

# Maths - I

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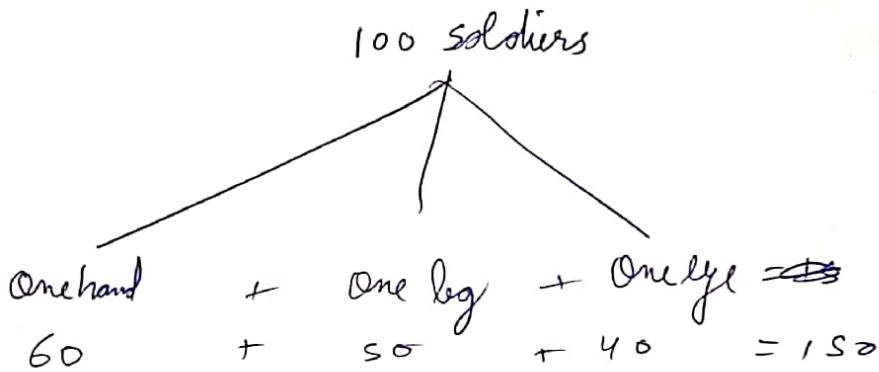
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# Sets, Number & Interval

~~Sets Groups of well defined things~~



$$100 \Rightarrow (L+E) + (H+E) - (L+H) + (L+H+E) \neq L+E+H$$

$$\Rightarrow L+E+H$$

Sets - Set is a collection of well defined objects

- e.g.
1. Collection of all natural numbers
  2. Collection of all rivers/lakes
  3. Collection of all days in a week

Q Verify whether the following are sets or not

- |   |               |
|---|---------------|
| 1. Collection of all boys in a class - <del>yes</del> | yes           |
| 2. Collection of all handsome boys in a class -       | <del>no</del> |
| 3. Collection of most dangerous animals of world      | <del>no</del> |
| 4. Collection of all even integers                    | yes           |
| 5. A, B, C, Z   |               |
| 6. <del>ab</del>                                      |               |

(1)

\*  $A, B, C \dots Z \Rightarrow$  set denotation

$a, b, c \dots, 1, 2 \dots \Rightarrow$  Elements

\*  $\forall$  <sup>Element</sup>  
 $a \in A$  belongs to set A

$a \in A$

belongs to

~~★~~ Element  $a$  not belongs to  $A$

$a \in A$

Q Set A is a collection of  $n$  less than 7.  
natural

- ① SEA ✓
  - ② SEA ✗
  - ③ ØEA ✗ ✓
  - ④ 2EA ✓
  - ⑤ 10EA ✓

## Methods to write a set

## Roster or Tolosa method

Property or set builder form

1. Write down the property or rule which gives us ~~sets~~ elements of a set

e.g.  $A = \{x : P(x)\}$   
 Elements       $\downarrow$       Property  
 Such that

$A = \{x : \text{All Alphabits in}$   
 $\text{english vocabulary}\}$

2. Order does not matter here

e.g.  $A = \{a, b, c\}$  and

$A = \{b, a, c\}$  are some

2

## Roster / Tabular

## Property / builder form

3. Elements are not generally repeated here.

e.g.  $A = \{S, C, H, O, L\}$  and  
 $A = \{S, C, H, O, L\}$  are  
 some, duplicates are not counted

$B = \{x : x \text{ is prime no. less than } 10\}$

$C = \{x : x \text{ is a odd no. and } 1 \leq x \leq 10\}$

Q convert in set builder form

1.  $A = \{3, 6, 9, 12\}$

$A = \{x : x \text{ is double multiple of } 3 \text{ and } 3 \geq x \leq 12\}$  or  
 $A = \{x : x \in 3n \text{ and } n \in N, 1 \leq n \leq 4\}$

2.  $A \cup B = \{2, 4, 8, 15, 32\}$

~~$B = \{x : x \text{ is a natural no. less than } 6\}$~~

~~$B = \{2^x : x \text{ is a natural no. less than } 6\}$~~

~~$B = \{x : x \in "02" \text{ and } n \text{ is a natural no. less than } 6\}$~~

3.  $C = \{2, 4, 6, \dots\}$

$C = \{x : x \text{ is even no. and } x \geq 2\}$

Q convert in roster / tabular form

1.  $A = \{x : x \text{ is an integer and } -\frac{1}{2} < x < \frac{9}{2}\}$

$A = \{0, 1, 2, 3, 4\}$

2.  $B = \{x : x \text{ is a month of a year & not having } 31 \text{ day}\}$

$B = \{\text{February, April, June, September, November}\}$

3.  $C = \{x : x \text{ is a consonant in English which proceeds R}\}$

~~$C = \{l, m, n, p, t, s, r, v, w, x, f, g, h, j, z\}$~~

$C = \{b, c, d, f, g, h, j, z\}$

(3)

Q let A be the set of Natural nos. and  $x, y$  be any two elements of A.

Then:-

- a)  $x-y \in A$  X
  - b)  $x+y \in A$  ✓
  - c)  $xy \in A$  ✓
  - d)  $\frac{x}{y} \in A$  X
- 

Cardinality :- ① Number of distinct elements of a set finite set.

② It is denoted by  $n(A)$  or  $|A|$  for set A

e.g.  $A = \{a, b, c, d\} - 4$

$B = \{1, 2, 3, 4, 4, 5\} - 5$

$C = \{\} - 0$

$|A| = 4$

4

5

$|B| = 5$

$|C| = 0$

Q Find Cardinality

1.  $P = \{a, d, e, l, f\} |5|$

2.  $Q = \{a, a, b, d, d\} |4|$

3.  $C = \{a, \{c, d\}, k\} |3|$

4.  $D = \{a, \{3\}, k\} |3|$

5.  $E = \{n, \{q, \{r, s\}\}, t\} |3| \quad \text{and } n(E) = 3$

①

## 11 Practice Set

Q1. Set or not?

1. collection of natural nos. between 2 & 20. Yes ✓
2. collection of ~~not~~ nos. which satisfy  $x^2 - 5x + 6 = 0$ . Yes ✓
3.  $f'$  prime nos. between 2 & 100 Yes ✓
4. " all intelligent women in Udaipur. No ✓

Q2. Write in tabular form:

1.  $A = \{x : x \text{ is a prime} < 10\}$

Q 2.  $A = \{2, 3, 5, 7\}$  ✓

2.  $B = \{x : x = 3\lambda, x \in I, 1 \leq \lambda \leq 3\}$

$B = \{3, 6, 9\}$  ✓

Q3. Write in set builder form:

Q1. set of all rational nos.

$A = \{x : x \text{ is a rational number}\}$  ✓

2.  $\{2, 5, 10, 17, 26, 37, \dots\}$

$B = \{x : x = 2+y, y \in N, 3 \leq y, y \text{ is an odd number}\}$

$B = \{x : x = y^2+1, y \in N\}$  ✓

$x \in R \cap Q$   
Rational

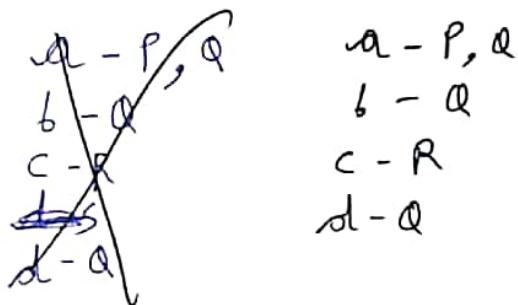
(5)

# Types of Sets

Set name	Definition	Example
1 Null/Void/Empty set	<p>a) set having no elements.  <del>is card</del></p> <p>b) <math>A = \{\}</math> or <math>A = \phi</math> (b)</p> <p>c) cardinality = 0</p> <p>d) <math>\emptyset \neq \{\emptyset\}</math></p>	$A = \{x : x \in N, x < 0\}$ $A = \{x : x \in W, x < 0\}$ $B = \{x : x \neq x\}$ $C = \text{set of all months having 31 days.}$
2 Singleton Set	<p>a) set having only one element.</p> <p>b) cardinality = 1</p> <p>c)</p> <p>d).</p>	$A = \{\text{set of all months having only 28 days.}\}$ $B = \{1, 1, 1, 1\}$ $C = \{x : x \in W, x < 1\}$ $D = \{\emptyset\}$
3. Finite Set	<p>a) set having a <del>fixed</del><sup>finite</sup> no. of elements</p> <p>b) All Null &amp; Singleton sets lie in finite set</p>	$A = \text{set of days of a week.}$ $B = \{5, 7, 9\}$ $C = \text{set of all lakes in India.}$
4. Infinite Set	<p>a) set having infinite number of elements.</p>	$A = \{1, 2, 3, \dots\}$ $B = \{x : x \text{ is prime no.}\}$

Q Match the following:-

- a) A =  $\{x : x^2 = 4, x \text{ is odd}\}$       P) Null set  $\emptyset$
- b) B =  $\{\text{Men living presently in Udaipur}\}$       Q) Finite set  $\{ \}$
- c) C = Set of all points on a line      R) Infinite set  $\mathbb{Z}$
- d) D. Set of solutions of  $x^2 - 16 = 0$       S) Singleton



Q. In rule method, the null set is represented by

- a)  $\{ \}$   
b)  $\emptyset$   
c)  $\{x : x = x\}$   
d)  $\{x : x \neq x\}$

Q A set having at least 1 element is called non-empty or non-vaid set.

(7)

## Equivalent Sets

- Only Cardinal numbers are same.  
(Some cardinality)
- Elements can be same or different.
- denoted by ' $\equiv$ ' or ' $\sim$ '
- e.g.  $A = \{1, 2, 3\}$   
 $B = \{a, b, c\}$   
 $C = \{m, n, o, p\}$   
 $D = \{3, 8, 9, f\}$   
 $A \equiv B \equiv C \equiv D$

## Equal Sets/Identical sets

- Cardinal number and the elements in sets are same.
- Order of elements does not matter
- denoted by ' $=$ '
- $Q = \{A, L, L, O, Y\}$   
 $P = \{L, O, Y, A, L\}$   
 $R = \{O, L, A, Y\}$   
 $P = Q = R$

~~B~~ All equal sets are always equivalent. ~~sets~~  
But equivalent sets may need not be equal sets.

H.W.

- D.Y.S - 1  
 $\{4, 5, 13, 3\}$
- O-1  
 $\{1, 2, 13, 14, 15, 16, 18\}$
- O-2  
 $\{10, 11\}$
- O-3  
 $\{10\}$

(8)

DYS - 1

- A-P, B-R, C-Q, D-S
- D-R, C-Q, B-S, A-P
- 3.D) 13. all x
- O-1  
1. B) 2. A) 13. ~~B~~ C)  
14 C) 15 D) 16. D)
18. A)
- O-2  
10. B, C, D 11. A, C, D
- O-3  
10. A-S, B-P, C-Q, D-R

## Subset

1. If every element of set - A is an element of set - B then, set - A is called subset of set - B and set - B is called superset of set - A.

Eg.  $J = \{R, L, C, GP\}$

$$A = \{G, P\}$$

$$B = \{R, G\}$$

$$C = \{R, L, G\}$$

$$D = \{R, L, G, C\}$$

sets  $A, B, C, D$  are subsets of  $J$  &  $J$  is a superset of  $A, B, C, D$ .

Eg 2. subset is a subset of Power set

→ Subsets are denoted by ' $\subseteq$ '

Eg.  $A \subseteq B$

A is subset of B

B is superset of A

Eg.  $A = \{a\}$

$$\text{Subsets} = \{\}, \{a\} \\ = 2^1 = 2$$

$$\left| \begin{array}{l} B = \{a, b\} \\ B \text{ subsets} = \{\}, \{a\}, \{b\}, \{a, b\} \\ = 2^2 = 4 \end{array} \right.$$

$$C = \{a, b, c\}$$

$$C = \{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \\ = 2^3 = 8$$

$$\text{Formula} = 2^{\text{cardinality}}$$

(9)

\* Equal sets are always subsets of each other

$$L = \{a, b, b, c, c, c, a\}$$

$$M = \{a, b, c\}$$

$$M \subseteq L \quad L \subseteq M$$

### # Proper Subset

- If set A is a subset of set B and  $A \neq B$  then, A is a proper subset of B.
- It is denoted by ' $\subset$ '

Eg.  $B = \{a, c\}$

$$\underbrace{\{\}, \{a\}, \{c\}, \{a, c\}}_{\text{Proper subsets of } B}$$

Subsets of B

Eg. 2.  $T = \{Munni, Chandi, Sheela\}$

Proper Subsets =  $\{\}, \{Munni\}, \{Chandi\}, \{Sheela\}, \{Munni, Chandi\}, \{Munni, Sheela\}, \{Chandi, Sheela\}$

Subsets = P. Subsets <sup>and</sup>  $\{Munni, Chandi, Sheela\}$

\* Every set is a subset of itself

\* Empty set is a subset of every set

\* If  $n(A) = m$   
no. of elements  
in A

Then Total no. of subsets of A =  $2^m$

(10) no. of proper subsets of A =  $2^m - 1$

Q1. Find total no. of subsets & Proper subsets

①  $A = \{l, m\}$

$$|A| = 2$$

$$\text{no. of subsets} = 2^{|A|}$$
$$= 2^2$$
$$\boxed{= 4}$$

$$\text{no. of sub/proper subsets} = 2^{|A|} - 1$$
$$= 4 - 1$$
$$\boxed{= 3}$$

②  $B = \{P, Q, R, S, T\}$

$$|B| = 5$$

$$\text{no. of subsets} = 2^{|B|}$$
$$= 2^5$$
$$\boxed{= 32}$$

$$\text{no. of Proper subsets} = 32 - 1$$
$$\boxed{= 31}$$

Q2. Two finite sets have  $m$  &  $n$  elements. The total no. of subsets of the first set is 48 more than the total no. of subsets of the second set. The value of  $m$  &  $n$  are?

a) 7, 6

b) 6, 3    ✓ c) 6, 4    d) 7, 4

$$2^m + 48 = 2^n$$

$$2^m - 2^n = -48$$

$$48 = 2^n - 2^m$$

$$2^n(2^{m-n} - 1) = 2^4 \times 3$$

$\cancel{2^{m-n}}$

$$2^n = 2^4$$

$$n = 4$$

$$2^{m-n} - 1 = 3$$

$$2^{m-4} = 4$$

$$2^{m-4} = 2^2$$

$$m - 4 = 2$$
$$\boxed{m = 6}$$

(11)

$$2^n = 3$$

$$2^{m-n} - 1 = 2^4$$

$$2^{m-n} = 2^4 + 1$$

$$2^n = 3$$

$$2^{m-n} = 17$$

(Affect)

## # Power Set

→ It is a set containing all the subsets of set A

→ It is denoted by  $P(A)$

e.g.  $A = \{a, b\}$

Subsets =  $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$   
 $P(A) = \{\{\emptyset\}, \{a\}, \{b\}, \{a, b\}\}$

e.g.  $X = \{3, 7, 11\}$

$$P(X) = \{\{\emptyset\}, \{3\}, \{7\}, \{11\}, \{3, 7\}, \{3, 11\}, \{7, 11\}, \{3, 7, 11\}\}$$

①  $3 \in X \checkmark$

②  $\{3\} \in P(X) \checkmark$

③  $\{3\} \subset X \checkmark$

④  $7 \in X \checkmark$

⑤  $\{7\} \in P(X) \checkmark$

⑥  $\{7\} \subset X \checkmark$

⑦  $\{3, 7\} \subset X \checkmark$

⑧  $\{3, 7\} \in P(X) \checkmark$

(12)

$$Q \quad W = \{1, 2\}$$

$$P(W) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

- A)  $\{1\} \subset W \checkmark$
- B)  $\{1\} \in P(W) \times$
- C)  $\{1\} \in P(W) \checkmark$
- D)  $\{1, 2\} \in P(W) \checkmark$
- E)  $\{1, 2\} \subset P(W) \times$
- F)  $\{1, 2\} \subseteq W \times$

$$Q \quad Z = \{\{1\}, 3, \{1, 5\}\}$$

$$P(Z) = \{\emptyset, \{\{1\}\}, \{3\}, \{\{1, 5\}\}, \{\{1\}, 3\}, \{\{1\}, \{1, 5\}\}, \{3, \{1, 5\}\}, \{\{1\}, 3, \{1, 5\}\}\}$$

- A)  $\{\{1\}\} \in Z \times$
- B)  $\{\{1\}\} \subset Z \checkmark$
- C)  $3 \in Z \checkmark$
- D)  $\{1\} \in Z \checkmark$
- E)  $\{3\} \in Z \times$
- F)  $\{3\} \subset Z \checkmark$

(B)

M.W. ~~04-04-~~  
09-04-2024

DYS-1 (Q6, Q7, B, Q9, Q11, Q12)

DYS-2 (Q3)

DYS-3 (Q8, Q10, Q11; Q12)

O-1 (~~Q1, 2~~, (Q17, Q19)

O-2 (Q12)

Answers

DYS-1

Q6 - 8

Q7 - 2

Q8 - 7

Q9 - A iii I  
B ii II  
C ii III  
D iv IV

~~Q10~~ Q11 - 3

Q12 - 2

DYS-3

Q8 - D

Q10 - D

Q11 - B

Q12 - B

O-1

Q17 - A

Q19 - A

O-2

Q12 - A, B, C, D

(14)

\* If set A is an empty set, then

$P(A)$  has 1 element

e.g.  $A = \{ \}$  or  $\emptyset$

$$P(A) = \{\{\}\} \text{ or } \{\emptyset\}$$

1. Power set is always non-empty and its minimum value is 1.

Q1 Determine the power set of the following :-

①  $A = \{a\}$

$$P(A) = \{\emptyset, \{a\}\}$$

②  $B = \{\emptyset, \{\emptyset\}\}$

$$P(A) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

Q2. Which of the following cannot be the number of elements in the power set of any finite set.

- a) 26      b) 32      c) 8      d) 16

Q3. write power set of  $\{0\}$ .

$$P(\{0\}) = \{\emptyset, \{0\}\}$$

# Universal Set  $\rightarrow$  A set contains all the elements occurs in the discussion.

\* It is denoted by ~~U~~ 'U'

e.g.  $A = \{1, 2, 3\}$

$$B = \{1, 3, 5, 7\}$$

$$C = \{2, 4, 6\}$$

$$U = \{1, 2, 3, 4, 5, 6, 7\}$$

Set A, B, C are subsets of universal set.

(15)

$$Q = \{1, 2, 3, 7, 6, 9, 10, 11\}$$

$$A = \{1, 2, 3, 7\}$$

elements cannot repeat.

$$B = \{6, 7, 10, 11\}$$

Q find  $\theta$   ~~$P_{\text{eff}}(P)$~~   $P(P(\phi)) \neq P(r(P(\theta\phi)))$

if  $P(\emptyset) = \{\emptyset\}$

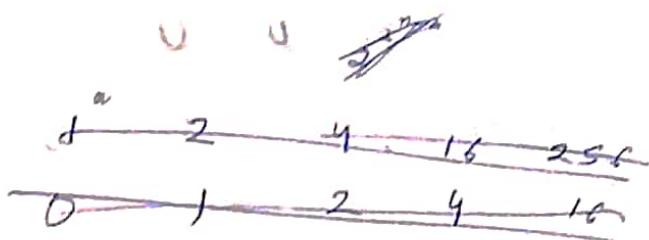
$$\phi = f(\phi)$$

$$\mathbb{P} \ P(\phi) = \{\phi\}$$

$$P(\{\phi\}) = \{\phi, \{\phi\}\}$$

$$P(P(\{ \phi \})) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\{\phi, \{\phi\}\}\}$$

\* In any particular discussion, no element can exist out of universal set. It should be noted that universal set is not unique, it may differ in problems.



## # Operation on sets

1. Venn Diagram :- It is a visual representation of different relations between sets.

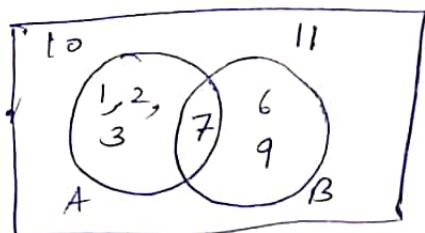
 → shows universal set

 → ~~united set~~ set

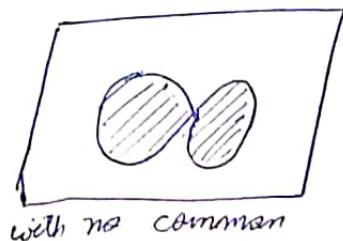
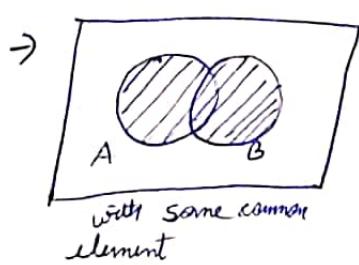
e.g.  $U = \{1, 2, 3, 7, 6, 5, 10, 11\}$

$A = \{1, 2, 3, 7\}$

$B = \{6, 7, 9\}$



2. Union ( $\cup$ ) - It contains all the elements which are either in set A or set B or in both.  
- It is denoted by  $A \cup B$

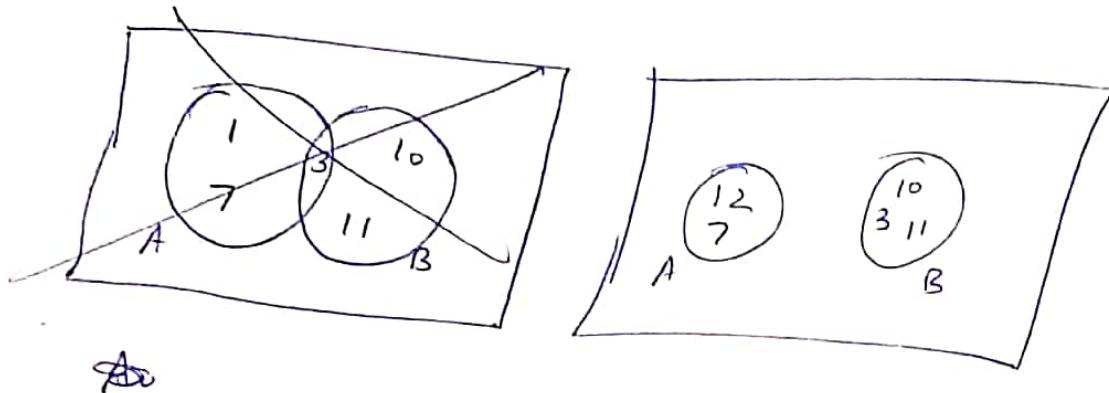


shaded area shows  $A \cup B$

$$Q \ A = \{1, 2, 7\}$$

$$B = \{3, 10, 11\}$$

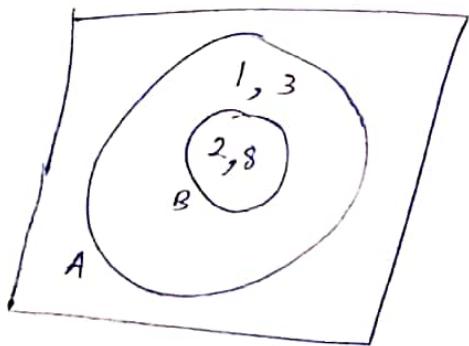
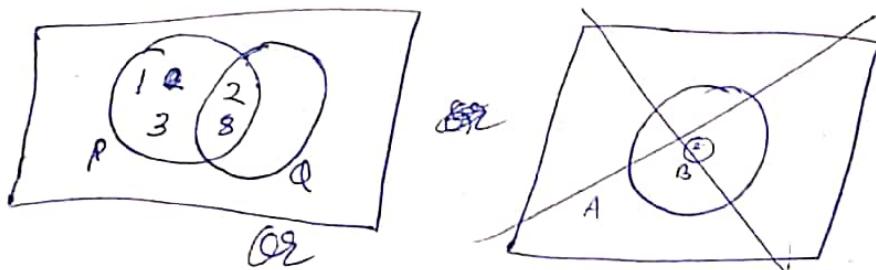
$$A \cup B = \{1, 2, 3, 7, 10, 11\}$$



$$Q2. \ P = \{1, 2, 3, 8\}$$

$$Q = \{2, 8\}$$

$$P \cup Q = \{1, 2, 3, 8\}$$



(18)

H.W. 12-04-23

~~DYS~~

~~DYS~~-1 (Q2,)

~~DYS~~-2 (

O-2 (9, 13)

JA (Q2, 4)

DYS -1

(Q2. c)

O-2

(Q9) ABD

(Q10) BCD

J-A

(Q2 3)

(Q4 2).

Q  $A = \{\{a, b\}, c\}$

elements in  $c$

$2^2=4$  in  $P(c)$

$$P(c) = \{\{\}, \{c\}, \{a, b\}, \{a, b, c\}\}$$

Q  $C = \{1, \{2\}\}$

$$P(C) = \{\{\}, \{1\}, \{1, \{2\}\}, \{\{2\}, 1\}\}$$

$$\{\{2\}\} \in P(C) \checkmark$$

$$\{\{2\}\} \subset P(C) \times$$

$$\{\{2\}\} \in C \times$$

Q If  $\phi$  is null set, then -

(a)  $\phi \in \{\{\phi\}, \{\phi, \{\phi\}\}\} \times$

(b)  $\{\phi\} \subseteq \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\} \checkmark$

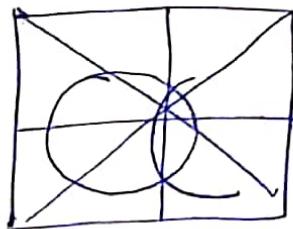
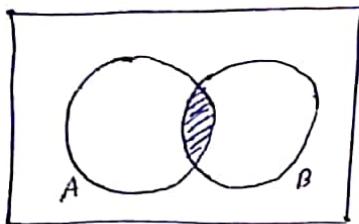
(c)  $\{\phi, \{\phi\}\} \subseteq \{\{\phi, \{\phi, \{\phi\}\}\}\} \times$

d) none  $\times$

(20)

3. Intersection - It contains all elements which are present in set-A and set-B both.

→ It is denoted by  $A \cap B$



e.g.  $A = \{1, 2, 7, 11\}$   
 $B = \{1, 2, 3, 5, 11, 55\}$

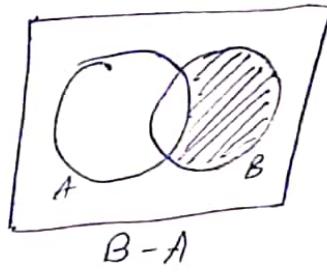
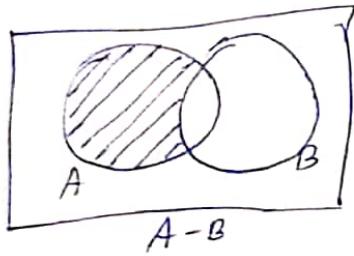
$$A \cap B = \{1, 2, 11\}$$

Property	union	intersection
① Commutative	$A \cup B = B \cup A$ $A \cup B \cup C = B \cup C \cup A$	$A \cap B = B \cap A$
② Associative	$A \cup (B \cup C) = (A \cup B) \cup C$	$A \cap (B \cap C) = (A \cap B) \cap C$
③ Identity	$A \cup A = A$	$A \cap A = A$
④ Law of identity - compare with $\emptyset$ and $U$	$A \cup U = U$ $A \cup \emptyset = A$	$A \cap U = A$ $A \cap \emptyset = \emptyset$

Note - For two sets  $A \& B \rightarrow A \cup B \rightarrow A \cap B$   
 3 sets  $A, B \& C \rightarrow A \cup B \cup C \rightarrow A \cap B \cap C$   
~~n sets  $A_1, A_2, A_3, \dots, A_n \rightarrow \bigcup_{i=1}^n A_i$~~

$$A \rightarrow \bigcap_{i=1}^n A_i$$

4. Difference of sets - It contains all the elements which are in set - A and not present in set - B.  
 $\rightarrow$  It is denoted by  $A - B$



e.g.  $A - B = A - (A \cap B)$

$A - B \neq B - A$  (in some cases  ~~$A - B = B - A$~~ )

Q. find difference  $A - B \& B - A$

$$A = \{1, 2, 7, 9\}$$

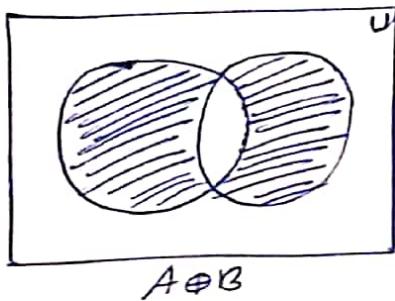
$$B = \{-1, 2, 3, 4, 11\}$$

$$A - B = \{7, 9\}$$

$$B - A = \{-1, 3, 4, 11\}$$

## 5. Symmetric Difference of 2 sets:-

- It is denoted by  $A \Delta B$  or  $A \oplus B$
- For two sets  $A$  and  $B$  it is the part of the sets  $A \Delta B$  except  $A \cap B$ .



$$A \Delta B = B \Delta A$$

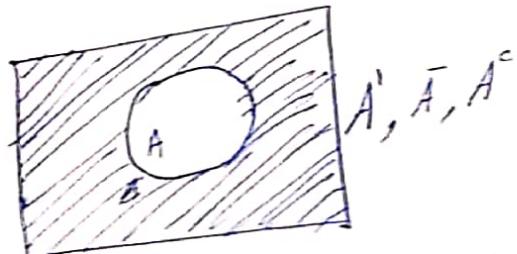
$$A \Delta A = \emptyset$$

$$\text{Q } A = \{1, 2, 3\}$$

$$B = \{-1, 1, 3, 5\}$$

$$A \Delta B = B \Delta A = \{-1, 5, 2\}$$

6. Complimentary of set - A set containing all the elements of Universal set which are not present in set-A.



→ It is denoted by  $\bar{A}, A^c$

$$\rightarrow \bar{A} = U - A$$

$$\text{Q } U = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A = \{1, 2, 3\}$$

$$\bar{A} = \{4, 5, 6, 7\}$$

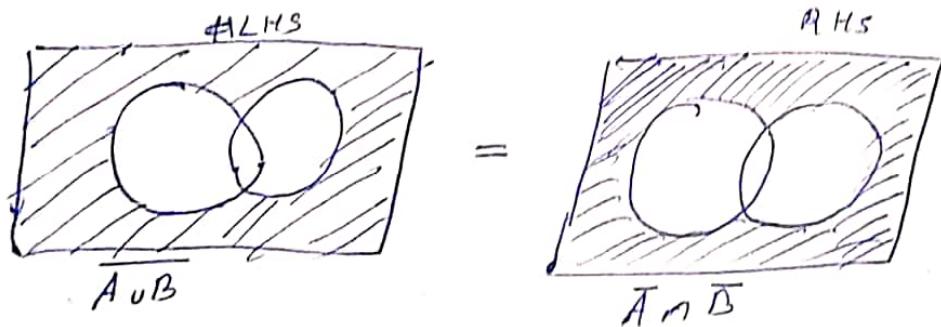
Note:-

- ①  $\phi' = V$
- ②  $V' = \phi$
- ③  $A \cup A' = V$
- ④  $A \cap A' = \phi$
- ⑤  $\bar{\bar{A}} = A$  (Even times complementry = 1)
- ⑥  $\bar{\bar{\bar{A}}} = \bar{A}$  (odd times = N)

7. DeMorgan's principle  $\rightarrow$  For 2 sets  $A$  &  $B$  (valid for more sets)

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$



$$\overline{\bigcup_{i=1}^n A_i} = \bigcap_{i=1}^n A_i'$$

7. Disjoint Sets - For any two sets  $A$  and  $B$  if  $A \cap B = \phi$

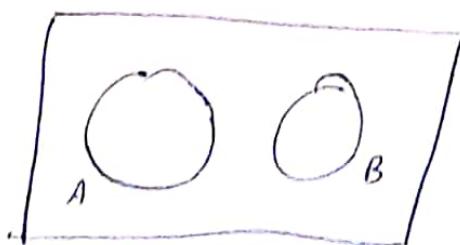
then set  $A$  and set  $B$  are called disjoint sets.

$$\text{eg. } A = \{1, 2, 3, 4\}$$

$$B = \{P, \emptyset, R\}$$

$$A \cap B = \phi$$

$A$  &  $B$  are disjoint sets.



(24)

DYS-1 (Q1, 10, all except Q13)

~~DYS-2 (Q1, 2, 3, 4, 5, 6, 7)~~

DYS-2 (Q1(1-5), 2, 3, 6, 7)

DYS-3 (Q2, 3, 4, 5, 6, 7)

O-1 (Q3, 4, 5, 6)

O-3 (Q8, 9)

DYS-1

Q1. All are right

(Q10. C)

O-1

(Q3. B)

(Q4. A)

(Q5. A)

(Q6. A)

DYS-2

Q1. All except ~~viii~~ viii)

Q2. A-R, B-P, C-Q, D-S

Q3. 2

Q6. 2

Q7. 4

O-3

Q8. A-P, B-Q, C-R, D-S

Q9. A-Q, B-P, C-R, D-S

DYS-3

Q2. C)

Q3. B)

Q4. B)

Q5. B)

Q6. C)

Q7. C)

$$Q2 \cup = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 5\}$$

$$B = \{2, 4, 6, 7\}$$

$$C = \{2, 3, 4, 8\}$$

$$\textcircled{1} \quad \bar{A} = \{4, 6, 7, 8, 9, 10\}$$

$$\textcircled{2} \quad \overline{(A-B)} = \{2, 4, 6, 7\}$$

$$\textcircled{2} \quad \overline{(A-B)} = \{4, 2, 6, 7, 8, 9, 10\}$$

$$\textcircled{3} \quad \bar{A} = \{1, 2, 3, 5\}$$

$$\textcircled{4} \quad \overline{B \cup C} = \overline{B} \cap \overline{C} = \{1, 9, 10\}$$

$$\textcircled{5} \quad \overline{C \cap A} = \{1, 4, 5, 6, 7, 8, 9, 10\}$$

$$Q2. \quad X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 3, 5, 7, 9\}$$

$$C = \{2, 4, 8, 10\}$$

$$\textcircled{1} \quad A \cap (B \cup C) = A \cap \{1, 2, 3, 4, 5, 7, 8, 9, 10\} = \{1, 2, 3, 4, 5\}$$

$$\textcircled{2} \quad (A \cap B) \cup (A \cap C) = \{1, 3, 5\} \cup \{2, 4\} = \{1, 2, 3, 4, 5\}$$

$$\textcircled{3} \quad (A \cup B \cup C)^c = \{6\}$$

(26)

Q3. If  $\mathbb{N} = \{x : x \in \mathbb{N}\}$ , then set  $3\mathbb{N} \cap 7\mathbb{N}$ .

