Maths - 2

Ch-3 Quadratic Equation

Ch-4 Logarithm

MS-2 (ch-3)

Q24. $x_1 = 9x_2$

$x_1 + x_2 = 3a+2$

$10x_2 = 3a+2$

$x_2 = \frac{3a+2}{10}$

$x_1 x_2 = a^2$

$\cancel{(3x_2)^2 = a^2}$

$\cancel{3x_2 = a}$

$\cancel{9 \times \frac{9a^2+4+12a}{100} = a^2}$

$\cancel{81a^2+36+108a = 100a^2}$

$\cancel{19a^2-108a-36=0}$

$\cancel{x^2-x+2=0}$

$\cancel{x^2-2x+x+2}$

$\cancel{x(x-2)+1}$

$\cancel{x=2,-1}$

$19a^2-108a-36=0$

$a = \frac{108 \pm \sqrt{108^2+3736}}{38}$

$a =$

108
108
864
10800
36
720
76
684
2736
3736

11664

11664

34.

$3x_2 = a$

$\frac{9a+6}{10} = a$

$9a+6 = 10a$

$6 = a$

$x_2 = 2$

$\boxed{\text{roots} - 18, 2}$

$\boxed{-\frac{18}{17}, \frac{2}{17}}$

$108+x = 38\times 6$

$x = \frac{38\times 6}{108}$

54

65
5
325

63
3
189

626 64
2 4
256

104

3

104

①

Condition for common roots

Types

- Case 1 → Only 1 common root
- Case 2 → Both common roots
- Case 3 → At least one root common
  - Case 1
  - Case 2
  - union
- Case 4 → Quic question: one root common between a ~~com~~ cubic & outside

Case 1 → Only 1 common root

$$a_1x^2+b_1x+c_1=0 \qquad a_2x^2+b_2x+c_2=0$$

Roots of $a_1x^2+b_1x+c_1=0$: α, β

Roots of $a_2x^2+b_2x+c_2=0$: α, γ

$\therefore\ a_1\alpha^2+b_1\alpha+c_1=0$ (multiply by a_2) $\qquad a_2\alpha^2+b_2\alpha+c_2=0$ (multiply by a_1)

$$a_1a_2\alpha^2+a_2b_1\alpha+c_1a_2=0 \quad \text{(subtract)} \quad a_1a_2\alpha^2+a_1b_2\alpha+c_2a_1=0$$

$$(a_2b_1-a_1b_2)\alpha+a_1c_2+a_2c_1=0$$

Q find λ if $x^2 - \lambda x - 21 = 0$ & $x^2 - 3\lambda x + 35 = 0$ have one root common.

$\alpha^2 - \lambda\alpha - 21 = 0$ --- ① $\alpha^2 - 3\lambda\alpha + 35 = 0$ --- ②

~~$2\alpha^2$~~ ② − ①

$-2\lambda\alpha + 56 = 0$

$2\lambda\alpha = 56$

$\boxed{\lambda\alpha = \frac{56}{2} = \frac{28}{\cancel{2}}}$

$\alpha = \frac{28}{\lambda}$

put in ①

$\left(\frac{28}{\lambda}\right)^2 - \lambda \times \frac{28}{\lambda} - 21 = 0$

$\frac{(28)^2}{\lambda^2} - 49 = 0$

$\frac{28}{\lambda^2} = 49$

$\frac{28^2}{49^2} = \lambda^2$

$\lambda = \frac{28}{\pm 7}$

$\boxed{\lambda = \pm 4}$

Q find a if $x^2+(a^2-2)x-2a^2=0$ & x^2-3x+2 have only one root common.

$\alpha^2+(a^2-2)\alpha-2a^2=0$ ② $\alpha^2-3\alpha+2=0$ ①

② − ①

$a^2-2\alpha+3\alpha-2a^2-2=0$

$-a^2+\alpha-2=0$

$\cancel{a^2=\alpha-2}$

$\cancel{a=\sqrt{\alpha-2}}$

$\alpha=a^2+2$

put in ①

$(a^2+2)^2+3(a^2+2)+2=0$

$a^4+4+4a^2+3a^2+6+2=0$

$a^4+7a^2+12=0$

$a^2=\frac{-7\pm\sqrt{49-48}}{2}$

$a^2=\frac{-7\pm1}{2}$

$a^2=-4,-3$

X

$a^2\alpha-2\alpha+3\alpha-2a^2-2=0$

$a^2\alpha+\alpha-2a^2-2=0$

$\alpha(a^2+1)-2(a^2+1)=0$

$(a^2+1)(\alpha-2)=0$

+V

$\alpha-2=0$

$\alpha=2$

so for every value of a, equation satisfies.

$\boxed{a\in R}$

Q If $x^2+Px+Q=0$ & $x^2+Qx+P=0$

$P \neq Q$ and one common root.

$\alpha^2+P\alpha+Q=0$ $\qquad$ $\alpha^2+Q\alpha+P=0$

(2) - (1)

$P\alpha-Q\alpha+Q-P=0$

$(P-Q)\alpha+Q-P=0$

$P-Q\,\alpha = P-Q$

$\alpha = \frac{P-Q}{P-Q}$

$\alpha=1$

$(1)^2+P+Q=0$

$\boxed{P+Q=-1}$

Case - 2 two roots common -

$a_1x^2+b_1x+c_1=0$ $\qquad$ $a_2x^2+b_2x+c_2=0$

$\alpha \quad \beta$ $\qquad$ $\alpha \quad \beta$

$\alpha+\beta = \frac{+b_1}{a_1} = \frac{+b_2}{a_2}$

$= \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\alpha\beta = \boxed{\frac{c_1}{c_2} = \frac{a_1}{a_2} = \frac{b_1}{b_2}}$

Q If $a, b, c \in R$ & eq $ax^2+bx+c=0$ & $x^2+2x+9=0$ have both roots as common, then find $a:b:c$

$\cancel{\frac{a}{1} = \frac{b}{2} = \frac{c}{9}}$

$\cancel{a:b:c}$

$\cancel{\frac{18a}{9}}$

$\cancel{\frac{18a}{18} = \frac{9b}{18} = \frac{2c}{18}}$

$\cancel{a:b:c}$

$\cancel{\boxed{18:9:2}}$

$$\frac{1}{a} = \frac{2}{b} = \frac{9}{c}$$

$a:b:c$

$\boxed{1:2:9}$

Q. $2x^2+x+k=0$ & $x^2+\frac{x}{2}-1=0$ have 2 common roots find k

$$\frac{2}{1} = \frac{1}{\frac{1}{2}} = \frac{k}{-1}$$

$$\frac{2}{1} = \frac{-k}{1}$$

$\boxed{-2=k}$

Q. find k $x^2+2kx+1=0$ & $x^2+x+1=0$ have 1 common root

$D<0$, so roots are in pair

as one is common, other will be common as well.

$$\frac{2k}{1} = 1$$

$2k=1$

$\boxed{k=\frac{1}{2}}$

Case 3 At least one root common.

Q Possible values of 'a' for which $x^2+ax+1=0$ & $x^2+x+a=0$ have at least one common root

Case 1 $x^2-x^2+ax-x+1-a=0$

$x(a-1)-1(a-1)=0$

$(a-1)(x-1)=0$

$\boxed{a=1}$ $\longrightarrow$ $x-1=0$

$x=1$

$(1)^2+a(1)+1=0$

$a+2=0$

$\boxed{a=-2}$

Case 2 $\frac{1}{1}=\frac{a}{1}=\frac{1}{a}$

$\frac{1}{a}=1$ $a^2=1$

$\boxed{a=1}$ $a=\pm1$

$\boxed{a=-1}$ not satisfy

Case 1 $\cup$ case 2 $= a \in \{1, -2\}$

Case 4 QuCu - One common root between a quadratic & cubic

Q $x^3-3x^2+(2k-1)x+3=0$ & $2kx+1-x^2=0$ have one common ~~same~~ root. find k.

$x^3-3x^2+(2k-1)x+3-2kx-1+x^2=0$

$x^3-2x^2+(2k-1-2k)x+2=0$

$x^3-2x^2-x+2=0$

$\boxed{x=1}$

$x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$\boxed{x=2,-1,1}$

$2k+1-1=0$

$2k=0$

$k=0$

$-2k+1+1=0$

$-2k=-2$

$k=1$

$4k+1-4=0$

$k=\frac{3}{4}$

7

① H.W. 20-05-2024

DYS-4 {3, 4, 5, 6, 7}

O-1 {Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Q If the eq $ax^2+bx+c=0$ & $x^3+3x^2+3x+2=0$ have 2 common roots. then -

a) $a=b\neq c$

B) $a=-b=c$

c) $a\neq b\neq c$

✓D) $a+b+c=3$

$-1+3-3+2$

$-8+12-6+2$

$(x+2)=0$

x^2+x+1

$x=\frac{-1\pm\sqrt{1-4}}{2}$

x^2+x+1

ax^2+bx+c

$\frac{1}{a}=\frac{1}{b}=\frac{1}{c}$

$a=b=c$

$a=1$

$1+1+1=3$

Q If $x^3+1=0$ & $ax^2+bx+c=0$ $a,b,c\in R$. have 2 common roots. then $a+b=?$

$x^3+1=0$

$x^3=-1$

$\boxed{x=-1}$

$\cancel{\alpha+\beta=-2}$

$\cancel{\alpha\beta=1}$

$\cancel{x^2+2x+1}$

$(x+1)$ is a factor

$x+1 \overline{)\,x^3+1\,}(x^2-x+1$

$-x^3-x^2$

$-x^2+1$

$+x^2+x$

$x+1$

$\cancel{b=2}$

$\cancel{c=1}$

$\cancel{a=1}$

$\cancel{a+b=2+1}$

$\cancel{=3}$

$\cancel{\boxed{D}}$

x^2-x+1

$\cancel{x=+}D<0$

$x^2-x+1=ax^2+bx+c$

$a=1$

$b=-1$

$\boxed{a+b=0}$

$\boxed{B}$

Quadratic expression and its graphs.

$ax^2+bx+c \ (a,b,c \in R), a \neq 0$

① $a<0$ concave down

$a>0$ concave up

② $D>0$ Roots real & unequal $\rightarrow$ Graph cuts X axis at 2 diff points

$D=0$ Roots are equal $\rightarrow$ Graph touch x-axis

$D<0$ Graph does not cut x-axis.

③ Vertex : $(x, y) = \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

Proof:- $y = ax^2+bx+c$

$$y = a\left[x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$y = \left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2+4ac}{4a^2}\right]$$

$$y = a\left[\left(x+\frac{b}{2a}\right)^2-\left(\frac{-D}{4a^2}\right)\right]$$

$$y = a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a}$$

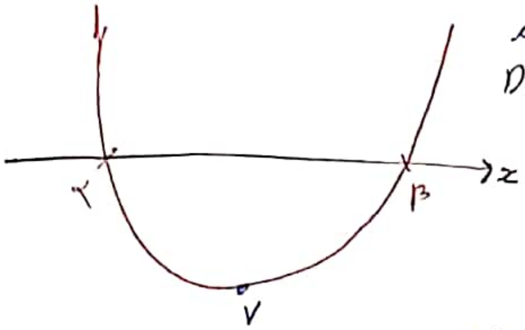
$$\left(y+\frac{D}{4a}\right) = a\left(x+\frac{b}{2a}\right)^2$$

$$\boxed{x = \frac{-b}{2a}}$$

$$\boxed{y = \frac{-D}{4a}}$$

(4) Graphs

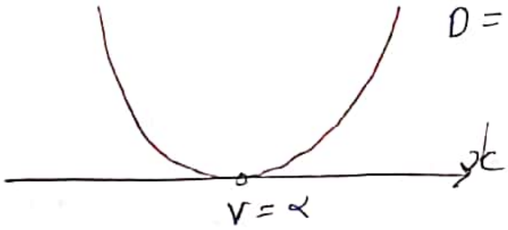
(1)



$a > 0$
$D > 0$

$y = \oplus, \ominus, 0$

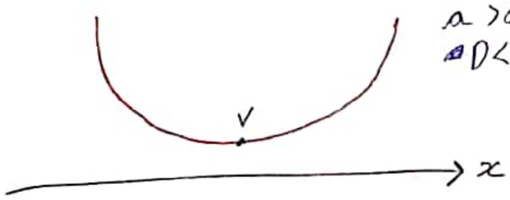
(2)



$a > 0$
$D = 0$

$y = \oplus, 0$

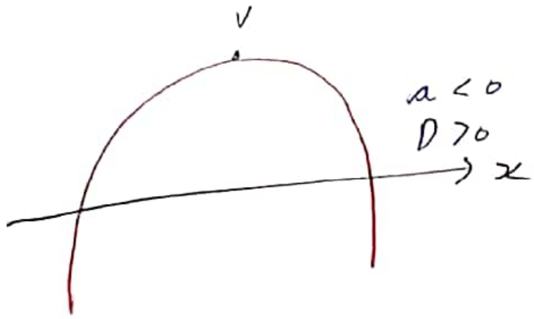
(3)



$a > 0$
$D < 0$

$y = \oplus$

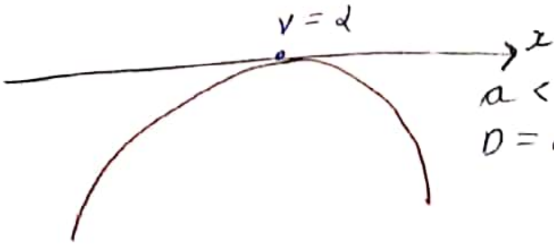
(4)



$a < 0$
$D > 0$

$y = \oplus, \ominus, 0$

(5)

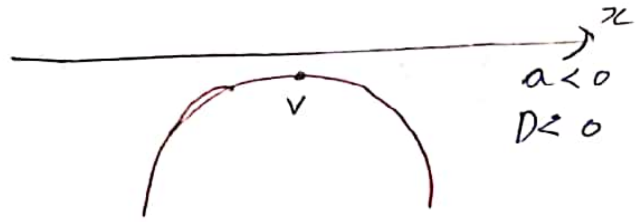


$y = \ominus, 0$

$a < 0$
$D = 0$

(5)

(6)



$y = \ominus$

$a<0$

$D<0$

$a, b, c \in R$

(1) $ax^2+bx+c>0$ $(a>0, \& D<0)$ Quadratic always $\oplus$ve

(2) $ax^2+bx+c<0$ $(a<0, \& D<0)$ Quadratic is always $\ominus$

(3) $ax^2+bx+c \geq 0$ $(a>0, \& D \leq 0)$ Quadratic is $\oplus$ or 0

Q Find 'a' for which $ax^2+3x+4>0$ $x \in R$

$a>0, D<0$

$9-\cancel{4a}16a<0$

$16a>9$

$a>9/16$ $\cup$ $a>0$

$\boxed{a>\frac{9}{16}}$ $a \in \left(\frac{9}{16}, \infty\right)$

Q. $ax^2+2ax+\frac{1}{2}<6$

$a<0, D<0$

$b^2-4ac<0$

$\cancel{4}4a^2-2a<0$

$2a(2a-1)<0$

$a \in (0, 1/2) \cup a<0$

$\boxed{a \in \phi}$

+ 0 − 1/2 +

(12)

Scanned with CamScanner

Q $kx^2+x+k>0$

$a>0$

$D<0$

$1-4k^2<0$

$\cancel{4k^2>1}$

$\cancel{k^2>\frac{1}{4}}$

$\cancel{k>\frac{1}{2}}$

$4k^2-1>0$

$+ \quad -1/2 \quad - \quad 1/2 \quad +$

$k \in [(-\infty,-\frac{1}{2}) \cup (\frac{1}{2},\infty)] \cap (0,\infty)$

$\boxed{k \in (\frac{1}{2},\infty)}$ ✓

H.W. 21-05-2024

DYS-8 [4, 11] − {5}

O-1 [11, 18] − {15, 16, 17}

O-2 [1, 6] − {4}

Range of A Quadratic (values of y)

Range $\in [y_{min}, y_{max}]$

Type 1:- $x \in R$

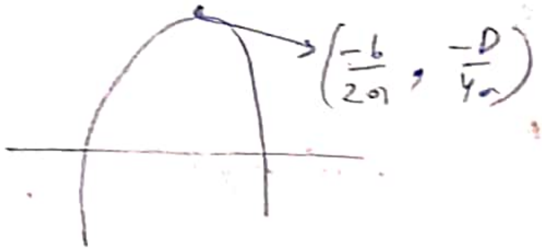
$a > 0$

Range $\in \left[\frac{-D}{4a}, \infty\right)$

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$a < 0$

Range $\in \left(-\infty, \frac{-D}{4a}\right]$



Type-2:- x is restricted.

