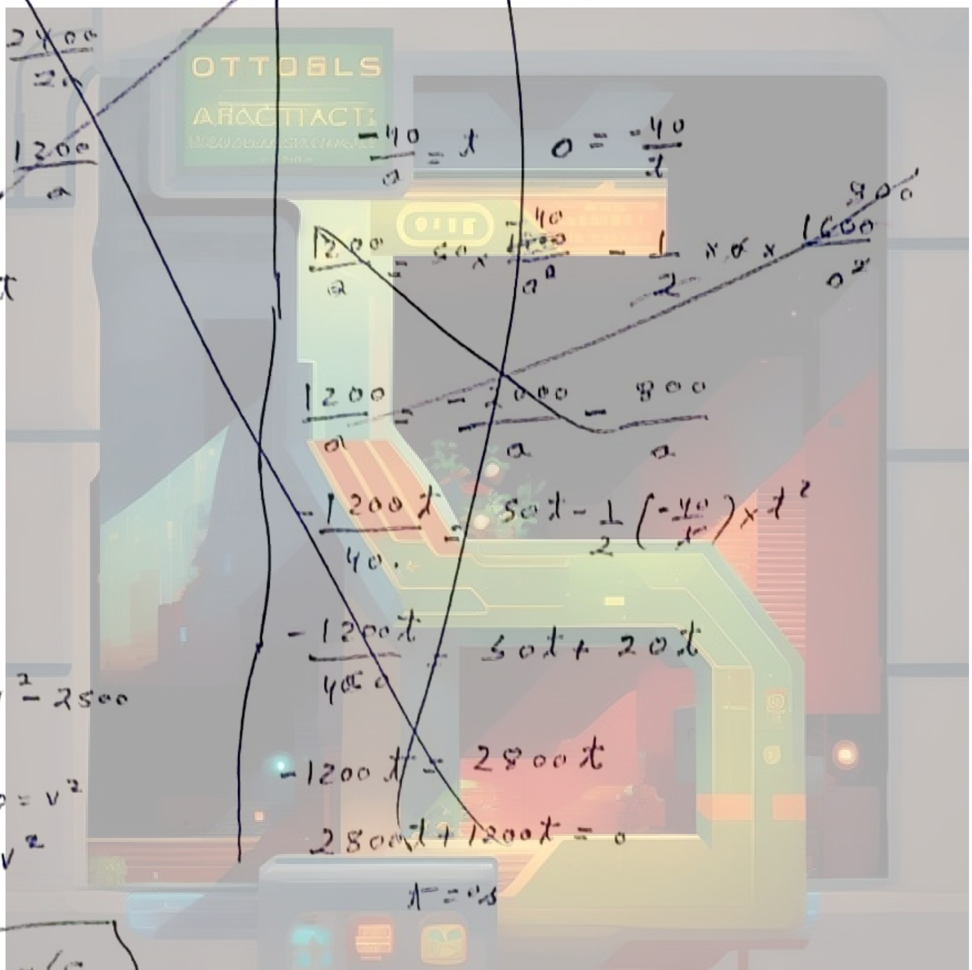
$\frac{-1200t}{40} = 50t - \frac{1}{2}\left(\frac{-40}{t}\right) \times t^2$

