Maths-2

Ch-3 Quadratic Equation

Ch-4 Logarithm

Ch-5 Sequence & Series

Ch-6 Trigonometry

15-2 (ch-3)

Q24. $x_1 = 9x_2$

$x_1 + x_2 = 3a+2$

$10x_2 = 3a+2$

$x_2 = \frac{3a+2}{10}$

$x_1 x_2 = a^2$

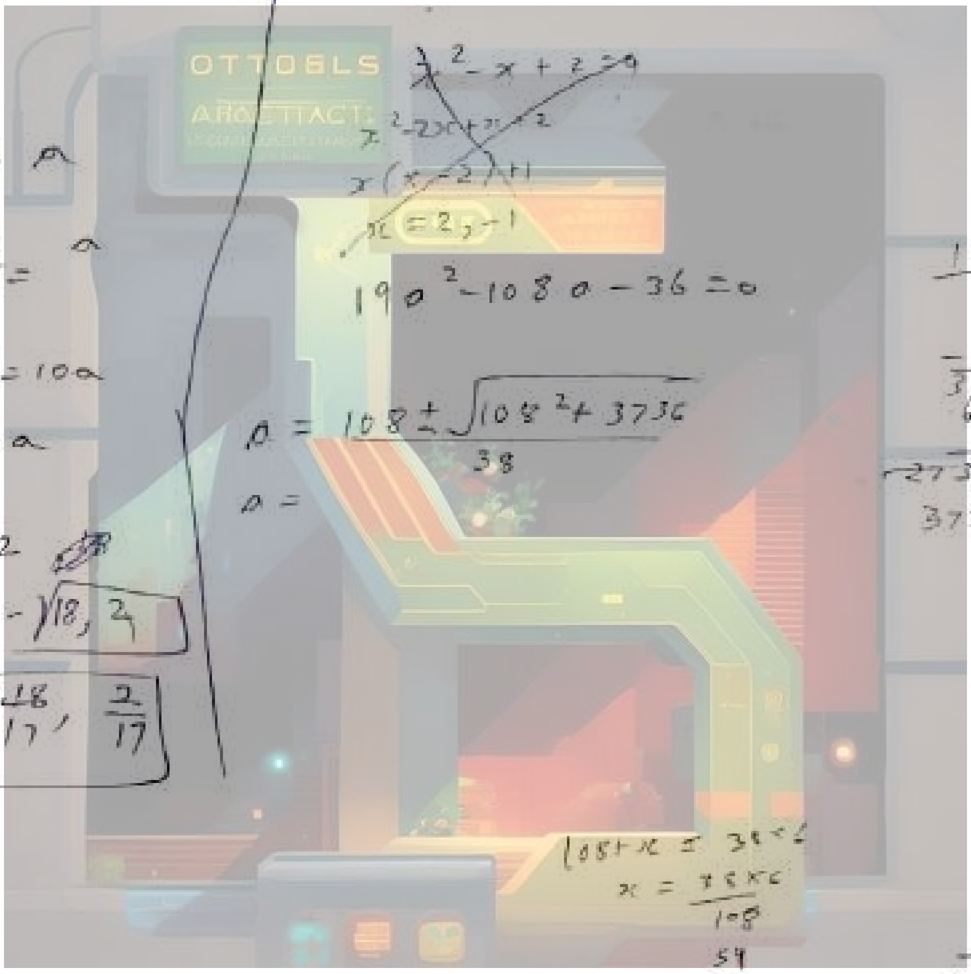
$\cancel{(3x_2)^2 = a^2}$

$\cancel{3x_2 = a}$

$\cancel{9 \times \frac{9a^2+4+12a}{100} = a^2}$

$\cancel{81a^2+36+108a = 100a^2}$

$\cancel{19a^2-108a-36=0}$



$\cancel{x^2-x+2=0}$

$\cancel{x^2-2x+x+2}$

$\cancel{x(x-2)+1}$

$\cancel{x=2,-1}$

$19a^2-108a-36=0$

$a = \frac{108 \pm \sqrt{108^2+3736}}{38}$

$a =$

$3x_2 = a$

$\frac{9a+6}{10} = a$

$9a+6 = 10a$

$6 = a$

$x_2 = 2$

roots $- \boxed{18, 2}$

$\boxed{-\frac{18}{17}, \frac{2}{17}}$

$108 \times x = 38 \times 6$

$x = \frac{38 \times 6}{108}$

54

$\begin{array}{r} 108 \\ \times 108 \\ \hline 864 \\ 10800 \\ \hline \end{array}$

36

19

720

$\begin{array}{r} 720 \\ - 36 \\ \hline 684 \\ \end{array}$

11664

4

$\begin{array}{r} 2736 \\ 3736 \\ \end{array}$

11664

34

$\begin{array}{r} 65 \\ \times 5 \\ \hline 325 \end{array}$

$\begin{array}{r} 63 \\ \times 3 \\ \hline 189 \end{array}$

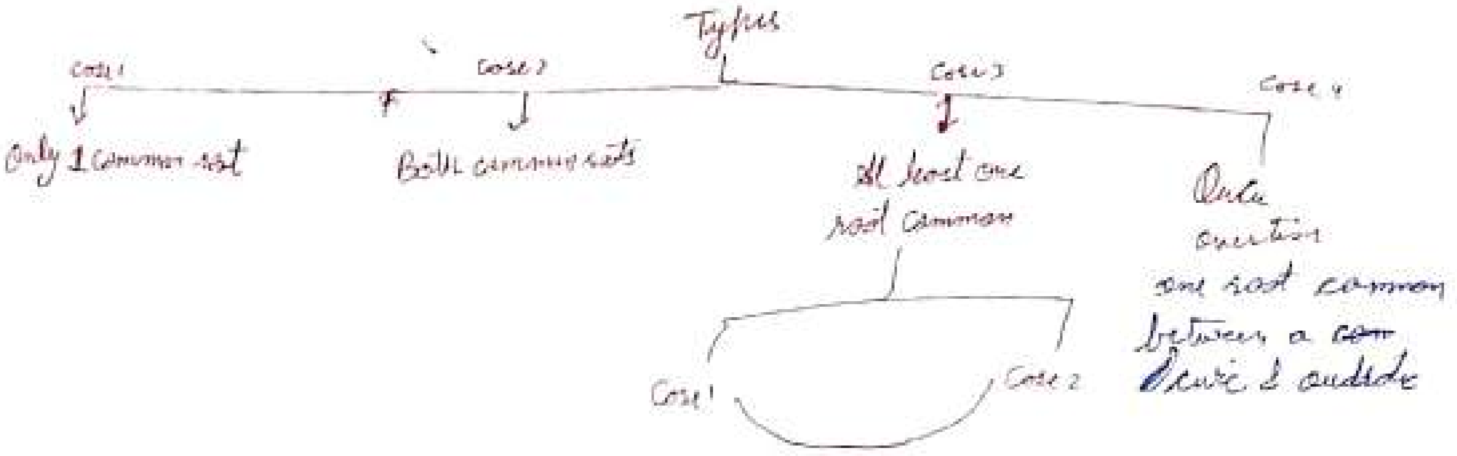
$\begin{array}{r} 64 \\ \times 4 \\ \hline 256 \end{array}$

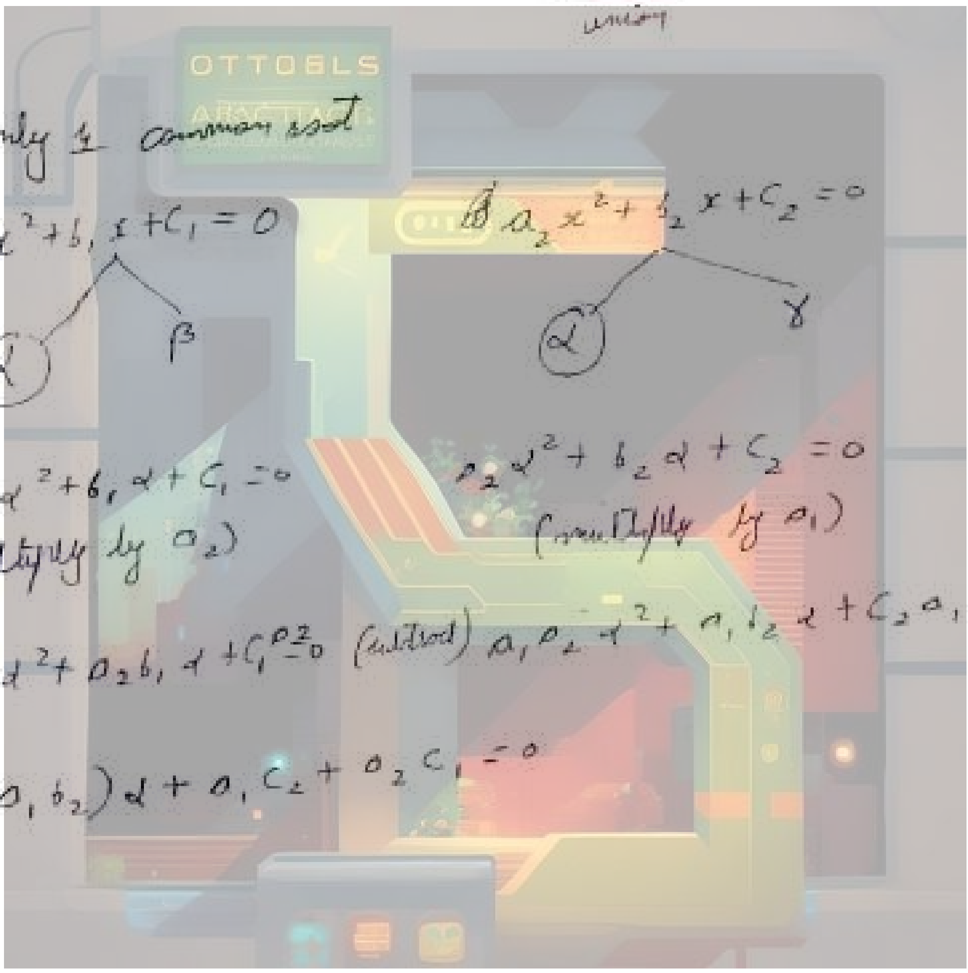
~~104~~

104

3

Condition for common roots





Case 1 → Only 1 common root

$a_1x^2+b_1x+c_1=0$ ⟶ α, β $a_2x^2+b_2x+c_2=0$ ⟶ α, γ

∴ $a_1\alpha^2+b_1\alpha+c_1=0$ (multiply by a_2) $a_2\alpha^2+b_2\alpha+c_2=0$ (multiply by a_1)

$a_1a_2\alpha^2+a_2b_1\alpha+c_1a_2=0$ (subtract) $a_1a_2\alpha^2+a_1b_2\alpha+c_2a_1=0$

$(a_2b_1-a_1b_2)\alpha+a_1c_2+a_2c_1=0$

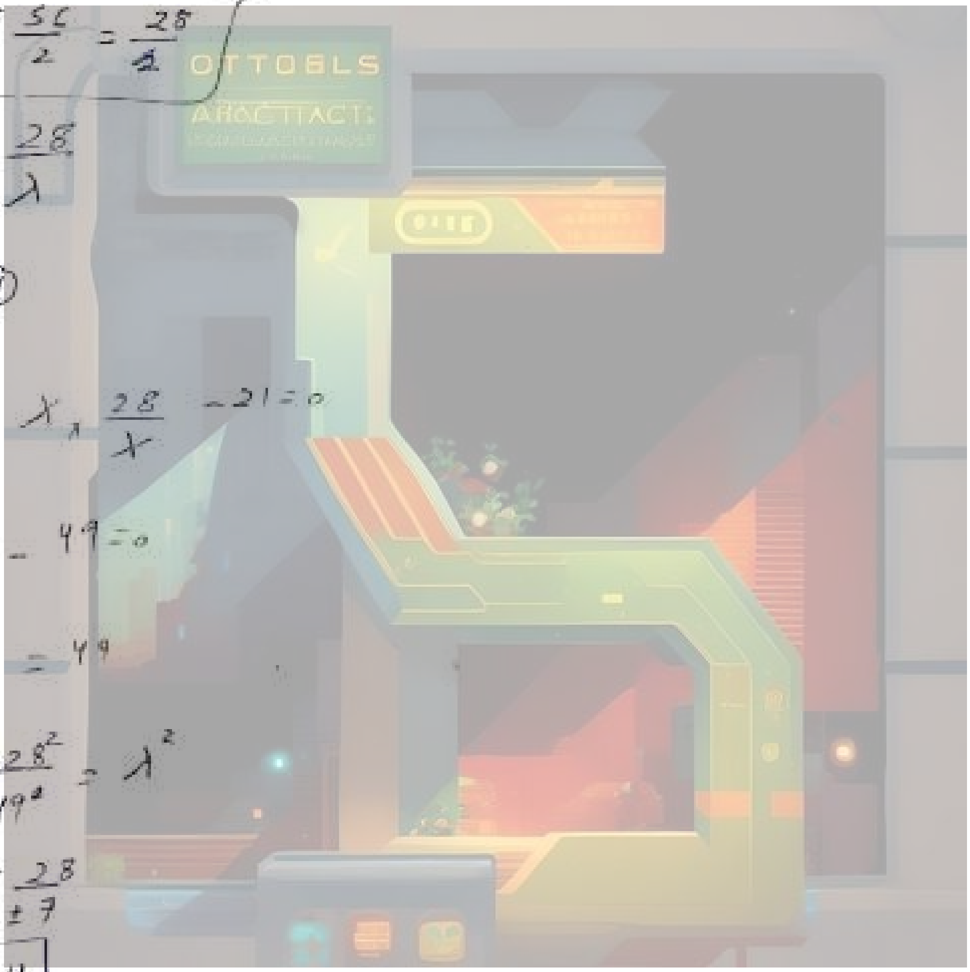
Q find λ if $x^2-\lambda x-21=0$ & $x^2-3\lambda x+35=0$
have one root common.

$\alpha^2-\cancel{\alpha}\lambda\alpha-21=0$ --① $\alpha^2-3\lambda\alpha+35=0$ -- ②

$\cancel{2\alpha^2}$: ② − ①

$-2\lambda\alpha+56=0$

$2\lambda\alpha=56$



$\boxed{\lambda\alpha=\frac{56}{2}=\frac{28}{1}}$

$\alpha=\frac{28}{\lambda}$

put in ①

$\left(\frac{28}{\lambda}\right)^2-\lambda\times\frac{28}{\lambda}-21=0$

$\frac{(28)^2}{\lambda^2}-49=0$

$\frac{28}{\lambda^2}=49$

$\frac{\cancel{28^2}\,28^2}{49}=\lambda^2$

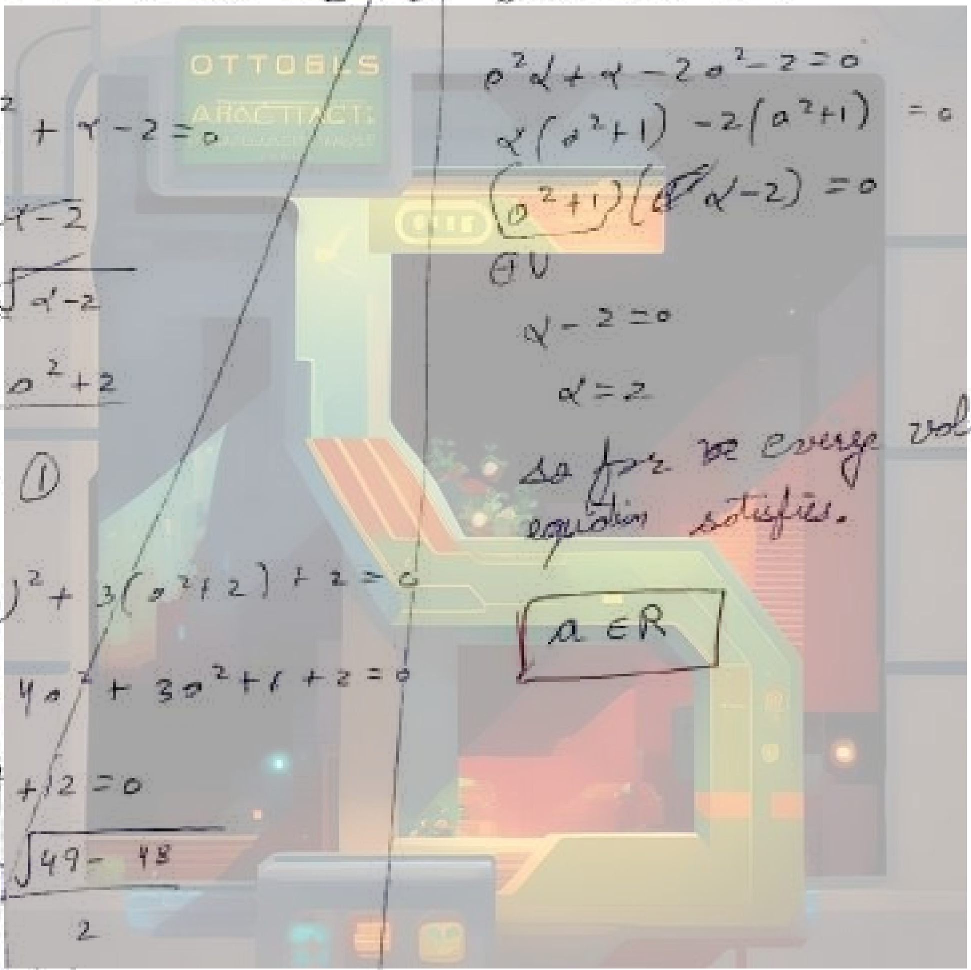
$\lambda=\frac{28}{\pm7}$

$\boxed{\lambda=\pm4}$

Q find a if $x^2+(a^2-2)x-2a^2=0$ & x^2-3x+2 have only one root common.

$\alpha^2+(a^2-2)\alpha-2a^2=0$ ② $\alpha^2-3\alpha+2=0$ ①

② − ①



$a^2\alpha-2\alpha+3\alpha-2a^2-2=0$

$-a^2+\alpha-2=0$

$a^2=\alpha-2$

$a=\sqrt{\alpha-2}$

$\alpha=a^2+2$

Put in ①

$(a^2+2)^2+3(a^2+2)+2=0$

$a^4+4+4a^2+3a^2+6+2=0$

$a^4+7a^2+12=0$

$a^2=\frac{-7\pm\sqrt{49-48}}{2}$

$a^2=\frac{-7\pm1}{2}$

$a^2=-4,-3$

X

$a^2\alpha-2\alpha+3\alpha-2a^2-2=0$

$a^2\alpha+\alpha-2a^2-2=0$

$\alpha(a^2+1)-2(a^2+1)=0$

$(a^2+1)(\alpha-2)=0$

⊖V

$\alpha-2=0$

$\alpha=2$

so for ~~xe~~ every value of a, equation satisfies.

$\boxed{a\in R}$

① If ~~o~~ $x^2 + P~~o~~x + Q = 0$ & $x^2 + Qx + P = 0$

$P \neq Q$ and one common root.

$\alpha^2 + ~~P~~ P\alpha + Q = 0$ ~~D~~ $\alpha^2 + Q\alpha + P = 0$

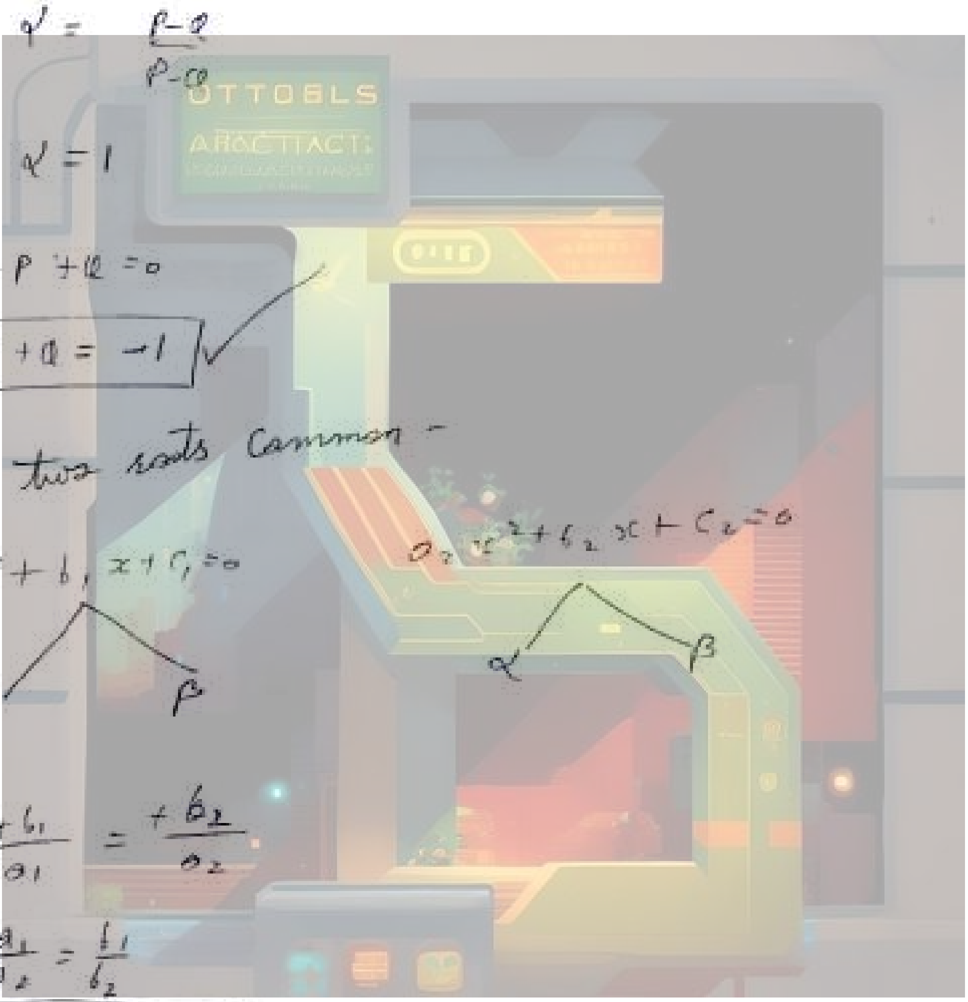
② − ①

$P\alpha - Q\alpha + Q - P = 0$

$(P - Q)\alpha + Q - P = 0$

$P - Q\alpha = P - Q$

$\alpha = \frac{P-Q}{P-Q}$



$\alpha = 1$

$(1)^2 + P + Q = 0$

$\boxed{P + Q = -1}$ ✓

Case - 2 two roots Common -

$a_1x^2 + b_1x + c_1 = 0$ $a_2x^2 + b_2x + c_2 = 0$

α β α β

$\alpha + \beta = \frac{+b_1}{a_1} = \frac{+b_2}{a_2}$

$= \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\alpha\beta = \boxed{\frac{c_1}{c_2} = \frac{a_1}{a_2} = \frac{b_1}{b_2}}$

Q If a, b, c ∈ R & eq $ax^2+bx+c=0$ & $x^2+2x+9=0$ have both roots as common, then find a : b : c

$\cancel{\frac{a}{1}=\frac{b}{2}=\frac{c}{9}}$

~~a : b : c~~

$\cancel{\frac{18a}{9}}$

$\cancel{\frac{18a}{15}=\frac{9b}{18}=\frac{2c}{18}}$

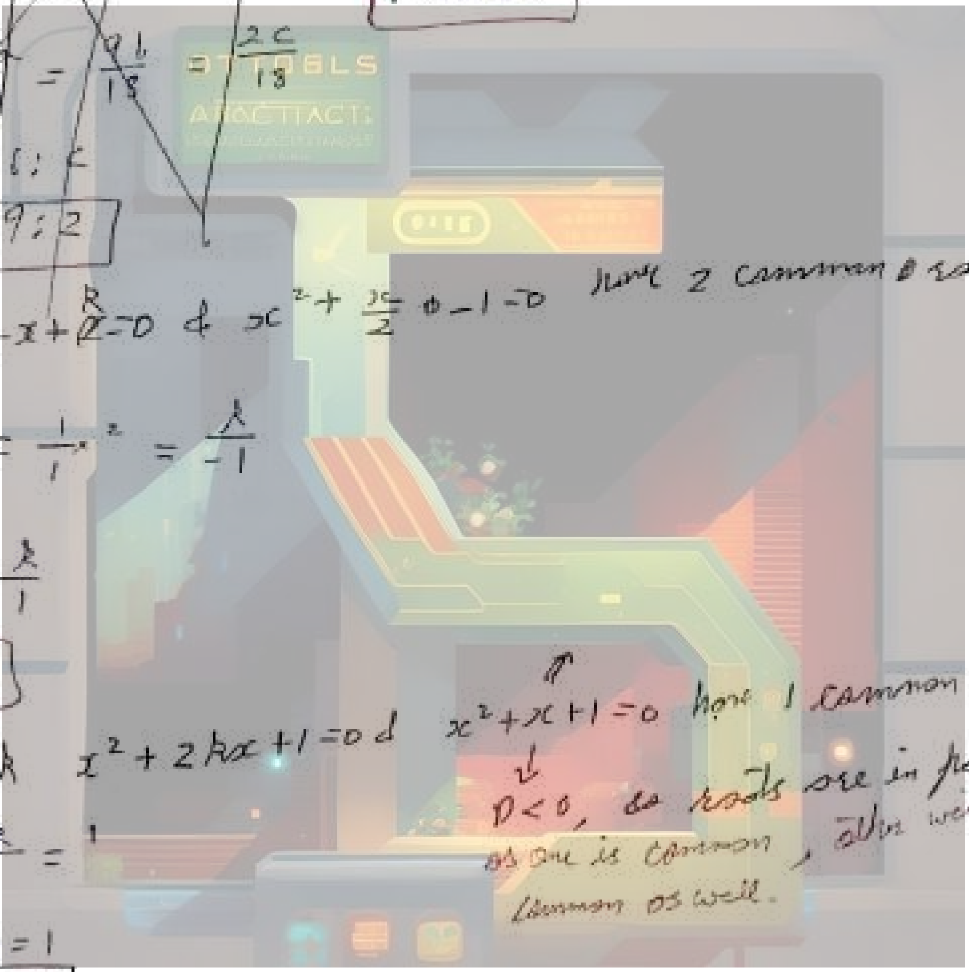
$\cancel{a : b : c}$

$\cancel{\boxed{18 : 9 : 2}}$

$\frac{1}{a}=\frac{2}{b}=\frac{9}{c}$

a : b : 9

$\boxed{1 : 2 : 9}$



Q. $2x^2+x+k=0$ & $x^2+\frac{x}{2}-1=0$ have 2 common roots find k

$\frac{2}{1}=\frac{1}{1/2}=\frac{k}{-1}$

$\frac{2}{1}=-\frac{k}{1}$

$\boxed{-2=k}$

Q. find k $x^2+2kx+1=0$ & $\overset{\uparrow}{x^2+x+1=0}$ have 1 common root

$\downarrow$ D<0, so roots are in pair as one is common, other will be common as well.

$\frac{2k}{1}=1$

$2k=1$

$\boxed{k=\frac{1}{2}}$

Case 3 At least one root common.

Q Possible values of 'a' for which $x^2+ax+1=0$ & $x^2+x+a=0$ have at least one common root

Case 1 $x^2-x^2+ax-x+1-a=0$

$x(a-1)-1(a-1)=0$

$(a-1)(x-1)=0 \longrightarrow x-1=0$

$\boxed{a=1}$

$x=1$

$(1)^2+a(1)+1=0$

$a+2=0$

$\boxed{a=-2}$

Case 2 $\frac{1}{1}=\frac{a}{1}=\frac{1}{a}$

$\frac{1}{a}=1$

$a^2=1$

$a=\pm1$

$\boxed{a=-1}$ not satisfy

$\boxed{a=1}$

Case 1 $\cup$ case 2 $=$ $\boxed{a\in\{1,-2\}}$

Type 4 Cubic - one common root between a quadratic & cubic

Q $x^3-3x^2+(2k-1)x+3=0$ & $2kx+1-x^2=0$ have one common root. find k.

$x^3-3x^2+(2k-1)x+3-2kx-1+x^2=0$

$x^3-2x^2+(2k-1-2k)x+2=0$

$x^3-2x^2-x+2=0$

$\boxed{x=1}$

$x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$\boxed{x=2,-1,1}$

$2k+1-1=0$

$2k=0$

$k=0$

$-2k+1+1=0$

$-2k=-2$

$k=1$

$4k+1-4=0$

$k=\frac{3}{4}$

① H.W. 20-05-2024

DYS-4 {3, 4, 5, 6, 7}

O-1 {Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Q If the eq $ax^2+bx+c=0$ & $x^3+3x^2+3x+2=0$ have 2 common roots, then—

a) $a=b\neq c$

B) $a=-b=c$

c) $a\neq b\neq c$

✓D) $a+b+c=3$

$-1+3-3+2$

$-8+12-6+2$

$(x+2)=0$

x^2+x+1

$x=\frac{-1\pm\sqrt{1-4}}{2}$

x^2+x+1

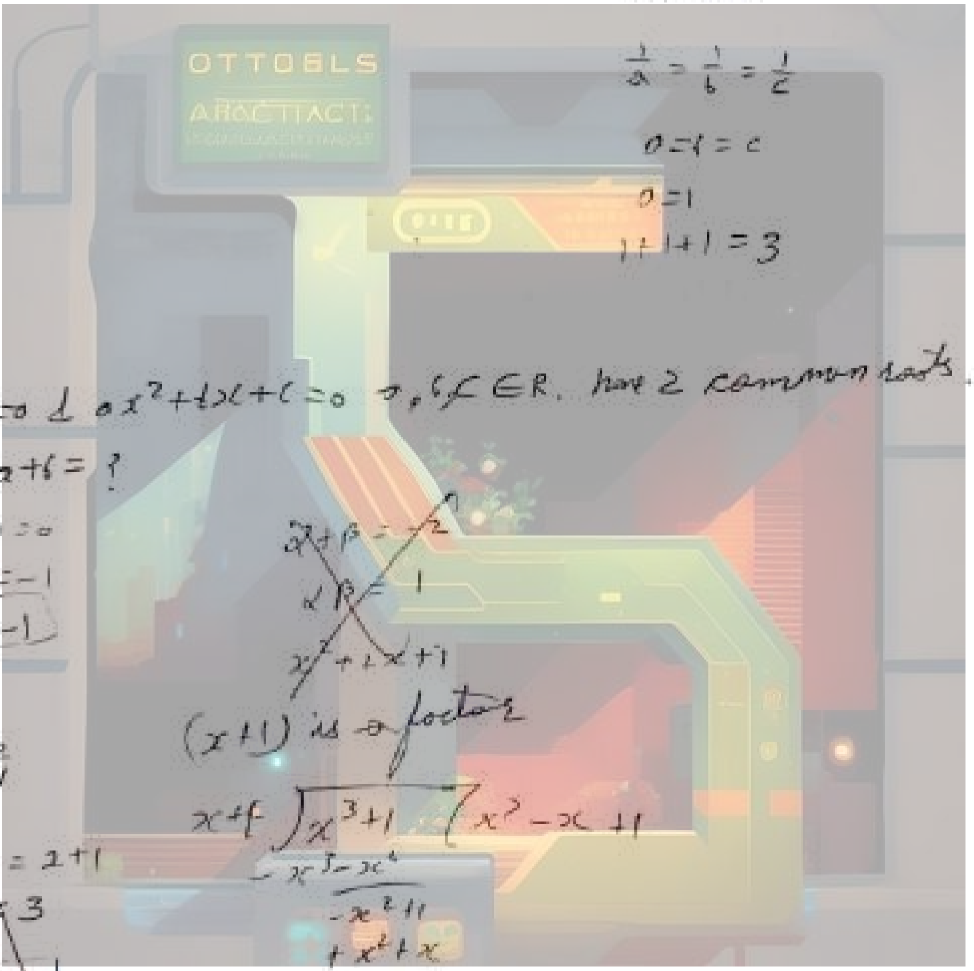
ax^2+bx+c

$\frac{1}{a}=\frac{1}{b}=\frac{1}{c}$

$a=b=c$

$a=1$

$1+1+1=3$



① If $x^3+1=0$ & $ax^2+bx+c=0$ $a,b,c\in R$. have 2 common roots. then $a+b=$?

$x^3+1=0$

$x^3=-1$

$\boxed{x=-1}$

~~$\alpha+\beta=-2$~~

~~$\alpha\beta=1$~~

~~x^2+2x+1~~

$(x+1)$ is a factor

$x+1\,\overline{)\,x^3+1\,}\,(x^2-x+1$

$-x^3-x^2$

$-x^2+1$

$+x^2+x$

$x+1$

~~$b=2$~~

~~$c=1$~~

~~$a=1$~~

~~$a+b=2+1$~~

~~$=3$~~

~~$\boxed{D}$~~

x^2-x+1

$x=\alpha\beta<0$

$x^2-x+1=ax^2+bx+c$

$a=1$

$b=-1$

$\boxed{a+b=0}$

$\boxed{B}$

Quadratic expression and its graphs.

$ax^2+bx+c \ (a,b,c \in R), a \neq 0$

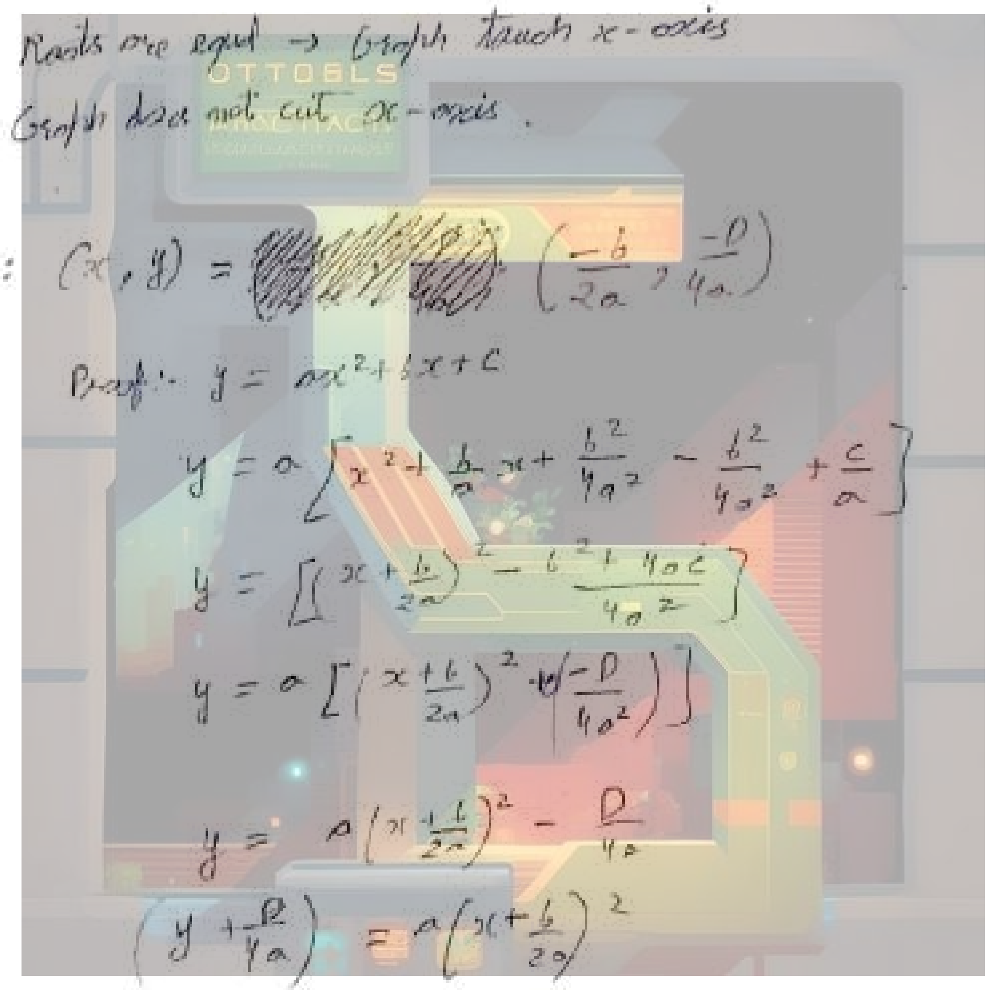
① $a<0$ concave down

$a>0$ concave up

② $D>0$ Roots real & unequal → Graph cuts x axis at 2 diff points

$D = 0$ Roots are equal → Graph touch x-axis

$D<0$ Graph does not cut x-axis.



③ Vertex : $(x, y) =$ [redacted] $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

Proof:- $y = ax^2+bx+c$

$$y = a\left[x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right]$$

$$y = \left[\left(x+\frac{b}{2a}\right)^2 - \frac{b^2+4ac}{4a^2}\right]$$

$$y = a\left[\left(x+\frac{b}{2a}\right)^2 + \left(\frac{-D}{4a^2}\right)\right]$$

$$y = a\left(x+\frac{b}{2a}\right)^2 - \frac{D}{4a}$$

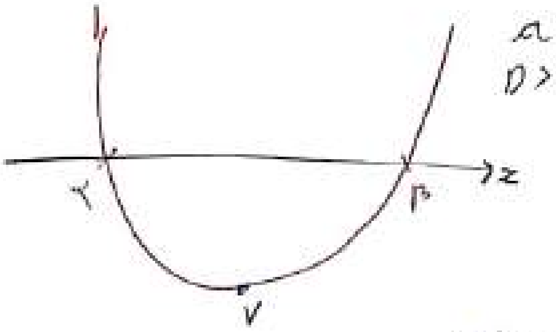
$$\left(y+\frac{D}{4a}\right) = a\left(x+\frac{b}{2a}\right)^2$$

$$\boxed{x = \frac{-b}{2a}}$$

$$\boxed{y = \frac{-D}{4a}}$$

④ Graphs

①



$a > 0$
$D > 0$

$y = \oplus, \ominus, 0$

②

$a > 0$
$D = 0$

$y = \oplus, 0$

③

$a > 0$
$D < 0$

$y = \oplus$

④

$a < 0$
$D > 0$

$y = \oplus, \ominus, 0$

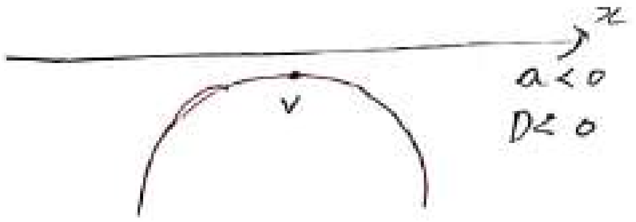
⑤

$a < 0$
$D = 0$

$y = \ominus, 0$

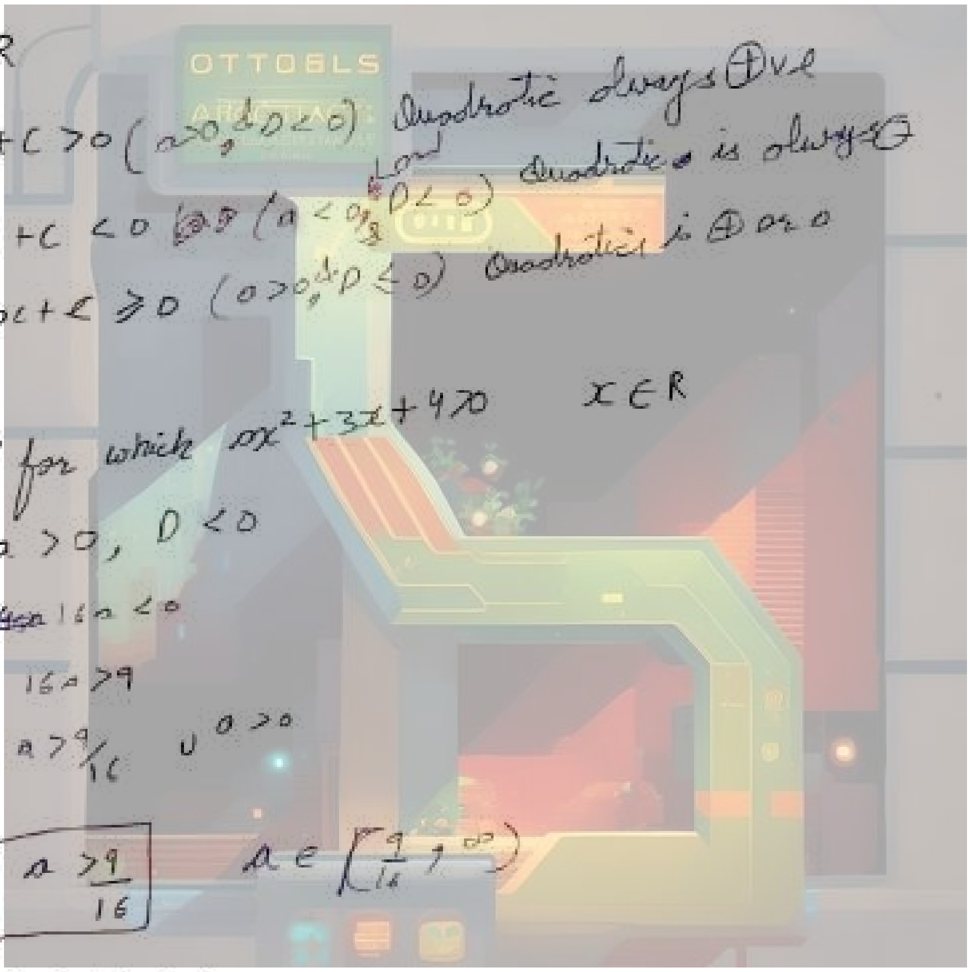
⑤

⑥



$y = ⊖$

$a<0$
$D<0$



$a, b, c \in R$

① $ax^2+bx+c>0$ ($a>0$, & $D<0$) Quadratic always ⊕ve

② $ax^2+bx+c<0$ ~~$a>0$~~ ($a<0$, and $D<0$) Quadratic is always ⊖

③ $ax^2+bx+c \geqslant 0$ ($a>0$, & $D \leq 0$) Quadratic is ⊕ or 0

Q Find 'a' for which $ax^2+3x+4>0$ $\quad x \in R$

$a>0, \; D<0$

$9-$ ~~$4a$~~ $16a<0$

$16a>9$

$a>\frac{9}{16} \cup a>0$

$\boxed{a>\frac{9}{16}}$ $\quad a \in \left[\frac{9}{16}, \infty\right)$

Q. $ax^2+2ax+\frac{1}{2}<0$

$a<0, \; D<0$

$b^2-4ac<0$

~~2~~$4a^2-2a<0$

$2a(2a-1)<0$

$a \in (0, 1/2) \cup a<0$

$\boxed{a \in \phi}$

⑫

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Q $kx^2 + x + k > 0$

$a > 0$
$D < 0$

$1 - 4k^2 < 0$

~~$4k^2 > 1$~~
~~$k^2 > \frac{1}{4}$~~
~~$k > \frac{1}{2}$~~

$4k^2 - 1 > 0$

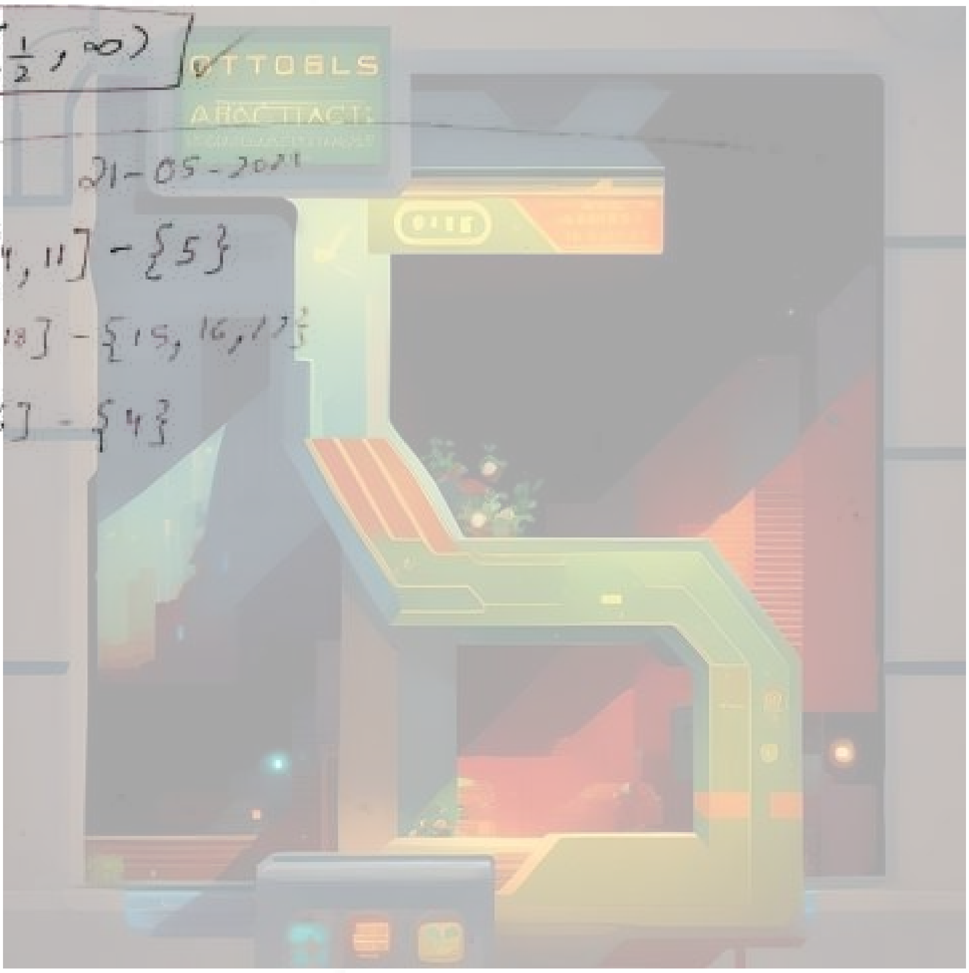
$+ \quad -1/2 \quad k^2 \quad - \quad 1/2 \quad +$

$k \in \left[(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)\right] \cap (0, \infty)$

$\boxed{k \in (\frac{1}{2}, \infty)}$ ✓

H.W. 21-05-2021

DYS-8 [4, 11] − {5}
O-1 [11, 18] − {15, 16, 17}
O-2 [1, 6] − {4}



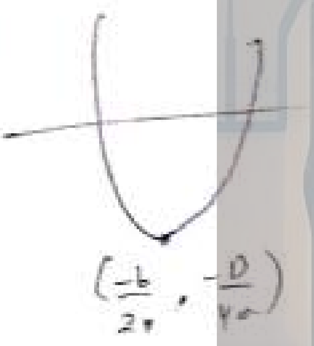
Range of A Quadratic (values of y)

Range ∈ $[y_{min}, y_{max}]$

Type 1:- $x \in R$

$a > 0$

Range ∈ $\left[\frac{-D}{4a}, \infty\right)$



$a < 0$

Range ∈ $\left(-\infty, \frac{-D}{4a}\right]$

Type-2:- x is restricted.

Case 1

when $x = \frac{-b}{2a}$ lies in $[x_1, x_2]$

$f\left(\frac{-b}{2a}\right), f(x_1), f(x_2)$

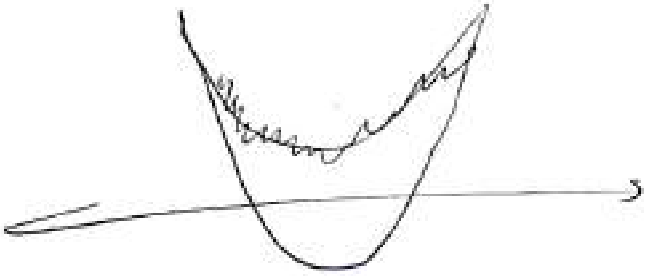
Case 2:

when $x = \frac{-b}{2a}$ dont lie in $[x_1, x_2]$

check :- $f(x_1)$, & $f(x_2)$

⑭

Q Draw the graph of $x^2 - 5x + 6 = 0$
① find Minimum value & point where min value occurs.
② Range of quadratic.



① min^m value $= \frac{-D}{4a}$ $= \frac{-1}{4}$ $\boxed{= -1/4}$

$x_{min} = \frac{5}{2}$

$\boxed{x_{min} = \frac{5}{2}}$

② Range = $[y_{min}, y_{max}]$

$y_{min} = -1/4$

$y_{max} = \infty$

$\boxed{\text{Range } [-1/4, \infty)}$

Q Draw graph of $x^2 + x + 1 = 0$
① find min value & Point
② Range

15

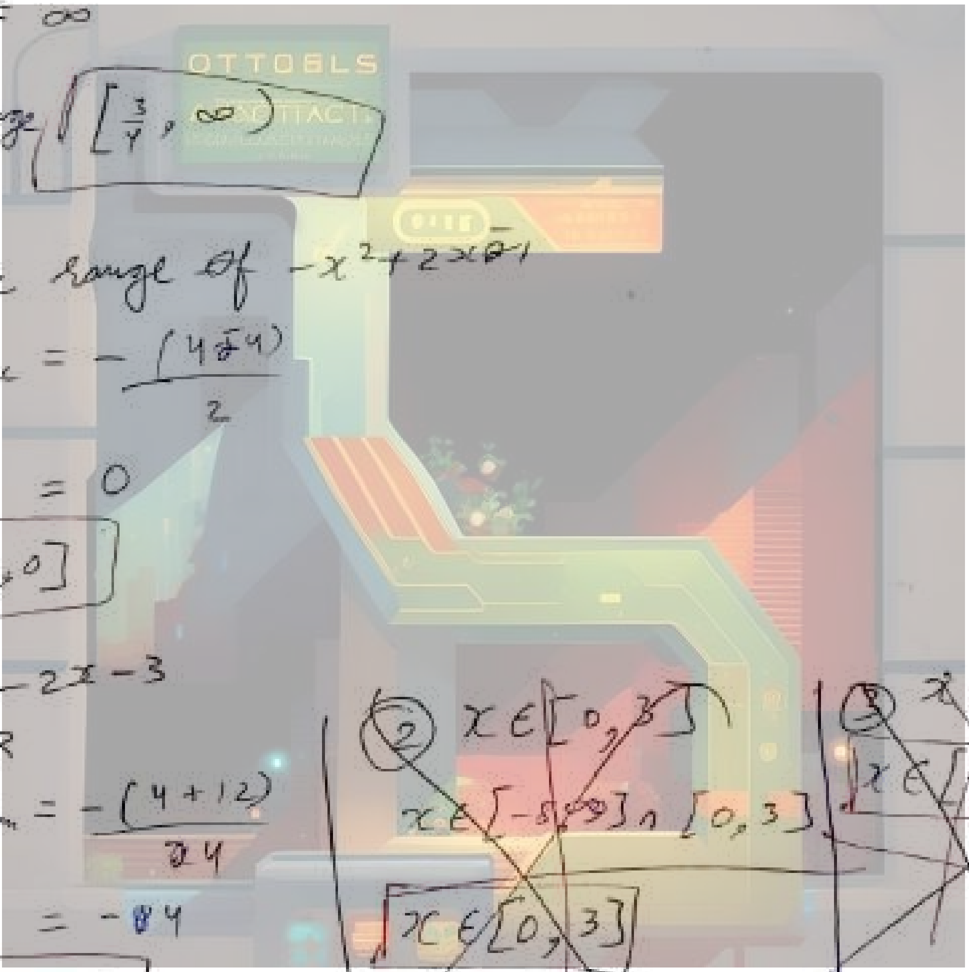
① $y_{min} = \frac{-D}{4a}$

$= \frac{4-1}{4}$

$\boxed{= \frac{3}{4}}$

$x_{min} = \boxed{\frac{-1}{2}}$

$x_{min} = \frac{-b}{2a}$



② $y_{max} = \infty$

Range $\boxed{\left[\frac{3}{4}, \infty\right)}$

Q find the range of $-x^2+2x-1$

$f_{max} = -\frac{(4-4)}{2}$

$= 0$

$\boxed{(-\infty, 0]}$

Q $y = x^2-2x-3$

① $x \in R$

$y_{min} = -\frac{(4+12)}{4}$

$= -4$

$\boxed{[-4, \infty)}$

② $x \in [0, 3]$

$x \in [-5, \infty] \cap [0, 3]$

$\boxed{x \in [0, 3]}$

③ $x \in [-2, 0]$

$\boxed{x \in [-2, 0]}$

⑯

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② $x \in [0, 3]$

$y(x)|_{x=\cancel{0}0} = 0 - 0 - 3$

$= -3$

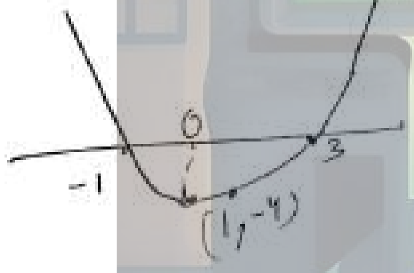
$y|_{x=3} = 9 - 6 - 3$

$= 0$

$\boxed{[-\cancel{3}4, 0]}$

$y|_{x=1} = 1 - 2 - 3$

$= -4$



③ $x \in [-2, 0]$

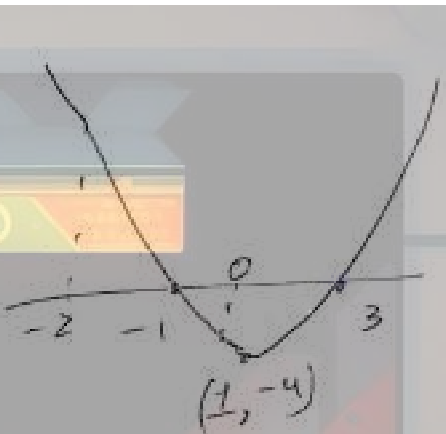
$f(-2) = 4 + 4 - 3$

$= 8 - 3$

$= 5$

$f(0) = -3$

$\boxed{[-3, 5]}$



Q $y = f(x) = x^2 - 5x + 6$

$y_{min} = \frac{-(25 - 24)}{4}$

$\boxed{= \frac{-1}{4}} = -0.25$

$x_{min} = \boxed{\frac{5}{2}} = 2.5$

① $[-3, 0]\ x \in x$

$f(-3) = 9 + 15 + 6$

$= 30$

$f(0) = 6$

$\boxed{y\cancel{x} \in [6, 30]}$

② $x \in [1, 5]$

$f(1) = 2$

$f(5) = 6$

$f(2.5) = -1/4$

$\boxed{y \in [-1/4, 6]}$

③ $x \in [3, 4]$

$f(3) = 0$

$f(4) = 16 - 20 + 6$

$= 2$

$\boxed{y \in [0, 2]}$

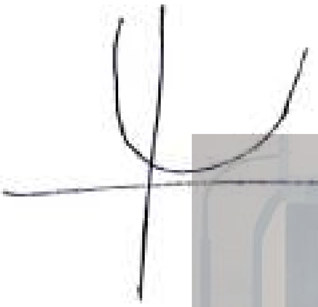
Commenting of Signs of a, b, c

$y = ax^2 + bx + c$

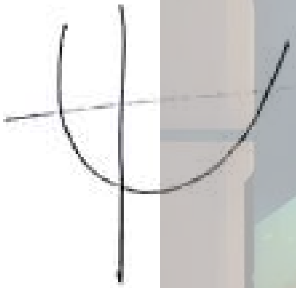
① $a > 0$ concave up

$a < 0$ concave down

② $c \to$ cut on y-axis



c ⊕ve



c ⊖ve

③ $b \to$ no fixed rule

① Comment on the signs of a, b, c

a = ⊕ve → concave up

c = ⊖ve → cut y-axis below x-axis.

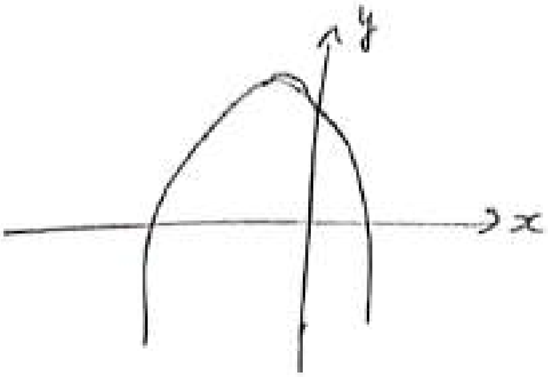
$\frac{-b}{2a} < 0$

⊕ve so $-b = \ominus$ ve

$\boxed{b = \oplus ve}$

Q Comment on a, b, c signs

①



$\boxed{a = ⊖ve}$

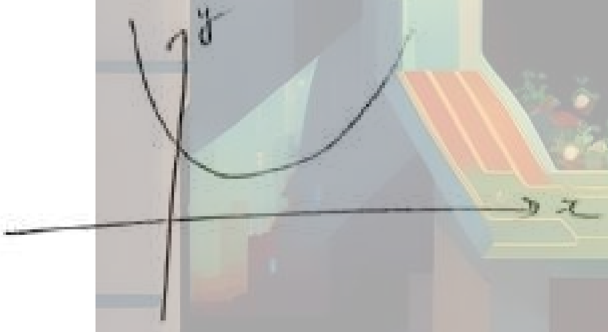
$\boxed{C = ⊕ve}$

$-\frac{b}{2a} = ⊖ve$

[illegible] ⊖ve

$\boxed{a = \text{[illegible]} ve}$ $\boxed{b = ⊖ve}$

②

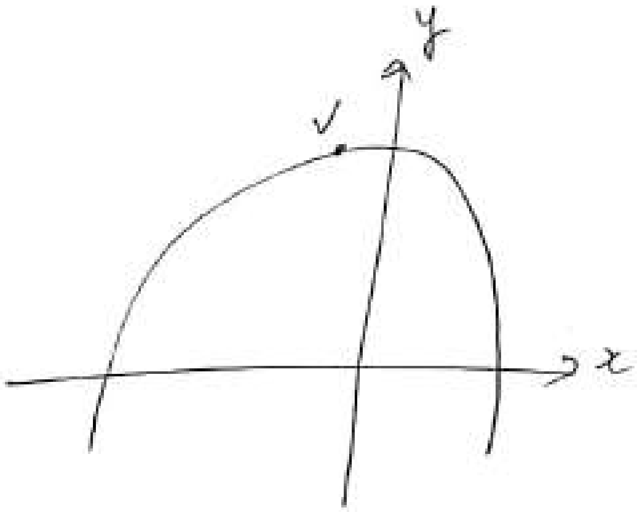


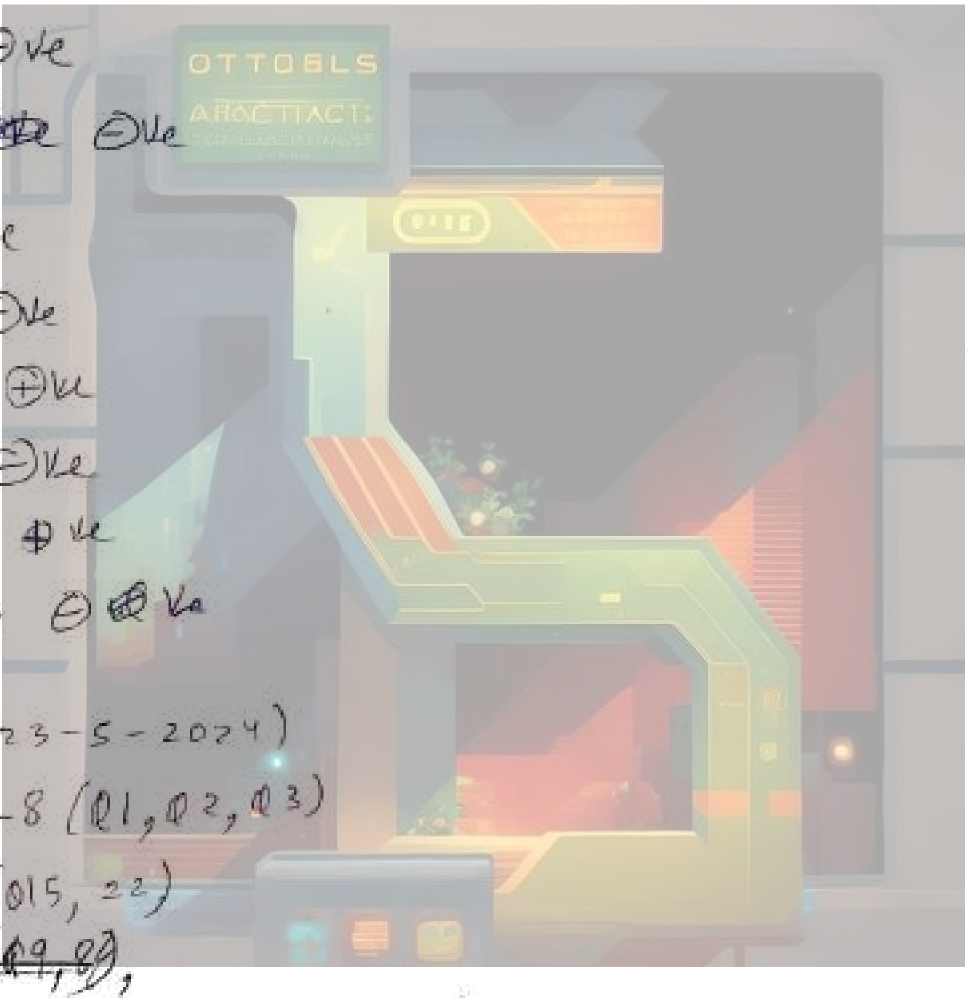
a = ⊕ve

c = ⊕ve

b = ⊖ve

Q





i) a $\ominus$ve

ii) b ~~$\oplus$ve~~ $\ominus$ve

iii) c $\oplus$ve

iv) bc $\ominus$ve

v) c−a $\oplus$ve

vi) ab^2 $\ominus$ve

vii) abc $\oplus$ve

viii) $\frac{a+b}{c}$ $\ominus$ ~~$\oplus$~~ ve

H.W. (23-5-2024)

~~Q~~ DYS-8 (Q1, Q2, Q3)

O-1 (Q15, 22)

~~O-2 (Q9, 8)~~,

O-2 (Q8, 9, 11, 16, 17, 18, 19,)

J-M (Q2, 3, 4, 5, 6, 7, 14)

Q $y = ax^2 + bx + c$, & $c < 0$ does not have any real roots then comment on the signs of – $a < 0$, $c < 0$

Ⓐ $c(a+b+c)$

Ⓑ $c(a-b+c)$

Ⓒ $4a+2b+c$

$\frac{-D}{+4a} < 0$

Ⓐ For $x = 1$

$y = a+b+c$

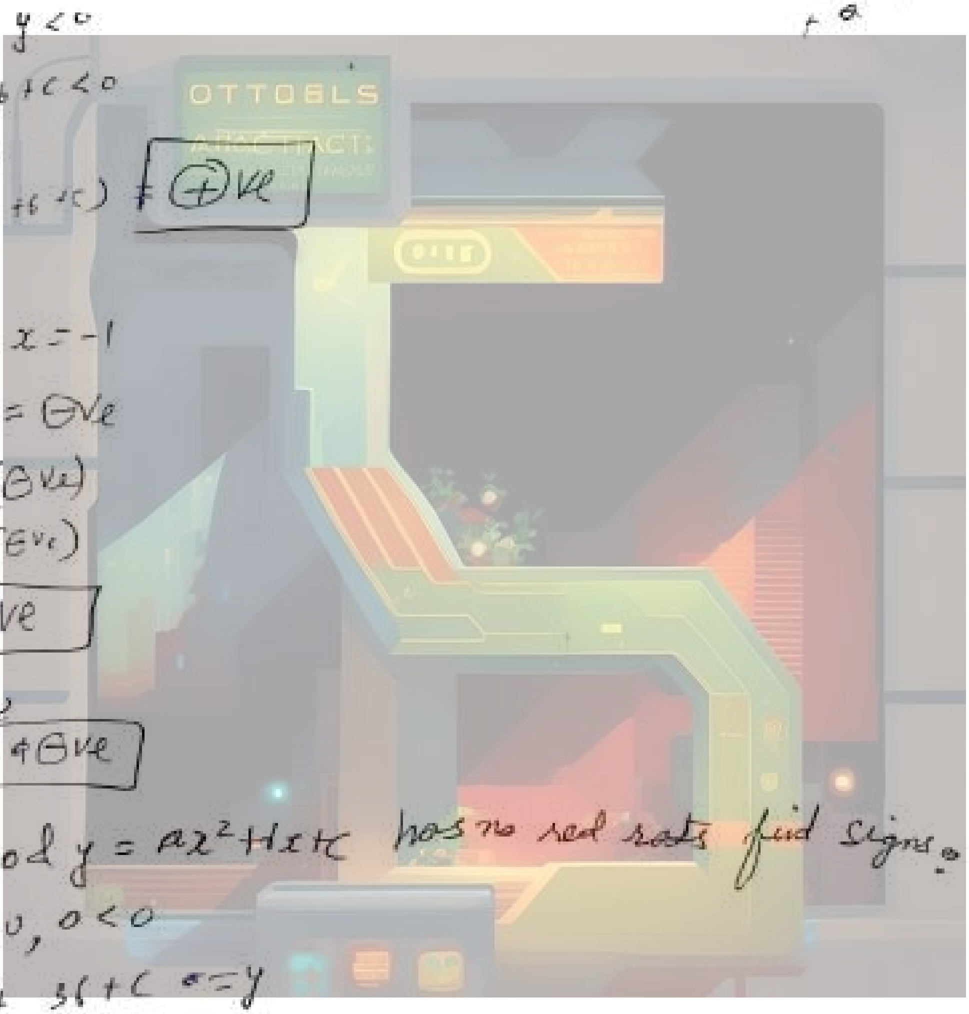
since $y < 0$

$a+b+c < 0$

$c < 0$

$c(a+b+c) = \boxed{\oplus ve}$

$\frac{-b}{+a} > 0$



Ⓑ for $x = -1$

$y = \ominus ve$

$c(\ominus ve)$

$\ominus(\ominus ve)$

$\boxed{\oplus ve}$

Ⓒ $x = 2$

$\boxed{y = 4\ominus ve}$

Q If $c < 0$ & $y = ax^2 + bx + c$ has no real roots find signs.

$c < 0$, $a < 0$

Ⓐ $9a + 3b + c = y$

for $x = 3$

$\boxed{y = \ominus ve}$

Ⓑ $a + 2b + 4c$

for $x = 1/2$

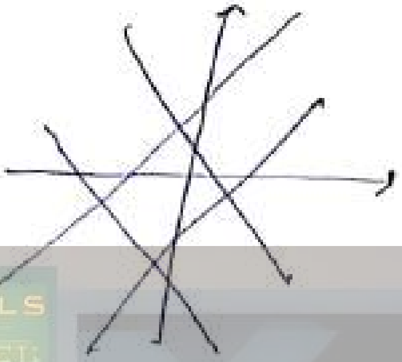
$\ominus ve$

Range of Linear, $\frac{\text{Linear}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Linear}}$ & $\frac{\text{Quad}}{\text{Quad}}$

① Linear –

$y = ax+b$

$y \in R$ always



eg. $y = 2x+3$

Range – $(-\infty, \infty)$

$y = \sqrt{2}x - \frac{7}{2}$

Range = $(-\infty, \infty)$

② $\frac{\text{Linear}}{\text{Linear}}$ – $y = \frac{ax+b}{cx+d}$

Range → $y \in R - \left\{\frac{a}{c}\right\}$

eg – $y = \frac{2x-3}{x+2}$

$y \in R - \left\{\frac{2}{1}\right\}$

$y \in R - \{2\}$

$y \in (-\infty, 2) \cup (2, \infty)$

Q <u>find ranges</u>

① $y = \frac{3x+1}{2x-1}$

$y \in R - \{\frac{3}{2}\}$

② $y = \frac{8x}{2x-1}$ ~~[illegible]~~

$y \in R - \{\frac{1}{2}\}$

③ $y = \frac{5x-2}{7-4x}$

$y \in R - \{\frac{5}{-4}\}$

④ $y = \frac{1}{3x+4}$

$y \in R - \{0\}$

③ $\frac{\text{Quad}}{\text{Linear}}$, $\frac{1}{\text{Quad}}$, $\frac{\text{Quad}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Quad}}$

<u>Process</u>:- Do cross Multiply

case-I

Case I - when leading cofficient $\neq 0$ then apply $D \geq 0$ & solve inequality in y.

<u>Case 2</u>: when leading coffecient $= 0$, If any value of x is comming then no problem. otherwise exclude.

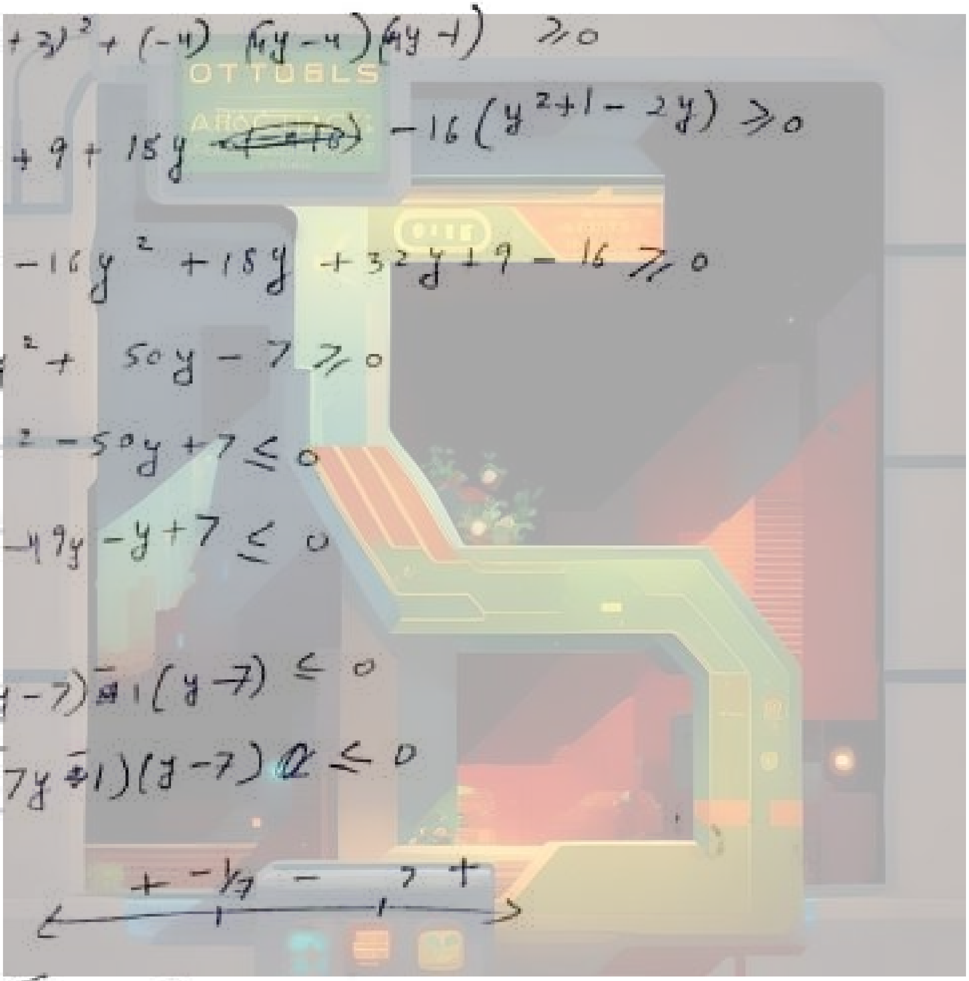
Q find range of $y = \frac{x^2-3x+4}{x^2+3x+4}$ $x \in R$

$yx^2 + 3yx + 4y = x^2 - 3x + 4$

$(y-1)x^2 + (3y+3)x + (4y-4)$

Case I $y-1 \neq 0$

$D \geq 0$



$(3y+3)^2 + (-4)(4y-4)(4y-1) \geq 0$

$9y^2 + 9 + 18y - 16(y^2+1-2y) \geq 0$

$9y^2 - 16y^2 + 18y + 32y + 9 - 16 \geq 0$

$-7y^2 + 50y - 7 \geq 0$

$7y^2 - 50y + 7 \leq 0$

$7y^2 - 49y - y + 7 \leq 0$

$7y(y-7) - 1(y-7) \leq 0$

$(7y-1)(y-7) \leq 0$

$+ \quad -\frac{1}{7} \quad - \quad 7 \quad +$

$y \in [+\frac{1}{7}, 7]$

Case 2 :- $y-1=0$

$y=1$

Put in Question

$\cancel{x^2}+3x+4-\cancel{x^2}+3x-4=0$

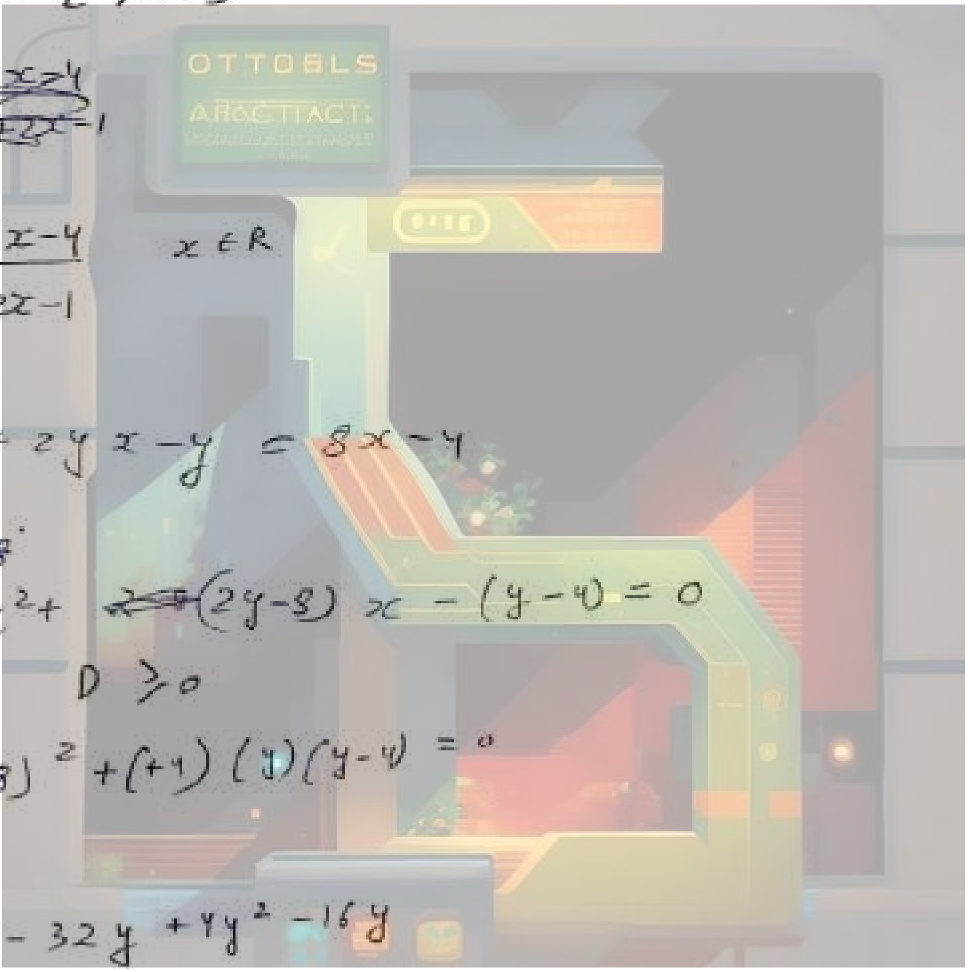
$6x=0$

$x=0$

$\therefore$ Value of x coming in Case 2 we need ~~to~~ to exclude $y=1$

Hence, $y \in [\frac{1}{7}, 7]$

~~Q. $y=\frac{8x-4}{x^2+2x-1}$~~



Q $y=\frac{8x-4}{x^2+2x-1}$ $x \in R$

Case 1 :-

$yx^2+2yx-y=8x-4$

~~$y=8$~~

$yx^2+\cancel{2}(2y-8)x-(y-4)=0$

$D \geq 0$

$(2y-8)^2+(+4)(y)(y-4)=0$

~~$2y$~~

$4y^2+64-32y+4y^2-16y$

$8y^2-48y+64 \geq 0$

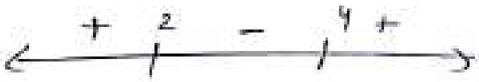
~~$2y^2$~~

$y^2-6y+8 \geq 0$

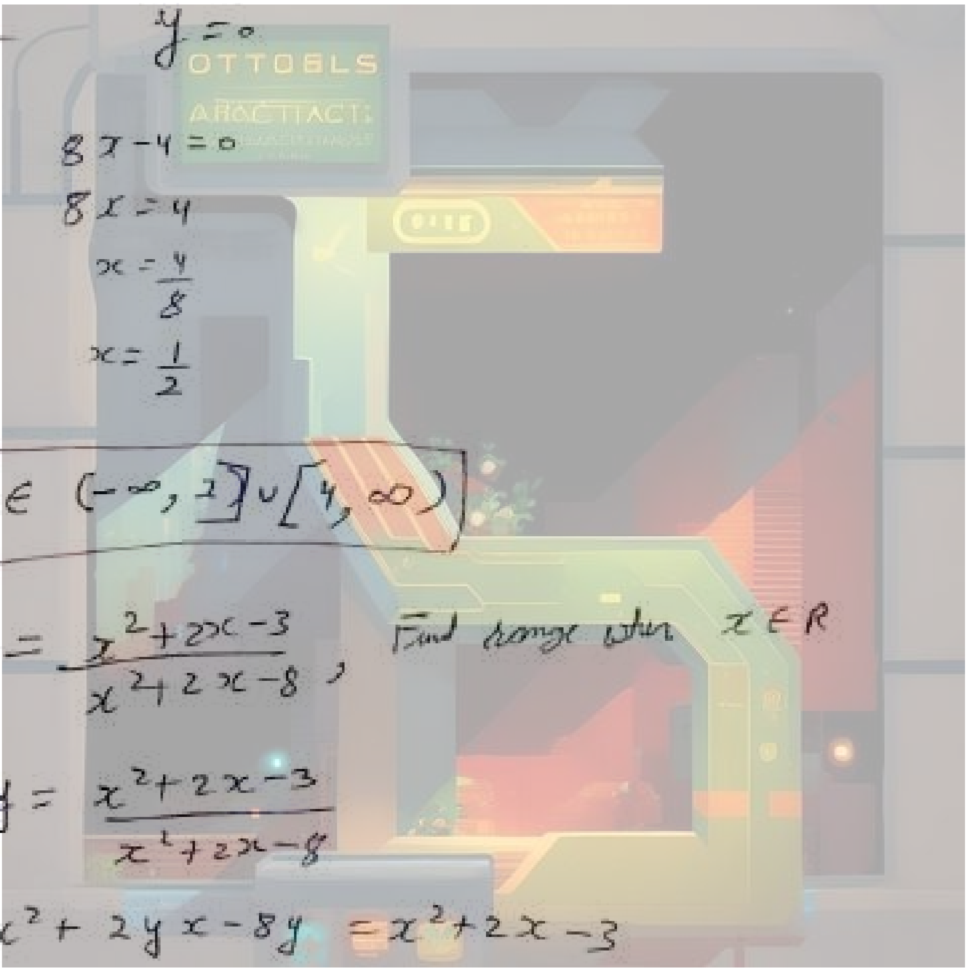
$y^2-4y-2y+8 \geq 0$

$y(y-4)-2(y-4)\geq 0$

$(y-2)(y-4)\geq 0$



$y\in(-\infty,2]\cup[4,\infty)$



case 2 $y=0$

$8x-4=0$

$8x=4$

$x=\frac{4}{8}$

$x=\frac{1}{2}$

$$\boxed{y\in(-\infty,2]\cup[4,\infty)}$$

l $y=\frac{x^2+2x-3}{x^2+2x-8}$, Find range when $x\in R$

$y=\frac{x^2+2x-3}{x^2+2x-8}$

$yx^2+2yx-8y=x^2+2x-3$

$yx^2-x^2+2yx-2x-8y+3=0$

$(y-1)x^2+(2y-2)x-(8y-3)=0$

Case 1

$D \geq 0$

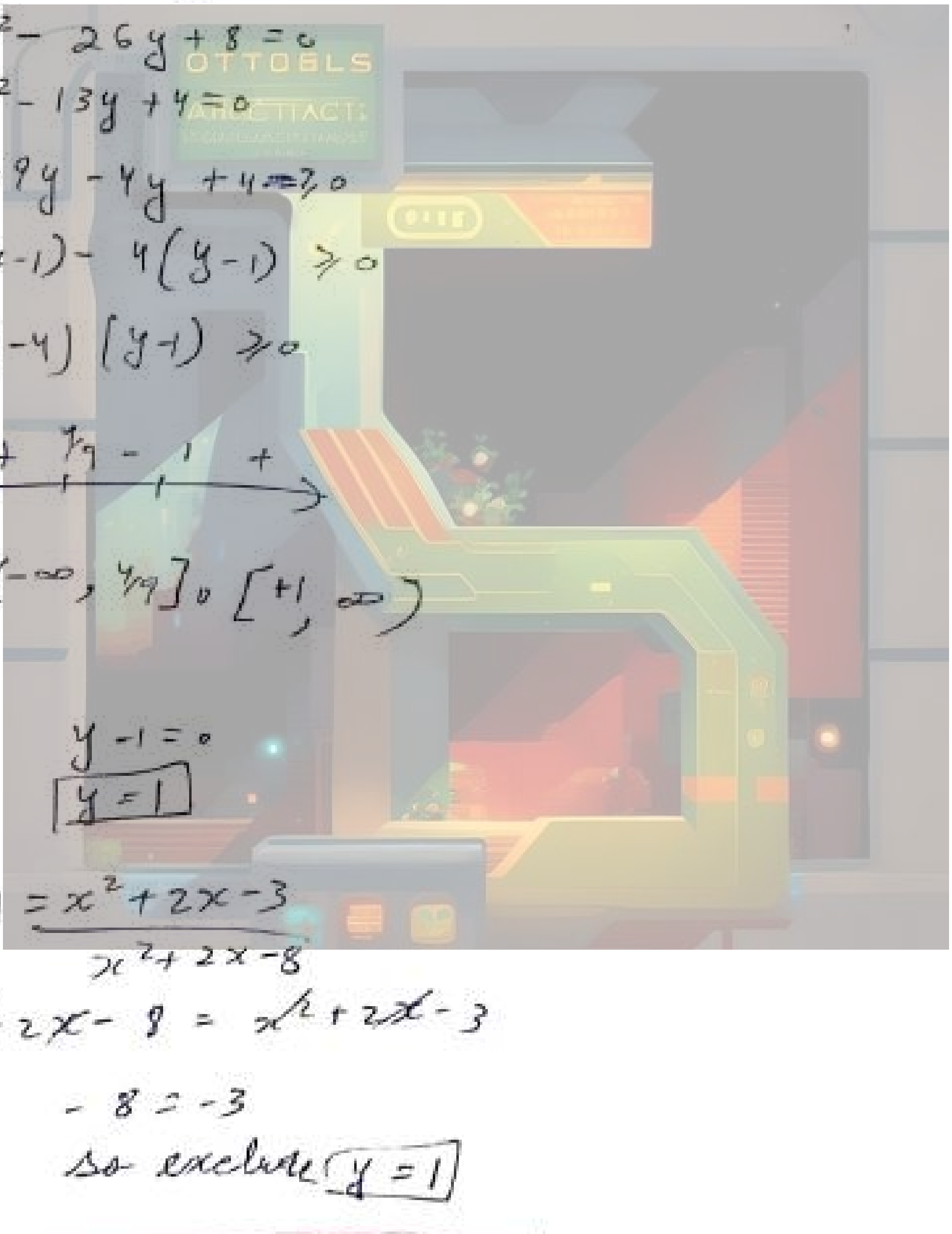
$b^2 - 4ac \geq 0$

$(2y-2)^2 + (+4)(y-1)(8y-3)$

$4y^2 + 4 - 8y + 4(8y^2 - 3y - 8y + 3)$

$4y^2 + 4 - 8y + 32y^2 - 44y + 12$

$36y^2 - 52y + 16 = 0$



$18y^2 - 26y + 8 = 0$

$9y^2 - 13y + 4 = 0$

$9y^2 - 9y - 4y + 4 \geq 0$

$9y(y-1) - 4(y-1) \geq 0$

$(9y-4)(y-1) \geq 0$

$$\xleftrightarrow{\quad + \quad 4/9 \quad - \quad 1 \quad + \quad}$$

$y \in (-\infty, 4/9] \cup [+1, \infty)$

Case 2

$y - 1 = 0$

$\boxed{y = 1}$

$1 = \dfrac{x^2 + 2x - 3}{x^2 + 2x - 8}$

$\cancel{x^2} + 2x - 8 = \cancel{x^2} + \cancel{2x} - 3$

$-8 = -3$

so exclude $\boxed{y = 1}$

$$\boxed{y \in (-\infty, 4/9] \cup (1, \infty)}$$

H.W. 24-05-2024

DYS-9 [All] ✓

O-1 {23}

~~O-2 {~~

J-M {1 , 13}

J-A {3, 4}

O-4 {1} ✓

O-3 {7, 8, 1, 2, 3}

~~[illegible]~~

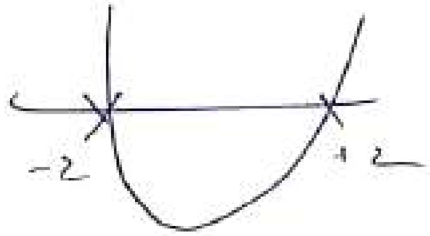
O-2 {7} ✓

Location of roots (Real nos.) (Type 1)

① Roots are equal in magnitude & opposite in sign.

$$\boxed{b=0 \ \& \ D>0}$$

eg.



$$(x-2)(x+2)=0$$

$$x^2-4=0$$

↳ Hence $b=0$

② Only 1 root is always zero

$$\boxed{c=0}$$

let ~~α~~ roots $\alpha, 0$

~~③ Roots are re~~

$$(x-\alpha)(x-0)=0$$

$$x^2-\alpha x=0$$

↳ Hence $c=0$

③ Roots are reciprocal to each other

$$\boxed{a=c \ \& \ D\geq 0}$$

$$(x-\alpha)\left(x-\frac{1}{\alpha}\right)=0$$

$$x^2-x\left(\alpha+\frac{1}{\alpha}\right)+1=0$$

$$\boxed{a=c}$$

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Scanned with CamScanner

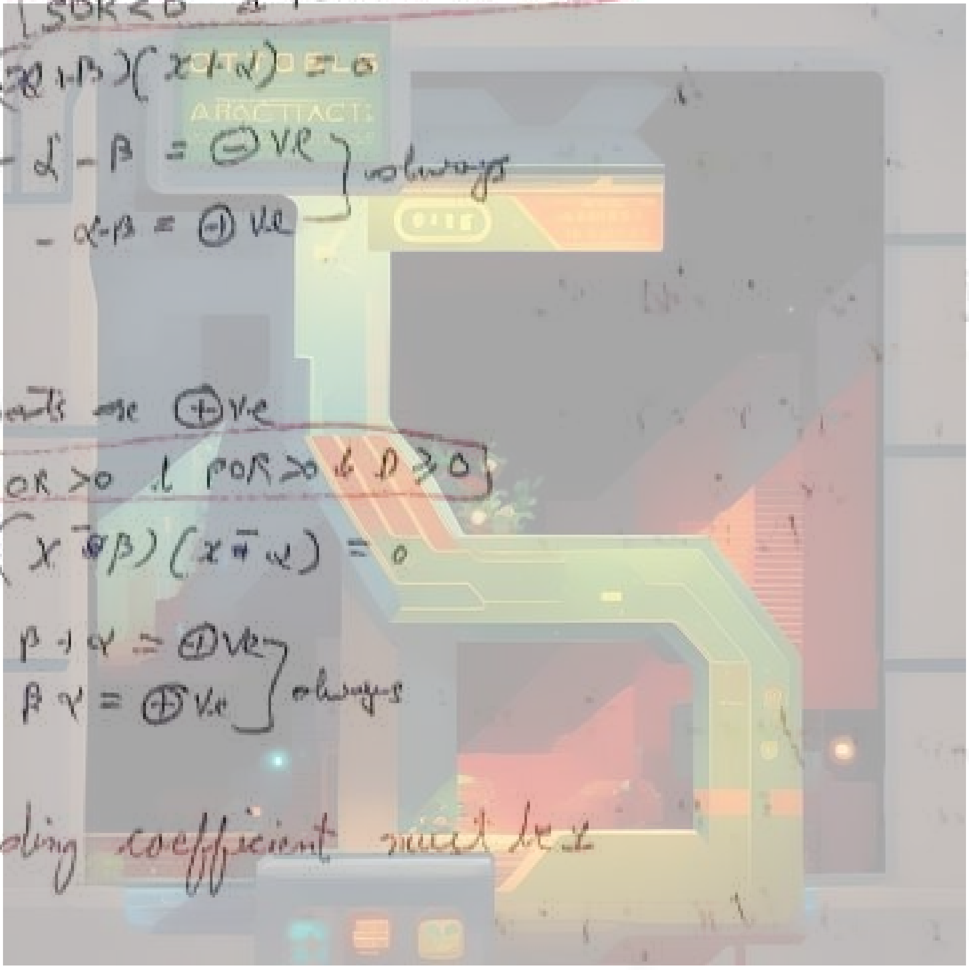
④ Roots are of opposite sign

$\boxed{POR<0 \ \& \ D>0}$

$(x-\alpha)(x+\beta)=0$

$SOR \Rightarrow \alpha-\beta$ (any can be bigger)