Case 1

when $x = \frac{-b}{2a}$ lies in $[x_1, x_2]$

$\downarrow$

$f\left(\frac{-b}{2a}\right), f(x_1), f(x_2)$

Case 2

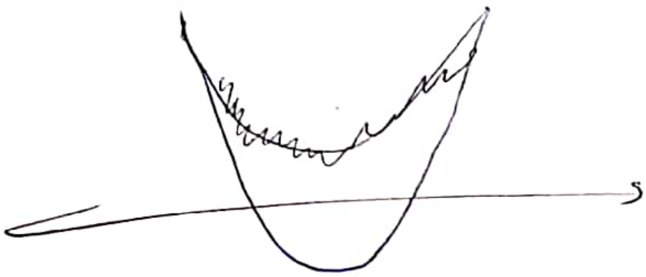
when $x = \frac{-b}{2a}$ don't lie in $[x_1, x_2]$

check :- $f(x_1)$, & $f(x_2)$

Q Draw the graph of $x^2 - 5x + 6 = 0$

(1) find Minimum value & point where min value occurs.

(2) Range of quadratic.



(1) min value $= \frac{-D}{4a}$

$= \frac{-1}{4}$

$\boxed{= -1/4}$

$x_{min} = \frac{5}{2}$

$\boxed{x_{min} = \frac{5}{2}}$

(2) Range $\in [y_{min}, y_{max}]$

$y_{min} = -1/4$

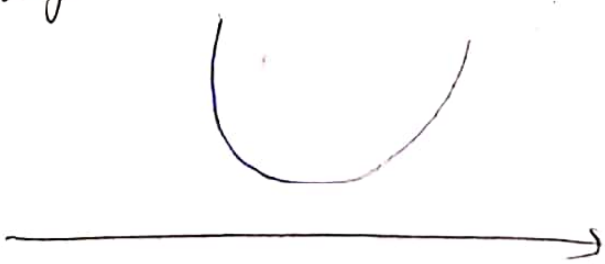
$y_{max} = \infty$

$\boxed{\text{Range } [-1/4, \infty)}$

Q Draw graph of $x^2 + x + 1 = 0$

(1) find min value & Point

(2) Range



① $y_{min} = \frac{-D}{4a}$

$= \frac{4-1}{4}$

$\boxed{= \frac{3}{4}}$

$x_{min} = \boxed{\frac{-1}{2}}$

$x_{min} = \frac{-b}{2a}$

② $y_{max} = \infty$

Range $\boxed{[\frac{3}{4}, \infty)}$

Q find the range of $-x^2+2x-1$

$y_{max} = -\frac{(4-4)}{2}$

$= 0$

$\boxed{(-\infty, 0]}$

Q $y = x^2 - 2x - 3$

① $x \in R$

$y_{min} = -\frac{(4+12)}{4}$

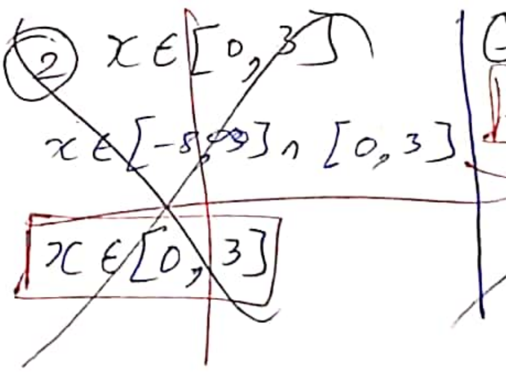
$= -4$

$\boxed{[-4, \infty)}$

② $x \in [0, 3]$

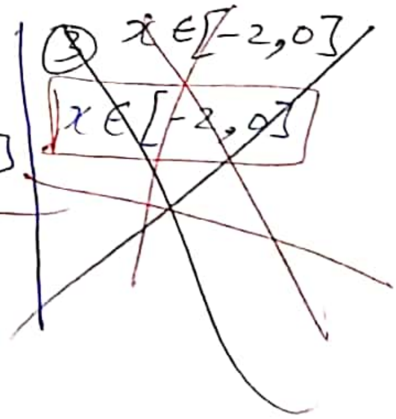
$x \in [-\infty, \infty] \cap [0, 3]$

$\boxed{x \in [0, 3]}$



③ $x \in [-2, 0]$

$\boxed{x \in [-2, 0]}$



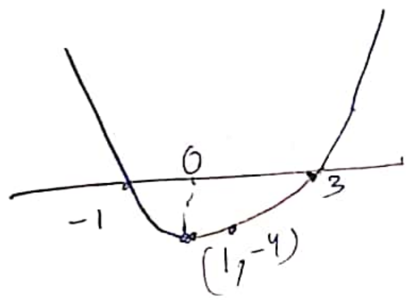
② $x \in [0,3]$

$y|_{x=0} = 0-0-3$
$= -3$

$y|_{x=3} = 9-6-3$
$= 0$

$\boxed{[-4,0]}$

$y|_{x=1} = 1-2-3$
$= -4$

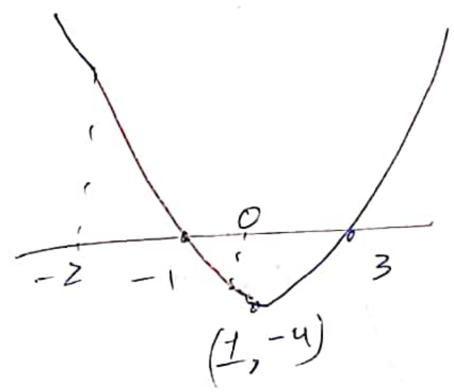


③ $x \in [-2,0]$

$f(-2) = 4+4-3$
$= 8-3$
$= 5$

$f(0) = -3$

$\boxed{[-3,5]}$



Q $y = f(x) = x^2 - 5x + 6$

$y_{min} = \frac{-(25-24)}{4}$

$\boxed{= \frac{-1}{4}} = -0.25$

$x_{min} = \boxed{\frac{5}{2}} = 2.5$

① $[-3,0]$ $x \in x$

$f(-3) = 9+15+6$
$= 30$

$f(0) = 6$

$\boxed{y \in [6,30]}$

② $x \in [1,5]$

$f(1) = 2$

$f(5) = 6$

$f(2.5) = -1/4$

$\boxed{y \in [-1/4, 6]}$

③ $x \in [3,4]$

$f(3) = 0$

$f(4) = 16-20+6$
$= 2$

$\boxed{y \in [0,2]}$

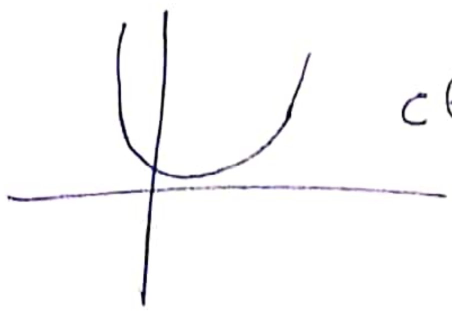
Commenting of signs of a, b, c

$y = ax^2 + bx + c$

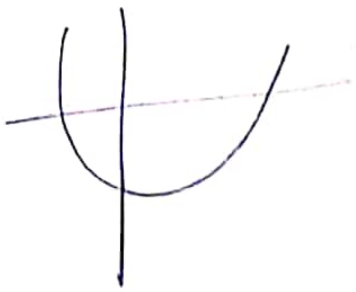
① $a > 0$ concave up

$a < 0$ concave down

② $c \rightarrow$ cut on y-axis



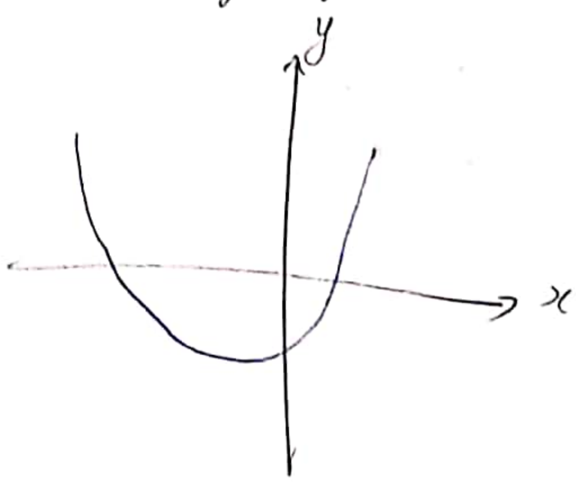
c ⊕ve



c ⊖ve

③ $b \rightarrow$ no fixed rule

Q Comment on the signs of a, b, c



$a = ⊕ve \rightarrow$ concave up

$c = ⊖ve \rightarrow$ cut y-axis below x-axis.

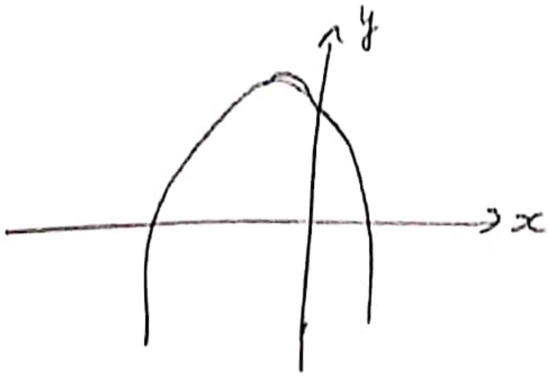
$$\frac{-b}{2a} < 0$$

⊕ve so $-b = ⊖ve$

$$\boxed{b = ⊕ve}$$

Q Comment on a, b, c signs

①



$\boxed{a = \ominus \text{ve}}$

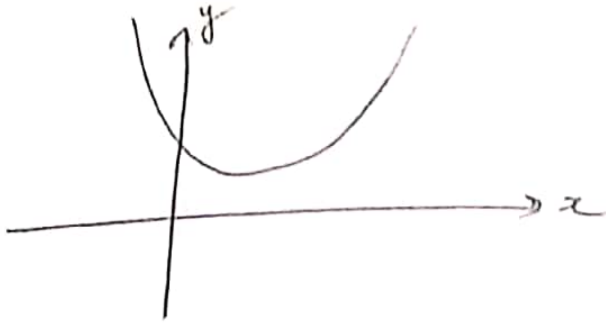
$\boxed{C = \oplus \text{ve}}$

$\frac{-b}{2a} = \ominus \text{ve}$

$2a \rightarrow a$ is $\ominus$ve

$\boxed{b = \ominus \text{ve}}$ $\boxed{b = \ominus \text{ve}}$

②

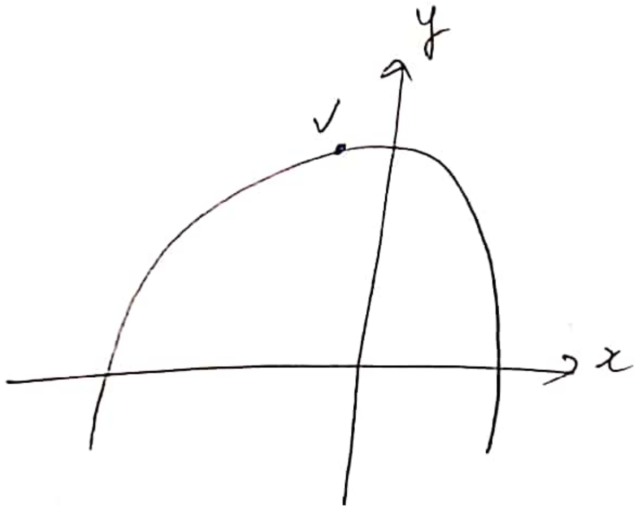


$a = \oplus \text{ve}$

$c = \oplus \text{ve}$

$b = \ominus \text{ve}$

Q



i) a ⊖ve

ii) b ~~⊕ve~~ ⊖ve

iii) c ⊕ ve

iv) bc ⊖ve

v) c−a ⊕ve

vi) ab^2 ⊖ve

vii) abc ⊕ ve

viii) $\frac{a+b}{c}$ ⊖ ~~⊕~~ ve

H.W. (23-5-2024)

~~O-1~~ DYS-8 (Q1, Q2, Q3)

O-1 (Q15, 22)

~~O-2 (Q9, 8)~~,

O-2 (Q8, 9, 11, 16, 17, 18, 19,)

J-M (Q2, 3, 4, 5, 6, 7, 14)

Q $y = ax^2 + bx + c$, & $c < 0$ does not have any real roots then comment on the signs of - $a < 0, bc < 0$

(A) $c(a+b+c)$

(B) $c(a-b+c)$

(C) $4a + 2b + c$

$\frac{+D}{+4a} < 0$

(A) For $x = 1$

$y = a + b + c$

where $y < 0$

$a + b + c < 0$

$c < 0$

$c(a+b+c)$ $\boxed{\oplus ve}$

$\frac{+b}{+a} > 0$

(B) for $x = -1$

$y = \ominus ve$

$c(\ominus ve)$

$\ominus(\ominus ve)$

$\boxed{\oplus ve}$

(C) $x = 2$

$\boxed{y = 4 \ominus ve}$

Q If $c < 0$ & $y = ax^2 + bx + c$ has no real roots find signs.

$c < 0, a < 0$

(A) $9a + 3b + c = y$

for $x = 3$

$\boxed{y = \ominus ve}$

(B) $a + 2b + 4c$

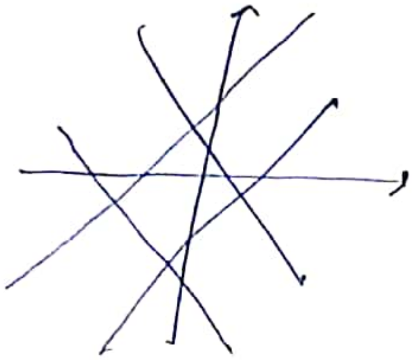
$\ominus ve$ for $x = 1/2$

Range of Linear, $\frac{\text{Linear}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Linear}}$ & $\frac{\text{Quad}}{\text{Quad}}$.

