Units & Dimentions

Physical Quantity - Can be measured eg. time, distance

Fundamental Quantity (7)

1 → Mass → kg
2 → Length - m
3 → Time - second
4 → Temperature - K
5 → Electric Current - Ampere
6 → Luminous Intensity - Candela (cd)
7 → Amount of Substance - mole

Derived Quantity

eg. → speed = $\frac{\text{distance}}{\text{time}}$ (m/s)

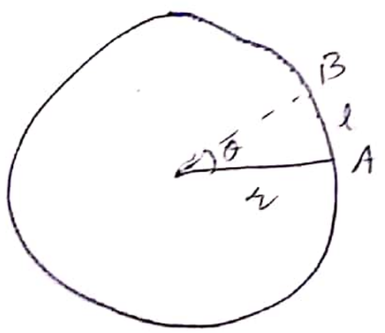
→ velocity
→ momentum
→ density (kg/m^3)

$1\ g/cc = 10^3\ kg/m^3$
= water density (4°C)

water
$0°C \xrightarrow[\text{contract}]{} 4°C \xrightarrow[\text{expand}]{}$

Supplementary units

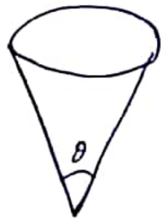
1. Plane Angle
SI → radian



$\theta = \frac{l}{r}$

2. Solid Angle

SI ⇒ Ste-radian



Practical Units

1. Light Year
2. Horse Power (HP)

   1 HP = 746 W
3. mile (1 mile = 1·6 km)

Improper Units

1. kg_f = 1 kg-wt = 9·8 N

   eg. 60 kg_f = 60 × 10 = 600 N

Dimensions → Dimensions of a physical quantity are the powers to which fundamental quantities must be raised to represent a given physical quantity.

eg. Dimensions of Mass - $[M^1]$

Length - $[L]$

Time - $[T]$

Temperature - $[\theta]$

Current - $[A]$

dimensions of speed $= \frac{dist}{time} = \frac{[L^1]}{[T^1]} = [L T^{-1}]$

~~density~~ energy =

work $= F \times S = [MLT^{-2}][L]$

$= [ML^2T^{-2}]$

Pressure $= \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

Force $= ma = M \times LT^{-2} = [MLT^{-2}]$

Applications of Dimensional Analysis

(1) To find dimensions of physical constants

$F = \frac{GMm}{d^2}$

$N = \frac{G\,kg^2}{m^2}$

$G = N\,m^2\,kg^{-2}$

$G = [M^{-1}L^3T^{-2}]$

$E = h\nu \rightarrow$ frequency
$\downarrow$
Plank's constant

$\nu = \frac{1}{\cancel{f}\, \rho t}$

$$ML^2T^{-2} = h\,T^{-1}$$

$$h = [ML^2T^{-1}]$$

2. <u>To check if a equation is dimentionaly correct</u>

eg. $F = ma^2$

$MLT^{-2} = M\,[LT^{-2}]^2$

$MLT^{-2} \neq ML^2T^{-4}$ ($F = ma^2$ is wrong as dimensions are diff)

$$F = \frac{mv^2}{r}$$

$$= \frac{M\,[LT^{-1}]^2}{L}$$

$$= \frac{ML^{\cancel{2}}T^{-2}}{\cancel{L}}$$

$MLT^{-2} = MLT^{-2}$ (correct equ)

$S = ut + \frac{1}{2}at^2$ (numbers are dimension less)

$([L]+[L] = [L])$

$L = LT^{-1} \times T + \frac{1}{2}[LT^{-2}][T^2]$

$= L + \cancel{f}L$

$[L] = [L]$ ~~correct~~ dimensonally correct

(4)

3. Derive new equations

$$E = mc^2$$

$$E = f(m, c)$$

$$E = K\, m^x c^y \quad (K \text{ is constant})$$

$$ML^2T^{-2} = K\, M^x [LT^{-1}]^y$$

$$ML^2T^{-2} = K\, M^x L^y T^{-y}$$

$$x = 1$$
$$y = 2$$

$$E = K\, mc^2$$

$K = 1$ (by experiments)

$$\boxed{E = mc^2}$$

Q If velocity, force & time are taken as funds. quan. find dimensions of mass & energy

$\text{vel} = \frac{\text{dis}}{\text{time}}$

$\text{force} = \text{mass} \times \frac{\text{dis}}{\text{time}^2} \times \frac{\text{vel}}{\text{time}}$

$$\frac{MLT^{-2} \times T}{LT^{-1}}$$

M

$$m = f(v, f, t)$$

$$m = K\, v^x F^y t^z$$

$$m = K\, [LT^{-1}]^x [MLT^{-2}]^y [T]^z$$

$$M^1 = K\, M^y L^{x+y} T^{-x-2y+z}$$

$y = 1$
$x + y = 0$

5

$$E = f(v, f, t)$$

$$ML^2T^{-2} = M^y L^{x+y} T^{-x-2y+z}$$

$$\boxed{y = 1}$$

$$x + y = 2$$

$$\boxed{x = 1}$$

$$-x - 2y + z = -2$$

$$-1 - \cancel{2} + z = -\cancel{2}$$

$$\boxed{z = 1}$$

$$\boxed{E = K\, v^1 F^1 T^1}$$

Q if momentum, area & time are taken

$E = ?$

area $= L^2$

Time $= T^1$

momentum $= MLT^{-1}$

$$ML^2T^{-2} = K P^x A^y T^z$$

$$ML^2T^{-2} = K [MLT^{-1}]^x [L^2]^y [T]^z$$

$$= K M^x L^{x+2y} T^{-x+z}$$

$$\boxed{x = 1}$$

$$x + 2y = 2$$

$$2y = 1$$

$$\boxed{y = 1/2}$$

$$z = 1 + -2$$

$$\boxed{z = -1}$$

$$\boxed{E = K\, P A^{1/2} T^{-1}}$$

Q

$$P = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$D = \frac{M}{L^3} = ML^{-3}$$

$$E = ML^2T^{-2}$$

$$T' = K\left(ML^{-1}T^{-2}\right)^x \left[ML^{-3}\right]^y \left[ML^2T^{-2}\right]^z$$

$$T' = K\ M^{x+y+z}\ L^{-x-3y+2z}\ T^{-2x-2z}$$

$$-2x-2z = 1$$

$$-2z = 2x$$

$$\boxed{z = -x}$$

$$z = \frac{1+2x}{2}$$

$$-x-3y+2x = 0$$

$$-3y = 3x$$

$$\boxed{x = -y}$$

$$-y+y+z = 0$$

$$\boxed{z=0}$$

$$\boxed{x=0}$$

$$\boxed{y=0}$$

$$-x-3y+2\left(\frac{1+2x}{2}\right)$$

$$-x+3y+1+2x$$

$$x+3y+1=0$$

$$\boxed{x = -1-3y}$$

$$-1-3y+y+\frac{1+2(-1-3y)}{2}$$

$$-1-3y+y+\frac{1-2-6y}{2}$$

$$\frac{-2-4y+1-2-6y}{2}$$

$$\frac{-3-10y}{2} = 0$$

$$y = \frac{3}{-10}$$

$$\boxed{y = \frac{-3}{10}}$$

$$x = -1-3\left(\frac{-3}{10}\right)$$

$$= -1+\frac{9}{10}$$

$$= \frac{-10+9}{10}$$

$$\boxed{x = \frac{-1}{10}}$$

$$z = \frac{1-\frac{2}{10}}{2}$$

$$z = \frac{\frac{8}{10}}{2}$$

$$\boxed{z = \frac{4}{10} = \frac{2}{5}}$$

7

$$T = K\, P^{x} d^{y} E^{z}$$

$$M^0 L^0 T^1 = [ML^{-1}T^{-2}]^{x}\ [ML^{-3}]^{y}\ [ML^{2}T^{-2}]^{z}$$

$$M^0 L^0 T^1 = M^{x+y+z}\ L^{-x-3y+2z}\ T^{-2x-2z}$$

$$x + y + z = 0$$

$$-x - 3y + 2z = 0$$

$$-2x - 2z = 1$$

$$-x = \frac{1+2z}{2}$$

$$-x - 3y + 2z = 0$$

$$\frac{1+2z}{2} - 3y + 2z$$

$$1 + 2z - 6y + 4z = 0$$

$$1 - 6y + 6z = 0$$

$$y = \frac{6z+1}{6}$$

$$\frac{6z+1}{6} + \frac{-1-2z}{2} + z = 0$$

$$\frac{6z + 1 - 3 \cancel{-6z} + 6z}{6}$$

$$\frac{6z-2}{6} = 0$$

$$6z = 2$$

$$\boxed{z = \frac{1}{3}}$$

$$x = \frac{-1 - \frac{2}{3}}{2}$$

$$x = \frac{-3-2}{6}$$

$$\boxed{x = \frac{-5}{6}}$$

$$y = \frac{2+1}{6}$$

$$\boxed{y = \frac{1}{3}}$$

$$\boxed{t = K P^{-5/6} d^{1/3} E^{1/3}}$$

System of units

FPS (Foot, Pound, Second) (British system)

CGS (Gausion system) (centimeter, gram, second)

MKS (Metric system) (Meter, kg, Second)

$1N = 10^5$ dyne (C.G.S)

$1J = 10^7$ erg (C.G.S)

Trignometry

__Formulas__

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
5. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
6. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
7. $\cos(A-B) = \cos A \cdot \cos B + \sin A \sin B$

8. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

9. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

10. $\sin 2\theta = 2 \sin\theta \cos\theta$

11. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

12. $1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$

13. $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

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eg. $\sin 150^\circ = \sin(90+60)$

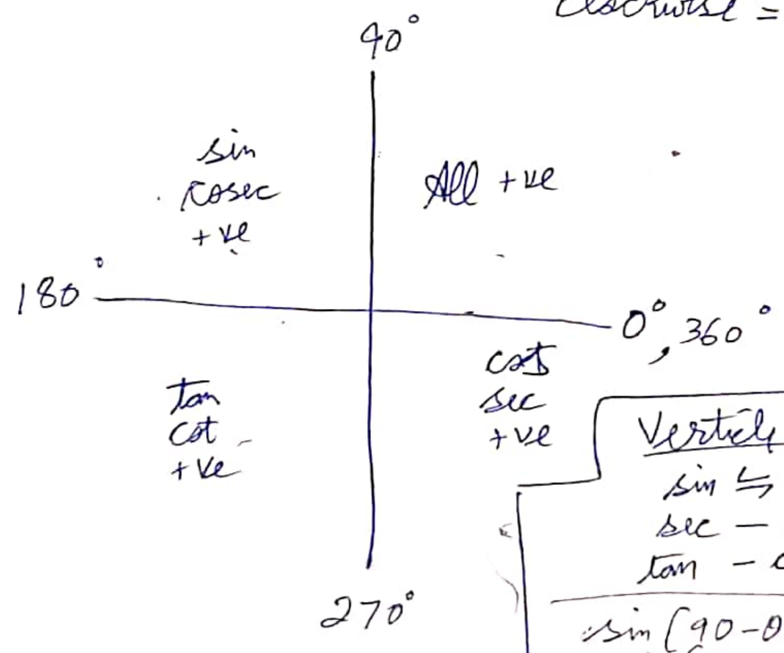
$\sin(A+B) = \sin 90 \cos 60 + \sin 60 \cos 90$

