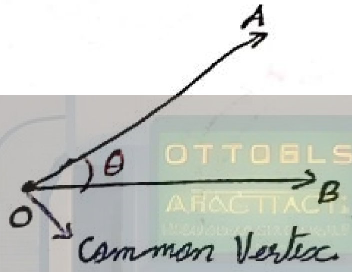
!! Trigonometry !!

त्रि side measurement

Angle - A figure formed by rays with common vertex is called an Angle.
Denoted by $\angle AOB$



$\angle AOB = \theta$

OA & OB are rays

→ There are 3 imp system for the measurement of an angle.

① Sexagesimal System (British)
② Radian System (Circular)
③ Grade System (French)

① Sexagesimal (Degree °)

→ A right Angle is divided into 90 equal parts.
→ Each degree is divided into 60 equal parts called minutes.
→ Each minute is divided into 60 equal parts called seconds.

1 right Angle ⇒ 90°

$1' = \left(\frac{1}{60}\right)^\circ \qquad 1^\circ = 60'$

$1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ \qquad 1^\circ = 3600''$

Q Convert the following in degrees.

① $8^\circ 30'$

$8^\circ + \frac{1}{2}^\circ = (8.5)^\circ$

② $12^\circ 23' 35''$

$12^\circ + \frac{23}{60}^\circ + \frac{35}{3600}^\circ$

$12^\circ + 0.38^\circ + 0.009^\circ$

12.389°

Q Convert $\frac{1}{2}^{\circ}$ in minutes

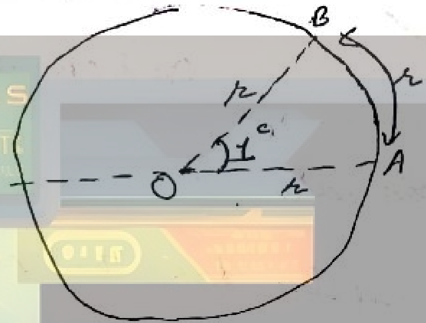
$1^{\circ} = 60$ min

$\frac{1}{2} \times 60 = \frac{60}{2} = \frac{30}{1} = 30'$ ~~minutes~~

② Radian System

→ Angle subtended at the center of a circle by an arc whose length is equal to the radius of the circle

$\theta = \frac{l}{r} = \frac{r}{r} = 1^{c}$



③ Grade System (G)

→ A right Angle is Divided into 100 equal parts called a Grade

→ Each Grade is divided to 100 minutes

→ Each minute is divided into 100 seconds.

1 right angle $= 100^{g}$

$1^{g} = 100'$

$1' = 100''$

Relation in Degree, Grade & Radian

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Q convert $\frac{\pi}{6}$ radian to degree

$R = \frac{\pi}{6}$

$\frac{D}{90} = \frac{2}{\cancel{\pi}} \times \frac{\cancel{\pi}}{6}$

$D = \frac{90}{3}$

$D = 30^{\circ}$

Q Convert following in radian.

(1) 0°

$$\frac{0}{90} = \frac{2R}{\pi}$$

$$\boxed{R = 0^c}$$

(2) 30°

$$\frac{30\times\pi}{90\times 2} = \boxed{\frac{\pi}{6}^{c}}$$

(3) 180°

$$\frac{\cancel{180}^{2}}{90}\times\frac{\pi}{\cancel{2}} = \boxed{\pi^{c}}$$

(4) 150°

$$\frac{\cancel{150}^{5}\times\pi}{\cancel{90}_{3}\times 2} = \boxed{\frac{5\pi}{6}^{c}}$$

(5) -56°

$$-\frac{\cancel{56}^{\cancel{28}\,14}\times\pi}{\cancel{2}\times\cancel{90}_{45}} = \boxed{\frac{-14\pi}{45}^{c}}$$

radian → Deg

(6) $\frac{9\pi}{5}$

$$\cancel{\frac{90\times 9\pi}{5} = \frac{810}{5}\pi}$$

$$\frac{\cancel{90}^{18}\times 2}{\cancel{\pi}}\times\frac{9\cancel{\pi}}{\cancel{5}} = \boxed{324^\circ}$$

(7)

$$\frac{-5\cancel{\pi}}{\cancel{6}_{3}}\times\frac{\cancel{90}^{30}\times 2}{\cancel{\pi}} = \boxed{-150^\circ}$$

(8)

$$-3\times\frac{2\times 90}{\pi} = \boxed{\frac{-540^\circ}{\pi}}$$

$$= \frac{-540}{22}\times 7$$

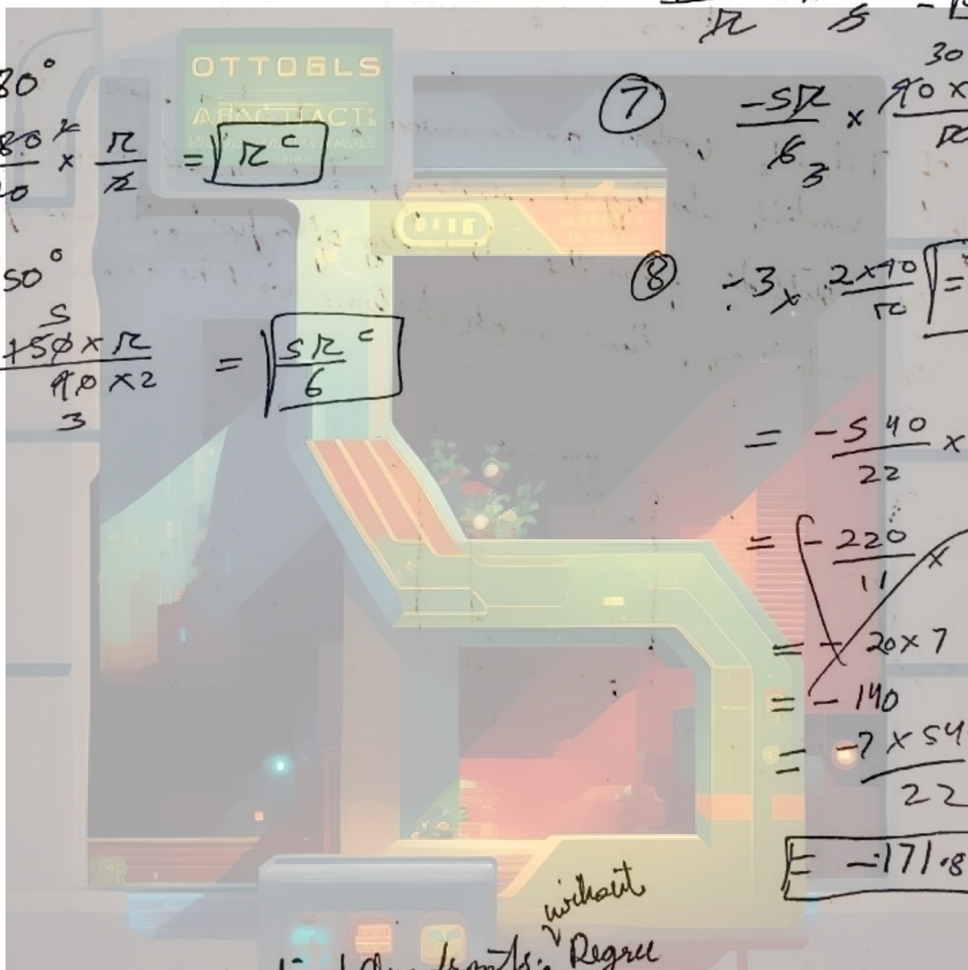
$$= \cancel{\frac{-220}{11}\times 7}$$

$$= \cancel{-20\times 7}$$

$$= \cancel{-140}$$

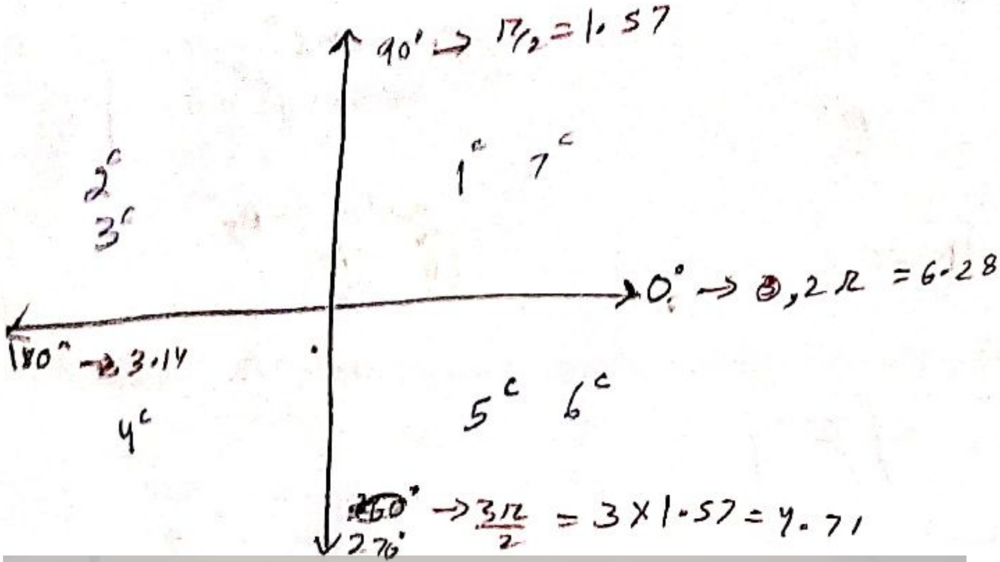
$$= \frac{-7\times 540}{22}$$

$$\boxed{= -171.8^\circ}$$

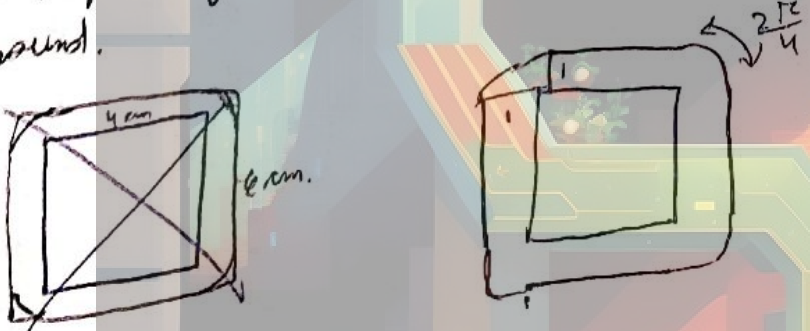


Q Mark Angles & find Quadrants, without Degree

$1^c, 2^c, 3^c, 4^c, 5^c, 6^c, 7^c$



Q A student of $TN_1 M_2$ walks on a road, A dog runs behind him then the student started running. The student runs at a distance of 1 cm outside from the boundary of a park. Assume park is square of side 4 cm. How much distance will he cover in 1 round.



$(4\times4) + \frac{2\pi}{4}\times4$

$2\pi + 16$

$16 + 6.28$

$\boxed{22.28}$

H.W.

Ch-5 - O-4

Ch-6 DYS-1

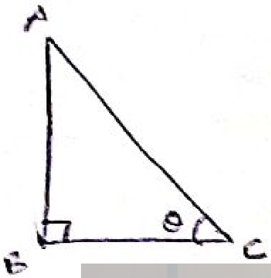
DYS-2

H.W. 11-7-24

Ch-5 J-M (Q1-10)

Note - In trignometry, we study some ratios of sides of right angle triangle wrt to its acute angle

→ we restrict our discussion to acute angles only. However these ratios can be extended to other angles also.



(sine) $\sin\theta = \frac{AB}{AC}$

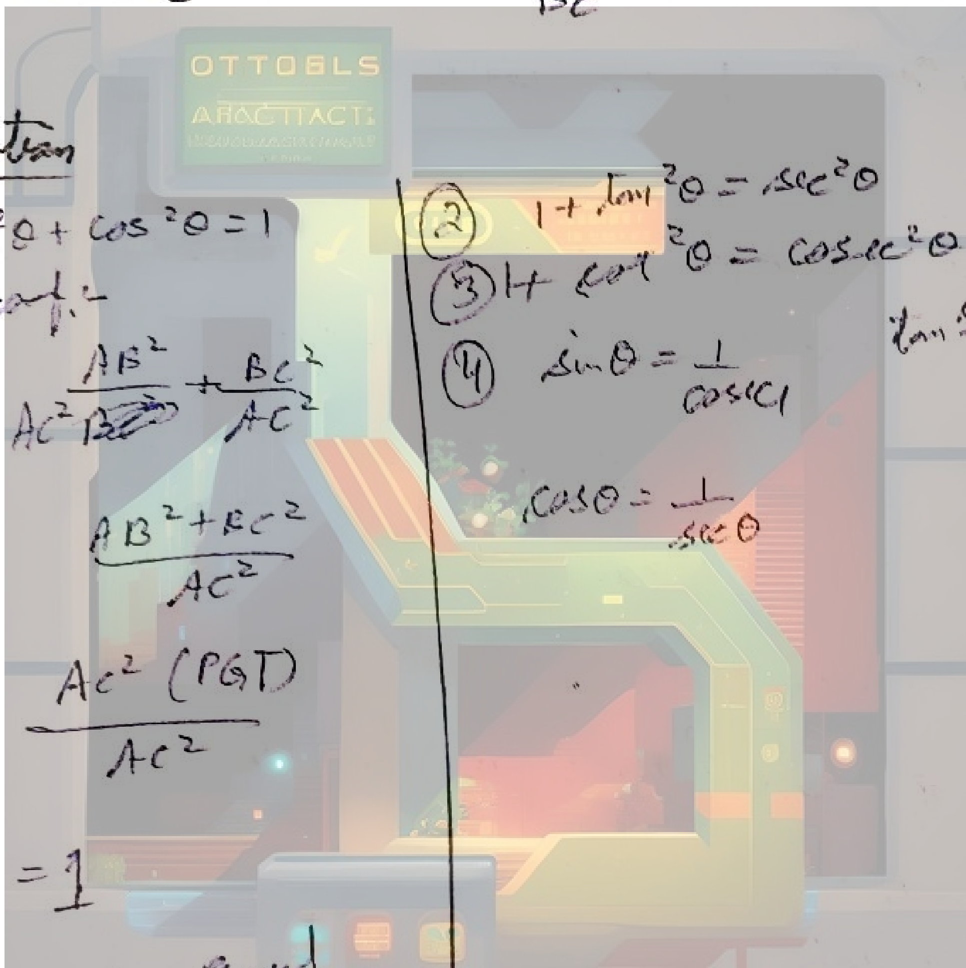
(cosine) $\cos\theta = \frac{BC}{AC}$

(tangent) $\tan\theta = \frac{AB}{BC}$

(co-secant) $\operatorname{cosec}\theta = \frac{AC}{AB}$

(secant) $\sec\theta = \frac{AC}{BC}$

(cotangent) $\cot\theta = \frac{BC}{AB}$



Observation

① $\sin^2\theta + \cos^2\theta = 1$

Proof:-

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$\frac{AB^2 + BC^2}{AC^2}$$

$$\frac{AC^2 \text{ (PGT)}}{AC^2}$$

$$= 1$$

Hence Proved

② $1 + \tan^2\theta = \sec^2\theta$

③ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

④ $\sin\theta = \frac{1}{\operatorname{cosec}}$

$\cos\theta = \frac{1}{\sec\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

General Mistakes :-

1. $\sin^2 A \neq \sin A^2$
2. $(\sin A)^2 \neq \sin A^2$
3. $\sin A + \sin B \neq \sin(A+B)$
4. $\sin\left(\frac{A}{2}\right) \neq \frac{\sin A}{2}$

Imp Results :-

1. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\cos^2\theta$

Proof :-

$$(\sin^2\theta + \cos^2\theta)^2 = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$$

$$1 - 2\sin^2\theta\cos^2\theta = \sin^4\theta + \cos^4\theta$$

Hence, Proved

2. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

3. $\tan\theta = \cot\theta - 2\cot 2\theta$

Q find value of $\sec\theta$ & $\tan\theta$ if $\sec\theta - \tan\theta = 4$

$$\sec\theta - \tan\theta = 4$$

$$1 = \sec^2\theta - \tan^2\theta$$

$$1 = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$$

$$\sec\theta + \tan\theta = \frac{1}{4}$$

$$2\sec\theta = \frac{17}{4}$$

$$\boxed{\sec\theta = \frac{17}{8}}$$

$$\boxed{\tan\theta = \frac{-15}{8}}$$

$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$ same for cot cosec

Quadrants :-

90°

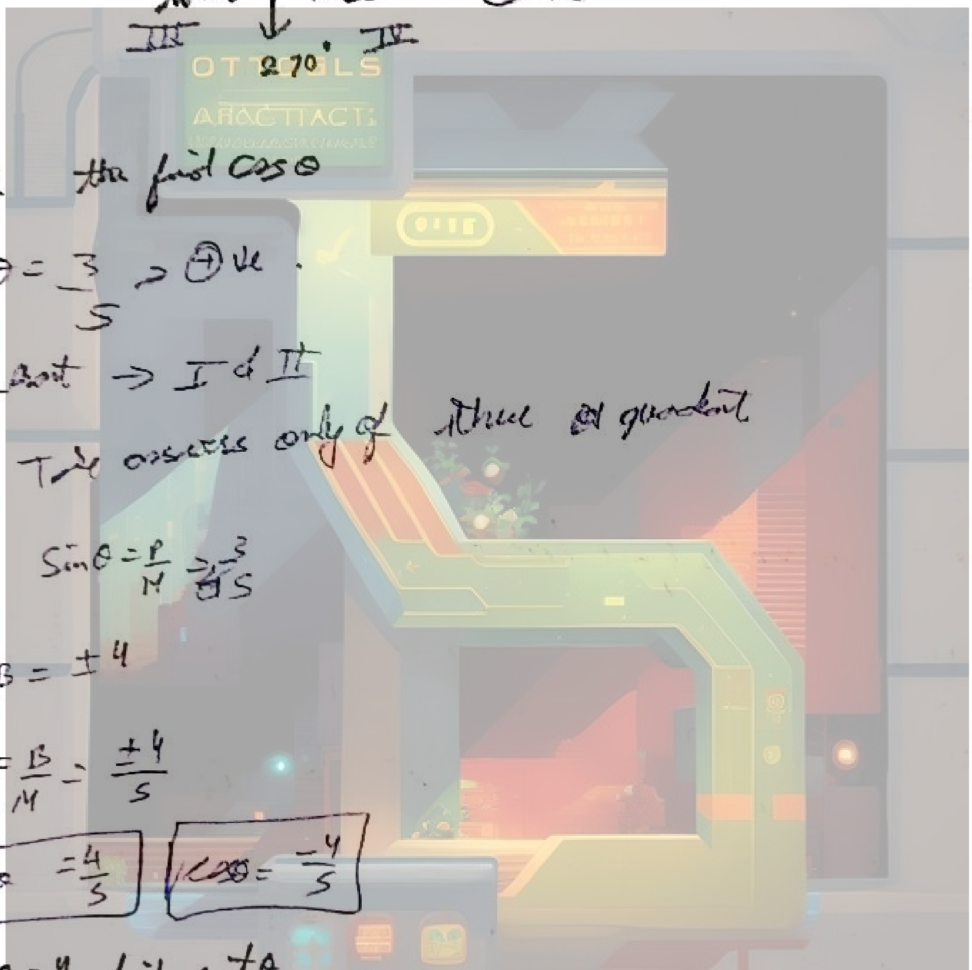
II: $\sec\theta$, $\operatorname{cosec}\theta$ | I: all θ

180° — 0°, 360°

III: $\cot\theta$, $\tan\theta$ | IV: $\sec\theta$, $\cos\theta$

270°

⊕ ve



Q $\sin\theta = \frac{3}{5}$ then find $\cos\theta$

$\sin\theta = \frac{3}{5} \rightarrow$ ⊕ ve

Quadrant $\rightarrow$ I & II

Take answers only of these quadrant

$\sin\theta = \frac{P}{H} \Rightarrow \frac{3}{5}$

$B = \pm 4$

$\cos\theta = \frac{B}{H} = \frac{\pm 4}{5}$

$\boxed{\cos\theta = \frac{4}{5}}$ $\boxed{\cos\theta = \frac{-4}{5}}$

Q If $\cos\theta = \frac{4}{5}$ find $\cot\theta$

$\cos\theta = \frac{4}{5}$

Quadrant $\rightarrow$ I & IV

$\frac{B}{H} = \frac{4}{5}$ $P = \pm 3$

$\cot\theta = \frac{\pm 4}{3}$

$\boxed{\cot\theta = \frac{4}{3}}$, $\boxed{\frac{-4}{3}}$

Q If $\tan\theta = \frac{1}{\sqrt{7}} = \frac{P}{B}$

$H^2 = 1 + 7$

$H = \pm 2\sqrt{2}$

$\operatorname{cosec}\theta = \boxed{\pm 2\sqrt{2}}$ $\qquad \sec\theta = \pm\frac{2\sqrt{2}}{\sqrt{7}}$

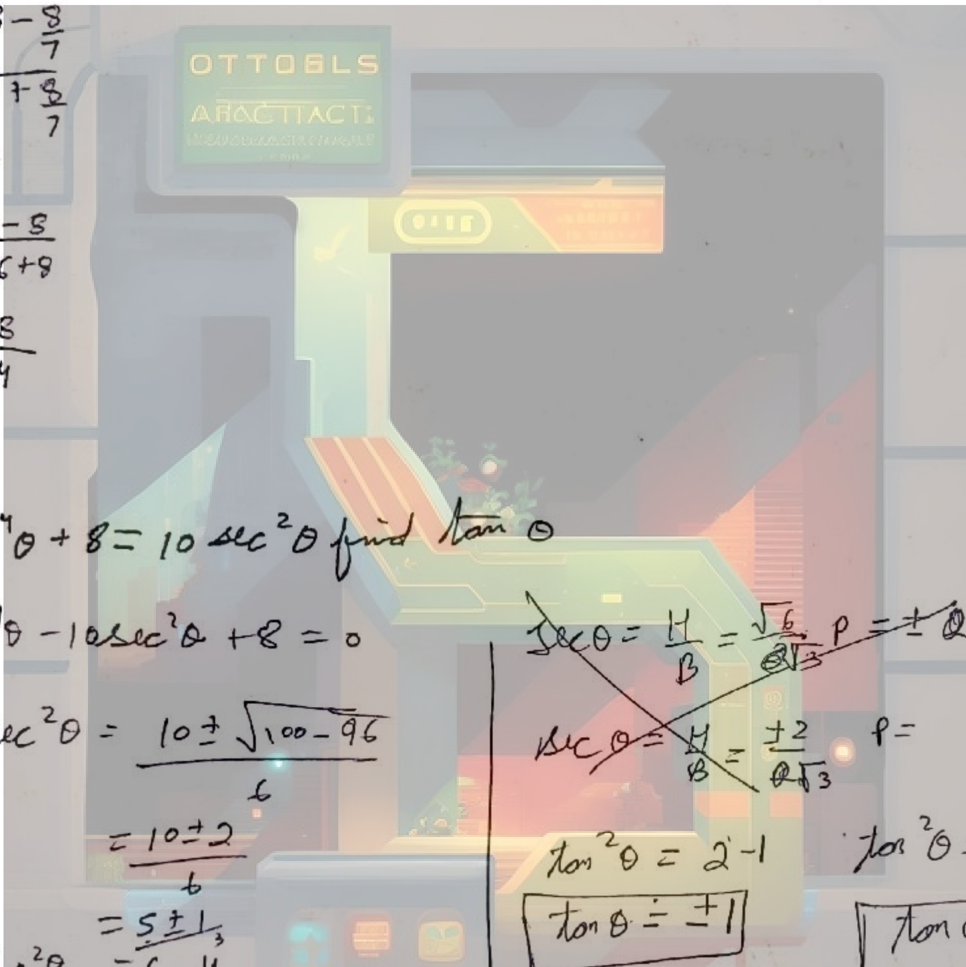
$\operatorname{cosec}^2\theta = 8$ $\qquad \sec^2\theta = 8/7$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{56-8}{56+8}$$

$$= \frac{48}{64}$$

$$\boxed{= \frac{3}{4}}$$



Q $3\sec^4\theta + 8 = 10\sec^2\theta$ find $\tan\theta$

$3\sec^4\theta - 10\sec^2\theta + 8 = 0$

$$\sec^2\theta = \frac{10 \pm \sqrt{100-96}}{6}$$

$$= \frac{10 \pm 2}{6}$$

$$= 5 \pm \frac{1}{3}$$

$\sec^2\theta = \frac{6}{3}, \frac{4}{3}$

$\sec\theta = \pm\frac{\sqrt{6}}{\sqrt{3}}, \pm\frac{2}{\sqrt{3}}$

I & IV Quad

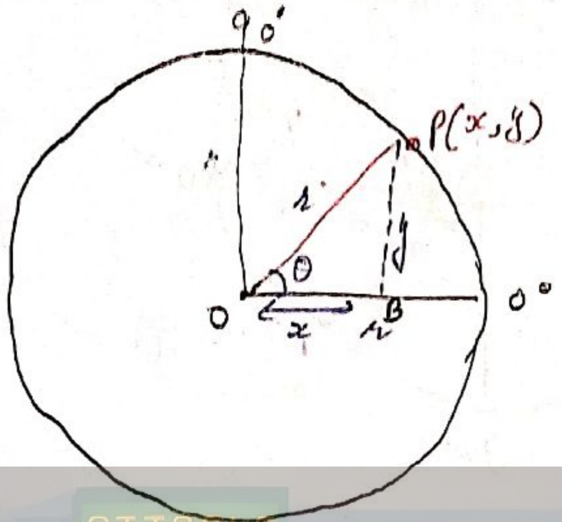
$\cancel{\sec\theta = \frac{H}{B} = \frac{\sqrt{6}}{\sqrt{3}}, P = \pm\sqrt{5}}$

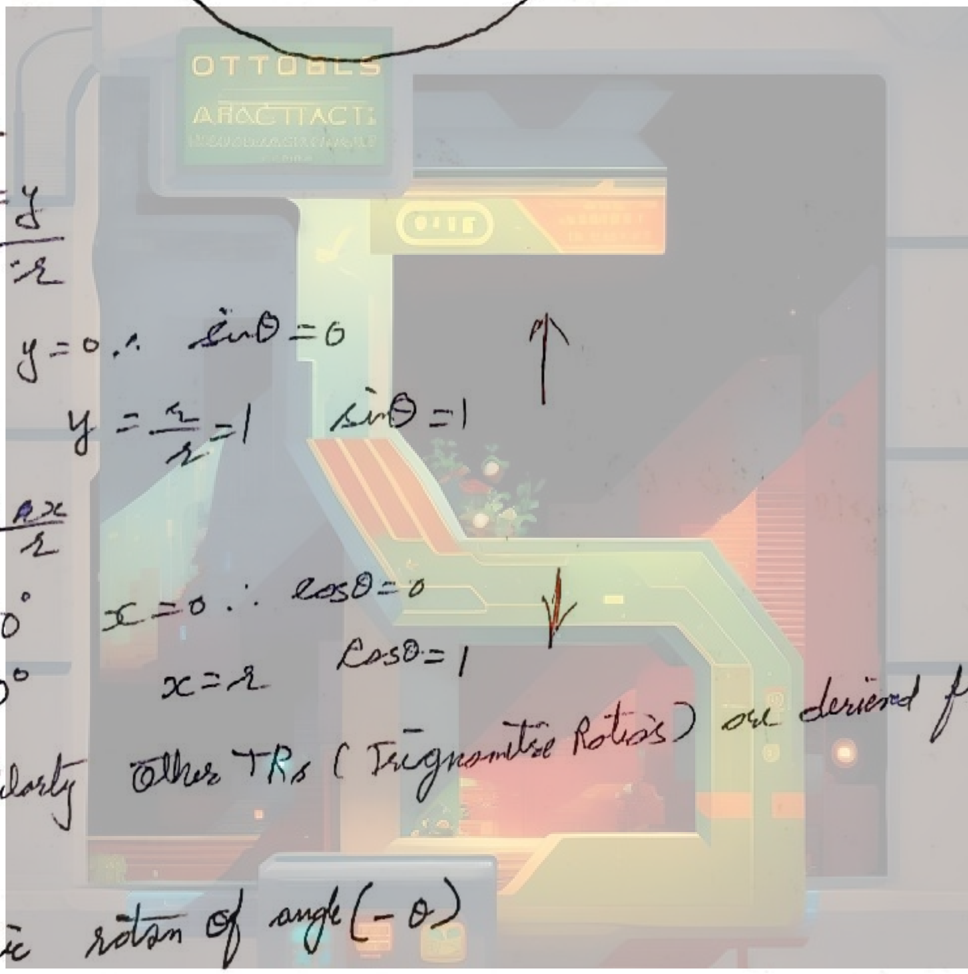
$\cancel{\sec\theta = \frac{H}{B} = \frac{\pm 2}{\sqrt{3}}}$ $P =$

$\tan^2\theta = 2 - 1$ $\qquad \tan^2\theta = \frac{1}{3}$

$\boxed{\tan\theta = \pm 1}$ $\qquad \boxed{\tan\theta = \pm\frac{1}{\sqrt{3}}}$

Trignometric Table :-





$\triangle OPB$:-

$\sin\theta = \frac{y}{r}$

$\theta \to 0°$ $y = 0 \therefore \sin\theta = 0$ ↑

$\theta \to 90°$ $y = r = \frac{r}{r} = 1$ $\sin\theta = 1$

$\cos\theta = \frac{x}{r}$

$\theta \to 90°$ $x = 0 \therefore \cos\theta = 0$ ↓

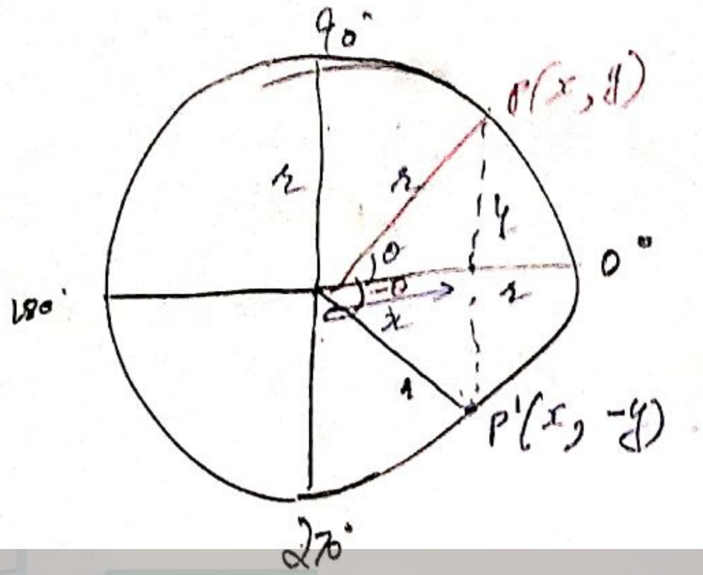
$\theta \to 0°$ $x = r$ $\cos\theta = 1$

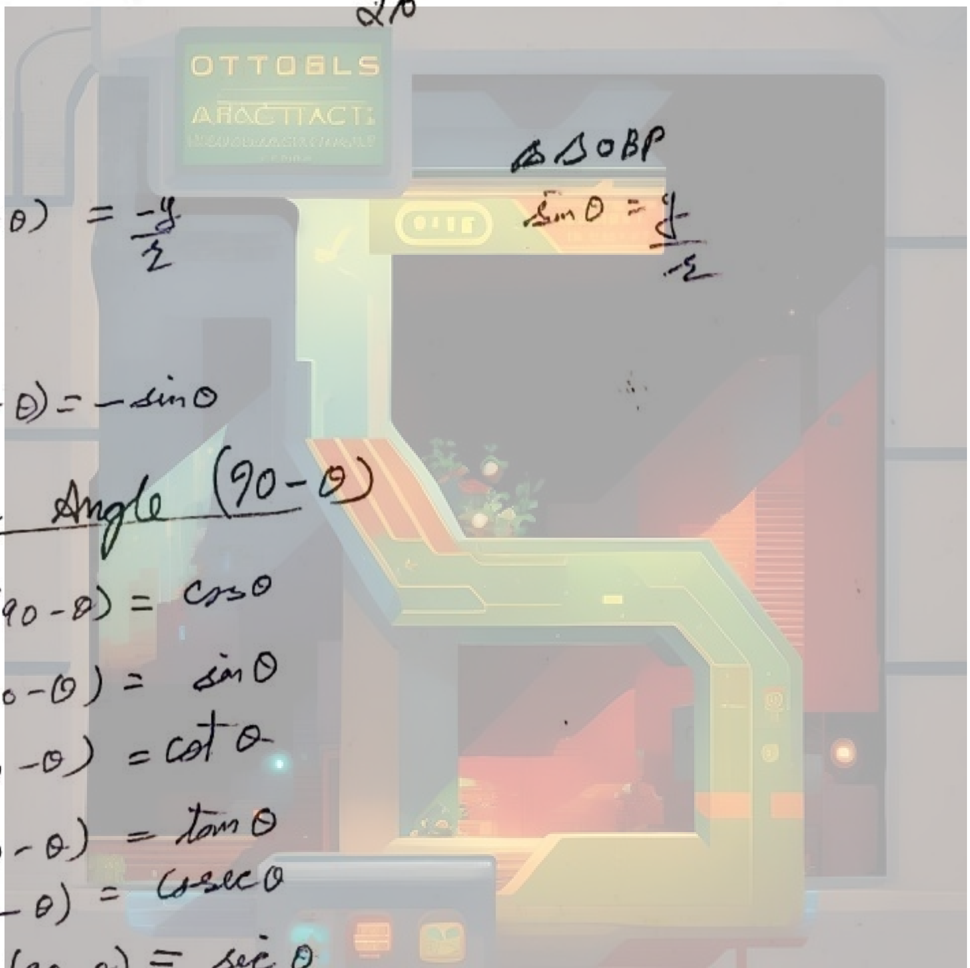
$\therefore$ Similarly other TRs (Trignometric Ratios) are derived from these two

Trignometric ratios of angle $(-\theta)$

1. $\sin(-\theta) = -\sin\theta$
2. $\cos(-\theta) = \cos\theta$
3. $\tan(-\theta) = -\tan\theta$
4. $\cot(-\theta) = -\cot\theta$
5. $\sec(-\theta) = \sec\theta$
6. $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Proof:-





$\Delta OBP'$

$\sin(-\theta) = \frac{-y}{r}$

ΔOBP

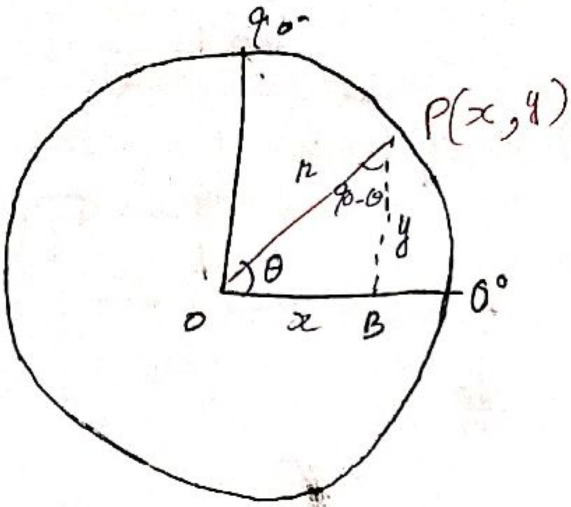
$\sin\theta = \frac{y}{r}$

$\sin(-\theta) = -\sin\theta$

TR of Angle $(90-\theta)$

1. $\sin(90-\theta) = \cos\theta$
2. $\cos(90-\theta) = \sin\theta$
3. $\tan(90-\theta) = \cot\theta$
4. $\cot(90-\theta) = \tan\theta$
5. $\sec(90-\theta) = \csc\theta$
6. $\csc(90-\theta) = \sec\theta$

Proof:-



$\sin(90-\theta) = \frac{x}{r}$ $\quad \cos\theta = \frac{x}{r}$

$\sin(90-\theta) = \cos\theta$

sin cosec (+)	All (+)
tan cot (+)	secθ cosθ (+)

$\sin \xleftrightarrow{90/270} \cos$

$\tan\theta \xleftrightarrow{90/270} \cot$

$\sec \xleftrightarrow{90/270} \text{cosec}$

0°/180°/360°
No change.