$\frac{-1200t}{400} = 50t + 20t$

$-1200t = 2800t$

$2800t + 1200t = 0$

$t = 0\ s$



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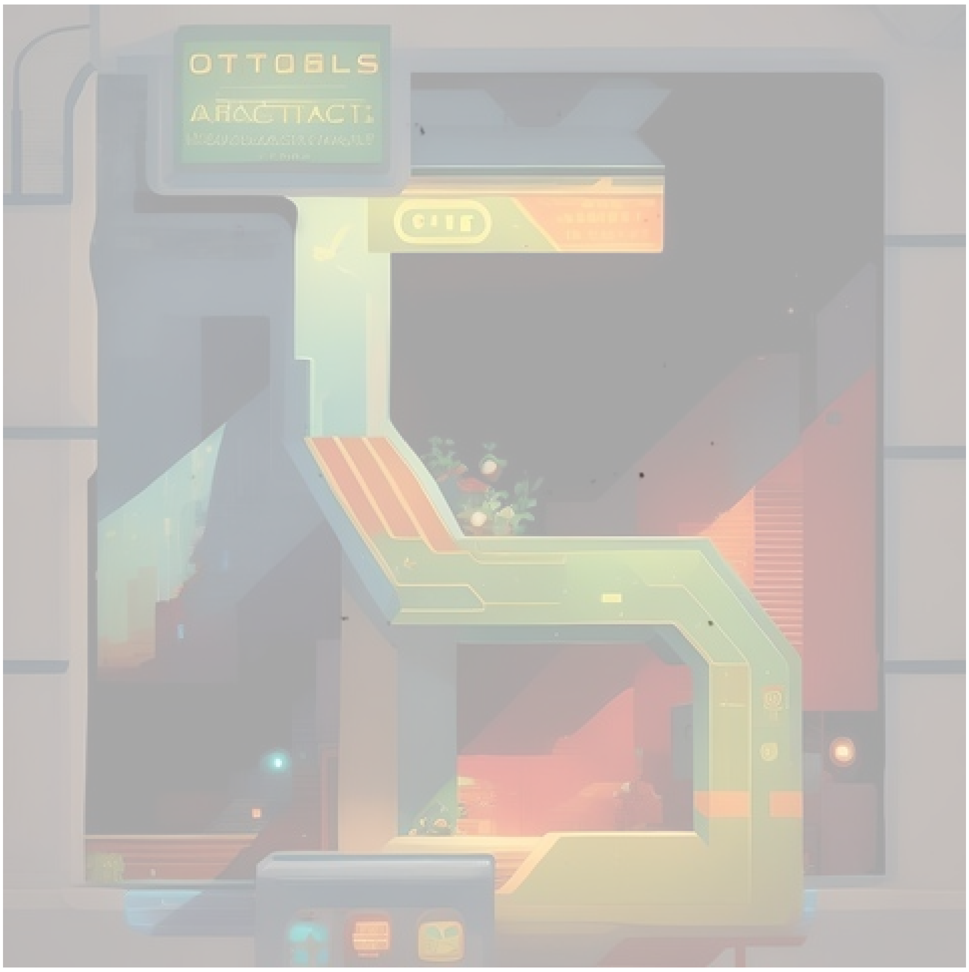

