Physics - 2

Ch-3 Projectile Motion (Continue)

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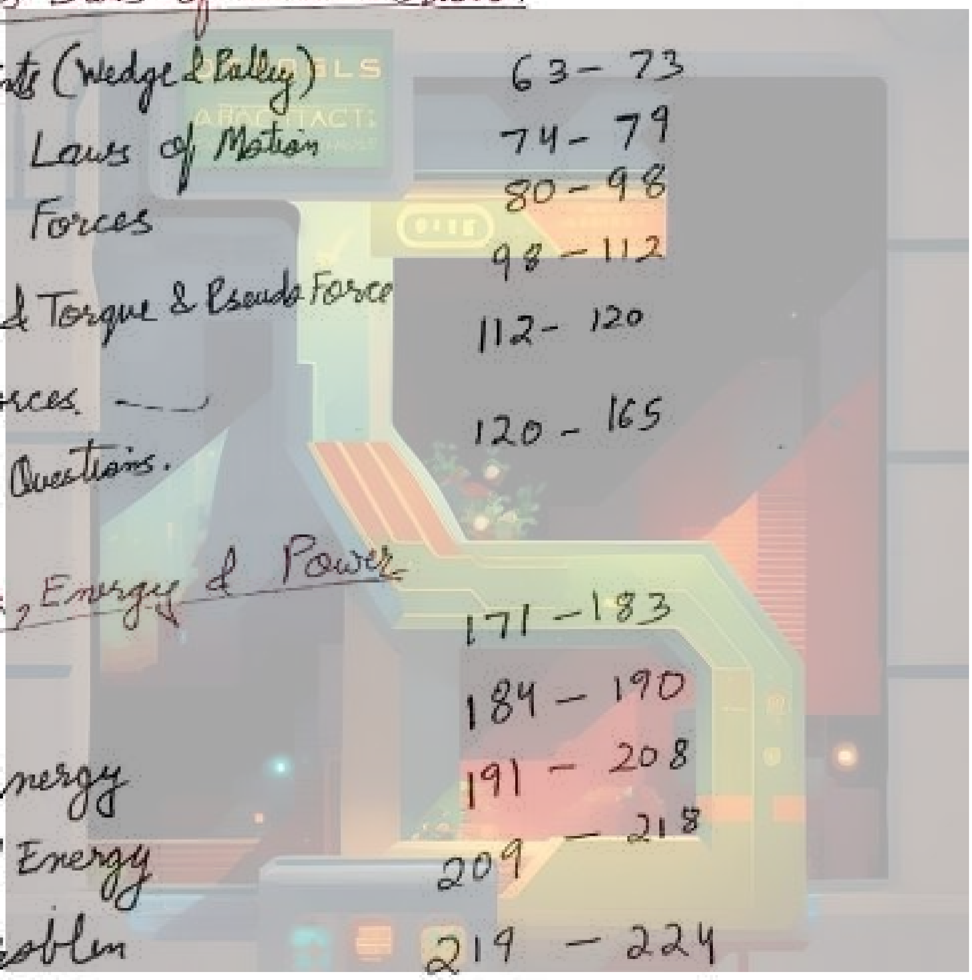

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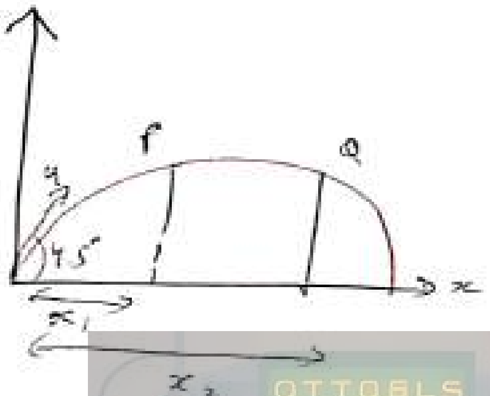
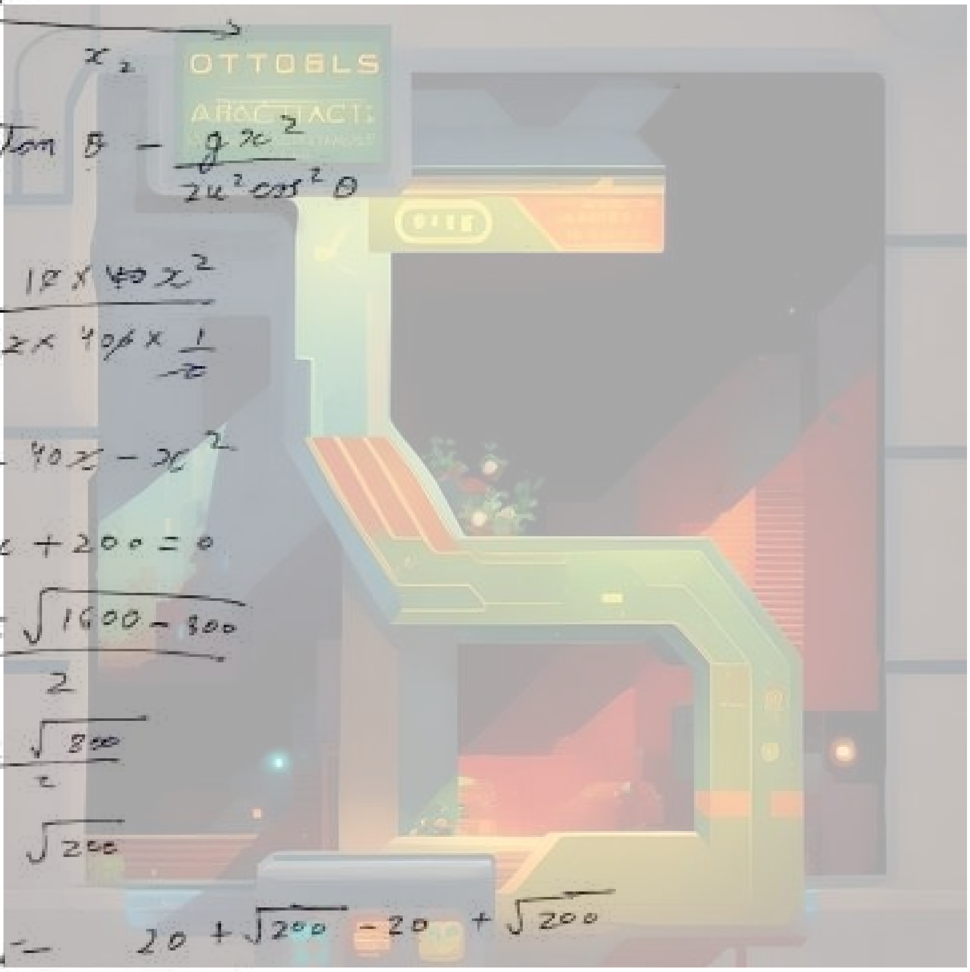
Q A particle fired with a velocity 20 m/s from a gun adjusted for maximum range. It passes through P & Q whose heights above horizontal are 5 m each. find separation between P & Q?





$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$5 = x - \frac{10 \times x^2}{2 \times 400 \times \frac{1}{2}}$$

$$200 = 40x - x^2$$

$$x^2 - 40x + 200 = 0$$

$$x = \frac{40 \pm \sqrt{1600 - 800}}{2}$$

$$x = \frac{40 \pm \sqrt{800}}{2}$$

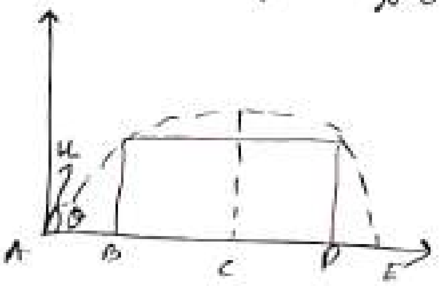
$$x = 20 \pm \sqrt{200}$$

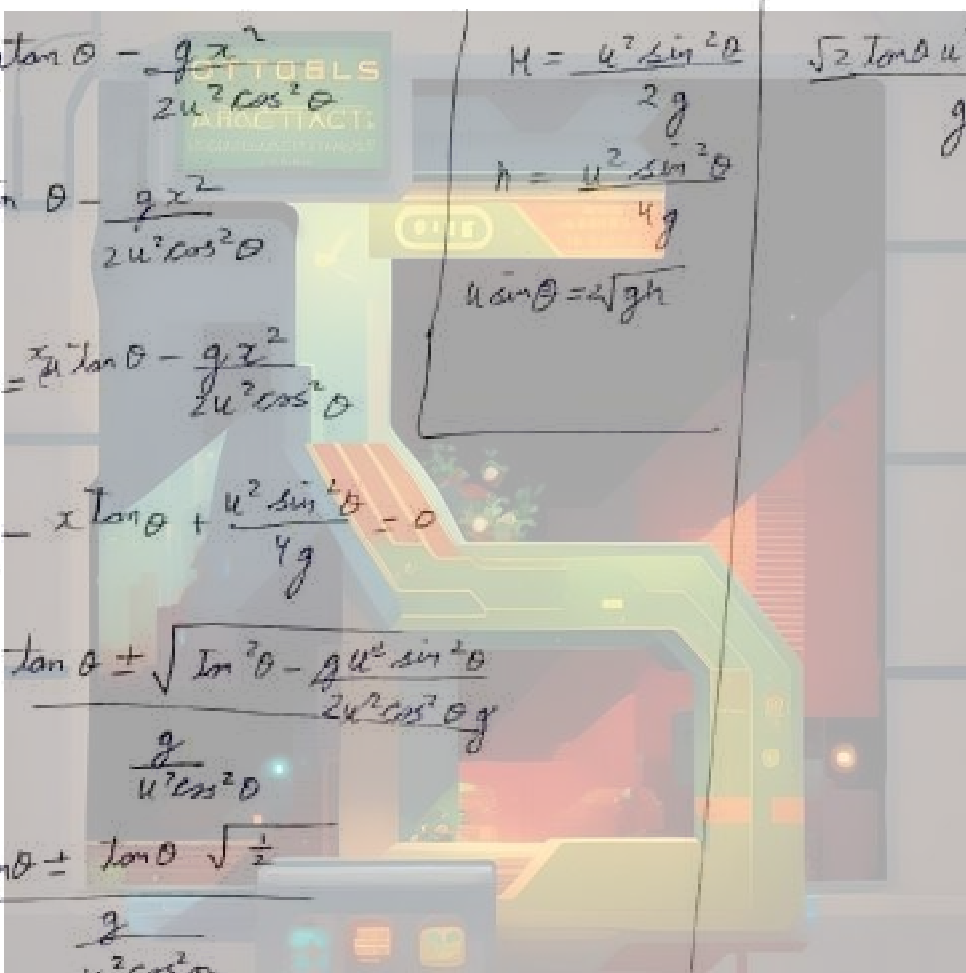
separation = $20 + \sqrt{200} - 20 + \sqrt{200}$

$$\boxed{= 20\sqrt{2}\ m}$$

Q bird height – h
max height – 2h

find ratio of ~~speed~~ velocity of bird & ~~s~~ horizontal velocity of the stone thrown





$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$\frac{\sqrt{2}\tan\theta\, u^2\cos^2\theta}{g}$$

$$h = \frac{u^2\sin^2\theta}{4g}$$

$$u\sin\theta = 2\sqrt{gh}$$

$$h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{u^2\sin^2\theta}{4g} = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{gx^2}{2u^2\cos^2\theta} - x\tan\theta + \frac{u^2\sin^2\theta}{4g} = 0$$

$$x = \frac{\tan\theta \pm \sqrt{\tan^2\theta - \frac{g u^2\sin^2\theta}{2u^2\cos^2\theta\, g}}}{\frac{g}{u^2\cos^2\theta}}$$

$$x = \frac{\tan\theta \pm \tan\theta\sqrt{\frac{1}{2}}}{\frac{g}{u^2\cos^2\theta}}$$

$$\frac{\tan\theta + \frac{\tan\theta}{\sqrt{2}}}{\frac{g}{u^2\cos^2\theta}} - \frac{\tan\theta - \frac{\tan\theta}{\sqrt{2}}}{\frac{g}{u^2\cos^2\theta}}$$

$$\frac{\tan\theta\, u^2\cos^2\theta}{\sqrt{2}g} + \frac{\tan\theta\, u^2\cos^2\theta}{\sqrt{2}g}$$

(2)

$$h = ut + \frac{1}{2} g t^2$$

$$\Rightarrow g t^2 + 2ut - 2h = 0$$

$$g t^2 + 2ut - \frac{u^2 \sin^2\theta}{2g} = 0$$

$$t = \frac{-2u \pm \sqrt{4u^2 + 2g u^2 \sin^2\theta}}{2g}$$

$$\frac{-2u + \sqrt{4u^2 + 2g u^2 \sin^2\theta} + 2u + \sqrt{4u^2 + 2g u^2 \sin^2\theta}}{2g}$$

$$\frac{\sqrt{4u^2 + 2g u^2 \sin^2\theta}}{g}$$

$$\text{Speed} = \frac{\frac{\sqrt{2} \tan\theta \, u^2 \cos^2\theta}{g}}{\frac{\sqrt{4u^2 + 2g u^2 \sin^2\theta}}{g}}$$

$$= \frac{\sqrt{2} \tan\theta \, u^2 \cos^2\theta}{\sqrt{4u^2 + 2g \cdot 4gh}}$$

$$= \frac{\sqrt{2} \tan\theta \, u^2 \cos^2\theta}{\sqrt{\frac{4u^2 h}{\sin^2\theta} + 8g^2 h}}$$ (vertical velocity

$$= \frac{\sqrt{2} \tan\theta \, u^2 \cos^2\theta}{\sqrt{8gh + 8g^2 h}}$$

$$= \frac{\tan\theta \, u^2 \cos^2\theta}{2\sqrt{gh}}$$

$$\frac{\text{Speed}}{u \cos\theta}$$

$$\boxed{\frac{\tan\theta \, u \cos\theta}{2\sqrt{gh}}}$$

$$\frac{u \sin\theta}{2\sqrt{gh}}$$

$$\frac{2\sqrt{gh}}{2\sqrt{gh}} = \boxed{\sqrt{2}\sqrt{h}}$$

$$\boxed{= \frac{2\sqrt{2}}{2+\sqrt{2}}}$$

Q2

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$2h = \frac{u^2 \sin^2\theta}{2g}$$

$$u^2 \sin^2\theta = 4gh$$

$$\boxed{u \sin\theta = 2\sqrt{gh}}$$

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$h = u\sin\theta\, t - \frac{1}{2} g t^2$$

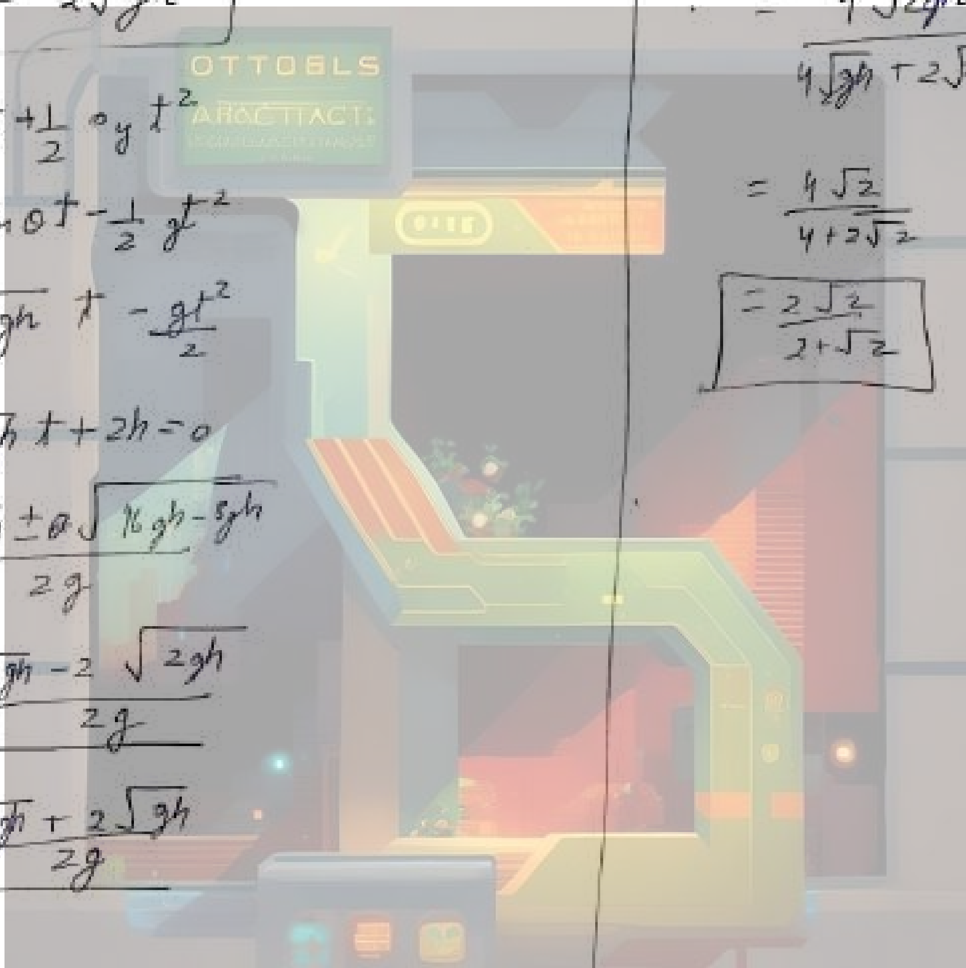
$$h = 2\sqrt{gh}\, t - \frac{gt^2}{2}$$

$$gt^2 - 4\sqrt{gh}\, t + 2h = 0$$

$$t = \frac{4\sqrt{gh} \pm \sqrt{16gh - 8gh}}{2g}$$

$$t_1 = \frac{4\sqrt{gh} - 2\sqrt{2gh}}{2g}$$

$$t_2 = \frac{4\sqrt{gh} + 2\sqrt{gh}}{2g}$$



$$AD = AB + BD$$

$$u\cos\theta\, t_2 = u\cos\theta\, t_1 + v_B t_2$$

$$u\cos\theta\,(t_2 - t_1) = v_B t_2$$

$$\frac{v_B}{u\cos\theta} = \frac{t_2 - t_1}{t_2}$$

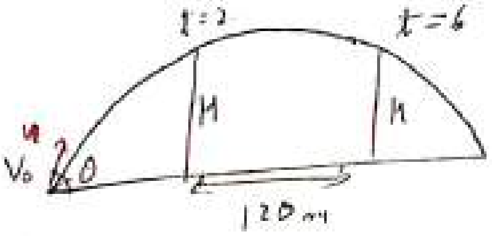
$$= \frac{4\sqrt{2gh}}{4\sqrt{gh} + 2\sqrt{2gh}}$$

$$= \frac{4\sqrt{2}}{4 + 2\sqrt{2}}$$

$$\boxed{= \frac{2\sqrt{2}}{2 + \sqrt{2}}}$$

Q

A projectile crosses 2 walls of height H. find



a) time of flight

b) H = ?

c) Max height attained = ?

d) Range, R = ?

e) θ = ?

f) V_0 = ?

$g = 10$

$u = V_0$

$u\cos\theta = \frac{120}{4} = 30$

$\boxed{u\cos\theta = 30}$

$S = ut + \frac{1}{2}at^2$

$H = u\sin\theta(t) + 5t^2$

~~$5t^2 + u\sin\theta t - H = 0$~~

~~$t = \frac{-u\sin\theta \pm \sqrt{u^2\sin^2\theta + 20H}}{10}$~~

$u\sin(2) + 5(2)^2 = u\sin(6) + 5(6)^2$

$2u\sin\theta - 20 = 6u\sin\theta - 180$

$0 = 4u\sin\theta + 200$

$0 = u\sin\theta + 50$

$\boxed{u\sin\theta = +40}$

$\boxed{u = \frac{40}{\sin\theta}}$

$T = \frac{2u\sin^2\theta}{2\times g}$

$= \frac{2\times 40}{2\times 10}$

$\boxed{\cancel{T = 4s}}$

$\boxed{= 8s}$ a)

$H = \frac{u^2\sin^2\theta}{2g}$

$H = \frac{1600}{2\times 10}$

$\boxed{H = 80m}$ c) max height

$H = 40(2) - 5(4)$

$H = 80 - 20$

$\boxed{H = 60}$ b)

$\frac{40}{\sin\theta} \times \cos\theta = 30$

$\frac{40}{30} = \tan\theta$

$\tan\theta = \frac{4}{3}$

$\boxed{\theta = 60^\circ}$ e)

Scanned with CamScanner

$u\cos\theta = 30$

$u \times \frac{3}{5} = 30$

$u = \frac{30 \times 5}{3}$

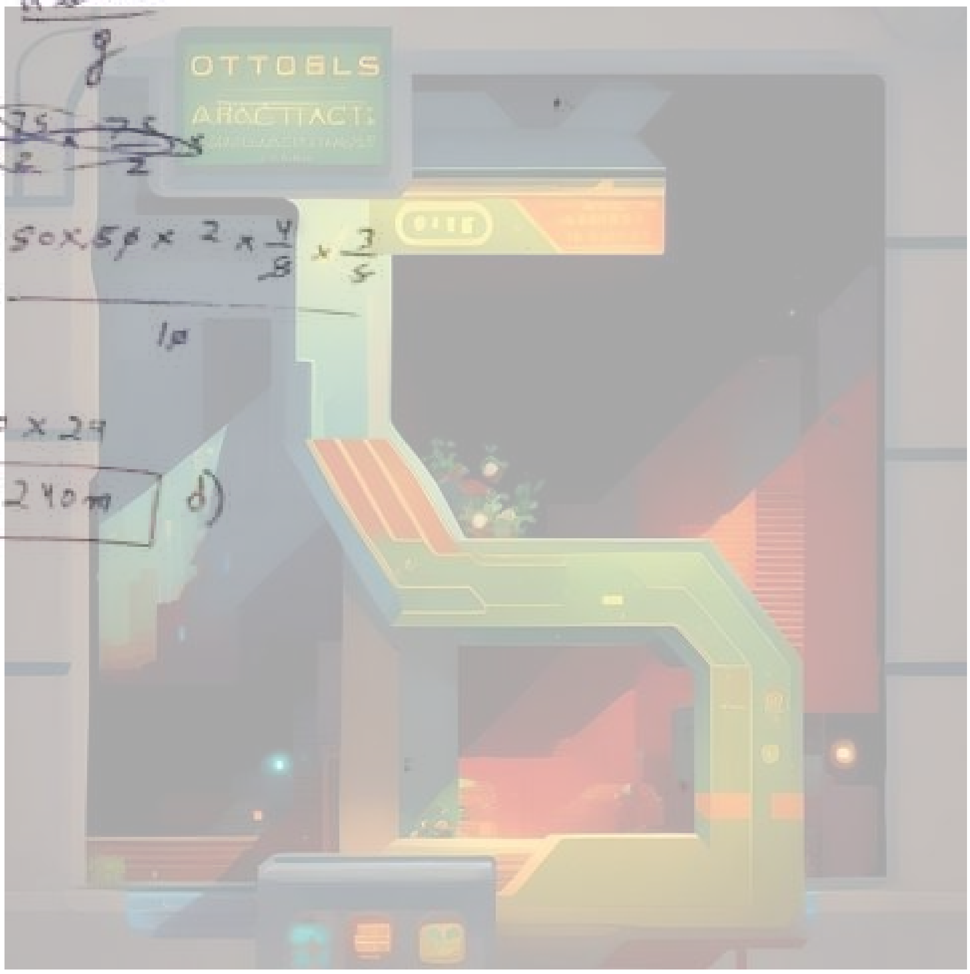
$\boxed{u = 50\ m/s}$ b)

$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{50 \times 50 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10}$

$R = 10 \times 24$

$\boxed{R = 240 m}$ d)



~~Hori~~ Horizontal

Projection From A Height

Horizontal

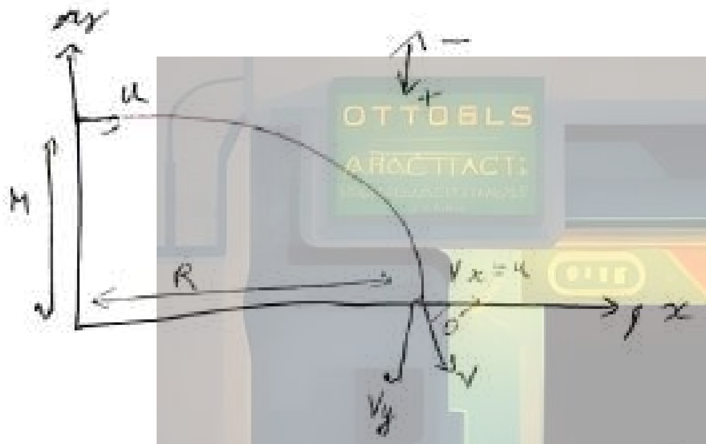
$U_x = u$

$a_x = 0$

Vertical

$U_y = 0$

$a_y = g$



Time of flight $=(T)$:-

$S_y = U_y t + \frac{1}{2} a_y t^2$

$H = \frac{1}{2} g T^2$

$$\boxed{T = \sqrt{\frac{2H}{g}}}$$

Range $(R) = U_x T$

$$\boxed{R = u\sqrt{\frac{2H}{g}}}$$

$V_y^2 = u_y^2 + 2 g S_y$

$V_y^2 = 0 + 2gH$

$V_y = \sqrt{2gH}$ (Vertical component of velocity on hitting ground)

$$\boxed{V_y = \sqrt{2gh}}$$

$\theta = \tan$

$$\boxed{\theta = \tan^{-1}\frac{\sqrt{2gH}}{u}}$$

$$\boxed{\vec{V} = u\hat{i} + \sqrt{2gh}\,\hat{j}}$$

Q A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find

i) time to reach ground

ii) The horizontal distance from foot of hill to ground

iii) The velocity with which it hits the ground. ($g = 9.8$)

$$T = \sqrt{\frac{2H}{g}}$$

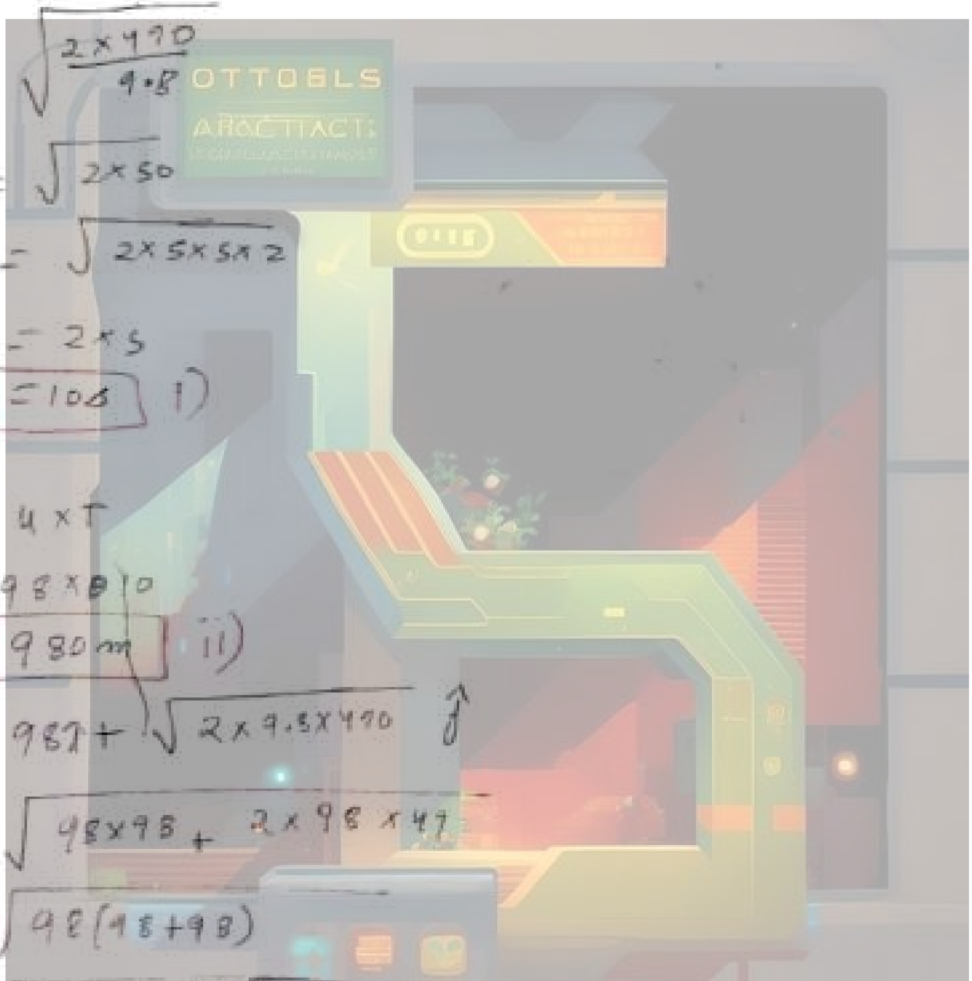
$$T = \sqrt{\frac{2 \times 490}{9.8}}$$

$$T = \sqrt{2 \times 50}$$

$$T = \sqrt{2 \times 5 \times 5 \times 2}$$

$$T = 2 \times 5$$

$$\boxed{= 10\,s} \quad \text{i)}$$



$$R = u \times T$$

$$= 98 \times 10$$

$$\boxed{= 980\,m} \quad \text{ii)}$$

$$\vec{v} = 98\hat{i} + \sqrt{2 \times 9.8 \times 490}\ \hat{j}$$

$$v = \sqrt{98 \times 98 + 2 \times 98 \times 49}$$

$$v = \sqrt{98(98 + 98)}$$

$$v = \sqrt{98 \times 98 \times 2}$$

$$\boxed{v = 98\sqrt{2}\ m/s} \quad \text{iii)}$$

Q.

$|V_x| = |V_y|$

find time taken to reach point B.

$T = \sqrt{\frac{2H}{g}}$

$V_y = \sqrt{2gH}$

$30 = \sqrt{2 \times 10 \times H}$

$900 = 20H$

$H = \frac{900}{20}$

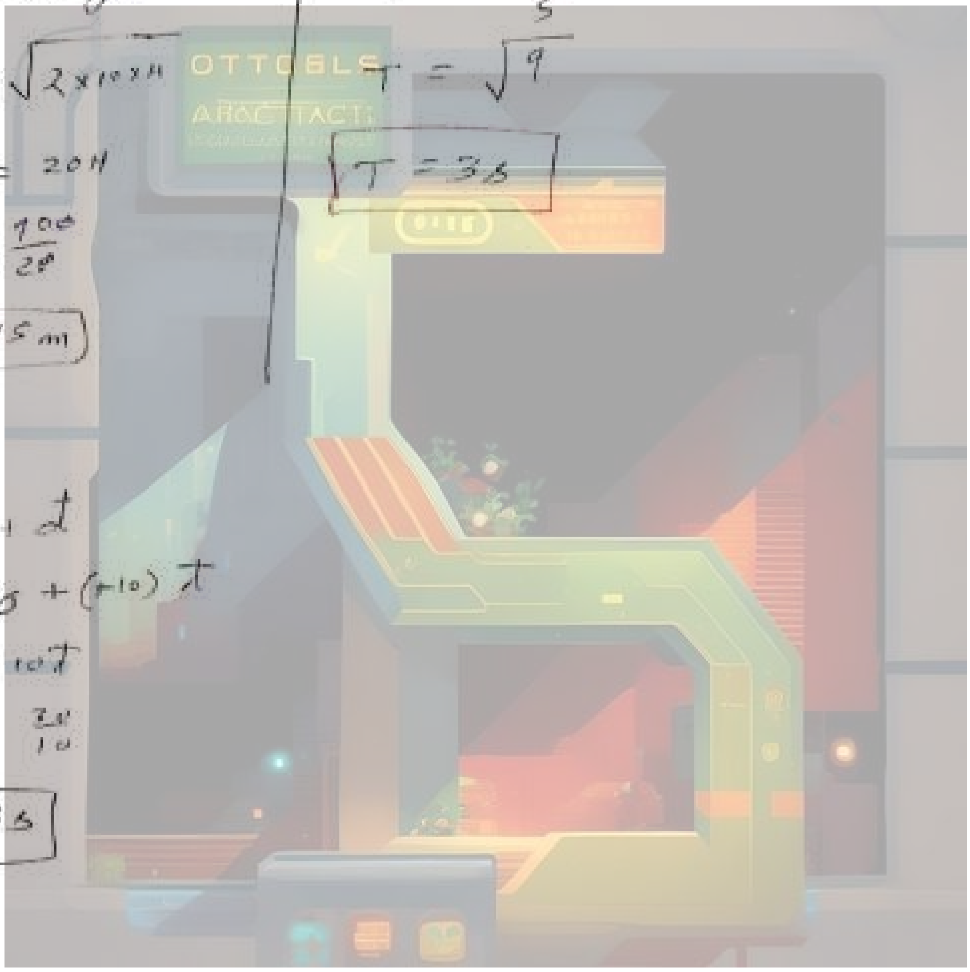
$\boxed{H = 45\,m}$

$T = \sqrt{\frac{2 \times 45}{10}}$

$T = \sqrt{\frac{45}{5}}$

$T = \sqrt{9}$

$\boxed{T = 3s}$



Q2.

$v = u + at$

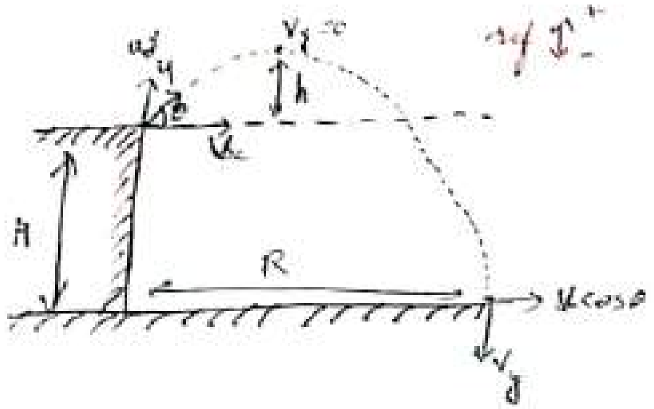
$30 = 0 + (+10)t$

$30 = 10t$

$t = \frac{30}{10}$

$\boxed{t = 3s}$

Projection at an angle from a height



Time of flight :-

i.e. $s = ut + \frac{1}{2} g t^2$

$-H = u\sin\theta\, T - \frac{1}{2} g t^2$

$T = ?$

Range (R) = $u\cos\theta\, T$

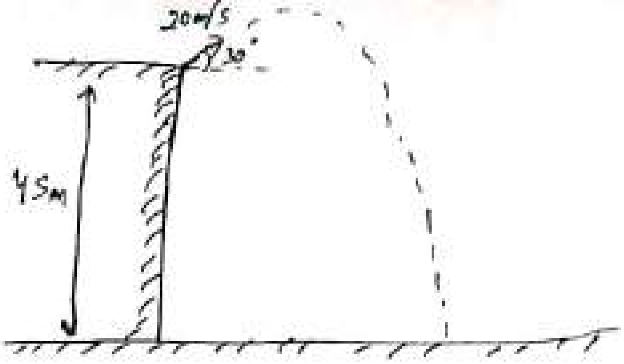
$h = \frac{u^2 \sin^2\theta}{2g}$

$v_y^2 = (u\sin\theta)^2 + 2gH$

$v_y = \sqrt{}$

$v_y = \sqrt{u^2\sin^2\theta + 2gH}$

Q



find a) T
b) R
c) speed (V)

$S = ut + \frac{1}{2} at^2$

$-45 = u\sin\theta\, t - 5t^2$

$5t^2 - 20 \times \frac{1}{2} t - 45 = 0$

$t^2 - 2t - 9 = 0$

$t = \frac{2 \pm \sqrt{4 + 36}}{2}$

$t = \frac{2 \pm \sqrt{40}}{2}$

$t = 1 \pm \sqrt{10}$

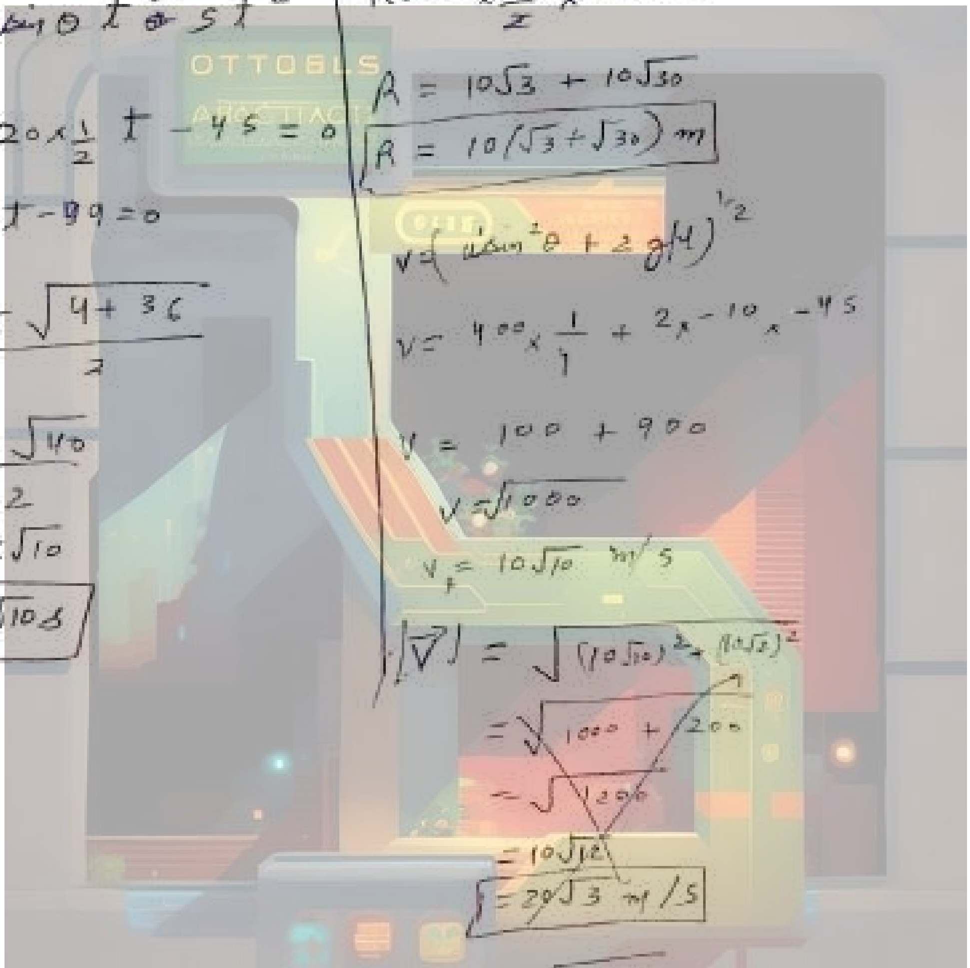
$\boxed{t = 1 + \sqrt{10}\, s}$

$R = u\cos\theta\, t$

$R = \cancel{20}^{10} \times \frac{\sqrt{3}}{\cancel{2}} \times (1 + \sqrt{10})$

$R = 10\sqrt{3} + 10\sqrt{30}$

$\boxed{R = 10(\sqrt{3} + \sqrt{30})\, m}$



$v = (u^2\sin^2\theta + 2gH)^{1/2}$

$v = 400 \times \frac{1}{4} + 2 \times -10 \times -45$

$v = 100 + 900$

$v = \sqrt{1000}$

$v_y = 10\sqrt{10}\ m/s$

$|\vec{V}| = \sqrt{(10\sqrt{10})^2 + (10\sqrt{3})^2}$

$= \sqrt{1000 + 200}$

$= \sqrt{1200}$

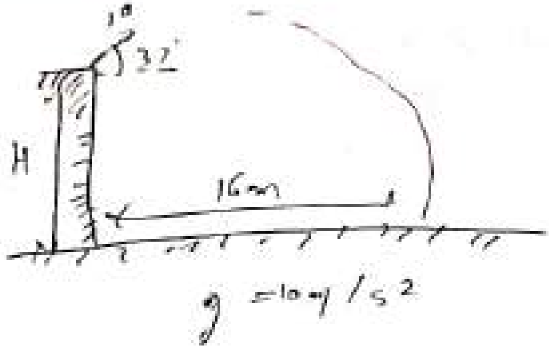
$= 10\sqrt{12}$

$= 20\sqrt{3}\ m/s$

$= \sqrt{1300}$

$\boxed{= 10\sqrt{13}\ m/s}$

Q



$g = 10 m/s^2$

u

$-H = 6t - 5t^2$

$5t^2 - 6t - H = 0$

$t = \frac{6 \pm \sqrt{36 + 20H}}{2}$

$t = 3 \pm \sqrt{9 + 5H}$

$16 = u\cos\theta\, t$

$16 = 8t$

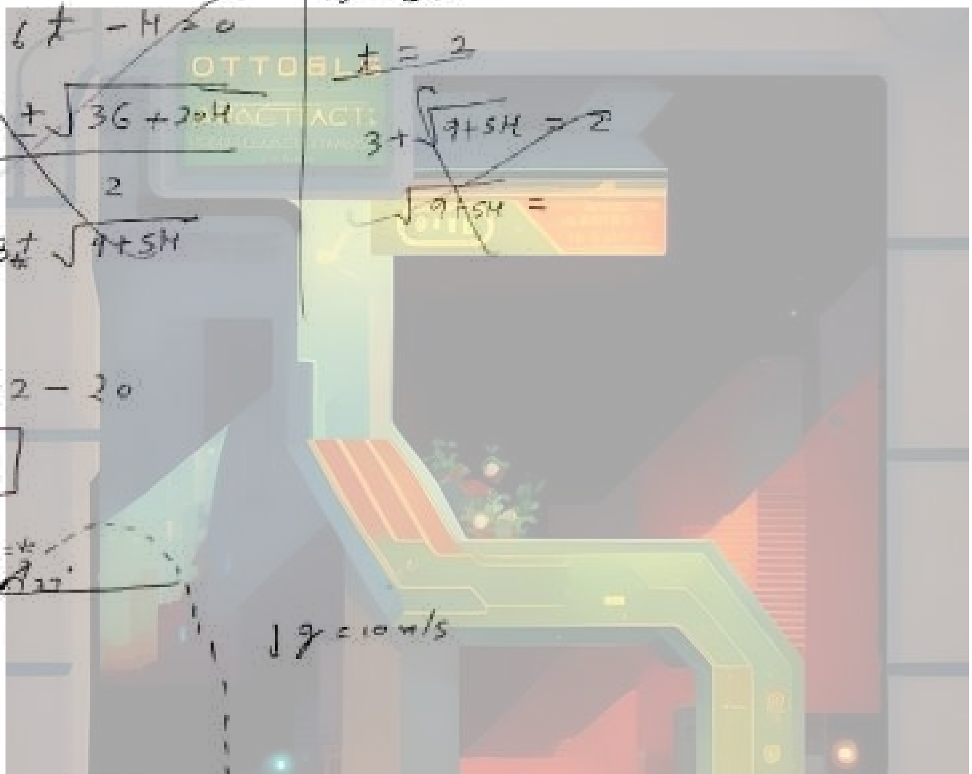
$t = 2$

$3 + \sqrt{9+5H} = 2$

$\sqrt{9+5H} =$

$-H = 12 - 20$

$\boxed{H = 8m}$



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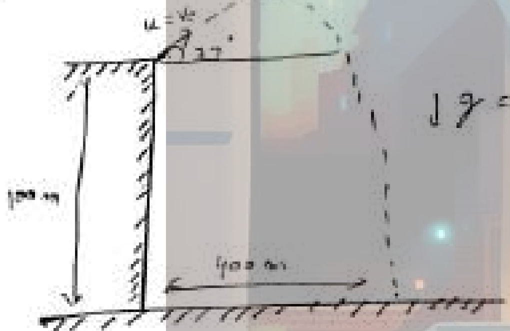

$g = 10 m/s$

$400 = u \times \frac{4}{5} \times t$

$500 = ut$

$\frac{500}{u} = t$

$-100 = u \times \frac{3}{5} \times \frac{500}{u} - \frac{1}{2} \times 10 \times \frac{500}{u} \times \frac{500}{u}$

$-100 = 300 - \frac{1250000}{u^2}$

$-100u^2 = 300u^2 - 12500$

$12500 = 400u^2$

$\frac{12500}{4z} = u^2$

$\cancel{u = \frac{5\sqrt{5}}{\sqrt{2}} m/s}$

$\cancel{u^2 = \frac{6250}{2}}$

$u = \frac{25\sqrt{10}}{2}$

$u^2 = \frac{6250}{2}$

$u^2 = 3125$

$\boxed{u^2 = 25\sqrt{5}}$

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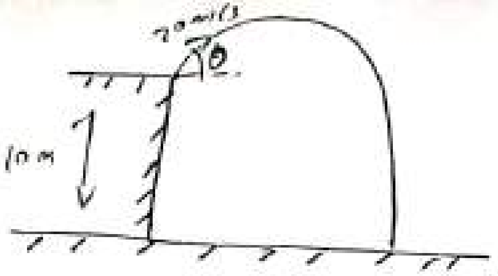
Q



$-10 = 20\sin\theta\, t - 5t^2$

$-2 = 4\sin\theta\, t - t^2$

$t^2 - 4\sin\theta\, t - 2 = 0$

$t = \frac{4\sin\theta + \sqrt{16\sin^2\theta + 8}}{2}$

$t = 2\sin\theta + \sqrt{4\sin^2\theta + 2}$

$0 = 2\cos\theta + \frac{1}{2\sqrt{4\sin^2\theta + 2}}$

$0 = 4\cos\theta\sqrt{4\sin^2\theta + 2} + 1$

$0 =$

$1 = 16\cos^2\theta\,(4\sin^2\theta + 2)$

$\frac{1}{16}\sec^2\theta = 64\sin^2\theta + 2$

~~R = ucos θ t~~

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$-10 = x\tan\theta - \frac{10x^2}{2\times 400\times\cos^2\theta}$

$-10 = x\tan\theta\left[1 - \frac{x}{R}\right]$

$-10 = x\tan\theta - \frac{x^2}{80\cos^2\theta}$

$-800\cos^2\theta = 80x\sin\theta\cos\theta - x^2$

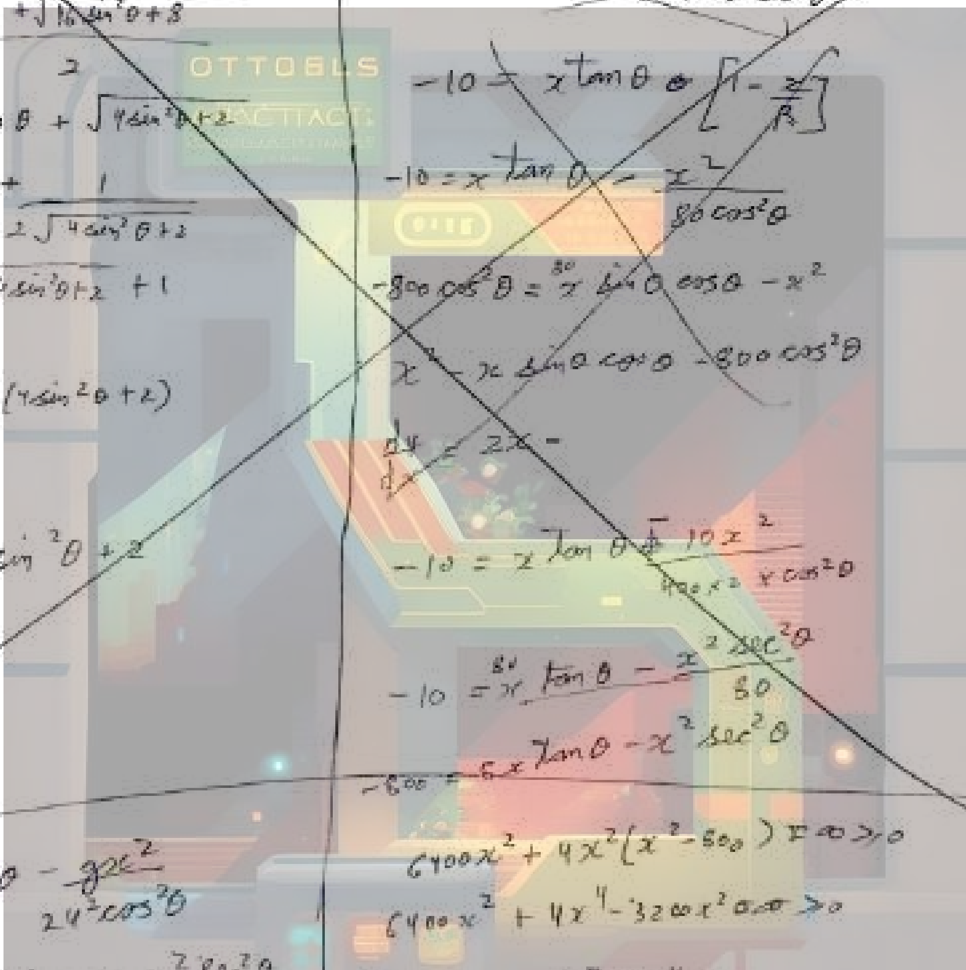
$x^2 - x\sin\theta\cos\theta - 800\cos^2\theta$

$\frac{dy}{dx} = 2x -$

$-10 = x\tan\theta - \frac{10x^2}{400\times 2\times\cos^2\theta}$

$-10 = x\tan\theta - \frac{x^2\sec^2\theta}{80}$

$-800 = 80x\tan\theta - x^2\sec^2\theta$



$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$-10 = x\tan\theta - \frac{10x^2\sec^2\theta}{800}$

$-10 = x\tan\theta - \frac{x^2(1+\tan^2\theta)}{80}$

$-800 = 80x\tan\theta - x^2 + x^2\tan^2\theta$

$x^2\tan^2\theta + 80x\tan\theta - (x^2 - 800) = 0$

$\tan\theta \quad b^2 - 4ac \geq 0$

$6400x^2 + 4x^2(x^2 - 800) \geq 0$

$6400x^2 + 4x^4 - 3200x^2 \geq 0$

$9600 - 3200x^2 + 4x^4 \geq 0$

$2400 \leq R^2$

$\boxed{R \leq 20\sqrt{6}}$

~~2400 tan²θ~~

$\tan\theta = \frac{1600\sqrt{6} \pm \sqrt{9600x^2 - 4x^4}}{4800}$

$$\tan\theta = \frac{1600\sqrt{6} + \sqrt{9600 \times 2400 + 4 \times 2400}}{2400 \times 2}$$

$$\tan\theta = \frac{\frac{2}{3}\sqrt{6} + \sqrt{9600 + 4}}{2}$$

$$\tan\theta = \frac{1}{3}\sqrt{6} + \sqrt{2401}$$

$$\theta = \tan^{-1}\left[\frac{\sqrt{6}}{3} + \sqrt{2401}\right]$$

$x = 20\sqrt{6}$

$x^2 = 400 \times 6$

$= 2400$

$$x^2 \tan^2\theta + 80x\tan\theta - (x^2 - 800) = 0$$

$$2400\tan^2\theta + 81600\tan\theta - \underbrace{(2400 - 800)}_{= 1600}$$

$$2400\tan^2\theta + 1600\sqrt{6}\tan\theta - 1600 = 0$$

$$\cancel{3\tan^2\theta + 2\tan\theta - 2 = 0}$$

$$\cancel{\tan\theta = \frac{-2 \pm \sqrt{4 + 24}}{2}}$$

$$\cancel{\tan\theta = \frac{-2 + \sqrt{70}}{2}}$$

$$3\tan^2\theta + 2\sqrt{6}\tan\theta - 2 = 0$$

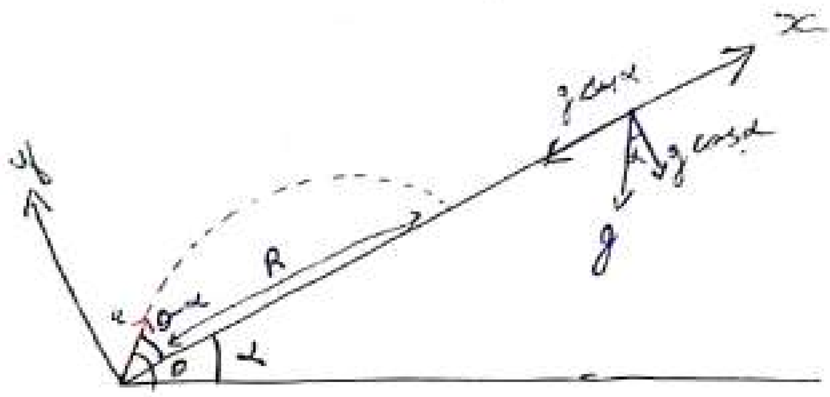
$$\tan\theta = \frac{-2\sqrt{6} \pm \sqrt{24 + 24}}{2 \times 3}$$

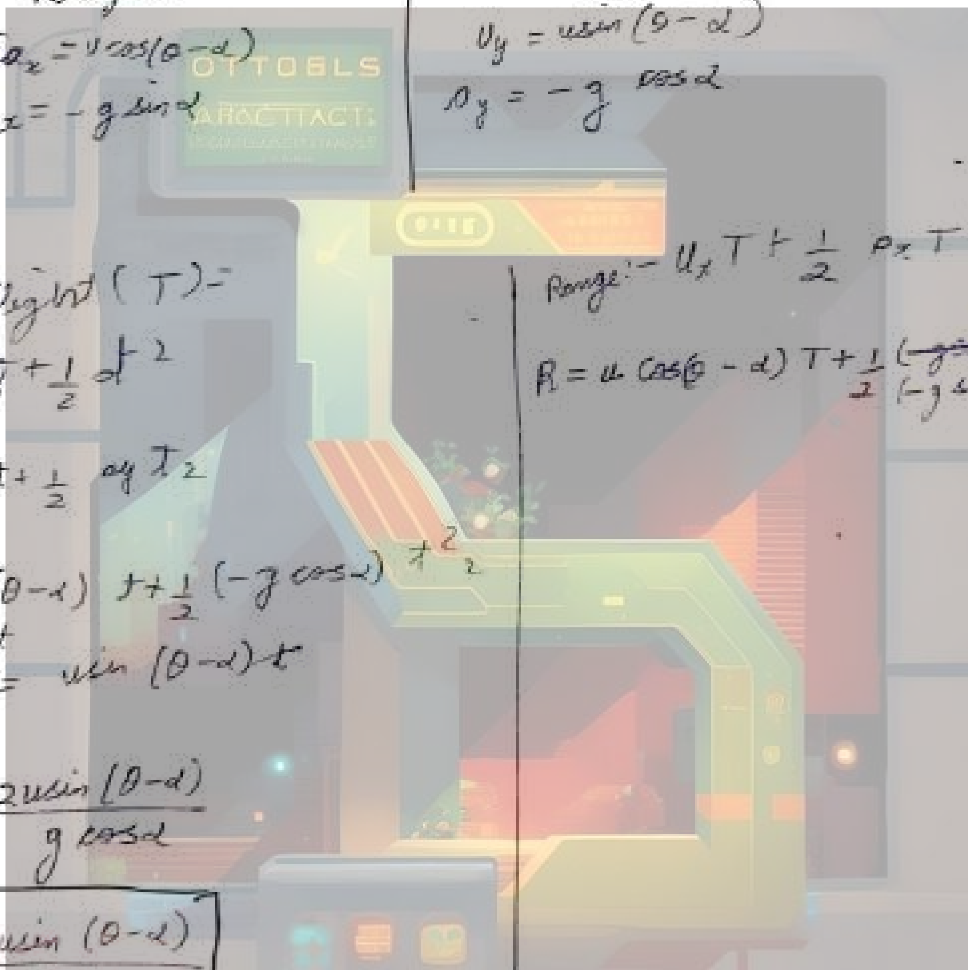
$$\cancel{\tan\theta = \frac{-\sqrt{6} \pm 12\sqrt{2}}{3}}$$

$$\tan\theta = \frac{-\sqrt{6} \pm \sqrt{12}}{3}$$

$$\boxed{\tan\theta = \frac{-\sqrt{6} \pm 2\sqrt{3}}{3}}$$

Projection of the inclined plane





Horizontal

$u_x = u\cos(\theta-\alpha)$

$a_x = -g\sin\alpha$

Verticle

$u_y = u\sin(\theta-\alpha)$

$a_y = -g\cos\alpha$

Time of flight (T) =

$s = ut + \frac{1}{2}at^2$

$0 = u_y t + \frac{1}{2} a_y t^2$

$0 = u\sin(\theta-\alpha)\, t + \frac{1}{2}(-g\cos\alpha)\, t^2$

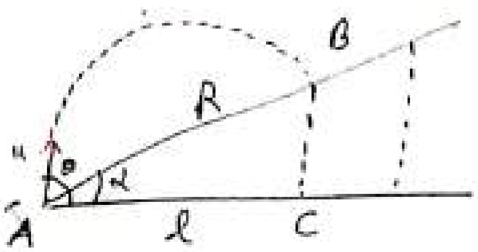
$\frac{g\cos\alpha}{2} t^2 = u\sin(\theta-\alpha)\, t$

$t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$

$$\boxed{t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}}$$

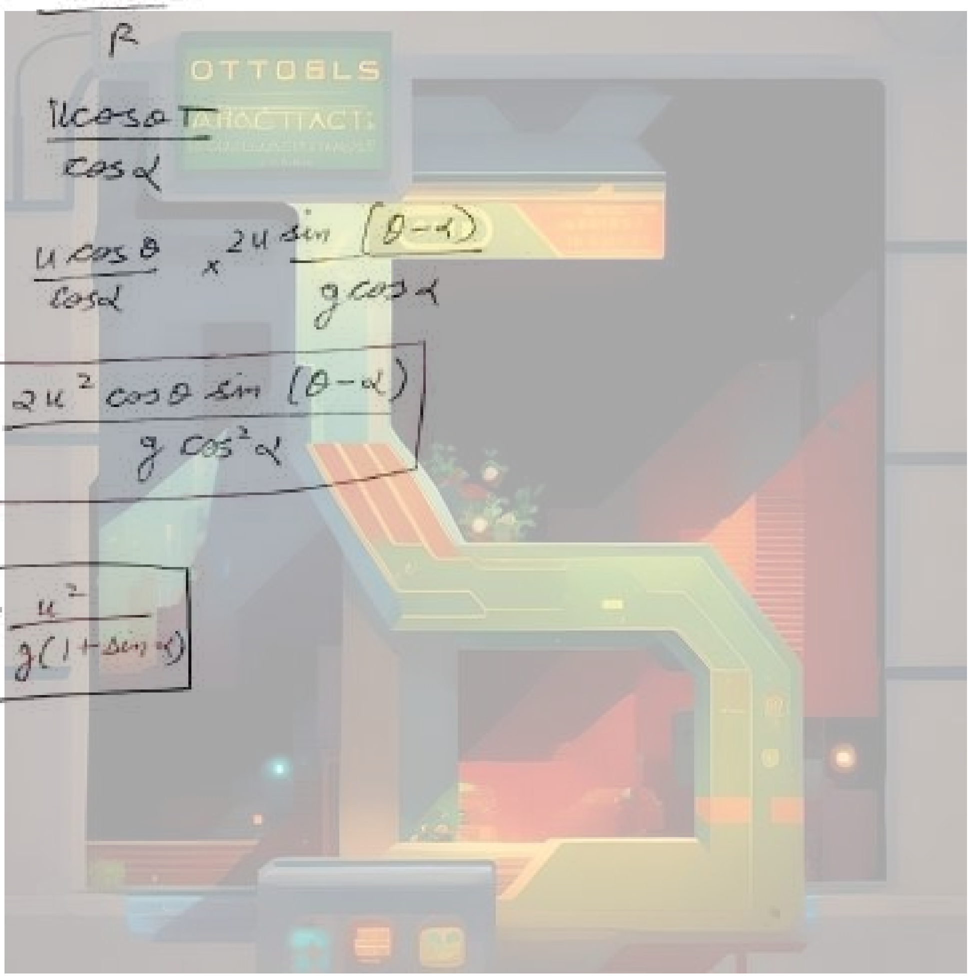
Range :- $u_x T + \frac{1}{2} a_x T^2$

$R = u\cos(\theta-\alpha)\, T + \frac{1}{2}(\cancel{-g\cos\alpha})(-g\sin\alpha)\, T^2$



$l = u\cos\theta\, T$ & $\cos\alpha = \frac{l}{R}$

$$\cos\alpha = \frac{u\cos\theta\, T}{R}$$



$$R = \frac{u\cos\theta\, T}{\cos\alpha}$$

$$R = \frac{u\cos\theta}{\cos\alpha} \times \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

$$\boxed{R = \frac{2u^2\cos\theta\sin(\theta-\alpha)}{g\cos^2\alpha}}$$

$$\boxed{R_{max} = \frac{u^2}{g(1+\sin\alpha)}}$$

JMG(Ch-6)

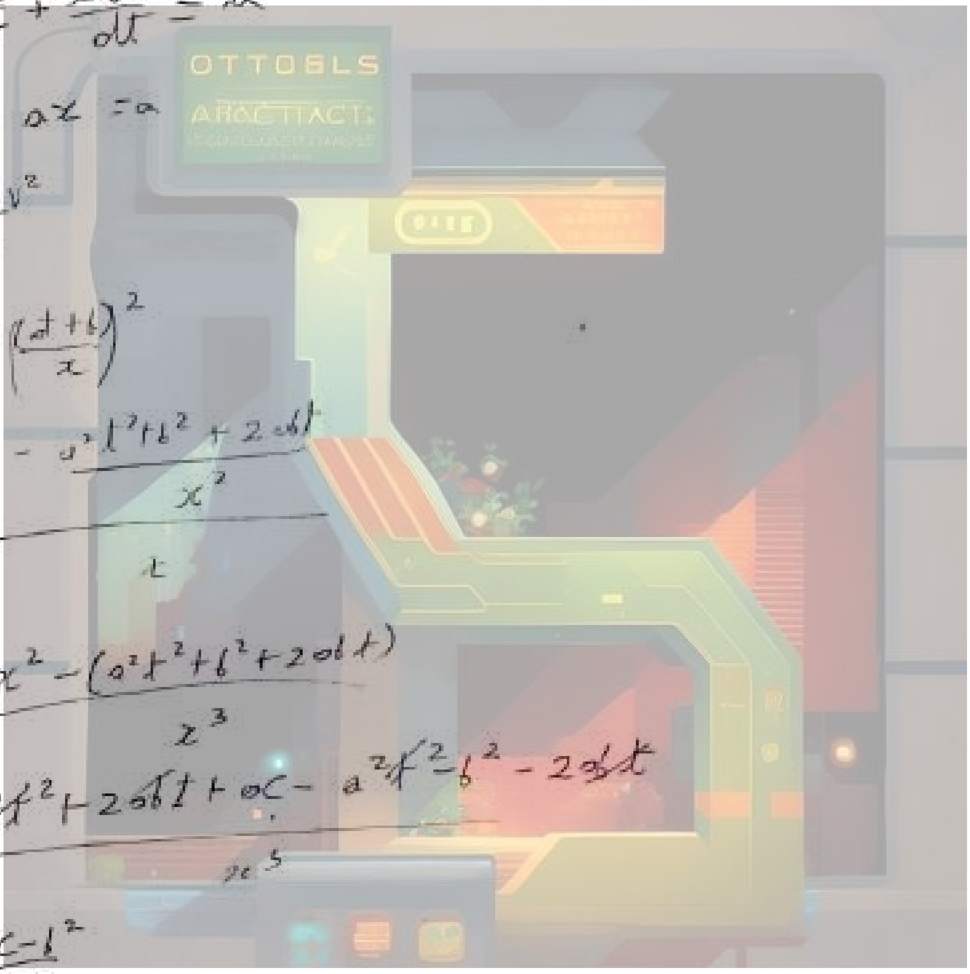
Q8. $x^2 = at^2 + 2bt + c$

$\frac{d}{dt}(x^2) = \frac{d}{dt}(at^2+2bt+c)$

$2x \cdot \frac{dx}{dt} = 2at + 2b$

$x v = at + b$

$v \cdot \frac{dx}{dt} + x\frac{dv}{dt} = a$



$v^2 + ax = a$

$A = \frac{a - v^2}{x}$

$A = a - \left(\frac{at+b}{x}\right)^2$

$A = \frac{a - \frac{a^2t^2+b^2+2abt}{x^2}}{x}$

$A = \frac{ax^2 - (a^2t^2+b^2+2abt)}{x^3}$

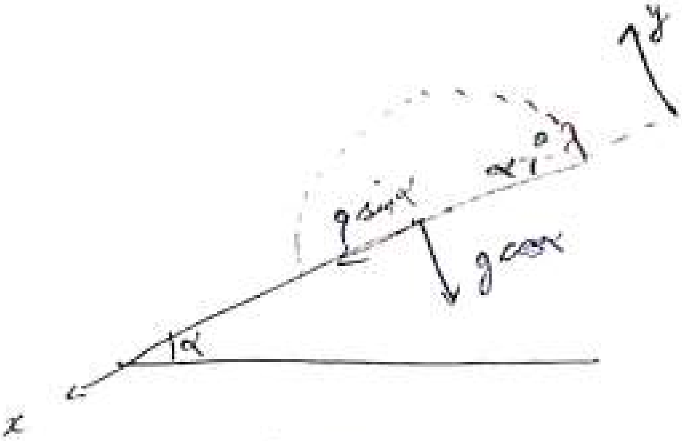
$A = \frac{a^2t^2+2abt+ac - a^2t^2 - b^2 - 2abt}{x^3}$

$A = \frac{ac-b^2}{x^3}$

$A \propto x^{-3}$

$\boxed{n = 3}$

Projection down the inclined plane



Horizontal

$U_x = u\cos(\theta+\alpha)$

$a_x = g\sin\alpha$

Vertical

$U_y = u\sin(\theta+\alpha)$

$a_y = -g\cos\alpha$

Time of flight (T) :-

$S_y = 0 = U_y T + \frac{1}{2} a_y T^2$

$0 = u\sin(\theta-\alpha) T + \frac{1}{2}(-g\cos\alpha)T^2$

$$\boxed{T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}}$$

$R = U_x T + \frac{1}{2} a T^2$

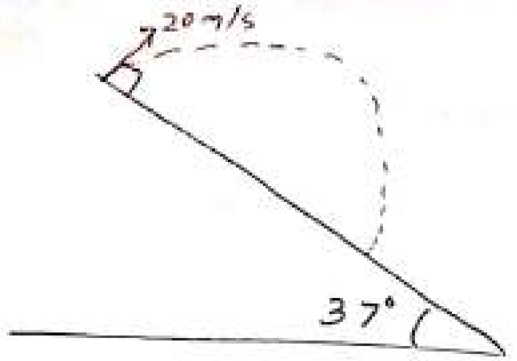
$R = u\cos(\theta+\alpha) T + \frac{1}{2} g\sin\alpha T^2$

$$\boxed{R = u\cos(\theta+\alpha) T + \frac{1}{2} g\sin\alpha T^2}$$

$$\boxed{R = \frac{2u^2\sin(\theta+\alpha)\cos\theta}{g\cos^2\alpha}}$$

$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$

Q



$$\cancel{R = \frac{2u^2 \sin(\theta + \alpha)\cos\theta}{g\cos^2\alpha}}$$

$$\cancel{R = \frac{2\times 20\times 20 \times \cos 37 \times 1}{10\times(\cos 37)^2}}$$

$$\cancel{R = \frac{80 \times \frac{4}{5}}{\frac{16}{25}}}$$

$$\cancel{R = \frac{80\times 4\times 25}{4\times 16\times 5}}$$

$$\cancel{R = 100}$$

$$R = \frac{2u^2 \sin(\theta + \alpha)\cos\theta}{g\cos^2\alpha}$$

$$R = \frac{2\times 20\times 20\times 1\times \frac{3}{5}}{10\times\frac{16}{25}}$$

$$R = \frac{80\times 3\times 25}{16\times 5}$$

$$R = 25\times 3$$

$$\boxed{= 75\,m}$$

Q

$$T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$$

$$= \frac{2\times 20\times 1}{2\times 10\times\frac{4}{5}}$$

$$\boxed{T = 5\,s}$$

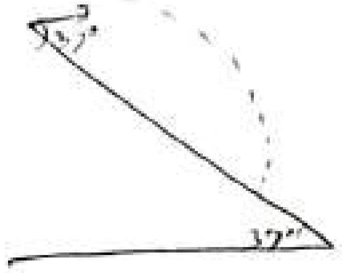
$$s = ut + \frac{1}{2}gt^2$$

$$R = u\cos(\theta+\alpha)T - \frac{1}{2}g\sin\alpha\times t^2$$

$$R = 20\times 0 - 5\times\frac{3}{5}\times 5\times 5$$

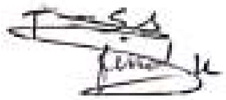
$$\boxed{R = -75\,m}$$

Q

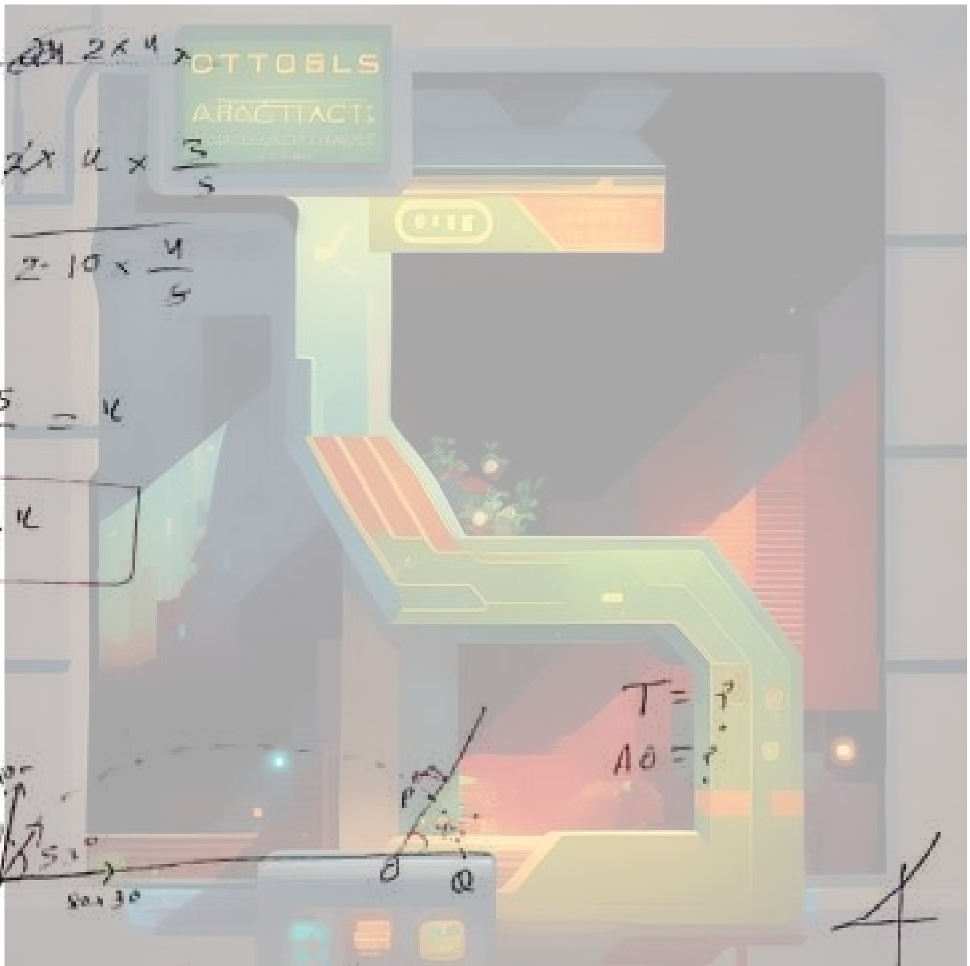


$T = 5\,s$

$u = ?$



$$T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$$



$\cancel{T = 2u\ 2\times u\times}$

$$5 = \frac{2\times u\times\frac{3}{5}}{2\cdot 10\times\frac{4}{5}}$$

$$\frac{20\times 5}{3} = u$$

$$\boxed{\frac{100}{3} = u}$$

Q

$T = ?$

$AO = ?$

$\vec{v} = u\cos 45^\circ\hat{i} + u\sin 45^\circ\hat{j}$

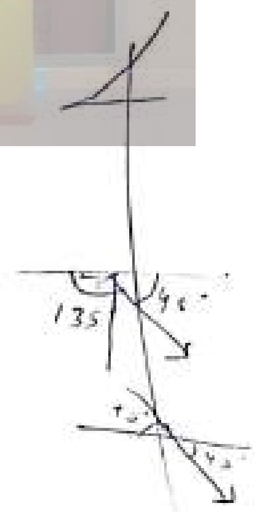
$\vec{v} = u\cos 53^\circ\hat{i} + u\sin 53 - gt\hat{j}$

$$\frac{50}{\sqrt{2}} = 50\times\frac{4}{5} - 10t$$

$$10t = 40 - \frac{50}{\sqrt{2}}$$

$$t = 4 - \frac{5}{\sqrt{2}}$$

$t = 4\sqrt{2} - 5\,s$



Velocity at P = $\vec{v}$

$\vec{v} = \frac{v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}$

$\vec{u} = 30\hat{i} + 40\hat{j}$

$\frac{v}{\sqrt{2}} = 30$

$\underline{v = 30\sqrt{2}}$

$v(\text{in } y) = \frac{30\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$= -30$

using $v - u = at$

$\frac{-30-40}{-10} = t$

$\frac{-70}{-10} = t$

$\boxed{t = 7}$

$\boxed{t = 7s}$

~~PQ~~

$s = ut + \frac{1}{2}at^2$

$PQ = 40(7) + \frac{1}{2}(-10)(7)^2$

$= 280 - 245$

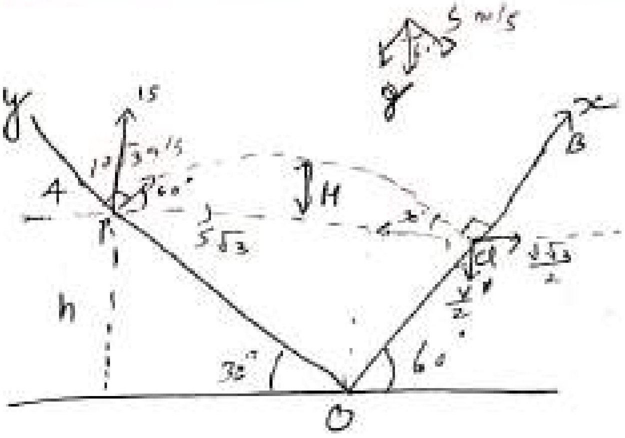
$= 35 = OQ$

$AQ = 30 \times 7 = 210$

$AO = AQ - PQ = 210 - 35$

$\boxed{= 175\ m}$

Q



a) Time of flight
b) velocity at OB.
c) $h = ?$
d) Max height attained by particle from O
e) distance PQ.

$\frac{V}{2} = 5\sqrt{3}$

$\boxed{V = 10\sqrt{3}}$ 1)

$\frac{V\sqrt{3}}{2} = \frac{10\sqrt{3}\times\sqrt{3}}{2} = \frac{30}{2} = 15$

$\frac{V\sqrt{3}}{2} = 5\sqrt{3}$

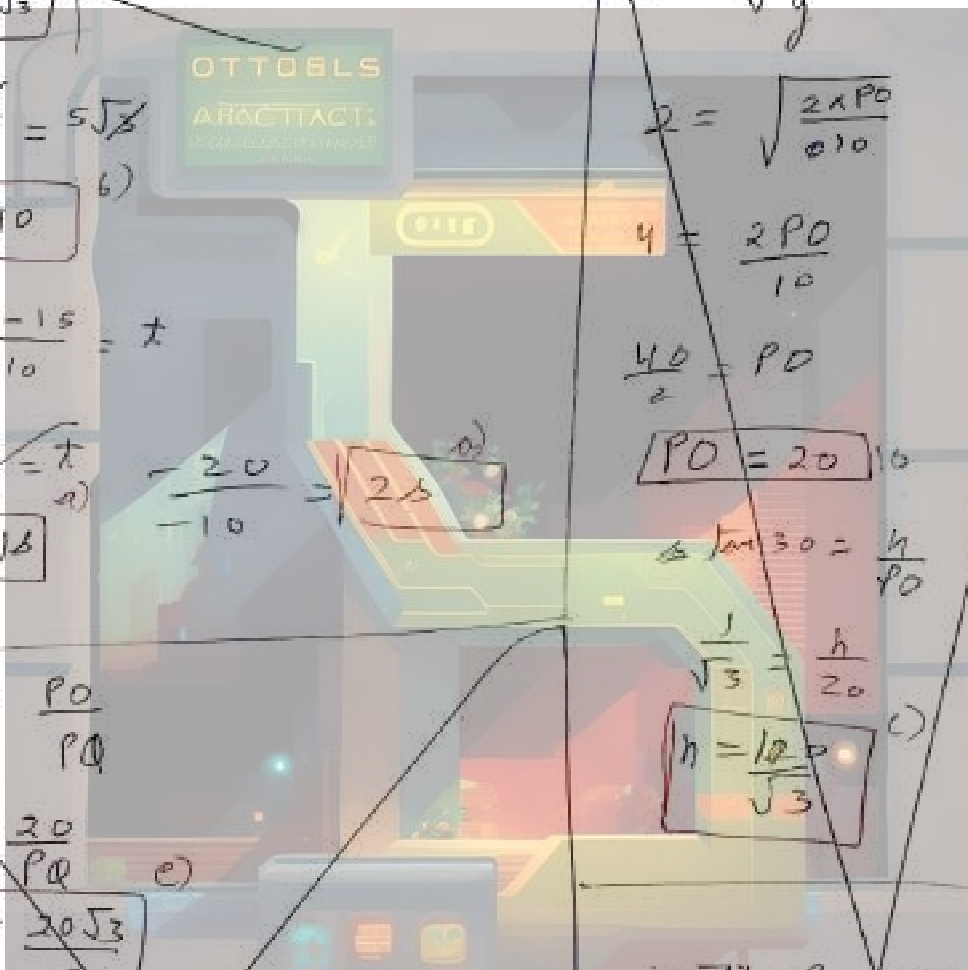
$\boxed{V = 10}$ b)

$\frac{-5-15}{-10} = t$

$\frac{-20}{-10} = \boxed{2s}$ a)

$\frac{10}{10} = t$ a)

$\boxed{\cancel{t = 1s}}$



$T = \sqrt{\frac{2H}{g}}$

$2 = \sqrt{\frac{2\times PO}{10}}$

$4 = \frac{2PO}{10}$

$\frac{40}{2} = PO$

$\boxed{PO = 20}$

$\tan 30 = \frac{h}{PO}$

$\frac{1}{\sqrt{3}} = \frac{h}{20}$

$\boxed{h = \frac{20}{\sqrt{3}}}$ c)

$\tan 60° = \frac{PO}{PQ}$

$\sqrt{3} = \frac{20}{PQ}$ e)

$\boxed{PQ = \frac{20\sqrt{3}}{3}}$

$H = \frac{u^2\sin^2\theta}{2g}$

$H = \frac{10\sqrt{3}\times 10\sqrt{3}\times\frac{3}{4}}{10\times 10}$

$H = \frac{9}{4}$

height $= \frac{9}{4} + \frac{20}{\sqrt{3}}$

$\boxed{= \frac{9\sqrt{3} + 80}{4\sqrt{3}}\ m}$ d)

(22)

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$-OP = 0 + \frac{1}{2}(-5)(2)^2$

$S_y = V_y T + \frac{10}{2} y T^2$

$\boxed{OP = 10}$

$S_x = V_x T + \frac{1}{2} a_x T^2$

$OQ = 10\sqrt{3}(2) + \frac{1}{2}(-5\sqrt{3})(2)^2$

$= 20\sqrt{3} - 10\sqrt{3}$

$\boxed{OQ = 10\sqrt{3}\,m}$

$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{100 + 300}$

$\boxed{PQ = 20m}$

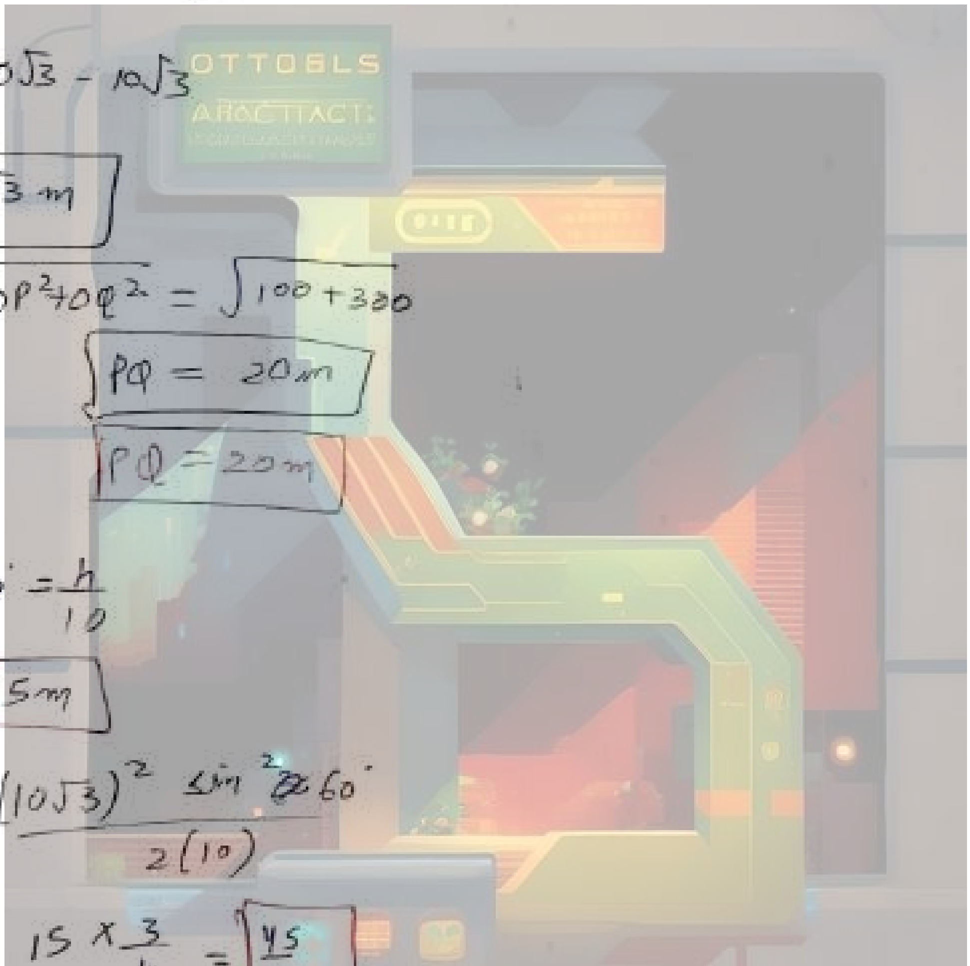
$\boxed{PQ = 20m}$

$\sin 30^\circ = \frac{h}{10}$

$\boxed{= 5m}$

$H = \frac{(10\sqrt{3})^2 \sin^2 60^\circ}{2(10)}$

$H = 15 \times \frac{3}{4} = \boxed{\frac{45}{4}}$



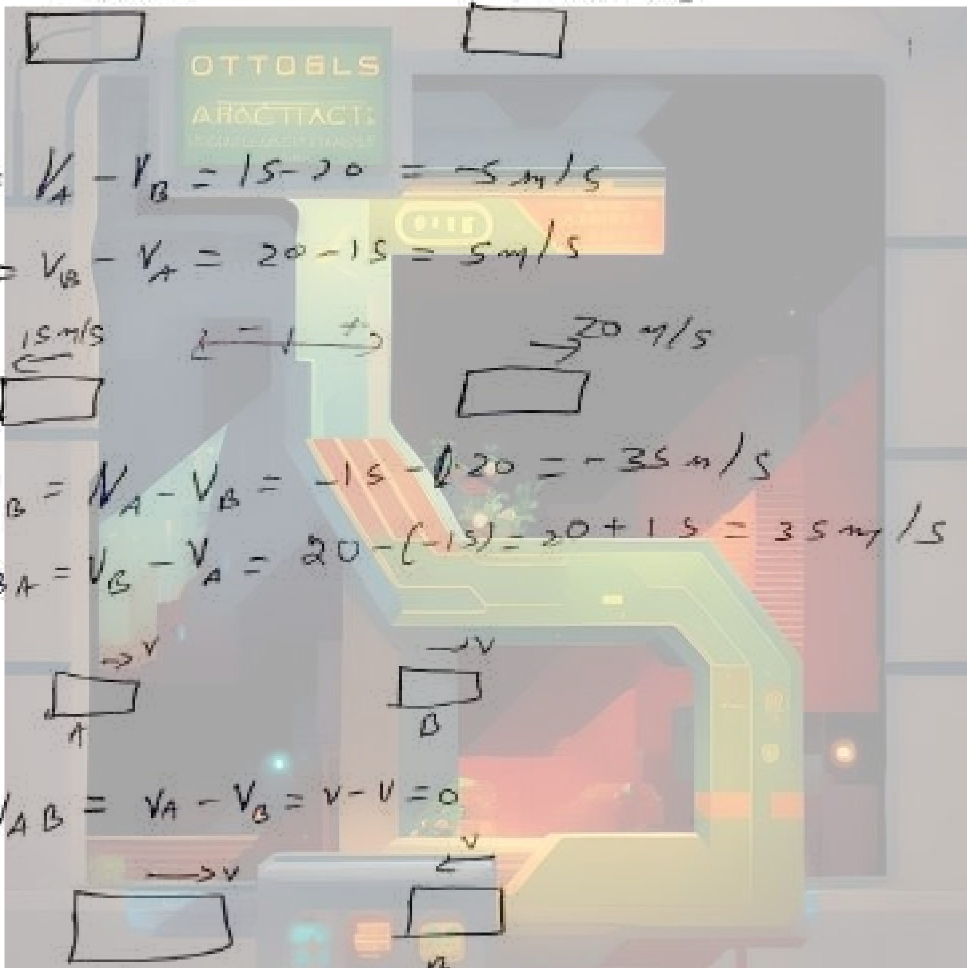
Relative Velocity

$$\boxed{\vec{V}_{obj,\,observer} = \vec{V}_{obj} - \vec{V}_{observer}}$$

→ Velocity of Object with respect to observer.

→ If distance increases, velocity of separation else velocity of approach

eg.



eg ①

$\xrightarrow{15}$ m/s $\longrightarrow$ 20 m/s

$V_{AB} = V_A - V_B = 15 - 20 = -5$ m/s

$V_{BA} = V_B - V_A = 20 - 15 = 5$ m/s

eg ②

15 m/s $\longleftarrow$ $\longrightarrow$ 20 m/s

$V_{AB} = V_A - V_B = -15 - 20 = -35$ m/s

$V_{BA} = V_B - V_A = 20 - (-15) = 20 + 15 = 35$ m/s

eg ③.

$\rightarrow v$ A $\rightarrow v$ B

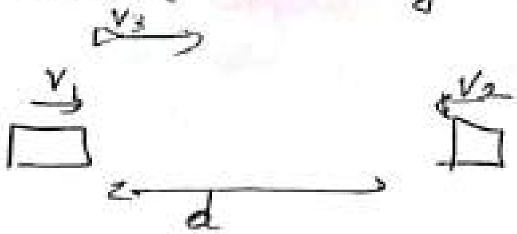
$V_{AB} = V_A - V_B = v - v = 0$

④ -

$\rightarrow v$ A $\leftarrow v$ B

$V_{AB} = V_A - V_B = v - (-v) = v + v = 2v$

Q find distance travelled by bird till car meets.



$V_{AB} = V_1 - (-V_2) = V_1 + V_2$

$\text{Time} = \frac{\text{distance}}{\text{speed}}$

$\boxed{\text{Time} = \frac{d}{V_1 + V_2}}$

distance by bird = speed × time

$= V_3 \times \frac{d}{V_1 + V_2}$

$\boxed{= \frac{V_3 d}{V_1 + V_2}}$ ✓

Q

$V_1 + V_2 = V_3$

$V_1 = \frac{l}{t_1}$

$V_2 = \frac{l}{t_2}$

$V_3 = \frac{l}{t_1} + \frac{l}{t_2}$

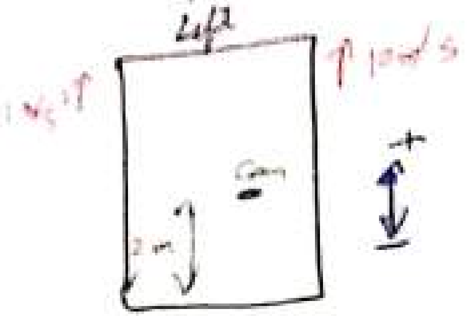
$\frac{l}{V_3} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$

$= \frac{t_2 l + t_1 l}{t_1 + t_2}$

$\frac{l}{V_3} = t_3 = \frac{l}{\frac{l(t_1 + t_2)}{t_1 + t_2}}$

$\boxed{= \frac{t_1 t_2}{t_1 + t_2}}$ ✓

Q A coin is dropped inside lift. Find time it takes to reach the floor ($g = 10\ m/s^2$)



$u_{relative} = u_{c,l} = u_c - u_l = 10 - 10 = 0\ m/s$

$a_{rel} = a_{c,l} = a_c - a_l = -10 - (+1) = -11\ m/s^2$

$s_{rel} = -2\ m$

$s = ut + \frac{1}{2} a t^2$

$s_{rel} = u_{rel} t + \frac{1}{2} a_{rel} t^2$

$-2 = -\frac{11}{2} t^2$

$\frac{4}{11} = t^2$

$\boxed{t = \frac{2}{\sqrt{11}}\ s}$

Q

$a_{rel} = -8$

$u_{rel} = 0$

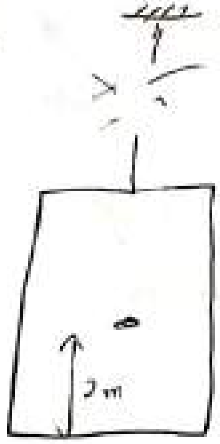
$s_{rel} = -2$

$+2 = \frac{1}{2} \times 8 t^2$

$\frac{4}{8} = \frac{1}{2} = t^2$

$\boxed{t = \frac{1}{\sqrt{2}}\ s}$

Q



~~a~~ = $V_{cl} = 0$ m/s

$a_{c,l} = -10 - (-10) = -10 + 10 = 0$

$-2 = 0 + \frac{1}{2}(0)t^2$

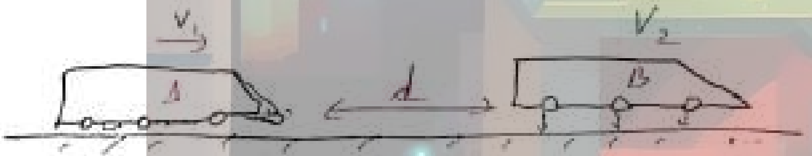
$-2 = 0t^2$

$\frac{-2}{0} = t^2$

$t^2 = \infty$

$\boxed{t = \infty\ s}$ The coin will never reach the floor of lift.

