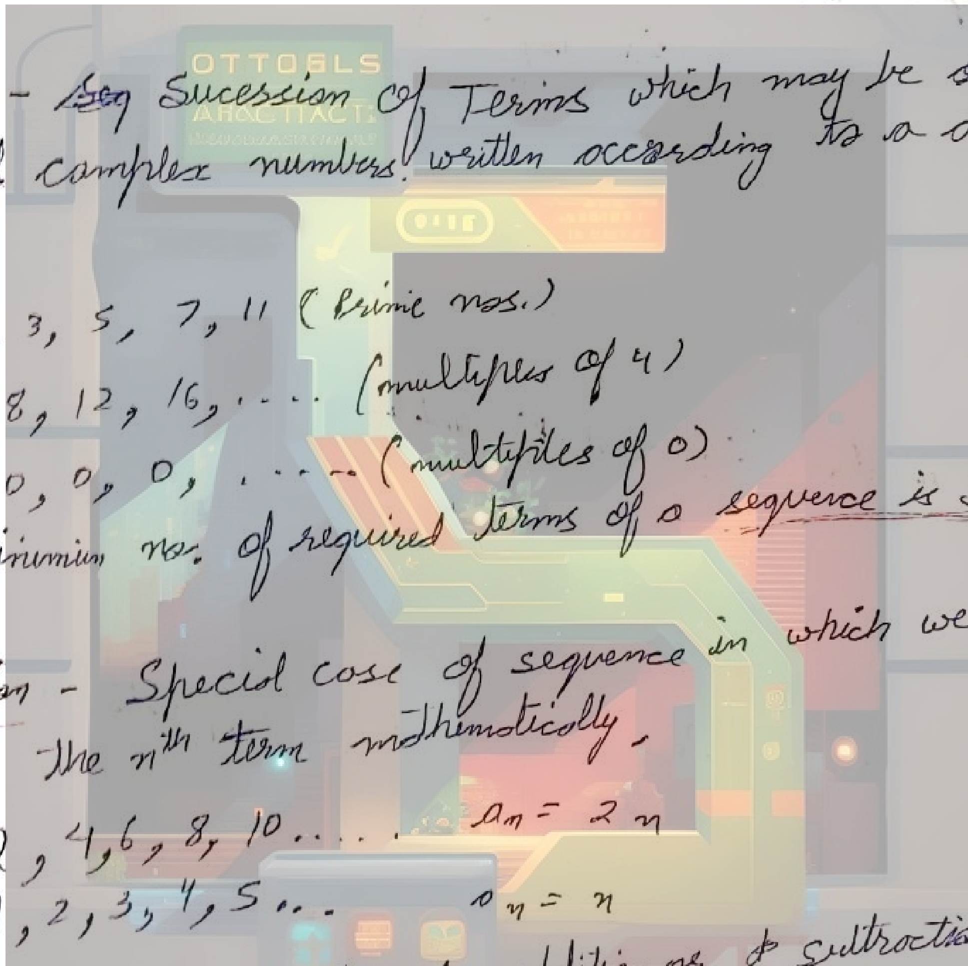
!! Sequence & Series !!

A.P. (Arithmetic Progression)
G.P (Geometric Progression)
H.P (Harmonic Progression)
A.G.P (Arithmetic Geometric progression)

Special Sequences - 4 types



Sequence - ~~Seq~~ Sucession of Terms which may be ~~a~~ algebric, real & complex numbers written according to a definite rule.

eg. 2, 3, 5, 7, 11 (Prime nos.)
4, 8, 12, 16, ... (multiples of 4)
0, 0, 0, 0, ... (multiples of 0)

→ Minimum no. of required terms of a sequence is 3.

Progression - Special case of sequence in which we can express the n^{th} term mathematically.

eg. 2, 4, 6, 8, 10 ..... $a_n = 2n$
1, 2, 3, 4, 5 ... $a_n = n$

Series - If we put sign of addition or ~~&~~ subtraction between the terms of sequence then it is called series.

eg. 2 + 3 + 5 + 7 + 11 + ...
0 + 7 + 26 + ...
1 − 3 + 9 − 27 + ...

Q find out the 2^{nd} & 4^{th} terms of the sequence.

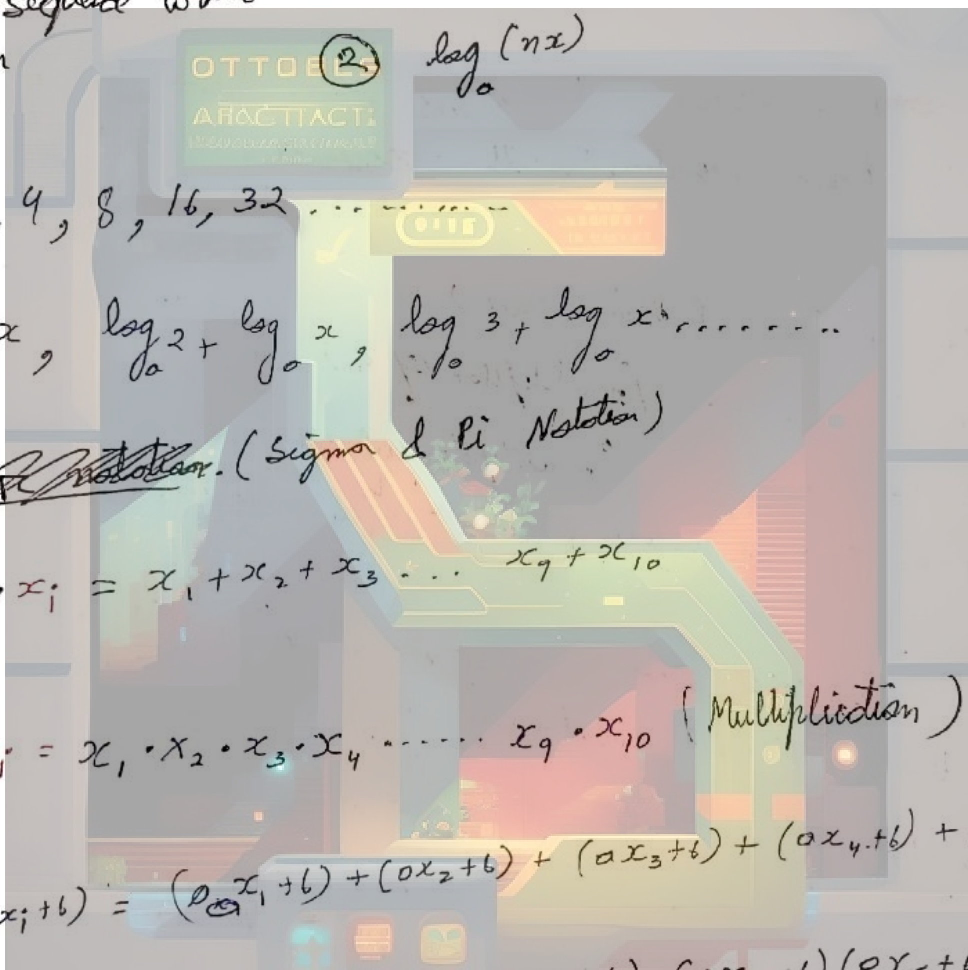
$$T_n / a_n = \left\{\frac{-1}{n}\right\}^n$$

$a_1 = \left(\frac{-1}{1}\right)^1$ $\quad a_2 = \left\{\frac{-1}{2}\right\}^2$ $\quad a_4 = \left\{\frac{-1}{4}\right\}^4$

$a_1 = -1$ $\quad a_4 = \frac{1}{256}$

$a_2 = \frac{1}{4}$

Q write sequence whose n^{th} terms are



(1) 2^n (2) $\log_a (nx)$

(1) $2, 4, 8, 16, 32 \ldots\ldots\ldots$

(2) $\log_a x, \log_a 2 + \log_a x, \log_a 3 + \log_a x \ldots\ldots$

~~Σ & Π notation~~ (Sigma & Pi Notation)

$$\sum_{i=1}^{10} x_i = x_1 + x_2 + x_3 \ldots x_9 + x_{10}$$

$$\prod_{i=1}^{10} x_i = x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdots\cdots x_9 \cdot x_{10}$$ (Multiplication)

$$\sum_{i=1}^{5} (ax_i + b) = (ax_1 + b) + (ax_2 + b) + (ax_3 + b) + (ax_4 + b) + (ax_5 + b)$$

$$\prod_{i=1}^{5} (ax_i + b) = (ax_1 + b)(ax_2 + b)(ax_3 + b)(ax_4 + b)(ax_5 + b)$$

Arithmetic Progression (A.P.)

→ Sequence Having two consecutive terms have the constant difference.

→ $a, \; a+d, \; a+2d, \; a+3d \; \dots \; a+nd$

$T_1 \quad T_2 \quad T_3 \quad T_4$

$$\boxed{T_n = a+(n-1)d}$$

a = first term

d = common difference

Common Difference (C.D.) can be $\oplus$ve, $\ominus$ve or zero

CD > 0 Increasing AP

CD < 0 Decreasing AP

CD = 0 Constant AP

Q1. If 6^{th} term & 11^{th} term of AP are 17 & 32 respectively find the 20^{th} term.

Q2. In an AP $T_2 + T_5 - T_3 = 10$

$T_2 + T_9 = 17$

find T_1 & CD.

Q3. If p^{th}, q^{th} & R^{th} Terms of an AP are a, b, c respectively then find $a(Q-R) + b(R-P) + c(P-Q)$

Q4. If 11 times the 11^{th} term of an AP = 9 times the 9^{th} term then find 20^{th} term.

Q1. $17 = a + 5d$

$32 = a + 10d$

$15 = 5d$

$\boxed{d = 3}$

$a = 17 - 15$

$\boxed{a = 2}$

$T_{20} = 2 + 19 \times 3$

$\boxed{= 59}$

Q2.

$2a + 6d = 20$

$2a + 9d = 17$

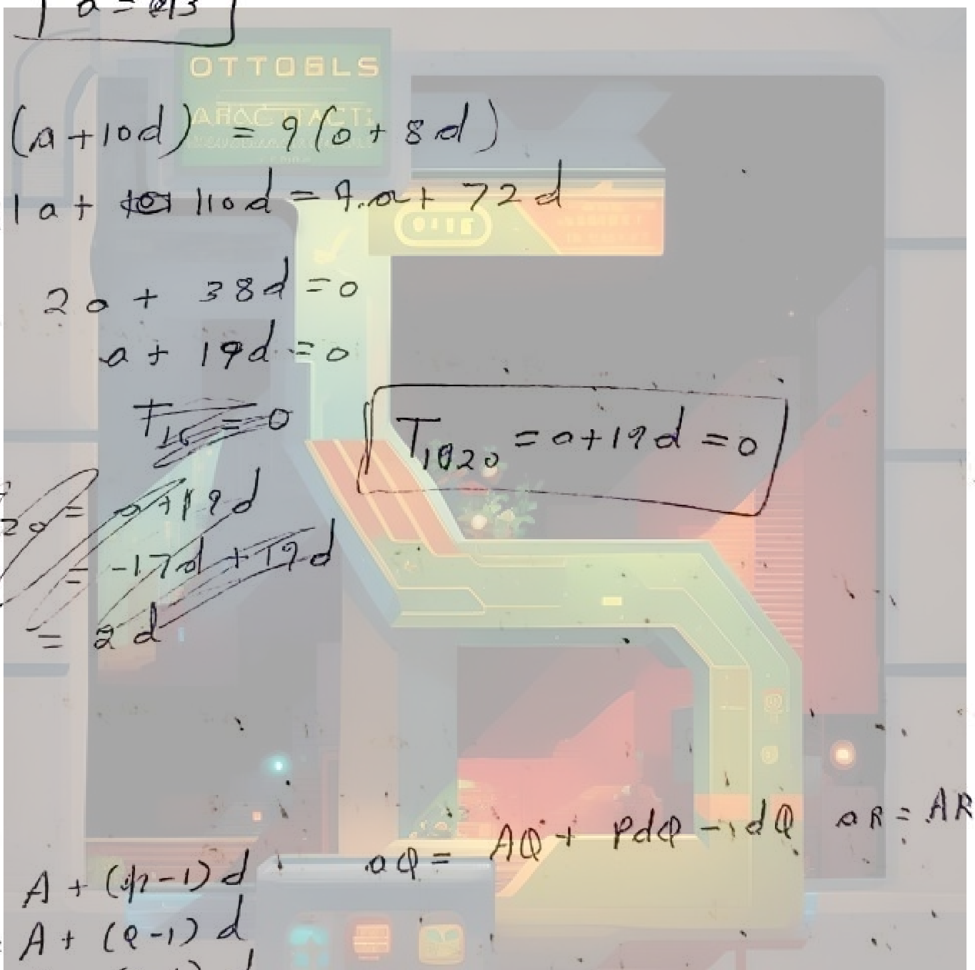
$-3d = 3$

$\boxed{d = -1}$

$a + 3(-1) = 10$

$a - 3 = 10$

$\boxed{a = 13}$



Q4. $11(a + 10d) = 9(a + 8d)$

$11a + 110d = 9a + 72d$

$2a + 38d = 0$

$a + 19d = 0$

$\cancel{T_{20} = 0}$

$\boxed{T_{20} = a + 19d = 0}$

$\cancel{T_{20} = a + 19d}$

$\cancel{= -17d + 19d}$

$\cancel{= 2d}$

Q3. $a = A + (p-1)d \quad aQ = AQ + PdQ - dQ \quad aR = AR + PdR - dR$

$b = A + (Q-1)d$

$c = A + (R-1)d$

$AQ + PdQ - dQ - AR - PdR + dR + AR - AP + QdR - dR$

$- QdP + dP - AQ + dQ + PdR - dR$

$\boxed{= 0}$

Some Facts

→ K^{th} term from the end of an AP

$$T_K = l + (K-1)(-d)$$

$n, (n-1), (n-2) \ldots 3, 2, \underset{\downarrow}{1} \Rightarrow -d$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad l$

→ If m^{th} term is n & n^{th} term is m then $(m+n)^{th}$ term $= 0$

→ Sum of K^{th} term from beginning & K^{th} term from end is always constant

→ If each term of an AP is added or subtracted or multiplied or divided by the same non-zero number then the resulting sequence is also an AP.

→ Assuming Terms in AP.

3 terms ⇒ $(a-d), a, (a+d)$

4 terms ⇒ $(a-3d), (a-d), (a+d), (a+3d)$

5 terms ⇒ $(a-2d), (a-d), a, (a+d), (a+2d)$

6 terms ⇒ $(a-5d), (a-3d), (a-d), (a+d), (a+3d), (a+5d)$

→ If a, b & c are consecutive terms in AP.

$$\boxed{a + c = 2b}$$

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H.W.

DYS-1

DYS-2

Summation of an A.P.

$S_n = a + (a+d) + (a+2d) + (a+3d) \ldots\ldots (a+(n-2)d) + (a+(n-1)d)$

$+S_n = a+(n-1)d + a+(n-2)d \ldots\ldots (a+d) + (a)$

$2S_n = 2a+(n-1)d + 2a+(n-1)d + 2a+(n-1)d \ldots\ldots + 2a+(n-1)d$

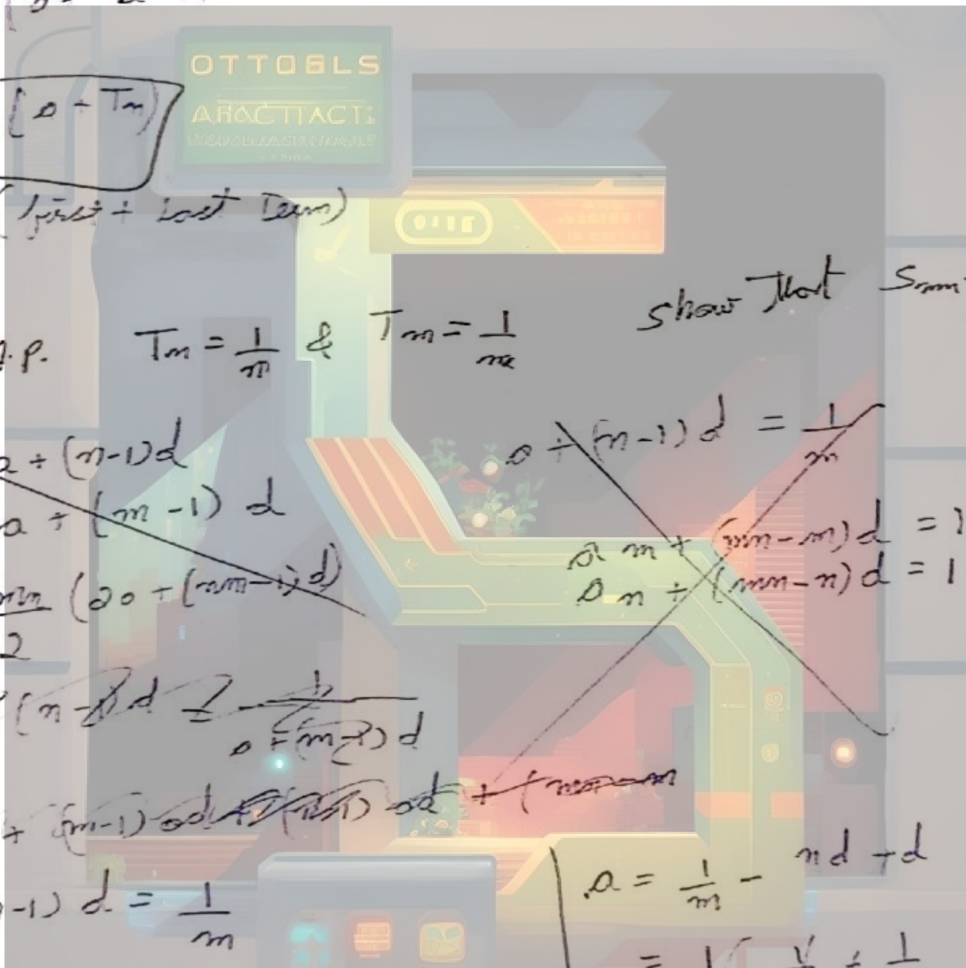
$= n(2a+(n-1)d)$

$$\boxed{S_n = \frac{n}{2}(2a+(n-1)d)}$$

$S_n = \frac{n}{2}(a + a + (n-1)d)$

$$\boxed{S_n = \frac{n}{2}(a + T_n)}$$

$S_n = \frac{n}{2}$ (first + last term)



Q In an A.P. $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$ show that $S_{mn} = \frac{1}{2}(mn+1)$

~~$T_n = a+(n-1)d$~~

~~$T_m = a+(m-1)d$~~

~~$S_{mn} = \frac{mn}{2}(2a+(mn-1)d)$~~

~~$a+(n-1)d = \frac{1}{a+(m-1)d}$~~

~~$a^2 + (m-1)ad + (n-1)ad + (mn-m-n)$~~

$a+(n-1)d = \frac{1}{m}$

$a+(m-1)d = \frac{1}{n}$

$(n-1)d - (m-1)d = \frac{1}{m} - \frac{1}{n}$

$d(n-1-m+1) = \frac{n-m}{mn}$

$d(n-m) = \frac{n-m}{mn}$

$$\boxed{d = \frac{1}{mn}}$$

~~$a + (n-1)d = \frac{1}{m}$~~

~~$am + (mn-m)d = 1$~~

~~$an + (mn-n)d = 1$~~

$a = \frac{1}{m} - nd + d$

$= \frac{1}{m} - \frac{1}{m} + \frac{1}{mn}$

~~$= \frac{2}{m} - \frac{1}{mn}$~~

~~$a = \frac{2n-1}{mn}$~~

$a = \frac{1}{mn}$

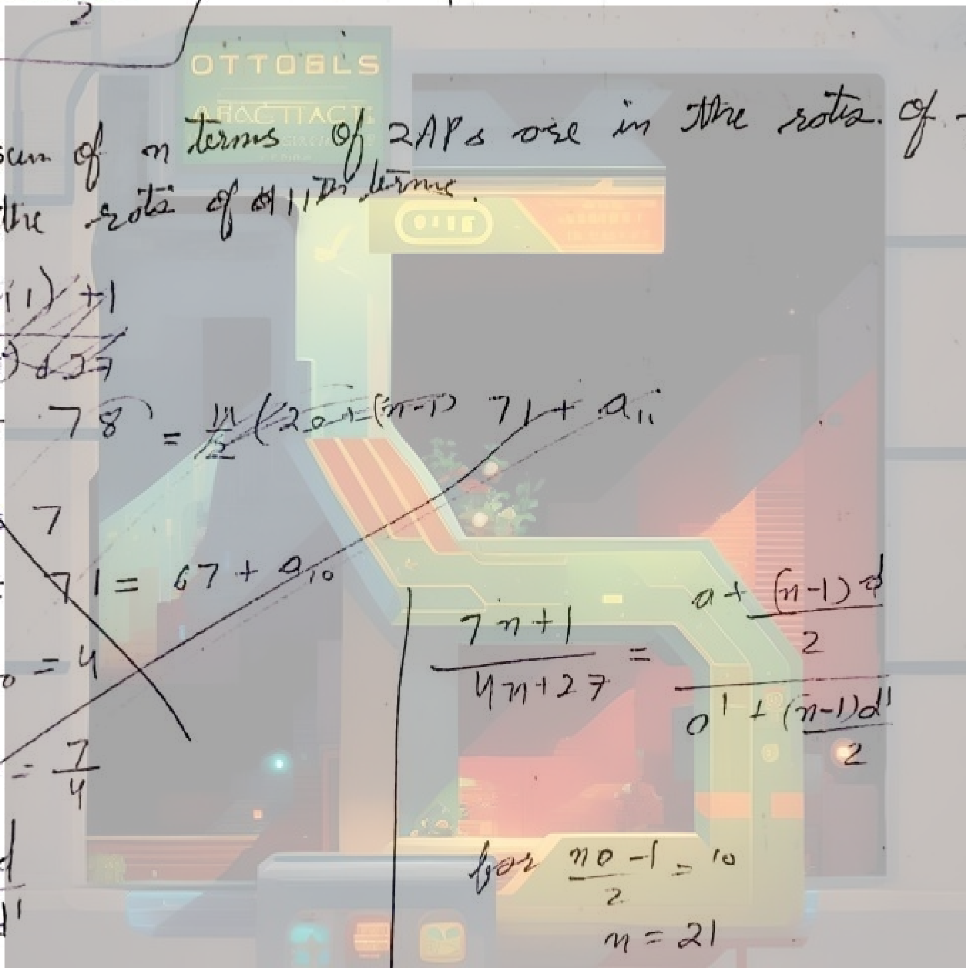
$$S_{mn} = \frac{mn}{2}\left(2a + (mn-1)d\right)$$

$$= \frac{mn}{2}\left(2 \times \frac{1}{mn} + mn \times \frac{1}{mn} - \frac{1}{mn}\right)$$

$$= \frac{mn}{2}\left(\frac{2}{mn} - \frac{1}{mn} + 1\right)$$

$$= \frac{mn}{2}\left(\frac{1+mn}{mn}\right)$$

$$\boxed{= \frac{1+mn}{2}}$$



Q The sum of n terms of 2APs are in the ratio of $7n+1 : 4n+27$. Find the ratio of 11th terms.

$$\cancel{\frac{7(11)+1}{4(11)+27}}$$

$$S_{11} = 78 = \frac{11}{2}\left(2a + (n-1)\right) \quad 71 + a_{11}$$

$$a_{11} = 7$$

$$S_{11} = 71 = 67 + a_{10}$$

$$a_{10} = 4$$

$$\frac{a_{11}}{a_{10}} = \frac{7}{4}$$

$$\frac{a+10d}{a'+10d'}$$

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}\left(2a+(n-1)d\right)}{\frac{n}{2}\left(2a'+(n-1)d'\right)}$$

$$= \frac{a + \frac{(n-1)d}{2}}{a' + \frac{(n-1)d'}{2}}$$

$$\frac{7n+1}{4n+27} = \frac{a + \frac{(n-1)d}{2}}{a' + \frac{(n-1)d'}{2}}$$

for $\frac{n-1}{2} = 10$

$$n = 21$$

$$\frac{148}{111} = \boxed{\frac{4}{3} = \frac{T_{11}}{T'_{11}}}$$

Note → If the sum of an A.P ~~...~~ S_n is given

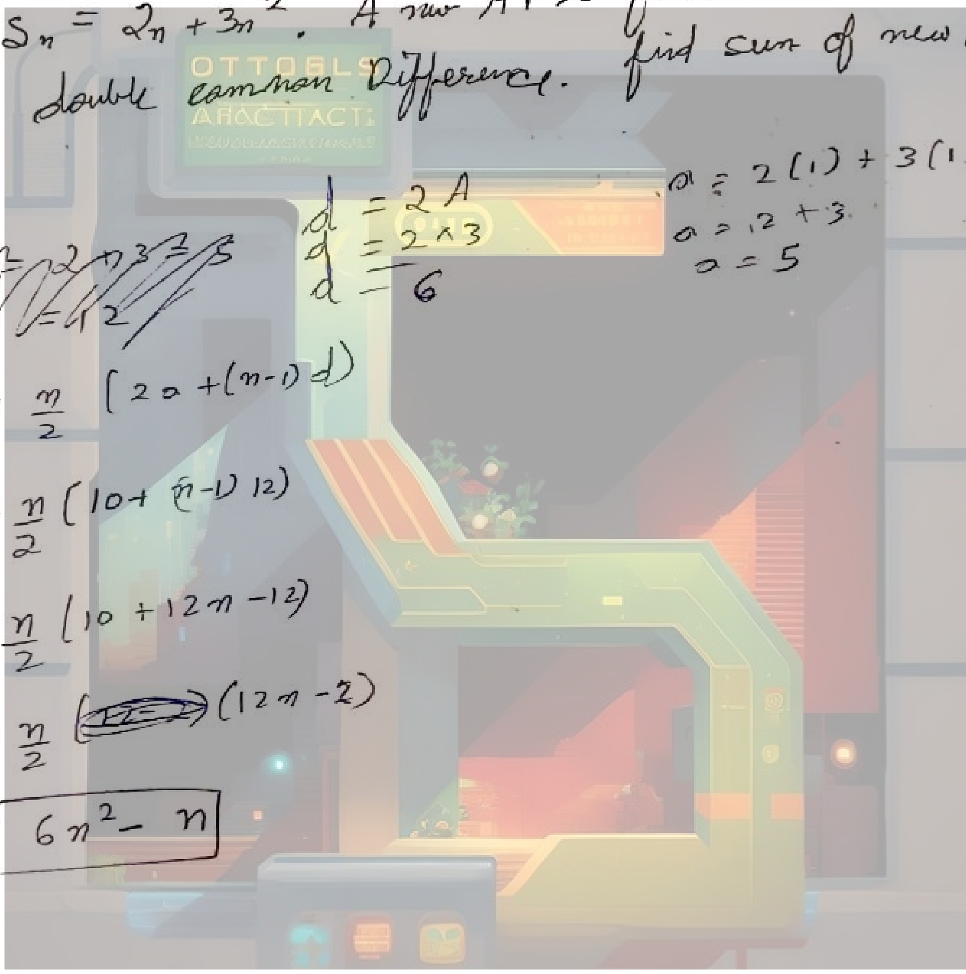
$T_n = S_n - S_{(n-1)}$

→ In an A.P If S_n is a quadratic of n ~~...~~ and T_n is in liner in n Then the series will be an A.P..

$T_n = An + B$ $\qquad\qquad$ $S_n = An^2 + Bn + C$

$\downarrow$ AP with common differed A $\qquad$ ↳ AP with common differce $= 2A$

Q Given $S_n = 2n + 3n^2$. A new AP is formed with same first term & double common Difference. find sum of new ~~...~~ AP.



$a = 2(1) + 3(1)^2$

$a = 2 + 3$

$a = 5$

$d = 2A$

$d = 2 \times 3$

$d = 6$

~~$d = 12$~~

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$= \frac{n}{2}\left(10 + (n-1)12\right)$

$= \frac{n}{2}\left(10 + 12n - 12\right)$

$= \frac{n}{2}(12n - 2)$

$\boxed{= 6n^2 - n}$

Geometric Progression

→ Collection of non zero terms in which each consecutive term bears the <u>same constant ratio.</u>

$a, ar, ar^2, ar^3, ar^4, ar^5 \ldots ar^{n-1}$

$T_n = a r^{(n-1)}$

$a \rightarrow$ First Term

$r \rightarrow$ common ratio, $r \neq 0$

$r > 0$: $r \in (0,1)$ ↓ ~~de~~decrease; $r \in (1, \infty)$ ↑ Increase

$r < 0$: $r \in (-1, 0)$ ↓ ~~decrease in O/V~~; $r \in (-\infty, -1)$ ↑

Note - Common Ratio can be $\frac{1}{r}$, ~~∞~~

Q $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots$

find 20^{th} term & n^{th} term.

