

Units & Dimensions

Physical Quantity - Can be measured e.g. time, distance

Fundamental Quantity (7)

- 1 → Mass → kg
- 2 → Length → m
- 3 → Time - second
- 4 → Temperature - K
- 5 → Electric Current - Ampere
- 6 → Luminous Intensity - Candela (cd)
- 7 → Amount of Substance - mole

Derived Quantity

e.g. → speed = $\frac{\text{distance} (\text{m/s})}{\text{time}}$

- velocity
- momentum
- density (kg/m^3)

$$1 \text{ g/cc} = 10^3 \text{ kg/m}^3$$

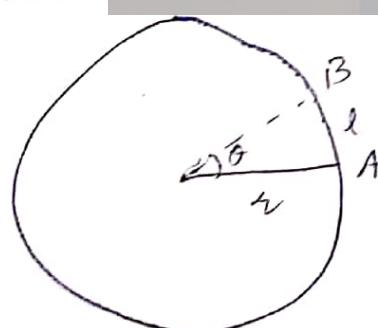
= water density (4°C)

water
 $0^\circ\text{C} \xrightarrow{\text{contract}} 4^\circ\text{C} \xrightarrow{\text{expand}}$

Supplementary units

1. Plane Angle

SI → radian



$$\theta = \frac{l}{r}$$

(1)

2. Solid Angle

SI \rightarrow steradian

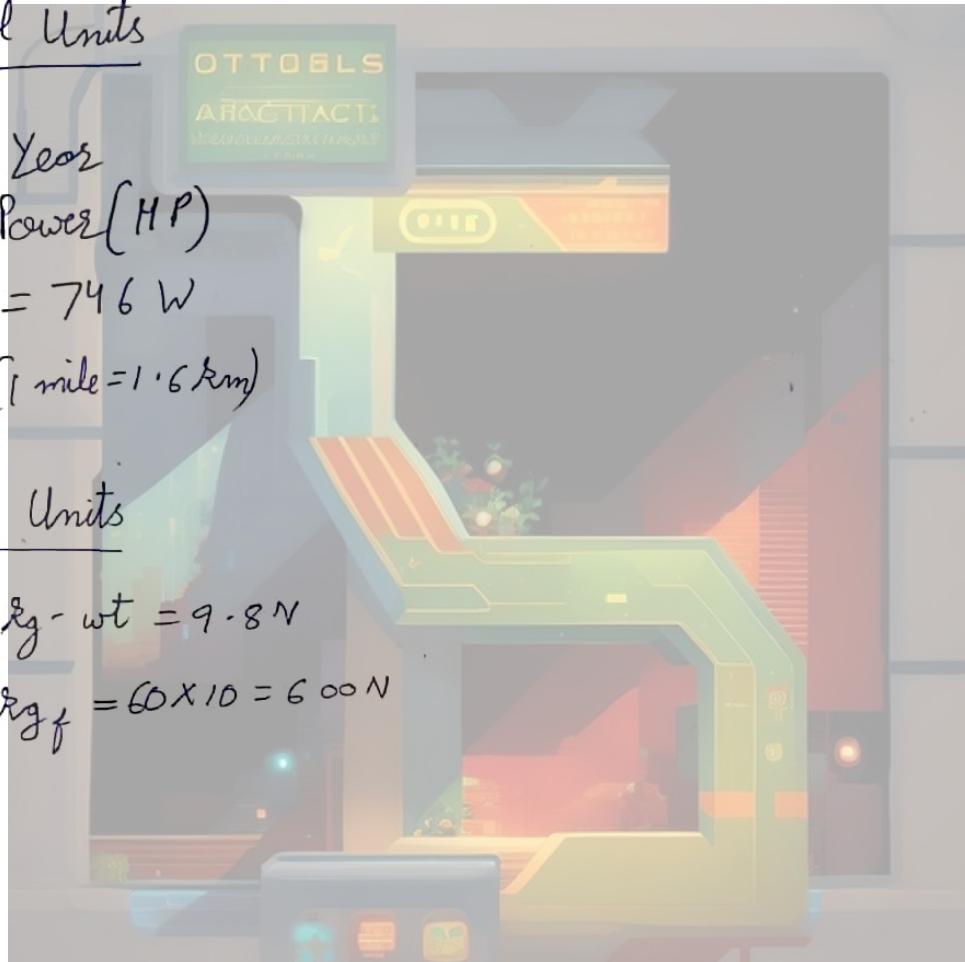


Practical Units

1. Light Year
2. Horse Power (HP)
3. mile ($1 \text{ mile} = 1.6 \text{ km}$)

Imperial Units

$$1. \text{ } Rg_f = 1 \text{ } Rg - \text{wt} = 9.8 \text{ N}$$
$$\text{eg. } 60 \text{ } Rg_f = 60 \times 10 = 600 \text{ N}$$



(2)

Dimensions → Dimensions of a physical quantity are the powers to which fundamental quantities must be raised to represent a given physical quantity.

e.g. Dimensions of Mass - [M]

Length - [L]

Time - [T]

Temperature - [θ]

Current - [A]

$$\text{Dimensions of speed} = \frac{\text{dist}}{\text{time}} = \frac{[L]}{[T]} = [L T^{-1}]$$

Density energy =

$$\text{work} \hat{=} F \times S = [MLT^{-2}] [L] \\ = [ML^2 T^{-2}]$$

$$\text{Pressure} = \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1} T^{-2}]$$

$$\text{Force} = m a = M \times L T^{-2} = [MLT^{-2}]$$

Applications of Dimensional Analysis

① To find dimensions of physical constants

$$F = \frac{G m_1 m_2}{d^2}$$

$$G = [m^{-1} L^3 T^{-2}]$$

$$N = \frac{G m^2}{L^2}$$

$$G = N \cdot m^2 \cdot kg^{-2}$$

(3)

$$E = h \nu \rightarrow \text{frequency}$$

↓

Planck's constant

$$\nu = \frac{1}{f \tau t}$$

$$ML^2T^{-2} = h T^{-1}$$

$$h = [ML^2T^{-1}]$$

2. To check if a equation is dimensionally correct

e.g. $F = ma^2$

$$MLT^{-2} = M [LT^{-2}]^2$$

$$MLT^{-2} \neq ML^2T^{-4} \quad (F = ma^2 \text{ is wrong as dimensions are diff})$$

$$F = \underline{m} \underline{v}^2$$

$$= \underline{M} \underline{[LT^{-1}]}^2$$

$$= \underline{ML^2T^{-2}}$$

$$\underline{MLT^{-2}} = \underline{MLT^{-2}} \quad (\text{correct eqn})$$

$$S = ut + \frac{1}{2} at^2 \quad (\text{numbers are dimension less})$$

$$(\underline{[L]} + \underline{[L]} = \underline{[L]})$$

$$L = LT^{-1} \times T + \frac{1}{2} [LT^{-2}] [T^2]$$

$$= L + \cancel{\frac{1}{2} L}$$

$$[L] = [L] \quad \text{correct dimensionally correct}$$

3. Derive new equations

$$E = mc^2$$

$$E = f(m, c)$$

$$E = f \propto K m^x c^y \quad (K \text{ is constant})$$

$$MLT^{-2} = K M^x [LT^{-1}]^y$$

$$MLT^{-2} = K M^x L^y T^{-y}$$

$$\begin{aligned} x &= 1 \\ y &= 2 \\ E &= K m c^2 \\ K &= 1 \quad (\text{from experiments}) \\ E &= mc^2 \end{aligned}$$

Q if velocity, force & time are taken as funda-qns. find dimensions of mass & energy

$$vel = \frac{\text{dis}}{\text{time}}$$

$$\text{force} = \text{mass} \times \frac{\text{dis}}{\text{time}^2} \times \frac{\text{vel}}{\text{time}}$$

$$\frac{MLT^{-2}}{LT^{-1}}$$

$$m = f(2, f, t)$$

$$m = K v^x F^y t^z$$

$$m = K [LT^{-1}]^x [MLT^{-2}]^y [F]^z$$

$$m = K L^x M^y L^{x+y} T^{z-y}$$

$$\begin{aligned} y &= 1 \\ x+y &= 0 \end{aligned}$$

(5)

$$E = f(v, f, t)$$

$$ML^2T^{-2} = M^y L^{x+y} T^{-x-2y+2}$$

$$\boxed{y=1}$$

$$\boxed{x=1}$$

$$-x - 2y + z = -2$$

$$-1/2 + 2 = -1/2$$

$$\boxed{z=1}$$

$$\boxed{E = K \cancel{v} v^y F^x T^z}$$

Q if momentum, area & time are taken

$$E = ?$$

$$ML^2T^{-2} = K P^x A^y T^z$$

$$ML^2T^{-2} = K [MLT^{-1}]^x [L^2]^y [T^z]^z$$

$$= K M^x L^{x+2y} T^{-x+2z}$$

$$\text{area} = L^2$$

$$\text{Time} \cancel{T} = T^1$$

$$\text{momentum} = MLT^{-1}$$

$$\boxed{x=1}$$

$$x+2y=2$$

$$2y = 1$$

$$\boxed{y = 1/2}$$

$$\cancel{z=1}$$

$$\boxed{z = 1 + -2}$$

$$\boxed{z = -1}$$

$$\boxed{E = K \cancel{P} P A^{1/2} T^{-1}}$$

⑥

$$\rho = \frac{MLT^{-2}}{L^2} = M L^{-1} T^{-2}$$

$$D = \frac{M}{L^3} = ML^{-3}$$

$$T' = K \left(ML^{-1} T^{-2} \right)^x \left[ML^{-3} \right]^y \left[ML^2 T^{-2} \right]^z$$

$$T' = K M^{x+y+z} L^{-x-3y+2z} T^{-2x-2z}$$

$$E = ML^2 T^{-2}$$

$$-2x - 2z = 1$$

$$-2z = 2x \quad z = \frac{1+2x}{2}$$

$$-x - 3y + 2x = 0$$

$$-3y = x \quad x = -3y$$

$$-y + y + 2 = 0$$

$$z = 0$$

$$x = 0$$

$$x = -1 - 3 \left(\frac{-3}{10} \right)$$

$$= -1 + \frac{9}{10}$$

$$= \frac{-10+9}{10}$$

$$x = \frac{-1}{10}$$

$$z = 1 - \frac{2}{10}$$

$$z = \frac{8}{10}$$

$$z = \frac{4}{10} \quad \frac{2}{5}$$

$$-x - 3y + 2 \left(\frac{1+2x}{2} \right)$$

$$-x + 3y + 1 + 2x$$

$$x + 3y + 1 = 0$$

$$x = -1 - 3y$$

$$-1 - 3y + y + 1 + 2(-1 - 3y)$$

$$-1 - 3y + y + \frac{1 + 2(-1 - 3y)}{2}$$

$$-2 - 24y + 1 - 2 - 6y$$

$$\frac{-1 - 3 - 10y}{2} = 0$$

$$y = \frac{3}{10}$$

$$y = \frac{-3}{10}$$

7)

$$T = K P^x d^y E^z$$

$$M^o L^o T^l = [ML^{-1} T^{-2}]^x [ML^{-3}]^y [ML^2 T^{-2}]^z$$

$$M^o L^o T^l = M^{x+y+z} L^{-x-3y+2z} T^{-2x-2z}$$

$$x + y + z = 0$$

$$-x - 3y + 2z = 0$$

$$-2x - 2z = 1$$

$$-x = \frac{1+2z}{2}$$

$$-x - 3y + 2z = 0$$

$$\frac{1+2z}{2} - 3y + 2z$$

$$1+2z - 6y + 4z = 0$$

$$1-6y + 6z = 0$$

$$y = \frac{6z+1}{6}$$

$$\frac{6z+1}{6} + \frac{-1-2z}{2} + z = 0$$

$$\frac{6z+1 - 3 + 6z + 6z}{6} = 0$$

$$\frac{6z-2}{6} = 0$$

$$6z = 2$$

$$z = \frac{1}{3}$$

$$x = -1 - \frac{2}{3}$$

$$x = \cancel{-1} - \frac{-3-2}{6}$$

$$x = \frac{-5}{6}$$

$$y = \frac{2+1}{6}$$

$$y = \frac{1}{3}$$

$$t = K P^{\frac{-5}{6}} d^{\frac{1}{3}} E^{\frac{-1}{3}}$$

(8)

System of units

FPS (Foot, Pound, Second) (British system)

CGS (Gaussian system) (centimeter, gram, second)

MKS (Metric system) (meter, kg, second)

$$1 \text{ N} = 10^5 \text{ dyne (c.g.s)}$$

$$1 \text{ J} = 10^7 \text{ erg (G.G.s)}$$

Formulas

$$1. \sin^2 \theta + \cos^2 \theta = 1$$

$$2. 1 + \tan^2 \theta = \sec^2 \theta$$

$$3. 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$4. \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$5. \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$6. \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$7. \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$8. \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$9. \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$10. \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\begin{aligned} 11. \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \end{aligned}$$

$$12. 1 + \cos \theta = 2 \cos^2 \left(\frac{\theta}{2} \right)$$

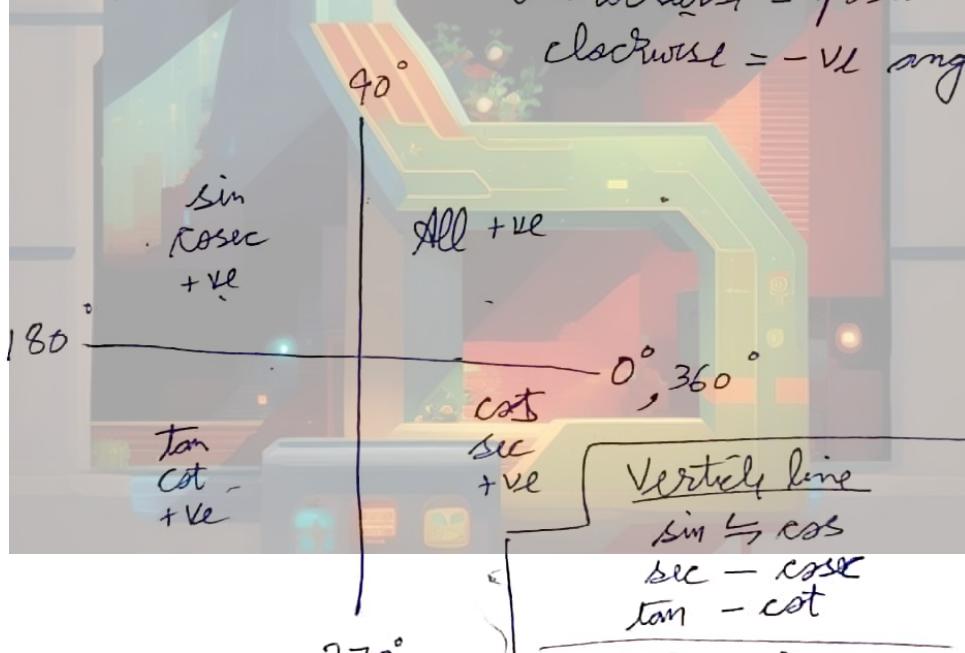
$$13. 1 - \cos \theta = 2 \sin^2 \left(\frac{\theta}{2} \right)$$

