Maths - 3

6 Trigonometric Ratios

7. Trigonometric Equations

8. Solution of Triangles

9. Determinants

10. Co-ordinate geometry

Trignometry

Sum & Difference formula

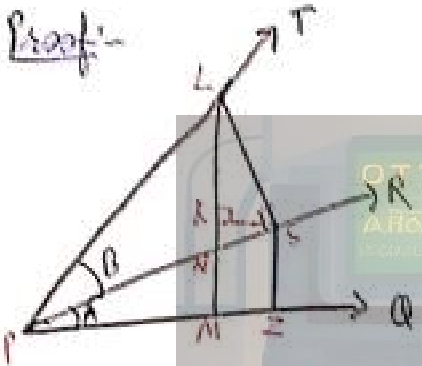
① $\sin(A+B) = \sin A \cos B + \cos A \sin B$

② $\sin(A-B) = \sin A \cos B - \cos A \sin B$

③ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

④ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

Proof:-



$\sin A = \frac{SZ}{PS}$ $\cos A = \frac{LK}{LS}$

$\sin B = \frac{LS}{PL}$ $\cos B = \frac{PS}{PL}$

$\sin(A+B) = \sin(\angle TPQ) = \frac{LM}{PL}$

$= \frac{LK+KM}{PL}$

$= \frac{LK}{PL} + \frac{SZ}{PL}$

$= \frac{LK}{LS} \times \frac{LS}{PL} + \frac{SZ}{PS} \times \frac{PS}{PL}$

$= \cos A \sin B + \sin A \cos B$

Hence, Proved

⑤ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Proof:- $\frac{\sin(A+B)}{\cos(A+B)} = \tan(A+B)$

$\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$

divide by $\cos A \cos B$

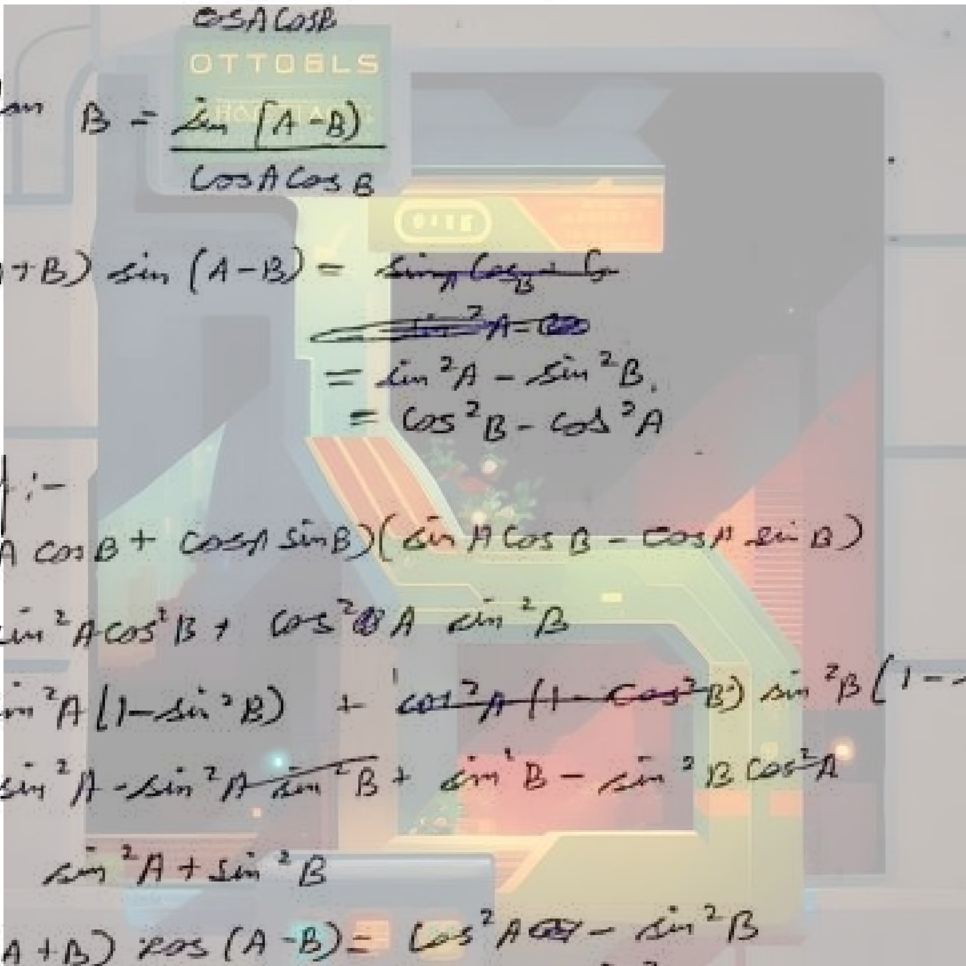
$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(7) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

(8) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(9) $\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A \cos B}$



(10) $\tan A - \tan B = \dfrac{\sin(A-B)}{\cos A \cos B}$

(11) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$

Proof:-
$(\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$

$\sin^2 A \cos^2 B + \cos^2 A \sin^2 B$

$\sin^2 A(1 - \sin^2 B) + \sin^2 B(1 - \cos^2 A)$

$\sin^2 A - \sin^2 A \sin^2 B + \sin^2 B - \sin^2 B \cos^2 A$

$\sin^2 A + \sin^2 B$

(12) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$

(7)

Transform Product into sum or difference.

① $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

Proof

$\sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$2\sin A\cos B$.

② $2\sin B\cos A = \sin(A+B) - \cancel{\sin(A-B)}\ \sin(A-B)$

③ $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

④ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Transform the sum or difference into product

① $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Proof:-

$A = C + D$

$B = C - \cancel{B}D$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

② $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

③ $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

④ $\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$

$= -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

Q find the values of (1) $\sin 15^\circ$ (2) $\cos 15^\circ$ (3) $\tan 15^\circ$ (4) $\cot 15^\circ$ (5) $\sin 75^\circ$ (6) $\cos 75^\circ$ (7) $\tan 75^\circ$ (8) $\cot 75^\circ$

(1) $\sin(45^\circ - 30^\circ) = \sin 15^\circ = \sin 45 \cos 30 - \cos 45 \sin 30$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\boxed{\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

(2) $\cos(45^\circ - 30^\circ) = \cos 15^\circ = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \cancel{\cos} \sin 30^\circ$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\boxed{\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

(3) $\tan 15^\circ = \tan(45^\circ - 30^\circ) = \dfrac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\boxed{\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} \;\Big|\; = 2-\sqrt{3}}$$

(4) $\cot 15^\circ = \dfrac{\cot 45 \cot 30 + 1}{\cot 30 - \cot 45}$

$$\boxed{\cot 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} \;\Big|\; = 2+\sqrt{3}}$$

(5) $\sin 75^\circ = \sin(45^\circ+30^\circ) = \sin 45 \cos 30 + \sin 30 \cos 45$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$\boxed{\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

(6) $\cos 75^\circ = \cos(45^\circ+30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30$

$$\boxed{\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

(7) $\tan 75^\circ = \tan(45^\circ+30^\circ) = \dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45 \tan 30}$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\boxed{\tan 75^\circ = 2+\sqrt{3}}$$

(8) $\cot 75^\circ = \cot(45^\circ+30^\circ) \Rightarrow \cancel{\cos 45^\circ \cot 30^\circ}$

$$= \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot 30^\circ + \cot 45^\circ}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\boxed{\cot 75^\circ = 2-\sqrt{3}}$$

Q find values of following:-

(1) $\sin 99^\circ \cos 21^\circ + \cos 99^\circ \sin 21^\circ$

$= \sin(99^\circ + 21^\circ)$

$= \sin(120^\circ)$

$= \cos 30^\circ$

$\boxed{= \frac{\sqrt{3}}{2}}$

② $\sin\left((n+1)A\right)\sin\left((n+2)A\right)+\cos\left((n+1)A\right)\cos\left((n+2)A\right)$

$\cos\left((n+1)A-(n+2)A\right)$

$\cos\left[nA+A-nA-2A\right]$

$\cos\left[-A\right]$

$\boxed{\cos A}$

③ $\cos\frac{2\pi}{3}\cos\frac{\pi}{4}-\sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

④ $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$

⑤ $\sin^2(127.5^\circ)-\sin^2(112.5)$

⑥ $\cos^2\left(\frac{\theta-\phi}{2}\right)-\sin^2\left(\frac{\theta+\phi}{2}\right)$

⑦ If $\theta=7.5^\circ$, then $\dfrac{\sin 8\theta\cos\theta-\sin 6\theta\cdot\cos 3\theta}{\cos 2\theta\cos\theta-\sin 3\theta\sin 4\theta}$

⑧ If $A, B\ \&\ C$ angles are in A.P., then find value of $\dfrac{\sin A-\sin C}{\cos C-\cos A}$ is.

Ⓐ $\cot\left(\frac{A-C}{2}\right)$ Ⓑ $\cot B$ Ⓒ $\tan B$ Ⓓ None

③ $\cos\frac{2\pi}{3}\cos\frac{\pi}{4}-\sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$\cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$\cos\left(120+45\right)$

$\cos(165^\circ)$

$=\cos(180-15)$

$=\cos 15^\circ$

$\boxed{=\frac{\sqrt{3}+1}{2\sqrt{2}}}$

⑥

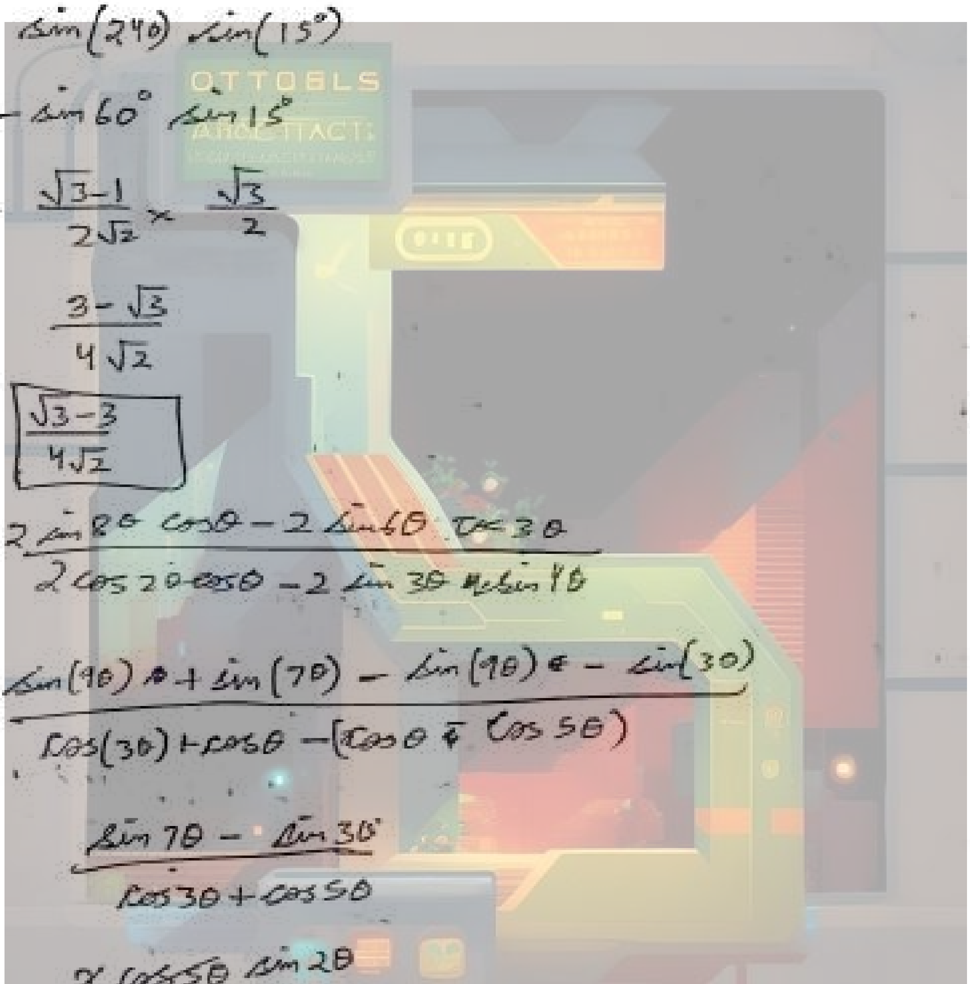
(4) $\sin(A+B)\ \sin(A-B)$

$\sin\left(\frac{\pi}{4}\right)\ \sin(A)$

$\frac{1}{\sqrt{2}} \times \sin A$

$\boxed{\frac{\sin A}{\sqrt{2}}}$

(5) $\sin(127.5+112.5)\ \sin(127.5-112.5)$



$\sin(240)\ \sin(15^\circ)$

$-\sin 60^\circ\ \sin 15^\circ$

$-\frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$\frac{3-\sqrt{3}}{4\sqrt{2}}$

$\boxed{\frac{\sqrt{3}-3}{4\sqrt{2}}}$

(7) $\frac{2\sin 8\theta\cos\theta - 2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cos\theta - 2\sin 3\theta\sin 4\theta}$

$\frac{\sin(9\theta) + \sin(7\theta) - \sin(9\theta) - \sin(3\theta)}{\cos(3\theta) + \cos\theta - (\cos\theta - \cos 5\theta)}$

$\frac{\sin 7\theta - \sin 3\theta}{\cos 3\theta + \cos 5\theta}$

$\frac{\cancel{2}\cancel{\cos 5\theta}\sin 2\theta}{\cancel{2}\cancel{\sin 4\theta}\cos 2\theta}$
$\cos 5\theta$

$\tan 2\theta = \tan 15^\circ = \boxed{2-\sqrt{3}}$

H.W ~~[illegible]~~

DYS − 6,7,8

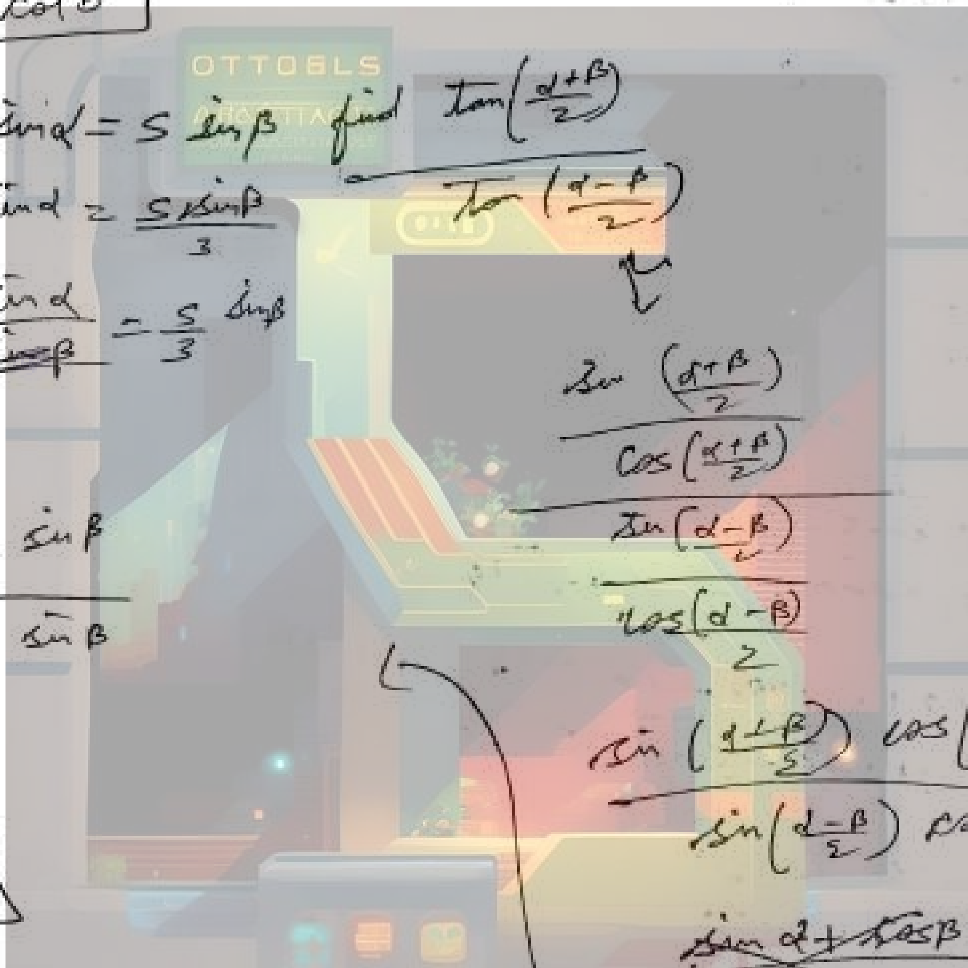
⑥ ~~cos θ~~

$\cos\theta$ ~~sin~~ $\cos\phi$

⑨ $$\frac{2\cos\left(\frac{A+C}{2}\right)\cancel{\sin\left(\frac{A-C}{2}\right)}}{2\sin\left(\frac{A+C}{2}\right)\cancel{\sin\left(\frac{A-C}{2}\right)}}$$

$$\cot\left(\frac{A+C}{2}\right)$$

$$\boxed{\cot B}$$



Q If $3\sin\alpha = 5\sin\beta$ find $\dfrac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$

$$\sin\alpha = \frac{5\sin\beta}{3}$$

$$\frac{\sin\alpha}{\sin\beta} = \frac{5}{3}$$

$$\frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)} \Big/ \frac{\sin\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$\frac{\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}$$

$$\cancel{\frac{\sin\alpha + \cos\beta}{\sin\alpha - \cos\beta}}$$

$$\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta}$$

$$\frac{\frac{8}{3}\sin\beta}{\frac{2}{3}\sin\beta}$$

$$\frac{8}{2}$$

$$\boxed{=4}$$

Q :- $(\sin 3A + \sin A)\sin A + (\cos 3A - \cos A)\cos A$

$\cancel{\sin 2A = 2\sin A\cos A}$

$\cancel{\sin 3A = \sin A\cos 2A + \sin 2A\cos A}$

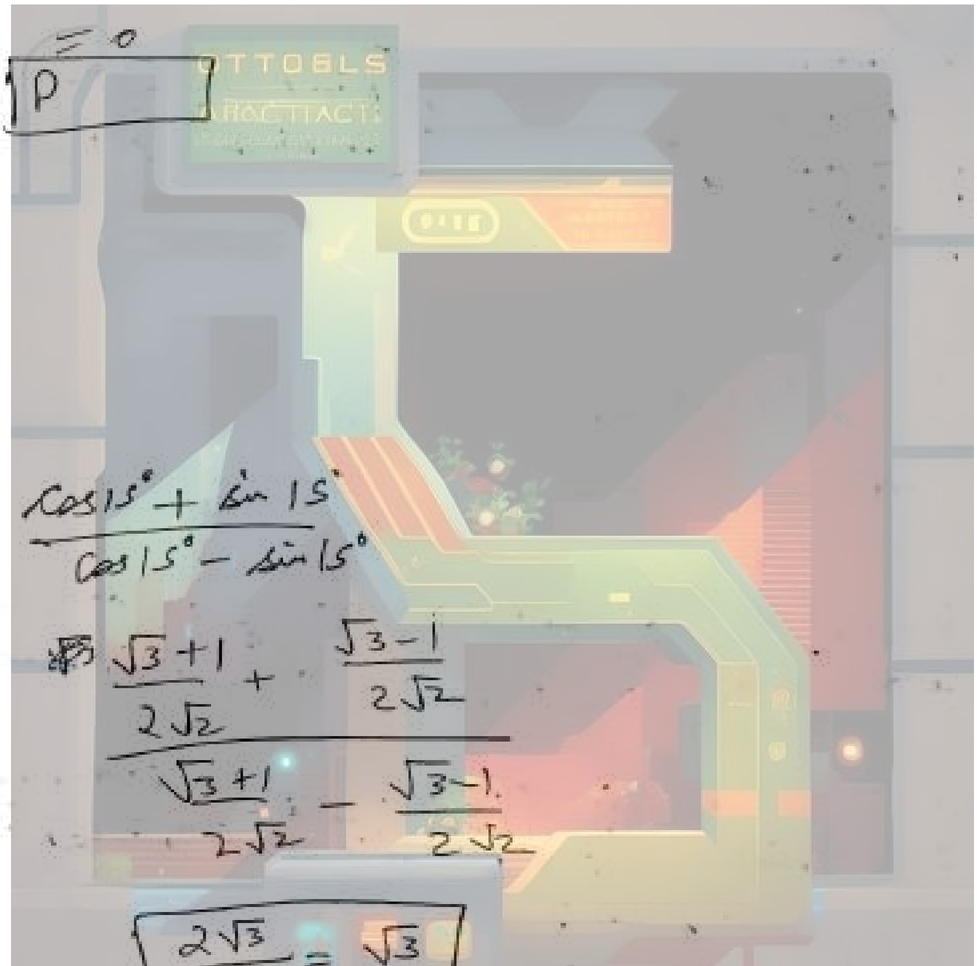
$\cancel{= \sin A(\cos^2 A - \sin^2 A) + \sin A\cos A\cos A \times 2 + \sin A}$

$\cancel{= \sin A\cos^2 A - \sin^3 A + 2\sin^2 A\cos A + \sin A}$

$2\sin 2A\cos A\sin A - 2\sin 2A\sin A\cos A$

$= 0$

$\boxed{0}$



Q $\dfrac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$

$$= \frac{\frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\boxed{\frac{2\sqrt{3}}{2} = \sqrt{3}}$$

Q If $\sin 2A = \lambda \sin 2B$, find $\dfrac{\tan(A+B)}{\tan(A-B)}$

$\cancel{2\sin A\cos A = 2\lambda \sin B\cos B}$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}\times\frac{1+\tan A\tan B}{\tan A-\tan B}$$

$$\frac{\sin 2A}{\sin 2B} = \lambda$$

$$\frac{\sin 2A+\sin 2B}{\sin 2A-\sin 2B} = \frac{\lambda+1}{\lambda-1}$$

$$\cancel{\frac{\sin(A+B)}{\cos A\cos B}\times\frac{\cos A\cos B}{\sin(A-B)}}$$

$$\cancel{\frac{\tan A+\tan B+1}{1-\tan A\tan B}\times\frac{\sin A\cos B+\cos A\sin B}{\sin A\cos B-\cos A\sin B}}$$

$$\frac{2\sin(A+B)\cos(A-B)}{2\sin(A-B)\cos(A+B)}$$

$$\boxed{\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}}$$

Q $\cot 16\cot 44 + \cot 44\cot 76 - \cot 76\cot 16$.

$$\cancel{\frac{2\cos 16\cos 44}{2\sin 16\sin 44}}$$

$$\cancel{\frac{\cos(60)+\cos(28)}{\cos(28)-\cos(60)} + \frac{\cos(120)+\cos(32)}{\cos(32)-\cos(120)} - \frac{\cos(92)+\cos(60)}{\cos(60)-\cos(92)}}$$

$$\cancel{\frac{\cos 16\cos 44\sin 76}{\sin 16\sin 44\sin 76} + \frac{\cos 44\cos 76\sin 16}{\sin 44\sin 76\sin 16} - \frac{\cos 76\cos 16\sin 44}{\sin 76\sin 16\sin 44}}$$

$$\cot(44+16) = \frac{1}{\sqrt3} = \frac{\cot 44\cot 16-1}{\cot 44+\cot 16}$$

$$\frac{\cot 44+\cot 16}{\sqrt3} + 1 + \frac{\cot 76+\cot 44}{\sqrt3} + 1 - \frac{\cot 16+\cot 76}{\sqrt3} - 1$$

$$\frac{0}{\sqrt3}+3 \Rightarrow \boxed{3}$$

Q ~~Prove~~

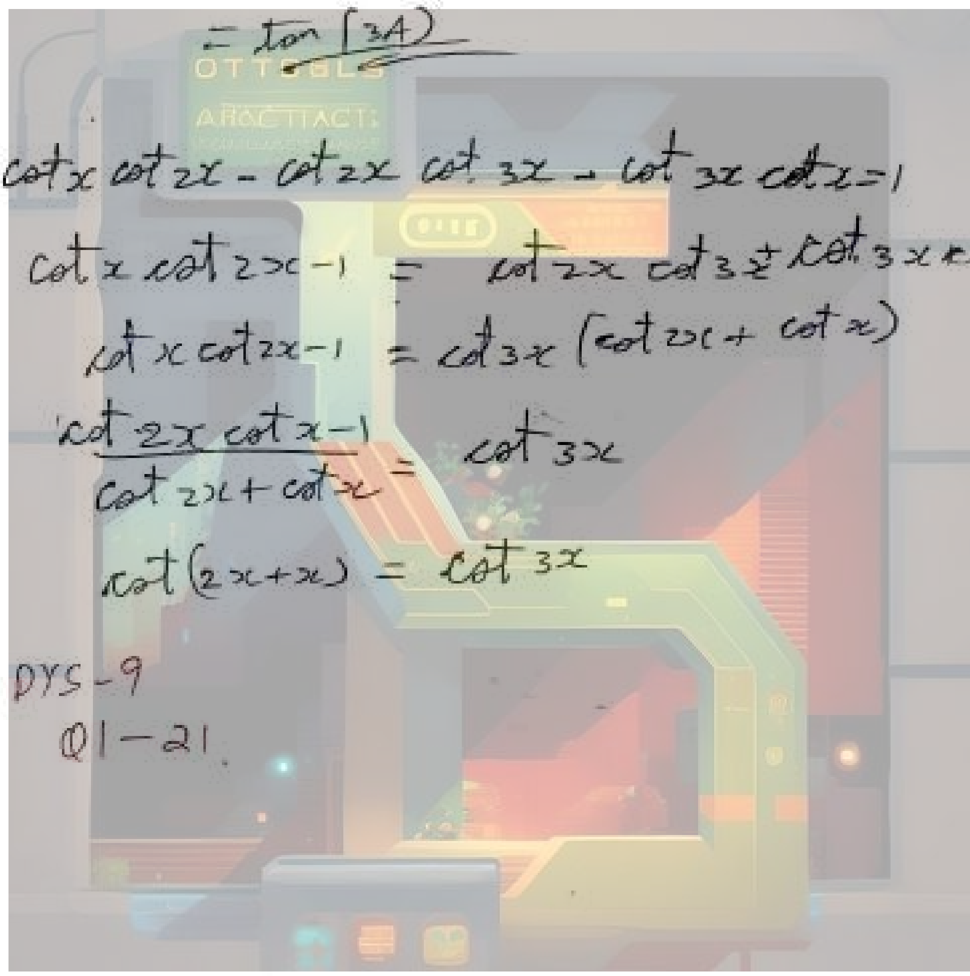
~~$\tan 70^\circ = \cot 70 + 2\cot 40$~~

Q $\tan 3A \cdot \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$

$\tan 2A + \tan A = \tan 3A(1 - \tan 2A + \tan A)$

$$\tan 3A = \frac{\tan 2A + \tan A}{1 - \tan 2A + \tan A}$$

$= \tan(2A + A)$



$= \tan(3A)$

Q $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

$\cot x \cot 2x - 1 = \cot 2x \cot 3x + \cot 3x \cot x$

$\cot x \cot 2x - 1 = \cot 3x(\cot 2x + \cot x)$

$$\frac{\cot 2x \cot x - 1}{\cot 2x + \cot x} = \cot 3x$$

$\cot(2x + x) = \cot 3x$

DYS-9

Q1-21

Sum of TR with more than 2 Angles.

(1) $\sin(A+B+C) = \sin[(A+B)+C]$

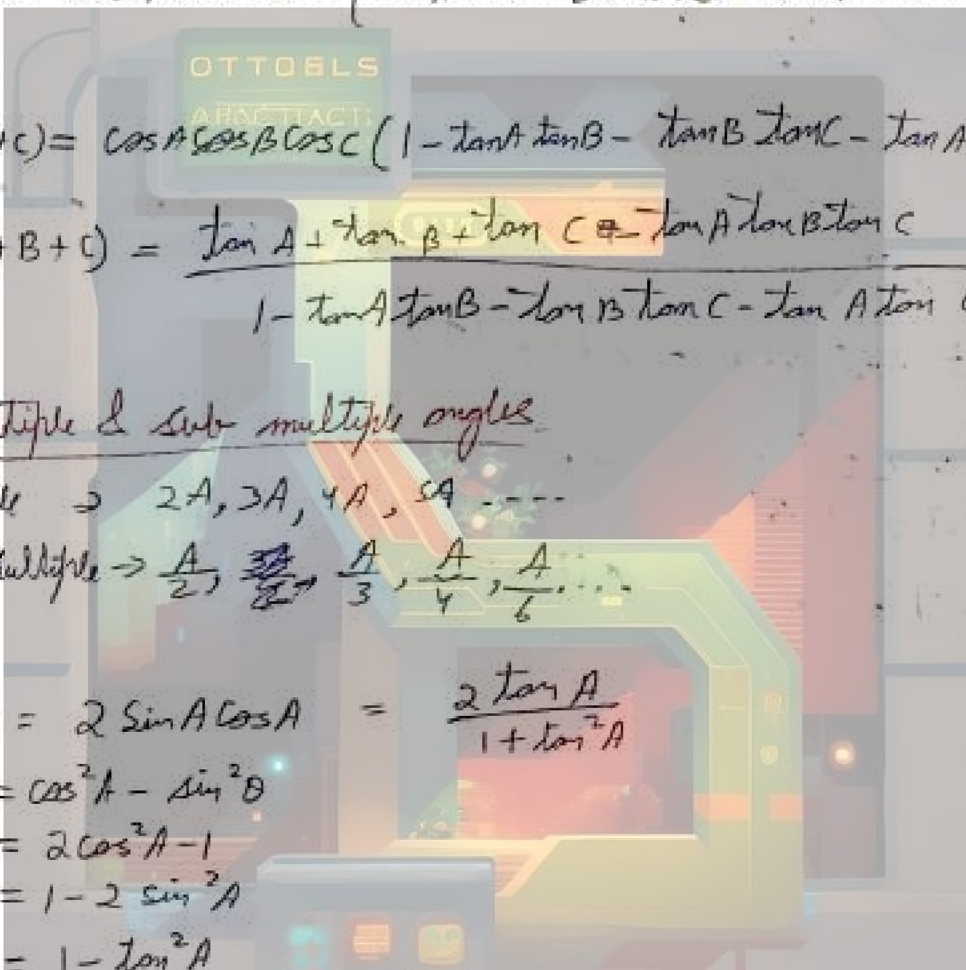
$= \sin(A+B)\cos C + \cos(A+B)\sin C$

$= (\sin A\cos B + \cos A\sin B)\cos C + (\cos A\cos B - \sin A\sin B)\sin C$

$= \sin A\cos B\cos C + \cos A\sin B\cos C + \sin C\cos A\cos B - \sin A\sin B\sin C$

$\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C + \sin C\cos A\cos B - \sin A\sin B\sin C$

$\sin(A+B+C) = \cos A\cos B\cos C(\tan A + \tan B + \tan C - \tan A\tan B\tan C)$



(2) $\cos(A+B+C) = \cos A\cos B\cos C(1 - \tan A\tan B - \tan B\tan C - \tan A\tan C)$

(3) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan A\tan C}$

TR of multiple & sub multiple angles

Multiple → $2A, 3A, 4A, 5A, \ldots$

Sub-Multiple → $\frac{A}{2}, \frac{A}{3}, \frac{A}{4}, \frac{A}{6}, \ldots$

(1) $\sin 2A = 2\sin A\cos A = \dfrac{2\tan A}{1+\tan^2 A}$

(2) $\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$= \dfrac{1-\tan^2 A}{1+\tan^2 A}$

(3) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

④ $\sin 3A = 3\sin A - 4\sin^3 A = \sin A(3 - 4\sin^3 A)$

$= 2\sin A\cos^2 A + \sin A - 2\sin^3 A$

Proof:-

$\sin 3A = \sin(2A + A)$

$= 2\sin A\cos A \cdot \cos A + \cancel{\cos}(1 - 2\sin^2 A)\sin A$

$= 2\sin A\cos A \cdot \cos A + \sin A - 2\sin^3 A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

⑤ $\cos 3A = 4\cos^3 A - 3\cos A$

⑥ $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Q $\sin A = \frac{4}{5}$ then. $\cos 2A = ?$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 1 - 2(\sin A)^2$

$= 1 - 2 \times \frac{16}{25}$

$= 1 - \frac{32}{25}$

$= \frac{25-32}{25}$

$\boxed{= \frac{-7}{25}}$

Q Prove $\cos^4 \frac{A}{2} - \sin^4 \frac{A}{2} = \cos A$

$\left(\cos^2 \frac{A}{2} + \sin^2 \frac{A}{2}\right)\left(\cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}\right)$

$\cancel{\left(\cos \frac{A}{2} + \cos \frac{A}{2} \sin \frac{A}{2}\right)\left(\cos \frac{A}{2} - \sin \frac{A}{2}\right)}$

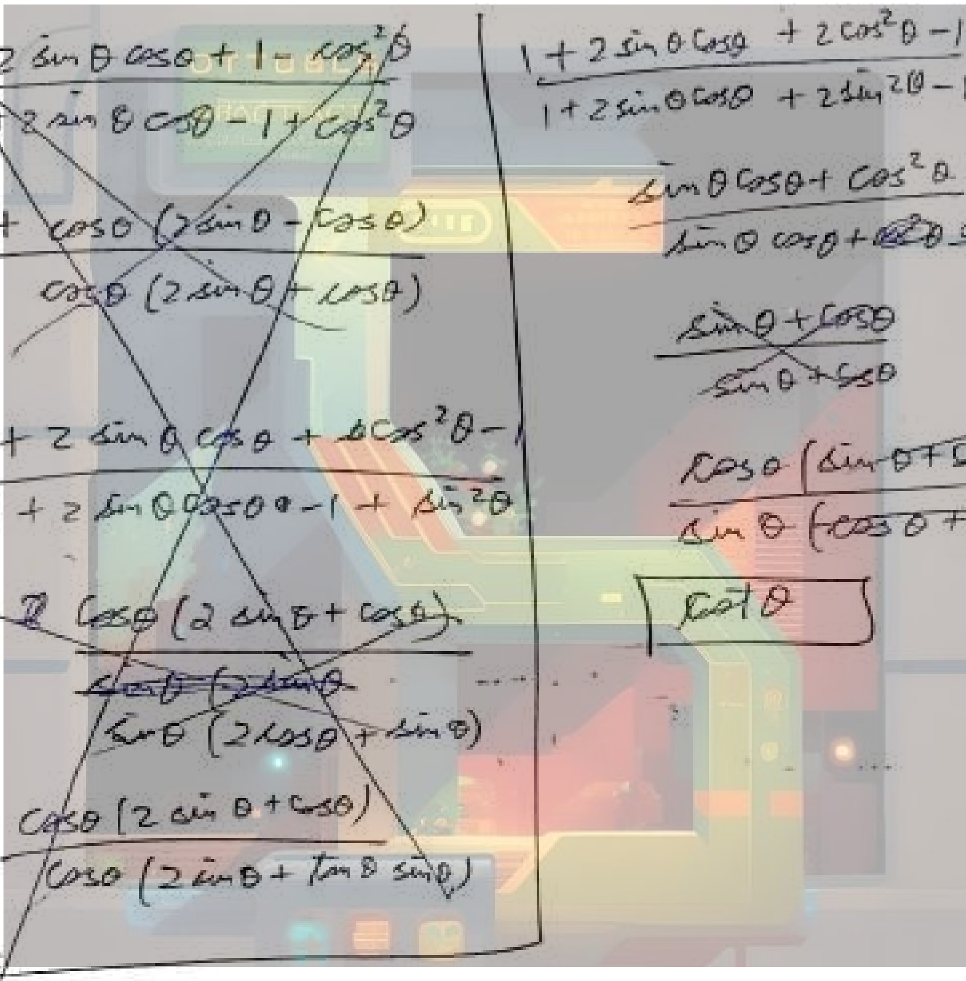
$$\cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right) = \cos 2 \times \frac{A}{2}$$

$$\boxed{= \cos A}$$

Q3. $\dfrac{1+\sin 2\theta + \cos 2\theta}{1+\sin 2\theta - \cos 2\theta}$

A) $\frac{\tan\theta}{2}$ B) $\frac{\cot\theta}{2}$

c) $\tan\theta$ ✓d) $\cot\theta$



$$= \frac{1+2\sin\theta\cos\theta + 1 - \cos^2\theta}{1+2\sin\theta\cos\theta - 1 + \cos^2\theta}$$

$$= \frac{2+\cos\theta(2\sin\theta - \cos\theta)}{\cos\theta(2\sin\theta + \cos\theta)}$$

$$= \frac{1+2\sin\theta\cos\theta + \cancel{b}\cos^2\theta - 1}{1+2\sin\theta\cos\theta - 1 + \sin^2\theta}$$

$$= \frac{\cancel{2}\cos\theta(2\sin\theta + \cos\theta)}{\cancel{\sin\theta(2\sin\theta)}\ \sin\theta(2\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta(2\sin\theta + \cos\theta)}{\cos\theta(2\sin\theta + \tan\theta\sin\theta)}$$

$=$

$$\frac{1+2\sin\theta\cos\theta + 2\cos^2\theta - 1}{1+2\sin\theta\cos\theta + 2\sin^2\theta - 1}$$

$$\frac{\sin\theta\cos\theta + \cos^2\theta}{\sin\theta\cos\theta + \sin^2\theta}$$

$$\frac{\cancel{\sin\theta + \cos\theta}}{\cancel{\sin\theta + \cos\theta}}$$

$$\frac{\cos\theta(\cancel{\sin\theta + \cos\theta})}{\sin\theta(\cancel{\cos\theta + \sin\theta})}$$

$$\boxed{\cot\theta}$$

Q 4. $\sqrt{2+\sqrt{2+2\cos 4\theta}}$

A) $2\sin\theta$ ✓B) $2\cos\theta$

C) $\sin\theta$ D) $\cos\theta$

~~$\sqrt{2+\sqrt{2+2\cos^2 2\theta-2\sin^2 2\theta}}$~~

~~$\sqrt{2+\sqrt{2(1+}}$~~

~~$\sqrt{2+\sqrt{2+2(2\cos^2\theta-1)}}$~~

~~$\sqrt{2+\sqrt{4\cos^2\theta}}$~~

~~$\sqrt{2+2\cos 2\theta}$~~

~~$\sqrt{2+2\cos^2\theta-1}$~~

~~$\sqrt{\sqrt{2}\cos^2\theta}$~~

~~$\sqrt{\sqrt{2}\cos\theta}$~~

~~$\cos\theta \times 2^{1/4}$~~

$\sqrt{2+\sqrt{2+2(\cos^2 2\theta-1)}}$

$\sqrt{2+\sqrt{4\cos^2 2\theta}}$

$\sqrt{2+2\cos 2\theta}$

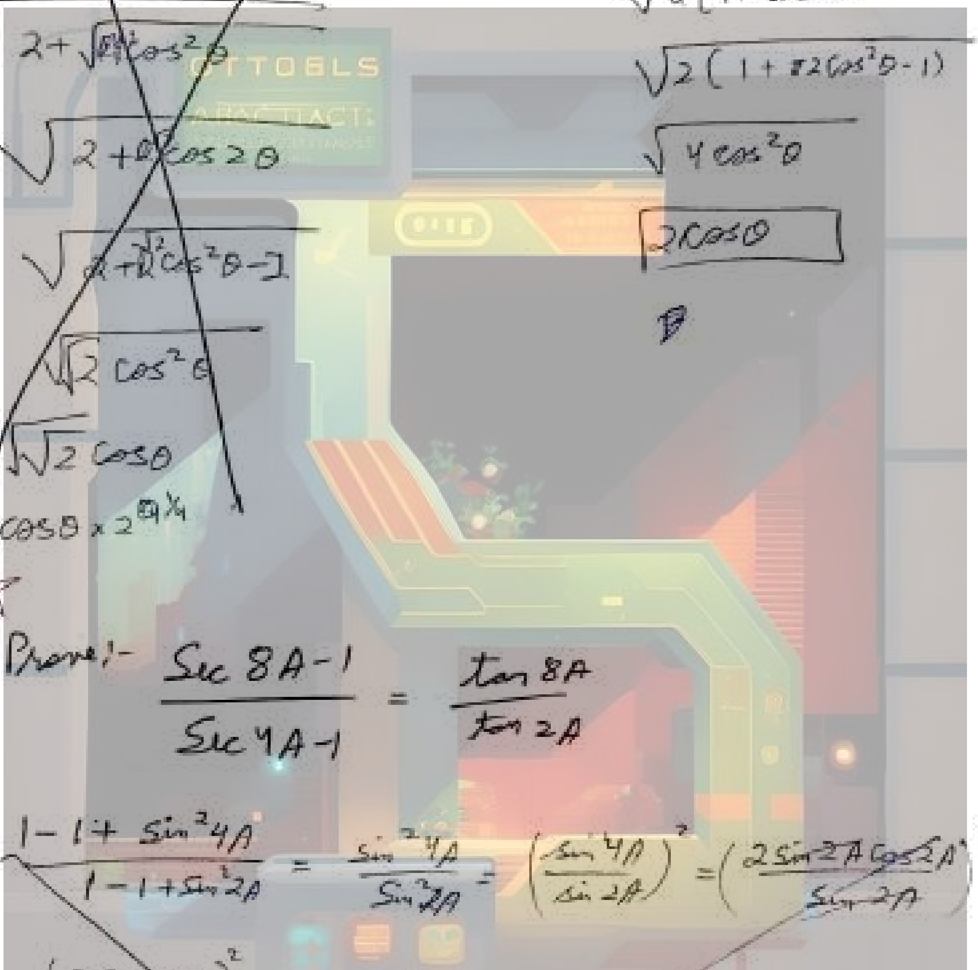
$\sqrt{2(1+\cos 2\theta)}$

$\sqrt{2(1+2\cos^2\theta-1)}$

$\sqrt{4\cos^2\theta}$

$\boxed{2\cos\theta}$

B



☆R

Q5. Prove:- $\dfrac{\sec 8A-1}{\sec 4A-1}=\dfrac{\tan 8A}{\tan 2A}$

$\dfrac{1-1+\sin^2 4A}{1-1+\sin^2 2A}=\dfrac{\sin^2 4A}{\sin^2 2A}=\left(\dfrac{\sin 4A}{\sin 2A}\right)^2=\left(\dfrac{2\sin 2A\cos 2A}{\sin 2A}\right)^2$

$=(2\cos 2A)^2$

$=(2\cos^2 A-2\sin^2 A)^2$

$=4\cos^4 A-4\sin^4 A$

$=4(\cos^4 A-\sin^4 A)$

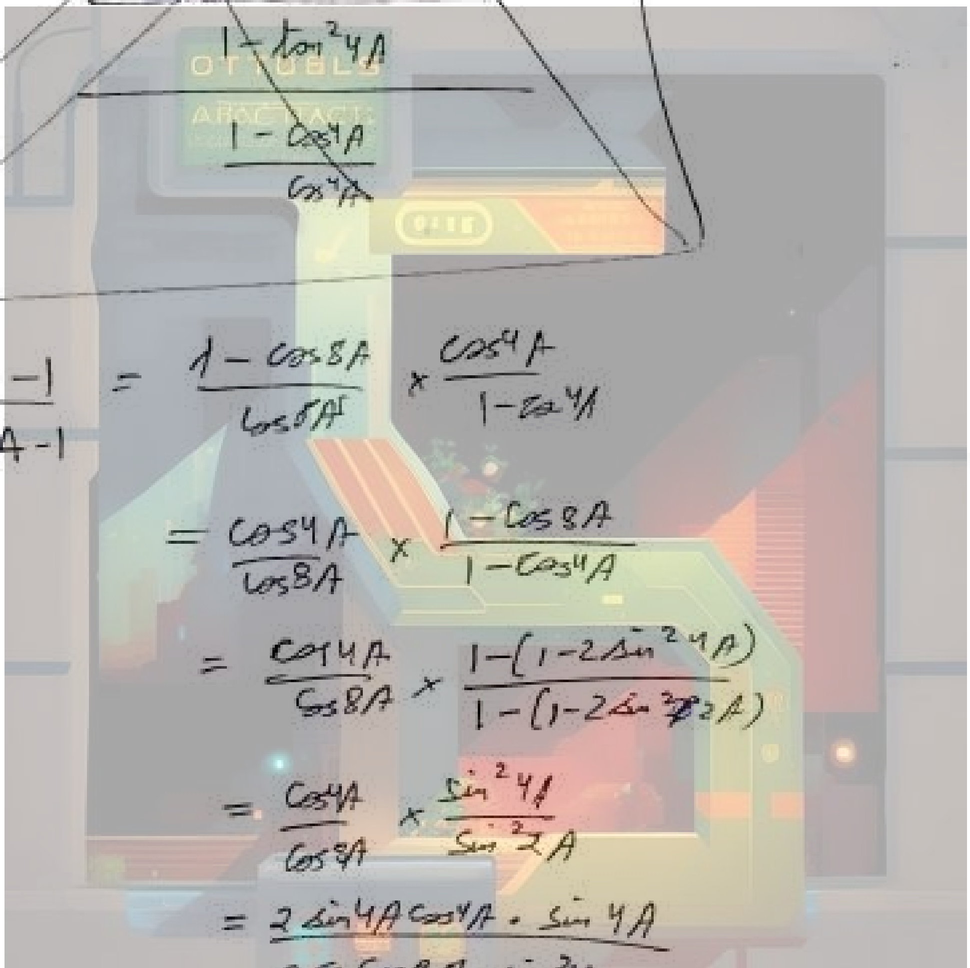
$=4(\cos^2 A+\sin^2 A)(\cos^2 A-\sin^2 A)$

$=4(\cos A-\sin A)(\cos A+\sin A)$

$=4\cos 2A$

$$\frac{\frac{1-\cos 8A}{\cos 8A(\cos}}{\frac{1-\cos 4A}{\cos 4A}} = \frac{\frac{1-\frac{1-\tan^2 4A}{1+\tan^2 4A}}{\frac{1-\tan^2 4A}{1+\tan^2 4A}}}{\frac{1-\cos 4A}{\cos 4A}}$$

$$= \frac{\frac{1+\tan^2 4A - 1 - \tan^2 4A}{1-\tan^2 4A}}{\frac{1-\cos 4A}{\cos 4A}}$$



$$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{1-\cos 8A}{\cos 8A} \times \frac{\cos 4A}{1-\cos 4A}$$

$$= \frac{\cos 4A}{\cos 8A} \times \frac{1-\cos 8A}{1-\cos 4A}$$

$$= \frac{\cos 4A}{\cos 8A} \times \frac{1-(1-2\sin^2 4A)}{1-(1-2\sin^2 2A)}$$

$$= \frac{\cos 4A}{\cos 8A} \times \frac{\sin^2 4A}{\sin^2 2A}$$

$$= \frac{2\sin 4A \cos 4A \cdot \sin 4A}{2\cos 8A \cdot \sin^2 2A}$$

$$= \tan 8A \times \frac{1}{2} \times 2\frac{\sin 2A \cos 2A}{\sin 2A \cdot \sin 2A}$$

$$\boxed{= \frac{\tan 8A}{\tan 2A}}$$

M.W. 22-7-24

DYS-10 (1-15)

Q find values of

(1) $\sin 22.5$ or $\sin \frac{\pi}{8}$

$\cancel{\sin 2\times 22.5 = 2\times \sin 22.5 \cos 22.5}$

(2) $\cos 22.5$

(3) $\tan 22.5$

$\cancel{\sin A}$

(1) $\frac{1-\cos 2A}{2} = \sin^2 A$

$$\sin A = \pm\sqrt{\frac{1-\cos 2A}{2}}$$

$$\sin 22.5 = \sqrt{\frac{1-\cos 45}{2}} \quad (\text{+ve as I st quad})$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$$

$$\boxed{\sin 22.5^\circ = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}$$

(2) $2\sin 45 = 2\sin 22.5 \cos 22.5$

$2\cdot\frac{1}{\sqrt{2}} = 2\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}\cos 22.5$ $\qquad$ $\cos 22.5 = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

$\cancel{\cos 22.5 = \frac{\sqrt{2\sqrt{2}}}{2\sqrt{(\sqrt{2}-1)2}}}$

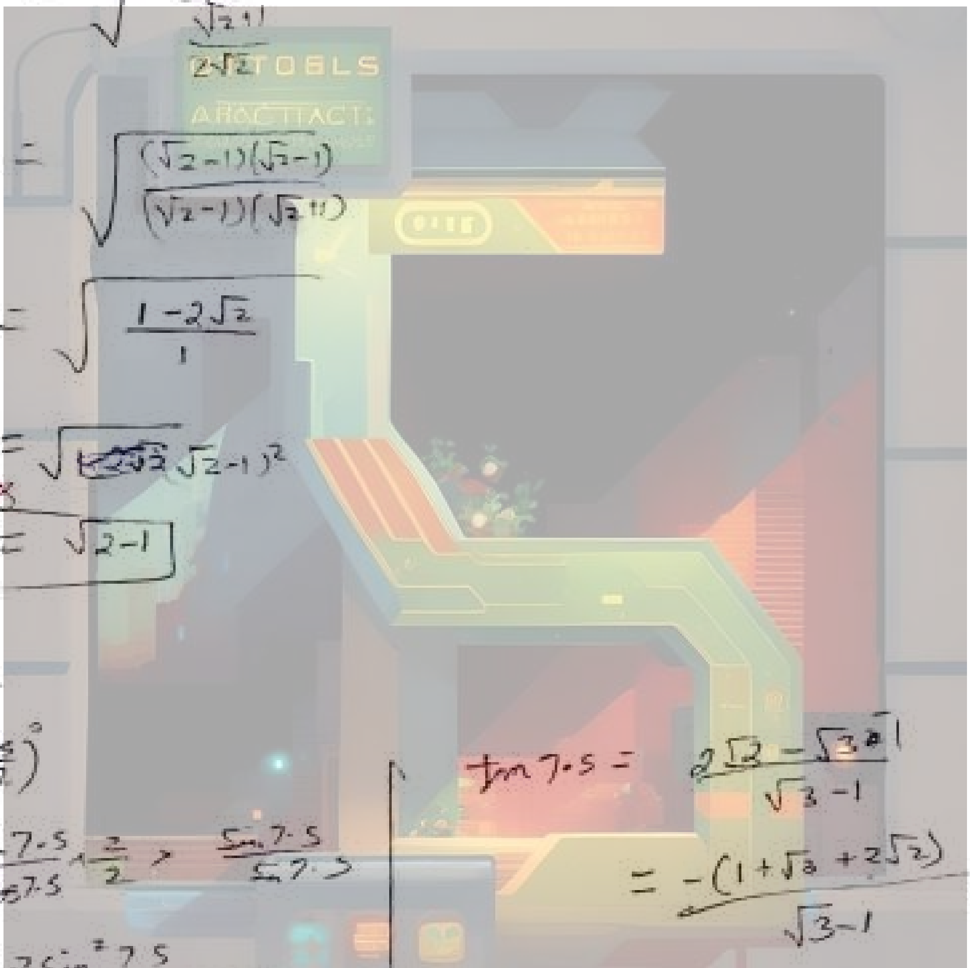
$\cancel{\cos 22.5 = \frac{\sqrt{\sqrt{2}}}{2\sqrt{\sqrt{2}-1}}}$

$\cancel{\cos 22.5 = \sqrt{\frac{\sqrt{2}}{4(\sqrt{2}-1)}}}$

(3) $\tan 22.5 = \frac{\sin 22.5}{\cos 22.5}$

$$= \frac{\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}}{\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}}$$

$$= \sqrt{\frac{\frac{\sqrt{2}-1}{2\sqrt{2}}}{\frac{\sqrt{2}+1}{2\sqrt{2}}}}$$



$$= \sqrt{\frac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)}}$$

$$= \sqrt{\frac{1-2\sqrt{2}}{1}}$$

$$= \sqrt{\cancel{1-2\sqrt{2}}\,(\sqrt{2}-1)^2}$$

$$\boxed{= \sqrt{2}-1}$$

(4) $\tan 7\frac{1}{2}$

$\tan\left(\frac{15}{2}\right)^\circ$

$$= \frac{2\sin 7.5}{\cos 7.5}\times\frac{2}{2} \times \frac{\sin 7.5}{\sin 7.5}$$

$$= \frac{2\sin^2 7.5}{2\sin 7.5\cos 7.5}$$

$$= \frac{1-\cos 2A}{\sin 2A}$$

$$= \frac{1-\cos 15^\circ}{\sin 15}$$

$$= \frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\tan 7.5 = \frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{-(1+\sqrt{3}+2\sqrt{2})}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{-(3+\sqrt{3}-2\sqrt{2}\sqrt{3}+\sqrt{3}+1-2\sqrt{2})}{2}$$

$$= \frac{-(4+2\sqrt{3}-2\sqrt{2}\sqrt{3}-2\sqrt{2})}{2}$$

$$= -(2+\sqrt{3}-\sqrt{2}\sqrt{3}-\sqrt{2})$$

$$= \sqrt{2}\sqrt{3}+\sqrt{2}-2-\sqrt{3}$$

$$= \sqrt{3}(\sqrt{2}-1)-\sqrt{2}(\sqrt{2}-1)$$

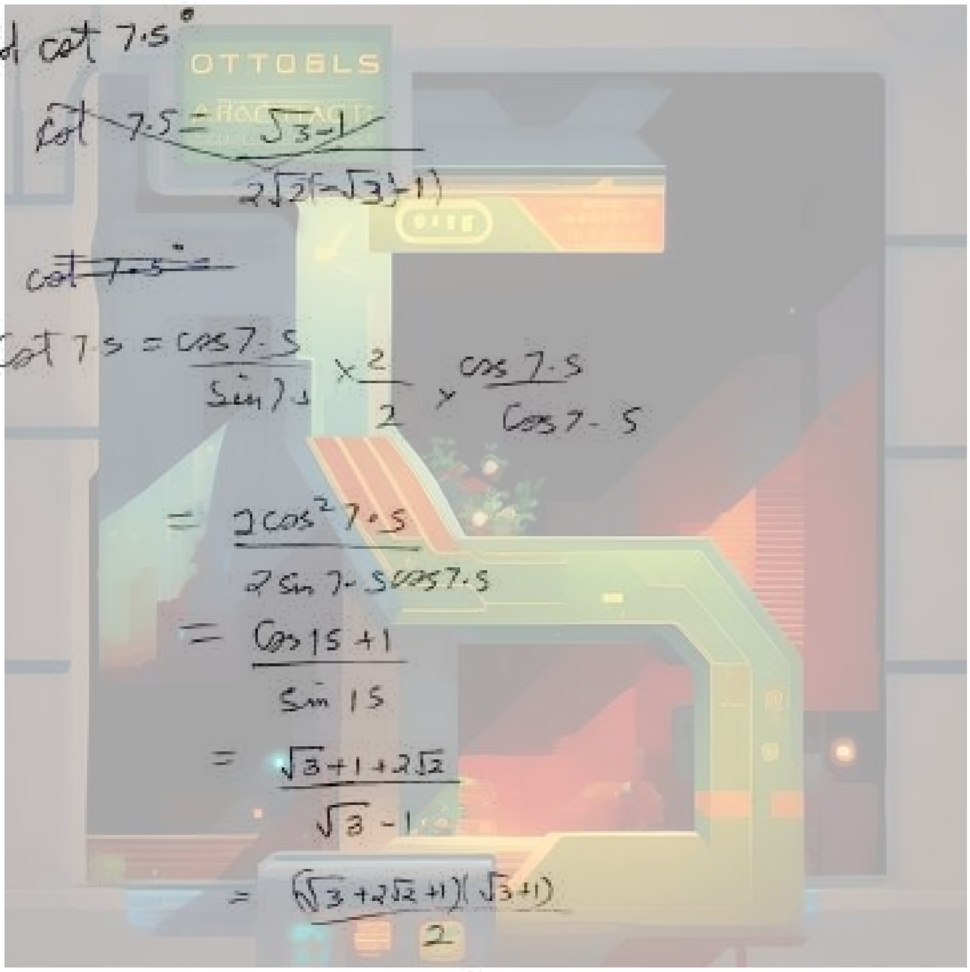
$$\boxed{\tan 7.5 = (\sqrt{3}-\sqrt{2})(\sqrt{2}-1)}$$

~~find cot 7.5~~

$\cancel{\cot 7.5 = \frac{1}{\tan 7.5} = \frac{1}{\sqrt{3}}}$

$$\tan 7.5^\circ = (\sqrt{3}-\sqrt{2}) \tan 22.5^\circ$$

$$\boxed{\frac{\tan 7.5^\circ}{\tan 22.5^\circ} = \sqrt{3}-\sqrt{2}}$$



Q find $\cot 7.5^\circ$

$\cancel{\cot 7.5 = \frac{\sqrt{3}-1}{2\sqrt{2}(-\sqrt{3})-1)}}$

$\cancel{\cot 7.5 =}$

$$\cot 7.5 = \frac{\cos 7.5}{\sin 7.5} \times \frac{2}{2} \times \frac{\cos 7.5}{\cos 7.5}$$

$$= \frac{2\cos^2 7.5}{2\sin 7.5 \cos 7.5}$$

$$= \frac{\cos 15 + 1}{\sin 15}$$

$$= \frac{\sqrt{3}+1+2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+2\sqrt{2}+1)(\sqrt{3}+1)}{2}$$

$$= \frac{3+\sqrt{3}+2\sqrt{2}\sqrt{3}+2\sqrt{2}+\sqrt{3}+1}{2}$$

$$= \frac{4+2\sqrt{3}+2\sqrt{2}+2\sqrt{2}\sqrt{3}}{2}$$

$$\boxed{\cos 7.5 = 2+\sqrt{3}+\sqrt{2}+\sqrt{3}\sqrt{2}}$$

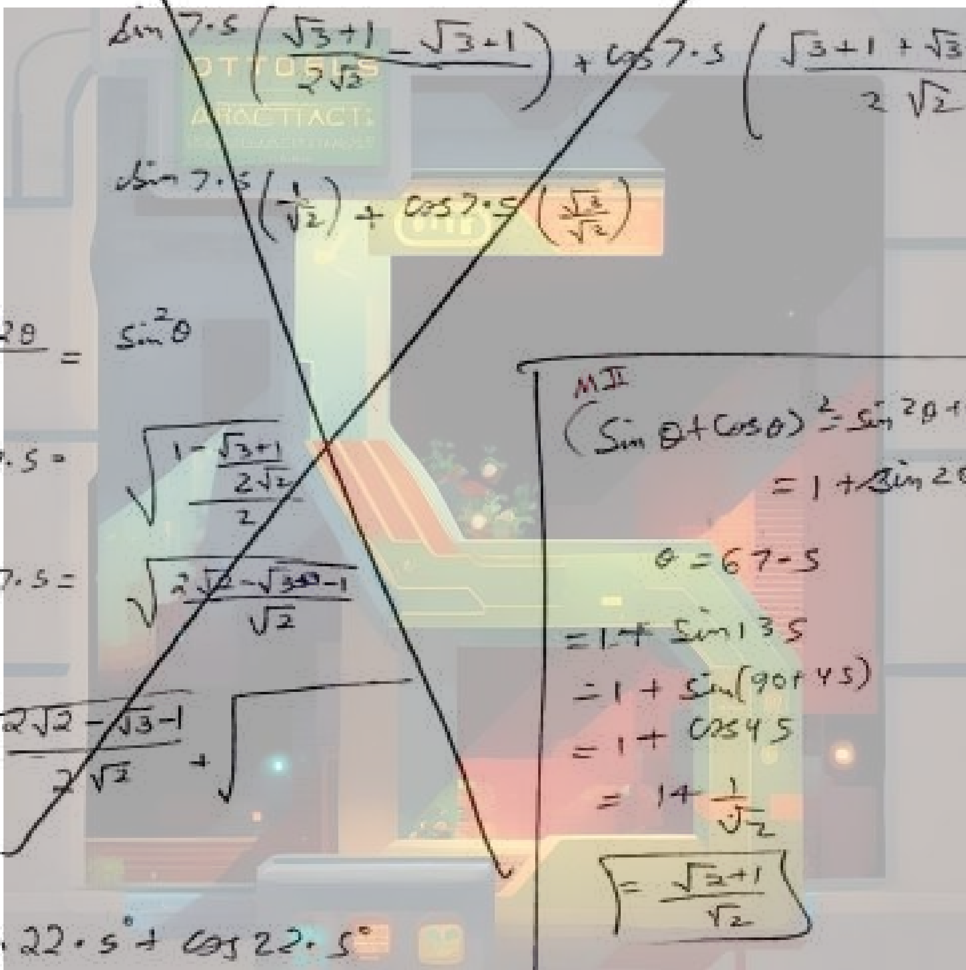
Q find $\cos 67\frac{1}{2}^\circ + \sin 67\frac{1}{2}$

$\cos\frac{135}{2} + \sin\frac{135}{2}$

$\cos(75 - 7.5) + \sin(75 - 7.5)$

$\cos 75\cos 7.5 + \sin 75\sin 7.5 + \sin 75\cos 7.5 - \sin 7.5\cos 75$

$\sin 7.5(\sin 75 - \cos 75) + \cos 7.5(\sin 75 + \cos 75)$



$\sin 7.5\left(\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos 7.5\left(\frac{\sqrt{3}+1+\sqrt{3}-1}{2\sqrt{2}}\right)$

$\sin 7.5\left(\frac{1}{\sqrt{2}}\right) + \cos 7.5\left(\frac{\sqrt{3}}{\sqrt{2}}\right)$

$\frac{1-\cos 2\theta}{2} = \sin^2\theta$

$\sin 7.5 = \sqrt{\frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{2}}$

$\sin 7.5 = \sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{\sqrt{2}}}$

$\sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{2\sqrt{2}}} + \sqrt{}$

$\sin 22.5^\circ + \cos 22.5^\circ$

$\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}} + \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

$$\boxed{\frac{\sqrt{\sqrt{2}-1} + \sqrt{\sqrt{2}+1}}{\sqrt{2\sqrt{2}}}}$$

M II

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta$

$= 1 + \sin 2\theta$

$\theta = 67.5$

$= 1 + \sin 135$

$= 1 + \sin(90 + 45)$

$= 1 + \cos 45$

$= 1 + \frac{1}{\sqrt{2}}$

$$\boxed{= \frac{\sqrt{2}+1}{\sqrt{2}}}$$

Q $\sqrt{3}\,\text{cosec}\,20^\circ - \sec 20^\circ$

$$\frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ}$$

$$\frac{\sqrt{3}}{\sin 20} - \frac{1}{\cos 20}$$

$$3\sin 20 - 4\sin^3 20 = \sin 60$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{\sin 20}{2}} - \frac{1}{\cos 20}$$

$$\frac{\sqrt{3}}{2} = 3t - 4t^3$$

$$4t^3 - 3t$$

$$\frac{\sin 60}{\cos 60 \sin 20} - \frac{1}{\cos 20}$$

$$8t^3 - 6t + \frac{\sqrt{3}}{2} = 0$$

$$\frac{\sin 60 \cos 20 - \sin 20 \cos 60}{\sin 20 \cos 20 \cdot \frac{1}{2}}$$

$$= \frac{2\sin 40}{\frac{\sin 40}{2}}$$

$$\boxed{= 4}$$

Q $\text{cosec}\,10^\circ - \sqrt{3}\sec 70^\circ$

$$\frac{1}{\sin 10} - \frac{\sqrt{3}}{\cos 10}$$

$$\frac{1}{\sin 10} - \frac{\cos 30}{\cos 10 \sin 30}$$

$$\frac{\cos 10 \sin 30 - \cos 30 \sin 10}{\sin 10 \cos 10 \times \frac{1}{2}}$$

$$\frac{2 \times 2\sin(30-10)}{\sin 10 \cos 10 \times 2}$$

$$\boxed{4}$$

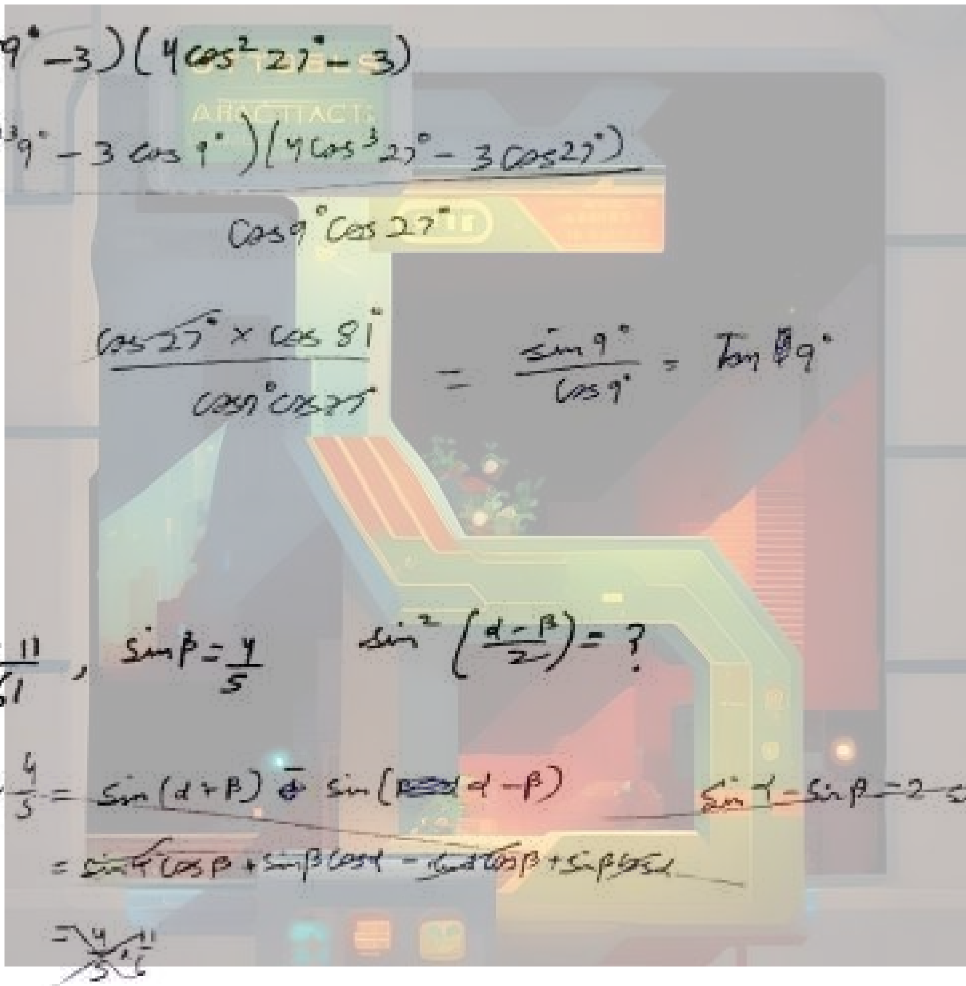
Q $-6\sin 40° + 8\sin^3 40°$

$-2(3\sin 40° - 4\sin^3 40°)$

$-2\sin(120°)$

$-2\cos 30$

$-2\times\frac{\sqrt{3}}{2} = \boxed{-\sqrt{3}}$



Q $(4\cos^2 9° - 3)(4\cos^2 27° - 3)$

$$\frac{(4\cos^3 9° - 3\cos 9°)(4\cos^3 27° - 3\cos 27°)}{\cos 9° \cos 27°}$$

$$\frac{\cos 27° \times \cos 81°}{\cos 9° \cos 27°} = \frac{\sin 9°}{\cos 9°} = \tan 9°$$

Q $\cos\alpha = \frac{11}{61}$, $\sin\beta = \frac{4}{5}$ $\quad \sin^2\left(\frac{\alpha-\beta}{2}\right) = ?$

$\cancel{2\times\frac{11}{61}\times\frac{4}{5} = \sin(\alpha+\beta) + \sin(\alpha-\beta)}$ $\quad \cancel{\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}$

$\cancel{= \sin\alpha\cos\beta + \sin\beta\cos\alpha - \sin\alpha\cos\beta + \sin\beta\cos\alpha}$

$\cancel{= \frac{4}{5}\times\frac{11}{6}}$

$\cos(\alpha-\beta) = 1 - 2\sin^2\left(\frac{\alpha-\beta}{2}\right)$

$\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1-\cos(\alpha-\beta)}{2}$

$= \frac{1-(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}{2}$

$= \frac{1-\left(\frac{11}{61}\times\frac{3}{5} + \frac{4}{5}\times\frac{60}{61}\right)}{2}$

$= \boxed{\frac{-136}{305}}$

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Important Points

(1) $\sin(60-\theta)\sin\theta\,\sin(60+\theta) = \frac{1}{4}\sin 3\theta$

(2) $\cos(60-\theta)\cos\theta\,\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

(3) ~~sin(60-θ)~~ $\tan(60-\theta)\tan\theta\,\tan(60+\theta) = \tan 3\theta$

proof :-

$\sin(60-\theta)\sin\theta\sin(60+\theta) = a$

$= (\sin^2 60 - \sin^2\theta)\sin\theta$

$= \frac{3-4\sin^2\theta}{4}(\sin\theta)$

$= \frac{3\sin\theta - 4\sin^3\theta}{4}$

$= \frac{\sin 3\theta}{4} = RHS.$

अति सिद्ध हुआ

(4) $\tan\theta + \tan(60+\theta) + \tan(120+\theta) = 3\tan 3\theta$

H.W. 23-7-24

DYS-10 (16-19)

DYS-11 (3, 9, 5, 6, 7)

DYS-12 (02, 3, 4, 06, ... 16)

O-1 (1-15)

Q find the values of

(1) $\sin 18°$

(2) $\cos 18°$

(3) $\tan 18°$

~~sin 18 =~~

~~$\sin 18 = \sin(\frac{90}{5})$~~

~~sin 18~~

~~$\sin 18 = \sin$~~

let $5\theta = 90°$

$$2\theta + 3\theta = 90°$$

$$2\theta = 90° - 3\theta$$

$$\sin 2\theta = \sin(90° - 3\theta)$$

$$\sin 2\theta = \cos 3\theta$$

$$2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta = 0$$

$$2\sin\theta\, c - 4c^3 + 3c = 0$$

$$c \neq 0$$

$$2s - 4c^2 + 3 = 0$$

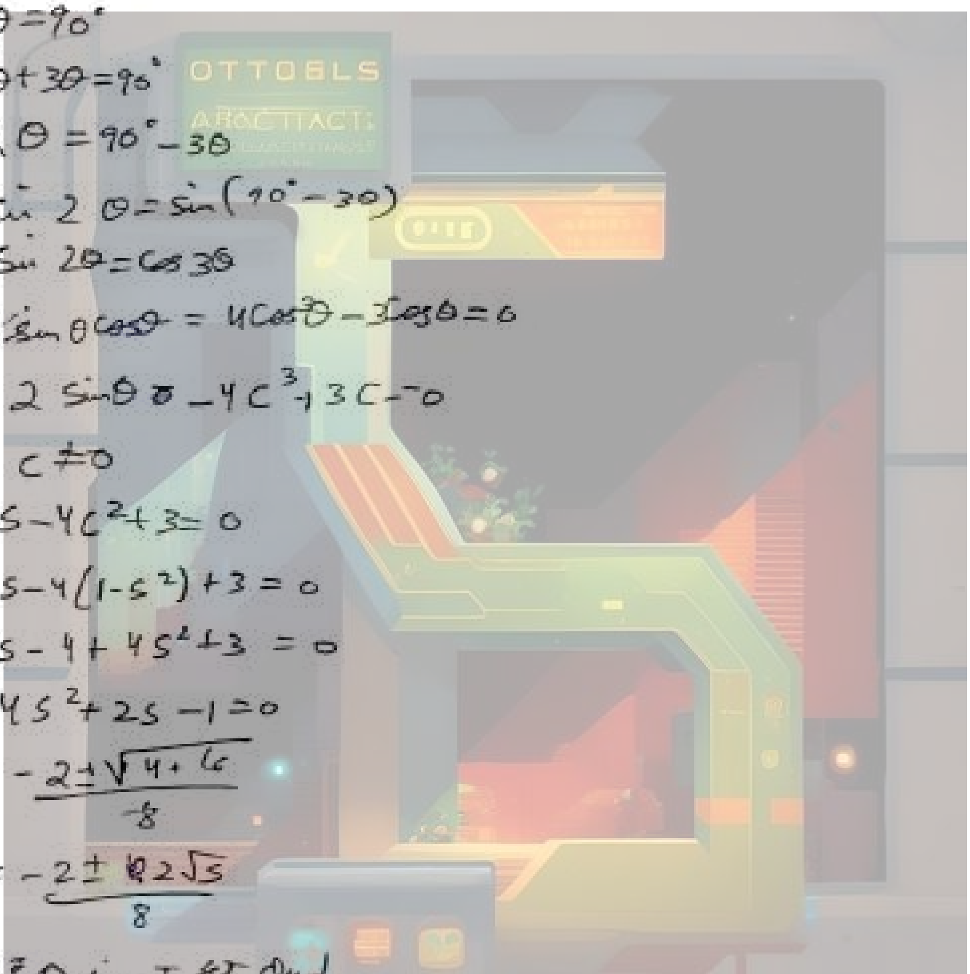
$$2s - 4(1 - s^2) + 3 = 0$$

$$2s - 4 + 4s^2 + 3 = 0$$

$$4s^2 + 2s - 1 = 0$$

$$s = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8}$$



$\sin\theta$ in I st Quad

$$s = \frac{-1 + \sqrt{5}}{4}$$

$$\boxed{\sin 18° = \frac{\sqrt{5} - 1}{4}}$$

(2) $\cos 18^\circ$

$\cos 5\theta = \cos 90^\circ$

$\cos 2\theta + \cos 3\theta = \cos 90^\circ$

$\cos 2\theta = \cos(90 - 3\theta)$

$\cos 2\theta = \sin 3\theta$

$1 - 2\sin^2\theta = 4\sin^3\theta - 3\sin\theta$

$4\sin^3\theta + 2\sin^2\theta - 3\sin\theta - 1 = 0$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\cos 18^\circ = \sqrt{1 - \sin^2 18}$

$\cos 18^\circ = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$

$= \sqrt{1 - \left(\frac{6 - 2\sqrt{5}}{16}\right)}$

$= \sqrt{\frac{16 - 6 + 2\sqrt{5}}{16}}$

$\cos 18 = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

$$\boxed{\cos 18^\circ = \frac{\sqrt{5+\sqrt{5}}}{2\sqrt{2}}}$$

(3) $\tan 18^\circ = \frac{\sin 18^\circ}{\cos 18^\circ}$

$= \frac{\sqrt{5}-1}{\cancel{4}\,2} \times \frac{2\sqrt{2}}{\sqrt{5+\sqrt{5}}}$

$\cancel{= \sqrt{10} - \sqrt{2}}$

$$\boxed{\tan 18^\circ = \frac{\sqrt{2}(\sqrt{5}-1)}{2\sqrt{5+\sqrt{5}}}}$$

~~(4) $\sin 36^\circ$~~

~~$\sin 2\theta = \sin 2\times 18^\circ$~~

~~$= 2\sin 18 \cos 18$~~

~~$= 2\times \frac{(\sqrt{5}-1)}{4}$~~

(4) $\cos 36^\circ$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 36 = \cancel{2}\ \frac{\sqrt{5}+5}{\cancel{8}4} - 1$$

$$= \frac{\sqrt{5}+5-4}{4}$$

$$\boxed{= \frac{\sqrt{5}+1}{4}}$$

(5) $\sin 36^\circ$

$$\sin 36 = \sqrt{1-\cos^2 36}$$

$$= \sqrt{1-\frac{(6+2\sqrt{5})}{16}}$$

$$= \sqrt{\frac{16-6-2\sqrt{5}}{16}}$$

$$= \frac{\sqrt{10-2\sqrt{5}}}{4}$$

$$\boxed{= \frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}}$$

Q find the value ~~of~~ sin 9° & cos 9°

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$\sin 9 = \sqrt{\frac{1-\cos 18}{2}}$$

$$\sin 9 = \sqrt{\frac{1-\frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}}{2}}$$

$$\boxed{\sin 9^\circ = \sqrt{\frac{2\sqrt{2}-\sqrt{5-\sqrt{5}}}{4\sqrt{2}}}}$$

$$\cos 9^\circ = \sqrt{\frac{1+\sin 18}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{5}-1}{4}}{2}}$$

$$\boxed{\cos 9^\circ = \sqrt{\frac{\sqrt{5}+3}{8}}}$$

Q ~~sin 18 + sin 54~~

$\sin\theta + \sin 3\theta$

$\sin\theta + 3\sin\theta - 4\sin^3\theta$

$4\sin\theta - 4\sin^3\theta$

$4(\sin\theta - \sin^3\theta)$

$4\sin 18(1 - \sin^2 18)$

$4\sin 18 \times \cos^2 18$

$4\frac{(\sqrt{5}-1)}{4} \times \frac{\sqrt{5}+5}{8}$

Q $\sin 18 + \sin 234$

$\sin 18 - \sin 54$

$\sin\theta - 3\sin\theta + 4\sin^3\theta$

$-2\sin\theta + 4\sin^3\theta$

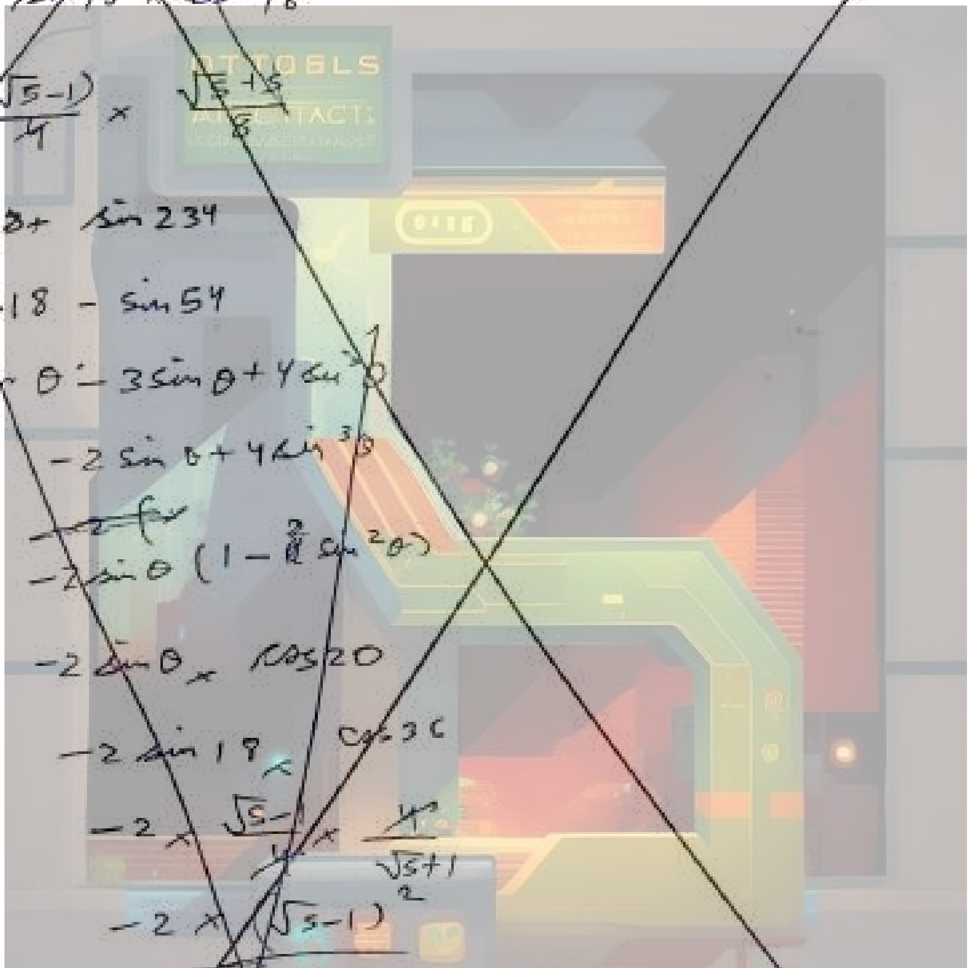
~~$-2\sin$~~

$-2\sin\theta(1 - 2\sin^2\theta)$

$-2\sin\theta \times \cos 2\theta$

$-2\sin 18 \times \cos 36$

$-2 \times \frac{\sqrt{5}-1}{4} \times \frac{4}{\sqrt{5}+1}$



$-2 \times \frac{(\sqrt{5}-1)^2}{4}$

$= \frac{-6 + 2\sqrt{5}}{2}$

$\boxed{\sqrt{5} - 3}$

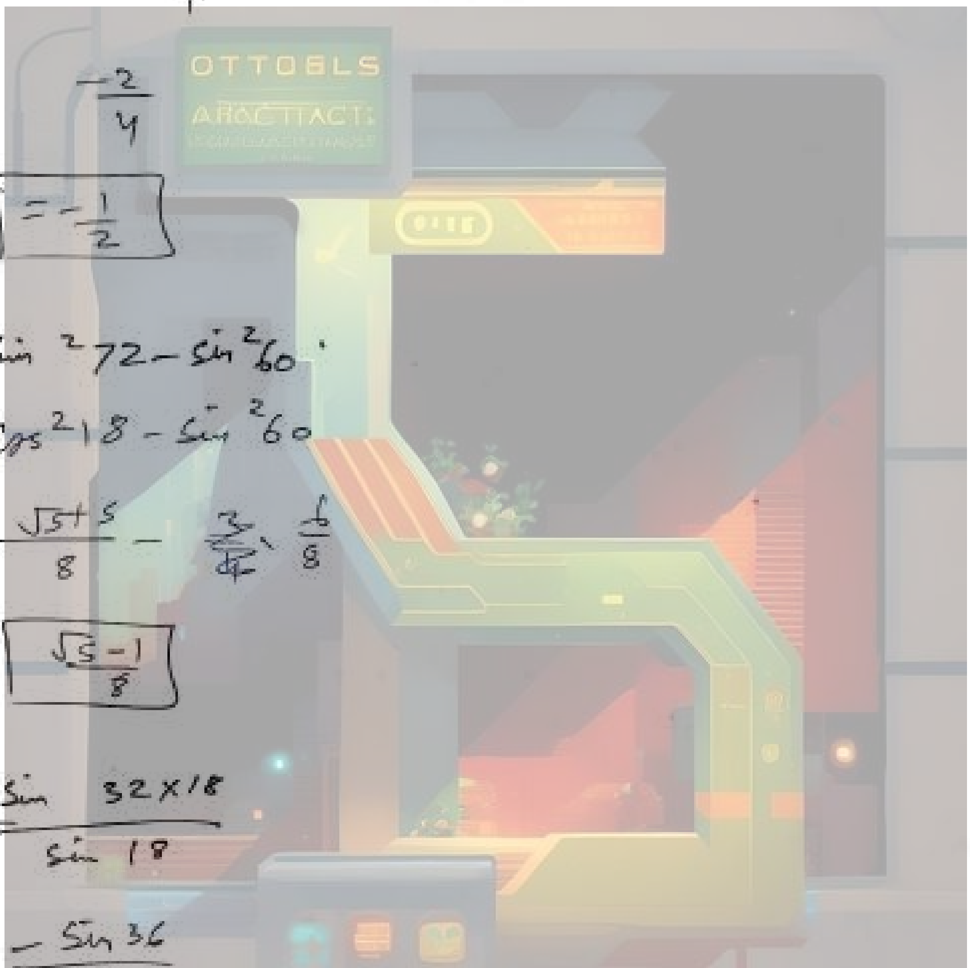
Q $\sin\frac{\pi}{10} + \sin\frac{17\pi}{10}$

$\sin 18 + \sin 234$

$\sin 18 - \sin 54$

$\sin 18 - \cos 36$

$\frac{\sqrt{5}-1 - \sqrt{5}-1}{4}$



$\frac{-2}{4}$

$\boxed{= -\frac{1}{2}}$

Q $\sin^2 72 - \sin^2 60$

$\cos^2 18 - \sin^2 60$

$\frac{\sqrt{5}+5}{8} - \frac{3}{\cancel{4}}, \frac{6}{8}$

$\boxed{\frac{\sqrt{5}-1}{8}}$

Q $\frac{\sin 2\times 18}{\sin 18}$

$\frac{\sin 36}{\sin 18}$

$\frac{\sqrt{\sqrt{5}+5}}{\cancel{2}\sqrt{2}} \times \frac{\cancel{4}\,\cancel{2}\sqrt{2}}{\sqrt{5}-1}$

$\boxed{= \frac{2\sqrt{5}+10}{1-\sqrt{5}}}$

Series Based Questions

① Product or addition Based series

→ Basic knowledge of angles ($15°, 18°, 22.5°, 36°$,)

→ Complimentory or Supplimentory Angles pairing search.