$r = \dfrac{\frac{1}{4}}{\frac{1}{2}}$ $\quad$ $r = \dfrac{\frac{1}{8}}{\frac{1}{4}}$

$= \frac{1}{4} \times \frac{2}{1}$ $\quad$ $= \frac{1}{8} \times \frac{4}{1}$

$= \frac{1}{2}$ $\quad$ $= \frac{1}{2}$

$a = \frac{1}{2}$

$T_{n^{th}} = \frac{1}{2} \times \frac{1}{2}^{n-1}$

$= \frac{1}{2}^{n}$

$= 2^{-n}$

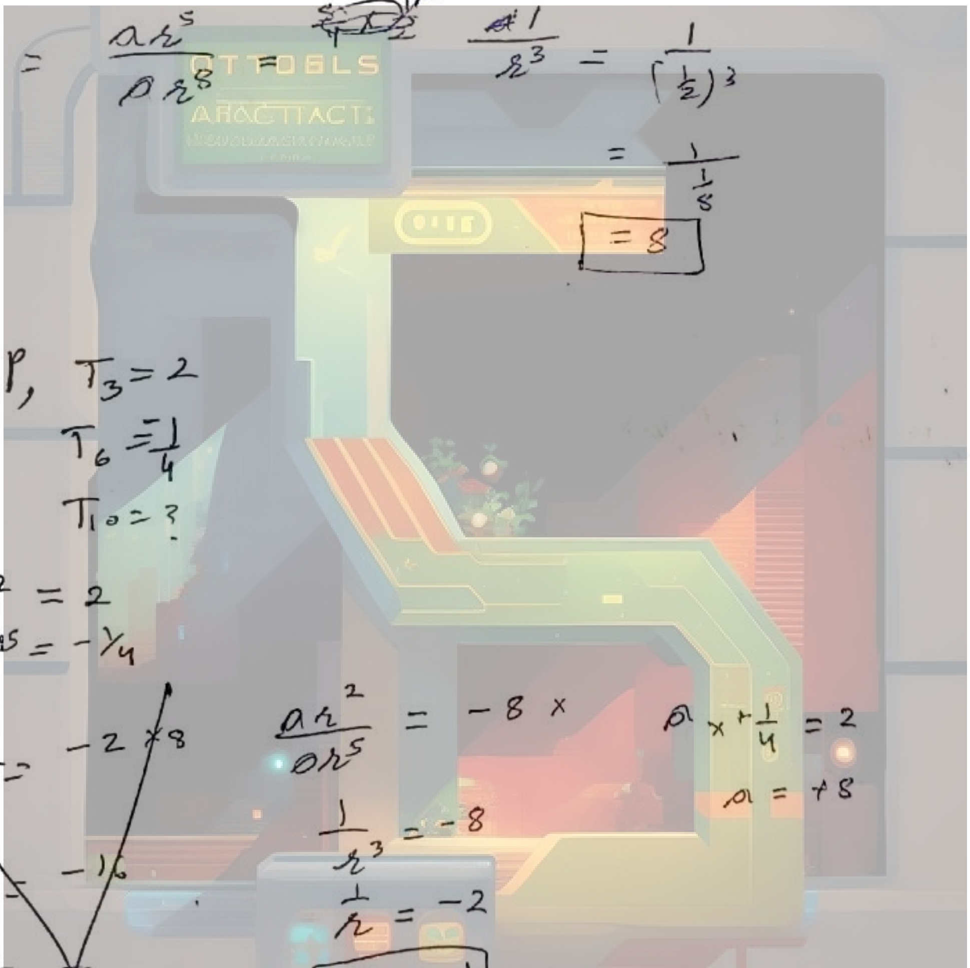
$\boxed{T_{20} = 2^{-20}}$

Q Find the ratio of 6^{th} term & 9^{th} term of the series.

$$\frac{5}{4}, \frac{5}{8}, \frac{5}{16} \ \ldots$$

$$r = \frac{\frac{5}{8}}{\frac{5}{4}}$$

$$= \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$$



$$\frac{T_6}{T_9} = \frac{ar^5}{ar^8} = \frac{1}{r^3} = \frac{1}{\left(\frac{1}{2}\right)^3}$$

$$= \frac{1}{\frac{1}{8}}$$

$$\boxed{= 8}$$

Q In GP, $T_3 = 2$

$T_6 = -\frac{1}{4}$

$T_{10} = ?$

$$ar^2 = 2$$

$$ar^5 = -\frac{1}{4}$$

$$\frac{ar^2}{ar^5} = -8 \qquad a \times \frac{1}{4} = 2$$

$$a = +8$$

$$\frac{1}{r^3} = -8$$

$$\frac{1}{r} = -2$$

$$\boxed{r = -\frac{1}{2}}$$

$$T_{10} = ar^9$$

$$= a \times \left(-\frac{1}{2}\right)^9$$

$$= +8 \times \frac{-1}{512}$$

$$= \frac{-1}{64}$$

Sum of G.P

$\cancel{S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}}$

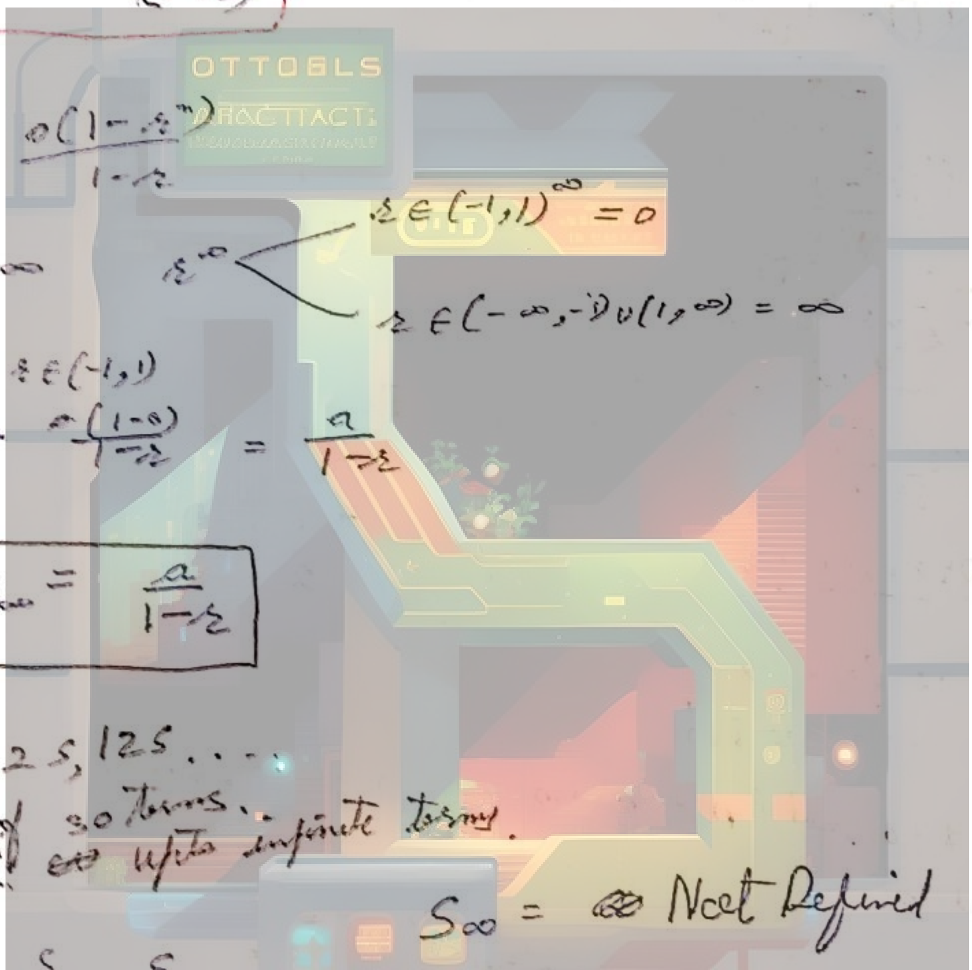
$S_n = a + ar + ar^2 + ar^3 + \dots \dots ar^{n-2} + ar^{n-1}$

$r S_n = ar + ar^2 + ar^3 + ar^4 + \dots \dots ar^{n-1} + ar^n$

$(1-r)S_n = a - ar^n$

$(1-r)S_n = a(1-r^n)$

$\boxed{S_n = \dfrac{a(1-r^n)}{(1-r)}}$ $(r \neq 1)$



* $S_n = \dfrac{a(1-r^n)}{1-r}$

$n \to \infty$ $\quad r^\infty$: $r \in (-1,1)^\infty = 0$; $r \in (-\infty,-1) \cup (1,\infty) = \infty$

for $r \in (-1,1)$

$S_\infty = \dfrac{a(1-0)}{1-r} = \dfrac{a}{1-r}$

$\boxed{S_\infty = \dfrac{a}{1-r}}$

Q $1, 5, 25, 125, \dots$

i) sum of 30 terms.

ii) Sum of ∞ upto infinite terms. $\quad S_\infty = \infty$ Not Defined

$r = \dfrac{5}{1} = 5$

$S_{30} = \dfrac{1(1-5^{30})}{1-5}$

$= \dfrac{1-5^{30}}{-4}$

$= \dfrac{5^{30}-1}{4}$

$= \dfrac{5^{30}}{4} - \dfrac{1}{4}$

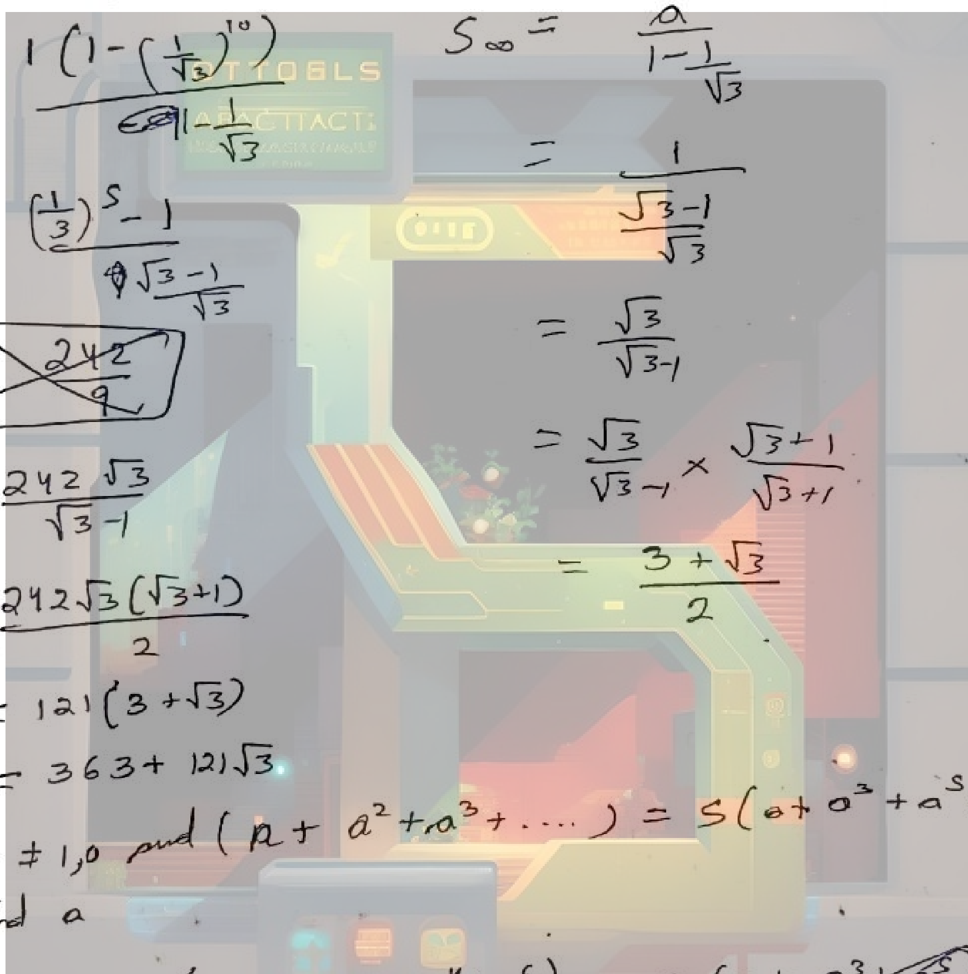
Q $1, \frac{1}{\sqrt3}, \frac{1}{3}, \frac{1}{3\sqrt3}$

$r = \dfrac{\frac{1}{\sqrt3}}{1}$

$= \frac{1}{\sqrt3}$

i) sum of 10 terms

ii) sum of infinite terms



$S_{10} = \dfrac{1\left(1-\left(\frac{1}{\sqrt3}\right)^{10}\right)}{1-\frac{1}{\sqrt3}}$

$= \dfrac{\left(\frac{1}{3}\right)^5 - 1}{\frac{\sqrt3-1}{\sqrt3}}$

$\cancel{\boxed{= \frac{242}{9}}}$

$= \dfrac{242\sqrt3}{\sqrt3-1}$

$= \dfrac{242\sqrt3(\sqrt3+1)}{2}$

$= 121(3+\sqrt3)$

$= 363 + 121\sqrt3$

$S_\infty = \dfrac{a}{1-\frac{1}{\sqrt3}}$

$= \dfrac{1}{\frac{\sqrt3-1}{\sqrt3}}$

$= \dfrac{\sqrt3}{\sqrt3-1}$

$= \dfrac{\sqrt3}{\sqrt3-1} \times \dfrac{\sqrt3+1}{\sqrt3+1}$

$= \dfrac{3+\sqrt3}{2}$

Q (i) If $a \neq 1, 0$ and $(a + a^2 + a^3 + \dots) = 5(a + a^3 + a^5 + \dots)$ find a

$(a + a^3 + a^5 \dots) + (a^2 + a^4 + a^6) = 5(a + a^3 + a^5)$

$a^2 + a^4 + a^6 \dots = 5$

$S_\infty = 5$

$\frac{a^2}{1-a^2} = 5$

$a^2 = 5 - 5a^2$

$6a^2 = 5$

$a^2 = \frac{5}{6}$

$\cancel{\boxed{a = \frac{\sqrt5}{\sqrt6}}}$

$a = \frac{\sqrt5}{\sqrt6}$

$\frac{a}{1-a} = 5\left(\frac{a}{1-a^2}\right)$

$a(1-a)(1+a) = 5a(1-a)$

$a(1-a)(1+a-5) = 0$

$1 + a - 5 = 0$

$a = 4$

but $a \in (-1, 1)$ so $a \in \phi$

(136)

Q $\left(5^{x}+5^{(x+1)}+5^{(x+2)}+\dots\right)=150\left(\frac{1}{5}+\frac{1}{5^3}+\frac{1}{5^5}+\dots\right)$

find ~~x~~ 'x',

$$\left(5^{x}+5^{(x+1)}+5^{(x+2)}\dots\right)=150\left(5^{-1}+5^{-3}+5^{-5}+5^{-7}\dots\right)$$

$a=5^{x}$ $\quad$ $r=5$ $\qquad\qquad$ $a=5^{-1}$ $\quad$ $r=5^{-2}$

$\cancel{\frac{5^{x}}{1-5}=150\left(\frac{\frac{1}{5}}{\frac{25-1}{25}}\right)}$

$\cancel{\frac{5^{x}}{-4}=150\times\left(\frac{5}{24}\right)}$

$\cancel{5^{x}=-600\times\frac{5}{24}}$

$\cancel{5^{x}=-300\times\frac{5}{12}}$

$\cancel{5^{x}=-\frac{250}{3}}$

$\cancel{\log_{5}\frac{-250}{3}}$

$\frac{5^{x}}{1-5}=150\left(\frac{1}{5}\times\frac{25}{24}\right)$

$\frac{5^{x}}{-4}=150\times\frac{5}{24}$

$5^{x}=-\frac{750}{6}$

$\boxed{x\in\phi}$

Q $11x^2-4x-2=0$ $\quad$ a, b

$10(1+a+a^2+a^3+\dots)(1+b+b^2+b^3\dots)$

$a=1$ $\quad$ $r=a$ $\qquad\qquad$ $a=1$ $\quad$ $r=b$

$10\left(\frac{1}{1-a}\right)\left(\frac{1}{1-b}\right)$

$\frac{10}{1-b-a+ab}$

$\frac{10}{1-(a+b)+ab}$

$\frac{10}{1-\frac{4}{11}-\frac{2}{11}}$

$\frac{10\times 11}{11-6}=\frac{110}{5}=\boxed{22}$

0 product

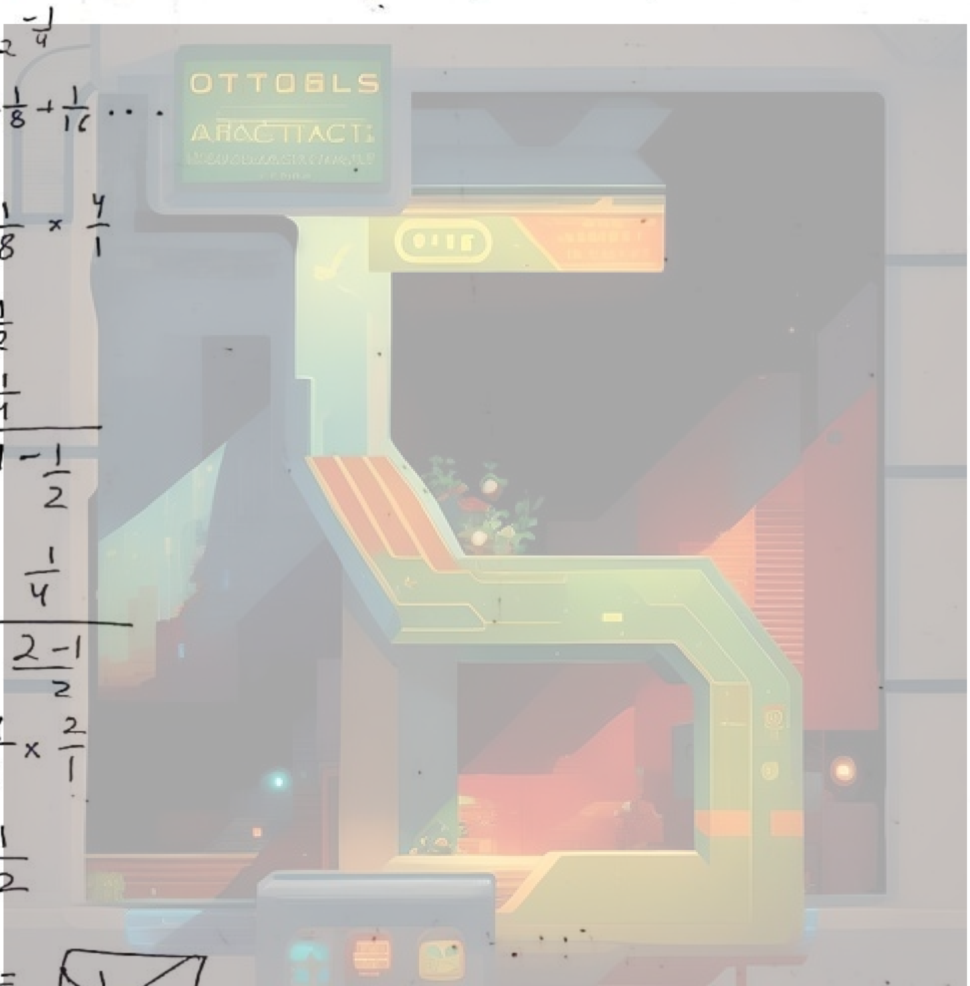
$$2^{1/4}\cdot 4^{1/16}\cdot 8^{1/48}\cdot 16^{\frac{1}{128}}\dots\infty$$

$$2^{1/4}\cdot 2^{1/8}\cdot 2^{\frac{1}{16}}$$

$$r = \frac{2^{1/8}}{2^{1/4}}$$

$$= 2^{\frac{1}{8}-\frac{1}{4}}$$

$$r = 2^{\frac{-1}{4}}$$



$$2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\dots}$$

$$r = \frac{1}{8}\times\frac{4}{1}$$

$$= \frac{1}{2}$$

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{2-1}{2}}$$

$$= \frac{1}{4}\times\frac{2}{1}$$

$$= \frac{1}{2}$$

$$2^{1/2} = \boxed{\cancel{\frac{1}{\sqrt{4}}}}$$

$$= \sqrt{2}$$

Q $(\ln a^2) + (\ln a^2)^2 + (\ln a^2)^3 \dots \infty = (\ln a) + (\ln a)^2 + (\ln a)^3$

$r = \ln a^2$

$a = \ln a^2$

$S_\infty = \dfrac{\ln a^2}{1-\ln a^2}$

$= \dfrac{2\ln a}{1-2\ln a}$

$= \dfrac{2\log_e a}{1-2\log_e a}$

$= \dfrac{2\log_e a}{\log_e e - 2\log_e a}$

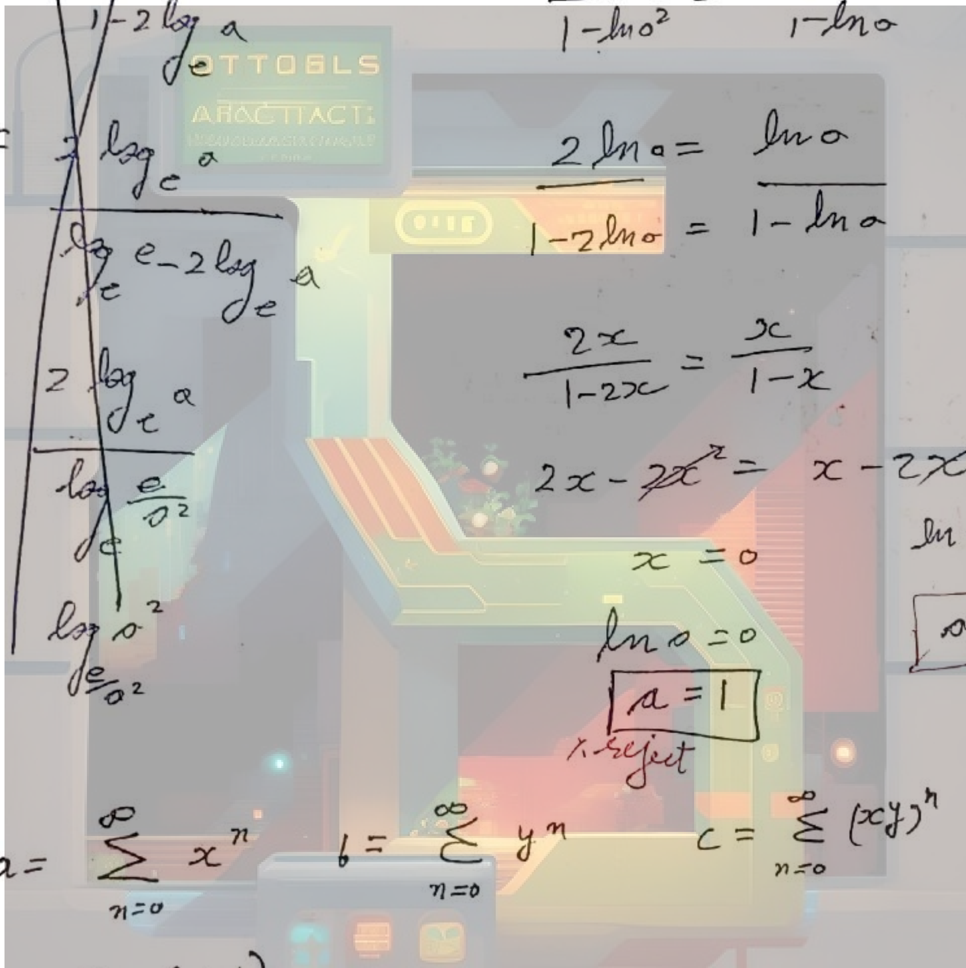
$= \dfrac{2\log_e a}{\log_e \frac{e}{a^2}}$

$= \log_{e/a^2} a^2$

$r = \ln a$

$a = \ln a$

$S_\infty = \dfrac{\ln a}{1-\ln a}$



$\dfrac{\ln a^2}{1-\ln a^2} = \dfrac{\ln a}{1-\ln a}$

$\dfrac{2\ln a}{1-2\ln a} = \dfrac{\ln a}{1-\ln a}$

$\dfrac{2x}{1-2x} = \dfrac{x}{1-x}$

$2x - \cancel{2x^2} = x - \cancel{2x^2}$

$x = 0$

$\ln a = 0$

$\boxed{a = 1}$

x-reject

$\ln a = +\frac{1}{4}$

$\boxed{a = e^{+1/4}}$

Q If $a = \sum_{n=0}^{\infty} x^n \quad b = \sum_{n=0}^{\infty} y^n \quad c = \sum_{n=0}^{\infty} (xy)^n$

$x, y \in (-1, 1)$

find $\left(\dfrac{a-1}{a}\right)\left(\dfrac{b-1}{b}\right) - \dfrac{(c-1)}{c}$

$x = -1$ ~~$y = -1$~~

a: $1 + (-1) + 1 - 1 \dots \infty$

$a = 0$

$x = 1$

$a = \infty$

$y = -1$

$a = 0$

$a = \infty$

$c = 0$

$c = \infty$

$a = \dfrac{1}{1-x}$

$b = \dfrac{1}{1-y}$

$c = \dfrac{1}{1-xy}$

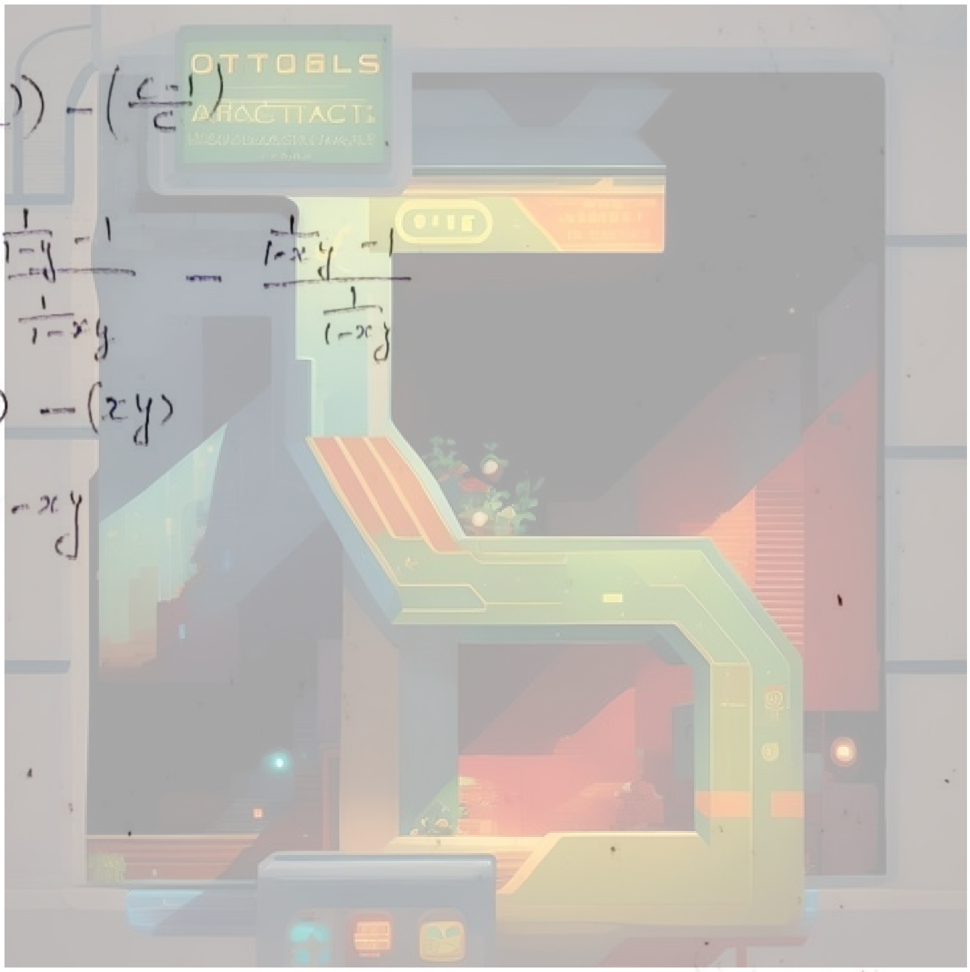
$$\left(\frac{a-1}{a}\right)\left(\frac{b-1}{b}\right) - \left(\frac{c-1}{c}\right)$$

$$\frac{\frac{1}{1-x} - 1}{\frac{1}{1-x}} \quad \frac{\frac{1}{1-y} - 1}{\frac{1}{1-xy}} - \frac{\frac{1}{1-xy} - 1}{\frac{1}{1-xy}}$$

$$(x)\ (y) - (xy)$$

$$xy - xy$$

$$\boxed{=0}$$



Q $7 + 77 + 777 + \dots n$

$7 + 7\times10+7 + 7\times100+7 \dots n$

$\underbrace{7\times100 + 7\times10 + 7}$

~~$7(1+10^1+7\times10^0$~~

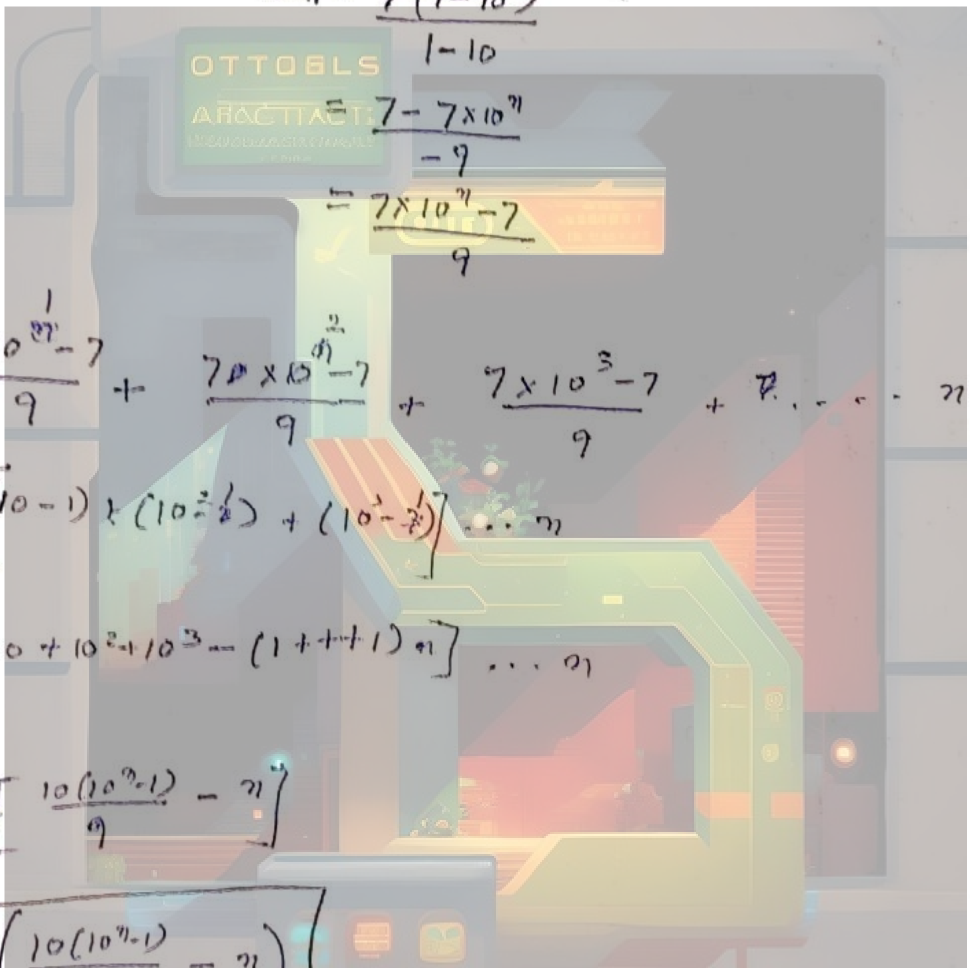
$\downarrow$

$7\times10^2 + 7\times10^1 + 7\times10^0$

So $a = 7$

$r = 10$

$S_n = \dfrac{7(1-10^n)}{1-10}$



$= \dfrac{7 - 7\times10^n}{-9}$

$= \dfrac{7\times10^n - 7}{9}$

$\dfrac{7\times10^1 - 7}{9} + \dfrac{7\times10^2 - 7}{9} + \dfrac{7\times10^3 - 7}{9} + \dots n$

$\dfrac{7}{9}\left[(10-1) + (10^2-1) + (10^3-1)\right] \dots n$

$\dfrac{7}{9}\left[10 + 10^2 + 10^3 \dots - (1+1+\dots+1)n\right] \dots n$

$\dfrac{7}{9}\left[\dfrac{10(10^n-1)}{9} - n\right]$

$\boxed{\dfrac{7}{9}\left(\dfrac{10(10^n-1)}{9} - n\right)}$

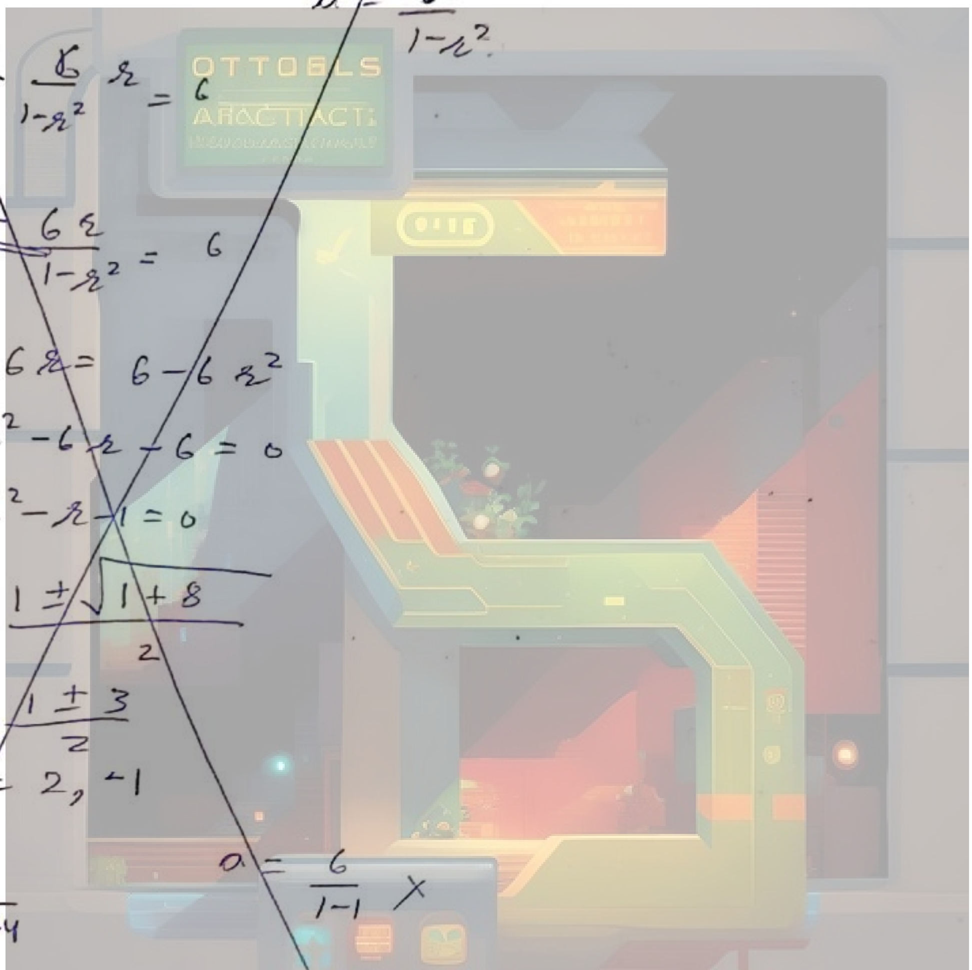
Q In a sequence of 4 nos., 1st 3 nos. are in G.P & last 3 are in A.P with CD -6 If 1st & last terms of the sequence are equal then find last term.