Ex. $\sin 150^\circ = \sin(90 + 60)$

$$\begin{aligned} \sin(A+B) &= \sin 90 \cos 60 + \sin 60 \cos 90 \\ &= 1 \times \frac{1}{2} + 0 \\ &= \frac{1}{2} \end{aligned}$$

~~ATSC~~ rule:-
ASTC

anticlockwise = positive angle
clockwise = -ve angle



Horizontal Line (some signs & ratio)

$$\begin{aligned} \sin(180-\theta) &= + \sin \theta \\ \sin(180+\theta) &= - \sin \theta \\ \cos(180-\theta) &= - \cos \theta \\ \tan(180+\theta) &= \tan \theta \end{aligned}$$

$$\begin{aligned} \sin(90-\theta) &= + \cos \theta \\ \sin(90+\theta) &= + \cos \theta \\ \cos(90-\theta) &= - \sin \theta \\ \cos(90+\theta) &= + \sin \theta \\ \tan(90-\theta) &= \cot \theta \\ \tan(90+\theta) &= - \cot \theta \end{aligned}$$

0

$$\text{eg. } \sin 150^\circ = \sin(90^\circ + 60^\circ) \\ = \cos 60^\circ \\ = \frac{1}{2}$$

$$\tan(270^\circ + \theta) = -\cot \theta$$

$$\text{eg. } \cos(120^\circ) = \cos(90^\circ + 30^\circ) \\ = -\sin 30^\circ$$

$$= -\frac{1}{\sqrt{2}}$$

Angles

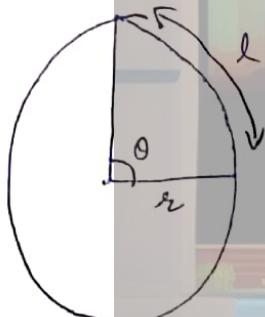
08-04-2024

① Sexagesimal system :- angle is measured in degrees (θ°)

$$1^\circ = 60' \text{ (arc minutes)}$$

$$1' = 60'' \text{ (arc seconds)}$$

② Circular system :- angle in radians (rad) (θ^c) (θ^c -no. superscript)



$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi^c = 360^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$180^\circ = \pi^c$$

$$90^\circ = \frac{\pi}{2}^c$$

$$45^\circ = \frac{\pi}{4}^c$$

$$30^\circ = \frac{\pi}{6}^c$$

$$60^\circ = \frac{\pi}{3}^c$$

* If θ is very small ($\theta < 5^\circ$)

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 \quad \text{or} \quad 1 - \frac{\theta^2}{2}$$

$$\tan \theta \approx \theta$$

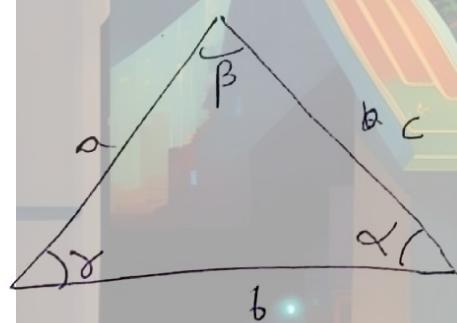
(θ must in radian)

$$\sin 37^\circ = \frac{3}{5} \quad \sin 53^\circ = \frac{4}{5}$$

$$\cos 37^\circ = \frac{4}{5} \quad \cos 53^\circ = \frac{3}{5}$$

$$\tan 37^\circ = \frac{3}{4} \quad \tan 53^\circ = \frac{4}{3}$$

Sine Rule



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

Cosine Rule:-

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

(D)

Binomial Theorem (Expression with 2 terms)

Factorial (!)

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$= 120$$

$$4! = 4 \times 3 \times 2 \times 1 = \cancel{2} 24$$

$$3! = 3 \times 2 \times 1 = 6$$

$$2! = 2 \times 1 = 2$$

$$1! = 1 = 1$$

$$0! = 1$$

$n!$ = Product of n natural numbers

$$= n \times (n-1) \times (n-2) \dots \times 3 \times 2 \times 1$$

$(x+y)$, $(4x-3y)$, $(a+b)^2$
Binomial Expression :-

Binomial Formula

$$(x+a)^n = {}^n C_0 x^n a^0 + {}^n C_1 x^{n-1} a^1 + {}^n C_2 x^{n-2} a^2 + \dots + {}^n C_n x^0 a^n$$

$${}^n C_r = \frac{n!}{r! (n-r)!}$$
] combination formula

$$\text{Lg. } (a+b)^2 = {}^2C_0 a^2 b^0 + {}^2C_1 a^1 b^1 + {}^2C_2 a^0 b^2$$

$$= \frac{2!}{0! 2!} a^2 + \frac{2!}{1! 1!} ab + \frac{2!}{2! 0!} b^2$$

$$= \frac{a^2}{1} + \frac{2}{1 \times 1} ab + \frac{b^2}{1}$$

$$= a^2 + 2ab + b^2$$

$$\text{Lg. 2. } (a+b)^3 = {}^3C_0 a^3 b^0 + {}^3C_1 a^2 b^1 + {}^3C_2 a^1 b^2 + {}^3C_3 a^0 b^3$$

$$= \frac{3!}{0! 3!} a^3 + \frac{3!}{1! 2!} a^2 b + \frac{3!}{2! 1!} a b^2 + \frac{3!}{3! 0!} b^3$$

$$= \frac{a^3}{1} + \frac{6}{2} a^2 b + \frac{6}{2} a b^2 + \frac{b^3}{1}$$

$$= a^3 + 3a^2 b + 3a b^2 + b^3$$

$$\text{Lg. 3. } (1+x)^6 = {}^6C_0 1^6 x^0 + {}^6C_1 1^5 x^1 + {}^6C_2 1^4 x^2 + {}^6C_3 1^3 x^3 +$$

$$+ {}^6C_4 1^2 x^4 + {}^6C_5 1^1 x^5 + {}^6C_6 1^0 x^6$$

$$= \frac{6!}{0! 6!} x^0 + \frac{6!}{1! 5!} x^1 + \frac{6!}{2! 4!} x^2 + \frac{6!}{3! 3!} x^3 + \frac{6!}{4! 2!} x^4 + \frac{6!}{5! 1!} x^5 + \frac{6!}{6! 0!} x^6$$

$$= 1 + \frac{720}{120} x^1 + \frac{\frac{360}{3240} \cdot 180 \cdot 90 \cdot 45 \cdot 15}{240} x^2 + \frac{\frac{720}{6 \times 6} \cdot 3240}{240} x^3 + \frac{720}{240} x^4 + \frac{720}{120} x^5 + x^6$$

$$= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$$

14

Binomial approximation :-

$$(1+x)^n$$

$$(1+x)^n \approx 1 + nx \quad x \ll 1$$

$x \ll 1$ (\ll means very small)

$$\text{eg. } (1+x)^2 = 1 + x^2 + 2x \approx 1 + 2x$$

$$(1+x)^3 = 1 + x^3 + 3x^2 + 3x \approx 1 + 3x$$

$x \ll 1$
 $x^2, x^3 \rightarrow$ higher orders of $x \rightarrow$ even smaller

$$\text{eg. } g_n = g \left[\frac{R}{(R+h)} \right]^2$$

$$R = 6400 \text{ km}$$

$$g = 9.8$$

~~$h = \text{height above}$~~

if $h \ll R$

$$R g_{nh} = g \left[\frac{R}{R(1+\frac{h}{R})} \right]^2$$

$$= g \left[\frac{1}{1+\frac{h}{R}} \right]^2$$

$$= g \left(1 + \frac{h}{R} \right)^{-2}$$

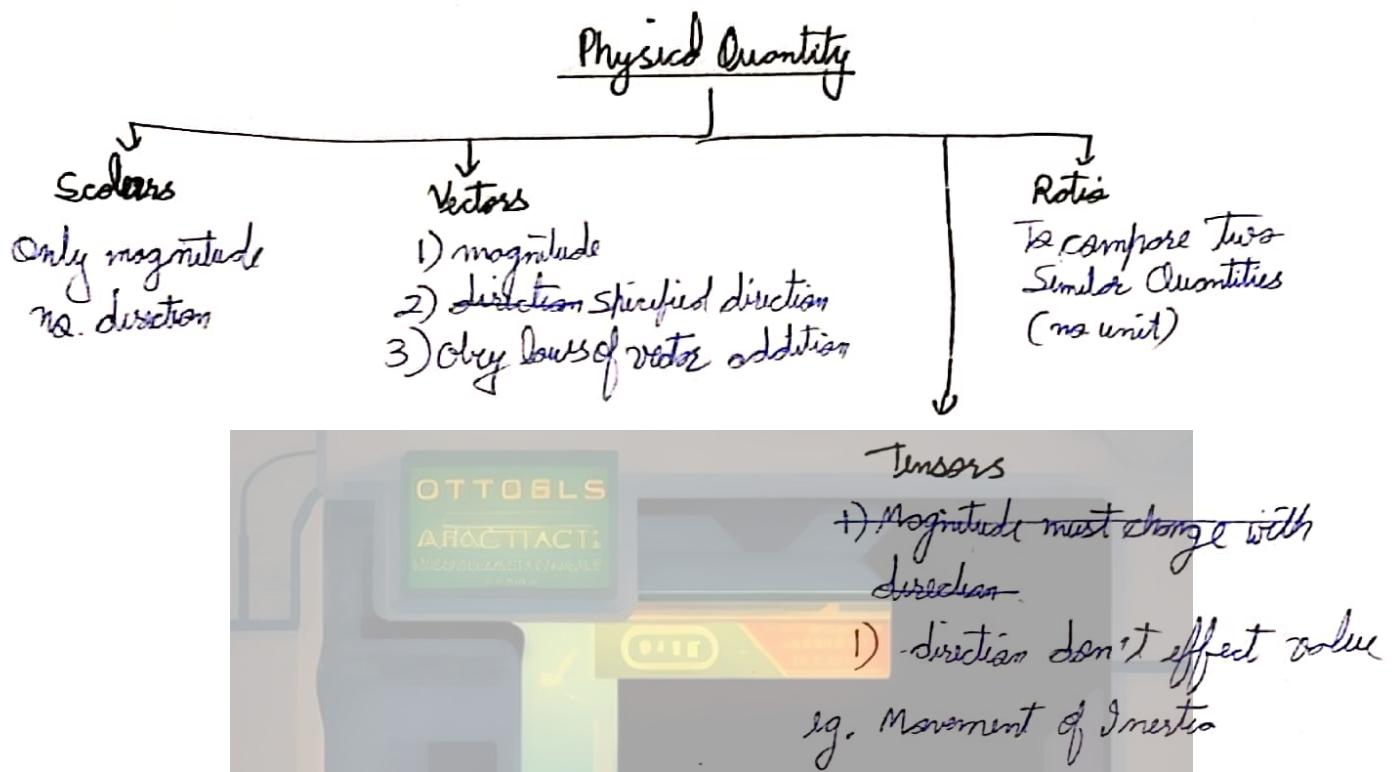
~~$g(1 -$~~

$$h \ll R$$

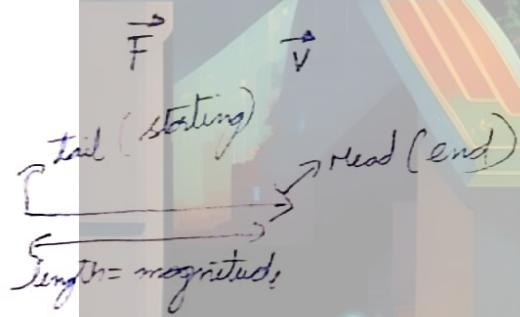
$$\begin{aligned} & \left| \frac{h}{R} \ll 1 \right. \\ & \left. \frac{h}{R} \right. \end{aligned}$$

$$g_h = g \left(1 - \frac{2h}{R} \right)$$

Vectors



Representation of a vector :-

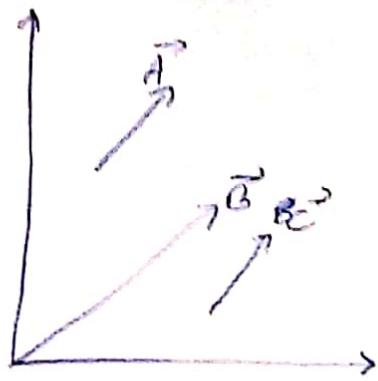


~~\vec{F}~~ (taking

→ For two vectors to be same, their magnitude & direction both should be equal.

$|\vec{A}|$ (taking only magnitude of the vector)

→ we can displace a vector parallel to itself without affecting it.

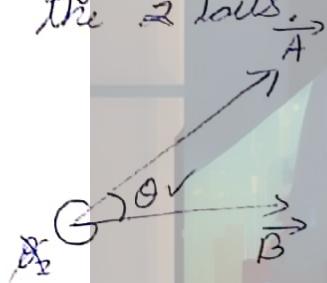


$$\vec{A} = \vec{C}$$

Angle between 2 vectors.

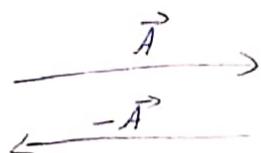
→ Join the 2 vectors by tail.

Angle between 2 vectors is the smaller angle between the 2 tails.



Negative of a vector

→ A vector of same magnitude but opposite in direction.



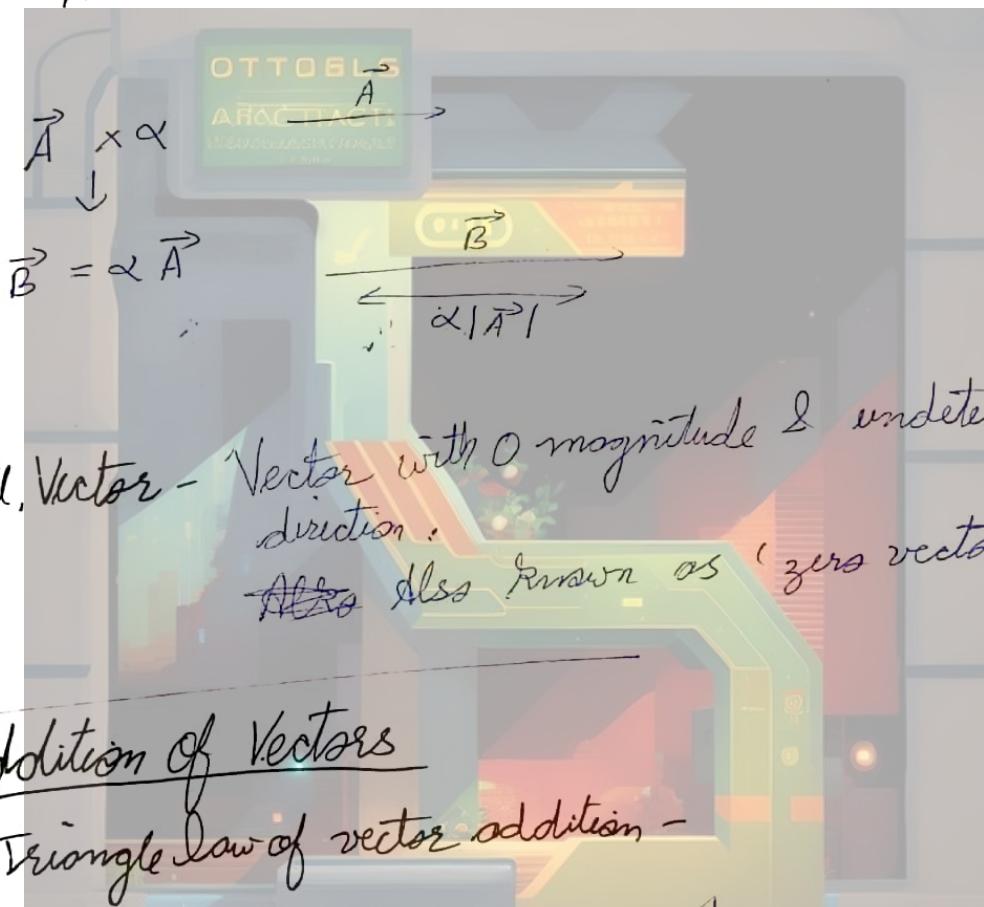
Multiply

① with Positive number

Multiplying vector \vec{A} with α will give new vector \vec{B} with magnitude α times & same direction.