$= 1 \times \frac{1}{2} + 0$

$= \frac{1}{2}$

~~ATSC~~ rule:-
ASTC

anticlockwise = positive angle
clockwise = -ve angle



Vertical line

sin ⇆ cos

sec — cosec

tan — cot

$\sin(90-\theta) = +\cos\theta$

$\sin(90+\theta) = +\cos\theta$

$\cos(90+\theta) = -\sin\theta$

$\cos(90-\theta) = +\sin\theta$

$\tan(90-\theta) = \cot\theta$

$\tan(90+\theta) = -\cot\theta$

Horizontal Line (same sign & ratio)

$\sin(180-\theta) = +\sin\theta$

$\sin(180+\theta) = -\sin\theta$

$\cos\theta(180-\theta) = -\cos\theta$

$\tan\theta(180+\theta) = \tan\theta$

eg $\sin 150° = \sin(90° + 60°)$

$= \cos 60°$

$= \frac{1}{2}$

~~$\tan(270+\theta) = -\cot\theta$~~

eg. $\cos(120°) = \cos(90+30)$

$= -\sin 30$

$\boxed{= -\frac{1}{2}}$

08-04-2024

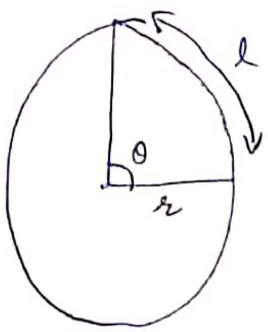
Angles

① Sexagesimal system :- Angle is measured in degrees ($\theta°$)

$1° = 60'$ (arc minutes)

$1' = 60''$ (arc seconds)

② Circular system :- Angle in radians (rad) (θ^c) (θ - no superscript)



$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi^c = 360°$

$1° = \frac{\pi}{180}$ rad

$180° = \pi^c$

$90° = \frac{\pi}{2}^c$

$45° = \frac{\pi}{4}^c$

$30° = \frac{\pi}{6}^c$

$60° = \frac{\pi}{3}^c$

★ If θ is very small $(\theta < 5^\circ)$

$$\boxed{\begin{aligned} \sin\theta &\approx \theta \\ \cos\theta &\approx 1 \quad \text{or} \quad 1 - \frac{\theta^2}{2} \\ \tan\theta &\approx \theta \end{aligned}}$$

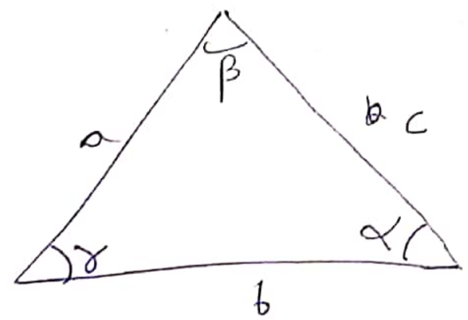
(θ must in radian)

$\sin 37^\circ = \frac{3}{5}$ $\sin 53^\circ = \frac{4}{5}$

$\cos 37^\circ = \frac{4}{5}$ ~~sin~~ $\cos 53^\circ = \frac{3}{5}$

$\tan 37^\circ = \frac{3}{4}$ $\tan 53^\circ = \frac{4}{3}$

sine Rule



$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

Cosine Rule:-

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

Binamiol Theorm (Expression with 2 terms)

Factorial (!)

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

$4! = 4 \times 3 \times 2 \times 1 = \cancel{20}24$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$\cancel{0}1! = 1 = 1$

$0! = 1$

$n! =$ Product of n natural numbers

$= n \times (n-1) \times (n-2) \ldots \times 3 \times 2 \times 1$

$$\underbrace{(x+y),\ (4x-3y),\ (a+b)^2}_{\text{Binamiol Expression :-}}$$

Binomial Formula

$$(x+a)^n = {}^nC_0\, x^n a^0 + {}^nC_1\, x^{(n-1)} a^1 + {}^nC_2\, x^{(n+2)} a^2 + {}^nC_n\, x^0 a^n$$

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$ combination formula

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eg. $(a+b)^2 = {}^2C_0\, a^2 b^0 + {}^2C_1\, a^1 b^1 + {}^2C_2\, a^0 b^2$

$$= \frac{\cancel{2!}}{0!\,\cancel{2!}} a^2 + \frac{2!}{1!\,1!} ab + \frac{\cancel{2!}}{\cancel{2!}\,0!} b^2$$

$$= \frac{a^2}{1} + \frac{2}{1\times1} ab + \frac{b^2}{1}$$

$$= a^2 + 2ab + b^2$$

eg 2. $(a+b)^3 = {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a^1 b^2 + {}^3C_3\, a^0 b^3$

$$= \frac{\cancel{3!}}{0!\,\cancel{3!}} a^3 + \frac{3!}{1!\,2!} a^2 b + \frac{3!}{2!\,1!} ab^2 + \frac{\cancel{3!}}{\cancel{3!}\,0!} ab^3$$

$$= \frac{a^3}{1} + \frac{6}{2} a^2 b + \frac{6}{2} ab^2 + \frac{b^3}{1}$$

$$= a^3 + 3a^2 b + 3ab^2 + b^3$$

eg. 3. $(1+x)^6 = {}^6C_0\, 1^6 x^0 + {}^6C_1\, 1^5 x^1 + {}^6C_2\, 1^4 x^2 + {}^6C_3\, 1^3 x^3 +$
${}^6C_4\, 1^2 x^4 + {}^6C_5\, 1^1 x^5 + {}^6C_6\, 1^0 x^6$

$$= \frac{6!}{0!\,6!} \times 1 + \frac{6!}{1!\,5!} x + \frac{6!}{2!\,4!} x^2 + \frac{6!}{3!\,3!} x^3 + \frac{6!}{4!\,2!} x^4 + \frac{6!}{5!\,1!} x^5 + x^6$$

$$= 1 + \frac{\overset{6}{\cancel{720}}}{120} x + \frac{\overset{360\ 180\ 90\ 45\ 15}{\cancel{720}}}{\underset{12\ 6\ 3}{\cancel{2\times 4!}}} x^2 + \frac{\overset{360\ 180\ 60\ 20}{720}}{\underset{3\ 3}{\cancel{6\times6}}} x^3 + \frac{720}{240} x^4 + \frac{720}{120} x^5 + x^6$$

$$\boxed{= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6}$$

Binomial approximation:-

$$\boxed{\begin{matrix}(1+x)^n \approx 1+nx \\ x<<1\end{matrix}}$$

$(1+x)^n$.

$x<<1$ (<< means very small)

eg. $(1+x)^2 = 1+x^2+2x \approx 1+2x$

$(1+x)^3 = 1+x^3+3x^2+3x \approx 1+3x$

$x<<1$

$x^2, x^3 \rightarrow$ higher orders of $x \rightarrow$ even smaller

eg. $g_h = g\left[\frac{R}{(R+h)}\right]^2$

$R = 6400$ km

$g = 9.8$

~~h = height above~~

if $h << R$

$$g_h = g\left[\frac{\cancel{R}}{\cancel{R}(1+\frac{h}{R})}\right]^2$$

$$= g\left[\frac{1}{1+\frac{h}{R}}\right]^2$$

$$= g\left(1+\frac{h}{R}\right)^{-2}$$

~~$= g(1-$~~

$h << R$

$\frac{h}{R} << 1$

$$g_h = g\left(1-\frac{2h}{R}\right)$$

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Vectors

Physical Quantity

Scalars
Only magnitude
no direction

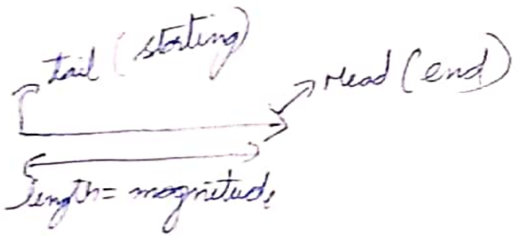
Vectors
1) magnitude
2) ~~direction~~ specified direction
3) obey laws of vector addition

Tensors
~~1) Magnitude must change with direction~~
1) direction don't effect value
eg. Movement of Inertia

Ratio
To compare two similar quantities
(no unit)

Representation of a vector :-

$\vec{F}$ $\vec{V}$

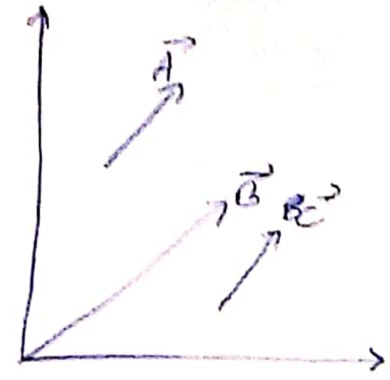


~~$[\vec{A}]$ (taking~~

→ For two vectors to be same, their magnitude & direction both should be equal.

$|\vec{A}|$ (taking only magnitude of the vector)

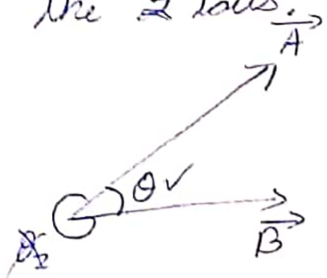
→ we can displace a vector parallel to itself without affecting it.



$\vec{A} = \vec{C}$

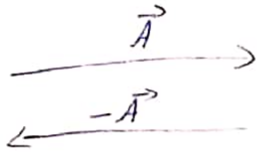
Angle between 2 vectors.

→ Join the 2 vectors by tail.
Angle between 2 vectors is the smaller vector between the 2 tails.



Negative of a vector

→ A vector of same magnitude but opposite in direction.



Multiply

① with Positive number

multiplying vector $\vec{A}$ with α will give new vector $\vec{B}$ with magnitude α times & same direction.

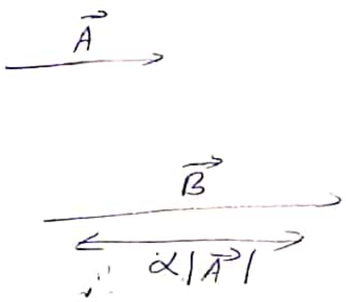
② with negative number

Opp direction & magnitude gets multiplied.

$\vec{A} \times \alpha$

$\downarrow$

$\vec{B} = \alpha \vec{A}$



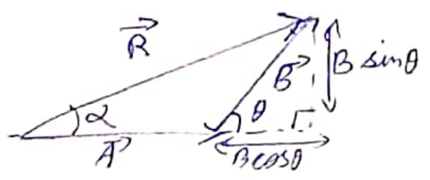
Null Vector - Vector with 0 magnitude & undetermined direction.