→ For any Angle > 360°, Break the Angle in multiple of 360° & remove the part of 360°

Q $\tan(150°) = \tan(180 - 30) = -\tan 30° = -\frac{1}{\sqrt{3}}$

Q $\cos(210)$ $\quad \cos(180+30) = \cancel{\sin 30}\ -\cos 30° = -\frac{\sqrt{3}}{2}$

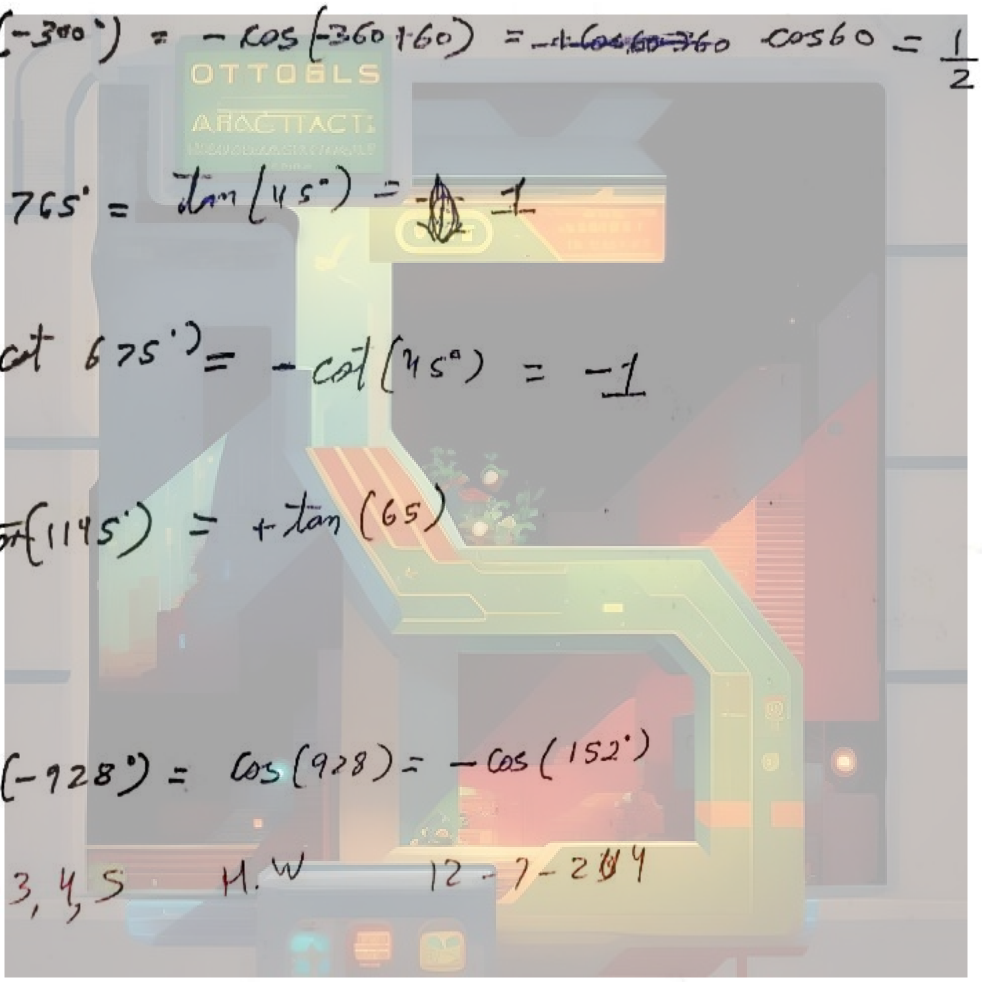
① $\tan(480^\circ) = \tan(120^\circ) = \tan(90+30) = -\cot 30$

$\boxed{= -\sqrt{3}}$

② $\cos(510) = \cos(150^\circ) = \cos(180-30) = -\cos(30)$

$\boxed{= -\frac{\sqrt{3}}{2}}$

③ $\sin(420^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$



② $\cos(-300^\circ) = -\cos(-360+60) = \cancel{-1 \cos 60 - 360}\ \cos 60 = \frac{1}{2}$

③ $\tan 765^\circ = \tan(45^\circ) = 1$

④ $\cot 675^\circ = -\cot(45^\circ) = -1$

⑤ $\tan(1145^\circ) = +\tan(65)$

⑥ $\cos(-928^\circ) = \cos(928) = -\cos(152^\circ)$

DYS - 3, 4, 5 H.W 12-7-2024

Q Solve the following.

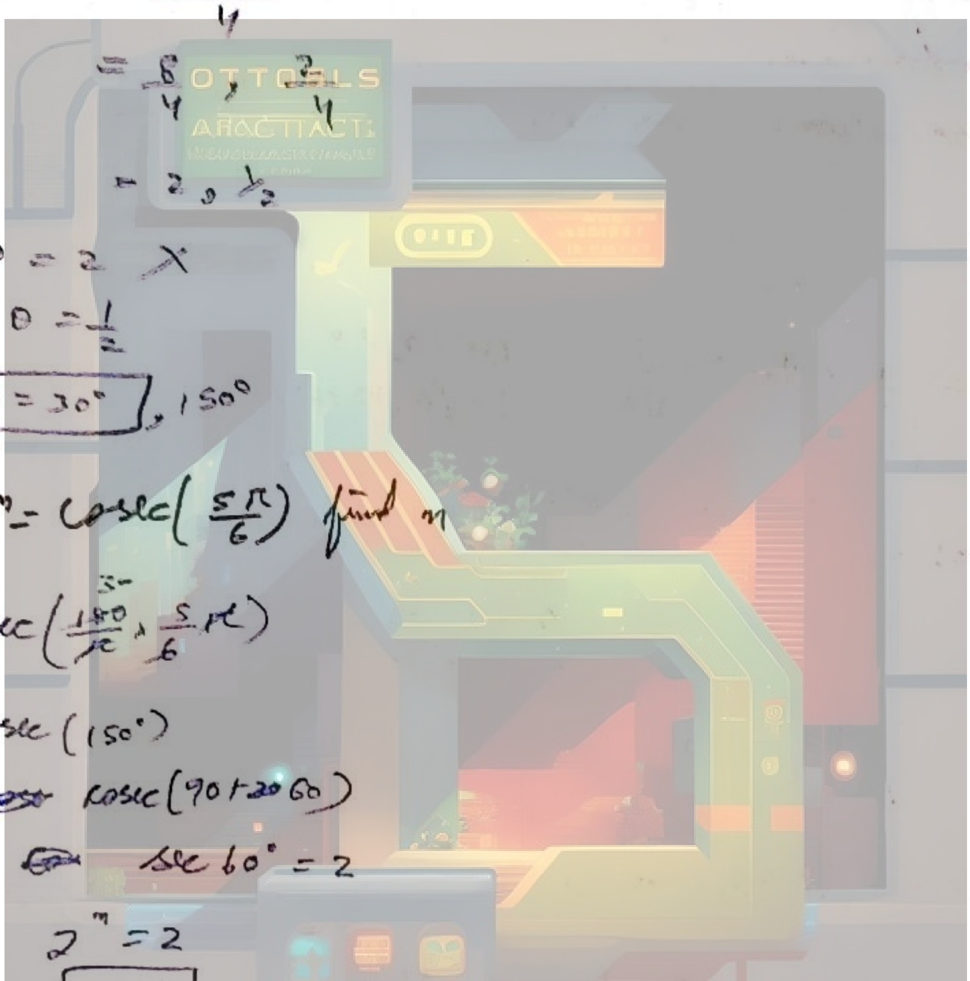
$(2\sin\theta-1)(\sin\theta-2)=0$ in $(0,\pi)$

$2\sin^2\theta-4\sin\theta-\sin\theta+2=0$

$2\sin^2\theta-5\sin\theta+2=0$

$\sin\theta=\frac{5\pm\sqrt{25-16}}{4}$

$=\frac{5\pm3}{4}$



$=\frac{8}{4},\frac{2}{4}$

$=2,\ \frac{1}{2}$

$\sin\theta=2$ ✗

$\sin\theta=\frac{1}{2}$

$\boxed{\theta=30^\circ}, 150^\circ$

Q If $2^n=\csc\left(\frac{5\pi}{6}\right)$ find n

$\csc\left(\frac{\cancel{180}^{30}}{\pi}\times\frac{5}{\cancel{6}}\pi\right)$

$\csc(150^\circ)$

~~cos~~ $\csc(90+\cancel{30}\,60)$

~~cos~~ $\sec 60^\circ=2$

$2^n=2$

$\boxed{n=1}$

Q. 3. $\sin\theta + \cos\theta = 0$ & θ lies in 4^{th} Quad find θ?

in 4^{th} Quad.

$\sin\theta = \ominus ve$

$\cos\theta = \oplus ve$

$-\sin\theta = \cos\theta$

~~$\sin\theta$~~ $45°$

$\theta = 270° + x$

~~$\frac{1}{\sqrt{3}}$~~

$\sin\theta = \ominus ve \; \cos\theta$

~~$\cos\theta$~~ $= \cos\theta$

~~$\cos$~~ $\cos(\cancel{180} x) = \cos(270° + x)$

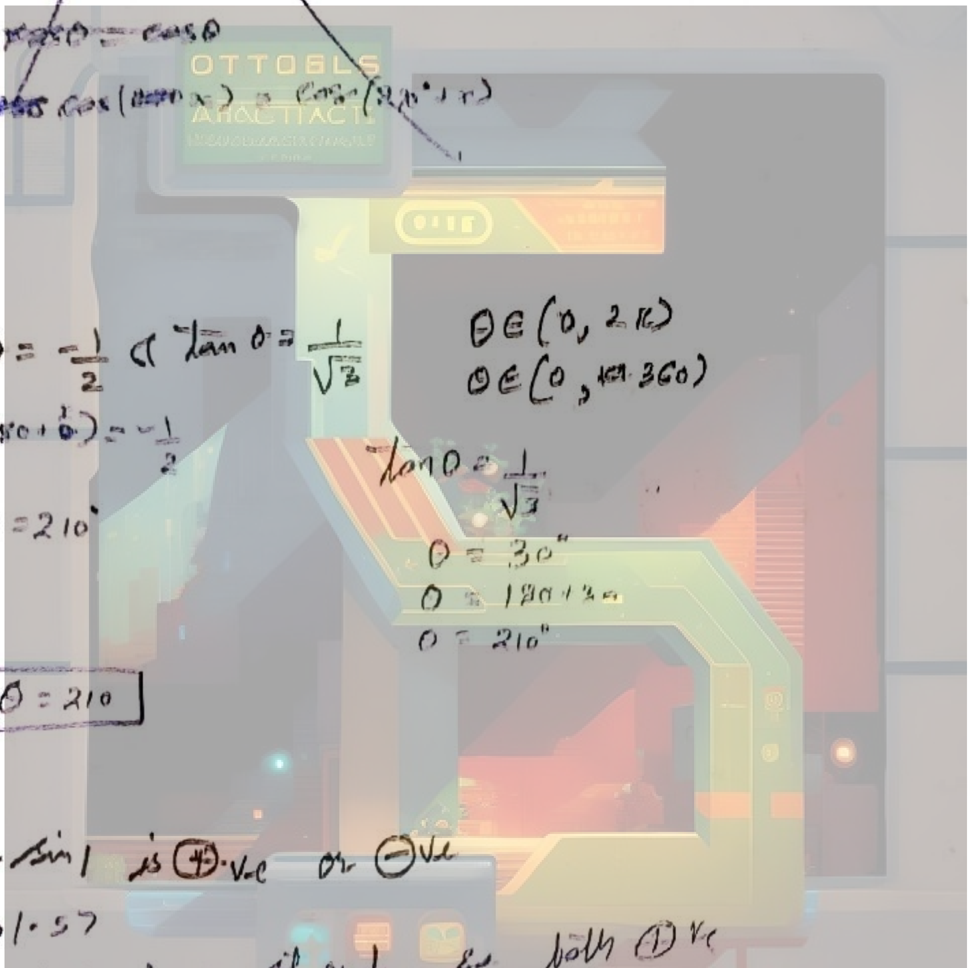
$\sin\theta = -\cos\theta$

$\tan\theta = -1$

~~$\tan\theta$~~ in 4^{th} Quad.

$\tan\theta = \ominus ve$

$\boxed{\theta = 45° \; -45°}$



Q 4. $\sin\theta = -\frac{1}{2}$ & $\tan\theta = \frac{1}{\sqrt{3}}$

$\theta \in (0, 2\pi)$

$\theta \in (0, 360)$

$\sin(180 + \theta) = -\frac{1}{2}$

$\theta = 210°$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

$\theta = 180 + 30$

$\theta = 210°$

$\boxed{\theta = 210}$

Q 5. $\cos 1 - \sin 1$ is $\oplus$ ve or $\ominus$ ve

$\frac{\pi}{2} \to 1.57$

So both in 1^{st} Quad, so both $\oplus$ ve

$\sin 1 > \cos 1$

$\ominus$ ve

$180-\theta$	θ
$180+\theta$	$-\theta$ or $(360-\theta)$

eg $\tan\theta = \frac{-1}{\sqrt{3}}$ $\quad \theta = ?$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\quad \theta = 30^\circ$

$\tan\theta \rightarrow$ ⊖ve — 2 nd $\quad 180^\circ - 30^\circ \Rightarrow 150^\circ$

— 4 th $\quad -\theta \Rightarrow -30^\circ$

$360-\theta \rightarrow 330^\circ$

Q find sign of the following.

(1) $\log_{\cos x} \sec x$

$\log_{\cos x} \frac{1}{\cos x}$

$\boxed{= -1}$ ⊖ve

(2) $\log_{\cot 30^\circ} \tan 60^\circ$

$\log_{\sqrt{3}} \sqrt{3}$

$\boxed{= 1}$ ⊕ve

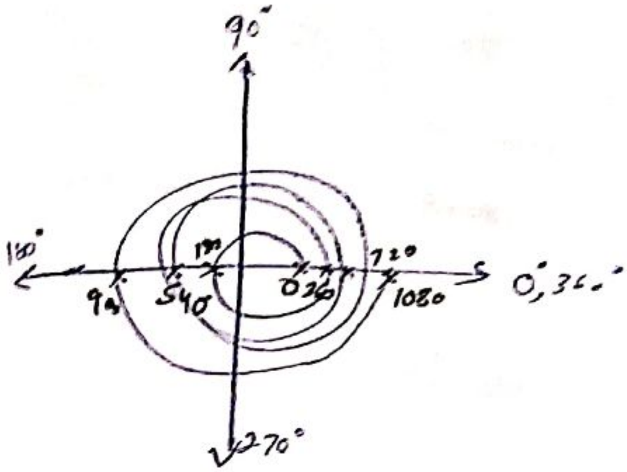
(3) $\log_{\sec 30^\circ} \tan 240^\circ$

$\log_{\sec 30^\circ} \tan 60^\circ$

$\log_{\frac{2}{\sqrt{3}}} \sqrt{3}$ $\quad \boxed{⊕ve}$

Q Note:

$\sin(n\pi) = \sin(180^\circ \times n) = 0$



① sin(integer multiple of π) = 0

② Cos(even integer multiple of π) = 1

③ cos(odd integer multiple of π) = −1

④ cos(odd integer multiple of $\frac{\pi}{2}$) = 0

Eg:- ① $n=1 \quad \sin\pi = \sin 180^\circ = 0$

$n=2 \quad \sin 2\pi = \sin 360^\circ = 0$

$n=3 \quad \sin 3\pi = \sin 540^\circ = \sin 180^\circ = 0$

$n=-3 \quad \sin(-3\pi) = -\sin 3\pi = -0 = 0$

$\sin 0^\circ = 0$

$\sin 180^\circ = 0$

$\sin 360^\circ = 0$

④ $n = 3 = \cos\left(\frac{3\pi}{2}\right) = \cos 270^\circ = \cos(180^\circ + 90^\circ) = -\cos 90^\circ = 0$

$n = 7 : \cos\left(\frac{7\pi}{2}\right) = \cos 630^\circ = \cos 270^\circ = 0$

$n = -9 = \cos\left(-\frac{9\pi}{2}\right) = \cos 810^\circ = \cos 90^\circ = 0$

③ $n = -3 = \cos(-3\pi) = \cos 3\pi = \cos 540^\circ = \cos 180^\circ = -\sin 90 = -1$

Q find x.

① $\sin(2022\pi) + x + 1 = 0$

$0 + x + 1 = 0$

$\boxed{x = -1}$

② $x^2 - 1 + \sec(2026\pi) = 1$

$x^2 = 1$

$\boxed{x = \pm 1}$

③ $\log_{10}(x-1)\left[\tan(556\pi) + 1\right] = 0$

$\log_{10}(x-1)[1] = 0$

$10^0 = x - 1$

$x - 1 = 1$

$\boxed{x = 2}$

Q find $\dfrac{\sin(505\pi)+\cos(2004\pi)}{\tan(88\pi)+\sec(550\pi)}$

$$\frac{0+1}{0+1} = \boxed{1}$$

Q two angle are complementary & of is 4 times as big as the other.

$4x = (90-x)$

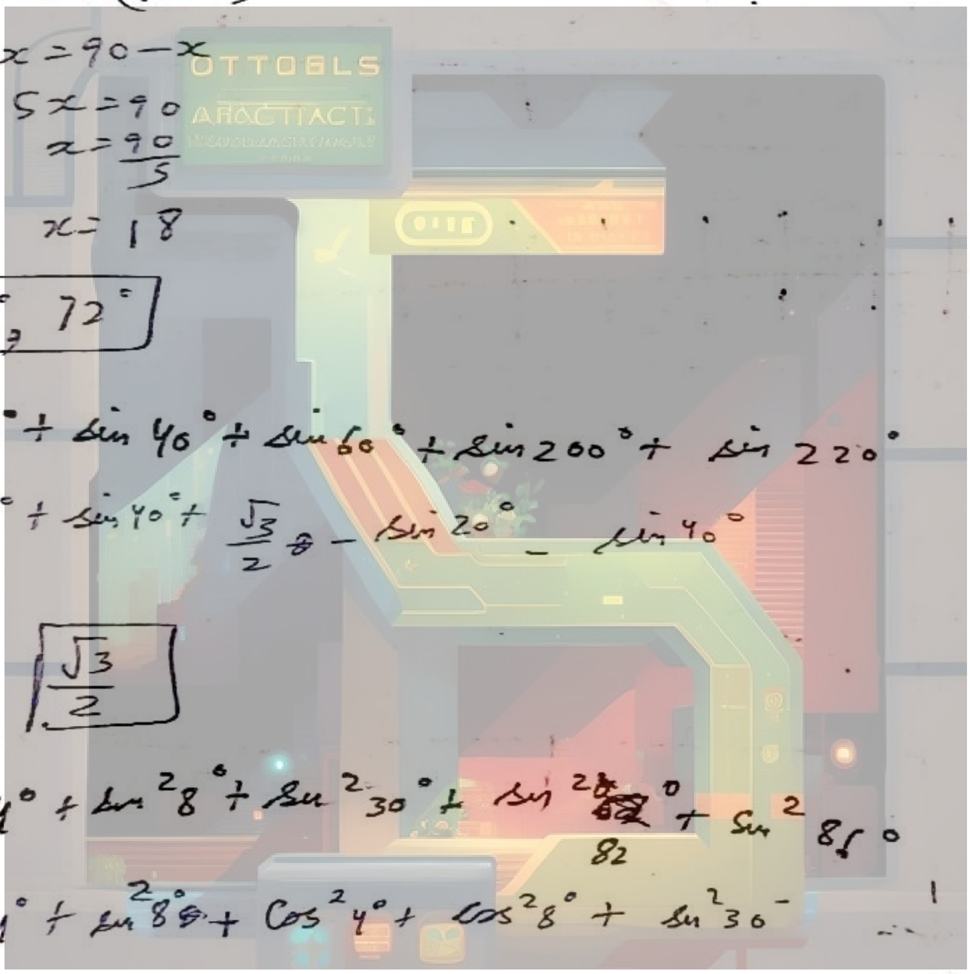
$4x = 90 - x$

$5x = 90$

$x = \dfrac{90}{5}$

$x = 18$

$\boxed{18^\circ, 72^\circ}$



Q $\sin 20^\circ + \sin 40^\circ + \sin 60^\circ + \sin 200^\circ + \sin 220^\circ$

$\sin 20^\circ + \sin 40^\circ + \frac{\sqrt{3}}{2} - \sin 20^\circ - \sin 40^\circ$

$\boxed{\dfrac{\sqrt{3}}{2}}$

Q $\sin^2 4^\circ + \sin^2 8^\circ + \sin^2 30^\circ + \sin^2 82^\circ + \sin^2 86^\circ$

$\sin^2 4^\circ + \sin^2 8^\circ + \cos^2 4^\circ + \cos^2 8^\circ + \sin^2 30$

$$2 + \left(\frac{1}{2}\right)^2 = 2 + \frac{1}{4} = \frac{8+1}{4} = \boxed{\frac{9}{4}}$$

Graphs of TF :-

~~① y = sin x~~

Domain - Possible values of x

Range - Possible values of y

Periodicity (Period) - repetitation ~~after~~ after a certain interval.

① $y = \sin x$

II Quad

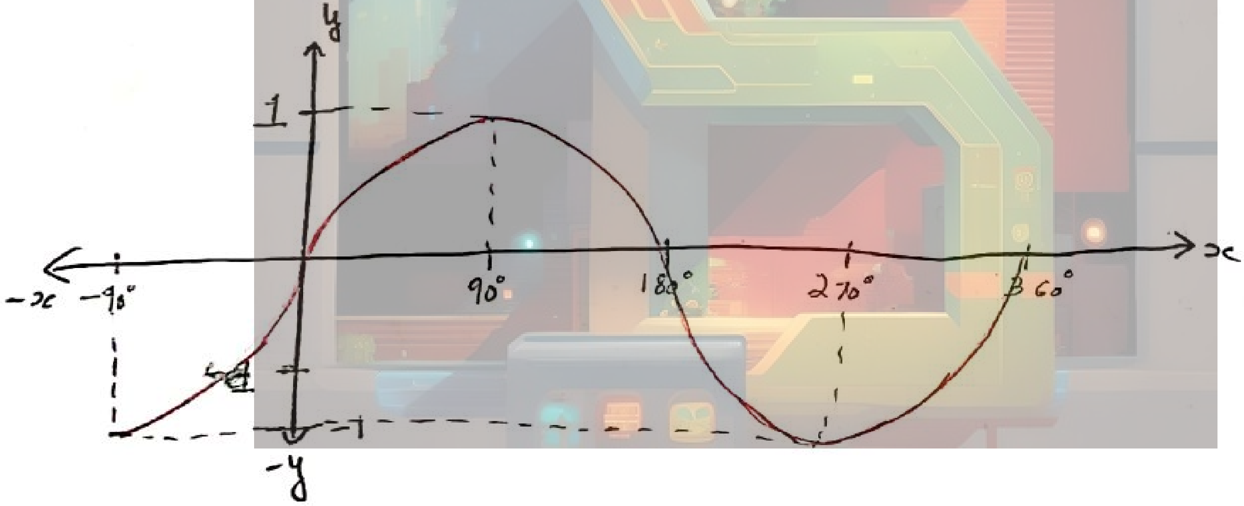
x	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
y	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

I Quad

IV

x	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
y	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0

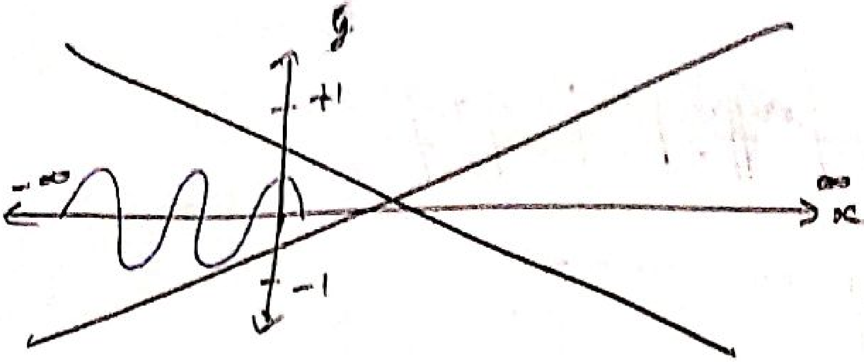
III



Domain:- $x \in R$

Range:- $y \in [-1, 1]$

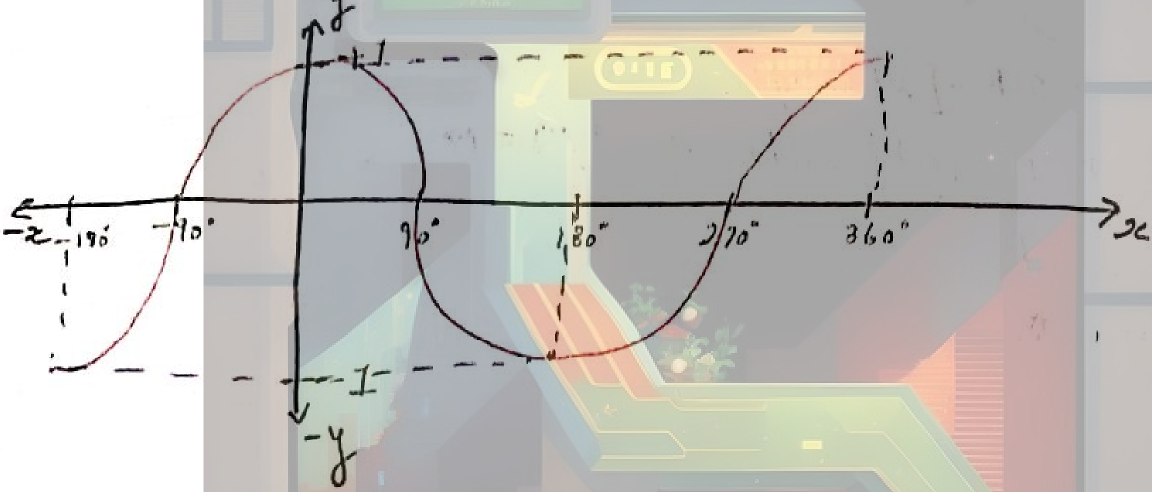
Period:- $2\pi/360°$



(2) $y = \cos x$

x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
y	1	$1/\sqrt{2}$	0	$1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0	$-1/\sqrt{2}$	$+1$

I II III IV



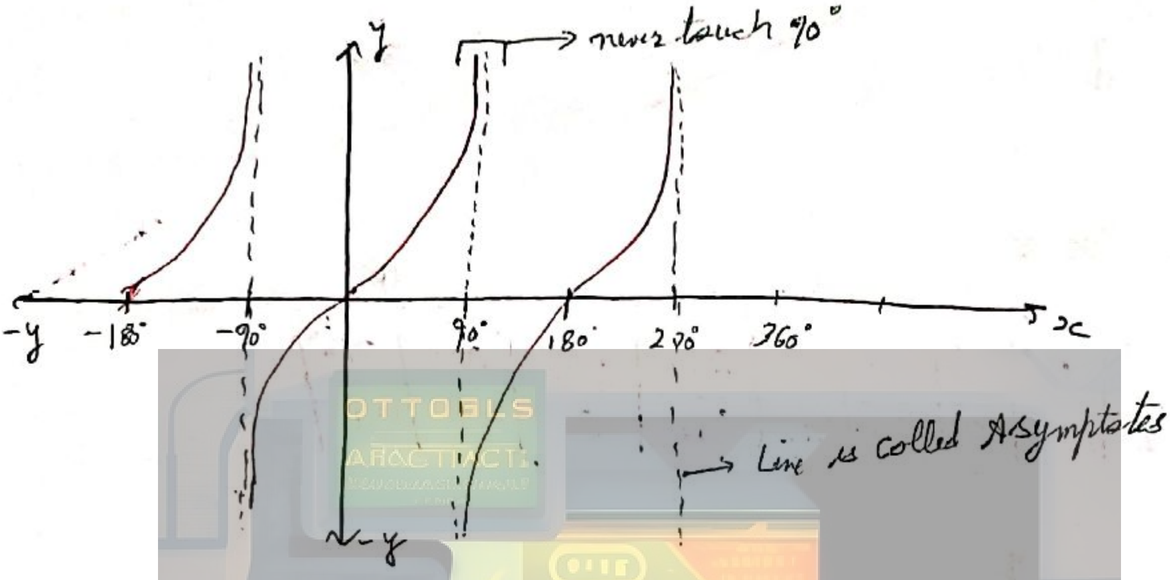
Domain:- $x \in R$

Range:- $y \in [-1, 1]$

Period:- $2\pi / 360°$

③ $y = \tan x$

x	$0°$	$90°$	$180°$	$270°$	$360°$	$450°$	$540°$	$630°$	$720°$
y	0	∞	0	∞	0	∞	0	∞	0



Domain :- ~~$x \in R - [$all multiples~~

$x \in R -$ (odd multiples of $90°$) / $R - \frac{(2n-1)\pi}{2}$

Range :- $y \in R$

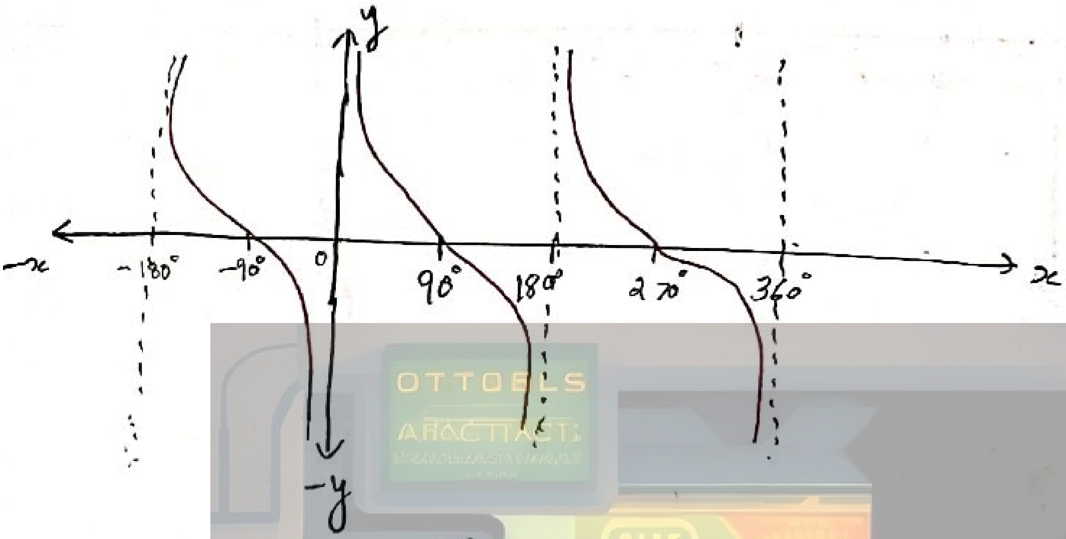
Period :- π or $180°$

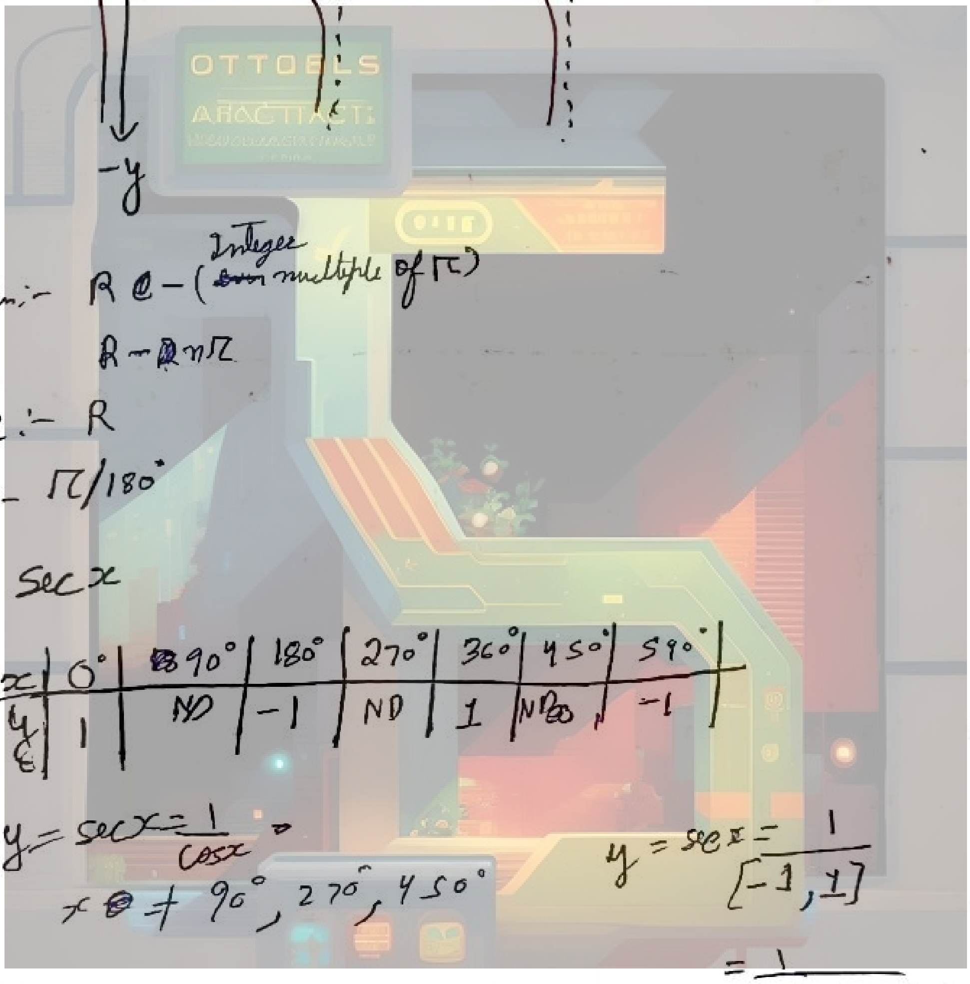
H.W 16-07-24

JM (sequence)
JA (Sequence)