② with negative number

Opp direction & magnitude gets multiplied.



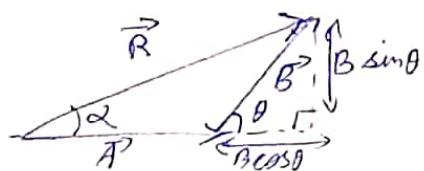
Null Vector - Vector with 0 magnitude & undetermined direction.

~~Also~~ also known as "zero vector"

Addition of Vectors

① Triangle Law of vector addition -

$$\vec{A} + \vec{B} = \vec{R}$$



$$\sin \theta = \frac{P}{x}$$

$$p = x \sin \theta$$

$$\cos \theta = \frac{b}{x}$$

$$b = x \cos \theta$$

PGT in

$$R^2 = (B \sin \theta)^2 + (A + B \cos \theta)^2$$

$$\begin{aligned} R^2 &= B^2 \sin^2 \theta + A^2 + B^2 \cos^2 \theta + 2AB \cos \theta \\ &= B^2 (\sin^2 \theta + \cos^2 \theta) + A^2 + 2AB \cos \theta \end{aligned}$$

$$R = \sqrt{B^2 + A^2 + 2AB \cos \theta}$$

Magnitude of resultant vector

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

for $\theta = 0^\circ$

$$\begin{cases} \theta = 0^\circ \\ R = A + B \end{cases}$$

OTTO BLS
ARCTIC
MOTORSPORTS

$$\begin{cases} \theta = 180^\circ \\ R = A - B \end{cases}$$

$$\begin{cases} \theta = 90^\circ \\ R^2 = \sqrt{A^2 + B^2} \end{cases}$$

$$\tan \alpha = \frac{B}{A} \quad |A - B| \leq R \leq |A + B|$$

e.g. Only the vectors of the some type can be added & subtracted & result is also same time.

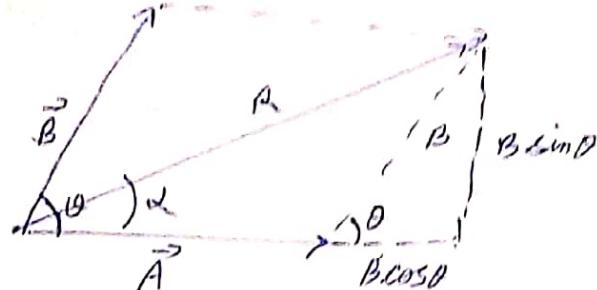
Vector addition is commutative i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Vector addition is associative i.e.

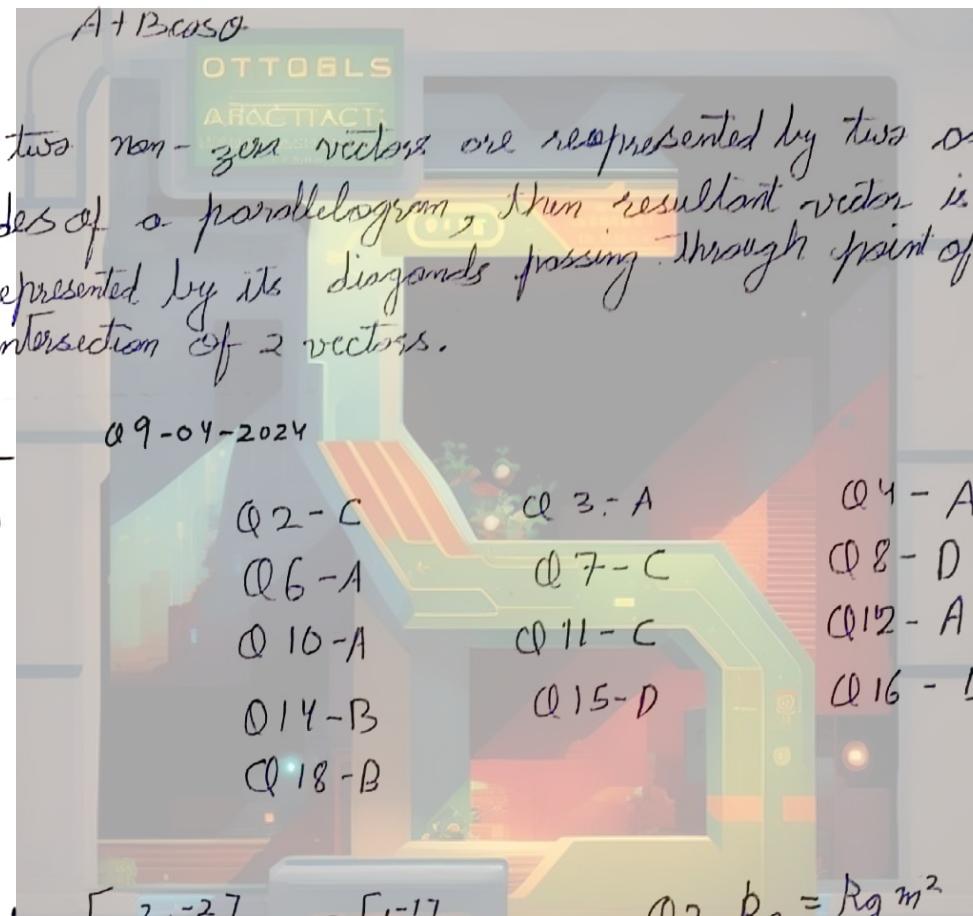
$$(\vec{P} + \vec{Q}) + \vec{R} = \vec{P} + (\vec{Q} + \vec{R}) = \vec{Q} + (\vec{P} + \vec{R})$$

II Parallelogram Law of Vector addition -



$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\tan \theta = \frac{B \sin \theta}{B \cos \theta}$$



Q-1

09-04-2024

Q4-D

Q2-C

Q3-A

Q4-A

Q5-B

Q6-A

Q7-C

Q8-D

Q9-D

Q10-A

Q11-C

Q12-A

Q13-D

Q14-B

Q15-D

Q16-B

Q17-C

Q18-B

S-Y

$$Q1-R = [ML^2T^{-2}] \quad a = [L^{-1}]$$

$$Q2-R_B = \frac{kg\ m^2}{K\ s^2}$$

$$Q3-t = a \sqrt{\frac{m}{K}}$$

$$Q4-w = K \sqrt{\frac{m}{s^2}}$$

$$Q5-R = 6$$

$$Q6-[MLT^{-1}]$$

$$Q7-i) 24.25$$

$$ii) -7.25$$

$$iii) \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}}$$

$$iv) \frac{1-\sqrt{3}}{2\sqrt{2}}$$

$$Q8-l = \frac{2\pi}{3} \times 6.4 \times 10^6 \text{ m}$$

$$Q9-a = 9.95$$

$$Q10-\frac{\pi}{100} \text{ m}$$

$$Q11.$$

Q1-3) Q2-4) Q3-~~4~~) Q4-2) Q5-2) ~~Q6-2~~
 Q10-3) Q11-4) Q16-4) Q21-C) Q22-C) Q23-B)

Q7
i) $\sin 74^\circ$

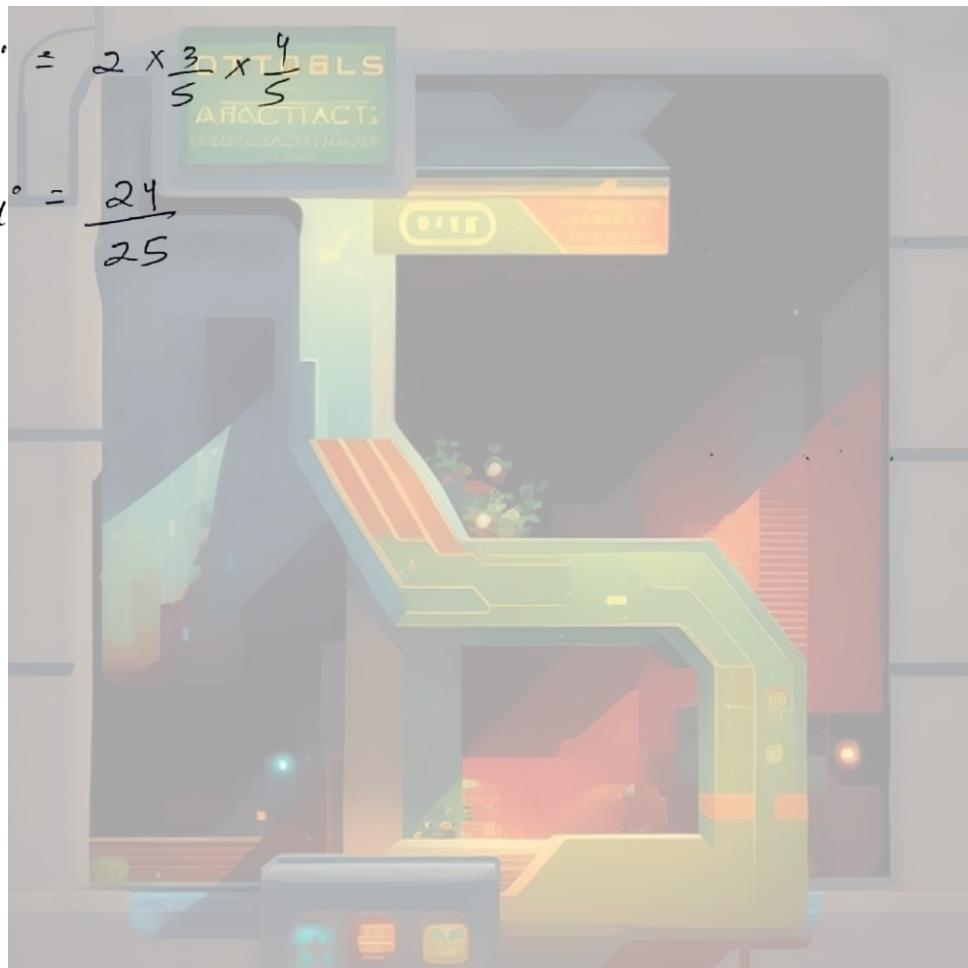
$\sin 2(37^\circ)$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 2(37^\circ) = 2 \sin 37^\circ \cos 37^\circ$$

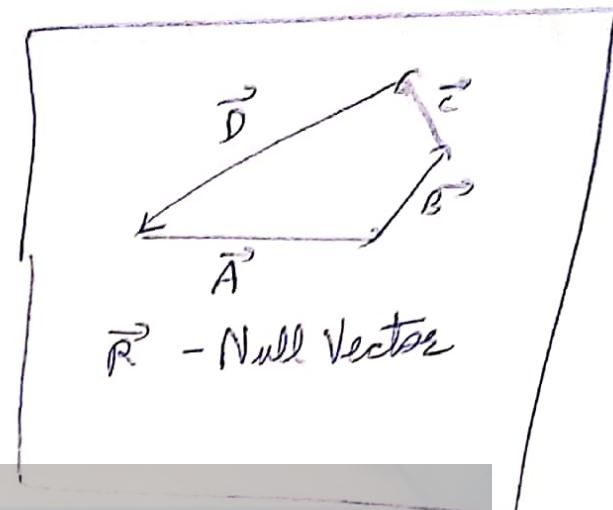
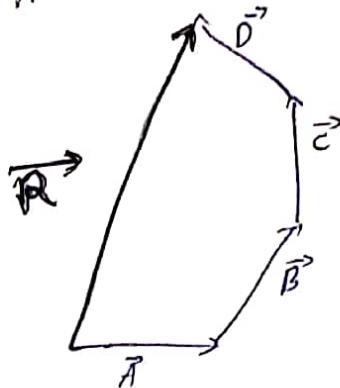
$$\sin 74^\circ = 2 \times \frac{3}{5} \times \frac{4}{5}$$

$$\sin 74^\circ = \frac{24}{25}$$

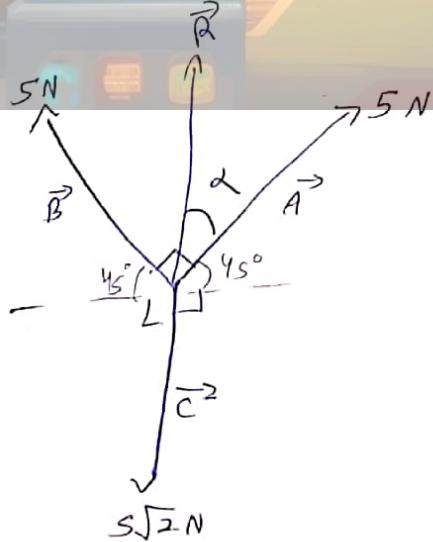
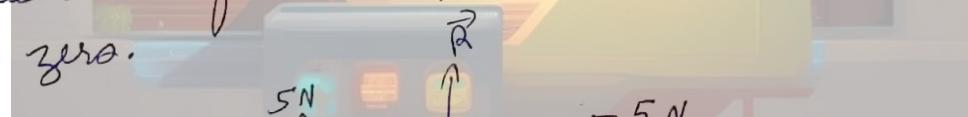


Polygon law of vector addition:

$$\vec{A} + \vec{B} + \vec{C} + \vec{D}$$



- ★ If all the vectors are arranged to form sides of a polygon in one order then their resultant is represented by closing side of polygon in opposite order.
- ★ If n vectors are represented by n sides of a polygon in cyclic manner, the resultant of these vectors is a null vector.
- ★ Resultant of two unequal vectors cannot be zero.
- ★ Resultant of 3-co-planer vectors may or may not be zero.



$$\vec{A} + \vec{B} + \vec{C} = (\vec{A} + \vec{B}) + \vec{C}$$

$$5\sqrt{2} + 5\sqrt{2} = 0$$

$$\alpha = 45^\circ$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$\vec{R} = \sqrt{25 + 25}$$

$$\vec{R} = 5\sqrt{2}$$

$$\tan \alpha = \frac{B}{A}$$

$$\tan \alpha = 1$$

$$\tan 45^\circ = 1$$

$$45^\circ = 45^\circ$$

$$\vec{R} + \vec{C} = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2}$$

$$= 0$$

Eg 2.