21, 42  $\boxed{\cancel{21N}}$

$$3\mathbb{N} = \{3x : x \in \mathbb{N}\}$$

$$3\mathbb{N} = \{3, 6, 9, 12, \dots\}$$

$$7\mathbb{N} = \{7x : x \in \mathbb{N}\}$$

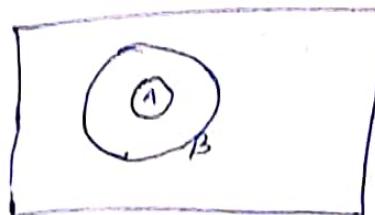
$$7\mathbb{N} = \{7, 14, 21, 28, 35, 42, \dots\}$$

$$3\mathbb{N} \cap 7\mathbb{N} = \{21, 42, 63, \dots\}$$

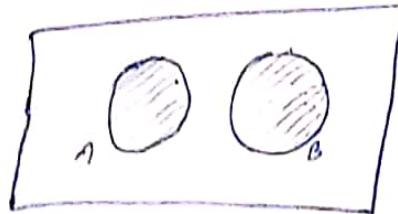
$$\text{LCM}(3\mathbb{N}, 7\mathbb{N}) \quad \boxed{= 21\mathbb{N}} \quad \text{LCM}(3, 7)\mathbb{N}$$

Q4. draw venn diagram

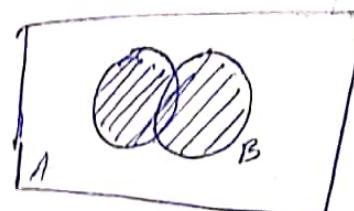
(1) A is a subset of B



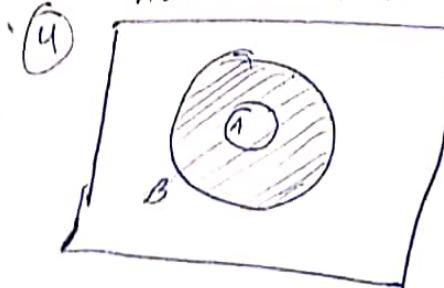
(2)  $A \cup B$ , when  $A \cap B = \emptyset$



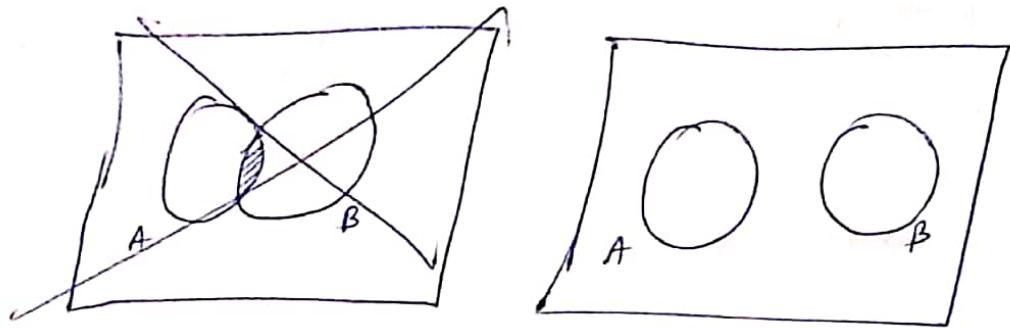
(3)  $A \cup B$  when  $A \cap B \neq \emptyset$



$A \cup B$  when  $A \subseteq B$

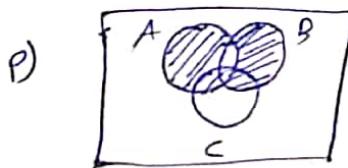


5)  $A \cap B$  when  $A \cap B \neq \emptyset = \phi$

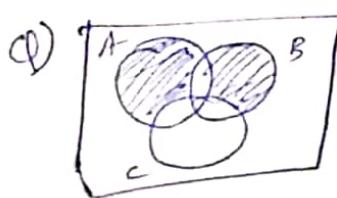


Q Match the column

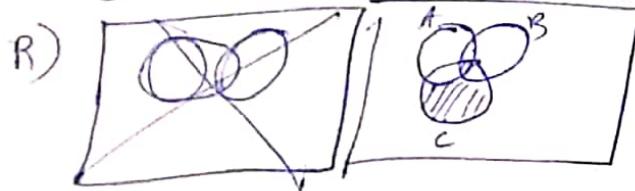
A)  $C - (A \Delta B)$



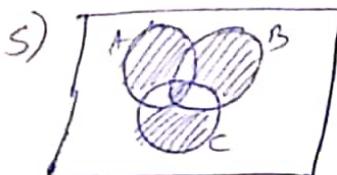
B)  $(A \Delta B) - C$



C)  $A \Delta B$

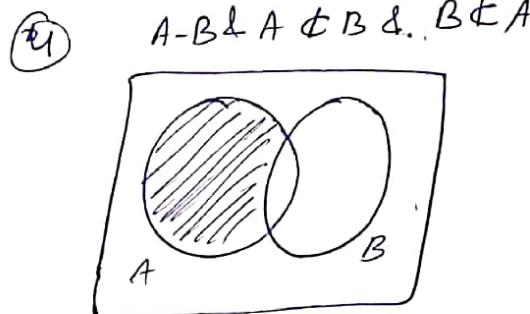
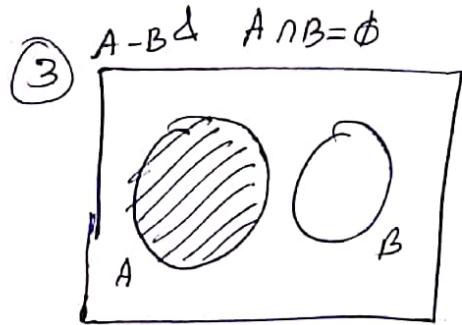
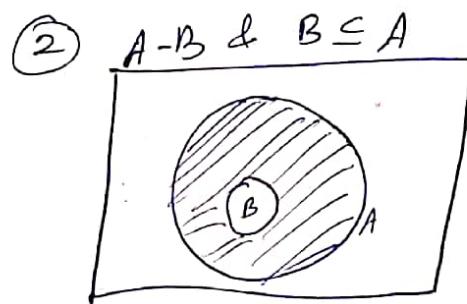
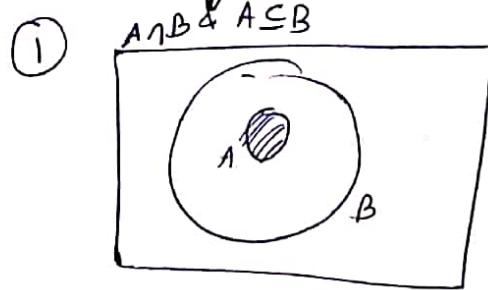


D)  $(A \Delta B) \Delta C$



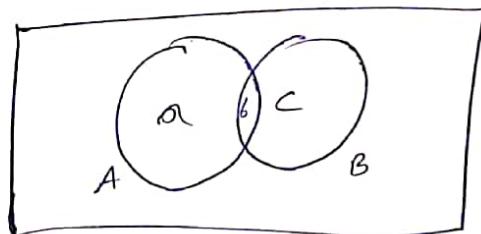
- A - R)
- B - Q)
- C - P)
- D - S)

Q Venn diagram



Q Some sum results for problem solving

1. For any two finite sets  $A$  &  $B$



$$n(A) = a + b$$

$$n(B) = b + c$$

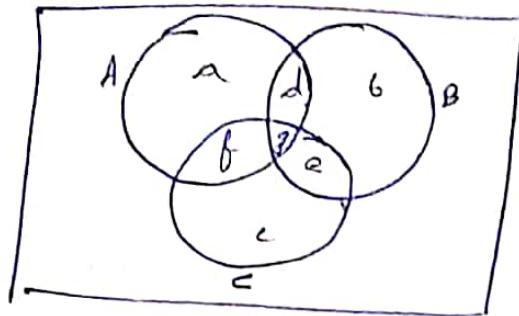
$$n(A \cup B) = a + b + c$$

$$n(A \cap B) = b$$

$$n(A \oplus A \cup B) = n(A) + n(B) - n(A \cap B)$$

2. elements only in set  $A = a$   
" " " set  $B = c$

3. for any 3<sup>finite</sup> sets,



$$\begin{aligned} n(A) &= a + d + f + g \\ n(B) &= b + d + e + g \\ n(C) &= c + e + f + g \end{aligned} \quad ] - n(A) + n(B) + n(C) = a + b + c + 2d + 2e + 2f + 3g$$

$$\begin{aligned} n(A \cap B) &= d + g \\ n(B \cap C) &= e + g \\ n(A \cap C) &= f + g \end{aligned} \quad ] n(A \cap B) + n(A \cap C) + n(B \cap C) = d + e + f + 3g$$

$$n(A \cap B \cap C) = g$$

$$n(A \cup B \cup C) = a + b + c + d + e + f + g$$

$$n(A \cup B \cup C) = n(A) + n(C) + n(B) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Only in set A = a  
 set B = b  
 set C = c

Only in set A & B = d

$$\begin{aligned} A \cap B \cap C &= f \\ B \cap C &= e \end{aligned}$$

M.W.

15-04-2024

O-1 (Q7)

Q-2 (Q1, 2, 4, 7, 8, 14, 15, 16)

O-3 (Q1, 2, 3)

JM

(Q1)

JA

(Q1, 3)

O-1

Q7 B)

O-2

Q1 - ACD

Q2 - ABCD

Q3 - BC

Q7 - ABC

Q8 - ABCD

Q14 - AB

Q15 - AD

Q16 - ABCD

O-3

Q1 - ABD

Q2 - ABCD

Q3 - ABCD

JM

Q1 - I

QJA

Q1 C)

Q3 2)

31

4. Number of elements belonging to exactly two of the sets. ~~A ∩ B~~

$$n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C) = \text{no. of elements in exactly two sets.}$$

5. Number of elements belonging to exactly one of the sets A, B and C is

$$n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(A \cap C) - 2n(B \cap C) + 3n(A \cap B \cap C)$$

$$\sum n(A) - 2 \sum n(A \cap B) + 3n(A \cap B \cap C)$$

6. no. of elements in

i. ~~n~~

$$n(\overline{A \cup B}) = n(\overline{A \cap B}) = n(v) - n(A \cap B)$$

$$n(\overline{A \cap B}) = n(\overline{A \cup B}) = n(v) - n(A \cup B)$$

$$\cancel{n}(\overline{A \cap B}) = n(\overline{A \cup B}) = \cancel{A \cap B} n(A - B) = n(A) - n(A \cap B)$$

$$\cancel{n}(\overline{A \cap B}) = n(\overline{A \cup B}) = n(B - A) = n(B) - n(A \cap B)$$

7. For two sets A & B

$A \cap B$  will be maximum when :-  $A = B$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- Q If  $n(v) = 700$ ,  
 $n(A) = 200$ ,  
 $n(B) = 240$   
 $n(A \cap B) = 100$   
 $n(F \cap B) = ?$

$$\begin{aligned} n(v) - n(\bar{A} \cup \bar{B}) &= 700 - n(A) - n(B) + n(A \cap B) \\ &= 700 - 200 - 240 + 100 \\ &= 800 - 340 \\ &= 360 \end{aligned}$$

$$\begin{aligned} n(v) - n(\bar{A} \cap \bar{B}) &= 700 - n(A \cap B) \\ &= 700 - 100 \\ &\boxed{= 600} \end{aligned}$$

Q2. An investigator interviewed 100 students for the preferences of 3 drinks - milk (M), coffee (C) & Tea (T)

$$\begin{array}{ll} n(M \cap C \cap T) = 10 & n(\text{only } M) = 12 \\ n(M \cap C) = 20 & \cancel{n(C)} \\ n(C \cap T) = 30 & n(\text{only } C) = 5 \\ n(M \cap T) = 25 & n(\text{only } T) = 8 \end{array}$$

$$\begin{aligned} n(M) &= 12 + 25 + 20 - 10 - 10 \\ &= 57 - 20 \\ &\boxed{= 37} + 10 = 47 \end{aligned} \quad \begin{aligned} n(T) &= 5 + 20 + 30 - 20 \\ &\boxed{= 35} + 10 = 45 \end{aligned}$$

$$\begin{aligned} n(C) &= 8 + 30 + 25 - 20 \\ &\boxed{= 43} + 10 = 53 \end{aligned}$$

$$\begin{aligned} n(M \cup C \cup T) &= \\ 25 - (35 + 37 + 43 - 20 - 30 - 25 + 10) &= \\ 25 - (115 - 75 + 10) &= \\ \cancel{25} 25 - 115 + 75 - 10 &= \\ 100 - 125 & \end{aligned}$$

~~25~~

~~M A C A T~~

$$115 - 40 - 60 - 25 + 30$$

$$115 - 125 + 30$$

$$145 - 125 = 20$$

$$\cancel{20} + \cancel{55} = -80$$

$25 +$

$$115 - 120 - 30 - 25 + 10$$

$$115 - 75 + 10$$

$$50 + 30 = 80$$

$$100 - 80 = \boxed{20}$$

$$80 - 80 = \boxed{30}$$

$$\boxed{130}$$

(34)

Q In a class of 55 students, the no. of students studying diff subjects are  
 $n(M) = 23$ ,  $n(P) = 24$ ,  $n(C) = 19$ ,  ~~$n(M \cap P \cap C)$~~ .

$$n(M \cap P) = 12$$

$$n(M \cap C) = 9$$

$$n(P \cap C) = 7$$

$$n(M \cap C \cap P) = 4$$

no. of exact one sub

$$23 + 24 + 19 - 24 - 18 - 14 + 12$$

$$+ 1 + 35 - 14$$

$$\boxed{22}$$

$$\leq n(A) - 2 \leq n(A \cap B) + 3 \leq n(A \cap B \cap C)$$

Q In udaipur city 25% women women own a cell phone, 15% women have a scooter & 65% have neither a phone nor a scooter. if 1500 women have both then find total no. of women in udaipur.



$$n(AP) = 25$$

$$n(S) = 15$$

$$n(P \cup S) = 65$$

$$P \cup S = 35$$

$$n(P \cap S) = 65$$

$$70\%$$

$$\text{Total} = x$$

$$m(c) = \frac{25x}{100}$$

$$m(s) = \frac{15x}{100}$$

$$m(\text{nothing}) = \frac{65x}{100}$$

$$x = \frac{25x}{100} + \frac{15}{100} - \frac{1500}{100} + \frac{65x}{100}$$

$$1500 = \frac{105x - 100x}{100}$$

$$x = 30000$$

### Cartesian Product of two sets

→ It is denoted by ~~set~~  $A \times B$  for sets A & B

→ It is a set of ordered pairs  $(a, b)$  where  $a$  belongs to A &  $b$  belongs to B.

$$\rightarrow A \times B = \{(a, b) : a \in A, b \in B\}$$

e.g.  $A = \{1, 2\}$   $B = \{p, q, r\}$

①  $A \times B = \{(1, p), (1, q), (1, r), (2, p), (2, q), (2, r)\}$

②  $B \times A = \{(p, 1), (p, 2), (q, 1), (q, 2), (r, 1), (r, 2)\}$

③  $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

④  $B \times B = \{(p, p), (p, q), (p, r), (q, p), (q, q), (q, r), (r, p), (r, q), (r, r)\}$

$$n(A) \times n(B) = n(A \times B)$$

Q)  $A = \{l, m\}$   
 $B = \{g, f\}$

$$A \times A \times A = \{(l, l), (l, m), (m, l), (m, m)\} \times \{l, m\}$$

$$= \cancel{\{(l, l, l), (l, l, m), (l, m, l), (l, m, m)\}}, \\ \cancel{\{(m, l, l), (m, l, m), (m, m, l), (m, m, m)\}}$$

$$(A \times B) \times A = \{(l, g), (l, f), (m, g), (m, f)\} \times A$$

$$= \{(l, l, l), (l, l, m), (l, m, l), (l, m, m), \\ (m, l, l), (m, l, m), (m, m, l), (m, m, m)\}$$

$$(A \times B) \times A = \{(l, g, l), (l, g, m), (l, f, l), (l, f, m), \\ (m, g, l), (m, g, m), (m, f, l), (m, f, m)\}$$

H.W.

14-04-2024

DYS-Q3(1, 9, 13, 14, 15, 16, 17, 18, 19, 20)

Q1 -

Q 9 - B

Q 13 - P - 30  
Q - 19  
R - 60

S - No

Q 14 4.

Q 15. 2

Q 16. 2

Q 17. 8

Q 18. B)

Q 19. A) B) D)

Q 20. A)

Note: ① if  $(a, b) = (x, y)$

$$\begin{aligned} a &= b \\ x &= y \end{aligned}$$

② Generally  $A \times B \neq B \times A$

③  $A \times B = \emptyset$ , Then either  $A = \{\}$  or  $B = \{\}$  or both are empty

④  $(A, B) \neq (B, A)$

⑤ If set A and set B have m elements common  
then number of elements common in  $A \times B$  is  $m^2$ .

$$n(A \cap B) = m$$

$$n(A \times B \cap B \times A) = m^2$$

e.g.  $A = \{1, 2\}$

$$B = \{1, 2, 3\}$$

$$A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$$

$$B \times A = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 2)\}$$

Q 1.  $A = \{1, 2\}$

$$B = \{1, 2, 3\}$$

find  $A \times B$

$$A \times B = \emptyset \text{ or } \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

Q 2.  $(2x-y, 2s) = (15, 2x+y)$

$$2x - y = 15$$

$$y = 2x - 15$$

$$\begin{aligned} 2x + y &= 2s \\ 2x + 2x - 15 &= 2s \end{aligned}$$

$$4x = 40$$

$$\boxed{4x = 10}$$

$$\begin{aligned} y &= 2x - 15 \\ \boxed{y = 5} \end{aligned}$$

Q3.  $a \in A$  &  $b \in B$   
find  $(a, b)$  such that  $a^2 < b$ .

$$A = \{2, 3\}$$

$$B = \{4, 16, 23\}$$

$$A \times B = \{(2, 4), (2, \checkmark 16), (2, \checkmark 23), (3, 4), (3, \checkmark 16), (3, \checkmark 23)\}$$

$$(a, b) = \boxed{\begin{array}{l} (2, 16) \\ (2, 23) \\ (3, 4) \\ (3, 16) \\ (3, 23) \end{array}}$$

Q4.  $A \times B = \{(2, 3), (2, 4), (5, 3), (5, 4)\}$   
find set  $A$  &  $B$

$$A = \{2, 5\}$$

$$B = \{3, 4\}$$

Given  $A \times B$  has 15 ordered pairs & set  $A$  has 5 elements  
find no. of elements in set  $B$

$$n(A) \times n(B) = 15$$

$$n(A) = 5$$

$$\boxed{n(B) = 3}$$

(4D)

Q6. If 2 sets  $A$  &  $B$  have 44 elements in common, then no. of elements common in  $A \times B$  &  $B \times A$  are

$$|A \cap B| = 44$$

$$\begin{aligned} |A \times B \cup B \times A| &= 44^2 \\ &= 44 \times 44 \\ &\quad \begin{array}{r} 1 \\ - 44 \\ \hline 176 \end{array} \\ &\quad \begin{array}{r} 1 \\ - 44 \\ \hline 176 \end{array} \\ &\quad \boxed{= 1936} \end{aligned}$$

Q7.  $A = \{P, Q, R\}$  find

a)  $n(A) = 3$

b)  $n(P(A)) = 8$

c)  $n(P(P(A))) = 2^3 = 8$

d)  $n(P(P(P(A))))$

$$\begin{aligned} &= 8^2 = 2^{256} \\ &= 8^1 = 4^{128} \\ &= 8^0 = 8^{64} \\ &= 8^{-1} = 16^{32} \\ &= 8^{-2} = 256^{16} \\ &= 8^{-3} = 256^8 \\ &= 8^{-4} = 256^4 \\ &= 8^{-5} = 256^2 \\ &= 8^{-6} = 256^1 \\ &= 8^{-7} = 256^0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} d &= n(P(P(P(A)))) = 2^{256} \\ &= 256^{16} \\ &= 65536^8 \end{aligned}$$

$$\begin{array}{r} 1111 \\ 655360 \\ \times 3 \\ \hline 196108 \\ 327680 \\ \hline 153600 \\ 25600 \\ \hline 1536 \\ 2560 \\ \hline 1536 \\ 12800 \\ 51200 \\ \hline 65536 \\ 313216 \\ 196080 \\ 3276800 \\ 3276800 \\ \hline 3932160000 \\ 4294967296 \end{array}$$

(41)

## Theory of Numbers

→ It helps us to study the relationship between different type of numbers such as prime nos., even nos.

Type	Example	Special Point
1. Natural Numbers (N)	$N = \{1, 2, 3, 4, \dots\}$	
2. Whole Numbers (W)	$W = \{0, 1, 2, 3, \dots\}$	
3. Integers ( $\mathbb{Z}/I$ )	$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ $I^+ = \{1, 2, 3, \dots\}$ $I^- = \{-3, -2, -1, \dots\}$	<ul style="list-style-type: none"> <li>* Non-negative integers - <math>\{0, 1, 2, 3, \dots\}</math></li> <li>* Non-positive integers - <math>\{\dots, -3, -2, -1, 0\}</math></li> <li>* Even integers - <math>\{0, -4, -2, 0, 2, 4, \dots\}</math></li> <li>* Odd integers - <math>\{\dots, -3, -1, 1, 3, \dots\}</math></li> </ul>
4. Prime Numbers	$\{2, 3, 5, 7, 11, 13, 17\}$	<ul style="list-style-type: none"> <li>* Natural numbers except 1 who have only two factors i.e. 1 &amp; number itself</li> <li>* 2 is the only even prime number</li> </ul>
5. Composite Numbers	$\{4, 6, 8, 9, 10, 12, 14\}$	<ul style="list-style-type: none"> <li>* Natural numbers except 1 who have more than 2 factors.</li> <li>* 4 is the smallest composite no.</li> </ul>

(6) co-prime numbers (relatively prime)	$(2, 3), (7, 8)$ $(10, 11), (12, 13)$ etc. $(11, 13), (17, 19)$	* HCF of two numbers = 1 * The numbers are not necessarily prime numbers * Any two consecutive nos. are co-prime. * All odd nos. consecutive are co-prime * Any two prime nos. are co-prime * Two prime numbers having difference of two. * Both must be prime.
(7) twin primes	$(3, 5), (5, 7), (17, 19)$ $(11, 13)$	
(8) rational numbers (Q)	$\frac{1}{2}, \frac{3}{5}, \frac{-9}{8}, \frac{7}{21}$	* Numbers in form $\frac{p}{q} (q \neq 0)$ * p & q are co-prime and don't have common factor
(9) Irrational nos. (Q or Q')	$\sqrt{2}, \sqrt{3}, \pi, e \dots$	* The numbers which are not rational.

10

10. Complex Numbers ( $z$ )  $\Rightarrow$  Numbers which are written in the form of  $a+ib$  ( $i$  stands for iota)

$$i = \sqrt{-1}$$

$a, b$  are real numbers

$$\text{eg. } 3+2i, -5+\frac{2}{3}i, 3(3+10i), -7+2i$$

$\rightarrow$  Two complex numbers  $a+ib$  and  $p+iq$  are equal. Then  $a=p, b=q$

$\rightarrow$  Conjugate of a complex number

$$(2+i\sqrt{3} \rightarrow 2-i\sqrt{3})$$

$$z = a+ib \rightarrow \bar{z} = a-ib$$

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Q. find conjugate

a)  $3+i2 \rightarrow 3-i2$

b)  $\sqrt{2}+i3 \rightarrow \sqrt{2}-i3$

c)  $i2-7 \rightarrow -7-i2$

$-3-i5 \rightarrow -3+i5$

$(8+\sqrt{3})+2i \rightarrow (8+\sqrt{3})-2i$

6  $\rightarrow$  6

$\rightarrow$  Two complex numbers  $a+ib$  &  $r+ic$  are conjugate to each other then

$$\begin{aligned} a &= r \\ b &= -c \end{aligned}$$

d)  $-3+ixy$  &  $x+y+4i$  are conjugate to each other, then  
find  $x$  &  $y$

$$\begin{aligned} x+y &= -3 \\ xy &= -4 \\ 2(-3-y)(y) &= -4 \\ -3y - y^2 &= -4 \\ y^2 + 3y - 4 &= 0 \\ y(y+4) - 1(y+4) &= 0 \\ (y-1)(y+4) &= 0 \\ y = 1 & \quad y = -4 \\ & | \quad x = -4 \\ & \boxed{-4, 1} \\ & \boxed{-2, -1} \end{aligned}$$

(11) Perfed nos.

$$x + y = -3$$

$$x \otimes y = -4$$

$$x = -3 - y$$

$$-3 - y (y) = -4$$

$$-3y - y^2 = -4$$

$$y^2 + 3y - 4 = 0$$

$$y^2 + 4y - y - 4 = 0$$

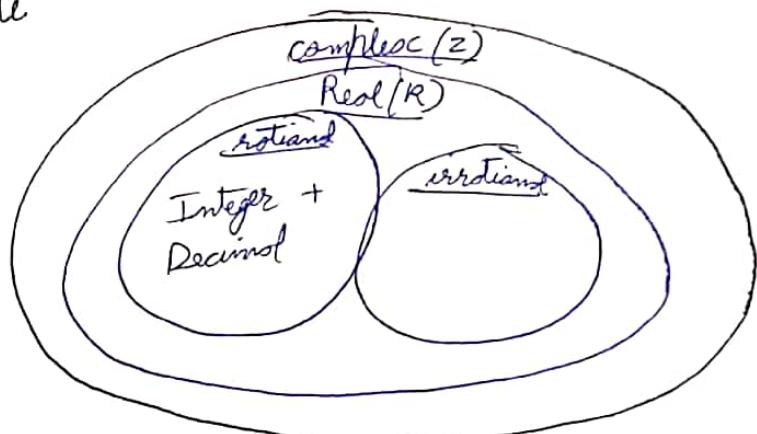
$$y(y+4) - 1(y+4)$$

$$(y-1)(y+4)$$

$$y = 1, y = -4$$

$$x = -4, x = 1$$

Note:-



(15)

11. Perfect number - Sum of all proper divisors of a number is the number itself then it is called a perfect number.

$$\text{eg. } 6 = \cancel{1} \cancel{2} \cancel{3} \quad 1+2+3=6$$

$6 \rightarrow$  perfect no.

$$28 = \cancel{1} \cancel{2} \cancel{4} \cancel{7} \cancel{14} \quad 1+2+4+7+14=28$$

$28 \rightarrow$  perfect no.

H.W. 18-04-2024

DYS-4 (Q1-Q6)

J-M (Q2,3)

DYS-4

Q1 :  $a=3, b=4$

Q2. i)  $x = \frac{2}{11}$

ii)  $x = \frac{16}{99}$

iii)  $x = \frac{419}{990}$

Q3. Rational

Q4. i) all rational ii) may or may not iii) all irrational

Q6. irrational

JEE-M

Q2. 4)

Q3. 3)

Note -

- ① Rational + Irrational = Irrational  
② Rational  $\times$  Irrational = Irrational (for rational  $\neq 0$ )  
③ Irrational  $\pm$  Irrational = Rational / Irrational

eg  $(2+\sqrt{3}) + (2-\sqrt{3}) \rightarrow$  rational  
 $(2+\sqrt{3}) + (2-\sqrt{5}) \rightarrow$  irrational

④ Irrational  $\times$  Irrational = Rational / Irrational  
 $\downarrow$   
 $\sqrt{2} \times \sqrt{2} \quad \cancel{\sqrt{2} \times \sqrt{2}} \rightarrow \sqrt{3} \times \sqrt{2}$

⑤  $\frac{\text{Rational}}{\text{Irrational}} = \text{Rational} / e \text{ Irrational}$   
 $\downarrow$   
 $\frac{\sqrt{2}}{\sqrt{2}}$

$$\pi \approx \frac{22}{7}$$

$$\pi = 3.1415$$

$$\frac{22}{7} = 3.14285$$

$$\frac{22}{7} > \pi$$

Investigation of a prime no.

Q. Investigate if 17 is a prime no. or not.

$$\sqrt{17} = 4. \text{ something}$$

$$\begin{array}{c} 2 \nmid 3 \\ \frac{1 \cdot 3 \text{ am}}{2} \times \end{array} \quad \begin{array}{c} 3 \nmid 1 \\ \frac{1 \cdot 3 \text{ am}}{3} \times \end{array}$$

So 17 is a prime.

eg 2.  $\sqrt{18}$   
 $\sqrt{18} = \sqrt{4 \cdot 9}$   
 $= 3\sqrt{2}$   
~~divisible by 3~~  
not prime

eg 3.  $\sqrt{571}$   
 $\sqrt{571} = 23$ , done  
~~23 is not divided by any prime smaller than it~~  
So. 571 is prime

$$\begin{array}{r} 1 \\ 24 \\ \overline{)24} \\ 96 \\ \overline{)48} \\ 0 \\ 576 \end{array}$$

eg 4.  $\sqrt{100} = 10$  is divided by 2 & 5  
So 100 is not prime.

Q If  ~~$x, y \in \mathbb{N}$~~  and  $xy = 10$  then find all ordered pairs  $(x, y)$

$$(1, 10) (2, 5) (5, 2) (10, 1) = 4$$

### Trick

$a, b \in \mathbb{W}$   
 $a + b - ab = 1$   
Add 2 subtract no. such that get common 3T STR

$$\begin{aligned} a + b - ab - 1 + 1 &= 1 \\ a - 1 + b - ab + 1 &= 1 \\ (a-1) + b(a-1) &= 0 \\ b(1-a) - (a-1)(1-a) &= 0 \\ (1-a)(b-1) &= 0 \end{aligned}$$

$$\boxed{\begin{array}{l} a = 1 \\ b = 1 \end{array}}$$

(48)

$$Q. \quad 7x + 7y - xy = 49$$

$$7x + y(7-x) = 49$$

$$7x + y(7-x) + 49 - 49 = 49$$

$$(7x - 49) + y(7-x) = 0$$

$$7(x-7) + y(7-x) = 0$$

$$y(7-x) - 7(7-x) = 0$$

$$\cancel{7}(7-x)(y-7) = 0$$

$$\boxed{\begin{array}{l} y = 7 \\ x = 7 \end{array}}$$

Q find all  $(x, y)$   $x, y \in \mathbb{N}$  such  $xy + 5x = 4y + 38$

$$5x - 4y + xy = 38$$

$$5x + y(-4+x) = 38$$

$$5x + y(x-4) + 20 - 20 = 38$$

$$y(x-4) - 5(x-4) = 38 - 20$$

$$(y+5)(x-4) = 18 \rightarrow$$

$$(1, 18)$$

$$(2, 9)$$

$$(3, 6)$$

$$(6, 3)$$

$$(9, 2)$$

$$(18, 1)$$

$$\boxed{(22, -4) (13, -3) (10, -2)}$$

$$\boxed{(7, 1) (6, 4) (5, 13)}$$

reflected as negative

Note - No. of integers b/w  $a$  &  $b$  =  $b-a-1$

e.g. B/w 2 & 7 =  $7-2-1$   
= 7-3  
 $\boxed{= 4}$

B/w 10 & 20 =  $20-10-1$   
= 20-11  
 $\boxed{= 9}$

B/w 11 & 501 =  $501-11-1$   
 $\boxed{= 489}$

Note :- no. of integers b/w  $a$  &  $b$  including  $a$  &  $b$  =  $b-a+1$

e.g. B/w 5 & 10 =  $10-5+1$   
 $\boxed{= 6}$

B/w 11 & 40 =  $40-11+1$   
= 40-10  
 $\boxed{= 30}$

B/w 207 & 509 =  $509-207+1$   
= 509-207  
 $\boxed{= 303}$

Q. How many integers in between -200 to 2500 (Both inclusive)  
a) are multiple of 3 or 5

$$\begin{aligned} 500+200+1 &= 700 \\ &= \frac{700}{3} \\ &= 233 + \frac{1}{3} + 1 (\text{zero}) \\ &= 378 + 1 - 1 \\ &\boxed{= 327} \end{aligned}$$

~~b) are multiples of 3 or 5 but not 15.~~

$$373 - \frac{700+1(\text{zero})}{15}$$

$$373 - 4\cancel{6}7$$

$$373 - 4\cancel{6}7$$

$$\boxed{327} - 4\cancel{6}7$$

$$\boxed{280}$$

~~457000~~

H.W.

19-04-2024

DYS-5 (01, 2, 3, 4, 5)

Q-1

(Q24)

Q-4

(Q3)

DYS-5

Q1 ~~6~~, 6

Q2. i) 2, 2)

~~i~~) 3

Q3. 3

Q4. 651

Q5. 293

Q-1

Q24. C)

~~Q1~~ Q-4

Q3. 7

$$\cancel{166} + \cancel{166} + 1$$

$$\underline{\underline{233}}$$

a) How many integers are between -200 to 500 (both inclusive)

- a) divisible by 3 or 5
- b) divisible by 3 or 5 but not 15.

①  $\frac{-200}{3} \leq x \leq \frac{500}{3}$

$$-66\frac{2}{3} \leq x \leq 166\frac{2}{3}$$

$$-66 \leq x \leq 166$$

~~$166 - (-66) + 1 = 233$  nos.~~

$$\frac{-200}{15} \leq x \leq \frac{500}{15}$$

$$-13 \leq x \leq 33$$

~~$33 + 13 + 1 = 47$~~

a) 3 or 5

$$\begin{aligned}3 \cup 5 &= n(3) + n(5) - n(3 \cap 5) \\&= 233 + 141 - 47 \\&= 374 - 47\end{aligned}$$

$$\boxed{= 327}$$

b)  $3 \Delta 5 = n(3) + n(5) - 2(n(3 \cap 5))$

$$= 327 - 47$$

$$\boxed{= 280}$$

Q Which is greater?

$$\textcircled{1} \quad \frac{8}{9} \text{ or } \frac{7}{8}$$

$$\frac{8}{9} > \frac{7}{8}$$

$$\begin{aligned}
 & \text{Simplifying the terms:} \\
 & \sqrt{13} - \sqrt{12} + \frac{\sqrt{13} + \sqrt{12}}{\sqrt{13} + \sqrt{12}} = 1 \\
 & \sqrt{14} - \sqrt{13} + \frac{\sqrt{14} + \sqrt{13}}{\sqrt{14} + \sqrt{13}} = 1
 \end{aligned}$$

$$\frac{1}{\sqrt{13} + \sqrt{12}} > \frac{1}{\sqrt{13} + \sqrt{14}}$$

$$\frac{14 - 13}{\sqrt{14} + \sqrt{13}}$$

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 156 \end{array}$$

$$\begin{array}{r}
 & 1 \\
 & 14 \\
 \times & 13 \\
 \hline
 & 42 \\
 140 & \hline
 182
 \end{array}$$

## Divisibility

- 2 → Last digit is even no.
- 3 → Sum of digits is divisible by 3
- 4 → Last 2 digits divisible by 4
- 5 → Last digit be 0 or 5
- 6 → Divisible by 2 & 3
- 8 → Last 3 digits should be divisible by 8
- 9 → Sum of digits divisible by 9
- 10 → Last digit be zero
- 11 → Sum of odd digits - sum of even digits be divisible by 11 or 0.

### Divisibility by 7

~~7~~       $\overbrace{3 \ 4 \ 3}^{\rightarrow 3 \times 2 = 6} \rightarrow 34 - 6 = 28 \rightarrow \text{divisible by 7}$

Only applicable for 3 digits.

## Intervals

- Subsets of real numbers
- Intervals are of 5 types.

Interval	Notation ( $B > A$ )	Definition
① Closed Interval	$[a, b]$ or $a \leq x \leq b$	All <u>real numbers</u> between $a$ & $b$ including $a$ and $b$ .
② open interval	$(a, b)$ or $a < x < b$	All <u>real numbers</u> between $a$ & $b$ excluding $a$ and $b$ .
③ open-closed interval -	$(a, b]$ or $a < x \leq b$	All <u>real numbers</u> between $a$ & $b$ including $b$ but not $a$ .
④ closed-open interval	$[a, b)$ or $a \leq x < b$	All <u>real numbers</u> between $a$ & $b$ including $a$ but not $b$ .
⑤ curly interval	$\{a, b\}$ or	only $a$ and $b$ .