① Linear -

$y = ax+b$

$y \in R$ always



eg. $y = 2x+3$

Range - $(-\infty, \infty)$

$y = \sqrt{2}x - \frac{7}{2}$

Range $= (-\infty, \infty)$

② $\frac{\text{Linear}}{\text{Linear}}$ - $y = \frac{ax+b}{cx+d}$

Range $\rightarrow y \in R - \left\{\frac{a}{c}\right\}$

eg - $y = \frac{2x-3}{x+2}$

$y \in R - \left\{\frac{2}{1}\right\}$

$y \in R - \{2\}$

$y \in (-\infty, 2) \cup (2, \infty)$

Q find ranges

(1) $y = \frac{3x+1}{2x-1}$

$y \in R - \{\frac{3}{2}\}$

(2) $y = \frac{x}{2x-1}$ ~~$\{\frac{1}{2}\}$~~

$y \in R - \{\frac{1}{2}\}$

(3) $y = \frac{5x-2}{7-4x}$

$y \in R - \{\frac{5}{-4}\}$

(4) $y = \frac{1}{3x+4}$

$y \in R - \{0\}$

(3) $\frac{\text{Quad}}{\text{Linear}}$, $\frac{1}{\text{Quad}}$, $\frac{\text{Quad}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Quad}}$

Process:- Do cross Multiply

case 1:- when leading coffecient $\neq 0$ then apply $D \geq 0$ & solve inequality in y.

case 2: when leading coffecient $= 0$, if any value of x is comming then no problem. otherwise exclude.

Q find range of $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ $x \in R$

$yx^2 + 3yx + 4y = x^2 - 3x + 4$

$(y-1)x^2 + (3y+3)x + (4y-4)$

Case 1 $y - 1 \neq 0$

$D \geq 0$

$(3y+3)^2 + (-4)(4y-4)(4y-1) \geq 0$

$9y^2 + 9 + 18y - 16(y^2 + 1 - 2y) \geq 0$

$9y^2 - 16y^2 + 18y + 32y + 9 - 16 \geq 0$

$-7y^2 + 50y - 7 \geq 0$

$7y^2 - 50y + 7 \leq 0$

$7y^2 - 49y - y + 7 \leq 0$

$7y(y-7) - 1(y-7) \leq 0$

$(7y-1)(y-7) \leq 0$

$+ \quad \frac{1}{7} \quad - \quad 7 \quad +$

$y \in \left[\frac{1}{7}, 7\right]$

Case 2 :- $y-1=0$

$y=1$

Put in Question

$\cancel{x^2}+3x+4-\cancel{x^2}+3x-4=0$

$6x=0$

$x=0$

$\therefore$ Value of x coming in Case 2 so we need to exclude $y=1$

Hence, $y \in [\frac{1}{7}, 7]$

~~Q. $y=\frac{8x-4}{x^2+2x-1}$~~

Q $y=\frac{8x-4}{x^2+2x-1}$ $x \in R$

Case 1 :-

$yx^2+2yx-y=8x-4$

$yx^2+(2y-8)x-(y-4)=0$

$D \geq 0$

$(2y-8)^2+(+4)(y)(y-4)=0$

$4y^2+64-32y+4y^2-16y$

$8y^2-48y+64 \geq 0$

$y^2-6y+8 \geq 0$

$y^2-4y-2y+8 \geq 0$

$$y(y-4)-2(y-4) \geq 0$$

$$(y-2)(y-4) \geq 0$$

$$y \in (-\infty, 2] \cup [4, \infty)$$

Case 2 $\quad y = 0$

$$8x-4=0$$

$$8x=4$$

$$x=\frac{4}{8}$$

$$x=\frac{1}{2}$$

$$\boxed{y \in (-\infty, 2] \cup [4, \infty)}$$

Q $\quad y = \dfrac{x^2+2x-3}{x^2+2x-8}$, Find range when $x \in R$

$$y = \frac{x^2+2x-3}{x^2+2x-8}$$

$$yx^2+2yx-8y = x^2+2x-3$$

$$yx^2-x^2+2yx-2x-8y+3=0$$

$$(y-1)x^2+(2y-2)x-(8y-3)=0$$

Case 1 $\quad D \geq 0$

$b^2 - 4ac \geq 0$

$(2y-2)^2 + (+4)(y-1)(8y-3)$

$4y^2 + 4 - 8y + 4(8y^2 - 3y - 8y + 3)$

$4y^2 + 4 - 8y + 32y^2 - 44y + 12$

$36y^2 - 52y + 16 = 0$

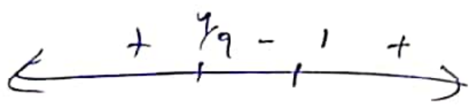
$18y^2 - 26y + 8 = 0$

$9y^2 - 13y + 4 = 0$

$9y^2 - 9y - 4y + 4 \geq 0$

$9y(y-1) - 4(y-1) \geq 0$

$(9y-4)(y-1) \geq 0$



$y \in (-\infty, 4/9] \cup [+1, \infty)$

Case 2 $\quad y - 1 = 0$

$\boxed{y = 1}$

$1 = \dfrac{x^2 + 2x - 3}{x^2 + 2x - 8}$

$\cancel{x^2} + \cancel{2x} - 8 = \cancel{x^2} + \cancel{2x} - 3$

$-8 \neq -3$

so exclude $\boxed{y = 1}$

$$\boxed{y \in (-\infty, 4/9] \cup (1, \infty)}$$

H.W. 24-05-2024

DYS-9 [All] ✓

O-1 {23}

~~O-2 {~~

J-M {1, 13}

J-A {3, 4}

O-4 {1} ✓

O-3 {7, 8, 1, 2, 3}

~~O-2 {~~

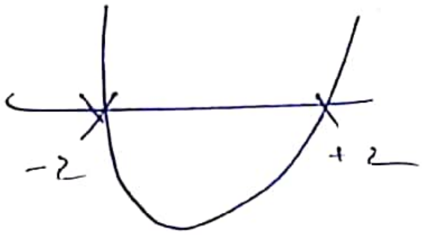
O-2 {7} ✓

Location of roots (Real nos.) (Type 1)

① Roots are equal in magnitude & opposite in sign.

$\boxed{b = 0 \quad \& \quad D > 0}$

eg.



$(x-2)(x+2) = 0$

$x^2 - 4 = 0$

↳ Hence $b = 0$

② Only 1 root is always zero

$\boxed{c = 0}$

let roots $\alpha, 0$

~~③ Roots are re~~

$(x-\alpha)(x-0) = 0$

$x^2 - \alpha x = 0$

↳ Hence $c = 0$

③ Roots are reciprocal to each other.

$\boxed{a = c \quad \& \quad D \geq 0}$

$(x-\alpha)\left(x - \frac{1}{\alpha}\right) = 0$

$x^2 - x\left(\alpha + \frac{1}{\alpha}\right) + 1 = 0$

$\boxed{a = c}$

(4) Roots are of opposite sign

$\boxed{POR < 0 \ \& \ D > 0}$

$(x-\alpha)(x+\beta) = 0$

SOR $\Rightarrow \alpha - \beta$ (any can be bigger)

POR $\Rightarrow \alpha(-\beta) = \underbrace{-\alpha\beta}_{\text{always } \ominus\text{ve}}$

(5) Both roots are $\ominus$ve

$\boxed{SOR < 0 \ \& \ POR > 0 \ \& \ D \geq 0}$

$(x+\beta)(x+\alpha) = 0$

$\left.\begin{array}{l} -\alpha - \beta = \ominus\text{ve} \\ -\alpha \cdot -\beta = \oplus\text{ve} \end{array}\right\}$ always

(6) Both roots are $\oplus$ve

$\boxed{SOR > 0 \ \& \ POR > 0 \ \& \ D \geq 0}$

$a(x-\beta)(x-\alpha) = 0$

$\left.\begin{array}{l} \beta + \alpha = \oplus\text{ve} \\ \beta\alpha = \oplus\text{ve} \end{array}\right\}$ always

leading coefficient must be 1

Q $f(x) = x^2 + 2(a-1)x + (a+5)$ find 'a'.

a) Roots are of opposite sign

$a + 5 < 0$

$\boxed{a < -5}$

$a \in (-\infty, -5)$

intersection

$4(a-1)^2 - 4(a+5) > 0$

$a^2 + 1 - 2a - a - 5 > 0$

$a^2 - 3a \cancel{+6}^{-4} > 0$

$\cancel{a^2 - 2a - a + 6 > 0}$

$\cancel{a(a-2) + 1(a}$

$\cancel{a = \frac{3 \pm \sqrt{9 - 24}}{2}}$

$a^2 - 4a + a - 4 > 0$

$a(a-4) + 1(a-4) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$a \in (-\infty, -1) \cup (4, \infty)$

$\boxed{a \in (-\infty, -5)}$

b) Roots equal in magnitude but opposite in sign

$a - 1 = 0$

$a = 1$

$a \in (-\infty, -1) \cup (4, \infty)$

$\boxed{a \in \phi}$

c) Both roots ⊕ve

SOR > 0

POR > 0

$D \geq 0$

SOR $2(1-a) > 0$	$a + 5 > 0$
$1 - a > 0$	$\boxed{a > -5}$
$\boxed{a < 1}$	$a \in (-\infty, -1] \cup [4, \infty)$

$\boxed{a \in (-5, -1]}$

d) Both roots ⊖ve

SOR < 0

$a - 1 > 0$

$\boxed{a > 1}$

$\boxed{a \in [4, \infty)}$

Q $f(x) = x^2 - (m-3)x + m$. find 'm'

a) Roots are of opposite sign

b) roots equal magnitude but opposite sign

c) Both roots are ⊕ve

d) Both roots are ⊖ve

$x^2 - (m-3)x + m$

$(m-3)^2 - 4m = D$

a) POR < 0

$D > 0$

$m^2 + 9 - 6m - 4m > 0$

$m^2 + 9 - 10m > 0$

$m^2 - 9m - m + 9 > 0$

$m(m-9) - 1(m-9) > 0$

$+ \quad 1 \quad - \quad 9 \quad +$

$(-\infty, 1) \cup (9, \infty)$

$m < 0$

$\boxed{m \in (-\infty, 0)}$

b) $b = 0 \quad D > 0$

$-(m-3) = 0$

$m = 3$

$\boxed{m \in \phi}$

c) SOR > 0

POR > 0

$D \geq 0$

$m > 0$

$m - 3 > 0$

$m > 3$

$\boxed{m \in [9, \infty)}$

d) $m > 0$

$m - 3 < 0$

$m < 3$

$\boxed{m \in (0, 1]}$

Q. If $f(x) = 3x^2 - 5x + p$ & $f(0)$ & $f(1)$ are of opposite signs find P.

p $\quad \cancel{3-5+p}$

$\cancel{-2+p}$

$f(0) \cdot f(1) < 0$

$p(3-5+p) < 0$

$p(p-2) < 0$

$p \in (0, 2)$

Location of Roots Type-2

$f(x) = ax^2 + bx + c$

⇒ Leading coeff must be 1.

① Both ~~two~~ roots of a quad are greater than a number 'd'.

$$\boxed{\begin{array}{l} D \geq 0 \\ f(d) > 0 \\ d < \frac{-b}{2a} \end{array}}$$

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Q) find "λ" for both roots of quadratic $x^2-6\lambda x+9\lambda^2-2\lambda+2=0$ are greater than 3

$36\lambda^2 - 36\lambda^2 + 8\lambda - 8 \geq 0$

$8(\lambda-1) \geq 0$

$\lambda - 1 \geq 0$

$\boxed{\lambda \geq 1}$

$9 - 18\lambda + 9\lambda^2 - 2\lambda + 2 > 0$

$9\lambda^2 - 20\lambda + 11 > 0$

$9\lambda^2 - 11\lambda - 9\lambda + 11 > 0$

$\lambda(9\lambda - 11) - 1(9\lambda - 11) > 0$

$(9\lambda - 11)(\lambda - 1) > 0$

intersection

$+ \quad 1 \quad - \quad 11/9 \quad +$

$\boxed{\lambda \in (-\infty, 1) \cup (11/9, \infty)}$

$3 + \frac{-6\lambda}{2} < 0$

$6 - 6\lambda < 0$

$(1-\lambda) < 0$

$\lambda - 1 > 0$

$\boxed{\lambda > 1}$

$\boxed{\lambda \in (11/9, \infty)}$

Q find 'K' for both the roots of the quadratic $(k+1)x^2 - 3kx + 4k = 0$ are greater than 1

$$x^2 - \frac{3k}{k+1}x + \frac{4k}{k+1} = 0$$

$\boxed{k \neq -1}$

$D \geq 0$

$$\frac{9k^2}{k^2+1+2k}$$

$$\left(\frac{3k}{k+1}\right)^2 - 4\left(\frac{4k}{k+1}\right) \geq 0$$

$$\frac{9k^2 - 16k^2 - 16k}{k^2+1+2k} \geq 0$$

$$\frac{7k^2+16k}{k^2+2k+1} \leq 0$$

$$\frac{k(7k+16)}{(k+1)^2} \leq 0$$

$-\frac{16}{7}$, -1, 0

$+ \quad - \quad - \quad +$

$$\boxed{\left[-\frac{16}{7}, -1\right) \cup (-1, 0]}$$

$(k+1) - 3k + 4 = 0$

$k + 1 - 3k + 4 = 0$

$\cancel{-2k + 5 = 0}$

$2k < 1$

$\boxed{k < \frac{1}{2}}$

$\cancel{2k - 5 < 0}$

$\cancel{\boxed{k < \frac{5}{2}}}$

$1 + \left(\frac{-3k}{k+1}\right) \times \frac{1}{2} \times \frac{k+1}{4k} < 0$

$1 + \frac{(-3k^2 - 3k)}{8k^2 + 8} < 0$

$\frac{8k^2 + 8 - 3k^2 - 3k}{8(k^2+1)} < 0$

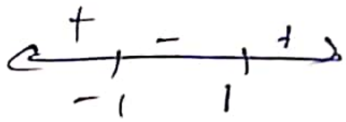
$\frac{5k^2 - 3k + 8}{k^2+1} < 0$

$5k^2 - 8k -$

$$\frac{-\left(\frac{-3k}{k+1}\right)}{2} > 1$$

$$\frac{3k}{2(k+1)} - 1 > 0$$

$$\frac{k-1}{k+1} > 0$$



$$(-\infty,-1) \cup (2,\infty)$$

$$\boxed{\left[-\frac{16}{7}, -1\right) \cup}$$

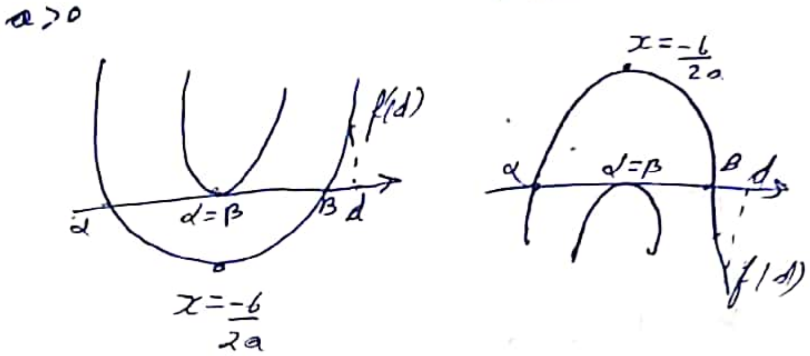
H.W. 25-05-2024

Type ②

② Both roots are less than any specific number 'd'.

$$\boxed{\begin{gathered} D \geq 0 \\ \frac{-b}{2a} < d \\ a f(d) > 0 \end{gathered}}$$ intersection

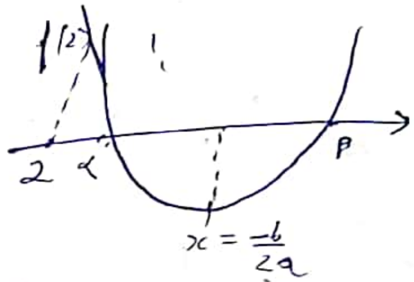
$a>0$ $\quad$ $a<0$



Q let $x^2-(m-3)x+m=0$ $(m\in R)$ be a quadratic equation find the value of m for which.

