Circles

Equation of Circle

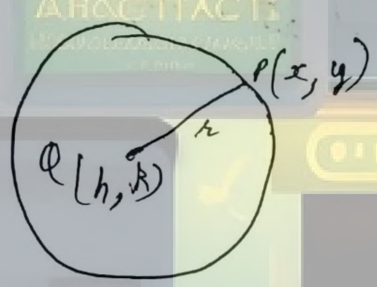
① Center - radius form (Central form) :-

$$\boxed{(x-h)^2+(y-k)^2=r^2} \quad \text{.①}$$

$(h, k) \Rightarrow$ Center

$r \Rightarrow$ radius

Proof :-



$PQ = r$

$$\sqrt{(x-h)^2+(y-k)^2} = r$$

$$(x-h)^2+(y-k)^2 = r^2$$

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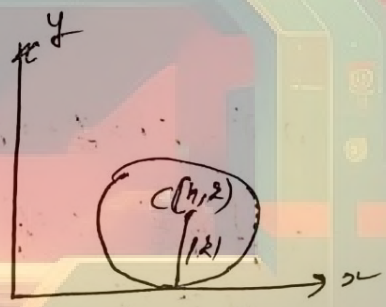
i) Circle touch x - axis

C_y = radius



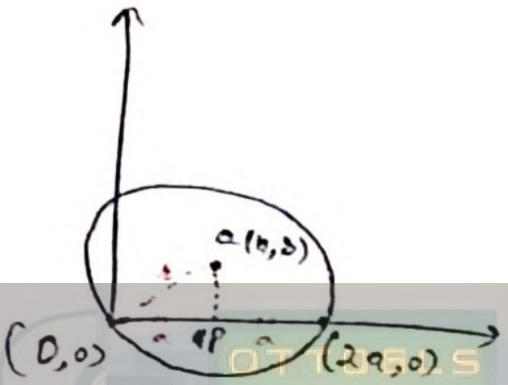
ii) Circle touch y - axis

C_x = radius

iii) Circle touch x & y axes both

$C_x = C_y$ = radius

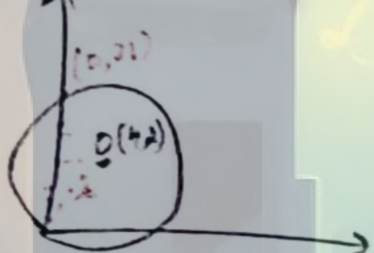
iv) when circle passes through origin & cut intercept '2a' on x axis.



$$\boxed{a^2 + k^2 = r^2}$$

$$\boxed{a = h}$$

v) when circle passes through origin & cut intercept '2b' on y axis



$$\boxed{h^2 + b^2 = r^2} \qquad \boxed{k = b}$$

vi) If radius of the circle is 0 then, circle is point circle.

(2) General form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$\left.\begin{array}{l}\text{coeff of } x^2 = \text{coeff of } y^2 \\ \& \\ \text{coeff of } xy = 0\end{array}\right\} \quad \begin{array}{l} a = b \\ h = 0\end{array}$$

RB

$$\boxed{x^2 + y^2 + 2gx + 2fy + c = 0} \quad (2)$$

Note:-

$$x^2+2gx+g^2+y^2+2fy+f^2+c-g^2-f^2=0$$

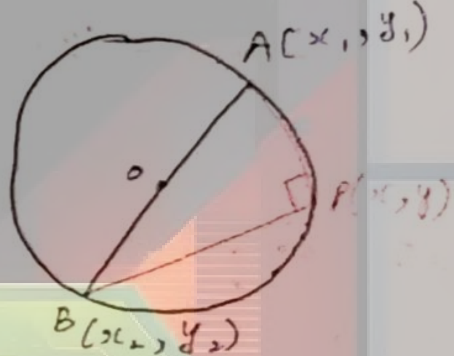
$$(x+g)^2+(y+f)^2=\left(\sqrt{g^2+f^2-c^2}\right)^2$$

$$\therefore (x-h)^2+(y-k)^2=r^2$$

$\boxed{\text{Center } (-g,-f)}$ $\boxed{r=\sqrt{g^2+f^2-c}}$

- >0 real circle
- <0 imaginary circle
- $=0$ point circle
- c is -ve real circle

③ **Diametric Form**



$$m_{AP}\cdot m_{BP}=-1$$

$$\frac{y-y_1}{x-x_1}\cdot\frac{y-y_2}{x-x_2}=-1$$

$$\boxed{(y-y_1)(y-y_2)+(x-x_1)(x-x_2)=0}$$

$A(x_1,y_1)$ & $B(x_2,y_2)$ are end points of diameter.

④ **Parametric form**

→ Assuming (x,y) in terms of $\sin\theta$ and $\cos\theta$ which satisfy the circle equation. $(\theta\in R)$

Eg. ① $x^2+y^2=4$

~~x = 2si~~

$x=2\cos\theta$ $y=2\sin\theta$

② $x^2+y^2=10$

$x=\sqrt{10}\cos\theta$ $y=\sqrt{10}\sin\theta$

③ $(x-3)^2 + (y-2)^2 = 1$

$x-3 = \cos\theta \qquad y-2 = \sin\theta$

$\boxed{x = 3+\cos\theta} \qquad \boxed{y = 2+\sin\theta}$

④ $(x-2)^2 + (y+4)^2 = 8$

~~$x-2 = \cos\theta$~~

$\dfrac{x-2}{2\sqrt{2}} = \cos\theta \qquad \dfrac{y+4}{2\sqrt{2}} = \sin\theta$

$\boxed{x = 2\sqrt{2}\cos\theta + 2} \qquad \boxed{y = 2\sqrt{2}\sin\theta - 4}$

General form -

$$\boxed{(x-h)^2 + (y-k)^2 = r^2 \to \begin{cases} x = h + r\cos\theta \\ y = k + r\sin\theta \end{cases}}$$

Q find the eqn of circle

① $r = 10$
Center $(-5,-6)$

$\boxed{(x+5)^2 + (y+6)^2 = 100}$

② Having center is POI of $2x-3y+4=0$ & $3x+4y-5$ & passes through $(0,0)$

Center:-

$8x - 12y + 16 = 0$
$9x + 12y - 15 = 0$

$17x = -1$

$x = \dfrac{-1}{17}$

$y = \dfrac{2x+4}{3}$

$y = 22/17$

center $\left(\dfrac{-1}{17}, \dfrac{22}{17}\right)$

$r = \sqrt{\dfrac{1+484}{17}}$

$r = \sqrt{\dfrac{485}{17\times17}}$

$r = \dfrac{1}{17}\sqrt{485}$

$$\boxed{\left(x+\frac{1}{17}\right)^2+\left(y-\frac{22}{17}\right)=\frac{485}{17^2}}$$

(3) which touches the x & y axes at distance 3 units from (0,0)

$r^2 = 9$

Centre (3,3) (3,−3) (−3,3) (−3,−3)

$$\boxed{(x\pm3)^2+(y\pm3)^2=9}$$

$(x+3)^2+(y+3)^2=9$

$(x-3)^2+(y+3)^2=9$

$(x+3)^2+(y-3)^2=9$

(4.) One end of diameter (6, −3) & centre (−3, −5)

$$\frac{x+6}{2}=-3 \qquad \frac{y-3}{2}=-5$$

$$x+6=-6 \qquad y-3=-10$$

$$x=-12 \qquad y=-7$$

other end (−7, −12)

$$\boxed{(x-6)(x+7)+(y-12)(y+3)=0}$$

H.W. 30-9-24

Q-1 [22, 23, 24, 25, 22] all left

Q-2 full

H.W. 31-10-24

JM full

Q find the centre and radius of the circle whose eqn is.

(1) $(x+3)^2+(y-4)^2=5$

(2) $x^2+y^2-6x+8y+14=0$

(3) $3x^2+3y^2-12x+6y+11=0$

(2) $\boxed{\text{Center} = (-6, -8)}$

$\text{radius} = \sqrt{36 + 64 - 14}$

$\boxed{\text{radius} = \sqrt{86}}$

(2) $\text{center } (3, -4)$

$r = \sqrt{9 + 16 - 14}$

$r = \sqrt{11}$

(3) $3x^2 + 3y^2 - 12x + 6y + 11 = 0$

$x^2 + y^2 - 4x + 2y + \frac{11}{3} = 0$

$\boxed{\text{Center} \equiv (4, -2)}$

$\text{radius}(r) = \sqrt{16 + 4 - \frac{11}{3}}$

$= \sqrt{\frac{49}{3}}$

$\boxed{r = \sqrt{\frac{49}{3}} = \frac{7}{\sqrt{3}}}$

(3) $x^2 + y^2 - 4x + 2y + \frac{11}{3} = 0$

$\text{Center} \equiv (2, -1)$

$r = \sqrt{4 + 1 - \frac{11}{3}}$

$r = \frac{2}{\sqrt{3}}$

(1) $(x+3)^2 + (y-4)^2 = 5$

$(x-h)^2 + (y-k)^2 = r^2$

$\boxed{r = \sqrt{5}}$

$\text{Center} = (h, k) = \boxed{(-3, 4)}$

Q for what values of K for the eqn $Kx^2 + Ky^2 - x - y + K = 0$ will be a real circle.

$g^2 + f^2 - c \geq 0$

$1 + 1 - K \geq 0$

$2 \geq K$

$K \leq 2$

$K \in (-\infty,$

$x^2 + y^2 - \frac{1}{K}x - \frac{1}{K}y + \frac{K}{K} = 0$

$\frac{1}{4K^2} + \frac{1}{4K^2} - \frac{K}{K} \geq 0$

$\frac{2}{K^2} \geq K$

$K^3 \leq 2$

$\boxed{K \leq \sqrt[3]{2}}$

$\frac{1}{2K^2} \geq 1$

$K^2 \leq \frac{1}{2}$

$K \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

(6)

Q find the area of Equilateral Δ inscribed in $x^2+y^2-2x=0$

$\cancel{r = \sqrt{g^2+f^2-c}}$

$\cancel{r = \sqrt{4}}$

$\cancel{r = 2}$

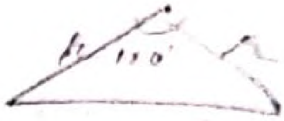
$\cancel{\Delta = \frac{1}{2} \times 2 \times 2 \times \frac{\sqrt{3}}{2}}$

$\cancel{\Delta = \sqrt{3}}$

$\cancel{\boxed{\Delta_{Total} = 3\sqrt{3}}}$

$r = \sqrt{1^2+0-0}$

$r = 1$

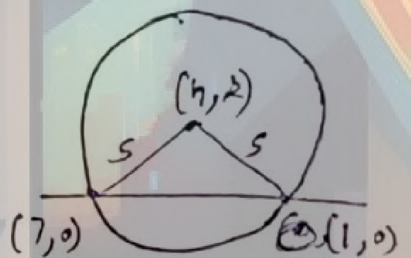


$\Delta = \frac{1}{2} \times 1 \times 1 \times \frac{\sqrt{3}}{2}$

$\Delta = \frac{\sqrt{3}}{4}$

$$\boxed{\Delta_{Total} = \frac{3\sqrt{3}}{4}}$$

Q find the eqn of the circle of radius 5 & cut x axis at A(1,0) & B(7,0)



$(h-1)^2 + k^2 = 25$ | $(h-7)^2 + k^2 = 25$

$h^2 + k^2 - 2h = 24$ | $h^2 + k^2 + 49 - 14h = 25$

$h^2 + k^2 - 14h = -24$

$12h = 48$

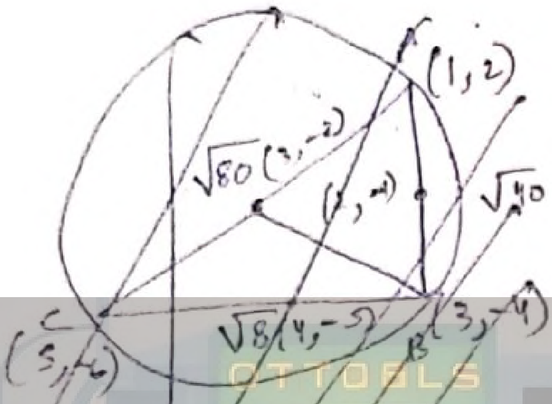
$h = 4$

$k = \pm 4$

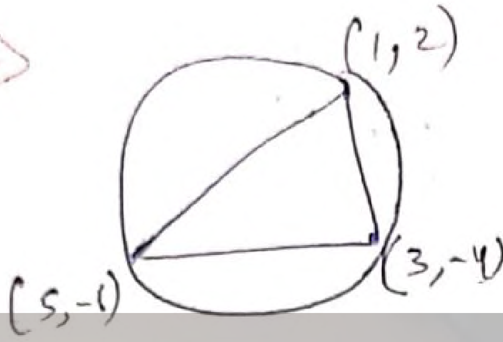
~~Center (4, 4)~~ Center $(\pm 4, \pm 4)$

$$\boxed{(x \pm 4)^2 + (y \pm 4)^2 = 25}$$

Q find the Eqⁿ of the circle passing through 3 points (1, 2) (3, -4) (5, -1)



M I



Sol ⊥ bisector eqⁿ for centre.

$m_{AB} = -3$

$m_{BC} = 1$

$m_{AC} = -2$

⊥ bisector of BC =

M II

$x^2 + y^2 + 2gx + 2fy + c = 0$

(1, 2) (3, -4) (5, -6)

$5 + 2g + 8f + c = 0$ | $6g - 8f + c = -25$

$2g + 8f + c = -5$ | $8g + 2c = -30$

$4g + c = -15$

$25 + 36 + 10g - 12f + c = 0$

$10g - 12f + c = -61$

Solve 3 eqⁿ

Power & Position of a point wrt circle

$S = 0$ or $x^2 + y^2 + 2gx + 2fy + c = 0$

S_1 or $h^2 + k^2 + 2gh + 2fk + c$

Power of circle.

•P(h,k)

$OP = \sqrt{(h+g)^2 + (k+f)^2}$

$OP^2 = h^2 + g^2 + 2hg + k^2 + f^2 + 2kf + c - c$

$OP^2 = r^2 + h^2 + k^2 + 2gh + 2fk + c$

$\boxed{OP^2 - r^2 = S_1}$

→ $S_1 > 0$ · $OP > r$
$S_1 > 0$ Point outside the circle.

→ $S_1 = 0$ $OP = r$
Point on the circumference

→ $S_1 < 0$ $OP < r$
Point inside the circle

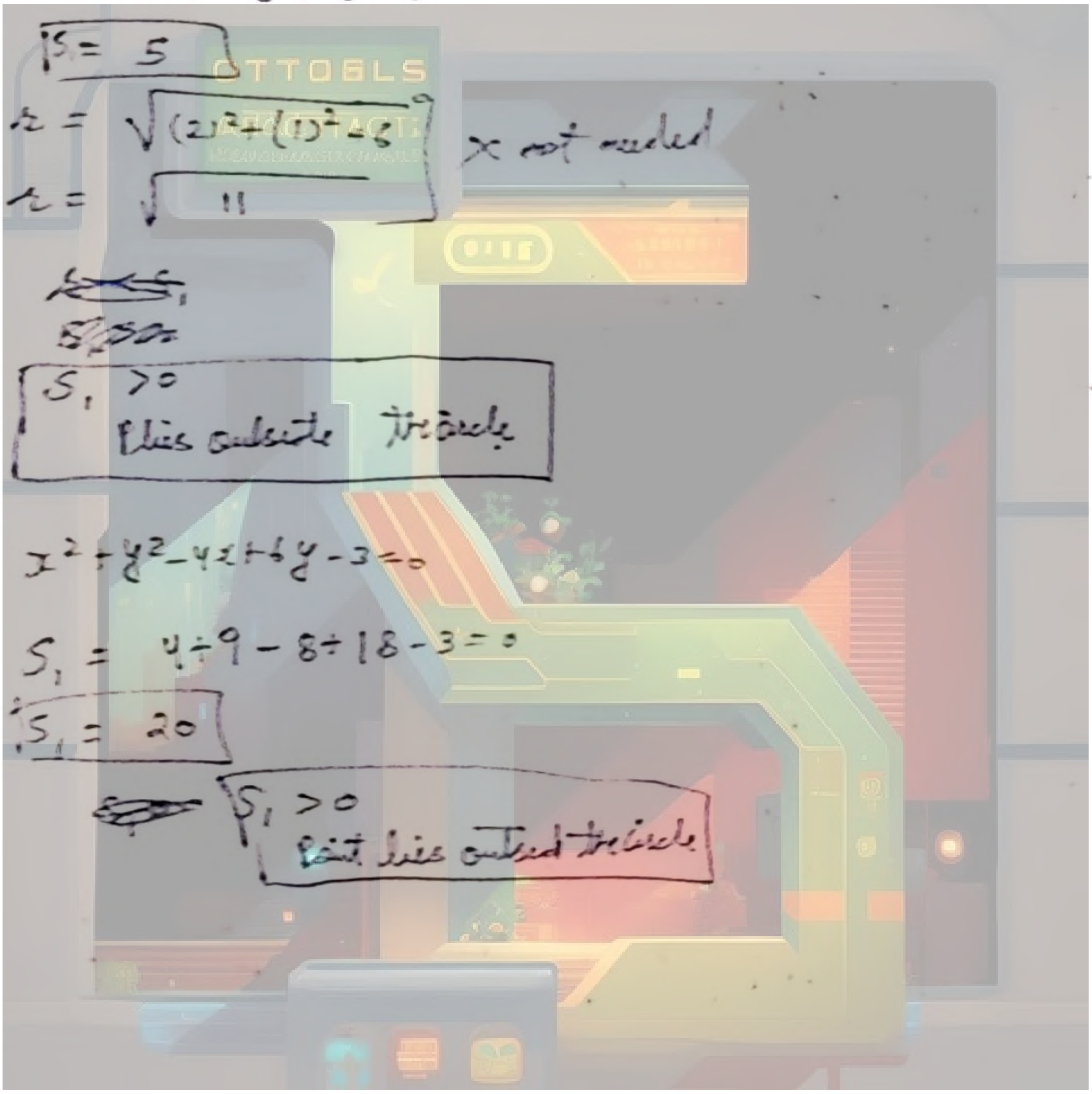
Note:- before applying the formula, make coefficient of x^2 & y^2 1.