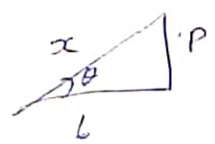
~~Also~~ Also known as 'zero vector'

Addition of Vectors

① Triangle law of vector addition -

$\vec{A} + \vec{B} = \vec{R}$





$\sin\theta = \frac{p}{x}$

$p = x\sin\theta$

$\cos\theta = \frac{b}{x}$

$b = x\cos\theta$

PGT

$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$

$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$
$$= B^2(\sin^2\theta + \cos^2\theta) + A^2 + 2AB\cos\theta$$

$$\boxed{R = \sqrt{B^2 + A^2 + 2AB\cos\theta}} \rightarrow \text{Magnitude of resultant vector}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

for $\boxed{\theta = 0^\circ \quad R = A + B}$ $\boxed{\theta = 180^\circ \quad R = A - B}$

$\boxed{\theta = 90^\circ \quad R = \sqrt{A^2 + B^2}} \rightarrow \tan\alpha = \frac{B}{A}$ $|A - B| \leq R \leq |A + B|$

eg. ✱ Only the vectors of the same type can be added & subtracted & result is also same type.

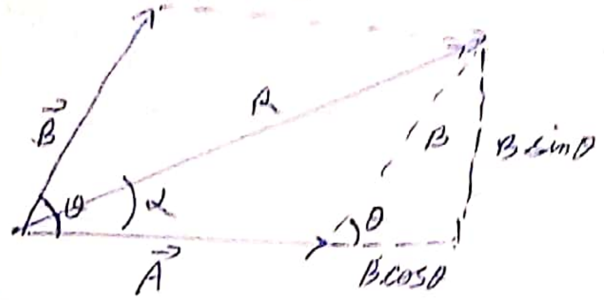
✱ Vector addition is commutative i.e

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

✱ Vector addition is associative i.e

$$(\vec{P} + \vec{Q}) + \vec{R} = \vec{P} + (\vec{Q} + \vec{R}) = \vec{Q} + (\vec{P} + \vec{R})$$

II Parallelogram Law of Vector addition -



$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

* If two non-zero vectors are represented by two adjacent sides of a parallelogram, then resultant vector is represented by its diagonals passing through point of intersection of 2 vectors.

O-1   09-04-2024

Q1 - D	Q2 - C	Q3 - A	Q4 - A
Q5 - B	Q6 - A	Q7 - C	Q8 - D
Q9 - D	Q10 - A	Q11 - C	Q12 - A
Q13 - D	Q14 - B	Q15 - D	Q16 - B
Q17 - C	Q18 - B		

S-1

Q1 - $k = [ML^2T^{-2}]$ $a = [L^{-1}]$ Q2 - $R_B = \frac{R_g m^2}{K s^2}$

Q3 - $t = a\sqrt{\frac{m}{k}}$ Q4 - $\omega = K\sqrt{\frac{mG}{r^3}}$ Q5 - $k = 6$

Q6 - $[MLT^{-1}]$ Q7 i) $24/25$ ii) $-7/25$ iii) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ iv) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

Q8 - $l = \frac{2\pi}{3} \times 6.4 \times 10^6\ m$
$\quad PK = \sqrt{3}\ 6.4 \times 10^6\ m$

Q9 - a - 9.95
$\quad$ b - 9.99

Q10. $\frac{\pi}{100}\ m$ QIT.

JEE - Mains

Q1-3) Q2-4) Q3-~~2~~4) Q4-2) Q5-2) ~~Q10~~

Q10-3) Q11-4) Q16-1) Q21-C) Q22-C) Q23-B)

Q7 i) $\sin 74^\circ$

$\underline{\sin 2(37)}$

$\sin 2\theta = 2 \sin\theta \cos\theta$

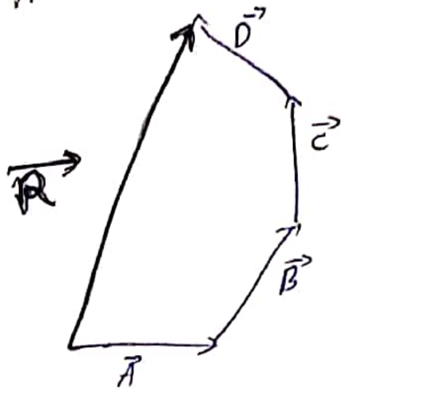
$\sin 2(37^\circ) = 2 \sin 37^\circ \cos 37^\circ$

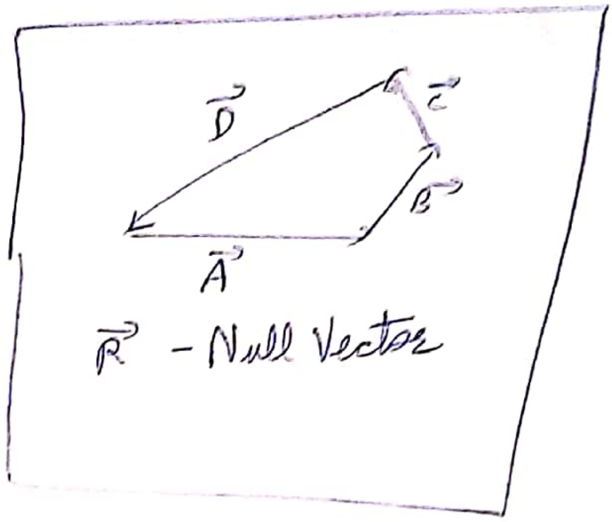
$\sin 74^\circ = 2 \times \frac{3}{5} \times \frac{4}{5}$

$\sin 74^\circ = \frac{24}{25}$

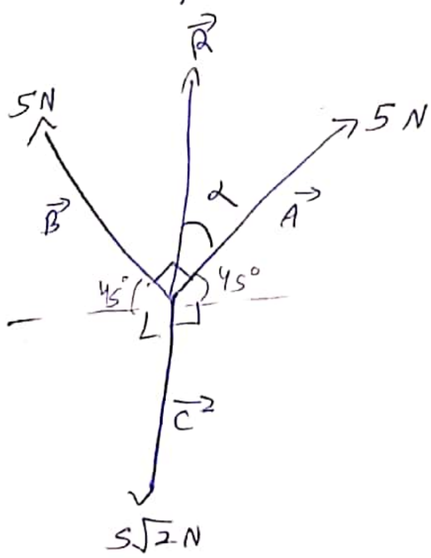
Polygon law of vector addition:

$\vec{A} + \vec{B} + \vec{C} + \vec{D}$





- ☆ If All the vectors are arranged to form sides of a polygon in one order then their resultant is represented by closing side of polygon in opposite order
- ☆ If n vectors are represented by n sides of a polygon in cyclic manner, the resultant of these vectors is a null vector.
- ☆ Resultant of two unequal vectors cannot be zero.
- ☆ Resultant of 3-co-planer vectors may or may not be zero.



$$\vec{A} + \vec{B} + \vec{C} = (\vec{A} + \vec{B}) + \vec{C}$$

$$\cancel{5\sqrt{2} + 5\sqrt{2} = 0}$$

$$\cancel{\alpha = 45^\circ}$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$\vec{R} = \sqrt{25 + 25}$$

$$\vec{R} = 5\sqrt{2}.$$

$$\tan\alpha = \frac{B}{A}$$

$$\tan\alpha = 1$$

$$\tan 45 = 1$$

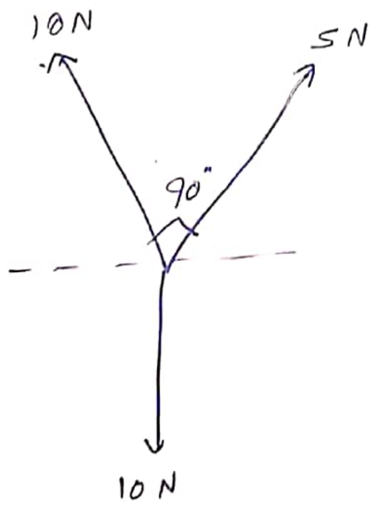
$$\alpha = 45$$

$$\vec{R} + \vec{C} = \sqrt{(5\sqrt{2})^2 - (5\sqrt{2})^2}$$

$$= 0$$

---

eg 2.



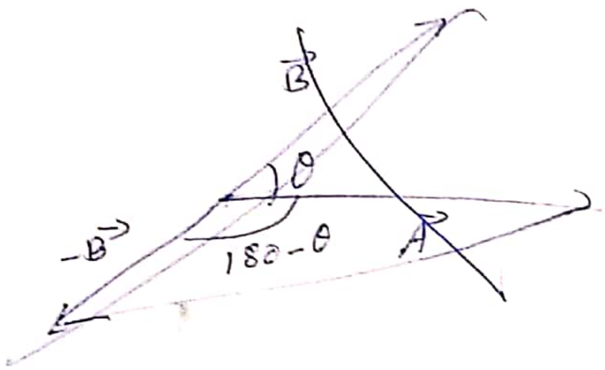
$$\vec{A} + \vec{B} + \vec{C} \neq 0$$

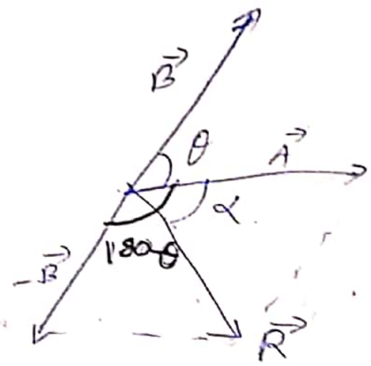
✱ The result of non-co-planer vectors cannot be zero

Subtraction of vectors

$\vec{A} - \vec{B}$

$\vec{A} + (-\vec{B})$





$\vec{A} - \vec{B} = \vec{R}$

$$\vec{R} = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$$

$$\vec{R} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

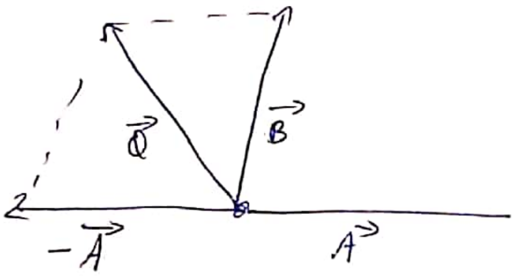
$$\tan\alpha = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)}$$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$ ✱ Non-commutative

$\vec{B} - \vec{A} = \vec{Q}$

$\vec{B} + (-\vec{A})$



$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

Q Given that $\vec{C} = \vec{A} + \vec{B}$.