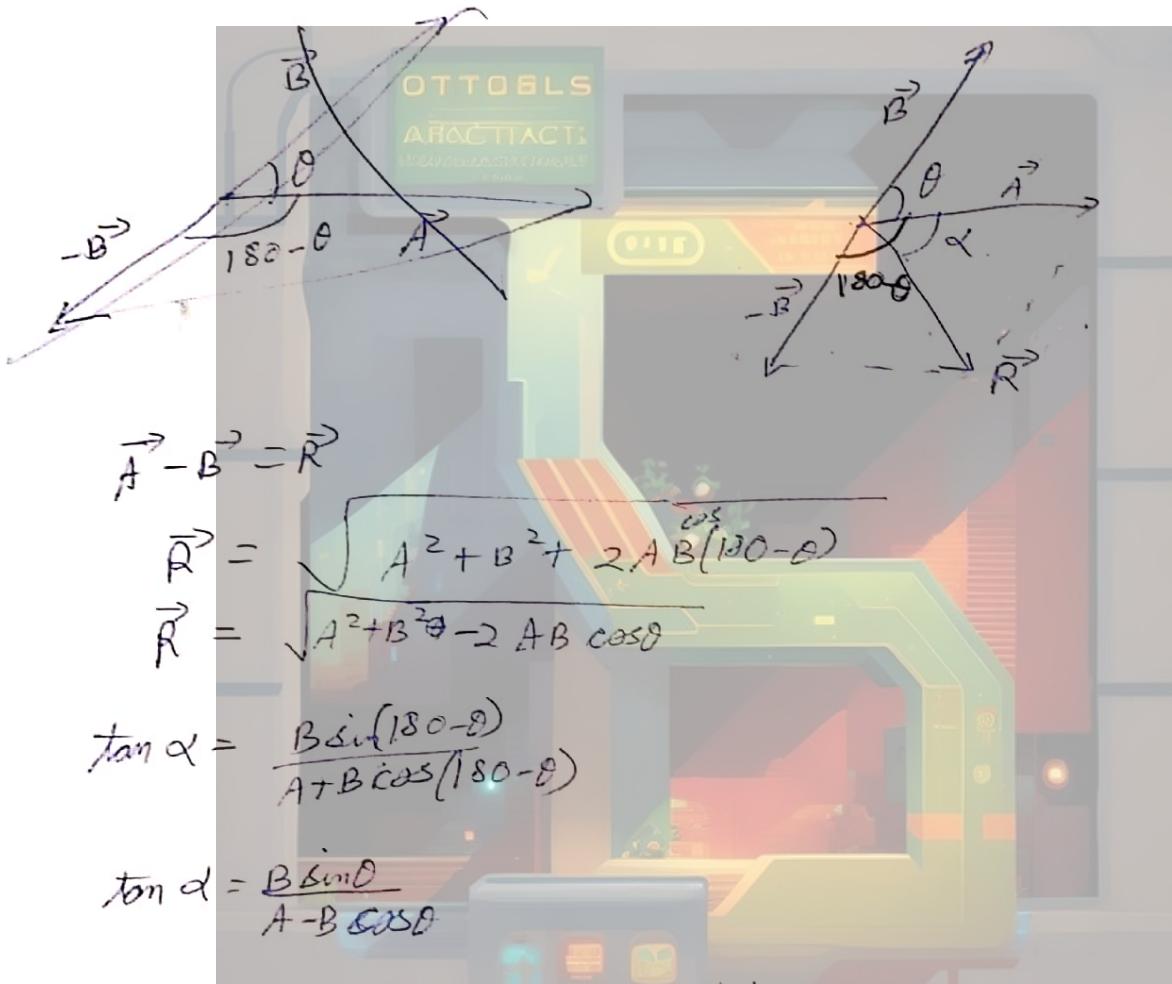
$$\vec{A} + \vec{B} + \vec{C} \neq 0$$

* The result of non-co-planar vectors cannot be zero

Subtraction of vectors

$$\vec{A} - \vec{B}$$

$$\vec{A} + (-\vec{B})$$

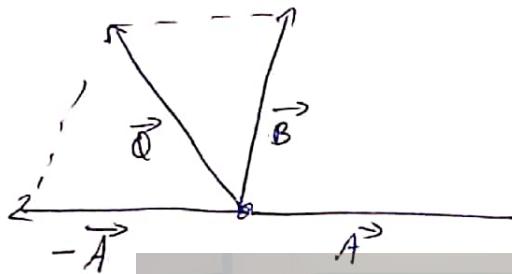


$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

* Non-commutative

$$\vec{B} - \vec{A} = \vec{0}$$

$$\vec{B} + (-\vec{A})$$



$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

Q Given that $\vec{C} = \vec{A} + \vec{B}$.

Also $|\vec{A}| = 12$, $|\vec{B}| = 5$, $|\vec{C}| = 13$ units

Find angle b/w \vec{A} & \vec{B}

$$|\vec{A}| = 13$$

$$A = 13 \text{ units}$$

$$B = \frac{13}{12} \text{ units}$$

$$C = \frac{13}{5} \text{ units}$$

$$\frac{13}{5} = \sqrt{(13)^2 + \left(\frac{13}{12}\right)^2 + 2 \times \frac{13}{12} \times 13 \times \cos \theta}$$

$$\frac{169}{25} = 169 + \frac{169}{144} + \frac{169}{6} \times \cos \theta$$

(25)

Ques.

$$C = \sqrt{A^2 + B^2 + 2AB \cos\theta}$$

$$13 = \sqrt{(12)^2 + 5^2 + (12)(5) \cos\theta}$$

$$169 = 144 + 25 + 120 \cos\theta$$

$$\cos\theta = 0$$

$$\theta = 90^\circ = \frac{\pi}{2}$$

A & B are Perpendicular / orthogonal

- Q2. The greatest least resultant of two forces acting at a point is 10 N & 6 N. If each is increased by 3 N, find the resultant of two forces when acting at a point at an angle of 90° with each other.

$$A + B = 10$$

$$A - B = 6$$

$$\underline{A = 6 + B}$$

$$6 + B + B = 10$$

$$2B = 4$$

$$\underline{B = 2}$$

$$\underline{A = 8}$$

$$A = 8 + 3 = 11$$

$$B = 2 + 3 = 5$$

$$\theta = 90^\circ$$

$$\begin{aligned} \vec{R} &= \sqrt{121 + 25} \\ \vec{R} &= \sqrt{146} \text{ N} \end{aligned}$$

$$\tan \alpha = \frac{5}{11}$$

Ans

(26)

Q. The maximum & minimum of two forces are in ratio 7:3
find force ratio.

$$A+B=7x$$

$$A-B=3x$$

$$A = 3x + B$$

$$3x + 2B = 7x$$

$$\begin{array}{r} A \\ B \end{array}$$

$$2B = 4x$$

$$\underline{\underline{B = 2x}}$$

$$\begin{array}{r} 3x + B \\ B \end{array}$$

$$\begin{array}{r} 3x + B \\ B \\ A = 5x \end{array}$$

$$\frac{A}{B} = \frac{5x}{2x} = \frac{5}{2}$$

$$\boxed{A:B = 5:2} \checkmark$$

$$c = \sqrt{2x^2 + 2x^2 \cos\theta}$$

$$c = 2x \sqrt{2\cos\theta}$$

$$x^2 \cancel{x^2} = 2x^2 + 2x^2 \cos\theta$$

$$-x^2 = 2x^2 \cos\theta$$

$$-\frac{1}{2} = \cos\theta$$

$$\boxed{\theta = 120^\circ} \checkmark$$

$$\boxed{\theta = 120^\circ} \checkmark$$

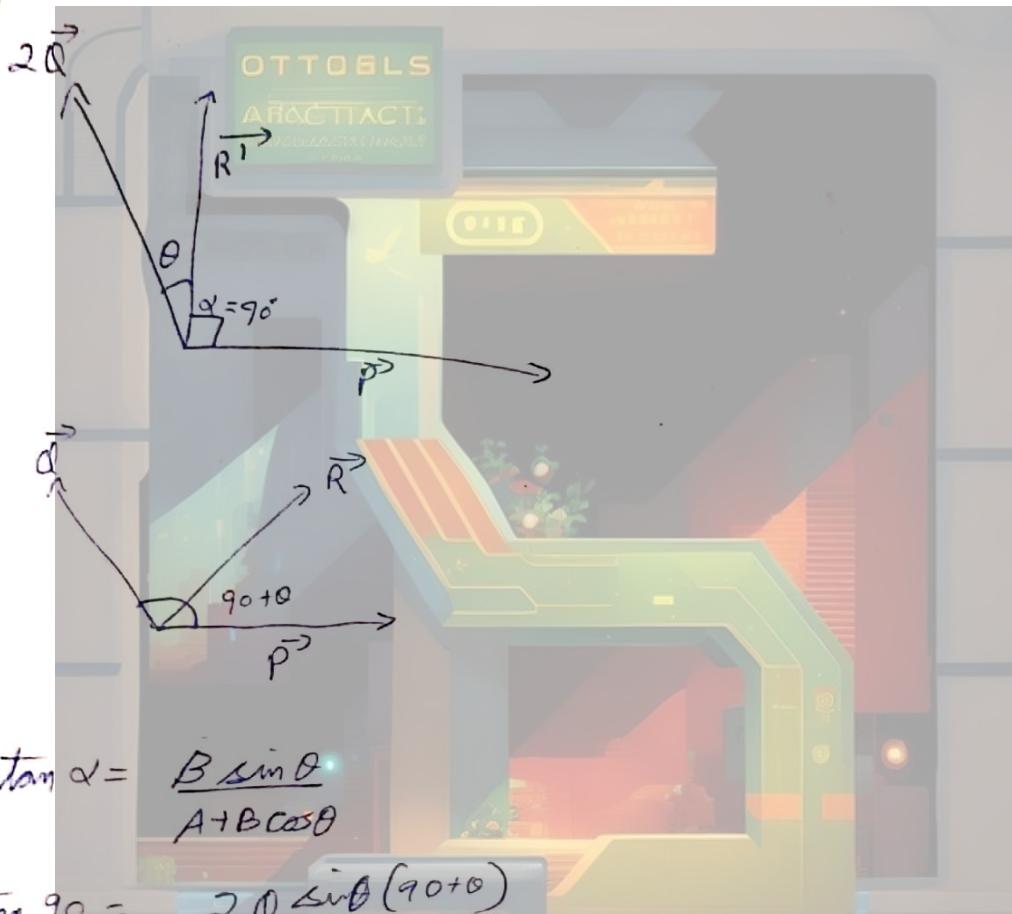
Q Two equal forces have their resultant equal to either. find angle.

(27)

$$\vec{P} + \vec{Q} = \vec{R}$$

$$\tan \theta = \frac{2Q \sin \theta}{P+Q}$$

Q The resultant of two vectors \vec{P} & \vec{Q} is \vec{R} . If the magnitude of \vec{Q} is doubled, the new vector is \perp to \vec{P} . find \vec{R}



$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan 90 = \frac{2Q \sin \theta (90+\theta)}{P + 2Q \cos (90+\theta)}$$

θ (as $\tan 90^\circ$ is undefined)

$$P + 2Q \cos(90+\theta)$$

$$\cos(90+\theta) = -\frac{P}{2Q}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos(\theta_0 + \theta)}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \frac{\theta}{2}}$$

$$\boxed{R = Q}$$

Q.6 The sum of magnitude of two forces at a point is 18 N and magnitude of resultant is 12 N. If the resultant makes an angle of 90° with the force of smaller magnitude, what are the magnitudes of two forces.

$A + B = 18 \rightarrow B = 18 - A$

$$|R| = 12 \text{ N}$$

$$\alpha = 90^\circ$$

$$\tan 90^\circ = \frac{A \sin \theta}{A + B \cos \theta} \Rightarrow 0$$

$$A + B \cos \theta = 0$$

$$\cos \theta = -\frac{A}{B}$$

$$\vec{R} = \vec{A} + \vec{B}$$

$$12 = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$144 = A^2 + B^2 + 2A^2$$

$$144 = B^2 - A^2$$

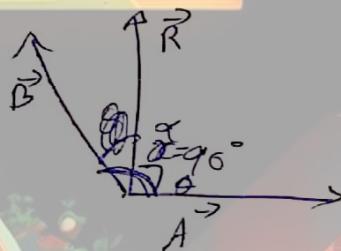
$$144 = (8-A)^2 - A^2$$

$$144 = 324 + A^2 - 36A - A^2$$

$$36A = 324 - 144$$

$$A = \frac{180}{36} = 5$$

$$\begin{array}{r} 118 \\ 144 \\ \hline 180 \\ 324 \end{array}$$



$$\boxed{A = 5 \text{ N}} \\ \boxed{B = 13 \text{ N}} \quad \checkmark$$

A vector \vec{B} which has a magnitude B , is added to \vec{A} which lies along the x -axis, the resultant lie along y -axis and has a magnitude that is twice of the magnitude of \vec{A} . The magnitude of \vec{A} is ____.

$$A + B \frac{\sin \theta}{\cos \theta}$$

$$\cos \sin \theta = \frac{-A}{B}$$

$$\textcircled{1} \quad 2x(2x)^2 = A^2 + B^2 + 2AB \cos \theta$$

$$4x^2 = x^2 + 64 + 2x4x\cancel{B} \times \frac{-A}{B}$$

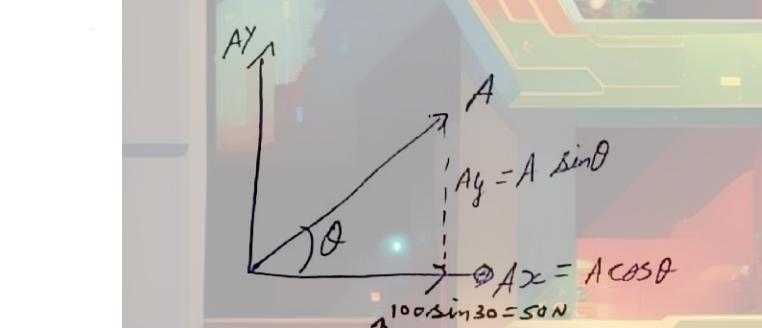
$$4x^2 = 64 - x^2$$

$$5x^2 = 64$$

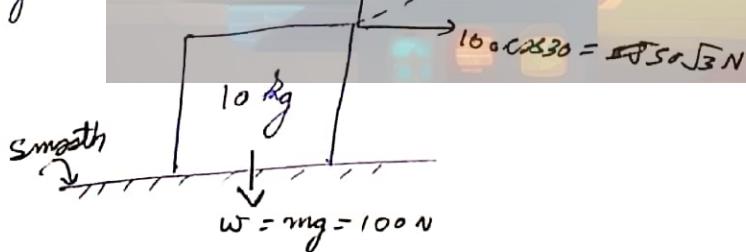
$$x^2 = \frac{64}{5}$$

$$\boxed{x = \frac{8}{\sqrt{5}}}$$

Components of Vectors



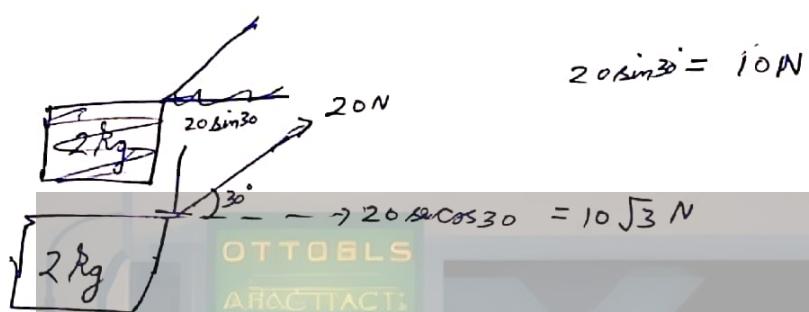
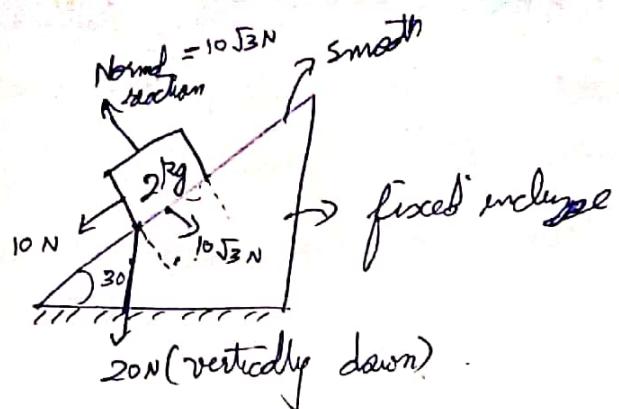
e.g.