$POR \Rightarrow \alpha(-\beta) = \underbrace{-\alpha\beta}_{\text{always } \ominus ve}$



⑤ Both roots are ⊖ve

$\boxed{SOR<0 \ \& \ POR>0 \ \& \ D\geq 0}$

$(x+\beta)(x+\alpha)=0$

$\left.\begin{aligned} -\alpha-\beta &= \ominus ve \\ -\alpha\cdot-\beta &= \oplus ve \end{aligned}\right\}$ always

⑥ Both roots are ⊕ve

$\boxed{SOR>0 \ \& \ POR>0 \ \& \ D\geq 0}$

$\alpha(x-\beta)(x-\alpha)=0$

$\left.\begin{aligned} \beta+\alpha &= \oplus ve \\ \beta\alpha &= \oplus ve \end{aligned}\right\}$ always

⊅ leading coefficient must be 1

Q $f(x) = x^2 + 2(a-1)x + (a+5)$ find 'a'.

a) Roots are of opposite sign

$a + 5 < 0$

$\boxed{a < -5}$

$a \in (-\infty, -5)$

$4(a-1)^2 - 4(a+5) > 0$

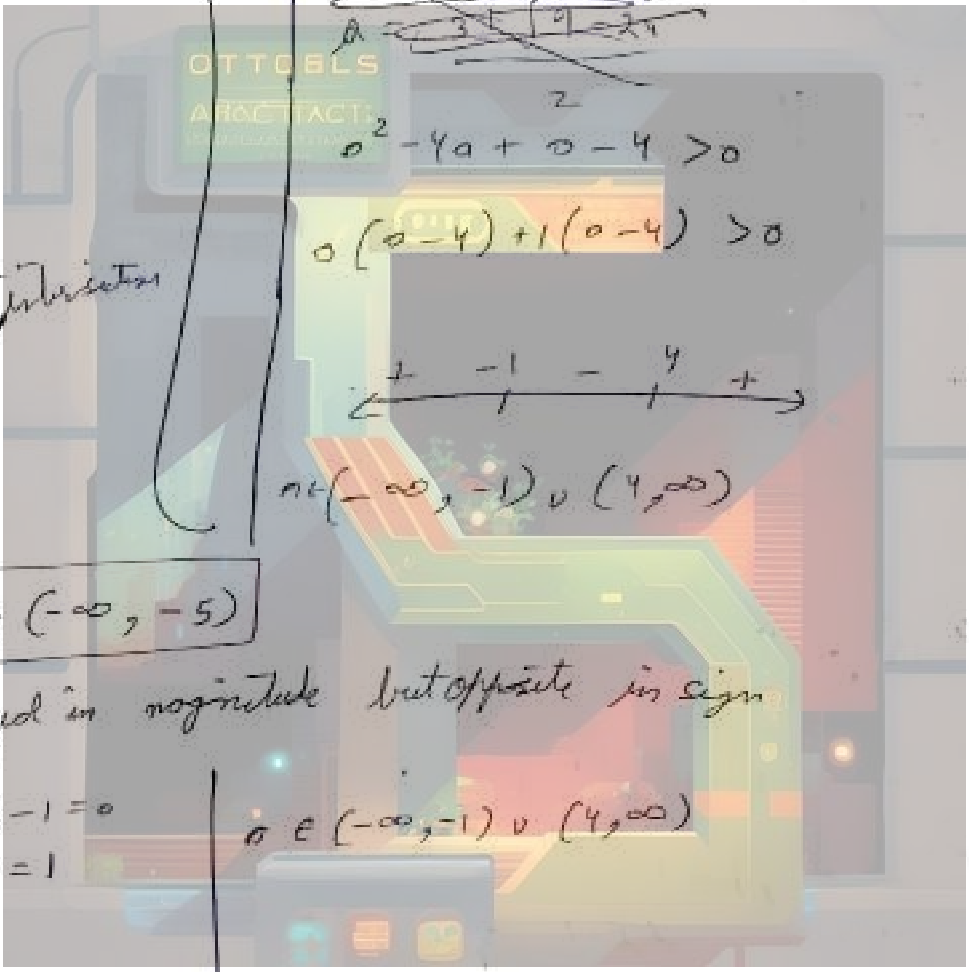
$a^2 + 1 - 2a - a - 5 > 0$

$a^2 - 3a \overset{-4}{\cancel{+6}} > 0$

~~$a^2 - 2a - a + 6 > 0$~~

~~$a(a-2) - 1(a$~~

~~$a = \frac{3 \pm \sqrt{9-24}}{2}$~~



$a^2 - 4a + a - 4 > 0$

$a(a-4) + 1(a-4) > 0$

$$\overset{+}{\longleftarrow} \; -1 \; \overset{-}{\text{———}} \; 4 \; \overset{+}{\longrightarrow}$$

$a \in (-\infty, -1) \cup (4, \infty)$

Intersection

$\boxed{a \in (-\infty, -5)}$

b) Roots equal in magnitude but opposite in sign

$a - 1 = 0$

$a = 1$

$a \in (-\infty, -1) \cup (4, \infty)$

$\boxed{a \in \phi}$

c) Both roots ⊕ve

SOR > 0

POR > 0

D ≥ 0

SOR $2(1-a) > 0$

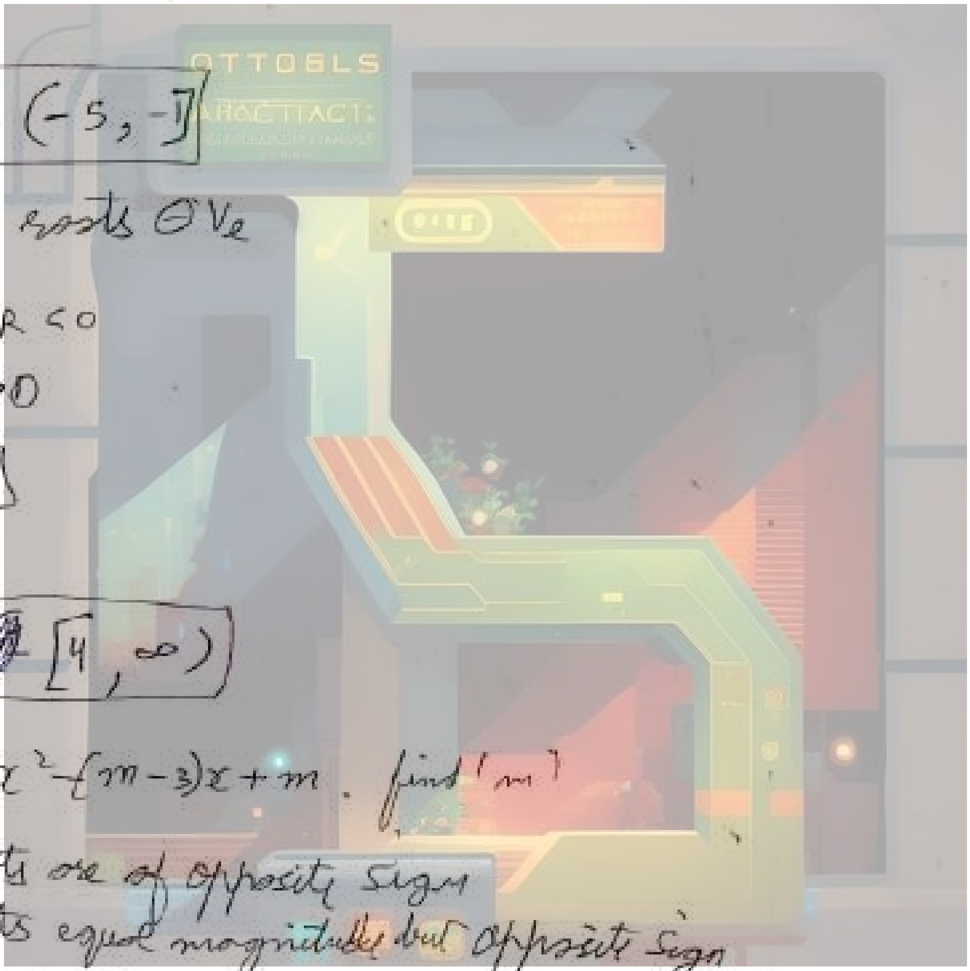
$1-a > 0$

$\boxed{a < 1}$

$a+5 > 0$

$\boxed{a > -5}$

$a \in (-\infty,-1] \cup [4,\infty)$



$\boxed{a \in (-5,-1]}$

d) Both roots ⊖ve

SOR < 0

$a-1 > 0$

$\boxed{a > 1}$

~~a∈~~

$\boxed{a \in [4,\infty)}$

Q $f(x)=x^2-(m-3)x+m$. find 'm'

a) Roots are of opposite sign
b) roots equal magnitude but opposite sign
c) Both roots are ⊕ve
d) Both roots are ⊖ve

$x^2-(m-3)x+m$

$(m-3)^2-4m=D$

a) $POR<0$

$D>0$

$m^2+9-6m-4m>0$

$m^2+9-10m>0$

$m^2-9m-m+9>0$

$m(m-9)-1(m-9)>0$

$\xleftarrow{\quad + \quad 1 \quad - \quad 9 \quad + \quad}\rightarrow$

$(-\infty,1)\cup(9,\infty)$

$m<0$

$\boxed{m\in(-\infty,0)}$

b) $b=0 \quad D>0$

$-(m-3)=0$

$m=3$

$\boxed{m\in\phi}$

c) $SOR>0$

$POR>0$

$D\geq0$

$\underline{m>0}$

$m-3>0$

$\underline{m>3}$

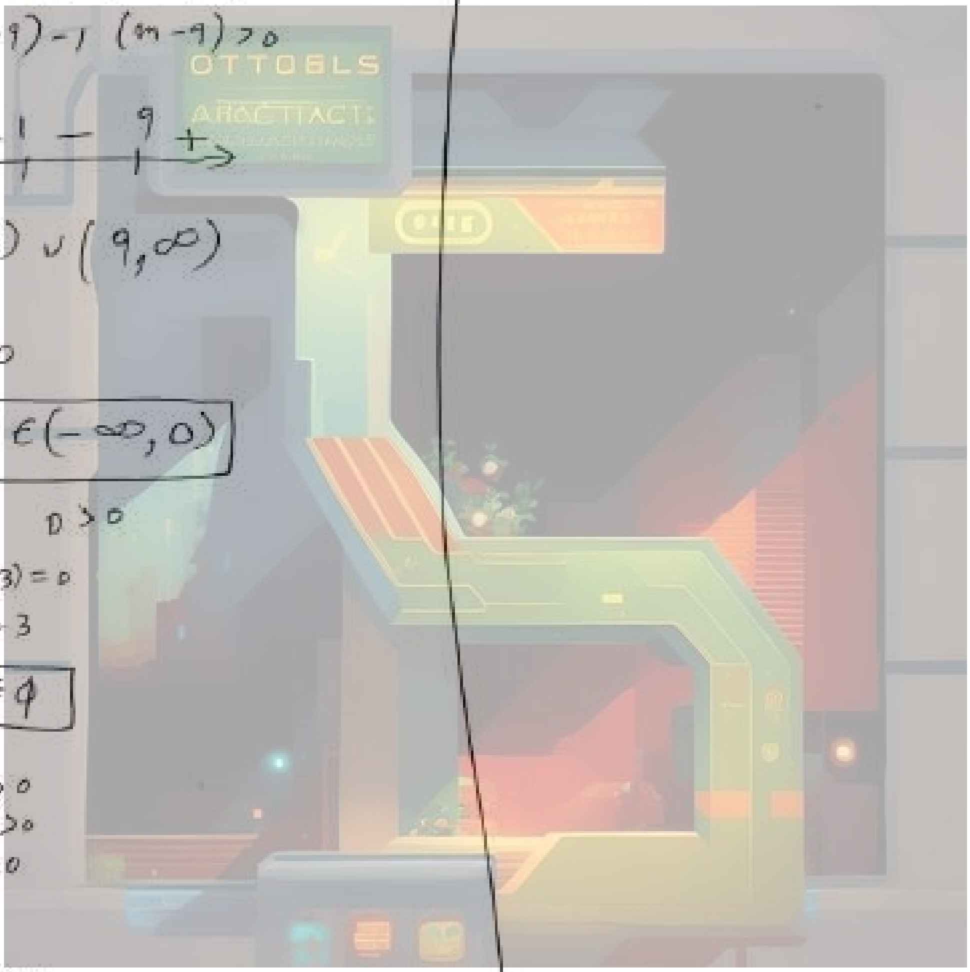
$\boxed{m\in[9,\infty)}$

d) $m>0$

$m-3<0$

$m<3$

$\boxed{m\in(0,1]}$



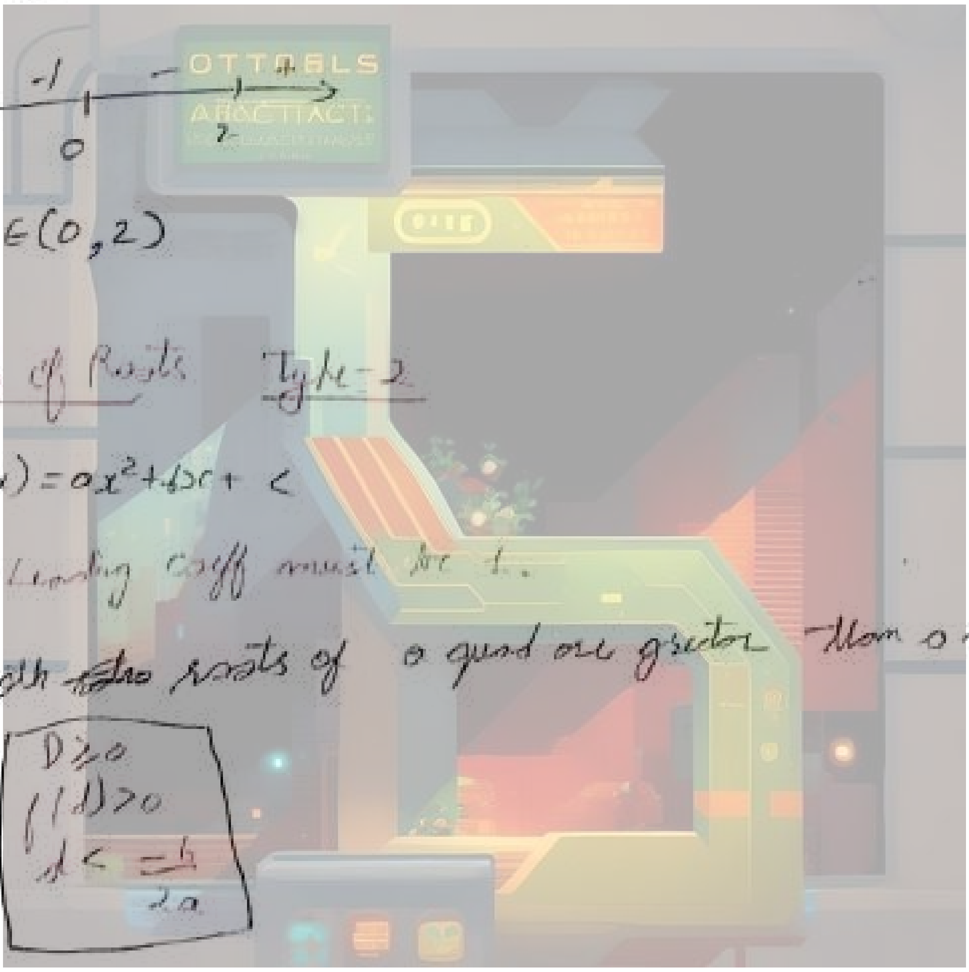
Q. ~~If~~ $f(x) = 3x^2 - 5x + p$ & $f(0)$ & $f(1)$ are of opposite signs find P.

p ~~$3-5+p$~~
~~$-2+p$~~

$f(0) \cdot f(1) < 0$

$p(3-5+p) < 0$

$P(P-2) < 0$



$P \in (0, 2)$

Location of Roots Type-2

$f(x) = ax^2 + bx + c$

⇒ Leading coeff must be 1.

① Both ~~the~~ roots of a quad are greater than a number 'd'?

$$\boxed{\begin{array}{l} D \geq 0 \\ f(d) > 0 \\ d < \frac{-b}{2a} \end{array}}$$

Q① find "λ" for both roots of quadratic $x^2-6\lambda x+9\lambda^2-2\lambda+2=0$ are greater than 3

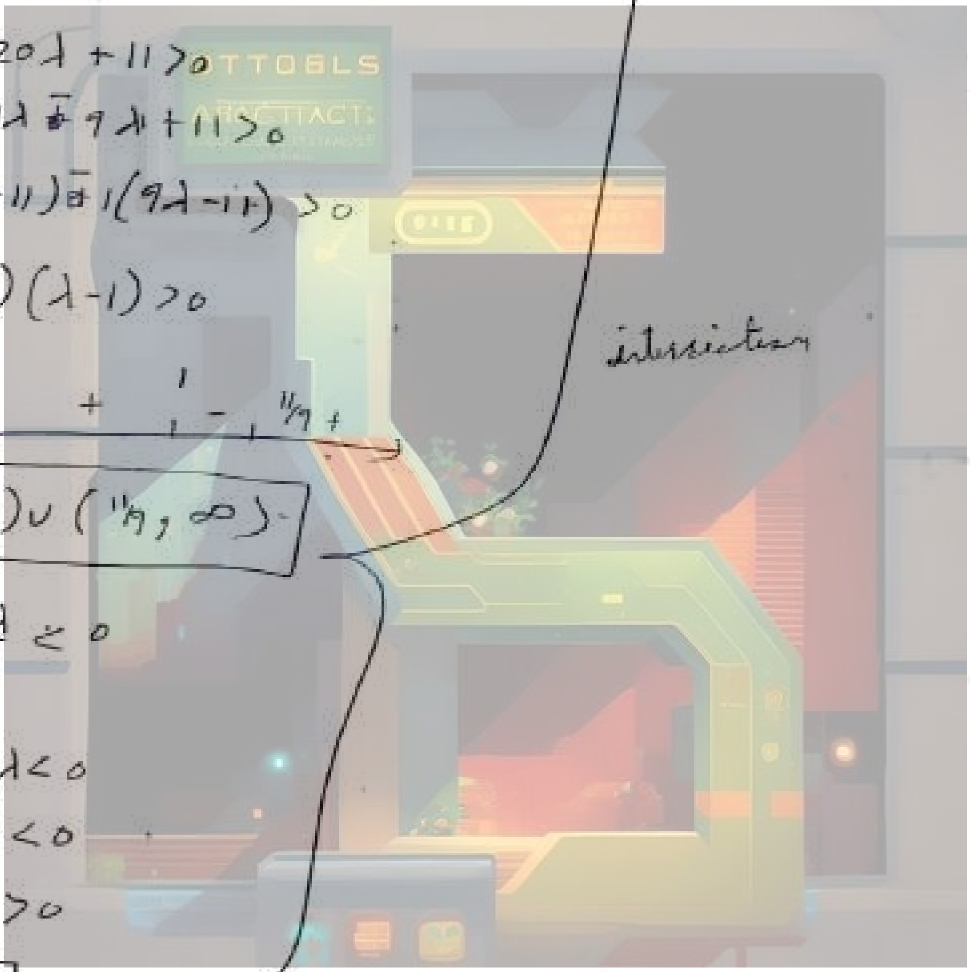
$36\lambda^2-36\lambda^2+8\lambda-8\geq 0$

$8(\lambda-1)\geq 0$

$\lambda-1\geq 0$

$\boxed{\lambda\geq 1}$

$9-18\lambda+9\lambda^2-2\lambda+2>0$



$9\lambda^2-20\lambda+11>0$

$9\lambda^2-11\lambda-9\lambda+11>0$

$\lambda(9\lambda-11)-1(9\lambda-11)>0$

$(9\lambda-11)(\lambda-1)>0$

intersection

$+\quad 1\quad -\quad 11/9\quad +$

$\boxed{\lambda\in(-\infty,1)\cup(11/9,\infty)}$

$3+\frac{-6\lambda}{2}<0$

$6-6\lambda<0$

$(1-\lambda)<0$

$\lambda-1>0$

$\boxed{\lambda>1}$

$\boxed{\lambda\in(11/9,\infty)}$ ✓

Q find 'K' for both the roots of the quadratic

$(k+1)x^2 - 3kx + 4k = 0$ are greater than 1

$x^2 - \frac{3k}{k+1}x + \frac{4k}{k+1} = 0$

$\boxed{k \neq -1}$

$D \geq 0$

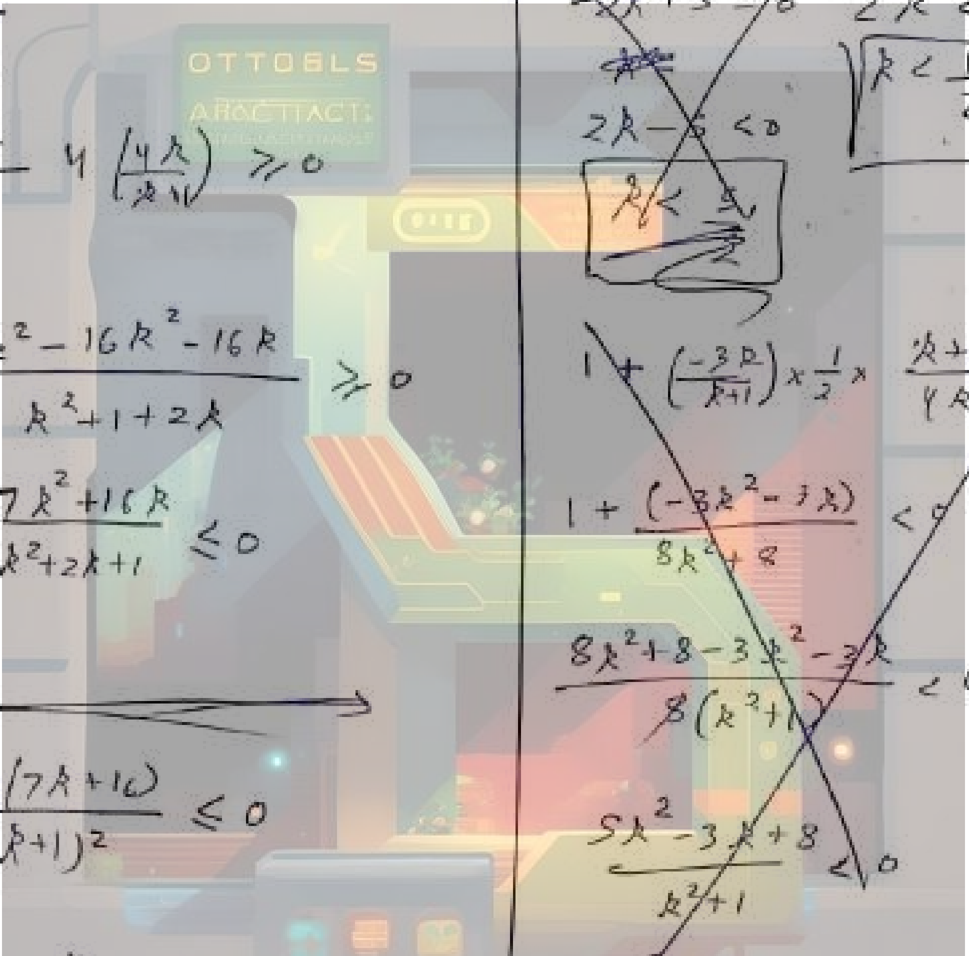
~~$\frac{9k^2}{k^2+1+2k}$~~

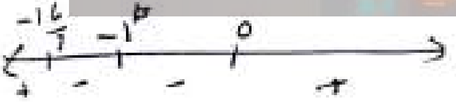
$\left(\frac{3k}{k+1}\right)^2 - 4\left(\frac{4k}{k+1}\right) \geq 0$

$\frac{9k^2 - 16k^2 - 16k}{k^2+1+2k} \geq 0$

$\frac{7k^2+16k}{k^2+2k+1} \leq 0$

$\frac{k(7k+16)}{(k+1)^2} \leq 0$





$\boxed{\left[-\frac{16}{9}, -1\right) \cup (-1, 0]}$

$(k+1) - 3k + 4 = 0$

$k+1-3k+4=0$

~~$-2k+5 \neq 0$~~

~~$2k-5 < 0$~~

~~$\boxed{k < \frac{5}{2}}$~~

$2k < 1$

$\boxed{k < \frac{1}{2}}$

~~$1 + \left(\frac{-3k}{k+1}\right) \times \frac{1}{2} \times \frac{k+1}{4k} \neq 0$~~

~~$1 + \frac{(-3k^2 - 3k)}{8k^2+8} < 0$~~

~~$\frac{8k^2+8-3k^2-3k}{8(k^2+1)} < 0$~~

~~$\frac{5k^2-3k+8}{k^2+1} < 0$~~

~~$5k^2 - 8k -$~~

(36)

$$\frac{-\left(\frac{-3k}{k+1}\right)}{2} > 1$$

$$\frac{3k}{2(k+1)} - 1 > 0$$

$$\frac{k-1}{k+1} > 0$$

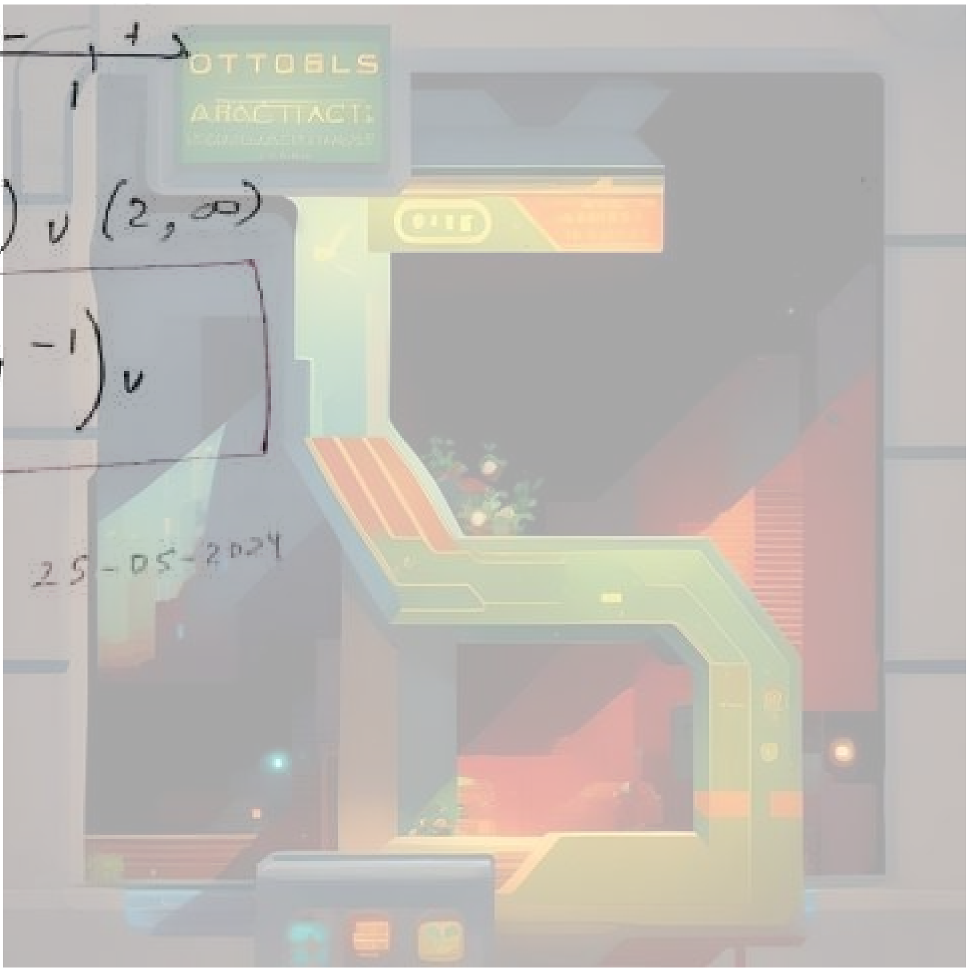
$+ \quad - \quad +$

$-1 \quad 1$

$(-\infty, -1) \cup (2, \infty)$

$$\boxed{\left[-\frac{16}{7}, -1\right) \cup}$$

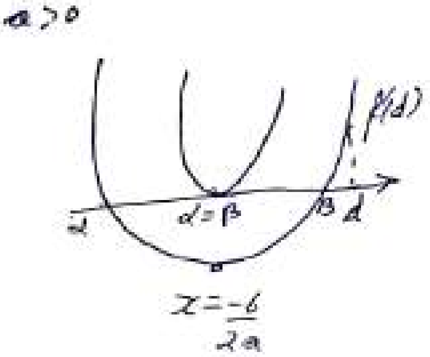
H.W. 25-05-2024

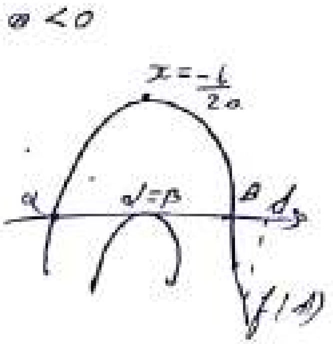


Type-②

(B) Both roots are less than any specific number 'd'.

$$\boxed{\begin{array}{c} D \geq 0 \\ \frac{-b}{2a} < d \\ af(d) > 0 \end{array}}$$ intersection

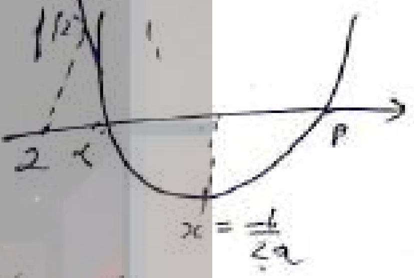




Q let $x^2-(m-3)x+m=0$ $(m \in R)$ be a quadratic equation find the value of m for which.

① both roots are greater than 2.

② both the roots are smaller than 2.



② $x^2-(m-3)x+m=0$

① $D = (m-3)^2-4m$

$= m^2+9-6m-4m$

$= m^2+9-10m$

~~m^2+10~~

$= m^2-9m-m+9$

$= m(m-9)-1(m-9)$

$= (m-1)(m-9) \geq 0$

...①

$\boxed{m \in (-\infty,1] \cup [9,\infty)}$

$\frac{m-3}{2} < 2$

$m-3 < 4$ ...②

$\boxed{m<7}$

$4-(m-3)\times 2+m > 0$

$4-2m+6+m > 0$

$10-m > 0$ ...③

$\boxed{m<10}$

①∩②∩③

$\boxed{m \in (-\infty,1]}$ ✓

② $\frac{m-3}{2} > 2$

$m-3 > 4$ ...②

$\boxed{m>7}$

$\boxed{m<10}$

①∩②∩③

$\boxed{m \in [9,10)}$ ✓

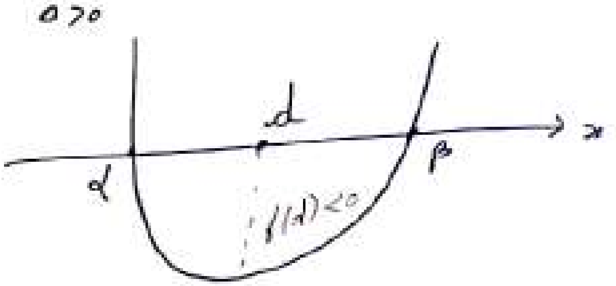
Type ③

Ⓑ Both roots lie on either side of specific no. d

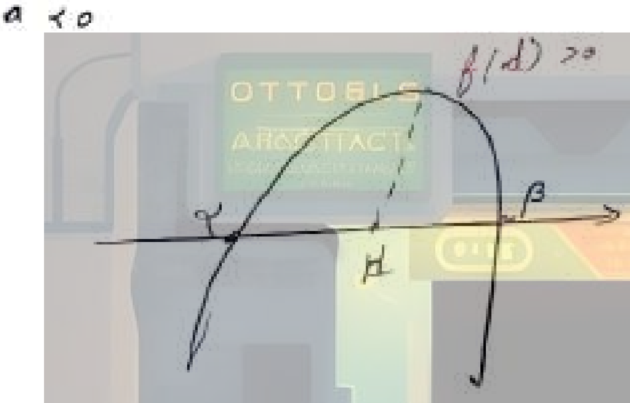
→ One root is greater than D & one is less

→ Specific no. d lies between the roots

a>0



a<0



$\boxed{a \cdot f(d) < 0}$

$$\left.\begin{array}{l} a>0,\ f(d)<0 \\ a<0,\ f(d)>0 \end{array}\right\}\ \text{so}\ \underline{af(d)<0}$$

Q ~~Ex~~ find all possible ~~of~~ values of a for which exactly one root of $x^2-(a+1)x+2a=0$ lie in interval (0, 3).

$f(0) = 2a$

$f(3) = 9 - 3a + 3 + 2a$

$= 6 - a$

$f(0) \cdot f(3) < 0$

$a(6-a) < 0$

$\underline{(-\infty, 0) \cup (6, \infty)}$

Check at the points of interval

put $a=0$

$x^2 - x = 0$

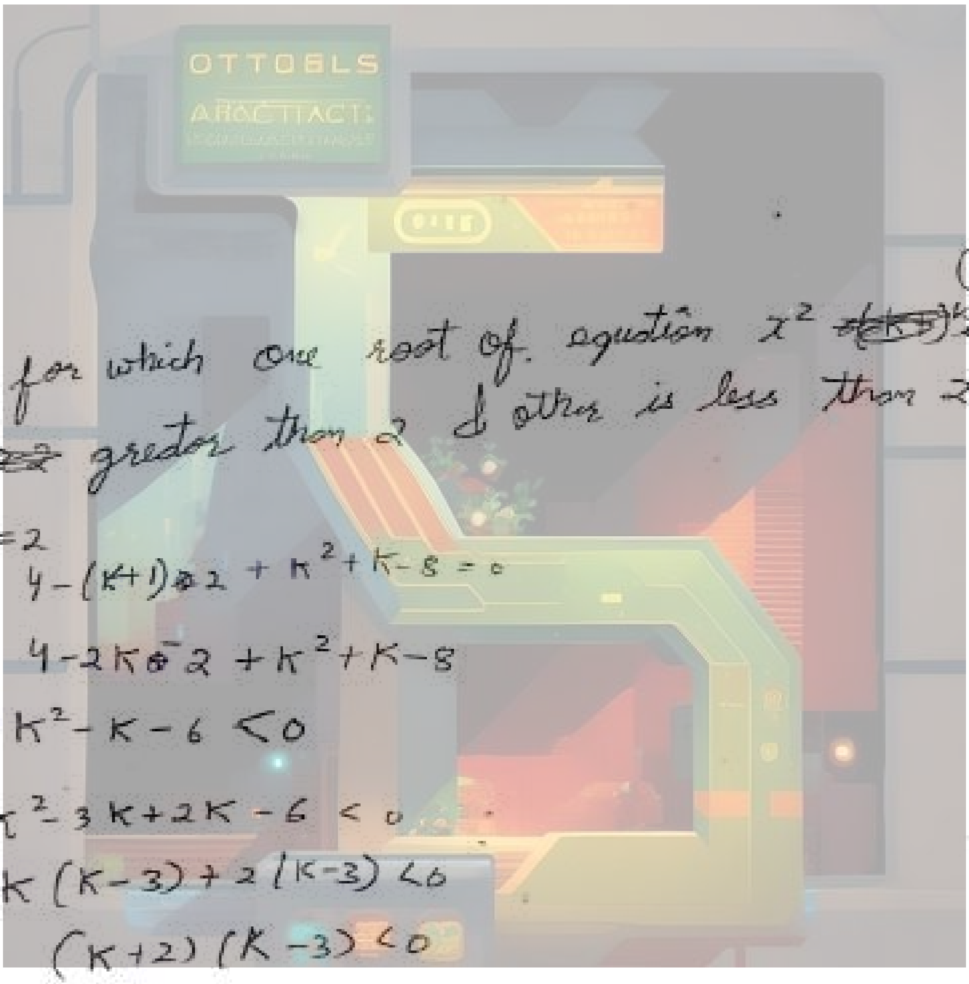
$x = 1, 0$ → 1 lie between in interval

so include 0 in answer

put $a = 3$

$D<0$ so no real roots

$\boxed{a \in (-\infty, 0] \cup (6, \infty)}$



Q find 'K' for which one root of equation $x^2 - \overset{(K+1)}{\cancel{(K+1)}}x + K^2 + K - 8 = 0$ is greater than 2 & other is less than 2.

$\alpha = 2$

$f(2) = 4 - (K+1)2 + K^2 + K - 8 = 0$

$= 4 - 2K - 2 + K^2 + K - 8$

$= K^2 - K - 6 < 0$

$K^2 - 3K + 2K - 6 < 0$

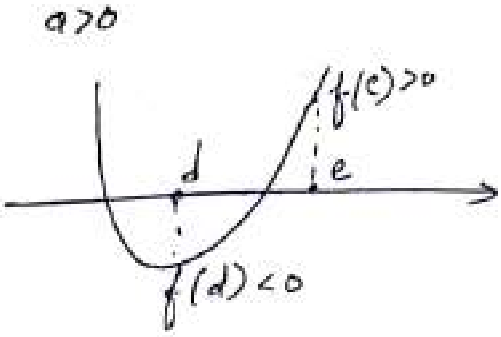
$K(K-3) + 2(K-3) < 0$

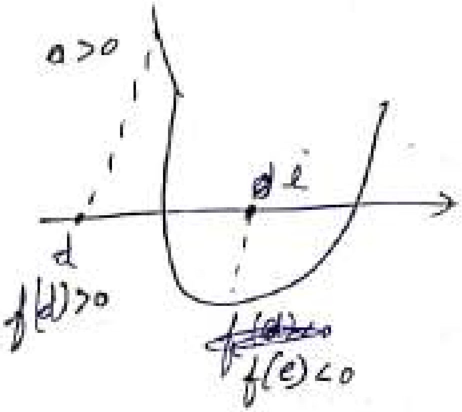
$(K+2)(K-3) < 0$

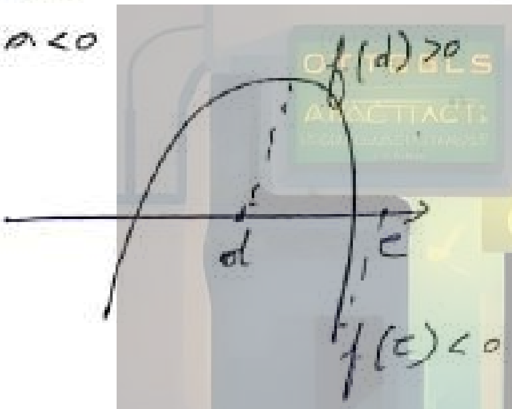
$\boxed{K \in (-2, 3)}$

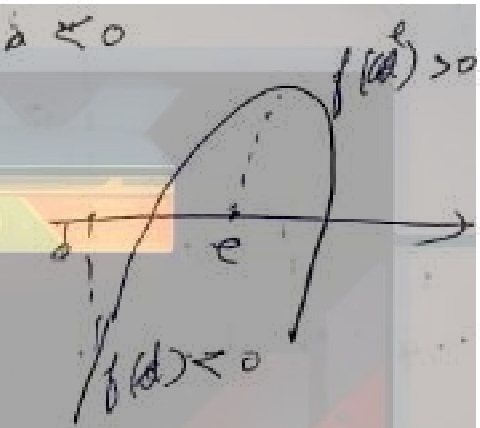
② Exactly one root lies in $(d<e)$ the interval (d,e)

Type-4









So $f(d)\cdot f(e)<0$

☆☆

$$\boxed{f(d)\cdot f(e)<0}$$

Note:- Check at the extreme points of Interval.

Pg 158

Q8. $(c-5)x^2 - 2cx + (c-4) = 0$

$-2x^2 - 6x - 1$

$2x^2 + 6x + 1$

$f(0) = c - 4$

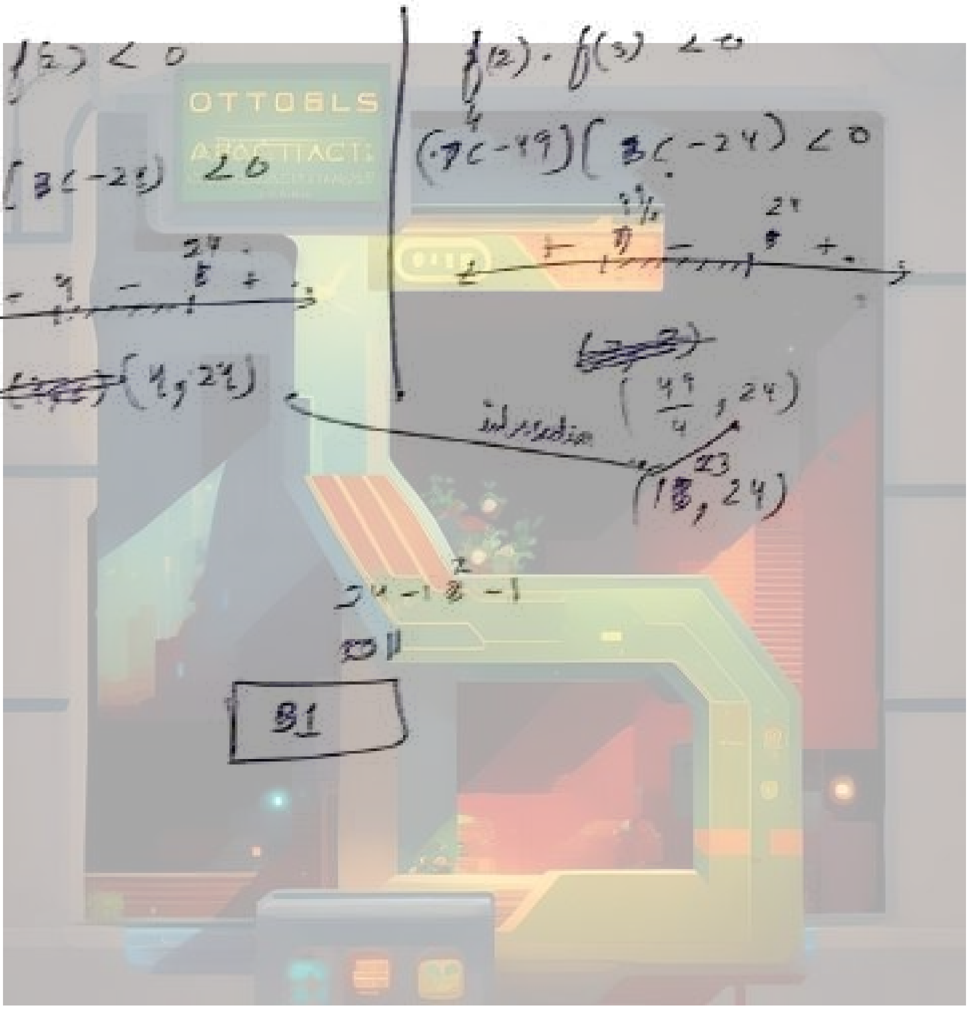
$f(2) = 4(c-5) - 2c + c - 4 = 0$

$f(2) = 4c - 20 - 8c + c - 4$

$= 3c - 24$

$f(3) = 9c - 45 - 6c + c - 4$

$= 4c - 49$



$f(0) \cdot f(2) < 0$

$(c-4)(3c-24) < 0$

$- \quad 4 \quad - \quad \frac{24}{3} \quad +$

~~(4, 8)~~ $(4, 24)$

$f(2) \cdot f(3) < 0$

$(4c-49)(3c-24) < 0$

$+ \quad \frac{49}{4} \quad - \quad \frac{24}{3} \quad +$

~~(2, 3)~~

$\left(\frac{49}{4}, 24\right)$

intersection

$(13, 24)$

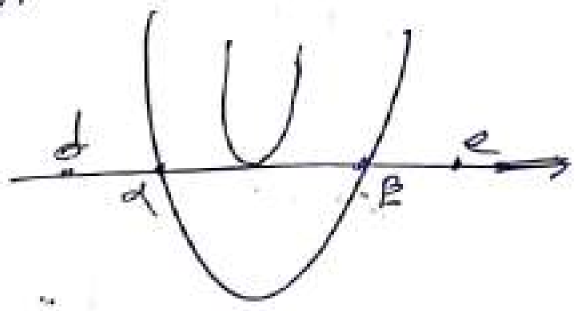
$24 - 13 - 1$

$= 11$

$\boxed{11}$

Type-5 Both the roots lie between number d & e (d<e)

$a>0$



$a<0$

Union of ① & ③

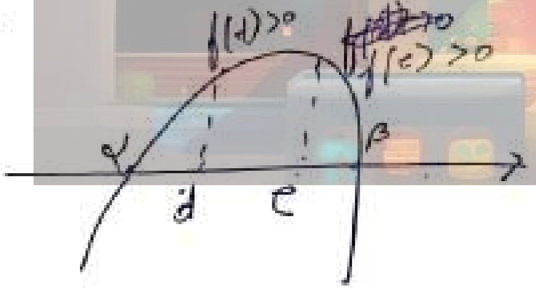
① $D \geq 0$

② $d < \frac{-b}{2a} < e$

③ $a f(d) > 0$

④ $a f(e) > 0$

Type-⑥ one root is greater than e & one is less than d



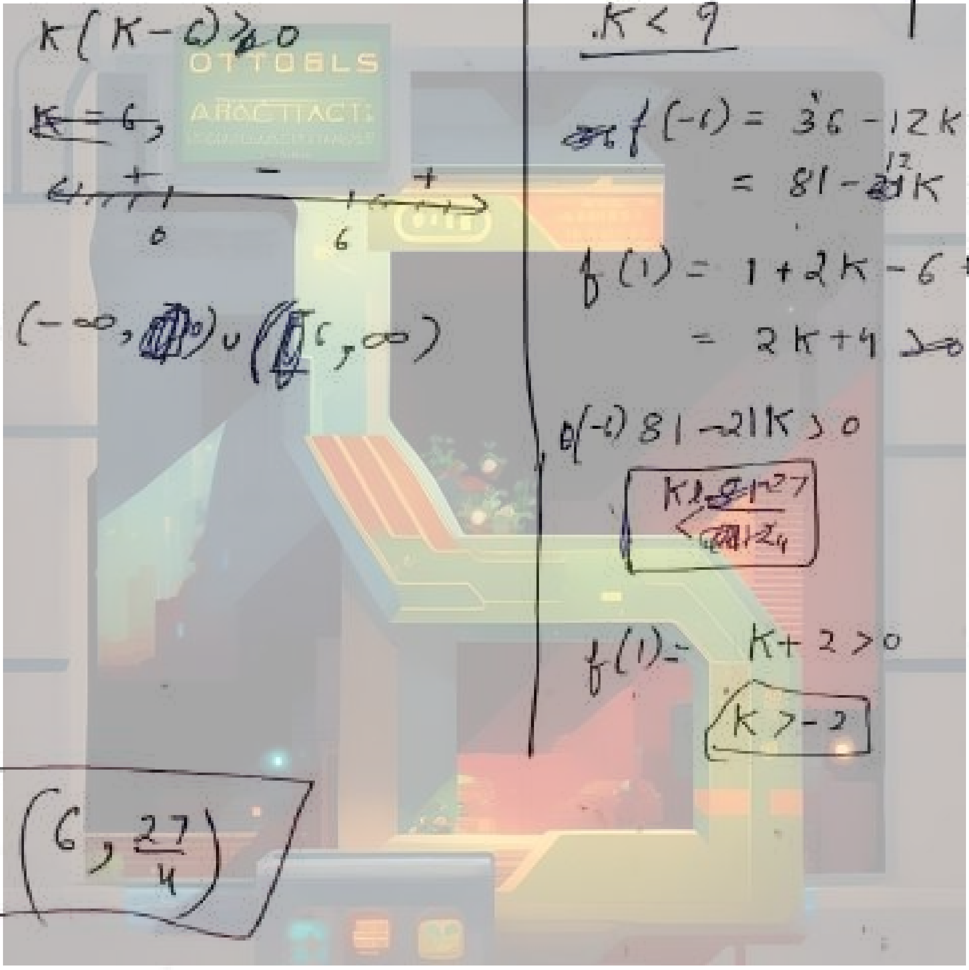
[redacted]

$a f(d) < 0$

$a f(e) < 0$

[redacted]

Q If α, β are the roots of $x^2+2(k-3)x+9=0$ if α,β belongs to $(-6,1)$ find k.



$4(k-3)^2-36 \geq 0$

$4k^2+36-24k-36 \geq 0$

$k^2-6k \geq 0$

$k(k-6) \geq 0$

~~$k=6$~~,

$$\overset{+}{\longleftarrow}\underset{0}{|}\overset{-}{\longrightarrow}\underset{6}{|}\overset{+}{\longrightarrow}$$

$k \in (-\infty, 0) \cup (6, \infty)$

$\frac{-2k+6}{2}$

$3-k$

$-6 < 3-k$

$k < 9$

$3-k > 1$

$k > 2$

$f(-1) = 36-12k+36+9=0$

$= 81-12k \geq 0$

$f(1) = 1+2k-6+9$

$= 2k+4 \geq 0$

$f(-1)\ 81-12k > 0$

$\boxed{k < \frac{81}{12} = \frac{27}{4}}$

$f(1) = k+2 > 0$

$\boxed{k > -2}$

$\boxed{\left(6, \frac{27}{4}\right)}$

(44)

Summary (Location of Roots)

① Both roots greater than D. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} > d\right)$

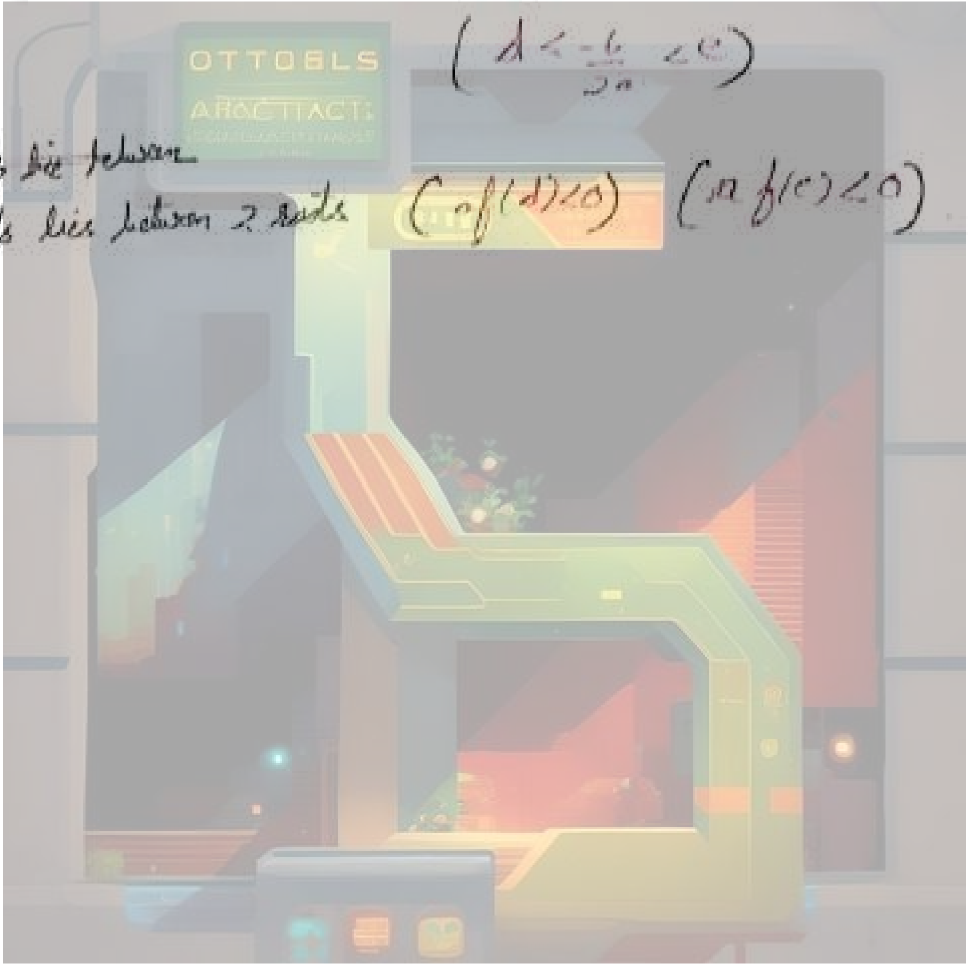
② Both roots lies than D. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} < d\right)$

③ 'd' lies between the roots. $(af(d) < 0)$

④ exactly one root lie in (d, e) $(f(d) \cdot f(e) < 0)$

⑤ Both roots lie between d and e $(D \geq 0)$ $(af(d) > 0)$ $(af(e) > 0)$
$\left(d < \frac{-b}{2a} < e\right)$

~~⑥ Both roots lie between~~
⑥ Both points lies between 2 roots $(af(d) < 0)$ $(af(e) < 0)$



Irrational Inequality

① Inequalities having √ sign.

② Direct Squaring is not allowed without checking

Q $\sqrt{2x-5} < 3$

$2x-5 \geq 0$ [under root quantity is always ⊕)

$2x-5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$x \in \left[\frac{5}{2}, \infty\right)$ — ①

$\underbrace{\sqrt{2x-5}}_{\oplus ve} < \underbrace{3}_{\oplus ve}$

Square both sides so we can square

$2x-5 < 9$

$2x < 14$

$x < 7$ — ②

$x \in (-\infty, 7)$

① ∩ ②

$\boxed{\left[\frac{5}{2}, 7\right)}$

Q $\sqrt{x+6} < x-6$

$x+6 \geq 0$

$x \geq -6$

$x \in [-6, \infty)$

case ① $x-6 < 0$

$x < 6$

$\oplus < \ominus$ not possible

$x \in \phi$

case ② $x-6 \geq 0$

$x \geq 0$

$x+6 < x^2+36-12x$

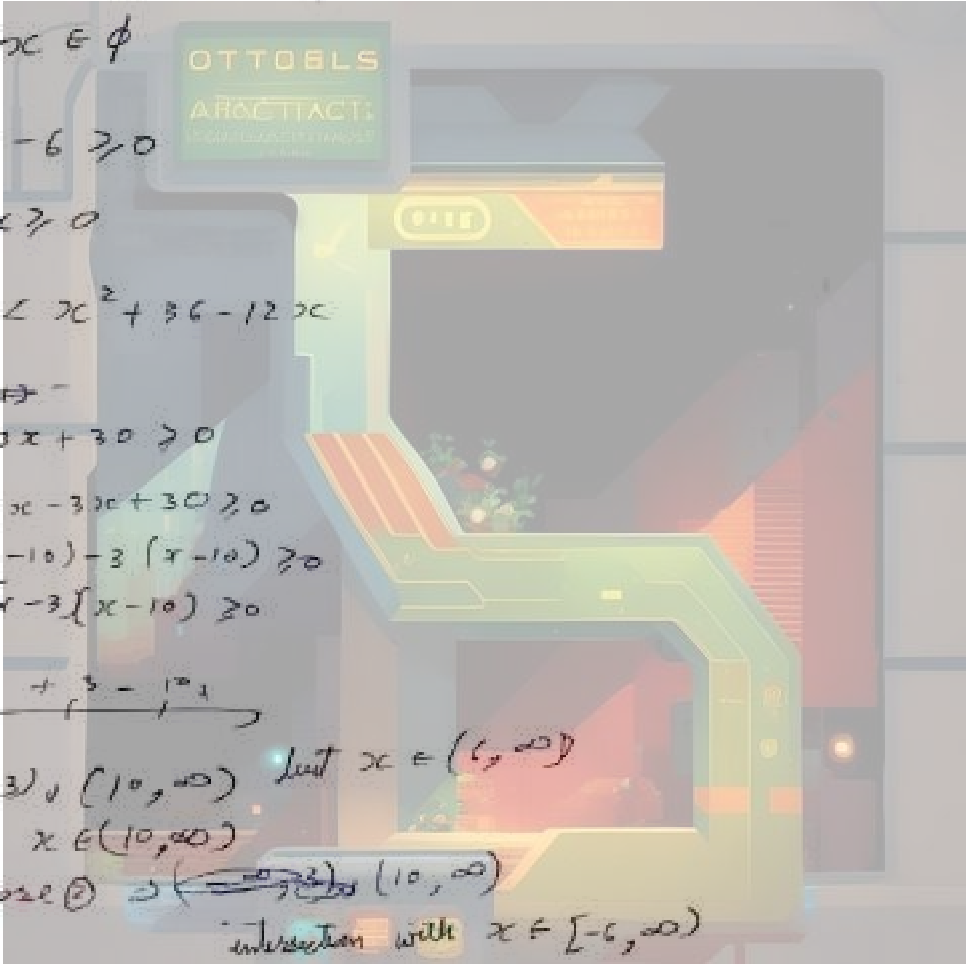
~~x^2+13~~ −

$x^2-13x+30 \geq 0$

$x^2-10x-3x+30 \geq 0$

$x(x-10)-3(x-10) \geq 0$

$(x-3)(x-10) \geq 0$



$$\xleftrightarrow{\;+\;\;3\;\;-\;\;10\;\;+\;}$$

$(-\infty, 3) \cup (10, \infty)$ but $x \in (6, \infty)$

$x \in (10, \infty)$

case ① ∪ case ② ⇒ ~~$(-\infty, 3)$~~ ∪ $(10, \infty)$

intersection with $x \in [-6, \infty)$

$\boxed{x \in (10, \infty)}$

Q $x+1 \geq \sqrt{5-x}$

$\sqrt{5-x} \leq x+1$

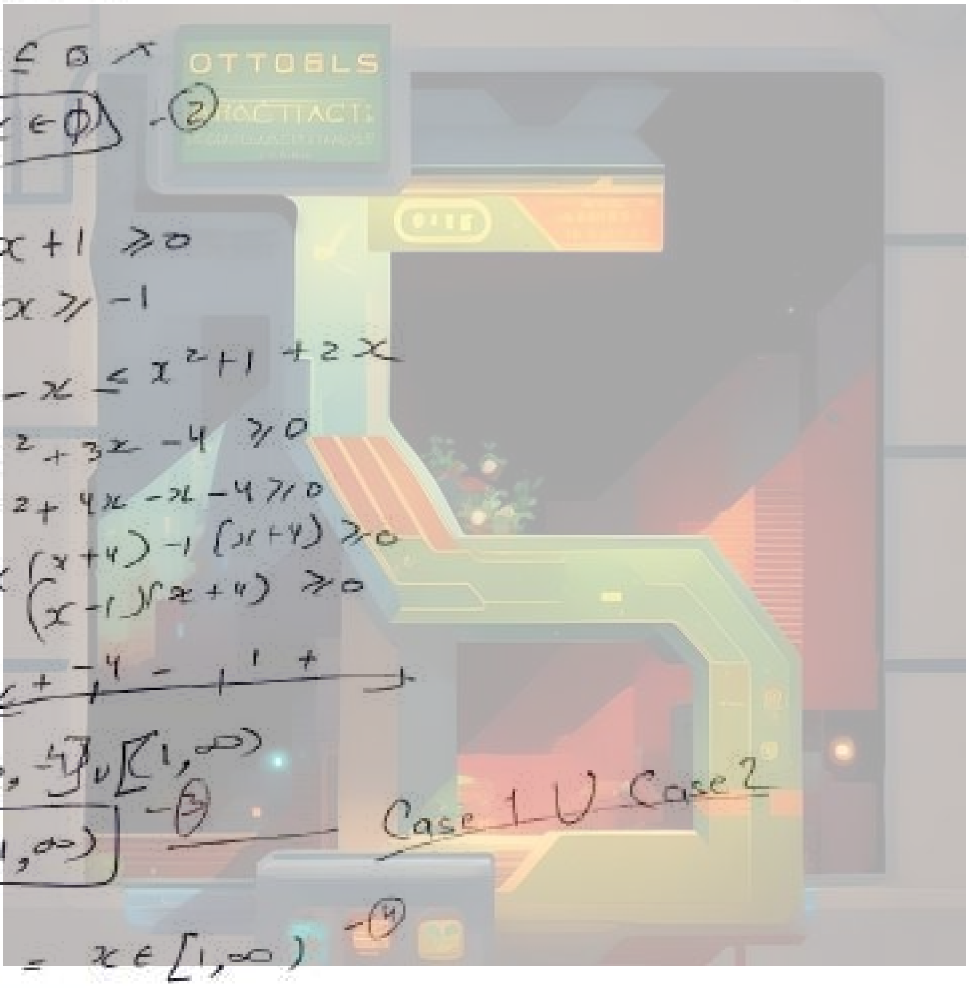
$5-x \geq 0$

$x-5 \leq 0$

$\boxed{x \leq 5}$ -①

Case ① $x+1 < 0$

$x < -1$



$\oplus \leq 0$ ×

$\boxed{x \in \phi}$ -②

Case ② $x+1 \geq 0$

$x \geq -1$

$5-x \leq x^2+1+2x$

$x^2+3x-4 \geq 0$

$x^2+4x-x-4 \geq 0$

$x(x+4)-1(x+4) \geq 0$

$(x-1)(x+4) \geq 0$

$+ \quad -4 \quad - \quad 1 \quad +$

$(-\infty,-4] \cup [1,\infty)$ -③

$\boxed{x \in [1,\infty)}$ Case 1 ∪ Case 2

② ∪ ③ = $x \in [1,\infty)$ -④

① $\boxed{[1,5]}$

Q $\sqrt{x+18} < 2 - x$

$x + 18 \geq 0$

$\boxed{x \geq -18}$

Case ① $2 - x < 0$

$-x < -2$

$x > 2$

$(+) < (-) \times$

$x \in \phi$

Case ② $2 - x > 0$

$x < 2$

$x + 18 < x^2 + 4 - 4x$

$x^2 - 5x - 14 > 0$

$x^2 - 7x + 2x - 14 > 0$

$x(x-7) + 2(x-7) > 0$

$(x+2)(x-7) > 0$

$\xleftarrow{\quad + \quad} -2 \quad - \quad 7 \xrightarrow{\quad + \quad}$

$(-\infty, -2) \cup (7, \infty)$

$(-\infty, -2) \cap [-18, \infty)$

$\boxed{x \in [-18, -2)}$

a

$$\sqrt{\frac{x-2}{1-2x}} > -1$$

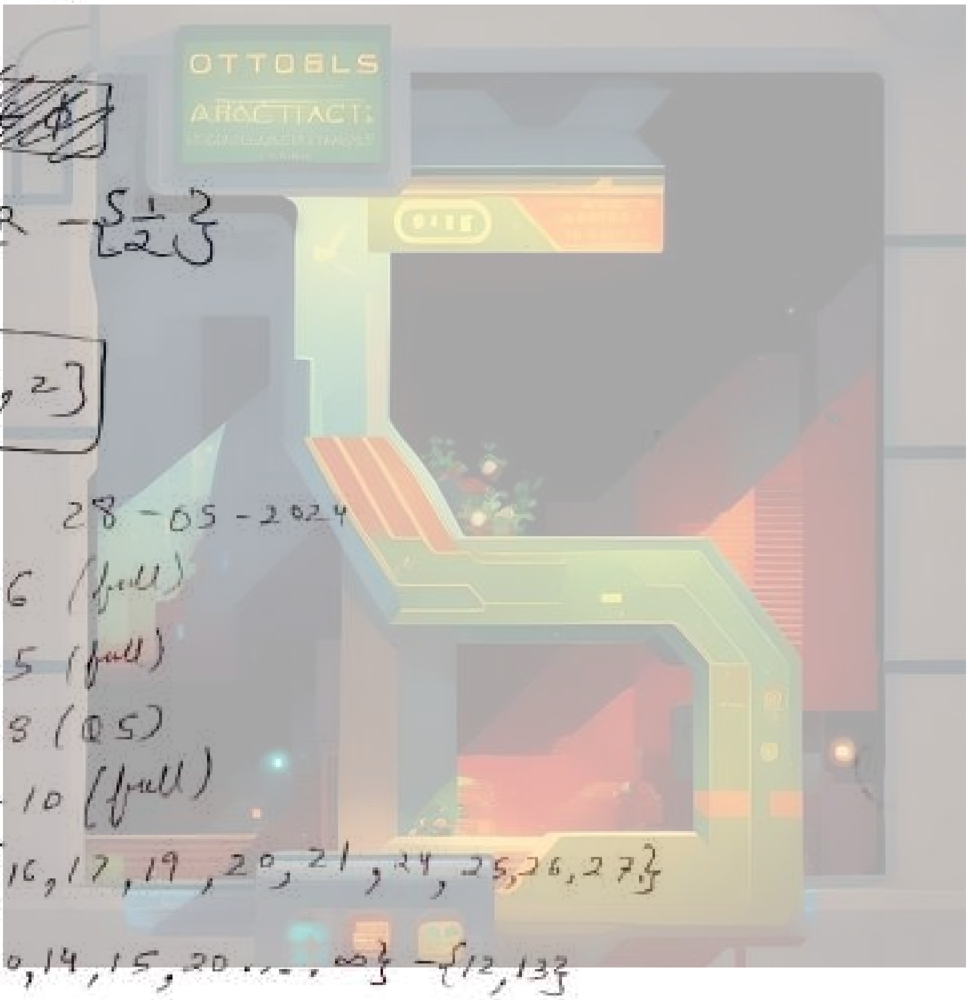
$$\frac{x-2}{1-2x} \geqslant 0$$

$$- \quad \frac{1}{2} \quad + \quad 2 \quad -$$

$$\left(\frac{1}{2}, 2\right]$$

$x \in R - \left\{\frac{1}{2}\right\}$

$$\boxed{\left(\frac{1}{2}, 2\right]}$$



H.W. 28-05-2024

DYS-6 (full)

DYS-5 (full)

DYS-8 (Q5)

DYS-10 (full)

O-1 {16, 17, 19, 20, 21, 24, 25, 26, 27}

O-2 {10, 14, 15, 20 . . . . ∞} − {12, 13}

~~O-3~~

Modulus Equality

① If $a = +ve$ ($a \to$ constant)

$|x| \le a \quad x \in [-a, a]$

$|x| < a \quad x \in (-a, a)$

$|x| \ge a \quad x \in (-\infty, -a] \cup [a, \infty)$

$|x| > a \quad x \in (-\infty, -a) \cup (a, \infty)$

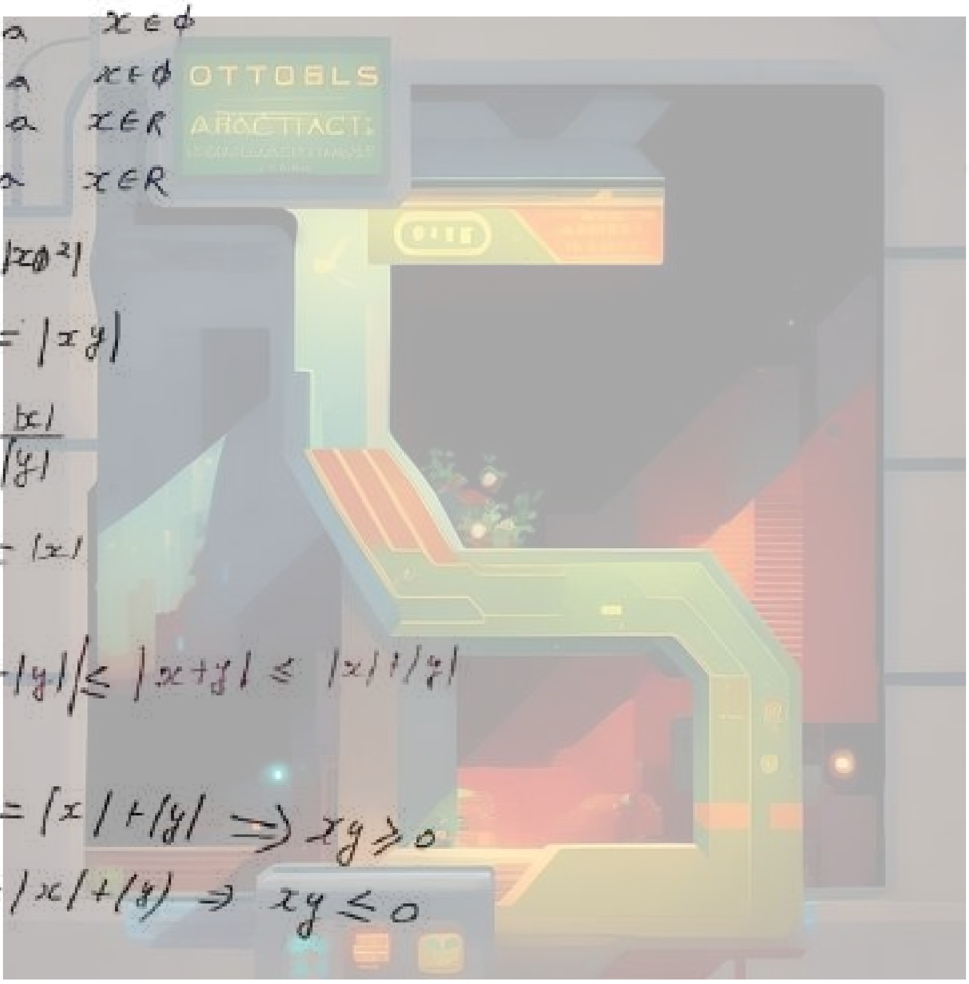
② If $a = -ve$ ($a \to$ constant)

$|x| \le a \quad x \in \phi$

$|x| < a \quad x \in \phi$

$|x| \ge a \quad x \in R$

$|x| > a \quad x \in R$



③ $|x|^2 = |x^2|$

④ $|x||y| = |xy|$

⑤ $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

⑥ $\sqrt{x^2} = |x|$

⑦ $\big||x| - |y|\big| \le |x+y| \le |x| + |y|$

⑧ $|x+y| = |x| + |y| \Rightarrow xy \ge 0$

$|x-y| = |x| + |y| \Rightarrow xy \le 0$

Q $|2x-7| = |x+3| + |x-10|$

$|x+3+2x-10| = |x+3| + |x-10|$

$(x+3)(x-10) \geq 0$

x 

$(-\infty, -3] \cup [10, \infty)$

Q $|x-2| + |x-7| = 5$

$(x-2)(x-7) \leq 0$

$\boxed{[2, 7]}$

Q $|x^2+6x+6| = |x^2+4x+9| + |2x-3|$

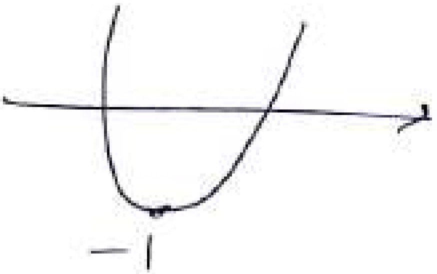
~~x^2+4x~~

$(x^2+4x+9)(2x-3) \geq 0$

$\boxed{\left[\frac{3}{2}, \infty\right)}$

Q-1

Q27.



as it is biggest -ve integer

$\frac{+p}{4} = -1$

$-p = -4$

$p = 4$

$2x^2 + 4x + 1$

$x = \frac{-4 \pm \sqrt{16 - 8}}{4}$

$= \frac{-4 \pm 2\sqrt{2}}{4}$

$= \frac{-2 \pm \sqrt{2}}{2}$

$x = \frac{-2 + \sqrt{2}}{2}$ × $x = \frac{-2 - \sqrt{2}}{2}$

$x \; \cancel{=} \;$

$x = \frac{(\sqrt{2} - 2)}{2}$ × $\frac{-(\sqrt{2} + 2)}{2}$

+ .

$2x^2 + px + 1 = -1$

$2x^2 + px + 2 = 0$

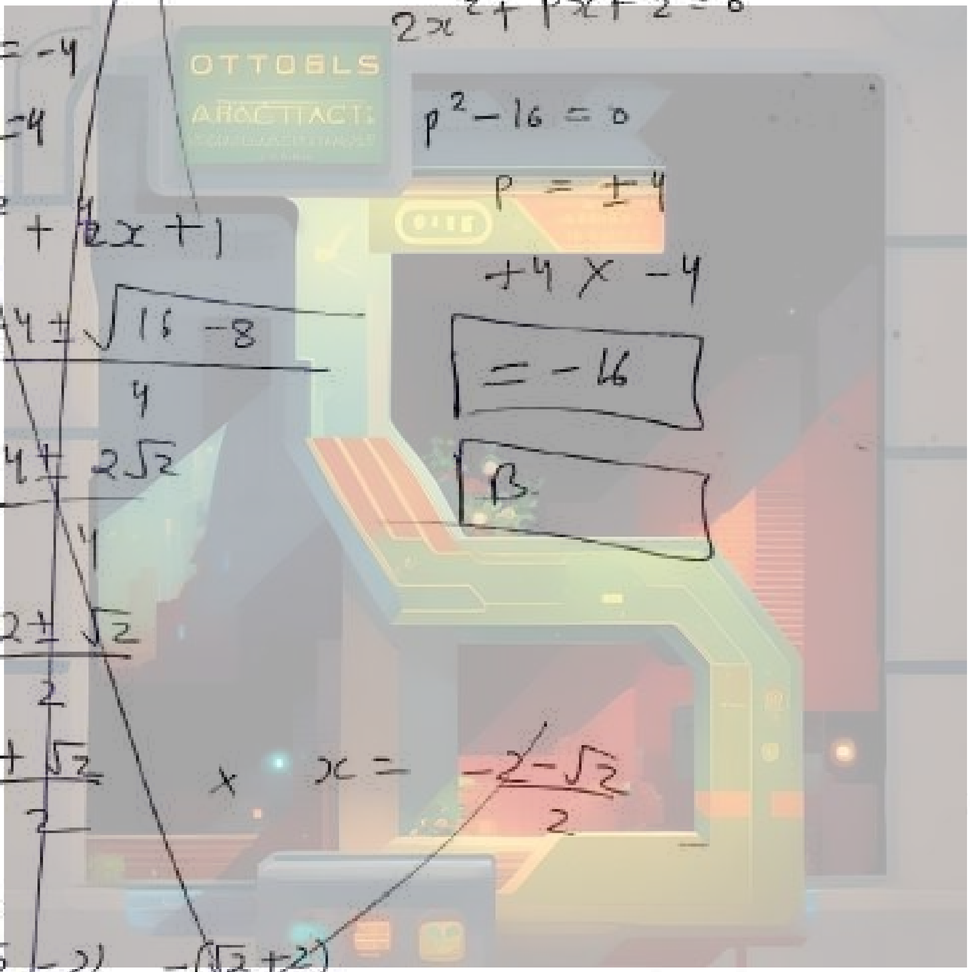
$p^2 - 16 = 0$

$p = \pm 4$

+4 × -4

$\boxed{= -16}$

$\boxed{B}$



Q find 'x'

$|x| \leq 2$

$|x| \leq a$

$x \in [-a, a]$

$\boxed{x \in [-2, 2]}$

Q $|x-3| \leq 2$

$x-3 \in [-2, 2]$

$\boxed{x \in [1, 5]}$

Q $|x| \geq 9$

~~$x \in [-9, 9]$~~

$\boxed{x \in (-\infty, -9] \cup [9, \infty)}$

Q $|x| < \sqrt{3}$

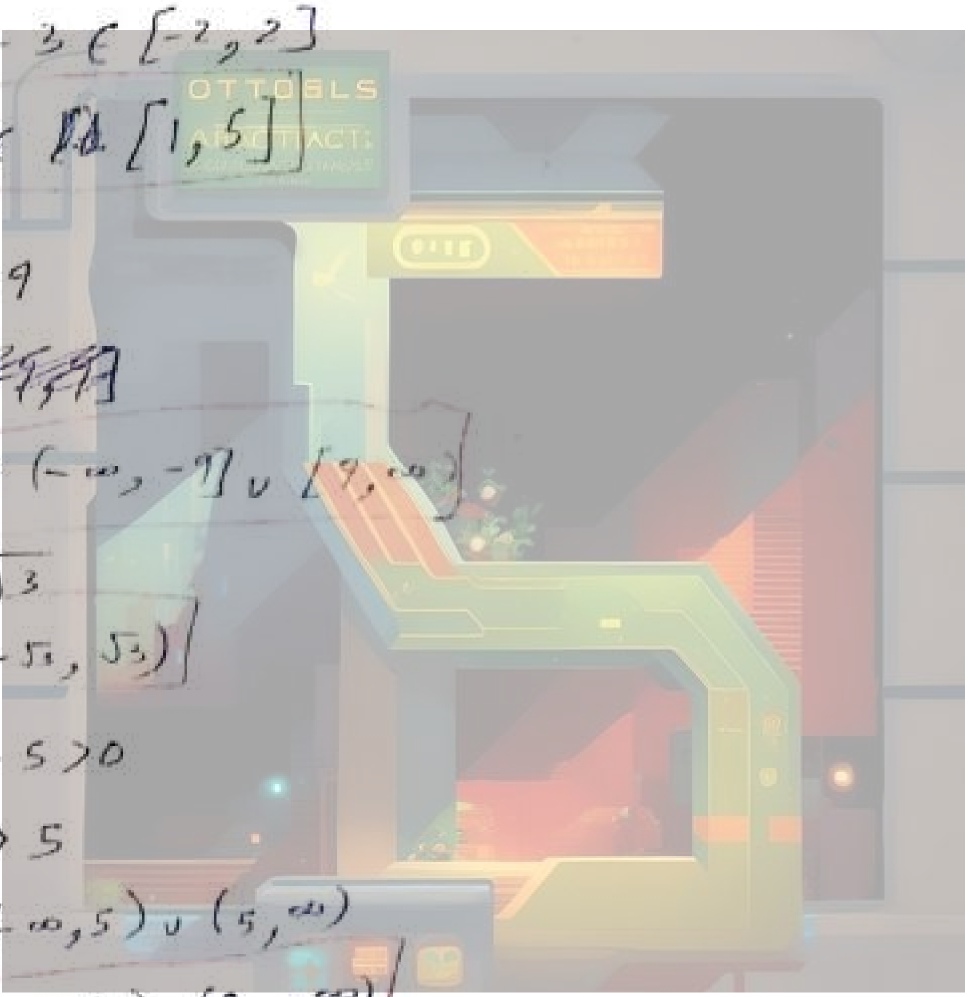
$\boxed{x \in (-\sqrt{3}, \sqrt{3})}$

Q $|2x| - 5 > 0$

$|2x| > 5$

$2x \in (-\infty, 5) \cup (5, \infty)$

$\boxed{x \in (-\infty, 5/2) \cup (5/2, \infty)}$



Q $|2-7x| < 8$

$2-7x \in (-8,8)$

$-7x \in (-10,6)$

~~$x \in \left(\frac{10}{7}, \frac{-6}{7}\right)$~~ $\boxed{x \in \left(-\frac{6}{7}, \frac{10}{7}\right)}$

Q no. of integerd valuesof x such that $4 \le |x-4| \le 10$

$|x-4| \le 10$

$x-4 \in [-10, 10]$

$x \in [-6, 14]$

$|x-4| \ge 4$

$x-4 \in (-\infty, -4] \cup [4, \infty)$

$x \in (-\infty, 0] \cup [8, \infty)$

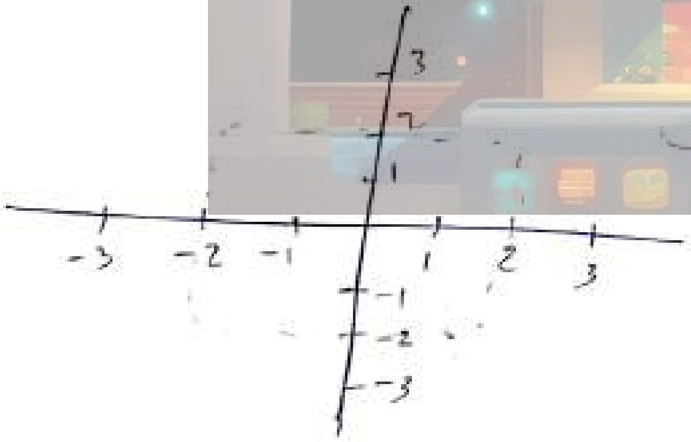
$\boxed{x \in [-6, 0] \cup [8, 14]}$

no. of integerd values $= 7+7$

$\boxed{= 14}$

Q $|x| \le 2$ & $|y| \le 2$

$x \in [-2, 2]$ $y \in [-2, 2]$



area $= 4 \times 4$

$\boxed{\text{Area} = 16}$ u

Q $||x-2|-1| \le 5$

$|x-2|-1 \in [-5, 5]$

$|x-2| \in [-4, 6]$

$-4 \le |x-2| \le 6$

$|x-2| \ge -4$

~~$|x-2|$~~

$x \in R$

$|x-2| \le 6$

$x-2 \in [-6, 6]$

$\boxed{x \in [-4, 8]}$

Q $|x-3| < x-3$

~~$|x-3| < x-3$~~

~~$x-3 \in (3-x, x-3)$~~

~~$x \in (6-x, x)$~~

~~$6-x \le x \le x$~~

~~$6-x < x$~~ ~~$x < x$~~

~~$6 < 2x$~~ ~~$0 < 2x$~~

~~$\boxed{3 < x}$~~ ~~$0 < R$~~

~~$x \in R$~~

$x-3 > 0$

$\boxed{x > 3}$

$|x-3| < x-3$

~~$x-3 < x-3$~~

$x-3 \in (3-x, x-3)$

~~$3 < x$~~

$x \in (6-x, x)$

$6-x < \boxed{x < x}$

$\boxed{x \in \phi}$

Theory of Equation

→ Deriving results for polynomials with degree 3 or more.

