Quadratic Equation

$x^2 - 2x + 1 = 0$

$x = 1, 1$

Solution - 1 solution [repeated roots are only counted once as solution]

Roots - 2 roots

Quadratic Equation - 2 degree polynomial (2 roots)

- $ax^2 + bx + c = 0$ ($a \to$ leading coefficient)
- $a \neq 0$, If $a = 0$ then Quadratic will be ~~zero~~ linear.
- no. of roots = degree of polynomial

Methods to find roots

(1) Shri-Dharacharya $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(2) factorisation (middle Term splitting)

(3) Perfect Square (leading coefficient = 1)

$\rightarrow \left(\dfrac{\text{coefficent of } x}{2}\right)^2 < \begin{matrix} + \\ - \end{matrix}$

eg (1) $x^2 - 5x + 6 = 0$

$\left(\dfrac{-5}{2}\right)^2 = \dfrac{25}{4} < \begin{matrix} + \\ - \end{matrix}$

$x^2 - 5x + \dfrac{25}{4} - \dfrac{25}{4} + 6 = 0$

$\left(x - \dfrac{5}{2}\right)^2 - \dfrac{1}{4} = 0$

$x - \dfrac{5}{2} = \dfrac{1}{4}$

$x - \dfrac{5}{2} = \pm\dfrac{1}{2}$

$x = \dfrac{1}{2} + \dfrac{5}{2}$

$\boxed{x = 3}$

$x = \dfrac{1}{2} - \dfrac{5}{2}$

$\boxed{x = -2}$

Relation in roots and coefficients/constants

$ax^2+bx+c=0$

roots:- α, β

$\alpha+\beta = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a}$

→ Difference of Roots - $\boxed{|\alpha-\beta| = \frac{\sqrt{D}}{|a|}}$

Proof:- $(\alpha+\beta)^2 - (\alpha-\beta)^2 = 4\alpha\beta$

$\left(\frac{-b}{a}\right)^2 - (\alpha-\beta)^2 = 4\frac{c}{a}$

$(\alpha-\beta)^2 = \frac{b^2}{a^2} - \frac{4c}{a}$

$(\alpha-\beta)^2 = \frac{b^2-4ac}{a^2}$

$|\alpha-\beta| = \frac{\sqrt{D}}{|a|}$

→ $ax^2+bx+c=0$, α & β are roots so they satisfy Q.E.

$\boxed{a\alpha^2+b\alpha+c=0}$ & $\boxed{a\beta^2+b\beta+c=0}$

Find some values using SoR & PoR (sum of roots & Product of roots)

① $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\alpha^2+\beta^2 = (\alpha-\beta)^2 + 2\alpha\beta$

② $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

③ $\alpha^3-\beta^3 = (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)$

Q $x^2 - 4x + 2 = 0$ (roots are α & β then find)

① $\alpha + \beta$

$\alpha + \beta = \frac{-b}{a}$

$\boxed{= 4}$

② $\alpha^2 + \beta^2$

$\alpha\beta = \boxed{2}$

$\alpha - \beta = \sqrt{\frac{16 - 8}{1}}$

$= 2\sqrt{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 16 - 4$

$\boxed{= 12}$

③ $\alpha^3 + \beta^3$

$\alpha^3 + \beta^3 = (4)^3 - 3(2)(4)$

$= 64 - 24$

$\cancel{28}$

$\boxed{= 40}$

④ $\frac{\alpha\beta}{\alpha + \beta}$

$\frac{2}{4} = \boxed{\frac{1}{2}}$

⑤ $\alpha - \beta = \sqrt{\frac{16 - 8}{1}}$

$\boxed{= 2\sqrt{2}}$

⑥ $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\frac{\beta^2 + \alpha^2}{(\alpha\beta)^2}$