④ $y = \cot x$

x	$0°$	$90°$	$180°$	$270°$	$360°$	$450°$	540
y	ND	0	ND	0	ND	0	ND





Domain :- $R - ($~~even~~ Integer multiple of $\pi)$

$R - n\pi$

Range :- R

Period :- $\pi / 180°$

⑤ $y = \sec x$

x	$0°$	$90°$	$180°$	$270°$	$360°$	$450°$	$540°$
y	1	ND	-1	ND	1	ND	-1

$y = \sec x = \frac{1}{\cos x}$

$x \neq 90°, 270°, 450°$

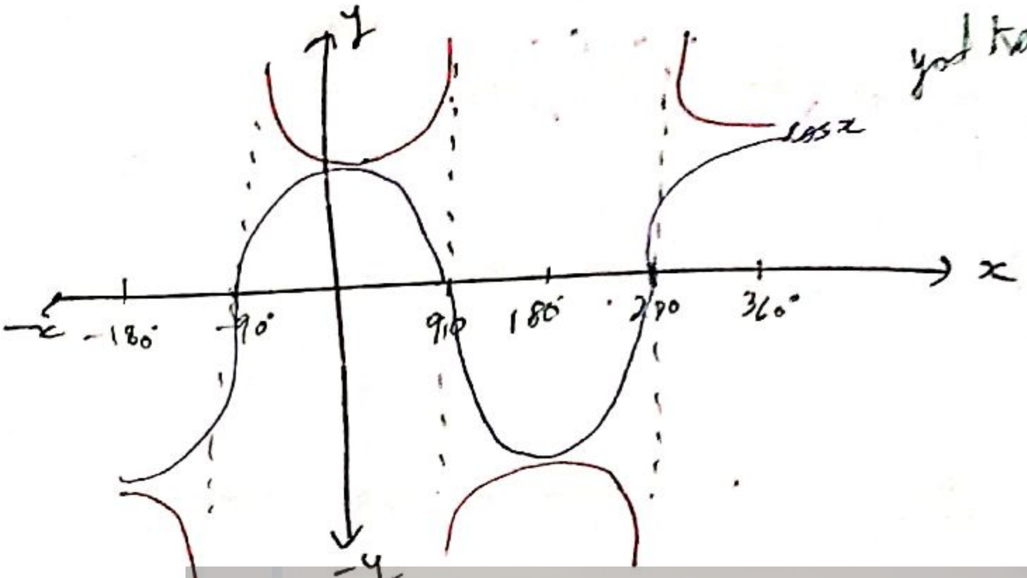
$y = \sec x = \frac{1}{[-1, 1]}$

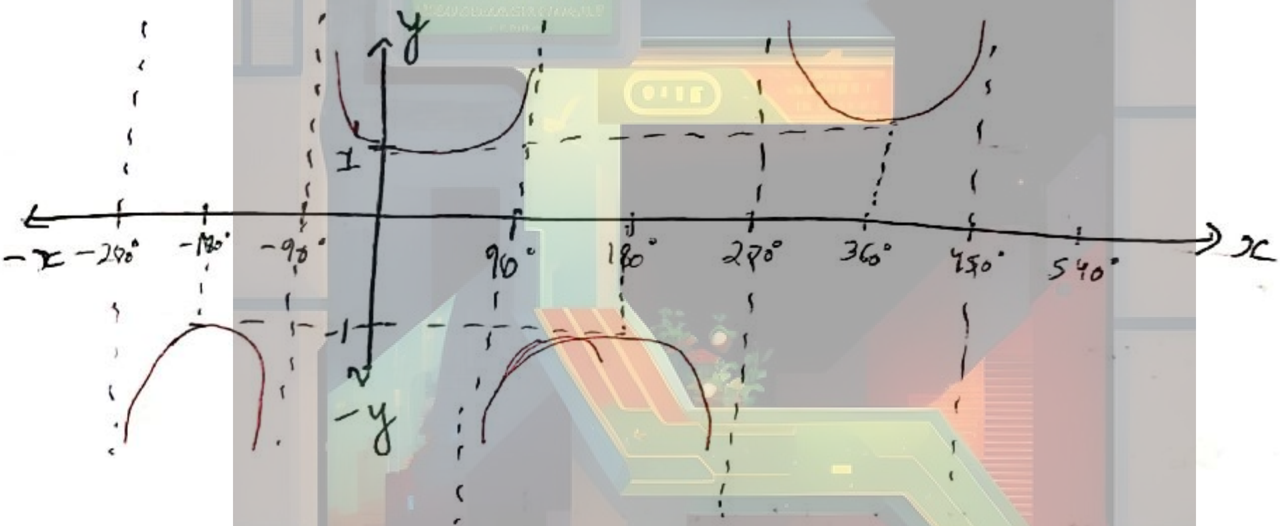
$= \frac{1}{[-1, 0) \cup (0, 1]}$

$\cancel{\geq [-1, \infty}$

$= (-\infty, -1] \cup [1, \infty)$

yad karo ki kya





Domain:- $x \in R - \{(2n-1)\frac{\pi}{2}\}$

Range:- $R - (-1, 1)$

Period:- 2π

(6) $y = \operatorname{cosec} x$

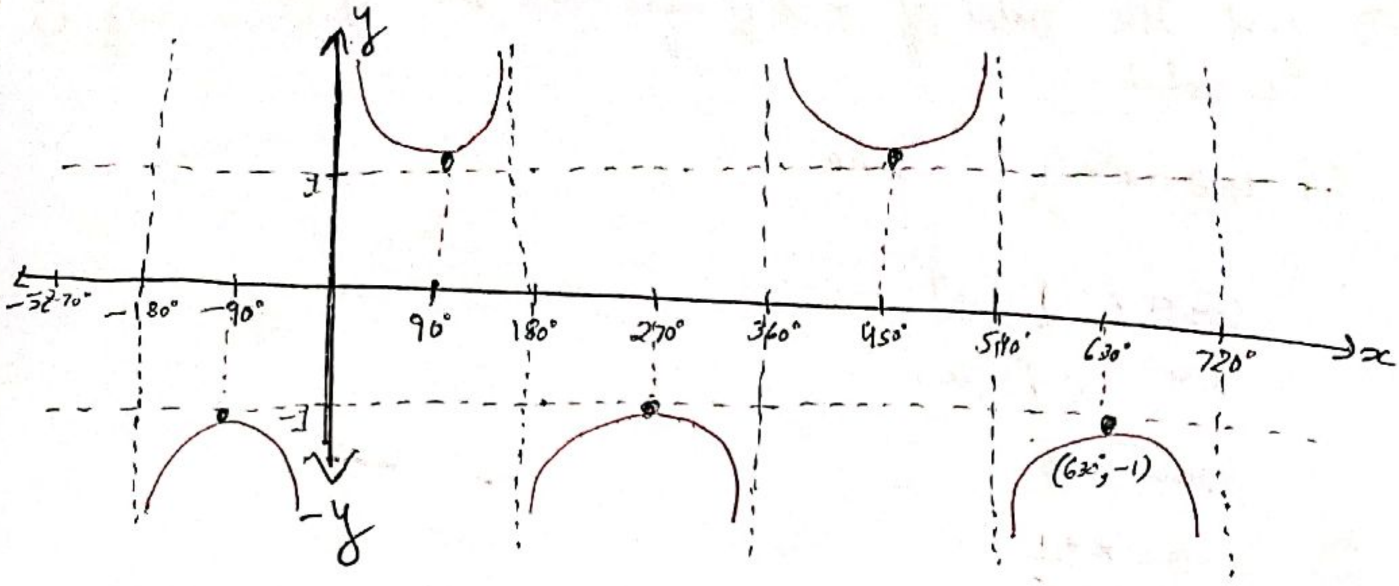
$y = \operatorname{cosec} x = \frac{1}{\sin x}$

$x \notin \{0, 180, 360\}$

$x \neq$ integer multiple of π

$y = \operatorname{cosec} x \in \frac{1}{[-1, 1]}$

$\operatorname{cosec} x \in (-\infty, -1] \cup [1, \infty)$



Domain:- $R - n\pi$

Range:- $R - (-1, 1)$

Period:- 2π

Conclusions:-

TF	Domain	Range	Period
① $y = \sin x$	$x \in R$	$y \in [-1, 1]$	2π
② $y = \cos x$	$x \in R$	$y \in [-1, 1]$	2π
③ $y = \tan x$	$x \in R - \{(2n-1)\frac{\pi}{2}\}$	$y \in R$	π
④ $y = \cot x$	$x \in R - n\pi$	$y \in R$	π
⑤ $y = \sec x$	$x \in R - \{(2n-1)\frac{\pi}{2}\}$	$y \in (-\infty, -1] \cup [1, \infty)$	2π
⑥ $y = \operatorname{cosec} x$	$x \in R - n\pi$	$y \in (-\infty, -1] \cup [1, \infty)$	2π

Q find the value of x & θ for which the equation is valid

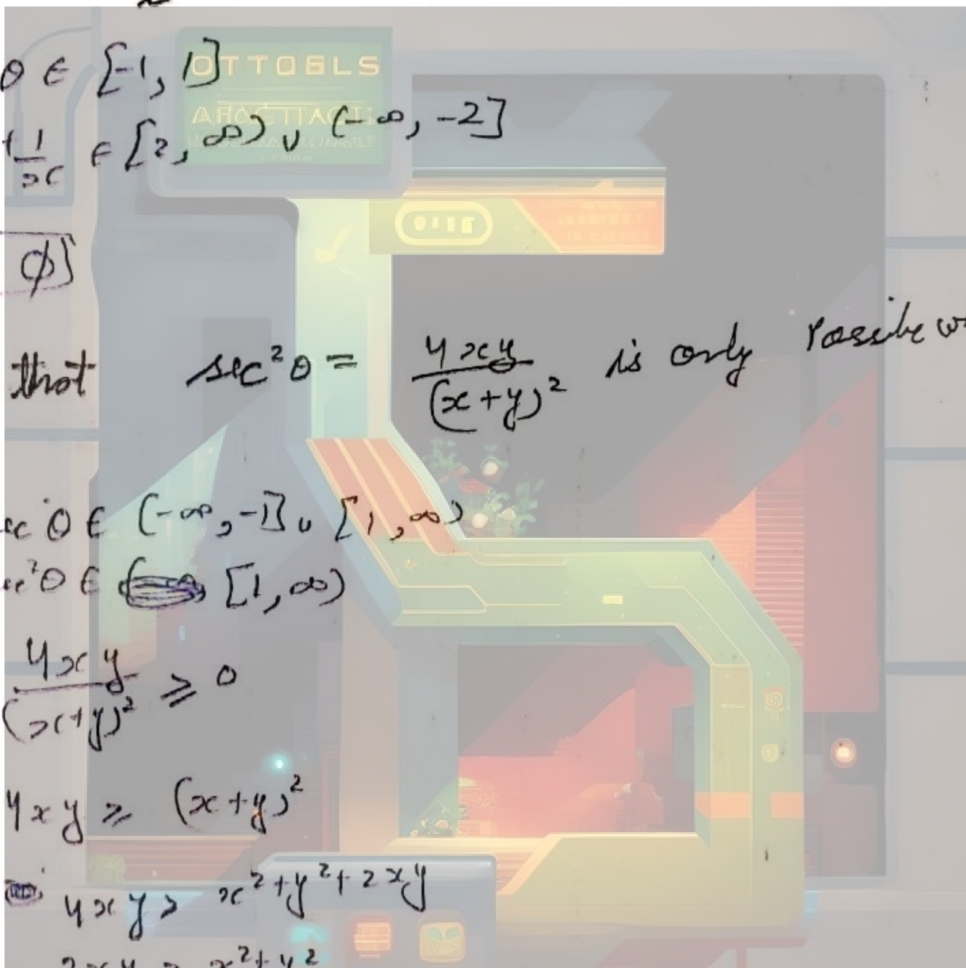
① $\cos\theta = x^2 + \frac{1}{x^2}$ $\qquad x \in R$

$\cos\theta \in [-1, 1]$

$x^2 + \frac{1}{x^2} \in [2, \infty)$

$\boxed{\text{solution} = \phi}$

② $\sin\theta = x + \frac{1}{x}$



$\sin\theta \in [-1, 1]$

$x + \frac{1}{x} \in [2, \infty) \cup (-\infty, -2]$

$\boxed{\theta \in \phi}$

Q Show that $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is only possible when $x = y$.

$\sec\theta \in (-\infty, -1] \cup [1, \infty)$

$\sec^2\theta \in [1, \infty)$

$\frac{4xy}{(x+y)^2} \geq 1$

$4xy \geq (x+y)^2$

$4xy \geq x^2 + y^2 + 2xy$

$2xy \geq x^2 + y^2$

$(x-y)^2 \leq 0$

$(x-y)^2 < 0$ ✗

$(x-y)^2 = 0$

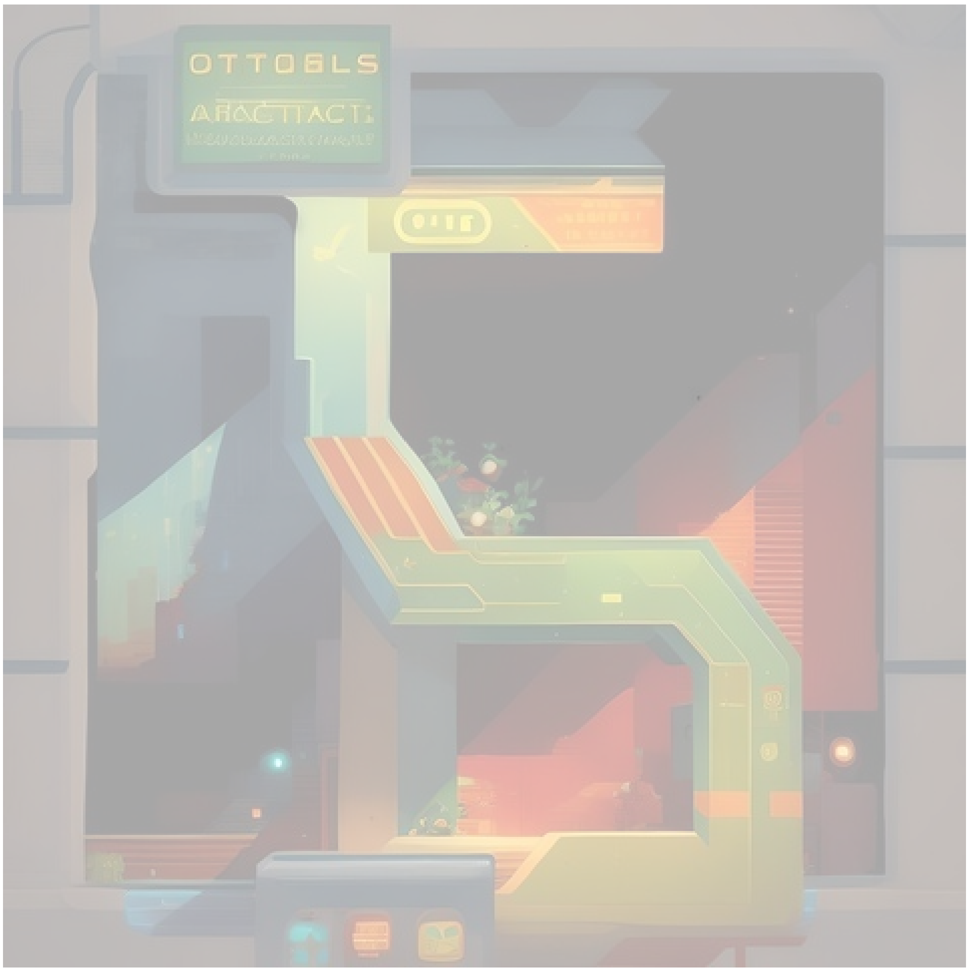
$\boxed{x = y}$ H.P.

Q find Solution $\sin(e^{x}) = 2^{x} + 2^{-x}$

$\sin(e^{x}) \in [-1, 1]$

$2^{x} + \frac{1}{2^{x}} \geq 2$

$\boxed{\phi}$



Trignometry

Sum & Difference formula

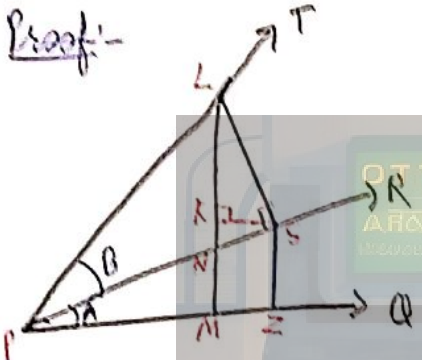
① $\sin(A+B) = \sin A \cos B + \cos A \sin B$

② $\sin(A-B) = \sin A \cos B - \cos A \sin B$

③ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

④ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

Proof:-



$\sin A = \frac{SZ}{PS}$ $\cos A = \frac{LK}{LS}$

$\sin B = \frac{LS}{PL}$ $\cos B = \frac{PS}{PL}$

$\sin(A+B) = \sin(\angle TPQ) = \frac{LM}{PL}$

$= \frac{LK + KM}{PL}$

$= \frac{LK}{PL} + \frac{SZ}{PL}$

$= \frac{LK}{LS} \times \frac{LS}{PL} + \frac{SZ}{PS} \times \frac{PS}{PL}$

$= \cos A \sin B + \sin A \cos B$

Hence, Proved

⑤ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Proof:- $\frac{\sin(A+B)}{\cos(A+B)} = \tan(A+B)$

$\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$

divide by $\cos A \cos B$

$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\cancel{\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}}$

$\cancel{\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}}$

(7) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

(8) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(9) $\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A \cos B}$

(10) $\tan A - \tan B = \dfrac{\sin(A-B)}{\cos A \cos B}$

(11) $\sin(A+B)\sin(A-B) = \cancel{\sin A \cos B}$

$\cancel{\sin^2 A - \cos}$

$= \sin^2 A - \sin^2 B$

$= \cos^2 B - \cos^2 A$

Proof:-

$(\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$

$\sin^2 A \cos^2 B + \cos^2 A \sin^2 B$

$\sin^2 A(1 - \sin^2 B) + \cancel{\cos^2 A(1 - \cos^2 B)}\ \sin^2 B(1 - \cos^2 A)$

$\sin^2 A - \sin^2 A \sin^2 B + \sin^2 B - \sin^2 B \cos^2 A$

$\sin^2 A + \sin^2 B$

(12) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$= \cos^2 B - \sin^2 A$

Transform Product into sum or difference.

(1) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

Proof

$\sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A\sin B}$

$2\sin A\cos B$.

(2) $2\sin B\cos A = \sin(A+B) - \cancel{\sin(A-B)}\ \sin(A-B)$

(3) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(4) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Transform the sum or difference into product.

(1) $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Proof:-

$A = C + D$

$B = C - D$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

(2) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

(3) $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

(4) $\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

$= -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Q find the values of ① $\sin 15^\circ$ ② $\cos 15^\circ$ ③ $\tan 15^\circ$ ④ $\cot 15^\circ$

⑤ $\sin 75^\circ$ ⑥ $\cos 75^\circ$ ⑦ $\tan 75^\circ$ ⑧ $\cot 75^\circ$

① $\sin(45^\circ - 30^\circ) = \sin 15^\circ = \sin 45 \cos 30 - \cos 45 \sin 30$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\boxed{\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

② $\cos(45^\circ - 30^\circ) = \cos 15^\circ = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \cancel{\cos} \sin 30^\circ$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\boxed{\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

③ $\tan 15^\circ = \tan(45^\circ - 30^\circ) = \dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\boxed{\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}} \boxed{= 2 - \sqrt{3}}$$

④ $\cot 15^\circ = \dfrac{\cot 45 \cot 30 + 1}{\cot 30 - \cot 45}$

$$\boxed{\cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}} \boxed{= 2 + \sqrt{3}}$$

(5) $\sin 75° = \sin(45°+30°) = \sin 45 \cos 30 + \sin 30 \cos 45$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\boxed{\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

(6) $\cos 75° = \cos(45°+30°) = \cos 45° \cos 30° - \sin 45° \cdot \sin 30$

$$\boxed{\cos 75° = \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

(7) $\tan 75° = \tan(45°+30°) = \dfrac{\tan 45° + \tan 30°}{1 - \tan 45 \tan 30}$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\boxed{\tan 75° = 2+\sqrt{23}}$$

(8) $\cot 75° = \cot(45°+30°) \Rightarrow \cancel{\cos 45° + \cot 30°}$

$$= \frac{\cot 45° \cot 30° - 1}{\cot 30° + \cot 45°}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\boxed{\cot 75° = 2-\sqrt{3}}$$

Q find values of following:-

(1) $\sin 99° \cos 21° + \cos 99° \sin 21°$

$= \sin(99°+21°)$

$= \sin(120°)$

$= \cos 30°$

$\boxed{= \frac{\sqrt{3}}{2}}$

② $\sin((n+1)A)\sin((n+2)A) + \cos((n+1)A)\cos((n+2)A)$

$\cos((n+1)A - (n+2)A)$

$\cos[\cancel{n}A + A - nA - 2A]$

$\cos[-A]$

$\boxed{\cos A}$

③ $\cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

④ $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$

⑤ $\sin^2(127.5^\circ) - \sin^2(112.5)$

⑥ $\cos^2\left(\frac{\theta-\phi}{2}\right) - \sin^2\left(\frac{\theta+\phi}{2}\right)$

⑦ If $\theta = 7.5^\circ$, then $\frac{\sin 8\theta\cos\theta - \sin 6\theta\cdot\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta}$

⑧ If A, B & C angles are in A.P., then find value of $\frac{\sin A - \sin C}{\cos C - \cos A}$ is.

(A) $\cot\left(\frac{A-C}{2}\right)$ (B) $\cot B$ (C) $\tan B$ (D) None

③ $\cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$\cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$\cos(120+45)$

$\cos(165^\circ)$

$= \cos(180-15)$

$= \cos 15^\circ$

$\boxed{= \frac{\sqrt{3}+1}{2\sqrt{2}}}$

⑥

(4) $\sin(A+B)\cdot\sin(A-B)$

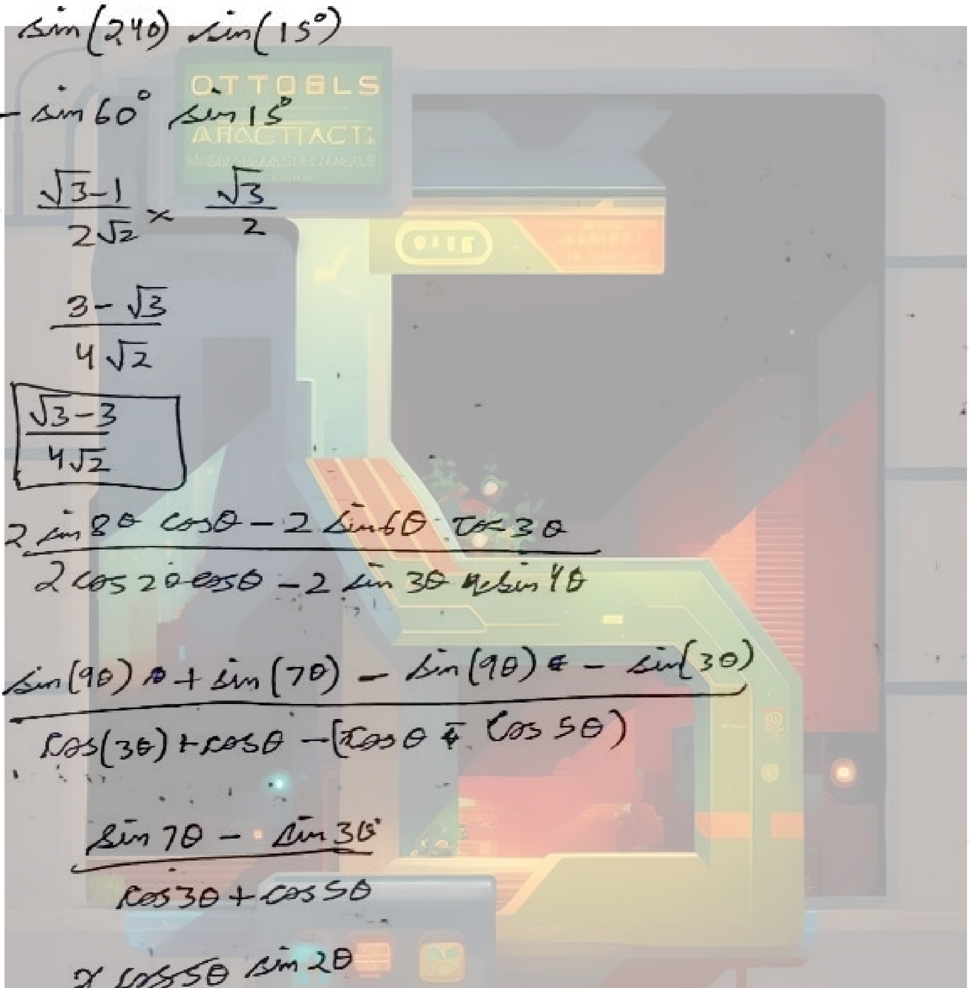
$\sin\left(\frac{\pi}{4}\right)\sin(A\Omega)$

$\frac{1}{\sqrt{2}}\times\sin A$

$\boxed{\frac{\sin A}{\sqrt{2}}}$

(5) $\sin(127.5+112.5)\ \sin(127.5-112.5)$

$\sin(240)\ \sin(15^\circ)$



$-\sin 60^\circ\ \sin 15^\circ$

$-\frac{\sqrt{3}-1}{2\sqrt{2}}\times\frac{\sqrt{3}}{2}$

$\frac{3-\sqrt{3}}{4\sqrt{2}}$

$\boxed{\frac{\sqrt{3}-3}{4\sqrt{2}}}$

(7) $\frac{2\sin 8\theta\cos\theta-2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cos\theta-2\sin 3\theta\sin 4\theta}$

$\frac{\sin(9\theta)+\sin(7\theta)-\sin(9\theta)-\sin(3\theta)}{\cos(3\theta)+\cos\theta-(\cos\theta+\cos 5\theta)}$

$\frac{\sin 7\theta-\sin 3\theta}{\cos 3\theta+\cos 5\theta}$

$\frac{\cancel{2}\cancel{\cos 5\theta}\sin 2\theta}{\cancel{2}\cancel{\sin 4\theta}\cos 2\theta}$

$\cos 5\theta$

$\tan 2\theta = \tan 15^\circ = \boxed{2-\sqrt{3}}$

H.W ~~Ex. ...~~

DYS - 6, 7, 8

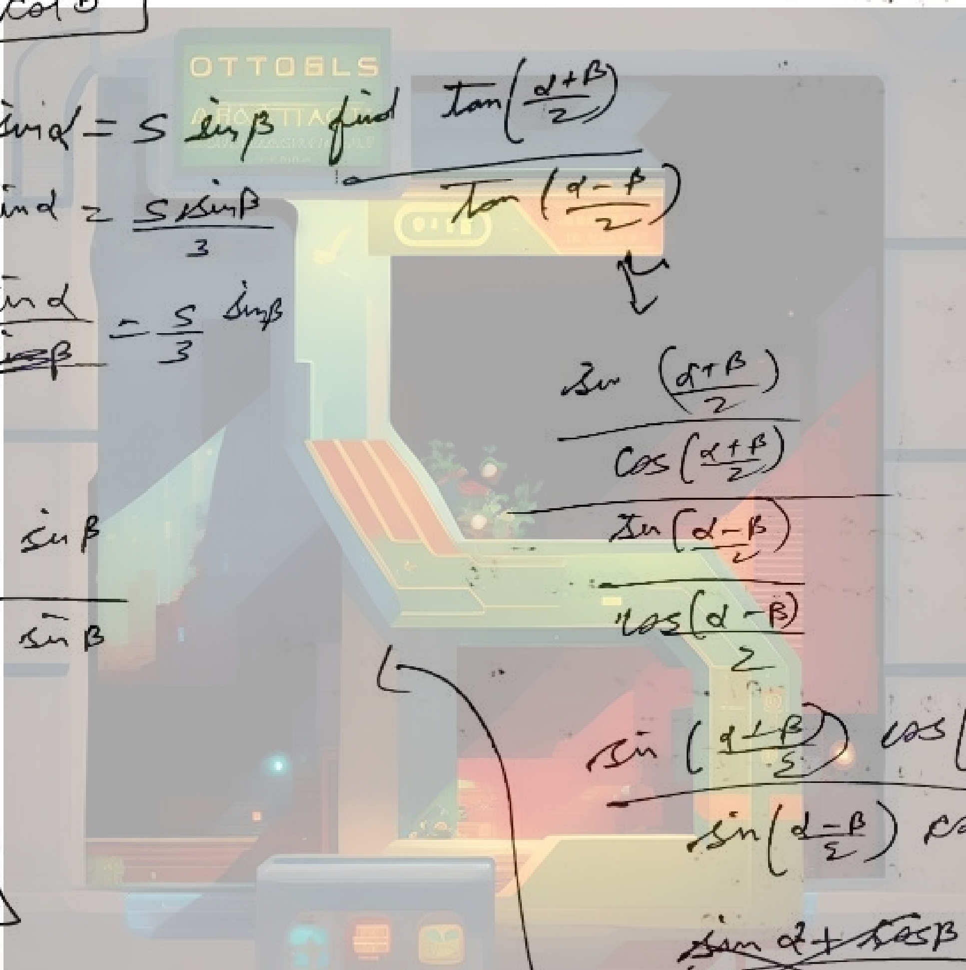
(6) ~~cos θ~~

$\cos\theta \cancel{\sin} \cos\phi$

(9) $\dfrac{2\cos\left(\frac{A+C}{2}\right) \cancel{\sin\left(\frac{A-C}{2}\right)}}{2\sin\left(\frac{A+C}{2}\right) \cancel{\sin\left(\frac{A-C}{2}\right)}}$

$\cot\left(\frac{A+C}{2}\right)$

$\boxed{\cot B}$



Q If $3\sin\alpha = 5\sin\beta$ find $\dfrac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$

$\sin\alpha = \dfrac{5\sin\beta}{3}$

$\dfrac{\sin\alpha}{\sin\beta} = \dfrac{5}{3}$

$\dfrac{\dfrac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}}{\dfrac{\sin\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}}$

$\dfrac{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}$

$\cancel{\dfrac{\sin\alpha + \cos\beta}{\sin\alpha - \cos\beta}}$

$\dfrac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta}$

$\dfrac{\frac{8}{3}\sin\beta}{\frac{2}{3}\sin\beta}$

$\dfrac{8}{2}$

$\boxed{=4}$

(8)

Q :- $(\sin 3A + \sin A)\sin A + (\cos 3A - \cos A)\cos A$

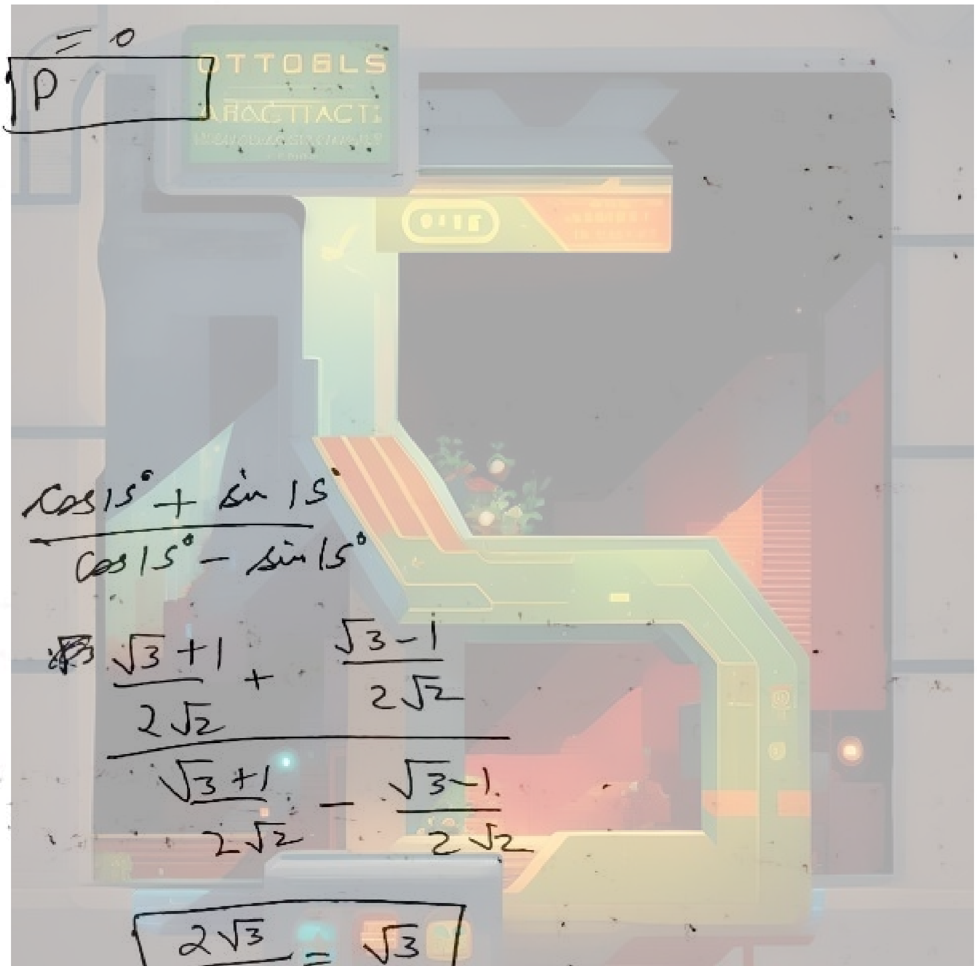
$$\cancel{\sin 2A = 2\sin A\cos A}$$

$$\cancel{\sin 3A = \sin A\cos 2A + \sin 2A\cos A}$$

$$\cancel{= \sin A(\cos^2 A - \sin^2 A) + \sin A\cos A\cos A \times 2 + \sin A}$$

$$\cancel{= 3\sin A\cos^2 A - \sin^3 A + 2\sin^2 A\cos^2 A + \sin A}$$

$$2\sin 2A\cos A\sin A - 2\sin 2A\sin A\cos A$$



$$\boxed{P} = 0$$

Q $\dfrac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$

$$\frac{\frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\boxed{\frac{2\sqrt{3}}{2} = \sqrt{3}}$$

Q If $\sin 2A = \lambda \sin 2B$, find $\frac{\tan(A+B)}{\tan(A-B)}$

~~2 sin A cos A = 2λ sin B cos B~~

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} \times \frac{1 + \tan A \tan B}{\tan A - \tan B}$$

$$\frac{\sin 2A}{\sin 2B} = \lambda$$

$$\frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} = \frac{\lambda + 1}{\lambda - 1}$$

$$\cancel{\frac{\sin(A+B)}{\cos A \cos B} \times \frac{\cos A \cos B}{\sin(A-B)}}$$

$$\frac{2 \sin(A+B) \cos(A-B)}{2 \sin(A-B) \cos(A+B)}$$

$$\cancel{\frac{\tan A \tan B + 1}{1 - \tan A \tan B} \times \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B}}$$

$$\boxed{\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda + 1}{\lambda - 1}}$$

Q $\cot 16 \cot 44 + \cot 44 \cot 76 - \cot 76 \cot 16$.

$$\cancel{\frac{2\cos 16 \cos 44}{2 \sin 16 \sin 44}}$$

$$\cancel{\frac{\cos(60) + \cos(28)}{\cos(28) - \cos(60)} + \frac{\cos(120) + \cos(32)}{\cos(32) - \cos(120)} - \frac{\cos(92) + \cos(60)}{\cos(60) - \cos(92)}}$$

$$\cancel{\frac{\cos 16 \cos 44 \sin 76}{\sin 16 \sin 44 \sin 76} + \frac{\cos 44 \cos 76 \sin 16}{\sin 44 \sin 76 \sin 16} - \frac{\cos 76 \cos 16 \sin 44}{\sin 76 \sin 16 \sin 44}}$$

$$\cot(44+16) = \frac{1}{\sqrt{3}} = \frac{\cot 44 \cot 16 - 1}{\cot 44 + \cot 16}$$

$$\frac{\cot 44 + \cot 16}{\sqrt{3}} + 1 + \frac{\cot 76 + \cot 44}{\sqrt{3}} + 1 - \frac{\cot 16 + \cot 76}{\sqrt{3}} + 1$$

$$\frac{0}{\sqrt{3}} + 3 \Rightarrow \boxed{3}$$

Q ~~Prove~~

~~$\tan 70^\circ = \cot 70 + 2\cot 40$~~

Q $\tan 3A \cdot \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$

$\tan 2A + \tan A = \tan 3A(1 - \tan 2A + \tan A)$

$\tan 3A = \dfrac{\tan 2A + \tan A}{1 - \tan 2A + \tan A}$

$= \tan(2A + A)$

$= \tan(3A)$

Q $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

$\cot x \cot 2x - 1 = \cot 2x \cot 3x + \cot 3x \cot x$

$\cot x \cot 2x - 1 = \cot 3x(\cot 2x + \cot x)$

$\dfrac{\cot 2x \cot x - 1}{\cot 2x + \cot x} = \cot 3x$

$\cot(2x + x) = \cot 3x$

DYS-9

Q1-21

Sum of TR with more than 2 Angles.