→ formula based series

* Sine Series – Angles in AP
* Cosine series – Angles in AP
* Product series of cos – Angles in GP

→ $\sin^4\theta + \cos^4\theta = 1 - 2s^2c^2$

$$= 1 - \frac{1}{2} \times 4s^2c^2$$

$$= 1 - \frac{(2\sin\theta\cos\theta)^2}{2}$$

$$\boxed{= 1 - \frac{\sin^2 2\theta}{2}}$$

→ $\sin^6\theta + \cos^6\theta = 1 - 3s^2c^2$

$$= 1 - \frac{3}{4} \times 4s^2c^2$$

$$\boxed{= 1 - \frac{3}{4}\sin^2 2\theta}$$

Q find values :-

① $\sin 36° \times \sin 2\times 36 \times \sin 3\times 36 \times \sin 4\times 36$

② $\tan\frac{\pi}{8}\ \tan\frac{3\pi}{8}\ \tan\frac{5\pi}{8}\ \tan\frac{7\pi}{8}$

③ $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

(4) $\sin^4\frac{\pi}{12} + \sin^4\frac{3\pi}{12} + \sin^4\frac{9\pi}{12} + \sin^4\frac{11\pi}{12}$

(5) $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$

(1) $\sin\frac{\pi}{5} \cdot \sin\frac{4\pi}{5} \cdot \sin\frac{2\pi}{5} \cdot \sin\frac{3\pi}{5}$

$\sin\frac{\pi}{5} \cdot \sin\left(\pi-\frac{\pi}{5}\right) \cdot \sin\frac{2\pi}{5} \sin\left(\pi-\frac{2\pi}{5}\right)$

$\left(\sin\frac{\pi}{5}\right)^2 \times \left(\sin\frac{2\pi}{5}\right)^2$

$\left(\sin\left(\frac{\pi}{5}\right)\right)^2 \times \cancel{\cos}\left(\cos\frac{2\pi}{5}\right)^2$

$\boxed{\sin^2 36 \cdot \cos^2 18}$

(2) $\tan\frac{\pi}{8} \cdot \tan\frac{7\pi}{8} \tan\frac{3\pi}{8} \tan\frac{5\pi}{8}$

$\tan\frac{\pi}{8} \tan\left(\pi-\frac{\pi}{8}\right) \times \tan\frac{3\pi}{8} \tan\left(\pi-\frac{3\pi}{8}\right)$

$-\left(\tan\frac{\pi}{8}\right)^2 \times -\left(\tan\frac{3\pi}{8}\right)^2$

$(\tan 22.5)^2 \times \cancel{\tan} \cot(22.5)^2$

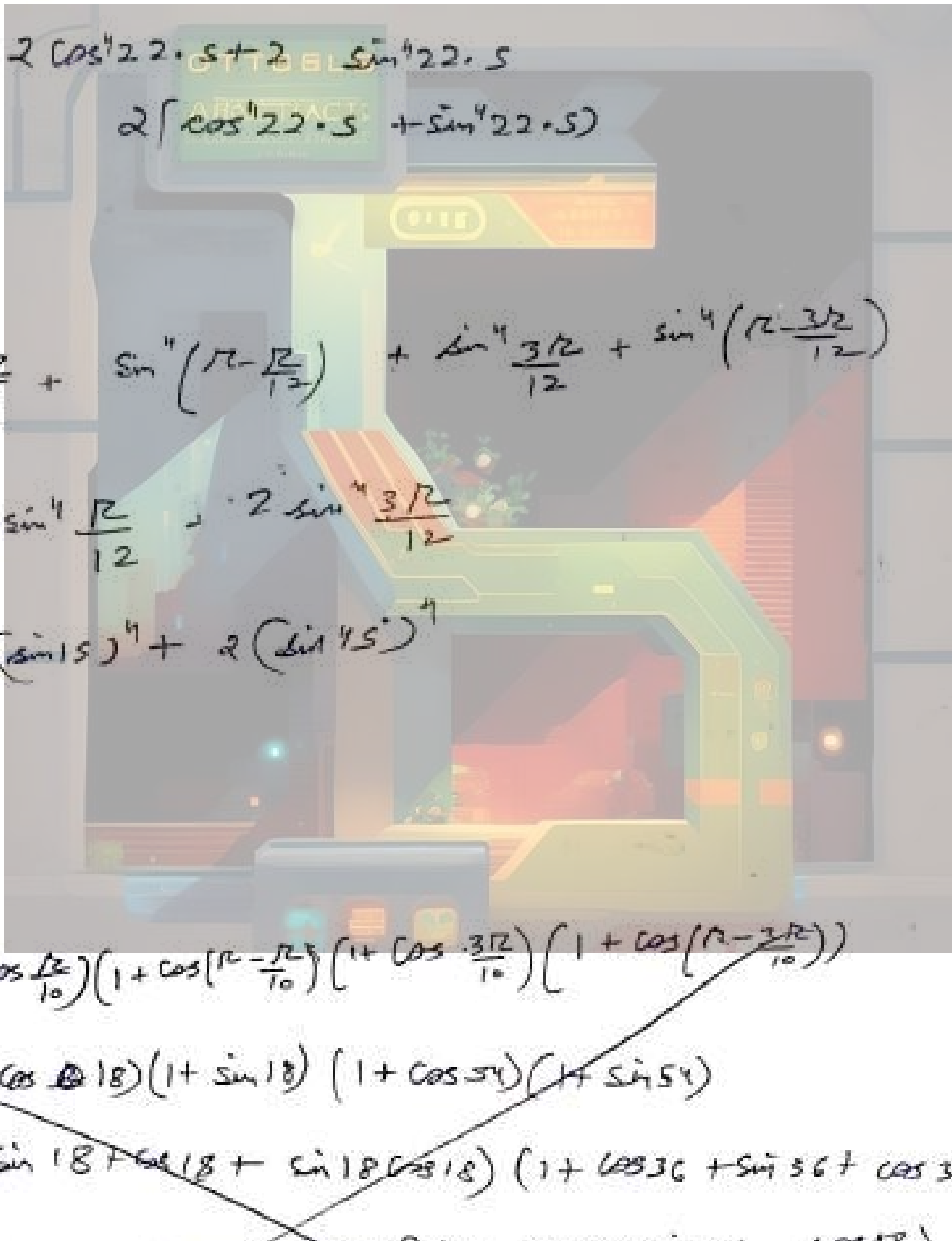
$\boxed{=1}$

(3) $\cos^4\frac{\pi}{8}+\cos^4\left(\pi-\frac{\pi}{8}\right)+\cos^4\left(\pi\frac{3\pi}{8}\right)+\cos^4\left(\pi-\frac{3\pi}{8}\right)$

$\cos^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{3\pi}{8}$

(4)

$2\cos^4 22.5+2\cos^4 67.5$



$2\cos^4 22.5+2\sin^4 22.5$

$2(\cos^4 22.5+\sin^4 22.5)$

(4) $\sin^4\frac{\pi}{12}+\sin^4\left(\pi-\frac{\pi}{12}\right)+\sin^4\frac{3\pi}{12}+\sin^4\left(\pi-\frac{3\pi}{12}\right)$

$2\sin^4\frac{\pi}{12}+2\sin^4\frac{3\pi}{12}$

$2(\sin 15)^4+2(\sin 45^\circ)^4$

(5) $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\left(\pi-\frac{\pi}{10}\right)\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\left(\pi-\frac{3\pi}{10}\right)\right)$

$(1+\cos 18)(1+\sin 18)(1+\cos 54)(1+\sin 54)$

$(1+\sin 18+\cos 18+\sin 18\cos 18)(1+\cos 36+\sin 36+\cos 36\sin 36)$

$\left(1+\sin 18+\cos 18+\frac{\sin 36}{2}\right)\left(1+\cos 36+\sin 36+\frac{\cos 18}{2}\right)$

Q5. $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\left(\pi-\frac{\pi}{10}\right)\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\left(\pi-\frac{3\pi}{10}\right)\right)$

$$\left(1+\cos\frac{\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)$$

$$\left(1^2-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$$

$$\sin^2\frac{\pi}{10}\times\sin^2\frac{3\pi}{10}$$

$$\sin^2 18\times\sin^2 54$$

$$\boxed{\sin^2 18\cos^2 36}$$

H.W. 25-7-21)

DYS-12 [17, 18]

DYS-13 [1, 4]

O-1 [16, 21]

J-M [1, 5] ∪ [9, 12]

Continued product of cosine with angles in GP $(r=2)$

$$\cos A\cdot\cos 2A\cdot\cos 4A\ldots\cos(2^{n-1}A)\times\frac{2\sin A}{2\sin A}$$

$$\frac{\sin 2A\cos 2A\ \cos 4A\ldots\cos(2^{n-1}A)}{2\sin A}\times\frac{2}{2}$$

$$\frac{\sin 4A\ \cos 4A\ldots\cos(2^{n-1}A)}{2^2\sin A}\times\frac{2}{2}$$

$$\boxed{=\frac{\sin(2^n A)}{2^n\sin A}}$$

Q find the values of the following:-

(1) $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$

$$= \frac{\sin(2^n\theta)}{2^n\sin\theta}$$

$$= \frac{\sin(8\times 20)}{8\times\sin 20}$$

$$= \frac{\sin\cancel{\cos 160}(180-20)}{8\sin 20}$$

$$= \frac{\sin 20}{8\sin 20}$$

$$\boxed{=\frac{1}{8}}$$

(2) $\cos\frac{16\pi}{10}\cos\frac{8\pi}{10}\cos\frac{4\pi}{10}\cos\frac{2\pi}{10}\cos\frac{\pi}{10}$

$$= \frac{\sin(2^n\theta)}{2^n\sin\theta}$$

$$= \frac{\sin(32\times 18)}{32\sin 18}$$

$$= \frac{-\sin 36}{32\sin 18}$$

$$\cancel{= \frac{-\sqrt{\frac{5-\sqrt{5}}{8}}}{32\times(\sqrt{5}-1)}}$$

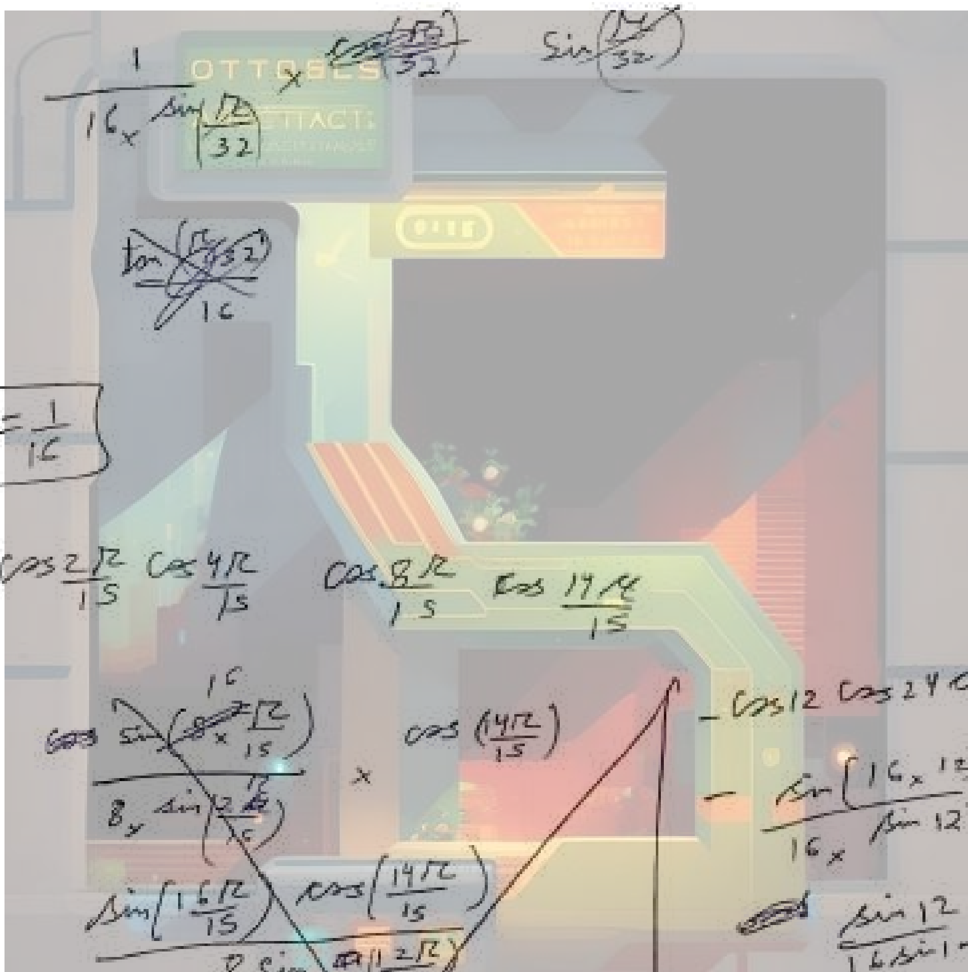
$$= \frac{-\cos 18}{16}$$

$$= \frac{-\sqrt{\frac{5+\sqrt{5}}{8}}}{16}$$

(3) $\cos\frac{\pi}{2^2}\cos\frac{\pi}{2^3}\cos\frac{\pi}{2^4}\cos\frac{\pi}{2^5}\cancel{\left(\cos\frac{\pi}{2^3}\right)}\quad \sin\left(\frac{\pi}{32}\right)$

$\downarrow$

$$\frac{\sin\left(2\cdot 16\times\frac{\pi}{2\cdot 32}\right)}{16\times\sin\frac{\pi}{32}}$$



$$\frac{1}{16\times\sin\left(\frac{\pi}{32}\right)}\times\cancel{\cos\left(\frac{\pi}{32}\right)}\quad\sin\left(\frac{\pi}{32}\right)$$

$$\cancel{\frac{\tan\left(\frac{\pi}{32}\right)}{16}}$$

$$\boxed{=\frac{1}{16}}$$

(4) $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$

$$\frac{\sin\left(16\times\frac{\pi}{15}\right)}{8\times\sin\left(\frac{2\pi}{15}\right)}\times\cos\left(\frac{14\pi}{15}\right)$$

$$\frac{\sin\left(\frac{16\pi}{15}\right)\cos\left(\frac{14\pi}{15}\right)}{8\sin\left(\frac{2\pi}{15}\right)}$$

$$\frac{-\left(\sin\frac{\pi}{15}+\cos\frac{\pi}{15}\right)}{8\times\cos\frac{3\pi}{15}\ \sin 36}$$

$$\frac{-(\sin 12+\cos 12)}{\sqrt{\frac{5-\sqrt{3}}{8}}\times 8}$$

$= \cos 12\cos 24\cos 48\cos 96$

$$=\frac{\sin(16\times 12)}{16\times\sin 12^\circ}$$

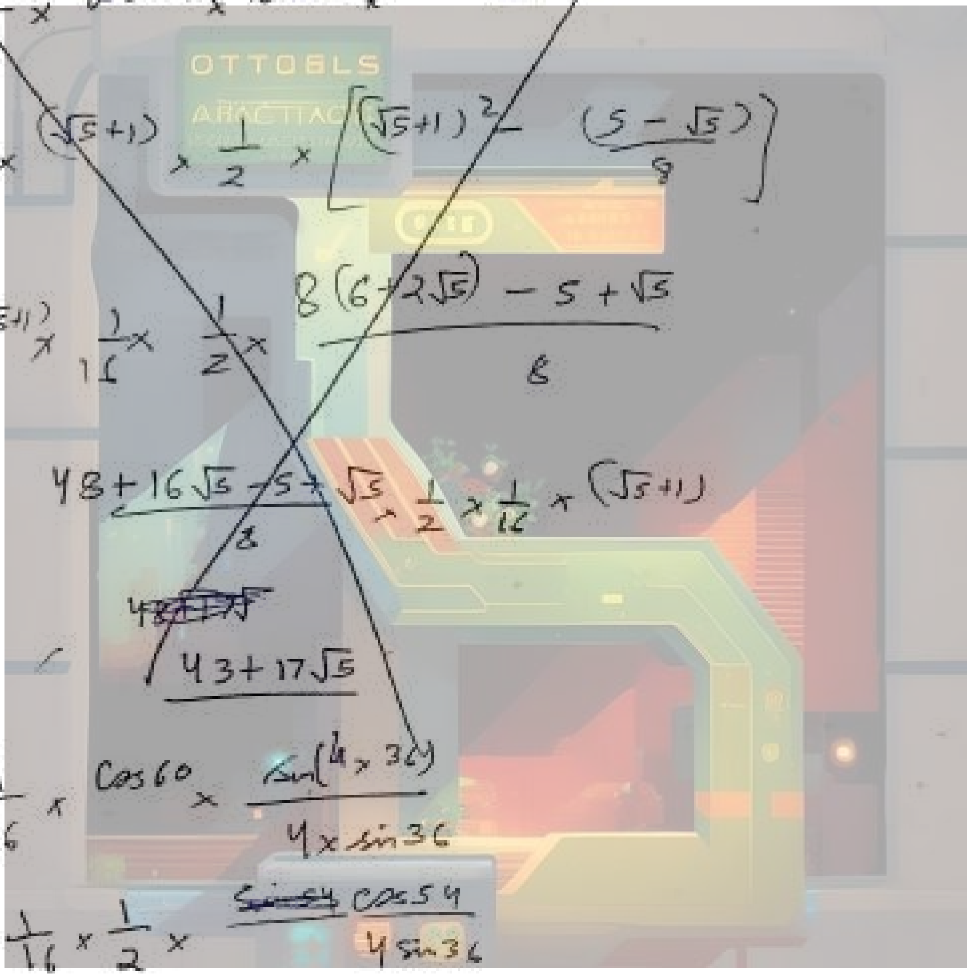
$$\frac{\sin 12}{16\sin 12}$$

$$\boxed{=\frac{1}{16}}$$

(5) $\underbrace{\cos\frac{\pi}{15}\cos\frac{2\pi}{15}}\cos\frac{4\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

$\cos\frac{7\pi}{15} \downarrow -\cos\frac{8\pi}{15}$

$$-\frac{\sin\left(16\times\frac{\pi}{15}\right)}{16\sin\left(\frac{\pi}{15}\right)}$$



$\frac{1}{16}\times\cos 36\times\cos 60\times\cos 2(36)$

$\frac{1}{16}\times(\sqrt5+1)\times\frac{1}{2}\times\left[(\sqrt5+1)^2 - \frac{(5-\sqrt5)}{8}\right]$

$(\sqrt5+1)\times\frac{1}{16}\times\frac{1}{2}\times\frac{8(6+2\sqrt5)-5+\sqrt5}{8}$

$\frac{48+16\sqrt5-5+\sqrt5}{8}\times\frac{1}{2}\times\frac{1}{16}\times(\sqrt5+1)$

$\cancel{43+17\sqrt5}$

$43+17\sqrt5$

$\frac{1}{16}\times\cos 60\times\frac{\sin(4\times36)}{4\times\sin 36}$

$\frac{1}{16}\times\frac{1}{2}\times\frac{\cancel{\sin 54}\cos 54}{4\sin 36}$

$\frac{1}{32}\times\frac{\sin 36}{4\sin 36}$

$\boxed{\frac{1}{128}}$ ✓

Q $\sin\frac{\pi}{14}\,\sin\frac{3\pi}{14}\,\sin\frac{5\pi}{14}\,\sin\frac{7\pi}{14}$

$$\cancel{\cos\frac{13\pi}{14}\cos\frac{11\pi}{14}\cos\frac{9\pi}{14}\cos\frac{7\pi}{14}}$$

$$\cos\frac{\cancel{6}^{\,3}\pi}{\cancel{14}_{\,7}}\;\underbrace{\cos\frac{\cancel{4}^{\,2}\pi}{\cancel{14}_{\,7}}\;\cos\frac{\cancel{2}^{\,1}\pi}{\cancel{14}_{\,7}}}\;\underbrace{\cos 0^\circ}_{1} = 0$$

$$\cancel{\frac{\sin\left(4\times\frac{\pi}{14}\times 2\right)}{4\times\sin\frac{2\pi}{14}}\times\frac{\sin\frac{4\pi}{7}}{4\sin\frac{2\pi}{14}}\times\cos\frac{5\pi}{14} = \frac{\sin\frac{4\pi}{14}}{2\times4\times\sin\frac{\pi}{14}\cos\frac{\pi}{14}}\times\cos\frac{\pi}{14}}$$

$$\cancel{\frac{\sin\left(\frac{4\pi}{7}\right)\cos\frac{8\pi}{7}}{4\times\sin\frac{\pi}{7}}}$$

$$\cancel{\frac{2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7}}{4\sin\frac{\pi}{7}}\times\cos\frac{3\pi}{7}}$$

$$\cancel{\frac{2\sin\frac{\pi}{7}\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\times\cos\frac{3\pi}{7}}{2^{4}\sin\frac{\pi}{7}}}$$

$$\cancel{\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}}$$

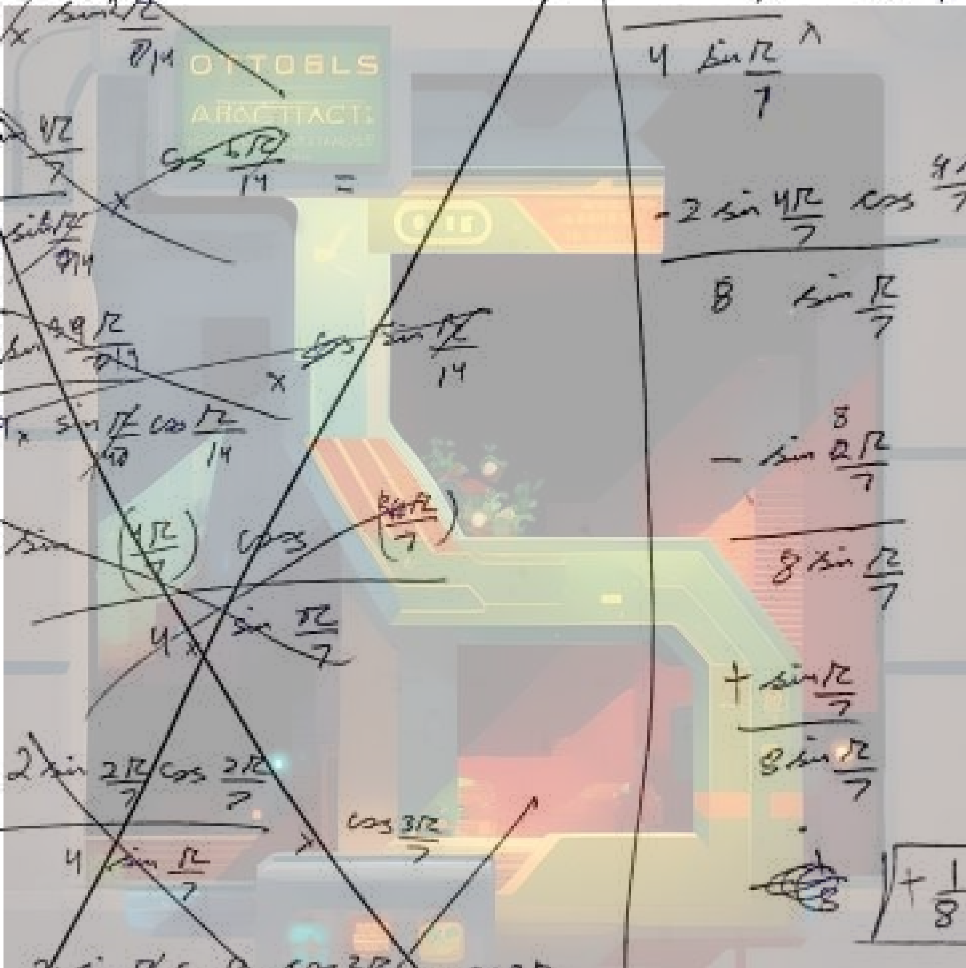
$$\frac{\sin\frac{4\pi}{7}}{4\sin\frac{\pi}{7}}\times\cos\frac{3\pi}{7}$$

$$\frac{-2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{8\sin\frac{\pi}{7}}$$

$$\frac{-\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}$$

$$\frac{+\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}}$$

$$\boxed{+\frac{1}{8}}$$



Conditional identities - Identities related to triangles.

$\rightarrow A+B+C=180^\circ/\pi$

$\rightarrow \frac{A}{2}+\frac{B}{2}+\frac{C}{2}=90^\circ$

If :- $A+B+C=180^\circ$ then :-

① $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

② $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

③ $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

④ $\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{A}{2}\tan\frac{C}{2}=1$

⑤ $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

⑥ $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Proof :-

① $\sin 2A+\sin 2B+\sin 2C$

$\sin 2A+2\sin(B+C)\cos(B-C)$

$\because A+B+C=180^\circ$

$B+C=180-A$

$2\sin A\cos A+2\sin(180-A)\cos(B-C)$

$2\sin A\cos A+2\sin A\cos(B-C)$

$2\sin(\cos A+\cos(B-C))$

$4\sin A\cos\left(\frac{180-C-C}{2}\right)\cos\left(\frac{180-B-B}{2}\right)$

$4\sin A\cos(90-C)\cos(90-B)$

$4\sin A\sin C\sin B.$

~~(5) $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$~~

Q $\dfrac{\sin 50 + \sin 100 + \sin 210}{\sin 25 * \sin 50 * \sin 105}$

$\dfrac{4 \sin 25 \sin 50 \sin 105}{\cancel{4 \cos} \sin A \sin B \sin C} = \boxed{4}$

Q $x+y+z = xyz$ · Prove

(1) $\dfrac{2x}{1-x^2} + \dfrac{2y}{1-y^2} + \dfrac{2z}{1-z^2} = \left(\dfrac{2x}{1-x^2}\right)\left(\dfrac{2y}{1-y^2}\right)\left(\dfrac{2z}{1-z^2}\right)$

~~$x = \tan A \quad y = \tan B \quad z = \tan C$~~

~~$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B + \tan 2C$~~

~~$\tan$~~

~~$x = \tan A \qquad y = \tan B \qquad z = \tan C$~~

~~$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$~~

~~$2A + 2B + 2C = 180$~~

~~$A+B+C = 90°$~~

~~$\tan A + \tan B + \tan C$~~

~~$\tan A \tan B + \tan$~~

$x = \tan A \quad \cdot \quad y = \tan B \qquad z = \tan C$

$A+B+C = \cancel{\quad} 180$

$\Rightarrow 2A + 2B + 2C = 360$

$2A + 2B = 360 - 2C$

$\tan(2A+2B) = \cancel{\tan} - \tan 2C$

$$\frac{\tan 2A + \tan 2B}{1 - \tan 2A \tan 2B} = -\tan 2C$$

$$\tan 2A + \tan 2B = -\tan 2C(1 - \tan 2A \tan 2B)$$

$$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B \tan 2C$$

$$\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2z}{1-z^2} = \left(\frac{2x}{1-x^2}\right)\left(\frac{2y}{1-y^2}\right)\left(\frac{2z}{1-z^2}\right)$$

अतः सिद्धम्

H.W. 26 - 7 - 24

DVS - 814 [1 - 5] y [11 - 15]

Maximizing & Minimizing :- (5 methods)

① Using Boundry :- (mainly for $\sin x$ & $\cos x$)

★ $\sin x \in [-1, 1]$ $\sin^2 x \in [0, 1]$

★ $\sin x \in [-\frac{1}{2}, \frac{4}{5}]$ find $\sin^2 x$

check at $-\frac{1}{2}$, $\frac{4}{5}$ & 0

$[0, \frac{16}{25}]$

★ $\sin x \in [-\frac{3}{4}, -\frac{1}{2}]$

$\sin^2 x \in [\frac{1}{4}, \frac{9}{16}]$

★ $\tan x \in (-\infty, \infty)$ $\tan^2 x \; [0, \infty)$

★ $\sec x \in (-\infty, -1] \cup [1, \infty)$ $\sec^2 x \in [1, \infty)$

- Addition, Subtraction, Multiplication & Division of two ranges is <u>not</u> <u>allowed.</u>

$$\cancel{[-1,1] + [-1,1] = [-2,2]}$$

$$\frac{(\sin x + \cos x)\sqrt{2}}{\sqrt{2}}$$

$$(\sin x \times \sin 45 + \cos x \cos 45)\sqrt{2}$$

$$\sqrt{2}\cos \cancel{(45 - x)}(x - 45)$$

valid.

$$\sqrt{2}\,[-1, 1]$$

$$= [-\sqrt{2}, \sqrt{2}]$$

→ we can add, subtract, multiply or ~~divide~~ divid any constant except 0. in range.

eg. ① $y = 2 + \sin x$

$$= 2 + [-1, 1]$$

$$= [1, 3]$$

$$y \in [1, 3]$$

② $y = 1000 \tan x$

$$= 1000\,[-\infty, \infty)$$

$$y \in (-\infty, \infty)$$

③ $y = \frac{\cos x}{\sqrt{3}}$

$$= \frac{[-1, 1]}{\sqrt{3}}$$

$$y \in \left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$$

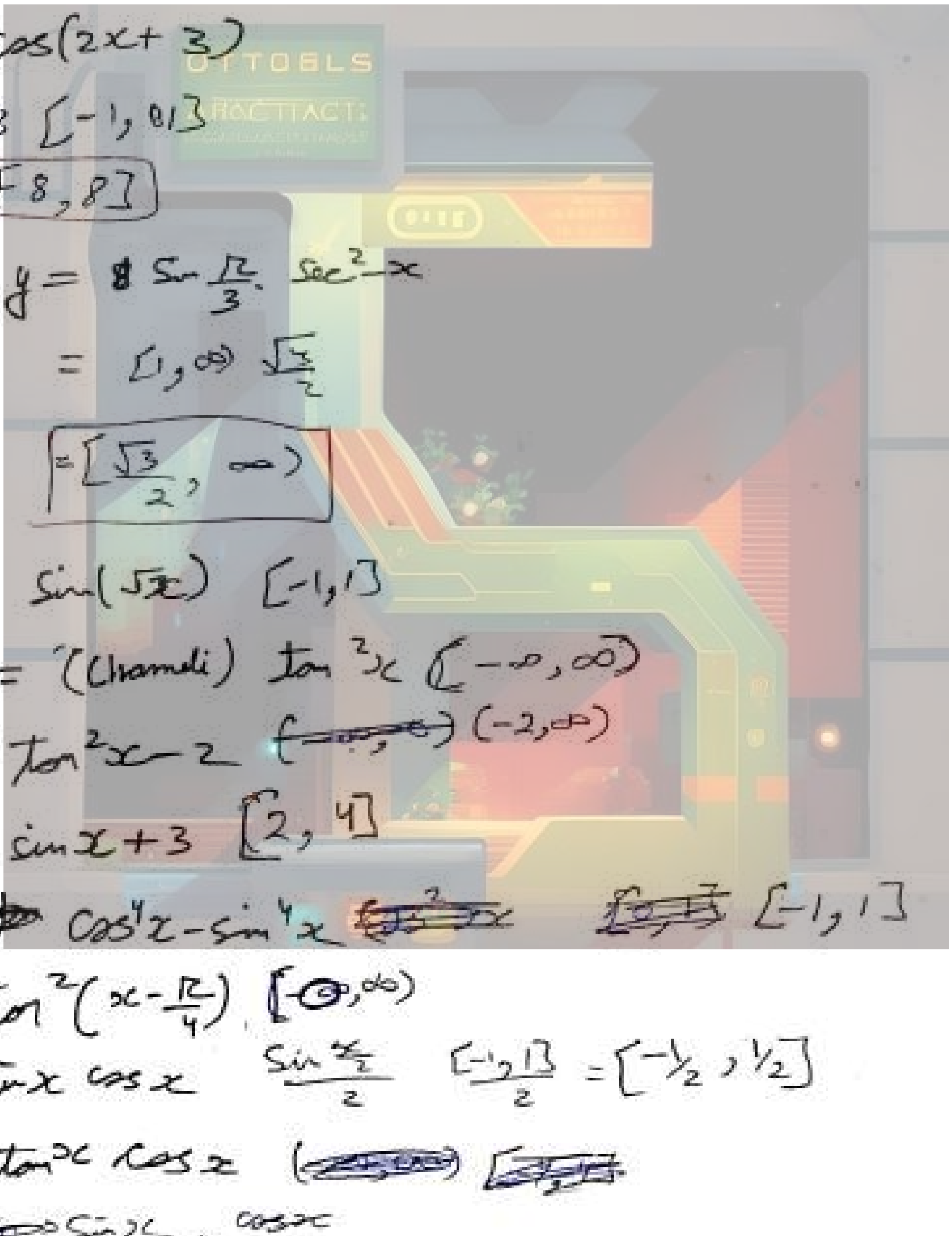
Q find range of

① $y = 4\sin(2x)$

$= 4[-1, 1]$

$\boxed{= [-4, 4]}$

Note :- No change in range will be observed if we change angle in any form.



② $y = 8\cos(2x + 3)$

$= 8\ [-1, 0, 1]$

$\boxed{= [-8, 8]}$

③ $y = 8\sin\frac{\pi}{3} \cdot \sec^2 x$

$= [1, \infty)\ \frac{\sqrt{3}}{2}$

$\boxed{= [\frac{\sqrt{3}}{2}, \infty)}$

④ $y = \sin(5x)\quad [-1, 1]$

⑤ $y = $ (Chamdi) $\tan^2 x\quad [-\infty, \infty)$

⑥ $y = \tan^2 x - 2\quad (-2, \infty)$

⑦ $y = \sin x + 3\quad [2, 4]$

⑧ $y = \cos^4 x - \sin^4 x\quad [-1, 1]$

⑨ $y = \tan^2(x - \frac{\pi}{4})\quad [0, \infty)$

⑩ $y = \sin x \cos x\quad \frac{\sin 2x}{2}\quad \frac{[-1, 1]}{2} = [-\frac{1}{2}, \frac{1}{2}]$

⑪ $y = 4\tan x \cos x$

$4 \frac{\sin x}{\cos x} \times \cos x$

$\cos x \neq 0$

$x \neq 90$

$x \neq -90$ ($\tan x$)

$4(-1, 1)$

$\boxed{(-4, 4)}$

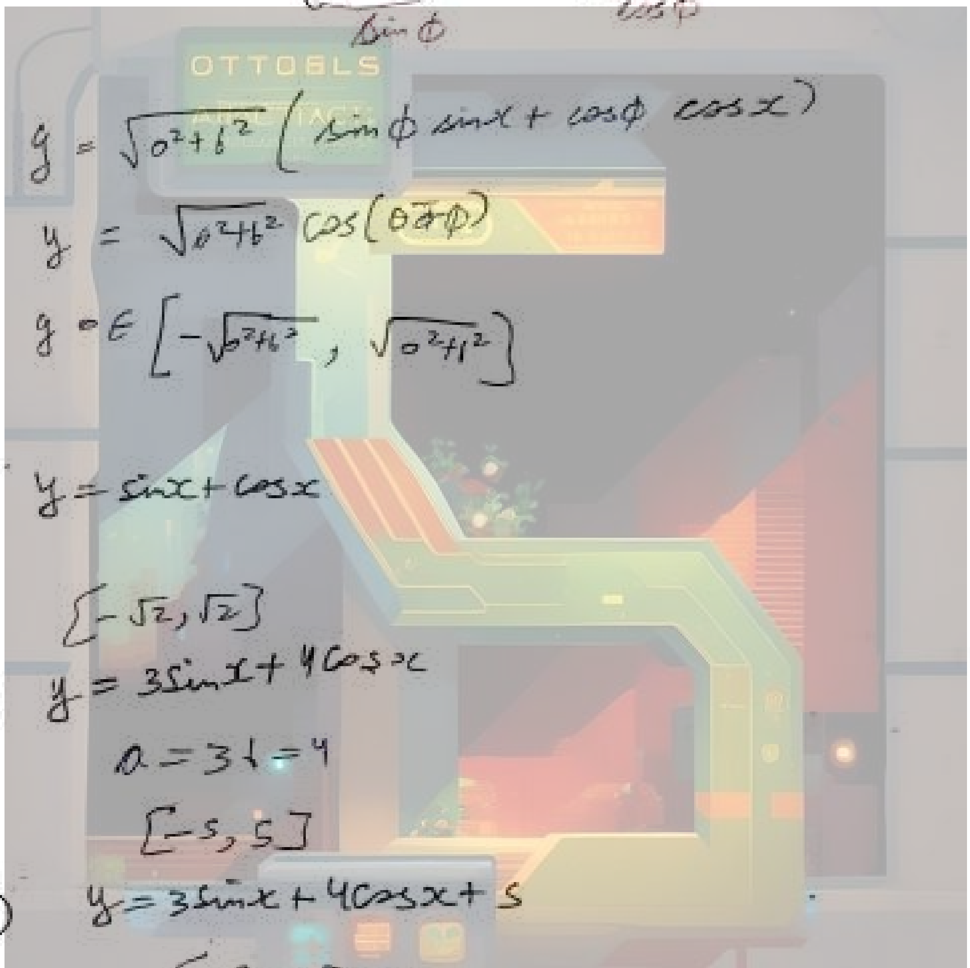
(2) $a\sin(x) + b\cos(x)$ form $a, b \in \mathbb{R}$.

must be same.

$y = a\sin x + b\cos x$

$y = \sqrt{a^2+b^2}\left(\frac{a\sin x}{\sqrt{a^2+b^2}}\right.$

$y = \sqrt{a^2+b^2}\left(\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\sin\phi}\sin x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\cos\phi}\cos x\right)$



$y = \sqrt{a^2+b^2}\left(\sin\phi\sin x + \cos\phi\cos x\right)$

$y = \sqrt{a^2+b^2}\cos(\theta \mp \phi)$

$y \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

Q (1) $y = \sin x + \cos x$

$[-\sqrt{2}, \sqrt{2}]$

(2) $y = 3\sin x + 4\cos x$

$a = 3 \; b = 4$

$[-5, 5]$

(3) $y = 3\sin x + 4\cos x + 5$

$[-5, 5] + 5$

$[0, 10]$

(4) $y = \log_2(\sin^2 x)$

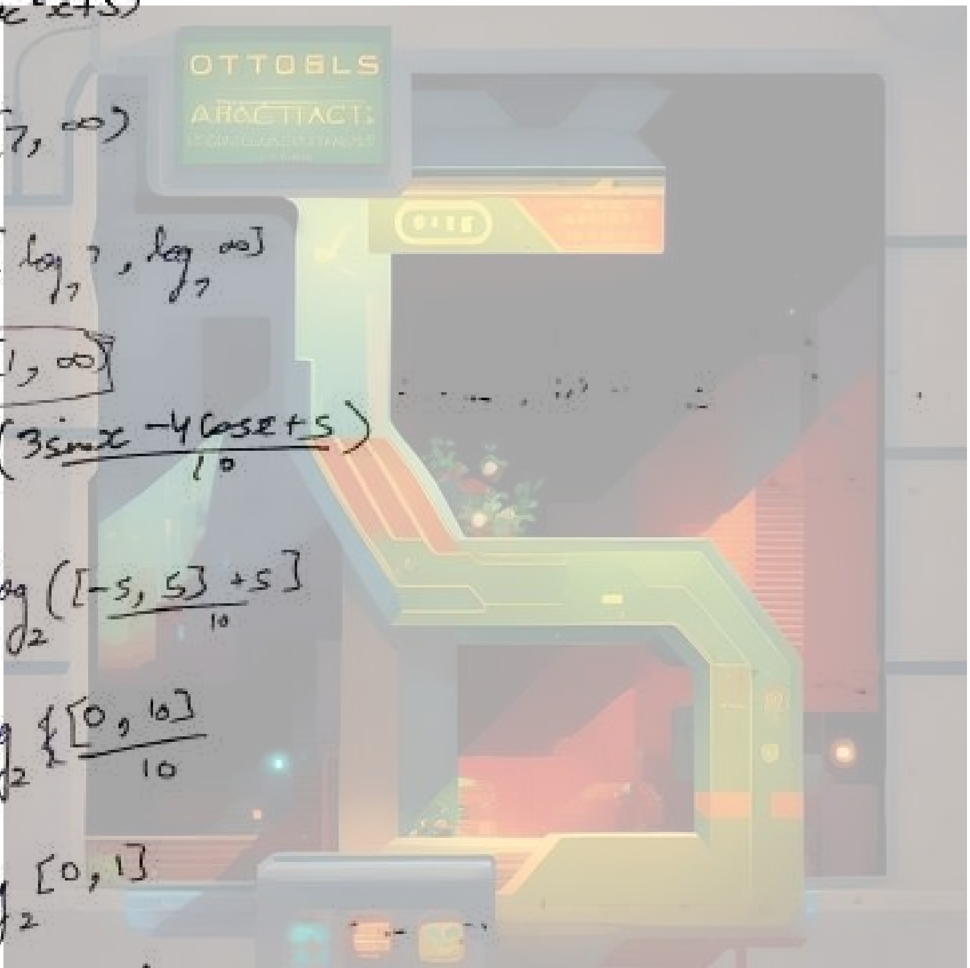
$y = \log_2[0,1]$

$\cancel{= [2^0, 2^1]}$ $\quad y \in [\log_2 0, \log_2 1]$

$\cancel{= [1,2]}$ $\quad \boxed{y \in (-\infty, 0]}$

$\cancel{= y \in (0,1]}$

(5) $y = \log_7(2\sec^2 x + 5)$



$y = \log_7[7, \infty)$

$\Rightarrow y \in [\log_7 7, \log_7 \infty)$

$\boxed{y \in [1, \infty)}$

(6) $y = \log_2\left(\frac{3\sin x - 4\cos x + 5}{10}\right)$

$y = \log_2\left(\frac{[-5, 5] + 5}{10}\right)$

$y = \log_2\left\{\frac{[0, 10]}{10}\right\}$

$y = \log_2[0, 1]$

$y \in [\log_2 0, \log_2 1]$

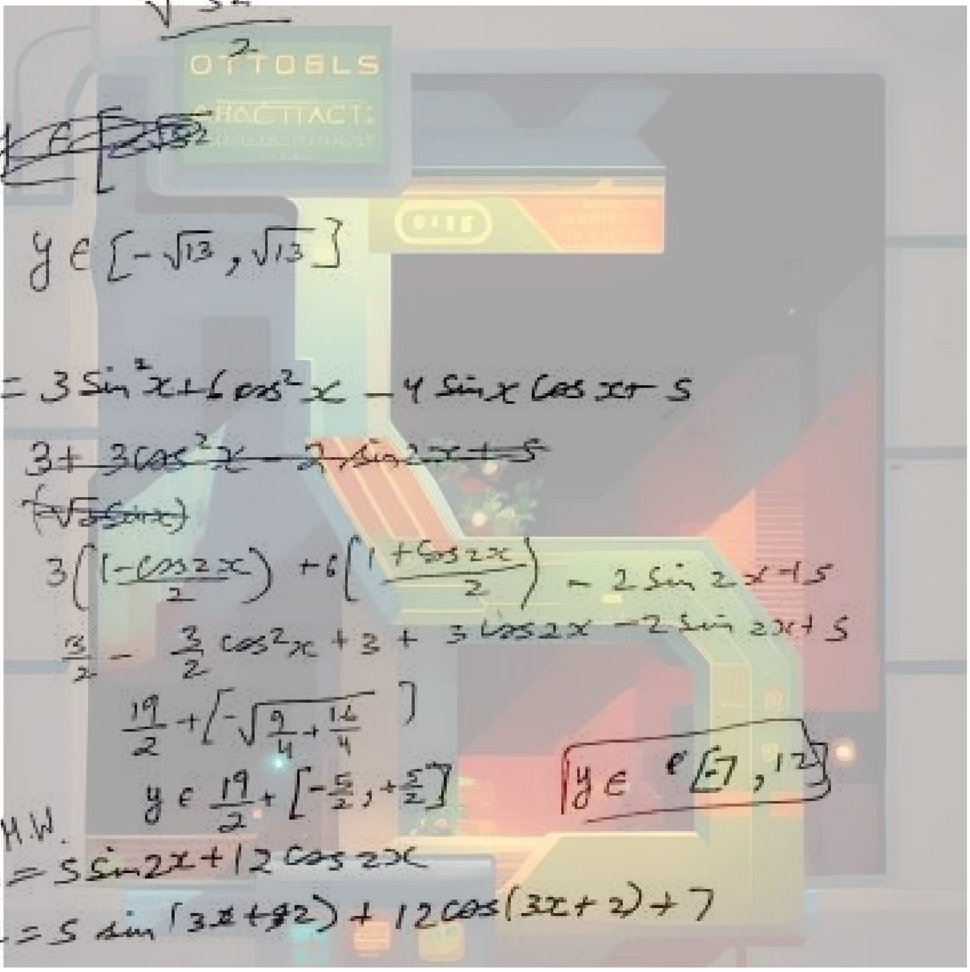
$\boxed{y \in (-\infty, 0]}$

(7) $y = \sin\left(x + \frac{\pi}{6}\right) + 3\cos x$

$\sin(30 + x) = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x + 3\cos x$

$\frac{7}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

$\sqrt{\frac{49}{4} + \frac{3}{4}}$



$\frac{\sqrt{52}}{2}$

$\cancel{y \in [\ \sqrt{52}}$

$y \in [-\sqrt{13}, \sqrt{13}]$

(8) $y = 3\sin^2 x + 6\cos^2 x - 4\sin x\cos x + 5$

$\cancel{3 + 3\cos^2 x - 2\sin 2x + 5}$

$3\left(\frac{1 - \cos 2x}{2}\right) + 6\left(\frac{1 + \cos 2x}{2}\right) - 2\sin 2x + 5$

$\frac{3}{2} - \frac{3}{2}\cos^2 x + 3 + 3\cos 2x - 2\sin 2x + 5$

$\frac{19}{2} + \left[-\sqrt{\frac{9}{4} + \frac{16}{4}}\ \right]$

$y \in \frac{19}{2} + \left[-\frac{5}{2}, +\frac{5}{2}\right]$ $\boxed{y \in [7, 12]}$

H.W.

(9) $y = 5\sin 2x + 12\cos 2x$

(10) $y = 5\sin(3x + 2) + 12\cos(3x + 2) + 7$

(11) $y = \sin\left(x + \frac{\pi}{6}\right) + 3\cos\left(x - \frac{\pi}{3}\right)$

(12) $y = 7\cos^2 x + 4\sin x\cos x + 3\sin^2 x$

③ Making of perfect square

① $y = \sin^2 x + 2\sin x + 4$

$y = \sin^2 x + 2\sin x + 1 + 3$

$= (\sin x + 1)^2 + 3$

$= [0, 2]^2 + 3$

$= [0, 4] + 3$

$\boxed{= [3, 7]}$

② $y = 2\cos^2 x - 4\cos x + 3$

$2\cos^2 x - 4\cos x + 1 + 2$

~~$\cos^2 x + \cos^2 x - 4\cos x + 2 + 1$~~

$\cos^2 x - 2\cos x + 1 + \frac{1}{2} + \frac{1}{2}$

$(\cos x - 1)^2 + \frac{1}{2} + \frac{1}{2}$

$[-2, -1]^2 + \frac{1}{2} + \frac{1}{2}$

$2 \times [1, 4] + \frac{1}{2} + \frac{1}{2}$

$\boxed{[1/2, 9/2]}$

③ $y = \sin^2 x - 20\cos x + 1$

$1 - \cos^2 x - 20\cos x + 1$

$\cos^2 x + 20\cos x - 2$

$\cos^2 x + 20\cos x + 100 - 98$

$(\cos x + 10)^2 - 98$

$[9, 11]^2 - 98$

$[81, 121] - 98$

~~$[-17, 19]$~~

$\boxed{[-19, 21]}$

(46)

(4) $y = \cos 2x + \sin x$

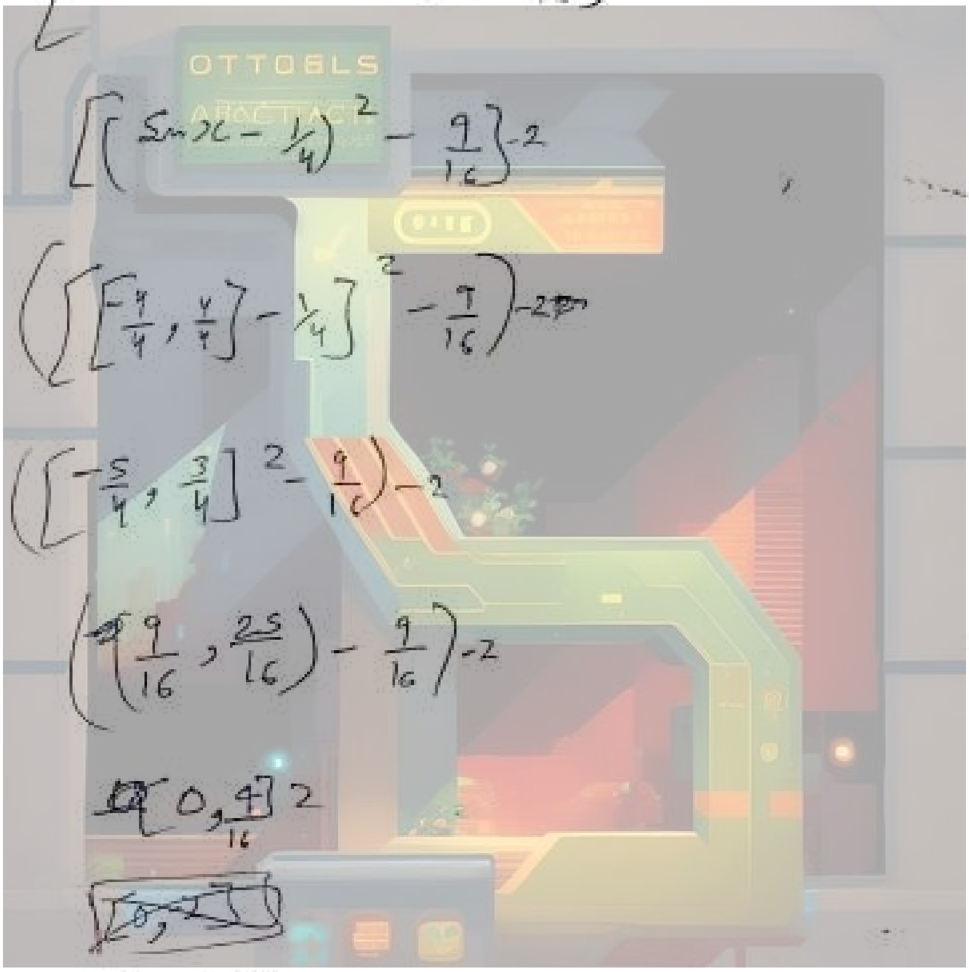
$\cancel{1 - 2\cos^2 x + \sin x}$

$1 - 2\sin^2 x + \sin x$

$2\sin^2 x - \sin x - 1$

$2\left[\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2}\right] - 2$

$\left[\sin^2 x - \frac{1}{2}\sin x + \left(\frac{1}{16}\right)^2 - \frac{9}{16}\right] - 2$



$\left[\left(\sin x - \frac{1}{4}\right)^2 - \frac{9}{16}\right] - 2$

$\left(\left[\left[-\frac{4}{4}, \frac{4}{4}\right] - \frac{1}{4}\right]^2 - \frac{9}{16}\right) - 2$

$\left(\left[-\frac{5}{4}, \frac{3}{4}\right]^2 - \frac{9}{16}\right) - 2$

$\left(\left(\frac{9}{16}, \frac{25}{16}\right) - \frac{9}{16}\right) - 2$

$\left[0, \frac{4}{16}\right] 2$

$\cancel{\boxed{[0, 2]}}$

$\left[-2, \frac{9}{8}\right]$

(4) using $AM \geq GM$

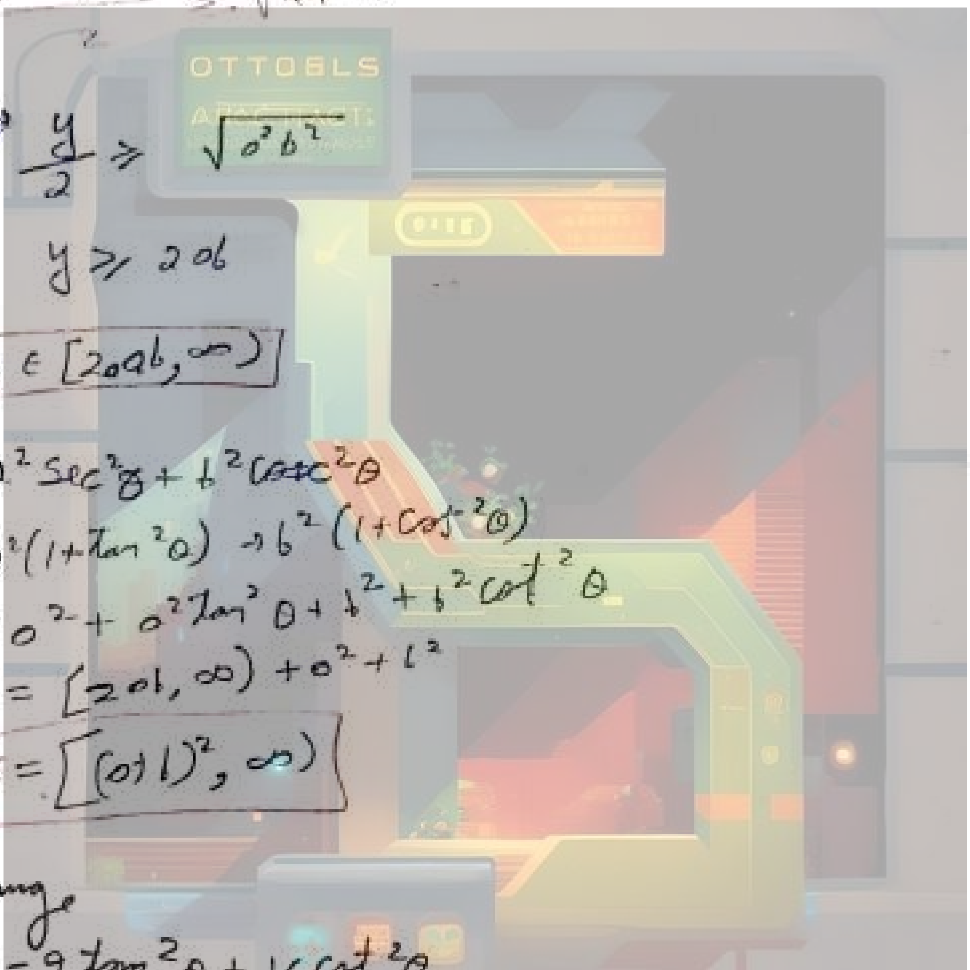
$\sin\theta \operatorname{cosec}\theta = 1$

$\cos\theta \sec\theta = 1$

$\cot\theta \tan\theta = 1$

Format - 1 $\quad y = \underbrace{a^2\tan^2\theta}_{\theta_1} + \underbrace{b^2\cot^2\theta}_{\theta_2}$

$\frac{\theta_1 + \theta_2}{2} \geq \sqrt{\theta_1 \theta_2}$



$\frac{y}{2} \geq \sqrt{a^2b^2}$

$y \geq 2ab$

$\boxed{y \in [2ab, \infty)}$

(2) $y = a^2\sec^2\theta + b^2\operatorname{cosec}^2\theta$

$= a^2(1+\tan^2\theta) + b^2(1+\cot^2\theta)$

$= a^2 + a^2\tan^2\theta + b^2 + b^2\cot^2\theta$

$= [2ab, \infty) + a^2 + b^2$

$\boxed{y \in [(a+b)^2, \infty)}$

Q find range

(1) $y = 9\tan^2\theta + 16\cot^2\theta$

$y \in (2\times 3\times 4 8, \infty)$

$y \in [24, \infty)$

② $y = 25\sec^2\theta + 16\operatorname{cosec}^2\theta$

$y \in [81, \infty)$

Format 2 & 3 :- $\sin\theta$ & $\operatorname{cosec}\theta$ / $\sec\theta$ & $\cos\theta$

& check holding of equality in AM-GM

① $y = 8\sec^2\theta + 18\cos^2\theta$ find range.

$= (2\sqrt{2}\sec\theta)^2 + (3\sqrt{2}\cos\theta)^2$

check. $2\sqrt{2}\sec\theta = 3\sqrt{2}\cos\theta$

$\cos^2\theta = \frac{2}{3}$

Possible.

$\frac{y}{2} \geq \sqrt{144}$

$y \geq 24$

$\boxed{y \in [24, \infty)}$

② $y = 18\sec^2\theta + 8\cos^2\theta$

$(3\sqrt{2}\sec\theta)^2 + (2\sqrt{2}\cos\theta)^2$

$3\sqrt{2}\sec\theta = 2\sqrt{2}\cos\theta$

$\cos^2\theta = \frac{3}{2}$ (not valid)

$y = 10\sec^2\theta + 8\sec^2\theta + 8\cos^2\theta$

$y = 10\sec^2\theta + [16, \infty)$

$y = [10, \infty) + [16, \infty)$

min $= 16 + 10$

$\boxed{= 26}$

③ find min value

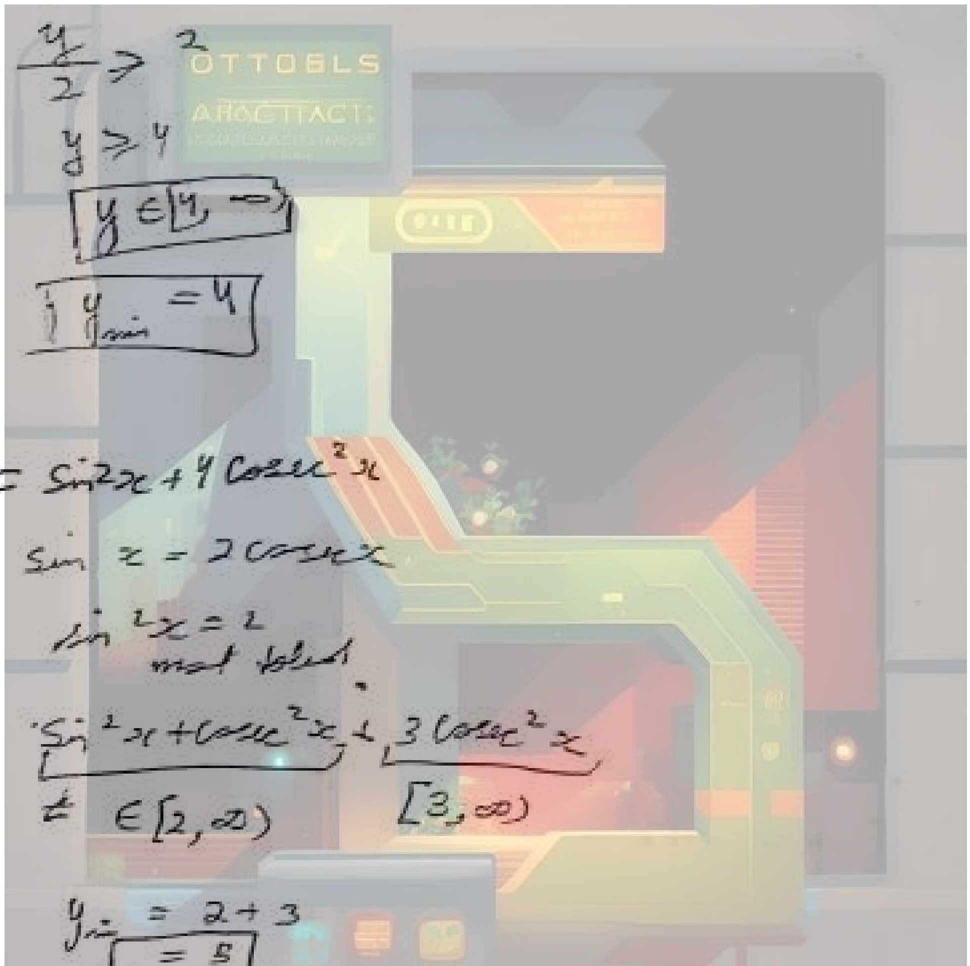
$y = 4\sin^2 x + \sec^2 x$

$2\sin x = \frac{1}{\sin x}$

$s^2 x = \frac{1}{2}$ $\quad \sin^2 x \, [0,1]$

$\cancel{\sin x = \frac{1}{\sqrt{2}}}$

$\sin x \in [-1, 1]$



$\frac{y}{2} \geq 2$

$y \geq 4$

$\boxed{y \in [4, \infty)}$

$\boxed{y_{min} = 4}$

④ $y = \sin^2 x + 4\operatorname{cosec}^2 x$

$\sin x = 2\operatorname{cosec} x$

$\sin^2 x = 2$

not valid.

$\underbrace{\sin^2 x + \operatorname{cosec}^2 x}_{t \in [2, \infty)} + \underbrace{3\operatorname{cosec}^2 x}_{[3, \infty)}$

$y_{min} = 2 + 3$

$\boxed{= 5}$

Type-5 (दुइ ओटा)

Q ① $x^2+y^2=4$ & $a^2+b^2=8$ find. Min & Max value of $ax+by$

$x^2+y^2=4$ $\qquad a^2+b^2=8$

$x = 2\cos\theta$ $\qquad a = 2\sqrt{2}\cos\phi$

$y = 2\sin\theta$ $\qquad b = 2\sqrt{2}\sin\phi$

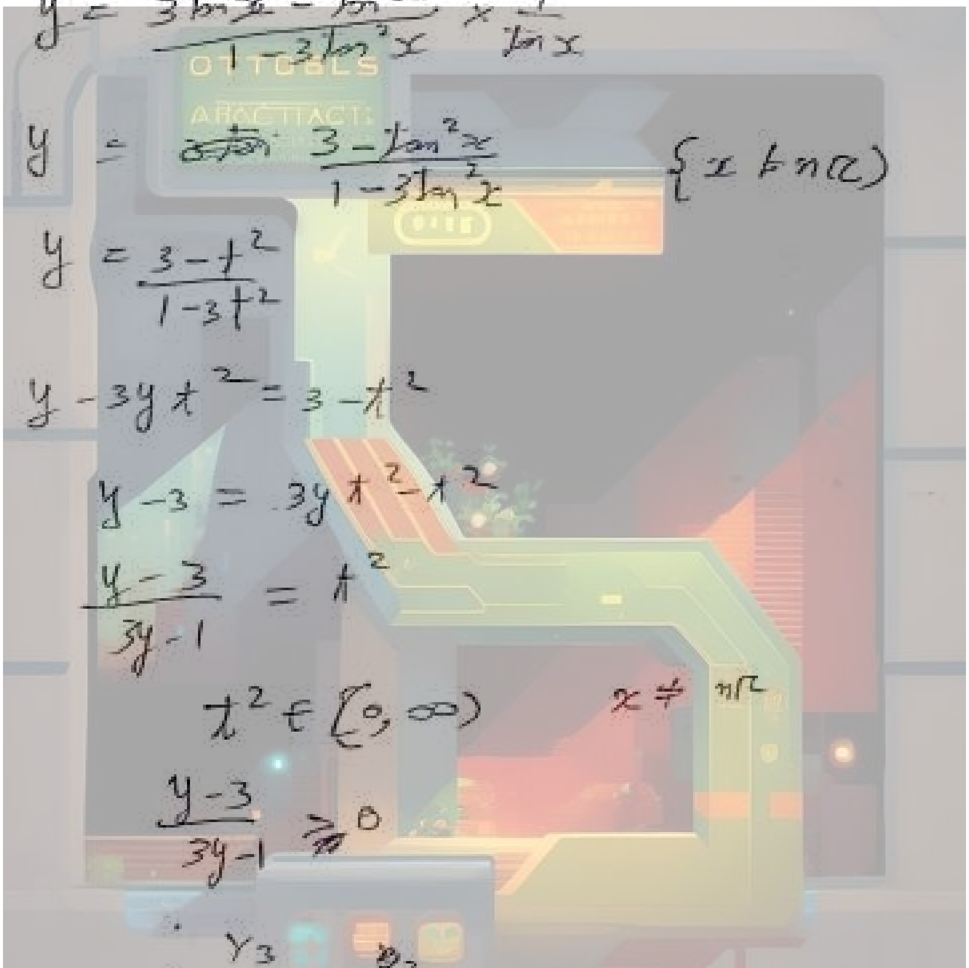
$$ax+by = 4\sqrt{2}(\cos\theta\cos\phi + \sin\theta\sin\phi)$$

$$= 4\sqrt{2}\cos(\theta-\phi)$$

$$= 4\sqrt{2}\,[-1, 1]$$

$$\boxed{= [4\sqrt{2}, 4\sqrt{2}]}$$

(2) $y = \dfrac{\tan 3x}{\tan x}$



$$y = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x} \times \frac{1}{\tan x}$$

$$y = \frac{3-\tan^2 x}{1-3\tan^2 x} \quad \{x \neq n\pi\}$$

$$y = \frac{3-t^2}{1-3t^2}$$

$$y - 3yt^2 = 3 - t^2$$

$$y - 3 = 3yt^2 - t^2$$

$$\frac{y-3}{3y-1} = t^2$$

$$t^2 \in [0, \infty) \qquad x \neq n\pi/2$$

$$\frac{y-3}{3y-1} \geq 0$$

$1/3 \quad 3$

$+ \quad - \quad +$

$\cancel{(-\infty, 1/3] \cup [3, \infty)}$

$$\boxed{(-\infty, 1/3) \cup [3, \infty)}$$

H.W. 30 - 7 - 24

DYS - 13 (5, 6, 7, 8, 9, 10)

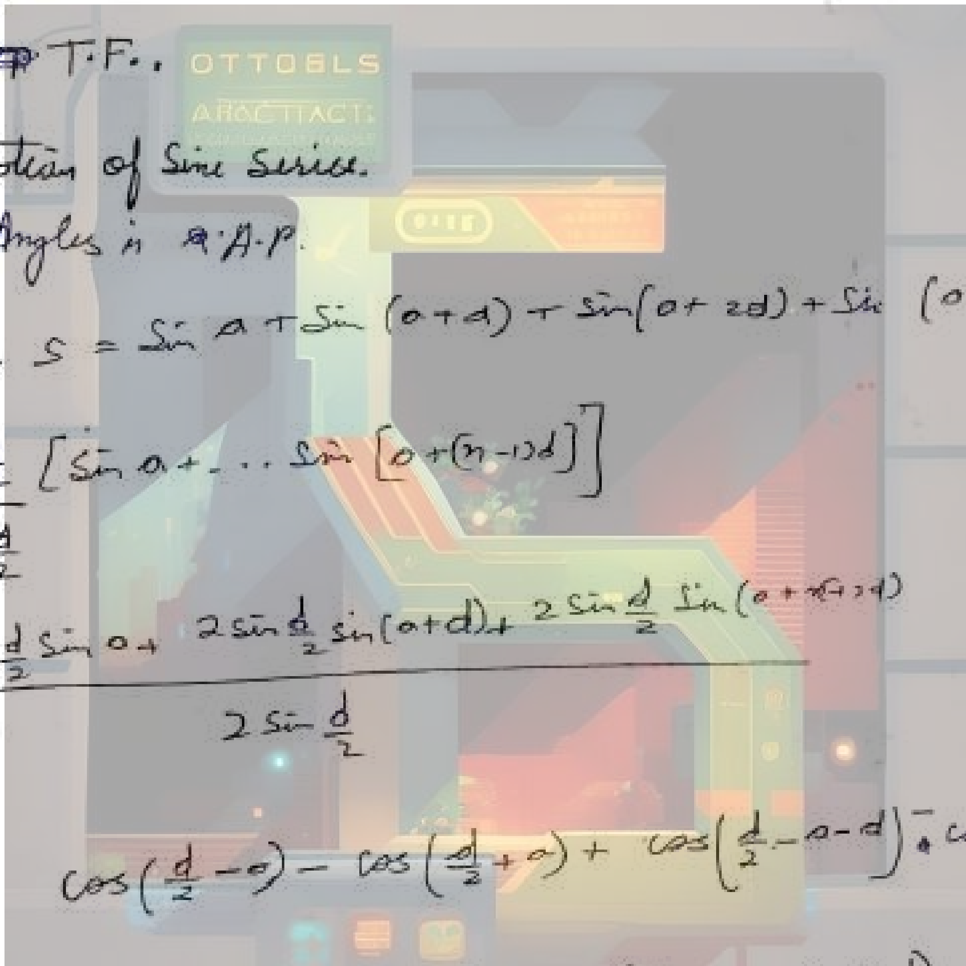
DYS - 14, 15

O - 1 (22 - 29)

J - M (6, 8)

Summation of Trigonometric series :-

→ Try to convert the given series in terms of Difference of 2 T.F..



→ Summation of Sine Series.

↳ Angles in A.P.

$$S = \sin a + \sin(a+d) + \sin(a+2d) + \sin(a+3d) + \dots \sin(a+(n-1)d)$$

$$S = \frac{2\sin\frac{d}{2}\left[\sin a + \dots \sin\left[a+(n-1)d\right]\right]}{2\sin\frac{d}{2}}$$

$$= \frac{2\sin\frac{d}{2}\sin a + 2\sin\frac{d}{2}\sin(a+d) + 2\sin\frac{d}{2}\sin(a+(n-1)d)}{2\sin\frac{d}{2}}$$

$$= \cos\left(\frac{d}{2}-a\right) - \cos\left(\frac{d}{2}+a\right) + \cos\left(\frac{d}{2}-a-d\right) - \cos\left(\frac{d}{2}+a+d\right)$$

$$+ \cos\left(\frac{d}{2}-a-(n-1)d\right) - \cos\left(\frac{d}{2}+a+(n-1)d\right)$$

$$= \cos\left(a-\frac{d}{2}\right) - \cancel{\cos\left(a+\frac{d}{2}\right)} + \cancel{\cos\left(a+\frac{d}{2}\right)} - \cancel{\cos\left(a+\frac{3d}{2}\right)} + \cos\left(a+\frac{3d}{2}\right)$$

$$= \cos\left(a-\frac{d}{2}\right) - \cos\left(\frac{d}{2}+a+(n-1)d\right)$$

$$S \cdot 2\sin\frac{d}{2} = 2\sin\left(\frac{a-\frac{d}{2}+\frac{d}{2}+a+(n-1)d}{2}\right)\sin\left(\frac{\frac{d}{2}+a+(n-1)d-a+\frac{d}{2}}{2}\right)$$

$$S \cdot 2\sin\frac{d}{2} = 2\sin\left(a+\frac{(n-1)d}{2}\right)\sin\left[\frac{nd}{2}\right]$$

$$S = \frac{\cancel{2}\sin\left(a+(n-1)\frac{d}{2}\right)\sin\left[\frac{nd}{2}\right]}{\cancel{2}\sin\frac{d}{2}}$$

$$\boxed{S = \frac{\sin\left(a+(n-1)\frac{d}{2}\right)\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

→ Summation Of Cosine Series:-

$$S = \cos a + \cos(a+d) \ldots\ldots \cos[a+(n-1)d]$$

$$\cancel{S = \frac{\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

$$\boxed{S = \frac{\cos\left(a+(n-1)\frac{d}{2}\right)\sin\left(\frac{nd}{2}\right)}{\sin\left(\frac{d}{2}\right)}}$$

Q find sum of following series.