① both roots are greater than 2.

② both the roots are smaller than 2.



② $x^2-(m-3)x+m=0$

① $D = (m-3)^2-4m$

$= m^2+9-6m-4m$

$= m^2+9-10m$

~~$= m^2+10-$~~

$= m^2-9m-m+9$

$= m(m-9)-1(m-9)$

$= (m-1)(m-9) \geq 0$

$+ \quad 1 \quad - \quad 9 \quad +$

$\boxed{m\in(-\infty,1]\cup[9,\infty)}$ ... ①

$\frac{m-3}{2} < 2$

$m-3<4$ ... ②

$\boxed{m<7}$

$4-(m-3)\times 2+m > 0$

$4-2m+6+m>0$

$10-m>0$

$\boxed{m<10}$ ... ③

$①\cap②\cap③$

$\boxed{m\in(-\infty,1]}$ ✓

② $\frac{m-3}{2} > 2$

$m-3>4$ ... ②

$\boxed{m>7}$

$\boxed{m<10}$

$①\cap②\cap③$

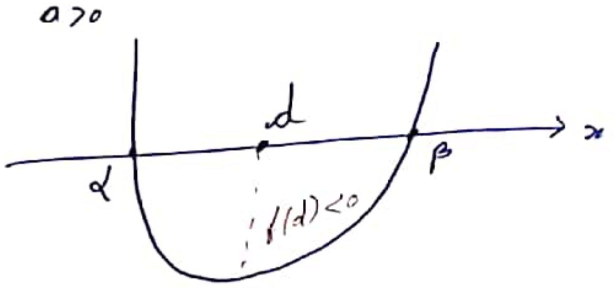
$\boxed{m\in[9,10)}$ ✓

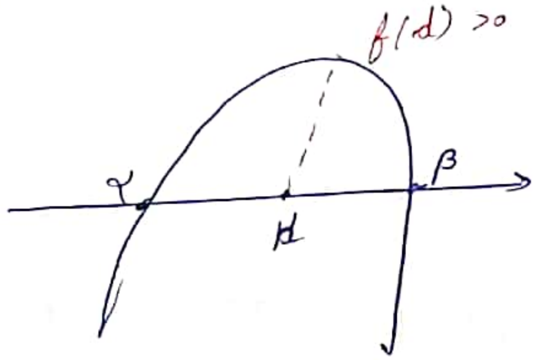
Type ③

(b) Both roots lie on either side of specific no. d

→ One root is greater than D & one is less

→ Specific no. d lies between the roots





$\boxed{a \cdot f(d) < 0}$

$a > 0,\ f(d) < 0$
$a < 0,\ f(d) > 0$] so $a f(d) < 0$

Q ~~Fi~~ find all possible values of a for which exactly one root of $x^2 - (a+1)x + 2a = 0$ lie in interval $(0, 3)$.

$f(0) = 2a$

$f(3) = 9 - 3a + 3 + 2a$

$= 6 - a$

$f(0) \cdot f(3) < 0$

$a(6 - a) < 0$

− 0 + 6 −

$(-\infty, 0) \cup (6, \infty)$

Check at the points of interval

put $a = 0$

$x^2 - x = 0$

$x = 1, 0$ → 1 lie ~~between~~ in interval

so include 0 in answer

put $a = 3$

$D < 0$ so no real roots

$\boxed{a \in (-\infty, 0] \cup (6, \infty)}$

Q find 'K' for which one root of equation $x^2 - (K+1)x + K^2 + K - 8 = 0$ is greator than 2 & other is less than 2.

$\alpha = 2$

$f(2) = 4 - (K+1)2 + K^2 + K - 8 = 0$

$= 4 - 2K - 2 + K^2 + K - 8$

$= K^2 - K - 6 < 0$

$K^2 - 3K + 2K - 6 < 0$

$K(K-3) + 2(K-3) < 0$

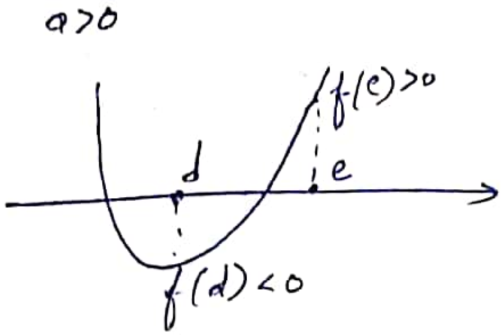
$(K+2)(K-3) < 0$

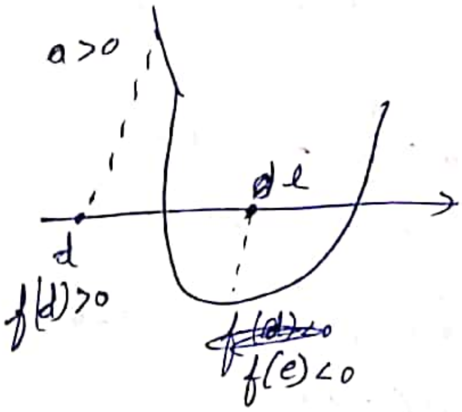
$+ \quad -2 \quad - \quad 3 \quad +$

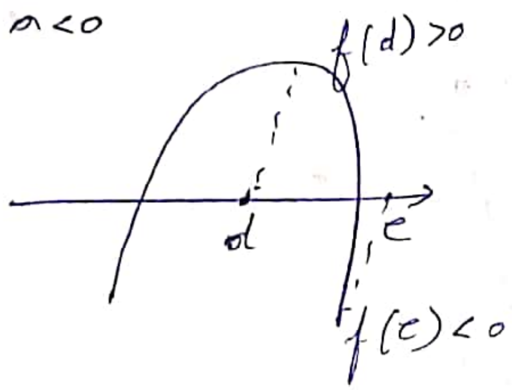
$\boxed{K \in (-2, 3)}$

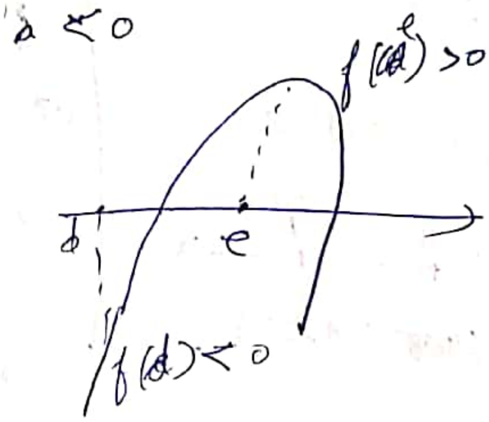
② Exactly one root lies in $(d<e)$ the interval (d,e)

Type-4









So $f(d)\cdot f(e)<0$

$$\boxed{f(d)\cdot f(e)<0}$$

Note:- Check at the extreme points of Interval.

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Q8. $(c-5)x^2 - 2cx + (c-4) = 0$

$-2x^2 - 6x - 1$

$2x^2 + 6x + 1$

$f(0) = c - 4$

$f(2) = 4(c-5) - 2c + c - 4 = 0$

$f(2) = 4c - 20 - 4c + c - 4$

$= 3c - 24$

$f(3) = 9c - 45 - 6c + c - 4$

$= 4c - 49$

$f(0) \cdot f(2) < 0$

$(c-4)(3c-24) < 0$

$(4, 24)$

$f(2) \cdot f(3) < 0$

$(4c-49)(3c-24) < 0$

$\left(\frac{49}{4}, 24\right)$

intersection

$(13, 24)$

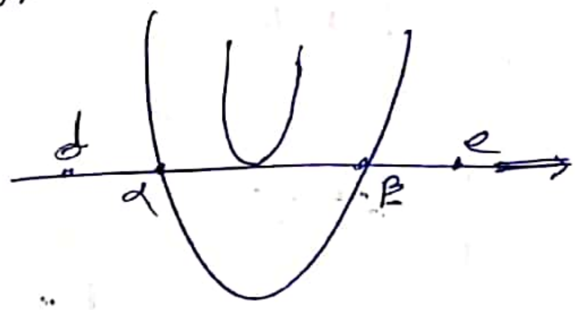
$24 - 13 - 1$

$= 11$

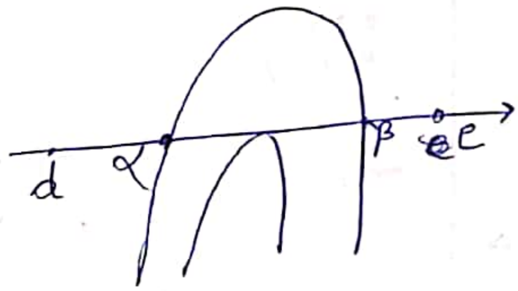
$\boxed{B1}$

Type - 5 Both the roots lie between number d & e ($d<e$)

$a>0$



$a<0$



Union of ① & ③

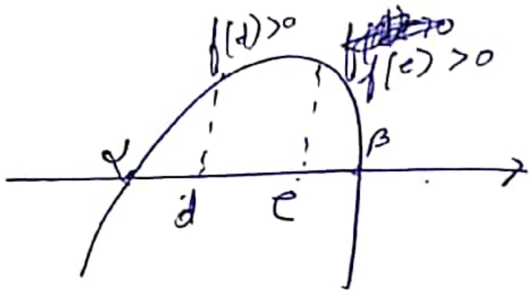
① $D \geq 0$

② $d < \frac{-b}{2a} < e$

③ $a f(d) > 0$

④ $a f(e) > 0$

Type - ⑥ One root is greater than e & one is less than d



$af(d) < 0$

$af(e) < 0$

Q If α, β are the roots of $x^2 + 2(K-3)x + 9 = 0$
if α, β belongs to $(-6, 1)$ find K.

$4(K-3)^2 - 36 > 0$

$4K^2 + 36 - 24K - 36 > 0$

$K^2 - 6K > 0$

$K(K-6) > 0$

~~$K = 6,$~~

$+ \quad - \quad +$

$0 \quad 6$

$K \in (-\infty, 0) \cup (6, \infty)$

$\frac{-2K+6}{2}$

$3 - K$

$-6 < 3 - K$

$K < 9$

$3 - K > 1$

$K > 2$

$f(-6) = 36 - 12K + 36 + 9 = 0$

$= 81 - 12K > 0$

$f(1) = 1 + 2K - 6 + 9$

$= 2K + 4 > 0$

$f(-6)\ 81 - 12K > 0$

$\boxed{K < \frac{27}{4}}$

$f(1) = K + 2 > 0$

$\boxed{K > -2}$

$$\boxed{\left(6, \frac{27}{4}\right)}$$

Scanned with CamScanner

Summary (Location of Roots)

1. Both roots greater than D. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} > d\right)$

2. Both roots less than D. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} < d\right)$

3. 'd' lies between the roots. $(af(d) < 0)$

4. exactly one root lies in (d, e) $(f(d) \cdot f(e) < 0)$

5. Both roots lie between d and e $(D \geq 0)$ $(af(d) > 0)$ $(af(e) > 0)$ $\left(d < \frac{-b}{2a} < e\right)$

~~6. Both roots lie between~~

6. Both points lies between 2 roots. $(af(d) < 0)$ $(af(e) < 0)$

Irrational Inequality

① Inequalities having $\sqrt{\ }$ sign.
② Direct Squaring is not allowed without checking

Q $\sqrt{2x-5} < 3$

$2x-5 \geq 0$ [under root quantity is always ⊕)

$2x-5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$x \in \left[\frac{5}{2}, \infty\right)$ — ①

$\underbrace{\sqrt{2x-5}}_{\oplus ve} < \underbrace{3}_{\oplus ve}$

Square both sides so we can square

$2x-5 < 9$

$2x < 14$

$x < 7$ — ②

$x \in (-\infty, 7)$

① ∩ ②

$\boxed{\left[\frac{5}{2}, 7\right)}$

Q $\sqrt{x+6} < x-6$

$x+6 \geq 0$

$x \geq -6$

$x \in [-6, \infty)$

Case ① $x-6 < 0$

$x < 6$

$\oplus < \ominus$ not possible

$x \in \phi$

Case ② $x-6 \geq 0$

$x \geq 0$

$x+6 < x^2+36-12x$

$\cancel{x^2+13}$ -

$x^2 - 13x + 30 > 0$

$x^2 - 10x - 3x + 30 \geq 0$

$x(x-10) - 3(x-10) \geq 0$

$(x-3)(x-10) \geq 0$

+ 3 − 10 +

$(-\infty, 3) \cup (10, \infty)$ but $x \in (6, \infty)$

$x \in (10, \infty)$

case ① ∪ case ② $\Rightarrow$ $\cancel{(-\infty, 3)} \cup (10, \infty)$

intersection with $x \in [-6, \infty)$

$\boxed{x \in (10, \infty)}$

Q $x+1 \geq \sqrt{5-x}$

$\sqrt{5-x} \leq x+1$

$5-x \geq 0$

$x-5 \leq 0$

$\boxed{x \leq 5}$ — (1)

Case (1) $x+1 < 0$

$x < -1$

$(+) \leq 0$ ✗

$\boxed{x \in \phi}$ — (2)

Case (2) $x+1 \geq 0$

$x \geq -1$

$5-x \leq x^2+1+2x$

$x^2+3x-4 \geq 0$

$x^2+4x-x-4 \geq 0$

$x(x+4)-1(x+4) \geq 0$

$(x-1)(x+4) \geq 0$

$+ \quad -4 \quad - \quad 1 \quad +$

$(-\infty,-4] \cup [1,\infty)$

$\boxed{x \in [1,\infty)}$ — (3)

Case 1 $\cup$ Case 2

(2) $\cup$ (3) $= x \in [1,\infty)$ — (4)

$\boxed{[1,5]}$

Q $\sqrt{x+18} < 2-x$

$x+18 \geq 0$

$\boxed{x \geq -18}$

Case (1) $2-x < 0$

$-x < -2$

$x > 2$

(1) < 0 ×

$x \in \phi$

Case (2) $2-x > 0$

$x < 2$

$x+18 < x^2+4-4x$

$x^2-5x-14 > 0$

$x^2-7x+2x-14 > 0$

$x(x-7)+2(x-7) > 0$

$(x+2)(x-7) > 0$

$+ \quad -2 \quad - \quad 7 \quad +$

$(-\infty,-2) \cup (7,\infty)$

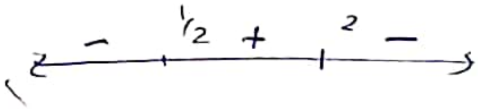
$(-\infty,-2) \cap [-18,\infty)$

$\boxed{x \in [-18,-2)}$

d

$$\sqrt{\frac{x-2}{1-2x}} > -1$$

$$\frac{x-2}{1-2x} \geq 0$$



$$\left(\frac{1}{2}, 2\right]$$

$$\cancel{x \in \phi}$$

$\underline{x \in R} - \left\{\frac{1}{2}\right\}$

$$\boxed{\left(\frac{1}{2}, 2\right]}$$

H.W. 28-05-2024

DYS-6 (full)