Q



$V_1 > V_2$ (initially)

A applies break for $a = -\alpha$ within distance is d, find min d to avoid collision. Speed of A must be equal to V_2 after relative distance d.

$v_3 = V_1 - V_2 = v_{rel}$

$a = -\alpha - 0 = -\alpha$

$S = d$

~~$d = v_3 t + 0 - \frac{1}{2}\alpha t^2$~~

$0 - V_3^2 = 2\times(-\alpha)\times d$

$$\boxed{\frac{-(V_1 - V_2)^2}{-2\alpha} = d}$$

Q A platform is moving upwards with a constant acceleration of $2 m/s^2$. At $t = 0$, a boy standing on platform throws a ball upwards with a relative speed of $8 m/s$. At this instant, platform was at height of 4 m from the ground and was moving with a speed of $2 m/s$. Find

a) when & where ball strikes the platform

b) Max height of ball from ground

c) Max height of ball from platform.

$a_{rel} = \cancel{8}\ -12 m/s^2$

$u_{rel} = 8 m/s$

$v = 0$

$\frac{-8}{-12} = t$

$t = \frac{2}{3}$ (to go up)

$\boxed{t = \frac{4}{3}}$ a)

$S = 2 \times \frac{4}{3} + \frac{1}{2} \times 2 \times \frac{4}{3} \times \frac{4}{3}$

$S = \frac{8 \times 3}{9} + \frac{16}{9}$

$S = \frac{40}{9} + 4$

$\boxed{S = 8.66 m}$ a)

$v + 100 = 2 \times 10 \times h$ (from ground) $+ 4$

$\frac{100}{20} + 4 = h$

$h = 5 + 4$

$\boxed{h = 9 m}$ b)

w.r.t platform, max ball height will be for velocity of ball = velocity of platform.

$U_{BP} = 8 m/s$

$V_{B\cdot P} = 0$

$a_B P = -12 m/s^2$

$S_B P = ?$

$v^2 - u^2 = 2as$

$0 + 64 = 2 \times 12 \times h$

$\frac{64}{24} = h$

$\boxed{h = \frac{8}{3} m}$ c)

(28)

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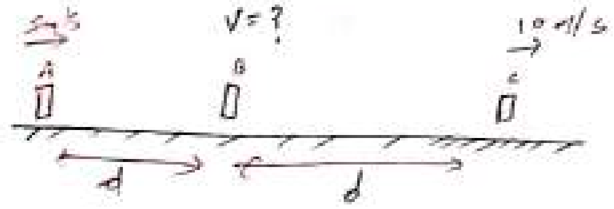
Q

~~when A catches B, sep~~
when B catches C, separation between A & C is 3d.
find speed of B.



Time taken by B to catch C, $t_1 = \frac{d-0}{v-10}$

A & C

→ change in separation

$10-5 = \frac{3d-2d}{T_1}$

$10-5 = \frac{d}{\frac{d}{v-10}}$

$10-5 = v-10$

$5+10 = v$

$\boxed{v = 15\ m/s}$

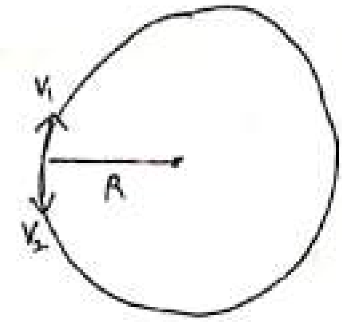
Q

find the speed with which A should chase C to catch it in 100 s.

$(v-10)\times 100 = 1000$

$\boxed{v = 20\ m/s}$

Q



After what time will they meet

$$\frac{2\pi R}{V_1+V_2}$$

rel Velocity $=$ ~~$V_1 - V_2$~~ $V_1 - (-V_2) = V_1 + V_2$

distance $= 2\pi R$ (circumference)

Time $= \frac{dis}{Time}$

$$\boxed{= \frac{2\pi R}{V_1+V_2}}$$

Q

$V_1 = 20$

$V_2 = 30$

$V_{rel} = 30 + 20$
$= 50$

distance $= 120 + 130$
$= 250$

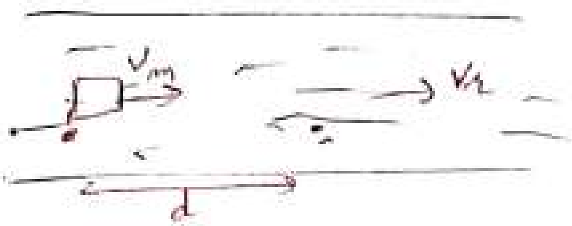
Time $= \frac{\text{distance}}{\text{~~Time~~ Speed}}$

$= \frac{250}{50}$

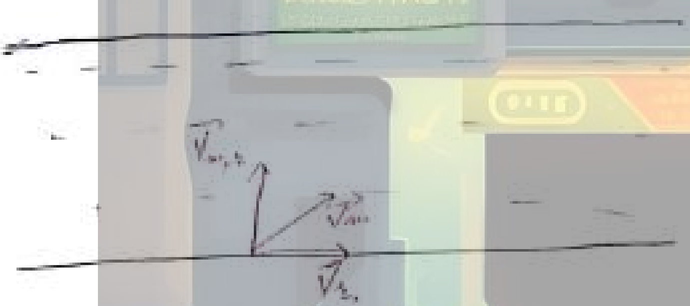
$$\boxed{= 5\,s}$$

river man problem.

upstream: against the river flow
downstream: with the river flow



$t_{down} = \dfrac{d}{V_m + V_r}$ (A → B) $\quad$ $T_{up} = \dfrac{d}{V_m - V_r}$ (B → A)

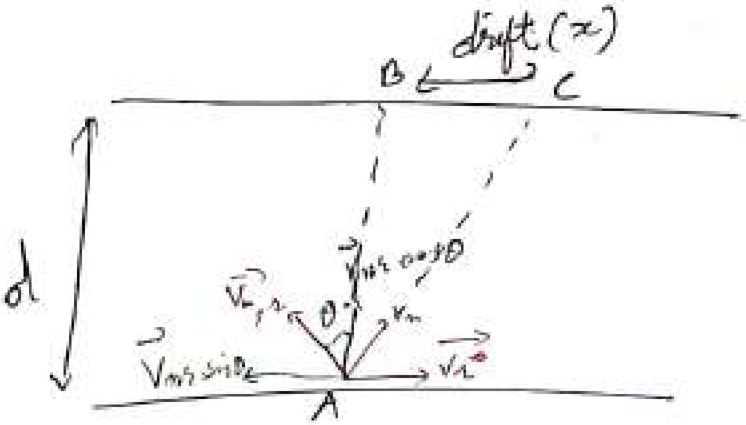


$\vec{V}_{m,r}$ = velocity of man w.r.t river (man generated v in still water)

$\vec{V}_r$ = Velocity ~~f~~ of flow of river

$\vec{V}_m$ = Net velocity of man

$\vec{V}_m = \vec{V}_{m,r} + \vec{V}_r$



Time Taken = $\boxed{\dfrac{d}{V_{mr}\cos\theta}}$

drift $(x) = (V_r - V_{mr}\sin\theta) \times t$

$$\boxed{x = \frac{(V_r - V_{mr}\sin\theta)\, d}{V_{mr}\cos\theta}}$$

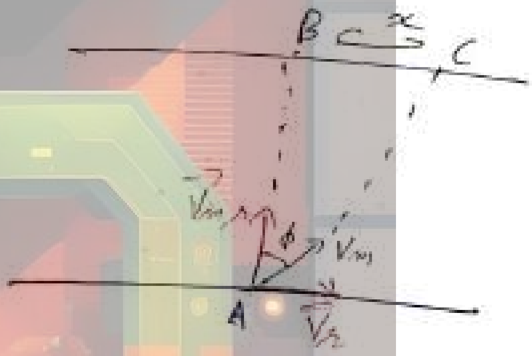
Case I :- For minimum time

$\cos\theta = 1$ (max)

$\theta = 0^\circ$

$\boxed{V_m = \sqrt{V_{mr}^2 + V_r^2}}$

man should swim ⊥ to river flow

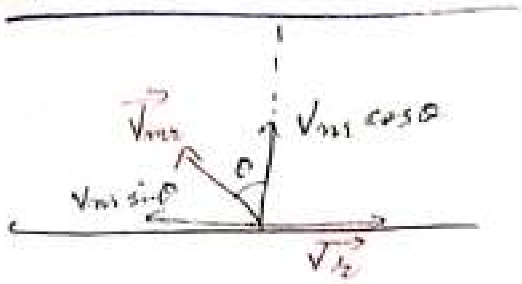


$x = V_r t$

$\boxed{x = \dfrac{V_r d}{V_{mr}}}$

$\boxed{\tan\phi = \dfrac{V_r}{V_{mr}}}$

Case 2.



To cross river with zero drift.

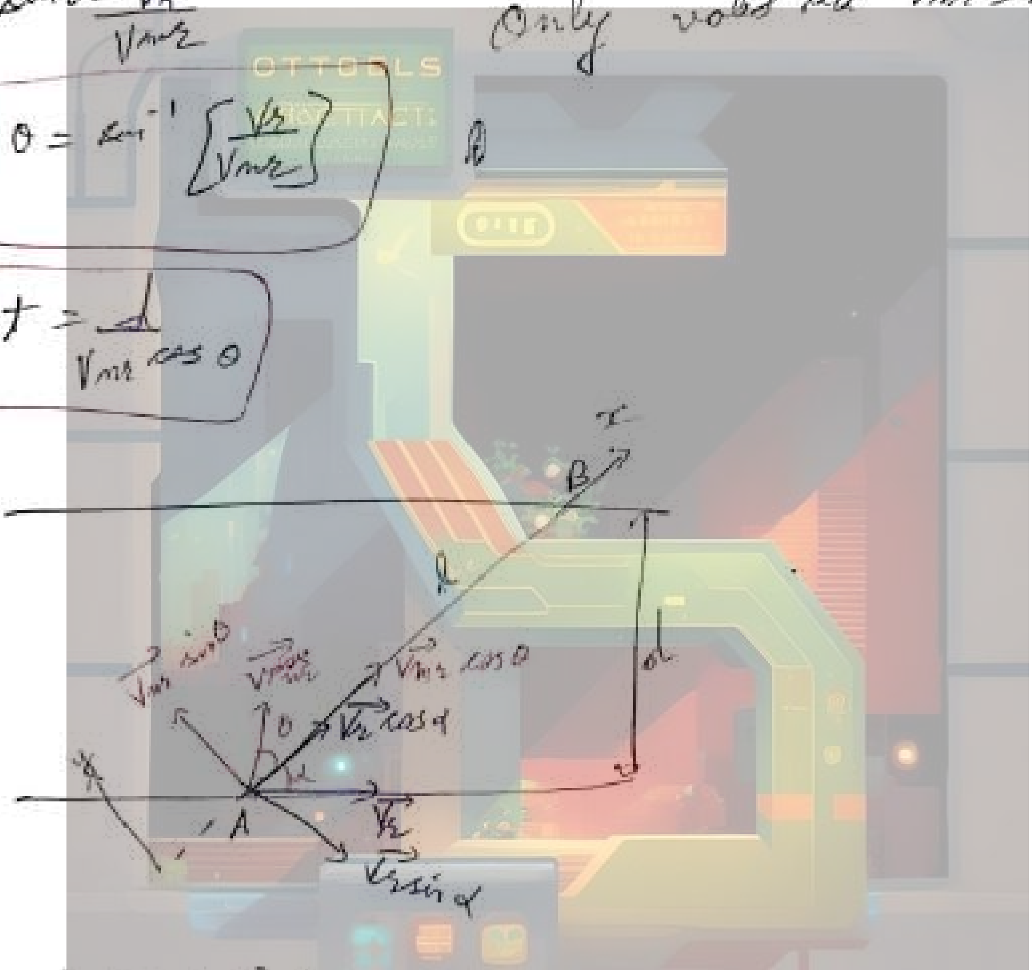
$V_r = V_{mr}\sin\theta$

$\sin\theta = \frac{V_r}{V_{mr}}$ Only valid till $V_{mr} > V_r$.

$$\boxed{\theta = \sin^{-1}\left[\frac{V_r}{V_{mr}}\right]}$$

$$\boxed{t = \frac{d}{V_{mr}\cos\theta}}$$

Q



$V_{mr}\sin\theta = V_r\sin\alpha$

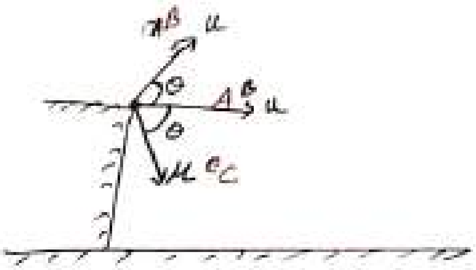
$$\boxed{\sin\theta = \frac{V_r}{V_{mr}}\sin\alpha}$$

→ Angle with which boat will be steered.

$$\boxed{t = \frac{l}{V_r\cos\alpha + V_{mr}\cos\theta}}$$

Extra Questions (Ch-1, 2, 3)

Q1.



A) $V_A > V_B > V_C$

B) $V_A = V_B = V_C$

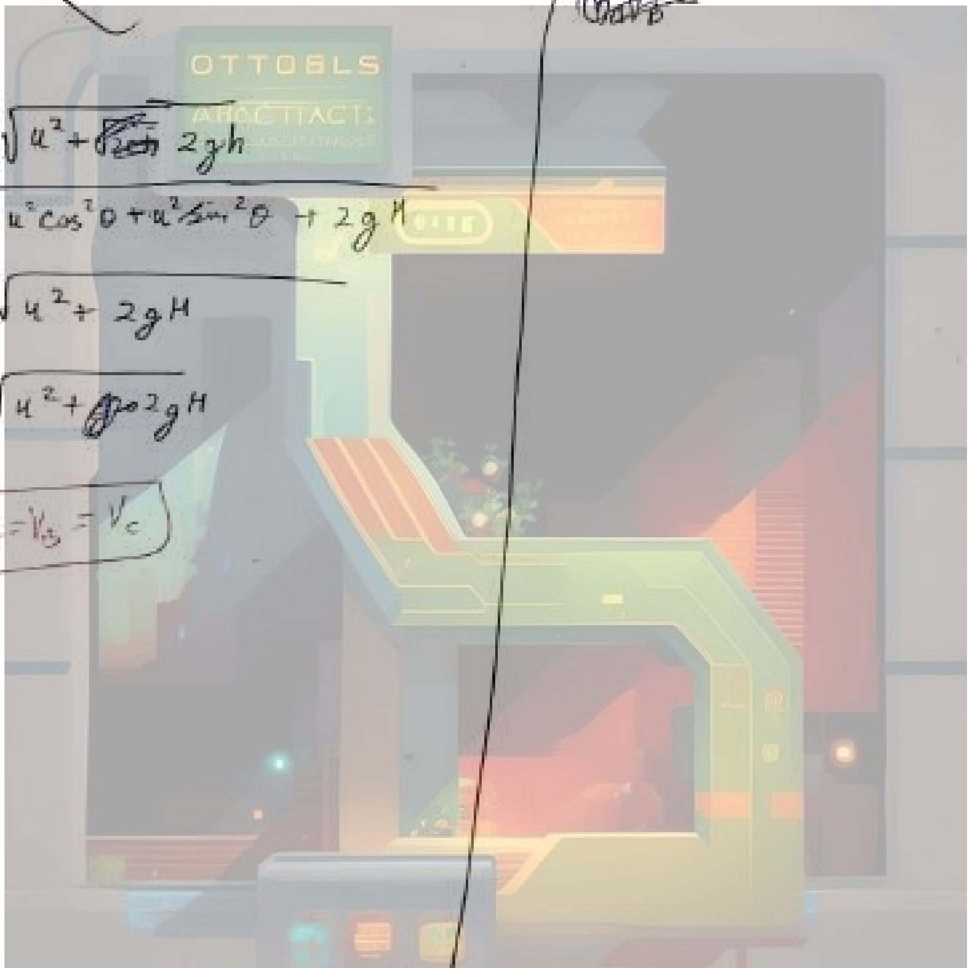
C) $V_C > V_B > V_A$

D) cannot find.

$$\cancel{V_{B_y} = \sqrt{2gh} = \sqrt{20h}}$$

$$\cancel{V_{A_y} = u\sin\theta + 10}$$

$$\cancel{V_C =}$$



$$\vec{V}_{BA} = \sqrt{u^2 + 2gh}$$

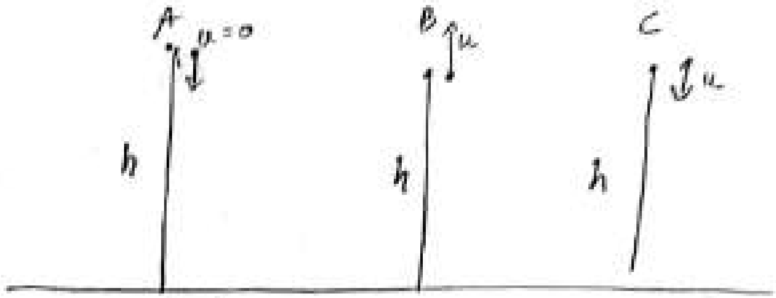
$$V_B = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + 2gH}$$

$$= \sqrt{u^2 + 2gH}$$

$$V_C = \sqrt{u^2 + 2gH}$$

$$\boxed{B)\ V_A = V_B = V_C}$$

Q2.



a) velocity relation

$V_A^2 = 2gH$

$V_B^2 = 2gH + u^2$

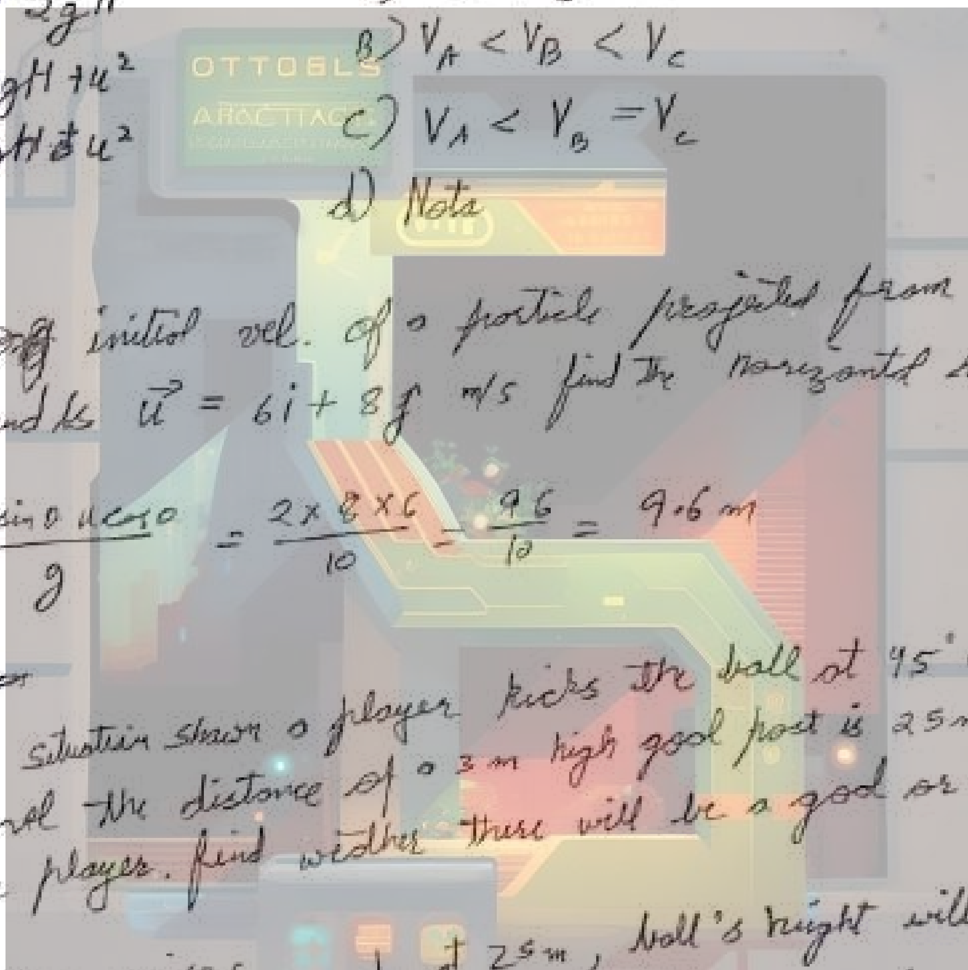
$V_C^2 = 2gH + u^2$

A) $V_C > V_B > V_A$

B) $V_A < V_B < V_C$

C) $V_A < V_B = V_C$

D) Nota



Q3, If initial vel. of a particle projected from ground is $\vec{u} = 6i + 8j$ m/s find the horizontal range

$R = \frac{u^2 \sin 2\theta}{g} \cdot \frac{2u\sin\theta\, u\cos\theta}{g} = \frac{2 \times 8 \times 6}{10} = \frac{96}{10} = 9.6$ m

Q4, In the situation shown a player kicks the ball at 45° with 20 m/s vel the distance of 0.3 m high goal post is 25 m from the player. find wether there will be a goal or not

$= 25 - \frac{10 \times 625}{2 \times 20 \times 20 \times \frac{1}{2}}$

$= 25 - \frac{625}{40}$

$= \frac{1000 - 625}{40}$

$= \frac{375}{40}$

$= \frac{93.\text{something}}{10}$

$y = 9.3$ something

at 25 m, ball's height will be 9.3

so there will be no goal.

Q5. A particle is thrown from ground at angle 30° with horizontal at 40 m/s find time for which it will be at the height more than 20 m

$20 = \cancel{40}^{20} \times \frac{1}{2} t^2 - 5t^2$

$\frac{\cancel{40}^{20} \times \cancel{40}^{16} \cdot \frac{1}{4}}{4} = 2 \times 10$

$5t^2 - 20t + 20 = 0$

$t^2 - 4t + 4 = 0$

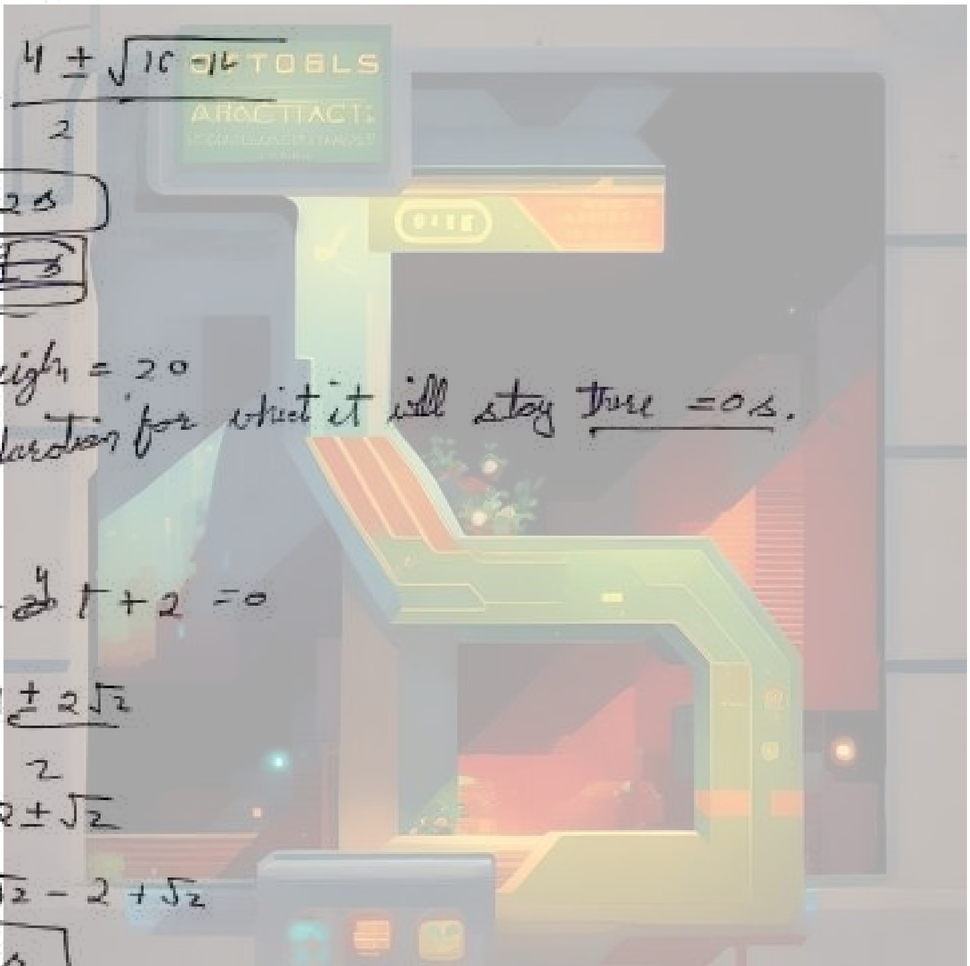
$t = \frac{4 \pm \sqrt{16 - 16}}{2}$

$\boxed{t = 2s}$

$\boxed{\cancel{t = 2 s}}$

Max height = 20

time duration for which it will stay there = 0 s.



$\cancel{\theta} t^2 - \cancel{2}^{4} t + 2 = 0$

$t = \frac{4 \pm 2\sqrt{2}}{2}$

$t = 2 \pm \sqrt{2}$

$2 + \sqrt{2} - 2 + \sqrt{2}$

$\boxed{2\sqrt{2}\ s}$

Q6. चोर is running away on a straight road at 9 m/s. A police man chases him on a motorcycle at 10 m/s if the distance between them is 100 m how long will it take for police to catch the thief.

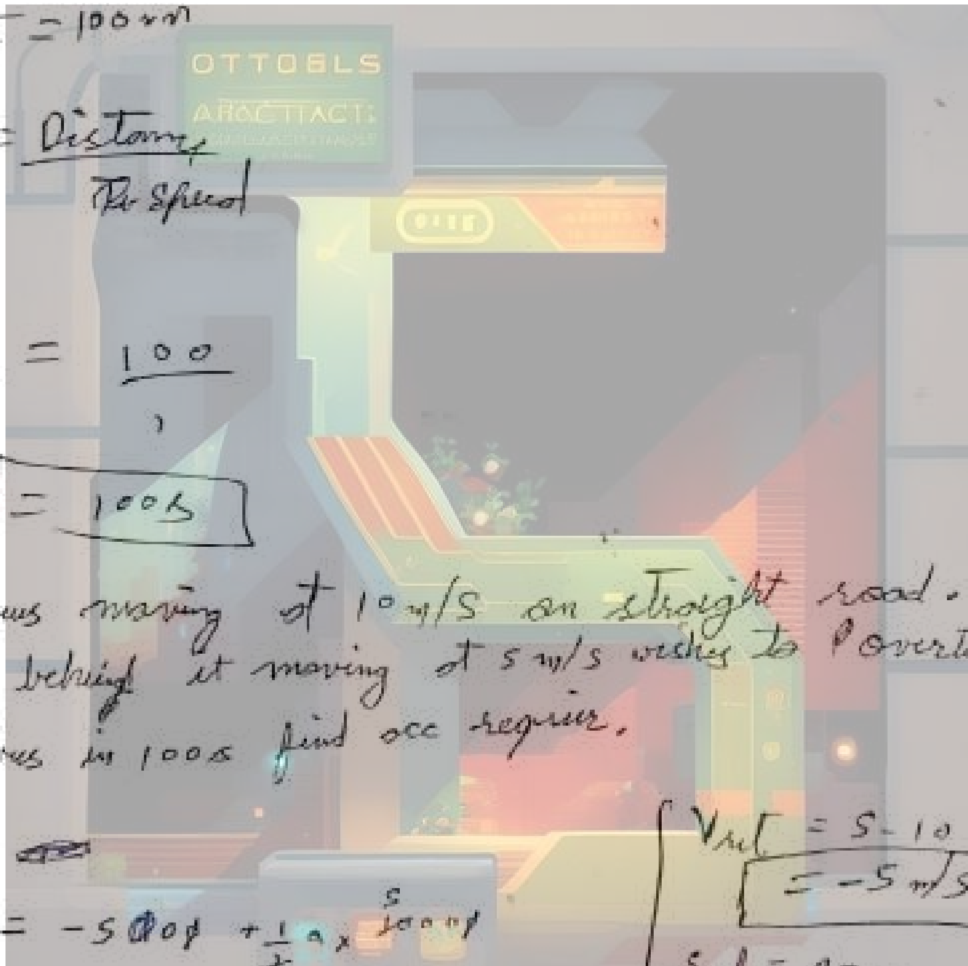
$V_t = 9$

$V_p = 10$

$V_{rel} = 10 - 9$

$= 1\ m/s$

& $S_{rel} = 100\ m$



$\text{Time} = \dfrac{\text{Distance}}{\text{Rel speed}}$

$\text{Time} = \dfrac{100}{1}$

$\boxed{= 100\,s}$

Q7. A bus moving at 10 m/s on straight road. a bikes 80 m behind it moving at 5 m/s wishes to overtake the bus in 100 s find acc require.

$V_{rel} = 5 - 10$

$\boxed{= -5\ m/s}$

$S_{rel} = 80\,m$

$t = 100\,s$

$86 = -5\cancel{00} + \frac{1}{2} a \times \cancel{100}^{5}$

$8 + 50 = a \times 500$

$a = \dfrac{58}{500}$

$a = \dfrac{58}{5 \times 100}$

$a = 11.6$

$\boxed{a = 0.116\ m/s^2}$

H.W.

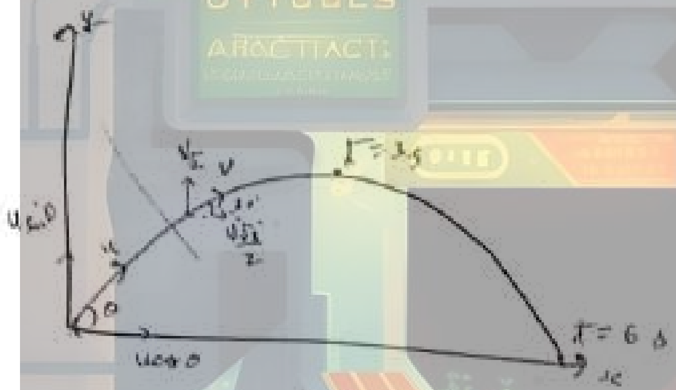
Q. A body is projected with velocity u at angle θ with horizontal. The body makes angle 30° with horizontal at $t = 2s$ & after 1s reaches max height.

a) angle of projection

b) speed of projection.

~~$v = u\sin\theta = 20$~~

~~$u\cos\theta = \ldots 20 \times \sqrt{3}$~~



$$T = \frac{2u\sin\theta}{g}$$

$$\frac{6 \times 10}{2} = u\sin\theta$$

$$30 = u\sin\theta$$

$$\boxed{u = \frac{30}{\sin\theta}}$$

$$\frac{v}{2} = u\sin\theta - gt$$

$$= 30 - 20$$

$$\frac{v}{2} = 10$$

$$v = 20$$

$$\frac{20\sqrt{3}}{2} = u\cos\theta$$

$$\cancel{20\sqrt{3}}$$

$$10\sqrt{3} = 30\cot\theta$$

$$\frac{30}{10\sqrt{3}} = \tan\theta$$

$$\sqrt{3} = \tan\theta$$

$$\boxed{\theta = 60^\circ}$$

$$u = \frac{30}{\sin 60}$$

$$= \frac{30}{\sqrt{3}} \times 2$$

$$\boxed{u = 20\sqrt{3}\, m/s}$$

Q1. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min. with a drift 120 m. If he crosses the river taking shortest route he takes 12.5 min. find velocity of boat w.r.t water.

$\theta = 0$

$600 = \frac{d}{V_{mr}}$

$120 = 600\, u_r$

$u_r = \frac{120}{600}$ m/s

$\boxed{u_r = \frac{1}{5}\ m/s}$

$\cancel{\sin\theta = \frac{V_{mr}\cos\theta}{u_r}}$

$\cancel{\sin\theta = V_{mr} \times 5}$

$\cancel{\sin\theta = 5 V_{mr}}$

$\cancel{750 = \frac{d}{V_{mr}\cos\theta}}$

$\cancel{750 = \frac{15 d}{V_{mr}}}$

$750 = \frac{d}{V_{mr}\cos\theta}$

$\frac{600}{750} = \cos\theta$

$\frac{4}{5}$

$\cos\theta = \frac{4}{5}$

$\cancel{750 = \frac{d}{V_{mr}} \times \frac{5}{4}}$

$\cancel{600 = \frac{d}{V_{mr}}}$

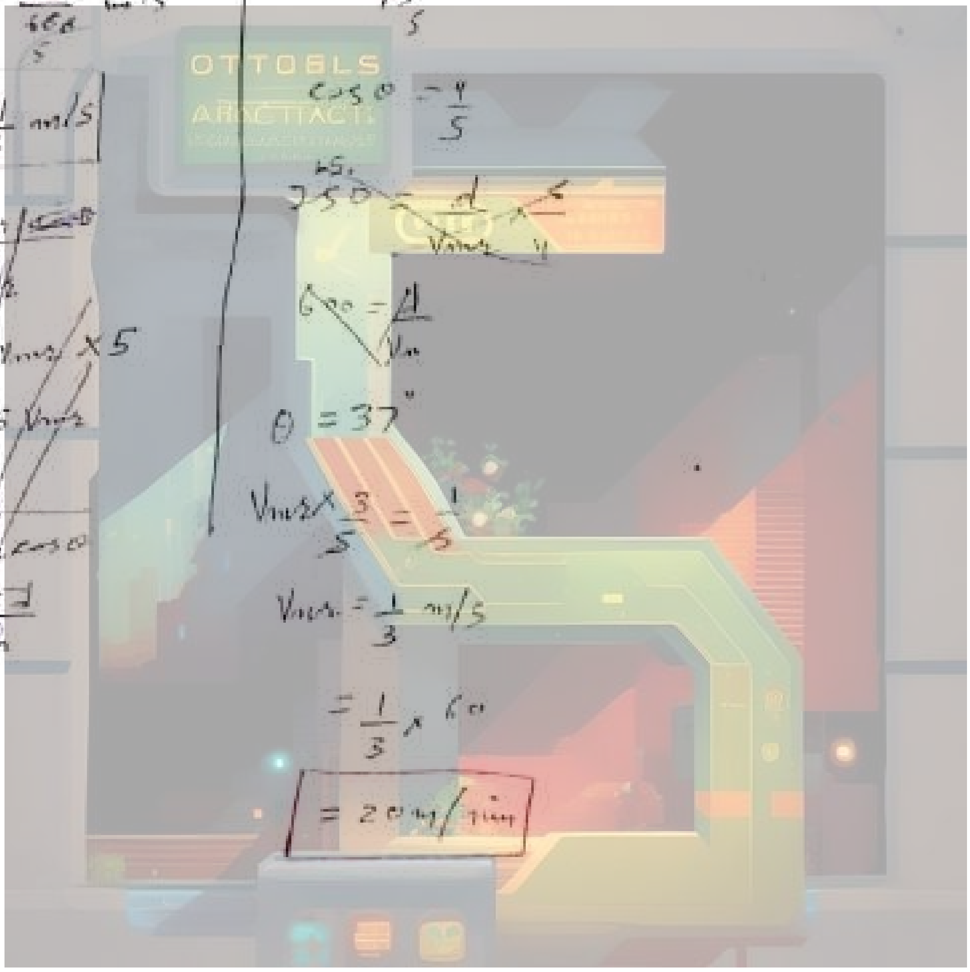
$\theta = 37^\circ$

$V_{mr} \times \frac{3}{5} = \frac{1}{5}$

$V_{mr} = \frac{1}{3}$ m/s

$= \frac{1}{3} \times 60$

$\boxed{= 20\ m/min}$



Q2. A man can swim at the rate of 5 km/hr in still water. A 1 km wide river flows at 3 km/h. The man wishes to swim across the river directly opposite to starting point.

a) along what direction should the man swim?

b) What should be his resultant velocity?

c) Find time taken to cross the river.

$d = 1\ km$

$V_r = 3\ km/h$

$V_{mr} = 5\ km/hr$

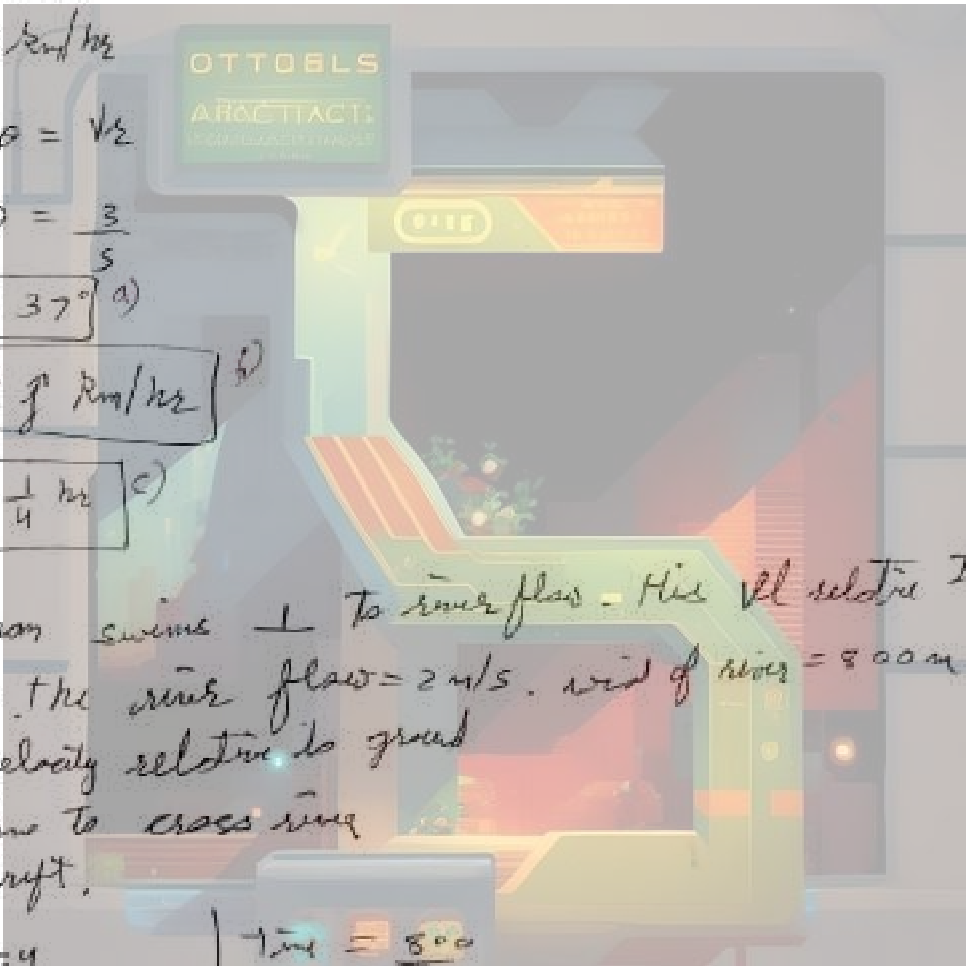
$V_{mr} \sin\theta = V_r$

$\sin\theta = \frac{3}{5}$

$\boxed{\theta = 37^\circ}$ a)

b) $\boxed{\vec{v} = 4\hat{j}\ km/hr}$ b)

$\boxed{Time = \frac{1}{4}\ hr}$ c)



Q3. A man swims $\perp$ to river flow. His vel relative to water is 4 m/s the river flow = 2 m/s. width of river = 800 m

a) velocity relative to ground

b) time to cross river

c) drift.

$V_{mr} = 4$

$V_r = 2$

$d = 800$

$V_m = \sqrt{16+4}$

$\boxed{= \sqrt{20}\ m/s}$ a)

$time = \frac{800}{4}$

$\boxed{= 200\ s}$ b)

drift = $\frac{distance}{~~Ising~~\ Speed}$

$= \frac{800}{\cancel{4}\ 2}$

$= \frac{800}{2}$

$\boxed{= 400\ m}$ c)

(40)

Q4. A swimmer crosses a 200 m wide river & return 10 minutes at a point away from starting point (downstream). Find velocity of man with respect to ground if he heads towards the bank & at right angles all the times.

$d = 200\,m$

$T = 300\,s$

$x = 150\,m$

$\theta = 90$

$V_{mr} = \frac{200}{300}$

$= \frac{2}{3}\,m/s$

$\cancel{\frac{225}{2} \times 3 = V_r}$

$\frac{150}{300} = \frac{1}{2} = V_r$

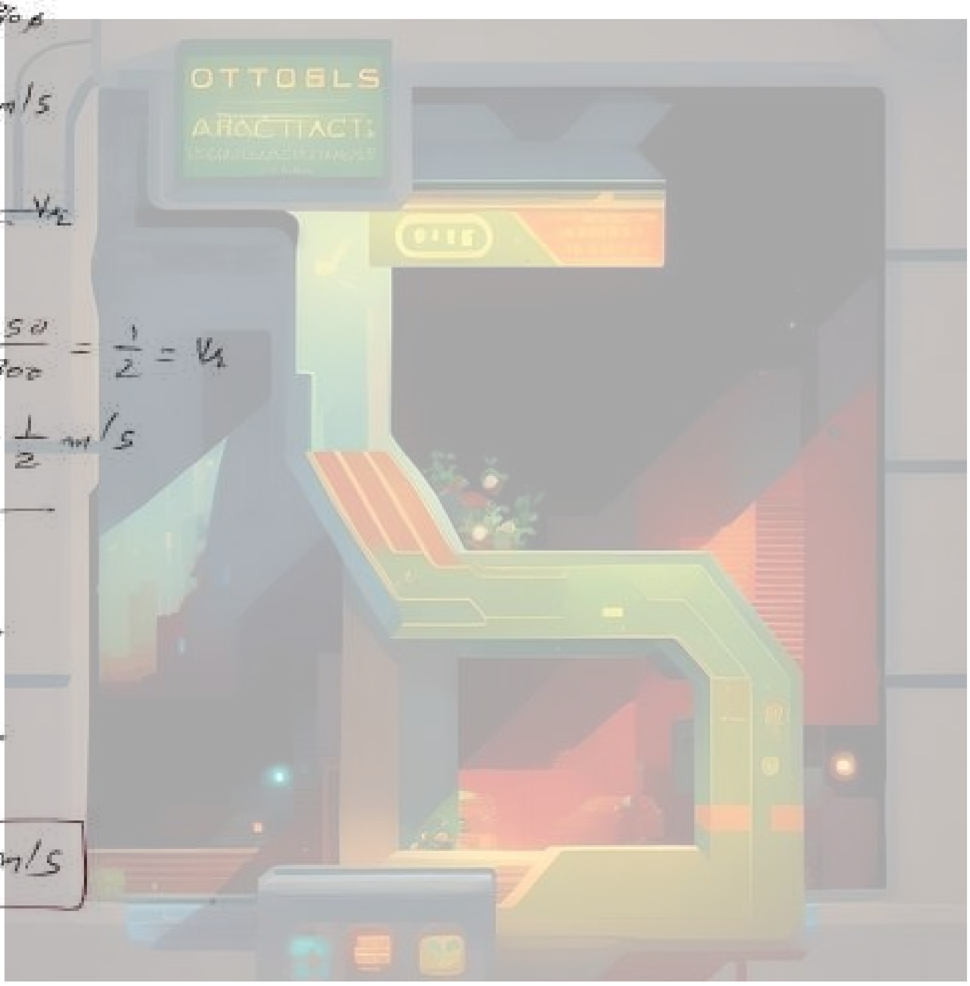
$V_r = \frac{1}{2}\,m/s$

$\sqrt{\frac{1}{4} + \frac{4}{9}}$

$\sqrt{\frac{9+16}{36}}$

$\sqrt{\frac{25}{36}}$

$\boxed{\frac{5}{6}\,m/s}$



Q6.

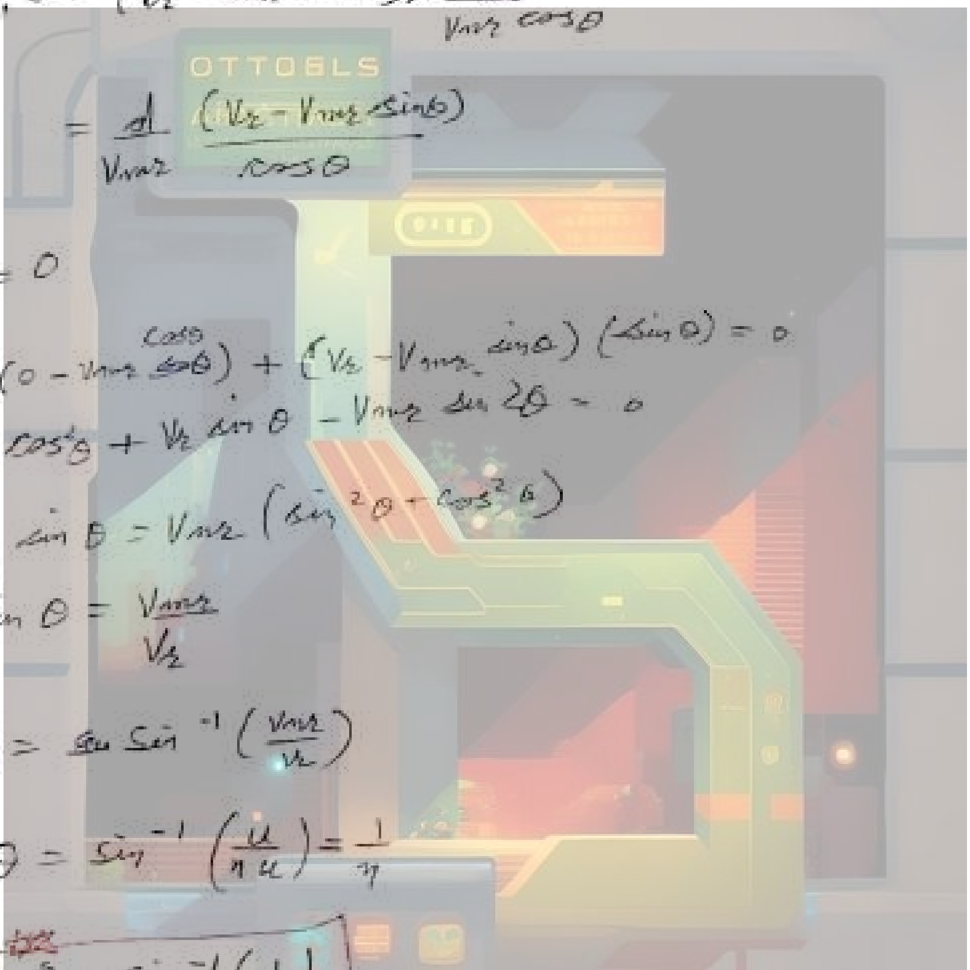
Q6. A boat moves relative to water with velocity v which is n times less than river flow u. At what angle to river flow must the boat move to minimise drifting?

$V_{mr} = \frac{u}{n}$

$V_r = u$

Time taken to cross river, $t = \frac{d}{V_{mr}\cos\theta}$

Drift, $x = (V_r - V_{mr}\sin\theta) \times \frac{d}{V_{mr}\cos\theta}$



$$= \frac{d}{V_{mr}} \frac{(V_r - V_{mr}\sin\theta)}{\cos\theta}$$

$$\frac{dx}{d\theta} = 0$$

$$\cos\theta(0 - V_{mr}\cos\theta) + (V_r - V_{mr}\sin\theta)(\sin\theta) = 0$$

$$-V_{mr}\cos^2\theta + V_r\sin\theta - V_{mr}\sin^2\theta = 0$$

$$V_r\sin\theta = V_{mr}(\sin^2\theta + \cos^2\theta)$$

$$\sin\theta = \frac{V_{mr}}{V_r}$$

$$\theta = \sin^{-1}\left(\frac{V_{mr}}{V_r}\right)$$

$$\theta = \sin^{-1}\left(\frac{u}{nu}\right) = \frac{1}{n}$$

$$\boxed{\theta = \sin^{-1}\left(\frac{1}{n}\right)}$$

Angle with river flow $= \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{n}\right)$

Q7. Two swimmers leave point A on the bank of river to reach point B right cross other bank. One of them crosses river along AB and other swims at right angles to river and then walks the drift distance to get to point B. find velocity u of his walking if both arrive at B at same time. ($V_r = 2$ km/hr, $V_{mr} = 2.5$ km/h)

$\sin\theta = \frac{\cancel{2}\,4}{\cancel{2.5}\,5}$

$\underline{\theta = 53^\circ}$

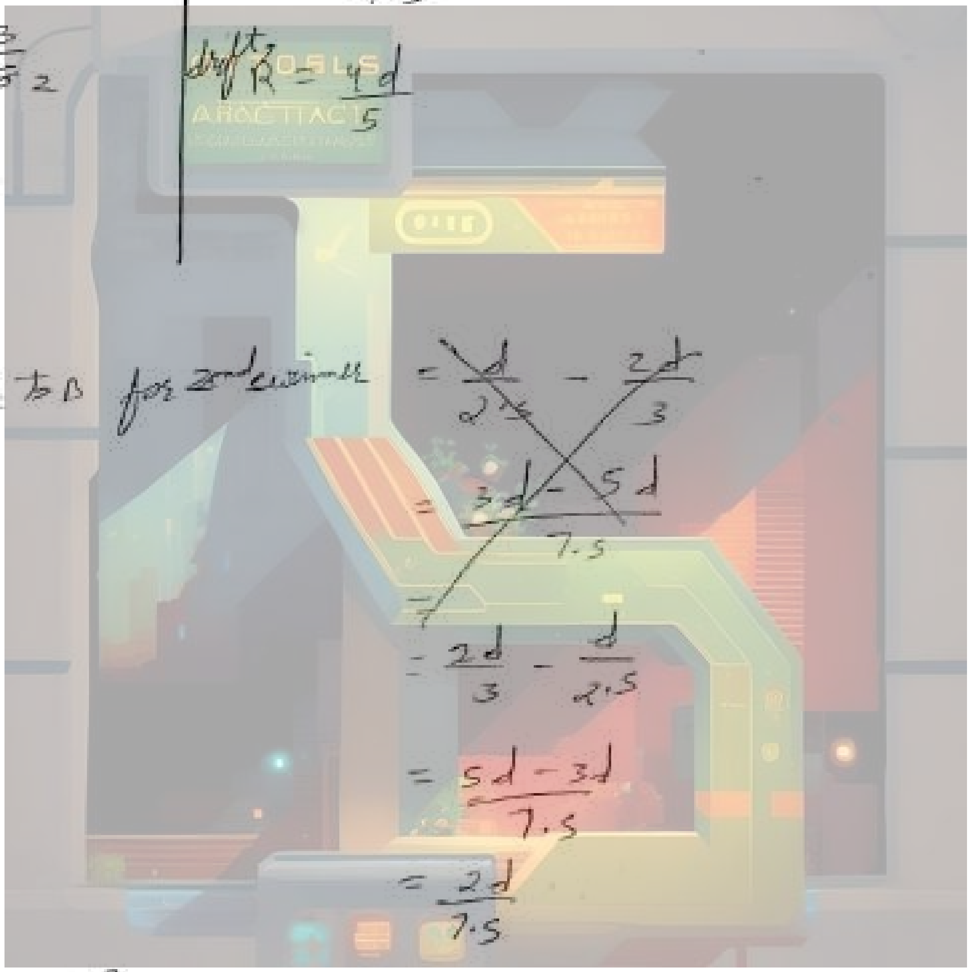
$T = \frac{d}{2.5 \times \frac{3}{5}}$

$T_1 = \frac{2d}{3}$

$T_2 = \frac{d}{2.5}$

$R = 2 \times \frac{d}{2.5}$

drift $R = \frac{4d}{5}$



Time to come to B for 2nd swimmer $= \cancel{\frac{d}{2.5} - \frac{2d}{3}}$

$= \cancel{\frac{3d - 5d}{7.5}}$

$= \frac{2d}{3} - \frac{d}{2.5}$

$= \frac{5d - 3d}{7.5}$

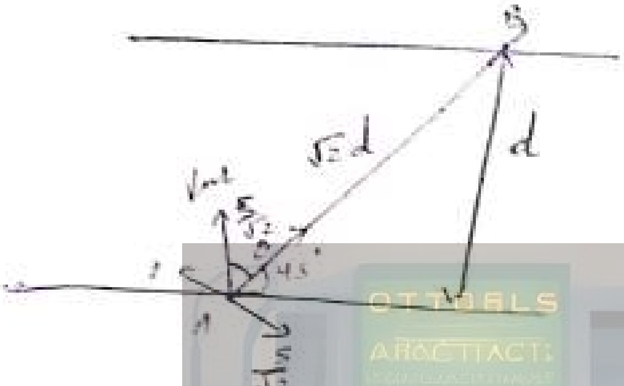
$= \frac{2d}{7.5}$

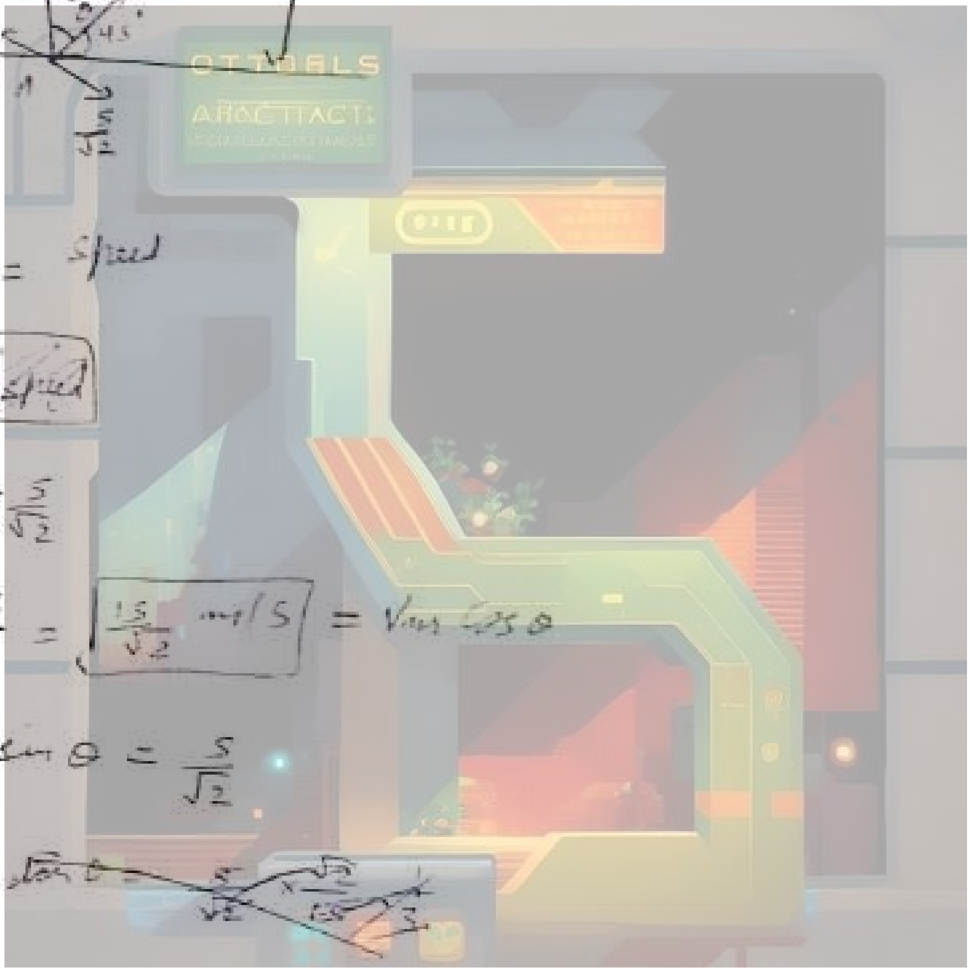
speed $= \frac{4d}{5} \times \frac{7.5}{2d}$

$= \frac{15}{5}$

$\boxed{= 3 \text{ km/h}}$

Q8. A swimmer crosses a river along the line making an angle of 45° with the direction of flow. Velocity of river water is 5 m/s. Swimmer takes 6 s to cross the river of width 60 m. Find velocity of swimmer w.r.t. water.





$$\frac{\sqrt{2}d}{6} = \text{speed}$$

$$\cancel{\boxed{10\sqrt{2} = \text{speed}}}$$

$$10\sqrt{2} - \frac{5}{\sqrt{2}}$$

$$\frac{20-5}{\sqrt{2}} = \boxed{\frac{15}{\sqrt{2}}\ m/s} = V_{mr}\cos\theta$$

$$V_{mr}\sin\theta = \frac{5}{\sqrt{2}}$$

$$\cancel{\tan\theta = \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{15} = \frac{1}{3}}$$

$$V_{mr}^2 = \frac{25}{2} + \frac{225}{2}$$

$$V_{mr}^2 = \frac{250}{2}$$

$$V_{mr} = \frac{5\sqrt{10}}{\sqrt{2}}$$

$$\boxed{V_{mr} = 5\sqrt{5}\ m/s}$$

H.W.

Pg 72 Q 29-45

Rain Man Problems

If rain is falling vertically with velocity $\vec{V_r}$ and ~~other~~ observer is moving horizontally with velocity $\vec{V_m}$, then velocity of rain w.r.t. man, $\vec{V_{r,m}} = \vec{V_r} - \vec{V_m}$

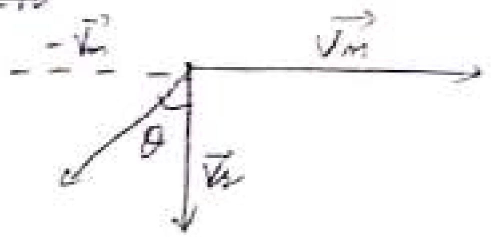
$$\vec{V_{r,m}} = \vec{V_r} - \vec{V_m} = \vec{V_r} + (-\vec{V_m})$$

Angle at which umbrella has to be held.

$$\boxed{\tan \theta = \frac{V_m}{V_r}}$$

$$V_{r,m} = \sqrt{V_m^2 + V_r^2}$$

Q Rain falls vertically at speed of 10 m/s. A man walks with a speed of 6 m/s on a horizontal road. find angle at which man should hold his umbrella to avoid getting wet?



$|\vec{V_r}| = 10\,m/s$

$\vec{V_r} = -10\hat{j}\,m/s$

$|\vec{V_m}| = 6\,m/s$

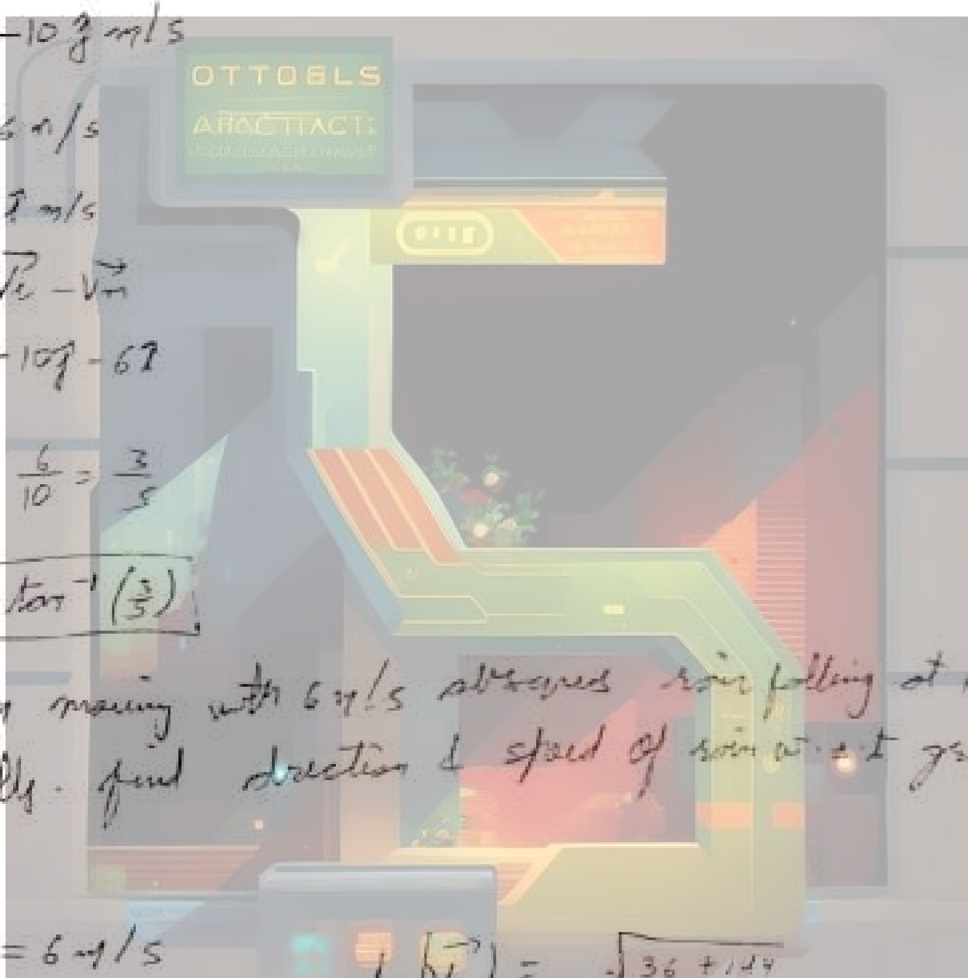
$\vec{V_m} = 6\hat{i}\,m/s$

$\vec{V_{r,m}} = \vec{V_r} - \vec{V_m}$

$= -10\hat{j} - 6\hat{i}$

$\tan\theta = \frac{6}{10} = \frac{3}{5}$

$\boxed{\theta = \tan^{-1}\left(\frac{3}{5}\right)}$



Q A man moving with 6 m/s observes rain falling at 12 m/s vertically. find direction & speed of rain w.r.t ground.

$v_m = 6\,m/s$

$v_{r,m} = 12$

$\vec{v_{r,m}} = -12\hat{j}$

$\vec{v_m} = 6\hat{i}$

$\vec{V_r} = \vec{V_m} + \vec{V_{r,m}}$

$= 6\hat{i} - 12\hat{j}$

$|\vec{V_r}| = \sqrt{36 + 144}$

$= \sqrt{180}$

$= \sqrt{3 \times 3 \times 2 \times 10}$

$= 3\sqrt{20}$

$\boxed{= 6\sqrt{5}\,m/s}$

$\cos\beta = \frac{-12}{6\sqrt{5}}$

$= \frac{-2}{\sqrt{5}}$

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\tan\theta = \frac{6}{12}$

$\tan\theta = \frac{1}{2}$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

Q3. Rain is falling with velocity of 20 m/s at an angle 30° with vertical. How fast should he move so that rain appears ⊥ to vertical to him?

$\vec{V_r} = 20\ m/s$

$\vec{V_r} = 20 \sin 30°\ \hat{i} - 20 \cos 30°\ \hat{j}$

$= 20 \times \frac{1}{2}\ \hat{i} - 20 \times \frac{\sqrt{3}}{2}\ \hat{j}$

$= 10\hat{i} - 10\sqrt{3}\ \hat{j}$

If rain is vertical,

$10 = V_m$

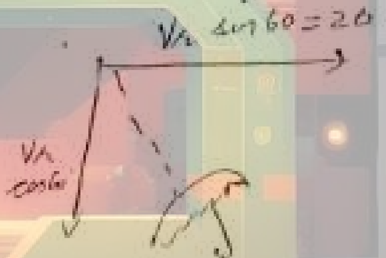
$\boxed{V_m = 10\ m/s}$

Q4. Nirjhar is standing on road has to hold his umbrella at 60° with vertical. He throws his umbrella and starts moving at 20 m/s. He finds that rain drops are hitting his head vertically. Find speed of rain drops w.r.t (a) road, (b) Nirjhar

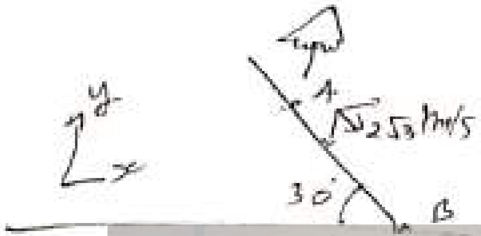
$\vec{V_r} \sin 60 = \frac{V_r \sqrt{3}}{2} = 20$

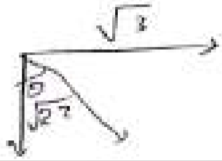
$\boxed{\vec{V_r} = \frac{40}{\sqrt{3}}}$ (a)

$V_r \cos 60 = \frac{40}{\sqrt{3}} \times \frac{1}{2} = \boxed{\frac{20}{\sqrt{3}}}$ (b)



Q The man is standing on a ramp with speed $2\sqrt{3}$ m/s and keeps umbrella vertical. Actual speed of rain is 5 m/s. At what angle should he keep his umbrella with vertical when he reaches & stops at B





$\vec{A} = \frac{2\sqrt{3}}{2}\hat{i} - 2\sqrt{3} \times \frac{\sqrt{3}}{2}\hat{j}$

$= \sqrt{3}\hat{i} - 3\hat{j}$

$\vec{R} = \sqrt{3}\hat{i} - \sqrt{22}\hat{j}$

$\frac{\sqrt{22}}{\sqrt{3}} = \tan\theta$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{22}}$

$\vec{A} = 3\hat{i} - \sqrt{3}\hat{j}$

$\vec{R} = 3\hat{i} - 4\hat{j}$

$\tan\theta = \frac{3}{4}$

$\boxed{\cancel{\theta = 30°}}$ $\boxed{\cancel{\theta = 37°}}$

$\boxed{\theta = 37°}$

Q Rain is falling with a speed 30 m/s. A person running in rain with a velocity of 5 m/s & wind is also blowing with a speed of 15 m/s (both from west). The angle with the vertical at which the person should hold his umbrella so that he may not get drenched is.

$V_{rm} = -20\hat{j} + 15 - 5\hat{i}$

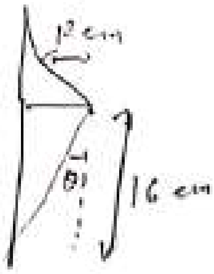
$= -20\hat{j} + 10\hat{i}$

$\tan\theta = \frac{10}{20}$

$= \frac{1}{2}$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

Q A wearing shot of extended length 12 cm is running in rain falling vertically with 10 m/s. find max speed with which man can run so that raindrops does not fall on his face. (length of face is 16 cm)



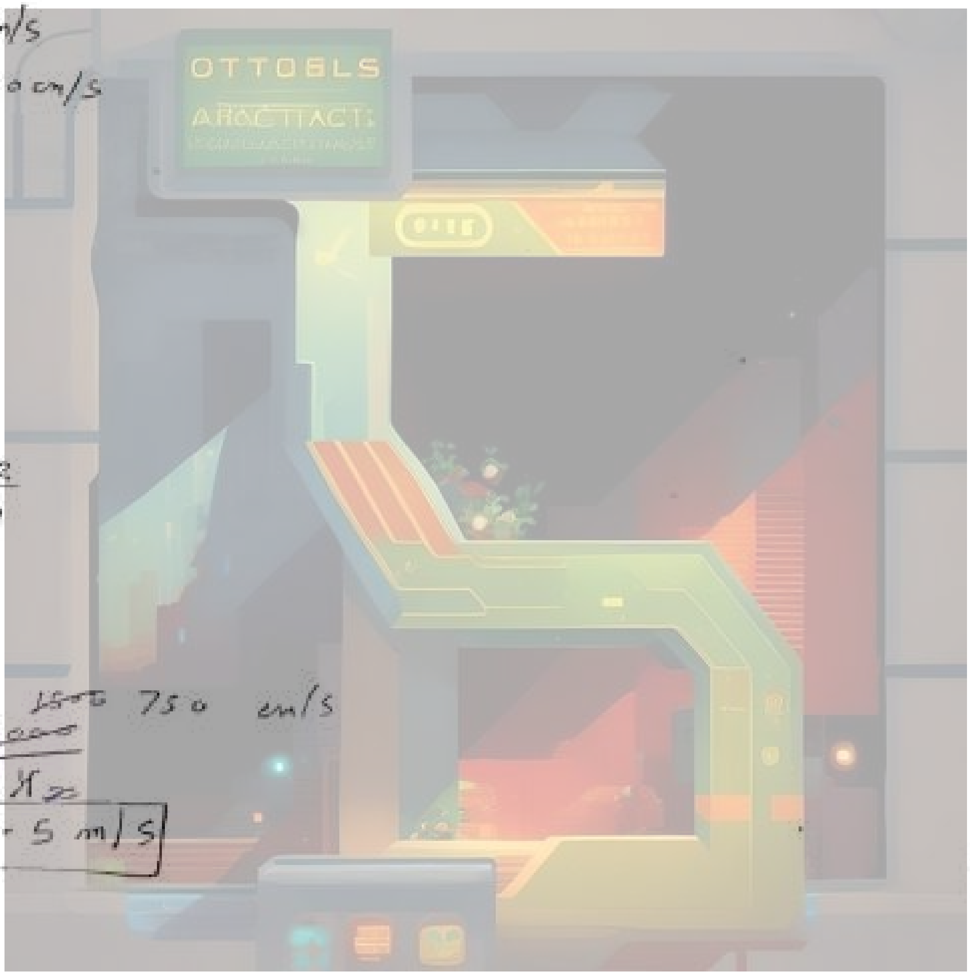
V_s (vertical) 10 m/s

$= 1000$ cm/s

$\tan\theta = \frac{12}{16}$

$\tan\theta = \frac{3}{4}$

$\theta = 37^\circ$



$\frac{V_m}{V_s(\text{vertical})} = \frac{3}{4}$

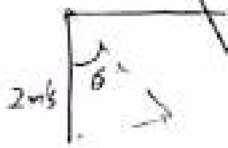
$\frac{V_m}{1000} = \frac{3}{4}$

$V_m = \frac{\cancel{3000}}{\cancel{4}}$ ~~1500~~ 750 cm/s

$\boxed{V_m = 7.5 \text{ m/s}}$

Q

a



$\tan\theta = \frac{\text{velocity}}{2}$

~~$\theta = \tan^{-1}$~~

$\frac{d\theta}{dt}$

$\frac{dv}{dt} = 2\,m/s^2, \; v = 2t$

~~$\frac{d\theta}{dv} \times \frac{dv}{dt} = \frac{d\theta}{dt}$~~

~~$= 2 \times \tan^{-1}\left(\frac{v}{2}\right)$~~

$\theta =$

$\frac{d\theta}{dt} = \frac{d\theta}{dv} \times \frac{dv}{dt}$

$= 2 \times \frac{\sec^{-2}\left(\frac{v}{2}\right)}{2}$

$\boxed{\frac{d\theta}{dt} = \left[\sec^{-1}(t)\right]^2}$

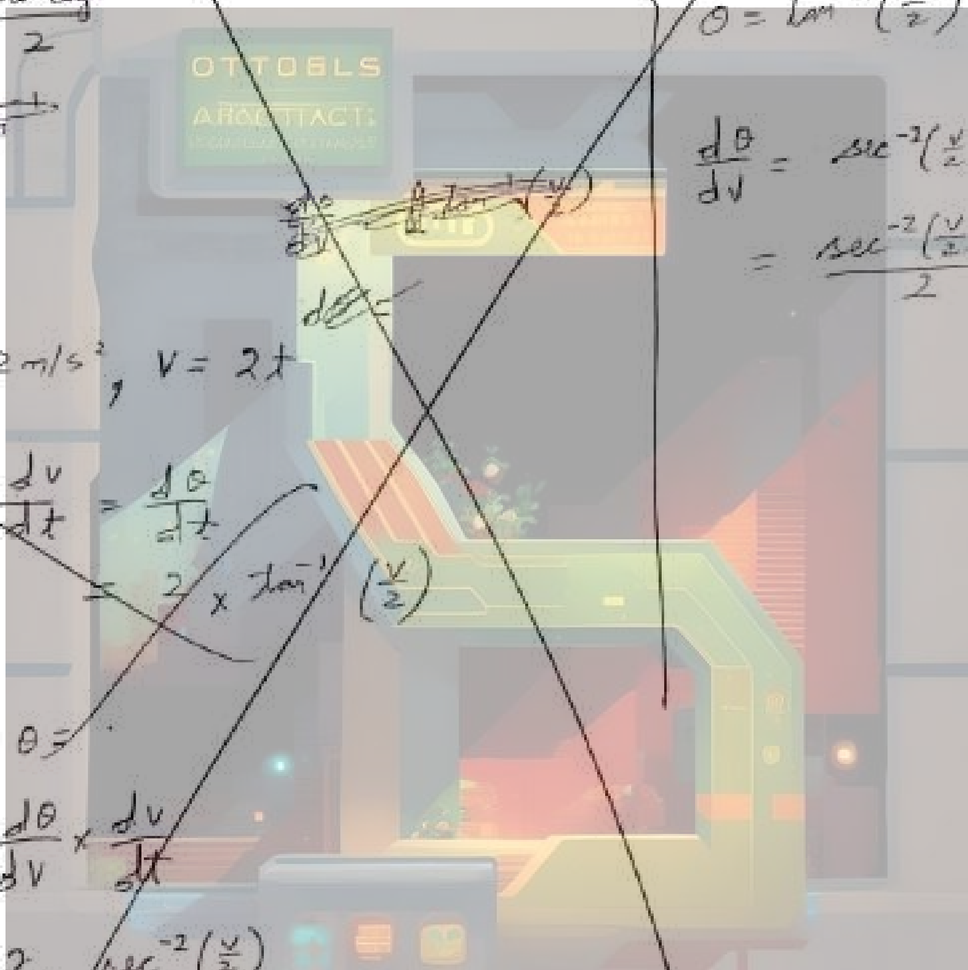
~~$\frac{d\theta}{dt} = \tan^{-1}\left(\frac{v}{2}\right)$~~

~~$d\theta =$~~

$\theta = \tan^{-1}\left(\frac{v}{2}\right)$

$\frac{d\theta}{dv} = \sec^{-2}\left(\frac{v}{2}\right) \times \frac{1}{2}$

$= \frac{\sec^{-2}\left(\frac{v}{2}\right)}{2}$



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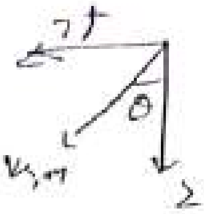
Q rain falling vertically with 2 m/s. A boy at rest accelerates at 2 m/s² along straight road. find rate of change of angle of umbrella?

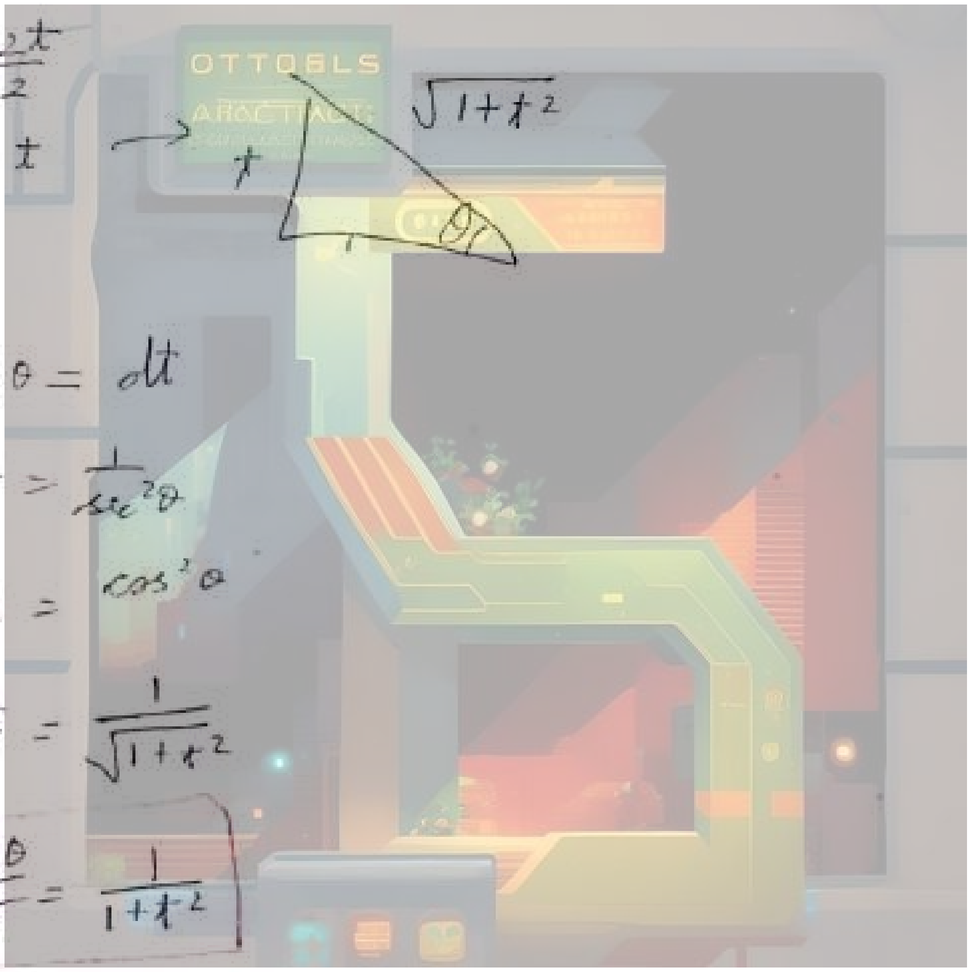
$V_r = -2\hat{j}$

at t $V_m = 2t\hat{i}$

$V_{rm} = V_r - V_m$

$V_{rm} = -2\hat{j} - 2t\hat{i}$





$\tan\theta = \frac{2t}{2}$

$\tan\theta = t$

$\sqrt{1+t^2}$

t

1

θ

$\sec^2\theta \, d\theta = dt$

$\frac{d\theta}{dt} = \frac{1}{\sec^2\theta}$

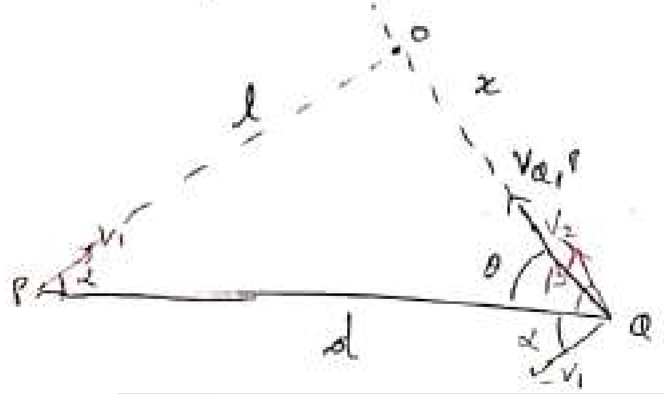
$\frac{d\theta}{dt} = \cos^2\theta$

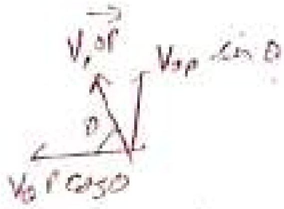
$\frac{d\theta}{dt} = \frac{1}{\sqrt{1+t^2}}$

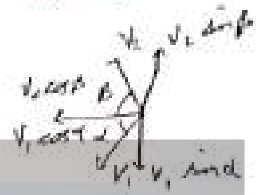
$$\boxed{\frac{d\theta}{dt} = \frac{1}{1+t^2}}$$

Shortest Distance Between two moving particles

(Distance of closest approach)







$V_{Q,P} = \vec{V_Q} - \vec{V_P}$

$\vec{V_{QP}} = \vec{V_2} - \vec{V_1} = \vec{V_2} + (-\vec{V_1})$

$V_{QP}\cos\theta = V_1\cos\alpha + V_2\cos\beta$ --- (1)

$V_{QP}\sin\theta = V_2\sin\beta + V_1\sin\alpha$ --- (2)

$(1)^2 + (2)^2$

$(V_{QP})^2 = (V_1\cos\alpha + V_2\cos\beta)^2 + (V_2\sin\beta + V_1\sin\alpha)^2$

$V_{QP} = ?$

$(2) \div (1)$

$\tan\theta = \dfrac{V_2\sin\beta + V_1\sin\alpha}{V_1\cos\alpha + V_2\cos\beta}$

$\theta = ?$

$\sin\theta = \dfrac{l}{d}$

$\boxed{l = d\sin\theta}$ (for particles to collide, $d = 0$)

$\cos\theta = \dfrac{x}{d}$

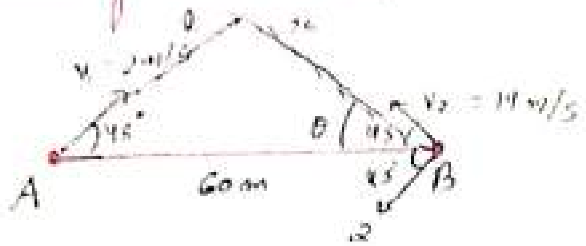
$x = d\cos\theta$

$time = \dfrac{x}{V_{QP}}$

$\boxed{= \dfrac{d\cos\theta}{V_{QP}}}$

Q1. Motion of A & B takes place in horizontal plane

a) find distance of closest approach
b) At what time is the separation b/w them minimum.



$V\cos\theta = \cos 45 \times 2 + \cos 45 \times 14$

$= \frac{2}{\sqrt{2}} + \frac{14}{\sqrt{2}}$

$= \frac{16}{\sqrt{2}}$

$= 8\sqrt{2}$

$V\sin\theta = \sin 45 \times 2 - \sin 45 \times 14$

$= 6\sqrt{2}$

$\tan\theta = \frac{6\sqrt{2}}{8\sqrt{2}}$

$= \frac{3}{4}$

$\boxed{\cancel{\theta = 45°}}$ $\boxed{\theta = 37°}$

a) $\cancel{\sin\theta = \frac{l}{60}}$ $\quad \sin 37 = \frac{l}{60}$

$\cancel{\frac{1}{\sqrt{2}} = \frac{l}{60}}$ $\quad \frac{3}{5} \times 60 = l$

$\cancel{\boxed{30\sqrt{2} = l}}$ a) $\quad \boxed{36 = l}$ a)

$\cancel{\frac{V}{\sqrt{2}} = 8\sqrt{2}}$

$\cancel{V = 8 \times 2}$

$\cancel{\boxed{V = 16\,m/s}}$

$\cancel{\frac{x}{\cos 45} = \frac{x}{60}}$

$\cancel{\boxed{30\sqrt{2} = x}}$

$\cancel{\text{Time} = \frac{\text{Distance}}{\text{Speed}}}$

$\cancel{= \frac{30\sqrt{2}}{16}}$

$\frac{4V}{5} = 8\sqrt{2}$

$V = \frac{5 \times 8\sqrt{2}}{4}$

$\boxed{V = 10\sqrt{2}}$

$\cos 37 = \frac{x}{60}$

$\frac{4 \times 60}{5} = x$

$\boxed{x = 48}$

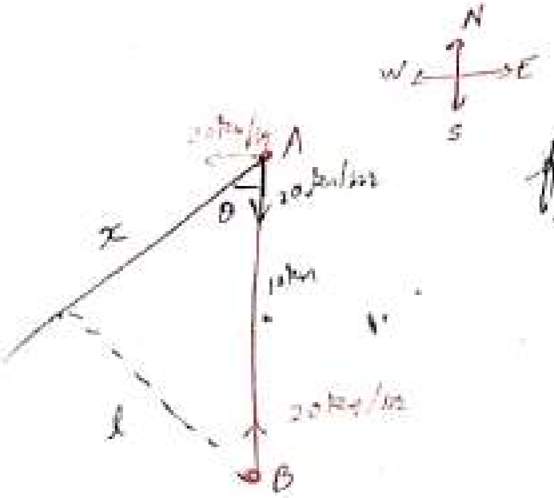
$\text{Time} = \frac{48}{10\sqrt{2}}$

$= \frac{48\sqrt{2}}{20}$

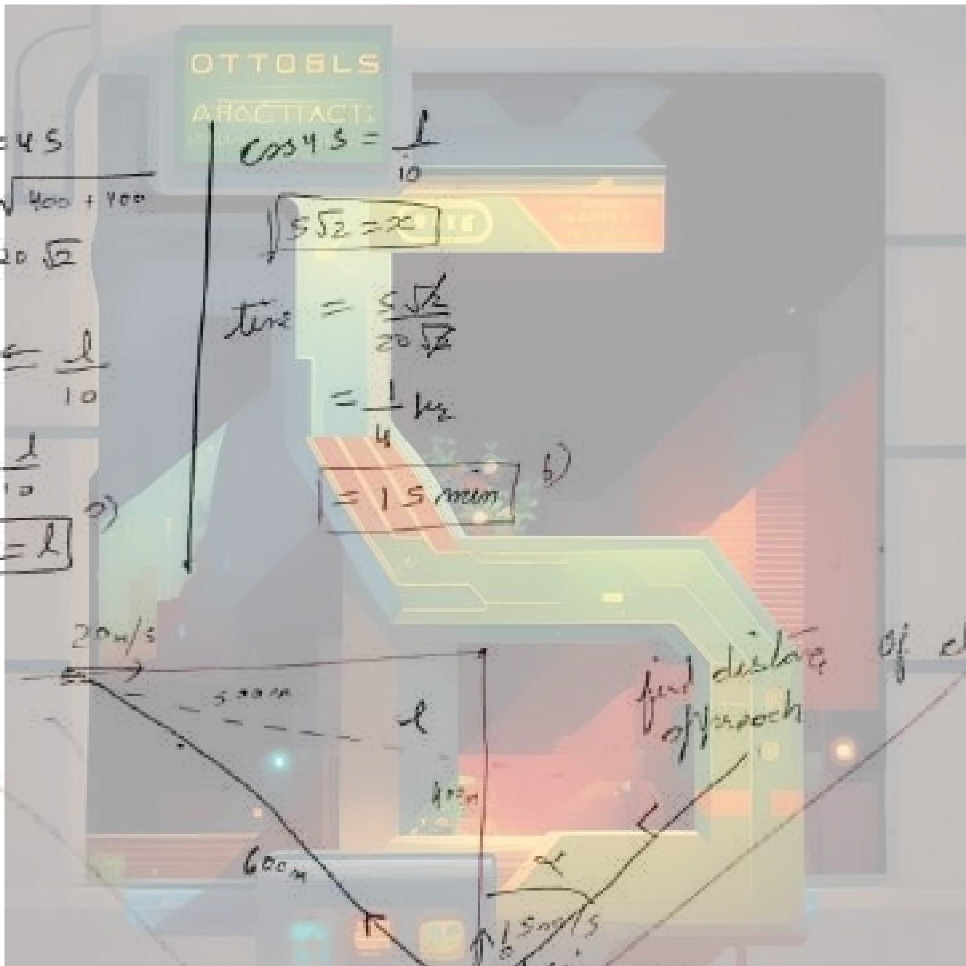
$= \frac{24\sqrt{2}}{10}$

$\boxed{= \frac{12\sqrt{2}}{5}}$ c)

Q 2.



find distance of closest approach and time



$\theta = 45$

$V = \sqrt{400 + 400}$

$= 20\sqrt{2}$

$\sin 45 = \dfrac{l}{10}$

$\dfrac{1}{\sqrt{2}} = \dfrac{l}{10}$

$\boxed{5\sqrt{2} = l}$ a)

$\cos 45 = \dfrac{l}{10}$

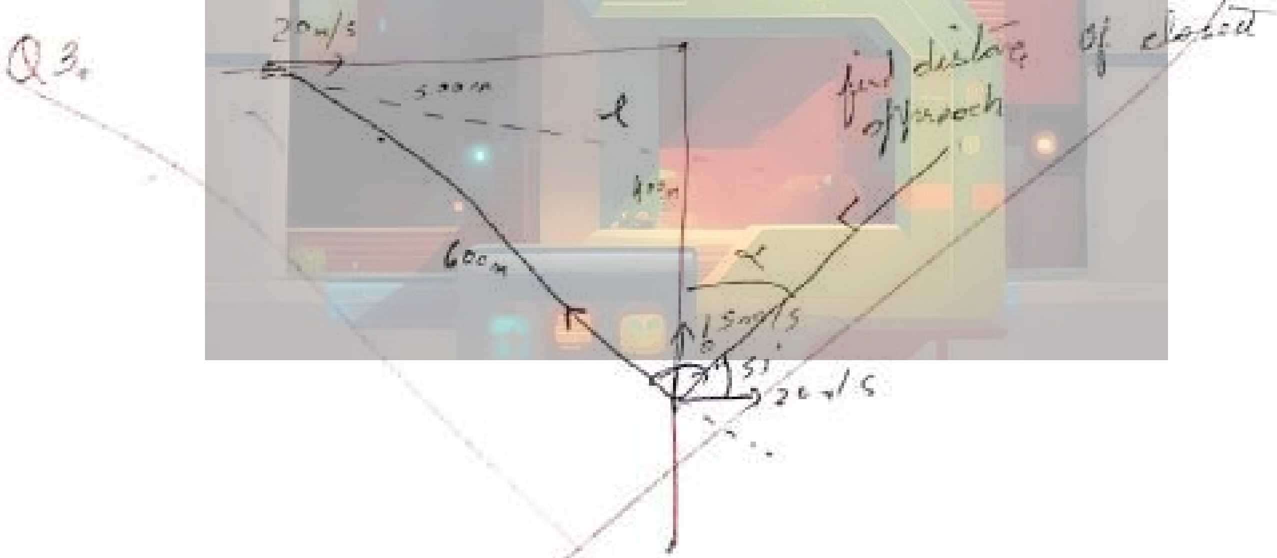
$\boxed{5\sqrt{2} = x}$

time $= \dfrac{5\sqrt{2}}{20\sqrt{2}}$

$= \dfrac{1}{4}$ hr

$\boxed{= 15 \text{ min}}$ b)

Q 3.



find distance of closest approach

$V = \sqrt{400 + 225}$

$= \sqrt{625}$

$\boxed{V = 25 \text{ m/s}}$

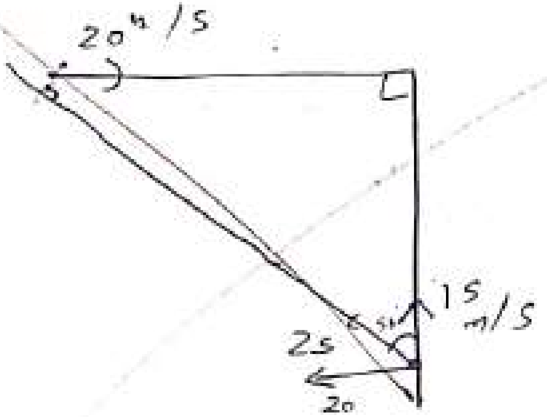
θ

$\alpha = 37^\circ$

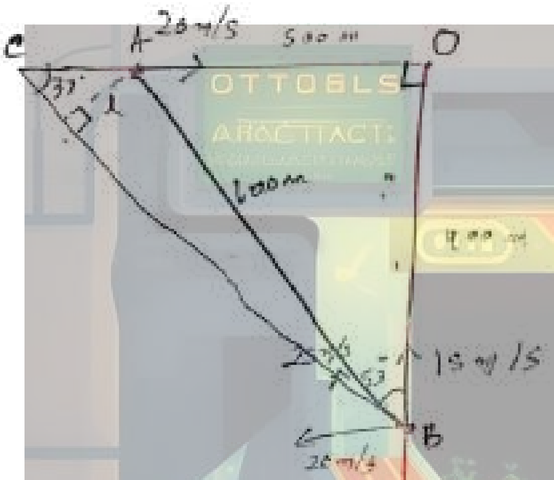
$15 \sin 37 = \sin 53 \times 20$

$15 \times \dfrac{3}{5} = \dfrac{4}{5} \times 20$

$9 =$



Q3.



find distance of closest approach

$$CA = \frac{1600-1500}{3}$$

$$= \frac{100}{3}$$

$$\sin 37 = l \times \frac{3}{100}$$

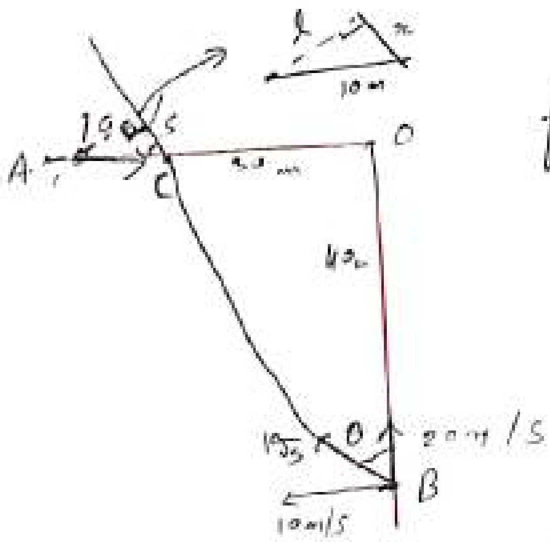
$$\frac{100}{3} \times \frac{3}{5} = \boxed{l = 20\,m}$$

$$\tan 53^\circ = \frac{x}{400}$$

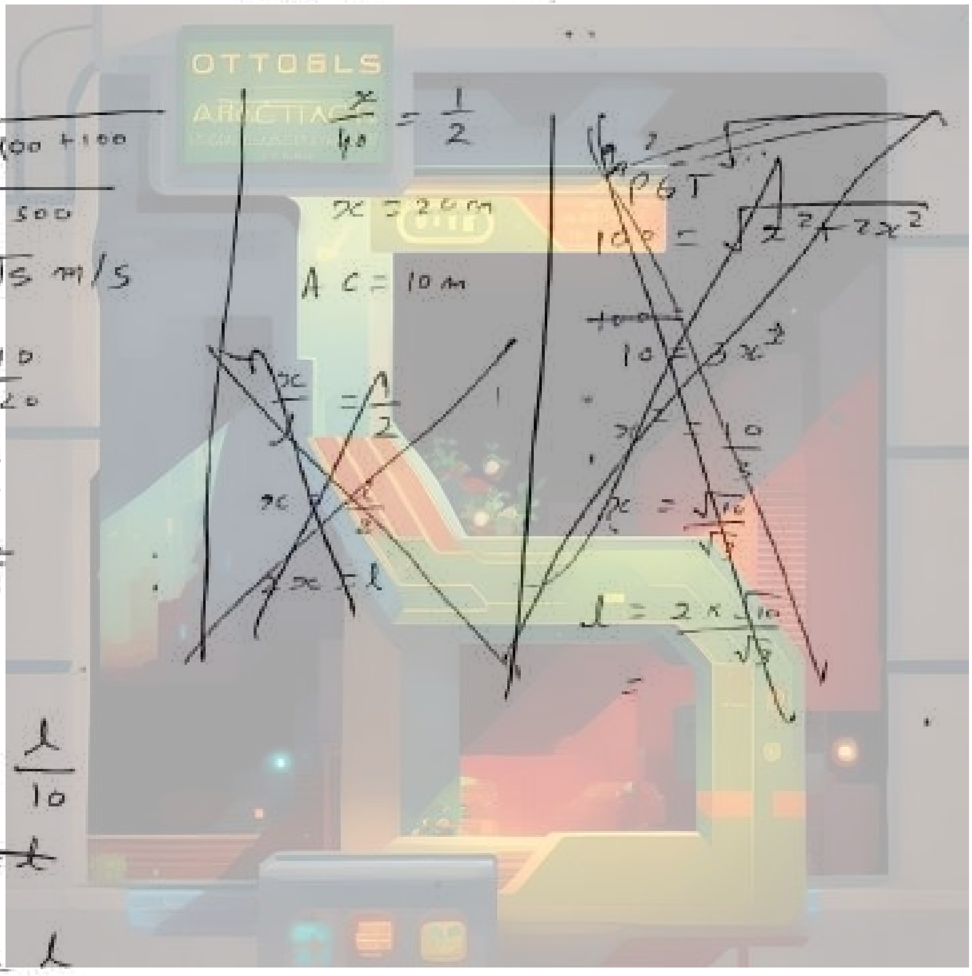
$$\frac{4}{3} = \frac{x}{400}$$

$$\boxed{\frac{1600}{3} = x}$$

Q



find distance of closest approach



$V_{AB} = \sqrt{400+100}$

$= \sqrt{500}$

$= 10\sqrt{5}\ m/s$

$\tan\theta = \frac{10}{20}$

$= \frac{1}{2}$

$\cos\theta = \frac{2}{\sqrt{5}}$

$\frac{x}{40} = \frac{1}{2}$

$x = 20\ m$

$AC = 10\ m$

$\frac{2}{\sqrt{5}} = \frac{l}{10}$

$\cancel{2\sqrt{5} = l}$

$\frac{10 \times 2}{\sqrt{5}} = l$

$\boxed{l = 4\sqrt{5}}$

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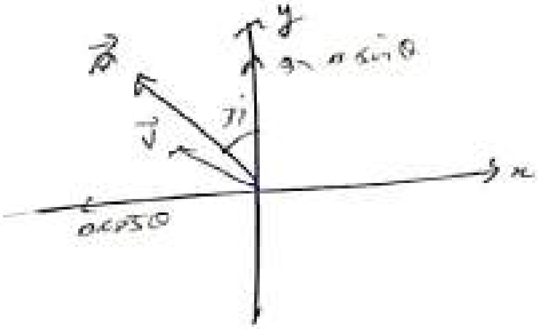
General 2-D motion equations

Q. A Particle with initial velocity $\vec{V_0} = (-2\hat{i} + 4\hat{j})$ undergoes constant acceleration of $3\ m/s^2$ at $\theta = 127°$ from +ve x-axis, find $\vec{v}$ at $t = 5s$



$\vec{a} = 3 \times \frac{3}{5}\ \hat{i} + 3 \times \frac{4}{5}\ \hat{j}$

$= \frac{-9}{5}\hat{i} + \frac{12}{5}\hat{j}$

$\vec{V} = -2\hat{i} + 4\hat{j}$

in x axis,

$v = -2$

$a = -\frac{9}{5}$

$t = 5$

$v = -2 + \frac{-9}{5} \times 5$

$= -2 - 9$

$\boxed{V = -11\ m/s}$

in y axis

$v = 4$

$a = \frac{12}{5}$

$v = 4 + \frac{12}{5} \times 5$

$v = 12 + 4$

$\boxed{= 16\ m/s}$

find $\boxed{\vec{V} = -11\hat{i} + 16\hat{j}}$

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OTTOBLS
ARACTIACT:

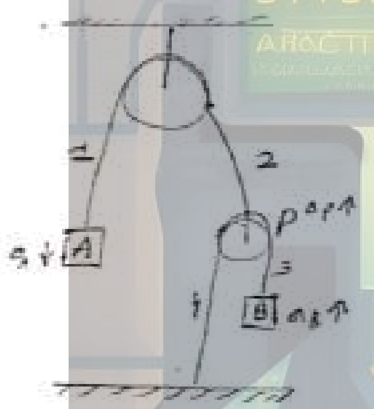
!! Chapter-4 !!