Q. find the power & position of $P(2,3)$ wrt given circles.

(1) $x^2+y^2-4x+2y-6=0$

$S_1 = (2)^2+(3)^2-4(2)+2(3)-6$

$= 4+9-8+6-6$



$\boxed{S_1 = 5}$

$r = \sqrt{(2)^2+(1)^2+6}$ x not needed

$r = \sqrt{11}$

$S_1 > 0$

P lies outside the circle

(2) $x^2+y^2-4x+6y-3=0$

$S_1 = 4+9-8+18-3=0$

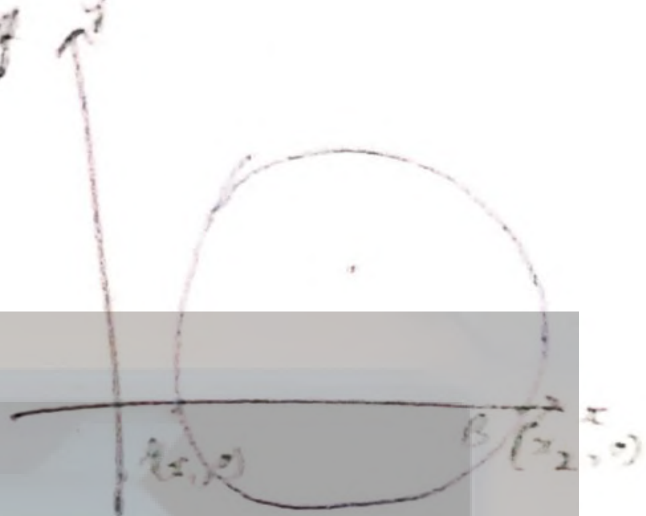
$\boxed{S_1 = 20}$

$S_1 > 0$

Point lies outside the circle

* Intercepts made by circle

case 1 – when circle make intercept of $2p$ on x-axis.



$AB = 2p$

$AB = |x_1 - x_2|$

$y = 0$

$x^2 + 2gx + c = 0$

$x_1 \qquad x_2$

$|x_1 - x_2| = \frac{\sqrt{D}}{|a|} = \frac{\sqrt{4g^2 - 4c}}{1}$

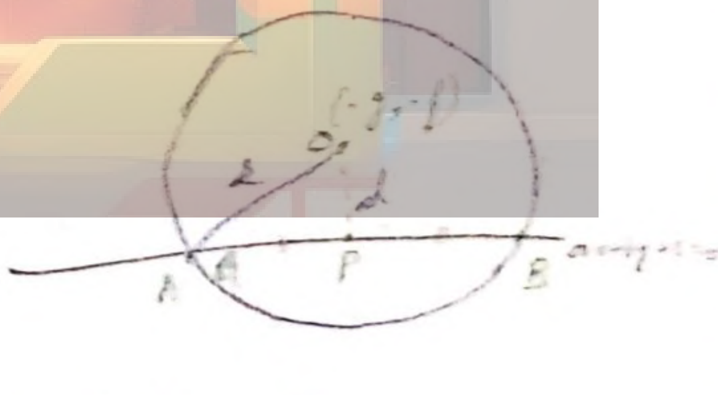
$\boxed{2p = 2\sqrt{g^2 - c}}$

case – when circle makes intercept $2p$ on y-axis

$\boxed{2p = 2\sqrt{f^2 - c}}$

Case 2 – On general line $ax + by + c = 0$

In $\triangle OAP$



$AO^2 = AP^2 + OP^2$

$AP = \sqrt{r^2 - d^2}$

$\boxed{AB = 2\sqrt{r^2 - d^2}}$

$d \rightarrow$ ⊥ distance of $(-g, -f)$ from AB.

Note when circle touch x-axis $\Rightarrow g^2 = c$
when circle touch y-axis $\Rightarrow f^2 = c$

Q find x intercept, y intercept ~~on~~ by circle.

$x^2+y^2-2x-4y-4=0$

$x = 2\sqrt{1+4}$ | $y = 2\sqrt{4+4}$

$\boxed{x = 2\sqrt{5}}$ | $\boxed{y = 4\sqrt{2}}$

Q find intercept made by $x-y=0$ on $x^2+y^2-9=0$

$r=3$

$d=0$

intercept $= 2\sqrt{9}$

$\boxed{= 6}$

Q $S_1 = x^2+y^2-4x+6y-3=0$

$S_2 = x^2+y^2+4x-6y-3=0$

$P(1,2)$

Point P lies

(1) inside S_1 & S_2

(2) Inside S_1 & outside S_2

(3) Outside S_1 & S_2

✓(4) Outside S_1 & Inside S_2

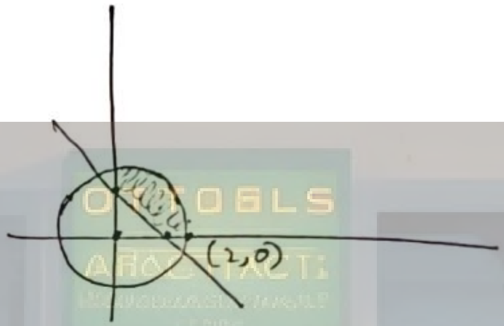
$S_1 = 1+4-4+12-3 = 10$

$S_2 = 1+4+4-12-3 = -6$

. Outside S_1
Inside S_2

$\boxed{4}$

Q for what values of the point $(a, a+1)$ lies inside region bounded by circle $x^2 + y^2 = 4$ & the line $x + y = 2$ in first Quadrant.



$S_1 = a^2 + a^2 + 1 + 2a - 4 = 0$

$S_1 = 2a^2 + 2a - 3 = 0$

$a = \frac{-2 \pm \sqrt{4 + 24}}{4}$

$a = \frac{-2 \pm \sqrt{28}}{4}$

$a = \frac{-2 \pm 2\sqrt{7}}{4}$

$a = \frac{-1 \pm \sqrt{7}}{2}$

$a = \frac{\sqrt{7} - 1}{2}$

$S_1 = \boxed{2a^2 + 2a - 3 > 0}$

$a + a + 1 - 2 \geq 0$ ($<$)

$2A \geq 1$ ($<$)

$a \geq \frac{1}{2}$ ($<$)

$r = \sqrt{4}$

$r = 2$

$S_1 = 2a^2 + 2a - 3 > 0$

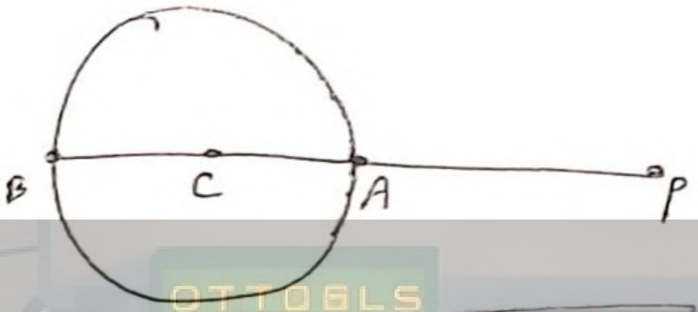
$\boxed{\cancel{a < \frac{1}{2}}}$

$2a + 1 - 2 \geq 0$

$\boxed{a > \frac{1}{2}}$

$\boxed{a \in \left(\frac{1}{2}, \frac{\sqrt{7} - 1}{2}\right)}$

* Greatest & least distance of a point from a circle.



$$\boxed{\begin{aligned} &\text{max distance} \Rightarrow PB = PC + r \\ &\text{min distance} \Rightarrow PA = |PC - r| \end{aligned}}$$

DYS-2

Q2. $P(7,3)$ $\quad x^2+y^2-8x-6y+16=0$

~~Power of P wrt circle, also~~

Center $C \equiv (-g, -f)$

$C(4,3)$

$\cancel{r = \sqrt{64+36-16} = \sqrt{84} = 2\sqrt{21}}$

$r = \sqrt{16+9-16} = 3$

$\cancel{max = PC + r =}$

$PC = \sqrt{9+0} = 3$

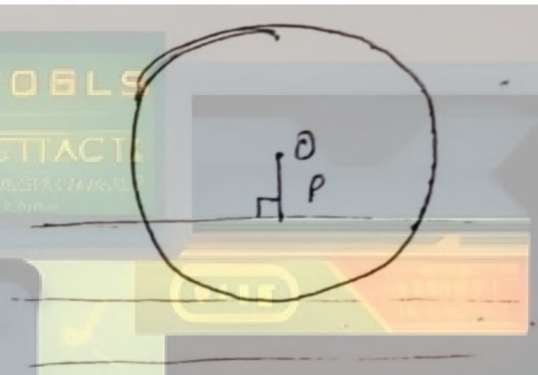
$$\boxed{\begin{aligned} max &= 6 \\ min &= 0 \end{aligned}}$$

Tangent line of circle (Imp)

Cond^n of tangency — Eq^n of tangent

Condition Of Tangency -

(1) TM I



- $OP > r$ — line outside circle
- $OP = r$ — tangent
- $OP < r$ — Secant
- $OP = 0$ — diameter

$OP \Rightarrow \perp$ length from center

(2) TM II

Line Eq^n $\Rightarrow$ $ax + by + c = 0$

Circle Eq^n $\Rightarrow$ $x^2 + y^2 = r^2$

$$x^2 + \left(\frac{ax + c}{b}\right)^2 = r^2$$

Quad in x

- real & diff roots — line is secant
- real & equal — tangent
- roots imaginary — outside

Q $x^2+y^2=25$ find position of following lines wrt circle

(1) $x+y+1=0$ (2) $x+y+5\sqrt{2}=0$ (3) $x+y+10=0$

(1) $y=-(x+1)$

$x^2+x^2+1+2x-25=0$

$2x^2+2x-24=0$

$D=\sqrt{b^2-4ac}$

$= \cancel{4}\sqrt{1+48} = 7$

$D = (+)ve$

$\boxed{\text{Secant}}$

(2) $x+y+5\sqrt{2}=0$

$y=-(x+5\sqrt{2})$

$x^2+x^2+50+10\sqrt{2}x-25\cancel{0}=0$

$2x^2+10\sqrt{2}x+25=0$

$\cancel{x^2+5\sqrt{2}x+}$

$D=\sqrt{200-200}=0$

$\boxed{\text{tangent}}$

(3) $y=-(x+10)$

$x^2+x^2+100+20x-25=0$

$2x^2+20x+75=0$

$\sqrt{400-600}=D$

$\boxed{\text{outside}}$

Q find eqⁿ of circle Centre $(6,1)$ & touches $5x+12y=3$.

$\perp$ dis $= \frac{30+12-3}{13} = 3$

$r=3$

$$\boxed{(x-6)^2+(y-1)^2=9}$$

$x^2+36-12x+y^2+1-2y-9=0$

$$\boxed{x^2+y^2-12x-2y+28=0}$$

Q1 find the eqⁿ of the tangent to the circle $x^2+y^2-6x+4y-12=0$ & which are

i) $\parallel$ to $4x-3y+6=0$

ii) $\perp$ to $12x-5y+9=0$

Q2. find 'c' if line $3x-4y-c=0$ will meet the circle $(x-2)^2+(y-4)^2=25$

Q1 Centre $(3,-2)$

$r=\sqrt{9+4+12}$

$r=5$

i) ~~3a-2b+c~~

i) $4x-3y+c_1=0$

$\frac{12+6+c_1}{5}=\pm 5$

$c_1=25-18$

$c_1=7$

$c_1=-25-18$

$c_1=-43$

$$\boxed{4x-3y+7=0}$$

$$\boxed{4x-3y-43=0}$$

ii) $-5x - 12y + c_1 = 0$

$5x + 12y - c = 0$

$15 + 24 - c = \pm 5 \times 13$

$-9 - c = 65$ $\quad$ $9 - c = -65$

$\cancel{-c = 26}$ $\quad$ $\cancel{c = 104}$

$\cancel{c = -26}$

$\cancel{\boxed{5x + 12y + 26 = 0}}$ $\quad$ $\cancel{\boxed{5x + 12y - 104 = 0}}$

$\boxed{5x + 12y + 74 = 0}$ $\quad$ $\boxed{5x + 12y - 56 = 0}$

Q2. Centre $(2, 4)$

$r = 5$

$3(2) - 4(4) - c \leq \pm 5 \times 5$

$\cancel{-7 - c = \pm 25}$

$c = -7 \pm 25$

$\cancel{\boxed{c = -32, 18}} \rightarrow$ Tangent

$-10 - c \leq 25$ $\quad$ $-10 - c \leq -25$

$c \geq -35$ $\quad$ $-c \leq -15$

$\therefore \boxed{c \geq 15}$

$\cancel{\boxed{c \in [15, \infty)}}$

$c \in [-35, 15]$

DYS-3

Q16. $x - 2y - 5 = 0$

$7x + y = 50$

$14x + 2y - 100 = 0$

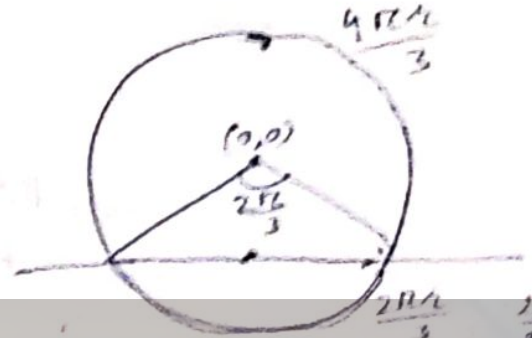
$15x = 105$

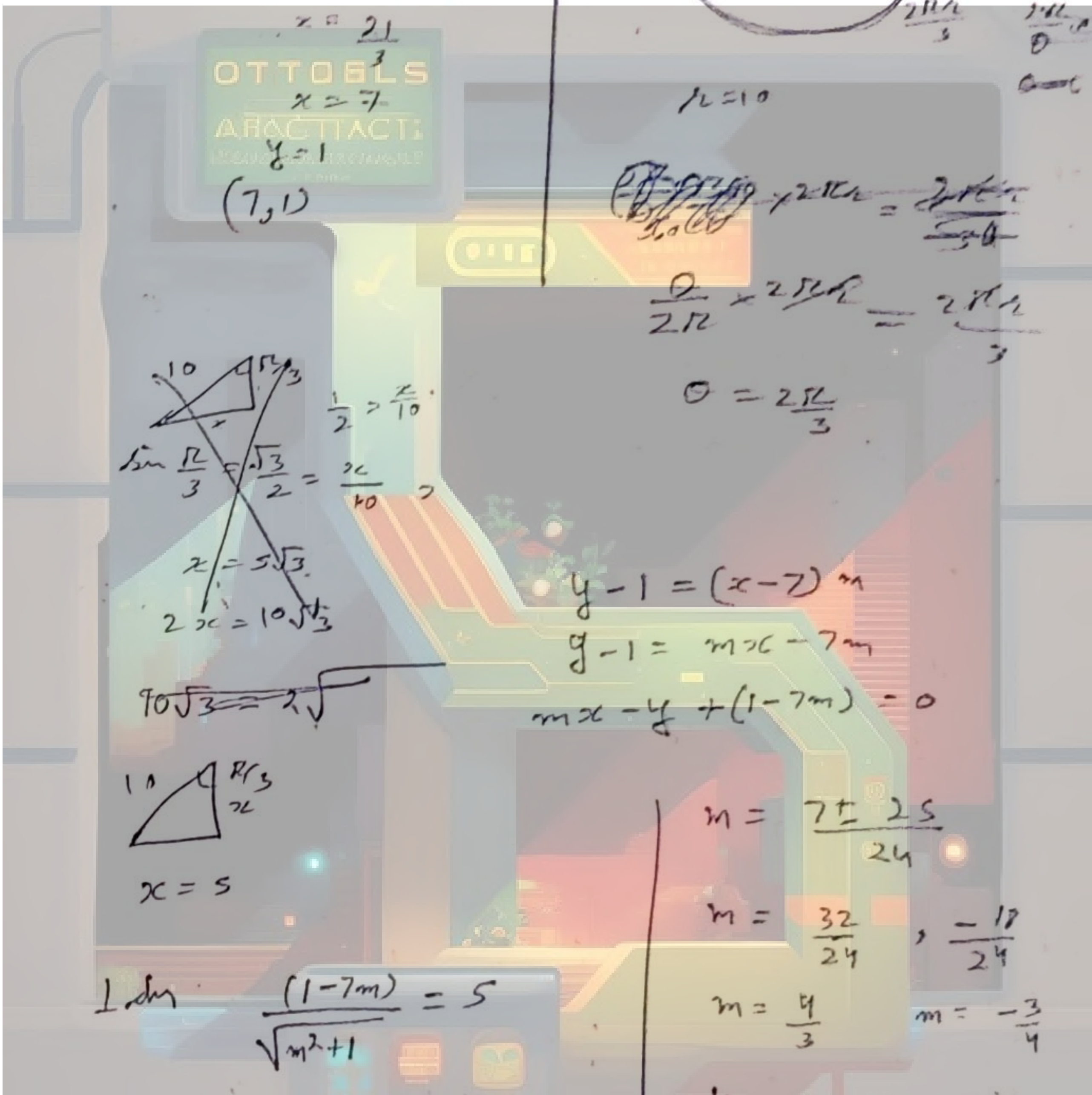
$x = \frac{21}{3}$

$x = 7$

$y = 1$

$(7, 1)$





$r = 10$

$\frac{\theta}{2\pi} \times 2\pi r = \frac{2\pi r}{3}$

$\theta = \frac{2\pi}{3}$

10, $\pi/3$, x

$\frac{1}{2} = \frac{x}{10}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} = \frac{x}{10}$