Q1. Match the column

- |                       |         |
|-----------------------|---------|
| (A) $[-3, 6]$         | (P) 7.1 |
| (B) <del>(5, 7)</del> | (Q) 0.9 |
| (C) $(-9, -3)$        | (R) -5  |
| (D) $[0, 8]$          | (S) 5   |
| (E) $[6, 9]$          | (T) 6.1 |
|                       | (U) 4.3 |

- (A) - Q S U
- (B) - T
- (C) - R
- (D) - P Q S T U
- (E) - P T

(55)

Infinite Intervals - It is of 4 types

①  $[a, \infty)$

All real nos from  $a$  to infinite  
All real nos. from  $a$  to  $\infty$  without  $a$

②  $(a, \infty)$

All real nos. from  $-\infty$  to  $b$  including  $b$

③  $[-\infty, b]$

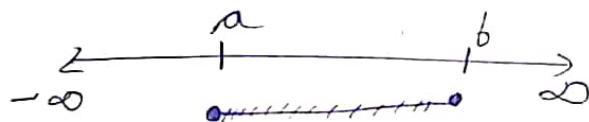
All nos from  $-\infty$  to  $b$  excluding  $b$   
real

④  $(-\infty, b)$

Representation of intervals in graphical form.

① Closed  $(b > a)$

$$[a, b]$$

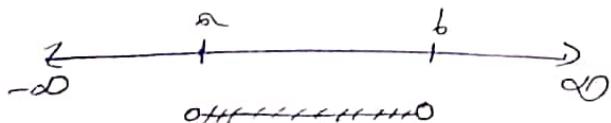


• - Include

○ - Exclude

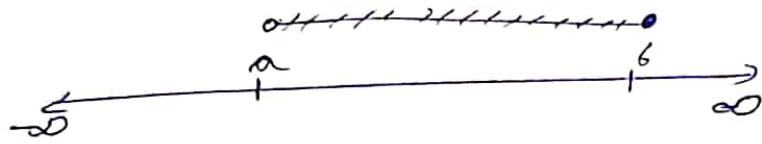
② open interval  $(b > a)$

$$(a, b)$$



③ Open-closed interval

$$(a, b]$$



④ Closed-open Interval

$$[a, b)$$

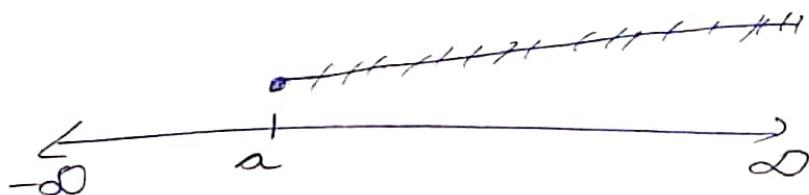


⑤ Empty interval

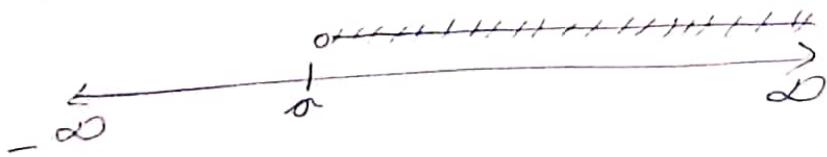
$$\{a, b\}$$



⑥  $[a, \infty)$

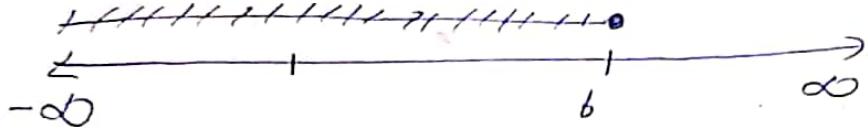


⑦  $(a, \infty)$



(57)

$$\textcircled{8} \quad (-\infty, b]$$

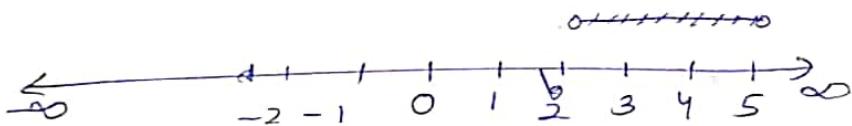


$$\textcircled{9} \quad (-\infty, b)$$

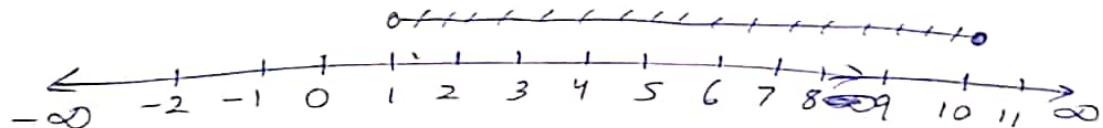


Q1. Represent the ~~sets~~ intervals on number line:

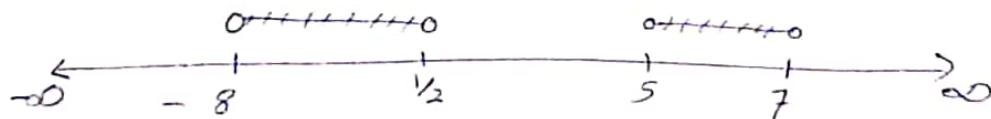
$$\textcircled{1} \quad (2, 5)$$



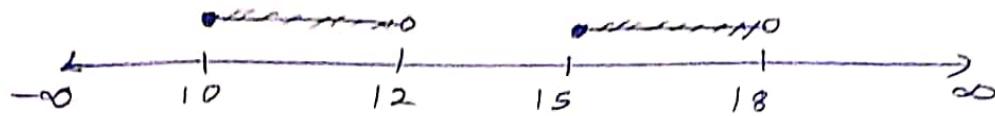
$$\textcircled{2} \quad (1, 10]$$



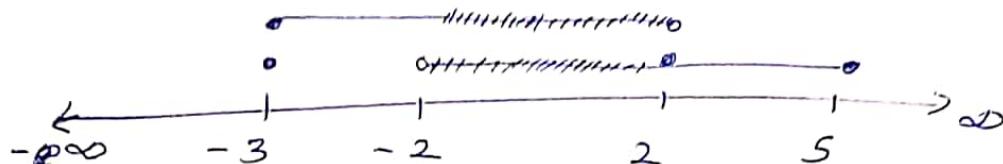
$$\textcircled{3} \quad (-8, \frac{1}{2}) \cup (5, 7)$$



$$\textcircled{4} \quad [10, 12) \cup [15, 18)$$

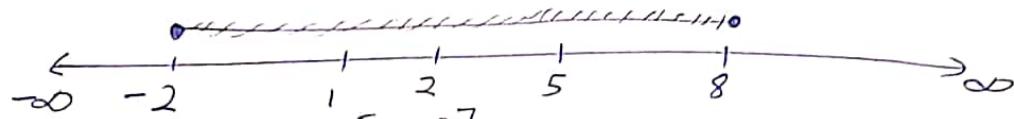


$$\textcircled{5} \quad (-2, 5] \cap [-3, 2)$$

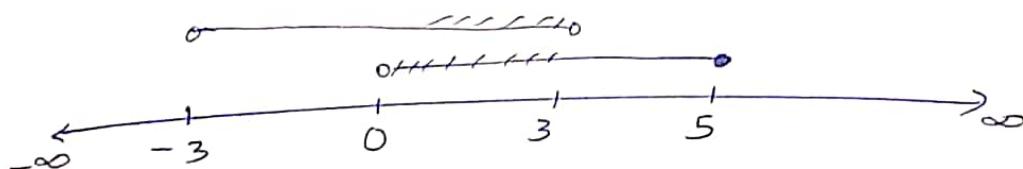


$$(-2, 2)$$

$$\textcircled{6} \quad [-2, 5] \cup (1, 2] \cup [5, 8]$$



$$\textcircled{7} \quad (0, 5] \cap (-3, 3)$$



$$(0, 3)$$

$$\textcircled{8} \quad ((-\infty, 5) \cup [4, 10]) - (-10, 10)$$

$$(-\infty, 10] - (-10, 10)$$



$$(-\infty, 10] \cup \{10\}$$

59

H.W. 20-4-2024

Q1. Represent  $[1, 4]$  or  $[-4, 1)$  on a number line.

Q2.  $(-8, -1] \cup [-3, 0]$  find.

Q3. 1. Total integers

2. Total +ve integers divisible by 3

3. Total -ve integers

4. Total real nos.

5. Total prime nos.

6. Total Irrational nos.

Q3. If  $x \in [1, 4)$ , which of the following lies in it.

(A)  $\frac{2}{3}$

(B)  $4 \cdot 1 - 0.7$

(C)  $4 \cdot 1 - 0.07$

(D)  $\sqrt{4 - 4\tau^2 + \tau^2}$

DYS-1 (Q13)

Q DYS-2 (Q4, Q5)

DYS-5 (Q6 - Q8)

DYS-6 (Q1 - 10)

Q-1

(Q8, 9, 10, 11, 23)

Q-2

(Q3, 5, 6)

Q-3

(Q7)

DYS-1

Q13-3

DYS-2

Q4 4

Q5. A-S, B-P, C-Q, D-R

DYS-5

Q6. -4

Q7 - 30

Q8 - 3

~~Q9~~ Q-1

Q8 - C

Q9 - B

Q10 - A

Q23 B

Q-2

Q3 AD  
Q5 CP

Q6. ABP

~~DYS-6~~

Q-3

Q7 - A-Q

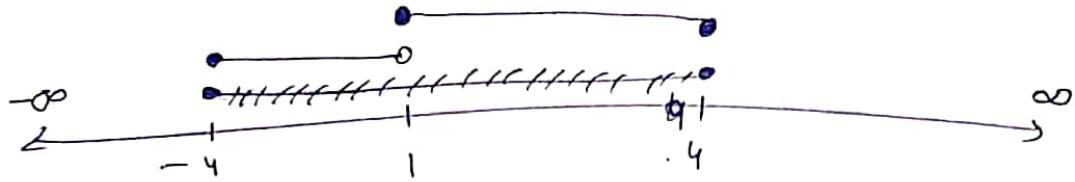
B-R

C-S

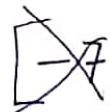
D-P

(60)

Q1.  $[1, 4] \cup [-4, 1)$



Q2.  $(-8, -1] \cup [-3, 10]$



$(-8, 10]$

1. Total integers =  $10 - (-8)$   
 $= 18$  ✓

2. Total integers divisible by 3

$$\begin{array}{r} -8 \leq x \leq 10 \\ 3 \quad 3 \\ \hline 3 + 3 = 6 \end{array}$$

Q3. ~~10~~ - (-8) - 1

$$\begin{array}{r} = 8 - 1 \\ \hline = 7 \end{array}$$

4. Infinite ✓

5. 4 ✓

6. Infinite ✓

Q3. A B C

DYS

013

(62)

(63)

(64)

(65)

(66)

# Fundamentals of Algebra

## Indices / Powers

Indices & Surds (Powers & Exponents).

$a^m$  → Exponent / Power  
 ↴ Base

$m \rightarrow$  One or no. integers

$a \rightarrow$  non-zero real no. or complex number

e.g.  $2^4, (-4)^6, (\sqrt{3})^9, (i\pi)^8, (2+3i)^{15}$  etc

Laws: ( $a \neq 0$ )

$$\textcircled{1} \quad a^0 = 1$$

$$\textcircled{2} \quad a^{-m} = \frac{1}{a^m}$$

$$\textcircled{3} \quad a^m \times a^n = a^{m+n}$$

$$\textcircled{4} \quad \frac{a^m}{a^n} = a^{m-n}$$

$$\textcircled{5} \quad (a^m)^n = a^{m \times n}$$

$$\textcircled{6} \quad \sqrt[c]{a^b} = a^{\frac{b}{c}}, c \in N, c \geq 2$$

Q1. find values

$$\textcircled{1} \quad \left( 256^{-\frac{1}{2}} \right)^{-\frac{1}{4}} \quad \text{?}$$

$$\frac{-1}{2} \times -\frac{1}{4} \times z = +\frac{1}{4}$$

$$\sqrt{\sqrt{256}} = \sqrt{16} = 4$$

$$\textcircled{2} \quad \left( 125^{\frac{1}{3}} + 64^{\frac{1}{3}} \right)^3$$

$$(\sqrt[3]{125} + \sqrt[3]{64})^3$$

$$(5+4)^3 = 9^3 = 729$$

$$\textcircled{3} \quad \left\{ \sqrt[5]{\left( \frac{1}{a} \right)^{-15}} \right\}^{-\frac{4}{3}}$$

$$\left\{ \sqrt[5]{a^{15}} \right\}^{-\frac{4}{3}}$$

$$\left( a^{\frac{15}{5}} \right)^{-\frac{4}{3}}$$

$$a^{\frac{15}{5} \times -\frac{4}{3}}$$

$$a^{-4}$$

$$\begin{array}{r} 3 \\ 8 \\ 5+2 \\ \hline 729 \end{array}$$

(68)

$$④ \frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{27}}$$

$$\begin{aligned} & x^{\frac{1}{3}} \times x^{\frac{5}{3}} \\ & x^{\frac{3}{5}} \\ & x^{\frac{1}{3} + \frac{5}{3} + \frac{27}{30} - \frac{3}{5}} \\ & x^{\frac{10+50+27-24}{30}} \end{aligned}$$

$$\begin{aligned} & x^{\frac{3}{2}} \times x^{\frac{5}{3}} \times x^{\frac{27}{30}} \\ & x^{\frac{3}{5}} \end{aligned}$$

$$\frac{3}{2} + \frac{5}{3} + \frac{27}{30} + \frac{3}{5}$$

$x$

$x^y$

$$y = \frac{3}{2} + \frac{5}{3} + \frac{27}{30} + \frac{3}{5}$$

$$y = \frac{45 + 50 + 77 + 18}{30}$$

$$y = \frac{196}{30}$$

$$⑤ \quad \begin{aligned} & \sqrt[5]{4} \sqrt[3]{\sqrt[3]{x^c}} = x^{\frac{1}{30k}} \\ & \left( \left( x^{\frac{1}{3}} \right)^{\frac{1}{4}} \right)^3 \\ & x^{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}} \\ & x^{\frac{1}{60}} \end{aligned}$$

$$\begin{aligned} & y = \frac{19}{3} \\ & x^{\frac{19}{3}} \end{aligned}$$

$$\begin{array}{r} 2 \\ 95 \\ 77 \\ 18 \\ \hline 190 \end{array}$$

$$\begin{aligned} & x^{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}} \\ & x^{\frac{20+15+12}{60}} \\ & x^{\frac{47}{60}} = x^{\frac{1}{30k}} \end{aligned}$$

$$\begin{aligned} & \frac{1}{60} = \frac{1}{30k} \\ & k = 2 \end{aligned}$$

$$\begin{aligned} & \frac{47}{60} = \frac{1}{30k} \\ & k = \frac{47}{2} \end{aligned}$$

$$Q2. \left( \frac{1}{100} \right)^{\frac{1}{2}} \cdot (125)^{\frac{1}{3}} + (64)^{\frac{1}{3}} \cdot (4)^{-2} + \left( (729)^{-\frac{1}{3}} \right)^{-3}$$

$$\begin{aligned}
 & 10000 \times \frac{1}{125} \times \frac{1}{125} \times \frac{1}{125} + 4 \times \frac{1}{16} + (729)^{\frac{1}{3}} \\
 & 2000 \times \frac{1}{125} \times \frac{1}{125} + \frac{1}{4} + 9 \\
 & 1000 \times \frac{1}{125} \times \frac{1}{125} + \frac{1}{4} + 9 \\
 & 25 \times \frac{1}{125} + \frac{1}{4} + 9 \\
 & 25 \times \frac{1}{625} + \frac{1}{4} + 9 \\
 & 5 \times \frac{1}{25} + \frac{1}{4} + 9 \\
 & 16 + \frac{1}{4} + 9
 \end{aligned}$$

$$10 \times \frac{1}{5} + \frac{1}{4} + 9$$

$$2 + \frac{1}{4} + 9$$

$$11 + \frac{1}{4}$$

$$\boxed{
 \begin{array}{c}
 45 \\
 \hline
 4
 \end{array}
 }$$

(70)

$$⑥ 3. \quad \frac{(2 \times 3)^{n+1} - (7 \times 3)^{n-1}}{3^{n+1} + 2 \times \left(\frac{1}{3}\right)^{1-n}}$$

$$\frac{2 \times (3)^{n+1} - (7 \times 3)^{n-1}}{3^{n+1} + 2 \times 3^{n-1}}$$

$$\frac{2 \times 3^n \times 3^1 - 7 \times 3^n \times 3^{-1}}{3^n \times 3^1 + 2 \times 3^n \times 3^{-1}}$$

$$\frac{2 \cancel{3}^n \left( 6 - \frac{7}{3} \right)}{\cancel{3}^n \left( 3 + \frac{2}{3} \right)}$$

$$\frac{6 - \frac{7}{3}}{3 + \frac{2}{3}}$$

$$\frac{\frac{18-7}{3}}{\frac{9+2}{3}} = \frac{18-7}{9+2} = \frac{11}{11} = \boxed{1}$$

(7)

$$\text{Q4. } \begin{aligned} a^x &= b \\ b^y &= c \\ c^z &= a \end{aligned}$$

prove  $xyz = 1$

$$a^x = c^y$$

$$a = c^{yz}$$

$$a^x = (c^y)^{xz} = c^{yz}$$

$$c^{xyz} = c^1$$

$$\boxed{xyz = 1}$$

Hence Proved

$$\text{Q5. } \begin{array}{ccccccc} 2 & ^{0.7} & 2 & & & & \infty \\ \swarrow & & \searrow & & & & \\ 2 & ^{0.5} & 2 & & & & \\ \downarrow & & \downarrow & & & & \\ 16 & & & & & & \end{array}$$

$$\begin{array}{ccc} 2 & ^{0.7} & \rightarrow 2 & ^{0.5} & \rightarrow 2 & ^{0.9} \rightarrow 0.9 \text{ to power } \infty = 1 \\ \uparrow & & & & & \\ 2 & ^{0.9} & & & & \\ \uparrow & & & & & \\ 2 & ^{0.5} & & & & \\ \uparrow & & & & & \\ 2 & ^{0.7} & & & & \end{array}$$

$$2^{0.7} \rightarrow 2^{0.5} \rightarrow 2^{0.9} \rightarrow \boxed{\underline{\underline{1}}}$$

$$(1)^{\infty} = 1$$

$$1^{\circ} = 1$$

$$(\text{value b/w } 0 \& 1)^{\infty} = 1$$

$$(\text{value b/w } 0 \& 1)^{\infty} = 0$$

$$(\text{value greater than } 1)^{\infty} = 1$$

$$(\text{value greater than } 1)^{\infty} = \infty$$

Note:- ① If  $a^x = a^y$  Then  $a^x = a^y$  ( $a \neq 0$ ) and  
if  $a^x = a^y$  then  $x = y$  ( $a \neq 0, 1, -1$ ) is not always true.  
 $\downarrow$   
not always

② Some base and different powers:

$$a^x = a^y$$

$$\text{Case 1}$$
  
 $a=1$

$$\text{Case 2}$$
  
 $a=-1$

$$\text{Case 3}$$
  
 $a=0$

$$\text{Case 4}$$
  
 $x=y$

⇒ verify in each case.

$$\text{Q} \quad (2x^2-1)^{5x+2} = (2x^2-1)^{x^2+6}$$

$$x^2+6 = 5x+2$$

$$Q \quad (2x^2 - 1)^{\frac{5x+2}{2}} = (2x^2 - 1)^{\frac{x^2+6}{2}}$$

sol case 1  $\rightarrow \alpha = 1$

$$2x^2 - 1 = 1$$

$$\begin{aligned} 2x^2 &= 2 \\ x^2 &= 1 \end{aligned}$$

$$x = \pm 1$$

$$\text{verify } \rightarrow x = +1 \rightarrow (2-1)^{\frac{5(1)+2}{2}} = (2-1)^{\frac{7}{2}}$$

$$1^7 = 1^7 \checkmark$$

$$x = -1 \rightarrow (2-1)^{-3} = (2-1)^7$$

$$\begin{aligned} 1^{-3} &= 1^7 \\ \frac{1}{1^3} &= 1^7 \end{aligned}$$

$$1 = 1 \checkmark$$

case 2  $\rightarrow \alpha = 0$

$$2x^2 - 1 = 0$$

$$\begin{aligned} 2x^2 &= 1 \\ x^2 &= \frac{1}{2} \end{aligned}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

$$\text{verify } \rightarrow x = \pm \frac{1}{\sqrt{2}} \rightarrow (2 \times \frac{1}{2} - 1)^{\frac{5}{\sqrt{2}} + 2} = (2 \times \frac{1}{2} - 1)^{\frac{1}{2} + c}$$

$$0^{\frac{5}{\sqrt{2}}} = 0^{\frac{1}{2} + c} \checkmark$$

$$x = \pm \frac{1}{\sqrt{2}} \rightarrow (2 \times \frac{1}{2} - 1)^{-\frac{5}{\sqrt{2}} + 2} = (2 \times \frac{1}{2} - 1)^{\frac{1}{2} + c}$$

$$0^{\frac{1}{2} + c} = 0^{\frac{1}{2} + c} \times (0^{-\frac{1}{2}} \text{ is not defined})$$

Case 3  $\rightarrow x = -1$

$$2x^2 + 1 = -1$$

$$2x^2 = 0$$

$$x^2 = 0$$

$$x = 0$$

Verify:-  $(0-1)^{0+2} = (0-1)^{0+4}$

$$-1^2 = -1^4$$

$$1 = 1 \checkmark$$

Case 4  $\rightarrow ax = y$

$$x^2 + 6 = 5x + 2$$

$$x^2 - 5x + 4 = 0$$

$$x^2 - 4x - x + 4 = 0$$

$$x(x-4) - 1(x-4) = 0$$

$$(x-1)(x-4) = 0$$

$$x=1, x=4$$

Verify:-  $x=1 \rightarrow$  verified already

$$x=4 \rightarrow (32-1)^{20+2} = (32-1)^{15+5}$$

$$\rightarrow 31^{22} = 31^{22} \checkmark$$

Find answer -  $x = 1, -1, 0, \frac{1}{2}, 4$  Answer.

Note:-

### ③ Some Power and Different Base

$$a^x = b^x$$

↓      ↓      ↓

Case-1      Case-2      Case-3

~~a & b~~ = 1      Power = 0      a & b = -1  
or  
 $a = b$       or  
 $a = -b$

Q find  $x$ ,  $(x+2)^{(x-3)} = (2x-5)^{(x-3)}$

Case 1  $\rightarrow a = 6$

$$x+2 = 2x-5$$

$$7 = x$$

verify

$$(7+2)^{(7-3)} = (14-5)^{(7-3)}$$

$$9^4 = 9^4 \checkmark$$

Case 2  $\rightarrow a(x-3) = 0$

$$x-3=0$$

$$x=3$$

Verify:-

$$(3+5)^{(3-3)} = (6-5)^{(3-3)}$$

$$8^0 = 1^0$$

$$1 = 1 \checkmark$$

Case 3  $\rightarrow x = -6$

$$x + 2 = 5 - 2x$$

$$\begin{array}{r} 3x = 3 \\ \underline{x = 1} \end{array}$$

Verify :-

$$(1+2)^{(-3)} = (2-5)^{(1-3)}$$

$$3^{-2} = -3^{-2}$$

$$\frac{1}{9} = \frac{1}{9} \quad \checkmark$$

$x = 1, 3, 7$  Answer

H.W.

23-04-2024

Q1. Find the values of  $x$

$$\textcircled{1} (4-x)^{x^3-4x} = (4-x)^{(x^2-4)x(x-1)}$$

$$\textcircled{2} (x^2-4)^{2x} = \cancel{3x}(x^2+2x)^{2x}$$

Case I - 8

## Surds

→ Any root of a<sup>n</sup> numbers which cannot be exactly found as a whole are surd numbers.

$$\sqrt[n]{a} \quad (a \text{ is a surd number})$$

n → degree of surd

e.g.  $\sqrt[2]{2}$ ,  $\sqrt[3]{7}$ ,  $\sqrt[4]{(+3)}$  → 15 degree  
 ↓              ↓              ↓  
 2 degree      3 degree      4 degree

→ If a is not rational then  $\sqrt[n]{a}$  is not a surd

e.g.  $\sqrt{2 + \sqrt{3}}$  is not a surd

- 1 term → simple surd
- 2 terms → Binomial surd
- 4 terms → Bi quadratic surd.

→ Two surds which differ only in sign which connects their terms are conjugate or complementary to each other.

e.g.  $2\sqrt{7} + 5\sqrt{3}$   
 ↓  
 conjugate  
 ↓

$-2\sqrt{7} + 5\sqrt{3}$  or  $2\sqrt{7} - 5\sqrt{3}$

→ Product of a surd and its conjugate is rational or irrational.

e.g.-1.  ~~$\sqrt{2} + \sqrt{3}$~~   $\rightarrow \sqrt{2} - \sqrt{3}$

$$(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$$

$$(\sqrt{2})^2 - (\sqrt{3})^2$$

$2 - 3 = 1$  (rational)

(if degree = 2, rational)

e.g.-2.  $\sqrt[3]{2} + \sqrt[3]{3} \rightarrow \sqrt[3]{28} - \sqrt[3]{3}$

$$(\sqrt[3]{2} + \sqrt[3]{3})(\sqrt[3]{2} - \sqrt[3]{3})$$

$$(\sqrt[3]{2})^2 - (\sqrt[3]{3})^2$$

$2^{2/3} - 3^{2/3} \rightarrow$  (irrational)

Q. Arrange the following in ascending order

$$\sqrt[3]{9}, \sqrt[4]{11}, \sqrt[6]{17}$$

$$9^{1/3}, 11^{1/4}, 17^{1/6} \quad (\text{LCM of } 3, 4 \text{ & } 6 \rightarrow 24)$$

$$9^8, 11^6, 17^4$$

$$81^4, 11^6, 17^4$$

$$81^{4/24}, 11^{6/24}, 17^{4/24}$$

$$\begin{array}{r} 11 \\ 11 \\ \hline 11 \\ 11 \\ \hline 121 \\ 11 \\ \hline 121 \\ 121 \\ \hline 1221 \\ 1221 \\ \hline 12221 \\ 1331 \end{array}$$

∴  ~~$\sqrt[3]{9} > \sqrt[4]{11} > \sqrt[6]{17}$~~

$$\sqrt[3]{9} > \sqrt[4]{11} > \sqrt[6]{17}$$

## Q Square Root

$$\rightarrow \sqrt{x^2} = |x| \quad (-x \text{ is not right}) \quad (x \text{ be } \oplus \text{ve})$$

$$(\sqrt{x})^2 = x$$

$\rightarrow$  It gives us only non-negative values

$\rightarrow$  All quantities in underroot must be positive

$\rightarrow$  Square Root of  $a + \sqrt{b}$  ( $a \geq 0, b \geq 0$ )

$$\sqrt{a + \sqrt{b}} = \sqrt{x} + \sqrt{y} \quad (x, y \geq 0)$$

Squaring

$$a + \sqrt{b} = x + y + 2\sqrt{xy}$$

$$a + \sqrt{b} = c + \sqrt{d} \rightarrow a = c, a^2 = d$$

$$\boxed{a = x + y} \rightarrow y = a - x$$

$$\sqrt{b} = 2\sqrt{xy}$$

$$\boxed{b = 4xy}$$

$$b = 4x(a-x)$$

$$b = 4ax - 4x^2$$

$$4x^2 - 4ax + b = 0$$

$$x = \frac{-(-4a) \pm \sqrt{16a^2 - 4(4)(b)}}{2(4)}$$

$$x = \frac{4a \pm \sqrt{16a^2 - 16b}}{8} = \frac{a \pm \sqrt{16a^2 - 16b}}{2} \quad (\oplus \text{ve because } x \text{ is positive})$$

$$y = \frac{a - \sqrt{16a^2 - 16b}}{2}$$

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$$\text{Q1. } \textcircled{1} \quad (y-x)^{x^{3-y}} = (y-x)^{(x^2-y)} (x-1)$$

Case 1:-  $a = 0$

$$y-x=0$$

$$x=4$$

verify

$$(y-4)^{4^3 - 4x^{x+1}} = (y-4)^{(4^2-4)(4-1)}$$

$$0^{4-16} = 0^{(12)(3)}$$

$$0^{48} = 0^{36}$$

$$0=0 \checkmark$$

case 2:-  $a = -1$

$$y-x=a-1$$

$$y+1=2L$$

$$x=5$$

$$\text{verify:- } (y-5)^{125-20} = (y-5)^{(25-4)(4)}$$

$$(-1)^{105} = (-1)^{\cancel{84}}$$

$$-1 = 1 \times$$

Case 3 :-  $a=0$

$$4-x=0$$

$$x=3$$

verify :-

$$(4-3)^{27-12} = (4-3)^{(1-4)(3-1)}$$

$$1^{15} = 1^{5 \times 2}$$

$$1^{15} = 1^{20}$$

$$1=1 \quad \checkmark$$

Case 4 :-  $x=4$

$$x^3-4x = (2x^2-4)(x-1)$$

$$x^3-4x = x^3 - x^2 - 4x + 4$$

$$\begin{aligned} x^2+4 &\neq 0 \\ x^2 &= -4 \\ x &= \pm 2 \end{aligned}$$

$$\begin{aligned} x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

verify  $x=2 \rightarrow (4-2)^{8-12} = (4-2)^{(4-4)(2-1)}$

$$2^0 = 2^0 \quad \checkmark$$

$$x=-2 \rightarrow (-2-2) (-4+8) = (-2)^{(4-4)(-3)}$$

$$6^0 = 6^0 \quad \checkmark$$

$$x = 2, -2, 3, 4,$$

(82)

$$\textcircled{2} \quad (x^2 - 4)^{2x} = (x^2 + 2x)^{2x}$$

case 1 :-  $a = 6$

$$x^2 - 4 = x^2 + 2x$$

$$x = \frac{-4}{2}$$

$$\underline{x = -2}$$

verify :-

$$((-2)^2 - 4)^{-2x^2} = ((-2)^2 + 2(-2))^{2(-2)}$$

$$(4 - 4)^{-4} = (4 + -4)^{-4}$$

$$0^{-4} = 0^{-4} \quad \cancel{\checkmark} \quad (0^{\frac{1}{0}} \text{ is not valid})$$

Case 2:-  $a = -6$

$$x^2 - 4 = -x^2 - 2x$$

$$2x^2 + x^2 = -2x + 4$$

$$2x^2 + 2x - 4 = 0$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$(x+2)(x-1)$$

$$\begin{aligned} x &= -2 \quad (\text{verified}) \\ x &= 1 \end{aligned}$$

~~Ans~~

Verify :-

$$\left( (1)^2 - 4 \right)^{2 \times 1} = \left( (1)^2 + 2(1) \right)^{2 \times 1}$$

$$(1-4)^2 = (1+2)^2$$

$$(-3)^2 = (3)^2$$

$$9 = 9 \checkmark$$

Case 3:- power = 0

$$\begin{aligned} 2x &= 0 \\ x &= \frac{0}{2} \\ x &= 0 \end{aligned}$$

Verify:-  $\left( (0)^2 - 4 \right)^{2 \times 0} = \left[ (0)^2 + 2(0) \right]^{2 \times 0}$

$$(0-4)^0 = (0+0)^0$$

$$-4^0 = 0^0 \quad X \quad (0^\circ \text{ is not defined})$$

$$\boxed{x = 1}$$

$$\textcircled{1} \text{ find } \sqrt{8+2\sqrt{15}}$$

$$\textcircled{2} \sqrt{37\sqrt{5}}$$

$$\textcircled{1} \quad \sqrt{8+2\sqrt{15}} =$$
  
$$\sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$
  
~~$$\sqrt{8+\sqrt{64-15}}$$~~

\textcircled{1} Method 1

$$(a+b) \approx a^2+b^2+2ab$$

$$\sqrt{8+2\sqrt{15}}$$

$$\sqrt{8+\sqrt{2\sqrt{5}\sqrt{3}}}$$

$$\sqrt{37+2\sqrt{5}\sqrt{3}}$$

$$\sqrt{(\sqrt{3})^2+(\sqrt{5})^2+2\sqrt{5}\sqrt{3}}$$

$$\sqrt{(\sqrt{3}+\sqrt{5})^2}$$

$$\underline{\underline{\sqrt{3}+\sqrt{5}}}$$

Method - 2

$$\sqrt{8+2\sqrt{15}}$$

$$\sqrt{8+\sqrt{4\sqrt{15}}}$$

$$\sqrt{8+\sqrt{60}}$$

$$\sqrt{\frac{8+\sqrt{64-60}}{2}} + \sqrt{\frac{8\sqrt{64-60}}{2}}$$

$$\sqrt{\frac{8+\sqrt{4}}{2}} + \sqrt{\frac{8\sqrt{4}}{2}}$$

$$\sqrt{\frac{8+2}{2}} + \sqrt{\frac{8-2}{2}}$$

$$\underline{\underline{\sqrt{5}+\sqrt{3}}}$$

$$\textcircled{2} \sqrt{3 + \sqrt{5}}$$

$$\sqrt{\frac{3 + \sqrt{9-5}}{2}} + \sqrt{\frac{3 - \sqrt{9-5}}{2}}$$
$$\sqrt{\frac{3 + \sqrt{4}}{2}} + \sqrt{\frac{3 - \sqrt{4}}{2}}$$

$\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \quad \textcircled{2}$

$$\sqrt{3 + \sqrt{5}}$$
$$\sqrt{(\sqrt{\frac{5}{2}})^2 + (\sqrt{\frac{1}{2}})^2} + 2 \cdot \frac{\sqrt{5}}{2} \cdot \frac{\sqrt{1}}{2}$$

$$\sqrt{\left(\frac{\sqrt{5}}{2} + \frac{\sqrt{1}}{2}\right)^2}$$

~~$$\frac{\sqrt{5}}{2} + \frac{\sqrt{1}}{2}$$~~  
~~$$\sqrt{\left(\frac{\sqrt{5}}{2} + \frac{\sqrt{1}}{2}\right)^2}$$~~

$\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \quad \textcircled{2}$

$$\frac{\sqrt{5}}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$\frac{\sqrt{5}+1}{\sqrt{2}}$

(86)

$$\textcircled{3} \quad \sqrt{7 - 3\sqrt{5}}$$

$$\frac{1}{\sqrt{2}} \sqrt{7 - 2\sqrt{3}\sqrt{9}}$$

$$\sqrt{7 - \sqrt{45}}$$

$$\sqrt{7 - \frac{2\sqrt{45}}{2}}$$

$$\frac{1}{\sqrt{2}} \sqrt{14 - 2\sqrt{45}}$$

$$\frac{1}{\sqrt{2}} \sqrt{9(\sqrt{9})^2 + (\sqrt{5})^2 - 2\sqrt{145}}$$

$$\frac{1}{\sqrt{2}} \sqrt{(8\sqrt{9} - \sqrt{5})^2}$$

$$\frac{1}{\sqrt{2}} (\sqrt{9} - \sqrt{5})$$

$$\frac{\sqrt{9}}{\sqrt{2}} - \frac{\sqrt{5}}{\sqrt{2}}$$

$$\boxed{\frac{\sqrt{9} - \sqrt{5}}{\sqrt{2}}} \quad \textcircled{3}$$

(87)

$$\begin{aligned}
 & \textcircled{1} \quad \sqrt{2\sqrt{3}-3} \\
 & \quad \cancel{\sqrt{-}(3-2\sqrt{3})} \\
 & \quad \cancel{\sqrt{-}(2)} \\
 & \quad \cancel{\sqrt{-}(9)} \\
 & \quad \sqrt{2\sqrt{3}-\cancel{9}}
 \end{aligned}$$