Newton's Laws of Motion & Friction

Constraint Motions:

→ The equations showing the relation of motion of bodies in which motion of one body is ~~con~~ constrained by the other are called constraint relations.

1. Pulley Constraint



Relation between acceleration of blocks

Δl_1 (change in length of rope 1) $= +x_A$
(जितना Block A निचे जाएगा)

$\Delta l_2 = -x_P$
(जितना pulley ऊपर जाएगा)

$\Delta l_1 + \Delta l_2 = 0$
(rope length remains same)

$+x_A - x_P = 0$

$$\boxed{x_A = x_P}$$

$\Delta l_3 = +x_P - x_B$
(जितना Pulley ऊपर जाएगा - जितना Block नीचे जाएगा)

$\Delta l_4 = x_P$

$$\Delta l_3 + \Delta l_4 = 0$$

$$x_p + x_p - x_B = 0$$

$$2x_p = x_p$$

$$2x_A = x_B$$

$$2V_A = \cancel{x_B}\, V_B$$

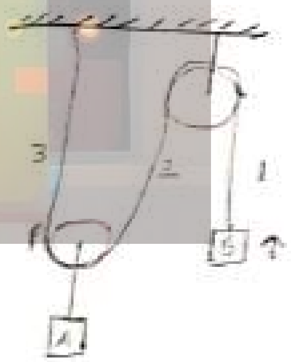
$$\boxed{2a_A = a_B}$$

Method II

Displacement of the block B will be $2x$
A will be x.

$$\boxed{2a_A = a_B}$$

Q Find relation between movement of blocks



~~$\Delta l = x_B$~~

$\cancel{\Delta l_2} = \cancel{x_A}$

$\Delta l_3 = +x_A$

$\Delta l_2 = +x_A$

$\Delta l_1 = -x_B$

$$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$$

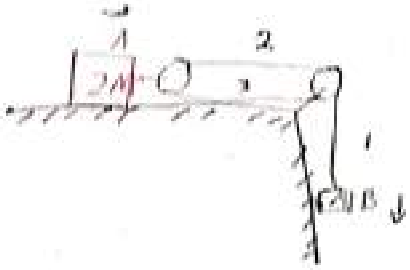
$$2x_A - x_B = 0$$

$$2x_A = x_B$$

$$2V_A = V_B$$

$$\boxed{2a_A = a_B}$$

Q find constraint relation between acceleration of A & B



$\Delta l_1 = x_B$

$\Delta l_2 = -x_A$

$\cancel{\Delta l_1 + \Delta l_2 = 0}$

$\cancel{x_B = x_A}$

$\Delta l_3 = -x_A$

$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$

$x_B - 2x_A = 0$

$\boxed{2x_A = x_B}$

$\boxed{2a_A = a_B}$

Q find constraint relation between acceleration of A and B.

$\Delta l_1 = \Delta l_3 = x_B$

$\Delta l_2 = -x_A$

$\Delta l_2 = -x_A$

$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$

$x_B - x_A - x_A = 0$

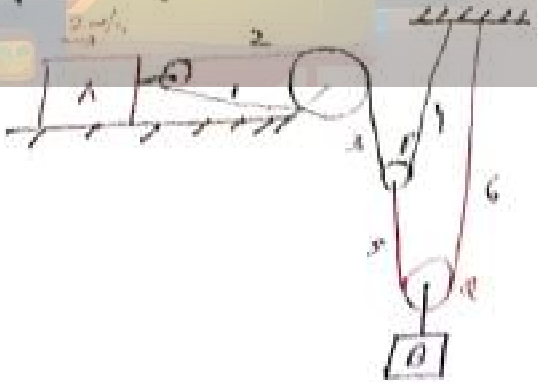
$x_B = 2x_A$

$2x_A = x_B$

$2V_A = V_B$

$\boxed{2a_A = a_B}$

Q find acceleration of block B, pulley P & Q. If acceleration of A is given



$\Delta l_1 = -x_A$

$\Delta l_2 = -x_A$

$\Delta l_3 = x_P$

$\Delta l_4 = x_P$

$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 = 0$

$-2x_A + 2x_P = 0$

$\boxed{x_P = x_A}$

$\Delta l_5 = +x_B \mp x_P$

$\Delta l_6 = +x_B$

$\Delta l_5 + \Delta l_6 = 0$

$+2x_B \mp x_P = 0$

$+x_P = +2x_B$

$x_A = 2x_P$

$V_A = 2V_B$

$\boxed{a_A = 2a_B}$

$a_A = 2\ m/s$

$2 = 2a_B$

$\boxed{a_B = 1\ m/s^2}$

Q Block A vel = 0.6 m/s to right, find V_B.

$\Delta l_1 = -x_A$

$\Delta l_2 = -x_{BA}$

$\Delta l_3 = -x_A$

$\Delta l_4 = x_B$

$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_3 = 0$

$-3x_A + x_B = 0$

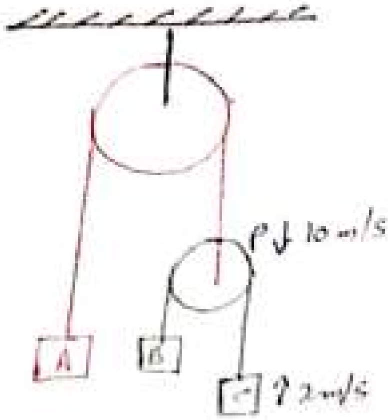
$x_B = 3x_A$

$V_B = 3V_A$

$V_B = 3(0.6)$

$\boxed{V_B = 1.8\ m/s}$

Q find velocities of A & B if velocity of P is 10 m/s downwards and velocity of C is 2 m/s upwards.



$V_A = -V_{P}$

$V_P = -10\hat{j}$

$\boxed{V_A = +10\hat{j}}$

$V_C = 2\hat{j}$

$V_B = ?$

$\boxed{\vec{V}_{B,P} = -\vec{V}_{C,P}}$

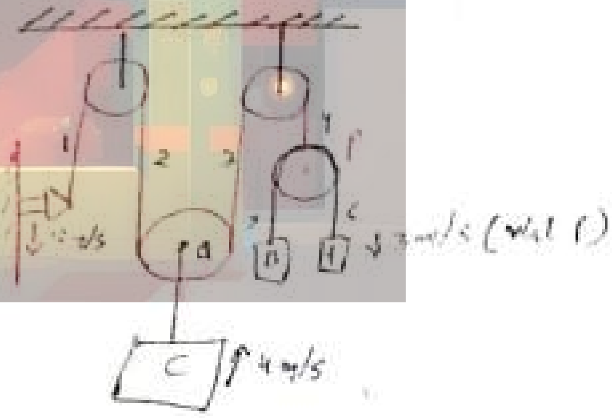
$\vec{V}_B - \vec{V}_P = -(\vec{V}_C - \vec{V}_P)$

$2\vec{V}_P = \vec{V}_B + \vec{V}_C$

$2(-10\hat{j}) = \vec{V}_B + 2\hat{j}$

$\boxed{-22\hat{j} = \vec{V}_B}$

Q ~~At an instant~~ Determine motion of B with ground



$\Delta l_1 = x_R$

$\Delta l_2 = -x_C$

$\Delta l_3 = -x_C$

$\Delta l_4 = x_P$

$x_R + x_P = 2x_C$

$x_P = 2(4) \mp 12$

$= 8 \mp 12$

$\boxed{x_P = \cancel{20}\,4\ m/s}$

$\cancel{x_{A,P} = x_A - x_P}$

$\cancel{= -3 - 20}$

$\cancel{= -23}$

$\cancel{x_{B,P} =}$

$x_{A,P} = -3$

$x_{B,P} = x_B - x_P = (-3)$

$\cancel{x_P = +3 + 20\,m}$

$\boxed{\cancel{x_B = 23}\ m/s}$

$-x_B = -3 - 4$

$\boxed{x_B = -7\ m/s}$

Method II (P mirdre Re based)

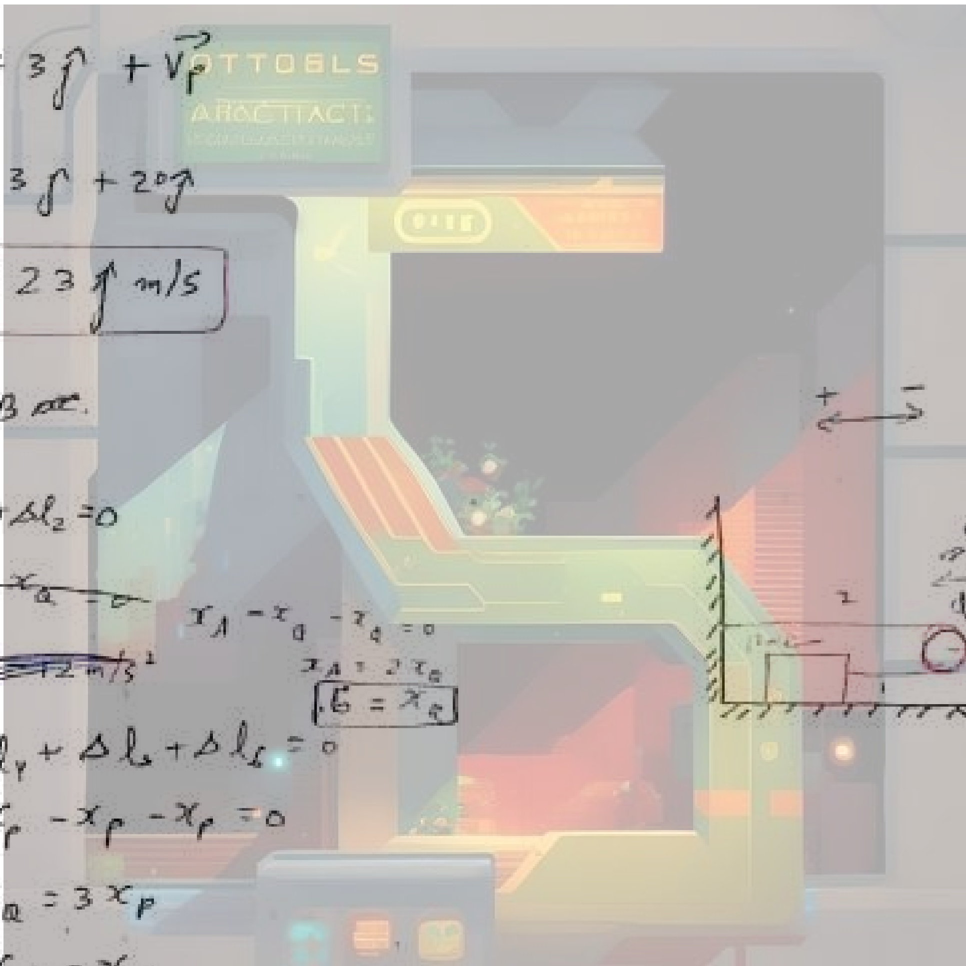
~~$\vec{V}_R = 12\hat{j}$~~

~~$\vec{V}_C = 4\hat{j}$~~

$\vec{V}_{A,P} = -3\hat{j}$

$\vec{V}_{B,P} = -\vec{V}_{A,P}$

$\vec{V}_B - \vec{V}_P = 3\hat{j}$

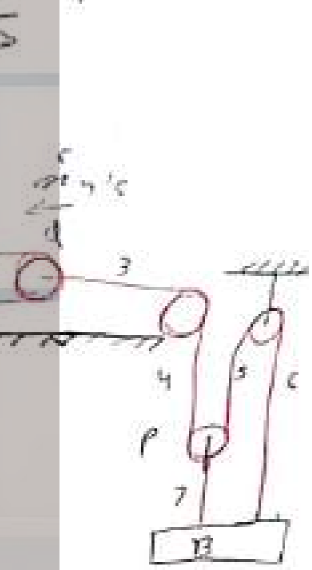


$\vec{V}_B = 3\hat{j} + \vec{V}_P$

$\vec{V}_B = 3\hat{j} + 20\hat{j}$

$\boxed{= 23\hat{j}\ m/s}$

Q find B acc.



$\Delta l_1 + \Delta l_2 = 0$

~~$x_A - x_Q = 0$~~

$x_A - x_Q - x_Q = 0$

$x_A = 2x_Q$

$\boxed{6 = x_Q}$

~~$x_Q = 12\ m/s^2$~~

$\Delta l_3 + \Delta l_4 + \Delta l_5 + \Delta l_6 = 0$

$x_Q - x_P - x_P - x_P = 0$

$x_Q = 3x_P$

$6 = 3x_P$

$x_P = \frac{6}{3}$

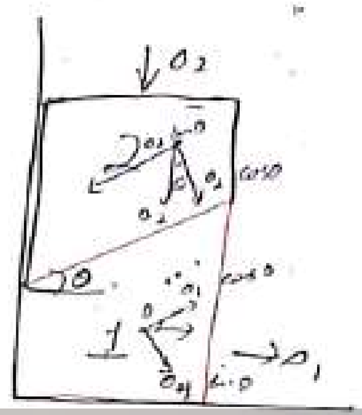
$\boxed{x_P = 2\ m/s^2}$

HW Q1-28 (O) 04-06-2024

Wedge constraint

→ Contact between the wedges is intact

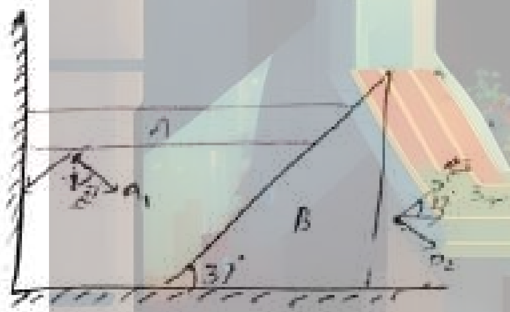
→ component of acceleration perpendicular to surface in contact is same for both.



$a_1 \sin\theta = a_2 \cos\theta$

$\boxed{a_2 = a_1 \tan\theta}$

Q find acceleration of A? ($\theta = 37°$)



$a_A \tan\theta = a_B$

$a_A \times \frac{3}{4} = 3$

$a_A = 3 \times \frac{4}{3}$

$\boxed{a_A = 4 \, m/s^2}$

Or

$a_1 = \cos 37° \times a$

$a_2 = \sin 37° \times 3$

$= 3 \times \frac{3}{5}$

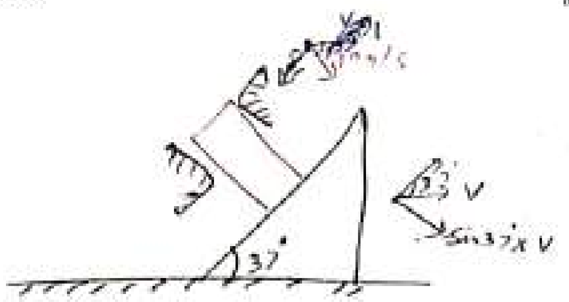
$= \frac{9}{5}$

$a_1 = a_2$

$a \times \frac{4}{5} = \frac{9}{5}$

$\boxed{a = \frac{9}{4} \, m/s^2}$

Q A rod is moving with speed 10 m/s . find V



$V_{O_1} = 10 \cos 37$ ∴ $V \sin 37° = 10$

$= 10 \times \frac{4}{5}$

$V \times \frac{3}{5} = 10$

$= 8$ m/s

$\boxed{V = \frac{50}{3} \; m/s}$

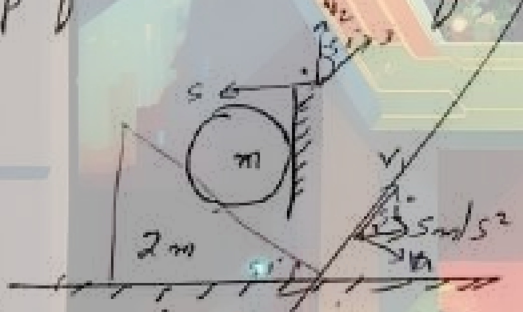
$V_2 = V \times \frac{3}{5}$

$V_2 = \frac{3V}{5}$

$V_1 = V_2$

$8 = \frac{3V}{5}$

Q (m = 1kg) find acceleration of sphere



$V_1 = 5 \times \sin 53$

$V_1 = \cos 53° \times 5$

$= 5 \times \frac{4}{5}$

$= 3 \, m/s^2$

$= 4 \, m/s$

$V = \sqrt{9 + 2 \times 30 \times \frac{4}{5}}$

$V = \sqrt{34 - 24}$

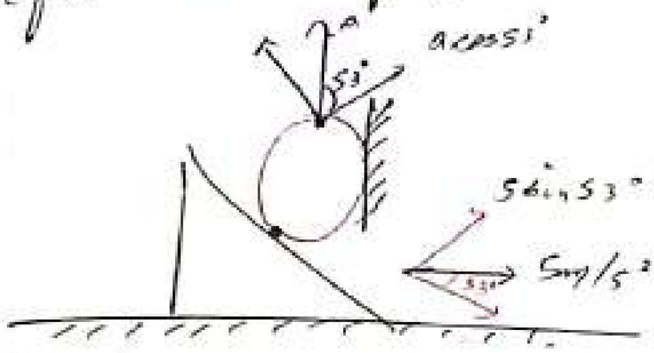
$V = \sqrt{10}$

$V_2 = 3 \cos 53°$

$V_2 = 3 \times \frac{4}{5}$

$V_2 = \frac{12}{5}$

Q (find acceleration of sphere



$a \cos 53° = 5 \sin 53°$

$a \times \frac{3}{5} = 5 \times \frac{4}{5}$

$a = \frac{4 \times 5}{3}$

$\boxed{a = \frac{20}{3}\ m/s^2}$

Pulley & Wedge constraint

Q Determine block acceleration wrt wedge.

$\Delta l_1 = -x_m$

$\Delta l_2 = x_m$

$x_M = x_m$

$a_M = a_m$

$\boxed{a_m = A \text{ (wrt wedge)}}$

$A_{\text{wrt ground}} = \sqrt{A^2 + A^2 + 2A^2 \cos(180 - \theta)}$

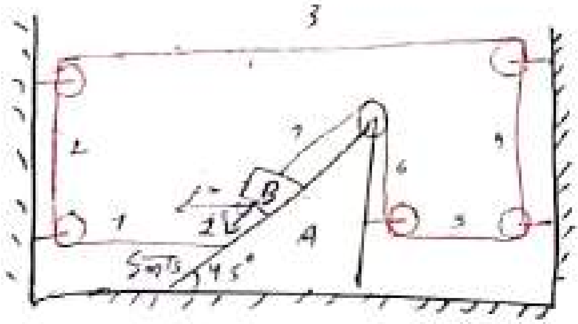
$= \sqrt{2A^2 - 2A^2 \cos\theta}$

$= \sqrt{2A^2 (1 - \cos\theta)}$

$= \sqrt{2A^2 \cdot 2\sin^2\frac{\theta}{2}}$

$\boxed{= 2A \sin\frac{\theta}{2}}$

Q. vel B wrt ground



$x_A = x_B$

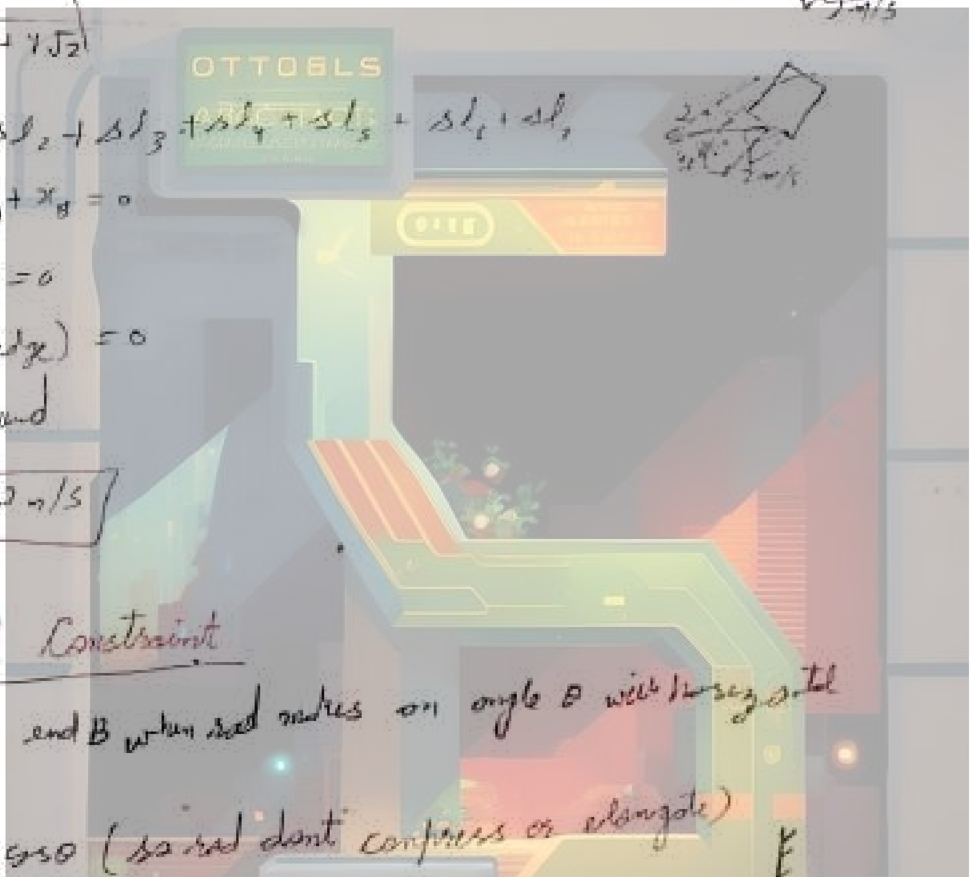
$V_A = V_B$

$V_B = 2(1) = 2\ m/s$

$V = \sqrt{8v^2 + v^2 + 2v^2 \cos 45°}$

$= \sqrt{8 + 8 \times \frac{1}{\sqrt{2}}}$

$= \sqrt{8 + 4\sqrt{2}}$



$\delta l_1 + \delta l_2 + \delta l_3 + \delta l_4 + \delta l_5 + \delta l_6 + \delta l_7$

$-x_A + x_A + x_B = 0$

$x_B = 0$

v_B (wrt wedge) $= 0$

wrt ground

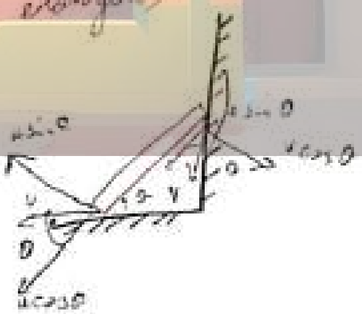
$\boxed{v_B = 2\ m/s}$

A General Constraint

Q Find vel of end B when rod makes an angle θ with horizontal

$v \sin\theta = u \cos\theta$ (so rod dont compress or elongate)

$\boxed{v = u \cot\theta}$

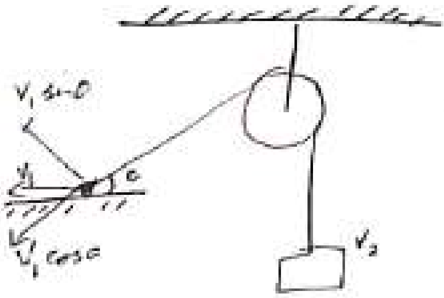


→ Component of velocity along the rod or string is equal for both ends.

Q find relation betw v_1 & v_2

(The component along string to remain same.)

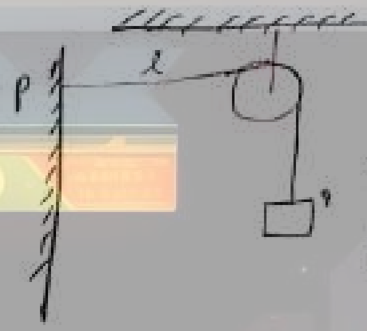
$\boxed{v_1 \cos\theta = v_2}$

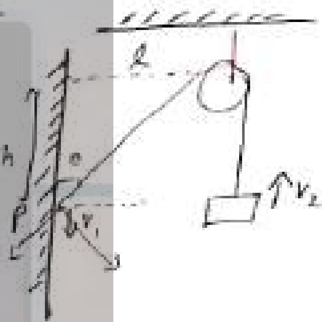


Q2. find relation betw v_1 & v_2 if distance moved by P is h.

$v_1 \cos\theta = v_2$

$\boxed{\dfrac{v_1 h}{\sqrt{h^2+l^2}} = v_2}$





Newton's Laws of Motion

→ cause of motion :- forces → dynamics

Balanced Forces — Net Force = 0

If body at rest → Remains at rest

Body is in motion → Continues in motion with same speed & direction.

→ Balanced Forces may lead to change in size or shape of the object.

Unbalanced Forces — Net Force ≠ 0

Body is at rest → Body will start moving in the direction of resultant force

Body is in motion →
- Direction of net force is same as motion of the body → Body will speed up
- Direction of net force opp. to motion of the body → Body will slow down & finally stop

~~Nelwon~~ Newton's First Law & Inertial Frame & non Inertial Frame.

→ Newton's First law / law of Inertia – defines a set of reference frames called inertial frames.

→ ~~Erder~~ Inertial Frame – Frames which do not have any acceleration.

→ Either the ~~Bo~~ frame is at rest or moving with a uniform velocity.

→ Newton's laws can be directly applied in such frames and dynamic equations can be written for objects in this frame. $\Sigma \vec{F} = m\vec{a}$

→ First Law – In the absence of external forces, when viewed from an inertial reference frame, every object continues to be in its state of rest or uniform motion.

Friction does not oppose the motion, It opposes the relative motion between two surfaces.

→ Inertial frames are also called as ~~tott~~ 'Galilean Frames'.

→ Any reference frame that moves with a constant relative velocity to an inertial frame is itself an inertial frame.

→ First law is a qualitative law. (does not talk about the quantity of forces)

→ Non-Inertial Frame – A frame of reference which is in accelerated motion with respect to a Inertial frame.

→ Newton's Laws cannot be ~~dead~~ directly ~~applied~~, ~~same as~~ applicable

→ Tendency of an object to resist any attempt to change its velocity is called Inertia.

→ Depends on mass,
more mass ↑ more Inertia ↑

Linear Momentum & Newton's second Law.

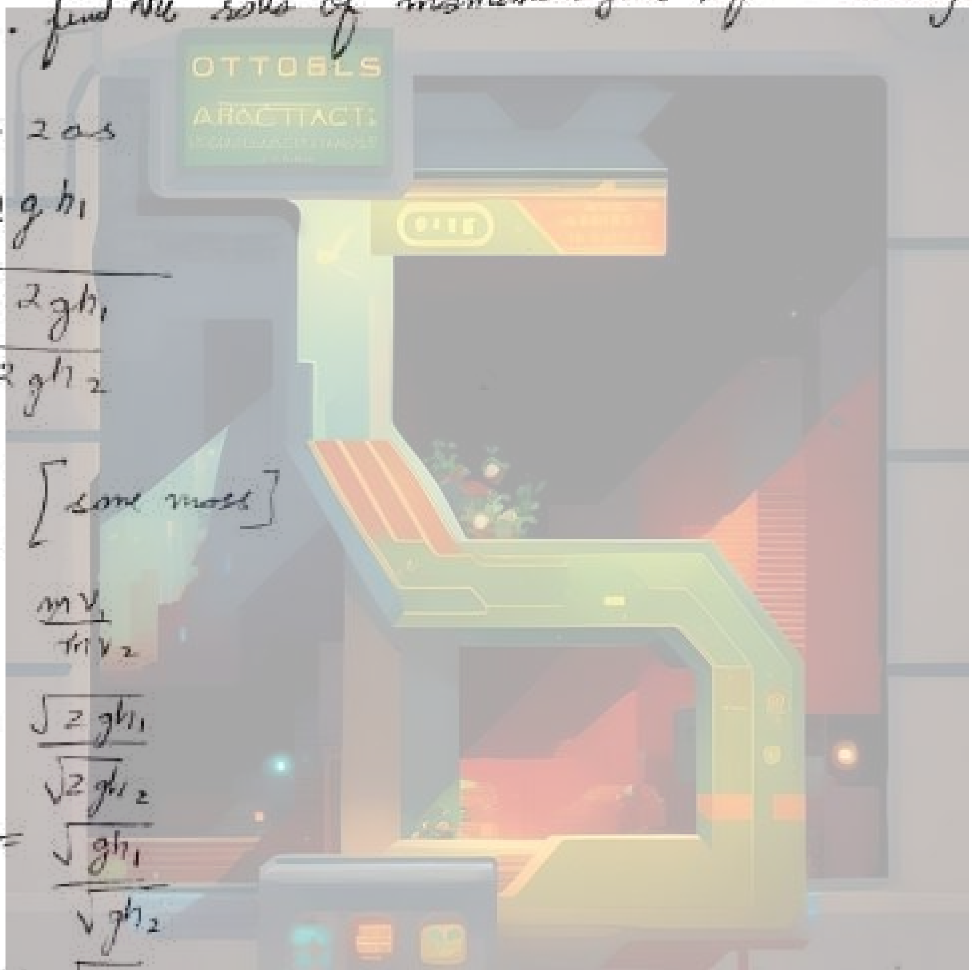
Linear Momentum (P) – The quantity of motion contained in the body.

$$\boxed{\vec{P} = m\vec{v}}$$

SI unit :- kg m/s or Ns

It is a vector quantity

Q Two identical bodies are allowed to fall from two different heights h_1 & h_2. find the ratio of momentum just before striking the ground.



$v^2 = u^2 + 2as$

$v_1^2 = 2gh_1$

$v_1 = \sqrt{2gh_1}$

$v_2 = \sqrt{2gh_2}$

$P_1 = mv_1$ [same mass]

$P_2 = mv_2$

$\frac{P_1}{P_2} = \frac{mv_1}{mv_2}$

$= \frac{\sqrt{2gh_1}}{\sqrt{2gh_2}}$

$= \frac{\sqrt{gh_1}}{\sqrt{gh_2}}$

$= \sqrt{\frac{h_1}{h_2}}$

$P_1 : P_2$

$\sqrt{h_1} : \sqrt{h_2}$

Q A ball of mass m is dropped from a height h on a smooth elastic floor, ~~such~~ such that it rebounds with same speed. What is the change in momentum of ball before & after striking the floor is : (take vertically downward as positive)

$v^2 = u^2 + 2as$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$P_1 = m\sqrt{2gh}$

$v_2 = (-\sqrt{2gh})$

$P_2 = -m\sqrt{2gh}$

$|P_1| = m\sqrt{2gh}$

$|P_2| = m\sqrt{2gh}$

$\boxed{|P_2| - |P_1| = 0}$

b) find magnitude of change in momentum

$P_2 - P_1$

$-m\sqrt{2gh} - m\sqrt{2gh}$

$-2m\sqrt{2gh}$

$\boxed{|P_2 - P_1| = 2m\sqrt{2gh}}$

Newton's Second Law

→ When viewed from an inertial frame of reference, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$a \propto \frac{1}{m}$ $\qquad a \propto F$

→ Rate of change of momentum is directly proportional to net unbalanced force acting on it.

$P_i = mu$

$P_f = mv$

$\Delta P = mv - mu$

$\boxed{\frac{\Delta P}{t} \propto F}$

$F \propto \frac{m(v-u)}{t}$

$F \propto ma$

$F = kma \quad (k=1)$

$\boxed{F = ma}$

$\boxed{F = \frac{dP}{dt}}$ → Slope of P-T graph

$$\underbrace{\int_{p_i}^{p_f} dP}_{\text{change in momentum}} = \underbrace{\int_0^t F\,dt}_{\text{area under } F\text{-}t \text{ graph}}$$

Impulse (J):- It is the change in momentum of a body.

$J = \Delta P = mv - mu = Ft$

SI unit:- Ns or kg m/s

Q. A machine gun has mass 5kg. It fires 50g bullets at the rate of 30 bullets per minute at a speed 400 m/s. What force is required to keep gun in position.

$F = m\frac{(v-u)}{t}$

$= \frac{50}{1000}\left[\frac{400-0}{2}\right]$

$= \frac{5}{100} \times 200$

$= 5 \times 2$

$\boxed{= 10N}$

Q. A dish of mass 10g is kept horizontally in air by firing 5g bullets 10/s. If bullets rebound with same speed, with what speed are bullets fired ($g = 9.8/s^2$)

Force to keep dish in air $= \frac{10}{1000} \times 9.8$

$= \frac{98}{1000}$ N

~~$F = m\frac{(v-u)}{t}$~~

~~$= 5 \times \frac{(-u)}{1} \times 10$~~

~~$\frac{98}{1000} = -50u$~~

$F = m\frac{(v-u)}{t}$

$\frac{98}{1000} = \frac{5(v-(-v))}{1000} \times 10$

$\frac{98}{1000} = 50 \times 2v$

$\frac{98}{1000} = \frac{100v}{1000}$

$v = \frac{98}{100}$

$\boxed{v = 0.98\,m/s}$

Q A body of mass 4 kg moving on horizontal surface with initial velocity 6 m/s comes to rest after 3 s. If one wants to keep moving the body with same speed of 6 on same surface, find required force.

$u = 6$
$v = 0$
$t = 3s$

$a = \frac{v-u}{t}$

$= \frac{-6}{3}$

$= -2$

• to keep moving, $a = 2$ to be applied

$F = ma$

$F = 4 \times 2$

$\boxed{F = 8N}$

Newton's Third Law

→ To Every Action there is a equal & opposite reaction.

→ Action & Reaction are equal in magnitude, opposite in direction and acts on two different bodies

→ if two forces are acting on the same object, even if they are equal in magnitude and opposite in direction, cannot be an action - reaction pair.

Free Body Diagram

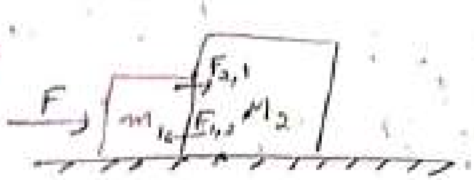
→ diagram of a body showing all the forces on it along with direction & magnitude.

→ Consider only the forces applied on that body & not the forces the body applies on any other body.

Types of Forces

1. Contact forces - The force which acts between two bodies in contact are called contact forces.

$|F_{21}| = |F_{12}| = F_c$



$F = (m_1 + m_2)\, a$

$a = \dfrac{F}{(m_1+m_2)}$

for m_2,

$F_c = m_2 a$

$$\boxed{F_c = \frac{m_2 F}{m_1+m_2}}$$

for m_1,

$F - F_c = m_1 a$

$F - F_c = \dfrac{m_1 F}{m_1 + m_2}$

$F_c = F\left[1 - \dfrac{m_1}{m_1+m_2}\right]$

$$\boxed{F_c = \frac{m_2 F}{m_1+m_2}}$$

$\boxed{\Sigma \vec{F} = m\vec{a}} \rightarrow$ Dynamic Equation

Q find acceleration & contact force b/w A & B.

$F = ma$

$10 = 5 \times a$

$a = \dfrac{10}{5}$

$\boxed{a = 2\ m/s^2}$

for, b

$F_c = 3 \times 2$

$\boxed{F_c = 6\ N}$.

for, a

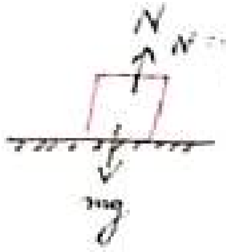
$F - F_c = 2 \times 2$

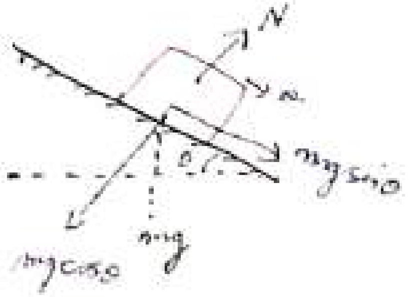
$10 - F_c = 4$

$F_c = 10 - 4$

$\boxed{F_c = 6\ N}$

2. <u>Normal Force & Weight of body</u> – Normal force is a special type of contact force which always act $\perp$ to surface in contact



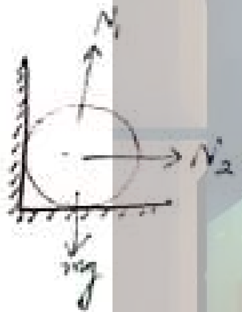


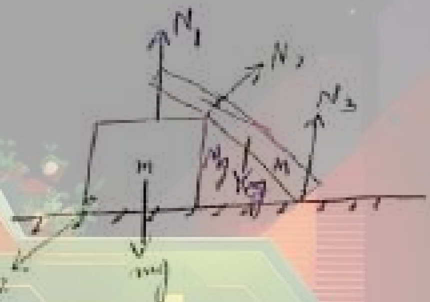
$$N = mg\cos\theta$$

$$mg\sin\theta = ma$$

$$\boxed{g\sin\theta = a}\ \text{(along incline)}$$

Eg





$N_1 = mg$

Note

→ when external force is applied on a object, the normal force shifts towards the direction of applied force.

→ on the verge of toppling, normal reaction passes through edge of the block.

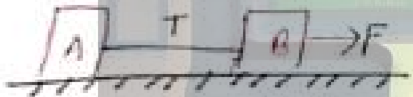
3. Tension Force - ~~The~~ The force with which elements of a string pull each other is called tension force.

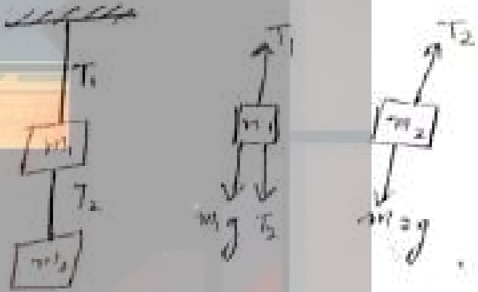
→ An Ideal string is considered to be massless, inextensible, pulls at any point on the string can pull but not push.

→ An Ideal pulley is assumed to be massless, frictionless. Action of pulley is to change the direction of force. Tension is same in the pulley on both sides of it.

→ Tension force is always directed away from point of contact.

Eg.





$T_2 = m_2 g$ (for body at rest)

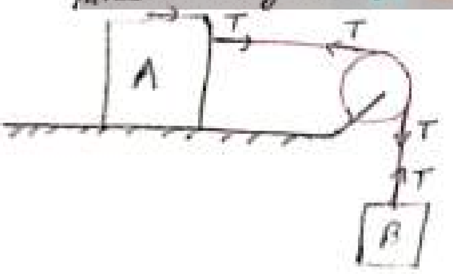
$T_2 = m_2 g$

$T_1 = m_1 g + T_2$

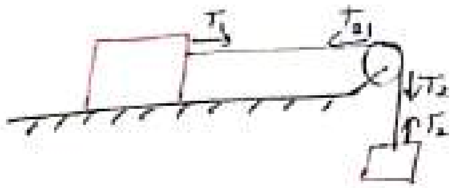
$T_1 = m_1 g + m_2 g$

$\boxed{T_1 = (m_1 + m_2) g}$

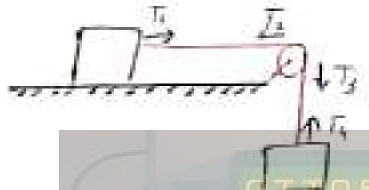
massless string and frictionless pulley & string

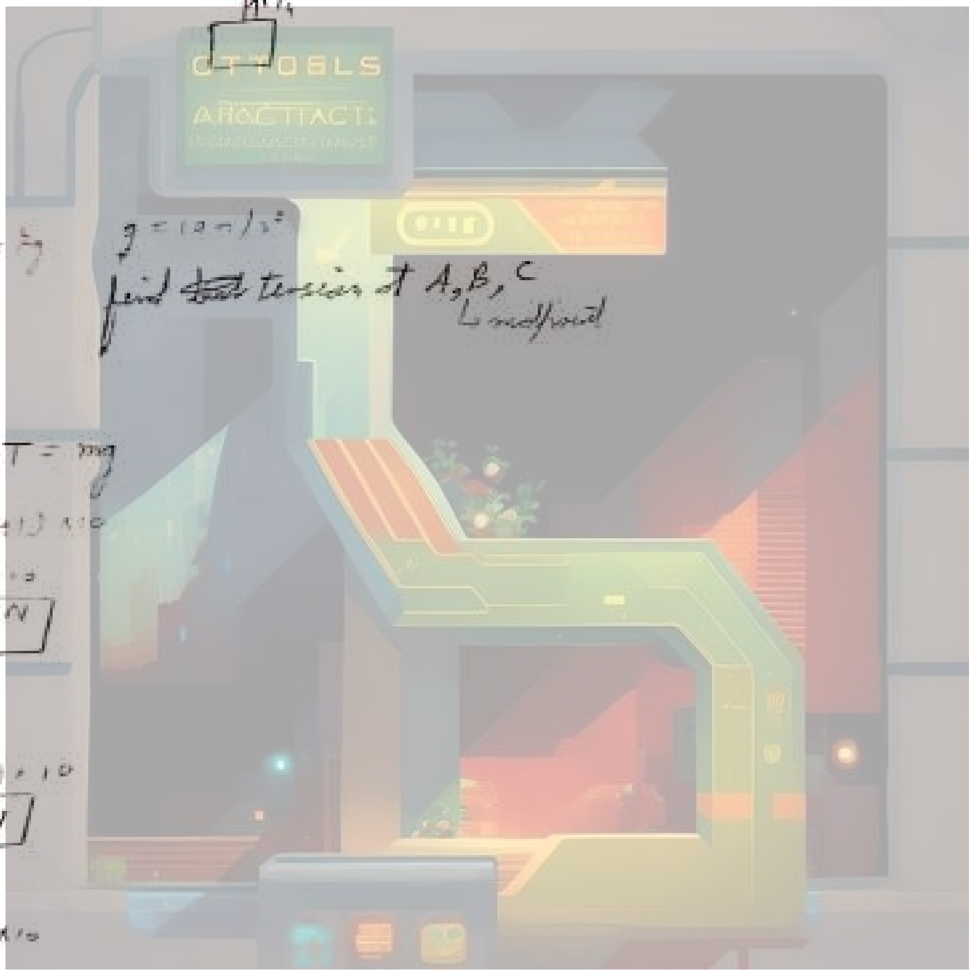


Massless String & pully is not frictionless



Massive String & Pulley is not frictionless





Eg

$g = 10\,m/s^2$

find ~~~~ tension at A, B, C
↳ midpoint

at A, $T = mg$

$T = (2+1) \times 10$

$= 3 \times 10$

$\boxed{T_A = 30N}$

$T_B = mg$

$= (1+1) \times 10$

$\boxed{T_B = 20N}$

$T_C = mg$

$= 1 \times 10$

$\boxed{T_C = 10N}$

Q A rope of uniform mass distribution of mass m & length l, find ~~[illegible]~~ tension at distance x from bottom.

length till x = x

mass = $\cancel{\frac{m}{l}(l-x)} = \frac{mx}{l}$

$T = mg$

$= \cancel{\frac{m(l-x)}{l}} g$

$\boxed{T = \frac{mxg}{l}}$

H.W.
Ch-3
S-1 (1-20)

Q Find acceleration of blocks & Tension in string connecting A.B.

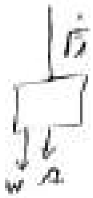
$\cancel{F = ma}$
$\cancel{16 = \frac{1}{3} \times a}$
$\cancel{a = \frac{8}{9}}$

$F = ma$
$16 = 8a$
$a = \frac{16}{8}$
$\boxed{a = 2}\ m/s^2$

$F_1 = 3 \times 2$
$\boxed{F_1 = 6N}$

Q With what min acceleration can a fireman slide down a rope whose breaking strength is of his $\frac{2}{3}$ weight.

$F_t - F = mga$

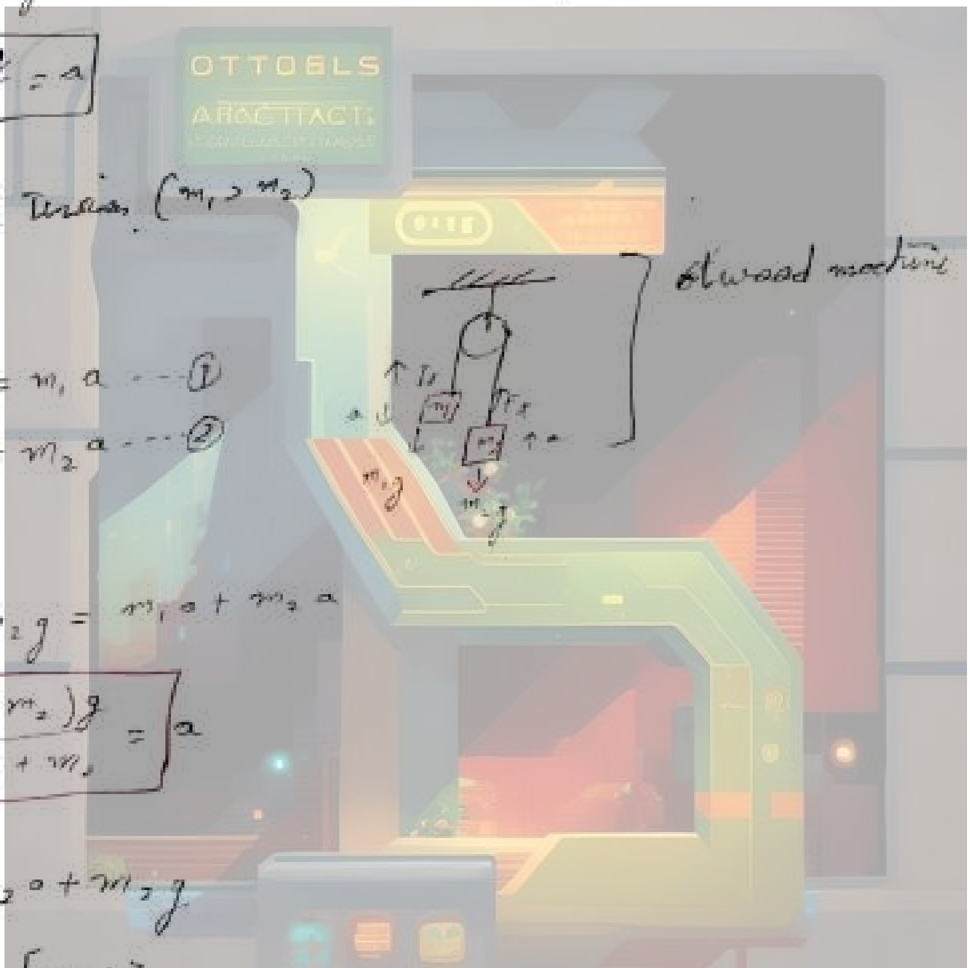


$W - \frac{2}{3}W = \frac{W}{g} \cdot a$

$1 - \frac{2}{3} = \frac{a}{g}$

$\frac{1}{3} = \frac{a}{g}$

$\boxed{\frac{g}{3} = a}$



Q Find acc & Tension ($m_1 > m_2$)

acc

Atwood machine

$m_1 g - F_t = m_1 a$ --- (1)

$F_t - m_2 g = m_2 a$ --- (2)

(1) + (2)

$m_1 g - m_2 g = m_1 a + m_2 a$

$\boxed{\frac{(m_1 - m_2)g}{m_1 + m_2} = a}$

$F_t = m_2 a + m_2 g$

$= m_2 [a + g]$

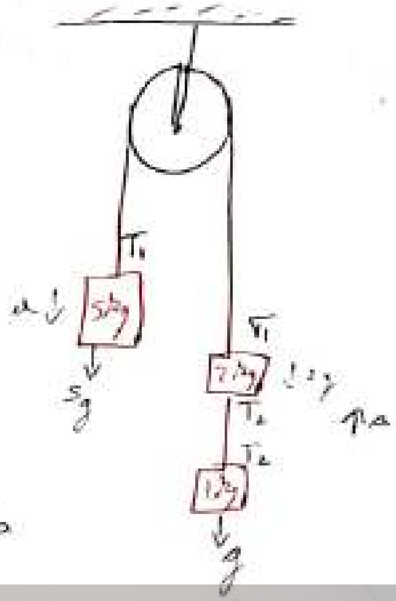
$= m_2 \left[\frac{(m_1 - m_2)g}{m_1 + m_2} + g\right]$

$= m_2 g$

$= m_2 g \left[\frac{m_1 - m_2 + m_1 + m_2}{m_1 + m_2}\right]$

$\boxed{F_t = \frac{2 m_1 m_2 g}{m_1 + m_2}}$

Q find acc & Tension



$5g - T_1 = 5a$ ---(1)

$T_1 - (2g + T_2) = 2a$ ---(2)

$T_2 - g = a$ ---(3)

(3) − (2)

$-g + 2g - T = 0$

$\boxed{T_1 = g}$

$g + T_2 = a$

$T_2 - g = a$

$-2g + 2T_2 = 2a$

$5g - g = a$

$\boxed{a = 4g}$

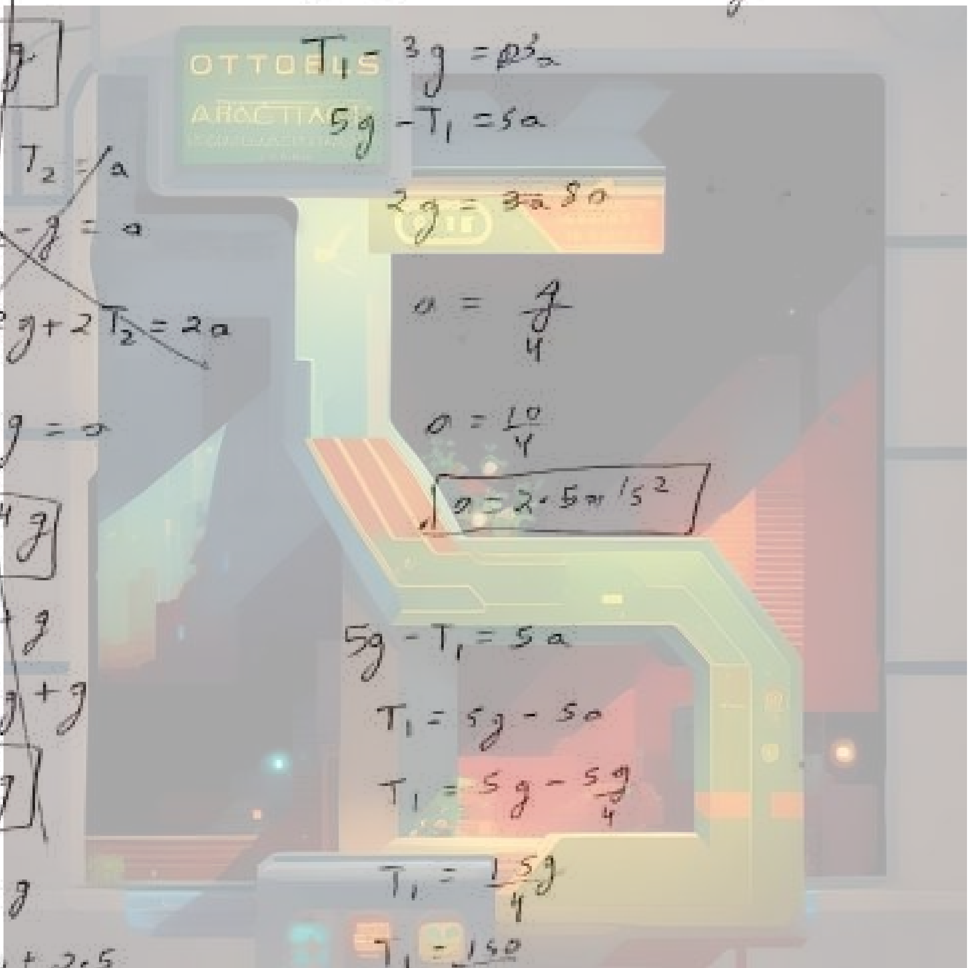
$T_2 = a + g$

$= 4g + g$

$\boxed{T_2 = 5g}$

(3) + (2)

$T_1 - 2g - g = 3a$



$T_1 - 3g = 3a$

$5g - T_1 = 5a$

$2g = 8a$

$a = \frac{g}{4}$

$a = \frac{10}{4}$

$\boxed{a = 2.5\,m/s^2}$

$5g - T_1 = 5a$

$T_1 = 5g - 5a$

$T_1 = 5g - \frac{5g}{4}$

$T_1 = \frac{15g}{4}$

$T_1 = \frac{150}{4}$

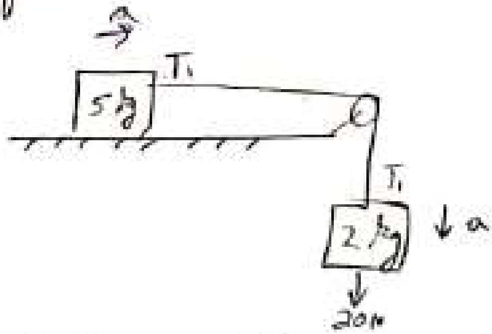
$\boxed{T_1 = \frac{75}{2}N}$

$T_2 = a + g$

$T_2 = 10 + 2.5$

$\boxed{T_2 = 12.5\,N}$

Q find acc & Tension.



$T_1 = 5a \quad \cdots ①$

$\cancel{T_1 - 20 = 2a \quad \cdots ②}$

$\cancel{20 = 3a}$

$\cancel{a = \frac{20}{3} \, m/s^2}$

$20 - T_1 = 2a$

$20 = 7a$

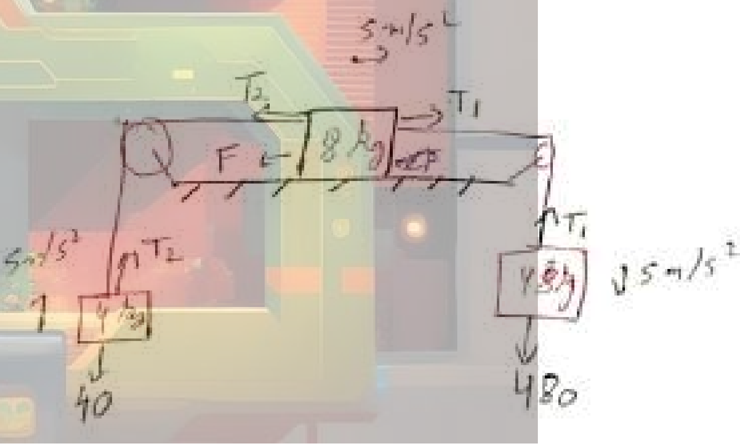
$\boxed{a = \frac{20}{7} \, m/s^2}$

$T = 5a$

$T = 5 \times \frac{20}{7}$

$\boxed{T = \frac{100}{7} \, N}$

Q. a = $5 \, m/s^2$
find friction of 8 kg block



for 48 kg

$480 - T_1 = 5 \times 48$

$480 - T_1 = 240$

$T_1 = 480 - 240$

$\boxed{T_1 = 240 \, N}$

for 4 kg

$T_2 - 40 = 5 \times 4$

$\cancel{T_2 = 45 \, N}$

$T_2 - 40 = 20$

$\boxed{T_2 = 60 \, N}$

for 8 kg

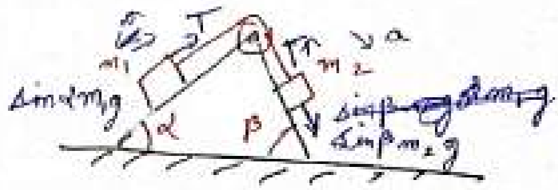
$T_1 - T_2 - F = 8 \times 5$

$240 - 60 - F = 40$

$F = 240 - 100$

$\boxed{F = 140 \, N}$

Q find acc.

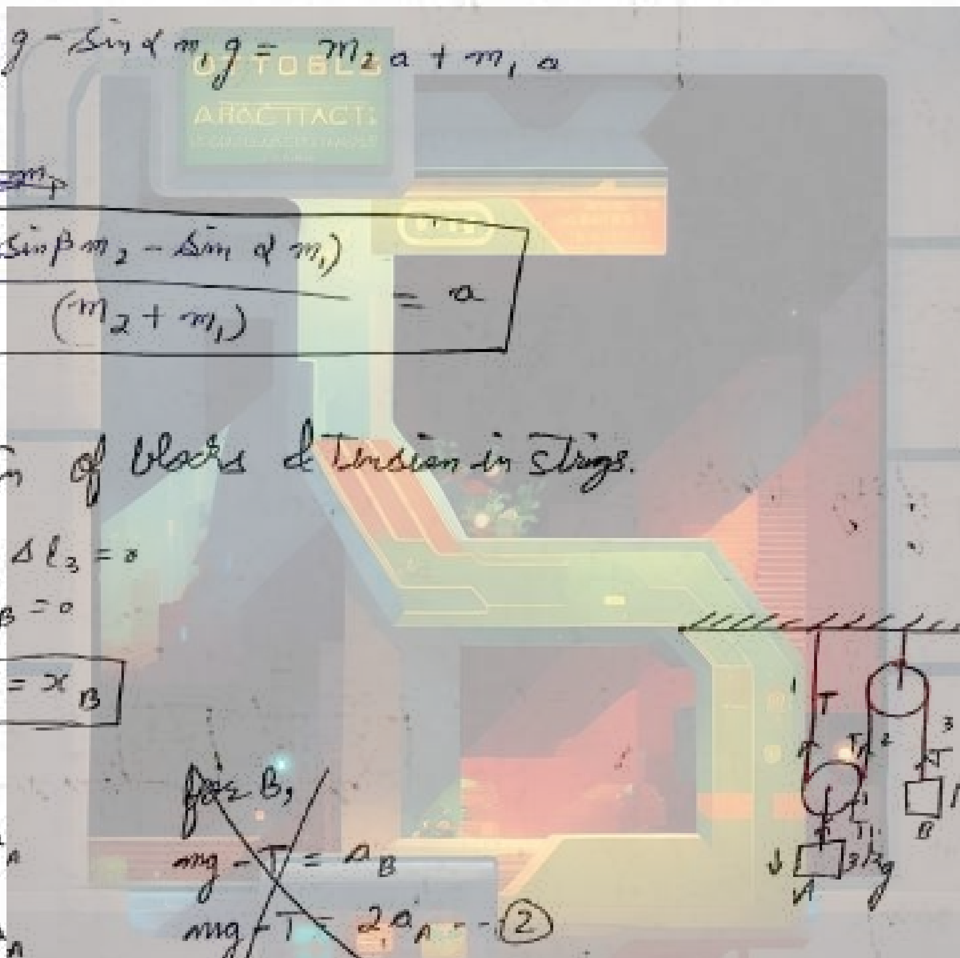


$\sin\beta\, m_2 g - T = m_2 a$ --- ①

$T - \sin\alpha\, m_1 g = m_1 a$ ---- ②

$\sin\beta\, m_2 g - \sin\alpha\, m_1 g = m_2 a + m_1 a$

$$\boxed{\frac{g(\sin\beta\, m_2 - \sin\alpha\, m_1)}{(m_2 + m_1)} = a}$$



Q acceleration of blocks & tension in strings.

$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$

$x_{B_A} + x_{B_A} - x_B = 0$

$\boxed{2x_A = x_B}$

for A,

$-T' + mg = 3a_A$

$mg - T' = 3a_A$

$mg - 2T = 3a_A$

$30 - 2T = 3a_A$ — ①

~~for B,~~

~~$mg - T = a_B$~~

~~$mg - T = 2a_A$ -- ②~~

~~$10 - T = 2a_A$~~

for B,

$T - mg = a_B$

$T - 10 = 2a$

$2T - 20 = 4a$ — ②

① + ②

$10 = 7a$

$\boxed{a_A = \frac{10}{7}}$

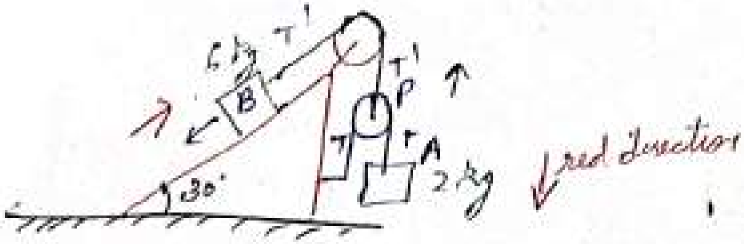
$\boxed{a_B = \frac{20}{7}}$

$T = \frac{20}{7} + 10$

$\boxed{T = \frac{90}{7}\ N}$

88

Q find acceleration ($m_1 > m_2$)



$x_B = x_P$

$x_P + x_P - x_A = 0$

$\boxed{2x_A = \cancel{2}\, x_B}$ $\boxed{T' = 2T}$

for A.

$T - 20 = 2a_A$ --- (1)

$2T - 40 = 4a_A$ --- (1)

for B.

$\cos 30\, mg$

$\frac{mg}{2} \leftarrow \boxed{B} \rightarrow T'$

$\cos 30\, mg$

$30 - T' = 6a_B$

$30 - 2T = 12 a_A$ --- (2)

(1) + (2)

$-10 = 16 a_A$

$\cancel{\boxed{a_A = \frac{-5}{8}\, m/s^2}}$

$\cancel{\boxed{a_B = \frac{-5}{7}\, m/s^2}}$ $\cancel{\boxed{a_A = \frac{-5}{14}\, m/s^2}}$

$a_A = \frac{-10}{7}$

$a_B = \frac{-5}{7}$

~~$T - 20 = -\frac{20}{7}$~~

~~$T = 20 - \frac{20}{7}$~~

~~$T = \frac{140 - 20}{7}$~~

~~$\boxed{T = \frac{120}{7}\, N}$~~

~~$\boxed{T' = \frac{240}{7}\, N}$~~

~~$T - 20 = 2 \times -\frac{5}{8}$~~

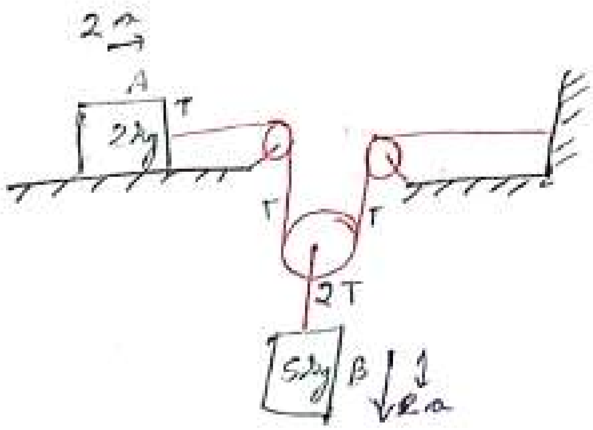
~~$T = 20 - \frac{5}{4}$~~

~~$T = \frac{80 - 5}{4}$~~

~~$\boxed{T = \frac{75}{4}\, N}$~~

~~$\boxed{T' = \frac{75}{2}\, N}$~~

Q. find acceleration.



$-x_A + 2x_B = 0$

$\boxed{2x_B = x_A}$

for A,

$T = \cancel{2}\,4a$ --- (1)

$2T = 8a$ --- (1)

for B

$50 - 2T = \cancel{10}\,5a$ --- (2)

(1) + (2)

$50 = \cancel{18}\,13a$

$\cancel{a = \frac{25}{7}}$ $a = \frac{50}{13}$

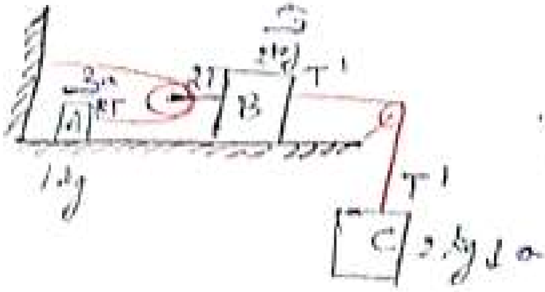
$\cancel{\boxed{a_A = \frac{25}{7}}}$

$\cancel{\boxed{a_B = \frac{50}{7}\ m/s^2}}$

$\boxed{a_B = \frac{50}{13}\ m/s^2}$

$\boxed{a_A = \frac{100}{13}\ m/s^2}$

Q find acceleration & Tension



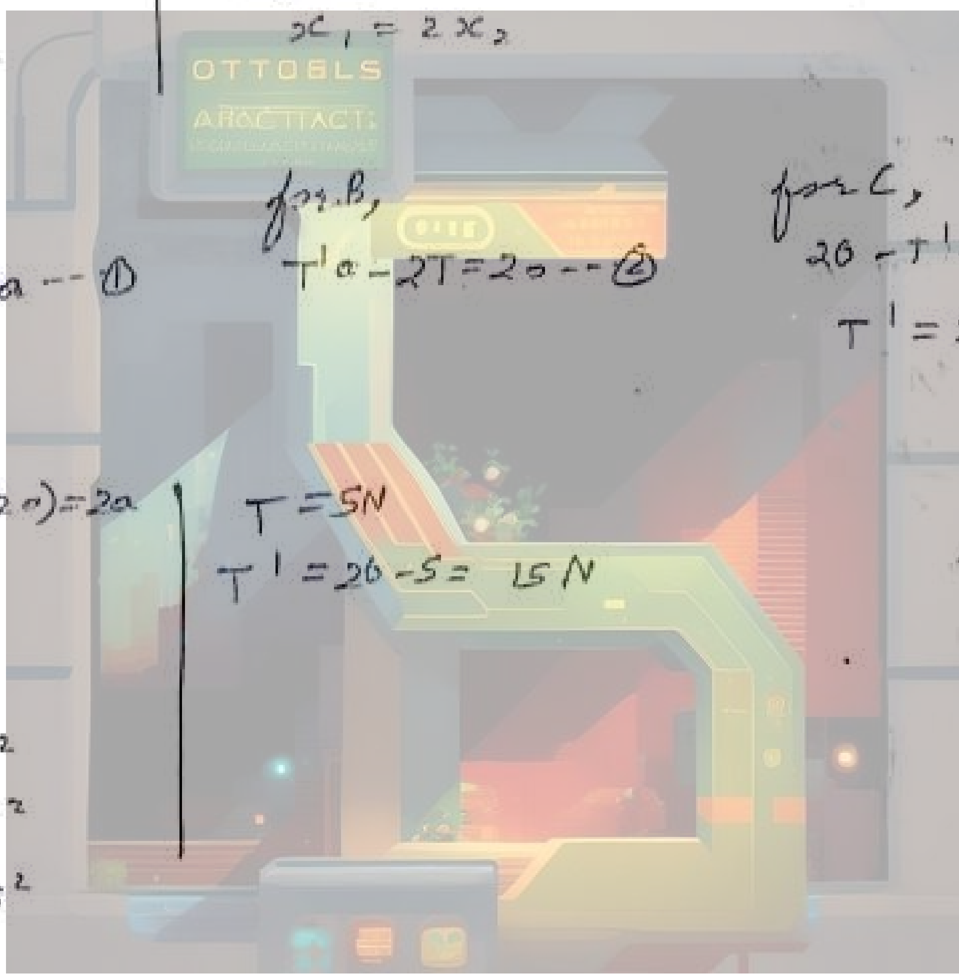
soln

$x_3 - x_2 = 0$

$x_3 = x_2$

$x_2 - x_1 + x_2 = 0$

$x_1 = 2x_2$



for A,

$T = 2a$ --- (1)

for B,

$T'a - 2T = 2a$ --- (2)

for C,

$20 - T' = 2a$

$T' = 20 - 2a$

$20 - 2a - 2(2a) = 2a$

$20 = 8a$

$a = \frac{5}{2}$

$T = 5N$

$T' = 20 - 5 = 15\,N$

$a_A = 5\ m/s^2$

$a_B = 5/2\ m/s^2$

$a_C = 5/2\ m/s^2$

Pg 103

Q25.

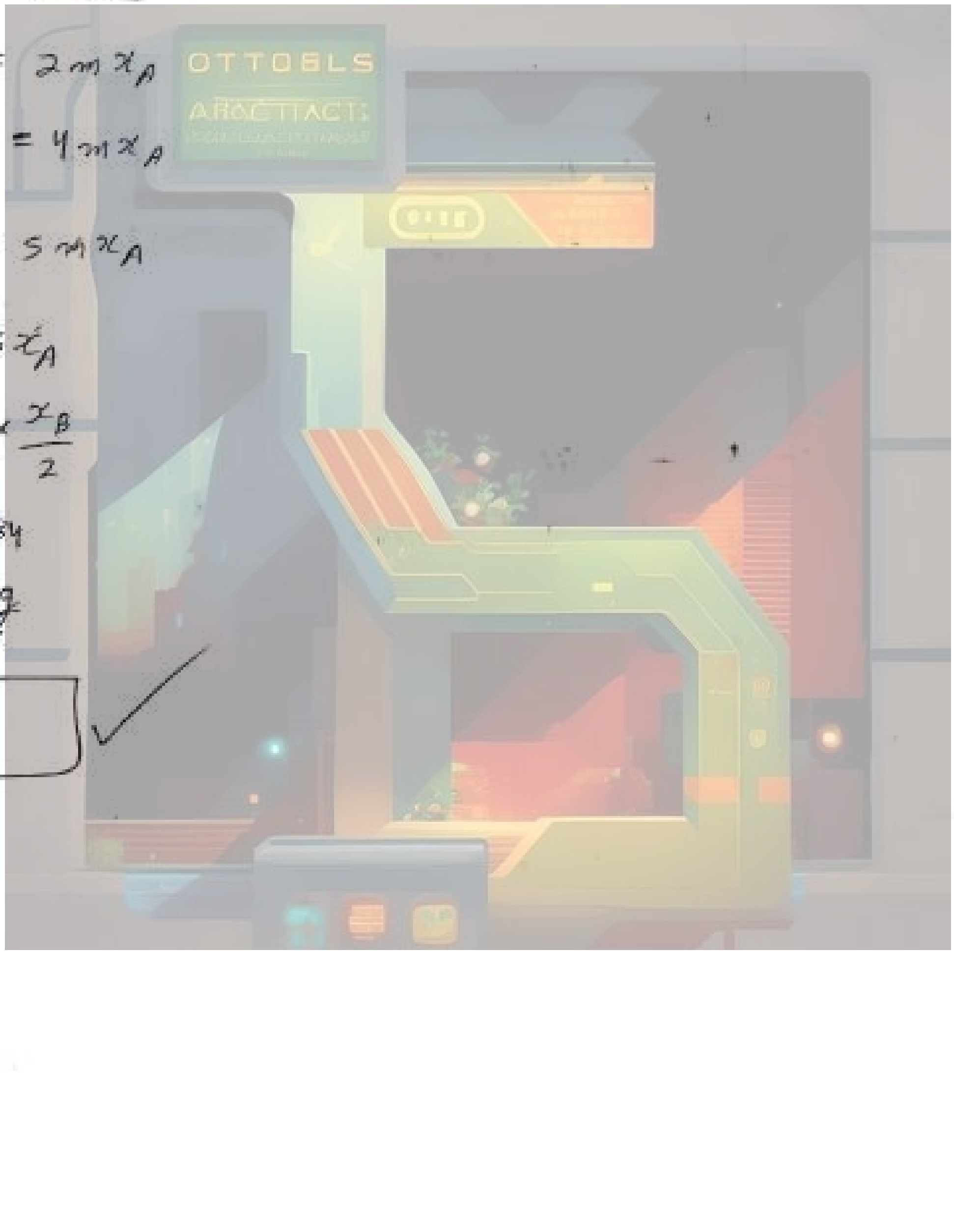
$x_A = x_P$

$-x_B + x_A + x_A = 0$

$2x_A = x_B$

$mg - 2T = m a x_A$

$\cancel{mg - 2T = 2m x_A}$



$T - mg = 2m x_A$

$2T - 2mg = 4m x_A$

$-mg = 5m x_A$

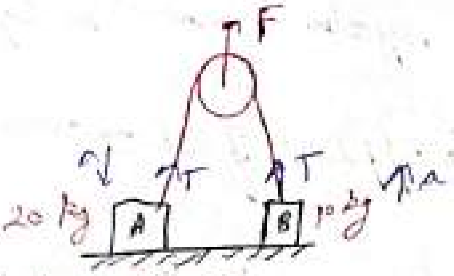
$-g = 5 x_A$

$-g = 5 \times \frac{x_B}{2}$

$x_B = \frac{2g}{5}$

$\boxed{C}$

1



find acc if F :-

a) 124 N   b) 294 N   c) 424 N

a) $T = 62\,N$

for A

$mg\ 200 - T = 20a$

for B

$T - 100 = 10a$

(1) + (2)

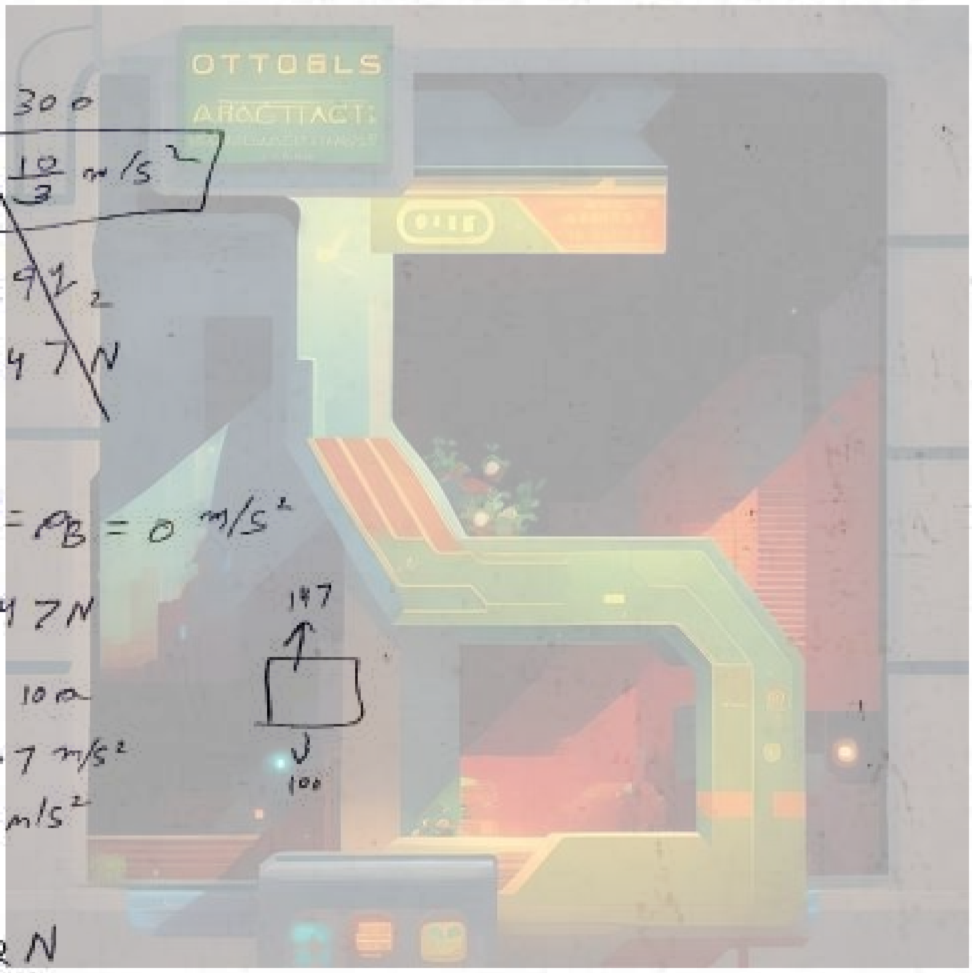
$100 = 30a$

$\boxed{a = \frac{10}{3}\ m/s^2}$

c) $T = 294/2$

$= 147\,N$

for A



a) acc of $A = a_B = 0\ m/s^2$

b) $T = 147\,N$

147
100

$147 - 100 = 10a$

$a_B = 4.7\ m/s^2$

$a_A = 0\ m/s^2$

c) $T\ 212\,N$

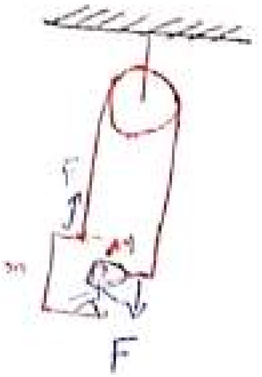
$12 = 20a$

$112 = 10a$

$a_A = \frac{3}{5}\ m/s^2 = 0.6\ m/s^2$

$a_B = 11.2\ m/s^2$

1)



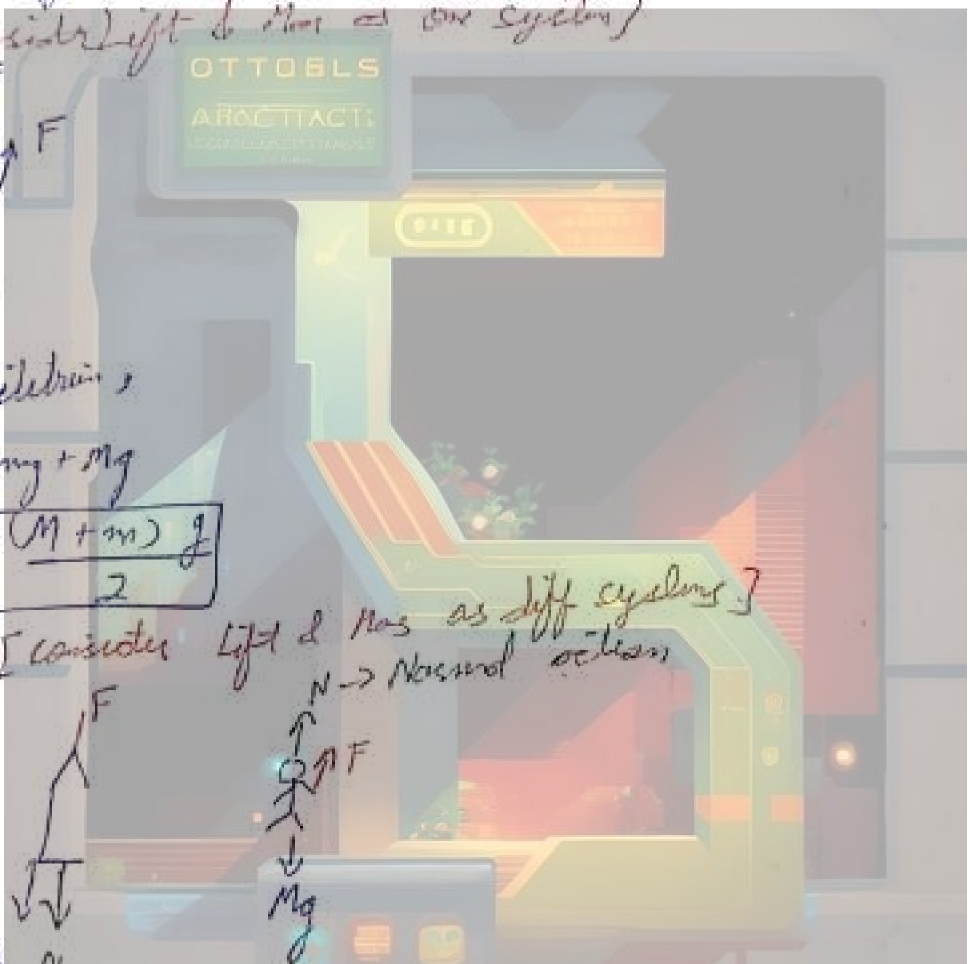
a) find F for which if system remains in equilibrium

b) If force F' (>F) is applied find acceleration of system

c) Find Normal Reaction on man in (b)

In Lift,

Method I [consider Lift & Man as one system]



F F

mg Mg

for equilibrium,

$2F = mg + Mg$

$$\boxed{F = \frac{(M+m)\,g}{2}}$$

Method II [consider Lift & Man as diff system]

F

N → Normal action

F

mg N → Normal reaction

Mg

$F = mg + N$

$F + N = Mg$

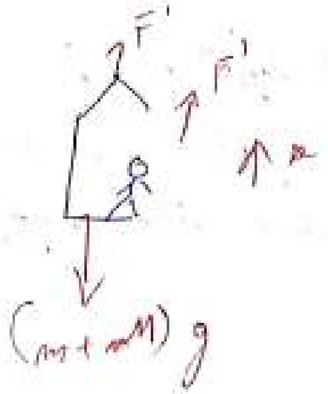
$N = Mg - F$

$F = mg + Mg - F$

$2F = (m+M)\,g$

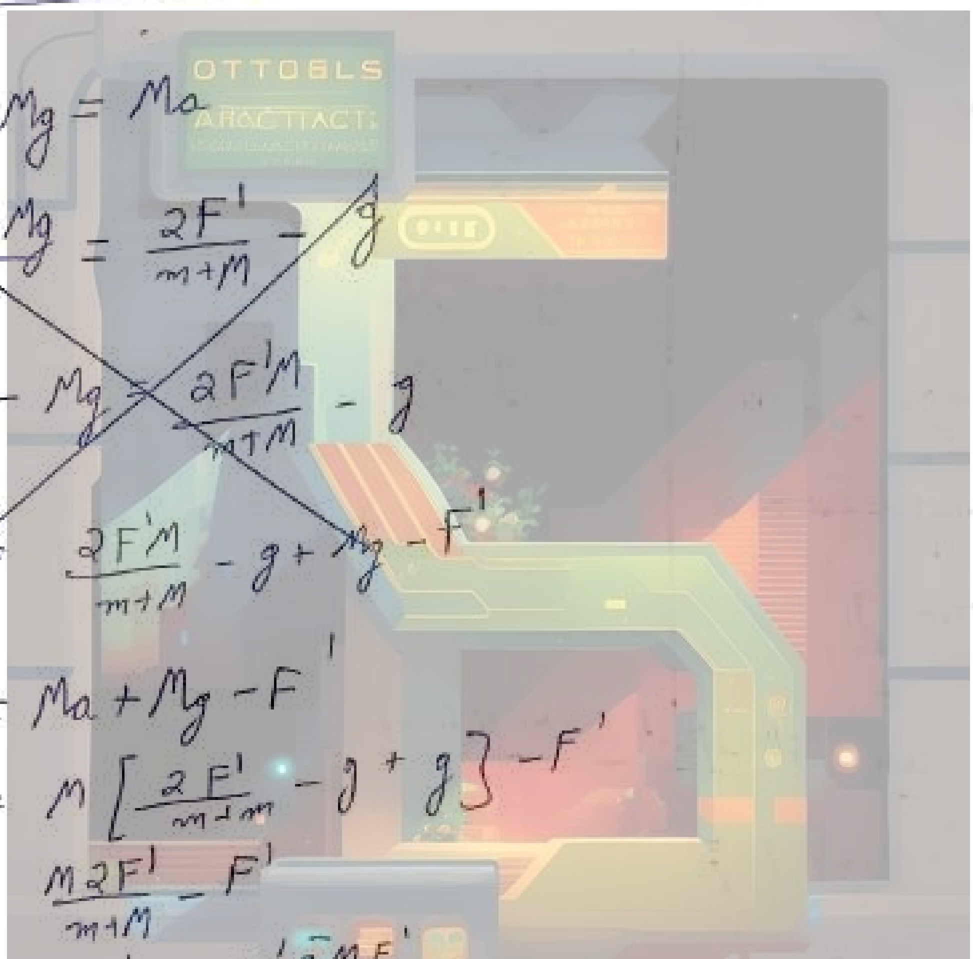
$$F = \frac{(m+M)\,g}{2}$$

b)



$$2F' - (m+M)g = (m+M)a$$

$$\boxed{a = \frac{2F'}{m+M} - g}$$



c) $N + F' - Mg = Ma$

$$\frac{N + F' - Mg}{M} = \frac{2F'}{m+M} - g$$

$$N + F' - Mg = \frac{2F'M}{m+M} - g$$

$$N = \frac{2F'M}{m+M} - g + Mg - F'$$

$$N = Ma + Mg - F'$$

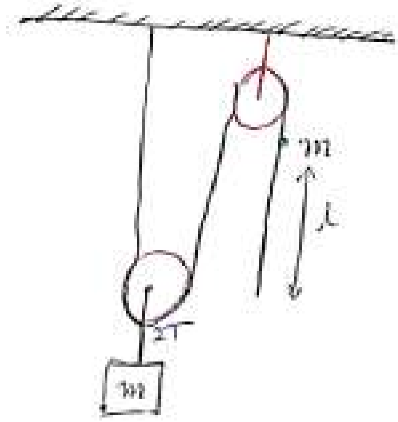
$$N = M\left[\frac{2F'}{m+M} - g + g\right] - F'$$

$$N = \frac{M2F'}{m+M} - F'$$

$$N = \frac{M2F' - mF' - MF'}{m+M}$$

$$\boxed{N = \frac{F'(\cancel{2}M - m)}{(M+m)}}$$

Q



friction between bead & light string is $mg/4$. System is released from rest. Find time taken by string to lost contact with string.