① $\sin 2^\circ + \sin 4^\circ + \sin 6^\circ + 8 \ldots \sin 180^\circ$

$$S = \frac{\sin\left(\frac{2+180}{2}\right) * \sin\left(90, \frac{2}{2}\right)}{\sin\left(\frac{2}{2}\right)}$$

$$= \frac{\sin 91^\circ * \sin 90^\circ}{\sin 1^\circ}$$

$$\boxed{= \frac{\cancel{\cos 1} \sin 91^\circ}{\sin 1^\circ}} = \cot 1^\circ$$

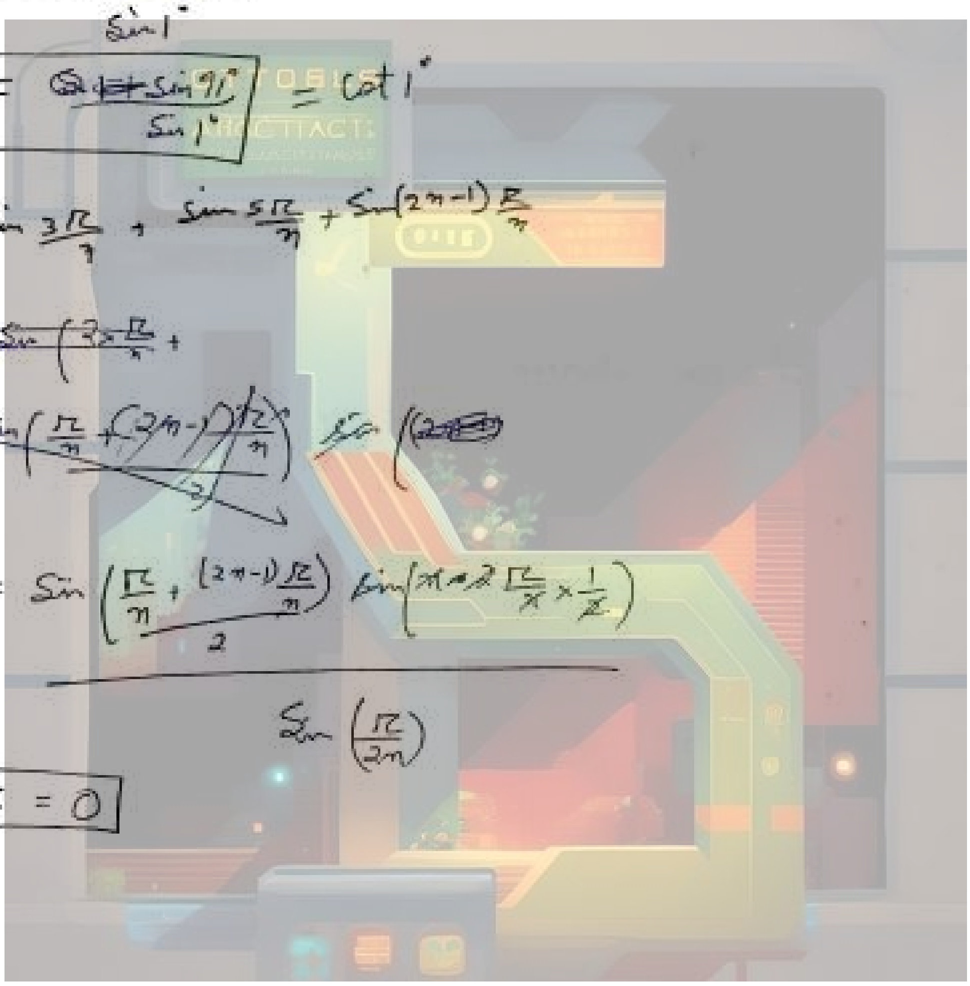
② $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \sin(2n-1)\frac{\pi}{n}$

$\cancel{S = \sin\left(2 \times \frac{\pi}{n} + \right.}$

$\cancel{S = \sin\left(\frac{\pi}{n} + \frac{(2n-1)\frac{\pi}{n}}{2}\right) \sin\left(\right.}$

$$S = \frac{\sin\left(\frac{\frac{\pi}{n} + (2n-1)\frac{\pi}{n}}{2}\right) \sin\left(\cancel{n} \times 2 \frac{\pi}{\cancel{n}} \times \frac{1}{\cancel{2}}\right)}{\sin\left(\frac{\pi}{2n}\right)}$$

$\boxed{S = 0}$

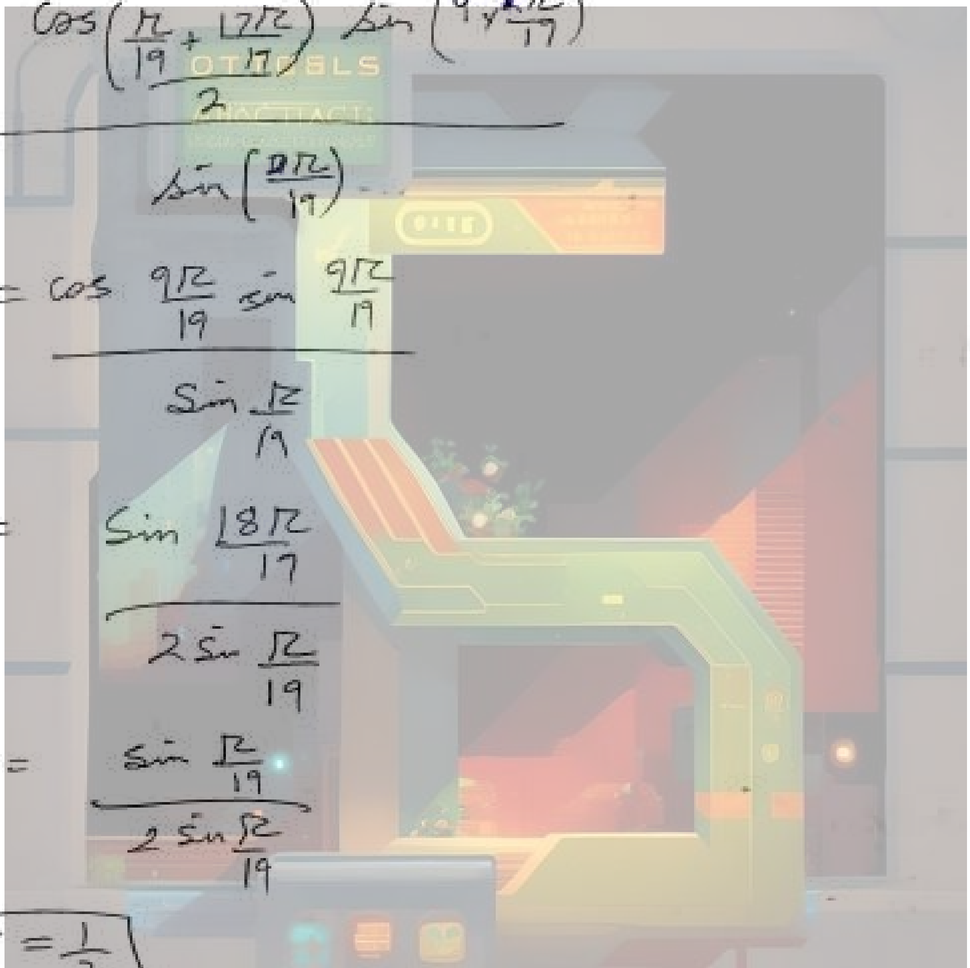


③ $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots \cos\frac{17\pi}{19}$

④ $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$

⑤ ** $\cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \cos^2\frac{3\pi}{n} + \ldots\ldots \cos^2\frac{(n-1)\pi}{n}$

⑥ $y = \dfrac{\cos x + \cos 2x + \cos 3x + \ldots\ldots \cos 7x}{\cancel{\cos}\ \sin x + \sin 2x + \sin 3x + \ldots \sin 7x}$



③ $S = \dfrac{\cos\left(\dfrac{\frac{\pi}{19} + \frac{17\pi}{19}}{2}\right) \sin\left(9 \times \frac{\pi}{19}\right)}{\sin\left(\frac{\cancel{2}\pi}{19}\right)}$

$S = \dfrac{\cos\frac{9\pi}{19} \sin\frac{9\pi}{19}}{\sin\frac{\pi}{19}}$

$S = \dfrac{\sin\frac{18\pi}{17}}{2\sin\frac{\pi}{19}}$

$S = \dfrac{\sin\frac{\pi}{19}}{2\sin\frac{\pi}{19}}$

$\boxed{S = \frac{1}{2}}$

④ $S = \dfrac{\cos\left(\frac{5\pi}{11}\right) \sin\left(5 \times \frac{\pi}{11}\right)}{\sin\frac{\pi}{11}}$

$S = \dfrac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$ $\boxed{S = \frac{1}{2}}$

(6) $$y = \frac{\dfrac{\cos(4x)\sin\left(7\times\frac{\pi}{2}\right)}{\sin\left(\frac{x}{2}\right)}}{\dfrac{\sin(4x)\sin\left(7\times\frac{\pi}{2}\right)}{\sin\left(\frac{x}{2}\right)}}$$

$$y = \cot 4x$$

(5) $$\cos^2\frac{\pi}{n} = \frac{1+\cos\frac{2\pi}{n}}{2}$$

$$\frac{1+\cos\frac{2\pi}{n}}{2} + \frac{1+\cos\frac{4\pi}{n}}{2} + \frac{1+\cos\frac{6\pi}{n}}{2} \cdots\cdots$$

$$S = \frac{(n-1) + \dfrac{\cos\left(\dfrac{\frac{2\pi}{n} + \frac{2(n-1)\pi}{n}}{2}\right)\sin\left((n-1)\times\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}}{2}$$

$$S = \frac{\cancel{\frac{2(n-1)}{2}} - \sin\left(\pi - \frac{\pi}{n}\right)}{\frac{2\sin\left(\frac{\pi}{n}\right)}{2}}$$

$$S = \frac{(n-1) - 1}{2}$$

$$\boxed{S = \frac{n-2}{2}}$$

→ Telescopic Series:-

Q $\operatorname{cosec} x + \operatorname{cosec} 2x + \operatorname{cosec} 4x + \dots\dots + \operatorname{cosec} 2^n x$

$$S = \frac{1}{\sin x} + \frac{1}{\sin 2x} + \frac{1}{\sin 4x} \dots\dots \frac{1}{\sin 2^n x}$$

$$S = \frac{\sin \frac{x}{2}}{\sin x \sin \frac{x}{2}} + \frac{\sin x}{\sin 2x \sin x} + \frac{\sin 2x}{\sin 4x \sin 2x} \dots\dots$$

$$S = \frac{\sin\left(x - \frac{x}{2}\right)}{\sin x \sin \frac{x}{2}} + \frac{\sin(2x - x)}{\sin 2x \sin x} + \frac{\sin(4x - 2x)}{\sin 4x \sin 2x} \dots\dots$$

$$S = \frac{\sin x \cos \frac{x}{2} - \cos x \sin \frac{x}{2}}{\sin x \sin \frac{x}{2}} + \frac{\sin 2x \cos x - \sin x \cos 2x}{\sin 2x \sin x} \dots\dots$$

$$S = \cot \frac{x}{2} - \cancel{\cot x} + \cancel{\cot x} - \cancel{\cot 2x} + \cancel{\cot 2x} - \cot 4x \dots\dots + \cot(2^{n-1}x) - \cot(2^n x)$$

$$\boxed{S = \cot \frac{x}{2} - \cot(2^n x)}$$

Q $S = \tan \frac{x}{2} \sec x + \tan \frac{x}{2^2} \cdot \sec \frac{x}{2} + \tan \frac{x}{2^3} \cdot \sec \frac{x}{2^2} \dots\dots$ n terms

$$= \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \frac{1}{\cos x} + \frac{\sin \frac{x}{2^2}}{\cos \frac{x}{2^2}} \frac{1}{\cos \frac{x}{2}} \dots\dots$$

$$= \frac{\sin\left(x - \frac{x}{2}\right)}{\cos \frac{x}{2} \cos x} + \frac{\sin\left(\frac{x}{2} - \frac{x}{2^2}\right)}{\cos \frac{x}{2} \cos x} \dots\dots$$

$$= \frac{\sin x \cos \frac{x}{2} - \cos x \sin \frac{x}{2}}{\cos \frac{x}{2} \cos x} + \dots\dots$$

$$= \tan x - \tan \frac{x}{2} + \tan \frac{x}{2} - \tan \frac{x}{2} \dots\dots$$

$$= \tan x - \tan\left(\frac{x}{2^n}\right)$$

H.W. 2-8-24

DYS-16 [1,6]


OTTOBLS
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OTTOBLS
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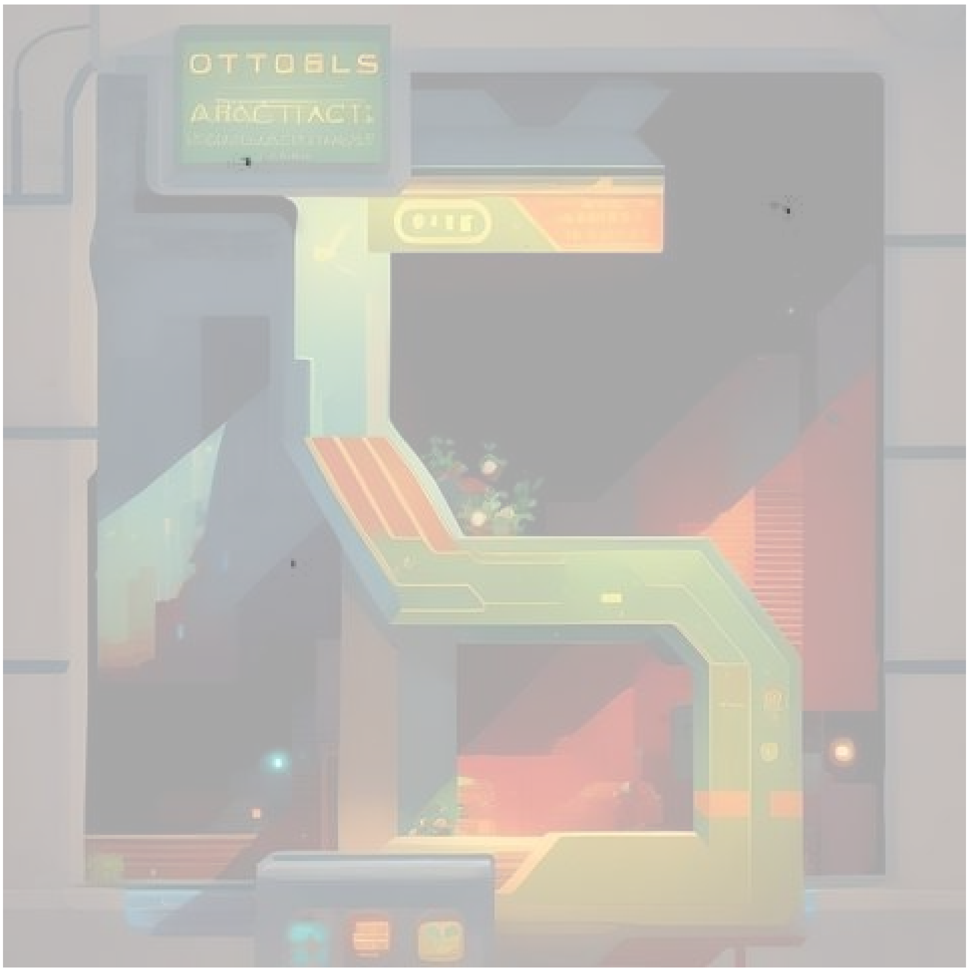


! Trigonometric Equation !

→ An Equation involving 1 or more than 1 TR of unknown angles

eg. $\cos\theta = \frac{1}{2}$

$\tan^2\theta - 4\tan\theta + 3 = 0$

$\sin^2\theta + \cos\theta - 1 = 0$

→ Types of Solutions :-

Principal

→ Solutions of TE lying in the interval $[0, 2\pi)$

General Sol

→ All such possible values of θ for which for which the given TE is satisfied.

→ It is given by a general formula

Particular Sol

→ Solutions which are lying in the Interval given in the question

→ General sol to be remembered.

① $\sin\theta = 0$	$\theta = n\pi$	$n \in I$
② $\cos\theta = 0$	$\theta = (2n-1)\frac{\pi}{2}$	$n \in I$
	$= (2n+1)\frac{\pi}{2}$	$n \in I$
③ $\tan\theta = 0$	$\theta = n\pi$	$n \in I$
④ $\cot\theta = 0$	$\theta = (2n-1)\frac{\pi}{2}$ or $(2n+1)\frac{\pi}{2}$	$n \in I$

(5) $\sec\theta = 0$ $\qquad \theta \in \phi$

(6) $\operatorname{cosec}\theta = 0$ $\qquad \theta \in \phi$

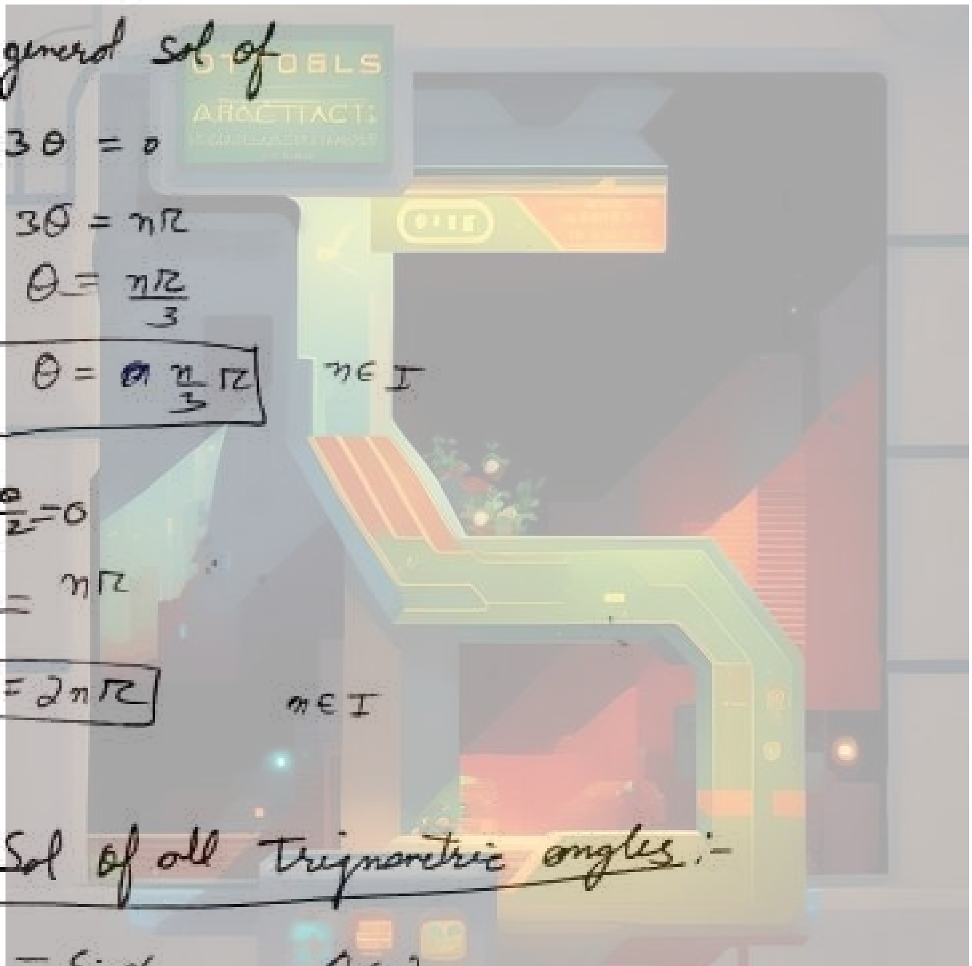
(7) $\sin(k\theta) = 0$ $\qquad k\theta = n\pi \qquad n \in I$

$k \in$ constant $\qquad \theta = \frac{n\pi}{k}$

(8) $\sin(a\theta + b) = 0$ $\qquad \theta = \frac{n\pi - b}{a} \qquad n \in I$

$a, b \in$ constant

Q1. find general sol of



(1) $\sin 3\theta = 0$

$3\theta = n\pi$

$\theta = \frac{n\pi}{3}$

$\boxed{\theta = \frac{n}{3}\pi} \qquad n \in I$

(2) $\tan\frac{\theta}{2} = 0$

$\frac{\theta}{2} = n\pi$

$\boxed{\theta = 2n\pi} \qquad n \in I$

General Sol of all trignometric angles :-

(1) $\sin\theta = \sin\alpha \qquad \theta = ?$

Proof :-

$\sin\theta - \sin\alpha = 0$

$2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\cos\frac{\theta+\alpha}{2} = 0 \qquad \sin\frac{\theta-\alpha}{2} = 0$

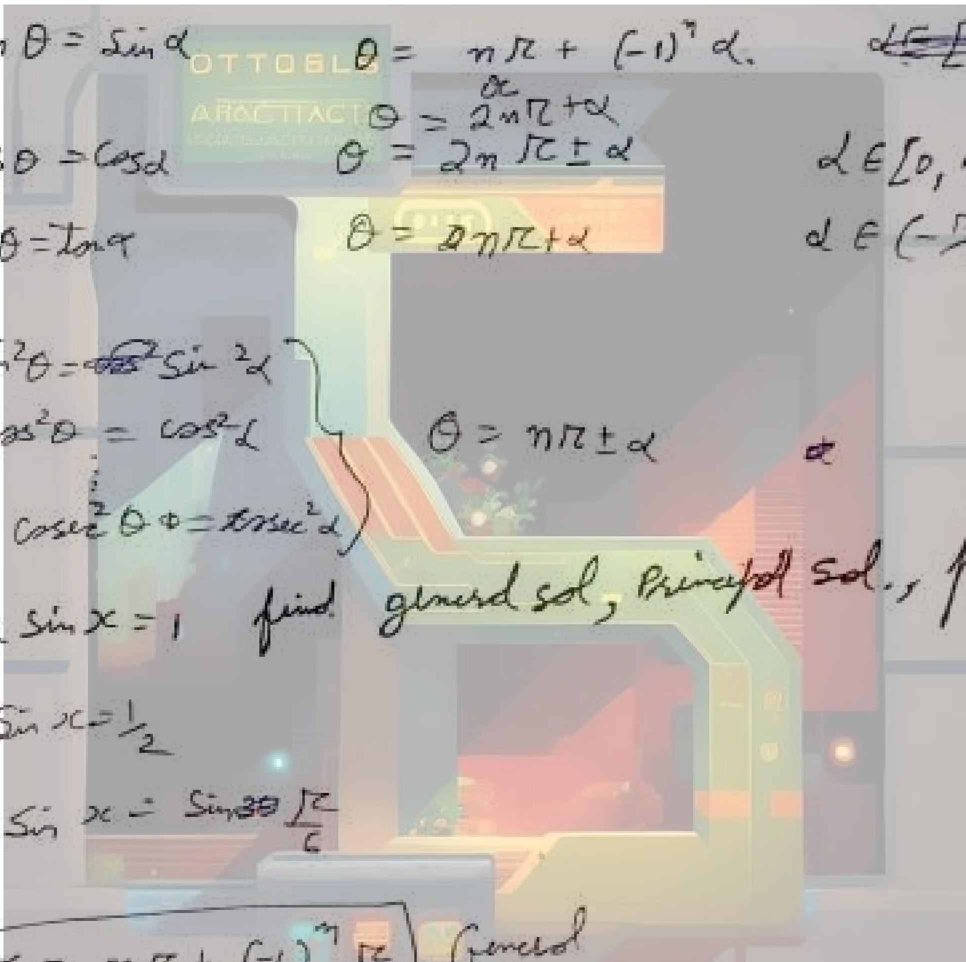
$\theta = 2n\pi + \pi - \alpha$ or $\theta = 2n\pi + \alpha$

$n=0 \quad \theta = \pi - \alpha \qquad\qquad \theta = \pi$

$n=1 \quad \theta = 3\pi - \alpha \qquad\qquad \theta = 2\pi + \alpha$

$n=2 \quad \theta = 5\pi - \alpha \qquad\qquad \theta = 4\pi + \alpha$

$$\boxed{\theta = n\pi + (-1)^n \alpha}$$



① $\sin\theta = \sin\alpha \qquad \theta = n\pi + (-1)^n \alpha. \qquad \cancel{\alpha \in [0, \pi]}\ \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\theta = 2n\pi + \alpha$

② $\cos\theta = \cos\alpha \qquad \theta = 2n\pi \pm \alpha \qquad \alpha \in [0, \pi]$

③ $\tan\theta = \tan\alpha \qquad \theta = 2n\pi + \alpha \qquad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

④ $\left.\begin{aligned} &\sin^2\theta = \sin^2\alpha \\ &\cos^2\theta = \cos^2\alpha \\ &\vdots \\ &\mathrm{cosec}^2\theta = \mathrm{cosec}^2\alpha \end{aligned}\right\} \quad \theta = n\pi \pm \alpha \qquad \alpha$

Q2. $2\sin x = 1$ find general sol, Principal sol., particular sol in $(2\pi, 4\pi)$

$\sin x = \frac{1}{2}$

$\sin x = \sin \cancel{30} \frac{\pi}{6}$

① $\boxed{x = n\pi + (-1)^n \frac{\pi}{6}}$ General

$n=0 \quad x = \frac{\pi}{6}$ ✓ $\qquad n=-1 \quad x = -\frac{7\pi}{6}$ ×

$n=1 \quad x = \frac{5\pi}{6}$ ✓

$n= \quad x = \frac{17\pi}{6}$ ×

② $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$ Principal

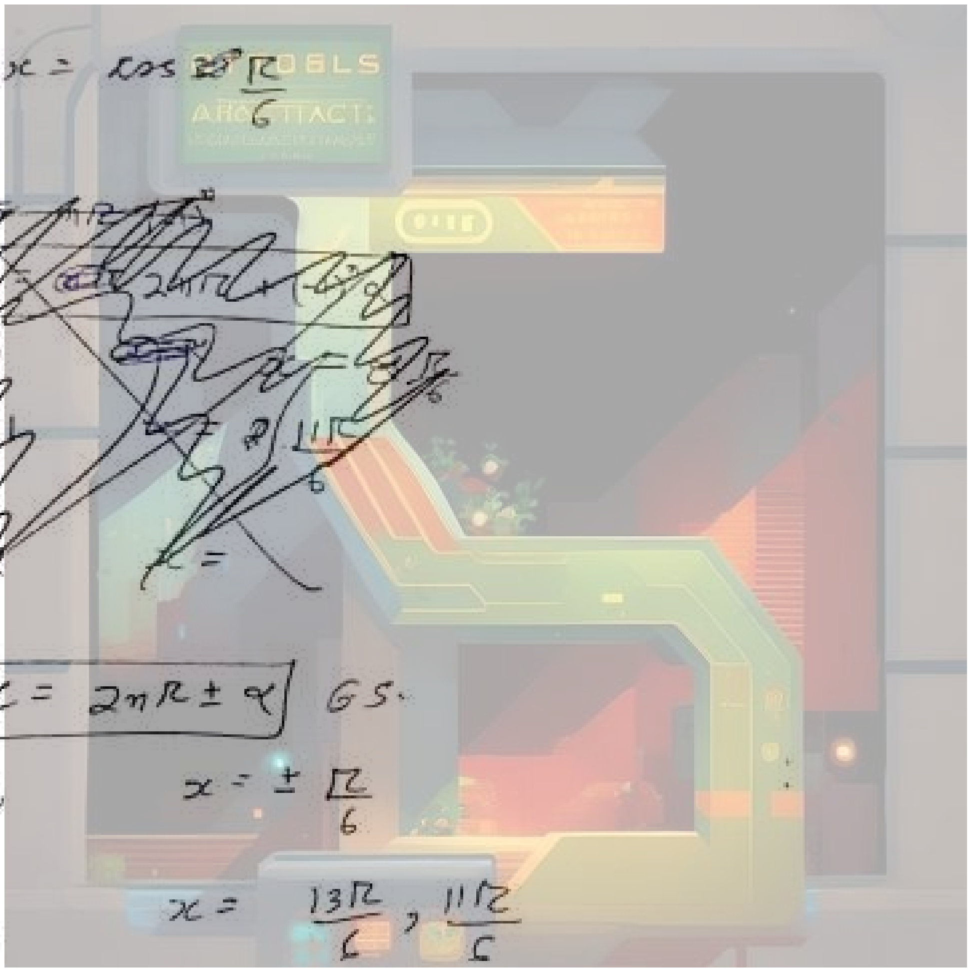
$n=3 \quad x = \frac{17\pi}{6}$ ✓ $\qquad n=2 \quad x = \frac{13\pi}{6}$ ✓

$n=4 \quad x = \frac{25\pi}{6}$ ✗

(3) $\boxed{x = \frac{17\pi}{6}, \frac{13\pi}{6}}$ Particular

Q3. a) $\cos x = \frac{\sqrt{3}}{2}$ find GS, Pr.S, Pa.S$\left(\frac{\pi}{2}, \frac{5\pi}{2}\right)$

$\cos x = \cos \frac{\pi}{6}$



$\boxed{x = 2n\pi \pm \alpha}$ GS.

$n=0 \quad x = \pm \frac{\pi}{6}$

$n=1 \quad x = \frac{13\pi}{6}, \frac{11\pi}{6}$

$n=2 \quad x = \frac{23\pi}{6}, \frac{25\pi}{6}$

$\boxed{P.S = x = \frac{\pi}{6}, \frac{11\pi}{6}}$

$\boxed{Pa.S \quad x = \frac{11\pi}{6}, \frac{13\pi}{6}}$

Q 4. $\tan x = -\sqrt{3}$ G.S, P.S

$\tan x = -\tan 60$

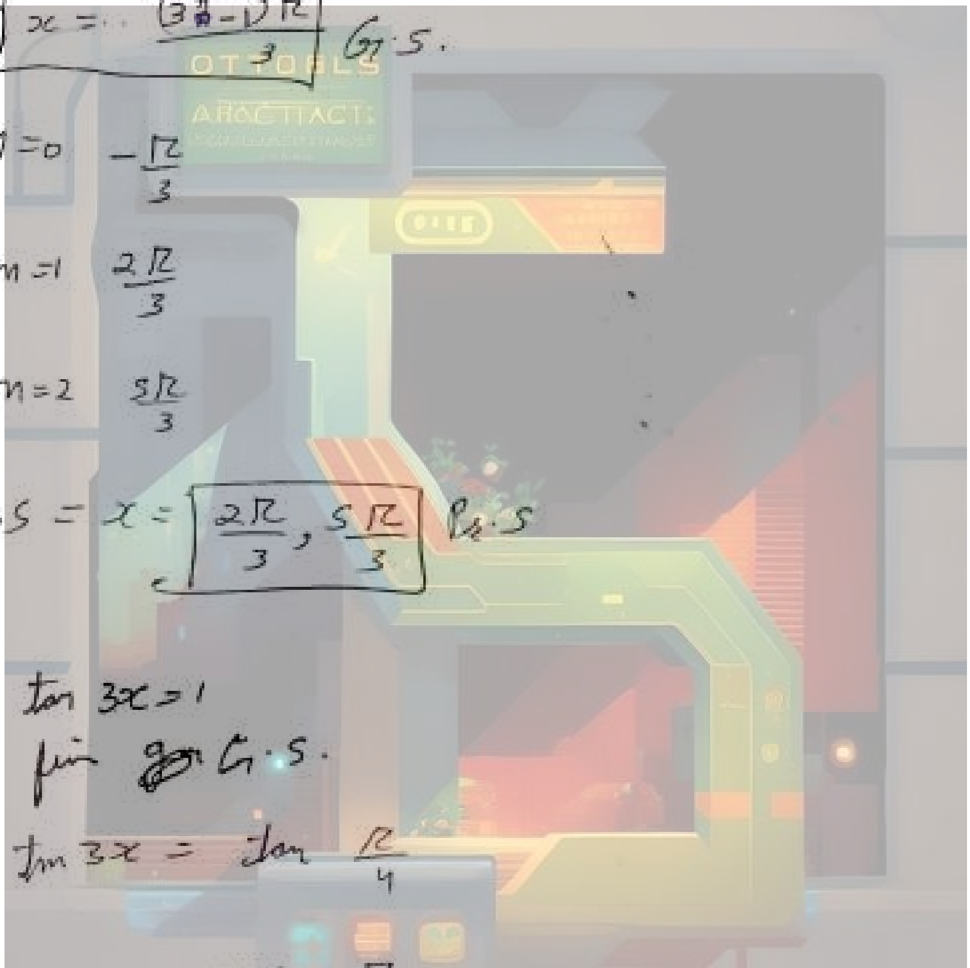
$\alpha = -60^\circ$

$\alpha = -\frac{\pi}{3}$

$x = n\pi + \alpha$

$x = n\pi - \frac{\pi}{3}$

$\boxed{x = \frac{(3n-1)\pi}{3}}$ G.S.



$n=0 \quad -\frac{\pi}{3}$

$n=1 \quad \frac{2\pi}{3}$

$n=2 \quad \frac{5\pi}{3}$

P.S $= x = \boxed{\frac{2\pi}{3}, \frac{5\pi}{3}}$ P.S

Q5. $\tan 3x = 1$
find gen G.S.

$\tan 3x = \tan \frac{\pi}{4}$

$3x = n\pi + \frac{\pi}{4}$

$3x = \frac{(4n-1)\pi}{4}$

$\boxed{x = \frac{(4n-1)\pi}{12}}$

Q6. $4\tan^2\theta = 3\sec^2\theta$ G.S.

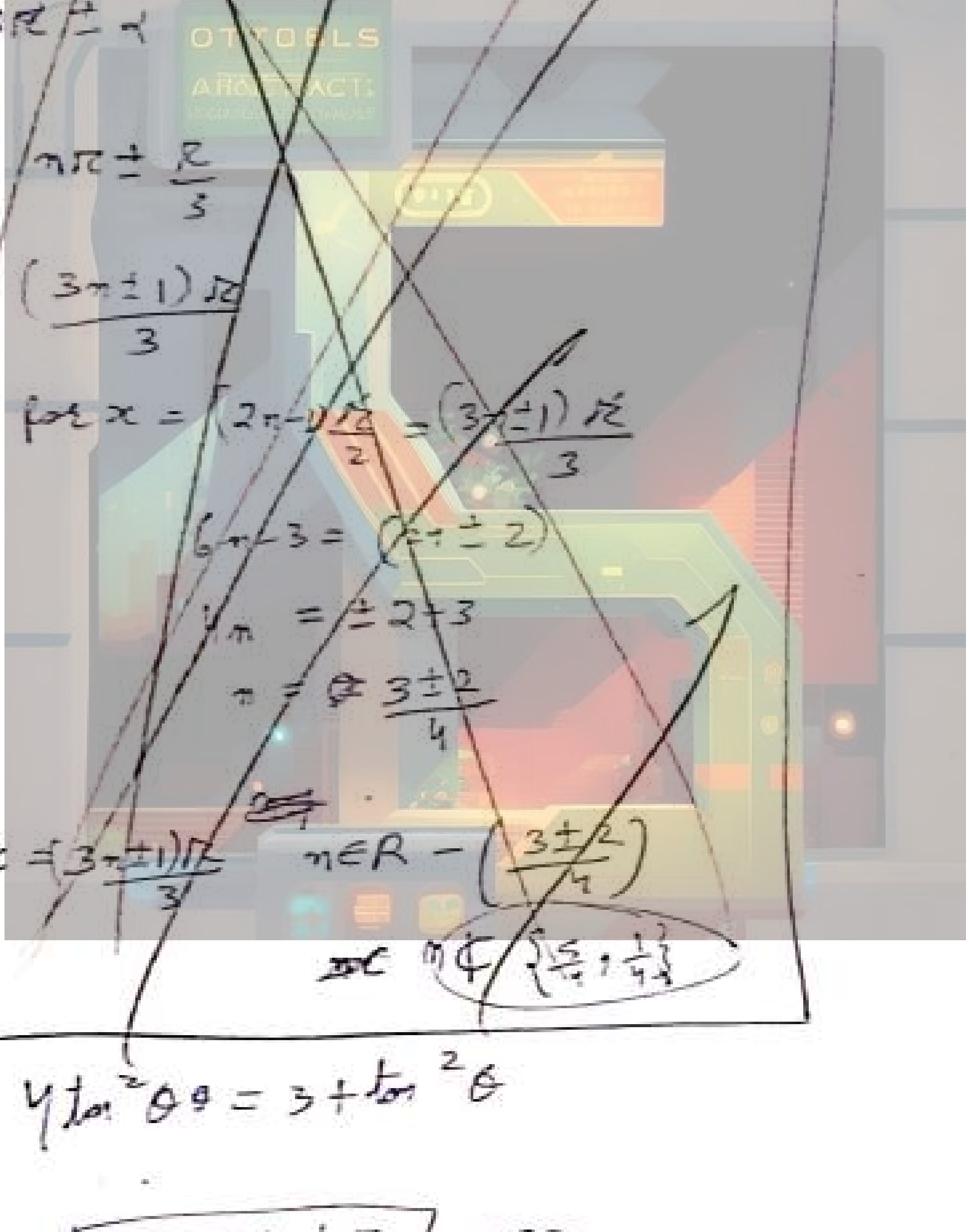
$$\frac{4\sin^2\theta}{\cos^2\theta} = \frac{3}{\cos^2\theta} \qquad \theta \neq (2n-1)\frac{\pi}{2}$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\theta\ \alpha = \frac{\pi}{3}$$

$$= n\pi \pm \alpha$$



$$x = n\pi \pm \frac{\pi}{3}$$

$$x = \frac{(3n \pm 1)\pi}{3}$$

$$\text{Let for } x = (2n-1)\frac{\pi}{2} = \frac{(3n \pm 1)\pi}{3}$$

$$6n - 3 = (2n \pm 2)$$

$$4n = \pm 2 + 3$$

$$n = \frac{3 \pm 2}{4}$$

$$x = \frac{(3n \pm 1)\pi}{3} \quad n \in R - \left(\frac{3 \pm 2}{4}\right)$$

$$n \notin \left\{\frac{5}{4}, \frac{1}{4}\right\}$$

$$4\tan^2\theta = 3 + \tan^2\theta$$

$$\boxed{\theta = n\pi \pm \frac{\pi}{3}} \quad n \in I$$

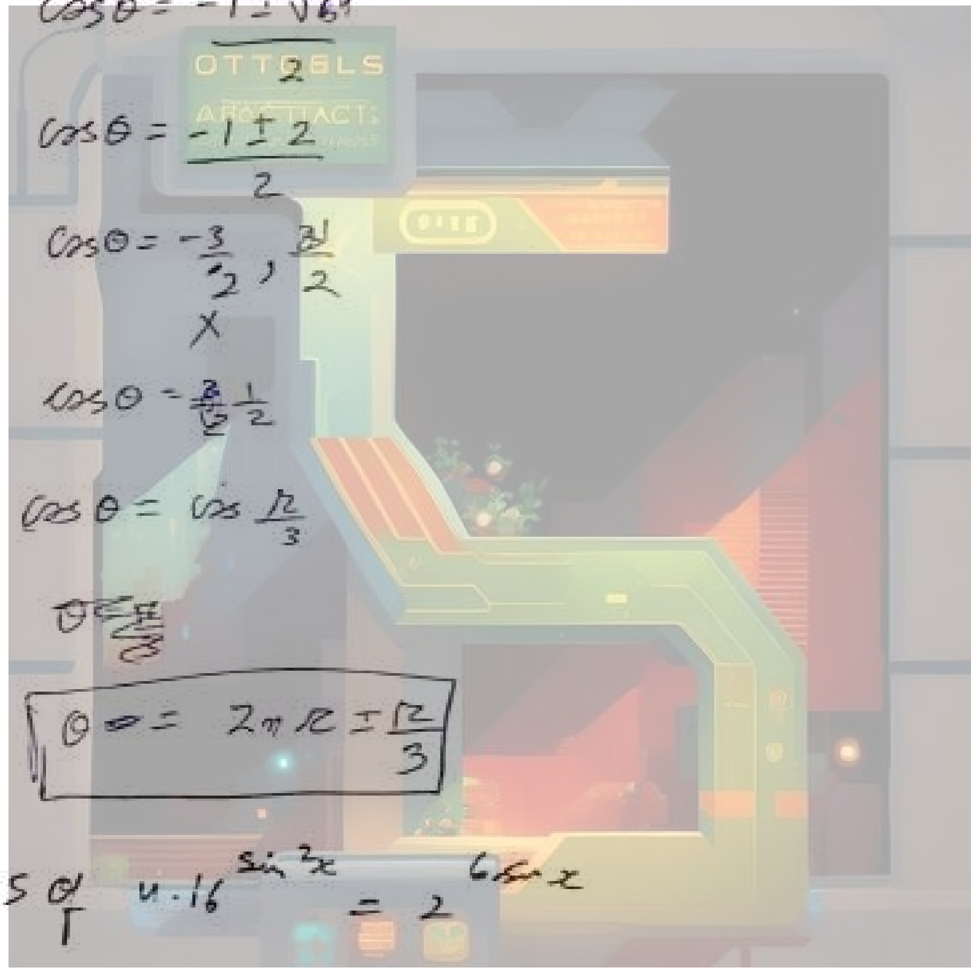
Q7. G.S. $\sin^2\theta - \cos\theta = \frac{1}{4}$

$$1 - \cos^2\theta - \cos\theta = \frac{1}{4}$$

$$\cos^2\theta + \cos\theta - \frac{3}{4} = 0$$

$$\cos\theta = \frac{-1 \pm \sqrt{1+3}}{2}$$

$$\cos\theta = \frac{-1 \pm \sqrt{4}}{2}$$



$$\cos\theta = \frac{-1 \pm 2}{2}$$

$$\cos\theta = \frac{-3}{2}, \frac{1}{2}$$

X

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos\frac{\pi}{3}$$

$$\boxed{\theta = 2n\pi \pm \frac{\pi}{3}}$$

Q Pr.5 Q1. $4 \cdot 16^{\sin^2 x} = 2^{6\sin x}$

$\cancel{\sin^2 x = \sin x}$

$\cancel{\sin x = 0 \qquad \sin x = 1}$

$\cancel{\sin x = \sin 0^\circ \qquad \sin x = \sin\frac{\pi}{2}}$

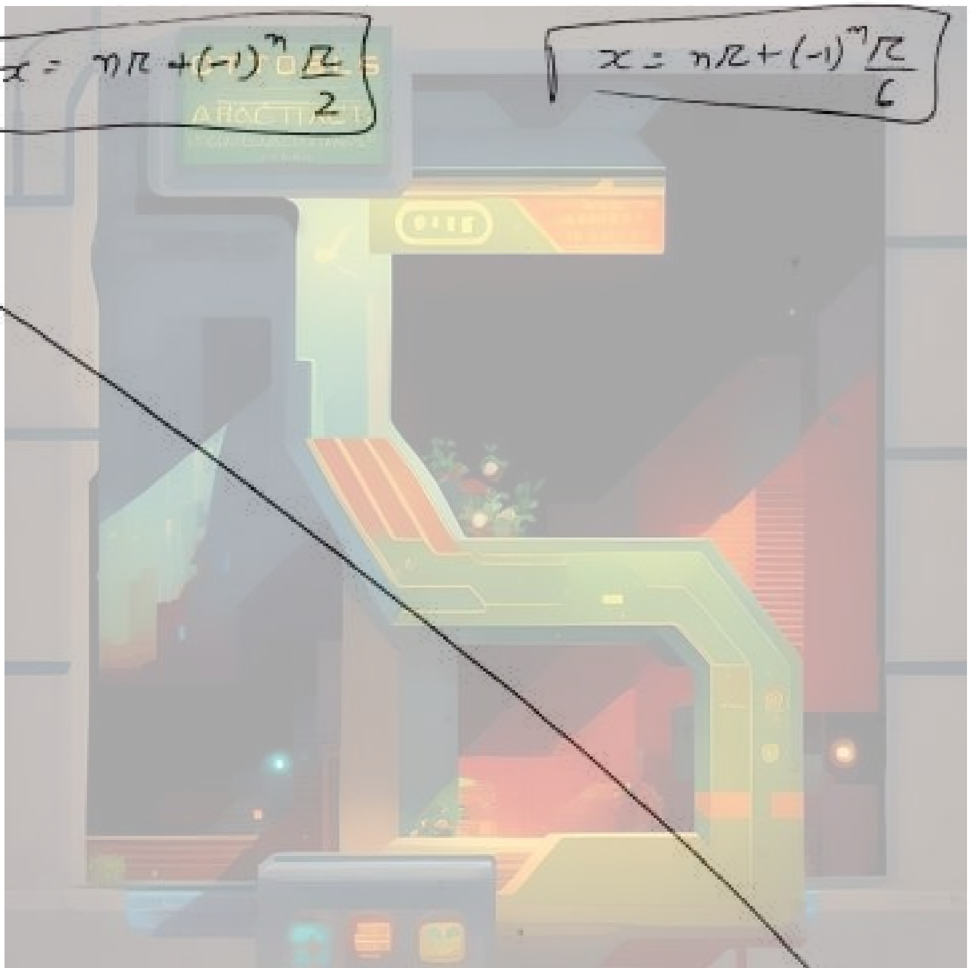
$$2^{4\sin^2 x + 2} = 2^{6\sin x}$$

$$4\sin^2 x - 6\sin x + 2 = 0$$

$\sin x = 1$ $\qquad\qquad$ $\sin x = \frac{1}{2}$

$\sin x = \sin\frac{\pi}{2}$ $\qquad\qquad$ $\sin x = \sin\frac{\pi}{6}$

$\boxed{x = n\pi + (-1)^n \frac{\pi}{2}}$ $\qquad\qquad$ $\boxed{x = n\pi + (-1)^n \frac{\pi}{6}}$



Q Gen. Sol of $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$

$\log_{1/2}|\sin x| = \log_{1/2}\dfrac{1/4}{|\cos x|}$

$|\sin x| = \dfrac{1/4}{|\cos x|}$

$\sin x = \dfrac{1/4}{\cos x}$ $\qquad$ $\sin x = -\dfrac{1/4}{\cos x}$

$|\sin x \cos x| = \dfrac{1}{4}$

$|\sin 2x| = \dfrac{1}{2}$

$\sin 2x = \dfrac{1}{2}$ $\qquad$ $\sin 2x = -\dfrac{1}{2}$

$\sin 2x = \sin 30$ $\qquad$ $\sin 2x = \sin(-30)$

$2x = n\pi + (-1)^n \dfrac{\pi}{6}$ $\qquad$ $2x = n\pi - (-1)^n \dfrac{\pi}{6}$

$(\sin 2x \geq 0)$ $\qquad$ $(\sin 2x \leq 0)$

Q Find most general values of θ

① $\sin\theta = -\dfrac{1}{2}$ & $\tan\theta = \dfrac{1}{\sqrt{3}}$

$\sin\theta = \sin(-30)$ $\qquad$ $\tan\theta = \tan(30)$

$\alpha = -30$ $\qquad$ $\alpha = 30$

$\theta = n\pi - (-1)^n \dfrac{\pi}{6}$ $\qquad$ $\theta = n\pi + \dfrac{\pi}{6}$

$n=0$ $\quad -\dfrac{\pi}{6}$ $\qquad$ $\dfrac{\pi}{6}$

$n=1$ $\quad \dfrac{7\pi}{6}$ $\qquad$ $\dfrac{7\pi}{6}$

$n=2$ $\quad \dfrac{11\pi}{6}$ $\qquad$ $\dfrac{13\pi}{6}$

$n=3$ $\quad \dfrac{19\pi}{6}$ $\qquad$ $\dfrac{19\pi}{6}$

$\cancel{\theta = 2n\pi + \alpha}$ $\boxed{\theta = (2n-1)\pi + \alpha}$ $n \in J$

$\cancel{\theta = n\pi + (-1)^n \alpha}$

(2) $\tan^2\alpha + 2\sqrt{3}\tan\alpha - 1 = 0$

$$\tan\alpha = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$$

$$= \frac{-2\sqrt{3} \pm 4}{2}$$

$$= -\sqrt{3} \pm 2$$

$\tan\alpha = -\sqrt{3} - 2$ $\tan\alpha = 2 - \sqrt{3}$

$\tan\alpha = -(\sqrt{3}+2)$ $\tan\alpha = \tan 15^\circ$

$\tan\alpha = -\frac{1}{\tan 15^\circ}$ $\alpha = n\pi + 15^\circ$

$\tan\alpha = \tan(-75)$

$\alpha = n\pi - 75^\circ$

$n=0$ $\cancel{-75}$ 15

$n=1$ $\alpha = 180n - 75$ $\alpha = 180n + 15$

$n=1$ $\alpha = 105$ $\alpha = 195$

$n=2$ $\alpha = 285$ $\alpha = 3875$

$\alpha = 180n + (\cancel{\;\;}) \; (-30 \pm 45)$

$\boxed{\alpha = n\pi + (-30 \pm 45)}$

(12)

Q. $1+\cos 3x - 2\cos 2x = 0$

$\cancel{1+\cos 3x}$

$1 + 4\cos^3 x - 3\cos x - 2(2\cos^2 x - 1) = 0$

$4\cos^3 x + 1 - 3\cos x - 4\cos^2 x + 2 = 0$

$4\cos^3 x - 4\cos^2 x - 3\cos x + 3 = 0$

$4\cos^2 x\,(c-1) - 3(c-1) = 0$

$(4c^2 - 3) = 0$ $\qquad$ $c - 1 = 0$

$c^2 = \frac{3}{4}$ $\qquad$ $c = 1$

$\cancel{\cos x = \frac{\sqrt{3}}{2}}$

$\cos^2 x = \cos^2 30$ $\qquad$ $\cos x = \cos 0°$

$\boxed{x = n\pi \pm 30°}$ $\qquad$ $\boxed{x = 2n\pi}$

imp points for solving T.E.

① Cancellation of terms which are in product is not allowed

eg. $\sin\theta\cos\theta = \sin\theta$

$\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(\cos\theta - 1) = 0$

$\sin\theta = 0$ $\qquad$ $\cos\theta = 1$

② Answer should not contain such values of angle which make any of the terms undefined in the given question

$\tan\theta, \sec\theta$: $\theta \notin \left\{(2n+1)\frac{\pi}{2}\right\}$

$\cot\theta, \csc\theta$: $\theta \neq \{n\pi\}$

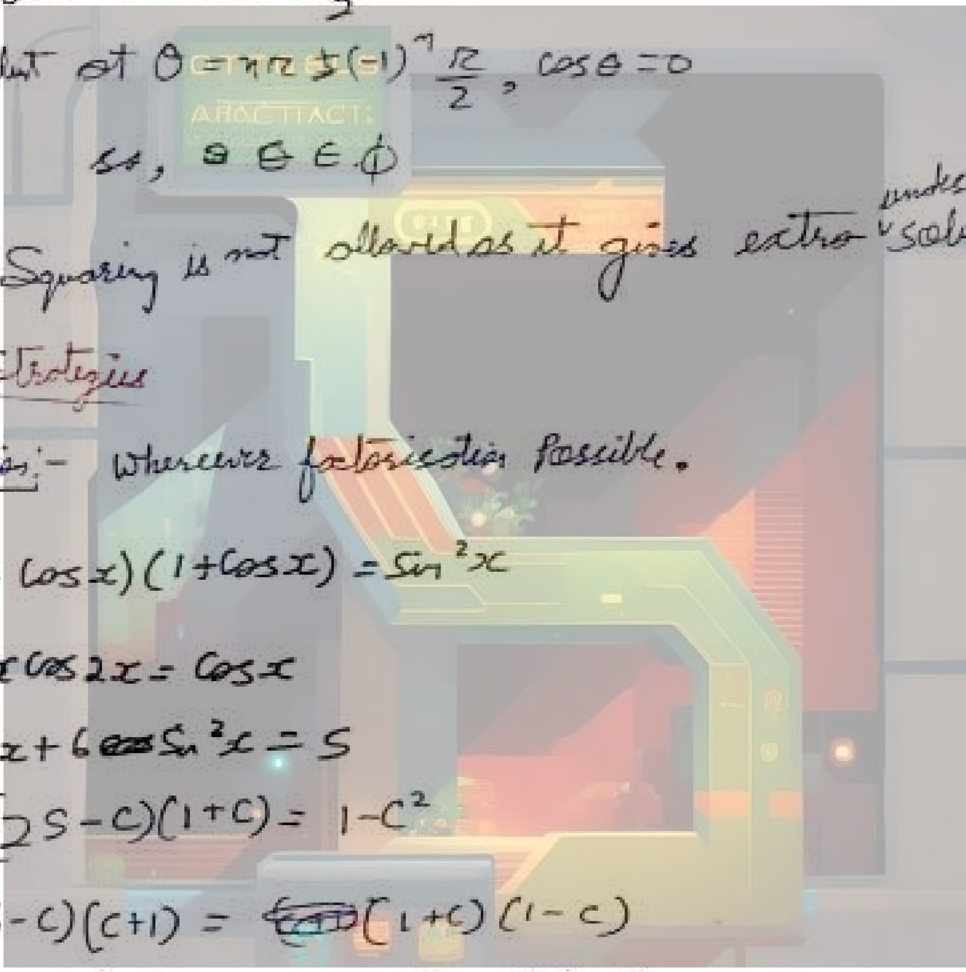
eg. $\frac{\sin 2\theta}{\cos\theta} = 2$

$= \frac{2\sin\theta\cos\theta}{\cos\theta} = 2$

$= \cancel{2}\sin\theta = 1 \quad \{\cos\theta \neq 0\}$

$\sin\theta = \sin\frac{\pi}{2}$

$\theta = n\pi + (-1)^n\frac{\pi}{2}$



but at $\theta = n\pi + (-1)^n\frac{\pi}{2}$, $\cos\theta = 0$

so, $\theta \in \phi$

(3) Direct Squaring is not allowed as it gives extra undesired solutions

Different Strategies

(1) factorisation:- wherever factorisation possible.

Q1. $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

Q2. $2\cos x\cos 2x = \cos x$

Q3. $2\sin^2 2x + 6\sin^2 x = 5$

Q1. $(2S - C)(1+C) = 1 - C^2$

$(2S - C)(C+1) = (1+C)(1-C)$

$(2S - C)(1+C) - (1+C)(1-C) = 0$

$(2S - C - 1 + C)(1+C) = 0$

$(2S - 1)(1+C) = 0$

$\cos x = -1$ $\quad$ $\sin x = \frac{1}{2}$

$\cos x = \cos\pi$ $\quad$ $\sin x = \sin\frac{\pi}{6}$

$\boxed{x = 2n\pi \pm \pi}$ or $\boxed{x = n\pi + (-1)^n\frac{\pi}{6}}$

(2) $2\cos x \cos 2x = \cos x$

$\cos x(2\cos 2x - 1) = 0$

~~$\cos x(\cos 2x)$~~

~~$\cos 2x = 0$~~ ~~$\cos 2x = 0$~~

$\cos x = 0$ $\qquad\qquad$ $\cos 2x = \frac{1}{2}$

$\cos x = \frac{\pi}{2}$ $\qquad\qquad$ $\cos 2x = \cos\frac{\pi}{3}$

$x = 2n\pi \pm \frac{\pi}{2}$ $\qquad\qquad$ $2x = 2n\pi \pm \frac{\pi}{3}$

$\boxed{x = 2n\pi \pm \frac{\pi}{2}}$ $\qquad\qquad$ $\boxed{x = n\pi \pm \frac{\pi}{6}}$

~~(3) $2\sin^2 2x + 6\sin^2 x = 5$~~

~~$2(2\sin x\cos x)^2 + 6\sin^2 x = 5$~~

~~$8\sin^2 x\cos^2 x + 6\sin^2 x = 5$~~

~~$2\sin^2 x(4\cos^2 x + 3) = 5$~~

(3) $2\sin^2 2x + 6\sin^2 x = 5$

$2\sin^2 2x + 6\sin^2 x - 5 = 0$

$8\sin^2 x\cos^2 x + 6\sin^2 x - 5 = 0$

$8\sin^2 x(1 - \sin^2 x) + 6\sin^2 x - 5 = 0$

$8\sin^2 x - 8\sin^4 x + 6\sin^2 x - 5 = 0$

$8\sin^4 x - 14\sin^2 x + 5 = 0$

$\sin^2 x = \frac{14 \pm \sqrt{196 - 160}}{16}$

$\sin^2 x = \frac{14 \pm 6}{16}$

$\sin^2 x = \frac{20}{16}$ ✗ $\qquad$ $\sin^2 x = \frac{1}{2}$

$\sin^2 x = \sin^2(45)$

$\alpha = \frac{\pi}{4}$

$\boxed{x = n\pi \pm \frac{\pi}{4}}$

$2(1 - \cos^2 2x) + 6\left(\frac{1 - \cos 2x}{2}\right) = 5$

$\cos 2x = t$

$2 - 2t^2 + 3 - 3t - 5 = 0$

$2t^2 + 3t = 0$

~~$\cos 2x = 0$~~

② reducing in Quad

Q1. $\cos 4x + 6 = 7\cos 2x$

$2\cos^2 2x - 1 + 6 = 7\cos 2x$

$2\cos^2 2x - 7\cos 2x + 5 = 0$

$\cos 2x = \dfrac{7 \pm \sqrt{49-40}}{4}$

$= \dfrac{7 \pm 3}{4}$

$\cos 2x = \dfrac{10}{4}$ ✗ $\qquad \cos 2x = \dfrac{4}{4}$

$\cos 2x = \cos 0^\circ$

$2x = 2n\pi \pm 0$

$\boxed{x = \dfrac{n\pi}{2}}$

Q2. $\sin^4 2x + \cos^4 2x = \sin 2x \cos 2x$

$(\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x\cos^2 2x = \sin 2x\cos 2x$

$\cancel{1 - 2(1-\cos^2 2x)\cos^2 2x = \ldots}$

$2x = \theta$

$1 - 2\cos^2\theta\sin^2\theta - \sin\theta\cos\theta = 0$

$2\cos^2\theta\sin^2\theta + \sin\theta\cos\theta - 1 = 0$

$\cos\theta\sin\theta = \dfrac{-1 \pm \sqrt{1+8}}{4}$

$= \dfrac{-1 \pm 3}{4}$

$\sin\theta\cos\theta = -1\ ,\ 1/2$

$\sin 4x = -2$

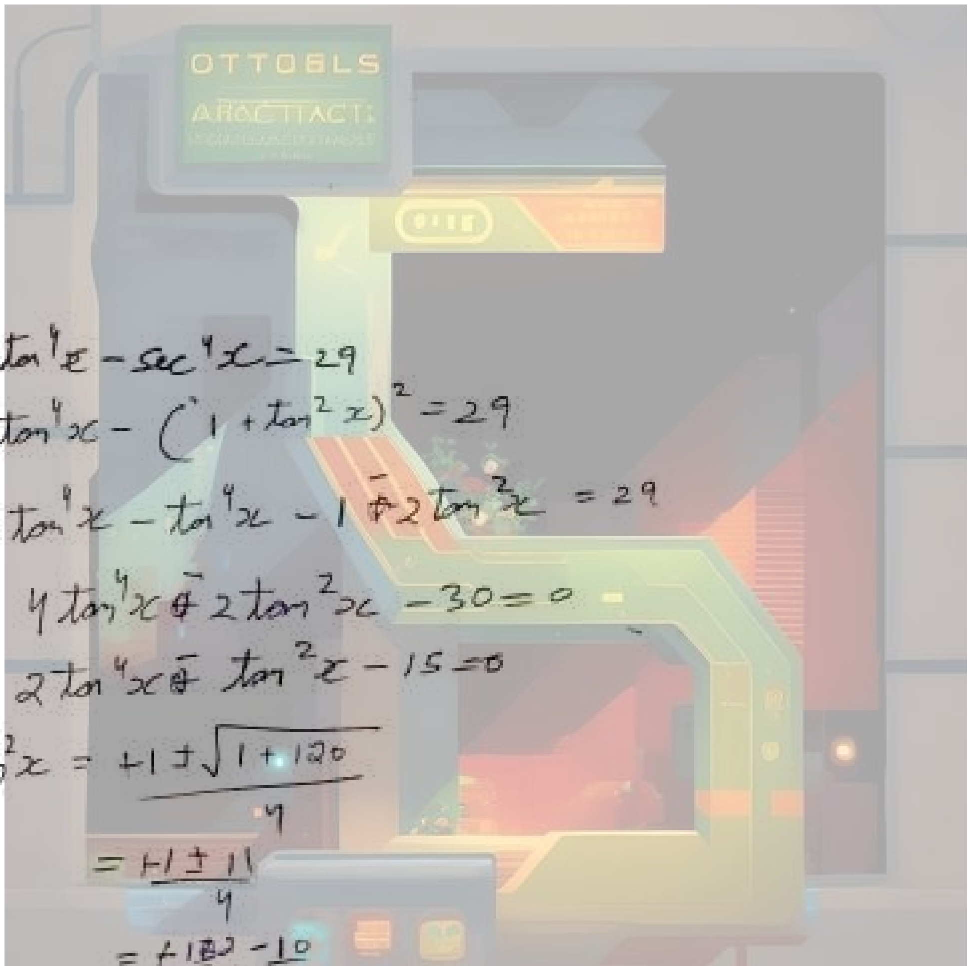
$\times$

$\sin 4x = 1$

$\sin 4x = \sin \frac{\pi}{2}$

$4x = n\pi + (-1)^n \frac{\pi}{2}$

$$\boxed{x = \frac{n\pi}{4} + (-1)^n \frac{\pi}{8}}$$



Q3. $5\tan^4 x - \sec^4 x = 29$

$5\tan^4 x - (1 + \tan^2 x)^2 = 29$

$5\tan^4 x - \tan^4 x - 1 - 2\tan^2 x = 29$

$4\tan^4 x - 2\tan^2 x - 30 = 0$

$2\tan^4 x - \tan^2 x - 15 = 0$

$\tan^2 x = \dfrac{+1 \pm \sqrt{1 + 120}}{4}$

$= \dfrac{+1 \pm 11}{4}$

$= \left(+\frac{12}{4}\right), -\frac{10}{4}$

~~$\tan^2 x = -\frac{5}{2}$~~

~~$\tan^2 x = \frac{12}{4}$~~

$\tan^2 x = -\frac{10}{4}$

$\times$

$\tan^2 x = 3$

$\tan^2 x = \tan^2\left(\frac{\pi}{3}\right)$

$$\boxed{x = n\pi \pm \frac{\pi}{3}}$$

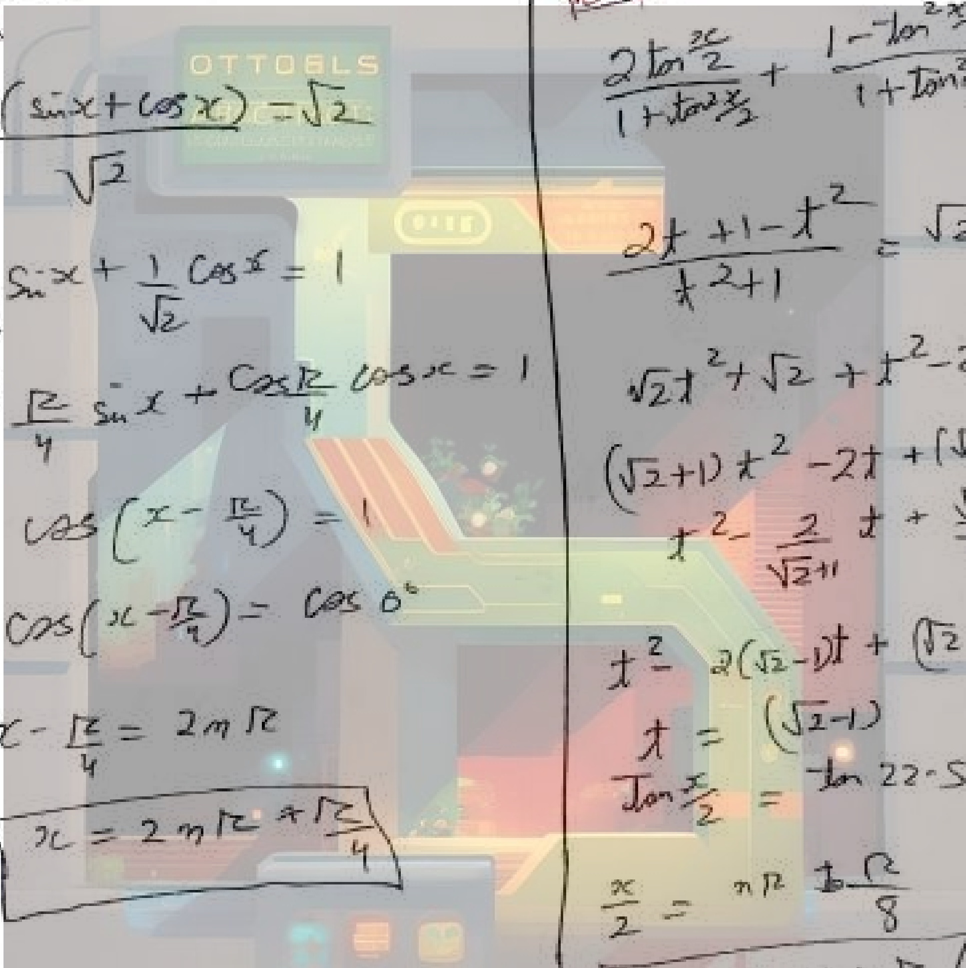
$x \notin \left\{ (2n+1)\frac{\pi}{2} \right\}$

~~③~~

③ $a\sin\theta + b\cos\theta = c$ form $\{a, b \in R\}$

M I: multiple & divide LHS by $\sqrt{a^2+b^2}$

M II: $\sin\theta = \dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ & $\cos\theta = \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ & form quad in $\tan\frac{\theta}{2}$



Q1. $\sin x + \cos x = \sqrt{2}$

M I

$$\frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{2}} = \sqrt{2}$$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = 1$$

$$\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x = 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos 0^\circ$$

$$x - \frac{\pi}{4} = 2n\pi$$

$$\boxed{x = 2n\pi + \frac{\pi}{4}}$$

M II

$$\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \sqrt{2}$$

$$\frac{2t+1-t^2}{t^2+1} = \sqrt{2}$$

$$\sqrt{2}t^2 + \sqrt{2} + t^2 - 2t - 1 = 0$$

$$(\sqrt{2}+1)t^2 - 2t + (\sqrt{2}-1) = 0$$

$$t^2 - \frac{2}{\sqrt{2}+1}t + \frac{\sqrt{2}-1}{\sqrt{2}+1} = 0$$

$$t^2 - 2(\sqrt{2}-1)t + (\sqrt{2}-1)^2 = 0$$

$$t = (\sqrt{2}-1)$$

$$\tan\frac{x}{2} = \tan 22.5$$

$$\frac{x}{2} = n\pi + \frac{\pi}{8}$$

$$\boxed{x = 2n\pi + \frac{\pi}{4}}$$

Q2. $\sqrt{3}\cos x + \sin x = 2$

$$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 1$$

$$\cos\frac{\pi}{6}\cos x + \sin x \sin\frac{\pi}{6} = 1$$

$$\cos\left(x - \frac{\pi}{6}\right) = 1$$

$$\cos\left(x - \frac{\pi}{6}\right) = \cos 0$$

$$x - \frac{\pi}{6} = 2n\pi + 0$$

$$\boxed{x = 2n\pi + \frac{\pi}{6}}$$

Q3. $\sin x + \cos x = \min\{1,\ a^2 - 4a + 6,\ 7\}$

$$\min = \frac{-D}{4a} = \frac{-(16-24)}{4} = 2$$

$$\sin x + \cos x = 1$$

$$\sin x \sin\frac{\pi}{4} + \cos x \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\boxed{x = 2n\pi + \frac{\pi}{4} \pm \frac{\pi}{4}}$$

Q 4. $1+\sin^3 x+\cos^3 x = \frac{3}{2}\sin 2x$

$(1)^3 + \sin^3 x + \cos^3 x = \frac{3}{2} \times 2\sin x \cos x$

$A^3+B^3+C^3 = 3ABC$

$A+B+C=0$

$\sin x + \cos x = -1$

$\sin x \sin \underset{\sqrt{2}}{\overset{45}{\frac{1}{}}} + \cos x \cos \underset{\sqrt{2}}{\overset{45}{\frac{1}{}}} = \frac{-1}{\sqrt{2}}$

$\cos\left(x - \overset{45}{\frac{1}{\sqrt{2}}}\right) = \cos(90+45^\circ)$

$\boxed{x = 2m\pi \pm 135 + \overset{45}{\frac{1}{\sqrt{2}}}}$

Q5. $3\cos x + 4\sin x = 5$

$\frac{3\cos x}{5} + \frac{4\sin x}{5} = 1$

$\cos x \cos 53 + \sin x \sin 53 = 1$

$\cos(x-53) = \cos 90^\circ$

$x - 53 = 2m\pi \pm 90$

$\boxed{x = 2m\pi \pm 90 + 53}$

④ Sin ⟷ Product

& Cos.

Q1. $\cos 3x + \sin 2x - \sin 4x = 0$

Q2. $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Q3. $\sin 5x \cos 3x = \sin 6x \cos 2x$

Q4.** $\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

Q5.** $8\cos x \cos 2x \cos 4x = \frac{\sin 6x}{\sin x}$

Q1. $\cos 3x + 2\cos(3x)\sin(-x) = 0$

$\cos 3x(1 - 2\sin x) = 0$

$\cos 3x = 0$

$\cos 3x = \cos\frac{\pi}{2}$

$3x = 2n\pi \pm \frac{\pi}{2}$

$\boxed{x = \frac{2n\pi}{3} \pm \frac{\pi}{6}}$

$1 - 2\sin x = 0$

$\sin x = \frac{1}{2}$

$\sin x = \sin\frac{\pi}{6}$

$\boxed{x = n\pi + (-1)^n\frac{\pi}{6}}$

Q2. $(\cos\theta + \cos 7\theta) + (\cos 3\theta + \cos 5\theta) = 0$

$(2\cos 4\theta \cos 3\theta) + (2\cos 4\theta \cos\theta) = 0$

$2\cos 4\theta(\cos 3\theta + \cos\theta) = 0$

$2\cos 4\theta(2\cos 2\theta \cos\theta) = 0$

$4\cos 4\theta \cos 2\theta \cos\theta = 0$

$\cos\theta = \cos\frac{\pi}{2}$ $\quad$ $\cos 2\theta = \cos\frac{\pi}{2}$ $\quad$ $\cos 4\theta = \cos\frac{\pi}{2}$

$\boxed{\theta = 2n\pi \pm \frac{\pi}{2}}$ or $\boxed{\theta = n\pi \pm \frac{\pi}{4}}$ or $\boxed{\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}}$

Q3. $2\sin 5x \cos 3x = 2\sin 6x \cos 2x$

$\sin 8x + \sin 2x = \sin 8x + \sin 4x$

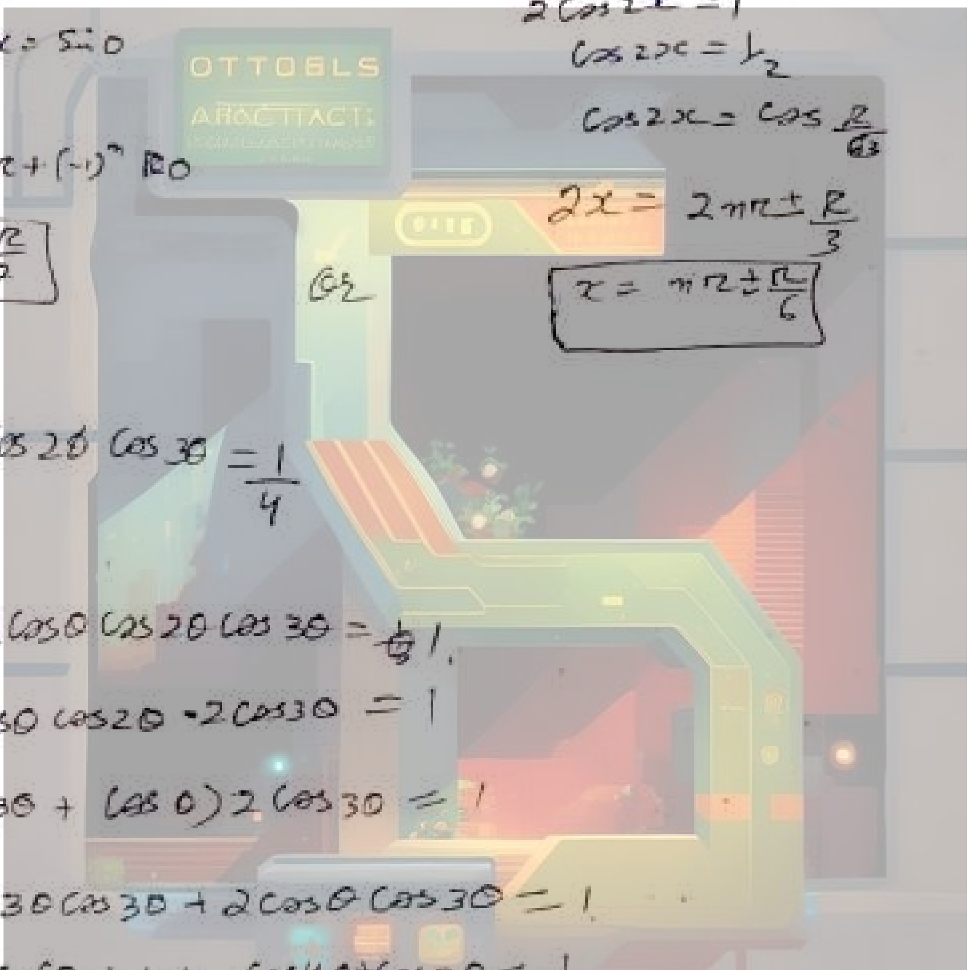
$\sin 2x = 2\sin 2x \cos x$

$2\sin 2x \cos 2x - \sin 2x = 0$

$\sin 2x(2\cos 2x - 1) = 0$

$\sin 2x = 0$

$\sin 2x = \sin 0$



$2x = n\pi + (-1)^n 0$

$\boxed{x = \frac{n\pi}{2}}$

$2\cos 2x - 1 = 0$

$2\cos 2x = 1$

$\cos 2x = \frac{1}{2}$

$\cos 2x = \cos\frac{\pi}{3}$

$2x = 2n\pi \pm \frac{\pi}{3}$

$\boxed{x = n\pi \pm \frac{\pi}{6}}$

Q4. $\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$

$4\cos\theta \cos 2\theta \cos 3\theta = 1$

$2\cos\theta \cos 2\theta \cdot 2\cos 3\theta = 1$

$(\cos 3\theta + \cos\theta)2\cos 3\theta = 1$

$2\cos 3\theta \cos 3\theta + 2\cos\theta \cos 3\theta = 1$

$\cos 6\theta + 1 + \cos 4\theta + \cos 2\theta = 1$

$\cos 6\theta + \cos 4\theta + \cos 2\theta = 0$

$\frac{\cos 4\theta \sin 3\theta}{\sin\theta} = 0$

$\cos 4\theta = 0$ $\qquad$ $\sin 3\theta = 0$ $\quad \{\theta \neq \frac{n\pi}{2}\}$

$\cos 4\theta = \cos\frac{\pi}{2}$ $\qquad$ $\sin 3\theta = \sin 0$

$\boxed{\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}}$ $\qquad$ $\boxed{\theta = \frac{n\pi}{3}}$

Q5. $8\cos x \cos 2x \cos 4x = \dfrac{\sin 6x}{\sin x}$ $\quad - \{x \neq n\pi\}$

2

$2 \div 4 \times 2 \sin x \cos x \ \cos 2x \cos 4x = \sin 6x$

$4\ 2 \times 2 \sin 2x \ \cos 2x \cos 4x = \sin 6x$

$2 \sin 4x \cos 4x = \sin 6\ 8x$

$\sin 8x = \sin 6x$

$\sin 8x - \sin 6x = 0$

$2 \cos 7x \sin x = 0$

$\cos 7x = \cos \dfrac{\pi}{2}$ $\qquad \sin x = \sin 0$

$7x = 2n\pi \pm \dfrac{\pi}{2}$ $\qquad x = n\pi \pm 0(-1)^n$

$\boxed{x = n\pi}$ X

$\boxed{x = \dfrac{2}{7} n\pi \pm \dfrac{\pi}{14}}$

(5) $(\sin x \pm \cos x)(\sin x \cos x)$ format

(1) $(\sin x + \cos x)^2 = t$

$\sin x \cos x = \dfrac{t^2 - 1}{2}$

(2) $(\sin x \mp \cos x)^2 = t$

$\sin x \cos x = \dfrac{1 - t^2}{2}$

Q1. $\sin x + \cos x = 1 + \sin x \cos x$

$t = \dfrac{2 + t^2 - 1}{2}$

$t^2 + 1 = 2t$

$t^2 - 2t + 1 = 0$

$t = \dfrac{2 \pm \sqrt{4 - 4}}{2}$

$t = 1$

M.I.

$t = 1$

$\sin x \cos x = 0$

$\sin x = 0$ $\qquad \cos x = 0$

$\boxed{x = n\pi}$ $\qquad \boxed{x = 2n\pi \pm \dfrac{\pi}{2}}$

MII

$\sin x + \cos x = 1$

$$\frac{\sqrt{2}\,(\sin x + \cos x)}{\sqrt{2}} = 1$$

$$\frac{1}{\sqrt{2}} = \sin x \sin 45 + \cos x \cos 45$$

$\cos(x - 45) = \cos 45$

$x - 45 = 2n\pi \pm 45$

$\boxed{x = 2n\pi + 45 \pm 45}$

$x = 2n\pi + \frac{\pi}{2}$ $\qquad$ $x = 2n\pi$

(1) Boundry

→ mostly we use maximum & minimum values of T.R. (y & x values)

Q1. $\cos x + \cos 2x + \cos 3x = 3$

Q2. $\sin^4 x = 1 + \cos^2 y$

Q3. $\sin x + \cos x = \sqrt{\frac{y+1}{y}}$, $y > 0$ find x

Q4. $2^{\frac{1}{\sin^2 x}} \times \sqrt{y^2 - 2y + 2} = 2$ Find x in $[0, 2\pi]$

Q5. $2\cos^2\frac{x}{2}\sin^2 x = x^2 + \frac{1}{x^2}$ $\quad x \in [0, \frac{\pi}{2}]$

Q6. $\cos(\pi\sqrt{x-4}) \cdot \cos(\pi\sqrt{x}) = 1$

Q1. $\cos x + \cos 2x + \cos 3x = 3$

$\cos x = 1$ $\qquad$ $\cos 2x = 1$ $\qquad$ $\cos 3x = 1$

$\cos x = \cos 0$ $\qquad$ $\cos 2x = \cos 0$ $\qquad$ $\cos 3x = \cos 0$

$\boxed{x = 2n\pi}$ and $2x = 2n\pi$ $\qquad$ $3x = 2n\pi$

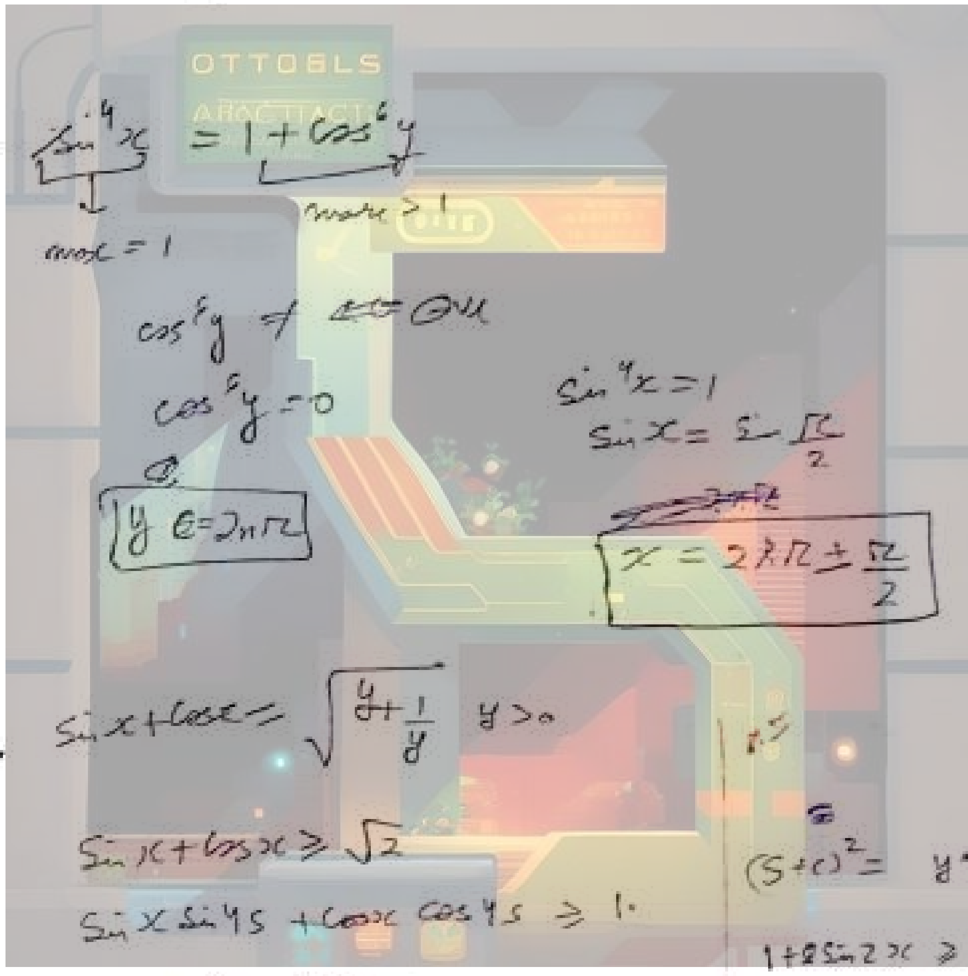
$\boxed{x = \frac{n\pi}{2}}$ and $\boxed{x = \frac{2n\pi}{3}}$

~~LCM = $2n\pi$~~ $\quad$ LCM $= 2\frac{n\pi}{3}(6, 3, 2)$ $\quad$ LCM $= 6$

$\boxed{2n\pi}$

Q2. ~~$\sin^4 x = 1 + \cos^6 y$~~

~~$\sin^4 x - \cos^6 y = 1$~~



Q2. $\underbrace{\sin^4 x}_{\max = 1} = \underbrace{1 + \cos^6 y}_{\max \geq 1}$

$\cos^6 y \neq$ ~~0~~

$\cos^6 y = 0$

$\boxed{y = 2n\pi}$

$\sin^4 x = 1$

$\sin x = \sin\frac{\pi}{2}$

~~$2n\pi$~~

$\boxed{x = 2n\pi \pm \frac{\pi}{2}}$

Q3. $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$ $\quad$ $y > 0$

$\sin x + \cos x \geq \sqrt{2}$

$\sin x \sin 45 + \cos x \cos 45 \geq 1.$

$\cos(x - 45) \geq 1$

~~$\cos$~~ - but cos max value $= 1$

$\cos(x - 45) = 1$

$x - 45 = 2n\pi$

$\boxed{x = 2n\pi + 45}$

$x =$

$(s + c)^2 = y + \frac{1}{y}$

$1 + 2\sin 2x \geq 2$

$\sin 2x = 1$

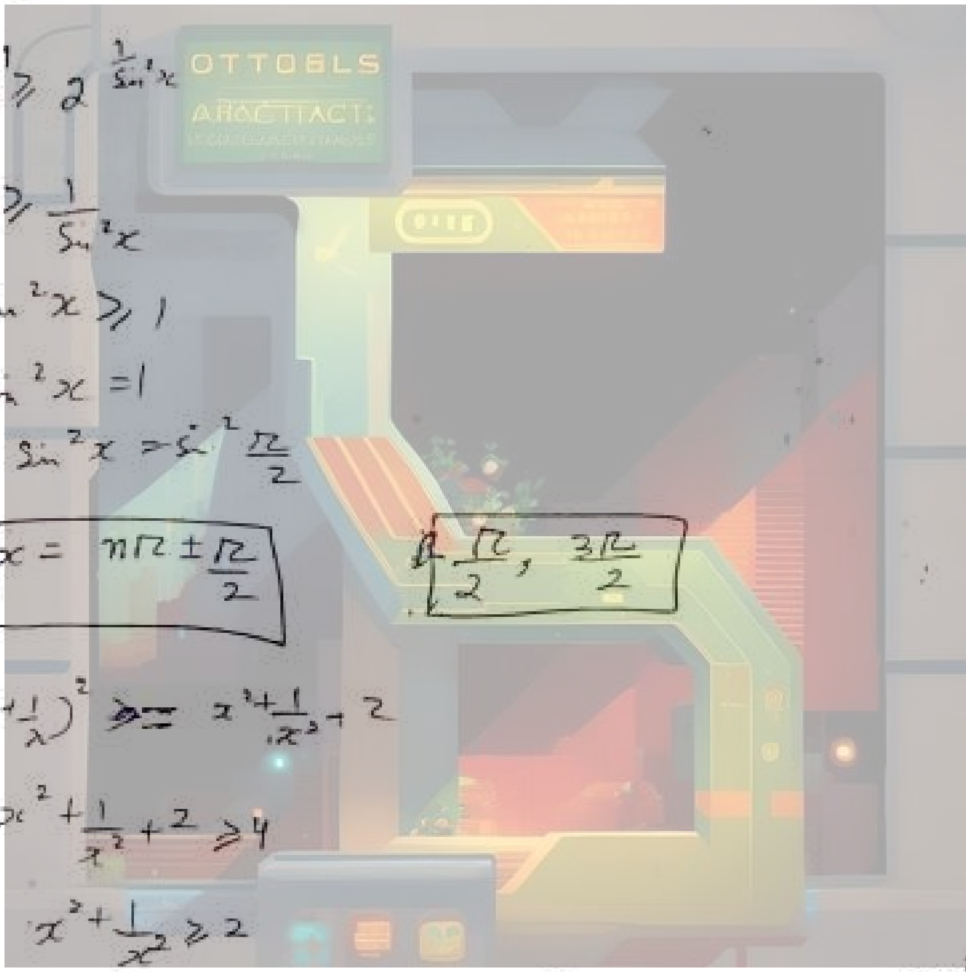
Q4.