$$F = ma$$

$$\frac{50\sqrt{3}}{10} = a = 5\sqrt{3} \text{ m/s}^2$$

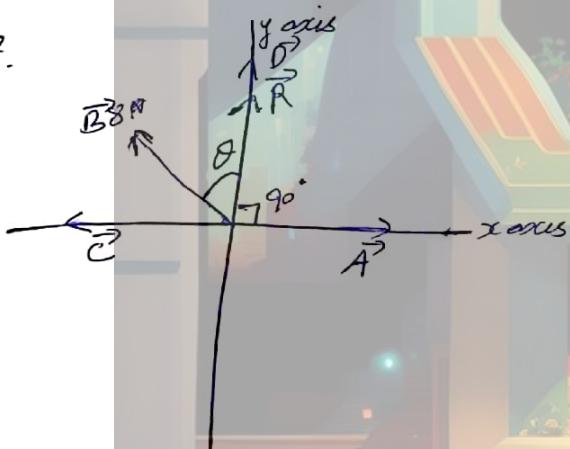
Q2.



$$F = ma$$

$$\frac{10}{2} = a = 5 \text{ m/s}^2$$

Ans 7.



$$|B| = 8$$

$$|R| = 2|A|$$

$$\vec{R} = B \sin \theta$$

$$\vec{R} = AB \cos \theta$$

$$\vec{A} + \vec{C} = 0$$

$$A = C$$

$$A = B \sin \theta$$

$$\vec{R} = R$$

$$B \cos \theta = 2B \sin \theta$$

$$AB \cos \theta = B \cos \theta$$

$$A^2 + R^2 = B^2 \sin^2 \theta + B^2 \cos^2 \theta$$

$$A^2 + R^2 = B^2$$

$$A^2 + R^2 = 64$$

$$A^2 + 4R^2 = 64$$

$$A^2 = \frac{64}{5}$$

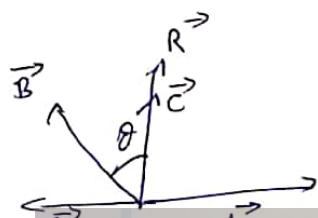
$$A = \frac{8}{\sqrt{5}}$$

(31)

RHS6.

$$|A| + |B| = 18 \quad B = 18 - A$$

$$R = 12$$



$$C = B \cos \theta$$
$$D = B \sin \theta$$

$D = A$ (as force is \perp & so they will cancel)

$$A = B \sin \theta$$

$$R = C = B \cos \theta$$

$$A^2 + R^2 = B^2 \sin^2 \theta + B^2 \cos^2 \theta$$

$$A^2 + R^2 = B^2$$

$$R^2 = B^2 - A^2$$

$$144 = (B-A)(B+A)$$

$$\frac{144}{18} = B-A$$

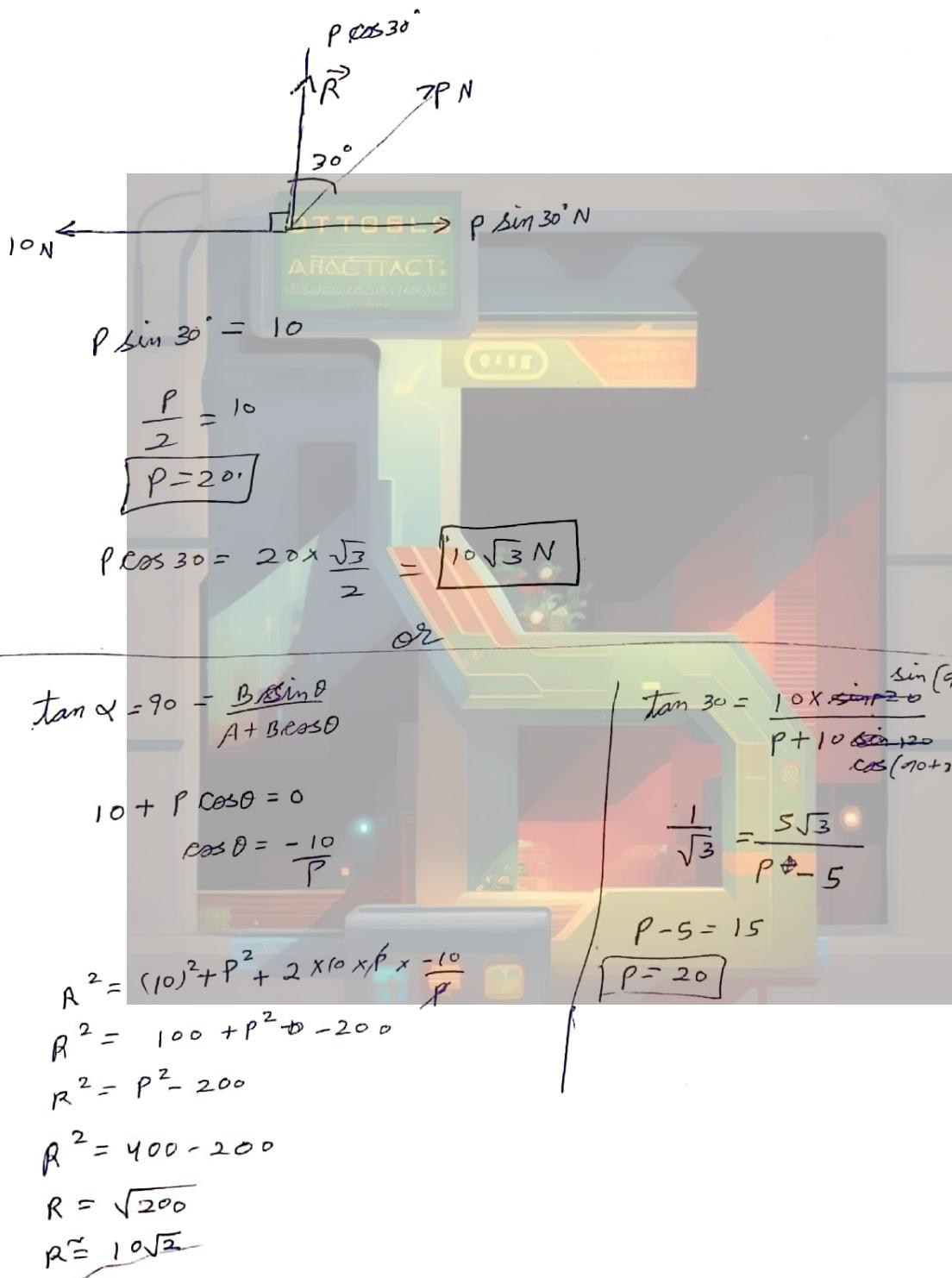
$$B-A = 8$$

$$B+A = 18$$

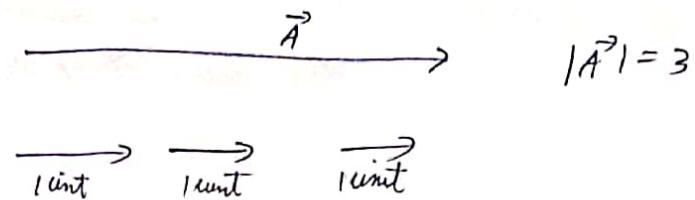
$$2B = 26$$

$$\boxed{\begin{aligned} B &= 13 \\ A &= 15 \end{aligned}}$$

Q8. Two horizontal forces of 10N & $P\text{N}$ act on a particle. The force of magnitude 7N acts west & the force of $P\text{N}$ acts on a bearing of 30° east of north as shown in figure. The resultant of these two forces acts due north. Find the magnitude of this resultant.



Unit Vector

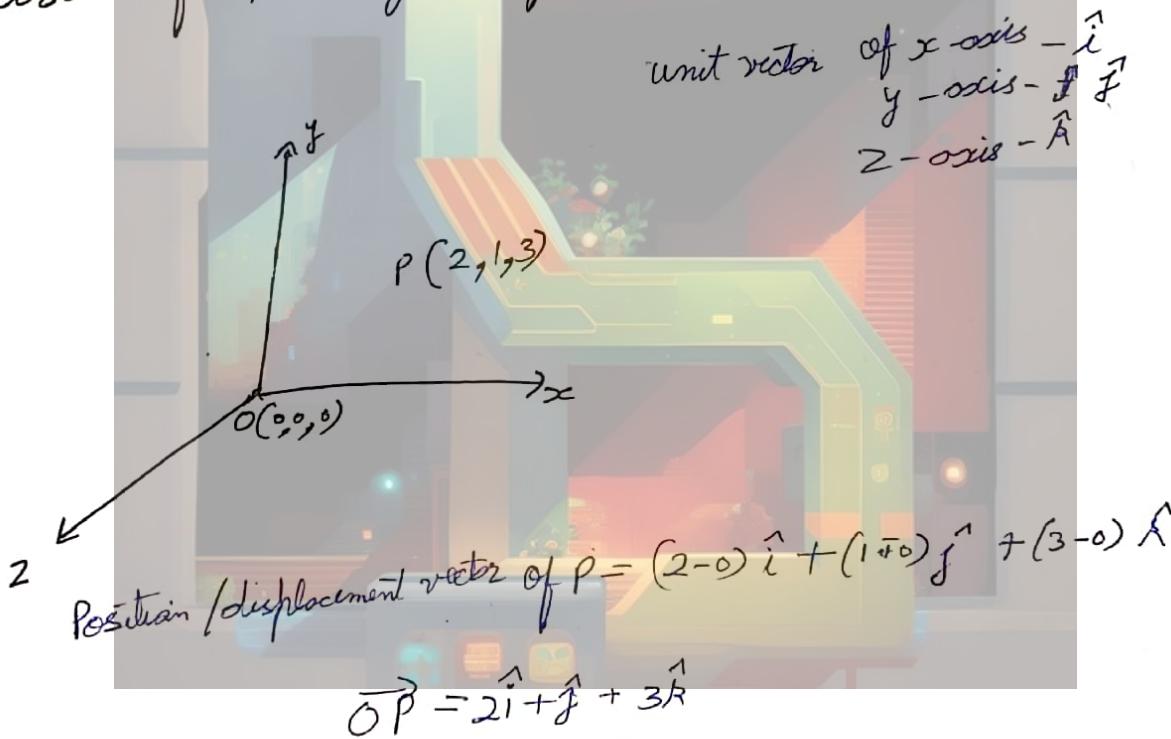


$$\text{unit vector of } \vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

dividing a vector by its magnitude, we get a vector of magnitude 1 & same direction as \vec{A} .

→ It has no dimensions and no units

→ Cartesian form/rectangular form

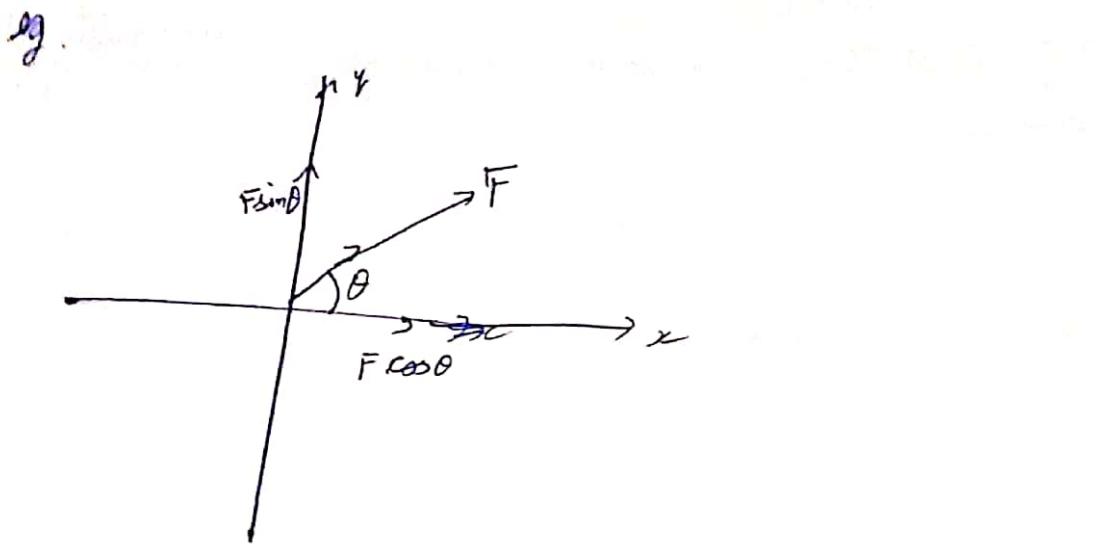


$$\begin{aligned} |\vec{OP}| &= \sqrt{2^2 + 1^2 + 3^2} \\ &= \sqrt{4 + 1 + 9} \\ &= \sqrt{14} \text{ units} \end{aligned}$$

$$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$|\vec{A}| = \sqrt{a^2 + b^2 + c^2}$$

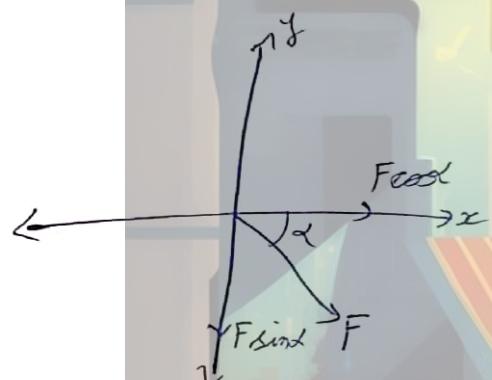
(34)



$$\boxed{\vec{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}}$$

OTTOBLS
ARCTIC
MECHANICAL
TECHNOLOGY

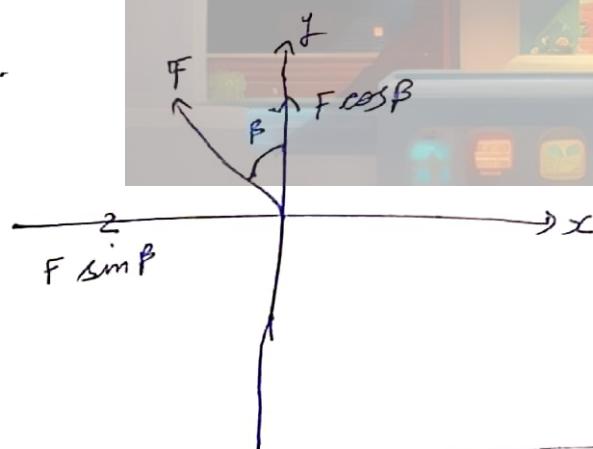
q2



$$\vec{F} = F \cos \alpha \hat{i} + F \sin \alpha \hat{j}$$

$$\boxed{= F \cos \alpha \hat{i} + F \sin \alpha \hat{j}}$$

q3



$$\boxed{\vec{F} = F \cos \beta \hat{i} - F \sin \beta \hat{j}}$$

Q9. find c if $0.3\hat{i} + 0.4\hat{j} + c\hat{k}$ is a unit vector.