Tension $T = mg/4$

$mg/2$

mg

$mg - \frac{mg}{2} = a$

$\cancel{mg}\ \frac{mg}{2} = a\,m$

~~$mg = 2a\,m$~~

$a_1 = \frac{g}{2}$

$mg/4$

mg

$mg - \frac{mg}{4} = a_2 m$

$g - \frac{g}{4} = a_2$

$a_2 = \frac{3}{4}g$

* rope will move with speed twice of block :-

$a_{rel} = \frac{3}{4}g - (-g)$

$= \frac{7}{4}g$

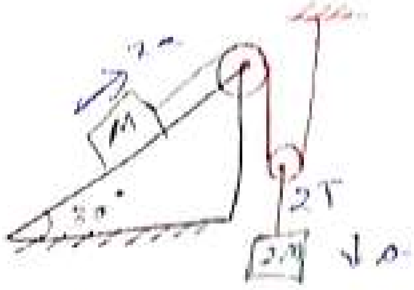
$s = ut + \frac{1}{2}at^2$

$l = 0 + \frac{1}{2} \times \frac{7}{4} g t^2$

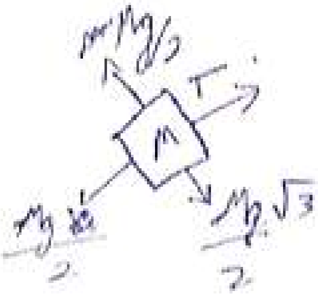
$\frac{8l}{7g} = t^2$

$\boxed{t = 2\sqrt{\frac{2l}{7g}}}$

Q.



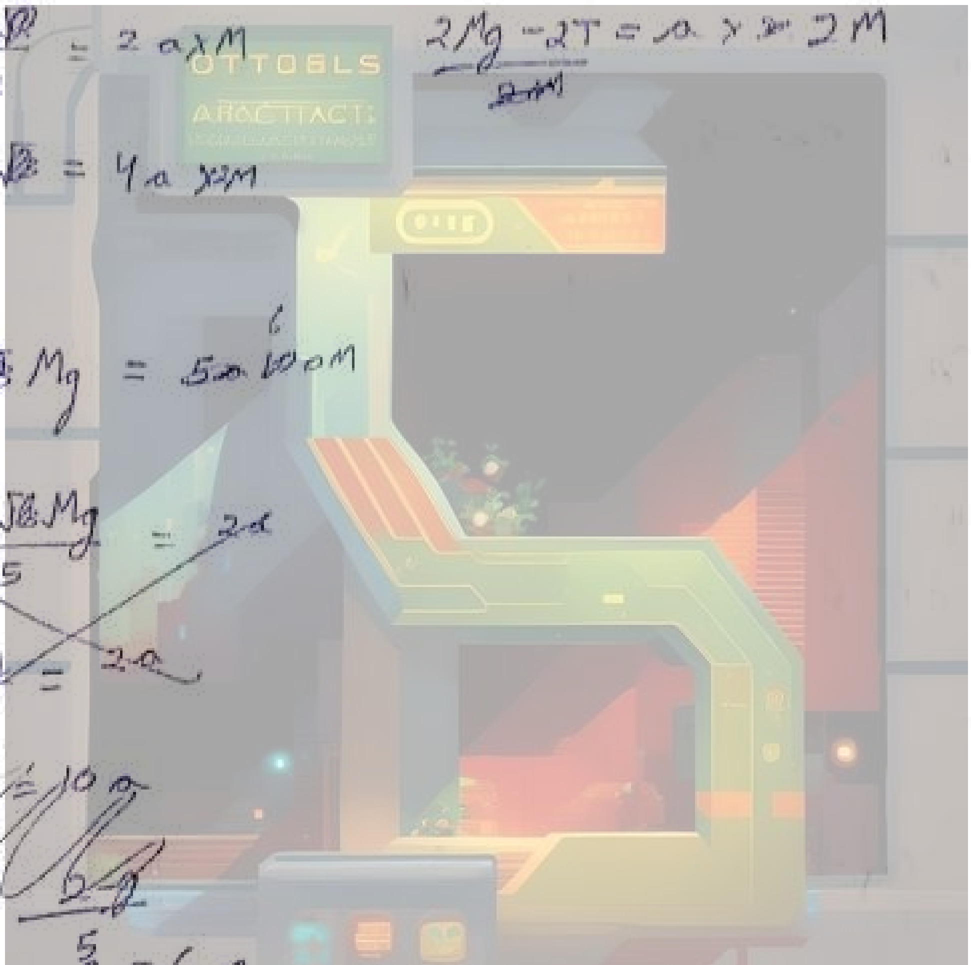
find acc of M



$T - \frac{Mg}{2} = 2a \times M$

$2Mg - 2T = a \times 2M$

$2T - Mg = 4a \times M$



$2Mg - Mg = 6aM$

$\frac{4Mg - 2\sqrt{3}Mg}{5} = 2a$

$\frac{2Mg}{5} = 2a$

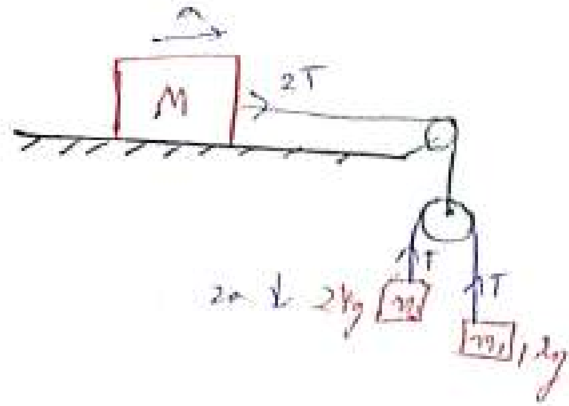
$2a = \frac{6g}{5}$

$2g - g = 6a$

$\frac{g}{6} = a$

$\boxed{2a = \frac{g}{3}}$

①



$a_{m_1} = 0$

$a_{2,p} = -a_{m_1 p}$

$a_{m_2} - a_{mp} = a_{mp} - a_{m_1}$

$a_{m_2} = 2a_p$

$a_{m_2} = 2a_M$

$T - 10 = 0$

$\boxed{T = 10}$

$20 - T = 2a$

$20 - 10 = 2a$

$\boxed{a = 5/2}$

$2T = Ma$

$\frac{20}{5/2} = aM$

$\frac{40}{5} = aM$

~~a = 8 m/s~~

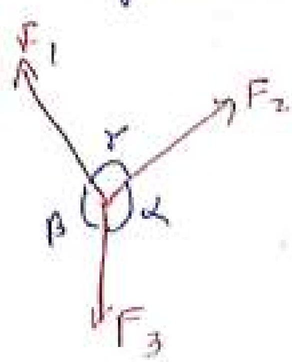
$\boxed{M = 8\ kg}$

Q <u>Static Equilibrium</u>

→ Vector Sum of all the ~~o~~forces acting on a body is zero.

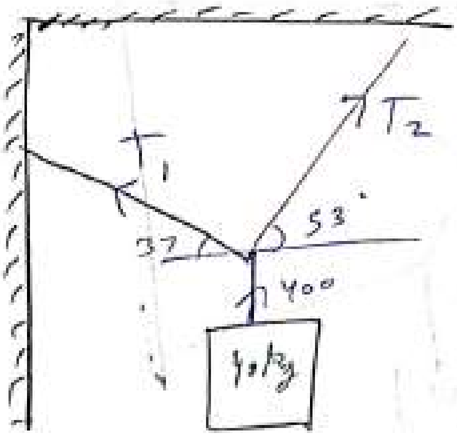
1) Lami's ~~rule~~ Theorem

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



2) Take component vectors of forces

Q find tension in three chords



M I

$$\frac{400}{\sin 90} = \frac{T_1}{\sin(90+53)} = \frac{T_2}{\sin(90+37)}$$

$$400 = \frac{T_1}{\cos 53}$$

$$400 \times \frac{3}{5} = T_1$$

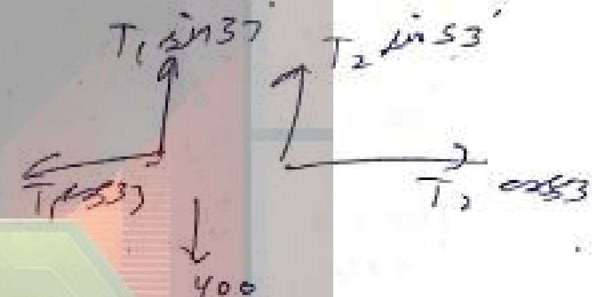
$$\boxed{T_1 = 240\,N}$$

$$400 = \frac{T_2}{\cos 37}$$

$$400 \times \frac{4}{5} = T_2$$

$$\boxed{T_2 = 320\,N}$$

M II



$$T_1 \cos 37 = T_2 \cos 53$$

$$T_1 = \frac{3T_2}{4}$$

$$T_1 \sin 37 + T_2 \sin 53 = 400$$

$$\frac{3T_2}{4} \times \frac{3}{5} + T_2 \times \frac{4}{5} = 400$$

$$\frac{9T_2}{20} + \frac{4T_2}{5} = 400$$

$$\frac{9T_2 + 16T_2}{20} = 400$$

$$25T_2 = 8000$$

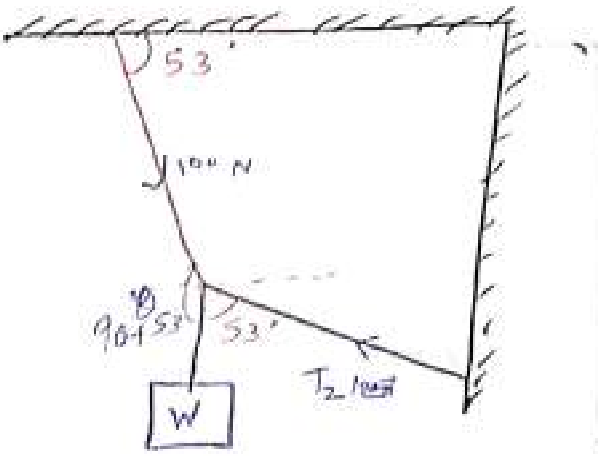
$$T_2 = \frac{8000}{25}$$

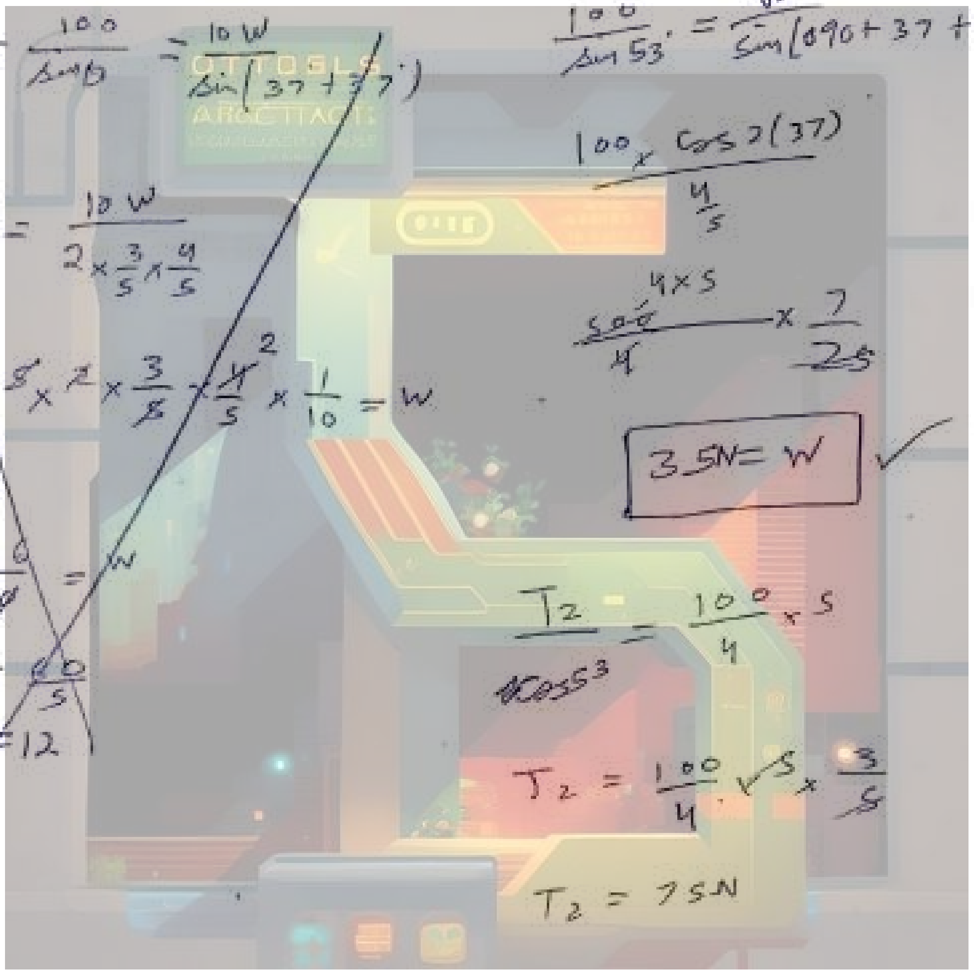
$$\boxed{= 320\,N}$$

$$T_1 = \frac{3}{4} \times 320$$

$$\boxed{= 240\,N}$$

Q max tension 100 N, which the ropes can withstand. find the maximum value of W.





$$\frac{100}{\sin 53} = \frac{100}{\sin\theta} = \frac{10W}{\sin(37+37)}$$

$$\frac{100}{4}\times 5 = \frac{10W}{2\times\frac{3}{5}\times\frac{4}{5}}$$

$$\frac{100}{\cancel{4}}\times \cancel{5}\times 2\times\frac{3}{\cancel{5}}\times\frac{\cancel{4}}{5}\times\frac{1}{10} = W$$

$$\frac{10\cancel{0}\times 6}{5\times 1\cancel{0}} = W$$

$$W = \frac{60}{5}$$

$$W = 12$$

$$\frac{100}{\sin 53} = \frac{W}{\sin(090+37+37)}$$

$$\frac{100\times\cos 2(37)}{\frac{4}{5}}$$

$$\frac{500}{4}^{4\times5}\times\frac{7}{25}$$

$$\boxed{35N = W}$$ ✓

$$\frac{T_2}{\cos 53} = \frac{100}{4}\times 5$$

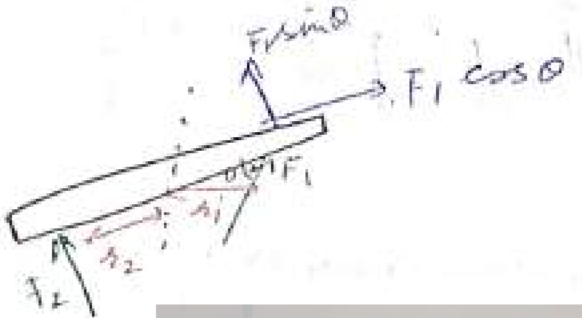
$$T_2 = \frac{100}{4}\times 5\times\frac{3}{5}$$

$$T_2 = 75N$$

(100)

Torque (τ)

→ Torque measures the turning effect of a force on the body.
→ Magnitude is given by the product of force & ⊥ distance from the axis of rotation.



$\tau_{F_1} = (F_1 \sin\theta)(r_1)$ (ACW) → Anti-clockwise

$\tau_{F_2} = F_2 r_2$ (CW)

$\tau_{net} = \tau_{F_1} - \tau_{F_2}$ (ACW)

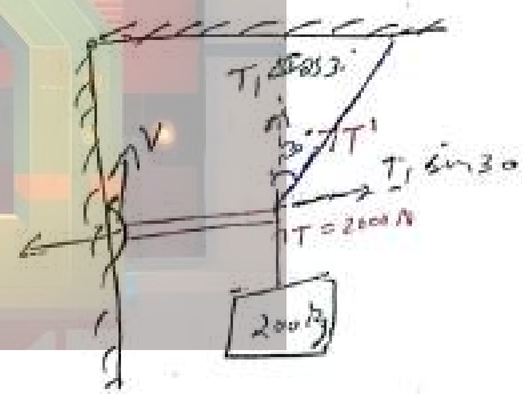
Q ~~for~~ find Tension T in string ($g = 10$)
find force exerted by ~~string and~~ rod on hinge

$\Sigma \vec{F} = 0$ translatory equilibrium

$T_1 \sin 30 = H$

$\frac{T_1}{2} = H$ --- ①

~~$T_1 = 2H$~~



$V + \frac{\sqrt{3}T'}{2} = 2000$ --- ②

rotational equilibrium

$\Sigma \vec{\tau} = 0 \quad \vec{\tau}_P = 0$

$T' \cos 30\, L = 2000 \times L$

$T\frac{\sqrt{3}}{2} = 2000$

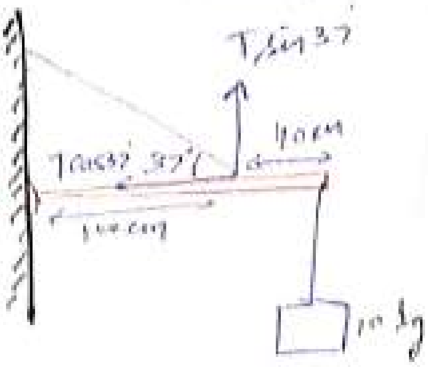
$\boxed{T' = \frac{4000}{\sqrt{3}}}$

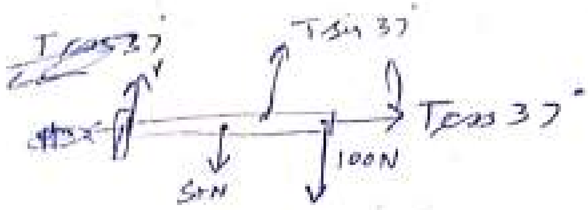
$\boxed{H = \frac{2000}{\sqrt{3}}}$

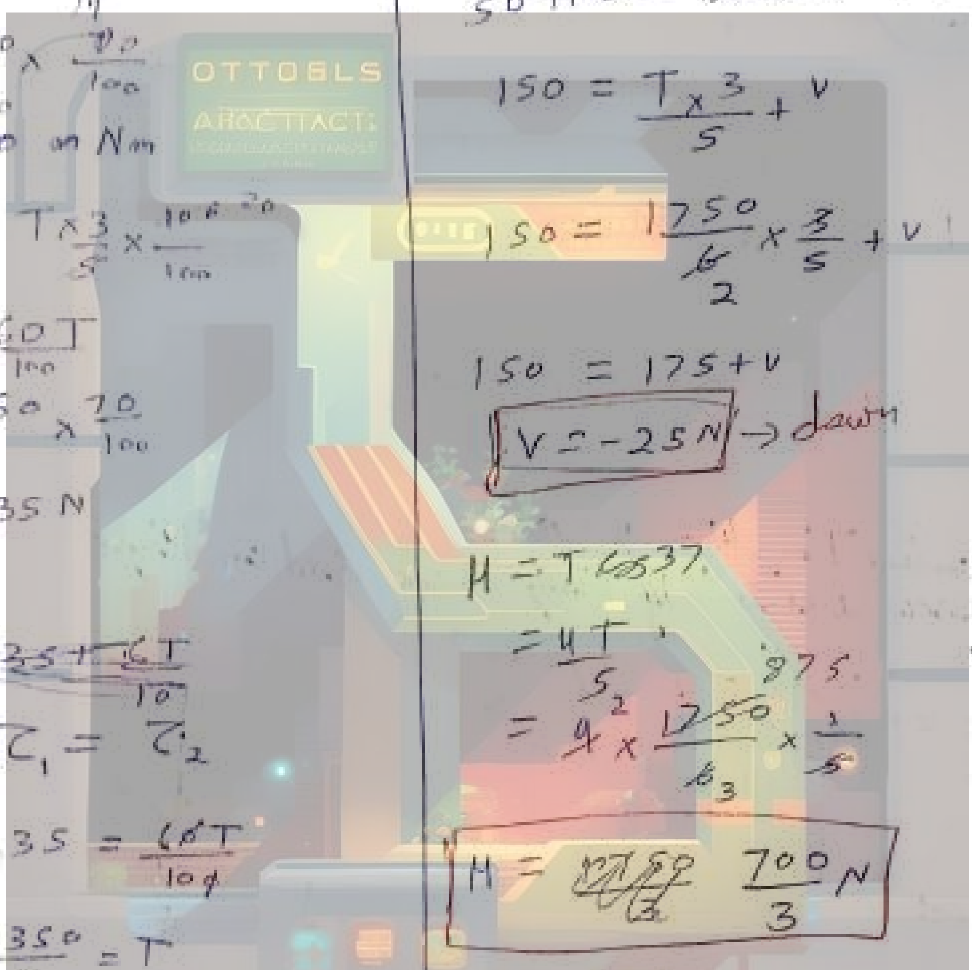
$\boxed{V = 0}$

Fnet on hinge → $\sqrt{H^2 + V^2} = \frac{2000}{\sqrt{3}}\,N$

Q Mass of rod = 5 kg
find tension & force exerted by hinge on rod.







$\tau_1 = 100 \times \frac{140}{100}$

$= 140$ Nm

$\tau_2 = T \times \frac{3}{5} \times \frac{100}{100}$

$= \frac{60T}{100}$

$\tau_3 = 50 \times \frac{70}{100}$

$= 35$ N

$\cancel{140 = 35 + \frac{6T}{10}}$

$\tau_3 + \tau_1 = \tau_2$

$140 + 35 = \frac{6T}{10}$

$\frac{1400 + 350}{6} = T$

$\boxed{T = \frac{1750}{6} \text{ N}}$

$50 + 100 = T \sin 37° + V$

$150 = T \times \frac{3}{5} + V$

$150 = \frac{1750}{6} \times \frac{3}{5} + V$

$150 = 175 + V$

$\boxed{V = -25 \text{ N}}$ → down

$H = T \cos 37$

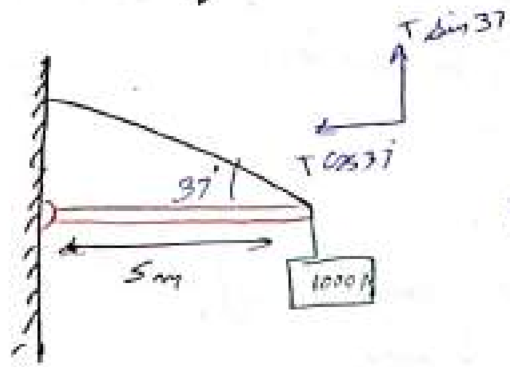
$= \frac{4T}{5}$

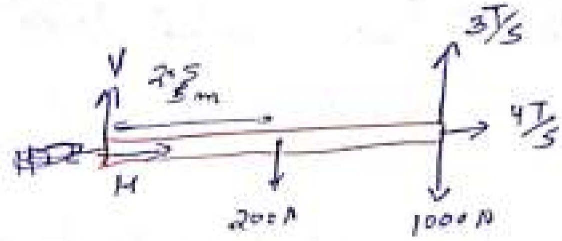
$= 4 \times \frac{1750}{6} \times \frac{1}{5}$

$\boxed{H = \frac{700}{3} \text{ N}}$

~~Force hinge~~

weight of rod = 200 N
find tension & force exerted by hinge on rod.





$\tau_1 = \frac{3T}{5} \times 5$

$= 3T$ Nm

~~$\tau_2 = \frac{4T}{5} \times 5$~~

~~$= 4T$ Nm~~

$\tau_2 = 1000 \times 5$

$= 5000$ Nm

$\tau_3 = 200 \times 2.5$

$= 500$ Nm

$\tau_3 + \tau_2 = \tau_1$

~~$500 + 4T = 3T$~~

~~$500 =$~~

$5000 + 500 = 3T$

$\frac{5500}{3} = T$

$\boxed{T = \frac{5500}{3} N}$ ✓

$200 + 1000 = V + \frac{3T}{5}$

$1200 - 1100 = V$

$\boxed{V = 100 N}$ ✓

$H = \frac{4T}{5}$

$\boxed{H = \frac{4400}{3} N}$ ✓

force on rod by hinge $= \sqrt{H^2 + V^2}$

$= \sqrt{10000 + \frac{19360000}{9}}$

$= \sqrt{\frac{90000 + 19360000}{9}}$

$= \frac{100}{3}\sqrt{1945}$ N

Pseudo Force

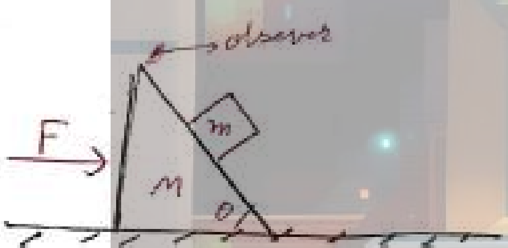
→ Apply a pseudo force on an object if and only if it is placed on another object (non-inertial frame) accelerating wrt some inertial frame of reference.

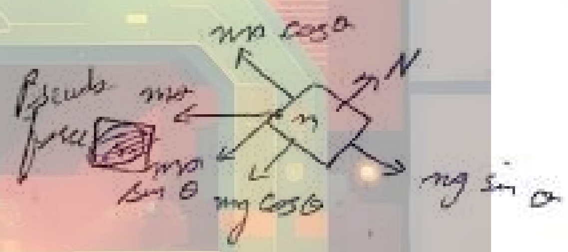
→ The direction of Pseudo force must be opposite to direction of acceleration of non-inertial frame.

→ Pseudo Force = mass of Body × acceleration of non-inertial frame.

→ After applying Pseudo force on a body, all equations and results associated with it become relative to respective non-inertial frame.

Q. All surfaces are smooth.
find F such that m remains at rest wrt wedge





$F = (m+M)a$

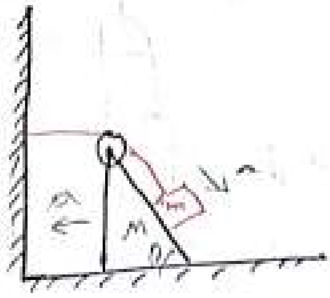
$a = \dfrac{F}{m+M}$

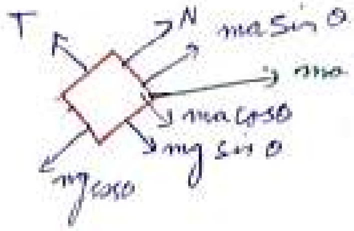
$ma\cos\theta = mg\sin\theta$ (at rest)

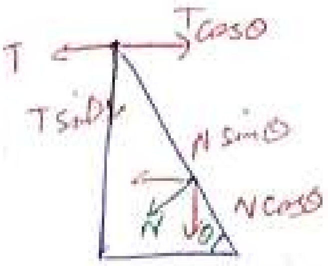
$a = g\tan\theta$

$$\boxed{F = (m+M)\,g\tan\theta}$$

Q find acceleration of M.







$N + ma\sin\theta = mg\cos\theta$ (block has no vertical velocity)

$N = mg\cos\theta - ma\sin\theta$ --- (1)

$mg\cos\theta + mg\sin\theta - T = am$ (block's horizontal vel)

$T = mg\cos\theta + mg\sin\theta - ma$ --- (2)

$T + N\sin\theta - T\cos\theta = Ma$ --- (3)

$mg\cos\theta + mg\sin\theta - ma + mg\cos\theta\sin\theta - mg\sin^2\theta - mg\cos^2\theta - mg\sin\theta\cos\theta$

$+ ma\cos\theta = Ma$

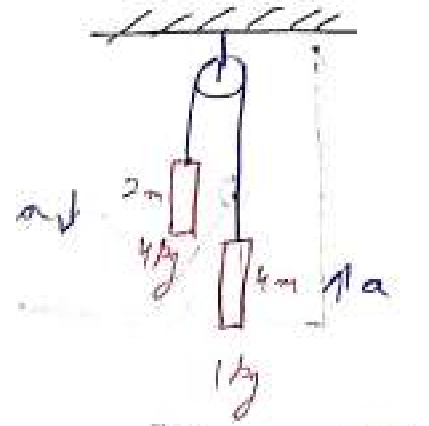
$mg\cos\theta + mg\sin\theta - ma - mg + ma\cos\theta = Ma$

$mg\sin\theta - mg = Ma + ma - 2ma\cos\theta$

$$a = \frac{mg\sin\theta - mg}{M + m - 2m\cos\theta}$$

$$\boxed{a = \frac{g(m\sin\theta - m)}{M + m - 2m\cos\theta}}$$

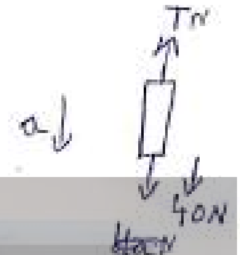
Q released from rest. find time to across each other



~~$40N + 4a - T = 4a$~~

~~$40N = T$~~

~~$10 - T = a$~~



$40 - T = 4a$

$T - 10 = a$

$\boxed{40 - 10 = 5a}$

$30 = 5a$

$\boxed{a = 6\ m/s^2}$

$a_{rel} = 12\ m/s^2$

$s_{rel} = 6\ m$

$u = 0$

$6 = \frac{1}{2} \times 12 t^2$

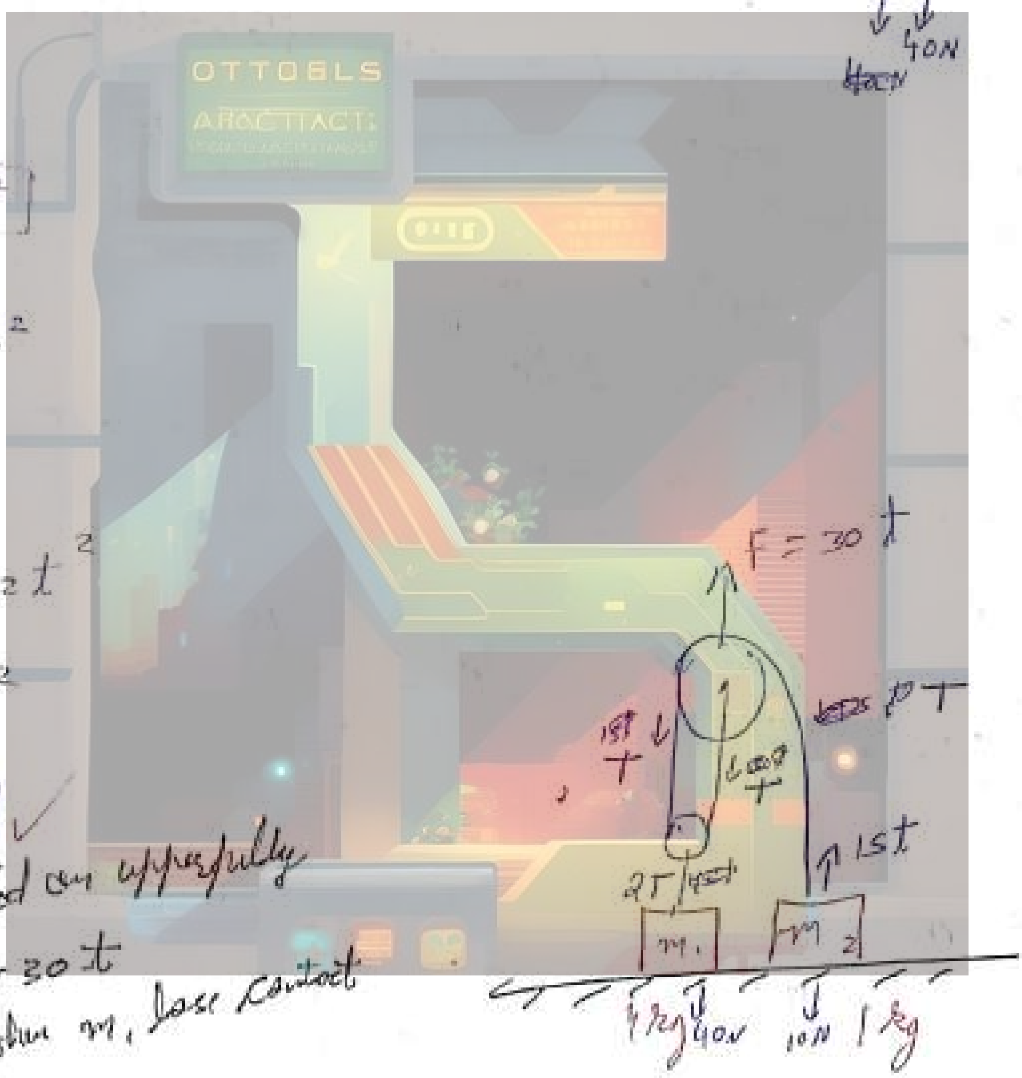
$\frac{12}{12} = t^2$

$\boxed{t = 1\ s}$

Q F is applied on upper pulley

$F = 30t$

find t when m_1 lose contact



$3T = 30t$

$T = 10t$

~~$45t = 40$~~

~~$t = \frac{8}{9}$~~

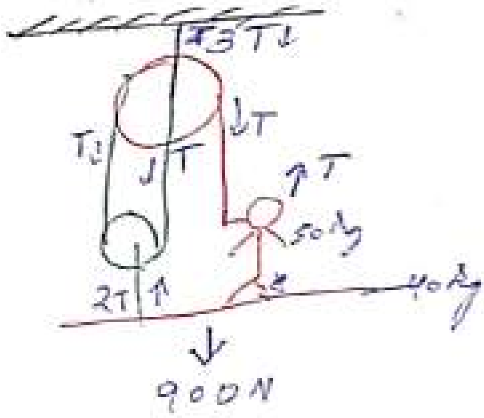
$2T = 40$

$20t = 40$

$\boxed{t = 2\ s}$

(100)

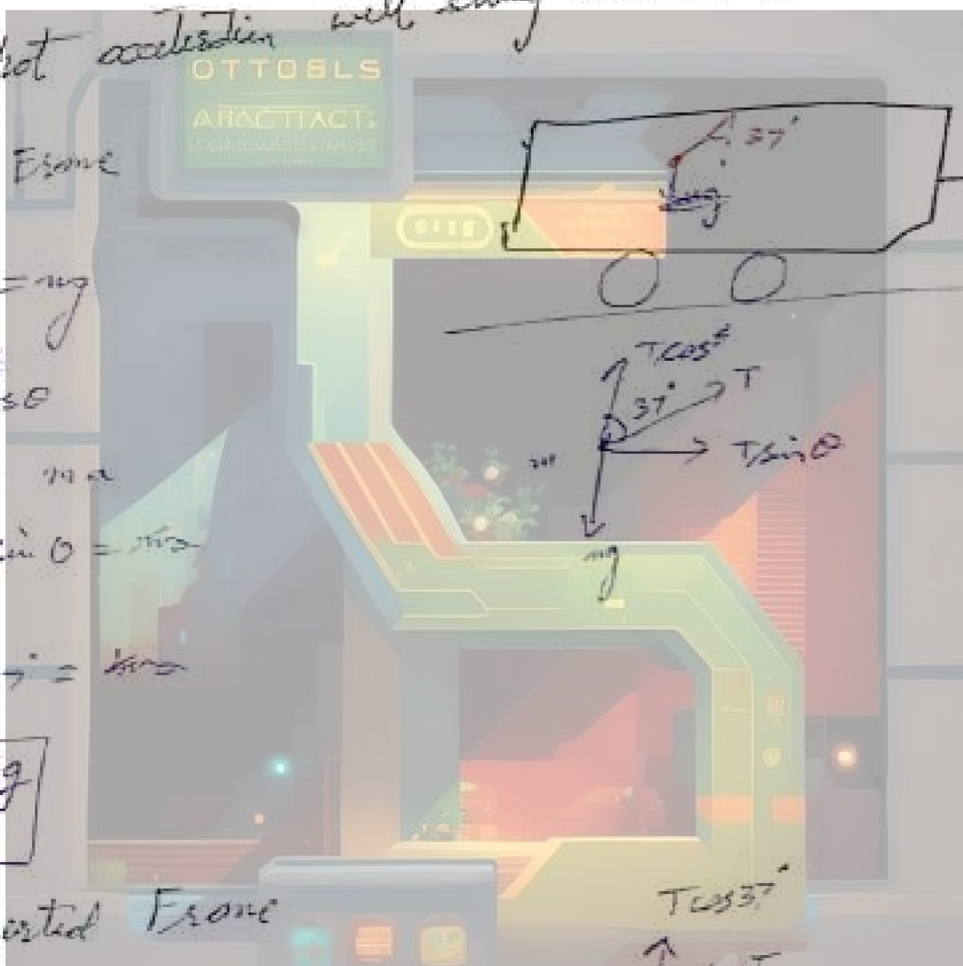
Q force man to apply to keep plat form in equation



$900\,N = \cancel{3T}\ 2T + T$

$\boxed{T = 300\,N}$

Q what accelerat acceleration will string make 37° with vertical



a) Ground Frame

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

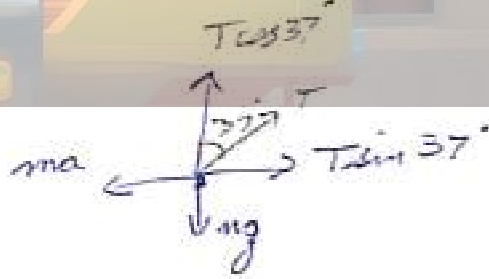
$T\sin\theta = ma$

$\frac{mg}{\cos\theta} \times \sin\theta = ma$

$g\tan 37° = a$

$\boxed{a = \frac{3g}{4}}$

b) Non-Inertial Frame



$T\sin\theta = ma$

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

$\frac{mg}{\cos\theta} \times \sin\theta = ma$

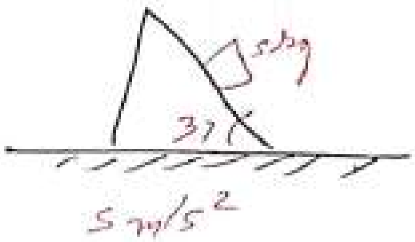
$g\tan\theta = a$

$\boxed{a = \frac{3g}{4}}$

Q Inclined plane is ~~moved~~ moved to right $5\,m/s^2$.
find force exerted by 5 kg block on inclined Plane.

~~$N = 5 \times \frac{3}{5}$~~

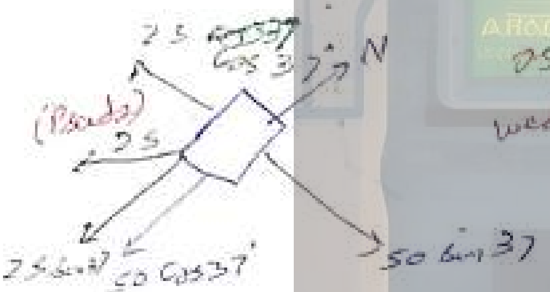
~~$N = 3$ Newton~~

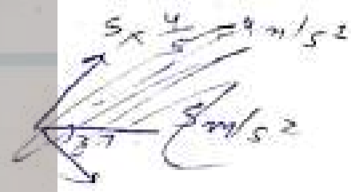


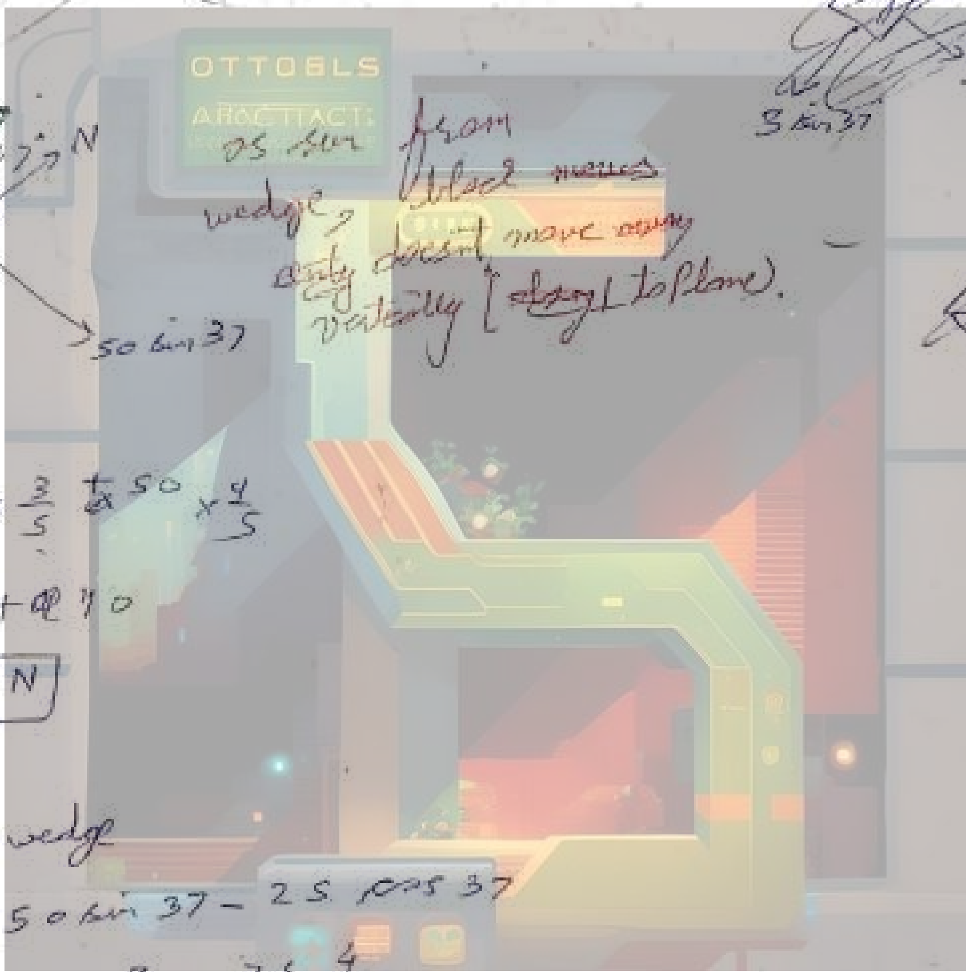
~~N~~

~~$N - 3 = 5 \times 4$~~

~~$N = ...$~~







from wedge, block moves only doesn't move away vertically [~~along~~ $\perp$ to plane].

$N = 25 \times \frac{3}{5} + 50 \times \frac{4}{5}$

$= 15 + 40$

$\boxed{= 55\,N}$

acceleration as seen from wedge

$5a = 50 \sin 37 - 25 \cos 37$

$= 50 \times \frac{3}{5} - 25 \times \frac{4}{5}$

$= 30 - 20$

$5a = 10$

$\boxed{a = 2\,m/s^2}$

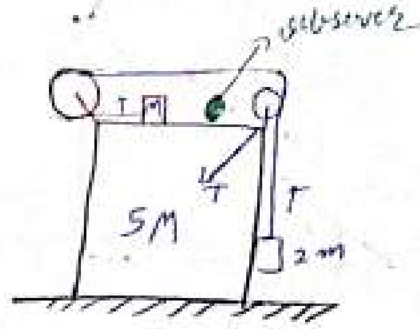
a_{net} (from ground) $= \sqrt{(2)^2 + (5)^2 + 2 \times 2 \times 5 \cos 37}$

$= \sqrt{4 + 25 + 20 \times \frac{4}{5}}$

$= \sqrt{45}$

$\boxed{= 3\sqrt{5}\,m/s^2}$

Q- find initial acceleration of wedge.



$N = 2Ma'$ --- (1)

$20Ma - T = 2Ma$ --- (2) non-inertial frame

$T + Ma' = Ma$ --- (3)

$T - N = 5Ma'$ Ground --- (4)

Put (1) in (4)

$T - 2Ma' = 5Ma'$

$T = 7Ma'$ --- (5)

put (5) in (3)

$8Ma' = Ma$

$8a' = a$ --- (6)

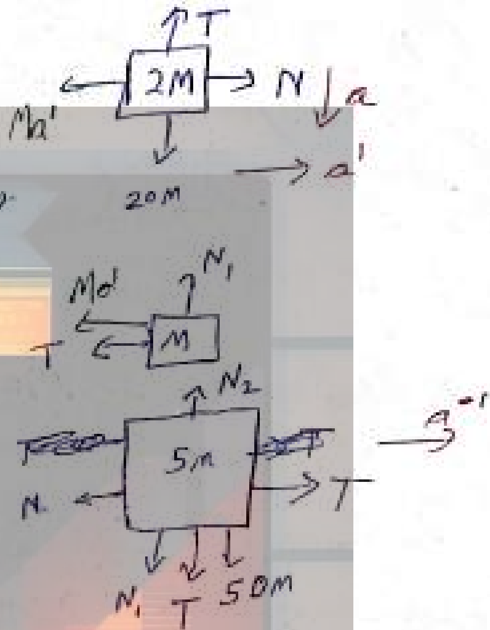
put (6) in (2) and (3)

$20M - 7Ma' = 2M(8a')$

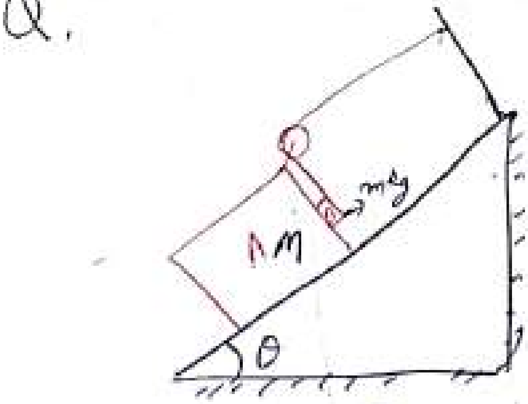
$20 - 7a' = 16a'$

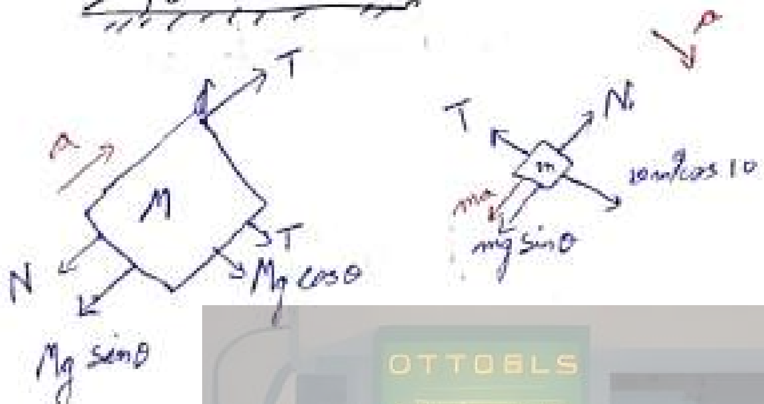
$20 = 23a'$

$\boxed{a' = \frac{20}{23}}$



Q. Find acceleration of A & B.





for A

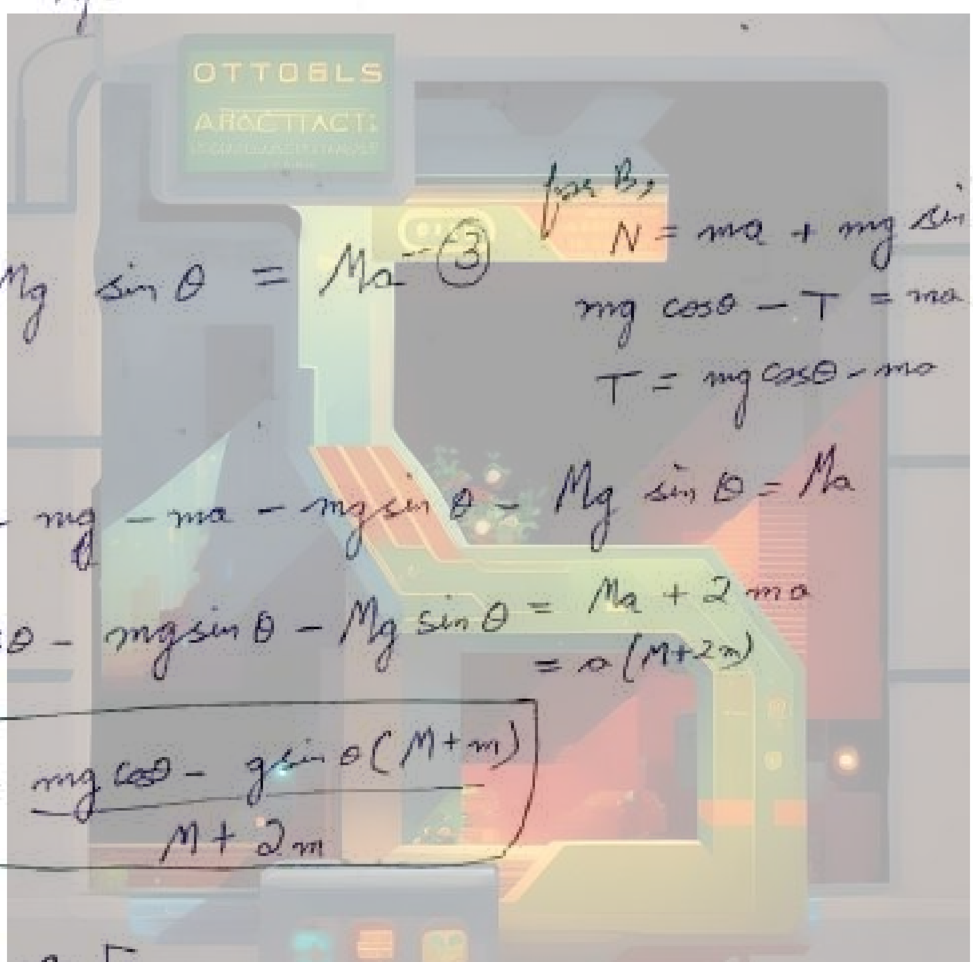
$$T - N - Mg\sin\theta = Ma \quad (3)$$

for B,

$$N = ma + mg\sin\theta \quad (1)$$

$$mg\cos\theta - T = ma$$

$$T = mg\cos\theta - ma \quad (2)$$



$$mg\cos\theta - ma - ma - mg\sin\theta - Mg\sin\theta = Ma$$

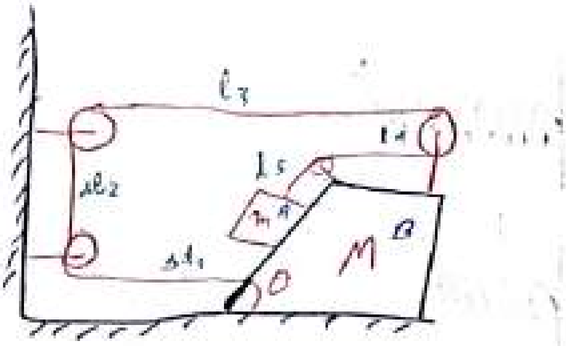
$$mg\cos\theta - mg\sin\theta - Mg\sin\theta = Ma + 2ma$$

$$= a(M+2m)$$

$$\boxed{a_A = \frac{mg\cos\theta - g\sin\theta(M+m)}{M+2m}}$$

$$a_B = a_A\sqrt{2}$$

Q find acceleration of M.



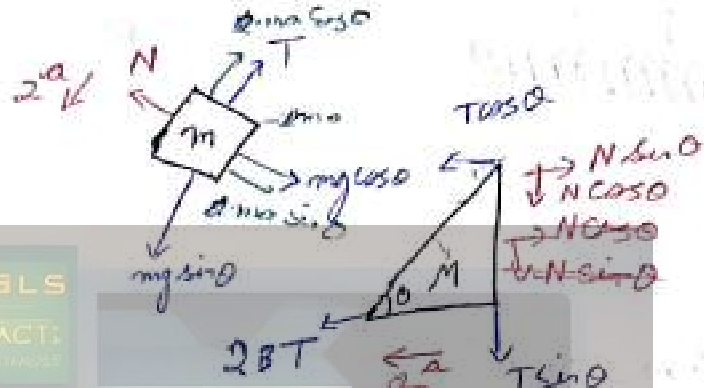
$x_A - x_B - x_B = 0$

$\cancel{x_A = 2x_B}$

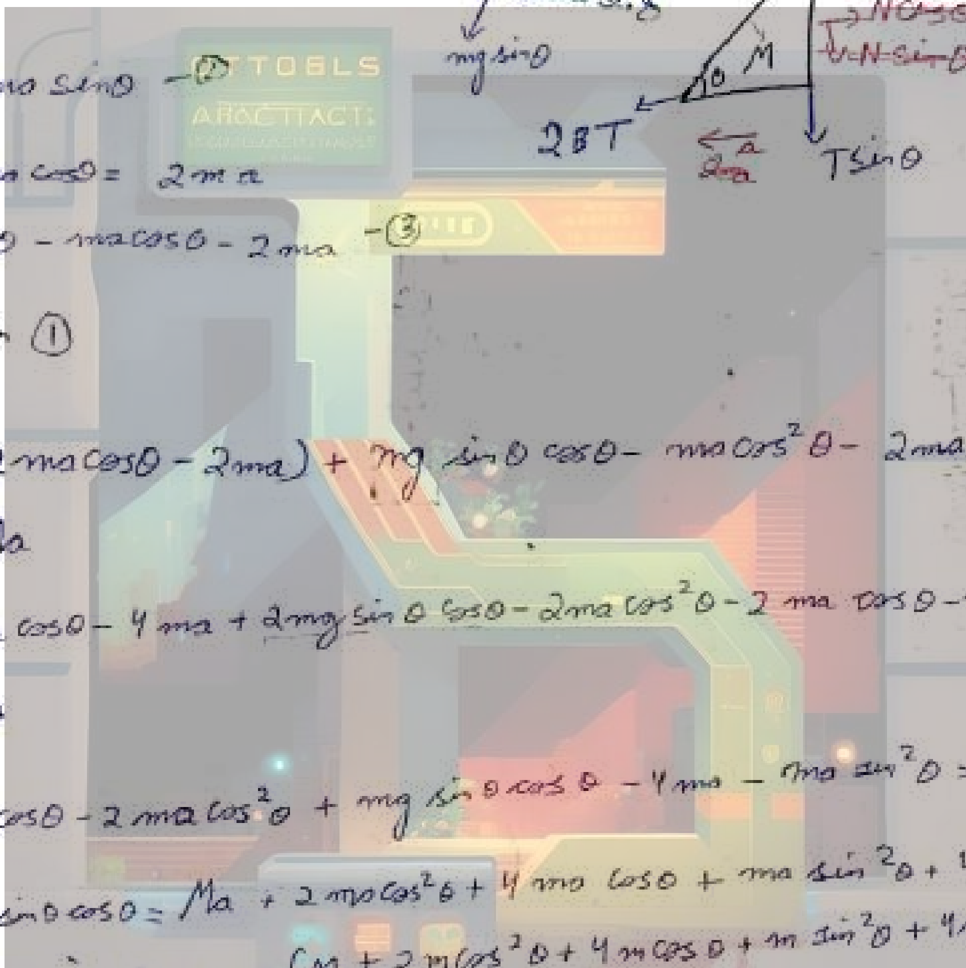
$x_A = 2x_B$

for M,

$2T + T\cos\theta - \cancel{N\cos\theta}\, N\sin\theta = Ma$ — (1)



for m,



$N = mg\cos\theta + ma\sin\theta$ — (2)

$mg\sin\theta - T - ma\cos\theta = 2ma$

$T = mg\sin\theta - ma\cos\theta - 2ma$ — (3)

put (3) & (2) in (1)

$2(mg\sin\theta - ma\cos\theta - 2ma) + mg\sin\theta\cos\theta - ma\cos^2\theta - 2ma\cos\theta - mg\cos\theta\sin\theta - ma\sin^2\theta = Ma$

$2mg\sin\theta - 2ma\cos\theta - 4ma + 2mg\sin\theta\cos\theta - 2ma\cos^2\theta - 2ma\cos\theta - mg\cos\theta\sin\theta - ma\sin^2\theta = Ma$

$2mg\sin\theta - 4ma\cos\theta - 2ma\cos^2\theta + mg\sin\theta\cos\theta - 4ma - ma\sin^2\theta = Ma$

$2mg\sin\theta + mg\sin\theta\cos\theta = Ma + 2ma\cos^2\theta + 4ma\cos\theta + ma\sin^2\theta + 4ma$

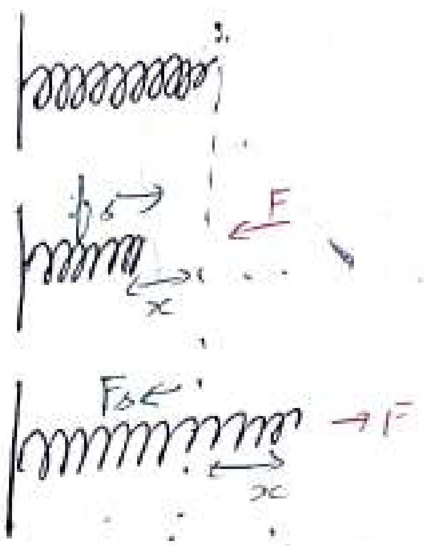
$2mg\sin\theta + mg\sin\theta\cos\theta = a(M + 2m\cos^2\theta + 4m\cos\theta + m\sin^2\theta + 4m)$

$2mg\sin\theta + mg\sin\theta\cos\theta = a(M + m\cos^2\theta + 4m\cos\theta + 5m)$

$mg(2\sin\theta + \sin\theta\cos\theta) = a(M + m\cos^2\theta + 4m\cos\theta + 5m)$

$$a = \frac{mg\sin\theta(2+\cos\theta)}{(M + m\cos^2\theta + 4m\cos\theta + 5m)}$$

Spring Force



Spring Force, $f_s \propto x$

$$\boxed{F_s = -Kx}$$

K = Spring Constant (stiffness of spring)

$$K \propto \frac{1}{\text{length of spring}}.$$

① Springs in series

$K_1 x_1 = K_2 x_2 = K_3 x_3 = mg$ (Tension at any Point is equal if springs are massless).

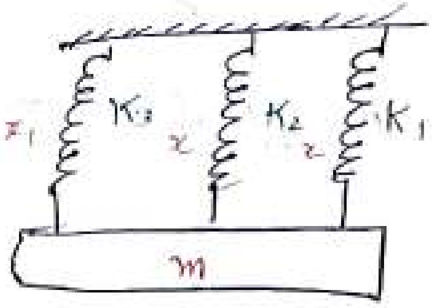
$K_{eq}(x_1 + x_2 + x_3) = mg$

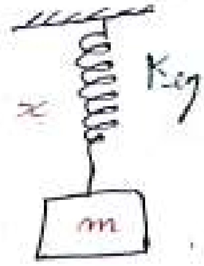
$$K_{eq}\left(\frac{mg}{K_1} + \frac{mg}{K_2} + \frac{mg}{K_3}\right) = mg$$

$$\cancel{\frac{1}{K_{eq}} = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$$

$$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}$$

(2) Springs in parallel



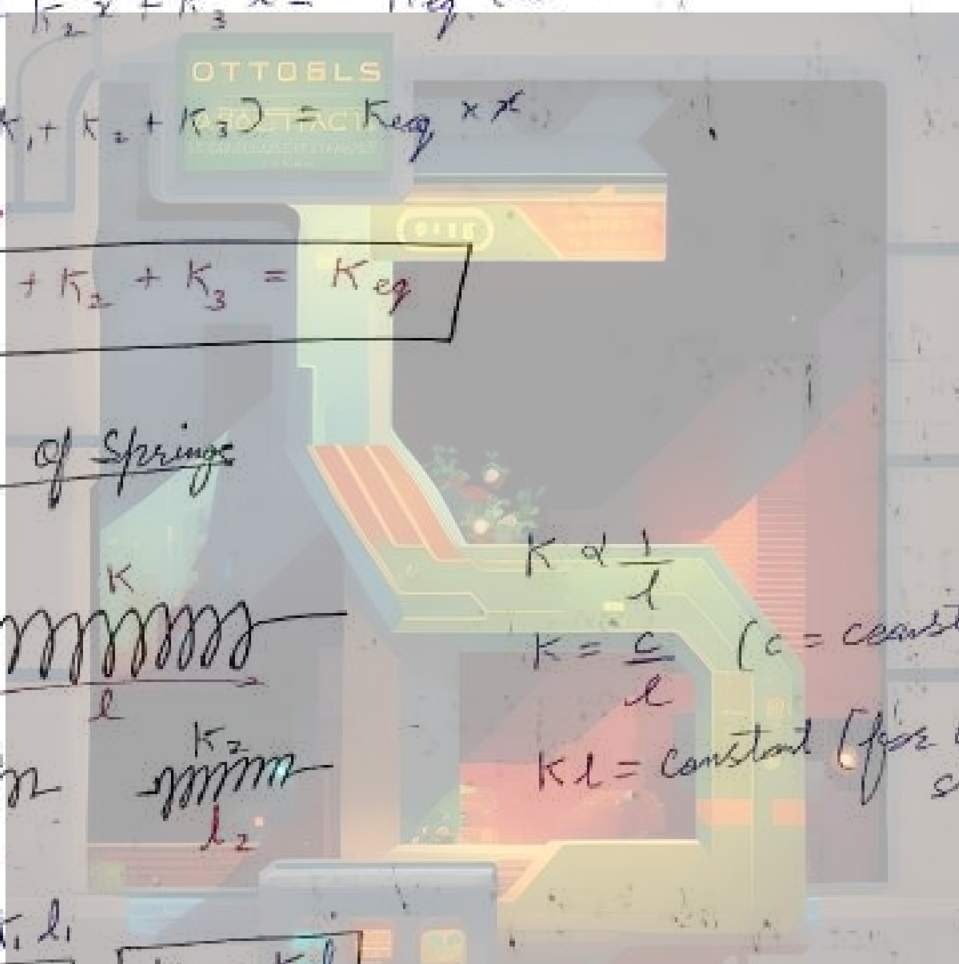


$$K_1 x + K_2 x + K_3 x = mg \qquad K_{eq}(x) = mg$$

$$K_1 x + K_2 x + K_3 x = K_{eq}(x)$$

$$x(K_1 + K_2 + K_3) = K_{eq} \times x$$

$$\boxed{K_1 + K_2 + K_3 = K_{eq}}$$



(3) Cutting of Springs

K l

K_1 l_1 K_2 l_2

$$K \propto \frac{1}{l}$$

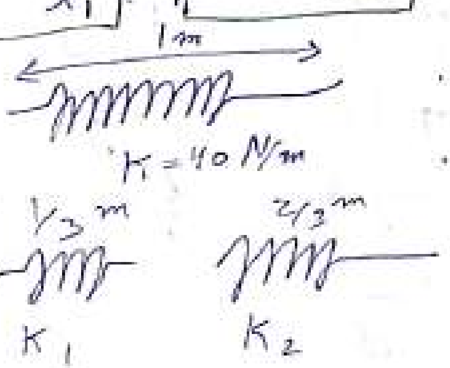
$$K = \frac{c}{l} \quad (c = \text{constant})$$

$$Kl = \text{constant (for an particular string)}$$

$$Kl = K_1 l_1$$

$$\boxed{K_1 = \frac{Kl}{l_1}} \qquad \boxed{K_2 = \frac{Kl}{l_2}}$$

eg.



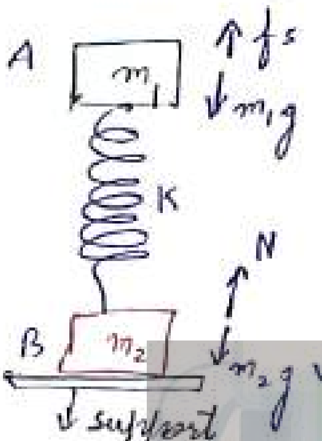
$$K_1 = \frac{40 \times 1}{1/3} = 120\ N/m$$

$$K_2 = \frac{40 \times 1}{2/3} = \frac{120}{2} = 60\ N/m$$

Breaking of Supports

→ The spring force do not change Instantaneously.

Q Find acceleration of blocks instantly after the support is removed.

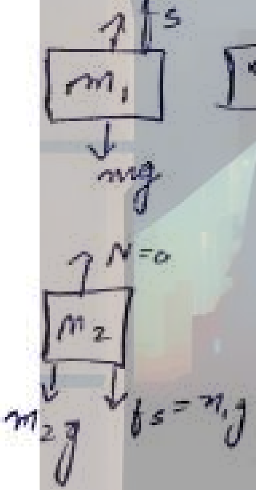


$\boxed{f_s = m_1 g}$

$N = m_2 g + f_s$

$\boxed{N = m_2 g + m_1 g}$

After removing is removed, $N = 0$

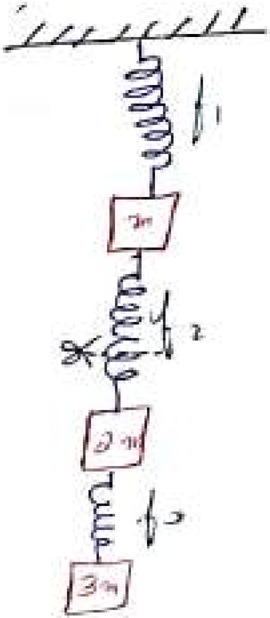


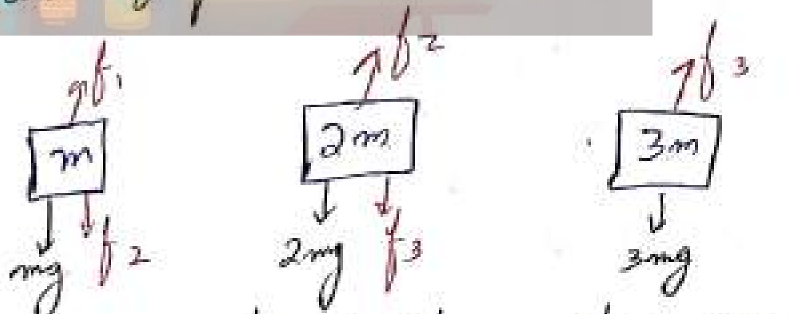
$\boxed{a_1 = 0}$

$m_2 g + m_1 g = m_2 a$

$\boxed{\dfrac{m_2 g + m_1 g}{m_2} = a}$

Q find initial acc of blocks if spring - 2 is cut.





$f_1 = mg + f_2$

$f_1 = 6\,mg$

$f_2 = 2mg + f_3$

$f_2 = 5mg$

$f_3 = 3\,mg$

$f_1 = 6mg$ | m | mg, $f_2 = 0$

$f_2 = 0$ | $2m$ | $2mg$, $f_3 = 3mg$

$f_3 = 3mg$ | $3m$ | $3mg$

when the spring 2 is cut, $f_2 = 0$

f_1 & f_3 = same

$6mg - mg = ma$

$5mg = ma$

$$\boxed{a_m = 5g \uparrow}$$

$3mg + 2mg = 2ma$

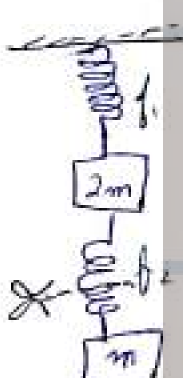
$5g = 2a$

$$\boxed{a_{m_2} = \frac{5g}{2} \downarrow}$$

$3mg - 3mg = 3ma$

$$\boxed{a_{m_3} = 0}$$

Q Find acc of blocks after spring 2 is cut



f_1 | $2m$ | $2mg$, f_2

f_2 | m | mg

$2mg + mg = f_1$

$f_1 = 3mg$

$f_2 = mg$

After cutting spring 2, $f_2 = 0$

$f_1 = 3mg$ | $2m$ | a_1 | $2mg$, 0

0 | m | a_2 | mg

$3mg - 2mg = 2ma_1$

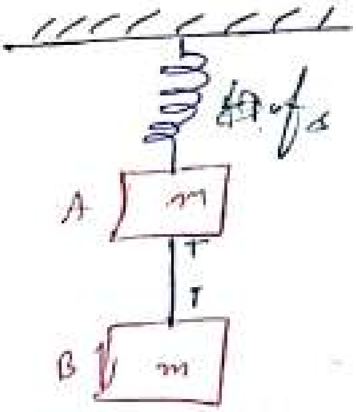
$g = 2a_1$

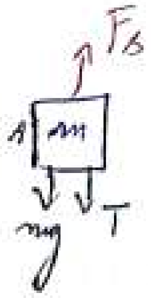
$$\boxed{a_1 = \frac{g}{2}} \uparrow m/s^2$$

$mg = ma_2$

$$\boxed{a_2 = g \downarrow} m/s^2$$

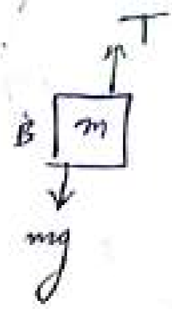
Q acc of Blocks after spring is cut





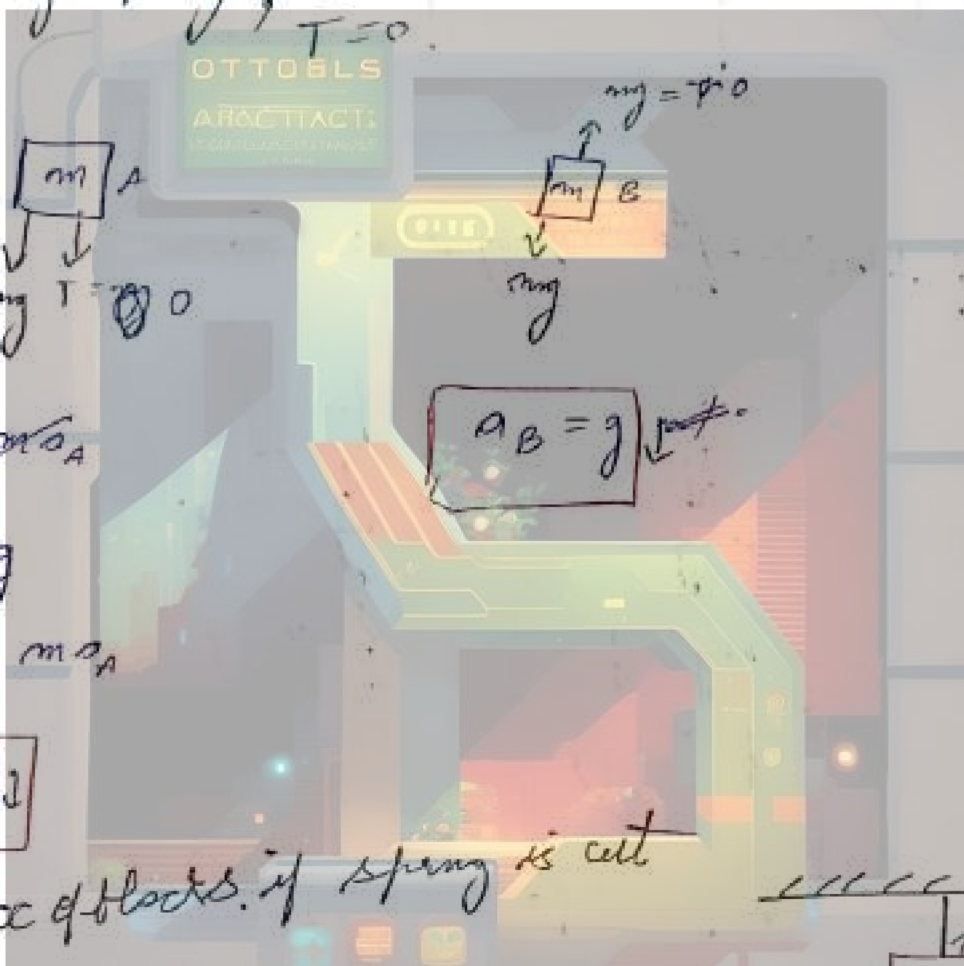
$mg + T = F_s$

$2mg = F_s$



$T = mg$

After cutting spring, $F_s = 0$, $T = 0$



$\cancel{2mg = m a_A}$

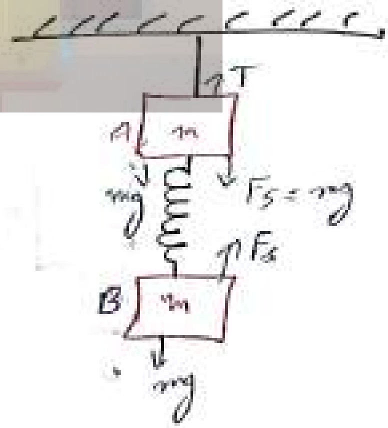
$\cancel{a_A = 2g}$

$mg = m a_A$

$\boxed{a_A = g \downarrow}$

$\boxed{a_B = g \downarrow}$

Q find acc of blocks if spring is cut



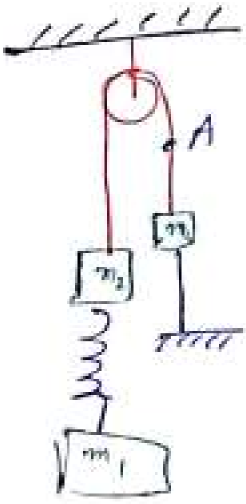
$F_s = 0$

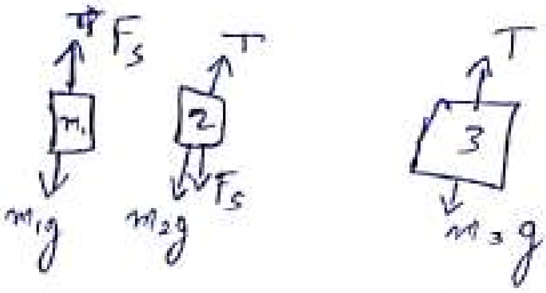
$mg = ma$

$a_B = g$

$a_A = 0$

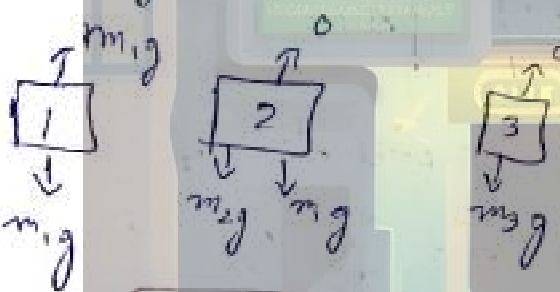
Q find acc if string is cut at A?





$F_s = m_1 g \quad m_2 g + m_1 g = m_3 g \quad T = m_3 g$

After cutting. $T = 0$



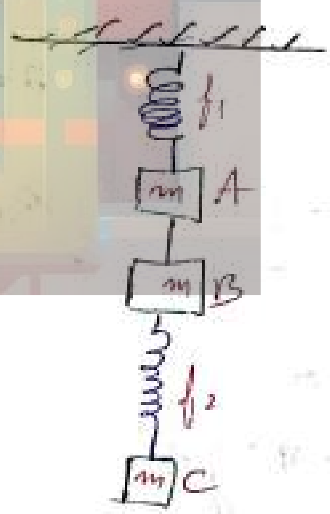
$\boxed{a_1 = 0}$ $\quad \boxed{a_2 = \frac{(m_1+m_2)}{m_2} g \uparrow}$ $\quad \boxed{a_3 = g} \downarrow$

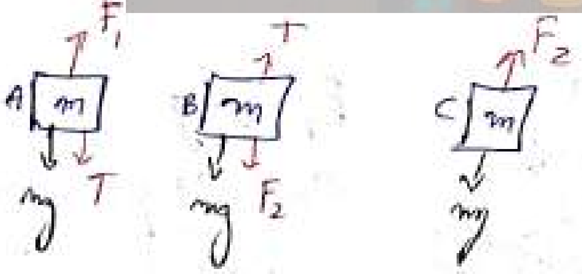
Q find acc of blocks

a) spring f_1 is cut

b) ~~string~~ string is cut

c) spring f_2 is cut.





$F_1 = 3mg \quad T = 2mg \qquad F_2 = mg$

a) $f_1 = 0$

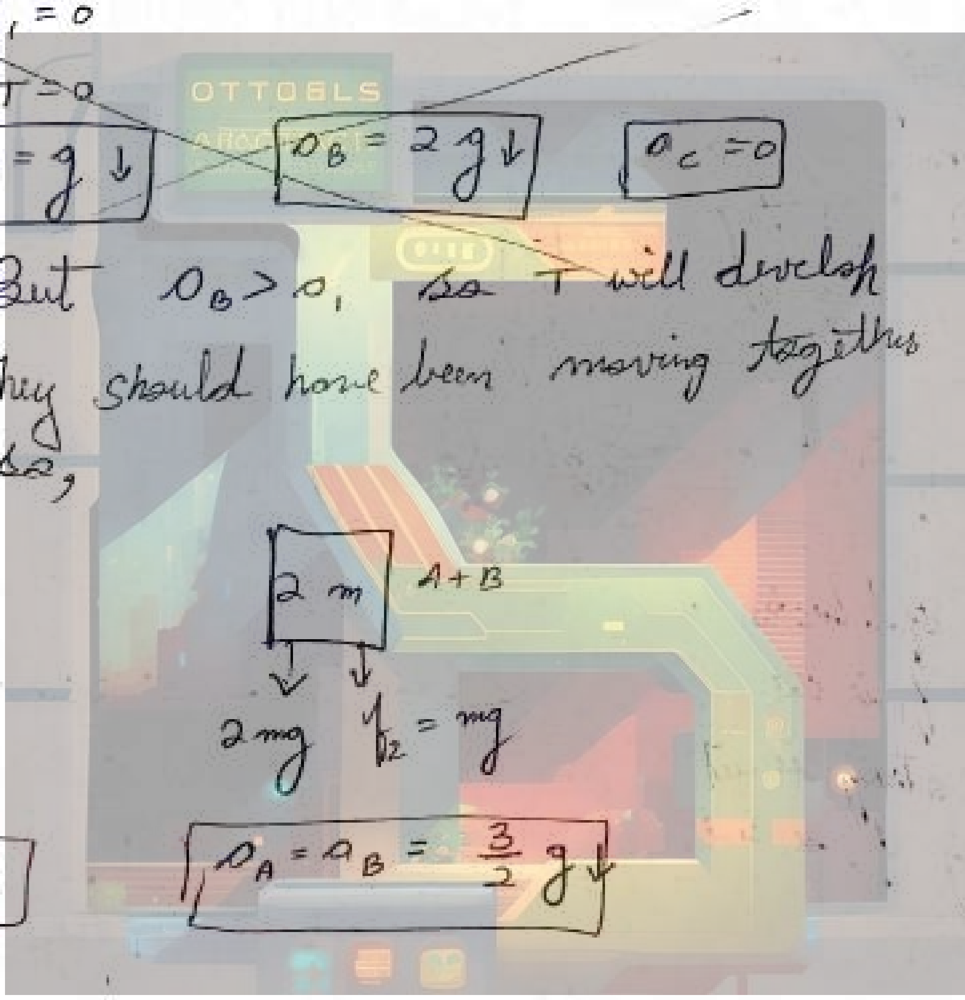
$3mg = m a_A$

$\boxed{a_A = 3g}$

c) $f_2 = 0$

$a_C = g$

a) $f_1 = 0$



$T = 0$

$\boxed{a_A = g \downarrow}$ $\boxed{a_B = 2g \downarrow}$ $\boxed{a_C = 0}$

~~b) T = 0~~ But $a_B > a_1$ so T will develop

They should have been moving together

So,

$\boxed{2m}$ A+B

$2mg$ $f_2 = mg$

$\boxed{a_C = 0}$ $\boxed{a_A = a_B = \frac{3}{2} g \downarrow}$

b) $T = 0$

$\boxed{a_A = 2g \uparrow}$

$\boxed{a_B = 2g \downarrow}$

$\boxed{a_C = 0}$

c) $f_2 = 0$

$\boxed{a_C = g} \downarrow$

$F_1 = 3mg$

$a_A = a_B$, $F_1 = 3mg$

$\boxed{A+B}$

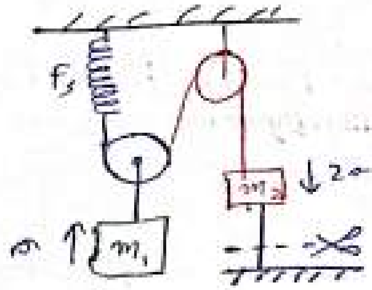
$2mg$

~~$a_A = a_B = g \downarrow$~~

$a_A = a_B = \frac{3mg - 2mg}{2mg}$

$\boxed{a_A = a_B = \frac{g}{2} \uparrow}$

Q find initial acceleration of m_2 $(m_1 > 2m_2)$ $\left(\frac{m_1 - 2m_2}{2m_2} g\right)$ Ans



$2T + \cancel{F_s}$ $\uparrow$ m_1 $\downarrow m_1 g$

$\uparrow T$ m_2 $\downarrow m_2 g$ $\downarrow T_2$

$F_s = T$

$m_1 g = 2T$

$T = \frac{m_1 g}{2}$

$T_2 = \frac{m_1 g}{2} - m_2 g$

~~After cutting~~ $\cancel{T_2 = 0}$ $\cancel{T = m_1 g}$

After cutting $T_2 = 0$

$2T = \cancel{m_1 g}\ 0$

m_1 $\downarrow m_1 g$

$\uparrow \frac{m_1 g}{2}$ m_2 $\downarrow m_2 g$

$\frac{m_1 g}{2} - m_2 g = m_2 a$

$\cancel{a_{m_1} = g}$

$\boxed{\cancel{a_{m_1} = a_1}}$

$\boxed{a_{m_1} = \frac{a_{m_2}}{2}}$

$\cancel{m_1 - m_2}$

$\boxed{\frac{(m_1 - 2m_2)g}{2m_2} = a}$

Friction

Static Friction (greater than Dynamic)

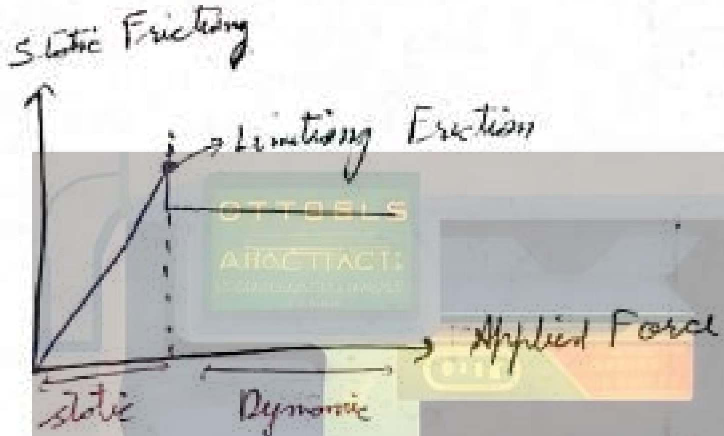
→ Self-Adjusting
(equal to Force applied)

→ Max-Value → Limiting Friction

Dynamic Friction (Kinetic Friction)

~~greater than Dynamic~~

Sliding > Rolling



Laws of Limiting Friction :-

→ The Limiting Friction is directly proportional to normal reaction by surface in contact.

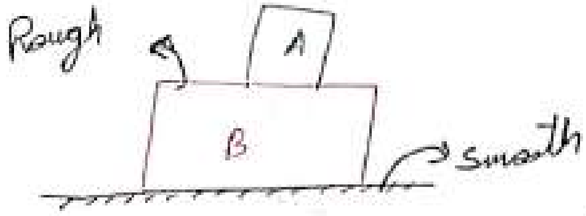
$f_l \propto N$

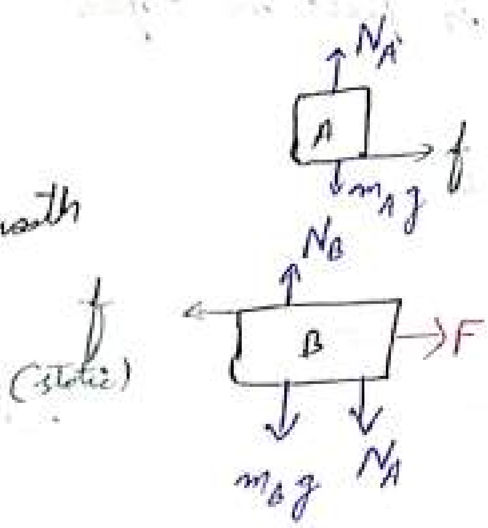
→ depends upon nature of surfaces in contact.

$$\boxed{f_l = \mu N}$$

μ = Coefficient of friction between 2 surfaces.

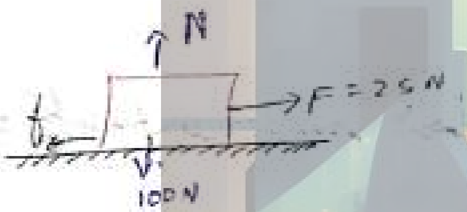
→ The Direction of Limiting Friction force is opposite to the direction in which the body is on the verge of starting its motion.





$a_{A_{max}} = \frac{f_e}{m_A}$

Q. Weight 100 N just begins to move at 25 N horizontal force. find coefficient of friction.



$N = 100\,N$

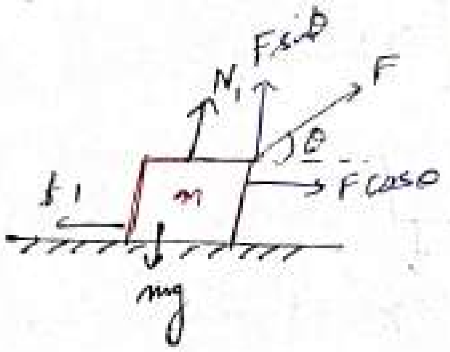
$f = 25\,N$

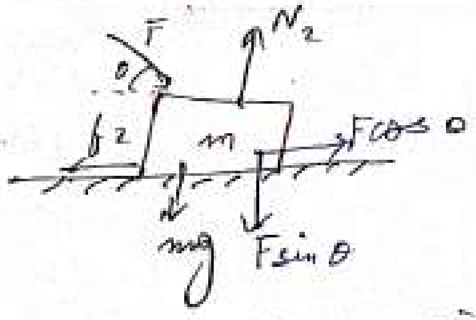
$25 = \mu(100)$

$\mu = \frac{25}{100}$

$\boxed{\mu = \frac{1}{4} = 0.25}$

is It easy to pull or push an object.





$N_1 = mg - F\sin\theta$

$N_2 = mg + F \sin\theta$

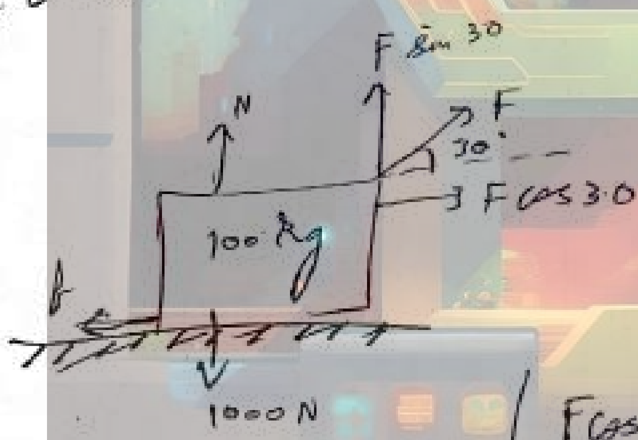
$N_2 > N_1$

$f_2 > f_1 \quad (f \propto N)$

Thus it is easier to Pull an object.

~~(f of object)~~

Q $M = 100\,kg$ $\theta = 30$ $\mu = 0.3$ find F so block moves uniformly on surface.