① $\sin(A+B+C) = \sin[(A+B)+C]$

$$= \sin(A+B)\cos C + \cos(A+B)\sin C$$

$$= (\sin A\cos B + \cos A\sin B)\cos C + (\cos A\cos B - \sin A\sin B)\sin C$$

$$= \sin A\cos B\cos C + \cos A\sin B\cos C + \sin C\cos A\cos B - \sin A\sin B\sin C$$

$$\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C + \sin C\cos A\cos B - \sin A\sin B\sin C$$

$$\sin(A+B+C) = \cos A\cos B\cos C\,(\tan A + \tan B + \tan C - \tan A\tan B\tan C)$$

② $\cos(A+B+C) = \cos A\cos B\cos C\,(1 - \tan A\tan B - \tan B\tan C - \tan A\tan C)$

③ $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan A\tan C}$

TR of multiple & sub multiple angles

Multiple $\rightarrow$ $2A, 3A, 4A, 5A, \ldots$

Sub-Multiple $\rightarrow$ $\frac{A}{2}, \frac{3A}{2}, \frac{A}{3}, \frac{A}{4}, \frac{A}{6}, \ldots$

① $\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$

② $\cos 2A = \cos^2 A - \sin^2\theta$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

③ $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

(4) $\sin 3A = 3\sin A - 4\sin^3 A = \sin A(3 - 4\sin^2 A)$

$= 2\sin A\cos^2 A + \sin A - 2\sin^3 A$

Proof:-

$\sin 3A = \sin(2A + A)$

$= 2\sin A\cos A \cdot \cos A + \cancel{\cos}(1 - 2\sin^2 A)\sin A$

$= 2\sin A\cos A \cdot \cos A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

(5) $\cos 3A = 4\cos^3 A - 3\cos A$

(6) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Q $\sin A = \frac{4}{5}$ then, $\cos 2A = ?$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 1 - 2(\sin A)^2$

$= 1 - 2 \times \frac{16}{25}$

$= 1 - \frac{32}{25}$

$= \frac{25 - 32}{25}$

$\boxed{= \frac{-7}{25}}$

Q Prove $\cos^4 \frac{A}{2} - \sin^4 \frac{A}{2} = \cos A$

$\left(\cos^2 \frac{A}{2} + \sin^2 \frac{A}{2}\right)\left(\cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}\right)$

$\cancel{\left(\cos \frac{A}{2} + \cos \frac{A}{2} \sin \frac{A}{2}\right)\left(\cos \frac{A}{2} - \sin \frac{A}{2}\right)}$

$$\cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right) = \cos 2 \times \frac{A}{2}$$

$$\boxed{= \cos A}$$

Q3. $\dfrac{1+\sin 2\theta + \cos 2\theta}{1+\sin 2\theta - \cos 2\theta}$ A) $\frac{\tan\theta}{2}$ B) $\frac{\cot\theta}{2}$ c) $\tan\theta$ ✓d) $\cot\theta$

$$= \cancel{\frac{1+2\sin\theta\cos\theta + 1-\cos^2\theta}{1+2\sin\theta\cos\theta - 1+\cos^2\theta}}$$

$$= \cancel{\frac{2+\cos\theta(2\sin\theta - \cos\theta)}{\cos\theta(2\sin\theta+\cos\theta)}}$$

$$= \cancel{\frac{1+2\sin\theta\cos\theta + \cos^2\theta - 1}{1+2\sin\theta\cos\theta - 1 + \sin^2\theta}}$$

$$= \cancel{\frac{\cos\theta(2\sin\theta+\cos\theta)}{\sin\theta(2\cos\theta+\sin\theta)}}$$

$$= \cancel{\frac{\cos\theta(2\sin\theta+\cos\theta)}{\cos\theta(2\sin\theta + \tan\theta\sin\theta)}}$$

$=$

$$\frac{1+2\sin\theta\cos\theta + 2\cos^2\theta - 1}{1+2\sin\theta\cos\theta + 2\sin^2\theta - 1}$$

$$\frac{\sin\theta\cos\theta + \cos^2\theta}{\sin\theta\cos\theta + \sin^2\theta}$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta}$$

$$\frac{\cos\theta(\cancel{\sin\theta+\cos\theta})}{\sin\theta(\cancel{\cos\theta+\sin\theta})}$$

$$\boxed{\cot\theta}$$

Q 4. $\sqrt{2+\sqrt{2+2\cos 4\theta}}$

A) $2\sin\theta$ ✓B) $2\cos\theta$

C) $\sin\theta$ D) $\cos\theta$

$\sqrt{2+\sqrt{2+2\cos^2 2\theta-2\sin^2 2\theta}}$

$\sqrt{2+\sqrt{2(1+}}$

$\sqrt{2+\sqrt{2+2(2\cos^2\theta-1)}}$

$\sqrt{2+\sqrt{4\cos^2\theta}}$

$\sqrt{2+2\cos 2\theta}$

$\sqrt{2+2\cos^2\theta-1}$

$\sqrt{\sqrt{2}\cos^2\theta}$

$\sqrt{\sqrt{2}\cos\theta}$

$\cos\theta\times 2^{1/4}$

$\sqrt{2+\sqrt{2+2(2\cos^2 2\theta-1)}}$

$\sqrt{2+\sqrt{4\cos^2 2\theta}}$

$\sqrt{2+2\cos^2 2\theta}$

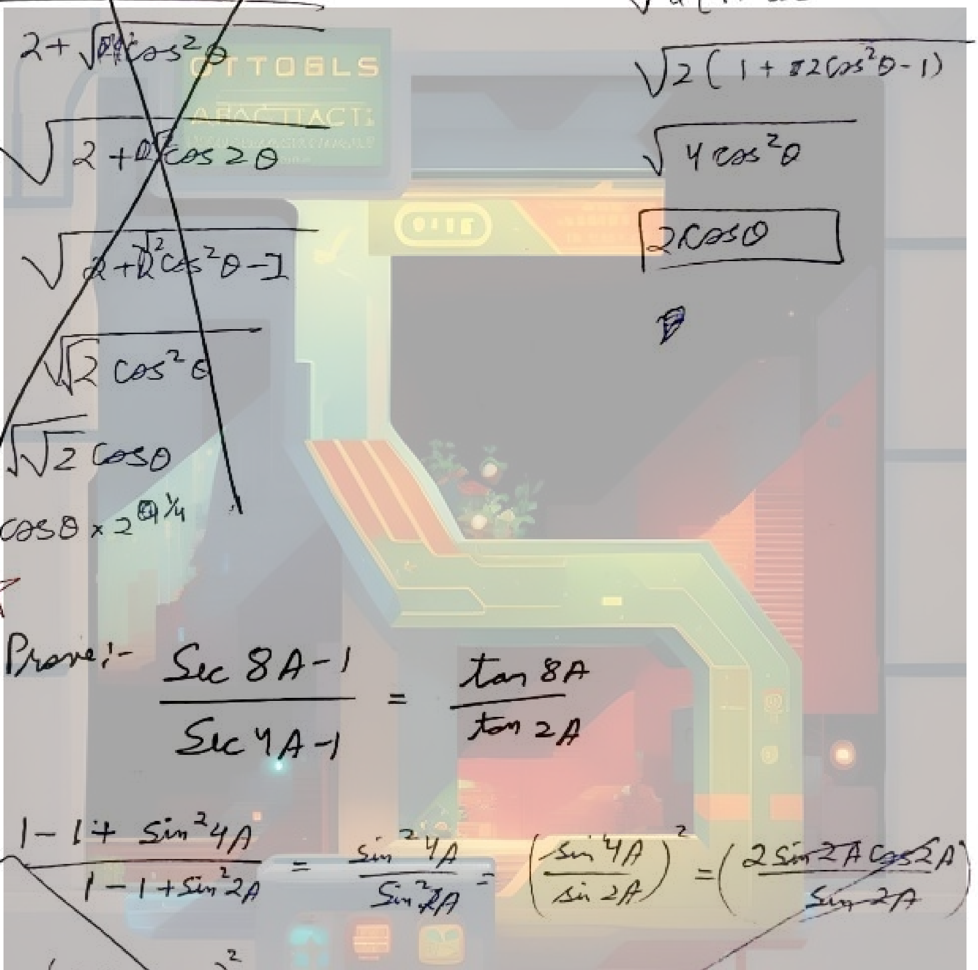
$\sqrt{2(1+\cos 2\theta}$

$\sqrt{2(1+2\cos^2\theta-1)}$

$\sqrt{4\cos^2\theta}$

$\boxed{2\cos\theta}$

B



Q5. Prove:- $\dfrac{\sec 8A-1}{\sec 4A-1}=\dfrac{\tan 8A}{\tan 2A}$

$\dfrac{1-1+\sin^2 4A}{1-1+\sin^2 2A}=\dfrac{\sin^2 4A}{\sin^2 2A}=\left(\dfrac{\sin 4A}{\sin 2A}\right)^2=\left(\dfrac{2\sin 2A\cos 2A}{\sin 2A}\right)^2$

$=(2\cos 2A)^2$

$=(2\cos^2 A-2\sin^2 A)^2$

$=4\cos^4 A-4\sin^4 A$

$=4(\cos^4 A-\sin^4 A)$

$=4(\cos^2 A+\sin^2 A)(\cos^2 A-\sin^2 A)$

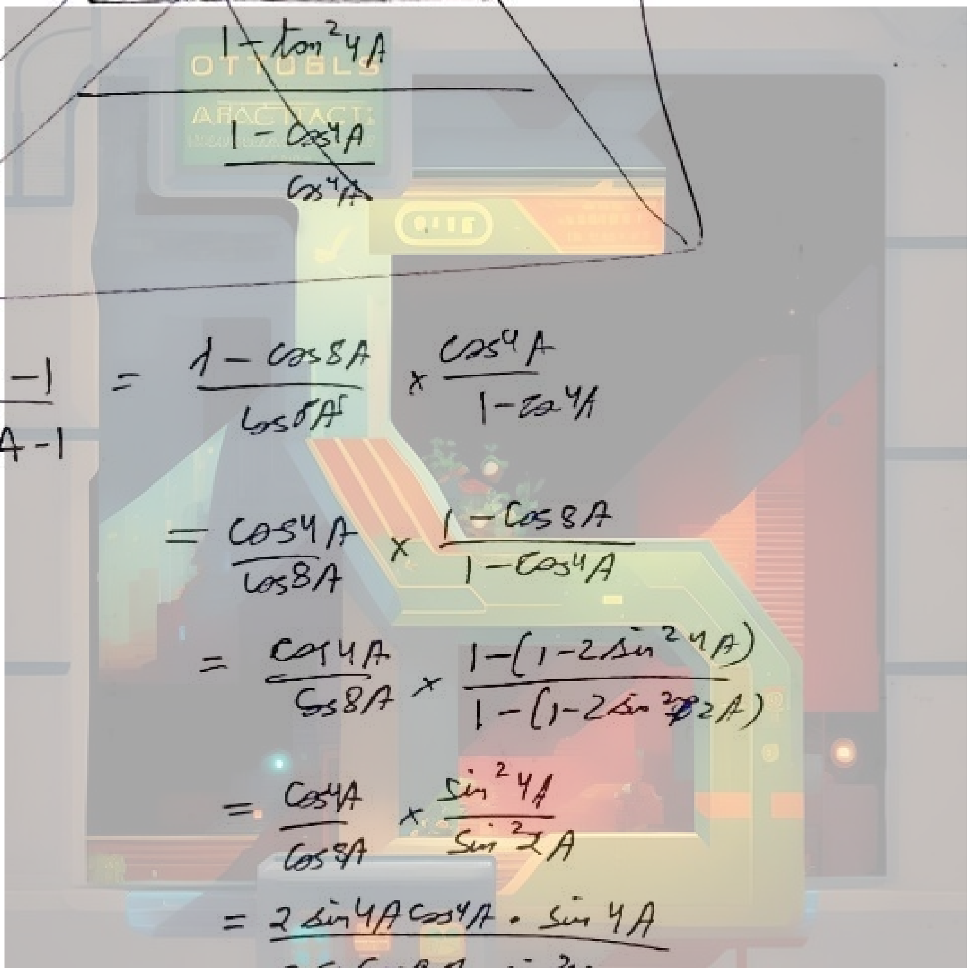
$=4(\cos A-\sin A)(\cos A+\sin A)$

$=4\cos 2A$

$$\frac{\frac{1-\cos 8A}{\cos 8A}}{\frac{1-\cos 4A}{\cos 4A}} \qquad \frac{1-\frac{1-\tan^2 4A}{1+\tan^2 4A}}{\frac{1-\tan^2 4A}{1+\tan^2 4A}}$$

$$= \frac{}{\frac{1-\cos 4A}{\cos 4A}}$$

$$= \frac{\frac{1+\tan^2 4A - 1 - \tan^2 4A}{1-\tan^2 4A}}{\frac{1-\cos 4A}{\cos 4A}}$$



$$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{1-\cos 8A}{\cos 8A} \times \frac{\cos 4A}{1-\cos 4A}$$

$$= \frac{\cos 4A}{\cos 8A} \times \frac{1-\cos 8A}{1-\cos 4A}$$

$$= \frac{\cos 4A}{\cos 8A} \times \frac{1-(1-2\sin^2 4A)}{1-(1-2\sin^2 2A)}$$

$$= \frac{\cos 4A}{\cos 8A} \times \frac{\sin^2 4A}{\sin^2 2A}$$

$$= \frac{2\sin 4A\cos 4A \cdot \sin 4A}{2\cos 8A \cdot \sin^2 2A}$$

$$= \tan 8A \times \frac{1}{2} \times 2\,\frac{\sin 2A \cos 2A}{\sin 2A \cdot \sin 2A}$$

$$\boxed{= \frac{\tan 8A}{\tan 2A}}$$

H.W. 22-7-24

DYS-10 (1-15)

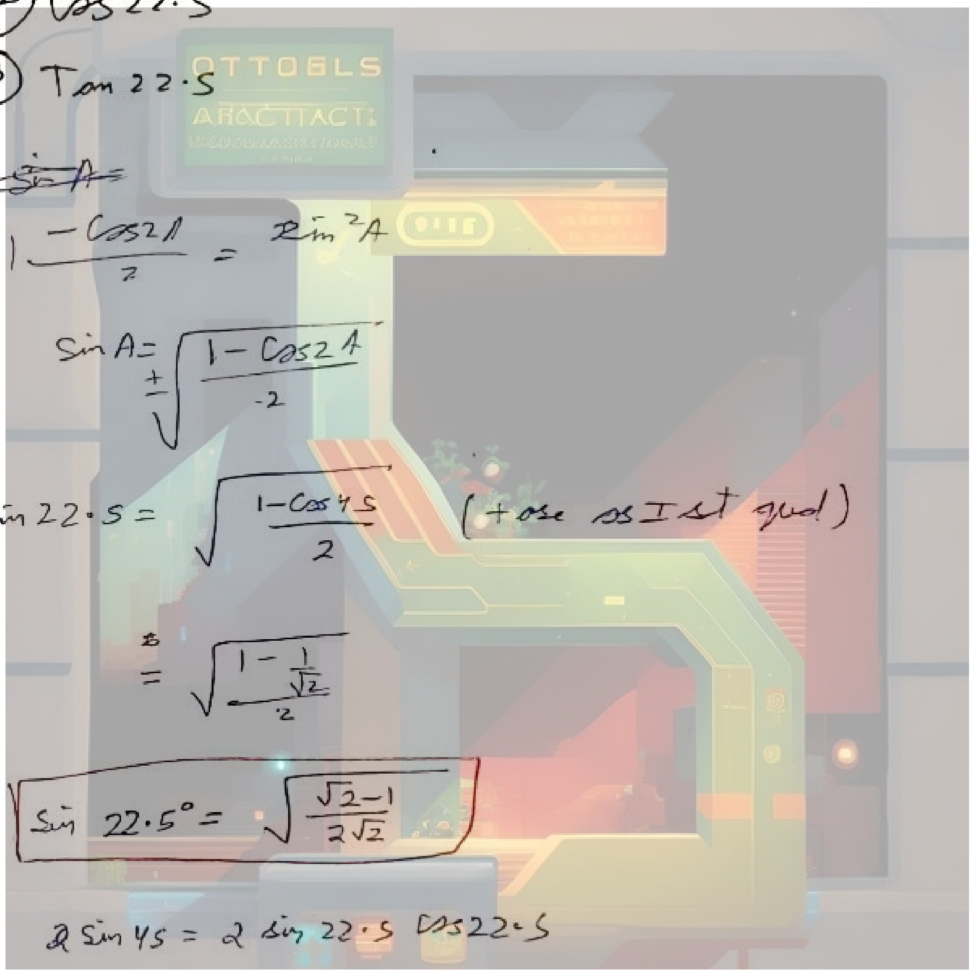
Q find values of

(1) $\sin 22.5$ or $\sin \frac{\pi}{8}$

~~$\sin 2\times 22.5 = 2\times \sin 22.5 \cos 22.5$~~

(2) $\cos 22.5$

(3) $\tan 22.5$



~~$\sin A =$~~

(1) $\frac{1-\cos 2A}{2} = \sin^2 A$

$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}}$

$\sin 22.5 = \sqrt{\frac{1-\cos 45}{2}}$ (+ve as Ist quad)

$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$

$\boxed{\sin 22.5^\circ = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}$

(2) $2\sin 45 = 2\sin 22.5 \cos 22.5$

~~$\frac{2}{\sqrt{2}} = 2\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}\cos 22.5$~~ $\cos 22.5 = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

~~$\cos 22.5 = \frac{\sqrt{2\sqrt{2}}}{2\sqrt{(\sqrt{2}-1)2}}$~~

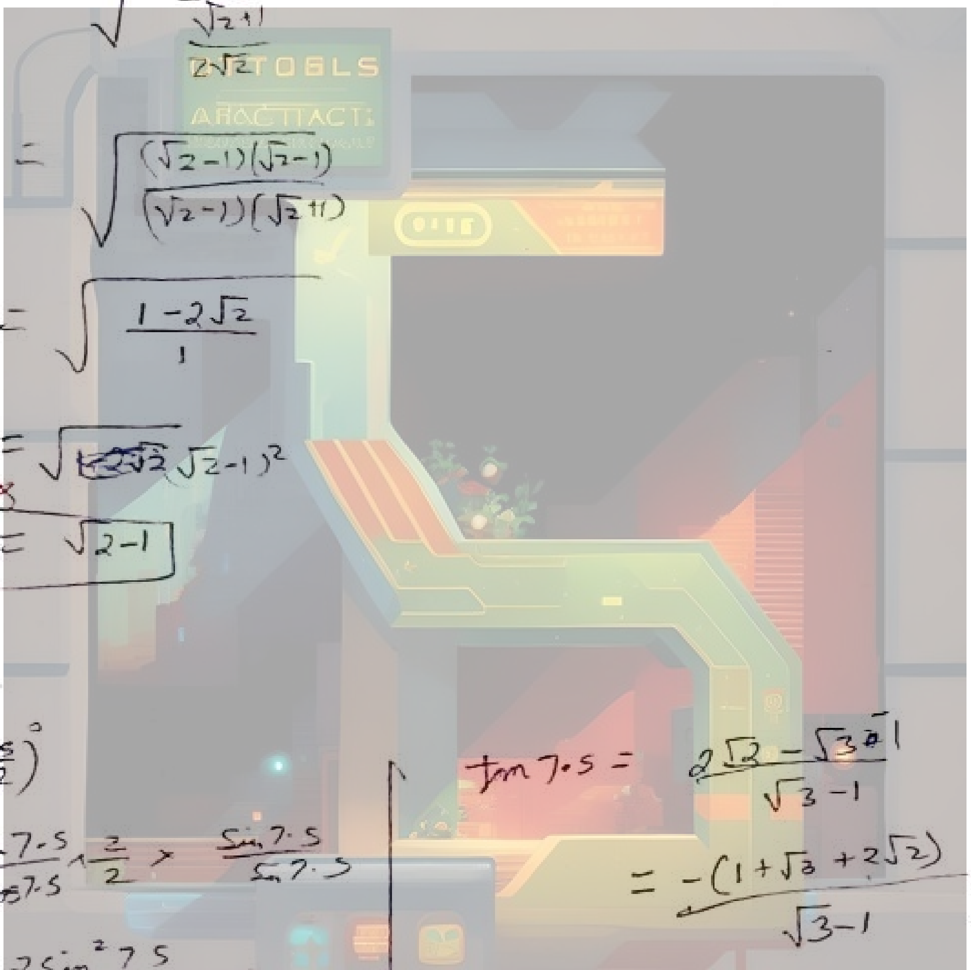
~~$\cos 22.5 = \frac{\sqrt{\sqrt{2}}}{2\sqrt{\sqrt{2}-1}}$~~

~~$\cos 22.5 = \sqrt{\frac{\sqrt{2}}{4(\sqrt{2}-1)}}$~~

(3) $\tan 22.5 = \dfrac{\sin 22.5}{\cos 22.5}$

$$= \frac{\sqrt{\dfrac{\sqrt{2}-1}{2\sqrt{2}}}}{\sqrt{\dfrac{\sqrt{2}+1}{2\sqrt{2}}}}$$

$$= \sqrt{\frac{\dfrac{\sqrt{2}-1}{2\sqrt{2}}}{\dfrac{\sqrt{2}+1}{2\sqrt{2}}}}$$



$$= \sqrt{\frac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)}}$$

$$= \sqrt{\frac{1-2\sqrt{2}}{1}}$$

$$= \sqrt{\cancel{1-2\sqrt{2}}(\sqrt{2}-1)^2}$$

$$\boxed{= \sqrt{2}-1}$$

(4) $\tan 7\frac{1}{2}$

$\tan\left(\frac{15}{2}\right)^\circ$

$$= \frac{\sin 7.5}{\cos 7.5} \times \frac{2}{2} \times \frac{\sin 7.5}{\sin 7.5}$$

$$= \frac{2\sin^2 7.5}{2\sin 7.5\cos 7.5}$$

$$= \frac{1-\cos 2A}{\sin 2A}$$

$$= \frac{1-\cos 15^\circ}{\sin 15}$$

$$= \frac{1-\dfrac{\sqrt{3}+1}{2\sqrt{2}}}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\tan 7.5 = \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{-(1+\sqrt{3}+2\sqrt{2})}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{-(3+\sqrt{3}-2\sqrt{2}\sqrt{3}+\sqrt{3}+1-2\sqrt{2})}{2}$$

$$= \frac{-(4+2\sqrt{3}-2\sqrt{2}\sqrt{3}-2\sqrt{2})}{2}$$

$$= -(2+\sqrt{3}-\sqrt{2}\sqrt{3}-\sqrt{2})$$

$$= \sqrt{2}\sqrt{3}+\sqrt{2}-2-\sqrt{3}$$

$$= \sqrt{3}(\sqrt{2}-1)-\sqrt{2}(\sqrt{2}-1)$$

$$\boxed{\tan 7.5 = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)}$$

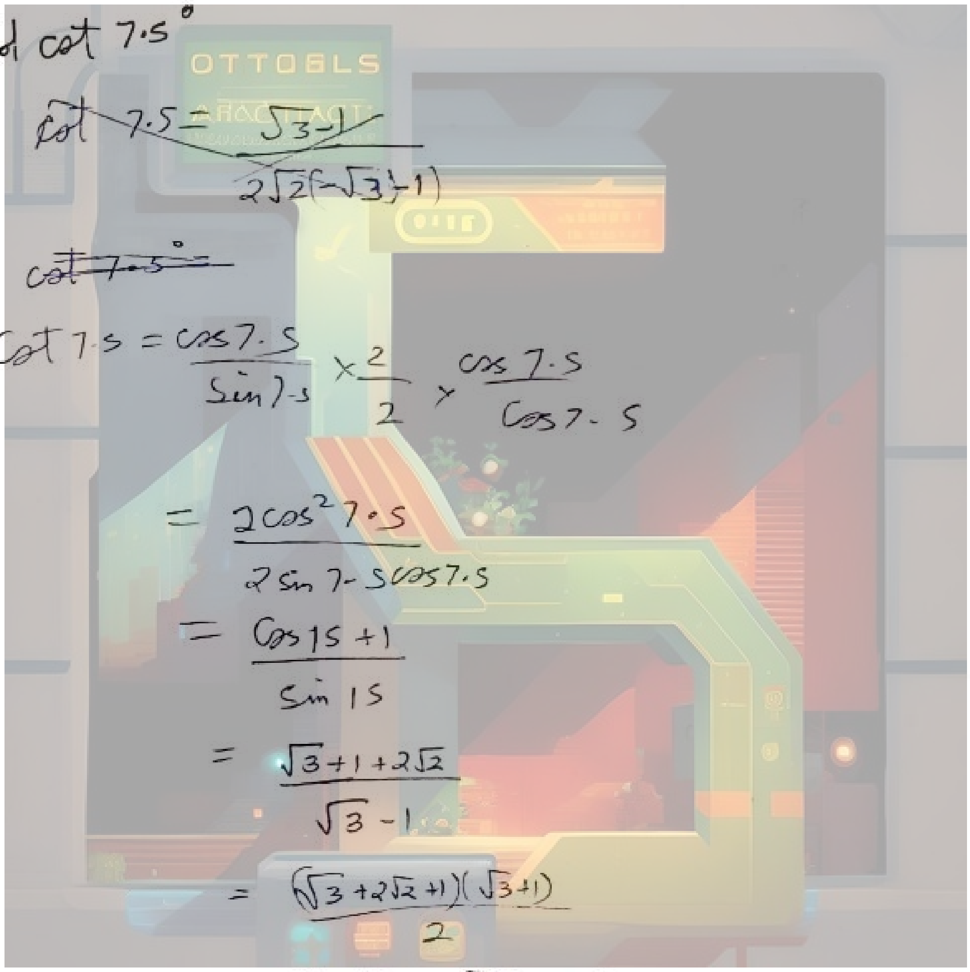
~~find cot 7.5~~

$$\cancel{\cot 7.5 = \frac{1}{\tan 7.5} = \frac{1}{\sqrt{3}}}$$

$$\tan 7.5^\circ = (\sqrt{3}-\sqrt{2}) \tan 22.5^\circ$$

$$\boxed{\frac{\tan 7.5^\circ}{\tan 22.5^\circ} = \sqrt{3}-\sqrt{2}}$$

Q find $\cot 7.5^\circ$



$$\cancel{\cot 7.5 = \frac{\sqrt{3}-1}{2\sqrt{2}(-\sqrt{3})-1)}}$$

$$\cancel{\cot 7.5 =}$$

$$\cot 7.5 = \frac{\cos 7.5}{\sin 7.5} \times \frac{2}{2} \times \frac{\cos 7.5}{\cos 7.5}$$

$$= \frac{2\cos^2 7.5}{2\sin 7.5 \cos 7.5}$$

$$= \frac{\cos 15 + 1}{\sin 15}$$

$$= \frac{\sqrt{3}+1+2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+2\sqrt{2}+1)(\sqrt{3}+1)}{2}$$

$$= \frac{3+\sqrt{3}+2\sqrt{2}\sqrt{3}+2\sqrt{2}+\sqrt{3}+1}{2}$$

$$= \frac{4+2\sqrt{3}+2\sqrt{2}+2\sqrt{2}\sqrt{3}}{2}$$

$$\boxed{\text{Cos } 7.5 = 2+\sqrt{3}+\sqrt{2}+\sqrt{3}\sqrt{2}}$$

Q find $\cos 67\frac{1}{2}^\circ + \sin 67\frac{1}{2}$

$\cos \frac{135}{2} + \sin \frac{135}{2}$

$\cos(75 - 7.5) + \sin(75 - 7.5)$

$\cos 75 \cos 7.5 + \sin 75 \sin 7.5 + \sin 75 \cos 7.5 - \sin 7.5 \cos 75$

$\sin 7.5(\sin 75 - \cos 75) + \cos 7.5(\sin 75 + \cos 75)$

$\sin 7.5\left(\frac{\sqrt{3}+1}{2\sqrt{2}} - \sqrt{3}+1\right) + \cos 7.5\left(\frac{\sqrt{3}+1+\sqrt{3}-1}{2\sqrt{2}}\right)$

$\sin 7.5\left(\frac{1}{\sqrt{2}}\right) + \cos 7.5\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$\sin 7.5 = \sqrt{\frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{2}}$

$\sin 7.5 = \sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{2}}}$

$\sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{2\sqrt{2}}} + \sqrt{}$

M II

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta$

$= 1 + \sin 2\theta$

$\theta = 67.5$

$= 1 + \sin 135$

$= 1 + \sin(90 + 45)$

$= 1 + \cos 45$

$= 1 + \frac{1}{\sqrt{2}}$

$\boxed{= \frac{\sqrt{2}+1}{\sqrt{2}}}$

$\sin 22.5^\circ + \cos 22.5^\circ$

$\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} + \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