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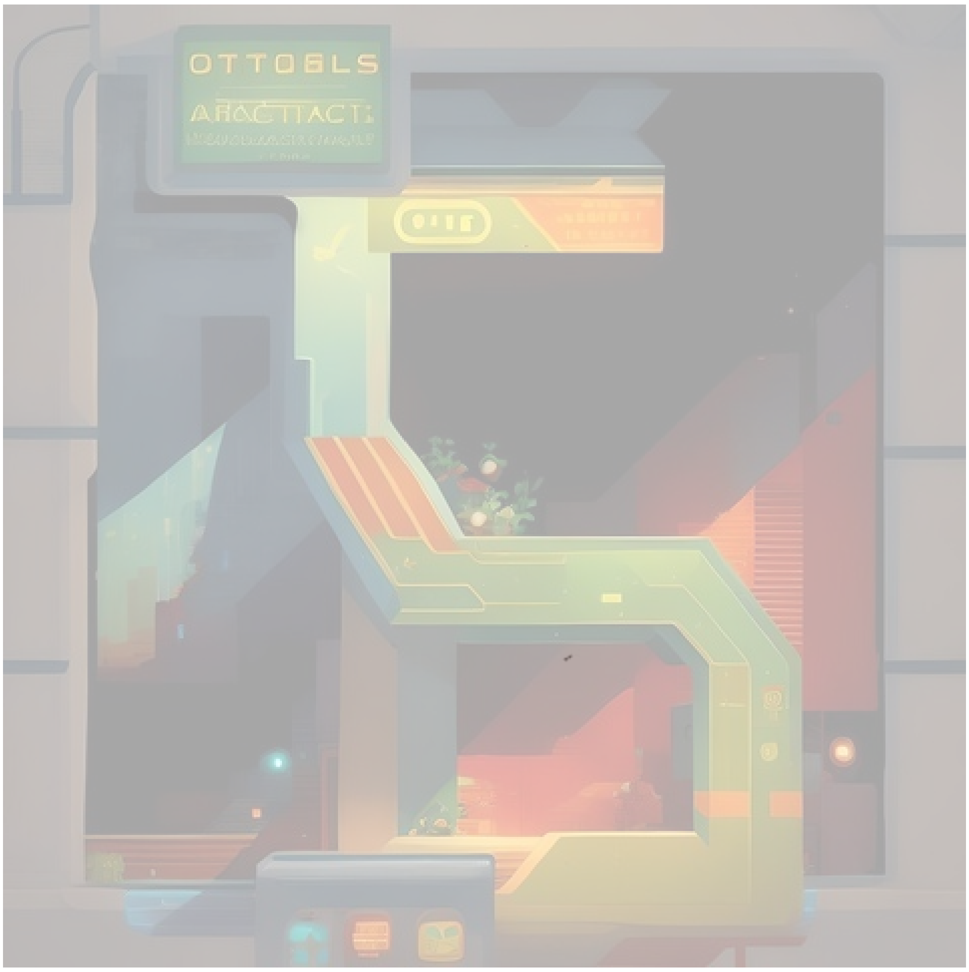

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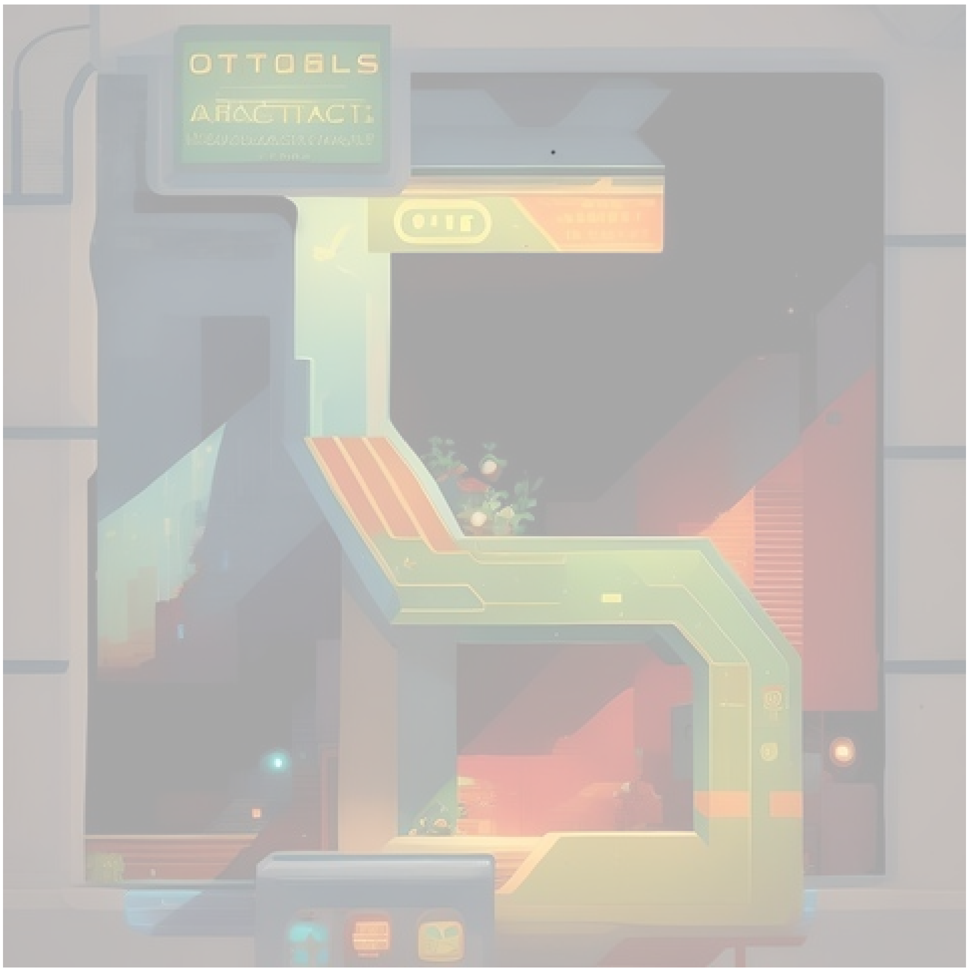