$a + ar + ar^2 + a$

$ar^2 - ar = 6$ $\quad$ $a - ar^2 = 6$

$\cancel{ar(r-1) = 6}$ $\quad$ $a(1-r^2) = 6$

$a = \frac{6}{1-r^2}$



$\frac{6}{1-r^2} \times r^2 - \frac{6}{1-r^2} r = 6$

$\frac{6r^2}{1-r^2} - \frac{6r}{1-r^2} = 6$

$6r^2 - 6r = 6 - 6r^2$

$12r^2 - 6r - 6 = 0$

$2r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{1+8}}{2}$

$= \frac{1 \pm 3}{2}$

$= 2, -1$

$a = \frac{6}{1-4}$ $\quad$ $a = \frac{6}{1-1}$ ×

$a = \frac{6}{-3}$

$= -2$

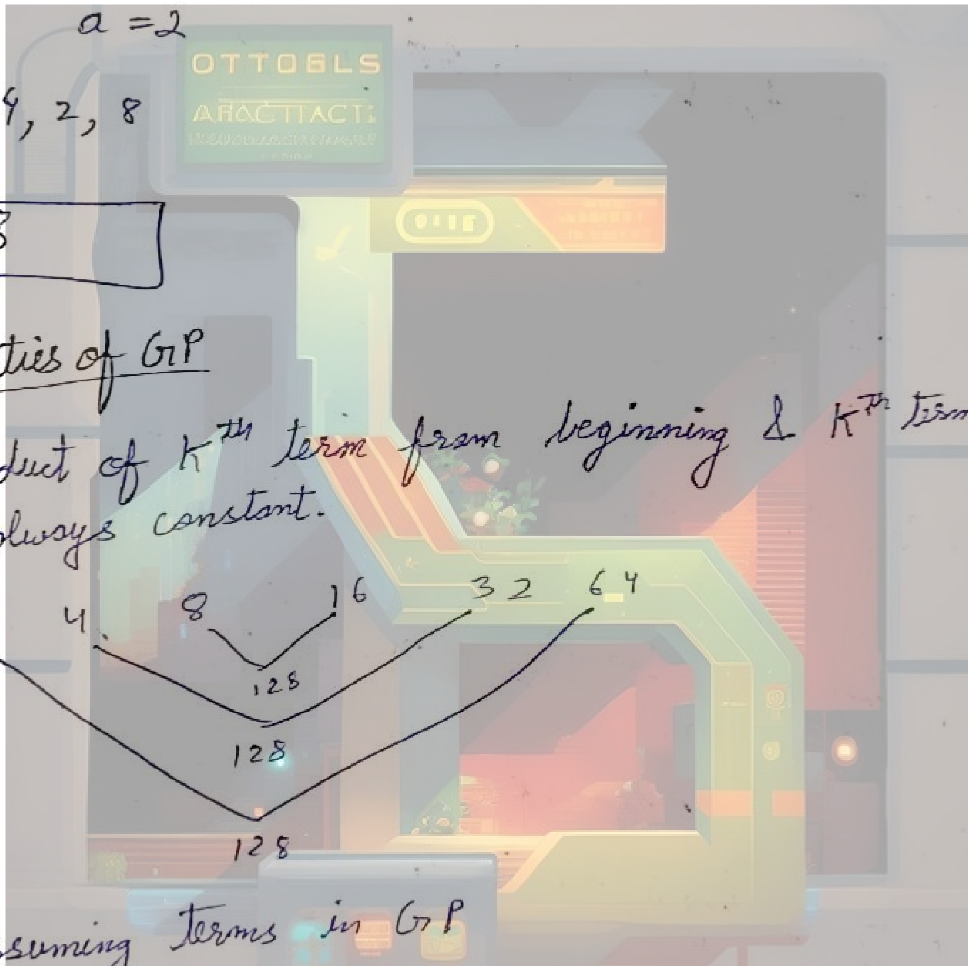
$\underbrace{a+6,\ a-6,\ a}$, $a+6$

GP

$$\frac{a-6}{a+6} = \frac{a}{a-6}$$

$$a^2 + 36 - 12a = a^2 + 6a$$

$$36 = 18a$$



$$a = 2$$

$8, -4, 2, 8$

$\boxed{8}$

Properties of GP

(1) Product of K^{th} term from beginning & K^{th} term from end is always constant.

2 4 8 16 32 64

128

128

128

(2) Assuming terms in GP

3: $\frac{a}{r}, a, ar$

4: $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

5: $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

6: $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

③ If each term of GP is raised to same power, multiplied or divided then the resulting series ~~w~~ is also a GP.

$2, 4, 8, 16, 32 \ldots\ldots$

$2^{1/8}, 4^{1/8}, 8^{1/8}, 16^{1/8}, 32^{1/8} \ldots\ldots$

$\dfrac{4^{1/8}}{2^{1/8}} = 2^{1/8}$ $\dfrac{8^{1/8}}{4^{1/8}} = 2^{1/8}$

④ If Three Terms A, B & C are in GP then $b^2 = ac$

a, b, c

$\dfrac{b}{a} = \dfrac{c}{b}$

$b^2 = ac$

⑤ For $a_1, a_2, a_3 \ldots a_n$ are +ve real nos. & They are in GP then

$\log a_1, \log a_2, \log a_3, \log a_4 \ldots \log a_n$ are in AP

eg. $2, 4, 8 \ldots$ are in GP

$\log 2, \log 4, \log 8$ are in AP

Hormonic Progression (H.P.)

→ A non zero sequence is said to be in H.P if resiprocal of its terms are in AP.

eg. $a_1, a_2, a_3, a_4 \ldots\ldots$ HP

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4} \ldots\ldots$ AP

→ standard form.

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} \ldots\ldots\ldots \frac{1}{a+(n-1)d}$ .. HP

Note:- ① No term of HP can be zero

② Resiprocal of every H.P. is A.P. but resiprocal of every A.P. can or cannot be H.P.

③ If a, b, c are in H.P then

$a, b, c \rightarrow HP$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow AP$

$2 \times \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

$\frac{2}{b} = \frac{a+c}{aBc}$

$$\boxed{b = \frac{2ac}{a+c}}$$

(4) No general formula for finding the sum of H.P.

Q Determine the 4^{th} & 8^{th} term of H.P. 6, 4, 3 ....

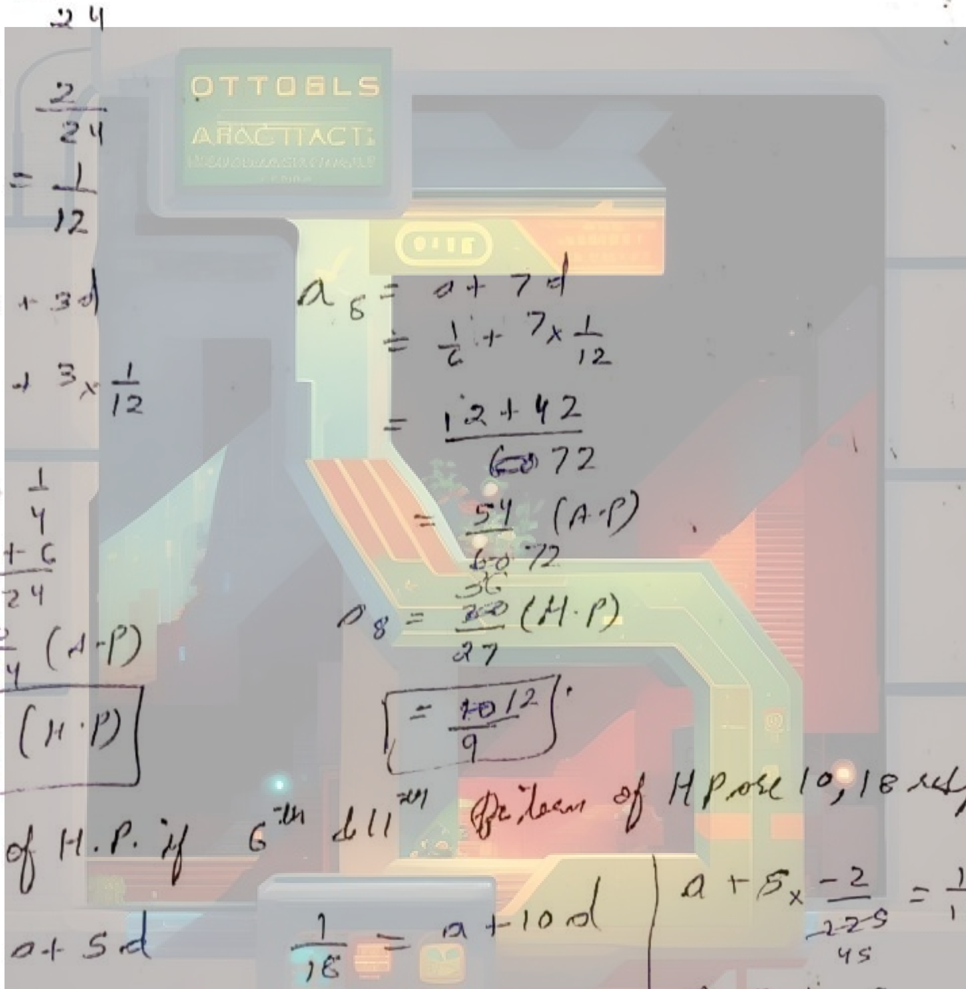
$\frac{1}{6}, \frac{1}{4}, \frac{1}{3} \dots$ A.P

$\frac{1}{4} - \frac{1}{6} = C.D$

$d = \frac{6-4}{24}$

$= \frac{2}{24}$

$= \frac{1}{12}$



$a_4 = a + 3d$

$= \frac{1}{6} + 3 \times \frac{1}{12}$

$= \frac{1}{6} + \frac{1}{4}$

$= \frac{4+6}{24}$

$= \frac{10}{24}$ (A.P)

$\boxed{a_4 = \frac{24}{10} \text{ (H.P)}}$

$a_8 = a + 7d$

$= \frac{1}{6} + 7 \times \frac{1}{12}$

$= \frac{12+42}{\cancel{60}\,72}$

$= \frac{54}{\cancel{60}\,72}$ (A.P)

$a_8 = \frac{36\,\cancel{30}}{27}$ (H.P)

$\boxed{= \frac{\cancel{10}\,12}{9}}$

Q find 16^{th} term of H.P. if 6^{th} & 11^{th} term of H.P are 10, 18 respectively

$\frac{1}{10} = a + 5d$

$\frac{1}{18} = a + 10d$

$\frac{1}{18} - \frac{1}{10} = 5d$

$\frac{-8}{180} = 5d$

$\frac{-8}{900} = d$

$d = \frac{-2}{225}$

$a + 5 \times \frac{-2}{\cancel{225}\,45} = \frac{1}{10}$

$a = \frac{1}{10} + \frac{2}{45}$

$a = \frac{\cancel{47}\,13}{\cancel{450}\,90}$

$\cancel{a_{16} = \frac{47}{450} - 15 \times \frac{+2}{225}}$

$\cancel{a_{16} = \frac{47-60}{450}}$

$\cancel{= \frac{-13}{450}}$

$a_{16} = \frac{13}{90} - 15 \times \frac{2}{225}$

$\boxed{a_{16} = 90}$

Q If a, b, c, d, e are 5 nos, a, b, c in AP.

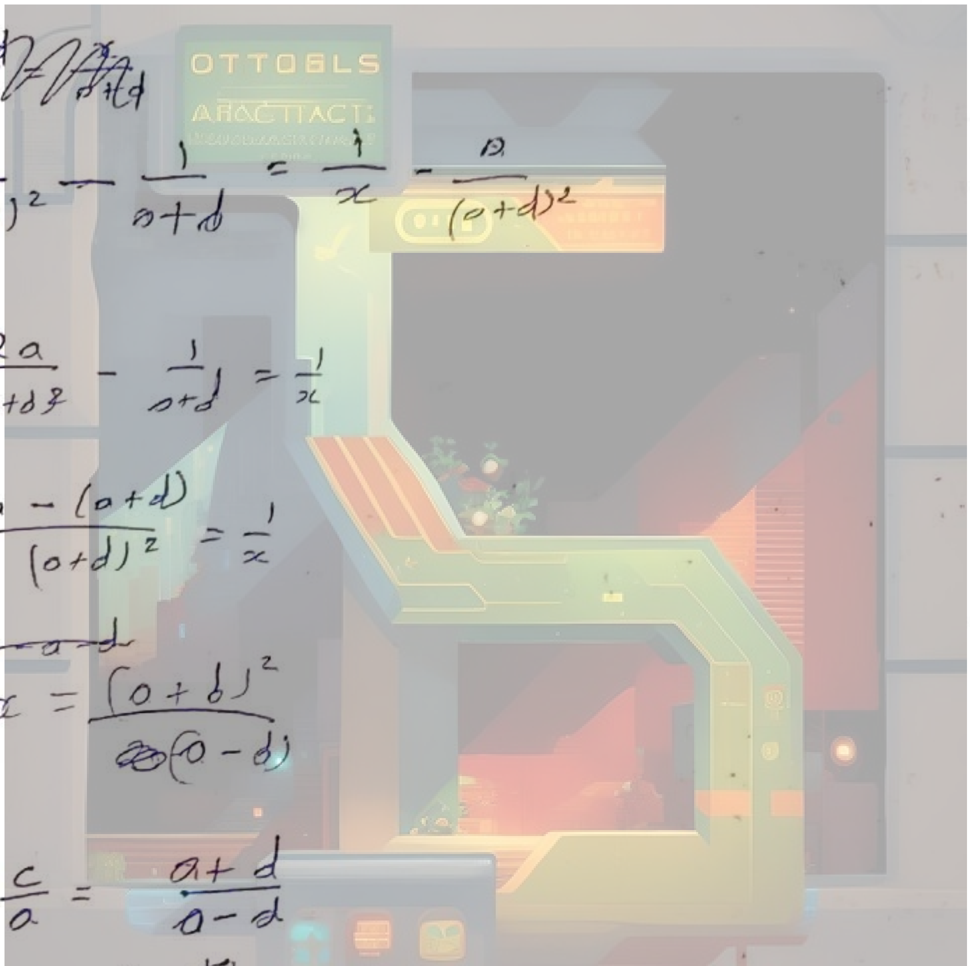
b, c, d in GP.

c, d, e in HP.

a) Prove a, c, e in GP

b) $e = \dfrac{(2b-a)^2}{a}$

$$a-d,\ a,\ a+d,\ \frac{(a+d)^2}{a},\ \frac{(a+d)^2}{a-d}$$



$$\frac{a}{(a+d)^2} - \frac{1}{a+d} = \frac{1}{x} - \frac{a}{(a+d)^2}$$

$$\frac{2a}{(a+d)^2} - \frac{1}{a+d} = \frac{1}{x}$$

$$\frac{2a-(a+d)}{(a+d)^2} = \frac{1}{x}$$

$$x = \frac{(a+d)^2}{(a-d)}$$

a) $\dfrac{c}{a} = \dfrac{a+d}{a-d}$

$$\frac{e}{c} = \frac{(a+d)^2}{a-d} \times \frac{1}{a+d}$$

$$\frac{e}{c} = \frac{a+d}{a-d}$$

$\dfrac{c}{a} = \dfrac{e}{c}$ so a, c, e are in GP.

b) $\dfrac{(2b-a)^2}{a}$

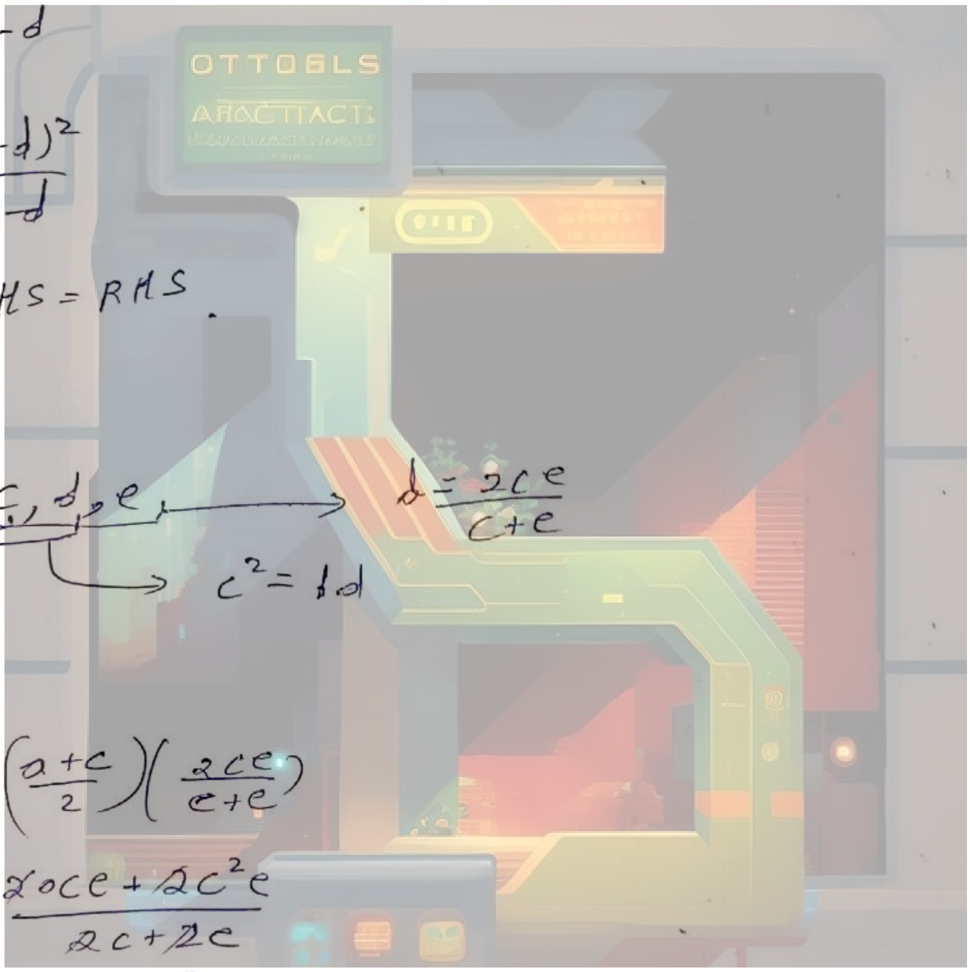
$\cancel{2a-d+d}$

$\dfrac{(2a-a+d)^2}{a-d}$

$= \dfrac{(a+d)^2}{a-d}$

$e = \dfrac{(a+d)^2}{a-d}$

Hence LHS = RHS.



Method II

a, b, c, d, e $\longrightarrow$ $d = \dfrac{2ce}{c+e}$

$\longrightarrow c^2 = bd$

$b = \dfrac{a+c}{2}$

$c^2 = \left(\dfrac{a+c}{2}\right)\left(\dfrac{2ce}{c+e}\right)$

$= \dfrac{2ace + 2c^2e}{2c+2e}$

$= \dfrac{ace + c^2e}{c+e}$

$c^2(c+e) = (a+c)ce$

$c^2e + c^3 = ace + c^2e$

$\cancel{c^2 e = ae}$

$\cancel{a = c^2}$

$c^2 = ae$

Thus, a, c & e are in GP

b) $e = \frac{(2b-a)^2}{a}$

$a e = (2b-a)^2$

$c^2 = (2b-a)^2$

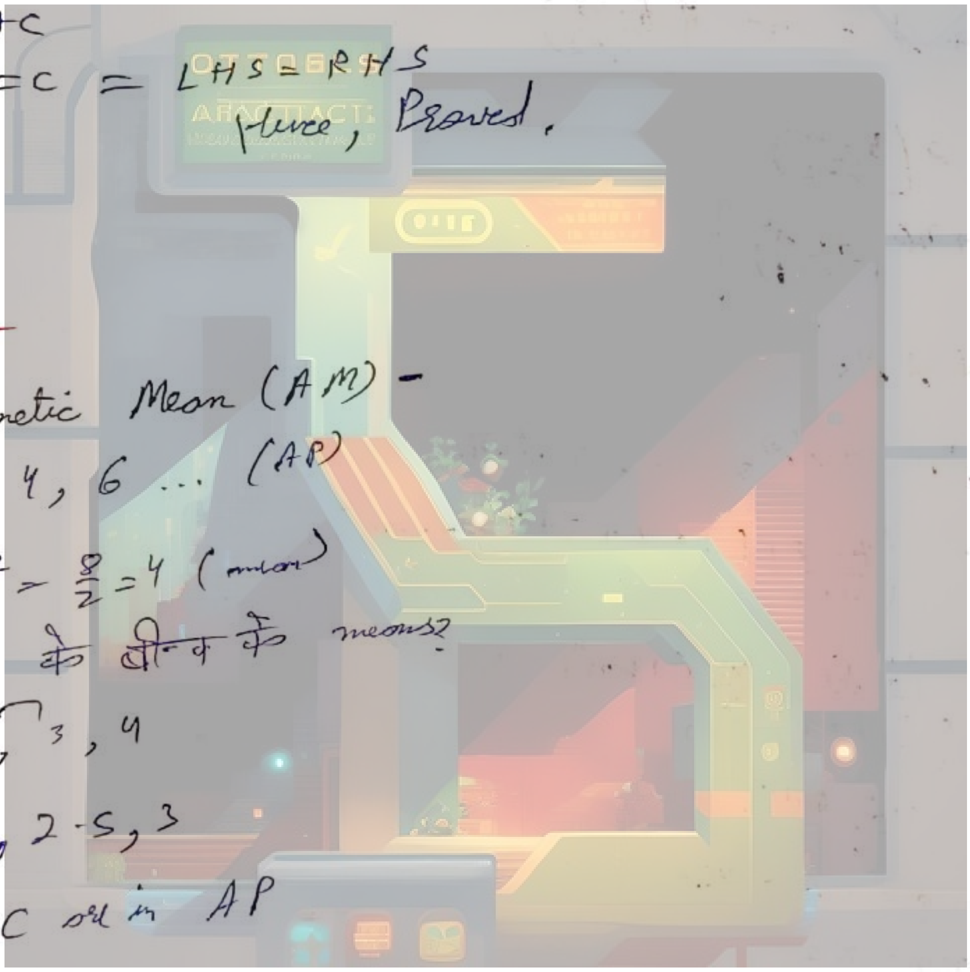
$c = 2b - a = LHS$

$b = \frac{a+c}{2}$

$2b = a + c$

$2b - a = c = LHS = RHS$

Hence, Proved.



Mean :-

① Airthmetic Mean (AM) -

$2, 4, 6 \ldots$ (AP)

$\frac{6+2}{2} = \frac{8}{2} = 4$ (mean)

2, 4 के बीच के means?

$2, 3, 4$

$2, 2.5, 3$

a, b, c are in AP

$b = \frac{a+c}{2}$

Find means b/w a & b?

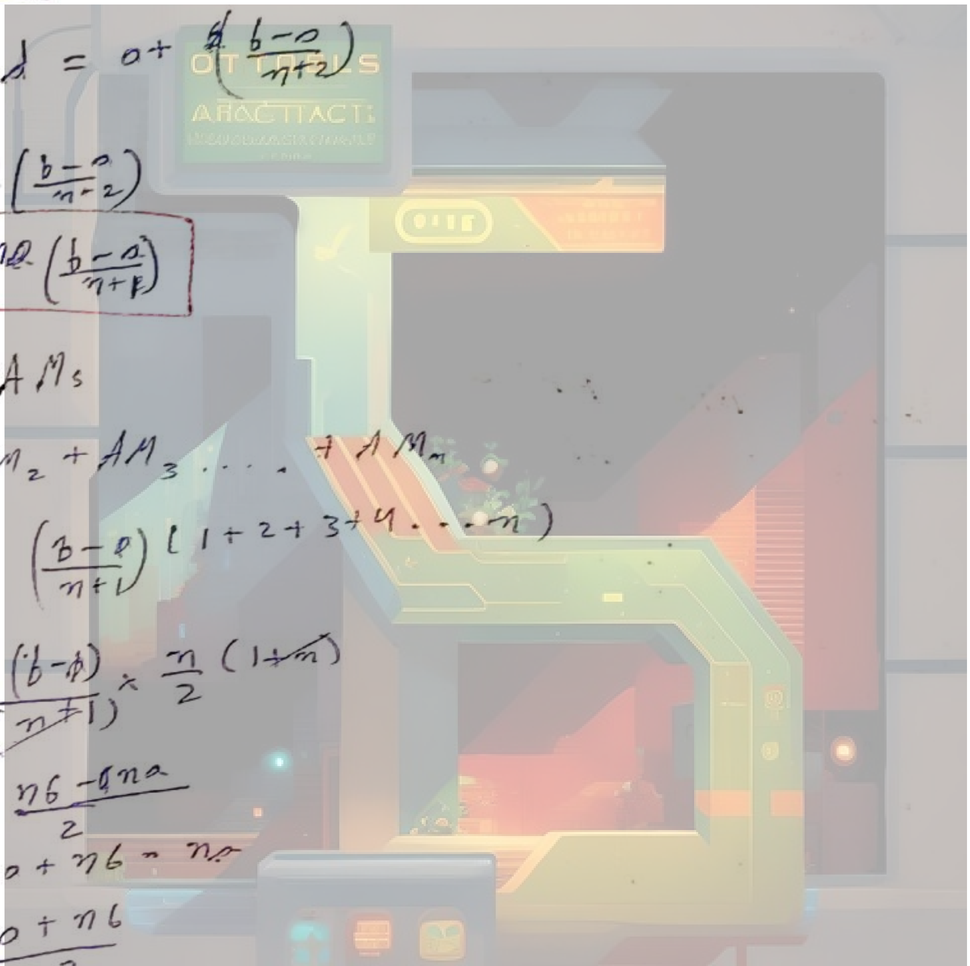
$$\underbrace{a, AM_1, AM_2, AM_3, AM_4 \ldots\ldots AM_n}_{n}, b \ (AP)$$