$\boxed{\frac{\sqrt{\sqrt{2}-1} + \sqrt{\sqrt{2}+1}}{\sqrt{2\sqrt{2}}}}$

Q $\sqrt{3}\operatorname{cosec}20° - \sec 20°$

$$\cancel{\frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}}$$

$$\cancel{3\sin 20 - 4\sin^3 20 = \sin 60}$$

$$\cancel{\frac{\sqrt{3}}{2} = 3t - 4t^3}$$

$$\cancel{4t^3 - 3t}$$

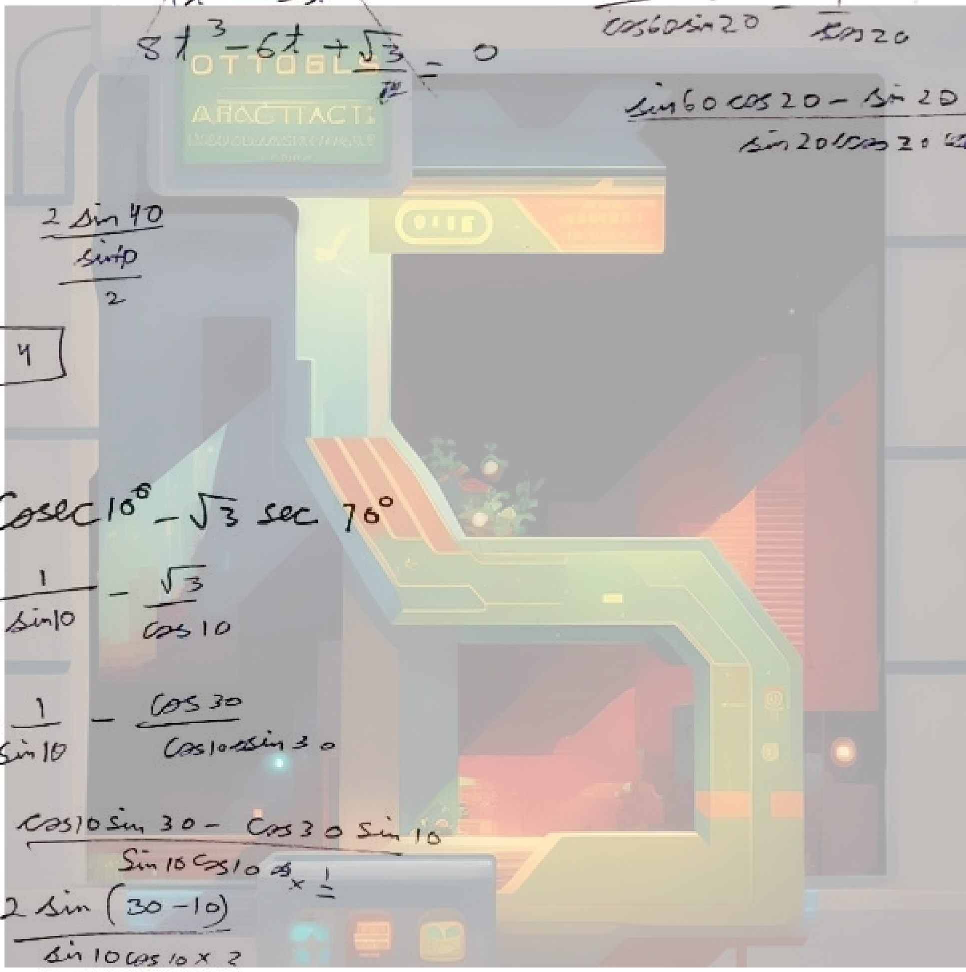
$$\cancel{8t^3 - 6t + \sqrt{3} = 0}$$

$$\frac{\sqrt{3}}{\sin 20} - \frac{1}{\cos 20}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{\sin 20}{2}} - \frac{1}{\cos 20}$$

$$\frac{\sin 60}{\cos 60 \sin 20} - \frac{1}{\cos 20}$$

$$\frac{\sin 60\cos 20 - \sin 20\cos 60}{\sin 20\cos 20 \cdot \frac{1}{2}}$$



$$= \frac{2\sin 40}{\frac{\sin 40}{2}}$$

$$\boxed{= 4}$$

Q $\operatorname{cosec}10° - \sqrt{3}\sec 70°$

$$\frac{1}{\sin 10} - \frac{\sqrt{3}}{\cos 10}$$

$$\frac{1}{\sin 10} - \frac{\cos 30}{\cos 10 \sin 30}$$

$$\frac{\cos 10\sin 30 - \cos 30\sin 10}{\sin 10\cos 10 \times \frac{1}{2}}$$

$$\frac{2\times 2\sin(30-10)}{\sin 10\cos 10\times 2}$$

$$\boxed{4}$$

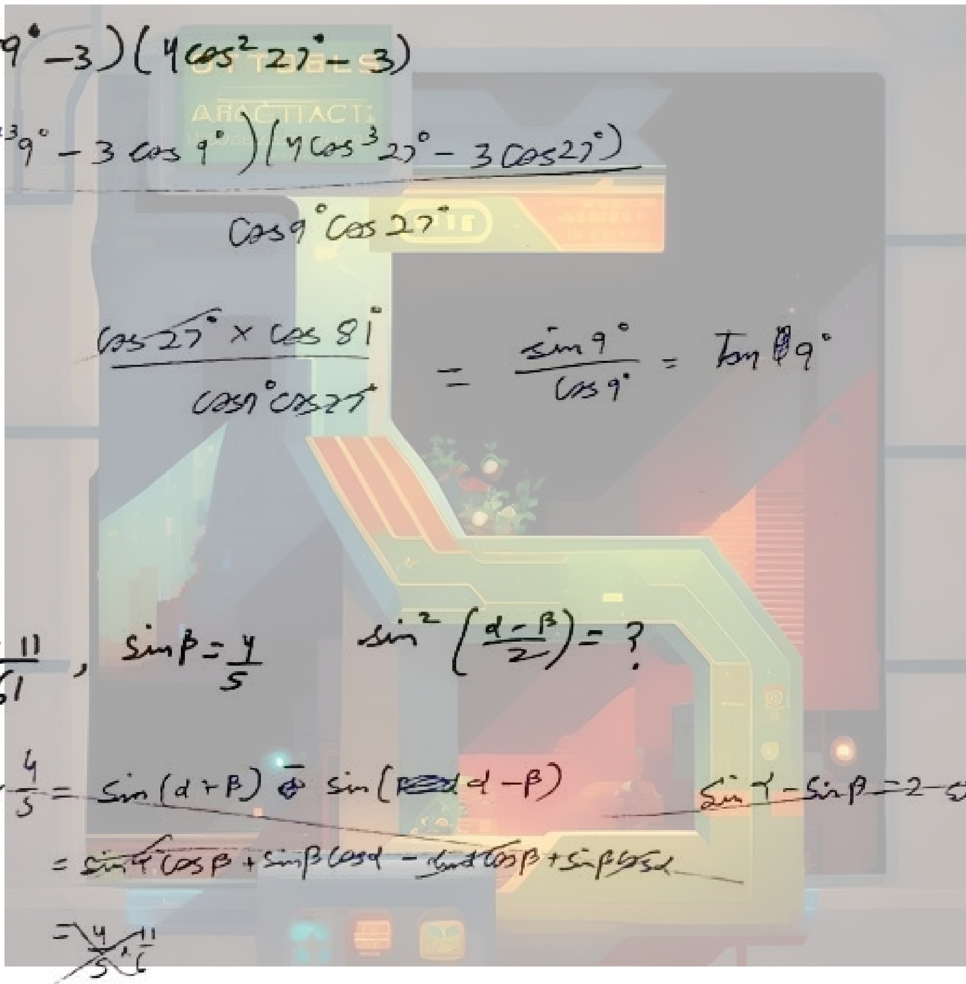
Q $-6\sin 40^\circ + 8\sin^3 40^\circ$

$-2(3\sin 40^\circ - 4\sin^3 40^\circ)$

$-2\sin(120^\circ)$

$-2\cos 30$

$-2 \times \frac{\sqrt{3}}{2} = \boxed{-\sqrt{3}}$



Q $(4\cos^2 9^\circ - 3)(4\cos^2 27^\circ - 3)$

$$\frac{(4\cos^3 9^\circ - 3\cos 9^\circ)(4\cos^3 27^\circ - 3\cos 27^\circ)}{\cos 9^\circ \cos 27^\circ}$$

$$\frac{\cos 27^\circ \times \cos 81^\circ}{\cos 9^\circ \cos 27^\circ} = \frac{\sin 9^\circ}{\cos 9^\circ} = \tan 9^\circ$$

Q $\cos\alpha = \frac{11}{61}$, $\sin\beta = \frac{4}{5}$ $\quad \sin^2\left(\frac{\alpha-\beta}{2}\right) = ?$

$\cancel{2 \times \frac{11}{61} \times \frac{4}{5} = \sin(\alpha+\beta) + \sin(\alpha-\beta)}$ $\quad \cancel{\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}$

$\cancel{= \sin\alpha\cos\beta + \sin\beta\cos\alpha - \sin\alpha\cos\beta + \sin\beta\cos\alpha}$

$\cancel{= \frac{4}{5} \cdot \frac{11}{6}}$

$$\cos(\alpha-\beta) = 1 - 2\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1-\cos(\alpha-\beta)}{2}$$

$$= \frac{1-(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}{2}$$

$$= \frac{1-\left(\frac{11}{61}\times\frac{3}{5} + \frac{4}{5}\times\frac{\overset{12}{\cancel{60}}}{61}\right)}{\cancel{2}}$$

$$\boxed{= -\frac{136}{305}}$$

(22)

Important Points

(1) $\sin(60-\theta)\sin\theta\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

(2) $\cos(60-\theta)\cos\theta\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

(3) $\cancel{\sin(60-\theta)}$ $\tan(60-\theta)\tan\theta\tan(60+\theta) = \tan 3\theta$

proof :-

$\sin(60-\theta)\sin\theta\sin(60+\theta) = a$

$= (\sin^2 60 - \sin^2\theta)\sin\theta$

$= \frac{3-4\sin^2\theta}{4}(\sin\theta)$

$= \frac{3\sin\theta - 4\sin^3\theta}{4}$

$= \frac{\sin 3\theta}{4} =$ RHS.

अति सिद्धम

(4) $\tan\theta + \tan(60+\theta) + \tan(120+\theta) = 3\tan 3\theta$

H.W. 23-7-24

DYS-10 (16-19)

DYS-11 (8, 9, 5, 6, 7)

DYS-12 (Q2, 3, 4, 6, ... 16)

O-1 (1-15)

Q find the values of

(1) $\sin 18^\circ$

(2) $\cos 18^\circ$

(3) $\tan 18^\circ$

~~sin 18 =~~ ~~sin(90 − 25)~~

~~sin 18 =~~ ~~sin~~

let $5\theta = 90^\circ$

$2\theta + 3\theta = 90^\circ$

$2\theta = 90^\circ - 3\theta$

$\sin 2\theta = \sin(90^\circ - 3\theta)$

$\sin 2\theta = \cos 3\theta$

$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta = 0$

$2\sin\theta - 4C^3 + 3C = 0$

$C \neq 0$

$2S - 4C^2 + 3 = 0$

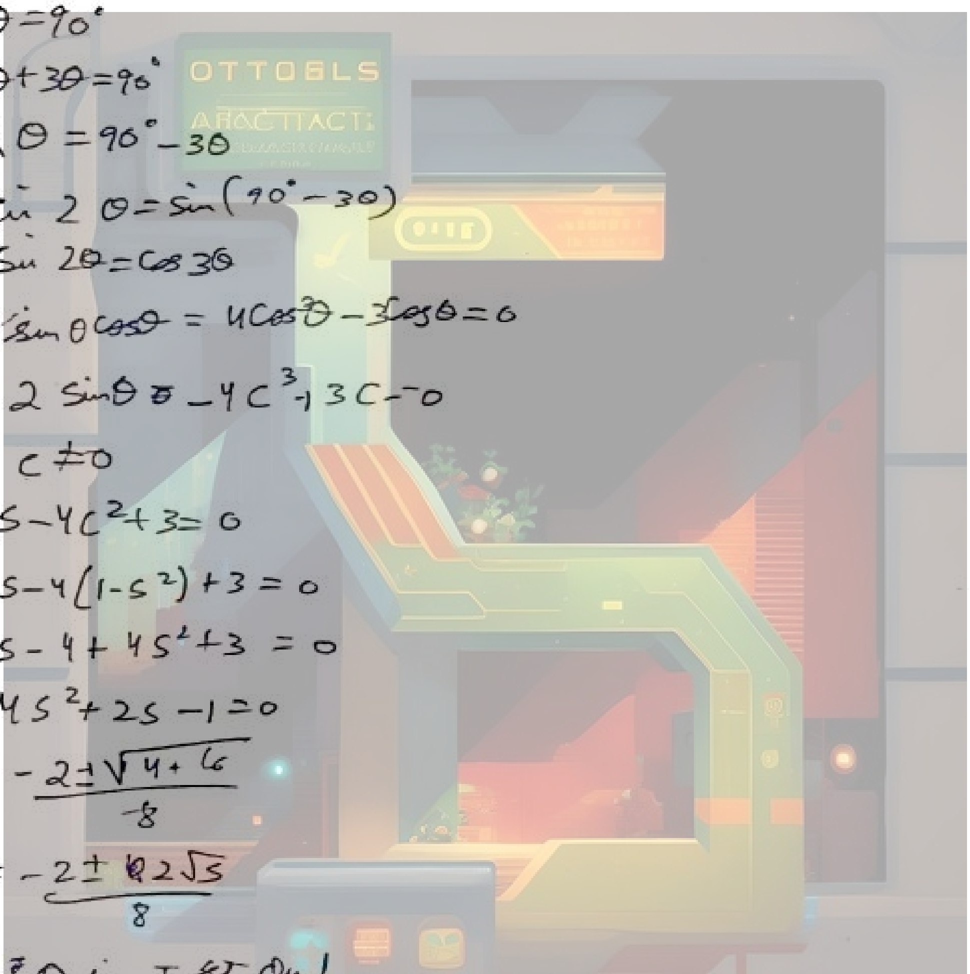
$2S - 4(1 - S^2) + 3 = 0$

$2S - 4 + 4S^2 + 3 = 0$

$4S^2 + 2S - 1 = 0$

$S = \dfrac{-2 \pm \sqrt{4 + 16}}{8}$

$= \dfrac{-2 \pm 2\sqrt{5}}{8}$



S is θ in I st Qud

$S = \dfrac{-1 + \sqrt{5}}{4}$

$$\boxed{\sin 18^\circ = \frac{\sqrt{5} - 1}{4}}$$

(2) $\cos 18^\circ$

$\cos 5\theta = \cos 90^\circ$

$\cos 2\theta + \cos 3\theta = \cos 90^\circ$

$\cos 2\theta = \cos(90 - 3\theta)$

$\cos 2\theta = \sin 3\theta$

$1 - 2\sin^2\theta = 4\sin^3\theta - 3\sin\theta$

$4\sin^3\theta + 2\sin^2\theta - 3\sin\theta - 1 = 0$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\cos 18^\circ = \sqrt{1 - \sin^2 18}$

$\cos 18^\circ = \sqrt{1 - \left(\frac{\sqrt5 - 1}{4}\right)^2}$

$= \sqrt{1 - \frac{(6 - 2\sqrt5)}{16}}$

$= \sqrt{\frac{16 - 6 + 2\sqrt5}{16}}$

$\cos 18 = \frac{\sqrt{10 + 2\sqrt5}}{4}$

$\boxed{\cos 18^\circ = \frac{\sqrt{5 + \sqrt5}}{2\sqrt2}}$

(3) $\tan 18^\circ = \frac{\sin 18^\circ}{\cos 18^\circ}$

$= \frac{\sqrt5 - 1}{\cancel{4}\,2} \times \frac{2\sqrt2}{\sqrt{5 + \sqrt5}}$

~~$= \sqrt{10} - \sqrt2$~~

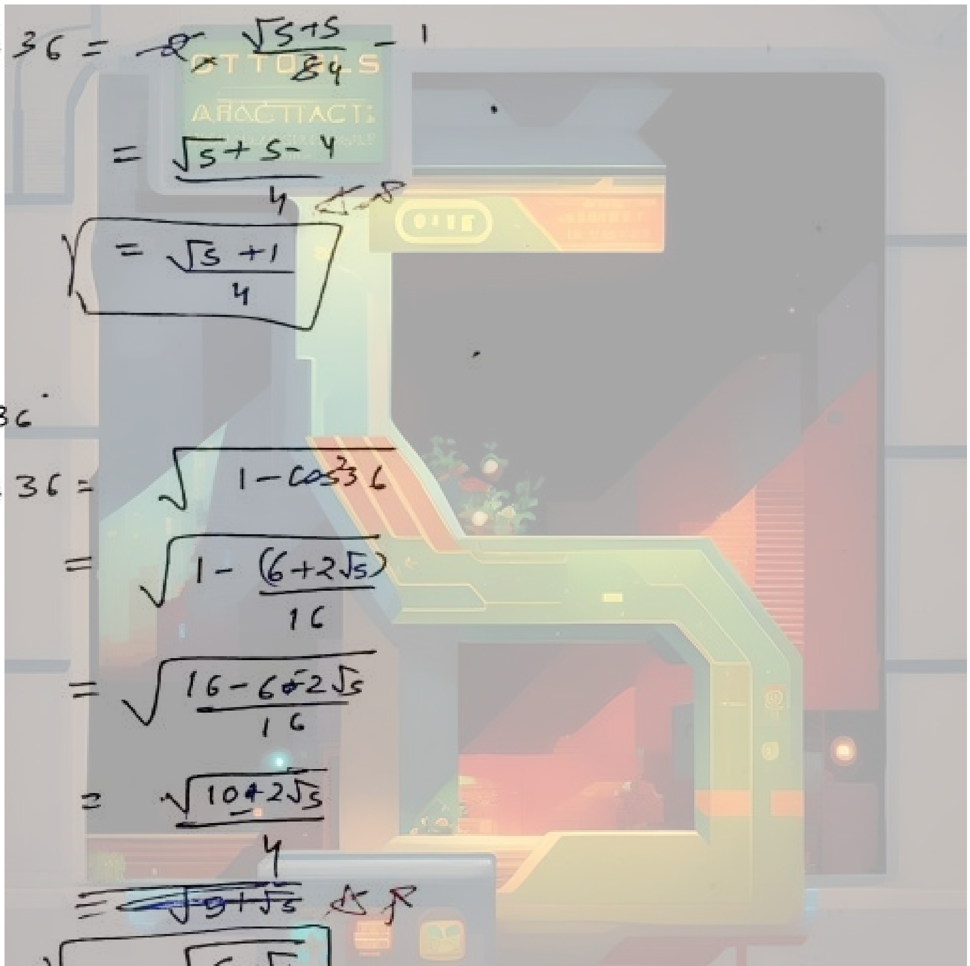
$\boxed{\tan 18^\circ = \frac{\sqrt2(\sqrt5 - 1)}{2\sqrt{5 + \sqrt5}}}$

(4) ~~Sin 36°~~

$$\cancel{\sin 2\theta = \sin 2\times 18^\circ}$$
$$\cancel{= 2\sin 18 \cos 18}$$
$$\cancel{= 2\times \frac{(\sqrt{5}-1)}{4_2}\times\sqrt{\ }}$$

(4) Cos 36°

$$\cos 2\theta = 2\cos^2\theta - 1$$



$$\cos 36 = \cancel{2}\,\frac{\sqrt{5}+5}{\cancel{8}4} - 1$$
$$= \frac{\sqrt{5}+5-4}{4}$$
$$\boxed{= \frac{\sqrt{5}+1}{4}}$$

(5) Sin 36°

$$\sin 36 = \sqrt{1-\cos^2 36}$$
$$= \sqrt{1-\frac{(6+2\sqrt{5})}{16}}$$
$$= \sqrt{\frac{16-6-2\sqrt{5}}{16}}$$
$$= \frac{\sqrt{10-2\sqrt{5}}}{4}$$
$$\cancel{= \sqrt{9+\sqrt{5}}}$$
$$\boxed{= \frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}}$$

Q find the value ~~of~~ $\sin 9^\circ$ & $\cos 9^\circ$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\sin 9 = \sqrt{\frac{1-\cos 18}{2}}$$

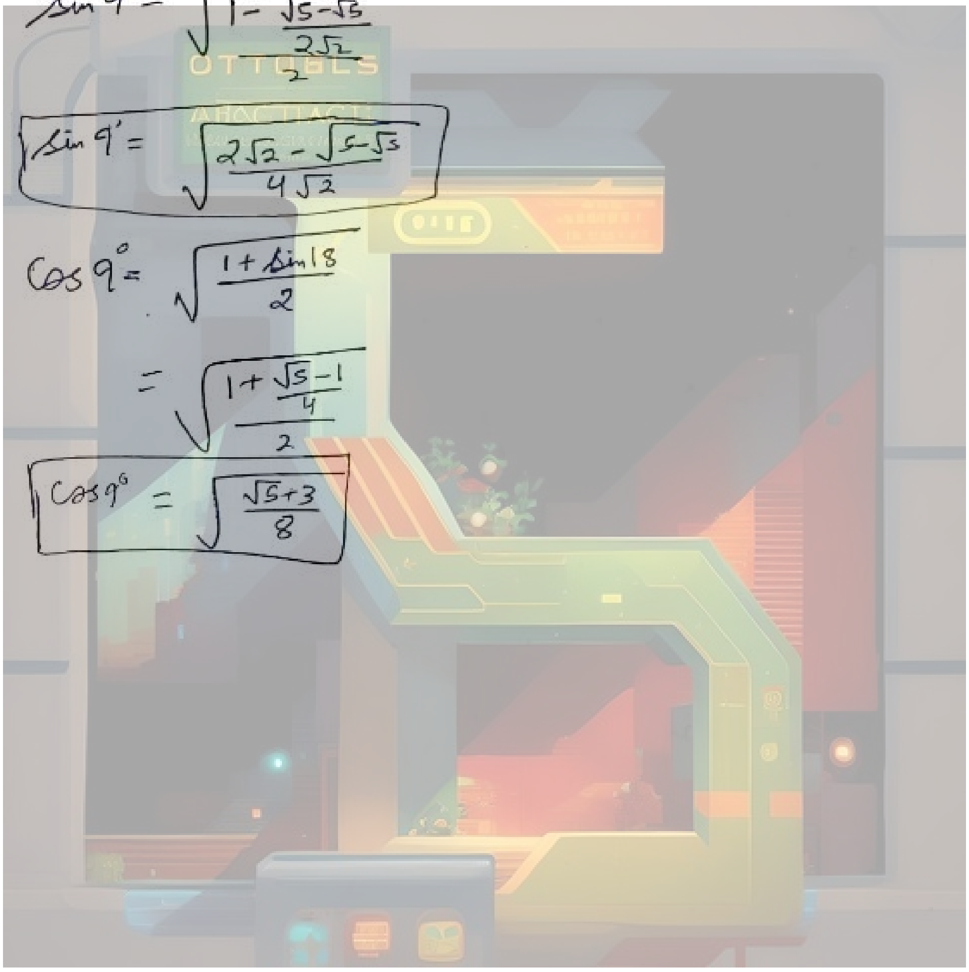
$$\sin 9 = \sqrt{\frac{1-\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}}{2}}$$

$$\boxed{\sin 9^\circ = \sqrt{\frac{2\sqrt{2}-\sqrt{5-\sqrt{5}}}{4\sqrt{2}}}}$$

$$\cos 9^\circ = \sqrt{\frac{1+\sin 18}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{5}-1}{4}}{2}}$$

$$\boxed{\cos 9^\circ = \sqrt{\frac{\sqrt{5}+3}{8}}}$$



Q ~~cos sin 18 + sin 54~~

$\sin\theta + \sin 3\theta$

$\sin\theta + 3\sin\theta - 4\sin^3\theta$

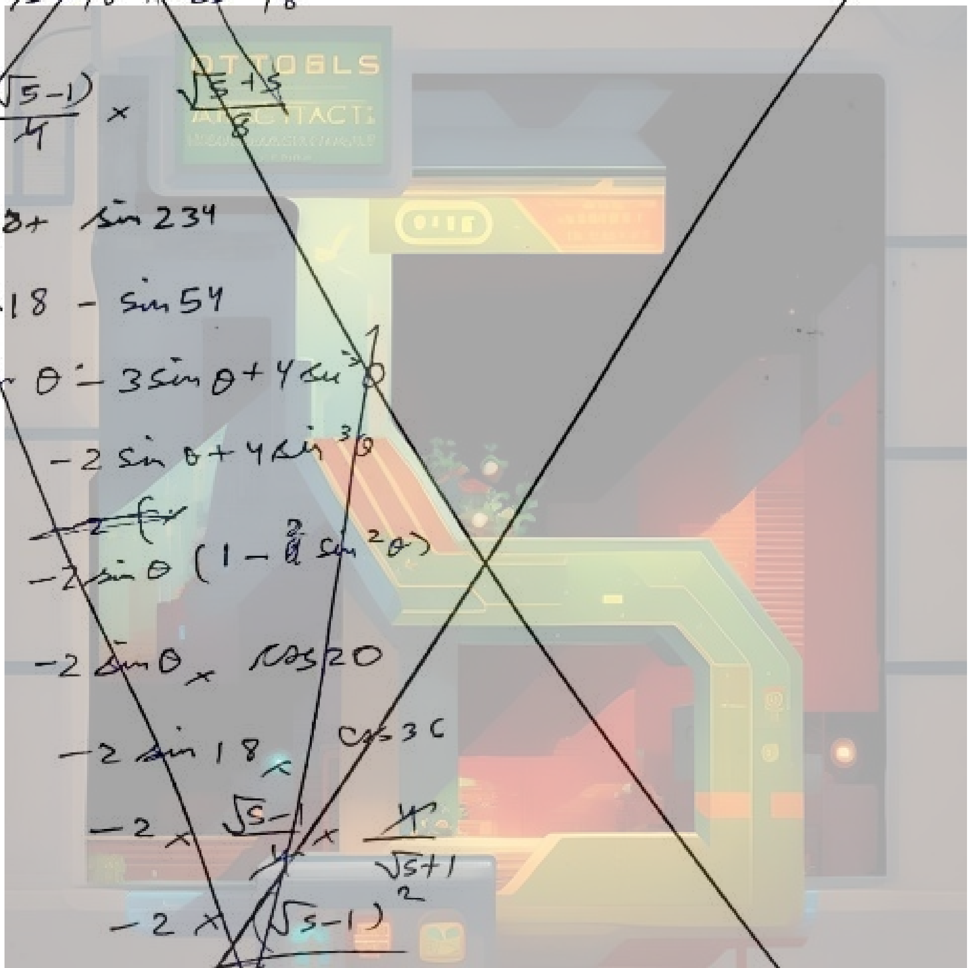
$4\sin\theta - 4\sin^3\theta$

$4(\sin\theta - \sin^3\theta)$

$4\sin 18(1 - \sin^2 18)$

$4\sin 18 \times \cos^2 18$

$4\frac{(\sqrt5 - 1)}{4} \times \frac{\sqrt{5+\sqrt5}}{8}$



Q $\sin 18 + \sin 234$

$\sin 18 - \sin 54$

$\sin\theta - 3\sin\theta + 4\sin^3\theta$

$-2\sin\theta + 4\sin^3\theta$

~~$-2($~~

$-2\sin\theta(1 - 2\sin^2\theta)$

$-2\sin\theta \times \cos 2\theta$

$-2\sin 18 \times \cos 36$

$-2 \times \frac{\sqrt5 - 1}{4} \times \frac{4}{\sqrt5 + 1}$

$-2 \times \frac{(\sqrt5 - 1)^2}{4}$

$= \frac{-6 + 2\sqrt5}{2}$

$\boxed{\sqrt5 - 3}$

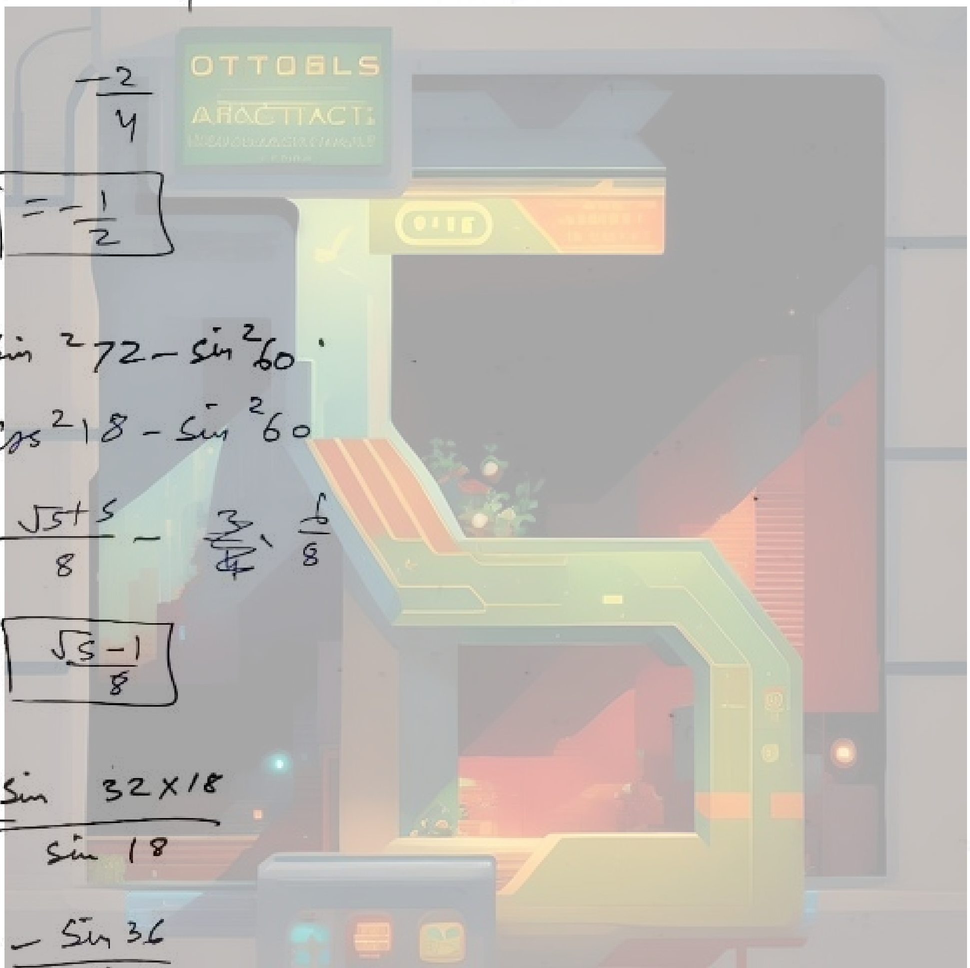
Q $\sin\frac{\pi}{10} + \sin\frac{13\pi}{10}$

$\sin 18 + \sin 234$

$\sin 18 - \sin 54$

$\sin 18 - \cos 36$

$\frac{\sqrt{5}-1 - \sqrt{5}-1}{4}$



$\frac{-2}{4}$

$\boxed{=-\frac{1}{2}}$

Q $\sin^2 72 - \sin^2 60$

$\cos^2 18 - \sin^2 60$

$\frac{\sqrt{5}+5}{8} - \cancel{\frac{3}{4}}\ \frac{6}{8}$

$\boxed{\frac{\sqrt{5}-1}{8}}$

Q $\frac{\sin 32\times 18}{\sin 18}$

$-\frac{\sin 36}{\sin 18}$

$-\frac{\sqrt{\sqrt{5}+5}}{\cancel{2}\sqrt{2}} \times \frac{4\cancel{2}\sqrt{2}}{\sqrt{5}-1}$

$\boxed{= \frac{2\sqrt{5}+10}{1-\sqrt{5}}}$

Series Based Questions

① Product or addition Based series

→ Basic knowledge of angles ($15^\circ, 18^\circ, 22.5^\circ, 36^\circ$,)

→ Complimentary or Supplimentary Angles pairing search.