DYS-5 (full)

DYS-8 (Q5)

DYS-10 (full)

O-1 {16, 17, 19, 20, 21, 24, 25, 26, 27}

O-2 {10, 14, 15, 20 ...., ∞} − {12, 13}

~~O-3~~

Modulus Equality

① If $a = \oplus ve$ ($a \to$ constant)

$|x| \le a \quad x \in [-a, a]$

$|x| < a \quad x \in (-a, a)$

$|x| \ge a \quad x \in (-\infty, -a] \cup [a, \infty)$

$|x| > a \quad x \in (-\infty, -a) \cup (a, \infty)$

② If $a = \ominus ve$ ($a \to$ constant)

$|x| \le a \quad x \in \phi$

$|x| < a \quad x \in \phi$

$|x| \ge a \quad x \in R$

$|x| > a \quad x \in R$

③ $|x|^2 = |x^2|$

④ $|x||y| = |xy|$

⑤ $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

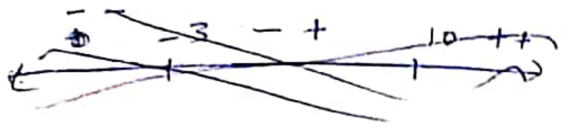
⑥ $\sqrt{x^2} = |x|$

⑦ $\big||x| - |y|\big| \le |x+y| \le |x| + |y|$

⑧ $|x+y| = |x| + |y| \Rightarrow xy \ge 0$

$|x-y| = |x| + |y| \Rightarrow xy \le 0$

Q $|2x-7| = |x+3| + |x-10|$



$|x+3+x-10| = |x+3| + |x-10|$

$(x+3)(x-10) \geq 0$

x : $+ \quad -3 \quad - \quad 10 \quad +$

$(-\infty, -3] \cup [10, \infty)$

Q $|x-2| + |x-7| = 5$

$(x-2)(x-7) \leq 0$

$+ \quad 2 \quad - \quad 7 \quad +$

$\boxed{[2, 7]}$

Q $|x^2+6x+6| = |x^2+4x+9| + |2x-3|$

$\cancel{x^2+4x}$

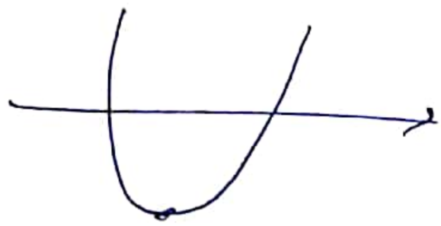
$(x^2+4x+9)(2x-3) \geq 0$

$+ \quad - \quad \frac{3}{2} \quad +$

$\boxed{\left(\frac{3}{2}, \infty\right)}$

Q-1

Q27.



-1

as it is biggest -ve integer

$\frac{-p}{4} = -1$

$-p = -4$

$p = 4$

$2x^2 + 4x + 1$

$x = \frac{-4 \pm \sqrt{16-8}}{4}$

$= \frac{-4 \pm 2\sqrt{2}}{4}$

$= \frac{-2 \pm \sqrt{2}}{2}$

$x = \frac{-2+\sqrt{2}}{2}$ × $x = \frac{-2-\sqrt{2}}{2}$

~~$x = -p$~~

$x = \frac{(\sqrt{2}-2)}{2}$ × $\frac{-(\sqrt{2}+2)}{2}$

$+$

$2x^2 + px + 1 = -1$

$2x^2 + px + 2 = 0$

$p^2 - 16 = 0$

$p = \pm 4$

$+4 \times -4$

$\boxed{= -16}$

$\boxed{B}$

Q find 'x'

$|x| \leq 2$

$|x| \leq a$

$x \in [-a, a]$

$\boxed{x \in [-2, 2]}$

Q $|x-3| \leq 2$

$x-3 \in [-2, 2]$

$\boxed{x \in [1, 5]}$

Q $|x| \geq 9$

$\boxed{x \in (-\infty, -9] \cup [9, \infty)}$

Q $|x| < \sqrt{3}$

$\boxed{x \in (-\sqrt{3}, \sqrt{3})}$

Q $|2x| - 5 > 0$

$|2x| > 5$

$2x \in (-\infty, 5) \cup (5, \infty)$

$\boxed{x \in (-\infty, 5/2) \cup (5/2, \infty)}$

Q $|2-7x| < 8$

$2-7x \in (-8, 8)$

$-7x \in (-10, 6)$

$\cancel{x \in \left(\frac{10}{7}, \frac{-6}{7}\right)}$ $\boxed{x \in \left(-\frac{6}{7}, \frac{10}{7}\right)}$

Q no. of integral values of x such that $4 \leq |x-4| \leq 10$

$|x-4| \leq 10$

$x-4 \in [-10, 10]$

$x \in [-6, 14]$

$|x-4| \geq 4$

$x-4 \in (-\infty, -4] \cup [4, \infty)$

$x \in (-\infty, 0] \cup [8, \infty)$

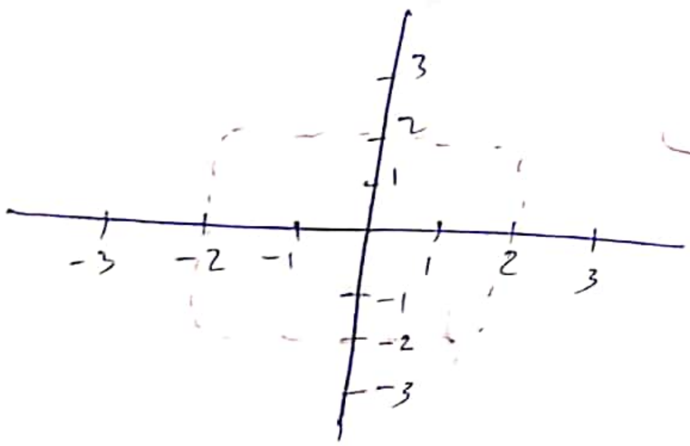
$\boxed{x \in [-6, 0] \cup [8, 14]}$

no. of integral values $= 7+7$

$\boxed{= 14}$

Q $|x| \leq 2$ & $|y| \leq 2$

$x \in [-2, 2]$ $y \in [-2, 2]$



area $= 4 \times 4$

$\boxed{\text{Area} = 16}$

Q $||x-2|-1| \le 5$

$|x-2|-1 \in [-5, 5]$

$|x-2| \in [-4, 6]$

$-4 \le |x-2| \le 6$

$|x-2| \geq -4$

$\cancel{|x-2| \geq 2}$

$x \in R$

$|x-2| \le 6$

$x-2 \in [-6, 6]$

$\boxed{x \in [-4, 8]}$

Q $|x-3| < x-3$

$|x-3| < x-3$

$x-3 \in (3-x, x-3)$

$x \in (6-x, x)$

$6-x \le x \le x$

$6-x < x$

$6 < 2x$

$\boxed{3 < x}$

$x < x$

$0 < 2x$

$\cancel{0 < 2 \in R}$

$x \in R$

$x-3 > 0$

$\boxed{x > 3}$

$|x-3| < x-3$

$\cancel{x-3 < x-3}$

$x-3 \in (3-x, x-3)$

$\cancel{3-x >}$

$x \in (6-x, x)$

$6-x < \boxed{x < x}$

$\boxed{x \in \phi}$

Theory of Equation

→ Deriving results for polynomials with degree 3 or more.

Let $a \neq 0$

(1) $ax^2+bx+c=0$ — α, β

$$\alpha+\beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

(2) $ax^3+bx^2+cx+d=0$ — α, β, γ

$$\alpha+\beta+\gamma = \frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

(3) $ax^4+bx^3+cx^2+dx+e$ — α, β, γ, δ

$$\alpha+\beta+\gamma+\delta = \frac{-b}{a} \quad \text{(sum)}$$

$$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\delta = \frac{c}{a} \quad \text{(2-2 sum)}$$

$$\alpha\beta\gamma+\beta\gamma\delta+\alpha\beta\delta+\alpha\gamma\delta = -\frac{d}{a} \quad \text{(3-3 sum)}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$

general

$$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + a_3 x^{n-3} \dots + a_n$$

(signs above terms: $+$, $-$, $+$, $-$)

a_n: $\ominus$ for odd n, $\oplus$ for even n

$$\boxed{\text{Sum of Roots} = \frac{-a_1}{a_0}}$$

$$2\text{-}2 \text{ sum} = \frac{a_2}{a_0}$$

$$3\text{-}3 \text{ sum} = \frac{-a_3}{a_0}$$

$$4\text{-}4 \text{ sum} = \frac{a_4}{a_0}$$

$$\boxed{\text{product} = (-1)^n \frac{a_n}{a_0}}$$

Q $2x^3 - 5x^2 + 4x - 1 = 0$ have roots $\alpha, \beta \,\&\, \gamma$

find ① $\alpha^2 + \beta^2 + \gamma^2$

② $\alpha^3 + \beta^3 + \gamma^3$

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \alpha\gamma)$$

$$\left(\frac{5}{2}\right)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(2)$$

$$\frac{25 - 16}{4} = \alpha^2 + \beta^2 + \gamma^2$$

$$\boxed{\frac{9}{4} = \alpha^2 + \beta^2 + \gamma^2}$$

(2) $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\alpha\gamma)$

$$\alpha^3+\beta^3+\gamma^3-\frac{1}{2} = \left(\frac{5}{2}\right)\left(\frac{9}{4}+2\right)$$

$$\cancel{\alpha^3+\beta^3+\gamma^3 = \frac{85}{8}+\frac{1}{2}}$$

$$\cancel{= \frac{85+4}{8}}$$

$$\cancel{= \frac{89}{8}}$$

$$\alpha^3+\beta^3+\gamma^3 = \frac{5}{2}\times\frac{1}{4}+\frac{1}{2}$$

$$\cancel{= \frac{5+4}{8}}$$

$$\cancel{= \frac{9}{8}}$$

$$= \frac{5}{8}+\frac{12}{8}$$

$$\boxed{= \frac{17}{8}}$$

Q $x^3-6x^2+10x-3=0$

$$\left(\alpha-\frac{1}{\beta\gamma}\right)\left(\beta-\frac{1}{\alpha\gamma}\right)\left(\gamma-\frac{1}{\alpha\beta}\right)$$

$$\frac{(\alpha\beta\gamma-1)}{\beta\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\beta}$$

$$\frac{(3-1)^3}{(3)^2} = \boxed{\frac{8}{9}}$$

Q $x^3-3x^2+2x+1=0$

① $(\alpha-2)(\beta-2)(\gamma-2)$

$\alpha+\beta+\gamma-6$

$\boxed{=-3}$

~~$\alpha\beta+$~~

$\alpha\beta-2\alpha-2\beta+4+\beta\gamma-2\beta-2\gamma+4+\alpha\gamma-2\alpha-2\gamma+4$

$\alpha\beta+\alpha\gamma+\beta\gamma+12-4(\alpha+\beta+\gamma)$

$-2+12-4(+3)$

~~$-2+12++2$~~

$-2+12+12$

$\boxed{=22}$

$\alpha\beta\gamma-2\alpha\gamma-2\beta\gamma+4\gamma-2\alpha\beta+4\alpha+4\beta-8$

$\alpha\beta\gamma-2(\alpha\beta+\beta\gamma+\alpha\gamma)+4(\alpha+\beta+\gamma)-8$

$-1-4+12-8$

$\boxed{\cancel{+25}}$ $\boxed{-1}$

~~$t^3+3+22+2$~~

$\boxed{t^3+3t^2+22t+1}$

60

② cubic with roots $\frac{\alpha+1}{2}, \frac{\beta+1}{2}, \frac{\gamma+1}{2}$

$$\frac{x+1}{2} = t$$

$$x + 1 = 2t$$

$$x = 2t - 1$$

$$(2t-1)^3 - 3(2t-1)^2 + 2(2t-1) + 1 = 0$$

$$8t^3 - 3 - 4t^2 + 2t - 12t^2 - 3 + 6t + 4t - 2 + 1 = 0$$

$$\boxed{8t^3 - 16t^2 + 12t - 7 = 0}$$

Q $x^3 - x + 1 = 0$ have roots α, β & γ

find

① Eq with roots are $\frac{\alpha+\beta}{\gamma^2}, \frac{\beta+\gamma}{\alpha^2}, \frac{\alpha+\gamma}{\beta^2}$

$$\alpha + \beta + \gamma = 0$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = -1$$

$$\alpha\beta\gamma = -1$$

$$\frac{\alpha+\beta}{\gamma^2}$$

$$\alpha+\beta+\gamma=0$$

$$\alpha+\beta=-\gamma$$

$$\frac{-\gamma}{\gamma^2} = -\frac{1}{\gamma}, -\frac{1}{\alpha}, -\frac{1}{\beta} \text{ are roots.}$$

$$-\frac{1}{x} = t$$

$$x = -\frac{1}{t}$$

$$\left(-\frac{1}{t}\right)^3 - \left(-\frac{1}{t}\right)+1=0$$

$$\frac{-1}{t^3}+\frac{1}{t}+1$$

$$\boxed{t^3+t^2-1=0}$$

(2) Value of $\frac{\beta\gamma}{(1-\beta)(1-\gamma)} + \frac{\alpha\gamma}{(1-\alpha)(1-\gamma)} + \frac{\alpha\beta}{(1-\alpha)(1-\beta)}$

α, β, γ are roots

$\alpha^3 - \alpha + 1 = 0$

$1-\alpha = -\alpha^3$

$1-\beta = -\beta^3$

$1-\gamma = -\gamma^3$

$$\frac{\beta\gamma+1+\alpha\gamma+1+\alpha\beta+1}{(1-\beta)(1-\gamma)(1-\alpha)}$$

$$= \frac{2}{(1-\beta)(1-\gamma)(1-\alpha)}$$

$$= \frac{2}{-(\alpha\beta\gamma)^3}$$

$$= \frac{2}{1}$$

$\boxed{=2}$ $\boxed{=2}$

$$\boxed{\begin{aligned}&\sum\alpha = \alpha+\beta+\gamma\\ &\sum\alpha^2 = \alpha^2+\beta^2+\gamma^2\end{aligned}}$$

H.W. 30-5-2024

Q-1 {28, 29, 30} *

Q-3 {4-6, 9, 10}

Q J-M {8, 9, 10, 11, 12}

General quadratic equation in two variable.

$$f(x,y) = ax^2+by^2+2hxy+2gx+2fy+c$$

$\hookrightarrow$ two linear factors when

$$\boxed{abc+2fgh-af^2-bg^2-ch^2=0}$$

Proof:- consider quadratic in x

$$\underbrace{ax^2}_{x^2}+\underbrace{(2hy+2g)\,x}_{x}+\underbrace{by^2+2fy+c}_{\text{constant}}=0$$

$$x = \frac{-(2hy+2g) \pm \sqrt{(2hy+2g)^2-4(a)(by^2+2fy+c)}}{2a}$$

~~$x = -2hy - 2g$~~

Q find weather $x^2+2xy+2x+6y-3=0$ have two of linear factors or not.

$2\times 3-9+3 = 0$

So it ~~can't~~ resolved in 2 linear factors.