$x = 5\sqrt{3}$

$\cancel{2x = 10\sqrt{3}}$

$\cancel{10\sqrt{3} = 2\sqrt{\ }}$

10, $\pi/3$, x

$x = 5$

$y - 1 = (x - 7)m$

$y - 1 = mx - 7m$

$mx - y + (1 - 7m) = 0$

1 dis $\frac{(1-7m)}{\sqrt{m^2+1}} = 5$

$25(m^2+1) = 49m^2 + 1 - 14m$

$24m^2 - 14m - 24 = 0$

$12m^2 - 7m - 12 = 0$

$m = \frac{7 \pm \sqrt{49+576}}{24}$

$m = \frac{7 \pm 25}{24}$

$m = \frac{32}{24}, \frac{-18}{24}$

$m = \frac{4}{3}$, $m = -\frac{3}{4}$

$\frac{4}{3}x - y + \left(\frac{3-28}{3}\right) = 0$

$\boxed{4x - 3y - 25 = 0}$

$-3x - 4y + 25 = 0$

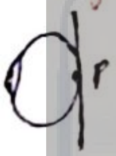
$\boxed{3x + 4y - 25 = 0}$

Equation of Tangent

① Point form

Point $P(x, y)$ lies

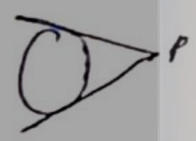
① On circle
1 tangent



$T = 0$

② inside circle
No Tangent

③ outside circle.
pair of tangent
(2 tangent)

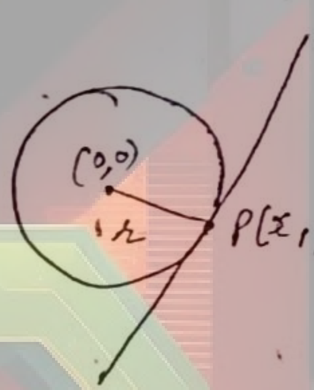


$SS_1 = T^2$

Note. – Meaning of $T = 0$ Track when point on circle
→ Replace in circle equation.

$x^2 \longrightarrow x x_1$
$y^2 \longrightarrow y y_1$
$x = \frac{x + x_1}{2}$
$y = \frac{y + y_1}{2}$
$xy \longrightarrow \frac{x y_1 + x_1 y}{2}$

सबूत



Of ⊥ line.

$m_{OP} \; m_L = -1$

$\frac{y_1}{x_1} \cdot m_L = -1$

$m_L = \frac{-x}{y_1}$

$r(x_1, y_1)$

$\longrightarrow y - y_1 = \frac{-x_1}{y_1}(x - x_1)$

$y y_1 - y_1^2 = -x x_1 + x_1^2$

$x x_1 + y_1 y = \underbrace{x_1^2 + y_1^2}$

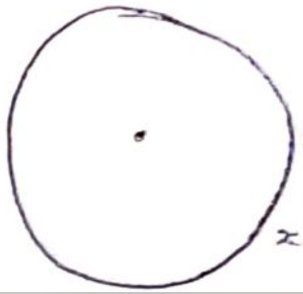
x_1, y_1 lies on circle

$x x_1 + y y_1 = r^2$

अतः सिद्ध हुआ

② Slope Form

$y = mx + c$ --- ①



$x^2 + y^2 = a^2$ --- ②

by ① & ②

$x^2 + (mx+c)^2 = a^2$

$x^2 + m^2x^2 + 2mcx + c^2 - a^2 = 0$

$\boxed{D = 0}$

$4m^2c^2 - 4(1+m^2)(c^2 - a^2) = 0$

$\cancel{4m^2c^2} - 4c^2 + 4a^2 - \cancel{4m^2c^2} + 4a^2m^2 = 0$

$\boxed{a^2(1+m^2) = c^2}$ only for $x^2+y^2=a^2$ circle i.e. centre $(0,0)$

tangent $\Rightarrow$ $\boxed{y = mx \pm a\sqrt{1+m^2}}$ a = radius.

point outside circle

③ For $(x-h)^2 + (y-k)^2 = a^2$

tangent $\Rightarrow$ $\boxed{(y-k) = m(x-h) \pm \underbrace{a\sqrt{1+m^2}}}$ → ye constant nahi hoi hona

Q find the eqn. of tangent in point & slope form for:-

① circle:- $(x-4)^2 + (y-6)^2 = 25$ & $P(1,2)$

② Circle:- $x^2 + y^2 - 2x + 4y = 0$ & $Q(0,1)$.

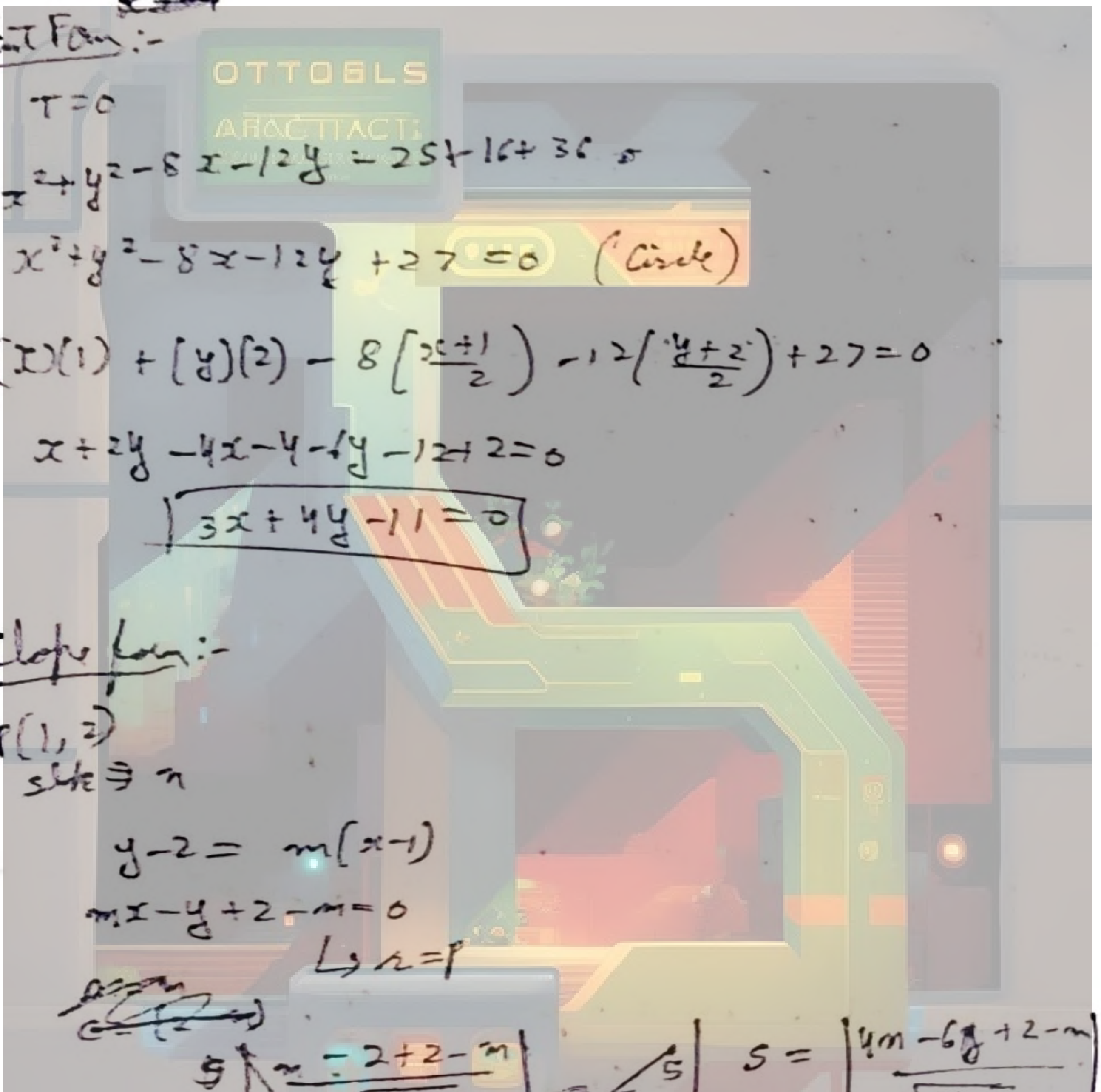
Note - If only 1 value of m comes either $m \to \infty$ so $\perp$ to x-axis

21

$$(1-4)^2 + (2-6)^2 - 25 =$$

$$9 + 16 - 25 = 0$$

on circumference.



Point Form:-

$T = 0$

$$x^2 + y^2 - 8x - 12y - 25 + 16 + 36 = 0$$

$$x^2 + y^2 - 8x - 12y + 27 = 0 \quad \text{(Circle)}$$

$$(x)(1) + (y)(2) - 8\left(\frac{x+1}{2}\right) - 12\left(\frac{y+2}{2}\right) + 27 = 0$$

$$x + 2y - 4x - 4 - 6y - 12 + 27 = 0$$

$$\boxed{3x + 4y - 11 = 0}$$

Slope Form:-

$P(1, 2)$

slope $= m$

$$y - 2 = m(x - 1)$$

$$mx - y + 2 - m = 0$$

$\hookrightarrow h = P$

$$\cancel{5 = \left|\frac{m - 2 + 2 - m}{\sqrt{m^2+1}}\right| = 5}$$

$$\cancel{m = \frac{-3}{4}}$$

$$\cancel{\boxed{3x + 4y - 11 = 0}}$$

$$5 = \left|\frac{4m - 6 + 2 - m}{\sqrt{1+m^2}}\right|$$

$$m = \frac{-3}{4}$$

$$3x + 4y - 11 = 0$$

(2) lies outside

Point form:-

~~$SS_1 = T^2$~~

$SS_1 = T^2$

$(x^2+y^2-2x+4y)(0+1-0+4) = \left(0\cdot x+1\cdot y-2\left(\frac{x+0}{2}\right)+4\left(\frac{y+1}{2}\right)\right)^2$

Too lenthy

~~Slope form:-~~

~~$x^2+y^2-2x+4y=0$~~

~~centre $=(1,-2)$~~

~~$r=\sqrt{5}$~~

~~$y+2 = m(x-1)\pm\sqrt{5}\sqrt{1+m^2}$~~

Slope Form:- (Galat Hai)

$x^2+y^2-2x+4y=0$

$(x-1)^2+(y+2)^2=5$

$y+2 = m(x-1)\pm\sqrt{5}\sqrt{1+m^2}$

put $(0,1)$

$3 = -m\pm\sqrt{5+5m^2}$

$m+3 = \pm\sqrt{5+5m^2}$

square

$m^2+9+6m = 5+5m^2$

$2m^2-3m-2=0$

$m = 2, \frac{-1}{2}$

Put in this

$y+2 = 2x-2\pm 5$

$\boxed{2x-y+1=0}$ $\quad 2x-y-9=0$ ✗

$y+2 = -\frac{1}{2}x+\frac{1}{2}\pm\frac{5}{2}$

$2y+4 = -x+1\pm 5$

$\boxed{x+2y-2=0}$ $\quad x+2y+8=0$ ✗

H.W. 7-10-24

DYS 1 - 1, 2, ~~3~~ (Q1

DYS - 3 (Q1, 2, 3, 4, 5, ~~6~~, 7, 8, 9, 10)

Note:- when slope is given then we use formula

$$y = mx \pm a\sqrt{1+m^2}$$

or

$$y - k = m(x-h) \pm a\sqrt{1+m^2}$$

Q $x^2+y^2-2x+4y=0$ $(0,1)$

let tangent is $y-1 = m(x-0)$

$$mx - y + 1 = 0$$

~~r~~ $= \sqrt{5}$

It is tangent, so $\boxed{r = p}$

$$\sqrt{5} = \left|\frac{m(1)-(-2)+1}{\sqrt{1+m^2}}\right|$$

$$\sqrt{5} = \left|\frac{m+3}{\sqrt{1+m^2}}\right|$$

$$5+5m^2 = m^2+9+6m$$

$$2m^2-3m-2=0$$

$$m = 2, -1/2$$

$$\boxed{2x-y+1=0}$$

$$-\frac{1}{2}x - y + 1 = 0$$

$$\boxed{x+2y-2=0}$$

Q1. find the point of contact (Point of tangent) of $3x-4y-15=0$ for $x^2+y^2-4x-8y-5=0$

Q2. find the eq^n of the tangent drawn to the circle $x^2+y^2-6x+4y-3=0$ at $(7,4)$ outside the circle.

Q2. $S_1 = 49+16-42+16-3 = 36.$

Outside.

let tangent $= y-4=(x-7)m$

$mx - y + (4-7m) = 0$

center $(3,-2)$

$\boxed{r=p}$

$$\sqrt{9+4+3} = \left| \frac{3m+2+4-7m}{\sqrt{1+m^2}} \right|$$

$$16+16m^2 = (6-4m)^2$$

$$\cancel{16m^2}\ 16+\cancel{16m^2} = \cancel{16m^2} - 48m + 36$$

$$48m = 20$$

$$m = \frac{5}{12}, \quad m \to \infty$$

$$\frac{5}{12}x - y + \left(4 - \frac{35}{12}\right) = 0$$

$$\frac{5}{12}x - y + \frac{13}{12} = 0$$

$$\boxed{5x - 12y + 13 = 0}$$

$$\boxed{x = 7}$$

Q1 MI

$3x - 4y - 15 = 0$

$x = \dfrac{4y+15}{3}$

in circle

$$\frac{16y^2 + 225 + 120y}{9} + y^2 - \frac{(16y+60)}{3} - 8y - 5 = 0$$

$$16y^2 + 120y + 225 + 9y^2 - 48y - 180 - 72y - 45 = 0$$

$25y^2 = 0$

$y = 0$

$x = 5$

$\boxed{(5, 0)}$

MII

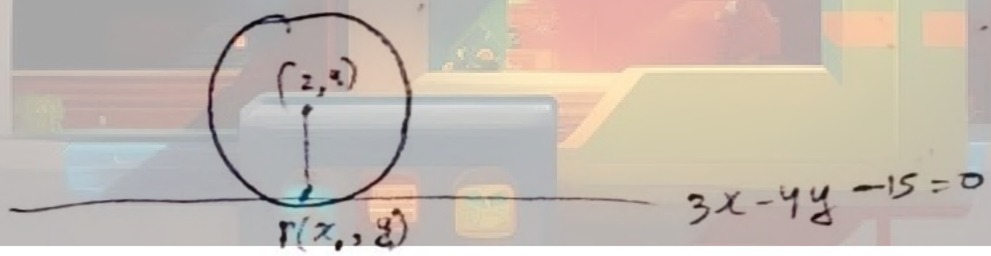
At pt = $P(x_1, y_1)$ write tangent eqn.

$$xx_1 + yy_1 - 2(x + x_1) - 4(y + y_1) - 5 = 0$$

$$xx_1 + yy_1 - 2x - 2x_1 - 4y - 4y_1 - 5 = 0$$

$$(x_1 - 2)x + (y_1 - 4)y - 2x_1 - 4y_1 - 5 = 0$$

MIII



$$\frac{(x_1 - 2)}{3} = \frac{(y_1 - 4)}{-4} = \frac{-3x_1 + 4y_1 + 15}{5} = \lambda$$

$x_1 - 2 = 3\lambda$ | $y_1 - 4 = -4\lambda$

$x_1 = 3\lambda + 2$ | $y_1 = 4 - 4\lambda$

$9\lambda + 6 - 16 + 16\lambda - 15 = 0$

$25\lambda - 10 - 15 = 0$

$\lambda = 1$

$x_1 = 3 + 2 = 5$

$y_1 = 4 - 4(1) = 4 - 4 = 0$

$\boxed{(5, 0)}$

Normal of Circle :-

→ line ⊥ to tangent at Point of contact.