H.W - 25-04-2024

- ①  $\sqrt{2\sqrt{2}-3}$
- ②  $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}}+\sqrt{7+\sqrt{48}}}$
- ③  $\sqrt[4]{17+12\sqrt{2}}$
- ④  $x = \sqrt{3+2\sqrt{3}} \quad \text{find} \quad x^3 - x^2 - 11x + 4$
- ⑤  $x = \frac{1}{2+\sqrt{3}} \quad \text{find} \quad x^3 - x^2 - 11x + 4$

$$x = \sqrt{3+2\sqrt{2}}$$

$$x^3 - x^2 - 11x + 4$$

$$x = \sqrt{3+\sqrt{9-12}}$$

$$x = \frac{1}{\sqrt{2}} \sqrt{4\sqrt{3} + 2\sqrt{9}}$$

$$x = \sqrt{(\sqrt{2})^2 + (\sqrt{1})^2 + 2\sqrt{2}\sqrt{1}}$$

$$= (\sqrt{2} + \sqrt{1})$$

$$(\sqrt{2} + \sqrt{1})^3 - (\cancel{(\sqrt{2} + \sqrt{1})})^2 - 11(\sqrt{2} + \sqrt{1}) + 4$$

$$(\sqrt{2} + \sqrt{1})^3 - 3\cancel{\sqrt{3}} - 11\sqrt{2} - 11\sqrt{1} + 4$$

$$(\sqrt{2})^3 + (\sqrt{1})^2 + 3\sqrt{2}(\sqrt{2} + \sqrt{1}) - 3 - \sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$$

$$(\sqrt{2})^3 + \cancel{(\sqrt{1})} + 3 - 3 - \sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$$

$$(\sqrt{2})^3 + 12 - 3 - \sqrt{2} - 11$$

$$(\sqrt{2})^3 - 2 - 12\sqrt{2}$$

$$Q \sqrt{3+2\sqrt{2}} \text{ find } \begin{cases} 1) x^3 - x^2 - 11x + 4 \\ 2) 2x^3 - x^2 - 8x + 120 \end{cases}$$

$$\textcircled{1} \textcircled{2} \quad \sqrt{3+2\sqrt{2}}$$

$$\sqrt{(\sqrt{2})^2 + (\sqrt{1})^2 + 2\sqrt{2}\sqrt{1}}$$

$$x = \sqrt{2} + 1$$

$$(x-1) = \sqrt{2}$$

$$2 = x^2 + 1 - 2x$$

$$x^2 - 1 - 2x = 0$$

$$x^2 - 2x - 1 = 0$$

$$2x^2 - 4x - 2 = 0$$

$$2x^3 - x^2 - 8x + 120$$

$$x(2x^2 - x - 8) + 120$$

$$x(x^2 - 2x - 1 + x^2 + x - 7) + 120 = 0$$

$$x(0x^2 + x - 7) + 120 = 0$$

$$x^3 + x^2 - 7x + 120 = 0$$

$$x^3 + x^2 - 8x - 1 + 121 - 5x = 0$$

$$x^3 + 0 + 121 - 5x = 0$$

$$x^3 + 121 - 5x = 0$$

$$x^3 - 5x + 121 = 0$$

$$x(2x^2 - 4x - 2 - 6 + 3x) + 120 \quad \text{or}$$

$$x(0 - 6 + 3x) + 120 = 0$$

$$3x^2 - 6x + 120 = 0$$

$$3(x^2 - 2x + 40) = 0$$

$$3(x^2 - 2x - 1 + 41) = 0$$

$$3(41)$$

90 123

$$\textcircled{1} \quad x^3 - x^2 - 11x + 4$$

$$x(x^2 - x - 11) + 4$$

$$x(x^2 - 2x - 1 - 10 + x) + 4$$

$$x(x - 10) + 4$$

$$x^2 - 10x + 4$$

$$x^2 - 2x - 1 + 5 - 8x$$

$$0 + 5 - 8x$$

$$5 - 8x$$

$$5 - 8(\sqrt{2} + 1)$$

$$5 - 8\sqrt{2} - 8$$

$$\boxed{-8\sqrt{2} - 3}$$

$$\textcircled{1} \quad \sqrt{2\sqrt{2}-3}$$

$$\sqrt{-(3-2\sqrt{2})}$$

$$\sqrt{-(\sqrt{2})^2 - (\sqrt{1})^2 - 2\sqrt{2}\sqrt{1}} = \sqrt{-(\sqrt{2} + \sqrt{1})^2}$$

~~$\sqrt{2\sqrt{2}-3}$~~

$$\textcircled{2} \quad \sqrt{3+\sqrt{3} + \sqrt{2+\sqrt{3} + \sqrt{7+\sqrt{48}}}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{2+\sqrt{3} + \sqrt{3+4+2\sqrt{24}\sqrt{3}}}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{2+\sqrt{3} + \sqrt{3+4}}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{4+2\sqrt{3}\sqrt{1}}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{3+12\sqrt{3}\sqrt{1}}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{(\sqrt{3})^2 + 1^2 + 2\sqrt{3}\sqrt{1}}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{(\sqrt{3}+1)^2}}$$

$$\sqrt{3+\sqrt{3} + \sqrt{3+1}}$$

$$\sqrt{4+2\sqrt{3}}$$

$$\sqrt{(\sqrt{3}+1)^2}$$

$$\sqrt{3+1} \quad \text{Q2}$$

$$③ \sqrt[4]{17+12\sqrt{2}}$$

$$\sqrt[4]{17+2\sqrt{2}\sqrt{36}}$$

$$\sqrt[4]{17+2\sqrt{72}}$$

$$\sqrt[4]{9+8+2\sqrt{9\sqrt{8}}}$$

$$\sqrt[4]{(\sqrt{9})^2+(\sqrt{8})^2+2\sqrt{9}\sqrt{8}}$$

$$\sqrt[4]{(\sqrt{9}+\sqrt{8})^2}$$

$$\sqrt{\sqrt{9}+\sqrt{8}}$$

$$\sqrt{3+\sqrt{8}}$$

$$\sqrt{3+2\sqrt{2}}$$

$$\sqrt{(\sqrt{2})^2+1^2+2\sqrt{2}\sqrt{1}}$$

$$\sqrt{(\sqrt{2}+1)^2}$$

$$\boxed{|\sqrt{2}+1 \quad ③|}$$

$$④ x = \frac{1}{2+\sqrt{3}}, \text{ find } x^3 - x^2 - 11x + 4$$

$$x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{2-\sqrt{3}}{4-3}$$

$$x = 2-\sqrt{3}$$

$$x = 2 - \sqrt{3}$$

$$\sqrt{3} = 2 - x$$

$$(\sqrt{3})^2 = (2 - x)^2$$

$$3 = 4 + x^2 - 4x$$

$$x^2 - 4x + 1 = 0$$

$$3x^2 - 12x + 3 = 0$$

$$x^3 - 2x^2 - 11x + 4$$

$$x(x^2 - x - 11) + 4$$

$$x(x^2 - 4x + 1 + 3x - 12) + 4$$

$$x(0 + 3x - 12) + 4$$

$$3x^2 - 12x + 4$$

$$3x^2 - 12x + 3 + 1 \cancel{+ 4}$$

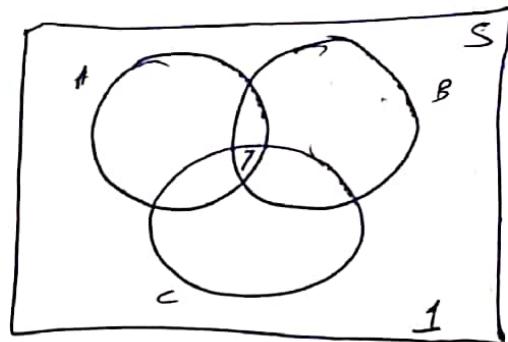
$$0 + 1 \cancel{+ 4}$$

$$\boxed{= 1 \quad 0}$$

(94)

Q-3

- Q4. ✓ A)  $A = B = C$   
✓ B)  $A = B = C$  (or solving)  
✓ C)  $A - B = (A \cup B \cup C) - (A \cap B \cap C)$   
 $\emptyset = \emptyset$  for  $(A \cap B \cap C)$  be max  
✗ D)



Q5. ✓ A)

✓ B)

✓ C)

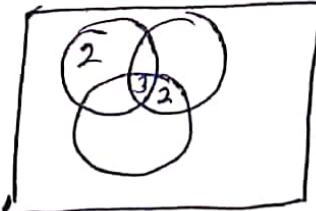
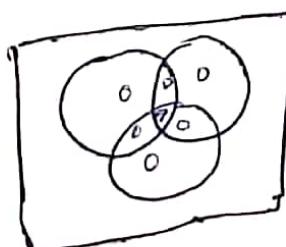
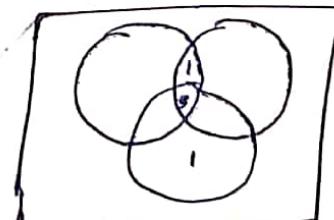
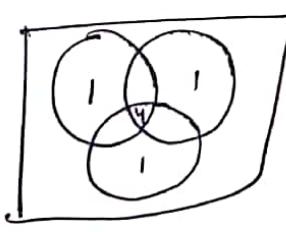
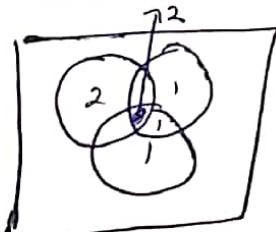
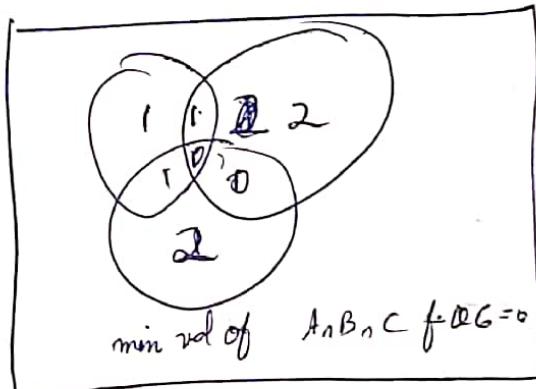
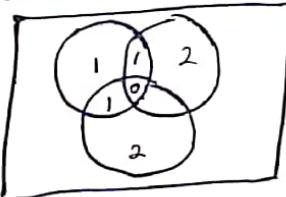
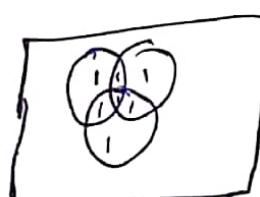
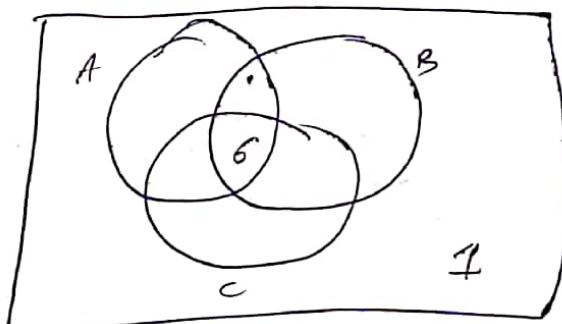
✗ D)

Q6. ✓ A)

✓ B)

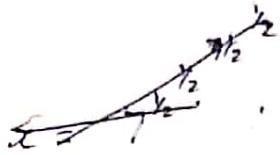
C)

✓ D)



7 cases

$$Q \quad x = \sqrt{7} \sqrt{7+ \sqrt{7+ \sqrt{7+ \sqrt{7+ \dots}}}}$$



$$x = \sqrt{7+x}$$

$$x^2 = 7x$$

$$\frac{x^2}{x} = 7$$

$$\boxed{x=7}, \quad \boxed{x=0} \quad \sqrt{7} \text{ is greater than } 0.$$

$$Q \quad x = \sqrt{6+\sqrt{6+\sqrt{6+\dots}}}$$

$$x = \sqrt{6+x}$$

$$x^2 = 6 + x$$

~~$$x^2 - 6 = x$$~~

~~$$x^2 - 6 = 0$$~~

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$x-3 = 0$$

$$x+2 = 0$$

$$\boxed{x=3} \checkmark$$

~~$$\boxed{x=-2} \checkmark$$~~

Q

$$\sqrt{\frac{1}{\sqrt{10}+\sqrt{9}}} + \sqrt{\frac{1}{\sqrt{11}+\sqrt{10}}} + \sqrt{\frac{1}{\sqrt{12}+\sqrt{11}}} = \sqrt{a\sqrt{b}-b}$$

~~$$\sqrt{11} + \sqrt{10} + \sqrt{10} + \sqrt{9}$$~~
~~$$\sqrt{10} + \sqrt{11} + \sqrt{10} + \sqrt{9} + \sqrt{11} + \sqrt{9} + \sqrt{10}$$~~

~~$$\frac{\sqrt{9} + 2\sqrt{10} + \sqrt{11}}{10 + \sqrt{11} + \sqrt{9} + \sqrt{11} + \sqrt{9} + \sqrt{10}}$$~~

~~$$\frac{3 + 2\sqrt{11}}{10 + 8}$$~~

$$\frac{\sqrt{10} - \sqrt{9}}{\cancel{\sqrt{10} - \sqrt{9} + \sqrt{11} - \sqrt{10} + \sqrt{10^2} - \sqrt{11}}} + \frac{\sqrt{11} - \sqrt{10}}{\cancel{\sqrt{10} - \sqrt{9} + \sqrt{11} - \sqrt{10} + \sqrt{10^2} - \sqrt{11}}} + \frac{\sqrt{12} - \sqrt{11}}{\cancel{\sqrt{10} - \sqrt{9} + \sqrt{11} - \sqrt{10} + \sqrt{10^2} - \sqrt{11}}}$$

$$\sqrt{12} - \sqrt{9}$$

$$\sqrt{12} - 3$$

$$2\sqrt{12} - 3 = a\sqrt{b} - b$$

$$b = 3$$

$$a = 2$$

~~$$2 + a + b$$~~

$$= 3 + 2$$

$$\boxed{= 5}$$

$$\textcircled{1} \quad \sqrt{x} + \frac{1}{\sqrt{x}} = 3$$

$$x^{\frac{3+1}{x^3}} = ?$$

$$\textcircled{2} \quad \sqrt{8+2\sqrt{15}} + \sqrt{8-2\sqrt{15}} = \sqrt{20}$$

$$\textcircled{3} \quad \sqrt{2x+3} - \sqrt{3x-5} = 1$$

$$\textcircled{4} \quad \sqrt{1988^{\sqrt{x}}} = (1988)^{\frac{1}{\sqrt{x}}}$$

Class 9 & 10 <sup>in</sup> polynomials, factorisation, formula (square cube)

remainder theorem

factor theorem

find & using factors

# SOS (Sum of Squares)

$$x^2 \geq 0$$

$$(x-a)^2 \geq 0$$

$$(x+a)^2 \geq 0$$

$$(x-a)^2 + (x+b)^2 + (x-c)^2 \cancel{+ (x+d)^2} \geq 0 \geq 0$$

$$\left\{ \begin{array}{l} (x-a)^2 + (y-b)^2 + (z+c)^2 = 0 \text{ if } \\ \quad x=a \\ \quad y=b \\ \quad z=-c \end{array} \right.$$

$$\sqrt{x} \geq 0$$

$$\sqrt{x-a} \geq 0$$

$$\sqrt{x+a} \geq 0$$

$$\sqrt{x-a} + \sqrt{y-b} + \sqrt{z+c} = 0$$

$$\text{if } x=a$$

$$y=b$$

$$z=-c$$

Q1. find  $x, y, z$  for

$$\textcircled{1} \quad \sqrt{3x-2} + \sqrt{y-2} = 6$$

$$\boxed{x = \frac{2}{3}, y = 2}$$

$$\textcircled{2} \quad \sqrt{x+2} + (y-3)^2 + (z+4)^2 = 6$$

$$\boxed{x = -2, y = 3, z = -4}$$

$$③ (P-2)^2 + (Q-100)^2 + (R-3)^2 = 0$$

$$\boxed{P=2 \\ Q=100 \\ R=3}$$

$$④ \sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}, x \in \mathbb{R}$$

$$(x+1) + (x-1) - 2\sqrt{(x+1)(x-1)}$$

$$2x - 2\sqrt{x^2-1} = 4x-1$$

$$2x - 4x + 1 = 2\sqrt{x^2-1}$$

$$\frac{-2x+1}{2} = \sqrt{x^2-1}$$

$$\frac{4x^2+1+4x}{4} = x^2-1$$

$$8x^2+1+4x = 4x^2-4$$

$$4x = -5$$

$$\boxed{x = \frac{-5}{4}}$$

100

$$(x+1) + (x-1) - 2\sqrt{x^2-1} = 4x-1$$

$$x + x + 1 - 2 - 2\sqrt{x^2-1} = 4x-1$$

$$\frac{-2x+1}{2} = \sqrt{x^2-1}$$

$$\frac{1-2x}{2} = \sqrt{x^2-1}$$

$$\frac{1+4x^2-4x}{4} = x^2-1$$

$$4x^2 - 4x + 1 = 4x^2 - 4$$

$$-4x = -4 - 1$$

$$-4x = -5$$

$$\boxed{x = \frac{5}{4}} \quad \text{Rejected}$$

no value of  $x$

H.W. 270-09-2024

Q1.  $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2} + 2x \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}$

$$(3)^2 = x + \frac{1}{x} + 2$$

$$(3)^2 = x + \frac{1}{x} + 2$$

$$9 - 2 = x + \frac{1}{x}$$

$$\boxed{x + \frac{1}{x} = 7}$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot x \cdot \frac{1}{x} \quad \left(x + \frac{1}{x}\right)$$

$$7^3 = x^3 + \frac{1}{x^3} + 3(7)$$

$$343 = x^3 + \frac{1}{x^3} + 21$$

$$343 - 21 = x^3 + \frac{1}{x^3}$$

$$\boxed{x^3 + \frac{1}{x^3} = 322}$$

$$\textcircled{2} \quad \sqrt{8+2\sqrt{15}} + \sqrt{\cancel{3+}8-2\sqrt{15}} - \sqrt{20}$$

$$\sqrt{\cancel{5+}3+2\sqrt{15}} + \sqrt{8-5+3-2\sqrt{15}} - \sqrt{20}$$

$$\sqrt{(\sqrt{5})^2 + (\sqrt{3})^2 + 2\sqrt{5}\sqrt{3}} + \sqrt{(\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}\sqrt{3}} - \sqrt{20}$$

$$\sqrt{(\sqrt{5}+\sqrt{3})^2} + \sqrt{(\sqrt{5}-\sqrt{3})^2} - \sqrt{20}$$

$$(\sqrt{5}+\sqrt{3}) + (\sqrt{5}-\sqrt{3}) - \sqrt{20}\sqrt{5}$$

$$\sqrt{5} + \cancel{\sqrt{3}} + \sqrt{5} - \cancel{\sqrt{3}} - 2\sqrt{5}$$

$$2\sqrt{5} - 2\sqrt{5}$$

$$\boxed{= 0}$$

$$\textcircled{3} \quad \sqrt{2x+3} - \sqrt{3x-5} = 1$$

$$\sqrt{2x+3} = 1 + \sqrt{3x-5}$$

$$2x+3 = 1 + 3x-5 + 2\sqrt{3x-5}$$

$$\frac{2x+3-1+5-3x}{2} = \sqrt{3x-5}$$

$$\frac{7-3x}{2} = \sqrt{3x-5}$$

$$\frac{49 + 9x^2 - 42x}{4} = 3x - 5$$

$$49 + 9x^2 - 42x = 12x - 20$$

$$9x^2 - 42x - 12x + 49 + 20 = 0$$

$$9x^2 - 54x + 69 = 0$$

$$3x^2 - 18x + \cancel{23} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+6 \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{18 \pm \sqrt{324 - 276}}{6}$$

$$= \frac{18 \pm \sqrt{72}}{6}$$

$$\boxed{x = \frac{18 \pm \sqrt{72}}{6}}$$

$$\begin{array}{r} & 18 \\ \sqrt{ } & 18 \\ \hline & 144 \\ & 180 \\ \hline & 324 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \\ 122 \\ \hline 72 \\ 1 \\ 13 \\ 5 \\ \hline 65 \\ 14 \\ 9 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 24 \\ + 24 \\ \hline 48 \end{array}$$

(104)

Q4.

KB

$$\sqrt{1988^{\frac{\sqrt{x}}{2}}} = 1988^{\frac{1}{\sqrt{x}}}$$

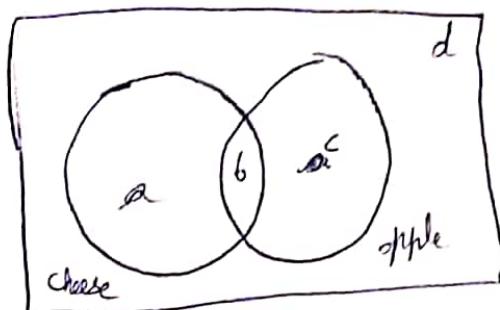
$$1988^{\frac{\sqrt{x}}{2}} = 1988^{\frac{1}{\sqrt{x}}}$$

$$\frac{\sqrt{x}}{2} = \frac{1}{\sqrt{x}}$$

$$\boxed{\sqrt{x} = 2}$$

Q-4

Q2.



$$a + b = 63$$

$$b + c = 76$$

$$a + b + c = 139$$

$$a + b + c + d = 100$$

~~$$a+b+c+d=100$$~~

$$b = 39 + d$$

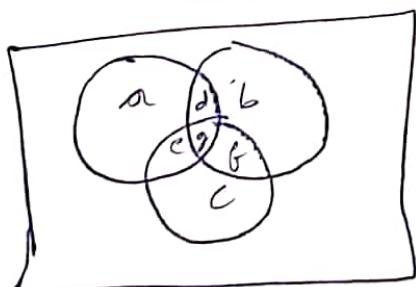
$$b = 39 \text{ (min)}$$

$$b = 63 \text{ (max)}$$

no. of values =  $\left[ \begin{matrix} 62 & 39 \\ 39 & 64 \end{matrix} \right]$

$$\begin{aligned} &= 64 - 39 + 1 \\ &= 25 \end{aligned}$$

Q38.



$$d + e + f = ?$$

$$e + f + c = 90$$

$$b + d + f = 120$$

$$a + d + e = 170$$

$$\begin{aligned} &\textcircled{1} + \textcircled{2} + \textcircled{3} \\ a + b + c + 2(d + e + f) &= 380 - \textcircled{1} \\ a + b + c + d + e + f + g &= 300 - \textcircled{2} \end{aligned}$$

4-5

$$\boxed{d + e + f = 80} + 30 = 110$$

106

(Q9. [1, 2000])

✓, ✓, 3, ✓, 5, ✓, 6, ✓, 7, ✓, 8, ✓, 9, ✓, 10, ✓, 11, ✓, 12, ✓, 13 (6)

✓, ✓, 15, ✓, 16, ✓, 17, ✓, 18, ✓, 19, ✓, 20, ✓, 21, ✓, 22, ✓, 23, ✓, 24, ✓, 25, ✓, 26 (6)

$$\frac{2000}{13} = 153 \times 6$$
$$= 918$$

✓1990  
✓1991  
✓1992  
✓1993  
✓1994  
1995  
1996  
✓1997  
✓1998 (6)  
✓1999  
✓2000

918 + 6  $\Rightarrow$  924  
Wrong in module

~~2000~~ 5  
13 x

16

$$\begin{array}{r} 13) 1989 \\ -13 \\ \hline 68 \\ -65 \\ \hline 39 \\ -39 \\ \hline 42 \\ -42 \\ \hline 55 \end{array}$$

(107)

factorisation, factor theorem & cyclic expression

$$\textcircled{1} \quad a^2 - b^2 = (a+b)(a-b)$$

$$\textcircled{2} \quad (a+b)^2 = a^2 + b^2 + 2ab$$

$$\textcircled{3} \quad (a-b)^2 = a^2 + b^2 - 2ab$$

~~∴~~

$$\textcircled{4} \quad \textcircled{2} - \textcircled{3}$$

$$(a+b)^2 - (a-b)^2 = 2ab + 2ab$$

$$\textcircled{5} \quad (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$\textcircled{6} \quad (a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

$$\textcircled{7} \quad a^3 + b^3 = a+b (a^2 + b^2 - ab)$$

$$\textcircled{8} \quad a^3 - b^3 = (a-b) (a^2 + b^2 + ab)$$

$$\textcircled{9} \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$$

↓

$$= a^2 + b^2 + c^2 + 2abc \left( \frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right)$$

$$\textcircled{10} \quad a^2 + b^2 + c^2 - ab - bc - ac$$

$$= \frac{1}{2} \left[ 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac \right]$$

$$= \frac{1}{2} \left[ a^2 + b^2 + c^2 + b^2 + c^2 + c^2 + a^2 - 2ab - 2bc - 2ac \right]$$

$$= \frac{1}{2} \left[ a^2 + b^2 - 2ab + a^2 + b^2 + c^2 - 2ac + b^2 + c^2 - 2bc \right]$$

$$= \frac{1}{2} [(a-b)^2 + (b-c)^2 + (a-c)^2]$$

⇒ Always greater or equal to zero.

$$(11) \quad a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$(12) \quad a^3 + b^3 + c^3 - 3abc = 0 \text{ if } (a+b+c) = 0$$

$$\text{or } \left[ \frac{1}{2} [(a-b)^2 + (b-c)^2 + (a-c)^2] \right] = 0$$

↓

$$\text{so, } a = b = c$$



$$\boxed{a^3 + b^3 + c^3 - 3abc = 0}$$

$$a = b = c$$

$$(a+b+c) = 0$$

$$(13) \quad a^4 - b^4 = (a^2)^2 - (b^2)^2$$

$$= (a^2 - b^2)(a^2 + b^2)$$

$$= (a-b)(a+b)(a^2 + b^2)$$

$$(14) \quad a^4 + 1 + a^2 = a^4 + 2a^2 + 1 + a^2 - 2a^2$$

$$= (a^2 + 1)^2 - a^2$$

$$= (a^2 + 1 + a)(a^2 + 1 - a)$$

Q find value of the following:-

①  $\frac{(x-y)^2}{(x-y)^3 + (y-z)^3 + (z-x)^3}$

$$\frac{(x-y)^3 + (y-z)^3 + (z-x)^3}{3[(x-y)(y-z)(z-x)]}$$

②  $\frac{(2.75)^3 - (2.25)^3}{(2.75)^2 + 2.75 \times 2.25 + (2.25)^2}$

③ if  $p = 2-\alpha$   
 $\alpha^3 + 6\alpha p + p^3 = 8$

④ DYS-2  
 Q1, 2, 3, 6, 4, 5  
 DYS-1 (all)

$$\textcircled{1} \quad A = x-y$$

$$B = y-z$$

$$C = z-x$$

$$A+B+C = (x-y) + (y-z) + (z-x)$$

$$= x-y+y-z+z-x$$

$$= 0$$

$$A^3 + B^3 + C^3 = 3ABC$$

$$(x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$$

Given Eqn

$$\frac{3(x-y)^3 + (y-z)^3 + (z-x)^3}{3(x-y)(y-z)(z-x)}$$

$$\frac{3(x-y)(y-z)(z-x)}{3(x-y)(y-z)(z-x)}$$

$$\boxed{=} 1$$

$$\textcircled{2} \quad \text{let } A = 2^{\circ} 75 \\ B = 2^{\circ} 25$$

Given :-

$$\frac{A^3 - B^3}{A^2 + AB + B^2}$$

$$= \frac{(A-B)(A^2 + B^2 + AB)}{A^2 + B^2 - AB}$$

$$= A - B$$

$$= 2.75 - 2.25$$

$$= 0.50$$

$$\boxed{= \frac{1}{2}}$$

$$Q3. \quad p = 2 - a$$

$$\text{find } a^3 + 6ab + p^3 - 8$$

$$p + a = 2$$

$$(p+a)^3 = (2)^3$$

$$p^3 + a^3 + 3pa(p+a) = 8$$

$$p^3 + a^3 - 8 = -3pa(p+a)$$

$$-3pa(p+a) + 6pa$$

$$+ 3^p(2 - (p+a))$$

$$+ 3(a-p-a)$$

$$\boxed{6 - 3p - 3a}$$

DYS-1

$$\text{Q6, iii)} \quad \frac{\sqrt{6+2\sqrt{3}+2\sqrt{2}+2\sqrt{6}} - 1}{\sqrt{5+2\sqrt{6}}}$$

$$\frac{\sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + (\sqrt{1})^2 + 2\sqrt{2}\sqrt{1} + 2\sqrt{3}\sqrt{1} + 2\sqrt{3}\sqrt{2}}} {\sqrt{(\sqrt{3} + \sqrt{2} + \sqrt{1})^2 - 1}}$$

$$\frac{\sqrt{3+\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$$

$$= 1$$

DYS-2

$$\text{Q6 i)} \quad 8a^6 + 5a^3 + 1$$

~~2a~~ ~~a~~

$$(2a^2)^3 + 6a^3 - a^3 + 1$$

$$(2a^2)^3 + (-a)^3 + (1)^3 + 6a^3$$

$$A^3 + B^3 + C^3 - 3ABC = (A+B+C)(a^2 + b^2 + c^2 + ab + bc + ac)$$

$$\boxed{(2a^2 - a + 1)(4a^4 + a^2 + 1 - 2a^3 - a + 2a^2)}$$

$$Q \text{ if } 4x^2 + 3y^2 + 16z^2 - 4x + 12y - 24z + 14 = 0 \\ x, y, z = ?$$

$$(2x)^2 + (3y)^2 + (4z)^2 - (2)(4x) +$$

$$(2x)^2 + (3y)^2 + (4z)^2 - 4x + 12y - 24z + 14 = 0$$

$$(2x)^2 + (3y)^2 + (4z)^2 - 4(2x + 3y + 6z) + 14 = 0$$

$$4x^2 - 4x + 9y^2 + 12y + 16z^2 - 24z + 14 = 0$$

$$(2x)^2 - (2)(2x) + 1^2 + (3y)^2 + (2)(3y)(2) + 2^2 + (4z)^2 - (2)(4z) + 3^2 = 0$$

$$(2x - 1)^2 + (3y + 2)^2 + (4z - 3)^2 = 0$$

$$2x - 1 = 0$$

$$2x = 1$$

$$\boxed{x = \frac{1}{2}}$$

$$3y + 2 = 0$$

$$3y = -2$$

$$\boxed{y = -\frac{2}{3}}$$

$$4z - 3 = 0$$

$$4z = 3$$

$$\boxed{z = \frac{3}{4}}$$

Factorisation:

Method :- Conversion into perfect square.

$$Q - x^4 + 3x^2y^2 + 4y^4$$

$$(x^2)^2 + (2y)^2 + 4x^2y^2 - x^2y^2$$

$$(x^2 + 2y^2)^2 - x^2y^2$$

$$(x^2 + 2y^2 + xy^2)(x^2 + 2y^2 - xy)$$

$$Q \quad 4x^4 + 81$$

$$(2x^2 - 9)^2 + 36x^2 - 36x^2$$

$$(2x^2 + 9)^2 - (6x)^2$$

$$\boxed{(2x^2 + 9 - 6x)(2x^2 + 9 + 6x)}$$

Method - 2 - Factor Theorem

$$6x^3 - 5x^2 - 3x + 2$$

$$\begin{array}{r} x=1 \\ 6 \end{array}$$

$$\cancel{-6} - 5 + 3 + 2$$

$$48 - 20 - 6 + 2$$

$$6 - 5 - 3 + 2$$

$$-8 + 8 = 0$$

$$(x-1)$$

$$6x^2 + 4x - 2$$

$$6x^2 + 4x - 3x - 2$$

$$2(3x+2) - 1(3x+2)$$

$$\cancel{2}(x-1)(3x+2)$$

$$\cancel{\text{or}} \quad \boxed{x=1} \quad \boxed{x = -2/3}$$

$$(2x-1)(3x+2)(x-1)$$

Method-3

$$6x^3 - 5x^2 - 3x + 2 = (x-1)(6x^2 + \cancel{6}x - 2)$$

$$\cancel{6x^2} \cdot 6x^2 - 6x^2 = -5x^2$$

$$6x^2 = \cancel{6x^2} - 5x^2$$

$$6x^2 = x^2$$

$b=1$

$$= (x-1)(6x^2 + x^2 - 2)$$

Q  $6x^3 + 11x^2 + 6x + 1$

$$-6 + 11 - 6 + 1$$

$$(x+1)(6x^2 + 6x + 1)$$

$$6x + 6x^2 = 11x^2$$

$$6x^2 = 5x^2$$

$$\underline{b=5}$$

$$(x+1)(6x^2 + 5x + 1)$$

$$\boxed{(x+1)(3x+1)(2x+1)}$$

Q Factorise

①  $x^3 + y^3 + z^3 - 6xyz$

②  $(x-y)^3 + (y-z)^3 + (z-x)^3$

$$\textcircled{2} \quad [3)(x-y)(y-z)(z-x)]$$

$$\textcircled{1} \quad (x+y+z)(x^2+y^2+z^2+xy+yz+xz)$$

$$(x)^3 + (y)^3 + (z)^3 + 3(xz+xy+yz) \\ A^3 + B^3 + C^3 - 3ABC = (A+B+C)(A^2+B^2+C^2 - AB - BC - AC)$$

$$[(x+y+z)(x^2+y^2+z^2-xy-yz-xz)]$$

### (3) Cyclic Expression & its factors

→ Expression will be unchanged when variables interchanged

Eg.  $x+y+z$  is cyclic  
 $\downarrow \downarrow \downarrow$   
 $y+z+x$

Eg 2  $x-y+z$  not cyclic  
 $\downarrow$   
 $y-z+x$

Eg 3.  $x^2+y^2+z^2+xy+yz+zx$  is cyclic

Eg 4.  $x(y-z)+y(z-x)+z(x-y)$

$y(z-x)+z(x-y)+x(y-z)$  is cyclic

Note - In cyclic Expression if  $(x-y)$  is a factor then  $(y-z)$  and  $(z-x)$  are also factors.

$$Q \quad x^2(y-z) + y^2(z-x) + z^2(x-y)$$

$$x^2(z-x) + x^2(z-x) + z^2(x-y)$$

$x^2(z-x) + z^2(x-y) = 0$  so  $(x-y)$  is a factor and it is cyclic so  $\boxed{(x-y)(y-z)(z-x)}$  are also factors.

~~$$Q \quad 2xyz + x^2y + y^2z + z^2x + xy^2 + y^2z + z^2x$$

$$2xyz + x^2(y+z) + y^2(z+x) + z^2(x+y) \text{ cyclic}$$

$$x=y$$~~

~~Ex~~  
~~Ex~~  $x = -y$  Eqn is cyclic  
~~Ex~~  $(x+y)$  is factor  
~~Ex~~  $(y+z)$  is a factor as well  
 $(z+x)$  is a factor as well

~~DYS~~ H. W.