Let $a \neq 0$

① $ax^2+bx+c=0 \begin{array}{l}\nearrow \alpha\\ \searrow \beta\end{array}$

$$\alpha+\beta=\frac{-b}{a}$$

$$\alpha\beta=\frac{c}{a}$$

② $ax^3+bx^2+cx+d=0 \begin{array}{l}\nearrow \alpha\\ \rightarrow \beta\\ \searrow \gamma\end{array}$

$$\alpha+\beta+\gamma=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{-d}{a}$$

③ $ax^4+bx^3+cx^2+dx+e \begin{array}{l}\nearrow \alpha\\ \rightarrow \beta\\ \searrow \gamma\\ \searrow \delta\end{array}$

$$\alpha+\beta+\gamma+\delta=\frac{-b}{a}$$ (sum)

$$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\delta=\frac{c}{a}$$ (2-2 sum)

$$\alpha\beta\gamma+\beta\gamma\delta+\alpha\beta\delta+\alpha\gamma\delta=\frac{-d}{a}$$ (3-3 sum)

$$\alpha\beta\gamma\delta=\frac{e}{a}$$

general

$$f(x) = \overset{+}{a_0} x^n + \overset{-}{a_1} x^{n-1} + \overset{+}{a_2} x^{n-2} + \overset{-}{a_3} x^{n-3} \ldots\ldots + a_n$$

⊖ for odd n

⊕ for even n

$$\boxed{\text{Sum of Roots} = -\frac{a_1}{a_0}}$$

$$2\text{-}2 \text{ sum} = \frac{a_2}{a_0}$$

$$3\text{-}3 \text{ sum} = \frac{-a_3}{a_0}$$

$$4\text{-}4 \text{ sum} = \frac{a_4}{a_0}$$

$$\boxed{\text{product} = \frac{(-1)^n a_n}{a_0}}$$

Q $2x^3 - 5x^2 + 4x - 1 = 0$ have roots α, β & γ

find ① $\alpha^2+\beta^2+\gamma^2$

② $\alpha^3+\beta^3+\gamma^3$

$$(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2 + 2(\alpha\beta+\beta\gamma+\alpha\gamma)$$

$$\left(\frac{5}{2}\right)^2 = \alpha^2+\beta^2+\gamma^2 + 2(2)$$

$$\frac{25-16}{4} = \alpha^2+\beta^2+\gamma^2$$

$$\boxed{\frac{9}{4} = \alpha^2+\beta^2+\gamma^2}$$

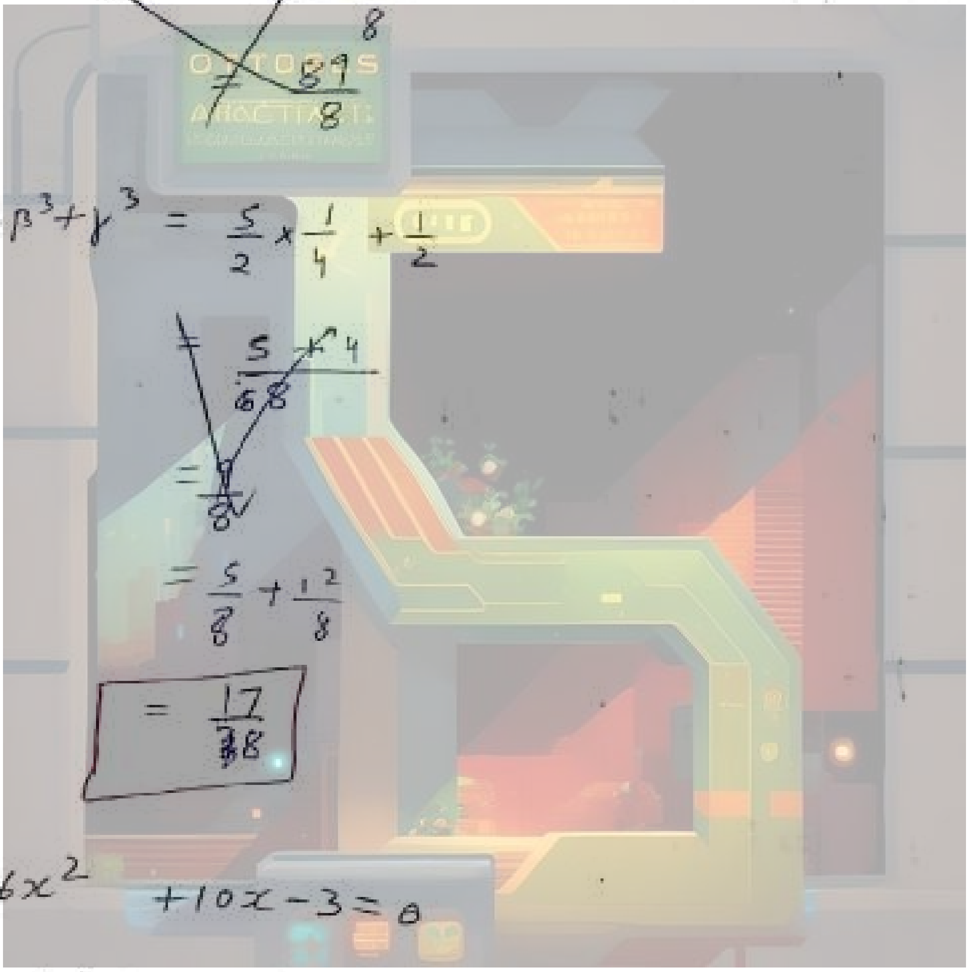
② $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\alpha\gamma)$

$\alpha^3+\beta^3+\gamma^3 - \frac{-1}{2} = \left(\frac{5}{2}\right)\left(\frac{9}{4}+2\right)$

$\alpha^3+\beta^3+\gamma^3 = \frac{85}{8}+\frac{1}{2}$

$= \frac{85+4}{8}$

$= \frac{89}{8}$



$\alpha^3+\beta^3+\gamma^3 = \frac{5}{2}\times\frac{1}{4}+\frac{1}{2}$

$= \frac{5+4}{8}$

$= \frac{9}{8}$

$= \frac{5}{8}+\frac{12}{8}$

$\boxed{= \frac{17}{8}}$

Q $x^3-6x^2+10x-3=0$

$\left(\alpha-\frac{1}{\beta\gamma}\right)\left(\beta-\frac{1}{\alpha\gamma}\right)\left(\gamma-\frac{1}{\alpha\beta}\right)$

$\frac{(\alpha\beta\gamma-1)}{\beta\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\beta}$

$\frac{(3-1)^3}{(3)^2} = \boxed{\frac{8}{9}}$

Q $x^3-3x^2+2x+1=0$

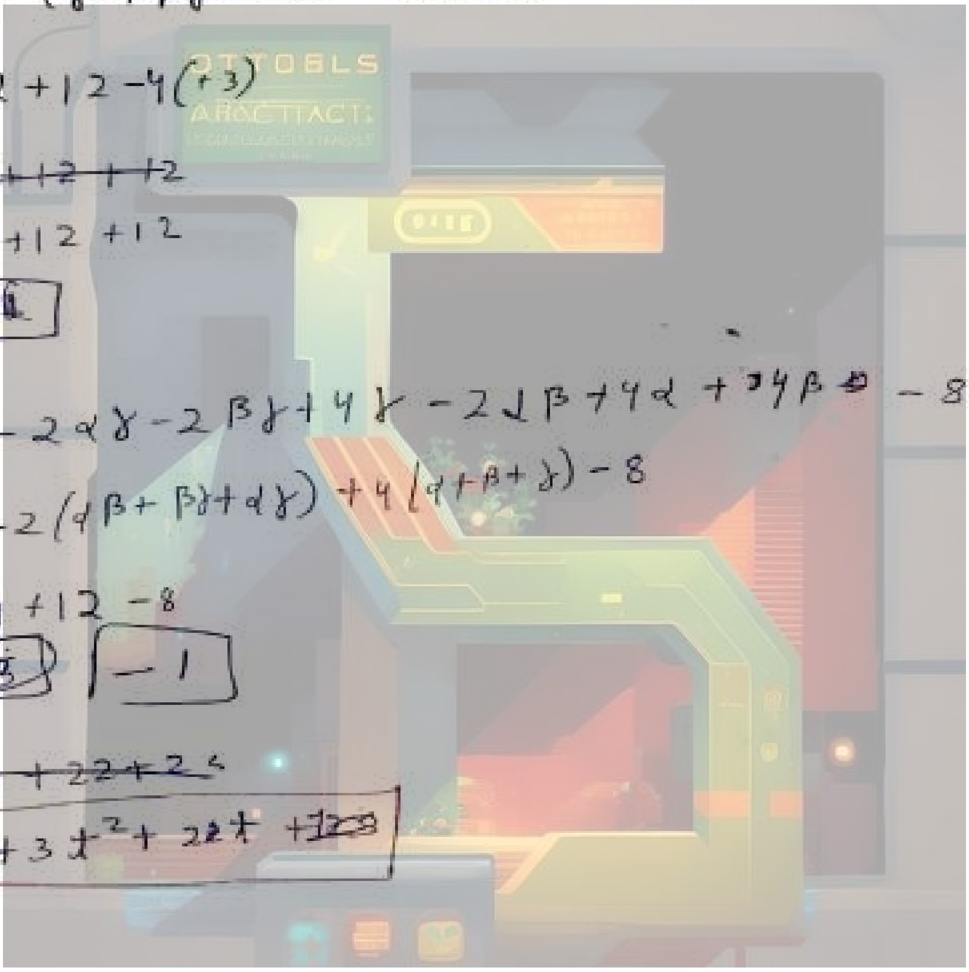
① $(\alpha-2)(\beta-2)(\gamma-2)$

$\alpha+\beta+\gamma-6 =$

$\boxed{=-3}$

$\alpha\beta-2\alpha-2\beta+4+\beta\gamma-2\beta-2\gamma+4+\alpha\gamma-2\alpha-2\gamma+4$

$\alpha\beta+\alpha\gamma+\beta\gamma+12-4\cancel{4}(\alpha+\beta+\gamma)$



$-2+12-4(+3)$

$\cancel{-2+12+12}$

$-2+12+12$

$\boxed{=22}$

$\alpha\beta\gamma-2\alpha\gamma-2\beta\gamma+4\gamma-2\alpha\beta+4\alpha+4\beta-8$

$\alpha\beta\gamma-2(\alpha\beta+\beta\gamma+\alpha\gamma)+4(\alpha+\beta+\gamma)-8$

$-1-4+12-8$

$\boxed{\cancel{-29}}$ $\boxed{-1}$

$\cancel{x^3+3+22+2}$

$\boxed{x^3+3x^2+22x+\cancel{123}}$

⑥

② cubic wth roots $\frac{\alpha+1}{2}, \frac{\beta+1}{2}, \frac{\gamma+1}{2}$

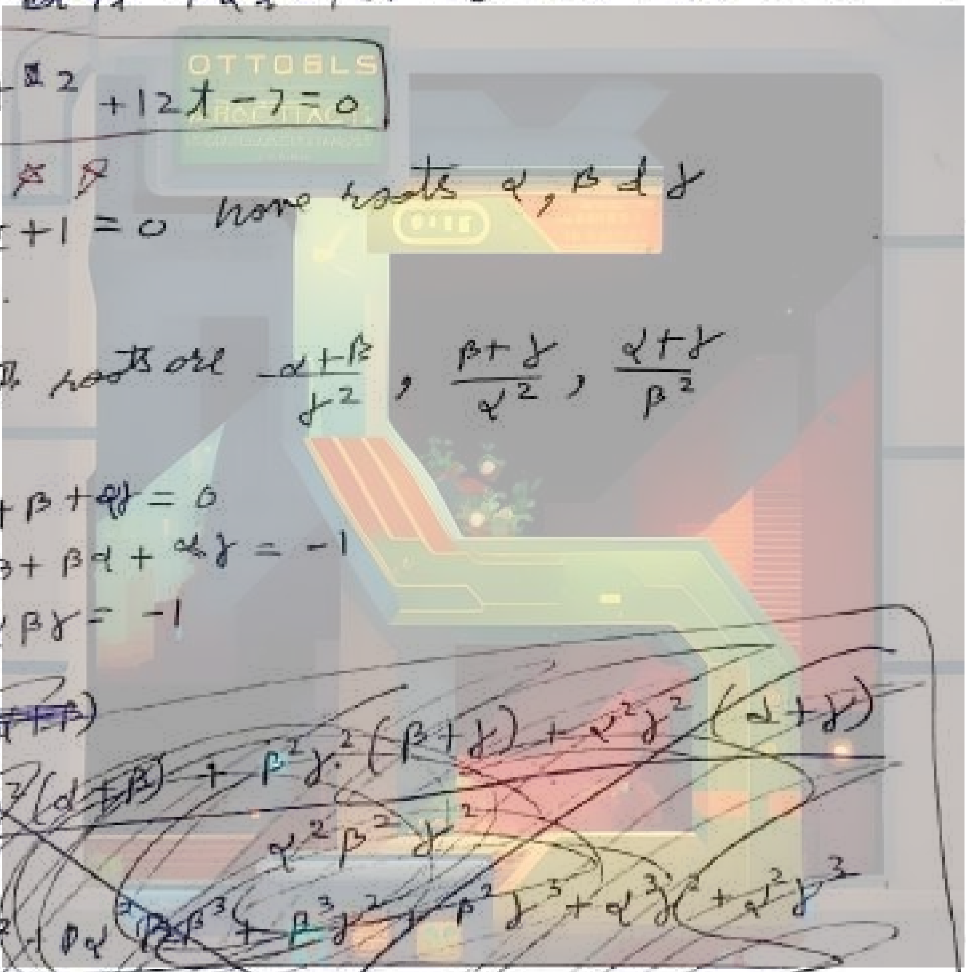
$\frac{x+1}{2} = t$

$x + 1 = 2t$

$x = 2t - 1$

$(2t-1)^3 - 3(2t-1)^2 + 2(2t-1) + 1 = 0$

$8t^3 - 3 - \cancel{\alpha}\, 4t^2 + 2t - 12t^2 - 3 + 6t + 4t - 2 + 1 = 0$



$\boxed{8t^3 - 16t^2 + 12t - 7 = 0}$

Q $x^3 - x + 1 = 0$ have roots α, β & γ

find.

① Eq with roots are $\frac{\alpha+\beta}{\gamma^2}, \frac{\beta+\gamma}{\alpha^2}, \frac{\alpha+\gamma}{\beta^2}$

$\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\alpha + \alpha\gamma = -1$

$\alpha\beta\gamma = -1$

~~$\frac{\alpha^2\beta^2(\alpha+\beta) + \beta^2\gamma^2(\beta+\gamma) + \alpha^2\gamma^2(\alpha+\gamma)}{\alpha^2\beta^2\gamma^2}$~~

~~$\frac{\alpha^3\beta^2 + \beta\alpha^2\beta^3 + \beta^3\gamma^2 + \beta^2\gamma^3 + \alpha^3\gamma^2 + \alpha^2\gamma^3}{(\alpha\beta\gamma)^2}$~~

$$\frac{\alpha+\beta}{\gamma^2}$$

$$\alpha+\beta+\gamma=0$$

$$\alpha+\beta=-\gamma$$

$$\frac{-\gamma}{\gamma^2} = -\frac{1}{\gamma}, -\frac{1}{\alpha}, -\frac{1}{\beta}$$ are roots.

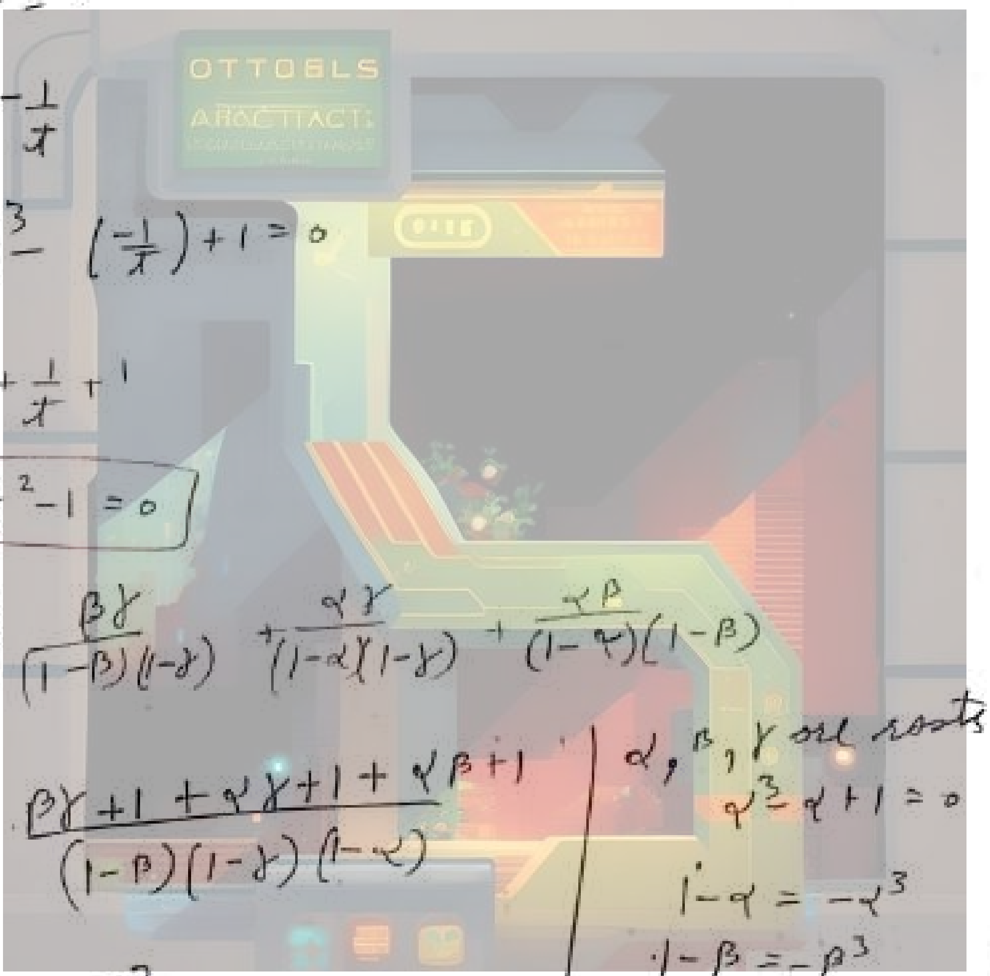
$$-\frac{1}{x} = t$$

$$x = -\frac{1}{t}$$

$$\left(\frac{-1}{t}\right)^3 - \left(\frac{-1}{t}\right)+1=0$$

$$\frac{-1}{t^3}+\frac{1}{t}+1$$

$$\boxed{t^3+t^2-1=0}$$



② Value of $\frac{\beta\gamma}{(1-\beta)(1-\gamma)}+\frac{\alpha\gamma}{(1-\alpha)(1-\gamma)}+\frac{\alpha\beta}{(1-\alpha)(1-\beta)}$

$$\frac{\beta\gamma+1+\alpha\gamma+1+\alpha\beta+1}{(1-\beta)(1-\gamma)(1-\alpha)}$$

α, β, γ are roots

$$\alpha^3-\alpha+1=0$$

$$1-\alpha=-\alpha^3$$

$$1-\beta=-\beta^3$$

$$1-\gamma=-\gamma^3$$

$$=\frac{2}{(1-\beta)(1-\gamma)(1-\alpha)}$$

$$=\frac{2}{-(\alpha\beta\gamma)^3}$$

$$=\frac{2}{1}$$

(G2) $\boxed{=2}$ $\boxed{=2}$

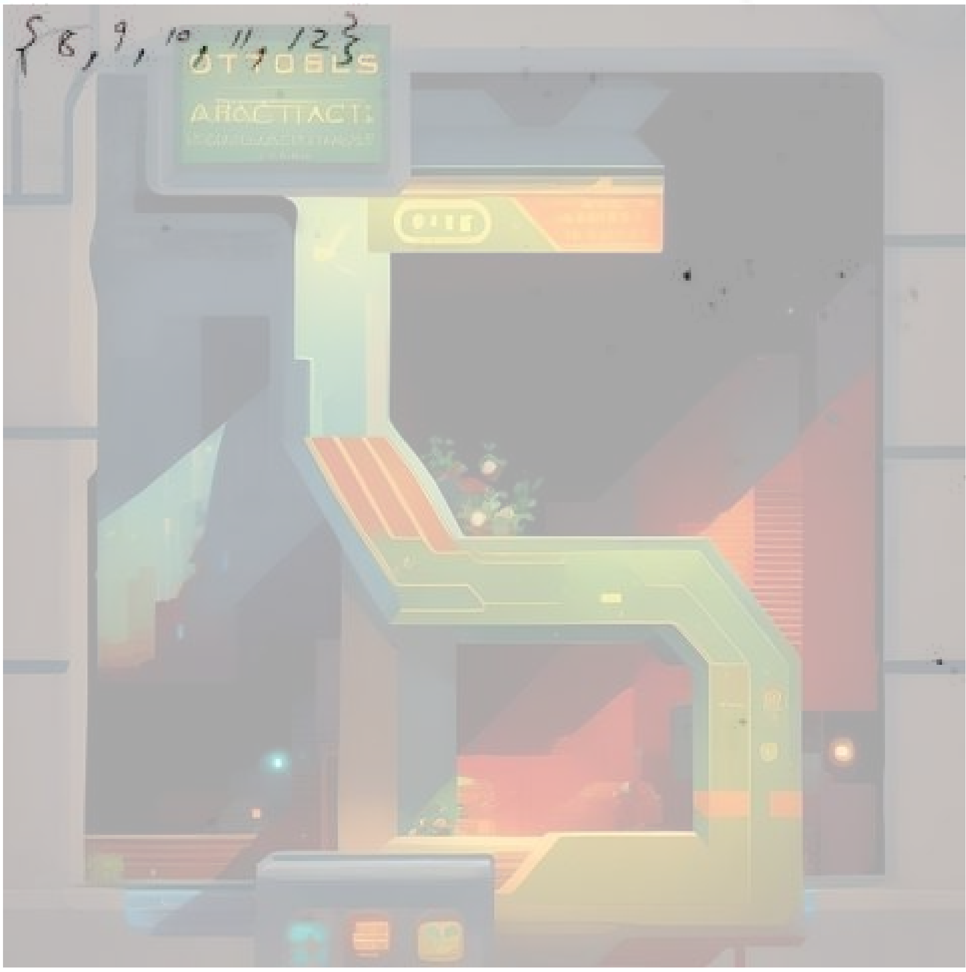
Q $\boxed{\begin{aligned}&\sum \alpha = \alpha+\beta+\gamma\\ &\sum \alpha^2 = \alpha^2+\beta^2+\gamma^2\end{aligned}}$

H.W. 30-5-2024

O-1 {28, 29, 30} *

O-3 {4-6, 9, 10}

Q J-M {8, 9, 10, 11, 12}



General quadratic equation in two variable

$$f(x,y) = ax^2+by^2+2hxy+2gx+2fy+c$$

↳ two linear factors when

$$\boxed{abc+2fgh-af^2-bg^2-ch^2=0}$$

Proof:- consider quadratic in x

$$\underbrace{ax^2}_{x^2} + \underbrace{(2hy+2g)x}_{x} + \underbrace{by^2+2fy+c}_{\text{constant}} = 0$$

$$x = \frac{-(2hy+2g) \pm \sqrt{(2hy+2g)^2-4(a)(by^2+2fy+c)}}{2a}$$

~~$x = -2hy - 2g$~~

Q find weather $x^2+2xy+2x+6y-3=0$ have two of linear factors or not.

$2\times 3 - 9 + 3 = 0$

So it ~~not~~ resolved in 2 linear factors.

Q $x^2+2xy+2x+ky^2+k=0$ find k if the above equation has two linear factors

$a=1$	$k^2-k-k=0$
$b=k$	$k^2-2k=0$
$c=k$	$\boxed{k=0,2}$
$h=1$	
$g=1$	
$f=0$	

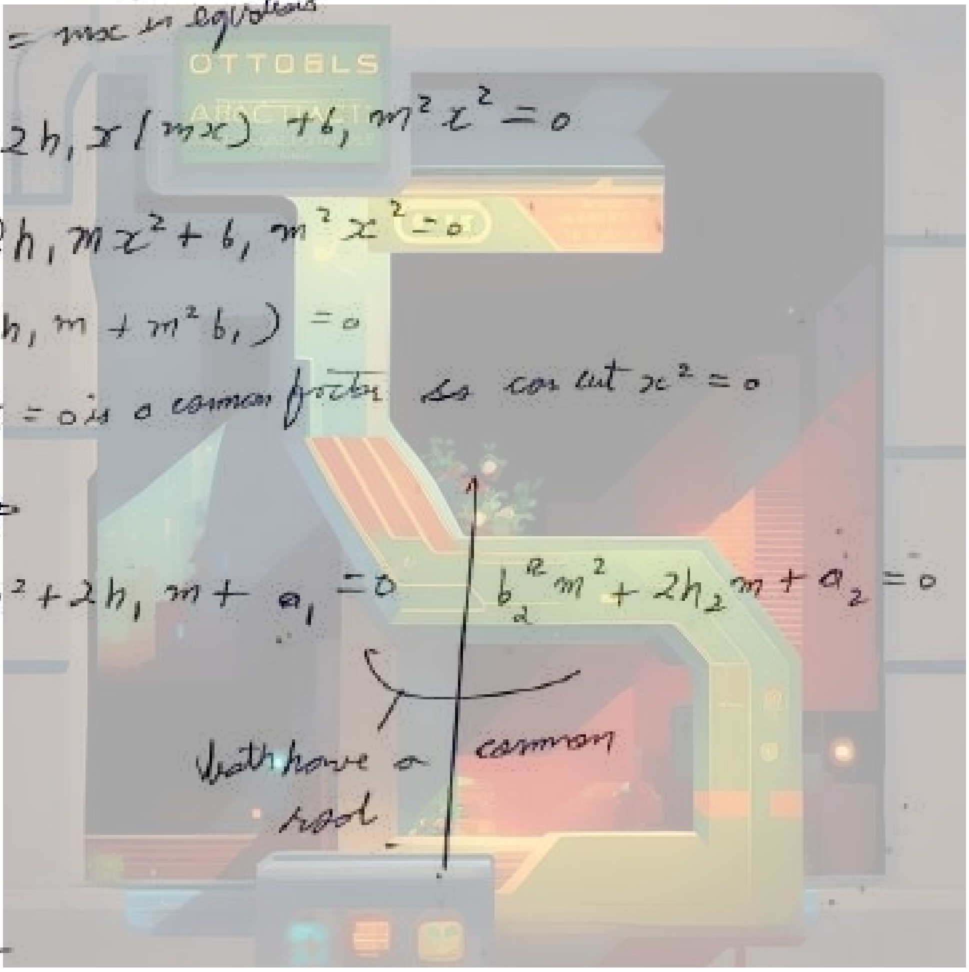
Type-2 - when two homogeneous equation have ~~common~~ common linear factors.

Homogenity → when degree of all terms is same (xy degree $= 2$)

$a_1x^2 + 2h_1xy + b_1y^2 = 0$ $\qquad$ $a_2x^2 + 2h_2xy + b_2y^2 = 0$

Assume $y - mx = 0$ is a common factor

$x = 0$ is a factor



put $y = mx$ in equations

$a_1x^2 + 2h_1x(mx) + b_1m^2x^2 = 0$

$a_1x^2 + 2h_1mx^2 + b_1m^2x^2 = 0$

$x^2(a_1 + 2h_1m + m^2b_1) = 0$

we know $x = 0$ is a common factor so can cut $x^2 = 0$

~~$a_1 + 2h_1$~~

$b_1m^2 + 2h_1m + a_1 = 0$ $\qquad$ $b_2^2m^2 + 2h_2m + a_2 = 0$

both have a common root

Method 2:-

$m^2b_1 + 2h_1m + a_1 = 0$

$m^2b_2 + 2h_2m + a_2 = 0$

$$\frac{m^2}{\begin{matrix}2h_1 & a_1\\ 2h_2 & a_2\end{matrix}} = \frac{-m}{\begin{matrix}b_1 & a_1\\ b_2 & a_2\end{matrix}} = \frac{1}{\begin{matrix}b_1 & 2h_1\\ b_2 & 2h_2\end{matrix}}$$

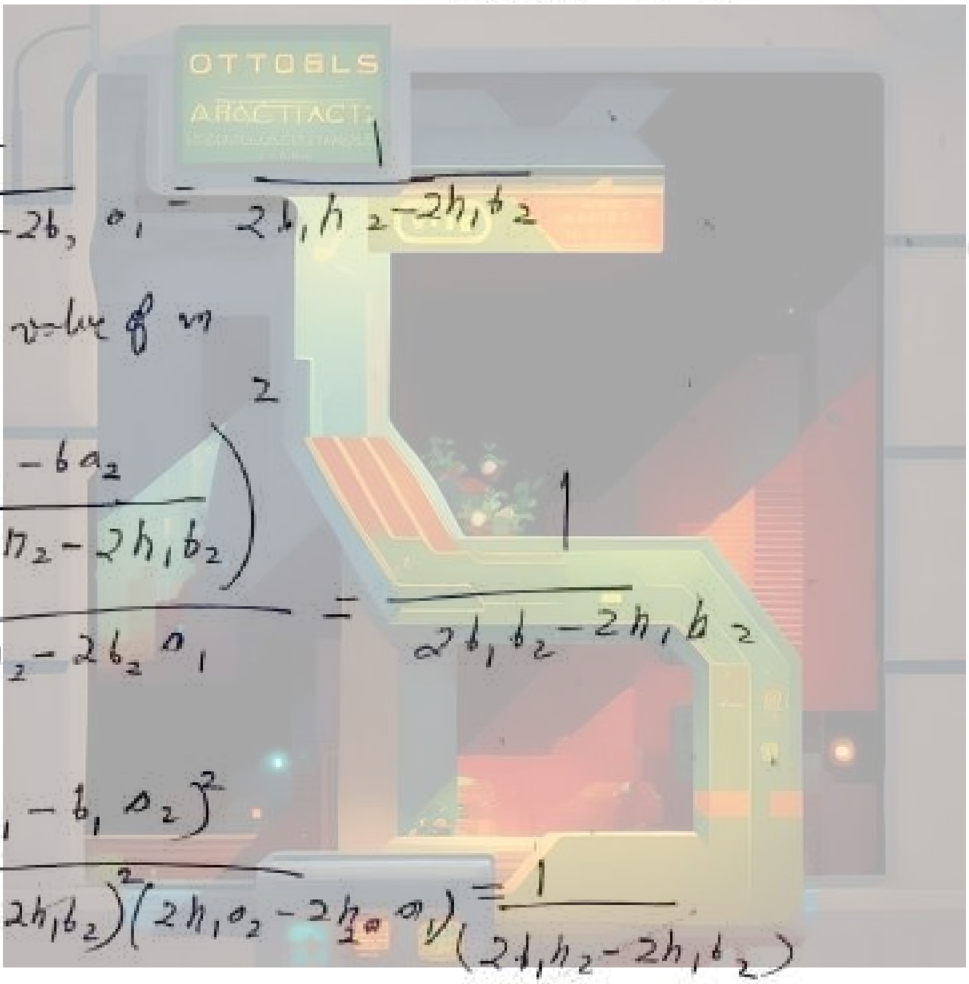
↓

Cross multiply

$$\frac{m^2}{2h_1a_2-2h_2a_1} = \frac{-m}{b_1a_2-b_2a_1} = \frac{1}{2b_1h_2-2h_1b_2}$$

~~$m = b_1a_2-b_2a_1$~~

$$m = \frac{b_2a_1-b_1a_2}{2b_1h_2-2h_1b_2}$$



Now -

$$\frac{m^2}{2b_1a_2-2b_2a_1} = \frac{1}{2b_1h_2-2h_1b_2}$$

Put value of m

$$\frac{\left(\frac{b_2a_1-ba_2}{2b_1h_2-2h_1b_2}\right)^2}{2h_1a_2-2b_2a_1} = \frac{1}{2b_1b_2-2h_1b_2}$$

$$\frac{(b_2a_1-b_1a_2)^2}{(2b_1h_2-2h_1b_2)^2(2h_1a_2-2h_2a_1)} = \frac{1}{(2b_1h_2-2h_1b_2)}$$

$$\frac{(b_2a_1-b_1a_2)^2}{(2b_1h_2-2h_1b_2)(2h_1a_2-2h_2a_1)} = 1$$

$$(b_2a_1-b_1a_2)^2 = (2b_1h_2-2h_1b_2)(2h_1a_2-2h_2a_1)$$

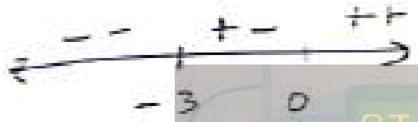
DYS-11 - Complete H.W.

~~[illegible]~~ J-A - Complete

J-M

Q16.

$$\frac{|x+3|-1}{|x|-2} \geq 0$$



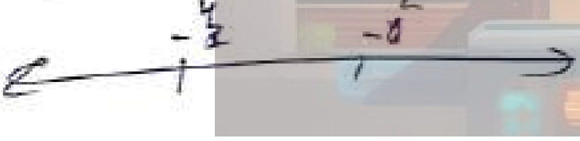
Case ①

$x \in (-\infty, -3)$

$$\frac{-(x+3)-1}{-x-2} \geq 0$$

$$\frac{[x+3+1]}{[x+2]} \geq 0$$

$$\frac{x+4}{x+2} \geq 0$$



$x \in (-\infty, -4] \cup (-2, \infty)$

$\boxed{x \in (-\infty, -4)}$

Case ②

$x \in [-3, 0]$

$$\frac{x+3-1}{-(x+2)} \geq 0$$

$$\frac{\cancel{x+2}}{-(\cancel{x+2})} \geq 0$$

$$\ominus \geq 0$$

$x \in \phi$

Case ①

$x \in (0, \infty)$

$$\frac{x+3-1}{x-2} \geq 0$$

$$\frac{x+2}{x-2} \geq 0$$

$x \in (-\infty, -2] \cup (2, \infty)$

$\boxed{x \in (2, \infty)}$

$x \in [-6, -2) \cup (-2, 2) \cup (2, 3]$

$\boxed{x \in [-6, -4] \cup (2, 3]}$

Part 2 - $x^2-7|x|+9 \le 0$

$x^2 = |x|^2$

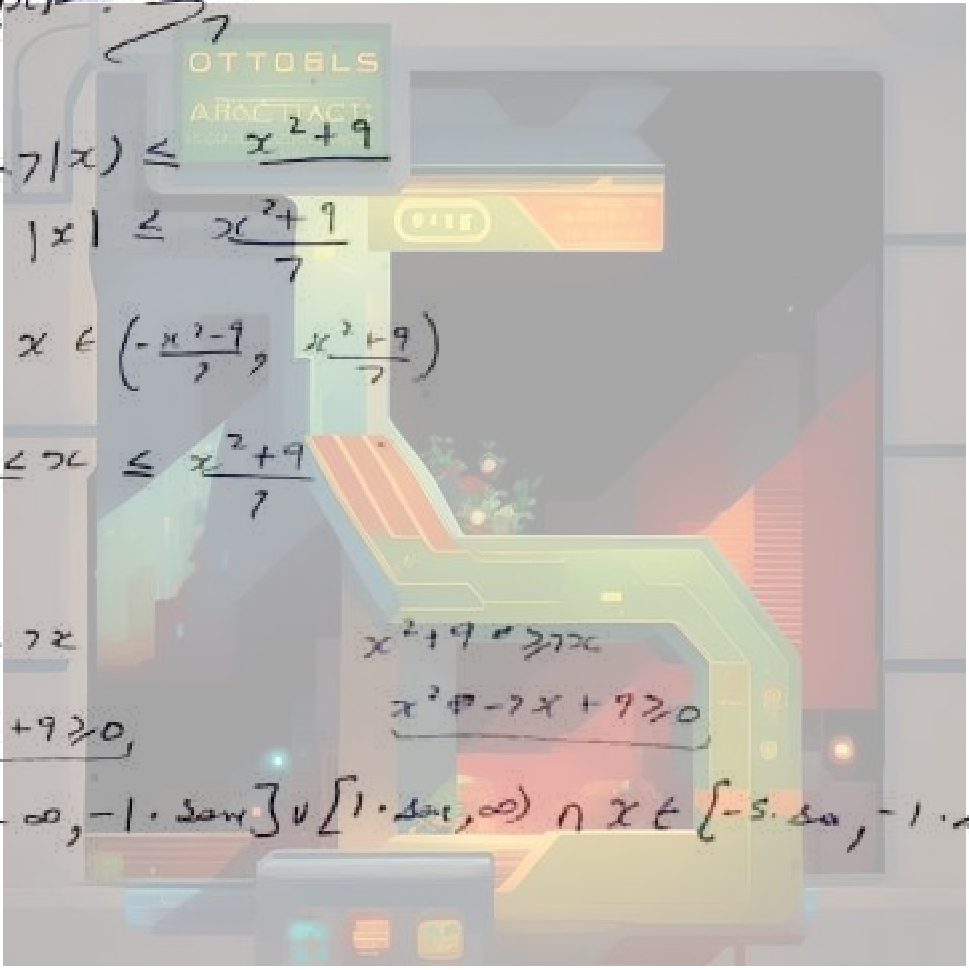
$|x|^2-7|x|+9 \le 0$

~~$t^2-7t+9 \le 0$~~

~~$t \le 5$~~

~~$+7|x| = x^2+9$~~

~~$|x| \le \frac{x^2+9}{7}$~~



$-7|x| \le \frac{x^2+9}{}$

$|x| \le \frac{x^2+9}{7}$

$x \in \left(\frac{-x^2-9}{7}, \frac{x^2+9}{7}\right)$

$\frac{-x^2-9}{7} \le x \le \frac{x^2+9}{7}$

$-x^2-9 \le 7x$ $\qquad$ $x^2+9 \ge 7x$

$\underbrace{x^2+7x+9 \ge 0,}$ $\qquad$ $\underbrace{x^2-7x+9 \ge 0}$

$x \in (-\infty, -1.\text{sam}] \cup [1.\text{sam}, \infty) \cap x \in [-5.\text{sa}, -1.\text{sam}] \cup [1.\text{sa}, 5.\text{sam}]$

$x \in [-5.\text{sam}, -1.\text{sam}] \cup [1.\text{sam}, 5.\text{sam}]$

Q 10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

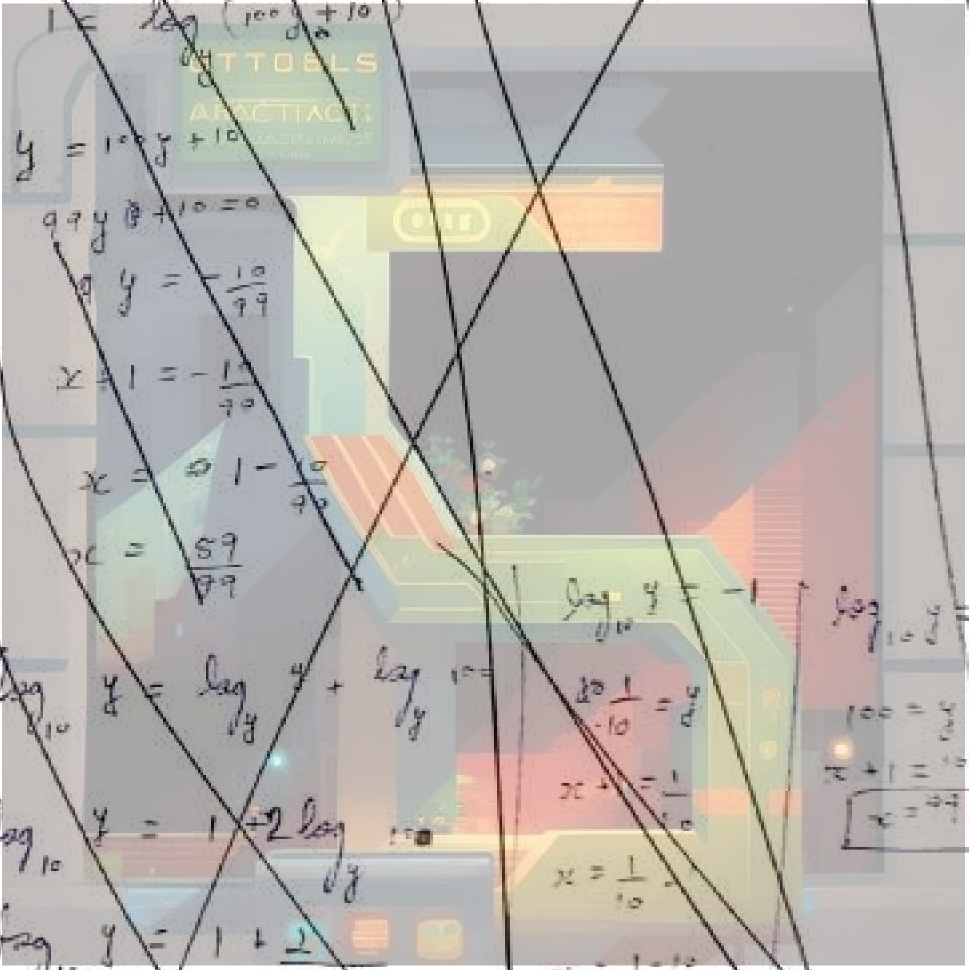
$x+1 = y$

$y^{\log_{10} y} = 100y$

take $\log_y$ both sides

$\log_{10} y = \log_y 100y$

$1 = \log(100y + 10)$



$y = 100y + 10$

$99y + 10 = 0$

$y = -\frac{10}{99}$

$x + 1 = -\frac{10}{99}$

$x = 1 - \frac{10}{99}$

$x = \frac{59}{99}$

$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$\log_{10} y = 1 + \frac{2}{\log_{10} y}$

$z = 1 + \frac{2}{z}$

$z^2 = z + 2$

$z^2 - z - 2 = 0$

$z = 2, -1$

$\log_{10} y = -1$

$\frac{1}{10} = y$

$x + 1 = \frac{1}{10}$

$x = \frac{1}{10} - 1$

$x = \frac{1-10}{10}$

$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$100 = y$

$x + 1 = 100$

$\boxed{x = 99}$

!! Logarithm !!

→ Every ⊕ve real number N can be expressed in exponential form as.

$$a^{x} = N$$

a → ⊕ve real ~~no~~ number > 0 but ≠ 1

x → Exponent

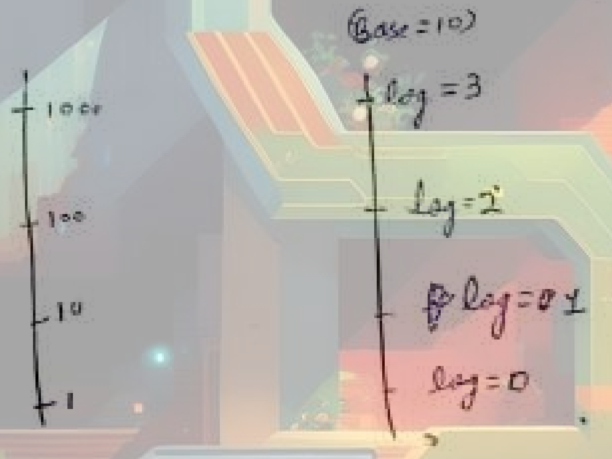
eg. $2^{2} = 4$, $\left(\frac{1}{2}\right)^{4} = \frac{1}{16}$ etc

Reason I

$2^{?} = 4$, $\left(\frac{1}{2}\right)^{?} = \frac{1}{16}$ $(8.2)^{?} = 16.32$

To find ?.

Reason II



For the above reasons we introduced log. It is expressed as

$$\log_{a} N = x$$

a → Base
x → exponent

[a की Power को कितने से Raise करें की N aa jaye.]

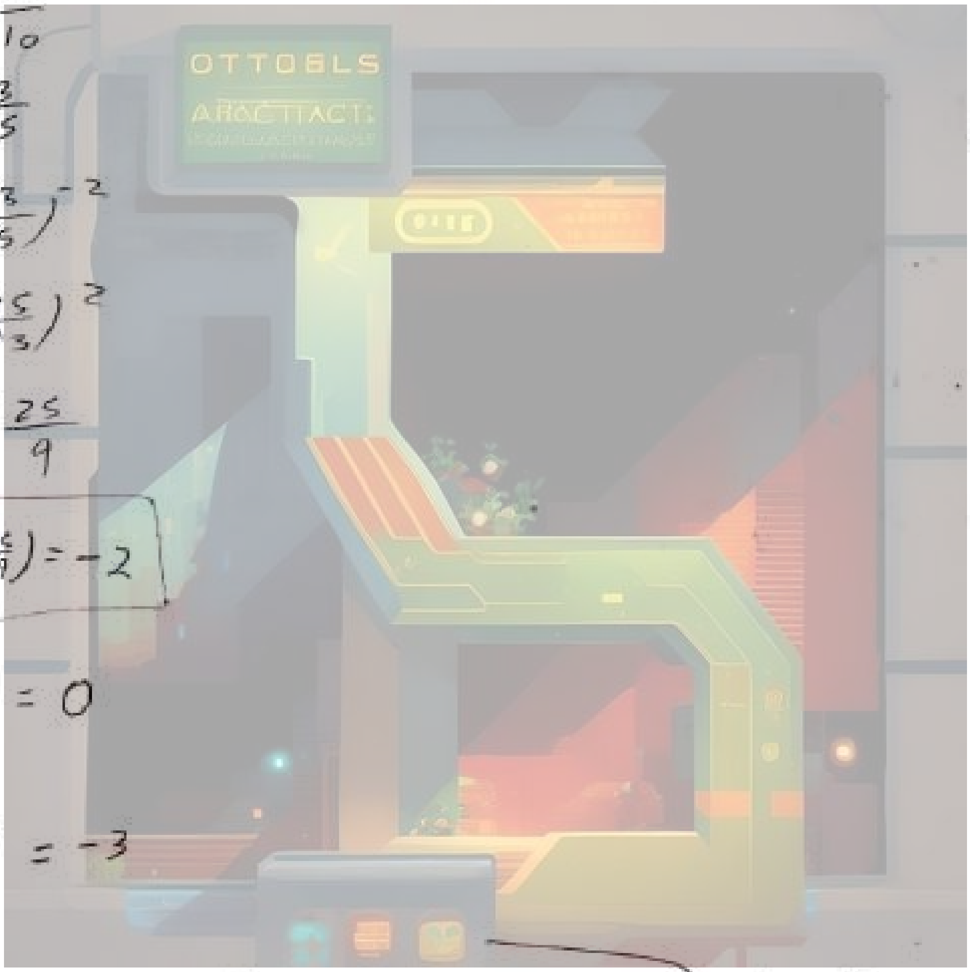
(74)

eg. $\log_2 8 = 3$ [2 की Power को कितने Raise करें कि 8 aaye]

$\log_6 216 = 3$

$\log_2 \frac{1}{16} = -4$

$\log_{0.6}\left(\frac{25}{9}\right) = \cancel{}$



$0.6 = \frac{6}{10}$

$= \frac{3}{5}$

$= \left(\frac{3}{5}\right)^{-2}$

$= \left(\frac{5}{3}\right)^{2}$

$= \frac{25}{9}$

$\boxed{\log_{0.6}\left(\frac{25}{9}\right) = -2}$

$\log_{\frac{1}{2}}(1) = 0$

$\log_{\frac{1}{3}} 27 = -3$

$\log_1\left(\frac{1}{2}\right) =$ Not Defined

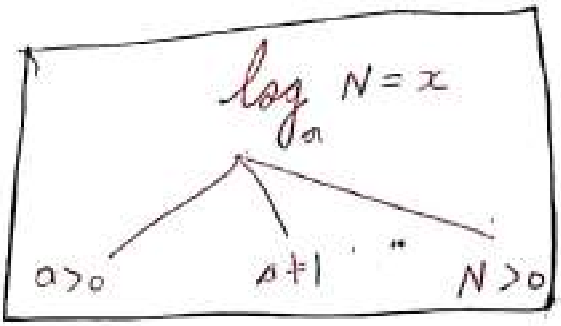
$\log_{(-2)}(16) = 4$ (Wrong)

$\cancel{\sqrt{}}16^{1/4} = \sqrt{4} = |\pm 2| = 2 \neq -2$

Thus Base is ⊕ve

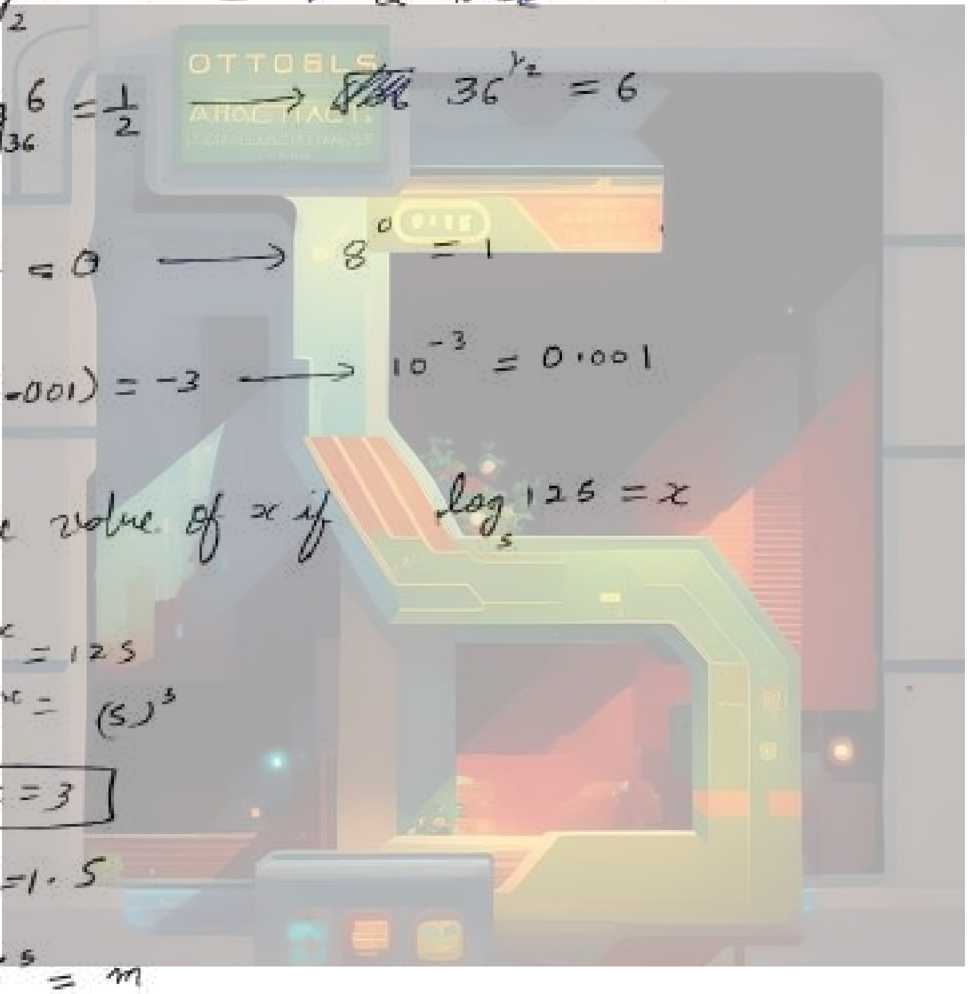
$\neq 1$

⑲



Q Conversion of Logarithm form in exponential form.

① $\log_2 32 = 5 \longrightarrow 2^5 = 32$



② $\log_{36} 6 = \frac{1}{2} \longrightarrow 36^{1/2} = 6$

③ $\log_8 1 = 0 \longrightarrow 8^0 = 1$

④ $\log_{10}(0.001) = -3 \longrightarrow 10^{-3} = 0.001$

Q find the value of x if $\log_5 125 = x$

$5^x = 125$

$5^x = (5)^3$

$\boxed{x = 3}$

Q $\log_2 m = 1.5$

$2^{1.5} = m$

$2^{3/2} = m$

$\sqrt{2^3} = m$

$\sqrt{8} = m$

$\boxed{m = 2\sqrt{2}}$

Note - For some number different bases gives different answers.

Q Find log

① 32 (base 1/2)

$\log_{1/2} 32 = x$

$\left(\frac{1}{2}\right)^x = 32$

$\frac{1}{2^x} = 32$

$\frac{1}{2^x} = 2^5$

$2^{-x} = 2^5$

$-x = 5$

$\boxed{x = -5}$

② 32 (base 2)

$\log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$\boxed{x = 5}$

③ $3\sqrt{3}$ (base 3)

$\log_3 3\sqrt{3} = x$

$3^x = 3\sqrt{3}$

$3^x = \sqrt{27}$

$3^x = \cancel{27}[(3)^3]^{1/2}$

$3^x = 3^{3/2}$

$\boxed{x = 3/2}$

④ $3\sqrt{3}$ (base 1/3)

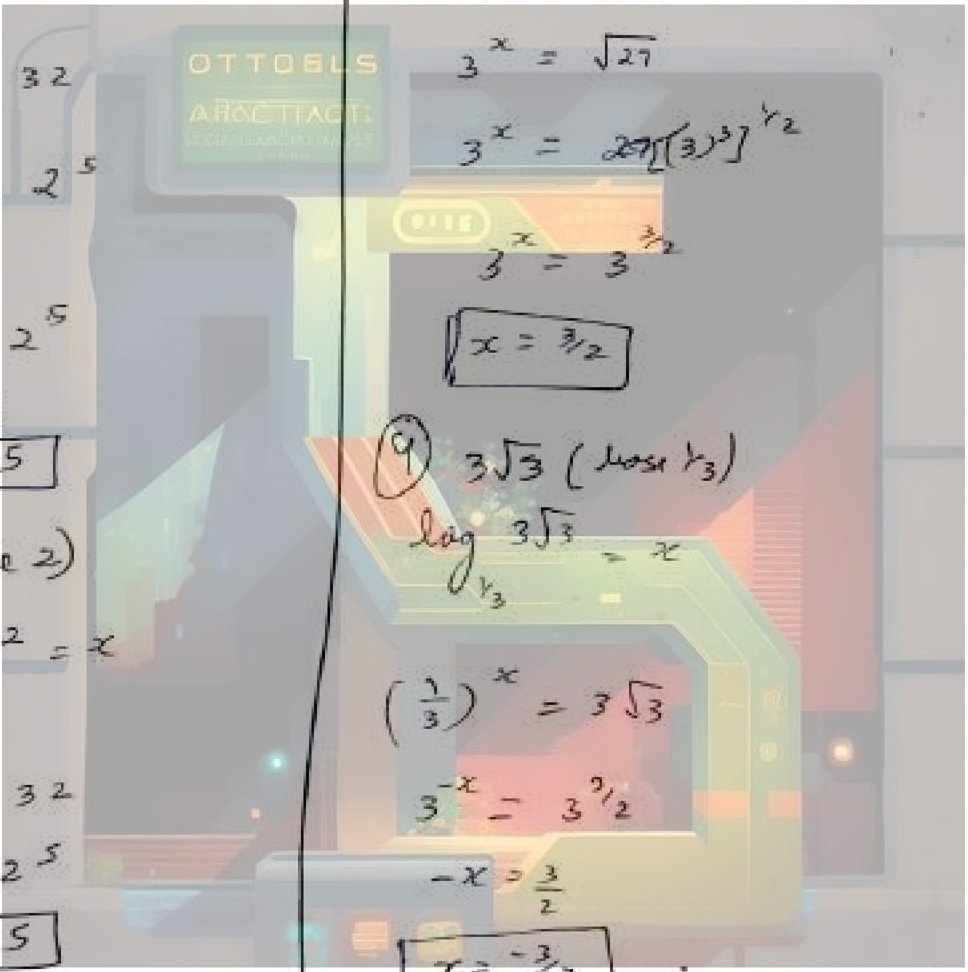
$\log_{1/3} 3\sqrt{3} = x$

$\left(\frac{1}{3}\right)^x = 3\sqrt{3}$

$3^{-x} = 3^{3/2}$

$-x = \frac{3}{2}$

$\boxed{x = -3/2}$



Note :-

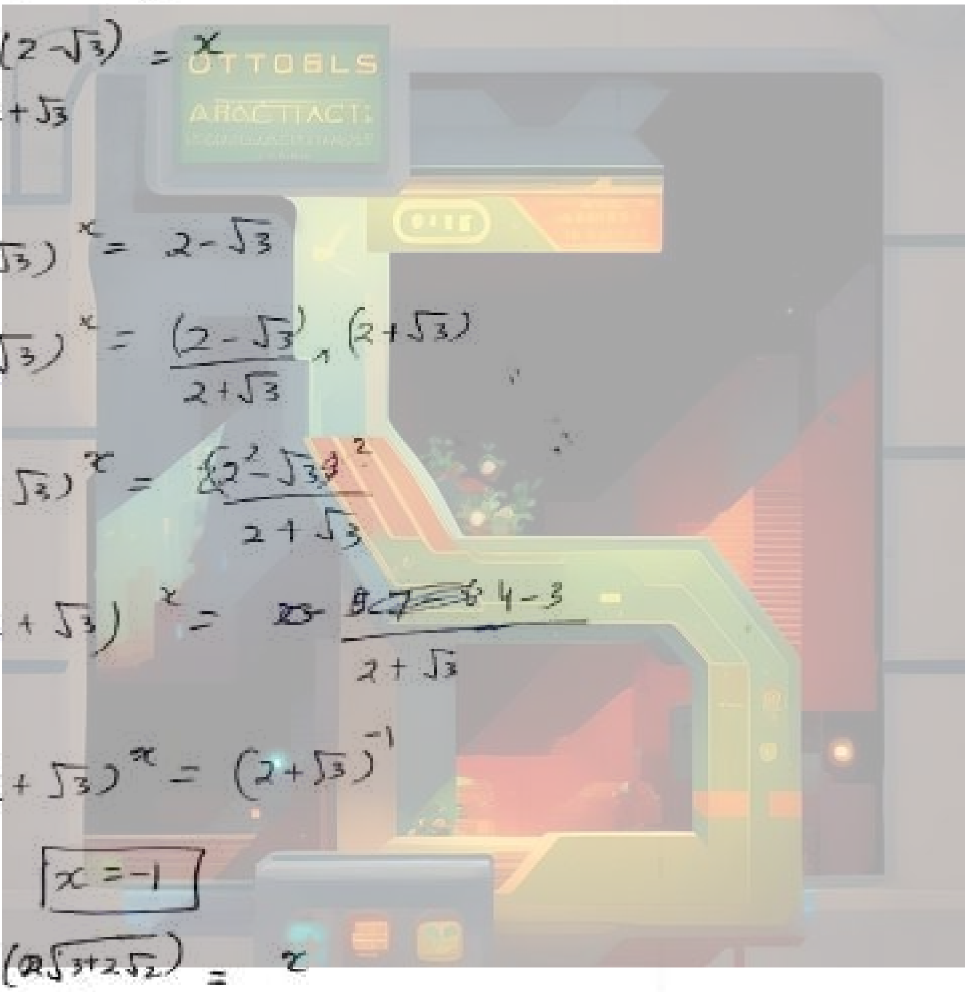
① $\log_a 1 = 0 \quad (a > 0, a \neq 1)$

② $\log_N N = 1$

③ $\log_{\frac{1}{N}} N = -1$ or $\log_N \frac{1}{N} = -1 \quad (N \geq 0, N \neq 1)$

Q find value of

① $\log_{2+\sqrt{3}} (2-\sqrt{3}) = x$



$(2+\sqrt{3})^x = 2-\sqrt{3}$

$(2+\sqrt{3})^x = \frac{(2-\sqrt{3})}{2+\sqrt{3}} \times (2+\sqrt{3})$

$(2+\sqrt{3})^x = \frac{\{2^2-\sqrt{3}\}^2}{2+\sqrt{3}}$

$(2+\sqrt{3})^x = \frac{\cancel{2-3}\ \cancel{87}\ 4-3}{2+\sqrt{3}}$

$(2+\sqrt{3})^x = (2+\sqrt{3})^{-1}$

$\boxed{x = -1}$

② $\log_{(1+\sqrt{2})} (\cancel{2}\sqrt{3+2\sqrt{2}}) = x$

$(1+\sqrt{2})^x = (\sqrt{2}+1)^1$

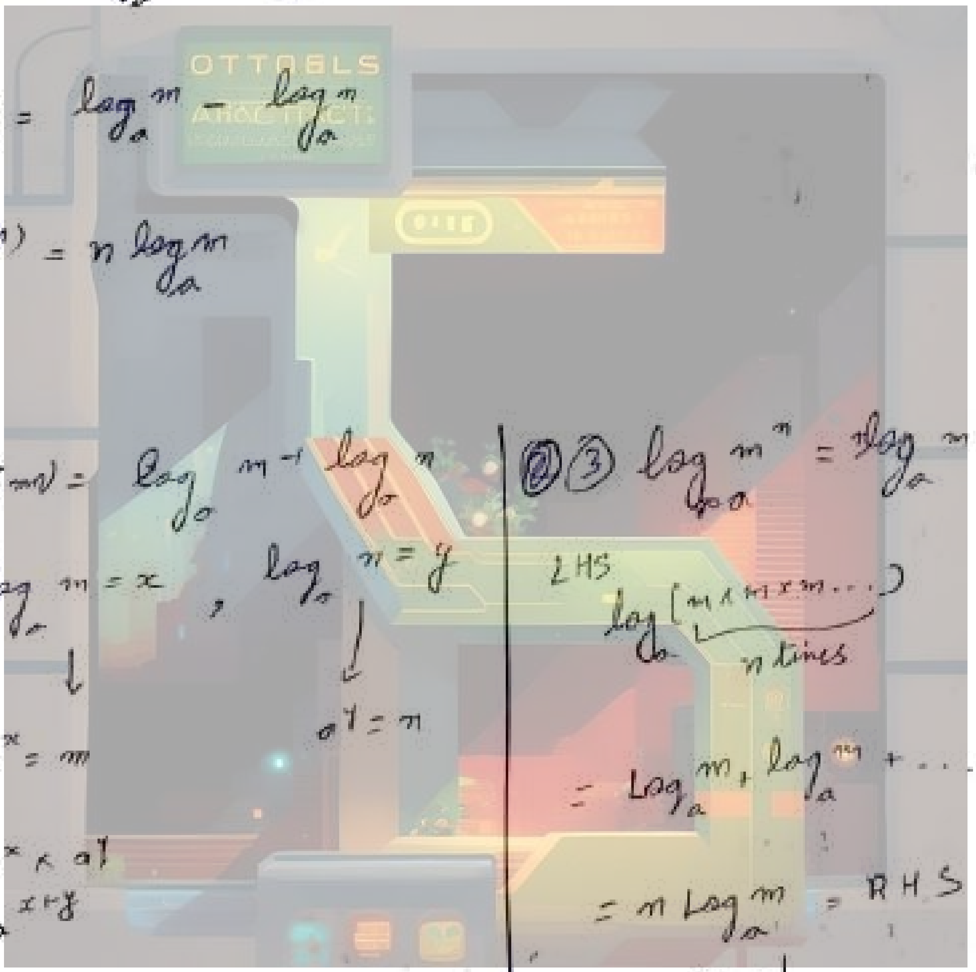
$\boxed{x = 1}$

H.W. 03-06-2024

DYS-1 {Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Principle Properties of log $\{a>0, \neq 1;\ m \,\&\, n>0\}$

① $\log_a(mn) = \log_a m + \log_a n$ (can put more-than two-terms)



② $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

③ $\log_a(m^n) = n\log_a m$

Proof :-

① $\log_a(mn) = \log_a m + \log_a n$

let $\log_a m = x$, $\log_a n = y$

$\downarrow$ $\qquad$ $\downarrow$

$a^x = m$ $\qquad$ $a^y = n$

$\therefore mn = a^x \times a^y$

$= a^{x+y}$

$\log_a(mn) = x+y$

$= \log_a m + \log_a n$

③ $\log_a m^n = n\log_a m$

LHS

$\log_a(\underbrace{m \times m \times m \ldots}_{n \text{ times}})$

$= \log_a m + \log_a m + \ldots n$ times

$= n\log_a m$ = RHS

Hence Proved

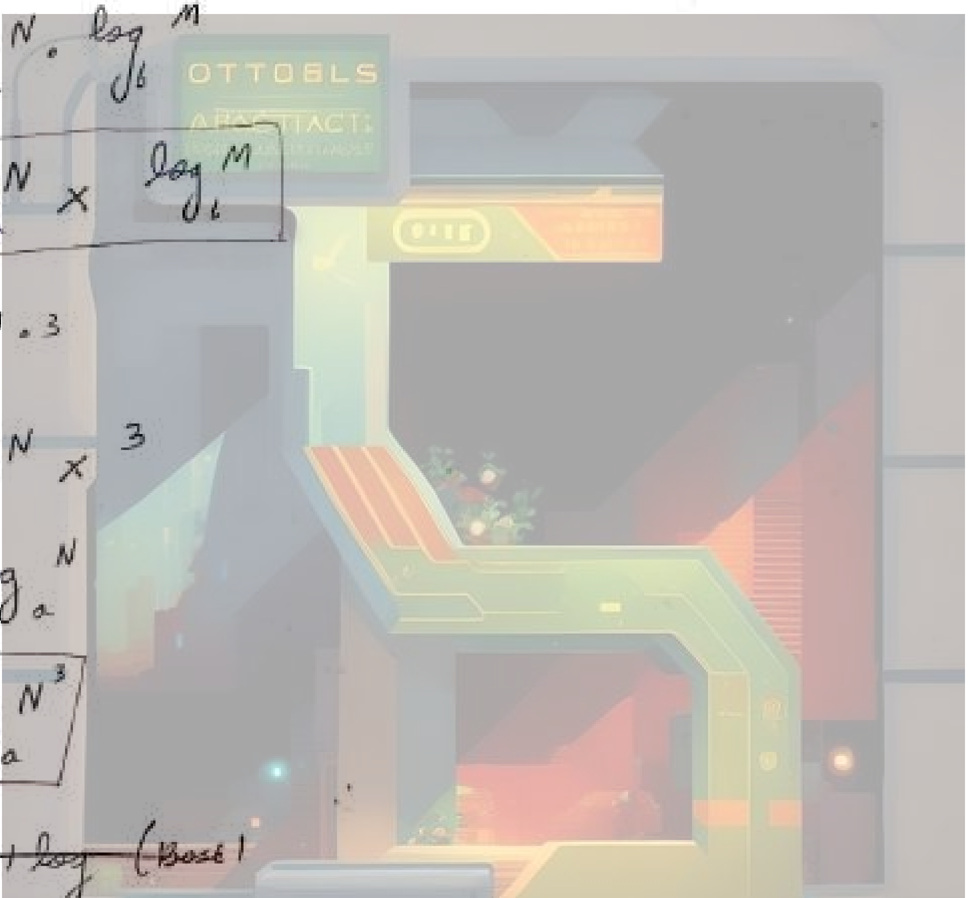
Note -

① $\log_{1/2} \log_4 2$

$\log_4 2 = \frac{1}{2}$

$\log_{1/2}\left(\frac{1}{2}\right) = x$

$\boxed{x=1}$



② $\log_a N \cdot \log_b M$

$\boxed{\log_a N \times \log_b M}$

③ $\log_a N \cdot 3$

$\log_a N \times 3$

$3\log_a N$

$\boxed{\log_a N^3}$

~~④ log 3 + log (Base 1~~

Q Find value (Base 10)

① $\log 3 + \log 5$

$\log_{10} (3\times5)$

$\boxed{\log_{10} 15}$

② $\log 6 - \log 2$

$\log\left(\frac{6}{2}\right)$

$\log_{10} 3$

③ $3\log^{4}$

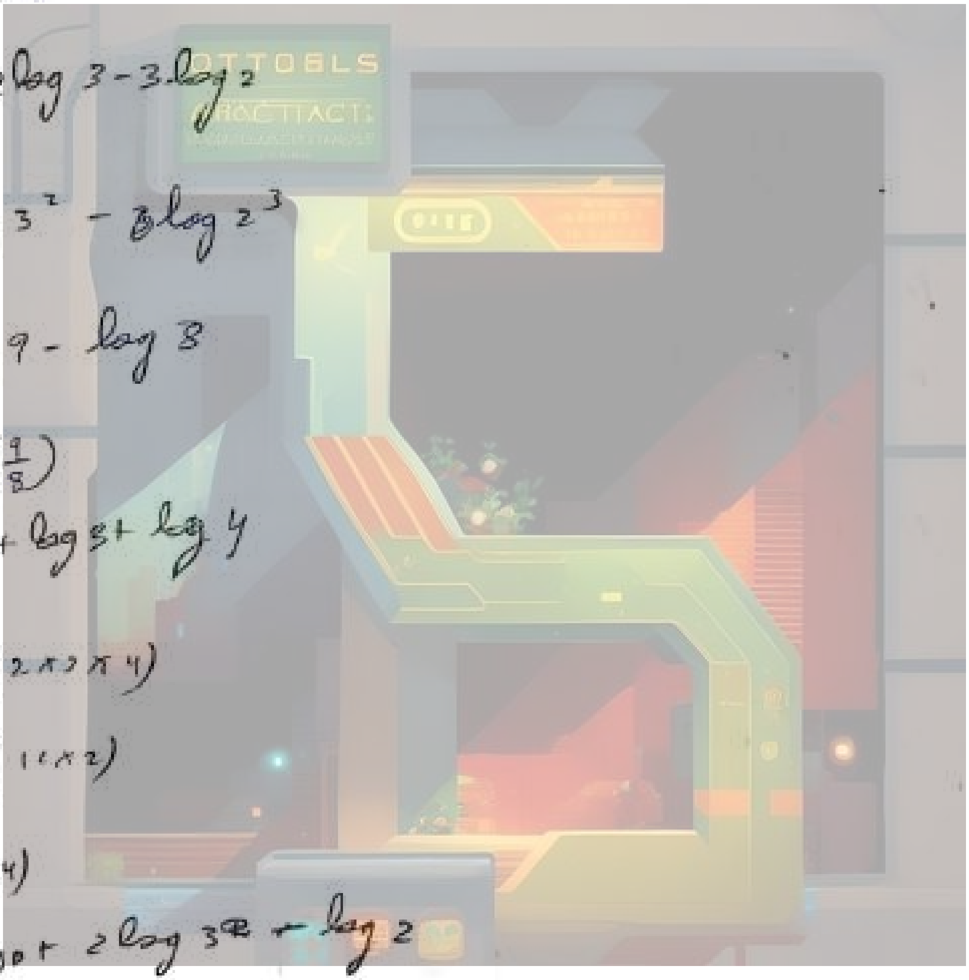
$\log(4^{3})$

~~log(64)~~

$\log_{10} 64$

④ $\log 2\,36 - \log 1$

$\log_{10} 236$



⑤ ~~2log~~ $2\log 3 - 3\log 2$

$\log 3^{2} - \cancel{3}\log 2^{3}$

$\log 9 - \log 8$

$\log_{10}\left(\frac{9}{8}\right)$

(60) $\log 2 + \log 3 + \log 4$

$\log(2\times3\times4)$

$-\log(12\times2)$

$\log_{10}(24)$

⑦ $5\log 10 + 2\log 3^{\cancel{2}} - \log 2$

$\log 10^{5} + 2\log 3^{2} - \log^{2}$

$\log 10^{5} + \log^{9} - \log^{2}$

$\log 10^{5} + \log\frac{9}{2}$

$\log_{10}(4.5\times10^{5})$

Properties

④ fundamental log. Identity

$$\boxed{a^{(\log_a N)} = N}$$

→ in power

Proof:- $a^{(\log_a N)} = N$

Taking $\log_a$ both sides

$\log_a \left(a^{\log_a N}\right) = \log_a N$

$\log_a N . \log_a a = \log_a N$

$\cancel{\log_a} N = \cancel{\log_a} N$

$N = N$

Hence, Proved

⑤ Base Changing Theorem

$$\boxed{\frac{\log_a m}{\log_a n} = \log_n m}$$

Proof:-

$\log_n m = p \Leftrightarrow n^p = m$

$\log_a m = q \Leftrightarrow a^q = m$

$\log_a n = r \Leftrightarrow a^r = n$

$n^p = a^q$

$(a^r)^p = a^q$

$a^q = a^{rp}$

$q = rp$

$\cancel{\log} \frac{q}{r} = p$

$\frac{\log_a m}{\log_a n} = \log_n m$

⑥ DDF

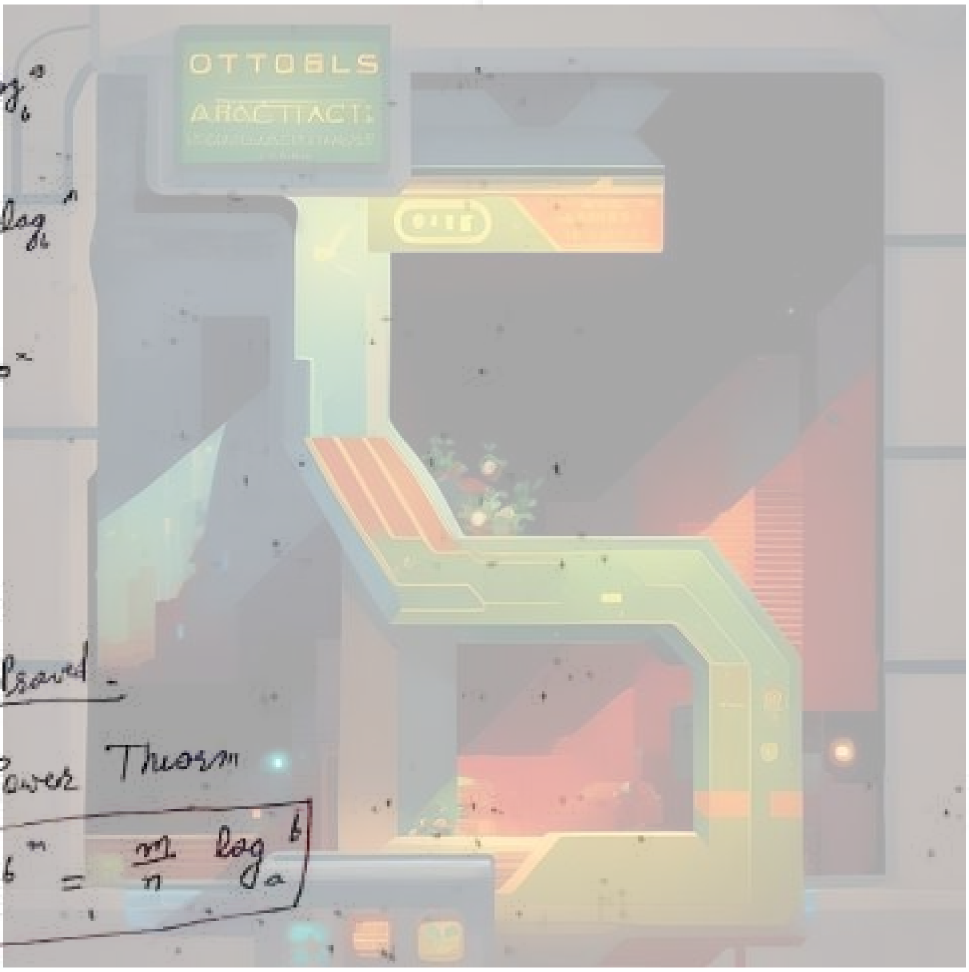
$$\boxed{a^{\log_b c} = \cancel{a^{\log}}\, c^{\log_b a}}$$

Proof

$\log_b c = x \iff b^x = c$

L.H.S

a^x



RHS

$c^{\log_b a}$

$b^{x \log_b a}$

$b^{\log_b a^x}$

a^x

LHS = RHS

Hence, Proved.

⑦ Base - Power Theorm

$$\boxed{\log_{a^n} b^m = \frac{m}{n} \log_a b}$$

Q find value (Base 10)

① $2^{\log_2 5}$

$\boxed{5}$

② $\pi^{\log_\pi 60}$

$\boxed{60}$

③ $25^{\log_5 8}$

$5^{2\log_5 8}$

$5^{\log_5 8^2}$

8^2

$\boxed{64}$

④ $\left(\frac{1}{16}\right)^{\log_2 \pi}$

$2^{\log_2 \pi^{-4}}$

$\boxed{\pi^{-4}}$

⑤ ~~$\log 8$~~ $\log_{64} 8$

$\log_{2^6} 2^3$

$\frac{3}{6}\log_2 2$

$\frac{3}{6}$

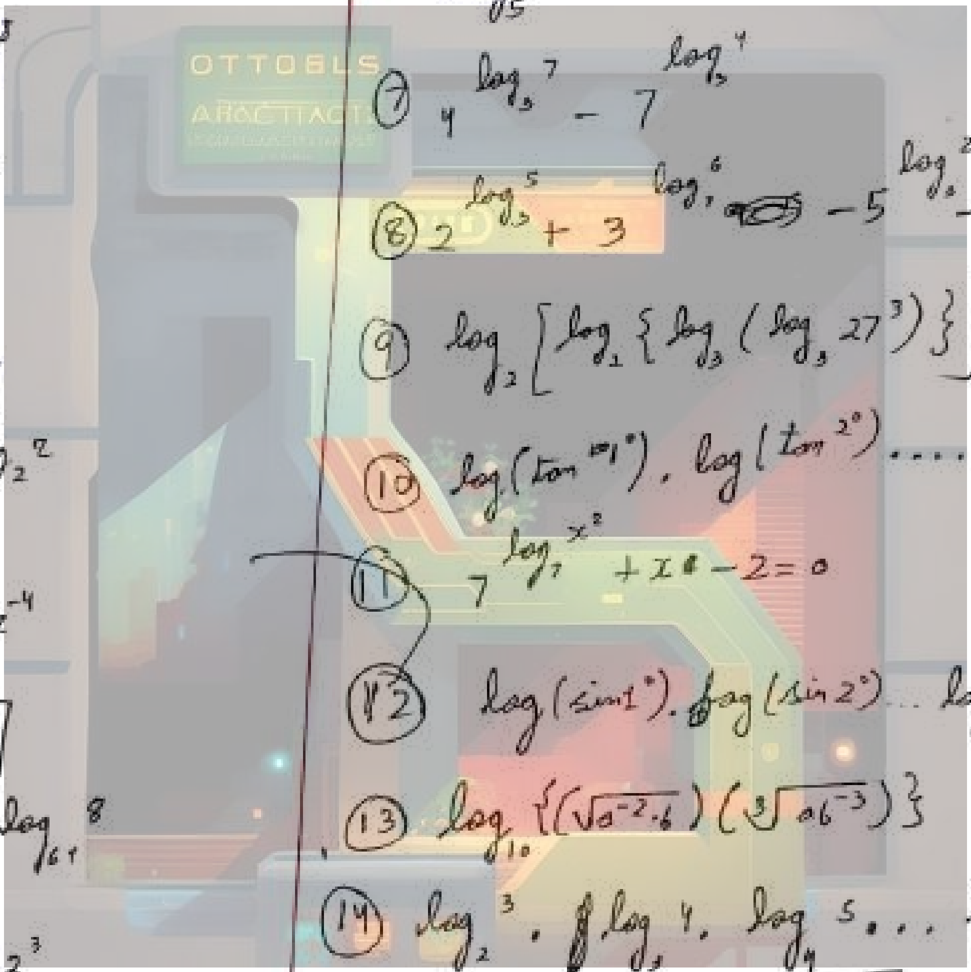
$\boxed{\frac{1}{2}}$

⑥ $\log_3 2 \times \log_4 3 \times \log_5 4$

$\log_4 3^{\log_3 2} \times \log_5 4$

$\log_5 4^{\log_4 3^{\log_3 2}}$

$\log_5 2$



⑦ $4^{\log_3 7} - 7^{\log_3 4}$

⑧ $2^{\log_3 5} + 3^{\log_7 6}$ $- 5^{\log_0 2} - 6^{\log_7 3}$

⑨ $\log_2\left[\log_2\left\{\log_3\left(\log_3 27^3\right)\right\}\right]$

⑩ $\log(\tan 1^\circ) \cdot \log(\tan 2^\circ) \ldots\ldots \log(\tan 89^\circ)$

⑪ $7^{\log_7 x^2} + x - 2 = 0$

⑫ $\log(\sin 1^\circ) \cdot \log(\sin 2^\circ) \ldots \log(\sin 90^\circ)$

⑬ $\log_{10}\left\{\left(\sqrt{a^{-2} \cdot b}\right)\left(\sqrt[3]{ab^{-3}}\right)\right\}$

⑭ $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots \log_n (n+1) = 5$

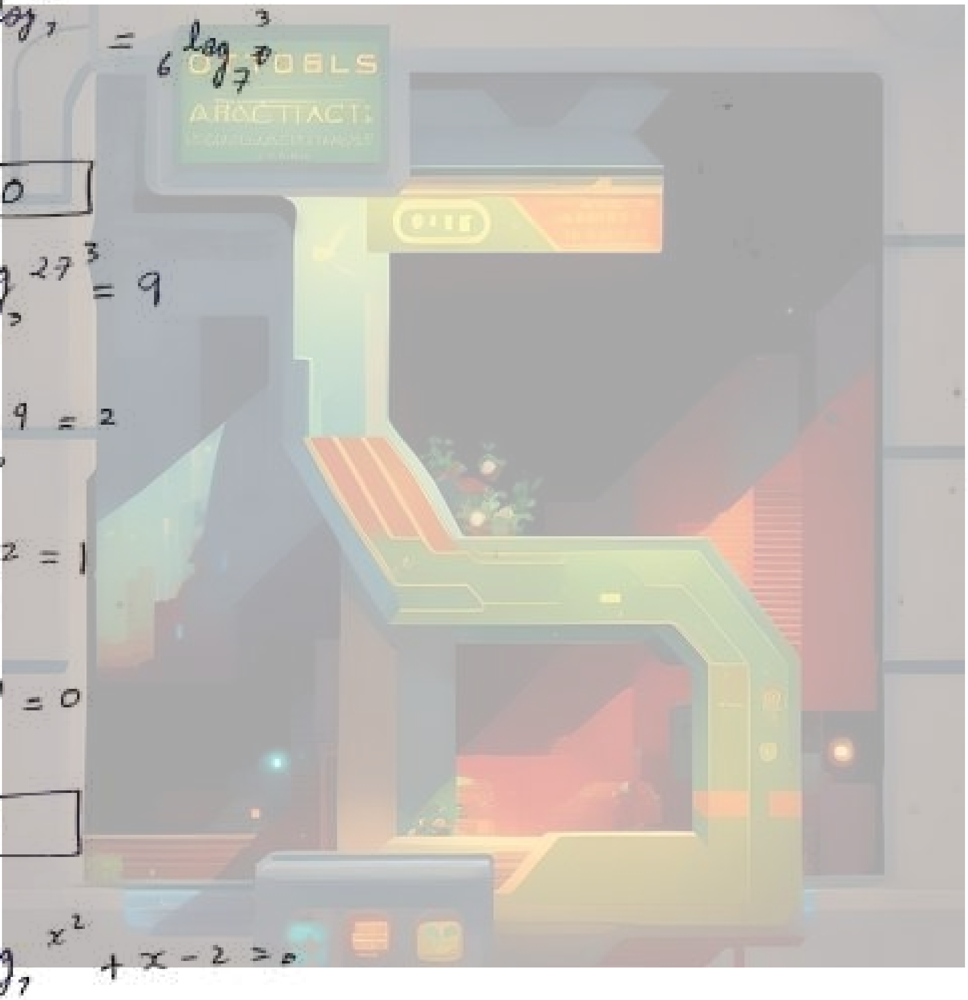
⑮ Prove: $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$

Q.7 $4^{\log_3 7} - 7^{\log_3 4}$

$4^{\log_3 7} = 7^{\log_3 4}$

$\boxed{0}$

Q8. $2^{\log_3 5} = 5^{\log_3 2}$



$3^{\log_7 4} = 6^{\log_7 3}$

$\boxed{0}$

Q9. $\log_3 27^3 = 9$

$\log_3 9 = 2$

$\log_2 2 = 1$

$\log_2 1 = 0$

$\boxed{0}$

Q11. $7^{\log_7 x^2} + x - 2 = 0$

$x^2 + x - 2 = 0$
$x^2 + 2x - x - 2 = 0$
$x(x+2) - 1(x+2) = 0$
$\boxed{x = 1, -2}$

~~⑬ $10^x = (\sqrt{a^{-2}\cdot b})(\sqrt[3]{a\, b^{-3}})$~~

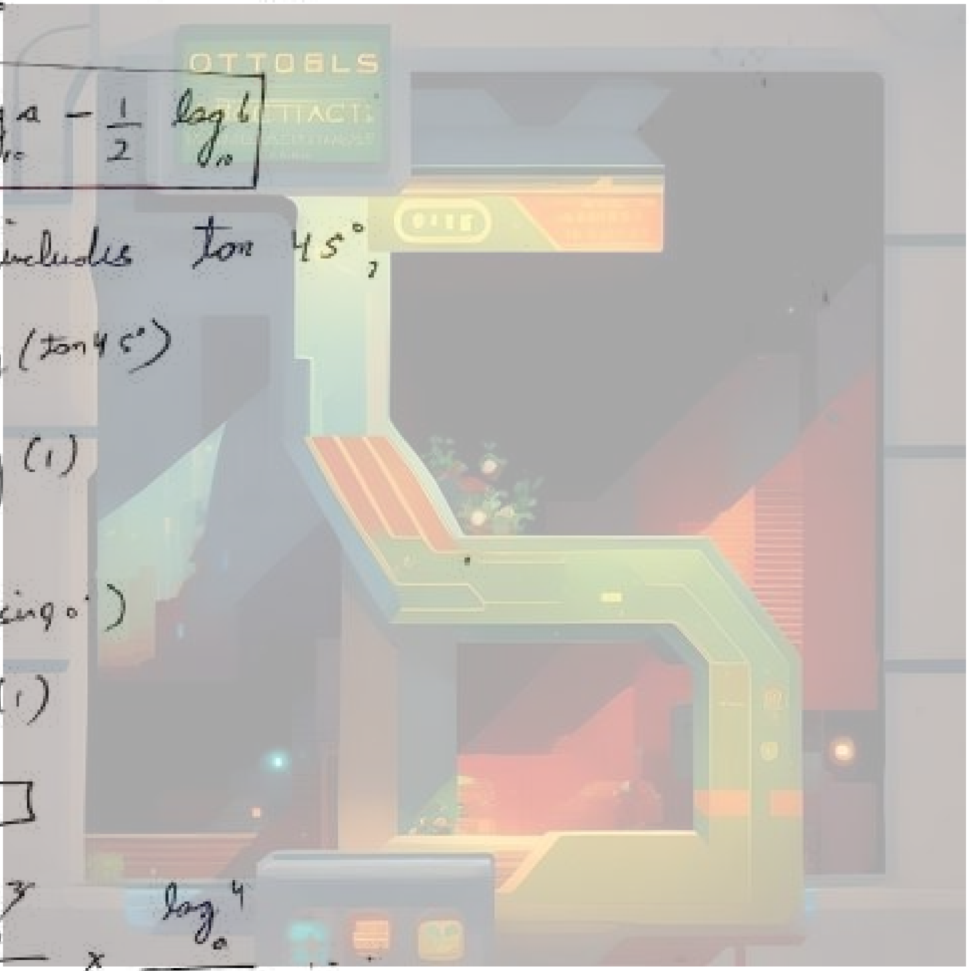
~~$10^x = a^{-1}\cdot b^{1/2}\cdot a^{1/3}\cdot b^{-1}$~~

~~$10^x = a^{-2/3}\cdot b^{-1/2}$~~

⑬ $10^x = a^{-2/3}\cdot b^{-1/2}$

$\log_{10}(a^{-2/3}) + \log_{10}(b^{-1/2})$

$\boxed{-\frac{2}{3}\log_{10} a - \frac{1}{2}\log_{10} b}$



⑩ It includes tan 45°,

$\log(\tan 45^\circ)$

$= \log(1)$

$\boxed{= 0}$

⑫ $\lg(\sin 90^\circ)$

$\log(1)$

$\boxed{0}$

⑭ $\frac{\log_a 3}{\log_a 2} \times \frac{\log_a 4}{\log_a 3}$

$\frac{\lg_a 4}{\lg_a 2} \ldots$

$\frac{\log_a (n+1)}{\log_a 2} = \boxed{\log_2 (n+1)}$

(86)

~~Q15. LHS~~

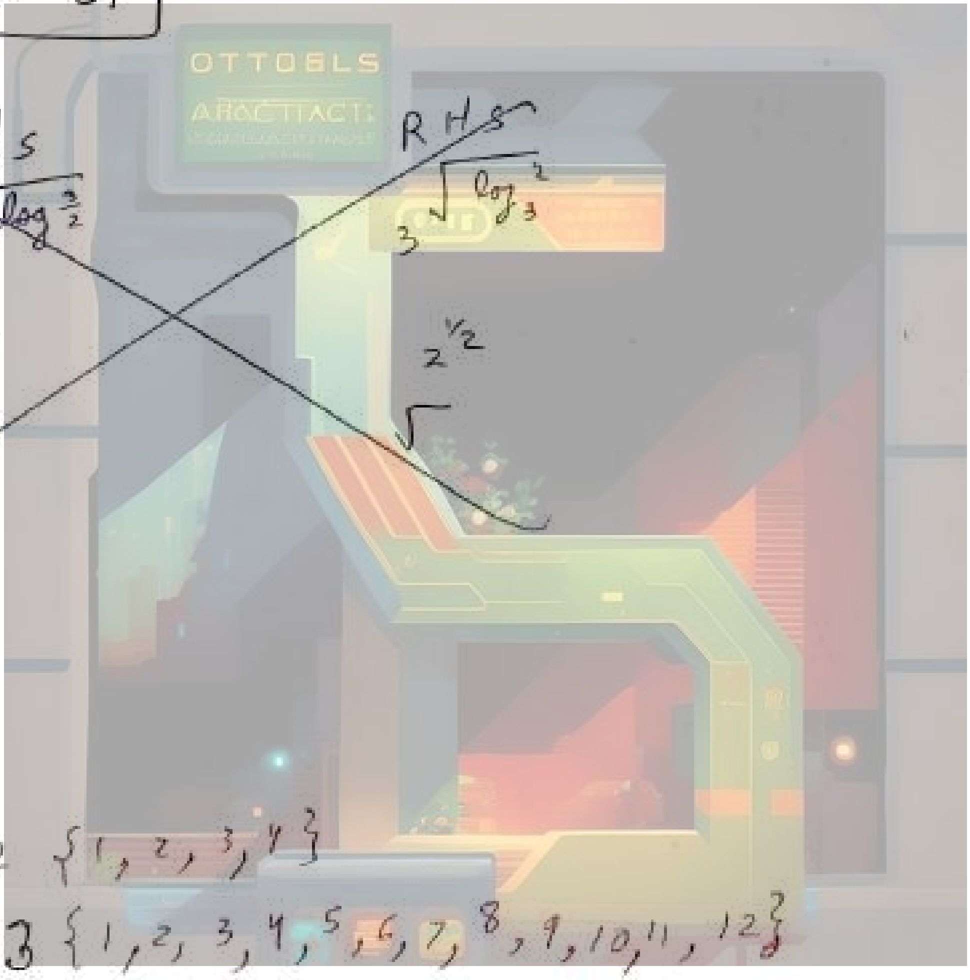
~~$2^{\sqrt{\log_2 3}}$~~

$\log_2 (n+1) = 5$

$2^5 = n + 1$

$3^2 = n + 1$

$\boxed{n = 31}$



~~Q15~~

~~LHS~~ ~~RHS~~

~~$2^{\sqrt{\log_2 3}}$~~ ~~$3^{\sqrt{\log_3 2}}$~~

~~$3^{1/2}$~~ ~~$2^{1/2}$~~

~~$\sqrt{3}$~~ ~~$\sqrt{}$~~

H.W.