→ formula based series

- ☆ Sine Series – Angles in AP
- ☆ Cosine series – Angles in AP
- ☆ Product series of cos – Angles in GP

→ $\sin^4\theta + \cos^4\theta = 1 - 2S^2C^2$

$$= 1 - \frac{1}{2} \times 4S^2C^2$$

$$= 1 - \frac{(2\sin\theta\cos\theta)^2}{2}$$

$$\boxed{= 1 - \frac{\sin^2 2\theta}{2}}$$

→ $\sin^6\theta + \cos^6\theta = 1 - 3S^2C^2$

$$= 1 - \frac{3}{4} \times 4S^2C^2$$

$$\boxed{= 1 - \frac{3}{4}\sin^2 2\theta}$$

Q find values :-

① $\sin 36^\circ \times \sin 2\times36 \times \sin 3\times36 \times \sin 4\times36$

② $\tan\frac{\pi}{8} \quad \tan\frac{3\pi}{8} \quad \tan\frac{5\pi}{8} \quad \tan\frac{7\pi}{8}$

③ $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

(4) $\sin^4\frac{\pi}{12} + \sin^4\frac{3\pi}{12} + \sin^4\frac{9\pi}{12} + \sin^4\frac{11\pi}{12}$

(5) $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$

(1) $\sin\frac{\pi}{5} \cdot \sin\frac{4\pi}{5} \cdot \sin\frac{2\pi}{5} \cdot \sin\frac{3\pi}{5}$

$\sin\frac{\pi}{5} \cdot \sin\left(\pi-\frac{\pi}{5}\right) \cdot \sin\frac{2\pi}{5} \sin\left(\pi-\frac{2\pi}{5}\right)$

$\left(\sin\frac{\pi}{5}\right)^2 \times \left(\sin\frac{2\pi}{5}\right)^2$

$\left(\sin\left(\frac{\pi}{5}\right)\right)^2 \times \cancel{\cos}\left(\cos\frac{2\pi}{5}\right)^2$

$\boxed{\sin^2 36 \cdot \cos^2 18}$

(2) $\tan\frac{\pi}{8} \cdot \tan\frac{7\pi}{8} \tan\frac{3\pi}{8} \tan\frac{5\pi}{8}$

$\tan\frac{\pi}{8} \tan\left(\pi-\frac{\pi}{8}\right) \times \tan\frac{3\pi}{8} \tan\left(\pi-\frac{3\pi}{8}\right)$

$-\left(\tan\frac{\pi}{8}\right)^2 \times -\left(\tan\frac{3\pi}{8}\right)^2$

$\frac{(\tan 22.5)^2}{\cancel{\tan}} \times \cancel{\tan} \cot(22.5)^2$

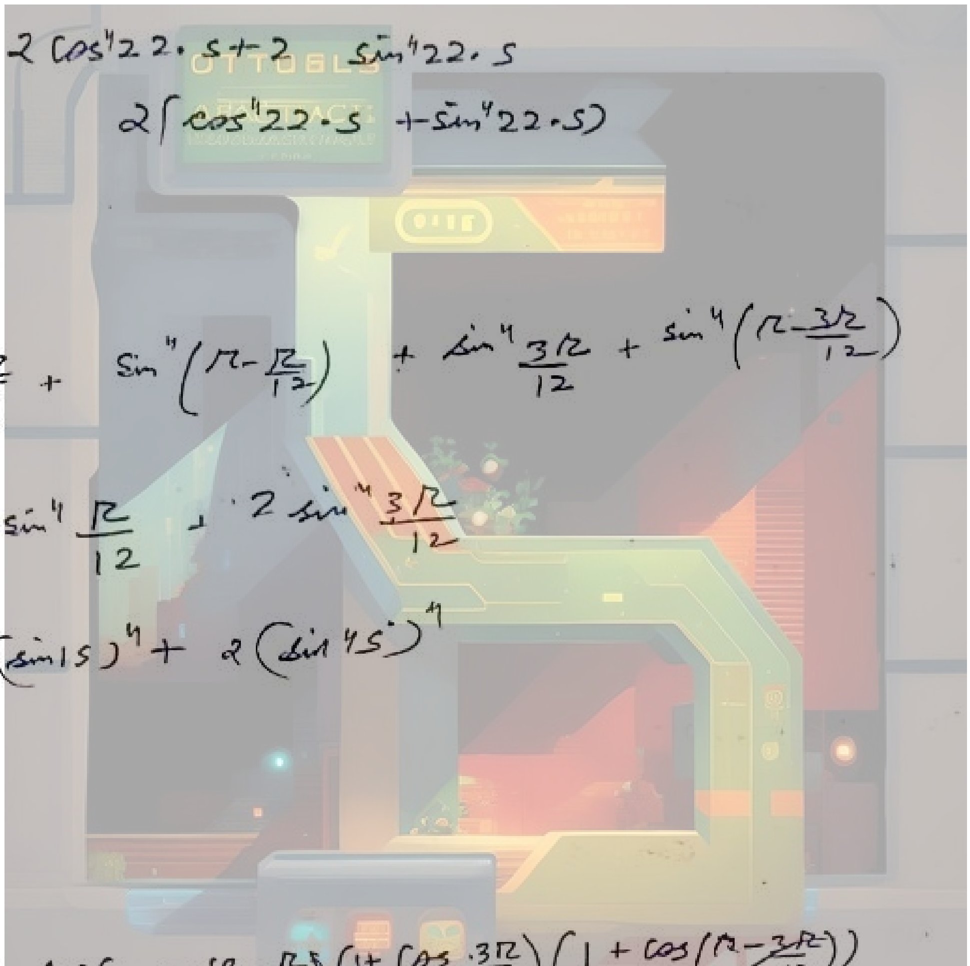
$\boxed{=1}$

(3) $\cos^4\frac{\pi}{8}+\cos^4\left(\pi-\frac{\pi}{8}\right)+\cos^4\left(\frac{3\pi}{8}\right)+\cos^4\left(\pi-\frac{3\pi}{8}\right)$

$\cos^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{3\pi}{8}$

(4) ~~4 cos~~

$2\cos^4 22.5+2\cos^4 67.5$



$2\cos^4 22.5+2\sin^4 22.5$

$2(\cos^4 22.5+\sin^4 22.5)$

(4) $\sin^4\frac{\pi}{12}+\sin^4\left(\pi-\frac{\pi}{12}\right)+\sin^4\frac{3\pi}{12}+\sin^4\left(\pi-\frac{3\pi}{12}\right)$

$2\sin^4\frac{\pi}{12}+2\sin^4\frac{3\pi}{12}$

$2(\sin 15)^4+2(\sin 45)^4$

(5) $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\left(\pi-\frac{\pi}{10}\right)\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\left(\pi-\frac{3\pi}{10}\right)\right)$

$(1+\cos 18)(1+\sin 18)(1+\cos 54)(1+\sin 54)$

$(1+\sin 18+\cos 18+\sin 18\cos 18)(1+\cos 36+\sin 36+\cos 36\sin 36)$

$\left(1+\sin 18+\cos 18+\frac{\sin 36}{2}\right)\left(1+\cos 36+\sin 36+\frac{\cos 18}{2}\right)$

Q5. $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\left(\pi-\frac{\pi}{10}\right)\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\left(\pi-\frac{3\pi}{10}\right)\right)$

$$\left(1+\cos\frac{\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)$$

$$\left(1^2-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$$

$$\sin^2\frac{\pi}{10}\times\sin^2\frac{3\pi}{10}$$

$$\sin^2 18\times\sin^2 54$$

$$\boxed{\sin^2 18\cos^2 36}$$

H.W. 25-7-21)

DYS-12 [17, 18]

DYS-13 [1, 4]

O-1 [16, 21]

J-M [1, 5] $\cup$ [9, 12]

Continued product of cosine with angles in GP ($r=2$)

$$\cos A\cdot\cos 2A\cdot\cos 4A\ldots\cos(2^{n-1}A)\times\frac{2\sin A}{2\sin A}$$

$$\frac{\sin 2A\cos 2A\ \cos 4A\ldots\cos(2^{n-1}A)}{2\sin A}\times\frac{2}{2}$$

$$\frac{\sin 4A\ \cos 4A\ldots\cos(2^{n-1}A)}{2^2\sin A}\times\frac{2}{2}$$

$$\boxed{=\frac{\sin(2^n A)}{2^n\sin A}}$$

Q find the values of the following :-

(1) $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$

$$= \frac{\sin(2^n\theta)}{2^n \sin\theta}$$

$$= \frac{\sin(8\times 20)}{8\times \sin 20}$$

$$= \frac{\sin \cancel{(160)}(180-20)}{8 \sin 20}$$

$$= \frac{\sin 20}{8 \sin 20}$$

$$\boxed{=\frac{1}{8}}$$

(2) $\cos\frac{16\pi}{10}\ \cos\frac{8\pi}{10}\ \cos\frac{4\pi}{10}\ \cos\frac{2\pi}{10}\ \cos\frac{\pi}{10}$

(2) $\cancel{\cos}\ \frac{\sin(2^n\theta)}{2^n\sin\theta}$

$$= \frac{\sin(32\times 18)}{32 \sin 18}$$

$$= \frac{-\sin 36}{32 \sin 18}$$

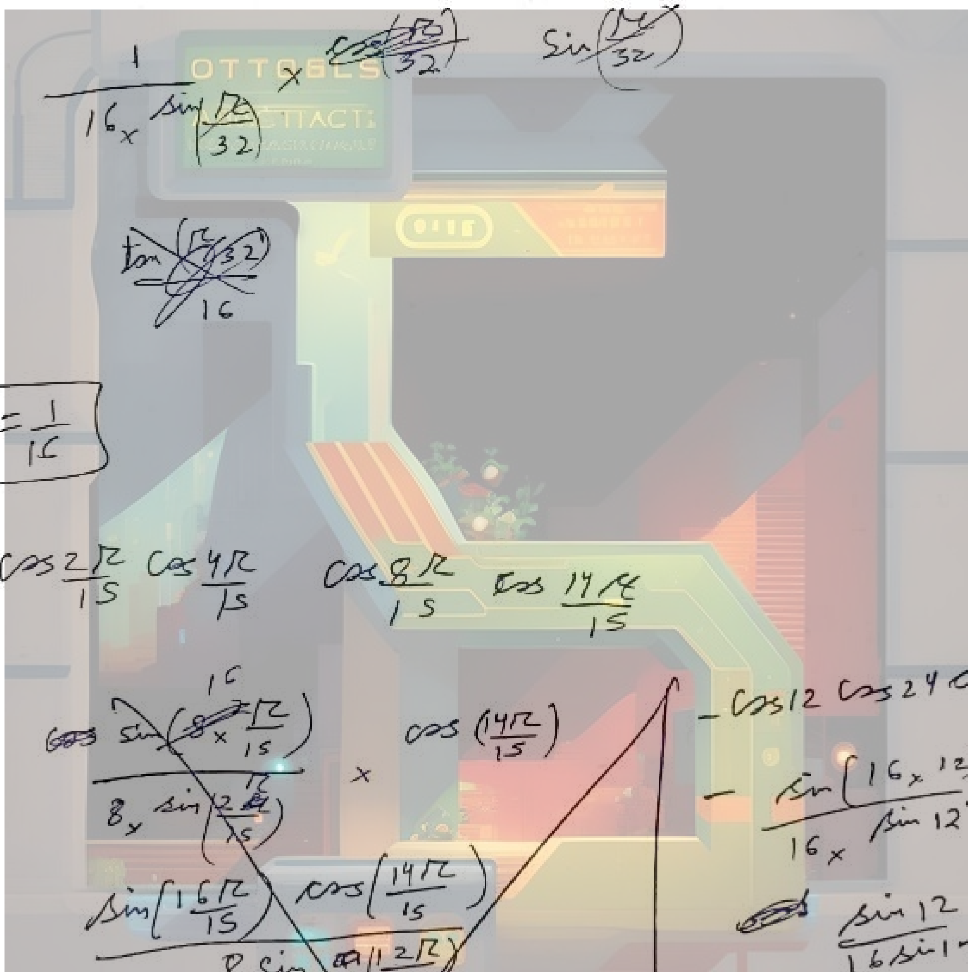
$$\cancel{= \frac{-\sqrt{\frac{5-\sqrt{5}}{8}}}{32\times(\sqrt{5}-1)}}$$

$$= \frac{-\cos 18}{16}$$

$$= \frac{-\sqrt{\frac{5+\sqrt{5}}{8}}}{16}$$

(3) $\cos\frac{\pi}{2^2}\cos\frac{\pi}{2^3}\cos\frac{\pi}{2^4}\cos\frac{\pi}{2^5}\cancel{\cos\frac{\pi}{2^3}}\quad \sin\left(\frac{\pi}{32}\right)$

$$\frac{\sin\left(\cancel{2}16\times\frac{\pi}{\cancel{2}32}\right)}{16\times\sin\frac{\pi}{32}}$$



$$\frac{1}{16\times\sin\left(\frac{\pi}{32}\right)}\times\cancel{\cos\left(\frac{\pi}{32}\right)}\quad\sin\left(\frac{\pi}{32}\right)$$

$$\cancel{\frac{\tan\left(\frac{\pi}{32}\right)}{16}}$$

$$\boxed{=\frac{1}{16}}$$

(4) $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$

$$\frac{\sin\left(\cancel{8}^{16}\times\frac{\pi}{15}\right)}{8\times\sin\left(\frac{2\pi}{15}\right)}\times\cos\left(\frac{14\pi}{15}\right)$$

$$\frac{\sin\left(\frac{16\pi}{15}\right)\cos\left(\frac{14\pi}{15}\right)}{8\sin\left(\frac{2\pi}{15}\right)}$$

$$\frac{-\left(\sin\frac{\pi}{15}+\cos\frac{\pi}{15}\right)}{8\times\sin\frac{3\pi}{15}\ \sin 36}$$

$$\frac{-(\sin 12+\cos 12)}{\sqrt{\frac{5-\sqrt{5}}{8}}\times 8}$$

$-\cos 12\cos 24\cos 48\cos 96$

$$-\frac{\sin(16\times 12)}{16\times\sin 12^\circ}$$

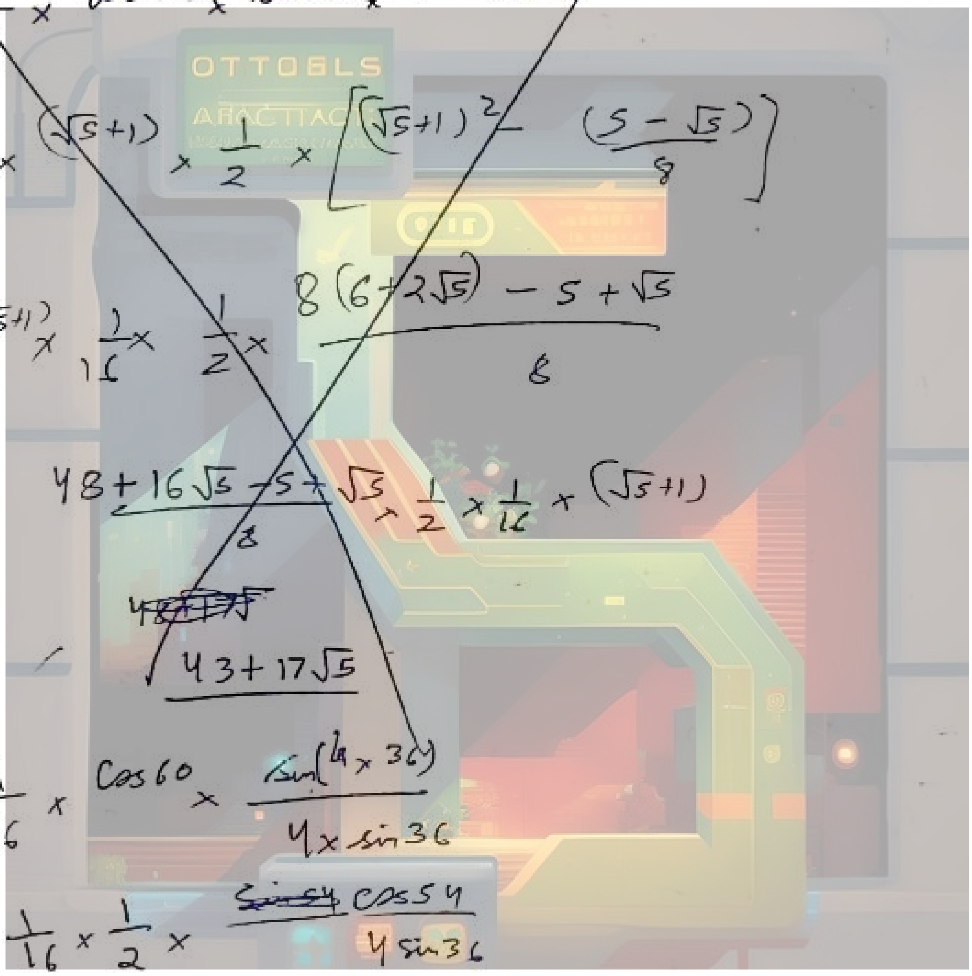
$$\frac{\sin 12}{16\sin 12}$$

$$\boxed{=\frac{1}{16}}$$

⑤ $\underbrace{\cos\frac{\pi}{15}\,\cos\frac{2\pi}{15}\,\cos\frac{4\pi}{15}}\,\cos\frac{3\pi}{15}\,\cos\frac{5\pi}{15}\,\cos\frac{6\pi}{15}\,\cos\frac{7\pi}{15}$

$\cos\frac{7\pi}{15} \downarrow -\cos\frac{8\pi}{15}$

$$\frac{-\sin\left(16\times\frac{\pi}{15}\right)}{16\sin\left(\frac{\pi}{15}\right)}$$



$$\cancel{\frac{1}{16}\times\cos 36\times\cos 60\times\cos 2(36)}$$

$$\frac{1}{16}\times(\sqrt{5}+1)\times\frac{1}{2}\times\left[(\sqrt{5}+1)^2 - \frac{(5-\sqrt{5})}{8}\right]$$

$$(\sqrt{5}+1)\times\frac{1}{16}\times\frac{1}{2}\times\frac{8(6+2\sqrt{5})-5+\sqrt{5}}{8}$$

$$\frac{48+16\sqrt{5}-5+\sqrt{5}}{8}\times\frac{1}{2}\times\frac{1}{16}\times(\sqrt{5}+1)$$

$$\cancel{48+17\sqrt{5}}$$

$$43+17\sqrt{5}$$

$$\frac{1}{16}\times\cos 60\times\frac{\sin(4\times 36)}{4\times\sin 36}$$

$$\frac{1}{16}\times\frac{1}{2}\times\frac{\cancel{\sin 54}\,\cos 54}{4\sin 36}$$

$$\frac{1}{32}\times\frac{\sin 36}{4\sin 36}$$

$$\boxed{\frac{1}{128}}$$

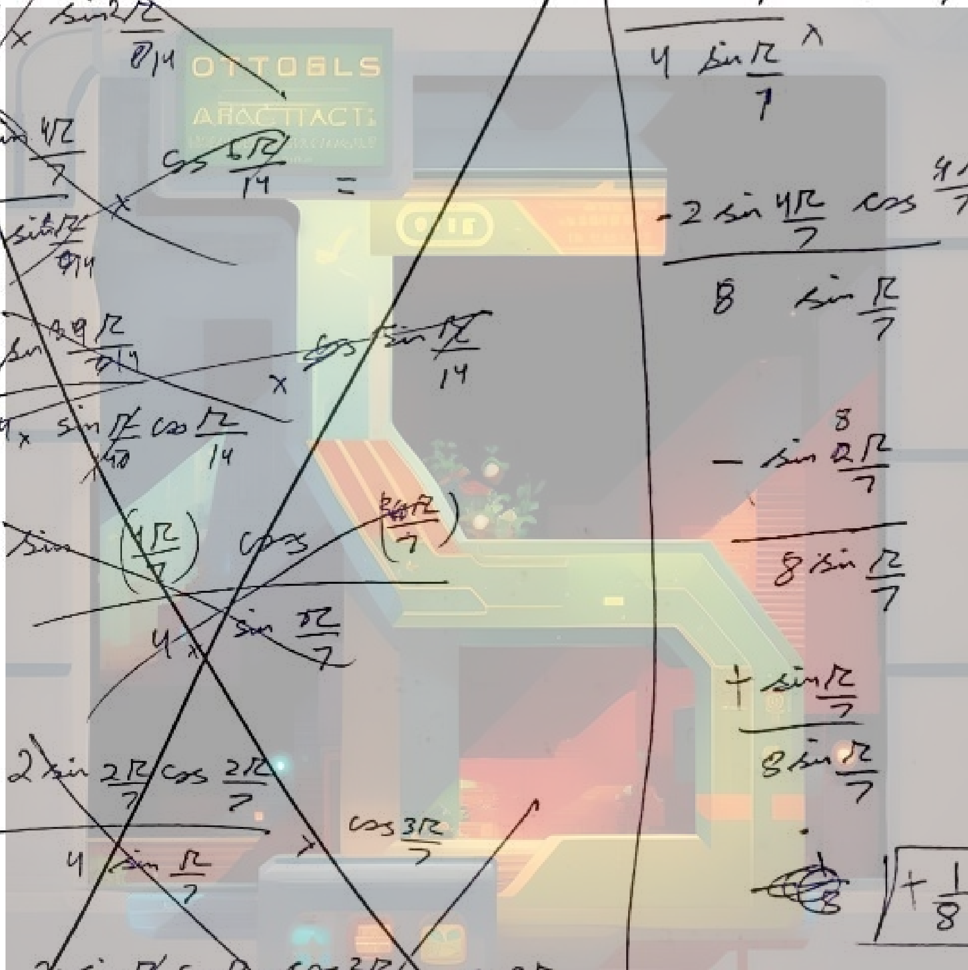
Q $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}$

$\cancel{\cos\frac{13\pi}{14}\cos\frac{11\pi}{14}\cos\frac{9\pi}{14}\cos\frac{7\pi}{14}}$

$\cos\frac{6\pi}{14}\cos\frac{4\pi}{14}\cos\frac{2\pi}{14}\cos 0^\circ = 0$

$\downarrow$

1



$\cancel{\dfrac{\sin\left(4\times\frac{\pi}{14}\times 2\right)}{4\times\sin\frac{2\pi}{14}}\times\cos\frac{6\pi}{14} =}$

$\cancel{\dfrac{\sin\frac{4\pi}{7}}{4\sin\frac{2\pi}{14}}\times\cos\frac{\pi}{14}}$

$\cancel{\dfrac{\sin\frac{2\pi}{14}}{2\times4\times\sin\frac{\pi}{14}\cos\frac{\pi}{14}}}$

$\cancel{\dfrac{\sin\left(\frac{4\pi}{7}\right)\cos\left(\frac{3\pi}{7}\right)}{4\times\sin\frac{\pi}{7}}}$

$\cancel{\dfrac{2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7}}{4\sin\frac{\pi}{7}}\times\cos\frac{3\pi}{7}}$

$\cancel{\dfrac{2\sin\frac{\pi}{7}\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}}{2\cdot4\sin\frac{\pi}{7}}}$

$\cancel{\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}}$

$\dfrac{\sin\frac{4\pi}{7}}{4\sin\frac{\pi}{7}}\times\cos\frac{3\pi}{7}$

$\dfrac{2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{8\sin\frac{\pi}{7}}$

$-\dfrac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}$

$+\dfrac{\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}}$

$\boxed{+\frac{1}{8}}$

Conditional identities - Identities related to triangles.

$\hookrightarrow A+B+C = 180^\circ/\pi$

$\hookrightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2} = 90^\circ$

If :- $A+B+C = 180^\circ$ then :-

(1) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$

(2) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$

(3) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(4) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{C}{2} = 1$

(5) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(6) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Proof:-

(1) $\sin 2A + \sin 2B + \sin 2C$

$\sin 2A + 2\sin(B+C)\cos(B-C)$

$\because A+B+C = 180^\circ$

$B+C = 180 - A$

$2\sin A\cos A + 2\sin(180-A)\cos(B-C)$

$2\sin A\cos A + 2\sin A\cos(B-C)$

$2\sin(\cos A + \cos(B-C))$

$4\sin A \cos\left(\frac{180-C-C}{2}\right)\cos\left(\frac{180-B-B}{2}\right)$

$4\sin A\cos(90-C)\cos(90-B)$

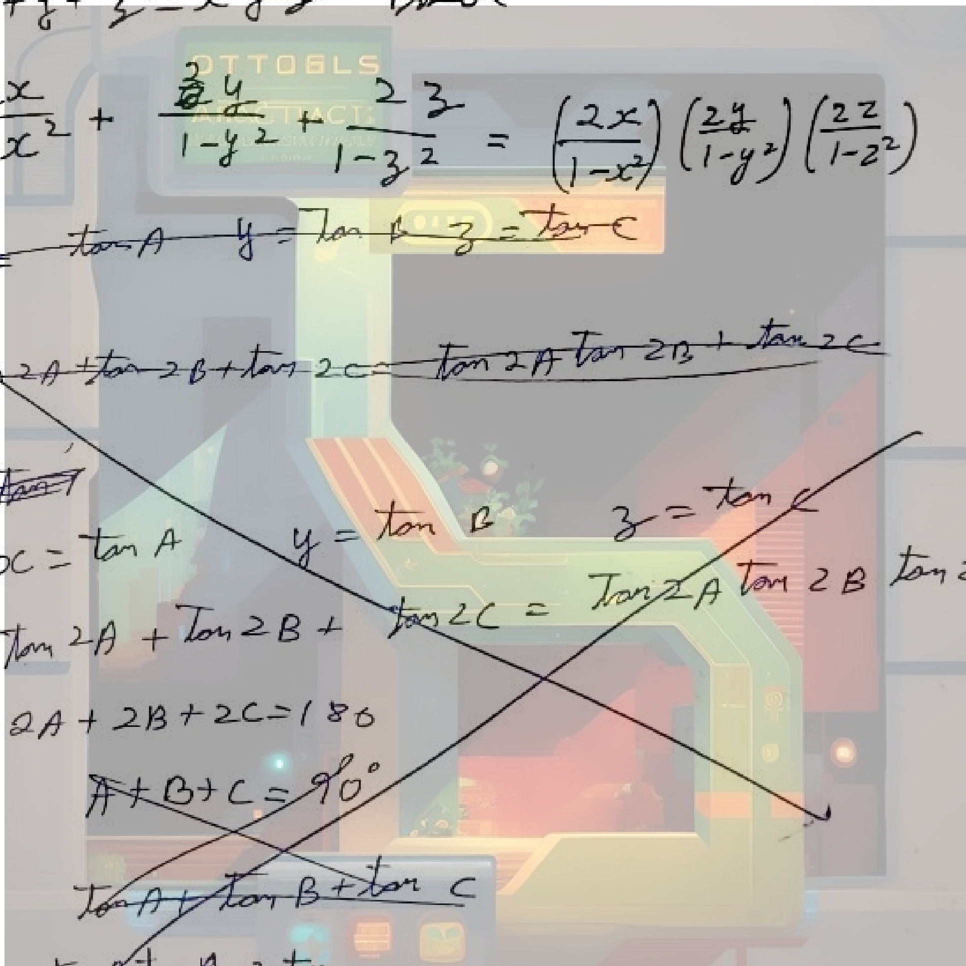
$4\sin A\sin C\sin B.$

~~③ $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$~~

Q $\frac{\sin 50 + \sin 100 + \sin 210}{\sin 25 \sin 50 \sin 105}$

$\frac{4 \sin 25 \sin 50 \sin 165}{4 \cancel{\cos} \sin A \sin B \sin C} = \boxed{4}$

Q $x + y + z = xyz$. Prove



① $\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \left(\frac{2x}{1-x^2}\right)\left(\frac{2y}{1-y^2}\right)\left(\frac{2z}{1-z^2}\right)$

~~$x = \tan A \quad y = \tan B \quad z = \tan C$~~

~~$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$~~

~~tan~~

$x = \tan A \quad y = \tan B \quad z = \tan C$

$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$

$2A + 2B + 2C = 180$

~~$A + B + C = 90°$~~

~~$\tan A + \tan B + \tan C$~~

$\tan A \tan B + \tan$

$x = \tan A \quad y = \tan B \quad z = \tan C$

$A + B + C = 180$

$2A + 2B + 2C = 360$

$2A + 2B = 360 - 2C$

$\tan(2A + 2B) = -\tan 2C$

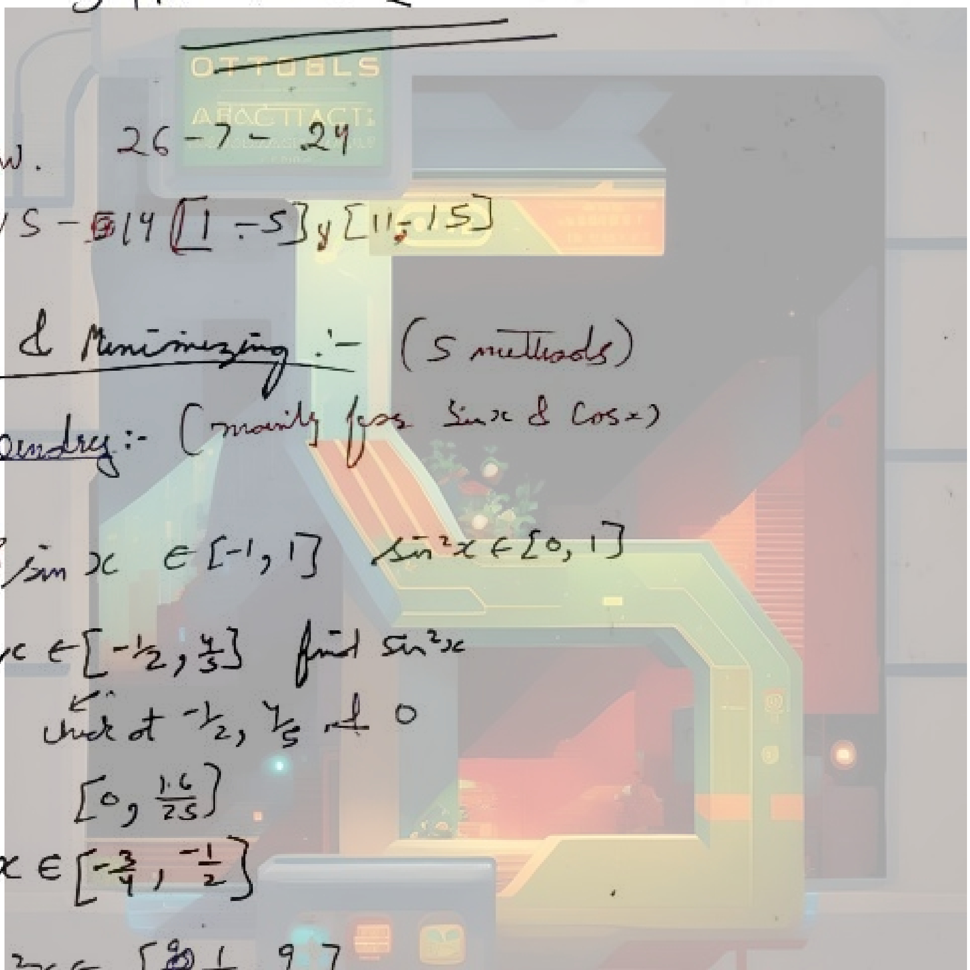
$$\frac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = -\tan 2C$$

$$\tan 2A + \tan 2B = -\tan 2C(1 - \tan 2A \tan 2B)$$

$$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$$

$$\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \left(\frac{2x}{1-x^2}\right)\left(\frac{2y}{1-y^2}\right)\left(\frac{2z}{1-z^2}\right)$$

अतः सिद्धम्



H.W. 26 - 7 - 24

DVS - 14 [1 - 5] , [11 - 15]

Maximizing & Minimizing :- (5 methods)

① Using Boundry :- (mainly for $\sin x$ & $\cos x$)

$\sin x \in [-1, 1]$ $\quad \sin^2 x \in [0, 1]$

$\sin x \in [-\frac{1}{2}, \frac{4}{5}]$ find $\sin^2 x$

check at $-\frac{1}{2}, \frac{4}{5}$ & 0

$[0, \frac{16}{25}]$

$\sin x \in [-\frac{3}{4}, -\frac{1}{2}]$

$\sin^2 x \in [\frac{1}{4}, \frac{9}{16}]$

$\tan x \in (-\infty, \infty)$ $\quad \tan^2 x \in [0, \infty)$

$\sec x \in (-\infty, -1] \cup [1, \infty)$ $\quad \sec^2 x \in [1, \infty)$

- Addition, Subtraction, Multiplication & Division of two ranges is <u>not</u> <u>allowed.</u>

$$\cancel{[-1,1] + [-1,1] = [-2,2]}$$

$$\frac{(\sin x + \cos x)\sqrt{2}}{\sqrt{2}}$$

$$(\sin x \cdot \sin 45 + \cos x \cos 45)\sqrt{2}$$

$$\sqrt{2}\cos \cancel{(45 - x)}(x - 45)$$

$$\sqrt{2}[-1, 1]$$

$$= [-\sqrt{2}, \sqrt{2}]$$

valid.

→ we can add, subtract, multiply or ~~divide~~ divide any constant except 0 in range.

eg. ① $y = 2 + \sin x$

$$= 2 + [-1, 1]$$

$$= [1, 3]$$

$$y \in [1, 3]$$

② $y = 1000 \tan x$

$$= 1000(-\infty, \infty)$$

$$y \in (-\infty, \infty)$$

③ $y = \frac{\cos x}{\sqrt{3}}$

$$= \frac{[-1, 1]}{\sqrt{3}}$$

$$y \in \left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$$

Q find range of

(1) $y = 4\sin(2x)$

$= 4\,[-1, 1]$

$\boxed{= [-4, 4]}$

Note :- No change in range will be observed if we change angle in any form.

(2) $y = 8\cos(2x + 3)$

$= 8\,[-1, 01]$

$\boxed{= [-8, 8]}$

(3) ~~$y\theta$~~ $y = \cancel{8}\sin\frac{\pi}{3} \cdot \sec^2 x$

$= [1, \infty)\ \frac{\sqrt{3}}{2}$

$\boxed{= \left[\frac{\sqrt{3}}{2}, \infty\right)}$

(4) $y = \sin(\sqrt{x})\quad [-1, 1]$

(5) $y \cancel{x} =$ (Chameli) $\tan^2 x\quad (-\infty, \infty)$

(6) $y = \tan^2 x - 2\quad \cancel{(-\infty, \infty)}\ (-2, \infty)$

(7) $y = \sin x + 3\quad [2, 4]$

(8) $y = \cancel{\quad} \cos^4 x - \sin^4 x\quad \cancel{\cos^2 2x}\quad \cancel{[0, 1]}\ [-1, 1]$

(9) $y = \tan^2\left(x - \frac{\pi}{4}\right)\quad [0, \infty)$

(10) $y = \sin x \cos x\quad \frac{\sin 2x}{2}\quad \frac{[-1, 1]}{2} = \left[-\frac{1}{2}, \frac{1}{2}\right]$

(11) $y = 4\tan x \cos x\quad (\cancel{\quad})\quad \cancel{\left[\frac{\quad}{2}\right]}$

$4\,\cancel{\tan x}\ \frac{\sin x}{\cancel{\cos x}} \times \cos x$

$\cos x \neq 0$

$x \neq 90$

$x \neq -90$ ($\tan x$)

$4\,(-1, 1)$

$\boxed{(-4, 4)}$

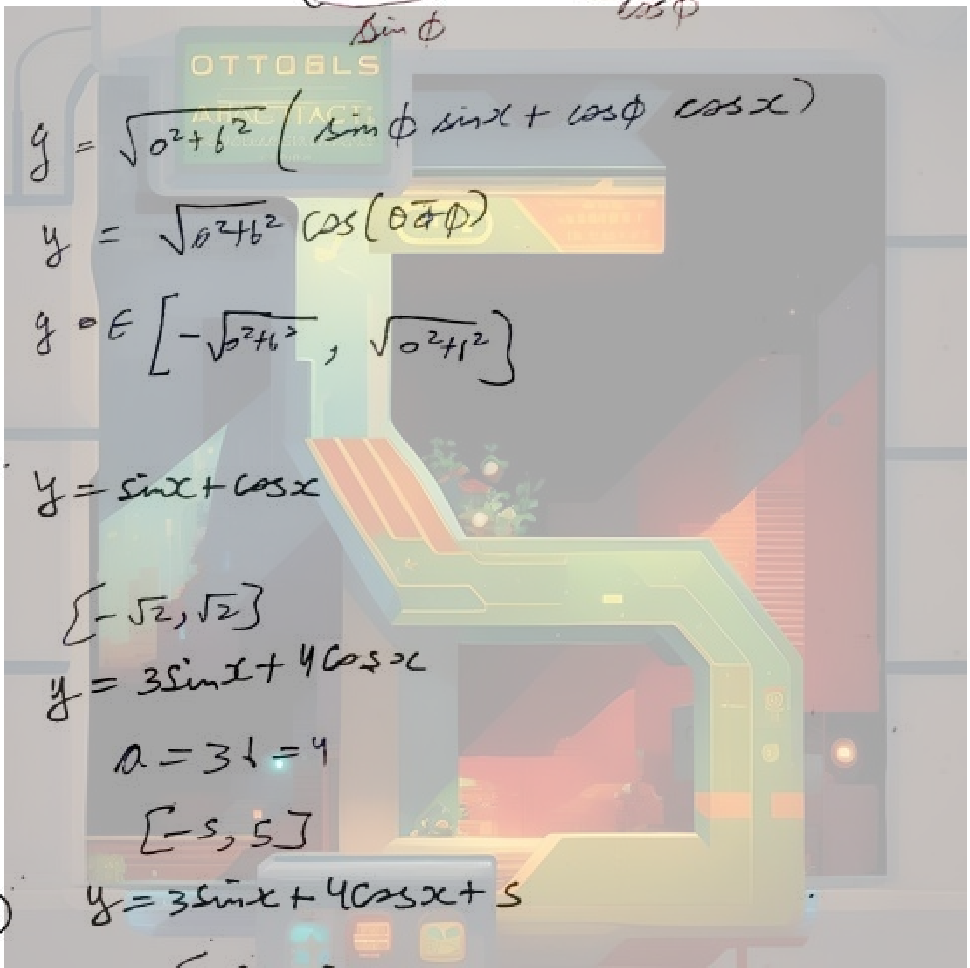
② $a\sin(x) + b\cos(x)$ form $a, b \in R$.

must be same.