→ Passes through Center. (Always)

Q find eq^n of tangent & normal to circle at

$x^2 + y^2 - 5x + 2y - 48 = 0$ at $(5, 6)$

tangent ⇒ $y - 6 = m(x - 5)$

$y - 6 = mx - 5m$

$mx - y + (6 - 5m) = 0$

Center $(5/2, -1)$ $r =$

$\sqrt{\frac{25}{4} + 1 + 48} = \left| \frac{5m - 6 + 6 - 5m}{\sqrt{1 + m^2}} \right|$

$\frac{221}{4}(1 + m^2) = 0$

$m^2 = -1$

$\sqrt{\frac{25}{4} + 1 + 48} = \left| \frac{\frac{5}{2}m + 1 + 6 - 5m}{\sqrt{1 + m^2}} \right|$

$\cancel{\frac{25}{4} + 9 x^2 + \frac{2}{2}\frac{25}{4}} =$

On circumference

$T = 0$

$$xx_1 + yy_1 - \frac{5}{2}(x + x_1) + (y + y_1) \cancel{-48} - 48 = 0$$

$$5x + 6y - \frac{5}{2}x - \frac{5}{2}\cancel{x}5 + y + 6 - 48 = 0$$

$$\frac{5}{2}x + 7y - \frac{109}{2} = 0$$

$$\boxed{5x + 14y - 109 = 0}$$

$m = -\frac{5}{14}$

normal $\Rightarrow$ $(5, 6)$ $m = \cancel{5}\ \frac{14}{5}$

$$y - 6 = (x - 5)\frac{14}{5}$$

$$5y - 30 = 14x - 70$$

$$\boxed{14x - 5y - 40 = 0}$$

Q Find the Eqn of normal $\parallel$ to $3x + 4y - 7 = 0$ for circle $x^2 + y^2 - 6x + 4y - 12 = 0$

Center $(3, -2)$

$m = \frac{-3}{4}$

$$y + 2 = (x - 3)\frac{-3}{4}$$

$$4y + 8 = -3x + 9$$

$$\boxed{3x + 4y - 1 = 0}$$

Q find the eqⁿ of tangent & normal to circle $x^2+y^2=25$ at $(4,3)$

on circumference.

$T=0$

$xx_1+yy_1-25=0$

$\boxed{4x+3y-25=0} \Rightarrow$ Tangent

Normal

centre $(0,0)$ ~~3/4~~ $m=\frac{3}{4}$

$y=\frac{3}{4}x$

$\boxed{3x-4y=0}$

Q find value of c if $3x+4y=4c$ is tangent to $x^2+y^2=25$.

~~$m_{normal}=\frac{4}{3}$~~

~~Centre $(0,0)$~~

~~normal $\Rightarrow$ $4x-3y=0$~~

~~$12x+16y=16c$~~

~~$12x-9y=0$~~

~~$7y=16c$~~

~~$y=\frac{16}{7}c$~~

~~$x=\frac{12}{7}c$ lie on circle~~

~~$\frac{144c^2}{49}+\frac{256c^2}{49}=25$~~

~~$400c^2=1225$~~

~~$c^2=\frac{1225}{400}$~~

~~$c=\frac{35}{20}$~~

~~$\boxed{c=\frac{7}{4}}$~~

Center $(0,0)$

$\perp$ dis $= r = 5$

$5=\left|\frac{-4c}{5}\right|$

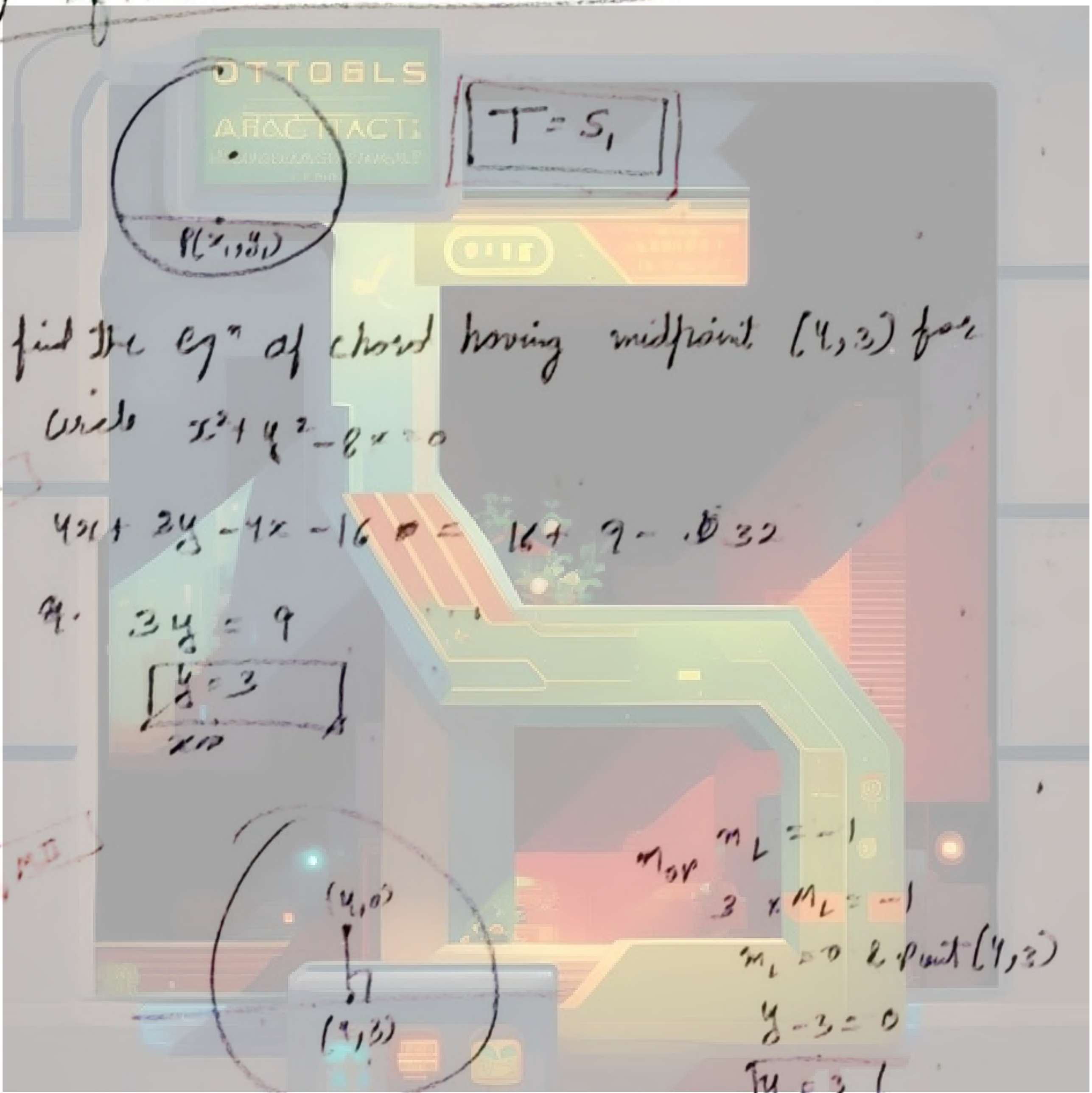
$\boxed{c=\pm\frac{25}{4}}$

[MII (

$x^2+y^2=a^2 \quad y=mx+c$

$$\boxed{c^2=a^2(1+m^2)}$$

Eqⁿ of chord with Mid Point (x_1, y_1).



$$\boxed{T=S_1}$$

$P(x_1, y_1)$

Q find the eqⁿ of chord having midpoint $(4,3)$ for circle $x^2+y^2-8x=0$

[M-I]

$4x+3y-4x-16 = 16+9-32$

$3y=9$

$$\boxed{y=3}$$

[M-II]

$m_{OP}\, m_L=-1$

$(4,0)$

$(4,3)$

$3 \times m_L=-1$

$m_L=0$ & point $(4,3)$

$y-3=0$

$$\boxed{y=3}$$

Q) find mid of chord which cuts the circle $x^2+y^2-2x+2y-2=0$ by line $y=x-1$.

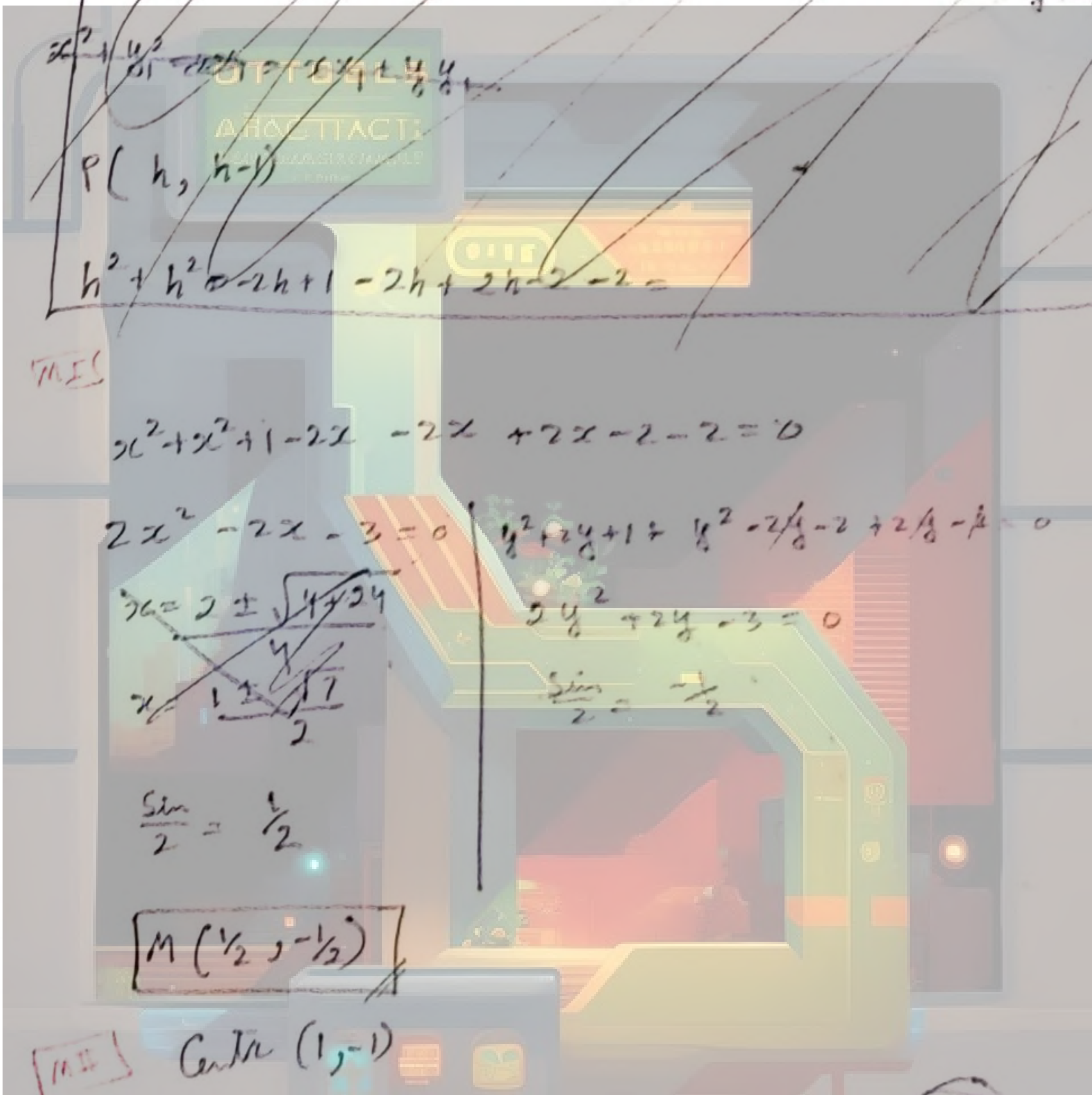
~~$P(x_1, y_1)$~~

~~$(x_1)^2+(y_1)^2-2x_1+2y_1-2 = xx_1+yy_1-x-x_1+y+y_1-2$~~

~~$x^2+y_1^2 \ldots + yy_1$~~

~~$P(h, h-1)$~~

~~$h^2+h^2-2h+1-2h+2h-2-2=$~~



M I

$x^2+x^2+1-2x-2x+2x-2-2=0$

$2x^2-2x-3=0$ | $y^2+2y+1+y^2-2y-2+2y-2=0$

$x = \frac{2 \pm \sqrt{4+24}}{4}$ | $2y^2+2y-3=0$

$x = \frac{1 \pm \sqrt{7}}{2}$ | $\frac{\text{Sum}}{2} = \frac{-1}{2}$

$\frac{\text{Sum}}{2} = \frac{1}{2}$

$\boxed{M\left(\frac{1}{2}, -\frac{1}{2}\right)}$

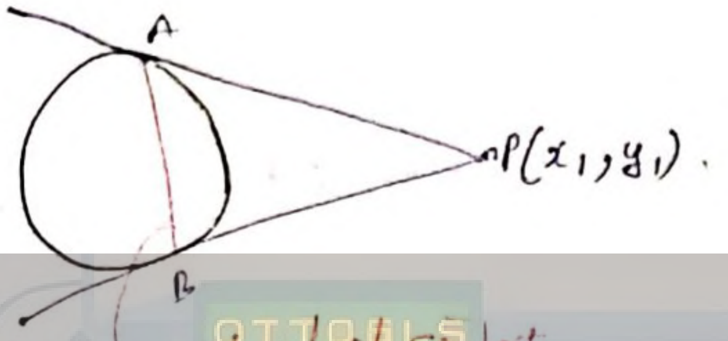
M II Centre $(1,-1)$

$\frac{x_1-1}{1} = \cancel{\frac{-x+y+1}{\sqrt{2}}} = \frac{1+1-1}{\sqrt{2}}$

$\boxed{x_1 = \frac{\sqrt{2}+1}{\sqrt{2}}, \frac{\sqrt{2}-1}{\sqrt{2}}}$

$x-y-1=0$

Chord of contact of Tangents from external Point $P(x_1, y_1)$



→ Chord of contact.

$\boxed{T = 0}$ Only when Point is external.

→ eq^n of AB.