$\frac{12}{4} = \boxed{\cancel{2}}$ $\boxed{3}$

⑦ $\alpha^2 - \beta^2$

$(\alpha + \beta)(\alpha - \beta)$

$(4)(2\sqrt{2})$

$\boxed{8\sqrt{2}}$

⑧ $\alpha^3\beta - \alpha\beta^3$

$\alpha\beta(\alpha^2 - \beta^2)$

$2(8\sqrt{2})$

$\boxed{16\sqrt{2}}$

$\alpha \quad 3x^2 + 7x + 3 = 0$

$\alpha + \beta = \frac{-7}{3}$

$\alpha\beta = \frac{3}{3} = 1$

$\alpha - \beta = \frac{\sqrt{13}}{3}$

(1) $\frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

$\frac{\beta^2 + \alpha^2}{\alpha\beta}$

$\frac{\frac{49}{9} - 2}{\cancel{2}1}$

$\frac{49-18}{\cancel{18}9}$

$\boxed{= \frac{31}{\cancel{18}9}}$ ✓

(2) $\alpha^2\beta + \alpha\beta^2$

$\alpha\beta(\alpha + \beta)$

$1\left(\frac{-7}{3}\right)$

$\cancel{\left(\frac{-7}{3}\right)}$ $\boxed{\frac{-7}{3}}$ ✓

(3) $\alpha^4\beta^7 + \alpha^7\beta^4$

$\alpha^4\beta^4(\alpha^3 + \beta^3)$

$\left(\frac{-7}{3}\right)^3 - 3\left(\frac{-7}{\cancel{3}}\right)$

$\frac{-343}{\cancel{3}27} + 7$

$\cancel{-343+21}$

$-3\cancel{2}$ $\quad \frac{-343+189}{27}$

$\boxed{\frac{-154}{2\cancel{0}7}}$ ✓

(4) $\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)^2$

$\left(\frac{\alpha^2 - \beta^2}{\alpha\beta}\right)^2$

$\left(\frac{\left(\frac{-7}{3}\right)\left(\frac{\sqrt{13}}{3}\right)}{1}\right)^2$

$\left(\frac{-7\sqrt{13}}{9}\right)^2$

$\frac{49 \times 13}{81}$

$\boxed{\frac{637}{81}}$

(5) $\frac{\alpha^3 - \beta^3}{\alpha^2 - \beta^2}$

$\frac{(\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta)}{-7\sqrt{13}}$

$\frac{(\sqrt{13})^3}{27} + \frac{\cancel{3} \times \sqrt{13}}{\cancel{3}}$

$\frac{(\sqrt{13})^3 + 27\sqrt{13}}{27(-7\sqrt{13})}$

$\frac{\cancel{\sqrt{13}}(\sqrt{13})^{\cancel{2}\,3} + 27\cancel{\sqrt{13}}}{-189\cancel{\sqrt{13}}}$

$\frac{13+27}{-\cancel{189}21}$

$\boxed{\frac{-40}{\cancel{189}21}}$

$27 \times 7 = 189$

49×13: 147, 490, 637

$49 \times 7 = 343$

$27 \times 7 = 189$

Q If x^2-x+1 has roots α, β find

① $\alpha^2-\alpha$

$\alpha(\alpha-1)$

$\alpha+\beta = 1$

$\alpha\beta = 1$

$\cancel{\alpha-\beta=}$

$\alpha(-\beta)$

$-\alpha\beta$

$\boxed{-1}$

Or

α is a root

$\alpha^2-\alpha+1=0$

$\alpha^2-\alpha=-1$

② $\alpha^{15}+\beta^{15}$

$\cancel{(1-\beta)^{15}+\beta^{15}}$

$\alpha^2-\alpha+1=0$

$\alpha^2-\alpha=-1$

$\alpha^3 - \alpha^2 = -\alpha$

$\alpha^3 = \alpha^2-\alpha$

$\alpha^3=-1$

$(\alpha^3)^5 = \alpha^{15}$

$\alpha^{15} = (-1)^5$

$\alpha^{15}=-1$

$\beta^{15}=-1$

$-1+(-1)$

$\boxed{-2}$

③ $\alpha^{2025}+\beta^{2025}$

$\alpha^3=-1$

$\alpha^{2025}=-1$

$\beta^{2025}=-1$

$-1+(-1)$

$\boxed{-2}$

Q find qud whose roots are

$\frac{1}{5+2\sqrt{6}}$ & $\frac{1}{5-2\sqrt{6}}$

$\alpha+\beta = \frac{5-2\sqrt{6}+5+2\sqrt{6}}{25-24}$

$= 10$

$\alpha\beta = 1$

$\boxed{x^2-10x+1=0}$

H.W. 16-05-2024

DYS-1 {Ol, 2, 3, 5, 6, 7, 8, 9, 10, 13, 12, 11, 14, 4}

[1, 14]