30-04-2024

DYS-2  
Q8 - i), ii), iii), iv), v), vi), vii), viii)

DYS-3

Q1, 2, 3, 4, 5, 7

$$Q = 2xyz + x^2y + y^2z + z^2x + xyz^2 + yz^2 + zx^2$$

It is cyclic

$$x = -y, \text{ value} = 0$$

$(x+y)$  is a factor

so  $(y+z)$  and  $(z+x)$  are also factors

$$\begin{aligned} E(x, y, z) &= 2xyz + x^2y + y^2z + z^2x + xyz^2 + yz^2 + zx^2 \\ &= A(x+y)(y+z)(z+x) \\ &\quad \downarrow \\ &\text{find } A. \\ &(A=1) \end{aligned}$$

Q6.

$$\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$$

$$a = 2+\sqrt{5} \quad b^3 = 2\theta - \sqrt{5}$$

$$(a^3)^{\frac{1}{3}} + (b^3)^{\frac{1}{3}} = a + b = t$$

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$t^3 = 4 + 3(4-5)(t)$$

$$t^3 = 4 - 3t$$

$$t^3 + 3t - 4 = 0$$

$$(t-1)$$

$$\begin{array}{r} \cancel{t^2+t+4} \\ \underline{-t^2-3t-4} \\ \hline \cancel{t^2} \end{array}$$

$$\begin{array}{r} -t^2+3t-4 \\ \underline{-t} \\ \hline 2t-4 \end{array}$$

$$t^2 - t + 2$$

$$t^2 + 2t - t + 2$$

$$t(t+2) - 1(t+2)$$

$$(t-1)(t+2)$$

$$\boxed{t=1, -2}$$

$$Q. \sqrt{2024 \times 2022 \times 2020 \times 2018 + 16}$$

$$\cancel{\sqrt{253 \times 1011 \times 2020 \times 2018 + 1}}$$

let  $x = 2021$

$$\sqrt{(x+3)(x+1)(x-1)(x-3) + 16}$$

$$\sqrt{(x^2 - 89)(x^2 - 1) + 16}$$

$$\sqrt{x^4 - 10x^2 + 9 + 16}$$

$$\sqrt{x^4 - 10x^2 + 25}$$

$$\sqrt{(x^2 - 5)^2}$$

$$x^2 - 5$$

$$2021^2 - 5$$

$$\boxed{4462446}$$

$$\begin{array}{r} 2021 \\ 2021 \\ \hline 2021 \\ 40420 \\ \hline 4042000 \\ 4441 \\ \hline 4962 \\ 4462441 \end{array}$$

$$Q2. (2+1)(2^2+1)(2^4+1)(2^8+1) - 2^{16}$$

$$\begin{aligned} & \cancel{(2+1)}(2-1) = \cancel{(2+1)}(2-1)(2^2+1)(2^4+1)(2^8+1) - 2^{16} \\ & \cancel{(2+1)} \end{aligned}$$

$$(2^2-1)(2^2+1)$$

$$(2^4-1)(2^4+1)$$

$$(2^8-1)(2^8+1)$$

$$(2^{16}-1) - 2^{16}$$

$$\boxed{-1}$$

Polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

$a_n \neq 0$  ( $a_n \rightarrow$  leading coefficient)

$n \in \text{whole nos.}$

$$a_n, a_{n-1}, a_{n-2}, \dots, a_0$$

$\not\rightarrow a_n = 1$  (monic polynomial)

monic polynomial  $\rightarrow$  coefficient is 1

monomial  $\rightarrow$  one term.

## Types of Polynomial

Name	Degree	Format
①. zero polynomial	Not-Defined	$f(x) = 0$ $\begin{aligned} f(x) &= 0x^0 \\ &= 0x^2 \\ &= 0x^3 \end{aligned} \quad \boxed{0}$
②. Non-zero polynomial	0	$f(x) = c \quad (c \neq 0)$
③. Linear	1	$f(x) = ax + b \quad (a \neq 0)$
④ Quadratic	2	$f(x) = ax^2 + bx + c \quad (a \neq 0)$
⑤ Cubic	3	$f(x) = ax^3 + bx^2 + cx + d \quad (a \neq 0)$
⑥ Bi-quadratic	4	$f(x) = ax^4 + bx^3 + cx^2 + dx + e$ $f(x) = ax^4 \quad (a \neq 0)$

\* Adding two polynomials may or not result in change in degree

$$(x^8 + x^7) + (x^8 - x^6) = 2x^8 + x^7 - x^6$$

$$(x^8 + x^7) - (x^8 - x^6) = x^7 + x^6 \text{ (change)}$$

~~total~~

Division in polynomial

$$P(x) = Q(x) \cdot d(x) + R(x)$$

Dividend      Quotient      Divisor      Remainder

- ① Degree of  $d(x) > \text{degree } r(x)$
- ② Degree of  $d(x) \leq P(x)$
- ③  $Q(x)$  &  $r(x)$  are unique
- ④ If  $d(x)$  is divisor of  $P(x)$  then  $K \cdot d(x)$  also be the divisor of  $P(x)$ . ( $K \neq 0$ )

Remainder Theorem

$$P(x) = Q(x)(ax+b) + R$$

$$ax+b=0$$

$$x = -\frac{b}{a}$$

$$\boxed{P(b/a) = R}$$

$$P\left(\frac{b}{a}\right) = Q\left(\frac{b}{a}\right)\left(ax\frac{b}{a} - b\right) + R$$

$$\boxed{P\left(\frac{b}{a}\right) = 0 + R}$$

Q1. find remainder if  $p(x) = x^3 - 6x^2 + 11x - 6$  is divided by

- (1)  $x$
- (2)  $2x - 4$
- (3)  $x - 4$
- (4)  $x^2 - 6x + 11$

(1)  $x = 0$

$$p(0) = 0 - 0 + 0 - 6$$

$$= -6$$

(2)  $2x - 4 = 0$

$$x = \frac{4}{2}$$

$$x = 2$$

$$\begin{aligned} p(2) &= 2^3 - 6 \times 2^2 + 11(2) - 6 \\ &= 8 - 24 + 22 - 6 \end{aligned}$$

$$= 0$$

(3)  $x - 4 = 0$

$$x = 4$$

$$64 - 96 + 44 - 6$$

$$108 - 102$$

$$6$$

(4)  $x^2 - 6x + 11 \neq 0$

$$\begin{aligned} 6 &+ \cancel{36 - 44} \\ 12x^2 - 11x &+ 12 + 72x \\ x^2 + 12x + 72x &- 44 \end{aligned}$$

(12)

$$\textcircled{1} \quad \begin{array}{r} x^2 - 6x + 11 \\ \cancel{x^3 - 6x^2 + 11x - 6} \\ \hline -x^3 + 6x^2 - 11x \\ \hline -6 \end{array}$$

Q2

$$x^2 - 6x + 11 = 0$$

$$x^3 - 6x^2 + 11x - 6$$

$$x(x^2 - 6x + 11) - 6$$

$$x(0) - 6$$

$$0 - 6$$

$$\boxed{-6}$$

Q2. find the constants.  $a, b$  &  $c$  such that

$$(2x^2 + 3x + 7)(ax^2 + bx + c) = 2x^4 + 11x^3 + 9x^2 + 13x - 35$$

$$\begin{array}{r} 2x^2 + 3x + 7 \\ \cancel{2x^4 + 11x^3 + 9x^2 + 13x - 35} \\ \hline - 2x^4 - 3x^3 - 7x^2 \\ \hline 8x^3 + 2x^2 + 13x - 35 \\ - 8x^3 - 12x^2 - 28x \\ \hline 14x^2 + 24x - 50 \end{array}$$

$$\begin{array}{r} 14x^2 + 41x - 25 \\ - 10x^2 - 15x - 35 \\ \hline + 10x + 15x + 35 \\ \hline 0 \end{array}$$

$$a = 1$$

$$b = +4$$

$$c = \cancel{+7} - 5$$

$$2x^4 = 2x^2 \times ax^2$$

$$2x^4 = 2ax^4$$

$$\boxed{2 \cdot b = 1}$$

$$2b + 3a = 11 \quad (\text{for } x^3)$$

$$2b + 3 = 11$$

$$\boxed{\begin{array}{l} 2b = 8 \\ b = 4 \end{array}}$$

~~$$3c + 7b = 9 \quad (\text{for } x^2)$$~~

$$3c + 2 \cancel{b} = 9$$

$$3c = 9 - 2 \cancel{b}$$

$$c = \cancel{9} - \frac{19}{3}$$

$$2c + 7a + 3b = 9$$

$$2c + 7 + 12 = 9$$

$$c = \frac{9 - 19}{2}$$

$$c = -\frac{10}{2}$$

$$\boxed{c = -5}$$

Q3. find remainder when  $P(x) = x^5 - 3x^3 + 2x^2 + 3x + 1$  is divided by  $x^2 - 1$

$$\begin{array}{r} & & & -2x \\ & & & \hline & & x^4 + 3 \\ & & & \end{array}$$

$$x^3 - 2x + 2$$

$$\boxed{R = x + 3}$$

~~M.E~~

$$\cancel{x^5 - 3x^3 + 2x^2 + 3x + 1}$$

$$\cancel{x^5}$$

$$x^2 \quad x^5$$

$$P(x) = Q(x)(x^2 - 1) + (ax + b)$$

$$\text{for } x = 1 \quad (x^2 - 1 = 0)$$

$$P(1) = 4$$

$$P(1) = 0 + (a + b)$$

$$(a + b) = 4 \quad \text{--- I}$$

$$\cancel{P(-1)} = 0 + (-a + b)$$

$$2 = b - a \quad \text{--- II}$$

I + II

$$2b = 6$$

$$\underline{b = 3}$$

$$a = 4 - b$$

$$a = 4 - 3$$

$$\underline{a = 1}$$

$$a x + b$$

$$\boxed{x + 3}$$

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Q4.  $(x-2)$  is factor of  $x^5 - 4x^3 + x + k$

$$x-2=0$$

$$x=2$$

$$32 - 32 + 2 + k = 0$$

$$\boxed{k = -2}$$

Q5.  $ax^3 + bx^2 + cx - 6$  has  $(x-1)$ ,  $(x-2)$  &  $(x-3)$  as factors.  
find  $a$ ,  $b$  &  $c$

$$x=1$$

$$a+b+c-6 = 0$$

$$\boxed{a+b+c=6}$$

$$a = \cancel{6} - b - c$$

$$x=2$$

$$8a + 4b + 2c - 6 = 0$$

$$\boxed{4a + 2b + c = 3}$$

$$x=3$$

$$27a + 9b + 3c - 6 = 0$$

$$\boxed{9a + 3b + c = 2}$$

$$9(6-b-c) + 36 + \frac{21-26}{3} = c$$

$$54 - 9b - 9c + 36 + \frac{21-26}{3} = 83$$

$$-6b$$

$$4(6-b-c) + 2b + c = 3$$

$$24 - 4b - 4c + 2b + c = 3$$

$$21 = +2b + 3c$$

$$\frac{21-21}{3} = c$$

M. W.

DYS-2'4 (Q1, 2, 3, 4, 5, 6, 7, 8)

DYS-3 (Q8)

$$Q5. \quad -a + b + c = -6$$

~~a~~

$$4a + 2b + c = 3$$

$$\underline{3a + b = -3}$$

$$-9a + 3b = +9$$

$$9a + 3b + c = 2$$

$$\boxed{c = 11}$$

$$a + b + 11 = 6$$

$$a + b = -5$$

$$a = -5 - b$$

$$-15 - 3b + b = -3$$

$$-15 - 2b = -3$$

$$-12 = 2b$$

$$\boxed{b = -6}$$

$$a = -5 - b$$

$$a = -5 + 6$$

$$\boxed{a = 1}$$

Q5.

Method II

$$\begin{aligned} ax^3 + bx^2 + cx - 6 &= (x-1)(x-2)(x-3) \\ &= (x^2 - 2x - x + 2)(x-3) \\ &= (x^2 - 3x + 2)(x-3) \\ &= x^3 - 3x^2 + 2x - 3x^2 + 9x - 6x \\ &= x^3 - 6x^2 + 11x - 6x \end{aligned}$$

$$\boxed{\begin{array}{l} a = 1 \quad [x^3] \\ b = -6 \quad [x^2] \\ c = 11 \quad [x] \end{array}}$$

Q If  $f(x)$  is a 4 degree polynomial having leading coefficient 1 such that

$$f(1) = 1$$

$$f(2) = 4$$

$$f(3) = 9$$

$$f(4) = 16$$

then find the value of  $f(5) = 25$

$$f(x) \equiv x^2$$

$$\cancel{x} \quad f(x) - x^2 = (x-1)(x-2)(x-3)(x-4)$$

$$x = 5$$

$$\begin{aligned} f(5) - 25 &= 4 \times 3 \times 2 \times 1 \\ &= 24 + 25 \end{aligned}$$

$$f(5) = 49$$

DYS-4

Q10

$$f(1) = 1$$

$$f(2) = 2$$

$$f(3) = 3$$

$$f(4) = 4$$

$$f(0) = 1$$

$$f(x) - x = A(x-1)(x-2)(x-3)(x-4)$$

$$f(0) - 0 = A(-1)(-2)(-3)(-4)$$

$$\frac{1}{A} = 24$$

$$\boxed{A = \frac{1}{24}}$$

$$f(s) - s = \frac{1}{24}(x-1)(x-2)(x-3)$$

$$\boxed{\cancel{f(s) = 0}} \quad \frac{1}{24}x^4 - 1$$

$$f(s) - s = 1$$

$$f(s) = 1 + s$$

$$\boxed{f(s) = 6}$$

$$Q9. \quad f(x) = (x-1)(x+2) \quad (\cancel{x+1})$$

$$f(x) - \cancel{4} = (x+1) + 4$$

$$f(x) - \cancel{28} = (x-2) + 28$$

$$\cancel{(x^2 + 2x - x - 2)} \quad (x + 0)(5)$$

$$\cancel{(x^2 + x - 2)(x + 5)}$$

$$\cancel{x^3 + 5x^2} + x^2 + 5x - 2x - 10$$

$$A(x^3 + \cancel{5x^2} + 3x - 10) =$$

$$A(8 + 24 + 6 - 10) = 28$$

$$A(2\beta) = 2\beta$$

$$A = 1$$

$$x^3 + 6x^2 + 3x - 10$$

DYS-4

$$Q9. \quad f(x) = (x-1)(x+2)(ax+b)$$

$$x = -1$$

$$y = (-2)(1)(b-a)$$

$$b-a=2$$

$$28 = (1)(4)(2a+6)$$

$$7 = 2a + b$$

$$z = b - a$$

$$F_2 = -6 \rightarrow \sigma$$

$$9\theta = 3\alpha$$

$$a = 3$$

$$\sqrt{b} = 5$$

$$(x-1)(x+2)(3x+5)$$

$$(x^2+x-2)(3x+5)$$

## Equations Reducible to quadratic

$a(\sqrt{st})^2 + b(\sqrt{st}) + c = 0$   
 $\sqrt{st} \rightarrow x^{\frac{2}{5}}, x^{\frac{1}{5}}, (\sqrt{s} + \sqrt{t})^x, x^2 + \frac{1}{x^2}, (x + \frac{1}{x})^2, 2^x$  etc

Ex. 1.  $x^{\frac{2}{5}} + x^{\frac{1}{5}} + 2 = 0$

$$1(x^{\frac{1}{5}})^2 + 1(2x)^{\frac{1}{5}} + 2$$

$$t = x^{\frac{1}{5}}$$

$$\begin{array}{c} t^2 + t + 2 \\ \cancel{t^2+2} \end{array}$$

Ex. 2.  $4^x + (2^2)^x - 7 = 0$  multiply

Ex. 3.  $(\sqrt{5} + \sqrt{3})^x + (\sqrt{5} - \sqrt{3})^x - 8 = 0$

- Method → 1. Assume  $\sqrt{st} = t$  such that first term have square, second term is linear and 3rd term is constant,  
 2. solve the quadratic in  $t$   
 3. get values of small  $t$  & replace  $t$  by  $\sqrt{st}$

$$Q1. \quad 5^{2x} - 6x5^{x+1} + 125 = 0$$

$$(5^x)^2 - 30(5^x) + 125 = 0$$

$$t = 5^x$$

$$t^2 - 30t + 125$$

$$t^2 - 25t - 5t + 125$$

$$t(t-25) - 5(t-25)$$

$$t = 5$$

$$5 = 5^x$$

$$\boxed{x=1}$$

$$t = 25$$

$$\cancel{5} 25 = 5^x$$

$$\boxed{\sqrt{x}=2}$$

$$Q2. \quad x^{2/3} + x^{1/3} - 2 = 0$$

$$(\sqrt[3]{x})^2 + 2(\sqrt[3]{x}) - \sqrt[3]{x} - 2 = 0$$

$$\sqrt[3]{x}(\sqrt[3]{x} + 2) - 1(\sqrt[3]{x} + 2)$$

$$(\sqrt[3]{x} - 1)(\sqrt[3]{x} + 2)$$

$$\sqrt[3]{x} = 1$$

$$\boxed{x=1}$$

$$\sqrt[3]{x} = -2$$

$$\boxed{x=-8}$$

$$Q3. \quad 4^x + 3 \cdot 2^{x+3} + 128 = 0$$

$$9(2^x)^2 + 24(2^x) + 128 = 0$$

$$(2^x)^2 + 16(2^x) + 8(2^x) + 128 = 0$$

$$2^x(2^x+16) + 8(2^x+16)$$

$$2^x + 8 = 0$$

$$2^x = -8$$

$$\begin{array}{l} 2^x = -8 \\ \cancel{2^x = -8} \\ \boxed{x = -3} \\ \text{not possible} \end{array}$$

$$2^x + 16 = 0$$

$$\cancel{2^x = -16}$$

not possible

$$Q4. \quad 4^x - 3 \cdot 2^{x+3} + 128$$

$$2^x = 8$$

$$\boxed{x = 3}$$

$$2^x = 16$$

$$\boxed{x = 4}$$

M.W. 3-5-24

DYS-5

(Q2,3), Q5, Q7, (Q15-f)

DYS-4

(Q8,12)

$$Q \quad 3 \cdot 4^x + 2 \cdot 9^x - 5 \cdot 6^x = 0$$

$$3 \cdot 2^{2x} + 2 \cdot 3^{2x} - 5 \cdot 2^x 3^x$$

$$\frac{3}{2}^x = \frac{3}{2}^x$$

$$\boxed{x=1}$$

$$\left(\frac{3}{2}\right)^x = 1$$

$$x=0$$

~~$$\frac{3 \cdot 2^{2x}}{3^x} + \frac{2 \cdot 3^{2x}}{3^x} - \frac{5 \cdot 2^x 3^x}{3^x}$$~~

~~$$\frac{3 \cdot 2^{2x}}{2^{2x}} + \frac{2 \cdot 3^{2x}}{2^{2x}} - \frac{5 \cdot 2^x 3^x}{2^{2x}}$$~~

~~3<sup>2x</sup>~~

$$\frac{3 \cdot 2^{2x}}{2^{2x}} + \frac{2 \cdot 3^{2x}}{2^{2x}} - \frac{5 \cdot 2^x 3^x}{2^{2x}}$$

$$3 + 2 \cdot \left(\frac{3}{2}\right)^{2x} - 5 \times \frac{7}{2} \left(\frac{3}{2}\right)^{2x}$$

$$t = \left(\frac{3}{2}\right)^x$$

$$2t^2 - 5t + 3$$

$$5 \pm \sqrt{24 - 24}$$

y

$$\frac{5+1}{4}$$

$$\frac{5-1}{4}$$

$$\boxed{t = \frac{3}{2}}$$

$$\frac{5-1}{4}$$

$$\boxed{t = 1}$$

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Note -

$$\left( x + \frac{1}{x} \right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\therefore x + \frac{1}{x} = t$$

$$t^2 = x^2 + \frac{1}{x^2} + 2$$

$$t^2 - 2 = x^2 + \frac{1}{x^2}$$

$$x + \frac{1}{x} = t$$

$$x - \frac{1}{x} = t$$

$$x^2 + \frac{1}{x^2} = t^2 - 2$$

$$x^2 + \frac{1}{x^2} = t^2 + 2$$

Q1.  $3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$

$$3(t^2 - 2) - 16(t) + 26 = 0$$

$$3t^2 - 6 - 16t + 26 = 0$$

$$3t^2 - 16t + 20 = 0$$

$$3t^2 - 10t - 10t + 20 = 0$$

$$3t(t-2) - 10(t-2)$$

$$(3t-10)(t-2)$$
$$\boxed{t=2}$$
$$\boxed{t=\frac{10}{3}}$$

$$x + \frac{1}{x} = 2t$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$x + \frac{1}{x} = \frac{10}{3}$$

$$x^2 + 1 = \frac{100}{9}$$

$$3x^2 - 10x + 3 = 0$$

$$x^2 - x - x + 1$$

$$x(x-1) - 1(x-1)$$

$$\boxed{x=1}$$

$$\frac{10 \pm \cancel{\sqrt{16+3}}}{5} 8$$

$$\frac{2}{6} = \frac{1}{3}$$

$$\boxed{x = \frac{1}{3}}$$

$$\boxed{x = 3}$$

DYS-~~ds~~

$$\textcircled{6} \quad 2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 4 = 0$$

$$2(t^2 + 2) - 3t - 4 = 0$$

$$2t^2 + 4 - 3t - 4 = 0$$

$$2t^2 - 3t = 0$$

$$t(2t - 3) = 0$$

$$t = 0$$

$$\cancel{t = \frac{3}{2}}$$

$$x - \frac{1}{x} = t$$

$$x - \frac{1}{x} = \frac{3}{2}$$

$$x^2 - 1 = \frac{3x}{2}$$

$$2x^2 - 3x - 2 = 0$$

$$\frac{3 \pm \sqrt{9+16}}{4}$$

$$\frac{3+5}{4}$$

$$\boxed{2}$$

$$\frac{3-5}{4}$$

$$\boxed{-\frac{1}{2}}$$

$$\begin{aligned} t &= 0 \\ \frac{x+1}{x} &= 0 \end{aligned}$$

$$x^2 - 1 = 0$$

$$x^2 = +1$$

$$\boxed{x = \pm 1}$$

$$Q \quad 2^{2x+1} - 7 \cdot 10^x + 5^{2x+1} = 0$$

$$2 \cdot 2^{2x} - 7 \cdot 2^x \cdot 5^x + 5 \cdot 5^{2x}$$

$$\frac{2 \cdot 2^{2x}}{5^{2x}} - \frac{7 \cdot 2^x \cdot 5^x}{5^{2x}} + \frac{5}{5^{2x}}$$

$$2 \cdot \left(\frac{2}{5}\right)^{2x} - 7 \cdot \left(\frac{2}{5}\right)^x + 5$$

$$t = \left(\frac{2}{5}\right)^x$$

$$2t^2 - 7t + 5$$
$$\frac{7 \pm \sqrt{49}}{4}$$

$$\frac{10}{4}$$

$$\frac{4}{4}$$

$$t = \frac{5}{2}$$
$$\left(\frac{2}{5}\right)^x = \frac{5}{2}$$

$$\boxed{x = -1} \checkmark$$

$$t = 1$$
$$\left(\frac{2}{5}\right)^2 = 1$$

$$\left(\frac{2}{5}\right)^x = \left(\frac{2}{5}\right)^0$$
$$\boxed{x = 0} \checkmark$$

Q15. F

$$(5^{2x} - 7^x) - 35(5^{2x} - 7^x) = 0$$

$$(5^{2x} - 7^x)(-34) = 0$$

$$5^{2x} = 7^x$$

$$25^x = 7^x$$

$$A \neq B$$

$$\not A \neq B$$

$$\text{Power} = \boxed{x=0}$$

Q  $(5+2\sqrt{6})^{\frac{x}{2}} + (5-2\sqrt{6})^{\frac{x}{2}} = 10$

$$\cancel{5-2\sqrt{6}} = 5-2\sqrt{6} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}} = \frac{25-4\cancel{6}}{5+2\sqrt{6}} = \frac{1}{5+2\sqrt{6}}$$

$$(5+2\sqrt{6})^{\frac{x}{2}} + \frac{1}{5+2\sqrt{6}}^{\frac{x}{2}} = 10$$

$$t = (5+2\sqrt{6})^{\frac{x}{2}}$$

$$t + \frac{1}{t} = 10$$

$$t^2 + 1 = 10t$$

$$t^2 - 10t + 1$$

$$t = \frac{10 \pm \sqrt{100-4}}{2}$$

$$= 5 \pm \sqrt{24}$$

$$t = 5 + 2\sqrt{6}$$

$$t = 5 - 2\sqrt{6}$$

$$(5+2\sqrt{6})^{\frac{x}{2}} = (5+2\sqrt{6})^1 \quad (5+2\sqrt{6})^{\frac{x}{2}} = \cancel{(5+2\sqrt{6})^1}$$

$$\frac{2t}{2} = 1$$

$$\boxed{x=2}$$

$$5+2\sqrt{6}$$

$$\frac{2t}{2} = -1$$

$$\boxed{x = -2}$$

(142)

~~DYS~~ H.W.

DYS-5 (Q180), Q12, Q13, Q8  
DYS-2 (Q7)

DYS-5

Q9  $(\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^{-x} - 2\sqrt{3} = 0$

$$\frac{\sqrt{3} - \sqrt{2} \times \sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{3 - 2}{\sqrt{3} + \sqrt{2}} = \frac{1}{\sqrt{3} + \sqrt{2}}$$

$$t = (\sqrt{3} + \sqrt{2})^x$$

$$t + \frac{1}{t} - 2\sqrt{3} = 0$$

$$t^2 + 1 = 2\sqrt{3}t$$

$$t^2 - 2\sqrt{3}t + 1$$

$$t = \frac{2\sqrt{3} \pm \sqrt{12 - 4}}{2}$$

$$= \frac{2\sqrt{3} \pm 2\sqrt{2}}{2}$$

$$= \sqrt{3} \pm \sqrt{2}$$

$$\sqrt{3} + \sqrt{2} \stackrel{x}{=} \sqrt{3} + \sqrt{2}$$

$$x = 1$$

$$\boxed{x = \pm 1}$$

$$(\sqrt{3} + \sqrt{2})^x = \sqrt{3} - \sqrt{2} = \frac{1}{\sqrt{3} + \sqrt{2}}$$

$$\boxed{x = -1}$$

$$Q \quad x \underbrace{(x+1)(x+2)(x+3)}_{(x^2+3x+2)} - 8 = 0$$

$$x(x+3)(x+1)(\cancel{x+2}) = 8$$

$$(x^2+3x)(x^2+3x+2) = 8$$

$$x^2 + 3x = t$$

$$t(t+2) = 8$$

$$t^2 + 2t - 8 = 0$$

$$D = \frac{-2 \pm \sqrt{4+32}}{2}$$

$$= \frac{-2 \pm 6}{2}$$

$$= \frac{-9}{2}, \frac{4}{2}$$

$$\underline{t = -4, 2}$$

$$x^2 + 3x = -4$$

$$x^2 + 3x + 4 = 0$$

$$x = \frac{-3 \pm \sqrt{9-16}}{2}$$

Not Possible as  $\sqrt{is \text{ negative}}$

$$\left. \begin{aligned} & x^2 + 3x - 2 \\ & x = \frac{-3 \pm \sqrt{9+8}}{2} \\ & = \frac{-3 \pm \sqrt{17}}{2} \end{aligned} \right\}$$

$$x = \boxed{\frac{-3 \pm \sqrt{17}}{2}}$$

$$\text{Q} \text{ no } 11 \quad x(x+1)(x+2)(x+3) = 24$$

$$t = x^2 + 3x$$

$$t(t+2) = 24$$

$$t^2 + 2t - 24 = 0$$

$$t = \frac{-2 \pm \sqrt{4+96}}{2}$$

$$= \frac{-2 \pm 10}{2}$$

$$= -6, 4$$

$$t = -6$$

$$x^2 + 3x + 6 = 0$$

$$x = \frac{-3 \pm \sqrt{9-24}}{2}$$

$$x$$

$$\left. \begin{array}{l} x^2 + 3x - 4 = 0 \\ x = \frac{-3 \pm \sqrt{9+16}}{2} \\ x = \frac{-3 \pm 5}{2} \\ x = -4, 1 \end{array} \right\}$$

$$\text{Q} \quad (x+1)(x+2)(x+3)(x+6) = 3x^2$$

$$(x^2 + 4x + 3)(x^2 + 8x + 12)$$

$$(x^2 + 7x + 6)(x^2 + 5x + 6) = 3x^2$$

~~$$(x^2 + x^2 + 6)(x + 7x) = 3x^2$$~~

$$\left( \frac{x^2 + 7x + 6}{x} \right) \left( \frac{x^2 + 5x + 6}{x} \right) = 3$$

$$x^2 + 6 \cancel{x} \Rightarrow 7$$

$$\left( x^2 + 7 + \frac{6}{x} \right) \left( x^2 + 5 + \frac{6}{x} \right) = 3$$

$$t = x + \frac{6}{x}$$

$$(t+7)(t+5) = 3$$

$$t^2 + 12t + 35 - 3 = 0$$

$$t^2 + 12t + 32 = 0$$

$$t = -12 \pm \frac{\sqrt{144 - 128}}{2}$$

$$= \frac{-12 \pm 4}{2}$$

$$= -8, -4$$

$$-8 = x + \frac{6}{x}$$

$$-8x = x^2 + 6$$

$$x^2 + 8x + 6 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{2}$$

$$= -8 \pm \sqrt{40}$$

$$x = -4 \pm \sqrt{10}$$

3)

$$\begin{aligned} -4 &= x^2 + 6 \\ x^2 + 4x + 6 & \\ x &= \frac{-4 \pm \sqrt{16 - 24}}{2} \end{aligned}$$

$$Q \quad (x+2)(x+3)(x+8)(x+12) = 4x^2$$

$$(x^2 + 14x + 24) / (x^2 + 11x + 24) = 4x^2$$

$$(x+14+24/x)(x+11+24/x)$$

$$t = x + \frac{24}{x}$$

$$(t+14)(t+11) = 4$$

$$t^2 + 25t + 154 = 4$$

$$t^2 + 25t + 150 = 0$$

$$t = -25 \pm \sqrt{625 - 600} \over 2$$

$$= -25 \pm 5 \over 2$$

$$= -15, -10$$

$$\textcircled{B} \quad x^2 + 24 = -15x$$

$$x^2 + 15x + 24 = 0$$

$$x = -15 \pm \sqrt{225 - 96} \over 2$$

$$x = -15 \pm \sqrt{129} \over 2$$

$$x^2 + 24 = -10x$$

$$x^2 + 10x + 24 = 0$$

$$x = -10 \pm \sqrt{100 - 96} \over 2$$

$$= -10 \pm 2 \over 2$$

$$= -4, -6$$

$$Q) x^4 - 2x^3 + 3x^2 - 2x = 0$$

$$\cancel{x(x^3 - 2x^2 + 3x - 2)} = 0$$

$$\boxed{x=0}$$

~~$$Q) x^4 - 2x^3 + 3x^2 - 2x + 1 = 0$$~~

$$Q) x^4 - 2x^3 + 3x^2 - 2x + 1 = 0$$

$$x^4 + 1 - 2(x^3 + x^2) + 3x^2 = 0$$

$$\frac{x^4 + 1}{x^2} - 2\left(\frac{x^3 + x^2}{x^2}\right) + \frac{3x^2}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2\left(x + \frac{1}{x}\right) + 3 = 0$$

$$x + \frac{1}{x} = t$$

$$(t^2 - 2) - 2t + 3 = 0$$

$$t^2 - 2t + 1 = 0$$

$$t = \frac{2 \pm \sqrt{4-0}}{2}$$

$$= 1$$

$$x^2 + 1 = x$$

$$x^2 - x + 1 = 0$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

Not Possible

DYS-5

Q14  $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

$$\frac{x^4+1}{x^2} - 2 \left( \frac{x^3-x}{x^2} \right) - 2 \frac{x^2}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2 \left( x - \frac{1}{x} \right) - 2 = 0$$

$$(t^2 + 2) - 2t - 2 = 0$$

$$t^2 - 2t = 0$$

$$t(t-2) = 0$$

$$t = 2$$

$$x^2 - 1 = 2x$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

$$t = 0$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\boxed{x = \pm 1}$$

Q  $(x-a)^4 + (x-b)^4 = c$

$$t = \frac{x-a+x-b}{2}$$

$$2t = 2x - (a+b)$$

$$2t + (a+b) = 2x$$

$$\boxed{x = t + \frac{(a+b)}{2}}$$

150

$$\text{Q} \quad (\underline{x-1})^4 + (x-7)^4 = 272$$

$$\left( t + \frac{8}{2} - 1 \right)^4 + \left( t + \frac{8}{2} - 7 \right)^4 = 272$$

$$(t+4-1)^4 + (t+4-7)^4 = 272$$

$$(t+3)^4 + (t-3)^4 = 272$$

~~t~~

$$\underbrace{(t^2+9+6t)}_{a^2}^2 + \underbrace{(t^2+9-6t)}_{b^2}^2 = 272$$

~~a~~

$$(a^2+b^2)^2 = a^4+b^4+2a^2b^2$$

$$(t^2+9+6t+t^2+9-6t)^2 = 272 + 2(t^2-9)^2$$

$$(2t^2+18)^2 = 272 + 2(t^4+81-18t^2)$$

$$4t^4 + 32t^2 + 72t^2 + 2t^4 + 2t^4 + 162 - 36t^2$$

$$3t^4 + 98t^2 + 36t^2 = 272$$

$$3t^4 + 18t^2 - 136 + 243 = 0$$

$$3t^4 + 18t^2 + 107 = 0$$

$$t^2 = \cancel{324} - 18 \pm \sqrt{324} -$$

$$4t^4 + 32t^2 + 72t^2 - 2t^4 - 162 + 36t^2 = 272$$

$$2t^4 + \frac{81}{162} + \frac{54}{108} t^2 = \frac{136}{272}$$

$$t^4 + 54t^2 - 545 = 0$$

$$t^4 + 55t^2 - t^2 - 55 = 0$$

$$t^2(t^2 + 54) - 55 = 0$$

$$(t^2 - 1)(t^2 + 55) = 0$$

$$t^2 - 1 = 0$$

$$t^2 = 1$$

$$\cancel{t = \pm 1}$$

$$\boxed{t = \pm 1}$$

### National Minor Test - I

Q18.  $\left(\underbrace{\frac{x-1}{x}}_{a^2}\right)^{b^2} + \left(\underbrace{1-\frac{1}{x}}_{b^2}\right)^{a^2} = x$

$$\cancel{a^2 + b^2} = \frac{x-1}{x} + x$$

$$a+b = x$$

$$2a = \frac{x^2 + x - 1}{2x}$$

$$a^2 + b^2 = \frac{x-1}{x} - 1 + \frac{1}{x}$$

$$D = 0 \text{ v/c}$$

$$(a+b)(a-b) = (x-1)$$

so, irrational.

$$(a-1)x = (x-1)$$

$$a-1 = \frac{x-1}{x}$$

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Integer type

Q5.

$$x^8 = 8$$

$$x^8 = t$$

$$x^t = 8$$

$$\Rightarrow \boxed{x = t^{\frac{1}{8}}}$$

$$(t^{\frac{1}{8}})^t = 8$$

$$t^{\frac{t}{8}} = 8$$

$$t^t = 8^8$$

$$\boxed{t=8}$$

DYS - 5

Q.B.  $(x-1)^4 + (x-5)^4 = 82$

$$t = x-3$$

$$x = t+3$$

$$(t+2)^4 + (t+2)^4 = 82$$

$$\cancel{t^2 + 4 + 4t + t^2 + 4 - 4t = 82}$$
$$\cancel{(2t^2 + 8t)^2 =}$$

~~t~~

$$(t^2 + 6^2)^2 - 2t^2 \cdot 6^2 = 2t^4 + 6^4$$

$$(2t^2 + 8)^2 - 2(t^2 + 4 - 2t)(t^2 + 4 + 2t) = 82$$

$$4t^4 + 64 + 32t^2 - 2(t^4 + 16 + 8t^2 - 4t^2) = 82$$

$$4t^4 + 64 + 32t^2 - 2t^4 - 32 - 8t^2 = 82$$

$$2t^4 + 24t^2 + 32 = 82$$

$$t^4 + 12t^2 - 28 = 0$$

$$t^4 + 14t^2 - 2t^2 - 28 = 0$$

$$t^2(t^2 + 14) - 2(t^2 + 14) = 0$$

$$t^2 = 2$$
$$t = \pm \sqrt{2}$$

$$\left| \begin{array}{l} t^2 = 14 \\ t = \pm 2 \\ \boxed{x = .5, 1} \end{array} \right.$$

## System of Equations

→ It comprises two or more equations which are satisfied by the same set of values of variable

$$\textcircled{Q} \quad x^2 - y^2 = 16$$

$$x + y = 8$$

$$x = 8 - y$$

~~2~~

$$(8-y)^2 - y^2 = 16$$

$$\textcircled{P} \quad y^2 + 64 - 16y - y^2 = 16$$

$$16y = 64 - 16$$

$$y = \frac{48}{16}$$

$$\boxed{y = 3}$$

$$\boxed{x = 5}$$

$$\text{Q4. } \begin{cases} x^3 - y^3 = 1 \\ x - y^3 = 7 \end{cases}$$

$$(1+y)^3 - y^3 = 7$$

$$1 + y^3 + 3y(y+1) - y^3 = 7$$

$$3y(y+1) + 1 = 7$$

$$3y^2 + 3y = 86$$

$$y^2 + y - 2 = 0$$

$$y^2 + 2y - y - 2 = 0$$

$$\left. \begin{array}{l} y(y+2) - 1(y+2) \\ (y-1)(y+2) \end{array} \right\} \begin{array}{l} y=1 \\ y=-2 \end{array} \quad \begin{array}{l} x=2 \\ x=-1 \end{array}$$

(155)

M.W.