H.W. 8-10-24

DYS-4, 3 left

O-1 {Q1, 2, 3, 4, 5, 6, 7, 2, 8, 9, 10, 11}

Q find the eqn of chord of contact of tangents drawn from the point $(2, -3)$ to the circle

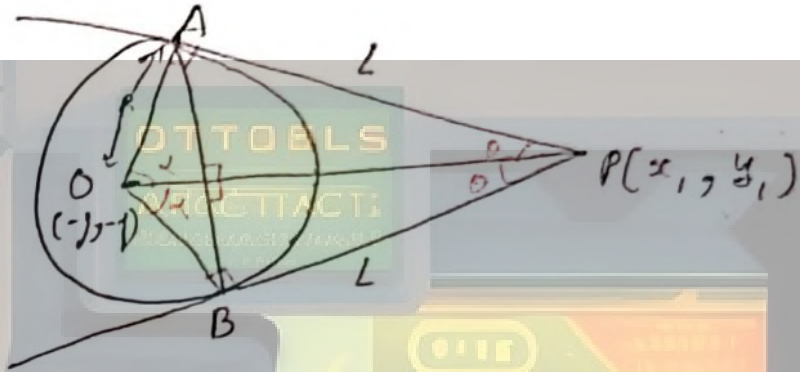
$$x^2 + y^2 + 4x - 6y - 12 = 0$$

$$2x - 3y + 2x + 4 - 3y + 9 - 12 = 0$$

$$\boxed{4x - 6y + 1 = 0}$$

System of tangents ~~&~~ & chord of ~~contact~~ contact

$$x^2+y^2+2gx+2fy+c=0$$



$$\triangle OPA \cong \triangle OPB \ \& \ \triangle OAQ \cong \triangle OBQ$$

1. Length of ~~Triangle~~ Tangent

$$OP^2 = R^2 + L^2$$

$$(x_1+g)^2 + (y_1+f)^2 = L^2 + g^2 + f^2 - c$$

$$x_1^2 + g^2 + y_1^2 + f^2 + 2gx_1 + 2fy_1 = L^2 + f^2 + g^2 - c$$

$$L^2 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

$$\therefore \boxed{L = \sqrt{S_1}}$$

2. Area of Triangle OPA

$$\boxed{ar(\triangle OPA) = \frac{1}{2} \times L \times R}$$

$$\boxed{\text{area Quadrilateral} = LR}$$

3. Angle b/w Tangents (2θ):

in ΔOPA

$$\boxed{\tan\theta = \frac{R}{L}}$$

4. Length of chord AB

$AB = 2AQ$ — ①

In ΔAQP:- $\sin\theta = \frac{AQ}{L}$

$AQ = L\sin\theta$ $\left[\sin\theta = \frac{R}{\sqrt{L^2+R^2}}\right]$

$$\boxed{AB = \frac{2LR}{\sqrt{L^2+R^2}}}$$

5. Length OQ:-

ΔAQO, $\cos\alpha = \frac{OQ}{R}$

~~$\sin\theta$~~

$OQ = R\sin\theta$

$$\boxed{OQ = \frac{R^2}{\sqrt{L^2+R^2}}}$$

6. Area of ΔPAB

~~[illegible]~~ $\cos\theta = \frac{L}{\sqrt{L^2+R^2}}$

$\text{Area} = \frac{1}{2}\times\cancel{2}\times\sin\theta \;\Big|\; L^2 \times \frac{L}{\sqrt{L^2+R^2}} \times \frac{R}{\sqrt{L^2+R^2}} = \frac{RL^3}{L^2+R^2}$

~~Area =~~

$$\boxed{\text{Ar } \Delta PAB = \frac{RL^3}{L^2+R^2}}$$

A circle is always available which passes through O, A, P & B. Where OP is diameter.

Eq^n of ~~concyclic~~ circle :-

$$\boxed{(x-x_1)(x+g)+(y-y_1)(y+f)=0}$$

Q1. find the length of tangent drawn from the point (1, 5) to the circle $2x^2+2y^2=3$.

Q2. find the length of chord of contact of tangents from (3, 4) to the circle $x^2+y^2=4$

Q3. Let A be the center of the circle $x^2+y^2-2x-4y-20=0$ & B (1, 7) & D (4, -2) are points of circle from which two tangents are drawn. which meets at C.

i) find the area of Quad ABCD

ii) find the Eq^n of circle passing through A B C & D

Q1. $S = \frac{2+50-3}{2} = \frac{49}{2}$

$$\boxed{L_T = \frac{7}{\sqrt{2}}}$$

Q2. $L = \sqrt{9+16-4} = \sqrt{21}$

$R = 2$

$$\text{Chord} = \frac{2\times2\times\sqrt{21}}{\sqrt{21+4}} = \boxed{\frac{4\sqrt{21}}{5}}$$

Q3 $x^2+y^2-2x-4y-20$

$A(1,2)$ $\quad B(1,7)$ $\quad \cancel{C(\ ,\ )}$ $\quad D(4,-2)$

$C(16,7)$

$BC \Rightarrow y-7 = \frac{(x-1)}{0}\cdot 0$

$y = 7$

$CD \Rightarrow y+2 = (x-4)\frac{3}{4}$

$4y+8 = 3x-12$

$3x-4y-20=0$

$\cancel{L = \sqrt{1+\frac{289}{16}-2+17-20} = \sqrt{\frac{289-\ }{16}} = \frac{15}{4}}$

$\cancel{R = 5}$

$\cancel{\text{area} = L\times 5R = \boxed{\frac{75}{4}}}$

$\cancel{(x-1)(x-1)+(y-\frac{\ }{4})}$

$L = \sqrt{256+49-32-28-20} = 15$

$R = 5$

i) $\boxed{\text{area} = 75}$

ii) $(x-16)(x-1)+(y-7)(y-2)=0$

$x^2-17x+16+y^2-9y+14=0$

$\boxed{x^2+y^2-17x-9y+30=0}$

Director Circle

→ It is a locus of point of intersection of two ⊥ tangents of a given curve.

$x^2+y^2=a^2$

↳ Director circle for black circle

Let $P(h, k)$

tangent : $y = mx \pm a\sqrt{1+m^2}$

$\cancel{k = m}$

$k = mh \pm a\sqrt{1+m^2}$

$\cancel{k^2 = m^2h^2}$

$k^2+m^2h^2-2hkm = a^2+a^2m^2$

$m^2(h^2-a^2) - 2hkm + k^2-a^2=0$

$m_1m_2 = -1$

$\frac{k^2-a^2}{h^2-a^2} = -1$

$k^2-a^2 = -h^2+a^2$

$h^2+k^2 = 2a^2$

$h^2+k^2 = (\sqrt{2}a)^2$

Locus :- $\boxed{x^2+y^2 = (\sqrt{2}a)^2}$ Director circle

$(x-h)^2+(y-k)^2 = a^2 \Rightarrow (x-h)^2+(y-k)^2 = (\sqrt{2}a)^2$

Note - Director circle is always concentric circle.

Q find the Eq^n of director circle for the circle

① $x^2+y^2=9$

② $x^2+y^2=\sqrt{2}\log_2^3$

③ $x^2+y^2=a^2+b^2$

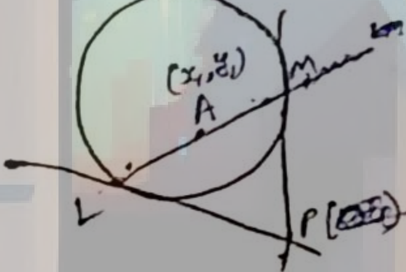
① $x^2+y^2=18$

② $x^2+y^2=\left(\sqrt{2}\times\sqrt{\sqrt{2}\log_2^3}\right)^2=\left(\sqrt{2\sqrt{2}\log_2^3}\right)^2$ ~~$=2\sqrt{2}$~~

$x^2+y^2=2\sqrt{2}\log_2^3$

③ $x^2+y^2=2a^2+2b^2$

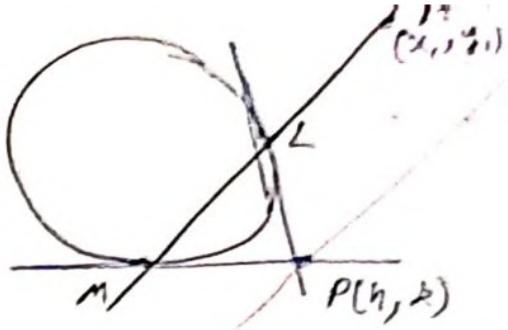
Pole & Polar



→ ~~Pole is point ... & Polar is ...~~

→ Let A is a point inside or outside the circle & LM be a chord drawn from A which intersect the circle at 2 points. (L & M)

→ If two Tangents are drawn from L & M then they intersect at point P.
Then locus of point P is called Polar of A & A is called Pole



Eqⁿ of Polar / Eqⁿ of locus of P(h, k) w.r.t. A

Eqⁿ :- $\boxed{T = 0}$

Q find the Eqⁿ of Polar w.r.t point A (3, -1) for circle

$2x^2 + 2y^2 - 3x + 5y - 7 = 0$

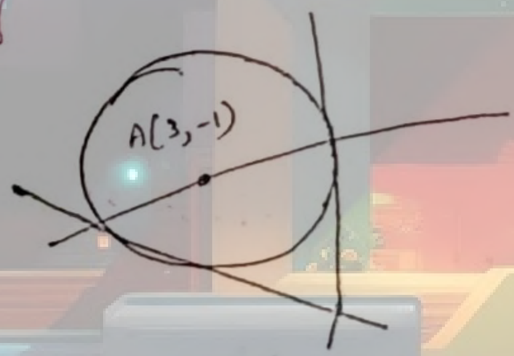
M I

$6x - 2y - \frac{3}{2}x - \frac{9}{2} + \frac{5}{2}y - \frac{5}{2} - 7 = 0$

$12x - 4y - 3x - 9 + 5y - 5 - 14 = 0$

$\boxed{9x + y - 28 = 0}$

M II



get P.O.I of two tangents then get locus.

H.W. 10-10-24

DYS - 5, 6, 7

O-1 {13, 15, 16, 17, 20, 21, 22, 24, 25, 26}

Q If $3x+5y+17=0$ is polar for circle $x^2+y^2+4x+6y+9=0$ then find pole

$P(x_1, y_1)$

Polar ⇒ $[T=0]_{(x_1, y_1)}$

$x_1x + y_1y + 2x + 2x_1 + 3y + 3y_1 + 9 = 0$

$(x_1+2)x + (y_1+3)y + (2x_1+3y_1+9) = 0$

Polar ⇒ $\lambda(3x + 5y + 17) = 0$

$x_1 = 3\lambda - 2$

$y_1 = 5\lambda - 3$

$17\lambda = 6\lambda - 4 + 15\lambda - \cancel{9} + \cancel{9}$

$17\lambda = 21\lambda - 4$

$4 = 4\lambda$

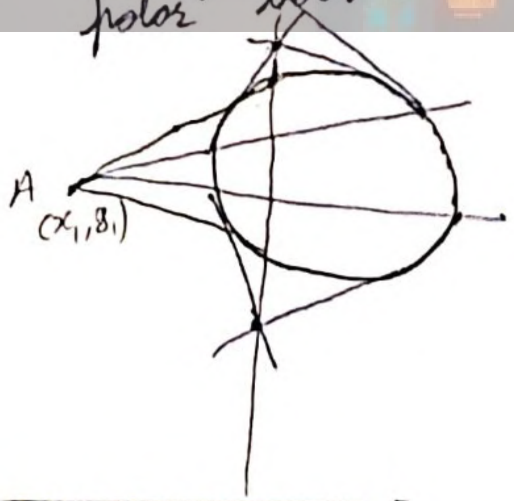
$\lambda = 1$

$x_1 = 1$

$y_1 = 2$

$\boxed{(1, 2)}$

Note :- (1) If Point A is outside then chord of contact & polar both are same



chord of contact

Polar

(2) when point is inside then chord of contact do not exist but Polar exists

(3) when Point A is on the circle then, Polar, chord of contact and tangent are on same point.

(4) Polar of A wrt circle passes through P then the Polar of P will pass through A. Hence, P & A are Conjugate Points of Each other.

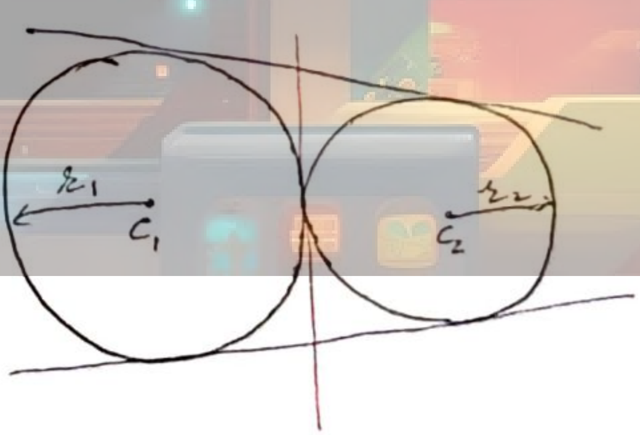
System of Circle:

(1) Externally Seperated

$$\boxed{C_1C_2 > r_1 + r_2}$$

4C.T (Common Tangent)

(2) Externall touch



3 CT

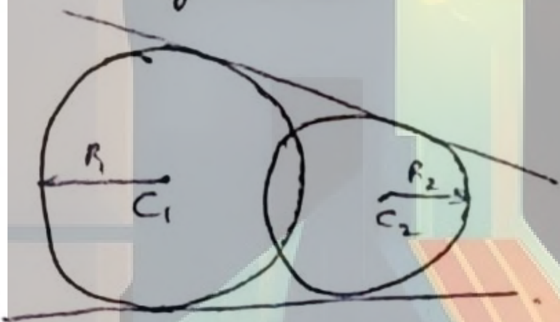
$$\boxed{C_1C_2 = r_1 + r_2}$$

(3) Internally Touch

1 C.T.

$$\boxed{C_1C_2 = |r_1 - r_2|}$$

(4) Intersecting circles



2 C.T.

$$\boxed{|r_1 - r_2| < C_1C_2 < r_1 + r_2}$$

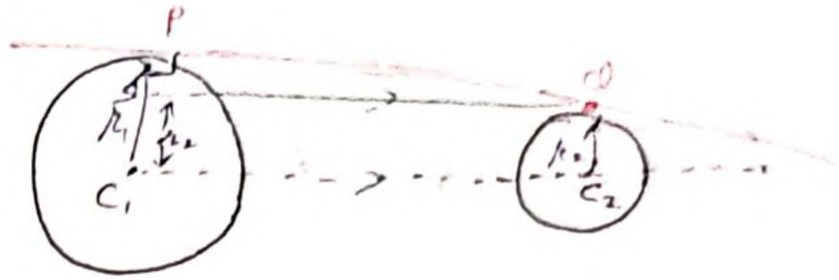
(5) Internally Seperated

0 C.T.

$$\boxed{C_1C_2 < |r_1 - r_2|}$$

Note:

(1)



$$PS = R_1 - R_2$$

in ΔPSQ

$$(C_1C_2)^2 = (R_1 - R_2)^2 + PQ^2$$

$$\cancel{PQ^2 = (C_1C_2)^2 - R_1^2 - R_2^2 + 2R_1R_2}$$

$$\cancel{PQ = \sqrt{(R_1 - R_2)^2}}$$

$$PQ = \sqrt{(C_1C_2)^2 - (R_1 - R_2)^2}$$

$$\boxed{PQ = \sqrt{D^2 - (R_1 - R_2)^2}}$$

$d = C_1C_2$

(2)

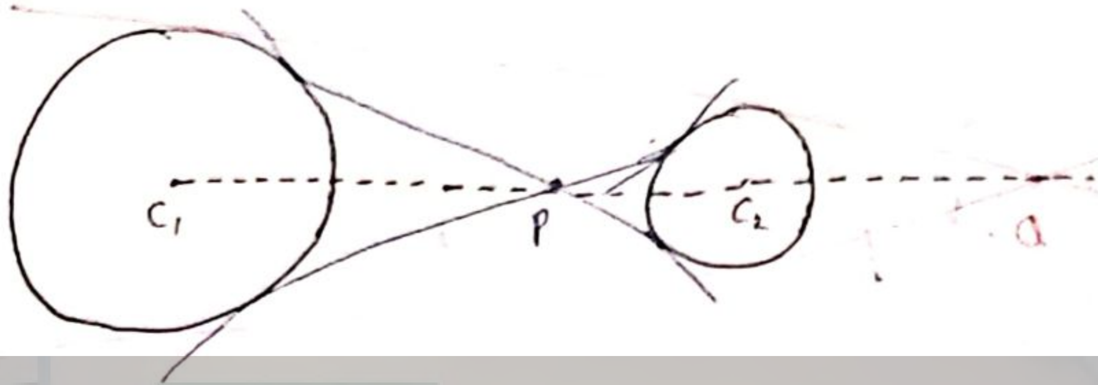
in ΔC_1C_2M

$$(C_1C_2)^2 = (r_1 + r_2)^2 + (C_1M)^2$$

$$PQ^2 = (C_1C_2)^2 - (r_1 + r_2)^2$$

$$\boxed{PQ = \sqrt{D^2 - (R_1 + R_2)^2}}$$

③.



$$\boxed{\frac{C_1P}{C_2P} = \frac{C_1Q}{C_2Q} = \frac{r_1}{r_2}}$$

→ P & Q are Harmonic Conjugate of each other wrt C_1 & C_2

→ for Tangent Eqn, use slope form i.e.

$$y - y_1 = m(x - x_1)$$

then, $r = p$

Q Find the range of r such that the circles $(x-1)^2 + (y-3)^2 = r^2$ & $(x-4)^2 + (y+1)^2 = 9$ have:-

① 4 CT ② 3 C.T. ③ intersect at 2 points
④ 1 C.T ⑤ no C.T.