~~Q find roots $a^2 + a - 44444 222222 = 0$~~

$\alpha\beta = -1$

~~$a = 0$~~ $a(a+1) = 4444444|2222222$

$\alpha\beta = -444$

$\boxed{a = 44444442222222}$

Sol

Not defined

$a(a+1) = 4$

0

444

y z

$\frac{a(a+1)}{y} = 1$

$4 = y \quad 2 = z$

$7y\,7z = a(a+1)$

$\frac{a(a+1)}{y} - 1 = 0$

$\frac{a(a+1) - y}{y} = 0$

Q find roots of $a^2 + a - 44444442222222$

Nature of roots

$ax^2+bx+c \ (a,b,c \in R)$

$D = b^2-4ac$

$D>0$
Distinct real roots

$D=0$
Equal real roots

$D<0$
Imaginary roots in pairs.

$D = b^2-4ac \ (a,b,c \in Q)$
↳ rational

Rational no.
Perfect Square

Perfect square
Not a perfect square
Roots are irrational in pair
$a+\sqrt{b}$ & $a-\sqrt{b}$

Q Find nature of roots

① $x^2+x+1=0$

$D=(1)^2-(4)(1)(1)$

$= 1-4$

$= -3$

Imaginary & Paired

② $2x^2-6x+3=0$

$D = 36-24$

$= 12$

Real, Irrational in Pair

③ $3x^2-4\sqrt{3}x+4=0$

~~Irrat~~

$D = 48-48$

$=0$

$\frac{4\sqrt{3}}{6}$

real & Equal, irrational

Q find 'm' if equation $x^2 - x + m^2 = 0$ have no real roots.

$D < 0$

$1 - 4m^2 < 0$

$-\frac{1}{2} \quad + \quad \frac{1}{2}$

$\boxed{m \in (-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)}$

$1 - 4m^2 = 0$

$4m^2 = 1$

$m^2 = \frac{1}{4}$

Q Value of 'a' for which roots of the eq. $(2a-5)x^2 - 2(a-1)x + 3 = 0$ are real & equal

$D = 0$

$(-2(a-1))^2 - (4)(3)(2a-5) = 0$

$4(a^2 + 1 - 2a) - 24a + 60 = 0$

$4a^2 + 4 - 8a - 24a + 60 = 0$

$4a^2 - 32a + 64 = 0$

$\cancel{2a^2 - 16a - 27 = 0}$

$\cancel{a = \frac{16 \pm \sqrt{256 + 216}}{4}}$

$\cancel{a = \frac{16 \pm \sqrt{472}}{4}}$

$a =$

$a^2 - 8a + 16 = 0$

$a = \frac{8 \pm \sqrt{64 - 64}}{2}$

$\boxed{a = 4}$

Q $x^2 + (2a\sqrt{a^2-3})x + 4 = 0$

$(2a\sqrt{a^2-3})^2 - (4)(4)(1) = 0$

$4a^2(a^2-3) - 16 = 0$

$4a^4 - 12a^2 - 16 = 0$

$2a^4 - 6a^2 - 8 = 0$

$a^4 - 3a^2 - 4 = 0$

$a^2 = \frac{3 \pm \sqrt{9+16}}{2}$

$a^2 = \frac{3+5}{2}$, $\cancel{\frac{3-5}{2}}$

$a^2 = \frac{8}{2}$, $\cancel{a^2 = \frac{-2}{2}}$

$\boxed{a = \pm 2}$ ✓, $\cancel{\boxed{a = i}}$

Q find 'c' if $(c-2)x^2 + 2(c-2)x + 2 = 0$ no real roots

$(-4)^2 - (4)(2)(-2)$

$\cancel{16 - 8(c-2)}$

for leading coefficient $\neq 0$

$D = 4c^2 + 16 - 16c - 8c + 16$

$D = 4c^2 - 24c + 32$

$D = c^2 - 6c + 8$

$c^2 - 4c - 2c + 8$

$c(c-4) - 2(c-4)$

$(c-2)(c-4)$

$(c-2)(c-4) < 0$

$+ \quad 2 \quad - \quad 4 \quad +$

$\boxed{(2, 4)}$

(228)