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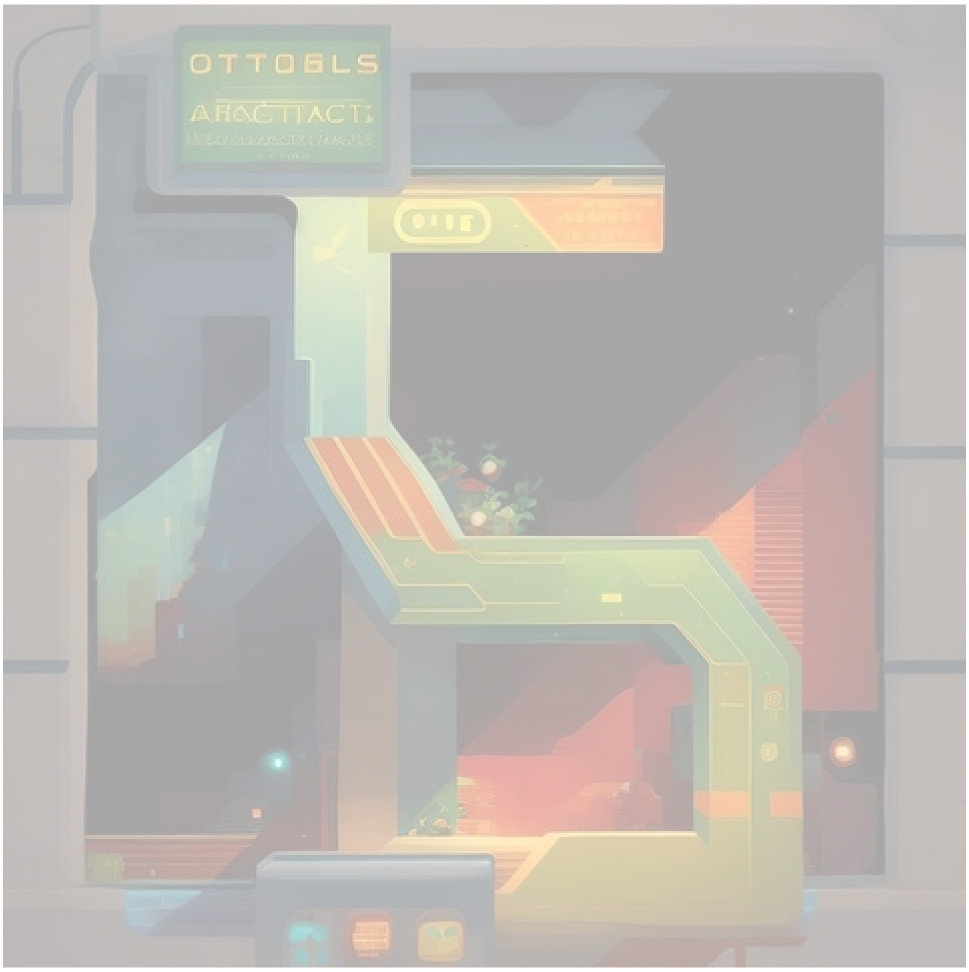

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