DYS-2 {1, 2, 3, 4}

DYS-3 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

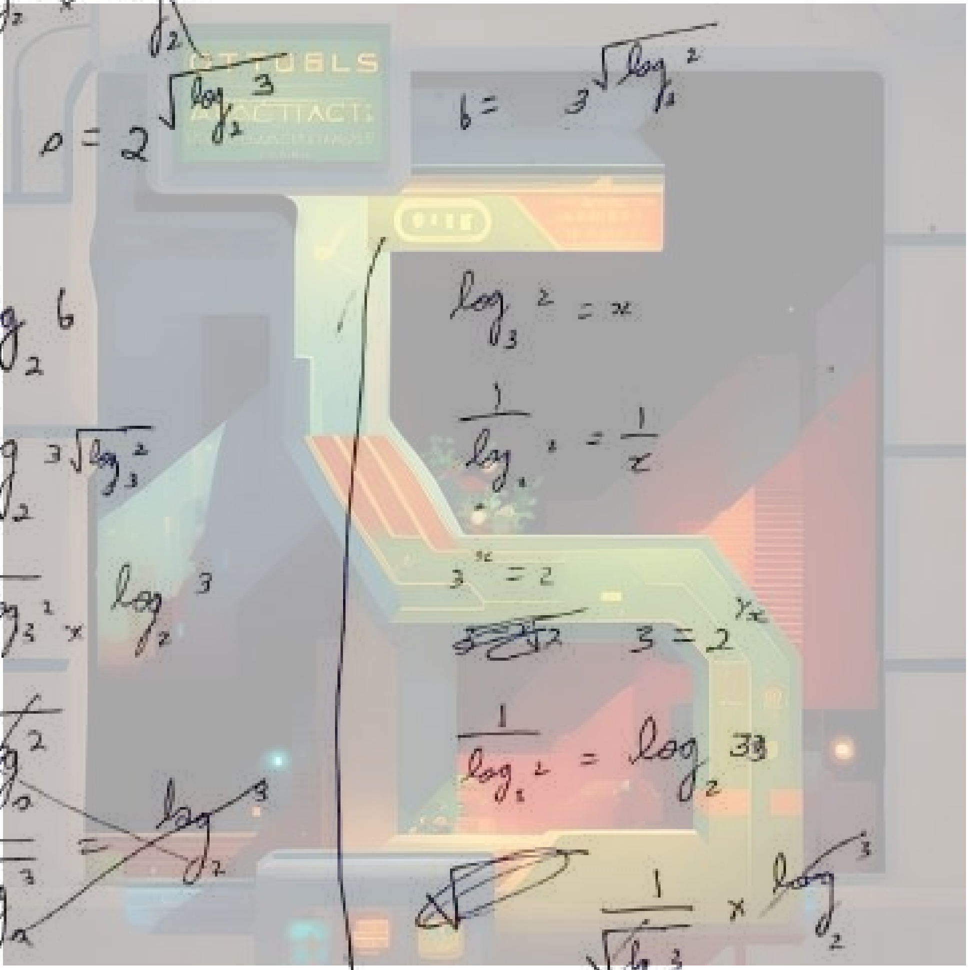
O-1 {1, 2, 3, 4, 5, 6}

~~Q15. $\log_2 a = \sqrt{\log_2 3}$~~

~~$a = 2^{\sqrt{\log_2 3}}$~~

~~$\log_2 2^{\sqrt{\log_2 3}}$~~

~~$\sqrt{\log_2 3} \times \log_2 2$~~



Q15. $a = 2^{\sqrt{\log_2 3}}$

$= \log_2 6$

$= \log_2 3^{\sqrt{\log_3 2}}$

$= \sqrt{\log_3 2} \times \log_2 3$

$= \dfrac{\sqrt{\log_2 2}}{\sqrt{\log_2 3}} = \cancel{\log_2 3}$

~~$3^x = 2$~~

$b = 3^{\sqrt{\log_3 2}}$

$\log_3 2 = x$

$\dfrac{1}{\log_2 3} = \dfrac{1}{x}$

$3^x = 2$

~~$3 = \sqrt[x]{2}$~~ $3 = 2^{1/x}$

$\dfrac{1}{\log_3 2} = \log_2 3$

$\dfrac{1}{\sqrt{\log_2 3}} \times \cancel{\log_2 3}$

$\sqrt{\log_2 3} = \log_2 b$

$2^{\sqrt{\log_2 3}} = b$

$\boxed{a = b}$

Hence, Ans 3rd

Antilog

$$\boxed{\text{Antilog}_a x = a^x}$$

$\log_a N = x$

$\text{Antilog}_a(\log_a N) = \text{Antilog}_a x$

DYS − 3

Q13 $\text{Antilog}_{64}\left(\frac{5}{6}\right)$

$(64)^{5/6}$

$(2)^5$

$\boxed{32}$

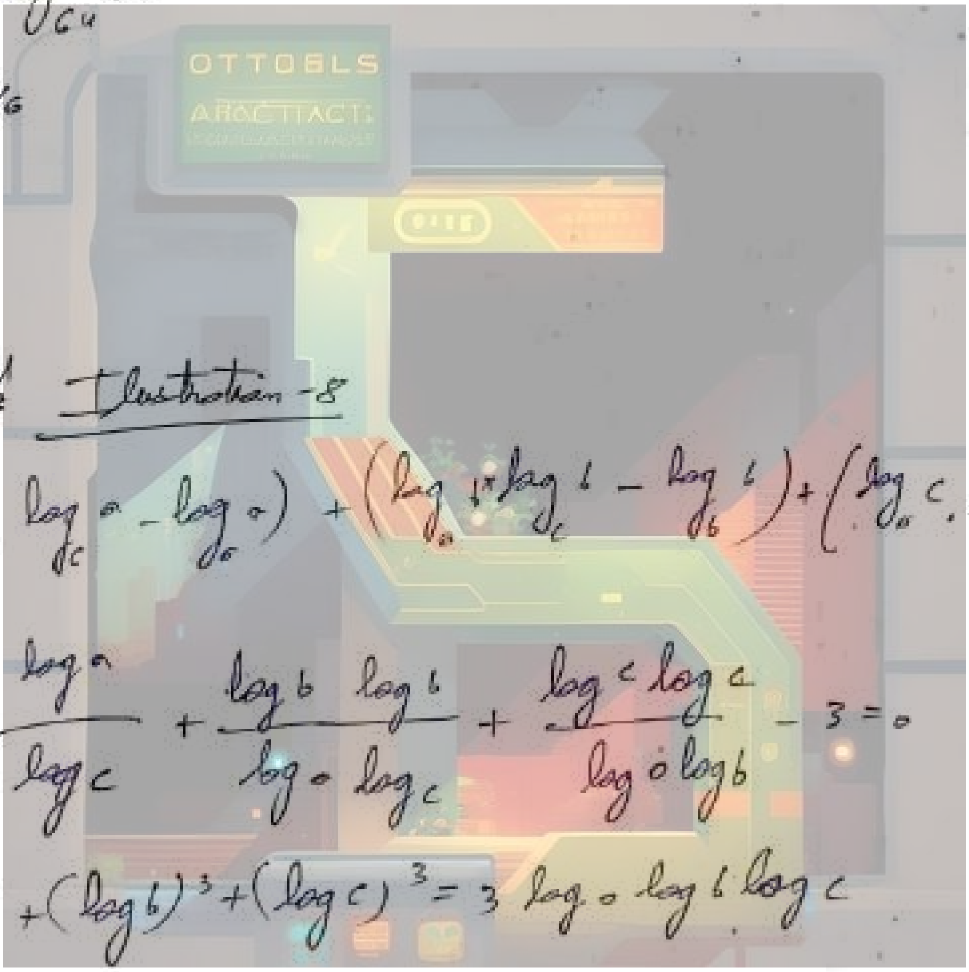


Illustration − 8

$\left(\log_b a \cdot \log_c a - \log_a a\right) + \left(\log_a b \cdot \log_c b - \log_b b\right) + \left(\log_a c \cdot \log_b c - \log_c c\right) =$

$\frac{\log a}{\log b} \times \frac{\log a}{\log c} + \frac{\log b \, \log b}{\log a \, \log c} + \frac{\log c \, \log c}{\log a \, \log b} - 3 = 0$

$(\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a \log b \log c$

$\log a + \log b + \log c = 0$

$\log(abc) = 0$

$\boxed{abc = 1}$

84

Scanned with CamScanner

Illustration 10

$\log_4 18 = x$

$4^x = 18$

$2^{2x} = 9^2$

thus irrational.

~~Q 2 $\log_2 (\log_2 x) + \log_{1/2}\left(\frac{3}{2} + \log_2 x\right) = 1$~~

~~$\log_2 \left((\log_2 x)^2 \times \frac{1}{\log_4 x^2}\right) = 1$~~

~~$\log_2 x \times \log_2 x \times \frac{2}{3 \log_2 x} = 2$~~

~~$\log_2 x = 2 \times \frac{3}{2}$~~

~~$\log_2 x = 3$~~

~~$2^3 = x$~~

~~$\boxed{8 = x}$~~

Q 3. $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \ldots \infty}}\right)$

$x = \sqrt{4 - \frac{x}{3\sqrt{2}}}$

$x^2 = 4 - \frac{x}{3\sqrt{2}}$

$3\sqrt{2}x^2 = 12\sqrt{2} - x$

$3\sqrt{2}x^2 + x - 12\sqrt{2} = 0$

$x = -1 \pm \sqrt{1 + 287}$

$x = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$x = \frac{16}{6\sqrt{2}}$

$6 + \log_{3/2} \frac{4}{9}$

$6 - 2$

$\boxed{4}$

~~$6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{16}{3\sqrt{2}}\right)$~~

~~$6 + \log_{3/2} \frac{8}{9} x^2$~~

~~$\left(\frac{3}{2}\right)^x = \frac{8}{9}$~~

~~$\left(\frac{2}{3}\right)^{-x} = \frac{8}{9}$~~

~~$6 - 2 + \log_{3/2} 2$~~

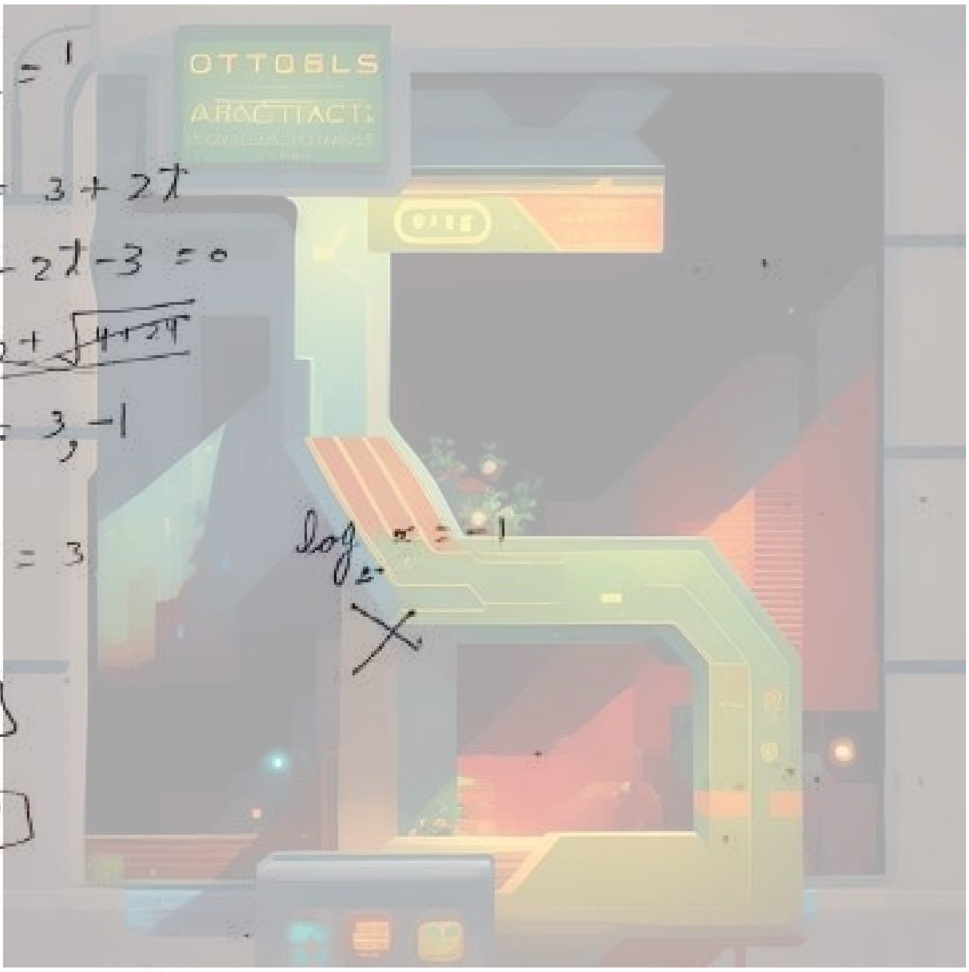
~~$4 + \sqrt{8}$~~

~~$\boxed{4 + 2\sqrt{2}}$~~

Q. $2\log_2(\log_2 x) + \log_{1/2}\left(\frac{3}{2} + \log_2 x\right) = 1$

$\log_2 t^2 + \log_2\left(\frac{1}{\frac{3}{2}+t}\right) = 1$

$\frac{t^2}{\frac{3+2t}{2}} = 1$



$\frac{2t^2}{3+2t} = 1$

$2t^2 = 3+2t$

$2t^2 - 2t - 3 = 0$

~~$t = 2 \pm \sqrt{4+24}$~~

$t = 3, -1$

$\log_2 x = 3$ $\cancel{\log_2 x = -1}$

$\boxed{x=8}$

$\boxed{x=8}$

H.W

J.A - {5, 6}

DYS-9 [9, 10]

Q

① $\cancel{2\log_2}\ 2^{\log_2 x^2} - 3x - 4 = 0$

$2^{\log_2 x^2} = 3x + 4$

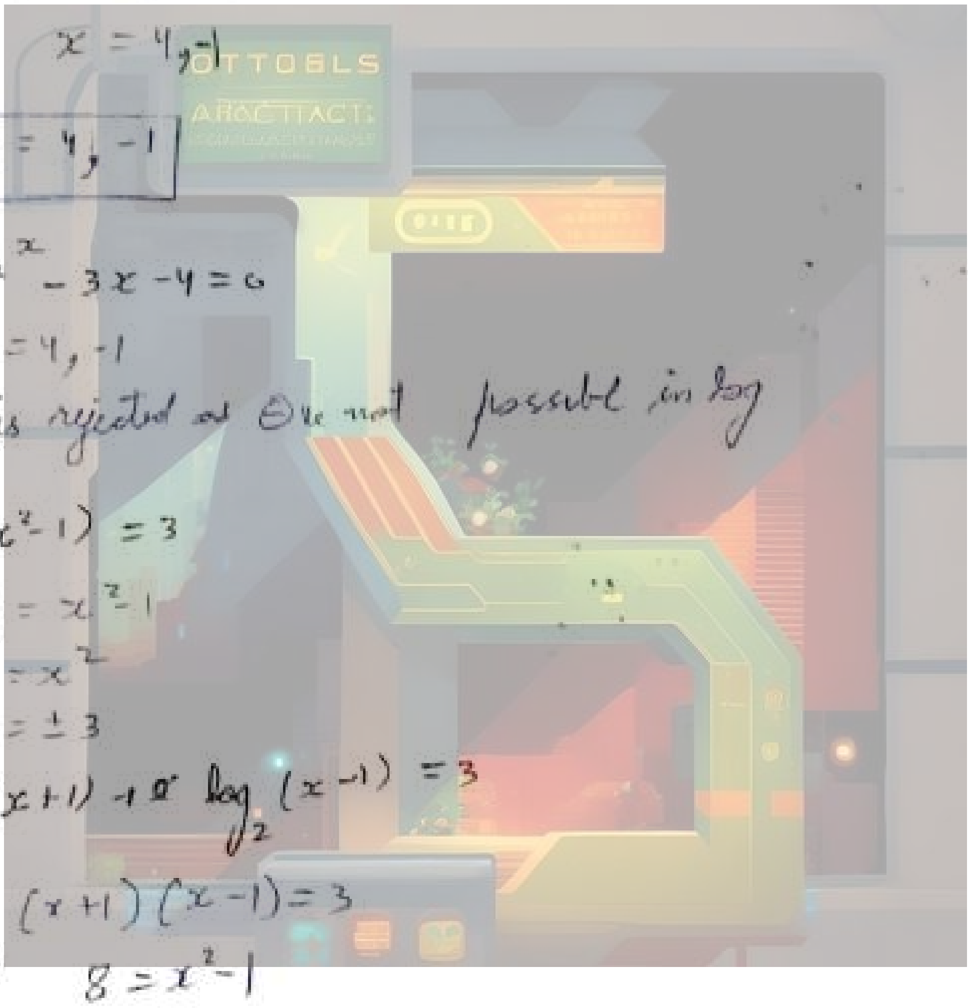
$\log_2^{3x+4} = \log_2 x^2$

$3x + 4 = x^2$

$x^2 - 3x - 4 = 0$

$x = 4, -1$

$\boxed{x = 4, -1}$



② $2^{2\log_2 x} - 3x - 4 = 0$

$x = 4, -1$

-1 is rejected as -ve not possible in log

③ $\log_2 (x^2 - 1) = 3$

$8 = x^2 - 1$

$9 = x^2$

$x = \pm 3$

④ $\log_2 (x+1) + \log_2 (x-1) = 3$

$\log_2 (x+1)(x-1) = 3$

$8 = x^2 - 1$

$x^2 + 9$

$x = \pm 3$

-3 is reject

$\boxed{x = 3}$

Q 5 $x^2 + 7^{\log_7 x} - 2 = 0$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$x = 1, -2$

-2 reject

$\boxed{x = 1}$

Q 7. $5^{(\log_5 x)^2} + x^{\log_5 x}$

$5^{y^2} + x^{y} = 1250$

$\log_5 x = y$

$5^y = x$

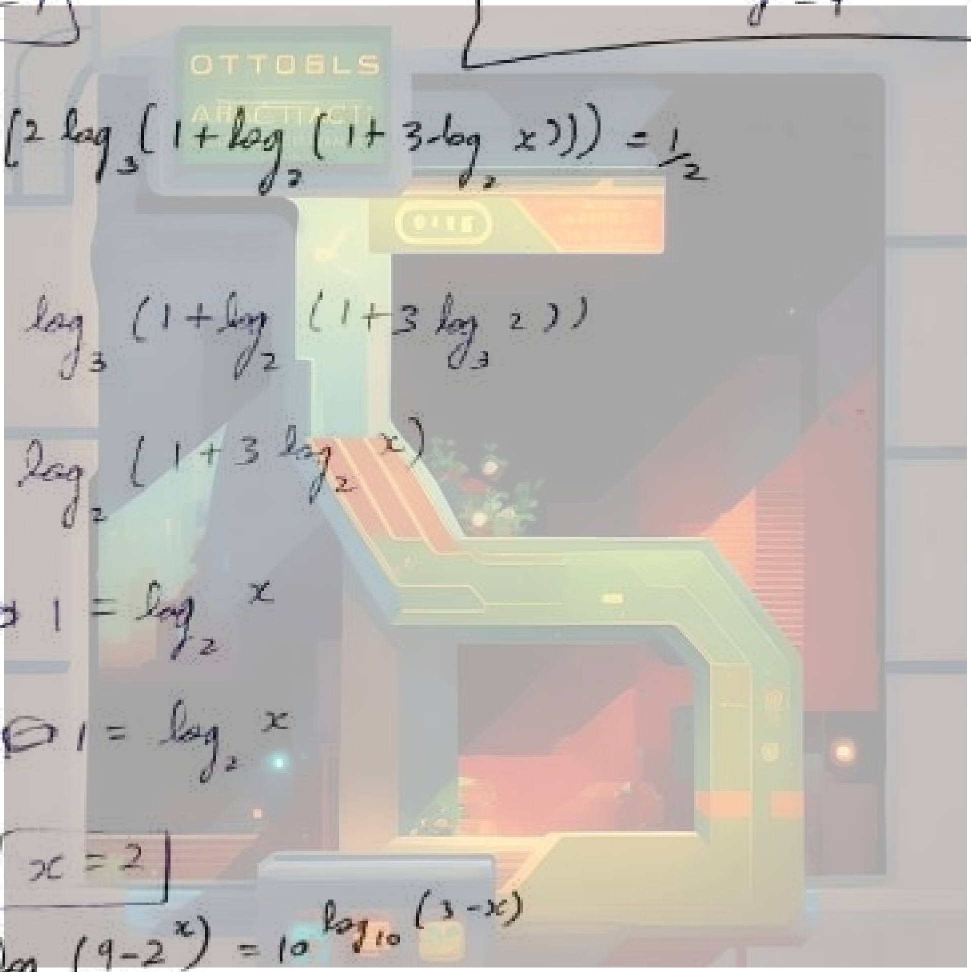
$5^{y^2} + 5^{y^2} = 1250$

$5^{y^2} = 625$

$5^{y^2} = 5^4$ $y = \pm 2$

$y^2 = 4$

$\boxed{x = 25, \frac{1}{25}}$



Q 6. $\log_4 (2 \log_3 (1 + \log_2 (1 + 3 \log_2 x))) = \frac{1}{2}$

$4^{1/2} = 2 \log_3 (1 + \log_2 (1 + 3 \log_3 2))$

$2 = \log_2 (1 + 3 \log_2 x)$

$1 = \log_2 x$

$1 = \log_2 x$

$\boxed{x = 2}$

Q 8. $\log_2 (9 - 2^x) = 10^{\log_{10} (3-x)}$

$2^{3-x} = 9 - 2^x$

$2^3 \times 2^{-x} = 9 - 2^x$

$8y = 9 - y$

$y = 8, 1$

$x = 3, 0$

$= 3$ reject

$\boxed{x = 0}$

Q9. $\log_7 \log_5 (\sqrt{x+5}+\sqrt{x} = 0$

$\sqrt{x+5} + \sqrt{x} = 5$

$5 = \sqrt{x+5} + \sqrt{x}$

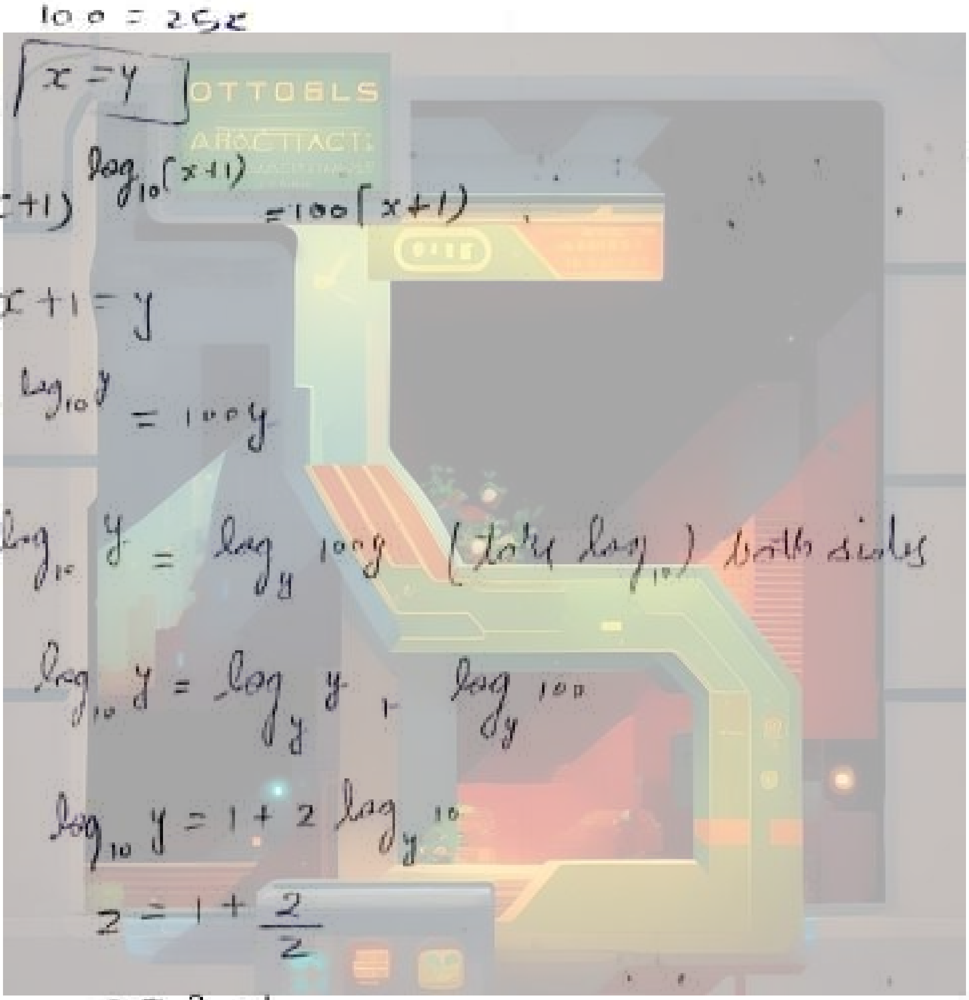
$25 = x+5+x+2\sqrt{(x+5)(x)}$

~~25=~~ $10 = x + \sqrt{x^2+5x}$

$10 - x = \sqrt{x^2+5x}$

$100 = 25x$

$\boxed{x = 4}$



Q10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$x+1 = y$

$y^{\log_{10} y} = 100y$

$\log_{10} y = \log_y 100y$ (take $\log_{10}$) both sides

$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$z = 1 + \frac{2}{z}$

$\underline{z = 2, -1}$

$\log_{10} y = -1$

$y = \frac{1}{10}$

$x+1 = \frac{1}{10}$

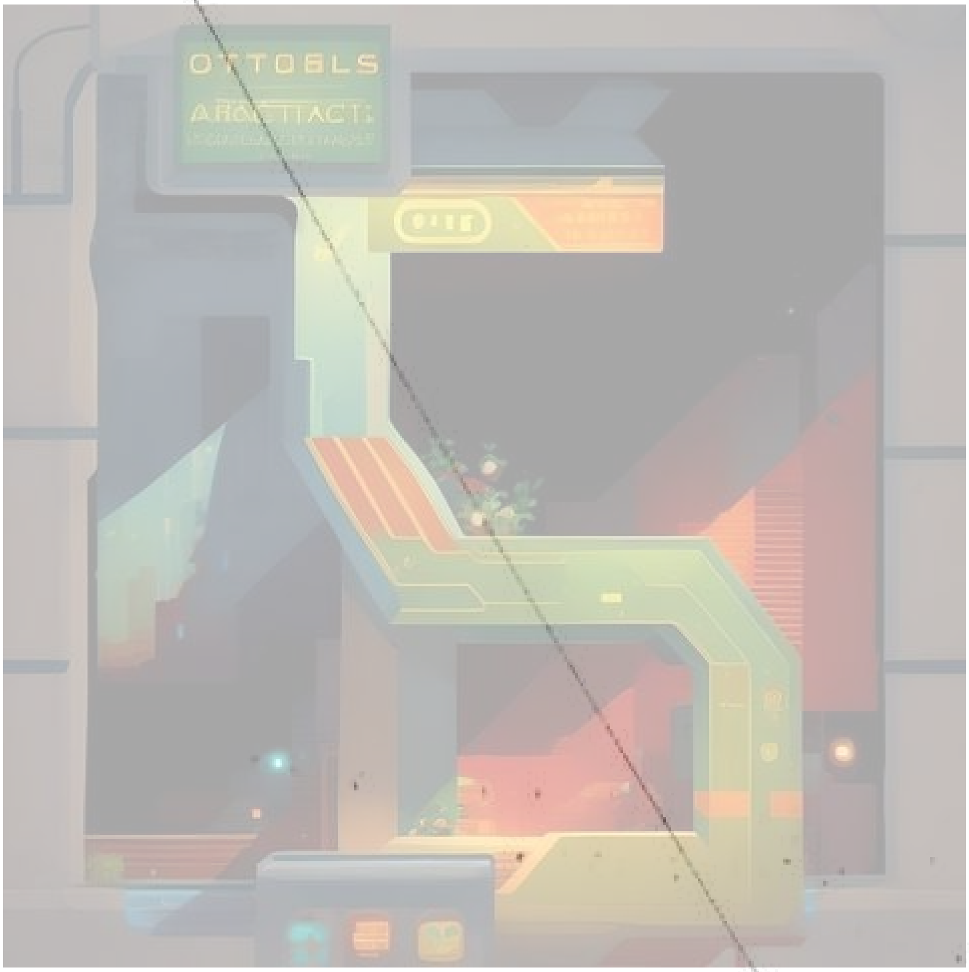
$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$y = 100$

$x+1 = 100$

$\boxed{x = 99}$



(95)

Q ⑪ $\log_{x-1}(4) = 1 + \log_2(x-1)$

$\log_y 4 = 1 + \log_2 y$

~~$y^{1+\log_2 y}$~~

$2\log_y 2 = 1 + \log_2 y$

$\log_2 y = z$

$\frac{2}{z} = 1 + z$

$2 = z + z^2$

$z^2 + z - 2 = 0$

$z = \frac{-1 \pm \sqrt{1+8}}{2}$

$z = \frac{-1 \pm 3}{2}$

$z = -2, 1$

$\log_2 y = -2$

$\frac{1}{4} = y$

$x - 1 = y$

$x - 1 = \frac{1}{4}$

$x = \frac{1}{4} + \frac{4}{4}$

$x = \frac{5}{4}$

$\boxed{x = 3, 5/4}$

$\log_2 y = 1$

$y = 2$

$x - 1 = 2$

$x = 3$

⑫ sum of values of x A) 1 , B) 4 C) 0 D) 3

$\log_{2x-1}(x^3 + 3x^2 - 13x + 10) = 2$

$(2x-1)^2 = x^3 + 3x^2 - 13x + 10$

$4x^2 + 01 - 4x = x^3 + 3x^2 - 13x + 10$

$x^3 - x^2 - 9x + 9 = 0$

$x = 1$

$(x-1)$

$x^2 - 9 = 0$

$x = \pm 3$

$x = 1, 3, -3$

~~$3 + -3 + 1$~~ but, −3 is reject as $2x - 1$ will be ⊖ve

~~1~~

~~A)~~

~~A~~

1 reject as $2x-1$ would be 1

$x = 3$

$\boxed{D)}$

⑬ $$5^{1+\log_4 x} + 5^{(\log_{1/4} x)-1} = \frac{26}{5}$$

$$5^{1+\log_4 x} + 5^{-(\log_4 x+1)} = \frac{26}{5}$$

$$5^y + \frac{1}{5^y} = \frac{26}{5}.$$

$$5^{2y}+1 = \frac{26 \times 5^y}{5}$$

$$5^{2y}$$

$$5 \times 5^{2y}+1 = 26 \times 5^y$$

$$\cancel{5 \times 5^{x^2}+1 = 26}$$

$$5 \times z^2+1 = 26 z$$

$$5z^2-26z+1=0$$

$$z = \frac{26 \pm \sqrt{676-20}}{2}$$

$$z = \frac{26 \pm \sqrt{656}}{2}$$

$$t + \frac{1}{t} = \frac{26}{5}$$

$$t^2+1 = \frac{26t}{5}$$

$$5t^2-26t+1=0$$

$$5_a t + \frac{1}{5t} = \frac{26}{5}$$

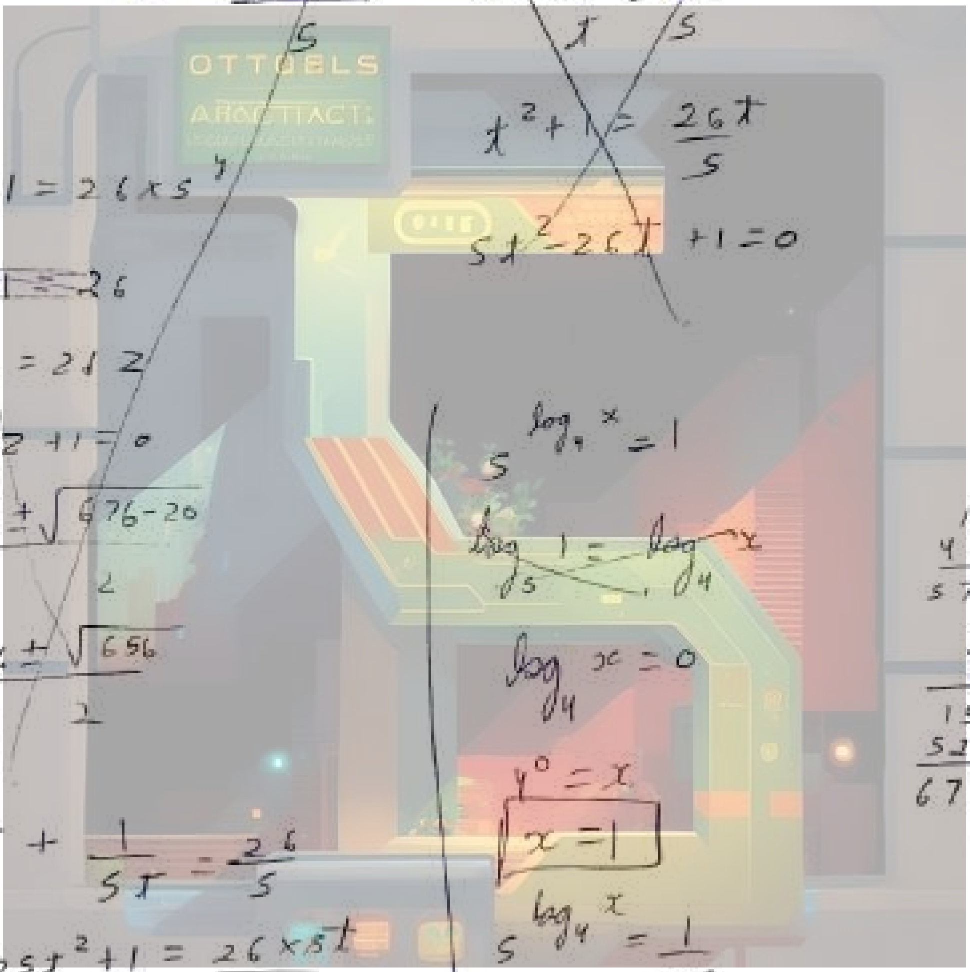
$$25t^2+1 = \frac{26 \times 5t}{5}$$

$$25t^2-26t+1$$

$$25t^2-25t-t+1$$

$$25t(t-1)-1(t-1)$$

$$t = 1,\ 1/25$$



$$5^{\log_4 x} = 1$$

$$\log_5 1 = \log_4 x$$

$$\log_4 x = 0$$

$$4^0 = x$$

$$\boxed{x=1}$$

$$5^{\log_4 x} = \frac{1}{25}$$

$$\log_4 x = -2$$

$$\boxed{\frac{1}{16} = x}$$

$$\boxed{x = 1,\ 1/16}$$

3
26
26
156
420
576
3
26
26
156
520
676

⑫ $(x-2)^{\log^2(x-2)+\log(x-2)^5-12} = 10^{2\log(x-2)}$

~~$(x-2)^{y^2}$~~

~~$x-2=y$~~

$y^{(\log y)+5(\log y)} + \log y^5 - 12 = 10^{2\log y}$

~~$\log y = 2$~~

~~$(x-2)^{\log y^2}$~~

$x-2=y$

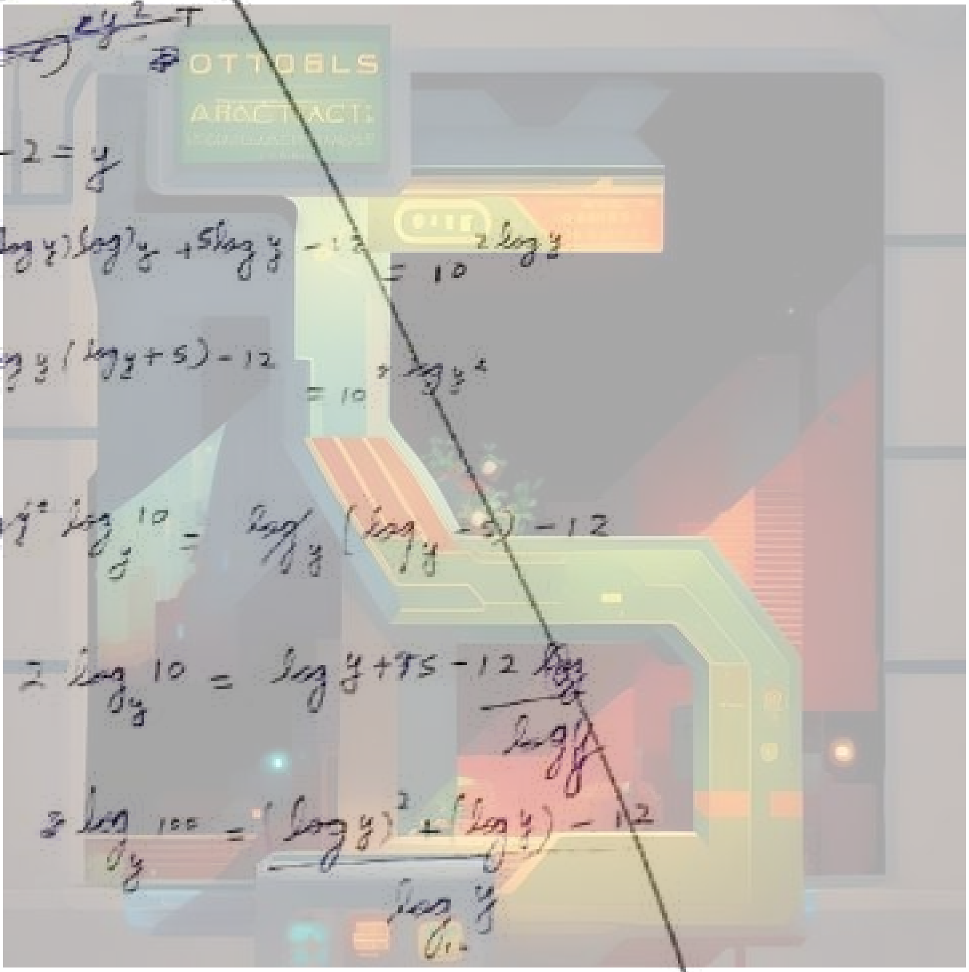
$y^{(\log y)(\log)_y + 5\log y - 12} = 10^{2\log y}$

$y^{\log y(\log y+5)-12} = 10^{2\log y^2}$

$2\log y^2 \log_y 10 = \log_y y(\log_y y + 5) - 12$

$2\log_y 10 = \log y + 5 - 12\frac{\log y}{\log y}$

$2\log_y 100 = \frac{(\log y)^2 + (\log y) - 12}{\log_{10} y}$



Q 12. $(x-2)^{\log(x-2)(\log(x-2)+\log(x-2)^5-12}=10^{2\log(x-2)}$

Note - can assume same base on both sides.

$y^{\log_{10} y \times \log_{10} y + 5\log_{10} y^{0} - 12} = 10^{\log_{10} y^2}$

$y^{\log_{10} y(\log_{10} y + 5)-12} = y^2$

$\log_{10} y(\log_{10} y+5)-12 = y^2$

$100 = \cancel{y}\, y(\log_{10} y+5)-12$

~~$100 = 5y + y\log_{10} y$~~

~~$100 =$~~

$2 = (\log_{10} y)^2 + 5\log_{10} y - 12$

$14 = z^2 + 5z$

$z^2+5z-14=0$

$z = \frac{-5 \pm \sqrt{25+56}}{2}$

$z = \frac{-5 \pm 9}{2}$

$z = -7, 2$

$\log_{10} y = 2$

$y = 100$

$x-2 = 100$

$\boxed{x = 102}$

$\log_{10} y = -7$

$\frac{1}{10^7} = y$

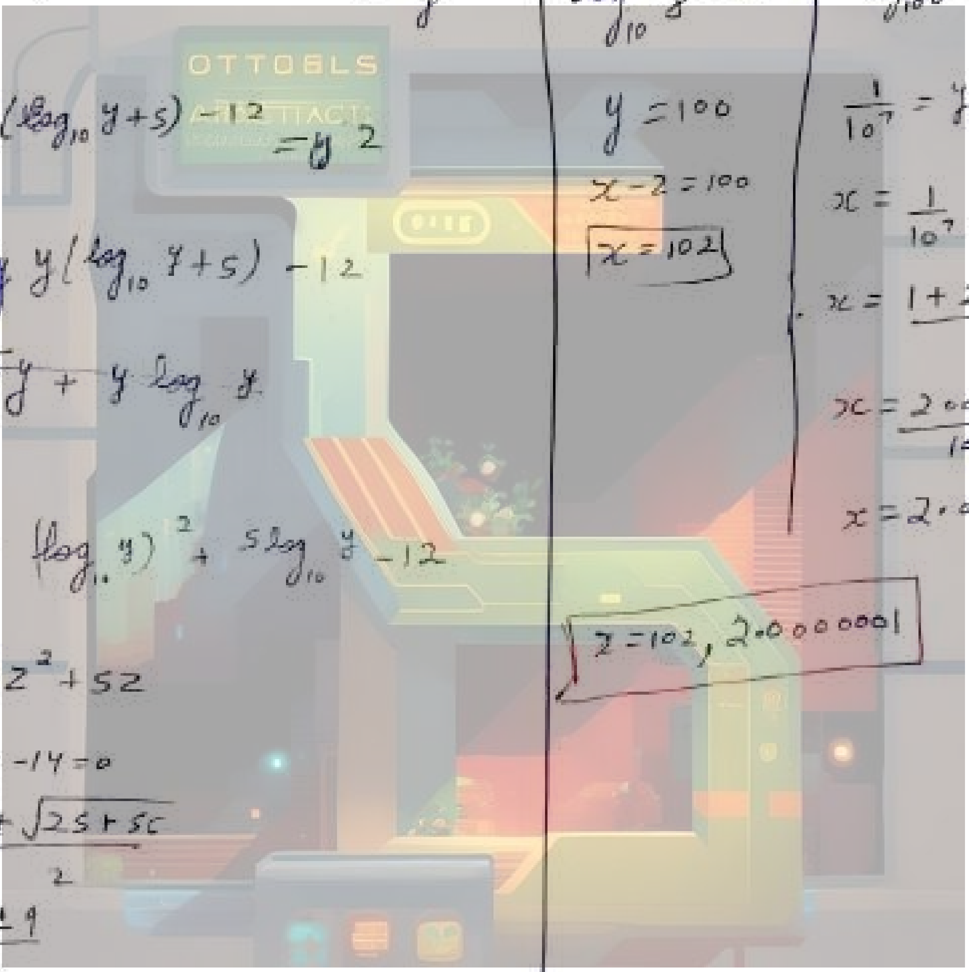
$x = \frac{1}{10^7} + 2$

$x = \frac{1+20000000}{10000000}$

$x = \frac{20000001}{10000000}$

$x = 2.0000001$

$\boxed{x = 102, 2.0000001}$



H.W. 07-06-2024

DYS-4 [10:]

Logarithmic

Logarithmic Inequalities

Constant Base

Variable Base

$\in (0,1)$ sign change

$(1,\infty)$ no sign change

case 1 Base $\in (0,1)$ $\xrightarrow[\text{union}]{}$ case 2 $\in (1,\infty)$

e.g. $\log_{1/2} x > \log_{1/2}(2x-1)$

$x < 2x-1$

$1 < x$

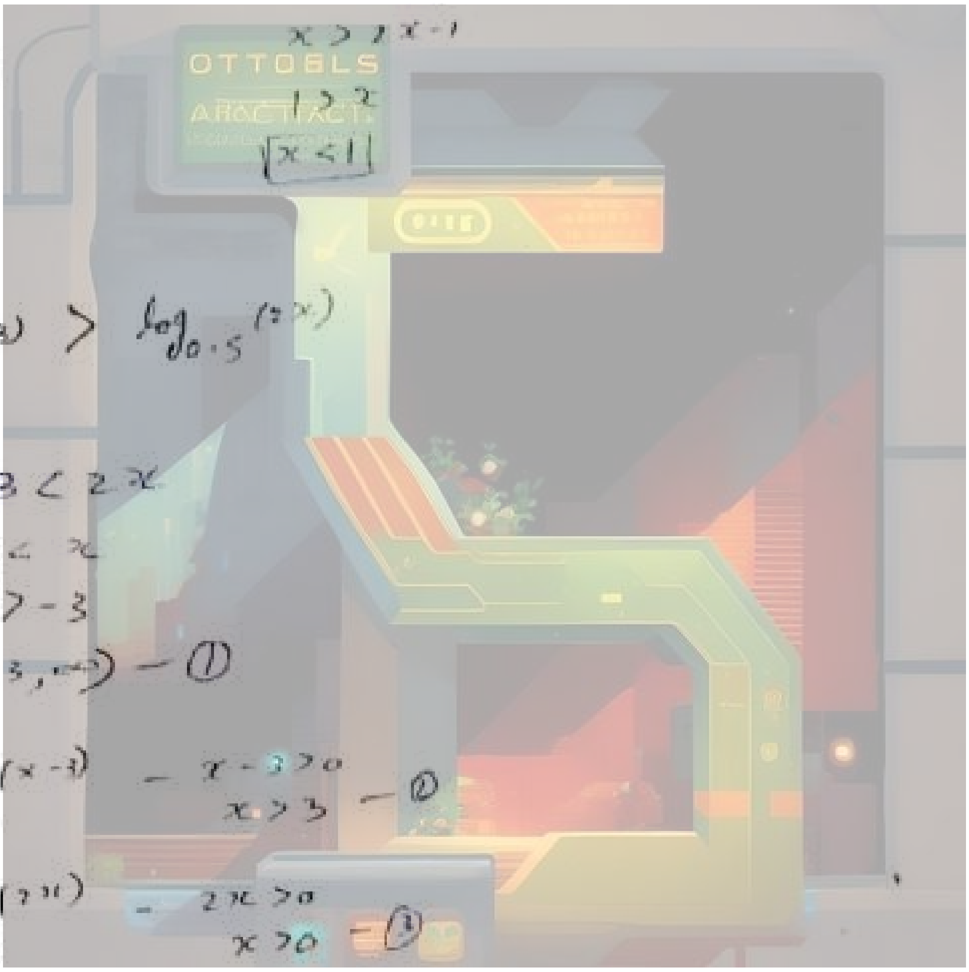
$\boxed{x > 1}$

e.g. $\log_5 x > \log_5 (2x-1)$

$x > 2x-1$

$1 > x$

$\boxed{x < 1}$



Q $\log_{0.5}(x-3) > \log_{0.5}(2x)$

$x-3 < 2x$

$-3 < x$

$x > -3$

$\therefore x \in (-3,\infty)$ — ①

For $\log_{0.5}(x-3)$ — $x-3>0$

$x>3$ — ②

$\log_{0.5}(2x)$ — $2x>0$

$x>0$ — ③

$\therefore$ ① $\cap$ ② $\cap$ ③

$\boxed{x \in (3,\infty)}$

Q2. $\log_7(x^2-3x) \geqslant \log_7(2x-6)$

$x^2-3x \geqslant 2x-6$

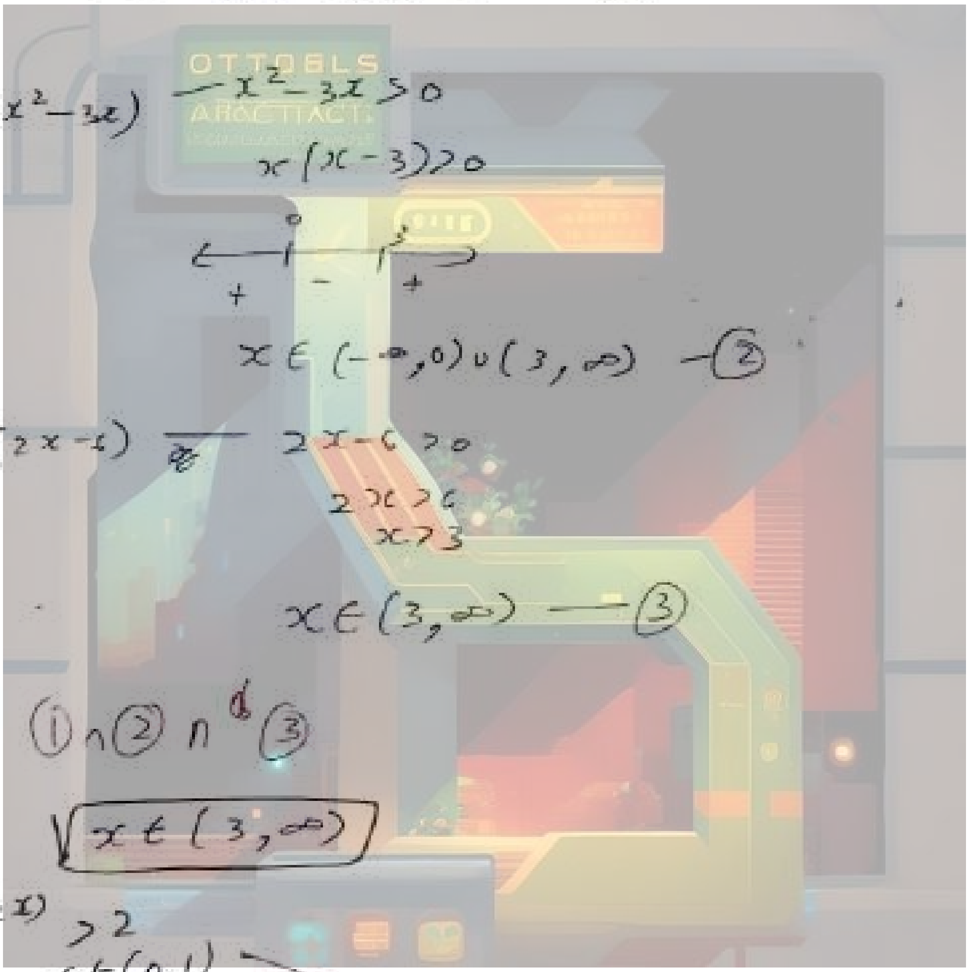
$x^2-5x+6 \geqslant 0$

$(x-2)(x-3) \geqslant 0$

$+ \quad - \quad +$

$2 \quad 3$

$x \in (-\infty,2] \cup [3,\infty)$ — ①



$\log_7(x^2-3x)$ — $x^2-3x>0$

$x(x-3)>0$

$0 \quad 3$

$+ \quad - \quad +$

$x \in (-\infty,0) \cup (3,\infty)$ — ②

$\log_7(2x-6)$ — $2x-6>0$

$2x>6$

$x>3$

$x \in (3,\infty)$ — ③

① ∩ ② ∩ ③

✓ $\boxed{x \in (3,\infty)}$

Q $\log_x(2x) > 2$

$x \in (0,1)$

Case 1 $2x < x^2$

$0 < x^2-2x$

$x(x-2) > 0$

$+ \quad - \quad +$

$0 \quad 2$

$x \in (-\infty,0) \cup (2,\infty)$

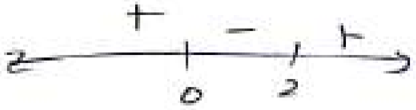
$\xrightarrow{\cap} x \in \phi$

Case 2:- $x \in (1, \infty)$

$2x > x^2$

$0 > x^2 - 2x$

$x(x-2) < 0$

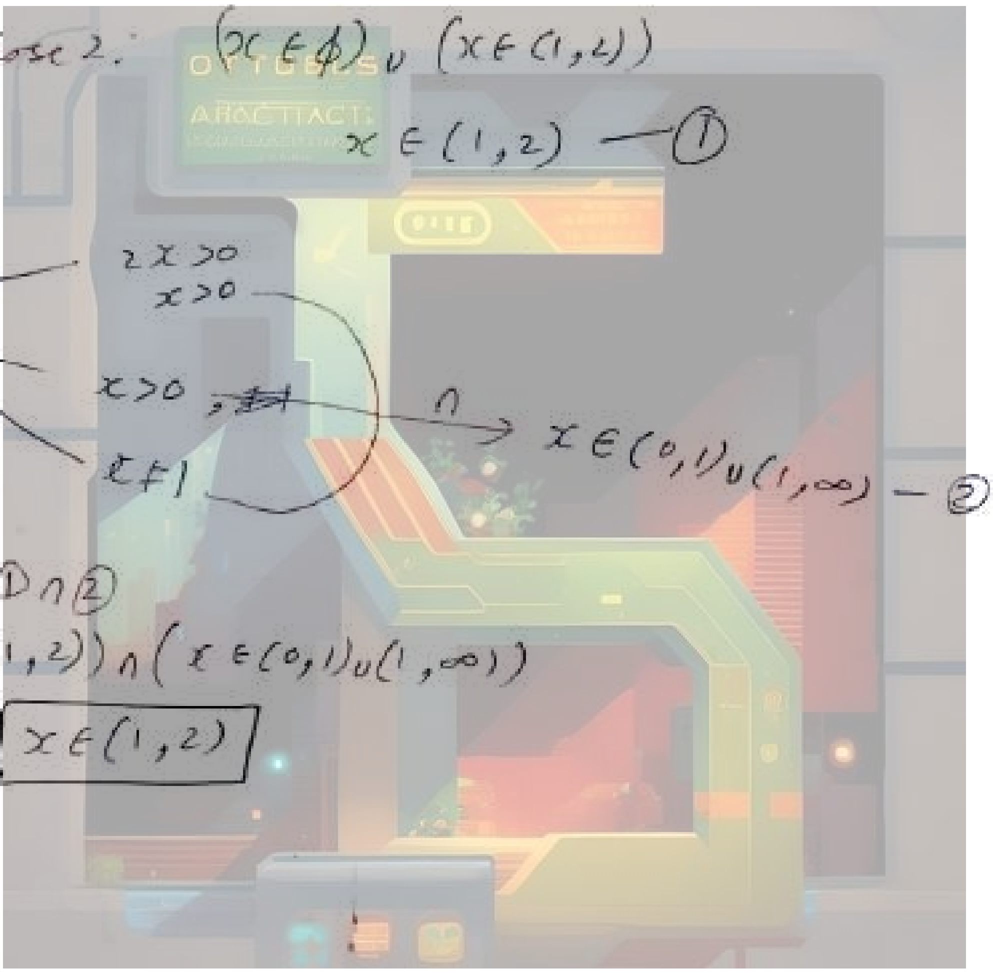


$x \in (0, 2)$

$\cap \rightarrow x \in (1, 2)$

Case 1 ∪ Case 2: $(x \in \phi) \cup (x \in (1,2))$

$x \in (1,2)$ — ①



$\log_x 2x$

$2x > 0$

$x > 0$

$x > 0$, $\cancel{x \neq 1}$

$x \neq 1$

$\cap \rightarrow x \in (0,1) \cup (1, \infty)$ — ②

① ∩ ②

$(x \in (1,2)) \cap (x \in (0,1) \cup (1,\infty))$

$\boxed{x \in (1,2)}$

(10.3)

Q ① $\log_{1/3}\left(\frac{1-2x}{x}\right) \le 0$

~~$\left(\frac{1}{3}\right)^0 \le \frac{1-2x}{3}$~~

~~$\frac{1-2x}{3} \le 1$~~

~~$1-2x \le 3$~~

~~$-2x \le 3-1$~~

~~$-2x = -2$~~

~~$2x = 2$~~

~~$x = \frac{2}{2}$~~

~~$x = 1$~~

~~$1-2x \le 3$~~

~~$-2x \le -2$~~

~~$2x \ge 2$~~

~~$x \ge 1$~~

~~$\log_{1/3}\left(\frac{1-2x}{3}\right) \longrightarrow 1-\frac{2x}{3} > 0$~~

~~$1 > \frac{2x}{3}$~~

~~$3 > 2x$~~

~~$x < \frac{3}{2}$~~

~~$\boxed{x \in [1, 1.5)}$~~

$\frac{1-2x}{x} \ge 1$

$\frac{1-2x-x}{x} \ge 0$

$\frac{1-3x}{x} \ge 0$

$-\quad 0 \quad + \quad \frac{1}{3} \quad -$

~~$(-\infty, 0) \cup (\frac{1}{3}, \infty)$~~

$x \in (0, \frac{1}{3}]$ — ①

$\log_{1/3}\left(\frac{1-2x}{x}\right) \longrightarrow \frac{1-2x}{x} > 0$

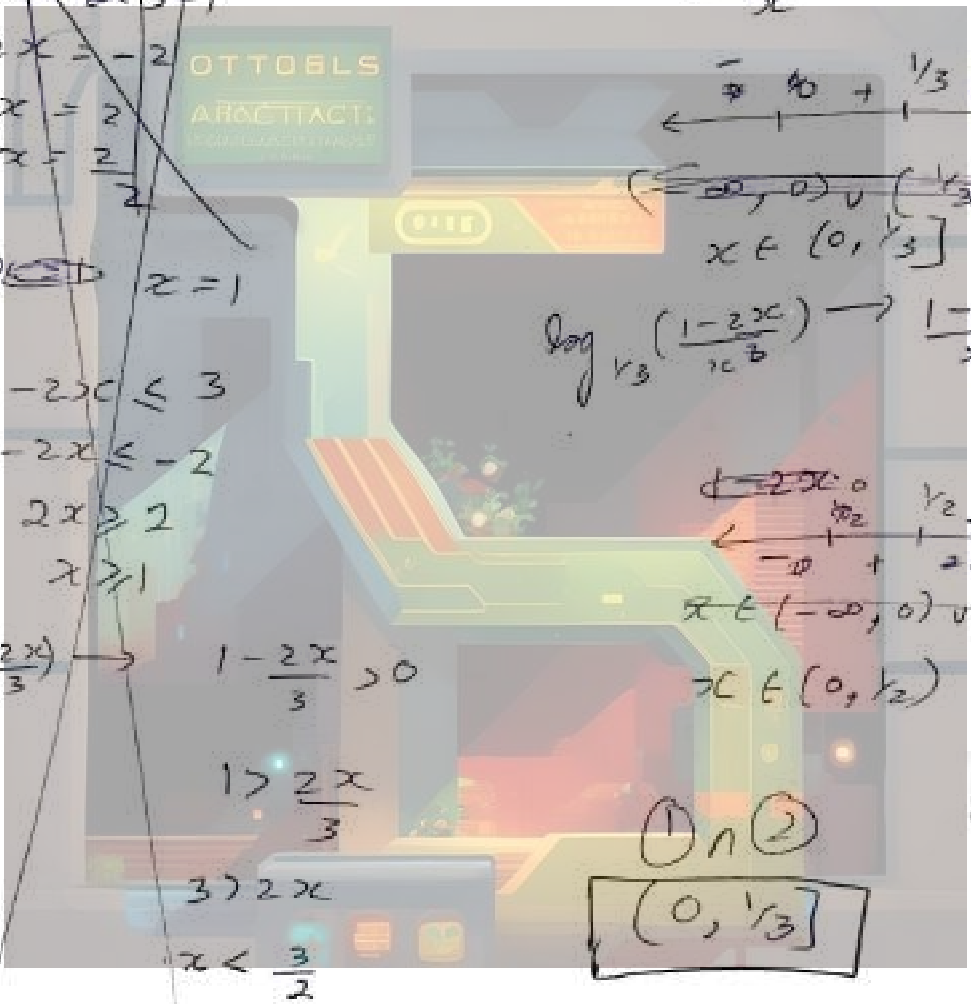
$-\quad 0 \quad + \quad \frac{1}{2} \quad -$

~~$x \in (-\infty, 0) \cup (\frac{1}{2}, \infty)$~~

$x \in (0, \frac{1}{2})$ — ②

① ∩ ②

$\boxed{(0, \frac{1}{3}]}$



② $\log_{2x}(x^2-5x+6) < 1$

Case 1 $2x \in (0,1)$

$x \in (0, \frac{1}{2})$

$x^2-5x+6 > 2x$

$x^2-7x+6>0$

$x^2-6x-x+6>0$

$(x-6)(x-1)>0$

$+ \quad 1 \quad - \quad 6 \quad +$

$x \in (-\infty,1) \cup (6,\infty)$

$x \in (0,\frac{1}{2})$ — ①

Case 2 $2x \in (1,\infty)$

$x \in (\frac{1}{2},\infty)$

$(x-6)(x-1) < 0$

$+ \quad 1 \quad - \quad 6 \quad +$

$x \in (1,6)$

$x \in (1,6)$ — ②

① ∪ ②

$x \in (0,\frac{1}{2}) \cup (1,6)$ — ③

$x^2-5x+6>0$

$x^2-3x-2x+6>0$

$x(x-3)-2(x-3)>0$

$(x-2)(x-3)>0$

$+ \quad 2 \quad - \quad 3 \quad +$

$x \in (-\infty,2) \cup (3,\infty)$

$\cap$

$2x>0$

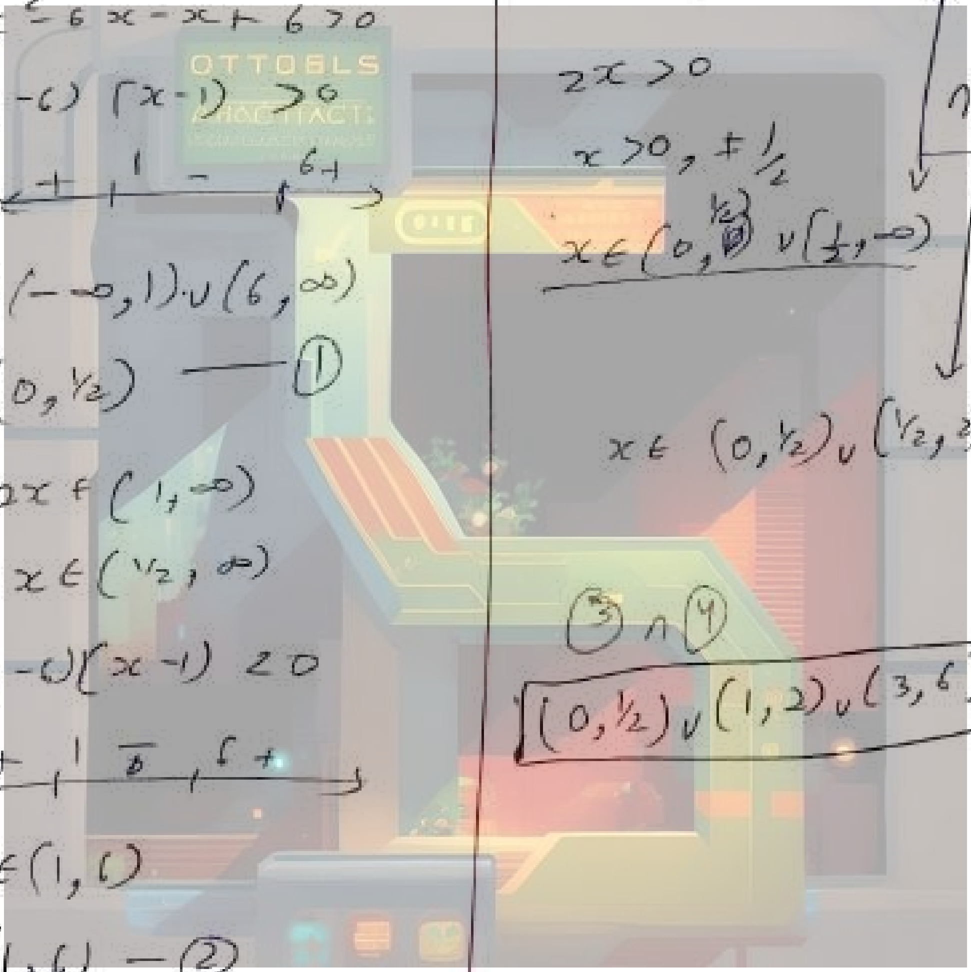
$x>0, \neq \frac{1}{2}$

$x \in (0,\frac{1}{2}) \cup (\frac{1}{2},\infty)$

$x \in (0,\frac{1}{2}) \cup (\frac{1}{2},2) \cup (3,\infty)$ — ④

③ ∩ ④

$\boxed{(0,\frac{1}{2}) \cup (1,2) \cup (3,6)}$



M.W. 08-06-2024

DYS-5 [1, 2, 3, 5, 7, 8, 9, 10]

O-1 [7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]

Q $\log_{0.5}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$

~~$\log_{0.5}(\ \ )$~~

$\log_6\left(\frac{x^2+x}{x+4}\right) > 1$

$\frac{x^2+x}{x+4} > 6$

$\frac{x^2+x}{x+4} - 6 > 0$

$\frac{x^2+x-6x-24}{x+4} > 0$

$\frac{x^2-5x-24}{x+4} > 0$

$\frac{x^2-8x+3x-24}{x+4} > 0$

$\frac{(x-8)(x+3)}{x+4} > 0$

$-4 \quad -3 \quad 8$

$- \quad + \quad - \quad +$

$x \in (-4,-3) \cup (8,\infty)$

$\frac{x^2+x}{x+4} > 0$

$\frac{x(x+1)}{x+4} > 0$

$-4 \quad -1 \quad 0$

$- \quad + \quad - \quad +$

$(-4,-1) \cup (0,\infty)$

$\frac{x^2+x}{x+4} > 1$

$\frac{x^2+x-x-4}{x+4} > 0$

$\frac{(x+2)(x-2)}{x+4} > 0$

$-4 \quad -2 \quad 2$

$- \quad + \quad - \quad +$

~~$x(x-3)$~~$+3(x-3)$

$x \in (-4,-2) \cup (2,\infty)$

$x \in (-4,-2) \cup (2,\infty)$

$x \in (-4,-3) \cup (8,\infty)$

$\boxed{x \in (-4,-3) \cup (8,\infty)}$

$\log_2 \log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$

$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 1$

$\log_7 \log_{1/3} (2x-3) < \frac{1}{2}\, x^{-1}\, \frac{1}{2}$

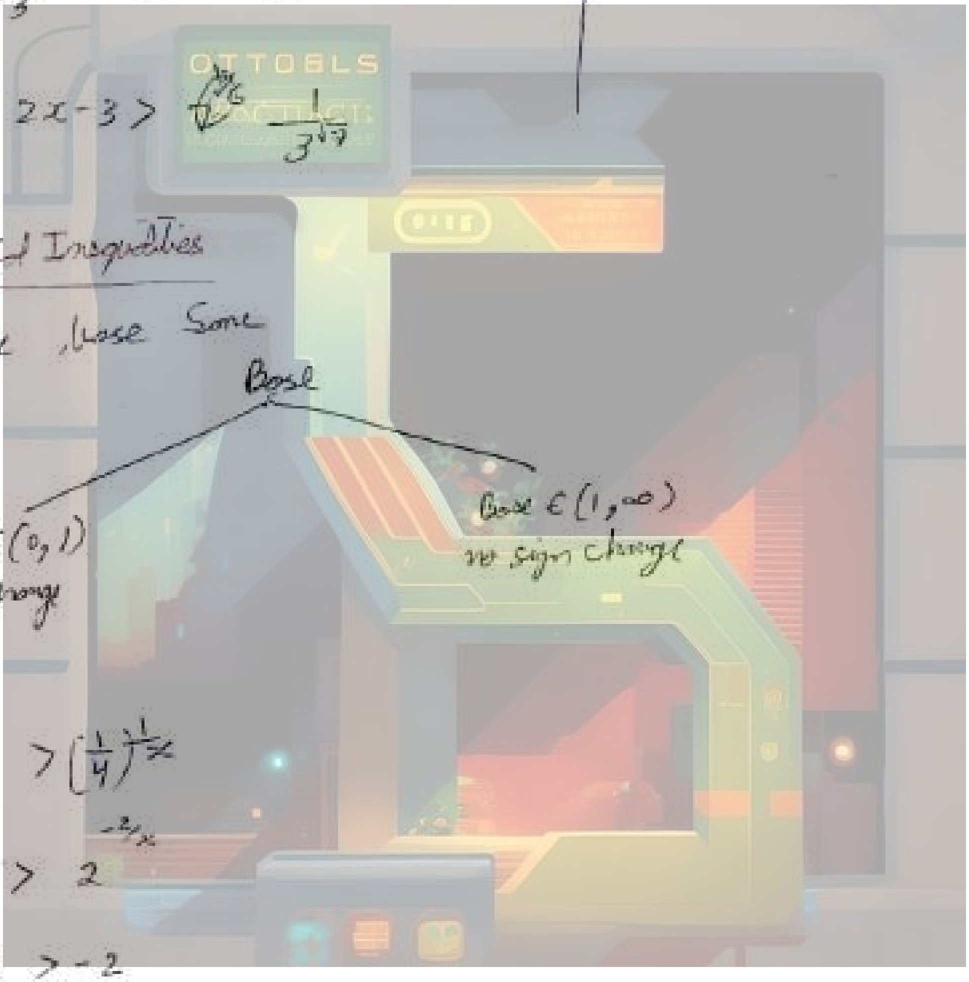
$\log_{1/3} (2x-3) < \sqrt{7}$

$2x-3 > 0$

$\log_{1/3} (2x-3) > 0$

$\log_7 \log_{1/3} (2x-3) > 0$

$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$



$\log\ 2x-3 > \frac{1}{3^{\sqrt{7}}}$

Exponential Inequalities

→ Make the base same

→

Base

Base $\in (0,1)$
Sign Change

Base $\in (1,\infty)$
no sign change

Q1. $2^{x+2} > \left(\frac{1}{4}\right)^{\frac{1}{x}}$

$2^{x+2} > 2^{-2/x}$

$x+2 > -\frac{2}{x}$

$x+2+\frac{2}{x} > 0$

$\frac{x^2+2x+2}{x} > 0$

$\frac{1}{x} > 0$

$\leftarrow\!\!-\!\!\underset{0}{|}\!\!-\!\!-$

$\boxed{x \in (0,\infty)}$

Q 2. $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

Q 3. $\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \frac{25}{4}$

$\frac{5}{2}\,\frac{2}{5} > \left(\frac{2}{5}\right)^{\frac{1}{2}\left(\frac{6-5x}{2+5x}\right)}$

$1 < \frac{1}{2}\left(\frac{6-5x}{2+5x}\right)$

$\frac{6-5x-2-5x}{2+5x} > 0$

$\frac{4\cancel{B}-10x}{2+5x} > 0$

$\frac{10x-4}{5x+2} < 0$

p~~a~~

$1 + \cancel{1}\,\frac{12-5x}{2+5x} < 0$

$\frac{2x+5x+12-5x}{2+5x} < 0$

~~12~~

$\frac{6+x}{5x+2} < 0$

$$\xleftarrow{\quad -6 \quad}\rightarrow$$

$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{2}{5}\right)^{-2}$

$\frac{5x-6}{5x-2} < 2$

$\frac{5x-6-10x-1}{5x-2} < 0$

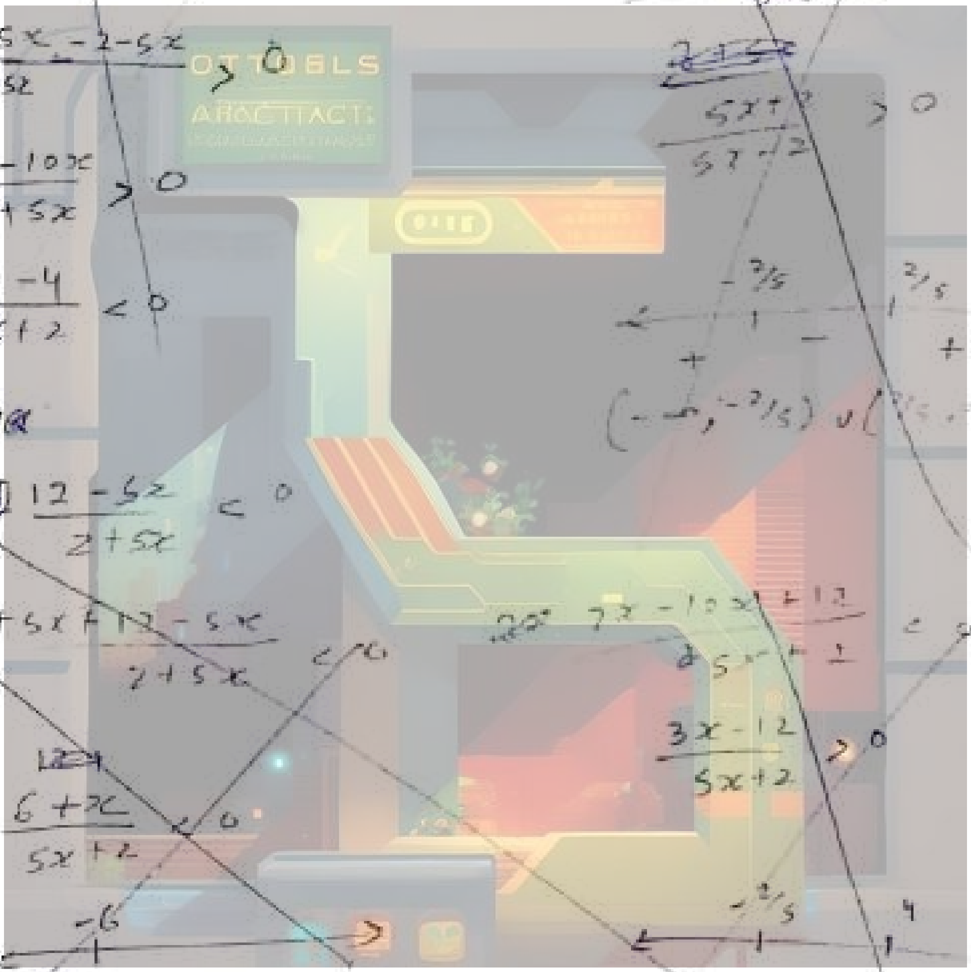
~~$2+5x$~~

$\frac{5x+2}{5x-2} > 0$

$$\xleftarrow{\quad -2/5 \qquad 2/5 \quad}\rightarrow$$

$+ \quad - \quad +$

$(-\infty, -2/5) \cup (2/5, \infty)$



$\frac{7x-10x+12}{5x+2} < 0$

$\frac{3x-12}{5x+2} > 0$

$$\xleftarrow{\quad -2/5 \qquad 4 \quad}\rightarrow$$

$(-\infty, -2/5) \cup (4, \infty)$

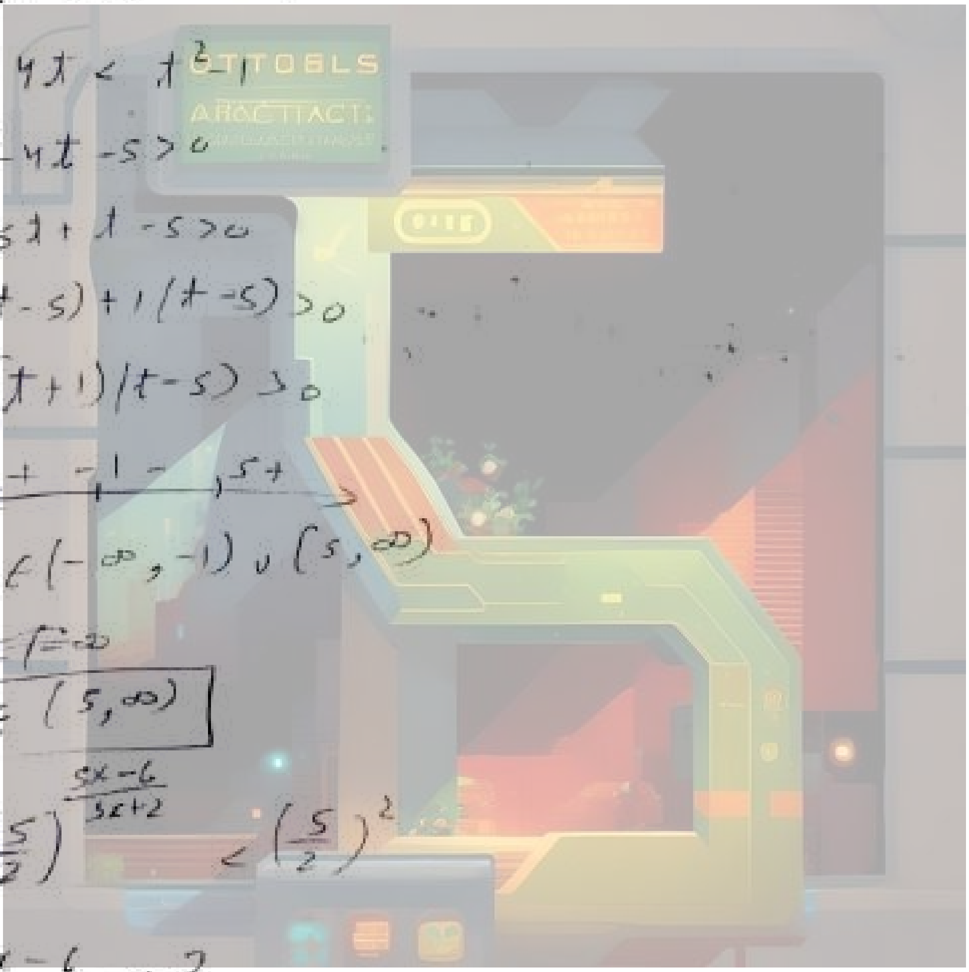
② $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

$\left(\frac{125}{100}\right)^{1-x} < \left(\frac{64}{100}\right)^{2(1+\sqrt{x})}$

$\left(\frac{5}{4}\right)^{1-x} < \left(\frac{5}{4}\right)^{-4(1+\sqrt{x})}$

$1-x < -4(1+\sqrt{x})$

$4(1+\sqrt{x}) < x-1$



$4+4t < t^2-1$

$t^2-4t-5>0$

$t-5t+t-5>0$

$t(t-5)+1(t-5)>0$

$(t+1)(t-5)>0$

$\xleftrightarrow{\;+\;\;-1\;\;-\;\;5\;\;+\;}$

$t\in(-\infty,-1)\cup(5,\infty)$

~~$x\in(-\infty$~~

$\boxed{x\in(5,\infty)}$

③ $2\left(\frac{5}{2}\right)^{\frac{5x-6}{5x+2}} < \left(\frac{5}{2}\right)^2$

$\frac{5x-6}{5x+2} < 2$

$\frac{5x-10}{5x+2} < 0$

$\frac{x+2}{5x+2} > 0$

$x\in(-\infty,-2)\cup(-2/5,\infty)$

DYS-6

⑧ $\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > 1$

$\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > \left(\frac{2}{3}\right)^{0}$

$\frac{|x|-1}{|x|+1} < 0$

$|x|-1 < 0$

$|x| < 1$

$\boxed{x \in (-1, 1)}$

H.W. ~~CC~~

DYS-6 -(Full) O-1(21, 22, 23, 24, 25, 26, 27, 28, 29)

O-1(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

O-2(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

C = 2.7

DYS-~~4~~ 5

④ $\left(\log_{10} 100x\right)^2 + \left(\log_{10} 10x\right)^2 + \log_{10} x < 14$

$\cancel{\frac{(\log 100x)^2}{(\log 10)^2} + \frac{(\log 10x)^2}{(\log 10)^2} + \frac{\log x}{\log 10}}$

$\cancel{\frac{(\log 100)^2 + (\log x)^2 + (\log 10)^2 + (\log x)^2 + \ldots}{\log 10 \; \log 10}}$

$\left(2 + \log_{10} x\right)^2 + \left(1 + \log_{10} x\right)^2 + \log_{10} x < 14$

$4 + y^2 + 4y + 1 + y^2 + 2y + y < 14$

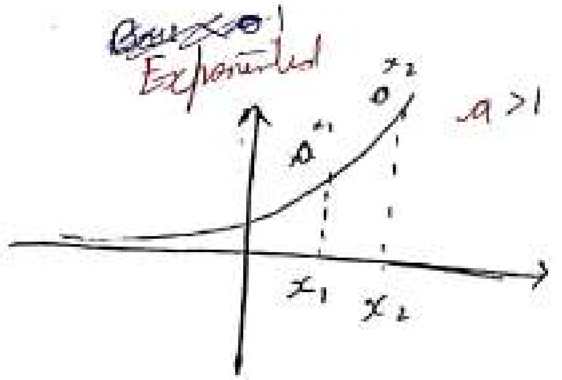
$2y^2 + 7y - 9 < 0$ $\quad 2y^2 + 9y - 2y - 9 < 0$

$-9/2 \qquad 1$

$\log_{10} x \in (\cancel{-\infty}\, -9/2, 1)$

$\boxed{x \in (10^{-9/2}, 10)}$ ✓

Exponential graphs and logarithmic graphs



$x_1 < x_2$

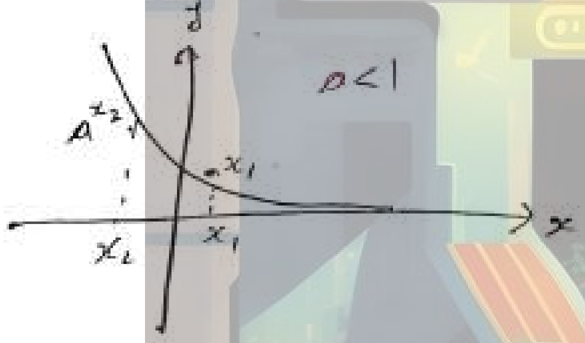
$a^{x_1} < a^{x_2}$

$a^{f(x)} > b$

$\log_a a^{f(x)} > \log_a b$

$a>1 \Rightarrow f(x) > \log_a b$

$0<a<1 \Rightarrow f(x) < \log_a b$



$x_2 < x_1$

$a^{x_2} > a^{x_1}$

Domain (Range of x) → $(-\infty, \infty)$

Range → $(0, \infty)$

$y = \log_a x$

$a>0 \quad x>0$

$a=2$: $x=2, y=1$, $x=4, y=2$, $x=8, y=3$, $x=16, y=4$

$a=4$: $x=4, y=1$, $x=16, y=2$, $x=64, y=3$, $x=256, y=4$

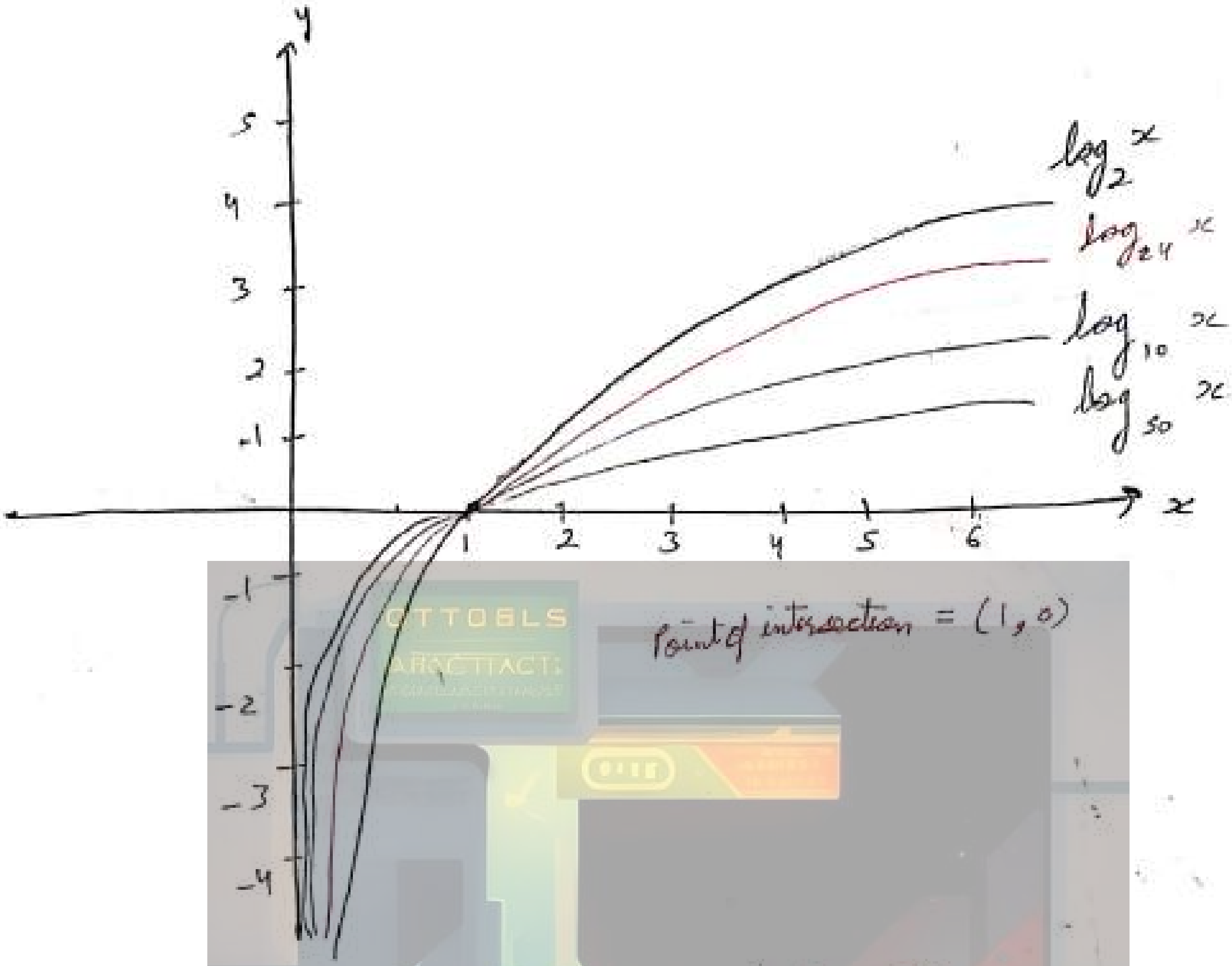
$a=10$: $x=10, y=1$, $x=100, y=2$, $x=10^3, y=3$, $x=10^4, y=4$

$a>0 \quad x<0$

$a=2$: $x=1/2$, $1/4$, $1/8$; $y=-1$, -2, -3

$a=4$: $x=1/4$, $1/16$, $1/64$; $y=-1$, -2, -3

$a=10$: $x=1/10$, $1/100$, $1/1000$; $y=-1$, -2, -3

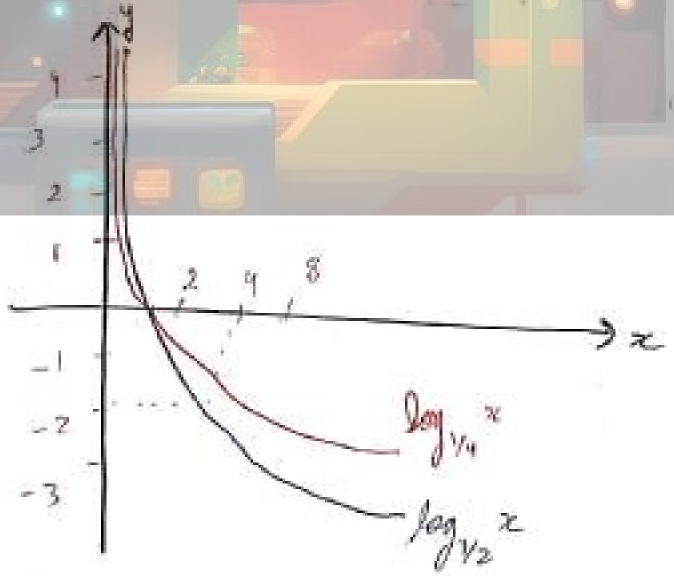


Point of intersection = $(1, 0)$

I draw graph $y = \log_{1/2} x$ $y = \log_{1/4} x$

$y =$	1	2	3	4
$x =$	$1/2$	$1/4$	$1/8$	$1/16$

$0 < b < 1$



Conclusions

→ Value of log can be ⊕ve & can be ⊖ve or zero.

→ Graph of log lies in Ist & IVth quadrant.

→ when Base 'a' and number 'x' lies on either side of 1 (1 ke aaju baaju) then value of log will be ⊖ve.
when base 'a' and number 'x' lies on the same side of one then the value of log will be ⊕ve.