~~C = 0.3~~

$$c = \sqrt{(0.3)^2 + (0.4)^2 + c^2}$$

$$| \vec{v} | = 0.09 + 0.16 + c^2$$

$$1 - 0.25 = c^2$$

$$0.75 = c^2$$

$$\frac{\sqrt{3}}{\sqrt{100}} = \frac{c}{\sqrt{3}}$$

OTTOSLS
ATRACTA
MUSICAL INSTRUMENTS

$$c = \frac{\sqrt{3}}{2}$$

Q10. find unit vector of $\underline{2\hat{i} + 2\hat{j} - 2\hat{k}}$.

$$|\vec{A}| = \sqrt{4+1+4}$$

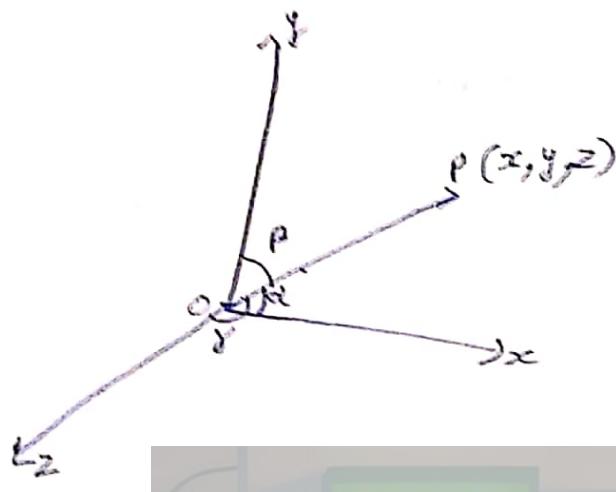
$$|\vec{A}| = \sqrt{9}$$

$$|\vec{A}| = 3$$

$$\hat{\vec{A}} = \frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}$$

$$\hat{\vec{A}} = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}$$

Direction cosines :-



angle with x -axis = α

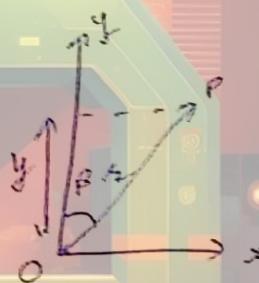
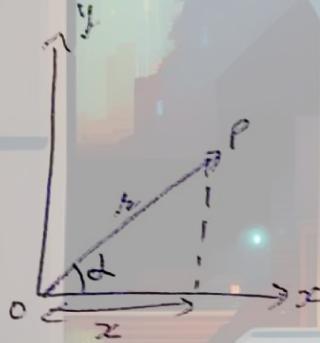
$$\text{OTTOOLS} \\ \text{SIN OF ANGLES} \\ = \beta$$

$$z = \gamma$$

$$\alpha + \beta + \gamma = 90^\circ$$

$$\overrightarrow{OP} = xi + yj + zk$$

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2} = \Sigma$$



$$\cos \alpha = \frac{x}{\Sigma}$$

$$\cos \beta = \frac{y}{\Sigma}$$

$$\cos \gamma = \frac{z}{\Sigma}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$$

$$= \frac{x^2 + y^2 + z^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$$\boxed{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1}$$

The sum of square of direction cosines of vector is always ~~one~~
equal to one.

Q what are direction cosines of $\hat{i} + \hat{j} + \hat{k}$?

$$\cos \alpha = \frac{x}{r} \quad x=1$$

$$y=1$$

$$z=1$$

$$r = \sqrt{1^2 + 1^2 + 1^2}$$

$$r = \sqrt{3}$$

$$\cos \alpha = \frac{x}{r}$$

$$\boxed{\cos \alpha = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos \beta = \frac{y}{r} = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos \gamma = \frac{z}{r} = \frac{1}{\sqrt{3}}}$$

Q2. if $\vec{P} = 4\hat{i} - 2\hat{j} + 6\hat{k}$, $\vec{Q} = \hat{i} - 2\hat{j} - 3\hat{k}$
find angle ($\vec{P} + \vec{Q}$) makes with x axis

$$\vec{P} + \vec{Q} = 5\hat{i} - 4\hat{j} + 3\hat{k}$$

$$r = \sqrt{25 + 16 + 9} \\ = \sqrt{50} \\ r = 5\sqrt{2}$$

$$\cos \alpha = \frac{5}{5\sqrt{2}} \\ \cos \alpha = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\alpha = 45^\circ$$

$$= \frac{45 \times \sqrt{2}}{180} = \frac{\pi}{4}$$

$\sin^{-1}\left(\frac{1}{2}\right)$ is angle re like
 $\sin \frac{1}{2}$ ago.

Q3. A bird moves with velocity 20 m/s in a direction making an angle of 60° with the eastern line & 60° with vertical upward. Write velocity in rectangular form.

$$\cos 60^\circ = \frac{1}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\frac{x}{r} = \frac{1}{2}$$

$$\frac{y}{r} = \frac{1}{2}$$

$$2x = r$$

$$2y = r$$

$$2x = 2\sqrt{2}$$

$$x = \sqrt{2}$$

$$\begin{matrix} x = 20 \\ y = 10 \\ z = 10 \end{matrix}$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{2x^2}$$

$$r = 2\sqrt{2}$$

$$\vec{v} = 10\hat{i} + 10\hat{j}$$

$$|\vec{V}| = 20 \text{ m/s}$$

$$\alpha = 60^\circ$$

$$\beta = 60^\circ$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\frac{1}{4} + \cos^2 \beta + \frac{1}{4} = 1$$

$$\cos^2 \beta = \frac{1}{2}$$

$$\cos \beta = \frac{1}{\sqrt{2}}$$

$$x \cos \alpha = \frac{1}{2} = \frac{x}{20}$$

$$x = 10$$

$$z = 10$$

$$y = 10\sqrt{2}$$

$$\boxed{\vec{V} = 10\hat{i} + 10\sqrt{2}\hat{j} + 10\hat{k}} \text{ m/s}$$

04. find the vector which must be added to the sum of 2 vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ to get resultant of unit vector along z axis.

$$\vec{V} = 2\hat{i} + 0\hat{j} + \hat{k}$$

$$\vec{V} - \text{unit vector along } z = -\hat{k}$$

$$-\hat{k} = 2\hat{i} + \hat{k} - \hat{k}$$

$$\boxed{\vec{R} = -2\hat{i}}$$

(40)

Q5. A particle is moving towards east with 5 m/s in ~~at~~ 10 seconds, its velocity becomes 5 m/s towards north. find average acceleration.

$$\vec{u} = 5\hat{i} \text{ m/s}$$

$$\vec{v} = 5\hat{j} \text{ m/s}$$

$$t = 10\text{ s}$$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t}$$

$$\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$$

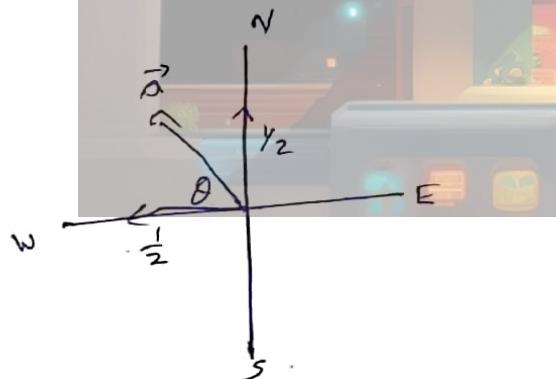
$$\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$$

$$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$r = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

$$|\vec{a}| = \frac{1}{\sqrt{2}} \text{ m/s}^2 \text{ (at an angle } 45^\circ \text{ west of north)}$$

$$\vec{a} = -\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$$



$$\text{or } \cos \theta = \frac{1}{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

Q6. A car is moving towards North with speed of ~~36~~ m/s. In 5s it turns towards left with its speed unchanged. find acc of car.

$$\vec{u} = \cancel{\frac{36}{5}} \hat{j} \text{ m/s}$$

$$\vec{v} = -36 \hat{i} \text{ m/s}$$

$$t = 5 \text{ s}$$

$$\vec{a} = \frac{-36\hat{i} - 36\hat{j}}{5}$$

$$\vec{u} = \frac{36^2 \times \frac{5}{18}}{5} = 10 \hat{j}$$

$$\vec{v} = -10 \hat{i}$$

$$t = 5 \text{ s}$$

$$\vec{a} = - \cancel{(36\hat{i} + 36\hat{j})}$$

$$\vec{a} = -12 \cancel{(\frac{36\hat{i} + 36\hat{j}}{5})}$$

$$|\vec{a}| = \sqrt{36^2 + 36^2}$$

$$(|\vec{a}|) = \sqrt{9+9}$$

$$|\vec{a}| = \frac{\sqrt{18}}{3\sqrt{2} \times 5}$$

$$|\vec{a}| =$$

$$\vec{a} = -\frac{10\hat{i}}{2} - \frac{10\hat{j}}{2}$$

$$\vec{a} = -\left(\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$\vec{a} = -2\hat{i} - 2\hat{j}$$

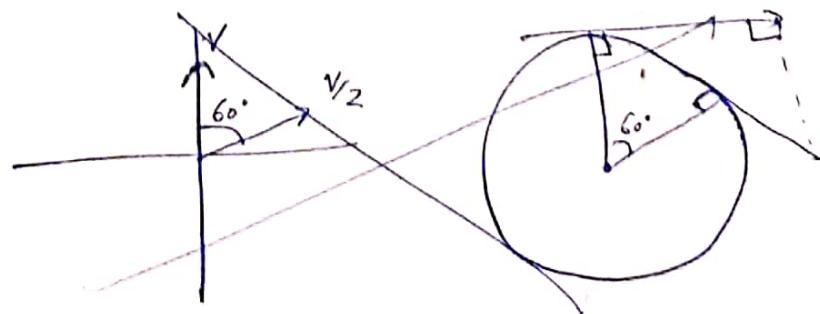
$$|\vec{a}| = \sqrt{4+4}$$

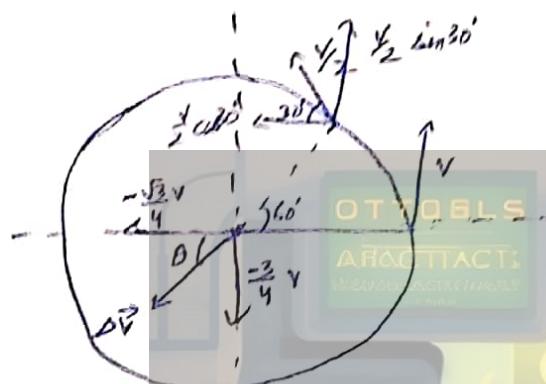
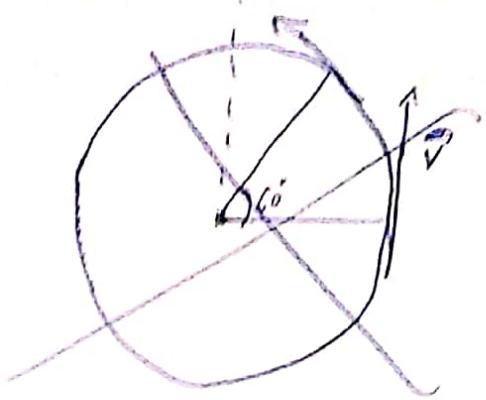
$$= \sqrt{8}$$

$$|\vec{a}| = 2\sqrt{2} \text{ m/s}^2$$

(45° towards south of west)

Q7. A particle is moving on a circular path with speed v at a particular instant. After some time, when it has described an angle 60° its speed becomes $\frac{v}{2}$ what is change in velocity.





$$\vec{v} - \vec{u}$$

$$\vec{u} = v\hat{j}$$

$$\vec{v} = \frac{v}{2} \sin 30^\circ (\hat{j}) + \frac{v}{2} \cos 30^\circ (-\hat{x})$$

$$\vec{v} = \frac{v}{4} \hat{j} - \frac{\sqrt{3}}{4} v \hat{x}$$

$$\Delta \vec{v} = \vec{v} - \vec{u}$$

$$= -\frac{\sqrt{3}}{4} v \hat{x} + \frac{v}{4} \hat{j} - v \hat{j}$$

$$\boxed{\Delta \vec{v} = -\frac{\sqrt{3}}{4} v \hat{x} - \frac{3}{4} v \hat{j}}$$

$$|\Delta \vec{v}| = \sqrt{\left(\frac{\sqrt{3}}{4} v\right)^2 + \left(-\frac{3}{4} v\right)^2}$$

$$\boxed{|\Delta \vec{v}| = \frac{\sqrt{3}v}{2}}$$

$$\tan \theta = \frac{\frac{3}{4} v}{\frac{\sqrt{3}}{4} v} = \sqrt{3}$$

$$\boxed{\theta = 60^\circ}$$

43

Q 8. A particle whose speed is 50 m/s moves along a line from A(2,1) to B(9,25) find its velocity in rectangular form.