Case 2:- leading coefficient $= 0$

$C = 2$

$(2-2)x^2 + 2(2-2)x + 2 = 0$

$0x^2 + 0x + 2 = 0$

$2 = 0$

not a real value

so Roots are imaginary

$\boxed{C \in [2, 4)}$

~~Does~~ This is because questions not mentioned that given is Quadratic equation. So leading coefficient can be zero making The Equation linear

If question mensions quadratic equation, take leading coefficient $\neq 0$.

DYS-2

Q11. $(K-12)x^2 + 2(K-12)x + 2 = 0$

find integral values of K for which quadratic equation posses no real roots.

$\cancel{(K-12)^2 - (4)(2K-24)}$

$D = 4(K^2 + 144 - 24K) - 8K + 96$

$D = 4K^2 + 576 - 96K - 8K + 96$

$D = 4K^2 - 104K + 672$

$D = K^2 - 26K + 168$

$(K-12)(K-14) < 0$

$(12, 14)$

so $\boxed{K = 13}$

Q If $x = 1 + 2i$

find $x^3 + x^2 - x + 22$

$x - 1 = 2i$

$(x-1)^2 = -2$

$x^2 + 1 - 2x = -2$

$x^2 - 2x + 1 = -2$

$3x^2 - 6x + 03 = -6$

$x(x^2 + x - 1) + 22$

$\cancel{x(x^2 + x - 1)}$

$x(x^2 - 2x + 1 + 3x - 2) + 22$

$x(3x - 4) + 22$

$3x^2 - 4x + 22$

$3x^2 - 6x + 3 + 19 + 2x$

$-6 + 19 + 2x$

$13 + 2x$

$x^2 - 2x + 3 = 0$

$x = 2 \pm \sqrt{4 -}$

$x = 1 + 2i$

$x - 1 = 2i$

$x^2 + 1 - 2x = -4$

$x^2 - 2x + 5 = 0$

$\cancel{x = 2 \pm \sqrt{4}}$

$3x^2 - 6x + 15 = 0$

$x(x^2 - 2x + 5 + 3x \cancel{+5} - 6) + 22$

$x(3x - 6) + 22$

$3x^2 - 6x + 22$

$3x^2 - 6x + 15 + 7 = 0$

$\boxed{7}$

DYQS-2

Q16. find the value of $x^3 - 3x^2 - 8x + 15$

$x = 3 + i$

$x - 3 = i$

$x^2 + 9 - 6x = -1$

$x^2 + 10 - 6x = 0$

$3x^2 + 30 - 18x = 0$

$x(x^2 - 3x - 8) + 15$

$x(x^2 - 6x + 10 + 3x - 18) + 15$

$x(3x - 18) + 15$

$3x^2 - 18x + 15$

$\boxed{-15}$ $\boxed{-15}$

H.W.

DVS-2 $[1,16] - [\{11,16\} \cup \{5\}]$

$a^2 + a = 44444442222222$

$\cancel{= 4(1111111)}$

$= 44444440000000 + 2222222$

$= 4444444 \times 10000000 + 2222222$

$= 4444444(9999999 + 1) + 2222222$

$= 4(1111111)[9(1111111) + 1] + 2(1111111)$

$\cancel{= 4P(9P+1) + 2P}$

$= 4y(9y+1) + 2y$

$= 36y^2 + 4y + 2y$

$a^2 + a = 36y^2 + 6y$

$a^2 - 36y^2 + a - 6y$

$(a+6y)(a-6y) - (a+6y)$

$a + 6y\ (a - 6y - 1) = 0$

$a + 6y = 0$

$a = -6y$

$a = -6(1111111)$

$a = -6666666$

$a - 6y - 1 = 0$

$a = 6y + 1$

$(a - 6y)(a + 6P + 1) = 0$

$a = 6y$

$\boxed{a = 6666666}$

$a = -6P - 1$

$a = -6666666 - 1$

$\boxed{a = -6666667}$

Q find the quadratic if 1 root is $5+2\sqrt{6}$ & coefficients of quadratic are rational.