$$b = a + (n+2-1)d$$

$$b = a + (n+1)d$$

$$b - a = (n+1)d$$

$$\boxed{d = \frac{b-a}{n+1}}$$



$$AM_1 = a + d = a + \left(\frac{b-a}{n+2}\right)$$

$$AM_2 = a + 2\left(\frac{b-a}{n+2}\right)$$

$$\boxed{AM_n = a + n\left(\frac{b-a}{n+1}\right)}$$

Sum of all AMs

$$AM_1 + AM_2 + AM_3 \ldots + AM_n$$

$$na + \left(\frac{b-a}{n+1}\right)(1+2+3+4\ldots n)$$

$$na + \frac{(b-a)}{\cancel{(n+1)}} \times \frac{n}{2}\cancel{(1+n)}$$

$$na + \frac{nb - na}{2}$$

$$\frac{2na + nb - na}{2}$$

$$\frac{na + nb}{2}$$

$$\boxed{\frac{n(a+b)}{2} = S_n \text{ of AMs}}$$

Q Insert 20 AMs btw 4 & 67

1) find AMs

(2) find Sum of AMs

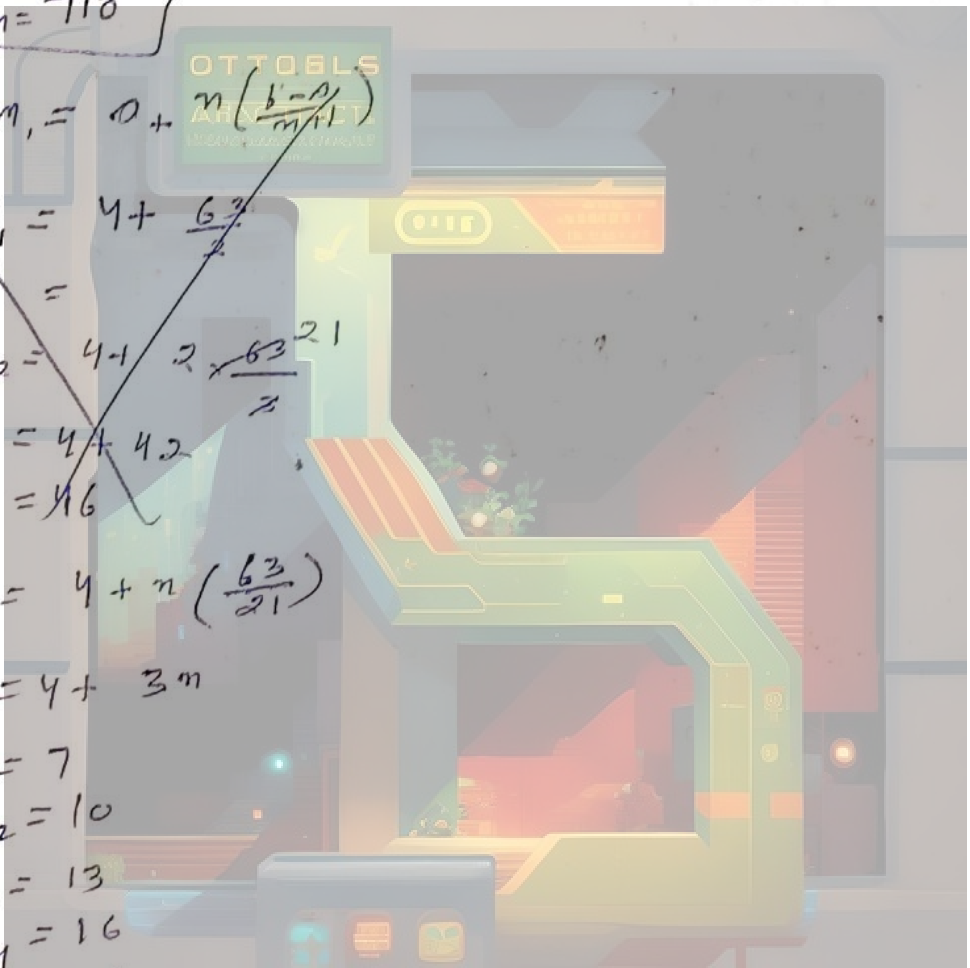
4, AM_1, AM_2 ... 67

$n = 20$

(2) $20\left(\frac{67+4}{2}\right)$

$= 10 \times 71$

$\boxed{S_n = 710}$



(1) $AM_1 = a + n\left(\frac{b-a}{n+1}\right)$

$AM_1 = 4 + \frac{63}{2}$

$=$

$AM_2 = 4 + 2 \times \frac{63}{2}$ 21

$= 4 + 42$

$= 46$

(1) $AM = 4 + n\left(\frac{63}{21}\right)$

$= 4 + 3n$

$AM_1 = 7$

$AM_2 = 10$

$AM_3 = 13$

$AM_4 = 16$

$AM_5 = 19$

$AM_6 = 22$

⋮

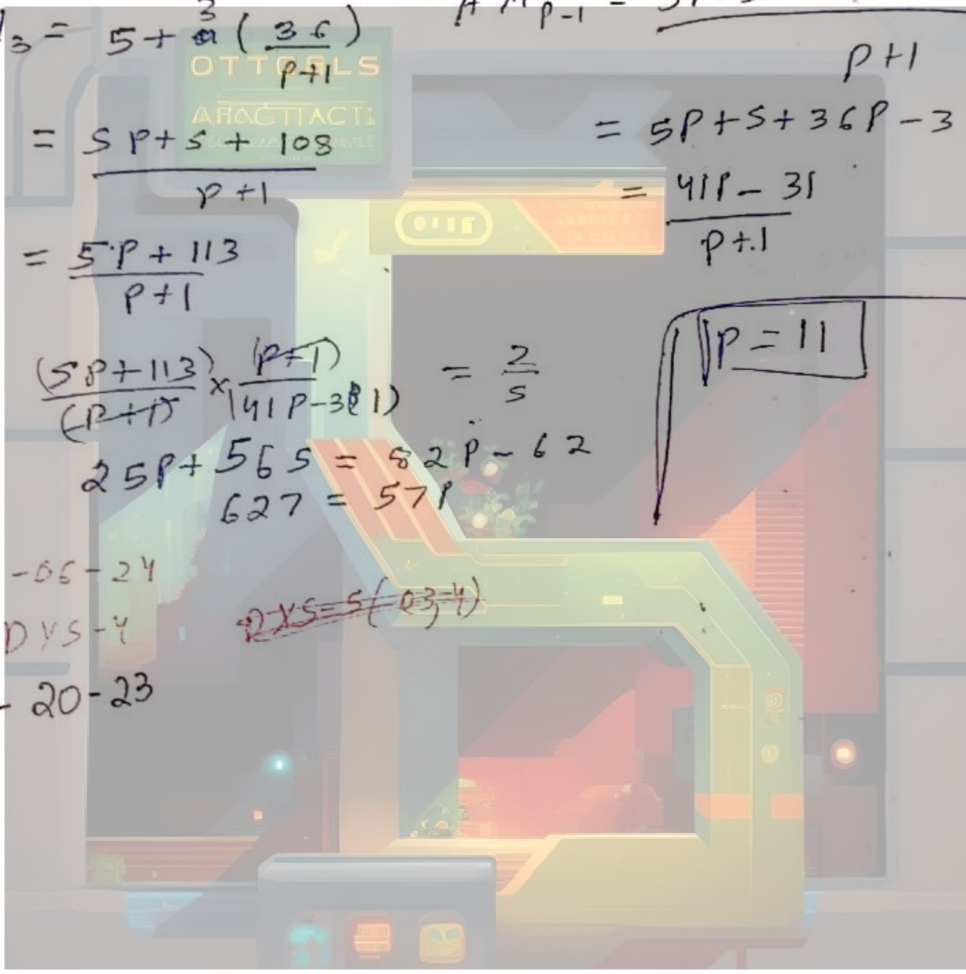
Q find 50 AMs b/w 1 & 99

$$S_m = \frac{99+1}{2} \times 50$$

$$= 50 \times 50$$

$$\boxed{= 2500}$$

Q If p AMs inserted b/w 5, 41, $\frac{AM_3}{AM_{p-1}} = \frac{2}{5}$ find 'p'



$$AM_3 = 5 + 3\left(\frac{36}{p+1}\right)$$

$$= \frac{5p+5+108}{p+1}$$

$$= \frac{5p+113}{p+1}$$

$$AM_{p-1} = \frac{5p+5+(p-1)(36)}{p+1}$$

$$= 5p+5+36p-36$$

$$= \frac{41p-31}{p+1}$$

$$\frac{(5p+113)}{(p+1)} \times \frac{(p+1)}{(41p-31)} = \frac{2}{5}$$

$$25p+565 = 82p-62$$

$$627 = 57p$$

$$\boxed{p = 11}$$

H.W. 25-06-24

PYS-3 DYS-4 ~~PYS=5(Q3-4)~~

Prac-20-23

Geometric mean :

$$a,\ \underbrace{GM_1,\ GM_2,\ GM_3,\ GM_4\ \ldots}_{n}\ b \qquad (\text{Total } n\ GM_s + 2\ (a,b))$$

$$b = ar^{(n+2)-1}$$

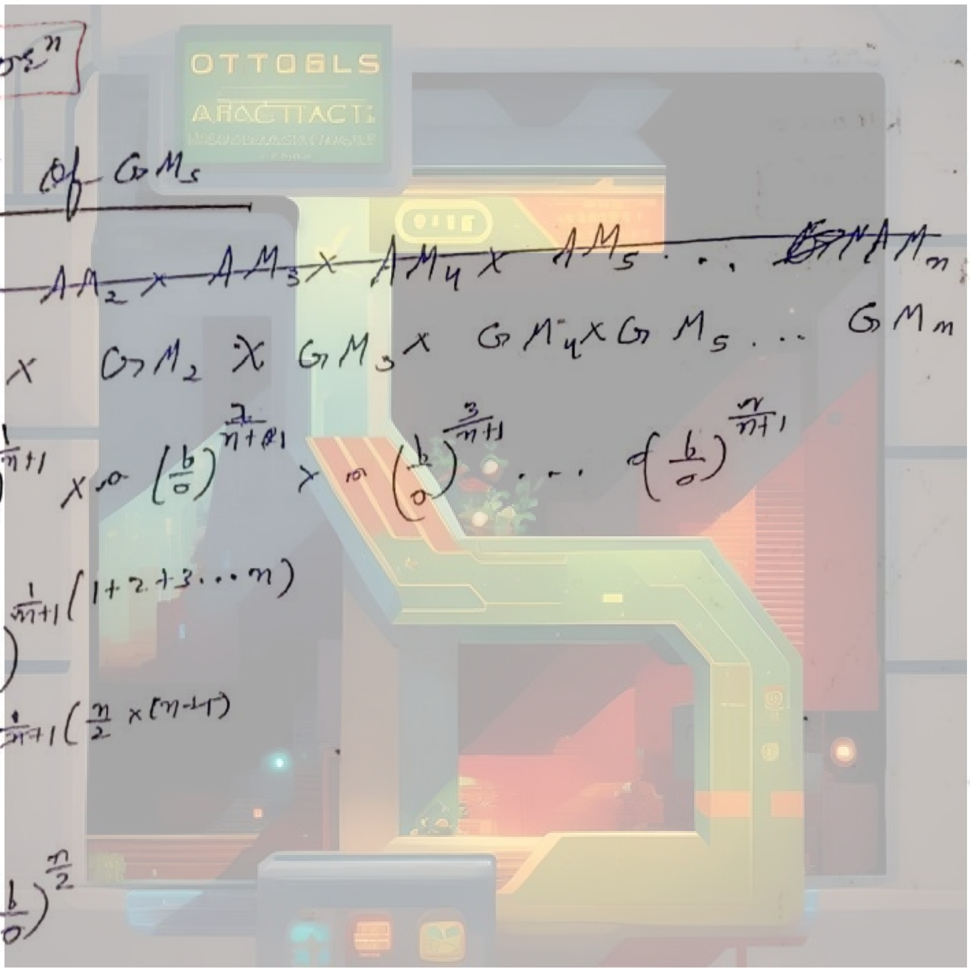
$$b = ar^{n+1}$$

$$\boxed{\left(\frac{b}{a}\right)^{\frac{1}{n+1}} = r}$$

$$GM_1 = ar$$

$$GM_2 = ar^2$$

$$\boxed{GM_n = ar^n}$$



Product of GM_s

$$\cancel{AM_1 \times AM_2 \times AM_3 \times AM_4 \times AM_5 \ldots AM_n}$$

$$GM_1 \times GM_2 \times GM_3 \times GM_4 \times GM_5 \ldots GM_n$$

$$a\left(\frac{b}{a}\right)^{\frac{1}{n+1}} \times a\left(\frac{b}{a}\right)^{\frac{2}{n+1}} \times a\left(\frac{b}{a}\right)^{\frac{3}{n+1}} \ldots a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

$$a^n \left(\frac{b}{a}\right)^{\frac{1}{n+1}(1+2+3\ldots n)}$$

$$a^n \left(\frac{b}{a}\right)^{\frac{1}{n+1}\left(\frac{n}{2}\times(n+1)\right)}$$

$$a^n \left(\frac{b}{a}\right)^{\frac{n}{2}}$$

$$\left(a \cdot \frac{\sqrt{b}}{\sqrt{a}}\right)^n$$

$$\boxed{(\sqrt{ab})^n}$$

Q Insert 4 GMs b/w 5 & 160. find product of GMs.

$$r = \left(\frac{160}{5}\right)^{\frac{1}{5}}$$

$$= (32)^{1/5}$$

$$\underline{r = 2}$$

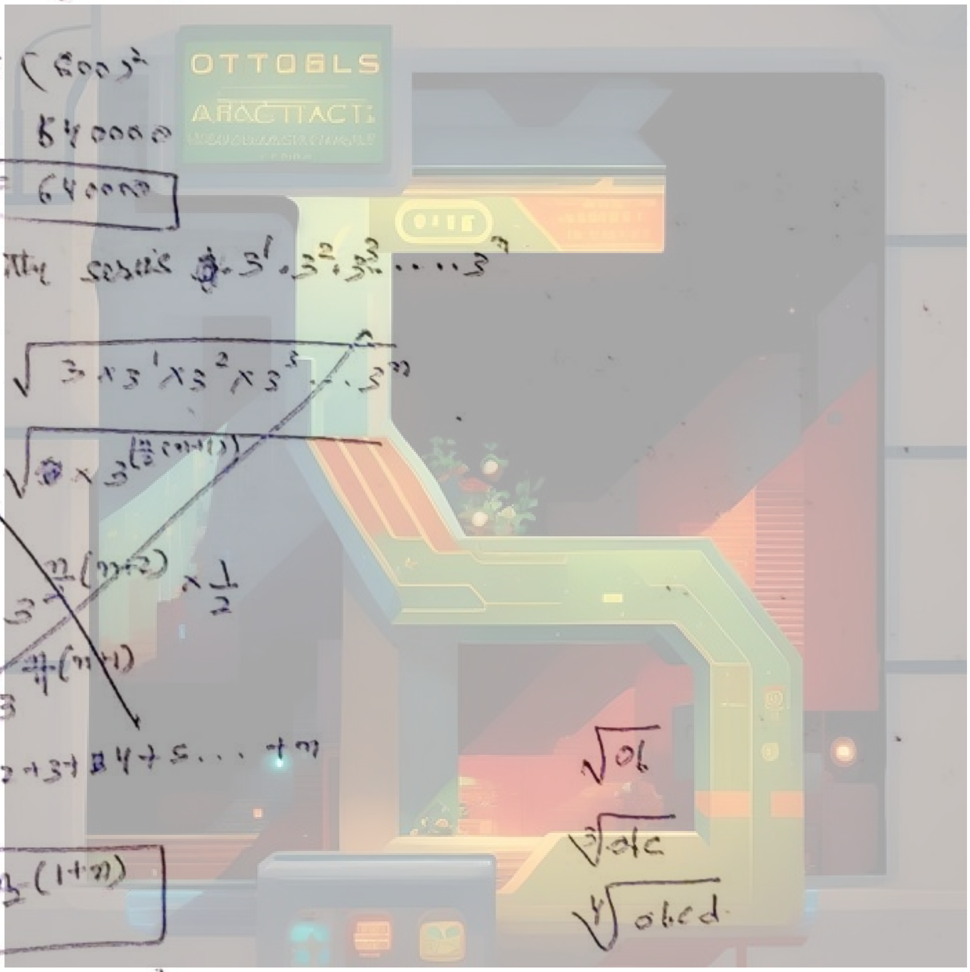
GMs → 10, 20, 40, ..., 80

Product = $\left(\sqrt{5\times160}\right)^4$

$$= (800)^2$$

$$= 640000$$

$$\boxed{= 640000}$$



Q find GM of the series 3, 3^1, 3^2, 3^3 ..... 3^n

$$\cancel{GM = \sqrt[n]{3\times3^1\times3^2\times3^3 ... 3^n}}$$

$$\cancel{= \sqrt{3\times3^{\left(\frac{n}{2}(n+1)\right)}}}$$

$$\cancel{= 3^{\frac{n(n+2)}{2}\times\frac{1}{2}}}$$

$$\cancel{= 3^{\frac{n}{4}(n+1)}}$$

$$3^{1+2+3+4+5...+n}$$

$$\boxed{3^{\frac{n}{2}(1+n)}}$$

$$\sqrt{ab}$$

$$\sqrt[3]{abc}$$

$$\sqrt[4]{abcd}$$

$$\left[3^{\frac{n}{2}(1+n)}\right]^{\frac{1}{n}}$$

$$\boxed{3^{\frac{(1+n)}{2}}}$$

Q find product of 30 GMs btw 2 & 8.

Product $= (\sqrt{ab})^{30}$

$= (2\times 8)^{15}$

$\boxed{= 16^{15}}$

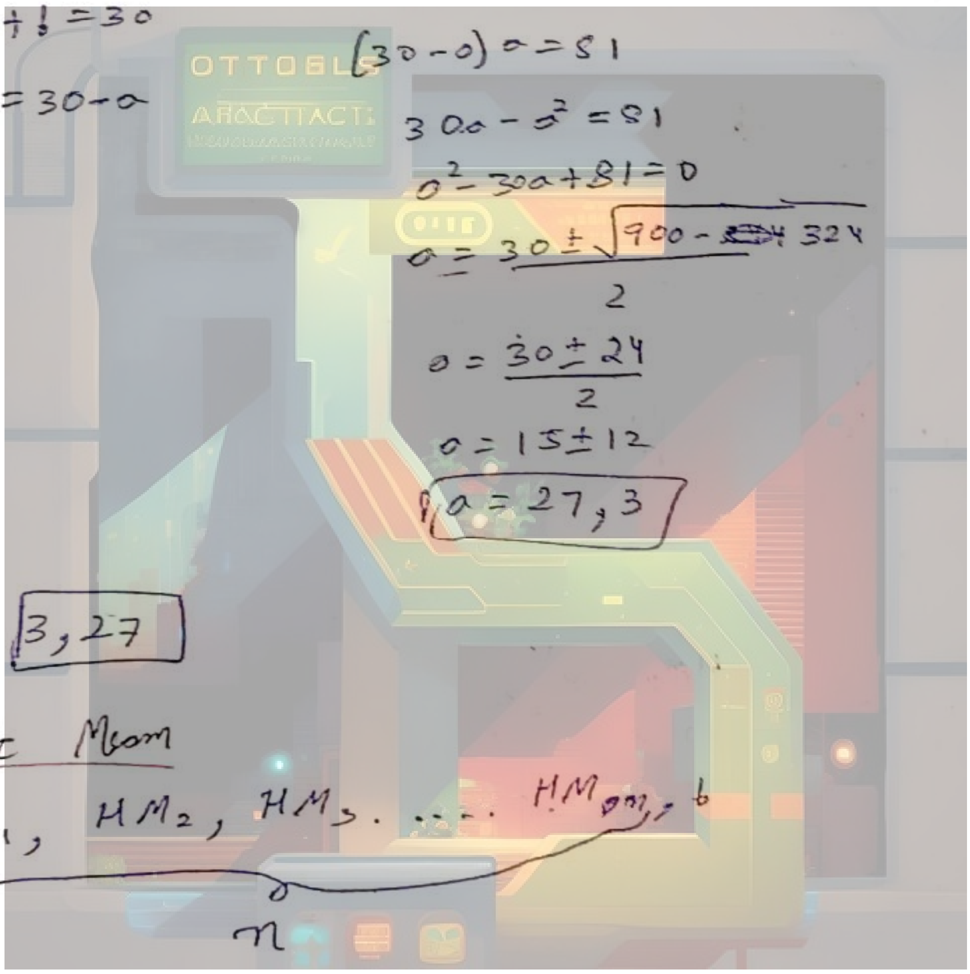
Q $AM(a,b)=15$
$GM(a,b)=9$. find a,b.

$\frac{a+b}{2} = 15$ $\qquad \sqrt{ab} = 9$

$ab = 81$

$a+b = 30$

$b = 30-a$



$(30-a)a = 81$

$30a - a^2 = 81$

$a^2 - 30a + 81 = 0$

$a = \frac{30 \pm \sqrt{900 - 324}}{2}$

$a = \frac{30 \pm 24}{2}$

$a = 15 \pm 12$

$\boxed{a = 27, 3}$

$b = \boxed{3, 27}$

Harmonic Mean

$\underbrace{a, HM_1, HM_2, HM_3 \ldots\ldots HM_n, b}_{n}$

$\frac{1}{a}, \frac{1}{HM_1}, \frac{1}{HM_2} \ldots\ldots \frac{1}{HM_3}, \frac{1}{b} \ldots\ldots$ A.P.

$\frac{1}{b} = \frac{1}{a} + (n+2-1)d$

$\frac{1}{b} - \frac{1}{a} = (n+1)d$

$\boxed{d = \frac{a-b}{ab(n+1)}}$

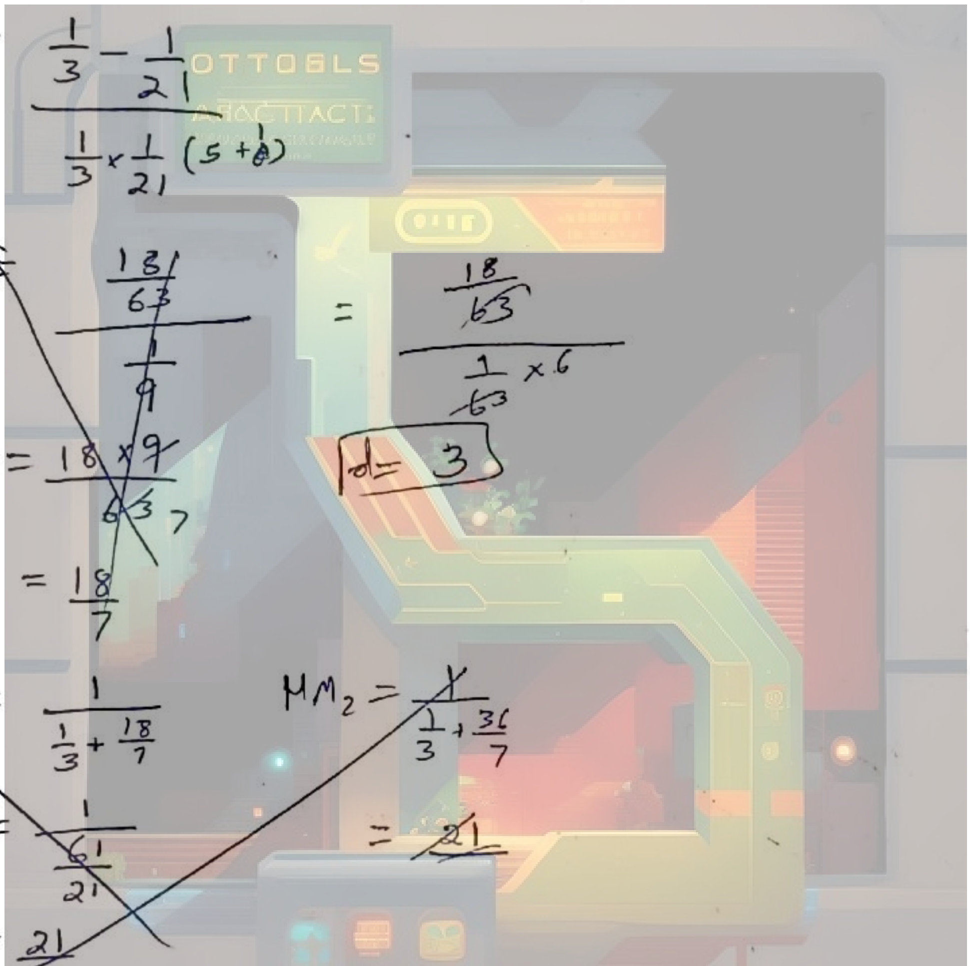
$AM_1 = a + (1)d$ $\quad \boxed{a = \frac{1}{a}}$

$HM_1 = \frac{1}{a+d}$

$HM_2 = \frac{1}{a+2d}$

$\boxed{HM_n = \frac{1}{a+nd}}$

Q Insert 5 HM btw $\frac{1}{3}$ & $\frac{1}{21}$.



$d = \dfrac{\frac{1}{3} - \frac{1}{21}}{\frac{1}{3} \times \frac{1}{21}(5+1)}$

$\cancel{\dfrac{\frac{18}{63}}{\frac{1}{9}}} = \dfrac{\frac{18}{63}}{\frac{1}{63} \times 6}$

$\cancel{= \frac{18 \times 9}{63_7}}$ $\quad \boxed{d = 3}$

$\cancel{= \frac{18}{7}}$

$\cancel{HM_1 = \dfrac{1}{\frac{1}{3} + \frac{18}{7}}}$ $\quad HM_2 = \dfrac{1}{\frac{1}{3} + \frac{36}{7}}$

$\cancel{= \dfrac{1}{\frac{61}{21}}}$ $\quad = \cancel{21}$

$\cancel{= \frac{21}{61}}$

$HM_1 = \frac{1}{a+6}$ $\quad HM_2 = \dfrac{1}{\frac{1}{3} + \frac{36}{3}}$

$= \frac{3}{18}$ $\quad = \frac{3}{36}$

$= \frac{1}{6}$ $\quad = \frac{1}{9}$

Q. HM of roots of eq $(5+\sqrt{2})x^2 - (4+\sqrt{3})x + (8+2\sqrt{3}) = 0$

$$\cancel{x = \frac{(4+\sqrt{3}) \pm \sqrt{16+3+8\sqrt{3}-160-40\sqrt{3}-32\sqrt{2}-8\sqrt{6}}}{10+2\sqrt{2}}}$$

$$HM = \frac{2\alpha\beta}{\alpha+\beta}$$

$$= \frac{2\,\frac{(8+2\sqrt{3})}{5+\sqrt{2}}}{\frac{4+\sqrt{3}}{5+\sqrt{2}}}$$

$$= \frac{16+4\sqrt{3}}{4+\sqrt{3}}$$

$$= \frac{4(4+\sqrt{3})(4-\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})}$$

$$\boxed{= 4}$$

H.W. 27-6-24
DYS-5 (2-13)

Q Insert 2 H.M btw 2.5 & 0.4

$\frac{5}{2}$, $\frac{2}{5}$

$$d = \frac{\frac{2}{5} + \frac{5}{2}}{3}$$

$$= \frac{4+25}{30}$$

$$= \frac{29}{30}$$

$$HM_1 = \frac{1}{\frac{2}{5} + \frac{29}{30}}$$

$$\boxed{= \frac{30}{41}}$$

$$HM_2 = \frac{1}{\frac{2}{5} + \frac{58}{30}}$$

$$= \frac{30}{70}$$

$$\boxed{= \frac{3}{7}}$$

AM, GM, HM (Inequality & Relation)

Properties:-

$a, b, c, d \in \oplus$ve Real no.

① a, b

$AM = \frac{a+b}{2}$ $GM = \sqrt[2]{ab}$ $HM = \frac{2}{\frac{1}{a}+\frac{1}{b}}$

a, b, c

$AM = \frac{a+b+c}{3}$ $GM = \sqrt[3]{abc}$ $HM = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

a, b, c, d

$AM = \frac{a+b+c+d}{4}$ $GM = \sqrt[4]{abcd}$ $HM = \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$

$a_1, a_2, a_3 \dots a_n$

$AM = \frac{a_1+a_2+a_3+a_4 \dots a_n}{n}$ $GM = \sqrt[n]{a_1 \times a_2 \times a_3 \times a_4 \dots a_n}$ $HM = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2} \dots \frac{1}{a_n}}$

② $AM \geq GM \geq HM$

$AM = GM = HM$ when $a_1 = a_2 = a_3 \dots = a_n$

Proof $AM \geq GM$

$\frac{a+b}{2} \geq \sqrt{ab}$

$(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab} \geq 0$

$(\sqrt{a} - \sqrt{b})^2 \geq 0$

always true

$GM \geq HM$

$\sqrt{ab} \geq \frac{2ab}{a+b}$

$\sqrt{ab}(a+b) - 2ab \geq 0$

$\sqrt{ab}\left(a+b-2\sqrt{ab}\right) \geq 0$

$(\sqrt{a} - \sqrt{b})^2 \geq 0$

Always true

(3) For any two ~~the~~ positive numbers their AM, GM & HM ~~are~~ makes a ~~G~~ GP

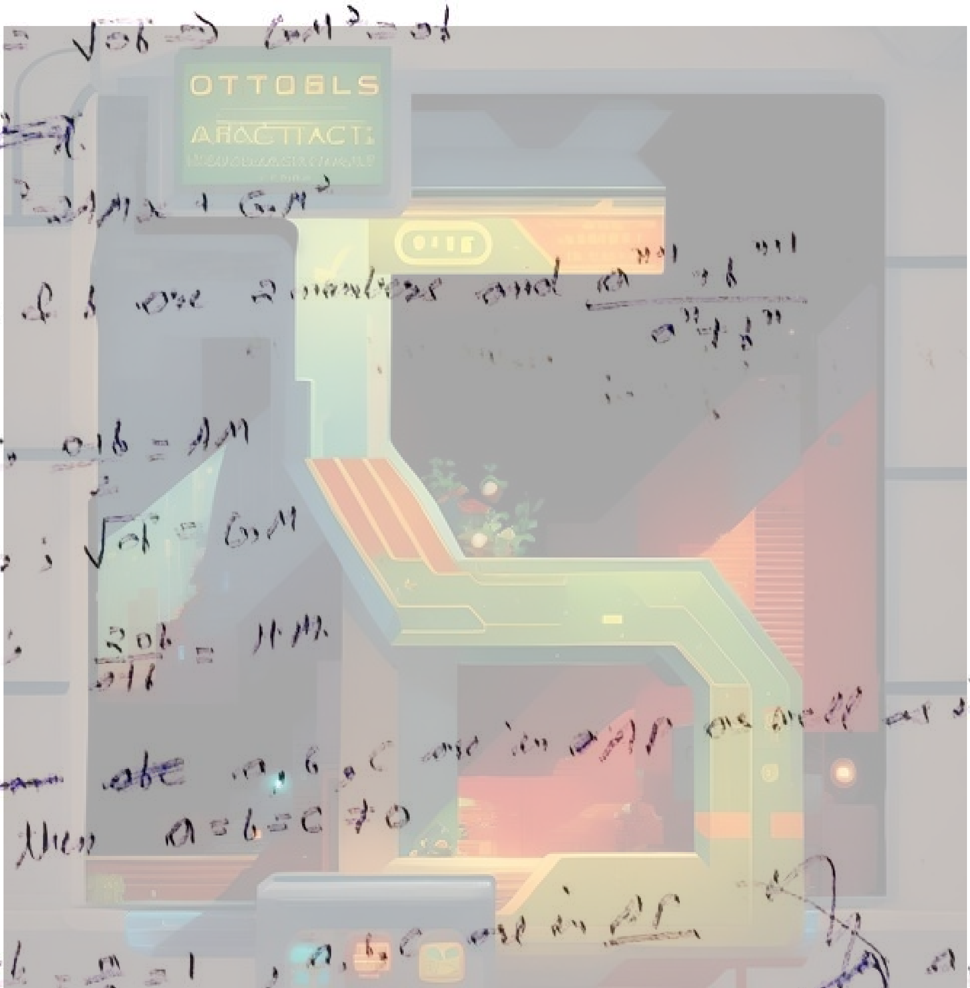
$\boxed{GM^2 = AM \times HM}$

(4) $x^2 - 2AMx + GM^2 = 0$

$x^2 - (a+b)x + ab = 0$

$AM = \frac{a+b}{2} \Rightarrow a+b = 2(AM)$

$GM = \sqrt{ab} \Rightarrow GM^2 = ab$



~~x^2~~

$x^2 - 2AMx + GM^2$

(5) If a & b are 2 numbers and $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is

$n=0$; $\frac{a+b}{2} = AM$

$n=-\frac{1}{2}$; $\sqrt{ab} = GM$

$n=-1$; $\frac{2ab}{a+b} = HM$

(6) If ~~are~~ a, b, c are in AP as well as in GP then $a=b=c \neq 0$

(7) i) $\frac{a-b}{b-c} = \frac{a}{a} = 1$, a, b, c are in AP

ii) $\frac{a-b}{b-c} = \frac{a}{b}$ or $\frac{a-b}{b-c} = \frac{b}{c} \Rightarrow ac - bc = b^2 - bc$, $ac = b^2$; a, b, c are in GP

iii) $\frac{a-b}{b-c} = \frac{a}{c}$; a, b, c are in HP

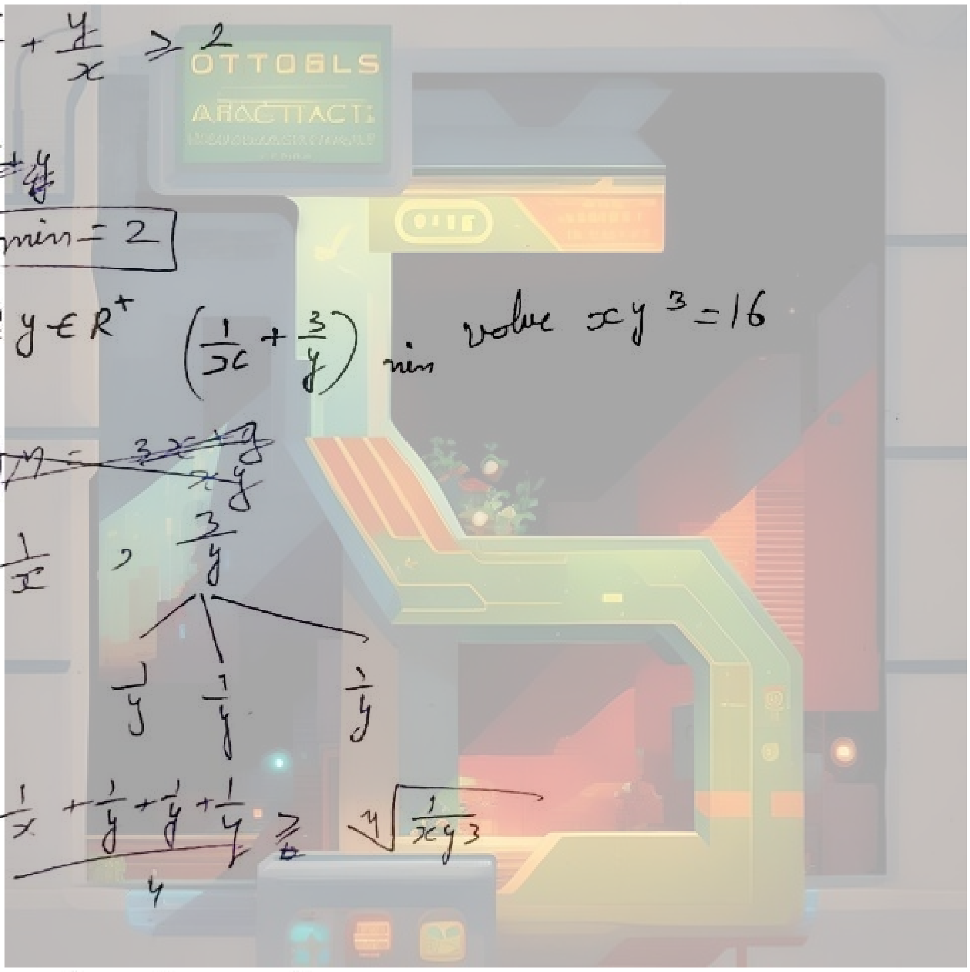
Inequalities

Q1. If x & $y \in R^+$ then $\frac{x}{y} + \frac{y}{x}$ min

$GM = \sqrt{\frac{x}{y} \times \frac{y}{x}} = \sqrt{1} = 1$

$AM = \dfrac{\frac{x}{y} + \frac{y}{x}}{2}$

$AM \geq GM$



$\frac{x}{y} + \frac{y}{x} \geq 2$

$\cancel{x^2 + y}$

$\boxed{\text{min} = 2}$

Q2. x & $y \in R^+$ $\left(\frac{1}{x} + \frac{3}{y}\right)$ min value $xy^3 = 16$