0 < Base < 1	Base > 1
$x \in (0,1)$ ⊕ve	$x \in (1,\infty)$ ⊕ve
$x \in (1,\infty)$ ⊖ve	$x \in (0,1)$ ⊖ve

for $\log_a x$

Q which of the following is ⊖ve

① $(\log_{1/5} 7)^{10}$ ⊕ve

② $\log_{1/5} 7 \cdot \log_{1/3} 2$ ⊕ve

③ $\log_3 1/2$ ⊖ve

④ $\log_2 8$ ⊕ve

Q. find Signs

① $\log_9(\log_4 5)$ ⊕ve

② $\log_5(\log_4 22)$ ⊕ve

③ $\log_{\log_{11} 5}\left(\frac{3}{7}\right)$ ⊕ve

Characteristic & Mantissa

↓ log's integer part

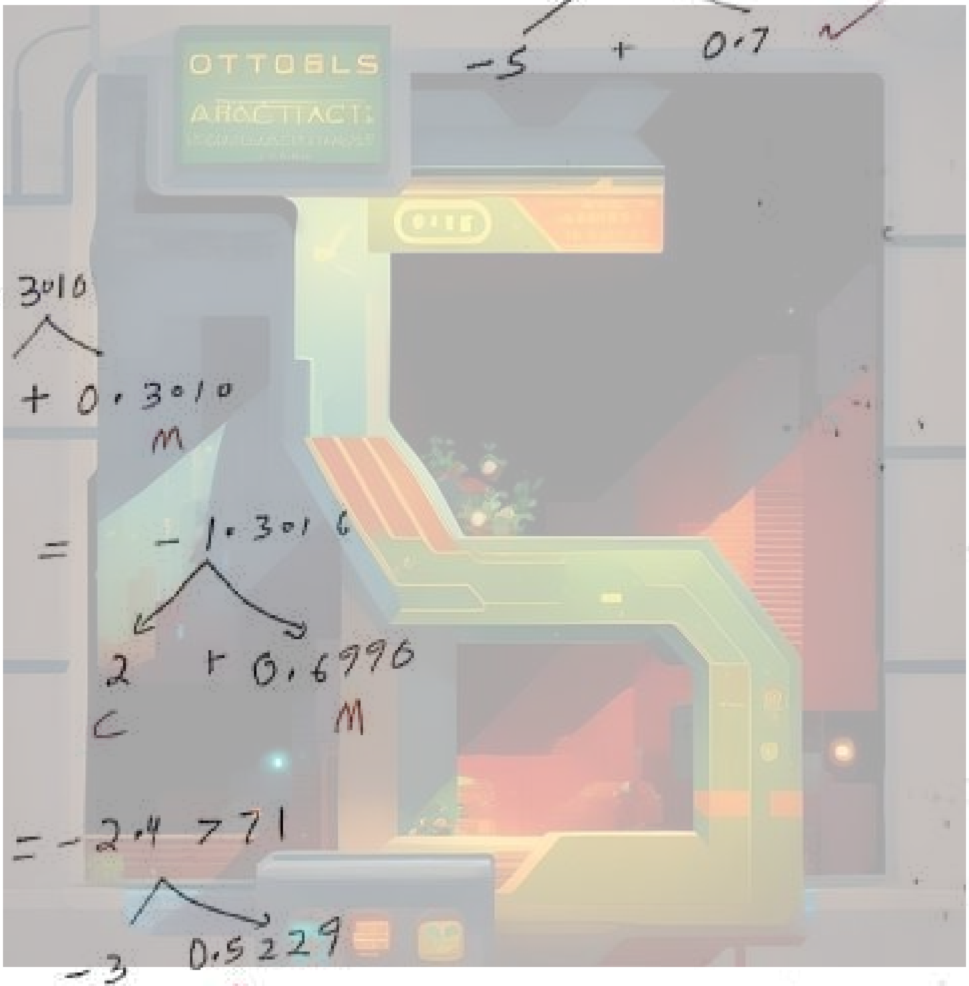
→ fractional part [0 - 1)

can be zero

$$\underset{1 \quad + \quad 0.7}{1.7}$$

$$\underset{-4 \quad + \quad -0.3}{-4.3} \quad ✗$$

$$\underset{-5 \quad + \quad 0.7}{-4.3} \quad ✓$$



$\log_{10} 20 = 1.3010$

$\underset{C}{1} + \underset{M}{0.3010}$

$\log_{10} 1/20 = -1.3010$

$\underset{C}{2} + \underset{M}{0.6990}$

$\log_{10} 1/300 = -2.4771$

$\underset{C}{-3} \quad \underset{M}{0.5229}$

Applications :-

→ Finding the number of digits using common log
Base - 10

$$\boxed{\text{No. of ~~gets~~ digits} = C + 1}$$

~~AB~~

* $\log_{10} 2 = 0.3010$
* $\log_{10} 3 = 0.4771$

Eg. ① 10^2 $\log_{10} 10^2 = 2$

$c = 2$ $m = 0$

no. of digits $= c+1$
$= 2+1$
$= 3$

② 10^3 $\log_{10} 10^3 = 3$

$c = 3$ $m = 0$

no. of digits $= c+1$
$= 3+1$
$= 4$

③ 10^4 $\log_{10} 10^4 = 4$

$c = 4$ $m = 0$

no. of digits $= c+1$
$= 4+1$
$= 5$

Q Find the no. of digits in 6^{50}

$\log_{10} 6^{50}$

$50 \log_{10} 6$

$50(\log_{10} 2 + \log_{10} 3)$

$50(0.3010 + 0.4771)$

$50(0.7781)$

38.905

$C = 38$ $M = 0.905$

no. of digits $= 38+1$
$\boxed{= 39}$

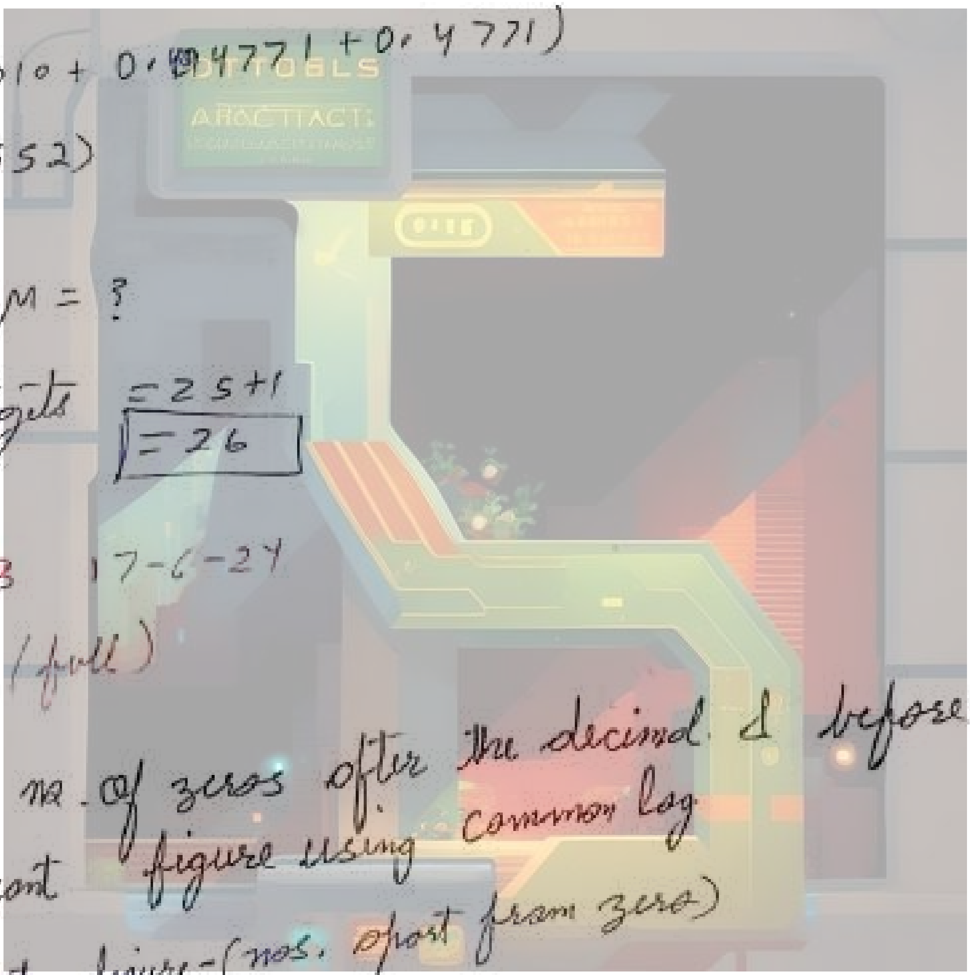
DYS 7

Q 2. 18^{20}

$\log_{10} 18^{20}$

$20 \log_{10} 18$

$20(\log_{10} 3 + \log_{10} 3 + \log_{10} 2)$



$20(0.3010 + 0.4771 + 0.4771)$

$20(1.2552)$

$C = 25 \quad M = ?$

no. of digits $= 25+1$

$\boxed{= 26}$

H.W. ~~Q-3~~ 17-6-24

Q-3 (full)

→ Finding no. of zeros after the decimal & before the significant figure using common log

Significant figure-(nos. apart from zero)

no. of zeros $= |C| - 1$

eg. 0.01

$\log_{10} 0.01 = -2$

$C = -2 \quad M = 0$

nos. of zeros $= 2 - 1$

$= 1$

0.0001

$\log_{10} 10^{-4} = -4$

$C = -4 \quad M = 0$

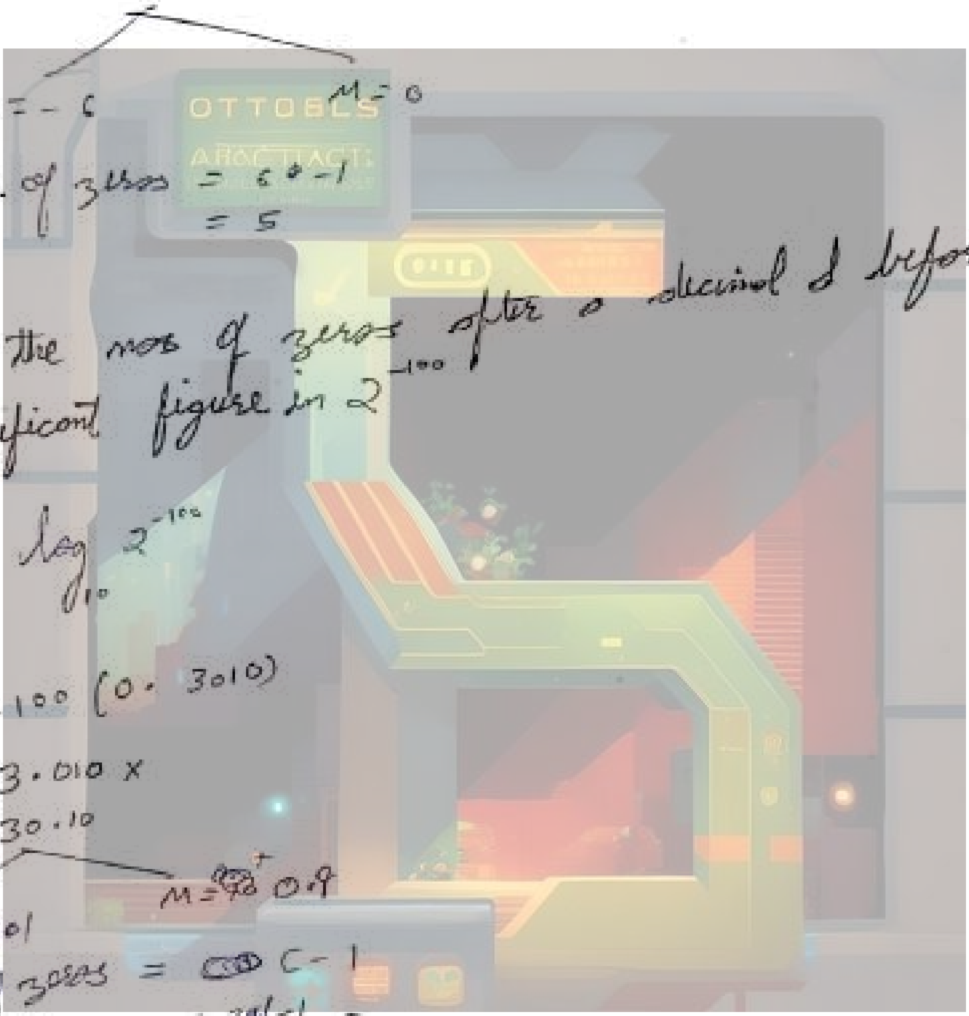
nos. of zeros = 4 - 1

= 3

0.000001

$\log_{10} 10^{-6} = -6$

$C = -6 \quad M = 0$



nos. of zeros = 6 - 1

= 5

Q Find the nos of zeros after a decimal & before a significant figure in 2^{-100}

$\log_{10} 2^{-100}$

$-100 (0.3010)$

$-3.010 \times$

-30.10

$C = -301 \quad M = 0.9$

no. of zeros = C - 1

= 301 - 1

$\boxed{= 30}$

DYS-7

Q7. $\log_{10} 3^{-100}$

$-100(0.4771)$

-47.71

$C = 48$ $m =$

$48 - 1$

$= ~~[illegible]~~ 47$

$\boxed{59}$

$\log_{10} 6^{15}$

$15(0.3010 + 0.4771)$

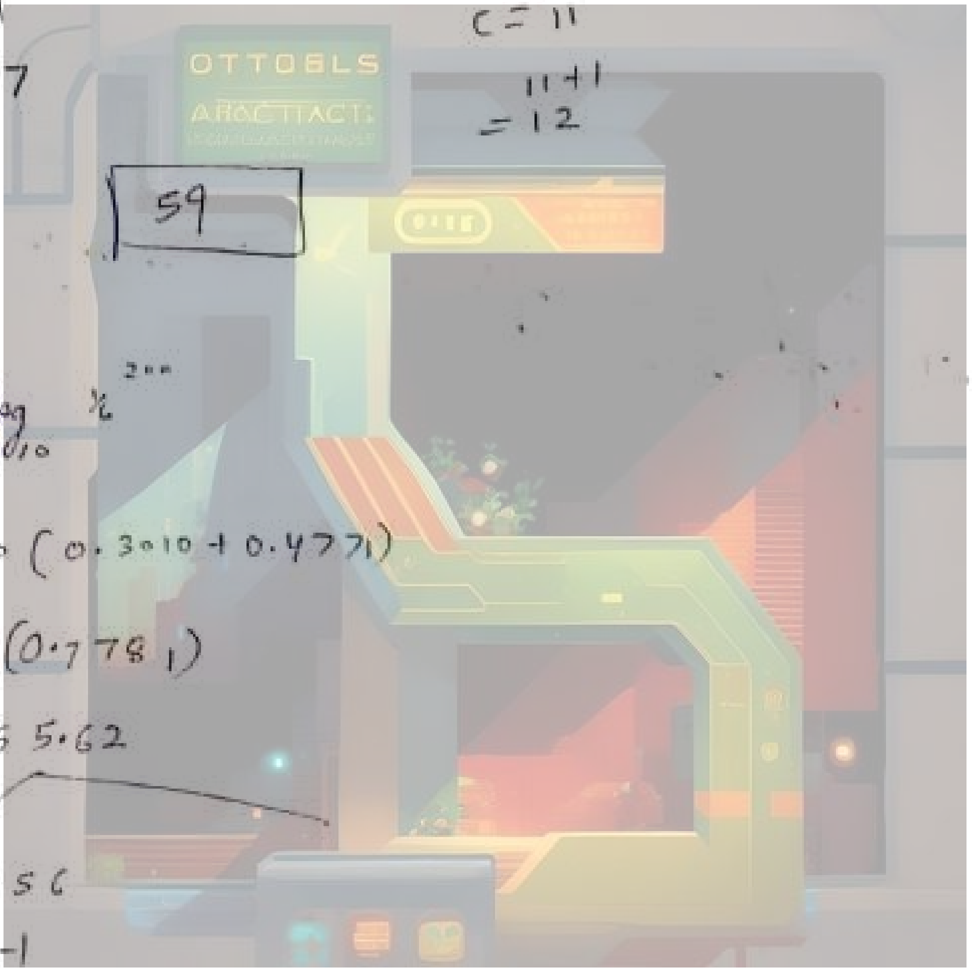
$15 \times (0.7781)$

11.6715

$C = 11$

$11 + 1$

$= 12$



Q3. $\log_{10} \frac{1}{6}^{200}$

$-200(0.3010 + 0.4771)$

$-200(0.7781)$

-155.62

$C = -156$

$156 - 1$

$\boxed{155}$

Q Evaluate: $\log_{10}(0.06)^6$ find characteristic & Mantissa

~~$6 \log_{10} 6 + \log_{10} 10^{-2}$~~

~~$6[(0.7781) - 2]$~~

~~$4.6686 - 12$~~

~~7.6686~~ -8.6686

~~$C = 2$~~
~~$M = 0.6656$~~

$C = -9$

$M = 0.3314$

$= \log_{10} \left(\frac{6}{10}\right)$

$6(0.7781 - 1)$

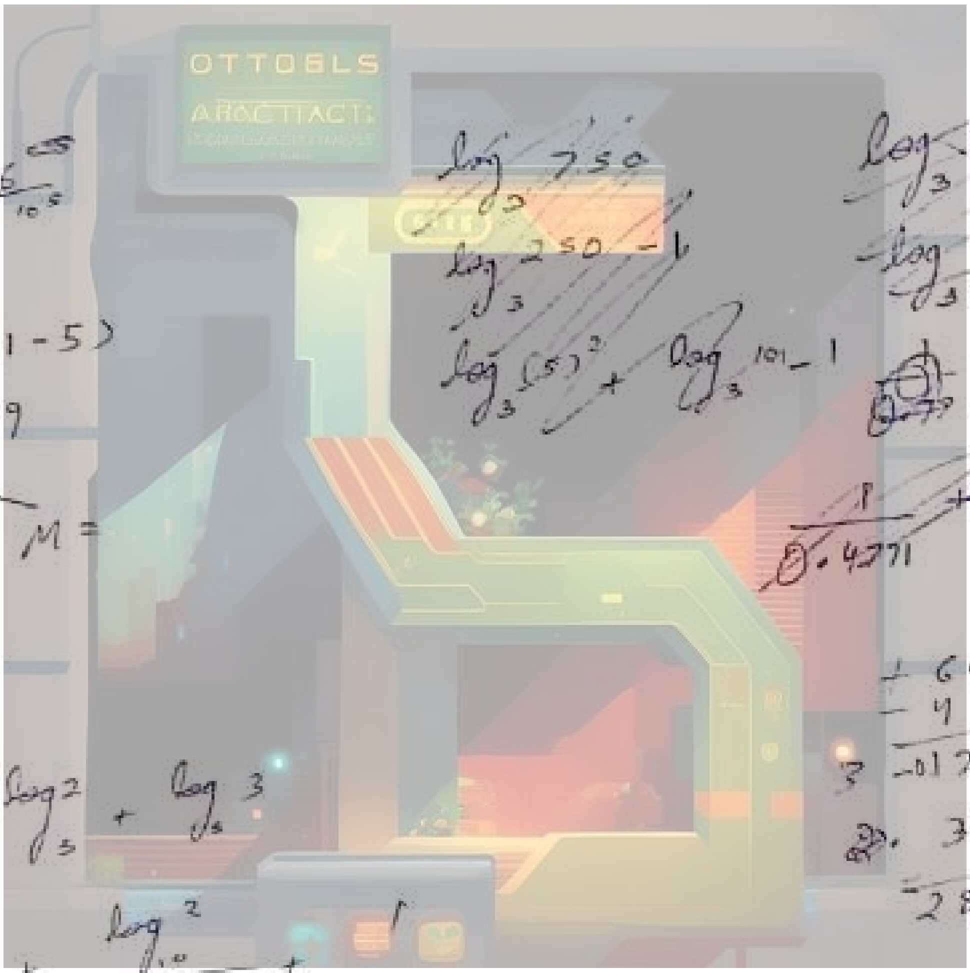
~~$6 \times (-2)$~~

$6(-1.2219) = -7.3301$

$C = -8$ $\quad M = 0.668$

DYS-7

Q 5.



$\log_{10} \frac{6}{10^5}$

$(0.7781 - 5)$

-4.2219

$C = -5$ $\quad M =$

~~$\log_3 750$~~

~~$\log_3 250 - 1$~~

~~$\log_3 (5)^2 + \log_3 10 - 1$~~

~~$\log_3 750$~~

~~$\log_3 (10)^2 + \log_3 75$~~

~~$\frac{1}{0.4771} + 3$~~

$\log_3 750$

$3\log_3 5 + \log_3 2 + \log_3 3$

$\frac{3\log_{10} 10/2}{\log_{10} 3} + \frac{\log_{10} 2}{\log_{10} 3} + 1$

$\frac{3(1-0.3010)}{0.4771} + \frac{0.3010}{0.4771} + 1$

$\frac{3 - 0.9030 + 0.3010 + 0.4771}{0.4771}$

$\frac{2.8751}{0.4771}$

$= 6.\ldots$

$\boxed{C = 6}$

$$\begin{array}{r} .6020 \\ -.4771 \\ \hline .1249 \end{array}$$

$$\begin{array}{r} 3.0000 \\ .1249 \\ \hline 2.8751 \end{array}$$

$\overline{2.8751}$

$6 + (-5)$

$\boxed{= 1}$

H.W. 18-6-29

Race 9-19

Q.4

Q.4/Quadratic?

J.A


OTTOBLS
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!! Sequence & Series !!

A.P. (Arithmetic Progression)
G.P (Geometric Progression)
HP (Harmonic Progression)
AGP (Arithmetic Geometric progression)
Special Sequences - 4 types

Sequence - ~~Seq~~ Succession of Terms which may be ~~..~~ algebric, real & complex numbers written according to a definite rule.

eg. 2, 3, 5, 7, 11 (Prime nos.)
4, 8, 12, 16, ... (multiples of 4)
0, 0, 0, 0, ... (multiples of 0)

→ Minimum no. of required terms of a sequence is 3.

Progression - Special case of sequence in which we can express the n^{th} term mathematically.

eg. 2, 4, 6, 8, 10..... $a_n = 2n$
1, 2, 3, 4, 5... $a_n = n$

Series - If we put sign of addition or & subtraction between the terms of sequence then it is called series.

eg. 2 + 3 + 5 + 7 + 11 + ...
0 + 7 + 26 + ...
1 - 3 + 9 - 27 + ...

Q find out the 2nd & 4th Terms of the Sequence.

$$T_n / a_n = \left\{\frac{-1}{n}\right\}^n$$

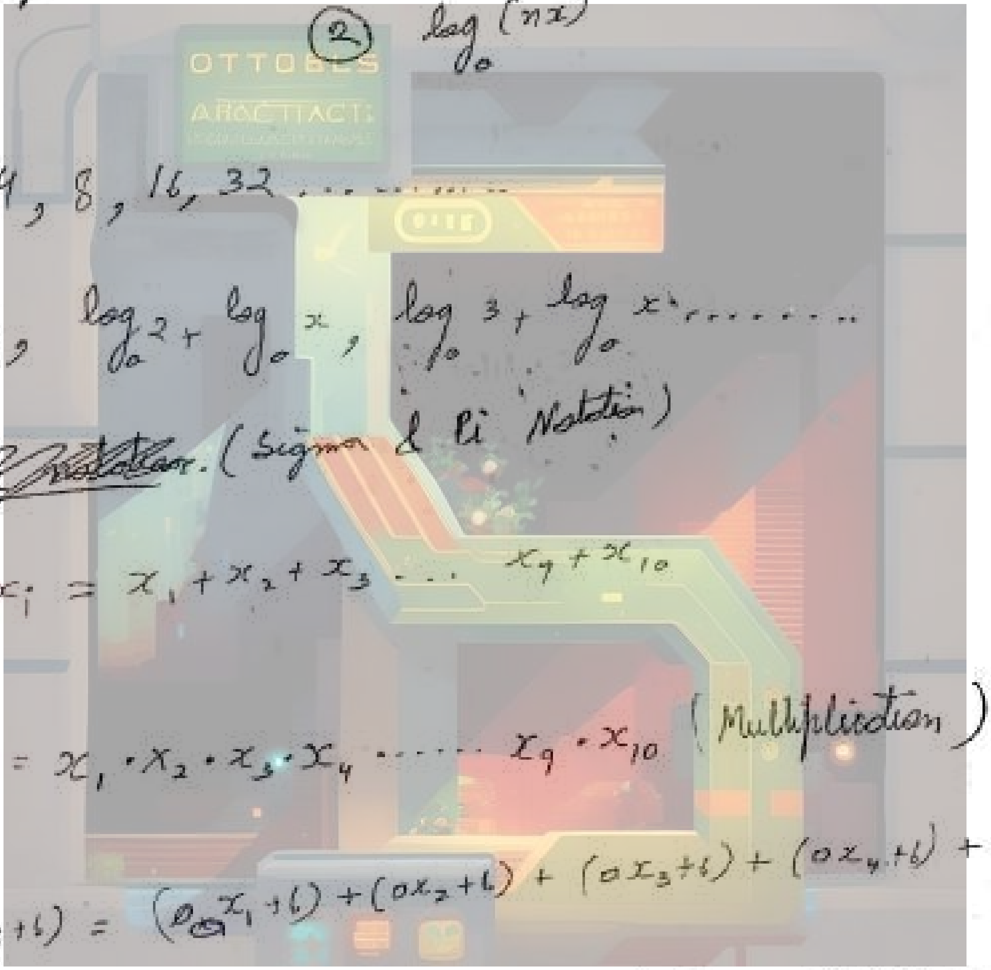
$a_1 = \left(\frac{-1}{1}\right)^1$ $a_2 = \left\{\frac{-1}{2}\right\}^2$ $a_4 = \left\{\frac{-1}{4}\right\}^4$

$a_1 = -1$ $a_4 = \frac{1}{256}$

$a_2 = \frac{1}{4}$

Q write sequence whose nth terms are

① 2^n ② $\log_a (nx)$



① $2, 4, 8, 16, 32 \ldots\ldots$

② $\log_a x, \log_a 2 + \log_a x, \log_a 3 + \log_a x \ldots\ldots$

. (Sigma & Pi Notation)

$$\sum_{i=1}^{10} x_i = x_1 + x_2 + x_3 \ldots x_9 + x_{10}$$

$$\prod_{i=1}^{10} x_i = x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdots x_9 \cdot x_{10}$$ (Multiplication)

$$\sum_{i=1}^{5} (ax_i + b) = (ax_1 + b) + (ax_2 + b) + (ax_3 + b) + (ax_4 + b) + (ax_5 + b)$$

$$\prod_{i=1}^{5} (ax_i + b) = (ax_1 + b)(ax_2 + b)(ax_3 + b)(ax_4 + b)(ax_5 + b)$$

Arithmetic Progression (A.P.)

→ Sequence Having two consecutive terms have the constant difference.

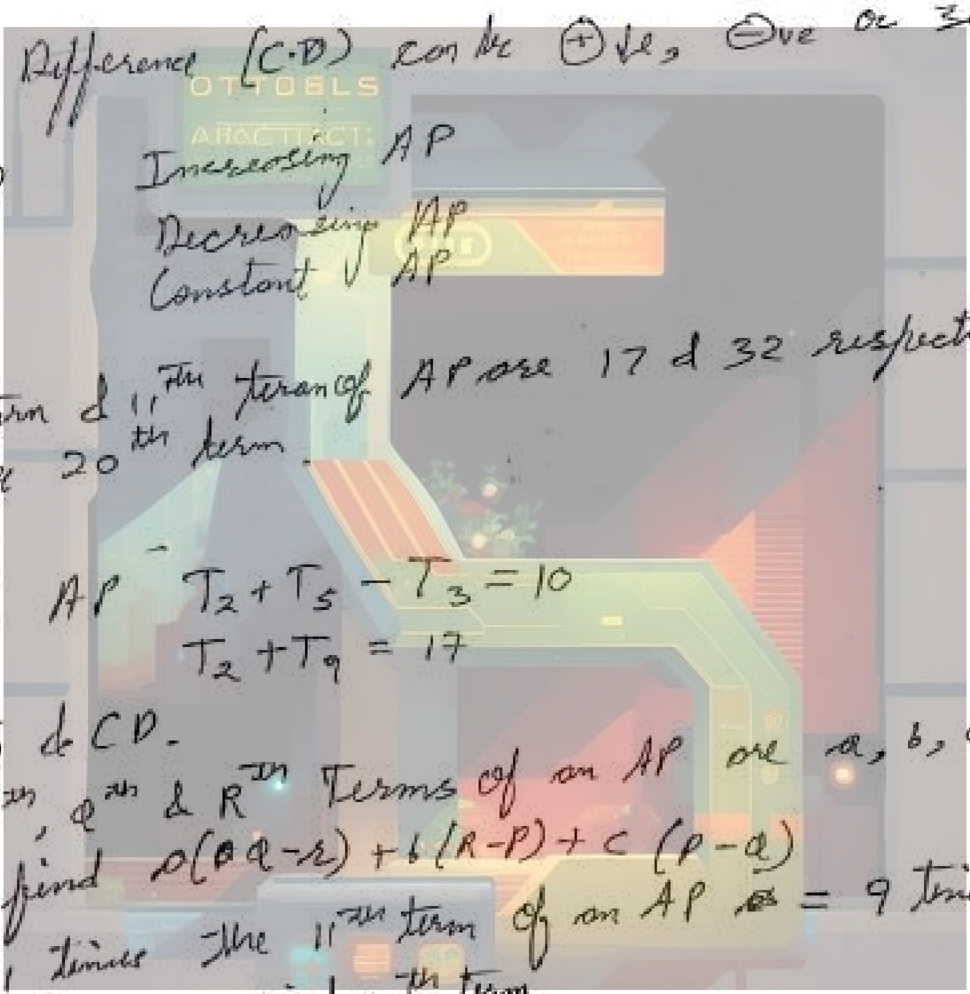
→ $a,\ a+d,\ a+2d,\ a+3d\ \ldots\ a+nd$

$T_1 \quad T_2 \quad T_3 \quad T_4$

$\boxed{T_n = a+(n-1)d}$

a = first term

d = common difference.



Common Difference (C.D) can be +ve, −ve or zero

$CD>0$ Increasing AP

$CD<0$ Decreasing AP

$CD=0$ Constant AP

Q1. If 6th term & 11th term of AP are 17 & 32 respectively find the 20th term.

Q2. In an AP $T_2+T_5-T_3=10$

$T_2+T_9=17$

find T_1 & CD.

Q3. If Pth, Qth & Rth Terms of an AP are a, b, c respectively then find $a(Q-R)+b(R-P)+c(P-Q)$

Q4. If 11 times the 11th term of an AP = 9 times the 9th term then find 20th term.

Q1. $17 = a+5d$

$32 = a+10d$

$15 = 5d$

$\boxed{d=3}$

$a = 17-15$

$\boxed{a=2}$

$T_{20} = 2+19\times 3$

$\boxed{=59}$

Q2.

$2a+63d = 2020$

$2a+9d = 117$

$-3d = 3$

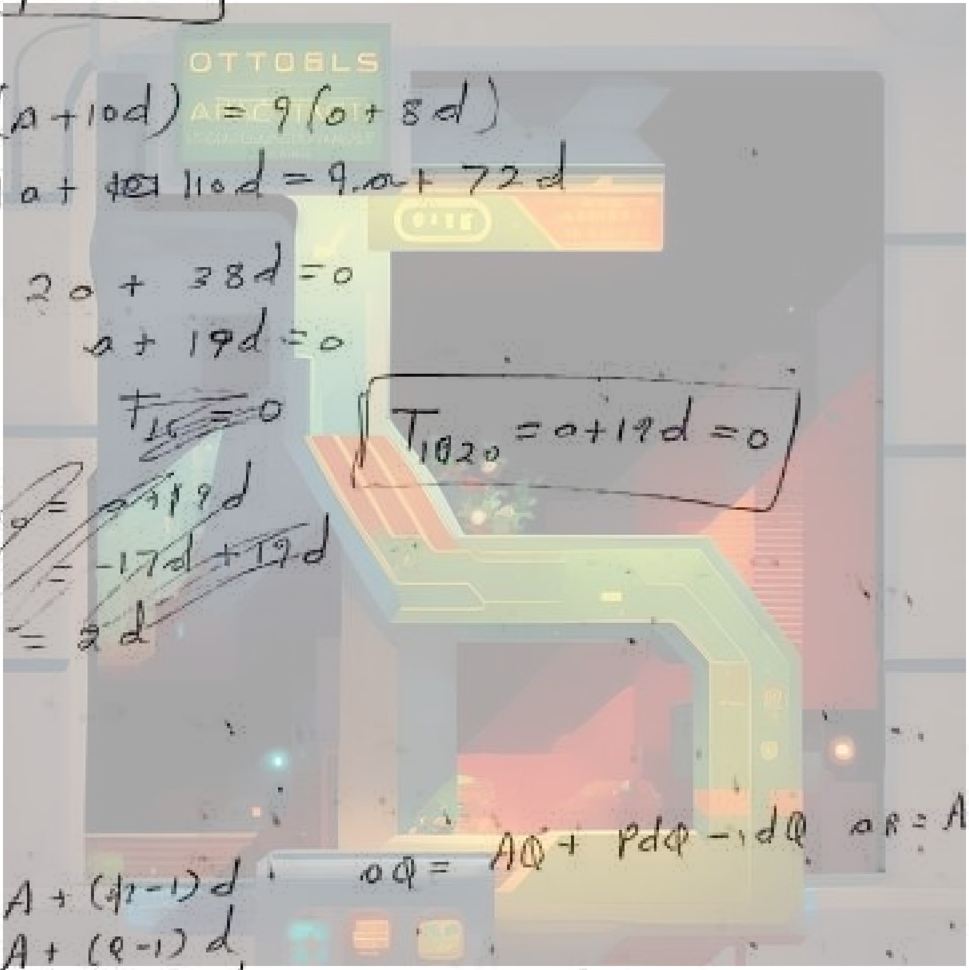
~~$d = 1$~~

$\boxed{d=-1}$

$a+3(-1) = 120$

$a-3 = 120$

$\boxed{a = 123}$



Q4. $11(a+10d) = 9(a+8d)$

$11a + 110d = 9a + 72d$

$2a + 38d = 0$

$a + 19d = 0$

~~$T_{15} = 0$~~

$\boxed{T_{1020} = a+19d = 0}$

~~$T_{20} = a+19d$~~

~~$= -17d + 19d$~~

~~$= 2d$~~

Q3. $a = A+(P-1)d$ $aQ = AQ + PdQ - 1dQ$ $aR = AR + PdR - dR$

$b = A+(Q-1)d$

$c = A+(R-1)d$

$AQ + PdQ - dQ - AR - PdR + dR + AR - AP + QdR - dR$

$- QdP + dP - AQ + dQ + PdR - dR$

$\boxed{= 0}$

Some Facts

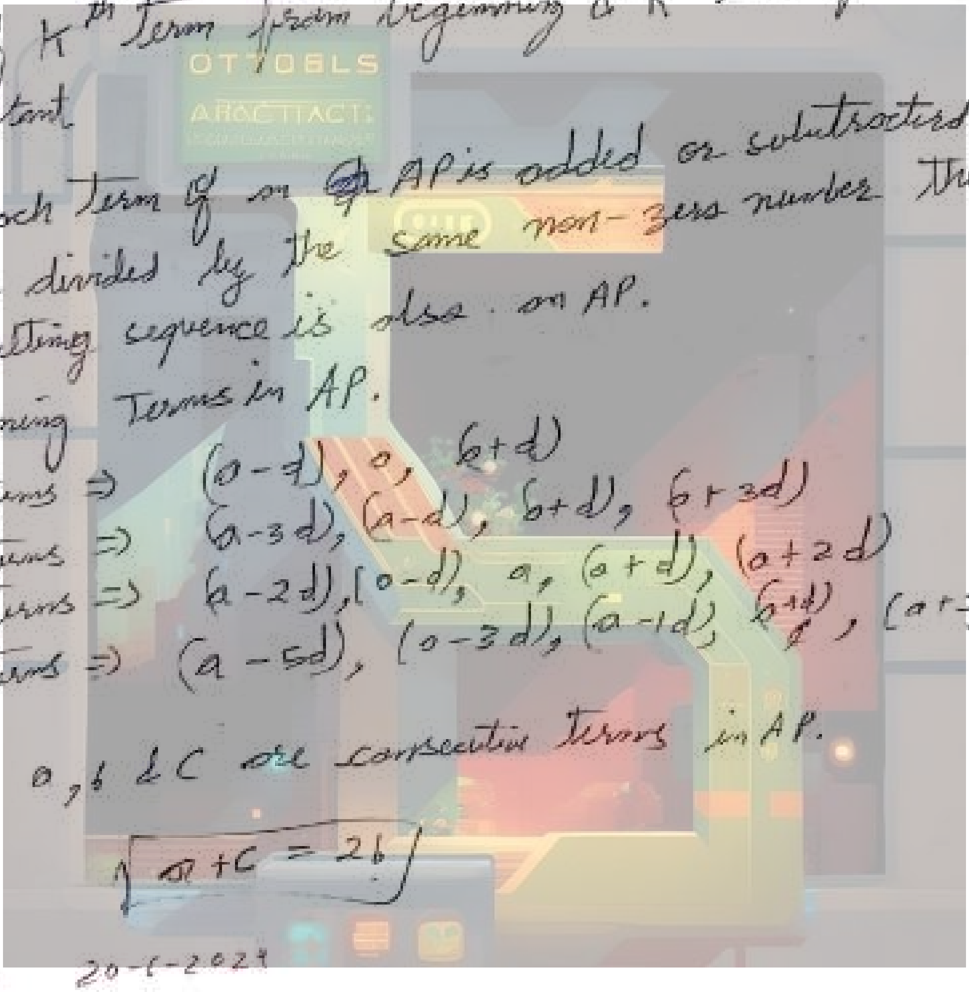
→ K^{th} term from the end of an AP

$$T_K = l + (K-1)(-d)$$

$$n, (n-1), (n-2) \dots 3, 2, \underset{l}{\overset{\downarrow}{1}} \Rightarrow -d$$

→ If m^{th} term is n & n^{th} term is m then $(m+n)^{th}$ term $= 0$

→ Sum of K^{th} term from beginning & K^{th} term from end is always constant



→ If each term of an AP is added or subtracted or multiplied or divided by the same non-zero number then the resulting sequence is also an AP.

→ Assuming Terms in AP.

3 terms ⇒ $(a-d), a, (a+d)$

4 terms ⇒ $(a-3d), (a-d), (a+d), (a+3d)$

5 terms ⇒ $(a-2d), (a-d), a, (a+d), (a+2d)$

6 terms ⇒ $(a-5d), (a-3d), (a-1d), (a+1d), (a+3d), (a+5d)$

→ If a, b & C are consecutive terms in AP.

$$\boxed{a + c = 2b}$$

H.W. 20-6-2024

DYS-1

DYS-2

Summation of an A.P.

$S_n = a + (a+d) + (a+2d) + (a+3d) \ldots\ldots (a+(n-2)d) + (a+(n-1)d)$

$+S_n = a + (n-1)d + a + (n-2)d \ldots\ldots (a+d) + (a)$

$2S_n = 2a + (n-1)d + 2a + (n-1)d + 2a + (n-1)d \ldots\ldots + 2a + (n-1)d$

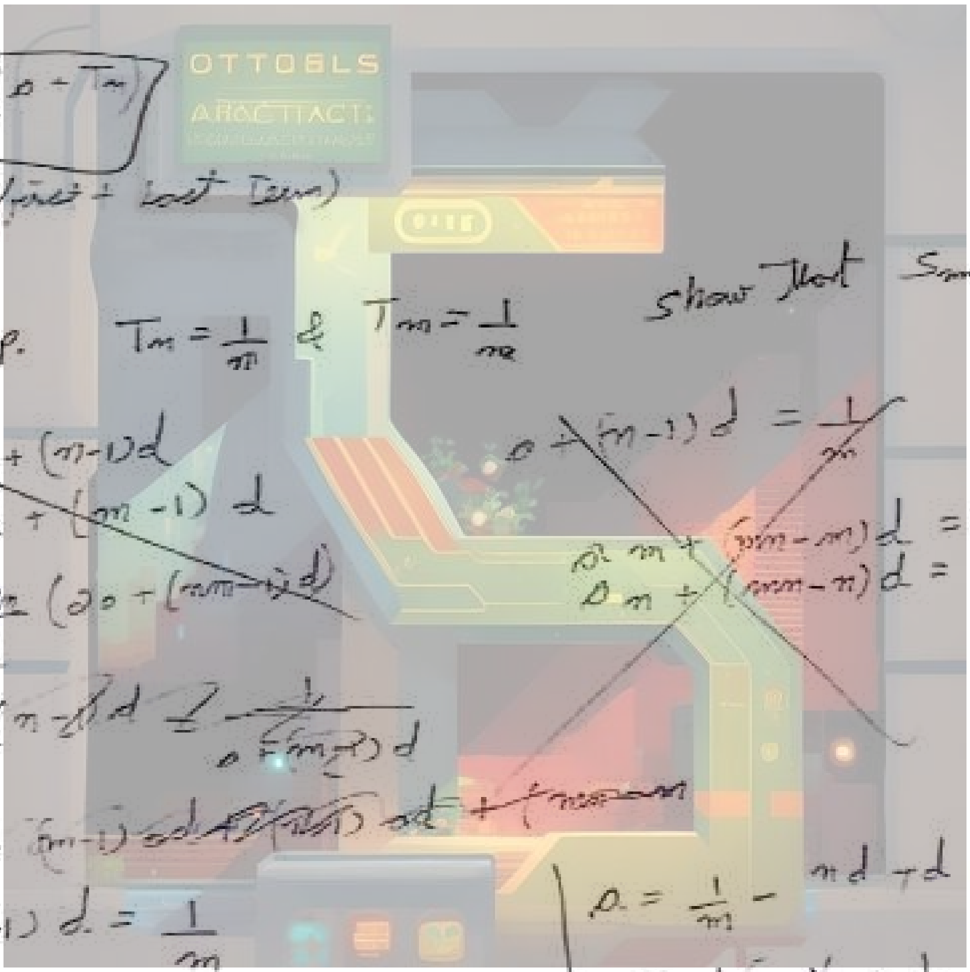
$= n(2a + n-1)d$

$$\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$$

$S_n = \frac{n}{2}(a + a + (n-1)d)$

$$\boxed{S_n = \frac{n}{2}(a + T_n)}$$

$S_n = \frac{n}{2}$ (first + last Term)



Q In an A.P. $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$ show that $S_{mn} = \frac{1}{2}(mn+1)$

~~$T_n = a + (n-1)d$~~

~~$T_m = a + (m-1)d$~~

~~$S_{mn} = \frac{mn}{2}(2a + (mn-1)d)$~~

~~$a + (n-1)d = \frac{1}{a + (m-1)d}$~~

~~$a^2 + (m-1)ad + (n-1)ad + (mn-m$~~

~~$a + (n-1)d = \frac{1}{m}$~~

~~$am + (mn-m)d = 1$~~

~~$an + (mn-n)d = 1$~~

$a + (n-1)d = \frac{1}{m}$

$a + (m-1)d = \frac{1}{n}$

$(n-1)d - (m-1)d = \frac{1}{m} - \frac{1}{n}$

$d(n-1-m+1) = \frac{n-m}{mn}$

$d(n-m) = \frac{n-m}{mn}$

$$\boxed{d = \frac{1}{mn}}$$

$a = \frac{1}{m} - nd + d$

$= \frac{1}{m} - \frac{1}{m} + \frac{1}{mn}$

~~$= \frac{2}{m} - \frac{1}{mn}$~~

~~$a = \frac{2n-1}{mn}$~~

$a = \frac{1}{mn}$

13

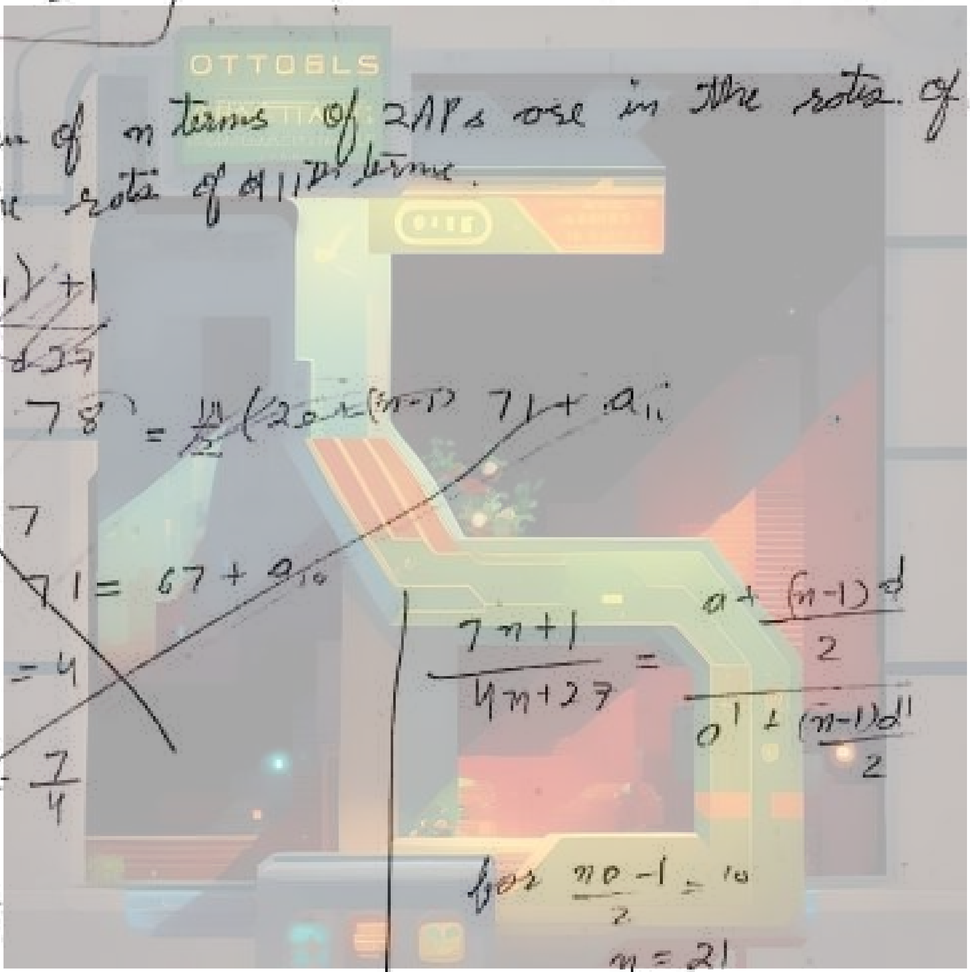
$$S_{mn} = \frac{mn}{2}\left(2a + (mn-1)d\right)$$

$$= \frac{mn}{2}\left(2 \times \frac{1}{mn} + mn \times \frac{1}{mn} - \frac{1}{mn}\right.$$

$$= \frac{mn}{2}\left(\frac{2}{mn} - \frac{1}{mn} + 1\right)$$

$$= \frac{mn}{2}\left(\frac{1+mn}{mn}\right)$$

$$\boxed{= \frac{1+mn}{2}}$$



Q The sum of n terms of 2 AP's are in the ratio of $7n+1 : 4n+27$. Find the ratio of 11th terms.

$$\frac{7(11)+1}{4(11)+27}$$

~~$S_{11} = 78 = \frac{11}{2}(2a+(n-1)$ $71 + a_{11}$~~

~~$a_{11} = 7$~~

~~$S_{11} = 71 = 67 + a_{10}$~~

~~$a_{10} = 4$~~

~~$\frac{a_{11}}{a_{10}} = \frac{7}{4}$~~

$$\frac{a+10d}{a'+10d'}$$

$$\frac{S_n}{S_n'} = \frac{\frac{n}{2}(2a+(n-1)d)}{\frac{n}{2}(2a'+(n-1)d')}$$

$$= \frac{a + \frac{(n-1)d}{2}}{a' + \frac{(n-1)d'}{2}}$$

$$\frac{7n+1}{4n+27} = \frac{a + \frac{(n-1)d}{2}}{a' + \frac{(n-1)d'}{2}}$$

for $\frac{n-1}{2} = 10$

$n = 21$

$$\frac{148}{111} = \boxed{\frac{4}{3} = \frac{T_{11}}{T_{11}'}}$$

Note → If the sum of an A·P ~~[illegible]~~ S_n is given

$T_n = S_n - S_{(n-1)}$

→ In an A·P If S_n is a quadratic of n ~~[illegible]~~ and ~~[illegible]~~ T_n is in linear in n Then the series will be an A·P.

$T_n = An + B$ $\quad$ $S_n = An^2 + Bn + C$

$\downarrow$ AP with commondifference A

$\llcorner$ AP with common difference = 2A

Q Given $S_n = 2n + 3n^2$. A new AP is formed with same first term & double common Difference. find sum of new ~~[illegible]~~ AP.

$d = 2A$
$d = 2 \times 3$
$d = 6$

$a = 2(1) + 3(1)^2$
$a = 2 + 3$
$a = 5$

~~$d = 12$~~

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$= \frac{n}{2}(10 + (n-1)12)$

$= \frac{n}{2}(10 + 12n - 12)$

$= \frac{n}{2}$ ~~[illegible]~~ $(12n - 2)$

$\boxed{= 6n^2 - n}$

Geometric Progression

→ Collection of non zero terms in which each consecutive term bears the <u>same constant ratio.</u>

$a, ar, ar^2, ar^3, ar^4, ar^5 \ldots ar^{n-1}$

$T_n = a r^{(n-1)}$

$a \rightarrow$ First Term

$r \rightarrow$ common ratio

$r \neq 0$

$r > 0$

$r \in (0,1)$ ↓ decrease

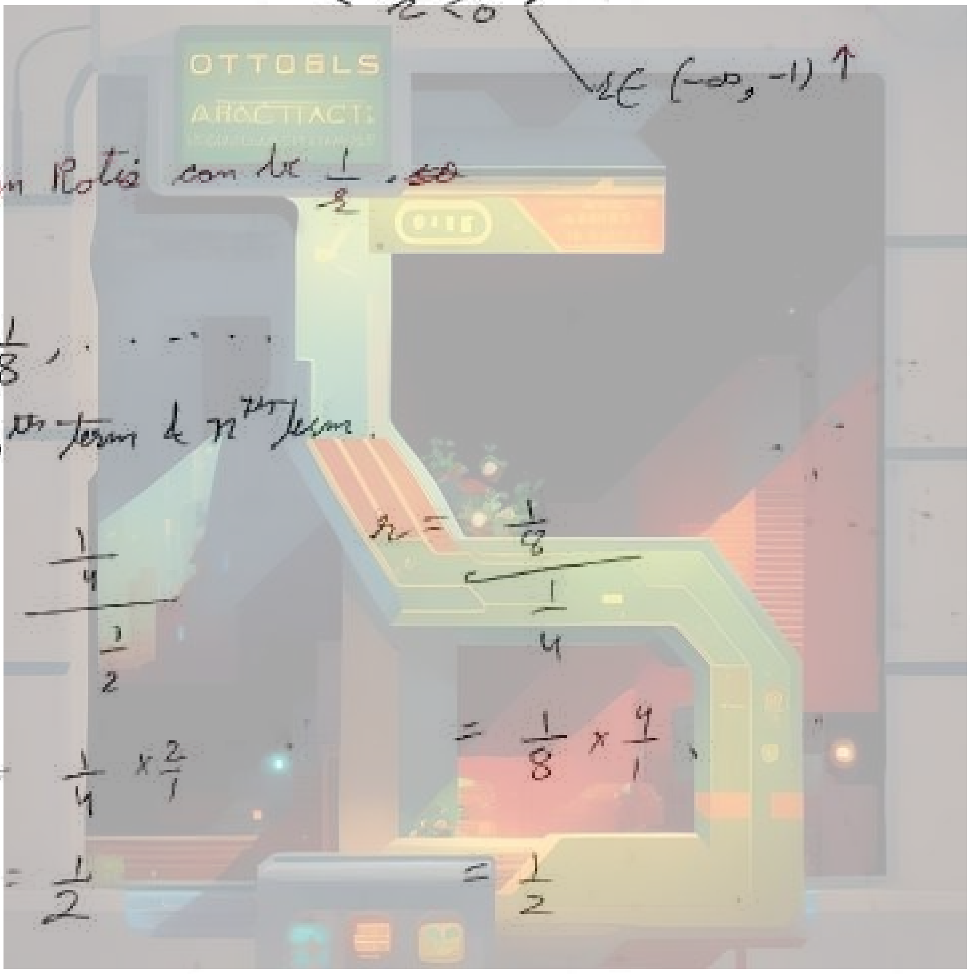
$r \in (1, \infty)$ ↑ Increase

$r < 0$

$r \in (-1, 0)$ ↓ ~~decrease in OH~~

$r \in (-\infty, -1)$ ↑

Note - Common Ratio can be $\frac{1}{2}$. ~~∞~~



Q $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots$

find 20^{th} term & n^{th} term.

$r = \dfrac{\frac{1}{4}}{\frac{1}{2}}$

$= \frac{1}{4} \times \frac{2}{1}$

$= \frac{1}{2}$

$r = \dfrac{\frac{1}{8}}{\frac{1}{4}}$

$= \frac{1}{8} \times \frac{4}{1}$

$= \frac{1}{2}$

$a = \frac{1}{2}$

$T_{n^{th}} = \frac{1}{2} \times \frac{1}{2}^{n-1}$

$= \frac{1}{2}^{n}$

$= 2^{-n}$

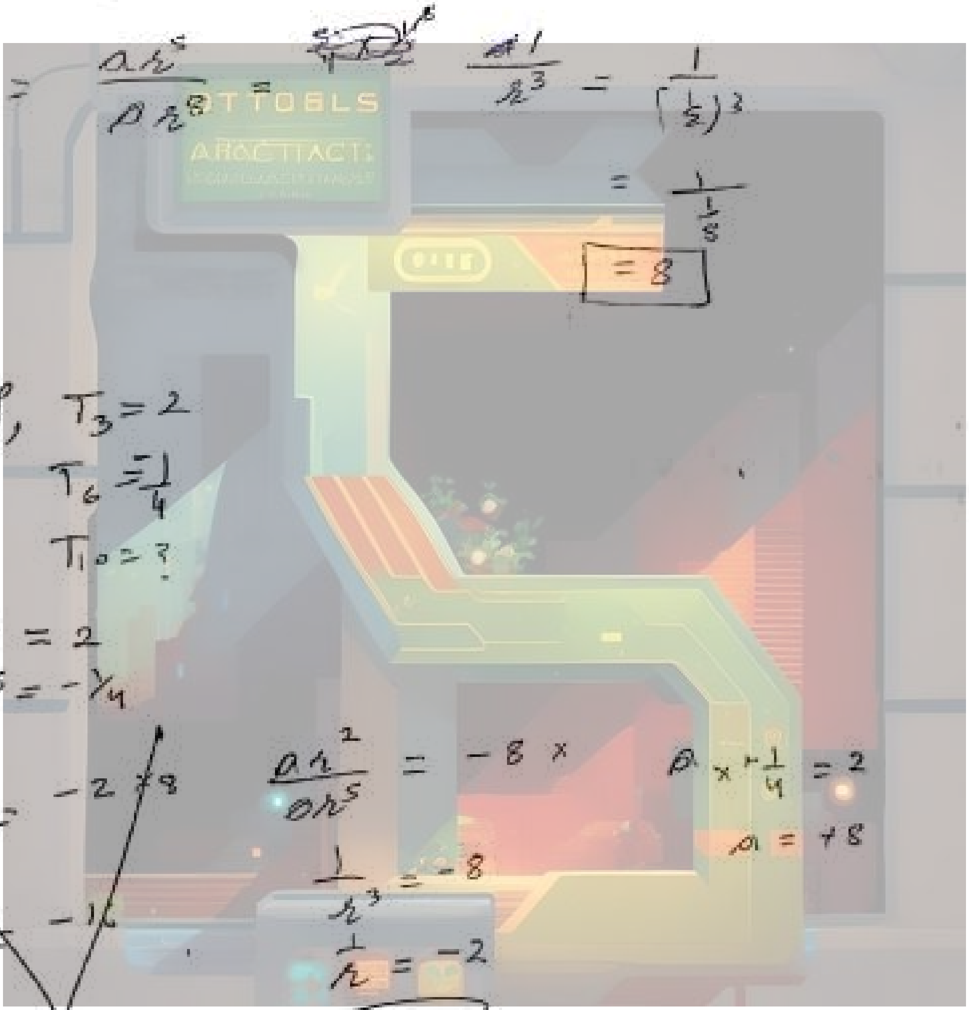
$\boxed{T_{20} = 2^{-20}}$

Q Find the ratio of 6^{th} term & 9^{th} term of the series.

$\frac{5}{4}, \frac{5}{8}, \frac{5}{16} \ldots$

$r = \frac{\frac{5}{8}}{\frac{5}{4}}$

$= \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$



$\frac{T_6}{T_9} = \frac{ar^5}{ar^8} = \cancel{\frac{1}{r^3}} \quad \frac{1}{r^3} = \frac{1}{(\frac{1}{2})^3}$

$= \frac{1}{\frac{1}{8}}$

$\boxed{= 8}$

Q In GP, $T_3 = 2$

$T_6 = -\frac{1}{4}$

$T_{10} = ?$

$ar^2 = 2$

$ar^5 = -\frac{1}{4}$

~~$\frac{ar^2}{ar^4} = -2 \times 8$~~

~~$\frac{1}{r^2} = -16$~~

~~$\cancel{r^2 =}$~~

~~$\cancel{\frac{1}{r^2}} \quad \frac{1}{r} =$~~

$\frac{ar^2}{ar^5} = -8 \times$

$\frac{1}{r^3} = -8$

$\frac{1}{r} = -2$

$\boxed{r = \frac{-1}{2}}$

$a \times \frac{1}{4} = 2$

$a = +8$

$T_{10} = ar^9$

$= a \times (\frac{-1}{2})^9$

$= +8 \times \frac{-1}{512}$

$= \frac{-1}{64}$

Sum of GP

~~$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$~~

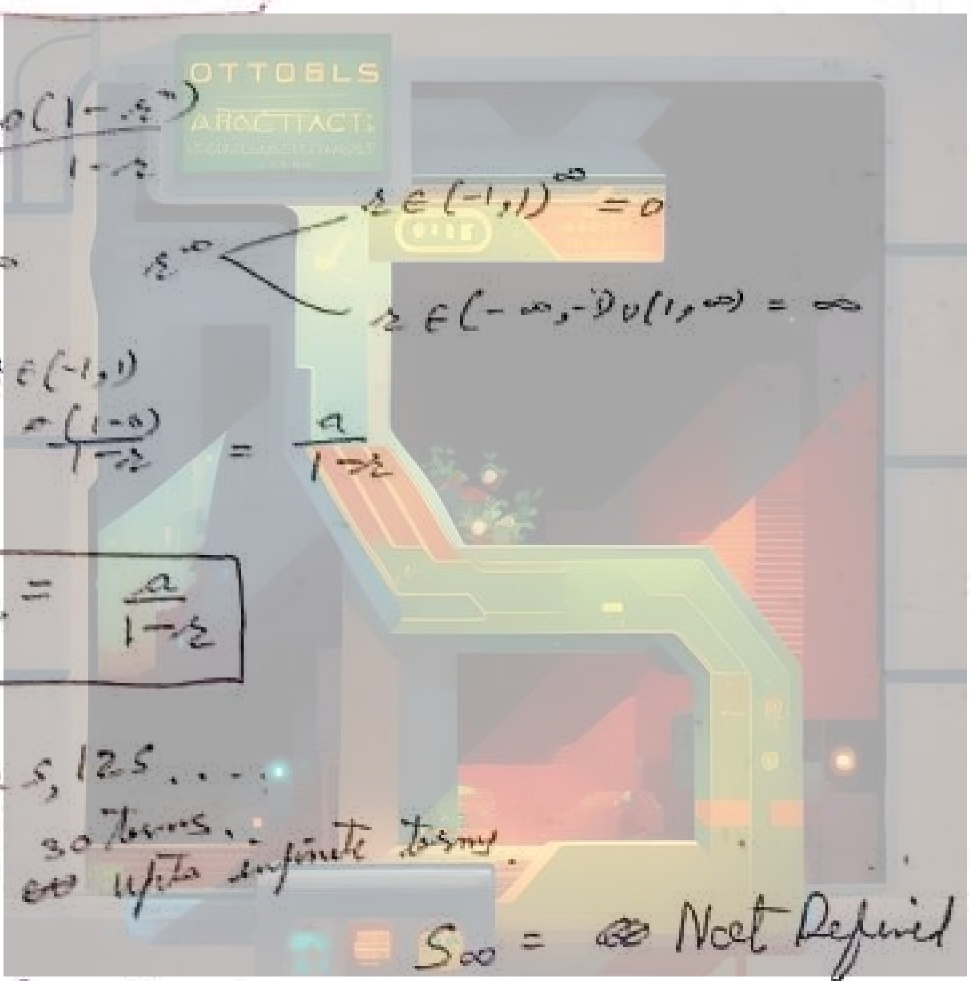
$S_n = a + ar + ar^2 + ar^3 + \dots\dots ar^{n-2} + ar^{n-1}$

$r S_n = ar + ar^2 + ar^3 + ar^4 + \dots\dots ar^{n-1} + ar^n$

$(1-r)S_n = a - ar^n$

$(1-r)S_n = a(1-r^n)$

$\boxed{S_n = \frac{a(1-r^n)}{(1-r)}}$ $(r \neq 1)$



$S_n = \frac{a(1-r^n)}{1-r}$

$n \to \infty$ $\quad r^n \begin{cases} r \in (-1,1)^{\infty} = 0 \\ r \in (-\infty,-1) \cup (1,\infty) = \infty \end{cases}$

for $r \in (-1,1)$

$S_\infty = \frac{a(1-0)}{1-r} = \frac{a}{1-r}$

$\boxed{S_\infty = \frac{a}{1-r}}$

Q $1, 5, 25, 125, \dots$

i) sum of 30 terms.

ii) sum of ~~∞~~ upto infinite terms.

$S_\infty =$ ~~∞~~ Not Defined

$r = \frac{5}{1} = 5$

$S_{30} = \frac{1(1-5^{30})}{1-5}$

$= \frac{1-5^{30}}{-4}$

$= \frac{5^{30}-1}{4}$

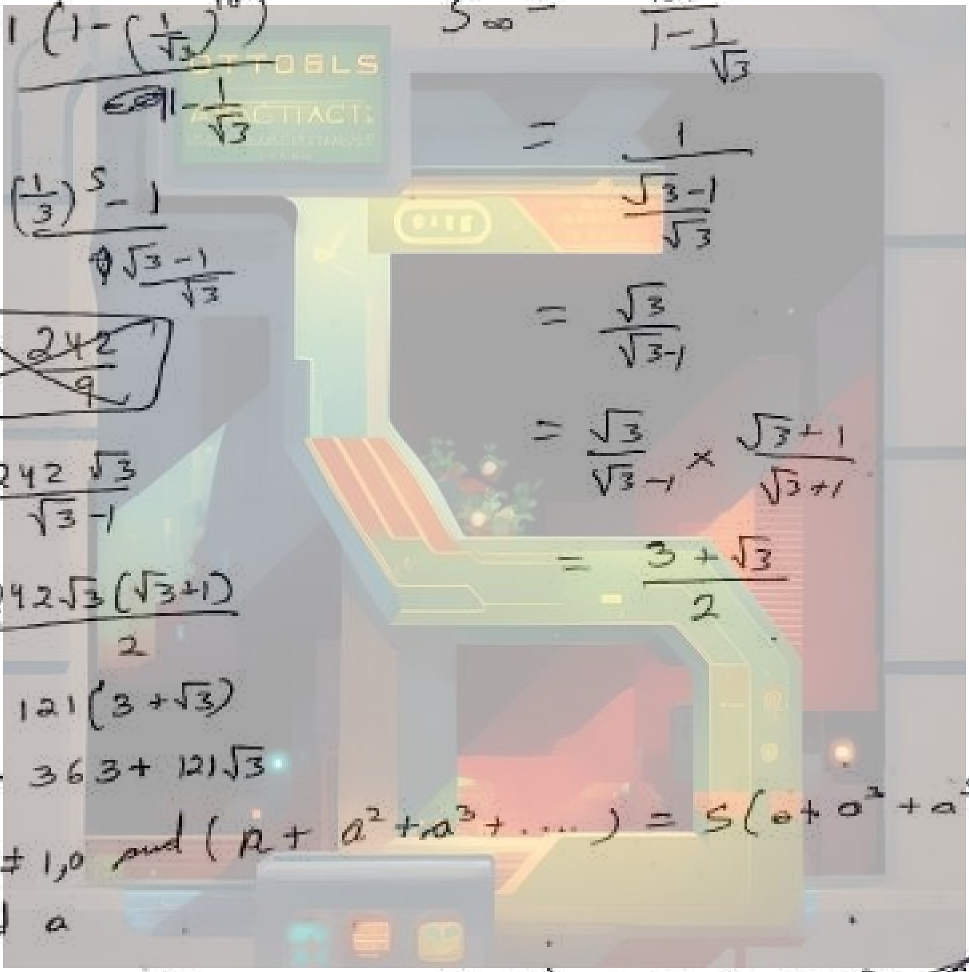
$= \frac{5^{30}}{4} - \frac{1}{4}$

Q $1, \frac{1}{\sqrt{3}}, \frac{1}{3}, \frac{1}{3\sqrt{3}}$

$r = \frac{\frac{1}{\sqrt{3}}}{1}$

$= \frac{1}{\sqrt{3}}$

i) ~~o~~ sum of 10 terms
ii) sum of infinite terms



$S_{10} = \frac{1\left(1-\left(\frac{1}{\sqrt{3}}\right)^{10}\right)}{1-\frac{1}{\sqrt{3}}}$

$= \frac{\left(\frac{1}{3}\right)^5 - 1}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$\cancel{\boxed{= \frac{242}{9}}}$

$= \frac{242\sqrt{3}}{\sqrt{3}-1}$

$= \frac{242\sqrt{3}(\sqrt{3}+1)}{2}$

$= 121(3+\sqrt{3})$

$= 363 + 121\sqrt{3}$

$S_\infty = \frac{a}{1-\frac{1}{\sqrt{3}}}$

$= \frac{1}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \frac{\sqrt{3}}{\sqrt{3}-1}$

$= \frac{\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{3+\sqrt{3}}{2}$

Q (i) If $a \neq 1, 0$ and $(a + a^2 + a^3 + \dots) = 5(a + a^2 + a^3 + \dots)$
find a

$(a + a^3 + a^5 \dots) + (a^2 + a^4 + a^6) = 5(a + a^3 + a^5)$

$a^2 + a^4 + a^6 \dots = 5$

$S_\infty = 5$

$\frac{a^2}{1-a^2} = 5$

$a^2 = 5 - 5a^2$

$6a^2 = 5$

$a^2 = \frac{5}{6}$

$a = \frac{\sqrt{5}}{\sqrt{6}}$

$\frac{a}{1-a} = 5\left(\frac{a}{1-a^2}\right)$

$a(1-a)(1+a) = 5a(1-a)$

$a(1-a)(1+a-5) = 0$

$1 + a - 5 = 0$

$a = 4$

but $a \in (-1, 1)$ so $a \in \phi$

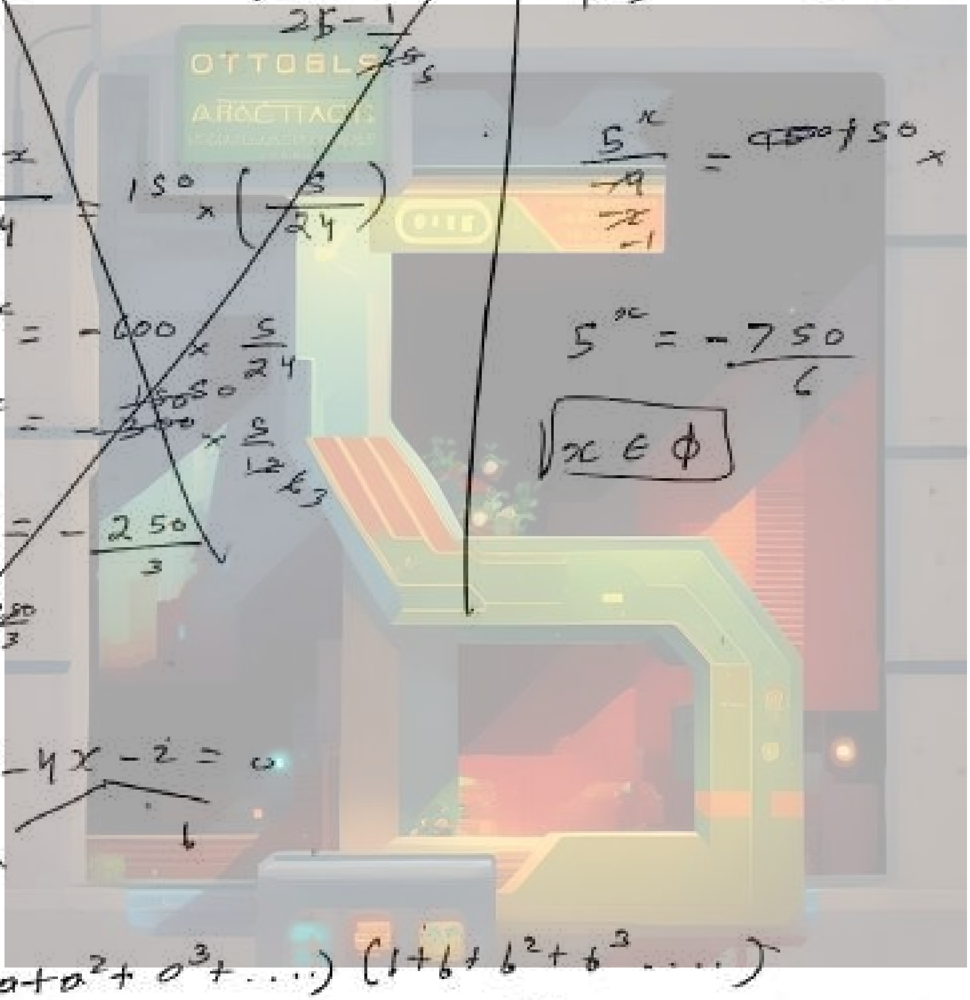
Q $(5^x + 5^{(x+1)} + 5^{(x+2)} + \dots) = 150\left(\frac{1}{5}+\frac{1}{5^3}+\frac{1}{5^5}+\dots\right)$

find 'x',

$(5^x + 5^{(x+1)} + 5^{(x+2)}\dots) = 150(5^{-1}+5^{-3}+5^{-5}+5^{-7}\dots)$

$a = 5^x$ $\quad$ $a = 5^{-1}$

$r = 5$ $\quad$ $r = 5^{-2}$



$\frac{5^x}{1-5} = 150\left(\frac{\frac{1}{5}}{\frac{25-1}{25}}\right)$

$\frac{5^x}{-4} = 150\times\left(\frac{5}{24}\right)$

$5^x = -600\times\frac{5}{24}$

$5^x = -300\times\frac{5}{12}$

$5^x = -\frac{250}{3}$

$\log_5 \frac{-250}{3}$

$\frac{5^x}{1-5} = 150\left(\frac{1}{5}\times\frac{25}{24}\right)$

$\frac{5^x}{-4} = 150\times\frac{5}{24}$

$5^x = -\frac{750}{6}$

$\boxed{x \in \phi}$

Q $11x^2-4x-2=0$ $\quad$ a, b

$10(1+a+a^2+a^3+\dots)(1+b+b^2+b^3\dots)$

$a=1$ $\quad$ $a=1$

$r=a$ $\quad$ $r=b$

$10\left(\frac{1}{1-a}\right)\left(\frac{1}{1-b}\right)$

$\frac{10}{1-b-a+ab}$

$\frac{10}{1-(a+b)+ab}$

$\frac{10}{1-\frac{4}{11}-\frac{2}{11}}$

$\frac{10\times 11}{11-6} = \frac{110}{5} = \boxed{22}$

) product

$2^{1/4} \cdot 4^{1/16} \cdot 8^{1/48} \cdot 16^{\frac{1}{128}} \ldots \infty$

$2^{1/4} \cdot 2^{1/8} \cdot 2^{\frac{1}{16}}$

$r = \frac{2^{1/8}}{2^{1/4}}$

$= 2^{\frac{1}{8} - \frac{1}{4}}$

$r = 2^{\frac{-1}{4}}$

$2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots}$

$r = \frac{1}{8} \times \frac{4}{1}$

$= \frac{1}{2}$

$S_\infty = \frac{\frac{1}{4}}{1 - \frac{1}{2}}$

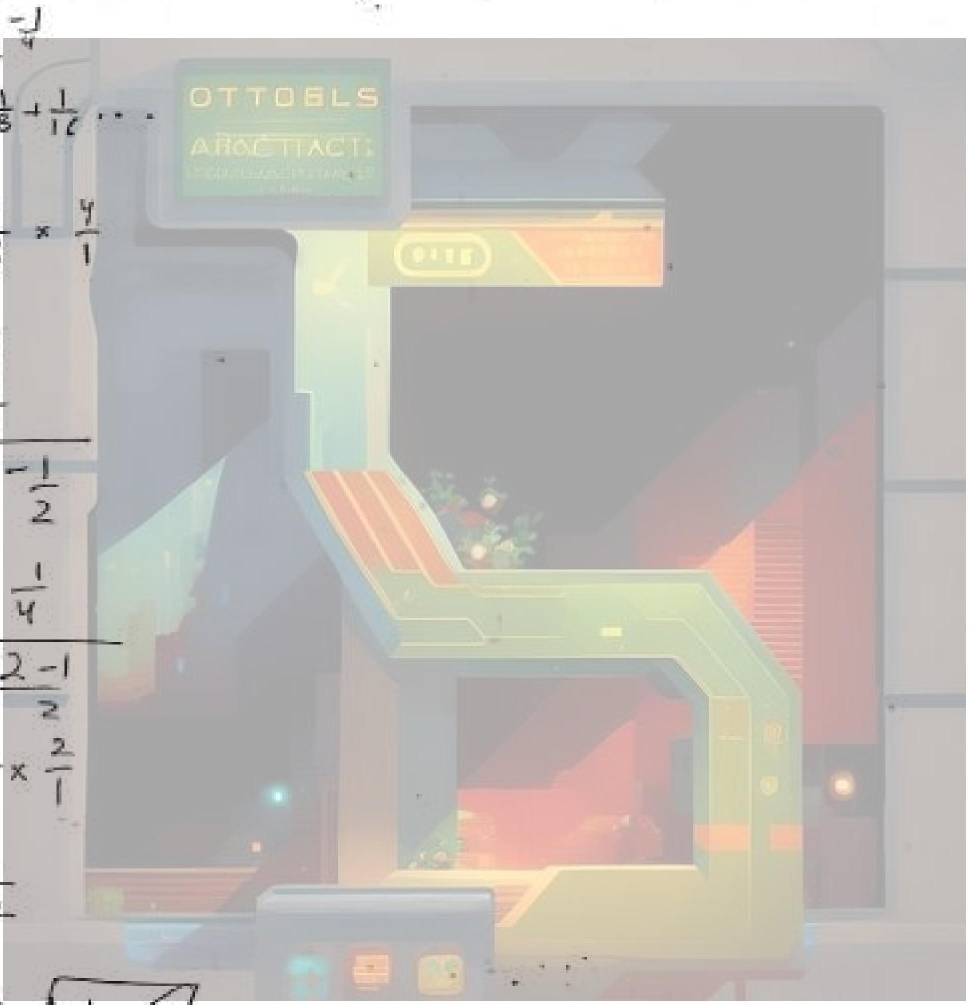
$= \frac{\frac{1}{4}}{\frac{2-1}{2}}$

$= \frac{1}{4} \times \frac{2}{1}$

$= \frac{1}{2}$

$2^{1/2} = \boxed{\cancel{\frac{1}{4}}}$

$= \sqrt{2}$



Q $(\ln a^2) + (\ln a^2)^2 + (\ln a^2)^3 \ldots \infty = (\ln a) + (\ln a)^2 + (\ln a)^3$

$r = \ln a^2$

$a = \ln a^2$

$S_\infty = \dfrac{\ln a^2}{1-\ln a^2}$

~~$= \dfrac{2\ln a}{1-2\ln a}$~~

~~$= \dfrac{2\log_e a}{1-2\log_e a}$~~

~~$= \dfrac{2\log_e a}{\log_e e - 2\log_e a}$~~

~~$= \dfrac{2\log_e a}{\log_e \frac{e}{a^2}}$~~

~~$= \log_{\frac{e}{a^2}} a^2$~~

$r = \ln a$

$a = \ln a$

$S_\infty = \dfrac{\ln a}{1-\ln a}$

$\dfrac{\ln a^2}{1-\ln a^2} = \dfrac{\ln a}{1-\ln a}$

$\dfrac{2\ln a}{1-2\ln a} = \dfrac{\ln a}{1-\ln a}$

$\dfrac{2x}{1-2x} = \dfrac{x}{1-x}$

$2x - \cancel{2x^2} = x - 2x^2$

$x = 0$

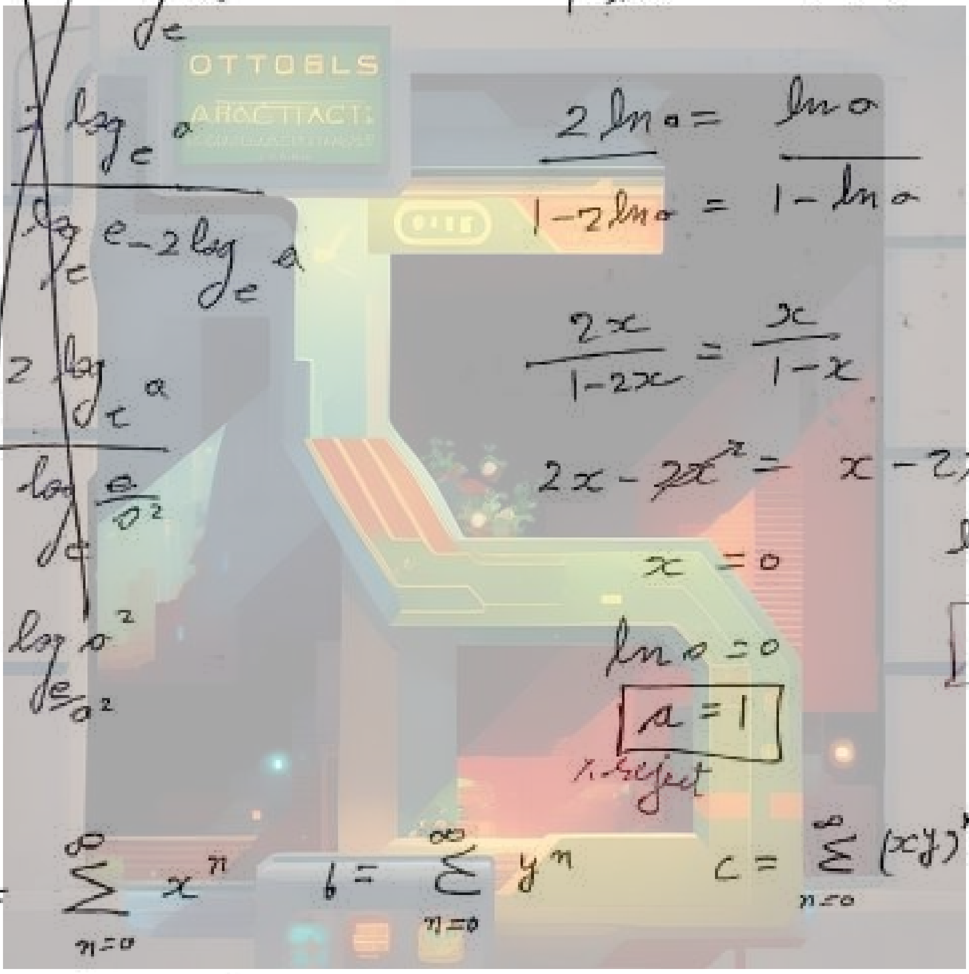
$\ln a = 0$

$\boxed{a = 1}$

x reject

$\ln a = +\frac{1}{4}$

$\boxed{a = e^{+1/4}}$



Q If $a = \sum_{n=0}^{\infty} x^n$ $b = \sum_{n=0}^{\infty} y^n$ $c = \sum_{n=0}^{\infty} (xy)^n$

$x, y \in (-1, 1)$

find $\left(\frac{a-1}{a}\right)\left(\frac{b-1}{b}\right) - \frac{(c-1)}{c}$

~~$x = -1$~~

~~$1, -1 + 1 - 1 \ldots \infty$~~

~~$a = 0$~~

~~$x = 1$~~

~~$a = \infty$~~

~~$y = -1$~~

~~$a = 0$~~

~~$a = \infty$~~

$c \neq 0$

$c = \infty$

$a = \frac{1}{1-x}$

$b = \frac{1}{1-y}$

$c = \frac{1}{1-xy}$

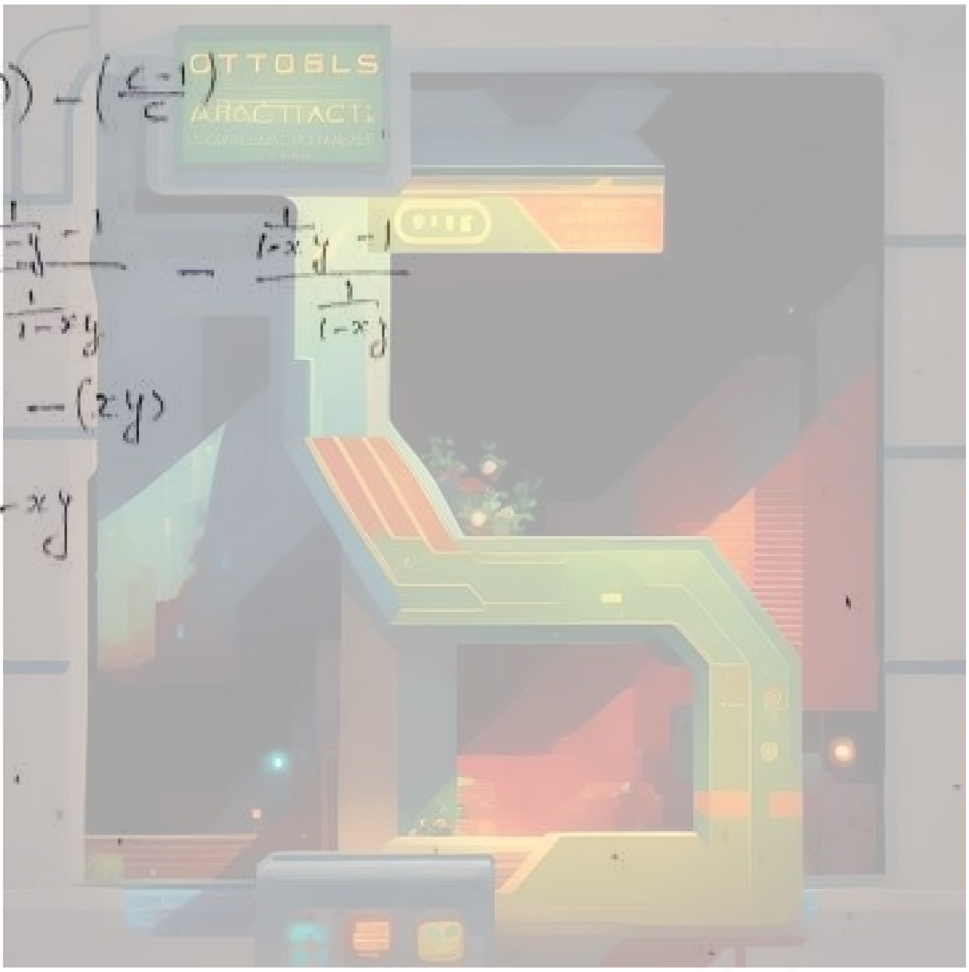
$\left(\frac{a-1}{a}\right)\left(\frac{b-1}{b}\right) - \left(\frac{c-1}{c}\right)$

$\frac{\frac{1}{1-x}-1}{\frac{1}{1-x}} \quad \frac{\frac{1}{1-y}-1}{\frac{1}{1-xy}} - \frac{\frac{1}{1-xy}-1}{\frac{1}{1-xy}}$

$(x)\ (y) - (xy)$

$xy - xy$

$\boxed{=0}$



(1/8)

Q $7+77+777+\dots n$

$7+7\times10+7+7\times100+7\dots n$

~~$7(1\times10^1+7\times10^0$~~

$\underbrace{7\times100+7\times10+7}$

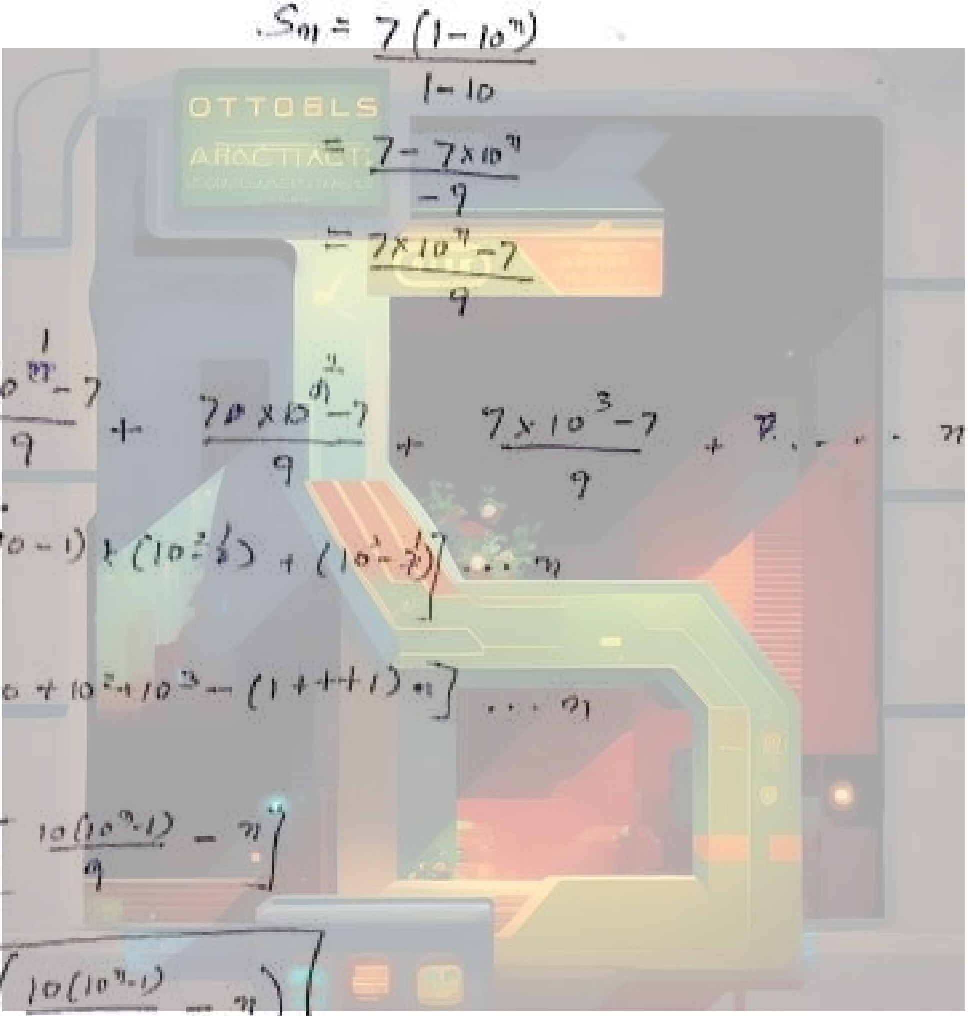
$\downarrow$

$7\times10^2+7\times10^1+7\times10^0$

$a=7$

$r=10$

$S_n=\frac{7(1-10^n)}{1-10}$



$=\frac{7-7\times10^n}{-9}$

$=\frac{7\times10^n-7}{9}$

$\frac{7\times10^{1}-7}{9}+\frac{7\times10^{2}-7}{9}+\frac{7\times10^3-7}{9}+\dots n$

$\frac{7}{9}\left[(10-1)+(10^2-1)+(10^3-1)\right]\dots n$

$\frac{7}{9}\left[10+10^2+10^3\dots-(1+1+1)n\right]\dots n$

$\frac{7}{9}\left[\frac{10(10^n-1)}{9}-n\right]$

$$\boxed{\frac{7}{9}\left(\frac{10(10^n-1)}{9}-n\right)}$$

Q In a sequence of 4 nos., 1st 3 nos. are in G.P & last 3 are in A.P with CD -6 If 1st & last terms of the sequence are equal then find last term.

$a + ar + ar^2 + a$

$ar^2 - ar = 6$ $\quad$ $a - ar^2 = 6$

~~$ar(r-1) = 6$~~ $\quad$ $a(1 - r^2) = 6$

$a = \frac{6}{1-r^2}$

$\frac{6}{1-r^2} \times r^2 - \frac{6}{1-r^2} r = 6$

$\frac{6r^2}{1-r^2} - \frac{6r}{1-r^2} = 6$

$6r^2 - 6r = 6 - 6r^2$

$12r^2 - 6r - 6 = 0$

$2r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{1+8}}{2}$

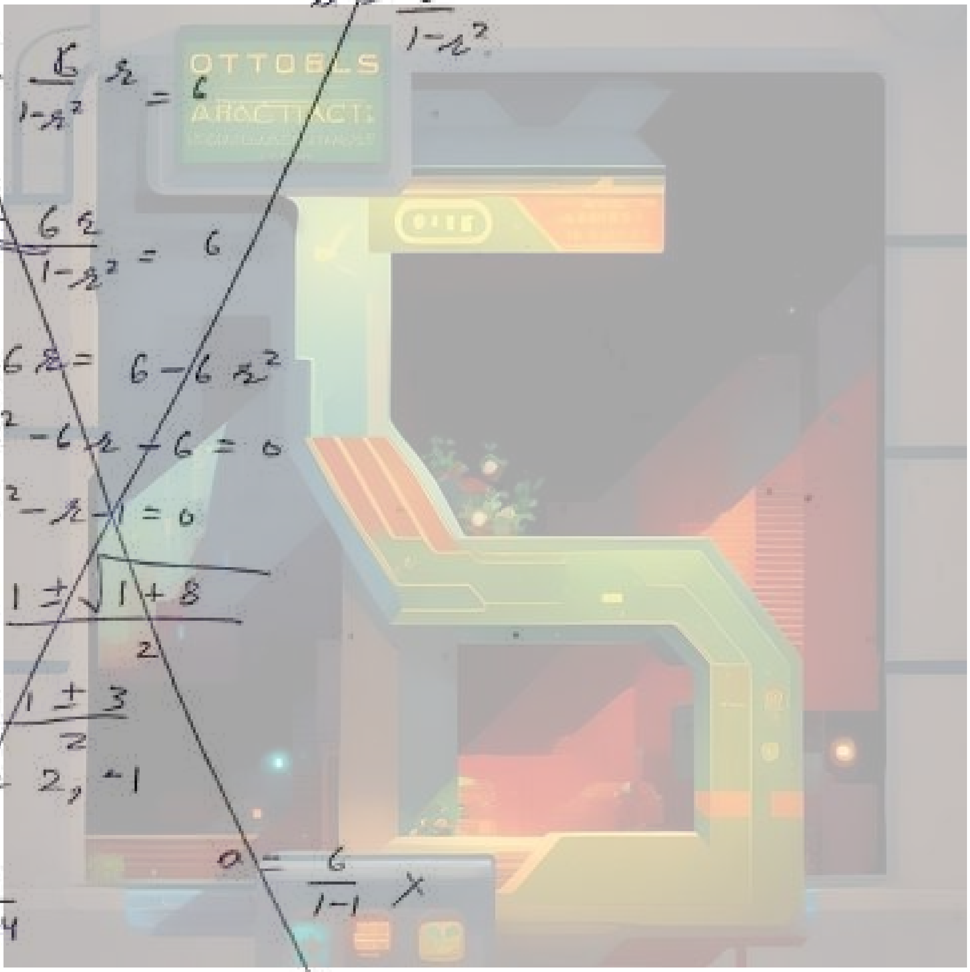
$= \frac{1 \pm 3}{2}$

$= 2, -1$

$a = \frac{6}{1-4}$ $\quad$ $a = \frac{6}{1-1}$ ×

$a = \frac{6}{-3}$

$= -2$



$\underbrace{a+6,\ a-6,\ a}$, $a+6$

GP

$$\frac{a-6}{a+6} = \frac{a}{a-6}$$

$$a^2+36-12a = a^2+6a$$

$$36 = 18a$$

$$a=2$$

$8, -4, 2, 8$

$\boxed{8}$

Properties of GP

① Product of K^{th} term from beginning & K^{th} term from end is always constant.