$$u = 2\hat{i} + \hat{j}$$

$$v = 9\hat{i} + 25\hat{j}$$

$$v - u = 7\hat{i} + 24\hat{j}$$

$$\vec{v} = 7\hat{i} + 24\hat{j}$$

Q 8

$$\vec{v} = \frac{7\hat{i} + 24\hat{j}}{\sqrt{(7)^2 + (24)^2}}$$

$$\vec{v} = \frac{7\hat{i} + 24\hat{j}}{25}$$

$\vec{v} \times 50$ = velocity

$$\boxed{\text{Velocity} = 14\hat{i} + 48\hat{j} \text{ m/s}}$$

Product of two vectors

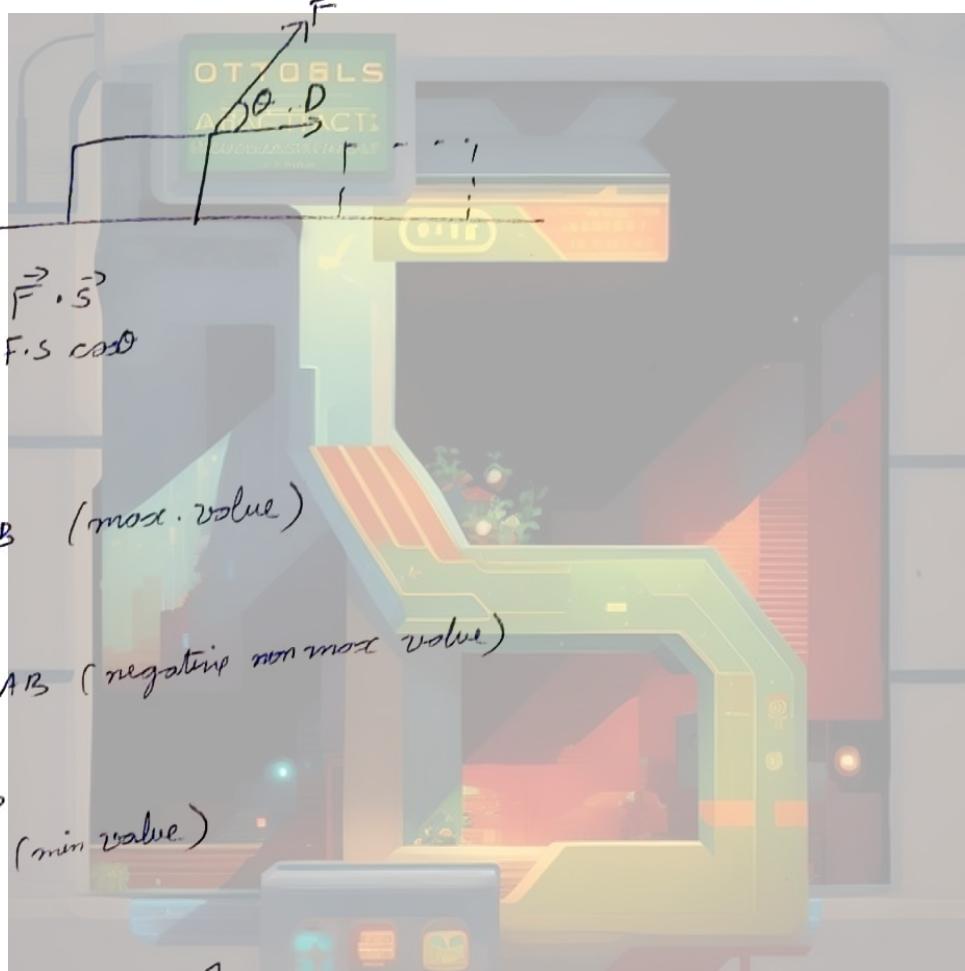
I) Scalar Product/dot product:-

$$\boxed{\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos\theta}$$

$$= AB \cos\theta$$


→ The scalar product is a way to multiply two vectors to get a scalar result

e.g.



$\theta = 0^\circ$
 $\cos 0^\circ = 1$
 $\vec{A} \cdot \vec{B} = AB$ (max. value)

$\theta = 180^\circ$
 $\cos 180^\circ = -1$
 $\vec{A} \cdot \vec{B} = -AB$ (negative non max. value)

$\theta = 90^\circ$
 $\cos 90^\circ = 0$
 $\vec{A} \cdot \vec{B} = 0$ (min. value)

$$\begin{aligned}\hat{i} \cdot \hat{i} &= 1 \\ \hat{j} \cdot \hat{j} &= 1 \\ \hat{k} \cdot \hat{k} &= 1\end{aligned}$$

$$\left| \begin{array}{l} \hat{i} \cdot \hat{j} = 0 \\ \hat{j} \cdot \hat{k} = 0 \\ \hat{k} \cdot \hat{i} = 0 \end{array} \right.$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})(B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

* To find angle between two vectors

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\boxed{\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}}$$

* $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (Commutative)

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

* distributive

$$\vec{A}(\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$* \vec{A} \cdot \vec{A} = (A)(A) \cos 0^\circ = A^2$$

$$|\vec{P} + \vec{Q}|^2 = (\vec{P} + \vec{Q})(\vec{P} + \vec{Q}) \\ = \vec{P} \cdot \vec{P} + \vec{P} \cdot \vec{Q} + \vec{Q} \cdot \vec{P} + \vec{Q} \cdot \vec{Q}$$

$$= P^2 + Q^2 + 2 PQ \cos \theta$$

$$|\vec{P} + \vec{Q}| = \sqrt{P^2 + Q^2 + 2 PQ \cos \theta}$$

$$Q \quad \vec{A} = 2\vec{i} + 3\vec{j} + \vec{k}$$

$$\vec{B} = 5\vec{i} + 2\vec{j} + 7\vec{k}$$

$$\vec{A} \cdot \vec{B} = 2 \times 5 + (-3) \times 2 + (-7) \times 1$$

$$= 10 - 6 - 7$$

$$= 10 - 13$$

$$\boxed{= -3}$$

$$Q2. \quad \vec{A} = \vec{i} + \vec{j} - \vec{k}$$

$$\vec{B} = \vec{i} - \vec{j} + \vec{k}$$

$$\vec{A} \cdot \vec{B} = ?$$

$$\vec{A} \cdot \vec{B} = 1 - 1 - 1$$

$$= -2 + 1$$

$$\boxed{= -1}$$

$$Q3. \quad \vec{A} = 2\vec{i} - 3\vec{j} + \vec{k}$$

$$\vec{B} = 3\vec{i} + 2\vec{j}$$

$$\vec{A} \cdot \vec{B} = 6 - 6 + 0$$

$$\boxed{= 0}$$

Q Find value of n , such that $(2\vec{i} - 3\vec{j} + \vec{k})$ is perpendicular to $\vec{i} + 2\vec{j} + n\vec{k}$.

$$\cos \theta = \frac{2 - 6 + n}{\sqrt{4+9+1} \times \sqrt{1+4+n^2}}$$

$$\cos \theta = \frac{n-4}{\sqrt{14} \times \sqrt{5+n^2}}$$

$$\cos \theta = \frac{n-4}{\sqrt{70+14n^2}}$$

$$\frac{n-4}{\sqrt{70+14n^2}} = 0$$
$$\boxed{n=4}$$

(47)

Q find angle between $(\vec{r} + \vec{j})$ & $(\vec{r} - \vec{j})$

$$\cos \theta = \frac{1-1}{AB}$$

$$\boxed{\theta = 90^\circ} \text{ as } 1-1=0$$

Q find angle b/w $(\vec{r} + \vec{j})$ & $(\vec{j} + \vec{k})$

$$\cos \theta = \frac{0+1+0}{AB}$$

$$\cos \theta = \frac{1}{\sqrt{1+1} \times \sqrt{1+1}}$$

$$\cos \theta = \frac{1}{\sqrt{2} \times \sqrt{2}}$$

$$\cos \theta = \frac{1}{2}$$

$$\boxed{\theta = 60^\circ}$$

Q A force $\vec{F} = (10\vec{i} - 3\vec{j} + 6\vec{k}) N$ acts on a body of mass 100g & displaces it from $(6\vec{i} + 5\vec{j} + 3\vec{k}) m$ to $(10\vec{i} - 2\vec{j} + 7\vec{k}) m$ find work done.

$$\text{displacement} = 4\vec{i} - 7\vec{j} + 10\vec{k} m$$

$$\text{work} = \vec{F} \cdot \vec{s}$$

$$= (10\vec{i} - 3\vec{j} + 6\vec{k}) \cdot (4\vec{i} - 7\vec{j} + 10\vec{k})$$

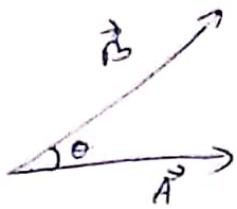
$$= 40 + 21 + 240$$

$$\boxed{= 285 J}$$

$$\boxed{= 121 J}$$

④ 48

II Cross Product / Vector Product



$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta$$

↑ direction of $\vec{A} \times \vec{B}$
(Right hand thumb rule)

- ① out of plane
- ② in plane

✗ not commutative

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB \sin \theta$$

✗ distributive

$$\vec{A}(\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\hat{i} \times \hat{i} = (1)(1) \sin 0^\circ = 0$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = (1)(1) \sin 90^\circ \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{j} = \hat{i}$$

$$\hat{k} \times \hat{i} = -\hat{j}$$

$$\hat{i} \times \hat{j} = \hat{j}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

✗ If cross product is 0, vectors are either equal or parallel
✗ If dot product is 0, vectors are \perp to each other

clockwise - (+ve)
& negative - (-ve)

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$\vec{A} \times \vec{B} =$

\hat{i}	\hat{j}	\hat{k}
A_x	A_y	A_z
B_x	B_y	B_z

3 by 3 determinant

$$\vec{A} \times \vec{B} = \hat{i} (A_y - B_z) - \hat{j} (A_x - B_z) + \hat{k} (A_x - B_y)$$

$$\boxed{\vec{A} \times \vec{B} = \hat{i} (A_y B_z - B_y A_z) - \hat{j} (A_x B_z - B_x A_z) + \hat{k} (A_x B_y - A_y B_x)}$$

Q $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\vec{A} \times \vec{B} =$$

\hat{i}	\hat{j}	\hat{k}
3	1	2
2	-2	4

$$\vec{A} \times \vec{B} = \hat{i} (4 - (-4)) - \hat{j} (12 - 4) + \hat{k} (-6 - 2)$$

$$\boxed{= 8\hat{i} - 8\hat{j} - 8\hat{k}}$$

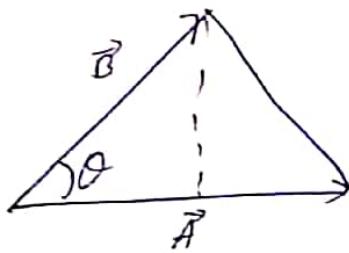
$$|\vec{A} \times \vec{B}| = \sqrt{8^2 + (-8)^2 + (-8)^2}$$

$$\boxed{= 8\sqrt{3}}$$

(30)

Uses of vector product

1) To calculate area of triangle:-



$$\text{area of triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} (A)(B \sin \theta)$$

$$= \frac{1}{2} |\vec{A} \times \vec{B}|$$

2) To calculate area of || gm:-



Q1. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$

$$\vec{\omega} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & -3 \\ -3 & 4 & 1 \end{pmatrix} \quad \text{find } |\vec{v}|$$

$$\vec{r} = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 2 & 1 \\ 0 & 4 & -3 \end{pmatrix}$$

$$\vec{v} = \vec{\omega} \times \vec{r} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & -3 \\ -3 & 4 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ -2 & 2 & 1 \\ 0 & 4 & -3 \end{pmatrix}$$

$$\vec{v} = 1(6-8) - 2(-3-0) + 2(4+2) = -2\vec{i} + 3\vec{j} + 4\vec{k}$$

$$|\vec{v}| = \sqrt{4+9+16}$$

$$= \sqrt{29}$$

~~$$|\vec{v}| = \sqrt{4+9+36}$$~~

~~$$|\vec{v}| = \sqrt{65}$$~~

~~$$= \sqrt{5 \times 13}$$~~

Q2. Calculate area of triangle determined by $\vec{A} = 3\hat{i} + \hat{j}$
 $\vec{B} = -3\hat{i} + 7\hat{j}$

$$1/2 \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ -3 & 7 & 0 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(0) - \hat{j}(0) + \hat{k}(21 + 12)$$

$$|\vec{A} \times \vec{B}| = \sqrt{(33)^2} = 33 \hat{k}$$

2
 Area = $\frac{33}{2}$
 $= 16.5 \text{ m}^2$

Q3. $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$
 $\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

$$\sqrt{p^2 + q^2 + r^2} = \sqrt{25 + 49 + 9} = \sqrt{83}$$

$$p^2 + q^2 + r^2 = 83$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 5 & 7 & 3 \end{vmatrix}$$

$$= \hat{i}(3p - 7q) - \hat{j}(6 - 5q) + \hat{k}(14 - 5p)$$

$$6 - 5Q = 0$$

$$Q = \frac{6}{5}$$

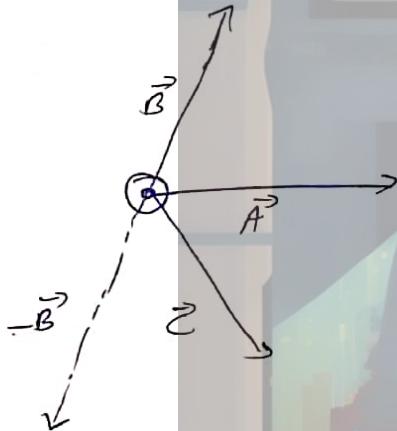
$$14\theta - 5h = 0$$

$$\begin{aligned} h &= 14 \\ h &= \frac{14}{5} \end{aligned}$$

Q3. What is the angle between $\vec{A} - \vec{B}$ and $\vec{A} \times \vec{B}$

$$\cos \theta = \frac{(\vec{A} \times \vec{B}) \cdot (\vec{A} - \vec{B})}{|\vec{A} - \vec{B}| |\vec{A} \times \vec{B}|}$$

$$= AB \sin \theta \text{ m.}$$



$$\therefore \theta = 90^\circ$$

$$Q4. \quad |\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

$$\sqrt{A^2 + B^2 + 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

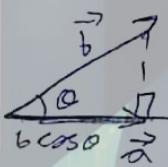
$$4AB \cos \theta = 0$$

$$\theta = 90^\circ$$

Projection of vector over another:-

$$I) \text{Scalar projection of } \vec{b} \text{ on } \vec{a} = b \cos \frac{\alpha}{a} = \frac{b \cos \alpha}{a} = \frac{\vec{a} \cdot \vec{b}}{a}$$

जिसपर projection
होती है।



$$II) \text{Vector projection} = \frac{\vec{a} \cdot \vec{b}}{a} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{a} \frac{\vec{a} \cdot \vec{a}}{a} = \frac{\vec{a}}{a^2} (\vec{a} \cdot \vec{b})$$

Q find scalar vector projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$$\frac{\vec{a} \cdot \vec{b}}{b} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$$

$$\text{vector } \frac{\vec{b}}{b} (\vec{a} \cdot \vec{b}) = \frac{5}{3}\hat{i} + \frac{10}{3}\hat{j} + \frac{5}{3}\hat{k}$$

① find scalar product of $\vec{A} = \vec{i} + \vec{j} + \vec{k}$ on $\vec{B} = \vec{i} + \vec{j}$
projection and vector projection

$$\frac{\vec{A} \cdot \vec{B}}{6} = \frac{1+1+0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$$

V.P. $\frac{\vec{B}}{6} \cdot (\vec{a} \cdot \vec{B}) = \frac{\vec{i} + \vec{j}}{2} \times \vec{z} = \boxed{\sqrt{2}}$

② $\vec{A} + \vec{B} = 2\vec{i}$

$$\vec{A} - \vec{B} = 4\vec{j}$$

$$- \cancel{\vec{A}^2 + \vec{B}^2 + 2AB \cos \theta} = -\vec{A}^2 + \vec{B}^2 + \cancel{2AB \cos \theta} = 2\vec{i}$$

$$- \cancel{\vec{A}^2 + \vec{B}^2 + 2AB \cos \theta} = 4\vec{j}$$

$$- 4AB \cos \theta = 2\vec{j}$$

$$\cos \theta = \frac{-1}{2AB}$$

$$\cos(90^\circ)$$

$$\vec{A} + \vec{B} = 2\vec{i}$$

$$\vec{A} - \vec{B} = 4\vec{j}$$

$$\cancel{2\vec{i}}$$

$$\cos \theta = 90^\circ$$

$$\text{Q1. } \vec{A} + \vec{B} = 2\vec{x}$$

$$\vec{A} - \vec{B} = 4\vec{y}$$

$$\theta = ?$$

- A) 127°
 B) 143°
 C) 53°
 D) 37°

$$\vec{A} + \vec{B} = 2\vec{x}$$

$$\vec{A} - \vec{B} = 4\vec{y}$$

$$2\vec{A} = 2\vec{x} + 4\vec{y}$$

$$\vec{A} = \vec{x} + 2\vec{y}$$

$$\vec{B} = \vec{x} - 2\vec{y}$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}\vec{B}|}$$

$$= \frac{1 - 4}{\sqrt{5} \times \sqrt{5}}$$

$$\cos\theta = \frac{-3}{5}$$

$$\cos(90 + \theta) = -\sin\theta$$

$$-\sin\theta = \frac{-3}{5}$$

$$\sin 37^\circ = \frac{3}{5}$$

$$90 + 37 = 127^\circ$$

A) 127°

Q2. Two forces P & Q are in ratio 1:2 if their resultant is at an angle $\tan \alpha = \frac{\sqrt{3}}{2}$ to vector P. Then angle between P & Q is.