$y^2 - 2y + 2$

$\min = \frac{-D}{4a} = \frac{4ac - b^2}{4a} = \frac{8-4}{4}$

$= 1$

$\frac{2}{2^{\sin^2 x}} \geqslant 1$



$2^1 \geqslant 2^{\frac{1}{\sin^2 x}}$

$1 \geqslant \frac{1}{\sin^2 x}$

$\sin^2 x \geqslant 1$

$\sin^2 x = 1$

$\sin^2 x = \sin^2 \frac{\pi}{2}$

$\boxed{x = n\pi \pm \frac{\pi}{2}}$ $\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

Q5. $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$x^2 + \frac{1}{x^2} + 2 \geqslant 4$

$x^2 + \frac{1}{x^2} \geqslant 2$

$\cos^2 \frac{\pi}{2} = 1$

$\cos^2 \frac{\pi}{2} - \cos^2 \frac{\pi}{2}$

$\sin^2 x = 1$

$\sin^2 x = \sin^2 \frac{\pi}{2}$

$\boxed{x = n\pi \pm \frac{\pi}{2}}$ and $\boxed{x = 2n\pi}$

$\cos^2 \frac{x}{2} = 1$

$\boxed{\frac{x}{2} = n\pi}$

Q6. ~~cos~~ cos(π ...) ...

either both (1) or (−1)
$\downarrow$
not possible

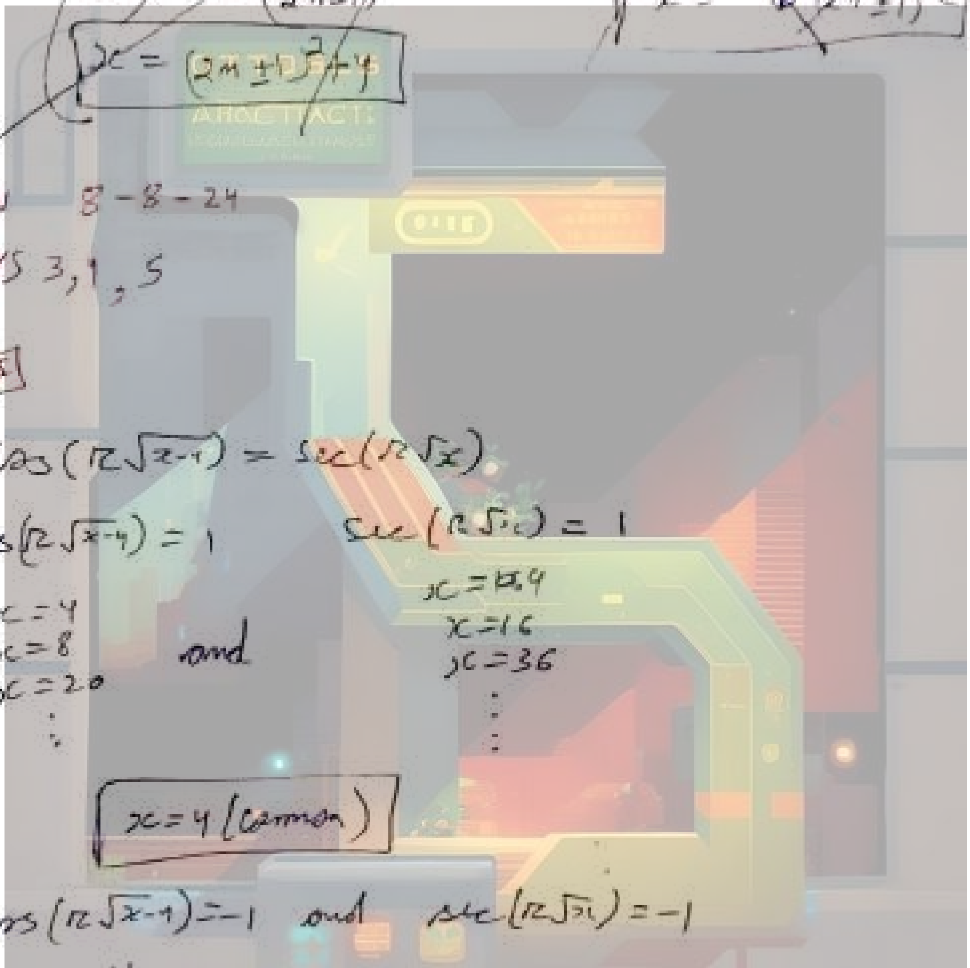
~~$\pi\sqrt{x-4}$~~ $\cos(\pi\sqrt{x-4}) = -1$ $\qquad$ $\cos(\pi\sqrt{x}) = -1$

$\cos(\pi\sqrt{x-4}) = \cos(\pi)$ $\qquad$ $\cos(\pi\sqrt{x}) = \cos\pi$

$\pi\sqrt{x-4} = (2n\pm1)\pi$ $\qquad$ $\pi\sqrt{x} = (2n\pm1)\pi$

$x-4 = (2n\pm1)^2$ $\qquad$ $\boxed{x = (2n\pm1)^2}$

$\boxed{x = (2n\pm1)^2+4}$



H.W 8 − 8 − 24

DYS 3, 4, 5

Q6. M II

$\cos(\pi\sqrt{x-4}) = \sec(\pi\sqrt{x})$

$\cos(\pi\sqrt{x-4}) = 1$ $\qquad$ $\sec(\pi\sqrt{x}) = 1$

$x = 4$ $\qquad\qquad$ $x = 4$
$x = 8$ $\qquad$ and $\qquad$ $x = 16$
$x = 20$ $\qquad\qquad$ $x = 36$
$\vdots$ $\qquad\qquad$ $\vdots$

$\boxed{x = 4 \text{ (common)}}$

$\cos(\pi\sqrt{x-4}) = -1$ and $\sec(\pi\sqrt{x}) = -1$

No common

System of T.E

Q1. find x, y if $\sin x \sin y = \frac{\sqrt{3}}{4}$ — ① $\cos x \cos y = \frac{\sqrt{3}}{4}$ — ②

② + ①

$$\sin x \sin y + \cos x \cos y = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\cos(x-y) = \cos\frac{\pi}{6}$$

$$x - y = 2n\pi \pm \frac{\pi}{6} \quad — ③$$

② − ①

$$\cos(x+y) = \cos\frac{\pi}{2}$$

$$x + y = 2k\pi \pm \frac{\pi}{2} \quad — ④$$

$$① + ② = \quad 2x = 2\pi(n+k) \pm \frac{\pi}{6} \pm \frac{\pi}{2}$$

$$\boxed{x = \cancel{2}\,\pi(n+k) \pm \frac{\pi}{12} \pm \frac{\pi}{4}}$$

Q2. $x + y = \frac{2\pi}{3}$ $\quad \frac{\sin x}{\sin y} = 2$

~~$\sin(x+y) = \sin\frac{2\pi}{3} \cdot \cos 30$~~ ~~$\sin x \sin 30 = \sin y$~~

$\boxed{\text{M I}}$ find y & put in eq to get A whole eq in x ~~$\sin(x-y) = \sin x \cos y - \cos x \sin y$~~

M II

$$\frac{\sin x + \sin y}{\sin x - \sin y} = \frac{3}{7} = \frac{\cancel{2}\sin\frac{(x+y)}{2}\cos\frac{(x-y)}{2}}{\cancel{2}\sin\frac{(x-y)}{2}\cos\frac{(x+y)}{2}} = 3$$

$$= \frac{\sin 60 \cos\left(\frac{x-y}{2}\right)}{\sin\left(\frac{x-y}{2}\right)\cos 60} = 3$$

$$\frac{\sqrt{3}}{3} = \tan\left(\frac{x-y}{2}\right)$$

$\tan\frac{x-y}{2} = \frac{1}{\sqrt{3}}$

$\frac{x-y}{2} = n\pi + \frac{\pi}{6}$

$x-y = 2n\pi + 60^\circ$

$2x = 2n\pi + 180^\circ$

$\boxed{x = n\pi + 90^\circ}$

Q $a^2+2a+\operatorname{cosec}^2\left(\frac{\pi}{2}(a+x)\right)$

(A) $a=1,\ \frac{x}{2}\in I$ (C) $a\in R,\ x\in\phi$

(B) $a=-1\ \frac{x}{2}\in I$ (D) a & x are finite but not possible to find.

$\boxed{M I}$

$a = \frac{-2\pm\sqrt{4-4\operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right]}}{2}$

$a = -1\pm\sqrt{1-\operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right]}$

$\boxed{a=-1}$

$\operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right] = 1$

$\frac{\pi}{2}(a+x) = n\pi\pm\pi$

$a+x = 2n\pm2$

$x = 2n\pm2+1$

$\boxed{B}$

$\boxed{M II}$

$a^2+2a+1+\operatorname{cosec}^2\left(\frac{\pi}{2}(a+x)\right) = 1$

$\underbrace{(a+1)^2}_{0} + \underbrace{\operatorname{cosec}^2\left[\frac{\pi}{2}(a+x)\right]}_{1} = 1$

Q4. $(\sin x-1)^2 + \cos^4 y + \left|\tan^2 z-\sqrt{3}\right| = 0$ find x, y, z

$\sin x = 1$ $\cos y = 0$ $\tan^2 z = \sqrt{3}$

$\boxed{x = n\pi+(-1)^n\frac{\pi}{2}}$ and $\boxed{y = n\pi}$ and

$\tan^2 z = 3$

$\boxed{z = n\pi\pm\frac{\pi}{3}}$

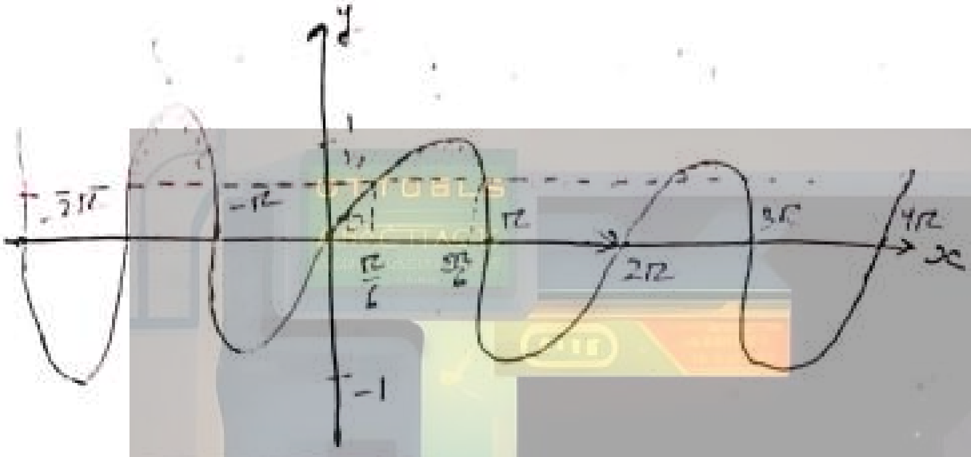
Trigonometric Inequalities

Q find the values of x for $\sin x > \frac{1}{2}$

i) when $x \in [0, 2\pi]$

ii) $x \in [-2\pi, 4\pi]$

iii) $x \in \mathbb{R}$

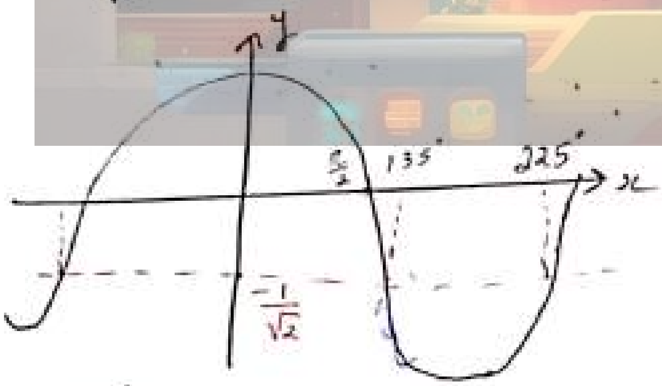


i) $\boxed{x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)}$

ii) $x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \cup \left(2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}\right) \cup \left(\frac{\pi}{6} - 2\pi, \frac{5\pi}{6} - 2\pi\right)$

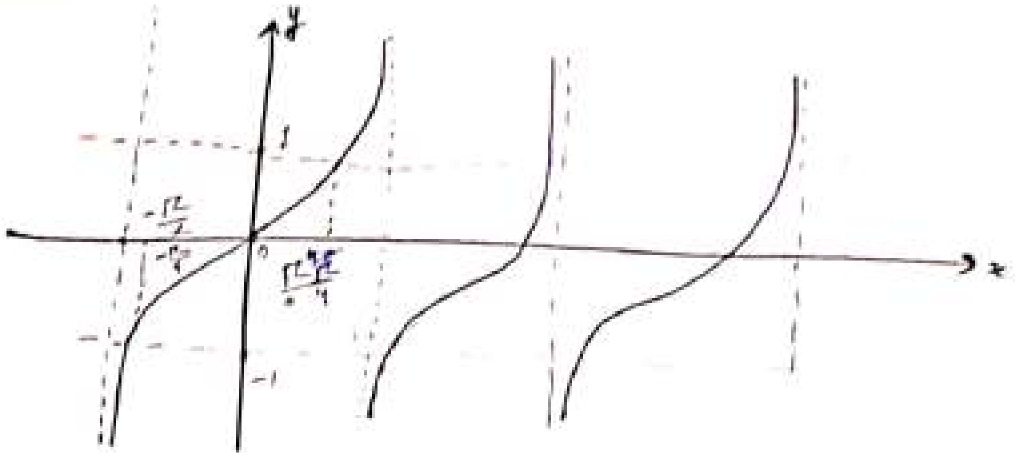
iii) $x \in \left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$, $n \in \mathbb{Z}$

Q2. $\cos x > -\frac{1}{\sqrt{2}}$



$x \in \left(2n\pi + 135^\circ, 2n\pi - 135^\circ\right)$

Q3. [illegible] ≤ 1



$\cancel{x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]}$

$x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]$

Q4. $\tan^2 x - (\sqrt{3} + 1)\tan x + \sqrt{3} < 0$

$\tan^2 x - \sqrt{3}\tan x - \tan x + \sqrt{3} < 0$

$\tan x(\tan x - \sqrt{3}) - 1(\tan x - \sqrt{3})$

$\tan x = \sqrt{3} \qquad \tan x = 1$

$\xleftarrow{\quad + \quad 1 \quad - \quad \sqrt{3} \quad + \quad} \qquad < 0$

$\tan x \in [1, \sqrt{3})$

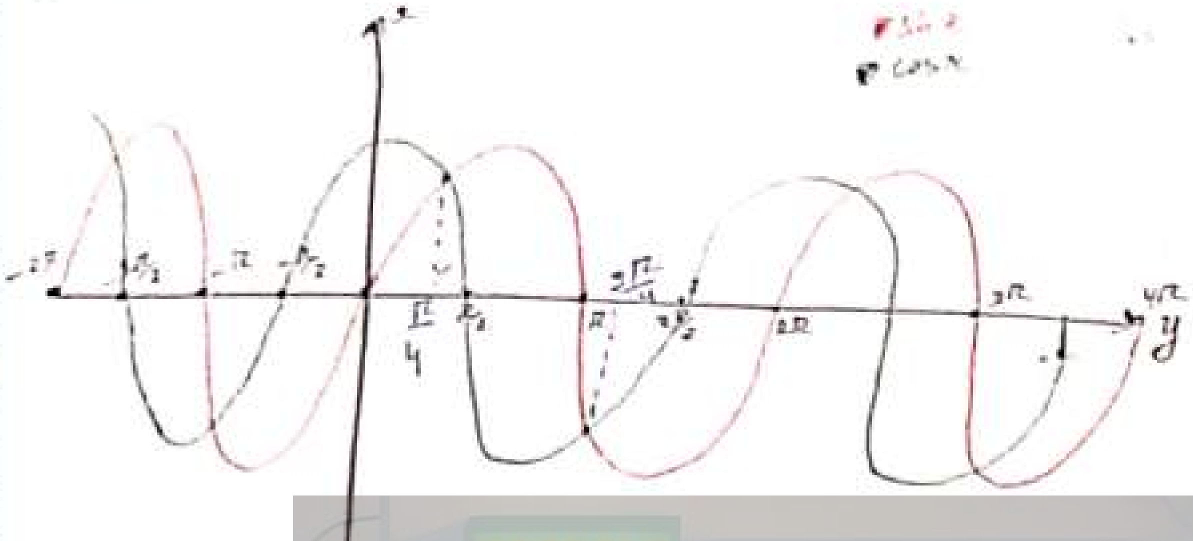
$\boxed{x \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}\right)}$

Q5. $|\tan x| < -1$

Asambhav

$\boxed{x \in \phi}$

Q $\sin x > \cos x \quad x \in [0, 2\pi]$



$$\boxed{x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)}$$

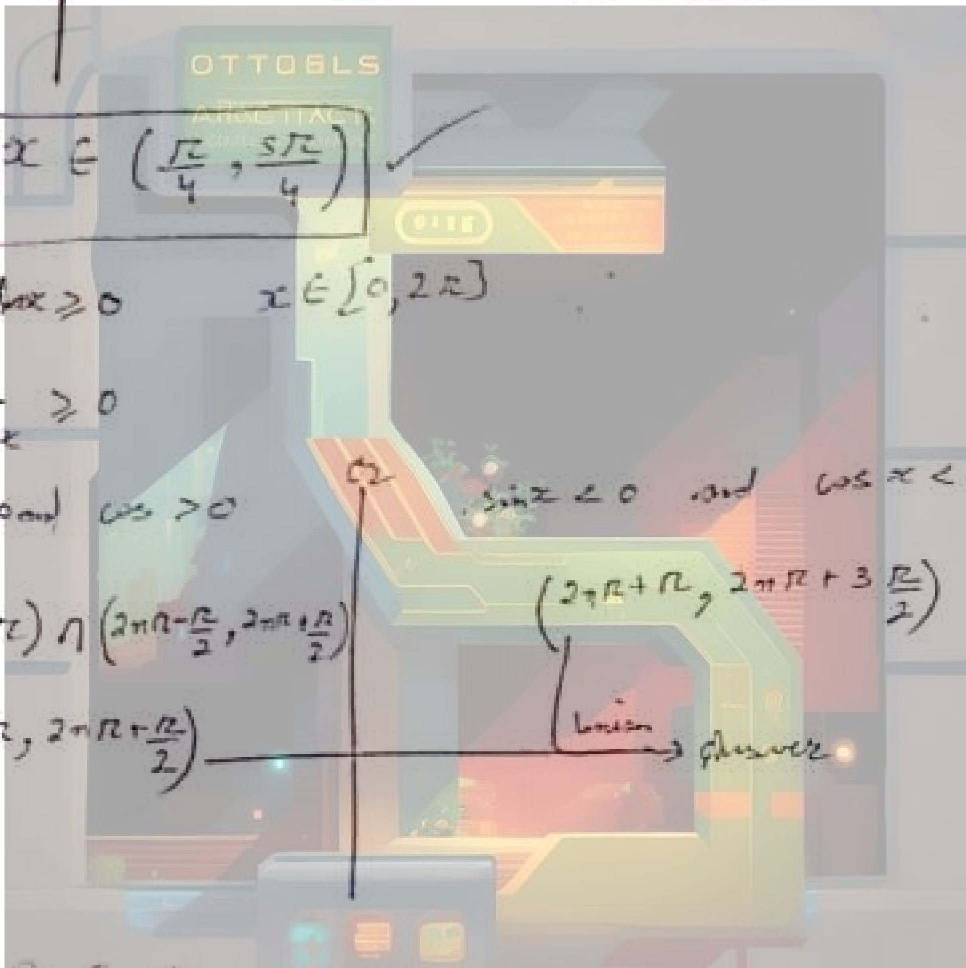
Q $\operatorname{cosec}^2 x - \tan x \geq 0 \qquad x \in [0, 2\pi]$

$$\frac{1}{\sin x \cos x} \geq 0$$

$\sin x > 0$ and $\cos > 0$ $\qquad$ $\sin x < 0$ and $\cos x < 0$

$\left(2n\pi + 0, 2n\pi + \pi\right) \cap \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right)$ $\qquad$ $\left(2n\pi + \pi, 2n\pi + 3\frac{\pi}{2}\right)$

$\left(2n\pi, 2n\pi + \frac{\pi}{2}\right)$ union → Answer



H.W. 9-8-24

DYS-6

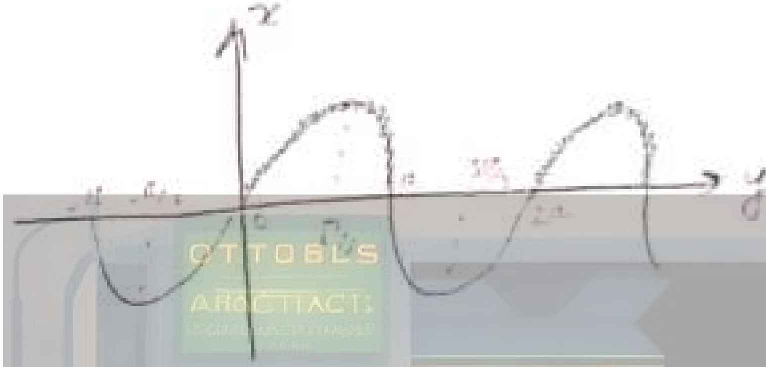
O-1 [1-15]

Q $\sqrt{\sin x} \le 1$ $\sin x \le 1$

↳ $\sin x \ge 0$

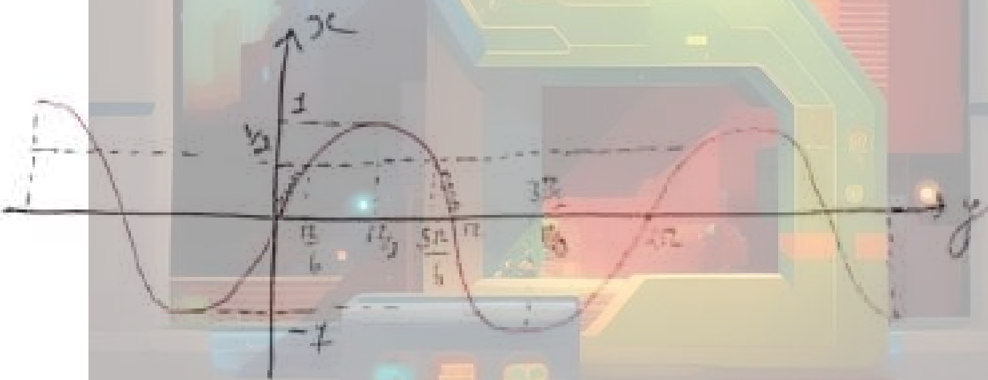
$\sin x \in [0, 1]$

$\boxed{x \in [2n\pi, 2n\pi + \pi]}$



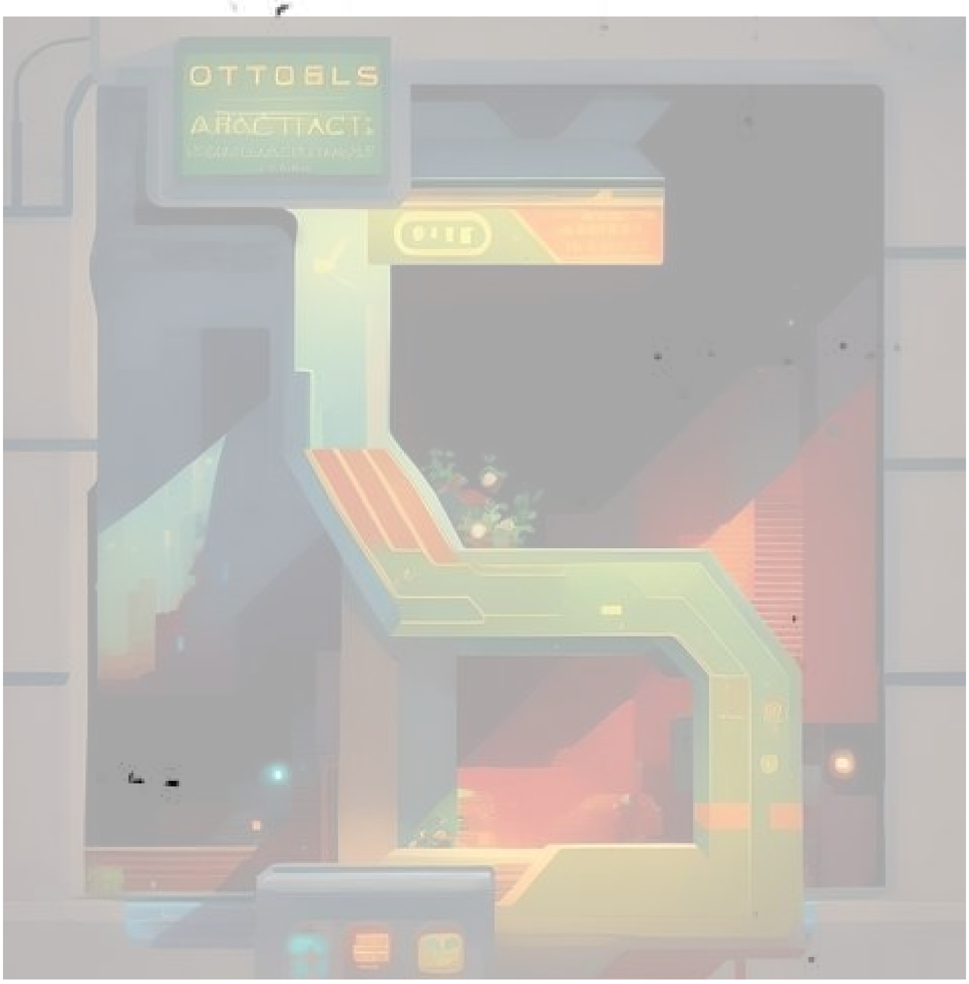
Q $\log_2\left(\sin \frac{x}{2}\right) < -1$ in $x \in [0, 2\pi]$

$\sin \frac{x}{2} < \frac{1}{2}$, $\sin \frac{x}{2} > 0$



$\frac{x}{2} \in \left(2n\pi, 2n\pi + \frac{\pi}{6}\right) \cup \left(2n\pi + \frac{5\pi}{6}, 2n\pi + \pi\right)$

$\boxed{x \in \left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right)}$




OTTOBLS
ARACTTACT:


OTTOBLS
ARACTIACT:


OTTOBLS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ARACTIACT:

! Fundamentals of Geometry !

① Similar & Congruent triangle :-

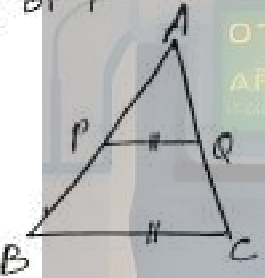
↓ Zooming Effect

↓ Sides Proportional

↳ Exactly Same shape & size (can overlap)

② Similar Polygons – BPT & Converse BPT

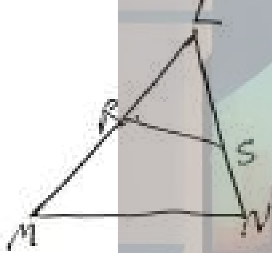
i) BPT



$PQ \parallel BC$

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

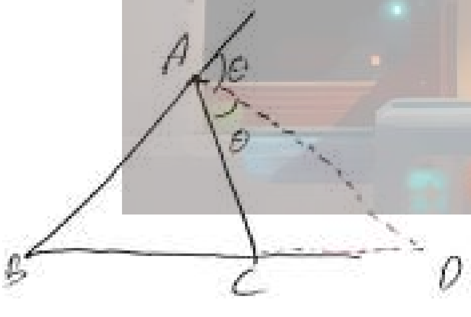
ii) Converse BPT



$$\frac{LR}{RM} = \frac{LS}{SN}$$

then, $RS \parallel MN$

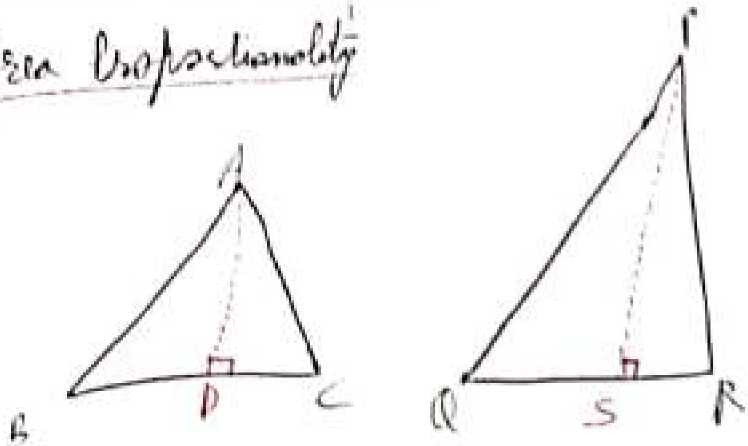
③ External Angle Bisector Theorem



If AD is angle bisector then.

$$\frac{AB}{AC} = \frac{BD}{CD}$$

(4) Area Proportionality

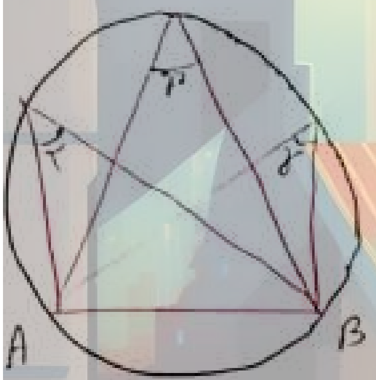


If ΔABC & ΔPQR are similar then—

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

(5) Circle Theorem:

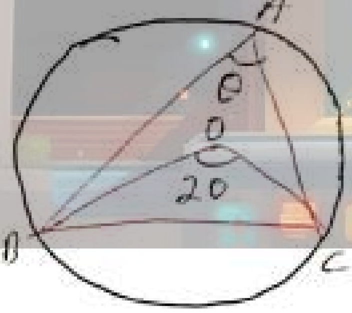
a)



for same chord AB (same segment angles)

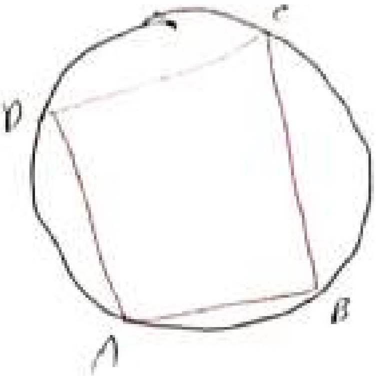
$\angle\alpha = \angle\beta = \angle\gamma$

b)



Angle at centre is double of that at circumference for same chord.

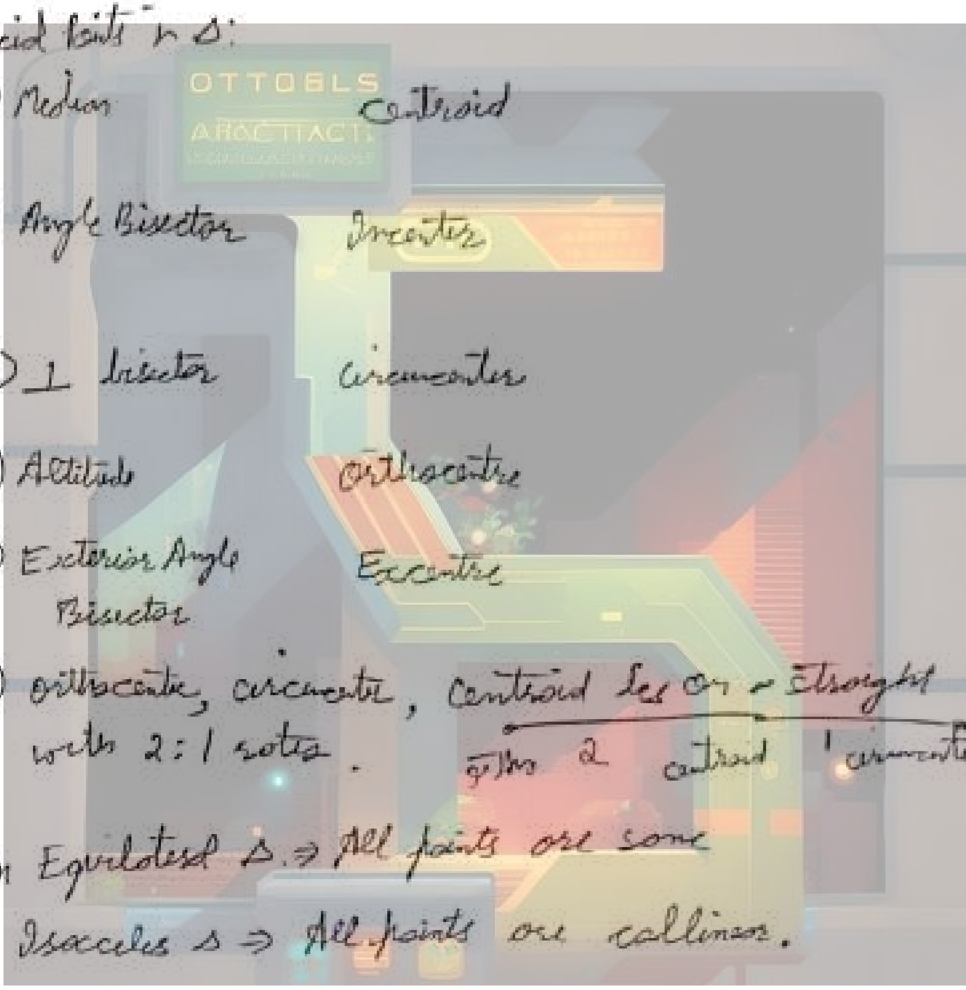
⑥ Cyclic Quadrilateral



$\angle A + \angle C = 180$
$\angle B + \angle D = 180°$

☆ Point of intersection of diagonals can or cannot be center of circle.

⑦ Special points in Δ:



a) Median — Centroid

b) Angle Bisector — Incentre

c) $\perp$ bisector — circumcenter

d) Altitude — Orthocentre

e) Exterior Angle Bisector — Excentre

f) orthocentre, circumcentre, centroid lies on a straight line with 2:1 ratio. ortho 2 centroid 1 circumcenter

☆ In Equilateral Δ ⇒ All points are same

☆ In Isosceles Δ ⇒ All points are collinear.

H.W. 10-5-24
Compound Angle (JM)
TE (O-1)

! Solution of Triangle !

→ Process of calculating the sides or angles of a triangle.
→ Here Angles are denoted by capital letters (A, B, ..., Z) & Sides are denoted by small letters (a, b, ... z).
→ Length opposite to above angles are are taken ~~respecto~~ respectily.

→ All the above 6 elements (3 angles + 3 sides) are not independent they are connected to each other somehow

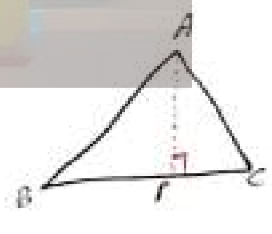
→ Questions are based of giving 1-2 elements & finding others

Basic Laws

1. Sine Law:- Sines of angles are proportional to the opposite Sides.

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

Proof:- $\sin B = \frac{AP}{c}$, $\sin C = \frac{AP}{b}$



$c \sin B = b \sin C$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

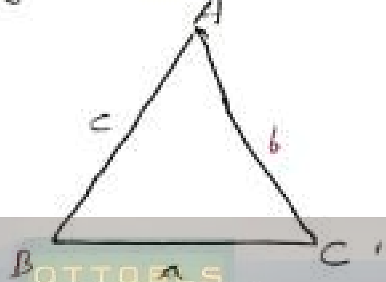
H.P.

Note:- If $a+b > c$

$\sin a + \sin b > \sin c$

2. <u>Cosine Rule:-</u> when all three sides of a triangle are known.

→ Any two sides & Included angle of the triangle are known

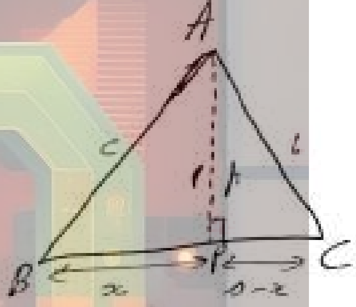


$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\boxed{\cos B = \frac{a^2+c^2-b^2}{2ac}}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

<u>Proof:-</u>



in $\triangle ABP$,

$p^2 + x^2 = c^2$

$p^2 = c^2 - x^2 \quad -(1)$

in $\triangle ACP$,

$p^2 + (a-x)^2 = b^2$

$p^2 = b^2 - (a-x)^2 \quad -(2)$

in $\triangle ABP$

$\cos B = \frac{x}{c}$

$x = c\cos B \quad -(3)$

$(1) = (2)$

$c^2 - x^2 = b^2 - (a-x)^2$

$c^2 - \cancel{x^2} = b^2 - a^2 - \cancel{x^2} + 2ax$

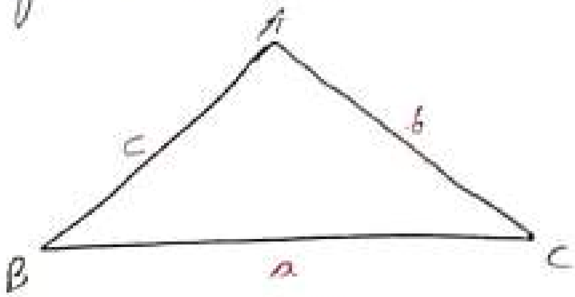
$a^2 + c^2 - b^2 = 2ax$

$a^2 + c^2 - b^2 = 2ac\cos B$

$$\frac{a^2+c^2-b^2}{2ac} = \cos B$$

अतः सिद्ध हुआ

3. <u>Projection Law:-</u> Any side of a triangle is equal to sum of projections of other two sides on it

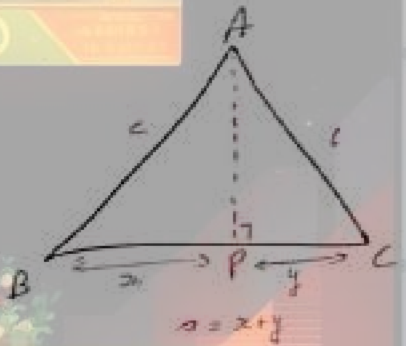


$$\boxed{a = b\cos C + c\cos B}$$

$$b = a\cos C + c\cos A$$

$$c = a\cos B + b\cos A$$

Proof:-



in ΔABP,

$$\cos B = \frac{x}{c}$$

$$c\cos B = x \quad -(1)$$

in ΔAPC,

$$\cos C = \frac{y}{b}$$

$$b\cos C = y \quad -(2)$$

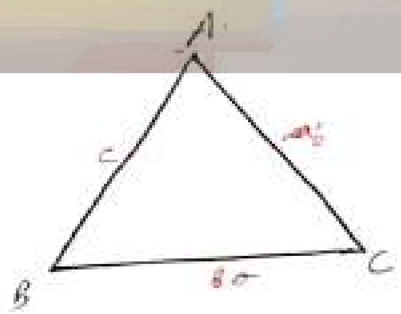
(1) + (2)

$$x + y = \cancel{a\cos} \; b\cos C + c\cos B$$

$$a = b\cos C + c\cos B$$

अतः सिद्धम

4. <u>Tangent Law (Napier's analogy):-</u>



$$\boxed{\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)}$$

$$\tan\left(\frac{A-C}{2}\right) = \frac{a-c}{a+c}\cot\left(\frac{B}{2}\right)$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

Proof:- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda$

$b = \sin B \, \lambda \quad -(1)$

$c = \lambda \sin C \quad -(2)$

by (1) & (2)

$\therefore \frac{b-c}{b+c} = \frac{\lambda \sin B - \lambda \sin C}{\lambda \sin B + \lambda \sin C}$

$$\frac{b-c}{b+c} = \frac{\cancel{\lambda}\left[2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)\right]}{\cancel{\lambda}\left[2\cos\left(\frac{B-C}{2}\right)\sin\left(\frac{B+C}{2}\right)\right]}$$

$$\frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)\cot\left(\frac{B+C}{2}\right)$$

$$\frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)\cot\left(\frac{180-A}{2}\right) \quad \{A+B+C=180\}$$

$$\frac{b-c}{b+c} = \tan\left(\frac{B-C}{2}\right)\cancel{\cot}\tan\left(\frac{A}{2}\right)$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\left(\frac{A}{2}\right)$$

अतः सिद्ध हुआ

Simplify

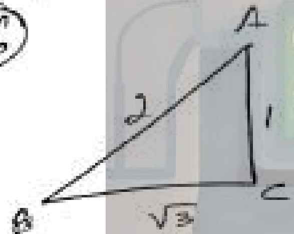
Q ① $a\sin(B-C) + b\sin(C-A) + c\sin(A-B)$

Q ② Prove:- $a\cos\left(\frac{B-C}{2}\right) = (b+c)\sin\frac{A}{2}$

Q ③ Prove:- $\sin(B-C) = \frac{b^2-c^2}{a^2}\sin A$

Q ④ In $\triangle ABC$, If $a\cos A = b\cos B$ then Prove, $a=b$

Q ⑤ Angles of a $\triangle$ are in ratio 4:1:1. find ratio of its greatest side & perimeter.

Q ⑥ 

Find $\angle C$

Q ⑦ In $\triangle ABC$, $(a^2-b^2-c^2)\tan A + (a^2-b^2+c^2)\tan B =$

Ⓐ $(a^2+b^2-c^2)\tan C$

Ⓑ $(b^2+c^2-a^2)\tan C$

Ⓒ $(a^2+b^2+c^2)\tan C$

Ⓓ None

Q1. $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda$

$a = \sin A\,\lambda$

$\lambda\sin A\sin(B-C) + \lambda\sin B\sin(C-A) + \lambda\sin C\sin(A-B)$

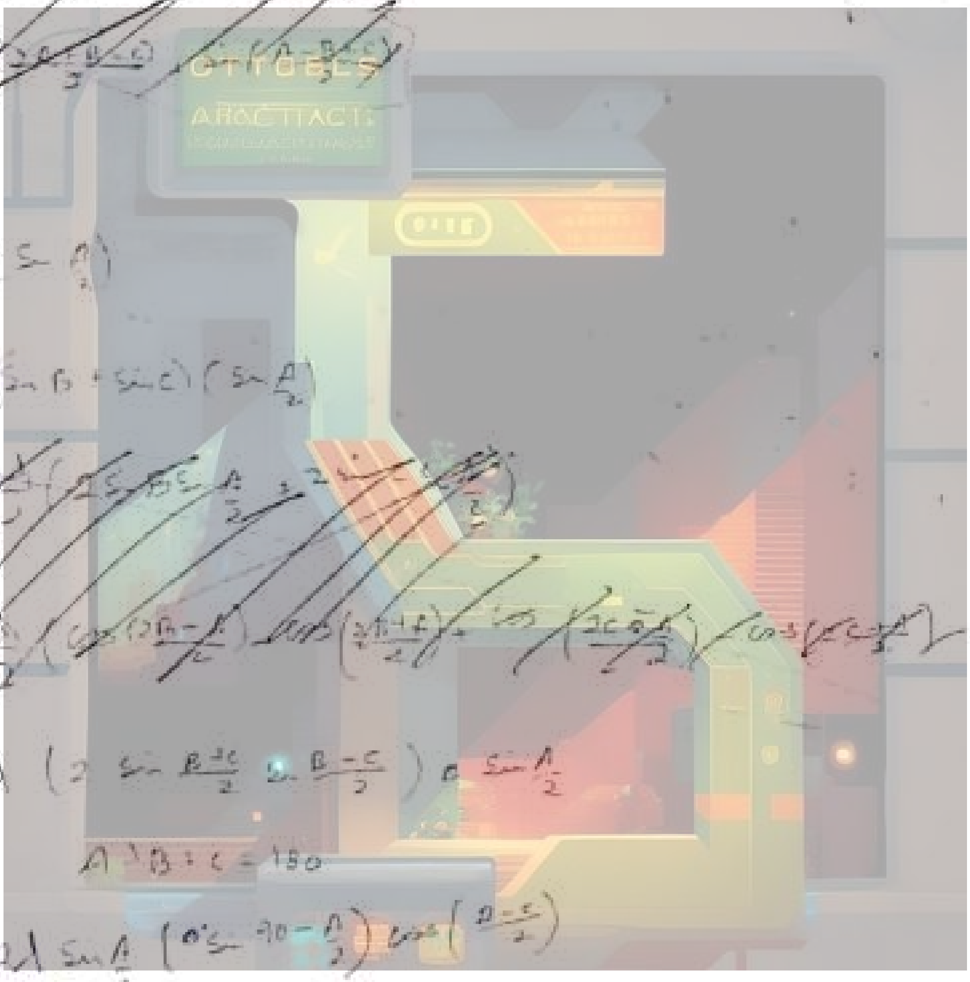
$\frac{\lambda}{2}\left[\cos(A-B+C) - \cos(A+B-C) + \cos(B-A+B-C) - \cos(-A+B+C) + \cos(A-A+B+C) - \cos(A-B+C)\right]$

$\boxed{=0}$

(2) LHS =

$a\cos\left(\frac{B-C}{2}\right)$

$= \lambda \sin A \cos\left(\frac{B-C}{2}\right)$



RHS =

$= (b+c)\left(\sin\frac{A}{2}\right)$

$= \lambda(\sin B + \sin C)\left(\sin\frac{A}{2}\right)$

$= \lambda\left(2\sin\frac{B+C}{2}\cos\frac{B-C}{2}\right)\sin\frac{A}{2}$

$A+B+C=180$

$= 2\lambda\sin\frac{A}{2}\left(\sin\left(90-\frac{A}{2}\right)\right)\cos\left(\frac{B-C}{2}\right)$

$= \lambda\, 2\sin\frac{A}{2}\cos\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$= \lambda\sin A\cos\left(\frac{B-C}{2}\right)$

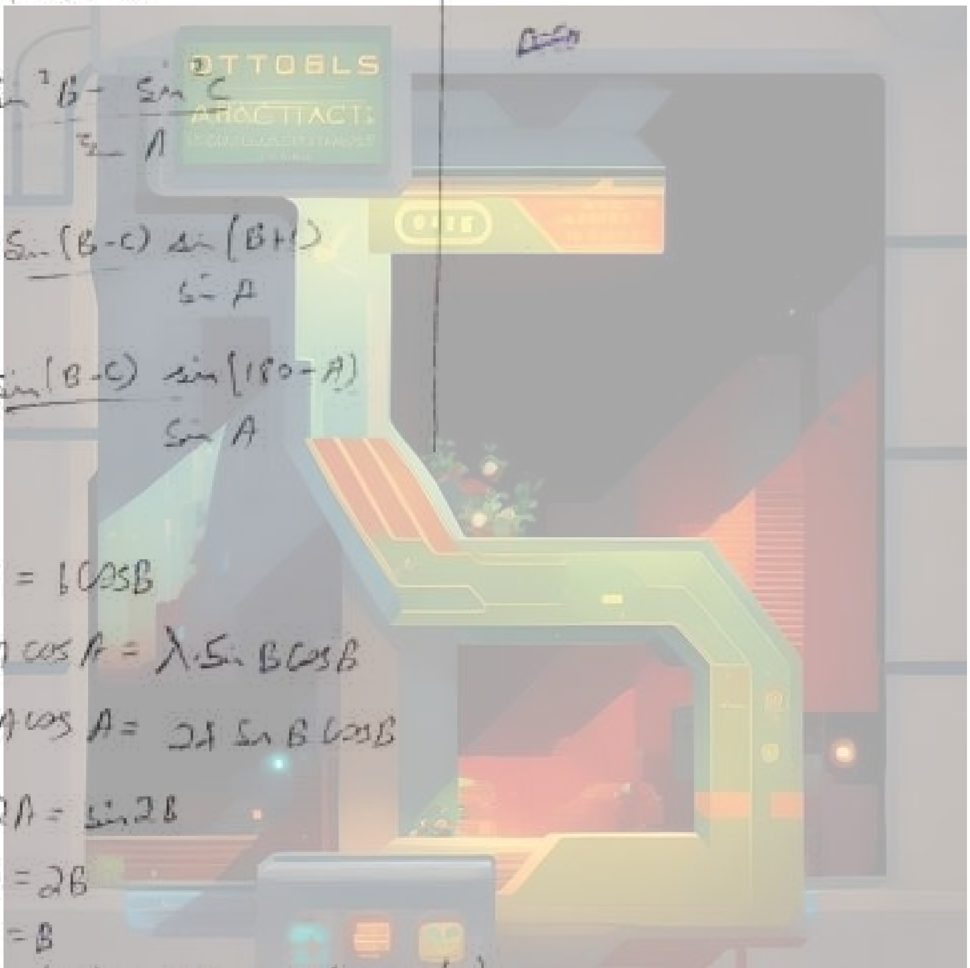
$\boxed{LHS = RHS}$

H.W
J-11, Q3 (T.E)

Q3 RHS =

$$\frac{b^2-c^2}{a^2}\sin A$$

$$= \frac{\lambda^2 \sin^2 B - \lambda^2 \sin^2 C}{\lambda^2 \sin^2 A} \cdot \sin A$$



$$= \frac{\sin^2 B - \sin^2 C}{\sin A}$$

$$= \frac{\sin(B-C)\sin(B+C)}{\sin A}$$

$$= \frac{\sin(B-C)\sin(180-A)}{\sin A}$$

$$= \frac{\sin(B-C)\sin A}{\sin A}$$

$$= \sin(B-C) = LHS$$

LHS = RHS

Q4. $a\cos A = b\cos B$

$\lambda \sin A \cos A = \lambda \sin B \cos B$

$2\lambda \sin A \cos A = 2\lambda \sin B \cos B$

$\sin 2A = \sin 2B$

$2A = 2B$

$A = B$

$a = b$ (sides opp. to equal angles)

Let angles be $4x, x, x$

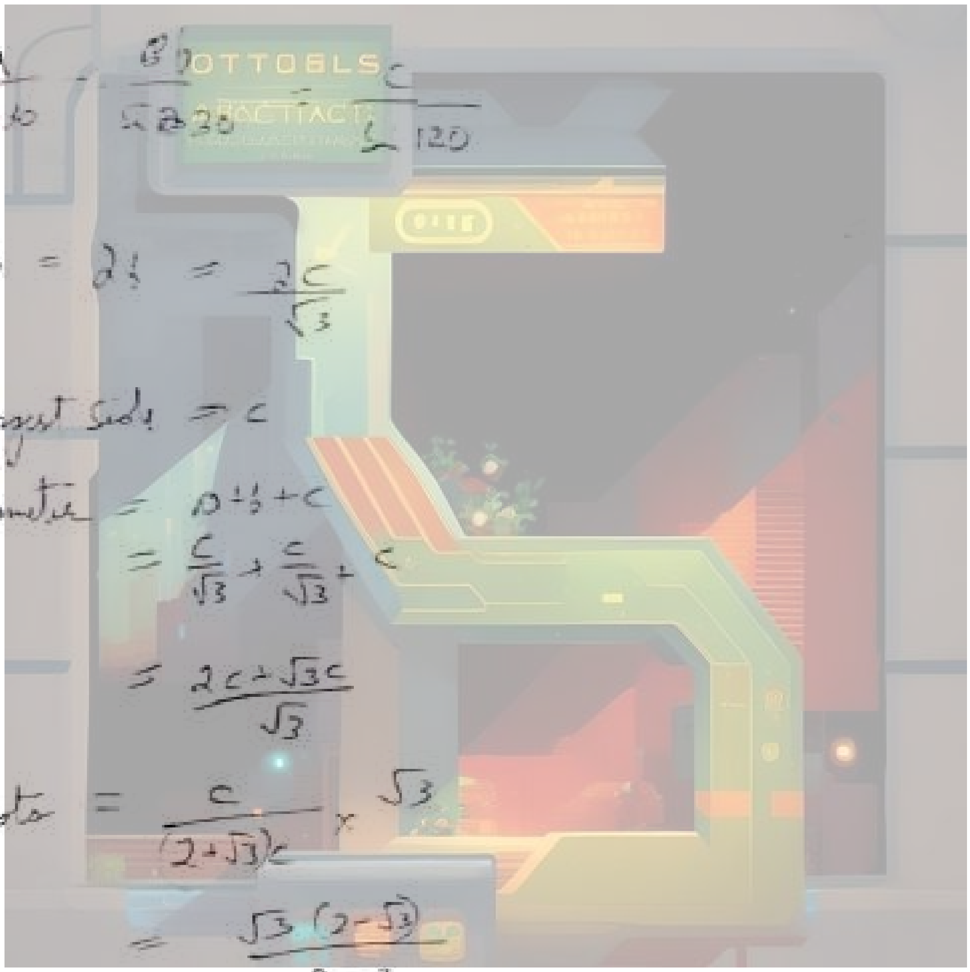
$$4x + x + x = 180$$

$$6x = 180$$

$$x = \frac{180}{6}$$

$$x = 30^\circ$$

~~largest side, s = s~~



$$\frac{a}{\sin 30} = \frac{b}{\sin 30} = \frac{c}{\sin 120}$$

$$2a = 2b = \frac{2c}{\sqrt{3}}$$

largest side $= c$

perimeter $= a+b+c$

$$= \frac{c}{\sqrt{3}} + \frac{c}{\sqrt{3}} + c$$

$$= \frac{2c+\sqrt{3}c}{\sqrt{3}}$$

ratio $= \frac{c}{(2+\sqrt{3})c} \times \sqrt{3}$

$$= \frac{\sqrt{3}(2-\sqrt{3})}{2-3}$$

$$= (\sqrt{3}-2)\sqrt{3}$$

$$\boxed{\cancel{=}\ 3-2\sqrt{3}}$$

$$\boxed{3-2\sqrt{3} : 1}$$

Q6

$\cos C = \frac{a^2+b^2-c^2}{2ba}$

$\cos C = \frac{3+1-4}{2\sqrt{3}}$

$\cos C = 0$

$\boxed{C = 90^\circ}$

Q7 $-\frac{b\sin A}{\cos A}(b^2+c^2-a^2) + (a^2+c^2-b^2)\frac{a\sin B}{\cos B}$

$= -\frac{\sin A \, 2abc\cos A}{\cos A} + \frac{\sin B \, 2ac\cos B}{\cos B}$

$= 2ac\sin B - 2bc\sin A$

~~$= 2c(a\sin B - b\sin A)$~~

$= 2 \times \sin B \times a \times \frac{\sin C \times b}{\sin B} - 2 \times \sin A \times 1 \times \frac{\sin C \times a}{\sin A}$

$= 2ab\sin C - 2ab\sin C$

$\boxed{= 0}$

Q5. In $\triangle ABC$, $B = 30^\circ$ & $c = \sqrt{3}\, b$ find $\angle A$

$$\frac{\sin C}{c} = \frac{1}{2b}$$

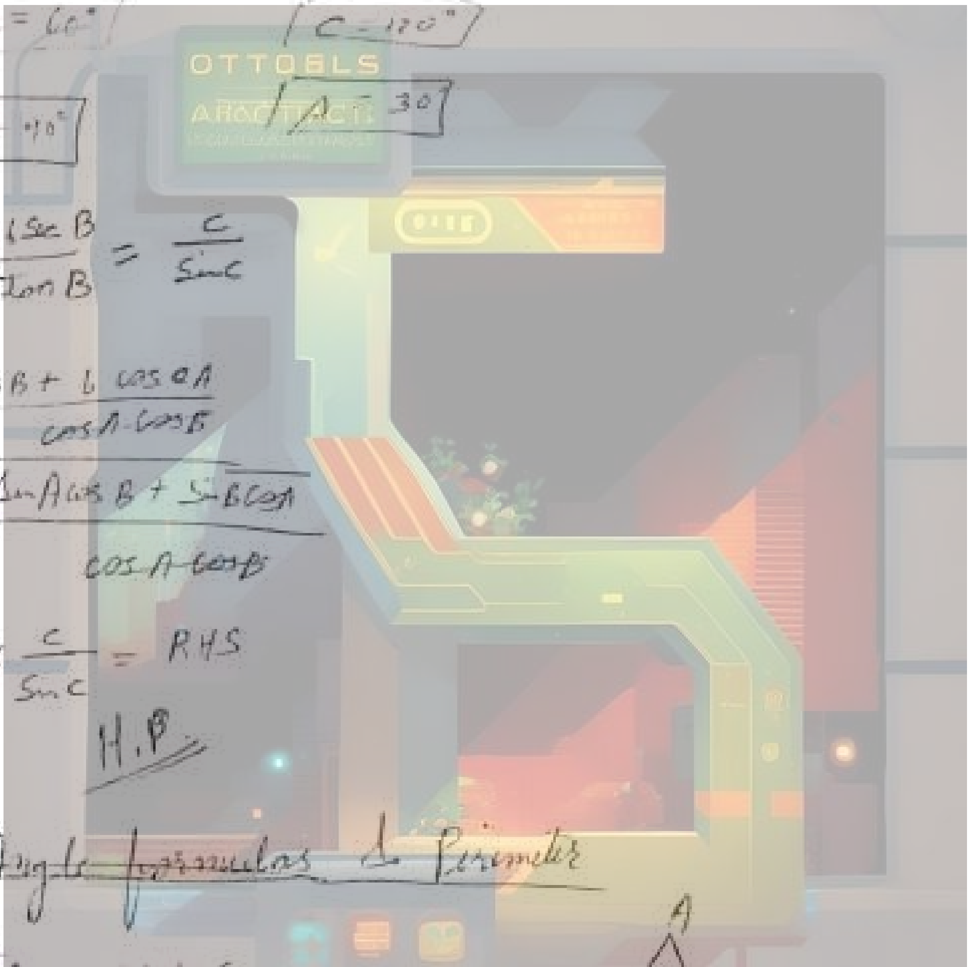
$$2b \sin C = c$$

$$2b \sin C = \sqrt{3}\, b$$

$$\sin C = \frac{\sqrt{3}}{2}$$

$\boxed{C = 60^\circ}$ $\qquad$ $\boxed{C = 120^\circ}$

$\boxed{A = 90^\circ}$ $\qquad$ $\boxed{A = 30^\circ}$



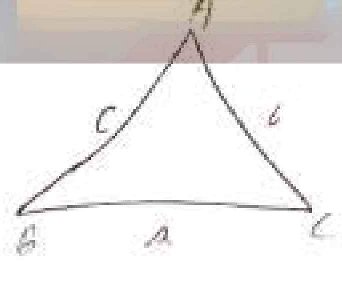
Q7. $$\frac{a \sec A + b \sec B}{\tan A + \tan B} = \frac{c}{\sin C}$$

$$= \frac{\dfrac{a \cos B + b \cos A}{\cos A \cos B}}{\dfrac{\sin A \cos B + \sin B \cos A}{\cos A \cos B}}$$

$$= \frac{c}{\sin C} = RHS$$

H.P

Half Angle formulas & Perimeter



Perimeter = $a+b+c$

Semi Perimeter $(s) = \dfrac{a+b+c}{2}$

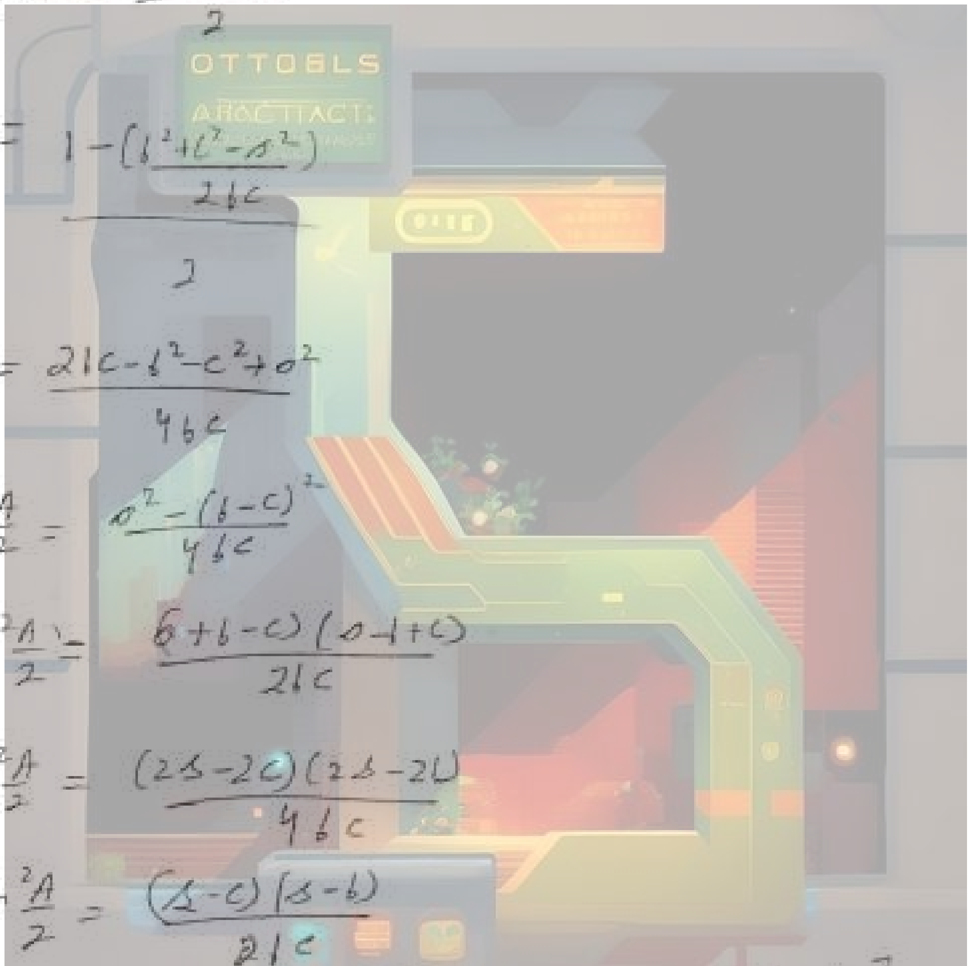
(1) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$

$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$

$$\boxed{\sin\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{ab}}}$$ (5)

Proof :-

$\sin^2\frac{A}{2} = \frac{1-\cos A}{2}$



$\sin^2\frac{A}{2} = \frac{1-\left(\frac{b^2+c^2-a^2}{2bc}\right)}{2}$

$\sin^2\frac{A}{2} = \frac{2bc-b^2-c^2+a^2}{4bc}$

$\sin^2\frac{A}{2} = \frac{a^2-(b-c)^2}{4bc}$

$\sin^2\frac{A}{2} = \frac{(a+b-c)(a-b+c)}{2bc}$

$\sin^2\frac{A}{2} = \frac{(2s-2c)(2s-2b)}{4bc}$

$\sin^2\frac{A}{2} = \frac{(s-c)(s-b)}{bc}$

$\sin\frac{A}{2} = \sqrt{\frac{(s-c)(s-b)}{bc}}$ [-ve reject as its only possible when $A > 180$]

(2) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ (6)

$\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$

$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

Proof:- $\cos^2\frac{A}{2} = \sqrt{1-\sin^2\frac{A}{2}}$

(3) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ (7)

$\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\Delta}{s(s-b)}$ $\Delta \rightarrow$ area

$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\Delta}{s(s-c)}$

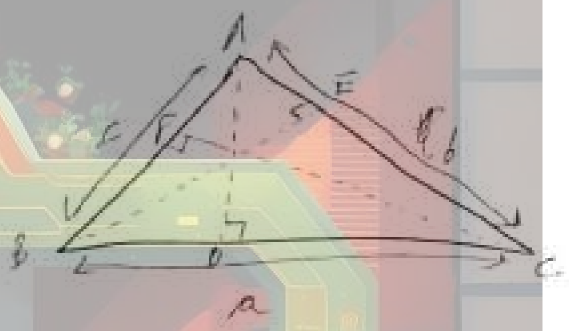
(4) Area of Triangle :-

(1) $\frac{1}{2} \times BC \times AD = \text{Area}$

$= \frac{1}{2} \times a \times AD$

$= \frac{1}{2} \times a \times c\sin B$

$= \frac{ac\sin B}{2}$



$$\boxed{\begin{aligned} \text{area} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}ac\sin B \end{aligned}}$$ (8)

(2) $\boxed{\text{area} = ab\sin\frac{C}{2}\cos\frac{C}{2}}$ (9)

$= ac\sin\frac{B}{2}\cos\frac{B}{2}$

$= bc\sin\frac{A}{2}\cos\frac{A}{2}$

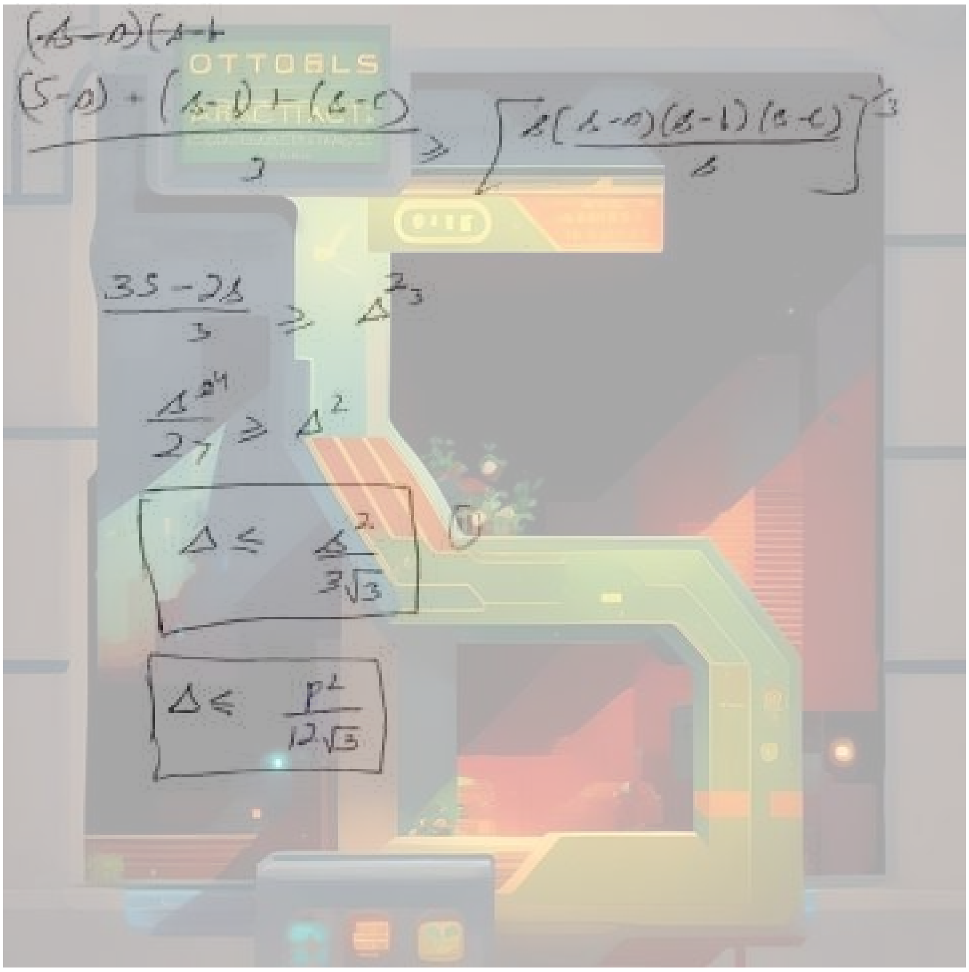
(3) $\Delta = bc \sin\frac{A}{2} \cos\frac{A}{2}$

$$= bc \sqrt{\frac{(s-b)(s-c)}{bc}} \sqrt{\frac{s(s-a)}{bc}}$$

$$\boxed{= \sqrt{s(s-a)(s-b)(s-c)}}$$

Note :- Relation in Area of Triangle & Perimeter of Triangle

AM ≥ GM



~~(s-a)(s-b~~

$$\frac{(s-a)+(s-b)+(s-c)}{3} \geq \left[\frac{s(s-a)(s-b)(s-c)}{s}\right]^{1/3}$$

$$\frac{3s-2s}{3} \geq \Delta^{2/3}$$

$$\frac{s^{4}}{27} \geq \Delta^{2}$$

$$\boxed{\Delta \leq \frac{s^2}{3\sqrt{3}}}$$

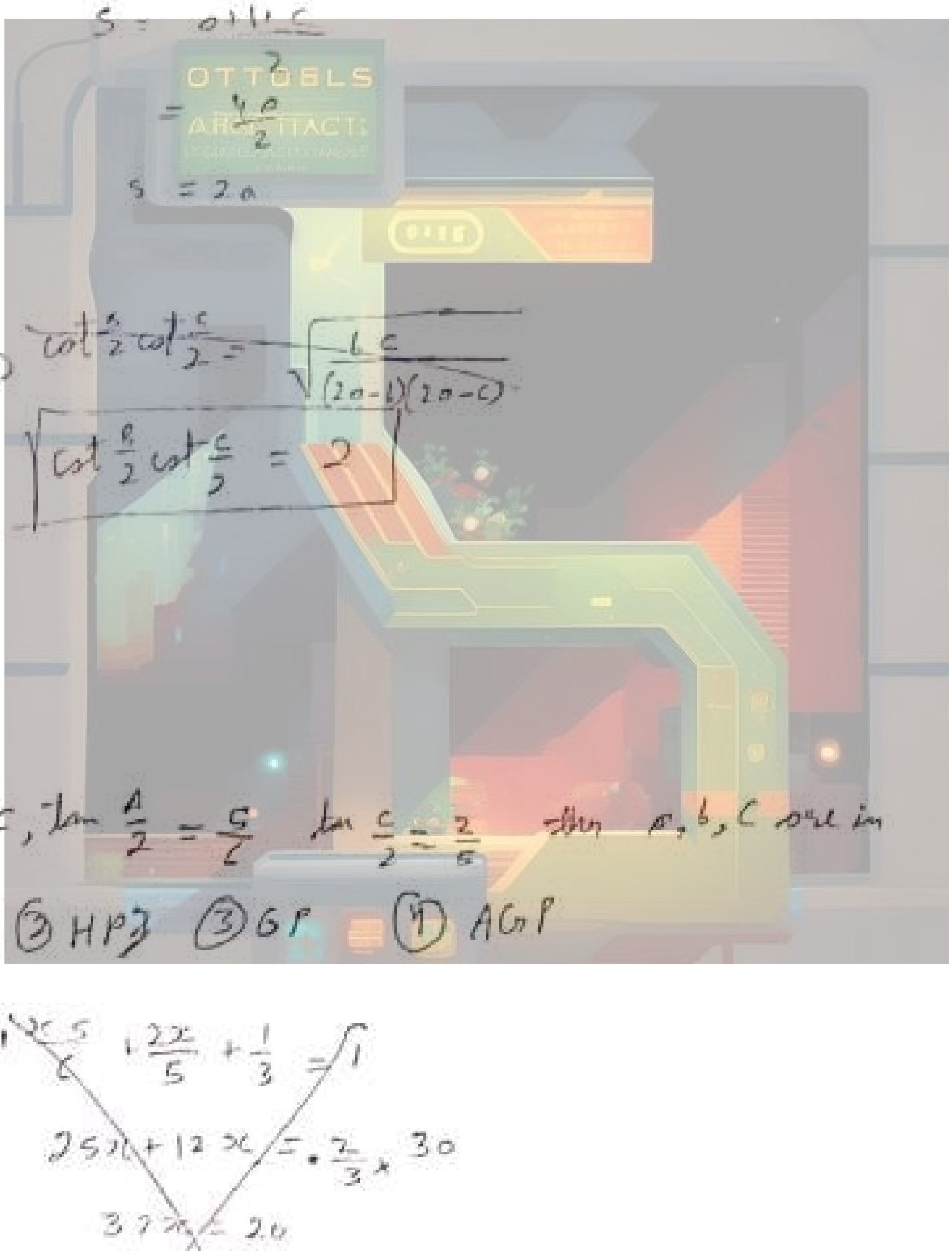
$$\boxed{\Delta \leq \frac{p^2}{12\sqrt{3}}}$$

Q10. If $b+c=3a$ find $\cot\frac{B}{2}\cot\frac{C}{2}$

$$\cot\frac{B}{2}=\sqrt{\frac{ac(s-b)\Delta}{(s-a)(s-b)\Delta}}$$

$$\cot\frac{C}{2}=\sqrt{\frac{ab(s-c)\Delta}{(s-a)(s-c)\Delta}}$$

$$\cot\frac{B}{2}\cot\frac{C}{2}=\sqrt{\frac{ac\times ab\times 4a^2}{(s-a)^2(s-b)(s-c)}}$$



$$s=\frac{a+b+c}{2}$$

$$=\frac{4a}{2}$$

$$s=2a$$

$$\cot\frac{B}{2}\cot\frac{C}{2}=\sqrt{\frac{bc}{(2a-b)(2a-c)}}$$

$$\boxed{\cot\frac{B}{2}\cot\frac{C}{2}=2}$$

Q11. In ΔABC, $\tan\frac{A}{2}=\frac{5}{6}$, $\tan\frac{C}{2}=\frac{2}{5}$ then a,b,c are in

(1) AP (2) HP (3) GP (4) AGP

$$\cancel{x\cdot\frac{5}{6}+\frac{2x}{5}+\frac{1}{3}=1}$$

$$\cancel{25x+12x=\frac{2}{3}\times 30}$$

$$\cancel{37x=20}$$

$$\cancel{x=\frac{20}{37}}$$

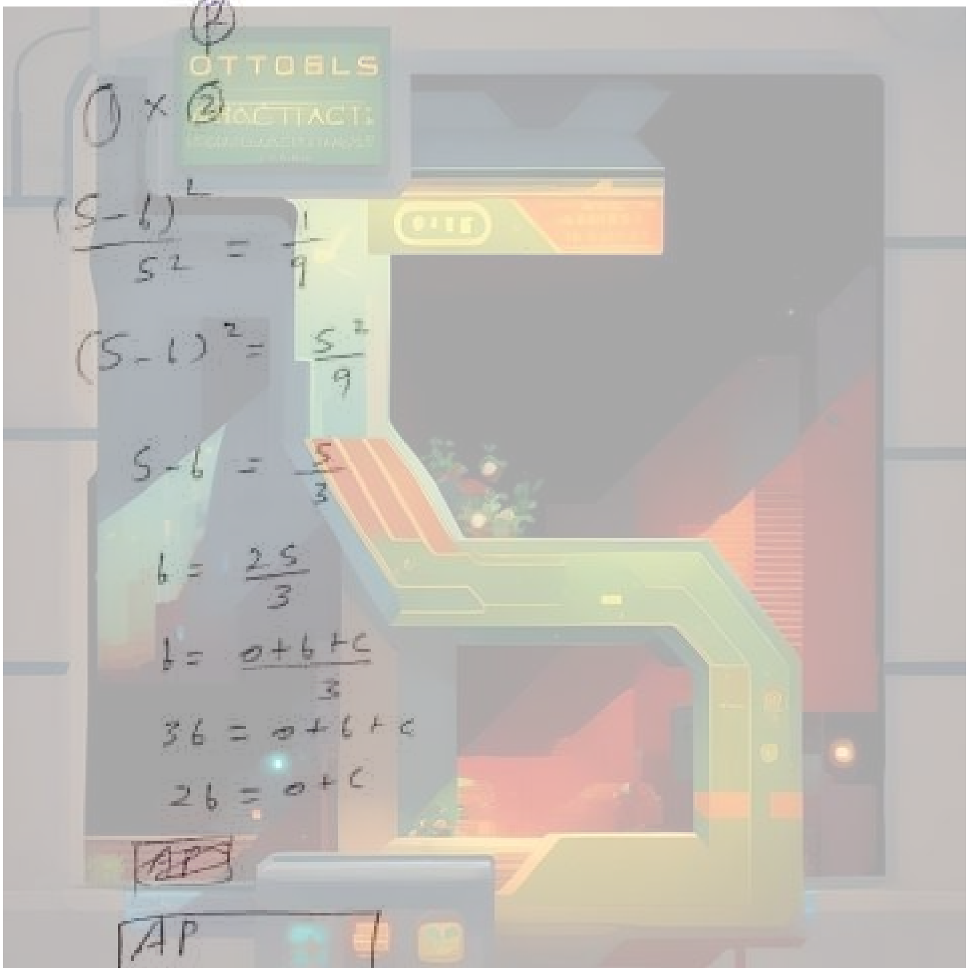
$$\cancel{\tan\frac{B}{2}=\frac{20}{37}}$$

$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{5}{6}$

$$\frac{(s-b)(s-c)}{s(s-a)} = \frac{25}{36} \quad \text{—(1)}$$

$$\frac{(s-a)(s-b)}{s(s-c)} = \frac{4}{25} \quad \text{—(2)}$$

~~So (1)/(2)~~



(1) × (2)

$$\frac{(s-b)^2}{s^2} = \frac{1}{9}$$

$$(s-b)^2 = \frac{s^2}{9}$$

$$s-b = \frac{s}{3}$$

$$b = \frac{2s}{3}$$

$$b = \frac{a+b+c}{3}$$

$$3b = a+b+c$$

$$2b = a+c$$

~~AP~~

$\boxed{AP}$

Q12. ΔABC, if $\tan\frac{A}{2}\tan\frac{B}{2} = \frac{1}{3}$ & $ab = 4$ then $\min(c) = ?$

$$\sqrt{\frac{(s-b)(s-c)}{s(s-a)} \times \frac{(s-a)(s-c)}{s(s-b)}} = \frac{1}{3}$$

$$\frac{s-c}{s} = \frac{1}{\cancel{9}\,3}$$

$3s - 3c = s$

$2s = 3c$

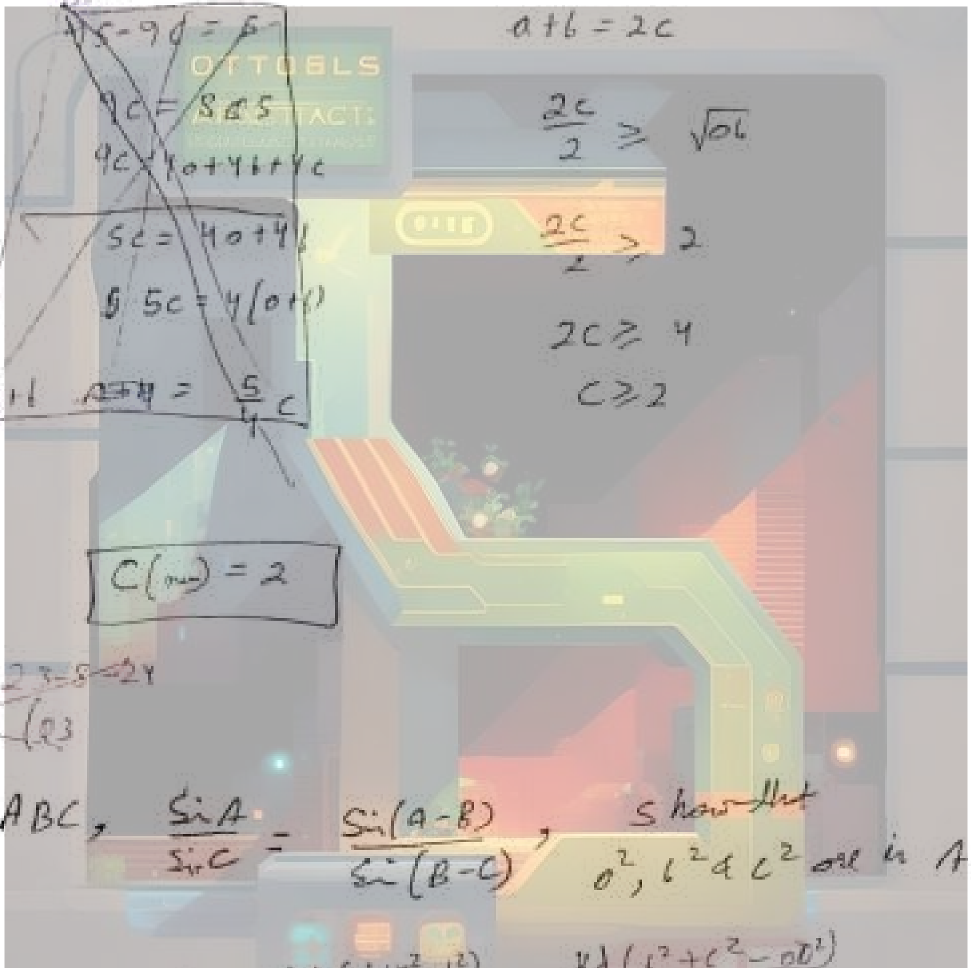
$a + b = 2c$

$\frac{2c}{2} \geq \sqrt{ab}$

$\frac{2c}{2} \geq 2$

$2c \geq 4$

$c \geq 2$



~~$9s - 9c = s$~~

~~$9c = 8s$~~

~~$9c = 4a + 4b + 4c$~~

~~$5c = 4a + 4b$~~

~~$5c = 4(a+b)$~~

~~$a + b = \frac{5}{4}c$~~

$$\boxed{C(\min) = 2}$$

H.W. 23-S-24
DYS-1 Q3

Q13. In ΔABC, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, show that a^2, b^2 & c^2 are in A.P.

$$\frac{a}{c} = \frac{b\lambda\frac{(a^2+c^2-b^2)}{ac} - b\lambda\frac{(b^2+c^2-a^2)}{bc}}{b\lambda\frac{(a^2+b^2-c^2)}{ab} - a\lambda\frac{(a^2+c^2-b^2)}{ac}}$$

~~$\frac{a}{c} = \frac{\frac{1}{2c}(2c)}{\frac{1}{2c}(2b^2)}$~~

~~$\frac{a}{c} = \frac{c}{b}$~~

~~$2b^2 = c^2$~~

$$\frac{a}{c} = \frac{a^2-b^2}{c^2} \times \frac{c^2}{b^2-c^2}$$

~~$c(b^2-c^2) = a(a^2-b^2)$~~

$b^2 - c^2 = a^2 - b^2$

$$\boxed{2b^2 = a^2 + c^2}$$

H.W. 24-8-24

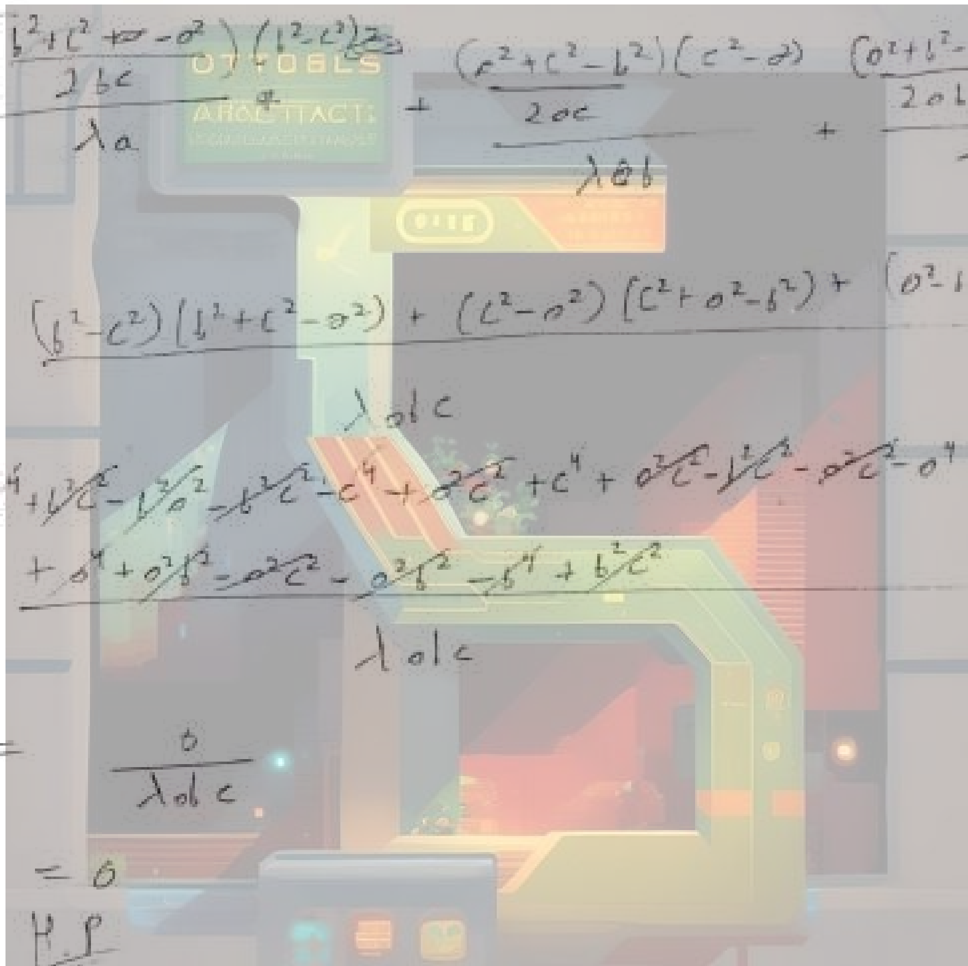
DYS-1 (Q1, 2, 4, 7, 8) & others

DYS-3

DYS-2 → (Q2, 8) $\underbrace{}_{\text{short}}$ note

DYS-1

Q5 $(b^2-c^2)\cot A + (c^2-a^2)\cot B + (a^2-b^2)\cot C = 0$ (Isosceles)



$$= \frac{\left(\frac{b^2+c^2-a^2}{2bc}\right)(b^2-c^2)}{\lambda a} + \frac{\frac{(a^2+c^2-b^2)(c^2-a^2)}{2ac}}{\lambda b} + \frac{\frac{(a^2+b^2-c^2)(a^2-b^2)}{2ab}}{\lambda c}$$

$$= \frac{(b^2-c^2)(b^2+c^2-a^2) + (c^2-a^2)(c^2+a^2-b^2) + (a^2-b^2)(a^2+b^2-c^2)}{\lambda abc}$$

$$= \frac{b^4 + \cancel{b^2c^2} - \cancel{b^2a^2} - \cancel{b^2c^2} - c^4 + \cancel{a^2c^2} + c^4 + \cancel{a^2c^2} - \cancel{b^2c^2} - \cancel{a^2c^2} - a^4 + \cancel{a^2b^2} + a^4 + \cancel{a^2b^2} - \cancel{a^2c^2} - \cancel{a^2b^2} - b^4 + \cancel{b^2c^2}}{\lambda abc}$$

$$= \frac{0}{\lambda abc}$$

$= 0$

H.P.