② Assuming terms in GP

3: $\frac{a}{r}, a, ar$

4: $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

5: $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

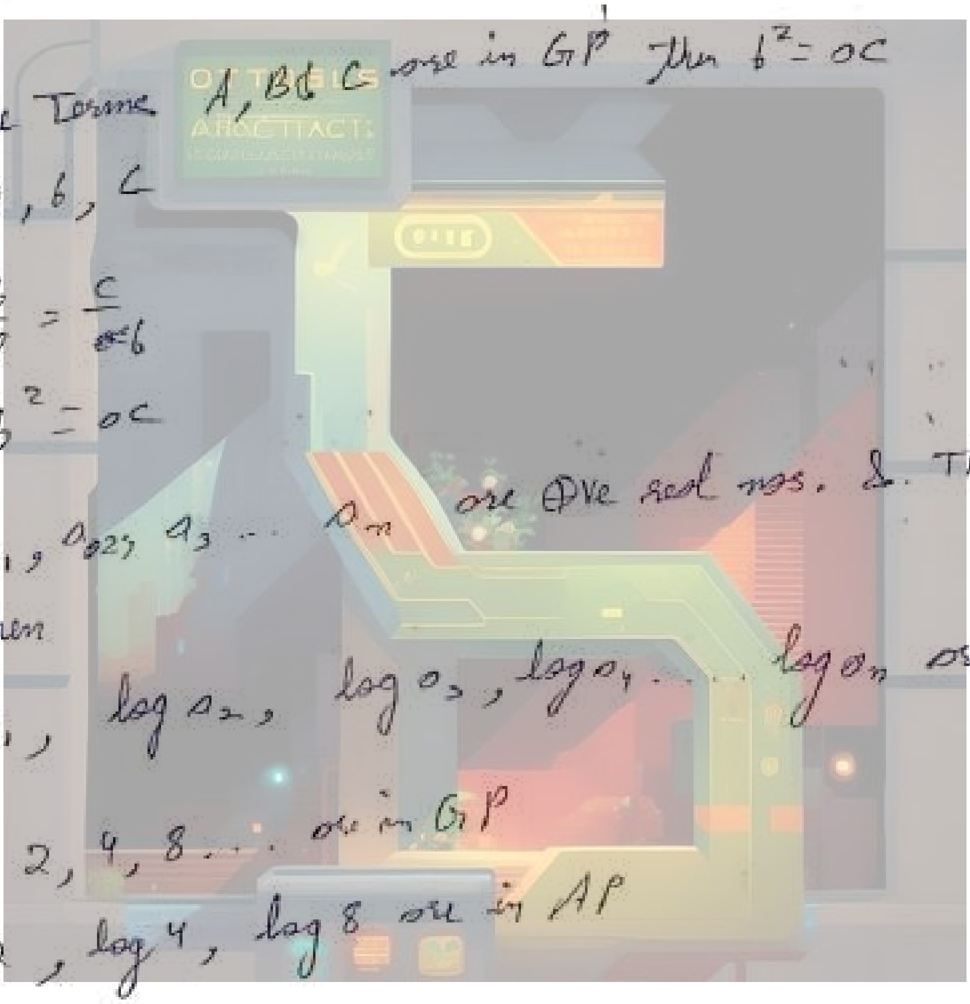
6: $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5$

③ If each term of GP is raised to same power, multiplied or divided then the resulting series is also a GP.

$2, 4, 8, 16, 32 \dots$

$2^{1/3}, 4^{1/3}, 8^{1/3}, 16^{1/3}, 32^{1/3} \dots$

$\frac{4^{1/3}}{2^{1/3}} = 2^{1/3}$ $\quad \frac{8^{1/3}}{4^{1/3}} = 2^{1/3}$



④ If Three Terms A, B & C are in GP then $b^2 = ac$

a, b, c

$\frac{b}{a} = \frac{c}{b}$

$b^2 = ac$

⑤ For $a_1, a_2, a_3 \dots a_n$ are +ve real nos. & They are in GP then

$\log a_1, \log a_2, \log a_3, \log a_4 \dots \log a_n$ are in AP

eg. $2, 4, 8 \dots$ are in GP

$\log 2, \log 4, \log 8$ are in AP

Hormonic Progression (H·P·)

→ A non zero sequence is said to be in H·P if resiprocal of its terms are in AP.

eg. $a_1, a_2, a_3, a_4 \dots$ HP

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4} \dots$ AP

→ standard form.

$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} \dots \frac{1}{a+(n-1)d}$ .. HP

Note:- ① No term of HP can ~~be~~ be zero

② Resiprocal of ~~a~~ every H·P. is A.P. but resiprocal of every A.P. can or cannot be H·P.

③ If a, b, c are in H.P then

a, b, c → HP

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow$ AP

$2 \times \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

$\frac{2}{b} = \frac{a+c}{ac}$

$\boxed{b = \frac{2ac}{a+c}}$

(9) No general formula for finding the sum of H.P.

Q Determine the 4th & 8th term of H.P. 6, 4, 3 ....

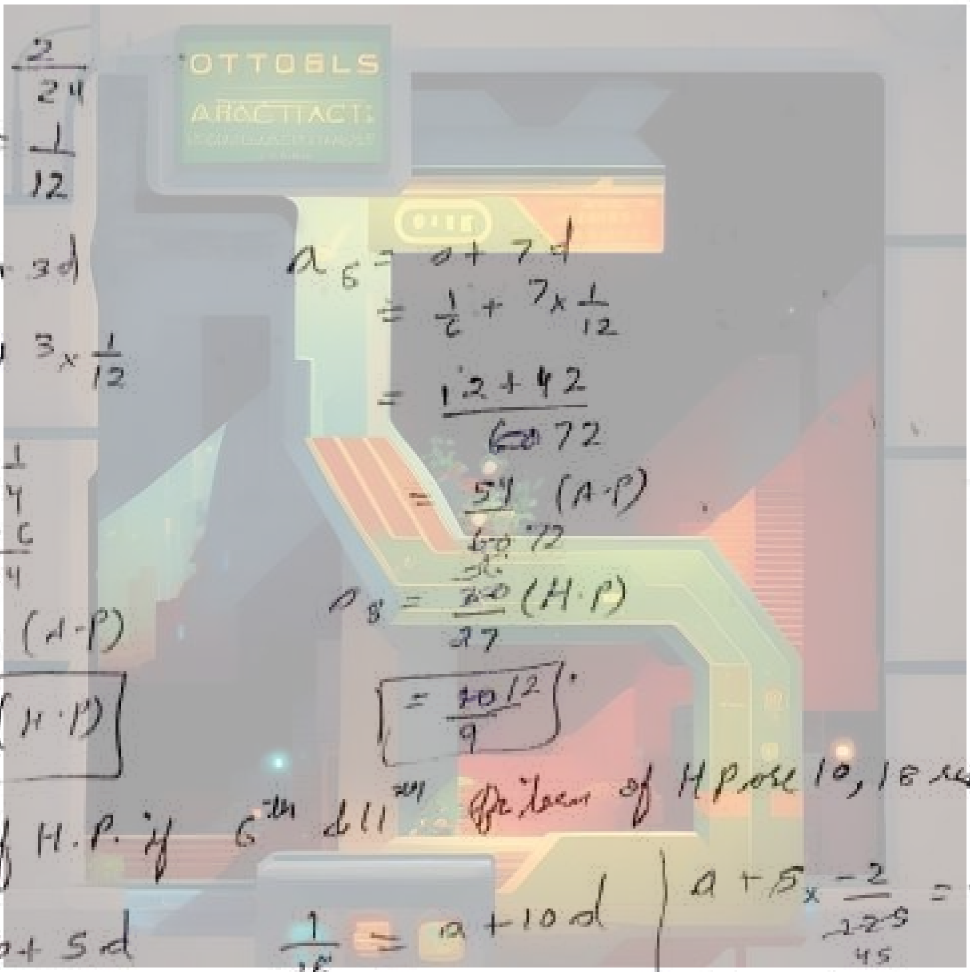
$\frac{1}{6}, \frac{1}{4}, \frac{1}{3} \; .... A.P$

$\frac{1}{4} - \frac{1}{6} = C.D$

$d = \frac{6-4}{24}$

$= \frac{2}{24}$

$= \frac{1}{12}$



$a_4 = a + 3d$

$= \frac{1}{6} + 3 \times \frac{1}{12}$

$= \frac{1}{6} + \frac{1}{4}$

$= \frac{4+6}{24}$

$= \frac{10}{24} (A.P)$

$\boxed{a_4 = \frac{24}{10} (H.P)}$

$a_8 = a + 7d$

$= \frac{1}{6} + 7 \times \frac{1}{12}$

$= \frac{12+42}{\cancel{60}\,72}$

$= \frac{54}{\cancel{60}\,72} (A.P)$

$a_8 = \frac{\cancel{30}\,20}{27} (H.P)$

$\boxed{= \frac{20\,12}{9}}$

Q find 16th term of H.P. if 6th & 11th terms of H.P are 10, 18 respectively.

$\frac{1}{10} = a + 5d$

$\frac{1}{18} = a + 10d$

$\frac{1}{18} - \frac{1}{10} = 5d$

$\frac{-8}{180} = 5d$

$\frac{-8}{900} = d$

$d = \frac{-2}{225}$

$a + 5 \times \frac{-2}{\cancel{225}\,45} = \frac{1}{10}$

$a = \frac{1}{10} + \frac{2}{45}$

$a = \frac{\cancel{49}\,\cancel{5}\,13}{\cancel{450}\,90}$

$\cancel{a_{16} = \frac{47}{450} - 15 \times \frac{+2}{225}}$

$\cancel{a_{16} = \frac{47-60}{450}}$

$\cancel{= \frac{-13}{450}}$

$a_{16} = \frac{13}{90} - 15 \times \frac{2}{225}$

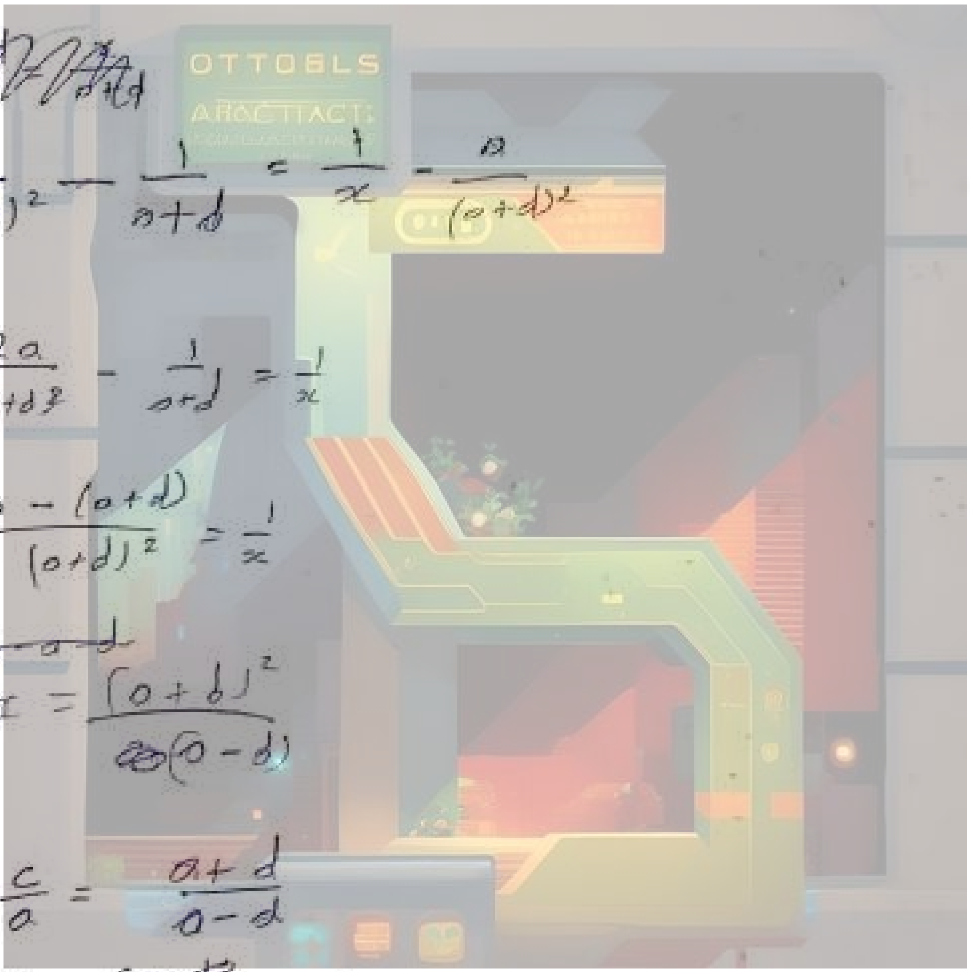
$\boxed{a_{16} = 90}$

① If a, b, c, d, e are 5 nos, a, b, c in AP.
b, c, d in GP.
c, d, e in HP.

a) Prove a, c, e in GP

b) $e = \frac{(2b-a)^2}{a}$

$$a-d,\ a,\ a+d,\ \frac{(a+d)^2}{a},\ \frac{(a+d)^2}{a-d}$$



$$\frac{a}{(a+d)^2} - \frac{1}{a+d} = \frac{1}{x} - \frac{a}{(a+d)^2}$$

$$\frac{2a}{(a+d)^2} - \frac{1}{a+d} = \frac{1}{x}$$

$$\frac{2a-(a+d)}{(a+d)^2} = \frac{1}{x}$$

~~2a − a − d~~

$$x = \frac{(a+d)^2}{(a-d)}$$

a) $\frac{c}{a} = \frac{a+d}{a-d}$

$$\frac{e}{c} = \frac{(a+d)^2}{a-d} \times \frac{1}{a+d}$$

$$\frac{e}{c} = \frac{a+d}{a-d}$$

$\frac{c}{a} = \frac{e}{c}$ so a, c, e are in GP.

b) $\frac{(2b-a)^2}{a}$

2~~a-d+d~~

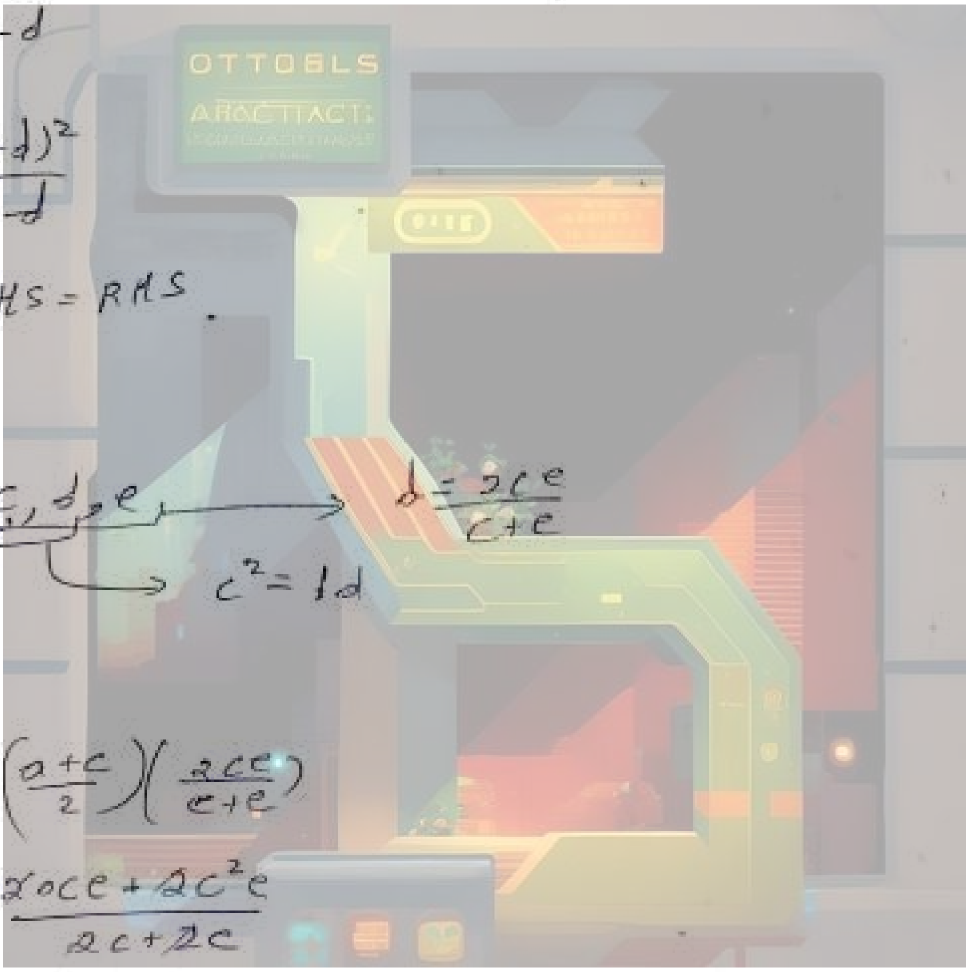
$\frac{(2a-a+d)^2}{a-d}$

$= \frac{(a+d)^2}{a-d}$

$e = \frac{(a+d)^2}{a-d}$

Hence LHS = RHS.

Method II



a, b, c, d, e $\longrightarrow d = \frac{2ce}{c+e}$

$\longrightarrow c^2 = bd$

$b = \frac{a+c}{2}$

$c^2 = \left(\frac{a+c}{2}\right)\left(\frac{2ce}{c+e}\right)$

$= \frac{\cancel{2}ace + 2c^2e}{2c+2e}$

$= \frac{ace+c^2e}{c+e}$

$c^2(c+e) = (a+c)ce$

$c^2e + c^3\cancel{e} = ace + c^2\cancel{e}$

~~$c^2 e = ae$~~

~~$a = c^2$~~

$c^2 = ae$

Thus, a, c d e are in GP

1) $c = \frac{(2b-a)^2}{a}$

$ac = (2b-a)^2$

$c^2 = (2b-a)^2$

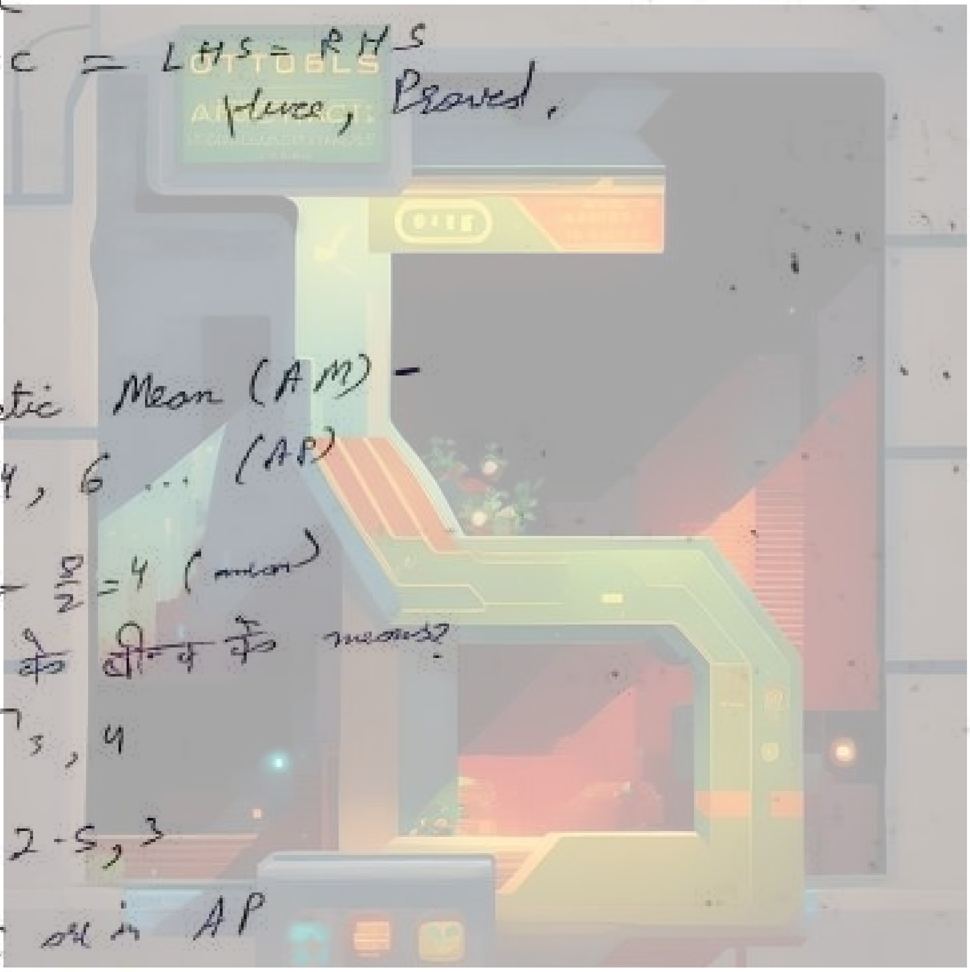
$c = 2b-a$ = LHS

$b = \frac{a+c}{2}$

$2b = a+c$

$2b - a = c$ = LHS = RHS

Hence, Proved.



Mean :-

① Arithmetic Mean (AM) -

2, 4, 6 ... (AP)

$\frac{6+2}{2} = \frac{8}{2} = 4$ (mean)

2, 4 के बीच के means?

2, 3, 4

2, 2·5, 3

a, b, c are in AP

$b = \frac{a+c}{2}$

Find means btw a & b?

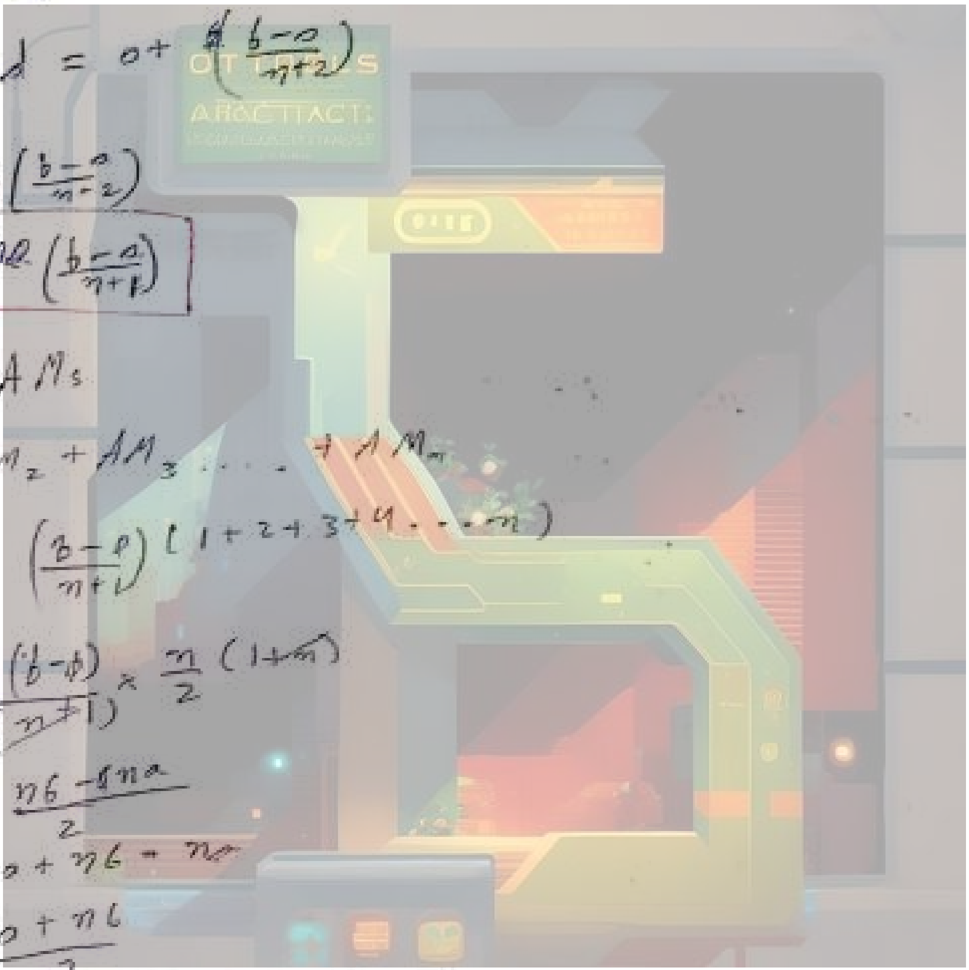
$$a, \underbrace{AM_1, AM_2, AM_3, AM_4 \ldots\ldots AM_n}_{n}, b \ (AP)$$

$b = a + (n+2-1)d$

$b = a + (n+1)d$

$b - a = (n+1)d$

$$\boxed{d = \frac{b-a}{n+1}}$$



$AM_1 = a + d = a + \left(\frac{b-a}{n+2}\right)$

$AM_2 = a + 2\left(\frac{b-a}{n+2}\right)$

$$\boxed{AM_n = a + n\left(\frac{b-a}{n+1}\right)}$$

Sum of all AMs

$AM_1 + AM_2 + AM_3 \ldots\ldots + AM_n$

$na + \left(\frac{b-a}{n+1}\right)(1+2+3+4 \ldots n)$

$na + \frac{(b-a)}{(n+1)} \times \frac{n}{2}(1+n)$

$na + \frac{nb - na}{2}$

$2na + nb - na$

$\frac{na + nb}{2}$

$$\boxed{\frac{n(a+b)}{2} = S_n \text{ of AMs}}$$

Q Insert 20 AMs betw 4 & 67

1) find AMs

② find sum of AMs

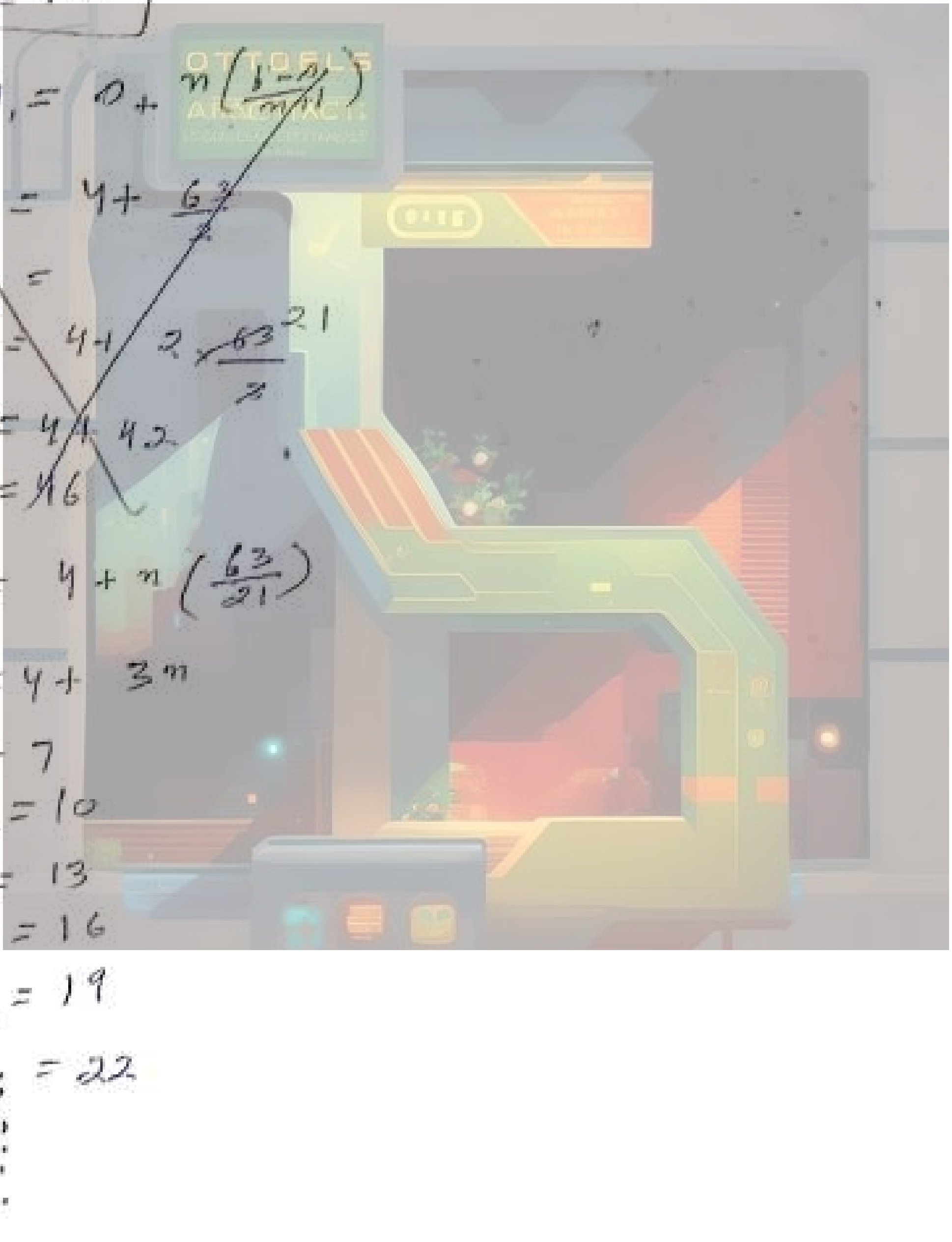
$4, AM_1, AM_2 \dots 67$

$n = 20$

② $20\left(\frac{67+4}{2}\right)$

~~20~~ $= 10 \times 71$

$\boxed{S_n = 710}$



~~① $AM_1 = a + n\left(\frac{b-a}{n+1}\right)$~~

~~$AM_1 = 4 + \frac{63}{2}$~~

~~$=$~~

~~$AM_2 = 4 + 2 \times \frac{\cancel{63}^{21}}{\cancel{2}}$~~

~~$= 4 + 42$~~

~~$= 46$~~

① $AM = 4 + n\left(\frac{63}{21}\right)$

$= 4 + 3n$

$AM_1 = 7$

$AM_2 = 10$

$AM_3 = 13$

$AM_4 = 16$

$AM_5 = 19$

$AM_6 = 22$

$\vdots$

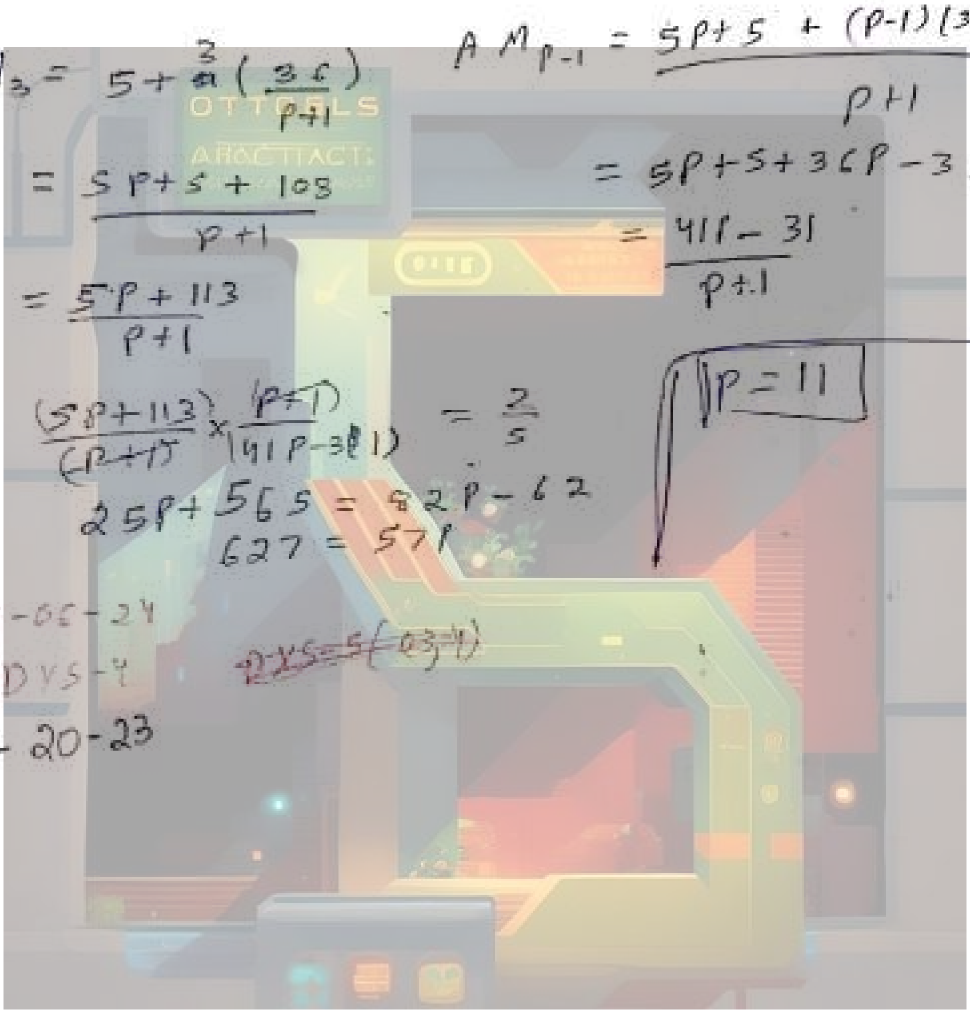
Q find sum of AMs btw 1 & 99

$$S_m = \frac{99+1}{2} \times 50$$

$$= 50 \times 50$$

$$\boxed{= 2500}$$

Q If P AMs inserted btw 5, 41, $\frac{AM_3}{AM_{p-1}} = \frac{2}{5}$
find 'p'



$$AM_3 = 5 + 3\left(\frac{36}{p+1}\right)$$

$$= \frac{5p+5+108}{p+1}$$

$$= \frac{5p+113}{p+1}$$

$$AM_{p-1} = \frac{5p+5+(p-1)(36)}{p+1}$$

$$= 5p+5+36p-36$$

$$= \frac{41p-31}{p+1}$$

$$\frac{(5p+113)}{\cancel{(p+1)}} \times \frac{\cancel{(p+1)}}{(41p-31)} = \frac{2}{5}$$

$$25p+565 = 82p-62$$

$$627 = 57p$$

$$\boxed{p = 11}$$

H.W 25-05-24
DYS-3 DYS-4 ~~DYS=5(03-4)~~
Prac - 20-23

Geometric mean :-

$$\underbrace{a,\ GM_1,\ GM_2,\ GM_3,\ GM_4\ \ldots\ b}_{n}$$ (Total n GMs + 2 (a, b))

$b = ar^{(n+2)-1}$

$b = ar^{n+1}$

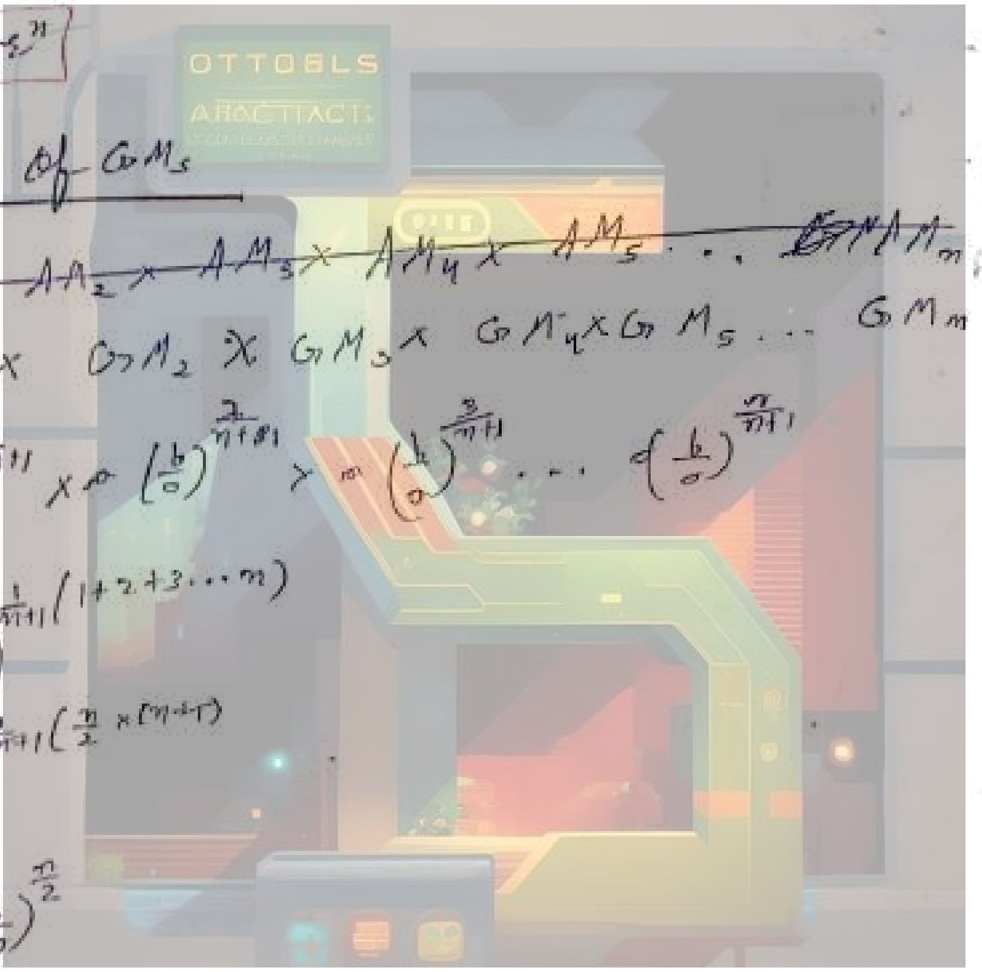
$$\boxed{\left(\frac{b}{a}\right)^{\frac{1}{n+1}} = r}$$

$GM_1 = ar$

$GM_2 = ar^2$

$$\boxed{GM_n = ar^n}$$

Product of GMs



~~$AM_1 \times AM_2 \times AM_3 \times AM_4 \times AM_5 \ldots GMAM_n$~~

$GM_1 \times GM_2 \times GM_3 \times GM_4 \times GM_5 \ldots GM_n$

$a\left(\frac{b}{a}\right)^{\frac{1}{n+1}} \times a\left(\frac{b}{a}\right)^{\frac{2}{n+1}} \times a\left(\frac{b}{a}\right)^{\frac{3}{n+1}} \ldots a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

$a^n \left(\frac{b}{a}\right)^{\frac{1}{n+1}(1+2+3\ldots n)}$

$a^n \left(\frac{b}{a}\right)^{\frac{1}{n+1}\left(\frac{n}{2}\times(n+1)\right)}$

$a^n \left(\frac{b}{a}\right)^{\frac{n}{2}}$

$\left(a \cdot \frac{\sqrt{b}}{\sqrt{a}}\right)^n$

$$\boxed{\left(\sqrt{ab}\right)^n}$$

Q Insert 4 G.Ms b/w 5 & 160. find product of G.Ms.

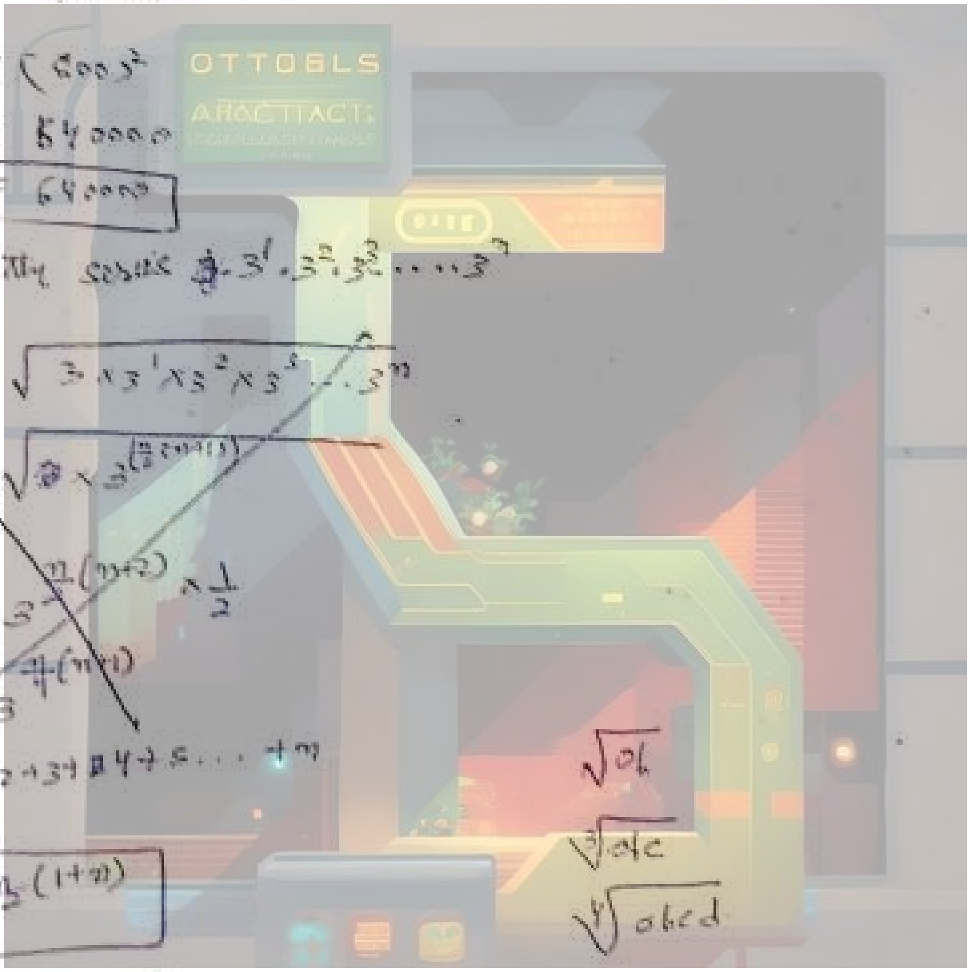
$r = \left(\frac{160}{5}\right)^{\frac{1}{5}}$

$= (32)^{1/5}$

$r = 2$

G.Ms → 10, 20, 40 ..., 80

Product = $\left(\sqrt{5\times160}\right)^4$



$= (800)^2$

$= 640000$

$\boxed{= 640000}$

Q find G.M of the series 3, $3^1, 3^2, 3^3 \ldots 3^n$

$G.M = \sqrt[n]{3\times3^1\times3^2\times3^3\ldots3^n}$

$= \sqrt{3\times3^{(\frac{n}{2}(n+1))}}$

$= 3^{\frac{n}{2}(n+2)\times\frac{1}{2}}$

$= 3^{\frac{n}{4}(n+1)}$

$3^{1+2+3+4+5\ldots+n}$

$\boxed{3^{\frac{n}{2}(1+n)}}$

$\left[3^{\frac{n}{2}(1+n)}\right]^{\frac{1}{n}}$

$\boxed{3^{\frac{(1+n)}{2}}}$

$\sqrt{ab}$

$\sqrt[3]{abc}$

$\sqrt[4]{abcd}$

Q find product of 30 GMs b/w 2 & 8.

$$\text{Product} = \left(\sqrt{ab}\right)^{30}$$

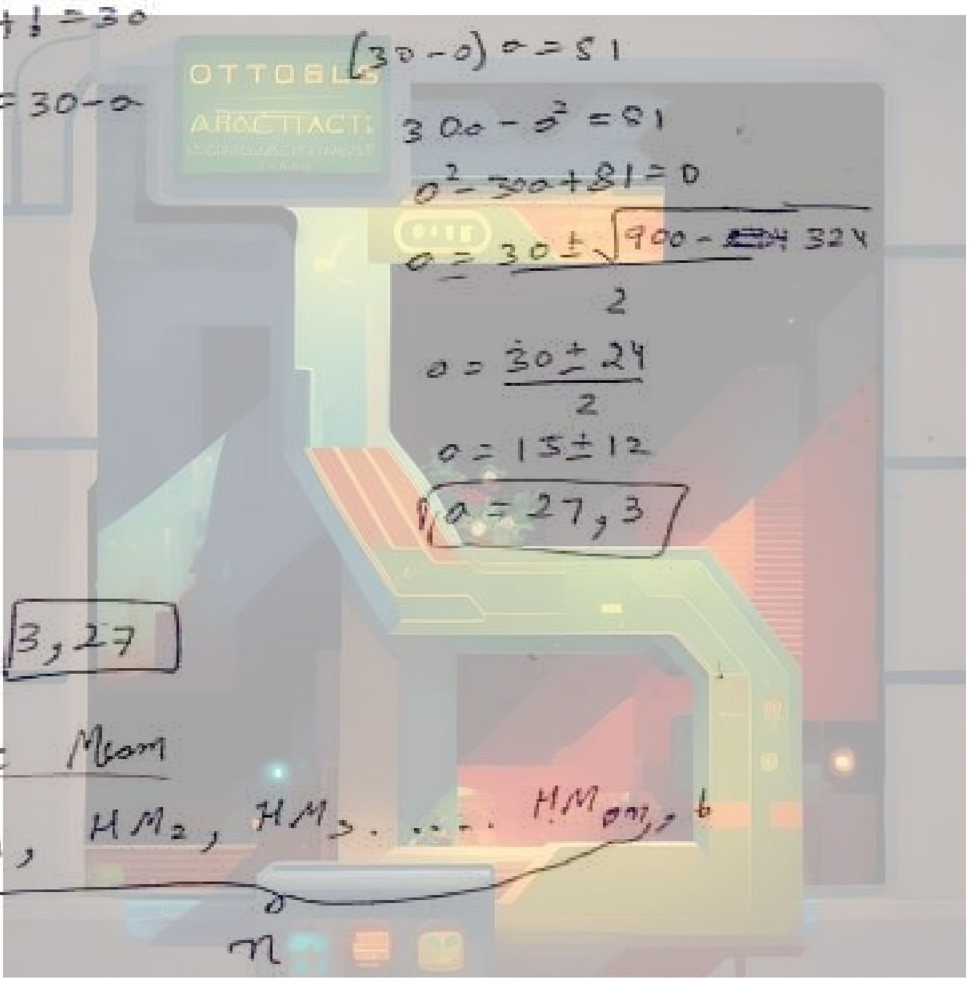
$$= (2\times 8)^{15}$$

$$\boxed{= 16^{15}}$$

Q $AM(a,b)=15$
$GM(a,b)=9$. find a, b.

$$\frac{a+b}{2}=15 \qquad \sqrt{ab}=9$$

$$a+b=30 \qquad ab=81$$



$$b=30-a \qquad (30-a)a=81$$

$$30a-a^2=81$$

$$a^2-30a+81=0$$

$$a=\frac{30\pm\sqrt{900-324}}{2}$$

$$a=\frac{30\pm 24}{2}$$

$$a=15\pm 12$$

$$\boxed{a=27,3}$$

$$\boxed{3,27}$$

[illegible]

Harmonic Mean

$$a,\ \underbrace{HM_1,\ HM_2,\ HM_3 \ldots\ldots HM_n}_{n},\ b$$

$$\frac{1}{a},\ \frac{1}{HM_1},\ \frac{1}{HM_2}\ \ldots\ldots\ \frac{1}{HM_3},\ \frac{1}{b}\ \ldots\ldots\ A.P.$$

$$\frac{1}{b}=\frac{1}{a}+(n+2-1)\,d$$

$$\frac{1}{b}-\frac{1}{a}=(n+1)d$$

$$\boxed{d=\frac{a-b}{ab(n+1)}}$$

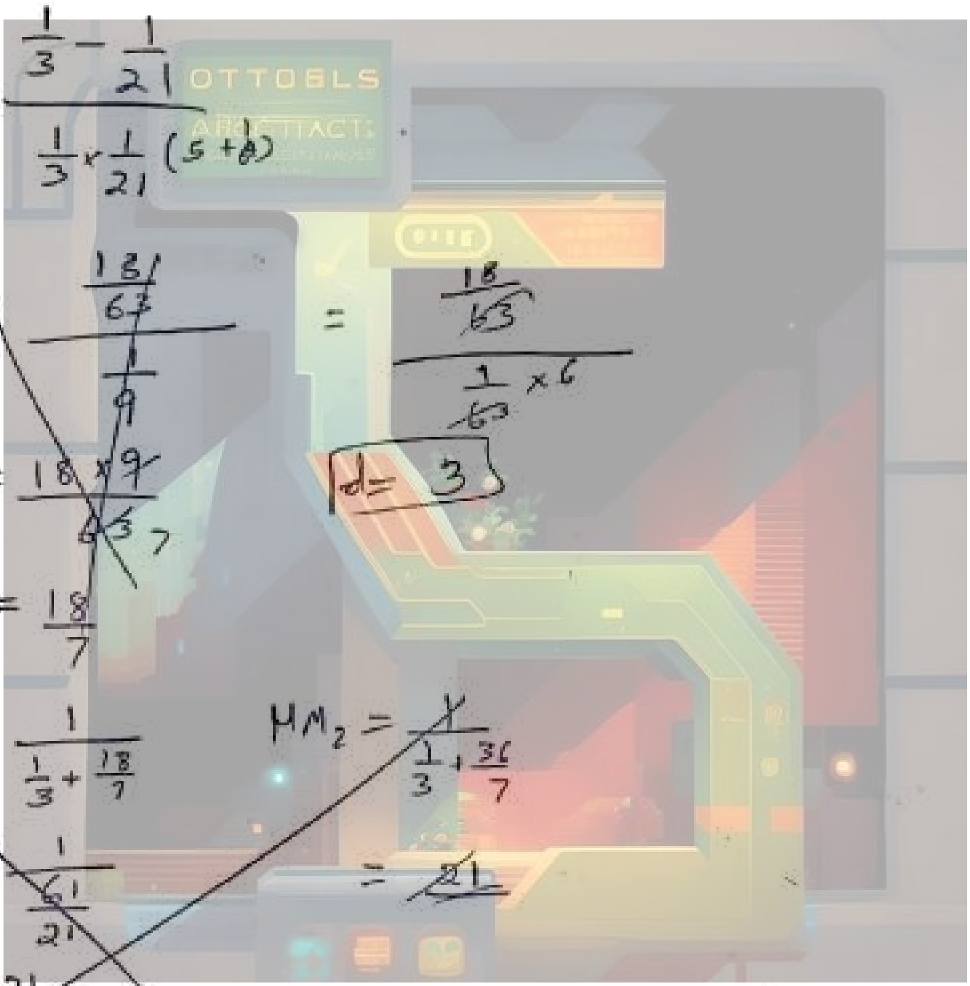
$AM_1 = a + (1)d$ $\boxed{a = \frac{1}{a}}$

$HM_1 = \frac{1}{a+d}$

$HM_2 = \frac{1}{a+2d}$

$$\boxed{HM_n = \frac{1}{a+nd}}$$

Q Insert 5 HM btw $\frac{1}{3}$ & $\frac{1}{21}$.



$d = \dfrac{\frac{1}{3} - \frac{1}{21}}{\frac{1}{3} \times \frac{1}{21} (5+1)} = \dfrac{\frac{18}{63}}{\frac{1}{63} \times 6}$

$\boxed{d = 3}$

~~$\dfrac{\frac{18}{63}}{\frac{1}{9}} = \frac{18 \times 9}{63\ 7} = \frac{18}{7}$~~

~~$HM_1 = \dfrac{1}{\frac{1}{3} + \frac{18}{7}} = \dfrac{1}{\frac{61}{21}} = \frac{21}{61}$~~

~~$HM_2 = \dfrac{1}{\frac{1}{3} + \frac{36}{7}} = 21$~~

$HM_1 = \frac{1}{a+6} = \frac{3}{18} = \frac{1}{6}$

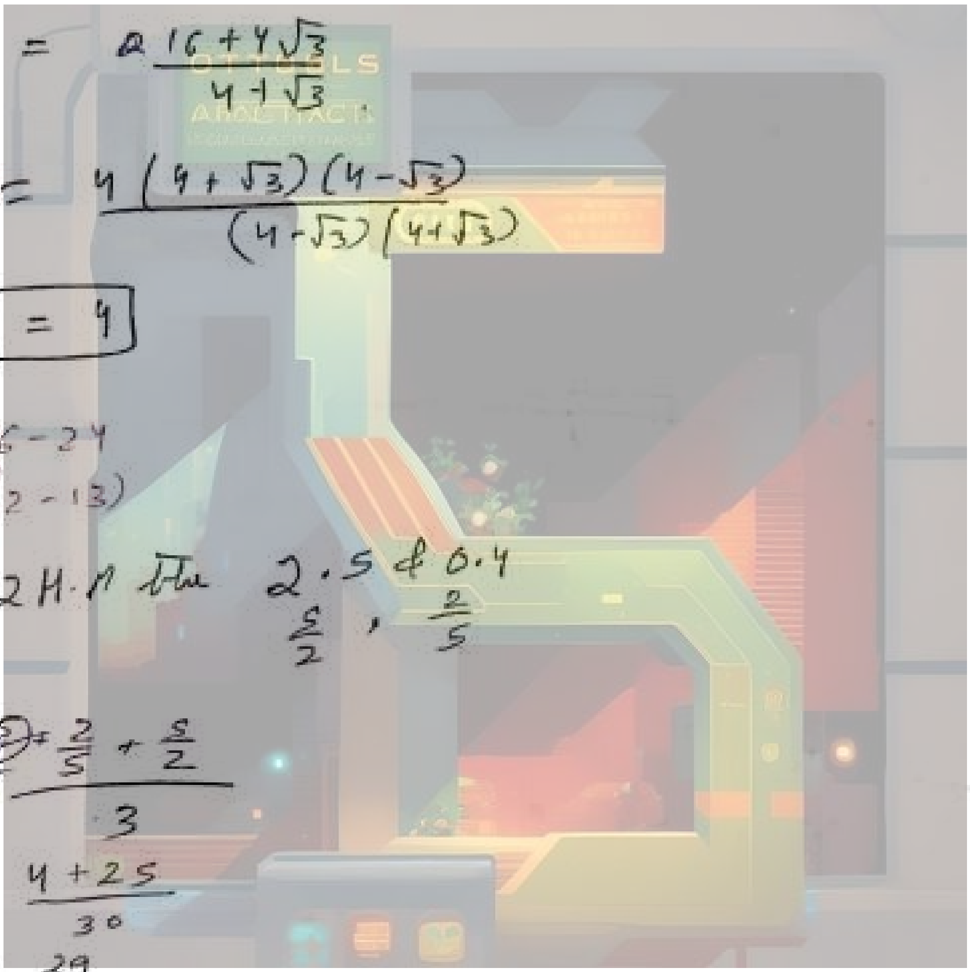
$HM_2 = \dfrac{1}{\frac{1}{3} + \frac{36}{7}} = \frac{3}{36} = \frac{1}{9}$

Q. HM of roots of eq $(5+\sqrt{2})x^2-(4+\sqrt{3})x+(8+2\sqrt{3})=0$

~~$x = \frac{(4+\sqrt{3})\pm\sqrt{16+3+8\sqrt{3}-160-40\sqrt{3}-32\sqrt{2}-8\sqrt{6}}}{10+2\sqrt{2}}$~~

$$HM = \frac{2\alpha\beta}{\alpha+\beta}$$

$$= \frac{2\,\frac{(8+2\sqrt{3})}{\cancel{5+\sqrt{2}}}}{\frac{4+\sqrt{3}}{\cancel{5+\sqrt{2}}}}$$



$$= \frac{16+4\sqrt{3}}{4+\sqrt{3}}$$

$$= \frac{4(4+\sqrt{3})(4-\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})}$$

$$\boxed{=4}$$

H.W. 27-6-24
DYS-5 (2-13)

Q Insert 2 H.M b/w 2.5 & 0.4

$\frac{5}{2}, \frac{2}{5}$

$$d = \frac{\frac{2}{5}+\frac{5}{2}}{3}$$

$$= \frac{4+25}{30}$$

$$= \frac{29}{30}$$

$$HM_1 = \frac{1}{\frac{2}{5}+\frac{29}{30}}$$

$$\boxed{=\frac{30}{41}}$$

$$HM_2 = \frac{1}{\frac{2}{5}+\frac{58}{30}}$$

$$= \frac{30}{70}$$

$$\boxed{=\frac{3}{7}}$$

AM, GM, HM (Inequality & Relation)

Properties:-

$a, b, c, d \in \oplus$ve Real no.

① a, b

$AM = \frac{a+b}{2}$ $\quad GM = \sqrt{ab}$ $\quad HM = \frac{2}{\frac{1}{a}+\frac{1}{b}}$

a, b, c

$AM = \frac{a+b+c}{3}$ $\quad GM = \sqrt[3]{abc}$ $\quad HM = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

a, b, c, d

$AM = \frac{a+b+c+d}{4}$ $\quad GM = \sqrt[4]{abcd}$ $\quad HM = \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$

$a_1, a_2, a_3 \dots a_n$

$AM = \frac{a_1+a_2+a_3+a_4 \dots a_n}{n}$ $\quad GM = \sqrt[n]{a_1 \times a_2 \times a_3 \times a_4 \dots a_n}$ $\quad HM = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2} \dots \frac{1}{a_n}}$

② $AM \geq GM \geq HM$

$AM = GM = HM$ when $a_1 = a_2 = a_3 \dots = a_n$

Proof $AM \geq GM$

$\frac{a+b}{2} \geq \sqrt{ab}$

$(\sqrt{a})^2 + (\sqrt{b})^2 - 2\sqrt{ab} \geq 0$

$(\sqrt{a} - \sqrt{b})^2 \geq 0$

always true

$GM \geq HM$

$\sqrt{ab} \geq \frac{2ab}{a+b}$

$\sqrt{ab}(a+b) - 2ab \geq 0$

$\sqrt{ab}(a+b-2\sqrt{ab}) \geq 0$

$(\sqrt{a} - \sqrt{b})^2 \geq 0$

Always true

③ For any ~~two~~ different numbers their AM, GM & HM ~~are~~ are in GP

$$\boxed{GM^2 = AM \times HM}$$

④ $x^2 - 2AMx + GM^2 = 0$

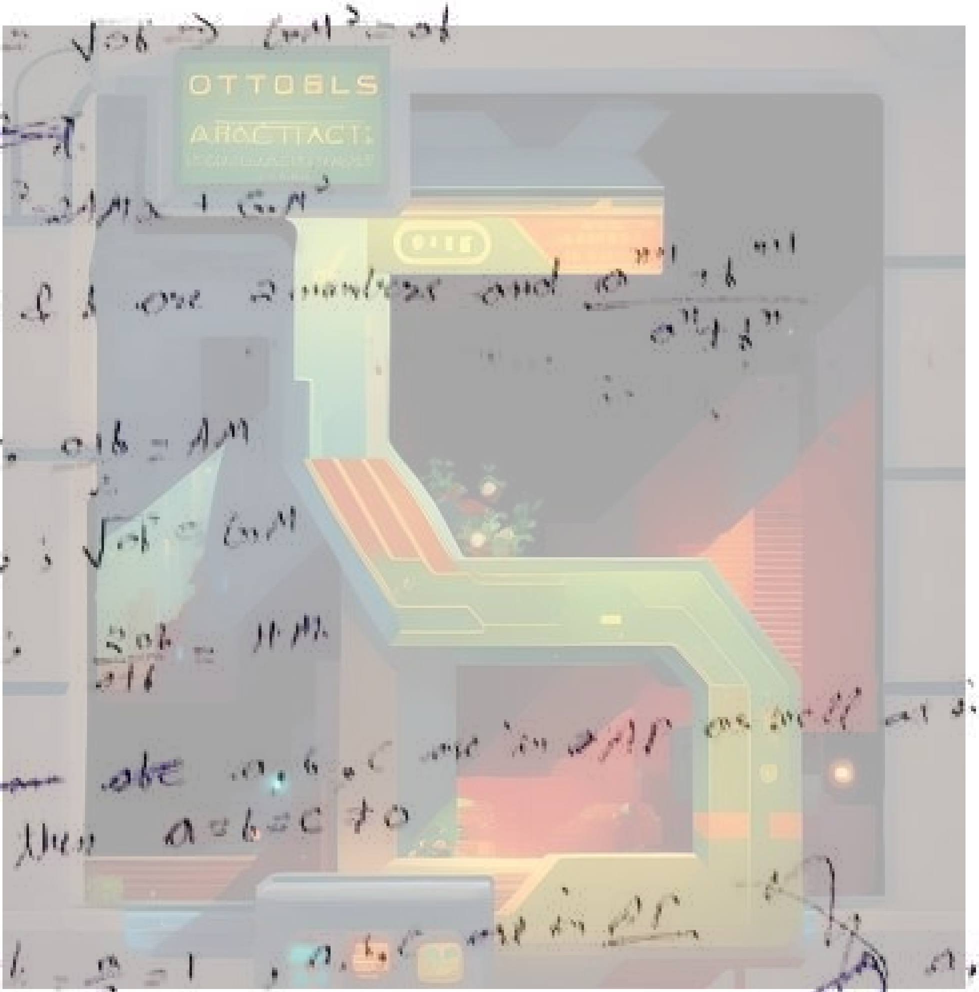
$x^2 - (a+b)x + ab = 0$

$AM = \frac{a+b}{2} \Rightarrow a+b = 2(AM)$

$GM = \sqrt{ab} \Rightarrow GM^2 = ab$

~~x~~

$x^2 - 2AMx + GM^2$



⑤ If a & b are 2 numbers and $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$n=0$; $\frac{a+b}{2} = AM$

$n=-\frac{1}{2}$; $\sqrt{ab} = GM$

$n=-1$; $\frac{2ab}{a+b} = HM$

⑥ If ~~a~~ a, b, c are in AP as well as in GP then $a=b=c \neq 0$

⑦ i) $\frac{a-b}{b-c} = \frac{a}{a} = 1$, a, b, c are in AP

ii) $\frac{a-b}{b-c} = \frac{a}{b}$ or $\frac{a-b}{b-c} = \frac{b}{c} \Rightarrow ac - bc = b^2 - bc$; $ac = b^2$; a, b, c are in GP

iii) $\frac{a-b}{b-c} = \frac{a}{c}$; a, b, c are in HP

Inequalities

Q1. If x & $y \in R^+$ then $\frac{x}{y} + \frac{y}{x}$ min

$GM = \sqrt{\frac{x}{y} \times \frac{y}{x}} = \sqrt{1} = 1$

$AM = \dfrac{\frac{x}{y} + \frac{y}{x}}{2}$

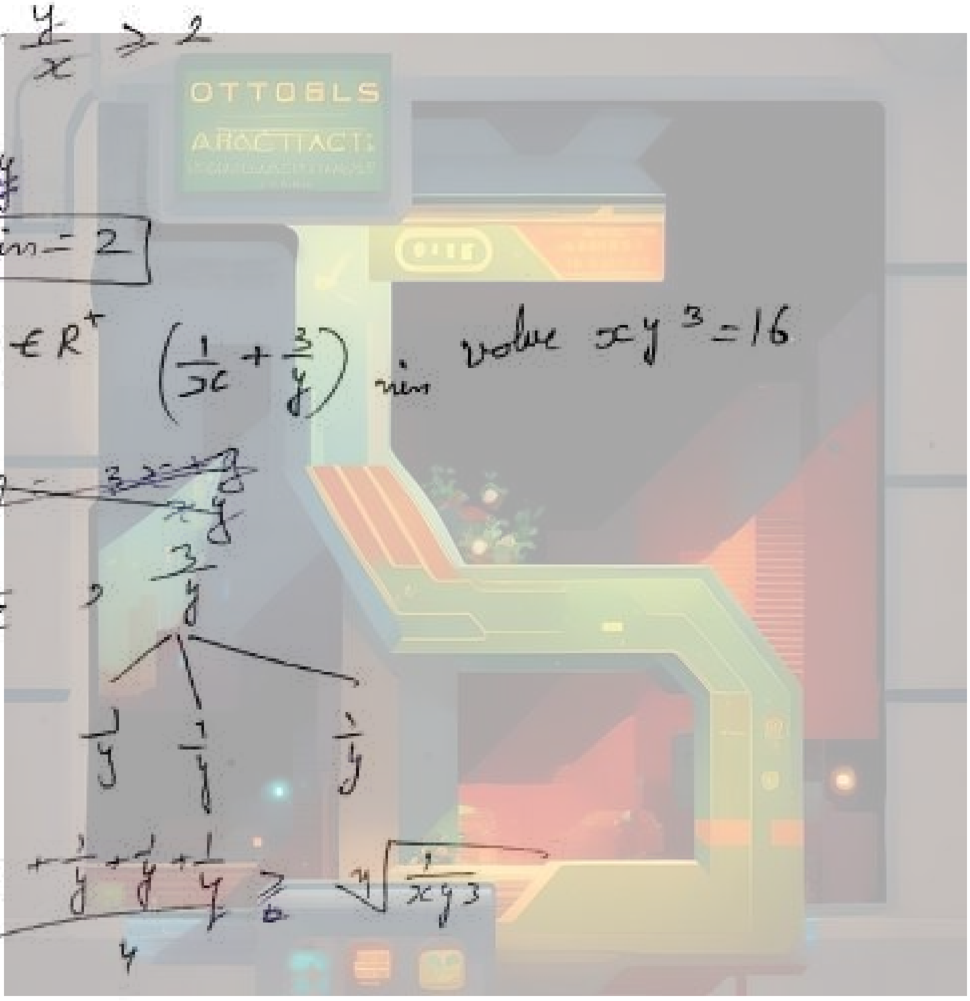
$AM \geq GM$

$\frac{x}{y} + \frac{y}{x} \geq 2$

~~$x^2 \geq y$~~

$\boxed{\min = 2}$

Q2. x & $y \in R^+$ $\left(\frac{1}{x} + \frac{3}{y}\right)$ min value $xy^3 = 16$



~~$AM = \frac{3x+y}{xy}$~~

$\frac{1}{x}$, $\frac{3}{y}$

$\frac{1}{y}$ $\frac{1}{y}$ $\frac{1}{y}$

$\dfrac{\frac{1}{x} + \frac{1}{y} + \frac{1}{y} + \frac{1}{y}}{4} \geq \sqrt[4]{\frac{1}{xy^3}}$

$\frac{1}{x} + \frac{3}{y} \geq 2$

$\boxed{\min = 2}$

Q $P+2Q+4R=9$ $P,Q,R \in I^+$

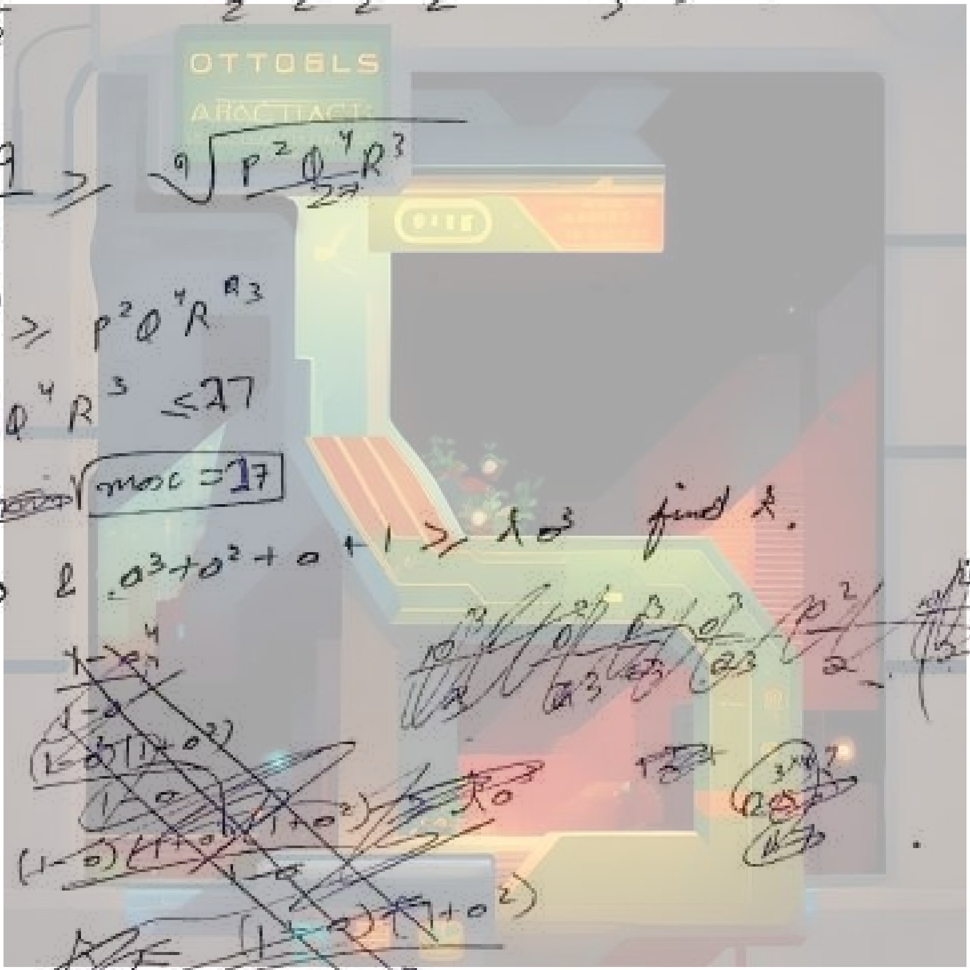
find $(P^2Q^4R^3)_{max}$

$$\cancel{3 \geq 2\sqrt[3]{PQR}}$$

$$\cancel{\frac{27}{8} \geq PQR}$$

$$\cancel{\max PQR = \frac{27}{8}}$$

$P \to \frac{P}{2}, \frac{P}{2}$ $2Q \to \frac{Q}{2}, \frac{Q}{2}, \frac{Q}{2}, \frac{Q}{2}$ $4R \to \frac{4}{3}R, \frac{4}{3}R, \frac{4}{3}R$



$$\frac{9}{9} \geq \sqrt[9]{\frac{P^2Q^4R^3}{27}}$$

$$1^9 \geq \frac{P^2Q^4R^3}{27}$$

$$P^2Q^4R^3 \leq 27$$

$$\boxed{\max = 27}$$

Q If $a>0$ & $a^3+a^2+a+1 \geq \lambda a^3$ find λ.

~~$1-a^4$~~

~~$(1-a)(1+a)(1+a^2)$~~

~~$\frac{a^3+a^2+a+1}{4} \geq \sqrt[4]{a^6}$~~

~~$a^3+a^2+a+1 \geq 4a^{3/2}$~~

H.W. [illegible] (28-6-24)

O-1 (01, 2-5, 7, 11, 12, 13, 14, 16)

$a^3 + b^3 - 1 = 13 ka^3$, $a > 0$

Not possible

Q $2x + 3y = 15$. (find x^2y max

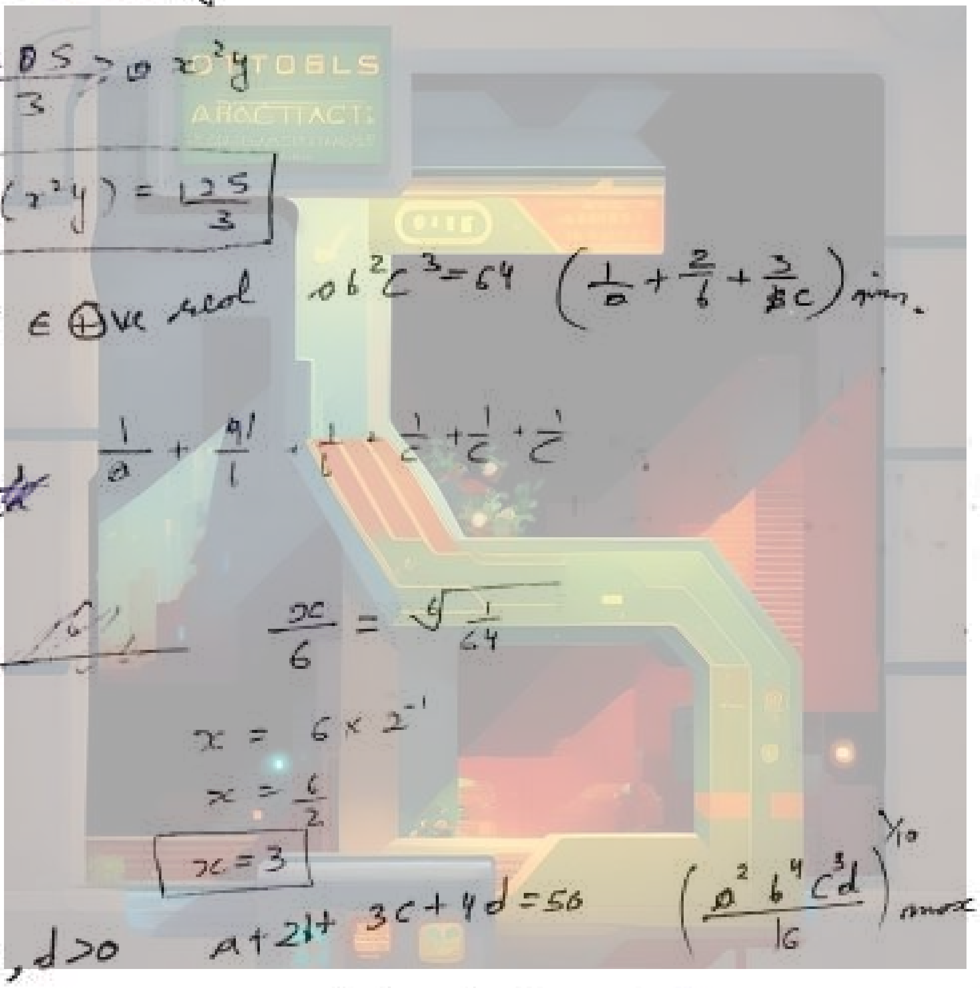
$$\frac{x + x + 3y}{3} = \sqrt[3]{3x^2y}$$

$2 = 5x+5$

$$5 \; \frac{15}{3} \geq \sqrt[3]{3x^2y}$$

$$125 \geq 3x^2y$$

$$\frac{125}{3} \geq x^2y$$



$$\boxed{\max(x^2y) = \frac{125}{3}}$$

Q $a, b, c \in \oplus$ve real $ab^2c^3 = 64$ $\left(\frac{1}{a} + \frac{2}{b} + \frac{3}{c}\right)$ min.

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{b} + \frac{1}{c} + \frac{1}{c} + \frac{1}{c}$$

$$\frac{x}{6} = \sqrt[6]{\frac{1}{64}}$$

$$x = 6 \times 2^{-1}$$

$$x = \frac{6}{2}$$

$$\boxed{x = 3}$$

Q $a, b, c, d > 0$ $a + 2b + 3c + 4d = 50$ $\left(\frac{a^2 b^4 c^3 d}{16}\right)^{1/10}_{\max}$

$$\frac{\frac{a}{2} + \frac{a}{2} + \frac{b}{2} + \frac{b}{2} + \frac{b}{2} + \frac{b}{2} + c + c + c + 4d}{10} \geq \sqrt[10]{\frac{a^2 b^4 c^3 d}{16}}$$

$$\boxed{5 =}$$

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Q $p, q, r \in R^+$ $pqr = 1$

$$\left[(1+p+p^2)(1+q+q^2)(1+r+r^2)\right]_{min}$$

$\cancel{1+}p(1+p)$

$$\frac{1+p+p^2}{3} \geq (1\cdot p\cdot p^2)^{1/3} \geq p$$

$$\frac{1+q+q^2}{3} \geq q$$

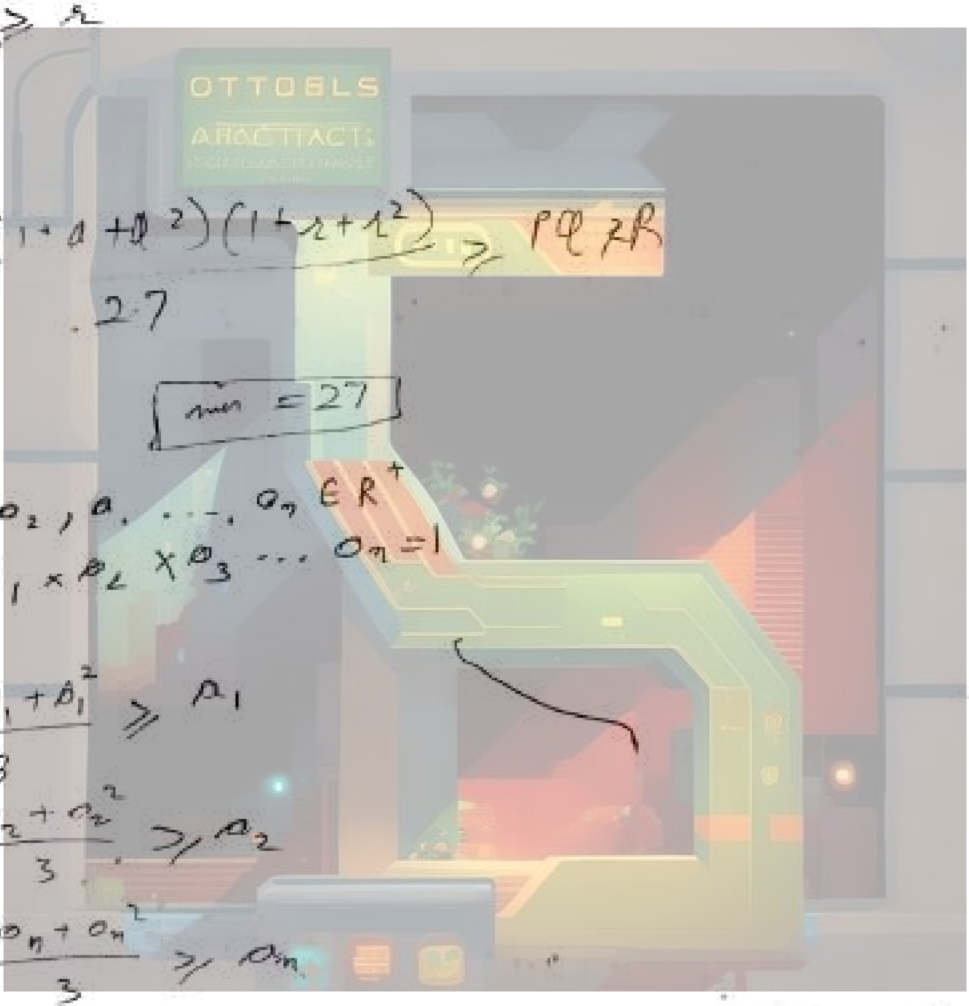
$$\frac{1+r+r^2}{3} \geq r$$

multiply

$$\frac{(1+p+p^2)(1+q+q^2)(1+r+r^2)}{27} \geq pqR$$

$$\boxed{min = 27}$$

Q If $a_1, a_2, a_3, \ldots, a_n \in R^+$
& $a_1 \times a_2 \times a_3 \ldots a_n = 1$



$$\frac{1+a_1+a_1^2}{3} \geq a_1$$

$$\frac{1+a_2+a_2^2}{3} \geq a_2$$

$$\frac{1+a_n+a_n^2}{3} \geq a_n$$

$$\frac{(1+a_1+a_1^2)(1+a_2+a_2^2)(1+a_n+a_n^2)}{3^n} \geq 1$$

$$\boxed{3^n = min}$$

Q $x > 0$, $\left(\frac{x^{10}}{1+x+x^2\dots x^{20}}\right)_{max}$

~~$\frac{x^{10}}{\frac{x^{20}-1}{x-1}}$~~

~~$x^{10}(x-1)$~~

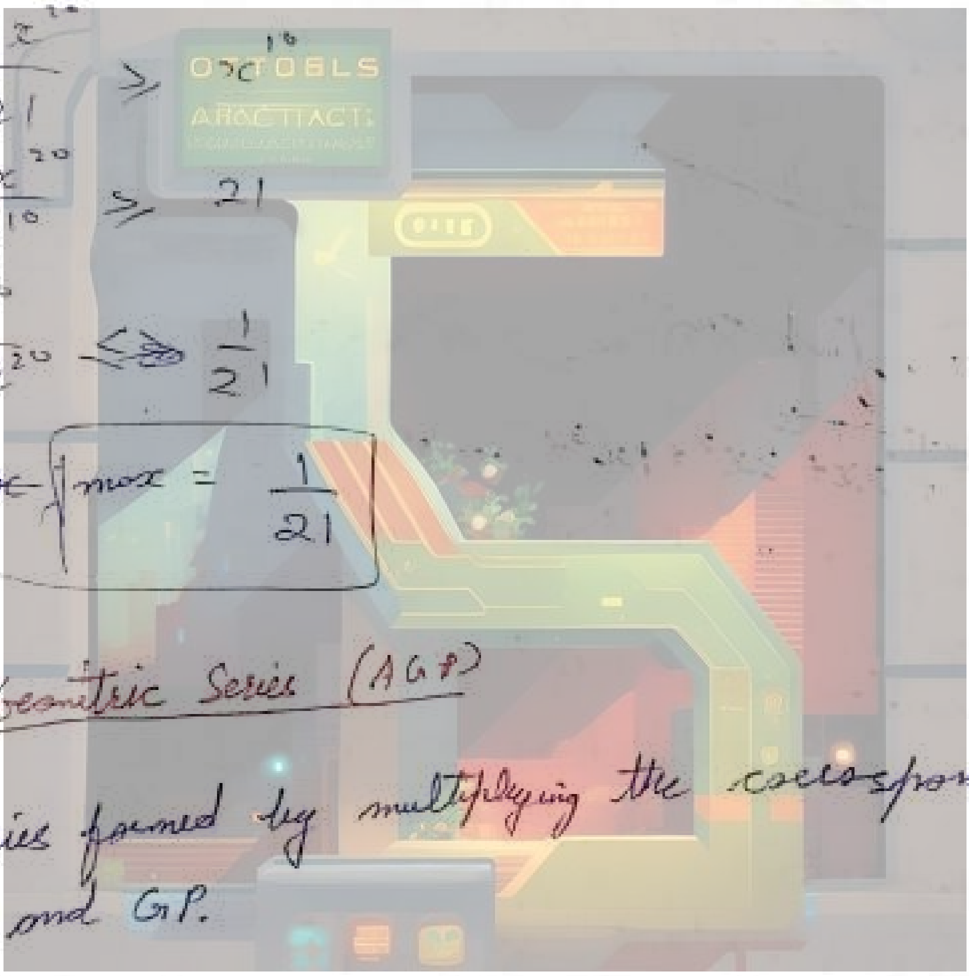
$\frac{1+\dots}{21} \geq x^{\frac{210}{21}}$

$\frac{1\dots+x^{20}}{21} \geq x^{10}$

$\frac{1\dots+x^{20}}{x^{10}} \geq 21$

$\frac{x^{10}}{1\dots+x^{20}} \leq \frac{1}{21}$

$\boxed{f_{max} = \frac{1}{21}}$



Aritho - Geometric Series (AGP)

→ A Series formed by multiplying the corresponding terms of AP and GP.

$ab, (a+d)br, (a+2d)br^2, \dots\dots (a+(n-1)d)br^{n-1}$

a → first term of AP

d → CD of AP

b → 1st term of GP

r → C.R of GP

$T_n = (a + (n-1)d)\, b r^{n-1}$

Sum of AGP :-

Process :-

① Find the C.R.

② Multiply the CR with given series & form a new series.

③ Subtract the given series & ~~to~~ the new series

④ After subtraction we either get AP or GP hence Apply sum.

⑤ In rare case we have to repeat the subtraction process two or 3 times If we don't get the A.P. or G.P. after first subtraction.

~~Q $|x| < 1$, find the sum of $1 +$~~

Q. $|x| < 1$. find sum

~~$1 + 2x + 3$~~

$1 + 2x + 3x^2 + 4x^3 + \dots \infty$

$S = 1 + 2x + 3x^2 + 4x^3 \dots \infty$

$CR = x$

$Sx = x + 2x^2 + 3x^3 + 4x^4 \dots \infty$

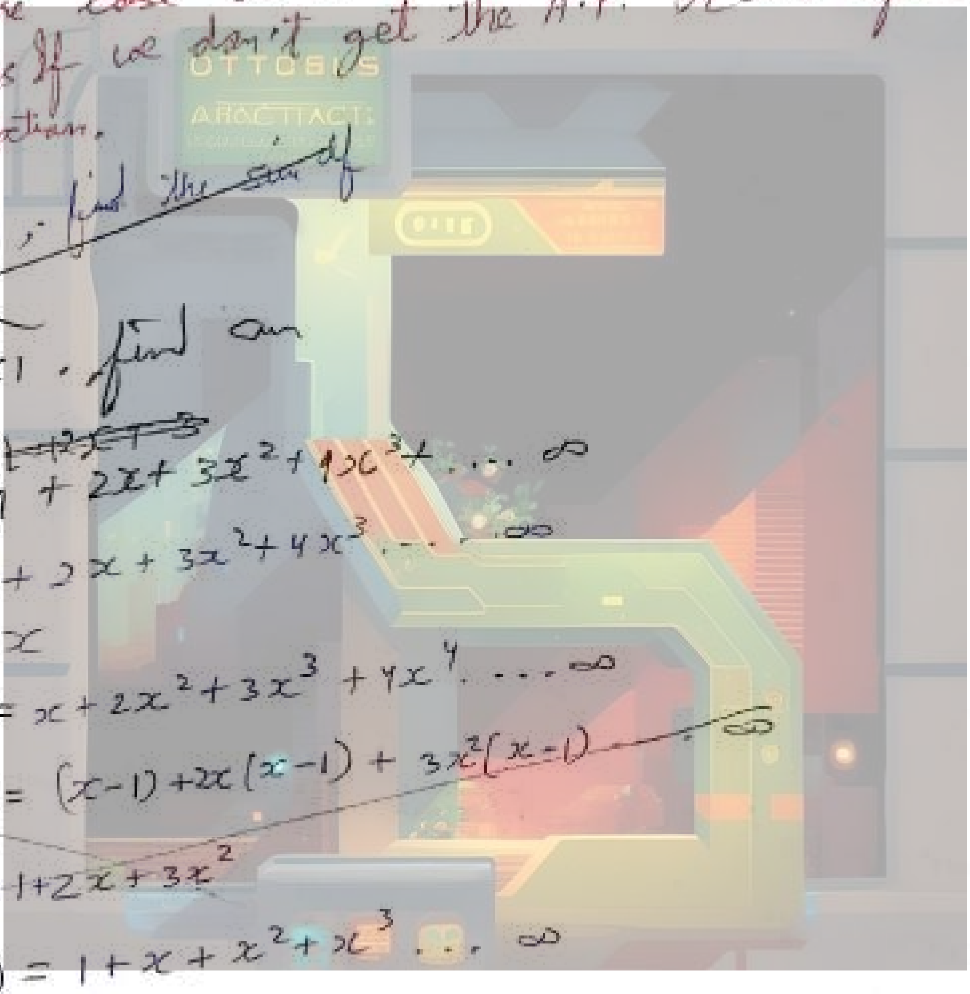
~~$Sx - S = (x-1) + 2x(x-1) + 3x^2(x-1) \dots \infty$~~

~~$S = 1 + 2x + 3x^2$~~

$S(1-x) = 1 + x + x^2 + x^3 \dots \infty$

$S(1-x) = \frac{1}{1-x}$

$S = \frac{1}{(1-x)^2}$



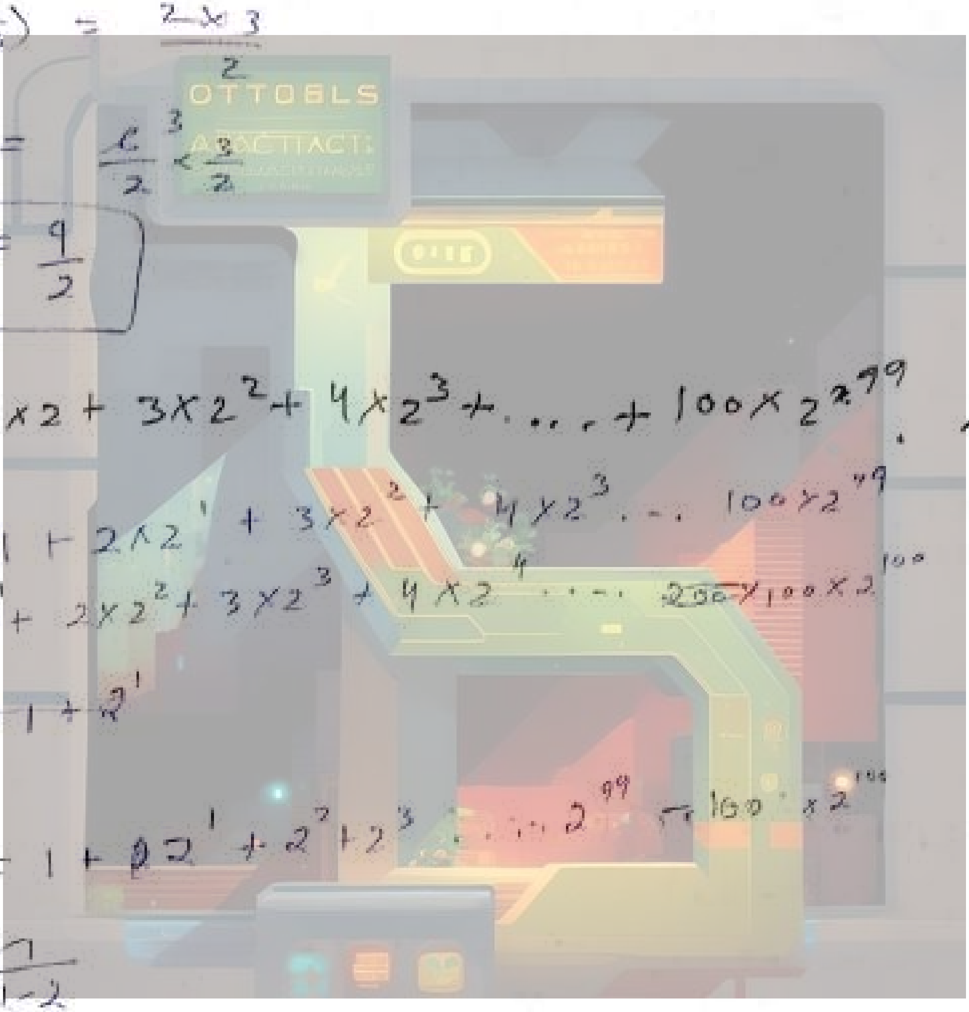
Q $S = 2 + \frac{4}{3} + \frac{6}{3^2} + \frac{8}{3^3} + \frac{10}{3^4} + \dots \infty$

$\frac{S_\infty}{3} = \frac{2}{3} + \frac{4}{3^2} + \frac{6}{3^3} + \frac{8}{3^4} \dots \infty$

$S - \frac{S}{3} = 2 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \frac{2}{3^4} \dots \infty$

$S\left(1 - \frac{1}{3}\right) = \frac{2}{1 - \frac{1}{3}}$

$S\left(\frac{2}{3}\right) = \frac{2 \times 3}{2}$



$S = \frac{\overset{3}{\cancel{6}}}{2} \times \frac{3}{2}$

$\boxed{S = \frac{9}{2}}$

Q $1 + 2\times2 + 3\times2^2 + 4\times2^3 + \dots + 100\times2^{99}$. sum.