$1000 = N + F \times \frac{1}{2}$

$1000 = \frac{2N + F}{2}$

$2000 = 2N + F$

$\frac{2000 - F}{2} = N$

$F\cos 30 = \mu \times N$

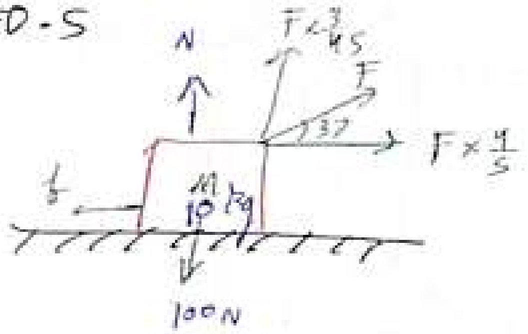
$\frac{F\sqrt{3}}{2} = 0.3 \times \frac{2000 - F}{2}$

$F\left(\frac{\sqrt{3} + 0.3}{0.3}\right) = \frac{6000}{10}$

$F = \frac{600}{\sqrt{3} + 0.3}$

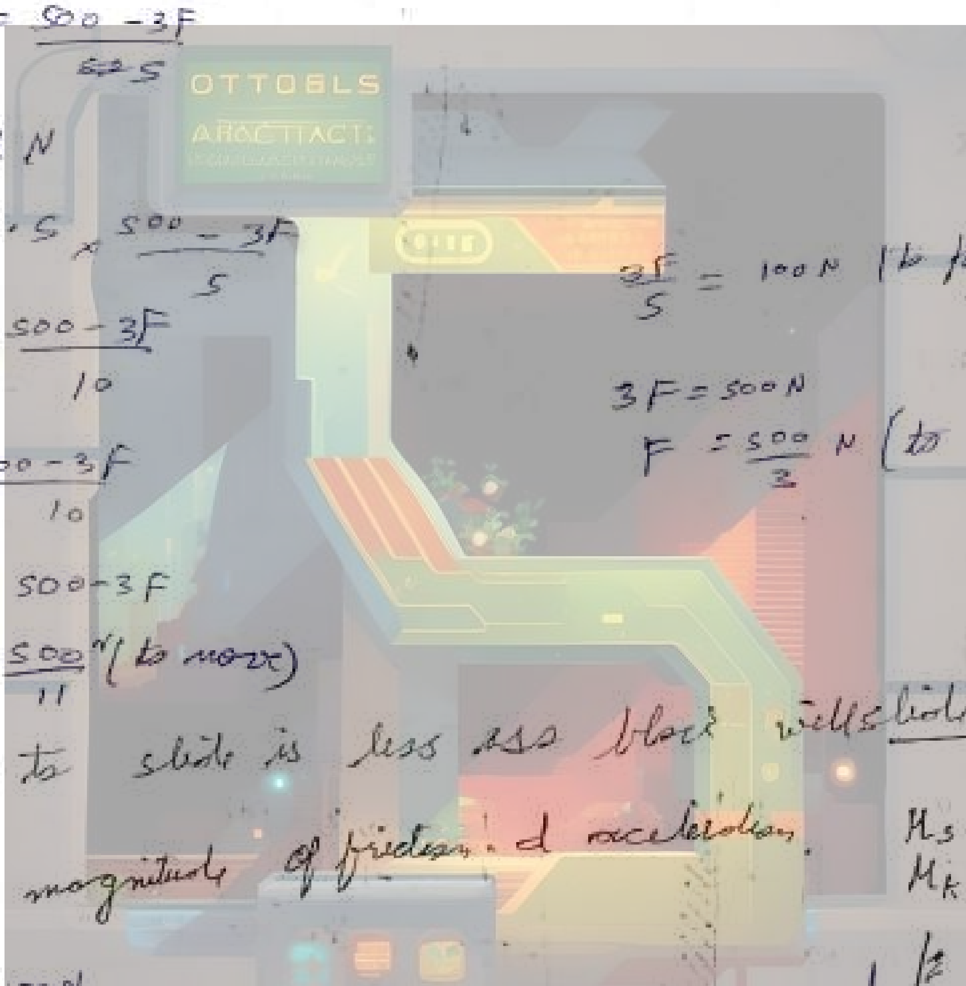
Q If Force is gradually increased from 0, will block first slide or lift.

$\mu = 0.5$



$N = 100 - \frac{3F}{5}$

$N = \frac{500 - 3F}{5}$



$f = \mu N$

$= 0.5 \times \frac{500 - 3F}{5}$

$f = \frac{500 - 3F}{10}$

$\frac{4F}{5} = \frac{500 - 3F}{10}$

$8F = 500 - 3F$

$F = \frac{500}{11}$ N (to move)

$\frac{3F}{5} = 100$ N (to lift)

$3F = 500$ N

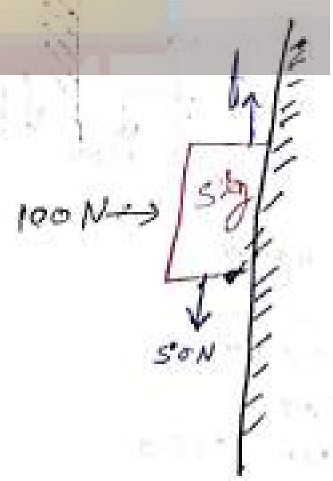
$F = \frac{500}{3}$ N (to lift)

force to slide is less so block will slide first.

Q Determine magnitude of friction and acceleration.

$\mu_s = 0.4$

$\mu_k = 0.3$



1) $N = 100$ N

$f_{max} = f_L = \mu_s \times N$

$= 0.40 \times 100$

$\boxed{f_L = 40 N} < 50$ N

So object will move

$f_k = \mu_k \times N$

$= 0.3 \times 100$

$\boxed{f = 30 N}$

$50 - 30 = 5 \times a$

$\frac{20}{5} = a$

$\boxed{a = 4 m/s^2 \downarrow}$

b) mass = 2 kg

$f_{max} = \mu_s \times 100$

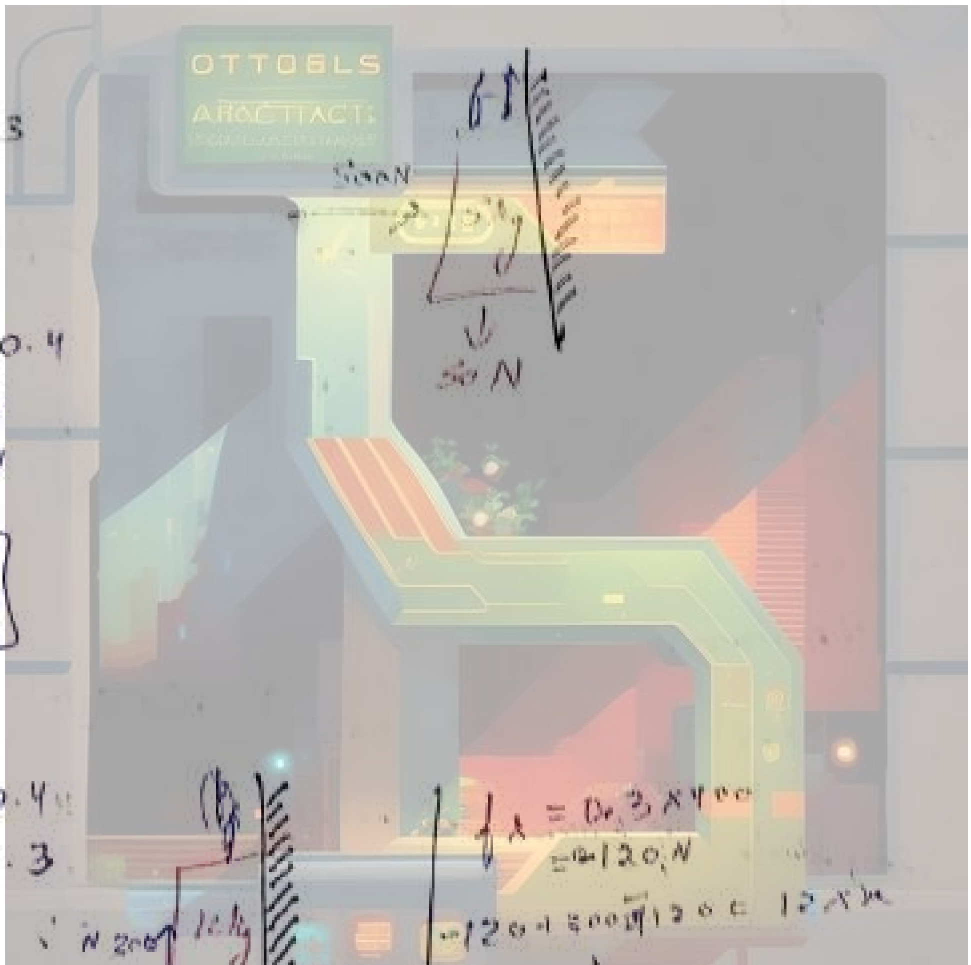
$= 40 N > 20$

the object will not move

$$\boxed{\begin{array}{l} f = 20 N \\ a = 0 \ m/s^2 \end{array}}$$

c) $\mu_s = 0.4$

$\mu_k = 0.3$



$N = 500$

$f_s = 500 \times 0.4$

$= 40 \times 5$

$= 200 N$

$$\boxed{\begin{array}{l} a = 0 \\ f = 50 N \end{array}}$$

Q

$\mu_s = 0.4$

$\mu_k = 0.3$

$N = 400 N$

$f_s = 0.4 \times 400$

$= 40 \times 4$

$= 160 N < 180$

will move

$f_k = 0.3 \times 400$

$= 120 N$

$-120 + 300 - 120 = 12 \times a$

$\cancel{\frac{300}{12} = 25}$

$$\cancel{\boxed{\begin{array}{l} a = 25 \ m/s^2 \uparrow \\ f = 120 N \end{array}}}$$

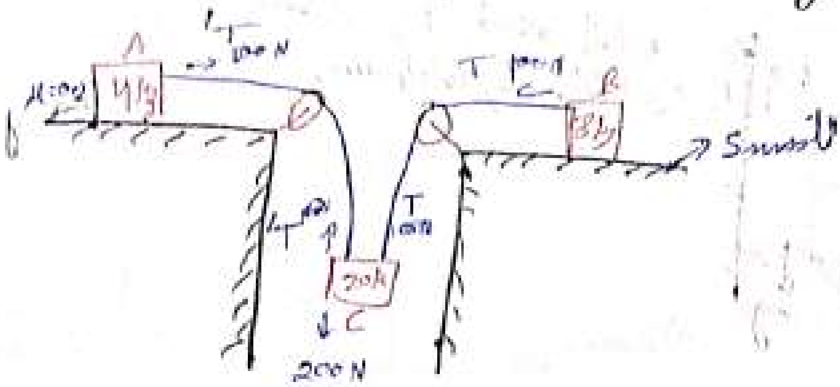
$300 - 240 = 12 a$

$a = \frac{60}{12}$

$$\boxed{a = 5 \ m/s^2 \uparrow}$$

$$\boxed{f = 120 N \downarrow}$$

Q find acceleration of blocks & tension in strings



$\boxed{T = 8a}$

$f_l = 0.2 \times 40$
$= 8N$

$T' - 8 = 4a$

$\cancel{T + T' = 20a}$

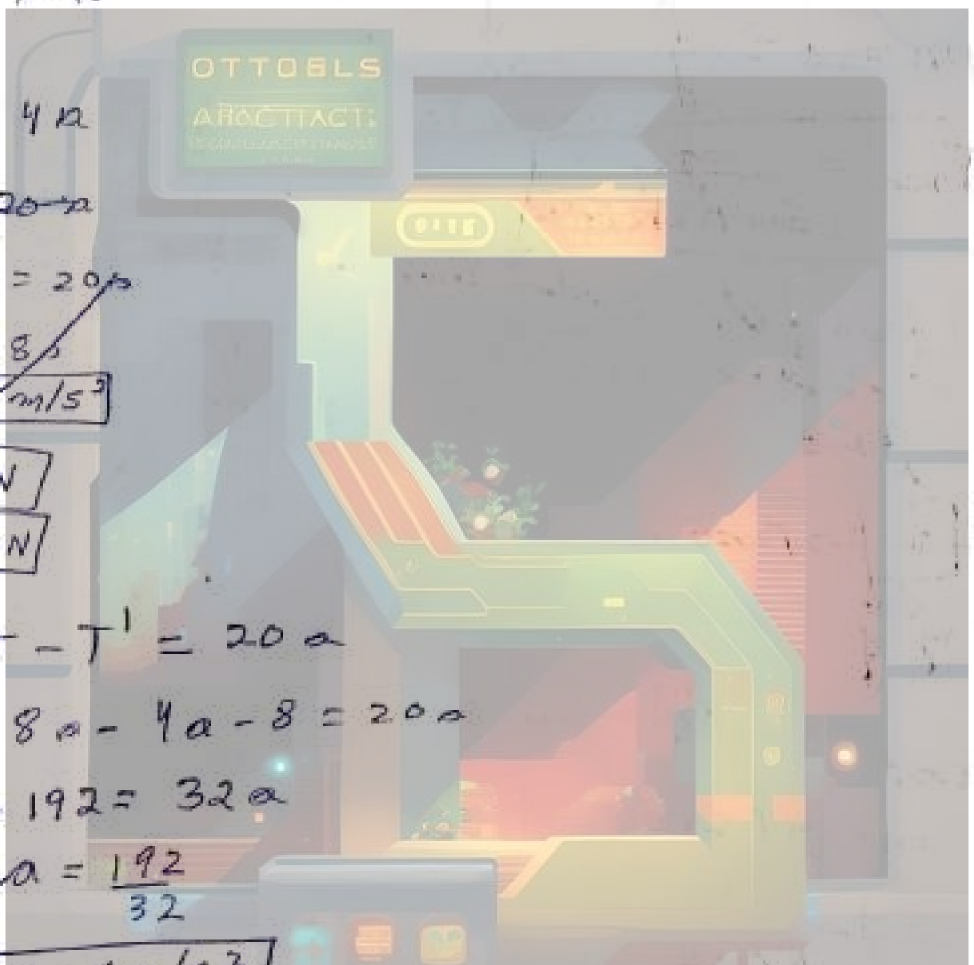
$\cancel{8a + 4a + 8 = 20a}$

$\cancel{8 = 8a}$

$\cancel{\boxed{a = 1\, m/s^2}}$

$\cancel{\boxed{T = 8N}}$

$\cancel{\boxed{T' = 12N}}$



$200 - T - T' = 20a$

$200 - 8a - 4a - 8 = 20a$

$192 = 32a$

$a = \frac{192}{32}$

$\boxed{a = 6\, m/s^2}$

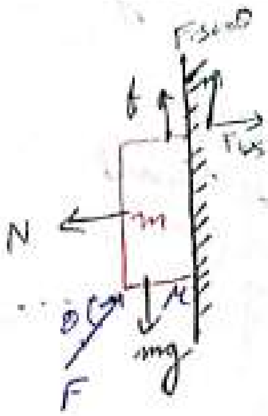
$T = 8 \times 6$

$\boxed{T = 48N}$

$T' = 24 + 8$

$\boxed{T' = 32N}$

Q



What range F block remains in static equilibrium.

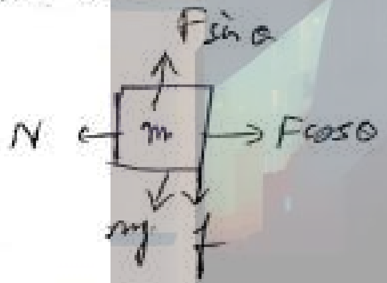
- F will be minimum when friction will be max

$f = \mu N$
$= F\mu \cos\theta$

$F\sin\theta + F\mu\cos\theta = mg$

$$\boxed{F_{min} = \frac{mg}{\sin\theta + \mu\cos\theta}}$$

As $F\sin\theta$ increases from f_{min}, block tends to move up & so friction acts downwards.



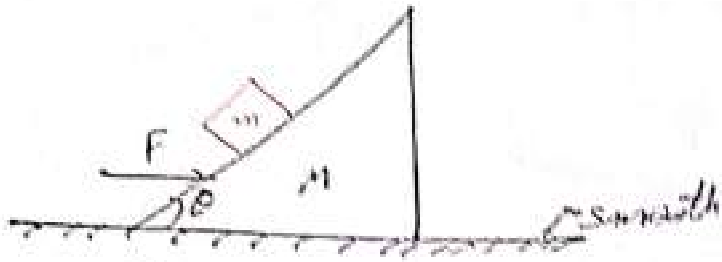
$f_L = \mu N$
$= \mu F\cos\theta$

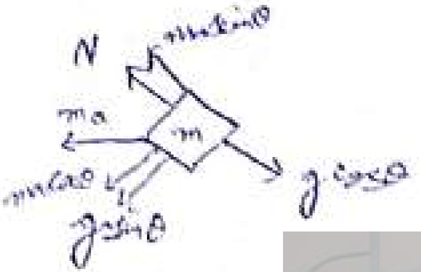
$F\sin\theta = mg - \mu F\cos\theta$

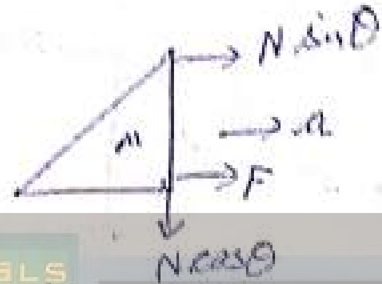
$F = \frac{mg}{\sin\theta - \mu\cos\theta}$

$$\boxed{F_{max} = \frac{mg}{\sin\theta - \mu\cos\theta}}$$

Q find F for friction between block & wedge is 0.



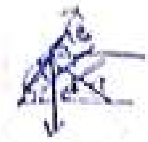


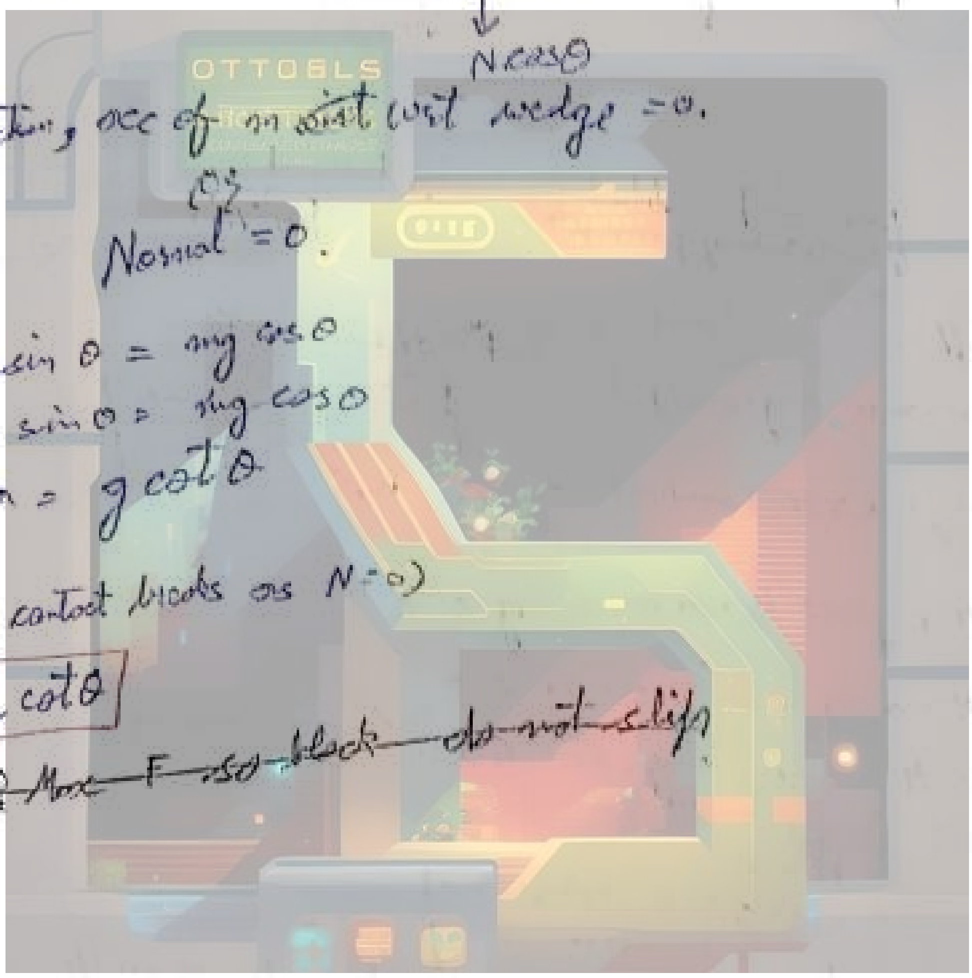


For zero friction, acc of m ~~w.r.t~~ w.r.t wedge = 0.

or

Normal = 0.





$\cancel{N}^{0} + ma\sin\theta = mg\cos\theta$

$ma\sin\theta = mg\cos\theta$

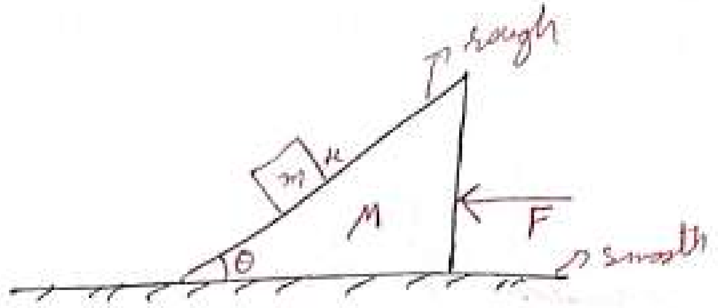
$a = g\cot\theta$

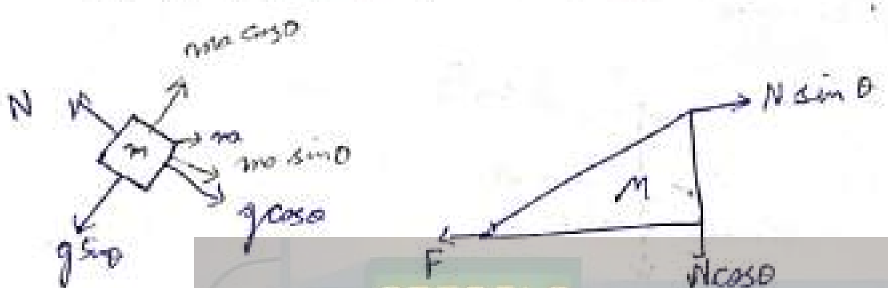
$F = Ma$ (contact breaks as $N = 0$)

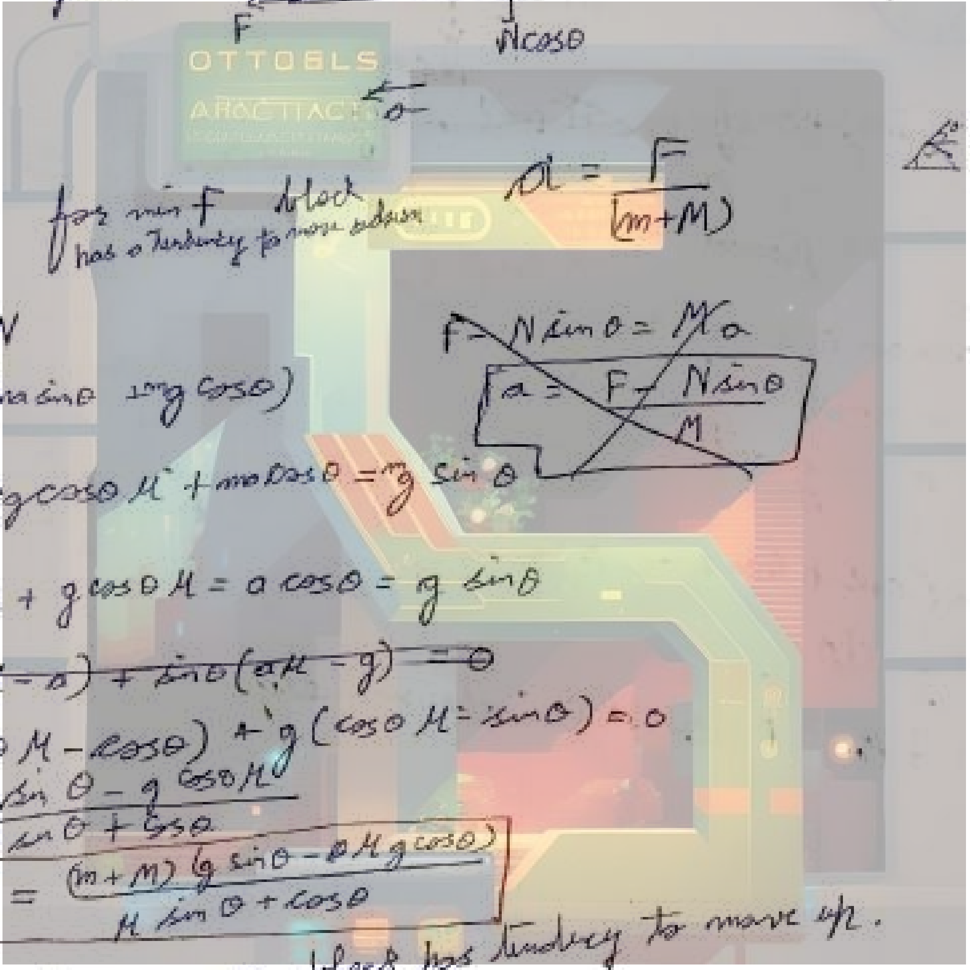
$\boxed{F = Mg\cot\theta}$

~~Q find min & Max F so block do not slip~~

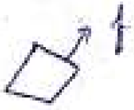
Q find min & Max F so block do not slip.







$$a = \frac{F}{(m+M)}$$



for min F block has a tendency to move down

$$f_l = \mu N = \mu(ma\sin\theta + mg\cos\theta)$$

$$F - N\sin\theta = Ma$$

$$\cancel{\boxed{a = \frac{F - N\sin\theta}{M}}}$$

$$ma\sin\theta\,\mu + mg\cos\theta\,\mu + ma\cos\theta = mg\sin\theta$$

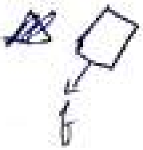
$$a\sin\theta\,\mu + g\cos\theta\,\mu = a\cos\theta = g\sin\theta$$

$$\cancel{\cos\theta(g\mu - a) + \sin\theta(a\mu - g) = 0}$$

$$a(\sin\theta\,\mu - \cos\theta) + g(\cos\theta\,\mu - \sin\theta) = 0$$

$$a = \frac{g\sin\theta - g\cos\theta\,\mu}{\mu\sin\theta + \cos\theta}$$

$$\boxed{F_{min} = \frac{(m+M)(g\sin\theta - \mu g\cos\theta)}{\mu\sin\theta + \cos\theta}}$$



for max F, block has tendency to move up.

$$f_l = \mu N = \mu(ma\sin\theta + mg\cos\theta)$$

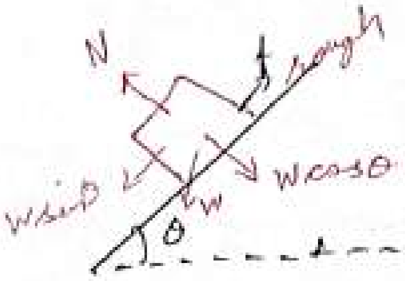
$$mg\sin\theta + ma\sin\theta\,\mu + mg\cos\theta\,\mu = ma\cos\theta$$

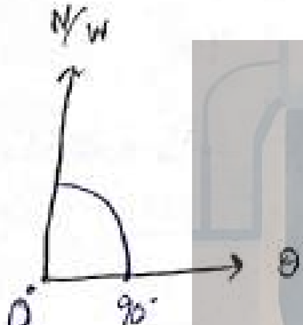
$$a = \frac{g\sin\theta + g\cos\theta\,\mu}{\cos\theta - \mu\sin\theta}$$

$$\boxed{F_{max} = \frac{(M+m)(g\sin\theta + \mu g\cos\theta)}{\cos\theta - \mu\sin\theta}}$$

Q A block of weight W rests on a rough wooden plank. The angle of incline of ~~plan~~ plank is gradually increased from 0° to 90°.

Draw. i) ratio of N/W vs θ graph
ii) ratio of f/W vs θ graph





$N = W\cos\theta$

$\frac{N}{W} = \cos\theta$

Till the block is static

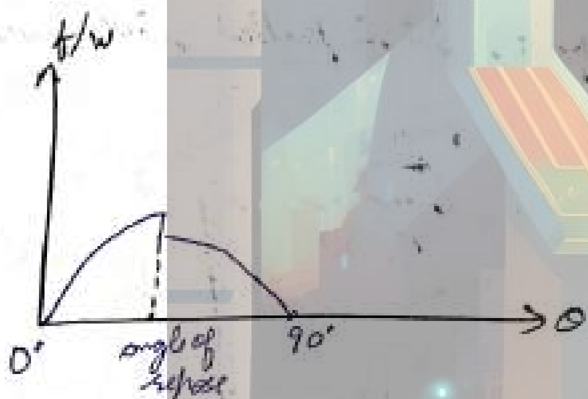
$f = W\sin\theta$

$\frac{f}{W} = \sin\theta$

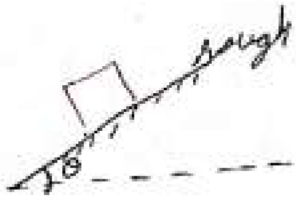
Block just begins to slide at θ = angle of repose

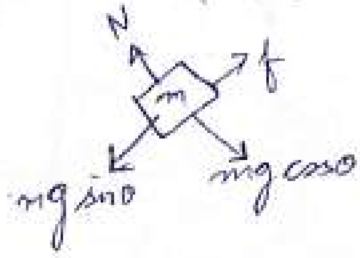
$f = \mu W\cos\theta$

$\frac{f}{W} = \mu\cos\theta \quad (\mu < 1)$



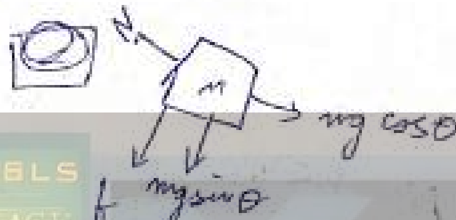
Q. A block slides down an incline plane of angle θ with constant velocity. It is then projected up the same plane with u velocity. How far up the incline will it move before comming to rest?





$mg\sin\theta = f$

$f + mg\sin\theta = ma$

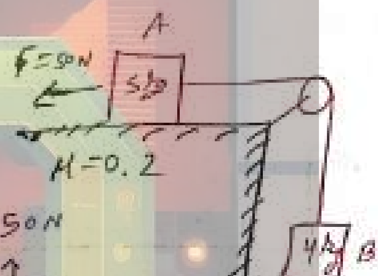


$\boxed{2g\sin\theta = a}$

$v = 0$
$u = u$
$a = -2g\sin\theta$
$v^2 = u^2 + 2as$

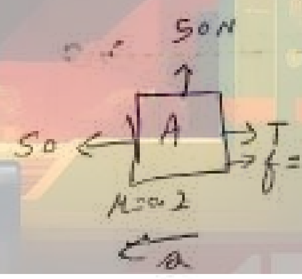
$$\boxed{S = \frac{+u^2}{4g\sin\theta}}$$

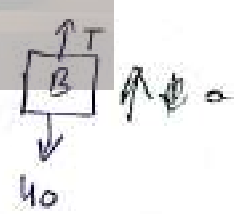
Q2. find acc & friction b/w A & surface. It is pulled with force 50N.



$f_L = 10N$

If system at rest,
$T = 40$ (Block-B)





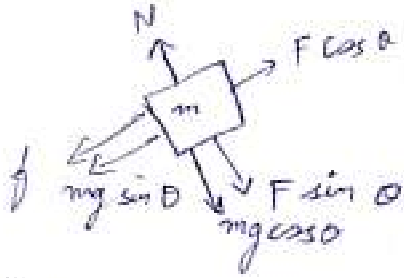
Block-A
$50 = 40 + f$
$f = 10$ (possible)
So, system is in rest

$\boxed{f = 10N}$

$\boxed{T = 40N}$

$\boxed{a = 0}$

Q Find max μ_s such that block do not go up the incline no matter how much F is.



$F\cos\theta = f + mg\sin\theta$

$N = F\sin\theta + mg\cos\theta$

$F\cos\theta = \mu F\sin\theta + \mu mg\cos\theta + mg\sin\theta$

$F(\cos\theta - \mu\sin\theta) = mg(\mu\cos\theta + \sin\theta)$

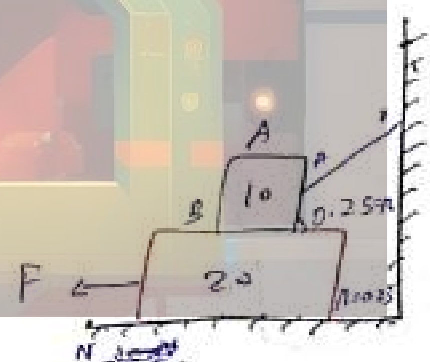
$$F = \frac{mg(\mu\cos\theta + \sin\theta)}{\cos\theta - \mu\sin\theta}$$

let F be infinite so,

$\cos\theta - \mu\sin\theta = 0$

$\boxed{\mu = \cot\theta}$

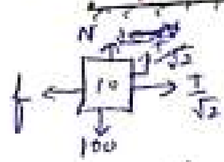
Q find F to pull ~~F~~ 20kg block, find tension in string.



$N + \frac{T}{\sqrt{2}} = 100$

$N = 100 - \frac{T}{\sqrt{2}}$

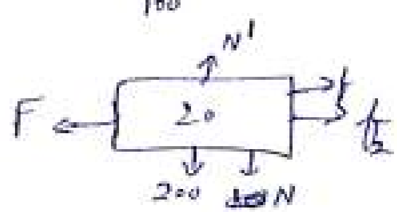
$\frac{T}{\sqrt{2}} = 0.25 \times \frac{100\sqrt{2} - T}{\sqrt{2}}$



$T = 25\sqrt{2} - 0.25T$

$\frac{5}{4}T = 25\sqrt{2}$

$\boxed{T = 20\sqrt{2}}$ ✓



$N = ~~20~~\ 100 - 20$

$\boxed{N = 80N}$

$f = \frac{20\sqrt{2}}{\sqrt{2}}$

$f = 20$

$N' = 200+80$

$N' = 280$

$f = 0.25 \times 280$

$f = 2.5 \times 28$

$f = 2\cancel{8}^{14} \times \frac{5}{\cancel{2}}$

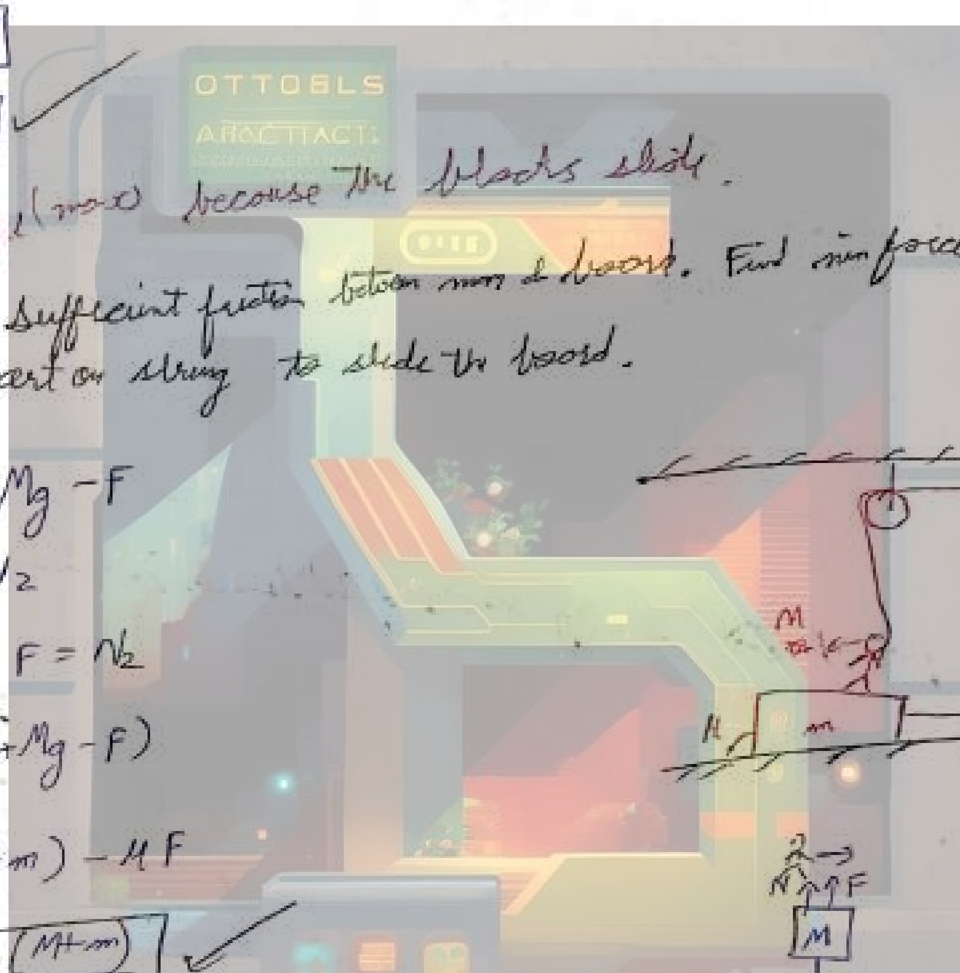
$\boxed{f = 70\,N}$

$F = f + 20$

$\boxed{\cancel{F = 70\,N}}$

$\boxed{F = 90\,N}$

we use $f_s(max)$ because the blocks slide.



(1) There is sufficient friction between man & board. Find min force required to exert on string to slide the board.

$N = Mg - F$

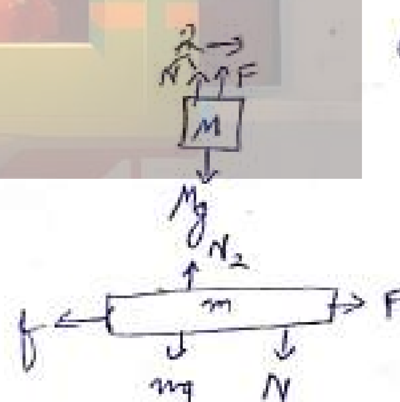
$mg + N = N_2$

$mg + Mg - F = N_2$

$f_s = \mu(mg + Mg - F)$

$F = \mu g(M+m) - \mu F$

$$\boxed{F \geq \frac{\mu g (M+m)}{1+\mu}}$$



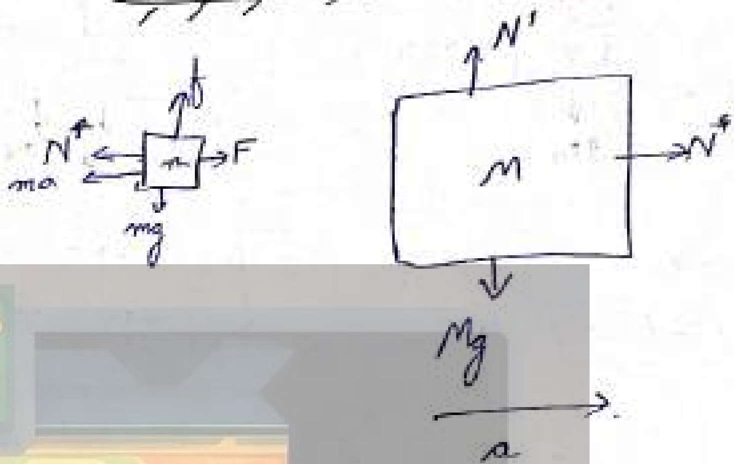
Q find min F so m do not slide

$f = mg = \mu N$

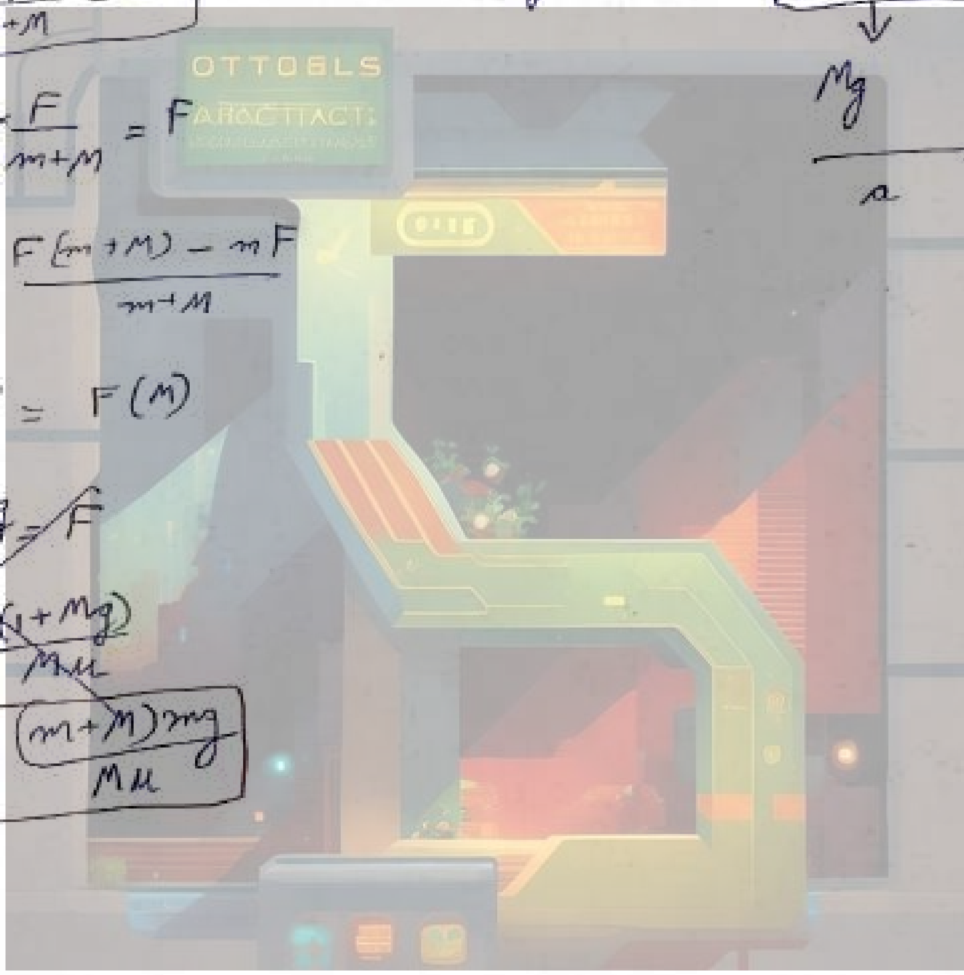
$N = \frac{mg}{\mu}$

$N + ma = F$

$\frac{mg}{\mu} + ma = F$



$\boxed{\frac{F}{m+M} = a}$



$\frac{mg}{\mu} + m \times \frac{F}{m+M} = F$

$\frac{mg}{\mu} = \frac{F(m+M) - mF}{m+M}$

$\frac{(m+M)(mg)}{\mu} = F(M)$

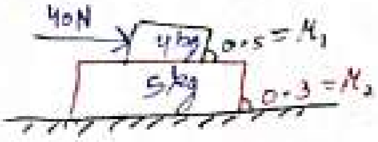
$\cancel{\frac{m + Mmg}{M\mu} = F}$

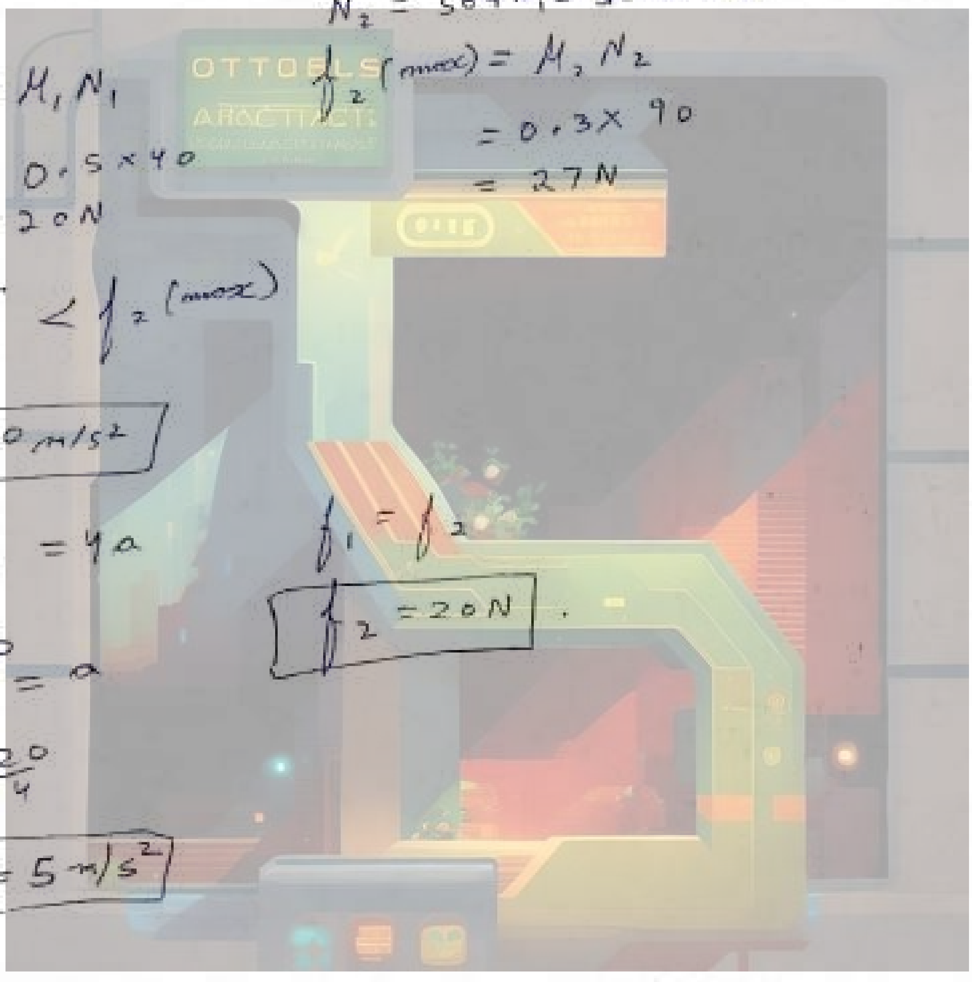
$\cancel{F = m\frac{(1+Mg)}{M\mu}}$

$\boxed{F = \frac{(m+M)mg}{M\mu}}$

Two Block System :-

eg.





$N_1 = 40$

$f_1(max) = \mu_1 N_1$

$= 0.5 \times 40$

$= 20N$

$N_2 = 50 + N_1 = 50 + 40 = 90$

$f_2(max) = \mu_2 N_2$

$= 0.3 \times 90$

$= 27N$

$f_{21}(max) < f_2(max)$

$\boxed{a_{5kg} = 0\ m/s^2}$

$40 - f_1 = 4a$

$\frac{40-20}{4} = a$

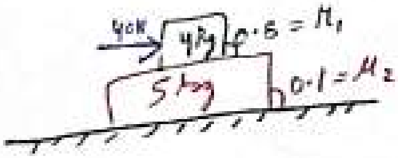
$a = \frac{20}{4}$

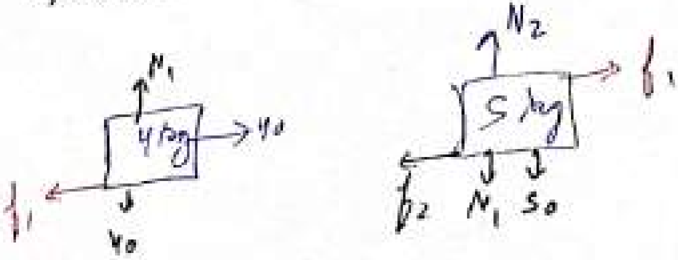
$\boxed{a_{4kg} = 5\ m/s^2}$

$f_1 = f_2$

$\boxed{f_2 = 20N}$.

eg 2.





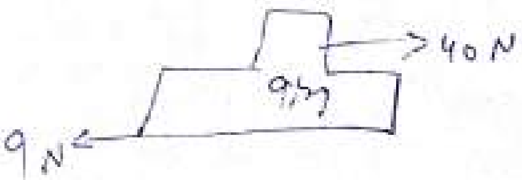
$N_1 = 40\,N$

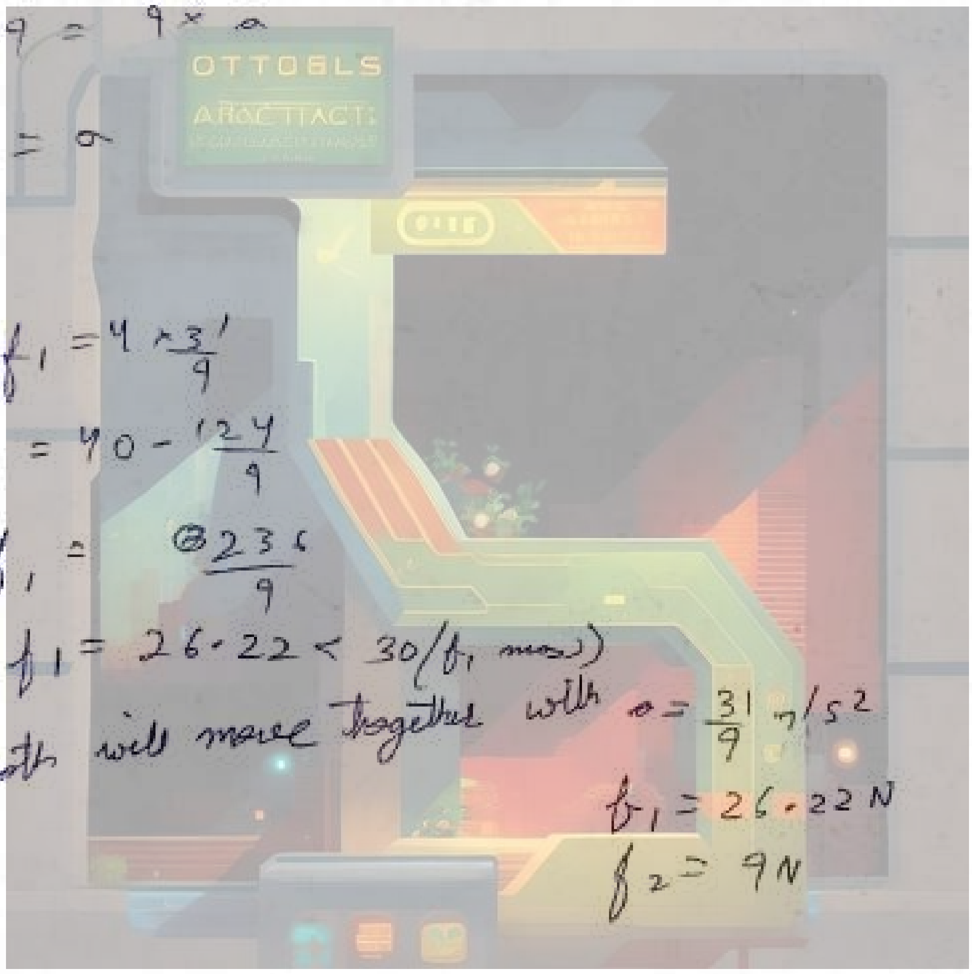
$f_1(max) = \mu_1 N_1$
$= 0.8 \times 40$
$= 32\,N$

$N_2 = 50 + 40 = 90\,N$

$f_2(max) = \mu_2 N_2$
$= 0.1 \times 90\,N$
$= 9\,N$

If both move together,





$40 - 9 = 9 \times a$

$\frac{31}{9} = a$

verify

$40 - f_1 = 4 \times \frac{31}{9}$

$f_1 = 40 - \frac{124}{9}$

$f_1 = \frac{236}{9}$

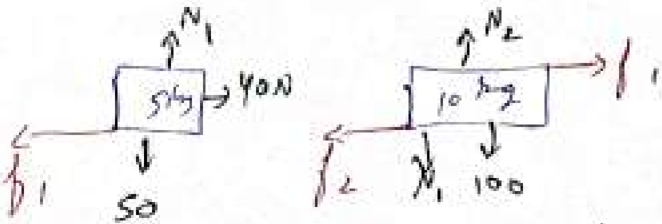
$f_1 = 26.22 < 30\,(f_1\ max)$

So, both will move together with $a = \frac{31}{9}\ m/s^2$

$f_1 = 26.22\,N$

$f_2 = 9\,N$

Q find acc. & friction forces.



$N_2 = N_1 + 100 = 150$

$N_1 = 50$

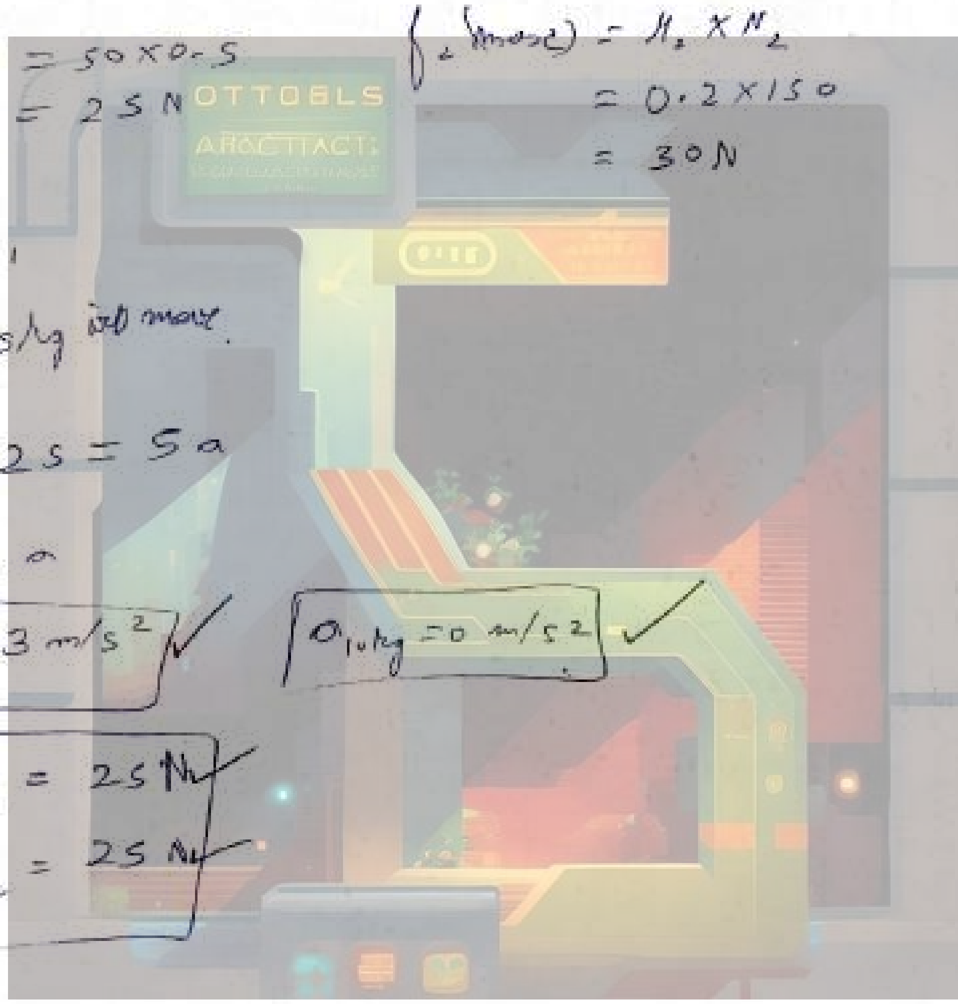
$f_1(max) = 50 \times 0.5$

$= 25\,N$

$f_2(max) = \mu_2 \times N_2$

$= 0.2 \times 150$

$= 30\,N$



$f_2 > f_1$

Only 5kg will move.

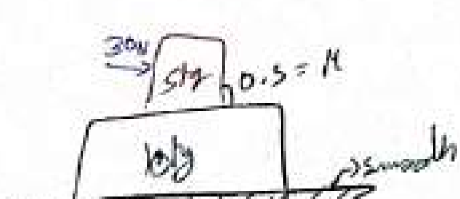
$40 - 25 = 5a$

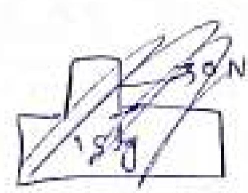
$\frac{15}{5} = a$

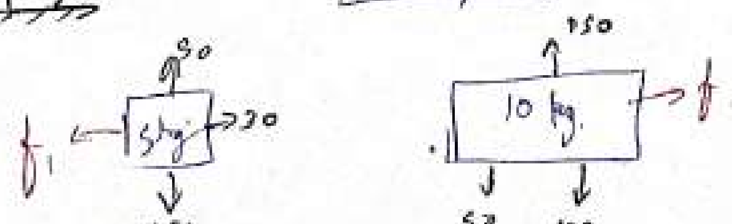
$\boxed{a_{5kg} = 3\ m/s^2}$ ✓ $\boxed{a_{10kg} = 0\ m/s^2}$ ✓

$\boxed{\begin{aligned} f_1 &= 25\,N \\ f_2 &= 25\,N \end{aligned}}$

Q







$f_1(max) = 25\,N$

If both move together

$$\frac{30}{15} = \boxed{a = 2\ m/s^2}$$ ✓

verify

$30 - f_1 = 5 \times 2$

$f_1 = 30 - 10$

$\boxed{f_1 = 20}$ ✓ (possible)

F applied force $= 50N$

If both move together

$\frac{50}{15} = \cancel{10}\ \frac{10}{3}$

Verify

$f_1 = 50 - \frac{100}{3} > f_1\ max$

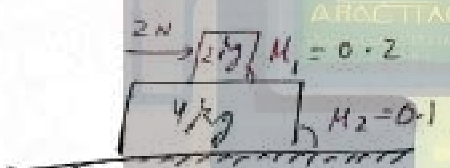
So will move separately

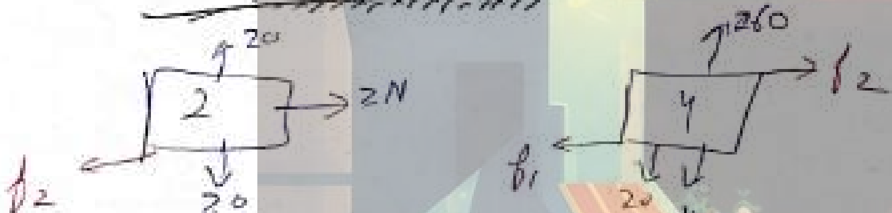
$f_1 = 25$

$\frac{50 - 25}{5} = a_{5kg}$

$$\boxed{\begin{aligned} a_{5kg} &= 5\ m/s^2 \checkmark \\ A_{10kg} &= \frac{25}{10} = 2.5\ m/s^2 \checkmark \end{aligned}}$$

Q find acc & friction





$f_2\ max = 0.2 \times 20$

$= 4N$

$f_1\ max = 0.1 \times 60$

$= 6N$

$\boxed{a_{4kg} = 0\ m/s^2}$ ✓

$A_{2kg} = 2 - 2$

$\boxed{= 0\ m/s^2}$

$$\boxed{\begin{aligned} f_2 &= 2N \\ f_1 &= 2N \end{aligned}}$$

If ground is smooth

If both move together

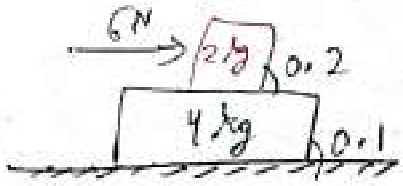
$\frac{2}{6} = \boxed{a = \frac{1}{3}}$ ✓

verify

$\frac{2}{3} = 2 - f_1$

$\boxed{f_1 = \frac{4}{3}}$ ✓ (possible)

Q find acc of blocked friction



$f_1(max) = 0.2 \times 20$

$= 4N$

$f_2(max) = 0.1 \times 60$

$= 6N$

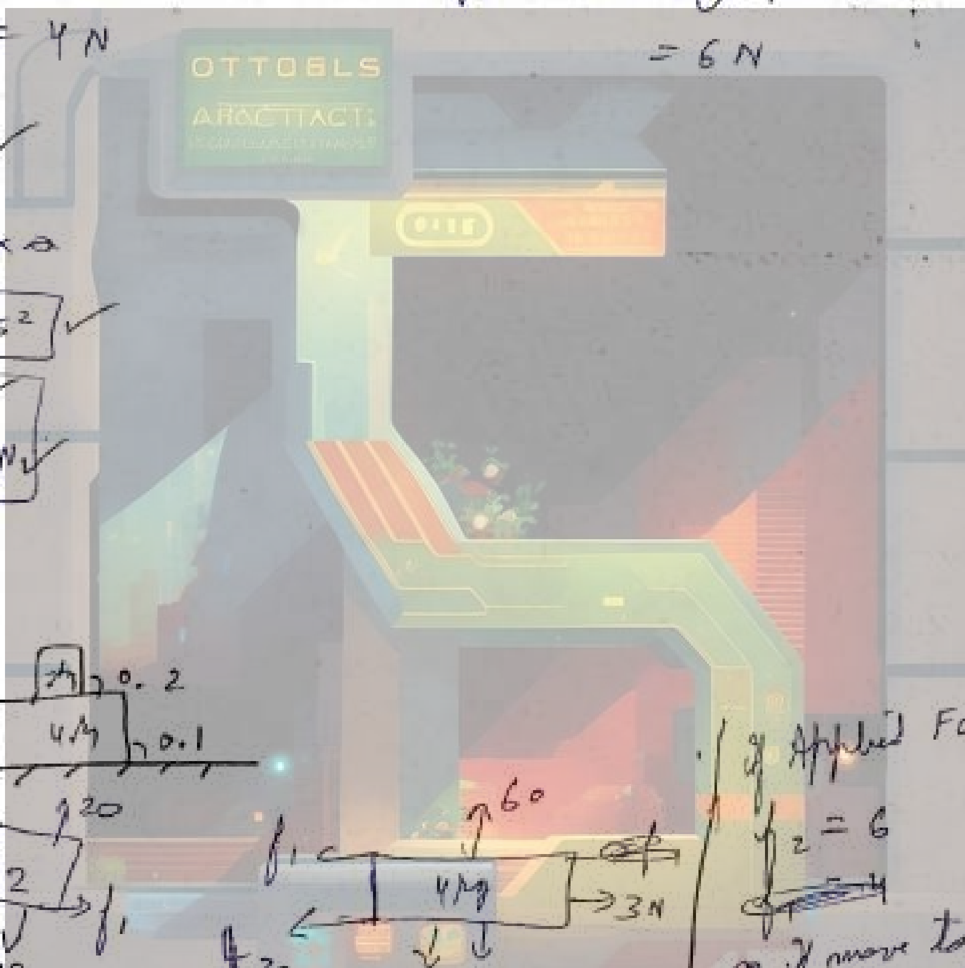
$\boxed{a_{4kg} = 0}$ ✓

$6 - 4 = 2 \times a$

$\boxed{a = 1\ m/s^2}$ ✓

$f_1 = 4N$ ✓

$f_2 = 4N$ ✓



Q

$f_1\ max = 4N$

$f_2\ max = 6N$

$3N < f_2\ max$

$a_{4kg} = 0$ ✓

$a_{2kg} = 0$ ✓

$f_2 = 3N$ ✓

$f_1 = 0N$ ✓

If Applied Force = 12N

$f_2 = 6$

$f_1 = 4$

If move together

$\frac{6}{6} = \boxed{a = 1\ m/s^2}$ ✓

Verify, $6 - f_1 = 4$

$\boxed{f_1 = 2}$ (possible)

$f_2\ 6N$ ✓

If Applied Force = 24.

If move Together

$$\frac{18}{6} = a$$

$$a = 3$$

Verify-

$f_1 = 6N$ (not possible)

So move seperately ✓

$\boxed{f_1 = 4N}$ ✓

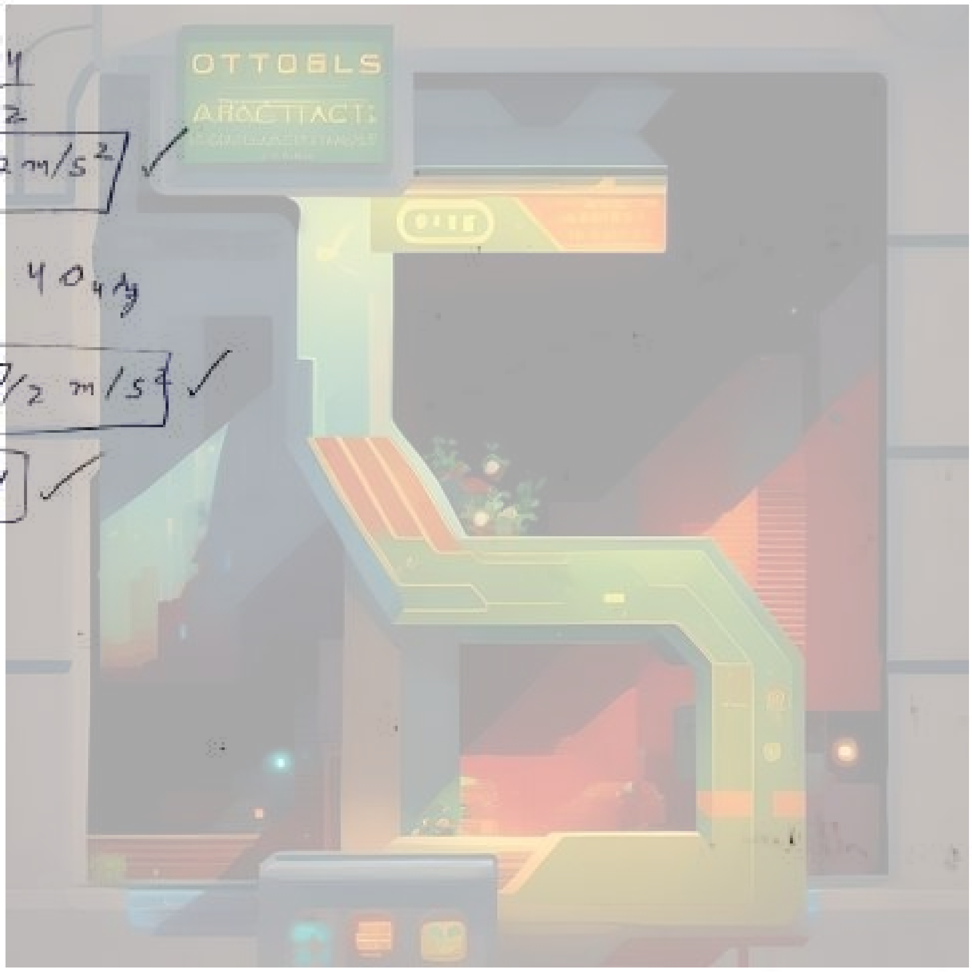
$$a_{2kg} = \frac{4}{2}$$

$\boxed{a_{2kg} = 2\, m/s^2}$ ✓

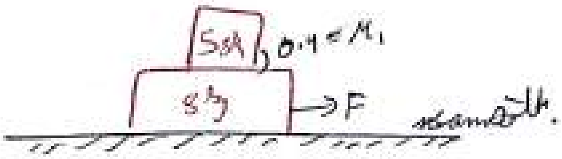
$$24 - 10 = 4 a_{4kg}$$

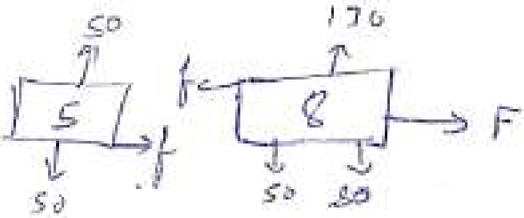
$\boxed{a_{4kg} = 7/2\, m/s^2}$ ✓

$\boxed{f_2 = 6N}$ ✓



Q find f $F^{(max)}$ if they move together.





$f_l = 0.4 \times 50$
$= 4 \times 5$
$= 20N$

to move together $f_l \le 20$

$f = 5 \times a$

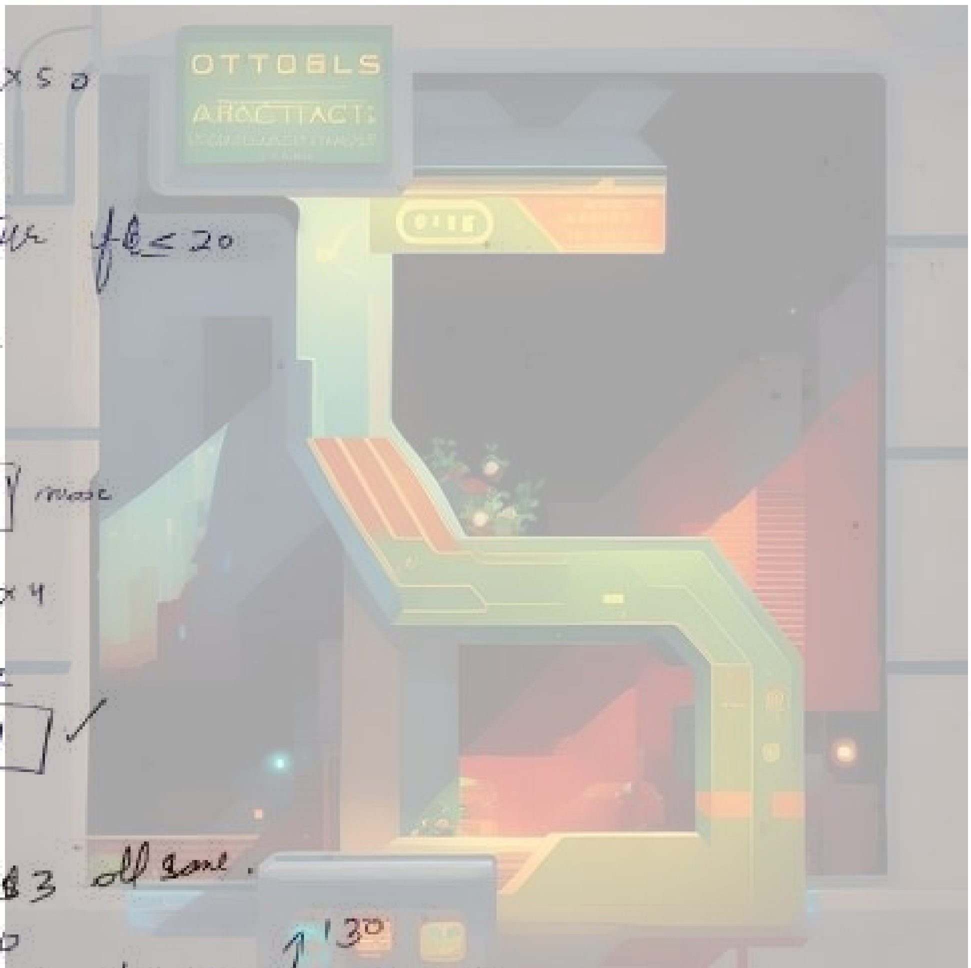
$20 = 5a$

$\boxed{a = 4 m/s^2}$ max

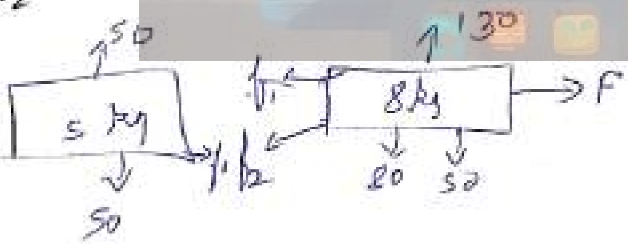
$F - 20 = 8 \times 4$

$F - 20 = 32$

$\boxed{F \le 52 N}$



Q. $\mu_2 = 0.3$ all same.



$f_1 (max) = 20$

$\boxed{a_{max} = 4 m/s^2}$

$f_{2max} = 0.3 \times 130$
$= 13 \times 3$
$= 39$

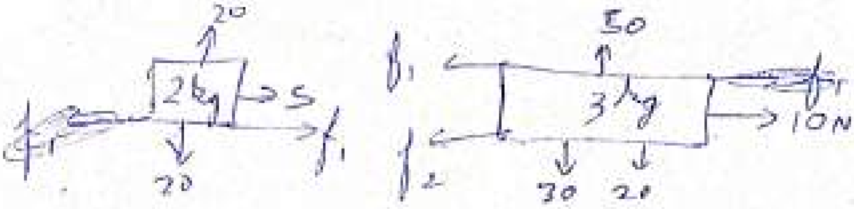
$F - 20 - 39 = 8 \times 4$

$F = 32 + 20 + 39$

$F = 52 + 39$

$\boxed{F = 91 N}$

Q Find acc & Friction

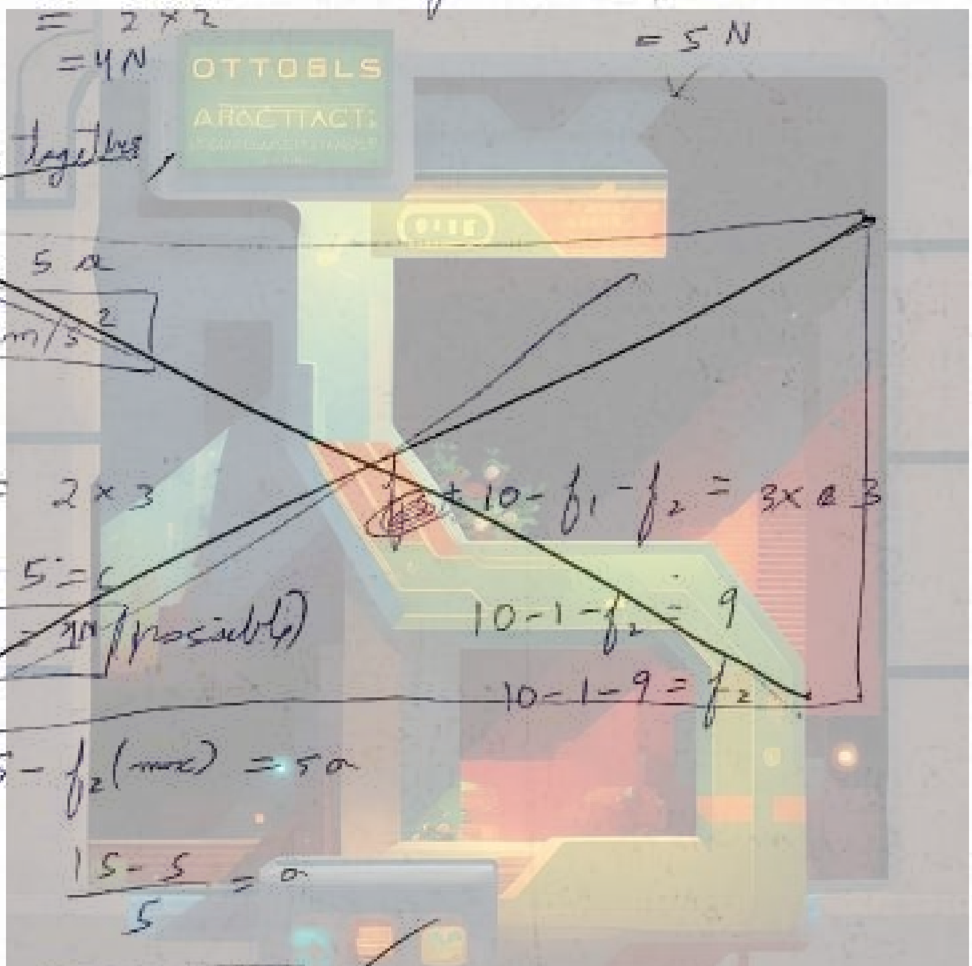


$f_1(max) = 0.2 \times 20$ $\qquad f_2(max) = 0.1 \times 50$

$= 2 \times 2$ $\qquad = 5N$

$= 4N$

If move together,



$15 = 5a$

$\boxed{a = 3 m/s^2}$

verify.

$f_1 + 5 = 2 \times 3$ $\qquad f_1 + 10 - f_1 - f_2 = 3 \times 3$

$f_1 + 5 = 6$ $\qquad 10 - 1 - f_2 = 9$

$\boxed{f_1 = 1N}$ (possible) $\qquad 10 - 1 - 9 = f_2$

$15 - f_2(max) = 5a$

$\frac{15 - 5}{5} = a$

$\boxed{a = 2 m/s^2}$ ✓

Verify:-

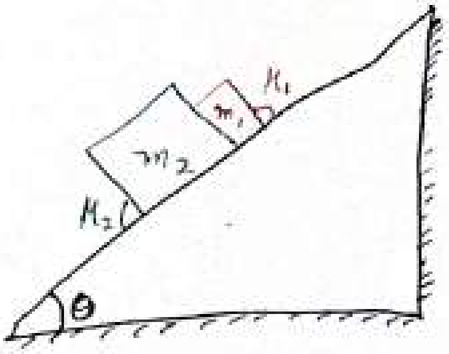
$f_1 + 5 = 2 \times 2$ $\qquad 10 + 1 - f_2 = 3 \times 2$

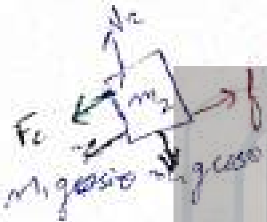
$f_1 = 4 - 5$ $\qquad 11 - f_2 = 6$

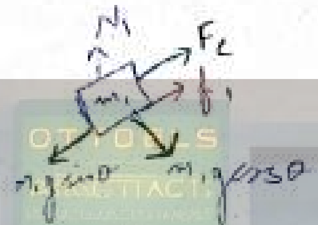
$\boxed{f_1 = -1N}$ ✓ $\qquad f_2 = 11 - 6$

$\boxed{f_2 = 5}$ ✓ (possible)

Blocks In Contact on an inclined plane





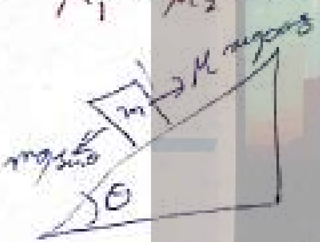


$N_1 = m_1 g\cos\theta$

$f_{1\,max} = \mu_1 N_1 = \mu_1 \; m_1 g\cos\theta$

$f_{2(max)} = m_2 g\cos\theta$

case 1:- $\mu_1 = \mu_2 = \mu$



$mg\sin\theta - \mu\, mg\cos\theta = ma$

$\boxed{a = g\sin\theta - \mu g\cos\theta}$

depend only on μ

$a_1 = a_2 = a$

$F_c = 0$

case 2:- $\mu_1 > \mu_2$

$a_2 > a_1$

$F_c = 0$

case 3:- $\mu_1 < \mu_2$

The blocks will move together

$a_1 = a_2 = a$

$F_c \neq 0$

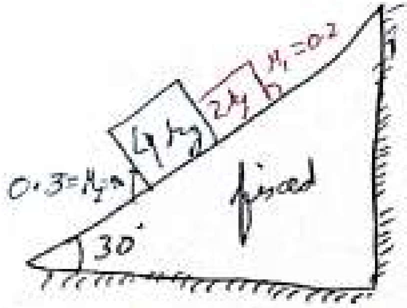
$m_2 g\sin\theta + F_c - \mu_2 g m_2\cos\theta = m_2 a$

$m_1 g\sin\theta - F_c - \mu_1 g m_1\cos\theta = m_1 a$

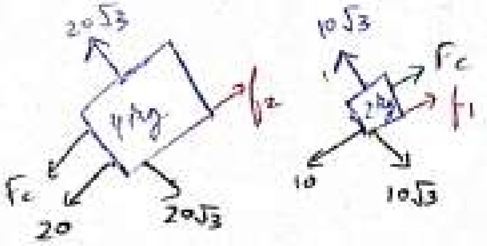
add

$\boxed{(m_1+m_2) g\sin\theta - (\mu_2 m_2 + \mu_1 m_1) g\cos\theta = (m_1+m_2) a}$

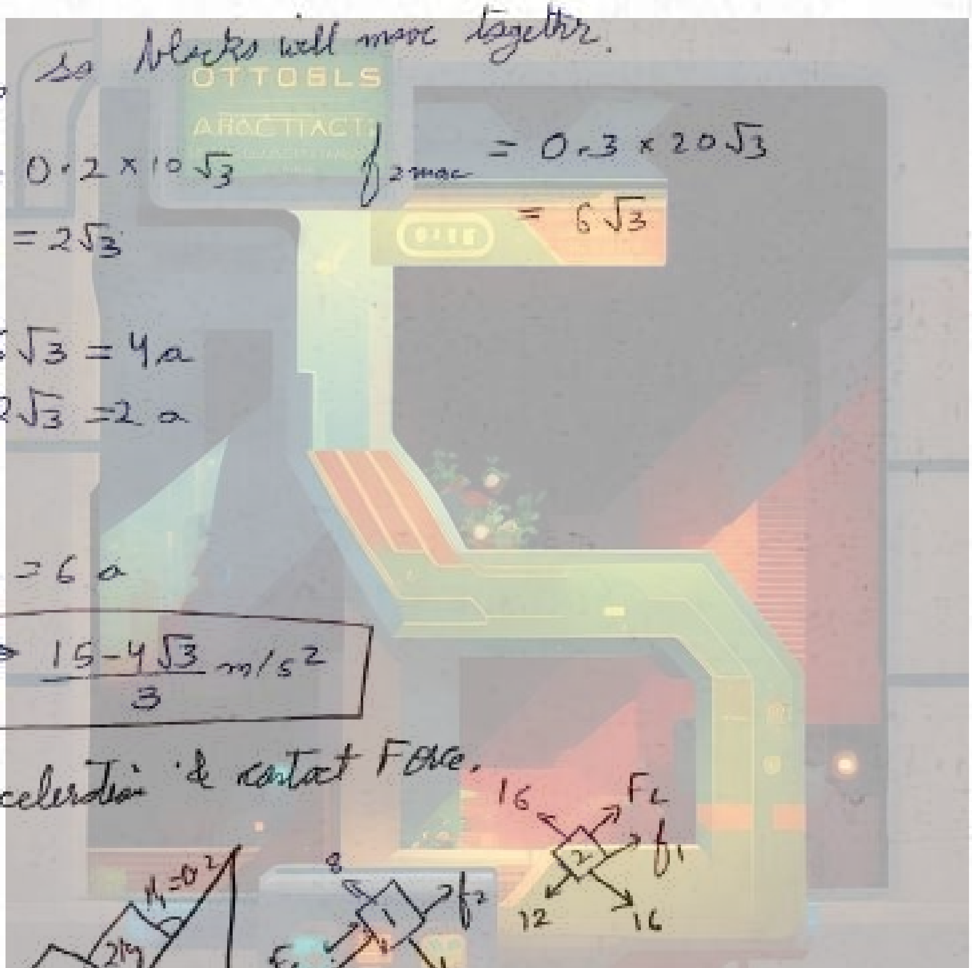
Q



Find the acceleration of blocks.



$\mu_2 > \mu_1$, so blocks will move together.



$f_{1\,max} = 0.2 \times 10\sqrt{3}$
$= 2\sqrt{3}$

$f_{2\,max} = 0.3 \times 20\sqrt{3}$
$= 6\sqrt{3}$

$20 + F_c - 6\sqrt{3} = 4a$

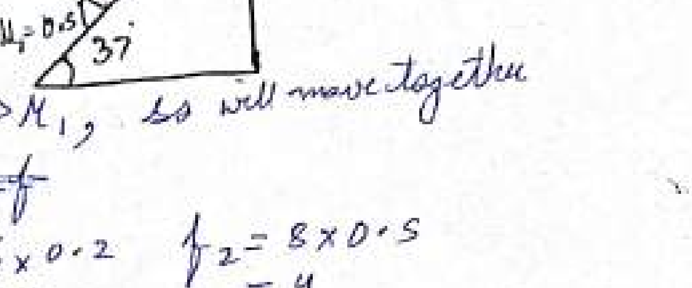
$10 - F_c - 2\sqrt{3} = 2a$

add

$30 - 8\sqrt{3} = 6a$

$$\boxed{a = \frac{15 - 4\sqrt{3}}{3}\ m/s^2}$$

Q2. Find acceleration & contact Force.



$\mu_2 > \mu_1$, so will move together

$\cancel{6 + F_c = f}$

$f_1 = 16 \times 0.2$
$= 3.2$

$f_2 = 8 \times 0.5$
$= 4$

$6 + F_c - 4 = a$

$12 - F_c - 3.2 = 2a$

Add

$18 - 7.2 = 3a$

$a = \frac{10.8}{3}$

$= \frac{108}{30}$

$= \frac{36}{10}$

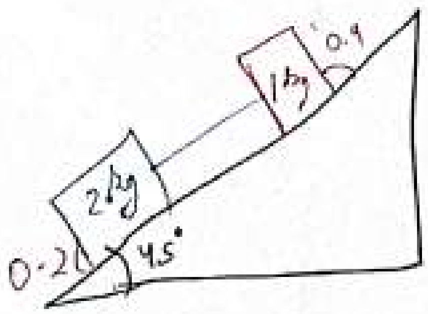
$\boxed{= 3.6\ m/s^2}$

$6 + F_c - 4 = 3.6$

$F_c = 3.6 - 2$

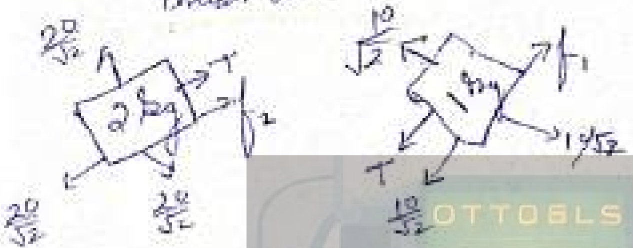
$\boxed{F_c = 1.6\ N}$

Q.



$\mu_1 > \mu_2$

Tension will act



$$f_{1\,max} = 0.4 \times \frac{10}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \boxed{2\sqrt{2}}$$ ✓

$$f_{2\,max} = \frac{10}{\sqrt{2}} \times 2 \times 0.2 = \boxed{2\sqrt{2}}$$ ✓

$$10\sqrt{2} - T - 2\sqrt{2} = 2a \qquad 5\sqrt{2} + T - 2\sqrt{2} = a$$

$$8\sqrt{2} - T = 2a \qquad 3\sqrt{2} + T = a$$

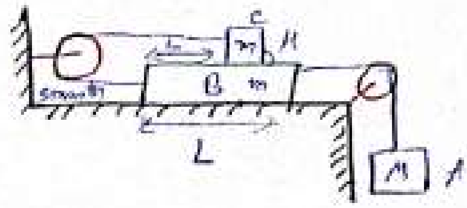
$$11\sqrt{2} = 3a$$

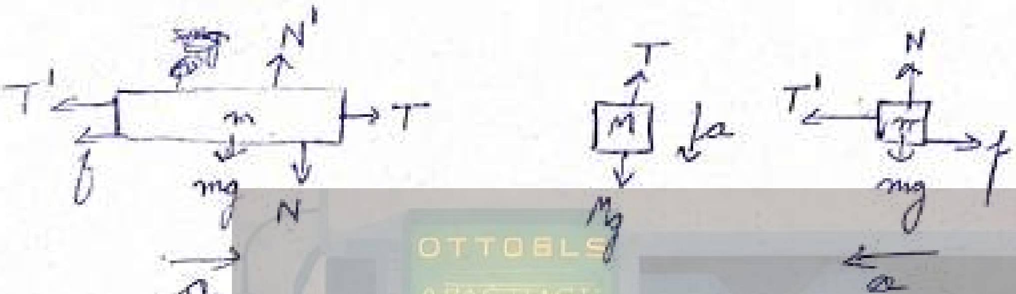
$$\boxed{a = \frac{11\sqrt{2}}{3}}$$ ✓

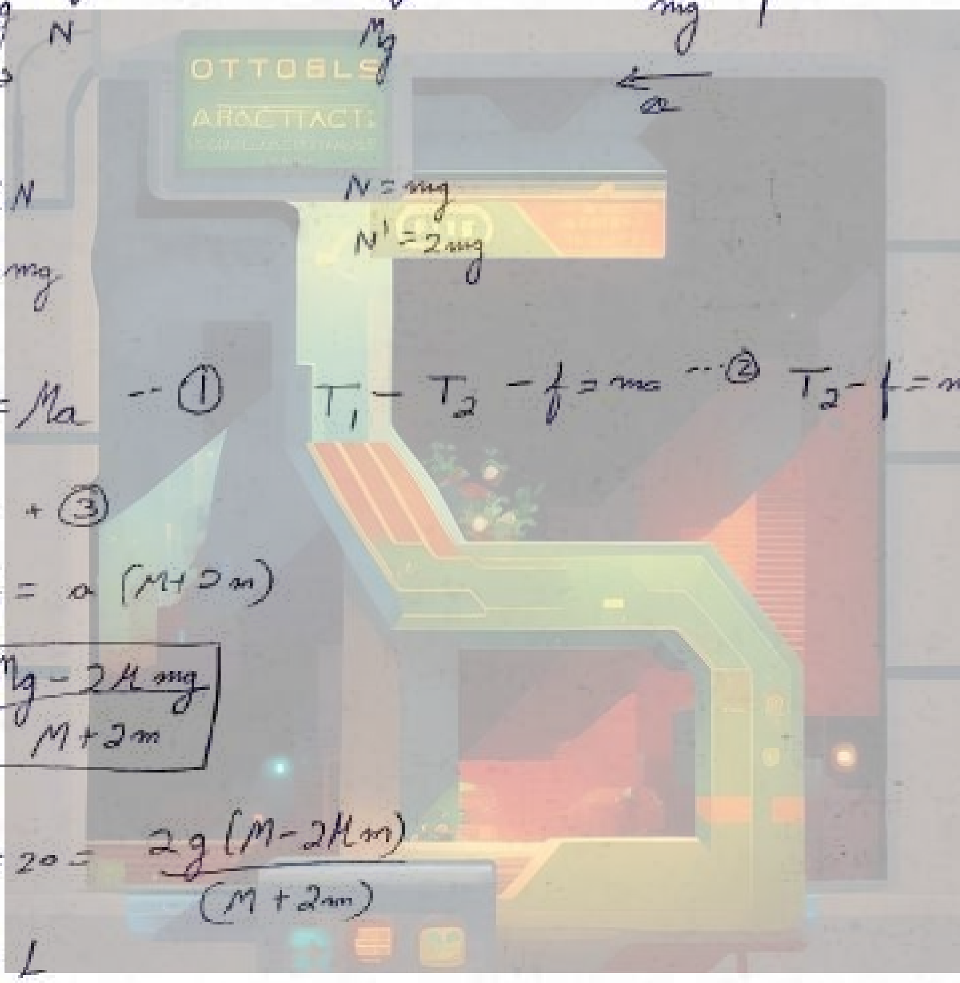
$$T = 8\sqrt{2} - 2a = 8\sqrt{2} - \frac{22\sqrt{2}}{3} = \frac{24\sqrt{2} - 22\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

Q i) find acceleration of A

ii) Speed of block C as it reaches left end of B.