$\alpha = 5+2\sqrt{6}$

$\beta = 5-2\sqrt{6}$

$\alpha+\beta = 5+2\sqrt{6}+5-2\sqrt{6}$

$= 10$

$\alpha\beta = (5+2\sqrt{6})(5-2\sqrt{6})$

$= 25-4\times 6$

$= 25-24$

$\boxed{x^2-10x+1} = 0$

Q find P & Q if the roots of the eq $x^2+Px+Q=0$ have roots P & Q.

$\cancel{\alpha+\beta = -P}$

$\cancel{\alpha\beta = Q}$

$P+Q = -P$

$PQ = Q$

$PQ - Q = 0$

$Q(P-1) = 0$

$\boxed{\begin{matrix} Q=0 \\ P=0 \end{matrix}}$ $\quad$ $\begin{matrix} P=1 \\ Q=-2 \end{matrix}$

$P, Q = (1,-2), (0,0)$

Q $x^2 + mx + 1 = 0$ find m if

a) On root is thrice of other

b) ratio of roots is $1/3$

c) Sum of roots is equal to POR.

a) $\alpha = 3\beta$

$\alpha + \beta = -m$

$4\beta = -m$

$\boxed{-\frac{4}{3}\sqrt{3}}$

$m = \boxed{\pm\frac{4}{\sqrt{3}}}$

$\alpha\beta = 1$

$3\beta(\beta) = 1$

$3\beta^2 = 1$

$\beta^2 = \frac{1}{3}$

$\beta = \frac{\sqrt{3}}{3}$

b) $\frac{\alpha}{\beta} = \frac{1}{3}$

$3\alpha = \beta$

$\alpha\beta = 1$

$3\alpha^2 = 1$

$\alpha = \frac{\sqrt{3}}{3}$

$\alpha + \beta = -m$

$\cancel{\frac{\sqrt{3}}{3}} + \beta = -m$

$4\alpha = -m$

$\cancel{\boxed{-\frac{4}{3}\sqrt{3} = m}}$

$\boxed{m = \pm\frac{4}{\sqrt{3}}}$

c) $\alpha + \beta = \alpha\beta$

$-m = 1$

$\boxed{m = -1}$

Symmetric Expression of α and β

If $(\alpha, \beta) = f(\beta, \alpha)$

Eg ① $f(\alpha, \beta) = \alpha^2 + \beta^2$

$f(\beta, \alpha) = \beta^2 + \alpha^2$
$= \alpha^2 + \beta^2$

So, symmetric

Eg ② $f(\alpha, \beta) = \alpha^2 - \beta^2$

$f(\beta, \alpha) = \beta^2 - \alpha^2 \neq \alpha^2 + \beta^2$

So, not symmetric

Q find which are symmetric

① $f(\alpha, \beta) = \alpha^4 - \beta^4$

$f(\beta, \alpha) = \beta^4 - \alpha^4 \neq \alpha^4 - \beta^4$

Not Symmetric

② $f(\alpha, \beta) = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$f(\beta, \alpha) = \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$

$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

Symmetric

③ $f(\alpha, \beta) = \frac{\alpha + \beta}{\alpha\beta}$

$f(\beta, \alpha) = \frac{\beta + \alpha}{\beta\alpha}$ Symmetric

$= \frac{\alpha + \beta}{\alpha\beta}$

Transformation of roots (valid for symmetric changes)

Eg ①. $x^2 - 4x + 5 = 0$

find quadratic whose roots are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$

$\frac{2}{\alpha}, \frac{2}{\beta} = \frac{2}{x} \to t$

$x = \frac{2}{t}$

(put in Quad)