① $C_1(1,3)$
$C_2(4,-1)$
$C_1C_2 = \sqrt{9+16} = 5$

$C_1C_2 > r_1 + r_2$

$5 > 3 + r$

$r < 2$

$\boxed{r \in (0,2)}$

② $C_1C_2 = r_1 + r_2$
$5 = 3 + r$
$\boxed{r = 2}$

③ $|3 - r| \le 5 \le r + 3$

$r \le 8$ & $r \ge 3$, $r \ge -2$ & $r \le 3$; $r \ge 2$

$r \in (2, 8)$

(13) (5) $5 < |3 - r|$

Case I

$r < 3$

$5 < 3 - r$

$r < -2$ ... ①

Case II

$r \geq 3$

$5 < r - 3$

$r > 8$ ... ②

Union

$\boxed{r \in R - [0, 8]}$

(14) (7) $5 = |3 - r|$

C I $r < 3$

$5 = 3 - r$

$r = -2$ ✗

C II $r > 3$

$5 = r - 3$

$\boxed{r = 8}$

(15)

Q find the no. of common tangents to $x^2+y^2=1$ & $x^2+y^2-2x-6y+6=0$.

Also, find the length of tangents too.

$C_1(0,0) \quad C_2(1,3)$

$C_1C_2 = \sqrt{1+9} = \sqrt{10} \simeq 3.$ something

$r_1 = 1$

$r_2 = 2$

$C_1C_2 > r_1 + r_2$

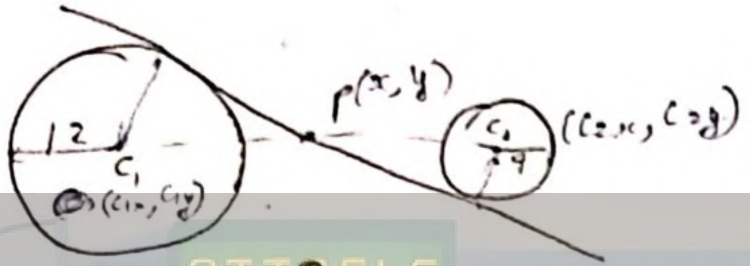
$\boxed{4 \text{ common tangents}}$

$L_{int} = \sqrt{10-9} = \boxed{1}$

$L_{ext} = \sqrt{10-1} = \boxed{3}$

Q1. $x^2+y^2=1$ & $(x-1)^2+(y-3)^2=4$. find eqⁿ of internal & external CT.

Q2.



$C_1C_2 = 35$ ① find $\frac{C_1P}{C_2P}$ ② ~~& C_2~~ C_1P

Q2. ① $\frac{C_1P}{C_2P} = \frac{r_1}{r_2} = \frac{12}{7} = \boxed{\frac{4}{3}}$

②

② $C_1P = 4x$

$C_2P = 3x$

$4x + 3x = 35$

$7x = 35$

$x = 5$

$C_1P = 5 \times 4$

$\boxed{C_1P = 20}$

Q1. $C_1(0,0)$ $r_1 = 1$

$C_2(1,3)$ $r_2 = 2$

$C_1C_2 = \sqrt{10}$

$C_1C_2 > R_1 + R_2$

$\frac{C_1Q}{C_2Q} = \frac{1}{2}$

$\cancel{Q_x = \frac{0-1}{-1} = \frac{1}{3}}$

$\cancel{Q_y = \frac{0-3}{-1} = -1}$

$\cancel{Q(-1/3, -1)}$

$Q(-1,-3)$

$y + 3 = mx + m$

$mx - y + m - 3 = 0$

$r = p$

$1 = \frac{|m-3|}{\sqrt{1+m^2}}$

$1 + m^2 = m^2 + 9 - 6m$

$6m = 8$

$m = \frac{4}{3}$

$\frac{4}{3}x - y + \frac{4}{3} - 3 = 0$

$\boxed{4x - 3y - 5 = 0}$

$\boxed{x = -1}$

$0 - 2P_x = P_x - 1$

$\frac{1}{3} = P_x$

$0 - 2P_y = P_y - 3$

$P_y = 1$

$P(1/3, 1)$

$y - 1 = m(x - \frac{1}{3})$

$r = p$

$1 = \frac{|-\frac{1}{3}m + 1|}{\sqrt{1+m^2}}$

$m^2 + 1 = \frac{m^2}{9} + 1 - \frac{2}{3}m$

$9m^2 + 9 = m^2 + 9 - 6m$

$8m^2 + 6m = 0$

$8m + 6 = 0$

$m = -\frac{3}{4}$

$y - 1 = \frac{-3}{4}(x - \frac{1}{3})$

$\frac{3}{4}x + y - 1 + \frac{1}{4} = 0$

$\boxed{3x + 4y - 3 = 0}$

$\boxed{y = 1}$

H.W 11-10-24

O-2 (1-10)

PYS - 9(1-9)

Family of Circles

(1) POI of two circles

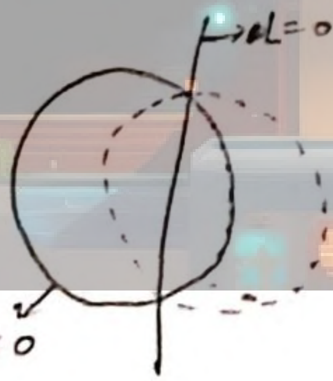
→ Family of Circles passes through POI of two circles.

→ Infinite no. of circles through both the POI.

$\boxed{S_1+\lambda S_2=0}$ $\lambda \in R-\{-1\}$

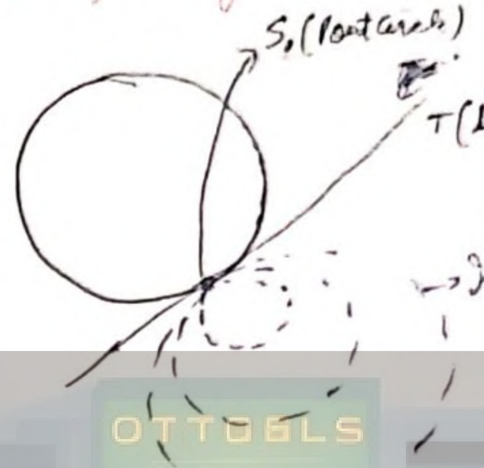
x^2 & y^2 ke terms cancel ho jayenge.

(2) POI of a line & a circle



$\boxed{S+\lambda L=0}$ $\lambda \in R$

③. Passing through Point of Contact & Tangent



$S_0 + \lambda T = 0$

$\lambda \in R$

Q Find the Eq^n of circle passes through

① POI of $x^2+y^2-8x-2y+7=0$ & $x^2+y^2-4x+10y+8=0$ & passes through $(2,3)$

$(1+\lambda)x^2+(1+\lambda)y^2-(8+4\lambda)x-(2-10\lambda)y+(7+8\lambda)=0$

$4+4\lambda+9+9\lambda-16-8\lambda-6-30\lambda+7+8\lambda=0$

$\lambda=-2$

$-x^2-y^2-22y-9=0$

$\boxed{x^2+y^2+22y+9=0}$

② POI of $x^2+y^2=1$ & $x^2+y^2-2x-4y+1=0$ & touches line $x+2y=0$

③ $(0,0)$ & touches the line $2x-y=4$ at point $(1,-2)$

① $x^2+y^2-8x-2y+7+\lambda(x^2+y^2-4x+10y+8)=0$

$P(2,3)$

$4+9-16-6+7+\lambda(4+9-8+30+8)=0$

$-2+\lambda(43)=0$

$\lambda=\frac{2}{43}$

$x^2+y^2-8x-2y+7+\frac{2}{43}(x^2+y^2-4x+10y+8)=0$

②

$x^2+y^2-1+\lambda(x^2+y^2-2x-4y+1)=0$

$(\lambda+1)x^2+(\lambda+1)y^2-2\lambda x-4\lambda y+(\lambda-1)=0$

Centre $(\lambda, 2\lambda)$ $\quad r=\sqrt{5\lambda^2-\lambda+1}$

Distance from centre $=r$

$5\lambda^2-\lambda+1=\frac{|\lambda+4\lambda|}{\sqrt{5}}$

$5\sqrt{5}\lambda^2-\sqrt{5}\lambda+1-5\lambda=0$

$5\sqrt{5}\lambda^2-(\sqrt{5}+5)\lambda+1=0$

$x^2+y^2+\left(\frac{-2\lambda}{\lambda+1}\right)x+\left(\frac{-4\lambda}{\lambda+1}\right)y+\left(\frac{\lambda-1}{\lambda+1}\right)=0$

$r=p$

$C\left(\frac{\lambda}{\lambda+1}, \frac{2\lambda}{\lambda+1}\right) \quad r=\sqrt{\frac{\lambda^2}{(\lambda+1)^2}+\frac{4\lambda^2}{(\lambda+1)^2}-\frac{(\lambda-1)(\lambda+1)}{(\lambda+1)^2}}$

$$\frac{\lambda^2+4\lambda^2-\lambda^2+1}{(\lambda+1)^2} = \left(\frac{\frac{\lambda}{\lambda+1}+\frac{4\lambda}{\lambda+1}}{\sqrt{5}}\right)^2$$

$$\cancel{\frac{4\lambda^2+1}{\lambda^2+2\lambda+1} = \frac{5\lambda}{\sqrt{5}\lambda+\sqrt{5}}}$$

$$\cancel{\sqrt{5}(\lambda+1)(4\lambda^2+1) = 5\lambda(\lambda^2+2\lambda+1)}$$

$$\frac{\lambda^2+4\lambda^2-\lambda^2+1}{\cancel{(\lambda+1)^2}} = \frac{25\lambda^2}{\cancel{5\lambda^2+10\lambda+5}}$$
$$5\cancel{(\lambda+1)^2}$$

$$20\lambda^2+5 = 25\lambda^2$$

$$5 = 5\lambda^2$$

$$\lambda^2 = 1$$

$$\lambda = \pm 1$$

$\lambda = -1$ asthambhav

$$\lambda = 1$$

$$\boxed{x^2+y^2-x-2y=0}$$

(3) $S_0 = \cancel{x(x+y)}\ (x-1)^2+(y-2)^2 = 0$

$S_0 = x^2+y^2-2x-4y+5=0$

$T = 2x-y-4=0$

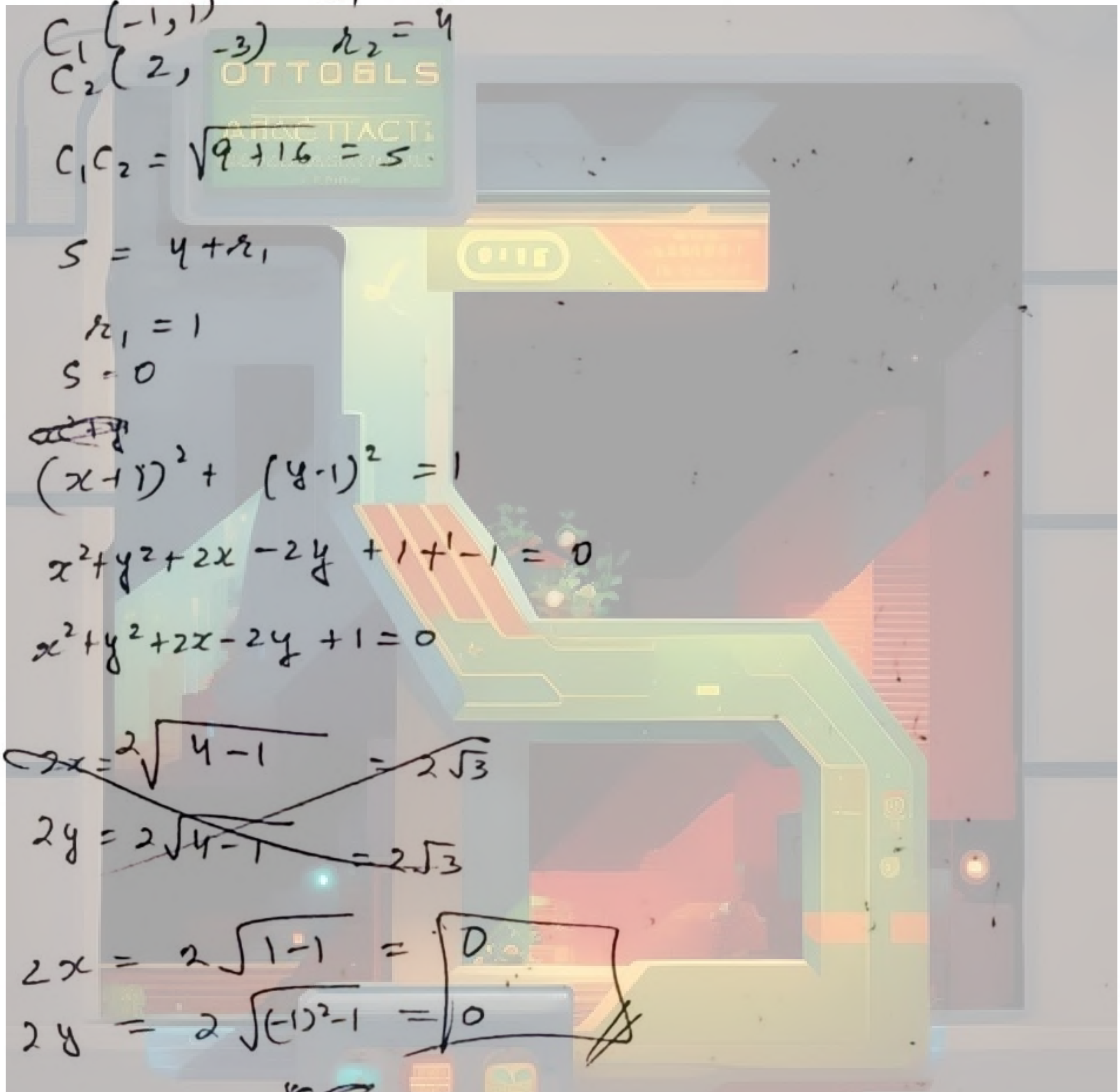
$$x^2+y^2-2(1-\lambda)x-(4+\lambda)y+(5-4\lambda)=0$$

$$5 = 4\lambda$$
$$\lambda = \frac{5}{4}$$

$$x^2+y^2+\frac{1}{2}x-\frac{21}{4}y=0$$

$$\boxed{4x^2+4y^2+2x-21y=0}$$

DYS - 9



Q 5. $C_1(-1,1)$ $\quad r_1 = r_1$

$C_2(2,-3)$ $\quad r_2 = 4$

$$C_1C_2 = \sqrt{9+16} = 5$$

$$5 = 4 + r_1$$

$$r_1 = 1$$

$$S = 0$$

$$\cancel{x^2+y^2}$$

$$(x+1)^2 + (y-1)^2 = 1$$

$$x^2+y^2+2x-2y+1+1-1=0$$

$$x^2+y^2+2x-2y+1=0$$

$$\cancel{2x = 2\sqrt{4-1} = 2\sqrt{3}}$$

$$\cancel{2y = 2\sqrt{4-1} = 2\sqrt{3}}$$

$$2x = 2\sqrt{1-1} = 0$$

$$2y = 2\sqrt{(-1)^2-1} = 0$$

Q6. $C_2(2,3)$ $r_2 = 5$
$C_1(h,k)$ $r_1 = 3$

$C_1C_2 = 2$

$(h-2)^2 + (k-3)^2 = 4 \rightarrow h^2+k^2+4h+6k+9=0$ — (1)

$(x-h)^2 + (y-k)^2 = 9$

$(h+1)^2 + (k+1)^2 = 9$

$h^2+k^2+2h+2k = 7$ — (2)

$h = \frac{4}{5}, \; k = \frac{7}{5}$

$$\boxed{5x^2+5y^2-8x-14y-32=0}$$

PYS-8
Q6 b) $x^2+y^2+4x-6y-12 + \lambda(x^2+y^2-5x+17y-19) = 0$

$(\lambda+1)x^2 + (\lambda+1)y^2 + (4-5\lambda)x - (6-17\lambda)y - (12+19\lambda) = 0$

$x^2+y^2+\left(\frac{4-5\lambda}{\lambda+1}\right)x - \left(\frac{6-17\lambda}{\lambda+1}\right)y - \frac{(12+19\lambda)}{(\lambda+1)} = 0$