Also $|\vec{A}| = 12$ $|\vec{B}| = 5$ $|\vec{C}| = 13$ units

Find angle btw $\vec{A}$ & $\vec{B}$

$|\vec{A}| = 13$ $A = 13$ units

$|\vec{B}|$ $B = \frac{13}{12}$ units

$C = \frac{13}{5}$ units

$$\frac{13}{5} = \sqrt{(13)^2 + \left(\frac{13}{12}\right)^2 + \cancel{2} \times \frac{13}{\cancel{12}_{6}} \times 13 \times \cos\theta}$$

$$\frac{169}{25} = 169 + \frac{169}{144} + \frac{169}{6} \times \cos\theta$$

Ans.

$$C=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$13=\sqrt{(12)^2+\cancel{2}(5)^2+(12)(5)\cos\theta}$$

$$169=144+25+120\cos\theta$$

$$\cos\theta=0$$

$$\theta=90^\circ=\frac{\pi}{2}^c$$

A & B are Perpendicular / orthogonal

Q2. The greatest & least resultant of two forces acting at a point is 10 N & 6 N. If each is increased by 3 N, find the resultant of two forces when acting at a point at an angle of 90° with each other.

$A+B=10$

$A-B=6$

$\underline{A=6+B}$

$6+B+B=10$

$2B=4$

$\underline{\underline{B=2}}$

$\underline{A=8}$

$A=8+3=11$

$B=2+3=5$

$\theta=90^\circ$

$\vec{R}=\sqrt{121+25}$

$\boxed{\vec{R}=\sqrt{146}\ N}$ ✓

$\tan\alpha=\frac{5}{11}$

Q. The maximum & minimum of two forces are in ratio 7:3 find force ratio.

$A+B=7x$

$A-B=3x$

$A=3x+B$

$3x+2B=7x$

$2B=4x$

$\underline{\underline{B=2x}}$

$\cancel{A} = \frac{3x+B}{\cancel{B}}$

$\underline{\underline{A=5x}}$

$\frac{A}{B}=\frac{5x}{2x}=\frac{5}{2}$

$\boxed{A:B=5:2}$ ✓

Q Two equal forces have their resultant equal to either. find angle.

$C=\sqrt{2x^2+2x^2\cos\theta}$

$C=x\sqrt{\cos\theta}$

$x^2=2x^2+2x^2\cos\theta$

$-x^2=2x^2\cos\theta$

$-\frac{1}{2}=\cos\theta$

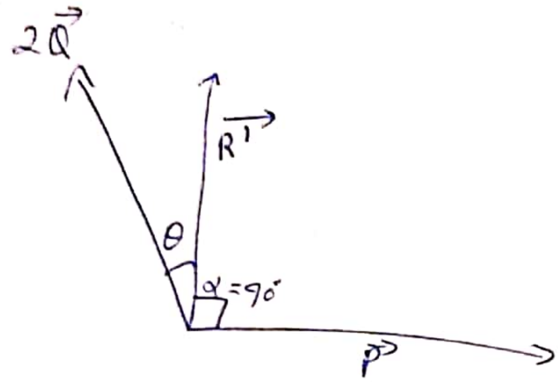
$\cancel{\boxed{\theta=150}}$

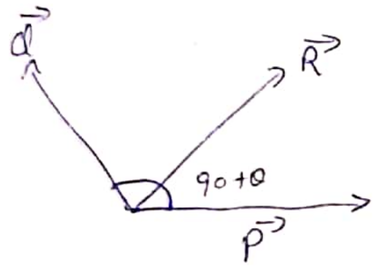
$\boxed{\theta=120^\circ}$ ✓

$\cancel{\vec{P}+\vec{Q}=\vec{R}}$

$\cancel{\tan 90 = \frac{2Q \sin\theta}{P+}}$

Q The resultant of two vectors $\vec{P}$ & $\vec{Q}$ is $\vec{R}$. If the magnitude of Q is doubled, the new vector is $\perp$ to $\vec{P}$, find $\vec{R}$





$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\tan 90 = \frac{2Q\sin(90+\theta)}{P+2Q\cos(90+\theta)}$$

0 (as $\tan 90°$ is undefined)

$P + 2\cos(90+\theta)$

$$\cos(90+\theta) = \frac{-P}{2Q}$$

$R = \sqrt{P^2+Q^2+2PQ\cos(90+\theta)}$

$R = \sqrt{P^2+Q^2+2PQ \times \frac{-P}{2Q}}$

$\boxed{R = Q}$

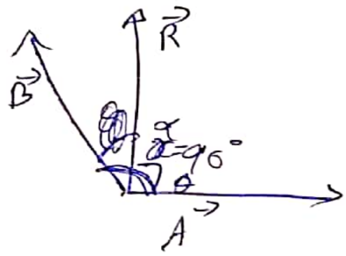
Q.6 The ~~sin~~ sum of magnitude of two forces at a point is 18 N and magnitude of resultant is 12 N. If the resultant makes an angle of 90° with the force of smaller magnitude, what are the magnitude of two forces.

$A + B = 18 \rightarrow B = 18 - A$

$|R| = 12N$

$\alpha = 90^\circ$

$\tan 90 = \frac{A\sin\theta}{A + B\cos\theta} \rightarrow 0$



$A + B\cos\theta = 0$

$\cos\theta = \frac{-A}{B}$

$\vec{R} = \vec{A} + \vec{B}$

$12 = \sqrt{A^2+B^2+2AB \times \frac{-A}{B}}$

$144 = A^2 + B^2 - 2A^2$

$144 = B^2 - A^2$

$144 = (18-A)^2 - A^2$

$144 = 324 + A^2 - 36A - A^2$

$36A = 324 - 144$

$A = \frac{180}{36}$

$\boxed{A = 5\ N \quad B = 13\ N}$ ✓

$18 \times 18 = 144 + 180 = 324$

Q 7. A vector $\vec{B}$ which has a magnitude 8. is added to $\vec{A}$ which lie along the x axis, the resultant lie along y-axis and has a magnitude that is twice of magnitude of $\vec{A}$. The magnitude of $\vec{A}$ is ____.

$$\tan 90 = \frac{AB\sin\theta}{A+B\cos\theta}$$

$$\cos\theta = \frac{-A}{B}$$

$$(2x)^2 = A^2+B^2+2AB\cos\theta$$

$$4x^2 = x^2+64+2\times A\times B\times\frac{-A}{B}$$

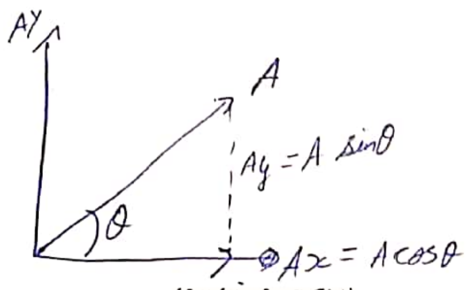
$$4x^2 = 64-x^2$$

$$5x^2 = 64$$

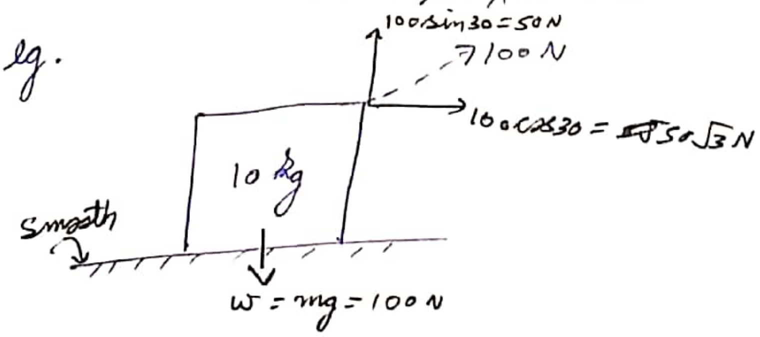
$$x^2 = \frac{64}{5}$$

$$\boxed{x = \frac{8}{\sqrt{5}}}$$

Components of Vectors



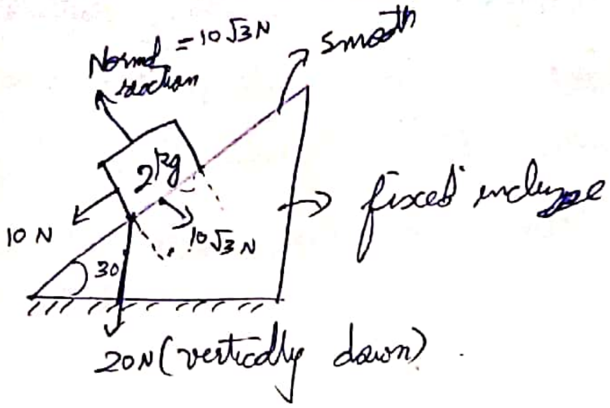
eg.



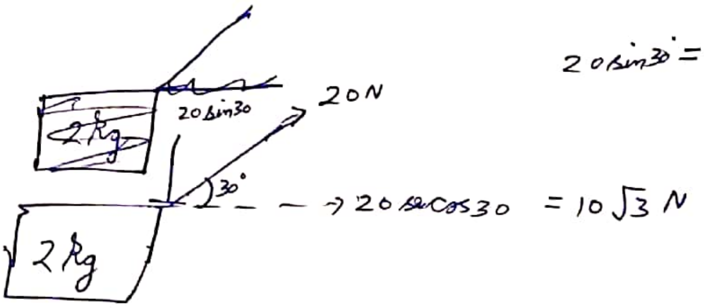
$$F = ma$$

$$\frac{50\sqrt{3}}{10} = a = 5\sqrt{3}\ m/s^2$$

Ans 2.



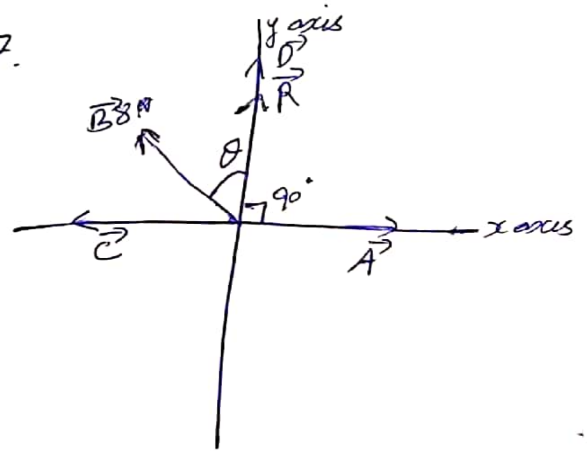
$20 \sin 30^\circ = 10N$



$F = ma$

$\frac{10}{2} = a = \boxed{5 \ m/s^2}$

Ans 7.



$|B| = 8$

$|R| = 2|A|$

$\vec{C} = B \sin\theta$

$\vec{D} = B\cos\theta$

$\vec{A} + \vec{C} = 0$

$A = C$

$A = B\sin\theta$

$\vec{D} = \vec{R}$

~~$B\cos\theta = 2B\sin\theta$~~

$R = B\cos\theta$

$A^2 + R^2 = B^2\sin^2\theta + B^2\cos^2\theta$

$A^2 + R^2 = B^2$

$A^2 + R^2 = 64$

$A^2 + 4A^2 = 64$

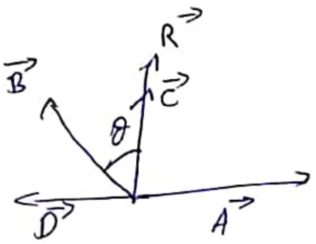
$A^2 = \frac{64}{5}$

$\boxed{A = \frac{8}{\sqrt{5}}}$

Ans6.