Q $x^2+2xy+2x+ky^2+k=0$ find k if the above equation has two linear factors

$a=1$
$b=k$
$c=k$
$h=1$
$g=1$
$f=0$

$k^2-k-k=0$
$k^2-2k=0$
$\boxed{k=0,2}$

Type-2 - when two homogeneous equation have ~~common~~ common linear factors.

Homogeneity → when degree of all terms is same (xy degree $= 2$)

$a_1x^2+2h_1xy+b_1y^2=0$ $\qquad$ $a_2x^2+2h_2xy+b_2y^2=0$

Assume $y-mx=0$ is a common factor

$x=0$ is a factor

put $y=mx$ in equations

$a_1x^2+2h_1x(mx)+b_1m^2x^2=0$

$a_1x^2+2h_1mx^2+b_1m^2x^2=0$

$x^2(a_1+2h_1m+m^2b_1)=0$

we know $x=0$ is a common factor so can cut $x^2=0$

~~a_1+2h_1~~

or $b_1m^2+2h_1m+a_1=0$ $\quad\Big|\quad$ $b_2m^2+2h_2m+a_2=0$

both have a common root.

Method 2:-

$m^2b_1+2h_1m+a_1=0$

$m^2b_2+2h_2m+a_2=0$

$$\frac{m^2}{\begin{matrix}2h_1 & a_1\\ 2h_2 & a_2\end{matrix}}=\frac{-m}{\begin{matrix}b_1 & a_1\\ b_2 & a_2\end{matrix}}=\frac{1}{\begin{matrix}b_1 & 2h_1\\ b_2 & 2h_2\end{matrix}}$$

cross multiply

$$\frac{m^2}{2h_1 a_2 - 2h_2 a_1} = \frac{-m}{b_1 a_2 - b_2 a_1} = \frac{1}{2b_1 h_2 - 2h_1 b_2}$$

$$\cancel{m = \frac{b_1 a_2 - b_2 a_1}{}}$$

$$m = \frac{b_2 a_1 - b_1 a_2}{2b_1 h_2 - 2h_1 b_2}$$

Now.

$$\frac{m^2}{2b_1 a_2 - 2b_2 a_1} = \frac{1}{2b_1 h_2 - 2h_1 b_2}$$

Put value of m

$$\frac{\left(\frac{b_2 a_1 - b a_2}{2b_1 h_2 - 2h_1 b_2}\right)^2}{2h_1 a_2 - 2b_2 a_1} = \frac{1}{2b_1 b_2 - 2h_1 b_2}$$

$$\frac{(b_2 a_1 - b_1 a_2)^2}{(2b_1 h_2 - 2h_1 b_2)^2 (2h_1 a_2 - 2h_2 a_1)} = \frac{1}{(2b_1 h_2 - 2h_1 b_2)}$$

$$\frac{(b_2 a_1 - b_1 a_2)^2}{(2b_1 h_2 - 2h_1 b_2)(2h_1 a_2 - 2h_2 a_1)} = 1$$

$$(b_2 a_1 - b_1 a_2)^2 = (2b_1 h_2 - 2h_1 b_2)(2h_1 a_2 - 2h_2 a_1)$$

DYS-11 - Complete H.W.

~~DYS~~ J-A - Complete

J-M

Q16.

$$\frac{|x+3|-1}{|x|-2} \geq 0$$

$-\,-$ $\quad$ $+\,-$ $\quad$ $+\,+$

$-3 \quad 0$

Case ①

$x \in (-\infty, -3)$

$$\frac{-(x+3)-1}{-x-2} \geq 0$$

$$\frac{[x+3+1]}{[x+2]} \geq 0$$

$$\frac{x+4}{x+2} \geq 0$$

$-4 \quad -2$

$x \in (-\infty, -4] \cup (-2, \infty)$

$\boxed{x \in (-\infty, -4)}$

Case ②

$x \in [-3, 0]$

$$\frac{x+3-1}{-(x+2)} \geq 0$$

$$\frac{\cancel{x+2}}{-(\cancel{x+2})} \geq 0$$

$-\,\geq 0$

$x \in \phi$

Case ③

$x \in (0, \infty)$

$$\frac{x+3-1}{x-2} \geq 0$$

$$\frac{x+2}{x-2} \geq 0$$

$+ \; -2 \; - \; 2 \; +$

$x \in (-\infty, -2] \cup (2, \infty)$

$\boxed{x \in (2, \infty)}$

$x \in [-6, -2) \cup (-2, 2) \cup (2, 3]$

$\boxed{x \in [-6, -4] \cup (2, 3]}$

Part 2 - $x^2 - 7|x| + 9 \le 0$

$$x^2 = |x|^2$$

$$|x|^2 - 7|x| + 9 \le 0$$

$$\cancel{t^2 - 7t + 9 \le 0}$$

$$\cancel{t = 7}$$

$$\cancel{+7|x| = x^2 + 9}$$

$$\cancel{|x| = \frac{x^2+9}{7}}$$

$$-7|x| \le x^2 + 9$$

$$|x| \le \frac{x^2+9}{7}$$

$$x \in \left(-\frac{x^2-9}{7}, \frac{x^2+9}{7}\right)$$

$$-\frac{x^2-9}{7} \le x \le \frac{x^2+9}{7}$$

$-x^2 - 9 \le 7x$ $\qquad\qquad$ $x^2 + 9 \ge 7x$

$\underbrace{x^2 + 7x + 9 \ge 0}$, $\qquad\qquad$ $\underbrace{x^2 - 7x + 9 \ge 0}$

$x \in (-\infty, -1.sau] \cup [1.sau, \infty) \cap x \in [-5.sa, -1.sau] \cup [1.sa, 5.sau]$

$x \in [-5.soie, -1.sau] \cup [1.sau, 5.sam]$

(72)

Q 10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$x+1 = y$

$y^{\log_{10} y} = 100y$

take $\log_y$ both sides

$\log_{10} y = \log_y 100y$

$1 = \log_y (100y + 10)$

$y = 100y + 10$

$99y + 10 = 0$

$y = -\frac{10}{99}$

$x + 1 = -\frac{10}{99}$

$x = 1 - \frac{10}{99}$

$x = \frac{59}{99}$

$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$\log_{10} y = 1 + \frac{2}{\log_{10} y}$

$z = 1 + \frac{2}{z}$

$z^2 = z + 2$

$z^2 - z - 2 = 0$

$z = 2, -1$

$\log_{10} y = -1$

$\frac{1}{10} = y$

$x + 1 = \frac{1}{10}$

$x = \frac{1}{10} - 1$

$x = \frac{1-10}{10}$

$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$100 = y$

$x + 1 = 100$

$\boxed{x = 99}$

!! Logarithm !!

→ Every ⊕ve real number N can be expressed in exponential form as.

$$a^x = N$$

$a \to$ ⊕ve real ~~no~~ number > 0 but $\neq 1$

$x \to$ Exponent

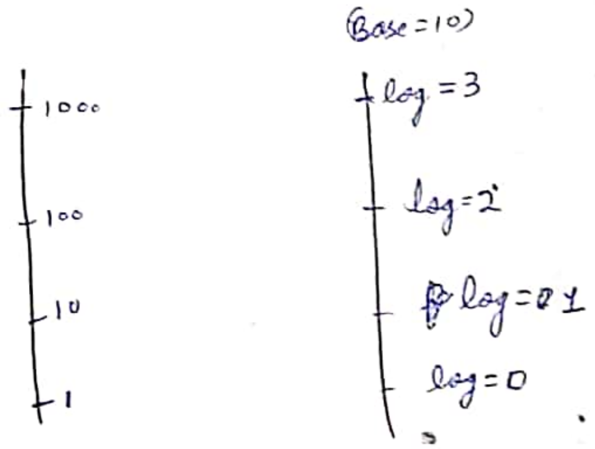
eg. $2^2 = 4$, $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ etc

Reason I

$2^y = 4$, $\left(\frac{1}{2}\right)^y = \frac{1}{16}$ $\quad (8.2)^y = 16.32$

To find y.

Reason II



For the above reasons we introduced log. It is expressed as

$$\log_a N = x$$

$a \to$ Base
$x \to$ exponent

[a की Power को कितने से Raise करें की N aa jaye.]

eg. $\log_2 8 = 3$ [2 की Power को कितने Raise करें कि 8 aaye]

$\log_6 216 = 3$

$\log_2 \frac{1}{16} = -4$

$\log_{0.6}\left(\frac{25}{9}\right) =$

$0.6 = \frac{6}{10}$

$= \frac{3}{5}$

$= \left(\frac{3}{5}\right)^{-2}$

$= \left(\frac{5}{3}\right)^{2}$

$= \frac{25}{9}$

$$\boxed{\log_{0.6}\left(\frac{25}{9}\right) = -2}$$

$\log_{\frac{1}{2}}(1) = 0$

$\log_{\frac{1}{3}} 27 = -3$

$\log_1\left(\frac{1}{2}\right) =$ Not Defined

$\log_{(-2)}(16) = 4$ (Wrong)

$16^{1/4} = \sqrt{4} = |\pm 2| = 2 \neq -2$

Thus Base is ⊕ve

$\neq 1$

$$\boxed{\log_a N = x \quad a>0 \quad a \neq 1 \quad N>0}$$

Q Conversion of Logarithm form in exponential form.

(1) $\log_2 32 = 5 \longrightarrow 2^5 = 32$

(2) $\log_{36} 6 = \frac{1}{2} \longrightarrow 36^{1/2} = 6$

(3) $\log_8 1 = 0 \longrightarrow 8^0 = 1$

(4) $\log_{10}(0.001) = -3 \longrightarrow 10^{-3} = 0.001$

Q find the value of x if $\log_5 125 = x$

$5^x = 125$

$5^x = (5)^3$

$\boxed{x = 3}$

Q $\log_2 m = 1.5$

$2^{1.5} = m$

$2^{3/2} = m$

$\sqrt{2^3} = m$

$\sqrt{8} = m$

$\boxed{m = 2\sqrt{2}}$

Note - For some number different bases gives different answers.

Q Find log

(1) 32 (base $\frac{1}{2}$)

$$\log_{1/2} 32 = x$$

$$\left(\frac{1}{2}\right)^x = 32$$

$$\frac{1}{2^x} = 32$$

$$\frac{1}{2^x} = 2^5$$

$$2^{-x} = 2^5$$

$$-x = 5$$

$$\boxed{x = -5}$$

(2) 32 (base 2)

$$\log_2 32 = x$$

$$2^x = 32$$

$$2^x = 2^5$$

$$\boxed{x = 5}$$

(3) $3\sqrt{3}$ (base 3)

$$\log_3 3\sqrt{3} = x$$

$$3^x = 3\sqrt{3}$$

$$3^x = \sqrt{27}$$

$$3^x = \cancel{27}[(3)^3]^{1/2}$$

$$3^x = 3^{3/2}$$

$$\boxed{x = 3/2}$$

(4) $3\sqrt{3}$ (base $\frac{1}{3}$)

$$\log_{1/3} 3\sqrt{3} = x$$

$$\left(\frac{1}{3}\right)^x = 3\sqrt{3}$$

$$3^{-x} = 3^{3/2}$$

$$-x = \frac{3}{2}$$

$$\boxed{x = -3/2}$$

Note :-

① $\log_a 1 = 0 \quad (a > 0, a \neq 1)$

② $\log_N N = 1$

③ $\log_{\frac{1}{N}} N = -1$ or $\log_N \frac{1}{N} = -1 \quad (N \geq 0, N \neq 1)$

Q find value of

① $\log_{2+\sqrt{3}} (2-\sqrt{3}) = x$

$$(2+\sqrt{3})^x = 2-\sqrt{3}$$

$$(2+\sqrt{3})^x = \frac{(2-\sqrt{3})}{2+\sqrt{3}} \times (2+\sqrt{3})$$

$$(2+\sqrt{3})^x = \frac{(2^2-\sqrt{3}^2)}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = \frac{4-3}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = (2+\sqrt{3})^{-1}$$

$$\boxed{x=-1}$$

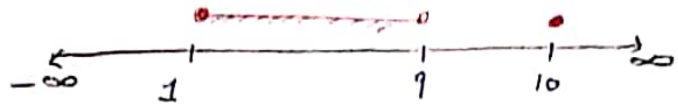
② $\log_{(1+\sqrt{2})} (\sqrt{3+2\sqrt{2}}) = x$

$$(1+\sqrt{2})^x = (\sqrt{2}+1)^1$$

$$\boxed{x=1}$$

H.W. 03-06-2024

DYS-1 {Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}



Principle Properties of log $\{a>0, \neq 1 \ ; \ m \ \& \ n > 0\}$

① $\log_a(mn) = \log_a m + \log_a n$ (can put more than two terms)

② $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

③ $\log_a(m^n) = n\log_a m$

Proof :-

① $\log_a(mn) = \log_a m + \log_a n$

let $\log_a m = x$, $\log_a n = y$

$\downarrow \qquad\qquad\qquad \downarrow$

$a^x = m \qquad\qquad a^y = n$

$\therefore mn = a^x \times a^y$
$= a^{x+y}$

$\log_a(mn) = x+y$

$= \log_a m + \log_a n$

③ $\log_a m^n = n\log_a m$

LHS

$\log_a(\underbrace{m \times m \times m \ldots}_{n \text{ times}})$

$= \log_a m + \log_a m + \ldots \text{ n times}$

$= n\log_a m = \text{RHS}$

Hence Proved

Note -

① $\log_{1/2} \log_4 2$

$\log_4 2 = \frac{1}{2}$

$\log_{1/2}\left(\frac{1}{2}\right) = x$

$\boxed{x = 1}$

② $\log_a N \cdot \log_b M$

$\boxed{\log_a N \times \log_b M}$

③ $\log_a N \cdot 3$

$\log_a N \times 3$

$3 \log_a N$

$\boxed{\log_a N^3}$

~~④ log 3 + log (Base 1~~

Q Find value (Base 10)

① $\log 3 + \log 5$

$\log_{10} (3 \times 5)$

$\boxed{\log_{10} 15}$

② $\log 6 - \log 2$

$\log \left(\frac{6}{2}\right)$

$\log_{10} 3$

(3) $3\log 4$

$\log (4^3)$

$\cancel{\log (64)}$

$\log_{10} 64$

(4) $\log 2\,36 - \log 1$

$\log_{10} 236$

(5) $\cancel{2\log}$ $2\log 3 - 3\log 2$

$\log 3^2 - \cancel{3}\log 2^3$

$\log 9 - \log 8$

$\log_{10} \left(\frac{9}{8}\right)$

(6) $\log 2 + \log 3 + \log 4$

$\log (2 \times 3 \times 4)$

$-\log (12 \times 2)$

$\log_{10} (24)$

(7) $5\log 10 + 2\log 3 - \log 2$

$\log 10^5 + 2\log 3^2 - \log 2$

$\log 10^5 + \log 9 - \log 2$

$\log 10^5 + \log \frac{9}{2}$

$\log_{10} (4.5 \times 10^5)$

Properties

④ fundamental log. Identity

$$\boxed{a^{(\log_a N) \to \text{in power}} = N}$$

Proof:- $a^{(\log_a N)} = N$

Taking $\log_a$ both sides

$$\log_a\left(a^{\log_a N}\right) = \log_a N$$

$$\log_a N \cdot \log_a a = \log_a N$$

$$\cancel{\log_a} N = \cancel{\log_a} N$$

$$N = N$$

Hence, Proved

⑤ Base Changing Theorem

$$\boxed{\frac{\log_a m}{\log_a n} = \log_n m}$$

Proof:-

$\log_n m = p \Leftrightarrow n^p = m$

$\log_a m = q \Leftrightarrow a^q = m$

$\log_a n = r \Leftrightarrow a^r = n$

$n^p = a^q$

$(a^r)^p = a^q$

$a^q = a^{rp}$

$q = rp$

$\frac{q}{r} = p$

$$\frac{\log_a m}{\log_a n} = \log_n m$$

⑥ DDF

$$\boxed{a^{\log_b c} = c^{\log_b a}}$$

Proof

$\log_b c = x \Leftrightarrow b^x = c$

L.H.S

a^x

RHS

$c^{\log_b a}$

$b^{x \log_b a}$

$b^{\log_b a^x}$

a^x

LHS = RHS

<u>Hence, Proved.</u>

⑦ Base - Power Theorm

$$\boxed{\log_{a^n} b^m = \frac{m}{n} \log_a b}$$

Q find value (Base 10)