$\cancel{AM = \frac{3x+y}{xy}}$

$\frac{1}{x}$, $\frac{3}{y}$

$\frac{1}{y}$ $\frac{1}{y}$ $\frac{1}{y}$

$\dfrac{\frac{1}{x} + \frac{1}{y} + \frac{1}{y} + \frac{1}{y}}{4} \geq \sqrt[4]{\frac{1}{xy^3}}$

$\frac{1}{x} + \frac{3}{y} \geq 2$

$\boxed{\text{min} = 2}$

Q $P+2Q+4R=9$ $\quad P, Q, R \in I^+$

find $(P^2Q^4R^3)_{max}$

$\cancel{3 \geq 2\sqrt[3]{PQR}}$

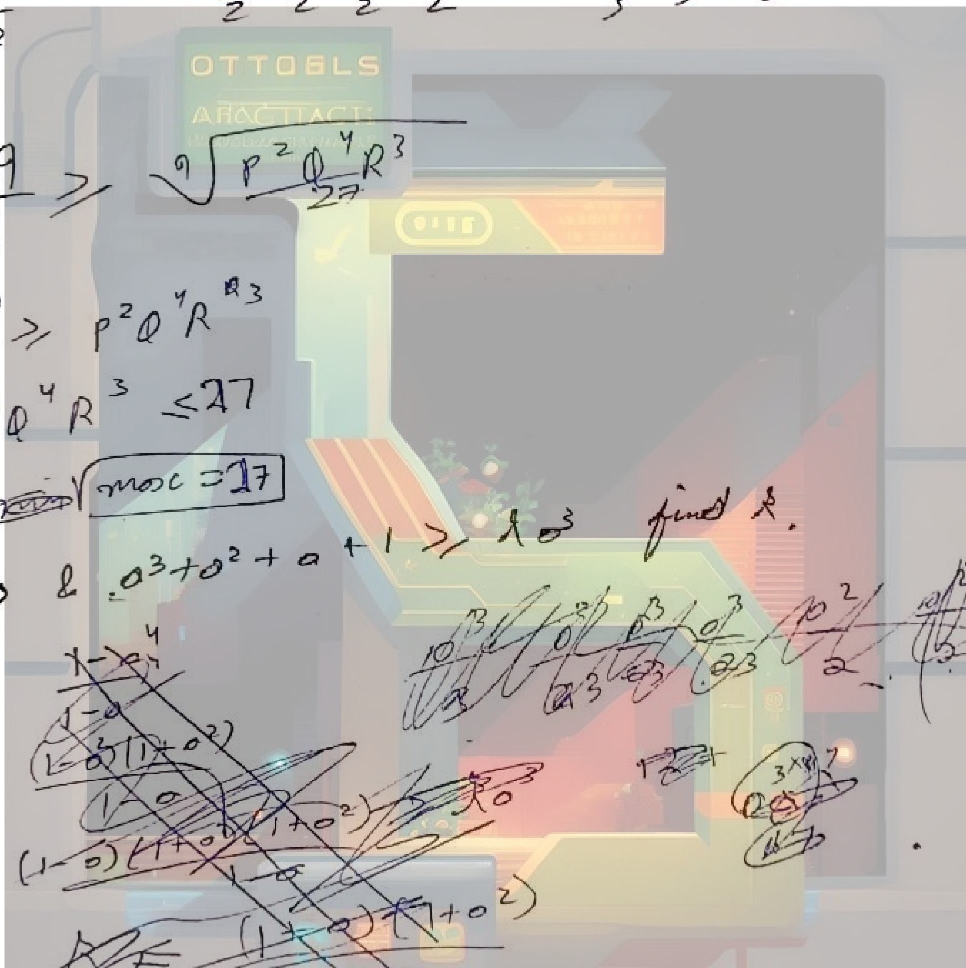
$\cancel{\frac{27}{8} \geq PQR}$

$\cancel{max\ PQR = \frac{27}{8}}$

$P \to \frac{P}{2}, \frac{P}{2}$

$2Q \to \frac{Q}{2}, \frac{Q}{2}, \frac{Q}{2}, \frac{Q}{2}$

$4R \to \frac{4}{3}R, \frac{4}{3}R, \frac{4}{3}R$



$\frac{9}{9} \geq \sqrt[9]{\frac{P^2Q^4R^3}{27}}$

$1^9 \geq P^2Q^4R^3$

$P^2Q^4R^3 \leq 27$

$\boxed{max = 27}$

Q If $a>0$ & $a^3+a^2+a+1 \geq \lambda a^3$ find λ.

$3+a^2+a+1 \geq 4\sqrt[4]{a^6}$

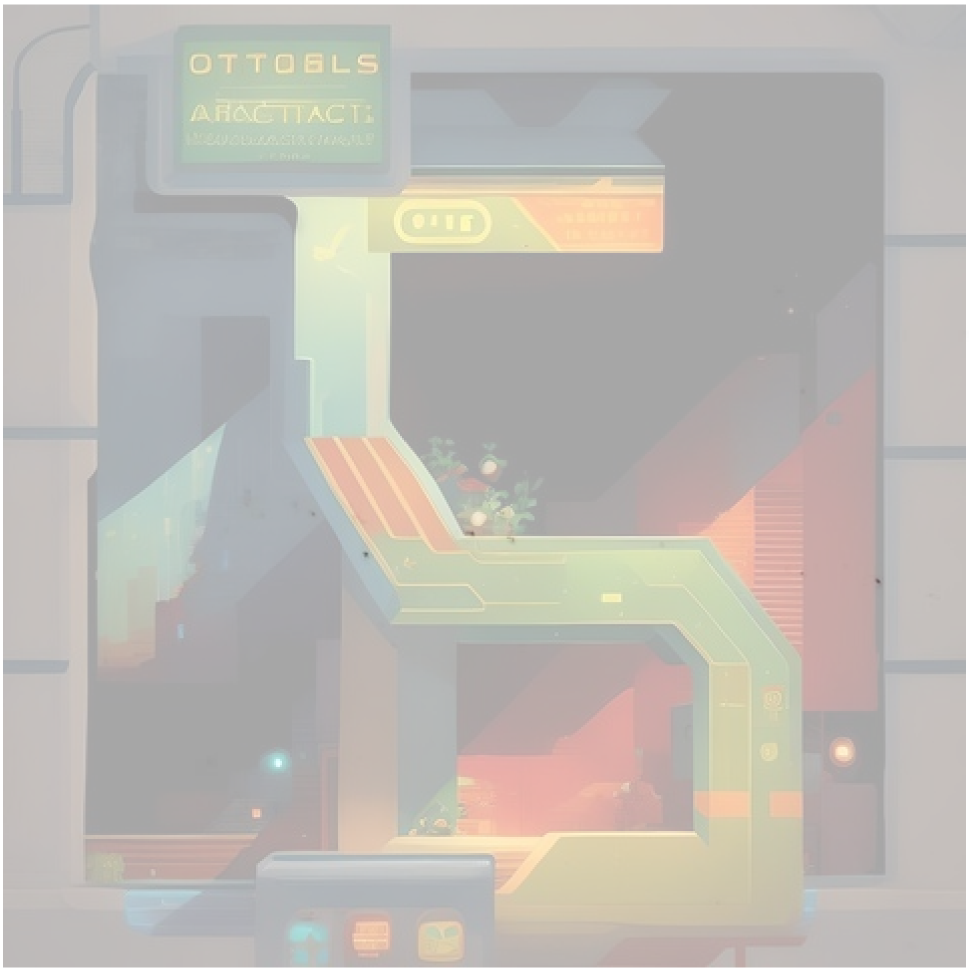
$4 a^{3/2}$

H.W. (28-6-24)

Q-1 (01, 2-5, 7, 11, 12, 13, 14, 15)

$a^3 + a^2 - 10a + 1 \geq ka^3$, $a > 0$

Not Possible



Q $2x + 3y = 15$. (first $\cancel{c}$) max x^2y

$$\frac{x + x + 3y}{3} = \sqrt[3]{3x^2y}$$

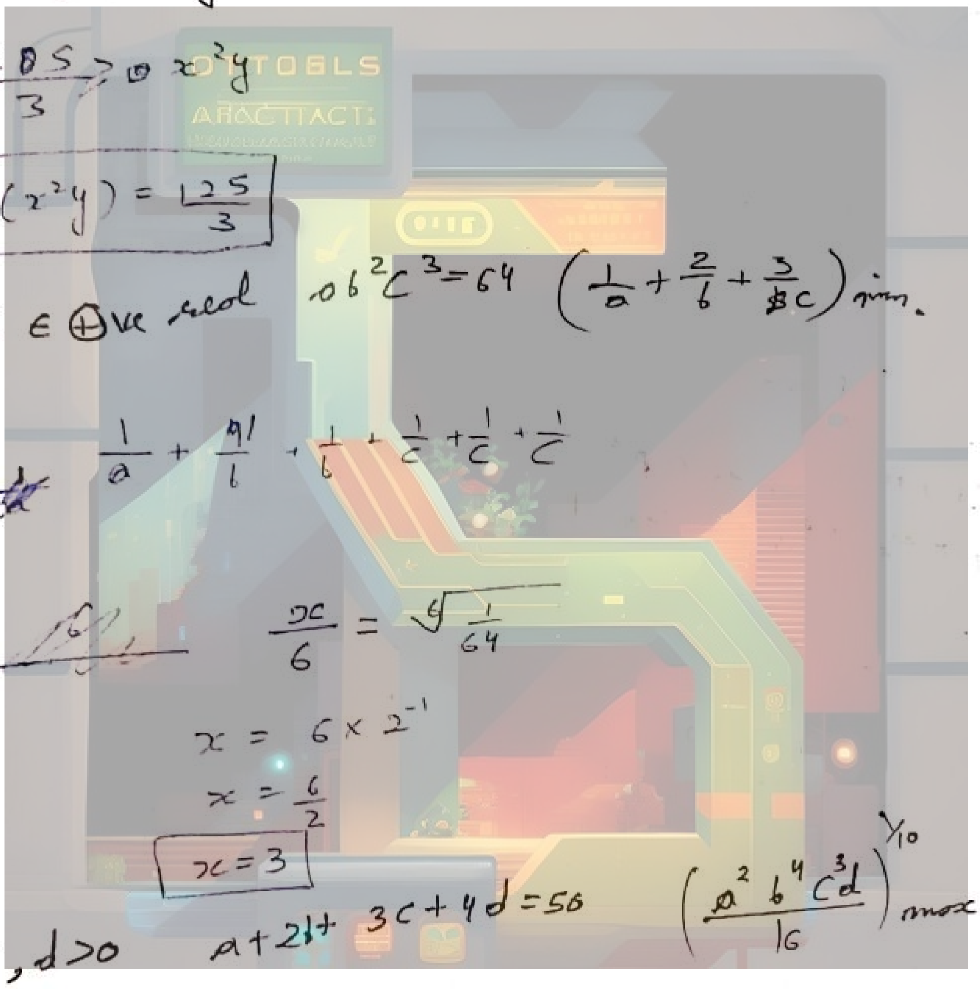
$$\cancel{2x = 5x+5}$$

$$5 \frac{15}{3} \geq \sqrt[3]{3x^2y}$$

$$125 \geq 3x^2y$$

$$\frac{125}{3} \geq x^2y$$

$$\boxed{\max(x^2y) = \frac{125}{3}}$$



Q $a, b, c \in \oplus$ve real $ab^2c^3 = 64$ $\left(\frac{1}{a} + \frac{2}{b} + \frac{3}{c}\right)$ min.

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{b} + \frac{1}{c} + \frac{1}{c} + \frac{1}{c}$$

$$\frac{x}{6} = \sqrt[6]{\frac{1}{64}}$$

$$x = 6 \times 2^{-1}$$

$$x = \frac{6}{2}$$

$$\boxed{x = 3}$$

Q $a, b, c, d > 0$ $a + 2b + 3c + 4d = 50$ $\left(\frac{a^2 b^4 c^3 d}{16}\right)^{1/10}$ max

$$\frac{\frac{a}{2} + \frac{a}{2} + \frac{b}{2} + \frac{b}{2} + \frac{b}{2} + \frac{b}{2} + c + c + c + 4d}{10} \geq \sqrt[10]{\frac{a^2 b^4 c^3 d}{16}}$$

$$\boxed{5 =}$$

Q $p, q, r \in R^+$ $\quad pqr = 1$

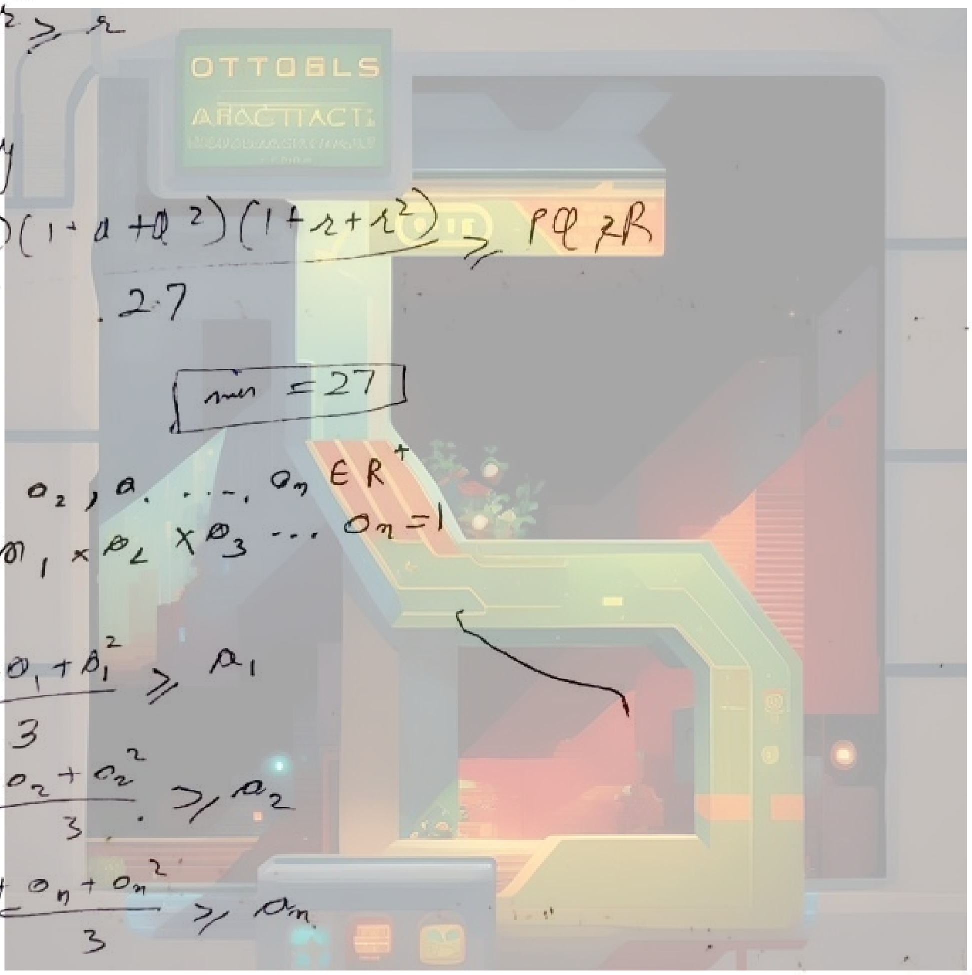
$$\left[(1+p+p^2)(1+q+q^2)(1+r+r^2)\right]_{min}$$

~~$1+p(1+p)$~~

$$\frac{1+p+p^2}{3} \geq (1\cdot p\cdot p^2)^{1/3} \geq p$$

$$\frac{1+q+q^2}{3} \geq q$$

$$\frac{1+r+r^2}{3} \geq r$$



multiply

$$(1+p+p^2)(1+q+q^2)(1+r+r^2) \geq pqr \cdot 27$$

$$\boxed{min = 27}$$

Q If $a_1, a_2, a_3, \dots, a_n \in R^+$
& $a_1 \times a_2 \times a_3 \dots a_n = 1$

$$\frac{1+a_1+a_1^2}{3} \geq a_1$$

$$\frac{1+a_2+a_2^2}{3} \geq a_2$$

$$\frac{1+a_n+a_n^2}{3} \geq a_n$$

$$\frac{(1+a_1+a_1^2)(1+a_2+a_2^2)(1+a_n+a_n^2)}{3^n} \geq 1$$

$$\boxed{3^n = min}$$

Q $x > 0$, $\left(\frac{x^{10}}{1+x+x^2\cdots x^{20}}\right)_{max}$

$\cancel{\frac{x^{10}}{\frac{x^{20}-1}{x-1}}}$

$\cancel{x^{10}(x-1)}$

$\frac{1+\cdots}{21} \geq x^{\frac{210}{21}}$

$\frac{1\ldots+x^{20}}{21} \geq x^{10}$

$\frac{1\ldots+x^{20}}{x^{10}} \geq 21$

$\frac{x^{10}}{1\ldots+x^{20}} \leq \frac{1}{21}$

$\boxed{max = \frac{1}{21}}$

Aritho - Geometric Series (AGP)

→ A series formed by multiplying the corresponding terms of AP and GP.

$ab, (a+d)br, (a+2d)br^2, \ldots\ldots (a+(n-1)d)br^{n-1}$

a → first term of AP

d → CD of AP

b → 1st term of GP

r → CR of GP

$T_n = (a+(n-1)d)\, b r^{n-1}$

Sum of AGP:-

Process :-

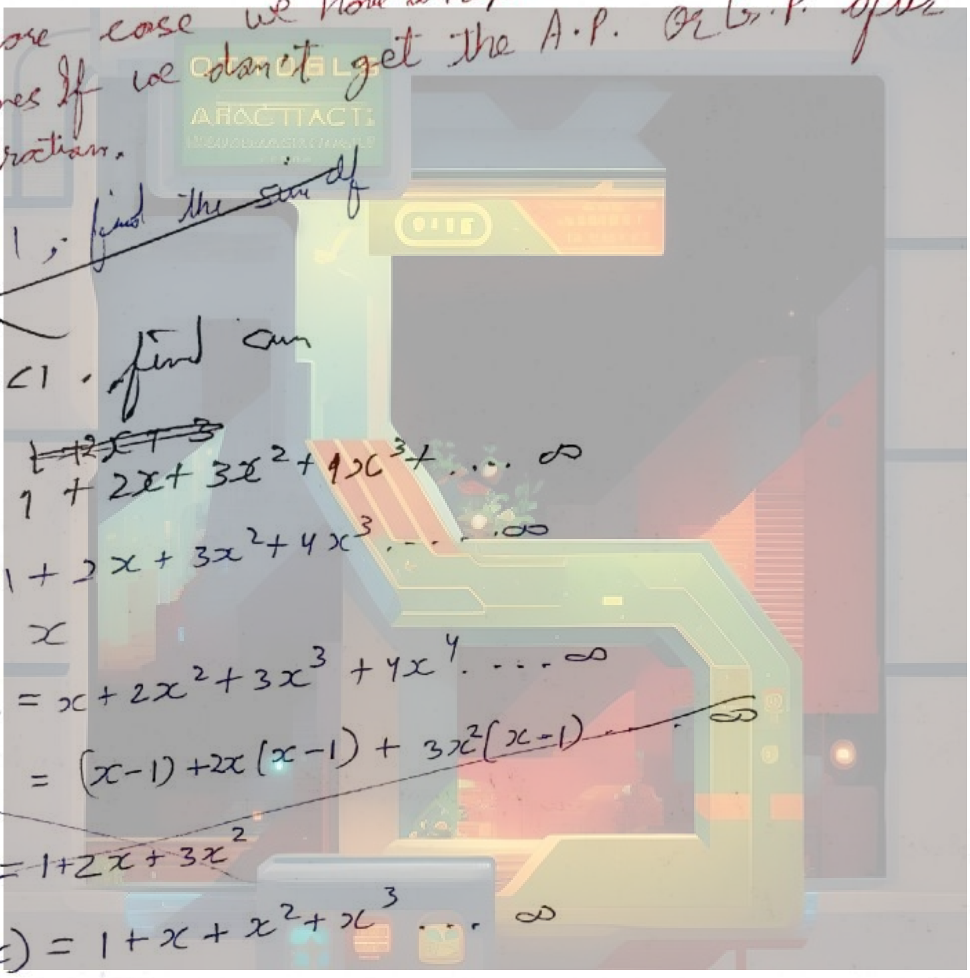
① Find the C.R.

② Multiply the CR with given series & form a new series.

③ Subtract the given series & ~~the~~ the new series

④ After subtraction we either get AP or GP hence Apply sum.

⑤ In rare case we have to repeat the subtraction process two or 3 times if we don't get the A.P. or G.P. after first subtraction.



Q ~~$|x|<1$, find the sum of $1+$~~

Q. $|x|<1$, find sum

~~$1+2x+3$~~

$1+2x+3x^2+4x^3+\dots\infty$

$1+2x+3x^2+4x^3\dots\infty$

$S = 1+2x+3x^2+4x^3\dots$

$CR = x$

$Sx = x+2x^2+3x^3+4x^4\dots\infty$

~~$Sx - S = (x-1)+2x(x-1)+3x^2(x-1)\dots\infty$~~

~~$S = 1+2x+3x^2$~~

$S(1-x) = 1+x+x^2+x^3\dots\infty$

$S(1-x) = \frac{1}{1-x}$

$S = \frac{1}{(1-x)^2}$

Q $S = 2 + \frac{4}{3} + \frac{6}{3^2} + \frac{8}{3^3} + \frac{10}{3^4} + \dots \infty$

$\frac{S}{3} = \frac{2}{3} + \frac{4}{3^2} + \frac{6}{3^3} + \frac{8}{3^4} \dots \infty$

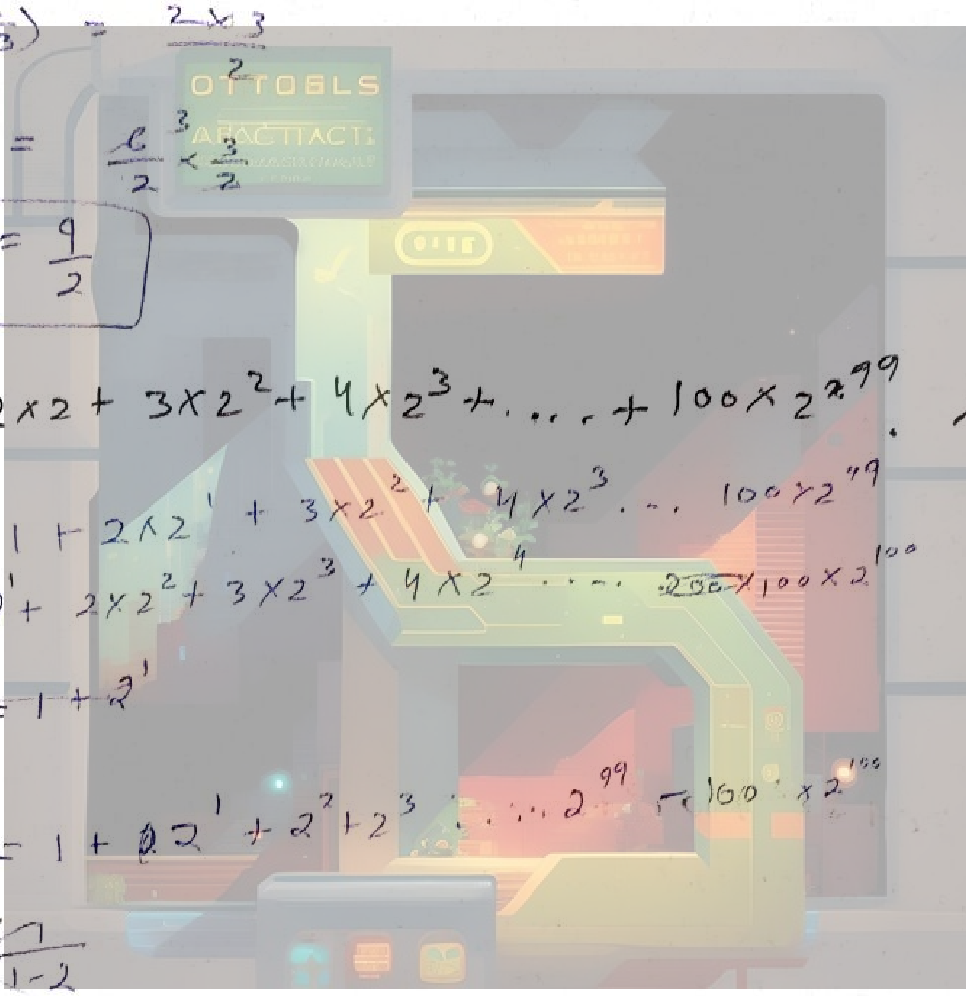
$S - \frac{S}{3} = 2 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} \dots \infty$

$S(1-\frac{1}{3}) = \frac{2}{1-\frac{1}{3}}$

$S(\frac{2}{3}) = \frac{2\times 3}{2}$

$S = \frac{6}{2} \times \frac{3}{2}$

$\boxed{S = \frac{9}{2}}$



Q $1 + 2\times 2 + 3\times 2^2 + 4\times 2^3 + \dots + 100\times 2^{99}$ sum.

$S = 1 + 2\times 2^1 + 3\times 2^2 + 4\times 2^3 \dots 100\times 2^{99}$

$2S = 2^1 + 2\times 2^2 + 3\times 2^3 + 4\times 2^4 \dots \cancel{200}\ 100\times 2^{100}$

$\cancel{S - 2S = 1 + 2^1}$

$\cancel{S}$

$S - 2S = 1 + 2^1 + 2^2 + 2^3 \dots 2^{99} - 100\times 2^{100}$

$\cancel{-S = \frac{1}{1-2}}$

$\cancel{-S = -\frac{1}{2}}$

$\cancel{\boxed{S = 1/2}}$

$-S = \frac{(1-2^{100})}{1-2} - 100\times 2^{100}$

$S = 1 - 2^{100} + 100\times 2^{100}$

$\cancel{S = 1 - 101\times 2^{100}}$

$\boxed{S = 99\times 2^{100} + 1}$

$\cancel{Q \quad 2 + \frac{4}{3} + \frac{6}{3^2} + \dots}$

Q $\frac{4}{7} - \frac{5}{7^2} + \frac{4}{7^3} - \frac{5}{7^4} + \frac{4}{7^5} - \frac{5}{7^6} \dots \infty$