$|A| + |B| = 18$ $\qquad B = 18 - A$

$R = 12$



$C = B\cos\theta$
$D = B\sin\theta$

$D = A$ (as force is $\perp$ & so they will cancle)

$A = B\sin\theta$

$R = C = B\cos\theta$

$A^2 + R^2 = B^2\sin^2\theta + B^2\cos^2\theta$

$A^2 + R^2 = B^2$

$R^2 = B^2 - A^2$

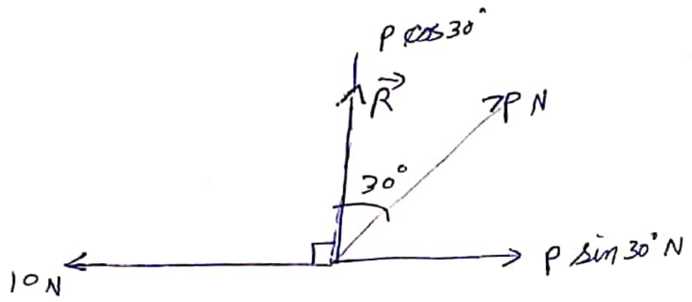
$144 = \cancel{18-A}(B-A)(B+A)$

$\dfrac{\cancel{144}^{72\,24\,8}}{\cancel{18}_{9\,3}} = B - A$

$B - A = 8$
$B + A = 18$
$2B = 26$

$\boxed{\begin{array}{l} B = 13 \\ A = 5 \end{array}}$

Q8. Two horizontal forces of 10N & P N act on a particle. The force of magnitude 70N acts west & the force of P N acts on a bearing of 30° east of north a shown in figure. The resultant of these two force acts due north. Find the magnitude of this resultant.



$P \sin 30^\circ = 10$

$\frac{P}{2} = 10$

$\boxed{P = 20}$

$P \cos 30 = 20 \times \frac{\sqrt{3}}{2} = \boxed{10\sqrt{3}\,N}$

or

$\tan \alpha = 90 = \frac{B \sin\theta}{A + B\cos\theta}$

$10 + P\cos\theta = 0$

$\cos\theta = \frac{-10}{P}$

$A^2 = (10)^2 + P^2 + 2 \times 10 \times P \times \frac{-10}{P}$

$R^2 = 100 + P^2 - 200$

$R^2 = P^2 - 200$

$R^2 = 400 - 200$

$R = \sqrt{200}$

$R = 10\sqrt{2}$

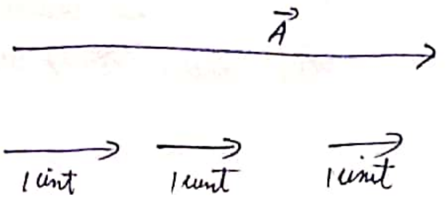
$\tan 30 = \frac{10 \times \sin(90+30)}{P + 10 \cos(90+30)}$

$\frac{1}{\sqrt{3}} = \frac{5\sqrt{3}}{P - 5}$

$P - 5 = 15$

$\boxed{P = 20}$

Unit Vector



$|\vec{A}| = 3$

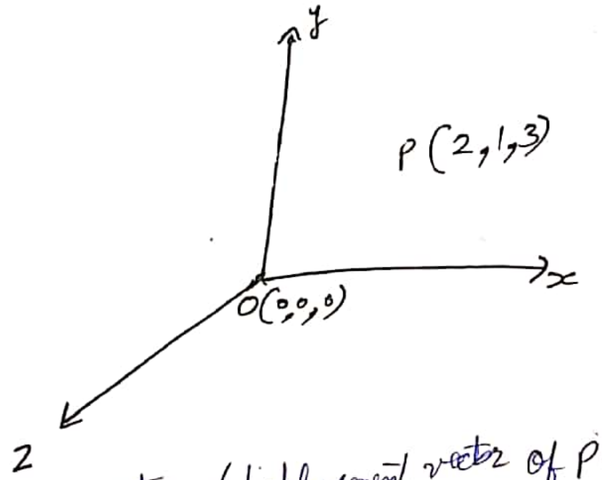
unit vector of $\vec{A} = \hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

dividing a vector by its magnitude, we get a vector of magnitude 1 & ~~as~~ same direction as $\vec{A}$.

→ It has no dimensions. ~~no~~ and no units

→ Cartesian form/rectangular form

unit vector of x-axis – $\hat{i}$
y-axis – $\hat{j}$
z-axis – $\hat{k}$



Position /displacement vector of $P = (2-0)\hat{i} + (1-0)\hat{j} + (3-0)\hat{k}$

$$\vec{OP} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$|\vec{OP}| = \sqrt{2^2+1^2+3^2}$$

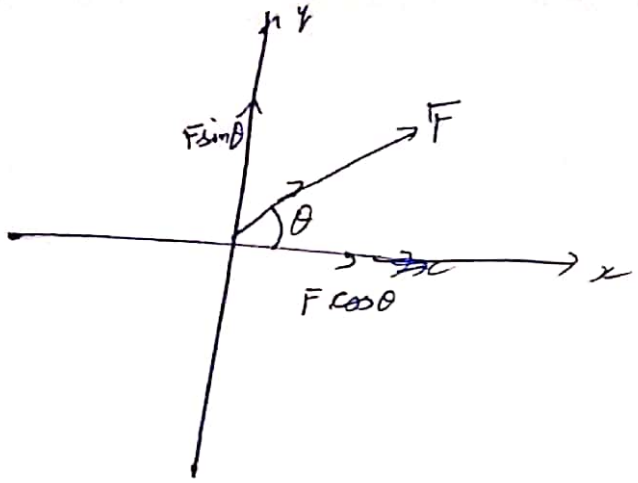
$$= \sqrt{4+1+9}$$

$$= \sqrt{14} \text{ units}$$

$$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$$

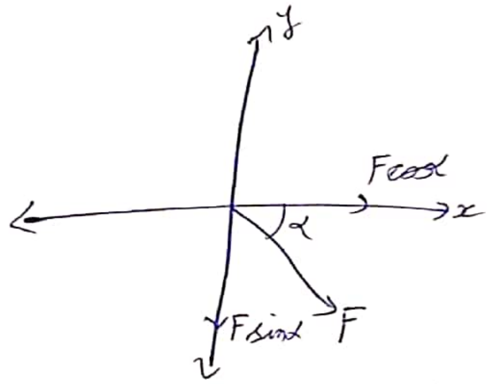
$$|\vec{A}| = \sqrt{a^2+b^2+c^2}$$

eg.



$$\boxed{\vec{F} = F\cos\theta\,\hat{i} + F\sin\theta\,\hat{j}}$$

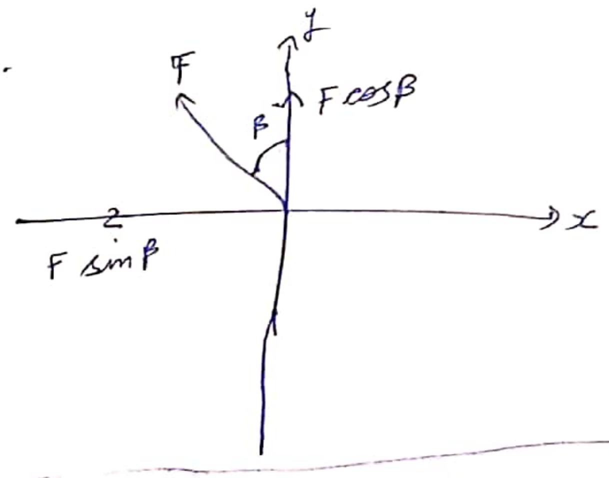
eg 2



$$\vec{F} = F\cos\alpha\,\hat{i} + F\sin\alpha\,(-\hat{j})$$

$$\boxed{= F\cos\alpha\,\hat{i} - F\sin\alpha\,\hat{j}}$$

eg 3.



$$\boxed{\vec{F} = \cancel{F\sin\beta}\ F\cos\beta\,\hat{j} - F\sin\beta\,\hat{i}}$$

Q9. find C if $0.3\hat{i} + 0.4\hat{j} + C\hat{k}$ is a unit vector.

~~C = 0.3~~

$$C = \sqrt{(0.3)^2 + (0.4)^2 + c^2}$$

$$1 = 0.09 + 0.16 + c^2$$

$$1 - 0.25 = c^2$$

$$0.75 = c^2$$

$$\frac{\cancel{7}\sqrt{3}}{\cancel{1}00} = c^2$$

$$\boxed{c = \frac{\sqrt{3}}{2}}$$

Q10. find unit vector of $\underbrace{2\hat{i} + \hat{j} - 2\hat{k}}_{\vec{A}}$.

$$|\vec{A}| = \sqrt{4+1+4}$$

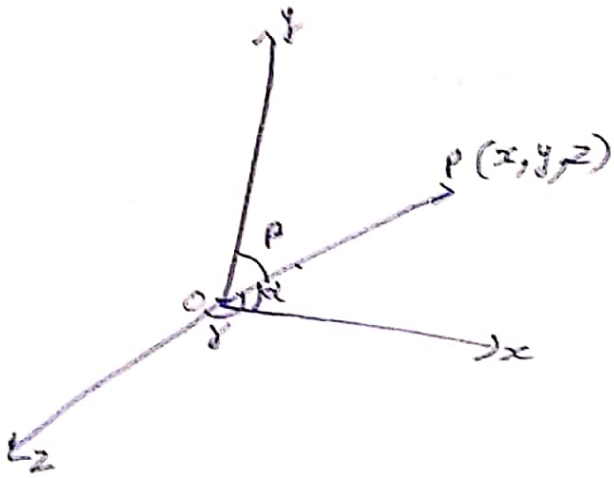
$$|\vec{A}| = \sqrt{9}$$

$$|\vec{A}| = 3$$

$$\hat{A} = \frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}$$

$$\boxed{\hat{A} = \tfrac{2}{3}\hat{i} + \tfrac{1}{3}\hat{j} - \tfrac{2}{3}\hat{k}}$$

Direction cosines :-

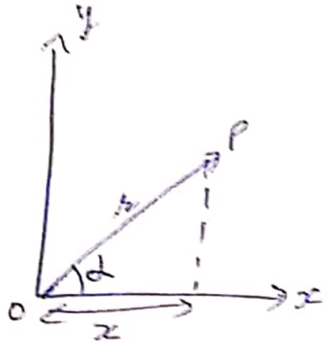


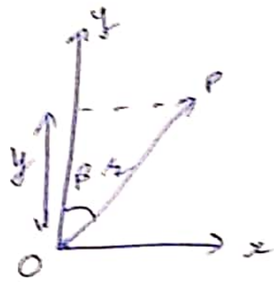
angle with x-axis = α
y = β
z = γ

α + β ≠ 90

$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{OP}| = \sqrt{x^2+y^2+z^2} = r$





$\cos\alpha = \frac{x}{r}$

$\cos\beta = \frac{y}{r}$

$\cos\gamma = \frac{z}{r}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$$

$$= \frac{x^2+y^2+z^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

The sum of square of direction cosines of a vector is always equal to one.