$y = a\sin x + b\cos x$

$$\cancel{y = \sqrt{a^2+b^2}\left(\frac{a\sin x}{\sqrt{a^2+b^2}}\right.}$$

$$y = \sqrt{a^2+b^2}\left(\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\sin\phi}\sin x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\cos\phi}\cos x\right)$$



$$y = \sqrt{a^2+b^2}\left(\sin\phi \sin x + \cos\phi \cos x\right)$$

$$y = \sqrt{a^2+b^2}\cos(\theta \mp \phi)$$

$$y \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$$

Q ① $y = \sin x + \cos x$

$[-\sqrt{2}, \sqrt{2}]$

② $y = 3\sin x + 4\cos x$

$a = 3\ b = 4$

$[-5, 5]$

③ $y = 3\sin x + 4\cos x + 5$

$[-5, 5] + 5$

$[0, 10]$

(4) $y = \log_2 (\sin^2 x)$

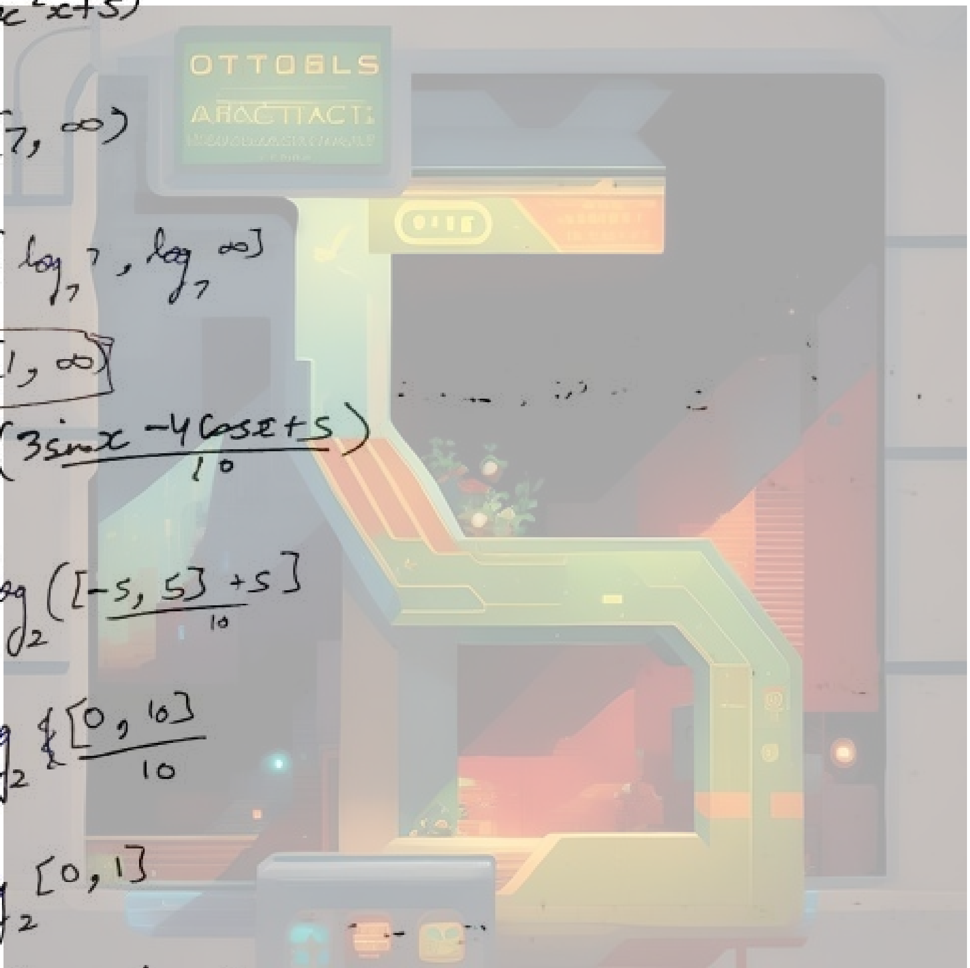
$y = \log_2 [0, 1]$

$= \cancel{[2^0, 2^1]}$ $\quad y \in [\log_2 0, \log_2 1]$

$= \cancel{[1, 2]}$ $\quad \boxed{y \in (-\infty, 0]}$

~~$= y \in [0, 1]$~~

(5) $y = \log_7 (2\sec^2 x + 5)$



$y = \log_7 [7, \infty)$

$= y \in [\log_7 7, \log_7 \infty)$

$\boxed{y \in [1, \infty)}$

(6) $y = \log_2 \left(\frac{3\sin x - 4\cos x + 5}{10} \right)$

$y = \log_2 \left(\frac{[-5, 5] + 5}{10} \right]$

$y = \log_2 \left\{ \frac{[0, 10]}{10} \right.$

$y = \log_2 [0, 1]$

$y \in [\log_2 0, \log_2 1]$

$\boxed{y \in (-\infty, 0]}$

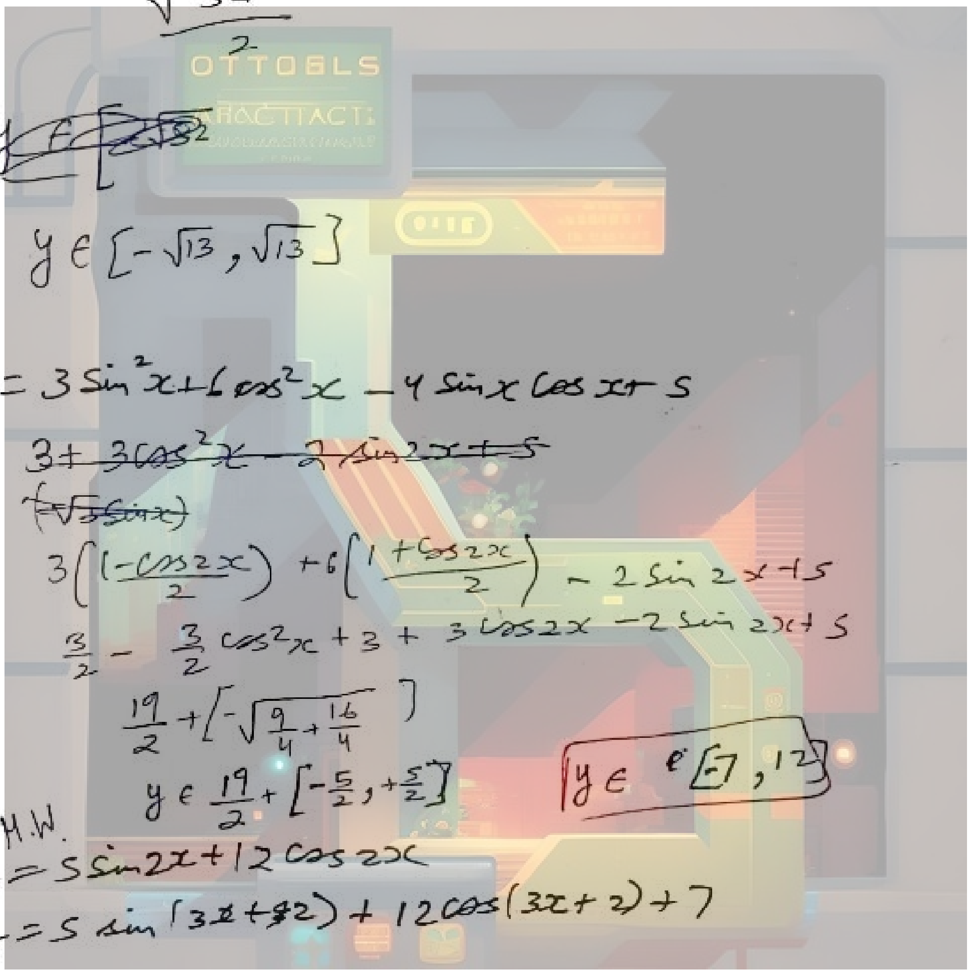
(7) $y = \sin\left(x + \frac{\pi}{6}\right) + 3\cos x$

$\sin(30 + x) = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x + 3\cos x$

$\frac{7}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

$\sqrt{\frac{49}{4} + \frac{3}{4}}$

$\frac{\sqrt{52}}{2}$



~~$y \in [\sqrt{52}$~~

$y \in [-\sqrt{13}, \sqrt{13}]$

(8) $y = 3\sin^2 x + 6\cos^2 x - 4\sin x\cos x + 5$

~~$3 + 3\cos^2 x - 2\sin 2x + 5$~~

~~$(-\sqrt{5}\sin x)$~~

$3\left(\frac{1 - \cos 2x}{2}\right) + 6\left(\frac{1 + \cos 2x}{2}\right) - 2\sin 2x + 5$

$\frac{3}{2} - \frac{3}{2}\cos 2x + 3 + 3\cos 2x - 2\sin 2x + 5$

$\frac{19}{2} + \left[-\sqrt{\frac{9}{4} + \frac{16}{4}}\right]$

$y \in \frac{19}{2} + \left[-\frac{5}{2}, +\frac{5}{2}\right]$ $\boxed{y \in [7, 12]}$

H.W.

(9) $y = 5\sin 2x + 12\cos 2x$

(10) $y = 5\sin(3x + 2) + 12\cos(3x + 2) + 7$

(11) $y = \sin\left(x + \frac{\pi}{6}\right) + 3\cos\left(x - \frac{\pi}{3}\right)$

(12) $y = 7\cos^2 x + 4\sin x\cos x + 3\sin^2 x$

③ Making ~~of~~ perfect square

① $y = \sin^2 x + 2\sin x + 4$

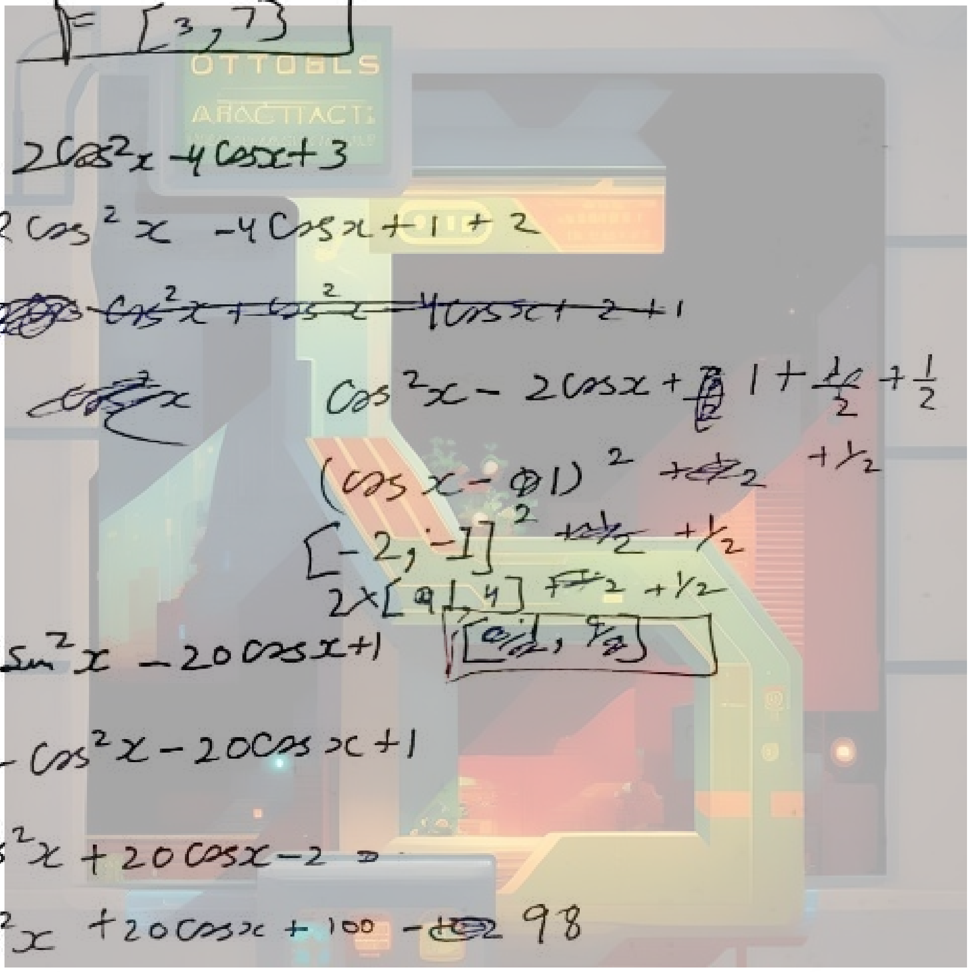
$y = \sin^2 x + 2\sin x + 1 + 3$

$= (\sin x + 1)^2 + 3$

$= [0, 2]^2 + 3$

$= [0, 4] + 3$

$\boxed{= [3, 7]}$



② $y = 2\cos^2 x - 4\cos x + 3$

$2\cos^2 x - 4\cos x + 1 + 2$

$\cancel{\cos^2 x + \cos^2 x - 4\cos x + 2 + 1}$

$\cos^2 x - 2\cos x + 1 + \frac{1}{2} + \frac{1}{2}$

$(\cos x - 1)^2 + \frac{1}{2}$

$[-2, -1]^2 + \frac{1}{2}$

$2 \times [1, 4] + \frac{1}{2}$

$\boxed{[\ldots, \tfrac{9}{2}]}$

③ $y = \sin^2 x - 20\cos x + 1$

$1 - \cos^2 x - 20\cos x + 1$

$\cos^2 x + 20\cos x - 2$

$\cos^2 x + 20\cos x + 100 - 98$

$(\cos x + 10)^2 - 98$

$[9, 11]^2 - 98$

$[81, 121] - 98$

$\boxed{\cancel{[-21, 19]}}$

$\boxed{[-19, 21]}$

46

(4) $y = \cos 2x + \sin x$

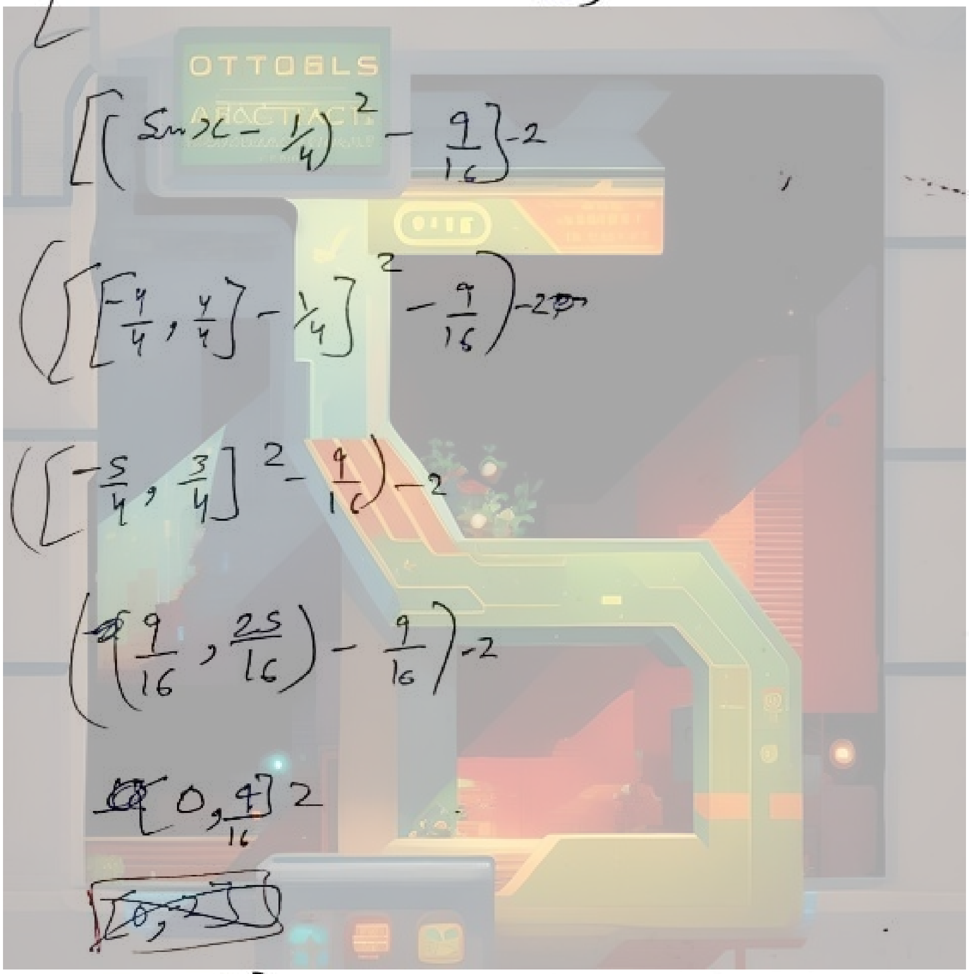
$\cancel{1 - 2\cos^2 x + \sin}$

$1 - 2\sin^2 x + \sin x$

$2\sin^2 x - \sin x - 1$

$2\left[\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2}\right] - 2$

$\left[\sin^2 x - \frac{1}{2}\sin x + \left(\frac{1}{4}\right)^2 - \frac{9}{16}\right] - 2$



$\left[\left(\sin x - \frac{1}{4}\right)^2 - \frac{9}{16}\right] - 2$

$\left(\left[\left[-\frac{4}{4}, \frac{4}{4}\right] - \frac{1}{4}\right]^2 - \frac{9}{16}\right) - 2$

$\left(\left[-\frac{5}{4}, \frac{3}{4}\right]^2 - \frac{9}{16}\right) - 2$

$\left(\left(\frac{9}{16}, \frac{25}{16}\right) - \frac{9}{16}\right) - 2$

$\left[0, \frac{4}{16}\right] 2$

$\cancel{[0, 2]}$

$\left[-2, \frac{9}{8}\right]$

(4) using AM $\geq$ GM

$\sin\theta \csc\theta = 1$

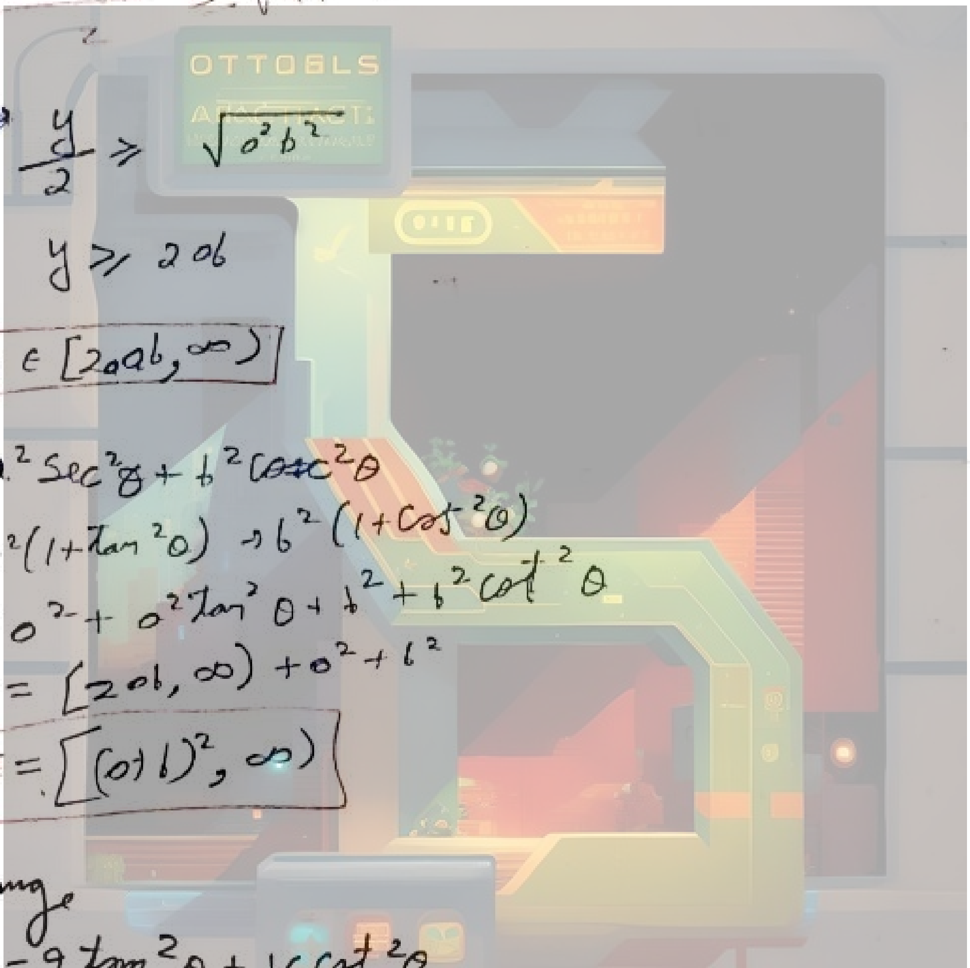
$\cos\theta \sec\theta = 1$

$\cot\theta \tan\theta = 1$

Format - 1

$$y = \underbrace{a^2\tan^2\theta}_{\theta_1} + \underbrace{b^2\cot^2\theta}_{\theta_2}$$

$$\frac{\theta_1 + \theta_2}{2} \geq \sqrt{\theta_1 \theta_2}$$



$$\frac{y}{2} \geq \sqrt{a^2 b^2}$$

$$y \geq 2ab$$

$$\boxed{y \in [2ab, \infty)}$$

(2) $y = a^2\sec^2\theta + b^2\csc^2\theta$

$= a^2(1+\tan^2\theta) + b^2(1+\cot^2\theta)$

$= a^2 + a^2\tan^2\theta + b^2 + b^2\cot^2\theta$

$= [2ab, \infty) + a^2 + b^2$

$$\boxed{y \in [(a+b)^2, \infty)}$$

Q find range

(1) $y = 9\tan^2\theta + 16\cot^2\theta$

$y \in (2 \times 3 \times 4, \infty)$

$y \in [24, \infty)$

② $y = 25\sec^2\theta + 16\,\mathrm{cosec}^2\theta$

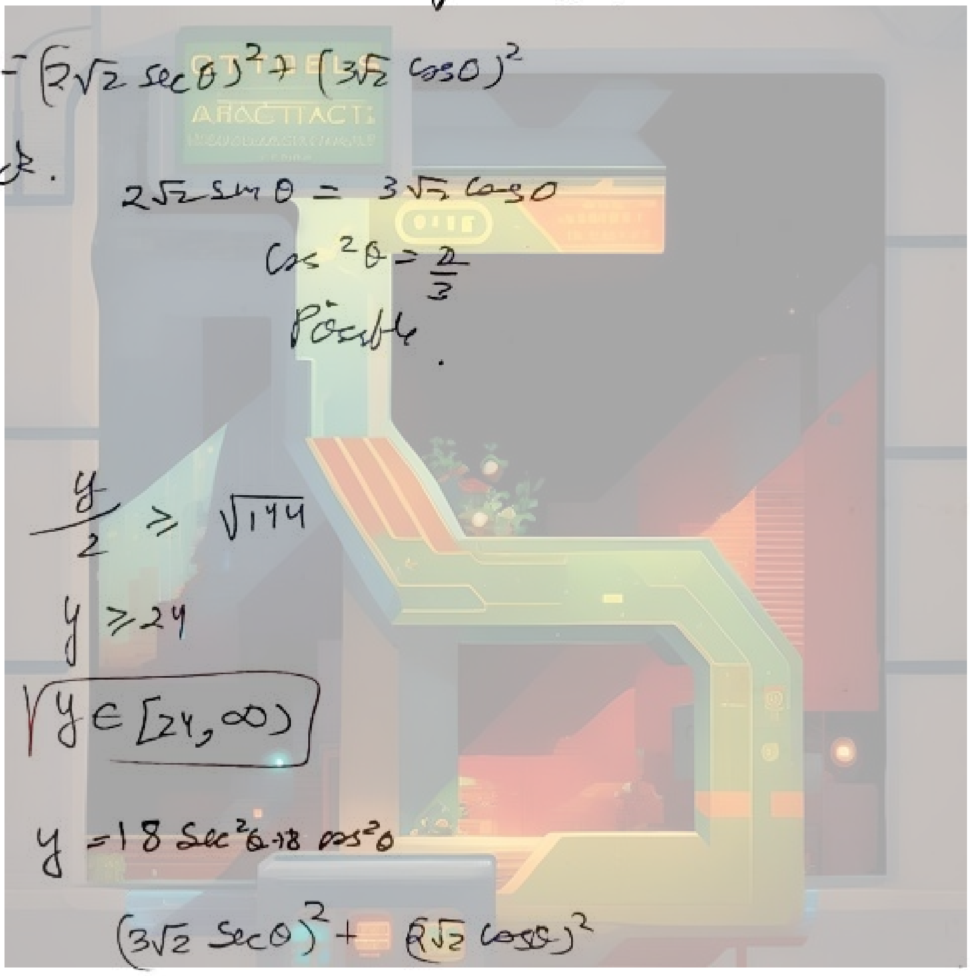
~~$y \in [40$~~.

$y \in [81, \infty)$

Format 2 & 3 :- $\sin\theta$ & $\mathrm{cosec}\,\theta$ / $\sec\theta$ & $\cos\theta$

& check holding of equality in AM-GM

① $y = 8\sec^2\theta + 18\cos^2\theta$ find range.



$= (2\sqrt{2}\sec\theta)^2 + (3\sqrt{2}\cos\theta)^2$

check.

$2\sqrt{2}\sec\theta = 3\sqrt{2}\cos\theta$

$\cos^2\theta = \frac{2}{3}$

Possible.

~~$8\,\sec$~~

$\frac{y}{2} \geq \sqrt{144}$

$y \geq 24$

$\boxed{y \in [24, \infty)}$

② $y = 18\sec^2\theta + 8\cos^2\theta$

$(3\sqrt{2}\sec\theta)^2 + (2\sqrt{2}\cos\theta)^2$

$3\sqrt{2}\sec\theta = 2\sqrt{2}\cos\theta$

$\cos^2\theta = \frac{3}{2}$ (not valid)

$y = 10\sec^2\theta + 8\sec^2\theta + 8\cos^2\theta$

$y = 10\sec^2\theta + [16, \infty)$

$y = [10, \infty) + [16, \infty)$

min $= 16 + 10$

$\boxed{= 26}$

③ find min value

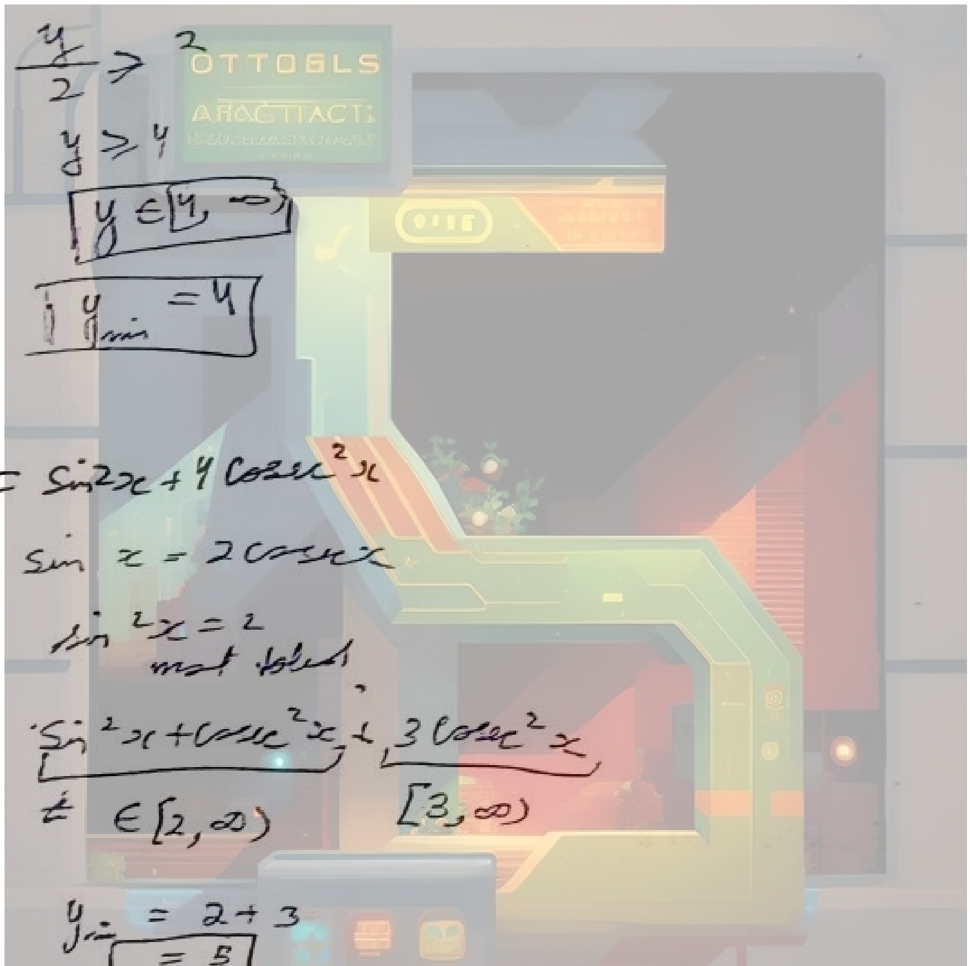
$$y = 4\sin^2 x + \operatorname{cosec}^2 x$$

$$2\sin x = \frac{1}{\sin x}$$

$$\sin^2 x = \frac{1}{2} \qquad \sin^2 x\ [0,1]$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$\sin x \in [-1,1]$$



$$\frac{y}{2} \geq 2$$

$$y \geq 4$$

$$\boxed{y \in [4,\infty)}$$

$$\boxed{y_{min} = 4}$$

④ $y = \sin^2 x + 4\operatorname{cosec}^2 x$

$$\sin x = 2\operatorname{cosec} x$$

$$\sin^2 x = 2$$

not valid

$$\underbrace{\sin^2 x + \operatorname{cosec}^2 x}_{\in [2,\infty)} + \underbrace{3\operatorname{cosec}^2 x}_{[3,\infty)}$$

$$y_{min} = 2+3$$

$$\boxed{= 5}$$

Type-5 ([illegible])

Q ① $x^2+y^2=4$ & $a^2+b^2=8$ find. Min & Max value of $ax+by$

$x^2+y^2=4$ $a^2+b^2=8$

$x = 2\cos\theta$ $a = 2\sqrt{2}\cos\phi$

$y = 2\sin\theta$ $b = 2\sqrt{2}\sin\phi$

$$ax+by = 4\sqrt{2}(\cos\theta\cos\phi + \sin\theta\sin\phi)$$

$$= 4\sqrt{2}\,\cancel{\phi}\cos(\theta-\phi)$$

$$= 4\sqrt{2}\,[-1,1]$$

$$\boxed{= [4\sqrt{2},\ 4\sqrt{2}]}$$

(2) $y = \dfrac{\tan 3x}{\tan x}$

$$y = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x} \times \frac{1}{\tan x}$$

$$y = \cancel{\tan}\ \frac{3-\tan^2 x}{1-3\tan^2 x} \qquad \{x \neq n\pi\}$$

$$y = \frac{3-t^2}{1-3t^2}$$

$$y - 3yt^2 = 3 - t^2$$

$$y - 3 = 3yt^2 - t^2$$

$$\frac{y-3}{3y-1} = t^2$$

$$t^2 \in [0,\infty) \qquad x \neq n\pi$$

$$\frac{y-3}{3y-1} \geq 0$$

$1/3$ $\quad$ 3

$+\quad -\quad +$

~~$(-\infty, 3] \cup [1/3, \infty)$~~

$$\boxed{(-\infty, 1/3) \cup [3, \infty)}$$

H.W. 30 - 7 - 24

PYS - 13 (5, 6, 7, 8, 9, 10)

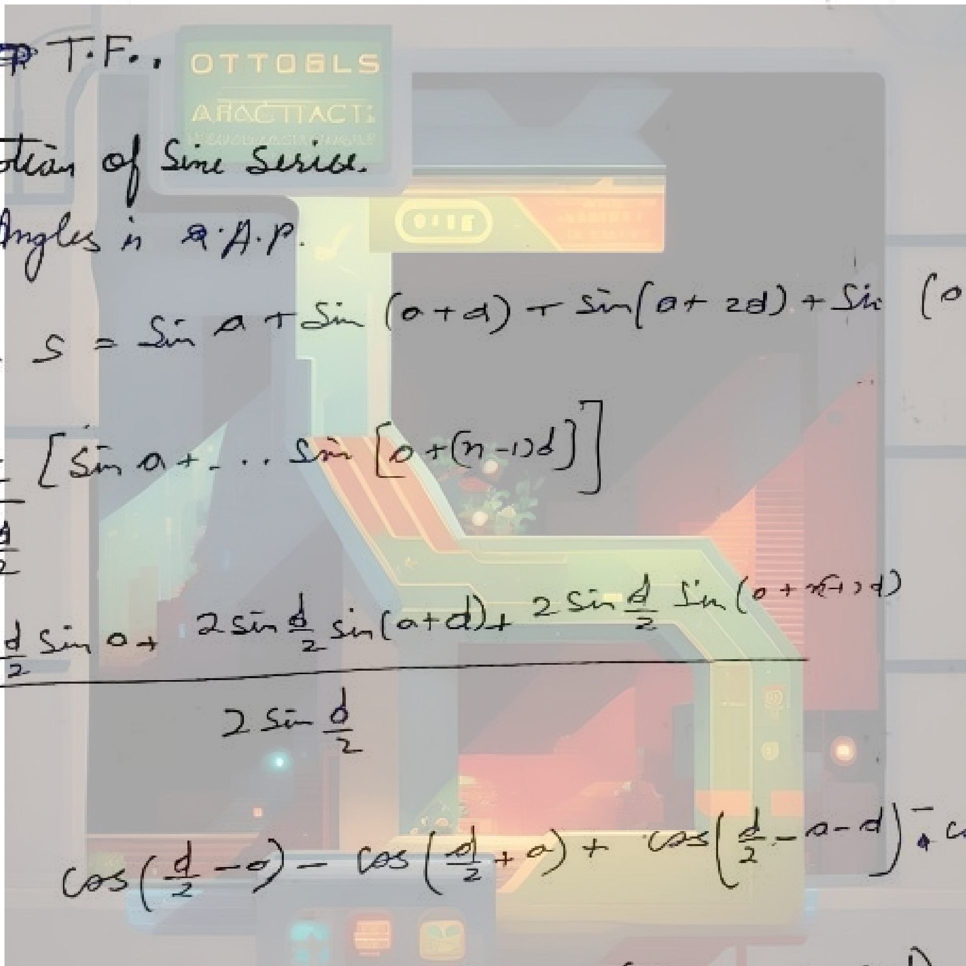
PYS - 14, 15

O - 1 (22 - 29)

J - M (6, 8)

Summation of Trigonometric series :-

→ Try to convert the given series in terms of Difference of 2 ~~TF~~ T.F.,



→ Summation of Sine Series.