$\left(\frac{2}{t}\right)^2 - 4\left(\frac{2}{t}\right) + 5 = 0$

$\frac{4}{t^2} - \frac{8}{t} + 5 = 0$

$\boxed{5t^2 - 8t + 4 = 0}$

Q 2. $x^2 - 3x + 2 = 0$

find Quad of roots $\alpha + 1$, $\beta + 1$

$\alpha+1, \beta+1 \Rightarrow x + 1 = t$

$x = t - 1$

$(t-1)^2 - 3(t-1) + 2 = 0$

$t^2 + 1 - 2t - 3t + 3 + 2 = 0$

$\boxed{\cancel{t^2 + 5t} = 0}$

$\boxed{t^2 - 5t + 6 = 0}$

Method II

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$x = 1, 2$

$\alpha + 1 = 1 + 1 = 2$

$\beta + 1 = 2 + 1 = 3$

$\alpha + \beta = 5$

$\alpha\beta = 6$

$\boxed{x^2 - 5x + 6 = 0}$

Q $x^2-x+1=0$ have roots α and β, find quadratic whose roots are -

① $3+\alpha,\ 3+\beta$

$x+3=y$

$x=y-3$

$(y-3)^2-(y-3)+1=0$

$y^2+9-6y-y+3+1=0$

$\boxed{y^2-7y+13=0}$

② $1-\frac{1}{\alpha},\ 1-\frac{1}{\beta}$

$1-\frac{1}{x}=y$

$1-y=\frac{1}{x}$

$x=\frac{1}{1-y}$

$\left(\frac{1}{1-y}\right)^2-\left(\frac{1}{1-y}\right)+1=0$

$\frac{1}{\cancel{1+y^2-2y}\ (1-y)^2}-\frac{1}{1-y}+1=0$

$(1-y)^2-(1-y)+1=0$

$y^2+1-2y-1-y+1=0$

$\boxed{y^2-3y+1=0}$

③ $\frac{2}{1+\alpha},\ \frac{2}{1+\beta}$

$\frac{2}{1+x}=y$

$\frac{2}{y}=1+x$

$x=\frac{2-y}{y}$

$\left(\frac{2-y}{y}\right)^2-\left(\frac{2-y}{y}\right)+1$

$\frac{y^2+4-4y}{y^2}-\frac{(2-y)}{y}+1$

$y^2+4-4y-2y+y^2+y^2$

$\boxed{3y^2-6y+4=0}$

④ $-\alpha,-\beta$

$-x=y$

$x=-y$

$(-y)^2-(-y)+1=0$

$\boxed{y^2+y+1=0}$

Q $\cancel{x^2+1}\ x^2-x+1=0$ have roots α & β

find Quadratic whose roots are.

(1) $\alpha+3\beta,\ 3\alpha+\beta$

(2) $(\alpha-\beta)^2,\ (\alpha+\beta)^2$

(1)

$\alpha+\beta=1$

$\alpha\beta=1$

$\alpha-\beta=\sqrt{-3}$

$R_1=\alpha+3\beta$

$R_2=3\alpha+\beta$

$R_1+R_2=\alpha+3\beta+3\alpha+\beta$

$=4\alpha+4\beta$

$=4(\alpha+\beta)$

$=4(1)$

$=4$

$R_1\times R_2=(\alpha+3\beta)(3\alpha+\beta)$

$=3\alpha^2+3\beta^2+\alpha\beta+9\alpha\beta$

$=3((\alpha+\beta)^2-2\alpha\beta)+10\alpha\beta$

$=3(1-2)+10$

$=-3+10$

$=7$

$x^2-(4)+7$

$\boxed{x^2-4x+7}$

(2)