$\boxed{\text{M II}}$

$b = \lambda \cos B$

$(\lambda^2 \sin^2 B - \lambda^2 \sin^2 C)\cot A$

$\lambda^2 [\sin(B+C)\sin(B-C)\cot A$

$\lambda^2 \sin(180-A)\ \sin(B-C)\ \frac{\cot A}{\sin A}$

$\lambda^2 \sin(B-C)\ \cancel{\cot A}\cos A + \lambda^2 \sin(C-A)\cos B + \lambda^2 \sin(A-B)\cos C$

$= 0$

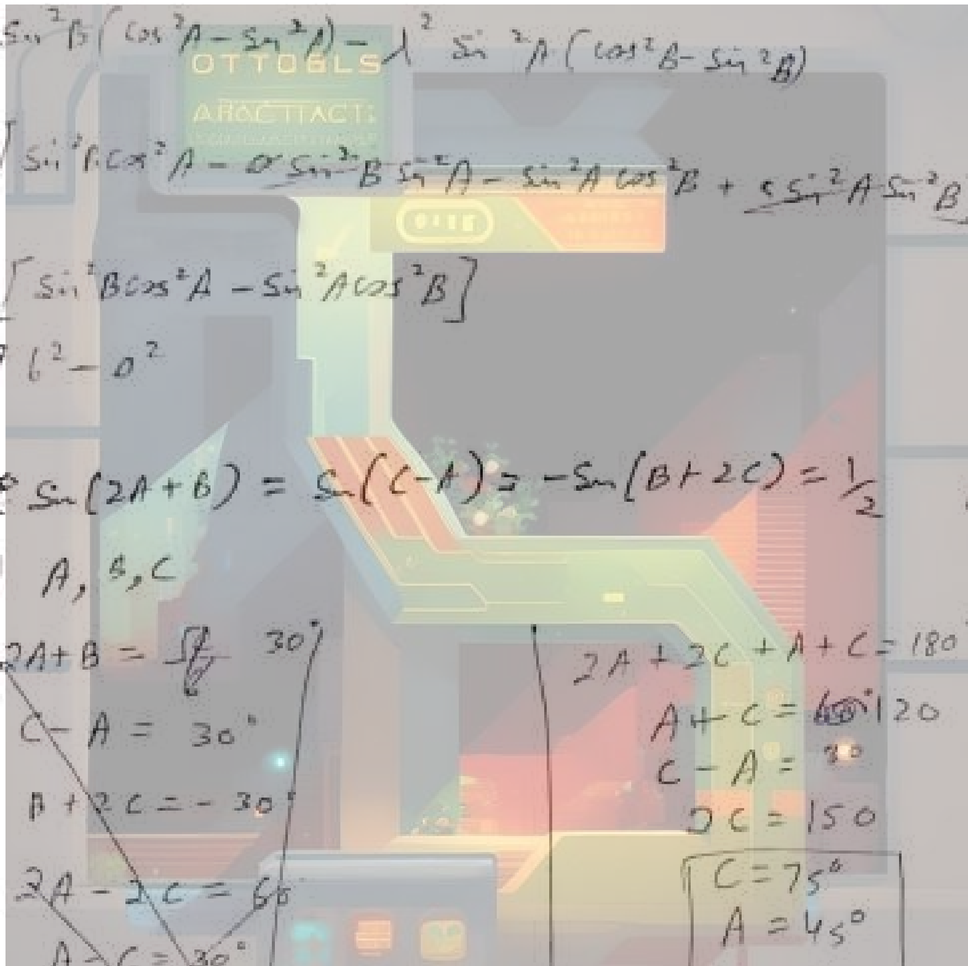
Q6. $b^2\cos 2A - a^2 \cos 2B = b^2 - a^2$

$\cancel{\lambda^2 \cos^2 B \cos \cdot 2 \cos B \cos A}$

$\cancel{\lambda^2 \cos^2 B \times 2 \sin A}$

$\cancel{\lambda^2 \cos^2 B(\cos^2 A - \sin^2 B) - \lambda^2 \cos A \cos}$

$= \lambda^2 \sin^2 B \cos 2A - \lambda^2 \sin^2 A \cos 2B$



$= \lambda^2 \sin^2 B(\cos^2 A - \sin^2 A) - \lambda^2 \sin^2 A(\cos^2 B - \sin^2 B)$

$= \lambda^2 [\sin^2 B\cos^2 A - \sin^2 B \sin^2 A - \sin^2 A \cos^2 B + \sin^2 A \sin^2 B]$

$= \lambda^2 [\sin^2 B\cos^2 A - \sin^2 A \cos^2 B]$

$= b^2 - a^2$

Q7. $\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}$ A, B, C in AP. find A, B, C

$2A+B = 30°$

$C-A = 30°$

$B+2C = -30°$

$2A-2C = 60$

$A-C = 30°$

$A = C$

$A+B+C = 180$

$2A+2C+A+C = 180$

$A+C = 60$

$A-C = +30$

$\boxed{A = 15°, \ C = 45°, \ B = 30°}$ X

$2A+2C+A+C = 180°$

$A+C = 120$

$C-A = 30$

$2C = 150$

$\boxed{C = 75°, \ A = 45°, \ B = 60°}$

DYS - 2

Q.4. A, B, C in AP

$$2\cos\left(\frac{A-C}{2}\right) = \frac{a+c}{\sqrt{a^2+c^2-ac}}$$

M I

LHS:

$$= 2\sqrt{\frac{\cos(A-C)+1}{2}}$$

$$= \sqrt{2}\sqrt{\cos A\cos C + \sin A \sin C + 1}$$

$$= \sqrt{2}\sqrt{\left(\frac{b^2+c^2-a^2}{2bc}\right)\left(\frac{a^2+b^2-c^2}{2ba}\right) + \lambda^2 ac + 1}$$

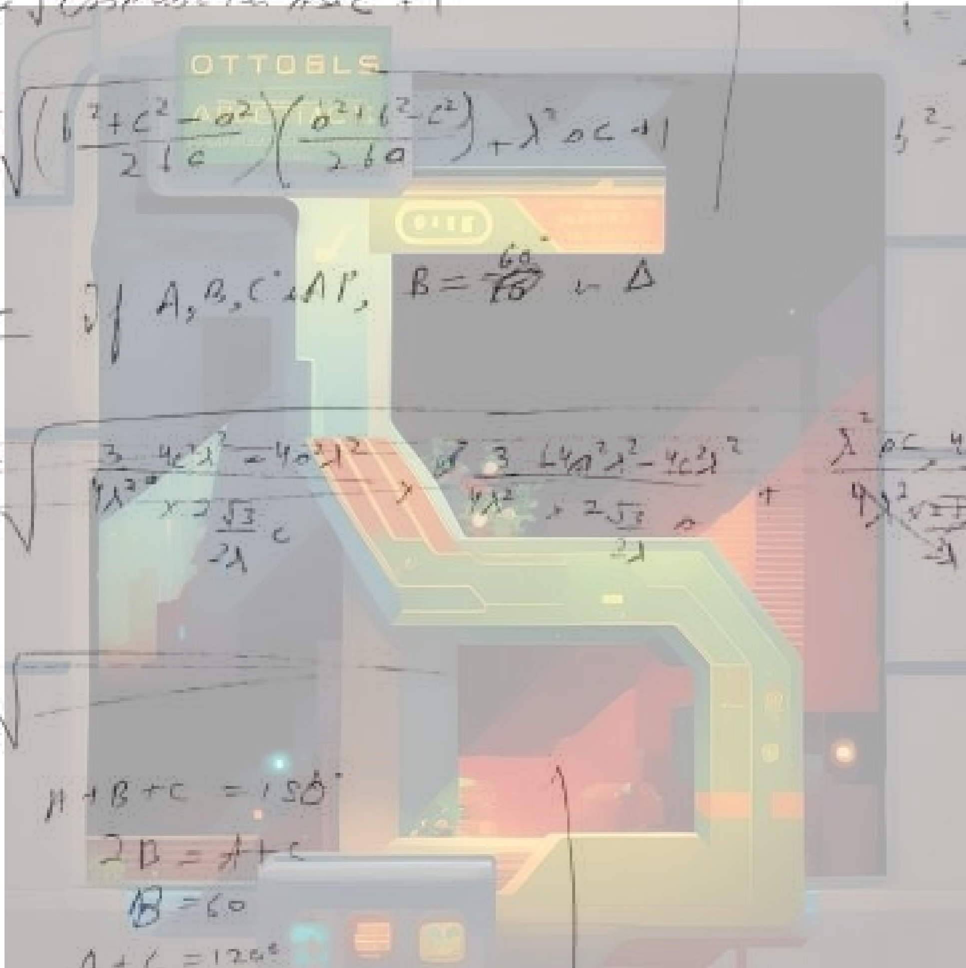
$\frac{\sin B}{b} = \lambda$

$\frac{\sqrt{3}}{2b} = \lambda$

$b = \frac{\sqrt{3}}{2\lambda}$

$b^2 = \frac{3}{4\lambda^2}$

Note:- If A, B, C in AP, $B = 60$ in Δ



$$= \sqrt{2}\sqrt{\frac{\frac{3}{4\lambda^2} \frac{4c^2\lambda^2 - 4a^2\lambda^2}{4\lambda^2}}{2\frac{\sqrt{3}}{2\lambda}c} \times \frac{\frac{3}{4\lambda^2} \frac{4a^2\lambda^2 - 4c^2\lambda^2}{4\lambda^2}}{2\frac{\sqrt{3}}{2\lambda}a} + \lambda^2 ac + 1}$$

$$= \sqrt{2}\sqrt{}$$

M II

$A + B + C = 180°$

$2B = A + C$

$B = 60$

$A + C = 120°$

RHS:

$$= \frac{\sin A + \sin C}{\sqrt{\sin^2 A + \sin^2 C - \sin A \sin C}}$$

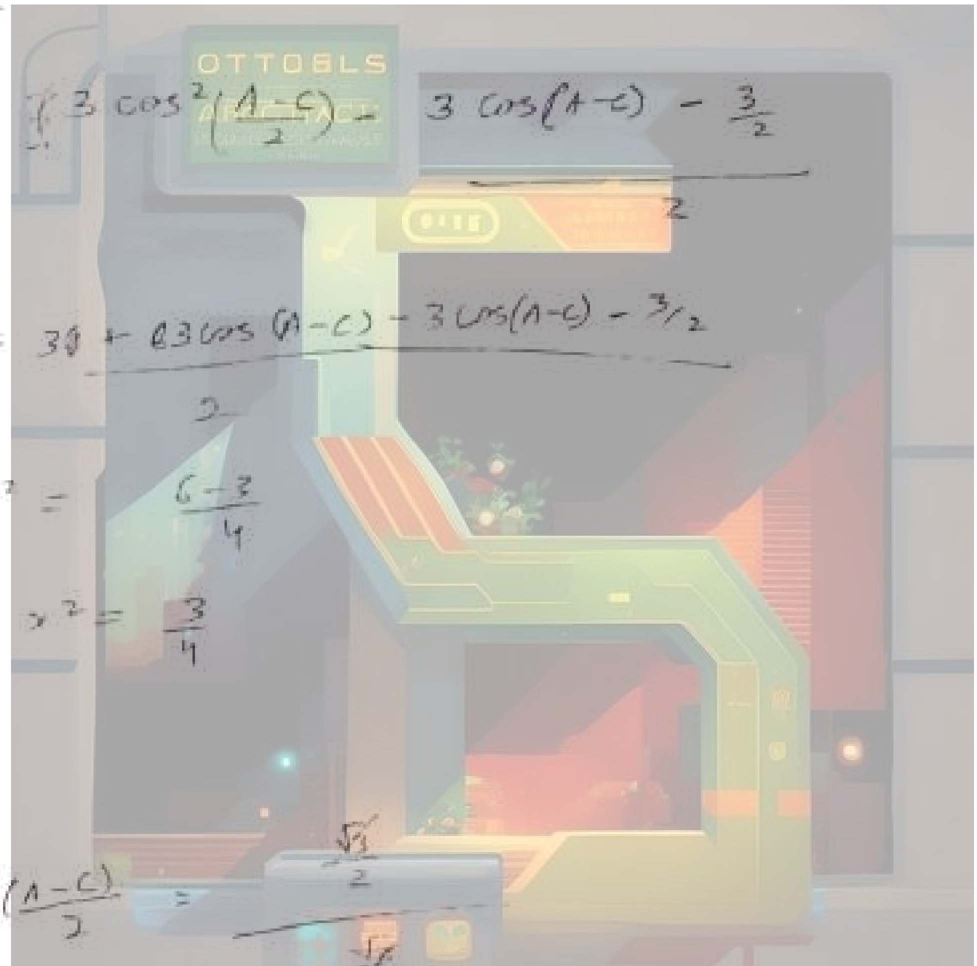
$$= \frac{\sqrt{3}\cos\left(\frac{A-C}{2}\right)}{\sqrt{\sin^2 A + \sin^2 C - \sin A \sin C}}$$

$\sqrt{2}$

$= 2\cos\frac{A-C}{2}$

$$\sqrt{(\sin A + \sin C)^2 - \frac{6\sin A \sin C}{2}}$$

$$x^2 = \left(2\sin\frac{A+C}{2}\cos\frac{A-C}{2}\right)^2 - \frac{3\cos(A-C) + 3\cos 120}{2}$$



$$x^2 = 3\cos^2\left(\frac{A-C}{2}\right) - \frac{3\cos(A-C) - \frac{3}{2}}{2}$$

$$x^2 = \frac{3 + 3\cos(A-C) - 3\cos(A-C) - 3/2}{2}$$

$$x^2 = \frac{6-3}{4}$$

$$x^2 = \frac{3}{4}$$

LHS

$$= 2\cos\frac{(A-C)}{2} = \frac{\frac{\sqrt{3}}{2}}{1/2}$$

= RHS

अतः सिद्ध

(122)

DVS2

Q8. If $9a^2 + 9b^2 = 19c^2$, find $\dfrac{\cot C}{\cot A + \cot B}$

$$= \frac{\dfrac{\cos C}{\sin C}}{\dfrac{\cos A}{\sin A} + \dfrac{\cos B}{\sin B}}$$

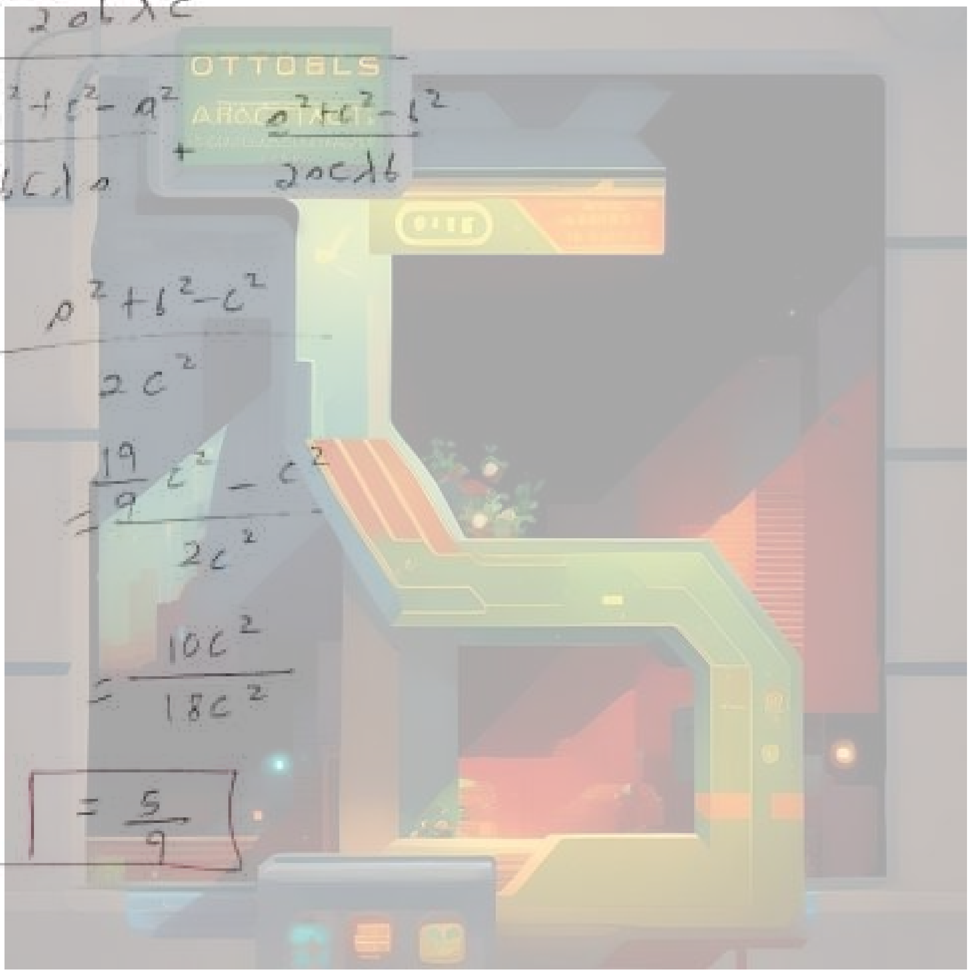
$$= \frac{\dfrac{a^2+b^2-c^2}{2ab\lambda c}}{\dfrac{b^2+c^2-a^2}{2bc\lambda a} + \dfrac{a^2+c^2-b^2}{2ac\lambda b}}$$

$$= \frac{a^2+b^2-c^2}{2c^2}$$

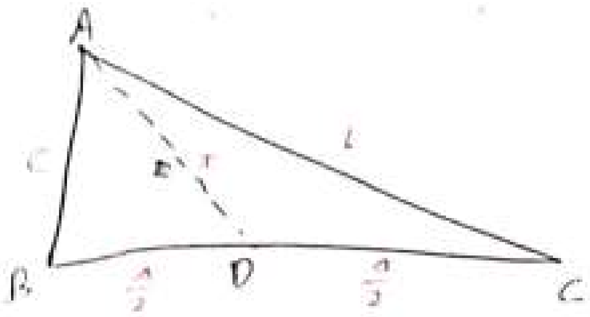
$$= \frac{\frac{19}{9}c^2 - c^2}{2c^2}$$

$$= \frac{10c^2}{18c^2}$$

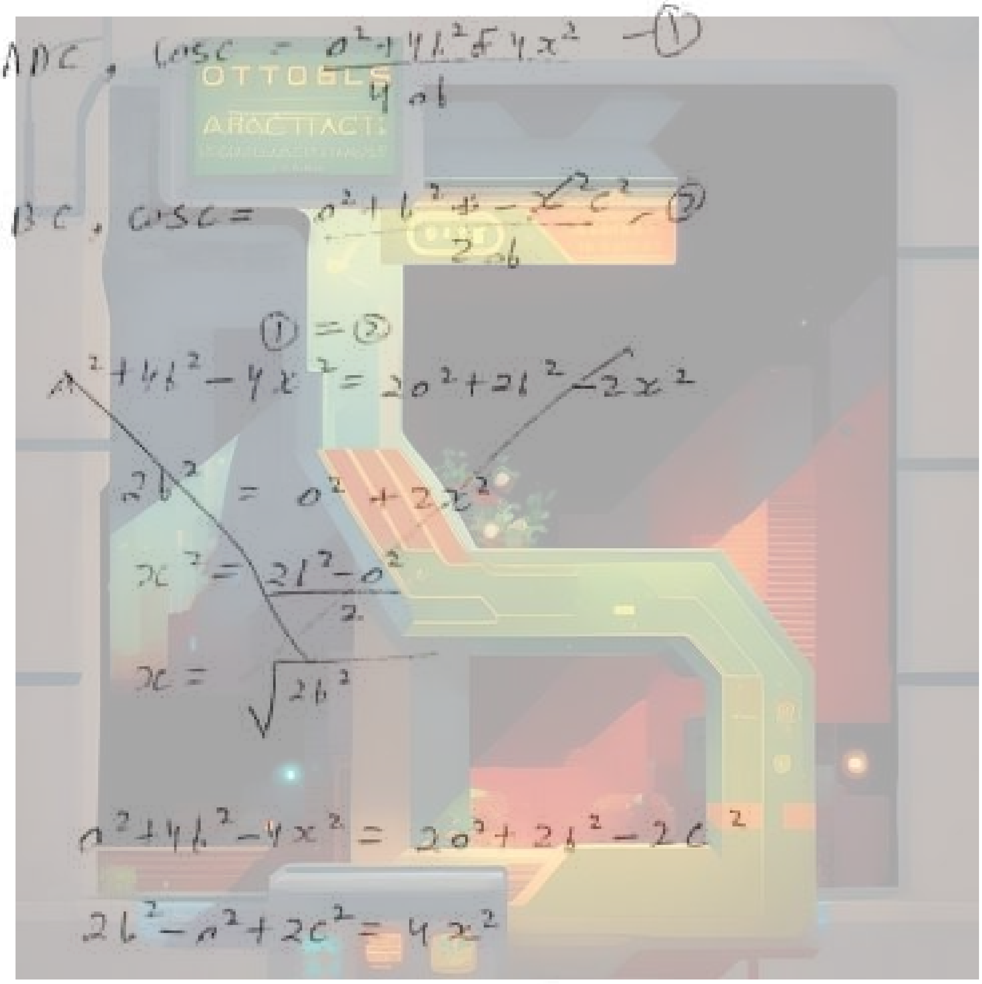
$$\boxed{= \frac{5}{9}}$$



Length of Median and Angle Bisector



x is median.



In $\triangle ADC$, $\cos C = \dfrac{a^2+4b^2-4x^2}{4ab}$ — (1)

In $\triangle ABC$, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$ — (2)

(1) = (2)

$\cancel{a^2+4b^2-4x^2 = 2a^2+2b^2-2x^2}$

$\cancel{2b^2 = a^2+2x^2}$

$\cancel{x^2 = \dfrac{2b^2-a^2}{2}}$

$\cancel{x = \sqrt{2b^2}}$

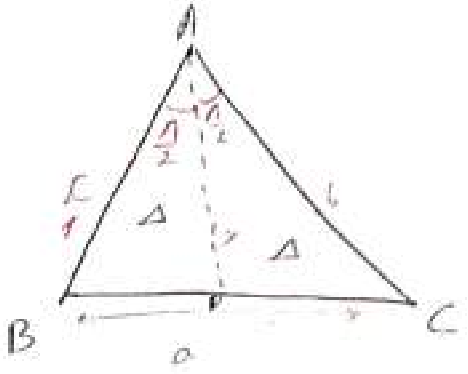
$a^2+4b^2-4x^2 = 2a^2+2b^2-2c^2$

$2b^2-a^2+2c^2 = 4x^2$

$x^2 = \dfrac{2b^2-a^2+2c^2}{4}$

$$\boxed{x = \dfrac{\sqrt{2b^2-a^2+2c^2}}{2}} \quad (12)$$

Length of Angle Bisector



$$\text{Area}(\Delta ADB) = \frac{1}{2} c x \sin\frac{A}{2} \quad -①$$

$$\text{area}(\Delta ADC) = \frac{1}{2} b x \sin\frac{A}{2} \quad -②$$

$$\text{area}(\Delta ABC) = \cancel{\frac{bc}{2} \sin A}$$

$$= \frac{1}{2} bc \sin A \quad -③$$

① + ② = ③

$$\frac{c x \sin\frac{A}{2} + b x \sin\frac{A}{2}}{2} = \frac{1}{2} bc \sin A$$

$$x\left(c \sin\frac{A}{2} + b \sin\frac{A}{2}\right) = 2bc \sin\frac{A}{2} \cos\frac{A}{2}$$

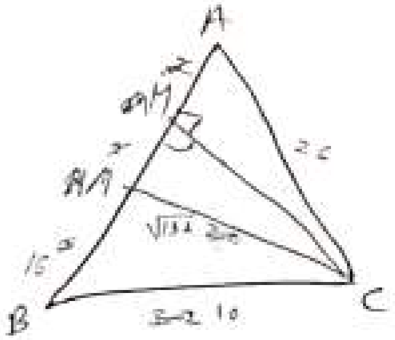
$$x(b+c) = 2bc \cos\frac{A}{2}$$

$$\boxed{x = \frac{2bc \cos\frac{A}{2}}{b+c}} \quad ⑬$$

Note :- Formula contains HM of b & c

Q in $\triangle$, CH & CM are lengths of altitude & Median to base AB. If $a=10$, $b=26$ & $c=32$ find HM.

$$CM = \frac{\sqrt{2a^2+2b^2-c^2}}{2}$$



$\cancel{CM = 30}$

$$\cos A = \frac{x}{26}$$

$$\cancel{\cos B = \frac{32-x}{10}}$$

$$\frac{10}{x} = \frac{26}{32-x}$$

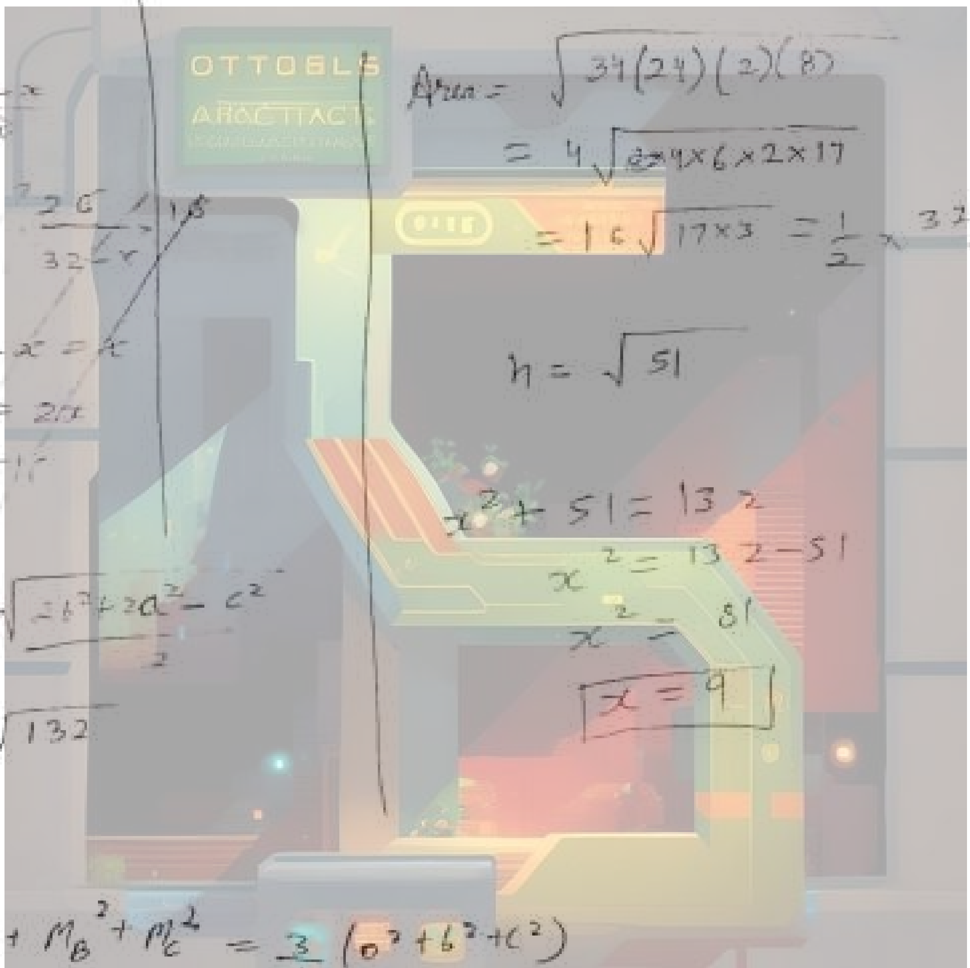
$$32 - x = x$$

$$32 = 2x$$

$$x = 16$$

$$CM = \frac{\sqrt{2b^2+2a^2-c^2}}{2}$$

$$CM = \sqrt{132}$$



$$\text{Area} = \sqrt{34(24)(2)(8)}$$

$$= 4\sqrt{2\times4\times6\times2\times17}$$

$$= 16\sqrt{17\times3} = \frac{1}{2}\times32\times h$$

$$h = \sqrt{51}$$

$$x^2 + 51 = 132$$

$$x^2 = 132 - 51$$

$$x^2 = 81$$

$$\boxed{x = 9}$$

Note:- $M_A^2 + M_B^2 + M_C^2 = \frac{3}{4}(a^2+b^2+c^2)$

Proof:-

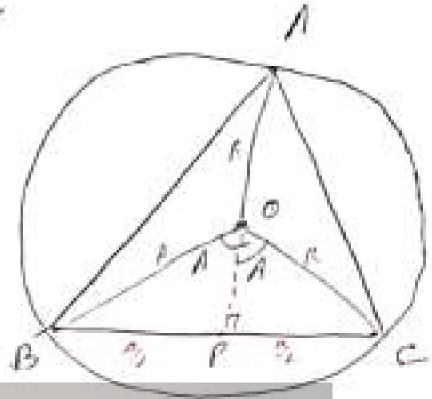
$$\frac{2a^2+2b^2-c^2 + 2b^2+2c^2-a^2 + 2a^2+2c^2-b^2}{4}$$

$$= \frac{3(a^2+b^2+c^2)}{4}$$

H.P.

** Circumcircle & in circle

Circumcircle :- A circle passes through the vertices of a triangle & radius is called circum-radius (R).



$\sin A = \frac{a}{2} \times \frac{1}{R}$

$\frac{\sin A}{a} = \frac{1}{2R}$

$\therefore \boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$ (14)

$\text{Area}(\Delta ABC) = \frac{1}{2} bc \sin A$

$\Delta = \frac{abc}{4R}$

$\boxed{R = \frac{abc}{4\Delta}}$ (15)

Q In ΔABC, P.T.

$s = 4R \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$

$s = 4R \sqrt{\frac{s(s-a)\, s(s-b)\, s(s-c)}{bc \times ab \times abc}}$

$s = \frac{4R}{abc}\, s\sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{4R}{abc}\, s\Delta$

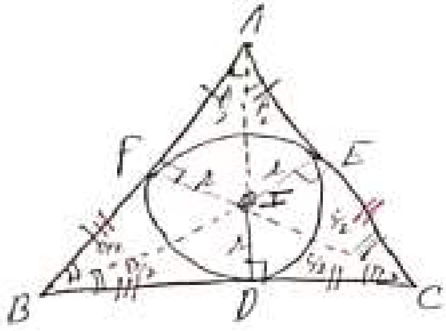
$s = 4 \times \frac{abc}{4\Delta} \times \frac{1}{abc} \times \Delta \times s$

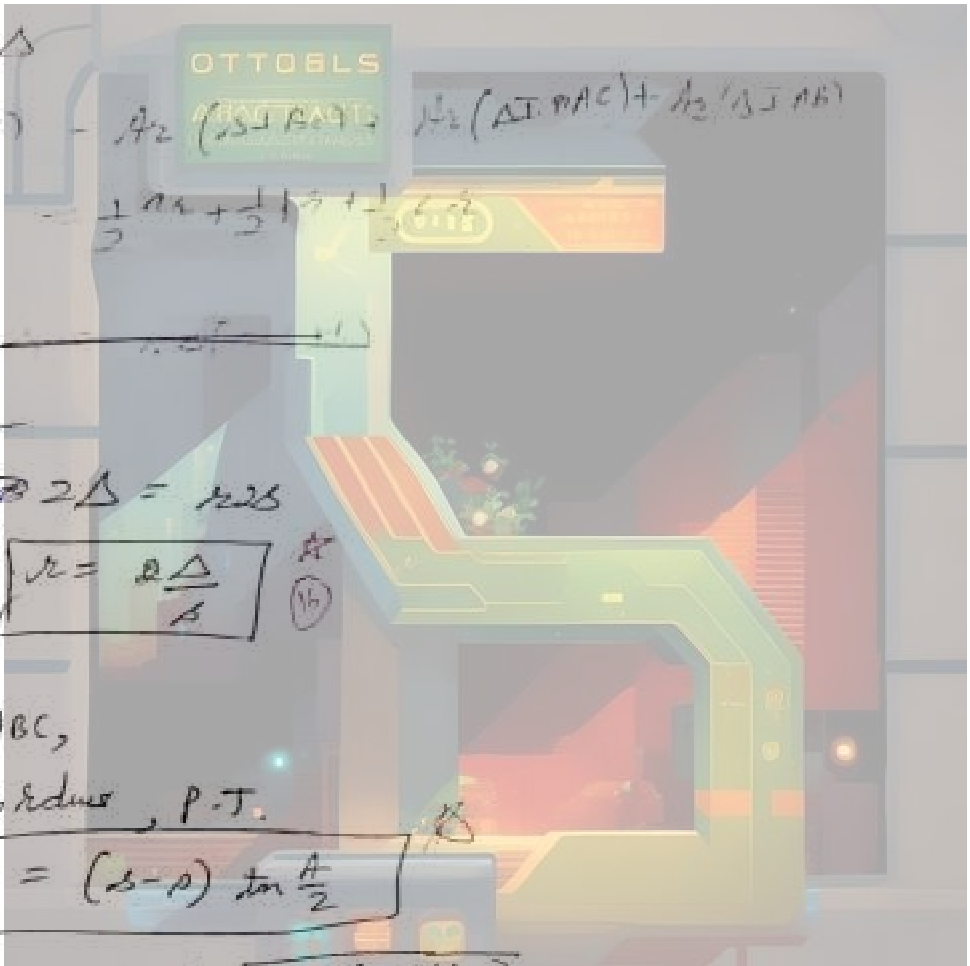
$s = s$

H.P

Incircle: The circle touching all the 3 sides of a triangle internally is called incircle and radius of this circle is called in-radius (r).

→ All the 3 sides of the Δ are $\perp$ to the in-radius





$\Delta = \frac{1}{2}\Delta$

$A(\Delta ABC) = A_2(\Delta IBC) + A_2(\Delta IAC) + A_2(\Delta IAB)$

$\Delta = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$

$2\Delta = r2s$

$$\boxed{r = \frac{\Delta}{s}}$$ ☆ (16)

① In a ΔABC,
r is in-radius, P.T.

$$\boxed{r = (s-a)\tan\frac{A}{2}}$$

$= (s-a)\sqrt{\frac{s(s-b)(s-c)(s-a)}{s^2(s-a)^2}}$

$= \frac{(s-a)}{s(s-a)} \times \Delta$

$= \frac{\Delta}{s}$

$= r$

H.P

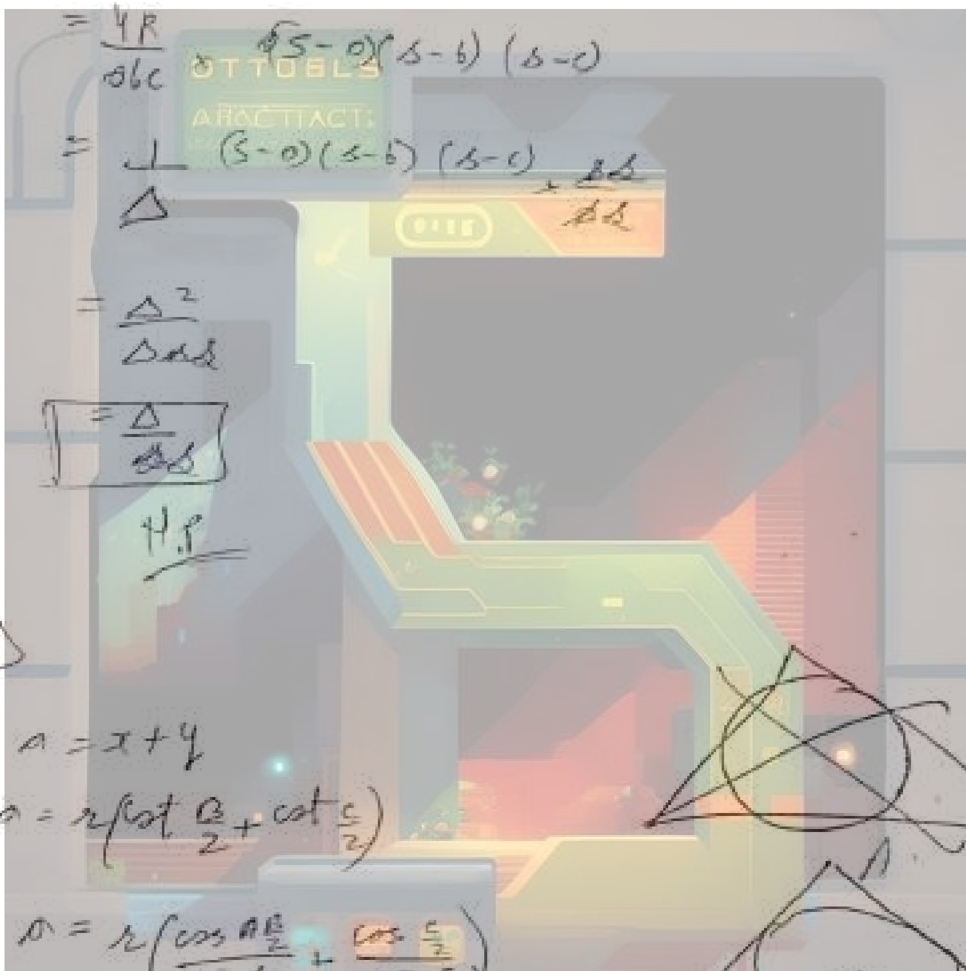
Relation in Circumradius & Inradius

$$\boxed{r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}} \quad (17)$$

Proof:- $\boxed{\text{M-I}}$

RHS

$$= 4R \times \sqrt{\frac{s(s-b)(s-c)(s-a)(s-c)(s-a)(s-b)}{bc \times ac \times ab}}$$



$$= \frac{4R}{abc}\, s(s-a)(s-b)(s-c)$$

$$= \frac{1}{\Delta}(s-a)(s-b)(s-c) \times \frac{s}{s}$$

$$= \frac{\Delta^2}{\Delta s}$$

$$\boxed{= \frac{\Delta}{s}}$$

H.P.

$\boxed{\text{M-II}}$

$$a = x + y$$

$$a = r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right)$$

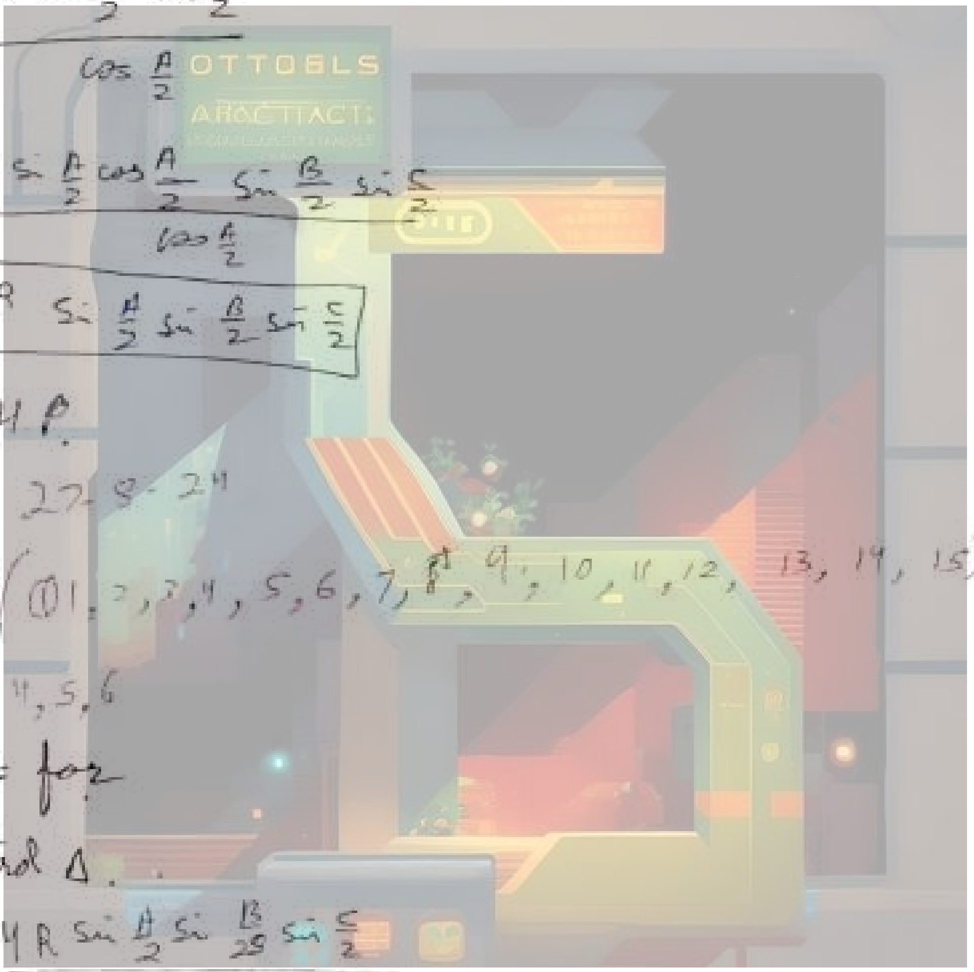
$$a = r\left(\frac{\cos\frac{B}{2}}{\sin\frac{B}{2}} + \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}}\right)$$

$$a = \frac{r\left(\cos\frac{B}{2}\sin\frac{C}{2} + \cos\frac{C}{2}\sin\frac{B}{2}\right)}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$\Delta = \frac{r \sin\frac{(B+C)}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$\Delta = \frac{r \cos\frac{A}{2}}{\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$r = \frac{\Delta \sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$$



$$r = \frac{4R \sin\frac{A}{2}\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}$$

$$\boxed{r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

H.P.

H.W 27-8-24

C-1 (①1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

DYS - 4, 5, 6

Q find $\frac{r}{R}$ for

① equilateral Δ.

$$\frac{r}{R} = \frac{4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}{R}$$

$$\frac{r}{R} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$A = B = C = 60°$

$$\frac{r}{R} = 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$\boxed{\frac{r}{R} = \frac{1}{2}}$$

(2) isosceles right angled Δ, $C = 90°$ $A = 45°$
$B = 45$

$$\frac{r}{R} = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\frac{r}{R} = 4 \times \frac{1}{\sqrt{2}} \times \sin^2 22.5$$

$$\frac{r}{R} = \frac{4}{\sqrt{2}} \times \frac{(1-\cos 45)}{2}$$

$$\frac{r}{R} = \sqrt{2} \times \frac{(\sqrt{2}-1)}{\sqrt{2}}$$

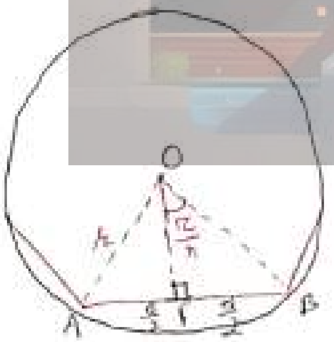
$$\boxed{\frac{r}{R} = \sqrt{2}-1}$$

Regular Polygon

→ All sides and all angles are equal

$$\boxed{\text{Sum of all int. angles} = (n-2)\pi}$$ n = no. of sides

(I) Perimeter & Area of regular polygon ~~...~~ inscribed in a circle.



$$\sin\left(\frac{\pi}{n}\right) = \frac{a/2}{r}$$

$$a = 2r\sin\left(\frac{\pi}{n}\right)$$

n = no. of side
r = radius

$$\boxed{\text{Perimeter} = 2rn\sin\left(\frac{\pi}{n}\right)}$$

$OP = r\cos\left(\frac{\pi}{n}\right)$

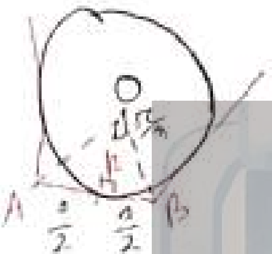
$\text{Area}(\Delta OAB) = \frac{1}{2} \times a \times OP$

$= \frac{1}{2} \times 2r\sin\left(\frac{\pi}{n}\right) \times r\cos\left(\frac{\pi}{n}\right)$

$$\text{area}(\Delta OAB) = \frac{r^2}{2}\sin\left(\frac{2\pi}{n}\right)$$

$$\therefore \boxed{\text{Area(Polygon)} = \frac{nr^2}{2}\sin\left(\frac{2\pi}{n}\right)} \quad (20)$$

(II) Area & Perimeter of ~~Irreg~~ regular Polygon circumscribing a circle.



$\tan\left(\frac{\pi}{n}\right) = \frac{a/2}{r}$

$a = 2r\tan\left(\frac{\pi}{n}\right)$

$$\boxed{\text{Perimeter} = 2nr\tan\left(\frac{\pi}{n}\right)} \quad (21)$$

~~Area = nr^2.~~

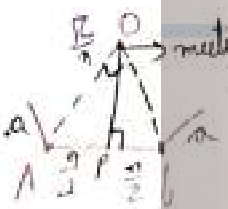
$\text{Area}(\Delta OAB) = \frac{1}{2}\times a\times r$

$= \frac{1}{2}\times 2r\tan\frac{\pi}{n}\times r$

$= r^2\tan\left(\frac{\pi}{n}\right)$

$$\therefore \boxed{\text{Area} = nr^2\tan\left(\frac{\pi}{n}\right)} \quad (22)$$

(III) Perimeter & Area of a regular Polygon of side 'a'.



$\tan\left(\frac{\pi}{n}\right) = \frac{a/2}{OP}$

$OP = \frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

~~$\tan\left(\frac{\pi}{n}\right)$~~

$\text{ar}(\Delta OAB) = \frac{1}{2}\times a\times\frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

$= \frac{a^2}{4}\cot\left(\frac{\pi}{n}\right).$

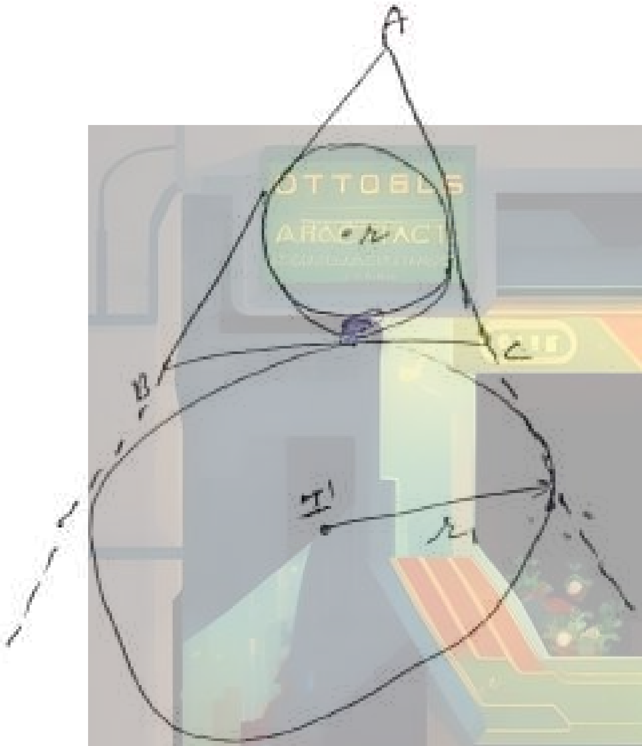
$$\therefore \boxed{\text{Area of Polygon} = \frac{na^2}{4}\cot\left(\frac{\pi}{n}\right)} \quad (23)$$

$$\boxed{\therefore \text{Perimeter} = n\times a} \quad (24)$$

If $a^2+b^2+c^2=8R^2 \rightarrow$ Right Angle Triangle

~~equilateral Triangle~~

Ex center :-



$r_1 = \frac{\Delta}{s-a}$ → exradius

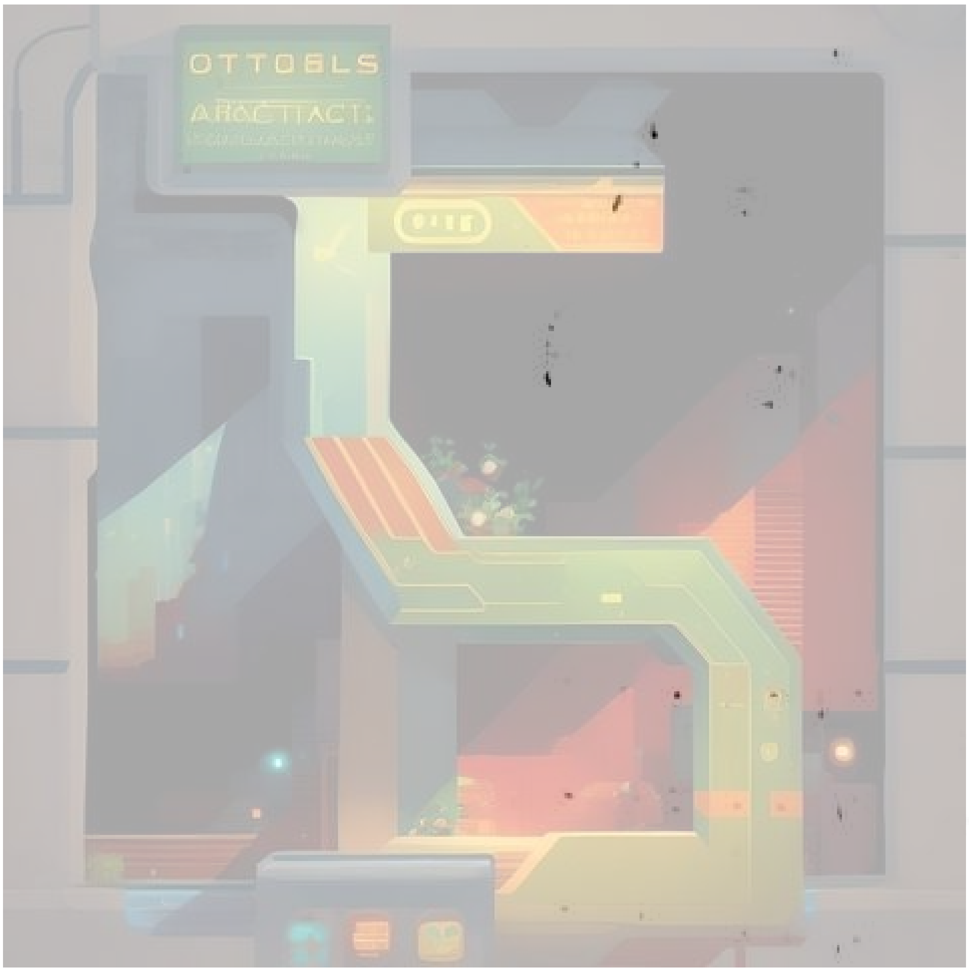
$r_2 = \frac{\Delta}{s-b}$

$r_3 = \frac{\Delta}{s-c}$

$r_1 = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$

$r_2 = 4R \cos\frac{A}{2} \sin\frac{B}{2} \cos\frac{C}{2}$

$r_3 = 4R \cos\frac{A}{2} \cos\frac{B}{2} \sin\frac{C}{2}$

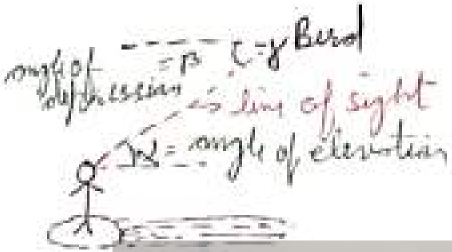

OTTOBLS
ARACTIACT:


OTTOBLS
ARACTTACT:


OTTOBLS
ARACTIACT:

! Heights & Distances !

i) Line of sight
ii) Angle of Elevation
iii) Angle of Depression.



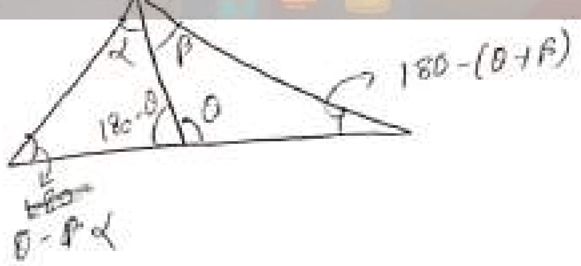
$\alpha_1, \alpha_2 \rightarrow$ angle of elevation.

I Results to Remember

① In a $\triangle ABC$, If $BD : DC = m : n$,
$\angle BAD = \alpha$
$\angle ADC = \theta$
$\angle CAD = \beta$

Then, $$\boxed{(m+n)\cot\theta = m\cot\alpha - n\cot\beta}$$ (m-n theorem)

Proof:-



Proof:- ΔABD -

$$\frac{\sin\alpha}{BD} = \frac{\sin(\theta-\alpha)}{AD} \quad \text{— ①}$$

ΔADC -

$$\frac{\sin\beta}{DC} = \frac{\sin(180-(\theta+\beta))}{AD} \quad \text{— ②}$$

① ÷ ②

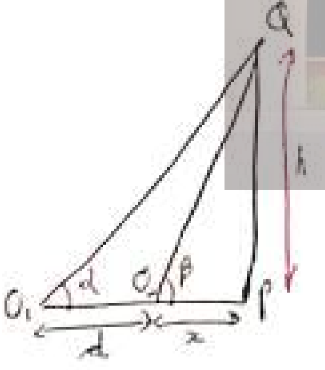
$$\frac{\frac{\sin\alpha}{BD}}{\frac{\sin\beta}{DC}} = \frac{\sin(\theta-\alpha)}{AD} \times \frac{AD}{\sin(\theta+\beta)}$$

$$\frac{n\sin\alpha}{m\sin\beta} = \frac{\sin\theta\cos\alpha - \cos\theta\sin\alpha}{\sin\theta\cos\beta + \cos\theta\sin\beta} \qquad \left\{\frac{BD}{DC} = \frac{m}{n}\right\}$$

cross multiply

② The angles of elevation of two objects are α & β respectively and the distance btw two objects is d.
Then,

$$\boxed{d = h(\cot\alpha - \cot\beta)}$$



Proof:-

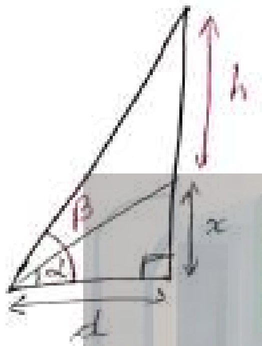
$$\cot\alpha = \frac{d+x}{h} \Rightarrow (x = h\cot\alpha - d) \quad \text{— ①}$$

$$\cot\beta = \frac{x}{h} \Rightarrow (x = h\cot\beta) \quad \text{— ②}$$

① = ②

$$h(\cot\alpha - \cot\beta) = d$$

Q A vertical tower stands on a horizontal plane and is ~~surround~~ surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom & the top of the flag staff are α & β respectively. Prove that the height of the tower is $\dfrac{h\cot\beta}{\cot\alpha-\cot\beta}$.



$$\cot\beta = \frac{d}{h+x} \Rightarrow d = (h+x)\cot\beta \quad —①$$

$$\cot\alpha = \frac{d}{x} \Rightarrow d = x\cot\alpha \quad —②$$

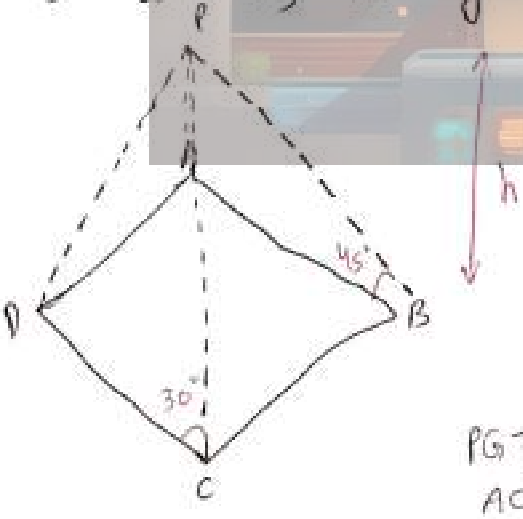
① = ②

$$h\cot\beta + x\cot\beta = x\cot\alpha$$

$$\Rightarrow x = \frac{h\cot\beta}{\cot\alpha-\cot\beta}$$

H.P

Q A vertical lamp post of height 9m stands at the corner of a rectangular field. The angle of elevation of its top from the farthest corner of field is 30° while from the ~~oth~~ other one corner, the angle is 45°. Find the area of the field.



ΔBAP –

$$\tan 45^\circ = \frac{9}{AB} \Rightarrow AB = 9\text{m}$$

ΔCAP –

$$\tan 30^\circ = \frac{9}{AC} \Rightarrow AC = 9\sqrt{3}\text{ m}$$

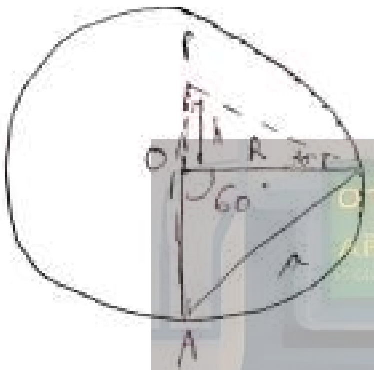
PGT in ΔABC –

$$AC^2 = AB^2 + BC^2$$

$$BC = 9\sqrt{2}\text{m}$$

$$\therefore \text{Area} = 9\times 9\sqrt{2} = \boxed{81\sqrt{2}\text{ m}^2}$$

Q A tower stands at the top centre of a circular path. A & B are 2 points on the boundary of the park such that AB = a and it subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30°. find h of tower in terms of a.



$\triangle BOP$ –

$\tan 30° = \frac{h}{R} \Rightarrow h = \frac{R}{\sqrt{3}}$

$\triangle OAB$ –

$\angle O = 60°$

Also, $\angle A = \angle B$ {angles opp to equal sides of $\triangle$}

$\therefore$ $60° + \angle A + \angle B = 180°$

$\angle A = \angle B = 60°$

$\therefore$ $a = R$

$$\boxed{h = \frac{a}{\sqrt{3}}}$$


OTTOBLS


OTTOBLS
ARACTIACT:

! Determinants !

- Determinant is a scalar value which can be a real no. or complex no.
- It is represented by two vertical lines & also btw these two lines we take the grid of nos, in form of rows & colums.
- Modulus & Determinants are diff.
- Determinants are only possible for square orders.

  eg. 1×1, 2×2, 3×3 .... $n\times n$ etc.

  where 1^{st} no. denotes no. of rows

  2^{nd} no. denotes no. of columns

- It is represented by $|A|$ or $det(A)$
- 1×1 order — $|3| = 3$

  $|-2| = 2$

  $det(-3) = -3$

2×2 order —

$$\begin{array}{cc} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} & \begin{array}{l} \to R_1 \\ \to R_2 \end{array} \\ C_1 \quad C_2 & \end{array}$$

$$\begin{array}{cc} \begin{vmatrix} a & b \\ c & d \end{vmatrix} & \begin{array}{l} \to R_1 \\ \to R_2 \end{array} \\ C_1 \quad C_2 & \end{array}$$

3×3 order -

$$\begin{array}{cc} \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} & \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \\ C_1 \quad C_2 \quad C_3 & \end{array}$$

Elements of a determinant

a_{ij} — i - no. of rows, j - no. of columns

$1\times1: \quad A = |a_{11}|$

$2\times2: \quad A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$3\times3: \quad A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Q1. make a ~~3×3~~ 2×2 determinant whose elements follow.

i) $a_{ij} = (i+j)^2$

$A = \begin{vmatrix} 4 & 9 \\ 9 & 16 \end{vmatrix}$

ii) $a_{ij} = \left(\frac{i}{j}\right)$

$A = \begin{vmatrix} 1 & 1/2 \\ 2 & 1 \end{vmatrix}$

Evaluation of determinants.

① 1×1 $\quad |3| = 3$

$|-2| = -2$

② 2×2 i) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1\times 4) - (2)(3)$

$= 4-6$

$= -2$

ii) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $ad - bc$

iii) $\begin{vmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{vmatrix}$ $-\sin^2\theta - \cos^2\theta = -1$

③ 3×3 — i) $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1(2-6) - 2(-1-6) + 3(-2-0)$

$= 2+8-6$

$= 4$

ii) $\begin{vmatrix} 1 & 3 & 6 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} + 6\begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix}$

$= 1(4-0) - 3(-2-0) + 6(0-2)$

$= 4 + 6 - 12$

$= -2$

Q2. find the values of

① $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \\ 2 & 1 & 3 \end{vmatrix} = 1(9-2) - 2(0-4) - 1(0-6)$

$= 7 + 8 + 6$

$\boxed{= 21}$

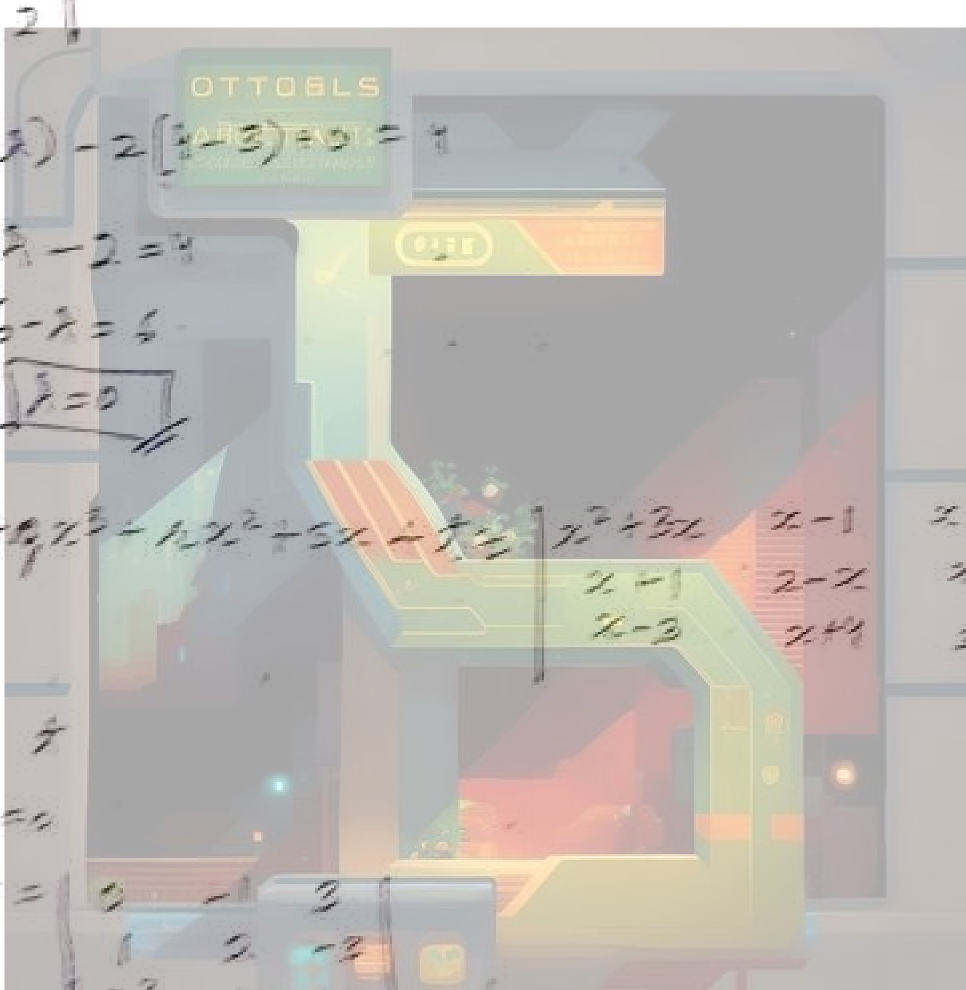
(2) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 3(1+6) + 4(1+4) + 5(3-2)$

$= 21 + 20 + 5$

$\boxed{= 46}$

Q. find λ if :-

$\begin{vmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & \lambda & 2 \end{vmatrix} = 4$



$1(6-\lambda) - 2(4-3) + 0 = 4$

$6 - \lambda - 2 = 4$

$6 - \lambda = 6$

$\boxed{\lambda = 0}$

Q. If $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & 2-x & x-3 \\ x-3 & x+4 & 3x \end{vmatrix}$

find t

at $x = 0$

$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 3 \end{vmatrix}$

$t = 1(-9) + 3(4+6)$

$= 30 - 9$

$\boxed{= 21}$

Q find value(s) of x if:

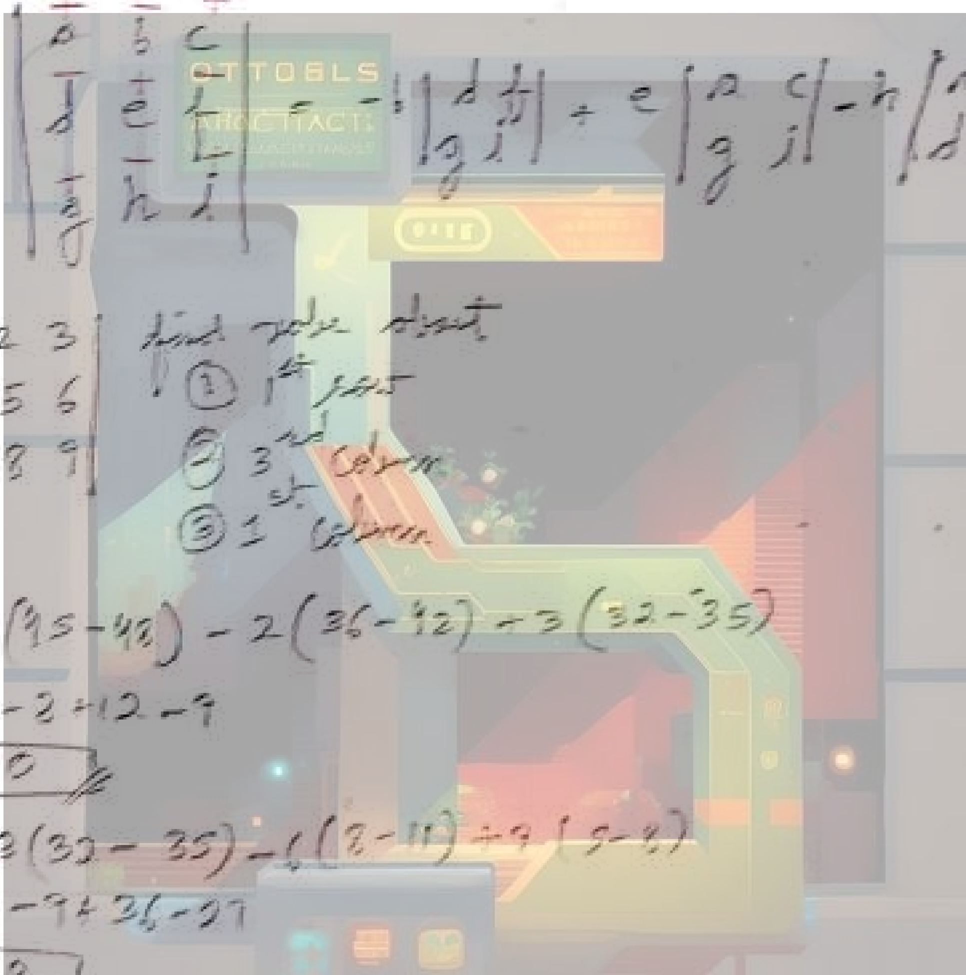
$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$x^2 - 36 = 36 - 36$$

$$x^2 = 36$$

$$\boxed{x = \pm 6}$$

Note - $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + e\begin{vmatrix} a & c \\ g & i \end{vmatrix} - h\begin{vmatrix} a & c \\ d & f \end{vmatrix}$



Q $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ find value about
(1) 1st row
(2) 3rd column
(3) 1st column

(1)
$$= 1(45-48) - 2(36-42) + 3(32-35)$$
$$= -3 + 12 - 9$$
$$\boxed{= 0}$$

(2) $= 3(32-35) - 6(8-14) + 9(5-8)$
$$= -9 + 36 - 27$$
$$\boxed{= 0}$$

~~(3) 1(18-24) - 4(~~

(3) $1(45-48) - 4(18-24) + 7(12-15)$

$$-3 + 24 - 21$$

$$\boxed{= 0}$$

Minor & Cofactor

→ Minor – Minor of an element is defined by deleting the elements of row/column in which it lies.

→ It is denoted by M_{ij}.

→ Co-factor – ~~It is denoted by~~
It is relted to minor

$$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$$

Q find minor & cofactor of each element in

$$\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix}$$

$a_{11} = 1$ $\quad a_{12} = 2$ $\quad a_{21} = 3$ $\quad a_{22} = 7$

$M_{11} = 7$ $\quad M_{12} = 3$ $\quad M_{21} = 2$ $\quad M_{22} = 1$

$C_{11} = (-1)^{1+1} \times 7 = 7$ $\quad C_{12} = -3$ $\quad C_{21} = -2$ $\quad C_{22} = 1$

Q find minor & co-factor of each element in

$$\begin{vmatrix} \cancel{0}\ 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix}$$

element	minor	Co-factor
$a_{11} = 1$	$\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$	1
$a_{12} = 2$	$\begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} = +2$	-2
$a_{13} = 3$	$\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = +1$	1
$a_{21} = 0$	$\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 0$	0
$a_{22} = 1$	$\cancel{\begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} = +2}$ $\begin{vmatrix} 1 & 3 \\ -1 & 3 \end{vmatrix} = 6$	$\cancel{2}$ 6
$a_{23} = 2$	$\cancel{\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = 1}$ $\begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 4$	$\cancel{-1}$ -4

$A_{31} = -1$ $\quad \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$ $\quad 1$

$A_{32} = 2$ $\quad \begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = 2$ $\quad -2$

$A_{33} = 3$ $\quad \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$ $\quad 1$

Note:- $C_{ij} = (-1)^{i+j} M_{ij}$

$i+j \in$ odd: $C_{ij} = -M_{ij}$

$i+j \in$ even: $C_{ij} = M_{ij}$

Q find minor & co-factor of 2nd raw in

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$a_{21} = 4$ $\quad M_{21} = \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = 18 - 24 = -6$ $\quad C_{21} = 6$

$A_{22} = 5$ $\quad M_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 9 - 21 = -12$ $\quad C_{22} = -12$

$A_{23} = 6$ $\quad M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$ $\quad C_{22} = 6$

Note:- Value of Determinant can be written as

$$\boxed{\begin{aligned} \Delta &= a_{11} M_{11} - a_{12} M_{12} + a_{13} M_{13} \\ \Delta &= a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} \end{aligned}}$$

eg. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$

$= -3 + 12 - 9$

$\boxed{= 0}$

Property of Minor & Co-factor

① $\Delta = \Sigma$ (Elements of any row) $\times$ (corresponding co-factors of that row)

$\Delta = \Sigma$ (Elements of any column) $\times$ (corresponding co-factors of that column)

② Σ (elements of any row) $\times$ (corresponding co-factors of any other row) $= 0$

Σ (elements of any column) $\times$ (corresponding co-factors of any other column) $= 0$

Q find values of following in $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$

① $pC_{21} + qC_{22} + rC_{23} = 0$

② $xC_{21} + yC_{22} + zC_{23} = \Delta$

③ $-qM_{12} + yM_{22} - mM_{32} = \Delta$

④ $qM_{12} - yM_{22} + mM_{32} = -\Delta$

⑤ $-xM_{21} + yM_{22} - zM_{23} = \Delta$

⑥ $xM_{21} - yM_{22} + zM_{23} = -\Delta$

Q $\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \delta & \theta & \pi \\ e & g & i \end{vmatrix}$

① $\alpha C_{21} + \beta C_{22} + \gamma C_{23} = 0$

② $e M_{11} - g M_{12} + i M_{13} = 0$

③ $\gamma C_{11} - \pi C_{12} + i C_{13} =$ ek colum, ek raw ishje, potondi

④ $-\delta M_{21} + \theta M_{22} - \pi M_{23} = \Delta$

Properties of Determinants:

① Value of det remains same if rows & colums are interchanged

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$$

② If any two rows/colums are interchanged, only the sign of the determinant changes.

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = - \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

③ If a det has two rows/columns identical, then its value becomes 0.

eg. $\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$

$$\begin{vmatrix} a & b & c \\ P & Q & R \\ a & b & c \end{vmatrix} = 0$$

(1) If all the elements of a row or column are multiplied by a same no., then the value of det is also multiplied by that no.

eg

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$$

$$\begin{vmatrix} 3 & 6 \\ 3 & 4 \end{vmatrix} = -6$$

Q find value of $\Delta = \begin{vmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 10 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 3 \end{vmatrix} \times 3\times10\times10\times10$$

Q If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$ Then value of, $\begin{vmatrix} 2a & 2b & 2c \\ 3d & 3e & 3f \\ 5g & 5h & 5i \end{vmatrix}\begin{matrix} \times\frac{1}{2} \\ \times\frac{1}{3} \\ \times\frac{1}{5} \end{matrix}$

$$\cancel{\Delta = \frac{1}{2}\times\frac{1}{3}\times\frac{1}{5}\times\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}} \qquad \Delta = 2\times3\times5\times2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\boxed{\Delta = 60}$$

$$\cancel{\Delta = \frac{1}{15}}$$

Q $\begin{vmatrix} x & 1 & x^{2013} \\ x^4 & 2 & x^{2016} \\ x^{10} & 3 & x^{2022} \end{vmatrix} \Rightarrow \underbrace{\begin{vmatrix} 1 & 1 & 1 \\ x^3 & 2 & x^3 \\ x^9 & 3 & x^9 \end{vmatrix}}_{=0} \times x \times x^{2013}$

$$\boxed{=0}$$

④ $\begin{vmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{vmatrix} \rightarrow \begin{vmatrix} a & a & a \\ b & b & b \\ c & c & c \end{vmatrix} \times a \times b \times c$

$\begin{array}{ccc} a & b & c \end{array}$

$\boxed{= 0}$

⑤ If all the elements of a row or column are zero, the value of the determinant will be zero.

eg. $\begin{vmatrix} \alpha & \delta & 0 \\ \beta & \theta & 0 \\ \gamma & \phi & 0 \end{vmatrix} = 0$

⑥ If any row/column is written in terms of addition & subtraction of any 2 elements then we can break the det. accordingly to that row or column. order is important

eg. $\begin{vmatrix} 1+2 & 3+4 \\ 5 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 5 & 6 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix}$

$\begin{vmatrix} \alpha+1 & 3 \\ \beta+1 & 4 \end{vmatrix} = \begin{vmatrix} \alpha & 3 \\ \beta & 4 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix}$

$\begin{vmatrix} 1 & a+b & p+q \\ 1 & c+d & r+s \\ 1 & e+f & t+u \end{vmatrix} = \begin{vmatrix} 1 & a & p+q \\ 1 & c & r+s \\ 1 & e & t+u \end{vmatrix} + \begin{vmatrix} 1 & b & p+q \\ 1 & d & r+s \\ 1 & f & t+u \end{vmatrix}$

$= \begin{vmatrix} 1 & a & p \\ 1 & c & r \\ 1 & e & t \end{vmatrix} + \begin{vmatrix} 1 & a & q \\ 1 & c & s \\ 1 & e & u \end{vmatrix} + \begin{vmatrix} 1 & b & p \\ 1 & d & r \\ 1 & f & t \end{vmatrix} + \begin{vmatrix} 1 & b & q \\ 1 & d & s \\ 1 & f & u \end{vmatrix}$

Q $\begin{vmatrix} \sqrt{13}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{26}+\sqrt{15} & 5 & \sqrt{10} \\ \sqrt{65}+3 & \sqrt{15} & 5 \end{vmatrix} = \begin{vmatrix} \sqrt{13} & \sqrt{5} & \sqrt{5} \\ \sqrt{26} & 5 & \sqrt{10} \\ \sqrt{65} & \sqrt{15} & 5 \end{vmatrix} + \begin{vmatrix} \sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$

$\cancel{= \sqrt{13}\times\sqrt{5}\times\sqrt{5} \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{25} \end{vmatrix} + \sqrt{3}\times\sqrt{5}\times\sqrt{5}}$

$$= \sqrt{13}\times\sqrt{5}\times\sqrt{5}\times \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{2} & \sqrt{5} & \sqrt{2} \\ \sqrt{5} & \sqrt{3} & \sqrt{5} \end{vmatrix} + \sqrt{3}\times\sqrt{5}\times\sqrt{5}\times \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$\boxed{=0}$

(7) In the Ques with Summation Operator (Σ) with det, we can apply it in any one of the row/colum.

eg. $D = \begin{vmatrix} r & x & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$

$$\sum_{r=1}^{n} D = \begin{vmatrix} \Sigma r & x & \frac{n(n+1)}{2} \\ 2\Sigma r - \Sigma 1 & y & n^2 \\ 3\Sigma r - \Sigma 2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & x & \frac{n(n+1)}{2} \\ n^2 & y & n^2 \\ \frac{n(3n-1)}{2} & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

$\boxed{=0}$

(8) If the value of det. of order $n \times n$ is Δ, then the value of det. made by using the co-factors of given data is Δ^{n-1}!

eg. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2 \quad (2\times 2)$

$\begin{vmatrix} 4 & -3 \\ -2 & 1 \end{vmatrix} = -2$ co-factor of a_{12}

$\therefore (-2)^{2-1} = (-2)^1 = -2$

Q If the value of a 3×3 det is -3, value of det. made by its co-factors is ?

$(-3)^{3-1} = (-3)^2 = 9$

Q $\begin{vmatrix} p & q & r \\ a & b & c \\ l & m & n \end{vmatrix} = 5$ & find $\begin{vmatrix} bn-mc & lc-an & am-bl \\ mr-qn & pn-lr & lq-pm \\ qc-br & ar-pc & pb-aq \end{vmatrix}$

$(5)^{3-1} = 5^2 = 25$

Q $\underbrace{\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc-a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix}}_{\Delta^2} \times \underbrace{\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}}_{\Delta}$

$\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}^3$

(4) The value of a det. is not altered by adding elements of any row (or column). The same ~~multiple~~ multiples of corresponding elements of any other row / column.

[~~first~~ 1 row must be left on which we do not perform any action]

eg.