Q what are direction cosines of $\hat{i} + \hat{j} + \hat{k}$?

~~$\cos\alpha = \frac{x}{r}$~~

$x = 1$
$y = 1$
$z = 1$

$$r = \sqrt{1^2 + 1^2 + 1^2}$$

$$r = \sqrt{3}$$

$$\cos\alpha = \frac{x}{r}$$

$$\boxed{\cos\alpha = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos\beta = \frac{y}{r} = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos\gamma = \frac{z}{r} = \frac{1}{\sqrt{3}}}$$

Q2. if $\vec{P} = 4\hat{i} - 2\hat{j} + 6\hat{k}$, $\vec{Q} = \hat{i} - 2\hat{j} - 3\hat{k}$

find angle $(\vec{P} + \vec{Q})$ makes with x axis

$$\vec{P} + \vec{Q} = 5\hat{i} - 4\hat{j} + 3\hat{k}$$

$$r = \sqrt{25 + 16 + 9}$$

$$= \sqrt{50}$$

$$r = 5\sqrt{2}$$

$$\cos\alpha = \frac{5}{5\sqrt{2}}$$

$$\boxed{\cos\alpha = \frac{1}{\sqrt{2}}} \qquad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\boxed{\alpha = 45^\circ} = \frac{45 \times \pi}{180_4} = \frac{\pi}{4}^c$$

$\sin^{-1}\left(\frac{1}{2}\right)$ kis angle ke like $\sin\frac{1}{2}$ hogo.

Q3. A bird moves with velocity 20 m/s in a direction making an angle of 60° with the eastern line & 60° with vertical upword. write velocity in a rectangular form.

$\alpha = 60^\circ$

$\gamma = 60^\circ$

$\cos 60^\circ = \frac{1}{2}$

$\frac{x}{r} = \frac{1}{2}$

$2x = r$

$2x = 2z$

$x = z$

$\cos 60^\circ = \frac{1}{2}$

$\frac{z}{r} = \frac{1}{2}$

$2z = r$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{2x^2}$

$r^2 = 2x^2$

~~r = 20~~

~~x = 10~~

~~z = 10~~

$$\boxed{\cancel{\vec{v} = 10\hat{i} + 10\hat{k}}}$$

Scanned with CamScanner

$|\vec{V}| = 20 \ m/s$

$\alpha = 60$

$\gamma = 60^\circ$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{4} + \cos^2\beta + \frac{1}{4} = 1$

$\cos^2\beta = \frac{1}{2}$

$\cos\beta = \frac{1}{\sqrt{2}}$

$\cos\alpha = \frac{1}{2} = \frac{x}{20}$

$x = 10$

$z = 10$

$y = 10\sqrt{2}$

$\boxed{\vec{V} = 10\hat{i} + 10\sqrt{2}\hat{j} + 10\hat{k}}$ m/s

---

Q4. find the vector which must be added to the sum of 2 vectors $\hat{i} + 2\hat{j} ~~+~~ - \hat{k}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ to get resultant of unit vector along z axis.

$\vec{V} = 2\hat{i} + 0\hat{j} + \hat{k}$

$\vec{V}$ - unit vector along z $= -\vec{R}$

$-\vec{R} = 2\hat{i} + \hat{k} - \hat{k}$

$\boxed{\vec{R} = -2\hat{i}}$

Q 5. A particle is moving towards east with 5 m/s in 10 seconds, its velocity becomes 5 m/s towards north. find average acceleration

$\vec{u} = 5\hat{i}$ m/s

$\vec{v} = 5\hat{j}$ m/s

$t = 10s$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{10}$$

$$\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$$

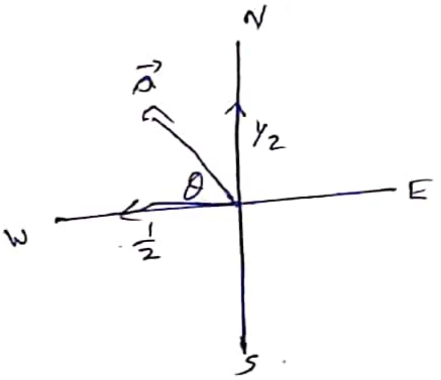
$$\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$$

$$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$r = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

$\boxed{|\vec{a}| = \frac{1}{\sqrt{2}} \text{ m/s}^2}$ (at an angle 45° west of north)

$$\vec{a} = -\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}$$



$a\cos\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\boxed{\theta = 45°}$

Q6. A car is moving towards North with speed of 36 km/h. In 5s it turns towards left with its speed unchanged. find acc of car.

$\vec{u} = 36\hat{j}$ m/s

$\vec{v} = -36\hat{i}$ m/s

$t = 5s$

$\vec{a} = \frac{-36\hat{j} - 36\hat{i}}{5}$

$\vec{a} = -\frac{(36\hat{i} + 36\hat{j})}{5}$

$\vec{a} = -12\frac{(3\hat{i} + 3\hat{j})}{5}$

$\cancel{|\vec{a}| = \sqrt{36^2 + 3}}$

$|\vec{a}| = \sqrt{9+9}$

$= \sqrt{18}$

$|\vec{a}| = 3\sqrt{2} \times \frac{-12}{5}$

$|\vec{a}| =$

$\vec{u} = 36 \times \frac{5}{18} = 10\hat{j}$

$\vec{v} = -10\hat{i}$

$t = 5s$

$\cancel{\vec{a} = -\frac{10}{2}\hat{i} - \frac{10}{2}\hat{j}}$

$\cancel{\vec{a} = -(\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j})}$

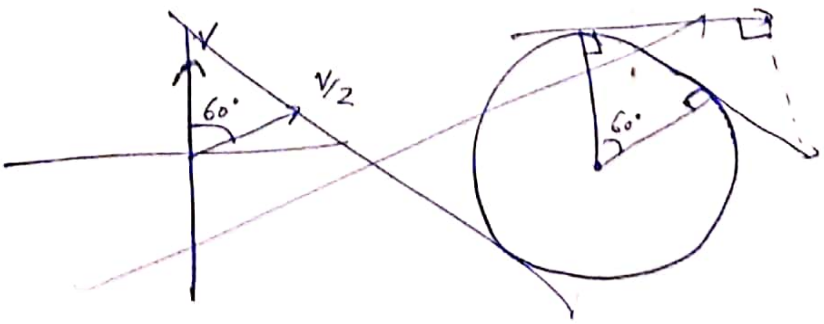
$\vec{a} = -2\hat{i} - 2\hat{j}$

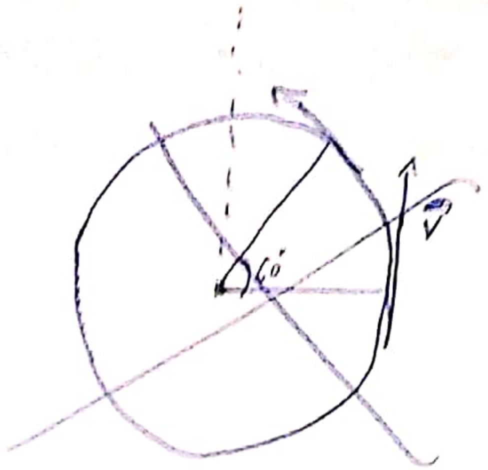
$|\vec{a}| = \sqrt{4+4}$

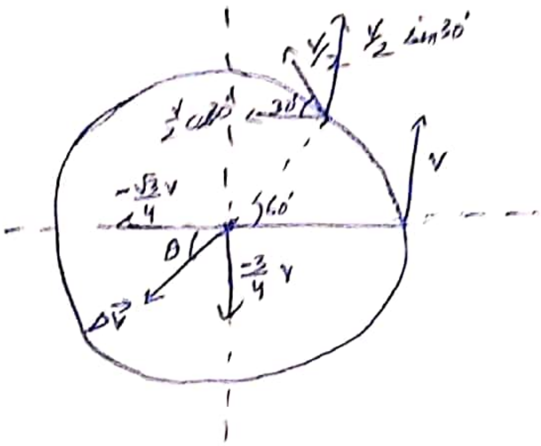
$= \sqrt{8}$

$\boxed{|\vec{a}| = 2\sqrt{2} \text{ m/s}^2}$ (45° towards South of west)

Q7. A particle is moving on a circular path with speed V at a particular instant after some time, when it has described an angle 60° its speed becomes $\frac{V}{2}$ what is change in velocity.







$\vec{V} - \vec{u}$

$\vec{u} = v\hat{j}$

$\vec{V} = \frac{v}{2}\sin 30^\circ (\hat{j}) + \frac{v}{2}\cos 30^\circ (-\hat{i})$

$\vec{V} = \cancel{1}\frac{v}{4}\hat{j} - \frac{-\sqrt{3}}{4} v\hat{i}$

$\Delta\vec{V} = \vec{V} - \vec{u}$

$= \frac{-\sqrt{3}}{4} v\hat{i} + \cancel{1}\frac{v}{4}\hat{j} - v\hat{j}$

$\boxed{\Delta\vec{V} = -\frac{\sqrt{3}}{4} v\hat{i} - \frac{3}{4} v\hat{j}}$

$|\Delta\vec{V}| = \sqrt{\left(\frac{-\sqrt{3}}{4}v\right)^2 + \left(-\frac{3}{4}v\right)^2}$

$\boxed{|\Delta\vec{V}| = \frac{\sqrt{3}v}{2}}$

$\tan\theta = \frac{\frac{3}{4}v}{\frac{\sqrt{3}}{4}v} = \sqrt{3}$

$\boxed{\theta = 60^\circ}$

Q 8. A particle whose speed is 50 m/s moves along a line from A(2,1) to B(9,25) find its velocity in rectangular form.

$$u = 2\hat{i} + \hat{j}$$

$$v = 9\hat{i} + 25\hat{j}$$

$$v - u = 7\hat{i} + 24\hat{j}$$

$$\vec{v} = 7\hat{i} + 24\hat{j}$$

$$\therefore \hat{v} = \frac{7\hat{i} + 24\hat{j}}{\sqrt{(7)^2 + (24)^2}}$$

$$\hat{v} = \frac{7\hat{i} + 24\hat{j}}{25}$$

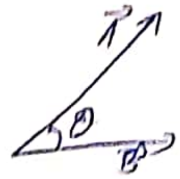
$$\hat{v} \times 50 = \text{velocity}$$

$$\boxed{\overrightarrow{\text{Velocity}} = 14\hat{i} + 48\hat{j} \text{ m/s}}$$

Product of two vectors

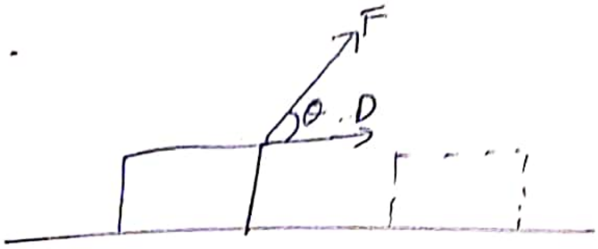
I) Scalar Product/dot product:-

$$\boxed{\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = AB\cos\theta}$$



→ The scalar product is a way to multiply two vectors to get a scalar result

eg.