$\alpha+\beta=1$

$\alpha\beta=1$

$R_1+R_2=(\alpha-\beta)^2+(\alpha+\beta)^2$

$=\alpha^2+\beta^2-\cancel{2\alpha\beta}+\alpha^2+\beta^2+\cancel{2\alpha\beta}$

$=2(\alpha^2+\beta^2)$

$=2(-1)$

$\boxed{=-2}$

$R_1\times R_2=(\alpha-\beta)^2(\alpha+\beta)^2$

$=\cancel{(\alpha^2+\beta^2-2\alpha\beta)(\alpha^2+\beta^2)}$

$=(\sqrt{-3})^2(1)^2$

$=-3\times1$

$\boxed{=-3}$

$\boxed{x^2+2x-3}$

Equation (vs) Identity

→ Equation which is true for every value of the variable is a identity

eg. $\sin^2\theta + \cos^2\theta = 1$ $(\theta \in R)$

$(a+b)^2 = a^2 + 2ab + b^2$ $(a, b \in R)$

→ In quadratic if it has more than 2 roots then it will be an identity

Q $(p-1)x^2 + (p^2-3p+2)x + (p^2-4p+3) = 0$ is an identity in x. find value of p

$(p-1)x^2 + (p^2-3p+2)x + (p^2-4p+3) = 0$

$p-1=0$ $\quad$ $p^2-3p+2=0$ $\quad$ $p=3,1$

$p=1$ $\quad$ $p=2,1$

$\boxed{p=1}$

for $p=1$

$\cancel{0\ (1-1)x^2+}$

$0x^2 + 0x + 0 = 0$

Q. find x if Quad in λ have more than 2 roots.

$x^2(\lambda^2-5\lambda-16) + x(\lambda^2+3\lambda+2) + \lambda^2 - 4 = 0$

$\cancel{x^2\lambda^2 - 5x^2\lambda + 6x^2 +}$

$x^2\lambda^2 - 5x^2\lambda + 6x^2 + x\lambda^2 + 3x\lambda + 2x + \lambda^2 - 4 = 0$

$\lambda^2(x^2+x+1) + \lambda(-5x^2+3x) + (6x^2+2x-4)$

$x \in \phi$

no common

$\boxed{x \in \phi}$

x^2+x+1

$x = -1 \pm \sqrt{\ }$

Q $\frac{a(x-b)(x-c)}{(a-b)(a-c)} + \frac{b(x-c)(x-a)}{(b-c)(b-a)} + \frac{c(x-a)(x-b)}{(c-a)(c-b)} = x$

~~$ax^2(1-b)$~~

let $x = a$

$a + 0 + 0 = a$ $\longrightarrow$ $\frac{a(\cancel{a-b})(\cancel{a-c})}{(\cancel{a-b})(\cancel{a-c})} + 0 + 0 = a \rightarrow a = a$

$x = b$

$0 + b + 0 = b$

$x = c$

$0 + 0 + c = c$

so $x = a, b, c$

so its a identity.

H.W. (18-05-24)

DYS-2 [17, 27] - {27}

DYS-3 (full) - {2, 15(a, b)}

DYS-4 {1, 2}

Race - {9, 10, 11, 12, 13, 14}

MS-2 (ch-3)

Q24. $x_1 = 9x_2$

$x_1 + x_2 = 3a+2$

$10x_2 = 3a+2$

$x_2 = \frac{3a+2}{10}$

$3x_2 = a$

$\frac{9a+6}{10} = a$

$9a+6 = 10a$

$6 = a$

$x_2 = 2$

roots $- \sqrt{18}, 2$

$\boxed{-\frac{18}{17}, \frac{2}{17}}$

$x_1 x_2 = a^2$

$\cancel{(3x_2)^2 = a^2}$

$\cancel{3x_2 = a}$

$\cancel{9 \times \frac{9a^2+4+12a}{100} = a^2}$

$\cancel{81a^2+36+108a = 100a^2}$

$\cancel{19a^2-108a-36=0}$

$\cancel{x^2 - x + 2 = 0}$

$\cancel{x^2-2x+x+2}$

$\cancel{x(x-2)+1}$

$\cancel{x = 2, -1}$

$19a^2 - 108a - 36 = 0$

$a = \frac{108 \pm \sqrt{108^2 + 3736}}{38}$

$a =$

108
108
864
10800
36
19
720
76
3684
2736
3736

11664

11664

34.