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$$

$R_1 \to R_1 + R_2$

$$\begin{vmatrix} 4 & 6 \\ 3 & 4 \end{vmatrix} = -2$$

Note :- (1) for 2×2,

$R_1 \to R_1 + \alpha R_2 \quad \alpha \in R$

$C_1 \to C_1 + \alpha C_2 \quad \alpha \in R$

for 3×3,

$R_1 \to R_1 + \alpha R_2 + \cancel{\alpha R_3}\ \beta R_3 \quad \alpha, \beta \in R$

$C_1 \to C_1 + \alpha C_2 + \beta C_3 \quad \alpha, \beta \in R$

(ii) 2×2, $R_1 \to x R_1 + \alpha R_2 \quad x, \alpha \in R$

value of det becomes x times

(iii) while applying the other property, atleast one row/column must be unchanged.

$$\begin{vmatrix} 5 & 8 & 10 \\ 77 & 71 & 46 \\ 8 & 7 & 4 \end{vmatrix} = 2 \times \begin{vmatrix} 5 & 8 & 5 \\ 77 & 71 & 23 \\ 8 & 7 & 2 \end{vmatrix}$$

$R_1 \to R_1 + 9R_3$

$$= 2 \times \begin{vmatrix} 77 & 11 & 23 \\ 77 & 11 & 23 \\ 8 & 7 & 2 \end{vmatrix}$$

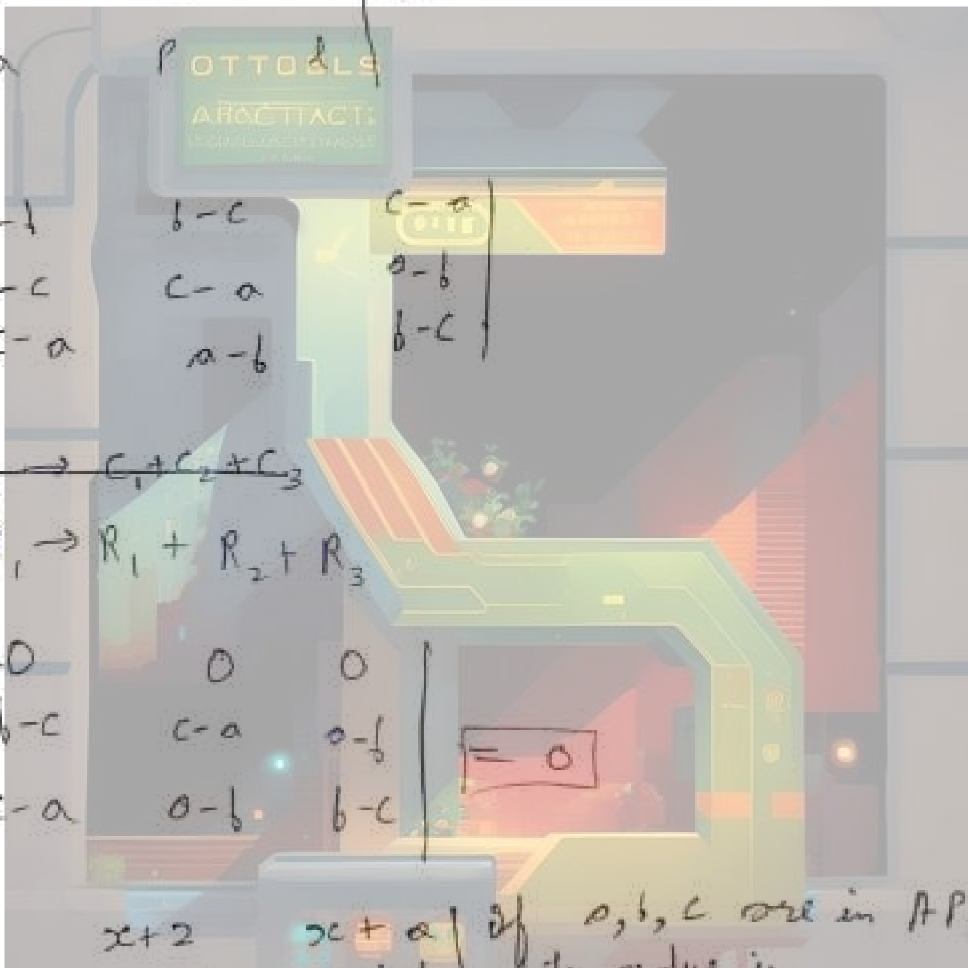
$\boxed{= 0}$

Q $\begin{vmatrix} a+b+c & p+q+r & l+m+n \\ a+b & p+q & l+m \\ a & p & l \end{vmatrix} = ?$

$R_1 \rightarrow R_1 - R_2$

$R_2 \rightarrow R_2 - R_3$

$\begin{vmatrix} c & r & n \\ b & q & m \\ a & p & l \end{vmatrix} =$



Q $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

$\cancel{C_1 \rightarrow C_1 + C_2 + C_3}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$\begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} \boxed{= 0}$

Q $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ If a, b, c are in AP, then its value is

(A) $a+b+c$ (C) 0

(B) $x+a+b+c$ (D) None.

$C_1 \rightarrow C_1 - C_2$

$C_2 \rightarrow C_2 - C_1$

$= -1 \times \begin{vmatrix} 1 & 1 & x+a \\ 1 & 1 & x+b \\ 1 & 1 & x+c \end{vmatrix} \boxed{= 0}$


OTTOBLS

Q1) If x, y & z are +ve integers then, find

$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

Q2. $a^2+b^2+c^2=-2$ find degree of

$$\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Q3. $\begin{vmatrix} 3^2+K & 4^2 & 3^2+3+K \\ 4^2+K & 5^2 & 4^2+4+K \\ 5^2+K & 6^2 & 5^2+5+K \end{vmatrix} = 0$ find 'K'

Q1. $\begin{vmatrix} \log_x x & \log_x y & \log_x z \\ \log_y x & \log_y y & \log_y z \\ \log_z x & \log_z y & \log_z z \end{vmatrix}$

$\log x = a$

$\log y = b$

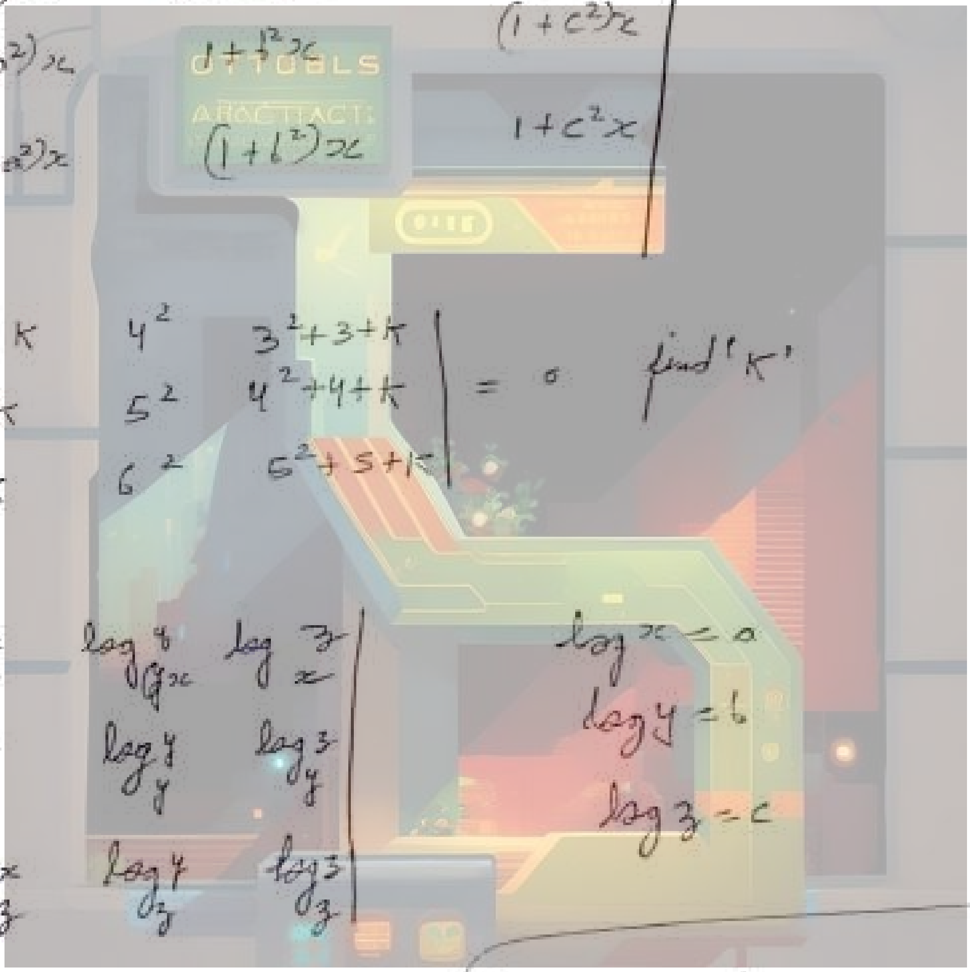
$\log z = c$

$\begin{vmatrix} \frac{a}{a} & \frac{b}{a} & \frac{c}{a} \\ \frac{a}{b} & \frac{b}{b} & \frac{c}{b} \\ \frac{a}{c} & \frac{b}{c} & \frac{c}{c} \end{vmatrix} \begin{matrix} \to \frac{1}{a} \\ \to \frac{1}{b} \\ \to \frac{1}{c} \end{matrix}$

$\begin{vmatrix} a & b & c \\ a & b & c \\ a & b & c \end{vmatrix} \times \frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}$

$0 \times \frac{1}{a} \times \frac{1}{b} \times \frac{1}{c}$

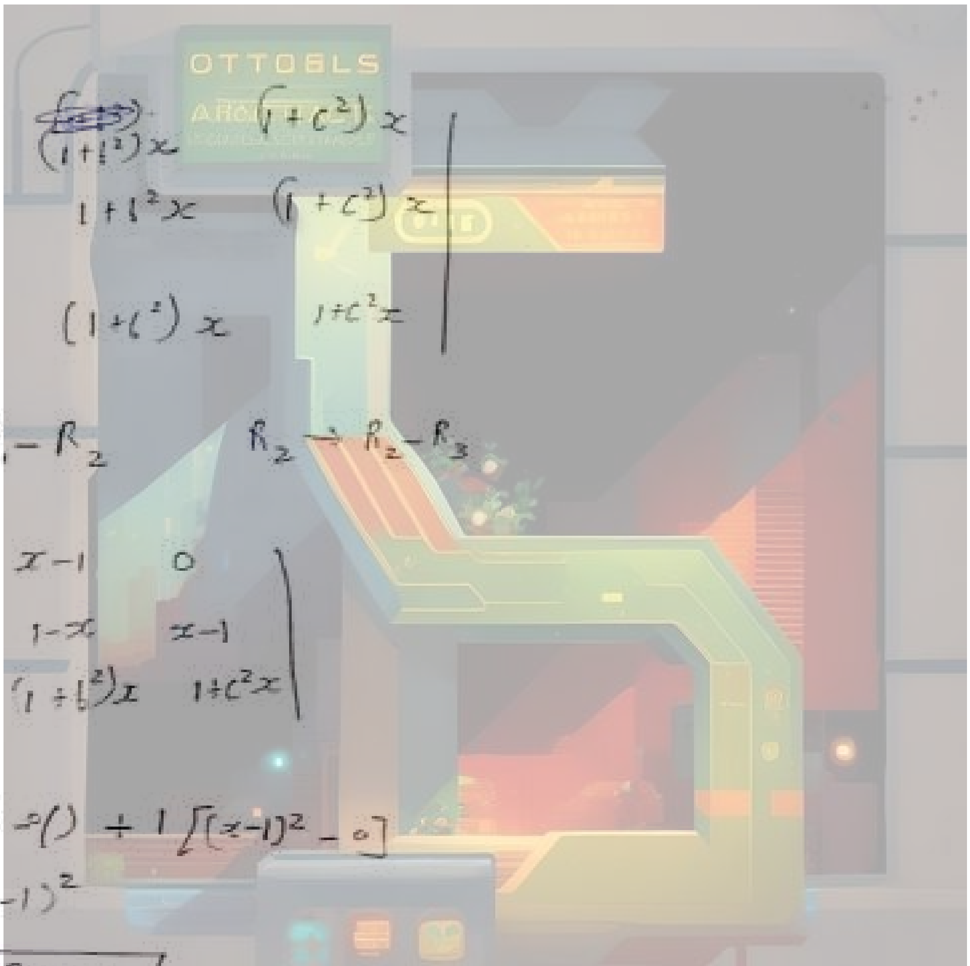
$\boxed{=0}$



Q2 $a^2+b^2+c^2+2=0$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+2x+x(a^2+b^2+c^2) & (1+b^2)x & (1+c^2)x \\ 1+2x+x(a^2+b^2+c^2) & 1+b^2x & (1+c^2)x \\ 1+2x+x\underbrace{(a^2+b^2+c^2)}_{-2} & (1+b^2)x & 1+c^2x \end{vmatrix}$$



$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+c^2)x & 1+c^2x \end{vmatrix}$$

$R_1 \to R_1 - R_2$ $\quad R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & x-1 & 0 \\ 0 & 1-x & x-1 \\ 0 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$= 0() - 0() + 1[(x-1)^2 - 0]$

$= (x-1)^2$

$\boxed{\text{2 Degree}}$

Q3 ~~$3^2+K \quad 4^2$~~

Q3.

$$= \begin{vmatrix} 3^2+K & 4^2 & (3^2+K)+3 \\ 4^2+K & 5^2 & (4^2+K)+4 \\ 5^2+K & 6^2 & (5^2+K)+5 \end{vmatrix} = 0$$

$$= \underbrace{\begin{vmatrix} 3^2+K & 4^2 & 3^2+K \\ 4^2+K & 5^2 & 4^2+K \\ 5^2+K & 6^2 & 5^2+K \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} 3^2+K & 4^2 & 3 \\ 4^2+K & 5^2 & 4 \\ 5^2+K & 6^2 & 5 \end{vmatrix}}_{\substack{R_3 \to R_3 - R_2 \\ R_2 \to R_2 - R_1}} = 0$$

$$= \begin{vmatrix} 9+K & 16 & 3 \\ 7 & 9 & 1 \\ 9 & 11 & 1 \end{vmatrix} = 0$$

$$+3(77-81) - (99+11K-144) + (9K+81-112) = 0$$

$\cancel{45+11K}$

$$(-45+11K) = 12 + 31 - 9K$$

$$11K - 45 = 43 - 9K$$

$$\cancel{20K = 88}$$

$$K$$

$$-12 + 45 - 11K - 31 + 9K = 0$$

$$32K = 32$$

$$\boxed{K = 1}$$

H.W. Ques

Q1. $\begin{vmatrix} 1+\sin^2\phi & \cos^2\phi & 2\sin 4\phi \\ \sin^2\phi & 1+\cos^2\phi & 2\sin 4\phi \\ \sin^2\phi & \cos^2\phi & 1+2\sin 4\phi \end{vmatrix} = 0$ $\quad R_1 \to R_1 + R_2$, $\quad R_2 \to R_2 + R_1$

Then, (A) $\phi = \frac{3\pi}{4}$ $\quad$ (C) $\phi = \frac{3\pi}{2}$

(B) $\phi = \frac{3\pi}{8}$ $\quad$ ✓(D) $\phi \in R$

Q2. Prove:-

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = 0$$

※ Factor Theorem

→ If the elements of the determinant are rational integral in x & for $x = a$ two rows (columns) become identical then $(x-a)$ is a factor of determinant.

Q1. Prove. $\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = m(x-a)(x-b)$

M I Direct Expand

M II

$x = a$: $\begin{vmatrix} a & a & a \\ m & m & m \\ b & a & b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ b & a & b \end{vmatrix} \times a m \times a = 0$

$x = b$: $\begin{vmatrix} a & a & b \\ m & m & m \\ b & b & b \end{vmatrix} = 0$

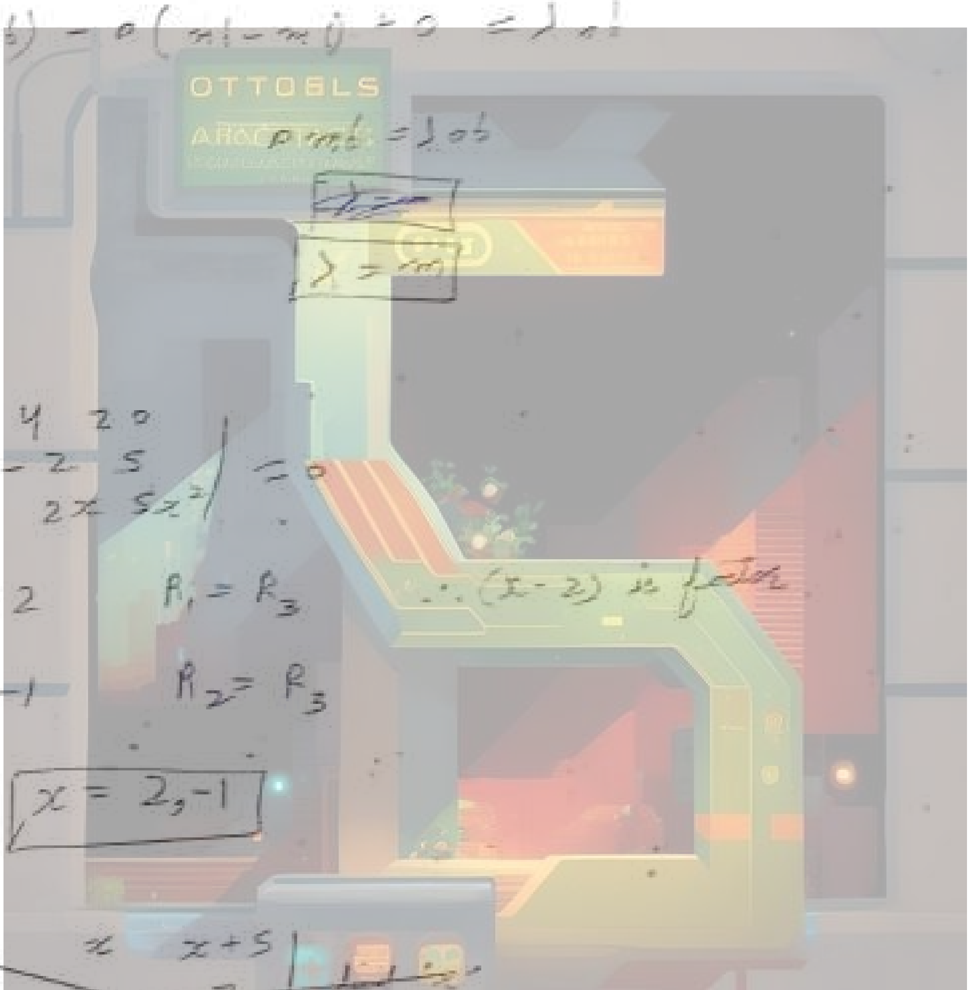
$(x-a)$ & $(x-b)$ are factors

$$\begin{vmatrix} a & a & x \\ m & m & m \\ b & x & b \end{vmatrix} = \lambda(x-a)(x-b)$$

Put $x=0$

$$\begin{vmatrix} a & 0 & 0 \\ m & m & m \\ b & 0 & b \end{vmatrix} = \lambda ab$$

$$a(mb) - 0(mb - m) + 0 = \lambda ab$$



$$amb = \lambda ab$$

$$\cancel{\lambda =}$$

$$\boxed{\lambda = m}$$

Q. $$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

$x=2$ $R_1 = R_3$ $\therefore (x-2)$ is factor

$x=-1$ $R_2 = R_3$

$$\boxed{x = 2, -1}$$

Q3. H.W

$$\begin{vmatrix} x^2 & x & x+5 \\ 4 & 2 & 7 \\ 9 & 3 & 8 \end{vmatrix}$$ find x

Q If $\begin{vmatrix} a^n & a^{n+1} & a^{n+2} \\ b^n & b^{n+1} & b^{n+2} \\ c^n & c^{n+1} & c^{n+2} \end{vmatrix} = (a-b)(b-c)(c-a)$. find n. $(a+b+c)$

$$a^n b^n c^n \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$C_2 \to C_2 - C_1$ $\quad$ $C_3 \to C_3 - C_1$

$$a^n b^n c^n \begin{vmatrix} 1 & a-1 & a^2-1 \\ 1 & b-1 & b^2-1 \\ 1 & c-1 & c^2-1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ $\quad$ $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c-1 & c^2-1 \end{vmatrix} \times a^n b^n c^n$$

$$= (a-b)(b-c)(b+c) - (b-c)(a-b)(a+b)$$

$$(a-b)(b-c)(c-a) = (a-b)(b-c)(c-a) \times a^n b^n c^n$$

$$a^n b^n c^n = 1$$

$$\boxed{n=0}$$

Q2. Prove $\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

~~at $a=b$, $\Delta=0$~~

~~$b=c$, $\Delta=0$~~

~~$a=c$, $\Delta=0$~~

$R_1 \to R_1 - R_2$

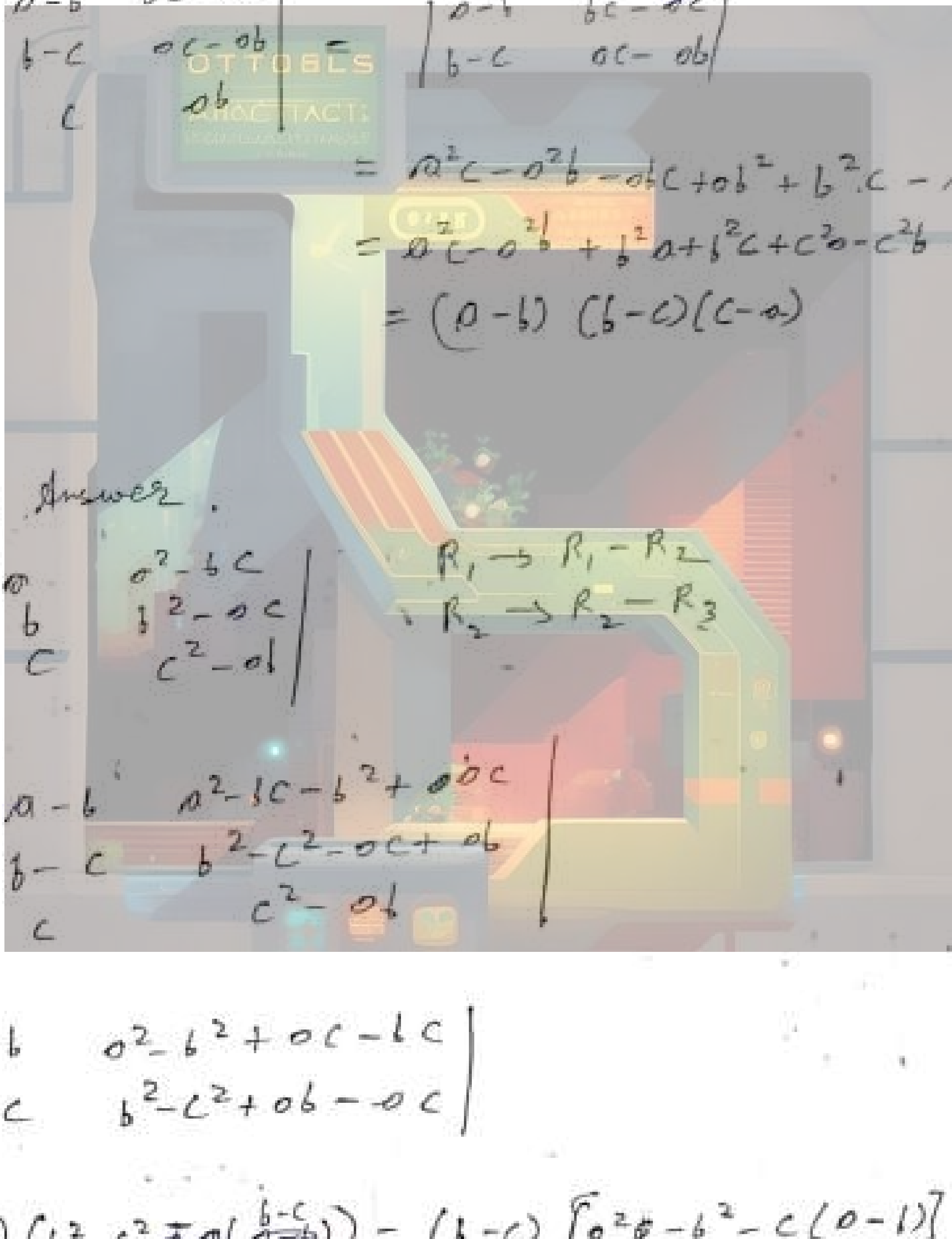
$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ac-ab \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} a-b & bc-ac \\ b-c & ac-ab \end{vmatrix}$$

$$= a^2c - a^2b - abc + ab^2 + b^2c - abc - bc^2 + ac^2$$

$$= a^2c - a^2b + b^2a + b^2c + c^2a - c^2b - 2abc$$

$$= (a-b)(b-c)(c-a)$$



H.W Q2 Answer.

$$= \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} \qquad \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$= \begin{vmatrix} 0 & a-b & a^2-bc-b^2+ac \\ 0 & b-c & b^2-c^2-ac+ab \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= \begin{vmatrix} a-b & a^2-b^2+ac-bc \\ b-c & b^2-c^2+ab-ac \end{vmatrix}$$

$$= (a-b)(b^2-c^2+a(b-c)) - (b-c)[a^2-b^2-c(a-b)]$$

$$= b^2(a-b) - c^2(a-b) - a(a-b)(b-c) - a^2(b-c) + b^2(b-c) + c(b-c)(a-b)$$

$$= (a-b)(b-c)(c-a) - b^2(c-a) - a^2(b-c) - b^2(a-b)$$

$$\boxed{=0}$$

Note :- special determinant values to remember.

① cyclic det

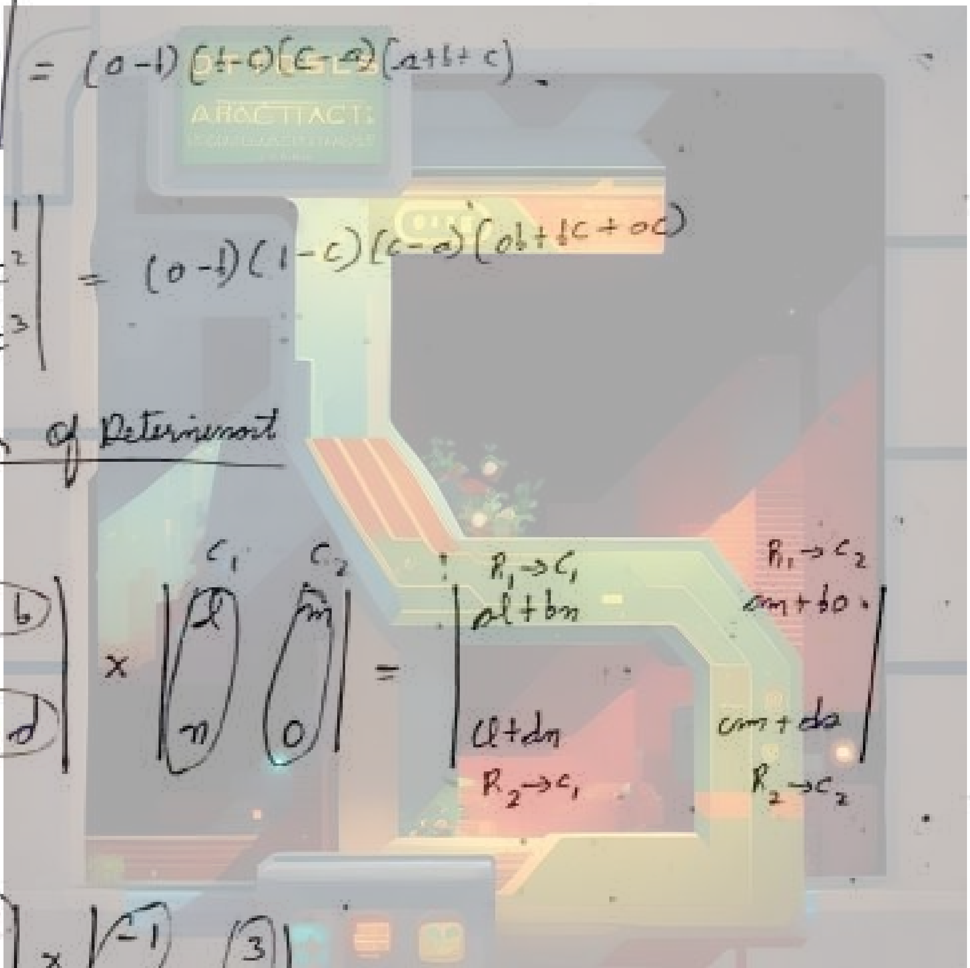
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$$

②

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

③

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$



④

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ac)$$

Multiplication of Determinant

2×2

$$\begin{array}{c} R_1 \\ \\ R_2 \end{array}\begin{vmatrix} a & b \\ c & d \end{vmatrix} \times \begin{array}{cc} C_1 & C_2 \\ \end{array}\begin{vmatrix} l & m \\ n & o \end{vmatrix} = \begin{vmatrix} \underset{R_1\to C_1}{al+bn} & \underset{R_1\to C_2}{am+bo} \\ \underset{R_2\to C_1}{cl+dn} & \underset{R_2\to C_2}{cm+do} \end{vmatrix}$$

Q

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \times \begin{vmatrix} -1 & 3 \\ 4 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} -1+8 & 3+4 \\ -3+16 & 9+8 \end{vmatrix} = \begin{vmatrix} 7 & 7 \\ 13 & 17 \end{vmatrix} = (17)(7)-(13)(7)$$

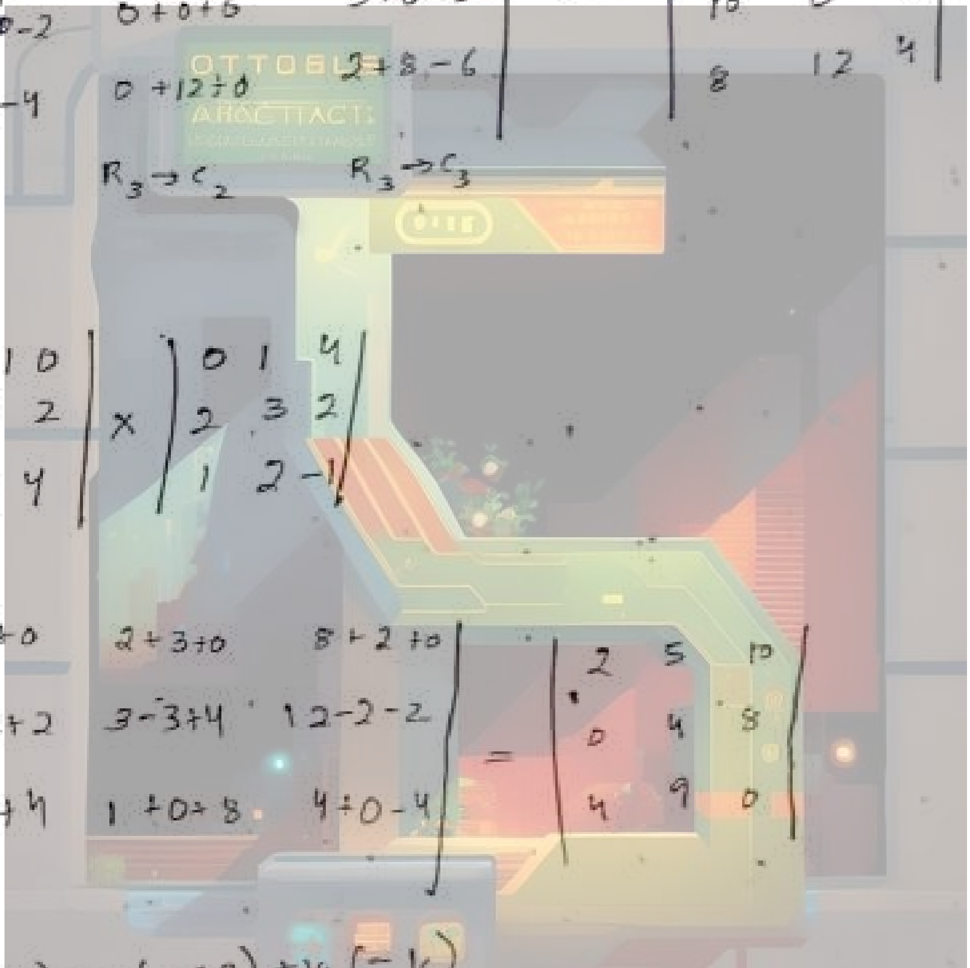
$$= (7)(4)$$

$$\boxed{=28}$$

3×3

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 0 & -1 \\ 2 & 4 & -2 \end{vmatrix} \times \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{vmatrix} 4 & 0 & 1 \\ 1 & 3 & 2 \\ 2 & 0 & 3 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} \overset{R_1\to C_1}{4+2+2} & \overset{R_1\to C_2}{0+6+0} & \overset{R_1\to C_3}{1+4+3} \\ \overset{R_2\to C_1}{12+0+0-2} & \overset{R_2\to C_2}{0+0+0} & \overset{R_2\to C_3}{3+0-3} \\ \underset{R_3\to C_1}{8+4-4} & \underset{R_3\to C_2}{0+12+0} & \underset{R_3\to C_3}{2+8-6} \end{vmatrix} = \begin{vmatrix} 8 & 6 & 8 \\ 10 & 0 & 0 \\ 8 & 12 & 4 \end{vmatrix}$$



Q H.W.

$$\begin{vmatrix} 2 & 1 & 0 \\ 3 & -1 & 2 \\ 1 & 0 & 4 \end{vmatrix} \times \begin{vmatrix} 0 & 1 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 0+2+0 & 2+3+0 & 8+2+0 \\ 0+-2+2 & 3-3+4 & 12-2-2 \\ 0+0+4 & 1+0+8 & 4+0-4 \end{vmatrix} = \begin{vmatrix} 2 & 5 & 10 \\ 0 & 4 & 8 \\ 4 & 9 & 0 \end{vmatrix}$$

$= 2(-72) - 5(-32) + 10(-16)$

$= -144 + 160 - 160$

$\cancel{-144}$ $\boxed{= -144}$

Q find both the determinants which gives the determinant by multiplication.

(1)
$$\begin{vmatrix} 1+1 & \alpha+\beta \\ \alpha+\beta & \alpha^2+\beta^2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ \alpha & \beta \end{vmatrix} \times \begin{vmatrix} 1 & \alpha \\ 1 & \beta \end{vmatrix}.$$

(2)
$$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

DYS-5 (Q5)

$$\begin{vmatrix} a^2+b^2+c^2 & bc+ac+ab & bc+ac+ab \\ bc+ac+ab & a^2+b^2+c^2 & bc+ac+ab \\ bc+ac+ab & bc+ac+ab & a^2+b^2+c^2 \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \times \begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

* System of linear equation

2 Variable

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

Parallel lines

$$\frac{a_1}{b_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

inconsistent

0 solⁿ

Coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Consistent

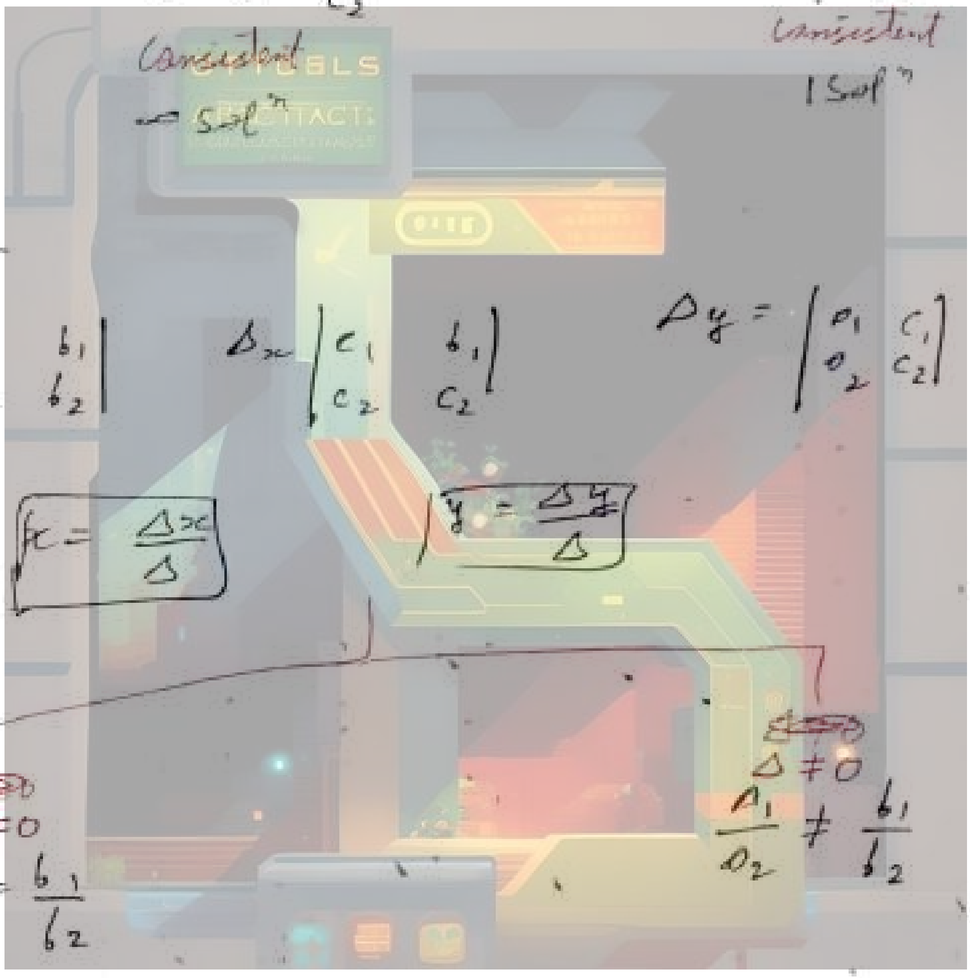
∞ sol^n

Intersect lines

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Consistent

1 sol^n



Proof :-

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & c_2 \end{vmatrix} \qquad \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$\boxed{x = \frac{\Delta_x}{\Delta}} \qquad \boxed{y = \frac{\Delta_y}{\Delta}}$$

$\Delta = 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$\Delta \neq 0$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

eg. $2x - 3y = 4$
$x + y = 3$

$$\Delta = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 5$$

$$\Delta_x = \begin{vmatrix} 4 & -3 \\ 3 & 1 \end{vmatrix} = 13$$

$$\Delta_y = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} = 2$$

$$x = \frac{\Delta_x}{\Delta} = \frac{13}{5}$$

$$y = \frac{\Delta_y}{y} = \frac{2}{5}$$

Proof:- (Δy)

$$(a_1x + b_1y = c_1) \times a_2$$
$$(a_2x + b_2y = c_2) \times a_1$$

$$a_1a_2x + a_2b_1y = a_2c_1 \quad — (1)$$
$$a_1a_2x + a_1b_2y = a_1c_2 \quad — (2)$$

$$(2) - (1)$$

$$(a_1b_2 - a_2b_1)y = a_1c_2 - a_2c_1$$
$$\Delta y = \Delta_y$$
$$\therefore \quad y = \frac{\Delta_y}{\Delta}$$

3 Variables

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \text{equation of planes}$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \qquad \Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

$$x = \frac{\Delta_x}{\Delta} \qquad y = \frac{\Delta_y}{y\Delta} \qquad z = \frac{\Delta_z}{z\Delta}$$

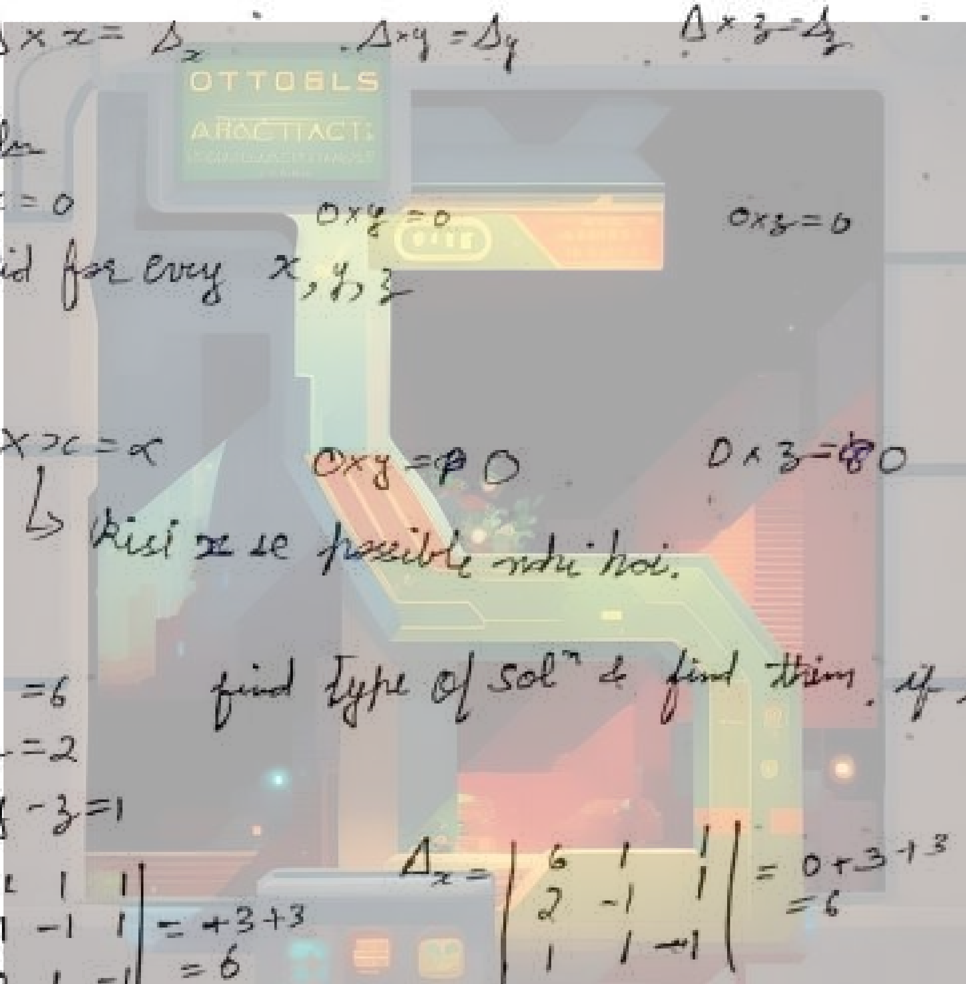
$\Delta \neq 0$ — 1 sol^n Consistent

$\Delta = 0$

- $\Delta_x = \Delta_y = \Delta_z = 0$ — ∞ sol^n — consistent
- At least one of $\Delta_x, \Delta_y, \Delta_z \neq 0$ — no sol^n — Inconsistent

Proof:-

$\Delta \times x = \Delta_x$ $\quad \Delta \times y = \Delta_y$ $\quad \Delta \times z = \Delta_z$



∞ soln

$0 \times x = 0$ $\quad 0 \times y = 0$ $\quad 0 \times z = 0$

valid for every x, y, z

no soln

$0 \times x = \alpha$ $\quad 0 \times y = \cancel{\phi}\, 0$ $\quad 0 \times z = \cancel{8}0$

↳ kisi x se possible nhi hoi.

Q $x + y + z = 6$ $\quad$ find type of sol^n & find them if exists.

$x - y + z = 2$

$2x + y - z = 1$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = +3+3 = 6 \qquad \Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 0+3+3 = 6$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} \qquad \Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$$

$\boxed{\Delta = 0,\ 1\ \text{sol}^n}$

$\boxed{x = 1}$ $\quad z + y = 5$

$z - y = 1$

$\boxed{z = 3}$ $\quad \boxed{y = 2}$

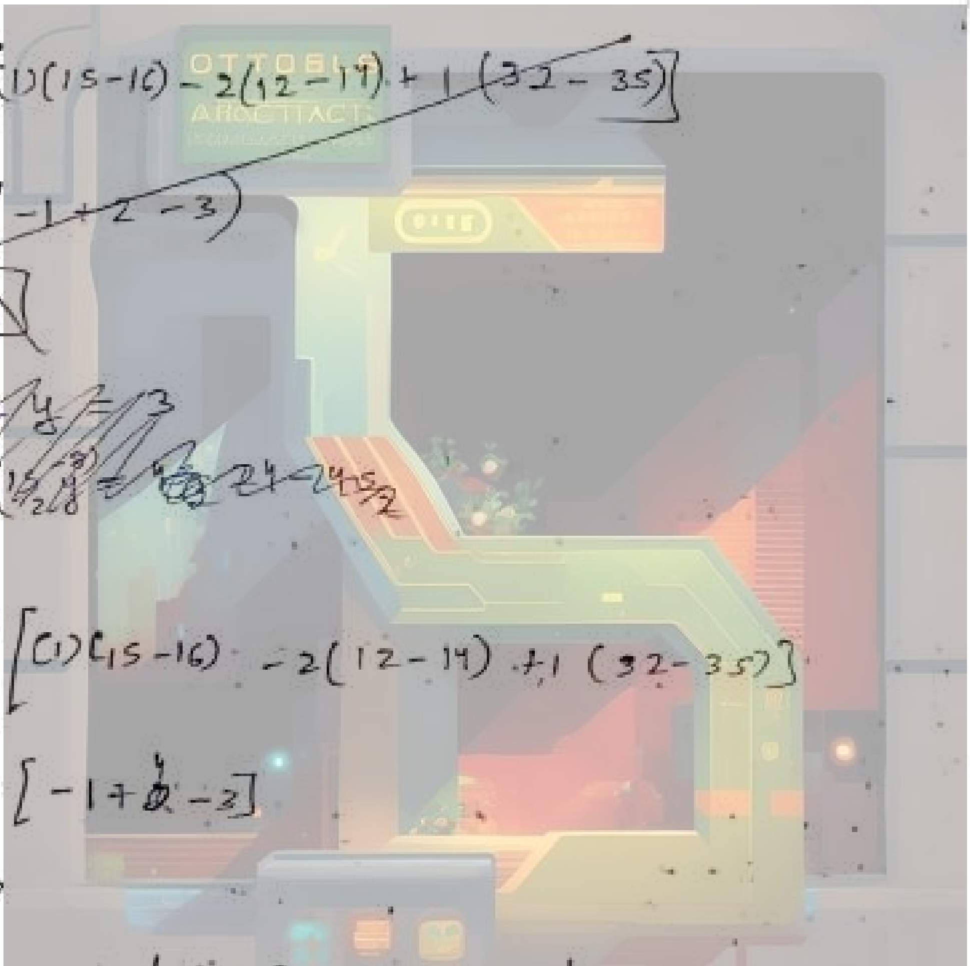
Q $x+2y+3z=6$

$4x+5y+6z=15$

$7x+8y+9z=24$

find type of solⁿ & find them if possible.

$$\Delta = \begin{vmatrix}1&2&3\\4&5&6\\7&8&9\end{vmatrix} = 3\begin{vmatrix}1&2&1\\4&5&2\\7&8&3\end{vmatrix}$$



$$\cancel{\Delta = 3\times\left[(1)(15-16)-2(12-14)+1(32-35)\right]}$$

$$\cancel{= 3(-1+2-3)}$$

$$\cancel{\boxed{\Delta=-6}}$$

$$\Delta = 3\left[(1)(15-16)-2(12-14)+1(32-35)\right]$$

$$\Delta = 3\left[-1+4-3\right]$$

$$\Delta = 0$$

$$\Delta_x = \begin{vmatrix}6&2&3\\15&5&6\\24&8&9\end{vmatrix} = 3\begin{vmatrix}2&2&3\\5&5&6\\8&8&9\end{vmatrix} = 0$$

$\Delta_x = 0$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 3 \\ 4 & 15 & 6 \\ 7 & 24 & 9 \end{vmatrix} = 3 \times 3 \begin{vmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 3 \end{vmatrix}$$

$$= 9\left[(1)(15-16) - 2(12-14) + 1(32-35)\right]$$

$$\boxed{=0}$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 6 \\ 4 & 5 & 15 \\ 7 & 8 & 24 \end{vmatrix} = 3 \begin{vmatrix} 1 & 2 & 2 \\ 4 & 5 & 5 \\ 7 & 8 & 8 \end{vmatrix}$$

$$= 0$$

$\Delta_z = 0$

$\Delta_x = \Delta_y = \Delta_z = 0$

$\boxed{\infty \text{ sol}^n}$

Put $x = \lambda$

put in ①

$\lambda + 2y + 3z = 6$

$2y + 3z = 6 - \lambda$

Put in ②

$5y + 6z = 15 - 4\lambda$

$$\boxed{\begin{aligned} y &= 3 - 2\lambda \\ z &= \lambda \\ x &= \lambda \end{aligned}} \quad \lambda \in I$$

Q $x + y + z = 3$

$x + 2y + 3z = 4$

$2x + 3y + 4z = 8$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = -1 + 2 - 1 = 0$$

$$\Delta_x = \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 3 \\ 8 & 3 & 4 \end{vmatrix} = 3(-1) - 1(-4) + 1(12-16)$$

$= -3 + 4 - 4$

$= -3$

$\boxed{\Delta = 0, \ \Delta_x \neq 0, \ \text{no sol}^n}$

Q $2x + Py + 6z = 8$ find P & Q if system of eqⁿ has
$x + 2y + Qz = 5$ ① no solⁿ ③ ∞ solⁿ
$x + y + 3z = 4$ ② Unique solⁿ

③ $\Delta = \begin{vmatrix} 2 & P & 6 \\ 1 & 2 & Q \\ 1 & 1 & 3 \end{vmatrix} = 0$

$2(6-Q) - P(3-Q) + 6(-1) = 0$

$12 - 2Q - 3P + PQ - 6 = 0$

$2Q + 3P - PQ = 6$ — ①

$\Delta = (P-2)(Q-3)$

$\Delta_y = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & Q \\ 1 & 4 & 3 \end{vmatrix} = 0$

$30 - 8Q - 24 + 8Q - 6 = 0$ — ③

$0 = 0$

$\Delta_y = 0$

∴ $P = 2$, in ①

$2Q + 6 - 2Q = 6$

$\Delta = 0$ ✓

$\Delta_x = \begin{vmatrix} 8 & P & 6 \\ 5 & 2 & Q \\ 4 & 1 & 3 \end{vmatrix} = 0$

$8(6-Q) - P(15-4Q) + 6(5-8) = 0$

$48 - 8Q - 15P + 4PQ + 30 - 48 = 0$

$8Q + 15P - 4PQ = 30$ — ②

$\Delta_x = (4Q-15)(P-2)$

$\Delta_z = \begin{vmatrix} 2 & P & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix}$

$\Delta_z = 6 + P - 8$

$\Delta_z = P - 2 = 0$ — ④

$P = 2$

$\Delta_z = (P-2)$

in ②

$8Q + 30 - 8Q = 30$ ✓

③ $\boxed{\text{∴ } P = 2, \infty \text{ sol}^n \quad Q \in R}$ iii)

② $\Delta \neq 0$

$P \neq 2,\ Q \neq 3$

$\boxed{P \in R - \{2\} \text{ and } Q \in R - \{3\}}$ ii)

① $\Delta = 0$

$P = 2$ or $Q = 3$

$\Delta_x, \Delta_y, \Delta_z \neq 0$

∴ $P = 2$, $\Delta_x = \Delta_y = \Delta_z = 0$

$\boxed{\text{∴ } Q = 3, P \in R - \{2\} \quad 0 \text{ sol}^n}$ i)

Q $2x - y + 2z = 2$ find λ for no solⁿ

$x - 2y + z = -4$

$x + y + \lambda z = 4$

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$2(-2\lambda - 1) - 1(\overset{\lambda - 1}{\cancel{1+2}}) + 2(1+2) = 0$$

$$-4\lambda - 2 - \lambda + 1 + 6 = 0$$

$$-5\lambda + 5 = 0$$

$$\boxed{\lambda = 1}$$

$$\Delta_x = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

$$= 2(-3) - 1(-8) + 2(-12)$$

$$= -6 + 8 - 24$$

$$\Delta_x \neq 0$$

$$\boxed{\lambda = 1}$$

Homogeneous equ in 3 variable

$$\left.\begin{aligned} a_1x + b_1y + c_1z = 0 \\ a_2x + b_2y + c_2z = 0 \\ a_3x + b_3y + c_3z = 0 \end{aligned}\right\}$$ 3 Homogeneous equ

$x=0, y=0, z=0$ must be a solⁿ

Δ

$\Delta \neq 0$ Trivial Solⁿ | $\Delta = 0$ non-trivial solⁿ

<u>Trivial Solⁿ</u> - Solutions where <u>all the values</u> of the variable is 0.

eg. (0, 0, 1) Non-Trivial
(0, 0, 0) Trivial

Proof:- $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$

~~$\Delta \neq 0$~~

$\Delta \times x = 0$ $\quad \Delta \times y = 0$ $\quad \Delta \times z = 0$

If $\Delta = 0$; $0 = 0$ so, we can take any x, y & z

If $\Delta \neq 0$;
let $\Delta = 2$

$2x = 0$ $\quad 2y = 0$ $\quad 2z = 0$

<u>Trivial Solⁿ</u>

Q $(1-\alpha)x + 6y + 6z = 0$

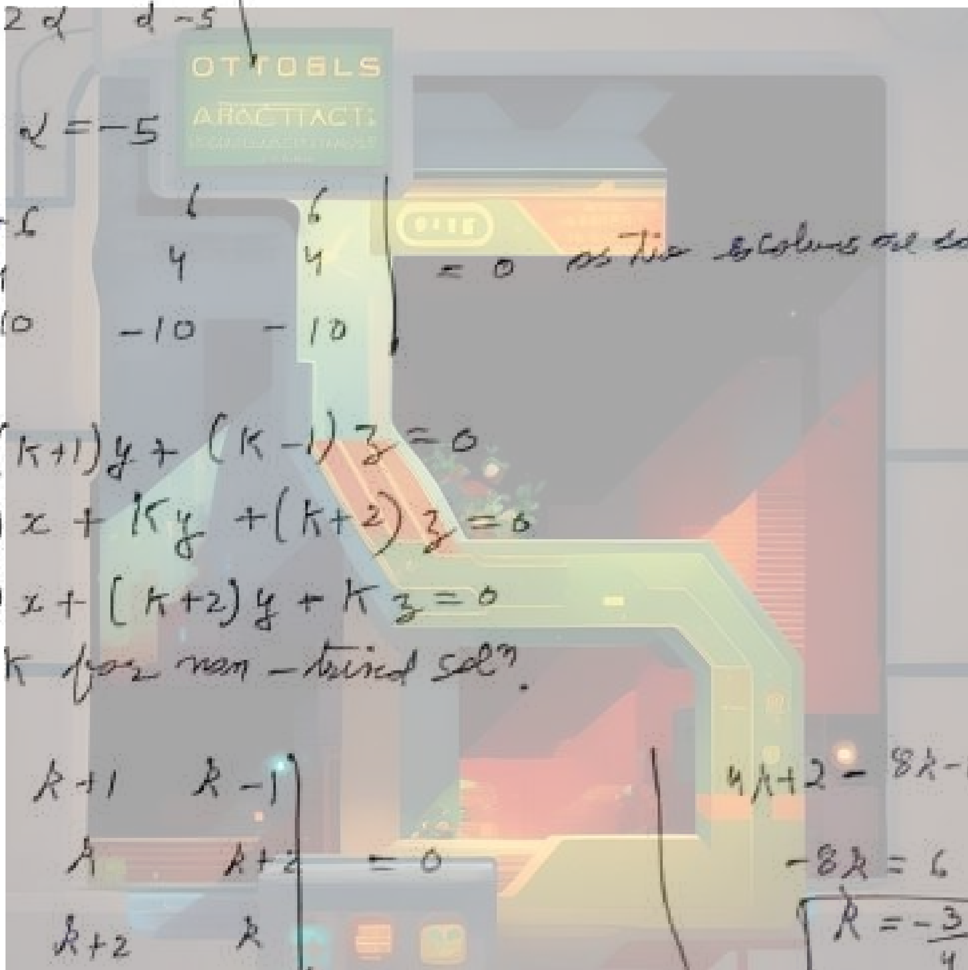
$4x - (\alpha+1)y + 4z = 0$

$2\alpha x + 2\alpha y + (\alpha-5)z = 0$

has non trivial sol^n find α

$\Delta = 0$

$$\begin{vmatrix} 1-\alpha & 6 & 6 \\ 4 & -(\alpha+1) & 4 \\ 2\alpha & 2\alpha & \alpha-5 \end{vmatrix} = 0$$



at $\alpha = -5$

$$\begin{vmatrix} 6 & 6 & 6 \\ 4 & 4 & 4 \\ -10 & -10 & -10 \end{vmatrix} = 0$$ as the 3 columns are same.

Q $kx + (k+1)y + (k-1)z = 0$

$(k+1)x + ky + (k+2)z = 0$

$(k-1)x + (k+2)y + kz = 0$

find k for non-trivial sol^n.

$$\begin{vmatrix} k & k+1 & k-1 \\ k+1 & k & k+2 \\ k-1 & k+2 & k \end{vmatrix} = 0$$

$4k+2 - 8k - 12 - 4k + 4 = 0$

$-8k = 6$

$\boxed{k = -\frac{3}{4}}$

$R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} -1 & 2 & k-1 \\ 1 & -2 & k+2 \\ -3 & 2 & k \end{vmatrix} = 0$$


OTTOBLS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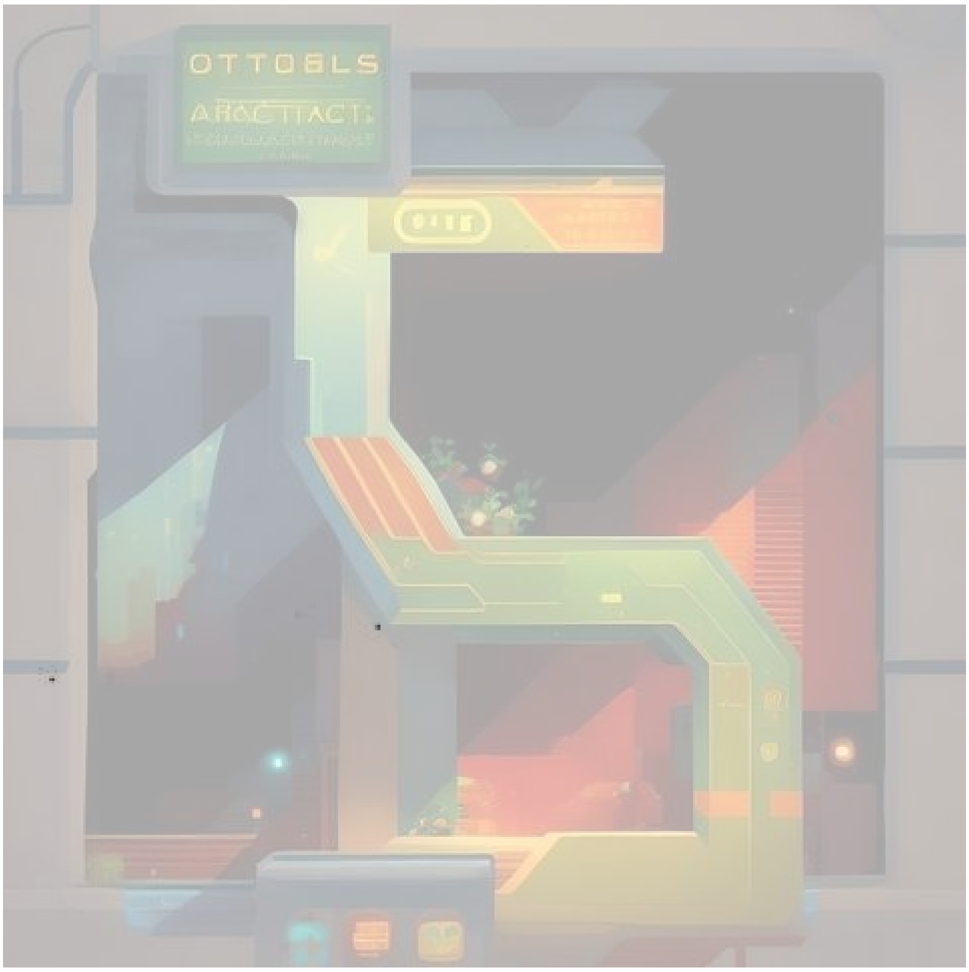
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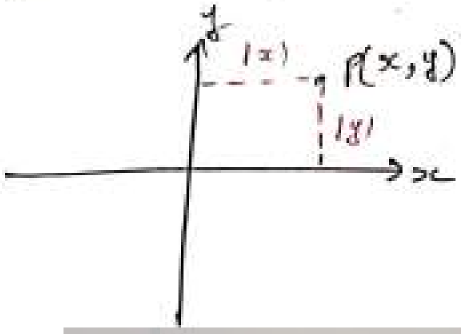
! Co-ordinate Geometry !

→ Algebra + Geometry = Co-ordinate Geometry



Dis. P from x-axis $= |y|$
Dis. P from y-axis $= |x|$

~~Dis P(x,y) & Q(x~~

Dis btw $P(x_1, y_1)$ & $Q(x_2, y_2)$ $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

Q find x if Dis btw $(x, 0)$ & $(7, 0) = 5$

$(49 + x^2 - 14x) = 5^2$

$\cancel{x^2 - 14x + 44 = 0}$

$\cancel{x = 7 \pm \sqrt{49 -}}$

$\cancel{x = \frac{14 \pm \sqrt{196-176}}{2}}$

$\cancel{x = \frac{14 \pm 2\sqrt{5}}{2}}$

$\cancel{\boxed{x = 7 \pm \sqrt{5}}}$

$\cancel{\boxed{x = 2, 12}}$

$x^2 - 14x + 22 = 0$

$x = \frac{14 \pm \sqrt{196 - 88}}{2}$

$x = \frac{14 \pm 10}{2}$

$\boxed{x = 2, 12}$

Q find the point where x & y are equal & which is equidistant from $(1,0)$ & $(0,3)$

(x, x)

$$(x-1)^2 + (x-0)^2 = (x-0)^2 + (x-3)^2$$

$$x^2+1-2x+x^2 = x^2+x^2+9-6x$$

$$1-2x = 9-6x$$

$$4x = 8$$

$$\boxed{x=2}$$

$$\boxed{(2, 2)}$$

Division of A Line Segment.

① Section formula

A) Internal Division

$m_1 : m_2$

$A(x_1, y_1)$ $P(x, y)$ $B(x_2, y_2)$

$$\boxed{P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)}$$

B) External Division

$m_1 : m_2$

$A(x_1, y_1)$ $B(x_2, y_2)$ $(P)(x, y)$

$$\boxed{P(x,y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)}$$

β If ratio is asked, we use $\lambda : 1$
If λ is $\oplus$ve then internal division
λ is $\ominus$ve then external division

Q find ratio in which x axis divides line joining $(2,-3)$ & $(5,6)$

$\lambda : 1$

$$y = 0 = \frac{6\lambda - 3}{\lambda + 1}$$

$$6\lambda - 3 = 0$$

$$\lambda = \frac{1}{2}$$

Trick - 2

$(2,-3)$ $(x,0)$ $(5,6)$
x_1, y_1 x, y x_2, y_2

ratio:

$$\boxed{y_1 - y : y - y_2}$$

$$\frac{1}{2} : 1$$

$\boxed{1:2}$ Internally

Q ② Sec. Midpoint formula

$m : n = 1 : 1$

$1 : 1$

$A(x_1, y_1)$ $P(x, y)$ $B(x_2, y_2)$

$$P(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

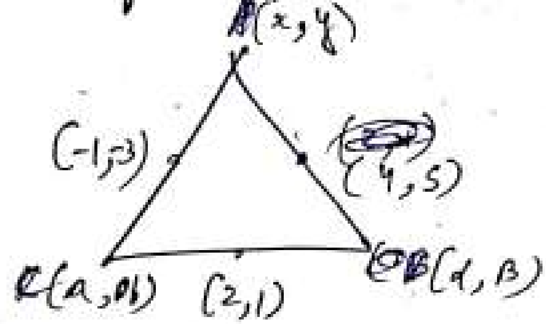
Q If $(2,3)$ is the mid point of line segment of $A(2,9)$ & $B(\alpha, \beta)$ then find α & β

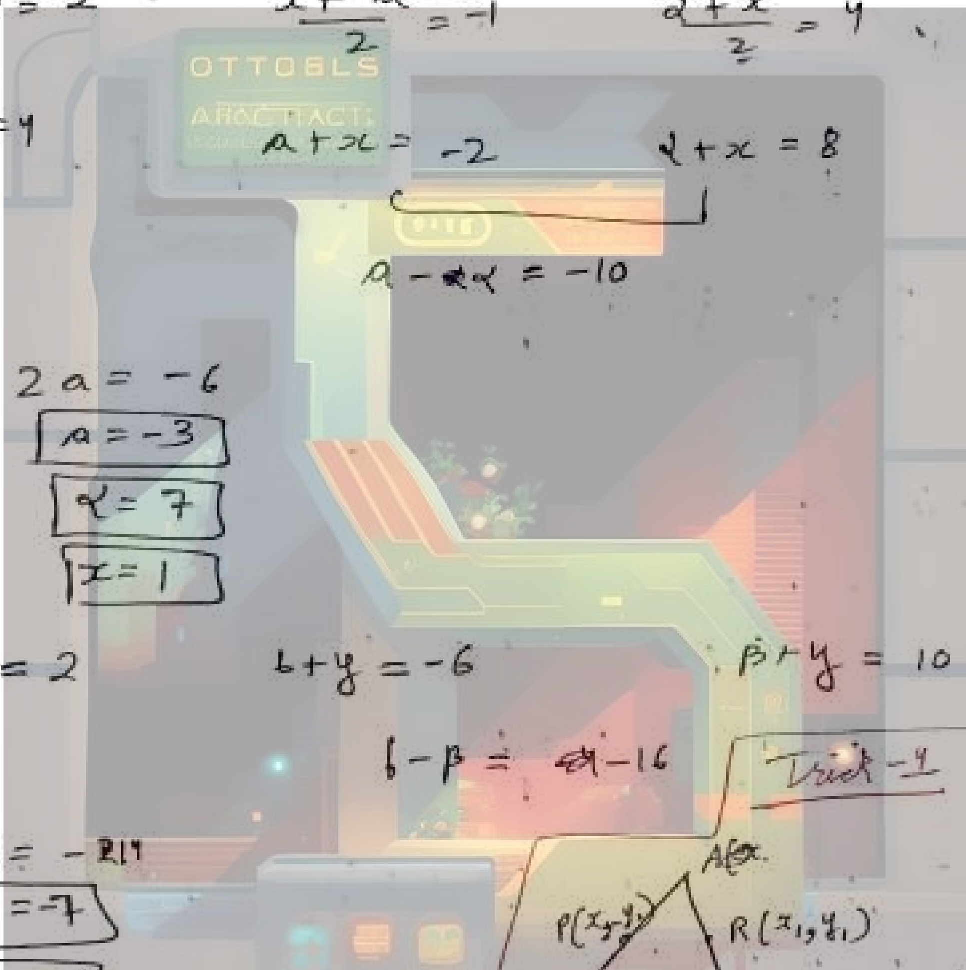
$$\frac{\alpha + 2}{2} = 2 \qquad \frac{\beta + 9}{2} = 3$$

$$\alpha + 2 = 4 \qquad \beta + 9 = 6$$

$$\boxed{\alpha = 2} \qquad \boxed{\beta = -3}$$

Q The midpoints of sides of Triangle are $(2,1)$ $(-1,-3)$ & $(4,5)$ find vertices of triangle.





$\frac{a+\alpha}{2} = 2$ $\quad$ $\frac{x+a}{2} = -1$ $\quad$ $\frac{\alpha+x}{2} = 4$

$a+\alpha = 4$ $\quad$ $a+x = -2$ $\quad$ $\alpha+x = 8$

$a - \alpha = -10$

$2a = -6$

$\boxed{a=-3}$

$\boxed{\alpha = 7}$

$\boxed{x=1}$

$b+\beta = 2$ $\quad$ $b+y=-6$ $\quad$ $\beta+y = 10$

$b-\beta = -16$

$2b = -14$

$\boxed{b=-7}$

$\boxed{\beta = 9}$

$\boxed{y=1}$

$\boxed{B(-3,-7),\ C(1,1)\ \&\ A(7,9)}$

Trick-4

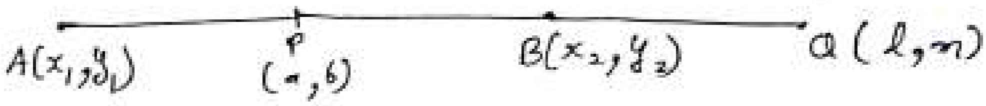
$A_x = P_x + R_x - Q_x$

$B_x = P_x + Q_x - R_x$

$C_x = R_x + Q_x - P_x$

Harmonic Conjugate

→ If P & Q are two points which divides the line segment AB internally & Externally in the same ratio $(m:n)$ then P & Q are Harmonic conjugate of each other with respect to A & B.



$\frac{AP}{PB} = \frac{m}{n}$ $\quad \frac{AQ}{BQ} = \frac{m}{n}$

P & Q are Harmonic conjugate wrt A & B.

→ AP, AQ & AB are in Harmonic series.

Proof:-

$$\frac{AP}{PB} = \frac{AQ}{BQ}$$

$$\frac{PB}{AP} = \frac{BQ}{AQ}$$

$$\frac{AB-AP}{AP} = \frac{AQ-AB}{AQ}$$

$$\frac{AB}{AP} - 1 = 1 - \frac{AB}{AQ}$$

$$\frac{AB}{AP} + \frac{AB}{AQ} = 2$$

$$\frac{1}{AP} + \frac{1}{AQ} = \frac{2}{AB}$$

Q3. Determine the ratio in which the point $P(3,5)$ divide the join of $A(1,3)$ & $B(7,9)$ find harmonic conjugate of P wrt AB.

$P\cancel{(x,y)}$ $\frac{AP}{BP} = \frac{\lambda}{1}$ (let division is internal)

$$\frac{7\lambda+1}{\lambda+1} = 3$$

$$7\lambda+1 = 3\lambda+3$$

$$4\lambda = 2$$

$\lambda = 1/2$ (Hence, internal division is True)

$$\frac{AP}{BP} = \frac{1}{2}$$

$$Q\cancel{B} = \left(\frac{7-2}{1-2}, \frac{9-6}{1-2}\right)$$

$$\boxed{Q(-5,-3)}$$

Q Find Harmonic conjugate of $R(2,4)$ wrt $P(2,2)$ & $Q(\cancel{3}2,5)$

$\lambda:1$

$$\frac{5\lambda+2}{\lambda+1} = 4$$

$5\lambda+2 = 4\lambda+4$

$\lambda = 2$

$2:1$

$$S = \left(\frac{2-4}{2-1}, \frac{2-10}{2-1}\right)$$

$$\boxed{S(-2,-8)}$$

Quadrilaterals :-

Quadrilateral
- Parallelogram
  - Rectangle
  - Rhombus
    - Square (from Rectangle and Rhombus)
- Trapezium
- Kite
- Cyclic Quadri.

→ Square is a special case of Rectangle with all sides equal.

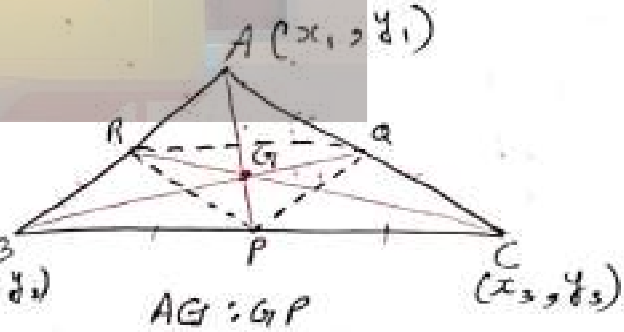
Diagonals & side based property :-

Parallelogram	opp sides equal	Diagonals Bisect
Rectangle	opp sides equal	Diagonals bisect & equal in length
Square	all sides equal	Diagonals bisect ~~at~~ at ⊥ & equal length.
Rhombus	all sides equal	Diagonals ⊥.

~~Points of a Triangle & :~~

Triangle (Points & Co-ordinates)

① Centroid (G) -



→ AP, BQ & CR are medians.

→ $\frac{AG}{GP} = \frac{BG}{GQ} = \frac{CG}{GR} = \frac{2}{1}$

AG : GP
2 : 1

→ $Area(BGC) = area(BGA) = area(\triangle CGA) = \frac{1}{3} area(\triangle ABC)$

→ $Area(PQR) = \frac{Area(\triangle ABC)}{4}$

→ $Ar(\triangle ABP) = Ar(\triangle ACP)$

$\rightarrow AP^2 + CR^2 + BQ^2 = \frac{3}{4}(a^2+b^2+c^2)$

$\rightarrow$ Centroid always lies inside triangle.

Ordinates of Centroid

$$\boxed{G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)}$$

Proof:-

1 : 1

$B(x_2, y_2)$ —— P —— $C(x_3, y_3)$

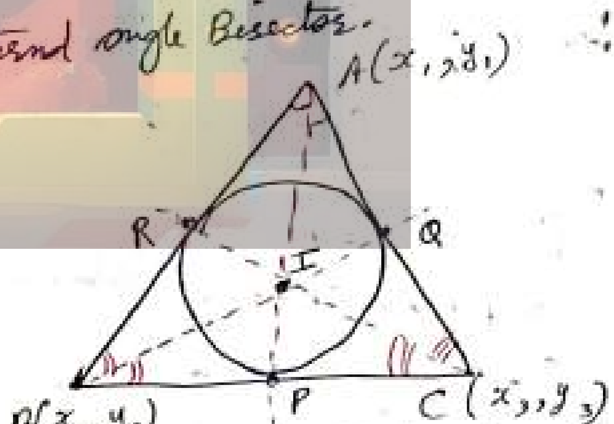
$$P = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

2 : 1

$A(x_1, y_1)$ —— G —— $P\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

$$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$ HP

(2) Incenter (I) $\rightarrow$ Intersection of Internal angle Bisectors.



$\rightarrow AP, BQ$ & CR are Angle Bisectors.

$\rightarrow \frac{BP}{PC} = \frac{c}{b}$, $\frac{AR}{RB} = \frac{b}{a}$, $\frac{CQ}{QA} = \frac{a}{c}$

$\rightarrow \frac{AI}{IP} = \frac{b+c}{a}$, $\frac{BI}{IQ} = \frac{c+a}{b}$

$\frac{CI}{IR} = \frac{a+b}{c}$

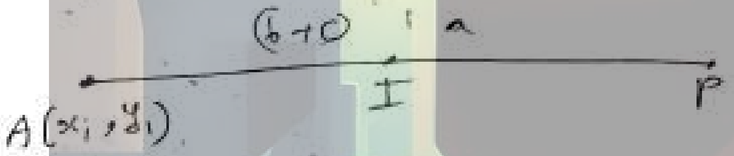
→ Incenter always lies inside a triangle.

Cordinates of Incenter

$$\boxed{I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)}$$

Proof:-

$$P = \left(\frac{cx_3+bx_2}{c+b}, \frac{cy_3+by_2}{c+b}\right)$$

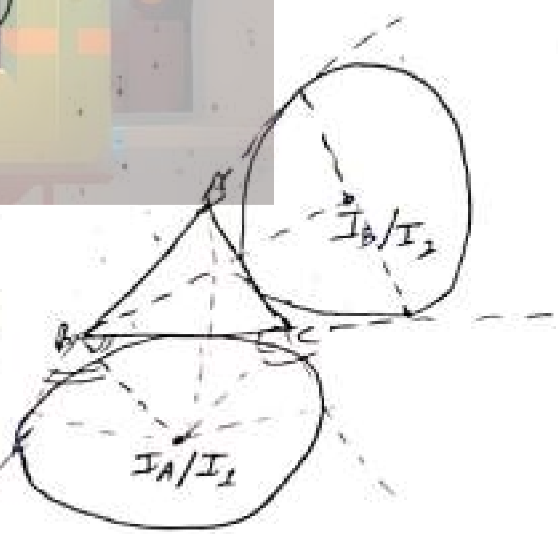


$$I = \left(\frac{(b+c)\frac{(cx_3+bx_2)}{b+c} + ax_1}{a+(b+c)}, \frac{(b+c)\frac{(cy_3+by_2)}{(b+c)} + ay_1}{a+(b+c)}\right)$$

$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

HP

Note:- Ex-center is related to Incenter



$$\boxed{I_A = \left(\frac{bx_2+cx_3-ax_1}{b+c-a}, \frac{by_2+cy_3-ay_1}{b+c-a}\right)}$$

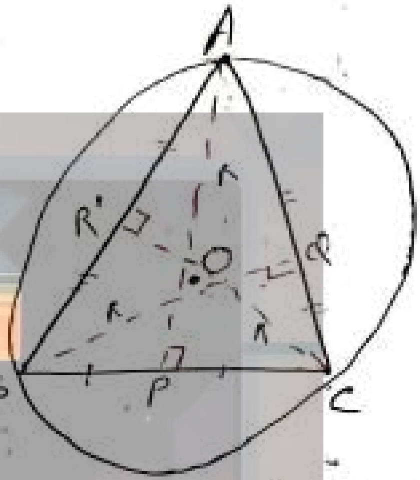
→ One Internal angle Bisector &
2 External angle bisector.

→ always outside the triangle.

$$I_B \equiv \left(\frac{ax_1 + cx_3 - bx_2}{a+c-b}, \frac{ay_1 + cy_3 - by_2}{a+c-b}\right)$$

$$I_c \equiv \left(\frac{ax_1 + bx_2 - cx_3}{a+b-c}, \frac{ay_1 + by_2 - cy_3}{a+b-c}\right)$$

(3) Circumcenter (O)- Intersection of ⊥ bisectors



→ ~~AP, BQ & CR are circumcenter~~

→ AP, BQ & CR' are ⊥ side bisectors.

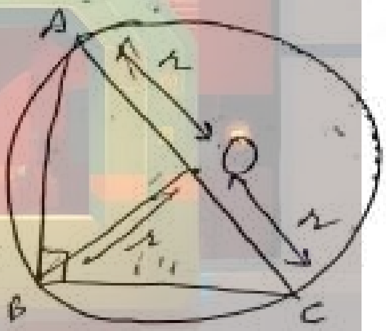
→ OA, OB & OC are circumRadius (R)

→ May or not go through the vertex.

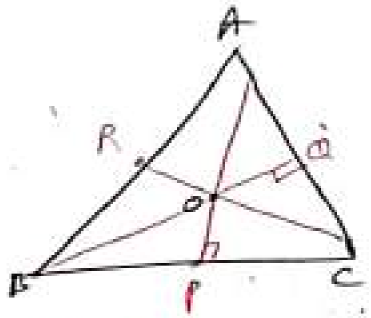
→ $R = \frac{abc}{4\Delta}$

→ For finding the circumcenter co-ordinates, we will solve any 2 equations of ⊥ side Bisectors.

Note:- for Right Angle Triangle, circumcenter is the mid point of Hypotenuse.



→ In Acute Angle Triangle it lies inside, For Right Angle Δ, It lies on Triangle and for Obtuse angle Triangle, It lies outside the triangle.

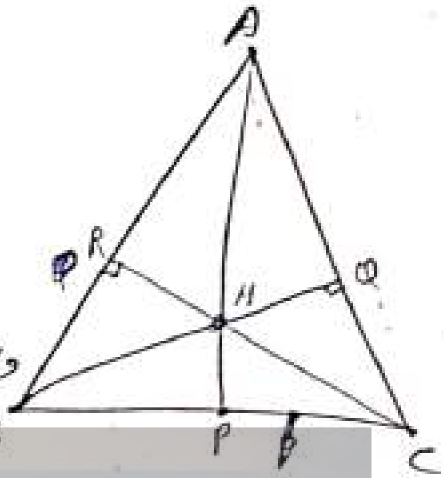


OP, OQ & OR are ⊥ bisector

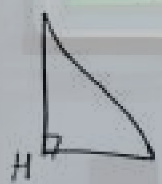
(4) Orthocenter (H) - Intersection of Altitude ($\perp$)

→ AP, BQ & CR are Altitudes

→ For finding the orthocenter, we will solve equation of any two altitude.



Note:- For A Right Angle Triangle, Orthocenter is the vertex where 90° angle lies.



→ For any acute Angle, orthocenter lies inside the triangle.
any Right Angle Triangle, orthocenter lies on vertex (90°)
any obtuse Angle Triangle, orthocenter lies outside the triangle.

Q. Two vertices of a $\triangle$ are $(-1, 4)$ & $(5, 2)$ If its Centroid is $(0, -3)$ find 3rd vertex.

$$\frac{x_1 + x_2 + x_3}{3} = 0$$

$$\frac{-1 + 5 + x_3}{3} = 0$$

$$x_3 = -4$$

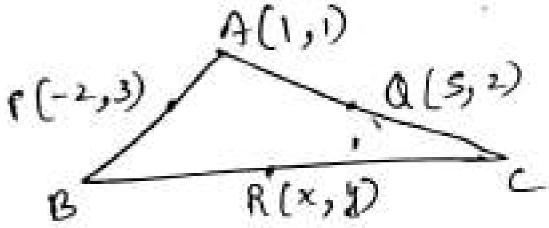
$$4 + 2 + y_3 = -3$$

$$4 + 2 + y_3 = -9$$

$$y_3 = -15$$

$$\boxed{(-4, -15)}$$

Q



If P, Q & R are mid points find.
Ans i) Vertices B & C
ii) Centroid of Δ

$\cancel{B = \left(\frac{x+1}{2}, \frac{y+1}{2}\right)}$

B

$\frac{x_B+1}{2} = -2$

$x_B + 1 = -4$

$x_B = -5$

$\frac{y_B+1}{2} = 3$

$y_B + 1 = 6$

$y_B = 5$

$\boxed{B \equiv (-5, 5)}$

$\frac{x_C+1}{2} = 5$

$x_C = 9$

$\frac{y_C+1}{2} = 2$

$y_C + 1 = 4$

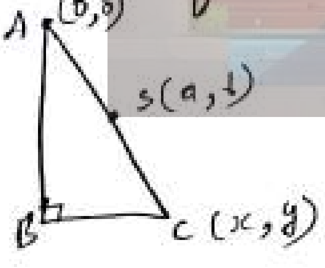
$y_C = 3$

$\boxed{C(9, 3)}$

ii) $G = \left(\frac{-5+1+9}{3}, \frac{1+3+5}{3}\right)$

$\boxed{G = \left(\frac{5}{3}, 3\right)}$

Q The orthocenter of Δ ABC is B & the circumcenter is S(a,b) if A is the origin. Find co-ordinates of C.