$\frac{(28-5)}{7^2} + \frac{(28-5)}{7^4} + \frac{(28-5)}{7^6} \dots \infty$

$a = (28-5)/7^2$

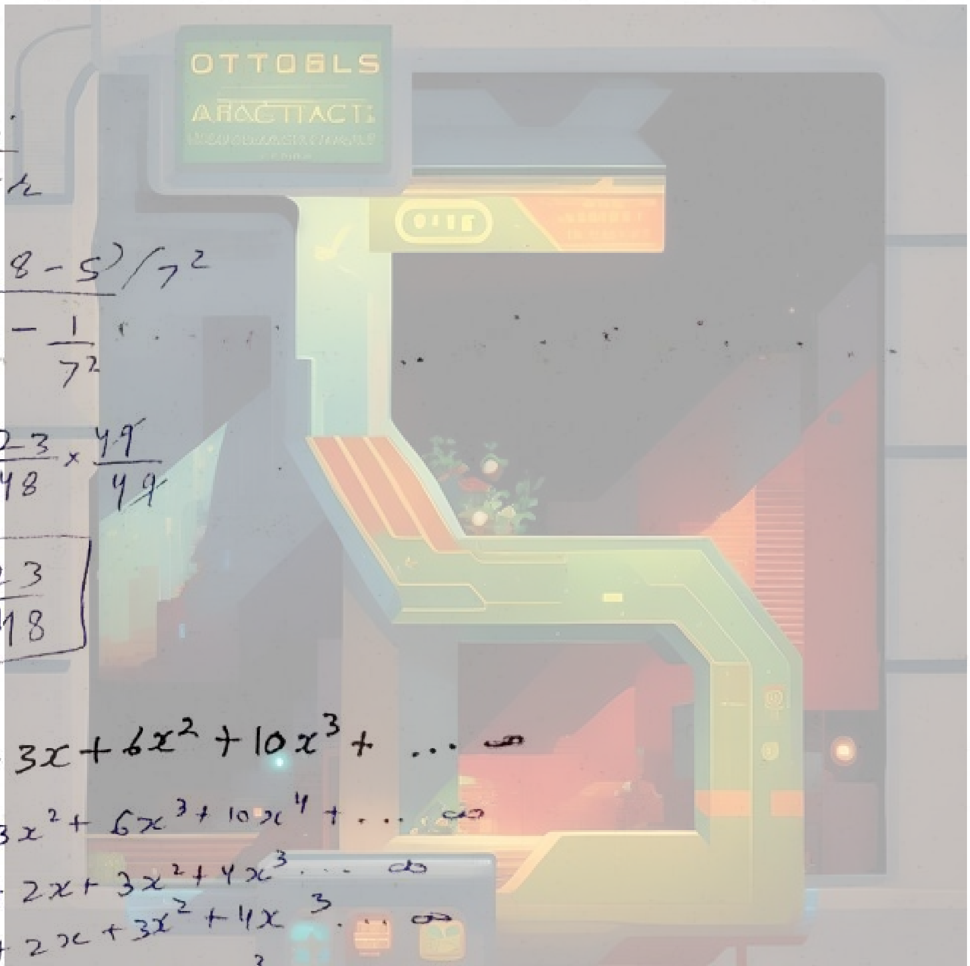
$r = 1/7^2$

$S_\infty = \frac{a}{1-r}$

$= \frac{(28-5)/7^2}{1 - \frac{1}{7^2}}$

$= \frac{23}{48} \times \frac{49}{49}$

$\boxed{= \frac{23}{48}}$



Q $S = 1 + 3x + 6x^2 + 10x^3 + \dots \infty$

$Sx = x + 3x^2 + 6x^3 + 10x^4 + \dots \infty$

$S - Sx = 1 + 2x + 3x^2 + 4x^3 \dots \infty$

$S(1-x) = 1 + 2x + 3x^2 + 4x^3 \dots \infty$

$Sx(1-x) = x + 2x^2 + 3x^3 \dots \infty$

$S(1-x)^2 = 1 + x + x^2 + x^3 \dots \infty$

$S(1-x)^2 = \frac{1}{1-x}$

$S = \frac{1}{(1-x)^3}$

Q Sum of 10 terms

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2 \dots$$

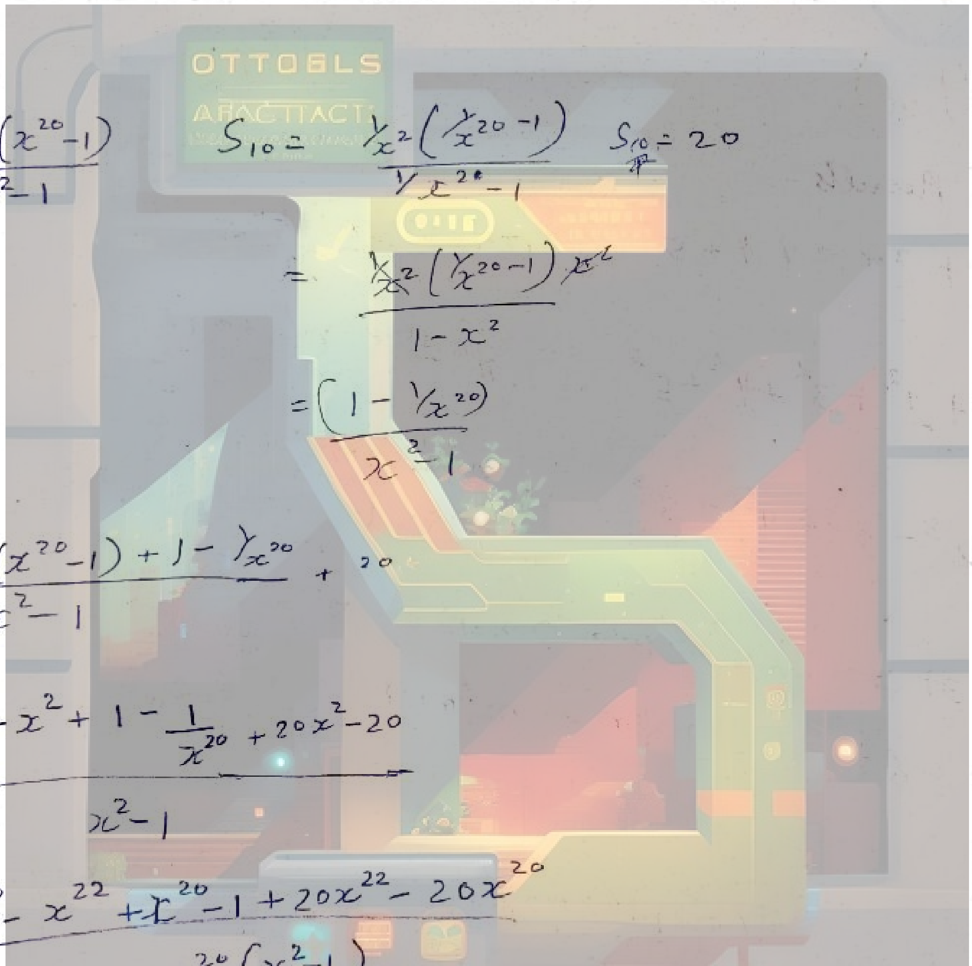
$$\cancel{\left(x+\frac{1}{x}\right)+\left(x^2+\frac{1}{x^2}\right)}$$

$$\left(x^2+\frac{1}{x^2}+2\right)+\left(x^4+\frac{1}{x^4}+2\right)+\left(x^6+\frac{1}{x^6}+2\right) \dots$$

$$x^2+x^4+x^6+x^8 \dots + \frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6} \dots + 2+2+2 \dots$$

$a = x^2$

$r = x^2$



$$S_{10} = \frac{x^2(x^{20}-1)}{x^2-1} \qquad S_{10} = \frac{1/x^2\left(1/x^{20}-1\right)}{1/x^2-1} \qquad S_{10} = 20$$

$$= \frac{1/x^2\left(1/x^{20}-1\right)x^2}{1-x^2}$$

$$= \frac{\left(1-1/x^{20}\right)}{x^2-1}$$

$$\frac{x^2(x^{20}-1)+1-1/x^{20}}{x^2-1}+20$$

$$\frac{x^{40}-x^2+1-\frac{1}{x^{20}}+20x^2-20}{x^2-1}$$

$$\frac{x^{60}-x^{22}+x^{20}-1+20x^{22}-20x^{20}}{x^{20}(x^2-1)}$$

$$\frac{x^{60}+19x^{22}-19x^{20}-1}{x^{20}(x^2-1)}$$

$$\cancel{\boxed{\frac{x^{40}+19x^2-19}{x^2-1}}}$$

Special Sequence

Type-1 :- Using Summation (Σ)

Type-2 :- Method of Difference (MOD)

Type-3 :- Splitting in denominator

Type-4 :- Splitting in Numerator

H.W- 01-7-24

DYS-6 (Full)

Type-1 :- Results:-

① $1+2+3+4+5 \dots\dots +n = \frac{n(1+n)}{2}$

② $1+2^2+3^2+4^2+5^2 \dots\dots n^2 = \frac{n(1+n)(2n+1)}{6}$

③ $1+2^3+3^3+4^3+5^3 \dots\dots n^3 = \left(\frac{n(n+1)}{2}\right)^2$

④ $1+3+5+ \dots\dots + (2n-1) = n^2$

⑤ $2+4+6+ \dots\dots 2n = n(n+1)$

Proof:-

② $1^2+2^2+3^2 \dots\dots \frac{n(n+1)(2n+1)}{6}$

$(x+1)^3 = x^3+1+3x+3x^2$

$(x+1)^3 - x^3 = 1+3x+3x^2$

~~$x=0$~~ ~~$1^3 - 0^3 = 1+0+0$~~

$x=1 \quad 2^3-1^3 = 1+3(1)+3(1)^2$

$x=2 \quad 3^3-2^3 = 1+3(2)+3(2)^2$

$x=3 \quad 4^3-2^3 = 1+3(3)+3(3)^3$

$x=n \quad (n+1)^3-n^3 = 1+3(n)+(3n)^2$

add de

(70)

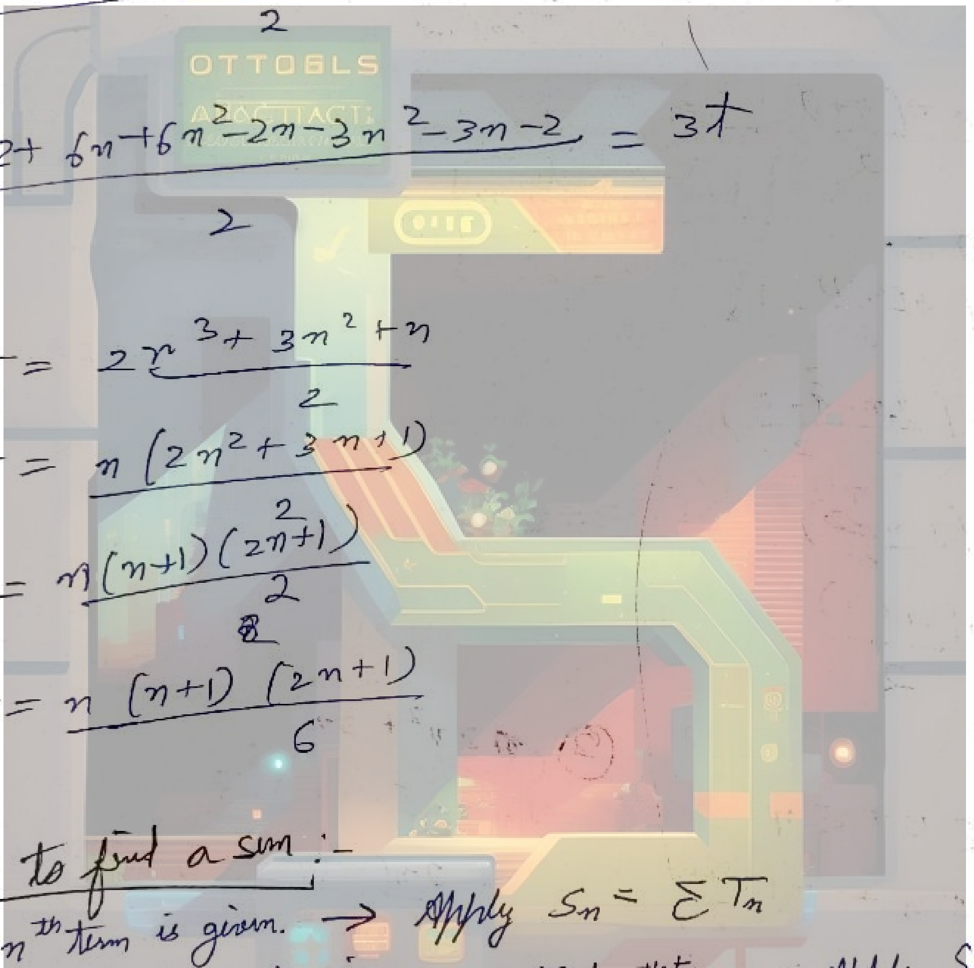
Sum

$$(n+1)^3 - 1 = n + 3(1+2+3+\dots+n) + 3\underbrace{(1^2+2^2+3^2+\dots n^2)}_{t}$$

$$(n+1)^3 - 1 = n + \frac{3n(n+1)}{2} + 3t$$

$$(n+1)^3 - 1 - n - \frac{3n(n+1)}{2} = 3t$$

$$\frac{2(n+1)^3 - 2n - 3n(n+1) - 2}{2} = 3t$$



$$\frac{2n^3 + 2 + 6n + 6n^2 - 2n - 3n^2 - 3n - 2}{2} = 3t$$

$$3t = \frac{2n^3 + 3n^2 + n}{2}$$

$$3t = \frac{n(2n^2 + 3n + 1)}{2}$$

$$3t = \frac{n(n+1)(2n+1)}{2}$$

$$t = \frac{n(n+1)(2n+1)}{6}$$

Method to find a sum :-

→ when n^{th} term is given. → Apply $S_n = \Sigma T_n$

→ when n^{th} term is not given:- → Find n^{th} term → Apply $S_n = \Sigma T_n$

Q find the sum of n terms whose n^{th} term is

(1) $\cancel{2n}$ $3n-1$

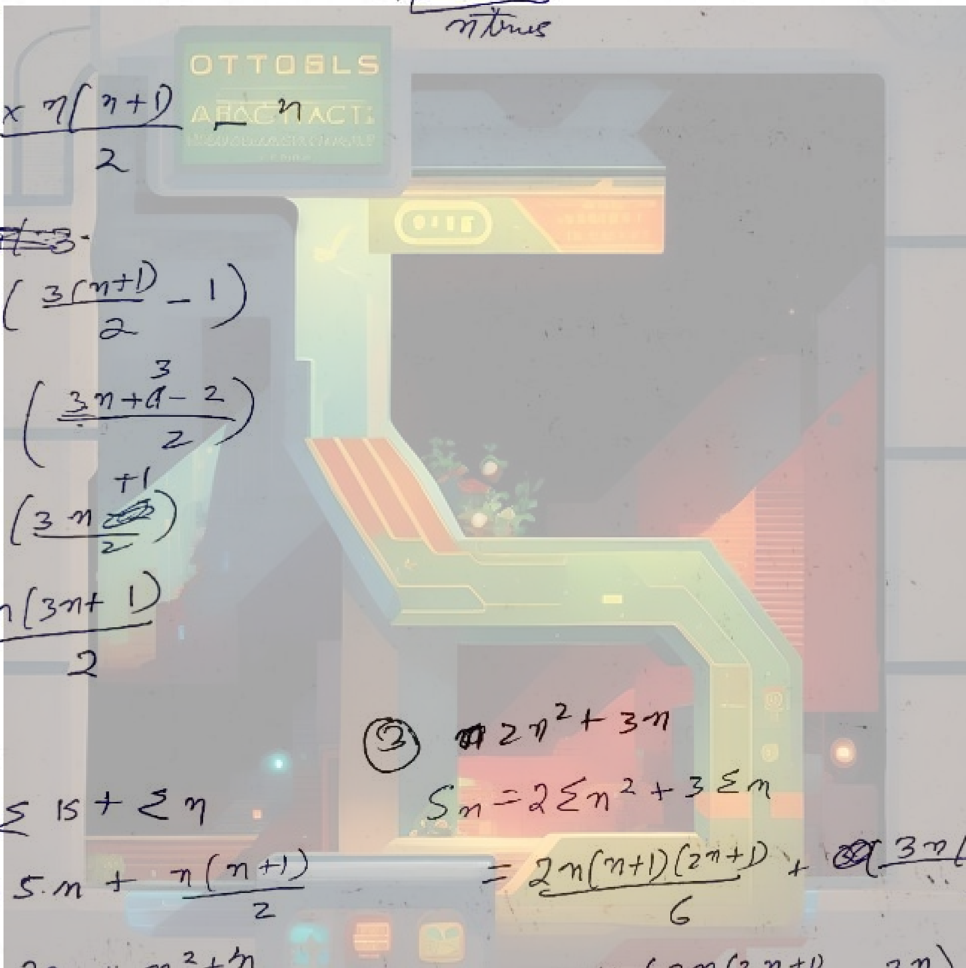
$2, 5, 8, 11, \ldots, 3n-1$

$S_n = \sum (3n-1)$

$= \sum 3n - \sum 1$

$= 3\sum n - \sum 1$

$= 3(1+2+3+\ldots n) - (\underbrace{1+1+1}_{n \text{ times}})$



$= \frac{3 \times n(n+1)}{2} - n$

$\cancel{= \frac{3n+3}{}}$

$= n\left(\frac{3(n+1)}{2} - 1\right)$

$= n\left(\frac{3n+\cancel{1}^{3}-2}{2}\right)$

$= n\left(\frac{3n\cancel{\;}^{+1}}{2}\right)$

$= \frac{n(3n+1)}{2}$

(2) $15+n$

$S_n = \sum 15 + \sum n$

$= 15 \cdot n + \frac{n(n+1)}{2}$

$= \frac{30n + n^2 + n}{2}$

$\boxed{= \frac{n(n+31)}{2}}$

(3) $2n^2 + 3n$

$S_n = 2\sum n^2 + 3\sum n$

$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}$

$= \frac{n+1}{2}\left(\frac{2n(2n+1)}{3} + 3n\right)$

$= \frac{n+1}{2} \frac{(4n^2+2n+9n)}{3}$

$= \frac{n+1}{6}(4n^2+11n)$

$\boxed{= \frac{n(n+1)(4n+11)}{6}}$

(4) $T_n = 3^n - 2^n$

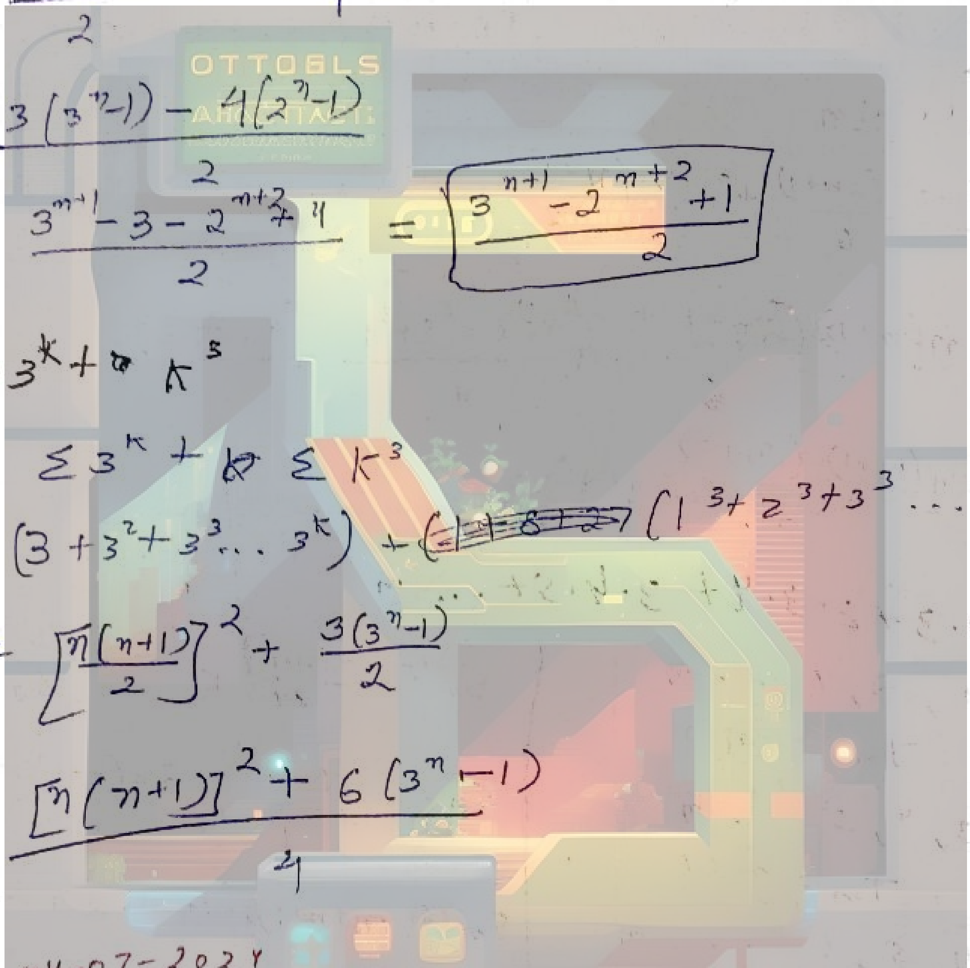
$S_n = \Sigma(3^n - 2^n)$

$= \Sigma 3^n - \Sigma 2^n$

$\cancel{= 2(3+9+27 \ldots 3^n) - 2(2+4+8+16 \ldots 2^n)}$

$= (3+3^2+3^3+3^3 \ldots 3^n) - (2+2^2+2^3 \ldots 2^n)$

$= \frac{3(3^n-1)}{2} - \frac{2(2^n-1)}{1}$



$= \frac{3(3^n-1) - 4(2^n-1)}{2}$

$= \frac{3^{n+1}-3-2^{n+2}+4}{2} = \boxed{\frac{3^{n+1}-2^{n+2}+1}{2}}$

(5) $T_n = 3^k + k^3$

$S_n = \Sigma 3^k + \Sigma k^3$

$= (3+3^2+3^3 \ldots 3^k) + \cancel{(1+8+27)}(1^3+2^3+3^3 \ldots n^3)$

$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3(3^n-1)}{2}$

$= \frac{[n(n+1)]^2 + 6(3^n-1)}{4}$

H. W. 04-07-2024

DYS-7 (Q2)

Q Find the Sum of following series.

① $1\times2 + 2\times3 + 3\times4 + 4\times5 + \ldots n$ terms.

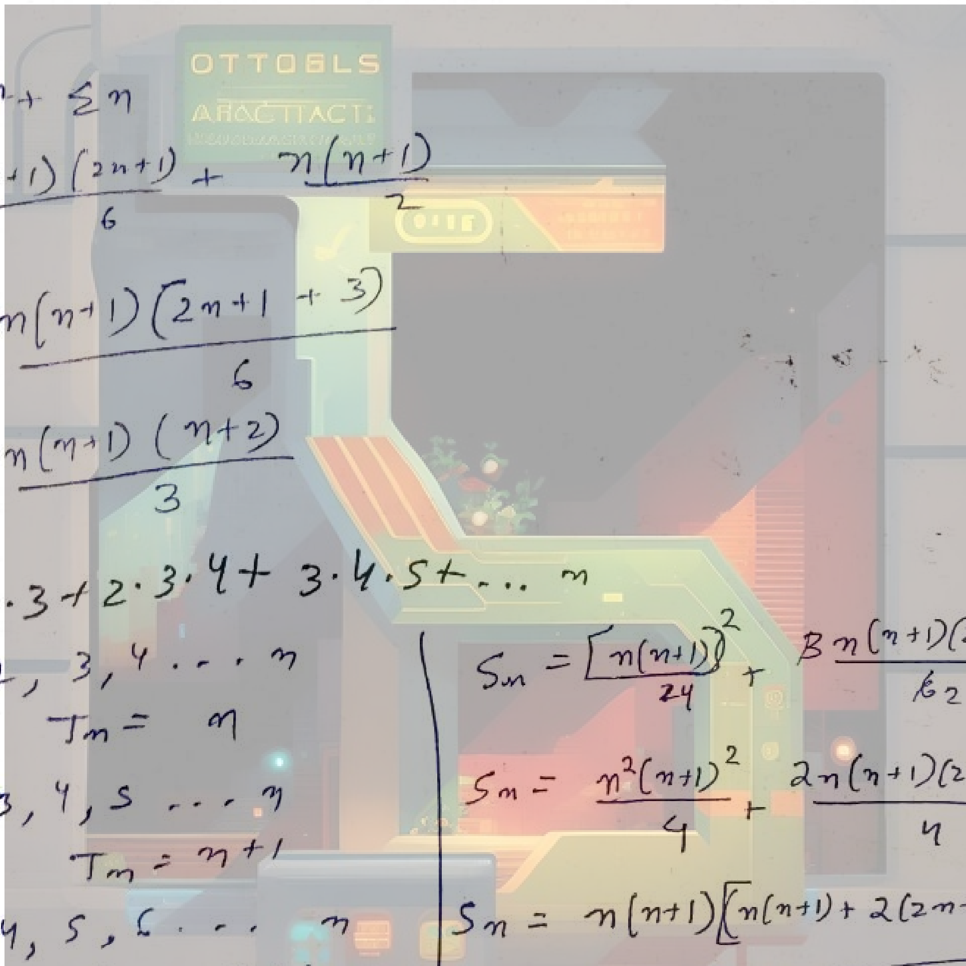
$1, 2, 3, 4 \ldots$

$T_n = n$

$2, 3, 4 \ldots$

$T_n = n+1$

$T_n = n(n+1)$



$S_n = \Sigma T_n$

$= \Sigma n^2 + \Sigma n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)(2n+1+3)}{6}$

$= \frac{n(n+1)(n+2)}{3}$

Q2. $1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + \ldots n$

$1, 2, 3, 4 \ldots n$

$T_n = n$

$2, 3, 4, 5 \ldots n$

$T_n = n+1$

$3, 4, 5, 6 \ldots n$

$T_n = n+2$

$T_n = n(n+1)(n+2)$

$S_n = \Sigma (n^2+n)(n+2)$

$S_n = \Sigma (n^3 + 3n^2 + 2n)$

$S_n = \Sigma n^3 + 3\Sigma n^2 + 2\Sigma n$

$S_n = \frac{[n(n+1)]^2}{24} + \frac{\cancel{3}\, n(n+1)(2n+1)}{\cancel{6}\,2} + \frac{\cancel{2}\times n(n+1)}{\cancel{2}}$

$S_n = \frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{4} + \frac{4n(n+1)}{4}$

$S_n = \frac{n(n+1)[n(n+1) + 2(2n+1) + 4(n+1)]}{4}$

Q3. $1\cdot3\cdot5 + 3\cdot5\cdot7 + 5\cdot7\cdot9 + \dots n$

~~1, 1, 3, 5~~

$1, 3, 5, \dots n$

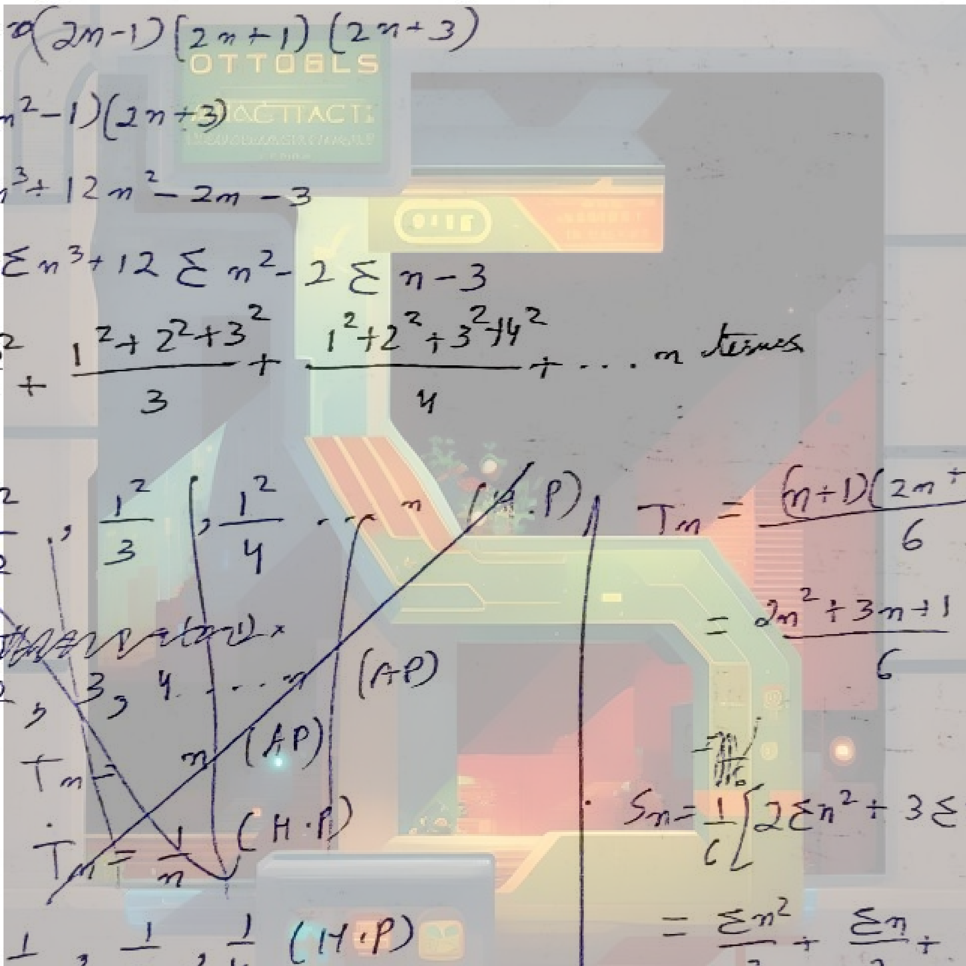
$T_n = 2n-1$

$3, 5, 7 \dots n$

$T_n = 2n+1$

$5, 7, 9 \dots$

$T_n = 2n+3$



$T_n = (2n-1)(2n+1)(2n+3)$

$= (4n^2-1)(2n+3)$

$= 8n^3 + 12n^2 - 2n - 3$

$S_n = 8\sum n^3 + 12\sum n^2 - 2\sum n - 3$

Q $1^2 + \frac{1^2+2^2}{2} + \frac{1^2+2^2+3^2}{3} + \frac{1^2+2^2+3^2+4^2}{4} + \dots$ n terms

$1^2, \frac{1^2}{2}, \frac{1^2}{3}, \frac{1^2}{4} \dots n$ (H.P)

$1, 2, 3, 4, \dots n$ (AP)

$T_n = n$ (AP)

$T_n = \frac{1}{n}$ (H.P)

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ (H.P)

$T_n = \frac{1}{n}$

$1^2, 1^2+2^2, 1^2+2^2+3^2, \dots$

$T_n = \sum n^2$

$T_n = \frac{n(n+1)(2n+1)}{6}$

$T_n = \frac{(n+1)(2n+1)}{6}$

$= \frac{2n^2+3n+1}{6}$

$S_n = \frac{1}{6}\left[2\sum n^2 + 3\sum n + n\right]$

$= \frac{\sum n^2}{3} + \frac{\sum n}{2} + \frac{n}{6}$

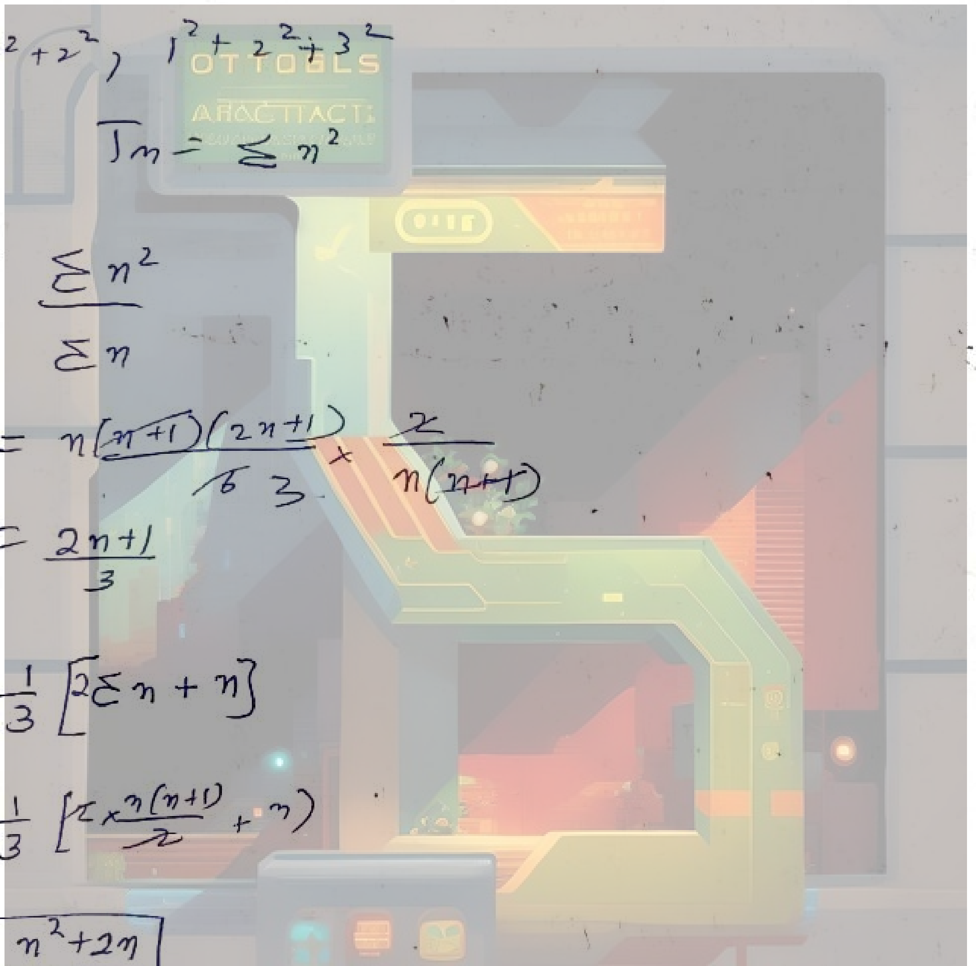
⑤ $1 + \frac{1^2+2^2}{1+2}, \frac{1^2+2^2+3^2}{1+2+3} + \frac{1^2+2^2+3^2+4^2}{1+2+3+4} \dots n$

$\frac{1}{1}, \frac{1}{1+2}, \frac{1}{1+2+3}$ (H.P)

$1, 1+2, 1+2+3$ (A.P)

$T_n = \Sigma n$

$T_n = \frac{1}{\Sigma n}$ (H.P)



$1, 1^2+2^2, 1^2+2^2+3^2$

$T_n = \Sigma n^2$

$T_n = \frac{\Sigma n^2}{\Sigma n}$

$= \frac{n\cancel{(n+1)}(2n+1)}{\cancel{6}\,3} \times \frac{\cancel{2}}{\cancel{n(n+1)}}$

$= \frac{2n+1}{3}$

$S_n = \frac{1}{3}\left[2\Sigma n + n\right]$

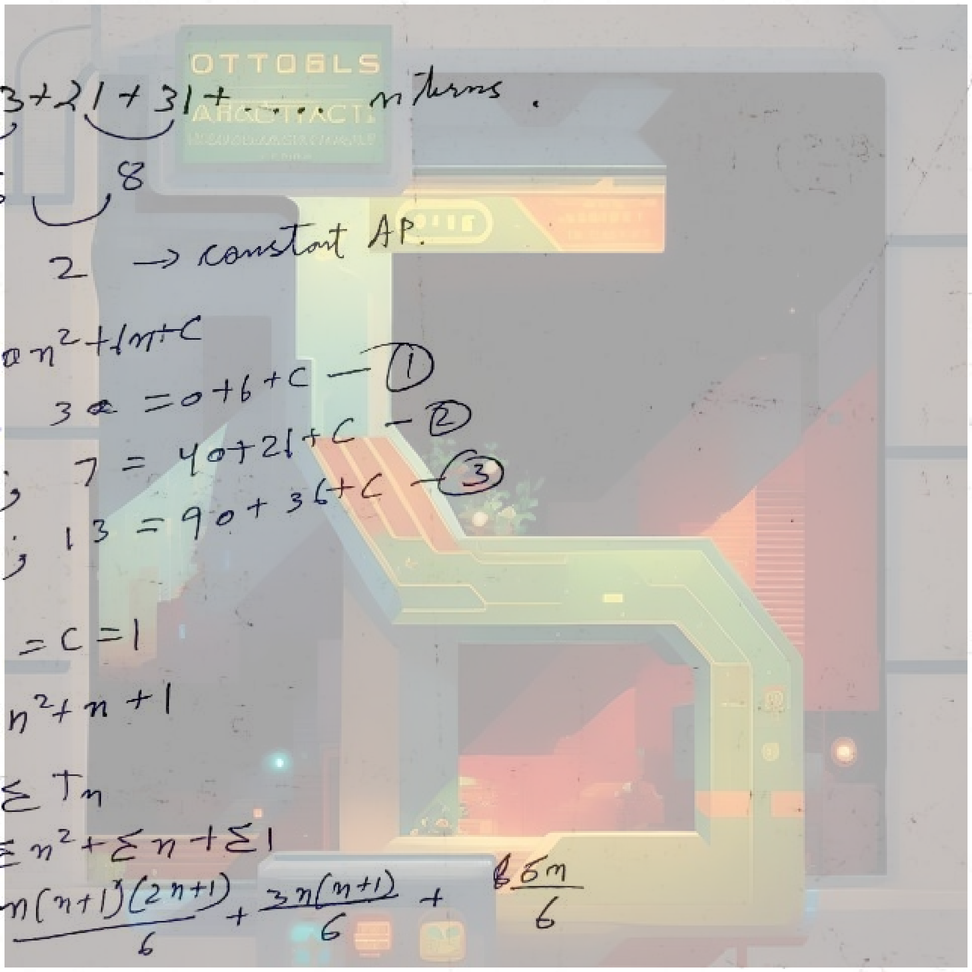
$= \frac{1}{3}\left[\cancel{2} \times \frac{n(n+1)}{\cancel{2}} + n\right)$

$\boxed{= \frac{n^2+2n}{3}}$

Type-3 :- Method of Difference

Difference

- AP constant
  - 1st order: $T_n = an+b$
  - 2nd order: $T_n = an^2+bn+c$
  - 3rd order: $T_n = an^3+bn^2+cn+d$
- GP
  - 1st order: $T_n = ar^{n-1}+b$
  - 2nd order: $T_n = ar^{n-1}+bn+c$



Q $3+7+13+21+31+\ldots\ldots$ n terms.

$$\underbrace{\underbrace{3+7}_{4}+\underbrace{13}_{6}+\underbrace{21}_{8}+31}$$

$4 \quad 6 \quad 8$

$2 \quad 2 \rightarrow$ constant AP.

$$T_n = an^2+bn+c$$

$n=1;\quad 3 = a+b+c$ — (1)

$n=2;\quad 7 = 4a+2b+c$ — (2)

$n=3;\quad 13 = 9a+3b+c$ — (3)

$$a=b=c=1$$

$$T_n = n^2+n+1$$

$$S_n = \Sigma T_n$$
$$= \Sigma n^2+\Sigma n+\Sigma 1$$
$$= \frac{n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{6}+\frac{6n}{6}$$
$$= \frac{2n^3+3n^2+n+3n^2+3n+6n}{6}$$
$$= \frac{2n^3+6n^2+10n}{6}$$
$$\boxed{= \frac{n^3+3n^2+5n}{3}}$$

Q2. $1 + 4 + 10 + 22 + \ldots\ldots n$ terms.