(1) $2^{\log_2 5}$

$\boxed{5}$

(2) $\pi^{\log_\pi 60}$

$\boxed{60}$

(3) $25^{\log_5 8}$

$5^{2\log_5 8}$

$5^{\log_5 8^2}$

8^2

$\boxed{64}$

(4) $\left(\frac{1}{16}\right)^{\log_2 \pi}$

$2^{\log_2 \pi^{-4}}$

$\boxed{\pi^{-4}}$

(5) ~~$\log_4 8$~~ $\log_{64} 8$

$\log_{2^6} 2^3$

$\frac{3}{6}\log_2 2$

$\frac{3}{6}$

$\boxed{\frac{1}{2}}$

(6) $\log_3 2 \times \log_4 3 \times \log_5 4$

$\log_4 3^{\log_3 2} \times \log_5 4$

$\log_5 4^{\log_4 3^{\log_3 2}}$

$\log_5 2$

(7) $4^{\log_3 7} - 7^{\log_3 4}$

(8) $2^{\log_3 5} + 3^{\log_7 6} - 5^{\log_3 2} - 6^{\log_7 3}$

(9) $\log_2\left[\log_2\left\{\log_3\left(\log_3 27^3\right)\right\}\right]$

(10) $\log(\tan 1^\circ) \cdot \log(\tan 2^\circ) \ldots \log(\tan 89^\circ)$

(11) $7^{\log_7 x^2} + x - 2 = 0$

(12) $\log(\sin 1^\circ) \cdot \log(\sin 2^\circ) \ldots \log(\sin 90^\circ)$

(13) $\log_{10}\left\{\left(\sqrt{a^{-2} \cdot b}\right)\left(\sqrt[3]{ab^{-3}}\right)\right\}$

(14) $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots \log_n (n+1) = 5$

(15) Prove: $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$

Q7 $4^{\log_3 7} - 7^{\log_3 4}$

$4^{\log_3 7} = 7^{\log_3 4}$

$\boxed{0}$

Q8. $2^{\log_3 5} = 5^{\log_3 2}$

$3^{\log_7 6} = 6^{\log_7 3}$

$\boxed{0}$

Q9. $\log_3 27^3 = 9$

$\log_3 9 = 2$

$\log_2 2 = 1$

$\log_2 1 = 0$

$\boxed{0}$

Q11. $7^{\log_7 x^2} + x - 2 = 0$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$\boxed{x = 1, -2}$

85

Scanned with CamScanner

~~13~~ $\cancel{10^x = (\sqrt{a^{-2} \cdot b})(\sqrt[3]{ab^{-3}})}$

$\cancel{10^x = a^{-1} \cdot b^{1/2} \cdot a^{1/3} \cdot b^{-1}}$

$\cancel{10^x = a^{-2/3} \cdot b^{-1/2}}$

13 $10^x = a^{-2/3} \cdot b^{-1/2}$

$\log_{10}(a^{-2/3}) + \log_{10}(b^{-1/2})$

$\boxed{\frac{-2}{3}\log_{10} a - \frac{1}{2}\log_{10} b}$

~~11~~ 10 It includes $\tan 45°$,

$\log(\tan 45°)$

$= \log(1)$

$\boxed{= 0}$

12 $\log(\sin 90°)$

$\log(1)$

$\boxed{0}$

14 $\frac{\log_a 3}{\log_a 2} \times \frac{\log_a 4}{\log_a 3} \dots$

$\frac{\log_a 4}{\log_a 2} \dots$

$\frac{\log_a (n+1)}{\log_a 2} = \boxed{\log_2 (n+1)}$

86

~~Q15. LHS~~

$\cancel{2^{\sqrt{\log \frac{3}{2}}}}$

$\log_2 (n+1) = 5$

$2^5 = n + 1$

$3^2 = n + 1$

$\boxed{n = 31}$

~~Q 15~~

~~LHS~~ ~~RHS~~

$\cancel{2^{\sqrt{\log \frac{3}{2}}}}$ $\cancel{3^{\sqrt{\log_3 2}}}$

$\cancel{3^{1/2}}$ $\cancel{2^{1/2}}$

$\cancel{\sqrt{3}}$ $\cancel{\sqrt{}}$

H.W.

DYS-2 {1, 2, 3, 4}

DYS-3 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

O-1 {1, 2, 3, 4, 5, 6}

Q15. $\cancel{\log_2 a = \sqrt{\log_2 3}}$

$\cancel{a = 2^{\sqrt{\log_2 3}}}$

$\cancel{\log_2 2^{\sqrt{\log_2 3}}}$

$\cancel{\sqrt{\log_2 3} \times \log_2 2}$

Q15. $a = 2^{\sqrt{\log_2 3}}$ $b = 3^{\sqrt{\log_3 2}}$

$= \log_2 b$

$= \log_2 3^{\sqrt{\log_3 2}}$

$= \sqrt{\log_3 2} \times \log_2 3$

$\cancel{= \frac{\sqrt{\log_2 2}}{\sqrt{\log_2 3}} = \log_2 3}$

$\log_3 2 = x$

$\frac{1}{\log_2 3} = \frac{1}{x}$

$3^x = 2$

$3 = 2^{1/x}$

$\frac{1}{\log_3 2} = \log_2 3$

$\frac{1}{\sqrt{\log_2 3}} \times \log_2 3$

$\sqrt{\log_2 3} = \log_2 b$

$2^{\sqrt{\log_2 3}} = b$

$\boxed{a = b}$

Hence, Proved

(88)

Antilog

$$\boxed{\text{Antilog}_a x = a^x}$$

$$\log_a N = x$$

$$\text{Antilog}_a(\log_a N) = \text{Antilog}_a x$$

DYS - 3

Q13 $\text{Antilog}_{64}\left(\frac{5}{6}\right)$

$$(64)^{5/6}$$

$$(2)^5$$

$$\boxed{32}$$

~~Illustrati~~ Illustration - 8

$$\left(\log_b a \cdot \log_c a - \log_a a\right) + \left(\log_a b \times \log_c b - \log_b b\right) + \left(\log_a c \cdot \log_b c - \log_c c\right) = 0$$

$$\frac{\log a}{\log b} \times \frac{\log a}{\log c} + \frac{\log b \ \log b}{\log a \ \log c} + \frac{\log c \log c}{\log a \log b} - 3 = 0$$

$$(\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a \log b \log c$$

$$\log a + \log b + \log c = 0$$

$$\log(abc) = 0$$

$$\boxed{abc = 1}$$

Illustration 10

$\log_4 18 = x$

$4^x = 18$

$2^{2x} = 9^2$

thus irrational.

Q $2\log_{2}(\log_2 x) + \log_{1/2}\left(\frac{3}{2} + \log_2 x\right) = 1$

$\log_2 \left((\log_2 x)^2 \times \frac{1}{\log_4 x^3}\right) = 1$

$\log_2 x \times \log_2 x \times \frac{1}{\frac{3}{2}\log_2 x} = 2$

$\log_2 x = 2 \times \frac{3}{2}$

$\log_2 x = 3$

$2^3 = x$

$\boxed{8 = x}$

Q 3. $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots\infty}}\right)$

$x = \sqrt{4 - \frac{x}{3\sqrt{2}}}$

$x^2 = 4 - \frac{x}{3\sqrt{2}}$

$3\sqrt{2}x^2 = 12\sqrt{2} - x$

$3\sqrt{2}x^2 + x - 12\sqrt{2} = 0$

$x = -1 \pm \sqrt{1 + 288}$

$x = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$x = \frac{16}{6\sqrt{2}}$

$6 + \log_{3/2} \frac{4}{9}$

$6 - 2$

$\boxed{4}$

$6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{16}{3\sqrt{2}}\right)$

$6 + \log_{3/2} \frac{8}{9} \times 2$

$\left(\frac{3}{2}\right)^x = \frac{8}{9}$

$\left(\frac{2}{3}\right)^{-x} = \frac{8}{9}$

$6 - 2 + \log_{1/2} 2$

$4 + \sqrt{8}$

$\boxed{4 + 2\sqrt{2}}$

Q.20

Q. $2\log_2(\log_2 x) + \log_{1/2}\left(\frac{3}{2} + \log_2 x\right) = 1$

$\log_2 t^2 + \log_2\left(\frac{1}{\frac{3}{2}+t}\right) = 1$

$\frac{t^2}{\frac{3+2t}{2}} = 1$

$\frac{2t^2}{3+2t} = 1$

$2t^2 = 3 + 2t$

$2t^2 - 2t - 3 = 0$

~~$t = 2 \pm \sqrt{4+24}$~~

$t = 3, -1$

$\log_2 x = 3$ $\qquad$ $\log_2 x = -1$ ✗

$\boxed{x = 8}$

$\boxed{x = 8}$

H.W

J.A - {5, 6}

DYS-4 [9, 10]

Q

(1) $\cancel{2\log_2} 2^{\log_2 x^2} - 3x - 4 = 0$

$2^{\log_2 x^2} = 3x + 4$

$\log_2 3x+4 = \log_2 x^2$

$3x + 4 = x^2$

$x^2 - 3x - 4 = 0$

$x = 4, -1$

$\boxed{x = 4, -1}$

(2) $2^{2\log_2 x} - 3x - 4 = 0$

$x = 4, -1$

-1 is rejected as $\ominus$ve not possible in log

(3) $\log_2 (x^2 - 1) = 3$

$8 = x^2 - 1$

$9 = x^2$

$x = \pm 3$

(4) $\log_2 (x+1) + \log_2 (x-1) = 3$

$\log_2 (x+1)(x-1) = 3$

$8 = x^2 - 1$

$x^2 + 9$

$x = \pm 3$

-3 is reject

$\boxed{x = 3}$

Q 5 $x^2+7^{\log_7 x}-2=0$

$x^2+x-2=0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$x=1,-2$

-2 reject

$\boxed{x=1}$

Q 7. $5^{(\log_5 x)^2}+x^{\log_5 x}$

$5^{y^2}+x^y=1250$

$\log_5 x=y$

$5^y=x$

$5^{y^2}+5^{y^2}=1250$

$5^{y^2}=625$

$5^{y^2}=5^4$

$y^2=4$ $y=\pm 2$

$\boxed{x=25, \frac{1}{25}}$

Q 6. $\log_4(2\log_3(1+\log_2(1+3\log_2 x)))=\frac{1}{2}$

$4^{1/2}=2\log_3(1+\log_2(1+3\log_3 2))$

$2=\log_2(1+3\log_2 x)$

$1=\log_2 x$

$1=\log_2 x$

$\boxed{x=2}$

Q 8. $\log_2(9-2^x)=10^{\log_{10}(3-x)}$

$2^{3-x}=9-2^x$

$2^3\times 2^x=9-2^x$

$8y=9-y$

$y=8,1$

$x=3,0$ $\boxed{x=0}$

3 reject

(93)

Q9. $\log_7 \log_5 (\sqrt{x+5}+\sqrt{x} = 0$

$\sqrt{x+5} + \sqrt{x} = 5$

$5 = \sqrt{x+5} + \sqrt{x}$

$25 = x+5+x+2\sqrt{(x+5)(x)}$

$\cancel{25} \; 10 = x + \sqrt{x^2+5x}$

$10 - x = \sqrt{x^2+5x}$

$100 = 25x$

$\boxed{x = 4}$

Q10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$x+1 = y$

$y^{\log_{10} y} = 100y$

$\log_{10} y = \log_y 100y$ (take $\log_{10}$) both sides

$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$z = 1 + \frac{2}{z}$

$\underline{z = 2, -1}$

$\log_{10} y = -1$

$y = \frac{1}{10}$

$x+1 = \frac{1}{10}$

$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$y = 100$

$x+1 = 100$

$\boxed{x = 99}$

Q (11) $\log_{x-1}(4) = 1 + \log_2(x-1)$

$\log_y 4 = 1 + \log_2 y$

$\cancel{y^{1+\log_2 y}}$

$2\log_y 2 = 1 + \log_2 y$

$\log_2 y = z$

$\frac{2}{z} = 1 + z$

$2 = z + z^2$

$z^2 + z - 2 = 0$

$z = \frac{-1 \pm \sqrt{1+8}}{2}$

$z = \frac{-1 \pm 3}{2}$

$z = -2, 1$

$\log_2 y = -2$

$\frac{1}{4} = y$

$x - 1 = y$

$x - 1 = \frac{1}{4}$

$x = \frac{1}{4} + \frac{4}{4}$

$x = \frac{5}{4}$

$\log_2 y = 1$

$y = 2$

$x - 1 = 2$

$x = 3$

$\boxed{x = 3, 5/4}$

(12) sum of values of x A) 1, B) 4 C) 0 D) 3

$\log_{2x-1}(x^3+3x^2-13x+10) = 2$

$(2x-1)^2 = x^3+3x^2-13x+10$

$4x^2+1-4x = x^3+3x^2-13x+10$

$x^3 - x^2 - 9x + 9 = 0$

$x = 1$

$(x-1)$

$x^2 - 9 = 0$

$x = \pm 3$

$x = 1, 3, -3$

$\cancel{3+-3+1}$

but, -3 is reject as $2x-1$ will be $\ominus$ve

$\cancel{\boxed{A}}$

$\cancel{\boxed{A}}$

1 reject as $2x-1$ would be 1

$x = 3$

$\boxed{D}$

(13)

$$5^{1+\log_4 x} + 5^{(\log_{1/4} x)-1} = \frac{26}{5}$$

$$5^{1+\log_4 x} + 5^{-(\log_4 x+1)} = \frac{26}{5}$$

$$5^{y} + \frac{1}{5^{y}} = \frac{26}{5}.$$

$$5^{2y}+1 = \frac{26\times 5^{y}}{5}$$

$$5^{2y}$$

$$5\times 5^{2y}+1 = 26\times 5^{y}$$

$$\cancel{5\times 5^{x^2}+1 = 26}$$

$$5\times z^2+1 = 26z$$

$$5z^2-26z+1=0$$

$$z = \frac{26\pm\sqrt{676-20}}{2}$$

$$z = \frac{26\pm\sqrt{656}}{2}$$

$$t+\frac{1}{t} = \frac{26}{5}$$

$$t^2+1 = \frac{26t}{5}$$

$$5t^2-26t+1=0$$

$$5t + \frac{1}{5t} = \frac{26}{5}$$

$$25t^2+1 = \frac{26\times 5t}{5}$$

$$25t^2-26t+1$$

$$25t^2-25t-t+1$$

$$25t(t-1)-1(t-1)$$

$$t = 1,\ 1/25$$

$$5^{\log_4 x} = 1$$

$$\cancel{\log_5 1 = \log_4 x}$$

$$\log_4 x = 0$$

$$4^0 = x$$

$$\boxed{x=1}$$

$$5^{\log_4 x} = \frac{1}{25}$$

$$\log_4 x = -2$$

$$\boxed{\frac{1}{16} = x}$$

$$\boxed{x = 1,\ 1/16}$$

$$\begin{array}{r} 26 \\ \times 26 \\ \hline 156 \\ 520 \\ \hline 676 \end{array}$$