$\cancel{\frac{-8+10\lambda}{(\lambda+1)}}$

$\frac{5\lambda-4}{2\lambda+2} + \frac{6-17\lambda}{2\lambda+2} = 0$

$\frac{2-12\lambda}{2\lambda+2} = 0$

$\boxed{\lambda = 1/6}$

$$\boxed{x^2+y^2+\frac{19}{7}x - \frac{19}{7}y - \frac{91}{7} = 0}$$

Q7. $(x-1)^2 + (y-1)^2 = 0 \longrightarrow S_0$

$2x - 3y + 1 = 0 \longrightarrow T$

$S_0 + \lambda T = 0$

$x^2 + y^2 - (2-2\lambda)x - (2+3\lambda) + (2+\lambda) = 0$

$(1-\lambda)^2 + \frac{(2+3\lambda)^2}{4} - (\lambda+2) = 13$

$4\lambda^2 + 4 - 8\lambda + 9\lambda^2 + 12\lambda + 4 - 4\lambda - 8 = 52$

$13\lambda^2 = 52$

$\lambda^2 = 4$

$\lambda = \pm 2$

$\boxed{x^2 + y^2 - 6x + 4\lambda = 0}$ $\boxed{x^2 + y^2 + 2x - 8y + 4 = 0}$

H W : 12-10-24

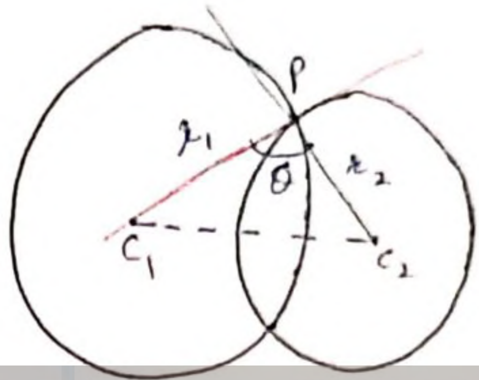
~~DYS - 8 (1, 9) double score~~

DYS - 8 R - {1, 9, 7, 6(c)} O-1 {12, 18, 19, 23, 27, 28, 29, 30}

DYS - 10 (9, 7) full

Angle Between Circles

→ The angle between the tangents of the two circles at POI at POI is called the angle of intersection of 2 circles.



$$\boxed{\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2 r_1 r_2}}$$ $d \to C_1C_2$

→ at 90°,

$\underline{r_1^2 + r_2^2 = d^2}$

Curves are orthogonal
↳ ⊥

~~$\cos\theta = \frac{g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2 - (g_1 - g_2)^2 + (f_1 - f_2)^2}{}$~~

For genera eq^n of circle

$r_1 = \sqrt{g_1^2 + f_1^2 - c_1}$ $\quad C_1(-g_1, -f_1)$

$r_2 = \sqrt{g_2^2 + f_2^2 - c_2}$ $\quad C_2(-g_2, -f_2)$

$d = \sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2}$

$r_1^2 + r_2^2 = d^2$

Hence, $g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2 = g_1^2 + g_2^2 - 2g_1g_2 + f_1^2 + f_2^2 - 2f_1f_2$

$-(c_1 + c_2) = -(2g_1g_2 + 2f_1f_2)$

$\boxed{c_1 + c_2 = 2(g_1g_2 + f_1f_2)}$ orthogonal

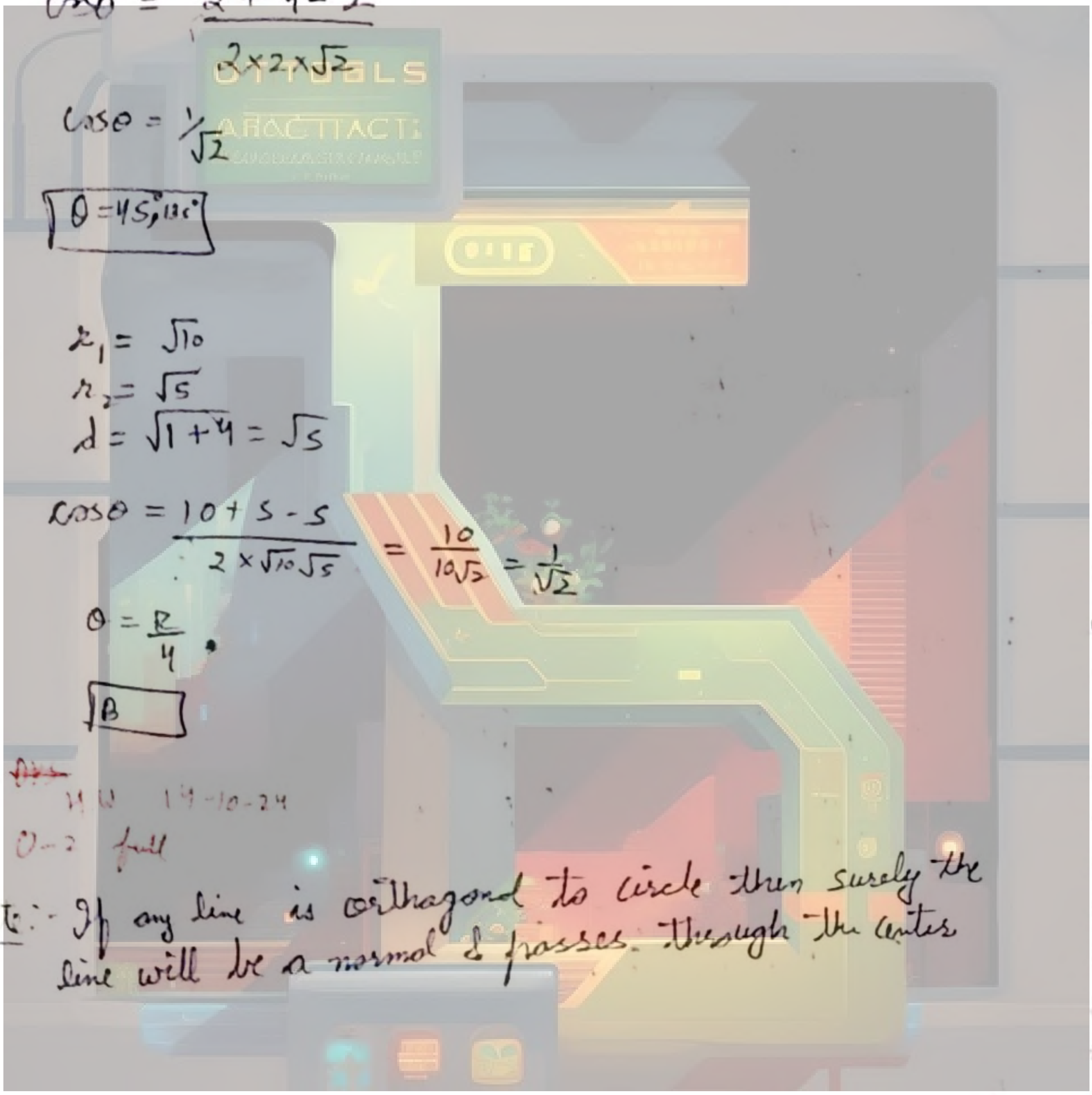
DYS-10

Q1.

$r_1 = \sqrt{2}$

$r_2 = 2$

$d = \sqrt{1+1} = \sqrt{2}$



$\cos\theta = \dfrac{2+4-2}{2\times 2\times\sqrt{2}}$

$\cos\theta = \dfrac{1}{\sqrt{2}}$

$\boxed{\theta = 45^\circ, 135^\circ}$

Q4.

$r_1 = \sqrt{10}$

$r_2 = \sqrt{5}$

$d = \sqrt{1+4} = \sqrt{5}$

$\cos\theta = \dfrac{10+5-5}{2\times\sqrt{10}\sqrt{5}} = \dfrac{10}{10\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$

$\boxed{B}$

~~Ans~~

H.W 14-10-24

Q-2 full

Note:- If any line is orthagonal to circle then surely the line will be a normal & passes through the center

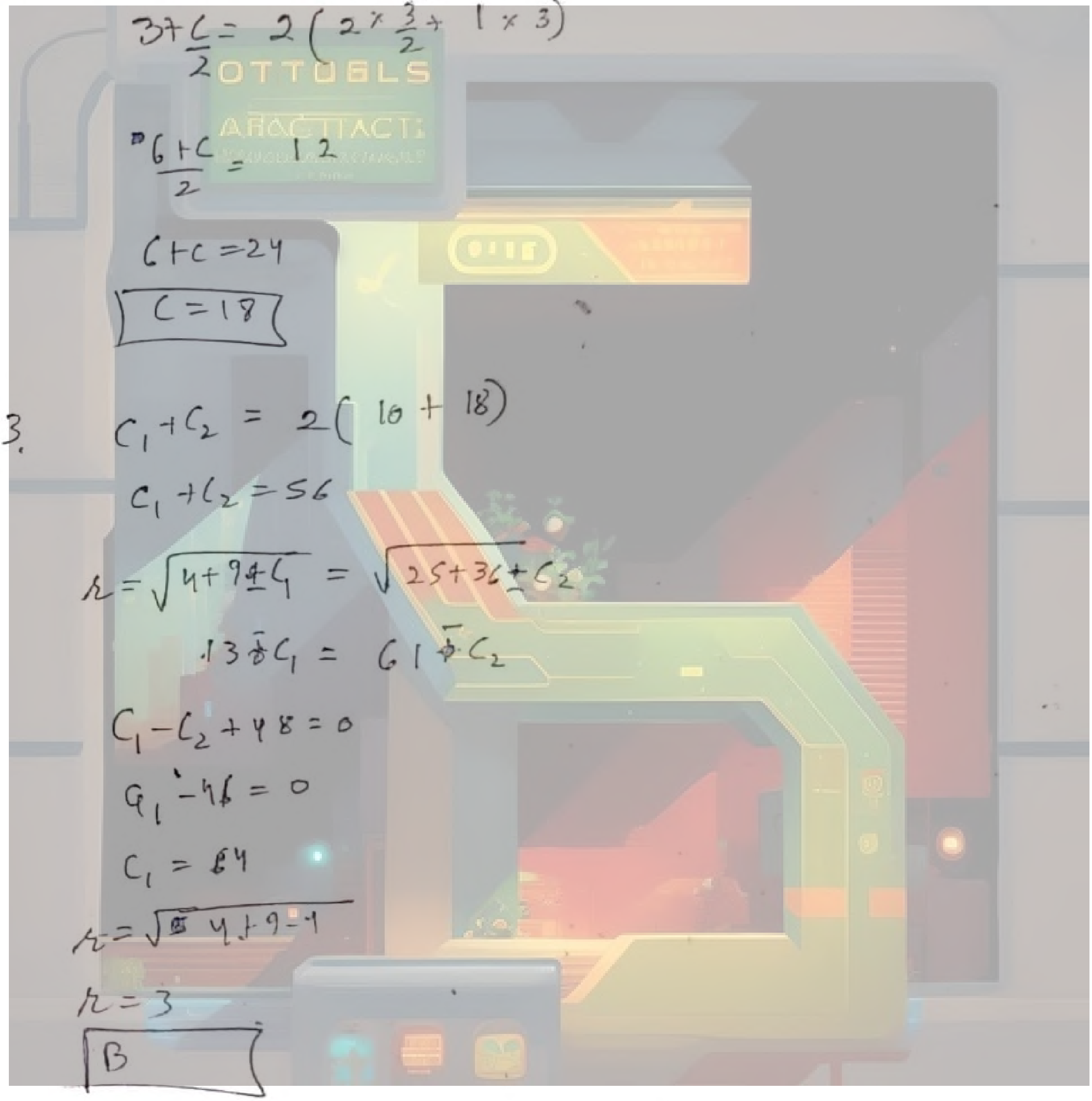
11YS-11

Q1. $\cancel{3+c = 2(2\times 3/2+1)}$

$\cancel{3+c = 8}$

$\cancel{\boxed{c=5}}$

Q1. $c_1+c_2 = 2(g_1g_2+f_1f_2)$



$3+\frac{c}{2} = 2(2\times\frac{3}{2}+1\times 3)$

$\frac{6+c}{2}=12$

$6+c=24$

$\boxed{c=18}$

Q3. $c_1+c_2=2(10+18)$

$c_1+c_2=56$

$r=\sqrt{4+9-c_1}=\sqrt{25+36-c_2}$

$13-c_1=61-c_2$

$c_1-c_2+48=0$

$c_1-4=0$

$c_1=4$

$r=\sqrt{4+9-4}$

$r=3$

$\boxed{B}$

Q.5.

$x^2+y^2+2gx+2fy+c=0$

$c+11=2(-2g-3f)$ and $c+21=2(-5g-2f)$

$-2g-3f-7=0$

$2g+3f+7=0$

$c+11=2(7)$

$\boxed{c=3}$

$24=-10g-4f$

$10g+4f+24=0$

$10g+15f+35=0$

$11f+11=0$

$f=-1$

$g=-2$

$$\boxed{x^2+y^2-4x-2y+3=0}$$

~~Q 6.~~ $\cancel{(g, f) \text{ on } x - y - 3x + 2y - 6 = 0}$

Q.7. $x^2+y^2+2gx+2fy+c=0$

$c_1-4=2(0,0)$

$c_1=4$

$-2g+2f+9=0$

$2f=2g-9$

$x=\frac{2y-9}{2}$

$x^2=\frac{4y^2+81-36}{4}$

$x^2+y^2+2gx+2gy-9y+4=0$

$\cancel{4y^2+81-36y+4y^2+}$

$x=-g$ (to cut g)

$x^2 + x^2 + 9x + 4 = 0$

$2x^2 + 9x + 4 = 0$

$x = \dfrac{-9 \pm \sqrt{81-32}}{4}$

$x = \dfrac{-9 \pm 7}{4}$

$x = -\dfrac{1}{2}, -4$

$y = -\dfrac{1}{2}, 4$

$\boxed{\left(-\dfrac{1}{2}, -\dfrac{1}{2}\right) \ (-4, 4)}$

Q8. a) $x^2 + y^2 + 2gx + 2fy + c = 0$

$gf + 3g - 2f - 6 = 0$

$g(f+3) - 2(f+3) = 0$

$g = 2, f = -3$

$\boxed{x^2 + y^2 + 4x - 6y = 0}$

$+8 = 2(-2k + 6k)$

$8 = 8k$

$\boxed{k = +1}$

* Radical Axis & Radical Centre

→ It is the locus of a point which moves such that the length of tangent from it is same for given circles.

$$\boxed{\sqrt{S_1} = \sqrt{S_2}}$$ Eqn of radical axis.

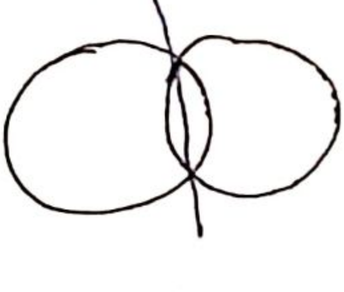
$$\boxed{S_1 - S_2 = 0}$$

Note :-

① when circles Touch each other

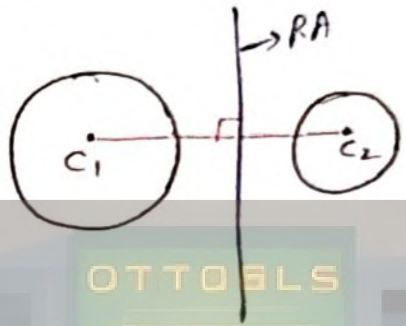
Radical Axis ⇒ Common Tangent

② Intersecting each other.

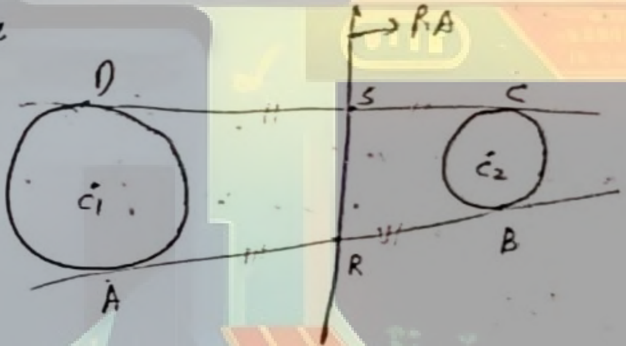


RA ⇒ common chord

③ Radical Axis is always $\perp$ to the line joining centers of the circle.