$f_l = \mu N$

$= \mu mg$

$N = mg$

$N' = 2mg$

$Mg - T = Ma \quad \text{--- (1)}$ $\quad T_1 - T_2 - f = ma \quad \text{--- (2)}$ $\quad T_2 - f = ma \quad \text{--- (3)}$

(1) + (2) + (3)

$Mg - 2f = a(M + 2m)$

$$\boxed{a = \frac{Mg - 2\mu mg}{M + 2m}}$$

ii) $a_{rel} = 2a = \dfrac{2g(M - 2\mu m)}{(M + 2m)}$

$S_{rel} = L$

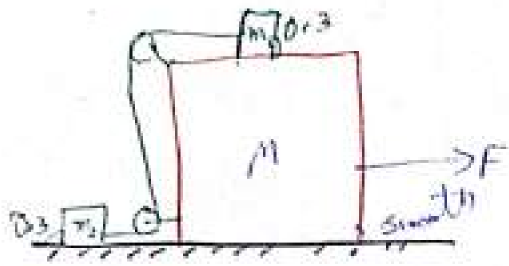
$u_{rel} = 0$

$v^2 = u^2 + 2as$

$v^2 = \dfrac{2g(M - 2\mu m)}{M + 2m} L$

$$v = \left[\frac{2g(M - 2\mu m)}{M + 2m} \times L\right]^{1/2}$$

Q find F for $f_1 = 2f_2$. Also find tension in string & acceleration.

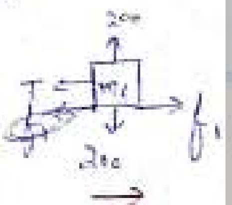


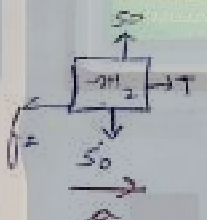
$m_1 = 20\,kg$

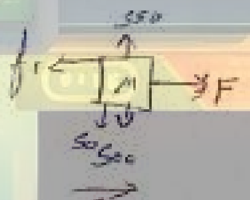
$m_2 = 5\,kg$

$M = 50\,kg$

for what F, $f_1 = 2f_2$







$f_{1\,max} = 0.3 \times 200 = 60$

$f_{2\,max} = 0.3 \times 50 = 15$

$f_1 = 2f_2$

$f_2 = 15$

$f_1 = 30 < f_{max}$

so no sliding btw m1 & M so will move together

$f_1 - T = 20a$ --- ①

$T - f_2 = 5a$ --- ②

$F - f_1 = 50a$ --- ③

$F - 30 = 50a$

$T - 15 = 5a$

$30 - T = 20a$

① + ②

$15 = 25a$

$\boxed{a = \frac{3}{5}}\ m/s^2$

in ③

$F - 30 = 50 \times \frac{3}{5}$

$F = 30 + 30$

$\boxed{F = 60N}$

$30 - T = 20 \times \frac{3}{5}$

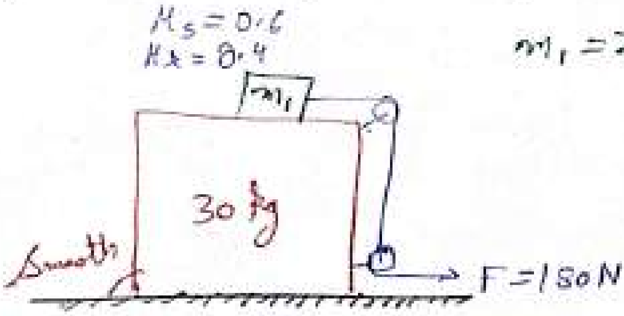
$30 - T = 12$

$T = 30 - 12$

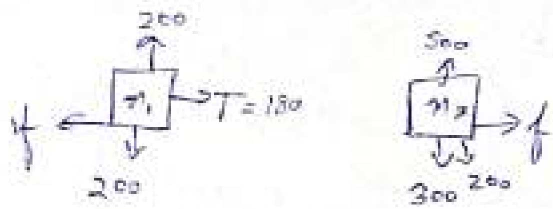
$\boxed{T = 18N}$

Q find a.

$m_1 = 20\,kg$



(2) $F = 400\,N$



$T - f = 20a$

~~$180 - 120 = 20a$~~

~~$\frac{60}{20} = a$~~

~~$a = 3$~~

~~but it will move so static friction will no longer act on it~~

~~$f_{s\,max} = 0.6 \times 200 = 120\,N$~~

~~$f_{k\,max} = 0.4 \times 200 = 80\,N$~~

~~$180 - 80 = 20a$~~

~~$\frac{100}{20} = a$~~

~~$\boxed{a = 5\,m/s}$~~

$f_{s\,max} = 0.6 \times 200 = 120\,N$

$f_{k\,max} = 0.4 \times 200 = 80\,N$

If move together,

$50a = 180$

$\boxed{a = \frac{18}{5}}$

verify

$\frac{18}{5} \times 20 = 180 - f$

$f = 108\,N < f_{s\,max}$

(2) $F = 400$

If move together

$50a = 400$

$a = 8$

verify $= 8 \times 20 = \cancel{180}\ 400 - f$

$= \cancel{f = 20\,N}$

$= 240 = f$ (not possible)

sliding occurs.

$f = 80\,N$.

$400 - 80 = 20a$

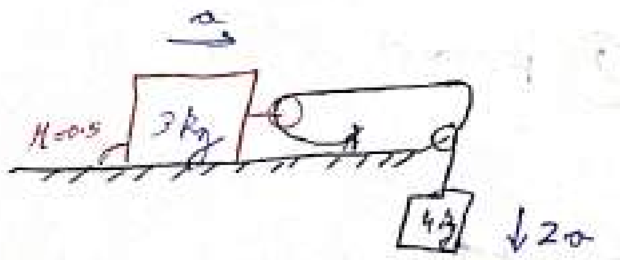
$\frac{320}{20} = a$

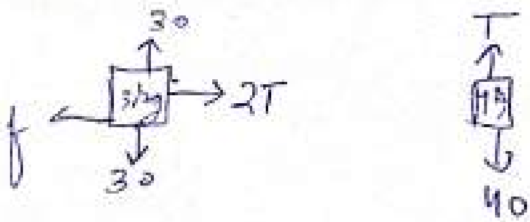
$\boxed{a = 16\,m/s^2}$

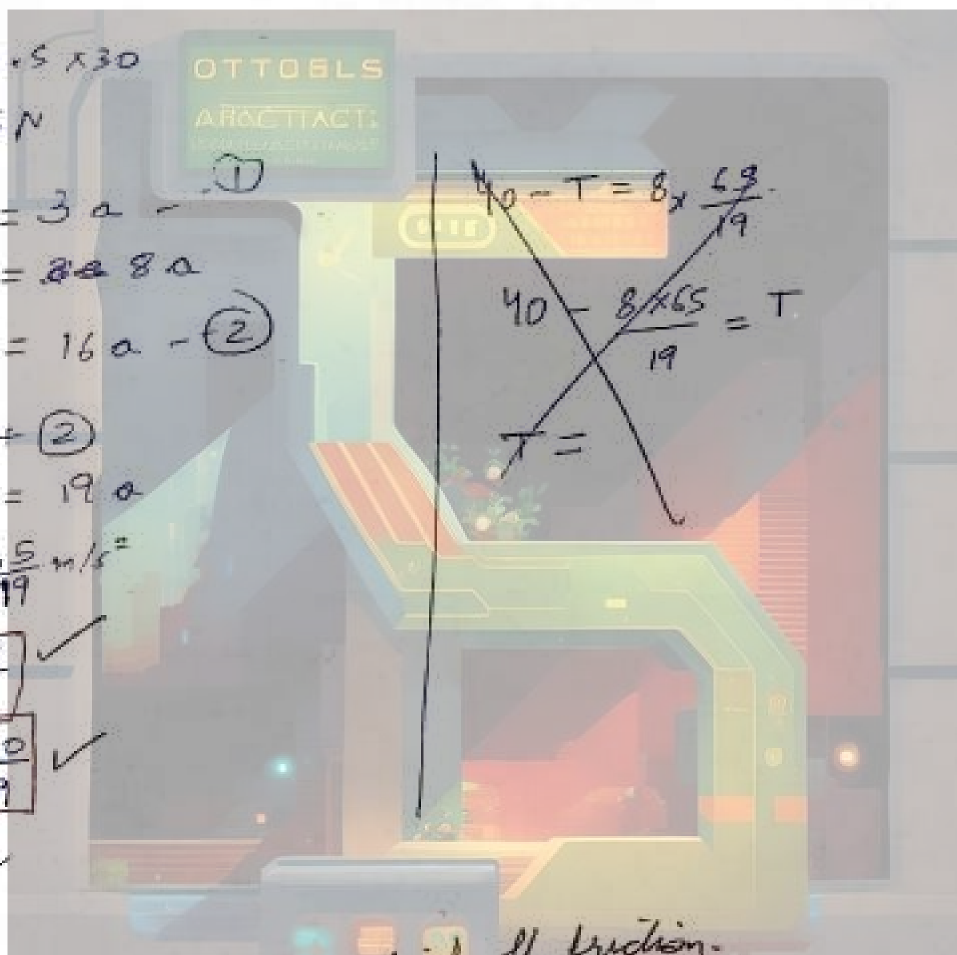
$a_{m_2} = \frac{80}{30}$

$\boxed{= \frac{8}{3}\,m/s^2}$

Q find acc of both & friction between ground & 3kg







$f_{max} = 0.5 \times 30$
$= 15\,N$

$2T - 15 = 3a$ — (1)
$40 - T = \cancel{8a}\ 8a$
$80 - 2T = 16a$ — (2)

(1) + (2)
$80 - 15 = 19a$
$a = \frac{65}{19}\ m/s^2$

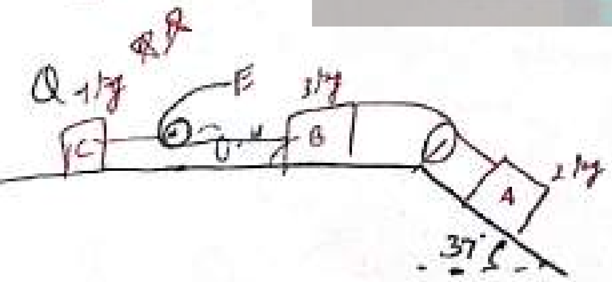
$\boxed{a_{3kg} = \frac{65}{19}}$ ✓

$\boxed{a_{4kg} = \frac{130}{19}}$ ✓

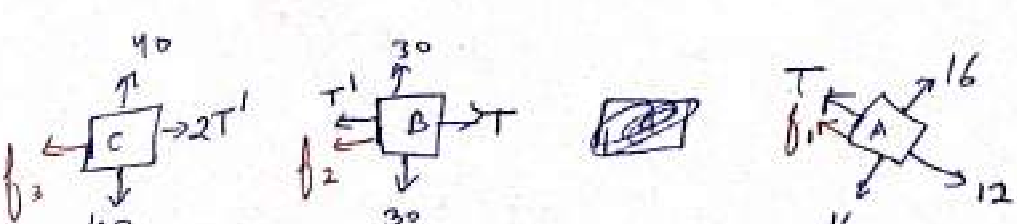
$\boxed{F = 15N}$ ✓

~~$40 - T = 8 \times \frac{65}{19}$~~
~~$40 - \frac{8 \times 65}{19} = T$~~
~~$T =$~~

Q find all friction.



$\mu_C = 0.6$
$\mu_B = 0.4$
$\mu_A = 0.6$



$f_{1max} = 0.6 \times 16 = \frac{96}{10}$ $\quad f_{2max} = 0.4 \times 30 = 12$ $\quad f_{3max} = 0.6 \times 40 = 24$

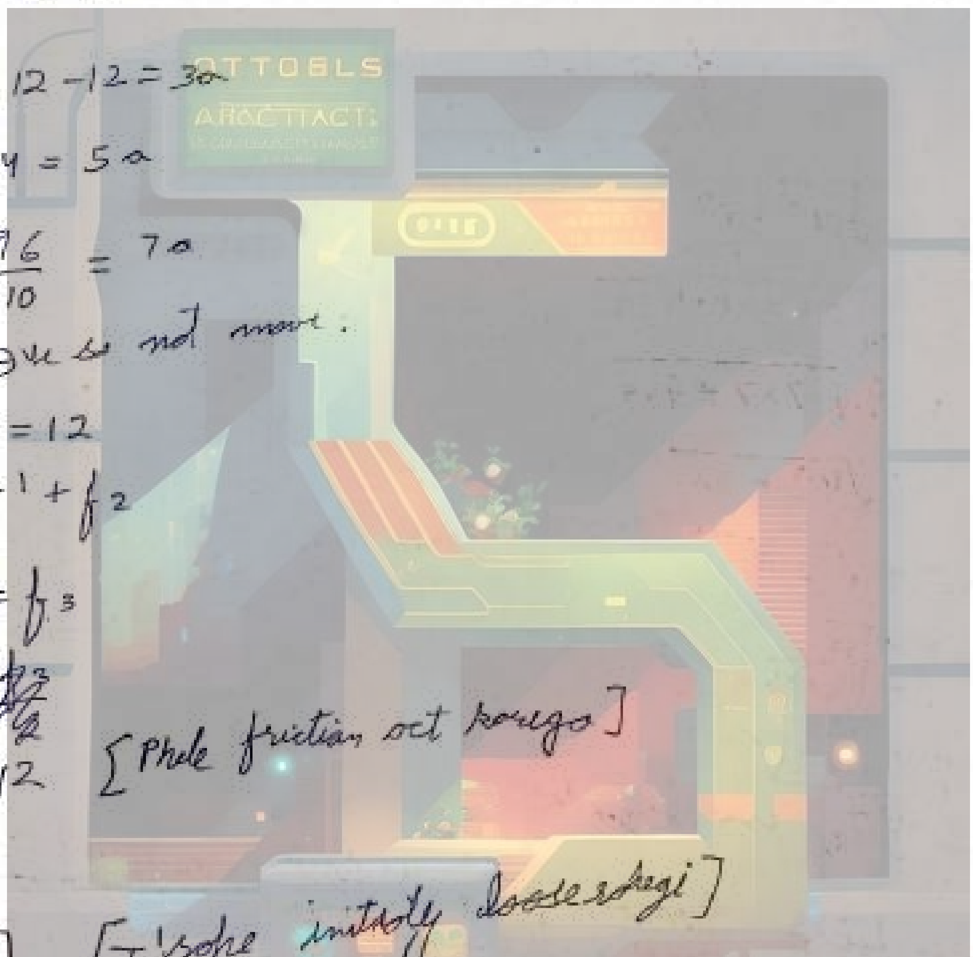
If system move

$12 - T - \frac{96}{10} = 2a$

$T - T' - 12 = 3a$

$2T' - 24 = 4a$

$T' = 2a + 12$



$T - 2a - 12 - 12 = 3a$

$T - 24 = 5a$

$12 - 24 - \frac{96}{10} = 7a$

$a = \ominus ve$ so not move.

$T + f_1 = 12$

$T = T' + f_2$

$2T' = f_3$

$\boxed{f_1 = 9.6}$

$T + 9.6 = 12$ [Phele friction act karego]

$T = 2.4$

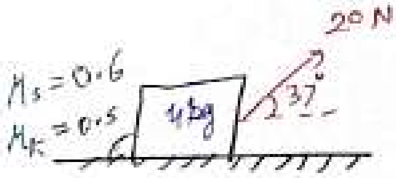
$\boxed{2.4 = f_2}$ [T' rope initially loose rahegi]

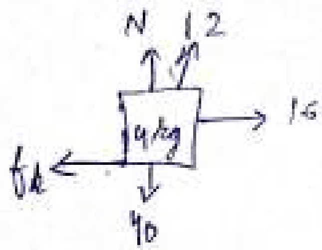
$T' = 0$

$\boxed{f_3 = 0}$

Q

find value & direction of ~~contact force~~ between block & surface. resultant of N & f





$N + 12 = 40$

$N = 28N$

$f_L = 0.6 \times 28$

$= 16.8$

$f = 16^N$

contact force $= \sqrt{N^2 + f^2}$

$= \sqrt{28 \times 28 + 16 \times 16}$

$= 4\sqrt{7 \times 7 + 4 \times 4}$

$= 4\sqrt{49 + 16}$

$= 4\sqrt{65}$

$= 4\sqrt{65}$

$= 4\sqrt{5 \times 13}$

$\boxed{= 4\sqrt{65}}$ ✓

at angle $\tan^{-1}(7/4)$ from Horizontal

Note:- Angle between ~~Contact force~~ resultant of f_L & Normal & Normal (vertical) = Angle of Friction.

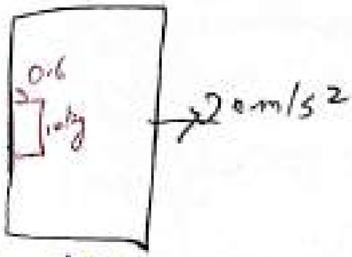
Angle of Friction:- The Angle of Friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction & normal reaction makes with normal reaction

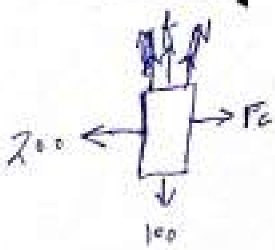
$\tan\theta = \frac{f_L}{N} = \frac{\mu N}{N} = \mu$

$\theta = \tan^{-1}(\mu)$

Q

find acc of Block, value of friction & contact force.





$F_c = 200$

$f_L = 0.6 \times 200$
$= 120\ N$

$\boxed{f = 100\ N}$

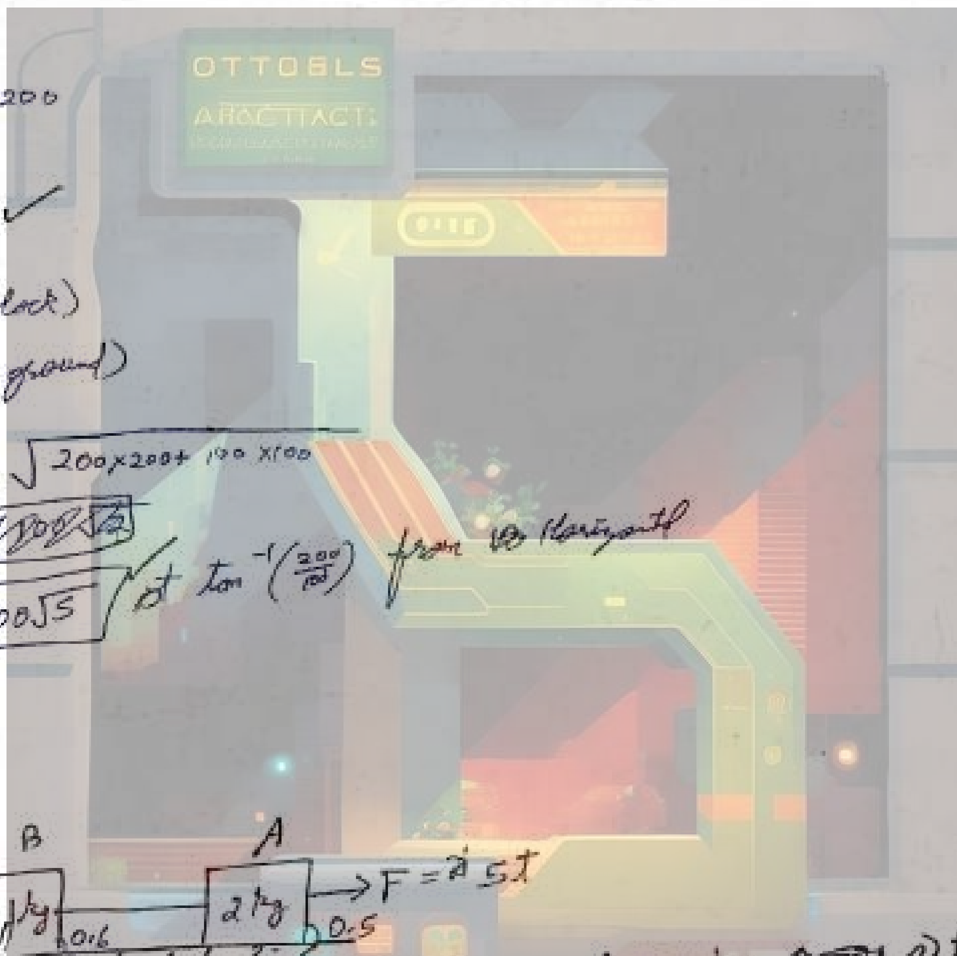
$a = 0$ (wrt block)

$\boxed{a = 20}$ (wrt ground)

contact force $= \sqrt{200 \times 200 + 100 \times 100}$

$\boxed{= 100\sqrt{5}}$ at $\tan^{-1}\left(\frac{200}{100}\right)$ from the horizontal

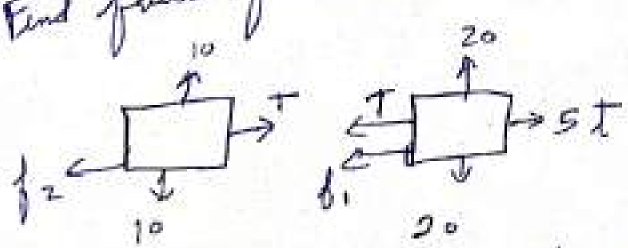
$\tan\theta = \frac{200}{100}$



Q

B
A
1kg
2kg
F = 5t
0.6
0.5

Find friction force between blocks & surface & tension
a) $t = 1\ s$
b) $t = 2\ s$
c) $t = 3\ s$
d) $t = 4\ s$



$f_1\ max = 0.5 \times 20$
$= 10\ N$

$f_2\ max = 10 \times 0.6$
$= 6\ N$

a) $F=5$

$\boxed{f_1=5}$, $T=0$

$\boxed{f_2=0}$

b) $F=10$

$\boxed{f_1=10}$, $T=0$

$\boxed{f_2=0}$

c) $F=15$

$\boxed{f_1=10}$, $T=5$

$\boxed{f_2=5}$

~~d) $F=20$~~

~~$\boxed{f_1=10}$, $T=10$~~

~~$f_2=6$ (max)~~

d) $F=\cancel{10}\ 20$

$f_1=10,\ f_2=6$ (max)

$a=\frac{4}{3}$

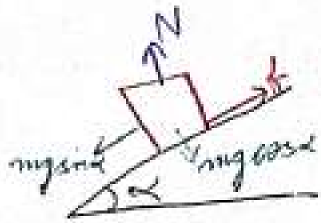
$T-6=\frac{4}{3}$

$T=\frac{4}{3}+6$

$\boxed{T=\frac{22}{3}}$

Angle of Repose (α)

→ It is the minimum angle of inclination of a plane with the horizontal such that a body placed on the plane just begins to slide down.



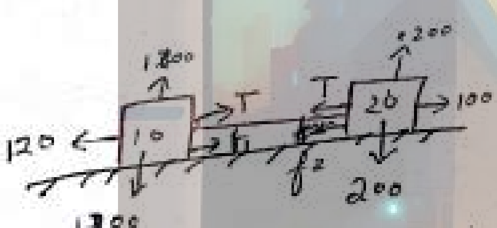
$N = mg\cos\alpha$

$f = \mu N = mg\sin\alpha$ [just begin to slide]

$\boxed{\mu = \tan\alpha}$

→ Angle of Repose does not depend on the mass of the object.

Q Find tension in the string.



$f_{1\,max} = 1200 \times 0.9 = 90\text{ N}$

$f_{2\,max} = 200 \times 0.3 = 60\text{ N}$

$F_{driving} = 120 - 100 = 20\text{ N}$

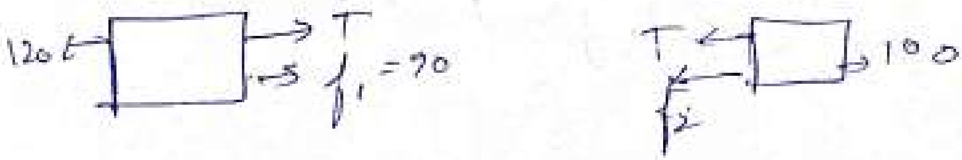
~~100 + 90 + T; T = 40; 120 - 40 = f1~~

$80 = f_1$

$F_{resisting} = 90 + 60 = 150$

Blocks will not move

Case I f_1 is limiting, f_2 is static



$T = 30$

$T + f_2 = 100$

$f_2 = 100 - 30$

$= 70 > f_{2\,max}$

not possible

Case II f_2 is limiting, f_1 is static

$100 = T + 60$

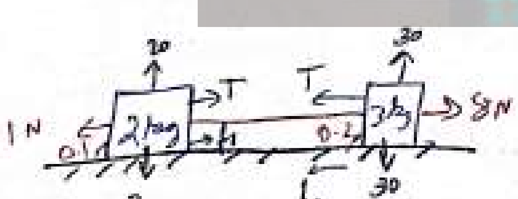
$T = 40$

$120 = T + f_1$

$80 = f_1$ (possible)

Thus, $\boxed{T = 40N}$ ✓

Q2. Find Tension



$f_{1\,max} = 20 \times 0.1$
$= 2N$

$f_{2\,max} = 30 \times 0.2$
$= 6N$

Net Driving $F = 8 - 1 = 7N$

Net resisting $F = 6 + 2 = 8$

at rest.

let f_2 = max

$8 - 6 = T$

$\boxed{T = 2}$

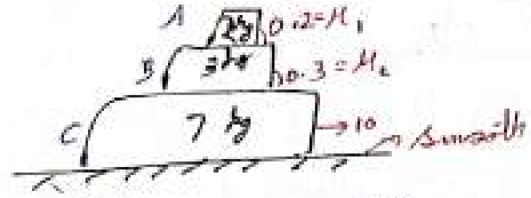
$T = f_1 + 1$

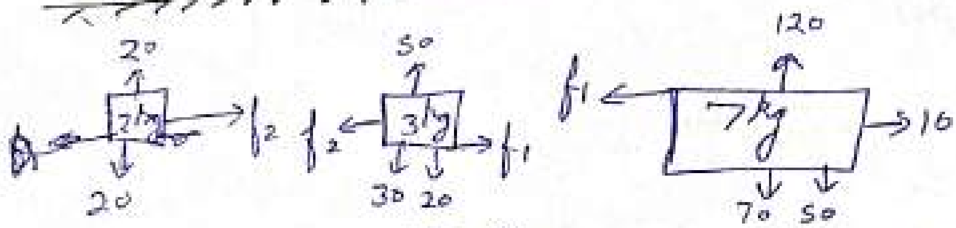
$2 = f_1 + 1$

$f_1 = 1$ (possible)

Three Block System

Q find acc & friction





Case 1:- If all move together.

$f_{2\,max} = 20 \times 0.2$
$= 4N$

$10 = 12a$

$\boxed{a = \frac{5}{6}\ m/s^2}$ ✓

$f_{1\,max} = 0.3 \times 50$
$= 15N$

Verify :-

i) $f_2 = 2 \times \frac{5}{6}$ ✓

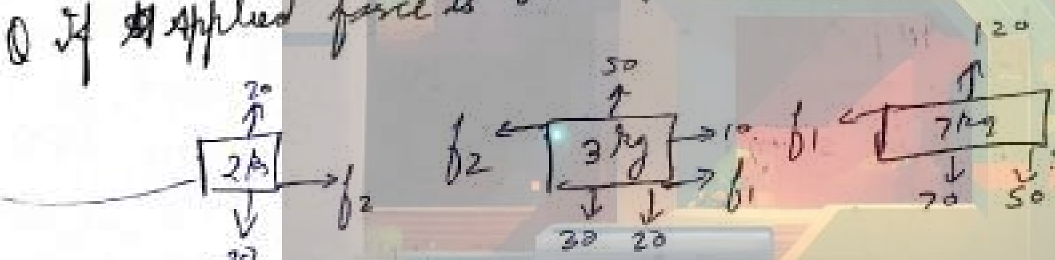
$\boxed{f_2 = \frac{5}{3}}$ (Possible)

ii) $f_1 - f_2 = 3 \times \frac{5}{6}$

$f_1 = \frac{5}{2} + \frac{5}{3}$

$\boxed{f_1 = \frac{25}{6}}$ (Possible) ✓

Q If Applied force is on 3kg block.



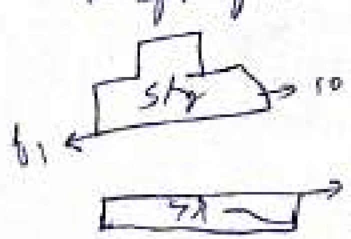
Case:- 1 All move together ✓

$10 = 12a$

$\boxed{a = \frac{5}{6}}$ ✓

Verify :-

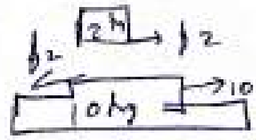
~~$10 + f_1 - f_2$~~



~~$f_1 = 10\,kg \times 10$~~ ~~(Possible)~~

$f_1 = 7 \times \frac{5}{6}$

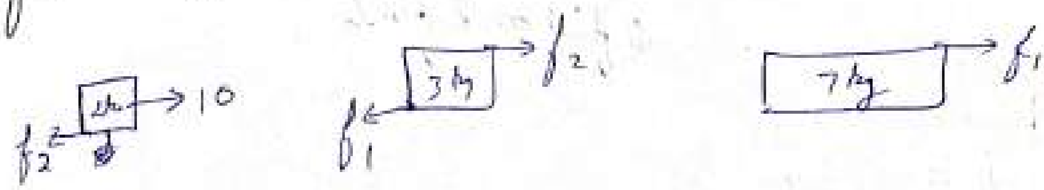
$\boxed{= \frac{35}{6}}$ (possible) ✓



$f_2 = 2 \times \frac{5}{6}$

$\boxed{= \frac{5}{3}}$ (Possible)

Q If force Applied on 2kg block.



Case 1:- If All move Together.

$a = \frac{5}{6}$

$f_2 = 10 \times \frac{5}{6}$

$= \frac{25}{3}$

$= 8.33$ ~~(Possible)~~

(not-Possible)

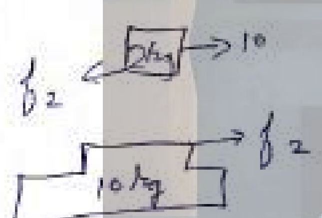
$f_1 = 7 \times \frac{5}{6}$

= (Possible)

$f_1 = \frac{35}{6}$

So, there will be sliding between 2kg & 3kg block.

$f_2 = 4N$ (sliding friction)



$a_{2kg} = \frac{10-4}{2}$

A $\boxed{= 3\,m/s^2}$ ✓

$a_{3kg,7kg} = \frac{4}{10}$

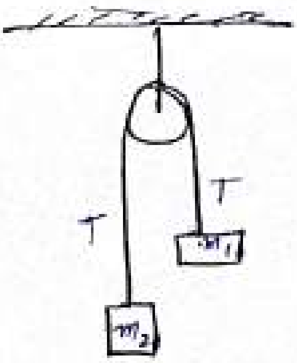
B, C $\boxed{= \frac{2}{5}\,m/s^2}$ ✓

$\boxed{f_1 = \frac{35}{6}N}$ ✓ $\boxed{f_2 = 4N}$ ✓

Shortcut to find Tension in string.

$$\boxed{T = \frac{\Sigma\left[(\text{coefficients of tension connected with block}) \times (g_{eff})\right]}{\sum_{i=1}^{n} \frac{(\text{coeff. of } T)^2}{m_i}}}$$

lg.

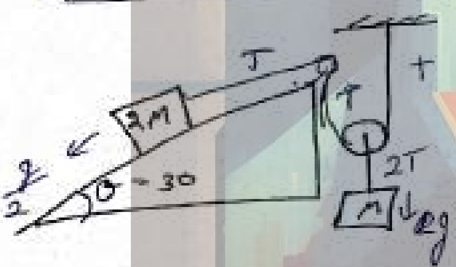


geff → component of g in direction of motion.
Coeff of T → ~~a~~ Number multiplied by T

$$T = \frac{1g + 1g}{\frac{(1)^2}{m_1} + \frac{(1)^2}{m_2}}$$

$$\boxed{= \frac{2g\, m_1 m_2}{m_1 + m_2}}$$

2.



$$T = \frac{\frac{g}{2} + 2g}{\frac{1}{2M} + \frac{4}{M}}$$

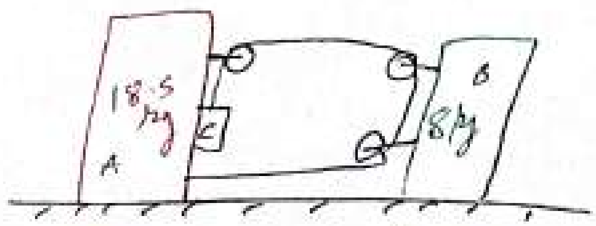
$$= \frac{5g}{2\left[\frac{9}{2M}\right]}$$

$$= \frac{5g \times 2M}{18}$$

$$\boxed{= \frac{5}{9} gM}$$

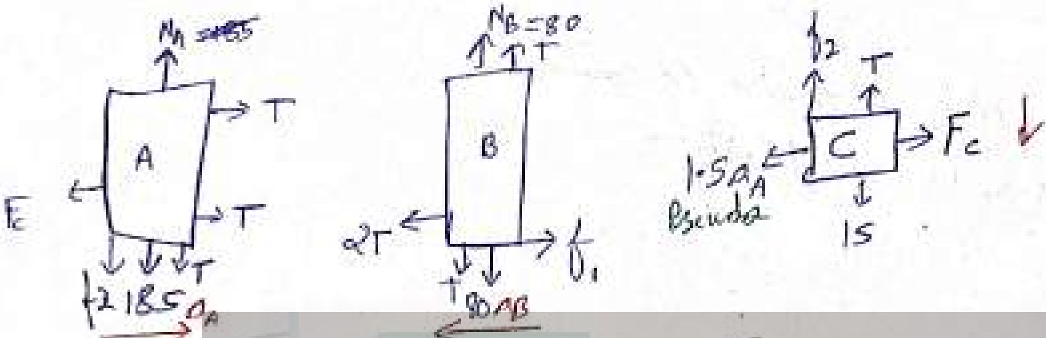
① find acc.

$M_C = 1.5\,kg$



$\mu_{AC} = \frac{1}{3}$

$\mu_B = 0.2$



$f_{1\,max} = N_B \times 0.2$
$= 80 \times 0.2$
$= 16\,N$

$f_{2\,max} = \frac{F_c}{3}$

$-x_A - x_B - x_A - x_B + x_C = 0$

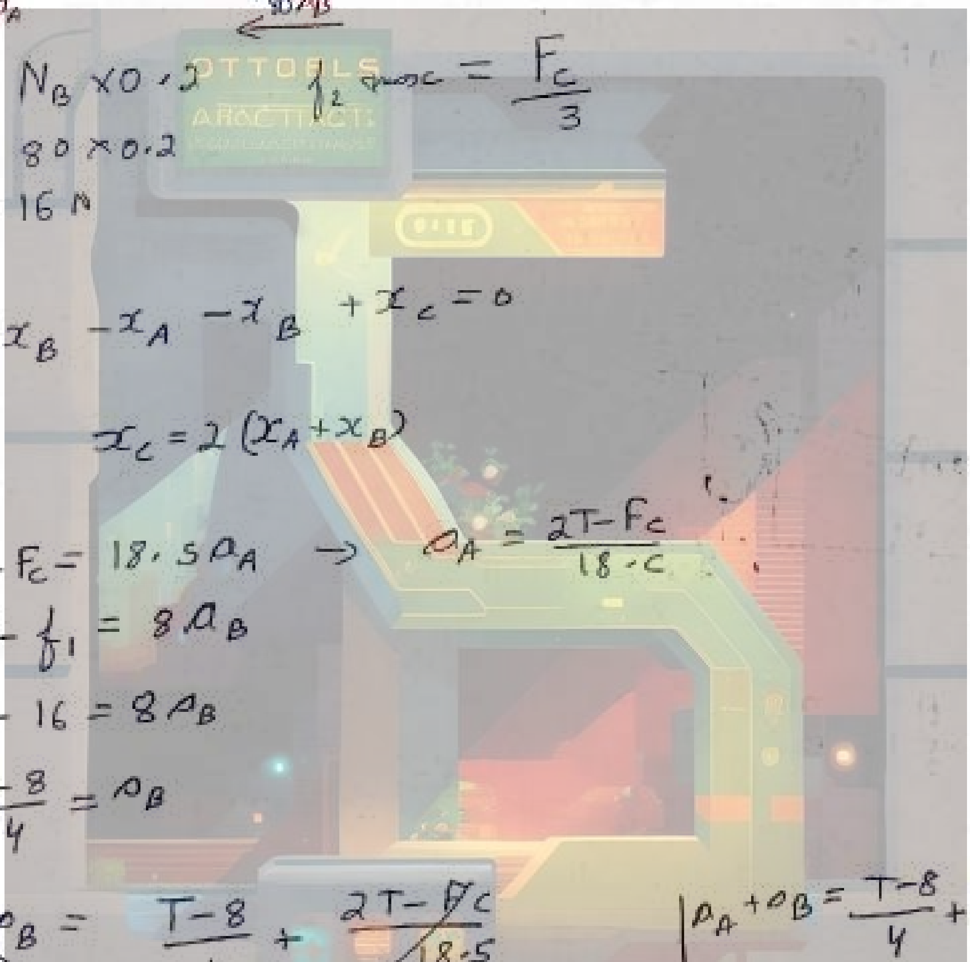
$x_C = 2(x_A + x_B)$

$2T - F_c = 18.5 a_A \rightarrow a_A = \frac{2T - F_c}{18.5}$

$2T - f_1 = 8 a_B$

$2T - 16 = 8 a_B$

$\frac{T-8}{4} = a_B$



~~$a_A + a_B = \frac{T-8}{4} + \frac{2T - F_c}{18.5}$~~

~~$= \frac{T-8}{4} + \frac{4T - 2F_c}{37}$~~

~~$= \frac{37T - 37 \cdot 8 + 16T - 8F_c}{148}$~~

~~$2(a_A + a_B) = \frac{21T - 8F_c - 296}{148\ 74} \times 2$~~

~~$a_C = \frac{15 - T - \frac{F_c}{3}}{1.5}$~~

~~$= 1.5$~~

$a_A + a_B = \frac{T-8}{4} + \frac{T}{10}$

$= \frac{10T - 80 + 4T}{40}$

$= \frac{14T - 80}{40}$

$= \frac{7T - 40}{20}$

$2(a_A + a_B) = \frac{7T - 40}{10}$

$F_c = 1.5\, a_A$

$f_2 = \dfrac{a_A}{2}$

$\cancel{15 - f_2 - T = a_c 1.5}$

$\cancel{15 - T - 2T - F_c}$

$2T - F_c = 18.5\, a_A$

$2T - 1.5 a_A = 18.5 a_A$

$2T = 20 a_A$

$T = 10 a_A$

$a_A = \dfrac{T}{10}$

$a_c = \dfrac{15 - \frac{T}{20} - T}{1.5}$

$= \dfrac{300 - T - 20T}{30}$

$= \dfrac{300 - 21T}{30}$

$\dfrac{300 - 21T}{30} = \dfrac{7T - 40}{10}$

$300 - 21T = 21T - 120$

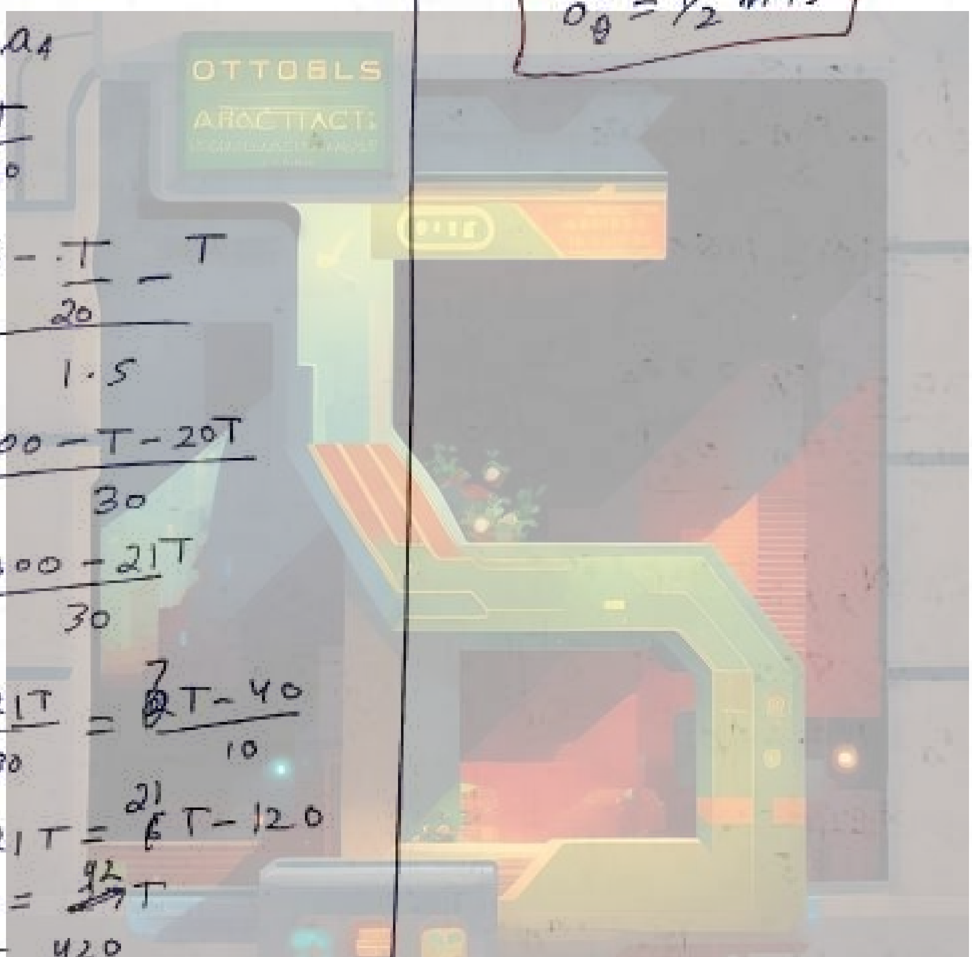
$420 = 42T$

$T = \dfrac{420}{42}$

$T = 10$

$a_A = \dfrac{10}{10}$

$\boxed{a_A = 1 m/s^2}$



$a_c = \dfrac{300 - 21 \times 10}{30}$

$= 10 - 7$

$\boxed{a_c = 3 m/s^2}$

$2 + 2a_B = \dfrac{70 - 40}{10}$

$2 + 2a_B = 3$

$2a_B = 1$

$\boxed{a_B = 1/2\ m/s^2}$

MII

$2(a_A + a_B) = a_C$

for A,

$N_A = 185 + f_1 + T$

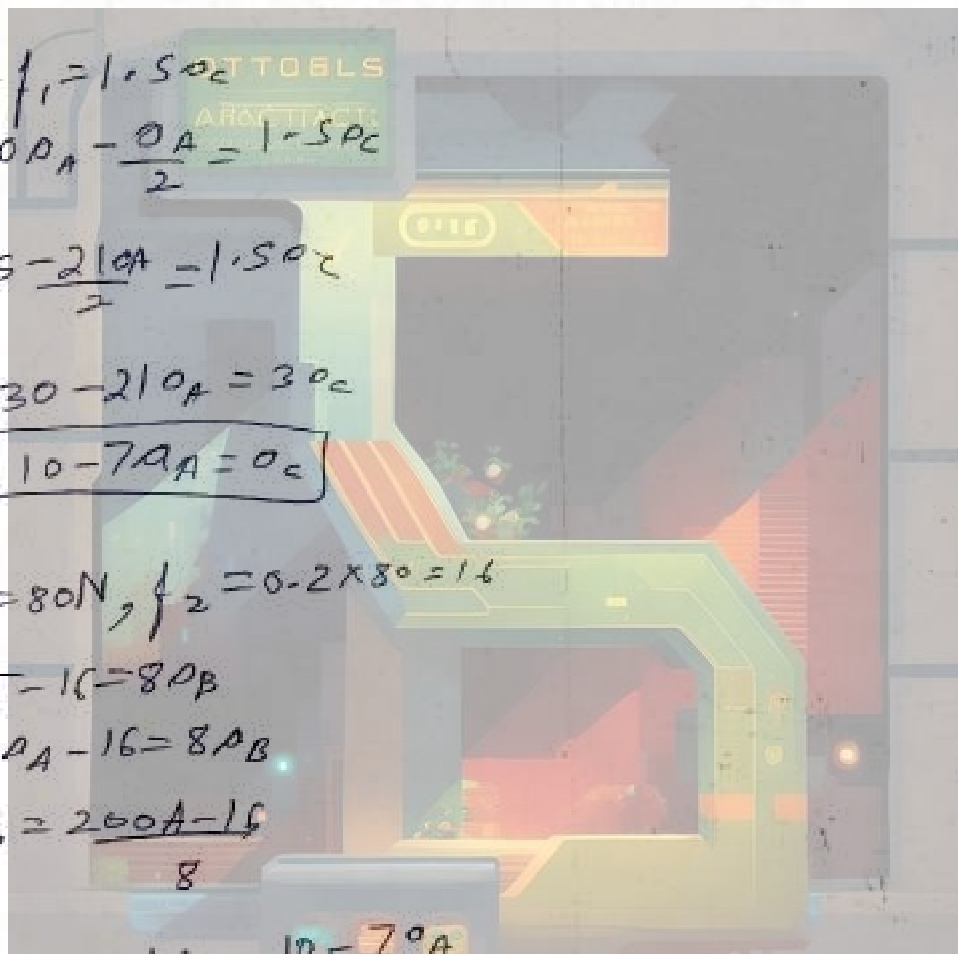
for $f_1 = \frac{1}{3}(1.5 a_A) = \frac{a_A}{2}$

~~$2T-A=$~~

$2T - F_C = 18.5 a_A$

$\boxed{T = 10 a_A}$

for C,



$15 - T - f_1 = 1.5 a_C$

$15 - 10 a_A - \frac{a_A}{2} = 1.5 a_C$

$15 - \frac{21 a_A}{2} = 1.5 a_C$

$30 - 21 a_A = 3 a_C$

$\boxed{10 - 7 a_A = a_C}$

for B,

$N_B = 80N, f_2 = 0.2 \times 80 = 16$

$2T - 16 = 8 a_B$

$20 a_A - 16 = 8 a_B$

$a_B = \frac{20 a_A - 16}{8}$

$2 a_A + \frac{20 a_A - 16}{4} = 10 - 7 a_A$

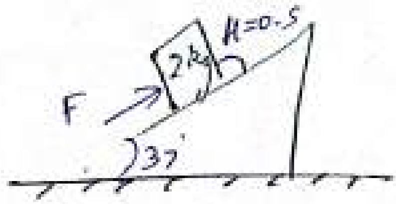
$\boxed{a_A = 1 \ m/s^2}$

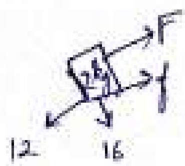
$a_C = 10 - 7$

$\boxed{a_C = 3 \ m/s^2}$

$\boxed{a_B = 0.5 m/s^2}$

Q find F so block is at rest





$f_{max} = 0.5 \times 16$
$= 8$

$F + f = 12$

$F = 12 - f$

$F = 12 - 0$
$= 12$

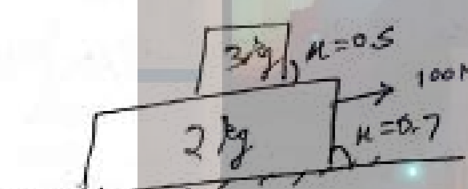
$F = 12 - 8$
$F = 4$

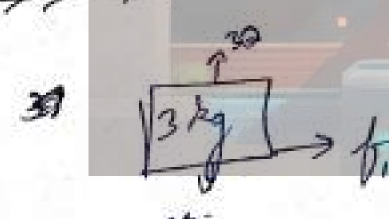
friction in opp dir

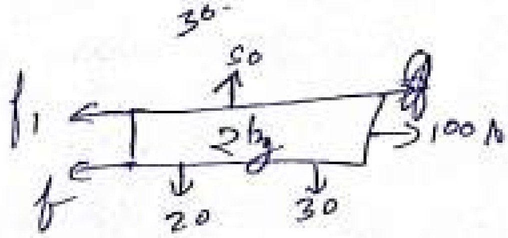
$F = 12 + f$
$= 12, 20$

$F \in [4, 20]$

Q2. find acc







$f_{max} = 50 \times 0.7$
$= 35$

$f_1 max = 30 \times 0.5$
$= 15$

if together,

$100 - 35 = 5a$

$a = 13$

slip,

$15 = 3a$

$\boxed{a = 5}$ m/s²

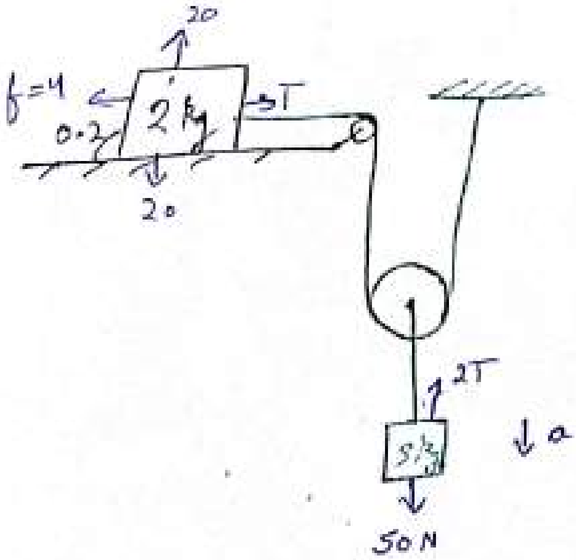
$100 - 15 - f = 2a$

$85 - 35 = 2a$

$a = \frac{50}{2}$

$\boxed{a = 25 \text{ m/s}^2}$

Q



find acc of blocks.

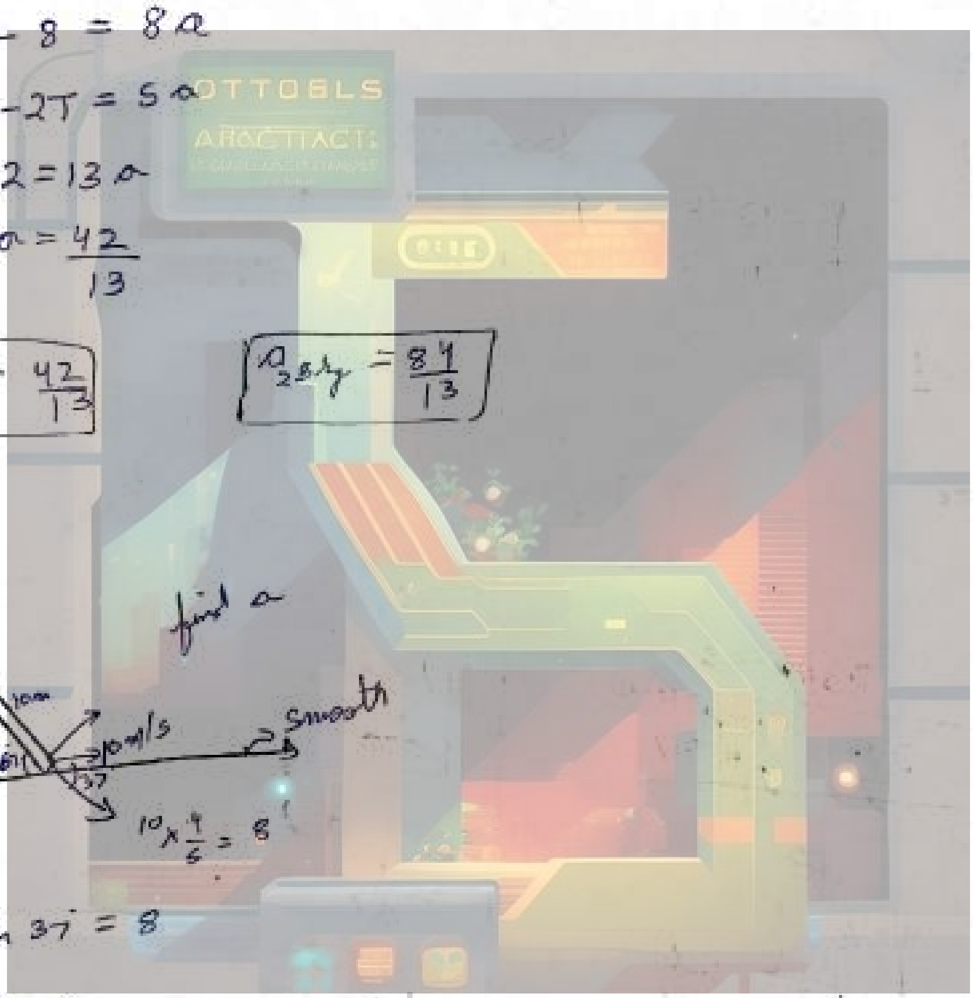
$2T - 8 = 8a$

$50 - 2T = 5a$

$42 = 13a$

$a = \frac{42}{13}$

$\boxed{a_{5kg} = \frac{42}{13}}$ $\boxed{a_{2kg} = \frac{84}{13}}$



Q

find a

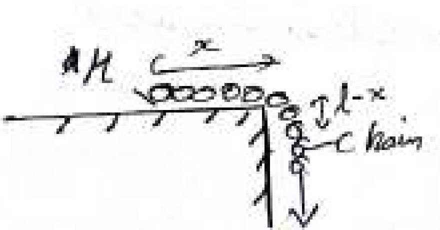
$10 \times \frac{4}{5} = 8$

$a \sin 37° = 8$

$a \times \frac{3}{5} = 8$

$\boxed{a = \frac{40}{3} \text{ m/s}}$

Q



Mass $- M$

length $- l$

minimum length to keep in equilibrium

$$\frac{M}{l} \times (l-x)\, g$$

$$f_{max} = \frac{M}{l} \times x \times g \times \mu$$

$$\frac{M}{l} x g \mu = \frac{M}{l} (l - x) g$$

$$x\mu = l - x$$

$$x(1+\mu) = l$$

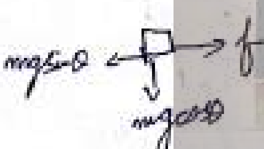
$$\boxed{x = \frac{l}{1+\mu}}$$

Q

$y = 2x^2$

$\mu = 0.5$

find max height the box can move without slipping.



$$f = mg\sin\theta$$

$$0.5 \times mg\cos\theta = mg\sin\theta$$

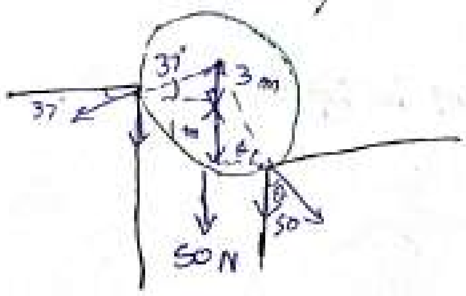
$$0.5 = \tan\theta$$

$$\tan\theta = 4x = 0.5$$

$$x = \frac{1}{8}$$

$$\boxed{y = \frac{1}{32}}$$

Q



find normal so ball remain in rest

mass - 5 kg

$\sin\theta = \frac{4}{5}$

$\theta = 53°$

$N = \cancel{50\cos 53°} = \cancel{50 \times \frac{3}{5}}$

$\cancel{\boxed{N = 30N}}$

$N' = 50 \times \frac{4}{5}$

$\boxed{N' = 40N}$

$\sin \cos 53° \times N = 50$

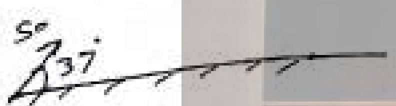
$N \times \frac{3}{5} = 50$

$\cancel{\boxed{N = \frac{250}{3}}}$

$N = 30$

Q

find time when velocity makes 67° with initial vel.



$u = 40\hat{i} + 30\hat{j}$

$V = 40\hat{i} + (30 - gt)\hat{j}$

$N \cos 30° = 40$

$V = \frac{80}{\sqrt{3}}$

$-V \sin 30° = 30 - gt$

$-\frac{40}{\sqrt{3}} = 30 - 10t$

$10t = \frac{30\sqrt{3} - 40}{\sqrt{3}}$

$\boxed{t = \frac{3\sqrt{3} + 4}{\sqrt{3}}}$

$\sin 30 \sin 37 - \cos 3 \cos 37 = \frac{1600 + 900 - 300t}{50 \times \sqrt{1600 + 900 + 100t^2 - 600t}}$

$\frac{3}{10} - \frac{4\sqrt{3}}{10} = \frac{2500 - 300t}{50 \times 10\sqrt{t^2 - 6t + 25}}$

$3 - 4\sqrt{3} = \frac{250 - 30t}{\sqrt{t^2 - 6t + 25}}$

$t^2 - 6t + 25 = \frac{(250 - 30t)^2}{3 - 4\sqrt{3}}$

Q

$F = -Kv$

$ma = -Kv$

$2\frac{dv}{dt} = -Kv$

$2 \times \frac{dv}{dx} = -K$

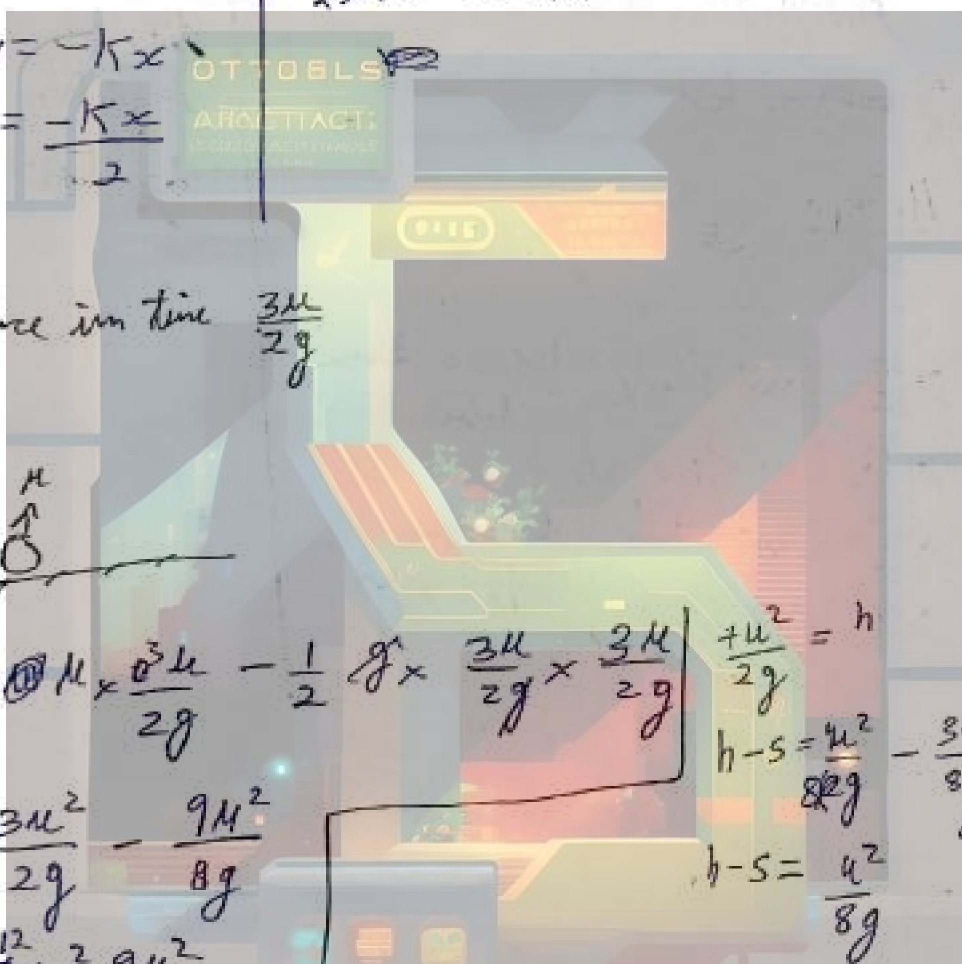
$2v = -Kx$

$v = \frac{-Kx}{2}$

$2 \times \frac{dv}{dt} = -Kv$

$\frac{2}{v} dv = -K\,dt$

$2 \ln v = -Kt$



Q find distance in time $\frac{3u}{2g}$

$s = u \times \frac{3u}{2g} - \frac{1}{2} g \times \frac{3u}{2g} \times \frac{3u}{2g}$

$= \frac{3u^2}{2g} - \frac{9u^2}{8g}$

$= \frac{12u^2 - 9u^2}{8g}$

$\boxed{s = \frac{3u^2}{8g}}$

$\frac{u^2}{2g} = h$

$h - s = \frac{u^2}{2g} - \frac{3u^2}{8g}$

$h - s = \frac{u^2}{8g}$

Distance $= \boxed{\frac{5u^2}{8g}}$

Important Result

(1) $a = \dfrac{(m_1 - m_2)\, g}{(m_1 + m_2)}$

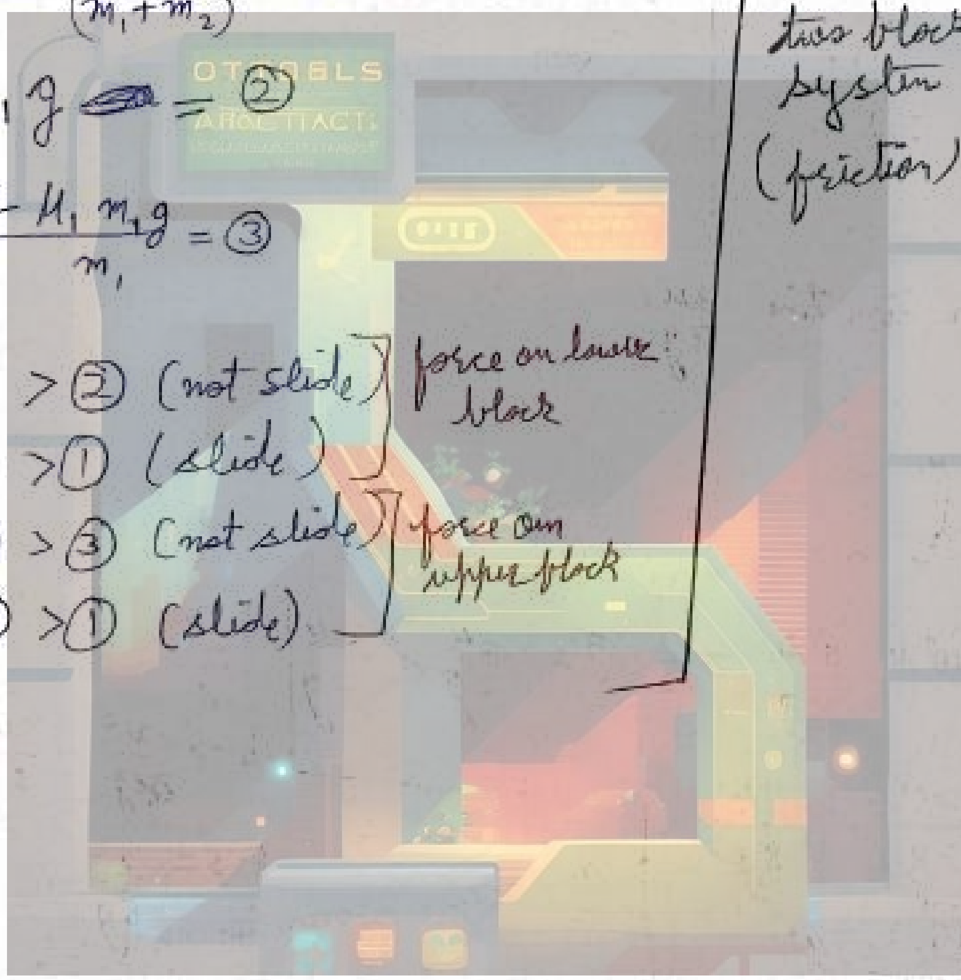
(2) $T = \dfrac{2 m_1 m_2\, g}{m_1 + m_2}$

atwood machine ($m_1 > m_2$)

$\dfrac{F - \mu_2 (m_1 + m_2) g}{(m_1 + m_2)} = (1)$

$\mu_1 g = (2)$

$\dfrac{F - \mu_1 m_2 g}{m_1} = (3)$



(1) > (2) (not slide)
(2) > (1) (slide)

force on lower block

(1) > (3) (not slide)
(3) > (1) (slide)

force on upper block

two block system (friction)


OTTOBLS
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! Ch-5 !

Work, Power & Energy

Work

→ Work is said to be done if and when ~~for~~ Applied force produces some displacement.

→ Whenever Force is Applied body is displaced, always atleast two ~~forces are~~ bodies are involved,

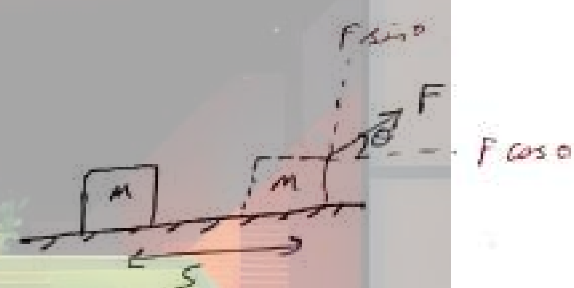
1. which is doing work (whose energy is decreasing)
2. On which the work is being done (whose energy is increasing)

→ work done by a constant force is the product of force in the direction of motion and magnitude of displacement.

$W = F\cos\theta \times s$

$W = Fs\cos\theta$

$W = \vec{F} \cdot \vec{S}$



→ It is a scalar quantity

→ SI unit :- Joule (J) - Nm

CGS Unit :- erg = 1 dyne cm

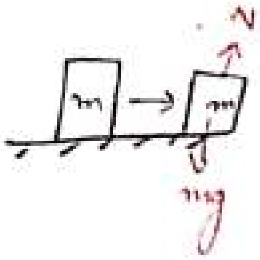
$1\,erg = 10^{-7}\,J$

Types of work :-

1. Zero Force :-

→ work done by a force is zero if the body is displaced perpendicular to the direction of the force.

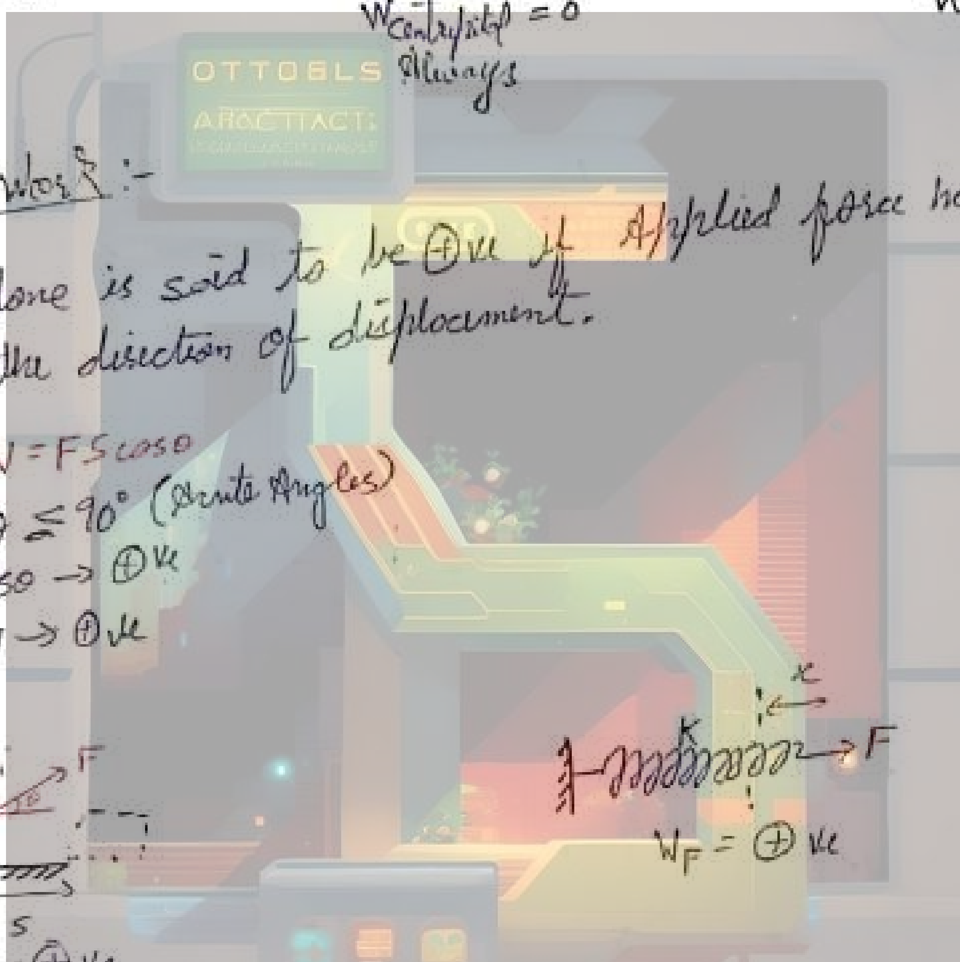
→ Work done by a force is zero if body suffers no displacement on application of force.



$W_{mg} = W_N = 0$

$W_{centripetal} = 0$ Always

$W_T = 0$



2. Positive Work :-

→ Work done is said to be (+)ve if Applied force has a component in the direction of displacement.

$W = FS\cos\theta$

$0^\circ \le \theta \le 90^\circ$ (Acute Angles)

$\cos\theta \to$ (+)ve

$W \to$ (+)ve

F

θ

s

$W_F =$ (+)ve

x

K

F

$W_F =$ (+)ve

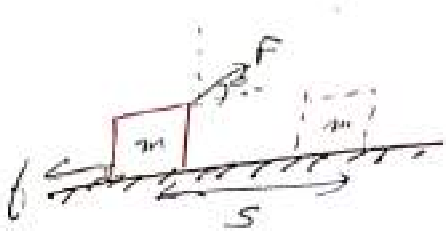
3. Negative Work :-

→ Work done is said to be negative if the applied force has a component in a direction opposite to that of displacement.

$90 < \theta \le 180$

$\cos\theta =$ (−)ve

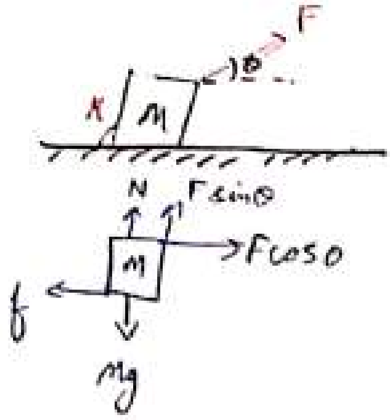
$W =$ (−)ve



$W_f =$ (−)ve

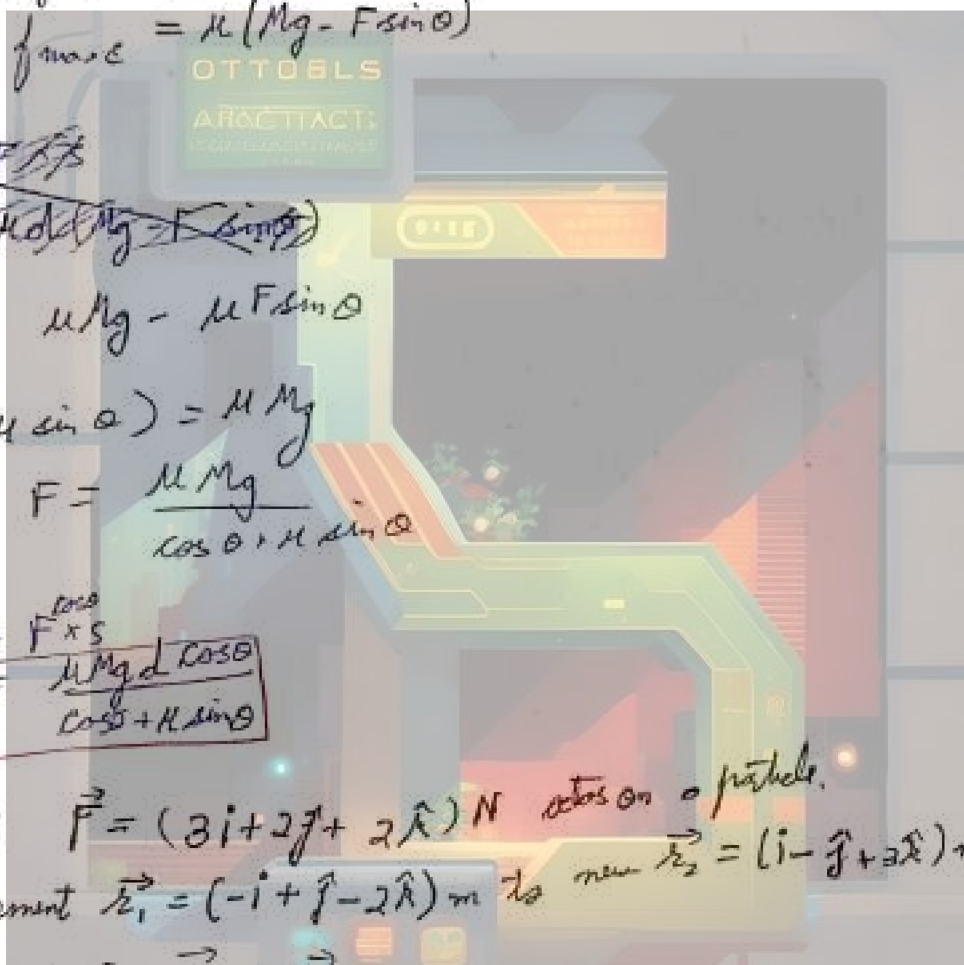
Q 1.

Q Travels with uniform velocity, find work done by applied force(F) during displacement d.



$N = Mg - F\sin\theta$

$f_{max} = \mu(Mg - F\sin\theta)$



$F\cos\theta = f_{max} = \mu(Mg - F\sin\theta)$

~~Work = F × s~~

~~$= \mu d(Mg - F\sin\theta)$~~

$F\cos\theta = \mu Mg - \mu F\sin\theta$

$F(\cos\theta + \mu\sin\theta) = \mu Mg$

$$F = \frac{\mu Mg}{\cos\theta + \mu\sin\theta}$$

$W_F = F\cos\theta \times s$

$$\boxed{W_F = \frac{\mu Mg\, d\cos\theta}{\cos\theta + \mu\sin\theta}}$$

Q2. constant $\vec{F} = (3\hat{i} + 2\hat{j} + 2\hat{k})\,N$ acts on a particle. displacement $\vec{r}_1 = (-\hat{i} + \hat{j} - 2\hat{k})\,m$ to new $\vec{r}_2 = (\hat{i} - \hat{j} + 3\hat{k})\,m$. find work.

Displacement, $\vec{s} = \vec{r}_2 - \vec{r}_1$

$= 2\hat{i} - 2\hat{j} + 5\hat{k}$

work $= \vec{F} \cdot \vec{s}$

$= 6 - 4 + 10$

$\boxed{W = 12\,J}$

Q3 Three constant forces act on a body $\vec{F_1} = (2\hat{i} - 3\hat{j} + 2\hat{k})$ N, $F_2 = (\hat{i} + \hat{j} - \hat{k})$
$F_3 = (3\hat{i} + \hat{j} - 2\hat{k})$ displaces from $(1,-1,2)$ to $(-1,-1,3)$ then to $(2,2,0)$ in metres. Find Total work done by Forces.

$$S = \hat{i} + 3\hat{j} - 2\hat{k}$$

$$F = 6\hat{i} - \hat{j} - \hat{k}$$

$$\text{Work} = 6 - 3 + 2$$

$$\boxed{= 5\,J}$$

Important Points on Work.

1. Work is said to be done by a force when its point of application moves by some distance.
2. Work is defined for an interval or displacement. There is no Term like instantaneous work.
3. For a particular displacement, work done by a force is independent of the type of motion i.e. weather it moves with constant velocity, acceleration or retardation etc.

eg.

$W_F = 2 \times 0.2$
$= 0.4\,J$

$W_F = 0.2 \times 2$
$= 0.4\,J$
$W_f = -0.2\,J$
$W_{Total} = 0.2\,J$

$W_F = 0.2 \times 2$
$= 0.4\,J$
$W_f = -0.4\,J$
$W_{Total} = 0\,J$

4. If a body is in dynamic equilibrium (moving with constant velocity) under the action of certain forces, then total work done on the body is zero but work done by individual forces may not be zero.

$|F_1| = |F_2|$

$W_{F_1} = F_1 S$

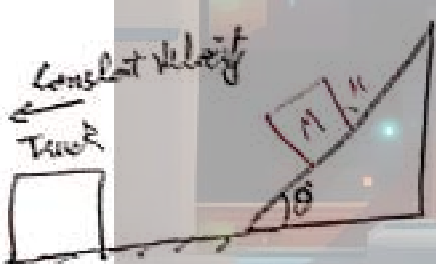
$W_{F_2} = -FS$

$W_{F_1} + W_{F_2} = 0$

5. Work done by a force is independent of time taken to do that work.

6. Work done by a force is frame dependent.

Q 4.



Q Find work done by friction in time t wrt inclined plane. Block is at rest wrt inclined plane. frame of truck & ground.

$s = vt$

$s = 0$ wrt truck

$\boxed{W = 0}$ wrt truck

$f = Mg\sin\theta$

$W = f s \cos(180-\theta)$

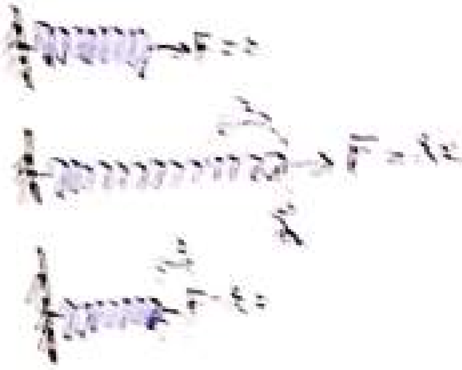
$= -vt\, Mg\sin\theta\cos\theta$

$\cancel{\boxed{= vt\, Mg\sin\theta}}$ ~~wrt ground~~

$\boxed{W = -vt\, Mg\sin\theta\cos\theta}$

wrt ground

Work done by variable force.



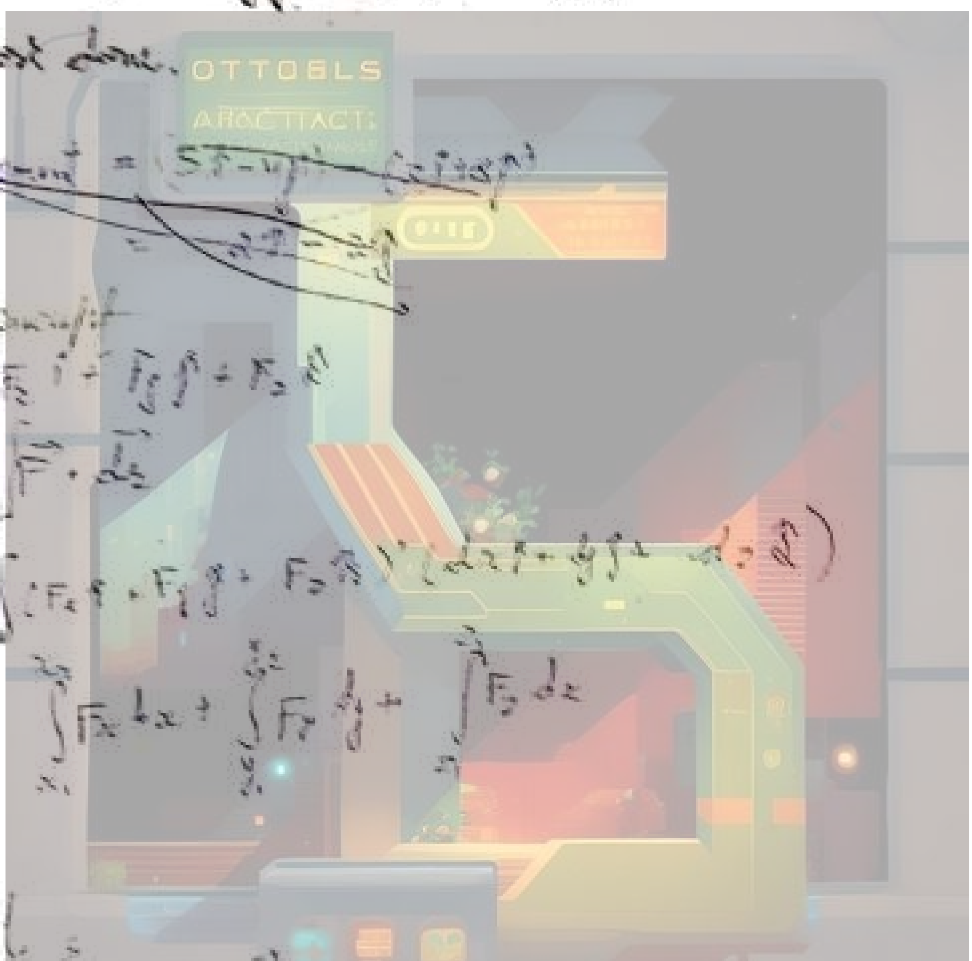
$dW = F\,dx$

$dW = kx\,dx$

$\int dW = \int kx\,dx$

$\boxed{W = \frac{1}{2}kx^2}$

Q5. force $F = (6x\hat{i} + 2y\hat{j})$ displaces body from $(3\hat{i}+8\hat{j})$ to $(5\hat{i}-4\hat{j})$ find work done.



~~Displacement $= (5\hat{i}-4\hat{j}) - (3\hat{i}+8\hat{j})$~~

~~$= 2\hat{i} - 12\hat{j}$~~

Concept

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$W = \int \vec{F}\cdot d\vec{r}$

$= \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$

$= \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz$

Sol.

$W = \int_3^5 6x\,dx + \int_8^{-4} 2y\,dy$

$W = 3(5)^2 - 3(3)^2 + (-4)^2 - (8)^2$

$W = 48 - 48$

$\boxed{W = 0}$

Q6. $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$ $\vec{r_1} = (2\hat{i} + 3\hat{j})$ $\vec{r_2} = (4\hat{i} + 6\hat{j})$

$$W = \int_2^4 3x^2\,dx + \int_3^6 2y\,dy$$

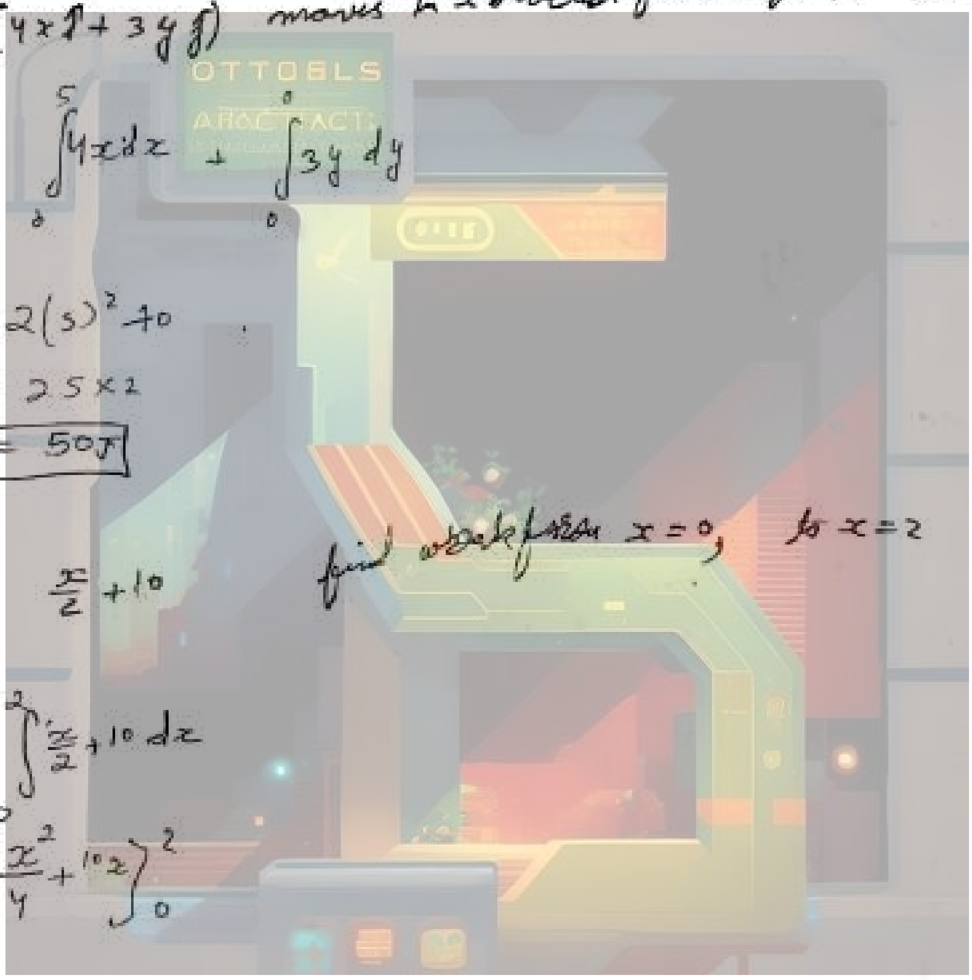
$$= \cancel{(2)^3 - (4)^3}\ (4)^3 - (2)^3 + (6)^2 - (3)^2$$

$$= 64 - 8 + (36 - 9)$$

$$= 56 + 27$$

$$\boxed{= 83\ J}$$

Q7. $\vec{F} = (4x\hat{i} + 3y\hat{j})$ moves in x direction from origin to $x = 5m$.



$$W = \int_0^5 4x\,dx + \int_0^0 3y\,dy$$

$$= 2(5)^2 + 0$$

$$= 25 \times 2$$

$$\boxed{= 50 J}$$

Q8. $F = \frac{x}{2} + 10$ find work from $x = 0$, to $x = 2$

$$W = \int_0^2 \frac{x}{2} + 10\,dx$$

$$= \left[\frac{x^2}{4} + 10x\right]_0^2$$

$$= \frac{4}{4} + 10(2)$$

$$= 20 + 1$$

$$\boxed{= 21 J}$$

Q9. $v = f\sqrt{x}$ (f = constant)

find work done for displacement $x=0$ to $x=d$

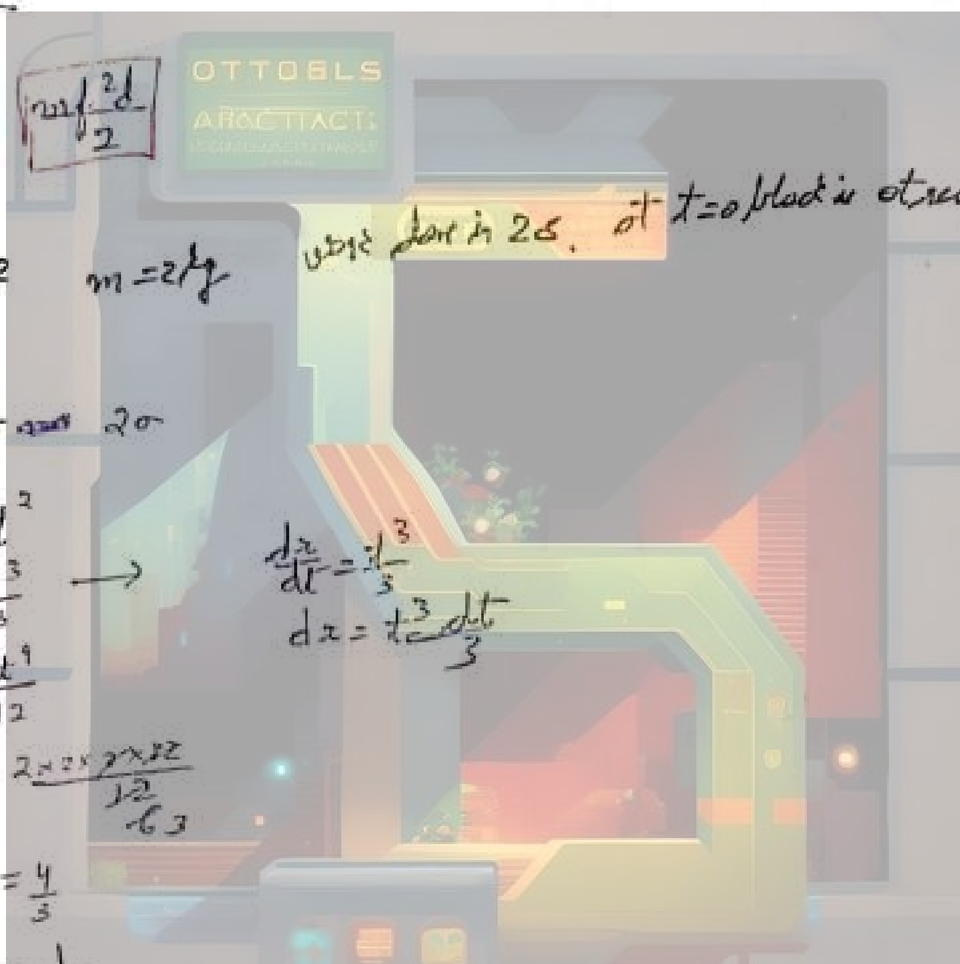
$a = v\frac{dv}{dx}$

$a = f\sqrt{x} \times \frac{f}{2\sqrt{x}}$

$a = \frac{f^2}{2}$

$F = \frac{mf^2}{2}$

Work done = $\boxed{\frac{mf^2 d}{2}}$



Q10. $F = 2t^2$ $m = 2kg$ work done in 2s. at $t=0$ block is at rest

$2t^2 = 2a$

$a = t^2$

$v = \frac{t^3}{3} \longrightarrow \frac{dx}{dt} = \frac{t^3}{3}$

$dx = \frac{t^3}{3}dt$

$x = \frac{t^4}{12}$

$x = \frac{2\times2\times2\times2}{12}$

$= \frac{4}{3}$

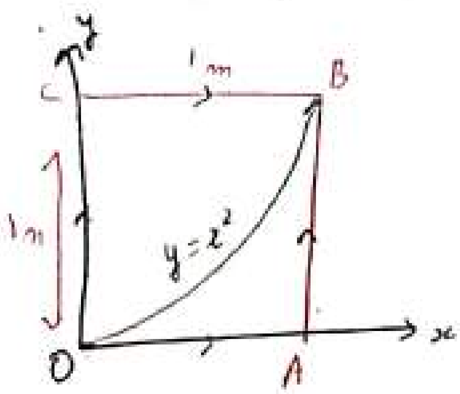
$W = \int_0^2 F \cdot dx$

$W = \int_0^2 F \times \frac{t^3}{3} dt$

$W = \int_0^2 \frac{2}{3} t^5 dt$

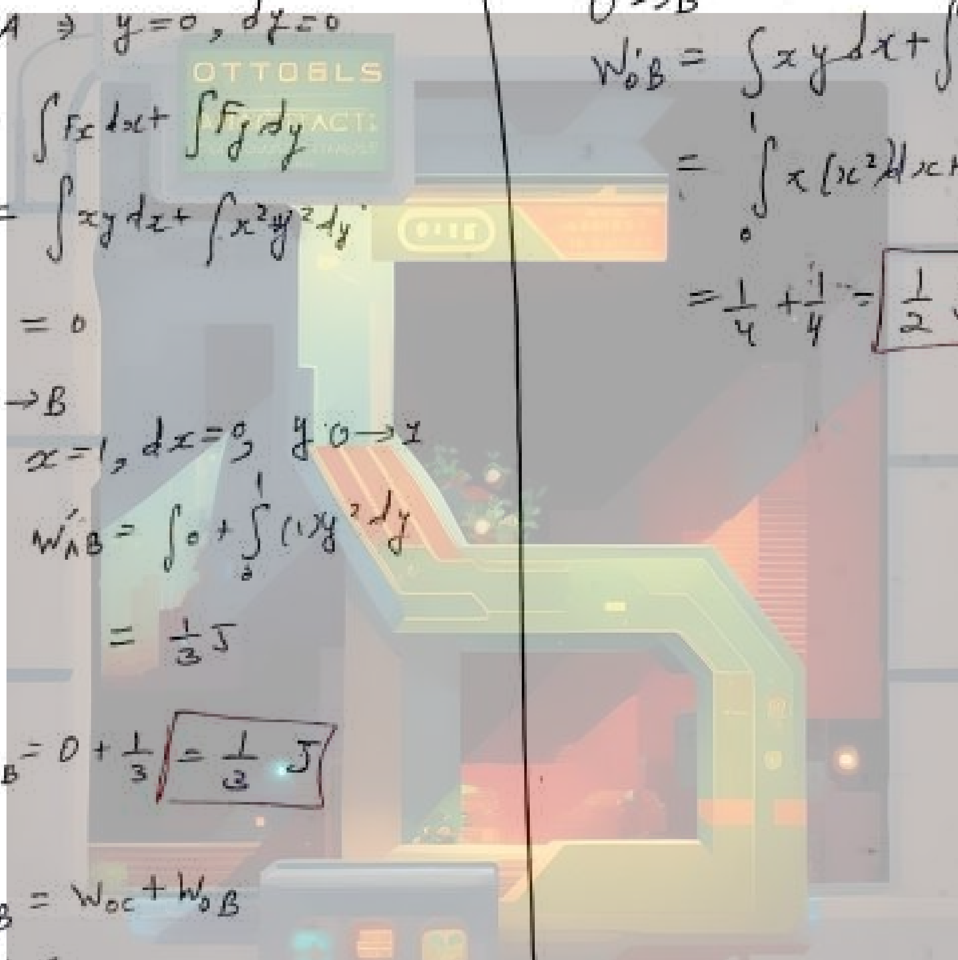
$W = \frac{t^6}{3\times 3} \Big|_0^2 = \left[\frac{t^6}{9}\right]_0^2 = \boxed{\frac{64}{9} J}$

Q11. $F = (xy\,i + x^2y^2\,j)$, work done along 3 paths. differently.