(12) $(x-2)^{\log^2(x-2)+\log(x-2)^5-12} = 10^{2\log(x-2)}$

$\cancel{(x-2)^{y^2}}$

$\cancel{x-2=y}$

$y^{\cancel{(\log y)(\log y)} + \log y^5 - 12} = 10^{2\log y}$

$\cancel{\log y = 2}$

$\cancel{(x-2)^{2y^2}}$

$x-2=y$

$y^{(\log y)(\log y)+5\log y-12} = 10^{2\log y}$

$y^{\log y(\log y+5)-12} = 10^{2\log y^2}$

$2\log y^2 \log_y 10 = \log_y(\log_y+5)-12$

$2\log_y 10 = \log y + 5 - \frac{12\log}{\log y}$

$2\log_y 100 = \frac{(\log y)^2+(\log y)-12}{\log_{10} y}$

Q 12. $(x-2)^{\log(x-2)(\log(x-2)+\log(x-2)^5-12)} = 10^{2\log(x-2)}$

Note - can assume same base on both sides.

$$y^{\log_{10} y \times \log_{10} y + 5\log_{10} y^{0} - 12} = 10^{\log_{10} y^2}$$

$$y^{\log_{10} y(\log_{10} y + 5) - 12} = y^2$$

$$\log_{10} y(\log_{10} y + 5) - 12 = 2$$

$$100 = y(\log_{10} y + 5) - 12$$

$$\cancel{100 = 5y + y\log_{10} y}$$

$$\cancel{100 =}$$

$$2 = (\log_{10} y)^2 + 5\log_{10} y - 12$$

$$14 = z^2 + 5z$$

$$z^2 + 5z - 14 = 0$$

$$z = \frac{-5 \pm \sqrt{25+56}}{2}$$

$$z = \frac{-5 \pm 9}{2}$$

$$z = -7, 2$$

$\log_{10} y = 2$

$y = 100$

$x - 2 = 100$

$\boxed{x = 102}$

$\log_{10} y = -7$

$\frac{1}{10^7} = y$

$x = \frac{1}{10^7} + 2$

$x = \frac{1 + 20000000}{10000000}$

$x = \frac{20000001}{10000000}$

$x = 2.0000001$

$\boxed{x = 102, 2.0000001}$

H.W. 07-06-2024

DYS-4 [10:]

~~Logarithmic~~

100

Logarithmic Inequalities

Constant Base

- $\in (0,1)$ — sign change
- $(1,\infty)$ — no sign change

Variable Base

case 1: Base $\in (0,1)$ $\xrightarrow{\text{union}}$ case 2: $\in (1,\infty)$

e.g. $\log_{1/2} x > \log_{1/2}(2x-1)$

$x < 2x-1$

$1 < x$

$\boxed{x > 1}$

e.g. $\log_{5} x > \log_{5}(2x-1)$

$x > 2x-1$

$1 > x$

$\boxed{x < 1}$

Q $\log_{0.5}(x-3) > \log_{0.5}(2x)$

$x-3 < 2x$

$-3 < x$

$x > -3$

$\therefore x \in (-3,\infty)$ — ①

For $\log_{0.5}(x-3)$ — $x-3>0$, $x>3$ — ②

$\log_{0.5}(2x)$ — $2x>0$, $x>0$ — ③

$\Rightarrow$ ① $\cap$ ② $\cap$ ③

$\boxed{x \in (3,\infty)}$

Q2. $\log_7(x^2-3x) \geq \log_7(2x-6)$

$x^2-3x \geq 2x-6$

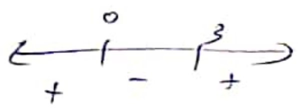
$x^2-5x+6 \geq 0$

$(x-2)(x-3) \geq 0$

$x \in (-\infty, 2] \cup [3, \infty)$ — (1)

$\log_7(x^2-3x)$ — $x^2-3x > 0$

$x(x-3) > 0$



$x \in (-\infty, 0) \cup (3, \infty)$ — (2)

$\log_7(2x-6)$ — $2x-6 > 0$

$2x > 6$

$x > 3$

$x \in (3, \infty)$ — (3)

(1) $\cap$ (2) $\cap$ (3)

$\boxed{x \in (3, \infty)}$

Q $\log_x(2x) > 2$

$x \in (0,1)$

Case 1 $2x < x^2$

$0 < x^2 - 2x$

$x(x-2) > 0$

$x \in (-\infty, 0) \cup [2, \infty)$

$\xrightarrow{\cap} x \in \phi$

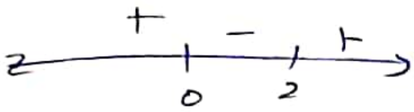
Case 2:-

$x \in (1, \infty)$

$2x > x^2$

$0 > x^2 - 2x$

$x(x-2) < 0$



$x \in (0, 2)$

$\xrightarrow{\cap} x \in (1, 2)$

Case 1 ∪ Case 2: $(x \in \phi) \cup (x \in (1,2))$

$x \in (1, 2)$ — ①

$\log_x 2x$

$2x > 0$

$x > 0$

$x > 0$, ~~~~

$x \neq 1$

$\xrightarrow{\cap} x \in (0,1) \cup (1, \infty)$ — ②

① ∩ ②

$(x \in (1,2)) \cap (x \in (0,1) \cup (1, \infty))$

$\boxed{x \in (1,2)}$

Q ① $\log_{1/3}\left(\frac{1-2x}{x}\right) \le 0$

$\cancel{\left(\frac{1}{3}\right)^0 \le \frac{1-2x}{3}}$

$\cancel{\frac{1-2x}{3} \le 1}$

$\cancel{1-2x \le 3}$

$\cancel{-2x \le 3-1}$

$\cancel{-2x = -2}$

$\cancel{2x = 2}$

$\cancel{x = \frac{2}{2}}$

$\cancel{x = 1}$

$\cancel{1-2x \le 3}$

$\cancel{-2x \le -2}$

$\cancel{2x \ge 2}$

$\cancel{x \ge 1}$

$\cancel{\log_{1/3}\left(\frac{1-2x}{3}\right) \to \frac{1-2x}{3} > 0}$

$\cancel{1 > \frac{2x}{3}}$

$\cancel{3 > 2x}$

$\cancel{x < \frac{3}{2}}$

$\cancel{\boxed{x \in [1, 1.5)}}$

$\frac{1-2x}{x} \ge 1$

$\frac{1-2x-x}{x} \ge 0$

$\frac{1-3x}{x} \ge 0$

$-$ 0 $+$ 1/3 $-$

$\cancel{(-\infty, 0) \cup (1/3, \infty)}$

$x \in (0, 1/3]$ — ①

$\log_{1/3}\left(\frac{1-2x}{x}\right) \to \frac{1-2x}{x} > 0$

0 1/2

$-$ $+$ $-$

$\cancel{x \in (-\infty, 0) \cup (1/2, \infty)}$

$x \in (0, 1/2)$ — ②

① ∩ ②

$\boxed{(0, 1/3]}$

② $\log_{2x}(x^2-5x+6) < 1$

Case 1 $\quad 2x \in (0,1)$

$x \in (0, \tfrac{1}{2})$

$x^2-5x+6 > 2x$

$x^2-7x+6>0$

$x^2-6x-x+6>0$

$(x-6)(x-1)>0$

$+ \quad 1 \quad - \quad 6 \quad +$

$x \in (-\infty,1)\cup(6,\infty)$

$x \in (0,\tfrac{1}{2})$ —— ①

Case 2 $\quad 2x \in (1,\infty)$

$x \in (\tfrac{1}{2},\infty)$

$(x-6)(x-1) < 0$

$+ \quad 1 \quad - \quad 6 \quad +$

$x \in (1,6)$

$x \in (1,6)$ — ②

① ∪ ②

$x \in (0,\tfrac{1}{2}) \cup (1,6)$ — ③

$x^2-5x+6>0$

$x^2-3x-2x+6>0$

$x(x-3)-2(x-3)>0$

$(x-2)(x-3)>0$

$+ \quad 2 \quad - \quad 3 \quad +$

$x \in (-\infty,2)\cup(3,\infty)$

$\cap$

$2x>0$

$x>0, \neq \tfrac{1}{2}$

$x \in (0,\tfrac{1}{2})\cup(\tfrac{1}{2},\infty)$

$x \in (0,\tfrac{1}{2})\cup(\tfrac{1}{2},2)\cup(3,\infty)$ ④

③ ∩ ④

$\boxed{(0,\tfrac{1}{2})\cup(1,2)\cup(3,6)}$

H.W. 08-06-2024

DYS-5 [1, 2, 3, 5, 7, 8, 9, 10]

O-1 [7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]

Q $\log_{0.5}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$

$\cancel{\log_{0.5}(\quad)}$

$\log_6\left(\frac{x^2+x}{x+4}\right) > 1$

$\frac{x^2+x}{x+4} > 6$

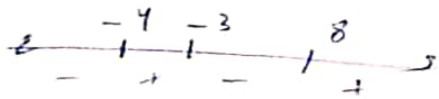
$\frac{x^2+x}{x+4} - 6 > 0$

$\frac{x^2+x-6x-24}{x+4} > 0$

$\frac{x^2-5x-24}{x+4} > 0$

$\frac{x^2-8x+3x-24}{x+4} > 0$

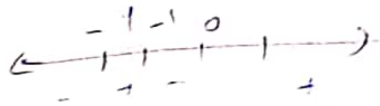
$\frac{(x-8)(x+3)}{x+4} > 0$



$x \in (-4,-3) \cup (8,\infty)$

$\frac{x^2+x}{x+4} > 0$

$\frac{x(x+1)}{x+4} > 0$

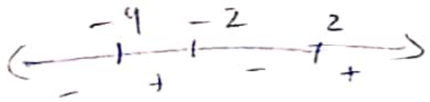


$(-4,-1) \cup (0,\infty)$

$\frac{x^2+x}{x+4} > 1$

$\frac{x^2+x-x-4}{x+4} > 0$

$\frac{(x+2)(x-2)}{x+4} > 0$



$x \in (-4,-2) \cup (2,\infty)$ $\cancel{x(x-8)+3(x-8)}$

$x \in (-4,-2) \cup (2,\infty)$

$x \in (-4,-3) \cup (8,\infty)$

$\boxed{x \in (-4,-3) \cup (8,\infty)}$

Q $\log_2 \log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$

$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 1$

$\log_7 \log_{1/3} (2x-3) < \frac{1}{2}$

$\log_{1/3} (2x-3) < \sqrt{7}$

$2x-3 > \frac{1}{3^{\sqrt{7}}}$

$2x-3 > 0$

$\log_{1/3} (2x-3) > 0$

$\log_7 \log_{1/3} (2x-3) > 0$

$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$

Exponential Inequalities

→ Make the base same

→

Base

Base $\in (0,1)$
Sign Change

Base $\in (1,\infty)$
no sign change

Q1. $2^{x+2} > \left(\frac{1}{4}\right)^{\frac{1}{x}}$

$2^{x+2} > 2^{-2/x}$

$x+2 > -\frac{2}{x}$

$x+2+\frac{2}{x} > 0$

$\frac{x^2+2x+2}{x} > 0$

$\frac{1}{x} > 0$

0

$\boxed{x \in (0,\infty)}$

Q 2. $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

Q 3. $\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \frac{25}{4}$

$\frac{2}{5} > \left(\frac{2}{5}\right)^{\frac{1}{2}\left(\frac{6-5x}{2+5x}\right)}$

$1 < \frac{1}{2}\left(\frac{6-5x}{2+5x}\right)$

$\frac{6-5x-2-5x}{2+5x} > 0$

$\frac{4-10x}{2+5x} > 0$

$\frac{10x-4}{5x+2} < 0$

$1 + \frac{12-5x}{2+5x} < 0$

$\frac{2x+5x+12-5x}{2+5x} < 0$

$\frac{6+x}{5x+2} < 0$

-6

$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{2}{5}\right)^{-2}$

$\frac{5x-6}{5x-2} < 2$

$\frac{5x-6-10x+4}{5x-2} < 0$

$\frac{5x+2}{5x-2} > 0$

$-2/5 \quad 2/5$

$+ \quad - \quad +$

$(-\infty, -2/5) \cup (2/5, \infty)$

$\frac{7x-10x+12}{5x+2} < 0$

$\frac{3x-12}{5x+2} > 0$

$-2/5 \quad 4$

$(-\infty, -2/5) \cup (4, \infty)$

② $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

$\left(\frac{125}{100}\right)^{1-x} < \left(\frac{64}{100}\right)^{2(1+\sqrt{x})}$

$\left(\frac{5}{4}\right)^{1-x} < \left(\frac{5}{4}\right)^{-4(1+\sqrt{x})}$

$1-x < -4(1+\sqrt{x})$

$4(1+\sqrt{x}) < x-1$

$4+4t < t^2-1$

$t^2-4t-5>0$

$t-5t+t-5>0$

$t(t-5)+1(t-5)>0$

$(t+1)(t-5)>0$

$+ \quad -1 \quad - \quad 5 \quad +$

$t \in (-\infty,-1) \cup (5,\infty)$

~~$x \in (-\infty$~~

$\boxed{x \in (5,\infty)}$

③ $\left(\frac{5}{2}\right)^{\frac{5x-6}{5x+2}} < \left(\frac{5}{2}\right)^2$

$\frac{5x-6}{5x+2} < 2$

$\frac{5x-10}{5x+2} < 0$

$\frac{x+2}{5x+2} > 0$

$x \in (-\infty,-2) \cup (-2/5,\infty)$

DYS-6

(8) $\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > 1$

$\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > \left(\frac{2}{3}\right)^{0}$

$\frac{|x|-1}{|x|+1} < 0$

$|x|-1 < 0$

$|x| < 1$

$\boxed{x \in (-1, 1)}$

H.W.

DYS-6-(Full) O-1(21, 22, 23, 24, 25, 26, 27, 28, 29)

O-2(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

C = 2.7

DYS-4 S

(4) $(\log_{10} 100x)^2 + (\log_{10} 10x)^2 + \log_{10} x < 14$

$\cancel{\frac{(\log 100x)^2}{(\log 10)^2} + \frac{(\log 10x)^2}{(\log 10)^2}} + \frac{\log x}{\log 10}$

$\cancel{\frac{(\log 100)^2 + (\log x)^2 + (\log 10)^2 + (\log x)^2 + \cdot}{\log 10 \cdot \log 10}}$

$(2 + \log_{10} x)^2 + (1 + \log_{10} x)^2 + \log_{10} x < 14$

$4 + y^2 + 4y + 1 + y^2 + 2y + y < 14$

$2y^2 + 7y - 9 < 0 \qquad 2y^2 + 9y - 2y - 9 < 0$

$-9/2 \qquad 1$

$\log_{10} x \in (-9/2, 1)$

$\boxed{x \in (10^{-9/2}, 10)}$

(110)