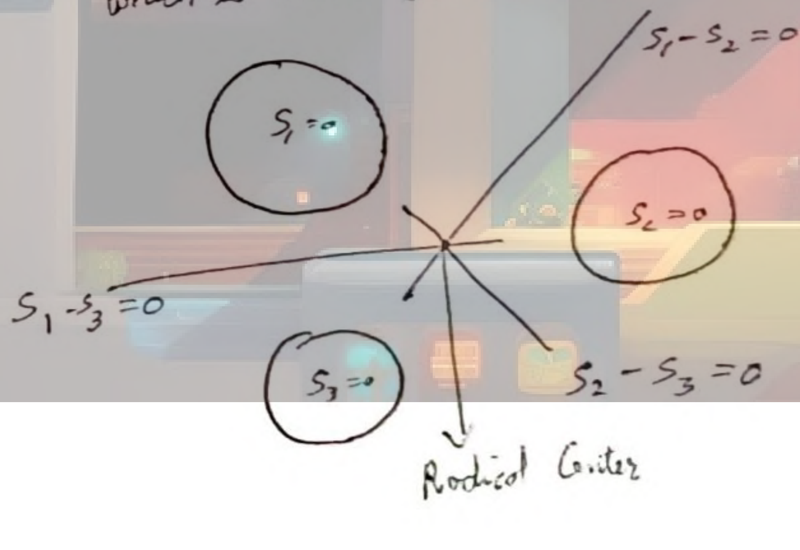


④ Radical Axis ~~bi~~ bisects only common tangents to two circle

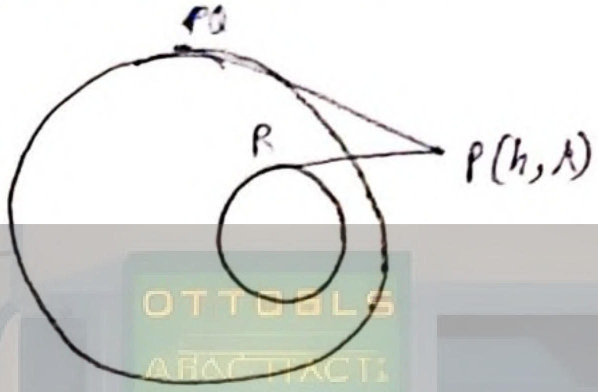


$DS = SC$
$AR = BR$

⑤ Radical Axes of 3 circles taking 2 at a time meet at a point which is called radical center.



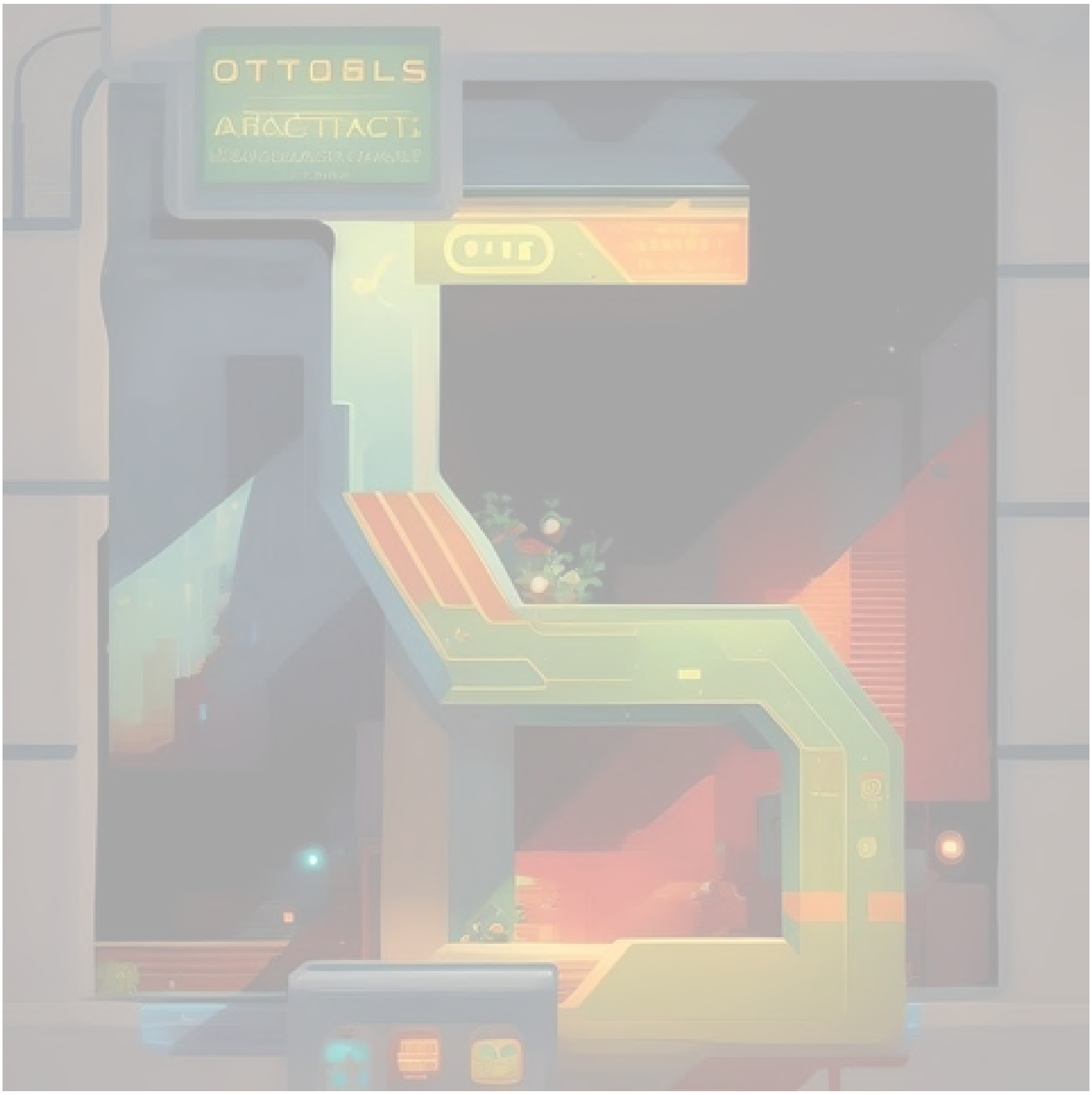
(6) If one circle lies in another circle (but not concentric) then radical axis exists

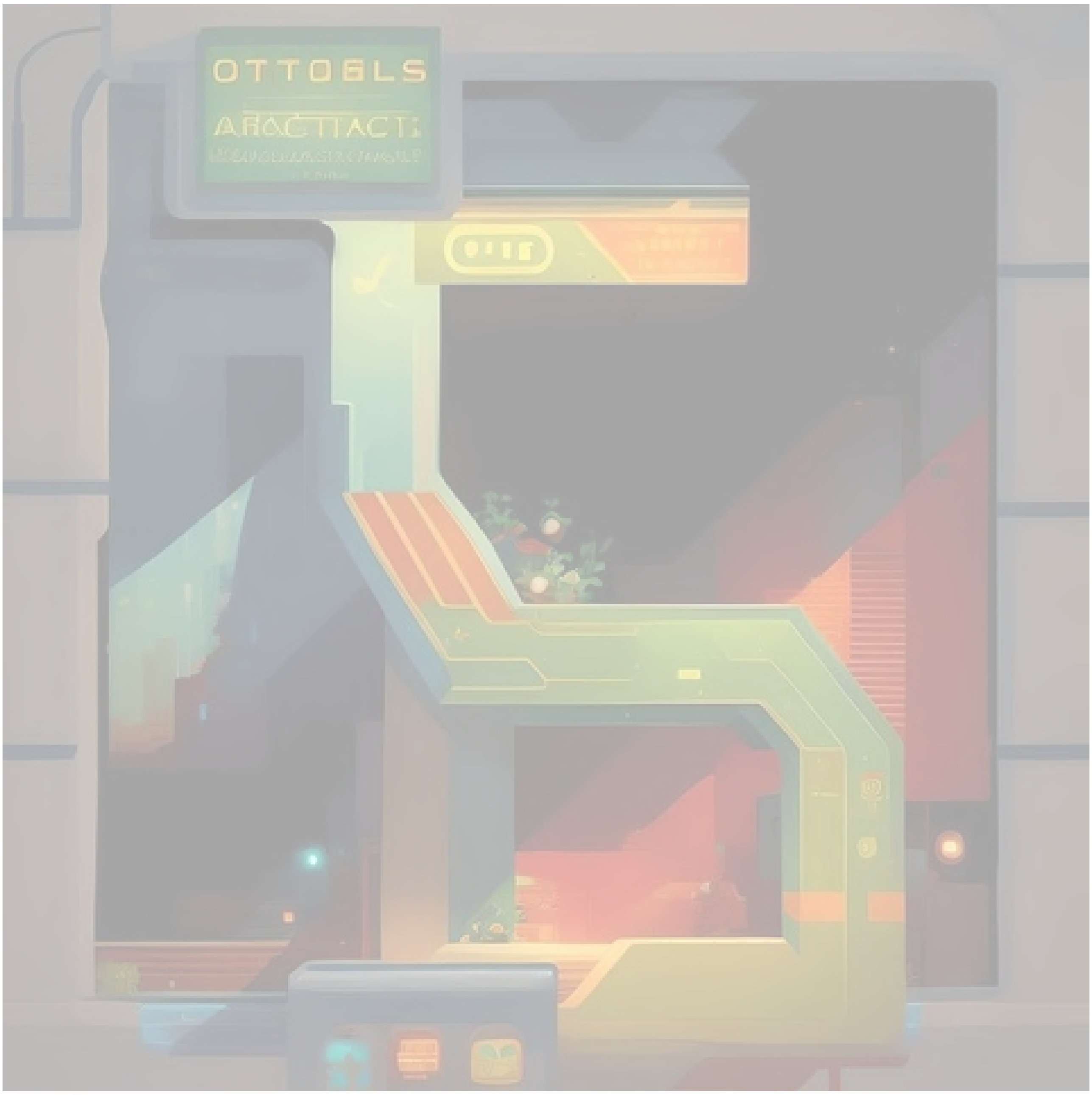


PQ can be equal to PR

→ not possible when concentric.

(7) When two circles are orthogonal to the 3rd circle then radical axis of both circles will pass through the center of the 3rd circle.

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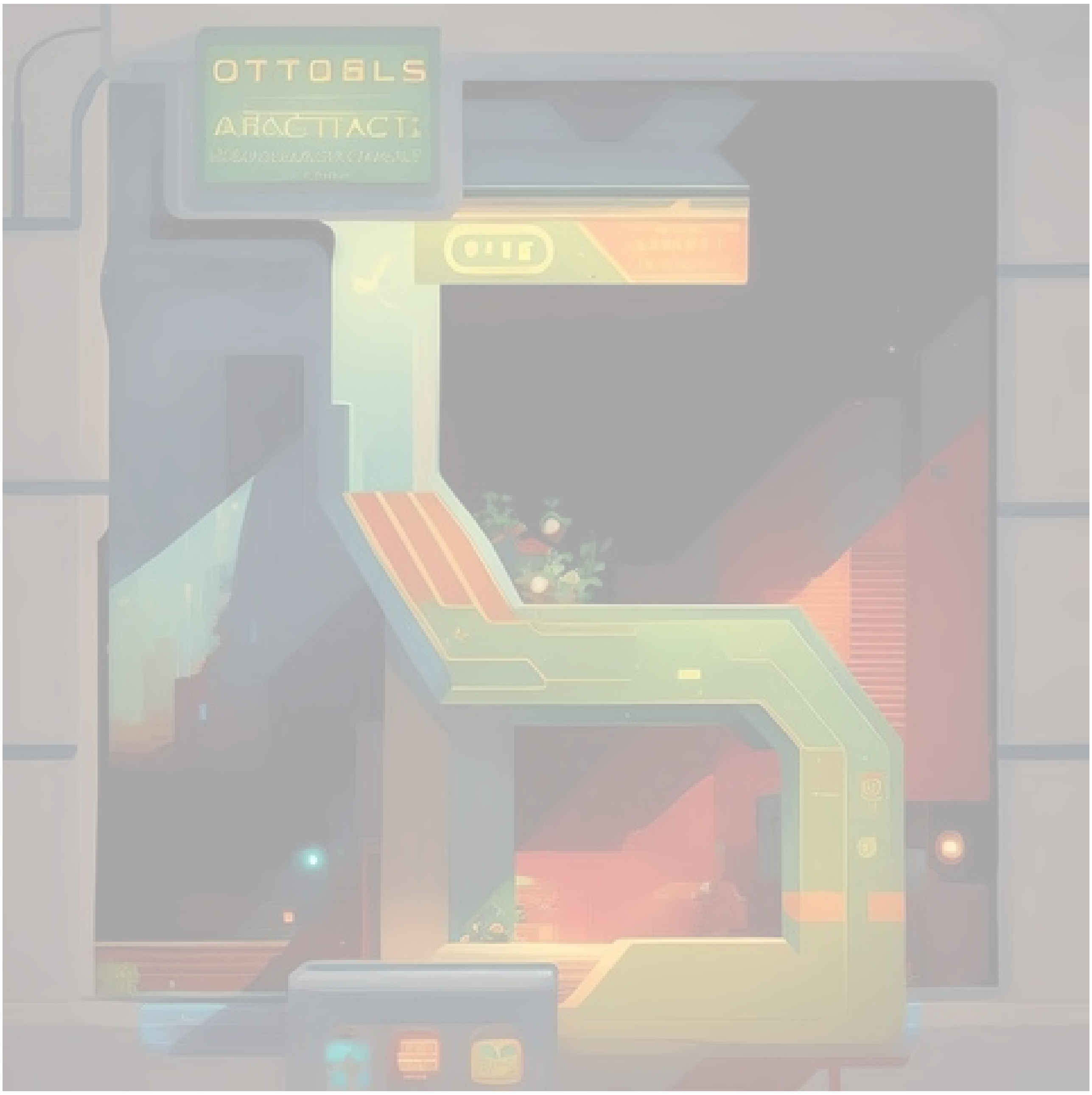

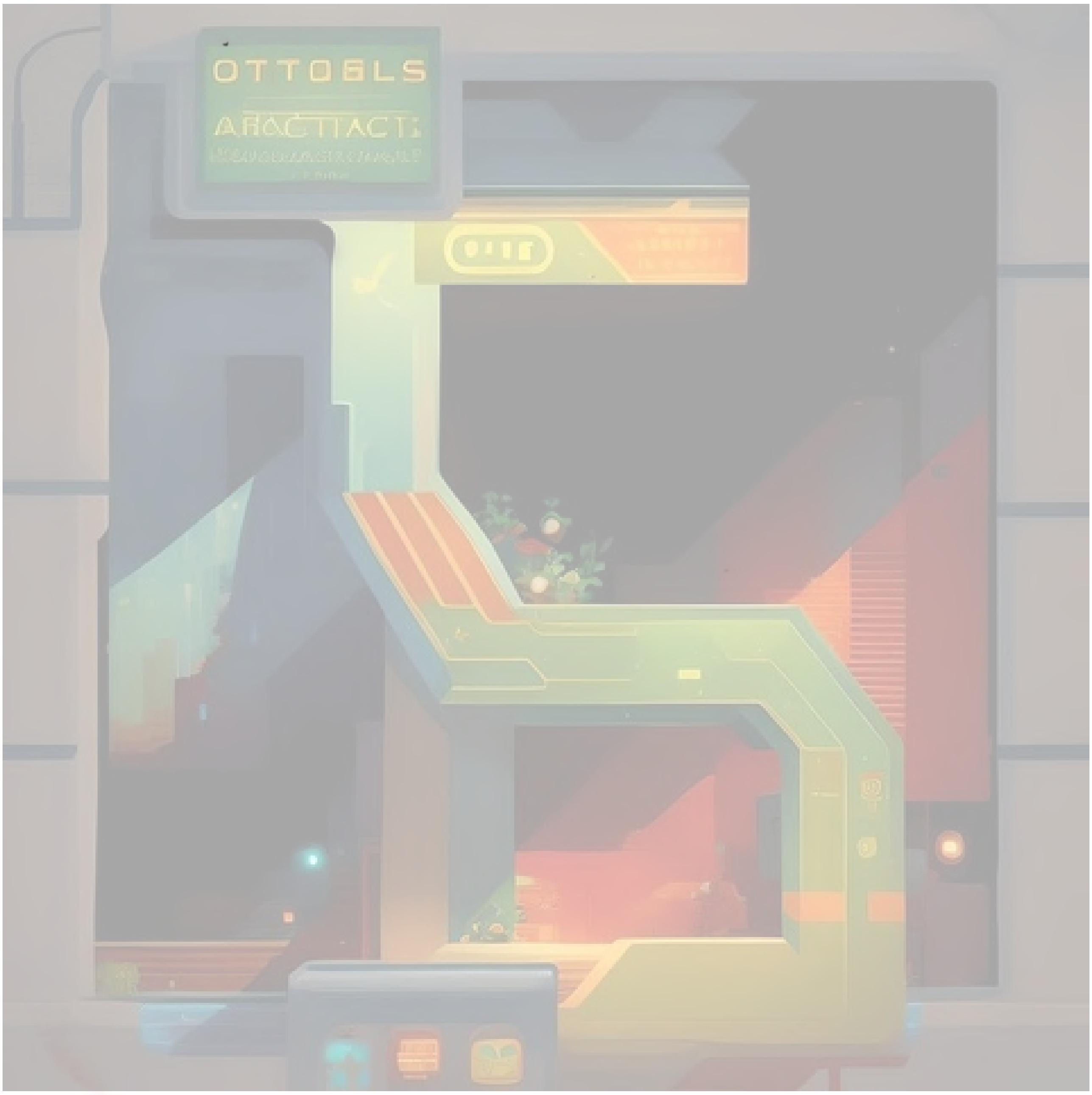
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