$S = 1 + 2\times2^1 + 3\times2^2 + 4\times2^3 \dots 100\times2^{99}$

$2S = 2^1 + 2\times2^2 + 3\times2^3 + 4\times2^4 \dots \cancel{200}\, 100\times2^{100}$

~~$S - 2S = 1 + 2^1$~~

~~$S =$~~

$S - 2S = 1 + 2^1 + 2^2 + 2^3 \dots 2^{99} - 100\times2^{100}$

~~$-S = \frac{1}{1-2}$~~

~~$-S = -\frac{1}{2}$~~

~~$\boxed{S = 1/2}$~~

$-S = \frac{(1 - 2^{100})}{1-2} - 100\times2^{100}$

$S = 1 - 2^{100} + 100\times2^{100}$

~~$S = 1 - 101\times2^{100}$~~

$\boxed{S = 99\times2^{100} + 1}$

~~Q $2+\frac{4}{3}+\frac{6}{3^2}+$~~

Q $\frac{4}{7}-\frac{5}{7^2}+\frac{4}{7^3}-\frac{5}{7^4}+\frac{4}{7^5}-\frac{5}{7^6}\dots\infty$

$$\frac{(28-5)}{7^2}+\frac{(28-5)}{7^4}+\frac{(28-5)}{7^6}\dots\infty$$

$a=(28-5)/7^2$

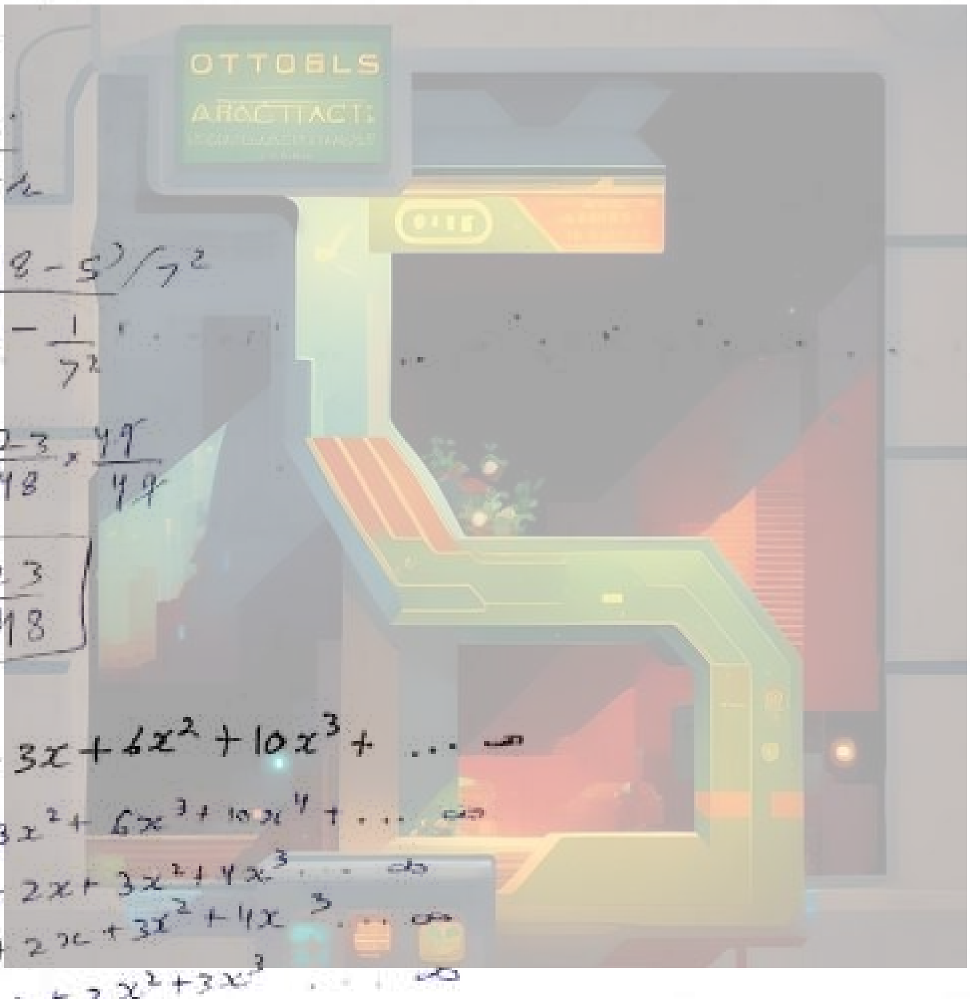
$r=1/7^2$

$$S_\infty=\frac{a}{1-r}$$

$$=\frac{(28-5)/7^2}{1-\frac{1}{7^2}}$$

$$=\frac{23}{48}\times\frac{49}{49}$$

$$\boxed{=\frac{23}{48}}$$



Q $S=1+3x+6x^2+10x^3+\dots\infty$

$Sx=x+3x^2+6x^3+10x^4+\dots\infty$

$S-Sx=1+2x+3x^2+4x^3\dots\infty$

$S(1-x)=1+2x+3x^2+4x^3\dots\infty$

$Sx(1-x)=x+2x^2+3x^3\dots\infty$

$S(1-x)^2=1+x+x^2+x^3\dots\infty$

$S(1-x)^2=\frac{1}{1-x}$

$S=\frac{1}{(1-x)^3}$

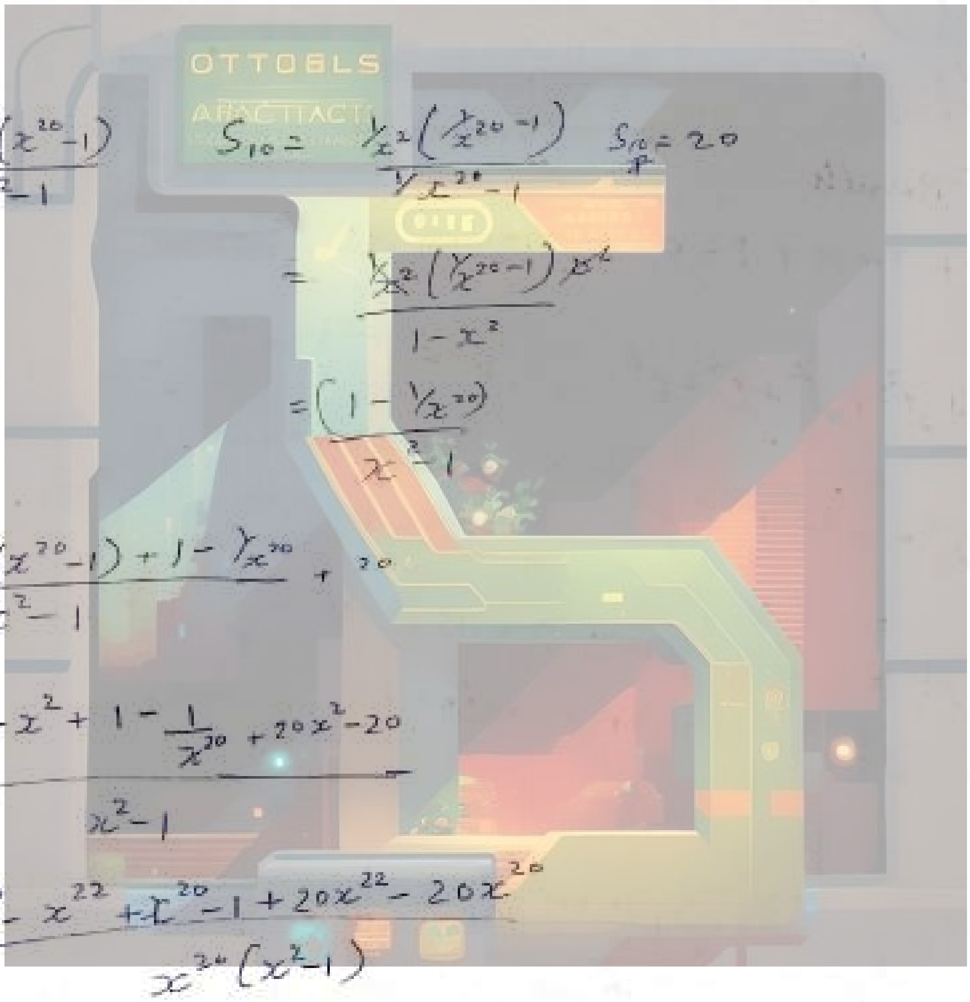
Q sin of 10 term

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2 \dots$$

~~$\left(x+\frac{1}{x}\right)+\left(x^2+\frac{1}{x^2}\right)$~~

$$\left(x^2+\frac{1}{x^2}+2\right)+\left(x^4+\frac{1}{x^4}+2\right)+\left(x^6+\frac{1}{x^6}+2\right)\dots$$

$$x^2+x^4+x^6+x^8\dots+\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6}\dots+2+2+2\dots$$



$a=x^2$

$r=x^2$

$$S_{10}=\frac{x^2(x^{20}-1)}{x^2-1} \qquad S_{10}=\frac{1/x^2\left(1/x^{20}-1\right)}{1/x^2-1} \qquad S_{10}=20$$

$$=\frac{1/x^2\left(1/x^{20}-1\right)x^2}{1-x^2}$$

$$=\frac{\left(1-1/x^{20}\right)}{x^2-1}$$

$$\frac{x^2(x^{20}-1)+1-1/x^{20}}{x^2-1}+20$$

$$\frac{x^{40}-x^2+1-\frac{1}{x^{20}}+20x^2-20}{x^2-1}$$

$$\frac{x^{60}-x^{22}+x^{20}-1+20x^{22}-20x^{20}}{x^{20}(x^2-1)}$$

$$\frac{x^{60}+19x^{22}-19x^{20}-1}{x^{20}(x^2-1)}$$

$$\boxed{\cancel{\frac{x^{40}+19x^2-19}{x^2-1}}}$$

Special Sequence

Type-1 :- Using Summation (Σ)

Type-2 :- Method of Difference (MOD)

Type-3 :- Splitting in denominator

Type-4 :- Splitting in Numerator

H.W- 01-7-24

DYS-6 (Full)

Type-1 :- Results:-

① $1+2+3+4+5 \ldots\ldots n = \frac{n(1+n)}{2}$

② $1+2^2+3^2+4^2+5^2 \ldots\ldots n^2 = \frac{n(1+n)(2n+1)}{6}$

③ $1+2^3+3^3+4^3+5^3 \ldots\ldots n^3 = \left(\frac{n(n+1)}{2}\right)^2$

④ $1+3+5+\ldots\ldots+(2n-1) = n^2$

⑤ $2+4+6+\ldots\ldots 2n = n(n+1)$

Proof:-

② $1^2+2^2+3^2 \ldots\ldots \frac{n(n+1)(2n+1)}{6}$

$(x+1)^3 = x^3+1+3x+3x^2$

$(x+1)^3 - x^3 = 1+3x+3x^2$

~~$x=0$~~ ~~$1^3-0^3 = 1+0+0$~~

$x=1 \quad 2^3-1^3 = 1+3(1)+3(1)^2$

$x=2 \quad 3^3-2^3 = 1+3(2)+3(2)^2$

$x=3 \quad 4^3-2^3 = 1+3(3)+3(3)^3$

$x=n \quad (n+1)^3-n^3 = 1+3(n)+(3n)^2$

ddd de

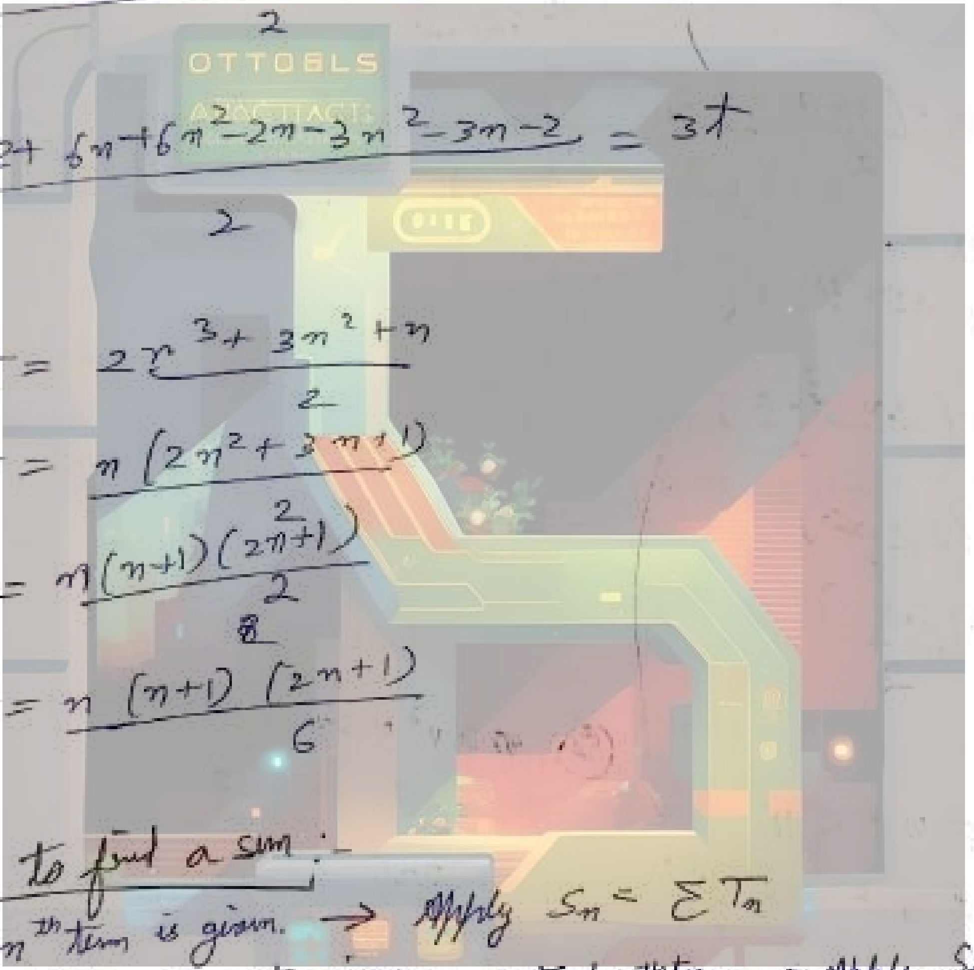
(70)

$$(n+1)^3-1 = n+3(1+2+3+\dots+n)+3\underbrace{(1^2+2^2+3^2+\dots n^2)}_{t}$$

$$(n+1)^3-1 = n+\frac{3n(n+1)}{2}+3t$$

$$(n+1)^3-1-n-\frac{3n(n+1)}{2}=3t$$

$$\frac{2(n+1)^3-2n-3n(n+1)-2}{2}=3t$$



$$\frac{2n^3+2+6n+6n^2-2n-3n^2-3n-2}{2}=3t$$

$$3t=\frac{2n^3+3n^2+n}{2}$$

$$3t=\frac{n(2n^2+3n+1)}{2}$$

$$3t=\frac{n(n+1)(2n+1)}{2}$$

$$t=\frac{n(n+1)(2n+1)}{6}$$

Method to find a sum:-

→ when n^{th} term is given. → Apply $S_n = \sum T_n$

→ when n^{th} term is not given:- → Find n^{th} term → Apply $S_n = \sum T_n$

Q find the sum of $\overline{n}$ terms whose n^{th} term is

① ~~2n~~ ~~2~~ $3n-1$

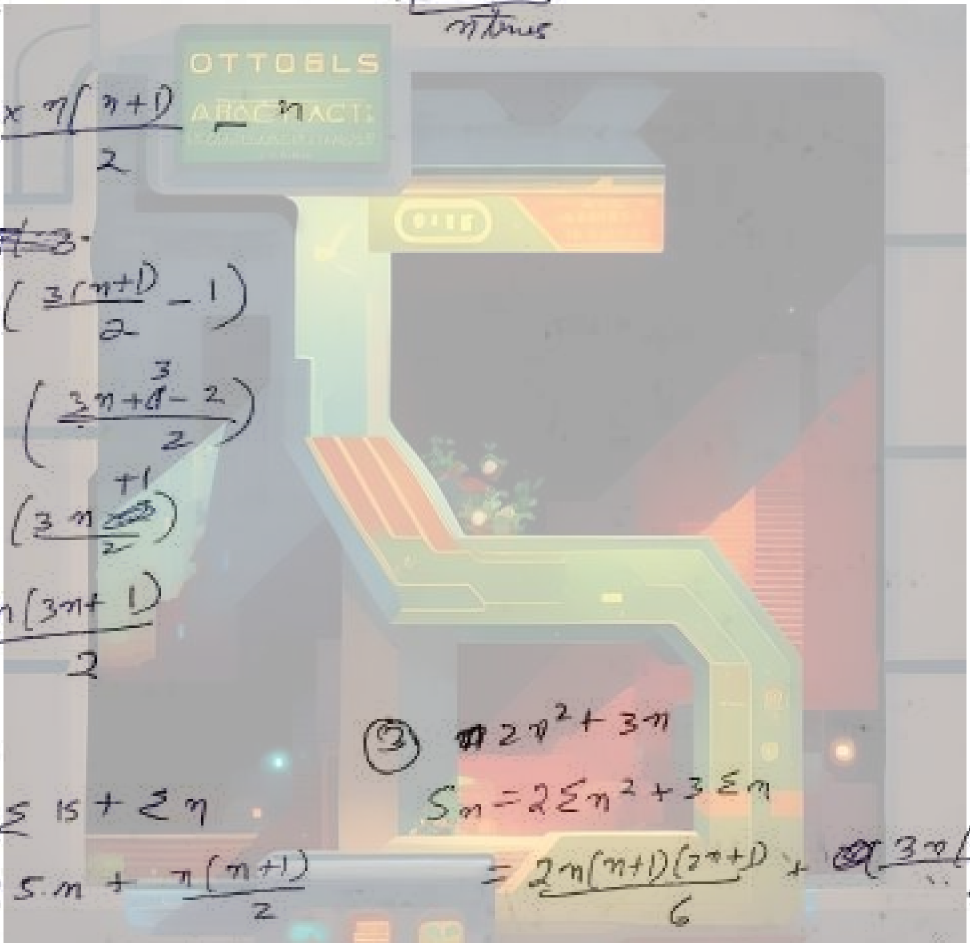
$2, 5, 8, 11 \ldots\ldots 3n-1$

$S_n = \Sigma(3n-1)$

$= \Sigma 3n - \Sigma 1$

$= 3\Sigma n - \Sigma 1$

$= 3(1+2+3+\ldots n) - (\underbrace{1+1+1}_{n\,times})$



$= \frac{3 \times n(n+1)}{2} - n$

~~$3n+3$~~

$= n\left(\frac{3(n+1)}{2} - 1\right)$

$= n\left(\frac{3n+\overset{3}{\cancel{1}}-2}{2}\right)$

$= n\left(\frac{3n\overset{+1}{\cancel{}}}{2}\right)$

$= \frac{n(3n+1)}{2}$

② $15+n$

$S_n = \Sigma 15 + \Sigma n$

$= 15 \cdot n + \frac{n(n+1)}{2}$

$= \frac{30n + n^2 + n}{2}$

$\boxed{= \frac{n(n+31)}{2}}$

③ ~~n~~ $2n^2+3n$

$S_n = 2\Sigma n^2 + 3\Sigma n$

$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}$

$= \frac{n+1}{2}\left(\frac{2n(2n+1)}{3} + 3n\right)$

$= \frac{n+1}{2} \frac{(4n^2+2n+9n)}{3}$

$= \frac{n+1}{6}(4n^2+11n)$

$\boxed{= \frac{n(n+1)(4n+11)}{6}}$

④ $T_n = 3^n - 2^n$

$S_n = \Sigma(3^n - 2^n)$

$= \Sigma 3^n - \Sigma 2^n$

~~$= \Sigma(3+9+27 \dots 3^n) - \Sigma(2+4+8+16 \dots 2^n)$~~

$= (3+3^2+3^2+3^3 \dots 3^n) - (2+2^2+2^3 \dots 2^n)$

$= \frac{3(3^n-1)}{2} - \frac{2(2^n-1)}{1}$

$= \frac{3(3^n-1) - 4(2^n-1)}{2}$

$= \frac{3^{n+1} - 3 - 2^{n+2} + 4}{2} = \boxed{\frac{3^{n+1} - 2^{n+2} + 1}{2}}$

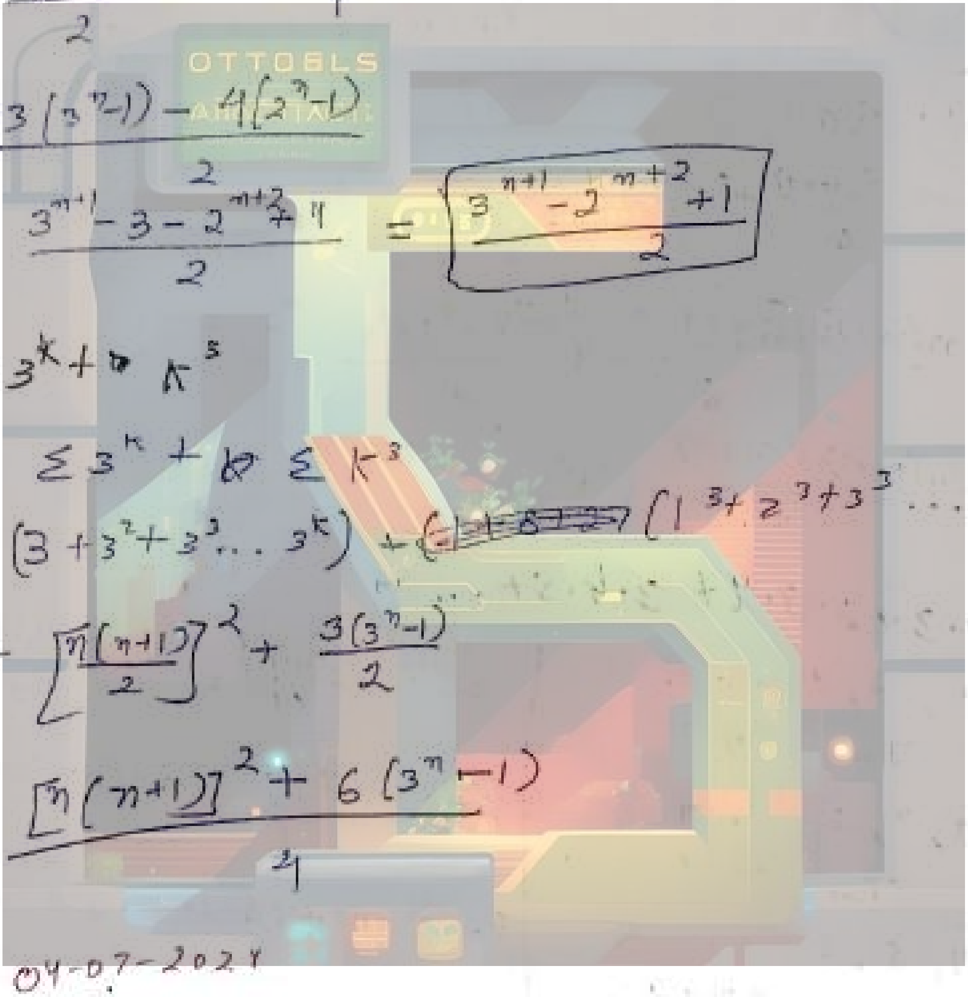
⑤ $T_n = 3^k + k^3$

$S_n = \Sigma 3^k + \Sigma k^3$

$= (3+3^2+3^3 \dots 3^k) + ~~(1+8+27)~~ (1^3+2^3+3^3 \dots n^3)$

$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3(3^n-1)}{2}$

$= \frac{[n(n+1)]^2 + 6(3^n-1)}{4}$



H. V. 04-07-2024

DYS-7 (Q2)

Q Find the Sum of following series.

① $1\times2+2\times3+3\times4+4\times5+\ldots n$ terms.

$1, 2, 3, 4 \ldots$

$T_n = n$

$2, 3, 4 \ldots$

$T_n = n+1$

$T_n = n(n+1)$

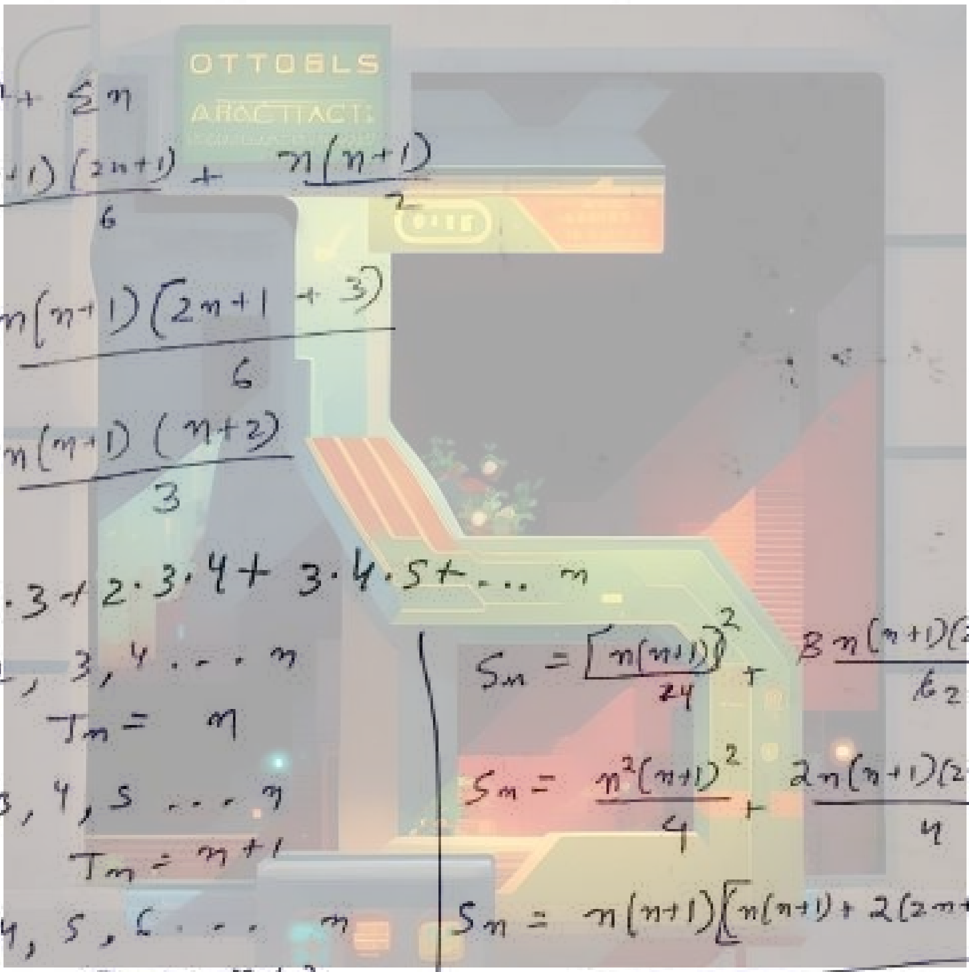
$S_n = \Sigma T_n$

$= \Sigma n^2 + \Sigma n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)(2n+1+3)}{6}$

$= \frac{n(n+1)(n+2)}{3}$



Q2. $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+\ldots n$

$1, 2, 3, 4 \ldots n$

$T_n = n$

$2, 3, 4, 5 \ldots n$

$T_n = n+1$

$3, 4, 5, 6 \ldots n$

$T_n = n+2$

$T_n = n(n+1)(n+2)$

$S_n = \Sigma(n^2+n)(n+2)$

$S_n = \Sigma(n^3+3n^2+2n)$

$S_n = \Sigma n^3 + 3\Sigma n^2 + 2\Sigma n$

$S_n = \left[\frac{n(n+1)}{2}\right]^2 + \cancel{3}\frac{n(n+1)(2n+1)}{\cancel{6}_2} + \cancel{2}\times\frac{n(n+1)}{\cancel{2}}$

$S_n = \frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{4} + \frac{4n(n+1)}{4}$

$S_n = \frac{n(n+1)\left[n(n+1)+2(2n+1)+4(n+1)\right]}{4}$

Q3. $1\cdot3\cdot5 + 3\cdot5\cdot7 + 5\cdot7\cdot9 + \dots n$

~~1, 3, 5~~

$1, 3, 5, \dots n$

$T_n = 2n-1$

$3, 5, 7 \dots n$

$T_n = 2n+1$

$5, 7, 9 \dots$

$T_n = 2n+3$

$T_n = n(2n-1)(2n+1)(2n+3)$

$= (4n^2-1)(2n+3)$

$= 8n^3 + 12n^2 - 2n - 3$

$S_n = 8\Sigma n^3 + 12\Sigma n^2 - 2\Sigma n - 3$

Q $1^2 + \frac{1^2+2^2}{2} + \frac{1^2+2^2+3^2}{3} + \frac{1^2+2^2+3^2+4^2}{4} + \dots n$ terms

~~$1^2, \frac{1^2}{2}, \frac{1^2}{3}, \frac{1^2}{4} \dots n$ (A.P)~~

~~$1, 2, 3, 4 \dots n$ (AP)~~

~~$T_n = n$ (AP)~~

~~$T_n = \frac{1}{n}$ (H.P)~~

$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ (H.P)

$T_n = \frac{1}{n}$

$1^2, 1^2+2^2, 1^2+2^2+3^2, \dots$

$T_n = \Sigma n^2$

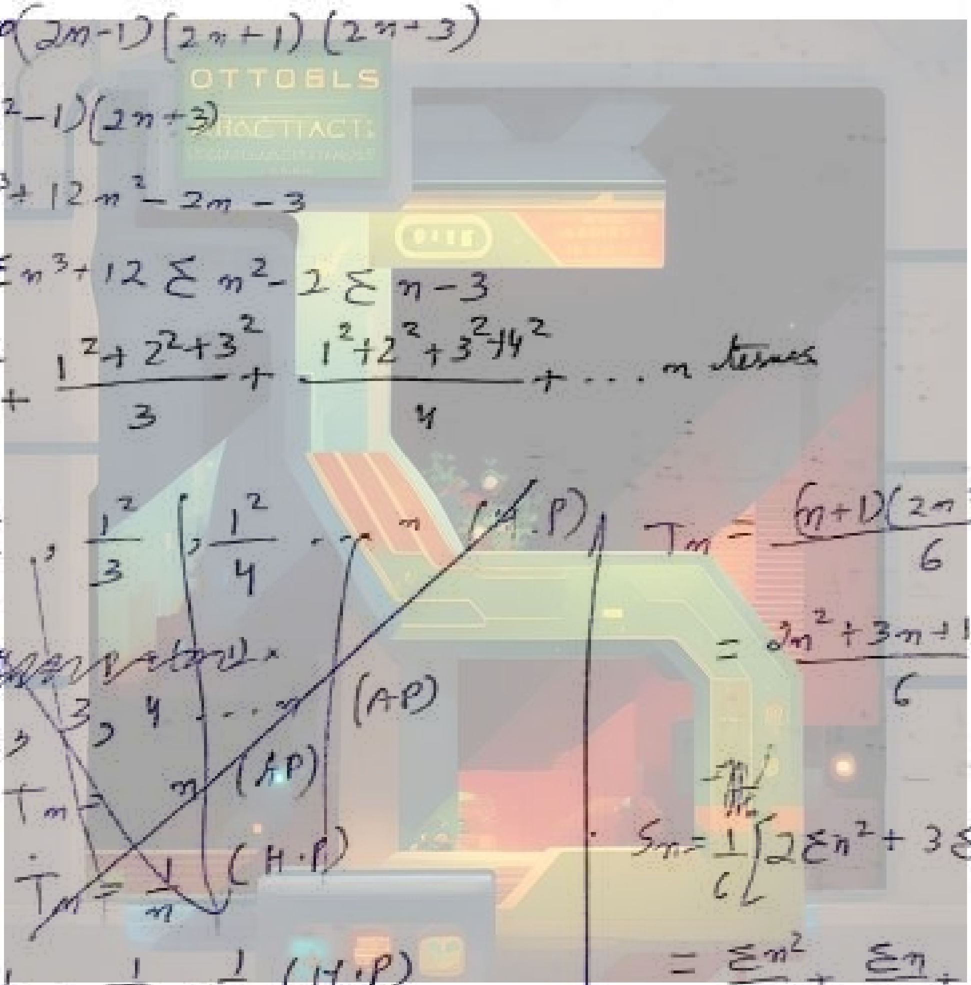
$T_n = \frac{n(n+1)(2n+1)}{6}$

$T_n = \frac{(n+1)(2n+1)}{6}$

$= \frac{2n^2+3n+1}{6}$

$S_n = \frac{1}{6}\left[2\Sigma n^2 + 3\Sigma n + n\right]$

$= \frac{\Sigma n^2}{3} + \frac{\Sigma n}{2} + \frac{n}{6}$



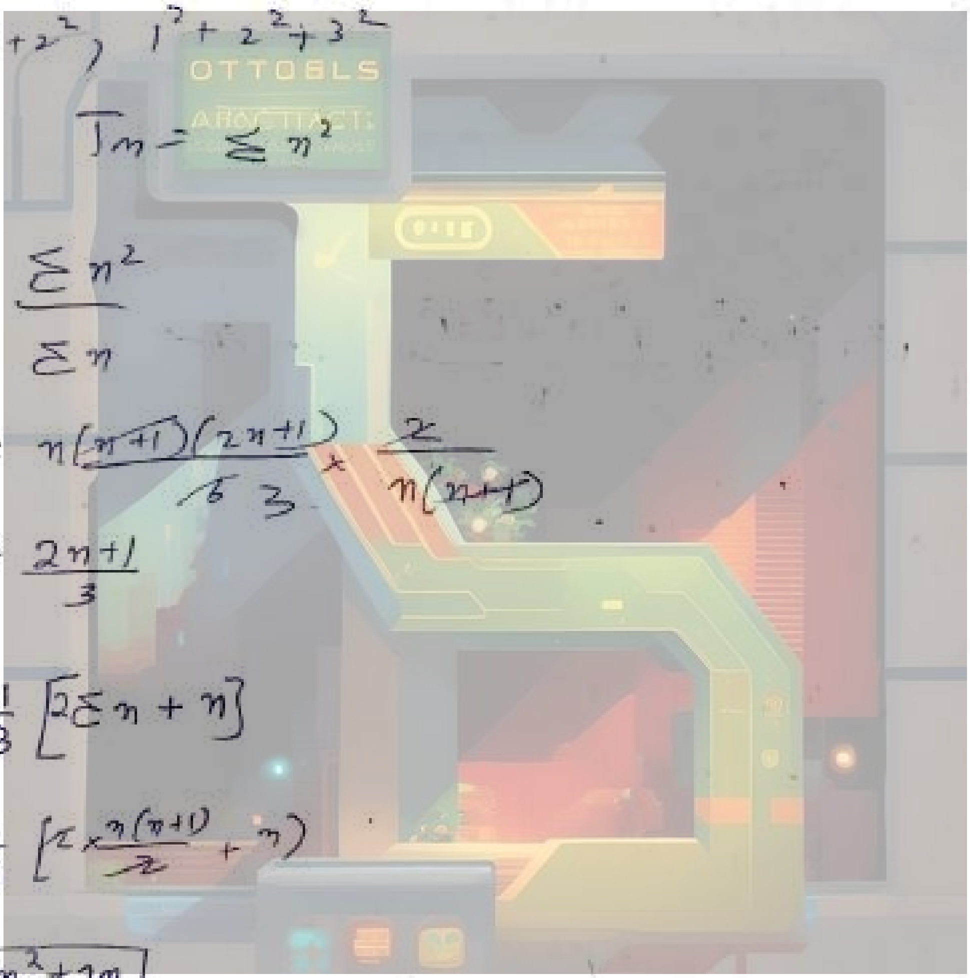
⑤ $1 + \frac{1^2+2^2}{1+2}, \frac{1^2+2^2+3^2}{1+2+3} + \frac{1^2+2^2+3^2+4^2}{1+2+3+4} \dots n$

$\frac{1}{1}, \frac{1}{1+2}, \frac{1}{1+2+3}$ (H.P)

$1, 1+2, 1+2+3$ (A.P)

$T_n = \Sigma n$

$T_n = \frac{1}{\Sigma n}$ (HP)



$1, 1^2+2^2, 1^2+2^2+3^2$

$T_n = \Sigma n^2$

$T_n = \frac{\Sigma n^2}{\Sigma n}$

$= \frac{n(\cancel{n+1})(2n+1)}{\cancel{6}\,3} \times \frac{\cancel{2}}{n(\cancel{n+1})}$

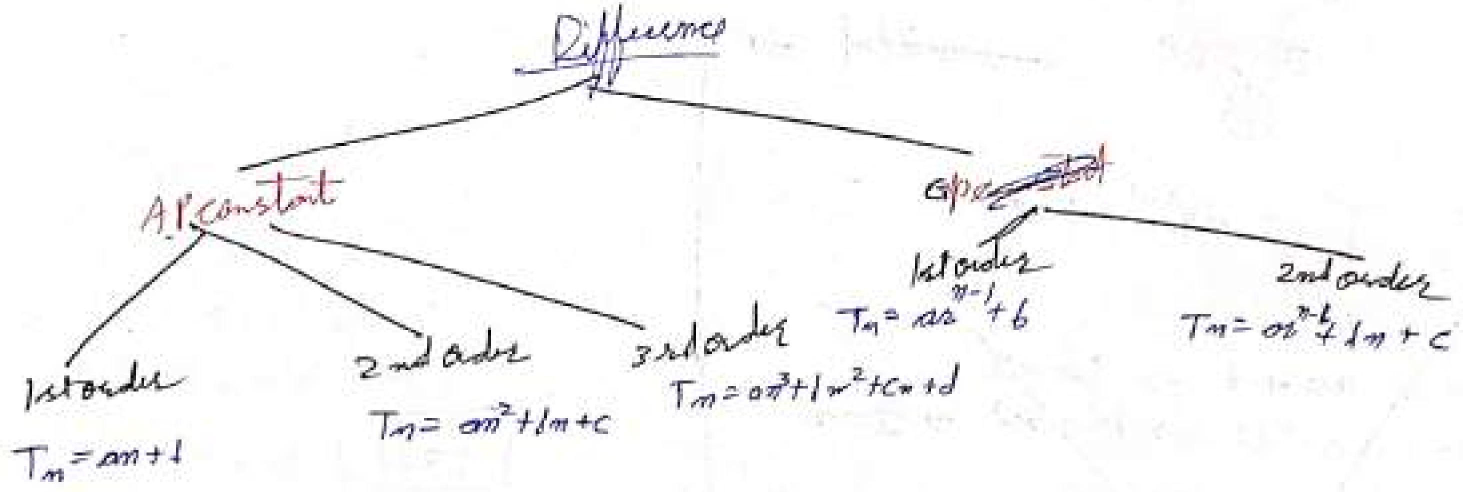
$= \frac{2n+1}{3}$

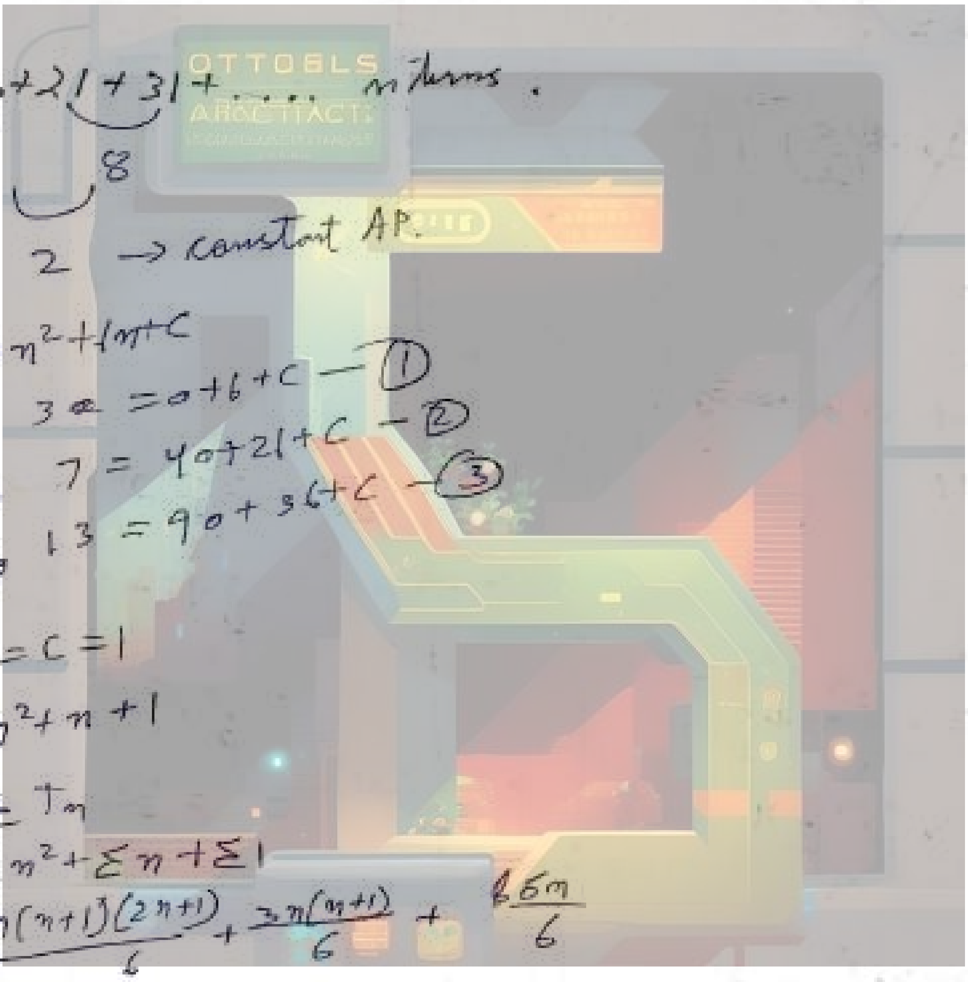
$S_n = \frac{1}{3}\left[2\Sigma n + n\right]$

$= \frac{1}{3}\left[\cancel{2} \times \frac{n(n+1)}{\cancel{2}} + n\right)$

$\boxed{= \frac{n^2+2n}{3}}$

Tyhe-3 :- Method of Difference





① $3+7+13+21+31+\ldots\ldots$ n terms.

$$\underbrace{\underbrace{4 \quad 6}_{2} \quad 8}_{2} \rightarrow \text{constant AP.}$$

$T_n = an^2+bn+c$

$n=1$; $3 = a+b+c$ — ①

$n=2$; $7 = 4a+2b+c$ — ②

$n=3$; $13 = 9a+3b+c$ — ③

$a = b = c = 1$

$T_n = n^2+n+1$

$$S_n = \Sigma T_n$$
$$= \Sigma n^2 + \Sigma n + \Sigma 1$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{6} + \frac{6n}{6}$$
$$= \frac{2n^3+3n^2+n+3n^2+3n+6n}{6}$$
$$= \frac{2n^3+6n^2+10n}{6}$$
$$\boxed{= \frac{n^3+3n^2+5n}{3}}$$

Q2. $\underbrace{1+4}_{3}\underbrace{+10}_{6}\underbrace{+22}_{12}+\ldots\ldots$ n terms.

$3 \quad 6 \quad 12 \rightarrow$ GP

~~3 6 3~~ ~~→ constant GP~~

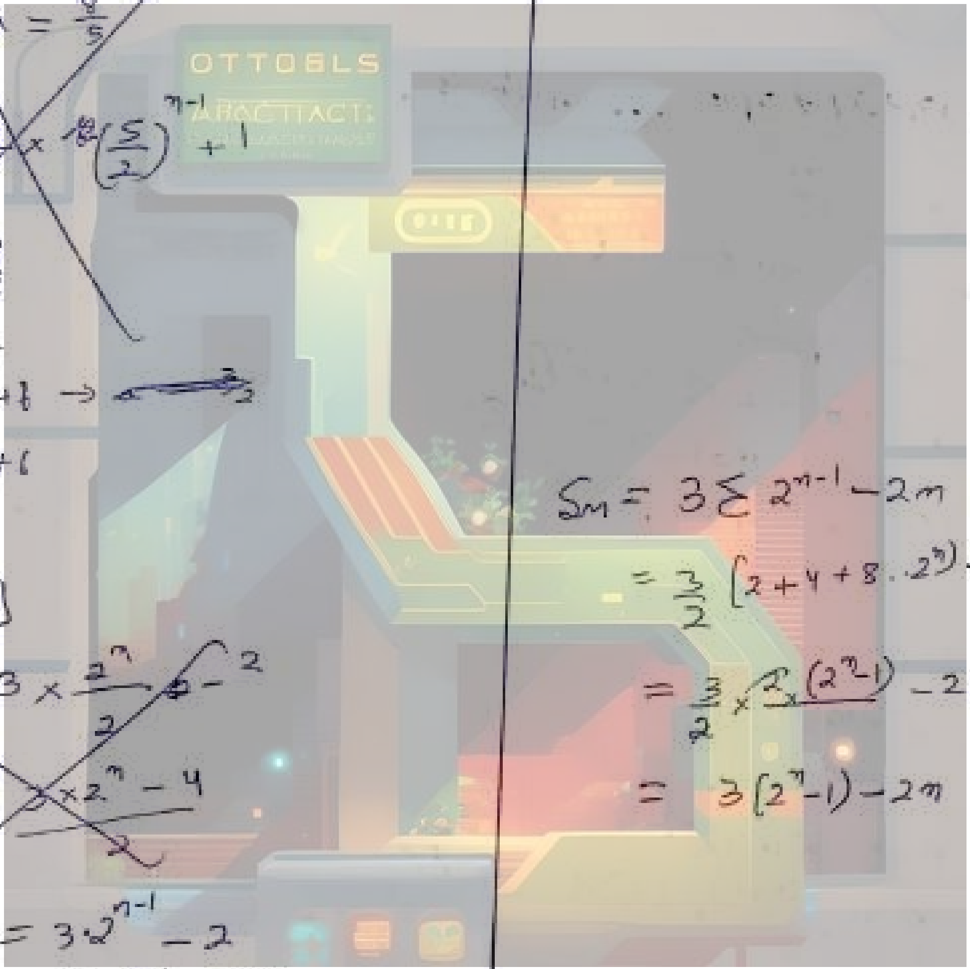
$T_n = ar^{n-1} + b$~~n~~

~~$1 = b$~~

~~$4 = ar + b \rightarrow 4 = ar$~~

~~$10 = ar^2 + b \rightarrow 10 = ar^2 \rightarrow \frac{5}{2} = r$~~

~~$a = \frac{8}{5}$~~



~~$T_n = \frac{8}{5} \times \left(\frac{5}{2}\right)^{n-1} + 1$~~

~~$T_n = \frac{8}{5}$~~

$1 = b + a$

$4 = a(2) + b \rightarrow$ ~~$a = \frac{3}{2}$~~

$4 = a2a + b$

$\boxed{3 = a}$

$\boxed{b = -2}$

~~$T_n = 3 \times \frac{2^n}{2} - 2$~~

~~$= \frac{3 \times 2^n - 4}{2}$~~

$T_n = 3 \cdot 2^{n-1} - 2$

~~$S_n = 3\Sigma 2^{n-1} - 2n$~~

~~$= 3(1+2+4 \ldots n) - 2n$~~

~~$= 3\frac{(2^n-1)}{1} - 2n$~~

~~$= 3 \times 2^n - 3 - 2n$~~

$S_n = 3\Sigma 2^{n-1} - 2n$

$= \frac{3}{2}(2+4+8 \ldots 2^n) - 2n$

$= \frac{3}{2} \times \cancel{2} \times (2^n-1) - 2n$

$= 3(2^n-1) - 2n$

Q $6 + 13 + 22 + 33 \ldots n$

$7 \quad 9 \quad 11$

$2 \quad 2$

$T_n = an^2 + bn + c$

$6 = a + b + c$

$13 = 4a + 2b + c$

$22 = 9a + 3b + c$

$9 = 5a + b$

$7 = 3a + b$

$2 = 2a$

$\boxed{a = 1}$

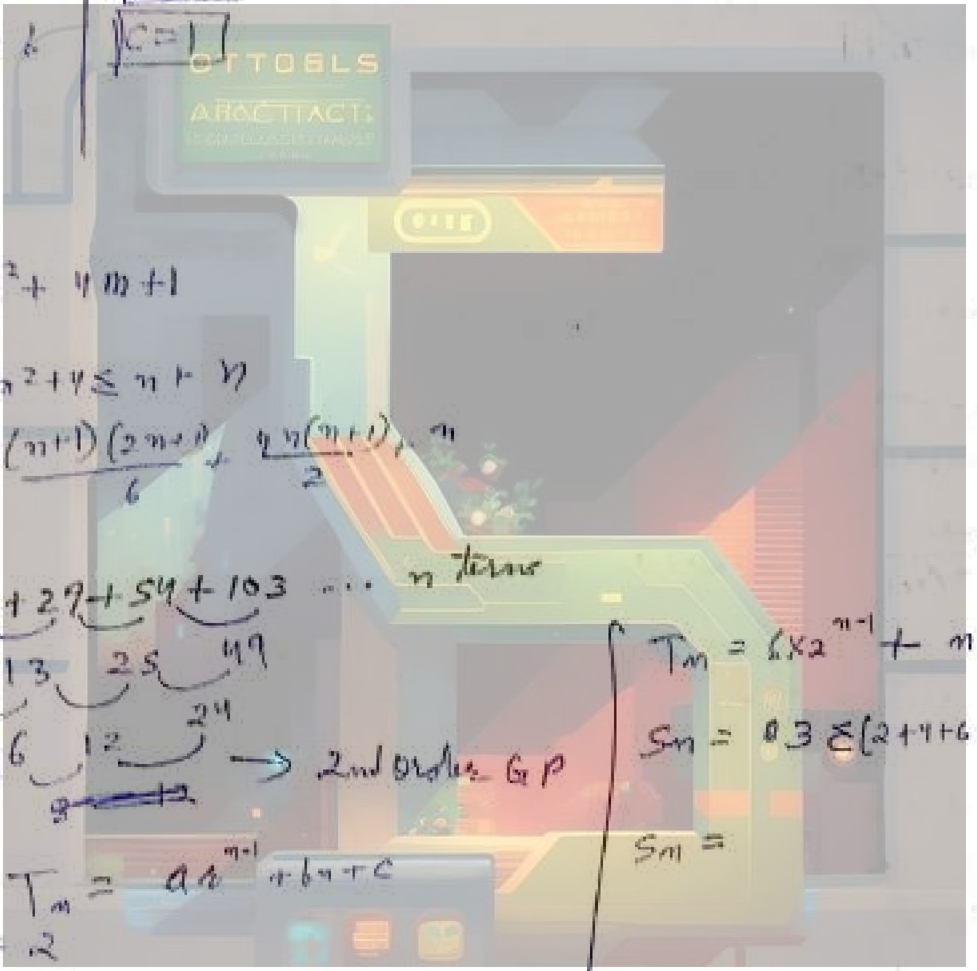
$\boxed{b = 4}$

$\boxed{c = 1}$

$T_n = n^2 + 4n + 1$

$S_n = \Sigma n^2 + 4\Sigma n + n$

$= \frac{n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n$



Q $9 + 16 + 29 + 54 + 103 \ldots$ n terms

$7 \quad 13 \quad 25 \quad 49$

$6 \quad 12 \quad 24 \rightarrow$ 2nd order GP

[illegible]

$T_n = ar^{n-1} + bn + c$

$r = 2$

$9 = a + b + c$

$16 = 2a + 2b + c$

$29 = 4a + 3b + c$

$13 = 2a + b$

$7 = a + b$

$\boxed{6 = a}$

$\boxed{b = 1}$

$\boxed{c = 2}$

$T_n = 6 \times 2^{n-1} + n + 2$

$S_n = 0\,3\,\Sigma(2 + 4 + 6 \cdots n) + \frac{n(n+1)}{2} + 2n$

$S_n =$

Type-3 ~~Splitting in denominator~~

Splitting the n^{th} term (denominator)

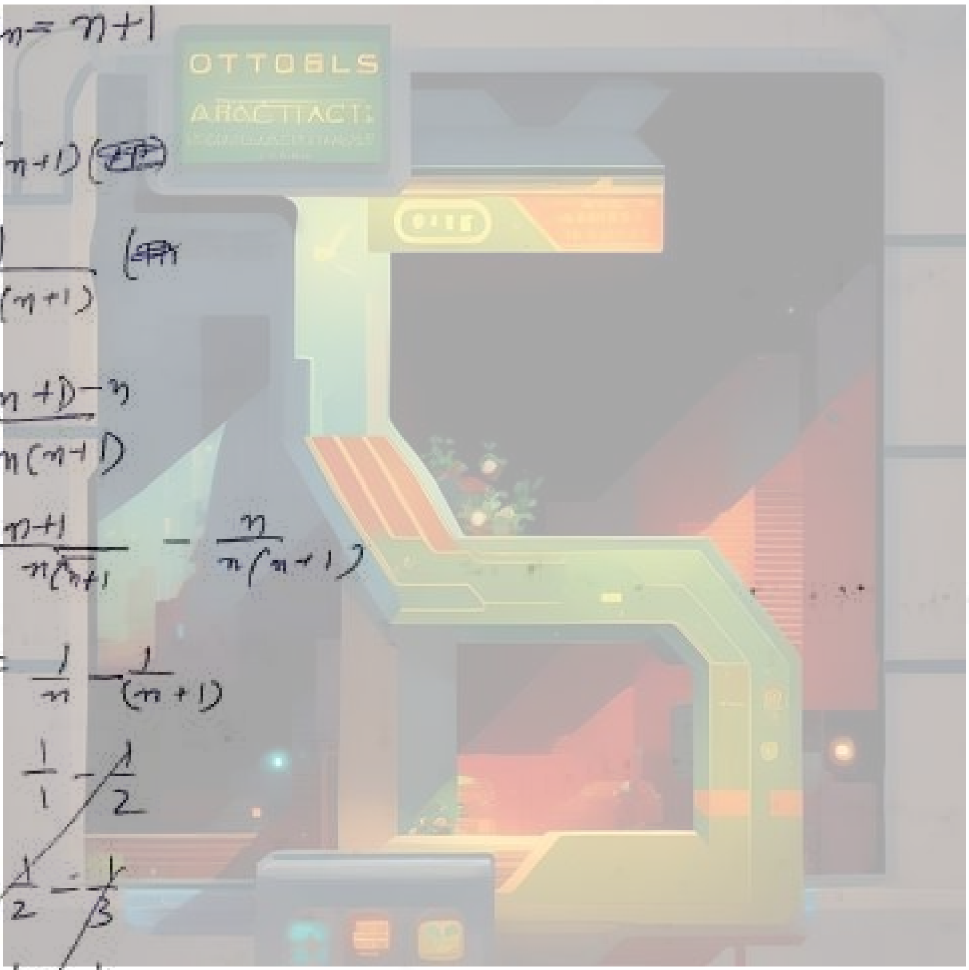
Q $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}\;\ldots\ldots n$

$1+2+3+4+5\ldots n$

$T_n = n$

$2+3+4+5+6$

$T_n = n+1$



$T_n = n(n+1)$ ([illegible])

$T_n = \frac{1}{n(n+1)}$ ([illegible]

$T_n = \frac{(n+1)-n}{n(n+1)}$

$= \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)}$

$= \frac{1}{n} - \frac{1}{(n+1)}$

$n=1 \quad \frac{1}{1} - \cancel{\frac{1}{2}}$

$n=2 \quad \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$

$n=3 \quad \cancel{\frac{1}{3}} - \frac{1}{4}$

$n=n \quad \cancel{\frac{1}{n}} - \frac{1}{n+1}$

$S_n = 1 - \frac{1}{n+1}$

$\boxed{= \frac{n}{n+1}}$

Q $\frac{1}{1\cdot4}+\frac{1}{\cancel{2\cdot3}\;4\cdot7}+\frac{1}{7\cdot10}+\cancel{\frac{1}{10\cdot}}\cdots n^{\text{terms}}$.

~~$T_n = n(n+\overset{3}{\cancel{4}})$~~

~~$= n(n+3)$~~

~~$T_n = \frac{1}{n(n+3)}$~~

~~$= \frac{(n+3)-n}{n(n+3}$~~

~~$= \frac{1}{n} - \frac{1}{n+3}$~~

~~$S_n = \frac{1}{1} - \frac{1}{n+3}$~~

~~$= \frac{n+3-1}{n+3}$~~

~~$\boxed{= \frac{n+2}{n+3}}$~~

$T_n = (3n-2)(3n+1)$

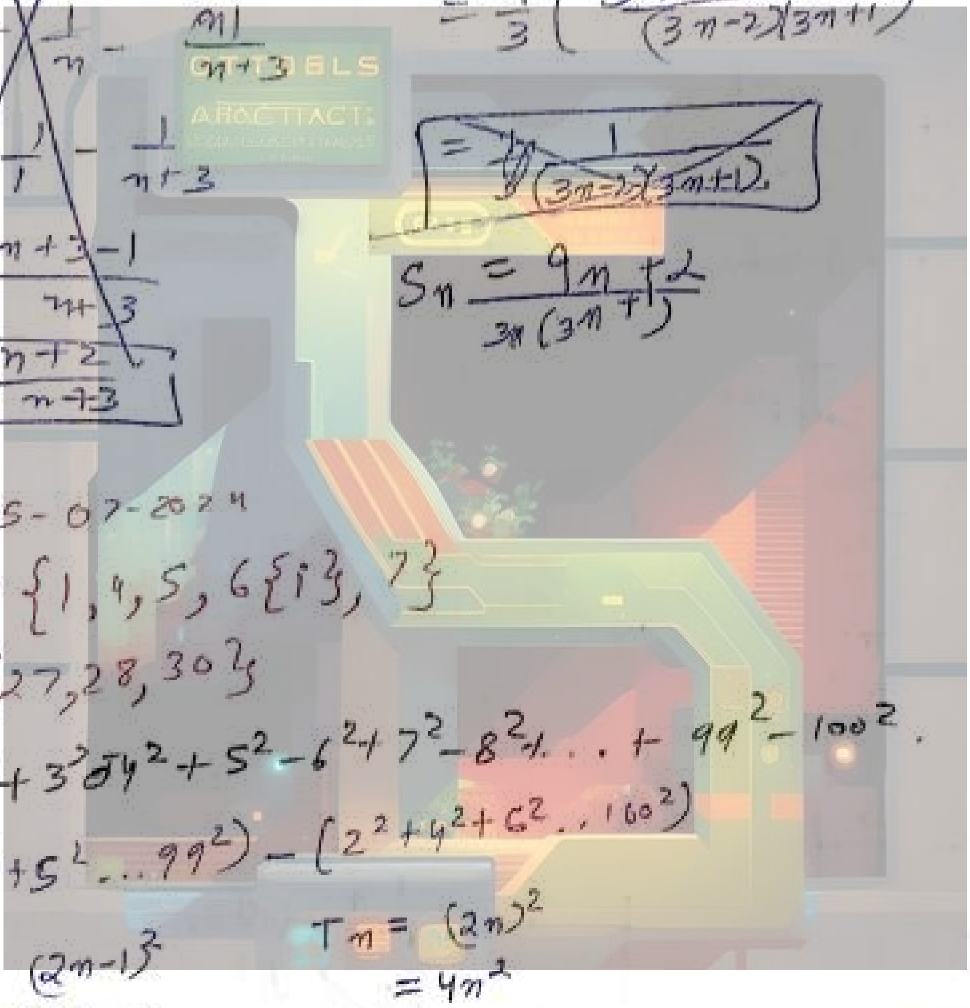
$= \frac{1}{3} \times \frac{3}{(3n-2)(3n+1)}$

$= \frac{1}{3}\left[\frac{(3n+1)\cancel{()}-3n-2)}{(3n-2)(3n+1)}\right.$

$= \frac{1}{3}\left(\frac{3n+1-3n+2}{(3n-2)(3n+1)}\right)$

~~$\boxed{= \cancel{\tfrac{1}{3}}\frac{1}{(3n-2)(3n+1)}}$~~

$S_n = \frac{9n+2}{3n(3n+1)}$



H.W. 05-07-2024

DXS 7 {1, 4, 5, 6{i}, 7}

01 {27, 28, 30}

Q $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 + \dots + 99^2 - 100^2$.

$(1^2+3^2+5^2 \dots 99^2) - (2^2+4^2+6^2 \dots 100^2)$

$T_n = (2n-1)^2$

$= 4n^2 - 4n + 1$

$S_n = 4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n$

$S_n = \frac{200(51)(101)}{6} - \frac{\overset{100}{\cancel{200}}(51)}{2} + 50$,

$= 100 \times 17 \times 101 - 5100 + 50$

$S_n = 171700 - 5100 + 50$

$\boxed{S_n = -5050}$

$T_n = (2n)^2$

$= 4n^2$

$S_n = \frac{4n(n+1)(2n+1)}{6}$

$S_n = \frac{200(51)(101)}{6}$

$S_n = 100 \times 101 \times 17$

$S_n = 171700$

Q Sum of ∞ terms ..

$\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\frac{1}{7\cdot10}$

$1+4+7 \ldots\ldots \infty$

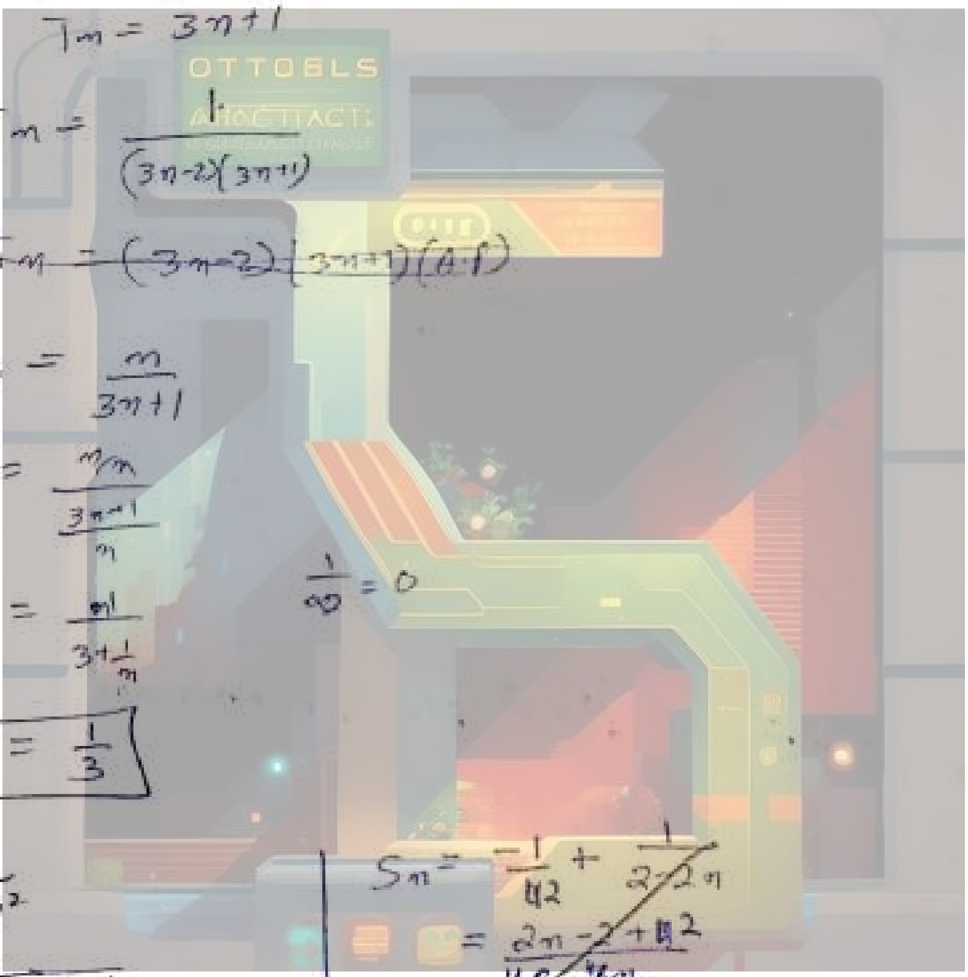
$T_n = \cancel{n+3}\ 1+(n-1)3$

$= 1+3n-3$

$= 3n-2$

$4+7+10$

$T_n = 3n+1$



$T_n = \frac{1}{(3n-2)(3n+1)}$

~~$T_n = (3n-2)(3n+1)(A\cdot1)$~~

$S_n = \frac{n}{3n+1}$

$S_n = \frac{n/n}{\frac{3n+1}{n}}$

$= \frac{1}{3+\frac{1}{n}}$ $\quad \frac{1}{\infty}=0$

$\boxed{= \frac{1}{3}}$

~~Q $T_n = \frac{n}{1-n^2}$~~

~~$= \frac{n}{(1+n)(1-n)}$~~

~~$= \frac{(1-n)-(1+n)}{-2(1+n)(1-n)}$~~

~~$= \frac{-1}{2(1+n)} + \frac{1}{2(1-n)}$~~

~~$= \frac{1}{2(1-n)} - \frac{1}{2(1+n)}$~~

~~start, $n=2$~~

~~$S_n = \frac{-1}{12} + \frac{1}{2-2n}$~~

~~$= \frac{2n-2+12}{48-48n}$~~

~~$\boxed{\cancel{= \frac{n+1}{8(1-n)}}}$~~

~~$\boxed{= \frac{n}{2(1-n)}}$~~

Q Find sum of the sequence upto infinite terms:-

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4}$$

~~$(1+1^2+1^4)$, $(1+2^2+2^4)$, $(1+3^2+3^4)$~~

~~$T_n = \frac{n(n^3-1)}{n-1}$~~

~~1, 2, 3~~

~~$T_n = n$~~

~~$T_n = n \times \frac{(n-1)}{n(n^3-1)}$~~

~~$= \frac{n-1}{n^3-1}$~~

~~$= \frac{n-1}{(n-1)(n^2+1+2n)}$~~

~~$T_n = \frac{1}{n^2+2n+1}$~~

$T_n = \frac{n}{1+n^2+n^4}$

~~$T_n = \frac{n}{(n^4+n^2+1)}$~~

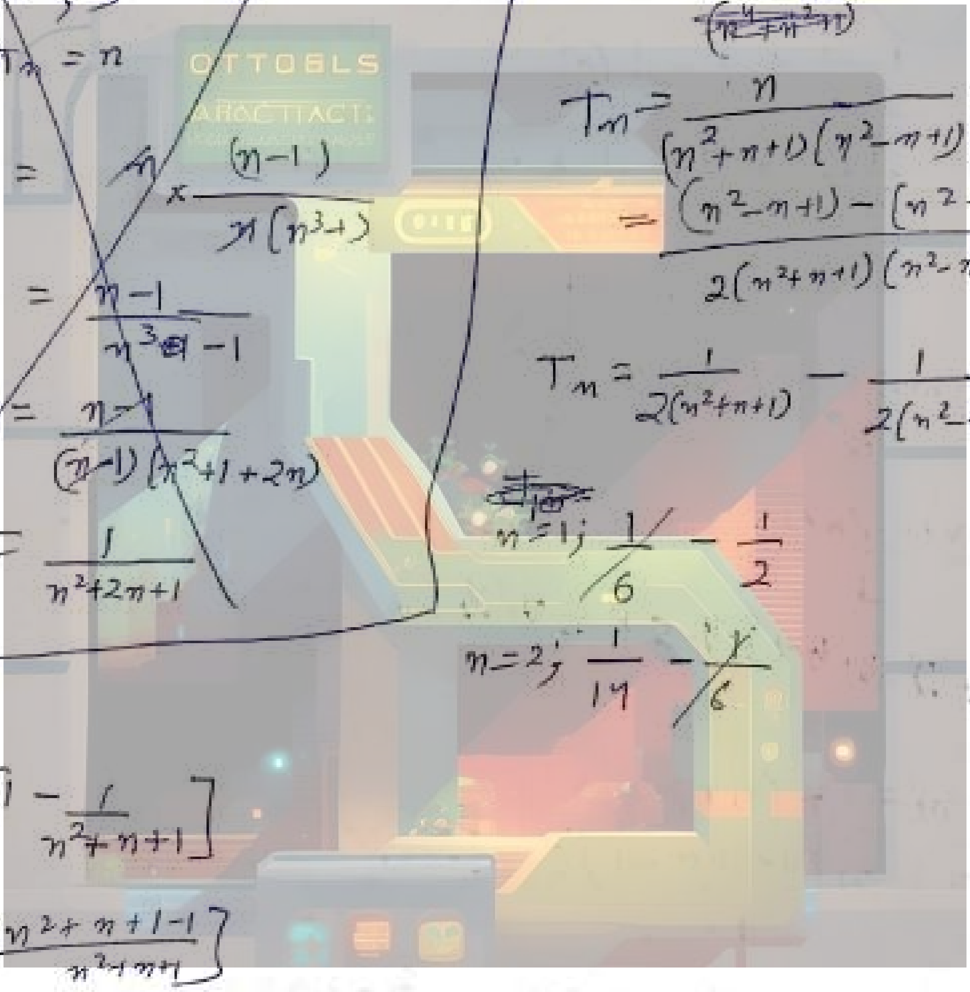
$T_n = \frac{n}{(n^2+n+1)(n^2-n+1)}$

$= \frac{(n^2-n+1)-(n^2+n+1)}{2(n^2+n+1)(n^2-n+1)}$

$T_n = \frac{1}{2(n^2+n+1)} - \frac{1}{2(n^2-n+1)}$

$n=1;\ \cancel{\frac{1}{6}} - \frac{1}{2}$

$n=2;\ \frac{1}{14} - \cancel{\frac{1}{6}}$



$S_n = \frac{1}{2}\left[1 - \frac{1}{n^2+n+1}\right]$

$S_n = \frac{1}{2}\left[\frac{n^2+n+1-1}{n^2+n+1}\right]$

$= \frac{1}{2}\left[\frac{n^2+n}{n^2+n+1}\right]$

$= \frac{1}{\frac{2}{n^2+n} + \frac{1}{n^2+n}}$

$\boxed{= \frac{1}{2}}$

Type-Y Using Difference in numerator

① $1\cdot2+2\cdot3+3\cdot4+4\cdot5+\ldots n$ terms

$$T_n = n(n+1)$$

$$T_n = \frac{1}{3}n(n+1)\left[(n+2)-(n-1)\right]$$

$$= \frac{n(n+1)(n+2)}{3} - \frac{(n-1)(n)(n+2)}{3}$$

$n=1;$ $\cancel{\frac{1}{3}\cdot1\cdot2\cdot3} - \frac{1}{3}(0)$

$n=2$ $\cancel{\frac{1}{3}\cdot2\cdot3\cdot4} - \cancel{\frac{1}{3}1\cdot2\cdot3}$

$n=n$ $\frac{n(n+1)(n+2)}{3} - 0$

$$S_n = \frac{n(n+1)(n+2)}{3}$$

① $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+\ldots n$ terms.

$$T_n = n(n+1)(n+2)$$

$$T_n = \frac{n(n+1)(n+2)}{4}\left[(n+3)-(n-1)\right]$$

$$T_n = \frac{n(n+1)(n+2)(n+3)}{4} - \frac{(n-1)(n)(n+1)(n+2)}{4}$$

$n=1;$ $\cancel{\frac{1\cdot2\cdot3\cdot4}{4}} - 0$

$n=2;$ $\frac{2\cdot3\cdot4\cdot5}{4} - \cancel{\frac{1\cdot2\cdot3\cdot4}{4}}$

$$S_n = \frac{n(n+1)(n+2)(n+3)}{4}$$

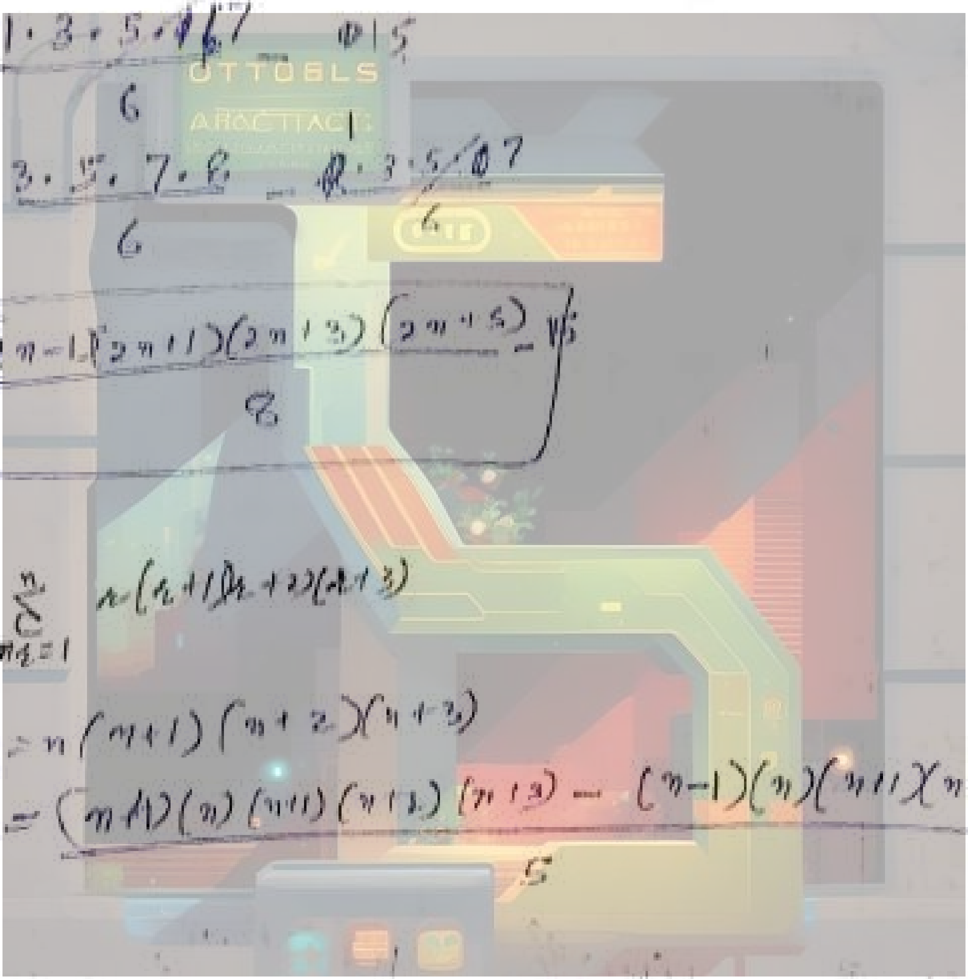
① $1\cdot3\cdot5 + 3\cdot5\cdot7 + 5\cdot7\cdot9 + \ldots n$

~~$T_n = n(2n+2)(n+1-1)$~~

$T_n = (2n-1)(2n+1)(2n+3)$

$$T_n = \frac{(2n-1)(2n+1)(2n+3)\left[\overset{5}{\cancel{2n+4}} - 2n+\cancel{6}\right]}{\overset{8}{\cancel{6}}}$$

$$T_n = \frac{(2n-1)(2n+1)(2n+3)\overset{5}{(2n+\cancel{4})}}{\overset{8}{\cancel{6}}} - \frac{(2n\overset{3}{-\cancel{0}})(2n-1)(2n+1)(2n+3)}{\cancel{6}\cancel{4}8}$$



$n=1;\quad \frac{1\cdot3\cdot5\cdot\cancel{4}\overset{7}{\cancel{6}}}{6} \quad \cancel{0}15$

$n=2;\quad \frac{3\cdot5\cdot7\cdot8}{6} - \frac{\cancel{1}\cdot3\cdot5\cdot\cancel{0}7}{6}$

$$\boxed{S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} - \frac{15}{8}}$$

Q DYS 7
Q6 ii) $\sum_{n=1}^{n} n(n+1)(n+2)(n+3)$

$T_n = n(n+1)(n+2)(n+3)$

$$T_n = \frac{(n+1)(n)(n+1)(n+2)(n+3) - (n-1)(n)(n+1)(n+2)(n+3)}{5}$$

$n=1;$

$$S_n = \boxed{\frac{n(n+1)(n+2)(n+3)(n+4)}{5}}$$

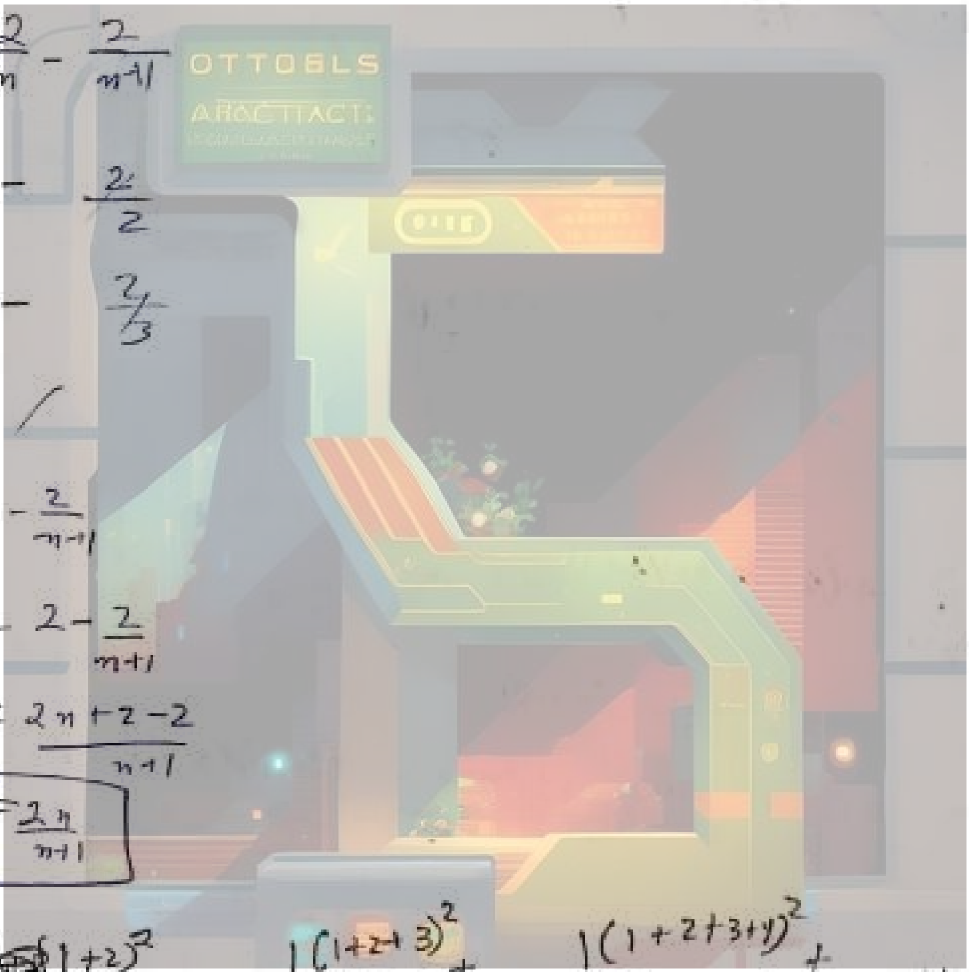
Q $S_p = 1+2+3+\dots p$ terms

$$\sum_{p=1}^{n} = \frac{1}{\cancel{8}S_p}$$

$$T_n = \frac{1}{\Sigma n}$$

$$= \frac{2}{n(n+1)}$$

$$t_n = \frac{2[(n+1)-n]}{n(n+1)}$$



$$T_n = \frac{2}{n} - \frac{2}{n+1}$$

$n=1;\quad 2 - \frac{2}{2}$

$n=2;\quad \frac{\cancel{2}}{\cancel{2}} - \frac{2}{3}$

/

$n=n;\quad \frac{2}{n} - \frac{2}{n+1}$

$$S_n = 2 - \frac{2}{n+1}$$

$$= \frac{2n+2-2}{n+1}$$

$$\boxed{S_n = \frac{2n}{n+1}}$$

Q $1 + \frac{(1+2)^2}{1+3} + \frac{1(1+2+3)^2}{1+3+5} + \frac{1(1+2+3+4)^2}{1+3+5+7} + \dots\dots n$ terms.

O-1

Q 16-30 - {27, 29, 30}

O-2

Q 1-15 (16-20)

O-3

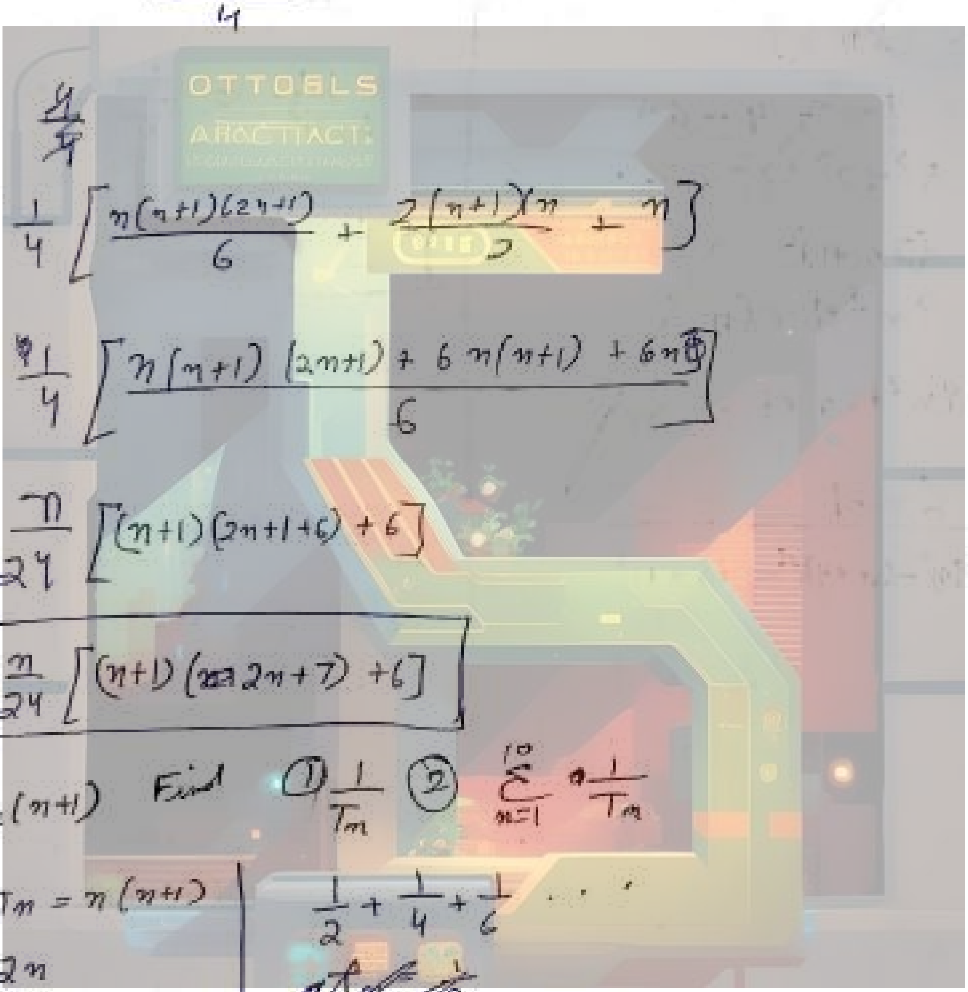
Q1-10

$$T_n = \frac{(\Sigma n)^2}{\cancel{n^2}\ \Sigma(2n-1)}$$

$$T_n = \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2}$$

$$= \frac{n^{\cancel{4}}(n^2+1+2n)}{4\cancel{n^2}}$$

$$T_n = \frac{n^2+2n+1}{4}$$



$$\cancel{\quad}$$

$$\frac{\cancel{4}}{\cancel{4}}$$

$$S_n = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2(n+1)(n}{2} + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1) + 6n(n+1) + 6n}{6}\right]$$

$$= \frac{n}{24}\left[(n+1)(2n+1+6) + 6\right]$$

$$\boxed{S_n = \frac{n}{24}\left[(n+1)(\cancel{\ }2n+7) + 6\right]}$$

Q1. $S_n = n(n+1)$ Find ① $\frac{1}{T_n}$ ② $\sum_{n=1}^{10} \frac{1}{T_n}$

$$S_n = \Sigma T_n = n(n+1)$$

$$T_n = 2n$$

$$\boxed{\frac{1}{T_n} = \frac{1}{2n}}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} \dots$$

$$\cancel{T_n = \frac{1}{2n}}$$

$$\cancel{S_n = \frac{1}{n(n+1)}}$$

$$\cancel{S_n = \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}}$$

$$\cancel{= 1 - \frac{1}{1024}}$$

$$\cancel{\boxed{= \frac{1023}{1024}}}$$

Q2. $\frac{5}{1^2\cdot4^2}+\frac{11}{4^2\cdot7^2}+\frac{17}{7^2\cdot10^2}\ \dots\ \infty$

$T_n = 5+(n-1)6\ \dots$

$= 5+6n-6$

$\boxed{=6n-1}$

$T_n = 1^2+4^2+7^2+\dots$

$=(1+(n-1)3)^2$

$=(1+3n-3)^2$

$=(3n-2)^2$

$=9n^2+4-6n$

$T_n = 4^2+7^2+10^2\dots$

$=(3n+1)^2$

$=9n^2+1+6n$

$=9n^2+6n+1$

$$T_n = \frac{6n-1}{(9n^2-6n+4)(9n^2+6n+1)}$$

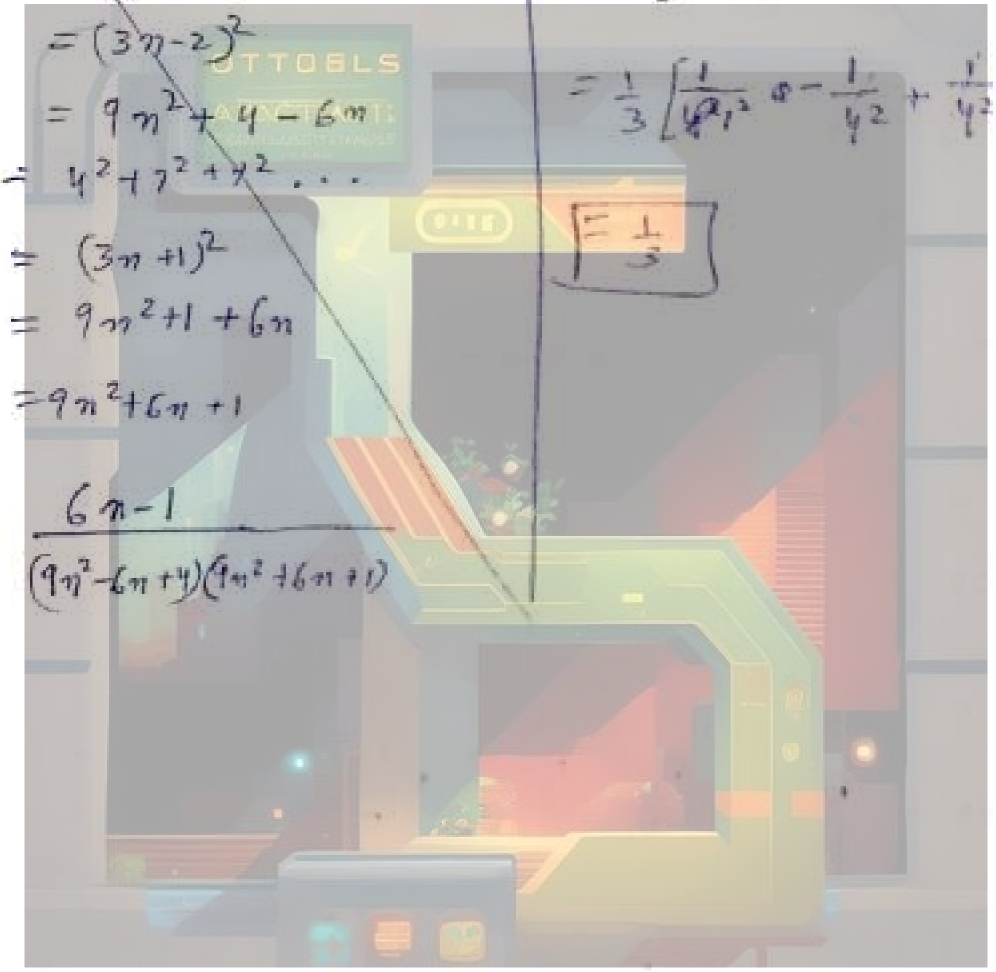
$$S = \frac{(4+1)}{4^2\cdot1^2}+\frac{(7+4)}{7^2\cdot4^2}+\frac{(10+7)}{10^2\cdot7^2}$$

$$=\frac{1}{3}\left[\frac{(4+1)(4-1)}{4^2\cdot1^2}+\frac{(7+4)(7-4)}{7^2\cdot4^2}+\frac{(10+7)(10-7)}{10^2\cdot7^2}\right]$$

$$=\frac{1}{3}\left[\frac{4^2-1^2}{4^2\cdot1^2}+\frac{7^2-4^2}{7^2\cdot4^2}+\frac{10^2-7^2}{10^2\cdot7^2}\right]$$

$$=\frac{1}{3}\left[\frac{1}{1^2}-\frac{1}{4^2}+\frac{1}{4^2}-\frac{1}{7^2}+\frac{1}{7^2}-\frac{1}{10^2}\right]$$

$\boxed{=\frac{1}{3}}$



Q $(1^2 - a_1) + (2^2 \mp a_2) + (3^2 - a_3) + \ldots + (n^2 - a_n) = \frac{1}{3} n (n^2 - 1)$

$a_7 = ?$

$T_n = S_n - S_{(n-1)}$

$$\underbrace{0 + 2}_{2} + \underbrace{6 + 12}_{\ldots}$$

$0 + 2 + 6 + 12$

$2 \quad 4 \quad 6$

$2 \quad 2$

$T_n = an^2 + bn + c$

$0 = a + b + c$

$2 = 4a + 2b + c$

$6 = 9a + 3b + c$

$4 = 5a + b$

$2 = 3a + b$

$2 = 2a$

$\boxed{a = 1}$

$\boxed{b = -1}$

$\boxed{c = 0}$

$\boxed{T_n = n^2 - n} \rightarrow$ ~~$a_1\ n^2 - a_n$~~

$1 - a_1 = 0$

$\boxed{a_1 = 1}$

$4 - a_2 = 2$

$\boxed{a_2 = 2}$

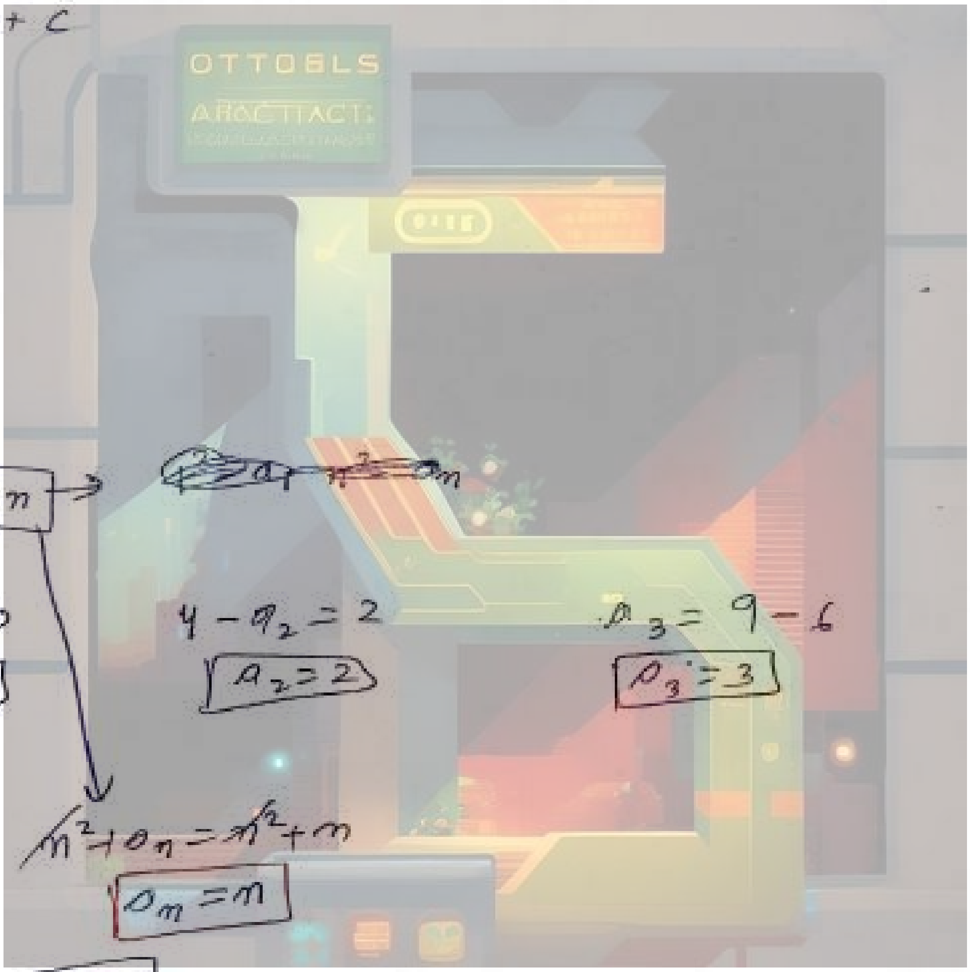
$a_3 = 9 - 6$

$\boxed{a_3 = 3}$

$\cancel{n^2} + a_n = \cancel{n^2} + n$

$\boxed{a_n = n}$

$\boxed{a_7 = 7}$




OTTOBLS
AROCTTACT: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ARACTIACT:


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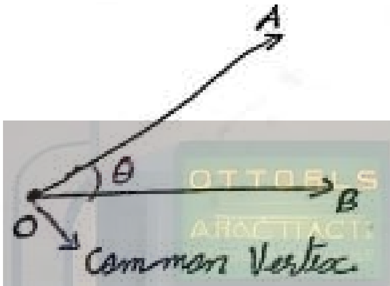
!! TRigonometry !!