① $W_{OAB} = W_{OA} + W_{AB}$

$O \to A \Rightarrow y = 0,\ dy = 0$



$W_{OA} = \int F_x\,dx + \int F_y\,dy$

$= \int xy\,dx + \int x^2y^2\,dy$

$= 0$

$A \to B$

$x = 1,\ dx = 0,\ y\ 0 \to 1$

$W'_{AB} = \int 0 + \int_0^1 (1)y^2\,dy$

$= \frac{1}{3}\,J$

$W_{OAB} = 0 + \frac{1}{3} \boxed{= \frac{1}{3}\,J}$

② $W_{OCB} = W_{OC} + W_{OB}$

$O \to C$

$x = 0,\ dx = 0$

$W_{OC} = \int 0 + \int 0 = 0$

$C \to B$

$y = 1\ \ dy = 0 \quad x\ 0 \to 1$

$W_{OB} = \int_0^1 x\,(x \cdot 1)\,dx + \int 0$

$= \frac{1}{2}\,J$

$\boxed{W_{OCB} = \frac{1}{2}\,J}$

③ W_{OB}

$O \to B$

$W'_{OB} = \int xy\,dx + \int x^2y^2\,dy$

$= \int_0^1 x(x^2)\,dx + \int_0^1 y(y^2)\,dy$

$= \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}\,J}$

Q12. A particle of mass 0.5 kg travels in a straight line with $v = ax^{3/2}$, $a = 5\ m^{-1/2}/s^{-1}$ what is word done from $x=0$ to $x=2$.

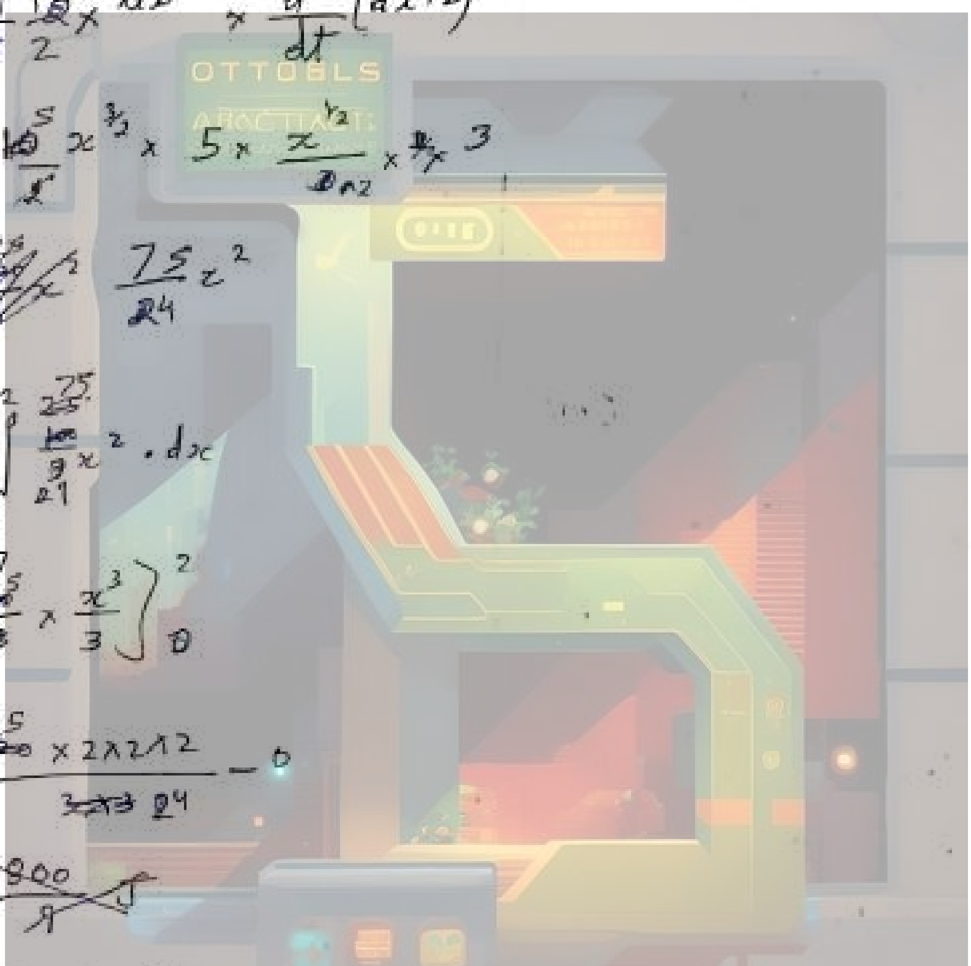
$$W = \int_0^2 F \cdot dx$$

$$F = ma$$

$$F = \cancel{0}\ \frac{a}{2}$$

$$F = \cancel{\frac{dv}{dt}}\ v\frac{dv}{dx} \times \frac{1}{2}$$

$$F = \cancel{\frac{d}{dt}}\ \frac{1}{2} \times ax^{3/2} \times \frac{d}{dt}(ax^{3/2})$$



$$F = \frac{\cancel{10}^{5}}{\cancel{2}} x^{3/2} \times 5 \times \frac{x^{1/2}}{\cancel{2} \times 2} \times \cancel{2} \times 3$$

$$F = \cancel{\frac{25}{3}x^2}\ \frac{75}{24}x^2$$

$$W = \int_0^2 \frac{75}{24} x^2 \cdot dx$$

$$= \left[\frac{75}{24} \times \frac{x^3}{3}\right]_0^2$$

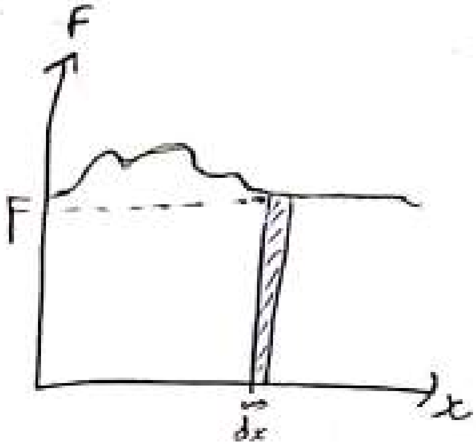
$$= \frac{25 \times 2 \times 2 \times 2}{3 \times 24} - 0$$

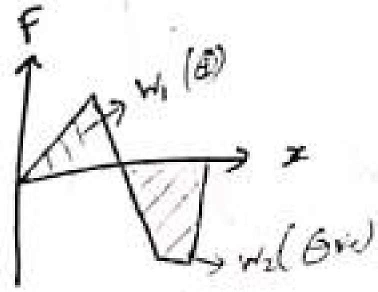
$$= \cancel{\frac{800}{9}\,J}$$

$$= \cancel{\frac{200}{9}\,J}$$

$$\boxed{= 50\,J}$$

Area under Force-Displacement graph.

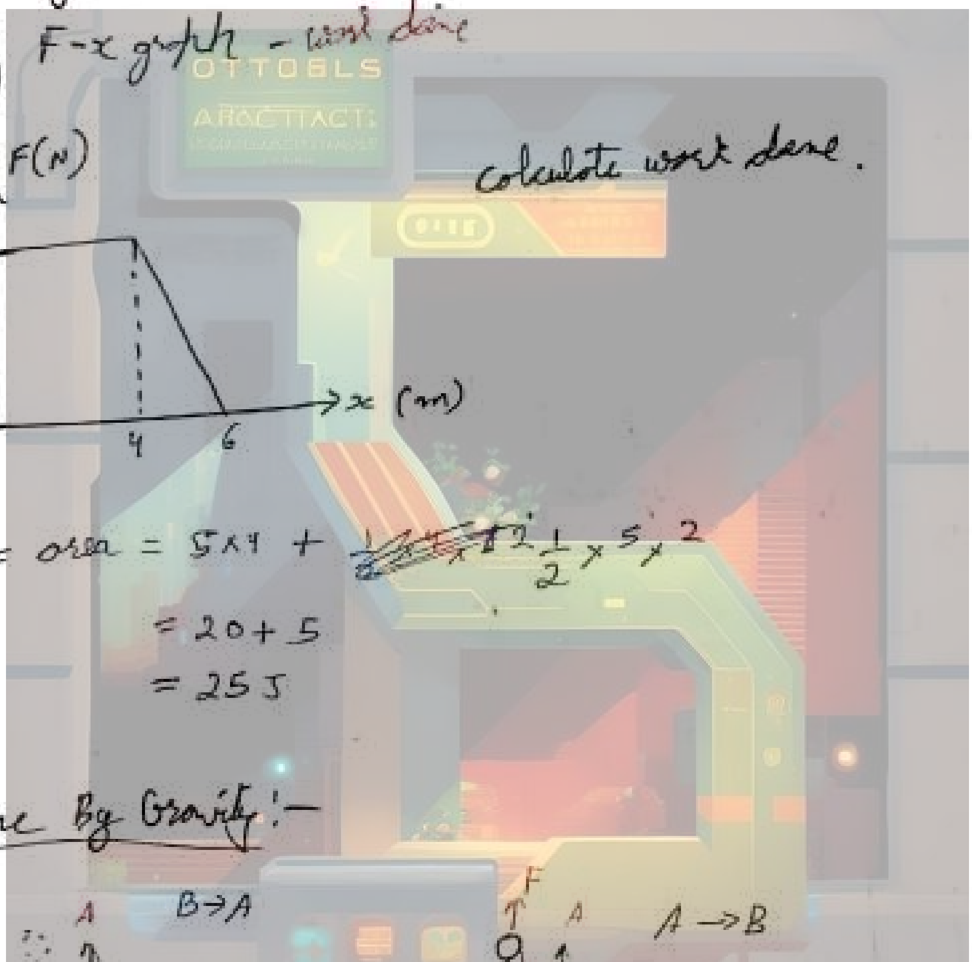




$F = W_1 - W_2$

$W = \int F\, dx$

Area of F-x graph = work done



Q13.

calculate work done.

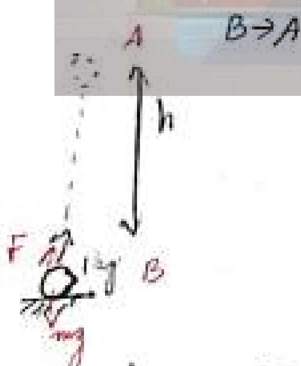
$W = \text{area} = 5 \times 4 + \cancel{\tfrac{1}{2} \times 2} \; \tfrac{1}{2} \times 5 \times 2$

$= 20 + 5$

$= 25\,J$

Work Done By Gravity :-

B→A



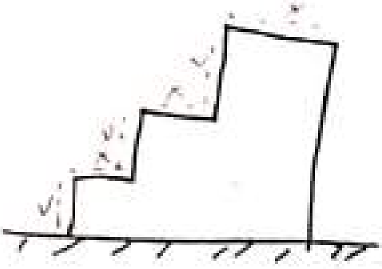
A→B

$W_F = mgh = 30\,J$

$W_{mg} = -mgh$

$W_F = -mgh = -30\,J$

$W_{mg} = mgh$



$W_F = mgh = 30\,J$

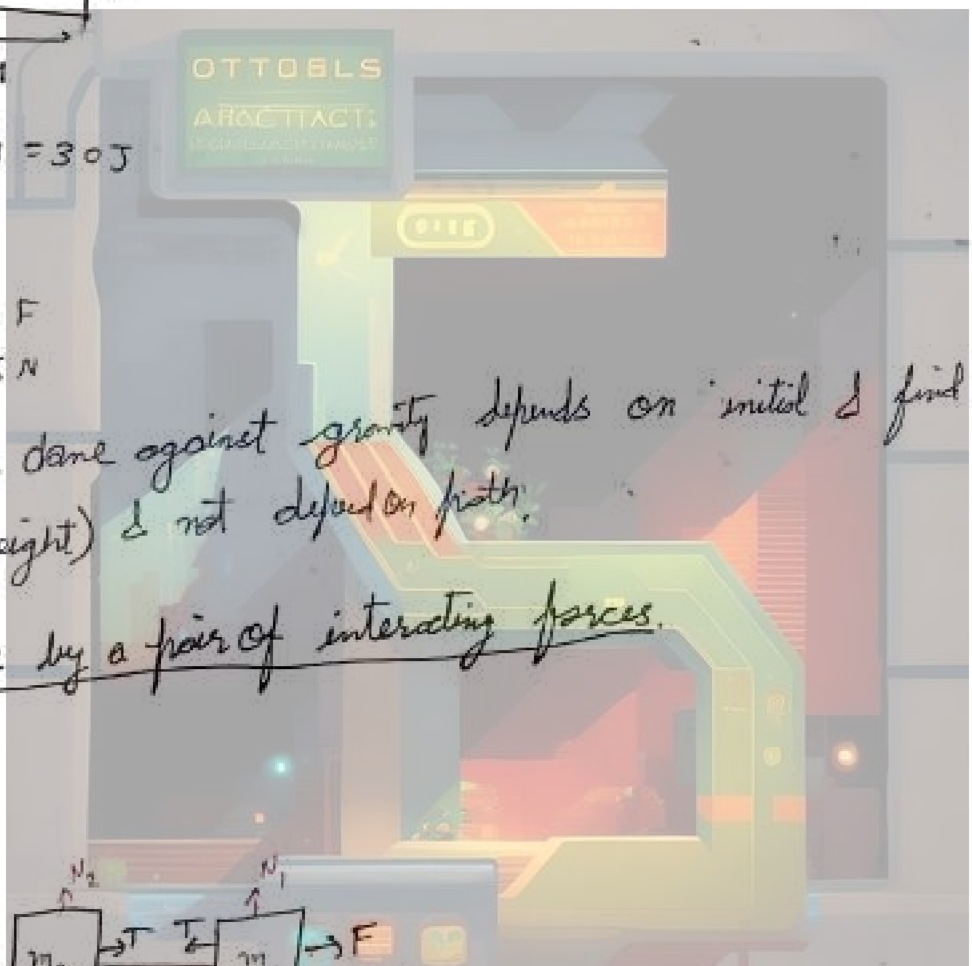
$W = mgh = 30\,J$

$\cancel{F}$

$30 = 5 \times F$

$F = 6\,N$

Note :- Work done against gravity depends on initial & final positions (vertical height) & not depend on path.



Work done by a pair of interacting forces.

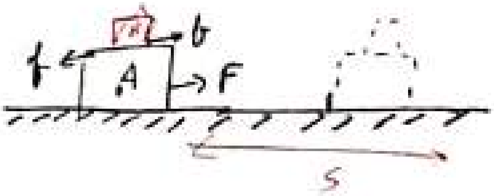
$W_{T(m_2)} = +TS$

$W_{T(m_1)} = -TS$

$W_{T(net)} = 0$

Note :- If there is no relative motion between 2 bodies, then work done by interacting forces is zero as a whole.

Static Friction



Both move together

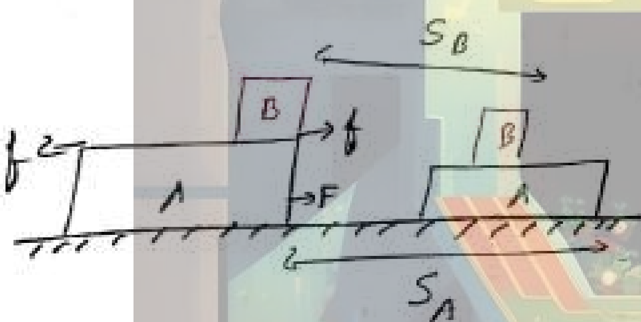
$(W_f)_B = +fS$

$(W_f)_A = -fS$

$W_f = 0$

work done by static friction on individual bodies can be ⊕ve & ⊖ve but Total work done by friction is 0.

Kinetic friction.



$W_{f(A)} = -fS_A$

$W_{f(B)} = fS_B$

$W_f = fS_B - fS_A$

$\because S_A > S_B$

$W_f = -ve$

Kinetic Friction does +ve work on one body & more −ve work on another body. So, total work done by kinetic friction is always −ve.

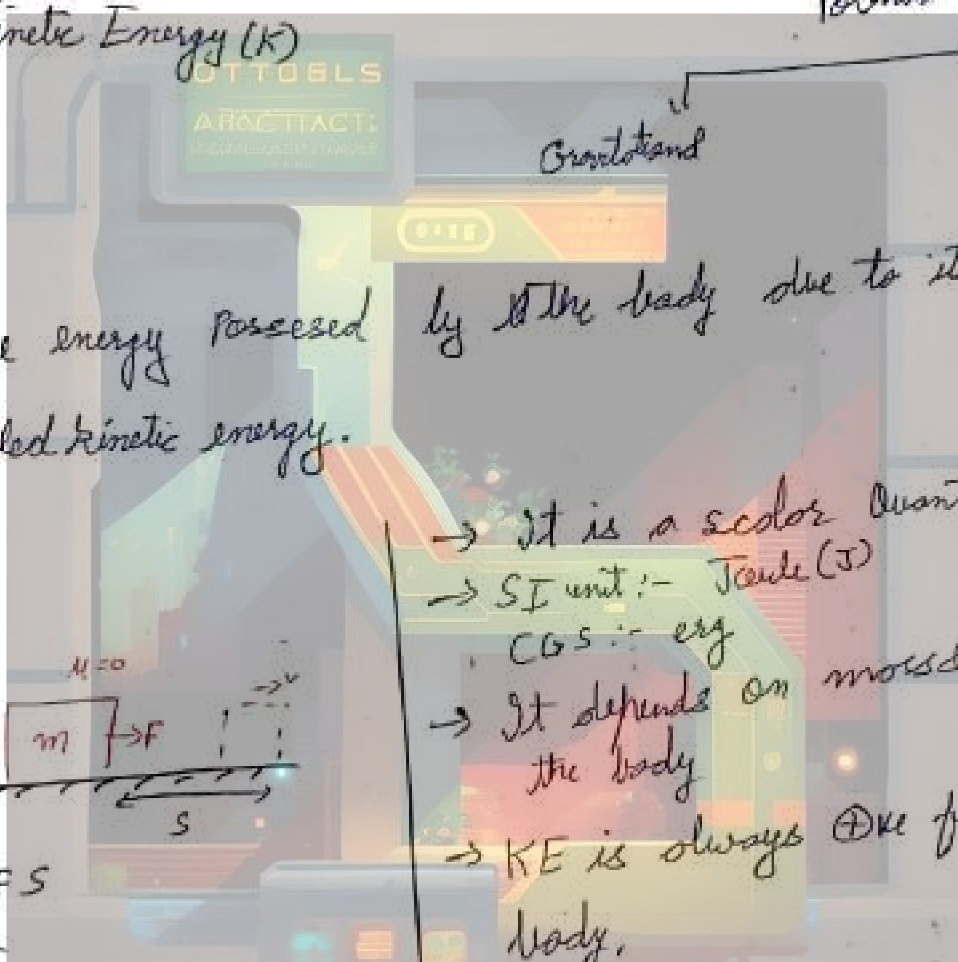
Note:- Total work done by friction ~~do~~ Do Not depend on choice of reference frame.

Energy → Ability to Do work.

Kinetic Energy :- (K) –

Energy
↓
Ability to Do work.
↓
Mechanical Energy

- Kinetic Energy (K)
- Potential Energy (U)
  - Gravitational
  - Elastic



→ The energy possessed by the body due to its motion is called kinetic energy.

→ It is a scalar Quantity

→ SI unit :- Joule (J)

CGS :- erg

→ It depends on mass & speed of the body

→ KE is always ⊕ve for a moving body.

$u=0$

$W = FS$

$\because F = ma$

$W = maS$

$\because v^2 = 2aS + 0$

$2aS = \frac{v^2}{2}$

$\boxed{W = \frac{1}{2} m v^2}$

~~Relation btw KE. & Momentum~~

Relation btw KE. & Momentum.

$K = \frac{1}{2}mv^2$; $p = mv$

$K = \frac{1}{2m}(mv)^2$

$\boxed{K = \frac{p^2}{2m}}$ RR

① Same Momentum

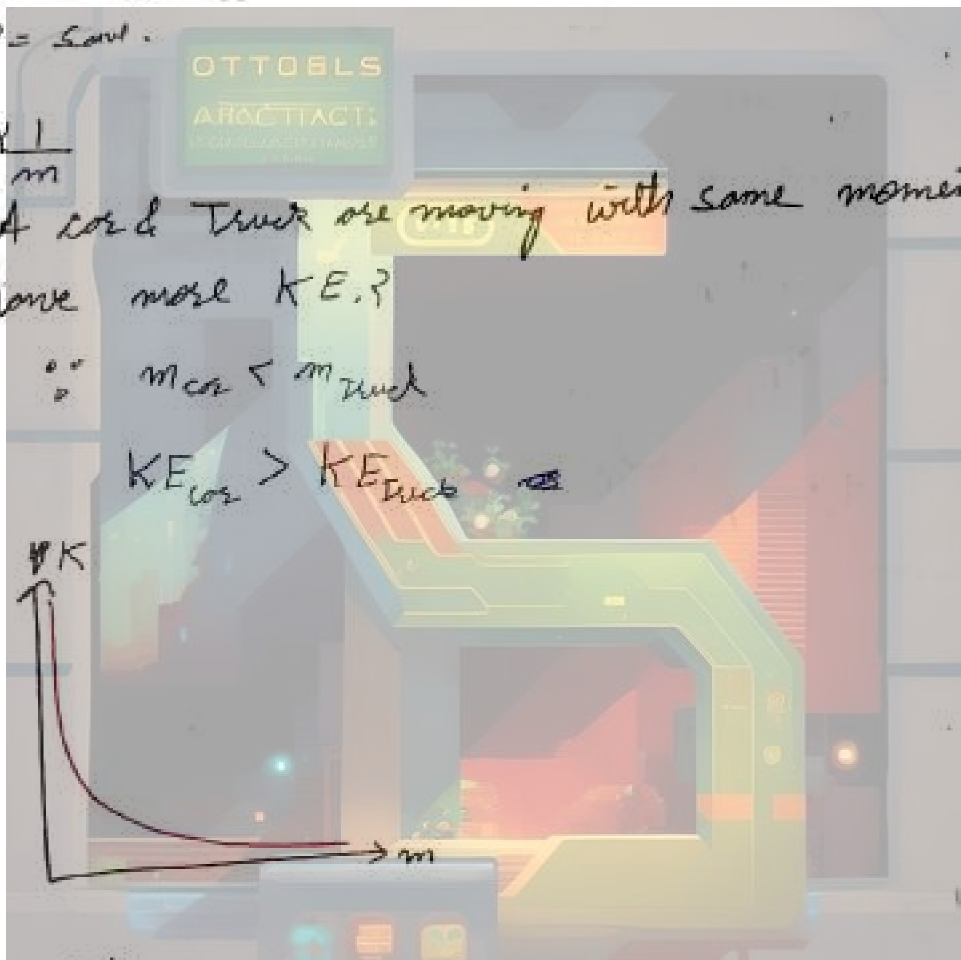
p = Same.

$K \propto \frac{1}{m}$

eg. A car & Truck are moving with same momentum which have more KE.?

$\because$ $m_{car} < m_{truck}$

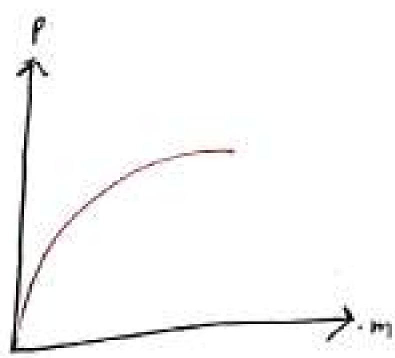
$KE_{car} > KE_{truck}$



② Same Kinetic energy

K = Same.

$p \propto \sqrt{m}$



eg. A car & Truck is moving with same KE, which has more momentum

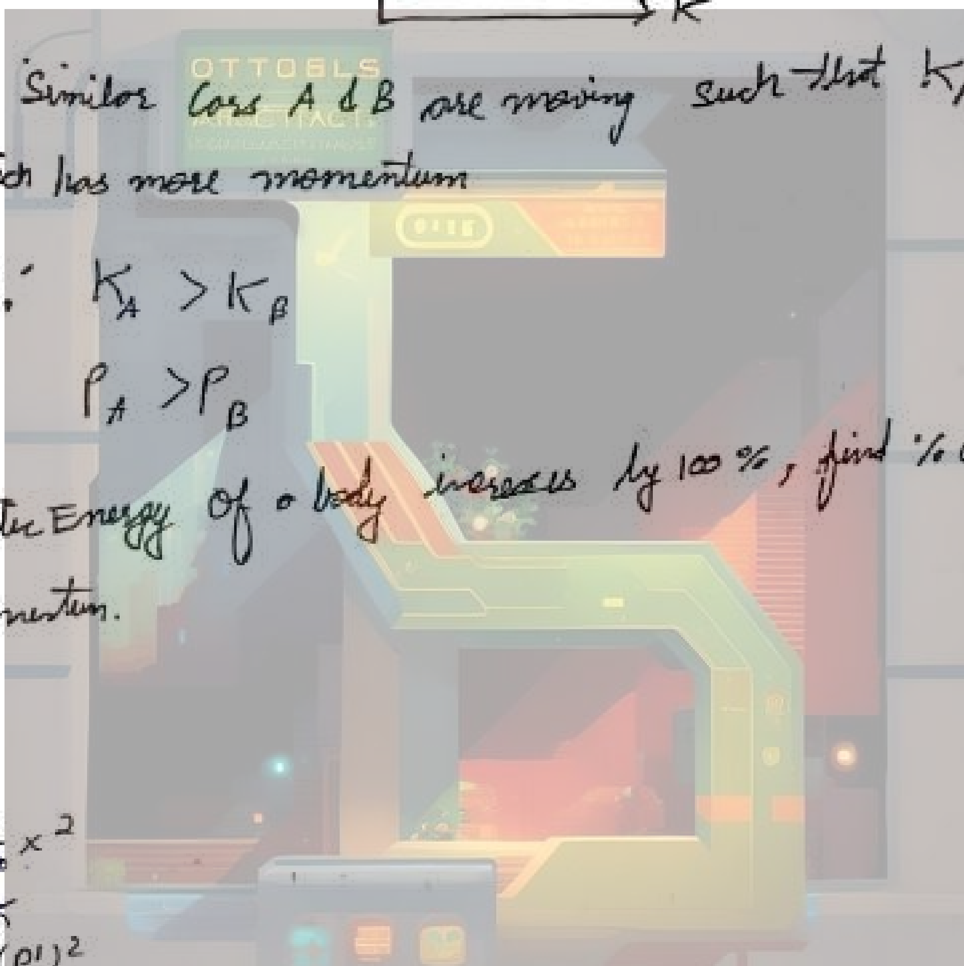
$\because m_{Truck} > m_{car}$

$\Rightarrow P_{Truck} > P_{car}$

(3) $m = \text{Same}$

$P^2 \propto K$

$P \propto \sqrt{K}$



eg Two Similar Cars A & B are moving such that $K_A > K_B$. Which has more momentum

$\because K_A > K_B$

$P_A > P_B$

Q14. If Kinetic Energy of a body increases by 100%, find % change in its momentum.

$K = \frac{P^2}{2m}$

$K' = \frac{K}{\cancel{100}} \times 2$

$= 2K$

$K' = \frac{(P')^2}{2m}$

$2K = \frac{(P')^2}{2m}$

$P' = \sqrt{2Km}$

$P = \sqrt{Km}$

$P' - P = \sqrt{2Km} - \sqrt{Km}$

$= (\sqrt{2}-1)\sqrt{Km}$

$\%\text{change} = \frac{(\sqrt{2}-1)\sqrt{Km}}{\sqrt{Km}} \times 100$

$= (\sqrt{2}-1) \times 100$

$\boxed{= 41.4\%}$

Q15. If Momentum of a body increases by 50%. keeping mass constant, find % change in ~~KE~~ KE.

$$KE = \frac{p^2}{2m}$$

$$KE' = \frac{\left(\frac{3p}{2}\right)^2}{2m}$$

$$KE' = \frac{9p^2}{8m}$$

$$\% \text{ Change} = \frac{\frac{9p^2}{8m} - \frac{4p^2}{8m}}{\frac{4p^2}{8m}} \times 100$$

$$= \frac{5}{4} \times 100$$

$$\boxed{= 125\%}$$

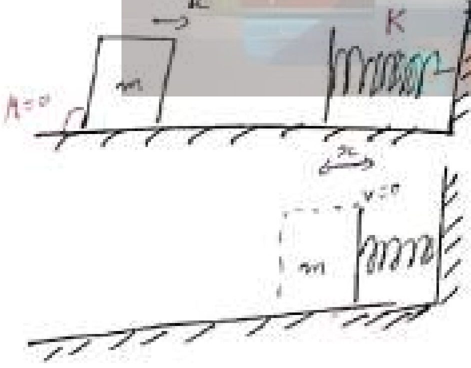
Work Kinetic Energy Theorem

$W = \Delta K$ [change in Kinetic energy]

$W = K_f - K_i$

$$\boxed{K_i \pm W = K_f}$$

eg. Find maximum compression of spring.

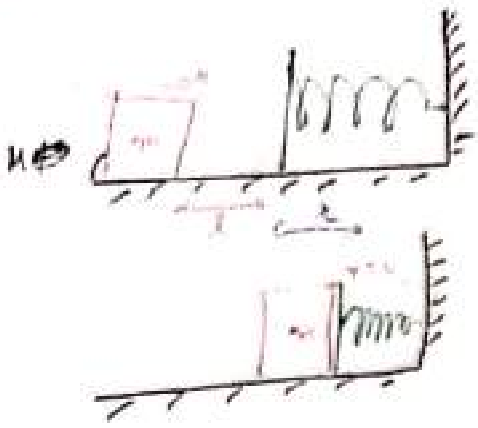


$$\frac{1}{2}mu^2 - \frac{1}{2}Kx^2 = \frac{1}{2}m(0)^2$$

$$mu^2 = Kx^2$$

$$\boxed{x = u\sqrt{\frac{m}{K}}}$$

Eg 2



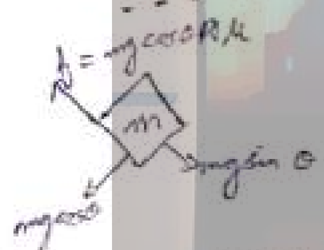
$\frac{1}{2}mu^2 - \mu mg(l+x) - \frac{1}{2}Kx^2 = \frac{1}{2}m(0)^2$

$\boxed{\frac{1}{2}mu^2 - \mu mg(l+x) - \frac{1}{2}Kx^2 = 0}$

Q16. write equation for max compression in the spring & distance by which block rebound up the inclined plane.

$\cancel{0 - mg\sin\theta}$

$0 - mg\sin\theta\, d - \mu mg\cos\theta\, d + \frac{1}{2}kx^2 = 0$

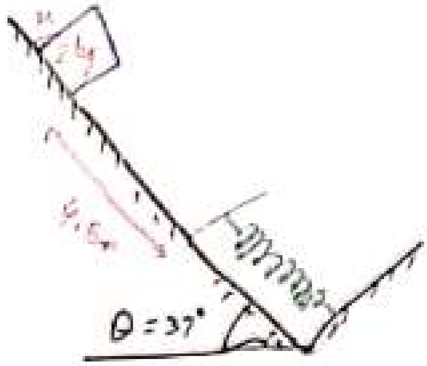


$W_f = -\mu mg\cos\theta(l+x)$

$W_g = +mg\sin\theta(l+x)$

$\underset{K_i}{0} + \underset{W_g}{mg\sin\theta(l+x)} - \underset{W_f}{\mu mg\cos\theta(l+x)} - \underbrace{\frac{1}{2}Kx^2}_{W_{spring}} = \underset{K_f}{0}$

Q17.



$x = 20\,cm = 0.2\,m = \frac{1}{5}\,m$

released = 1 m

a) find coefficient of friction b/w block & plane

b) find spring constant.

$g = 10\,m/s^2$

$0 - 12(1) - (16\mu)(1) + \frac{1}{2} \times K \times \frac{1}{25} = 0$

$\boxed{\frac{K}{50} = 16\mu + 12}$ — ①

$0 + 12(5) - 16\mu(5) - \frac{1}{2} \times K \times \frac{1}{25} = 0$

$\boxed{\frac{K}{50} = 60 - 80\mu}$ — ②

~~$80\mu - 60 = 16\mu + 12$~~

~~$54\mu = 72$~~

~~$\mu = \frac{72}{54}$~~

~~$\mu = \frac{4}{3}$~~

~~$K = \left[16 \times \frac{72}{54} + 12\right] \times 50$~~

~~$K = \frac{64 + 36}{3} \times 50$~~

~~$K = \frac{5000}{3}$~~

$60 - 80\mu = 16\mu + 12$

$48 = 96\mu$

$\mu = \frac{1}{2}$

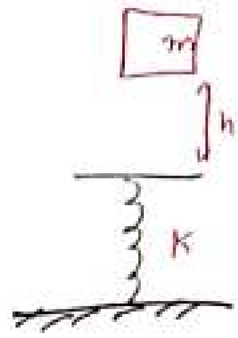
$\boxed{\mu = 0.5}$

$K = (8 + 12)\,50$

$= 20 \times 50$

$\boxed{= 1000}$ N/m

Q. find max compression. (write eqn)



$$0 + mg(h+x) - \frac{1}{2}kx^2 = 0$$

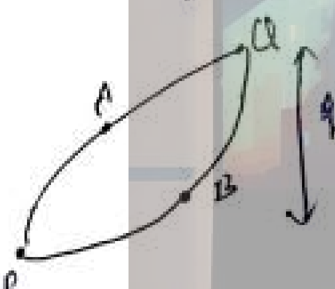
$$\boxed{\frac{1}{2}kx^2 = mg(h+x)}$$

Conservative & non conservative Forces

Conservative → The work done by conservative force on a particle moving between any two points is independent of the path taken by particle.

→ The work done by conservative force on a particle moving through any closed loop is zero.

eg. spring force, gravitational force etc.

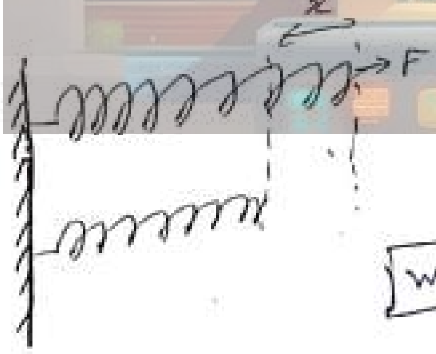


$W_{PAQ} = -mgh$

$W_{QBP} = mgh$

$\boxed{W_T = 0}$

eg 2.



$W_{S1} = -ve$

$W_{SP} = +ve$

$\boxed{W_T = 0}$

Non Conservative - The work done by non-conservative force is not only depends on initial and final positions but also on the path followed.

→ The work done by non conservative force on a particle moving in a closed loop is not zero.

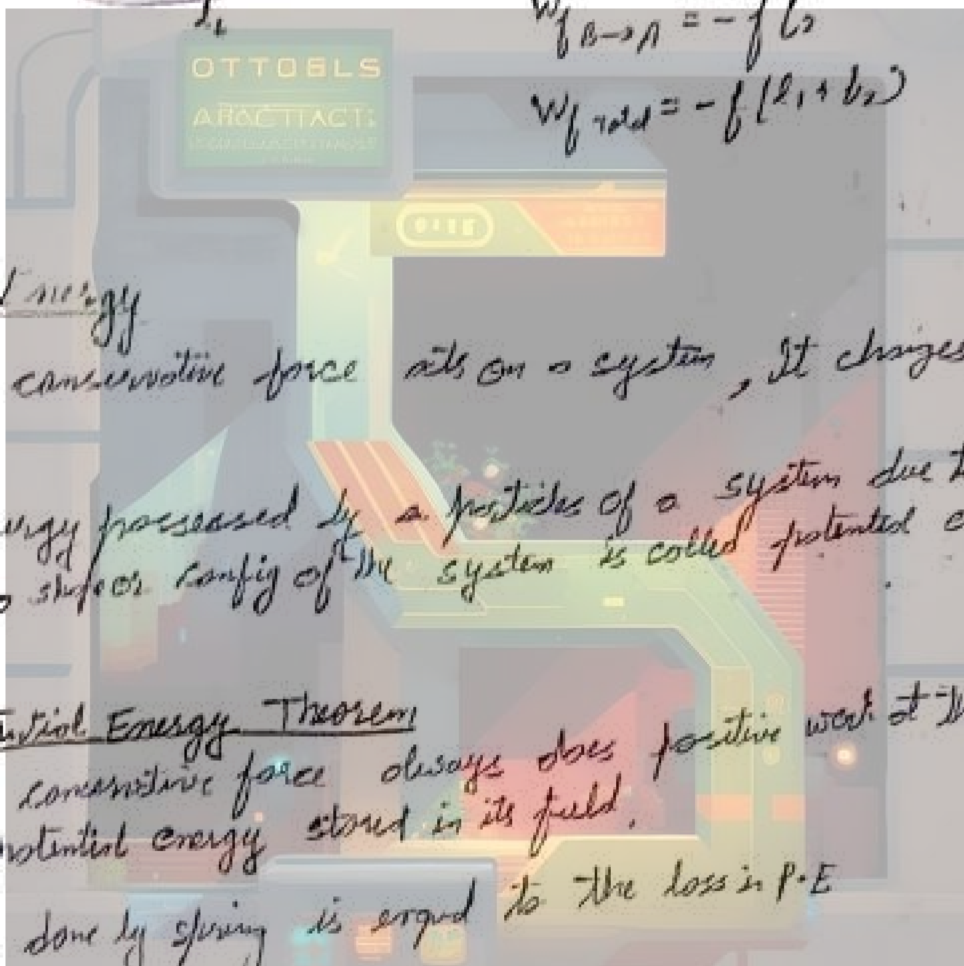
eg. friction force

$W_{friction}$

$W_{f\,A \to B} = -f l_1$

$W_{f\,B \to A} = -f l_2$

$W_{f\,total} = -f(l_1 + l_2)$



Potential Energy

→ When a conservative force acts on a system, It changes energy of system.

→ The energy possessed by a particles of a system due to their position, shape or config of the system is called potential energy.

Work - Potential Energy Theorem

→ The conservative force always does positive work at the expense of its potential energy stored in its field.

eg. Work done by spring is equal to the loss in P.E

$W_{conservative\ force} = -\Delta U \qquad \Delta K + \Delta U = 0$

Gravitational gravitational energy

1. Uniform gravity :-

for $h << R$ (radius of earth)

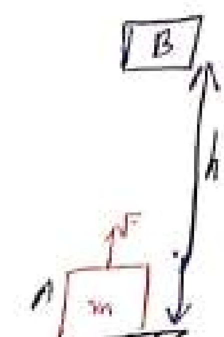
$W_F = + mgh$

$W_g = - mgh$

$-mgh = -\Delta U_g$

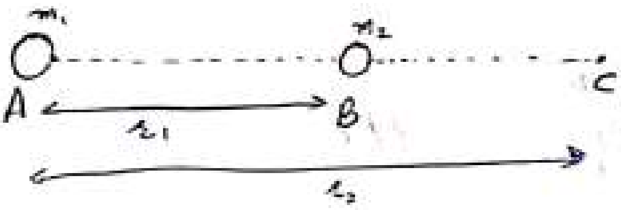
$\Delta U_g = mgh$

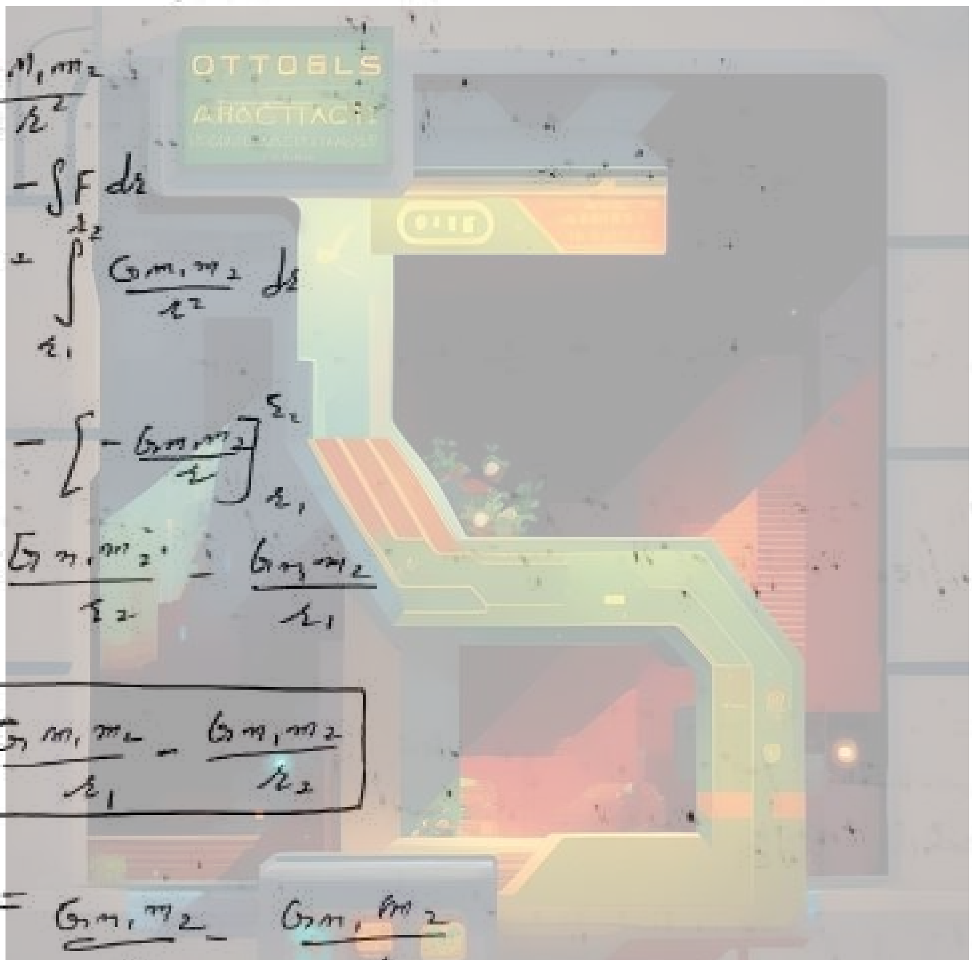
$\Delta U_g = mgh$



→ Can be $\oplus$ve, $\ominus$ve or zero
→ Depends on the choice of reference point/line.
→ Defined for only conservative forces.

2. Non-Uniform Gravity:-





$$F = \frac{Gm_1m_2}{r^2}$$

$$W_g = -\int F\,dr$$

$$= -\int_{r_1}^{r_2} \frac{Gm_1m_2}{r^2}\,dr$$

$$= -\left[-\frac{Gm_1m_2}{r}\right]_{r_1}^{r_2}$$

$$W_g = \frac{Gm_1m_2}{r_2} - \frac{Gm_1m_2}{r_1}$$

$$\boxed{\Delta U_g = \frac{Gm_1m_2}{r_1} - \frac{Gm_1m_2}{r_2}}$$

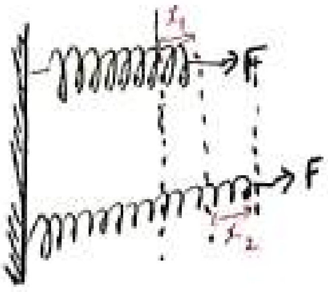
$$U_2 - U_1 = \frac{Gm_1m_2}{r_1} - \frac{Gm_1m_2}{r_2}$$

$r_2 \to \infty,\ U_2 = 0$

$$-U_1 = \frac{Gm_1m_2}{r_1} - 0$$

$$\boxed{U_1 = -\frac{Gm_1m_2}{r_1}}$$

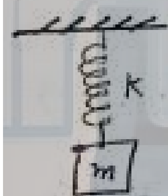
Elastic Potential Energy



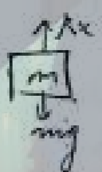
$$\Delta U_{sp} = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2$$

$$W_F = \frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2 \quad (\text{from } x_1 \to x_2)$$

Q.19. Find Maximum ~~length~~ elongation in spring & elongation in equilibrium position. Initially spring is in natural length when block is released.



At equilibrium position



$kx = mg$

$\boxed{x = \frac{mg}{k}}$ equilibrium position

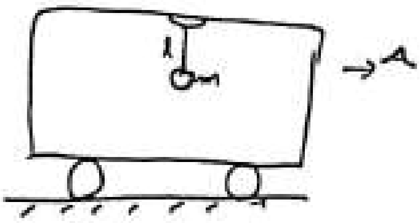
$K_i + W = K_f$

$0 + mg x_{max} - \frac{1}{2} k x_{max}^2 = 0$

$mg = \frac{1}{2} k x_{max}$

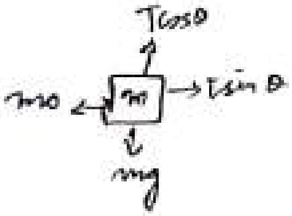
$$\boxed{x_{max} = \frac{2mg}{k}}$$

Q 20.



Find θ deflection θ from vertical
i) in equilibrium. ii) max deflection

i)



$T \sin\theta = ma$

$T = \frac{ma}{\sin\theta}$

$T\cos\theta = mg$

$\frac{ma}{\sin\theta}\cos\theta = mg$

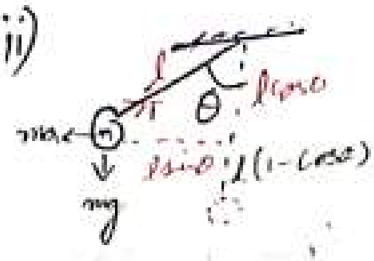
$\cot\theta = \cancel{mg}\ \frac{g}{a}$

$\boxed{\theta = \cot^{-1}\left(\frac{g}{a}\right)}$

Equilibrium.

$\theta = \tan^{-1}\left(\frac{a}{g}\right)$

ii)



$K_i \pm W = K_f$

$0 \underbrace{- mgl(1-\cos\theta)}_{W_g} + \underbrace{mal\sin\theta}_{W_{ma}} + \underbrace{0}_{W_T} = 0$

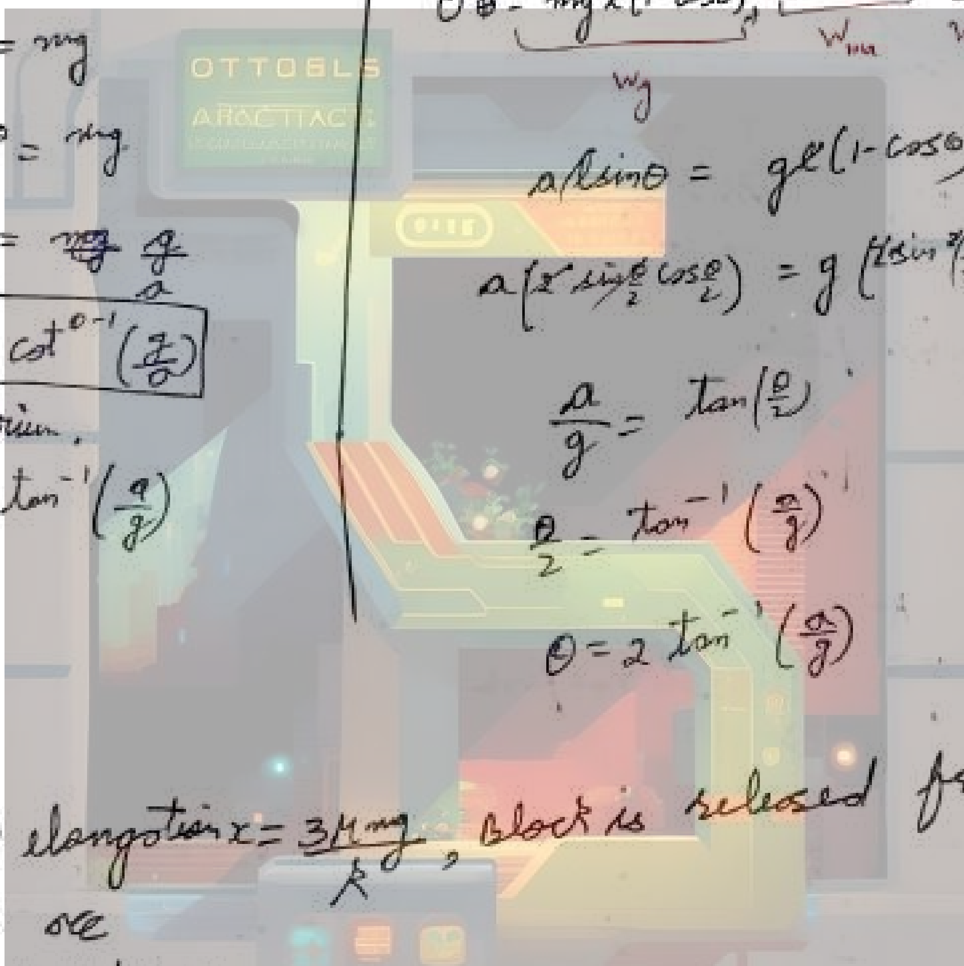
$al\sin\theta = gl(1-\cos\theta)$

$a\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = g\left(2\sin^2\frac{\theta}{2}\right)$

$\frac{a}{g} = \tan\left(\frac{\theta}{2}\right)$

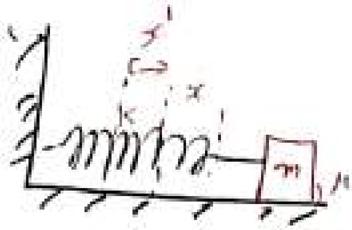
$\frac{\theta}{2} = \tan^{-1}\left(\frac{a}{g}\right)$

$\theta = 2\tan^{-1}\left(\frac{a}{g}\right)$



Q 21. Initial elongation $x = \frac{3\mu mg}{k}$, Block is released from rest.

i) initial acc
ii) max compression
iii) max speed.



i) $K_x \leftarrow \boxed{m} \rightarrow f$, N, mg

$f_r = \mu mg$

$K_x - f = ma$

$K_x \dfrac{3\mu mg}{K} - \mu mg = ma$

$\boxed{2\mu g = a}$ ✓

ii) $K_i \pm W = K_f$

$0 + \frac{1}{2}Kx^2 - \cancel{\mu mg}\,\frac{1}{2}K(x')^2 - \mu mg(x + x') = 0$

$\frac{1}{2}Kx^2 = \frac{1}{2}K(x')^2 + \mu mg(x + x')$

$\frac{1}{2}K(x^2 - (x')^2) = \mu mg\,[x + x']$

$\frac{1}{2}K\cancel{(x + x')}(x - x') = \mu mg\,\cancel{(x + x')}$

$\frac{1}{2}K(x - x') = \mu mg$

$x - x' = \dfrac{2\mu mg}{K}$

$x' = x - \dfrac{2\mu mg}{K}$

$\boxed{x' = \dfrac{\mu mg}{K}}$

~~$x + x' = \frac{4\mu mg}{2K}$~~

iii) ~~$0 + \frac{1}{2}Kx^2 - \mu mg x = \frac{1}{2}mv^2$~~

~~$\frac{1}{2}K \times \frac{3\mu mg}{K} - \mu mg \times \frac{3\mu mg}{K} = \frac{1}{2}mv^2$~~

~~$\frac{3\mu mg}{2K}$ μmg~~

~~$\frac{1}{2}mv^2 - \frac{1}{2}Kx^2 - \frac{\mu mg\, x}{2}$~~

~~$mv^2 = K\frac{9\mu^2 m^2 g^2}{K^2} - \frac{6(\mu mg)^2}{K}$~~

~~$mv^2 = \frac{3(\mu mg)^2}{K}$~~

~~$v^2 = \frac{3\mu^2 m g^2}{K}$~~

~~$v = \frac{\sqrt{3m}\,\mu g}{\sqrt{K}}$~~

$v = \sqrt{\frac{3m}{K}}\,\mu g$

iii) Fim equation bicdan.

$\cancel{Kx = \mu mg}$

$\cancel{K \times \frac{3\mu mg}{K} = \mu mg}$

$\cancel{3 = 1}$

$Ky = \mu mg$

$y = \frac{\mu mg}{K}$

~~from Block → x - y~~

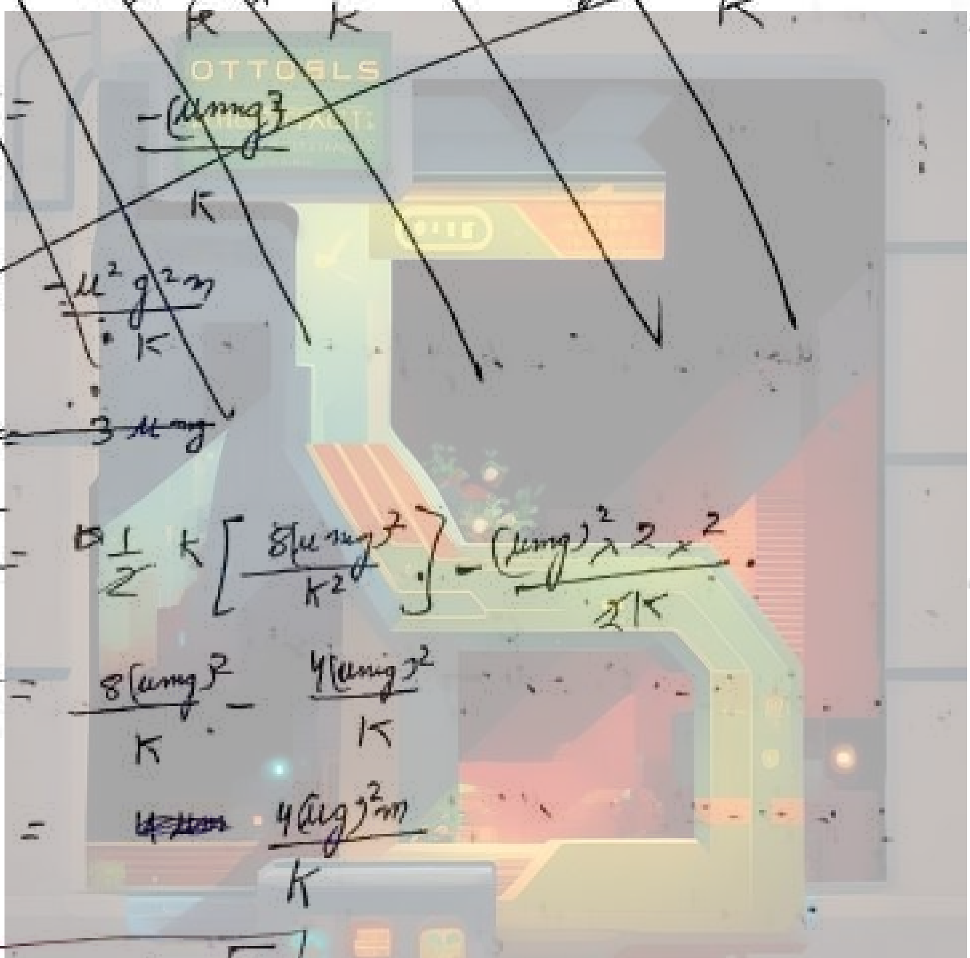
$\cancel{\frac{1}{2} m v^2 = -\frac{1}{2} K y^2 - \mu mg\, y + \frac{1}{2} k x^2}$

$\cancel{m v^2 = -K \frac{\mu mg}{K} \times \frac{\mu mg}{K} - 2\mu mg \frac{\mu mg}{K}}$

$\cancel{m v^2 = \frac{-(\mu mg)^2}{K}}$

$\cancel{v^2 = \frac{-\mu^2 g^2 m}{K}}$

$\cancel{m v^2 = 3\mu mg}$



$\frac{m v^2}{2} = \frac{1}{2} K \left[\frac{8(\mu mg)^2}{K^2}\right] - \frac{(\mu mg)^2 \times 2 \times 2}{2K}$

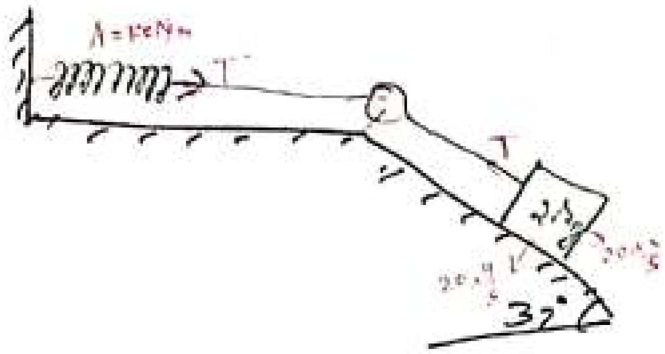
$m v^2 = \frac{8(\mu mg)^2}{K} - \frac{4(\mu mg)^2}{K}$

$v^2 = \frac{4(\mu g)^2 m}{K}$

$\boxed{V = 2\mu g\sqrt{\frac{m}{K}}}$

Q. Block is released from rest when spring in unstretched.

a) How far does the block move down before coming momentarily to rest. What is its ac at lowest point.



$\cancel{T = kx}$

$\cancel{T = 100x}$

$\cancel{12 - 100x = 2a}$

$\cancel{6 - 50x = a}$

$K_i \pm W = K_f$

$0 + 12x - \frac{1}{2} \times 100 \times x^2 = 0$

$12 - 50x = 0$

$12 = 50x$

$x = \frac{12}{50}$

$x = \frac{6}{25}$

$\boxed{x = 0.24}$

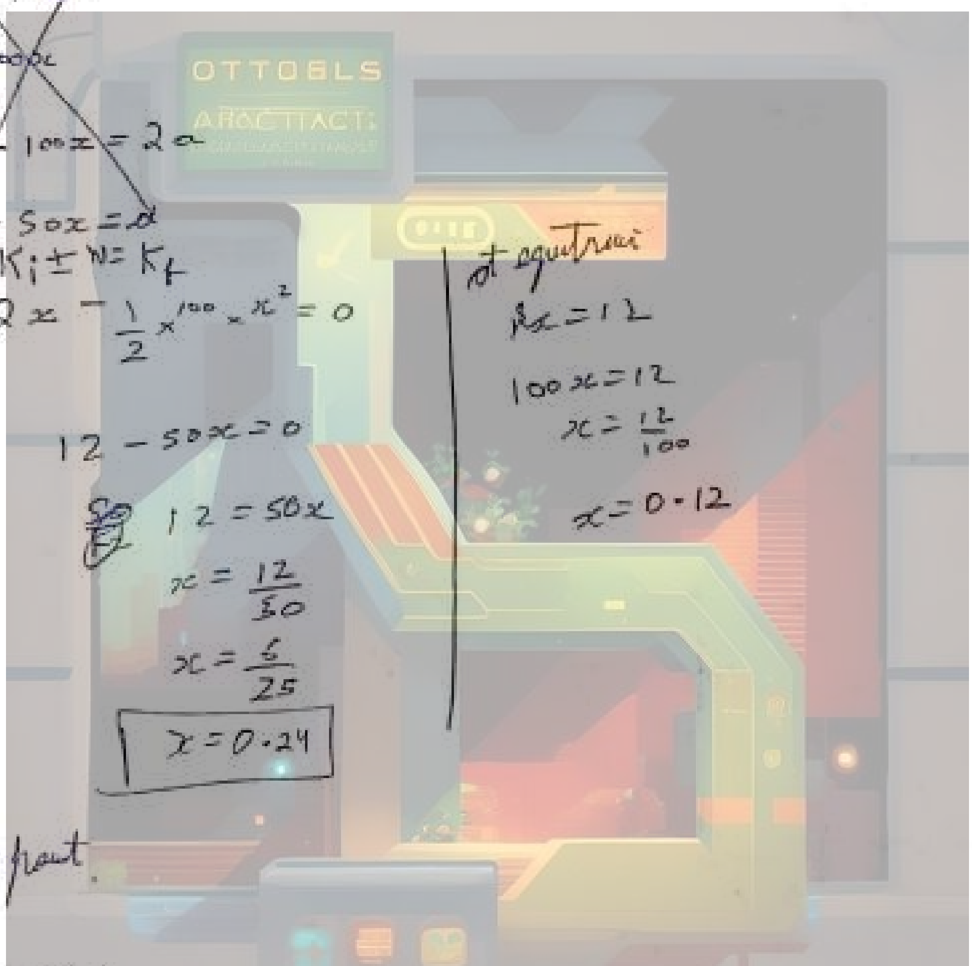
at equilibrium

$kx = 12$

$100x = 12$

$x = \frac{12}{100}$

$x = 0.12$



at lowest point

$kx - 12 = 2a$

$24 - 12 = 2a$

$12 = 2a$

$\boxed{a = 6 \, m/s^2}$

b) If surface is rough and block moves 0.2 m calculate coefficient of friction

$$0 + 12(0.2) - \frac{1}{2}(100)(0.04) - \mu(16)(0.2) = 0$$

$$2.4 - 2 - 3.2\mu = 0$$

$$3.2\mu = 2.4 - 2$$

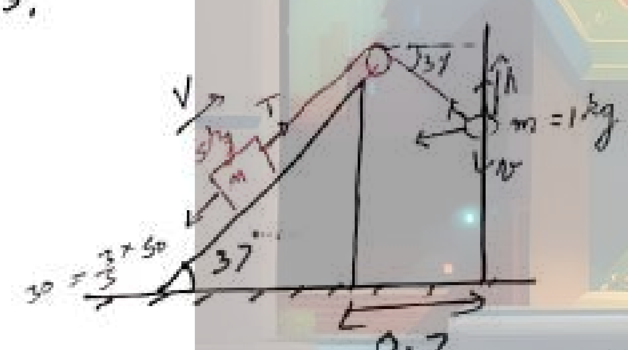
$$\mu = \frac{24 - 20}{32}$$

$$\mu = \frac{4}{32}$$

$$\mu = \frac{1}{8}$$

$$\boxed{\mu = 0.125}$$

Q 23.



find velocity of ring.

$N = \frac{5V}{3}$

$v^2 = \frac{25V^2}{9}$

$\sin 37° = \frac{3}{5} = \frac{h}{7} \times 8$

$h = \frac{3 \times 7}{8 \times 5}$

$h = \frac{21}{40}$

$\cos 37° = \frac{4}{5} = \frac{0.7}{L_2}$

$L_2 = \frac{7}{8}$

$L_2 - L_1 = \frac{7}{8} - \frac{7}{10}$

$= \frac{70 - 56}{80}$

$= \frac{14}{80}$

$K_i + w = K_f$

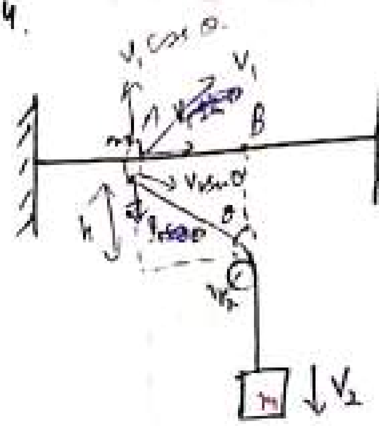
$\frac{21}{40} mg - \frac{7}{40} \sin 37° Mg = \frac{1}{2} mv^2 + \frac{M}{2} V^2$

$\frac{21}{40} \times 1 \times 10 - \frac{7}{40} \times \frac{3}{5} \times 5 \times 10 = \frac{1}{2} \times 1 \times \frac{25V^2}{9} + \frac{5}{2} \times V^2$

$\frac{21}{4} - \frac{21}{4} = \frac{25V^2}{18} + \frac{45V^2}{18}$

$$\boxed{\begin{array}{c} V = 0 \\ v = 0 \end{array}}$$

Q24.



$\theta = 60^\circ$

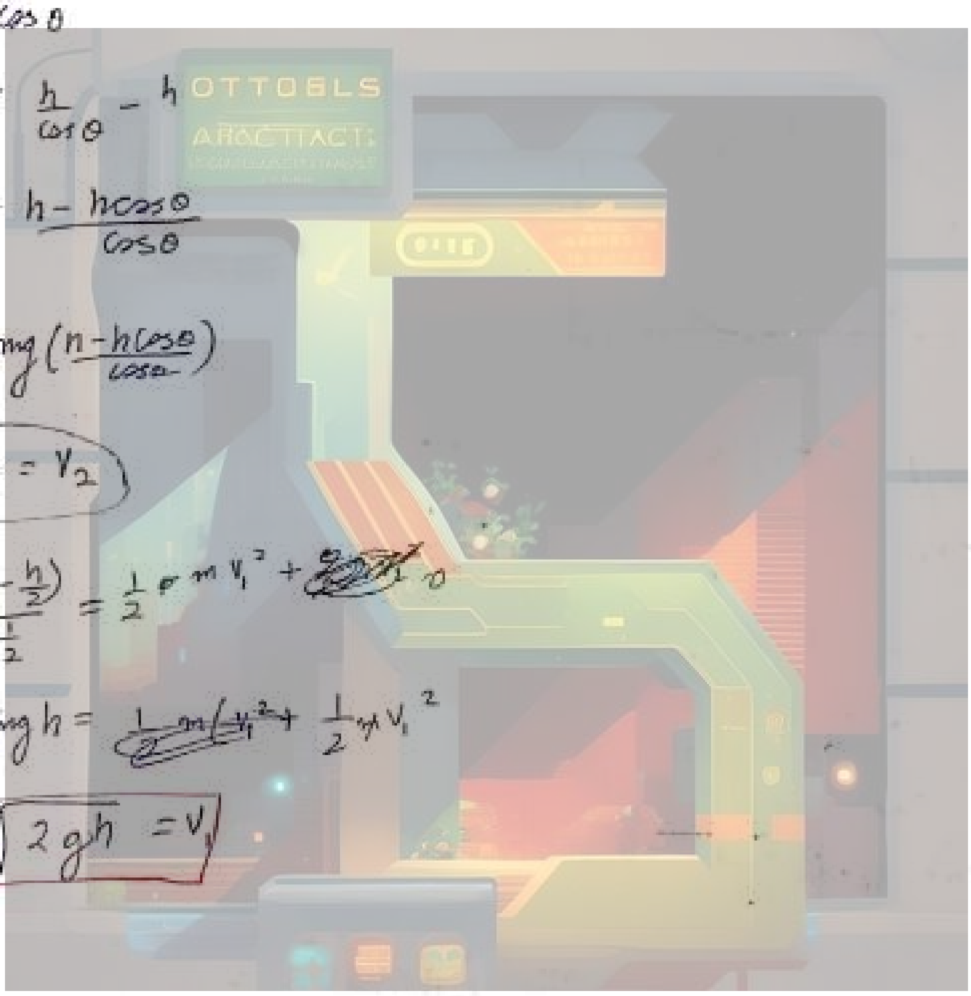
$$\cos\theta = \frac{h}{r}$$

$$r = \frac{h}{\cos\theta}$$

$$r - h = \frac{h}{\cos\theta} - h$$

$$= \frac{h - h\cos\theta}{\cos\theta}$$

$$W_{mg} = mg\left(\frac{h - h\cos\theta}{\cos\theta}\right)$$



$$V_1 \sin\theta = V_2$$

$$mg\left(\frac{h - \frac{h}{2}}{\frac{1}{2}}\right) = \frac{1}{2} m v_1^2 + \cancel{\ \ } 0$$

$$mgh = \cancel{\frac{1}{2} m v_1^2} + \frac{1}{2} m v_1^2$$

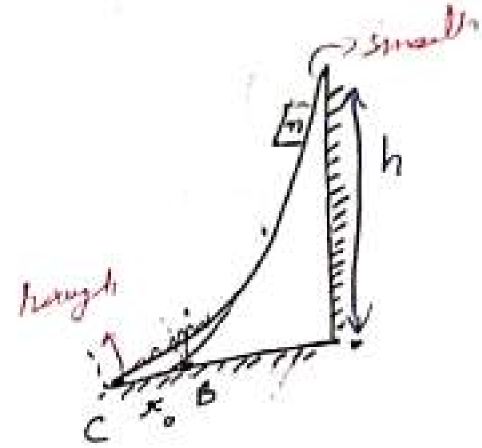
$$\boxed{\sqrt{2gh} = v_1}$$

Q25. find coefficient of friction if block stops btw B → C.

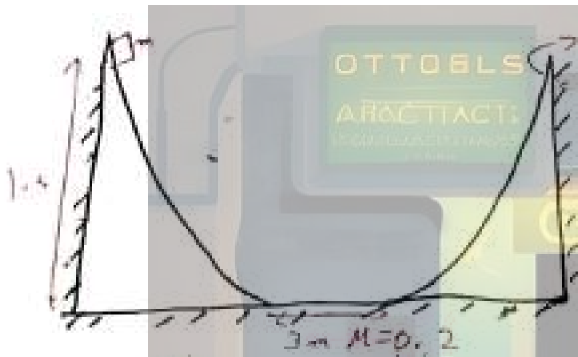
$$\overbrace{mgh}^{\text{gravity}} = \overbrace{\mu mg\,(x_0)}^{\text{friction}}$$

$$\boxed{\mu = \frac{h}{x_0}}$$

→ work done by gravity depends only on height.



Q26.



where does the block come to rest.

$$\overbrace{mgh}^{W_g} = \overbrace{\mu mg(x_0)}^{W_f}$$

x – Total distance travelled by block on rough surface before coming to rest.

$1.5 = 0.2\,(x_0)$

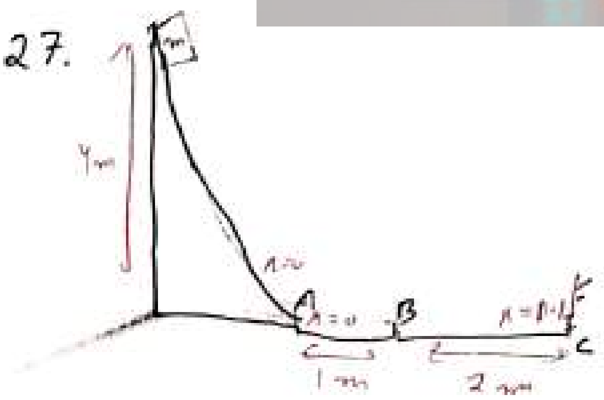
$x_0 = \frac{1.5}{0.2}$

$x_0 = \frac{15}{2}$

$x_0 = 7.5$

so block stops $\boxed{\text{at } 1.5\text{m from B}}$

Q27.



m = 0.1 kg
find total distance covered by block on horizontal surface before coming to rest.
$g = 10\,m/s^2$

$mgh = \mu m g x$

$\frac{4}{0.1} = x$

$x = 40$

~~$2n = 40$~~

$n = 20$ (n = no. of time move through horizontal)

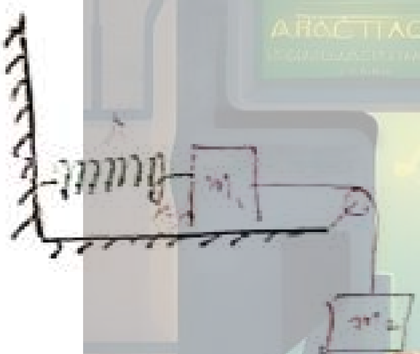
distance $= 60$

stops at B & not A

So distance travelled $= 60 - 1$

$\boxed{= 59\,m}$

Q28.



find μ if m_2 falls h before coming to rest.

$$m_2 g h - \frac{1}{2} k x^2 - \mu m_1 g (x) = 0 \quad (x = h)$$

$$m_2 g h = \frac{1}{2} k h^2 + \mu m_1 g h$$

$$\mu = \frac{m_2 g h - \frac{1}{2} k h^2}{m_1 g h}$$

$$\mu = \frac{10 m_2 - \frac{kh}{2}}{10 m_1}$$

$$\mu = \frac{20 m_2 - kh}{20 m_1}$$

$$\boxed{\mu = \frac{2g m_2 - kh}{2g m_1}}$$

Q spring initial stretched 5 cm, what work to stretch more by 5 cm

$k = 5\times10^{3}\ N/m$

$W = E_f - E_i$

$W = \frac{1}{2}k(0.1)^2 - \frac{1}{2}k(0.05)^2$

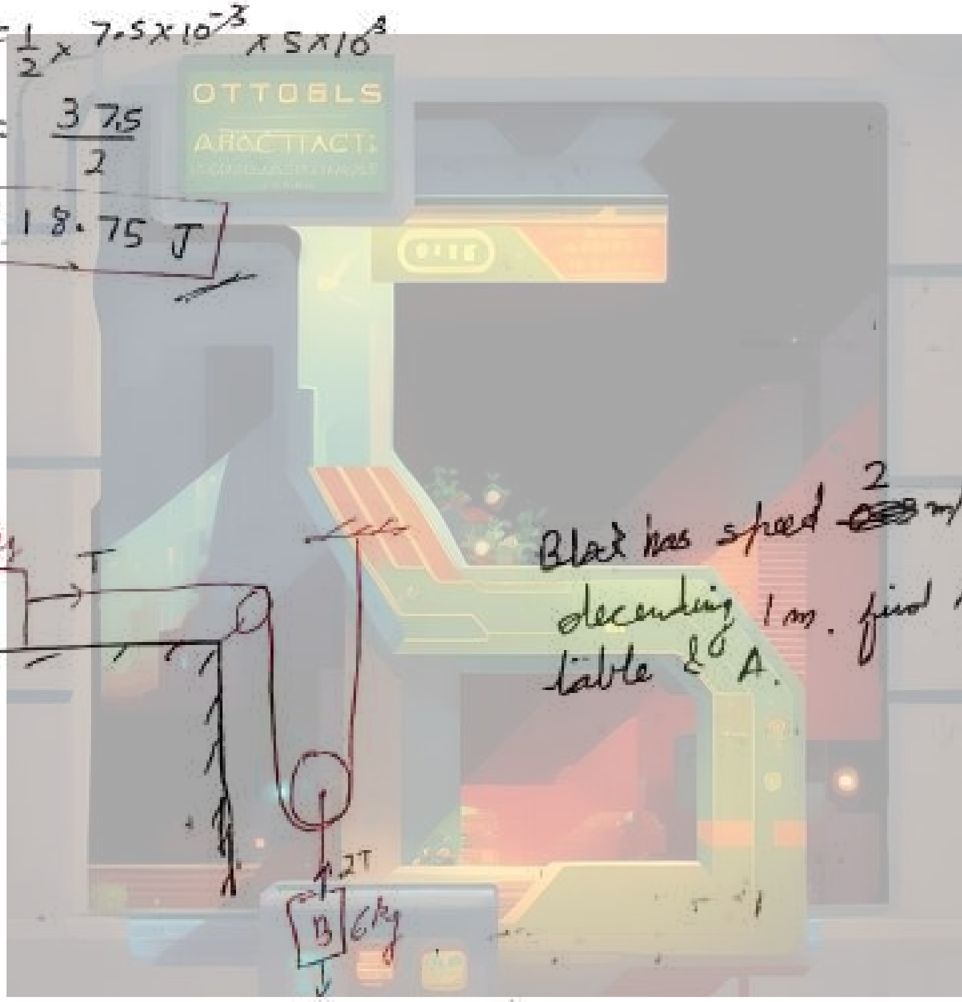
$W = \frac{1}{2}k(0.01 - 0.0025)$

$W = \frac{1}{2}k \times 0.0075$

$W = \frac{1}{2}\times 7.5\times10^{-3}\times 5\times10^{3}$

$W = \frac{37.5}{2}$

$\boxed{W = 18.75\ J}$



Q

Block has speed 2 m/s after descending 1 m. find k btw table & A.

$\overbrace{60(1)}^{W_g} - \overbrace{\mu(40)(2)}^{W_f} = \frac{1}{2}\times 6\times \cancel{0.09}\ 4^{2}$

~~$v^2 - u^2 = 2as$~~

~~$\frac{0.09}{2} = a$~~

~~$a = \frac{9}{200}$~~

$60 - 80\mu = 12$

$\frac{48}{80} = \mu$

$\mu = \frac{3}{5}$

$\therefore \mu = 0.6$

$K_i + W_f = K_f$

$$0 + \overbrace{60(1)}^{W_g} - \overbrace{\mu(40)(2)}^{W_f} = \overbrace{\frac{1}{2}\times 6\times 4^2}^{KE_A} + \overbrace{\frac{1}{2}\times 4\times 16}^{KE_B}$$

$60 - 80\mu = 12 + 32$

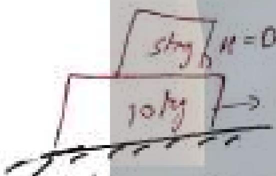
$80\mu = 16$

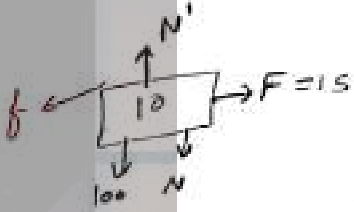
$\mu = \frac{16}{80}$

$\mu = \frac{1}{5}$

$\boxed{\mu = 0.2}$

Q 31. for Distance of 2m.





$$\overbrace{0}^{K_i} + \overbrace{15(2)}^{W_F} = \overbrace{\frac{1}{2}\times 15\times v^2}^{K_f}$$

$\frac{60}{15} = v^2$

$v^2 = 4$

$\boxed{v = 2\ m/s}$

$N = 50N$

$f_L = \mu N$

$= 0.5\times 50$

$= 25N$

Validate for 5kg.

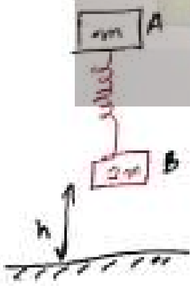
$f = 5\times 1 = 5N < f_L$

If both move together,

$15 = (5+10)a$

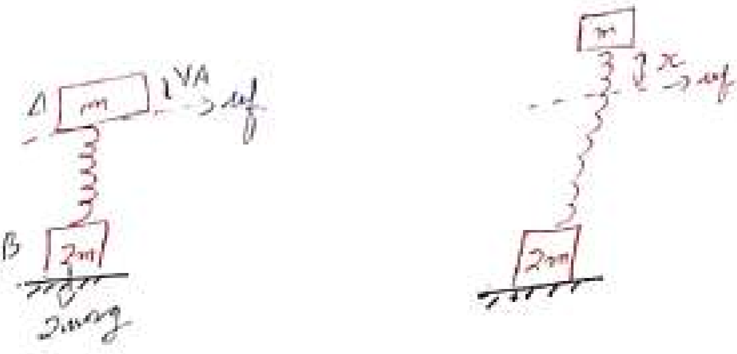
$\boxed{a = 1\ m/s^2}$

Q32. From what min h must the system be released that after a perfectly inelastic collision, B may be lifted off.



$v^2 = 0 + 2gh$

$v_A = \sqrt{2gh}$



To lift $2m$; $Kx = 2mg$

$$x = \frac{2mg}{k} \text{ (elongation)}$$

$$mgx + \frac{1}{2}Kx^2 + K_A = \frac{1}{2}mv_A^2$$

$$mg\left(\frac{2mg}{K}\right) + \frac{1}{2}K \times \frac{2m^2g^2}{K^2} \times 2 + K_A = \frac{1}{2}m(2gh)$$

$$\frac{4m^2g^2}{K} + K_A = mgh$$

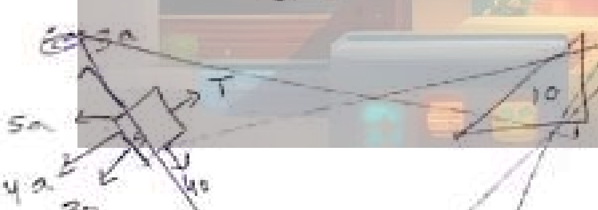
$K_A \geq 0$

So

$$\frac{4m^2g^2}{K} \leq mgh$$

$$\boxed{h \geq \frac{4mg}{K}}$$

Q33. find vel of A & B when A is about to touch ground.



$V_A = V_B$

V_A (wrt ground) $= \sqrt{v^2 + v^2 + 2v^2(-\cos 37)}$

$= \sqrt{2v^2 - 2v^2 \times \frac{4}{5}}$

$= v\sqrt{\frac{2}{5}}$

$V_A = V_B = V$

$K_i + W = K_f$

$5 \times 10 \times 2 = \frac{1}{2} \times 5 \times v^2 + \frac{1}{2} \times 10 \times v^2$

$200 = 15v^2$

$v^2 = \frac{200}{15}$

$\underbrace{5 \times 10 \times 2}_{W_g} = \underbrace{\frac{1}{2} \times 10 \times v^2}_{KE_B} + \underbrace{\frac{1}{2} \times 5 \times \frac{2v^2}{5}}_{KE_A}$

$100 = 5v^2 + v^2$

$v^2 = \frac{100}{6}$

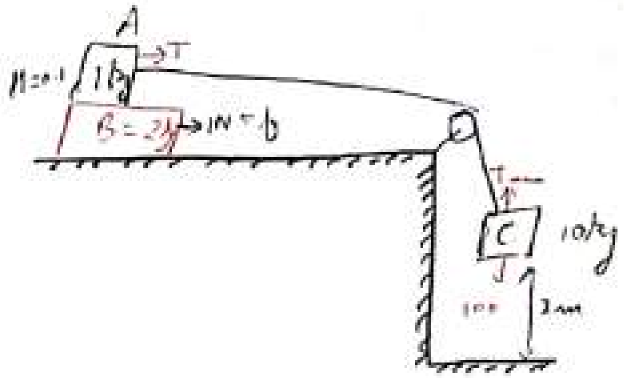
$$\boxed{v = \sqrt{\frac{50}{3}}}$$

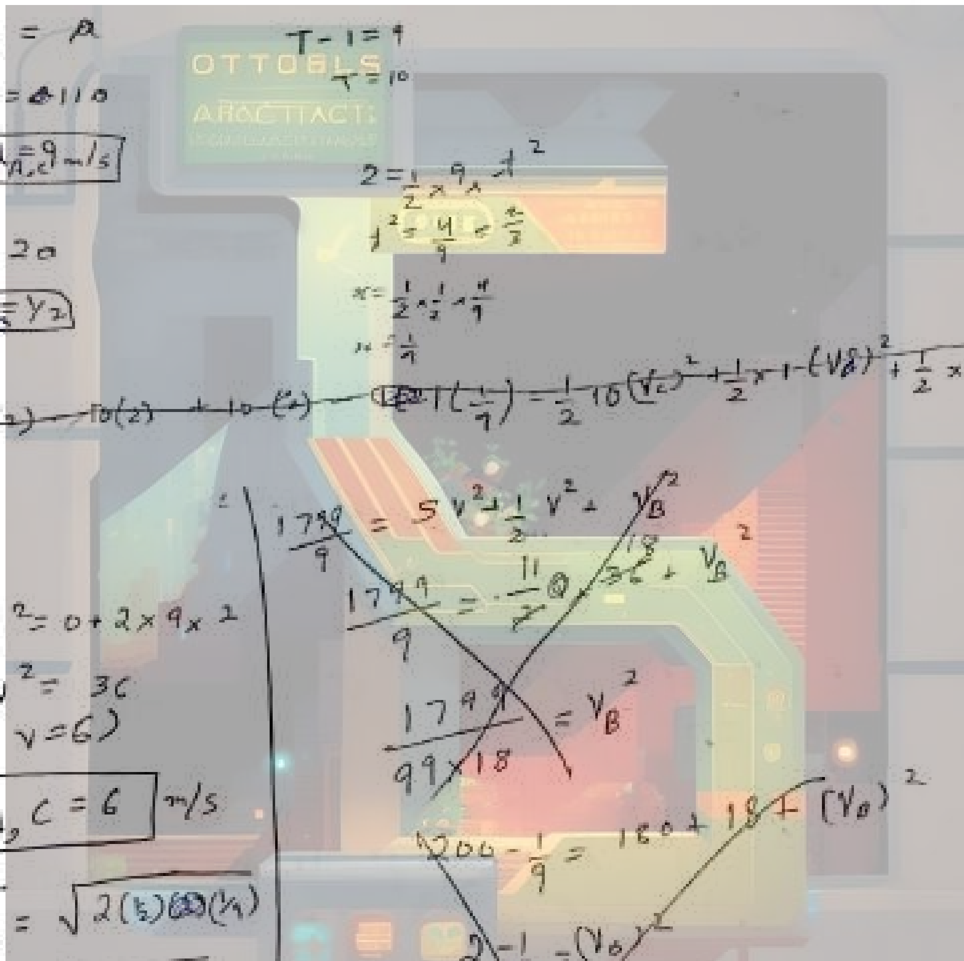
$V_B = 5\sqrt{\frac{2}{3}}$ m/s

$V_A = 2\sqrt{\frac{5}{3}}$ m/s

Q24.

find velocity of 3 blocks when C has descended 2m





$f_L = 1N$

$100 - T = 10a$

$T - 1 = a$

$99 = 11a$

$\boxed{A_{A,C} = 9 \, m/s}$

$T - 1 = 9$

$T = 10$

$1 = 2a$

$\boxed{a_B = 1/2}$

$2 = \frac{1}{2} \times 9 \times t^2$

$t^2 = \frac{4}{9}$

$x = \frac{1}{2} \times \frac{1}{2} \times \frac{4}{9}$

$x = \frac{1}{9}$

$\cancel{100(2) - 10(2) + 10(2) - 1\left(\frac{1}{9}\right) = \frac{1}{2} 10 (V_c)^2 + \frac{1}{2} \times 1 (V_A)^2 + \frac{1}{2} \times 2 \times (V_B)^2}$

$\cancel{\frac{1799}{9} = 5V^2 + \frac{1}{2}V^2 + V_B^2}$

$\cancel{\frac{1799}{9} = \frac{11}{2} \times 36 + V_B^2}$

$\cancel{\frac{1799}{99 \times 18} = V_B^2}$

$\cancel{200 - \frac{1}{9} = 180 + 18 + (V_B)^2}$

$\cancel{2 - \frac{1}{9} = (V_B)^2}$

$\cancel{\frac{17}{9} = (V_B)^2}$

$v^2 = 0 + 2 \times 9 \times 2$

$v^2 = 36$

$v = 6$

$\boxed{V_{A,C} = 6 \, m/s}$

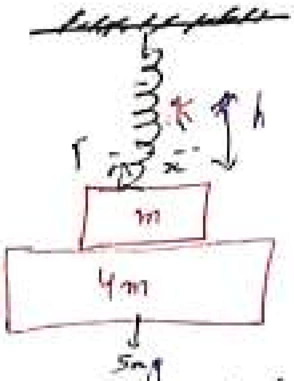
$V_B = \sqrt{2\left(\frac{1}{2}\right)\left(\frac{1}{9}\right)}$

$= \sqrt{1/9}$

$\boxed{V_B = \frac{1}{3} \, m/s}$

Q 35

initially at equilibrium. If 4m falls, how much will 4 m rise,



$5mgx$ = Spring Energy

$F = kx = 5mg$

$x = \frac{5mg}{k}$

$\underbrace{-mgh}_{W_g} + \underbrace{\frac{1}{2} \times k \times x^2}_{W_s} - \frac{1}{2} \times k \times (h-x)^2 = 0$

$\cancel{\frac{1}{2} \times k \times \frac{5mg}{k} \times \frac{5mg}{k} = mgh}$

$\cancel{\frac{5mg}{2k} = h}$

$(h^2 + x^2 - 2hx)$

$\frac{1}{2} \times k \times \frac{5mg}{k} \times \frac{5mg}{k} = \frac{1}{2} \times k \times \cancel{(h-x)^2} + mgh$

$\frac{25m^2g^2}{2k} = \frac{kh^2}{2} + \frac{k}{2}x^2 - \frac{2hxk}{2} + mgh$

$\cancel{\frac{25m^2g^2}{2k}} = \frac{kh^2}{2} + \cancel{\frac{25m^2g^2}{2k}} - h(5mg - mg)$

$h(4mg) = \frac{kh^2}{2}$

$\boxed{\frac{8mg}{k} = h}$

elongation from equilibrium position = max height from equilibrium position ∴

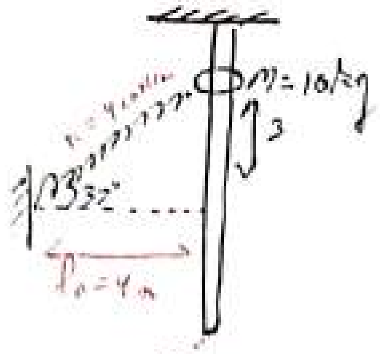
difference in equilibrium positions $= \frac{5mg}{k} - \frac{mg}{k} = \frac{4mg}{k}$

max height from equilibrium $= \frac{4mg}{k}$

initial distance from equilibrium $= \frac{4mg}{k}$

$\boxed{\max h = \frac{8mg}{k}}$

Q 36.



natural length = l_0

velocity of ring when spring horizontally.

elongation = ~~cos~~

$\cancel{\cos 37 = \frac{l}{4} = \frac{4}{5}}$

$\cancel{l = \frac{16}{5}}$

$\cancel{l - l^0 = 16}$

$\cos 37° = \frac{4}{l} = \frac{4}{5}$

$l = 5$

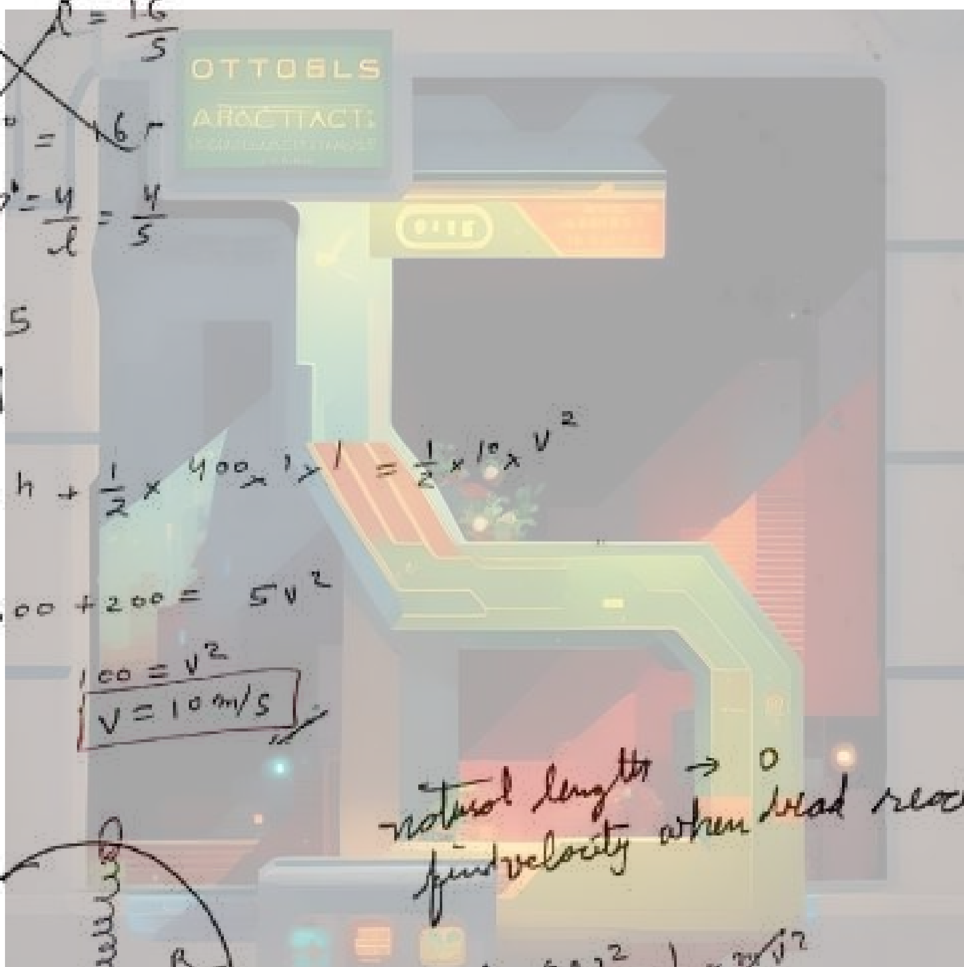
$\boxed{x = 1m}$

$10 \times g \times h + \frac{1}{2} \times 400 \times 1 \times 1 = \frac{1}{2} \times 10 \times v^2$

$300 + 200 = 5v^2$

$100 = v^2$

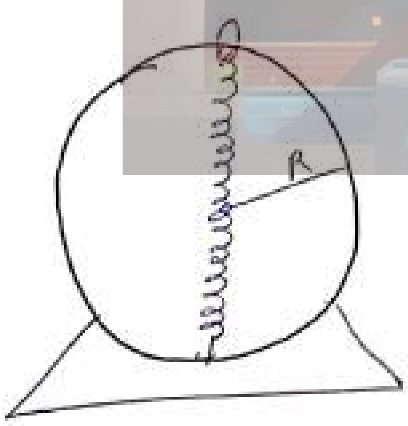
$\boxed{v = 10 m/s}$



natural length → 0

find velocity when bead reaches lowest position

Q 37.