Ans $\frac{x+0}{2} = a$

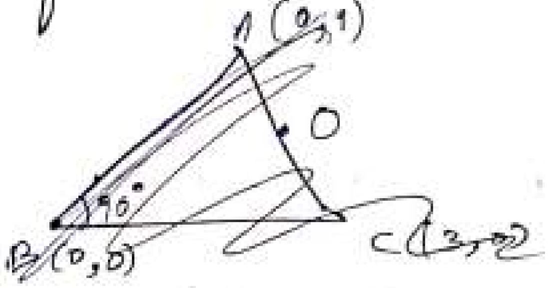
$x = 2a$

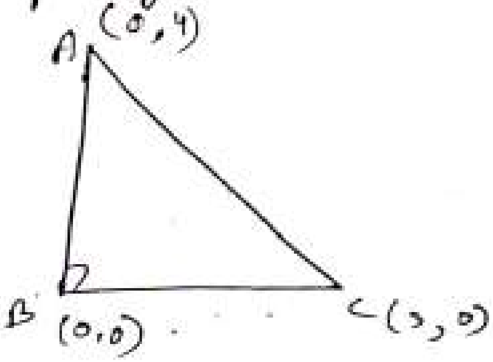
$\frac{y+0}{2} = b$

$y = 2b$

$\boxed{C(2a, 2b)}$

Q triangle with one vertex (0, 0) & sides 3, 4 cm ⊕ x & y axes. find circumcentre & orthocentre of Triangle

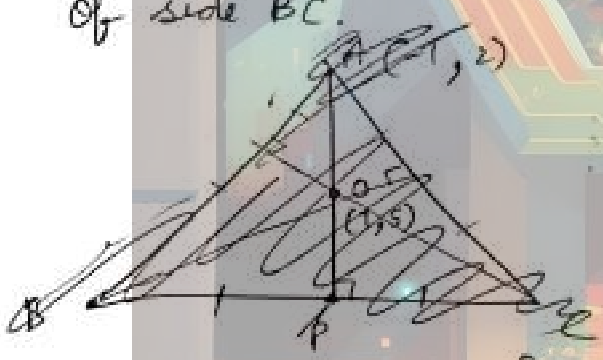


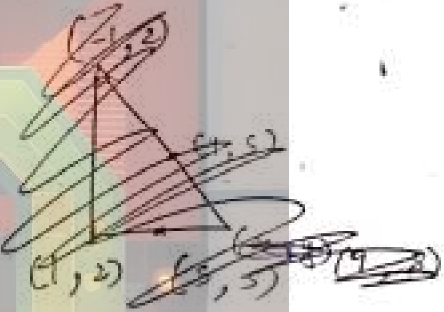


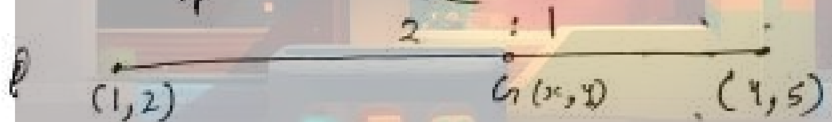
$\boxed{O = \left(\frac{3}{2}, 2\right)}$

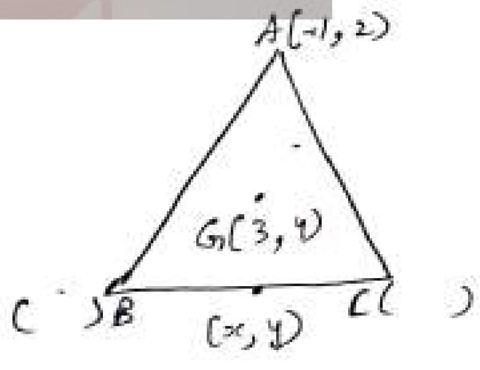
$\boxed{H = (0, 0)}$

Q orthocentre & circumcentre of a ΔABC are (1, 2) & (4, 5). If the co-ordinates of the vertex A are (−1, 2) find middle point of side BC.









$1 - x = 2x - 8$

$\boxed{x = 3}$

$2 - y = 2y - 10$

$\boxed{4 = y}$

$\frac{-1-3}{3-x} = 2$

$-4 = 6 - 2x$

$\boxed{x = 5}$

$-2 = 8 - 2y$

$\boxed{y = 3}$

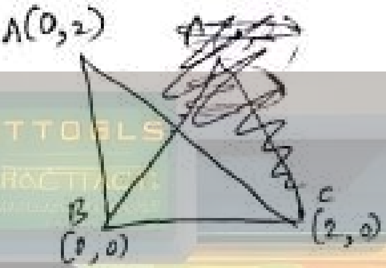
$\boxed{(5, 3)}$

H.W. 13-01-24

O-3 J-A [Determinants]

Q1 Find area of $\triangle$ formed by origin & POI of line $\frac{x}{6}+\frac{y}{8}=1$ & both axes.

Q2. If (0,0) (0,2) (2,0) are vertices of a $\triangle$ then, find its incenter.



An2. $\cancel{I \equiv (}$

$a = 2$

$c = 2$

$b = 2\sqrt{2}$

$I \equiv \left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$

$\boxed{I \equiv \left(\frac{2}{\sqrt{2}+2}, \frac{2}{\sqrt{2}+2}\right)}$

A1. at x axis, $(x, 0)$ $(6, 0)$

at y axis $(0, 8)$

$(0, 0)$

area $= \frac{1}{2} \times 6 \times 8$

$\boxed{= 24}$

Concurrency of 3 Lines :-

→ when all the given lines passes through a single point then the lines are called concurrent lines.

→ Solve any 2 equations of lines and put values of x & y in 3rd equation.

→ If it satisfies then the lines are concurrent.

Method - 2.

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
$$a_3x + b_3y + c_3 = 0$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$ then lines are concurrent.

Q If the lines $y - x = 5$, $3x + 4y = 1$ & $y = mx + 3$ are concurrent find m.

$$-x + y - 5 = 0$$
$$3x + 4y - 1 = 0$$
$$mx - y + 3 = 0$$

$$\begin{vmatrix} -1 & 1 & -5 \\ 3 & 4 & -1 \\ m & -1 & 3 \end{vmatrix} = 0$$

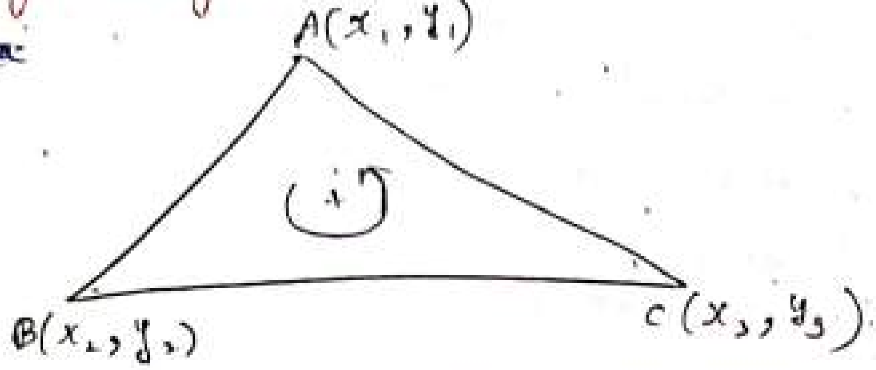
$$-11 - 9 - m + 15 + \overset{20}{4m} = 0$$

$$19m = 5$$

$$\boxed{m = \frac{5}{19}}$$

Area of Triangle

~~Area~~



$$\boxed{\text{Area} = \left|\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]\right|}$$

Coordinates :- Anticlockwise :- Area = +ve
Clockwise :- Area = −ve

Q find area of triangle of vertices. $(1,-1)\ (-1,1)\ (-1,-1)$

$$\text{area} = \frac{1}{2}\left[1(2)+(-1)(0)+(-1)(-2)\right]$$

$$= \frac{1}{2}\left[2+2\right]$$

$$\boxed{=2}$$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

→ mod

→ Det

200

Trick - 3 ~~Δ~~ R

$(1,-1), (-1,1), (-1,-1)$ $(1,-1)$

Subtract ~~both~~ ~~same~~ $(1,-1)$ from each

$(0,0)$ $(-2,2)$ $(-2,0)$

$x_1\ y_1$ $x_2\ y_2$

$$\boxed{\text{Area} = \frac{1}{2}|x_1 y_2 - x_2 y_1|}$$

Q find area of Δ taken vertices $A(4,4)$ $B(3,-2)$ $C(-3,16)$

$A(0,0)$ $B(-1,-6)$ $C(-7,12)$

$$\text{Area} = \frac{1}{2}\left|(-1)(12) - (-6)(-7)\right|$$

$$\frac{1}{2}\left|-12 - 42\right|$$

$$\boxed{= 27}$$

Note: → If Area of Δ Triangle ABC $= 0$ then, vertices A, B & C lies on a same line Hence they are co-linear.

Q prove that the points $A(a, b+c)$ $B(b, c+a)$ & $C(c, a+b)$ are co-linear.

area $= 0$

$$R_1 \to R_1 + R_2$$

$$\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix}$$

$= 0$

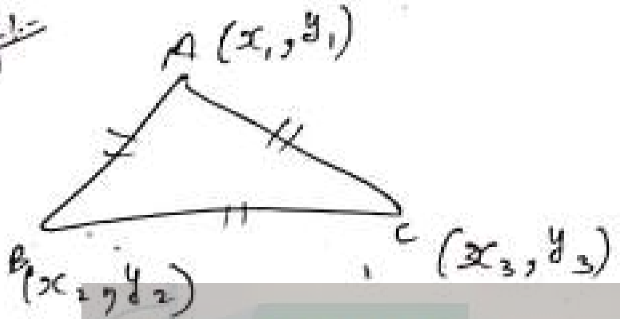
Thus, area $= 0$

HP

Type of Triangle (Trick)

→ (1) In an equilateral Triangle, co-ordinates of all the vertices can not be rational.

Proof:-



(1) $Ar = \frac{\sqrt{3}a^2}{4} = \underbrace{\frac{\sqrt{3}}{4}}_{\text{Irrational}}\underbrace{\left(\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right)^2}_{\text{Rational}}$

Area → Irrational

(2) Area $= \left|\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]\right|$

= rational

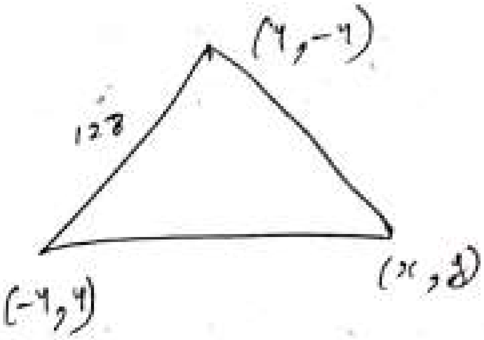
So, assumption is wrong.

Q If the points $(4,-4)$, $(-4,4)$ & (x,y) forms a equilateral $\triangle$, find x & y.

$64+64 = (4-x)^2+(y+4)^2$

$128 = 16+x^2-8x+y^2+16-8y$

$96 = x^2+y^2-8x+8y$



$$128 = (x+4)^2 + (y-4)^2$$

$$128 = 16 + 16 + x^2 + y^2 + 8x - 8y$$

$$x^2 + y^2 + 8x - 8y = x^2 + y^2 - 8x + 8y$$

$$16x = 16y$$

$$x = y$$

$$96 = x^2 + x^2 + 8x - 8x$$

$$96 = 2x^2$$

$$x^2 = 48$$

$$x = \pm\sqrt{48}$$

$$x = \pm 4\sqrt{3}$$

$$y = \pm 4\sqrt{3}$$

$(4\sqrt{3}, 4\sqrt{3})$ or $(-4\sqrt{3}, -4\sqrt{3})$

Trick-4

→ To find 3rd vertex, two given (equilateral)

$$\left(\frac{x_1 + x_2 \pm \sqrt{3}(y_2 - y_1)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_1 - x_2)}{2} \right)$$

→ Q No. of Points Having co-ordinates are Integers that lies in the interior of a triangle with vertices $(0,0)$ $(0,n)$ $(n,0)$ are $\boxed{\frac{(n-1)(n-2)}{2}}$

Q. find the no. of integral points which lies inside a triangle with vertices $(0,0)$, $(0,6)$ & $(6,0)$

$\boxed{= 10}$

Q find the area of triangle whose mid Points of the vertices are. $(0,0)$ $(3,0)$ & $(0,4)$

~~$(6,0)$ & $(0,8)$~~

~~Area $= \frac{1}{2}|48-0| - 0.48 - 24$~~

$\text{Area} = \frac{1}{2}|3\times4-0| = 6$ (area by mid Point)

$\text{Area}(\Delta) = 4\times6$

$\boxed{=24}$

Q If area of Δ is 5 & 2 vertices are $(2,1)$ & $(3,-2)$ & the third vertex lies on line $y = x+3$ then find cordinates of 3rd vertex.

$(2-x, 1-y)$ $(3-x, -2-y)$

$10 = +(2-x)(2+y) + (1-y)(3-x)$

$10 = 4+2y-2x-xy+3-3x-3y+xy$

$10 = |7-5x-y|$

$3 = |-5x-y|$

$-5x-y = 3$

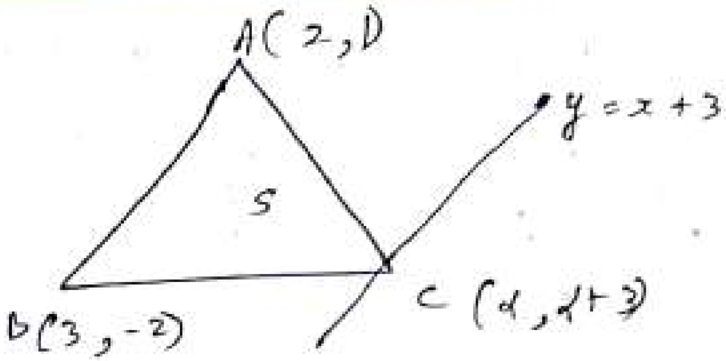
$+y-x = +3$

$x = 0$

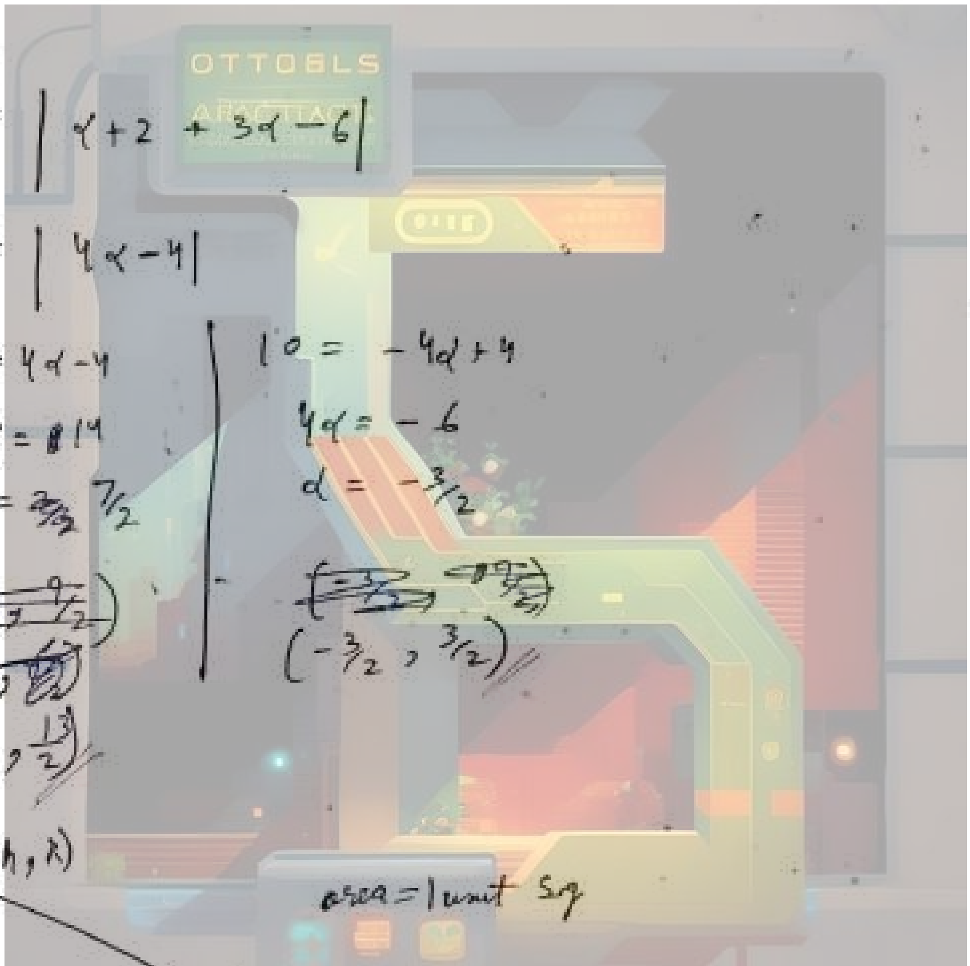
$y = 3$

$-6x = 6$

$x = -1$



$(0,0)\ (1,-3)\ (\alpha-2, \alpha+2)$



$$10 = |\alpha + 2 + 3\alpha - 6|$$

$$10 = |4\alpha - 4|$$

$10 = 4\alpha - 4$

$4\alpha = 14$

$\alpha = \cancel{3/3}\ 7/2$

$\cancel{(-3/2, -9/2)}$

$\cancel{(7/2, \ldots)}$

$(7/2, 13/2)$

$10 = -4\alpha + 4$

$4\alpha = -6$

$\alpha = -3/2$

$\cancel{(-3/2, -9/2)}$

$(-3/2, 3/2)$

Q

area = 1 unit sq

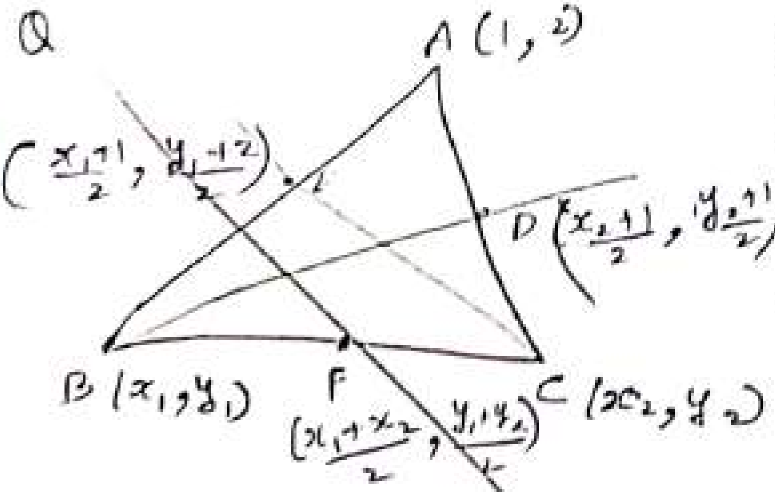
$$\frac{1}{2} \times 1 \times (k-1) = 1$$

$$k - 1 = 2$$

$$k = 3, -1$$

Q

If BD & CE are medians from B & C respectively with eqⁿ/s $x+y=5$ & $x=4$ find vertices B & C.



$\frac{x_2+1}{2} + \frac{y_2+2}{2} = 5$

$x_2 + y_2 + 2 = 10$

$x_2 + y_2 = \cancel{8}\ 7$

$x_2 = 4$

$y_2 = 4$

$\cancel{\frac{x_1+1}{2} + \frac{y_1+2}{2} =}$

$\frac{x_1+1}{2} = 4$

$x_1 + 1 = 8$

$x_1 = 7$

$x_1 = 7$

$x_1 + y_1 = 5$

$\therefore 7 + y_1 = 5$

$y_1 = -2$

$G_x = \frac{x_1 + x_2 + 1}{3}$

$\frac{3 - x_1 - x_2 - 1}{3} = 2\left(\frac{2x_1 + 2x_2 + 2 - 3x_1 - 3x_2}{6}\right)$

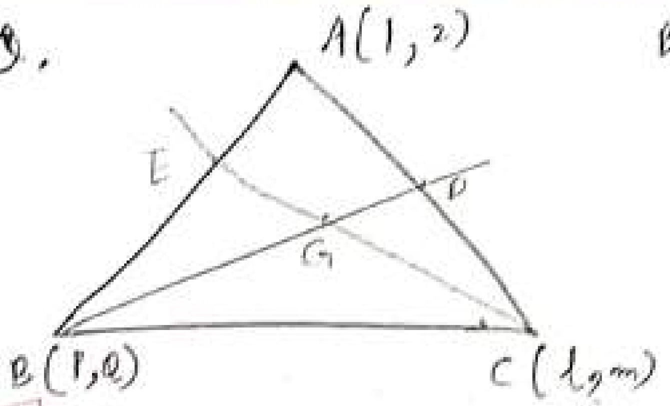
$\frac{2 - x_1 - x_2}{2} = \frac{2}{6}\left(-x_1 - x_2 + 2\right)$

$2 = 2$

$\frac{3x_1 - x_1 - x_2 - 1}{3} = 2\left[\frac{2x_1 + 2x_2 + 2 - 3x_2 - 3}{6}\right]$

$2x_1 - x_2 - 1 = 2x_1 - x_2 - 1$

Q.



BD & CE are medians satisfying eq.

$x + y = 5$

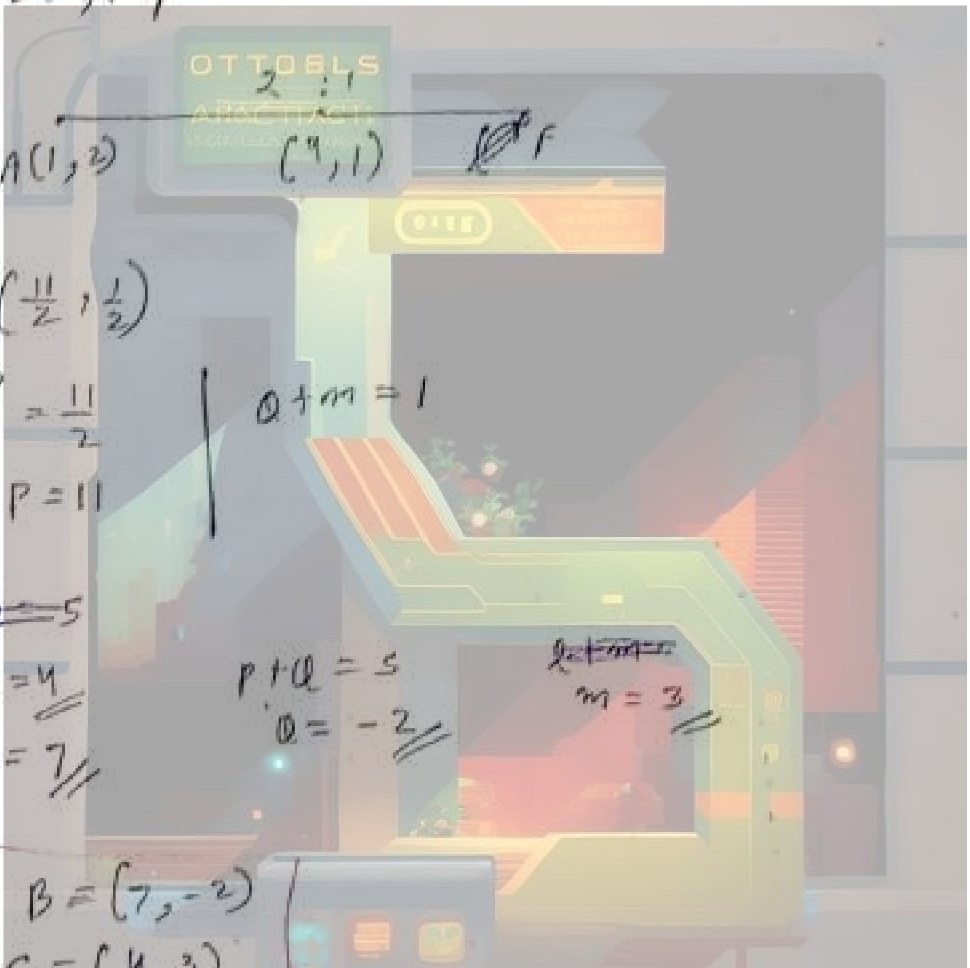
$x = 4$

find B & C.

MI

by solving Medians, $G(4, 1)$

So, F point is :-



2 : 1

A(1, 2) — (4, 1) — F

$F = \left(\frac{11}{2}, \frac{1}{2}\right)$

$\frac{l+p}{2} = \frac{11}{2}$

$l + p = 11$

$q + m = 1$

~~p + q = 5~~

$l = 4$

$p = 7$

$p + q = 5$

$q = -2$

~~l + m = ...~~

$m = 3$

$B = (7, -2)$

$C = (4, 3)$

MII

$\frac{p+1}{2} = 4$

$p + 1 = 8$

$p = 7$

$l = 4$

$\frac{l+1}{2} + \frac{m+2}{2} = 5$

$l + m = 7$

$m = 3$

$p + q = 5$

$q = -2$

H.W.

PYS-2, 3 (Co-ordinate)

O-1 (Q1, 2, 3, 4, 5)

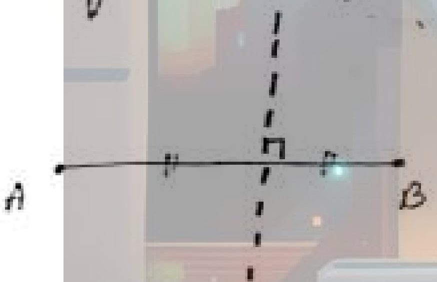
Locus

→ Path traced by a point $P(h, k)$ according to the given condition

eg. locus of a point which moves at a ~~equal~~ same distance from another point will be ~~~~ circumference of circle.

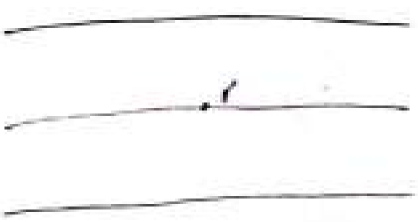
Locus → Circumference of a circle

eg2. Locus of a point which moves at a equal distance from two fixed points



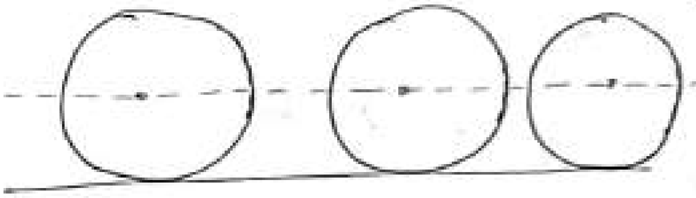
Locus :- Perpendicular Bisector of AB.

eg3. Locus of a point which moves at a fixed distance from 2 parallel lines.



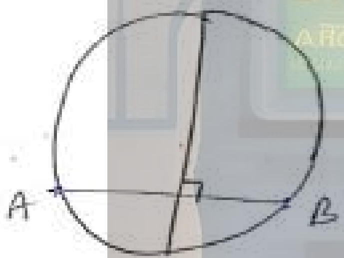
Locus :- Parallel lines between given two parallel lines.

eg4. locus of the center of a circle which moves on a flat surface.



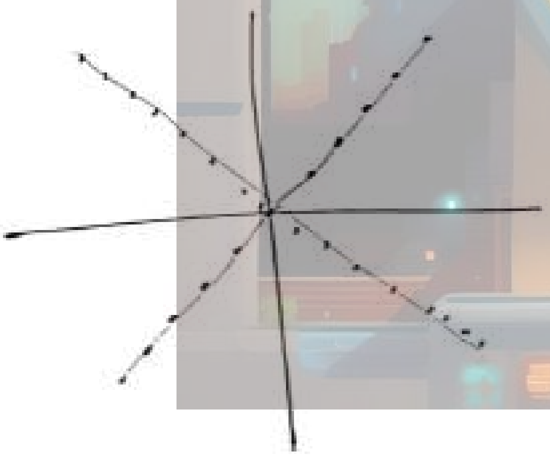
Locus:- A straight line passing through ~~surface~~ centers parallel to flat surface.

eg5. locus of a point which is inside a circle and equidistant from two fixed points on circumference.



Locus:- Diameter ~~or~~ Perpendicular (⊥) to line joining the given two points

eg6. Locus of a point which is at equal distance from two intersecting lines



Locus:- Angle Bisector of both given lines.

~~Note~~

~~Method to E~~

Method to Find Locus (Locus Equation)

→ Assume the point $P(h, k)$

→ Apply given condition of question.

→ Find the equation in the form of h & k

or

find relation in h & k

→ For generalisation, replace (h, k) with (x, y)

Q) Find the locus of a point

① which moves at a distance of 1 unit from origin

② which moves at a distance of 2 units from the y axis.

③ which moves at a equal distance from both the axes.

① Let $P(h, k)$

$P(h, k)$ —— $O(0, 0)$

$OP = 1$

$$\sqrt{(h-0)^2 + (k-0)^2} = 1$$

$$h^2 + k^2 = 1$$

$$\boxed{x^2 + y^2 = 1}$$

② at y axis, $x = 0$

↳ ~~(x, 0)~~ $(0, y)$

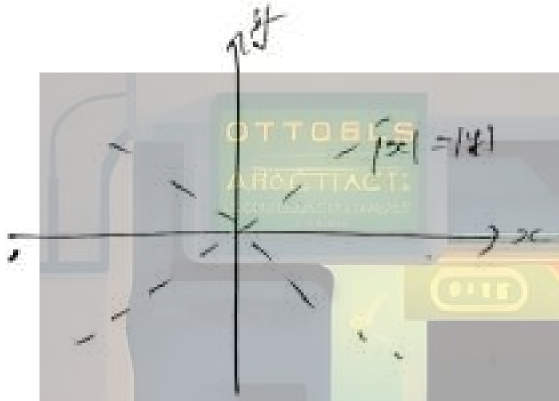
$P(h, k)$

dis = 2

$h = |2|$

~~x = 2, x = −2~~ $\boxed{x = 2,\ x = -2}$

③



$P(h, k)$

distance from x-axis = distance from y-axis.

$$|k| = |h|$$

$$\boxed{|x| - |y| = 0}$$

~~④ P which moves such that~~

④ Distance from the x-axis is 3 times the distance from y axis.

⑤ Sum of squares of its distances from the axes = 3.

⑥ its distance from the point (3, 0) is 3 times the distance from the point (0, 2).

④ $3|y| = 9|x|$

$3|x| = |y|$

$|3x| = |y|$

$3x = \pm y$

$\boxed{3x \pm y = 0}$

⑤ $|x|^2 + |y|^2 = 3$

$\boxed{x^2+y^2=3}$

⑥ $P(x, y)$

$\sqrt{(x-3)^2+(y-0)^2} = 3\sqrt{(x-0)^2+(y-2)^2}$

$x^2+9-6x+y^2 = 9(x^2+y^2+4-4y)$

$x^2+y^2-6x+9 = 9x^2+9y^2+36-36y$

$\boxed{8x^2+8y^2+6x-36y+27=0}$

~~⑦ find th~~

Q find the locus of centroid of a triangle whose vertices are $(a\cos t, a\sin t)$ $(b\sin t, -b\cos t)$ & $(1,0)$

$h = \frac{a\cos t+b\sin t+1}{3}$ $k = \frac{a\sin t-b\cos t}{3}$

$\cancel{3(h+k) = a\cos t + a\sin t + b\sin t - b\cos t + 1}$

$\cancel{h+k = \frac{a(\sin t+\cos t)+b(\sin t-\cos t)+1}{3}}$

$\cancel{x = \frac{a(\sin t+\cos t)+b(\sin t-\cos t)+1-3y}{3}}$

$3h-1 = a\cos t + b\sin t$ $\qquad$ $3k = a\sin t - b\cos t$

square & add

$$9h^2+1-6h+9k^2 = a^2\cos^2 t + b^2\sin^2 t + a^2\sin^2 t + b^2\cos^2 t$$
$$+ 2ab\sin t\cos t - 2ab\sin t\cos t$$

$$\boxed{9h^2+9k^2-6h+1 = a^2+b^2}$$

$\cancel{9h^2 \quad a^2+b^2+}$

H.W. 17-9-24

DYS-4

Slope/Gradient of a line

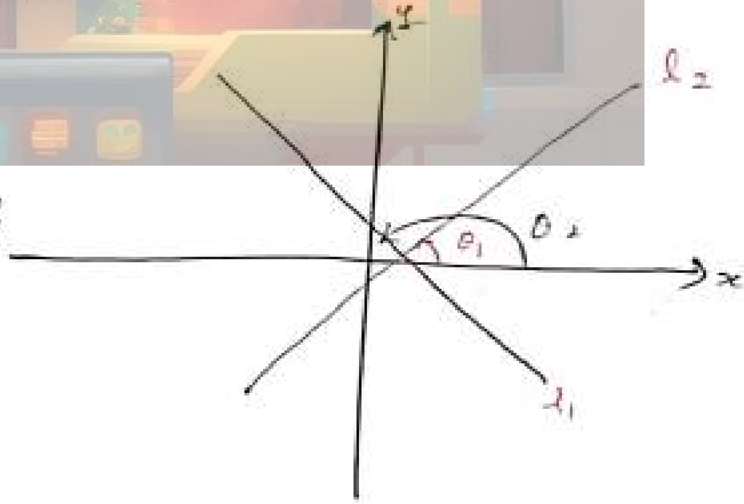
→ Angle made by any line with +ve side of x-axis in anti-clockwise direction is θ, then $\tan\theta$ is called slope.

→ It is denoted by 'm'.

$m(L_1) = \tan\theta_2$

$m(L_2) = \tan\theta_1$

$\theta \in [0, 180°) - \{90°\}$



Q find the slope of the line

① which makes 60° angle from +ve x-axis in anticlockwise direction

② which is ∥ to x-axis

① $\sqrt{3}$

② 0

③ ∥ to y-axis

③ not Defined

→ When two lines having slope m_1 & m_2 are given then

parallel → $m_1 = m_2$

Perpendicular → $m_1 m_2 = -1$

Slope of a line when any 2 points on it are given.

Slope = m = tan θ

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Q. If the line passing through ~~(A -2, 6) & (B,~~ $A(-2, 6)$ & $B(4, 8)$ is $\perp$ to the line passes through $P(8, 12)$ & $Q(x, 24)$ find x.

$$m(AB) = \frac{8-6}{4-(-2)} = \frac{2}{6} = \frac{1}{3} = m_2$$

$$m(PQ) = \frac{12}{x-8} = m_1$$

$$m_1 m_2 = -1$$

$$\frac{1}{3} \times \frac{12}{x-8} = -1$$

$$\frac{12}{3x-24} = -1$$

$$12 = 24 - 3x$$

$$3x = 12$$

$$\boxed{x = 4}$$

Equation of Straight Line

→ It is a linear Equation of Degree 1.

$ax + by + c = 0$

eg. $2x - 3y + 7 = 0$

$x + 2y = 0$

$x = 0$; $x = -8$; $y = 3$

→ Equation of y axis ⇒ $x = 0$

Eq^n of x-axis ⇒ $y = 0$

Eq^n of line ∥ to x-axis at distance 'a' ⇒ $y = \pm a$

Eq^n of line ∥ to y-axis at distance 'b' ⇒ $x = \pm b$

Q1. find the equation of line ~~to~~ parallel to x-axis & passes through (1, 2)

Q2. perpendicular to x-axis & passes through (-3, 4)

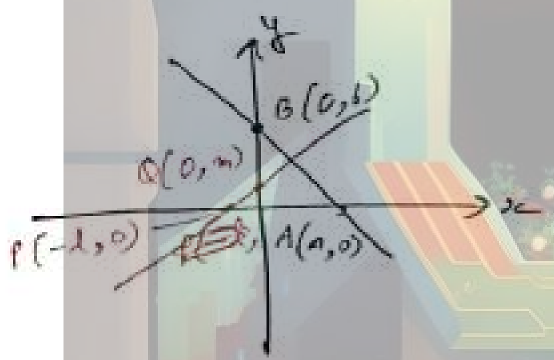
Q1. $y = 2$

Q2. $x = -3$

Intercept

→ It is a point where the given line cuts the x-axis or y-axis.

→ It can be ⊖ve & ⊕ve or 0.



x-intercept $= a$, y-intercept $= b$] AB

x-intercept $= -l$, y-intercept $= m$] PQ

Note :- all lines ‖ to x-axis will have x-intercept = not-Defined
all lines ‖ to y-axis have y-intercept = not-Defined

→ There are ∞ no. of lines with both x & y intercept = 0 (Pass through origin).

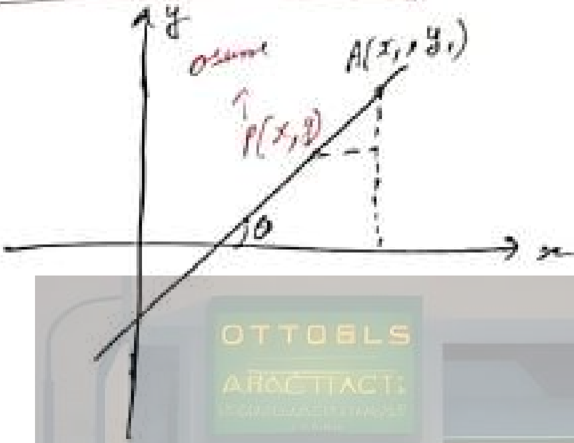
standard forms of straight lines.

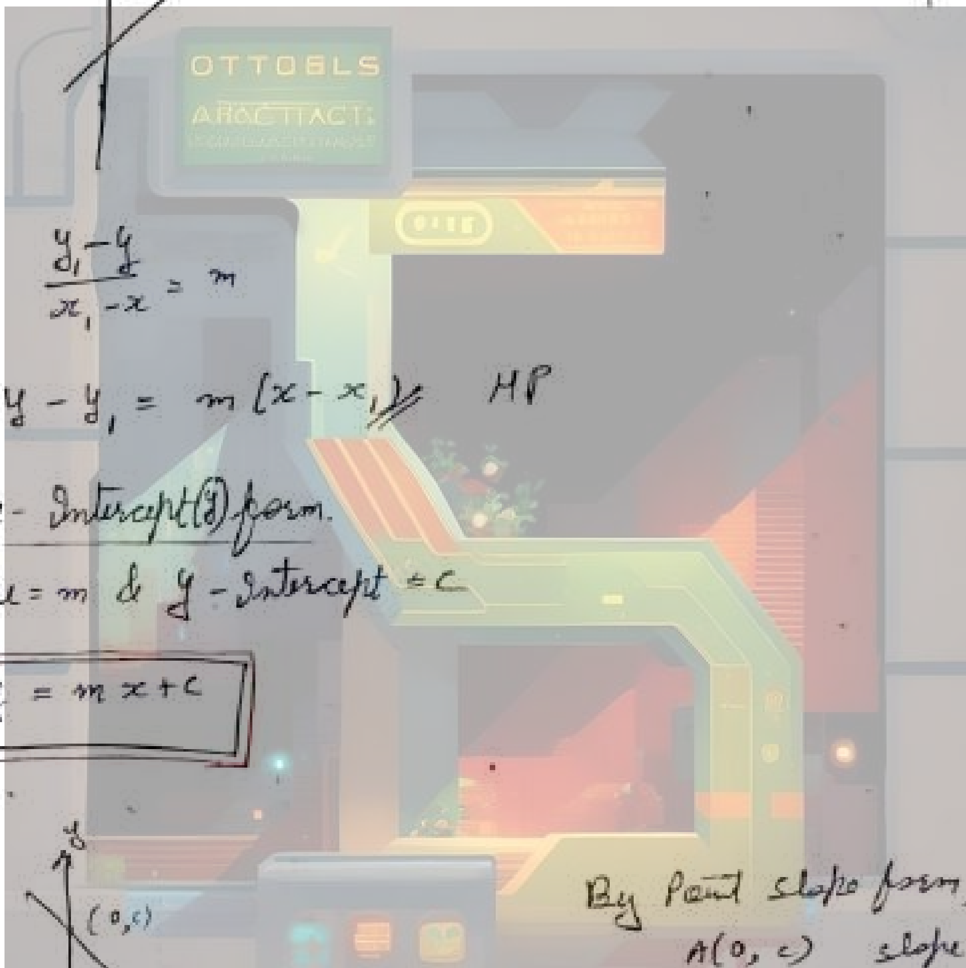
→ we need any two things for writing the eqⁿ of a line from slope, point, intercept.

① Point - Slope form

$A(x_1, y_1)$ & Slope $= m$

$$\boxed{y - y_1 = m(x - x_1)}$$





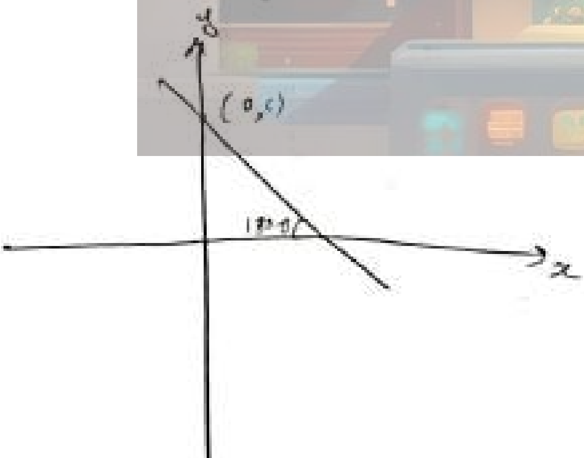
$$\frac{y_1 - y}{x_1 - x} = m$$

$$y - y_1 = m(x - x_1) \quad \text{MP}$$

② Slope-Intercept (y) form.

Slope $= m$ & y-Intercept $= c$

$$\boxed{y = mx + c}$$



By Point slope form,

$A(0, c)$ slope $= m$

$$y - c = m(x)$$

$$y = mx + c \quad \text{MP}$$

③ 2 Point form

$A(x_1, y_1)$ & $B(x_2, y_2)$

$$\boxed{y - y_1 = \underbrace{\left(\frac{y_2 - y_1}{x_2 - x_1}\right)}_{\text{slope}} (x - x_1)}$$

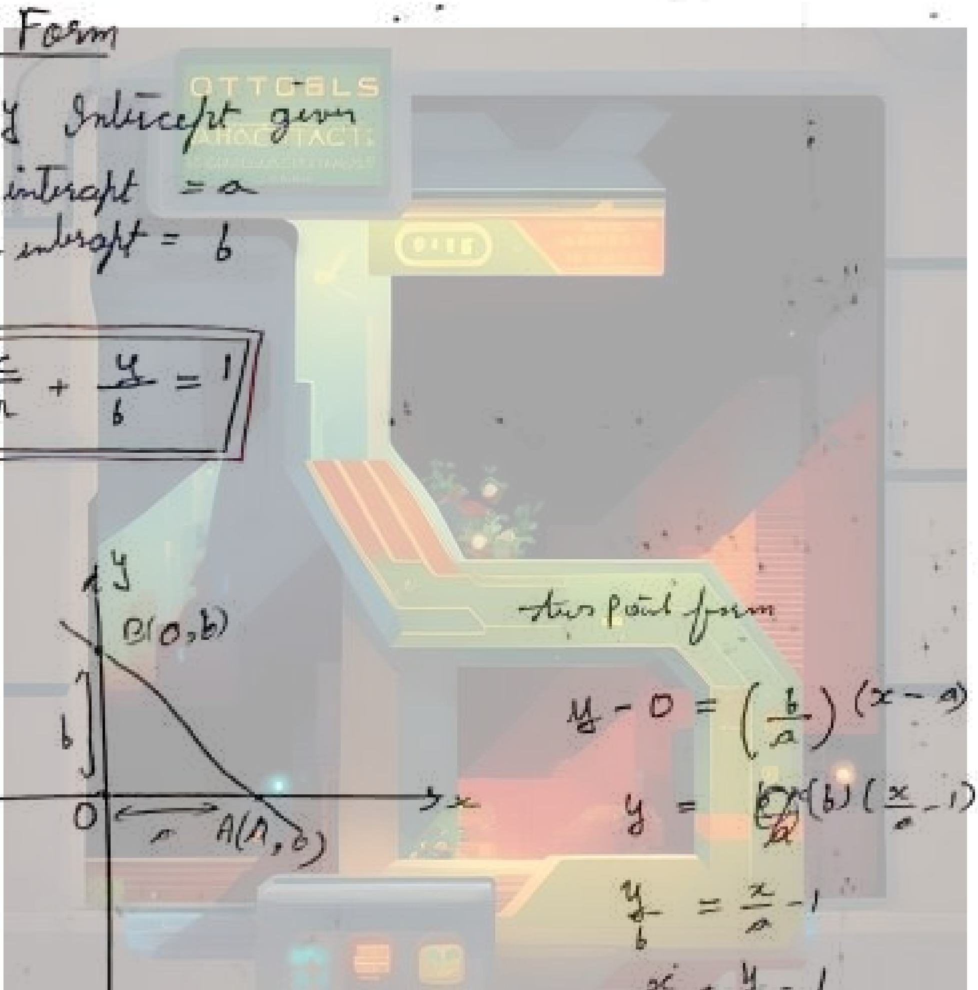
④ Intercept Form

↳ x & y Intercept given

x - intercept $= a$

y - intercept $= b$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



Proof:

two point form

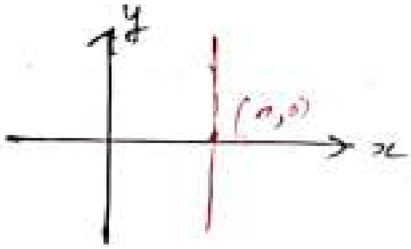
$$y - 0 = \left(\frac{b}{a}\right)(x - a)$$

$$y = (b)\left(\frac{x}{a} - 1\right)$$

$$\frac{y}{b} = \frac{x}{a} - 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Note:- area of $\triangle AOB = \sqrt{a^2 + b^2}$

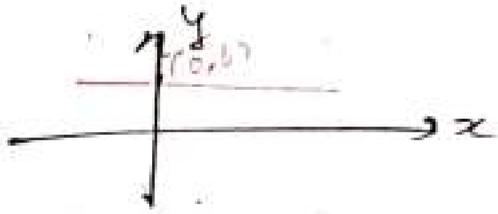


$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} + \frac{y}{\infty} = 1$$

$$\frac{x}{a} = 1$$

$x = a$ HP



$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{\infty} + \frac{y}{b} = 1$$

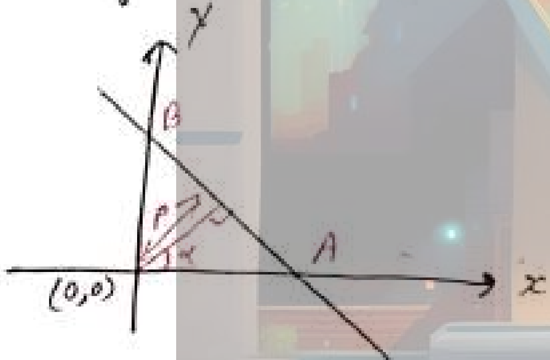
$y = b$ HP

H.W. 19-9-24

DYS-5 [1, 17]

(5) Normal Form (Made by intercept form)

→ when length of ~~Ratio~~ ⊥ from origin to the line is given & angle made by ⊥ with x axis is given



$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

$p \rightarrow$ +ve always

Proof:- $\frac{x}{OA} + \frac{y}{OB} = 1$ (Intercept form)

$OA = \frac{p}{\cos\alpha}$ $OB = \frac{p}{\sin\alpha}$

$$\frac{x\cos\alpha + y\sin\alpha}{p} = 1$$

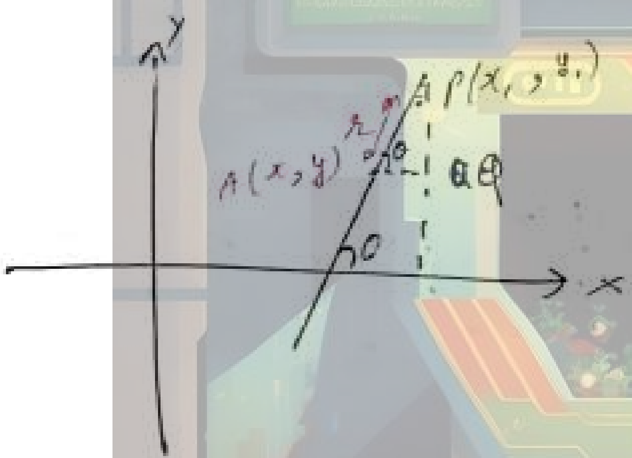
$x\cos\alpha + y\sin\alpha = p$ Hence Proved

⑥ Parametric form (Made by point slope)

→ Angle made by line with x-axis (+ve, anticlockwise) & a point $P(x_1, y_1)$ is given.

→ This form is mostly used for finding the co-ordinates of a point which are at same distance from another point

$$\boxed{\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \pm r}$$



~~Proof~~ Proof:-

$PQ = y_1 - y$

$AQ = x_1 - x$

$\cos\theta = \frac{x_1 - x}{r}$

$r = \frac{x_1 - x}{\cos\theta}$ H.P.

7. General Form

$$ax+by+c=0 \quad \{a \,\&\, b \text{ both} \neq 0\}$$

$$\boxed{\text{Slope} = -\frac{a}{b}}$$

Proof:-

$$ax+by+c=0$$

$$by = -ax-c$$

$$y = \underbrace{\left(-\frac{a}{b}\right)}_{m} x - c$$

$$m = -\frac{a}{b}$$

Q1. find equation of the line.

1. which passes through origin & makes angle 45° with +ve x axis in anticlock-wise direction
2. which makes an angle 60° with x-axis & y intercept of 5 in negative side.
3. which makes intercept 2 & −3 on X & Y axes respectively
4. which passes through 2 points A(1, 2) & B(3, 5)
5. Passes through (0, 0) with slope m.
6. Intersecting the x-axis at distance of 3 units to the left of the origin with slope = −2.
7. Passes through 2 points $(\sin\alpha, \cos\alpha)$ & $(\sin\beta, \cos\beta)$

① $m = \tan 45° = 1$

$(0, 0)$

$(y - 0) = 1(x - 0)$

$y = x$

$\boxed{x = y}$

② $m = \tan 60 = \sqrt{3}$

$(0, -5)$

$y = mx + c$

$y = \sqrt{3}x - 5$

$\boxed{\sqrt{3}x - y - 5 = 0}$

③ $\frac{x}{2} + \frac{y}{-3} = 1$

$\frac{x}{2} = \frac{3 + y}{3}$

$3x = 6 + 2y$

$\boxed{3x - 2y + 6 = 0}$

④ $(y - 2) = (x - 1)\left(\frac{3}{2}\right)$

$2y - 4 = 3x - 3$

$\boxed{3x - 2y + 1 = 0}$

⑤ $m = m$

$(0, 0)$

$(y - 0) = m(x - 0)$

$\boxed{y = mx}$

⑥ $(-3, 0)$

$m = -2$

$(y - 0) = (x + 3)(-2)$

$-2x - 6 = y$

$\cancel{2x + y + 6 = 0}$

$\boxed{2x + y + 6 = 0}$

⑦ $m = \frac{\cos\alpha - \cos\beta}{\sin\alpha - \sin\beta}$

$m = \frac{-\cancel{2}\sin\frac{\alpha+\beta}{2}\cancel{\sin\frac{\alpha-\beta}{2}}}{\cancel{2}\cancel{\sin\frac{\alpha-\beta}{2}}\cos\frac{\alpha+\beta}{2}}$

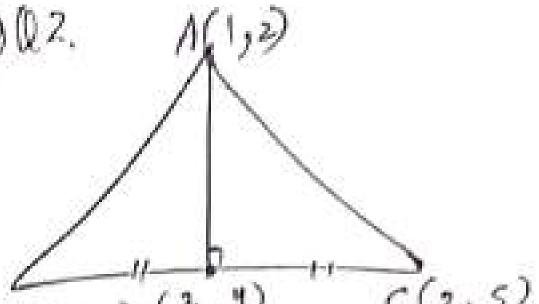
$m = -\tan\frac{\alpha+\beta}{2}$

$y - \cos\alpha = (x - \sin\alpha)\left(-\tan\frac{\alpha+\beta}{2}\right)$

$y - \cos\alpha = (\sin\alpha - x)\left(\tan\frac{\alpha+\beta}{2}\right)$

$y + \tan\left(\frac{\alpha+\beta}{2}\right)x - \cos\alpha - \sin\alpha\tan\left(\frac{\alpha+\beta}{2}\right)$

Q2.



A(1,2)

B(4,3) (3,4) C(2,5)

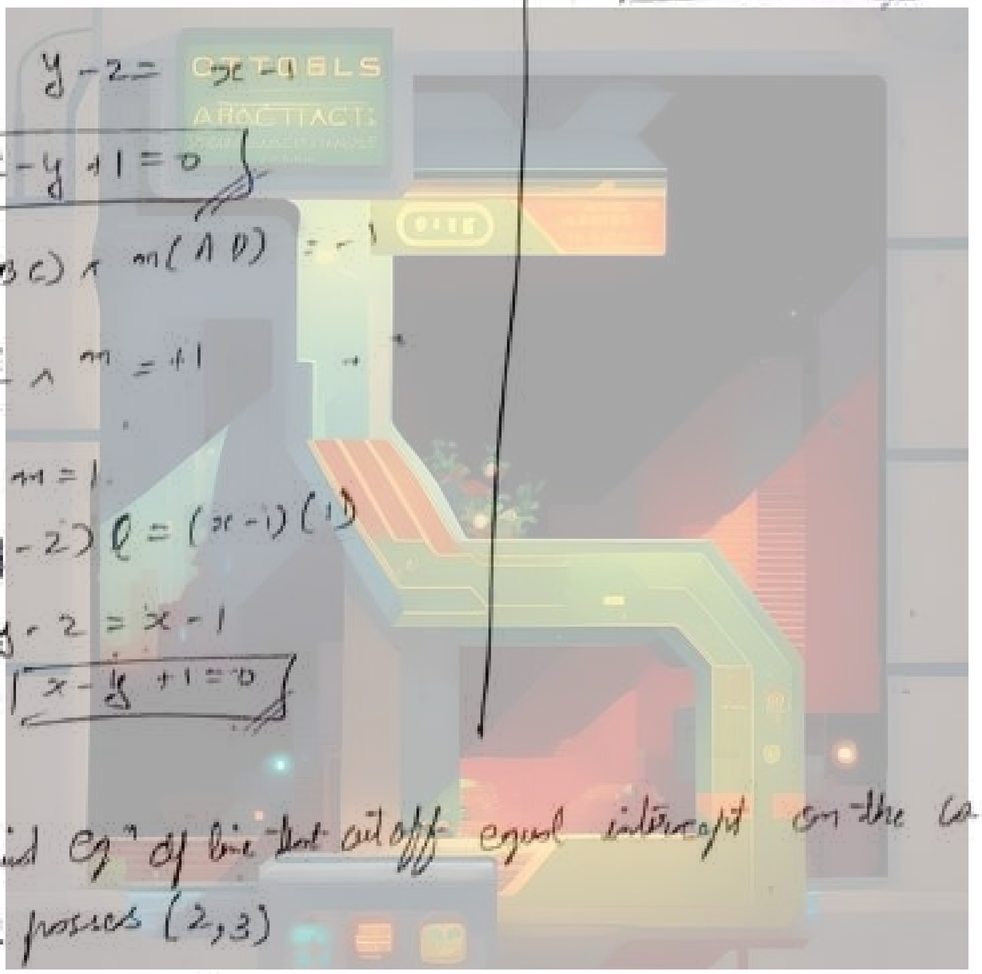
find equations
a) Median from A
b) Altitude from A
c) ⊥ side bisector from A.

a)

$(y-2) = (x-1)\left(\frac{2}{2}\right)$

$y-2 = x-1$

$\boxed{x-y+1=0}$



b) $m(BC) \times m(AD) = -1$

$\frac{2}{-2} \times m = -1$

$m = 1$

$(y-2) = (x-1)(1)$

$y-2 = x-1$

$\boxed{x-y+1=0}$

c) $m = 1$ (Df ⊥)

$\boxed{x-y+1=0}$

Q3. find eqn of line that cut off equal intercept on the co-ordinate axes & passes (2,3)

$(y-3) = (x-2)$

$y-3 = (x-2)(-1)$

$\boxed{\cancel{x-y+1=0}}$

$y-3 = 2-x$

$\boxed{x+y-5=0}$

H. W. 20-09-24

DYS-5 [10, ∞)

~~DYS-5~~

Broadcast

Q4. Find the equation of the line which passes through (3, 4) & have intercepts on axes.

① equal in magnitude but opp. in sign

② such that their sum = 14

① $m = 1$

$y - 4 = x - 3$

$\boxed{x - y + 1 = 0}$

~~② (0, x) (x-14, 0) (3, 4)~~

~~$\frac{4 - x}{3} = \frac{4}{-x-11}$~~

~~$-4x - 44 + x^2 + 11x = 12$~~

~~$x^2 + 7x - 56 = 0 \quad (x = h)$~~

~~$h = \frac{-7 \pm \sqrt{49 + 224}}{2}$~~

~~$h = \frac{-7 \pm 9\sqrt{3}}{2}$~~

~~$m = \frac{4 - h}{3}$~~

~~$(y - 4) = (x - 3)\left(\frac{4 - h}{3}\right)$~~

~~$(y - 4) = (x - 3)\left(\frac{8 + 7 \mp 9\sqrt{3}}{2}\right)$~~

② $\frac{x}{a} + \frac{y}{b} = 1$

$a + b = 14$

$\frac{x}{a} + \frac{y}{14 - a} = 1$

$\frac{3}{a} + \frac{4}{14 - a} = 1$

$42 - 3a + 4a = 14a - a^2$

$a^2 - 13a + 42 = 0$

$a = 7, 6$

$\boxed{\frac{x}{6} + \frac{x}{8} = 1}$

224

Q4. find line eq. when length of ⊥ from origin is 4 & its inclination from x-axis is 30°.

$x\cos\alpha + y\sin\alpha = p$

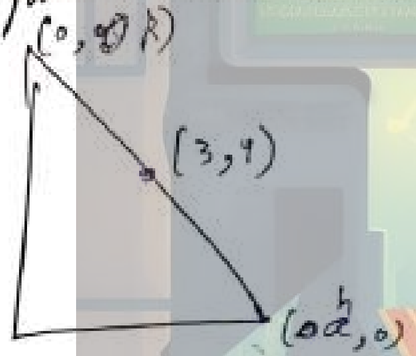
~~$\frac{x}{2} + \frac{y\sqrt{3}}{2} = 4$~~

~~$\boxed{x + \sqrt{3}y = 8}$~~

$\frac{\sqrt{3}x}{2} + \frac{y}{2} = 4$

$\boxed{\sqrt{3}x + y = 8}$

Q5. find the equation of the line which passes through A(3,4) & point A bisects the line segment btw both axes.



$\frac{h+0}{2} = 3$ | $k + 0 = 8$

$h = 6$ | $k = 8$

$(3,4)\ (6,0)\ \&\ (0,8)$

$m = \frac{4}{-3} = \frac{-4}{3}$

$(y-4) = (x-3)\frac{-4}{3}$

$3y - 12 = -4x + 12$

$\boxed{4x + 3y - 24 = 0}$

Conversion of line equation.

Q1. Convert the eqⁿ of the line $x+\sqrt{3}y=2$ in

① Slope intercept form & find Y intercept

② Intercept form & find X & Y intercept

③ Normal form & find length ⊥ drawn on it from origin.

① $x+\sqrt{3}y=2$

$$\frac{x}{\sqrt{3}}+y=\frac{2}{\sqrt{3}}$$

$$y=\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{3}}x$$

$$y=\left(\frac{-1}{\sqrt{3}}\right)x+\frac{2}{\sqrt{3}}$$

$$\boxed{y\text{-intercept}=\frac{2}{\sqrt{3}}}$$

② $\frac{x}{a}+\frac{y}{b}=1$

$x+\sqrt{3}y=2$

$$\frac{x}{2}+\frac{y}{2/\sqrt{3}}=1$$

$$\boxed{\begin{array}{l} a=2 \text{ (X intercept)} \\ b=2/\sqrt{3} \text{ (Y intercept)} \end{array}}$$

③ $\frac{x}{2}+\frac{\sqrt{3}}{2}y=1$ [multiply & Divide by $\sqrt{a^2+b^2}$

$\cos 60=1/2$

~~cos 30~~ $\sin 60=\frac{\sqrt{3}}{2}$

$x\cos 60+y\sin 60=1$

$\boxed{p=1}$ $\alpha=60°$

Q2. Same as previous in eq $x+y=-\sqrt{2}$.

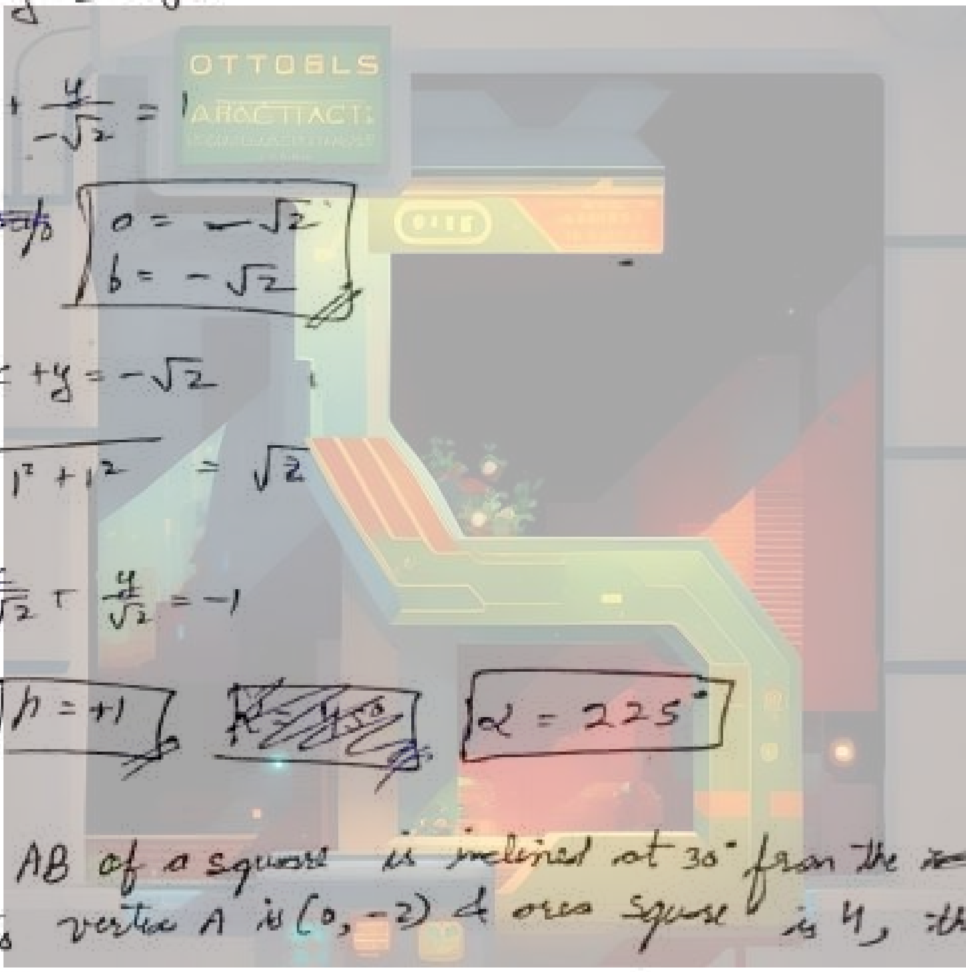
(1) $x+y=-\sqrt{2}$

$y=\cancel{0}-x-\sqrt{2}$

$y=(-1)x+(-\sqrt{2})$

$\boxed{y\text{ intercept}=(-\sqrt{2})}$

(2) $x+y=-\sqrt{2}$



$\frac{x}{-\sqrt{2}}+\frac{y}{-\sqrt{2}}=1$

~~intercepts~~ $\boxed{\begin{array}{l}a=-\sqrt{2}\\ b=-\sqrt{2}\end{array}}$

(3) $x+y=-\sqrt{2}$

$\sqrt{1^2+1^2}=\sqrt{2}$

$\frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}}=-1$

$\boxed{p=+1}$ $\cancel{\boxed{\;}}$ $\boxed{\alpha=225^\circ}$

Q3. Side AB of a square is inclined at 30° from the ~~x~~ x-axis. If its vertex A is $(0,-2)$ & area square is 4, then find vertex B.

~~$m(AB)=\frac{1}{\sqrt{3}}$~~

eq. $y+2=\frac{x}{\sqrt{3}}$

$x=\sqrt{3}y+2\sqrt{3}$

~~$\boxed{B(2\sqrt{3},0)}$~~

$B\left\{(\sqrt{3}+2\sqrt{3},1)\right.$

$B(\sqrt{3}h+2\sqrt{3},h)$

$AB=2$

~~$(\sqrt{3}+2)^2h^2+(h+2)^2=4$~~

~~$7h^2\,4\sqrt{3}\,h^2+h^2+4+4h=4$~~

~~$(8+4\sqrt{3})h^2+4h=0$~~

~~$h=0$, done~~

$3h^2+12+h^2+12h+4h=4$

$4h^2+4h+8=0$

$h^2+h-2=0$

$h=\frac{-1\pm\sqrt{1+8}}{2}$

$h=-2,1$

$$\frac{x - x_1}{\cos\theta} = \pm 2 \qquad B(x, y)$$

$$\frac{2(x-0)}{\sqrt{3}} = \pm 2 \qquad \Big| \qquad 2(y+2) = \pm 2$$

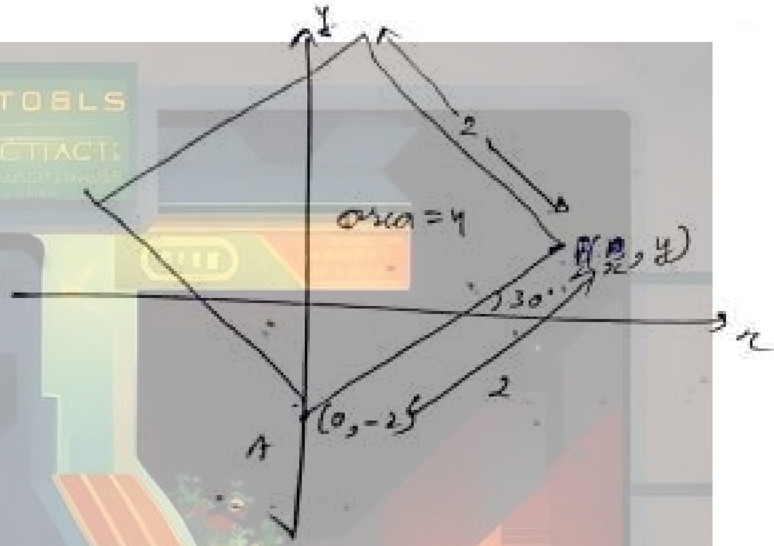
$$x = \frac{\pm 2\sqrt{3}}{2} \qquad \Big| \qquad y + 2 = \pm 1$$

$$\qquad \qquad \qquad y = \pm 1 - 2$$

$$x = \pm\sqrt{3}$$

$B(\sqrt{3}, -1)$

$B(-\sqrt{3}, -3)$



Equation of line ∥ or ⊥ to given line

$$ax + by + c = 0$$

∥ line

$$\boxed{ax + by + c_1 = 0}$$

$$m_1 = \frac{-a}{b}$$

⊥ line

$$\boxed{bx - ay + c_2 = 0}$$

$$m_2 = \frac{b}{a}$$

→ will find c_1 & c_2 by other given condition

Q find the ∥ line which passes through $(1, 2)$ to line $2x + 3y - 7 = 0$.

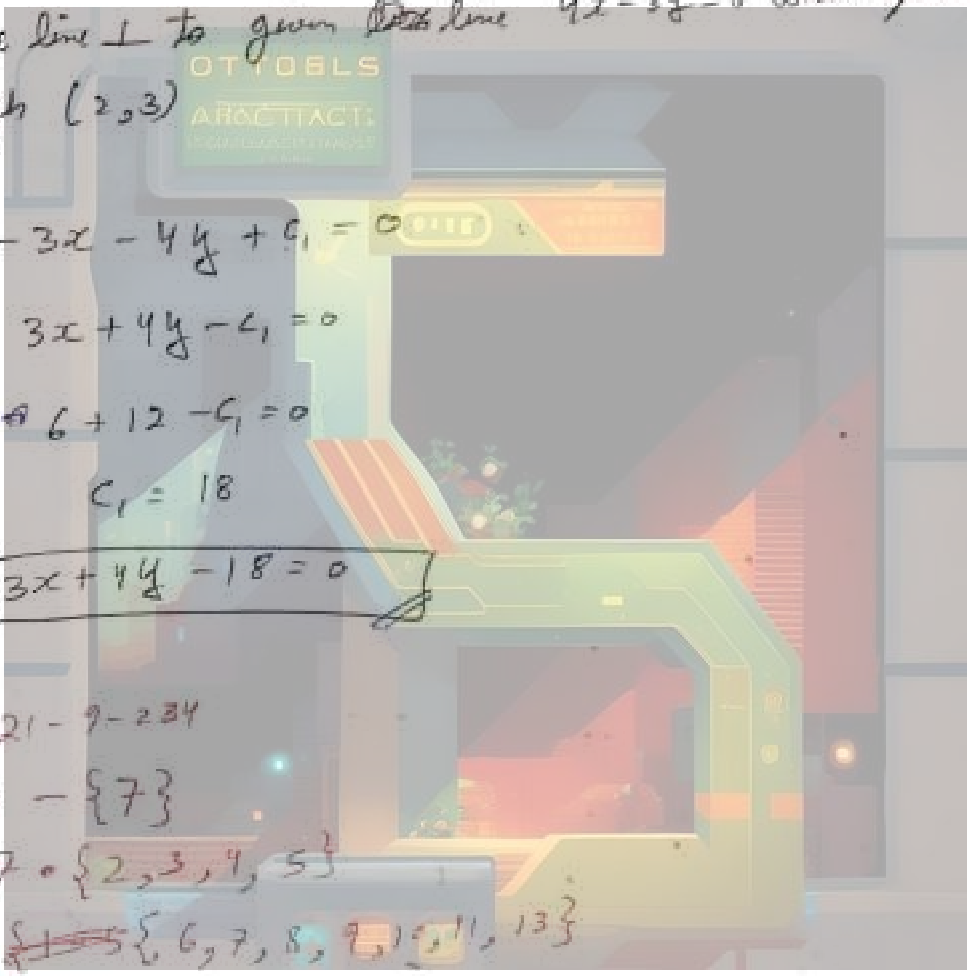
$$2x + 3y + c_1 = 0$$

$$2 + 6 + c_1 = 0$$

$$c_1 = -8$$

$$\boxed{2x + 3y - 8 = 0}$$

Q Find the line ⊥ to given ~~the~~ line $4x - 3y = 0$ which passes through $(2, 3)$



$$-3x - 4y + c_1 = 0$$

$$3x + 4y - c_1 = 0$$

$$6 + 12 - c_1 = 0$$

$$c_1 = 18$$

$$\boxed{3x + 4y - 18 = 0}$$

H.W. 21-9-234

DYS-6 — {7}

DYS-7 {2, 3, 4, 5}

O-1 ~~{1 — 5}~~ {6, 7, 8, 9, 10, 11, 13}

O-2 {1, 2, 4}

Angle b/w two lines

→ Always consider acute Angle

$$\boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|}$$

Proof:-

$$\theta + \theta_1 + 180 - \theta_2 = 180$$

$$\theta = \cancel{\theta_1}\ \theta_2 - \theta_1$$

$$\tan\theta = \cancel{\tan\theta_2 - \tan\theta_1}\ \tan(\theta_2 - \theta_1)$$

$$\tan\theta = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2 \tan\theta_1}$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$ → mod so $\tan\theta$ is 0° to 90°

2 lines are ∥

$$\theta = 0$$

$$0 = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$m_1 = m_2$$

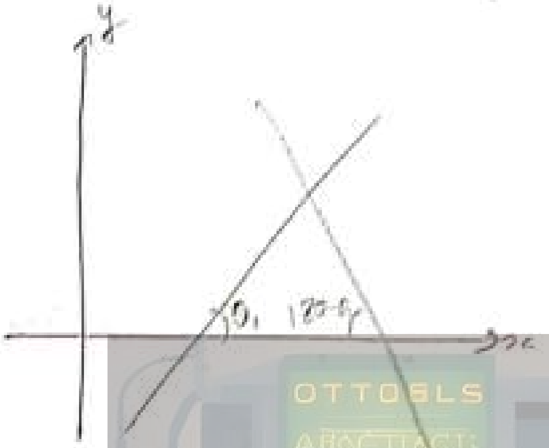
2 lines are ⊥

$$\theta = 90$$

$$\frac{1}{0} = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\therefore m_1 m_2 = -1$$

Note:- when two lines makes isosceles Δ with x axis & having slopes m_1 & m_2



$\theta_1 = 180 - \theta_2$

$\theta_1 + \theta_2 = 180$

$\tan 180 = \dfrac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$

$0 = \dfrac{m_1 + m_2}{1 - m_1 m_2}$

$\boxed{m_1 + m_2 = 0}$

Q. Find the angle betw given lines.

(1) $2x - 3y - 6 = 0$ & $3x + y + 1 = 0$

(2) $x = 9$ & $x - \sqrt{3}y + 7 = 0$

(1) $m_1 = \dfrac{-2}{-3} = \dfrac{2}{3}$ $\quad m_2 = \dfrac{-3}{1}$

$\tan\theta = \left| \dfrac{\frac{2}{3} + \frac{9}{3}}{10 - 2} \right|$ $\quad \tan\theta = \dfrac{11}{3}$

$\boxed{\theta = \tan^{-1}\left(\frac{11}{3}\right)}$

②

$\tan\theta = \tan(\theta_2 - \theta_1)$ | $\tan\theta_1 = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan(90 - 30)$ | $\theta_1 = 30°$

$\tan\theta = \tan 60$

$\boxed{\theta = 60°}$

Q2. If $A(-2,1)$, $B(2,3)$ & $C(-2,4)$ are three points, then find angle btw BA & BC.

$m(BA) = \frac{2}{4} = \frac{1}{2}$ $\quad m(BC) = \frac{1}{-4} = -\frac{1}{4}$

$$\tan\theta = \frac{\frac{1}{2} + \frac{1}{4}}{1 - \frac{1}{8}}$$

$$\tan\theta = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{6}{7}$$

$\boxed{\theta = \tan^{-1}\left(\frac{6}{7}\right)}$

H.W

Q3. Find the angle btw the lines $y = x + 5$ & $y = \sqrt{3}x - 4$.

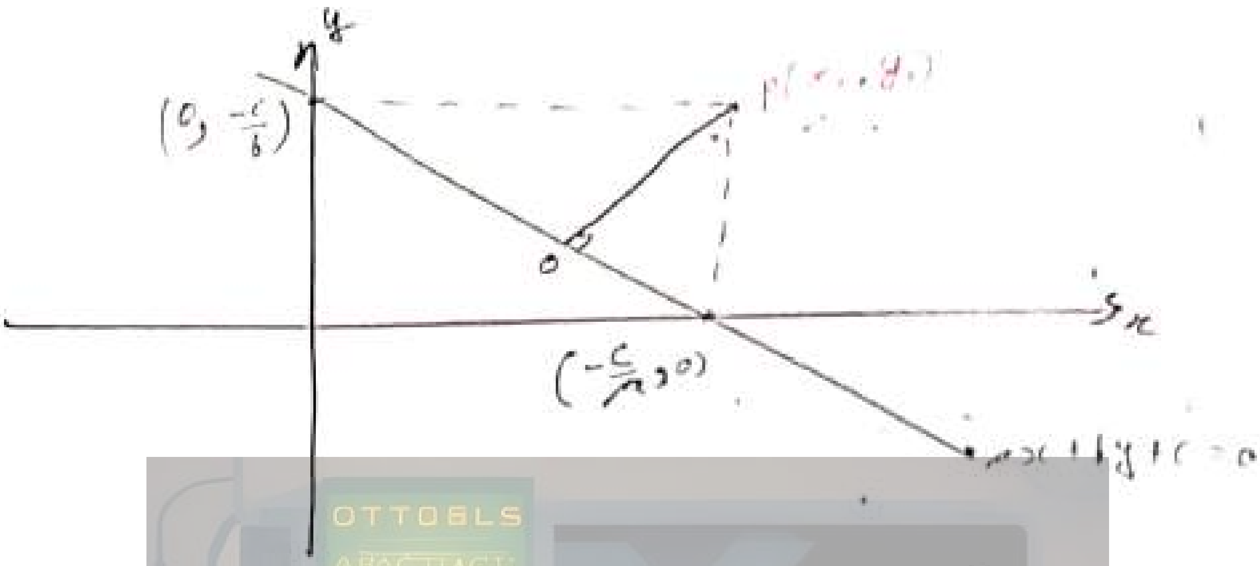
$x - y + 5 = 0$ $\quad \sqrt{3}x - y - 4 = 0$

$m_1 = 1$ $\quad m_2 = \sqrt{3}$

$$\tan\theta = \left|\frac{1-\sqrt{3}}{1+\sqrt{3}}\right| = \left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| = \tan 15°$$

$\boxed{\theta = 15°}$

$\perp$ length of $P(x_1, y_1)$ from line $ax+by+c=0$



$$\boxed{OP = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|}$$

Proof:- Area of $\Delta PAB = \frac{1}{2} \times AB \times OP = \frac{1}{2} \times OP \times \sqrt{\frac{c^2}{a^2} + \frac{c^2}{b^2}}$

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ 0 & -\frac{c}{b} & 1 \\ -\frac{c}{a} & 0 & 1 \end{vmatrix}$$

$$\frac{1}{2} \times c \sqrt{\frac{a^2+b^2}{a^2b^2}} \times OP = \frac{1}{2} \times c \left| \frac{c}{ab} + \frac{x_1}{b} + \frac{y_1}{a} \right|$$

$$\frac{\sqrt{a^2+b^2} \times OP}{ab} = \left| \frac{c + ax_1 + by_1}{ab} \right|$$

$$OP = \left| \frac{ax_1 + by_1 + c}{\cancel{ab}\sqrt{a^2+b^2}} \right|$$

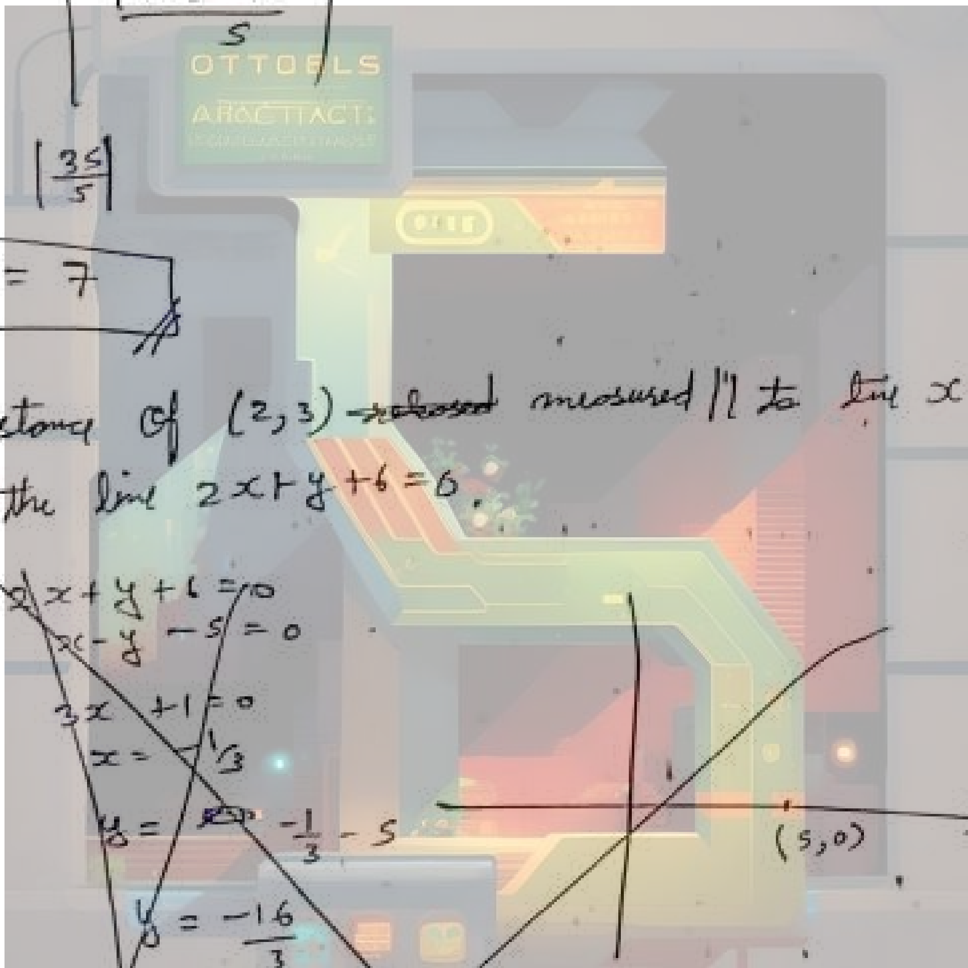
Q find the length of $\perp$ from point $(3,4)$ on the line
$3x+4y+10=0$

$$\perp = \left| \frac{(3)(3)+(4)(4)+10}{\sqrt{9+16}} \right|$$

$$\perp = \left| \frac{9+16+10}{5} \right|$$

$$\perp = \left| \frac{35}{5} \right|$$

$$\boxed{\perp = 7}$$



Q find distance of $(2,3)$ ~~related~~ measured ∥ to line $x-y=5$ from the line $2x+y+6=0$.