$3 \quad 6 \quad 12 \rightarrow GP$

~~$\rightarrow$ constant GP~~

$T_n = ar^{n-1} + b$

$1 = b$

$4 = ar + b \rightarrow 4 = ar$

$10 = ar^2 + b \rightarrow 10 = ar^2 \rightarrow \frac{5}{2} = r$

$a = \frac{8}{5}$

$T_n = \frac{8}{5} \times \left(\frac{5}{2}\right)^{n-1} + 1$

$T_n = \frac{8}{5}$

$1 = b + a$

$4 = a(2) + b \rightarrow$

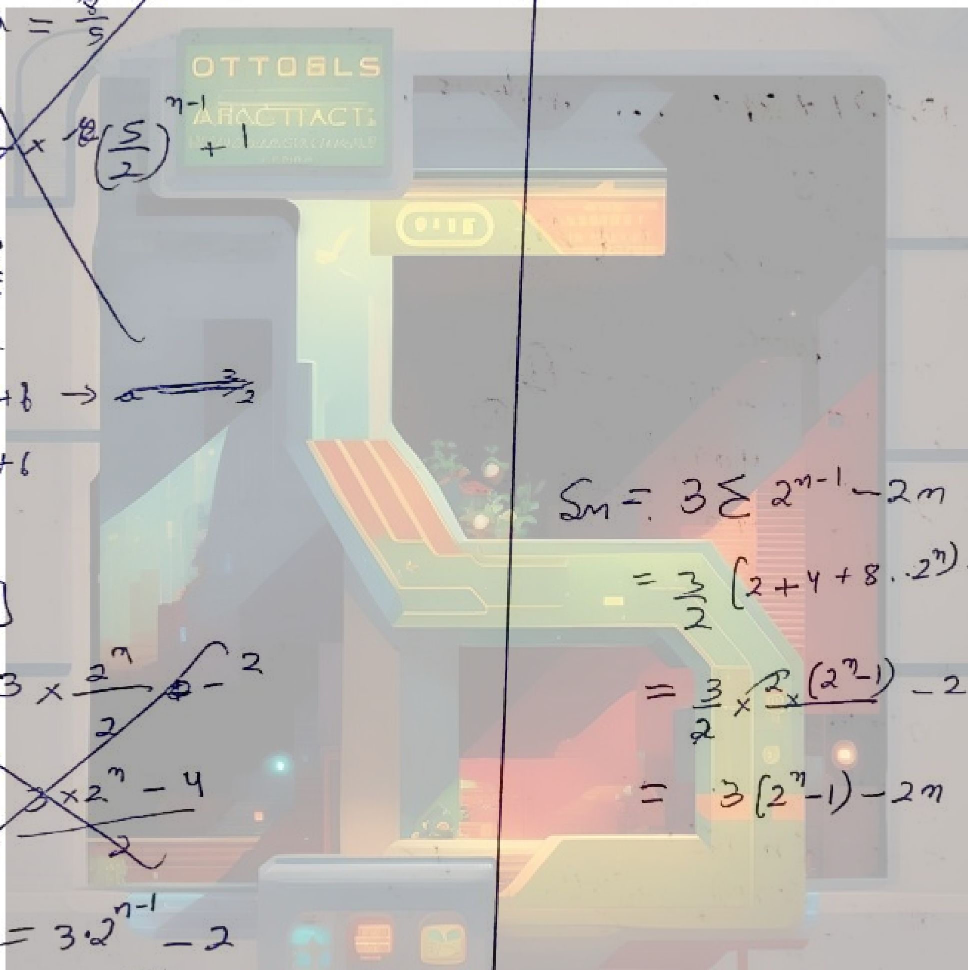
$4 = a2a + b$

$\boxed{3 = a}$

$\boxed{b = -2}$

$T_n = 3 \times \frac{2^n}{2} - 2$

$= \frac{3 \times 2^n - 4}{2}$



$$S_n = 3 \sum 2^{n-1} - 2n$$

$$= \frac{3}{2}\left(2 + 4 + 8 \ldots 2^n\right) - 2n$$

$$= \frac{3}{2} \times 2 \times (2^n - 1) - 2n$$

$$= 3(2^n - 1) - 2n$$

$T_n = 3 \cdot 2^{n-1} - 2$

$S_n = 3 \sum 2^{n-1} - 2n$

$= 3(1 + 2 + 4 \ldots n) - 2n$

$= 3\frac{(2^n - 1)}{1} - 2n$

$= 3 \times 2^n - 3 - 2n$

Q $6+13+22+33 \dots n$

$7 \quad 9 \quad 11$

$2 \quad 2$

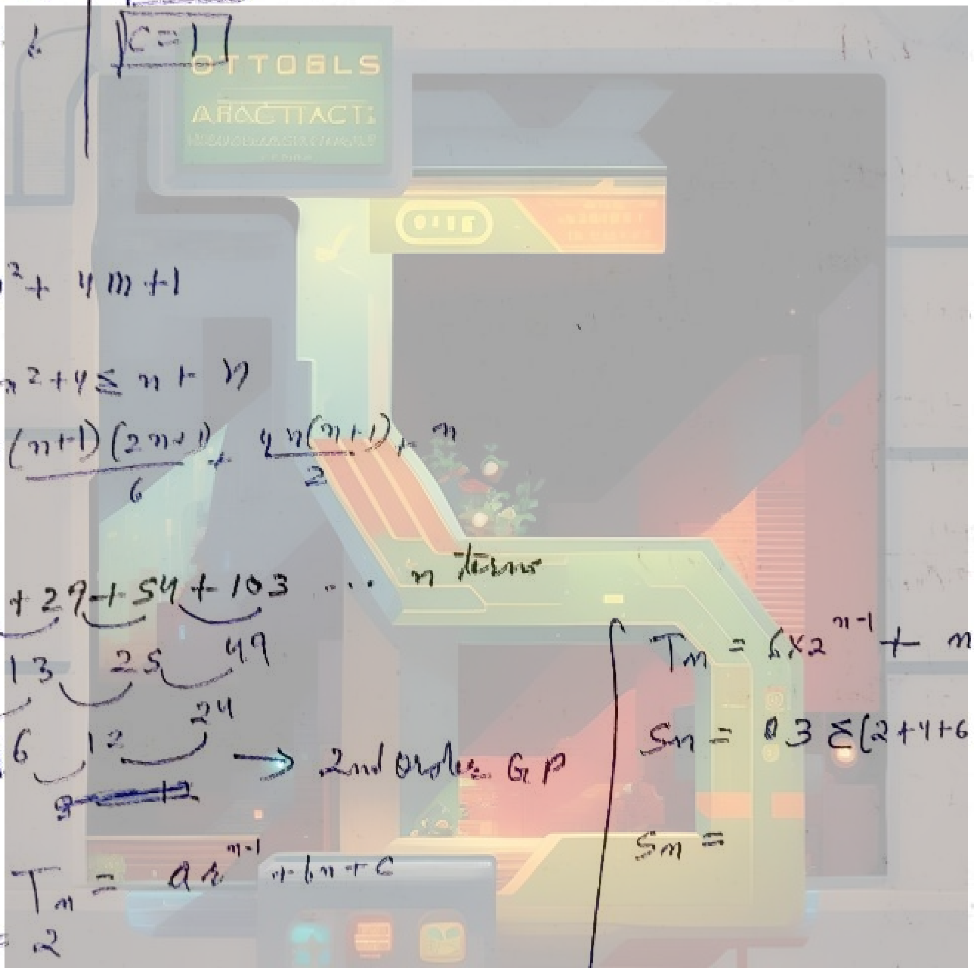
$T_n = an^2+bn+c$

$6 = a+b+c$

$13 = 4a+2b+c$

$22 = 9a+3b+c$

$9 = 5a+b$ | $\boxed{b=4}$



$7 = 3a+b$ | $\boxed{c=1}$

$2 = 2a$

$\boxed{a=1}$

$T_n = n^2+4n+1$

$S_n = \Sigma n^2 + 4\Sigma n + n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n$

Q $9+16+29+54+103 \dots$ n terms

$7 \quad 13 \quad 25 \quad 49$

$6 \quad 12 \quad 24 \rightarrow$ 2nd Order GP

$T_n = ar^{n-1}+bn+c$

$r = 2$

$9 = a+b+c$

$16 = 2a+2b+c$

$29 = 4a+3b+c$

$13 = 2a+b$

$7 = a+b$

$\boxed{6 = a}$

$\boxed{b = 1}$

$\boxed{c = 2}$

$T_n = 6\times 2^{n-1} + n+2$

$S_n = 3\,\Sigma(2+4+6 \cdots n) + \frac{n(n+1)}{2} + 2n$

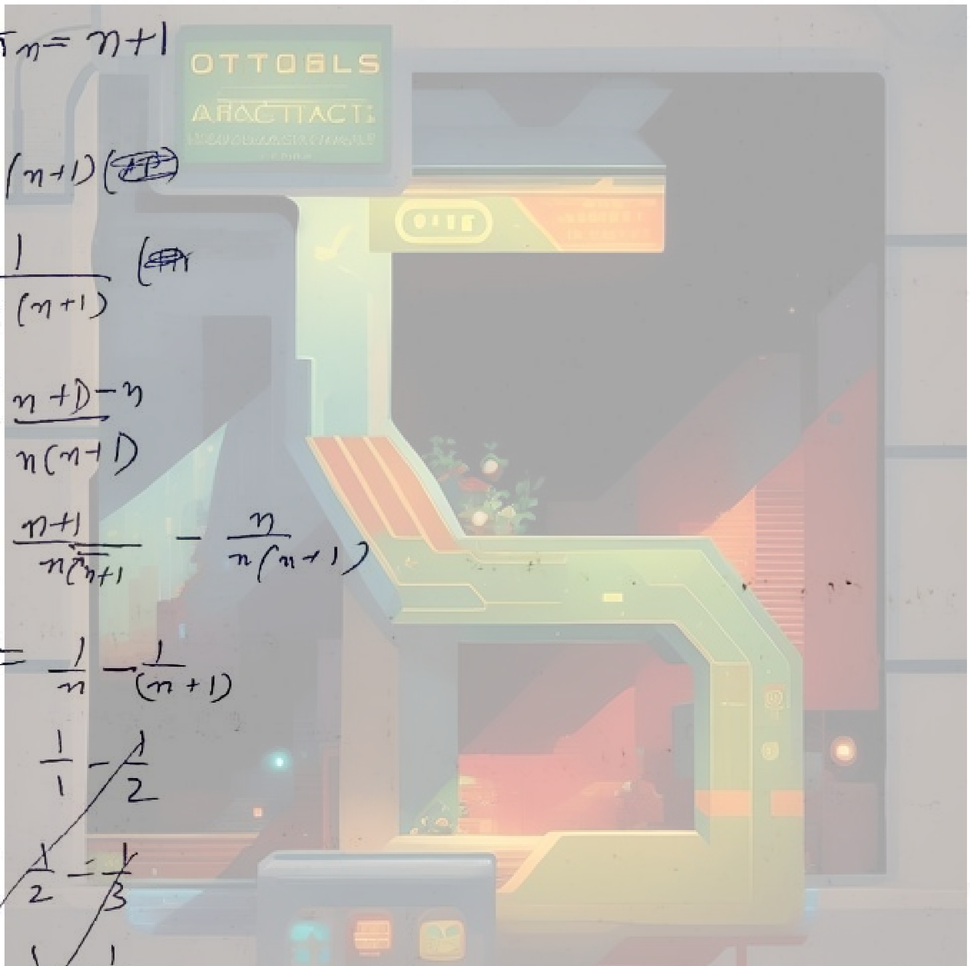
$S_n =$

Type-3 ~~Splitting in denominator~~
Splitting the n^{th} term (denominator)

Q $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}\ldots n$

$1+2+3+4+5\ldots n$
$T_n=n$

$2+3+4+5+6$
$T_n=n+1$



$T_n=n(n+1)$

$T_n=\frac{1}{n(n+1)}$

$T_n=\frac{(n+1)-n}{n(n+1)}$

$=\frac{n+1}{n(n+1)}-\frac{n}{n(n+1)}$

$=\frac{1}{n}-\frac{1}{(n+1)}$

$n=1 \quad \frac{1}{1}-\cancel{\frac{1}{2}}$

$n=2 \quad \cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}$

$n=3 \quad \cancel{\frac{1}{3}}-\frac{1}{4}$

$n=n \quad \cancel{\frac{1}{n}}-\frac{1}{n+1}$

$S_n=1-\frac{1}{n+1}$

$\boxed{=\frac{n}{n+1}}$

Q $\frac{1}{1\cdot 4} + \frac{1}{\cancel{2\cdot 3}\ 4\cdot 7} + \frac{1}{7\cdot 10} + \cancel{\frac{1}{10\cdot}} \cdots n$ Terms.

$\cancel{T_n = n(n+\cancel{4}3)}$

$\cancel{= n(n+3)}$

$\cancel{T_n = \frac{1}{n(n+3)}}$

$\cancel{= \frac{(n+3)-n}{n(n+3)}}$

$\cancel{= \frac{1}{n} - \frac{n1}{n+3}}$

$\cancel{S_n = \frac{1}{1} - \frac{1}{n+3}}$

$\cancel{= \frac{n+3-1}{n+3}}$

$\cancel{\boxed{= \frac{n+2}{n+3}}}$

$T_n = (3n-2)(3n+1)$

$= \frac{1}{3} \times \frac{3}{(3n-2)(3n+1)}$

$= \frac{1}{3}\left[\frac{(3n+1)(\cancel{0}-3n-2)}{(3n-2)(3n+1)}\right]$

$= \frac{1}{3}\left(\frac{3n+1-3n+2}{(3n-2)(3n+1)}\right)$

$\cancel{\boxed{= \frac{1}{\cancel{3}} \cdot \frac{1}{(3n-2)(3n+1)}}}$

$S_n = \frac{9n+2}{3n(3n+1)}$

H.W 05-07-2024

DXS 7 {1, 4, 5, 6{i}, 7}

01 {27, 28, 30}

Q $1^2\cancel{a} - 2^2 + 3^2 \cancel{-} 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + \cdots + 99^2 - 100^2$.

$(1^2 + 3^2 + 5^2 \ldots 99^2) - (2^2 + 4^2 + 6^2 \ldots 100^2)$

$T_n = (2n-1)^2$

$= 4n^2 - 4n + 1$

$T_n = (2n)^2$

$= 4n^2$

$S_n = 4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n$

$S_n = \frac{200(51)(101)}{6} - \frac{\overset{100}{\cancel{200}}(51)}{2} + 50$

$= 100 \times 17 \times 101 - 5100 + 50$

$S_n = 171700 - 5100 + 50$

$\boxed{S_n = -5050}$

$S_n = \frac{4n(n+1)(2n+1)}{6}$

$S_n = \frac{200(51)(101)}{6}$

$S_n = 100 \times 101 \times 17$

$S_n = 171700$

Q Sum of ∞ terms ..

$$\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}$$

$1+4+7 \ldots\ldots \infty$

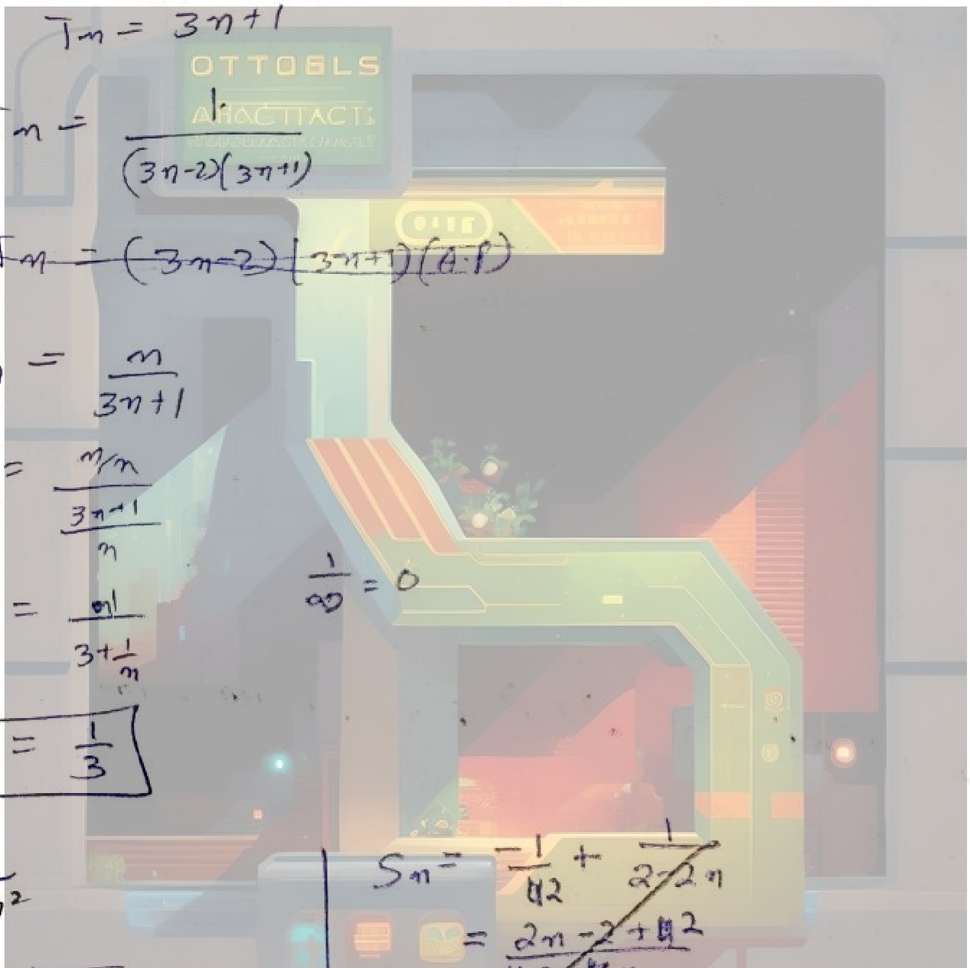
$T_n = \cancel{n+3}\ 1+(n-1)3$

$= 1+3n-3$

$= 3n-2$

$4+7+10$

$T_n = 3n+1$



$$T_n = \frac{1}{(3n-2)(3n+1)}$$

~~$T_n = (3n-2)(3n+1)(A\cdot P)$~~

$$S_n = \frac{n}{3n+1}$$

$$S_n = \frac{n/n}{\frac{3n+1}{n}}$$

$$= \frac{1}{3+\frac{1}{n}} \qquad \frac{1}{\infty}=0$$

$$\boxed{= \frac{1}{3}}$$

Q $T_n = \dfrac{n}{1-n^2}$

$= \dfrac{n}{n(1+n)(1-n)}$

$= \dfrac{(1-n)-(1+n)}{-2(1+n)(1-n)}$

$= \dfrac{-1}{2(1+n)} + \dfrac{1}{2(1\pm n)}$

$= \dfrac{1}{2(1-n)} - \dfrac{1}{2(1+n)}$

start, $n=2$

$S_n = \dfrac{-1}{42} + \dfrac{1}{2-2n}$

$= \dfrac{2n-2+42}{48-48n}$

$\cancel{\dfrac{n+1}{4(1-n)}}$

$$\boxed{= \frac{n}{2(1-n)}}$$

Q Find sum of the sequence upto infinite terms :-

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4}$$

$(1+1^2+1^4), (1+2^2+2^4), (1+3^2+3^4)$

$$T_n = \frac{n(n^3-1)}{n-1}$$

$1, 2, 3$

$T_n = n$

$$T_n = n \times \frac{(n-1)}{n(n^3-1)}$$

$$= \frac{n-1}{n^3-1}$$

$$= \frac{n-1}{(n-1)(n^2+1+2n)}$$

$$T_n = \frac{1}{n^2+2n+1}$$

$$T_n = \frac{n}{1+n^2+n^4}$$

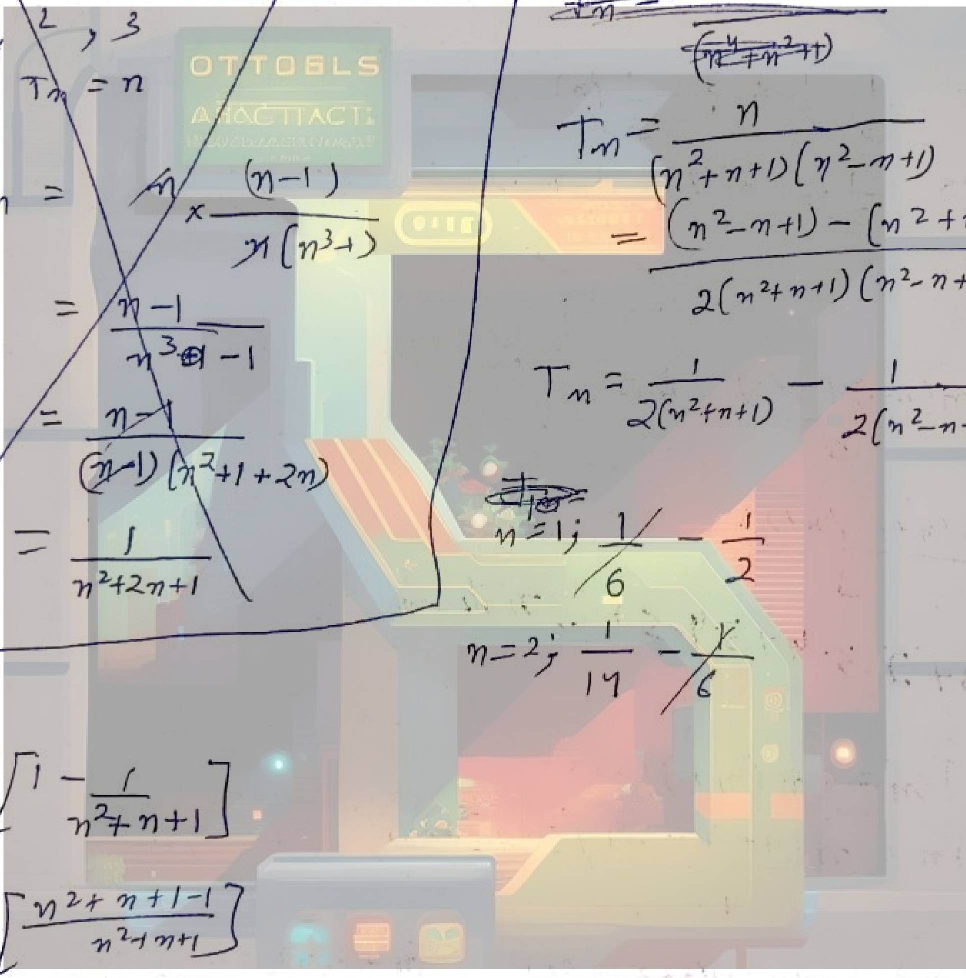
$$T_n = \frac{n}{(n^2+n+1)(n^2-n+1)}$$

$$= \frac{(n^2-n+1)-(n^2+n+1)}{2(n^2+n+1)(n^2-n+1)}$$

$$T_n = \frac{1}{2(n^2+n+1)} - \frac{1}{2(n^2-n+1)}$$

$n=1;\ \frac{1}{6} - \frac{1}{2}$

$n=2;\ \frac{1}{14} - \frac{1}{6}$



$$S_n = \frac{1}{2}\left[1 - \frac{1}{n^2+n+1}\right]$$

$$S_n = \frac{1}{2}\left[\frac{n^2+n+1-1}{n^2+n+1}\right]$$

$$= \frac{1}{2}\left[\frac{n^2+n}{n^2+n+1}\right]$$

$$= \frac{1}{\frac{2}{n^2+n} + \frac{1}{n^2+n}}$$

$$\boxed{= \frac{1}{2}}$$

Type-4 Using Difference in numerator

Q $1\cdot2+2\cdot3+3\cdot4+4\cdot5+\ldots n$ terms

$$T_n = n(n+1)$$

$$T_n = \frac{1}{3}n(n+1)\left[(n+2)-(n-1)\right]$$

$$= \frac{n(n+1)(n+2)}{3} - \frac{(n-1)(n)(n+2)}{3}$$

$n=1;\ \frac{1}{3}\cdot\cancel{1\cdot2\cdot3} - \frac{1}{3}(0)$

$n=2\ T_2 \Rightarrow \frac{1}{3}\cdot\cancel{2\cdot3\cdot4} - \cancel{\frac{1}{3}\cdot1\cdot2\cdot3}$

$n=n \quad \frac{n(n+1)(n+2)}{3} - 0$

$$S_n = \frac{n(n+1)(n+2)}{3}$$

Q $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+\ldots n$ terms.

$$T_n = n(n+1)(n+2)$$

$$T_n = \frac{n(n+1)(n+2)}{4}\cancel{\left[(n-1)(n+3)\right]}\left[(n+3)-(n-1)\right]$$

$$T_n = \frac{n(n+1)(n+2)(n+3)}{4} - \frac{(n-1)(n)(n+1)(n+2)}{4}$$

$n=1;\ \cancel{\frac{1\cdot2\cdot3\cdot4}{4}} - 0$

$n=2;\ \frac{2\cdot3\cdot4\cdot5}{4} - \cancel{\frac{1\cdot2\cdot3\cdot4}{4}}$

$$S_n = \frac{n(n+1)(n+2)(n+3)}{4}$$

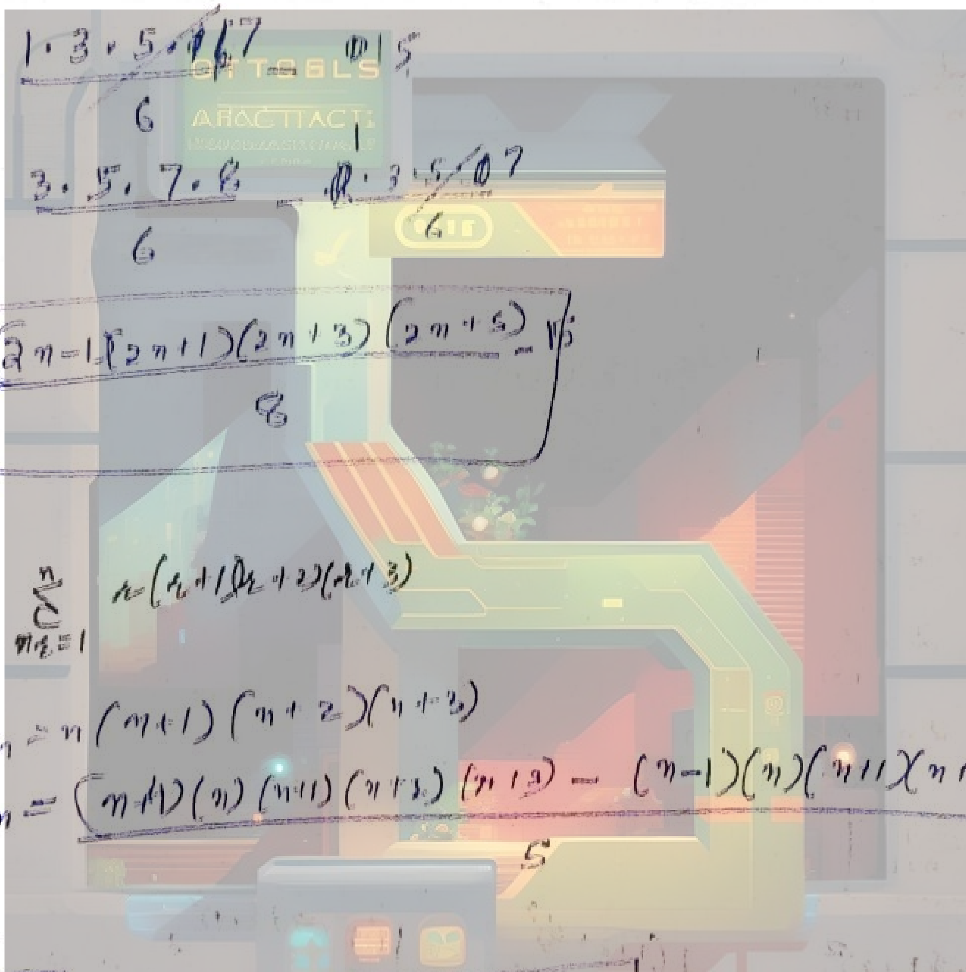
1) $1\cdot3\cdot5 + 3\cdot5\cdot7 + 5\cdot7\cdot9 + \dots n$