D YS-6

QS.

O-1 (001, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,  
17, 18)

$$Q6. \quad x^2 + y^2 + 6x + 2y = 0$$

$$x + y + 8 = 0$$

$$x = -(y + 8)$$

$$(y+8)^2 + y^2 - 6y - 48 + 2y = 0$$

$$y^2 + 16y + 64 - 6y - 48 + 2y$$

$$12y + 2y^2 + 16 = 0$$

$$y^2 + 6y + 8 = 0$$

$$y^2 + 4y + 2y + 8 = 0$$

$$y(y+4) + 2(y+4) = 0$$

$$(y+2)(y+4) = 0$$

$$\boxed{y = -2}$$

$$\boxed{x = -6}$$

$$\boxed{y = -4}$$

$$\boxed{x = -4}$$

Method 2

$$27. \quad \frac{x}{y} - \frac{y}{x} = \frac{5}{6}$$

$$x^2 - y^2 = 5$$

~~$$(x-y)^2 = x^2 + y^2 - 12$$~~

$$x^2 - y^2 = \frac{56xy}{6}$$

~~$$5 = 56xy/6$$~~
~~$$xy = \frac{5}{56}$$~~

$$\frac{30}{5} = 6 = xy$$

$$\frac{6}{y} = x$$

$$\frac{36}{y^2} - y^2 = 5$$

$$36 - y^4 = 5y^2$$

$$y^4 + 5y^2 - 36 = 0$$

$$y^4 + 9y^2 - 4y^2 - 36$$

$$y^2(y^2 + 9) - 4y(y^2 + 9)$$

$$(y^2 - 4)(y^2 + 9)$$

$$\boxed{y = \pm 2}$$

$$x = \frac{6}{\pm 4}$$

$$\boxed{x = \pm \frac{3}{2}}$$

$$\boxed{x = \pm 3}$$

$$y^2 = -3$$

$$\begin{array}{r} 3 \\ 8 \\ \hline 9 - 6 \\ \hline 2 \\ - 5 \\ \hline 2 \end{array} - \frac{4}{3}$$

$$Q8. \quad \frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{13}{6}$$

$$xy=5$$

$$\frac{(x+y)^2 + (x-y)^2}{x^2 - y^2} = \frac{13}{6}$$

$$x^2 + y^2 + 2xy + x^2 + y^2 - 2xy = \frac{13x^2 - 13y^2}{6}$$

$$12x^2 + 12y^2 = 13x^2 - 13y^2$$

$$2sy^2 = x^2$$

$$(sy)^2 = x^2$$

$$x = sy$$

$$sy^2 = 5$$

$$y^2 = 1$$

$$y = \pm 1$$

$$\boxed{x = \pm 5}$$

$$\frac{s+1}{s-1} + \frac{s-1}{s+1} = 1$$

$$\frac{6}{4} + \frac{4}{6}$$

$$\frac{36 + 16}{24}$$

$$\begin{array}{r} 82 \\ 29 \end{array} \overline{)26}^{13}$$

2

6

$$Q9 \quad \frac{1}{x+1} + \frac{1}{y} = \frac{1}{3}$$

$$\frac{1}{(x+1)^2} - \frac{1}{y^2} = \frac{1}{9}$$

$$y + x+1 = \cancel{xy + y} \quad | :3$$

$$3y + 3x + 3 = xy + y$$

$$2y + 3x + 3 = xy$$

$$x+1 = a$$

$$y = b$$

$$a+b = \frac{1}{3}$$

$$b = \frac{1}{3} - a$$

$$b = \frac{1}{3} - \frac{\sqrt{37}}{6}$$

$$b = \frac{2-\sqrt{37}}{6}$$

$$a^2 - b^2 = \frac{1}{4}$$

$$a^2 - \left(\frac{1}{3} - a\right)^2 = \frac{1}{4}$$

$$a^2 - \frac{1}{9} + a^2 - \frac{2 \cdot a}{3} = \frac{1}{4}$$

$$a^2 - \frac{1-6}{9} = \frac{1}{4}$$

$$a^2 - \frac{7}{9} = \frac{1}{4}$$

$$a^2 = \frac{1}{4} + \frac{7}{9}$$

$$a^2 = \frac{9+28}{36}$$

$$a^2 = \frac{37}{36}$$

$$a = \frac{\sqrt{37}}{6}$$

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$$\textcircled{10} \quad \frac{1}{y-1} - \frac{1}{y+1} = x$$

$$y+1 - y+1 = \frac{y^2-1}{x}$$

$$2x = y^2 - 1$$

$$y^2 = 2x + 1$$

$$y^2 = 8 + 1$$

$$y^2 = 9$$

$$\underline{y = \pm 3}$$

$$\boxed{(4, 3) (4, -3)}$$

$$\textcircled{10}. \quad x^2 + y^2 = 2s - 2xy$$

$$(x+y)^2 = s^2$$

$$\cancel{x+y=5}$$

$$\cancel{x=s-y}$$

$$\cancel{x=3}$$

$$\cancel{(3, 2)} \quad \cancel{(3, -2)}$$

$$x+y = \pm 5$$

$$\boxed{(3, 2) (-3, -2)}$$

$$y^2 - x - s = 0$$

$$\cancel{2x+1-x-s=0}$$

$$\underline{x = 4}$$

$$y(x+y) = 10$$

$$\cancel{sy=10}$$

$$\cancel{y=2}$$

$$xy + y^2 = 10$$

$$(s-y)y + y^2 = 10$$

$$\cancel{sy-y^2+y^2=10}$$

$$\cancel{sy=10}$$

$$\cancel{y=2}$$

$$y(\pm 5) = 10$$

$$\boxed{y = \pm 2}$$

$$\boxed{x = \pm 3}$$

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Method - 2

$$y(x+y) = 10$$

$$y^2(x+y)^2 = 100$$

$$(x+y)^2 = 25$$

$$y^2(25) = 100$$

$$y^2 = 4$$

$$\underline{y = \pm 2}$$

$$x = \pm 3$$

Q12. ~~Q12.~~ ~~2x + y~~  
Q12.  $2xy + y^2 - 4x - 3y + 2 = 0$

$$\cancel{-4x + y} (2x + y - 3) + 2 = 0$$

$$\cancel{xy - 2y^2 - 2x + 11y - 14} = 0$$

$$-5y^2 + 2sy - 30 = 0$$

$$-y^2 + sy - 6 = 0$$

$$y^2 - sy + 6 = 0$$

$$\cancel{y^2 - 6y + y + 6 = 0}$$

$$\cancel{y(y-6) + 1(y-6)}$$

$$\begin{array}{r} \cancel{2x + 4y - 2x} \\ \cancel{3x + 18 - 2x - 4y + 16} \\ \hline \cancel{x + 8 - 0} \\ 2xy + 6y^2 - 2x - 4y + 16 = 0 \end{array}$$

$$y^2 - 9y - 2y - 6 = 0$$

$$y(y-3) - 2(y-3)$$

$$(y-2)(y-3)$$

$$y = 2$$

x

$$y = 3$$

$$3x + 27 - 2x - 42 + 16$$

$$\boxed{x = \pm 1}$$

H.W.

$$\textcircled{1}, \frac{1}{x+1} + \frac{1}{y} = \frac{1}{3}$$

$$\frac{1}{(x+1)^2} - \frac{1}{y^2} = \frac{1}{4}$$

$$a+b = \frac{1}{3}$$

$$a^2 - b^2 = \frac{1}{4}$$

$$3a + 3b = 1$$

$$3a = 1 - 3b$$

$$3a = \frac{1-3b}{3}$$

$$\left(\frac{1-3b}{3}\right)^2 - b^2 = \frac{1}{4}$$

$$\left(\frac{1-3b}{3} + b\right) \left(\frac{1-3b}{3} - b\right) = \frac{1}{4}$$

$$\left(\frac{1-3b+3b}{3}\right) \left(\frac{1-3b-3b}{3}\right) = \frac{1}{4}$$

$$\frac{1}{3} \left(\frac{1-6b}{3}\right) = \frac{1}{4}$$

$$\frac{1-6b}{9} = \frac{1}{4}$$

$$1-6b = \frac{9}{4}$$

$$-6b = \frac{9-4}{4}$$

$$6b = \frac{-5}{4 \times 6}$$

$$b = \frac{-5}{24}$$

$$x+1 = \frac{13}{24}$$

$$24x + 24 = 13$$

$$x = \frac{13-24}{24}$$

$$x = \frac{-11}{24}$$

$$y = \frac{-5}{24}$$

Q-1

$$Q.C. \frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$$

$$\frac{1}{(a-b)(a-c)} - \frac{1}{(b-c)(a-b)} + \frac{1}{(a-c)(b-c)}$$

$$\cancel{(a-b)(a-c)(b-c)}$$

$$\cancel{6} - c - a + c + a - b = 0$$

$$x^{\circ} = 1$$

Q(1)

$$(a^n)^m = a^{nm}$$

$$mn = m^n$$

$$mn - m^n = 0$$

$$m \left( n - \frac{m^n}{m} \right) = 0$$

$$m(n - m^{n-1}) = 0$$

$m > 0$  given

$$n - m^{n-1} = 0$$

$$m^{n-1} = n$$

$$m^{\frac{n-1}{n-1}} = n^{\frac{1}{n-1}}$$

$$m = n^{\frac{1}{n-1}}$$



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H.W. 07-05-2024

DYS-6 Q1&3

DYS-7 (Q1, 2, 5)

O-1 (Q22, 23, 24, 25)

O-2 (Q1-5, 6, 8, 9, 10, 11, 12, 13, 14, 15)

DYS-7

Q5.

$$S > 3$$
$$\sqrt{14} > \sqrt{3}$$

$$\cancel{\sqrt{14}} - S > \sqrt{3} - 3$$

DVS-6  
Q14.

$$x^4 + y^4 = 82$$

$$xy = \pm 3$$

$$y = \frac{3}{x}$$

$$y^4 = \left(\frac{3}{x}\right)^2$$

$$= \left(\frac{9}{x^2}\right)^2$$

$$= \frac{81}{x^4}$$

$$x^4 + \frac{81}{x^4} = 82$$

$$x^8 + 81 = 82x^4$$

$$x^4 = a$$

$$a^2 - 82a + 81 = 0$$

$$a^2 - 81a - 1 + 81 = 0$$

$$a(a-81) - 1(a-81)$$

$$a=1$$

$$x^4 = 1$$

$$x = \pm 1$$

$$y = \pm 3$$

$$a = 81$$

$$x^4 = 81$$

$$x = \pm 3$$

$$y = \pm 1$$

$$(1, 3) (3, 1) \quad (-1, -3), (-3, -1)$$

Method - 2

$$(x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$$

$$(x^2 + y^2)^2 = 82 + 2x^2y^2$$

$$\underline{x^2 + y^2 = \pm 10}$$

put  $y = \frac{3}{x}$  & solve

### 3 Variables

$$Q1. \quad a+b=10$$

$$b+c=15$$

$$a+c=25$$

Find  $a, b, c$

$$b = 10 - a$$

$$10 - a + c = 15$$

$$c - a = 5$$

$$a + c = 25$$

$$2c = 30$$

$$\boxed{c = 15}$$

$$\cancel{a = 10} \quad a = 15 - 5$$

$$\boxed{a = 10}$$

$$b = 10 - 10$$

$$\boxed{b = 0}$$

$$Q2. \quad 2x + 3y + z = 1$$

$$\frac{2x-1}{1} = \frac{y+1}{-2} = \frac{z}{6}$$

$$-2x + 2 = y + 1$$

$$-(\cancel{2x+y=1}) \\ 2x + 3y + z = 1$$

$$-(2y + z = 0) \times 2$$

$$6y + 2z = -6$$

$$\begin{aligned} 2y &= -c \\ \boxed{y} &= -3 \end{aligned}$$

$$\begin{aligned} 2x + -9 + 6 &= 1 \\ 2x &= 1 + 3 \\ \boxed{x} &= 2 \end{aligned}$$

$$\begin{aligned} 6y + 6 &= -2z \\ 6y + 2z &= -6 \end{aligned}$$

$$-18 + 2z = -6$$

$$\begin{aligned} 2z &= -6 + 18 \\ \boxed{z} &= 6 \end{aligned}$$

$$\text{Mithal - 2} \\ \text{let } \frac{x-1}{1} = \frac{y+1}{-2} = \frac{z}{6} = k$$

put  $k$  in eq 1

Q13.

$$\begin{aligned}w + x + y &= -2 \\w + x + z &= 4 \\w + y + z &= 19 \\x + y + z &= 12\end{aligned}$$

$$\begin{aligned}w + y + z &= 19 \\-w - x - y &= -12\end{aligned}$$

$$z - x = 21$$

$$\underline{z = 21 + x} \rightarrow x = z - 21$$

$$6 + y = 21 + x$$

$$z - y = 6$$

$$\underline{z = 6 + y} \rightarrow y = z - 6 \quad \begin{aligned}y - x &= 21 - 6 \\y - x &= 15\end{aligned}$$

$$x - y = -15$$

$$z - 21 + z - 6 + x = 12$$

$$3z - 27 = 12$$

$$3z = 39$$

$$\boxed{z = 13}$$

$$\boxed{x = -8}$$

$$\boxed{y = 7}$$

$$Q4. \quad x+y+z=4$$

$$x^2+y^2+z^2=6$$

$$x^3+y^3+z^3=8$$

i)  ~~$x+y+z$~~

$$ii) xy+yz+xz=?$$

$$i. (4)^2 = 6 + 2(xy+yz+xz)$$

$$\frac{16-6}{2} = \frac{10}{2} = 5$$

$$xy+yz+xz=5$$

$$i) \frac{xy}{6} = 8 + 3xyz \quad (4)$$
$$\frac{56}{12} = xyz$$

$$xyz = \frac{14}{3}$$

$$iii) \frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$$

$$\frac{4 \times 3}{14} = \frac{12}{14} = \frac{6}{7}$$

$$i) xyz$$

$$8 - 3abc = (4)(6-5)$$

$$8 - 3abc = 4$$

$$3abc = 4$$

$$abc = \frac{4}{3}$$

$$iii) \frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$$

$$\frac{4 \times 3}{14} = \frac{12}{14} = \frac{6}{7}$$

$$\frac{x+y+z}{xyz} = \frac{4}{\frac{4}{3}} = 3$$

$$(iii) \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$\frac{xy+yz+xz}{xyz} = \frac{s}{\frac{4}{3}} = \boxed{\frac{15}{4}}$$

### Inequalities

→ Comparing comparability in real numbers.

→  $>$ ,  $<$ ,  $\geq$ ,  $\leq$

Strict  
Inequality

slack  
inequality

$$\text{eg. } 4 > -2 \quad (x \in \mathbb{R})$$

$$3 \geq 3 \quad (x \in \mathbb{R})$$

$$4 > 3 \quad (x \in \mathbb{R})$$

$$3 > 4 \quad (x \in \emptyset)$$

→ Properties -

(1) Addition, Subtraction, Multiplication of any constant.

$$\begin{array}{ccc}
 & \overbrace{5 > 2} & \\
 + \downarrow & & \downarrow - \\
 5 > 2 & & 5 > 2 \\
 5+2 > 2+2 & & 5-2 > 2-2 \\
 7 > 4 & & 3 > 0
 \end{array}$$

$$5 > 2 +$$

↓      ↓

Multiplication      Division

(3)  $5 > 2 \times 3$

$15 > 6$

$\frac{5}{1} > \frac{2}{1}$

$5 > 1$

⊖ve

$(-3) 5 > 2 \times (-3)$

$-15 > -2$  (not valid)

⊖ve

$\frac{5}{-1} > \frac{2}{-1}$

$-5 > -2$  (not valid)

→ If we multiply or divide any negative constant, then we have to reverse the sign of inequality.

→ Never multiply or divide any thing whose sign we don't know.

e.g -

$$9 > 2$$

$9x > 2x$  (not valid)

$x = 3$   
 $9(3) > 2(3)$   
 $27 > 6$

$x = -3$   
 $9(-3) > 2(-3)$   
 $-27 > -6$   
(wrong)

⊖e.g ② -  $9 > 2$   
 $9(x^2 + 2) > 2(x^2 + 2)$   $(x^2 + 2)$   
we know  $x^2$  is always  
⊕ve

→ Inequalities sign reverse when we take reciprocal both sides.

e.g  $\frac{3}{2} > 2$   
 $\frac{1}{3} < \frac{1}{2}$

→ Cross Multiplication is not valid until unless we don't know the sign.

Eg.  $\frac{2}{x} > 1$

$2 > x$  (wrong)

$$\frac{2}{x} - 1 > 0$$

$$\frac{2-x}{x} > 0 \quad (\text{right})$$

→ If we cancel anything in inequality then mention it (variables)

Eg.  $9 > \frac{2x}{2-x} \quad x \neq 2$

$9 > 1 \quad (\text{for } x \in R, x \neq 2)$

right

$$9 > \frac{(2-x)}{(2-x)}$$

wrong as - for  $x = 2$

$\cancel{x \in R}$

$9 > 1$

$9 > \frac{0}{0} \quad (\text{not defined})$

→ we can cancel out anything (constant & variables) which are in addition or subtraction in inequality.

Eg.  $x+2 > x-1$   
 $2 > -1 \quad (\text{right})$

$$2x > x(-1)$$

$$2 > -1 \quad (\text{wrong})$$

→

$$AB > AC \quad (0)$$



$$B > C \quad (\text{wrong})$$

$$AB - AC > 0$$

$$A(B-C) > 0 \quad (\text{right})$$

→ 2. Addition of two inequalities is valid with same sign side sign, but subtraction, multiplication & division is not valid.

e.g.,

$2 > 1$	$+ \frac{3 > 2}{5 > 3}$
	(right)

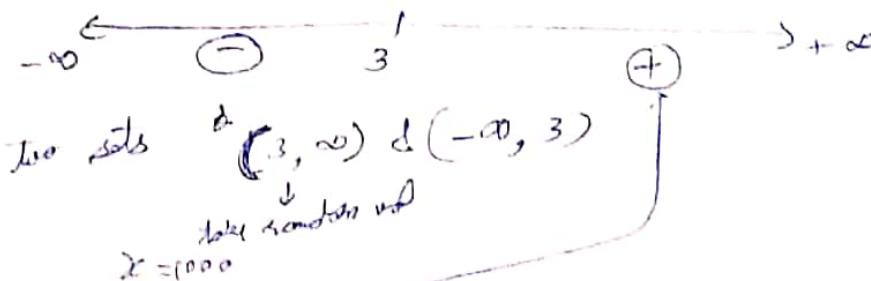
$\frac{2 > 1}{+ \frac{3 < 5}{5 \cdot 6}}$
(wrong)

Ex (2).  ~~$-3 < 1$~~        $-3 < 1$        $10 < 11$   
 ~~$+ 10 < 11$~~        $-10 < 10$  (wrong)

### Wavy Curve Method

Q.  $x - 3 > 0$

Factors  $\Rightarrow x - 3 = 0$   
 $\underline{x = 3}$

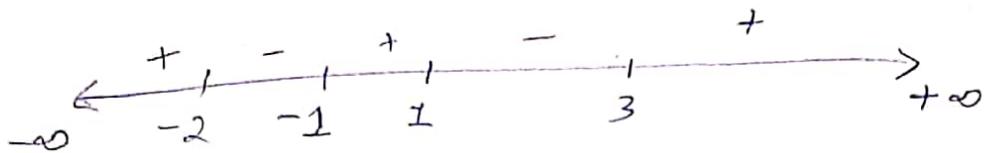


$x - 3 = 0$  V.L.  
 $\therefore (x - 3) \oplus V.L$  for  $x$  in  $(3, \infty)$

(17)

$$Q2. \quad (x-1)(x+1)(x+2)(x-3) \leq 0$$

$$\begin{aligned} x-1 &= 0 \\ x &= 1, -1, -2, 3 \end{aligned}$$



Intervals:  
let  $x = \underline{\text{_____}}$  in  $(3, \infty)$

$$(4-1)(4+1)(4+2)(4-3)$$

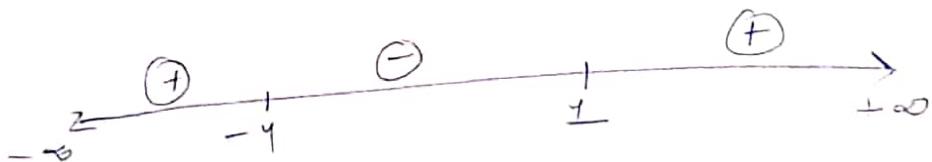
$$3 \times 5 \times 6 \times 1 = \oplus \vee$$

$x$  must be  $\oplus \vee$

$$\boxed{[-2, -1] \cup [1, 3]}$$

$$Q3. \quad (x+4)(x-1) < 0$$

$$x = 1, -4$$



$$x = \frac{20}{(1000)(1+1)} = \oplus \vee$$

$$\boxed{(-4, 1)}$$

$$Q4. (x-1)(x+3)(x+1)(x+2)(x-4)(x-5) \geq 0$$

$$x = 1, -3, -1, -2, 4, 5 \quad \text{∅}$$



but  $x=6$

$$5 \times 9 \times 7 \times 8 \times 2 \times 1 = \oplus \text{ve}$$

$$\boxed{x \in [-\infty, -3] \cup [-2, -1] \cup [1, 4] \cup [5, \infty)}$$

Note:-

$$\textcircled{1} \quad (x-a)(x-b) \leq 0 \rightarrow x \in [a, b]$$

$$\textcircled{2} \quad (x-a)(x-b) < 0 \rightarrow x \in (a, b)$$

$$\textcircled{3} \quad (x-a)(x-b) \geq 0 \rightarrow x \in (-\infty, a] \cup [b, \infty)$$

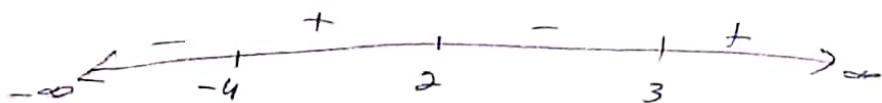
$$\textcircled{4} \quad (x-a)(x-b) > 0 \rightarrow x \in (-\infty, a) \cup (b, \infty)$$

$$Q5. (2-x)(4+x)(x-3) < 0$$

$$-(x-2)(4+x)(x-3) < 0$$

(-) multiply

$$(x-2)(x+4)(x-3) > 0$$



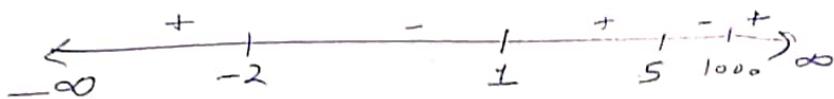
$$x \in (-4, 2) \cup (3, \infty)$$

$$Q6. (4-x)(x+4) \leq 0$$



$$x \in [4, \infty) \cup (-\infty, -4]$$

$$Q7. (x-1)(x+2)(5-x)(1000-x) > 0$$



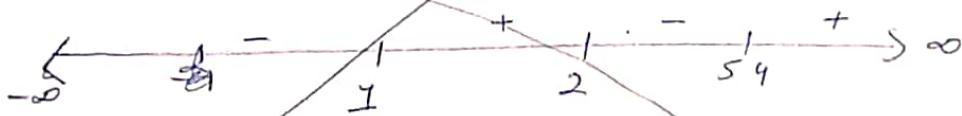
$$x \in (1000, \infty) \cup (1, 5) \cup (-\infty, -2)$$

~~$$Q8. (x^2-3x+2)(x^2+54x+54) > 0$$~~

~~$$(x^2-2x-x+2)(x^2-54x-x+54)$$~~

~~$$x(x-2)-1(x-2)$$~~

~~$$(x(x-54)-1)(x-54)$$~~



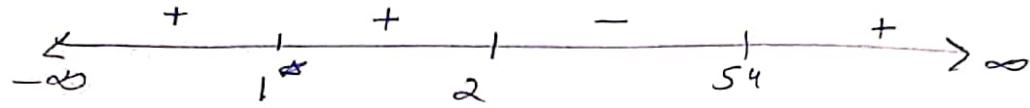
~~$$x \in (-54, \infty) \cup (1, 2)$$~~

Note:- ① Quantities which have whole even powers does not change sign and for whole odd powers we can ~~change~~ change sign

$$Q8. (x^2 - 3x + 2)(x^2 - 5x + 54) > 0$$

$$(x-2)(x-1)(x+1)(x-54) > 0$$

$$(x-2)(x-1)^2(x-54)$$



$$\boxed{x \in (-\infty, 1) \cup (1, 2) \cup (54, \infty)}$$

$$Q9. (x-1)^2(x+1)^3(x-4) < 0$$



$$(-1, 1) \cup (1, 4)$$

$$\boxed{x \in (-1, 1) \cup (1, 4)}$$

$$Q10. (x-1)^2(x+1)^3(x-2)^4(x+3)^5(x^2-36) > 0$$

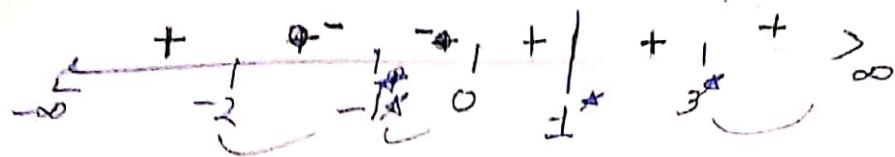
$$(x-1)^2(x+1)^3(x-2)^4(x+3)^5(x+6)(x-6) > 0$$

~~$(x-1)^2(x+1)^3(x-2)^4(x+3)^5(x^2-36) > 0$~~

$$\boxed{x \in (-\infty, -6) \cup (-3, -1) \cup (6, \infty)}$$



$$Q11. \quad (x-1)^2 (x+1)^4 (x) (x-3)^6 (x+2)^7 \geq 0$$



$$\cancel{[-2, -1] \cup [-1, 0] \cup [3, \infty)}$$

$$\cancel{x \in [-2, 0] \cup [3, \infty)}$$

$$\boxed{x \in \mathbb{R} \setminus (-\infty, -2] \cup [0, \infty) \setminus \{-1\}}$$

Note:-

① whenever we solve questions of odd & even whole powers, we will always check at the end points or factors values.

H.W.

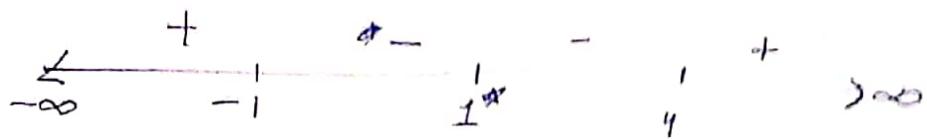
09-05-2024

$$0-3 (\cancel{0-7} - \cancel{1-7})$$

$$0-3 [1, 2] \cup \{3\} (1 \rightarrow 7)$$

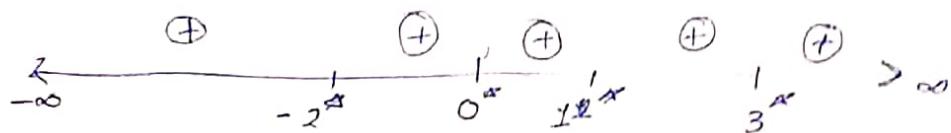
$$0-4 \{1, 2, 3, 4\}$$

$$Q. (x-1)^2(x+1)^3(x-4)^7 < 0$$



$$\boxed{x \in (-1, 1) \cup (1, 4)}$$

$$Q. x^4(x-2)^2(x+2)^3(x-3)^{10} \leq 0$$

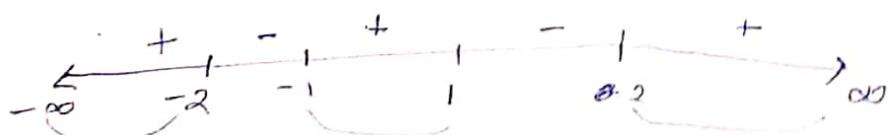


$$\cancel{\{x \in [-2, 2] \cup [0, 2, 3]\}} = \{x \in \{-2, 0, 1, 2, 3\}\}$$

$$\boxed{x \in \{-2, 0, 1, 2, 3\}}$$

Denominator based questions

$$Q1. \frac{6(x-1)(x-2)}{(x+1)(x+2)} \geq 0$$



$$x \in (-\infty, -2] \cup [-1, 1] \cup [2, \infty)$$

but  $x \notin \{-2, -1\}$

$$\boxed{x \in (-\infty, -2) \cup (-1, 1) \cup [2, \infty)}$$

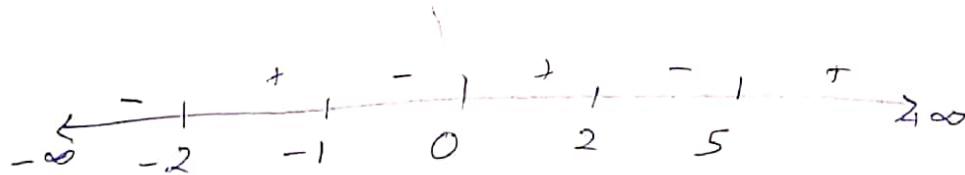
$$\text{Q2. } \frac{6x-5}{4x+1} < 0$$



$$x \notin \{-\frac{1}{4}\}$$

$$\boxed{x \in (-\frac{1}{4}, \frac{5}{6})}$$

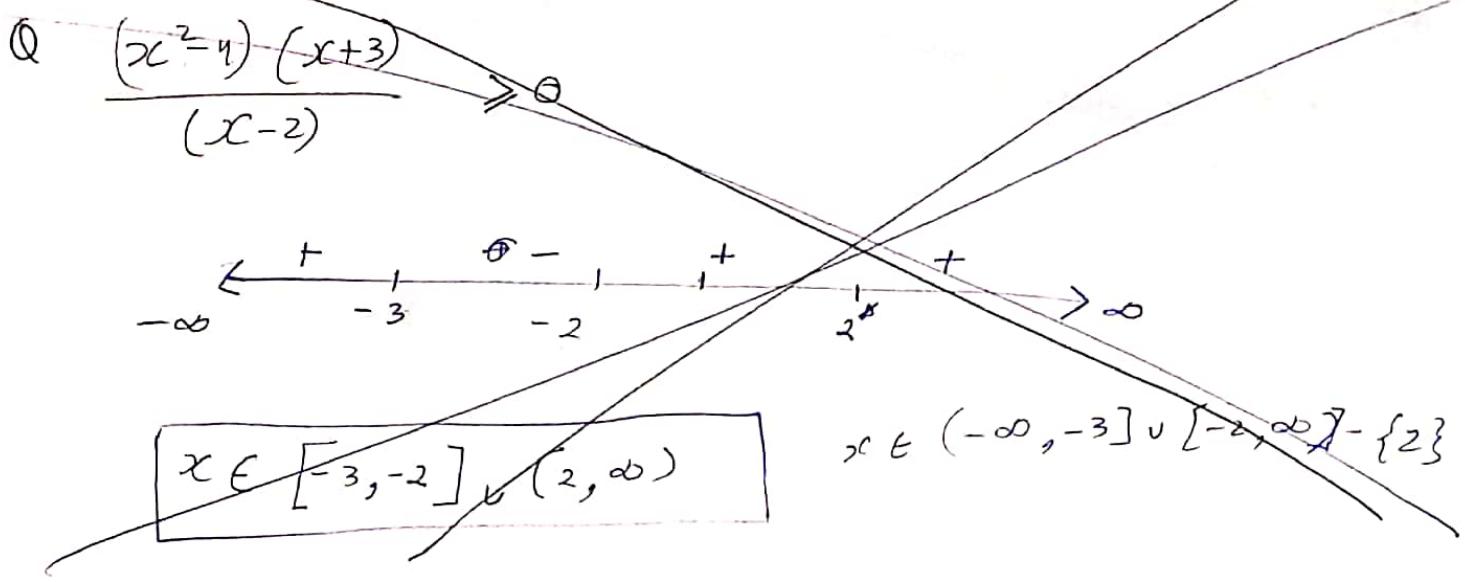
$$\text{Q3. } \frac{x(x-2)(x-5)}{(x+2)(x+1)} > 0$$



$$(-2, -1) \cup \cancel{(2, 5)} (0, 2) \cup (5, \infty)$$

$$x \notin \{-2, -1\}$$

$$\boxed{x \in (-2, -1) \cup (0, 2) \cup (5, \infty)}$$



Q ~~(x-2)~~  $\frac{(2x-1)(x-1)^2(x-2)^3}{(x-4)^4} > 0$

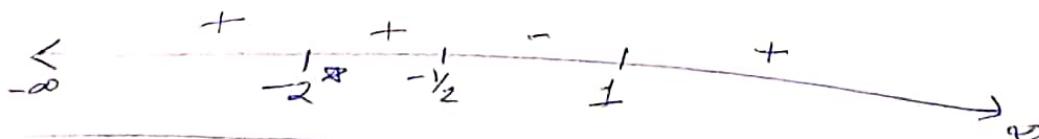


$$x \in (-\infty, 1/2) \cup (2, 4) \cup (4, \infty)$$

Q  $\frac{(x^2+4x+4)}{(2x^2-x-1)} < 0$

$$\frac{(x+2)(x+2)}{(2x+1)(x-1)} < 0$$

$$2x^2 - x - 1 \\ 2x^2 + 2x + x - 1 \\ 2x(x+1) + 1(x-1)$$



$$x \in (-1/2, 1)$$

$$x \notin \{-1/2, 1\}$$

$$Q \quad \frac{(x^2-4)(x+3)}{(x-2)} \geq 0$$

$$\frac{(x+2)(x-2)(x+3)}{(x-2)} \geq 0$$

$$(x+2)(x+3) \geq 0$$

$$x \neq 2$$



$$x \in (-\infty, -3) \cup (-2, \infty) - \{2\}$$

Note ① we can cancel out factors from numerators & denominators but we have to mention this also that

② ~~we can~~ following quantities are always positive -

1. quadratic equations with D  $\leq 0$  & a  $> 0$

$$D \leq 0 \text{ & } a > 0$$

2. quantities with whole even powers.