$\cancel{\frac{1}{2} \times k \times (R)^2 = \frac{1}{2} \times m v^2}$

$\cancel{\frac{kR(2R)^2}{m} = v^2}$

$\cancel{v = \sqrt{\frac{k}{m}}\ 2R}$

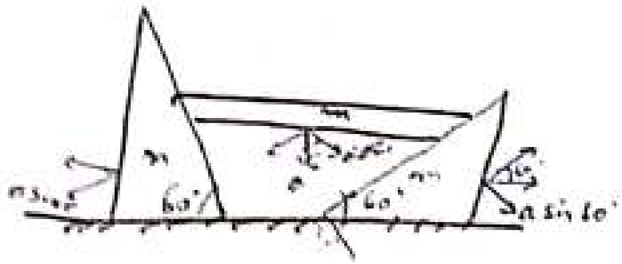
$0 + \frac{1}{2} m v^2 = \frac{1}{2} \times k \times (2R)^2 + \frac{mg \cdot 2R \times 2}{2}$

$v^2 = \frac{4R^2 k + 4mgR}{m}$

$\boxed{v = 2\sqrt{\frac{R^2 k + mgR}{m}}}$

Q 28.

find velocities when m decends h.



$N\cos 60^\circ = A\sin 60$

$\frac{N}{2} = \frac{\sqrt{3}A}{2}$

$N = \sqrt{3}A$

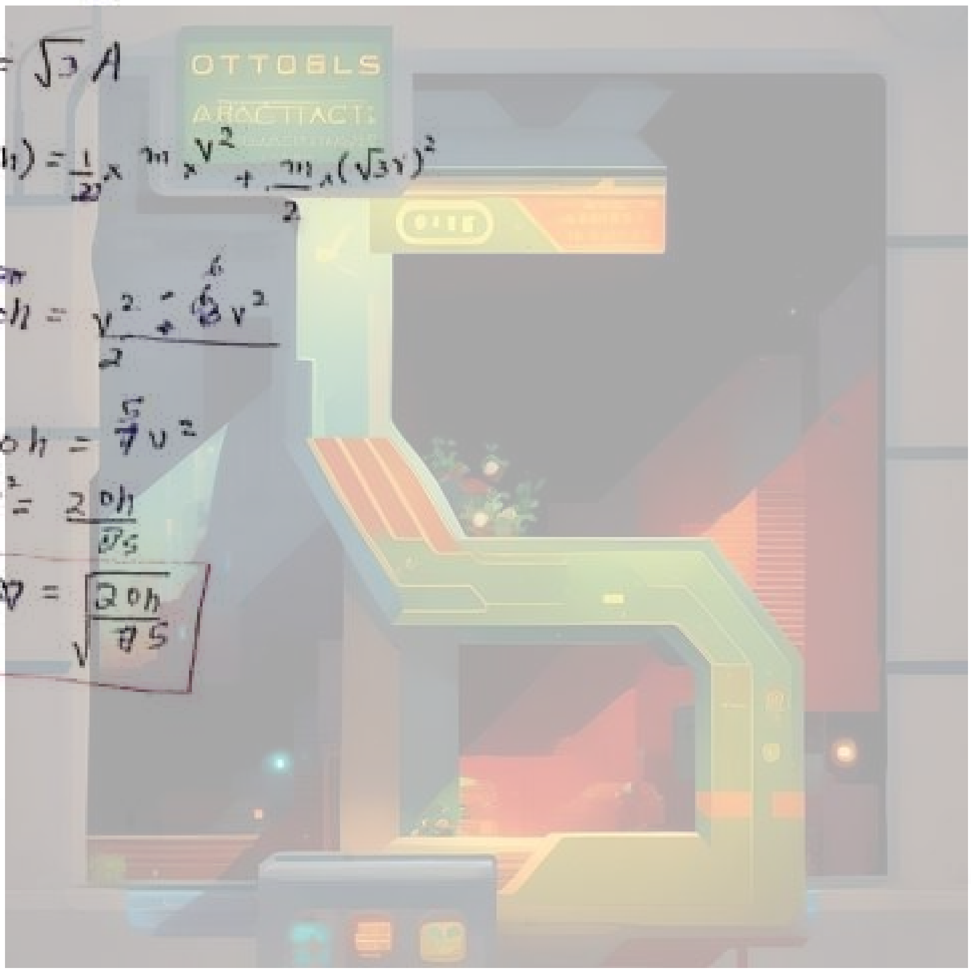
$m(10)(h) = \frac{1}{2} \times m \times v^2 + \frac{m}{2} \times (\sqrt{3}v)^2$

$10h = \frac{v^2 + 3v^2}{2}$

$20h = 5v^2$

$v^2 = \frac{20h}{5}$

$\boxed{V_{wedge} = v = \sqrt{\frac{20h}{5}}}$

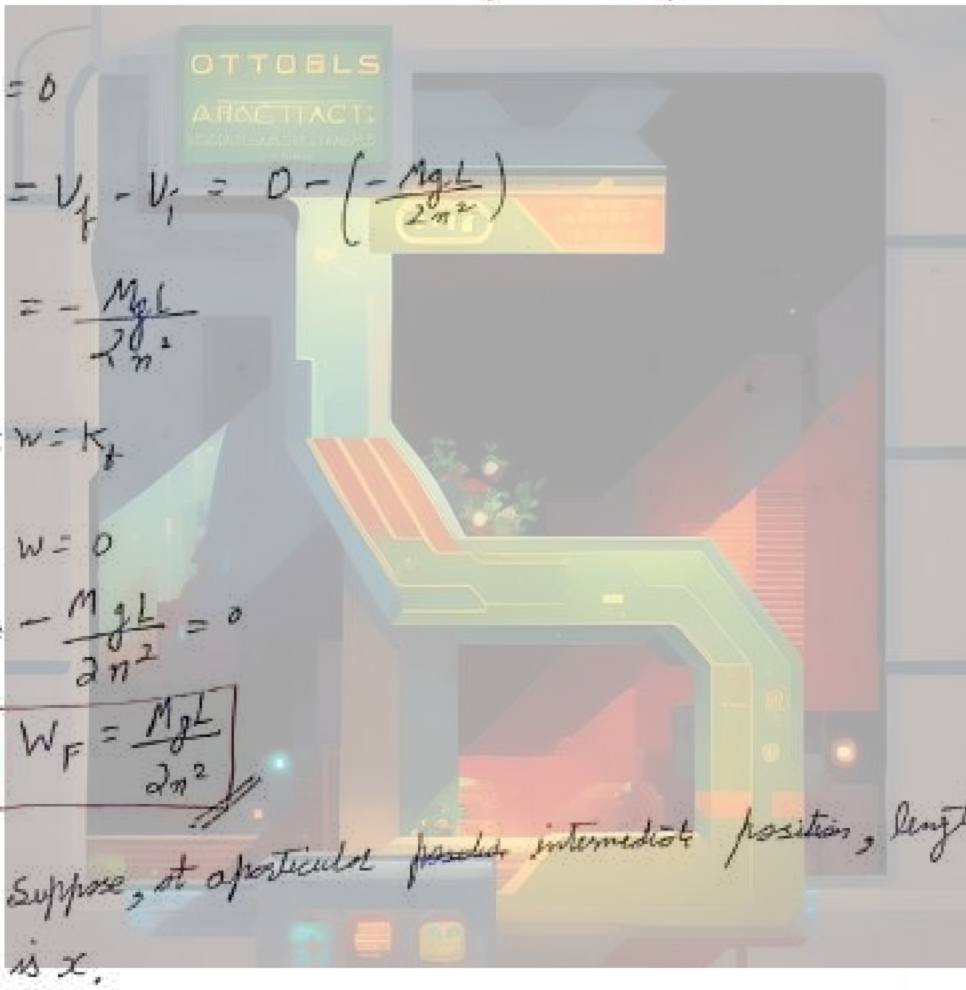


Q 39 A chain of mass M & length L is held on a smooth surface with $(1/n)^{th}$ of length hanging; calculate work done in pulling the chain on the table slowly.

M I

$$U_i = -\frac{M}{n} g \frac{L}{2n} = \frac{-MgL}{2n^2} \quad (\because U_i = -mgh)$$

h - centre of mass se distance



$$U_f = 0$$

$$\Delta U = U_f - U_i = 0 - \left(-\frac{MgL}{2n^2}\right)$$

$$= -\frac{MgL}{2n^2}$$

$$K_i \pm W = K_f$$

$$0 \pm W = 0$$

$$W_F - \frac{MgL}{2n^2} = 0$$

$$\boxed{W_F = \frac{MgL}{2n^2}}$$

M II

Suppose, at a particular ~~positi~~ intermediate position, length of hanging part is x.

$$F = mg$$

$$F = \frac{Mx}{L} g$$

$$W = \int_0^{L/n} F\,dx$$

$$W = \int_0^{L/n} \frac{Mxg}{L}\,dx$$

$$W = \left[\frac{Mgx^2}{2L}\right]_0^{L/n}$$

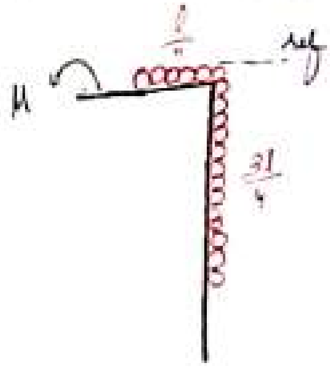
$$W = \frac{Mg}{2\cancel{L}} \frac{\cancel{L}}{n} \times \frac{L}{n}$$

$$\cancel{W = \frac{MgL}{2n^2}}$$

$$\boxed{W = \frac{MgL}{2n^2}}$$

~~Work Potential Energy theory~~

Q40. chain is on verge of slipping. find velocity when it was slipped.



let mass be M.

$f_l = \mu \frac{M}{4} g = \frac{3M g}{4}$

$\boxed{\mu = 3}$

$U_i = -\frac{3M}{4} \times g \times \frac{3l}{8}$

$= -\frac{9Mgl}{32}$

$U_f = -\frac{Mgl}{2}$

$-\Delta U = W_{g} = -(U_f - U_i) = U_i - U_f$

$= -\frac{9Mgl}{32} + \frac{Mgl}{2}$

$= \frac{16Mgl - 9Mgl}{32}$

$\boxed{W_g = \frac{7Mgl}{32}}$

for friction

$f = \mu \left(\frac{Mx}{l}\right) g$

$W_f = \int_{l/4}^{0} \mu \left(\frac{Mx}{l}\right) g \, dx$

$= \left[\mu \frac{Mx^2}{2l} g\right]_{l/4}^{0}$

$= -3 \times \frac{Mg}{2l} \times \frac{l}{4} \times \frac{l}{4}$

$\boxed{W_f = \frac{-3Mgl}{32}}$

$K_i + W = K_f$

$0 + W_g + W_f = \frac{1}{2} M v^2$

$\frac{7Mgl}{32} - \frac{3Mgl}{32} = \frac{1}{2} M v^2$

$\frac{7Mgl - 3Mgl}{32} = \frac{1}{2} M v^2$

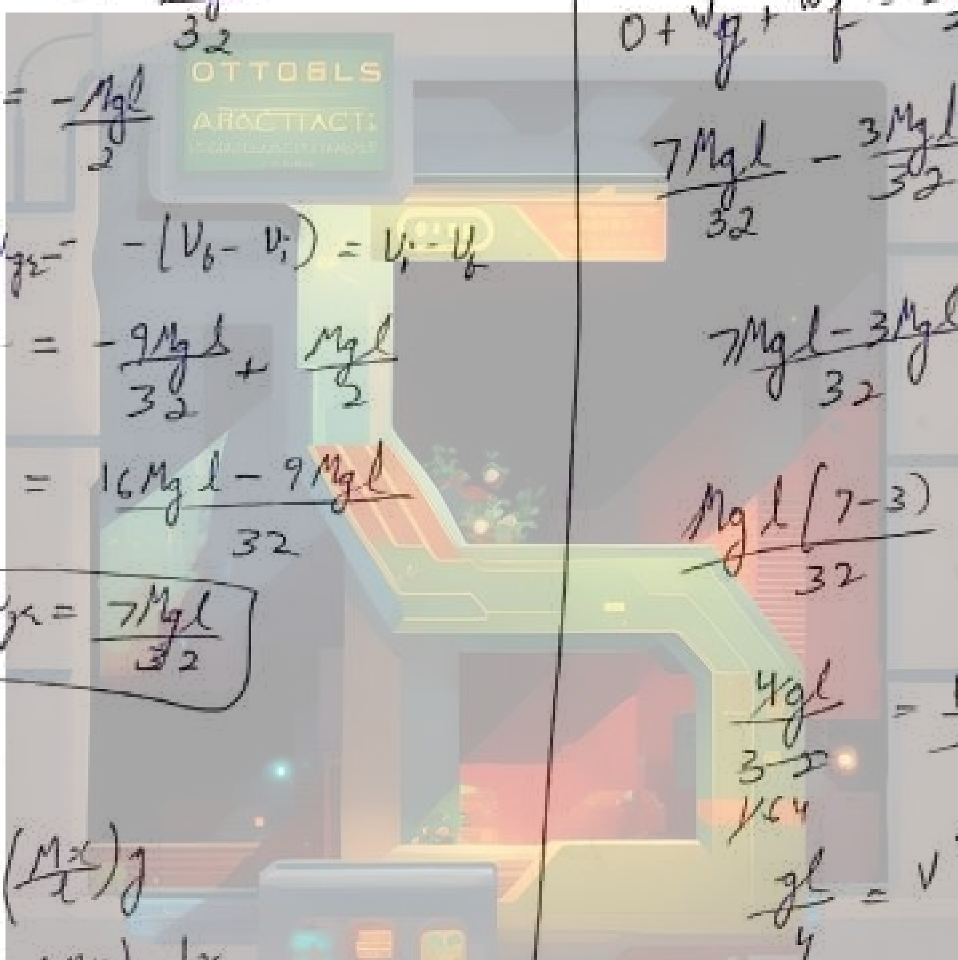
$\frac{Mgl(7-3)}{32} = \frac{1}{2} M v^2$

$\frac{4gl}{32} = \frac{v^2}{2}$

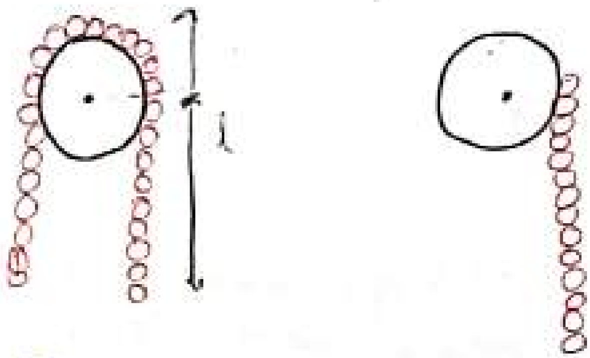
$\frac{gl}{4} = v^2$

$v = \sqrt{\frac{gl}{4}}$

$\boxed{v = \frac{1}{2}\sqrt{gl}}$



Q41.



Mass - M
find KE when chain becomes straight.

$$F_g = \frac{Mxg}{2l}$$

$$W_g = \int_l^0 \frac{Mx}{2l} g\, dx$$

$$= -\frac{Mg}{2l} \times \frac{1}{2} \times l \times l$$

$$\boxed{W_{g_1} = -\frac{Mgl}{4}}$$

$$F_g = \frac{Mg\,x}{2l}$$

$$W_{g_2} = \int_l^{2l} \frac{Mg}{2l} x\, dx$$

$$= \left[\frac{Mgx^2}{4l}\right]_l^{2l}$$

$$= \frac{Mg\,4l^2}{4l} - \frac{Mg\,l^2}{4l}$$

$$\boxed{W_{g_2} = \frac{3Mgl}{4}}$$

$$\frac{3Mgl}{4} - \frac{Mgl}{4} = \frac{1}{2} M v^2$$

$$\frac{2Mgl}{4} = \frac{1}{2} M v^2$$

$$v = \sqrt{gl}$$

$$KE = \frac{1}{2} M v^2$$

$$= \frac{1}{2} \times M \times (\sqrt{gl})^2$$

$$= \frac{1}{2} \times M \times gl$$

$$\boxed{= \frac{Mgl}{2}}$$

M II

$$U_i = -2\frac{M}{2} g \times \frac{l}{2}$$

$$= -\frac{Mgl}{2}$$

$$U_f = -Mgl$$

$$-\Delta U = U_i - U_f$$

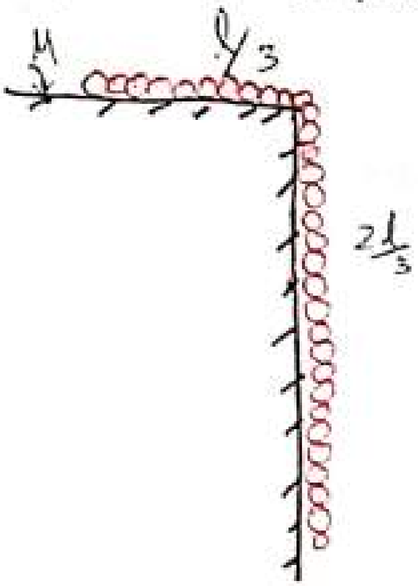
$$W_g = Mgl - \frac{Mgl}{2}$$

$$W_g = \frac{Mgl}{2}$$

$$0 + W_g = KE$$

$$\boxed{KE = \frac{Mgl}{2}}$$

Q Chain is on the verge of slipping.



a) $\mu = ?$

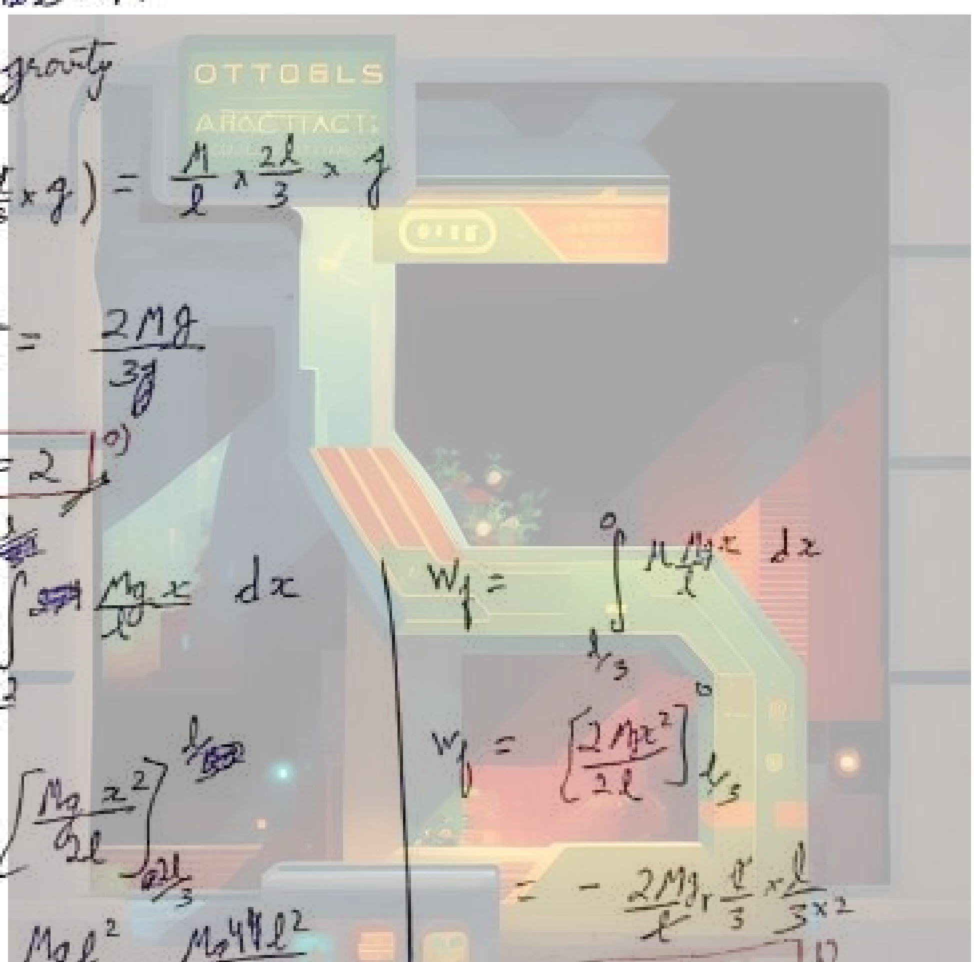
b) $= W_f, W_g$

c) $v = ?$

d) Work done at end A on slowly pulling the chain completely onto the table.

a) let mass $= M$

$f_l =$ gravity



$$\mu\left(\frac{M}{l}\times\frac{l}{3}\times g\right) = \frac{M}{l}\times\frac{2l}{3}\times g$$

$$\mu\frac{Mg}{3} = \frac{2Mg}{3}$$

$$\boxed{\mu = 2}$$ a)

b) $W_g = \int_{2l/3}^{l/3} \frac{Mg\,x}{l}\,dx$

$$= \left[\frac{Mg\,x^2}{2l}\right]_{2l/3}^{l/3}$$

$$= \frac{Mg\,l^2}{2l\,9} - \frac{Mg\,4l^2}{l\,9\times 2}$$

$$= \frac{Mgl}{2\,9} - \frac{4Mgl}{18}$$

$$\boxed{W_g = \frac{5Mgl}{18}\,J}$$ b)

$W_f = \int_{l/3}^{0} \mu\frac{Mg\,x}{l}\,dx$

$$W_f = \left[\frac{2Mx^2}{2l}\right]_{l/3}^{0}$$

$$= -\frac{2Mg}{l}\times\frac{l}{3}\times\frac{l}{3\times 2}$$

$$\boxed{W_f = \frac{-2Mgl}{18}\,J}$$ b)

(212)

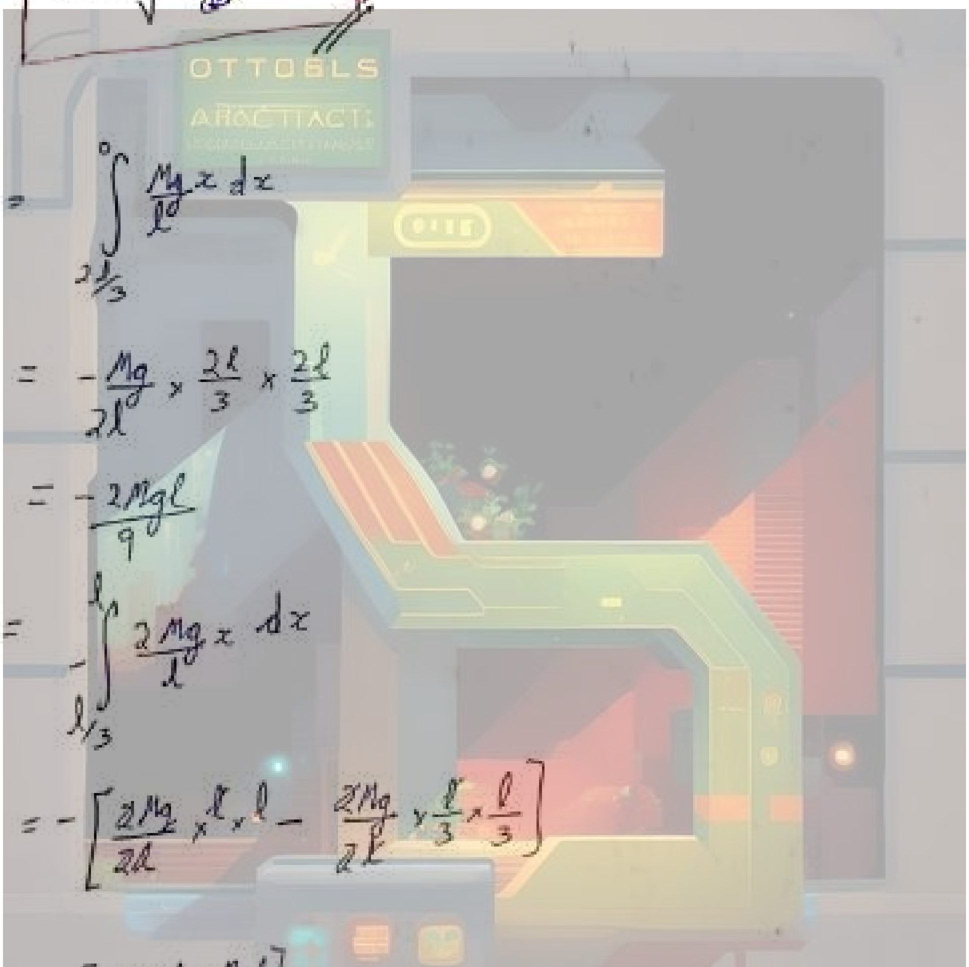
c) $K_i \pm W = K_f$

$$0 + \frac{5Mgl}{18} - \frac{2Mgl}{18} = \frac{1}{2} Mv^2$$

$$\frac{3Mgl}{18} = \frac{1}{2} Mv^2$$

$$\frac{2gl}{6} = v^2$$

$$\boxed{v = \sqrt{\frac{2gl}{6}}}$$ c)



d) $W_g = \int_{2l/3}^{0} \frac{Mg}{l} x \, dx$

$$= -\frac{Mg}{2l} \times \frac{2l}{3} \times \frac{2l}{3}$$

$$= -\frac{2Mgl}{9}$$

$W_f = -\int_{l/3}^{l} \frac{2Mg}{l} x \, dx$

$$= -\left[\frac{2Mg}{2l} \times l \times l - \frac{2Mg}{2l} \times \frac{l}{3} \times \frac{l}{3}\right]$$

$$= -\left[\frac{9Mgl - Mgl}{9}\right]$$

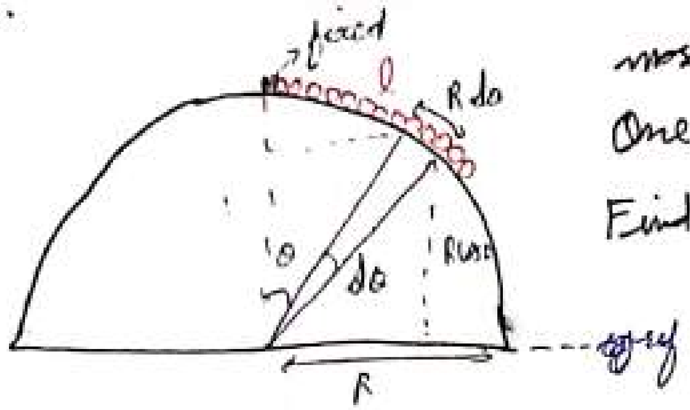
$$= -\frac{8Mgl}{9}$$

$K_i \pm W = K_f$

$$0 + W_{External} - \frac{2Mgl}{9} - \frac{8Mgl}{9} = 0$$

$$\boxed{W_F = \frac{10Mgl}{9} \text{ J}}$$ d)

Q 43.



mass m & length l.
One end is tied at top of hemisphere.
Find Gravitational P.E.

Taking a segment of chain as element from angle θ from vertical & angular width of segment is dθ

length of segment = $R\,d\theta$

mass of segment = $\frac{m}{l} \times R\,d\theta$

G.P.E of segment = $\frac{mR\,d\theta\,g}{l} \times R\cos\theta$

$= \frac{mg\,R^2\cos\theta\,d\theta}{l}$

G.P.E of chain = $\int_0^{l/R} \frac{mg\,R^2\cos\theta\,d\theta}{l}$

$= \frac{mg\,R^2}{l}\left[\sin\theta\right]_0^{l/R}$

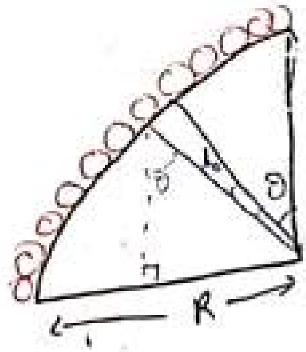
$\boxed{= \frac{mg\,R^2 \sin(l/R)}{l}}$

Q44.

$l = \pi R/2$

mass per unit length $= \rho$

determine velocity when chain leaves line with groove.



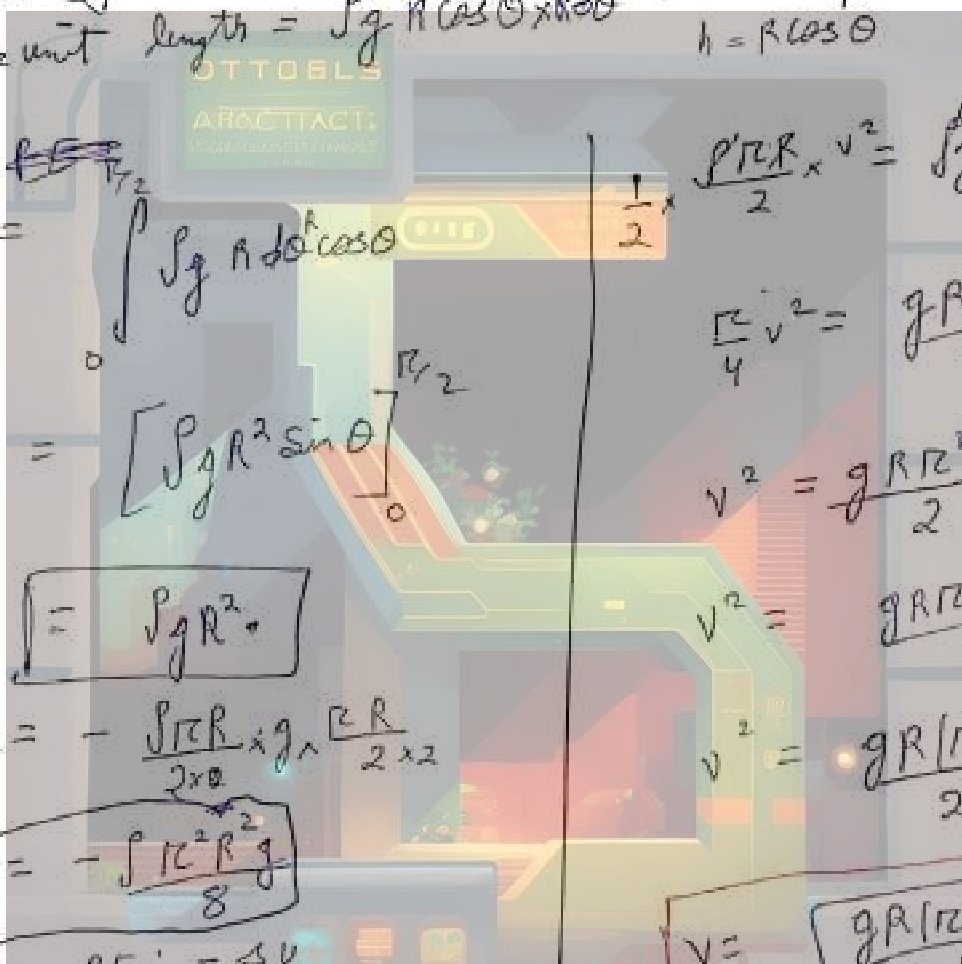
$\cancel{P.E' = \int \rho R d\theta}$ (per unit length)

$\cancel{P.E = \int_0^{\pi/2} \rho R d\theta}$

$\cancel{= [\rho R \theta]}$

P.E per unit length $= \rho g R\cos\theta \times R d\theta$ $\quad \cos\theta = \frac{h}{R}$

$h = R\cos\theta$



$$\cancel{P.E} \quad P.E = \int_0^{\pi/2} \rho g R d\theta R\cos\theta$$

$$= \left[\rho g R^2 \sin\theta\right]_0^{\pi/2}$$

$$\boxed{= \rho g R^2}$$

$$P.E_f = -\frac{\rho \pi R}{2\times 2} \times g \times \frac{\pi R}{2\times 2}$$

$$\boxed{= -\frac{\rho \pi^2 R^2 g}{8}}$$

$$PE_f - PE_i = \Delta U$$

$$\Delta U = -\frac{\rho \pi^2 R^2 g}{8} - \rho g R^2$$

$$-\Delta U = \rho g R^2 \left(\frac{\pi^2}{8} + 1\right)$$

$$\therefore \frac{1}{2} M v^2 = -\Delta U$$

$$\frac{1}{2} \times \frac{\rho \pi R}{2} \times v^2 = \rho g R^2 \left(\frac{\pi^2}{8} + 1\right)$$

$$\frac{\pi}{4} v^2 = \frac{g R \pi^2}{8} + gR$$

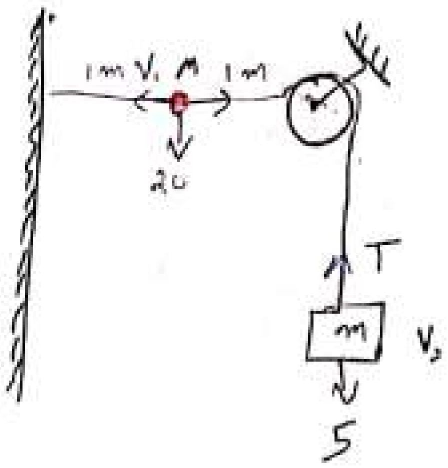
$$v^2 = \frac{g R \pi}{2} + \frac{4 g R}{\pi}$$

$$v^2 = \frac{g R \pi^2 + 8 g R}{2\pi}$$

$$v^2 = \frac{g R (\pi^2 + 8)}{2\pi}$$

$$\boxed{v = \sqrt{\frac{g R (\pi^2 + 8)}{2\pi}}}$$

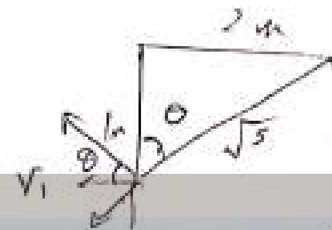
Q 45.

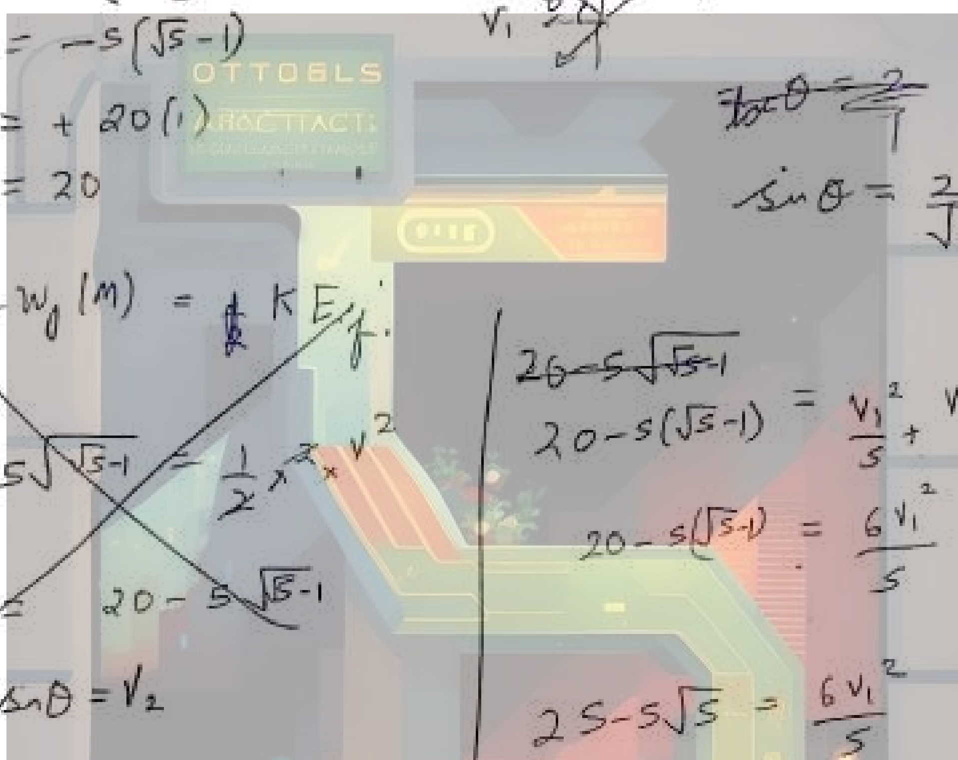


$M = 2kg$

$m = 0.5 kg$

find speed with which M strikes the wall.





$W_g(m) = -(\sqrt{5}-1)5$

$= -5(\sqrt{5}-1)$

$W_g(M) = +20(1)$

$= 20$

$\cancel{W_g(m) + W_g(M) = KE_f}$

$\cancel{20 - 5\sqrt{\sqrt{5}-1} = \frac{1}{2} \times 2 \times v^2}$

$\cancel{v^2 = 20 - 5\sqrt{\sqrt{5}-1}}$

$v_1 \sin\theta = v_2$

$\frac{2v_1}{\sqrt{5}} = v_2$

$K_E(m) = \frac{1}{2} \times \frac{1}{2} \times \left(\frac{2v_1}{\sqrt{5}}\right)^2$

$= \frac{4v_1}{5 \times 4}$

$= \frac{v_1^2}{5}$

$\cancel{\tan\theta = \frac{2}{1}}$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cancel{20 - 5\sqrt{\sqrt{5}-1}}$

$20 - 5(\sqrt{5}-1) = \frac{v_1^2}{5} + v_1^2$

$20 - 5(\sqrt{5}-1) = \frac{6v_1^2}{5}$

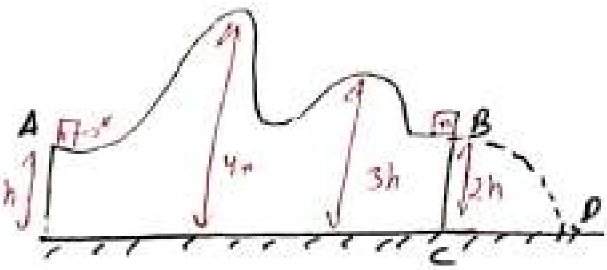
$25 - 5\sqrt{5} = \frac{6v_1^2}{5}$

$v^2 = \frac{5(25 - 5\sqrt{5})}{6}$

$v = \sqrt{\frac{25(5-\sqrt{5})}{6}}$

$\boxed{v = 5\sqrt{\frac{5-\sqrt{5}}{6}}}$ m/s

Q46.



fin min u for n to reach B.

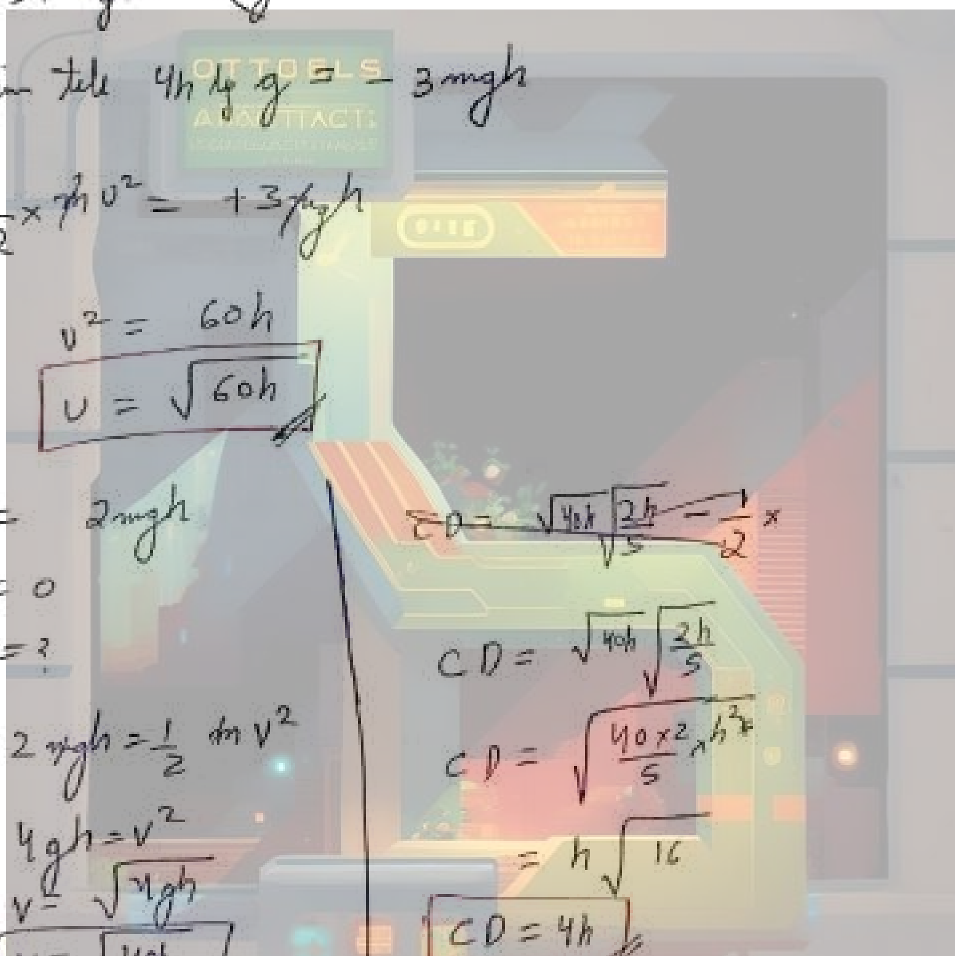
also find CD.

$$\cancel{PE_i = mgh}$$

$$\cancel{P.E_f = 2mgh}$$

$$\cancel{-\Delta U = PE_i - PE_f}$$

$$\cancel{= mgh - 2mgh}$$



W fo clim tile 4h by g = $-3mgh$

$$\frac{1}{2} \times \cancel{m} v^2 = +3 \cancel{m} gh$$

$$v^2 = 60h$$

$$\boxed{v = \sqrt{60h}}$$

$$W_g = 2mgh$$

$$K_i = 0$$

$$K_f = ?$$

$$0 + 2mgh = \frac{1}{2} m v^2$$

$$4gh = v^2$$

$$v = \sqrt{4gh}$$

$$\boxed{v = \sqrt{40h}}$$

$$2h = \frac{1}{2} \times 10 \times t^2$$

$$\sqrt{\frac{2h}{5}} = t$$

$$\cancel{CD = \sqrt{40h}\sqrt{\frac{2h}{5}} - \frac{1}{2} \times}$$

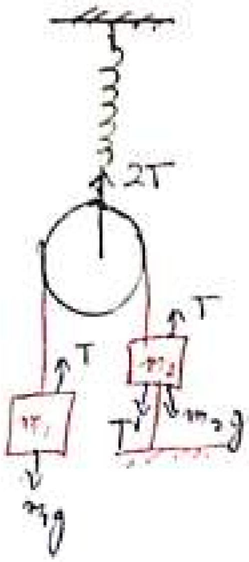
$$CD = \sqrt{40h}\sqrt{\frac{2h}{5}}$$

$$CD = \sqrt{\frac{40 \times 2}{5} \times h^2}$$

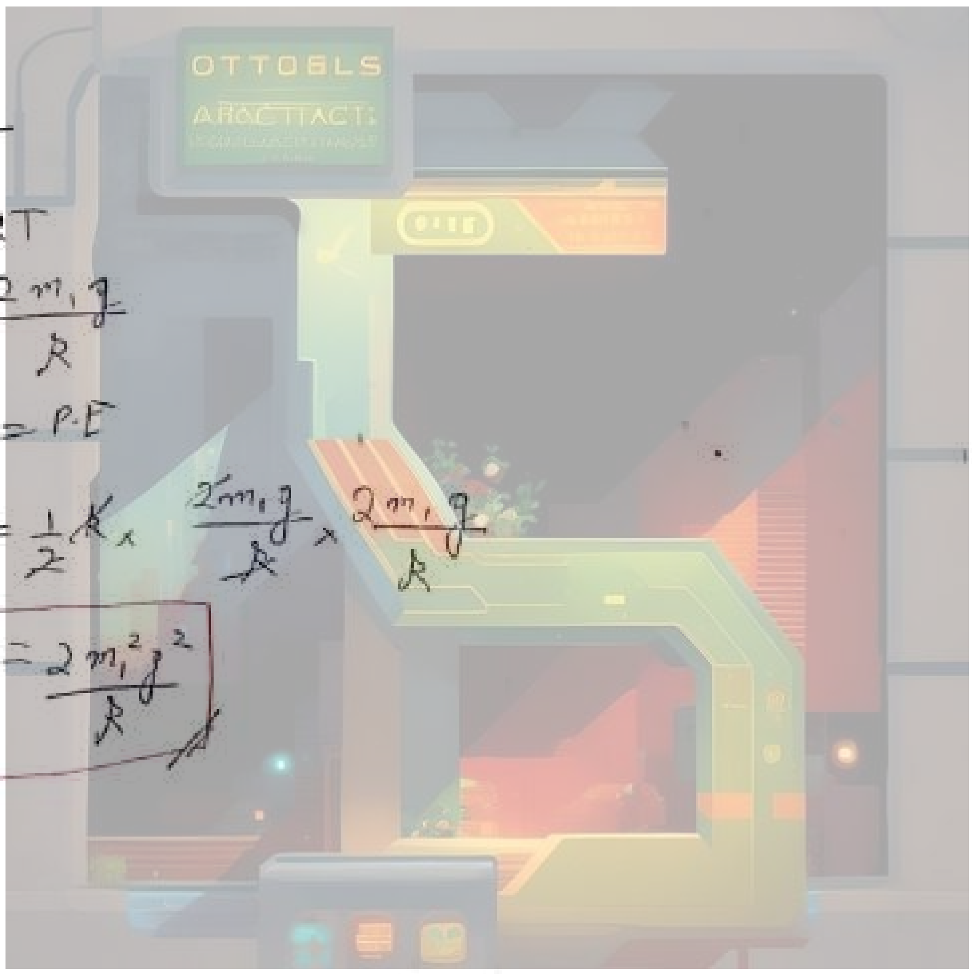
$$= h\sqrt{16}$$

$$\boxed{CD = 4h}$$

Q 47.



find P.E spring



$\cancel{m_2 g = T}$

$m_1 g = T$

$kx = 2T$

$x = \frac{2 m_1 g}{k}$

$\frac{1}{2} k x^2 = P.E$

$P.E = \frac{1}{2} k \times \frac{2 m_1 g}{k} \times \frac{2 m_1 g}{k}$

$\boxed{P.E = \frac{2 m_1^2 g^2}{k}}$

Power:

→ Work done per unit time is called Power.

or

Rate of doing work.

$$\boxed{\text{Power} = \frac{\text{Work}}{\text{Time}}}$$

→ SI unit:- watt

→ 1 W = 1 J/s

→ It is a scalar quantity.

→ Instantaneous Power = Fv

Power = Force × velocity

→ 1 HP = 746 W (horse power)

Power Delivered by Pump

$$P = \frac{dW}{dt}$$

$$P = \frac{1}{dt}\left[(dm)gh + \frac{1}{2}(dm)v^2\right]$$

$$\boxed{P = \frac{dm}{dt}\left[gh + \frac{1}{2}v^2\right]}$$

dm → mass of water delivered by pump in dt.

$\frac{dm}{dt}$ → rate of flow of water.

$$P = \frac{d(\text{volume})}{dt}\rho\left(gh + \frac{1}{2}v^2\right)$$

Q48. A Train has a constant speed of 40 m/s on a level road against resistive force of magnitude 3×10^4 N. Find power.

$$P = \frac{W}{t}$$

$$P = \frac{F \times S}{t}$$

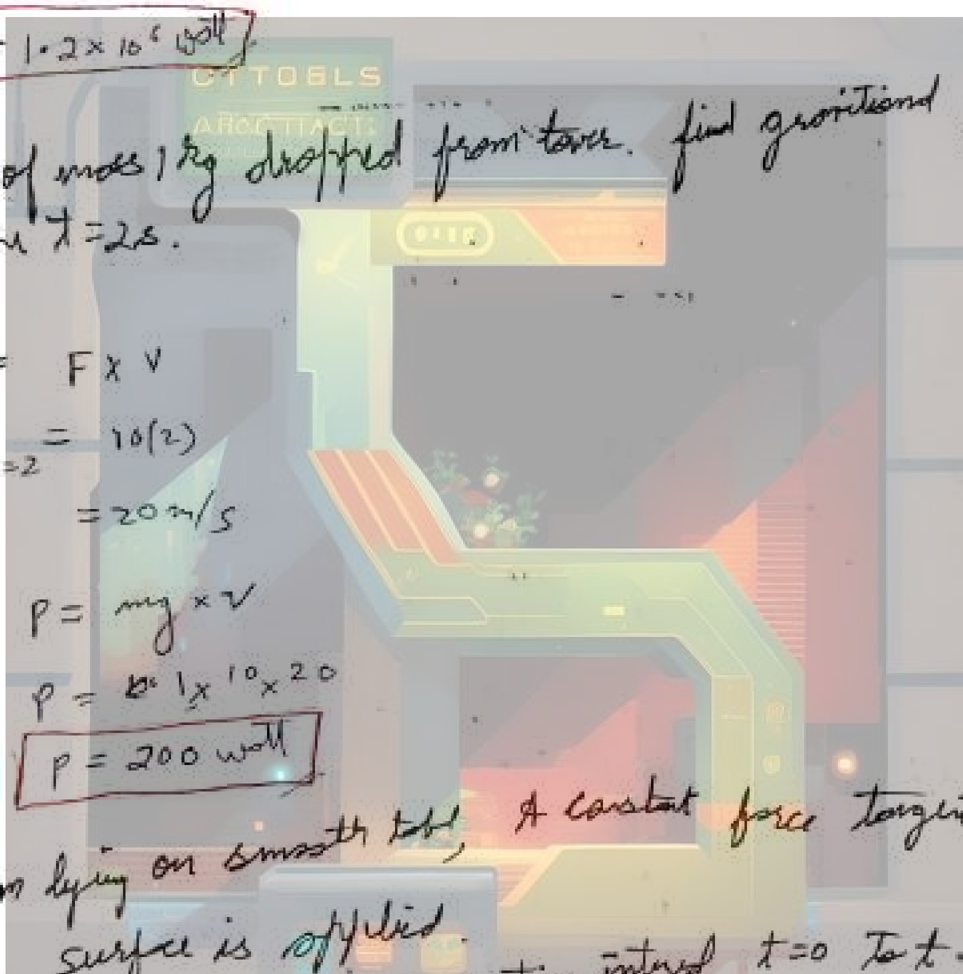
$$P = V \times F$$

$$P = 40 \times 3 \times 10^4$$

$$P = 120 \times 10^4$$

$$\boxed{P = 1.2 \times 10^6 \text{ watt}}$$

Q49. ball of mass 1 kg dropped from tower. find gravitational of Power at Time $t = 2s$.



$$P = F \times V$$

$$V|_{t=2} = 10(2)$$

$$= 20 \text{ m/s}$$

$$P = mg \times v$$

$$P = 1 \times 10 \times 20$$

$$\boxed{P = 200 \text{ watt}}$$

Q50. mass m lying on smooth table, A constant force tangential to the surface is applied.

i) average power over a time interval $t = 0$ To $t = t$

ii) instantaneous power as function of t,

ii) $P = FV$

$$P = F \times \frac{F}{m} t$$

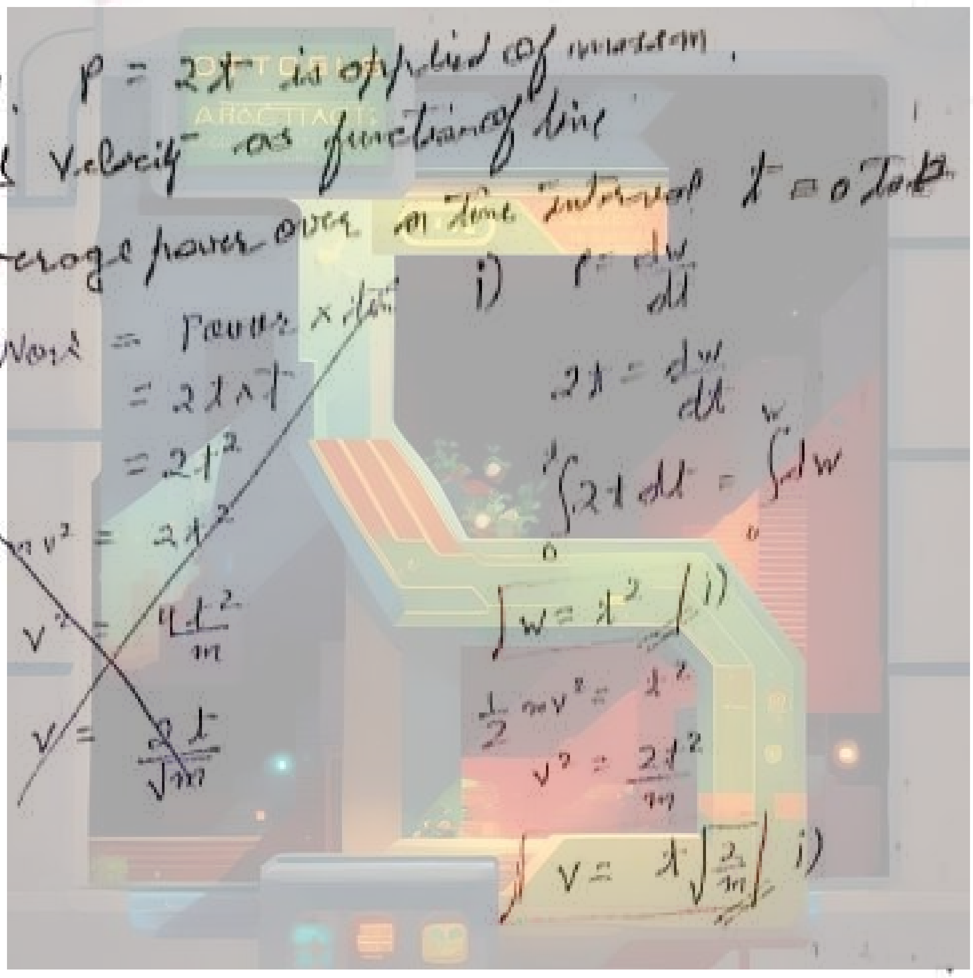
$$\boxed{P = \frac{F^2 t}{m}}$$ ii)

10 i) $P = \frac{Work}{Time}$ $\qquad s = \frac{1}{2} \times \frac{F}{m} t^2 \ (s = ut + \frac{1}{2}at^2)$

$P = \frac{Fs}{t}$ $\qquad s = \frac{Ft^2}{2m}$

$P = \frac{F}{t} \times \frac{Ft^2}{2m}$

$\boxed{P = \frac{F^2 t}{2m}}$ i)



Q51. $P = 2t$ is applied of motion.

i) KE & velocity as function of time

ii) average power over a time interval $t = 0$ to $t = t$.

~~i) KE = Work = Power × time~~

~~$= 2t \times t$~~

~~$= 2t^2$~~

~~$\frac{1}{2}mv^2 = 2t^2$~~

~~$v^2 = \frac{4t^2}{m}$~~

~~$v = \frac{2t}{\sqrt{m}}$~~

i) $P = \frac{dW}{dt}$

$2t = \frac{dW}{dt}$

$\int_0^t 2t\,dt = \int_0^W dW$

$W = t^2$

$\frac{1}{2}mv^2 = t^2$

$v^2 = \frac{2t^2}{m}$

$\boxed{v = t\sqrt{\frac{2}{m}}}$ i)

ii) $\frac{\text{Total work}}{\text{Time}} = \frac{t^2}{t}$

$\boxed{= t}$ ii)

Q52. An car of mass m accelerates from rest while engine supplies constant Power P. find its position & instantaneous velocity wrt time assuming automobile starts from rest.

$$p = \frac{dw}{dt} \cdot \frac{w}{t}$$

$$W = Pt$$

$$F \times s = Pt$$

$$ma \times s = Pt$$

$$a = \frac{Pt}{m s x}$$

$$v\frac{dv}{dx} = \frac{Pt}{mx}$$

$$v\,dv = \frac{Pt}{mx}dx$$

$$P = Fv$$

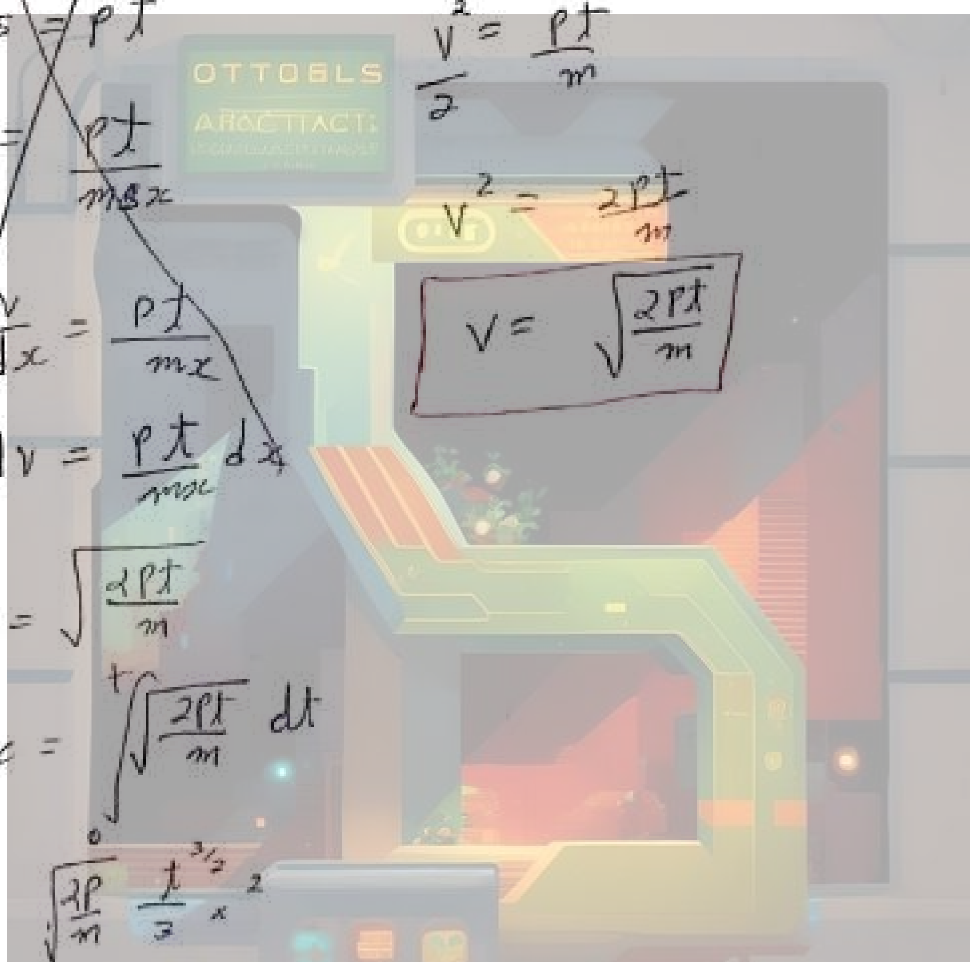
$$P = m\frac{dv}{dt}v$$

$$\int_0^t \frac{P}{m}dt = \int_0^v v\,dv$$

$$\frac{v^2}{2} = \frac{Pt}{m}$$

$$v^2 = \frac{2Pt}{m}$$

$$\boxed{v = \sqrt{\frac{2Pt}{m}}}$$



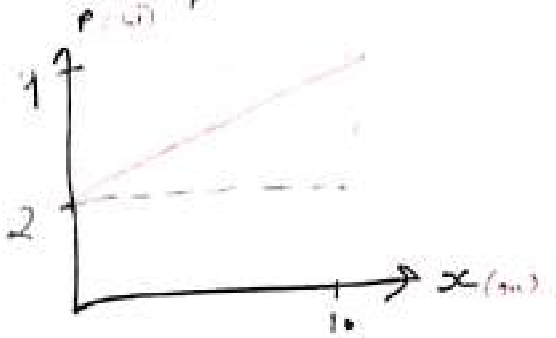
$$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$$

$$\int_0^s dx = \int_0^t \sqrt{\frac{2Pt}{m}}\,dt$$

$$s = \sqrt{\frac{2P}{m}}\,\frac{t^{3/2}}{3} \times 2$$

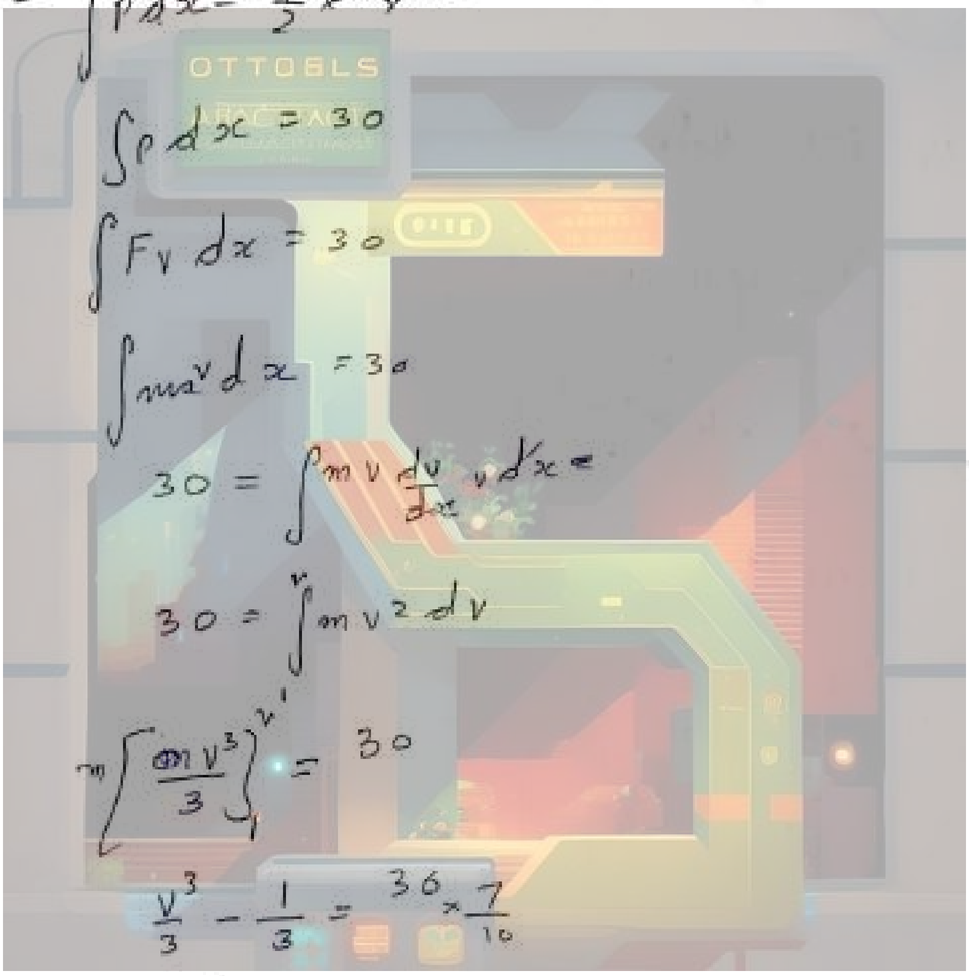
$$\boxed{s = \frac{2}{3}\sqrt{\frac{2P}{m}}\,t^{3/2}}$$

Q53. mass $= \frac{10}{7}$ kg, initial $x = 0$, initial $v = 0$



$P = \frac{dW}{dt}$

area $= \int P\,dx = \frac{1}{2} \times 6 \times 10$



$\int P\,dx = 30$

$\int F v\,dx = 30$

$\int m a v\,dx = 30$

$30 = \int m v \frac{dv}{dx} v\,dx$

$30 = \int_1^{v} m v^2\,dv$

$m\left[\frac{v^3}{3}\right]_1^{v} = 30$

$\frac{v^3}{3} - \frac{1}{3} = 30 \times \frac{7}{10}$

$\frac{v^3 - 1}{3} = 21$

$v^3 - 1 = 63$

$v^3 = 64$

$\boxed{v = 4\ m/s}$

Potential Energy Diagrams -

$$W_{conservative} = -\Delta U$$

$$dW = -dU$$

$$F dr = -dU$$

$$F = -\frac{dU}{dr}$$

$$\boxed{\vec{F}_{net} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]}$$

$\frac{\partial}{\partial x}$ = partial differentiation

$\frac{\partial U}{\partial x}$: differentiation of U wrt x keeping other variable (y, z) as constant

$\frac{\partial U}{\partial y}$: Differentiation of U wrt y keeping (x, z) as constants

$\frac{\partial U}{\partial z}$: Differentiation of U wrt z keeping (x, y) as constants

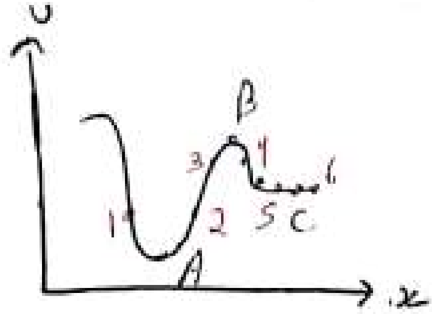
eg: $y = xz^2$

$$\frac{dy}{dx} = x\frac{d}{dx}(z^2) + z^2 \frac{d}{dx}(x)$$

$$\frac{\partial y}{\partial x} = z^2\frac{d}{dx}(x) = \boxed{z^2}$$

$$\frac{\partial y}{\partial z} = \frac{d}{dx}(z^2)(x)$$

$$\boxed{= 2xz}$$



A graph plotted between potential energy of a particle and its displacement from the mean position Center of the field is called potential energy diagram.

Point 1: slope $\frac{dU}{dx} < 0$, $F > 0$
Point 2: $\frac{dU}{dx} > 0$, $F < 0$
Stable Equilibrium $\frac{dU}{dx} = 0 \quad \frac{d^2U}{dx^2} < 0$

Force is always acting toward 'A'

Point 3: $\frac{dU}{dx} > 0$, $F < 0$
Point 4: $\frac{dU}{dx} < 0$, $F > 0$
Unstable equilibrium $\frac{dU}{dx} = 0 \quad \frac{d^2U}{dx^2} > 0$

Force is always acting away from 'B'

Point 5: $\frac{dU}{dx} = 0$
Point 6: $\frac{dU}{dx} = 0$
neutral equilibrium 'C'

Q 54. A particle moves along x-axis $U = 2x^3 - 3x^2 - 12x + 1$. Find co-ordinate of equilibrium position & its nature.
mass = 1 kg

$F = -\frac{dU}{dx}$

$F = -(6x^2 - 6x - 12)$

$F = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x = \frac{1 \pm \sqrt{1+8}}{2}$

$x = \frac{4}{2}, \frac{-2}{2}$

$\boxed{x = 2, -1}$

$\frac{d^2U}{dx^2} = 12x - 6$

$x = 2$

$12(2) - 6 = 24 - 6 = 18$

$\frac{d^2U}{dx^2} > 0$ so, stable equilibrium

$x = -1$

$12(-1) - 6 = -12 - 6 = -18$

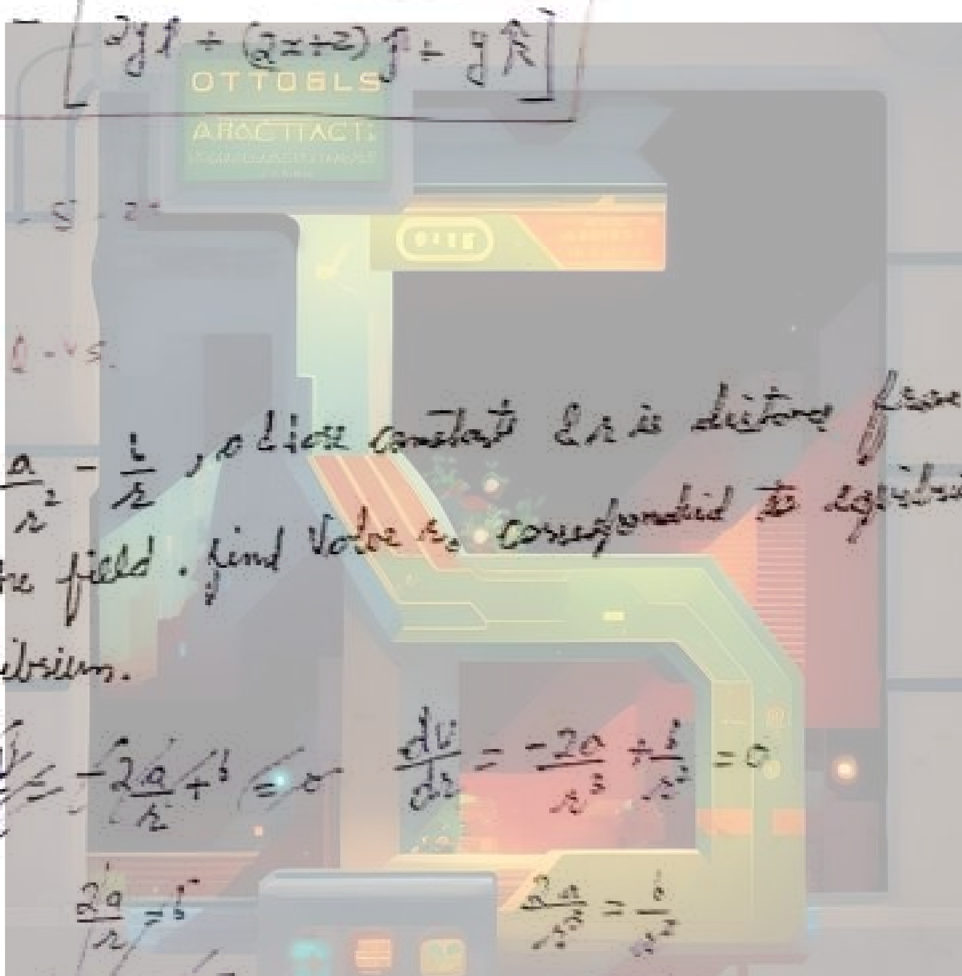
$\frac{d^2U}{dx^2} < 0$ so, unstable equilibrium.

Q.55. $V = (2xy + yz)$ Volt. find force along positive at $P(x, y, z)$

$$\vec{E}_x = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\vec{F} = -\left[\frac{\partial(2xy+yz)}{\partial x}\hat{i} + \frac{\partial(2xy+yz)}{\partial y}\hat{j} + \frac{\partial(2xy+yz)}{\partial z}\right]$$

$$\boxed{\vec{F} = -\left[2y\hat{i} + (2x+z)\hat{j} + y\hat{k}\right]}$$



H.W. 6-5-22

Ex 0-1
Q-1 to Q-45.

Q.56. $V = \frac{a}{r^2} - \frac{b}{r}$, a & b are constant & r is distance from the centre of the field. find value r_0 corresponding to equilibrium & type of equilibrium.

$\frac{dV}{dr} = -\frac{2a}{r} + b = 0$ $\qquad$ $\frac{dV}{dr} = \frac{-2a}{r^3} + \frac{b}{r^2} = 0$

$\frac{2a}{r} = b$ $\qquad$ $\frac{2a}{r^3} = \frac{b}{r^2}$

$\boxed{r_0 = \frac{2a}{b}}$ $\qquad$ $\boxed{r_0 = \frac{2a}{b}}$

$\frac{d^2V}{dr^2} =$

$$\frac{d^2V}{dr^2} = \frac{6a}{r^4} - \frac{2b}{r^3}$$

$$r_0 = \frac{2a}{b}$$

$$\frac{6a}{16a^4} \times b^4 - \frac{2b}{8a^3} \times b^3$$

$$\frac{6b^4 - 4b^4}{16a^3} = 0 \qquad \frac{3b^4}{8a^3} - \frac{b^4}{4a^3} > 0$$

$\boxed{\text{unstable}}$ $\qquad$ $\boxed{\text{Stable}}$

Q57. $V = V_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^6\right]$ where V_0 & a are constants

a) x at which $V = 0$

b) find F_x

c) x at which PE is min.

a) $V = V_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^6\right] = 0$

$\left(\frac{a}{x}\right)^{6+6} = 2\left(\frac{a}{x}\right)^6$

$\left(\frac{a}{x}\right)^6 = 2$

$\frac{a}{x} = 2^{1/6}$

$\boxed{x = a \times 2^{-1/6}}$ a)

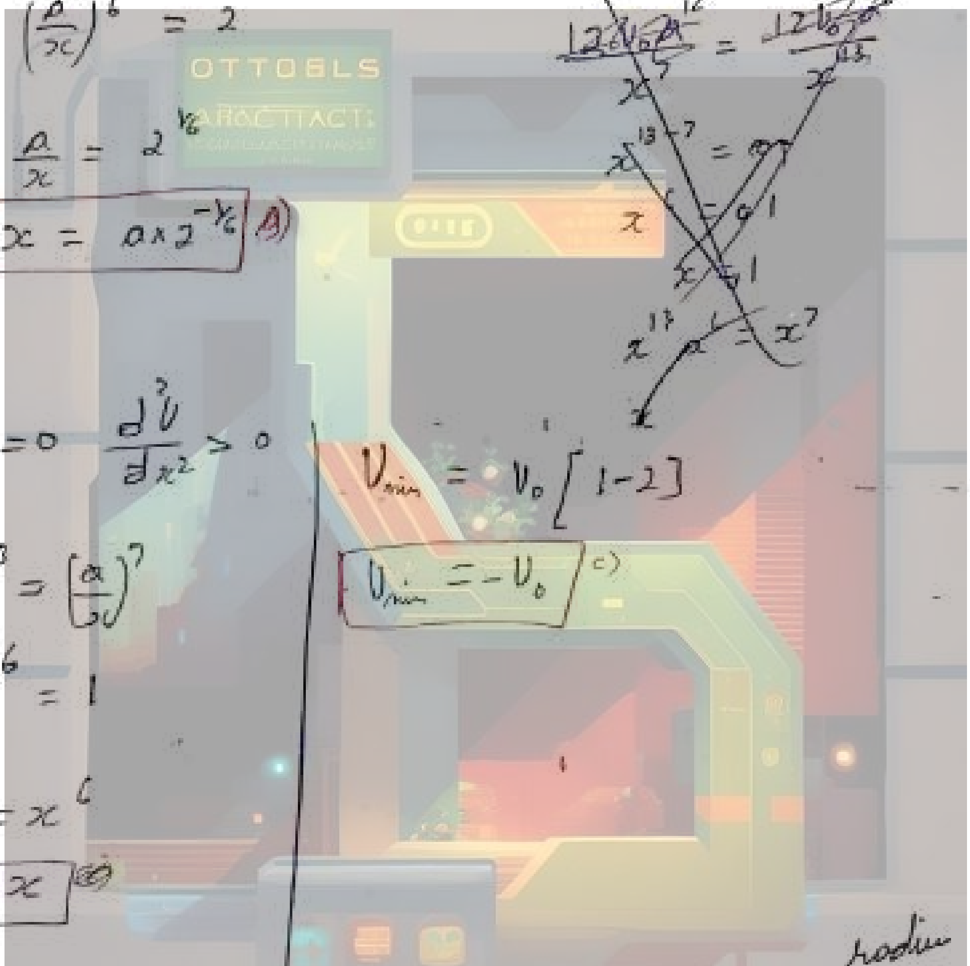
b) $\frac{dV}{dx} = \frac{-12V_0a^{12}}{x^{13}} + \frac{12V_0a^6}{x^7} = -F$

$\boxed{F = \frac{12V_0a^{12}}{x^{13}} - \frac{12V_0a^6}{x^7}}$ b)

c) ~~PE min, $\frac{dV}{dx} = 0$~~ $\frac{d^2V}{dx^2} > 0$

~~$\frac{12V_0a^{12}}{x^{13}} = \frac{12V_0a^6}{x^7}$~~

~~$x^{13-7} = a^6$~~



c) $\frac{dV}{dx} = 0$ $\frac{d^2V}{dx^2} > 0$

$\left(\frac{a}{x}\right)^{13} = \left(\frac{a}{x}\right)^7$

$\left(\frac{a}{x}\right)^6 = 1$

$a^6 = x^6$

$\boxed{a = x}$ c)

$V_{min} = V_0[1-2]$

$\boxed{V_{min} = -V_0}$ c)

Q58. $U = 2r^3$ J. body is moving in circular orbit radius $= 5$ m. find energy

$F_{centripetal} = \frac{mv^2}{r}$, mass = 2 kg find energy.

$F = -\frac{dV}{dr} = |-6r^2| = \frac{mv^2}{r}$

$mv^2 = +6r^3$

$E = \frac{1}{2}mv^2 = +3r^3 + V$

$= 3r^3 + 2r^3$

$= 5r^3$

$\boxed{= 625 \text{ J}}$

Q59. move along +x axis under $F(x)$ force. $V(x) = ax^3 - bx^2$

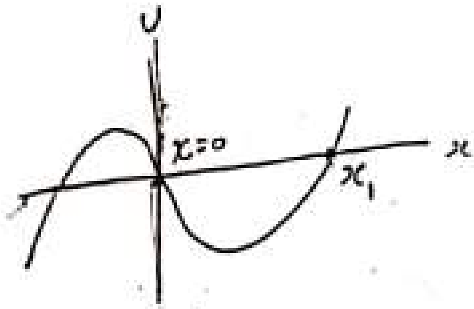
a) find $F(x)$

b) when energy = 0, particle can be between $x=0$ & $x=x_1$. find x_1

c) find Max KE.

a) $F(x) = \frac{-dV}{dx} = -[3ax^2 - b]$

$\boxed{F(x) = b - 3ax^2}$ ✓



b) $ma = b - 3ax^2$

$\frac{v\,dv}{dx} = \frac{b - 3ax^2}{m}$

$\int_0^v v\,dv = \int_0^x \frac{b - 3ax^2}{m}\,dx$

$\frac{v^2}{2} = \frac{bx}{m} - \frac{3ax^3}{3m}$

$\frac{v^2}{2} = \frac{3bx - 3ax^3}{3m}$

$v^2 = \frac{6bx - 6ax^3}{3m}$

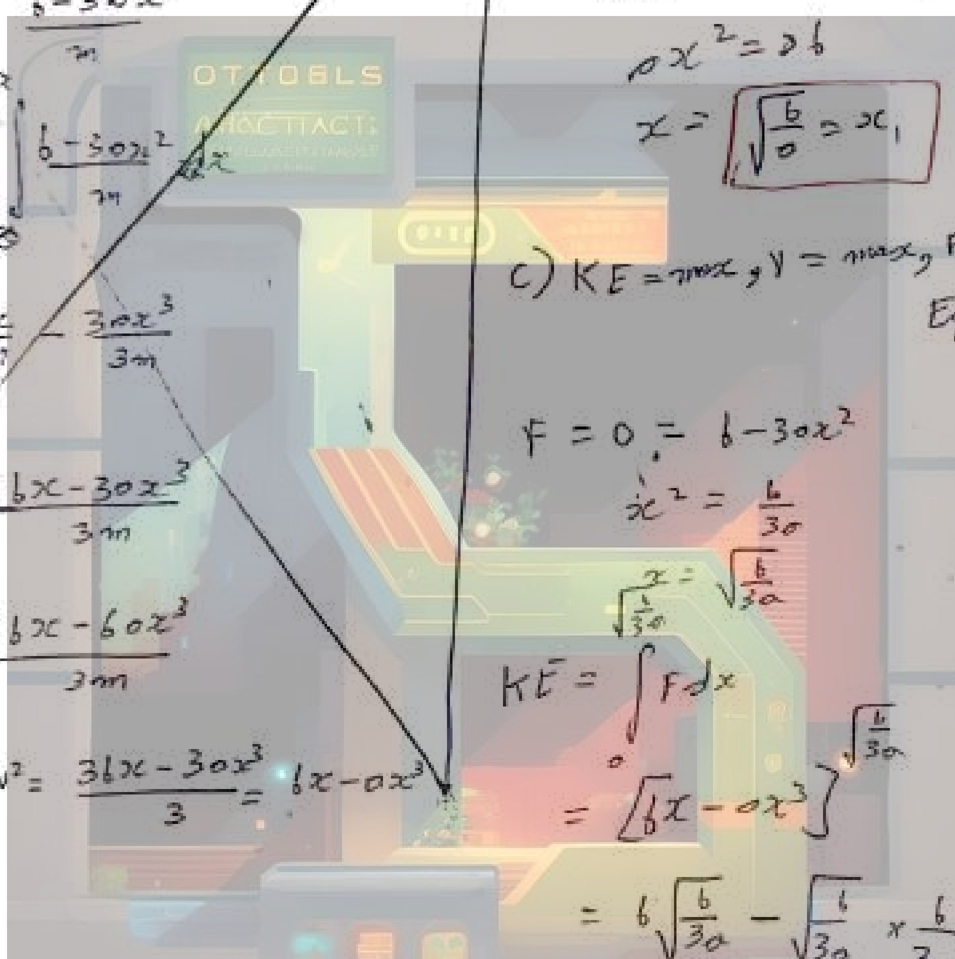
$KE = \frac{1}{2}mv^2 = \frac{3bx - 3ax^3}{3} = bx - ax^3$

b) $V = 0$

$ax^3 = bx$

$ax^2 = b$

$x = \boxed{\sqrt{\frac{b}{a}} = x_1}$



c) KE = max, V = max, F = 0 → Equilibrium.

$F = 0 = b - 3ax^2$

$x^2 = \frac{b}{3a}$

$x = \sqrt{\frac{b}{3a}}$

$KE = \int_0^{\sqrt{\frac{b}{3a}}} F\,dx$

$= \left[bx - ax^3\right]_0^{\sqrt{\frac{b}{3a}}}$

$= b\sqrt{\frac{b}{3a}} - \sqrt{\frac{b}{3a}} \times \frac{b}{3a} \times a$

$\boxed{= \sqrt{\frac{b}{3a}}\left(\frac{2b}{3}\right)}$

Q 60 mass m is taken from A → B

a) F is horizontal

b) ... tangential to surface

c) towards R.

a) $\Delta P = \frac{R}{2}$

$\boxed{W = FR}$ (displacement horizontal)

b) $W = \int_0^{\pi R/2} F\, ds$

$\boxed{W = \frac{F\pi R}{2}}$

c)

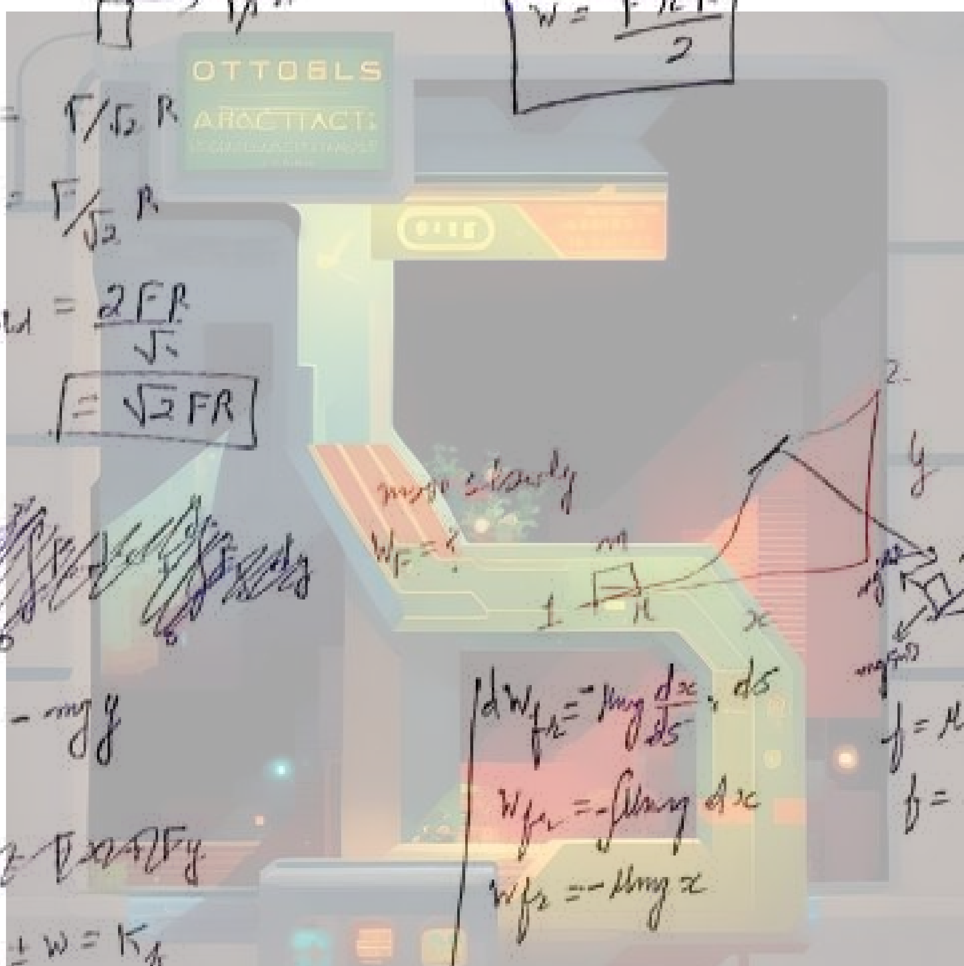
$F/\sqrt{2}$, $F/\sqrt{2}$

$W_x = F/\sqrt{2}\, R$

$W_y = \frac{F}{\sqrt{2}} R$

$W_{total} = \frac{2FR}{\sqrt{2}}$

$\boxed{= \sqrt{2} FR}$



Q 61.

move slowly

$W_F = ?$

$W_g = -mgy$

$dW_{fr} = -\mu mg \frac{dx}{ds} \cdot ds$

$W_{fr} = -\int \mu mg\, dx$

$W_{fr} = -\mu mg x$

$f = \mu mg \cos\theta$

$f = \mu mg \frac{dx}{ds}$

$K_i \pm W = K_f$

$W_F - W_g - W_{fr} = 0$

$W_F = mgy + \mu mg x$

$\boxed{W_F = mg(y + \mu x)}$