तीन side measurement

Angle- A figure formed by rays with common vertex is called an Angle.

Denoted by $\angle AOB$



$\angle AOB = \theta$

OA & OB are rays

→ There are 3 imp system for the measurement of an angle.

① Sexagesimal System (British)
② Radian System (Circular)
③ Grade System (French)

① Sexagesimal (Degree °)

→ A right Angle is divided into 90 equal parts.
→ Each degree is divided into 60 equal parts called minutes.
→ Each minute is divided into 60 equal parts called seconds.

1 right Angle ⇒ 90°

$1' = \left(\frac{1}{60}\right)^\circ$ $\quad 1^\circ = 60'$

$1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ$ $\quad 1^\circ = 3600''$

Q Convert the following in degrees.

① $8^\circ 30'$

$8^\circ + \frac{1}{2}^\circ = (8.5)^\circ$

② $12^\circ 23' 35''$

$12^\circ + \frac{23}{60}^\circ + \frac{35}{3600}^\circ$

$12^\circ + 0.38^\circ + 0.009^\circ$

12.389°

Q Convert $\frac{1}{2}^\circ$ in minutes

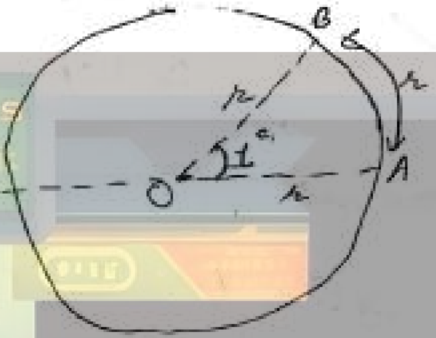
$1^\circ = 60$ min

$\frac{1}{2} \times 60 = \frac{60}{2} = \frac{30}{1} = 30'$

② Radian System

→ Angle subtended at the center of a circle by an arc whose length is equal to the radius of the circle

$\theta = \frac{l}{r} = \frac{r}{r} = 1^c$



③ Grade System (G)

→ A right Angle is Divided into 100 equal parts called a Grade

→ Each Grade is divided to 100 minutes

→ Each minute is divided into 100 seconds.

1 right angle $= 100^g$

$1^g = 100'$

$1' = 100''$

Relation in Degree, Grade & Radian

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Q convert $\frac{\pi}{6}$ radian to degree

$R = \frac{\pi}{6}$

$\frac{D}{90} = \frac{2}{\pi} \times \frac{\pi}{6}$

$D = \frac{90}{3}$

$D = 30^\circ$

Q Convert following in radian.

① 0°

$$\frac{0}{90} = \frac{2R}{\pi}$$

$$\boxed{R = 0^c}$$

② 30°

$$\frac{\cancel{30} \times \pi}{90 \times 2} = \boxed{\frac{\pi}{6}^c}$$

③ 180°

$$\frac{\overset{2}{\cancel{180}}}{90} \times \frac{\pi}{\cancel{2}} = \boxed{\pi^c}$$

④ 150°

$$\frac{\overset{5}{\cancel{150}} \times \pi}{\underset{3}{\cancel{90}} \times 2} = \boxed{\frac{5\pi}{6}^c}$$

⑤ -56°

$$\frac{-\overset{14}{\cancel{\overset{28}{\cancel{56}}}} \times \pi}{2 \times \underset{45}{\cancel{20}}} = \boxed{\frac{-14\pi}{45}^c}$$

radian → Deg

⑥ $\frac{9\pi}{5}$

$$\cancel{\frac{90 \times 9\pi}{5} = \frac{810}{5}\pi}$$

$$\frac{\overset{18}{\cancel{90}} \times 2}{\cancel{\pi}} \times \frac{9\cancel{\pi}}{\cancel{5}} = \boxed{324^\circ}$$

⑦

$$\frac{-5\cancel{\pi}}{\underset{3}{\cancel{6}}} \times \frac{\overset{30}{\cancel{90}} \times 2}{\cancel{\pi}} = \boxed{-150^\circ}$$

⑧

$$-3 \times \frac{2 \times 90}{\pi} \boxed{= \frac{-540^\circ}{\pi}}$$

$$= \frac{-540}{22} \times 7$$

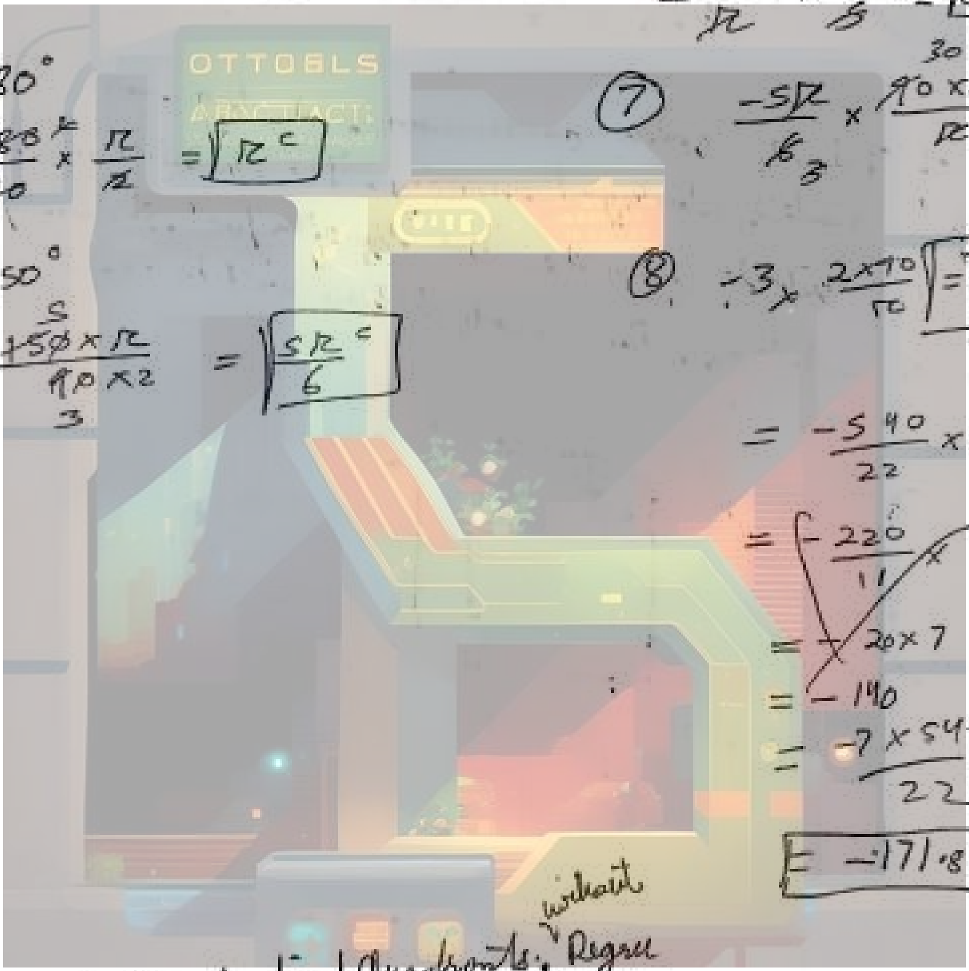
$$\cancel{= \frac{-220}{11} \times 7}$$

$$\cancel{= -20 \times 7}$$

$$\cancel{= -140}$$

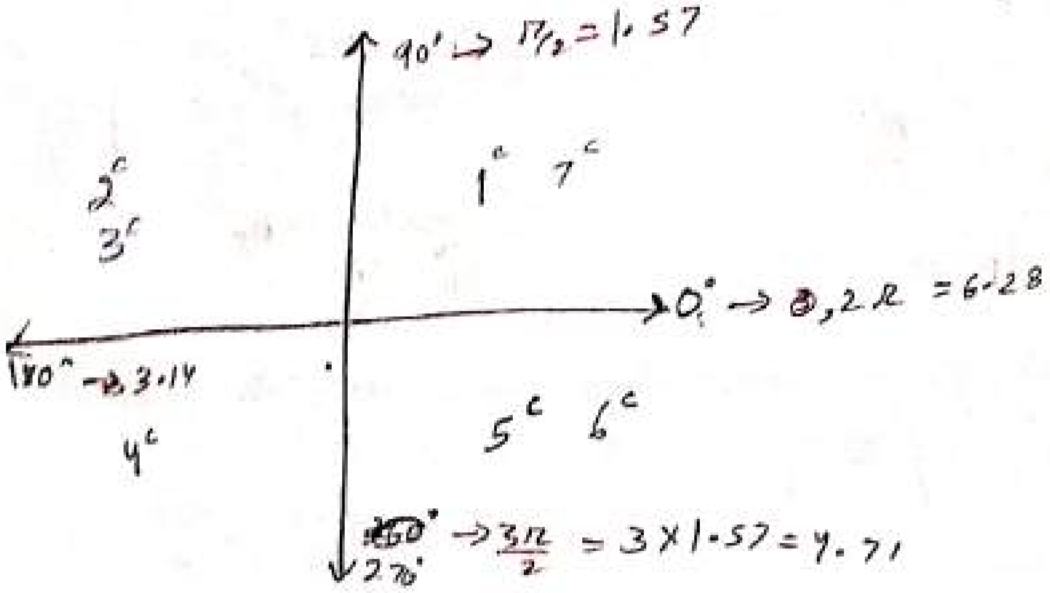
$$= \frac{-7 \times 540}{22}$$

$$\boxed{= -171.8^\circ}$$

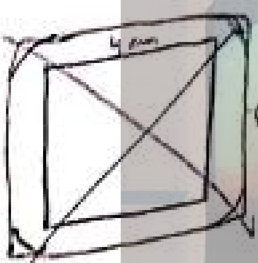


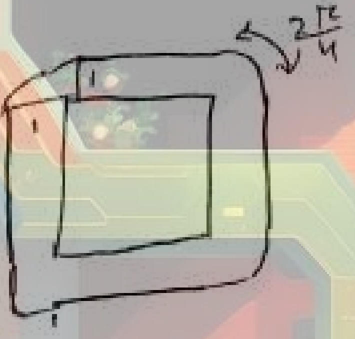
Q Mark Angles & find Quadrants; without Degree

$1^c, 2^c, 3^c, 4^c, 5^c, 6^c, 7^c$



Q A student of TN, M₂ walks on a road, A dog runs behind him then the student started running. The student runs at a distance of 1 cm outside from the boundary of a park. Assume park is square of side 4 cm. How much distance will he cover in 1 round.





$(4 \times 4) + \frac{2\pi}{4} \times 4$

$2\pi + 16$

$16 + 6.28$

$\boxed{22.28}$

H.W. . 11-7-24

Ch-5 ~~[illegible]~~ J-M (Q1-10)

H.W.

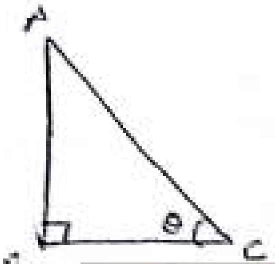
Ch-5 - O-4

Ch-6 DYS-1

DYS-2

Note - In trignometry, we study some ratios of sides of right angle triangle wrt to its acute angle

→ we restrict our discussion to acute angles only, however These ratios can be extended to other angles also.



(sine)
$\sin\theta = \frac{AB}{AC}$

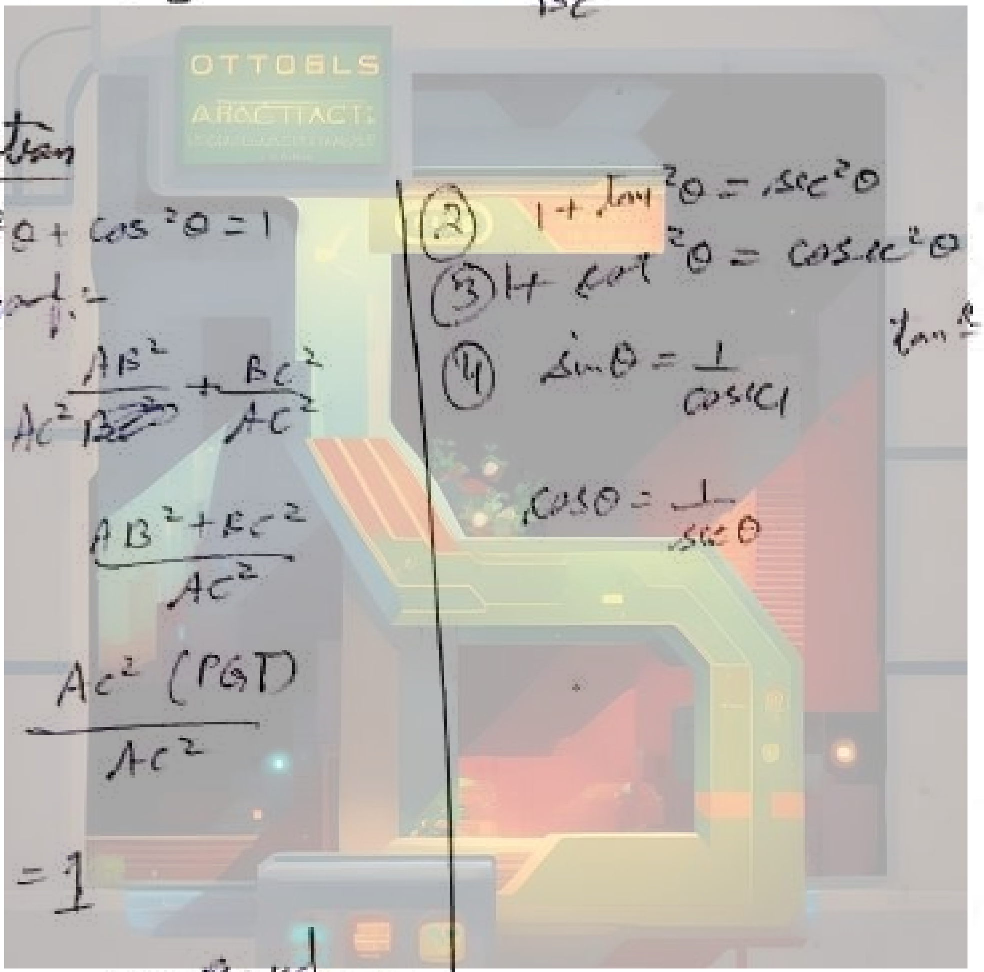
(cosine)
$\cos\theta = \frac{BC}{AC}$

(tangent)
$\tan\theta = \frac{AB}{BC}$

(co-secant)
$\operatorname{cosec}\theta = \frac{AC}{AB}$

(secant)
$\sec\theta = \frac{AC}{BC}$

(cotangent)
$\cot\theta = \frac{BC}{AB}$



Observation

① $\sin^2\theta + \cos^2\theta = 1$

Proof:-

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$\frac{AB^2 + BC^2}{AC^2}$$

$$\frac{AC^2 \text{ (PGT)}}{AC^2}$$

$= 1$

Hence Proved

② $1 + \tan^2\theta = \sec^2\theta$

③ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

④ $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

General Mistakes :-

① $\sin^2 A \neq \sin A^2$

② $(\sin A)^2 \neq \sin A^2$

③ $\sin A + \sin B \neq \sin(A+B)$

④ $\sin\left(\frac{A}{2}\right) \neq \frac{\sin A}{2}$

Imp Results:-

① $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\cos^2\theta$

Proof:-

$$(\sin^2\theta + \cos^2\theta)^2 = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$$

$$1 - 2\sin^2\theta\cos^2\theta = \sin^4\theta + \cos^4\theta$$

Hence Proved

② $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

③ $\tan\theta = \cot\theta - 2\cot 2\theta$

Q find value of $\sec\theta$ & $\tan\theta$ if $\sec\theta - \tan\theta = 4$

$\sec\theta - \tan\theta = 4$

$1 = \sec^2\theta - \tan^2\theta$

$1 = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$\sec\theta + \tan\theta = \frac{1}{4}$

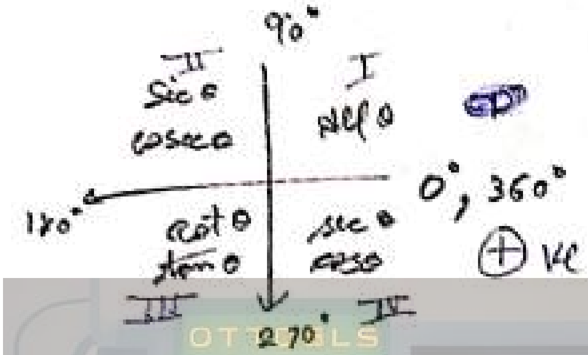
$2\sec\theta = \frac{17}{4}$

$\boxed{\sec\theta = \frac{17}{8}}$

$\boxed{\tan\theta = \frac{-15}{8}}$

$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$ same for cot case

Quadrants :-



Q $\sin\theta = \frac{3}{5}$ then find $\cos\theta$

$\sin\theta = \frac{3}{5} \to \oplus$ ve.

Quadrant → I & II

Take consider only of these of quadrant

$\sin\theta = \frac{P}{H} \Rightarrow \frac{3}{5}$

$B = \pm 4$

$\cos\theta = \frac{B}{H} = \frac{\pm 4}{5}$

$\boxed{\cos\theta = \frac{4}{5}}$ $\boxed{\cos\theta = \frac{-4}{5}}$

Q If $\cos\theta = \frac{4}{5}$ find $\cot\theta$

$\cos\theta = \frac{4}{5}$

Quadrant → I & IV

$\frac{B}{H} = \frac{4}{5}$ $P = \pm 3$

$\cot\theta = \frac{\pm 4}{5}$

$\boxed{\cot\theta = \frac{4}{3}}$, $\boxed{\frac{-4}{3}}$

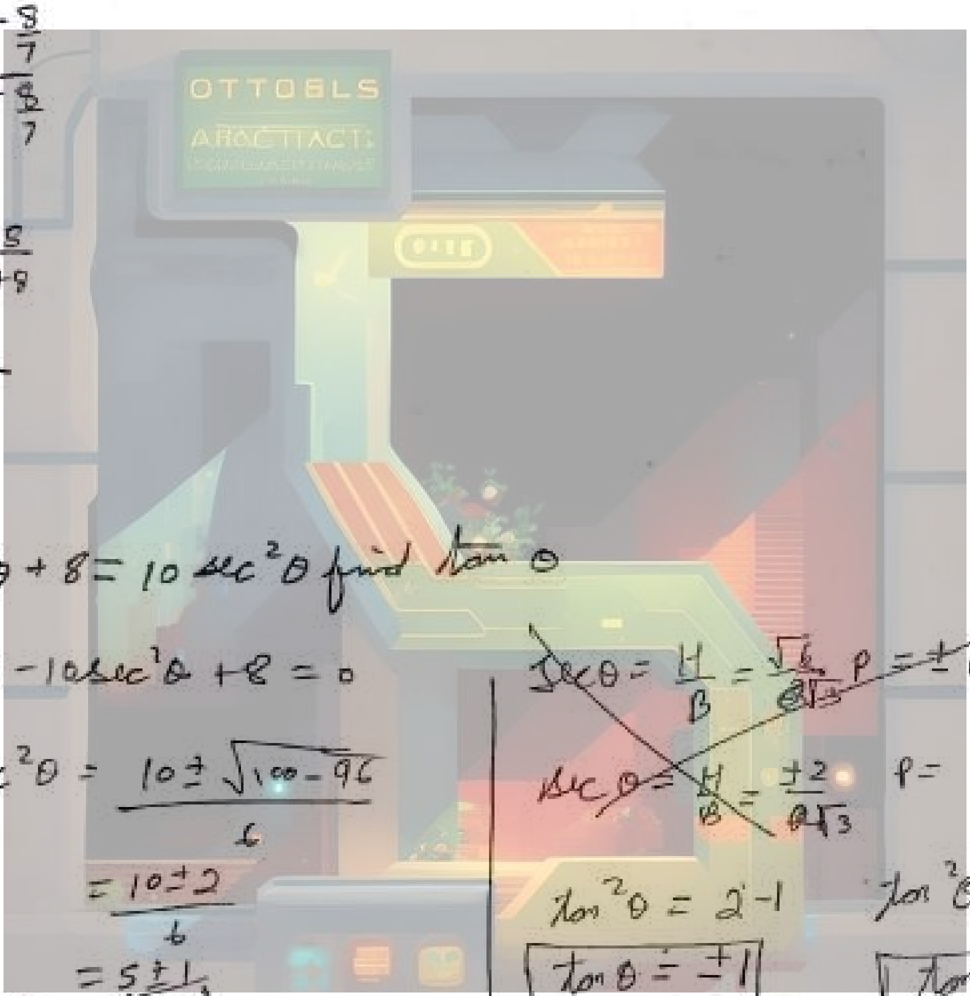
Q If $\tan\theta = \frac{1}{\sqrt{7}} = \frac{P}{B}$

$H^2 = 1 + 7$

$H = \pm 2\sqrt{2}$

$\operatorname{cosec}\theta = \boxed{\pm 2\sqrt{2}}$ $\qquad \sec\theta = \pm\frac{2\sqrt{2}}{\sqrt{7}}$

$\operatorname{cosec}^2\theta = 8$ $\qquad \sec^2\theta = 8/7$



$= \dfrac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$

$= \frac{56-8}{56+8}$

$= \frac{48}{64}$

$\boxed{= \frac{3}{4}}$

Q $3\sec^4\theta + 8 = 10\sec^2\theta$ find $\tan\theta$

$3\sec^4\theta - 10\sec^2\theta + 8 = 0$

$\sec^2\theta = \frac{10 \pm \sqrt{100-96}}{6}$

$= \frac{10 \pm 2}{6}$

$= \frac{5 \pm 1}{3}$

$\sec^2\theta = \frac{6}{3}, \frac{4}{3}$

$\sec\theta = \pm\frac{\sqrt{6}}{\sqrt{3}}, \pm\frac{2}{\sqrt{3}}$

I & IV Quad

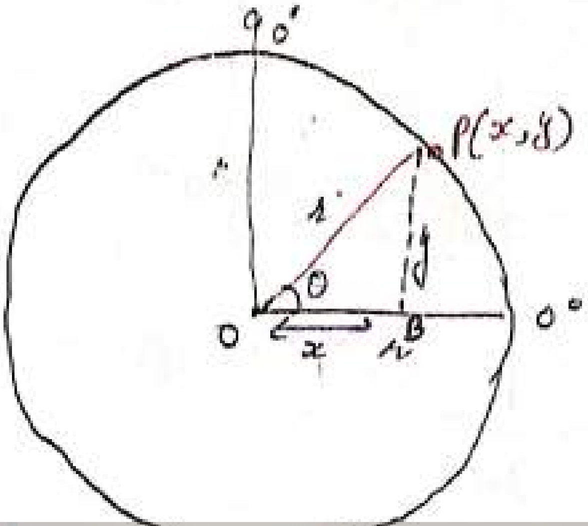
~~$\sec\theta = \frac{H}{B} = \frac{\sqrt{6}}{2\sqrt{3}}$ $P = \pm 2\sqrt{5}$~~

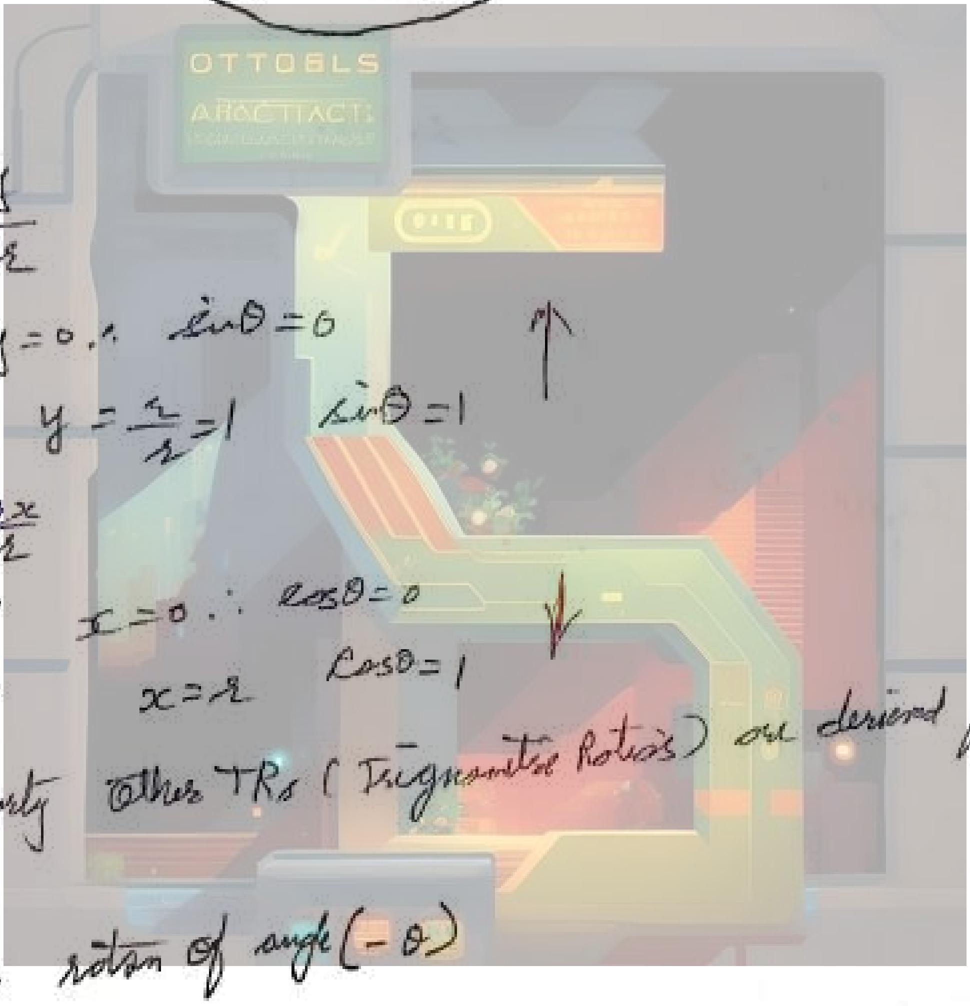
~~$\sec\theta = \frac{H}{B} = \frac{\pm 2}{2\sqrt{3}}$ $P =$~~

$\tan^2\theta = 2 - 1$ $\qquad \tan^2\theta = \frac{1}{3}$

$\boxed{\tan\theta = \pm 1}$ $\qquad \boxed{\tan\theta = \pm\frac{1}{\sqrt{3}}}$

Trignometric Table:-





ΔOPB:-

$\sin\theta = \frac{y}{r}$

$\theta \to 0°$ $y = 0$ ∴ $\sin\theta = 0$

$\theta \to 90°$ $y = \frac{r}{r} = 1$ $\sin\theta = 1$

$\cos\theta = \frac{x}{r}$

$\theta \to 90°$ $x = 0$ ∴ $\cos\theta = 0$

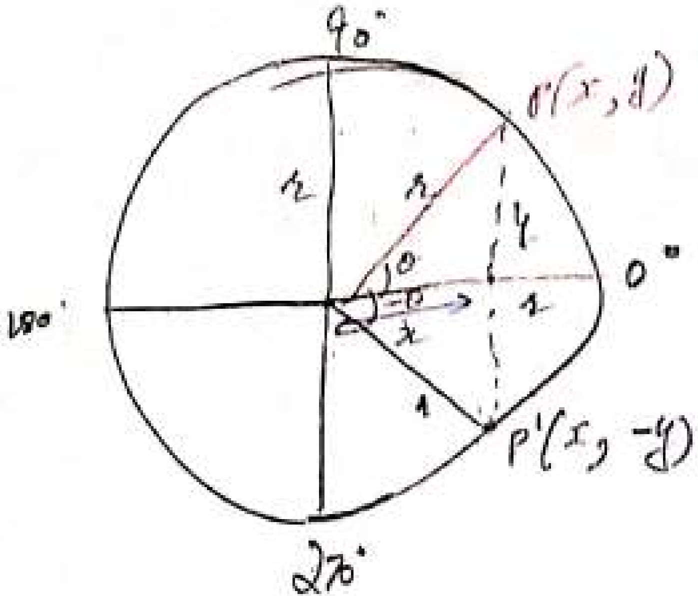
$\theta \to 0°$ $x = r$ $\cos\theta = 1$

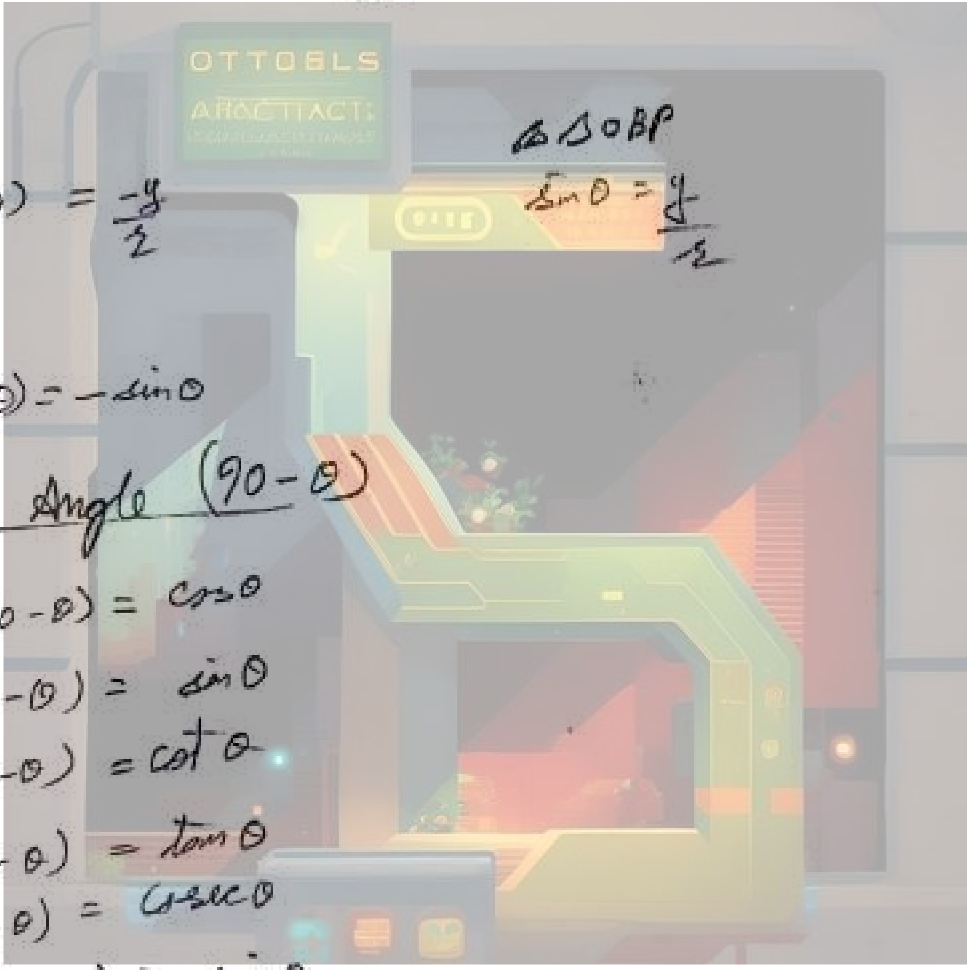
∴ Similarly other TRs (Trignometric Ratios) are derived from these two

Trignometric ratios of angle $(-\theta)$

① $\sin(-\theta) = -\sin\theta$
② $\cos(-\theta) = \cos\theta$
③ $\tan(-\theta) = -\tan\theta$
④ $\cot(-\theta) = -\cot\theta$
⑤ $\sec(-\theta) = \sec\theta$
⑥ $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Proof:-





△OBP'

$\sin(-\theta) = \frac{-y}{r}$

△OBP

$\sin\theta = \frac{y}{r}$

$\sin(-\theta) = -\sin\theta$

TR of Angle (90-θ)

① $\sin(90-\theta) = \cos\theta$

② $\cos(90-\theta) = \sin\theta$

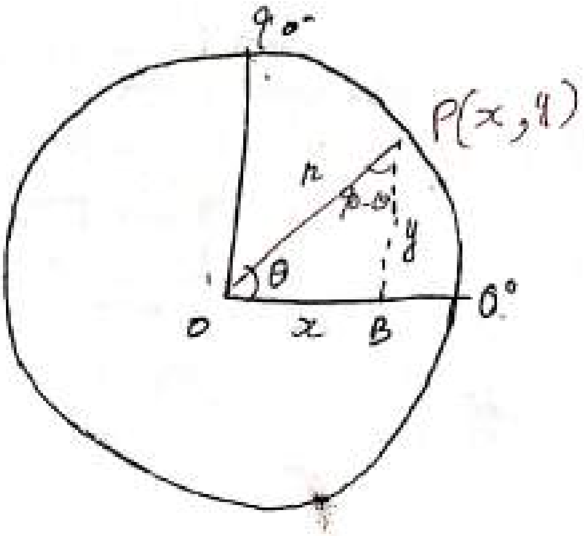
③ $\tan(90-\theta) = \cot\theta$

④ $\cot(90-\theta) = \tan\theta$

⑤ $\sec(90-\theta) = \csc\theta$

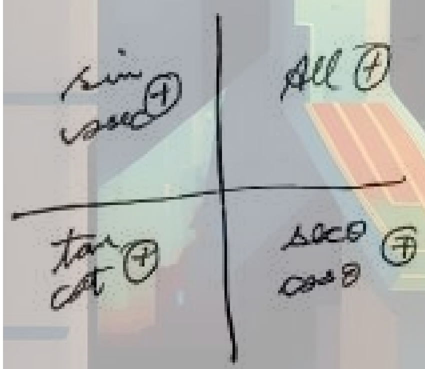
⑥ $\csc(90-\theta) = \sec\theta$

Proof:-



* $\sin(90-\theta) = \frac{x}{r}$ $\cos\theta = \frac{x}{r}$

$\sin(90-\theta) = \cos\theta$



$\sin \xleftrightarrow{90/270} \cos$

$\tan\theta \xleftrightarrow{90/270} \cot$

$\sec \xleftrightarrow{90/270} \text{cosec}$

0°/180°/360°
No change.

→ For any angle > 360°, Break the angle in multiple of 360° & remove the part of 360°

Q $\tan(150°) = \tan(180°-30) = -\tan 30° = \frac{-1}{\sqrt{3}}$

Q $\cos(210)$ $\cos(180+30) = -$ ~~sin 30~~ $-\cos 30° = \frac{-\sqrt{3}}{2}$

Q $\tan(480^\circ) = \tan(120^\circ) = \tan(90+30) = -\cot 30$

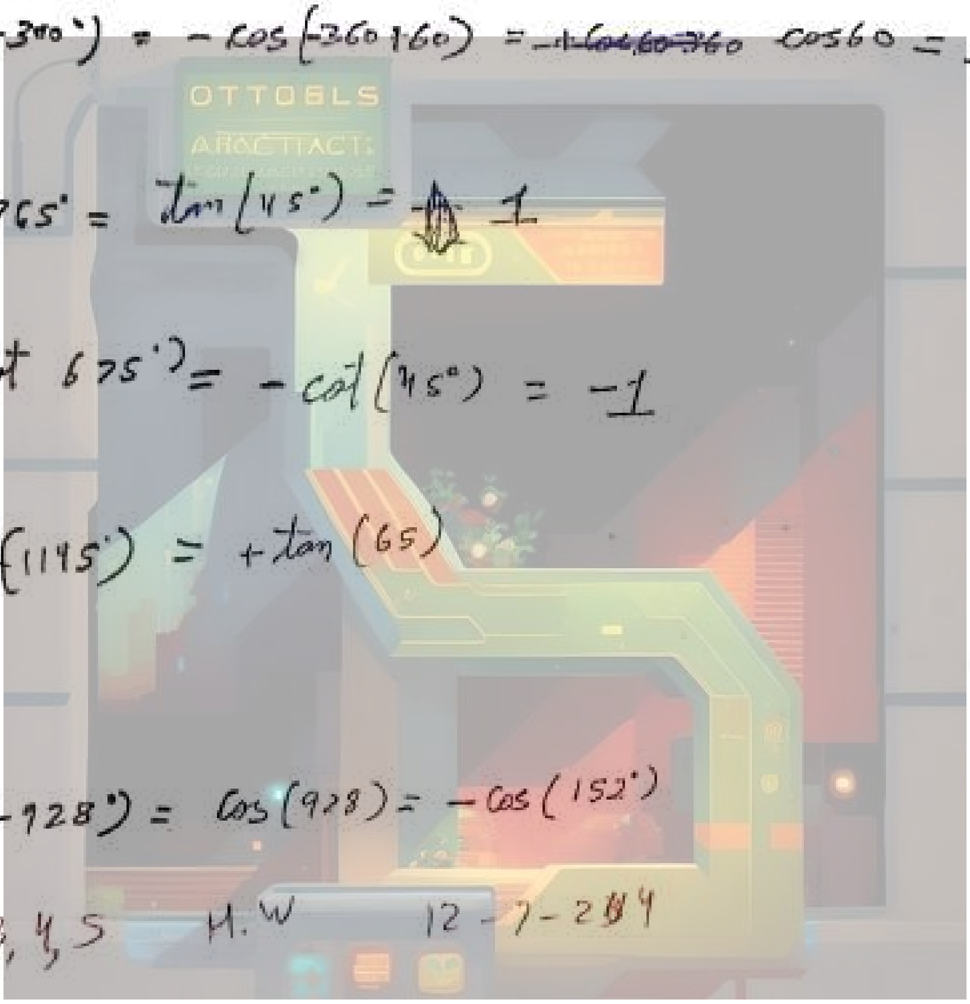
$\boxed{= -\sqrt{3}}$

Q $\cos(510) = \overset{\cos}{\cancel{\tan}}(150^\circ) = \cos(180-30) = -\cos(30)$

$\boxed{= -\frac{\sqrt{3}}{2}}$

Q $\sin(420^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$

② $\cos(-300^\circ) = -\cos(-360+60) = \cancel{-1\cos 60 - 360}\ \cos 60 = \frac{1}{2}$



③ $\tan 765^\circ = \tan(45^\circ) = 1$

④ $\cot 675^\circ = -\cot(45^\circ) = -1$

⑤ $\tan(1145^\circ) = +\tan(65)$

⑥ $\cos(-928^\circ) = \cos(928) = -\cos(152^\circ)$

DYS-3, 4, 5 H.W 12-7-2024

Q Solve the following.

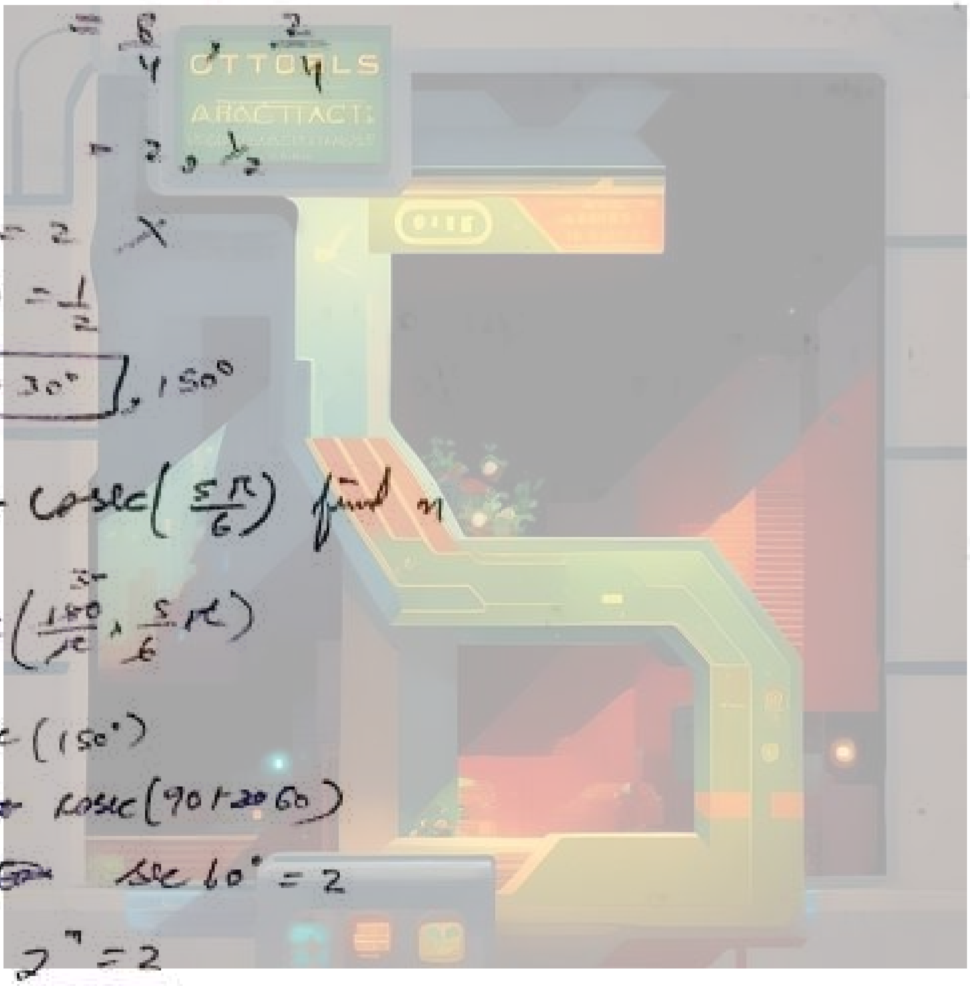
$(2\sin\theta-1)(\sin\theta-2)=0$ in $(0,\pi)$

$2\sin^2\theta-4\sin\theta-\sin\theta+2=0$

$2\sin^2\theta-5\sin\theta+2=0$

$\sin^2\theta = \frac{5\pm\sqrt{25-16}}{4}$

$= \frac{5\pm 3}{4}$



$= \frac{8}{4}, \frac{2}{4}$

$= 2, \frac{1}{2}$

$\sin\theta = 2$ ✗

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = 30^\circ}, 150^0$

Q If $2^n = \operatorname{cosec}\left(\frac{5\pi}{6}\right)$ find n

$\operatorname{cosec}\left(\frac{\overset{30}{180}}{\pi}\times\frac{5\pi}{6}\right)$

$\operatorname{cosec}(150^\circ)$

~~cosec~~ $\operatorname{cosec}(90+60)$

~~cos~~ $\sec 60^\circ = 2$

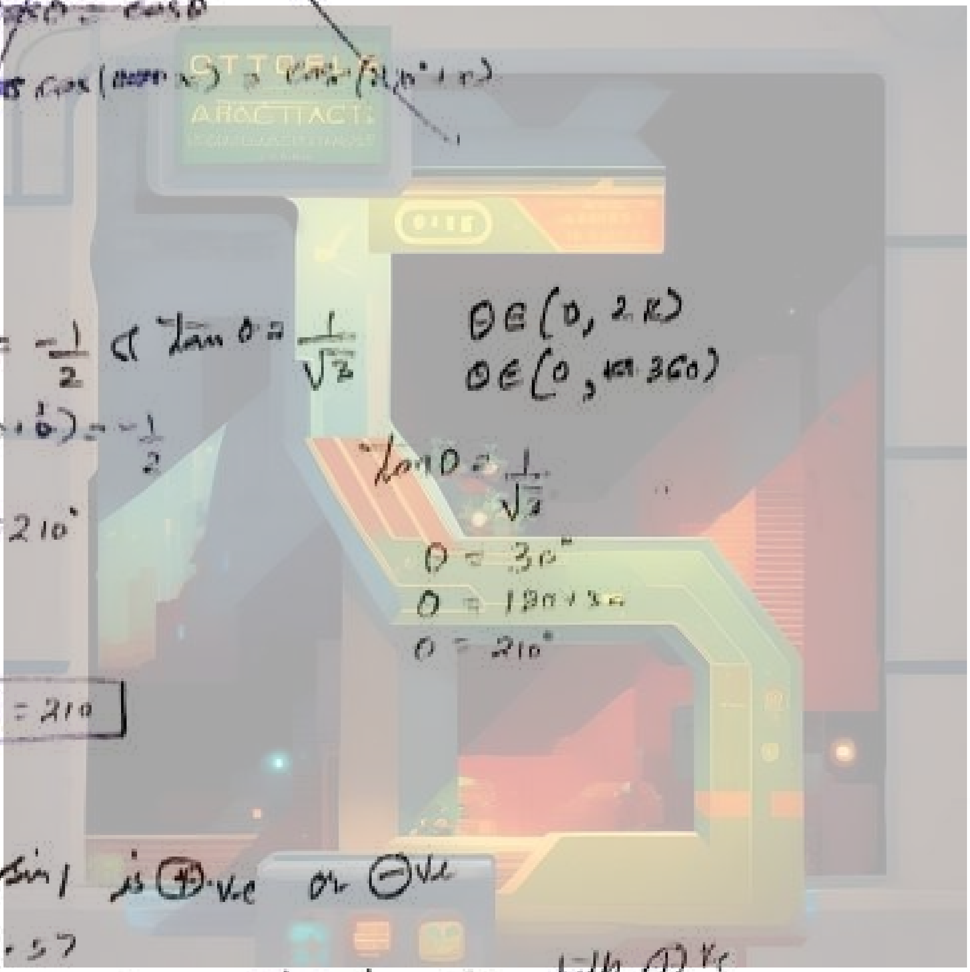
$2^n = 2$

$\boxed{n=1}$

Q.3. $\sin\theta + \cos\theta = 0$ & θ lies in 4th quad find θ?

~~in 4th quad.~~
~~$\sin\theta$ = ⊖ve~~
~~$\cos\theta$ = ⊕ve~~
~~∴ $\sin\theta = \cos\theta$~~
~~at θ = 45°~~
~~at $\theta = 270° + x$~~
~~$\frac{1}{\sqrt{3}}$ =~~
~~$\sin\theta$ = ⊖ve $\cos\theta$~~
~~$\sin\theta = \cos\theta$~~
~~$\cos(270+x) = \cos(270°+x)$~~

$\sin\theta = -\cos\theta$
$\tan\theta = -1$
~~$\tan\theta$ in~~ in 4th quad.
$\tan\theta$ = ⊖ve
$\boxed{\theta = \cancel{135°}\ -45°}$



Q4. $\sin\theta = \frac{-1}{2}$ & $\tan\theta = \frac{1}{\sqrt{3}}$ $\theta \in (0, 2\pi)$
$\theta \in (0, 360)$

$\sin(180+\theta) = -\frac{1}{2}$

$\theta = 210°$

$\tan\theta = \frac{1}{\sqrt{3}}$
$\theta = 30°$
$\theta = 180+30$
$\theta = 210°$

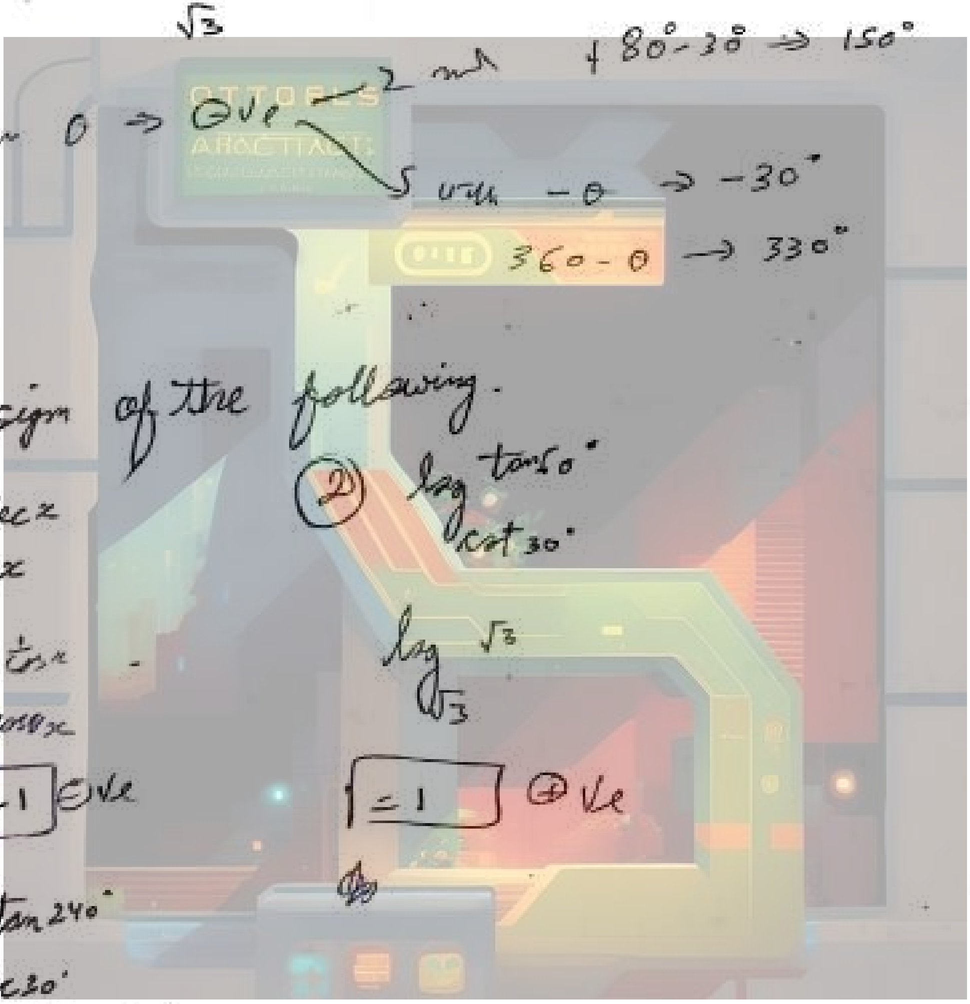
$\boxed{\theta = 210}$

Q5. cos 1 − sin 1 is ⊕ve or ⊖ve
$\frac{\pi}{2} \to 1.57$
so both in 1st quad, so both ⊕ve
sin 1 > cos 1
⊖ve

180-θ	θ
180+θ	-θ or (360-θ)

eg $\tan\theta = \frac{-1}{\sqrt{3}}$ θ = ?

$\tan\theta = \frac{1}{\sqrt{3}}$ θ = 30°



tan θ → ⊖ve → 2nd: $180° - 30° \Rightarrow 150°$

4th: $-\theta \to -30°$

$360-\theta \to 330°$

Q find sign of the following.

① $\log_{\cos x} \sec x$

$\log_{\cos x} \frac{1}{\cos x}$

$\boxed{= -1}$ ⊖ve

② $\log_{\cot 30°} \tan 60°$

$\log_{\sqrt{3}} \sqrt{3}$

$\boxed{= 1}$ ⊕ve

③ $\log_{\sec 30°} \tan 240°$

$\log_{\sec 30°} \tan 60°$

$\log_{\frac{2}{\sqrt{3}}} \sqrt{3}$ $\boxed{\oplus ve}$

Q Note:-

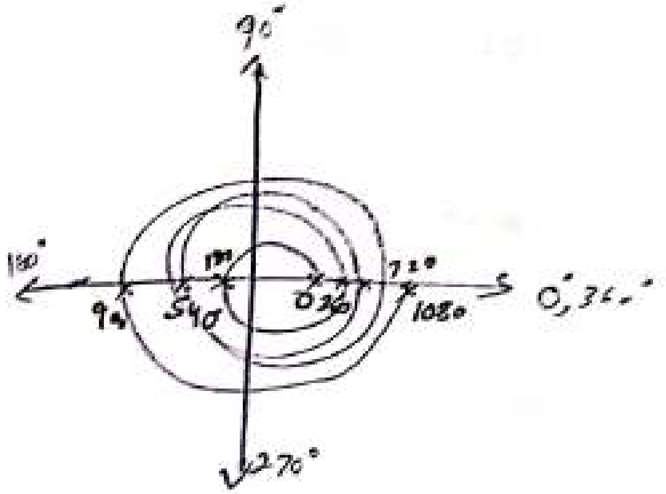
$\sin(n\pi) = \sin(180° \times n) = 0$

① $\sin(\text{integer multiple of } \pi) = 0$

② $\cos(\text{even integer multiple of } \pi) = 1$

③ $\cos(\text{odd integer multiple of } \pi) = -1$

④ $\cos(\text{odd integer multiple of } \frac{\pi}{2}) = 0$



Eg:- ① $n=1 \quad \sin\pi = \sin 180° = 0$

$n=2 \quad \sin 2\pi = \sin 360° = 0$

$n=3 \quad \sin 3\pi = \sin 540° = \sin 180° = 0$

$n=-3 \quad \sin(-3\pi) = -\sin 3\pi = -0 = 0$

$\sin 0° = 0$

$\sin 180° = 0$

$\sin 360° = 0$

④ $n=3 = \cos\left(\frac{3\pi}{2}\right) = \cos 270° = \cos(180° + 90°) = -\cos 90° = 0$

$n=7 : \cos\left(\frac{7\pi}{2}\right) = \cos 630° = \cos 270° = 0$

$n=-9 : \cos\left(-\frac{9\pi}{2}\right) = -\cos 810° = \cos 90° = 0$

③ $n=-3 = \cos(-3\pi) = \cos 3\pi = \cos 540° = \cos 180° = -\sin 90 = -1$

Q find x.

① $\sin(2022\pi) + x + 1 = 0$

$0 + x + 1 = 0$

$\boxed{x = -1}$

② $x^2 - 1 + \sec(2026\pi) = 1$

$x^2 = 1$

$\sqrt{\boxed{x = \pm 1}}$

③ $\log_{10}(x-1)\left[\tan(556\pi) + 1\right] = 0$

$\log_{10}(x-1)\left[1\right] = 0$

$10^0 = x - 1$

$x - 1 = 1$

$\sqrt{\boxed{x = 2}}$

Q find $\dfrac{\sin(505\pi) + \cos(2004\pi)}{\tan(88\pi) + \sec(550\pi)}$

$$\frac{0+1}{0+1} = \boxed{1}$$

Q two angle are complementary & of is 4 times as big as the other.

$4x = (90-x)$

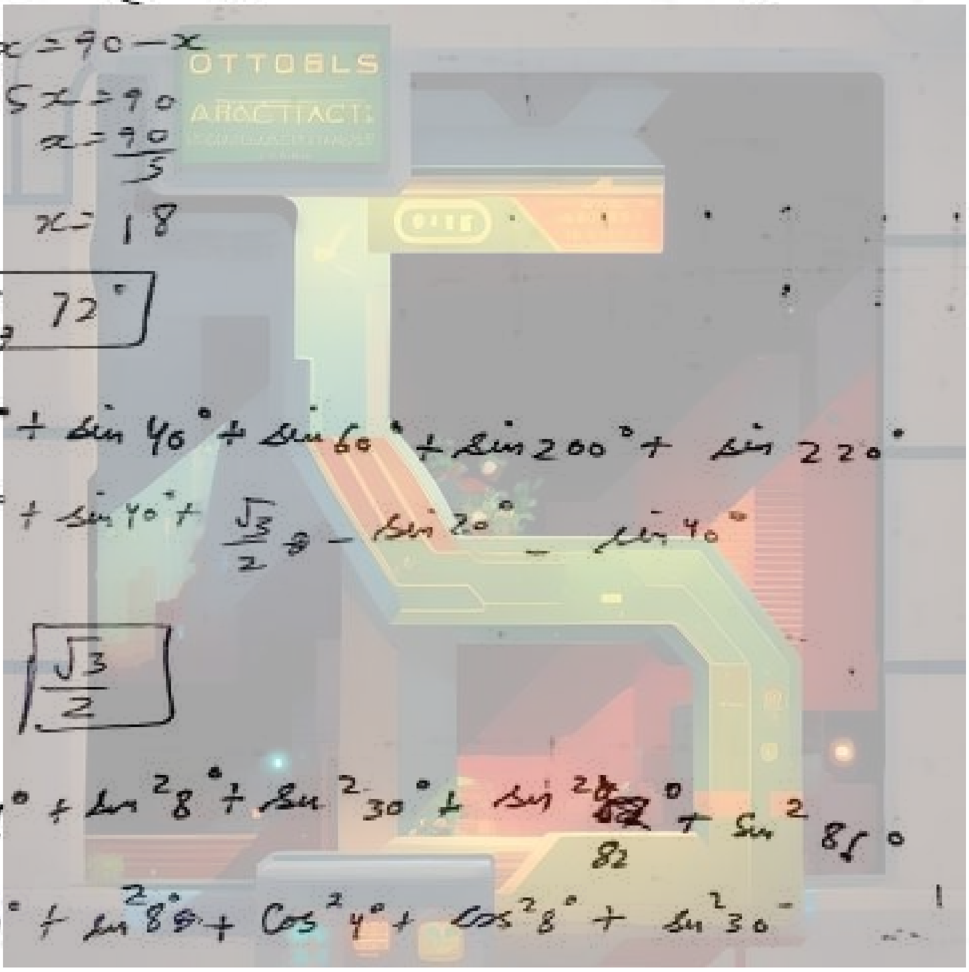
$4x = 90 - x$

$5x = 90$

$x = \frac{90}{5}$

$x = 18$

$\boxed{18^\circ, 72^\circ}$



Q $\sin 20^\circ + \sin 40^\circ + \sin 60^\circ + \sin 200^\circ + \sin 220^\circ$

$\sin 20^\circ + \sin 40^\circ + \frac{\sqrt{3}}{2} - \sin 20^\circ - \sin 40^\circ$

$\boxed{\frac{\sqrt{3}}{2}}$

Q $\sin^2 4^\circ + \sin^2 8^\circ + \sin^2 30^\circ + \sin^2 82^\circ + \sin^2 86^\circ$

$\sin^2 4^\circ + \sin^2 8^\circ + \cos^2 4^\circ + \cos^2 8^\circ + \sin^2 30^\circ$

$$2 + \left(\frac{1}{2}\right)^2 = 2 + \frac{1}{4} = \frac{8+1}{4} = \boxed{\frac{9}{4}}$$

Graphs of TF :-

~~① y = sin x~~

Domain - Possible values of x
Range - Possible values of y
Periodicity (Period) - repetitation ~~after~~ after a certain interval.

① $y = \sin x$

II Quad

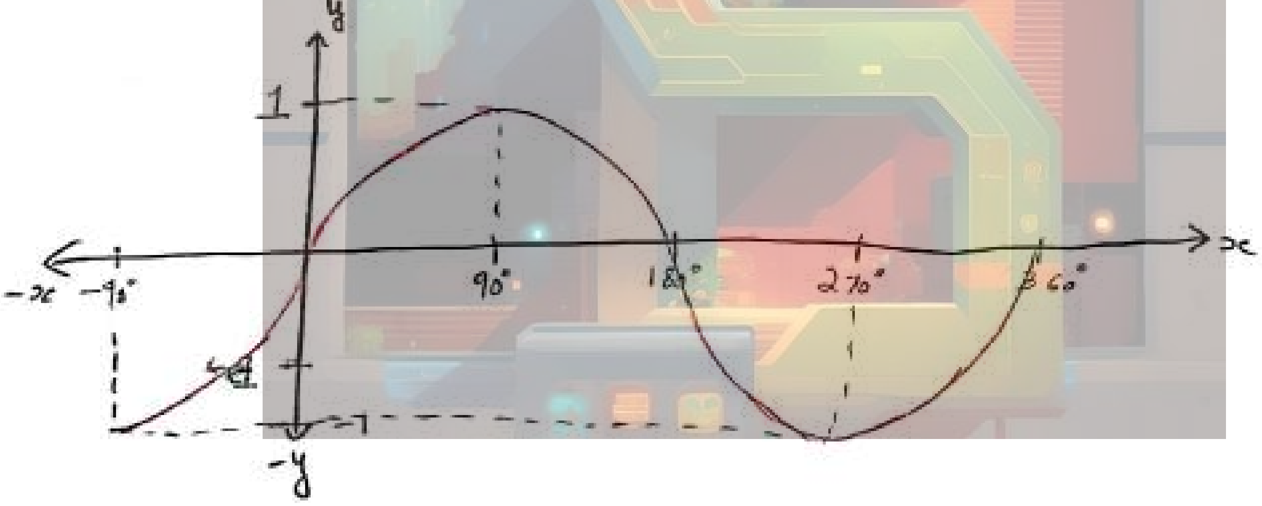
x	0°	30°	45°	60	90°	120°	135°	150°	180°
y	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

I Quad

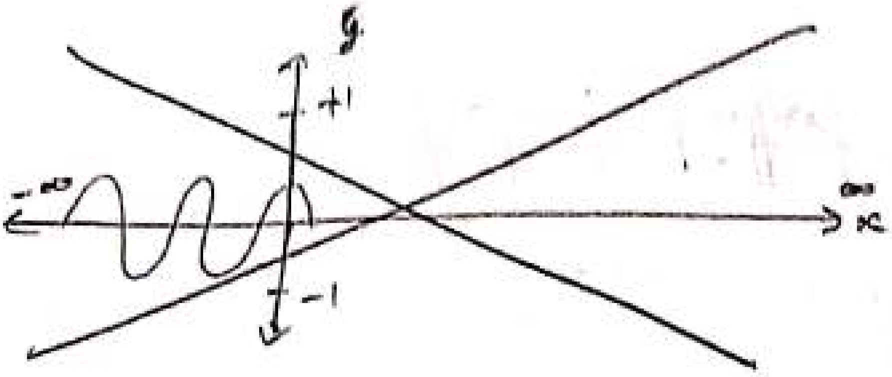
IV

x	210°	225°	240°	270°	300°	315°	330°	360°
y	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0

III



Domain :- $x \in R$
Range :- $y \in [-1, 1]$
Period :- $2\pi / 360°$

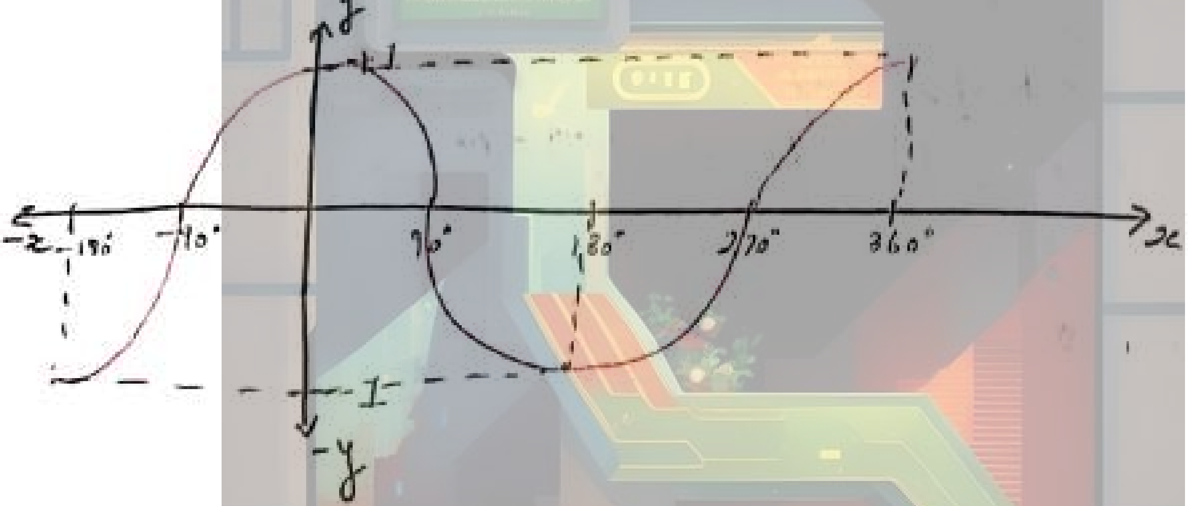


② $y = \cos x$

II　　　　　　　　IV

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	1	$1/\sqrt{2}$	0	$1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0	$-1/\sqrt{2}$	$+1$

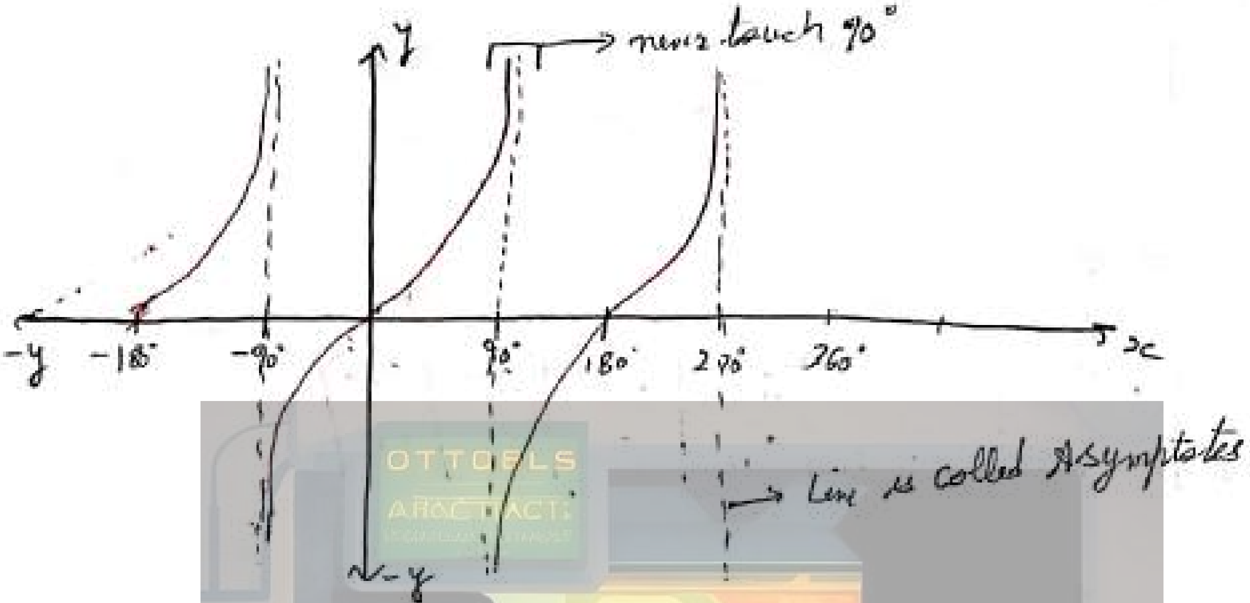
I　　　　　　　　III



Domain :- $x \in R$
Range :- $y \in [-1, 1]$
Period :- $2\pi/360°$

③ $y = \tan x$

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
y	0	∞	0	∞	0	∞	0	∞	0



Domain:- ~~$x \in R$ [all multiple~~
$x \in R$ - (odd multiple of 90°) / . $R - \frac{(2n-1)\pi}{2}$

Range:- $y \in R$

Period:- π or 180°

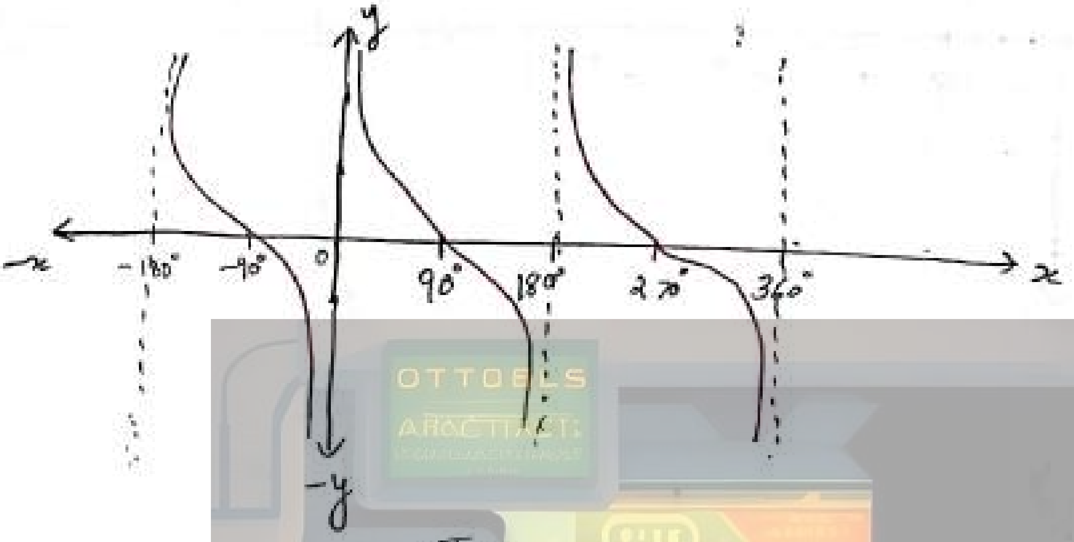
H.W 16-07-24

JM (sequence)

JA (Sequence)

④ $y = \cot x$

x	0°	90°	180°	270°	360°	450°	540°
y	ND	0	ND	0	ND	0	ND



Domain:- R ~~@~~ - (Integer multiple of π)

R - nπ

Range:- R

Period:- π/180°

⑤ $y = \sec x$

x	0°	90°	180°	270°	360°	450°	540°
y	1	ND	-1	ND	1	ND	-1

$y = \sec x = \frac{1}{\cos x}$

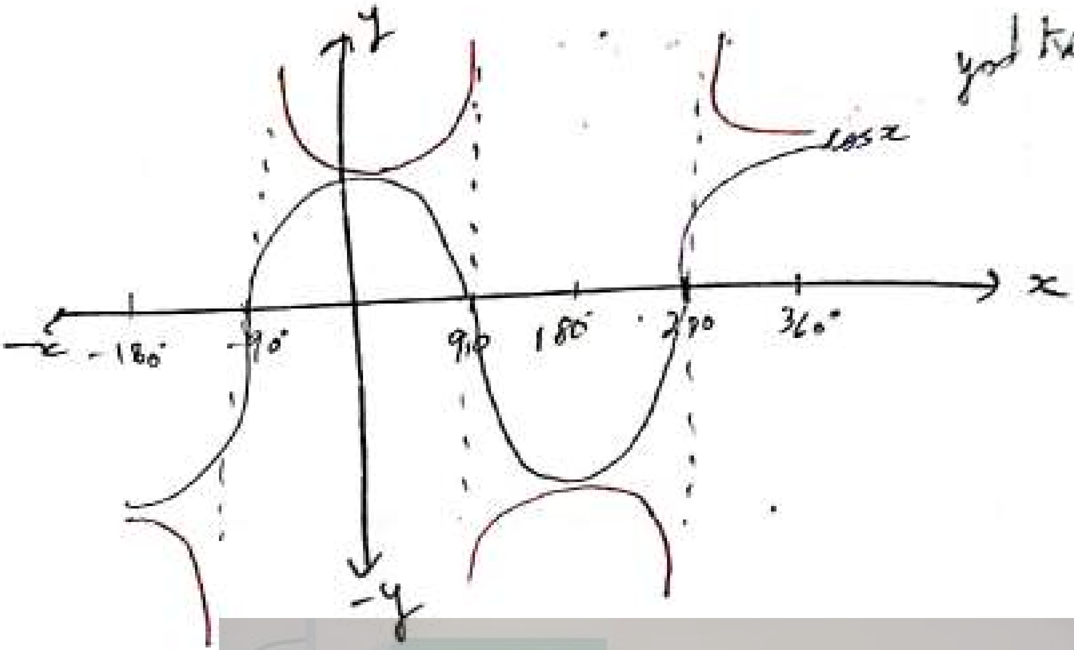
$x \neq 90°, 270°, 450°$

$y = \sec x = \frac{1}{[-1, 1]}$

$= \frac{1}{[-1, 0) \cup (0, 1]}$

~~= [1, ∞]~~

$= (-\infty, -1] \cup [1, \infty)$



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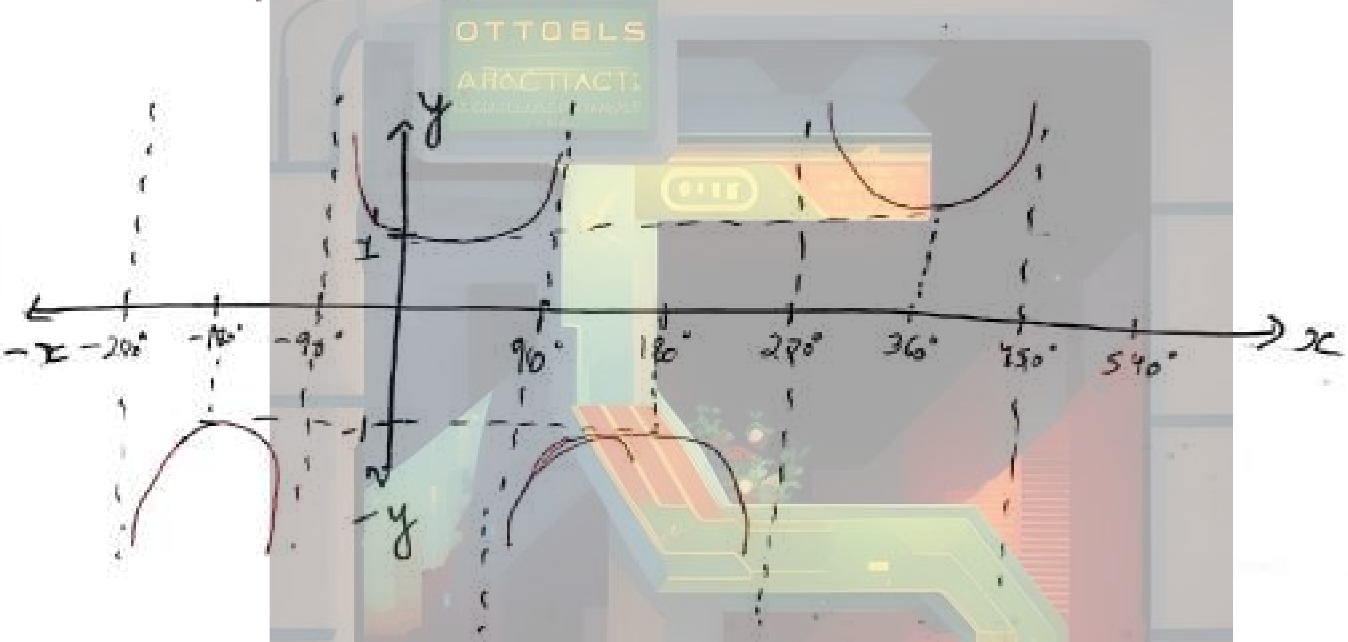


Domain:- $x \in R - \{(2n-1)\frac{\pi}{2}\}$

Range:- $R - (-1, 1)$

Period:- 2π

⑥ $y = \text{Cosec}\,x$

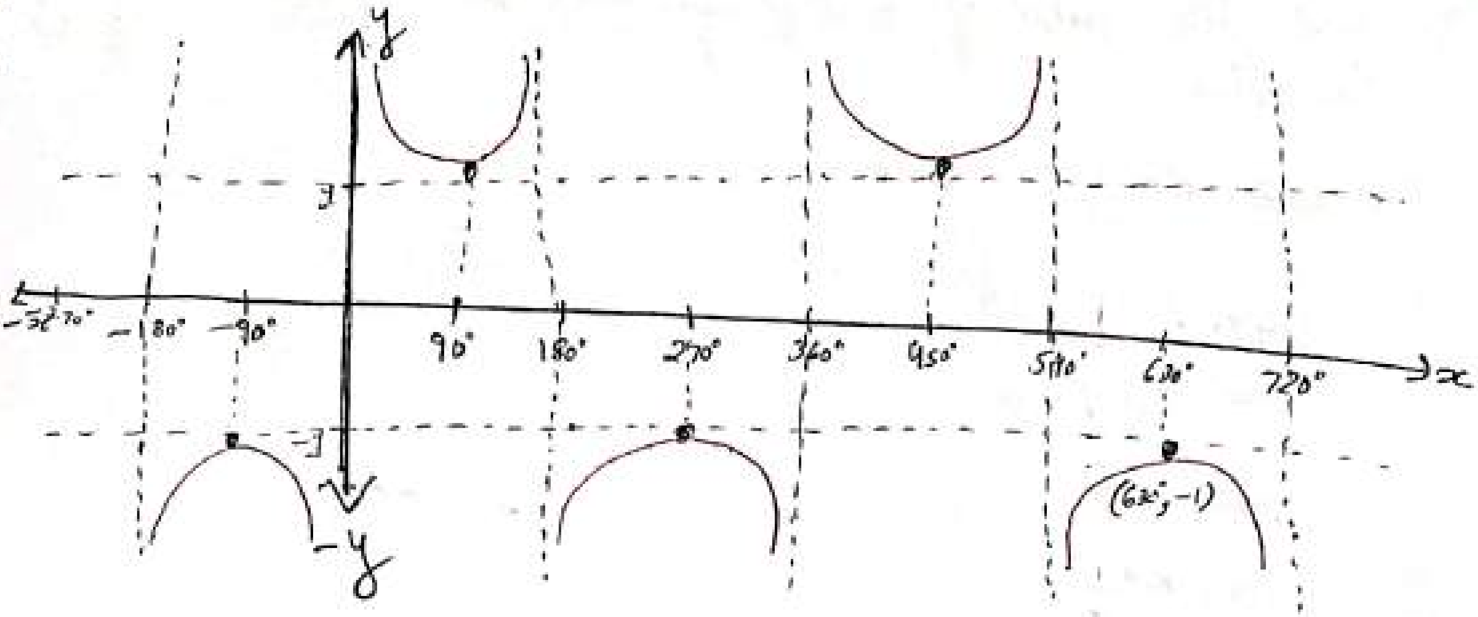
$y = \text{cosec}\,x = \frac{1}{\text{Sin}\,x}$

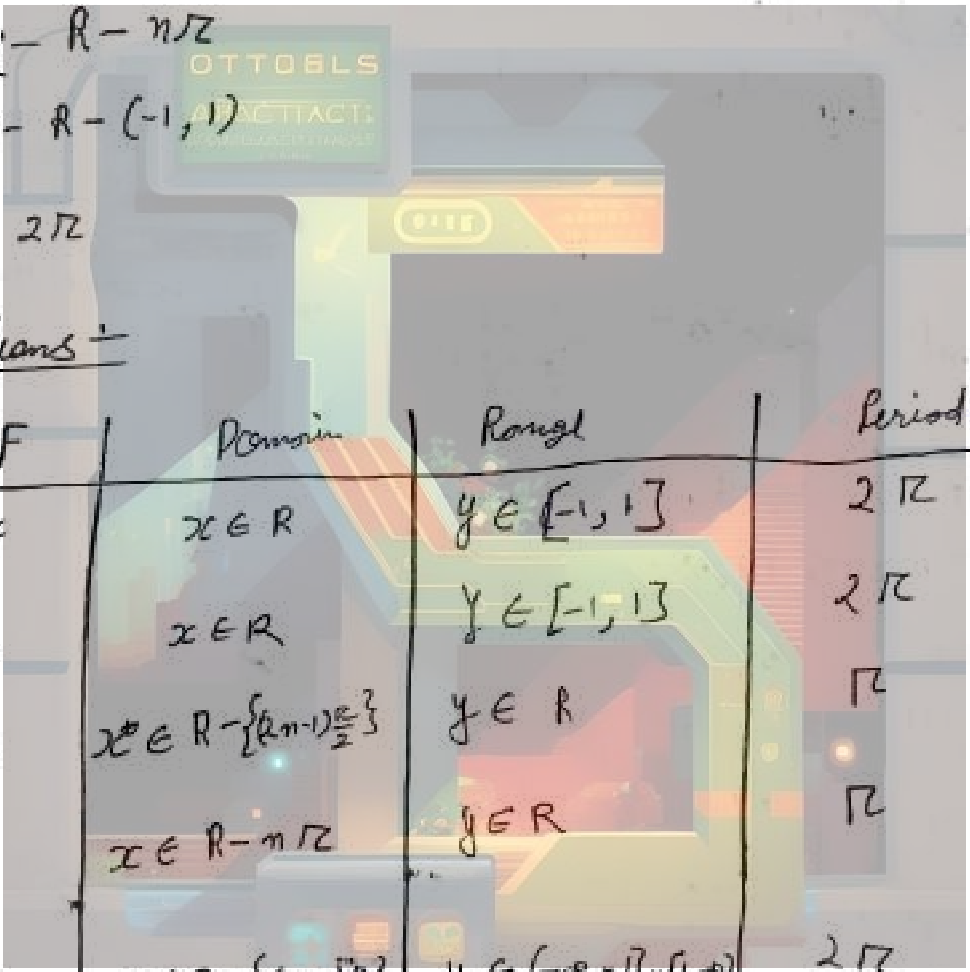
$x \notin \{0, 180, 360\}$

$x \neq$ integer multiple of π

$y = \text{cosec}\,x = \frac{1}{[-1, 1]}$

$\text{Cosec}\,x \in (-\infty, -1] \cup [1, \infty)$





Domain:- $R - n\pi$

Range:- $R - (-1, 1)$

Period:- 2π

Conclusions:-

	TF	Domain	Range	Period
①	$y = \sin x$	$x \in R$	$y \in [-1, 1]$	2π
②	$y = \cos x$	$x \in R$	$y \in [-1, 1]$	2π
③	$y = \tan x$	$x \in R - \{(2n-1)\frac{\pi}{2}\}$	$y \in R$	π
④	$y = \cot x$	$x \in R - n\pi$	$y \in R$	π
⑤	$y = \sec x$	$x \in R - \{(2n-1)\frac{\pi}{2}\}$	$y \in (-\infty, -1] \cup [1, \infty)$	2π
⑥	$y = \operatorname{cosec} x$	$x \in R - n\pi$	$y \in (-\infty, -1] \cup [1, \infty)$	2π

Q find the value of x & θ for which the equation is valid

① $\cos\theta = x^2 + \frac{1}{x^2}$ $\quad x \in R$

$\cos\theta \in [-1, 1]$

$x^2 + \frac{1}{x^2} \in [2, \infty)$

$\boxed{\text{Union} = \phi}$

② $\sin\theta = x + \frac{1}{x}$

$\sin\theta \in [-1, 1]$

$x + \frac{1}{x} \in [2, \infty) \cup (-\infty, -2]$

$\boxed{\theta \in \phi}$

Q Show that $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is only possible when $x = y$.

$\sec\theta \in (-\infty, -1] \cup [1, \infty)$

$\sec^2\theta \in [1, \infty)$

$\frac{4xy}{(x+y)^2} \geq 0$

$4xy \geq (x+y)^2$

$4xy \geq x^2 + y^2 + 2xy$

$2xy \geq x^2 + y^2$

$(x-y)^2 \leq 0$

$(x-y)^2 < 0$ ×

$(x-y)^2 = 0$

$\boxed{x = y}$ H.P

Q find Solution $\sin(e^x) = 2^x + 2^{-x}$

$\sin(e^x) \in [-1, 1]$

$2^x + \frac{1}{2^x} > 2$

$\boxed{\phi}$