3. Modulus.

$$Q \quad \frac{(x-1)(x+2)}{(x^2+1)(x^2+x+1)} < 0$$

$D = 1-4(1)(1) < 0$   
 $A = 0 \forall x$

$$D < 0, a > 0$$

Positive

$$(x-1)(x+2) \geq 0 \quad (x^2+1)(x^2+x+1)$$

$$(x-1)(x+2) < 0$$

$$x \in (-2, 1)$$

$$① \frac{3x-1}{(4x+1)(x^2)} \leq 0$$

$$3x-1 \leq 0$$

$$x \notin \left[-\frac{1}{4}, 0\right]$$

$$D = 0 - 4 \times 1 = 4$$



$$3x-1 \leq 0$$

$$3x \leq 1$$

$$x \leq \frac{1}{3}$$

$$x \in (-\infty, \frac{1}{3}] - \left\{-\frac{1}{4}, 0\right\}$$

$$\frac{3x-1}{4x+1} \leq 0$$

$$x \notin \{0, -\frac{1}{4}\}$$

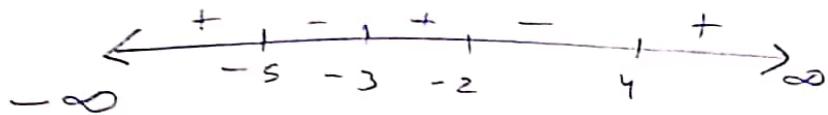


$$x \notin \left[-\frac{1}{4}, \frac{1}{3}\right] - \{0\}$$

(84)

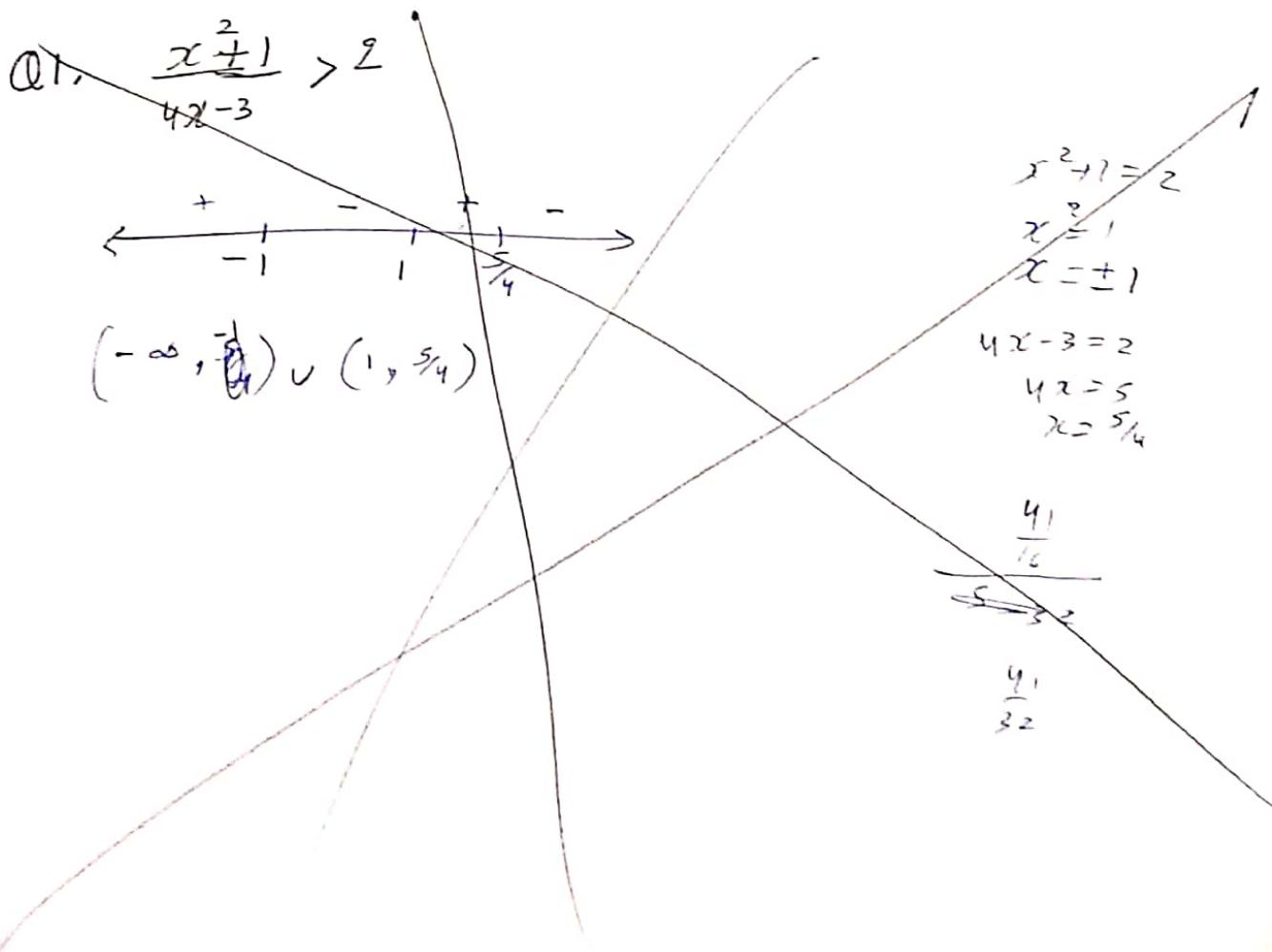
$$Q2. \frac{(x+2)(x+3)(x+5)}{(x-4)^3(x-6)^5} > 0$$

$$x \notin \{6, 4\}$$



$$x \in (-\infty, -5) \cup (-3, -2) \cup (4, \infty) - \{6\}$$

~~Q3~~ ~~Q4~~ ~~Q5~~ ~~Q6~~ ~~Q7~~ ~~Q8~~



$$Q1. \frac{x^2+1}{4x-3} > 2$$

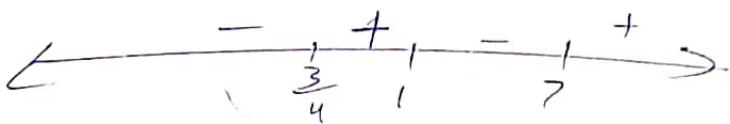
$$\frac{x^2+1 - 2(4x-3)}{4x-3} > 0$$

$$\frac{x^2 - 8x + 7}{4x-3} > 0$$

$$\frac{x^2 - 7x - x + 7}{4x-3} > 0$$

$$\frac{x(x-7) - 1(x-7)}{4x-3} > 0$$

$$\frac{(x-1)(x-7)}{4x-3} > 0$$



$$x \in \left( \frac{3}{4}, 1 \right) \cup \left( 7, \infty \right)$$

(1.86)

(2)

$$\frac{4x-3}{x^2+1} > 2$$

$$\frac{4x-3 - 2x^2 - 2}{x^2+1} > 0$$

$$4x - 2x^2 - 5 > 0$$

$$-2x^2 + 4x - 5 > 0$$

$$x = \frac{-4 \pm \sqrt{16 - 40}}{2}$$

$$x = -4 \pm$$

$$D = b^2 - 4ac$$

$$D = 0 - 4(1)(1)$$

D

$$D < 0, a > 0$$

always positive, never  $\leq 0$

$$x \in \emptyset$$

(3)

$$\frac{x}{x+1} > 2$$

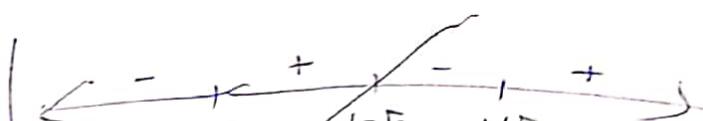
$$\frac{x - 2x - 2}{x+1} > 0$$

$$x = \frac{2 \pm \sqrt{4 + 8}}{2}$$

$$x = (\pm \sqrt{3})$$

$$x = 1 + \sqrt{3}, \quad 1 - \sqrt{3}, \quad -1$$

$$x \neq -1$$



$$x \in (-1, 1 - \sqrt{3}) \cup (1 + \sqrt{3}, \infty)$$

(187)

$$Q4. \frac{x+1}{(x-1)^2} < 1$$

$$\frac{x+1}{x^2+1-2x} < 1$$

~~0~~

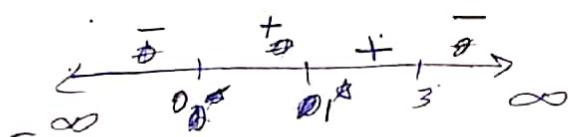
$$\frac{x+1 - (x-1)^2}{(x-1)^2} < 0$$

$$x+1 - x^2 + 1 - 2x < 0$$

~~$3x = 3x$~~

$$\frac{3x - x^2}{(x-1)^2} < 0$$

$$\frac{3x(3-x)}{(x-1)^2} < 0$$



$$x \in (0, 3) = \{1\}$$

$$x \in (-\infty, 0) \cup (3, \infty)$$

Method -2

$$x+1 < (x-1)^2$$

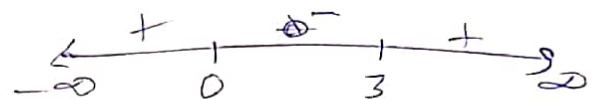
$$x+1 < x^2 - 2x + 1$$

$$x < x^2 - 2x$$

~~$x < x^2 - 2x$~~

$$0 < x^2 - 3x$$

$$x^2 - 3x > 0$$



$$(-\infty, 0) \cup (3, \infty) \in x$$

$$\text{Q3. } \frac{2x}{x+1} > 2$$

$$\frac{2x}{x+1} - \frac{2}{1} > 0$$

$$\frac{2x - 2(x+1)}{x+1} > 0$$

$$\frac{x-2x-2}{x+1} > 0$$

$$\frac{-x-2}{x+1} > 0$$

~~$x \neq -1$~~

$$\frac{-x-2}{x+1} > 0$$

$$\frac{x+2}{x+1} < 0$$

$$x \neq -1$$

$$\begin{array}{c} + \quad - \quad + \\ \xleftarrow{-\infty} -2 \quad -1 \quad \rightarrow \infty \end{array}$$

$$\boxed{x \in (-2, -1)}$$

MW. 10-08 - 2024

DYS-10 ~~601~~

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16\}$$

$$[1, 16] - \{13\}$$

Q13.  $\frac{1}{x^2-2} + \frac{1}{x-1} > \frac{1}{x}$

$$\frac{x-1 + x-2}{(x-2)(x-1)} > \frac{1}{x}$$

$$\frac{2x-3}{\cancel{x-2}(x-1)x} > \frac{1}{x}$$

$$\frac{x(2x-3)}{(x-2)(x-1)x} = \frac{\cancel{x}(x-2)(x-1)}{\cancel{x-2}(x-1)x} > 0$$

$$\frac{2x^2-3x-x^2+3x-2}{\cancel{x-2}(x-1)x} > 0$$

$$\frac{x^2-2}{(x-2)(x-1)x} > 0$$

$$\frac{(x-\sqrt{2})(x+\sqrt{2})}{(x-2)(x-1)x} > 0$$

$$\begin{array}{ccccccccc} - & + & - & + & - & + & \infty \\ \hline -\infty & -\sqrt{2} & 0 & \sqrt{2} & 1 & 2 & \infty \end{array}$$

$$\left\{ (-\sqrt{2}, 0) \cup (\sqrt{2}, 1) \cup (2, \infty) \right\}$$

Mu W. 21-05-2024

PYS-10 Q17

Q-1 Q19-20-21

Q-4 Q5, 6

## Mean

→ For any two positive real numbers,  $x \& y$  ( $y \geq x$ )

$$\text{AM (Arithmetic mean)} = \frac{x+y}{2}$$

$$\text{GM (Geometric mean)} = \sqrt{xy}$$

$$y \geq AM \geq GM \geq x$$

Ex ①.

$$x = 4$$

$$y = 16$$

$$AM = \frac{16+4}{2}$$

$$= \frac{20}{2}$$

$$= 10$$

$$GM = \sqrt{4 \times 16}$$

$$= \sqrt{64}$$

$$= 8$$

$$y \geq AM \geq GM \geq x$$

②.

$$x = 10$$

$$y = 90$$

$$AM = \frac{10+90}{2}$$

$$= 50$$

$$GM = \sqrt{10 \times 90}$$

$$= \sqrt{900}$$

$$= 30$$

③.

$$x = 10$$

$$y = 10$$

$$AM = \frac{10+10}{2}$$

$$= 10$$

$$GM = \sqrt{10 \times 10}$$

$$= \sqrt{100}$$

$$= 10$$

~~Note:-~~  $AM = GM$  when  $x=y$

→ In AM & GM, equality holds when  $x=y$

→ For 3 +ve quantities  $x, y, z$

$$AM = \frac{x+y+z}{3}$$

$$GM = \sqrt[3]{xyz}$$

→ For  $n$  quantities  $a_1, a_2, a_3$

$$AM = \frac{a_1 + a_2 + a_3 + a_4 + \dots + a_n}{n}$$

$$GM = \sqrt[n]{a_1 \times a_2 \times a_3 \times a_4 \times \dots \times a_n}$$

### Mmts

- Maximum / Minimum or largest / smallest in question
- Given all quantities are +ve
- Multiplication of quantities will be a constant

Q Find the ~~maximum~~<sup>minimum</sup> value of ①  $x + \frac{1}{x}$  ( $x$  is  $\oplus$ ve)

②  $x^2 + \frac{4}{x^2}$

①  $GM = \sqrt{x + \frac{1}{x}}$

$$\boxed{\sqrt{-1}} = \sqrt{-1}$$

$$AM = \frac{x + \frac{1}{x}}{2}$$
$$= \frac{x^2 + 1}{2x}$$



$$2 \leq \frac{x+1}{x}$$

minimum value = 2 ✓

②  $GM = \sqrt{x^2 + \frac{4}{x^2}}$

$$\boxed{\sqrt{4}} = 2$$

$$AM = \frac{x^2 + \frac{4}{x^2}}{2}$$

$$x^2 + \frac{4}{x^2} \geq 4$$

minimum = 4 ✓

Q2. find min value. ( $x > 0$ )

①  $x + \frac{1}{x} + 3$

~~AM =  $\frac{x + \frac{1}{x} + 3}{2}$~~

~~GM =  $\sqrt[3]{x \cdot (\frac{1}{x} + 3)}$~~

~~GM =  $\sqrt[3]{3}$~~

~~GM =  $\sqrt{3}$~~

$2\sqrt{3} \leq x + \frac{1}{x} + 3$

min  $2\sqrt{3}$

$x + \frac{1}{x}$ , min = 2

$2+3=5$

$\boxed{\text{min} = 5}$

~~$b = \frac{1}{x} \cdot 3$~~

$x = \frac{1}{x} = 3$  (not possible for  
any value of  $x$ )  
So cannot use AM & GM.

②  $x^2 + \frac{1}{x^2} + 7$

~~$x^2 + \frac{1}{x^2}$ , min = 9~~

~~$4+7=11$~~

~~AM =~~

$\frac{x^2 + \frac{1}{x^2}}{2} \geq 1$

$x^2 + \frac{1}{x^2} \geq 2$

$\boxed{2+7=9}$

$$Q3. f(x) = \frac{x^2 + 1 + 8/x}{2x}$$

find min value of  $x$  is  $\oplus$  ve

$$\boxed{x + \frac{1}{x} + 8}$$

$\min = 2$

$$2 + 8$$

$$\boxed{\min = 10}$$

$$Q4. \min \text{ value of } f(x) = (P+Q) \left( \frac{1}{P} + \frac{1}{Q} \right) \quad P & Q \in \mathbb{N} \oplus \text{ve}$$

~~$$\frac{P}{2} + \frac{Q}{2} + 2$$~~

~~$$\frac{P}{Q} + \frac{Q}{P} + 2$$~~

$$\frac{\frac{P}{Q} + \frac{Q}{P}}{2} \geq 1$$

$$\frac{P}{Q} + \frac{Q}{P} \geq 2$$

~~$$\frac{P}{Q} + \frac{Q}{P} + 2$$~~

~~$$\frac{P}{Q} + \frac{Q}{P} + 2$$~~

$$2 + 4$$

$$\boxed{P+Q = \min}$$

## Ratio & Proportion

### Ratio

- Comparison of quantities by the method of division.
- It says how many times one quantity is equal to the another quantity.
- two numbers in ratios are compared only when they have the same unit.
- a to b

a:b

~~or~~ %

~~a~~:  
 a → Antecedent  
 b → Consequent

### Proportion

- It is a equation which defines that two given ratios are equivalent to each other.

→ ∵ or =

→ 3 Types

① Direct proportion (~~a~~) ( $a \propto b$ )

② Inverse proportion ( $a \propto \frac{1}{b}$ )

③ Continued proportion

$$\frac{a}{b} = \frac{c}{d} \propto a:b :: c:d$$

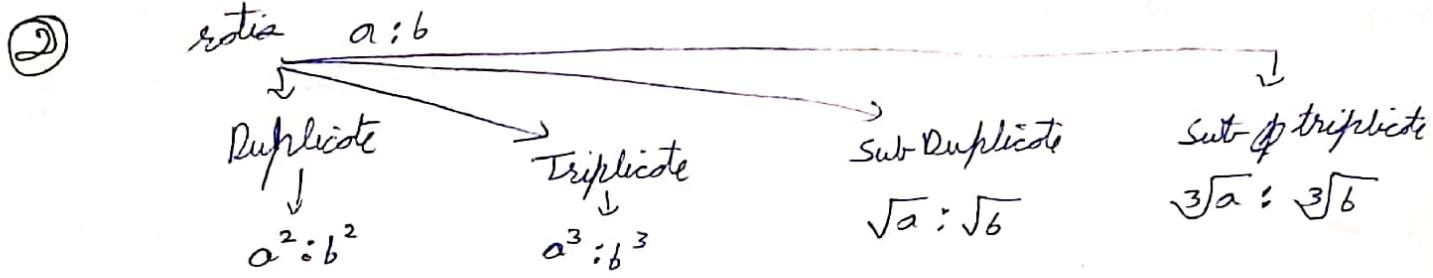
## Properties of ratio

$$\textcircled{1} \quad \frac{a}{b} = \frac{c}{d} = \frac{e}{f} \quad \text{then } \frac{a+d+f}{b+c+e} = \frac{a}{b}$$

$$\text{then:- } \frac{a+c+e}{b+d+f} = \frac{a}{b}$$

$$\text{eg. } \frac{1}{2} = \frac{3}{4} = \frac{3}{6}$$

$$\frac{1+2+3}{2+4+6} = \frac{6}{12} = \frac{1}{2}$$



Q1. Are the 2 ratios ~~8:10~~ & 7:10 in proportion or not

$$\textcircled{8} \quad \frac{8}{10} \neq \frac{7}{10}$$

not in proportion

② Properties of proportion -

$$① \underline{\text{Components}} - \frac{a}{b} = \frac{c}{d}$$

$$\text{components is } \frac{a+b}{b} = \frac{c+d}{d}$$

$$② \underline{\text{dividends}} \text{ of } \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+d}{b-d} = \frac{c}{d}$$

$$③ \underline{\text{componendo-dividendo}} \rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$④ \underline{\text{Alternando}} \rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a}{c} = \frac{b}{d}$$

$$⑤ \underline{\text{Invertendo}} \rightarrow \frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{b}{a} = \frac{d}{c}$$

~~(C)~~ reverse components - dividends -  $\frac{a+b}{c-d} = \frac{c+d}{c-d}$

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Q If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$  show that

$$\frac{a^3 b + 2c^2 e - 3ae^2 f}{b^4 + 2d^2 f - 3bf^3} = \frac{ace}{bdf}$$

~~$$\frac{a+c+e}{b+d+f} = \frac{a}{b}$$~~

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = t$$

$$a = ct, c = dt, e = ft$$

~~$$\frac{(bt)^3 b + 2(dt)^2 (ft) - 3(bt)(ft)^2 (ft)}{b^4 + 2d^2 f - 3bf^3}$$~~

~~$$\frac{(at)^3 b + 2(dt)^2 (ft) - 3(at)(ft)^2 (ft)}{b^4 + 2d^2 f - 3bf^3}$$~~

$$\frac{b^4 t^3 + 2d^2 f t^3 - 3b^2 f^2 t^3}{b^4 + 2d^2 f - 3bf^3}$$

$$\frac{t^3 (b^4 + 2d^2 f - 3bf^3)}{b^4 + 2d^2 f - 3bf^3}$$

$$t^3 = \frac{a}{b} \times \frac{b}{b} \times \frac{b}{b}$$

$$= \frac{a \times b \times e}{b \times d \times f}$$

$$= \frac{abe}{bdf}$$

$$\text{Q if } a:b = 1:2 \quad \frac{b-6a}{b-6a}$$

$$\frac{a}{b} = \frac{1}{2}$$

$$2a = b$$

$$\frac{2a-8a}{2a-6a} = \frac{-6a}{-4a}$$

$$\boxed{\frac{3}{2}}$$

$$\text{Q2. } \frac{x}{a} = \frac{y}{b} = \frac{z}{c} \text{ show } (a^2 + b^2 + c^2)(x^2 + y^2 + z^2) = (ax + by + cz)^2$$

$$x = at, y = bt, z = ct$$

$$(a^2 + b^2 + c^2)(a^2 t^2 + b^2 t^2 + c^2 t^2)$$

$$a^4 t^2 + b^2 a^2 t^2 + a^2 c^2 t^2 + a^2 b^2 t^2 + b^4 t^2 + b^2 c^2 t^2 + a^2 c^2 t^2 + b^2 c^2 t^2 + c^4 t^2$$

$$a^4 t^2 + b^4 t^2 + c^4 t^2 + 2a^2 c^2 t^2 + 2a^2 b^2 t^2 + 2b^2 c^2 t^2$$

$$t^2 (a^2 + b^2 + c^2)^2 = \text{LHS},$$

$$(a(at) + b(bt) + c(ct))^2$$

$$(a^2 t + b^2 t + c^2 t)^2$$

$$t^2 (a^2 + b^2 + c^2)^2 = \text{RHS}.$$

$$\underline{\text{LHS} = \text{RHS}}$$

$$\textcircled{Q} 4. \quad \frac{3x^4 + x^2 - 2x - 3}{3x^4 - x^2 + 2x + 3} = \frac{5x^4 + 2x^2 - 7x + 3}{5x^4 - 2x^2 + 7x - 3} \quad \text{find } x.$$

$$\frac{3x^4 + (x^2 - 2x - 3)}{3x^4 - (x^2 - 2x - 3)} = \frac{5x^4 + (2x^2 - 7x + 3)}{5x^4 - (2x^2 - 7x + 3)}$$

$$\frac{3x^4}{x^2 - 2x - 3} = \frac{5x^4}{2x^2 - 7x + 3}$$

$$6x^6 - 21x^5 + 9x^4 = 5x^6 - 10x^5 - 15x^4$$

$$x^6 - 21x^5 + 112x^4$$

$$x^6 - 11x^5 + 24x^4$$

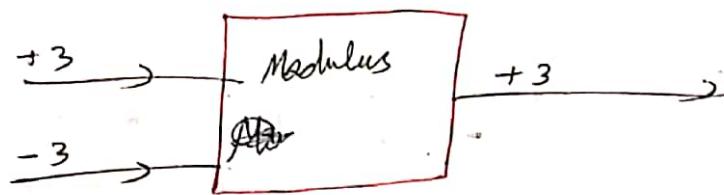
$$x^4(x^2 - 20x + 24) = 0$$

$$\begin{aligned} & x^4 = 0 \\ & x^2 = 20 \\ & x = \pm \sqrt{20} = \pm \sqrt{400 - 96} \\ & x = \pm 10 \pm \sqrt{76} \\ & x = \frac{11 \pm \sqrt{121 - 96}}{2} \end{aligned}$$

$$\boxed{x = 8, 3, 0}$$

$$\begin{aligned} 6x^2 - 14x + 9 &= 5x^2 - 10x \\ x^2 - 4x + 24 &= 0 \\ x = 2 \pm \sqrt{16 - 24} &= 2 \pm \sqrt{-8} \end{aligned}$$

## Modulus :-



- Also called absolute value
- $|x|$  (denotation)

$$f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

$$f(x) = |x-2| = \begin{cases} (x-2) & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$$

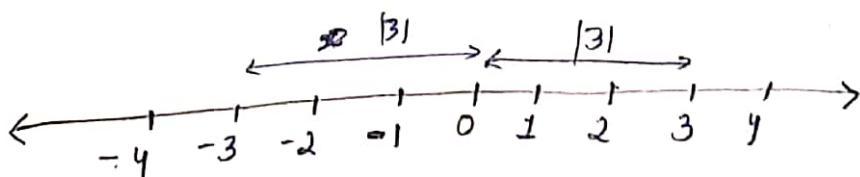
At  $x=2$  is zero  
 At  $x=1$ ?

$$f(x) = |x+3| = \begin{cases} (x+3) & x \geq -3 \\ -(x+3) & x < -3 \end{cases}$$

## Geometrical Representation -

$|x|=3$  → Distance 3 from origin.

$$x = 3, -3$$



•  $|x| = 5 \rightarrow$  Distance of  $x$  from origin is 5

$$x = \pm 5$$

•  $|x| = -2 \rightarrow$  Not Possible.

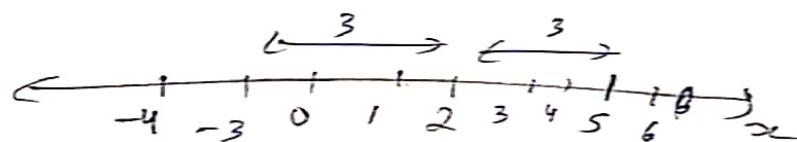
↳ Distance of any point from origin is  $(-2)$ .

So,  $x$  is empty set.

not possible.

•  $|x - 2| = 3$

↳ Distances of any point from 2 is 3

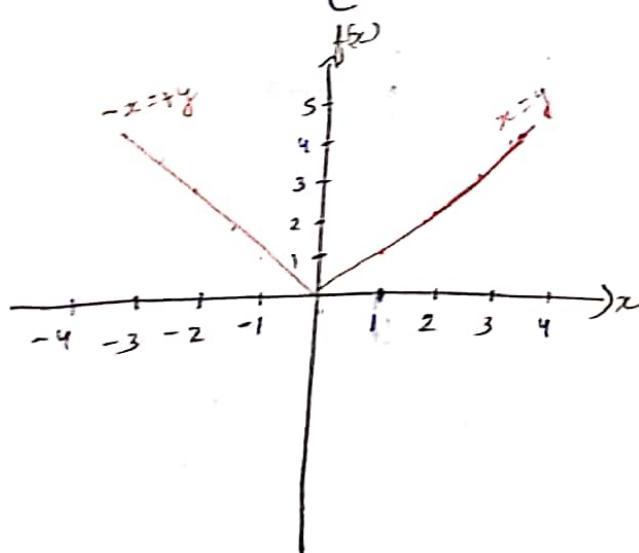


•  $|x + 5| = 7$

↳ Distance 7 from  $(-5)$ .

## \* Geographical Representation

$$f(x) = |x| \quad \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



$x$	0	1	2	3
$f(x)$	0	1	2	3

$-x$	-1	-2	-3
$f(x)$	1	2	3

Note:-

①  $|x| = | -x |$

$$|x-2| = |2-x|$$

$$|x+3| = |-x-3|$$

$$|x| \neq -|x|$$

②  $\sqrt{x^2} = |x|$

③ value of modulus cannot be negative

Q Find the value of ~~for~~  $x$ .

①  $|3x-2| = 2$

$$3x-2 = 2$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}} \checkmark$$

$$-3x+2 = 2$$

$$-3x = 0$$

$$\boxed{x = 0} \checkmark$$

②  $|8x+1| = 7$

②

$$\begin{aligned} 8x+1 &= 7 \\ x &= \frac{6}{8} \\ x &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} 8x+1 &= -7 \\ x &= \frac{-8}{8} \\ x &= -1 \end{aligned}$$

$$-8x-1 = 7$$

$$-8x = 8$$

$$\boxed{x = -1} \checkmark$$

③  $|x-7| = 0$

④  $|2x-3| = -3$  not possible

⑤  $\left| \frac{5x-10}{3} \right| = 4$

③

$$\begin{aligned} x-7 &= 0 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} -x+7 &= 0 \\ x &= 7 \end{aligned}$$

④  $x \in \emptyset$

⑤

$$\frac{5x-10}{3} = 4$$

$$5x-10 = 12$$

$$5x = 22$$

$$\boxed{x = \frac{22}{5}} \checkmark$$

$$\frac{5x-10}{3} = -4$$

$$5x-10 = -12$$

$$5x = -2$$

$$\boxed{x = -\frac{2}{5}} \checkmark$$

$$DYS-8(\text{full}) = \{x : x \in \text{Left}\} \checkmark$$

$$DYS-9(\text{full}) \checkmark$$

$$DYS-10 = \emptyset \checkmark$$

$$DYS-11 = \{1, 2, 3, 4, 5, 6\}$$

$$O-4 = \{7, 8, 9, 10\}$$

J-M, J-A full

& others left

$$DYS-11 \quad \{ [7, 21] - \{13, 7, 12 \}$$

Q

Hint:- with single modulus if constant is given in RHS or some positive quantity then we will solve modulus directly.

Q

$$\textcircled{1} \quad |x^2 - 3x + 2| = 5$$

$$x^2 - 3x + 2 = 5$$

$$x^2 - 3x - 3 = 0$$

$$x = \frac{3 \pm \sqrt{9+12}}{2}$$

$$x = \frac{3 \pm \sqrt{21}}{2}$$

$$\begin{aligned} x^2 - 3x + 2 &= -5 \\ x^2 - 3x + 7 &= 0 \\ x &= \frac{3 \pm \sqrt{9-40}}{2} \end{aligned}$$

$$x \checkmark$$

$$\textcircled{2} \quad |5x-4| = |2x-3|$$

$$5x-4 = 2x-3$$

$$\begin{cases} 3x = 1 \\ x = \frac{1}{3} \end{cases} \checkmark$$

$$5x-4 = -2x+3$$

$$\cancel{5x} - 7x = 7$$

$$\begin{cases} x = 1 \\ \end{cases} \checkmark$$

$$\textcircled{3} \quad |8x-1| = |x^2+1|$$

$$8x-1 = x^2+1$$

$$x^2 - 8x + 2 = 0$$

$$x = \frac{8 \pm \sqrt{64-8}}{2}$$

$$x = \frac{8 \pm \sqrt{56}}{2}$$

$$x = 4 \pm \sqrt{14}$$

$$8x-1 = -x^2-1$$

$$x^2 + 8x = 0$$

$$x(x+8) = 0$$

$$\begin{cases} x = 0 \\ \end{cases}$$

$$\begin{cases} x = -8 \\ \end{cases}$$

$$Q \quad |x+3| = 3|x-4|$$

~~207~~

$$2x+6 = 3x-12$$

$$\boxed{18 = x}$$

$$2x+6 = -3x+12$$

$$\begin{aligned} 5x &= 6 \\ x &= \frac{6}{5} \end{aligned}$$

$$Q \quad |x^2+x+1| = |x^2+x+2|$$

$$\cancel{x^2+x+1} = x^2+x+2$$

$$x^2+x+1 = x^2+x+2$$

$\cancel{x}$

No solution

$$x^2 + x + 1 = -x^2 - x - 2$$

$$2x^2 + 2x + 3 = 0$$

$$x = \frac{-2 \pm \sqrt{4-24}}{4}$$

$\cancel{x}$

$$Q \quad |x| = 3x$$

Case - 1

$$x \geq 0$$

$$x = 3x$$

$$x - 3x = 0$$

$$-2x = 0$$

$$\checkmark \boxed{x = 0}$$

Case - 2

$$x < 0$$

$$|x| = -x$$

$$x = -3x$$

$$x + 3x = 0$$

$$\begin{aligned} 4x &= 0 \\ \checkmark \boxed{x = 0} \end{aligned}$$

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$$Q |x^2 + 3x + 2| = x + 1$$

$$Q |x - 1| = x + 1$$

$$\text{Case 1 } x - 1 \geq 0$$

$$x \geq 1$$

$$x - 1 = x + 1$$

$$-1 = +1$$

$$x \in \emptyset$$

$$\text{Case 2 } x - 1 < 0$$

$$x - 1 < 0$$

$$x < 1$$

$$-(x - 1) = x + 1$$

$$-x + 1 = x + 1$$

$$2x = 0$$

$$\boxed{x = 0}$$

$$\boxed{x = 0}$$

$$Q |x^2 + 3x + 2| = -(x+1)$$

$$x^2 + 3x + 2 \geq 0$$

$$(x+2)(x+1) \geq 0$$

$$x^2 + 3x + 2 < 0$$

$$(x^2 + 2)(x+1) < 0$$



Case 1  $x < -2$

$$\left| (x+2)(x+1) \right| = (x+2)(x+1) \quad (\text{take } |x| = -x, \text{ as } x \in \mathbb{Q})$$

$$(x+2)(x+1) = -(x+1)$$

$$(x+2)(x+1) - (x+1)$$

$$(x+1)(x+2-1)$$

$$(x+1)(x+1)$$

$$x = -1, \quad x = -3$$

↓

satisfy

$$\boxed{x = -3}$$

Case 2-  $-2 \leq x \leq -1$

$$\left| (x+2)(x+1) \right| = -(x+2)(x+1) \quad (\text{take } |x| = -x \text{ as } x \in \mathbb{Q})$$

$$(x+2)(x+1) = -[-(x+1)]$$

$$(x+2)(x+1) - (x+1) = 0$$

$$(x+1)(x+2-1) = 0$$

$$(x+1)(x+1) = 0$$

$$\boxed{x = -1} \quad (\text{satisfy})$$

Case - 3     $x > -1$

$$(x+2)(x+1) = -(x+1)$$

$$x = -1, x = -3$$

(not satisfy)

$$\boxed{x \in \{-3, -1\}}$$

Combining answer from diff cases -

Q     $|x| = x$

Case 1

$$x \geq 0$$

$$x = x$$

$$x \in \mathbb{R}^+$$

$$x \in \mathbb{R}^+ \setminus \{0\}$$

Case 2     $x \leq 0$

$$x = -x$$

$$2x = 0$$

$$x = 0 \text{ (satisfy)}$$

$$\boxed{x \geq 0}$$

$$Q \quad |x+2| = -(x+1)$$

Case 1 ~~x > 0~~  $x > 0$

$$x+2 > 0$$

$$x > -2$$

$$x+2 = -(x+1)$$

$$x+2 = -x-1$$

$$\boxed{x = -\frac{3}{2}} \quad (\text{not satisfy})$$

Case 2  $x \leq 0$

$$x+2 = x+1$$

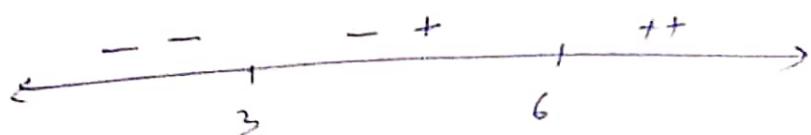
$$x+1 = x$$

$$+1 = x-x$$

$$x \in \emptyset$$

$$\boxed{x = -\frac{3}{2}}$$

$$Q \quad |x-6| + |x-3| = 1$$



Case  $x < 3$

$$-(x-6) - (x-3) = 1$$

$$-x+6 - x+3 = 1$$

$$-2x+9 = 1$$

$$9-1 = 2x$$

$$8 = 2x$$

$$x = 4 \quad (\text{not satisfy})$$

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Case 2  $3 \leq x \leq 6$

$$-(x-5) + (x-3) = 1$$

$$-x + 5 + x - 3 = 1$$

$$3 = 1$$

$$x \in \emptyset$$

Case 3  $x > 6$

$$(x-5) + (x-3) = 1$$

$$x - 5 + x - 3 = 1$$

$$2x - 9 = 1$$

$$2x = 10$$

$$x = 5 \text{ (not satisfy)}$$

$$\boxed{x \in \emptyset}$$

Q  $|2x-1| + |2x+3| = 6$

$$x = \frac{1}{2} \quad x = -\frac{3}{2}$$

$$\begin{array}{ccccccc} - & - & -\frac{3}{2} & - & + & & \\ \leftarrow & \hline & & & \rightarrow & & \\ & & & & & \frac{1}{2} & ++ \end{array}$$