$2x+y+6=0$
$x-y-5=0$
$3x+1=0$
$x=-\frac{1}{3}$

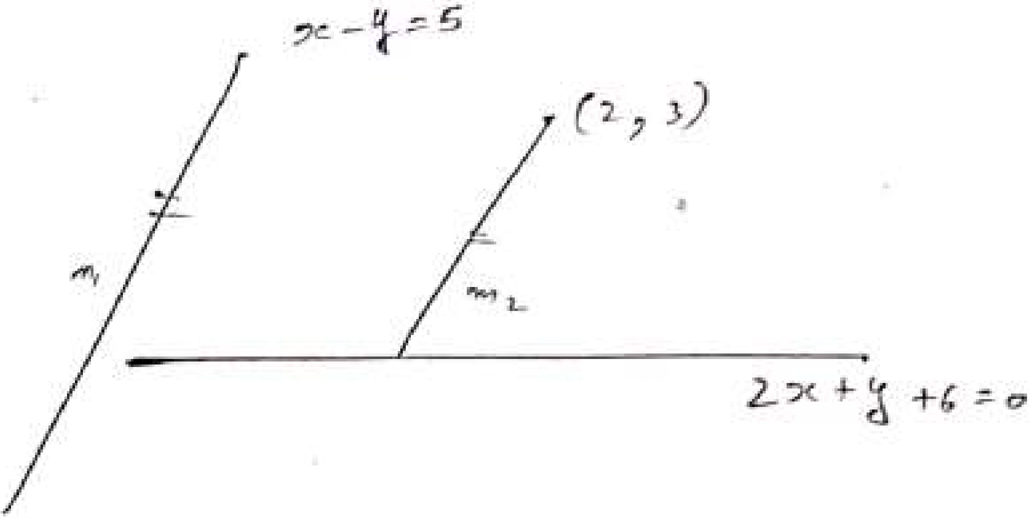
$y=-\frac{1}{3}-5$

$y=-\frac{16}{3}$

$\left(-\frac{1}{3}, -\frac{16}{3}\right)$

$\frac{49}{9}+\frac{625}{9}$

(5,0)



$m_1 = \frac{-1}{-1} = 1$

$m_2 = 1$

$y - 3 = x - 2$

$x - y + 1 = 0$

$2x + y + 6 = 0$

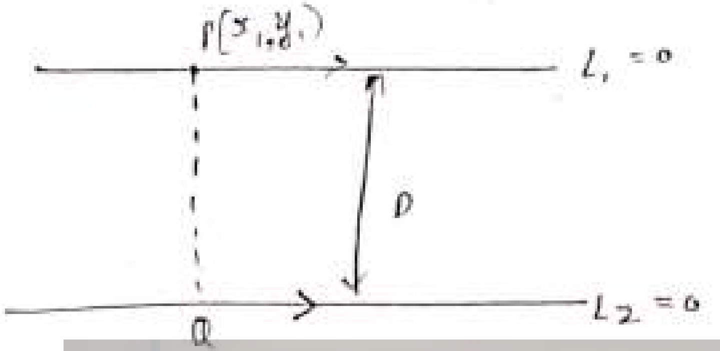
$3x + 7 = 0$

$x = -\frac{7}{3}$

$y = -\frac{4}{3}$

$Dis = \left(\frac{13}{3}\right)^2 + \left(\frac{13}{3}\right)^2 = \boxed{\frac{13\sqrt{2}}{3}}$

Distance between two parallel lines.



$L_1 : y = mx + c_1$

$L_2 : y = mx + c_2$

$y_1 = mx_1 + c_1$ —— (1)

$\therefore$ Distance of $P(x_1, y_1)$ from $L_2 = 0$

$$PQ = \left| \frac{mx_1 - y_1 + c_2}{\sqrt{m^2+1}} \right|$$

$$\boxed{PQ = \left| \frac{c_2 - c_1}{\sqrt{m^2+1}} \right|}$$

Note:- Before applying this formula, convert the line equation in slope intercept form.

① Find the Distance betw given ∥ line -

① $3x+4y = 9$ & $3x+4y = -1$

$\cancel{m = \frac{-3}{-4}}$

$4y = (-3)x + \frac{9}{4}$

$y = \left(\frac{-3}{4}\right)x + \frac{9}{4}$

$$\text{Dis} = \left| \frac{-\frac{1}{4} - \frac{9}{4}}{\sqrt{1+\frac{9}{16}}} \right| = \left| \frac{-10}{5} \right| = |-2| = \boxed{2}$$

$$\cancel{= \left| \frac{-\frac{37}{4}}{\frac{5}{4}} \right|}$$

$$\cancel{= \left| \frac{-37}{5} \right|}$$

$$\cancel{\boxed{= \frac{37}{5}}}$$

② $2x+4y = 15$ & $x+2y = 5$

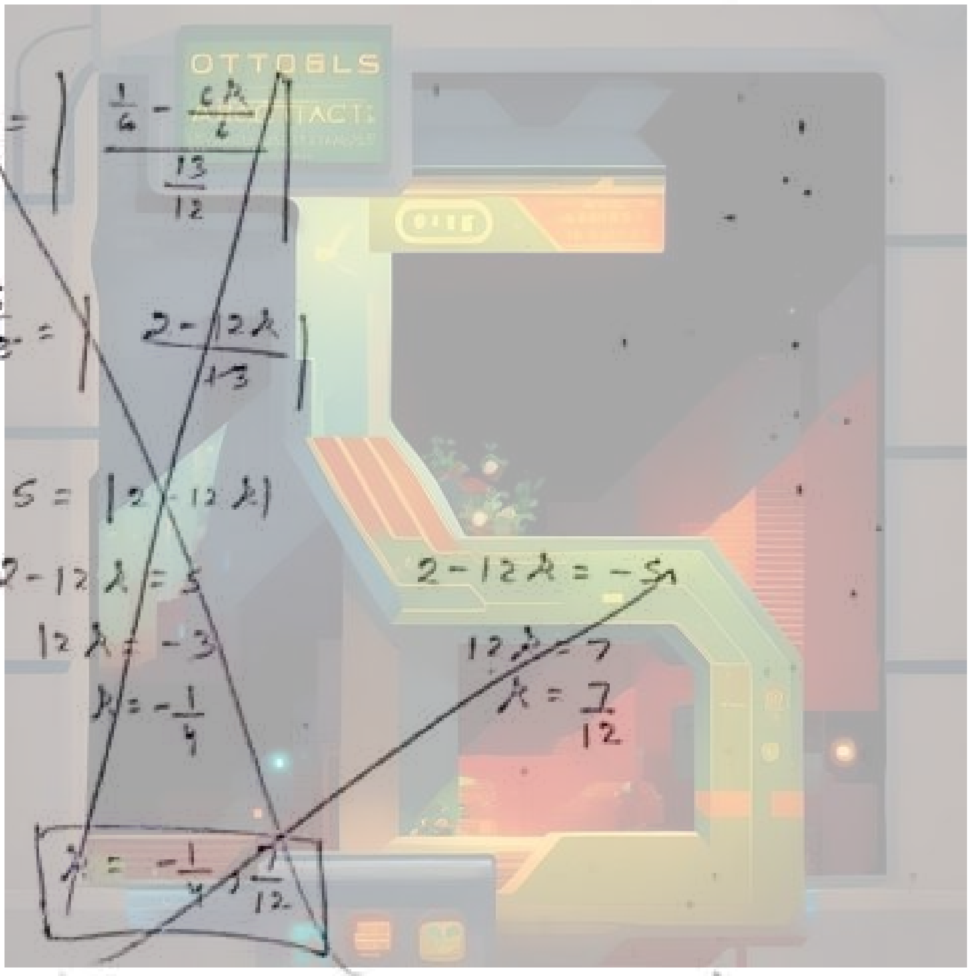
$y = \left(-\frac{1}{2}\right)x + \frac{15}{4}$ $\qquad$ $y = \left(-\frac{1}{2}\right)x + \frac{5}{2}$

$$\text{Dis} = \left| \frac{\frac{15}{4} - \frac{10}{4}}{\sqrt{1+\frac{1}{4}}} \right| = \left| \frac{\frac{5}{4}}{\frac{\sqrt{5}}{2}} \right| = \frac{5}{2\sqrt{5}} = \boxed{\frac{\sqrt{5}}{2}}$$

Q. The dis btw 2 ∥ lines $5x - 12y + 2 = 0$ & $5x - 12y + k = 0$ is $\frac{5}{13}$ units, find 'k'.

$$\left(\frac{5}{12}\right)x + \left(\frac{2}{12}\right) = y \qquad y = \left(\frac{5}{12}\right)x + \frac{k}{12}$$

$$y = \left(\frac{5}{12}\right)x + \left(\frac{1}{6}\right)$$



$$\frac{5}{13} = \left| \frac{\frac{1}{6} - \frac{6\lambda}{6}}{\frac{13}{12}} \right|$$

$$\frac{5}{13} = \left| \frac{2 - 12\lambda}{13} \right|$$

$$5 = |2 - 12\lambda|$$

$2 - 12\lambda = 5$ $2 - 12\lambda = -5$

$12\lambda = -3$ $12\lambda = 7$

$\lambda = -\frac{1}{4}$ $\lambda = \frac{7}{12}$

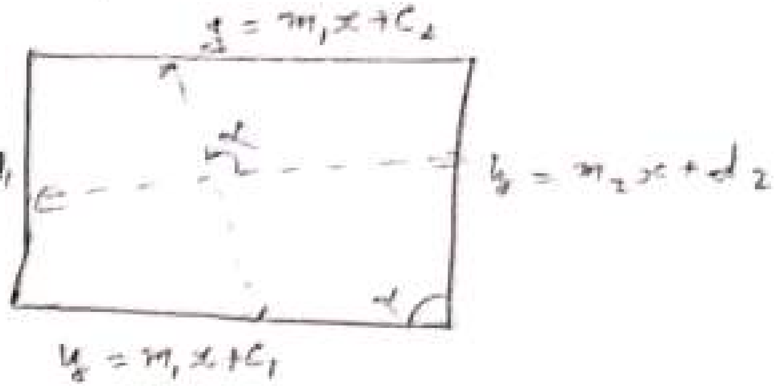
$$\boxed{\lambda = -\frac{1}{4}, \frac{7}{12}}$$

$$\pm\frac{5}{13} = \frac{k - 2}{13}$$

$$k - 2 = \pm 5$$

$$\boxed{k = 7, -3}$$

Note :- area of ||gram with all 4 side equations given



$$\boxed{\text{Area} = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|}$$

derived by

$$\text{area} = \frac{p_1\, p_2}{\sin\alpha}$$

Q find area of ||gm with sides

$3x - 2y + 33 = 0$

$x + 3y - 11 = 0$

$3x - 2y + 77 = 0$

$x + 3y + 44 = 0$

$y = \left(\frac{3}{2}\right)x + \frac{33}{2} = 0$

$y = \left(\frac{-1}{3}\right)x + \frac{11}{3} = 0$

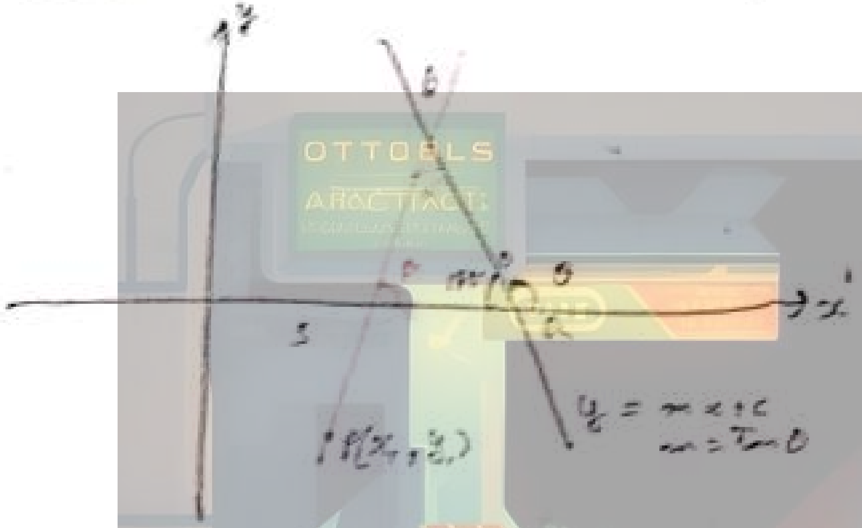
$y = \left(\frac{3}{2}\right)x + \frac{77}{2} = 0$

$y = \left(\frac{-1}{3}\right)x + \left(\frac{-44}{3}\right) = 0$

$\cancel{\text{area} = \left| \left(\frac{-33}{2}\right)\left(\frac{55}{3}\right) \right.}$ area $= \left| \dfrac{\frac{44}{2} \times \frac{55}{3}}{\frac{9}{6} + \frac{2}{6}} \right|$ area $= \left| \dfrac{44 \times 5 \times 11}{11} \right| = \boxed{220}$

DYS-7 (01, 7, 8, 9, 10)
DYS-8 (Full)
D-1 (...)

* Equation of line which makes angle α with another line $y = mx + c$



$\phi = \theta - \alpha$

Eqn of line

$$y - y_1 = \tan(\theta - \alpha)(x - x_1)$$

$$\boxed{y - y_1 = \tan(\theta \pm \alpha)(x - x_1)}$$

$\alpha \rightarrow$ angle two lines
$\theta \rightarrow$ angle of given line

Q find the eq^n of SL passing through (2, 3) & inclined at $\frac{\pi}{4}$ (45°) to given line ~~xxxx~~ $2x + 3y = 6$.

$\tan\theta = \frac{-2}{3}$

$\tan\beta = \dfrac{\frac{-2}{3} - 1}{1 + \frac{2}{3}}$

$\tan\beta = \frac{-5}{5}$

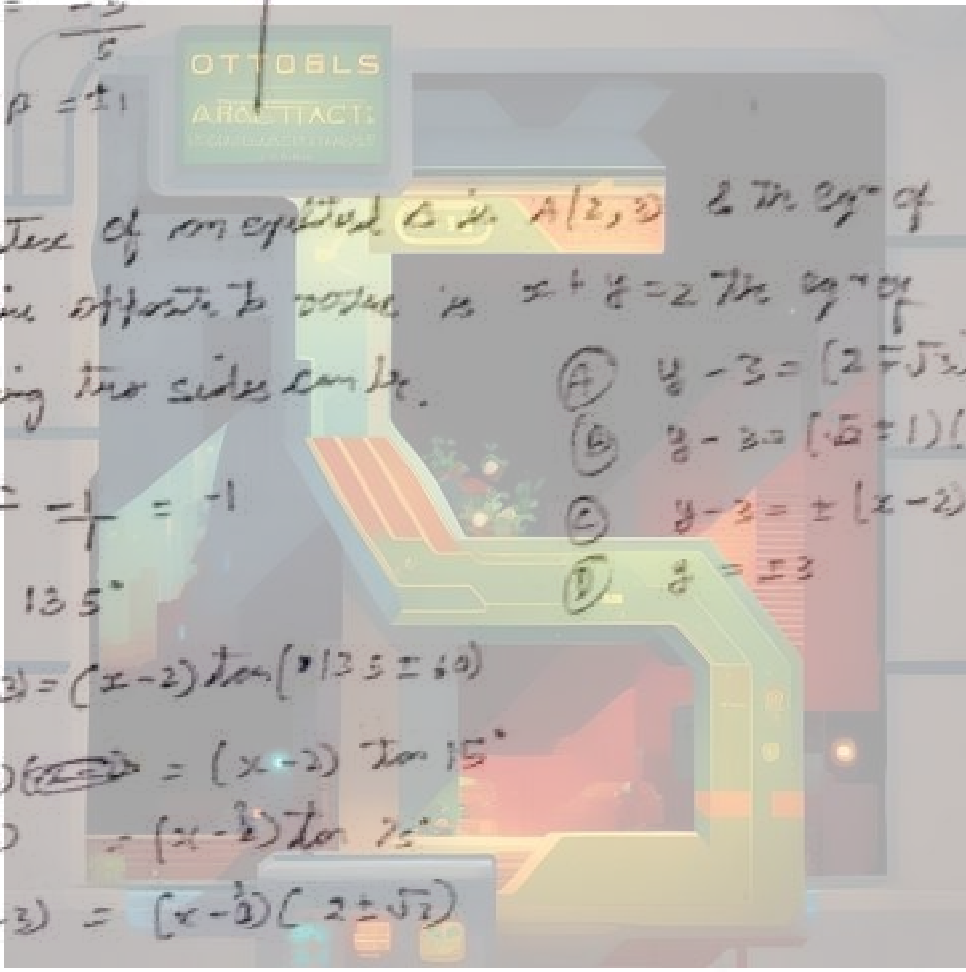
$\tan\beta = \pm 1$

eq^n

$y - 3 = (x - 2)(\pm 1)$

$y - 3 = 2 - x \qquad y - 3 = x - 2$

$\boxed{x + y - 5 = 0} \qquad \boxed{x - y + 1 = 0}$



Q One vertex of an equilateral Δ is A(2, 3) & the eq^n of the line opposite to vertex is $x + y = 2$. The eq^n of remaining two sides can be.

(A) $y - 3 = (2 \mp \sqrt{3})(x - 2)$
(B) $y - 3 = (\sqrt{3} \pm 1)(x - 2)$
(C) $y - 3 = \pm(x - 2)$
(D) $y = \pm 3$

$\tan\theta = -\frac{1}{1} = -1$

$\theta = 135°$

$(y - 3) = (x - 2)\tan(135 \pm 60)$

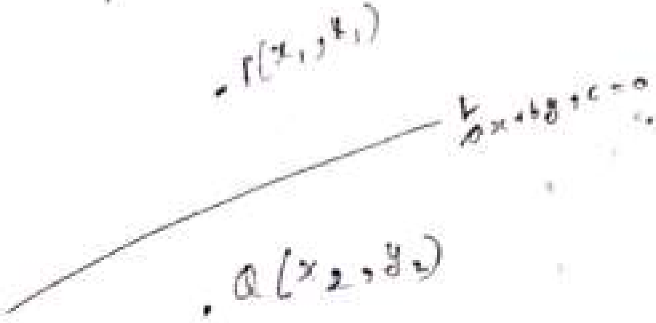
$(y - 3)$ ~~xxxx~~ $= (x - 2)\tan 15°$

$(y - 3) = (x - 2)\tan 75°$

$(y - 3) = (x - 2)(2 \pm \sqrt{3})$

$\boxed{A}$

Position of line w.r.t line.



$\Rightarrow$ $\dfrac{L(x_1, y_1)}{b} > 0$

$P(x_1, y_1)$ lies above the line

$\Rightarrow$ $\dfrac{L(x_2, y_2)}{b} < 0$

$Q(x_2, y_2)$ lies ~~above~~ below the line.

$\Rightarrow$ $\dfrac{L(x_1, y_1)}{b} = 0$

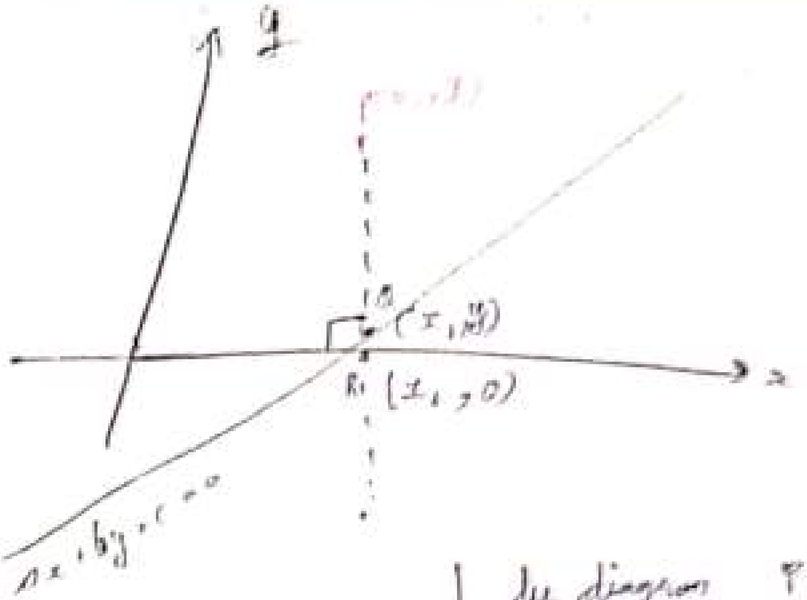
$P(x_1, y_1)$ lies on the line

$L(x_1, y_1) \times L(x_2, y_2)$

< 0 P & Q lies on opp sides

> 0 P & Q lies on same sides.

Proof:- Let $P(x_1, y_1)$ lie above the line $ax+by+c=0$



by diagram $PR > QR$

$y_1 > y$

$PR = y_1$

Q lies on the line

$ax_1 + by + c = 0$

$by = -(ax_1 + c)$

$y = \frac{-(ax_1 + c)}{b}$

$y_1 > -\frac{(ax_1 + c)}{b}$

$0 > -\frac{(ax_1 + by_1 + c)}{b}$

$\frac{ax_1 + by_1 + c}{b} > 0$

$\frac{L(x_1, y_1)}{b} > 0$

अतः

Proof 2:

$\frac{L(x_1, y_1)}{b} > 0 \qquad \frac{L(x_2, y_2)}{b} < 0 \qquad \frac{L(x_3, y_3)}{b} > 0$

$\frac{L(x_1, y_1)\, L(x_2, y_2)}{b^2} = \frac{(+)(-)}{(+)} = (-)$

Q find the position of $(3,-4)$ (1) wrt the line $3x-4y+5=0$
(2) wrt origin in above line.

(1) $L(3,-4)$

$= 3(3)-4(-4)+5$

$= 9+16+5$

$= 30$

$\frac{30}{-4}$ $0<0$

below line

(2) $L(3,-4) \cdot L(0,0)$

$30 \cdot 5 > 0$

~~above line~~

both same side.

H.W

$2y-4-24$

DYS-9

O-2 65-112

Q find position of $(3,4)$ & $(-9,1)$ in the line $7x+5y-9=0$

$L(3,4) \cdot L(-7,6)$

$(32)(-42) = \ominus ve$

On opp sides

Q If the point $(1,2)$ & $(3,4)$ are on opposite side of $3x-5y+a=0$ find a.

$L(1,2) \cdot L(3,4) < 0$

$(3-10+a)(9-20+a) < 0$

$(a-7)(a-11) < 0$

$\boxed{a \in [7,11)}$

Q If $P(2, 1/2)$ & ΔABC is $A(2,1)$ $B(0,0)$ & $C(3,0)$ then P lies in.

(A) ① Inside Δ (B) outside Δ (C) above or any side (D) P, A, B & C are co-linear

Area ≠ 0

$AB \Rightarrow y = \frac{1}{2}x$ $BC \Rightarrow y = 0$ $AC \Rightarrow y = (x-3)-1$

$x = 2y$ $y = x - 2$

$x - 2y = 0$ $x + y - 3 = 0$

$L(2, 1/2)$ $L'(2, 1/2)$ $L_2(2, 1/2)$

$= 1$ above $= -1/2$

above below

$\boxed{A}$

Reflection

$P(x, y)$

→ Reflection in x axis $Q(x, -y)$

→ Reflection in y axis $R(-x, y)$

→ Reflection in $y = x$ & $S(y, x)$

→ Reflection in $y = -x$ $T(-y, -x)$

→ Reflection in origin. $U(-x, -y)$

→ Take reflection in x axis & then in y-axis.

Q find the image of the point / reflection

① $(1,2)$ in x-axis
② $(-3,4)$ in y-axis
③ $(4,2)$ in $x=y$
④ $(-3,6)$ in $y=-x$
⑤ $(-2,4)$ in origin

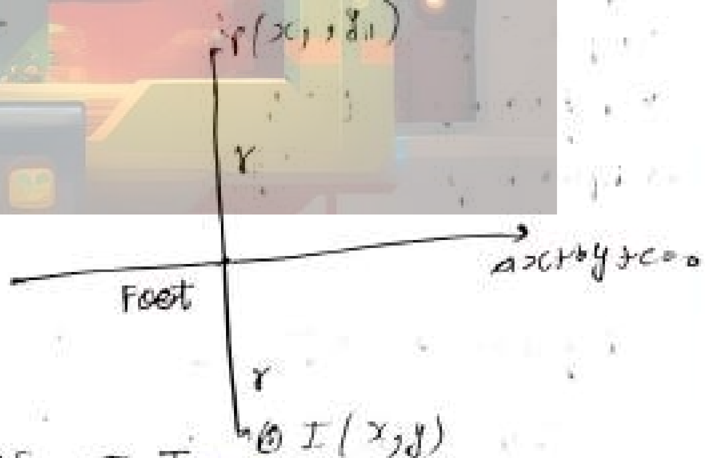
① $(1,-2)$
② $(3,4)$
③ $(2,4)$
④ $(-6,3)$
⑤ $(2,-4)$

* General formula of reflection in any line $ax+by+c=0$

$$\boxed{\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}}$$

Proof:- $m_{PI}=\frac{b}{a}$

$\tan\theta=\frac{b}{a}$ → $\cos\theta=\frac{a}{\sqrt{a^2+b^2}}$, $\sin\theta=\frac{b}{\sqrt{a^2+b^2}}$



$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=\pm r$$

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\frac{(ax_1+by_1+c)}{a^2+b^2}\ [\text{foot}]$$ Image

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-2\frac{(ax_1+by_1+c)}{a^2+b^2}\ [\text{image}]$$

Q find the foot & the image of the point $(2,3)$ in the line $3x+4y+7=0$

$I(x',y)$

$$\frac{x-2}{3} = \frac{-2(25)}{25} \quad\Bigg|\quad \frac{y-3}{4} = \frac{-2(25)}{25}$$

$x-2=-6 \quad | \quad y-3=-8$

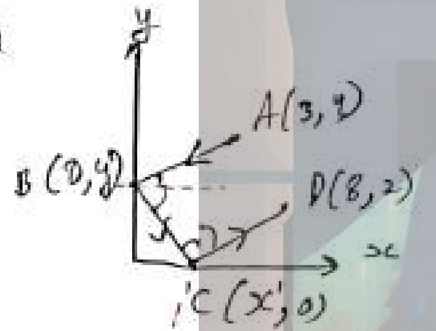
$x=-4 \quad | \quad y=-5$

$\boxed{I(-4,-5)}$

~~F(...)~~

$\boxed{F(-1,-1)}$

Q



find (x,y).

~~$BC \Rightarrow y = (x - x')\frac{-y'}{x'}$~~

$BC \Rightarrow y x' = x'y' - xy'$

$x'y + y'x = x'y'$

$$\frac{0-3}{y'x'} = \frac{y-4}{x'y'} = \frac{-2(3y'+4x'-x'y')}{(x')^2+(y')^2}$$

$A'(-3,4)$

$D'(8,-2)$

$A'D' \Rightarrow \frac{6}{-11} = -\frac{6}{11}$

$A'B' \Rightarrow y-4 = (x+3)\left(\frac{-6}{11}\right)$

$11y-44 = -6x-18$

$6x+11y-26=0$

$\boxed{y = \frac{26}{11}, \quad \frac{13}{3} = x}$

Q A ray of light along $x + \sqrt{3}y = \sqrt{3}$ is incidented on x axis & gets reflected, find the eqⁿ of reflect ray.

$\left(1, \frac{\sqrt{3}-1}{\sqrt{3}}\right)$ $(\sqrt{3}, 0)$

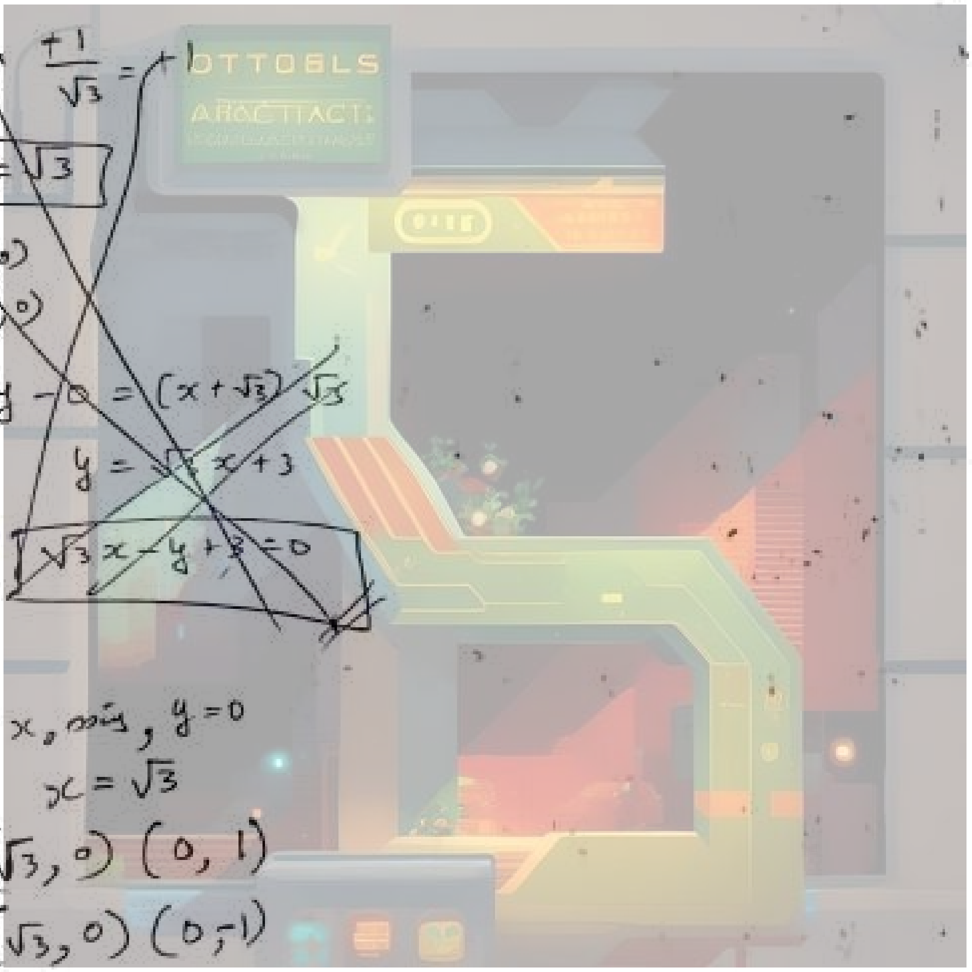
$A' = \left(-1, \frac{\sqrt{3}-1}{\sqrt{3}}\right)$ ~~$A' = (-\sqrt{3}, 0)$~~

~~$B' = (0, \sqrt{3}, 0)$~~

$B' =$

$m(L) \times \frac{+1}{\sqrt{3}} = +1$

$\boxed{m = \sqrt{3}}$



$(\sqrt{3}, 0)$

$A'(-\sqrt{3}, 0)$

line ⇒ $y - 0 = (x + \sqrt{3})\sqrt{3}$

$y = \sqrt{3}x + 3$

$\boxed{\sqrt{3}x - y + 3 = 0}$

at x axis, $y = 0$

$x = \sqrt{3}$

$(\sqrt{3}, 0)\ (0, 1)$

$(\sqrt{3}, 0)\ (0, -1)$

$m = \frac{1}{\sqrt{3}}$

$y - 0 = (x - \sqrt{3})\frac{1}{\sqrt{3}}$

$\sqrt{3}y = x - \sqrt{3}$

$\boxed{x - \sqrt{3}y - \sqrt{3} = 0}$

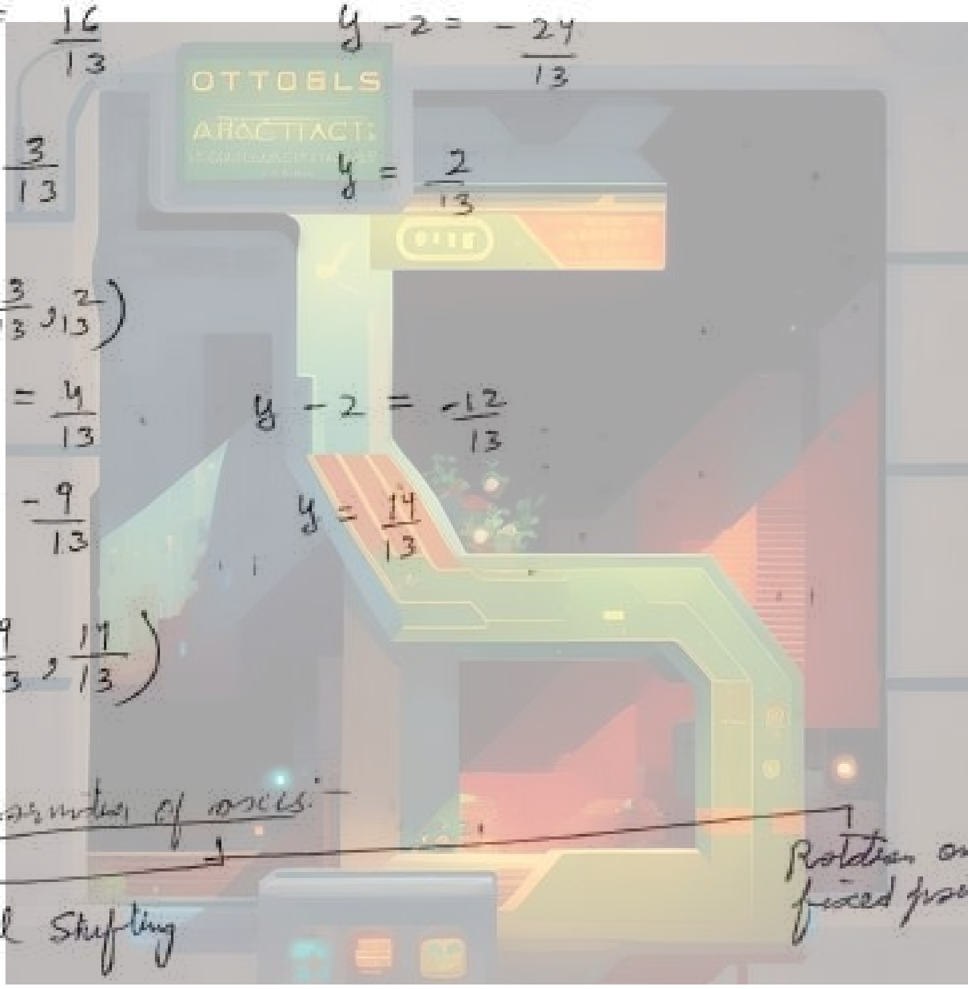
H.W.

Q Find the root & Image of point $P(-1,2)$ in the line mirror

$$2x-3y+4=0$$

$$\frac{-2(-2-6+4)}{4+9} = \frac{8}{13}$$

$$\frac{x+1}{2} = \frac{8}{13} \qquad \frac{y-2}{-3} = \frac{8}{13}$$



$$x+1 = \frac{16}{13} \qquad y-2 = -\frac{24}{13}$$

$$x = \frac{3}{13} \qquad y = \frac{2}{13}$$

$$I = \left(\frac{3}{13}, \frac{2}{13}\right)$$

$$x+1 = \frac{4}{13} \qquad y-2 = \frac{-12}{13}$$

$$x = \frac{-9}{13} \qquad y = \frac{14}{13}$$

$$F = \left(\frac{-9}{13}, \frac{14}{13}\right)$$

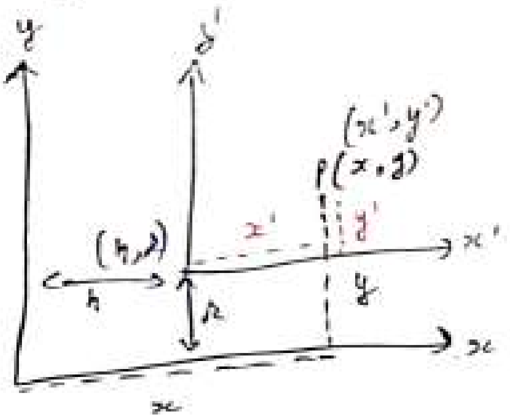
* Transformation of axes:

Parallel Shifting

Rotation on a fixed point

Parallel Shifting

Parallel Shifting



$\boxed{x = h + x'}$ $\boxed{y = y' + k}$

(h, k) jaha origin ko le gaye.

(x', y') new coordinate

(x, y) old coordinate

Q If axes are transformed from origin to the point $(-2, 1)$, then the new co-ordinates of $(4, -5)$ is -

(A) $(6, 4)$ (B) $(-2, 6)$ (C) $(6, -6)$ (D) $(2, -4)$

$x' = x - h$
$x' = 4 - (-2)$
$x' = 6$

$y' = y - k$
$y' = -5 - 1$
$y' = -6$

$6, -6$

$\boxed{C}$

H.W. 25-07-24

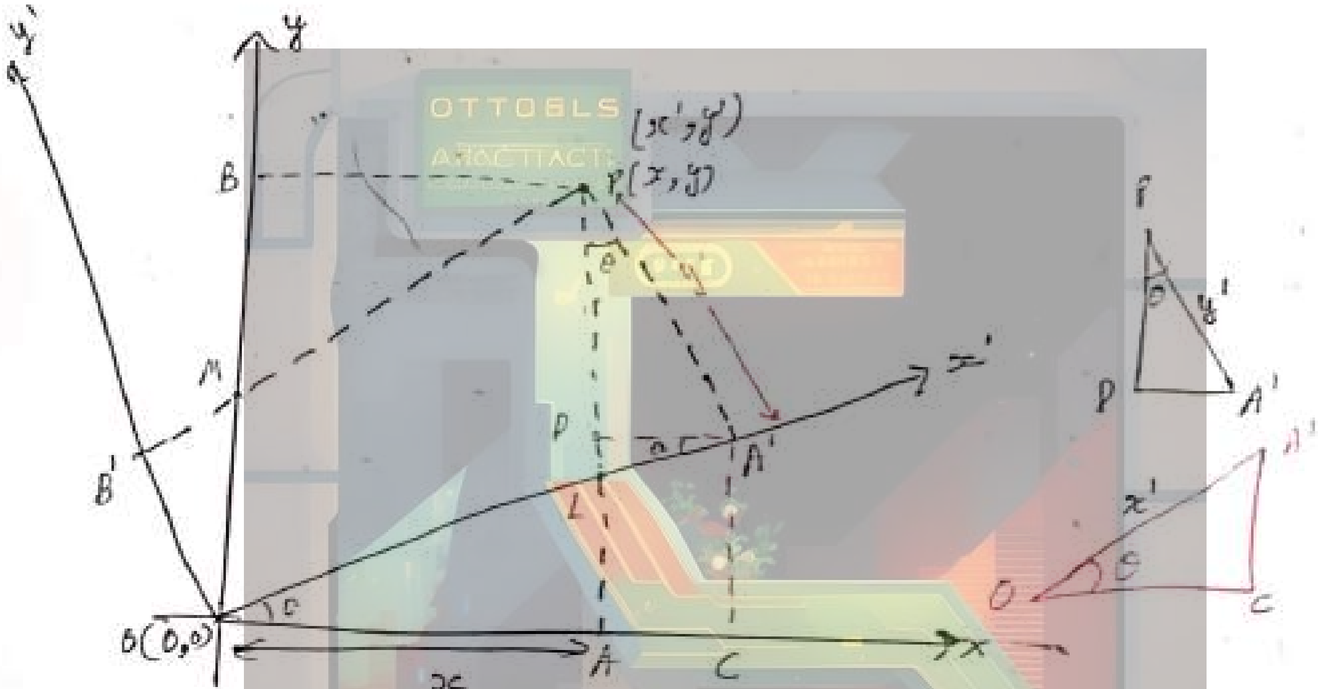
DYS-10 {1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

O-1 {15, 16, 20}

O-2 {9, 10, 6-8, 4-5}

O-4 {4, 6, 7, 8, 9, 10} + {2, 3}

(2) Rotation on fixed point → Here we consider rotation in anticlockwise dir. So in question if axis are rotated clockwise then we use $-\theta$ everywhere.



$OC = OA + AC$

$OC = x + AC$

$x = OC - AC$

$AP = AD + DP$

$y = AD + DP$

in $\Delta A'DP$

$\cos\theta = \frac{DP}{y'}$

$\sin\theta = \frac{AC}{y'}$

in $\Delta OCA'$

$\cos\theta = \frac{OC}{x'}$

$\sin\theta = \frac{AP}{x'}$

$x = x'\cos\theta - y'\sin\theta$

$y = x'\sin\theta + y'\cos\theta$

old / new	x	y
x'	$\cos\theta$	$\sin\theta$
y'	$-\sin\theta$	$\cos\theta$

$$\boxed{\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}}$$

$$\boxed{\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}}$$

Q Keeping the origin constant, axes are rotated at an angle of 30° in ACW. Find the co-ordinate of (2,1) wrt. the new ~~axes~~ axes.

$x' = x\cos\theta + y\sin\theta$

$x' = 2\cos 30 + \sin 30$

$x' = \sqrt{3} + \frac{1}{2}$

$y' = -x\sin\theta + y\cos\theta$

$y' = -2\sin 30 + \cos 30$

$y' = \frac{\sqrt{3}}{2} - 1$

$$\boxed{\left(\sqrt{3}+\frac{1}{2},\ \frac{\sqrt{3}}{2}-1\right)}$$

Q If the origin is taken to (1,1) & the axes are rotated by 45° ACW. Find the co-ordinates of (4,5) wrt find transformation.

$x_1 = 4-1$

$x_1 = 3$

$y_1 = 5-1$

$y_1 = 4$

(3, 4)

$x' = 3\cos 45 + 4\sin 45$

$x' = \frac{7}{\sqrt{2}}$

$y' = -3\sin 45 + 4\cos 45$

$y' = \frac{1}{\sqrt{2}}$

$$\boxed{\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}}$$

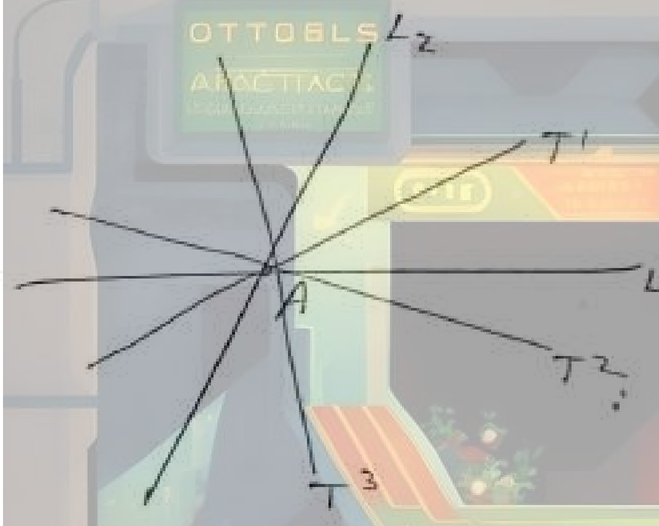
Family of Lines

→ A Group of similar lines having something in common.

eg. 1. all lines passing through (0, 0).

2. all lines ∥ to y-axis.

→ by solving L_1 & L_2 we get co-ordinates of A.



L_1, L_2, T_1, T_2, T_3 forms a family of ~~...~~ lines.

→ $L_1 + \lambda L_2 = 0$
$\lambda \in R$

Type: finding equations.

Q1. $L_1 \Rightarrow 3x + 4y + 6 = 0$
$L_2 \Rightarrow x + y + 2 = 0$] find eq^n of line which passes through POI of L_1 & L_2 &

i) through (2, −3)
ii) ∥ to the line $(x + 2y + 3) = 0$
iii) ⊥ to the line $2x - 3y + 1 = 0$
iv) at distance 2 unit from origin.

(8) $L_1 + \lambda L_2 = 0$

$(3x+4y+6) + \lambda(x+y+2) = 0$

$(\lambda+3)x + (\lambda+4)y + (2\lambda+6) = 0$

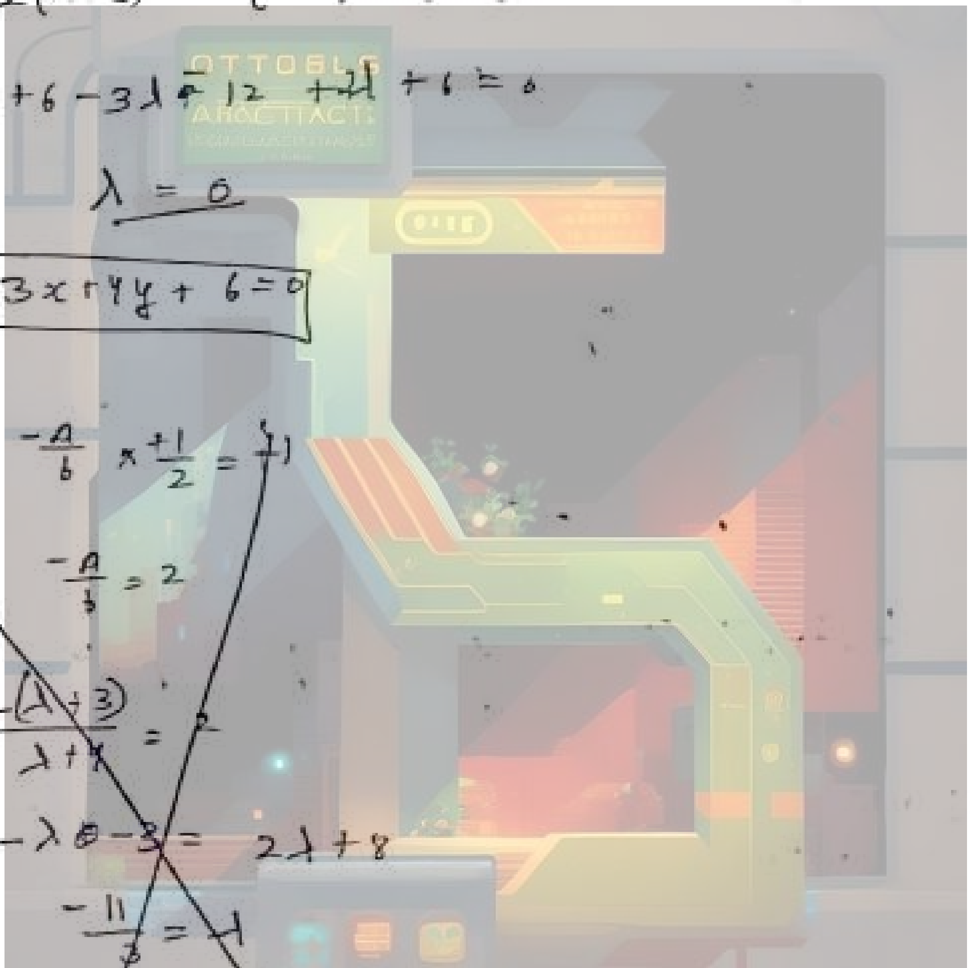
i) $(2,-3)$

$2(\lambda+3) - 3(\lambda+4) + (2\lambda+6) = 0$

$2\lambda + 6 - 3\lambda - 12 + 2\lambda + 6 = 0$

$\lambda = 0$

$\boxed{3x+4y+6=0}$



ii) $-\frac{a}{6}x + \frac{1}{2} = 2$

$-\frac{a}{3} = 2$

$\frac{-(\lambda+3)}{\lambda+4} = 2$

$-\lambda - 3 = 2\lambda + 8$

$-\frac{11}{3} = \lambda$

$-\frac{2}{3}x + \frac{1}{3}y + \frac{-4}{3} = 0$

$\boxed{2x-y+4=0}$

ii) $m_1 = m_2$

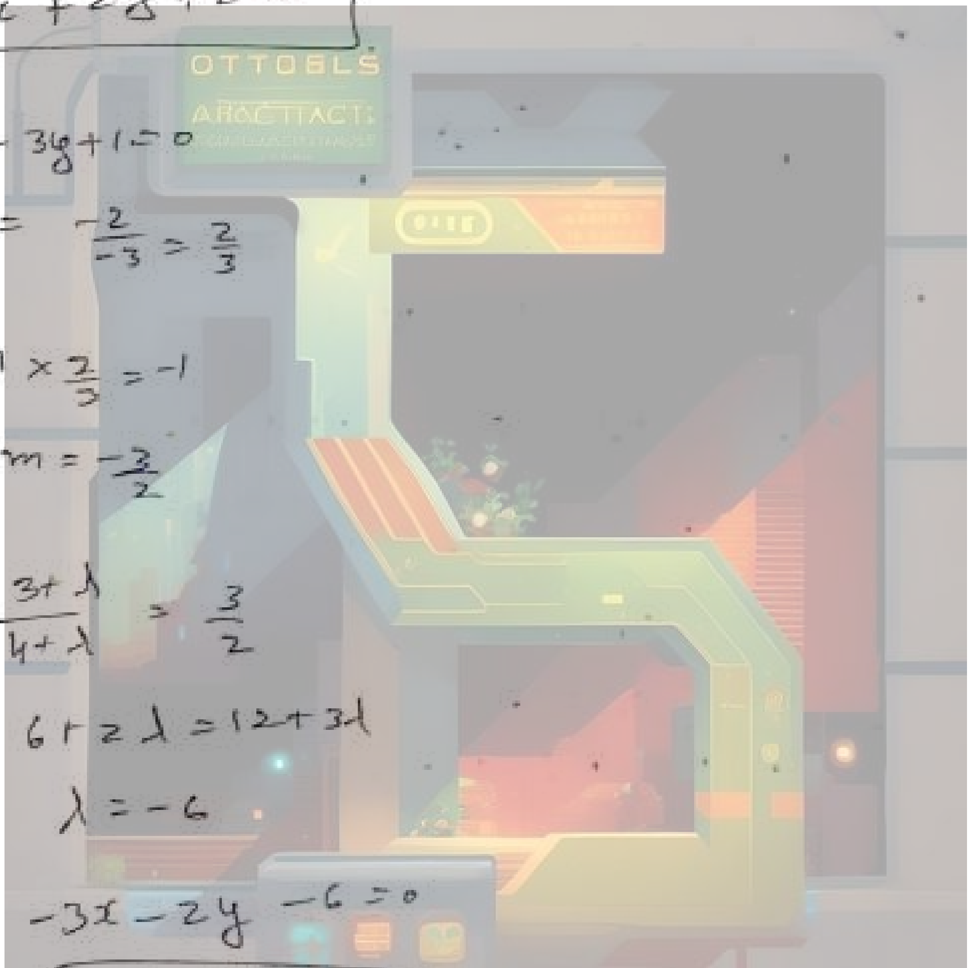
$x+2y+3=0$

$$\frac{+(3+\lambda)}{4+\lambda} = \frac{+1}{2}$$

$\cancel{2}\,6+2\lambda\,\cancel{0} = 4+\lambda$

$\lambda = -2$

$\boxed{x+2y+2=0}$



iii) $2x-3y+1=0$

$m' = \frac{-2}{-3} = \frac{2}{3}$

$m \times \frac{2}{3} = -1$

$m = -\frac{3}{2}$

$$\frac{3+\lambda}{4+\lambda} = \frac{3}{2}$$

$6+2\lambda = 12+3\lambda$

$\lambda = -6$

$-3x-2y-6=0$

$\boxed{3x+2y+6=0}$

iv) $\cancel{x}(3+\lambda)^2 + (4+\lambda)^2 = 94$

$\lambda^2+9+6\lambda+\lambda^2+16+8\lambda=9$

$2\lambda^2+14\lambda+\cancel{8}+16=0$

$\lambda^2+7\lambda+8=0$

$$\lambda = \frac{-7\pm\sqrt{49-32}}{2}$$

iv) $\lambda^2+6\lambda+9+\lambda^2+8\lambda+16=4$

$2\lambda^2+14\lambda+21=0$

$\lambda = \dfrac{-14\pm\sqrt{196-168}}{4}$

$\lambda = \dfrac{-7\pm\sqrt{7}}{4}$

iv) $2 = \left|\dfrac{6+2\lambda}{\sqrt{(3+\lambda)^2+(4+\lambda)^2}}\right|$

$6+2\lambda = 2\sqrt{2\lambda^2+14\lambda+25}$

$4\lambda^2+36+24\lambda = 8\lambda^2+56\lambda+100$

$4\lambda^2+32\lambda+64=0$

$\lambda^2+8\lambda+16=0$

$\lambda^2+4\lambda+4\lambda+16=0$

$\lambda(\lambda+4)+4(\lambda+4)=0$

$\lambda=-4$

$-x-2=0$

$\boxed{x=-2}$

H.W. 26-9-24

DYS-11 {2, 3, 10, 11}

O-1 {17, 20, 18, 25, 26, 27, 29}
imp

Type-2 finding fix point

Q Find the fixed point from which the given family of lines passes, $\lambda \in R, \theta \in R, a, b \in R$

① $(x-2y+1)+\lambda(x+y)=0$

② $x(\lambda+1)+y(2-\lambda)+5=0$

③ $ax(a+2b)+y(a+3b)-(a+b)=0$

④ $x(\cos\theta+\sin\theta)+y(\cos\theta-\sin\theta)-3(3\cos\theta+\sin\theta)=0$

① $x-2y+1=0$
$-x-y=0$

$-3y=-1$
$y=1/3$
$x=-1/3$

$\boxed{\left(-\frac{1}{3}, \frac{1}{3}\right)}$

② $(x+2y+5)+\lambda(x-y)=0$

$x+2y+5=0$
$2x-2y=0$

$3x=-5$
$x=-\frac{5}{3}$

$y=-\frac{5}{3}$

$\boxed{\left(-\frac{5}{3}, -\frac{5}{3}\right)}$

③ $\cancel{a}[x+y-1]+\frac{b}{a}[2x+3y-1]=0$

$2x+2y-2=0$
$2x+3y-1=0$
$y+1=0$
$y=-1$
$x=2$

$\boxed{(2,-1)}$

Note :- $L_1+\lambda L_2=0$
gives all the lines except L_2

for $L_2=0$; $\lambda=\infty$ not possible.

$L_2=0+(L_1+\lambda L_2=0)$

(9) $(x+y-9)+\frac{\cos\theta}{\sin\theta}(x-y-3)=0$

$x+y-9=0$

$x-y-3=0$

$2x=12$

$x=6$

$y=9-6$

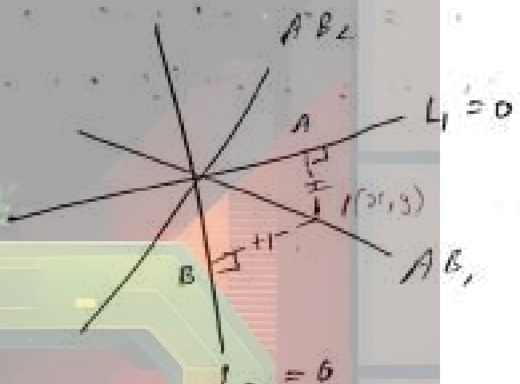
$y=3$

$\boxed{(6,3)}$

Eqⁿ of angle bisectors b/w 2 given lines:

$L_1: a_1x+b_1y+c_1=0$

$L_2: a_2x+b_2y+c_2=0$



$AB_1 \rightarrow$ locus of $P(x,y)$

$PA=PB$

$$\left|\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right|=\left|\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right|$$

$$\boxed{\left(\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}}\right)=\pm\left(\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}\right)}$$

↳ gives both AB_1 & AB_2

$$(a_1x+b_1y+c_1)=\underbrace{\pm\sqrt{\frac{a_1^2+b_1^2}{a_2^2+b_2^2}}}_{\lambda}(a_2x+b_2y+c_2)$$

Note:- Both angle bisectors are mutually $\perp$ to each other.

1) Identification of acute/obtuse $\angle$ bisector

→ Make C_1 & C_2 $\oplus$ve in eq^n of $\angle$ bisector

→ Check the sign of $\boxed{a_1 a_2 + b_1 b_2}$

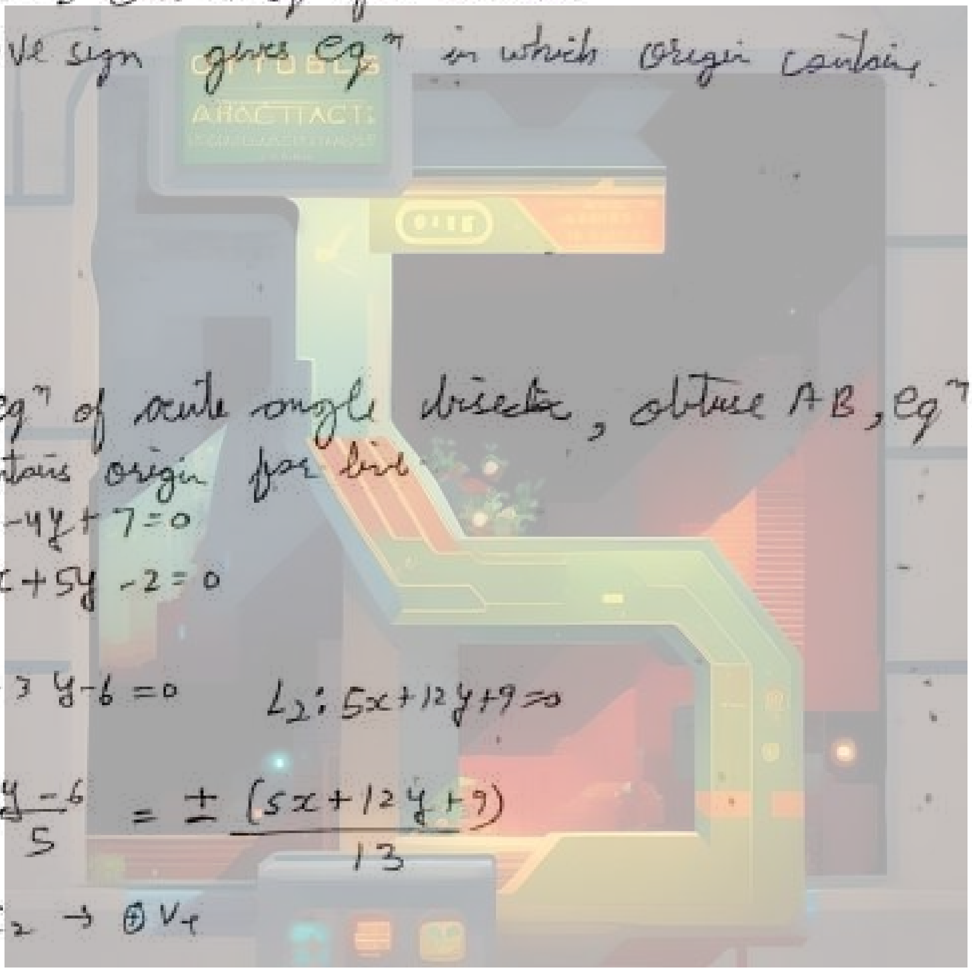
→ If it is $\ominus$ve then $\oplus$ve sign of eq^n gives acute angle bisector

→ If it is $\oplus$ve then $\ominus$ve sign gives acute bisector.

2) on which region of bisector, origin lie

→ make C_1 & C_2 $\oplus$ve in eq^n of $\angle$ bisector

→ More $\oplus$ve sign gives eq^n in which origin contains.



Q find the eq^n of acute angle bisector, obtuse AB, eq^n of AB that contains origin for line

① $L_1: 3x - 4y + 7 = 0$

$L_2: 12x + 5y - 2 = 0$

② $L_1: 4x + 3y - 6 = 0$ $\quad L_2: 5x + 12y + 9 = 0$

② $\dfrac{4x + 3y - 6}{5} = \pm \dfrac{(5x + 12y + 9)}{13}$

$C_1, C_2 \rightarrow \oplus$ve

$$\frac{-4x - 3y + 6}{5} = \pm \frac{(5x + 12y + 9)}{13} \quad \text{—} ①$$

$a_1 a_2 + b_1 b_2 = -20 - 36 < 0 = \ominus$ve

⇒ Put $\oplus$ in eq^n ① where C_1 & C_2 are $\oplus$ve

$-52x - 39y + 78 = 25x + 60y + 45$

$77x + 99y - 33 = 0$

$\boxed{7x + 9y - 3 = 0}$ (acute) (origin)

$$\frac{4x+3y-6}{5} = \frac{5x+12y+9}{13}$$

$$52x+39y-78 = 25x+60y+45$$

$$27x-21y-123=0$$

$$\boxed{9x-7y-41=0} \text{ (obtuse)}$$

(1)

$$\frac{3x-4y+7}{5} = \pm\frac{(-12x-5y+2)}{13}$$

$$a_1a_2+b_1b_2 = -36+20 < 0 \text{ -ve}$$

$$39x-52y+91 = -60x-25y+10$$

$$99x-27y+81=0$$

$$\boxed{11x-9y+27=0} \text{ (acute) (origin)}$$

$$39x-52y+91 = 60x+25y-10$$

$$\boxed{\cancel{-21x-77y+101=0}}$$

$$\boxed{21x+77y-101=0} \text{ (obtuse)}$$

Q find the eqⁿ of Acute Angle bisector btw the lines

① $L1:\ 3x-4y+10=0 \qquad L2:\ 12x+5y-1=0$

② $3x+4y-5=0$ & $12x+5y-7=0$

①

$$\frac{3x-4y+10}{5} = \pm\left(\frac{-12x-5y+1}{13}\right)$$

$$39x-52y+130 = -60x-25y+5$$

$$\boxed{99x-27y+125=0} \text{ (acute)}$$

$$\cancel{39x-52y+130 = 60x+25y-5}$$

②

$$\frac{-3x-4y+5}{5} = \pm\left(\frac{-12x-5y+7}{13}\right)$$

$$a_1a_2+b_1b_2>0$$

$$-39x-52y+65 = +60x+25y-35$$

$$\boxed{99x+77y-100=0} \text{ (acute)}$$

Q find the eqⁿ of bisector of $\angle ABC$ where A, B & C are $(-2,7)$, $(4,1)$ & $(-3,0)$

$AB \Rightarrow (y-1) = (x-4)(-1)$

$x+y-5=0$

$BC \Rightarrow y = (x+3)\left(\frac{1}{7}\right)$

$x-7y+3=0$

$$\frac{-x-y+5}{\sqrt{2}} = \pm\frac{(x-7y+3)}{5\sqrt{2}}$$

$$5x+5y-25 = x-7y+3$$

$$4x+12y-28=0$$

$$\boxed{x+3y-7=0}$$

H.W. 27-9-24

Pair of Straight line

① Homogenous Eqⁿ of (2) degree.

→ Always pass through origin (0,0).

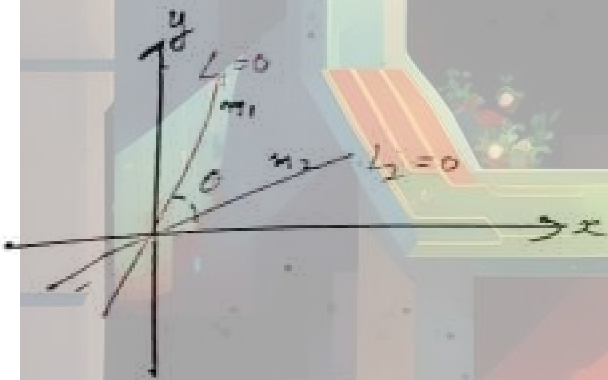
→ Degree of variables is same in each term

$$ax^2+2hxy+by^2=0$$

$$a+2h\left(\frac{y}{x}\right)+b\left(\frac{y}{x}\right)^2=0$$

$$bm^2+2hm+a=0 \qquad \left(\frac{y}{x}=m\right)$$

$$m_1+m_2=\frac{-2h}{b} \qquad m_1m_2=\frac{a}{b}$$



$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\boxed{\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}}$$

If $\boxed{\theta=0,\ h^2=ab}$ lines are coincident

If $\boxed{\theta=90,\ a+b=0}$ lines are ⊥.

If $a=b$ → slopes of lines are reciprocals. (m & $\frac{1}{m}$)

If $h=0$ → slopes are opposite sign same magnitude.

from eq ①

$D = 4h^2 - 4ab$

$D = 4(h^2 - ab)$

$h^2 - ab$

- >0 real & diff lines
- $=0$ real & coincide
- <0 lines are imaginary but POI is $(0,0)$.

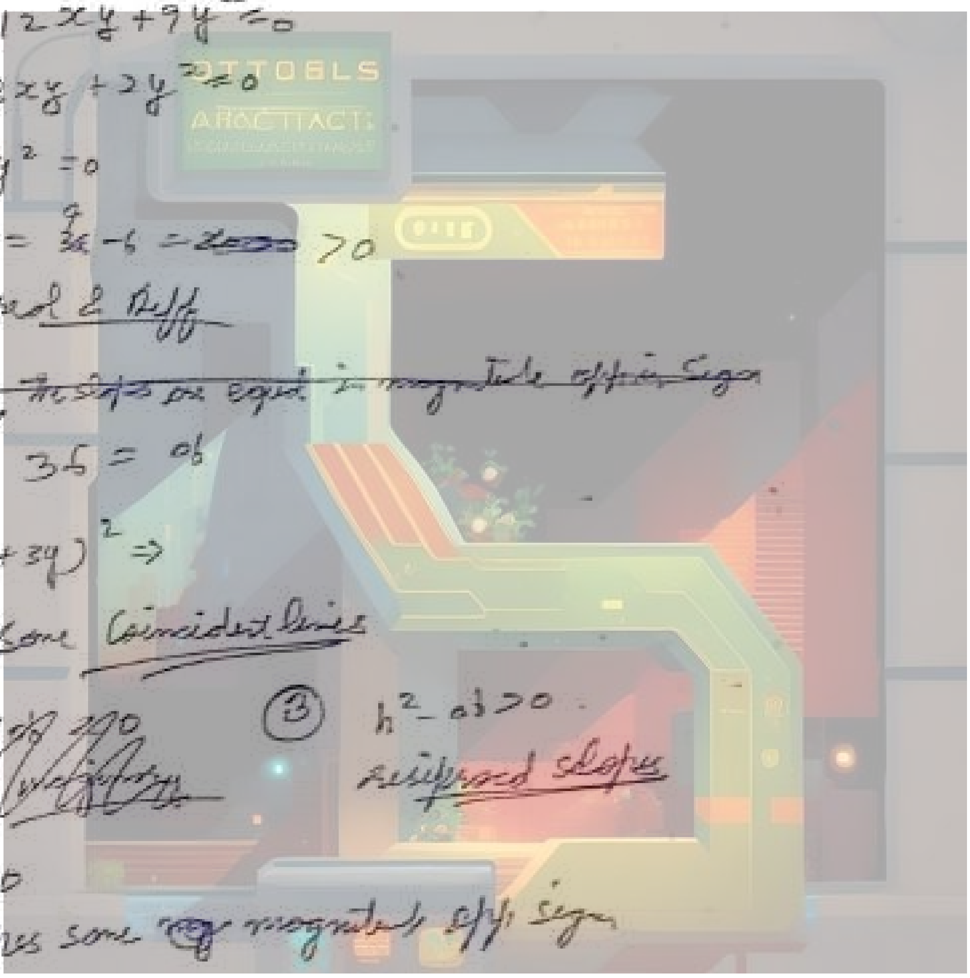
Q ~~Identify~~ Identify the lines.

① $2x^2 + 6xy + 3y^2 = 0$

② $4x^2 + 12xy + 9y^2 = 0$

③ $2x^2 + 8xy + 2y^2 = 0$

④ $4x^2 - 9y^2 = 0$



① $h^2 - ab = 9 - 6 = \cancel{3} > 0$

real & diff

② ~~the slopes are equal in magnitude opp in sign~~

② $h^2 = 36 = ab$

$(2x+3y)^2 \Rightarrow$

same Coincident lines

③ $h^2 - ab > 0$

reciprocal slopes

④ $h = 0$

Slopes same magnitude diff sign

Q solve ~~...~~ $= 0$ & find eq^n of lines

$x^2 - 5xy + 6y^2$

$bm^2 + 2mh + a = 0$

$6m^2 - 10m + 1 = 0$

$m = \frac{10 \pm \sqrt{100 - 36}}{12}$

$m = \frac{10 \pm 8}{12}$

$m = \frac{1}{6}, \frac{3}{2}$

$$x^2 - 2xy - 3xy + 6y^2 = 0$$

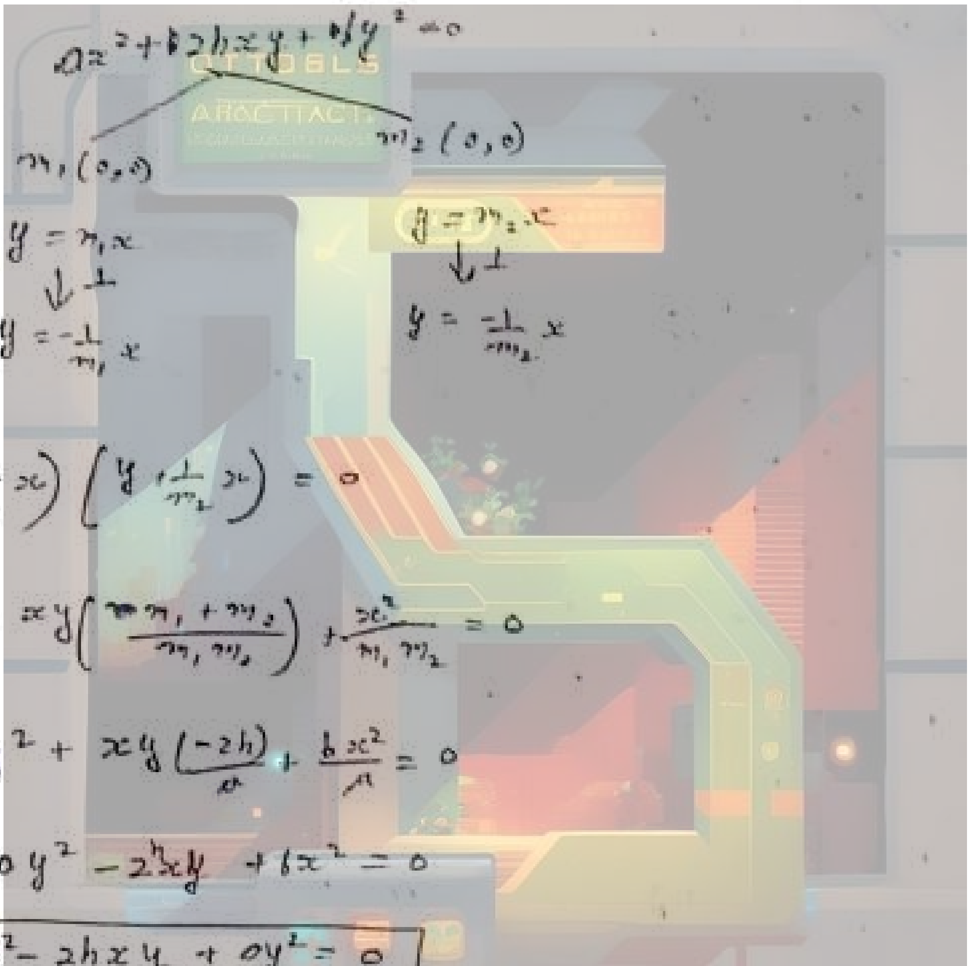
$$x(x-2y) - 3y(x-2y) = 0$$

$$\cancel{x(x-2y)}$$

$$(x-3y)(x-2y) = 0$$

$$\boxed{x - 3y = 0} \qquad \boxed{x - 2y = 0}$$

\# Pair of SOL $\perp$ to $ax^2 + 2hxy + by^2 = 0$



$$ax^2 + 2hxy + by^2 = 0$$

$m_1\ (0,0)$ $\qquad$ $m_2\ (0,0)$

$$y = m_1 x \qquad\qquad y = m_2 x$$

$$\downarrow \perp \qquad\qquad \downarrow \perp$$

$$y = -\frac{1}{m_1}x \qquad\qquad y = \frac{-1}{m_2}x$$

$$\left(y + \frac{1}{m_1}x\right)\left(y + \frac{1}{m_2}x\right) = 0$$

$$y^2 + xy\left(\frac{m_1 + m_2}{m_1 m_2}\right) + \frac{x^2}{m_1 m_2} = 0$$

$$y^2 + xy\frac{(-2h)}{a} + \frac{bx^2}{a} = 0$$

$$ay^2 - 2hxy + bx^2 = 0$$

$$\boxed{bx^2 - 2hxy + ay^2 = 0}$$

Q find ~~out~~ the combined eq^n of $\perp$ lines to

(1) $2x^2 + 6xy + 3y^2 = 0$ (2) $2x^2 + 3xy - 3y^2 = 0$ (3) $4x^2 - 9y^2 = 0$

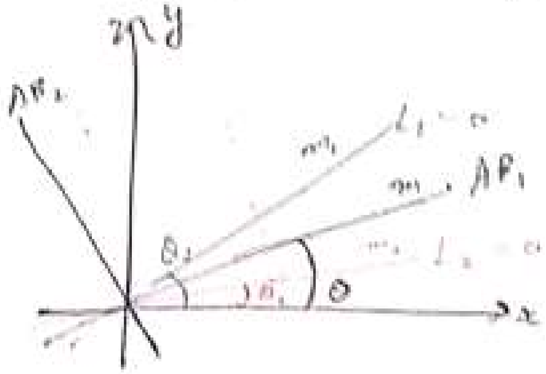
(1) $3x^2 - 6xy + 2y^2 = 0$

(2) $-3x^2 - 3xy + 2y^2 = 0$

(3) $-9x^2 + 4x^2 = 0$

~~Angle bisector~~

Combined eqn of angle bisector for homogeneous eqn



→ Always pass through (0,0) (∠ bisectors).

$$\theta - \theta_1 = \theta_2 - \theta$$

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

$$\frac{2m}{1-m^2} = \frac{m_1 + m_2}{1 - m_1 m_2}$$

$$\cancel{\frac{2m}{1-m^2} = -2h}$$

$$\frac{\frac{2y}{x}}{1 - \frac{y^2}{x^2}} = \frac{-2h}{b-a}$$

$$\frac{2xy}{x^2 - y^2} = \frac{2h}{a-b}$$

$$axy - bxy = hx^2 - hy^2$$

$$\boxed{hx^2 - (a-b)xy - hy^2 = 0}$$

$$\boxed{\frac{x^2 - y^2}{a-b} = \frac{xy}{h}}$$

Q find the Combined eqⁿ of ABs of

(1) $2x^2+3xy-y^2=0$

(2) $x^2-5xy+6y^2=0$

(1) $\frac{3}{2}x^2-3xy-\frac{3}{2}y^2=0$

$3x^2-6xy-3y^2=0$

$x^2-2xy-y^2=0$

(2) $-\frac{5}{2}x^2+\frac{5}{2}y^2+5xy$

$5x^2-8xy-5y^2=0$

$x^2-2xy-y^2=0$

General eqⁿ of 2nd degree representing POSL:

$ax^2+2hxy+2gx+2fy+by^2+c=0$

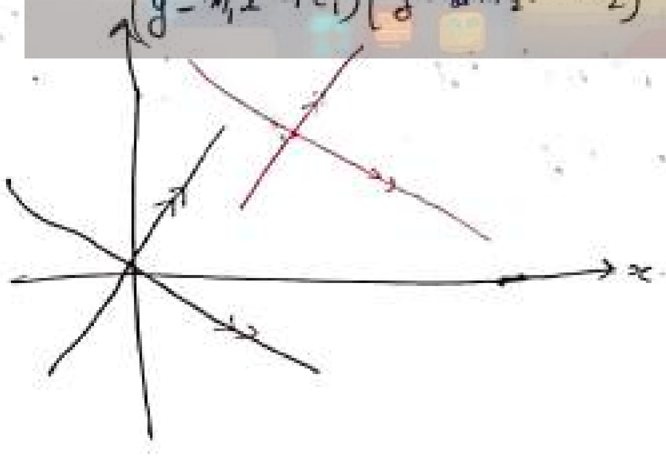
↳ condition for 2 lines.

$\Delta = abc+2fgh-ch^2-bg^2-af^2=0$

$$\Delta = \begin{vmatrix} a & f & g \\ -f & b & h \\ -g & -h & c \end{vmatrix} = 0$$

$$\underbrace{ax^2+by^2+2hxy}_{(y-m_1x)(y-m_2x)} + 2gx+2fy+c=0$$

$$(y-m_1x+c_1)(y-m_2x+c_2)=0$$



Q $3x^2+8xy-3y^2+2x-4y-1=0$

i) find both lines

ii) find their POI.

Sol $3x^2+8xy-3y^2$

$3x^2+9xy-xy-3y^2$

$\cancel{x(3x}$

$3x(x+3y)-y(x+3y)=0$

$3x-y=0 \qquad x+3y=0$

$(3x-y+c_1)=0 \qquad (x+3y+c_2)=0$

$(c_1+3c_2)x=2x \qquad (3c_1-c_2)y=-4y$

$c_1+3c_2=2$

$9c_1-3c_2=-12$

$10c_1=-10$

$c_1=-1$

$c_2=1$

$\cancel{(3x-y+1)(3}$

$\boxed{(3x-y-1)(x+3y+1)}$ i)

$9x-3y-3=0$

$x+3y+1=0$

$10x=2$

$x=1/5, \quad y=\frac{-2}{5}$

$\boxed{\left(\frac{1}{5}, \frac{-2}{5}\right)}$ ii)

Trick^5 ~~★ ★~~

for POI. Only POI is same, not lines

$$\frac{\partial f}{\partial x} = 0$$

$$\frac{\partial f}{\partial y} = 0$$

[AIT]

$$\frac{\partial f}{\partial x} = 6x + 8y + 2$$

$$\frac{\partial f}{\partial y} = 8x - 6y - 4 = 0$$

$$4x - 3y - 2 = 0$$

$$3x + 4y + 2 = 0$$

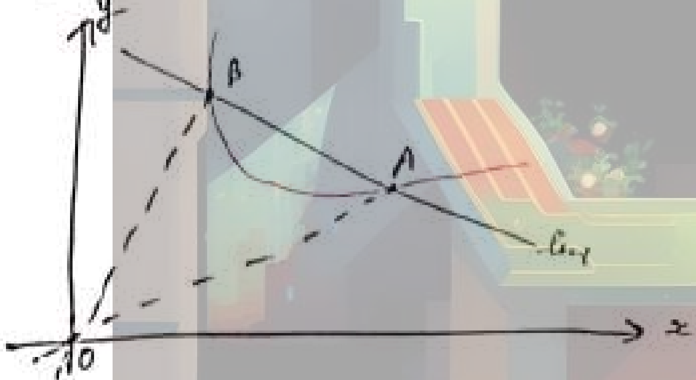
$$7x + y = 0$$

$$\boxed{\left(\frac{1}{5}, \frac{-2}{5}\right)}$$

Angle btw lines

$$\boxed{\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|}$$ Eh

Homogenisation



→ Trying to make degree same in each term.

Q $x^2 - y^2 - xy + 3x - 6y + 18 = 0$

$2x - y = 3$

Homogenise it. / find the eqⁿ of lines passing through (0, 0) & POI of line & curve

$2x - y = 3$

$1 = \frac{2x-y}{3}$ — ①

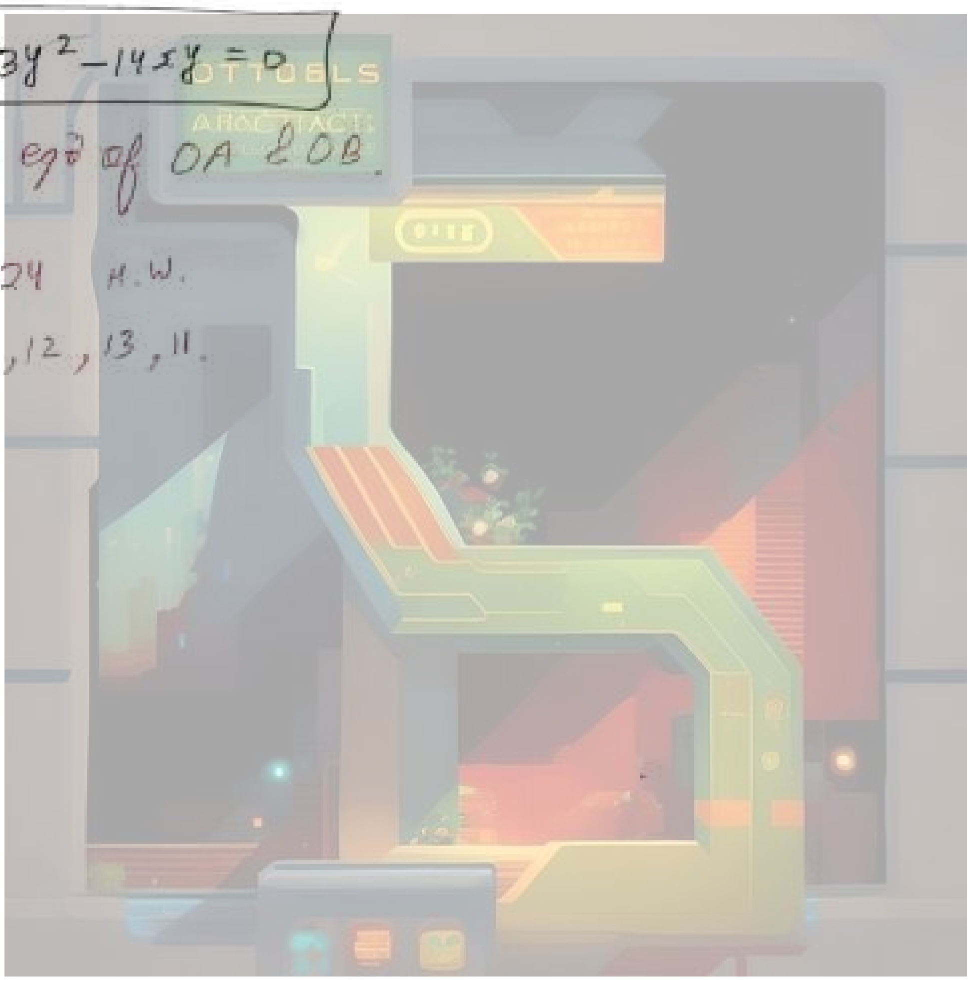
$x^2 - y^2 - xy + 3x(1) - 6y(1) + 18(1)^2 = 0$

$x^2 - y^2 - xy + 3x\left(\frac{2x-y}{3}\right) - 6y\left(\frac{2x-y}{3}\right) + 18 \cdot \frac{(2x-y)^2}{9}$

$x^2 - y^2 - xy + 2x^2 - xy \cancel{\quad} - 4xy + 2y^2 + 8x^2 + 2y^2 - 8xy = 0$

$\boxed{\therefore 11x^2 + 3y^2 - 14xy = 0}$

↳ eqⁿ of OA & OB.



28-9-24 H.W.

DYS 10, 12, 13, 11.