↳ Angles in A.P.

$$S = \sin a + \sin(a+d) + \sin(a+2d) + \sin(a+3d) + \ldots \sin(a+(n-1)d)$$

$$S = \frac{2\sin\frac{d}{2}\left[\sin a + \ldots \sin\left[a+(n-1)d\right]\right]}{2\sin\frac{d}{2}}$$

$$= \frac{2\sin\frac{d}{2}\sin a + 2\sin\frac{d}{2}\sin(a+d) + 2\sin\frac{d}{2}\sin(a+(n-1)d)}{2\sin\frac{d}{2}}$$

$$= \cos\left(\frac{d}{2}-a\right) - \cos\left(\frac{d}{2}+a\right) + \cos\left(\frac{d}{2}-a-d\right) - \cos\left(\frac{d}{2}+a+d\right)$$

$$+ \cos\left(\frac{d}{2}-a-(n-1)d\right) - \cos\left(\frac{d}{2}+a+(n-1)d\right)$$

$$= \cos\left(a-\frac{d}{2}\right) - \cancel{\cos\left(a+\frac{d}{2}\right)} + \cancel{\cos\left(a+\frac{d}{2}\right)} - \cos\left(a+\frac{3d}{2}\right) + \cos\left(a+\frac{3d}{2}\right)$$

$$= \cos\left(a-\frac{d}{2}\right) - \cos\left(\frac{d}{2}+a+(n-1)d\right)$$

$$S\,2\sin\frac{d}{2} = 2\sin\left(\frac{a-\frac{d}{2}+\frac{d}{2}+a+(n-1)d}{2}\right)\sin\left(\frac{\frac{d}{2}+a+(n-1)d-a+\frac{d}{2}}{2}\right)$$

$$S\times 2\sin\frac{d}{2} = 2\sin\left(a+\frac{(n-1)d}{2}\right)\sin\left(\frac{nd}{2}\right)$$

$$S = \frac{\cancel{2}\sin\left(a+(n-1)\frac{d}{2}\right)\sin\left(\frac{nd}{2}\right)}{\cancel{2}\sin\frac{d}{2}}$$

★★

$$\boxed{S = \frac{\sin\left(a+(n-1)\frac{d}{2}\right)\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

→ Summation Of Cosine Series:-

$$S = \cos a + \cos(a+d) \dots \cos[a+(n-1)d]$$

$$\cancel{S = \frac{\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

★★★

$$\boxed{S = \frac{\cos\left(a+(n-1)\frac{d}{2}\right)\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

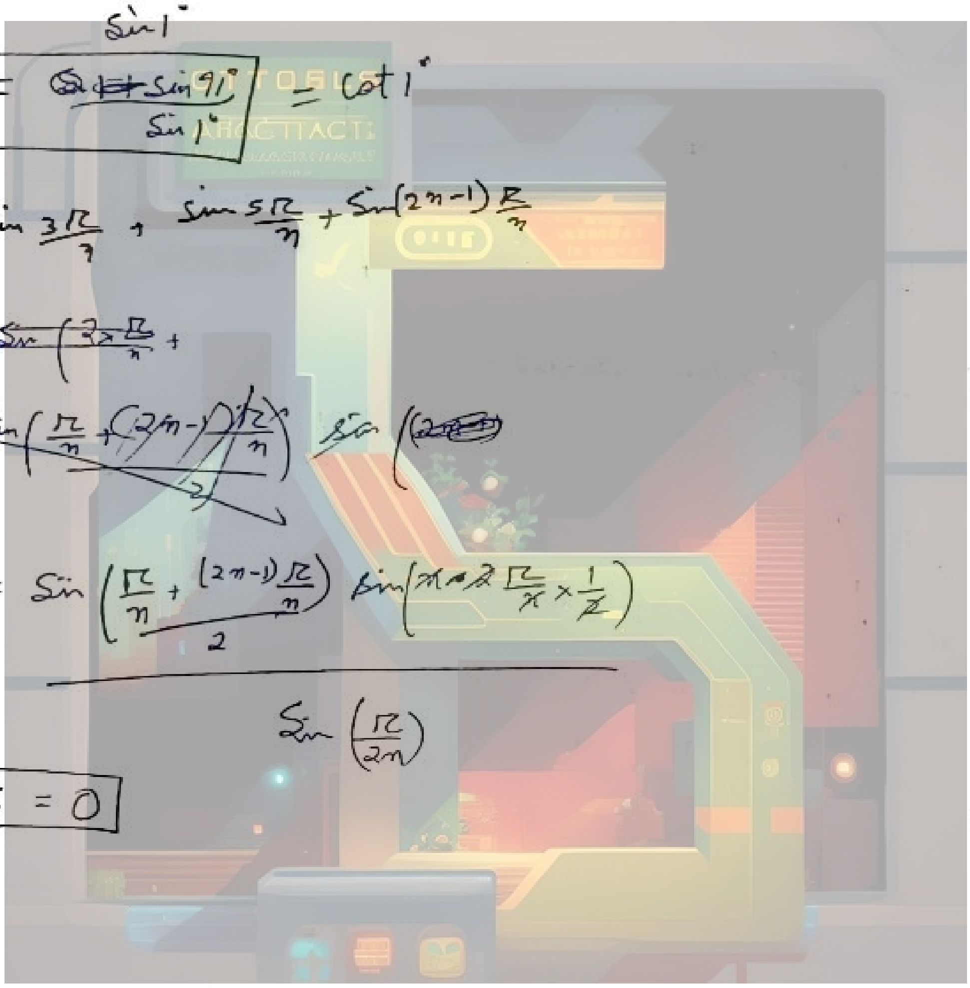
Q find sum of following series.

(1) $\sin 2^\circ + \sin 4^\circ + \sin 6^\circ + 8 \ldots \sin 180^\circ$

$$S = \frac{\sin\left(\frac{2+180}{2}\right) * \sin\left(90 \times \frac{2}{2}\right)}{\sin\left(\frac{2}{2}\right)}$$

$$= \frac{\sin 91^\circ * \sin 90^\circ}{\sin 1^\circ}$$

$$\boxed{= \frac{\cancel{\cos}\ \sin 91^\circ}{\sin 1^\circ}} = \cot 1^\circ$$



(2) $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \sin(2n-1)\frac{\pi}{n}$

$$\cancel{S = \sin\left(2 \times \frac{\pi}{n} + \right.}$$

$$\cancel{S = \sin\left(\frac{\frac{\pi}{n} + (2n-1)\frac{\pi}{n}}{2}\right) \sin\left(2n\right.}$$

$$S = \frac{\sin\left(\frac{\frac{\pi}{n} + (2n-1)\frac{\pi}{n}}{2}\right) \sin\left(\cancel{n} \times 2\frac{\pi}{\cancel{n}} \times \frac{1}{2}\right)}{\sin\left(\frac{\pi}{2n}\right)}$$

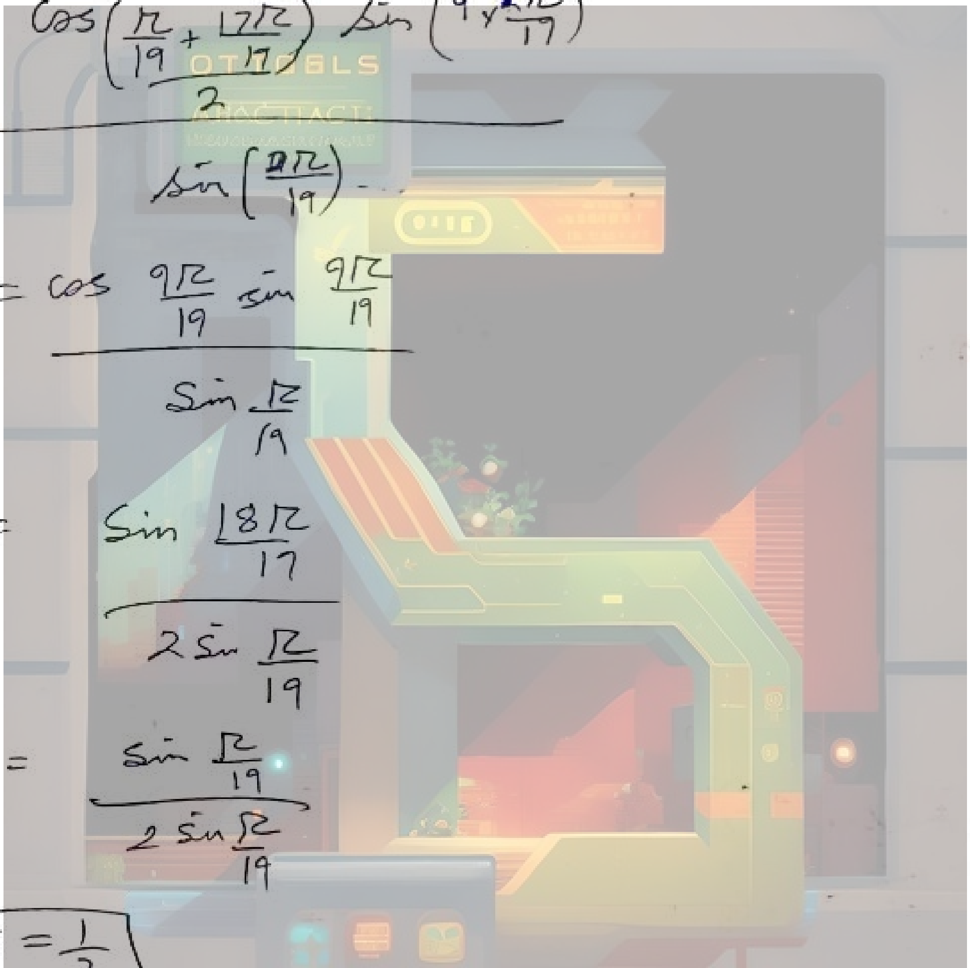
$$\boxed{S = 0}$$

③ $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots \cos\frac{17\pi}{19}$

④ $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

⑤ ** $\cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \cos^2\frac{3\pi}{n} + \ldots\ldots \cos^2\frac{(n-1)\pi}{n}$

⑥ $y = \dfrac{\cos x + \cos 2x + \cos 3x + \ldots\ldots \cos 7x}{\cancel{\cos}\ \sin x + \sin 2x + \sin 3x + \ldots \sin 7x}$



③ $S = \dfrac{\cos\left(\dfrac{\frac{\pi}{19} + \frac{17\pi}{19}}{2}\right) \sin\left(9 \times \frac{2\pi}{19}\right)}{\sin\left(\frac{2\pi}{19}\right)}$

$S = \dfrac{\cos\frac{9\pi}{19}\ \sin\frac{9\pi}{19}}{\sin\frac{\pi}{19}}$

$S = \dfrac{\sin\frac{18\pi}{17}}{2\sin\frac{\pi}{19}}$

$S = \dfrac{\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}}$

$\boxed{S = \frac{1}{2}}$

④ $S = \dfrac{\cos\left(\frac{\overset{5}{\cancel{20}}\pi}{11}\right) \sin\left(5 \times \frac{\pi}{11}\right)}{\sin\frac{\pi}{11}}$

$S = \dfrac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$ $\quad \boxed{S = \frac{1}{2}}$

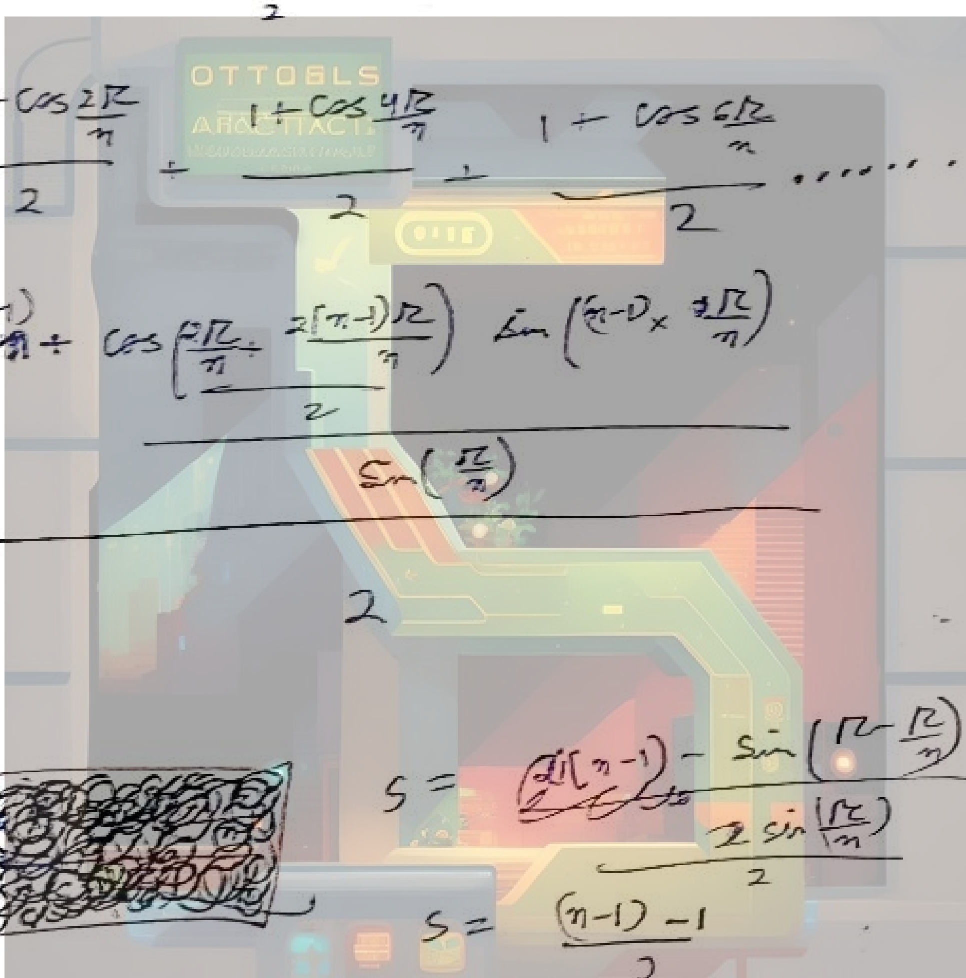
(6b)

$$y = \frac{\dfrac{\cos(4x)\sin\left(7\times\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}}{\dfrac{\sin(4x)\sin\left(7\times\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}}$$

$$y = \cot 4x$$

(5)

$$\cos^2\frac{\pi}{n} = \frac{1+\cos\frac{2\pi}{n}}{2}$$



$$\frac{1+\cos\frac{2\pi}{n}}{2} + \frac{1+\cos\frac{4\pi}{n}}{2} + \frac{1+\cos\frac{6\pi}{n}}{2} \cdots\cdots$$

$$S = \frac{(n-1) + \dfrac{\cos\left(\dfrac{\frac{2\pi}{n} + \frac{2(n-1)\pi}{n}}{2}\right)\sin\left((n-1)\times\frac{2\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}}{2}$$

$$S = \frac{(n-1) - \dfrac{\sin\left(\pi - \frac{\pi}{n}\right)}{2\sin\left(\frac{\pi}{n}\right)}}{2}$$

$$S = \frac{(n-1) - 1}{2}$$

$$\boxed{S = \frac{n-2}{2}}$$

→ Telescopic Series:-

Q $\operatorname{cosec} x + \operatorname{cosec} 2x + \operatorname{cosec} 4x + \ldots\ldots + \operatorname{cosec} 2^n x$

$$S = \frac{1}{\sin x} + \frac{1}{\sin 2x} + \frac{1}{\sin 4x} \ldots\ldots \frac{1}{\sin 2^n x}$$

$$S = \frac{\sin \frac{x}{2}}{\sin x \sin \frac{x}{2}} + \frac{\sin x}{\sin 2x \sin x} + \frac{\sin 2x}{\sin 4x \sin 2x} \ldots\ldots$$

$$S = \frac{\sin\left(x - \frac{x}{2}\right)}{\sin x \sin \frac{x}{2}} + \frac{\sin(2x - x)}{\sin 2x \sin x} + \frac{\sin(4x - 2x)}{\sin 4x \sin 2x} \ldots\ldots$$

$$S = \frac{\sin x \cos \frac{x}{2} - \cos x \sin \frac{x}{2}}{\sin x \sin \frac{x}{2}} + \frac{\sin 2x \cos x - \sin x \cos 2x}{\sin 2x \sin x} \ldots\ldots$$

$$S = \cot \frac{x}{2} - \cancel{\cot x} + \cancel{\cot x} - \cancel{\cot 2x} + \cancel{\cot 2x} - \cot 4x \ldots\ldots + \cot(2^{n-1}x) - \cot(2^n x)$$

$$\boxed{S = \cot \frac{x}{2} - \cot(2^n x)}$$

Q $S = \tan \frac{x}{2} \sec x + \tan \frac{x}{2^2} \cdot \sec \frac{x}{2} + \tan \frac{x}{2^3} \cdot \sec \frac{x}{2^2} \ldots$ n terms

$$= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \frac{1}{\cos x} + \frac{\sin \frac{x}{2^2}}{\cos \frac{x}{2^2}} \frac{1}{\cos \frac{x}{2}} \ldots\ldots$$

$$= \frac{\sin\left(x - \frac{x}{2}\right)}{\cos \frac{x}{2} \cos x} + \frac{\sin\left(\frac{x}{2} - \frac{x}{2^2}\right)}{\cos \frac{x}{2} \cos x} \ldots\ldots$$

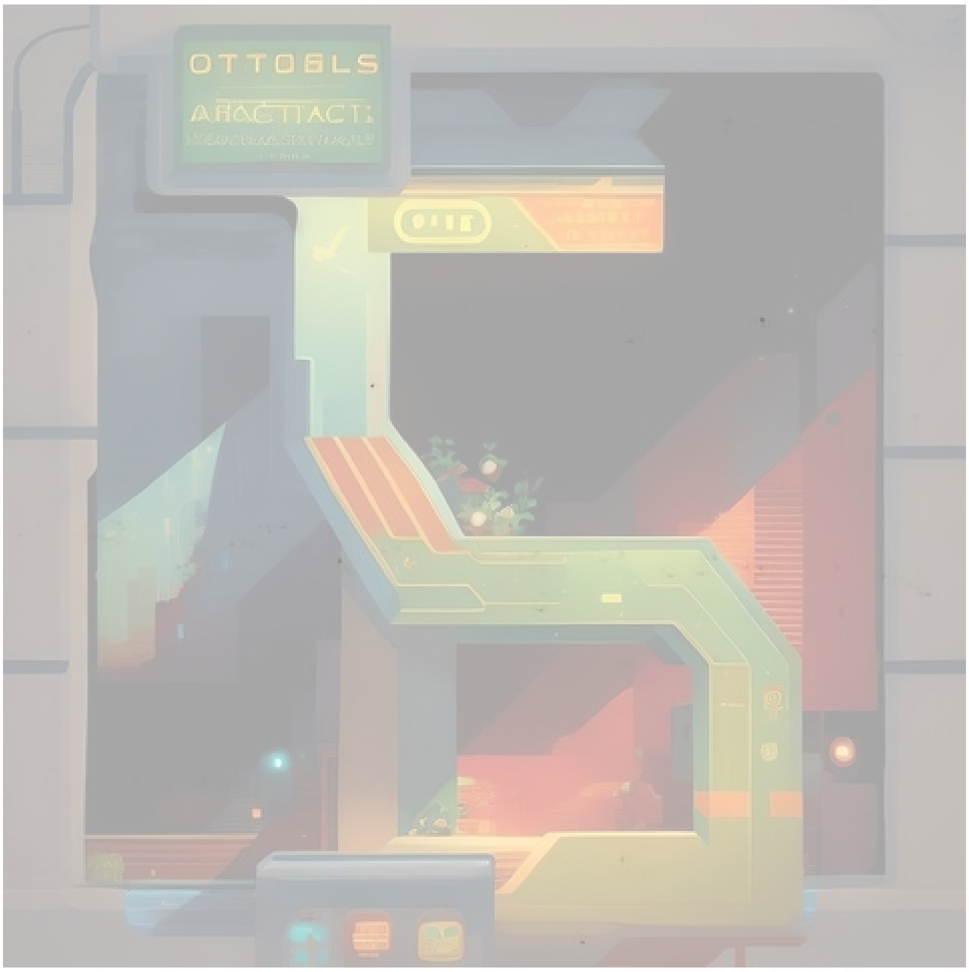
$$= \frac{\sin x \cos \frac{x}{2} - \cos x \sin \frac{x}{2}}{\cos \frac{x}{2} \cos x} + \ldots\ldots$$

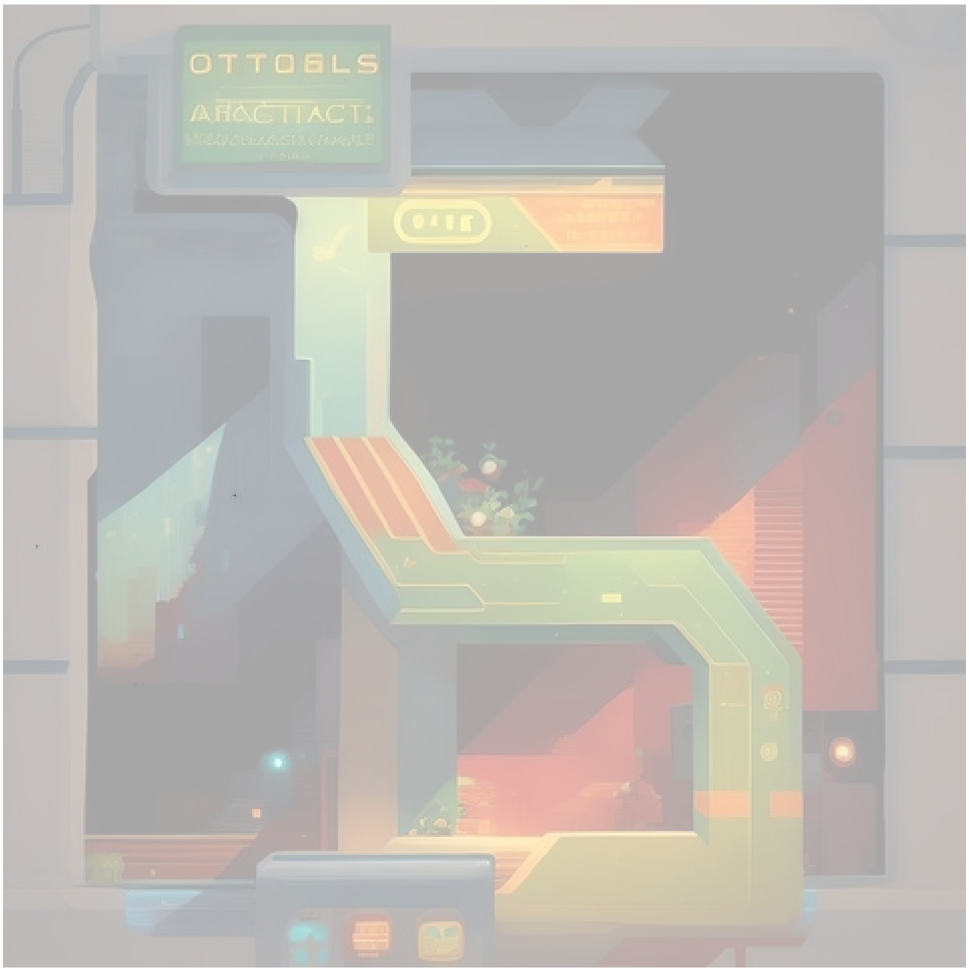
$$= \tan x - \tan \frac{x}{2} + \tan \frac{x}{2} - \tan \frac{x}{2} \ldots\ldots$$

$$= \tan x - \tan\left(\frac{x}{2^n}\right)$$

H.W. 2-8-24

DYS-16[1,6]

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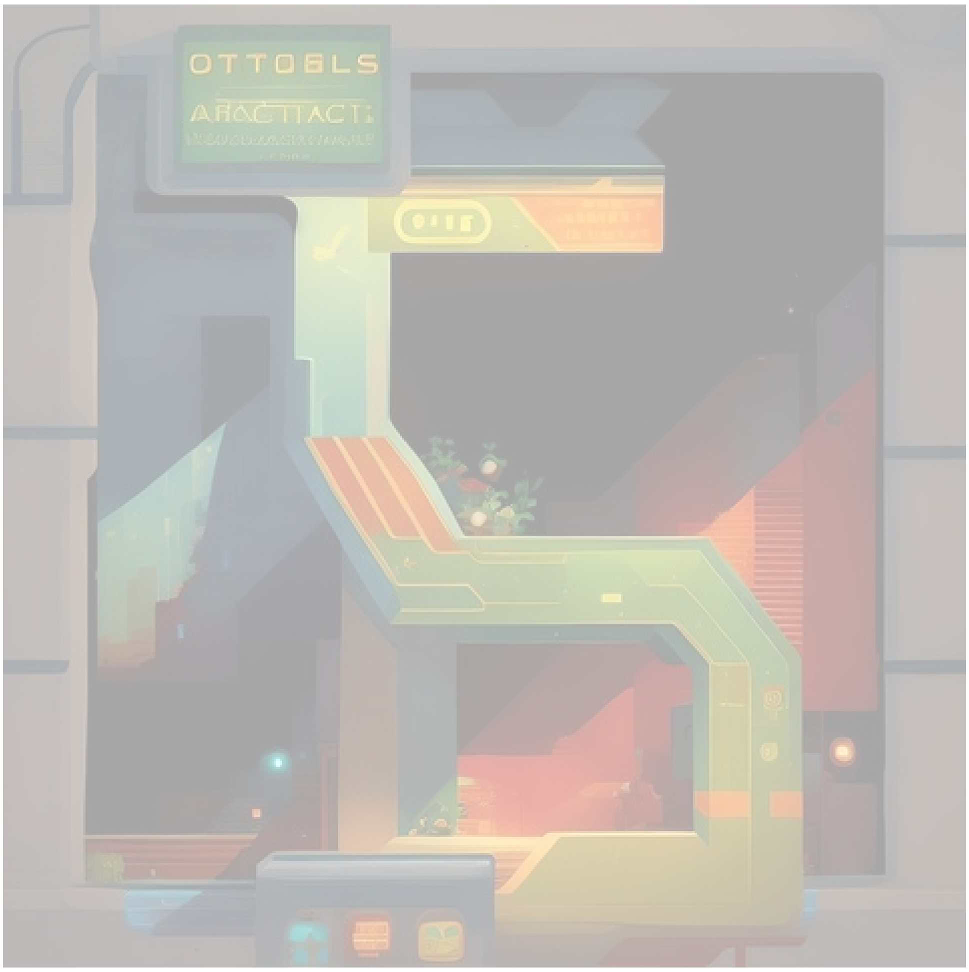

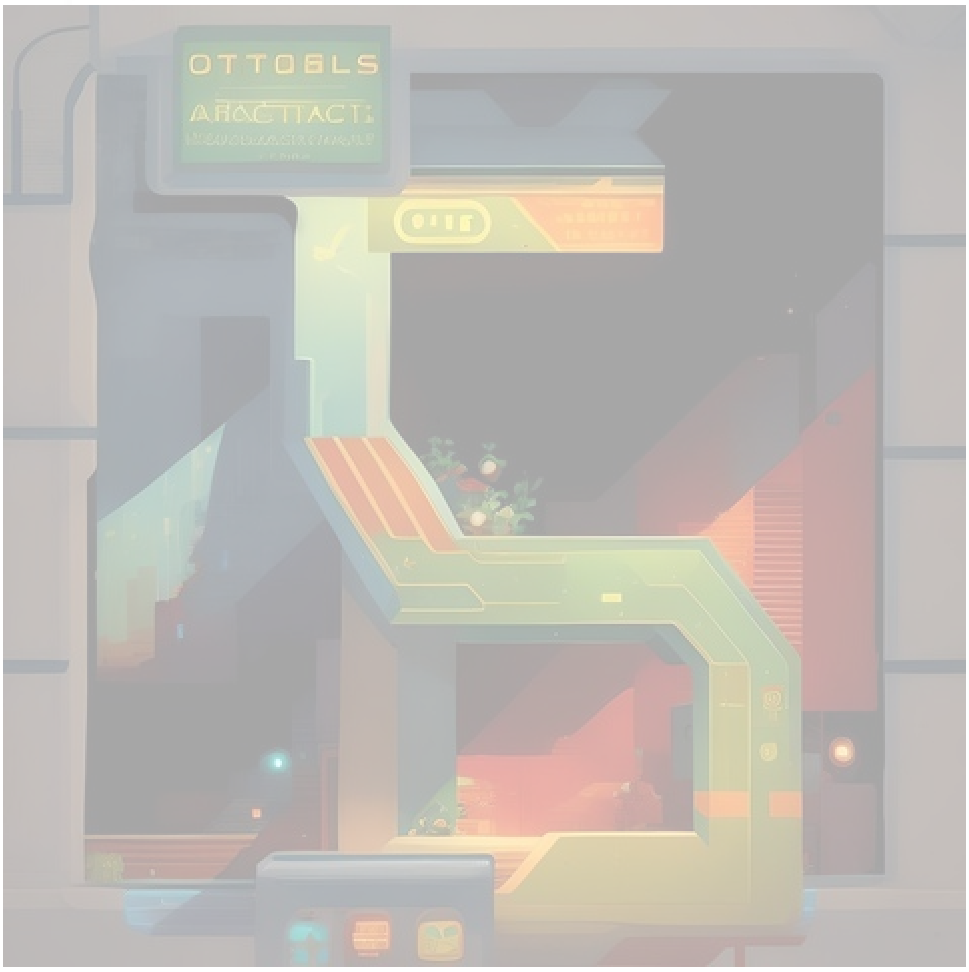

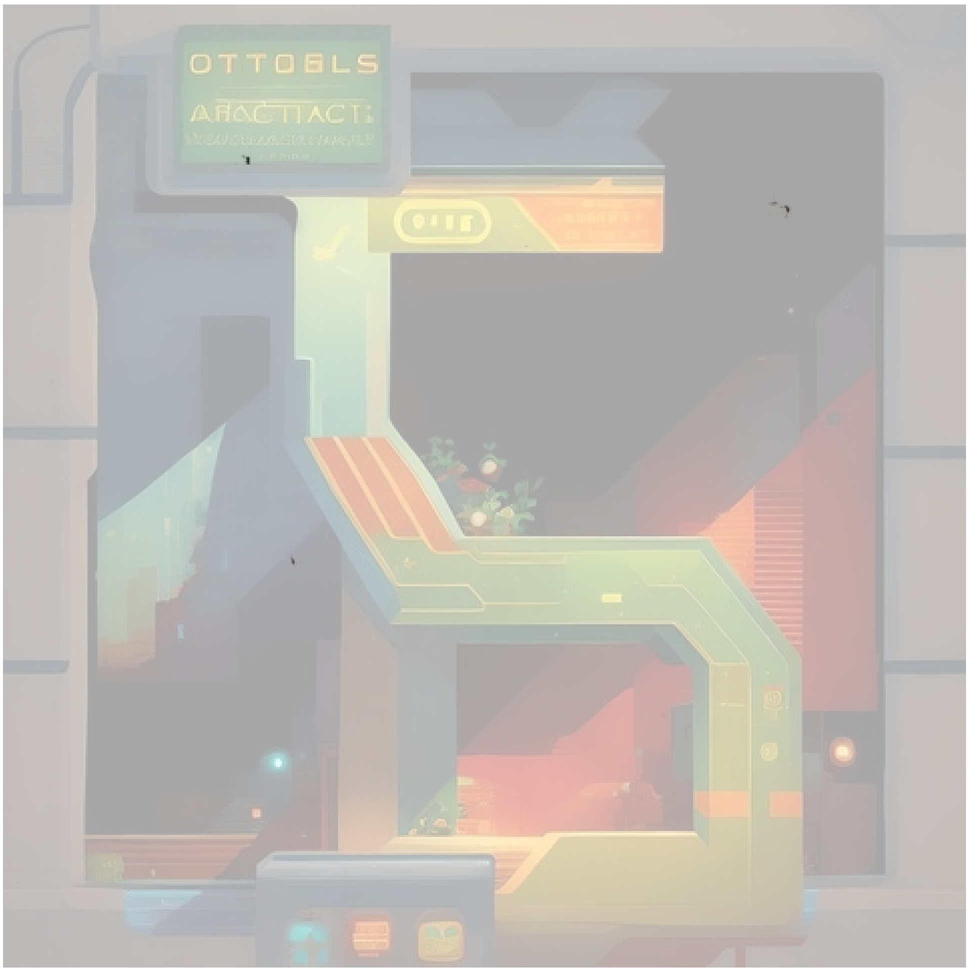
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