- A) $\tan^{-1}(\frac{1}{2})$ B) 45° C) 30° D) 60°

$$\tan \alpha = \frac{\sqrt{3}}{2} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\frac{P}{Q} = \frac{1}{2}$$

$$2P = Q \\ Q = 2P$$

$$\sqrt{3}P + \sqrt{3}Q \cos \theta = 2Q \sin \theta$$

$$\sqrt{3}P = 2Q \sin \theta - \sqrt{3}Q \cos \theta$$

$$\sqrt{3}(P + Q \cos \theta) = 2Q \sin \theta$$

$$\sqrt{3}(P + 2P \cos \theta) = 4P \sin \theta$$

$$\sqrt{3}P(1 + 2 \cos \theta) = 4P \sin \theta$$

$$\frac{1 + 2 \cos \theta}{\sin \theta} = \frac{4P}{\sqrt{3}P}$$

$$\frac{1}{\sin \theta} + \frac{2 \cos \theta}{\sin \theta} = \frac{4}{\sqrt{3}}$$

$$\frac{1}{\sin \theta} + 2 \cot \theta = \frac{4}{\sqrt{3}}$$

$$\boxed{\sqrt{3} + 2\sqrt{3} \cos \theta = 4 \sin \theta}$$

$$\sqrt{3} + 2\sqrt{3} \times \frac{1}{2} = 4 \times \frac{1}{2}$$

$$\sqrt{3} + 3 = 2x$$

$$\sqrt{3} + 2\sqrt{3} \times \frac{1}{2} = x^2 \times \frac{\sqrt{3}}{2}$$

$$1+1=2$$

$$LHS = RHS$$

$$\boxed{\int \theta = 60^\circ}$$

$$\boxed{D) 60}$$

$$Q3. \vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$$

find the angle between $(\vec{A} - \vec{B})$ & \vec{C}

$$A) \theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$B) \cos \theta = \frac{\sqrt{3}}{2}$$

$$C) \theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

D) none.

$$\vec{A} - \vec{B} = -1\hat{i} + 5\hat{j} + \hat{k}$$

$$\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Ans} \frac{-1 + 5 + 2}{\cancel{3\sqrt{3}} \times \cancel{1\sqrt{6}}} = \frac{6}{3\sqrt{18}}$$

$$= \frac{6}{3\sqrt{18}} = \frac{2}{\sqrt{18}}$$

$$\cos \theta = \frac{2}{3\sqrt{2}}$$

$$= \frac{\sqrt{2}}{3}$$

C

On Two forces $1\hat{i} + \hat{j} + \hat{k} N$ & $1 + 2\hat{j} + 3\hat{k} N$ acts on a particle and displace it from $(2, 3, 4)$ to point $(5, 4, 3)$.

$$\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{s} = 3\hat{i} + \hat{j} - \hat{k}$$

A) 5 J

B) 4 J

C) 3 J

D) None

$$W = 6 + 3 - 4$$

$$W = 5 J$$

A

Q5. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$

$$\vec{D} = -4\hat{i} + 2\hat{j} + 2\hat{k}$$

$$W_{23} = 6.5$$

A) 12

B) 0

C) 6

D) 4

$$W_{23} = -12 + 2c + 6$$

$$6 = -6 + 2c$$

$$12 = 2c$$

$$\boxed{c = 6}$$

$$\boxed{C}$$

Q6. A force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

A) $6\hat{i} - 8\hat{j} + 10\hat{k}$

B) 100

C) $10\sqrt{2}$

D) 10

$$6\hat{i} - 8\hat{j} + 10\hat{k} = m \times 1$$

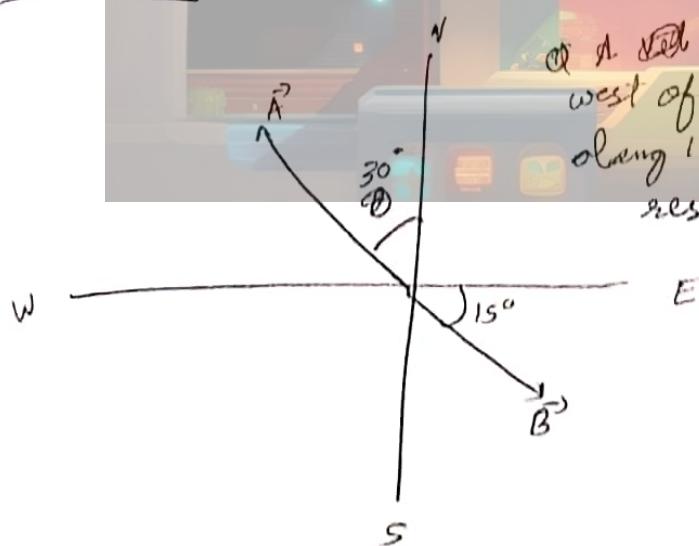
$$m = \sqrt{36 + 64 + 100}$$

$$= \sqrt{200}$$

$$= 10\sqrt{2} \text{ kg}$$

$$\boxed{C}$$

Q7.



a. If vector \vec{A} is directed 30° west of north & another vector \vec{B} along 15° south of east. Then resultant cannot be in

$\boxed{\text{South}}$

(57)

Q8. The sum of $\vec{F}_1 = 100\text{N}$, $\vec{F}_2 = 80\text{N}$ & $\vec{F}_3 = 60\text{N}$ is 0, angle between F_1 & F_2 is

A) 53
B) 143

C) 37
D) 127

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\sqrt{10000 + 6400 + 3600}$$

$$\sqrt{10000 + 6400 + 16000 \cos\theta} = 0$$

$$= \sqrt{32400 \cos\theta}$$

$$= \sqrt{16400 + 16000 \cos\theta}$$

$$= 100 \sqrt{164 + 160 \cos\theta}$$

$$= 20 \sqrt{41 + 40 \cos\theta} = -60$$

$$= \sqrt{41 + 40 \cos\theta} = -3$$

$$900 \times (41 + 40 \cos\theta) + 3600 =$$

$$41 + 40 \cos\theta = 9$$

$$\cos\theta = \frac{9 - 41}{40}$$

$$\cos\theta = \frac{-32}{40} = -\frac{4}{5}$$

$$\cos\theta = -\frac{4}{5}$$

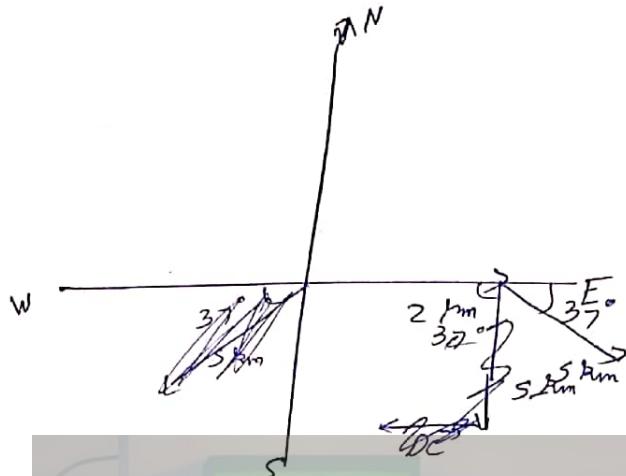
$$\cos(90 + 53^\circ) = -\frac{4}{5}$$

$$\theta = 90 + 53^\circ = 143^\circ$$

B

(60)

Q9. A sail boat sails 2 km east, 5 km 37° south of East and an unknown displacement. The final displacement is 6 km east, find unknown.



$$\vec{A} = 2\hat{i}$$

$$\begin{aligned}\vec{B} &= +5 \times \frac{4}{5} \hat{i} - \frac{3}{5} \times 5 \hat{j} \\ &= -4\hat{i} - 5\hat{j}\end{aligned}$$

$$\vec{C} = ?$$

$$\vec{A} + \vec{B} + \vec{C} = 6\hat{i}$$

$$2\hat{i} + 4\hat{i} + \cancel{-4\hat{i}} - 5\hat{j} = 6\hat{i}$$

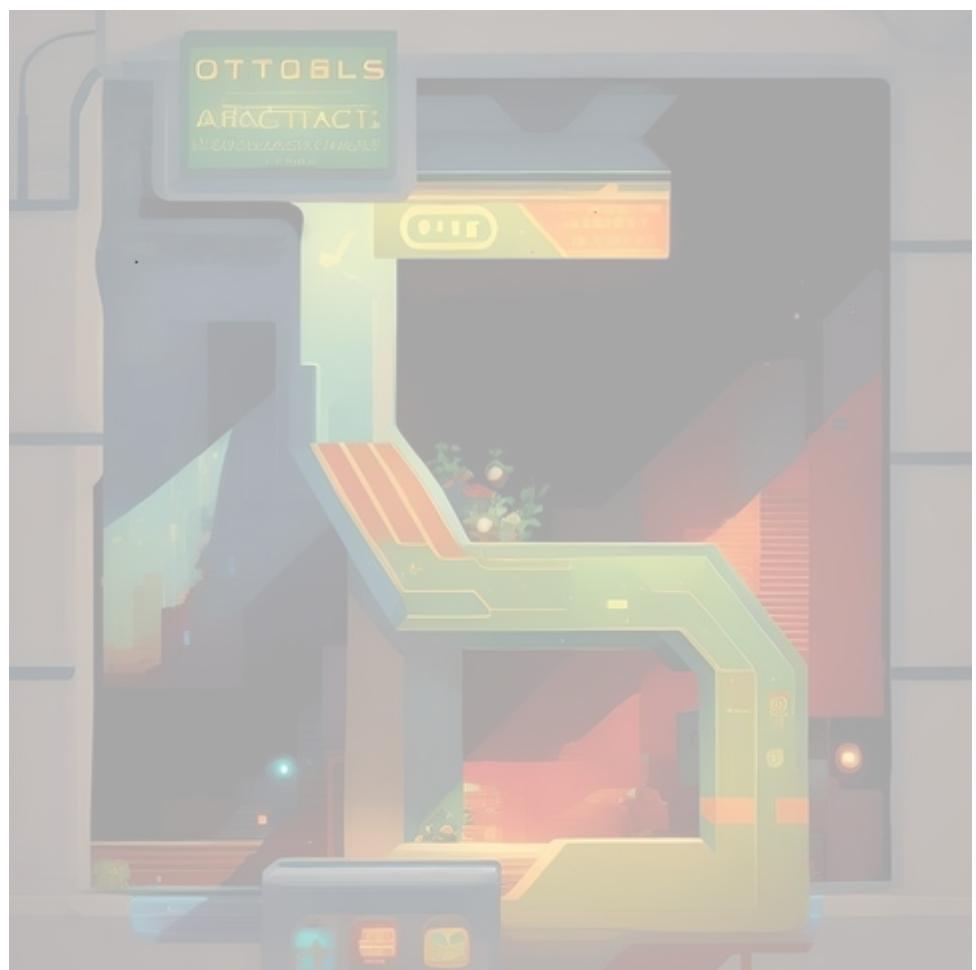
$$= 6\hat{i} + 2\hat{i} + 3\hat{j}$$

$$\boxed{\cancel{2\hat{i} + 3\hat{j}}}$$

$$\boxed{= 3\hat{j}} \quad \checkmark$$

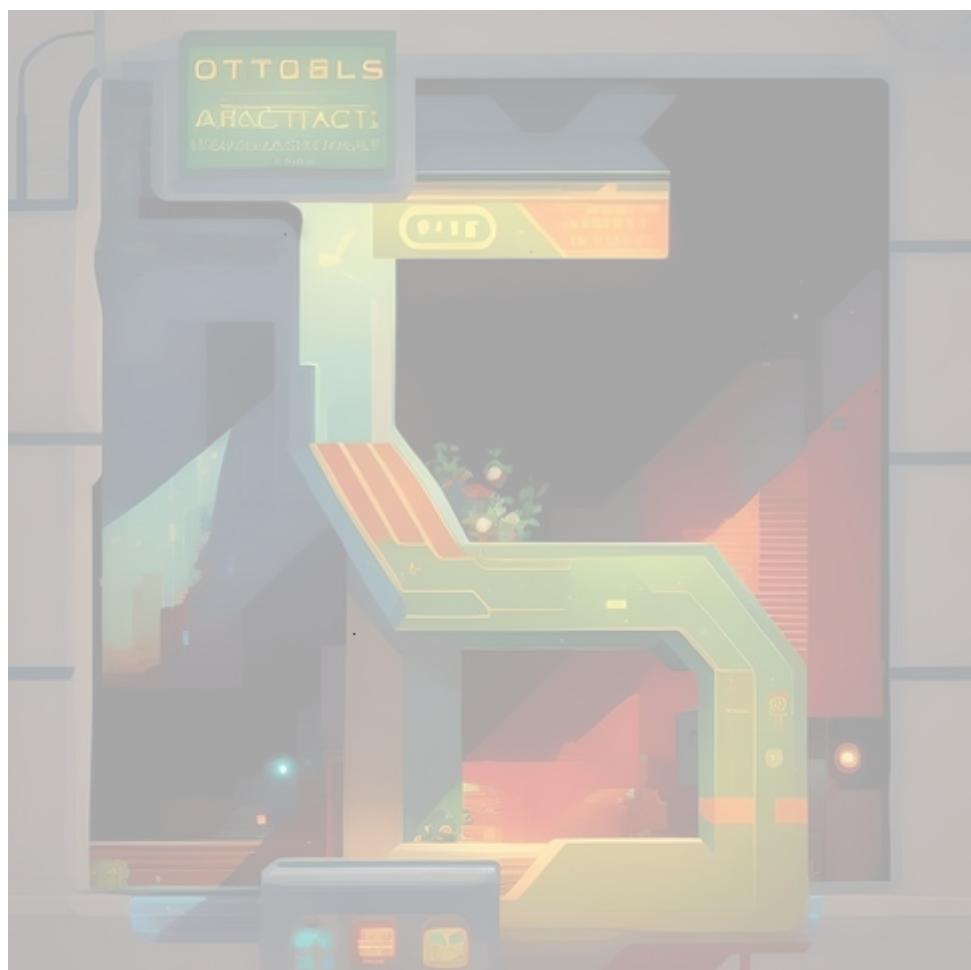


(62)



(63)





(65)