$W = \vec{F}\cdot\vec{S}$

$W = F.S\cos\theta$

☆ $\theta = 0$

$\cos 0^\circ = 1$

$\vec{A}\cdot\vec{B} = AB$ (max. value)

☆ $\theta = 180$

$\cos\theta = -1$

$\vec{A}\cdot\vec{B} = -AB$ (negative non max value)

☆ $\theta = 90$

$\cos\theta = 0$

$\vec{A}\cdot\vec{B} = 0$ (min value)

$\hat{i}\cdot\hat{i} = 1$	$\hat{i}\cdot\hat{j} = 0$
$\hat{j}\cdot\hat{j} = 1$	$\hat{i}\cdot\hat{k} = 0$
$\hat{k}\cdot\hat{k} = 1$	$\hat{j}\cdot\hat{k} = 0$

$\cancel{\vec{A} = A_x \hat{i} + B_z \hat{i} +}$

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})(B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

* To find angle between two vectors

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\boxed{\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}}$$

---

* Commutative

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

* distributive

$\vec{A}(\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

* $\vec{A} \cdot \vec{A} = (A)(A)\cos 0^\circ = A^2$

$|P+Q|^2 = (\vec{P} + \vec{Q})(\vec{P} + \vec{Q})$

$= \vec{P} \cdot \vec{P} + \vec{P} \cdot \vec{Q} + \vec{Q} \cdot \vec{P} + \vec{Q} \cdot \vec{Q}$

$= P^2 + Q^2 + 2PQ\cos\theta$

$|P+Q| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Q $\vec{A} = 2\hat{i} \mp 3\hat{j} + \hat{k}$

$\vec{B} = 5\hat{i} + 2\hat{j} \mp 7\hat{k}$

$\vec{A} \cdot \vec{B} = 2 \times 5 + (-3) \times 2 + -(7) \times 1$

$= 10 - 6 - 7$

$= 10 - 13$

$\boxed{= -3}$

Q2. $\vec{A} = \hat{i} + \hat{j} - \hat{k}$

$\vec{B} = \hat{i} - \hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = ?$

$\vec{A} \cdot \vec{B} = 1 - 1 - 1$

$= -2 + 1$

$\boxed{= -1}$

Q3. $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{B} = 3\hat{i} + 2\hat{j}$

$\vec{A} \cdot \vec{B} = 6 - 6 + 0$

$\boxed{= 0}$

Q Find value of n, such that $(2\hat{i} - 3\hat{j} + \hat{k})$ is perpendicular to $\hat{i} + 2\hat{j} + n\hat{k}$.

$$\cos\theta = \frac{2 - 6 + n}{\sqrt{4+9+1} \times \sqrt{1+4+n^2}}$$

$$\cos\theta = \frac{n - 4}{\sqrt{14} \times \sqrt{5 + n^2}}$$

$$\cos 90 = \frac{n - 4}{\sqrt{70 + 14n^2}}$$

$n - 4 = 0$

$\boxed{n = 4}$

Q find angle between $(\hat{i}+\hat{j})$ & $(\hat{i}-\hat{j})$

$$\cos\theta = \frac{1-1}{AB}$$

$\boxed{\theta = 90^\circ}$ as $1-1=0$

Q find angle b/w $(\hat{i}+\hat{j})$ & $(\hat{j}+\hat{k})$

$$\cos\theta = \frac{0+1+0}{AB}$$

$$\cos\theta = \frac{1}{\sqrt{1+1}\times\sqrt{1+1}}$$

$$\cos\theta = \frac{1}{\sqrt{2}\times\sqrt{2}}$$

$$\cos\theta = \frac{1}{2}$$

$$\boxed{\theta = 60^\circ}$$

Q A force $\vec{F} = (10\hat{i} - 3\hat{j} + 6\hat{k})$ N acts on a body of mass 100g & displaces it from $(6\hat{i} + 5\hat{j} - 3\hat{k})$ m to $(10\hat{i} - 2\hat{j} + 7\hat{k})$ m find work done.

displacement $= 4\hat{i} - 7\hat{j} + \cancel{9}\,10\hat{k}$ m

work $= F.S$

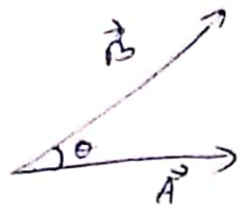
$= (10\hat{i} - 3\hat{j} + 6\hat{k}).(4\hat{i} - 7\hat{j} + \cancel{9}\,10\hat{k})$

$= 40 + 21 + \cancel{54}\,60$

$\boxed{\cancel{= 85\text{ J}}}$

$\boxed{= 121\text{ J}}$

II Cross Product / Vector Product



$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta \; \hat{n}$ ↳ direction of $\vec{A} \times \vec{B}$ (Right hand thumb rule)

$\odot$ out of plane

$\otimes$ in plane

* not commutative

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB\sin\theta$$

* distributive

$$\vec{A}(\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

$\hat{i} \times \hat{i} = (1)(1)\sin 0° = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = (1)(1)\sin 90 \; \hat{k}$

$\hat{i} \times \hat{j} = \hat{k}$

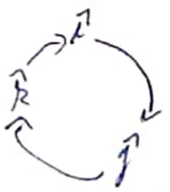
$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$



clockwise − (+ve)

negative − (−ve)

* If cross product is 0, vectors are either equal or parallel
* If dot product is 0, vectors are ⊥ to each other

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$ 3 by 3 determinant

$\cancel{\vec{A} \times \vec{B} = \hat{i}(A_y - B_z) - \hat{j}(A_x - B_z) + \hat{k}(A_x - B_y)}$

$$\boxed{\vec{A} \times \vec{B} = \hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_x B_z - B_x A_z) + \hat{k}(A_x B_y - A_x A_y)}$$

Q $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$\vec{A} \times \vec{B} = \hat{i}(4-(-4)) - \hat{j}(12-4) + \hat{k}(-6-2)$

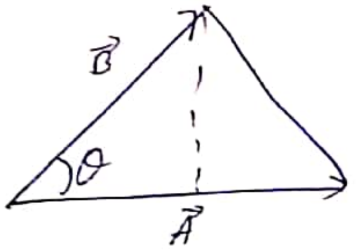
$$\boxed{= 8\hat{i} - 8\hat{j} - 8\hat{k}}$$

$|\vec{A} \times \vec{B}| = \sqrt{8^2 + (-8)^2 + (-8)^2}$

$$\boxed{= 8\sqrt{3}}$$

Uses of vector product

1) To Calculate area of triangle:-

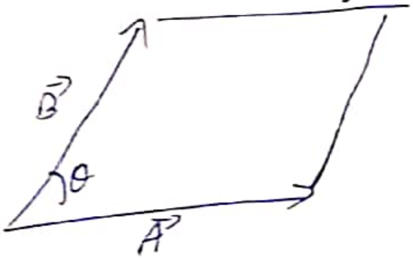


area of triangle $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times (A)(B \sin\theta)$$

$$= \frac{1}{2} |\vec{A} \times \vec{B}|$$

2) To calculate area of ∥gm:-



area $= |\vec{A} \times \vec{B}|$

Q1. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{r} = 4\hat{j} - 3\hat{k}$ find $|\vec{v}|$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$$

$\vec{v} = \hat{i}(6-8) - \hat{j}(-3-0) + \hat{k}(4+2)$

$= -2\hat{i} + 3\hat{j} + 6\hat{k}$

$|\vec{v}| = \sqrt{4+9+36}$

$|\vec{v}| = \sqrt{65}$

$= \sqrt{5 \times 13}$

$\vec{V} = \hat{i}(6-8) - \hat{j}(-3-0) + \hat{k}(4)$

$= -2\hat{i} + 3\hat{j} + 4\hat{k}$

$|\vec{V}| = \sqrt{4+9+16}$

$\boxed{= \sqrt{29}}$

Q2. Calculate area of triangle determined by $\vec{A} = 3\hat{i} + 4\hat{j}$
$\vec{B} = -3\hat{i} + 7\hat{j}$

$$\vec{Ar} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -3 & 7 & 0 \end{vmatrix}$$

$$\vec{Ar} = \hat{i}(0) - \hat{j}(0) + \hat{k}(21+12)$$

$$= 33\hat{k}$$

$$|\vec{Ar}| = \sqrt{(33)^2}$$

$$= 33$$

$$\text{Area} = \frac{33}{2}$$

$$\boxed{= 16.5\ m^2}$$

Q3. $\vec{A} = 2\hat{i} + p\hat{j} + q\hat{k}$

$\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

$$\sqrt{4+p^2+Q^2} = \sqrt{25+49+9}$$

~~$\cos\theta = 10\hat{i} + 7p\hat{j} + 3q\hat{k}$~~

$$p^2 + Q^2 + 4 = 83$$

$$p^2 + Q^2 = 79$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & p & Q \\ 5 & 7 & 3 \end{vmatrix}$$

$$= \hat{i}(3p - 7Q) - \hat{j}(6 - 5Q) + \hat{k}(14 - 5p)$$

$6 - 5q = 0$ $\qquad\qquad$ $14 - 5p = 0$

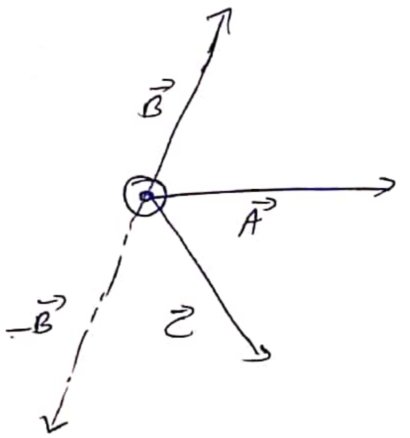
$\cancel{p = \beta}$

$\boxed{q = \frac{6}{5}}$ $\qquad\qquad$ $\boxed{p = \frac{14}{5}}$

Q3. What is the angle between $\vec{A} - \vec{B}$ and $\vec{A} \times \vec{B}$

$$\cancel{\cos\theta = \frac{(\vec{A} \times \vec{B}) \cdot (\vec{A} - \vec{B})}{|\vec{A} - \vec{B}|\,|\vec{A} \times \vec{B}|}}$$

$$\cancel{= AB \sin\theta\, \hat{n} \cdot}$$



$\boxed{\theta = 90^\circ}$

Q4. $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

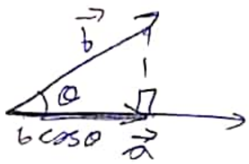
$$\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$$

$$4AB\cos\theta = 0$$
$$\theta = 90^\circ$$

Projection of vector over another:-

I) scalar projection of $\vec{b}$ on $\vec{a}$ $= b\cos\theta\,\frac{a}{a} = \frac{ab\cos\theta}{a} = \frac{\vec{a}\cdot\vec{b}}{a}$