$T_n = \cancel{n(n+2)(n+4)}$

$T_n = (2n-1)(2n+1)(2n+3)$

$T_n = \frac{(2n-1)(2n+1)(2n+3)\left[2n+5-2n+3\right]}{8}$

$T_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} - \frac{(2n-3)(2n-1)(2n+1)(2n+3)}{8}$



$n=1;\ \frac{1\cdot3\cdot5\cdot7}{6} - 15$

$n=2;\ \frac{3\cdot5\cdot7\cdot9}{6} - \frac{1\cdot3\cdot5\cdot7}{6}$

$$\boxed{S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} - 15}$$

Q) DYS 7
Q6 ii) $\sum_{n=1}^{n} n(n+1)(n+2)(n+3)$

$T_n = n(n+1)(n+2)(n+3)$

$T_n = \frac{(n+4)(n)(n+1)(n+2)(n+3) - (n-1)(n)(n+1)(n+2)(n+3)}{5}$

$n=1;$

$$S_n = \boxed{\frac{n(n+1)(n+2)(n+3)(n+4)}{5}}$$

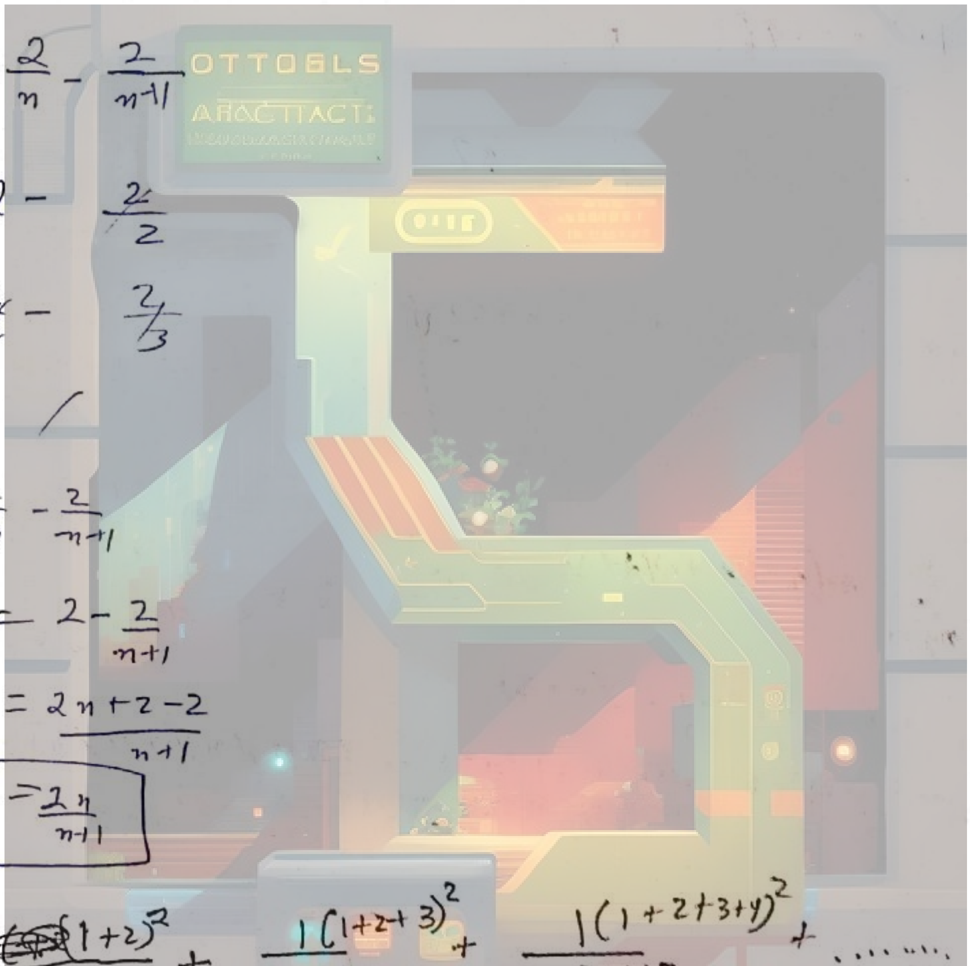
Q $S_p = 1 + 2 + 3 + \ldots p$ terms

$$\sum_{p=1}^{n} = \frac{1}{\cancel{8}S_p}$$

$$T_n = \frac{1}{\Sigma n}$$

$$= \frac{2}{n(n+1)}$$

$$t_n = \frac{2[(n+1) - n]}{n(n+1)}$$



$$T_n = \frac{2}{n} - \frac{2}{n+1}$$

$n=1; \quad 2 - \cancel{\frac{2}{2}}$

$n=2; \quad \cancel{\frac{2}{2}} - \frac{2}{3}$

$n=n; \quad \frac{2}{n} - \frac{2}{n+1}$

$$S_n = 2 - \frac{2}{n+1}$$

$$= \frac{2n + 2 - 2}{n+1}$$

$$\boxed{S_n = \frac{2n}{n+1}}$$

Q $1 + \frac{(1+2)^2}{1+3} + \frac{1(1+2+3)^2}{1+3+5} + \frac{1(1+2+3+4)^2}{1+3+5+7} + \ldots\ldots n$ terms.

O-1
Q 16-30 - {27, 29, 30}

O-2
Q 1-15 (16-20)

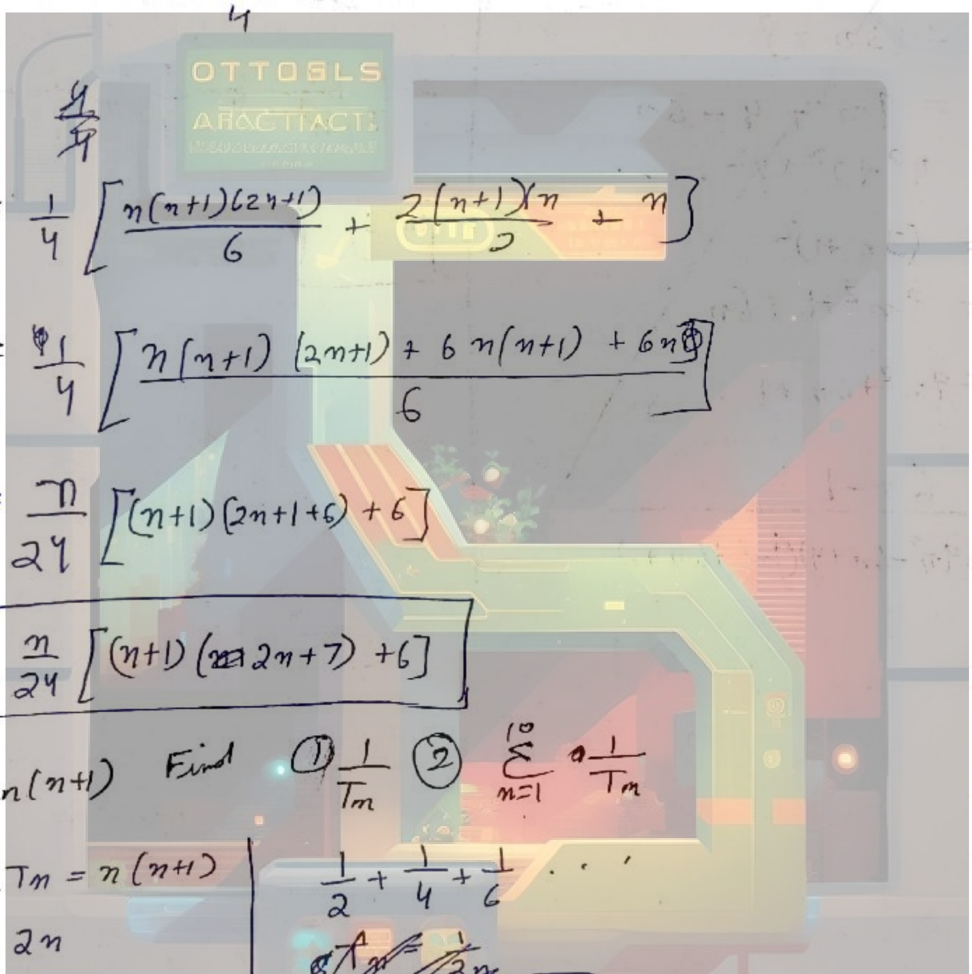
O-3
Q 1-10

$$T_n = \frac{(\Sigma n)^2}{\cancel{n^2}\ \Sigma(2n-1)}$$

$$T_n = \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2}$$

$$= \frac{\cancel{n^2}(n^2+1+2n)}{4\cancel{n^2}}$$

$$T_n = \frac{n^2+2n+1}{4}$$



$$S_n = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2(n+1)n}{2} + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1) + 6n(n+1) + 6n}{6}\right]$$

$$= \frac{n}{24}\left[(n+1)(2n+1+6)+6\right]$$

$$\boxed{S_n = \frac{n}{24}\left[(n+1)(2n+7)+6\right]}$$

Q1. $S_n = n(n+1)$ Find (1) $\frac{1}{T_n}$ (2) $\sum_{n=1}^{10} \frac{1}{T_n}$

$S_n = \Sigma T_n = n(n+1)$

$T_n = 2n$

$$\boxed{\frac{1}{T_n} = \frac{1}{2n}}$$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} \ldots$

$\frac{1}{T_n} = \frac{1}{2n}$

$\cancel{S_n = \frac{1}{n(n+1)}}$

$$\cancel{S_n = \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}}$$

$$\cancel{= 1 - \frac{1}{1024}}$$

$$\cancel{\boxed{= \frac{1023}{1024}}}$$

Q2. $\frac{5}{1^2 \cdot 4^2} + \frac{11}{4^2 \cdot 7^2} + \frac{17}{7^2 \cdot 10^2} \dots \infty$

$T_n = 5+(n-1)6 \dots$

$= 5+6n-6$

$\boxed{=6n-1}$

$T_n = 1^2+4^2+7^2+\dots$

$= (1+(n-1)3)^2$

$= (1+3n-3)^2$

$= (3n-2)^2$

$= 9n^2+4-6n$

$T_n = 4^2+7^2+x^2 \dots$

$= (3n+1)^2$

$= 9n^2+1+6n$

$= 9n^2+6n+1$

$T_n = \frac{6n-1}{(9n^2-6n+4)(9n^2+6n+1)}$

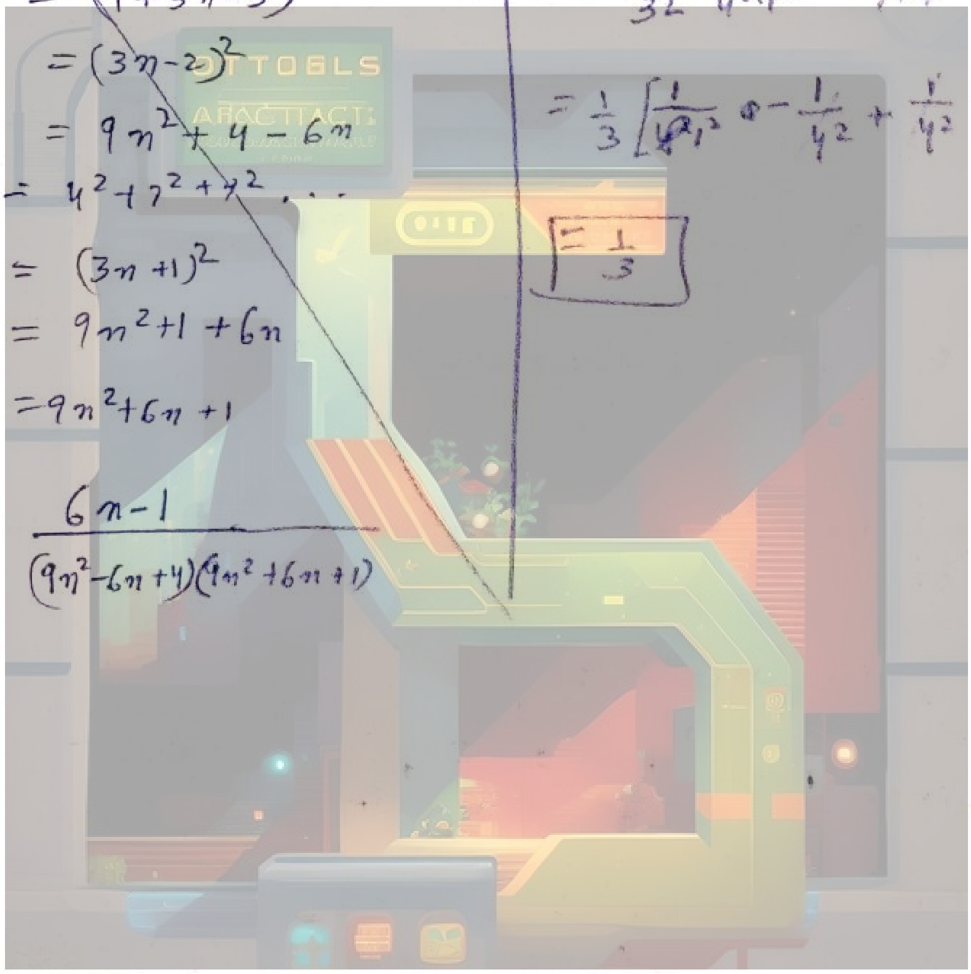
$S = \frac{(4+1)}{4^2 \cdot 1^2} + \frac{(7+4)}{7^2 \cdot 4^2} + \frac{(10+7)}{10^2 \cdot 7^2}$

$= \frac{1}{3}\left[\frac{(4+1)(4-1)}{4^2 \cdot 1^2} + \frac{(7+4)(7-4)}{7^2 \cdot 4^2} + \frac{(10+7)(10-7)}{10^2 \cdot 7^2}\right]$

$= \frac{1}{3}\left[\frac{4^2-1^2}{4^2 \cdot 1^2} + \frac{7^2-4^2}{7^2 \cdot 4^2} + \frac{10^2-7^2}{10^2 \cdot 7^2}\right]$

$= \frac{1}{3}\left[\frac{1}{1^2} - \frac{1}{4^2} + \frac{1}{4^2} - \frac{1}{7^2} + \frac{1}{7^2} - \frac{1}{10^2}\right]$

$\boxed{= \frac{1}{3}}$



Q $(1^2-a_1) + (2^2-a_2) + (3^2-a_3) + \dots + (n^2-a_n) = \frac{1}{3}n(n^2-1)$

$a_7 = ?$

$T_n = S_n - S_{(n-1)}$

$0 + 2 + 6 + 12$

$2 \quad 4 \quad 6$

$2 \quad 2$

$T_n = an^2 + bn + c$

$0 = a + b + c$

$2 = 4a + 2b + c$

$6 = 9a + 3b + c$

$4 = 5a + b$

$2 = 3a + b$

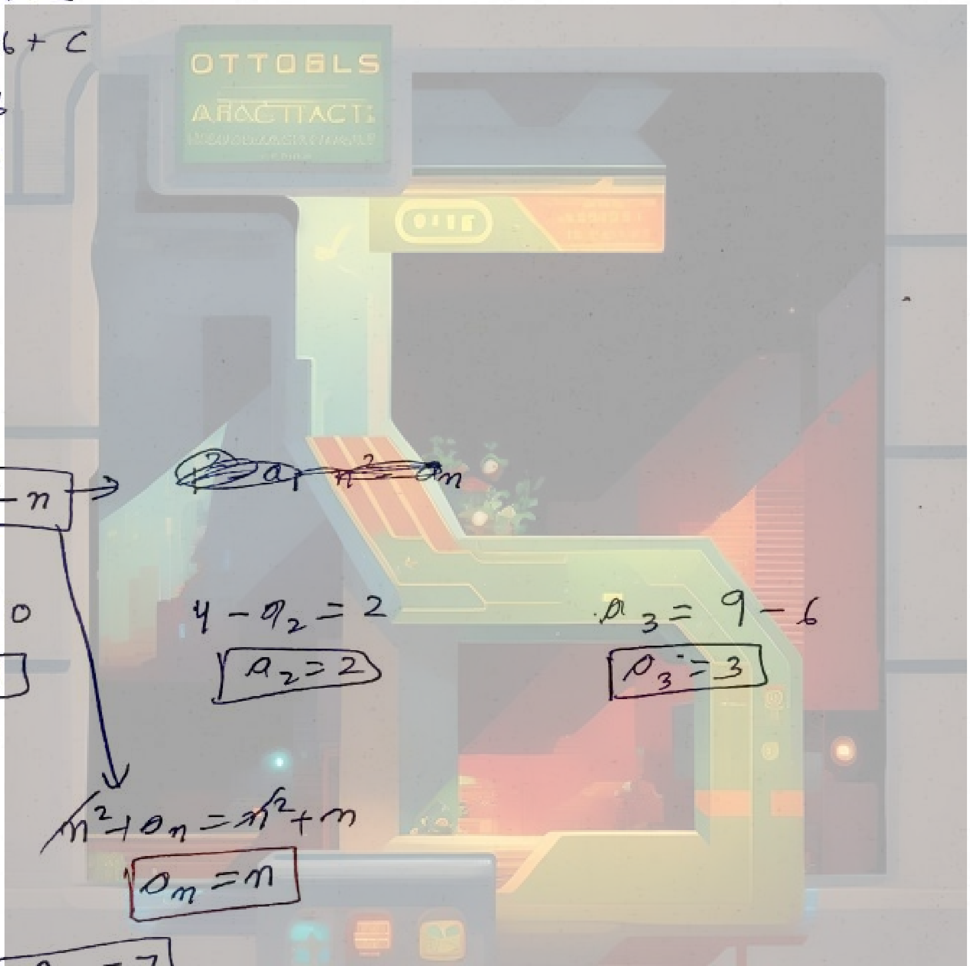
$2 = 2a$

$\boxed{a = 1}$

$\boxed{b = -1}$

$\boxed{c = 0}$

$\boxed{T_n = n^2 - n} \rightarrow \cancel{a_1}\ \cancel{n^2 - a_n}$



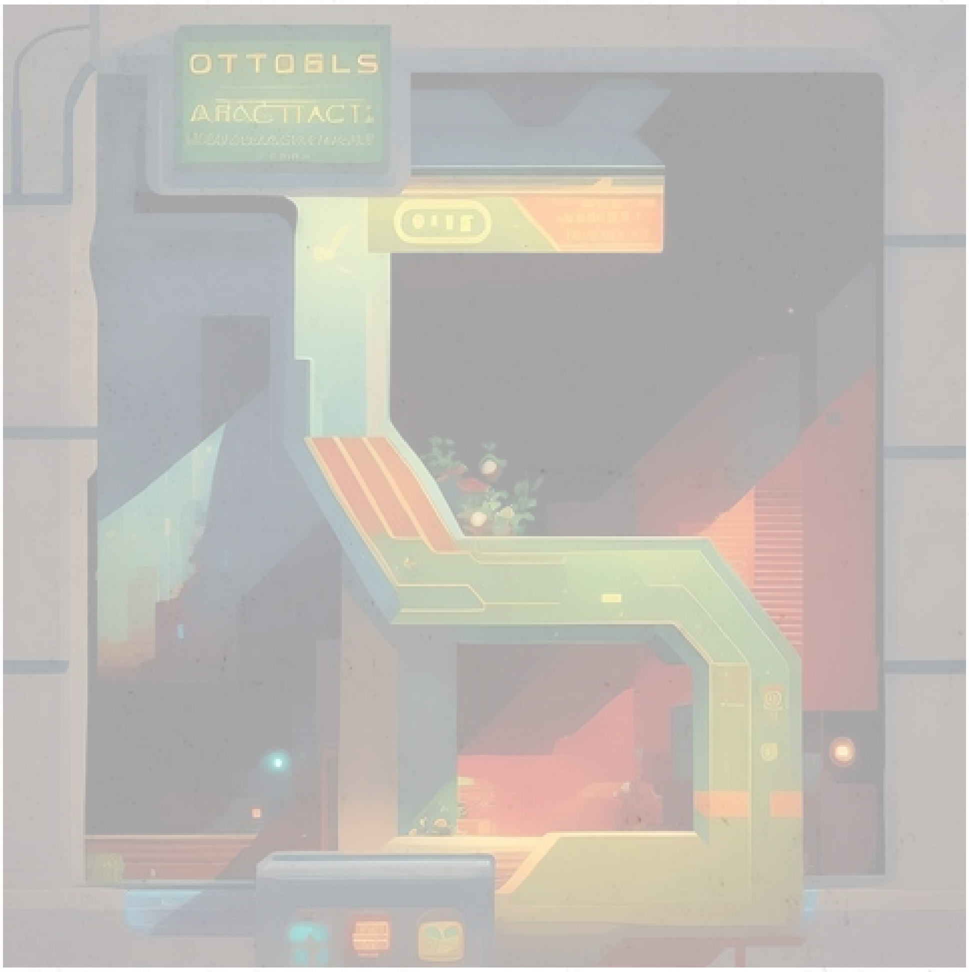
$1 - a_1 = 0$ $\quad$ $4 - a_2 = 2$ $\quad$ $a_3 = 9 - 6$

$\boxed{a_1 = 1}$ $\quad$ $\boxed{a_2 = 2}$ $\quad$ $\boxed{a_3 = 3}$

$\cancel{n^2} - a_n = \cancel{n^2} - n$

$\boxed{a_n = n}$

$\boxed{a_7 = 7}$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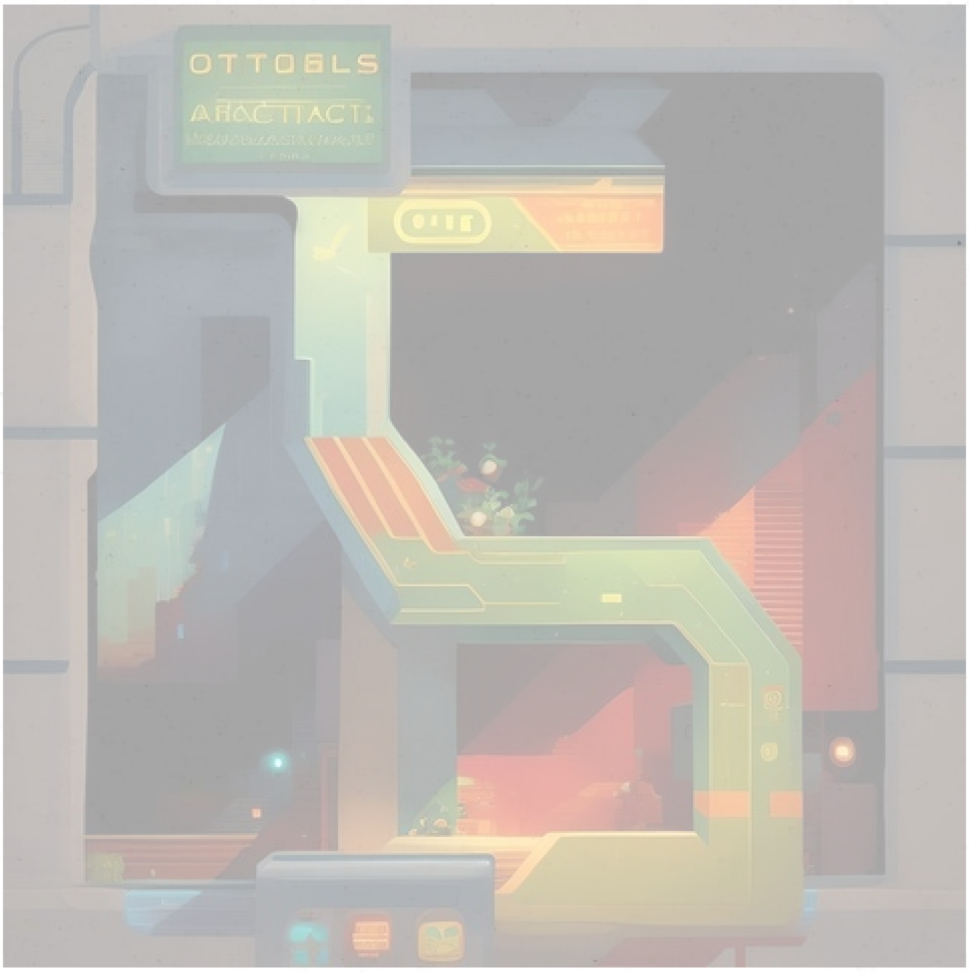

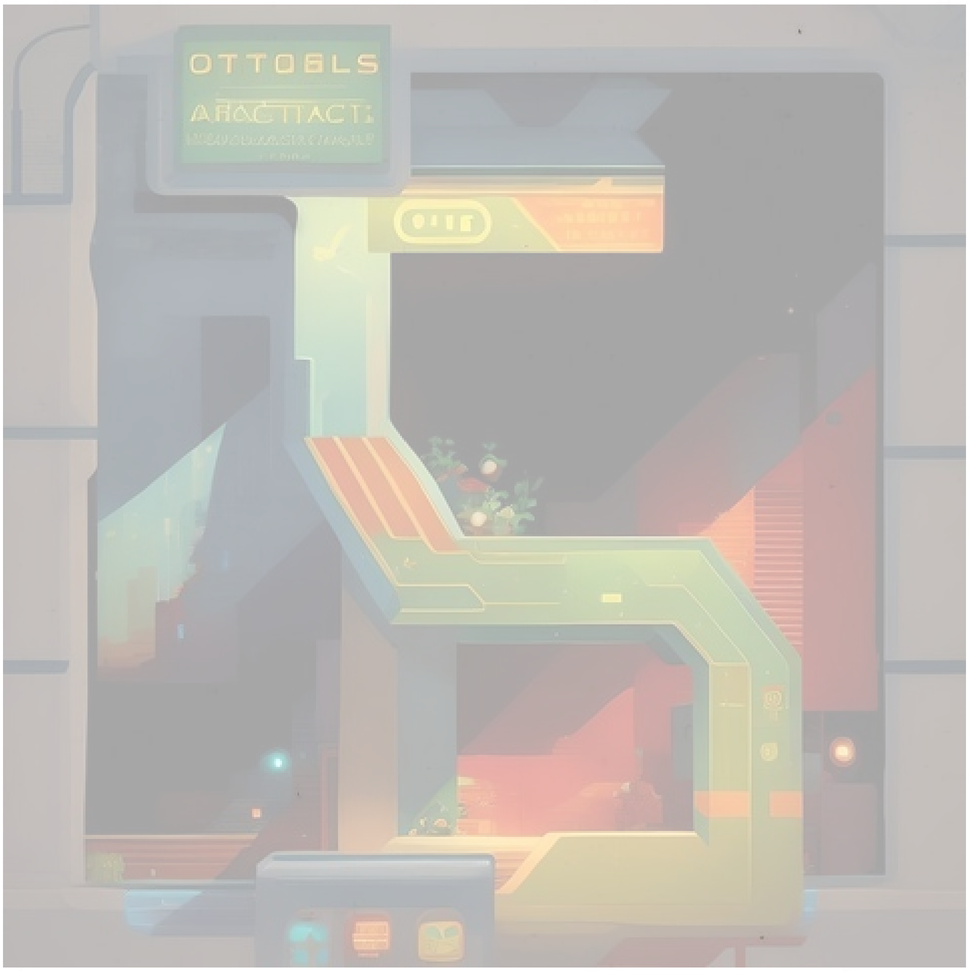

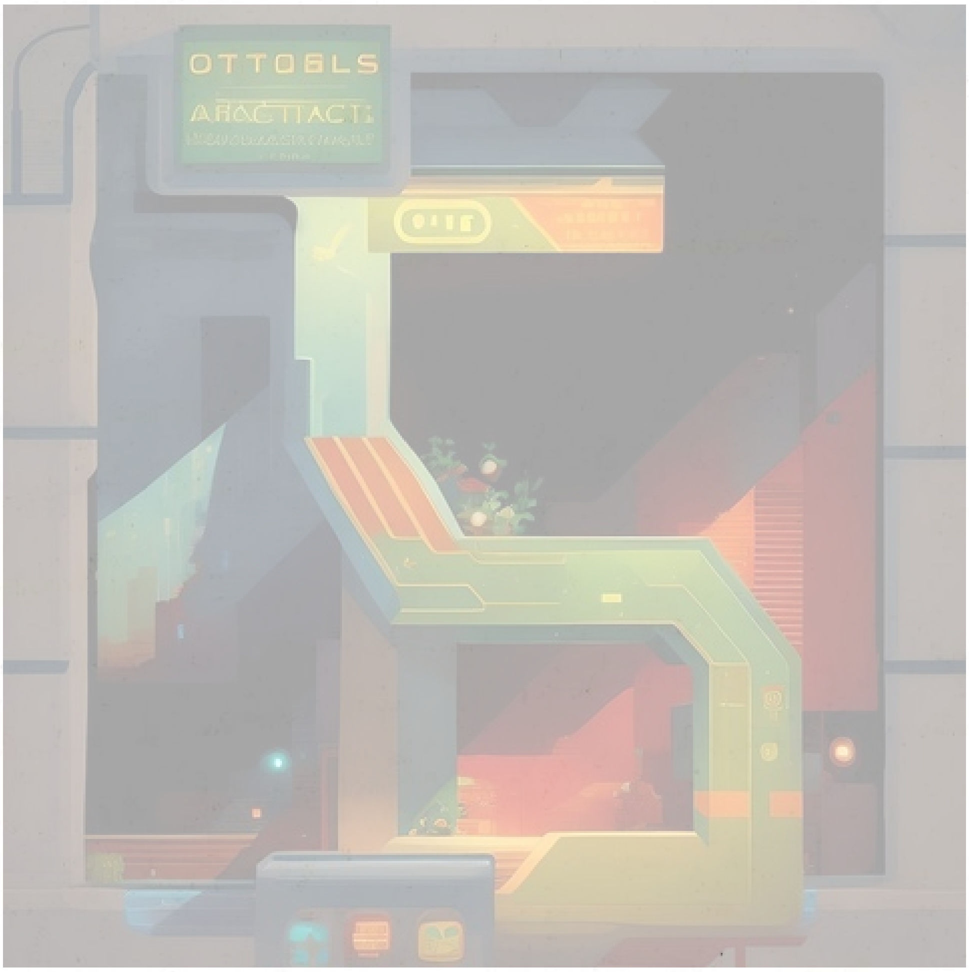


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