Case 1  $x < -\frac{3}{2}$

$$-(2x-1) - (2x+3) = 6$$

$$-2x + 1 - 2x - 3 = 6$$

$$-4x - 2 = 6$$

$$-4x = 8$$

$$\boxed{x = -2}$$

Case 2  ~~$x$~~   $-3\frac{1}{2} \leq x \leq \frac{1}{2}$

$$-(2x-1) + (2x+3) = 6$$

$$-2x+1 + 2x+3 = 6$$

$$4 = 6$$

$$x \in \emptyset$$

Case 3

$$x > \frac{1}{2}$$

$$(2x-1) + (2x+3) = 6$$

$$4x + 2 = 6$$

$$4x = 4$$

$$\boxed{x=1} \quad (\text{not satisfy})$$

$$\boxed{\cancel{x < -3}} \quad \boxed{\cancel{x > 1}}$$

$$\boxed{x \in \{-2, 1\}}$$

### Double Inequality

$$\textcircled{1} \quad -1 \leq \underbrace{8x-3}_{-81 \leq 8x-3} \leq 5$$

$$\downarrow \quad 8x-3 \leq 5$$

$$2 \leq 8x$$

$$8x \leq 8$$

$$\frac{1}{4} \leq x$$

$$x \leq 1$$

$$x \in \left[ \frac{1}{4}, \infty \right] \xrightarrow{\cap \text{ intersection}} x \in (-\infty, 1]$$

$$Q \quad 1 \leq \frac{x^2 - 5x - 15}{x^2 + x + 1} \leq 2$$

$$\hookrightarrow 0 \leq 0$$

~~$x^2 + x + 1 \leq x^2 - 5x - 15$~~

~~$0 \leq -6x - 16$~~

~~$16 + 6x \leq 0$~~

~~$0 \leq x^2 - 5x - 15$~~

~~$x^2$~~

$$x^2 + x + 1 \leq x^2 - 5x - 15$$

$$6x + 16 \leq 0$$

$$x \in \left[ \frac{-16}{6}, \infty \right)$$

$$x^2 - 5x - 15 \leq 2x^2 + 10x + 30$$

$$0 \leq x^2 - 5x - 15$$

$$0 \leq$$

$$Q \quad 1 \leq \frac{x^2 - 5x - 15}{x^2 + x + 1} \leq 2$$

$a > 0, D < 0$   
 $\oplus \vee \ell$

$$x^2 + x + 1 \leq x^2 - 5x - 15 \leq 2(x^2 + x + 1)$$

$$x^2 + x + 1 \leq x^2 - 5x - 15 \quad | \quad x - 5x - 15 \leq 2x^2 + 2x + 2$$

$$6x \leq -16$$

$$0 \leq x^2 + 7x + 17$$

$$x^2 + 7x + 17 \geq 0$$

$a > 0, D < 0$   
 Always  $\oplus \vee \ell$   
 $x \in \mathbb{R}$

$\oplus \cap$   
Intersection

$$\boxed{x \in (-\infty, -\frac{8}{3}]}$$

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# Quadratic Equation

$$x^2 - 2x + 1 = 0$$

$$x = 1, 1$$

Solution - 1 solution [repeated roots are only counted once as solution]  
 Roots - 2 roots

- Quadratic Equation - 2 degree polynomial (2 roots)
- $ax^2 + bx + c = 0$  ( $a \rightarrow$  leading coefficient)
- $a \neq 0$ , if  $a = 0$  then quadratic will be ~~zero~~ linear.
- no. of roots = degree of polynomial

## Methods to find roots

① Shri-Dharayya  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

② Factorisation (Middle Term Splitting)

③ Perfect Square (Leading coefficient = 1)

$$\rightarrow \left( \frac{\text{coefficient of } x}{2} \right)^2 \oplus$$

e.g. ①  $x^2 - 5x + 6 = 0$

$$0 \left( \frac{-5}{2} \right)^2 = \frac{25}{4} \quad \begin{matrix} \oplus \\ \ominus \end{matrix}$$

$$x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 6 = 0$$

$$\left( x - \frac{5}{2} \right)^2 - \frac{1}{4} = 0$$

$$x - \frac{5}{2} = \frac{1}{2}$$

$$x - \frac{5}{2} = \pm \frac{1}{2}$$

$$x = \frac{1}{2} + \frac{5}{2}$$

$$x = 3$$

$$x = \frac{1}{2} - \frac{5}{2}$$

$$x = -2$$

## Relation in roots and coefficients/ constants

$$ax^2 + bx + c = 0$$

roots:-  $\alpha, \beta$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha \beta = \frac{c}{a}$$

→ Difference of Roots -

$$|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$$

$$\text{Proof: } (\alpha + \beta)^2 - (\alpha - \beta)^2 = 4\alpha\beta$$

$$\left(\frac{-b}{a}\right)^2 - (\alpha - \beta)^2 = 4 \cdot \frac{c}{a}$$

$$(\alpha - \beta)^2 = \frac{b^2}{a^2} - \frac{4c}{a}$$

$$(\alpha - \beta)^2 = \frac{b^2 - 4ac}{a^2}$$

$$|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$$

→  $\alpha^2 + b\alpha + c = 0$ ,  $\alpha$  &  $\beta$  are roots as they satisfy Q.E.

→  $a\alpha^2 + b\alpha + c = 0$  &  $a\beta^2 + b\beta + c = 0$  & DOR (sum of roots & Product of roots)

Find some values using SOR & POR

$$(1) \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha^2 + \beta^2 = (\alpha - \beta)^2 + 2\alpha\beta$$

$$(2) \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$(3) \alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$$

Q  $x^2 - 4x + 2 = 0$  (roots are  $\alpha$  &  $\beta$  then find)

①  $\alpha + \beta$

$$\alpha + \beta = \frac{-b}{a}$$

$$= 4 \checkmark$$

②  $\alpha^2 + \beta^2$

$$\alpha\beta = \boxed{2}$$

$$\alpha - \beta = \sqrt{\frac{16 - 8}{1}}$$

$$= 2\sqrt{2}$$

$$\begin{aligned}\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 16 - 2 \cdot 4 \\ &= \boxed{8} \checkmark\end{aligned}$$

③  $\alpha^3 + \beta^3$

$$\begin{aligned}\alpha^3 + \beta^3 &= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) \\ &= 4(8 - 2 \cdot 2) \\ &= 64 - \cancel{28} \\ &= \boxed{40} \checkmark\end{aligned}$$

④  $\frac{\alpha\beta}{\alpha + \beta}$

$$\frac{2}{4} = \boxed{\frac{1}{2}} \checkmark$$

⑤  $\alpha - \beta = \sqrt{\frac{16 - 8}{1}}$

$$= \boxed{2\sqrt{2}} \checkmark$$

⑥  $\frac{1}{\alpha^2 + \beta^2}$

$$\frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$\frac{8}{4} = \boxed{\sqrt{2}} \boxed{3} \checkmark$$

⑦  $\alpha^2 - \beta^2$

$$(\alpha + \beta)(\alpha - \beta)$$

$$(4)(2\sqrt{2})$$

$$\boxed{8\sqrt{2}} \checkmark$$

⑧  $\alpha^3\beta - \alpha\beta^3$

$$\alpha\beta(\alpha^2 - \beta^2)$$

$$2(8\sqrt{2})$$

$$\boxed{16\sqrt{2}} \checkmark$$

$$Q \quad 3x^2 + 7x + 3 = 0$$

$$\textcircled{1} \quad \frac{\beta + \alpha}{\alpha \beta}$$

$$\alpha + \beta = -\frac{7}{3}$$

$$\frac{\beta^2 + \alpha^2}{\alpha \beta}$$

$$\alpha \beta = \frac{3}{3} = 1$$

$$\alpha - \beta = \frac{\sqrt{13}}{3}$$

$$\frac{\frac{49}{9} - 2}{21}$$

$$\frac{49 - 18}{27} =$$

$$= \boxed{\frac{31}{27}} \checkmark$$

$$\textcircled{2} \quad \alpha^2 \beta + \alpha \beta^2$$

$$\alpha \beta (\alpha + \beta)$$

$$1 \left( -\frac{7}{3} \right)$$

$$\cancel{\left( -\frac{7}{3} \right)} \boxed{-\frac{7}{3}} \checkmark$$

$$\textcircled{3} \quad \alpha^4 \beta^7 + \alpha^7 \beta^4$$

$$\alpha^4 \beta^4 (\alpha^3 + \beta^3)$$

$$\left( -\frac{7}{3} \right)^3 - 3 \left( -\frac{7}{3} \right)$$

$$\frac{-343}{27} + 7$$

$$\cancel{-343 + 21}$$

$$-343 - \frac{343 + 189}{27}$$

$$\boxed{\frac{-154}{27}} \checkmark$$

$$\begin{aligned} \textcircled{4} \quad & \left( \frac{\alpha - \beta}{\alpha \beta} \right)^2 \\ & \left( \frac{\alpha^2 - \beta^2}{\alpha \beta} \right)^2 \\ & \left[ \frac{\left( -\frac{7}{3} \right) \left( \frac{\sqrt{13}}{3} \right)}{1} \right]^2 \end{aligned}$$

$$\begin{array}{r} 1 \\ 27 \\ \hline 189 \end{array}$$

$$\left( \frac{-7\sqrt{13}}{9} \right)^2$$

$$\frac{49 \times 13}{81}$$

$$\boxed{\frac{637}{81}}$$

$$\begin{array}{r} 2 \\ 13 \\ \hline 147 \\ 490 \\ \hline 637 \end{array}$$

$$\textcircled{5} \quad \frac{\alpha^3 - \beta^3}{\alpha^2 - \beta^2}$$

$$\frac{(\alpha - \beta)^3 + 3\alpha \beta (\alpha - \beta)}{-7\sqrt{13}}$$

$$\begin{array}{r} 1 \\ 27 \\ \hline 49 \\ 7 \\ \hline 343 \end{array}$$

$$\begin{array}{r} 9 \\ 27 \\ \hline 7 \\ \hline 189 \end{array}$$

$$\frac{(\sqrt{13})^3}{27} + \beta \times \frac{\sqrt{13}}{3}$$

$$\frac{(\sqrt{13})^3 + 27\sqrt{13}}{27(-7\sqrt{13})}$$

$$\sqrt{3} \frac{(\sqrt{13})^2 + 27\sqrt{13}}{-189\sqrt{13}}$$

$$\begin{array}{r} 13 + 27 \\ - 189 \\ \hline -16921 \\ \boxed{-40} \\ \hline 18941 \end{array}$$

223

Q If  $x^2 - x + 1$  has roots  $\alpha, \beta$  find

$\textcircled{1} \quad \alpha^2 = \alpha$ $\alpha(\alpha-1)$ $\alpha + \beta = 1$ $\alpha\beta = 1$ <del><math>\alpha \neq \beta</math></del> $\alpha(-\beta)$ $-\alpha\beta$ <hr/> $\boxed{-1}$	$\textcircled{2} \quad \alpha^{15} + \beta^{15}$ <del><math>(1-\beta)^{15} + \beta^{15}</math></del> $\alpha^2 - \alpha + 1 = 0$ $\alpha^2 - \alpha = -1$ $\alpha^3 = \alpha^2 - \alpha$ $\alpha^3 = -1$ $(\alpha^3)^5 = \alpha^{15}$ $\alpha^{15} = (-1)^5$ $\alpha^{15} = -1$ $\beta^{15} = -1$ $-1 + (-1)$ <hr/> $\boxed{-2}$	$\textcircled{3} \quad \alpha^{2025} + \beta^{2025}$ $\alpha^3 = -1$ $\alpha^{2025} = -1$ $\beta^{2025} = -1$ $-1 + (-1)$ <hr/> $\boxed{-2}$
---	---	---

Q find qnd whose roots are

$$\frac{1}{5+2\sqrt{6}} \quad \& \quad \frac{1}{5-2\sqrt{6}}$$

$$\begin{aligned} \alpha + \beta &= \frac{5-2\sqrt{6} + 5+2\sqrt{6}}{25-24} \\ &= 10 \end{aligned}$$

$$\alpha\beta = 1$$

$$\boxed{x^2 - 10x + 1 = 0}$$

M.W. 16-05-2024

DYS-1  $\{0, 1, 2, 3, 5, 6, 7, 8, 9, 10, 13, 12, 11, 14, 4\}$

[1714]

Q find roots  $s^2 + a - 4444444222222 = 0$

~~$a \neq 0$~~   $a(a+1) = 4444444 | 2222222$

$\sqrt{a} = 4444444222222$

$a(a+1) = 4$

$\frac{a(a+1)}{y} = 1$

$\frac{a(a+1)}{y} - 1 = 0$

$\frac{a(a+1) - y}{y} = 0$

Q find roots of

~~$\alpha \beta = -1$~~

~~$\alpha \beta = -4444444222222$~~

~~Solve~~

~~NOT defined~~

~~0~~

4444444

y

2

$$4 = y \quad 2 = z \\ 7y + 7z = a(a+1)$$

$s^2 + a - 4444444222222 = 0$

## Nature of roots

$$ax^2 + bx + c \quad (a, b, c \in \mathbb{R})$$

$$D = b^2 - 4ac$$

$$D > 0$$

Distinct real roots

$$D = 0$$

Equal real roots

$$D < 0$$

Imaginary & distinct  
pairs.

$$D = b^2 - 4ac \quad (a, b, c \in \mathbb{Q})$$

Irrational

Rational no.

Perfect square

Perfect square

Not a perfect square

Roots are irrational in pair

$$a + \sqrt{b} \quad \& \quad a - \sqrt{b}$$

Q Find nature of roots

$$\textcircled{1} \quad x^2 + x + 1 = 0$$

$$D = (1)^2 - 4(1)(1) = 1 - 4$$

$$= -3$$

Imaginary & paired ✓

$$\textcircled{2} \quad 2x^2 - 6x + 3 = 0$$

$$D = 36 - 24 = 12$$

Real, Irrational  
in pair ✓

$$\textcircled{3} \quad 3x^2 - 4\sqrt{3}x + 4 = 0$$

~~Irish~~

$$D = 48 - 48 = 0$$

$$\frac{4\sqrt{3}}{6} \neq 0$$

Real & equal, irrational ✓

Q find the value of  $m$  if equation  $x^2 + 2x + m^2 = 0$  have real roots.

$$\Delta \leq 0$$

$$1 - 4m^2 \leq 0$$

$$1 - 4m^2 = 0 \Rightarrow 1 = 4m^2 \Rightarrow m^2 = \frac{1}{4}$$

$$\left[ m \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty) \right] \checkmark$$

$$1 - 4m^2 = 0 \\ 4m^2 \geq 1 \\ m^2 \geq \frac{1}{4}$$

Q Value of  $\alpha$  for which roots of the eq.  $(2\alpha+5)x^2 + 2(\alpha-1)x + 3 = 0$  are real & equal

$$\Delta = 0$$

~~$$(2\alpha+5)^2 - 4(2\alpha+5)(2\alpha-1) = 0$$~~

$$4(\alpha^2 + 10\alpha + 25) - 4(4\alpha^2 + 6\alpha - 5) = 0$$

$$4\alpha^2 + 40\alpha + 100 - 16\alpha^2 - 24\alpha + 20 = 0$$

$$12\alpha^2 - 16\alpha - 80 = 0$$

$$\alpha^2 - \frac{4}{3}\alpha - \frac{20}{3} = 0$$

$$12\alpha^2 - 16\alpha - 27 = 0$$

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\alpha = \frac{-(-16) \pm \sqrt{(-16)^2 - 4 \cdot 12 \cdot (-27)}}{2 \cdot 12}$$
$$\alpha = \frac{16 \pm \sqrt{256 + 1296}}{24}$$
$$\alpha = \frac{16 \pm \sqrt{1552}}{24}$$
$$\alpha = \frac{16 \pm 4\sqrt{39}}{24}$$
$$\alpha = \frac{4(4 \pm \sqrt{9})}{24}$$
$$\alpha = \frac{4(4 \pm 3)}{24}$$
$$\alpha = \frac{4 \cdot 7}{24} \quad \text{or} \quad \alpha = \frac{4 \cdot 1}{24}$$
$$\alpha = \frac{7}{6} \quad \text{or} \quad \alpha = \frac{1}{6}$$

$$\frac{24}{6}$$

$$\alpha^2 - 8\alpha + 16 = 0$$

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{\alpha = 4}$$

$$Q \quad x^4 + (2a - \sqrt{a^2 - b})x^2 + b = 0$$

$$(2a - \sqrt{a^2 - b})^2 - (4)(4)(b) = 0$$

$$4a^2 - 8a\sqrt{a^2 - b} + 16b = 0$$

$$4a^4 - 12a^2b + 16b = 0$$

$$2a^4 - 6a^2b + 8b = 0$$

$$a^4 - 3a^2b + 4b = 0$$

$$a^2 = \frac{-3 \pm \sqrt{9 + 16}}{2}$$

$$a^2 = \frac{-3 + 5}{2}, \quad a^2 = \frac{2}{2}$$

$$a^2 = \frac{2}{2}, \quad a^2 = \frac{-2}{2}$$

$\checkmark a = \pm 2$ ,  $\checkmark a = \pm i$

Q find 'c' if  $(c-2)x^2 + 2(c-2)x + 2 = 0$  no real roots

~~$$\begin{aligned} & 16 - 8c + c^2 \\ & 32 + 8c - 8c \\ & 16 - 8c + c^2 \\ & -4 \quad \boxed{N > 4} \end{aligned}$$~~

$$(c-2)(c-4) \neq 0$$

$$\frac{1}{c-2} = \frac{1}{4} \Rightarrow$$

$$\boxed{(2, 4)}$$

~~$$(-4)^2 - (4)(2)(-2)$$~~

~~$$16 - 8(c-2)$$~~

for leading coefficient to 0

$$D = 4c^2 + 16 - 16c - 8c + 16$$

$$D = 4c^2 - 24c + 32$$

$$D = c^2 - 6c + 8$$

$$c^2 - 4c - 2c + 8$$

$$c(c-4) - 2(c-4)$$

$$(c-2)(c-4)$$

(2, 4)

Case 2:- leading coefficient = 0

$$C=0 \quad 2$$

$$(2-2)x^2 + 2(2-2)x + 2 = 0$$

$$0x^2 + 0x + 2 = 0$$

$$2 = 0$$

not a real value

so Roots are imaginary

$$C \in [2, 4)$$

This is because question is not mentioned that given is quadratic equation. So leading coefficient can be zero making the equation linear.  
If question mentions quadratic equation, take leading coefficient ≠ 0.

Q DYS - 2

$$Q11. (K-12)x^2 + 2(K-12)x + 2 = 0$$

find integral values of K for which quadratic equation possess no real roots.

$$(K-12)^2 - (4)(2K-24) < 0$$

$$D = 4(K^2 + 144 - 24K) - 8K + 96 < 0$$

$$D = 4K^2 + 0.576 - 96K - 8K + 96 < 0$$

$$D = 4K^2 - 104K + 672 < 0$$

$$D = K^2 - 26K + 168 < 0$$

$$(K-12)(K-14) < 0$$



$$(12, 14)$$

so  $K = 13$

Q2)  $x = 1 + 2i$   
 find  $x^3 + x^2 - x + 22$

$$\begin{aligned}
 & x - 1 = 2i \\
 & (x - 1)^2 = -2 \\
 & x^2 + 1 - 2x = -2 \\
 & x^2 - 2x + 1 = -2 \\
 & 3x^2 - 6x + 03 = -6 \\
 & x(x^2 + x - 1) + 22 \\
 & \cancel{x(x+1)} \\
 & x(x^2 - 2x + 1 + 3x - 2) + 22 \\
 & x(3x - 4) + 22 \\
 & 3x^2 - 4x + 22 = \\
 & 3x^2 - 6x + 3 + 19 + 2x = 0 \\
 & -6 + 19 + 2x \\
 & 13 + 2x
 \end{aligned}$$
  

$$\begin{aligned}
 & x = 1 + 2i \\
 & x - 1 = 2i \\
 & x^2 - 2x + 3 = 0 \quad x^2 + 1 - 2x = -4 \\
 & x = 2 \pm \sqrt{4} - \\
 & x^2 - 2x + 5 = 0 \\
 & x = 2 \pm \sqrt{4} \\
 & 3x^2 - 6x + 15 = 0 \\
 & x(x^2 - 2x + 5 + 3x - 6) + 22 \\
 & x(3x - 6) + 22 \\
 & 3x^2 - 6x + 22 \\
 & 3x^2 - 6x + 15 + 7 = 22 \\
 & \boxed{7} \checkmark
 \end{aligned}$$

Q16. find the value of  $x^3 - 3x^2 - 8x + 15$   
 $x = 3 + i$

$$\begin{aligned}
 & x - 3 = i \\
 & x^2 + 9 - 6x = -1 \\
 & \underline{\cancel{x^2 + 10 - 6x = 0}} \\
 & \underline{\cancel{3x^2 + 30 - 18x = 0}} \\
 & x(x^2 - 3x - 8) + 15 = \\
 & x(x^2 - 6x + 10 + 3x - 18) + 15 \\
 & x(3x - 18) + 15 \\
 & 3x^2 - 16x + 15 \\
 & \boxed{-15} \quad \boxed{-15}
 \end{aligned}$$

(230)

H.W.

$$DVS-2 [1, 16] - \left[ \{11, 16\} \cup \{5\} \right]$$

$$\alpha^2 + \alpha = 44444442222222$$

$$= 4(111111)$$

$$= 44444440000000 + 2222222$$

$$= 4444444 \times 10000000 + 2222222$$

$$= 4444444(9999999+1) + 2222222$$

$$= 4(111111) [9(111111)+1] + 2(111111)$$

$$= 4P(9P+1) + 2P$$

$$= 4y(9y+1) + 2y$$

$$= 36y^2 + 4y + 2y$$

$$\alpha^2 + \alpha = 36y^2 + 6y$$

$$\alpha^2 - 36y^2 + \alpha - 6y$$

~~$$(\alpha + 6y)(\alpha - 6y) - (\alpha + 6y)$$~~

~~$$\alpha + 6y (\alpha - 6y - 1) = 0$$~~

~~$$\alpha + 6y = 0$$~~

~~$$\alpha = -6y$$~~

~~$$\alpha = -6(111111)$$~~

~~$$\alpha = -6666666$$~~

$$\alpha - 6y - 1 = 0$$

$$\alpha = 6y + 1$$

$$(\alpha - 6y)(\alpha + 6P + 1) = 0$$

$$\alpha = 6y$$

$$\boxed{\alpha = 6666666}$$

$$\alpha = -6P - 1$$

$$\alpha = -6666666 - 1$$

$$\boxed{\alpha = -6666667}$$

Q find the quadratic if 1 root is  $5+2\sqrt{6}$  & coefficients of quadratic are rational.

$$\cancel{x} - \alpha = 5+2\sqrt{6}$$
$$\beta = 5-2\sqrt{6}$$

$$\alpha + \beta = 5+2\sqrt{6} + 5 - 2\sqrt{6}$$
$$= 10$$

$$\alpha \beta = (5+2\sqrt{6})(5-2\sqrt{6})$$
$$= 25 - 4 \times 6$$
$$= 25 - 24$$

$$\boxed{x^2 - 10x + 1} = 0$$

Q find p & q if the roots of the eq  $x^2 + px + q = 0$  have ~~been~~ roots p & q.

~~$\alpha + \beta = -p$~~   
 ~~$\alpha \beta = q$~~

$$p+q = -p$$

$$pq = q$$

$$pq - q = 0$$

$$q(p-1) = 0$$

$$\boxed{\begin{aligned} q &= 0 \\ p &= 0 \end{aligned}}$$

$$\begin{aligned} p &= 1 \\ q &= -2 \end{aligned}$$

$$p, q = (1, -2), (0, 0)$$

Q)  $x^2 + mx + 1 = 0$  find  $m$  if

- a) One root is thrice of other
- b) Ratio of roots is  $\frac{1}{3}$
- c) Sum of roots is equal to PQR.

a)  $\alpha = 3\beta$

$$\begin{aligned} \alpha + \beta &\equiv -m \\ 4\beta &= -m \end{aligned}$$

$$\boxed{-\frac{4\sqrt{3}}{3}}$$

$$\begin{aligned} \sqrt{\beta} &= 1 \\ 3\beta(\beta) &= 1 \\ 3\beta^2 &= 1 \\ \beta^2 &= \frac{1}{3} \\ \beta &= \frac{\sqrt{3}}{3} \end{aligned}$$

$$\boxed{m = \pm \frac{4}{\sqrt{3}}}$$

b)  $\frac{\alpha}{\beta} = \frac{1}{3}$

$$\begin{aligned} 3\alpha &= \beta \\ \alpha\beta &= 1 \\ 3\alpha^2 &= 1 \\ \alpha &= \frac{\sqrt{3}}{3} \end{aligned}$$

$$\cancel{\frac{\sqrt{3}}{3} + \beta = -m}$$

$$\begin{aligned} \alpha + \beta &= -m \\ \cancel{\frac{\sqrt{3}}{3} + \beta} &= -m \\ 4\alpha &= -m \end{aligned}$$

$$\boxed{-\frac{4\sqrt{3}}{3} = m}$$

$$\boxed{m = \pm \frac{4}{\sqrt{3}}}$$

c)  $\alpha + \beta = \alpha\beta$

$$\begin{aligned} -m &= 1 \\ \boxed{m = -1} \end{aligned}$$

## Symmetric Expression of $\alpha$ and $\beta$

If  $(\alpha, \beta) = f(\beta, \alpha)$

Ex ①  $f(\alpha, \beta) = \alpha^2 + \beta^2$

$$f(\beta, \alpha) = \frac{\beta^2 + \alpha^2}{\alpha^2 + \beta^2}$$

So, symmetric

Ex ②  $f(\alpha, \beta) = \alpha^2 - \beta^2$

$$f(\beta, \alpha) = \beta^2 - \alpha^2 \neq \alpha^2 - \beta^2$$

So, not symmetric

Q find which are symmetric

①  $f(\alpha, \beta) = \alpha^4 - \beta^4$

$$f(\beta, \alpha) = \beta^4 - \alpha^4 \neq \alpha^4 - \beta^4$$

Not Symmetric

②  $f(\alpha, \beta) = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$f(\beta, \alpha) = \frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

Symmetric

③  $f(\alpha, \beta) = \frac{\alpha + \beta}{\alpha \beta}$

$$f(\beta, \alpha) = \frac{\beta + \alpha}{\beta \alpha} = \frac{\alpha + \beta}{\alpha \beta}$$

Symmetric

## Transformation of roots (valid for symmetric changes)

Eg 1.  $x^2 - 4x + 5 = 0$   
 find quadratic whose roots are  $\frac{2}{\alpha}$  and  $\frac{2}{\beta}$

$$\frac{2}{\alpha}, \frac{2}{\beta} = \frac{2}{x} \rightarrow t$$

$$x = \frac{2}{t}$$

$$\left(\frac{2}{t}\right)^2 - 4\left(\frac{2}{t}\right) + 5 = 0$$

(put in quad)

$$\frac{4}{t^2} - \frac{8}{t} + 5 = 0$$

$$4t^2 - 8t + 5 = 0$$

Q2.  $x^2 - 3x + 2 = 0$

find quad of roots  $\alpha + 1, \beta + 1$

$$\alpha + 1, \beta + 1 \rightarrow x + 1 = t$$

$$x = t - 1$$

$$(t - 1)^2 - 3(t - 1) + 2 = 0$$

$$t^2 - 2t + 1 - 3t + 3 + 2 = 0$$

$$t^2 - 5t + 6 = 0$$

$$t^2 - 5t + 6 = 0$$

Method II

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$x(x - 2) - 1(x - 2) = 0$$

$$x = 1, 2$$

$$\alpha + 1 = 1 + 1 = 2$$

$$\beta + 1 = 2 + 1 = 3$$

$$\alpha + \beta = 5$$

$$\alpha \beta = 6$$

$$x^2 - 5x + 6 = 0$$

Q  $x^2 - \alpha x + 1 = 0$  have roots  $\alpha$  and  $\beta$ , find quadratic whose roots are -

$$\textcircled{1} \quad 3+\alpha, 3+\beta$$

$$x+3=y$$

$$x=y-3$$

$$(y-3)^2 - (y-3) + 1 = 0$$

$$y^2 + 9 - 6y - y + 3 + 1 = 0$$

$$\boxed{y^2 - 7y + 13 = 0}$$

$$\textcircled{2} \quad 1 - \frac{1}{\alpha}, 1 - \frac{1}{\beta}$$

$$1 - \frac{1}{\alpha} = y$$

$$1-y = \frac{1}{\alpha}$$

$$x = \frac{1}{1-y}$$

$$\left(\frac{1}{1-y}\right)^2 - \left(\frac{1}{1-y}\right) + 1 = 0$$

$$\frac{1}{(1-y)^2} - \frac{1}{1-y} + 1 = 0$$

$$(1-y)^2 - (1-y) + 1 = 0$$

$$y^2 + 1 - 2y - 1 - y + 1 = 0$$

$$\boxed{y^2 - 3y + 1 = 0}$$

$$\textcircled{3} \quad \frac{2}{1+\alpha}, \frac{2}{1+\beta} = 0$$

$$\frac{2}{1+y} = y$$

$$2 \cdot \frac{2}{y} = 1+y$$

$$2c = \frac{2-y}{y}$$

$$\left(\frac{2-y}{y}\right)^2 - \left(\frac{2-y}{y}\right) + 1$$

$$\frac{y^2 + 4 - 4y}{y^2} - \frac{(2-y)}{y} + 1$$

$$y^2 + 4 - 4y - 2y + y^2 + y^2$$

$$\boxed{3y^2 - 6y + 4 = 0}$$

$$\textcircled{4} \quad -\alpha, -\beta$$

$$-x = y$$

$$x = -y$$

$$(-y)^2 - (-y) + 1 = 0$$

$$\boxed{y^2 + y + 1 = 0}$$

$$Q \quad \cancel{x^2 +} x^2 - x + 1 = 0 \quad \text{have roots } \alpha \text{ & } \beta$$

find quadratic whose roots are.

$$(1) \quad \alpha + 3\beta, 3\alpha + \beta$$

$$(2) \quad (\alpha - \beta)^2, (\alpha + \beta)^2$$

$$(1) \quad \alpha + \beta = 1$$

$$\alpha \beta = 1$$

$$\alpha - \beta = \sqrt{-3}$$

$$R_1 = \alpha + 3\beta$$

$$R_2 = 3\alpha + \beta$$

$$R_1 + R_2 = \alpha + 3\beta + 3\alpha + \beta$$

$$= 4\alpha + 4\beta$$

$$= 4(\alpha + \beta)$$

$$= 4(1)$$

$$= 4$$

$$R_1 \times R_2 = (\alpha + 3\beta)(3\alpha + \beta)$$

$$= 3\alpha^2 + 3\beta^2 + \alpha\beta + 9\alpha\beta$$

$$= 3((\alpha + \beta)^2 - 2\alpha\beta) + 10\alpha\beta$$

$$= 3(1-2) + 10$$

$$= -3 + 10$$

$$= 7$$

$$\therefore t^2 - (4) + 7$$

$$t^2 - 4t + 7$$

$$(2) \quad \alpha + \beta = 1$$

$$\alpha \beta = -1$$

$$R_1 + R_2 = (\alpha - \beta)^2 + (\alpha + \beta)^2$$

$$= \alpha^2 + \beta^2 - 2\alpha\beta + \alpha^2 + \beta^2 + 2\alpha\beta$$

$$= 2(\alpha^2 + \beta^2)$$

$$= 2(-1)$$

$$= -2$$

$$R_1 \times R_2 = (\alpha - \beta)^2 (\alpha + \beta)^2$$

$$= (\alpha^2 + \beta^2 - 2\alpha\beta)(\alpha^2 + \beta^2 + 2\alpha\beta)$$

$$= (\sqrt{-3})^2 (1)^2$$

$$= -3 \times 1$$

$$= -3$$

$$x^2 + 2x - 3$$

### Equation (vs) Identity

→ Equation which is true for every value of the variable is a identity

$$\text{eg. } \frac{\sin^2 \theta + \cos^2 \theta}{(\theta+1)^2} = 1 \quad (\theta, \theta+1 \in \mathbb{R})$$

→ In quadratic if it has more than 2 roots then it will be an identity

Q.  $(p-1)x^2 + (p^2 - 3p + 2)x + (p^2 - 4p + 3) = 0$

find value of  $p$  it is an identity in  $x$ .

$$(p-1)x^2 + (p^2 - 3p + 2)x + (p^2 - 4p + 3) = 0$$

$$(p-1)x^2 + (p^2 - 3p + 2)x + (p^2 - 4p + 3) = 0$$

$$p-1=0 \qquad \qquad p^2 - 3p + 2 = 0 \qquad \qquad p=3, 1$$

$$p=1 \qquad \qquad \qquad p=2, 1$$

$\boxed{p=1}$

for  $p=1$

$$(1-1)x^2 + 0x + 0 = 0$$

Q. find  $x$  if Quad in  $\lambda$  have more than 2 roots.

$$x^2(\lambda^2 - 5\lambda - 16) + x(\lambda^2 + 3\lambda + 2) + \lambda^2 - 4 = 0$$

~~$x^2 - 5x^2\lambda + 6x^2$~~

$$x^2\cancel{\lambda^2} - 5x^2\lambda + 6x^2 + x\cancel{\lambda^2} + x\cancel{3\lambda} + 2x + \lambda^2 - 4 = 0$$

$$x^2(-5x^2 + 3x) + (6x^2 + 2x - 4)$$

$$\lambda^2(x^2 + x + 1) + x(-5x^2 + 3x) + (6x^2 + 2x - 4)$$

$$\downarrow$$

$$x = x \in \phi$$

no common

$\boxed{x \in \phi}$

$$x^2 + x + 1$$

$$x = -1 \pm \sqrt{-3}$$

$$\textcircled{1} \quad \frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-a)(x-c)}{(b-c)(b-a)} + \frac{c(x-a)(x-b)}{(c-a)(c-b)} = x$$

$$\cancel{-ax^2(b-c)} \\ \left. \begin{array}{l} \text{let } xc = a \\ a+0+0=a \end{array} \right] \rightarrow \frac{a(x-b)(x-c)}{(a-b)(a-c)} + 0+0 = 0 \Rightarrow a=a$$

$$x=6$$

$$0+6+0=6$$

$$0c=c$$

$$0a+0+c=c$$

So  $x=a, b, c$   
as its a identity.

H.W. (18 - 05 - 24)

$$\text{DYS-2 } [17, 27] - \{27\}$$

$$\text{DYS-3 (full)} - \{2, 18, 6\}$$

$$\text{DYS-4 } \{1, 2\}$$

$$\text{base } \{9, 10, 11, 12, 13, 14\}$$