जिसपर projection ले रहे हैं



जिसपर projection लेना है।

II) vector projection $= \frac{\vec{a}\cdot\vec{b}}{a}\,\hat{a} = \frac{\vec{a}\cdot\vec{b}}{a}\cdot\frac{\vec{a}}{a} = \frac{\vec{a}}{a^2}(\vec{a}\cdot\vec{b})$

Q find scalar & vector projection of $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$ on $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$.

$$\frac{\vec{a}\cdot\vec{b}}{b} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$$

vector $\frac{\vec{b}}{b}(\vec{a}\cdot\vec{b}) = \frac{5}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{5}{3}\hat{k}$

① find scalar ~~product~~ projection and vector projection of $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on $\vec{B} = \hat{i} + \hat{j}$

$$\frac{\vec{A} \cdot \vec{B}}{b} = \frac{1+1+0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$$

V.P $\quad \frac{\vec{B}}{b} \cdot (\vec{a} \cdot \vec{B}) = \frac{\hat{i}+\hat{j}}{\cancel{2}} \times \cancel{2} = \boxed{\hat{i}+\hat{j}}$

Q4. $\vec{A} + \vec{B} = 2\hat{i}$

$\vec{A} - \vec{B} = 4\hat{j}$

$- A^2 + B^2 + 2AB\cos\theta = \cancel{A^2 + B^2 + 2AB\cos\theta}\ 2\hat{i}$

$A^2 + B^2 - 2AB\cos\theta = 4\hat{j}$

$-4AB\cos\theta = 2\hat{j}$

$\cos\theta = \frac{-1}{2AB}$

~~cos (90+~~

$\vec{A} + \vec{B} = 2\hat{i}$

$\vec{A} - \vec{B} = 4\hat{j}$

θ

$\cos\theta = 90^\circ$

Q1. $\vec{A}+\vec{B}=2\hat{i}$

$\vec{A}-\vec{B}=4\hat{j}$

$\theta = ?$

A) 127°

B) 143°

C) 53°

D) 37°

$$\vec{A}+\vec{B}=2\hat{i}$$
$$\vec{A}-\vec{B}=4\hat{j}$$

$$2\vec{A} = 2\hat{i}+4\hat{j}$$
$$\vec{A} = \hat{i}+2\hat{j}$$
$$\vec{B} = \hat{i}-2\hat{j}$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|AB|}$$

$$= \frac{1-4}{\sqrt{5}\times\sqrt{5}}$$

$$\cos\theta = \frac{-3}{5}$$

$$\cos(90+\theta) = -\sin\theta$$

$$-\sin\theta = -\frac{3}{5}$$

$$\sin 37^\circ = \frac{3}{5}$$

$$90+37=127^\circ$$

$$\boxed{A)\ 127^\circ}$$

Q2. two forces P & Q are in ratio 1:2 if their resultant is at an angle $\tan\alpha = \frac{\sqrt{3}}{2}$ to vector P Then angle between P & Q is.

A) $\tan^{-1}\left(\frac{1}{2}\right)$ B) 45° C) 30° D) 60°

$$\tan\alpha = \frac{\sqrt{3}}{2} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\frac{P}{Q} = \frac{1}{2}$$

$$2P = Q$$

$$Q = 2P$$

$$\sqrt{3}P + \sqrt{3}Q\cos\theta = 2Q\sin\theta$$

$$\sqrt{3}P = 2Q\sin\theta - \sqrt{3}Q\cos\theta$$

$$\sqrt{3}(P + Q\cos\theta) = 2Q\sin\theta$$

$$\sqrt{3}(P + 2P\cos\theta) = 4P\sin\theta$$

$$\sqrt{3}P(1 + 2\cos\theta) = 4P\sin\theta$$

$$\frac{1 + 2\cos\theta}{\sin\theta} = \frac{4\cancel{P}}{\sqrt{3}\cancel{P}}$$

$$\frac{1}{\sin\theta} + \frac{2\cos\theta}{\sin\theta} = \frac{4}{\sqrt{3}}$$

$$\frac{1}{\sin\theta} + 2\cot\theta = \frac{4}{\sqrt{3}}$$

$$\boxed{\sqrt{3} + 2\sqrt{3}\cos\theta = 4\sin\theta}$$

$$\sqrt{3} + \cancel{2}\sqrt{3} \times \frac{\sqrt{3}}{\cancel{2}} = \cancel{4}^{2} \times \frac{1}{\cancel{2}}$$

$$\sqrt{3} + 3 = 2 \times$$

$$\cancel{\sqrt{3}} + \cancel{2}\cancel{\sqrt{3}} \times \frac{1}{\cancel{2}} = \cancel{4}^{2} \times \frac{\cancel{\sqrt{3}}}{\cancel{2}}$$

$$1 + 1 = 2$$

LHS = RHS

$$\boxed{\theta = 60^\circ}$$

$\boxed{\text{D) } 60}$

Q3. $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$

find the angle between $(\vec{A} - \vec{B})$ & $\vec{C}$

A) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ B) $\cos\theta = \frac{\sqrt{3}}{2}$ C) $\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

D) none.

$\vec{A} - \vec{B} = -1\hat{i} + 5\hat{j} + \hat{k}$

$\vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$

$$\frac{-p + 5p + 2p}{3\sqrt{3} \times p\sqrt{6}}$$

$$\frac{6p}{3p\sqrt{18}} \quad \frac{2}{\sqrt{18}} = \frac{2}{3\sqrt{2}}$$

$$\cos\theta = \frac{2}{3\sqrt{2}}$$

$$= \frac{\sqrt{2}}{3}$$

$\boxed{C}$

Q4 Two forces $\hat{i} + \hat{j} + \hat{k}$ N & $\hat{i} + 2\hat{j} + 3\hat{k}$ N acts on a particle and displace it from $(2, 3, 4)$ to point $(5, 4, 3)$.

A) 5 J

B) 4 J

C) 3 J

D) None

$\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{S} = 3\hat{i} + \hat{j} - 1\hat{k}$

$W = 6 + 3 - 4$

$W = 5$ J

$\boxed{A}$

Q5. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$

$\vec{D} = -4\hat{i} + 2\hat{j} + 3\hat{k}$

work $= 6$ J

A) 12 B) 0 C) 6 D) 1

Work $= -12 + 2c + 6$

$6 = -6 + 2c$

$12 = 2c$

$\boxed{c = 6}$

$\boxed{C}$

Q6. A force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

$a = 1\ m/s^2$

find mass

$F = ma$

A) $6\hat{i} - 8\hat{j} + 10\hat{k}$

B) 100

C) $10\sqrt{2}$

D) 10

$6\hat{i} - 8\hat{j} + 10\hat{k} = m \times 1$

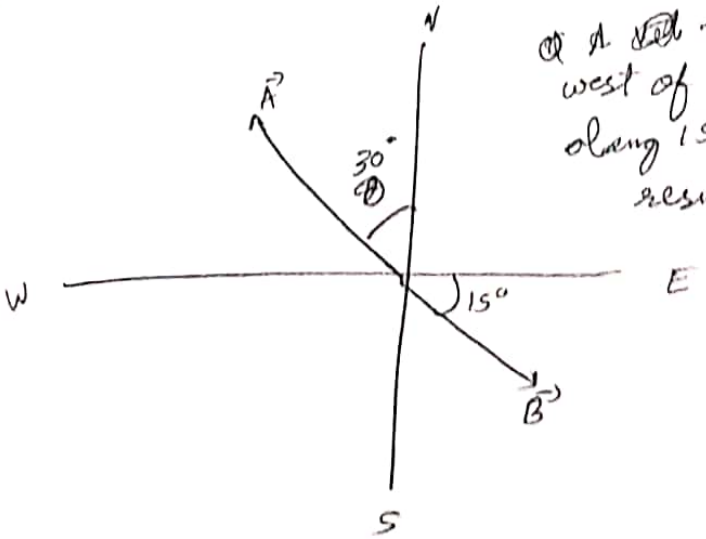
$m = \sqrt{36 + 64 + 100}$

$= \sqrt{200}$

$= 10\sqrt{2}$ kg

$\boxed{C}$

Q7.



Q A ~~vect~~ vector $\vec{A}$ is directed 30° west of north & another vector $\vec{B}$ along 15° south of east. There resultant cannot be in

$\boxed{\text{south}}$

Q 8. The sum of $\vec{F_1} = 100N$, $\vec{F_2} = 80N$ & $\vec{F_3} = 60N$ is 0, angle between F_1 & F_2 is

A) 53
B) 143
C) 37
D) 127

$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$\cancel{\sqrt{10000 + 6400 + 3600}}$

$\sqrt{10000 + 6400 + 16000 \cos\theta} = \cancel{08}$

$= \cancel{\sqrt{32400 \, Co}}$

$= \sqrt{16400 + 16000 \cos\theta}$

$= 100 \sqrt{164 + 160 \cos\theta}$

$= 20 \sqrt{41 + 40 \cos\theta} = -60$

$= \sqrt{41 + 40 \cos\theta} = -3$

$\cancel{900 \times (41 + 40 \cos\theta) + 3600 +}$

$41 + 40 \cos\theta = 9$

$\cos\theta = \frac{9 - 41}{40}$

$\cos\theta = \frac{-32}{40}$

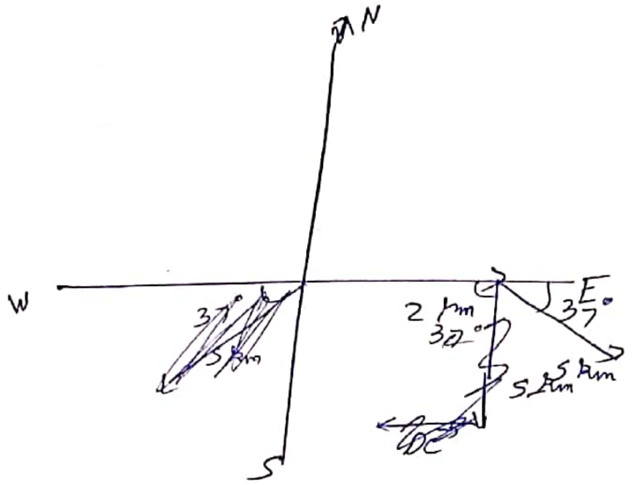
$\cos\theta = \frac{-4}{5}$

$\cos(90 + 53°) = \frac{-4}{5}$

$\theta = 90 + 53$
$= 143°$

$\boxed{B}$

Q9. A sail boat sails 2 km east, 5 km 37° south of East and an unknown displacement. The final displacement is 6 km east, find unknown.



$\vec{A} = 2\hat{i}$

$\vec{B} = +5 \times \frac{4}{5}\,\hat{i} - \frac{3}{5} \times 5\,\hat{j}$

$= -4\hat{i} - 5\hat{j}$

$\vec{C} = ?$

$\vec{A} + \vec{B} + \vec{C} = 6\hat{i}$

$\cancel{2\hat{i} + 4}\,\hat{i} - 3\hat{j} = 6\hat{i}$

$= \cancel{6\hat{i} + 2\hat{i}} + 3\hat{j}$

$\boxed{= 3\hat{j}}$