$108 + x = 38 \times 6$

$x = \frac{38 \times 6}{108}$

54

65
5
325

63
3
189

626 64
2 4
256

104

104

Scanned with CamScanner

Condition for common roots

Types

- Case 1 → Only 1 common root
- Case 2 → Both common roots
- Case 3 → At least one root common
  - Case 1
  - Case 2
  - union
- Case 4 → Rare question: one root common between a cubic & quadratic

Case 1 → Only 1 common root

$a_1x^2+b_1x+C_1=0$ with roots α, β

$a_2x^2+b_2x+C_2=0$ with roots α, γ

$\therefore\ a_1\alpha^2+b_1\alpha+C_1=0$ (multiply by a_2)

$a_2\alpha^2+b_2\alpha+C_2=0$ (multiply by a_1)

$a_1a_2\alpha^2+a_2b_1\alpha+C_1a_2=0$ (subtract) $a_1a_2\alpha^2+a_1b_2\alpha+C_2a_1=0$

$(a_2b_1-a_1b_2)\alpha+a_1C_2+a_2C_1=0$

Q find λ if $x^2 - \lambda x - 21 = 0$ & $x^2 - 3\lambda x + 35 = 0$ have one root common.

$\alpha^2 - \cancel{\alpha}\lambda\alpha - 21 = 0$ -- (1) $\qquad \alpha^2 - 3\lambda\alpha + 35 = 0$ -- (2)

$\cancel{2\alpha^2}$: (2) - (1)

$-2\lambda\alpha + 56 = 0$

$2\lambda\alpha = 56$

$$\boxed{\lambda\alpha = \frac{56}{2} = \frac{28}{\cancel{2}}}$$

$\alpha = \frac{28}{\lambda}$

put in (1)

$\left(\frac{28}{\lambda}\right)^2 - \lambda \times \frac{28}{\lambda} - 21 = 0$

$\frac{(28)^2}{\lambda^2} - 49 = 0$

$\frac{28}{\lambda^2} = 49$

$\frac{\cancel{28^2}\, 28^2}{49^2} = \lambda^2$

$\lambda = \frac{28}{\pm 7}$

$$\boxed{\lambda = \pm 4}$$

Q find a if $x^2+(a^2-2)x-2a^2=0$ & x^2-3x+2 have only one root common.

$$\alpha^2+(a^2-2)\alpha-2a^2=0 \quad (2) \qquad \alpha^2-3\alpha+2=0 \quad (1)$$

(2) − (1)

$a^2\alpha-2\alpha+3\alpha-2a^2-2=0$

$-a^2+\alpha-2=0$

$\cancel{a^2=\alpha-2}$

$\cancel{a=\sqrt{\alpha-2}}$

$\alpha=\underline{a^2+2}$

put in (1)

$(a^2+2)^2+3(a^2+2)+2=0$

$a^4+4+4a^2+3a^2+6+2=0$

$a^4+7a^2+12=0$

$a^2=\dfrac{-7\pm\sqrt{49-48}}{2}$

$a^2=\dfrac{-7\pm1}{2}$

$a^2=-4,-3$

X

$a^2\alpha-2\alpha+3\alpha-2a^2-2=0$

$a^2\alpha+\alpha-2a^2-2=0$

$\alpha(a^2+1)-2(a^2+1)=0$

$(a^2+1)(\alpha-2)=0$

+ V

$\alpha-2=0$

$\alpha=2$

so for every value of a equation satisfies.

$$\boxed{a\in\mathbb{R}}$$

Q If $x^2+Px+Q=0$ & $x^2+Qx+P=0$

$P\neq Q$ and one common root.

$\alpha^2+P\alpha+Q=0$ $\qquad$ $\alpha^2+Q\alpha+P=0$

② − ①

$P\alpha-Q\alpha+Q-P=0$

$(P-Q)\alpha+Q-P=0$

$P-Q\,\alpha = P-Q$

$\alpha = \frac{P-Q}{P-Q}$

$\alpha=1$

$(1)^2+P+Q=0$

$\boxed{P+Q=-1}$

Case - 2 two roots common -

$a_1x^2+b_1x+c_1=0$ $\qquad$ $a_2x^2+b_2x+c_2=0$

α, β $\qquad$ α, β

$\alpha+\beta = \frac{+b_1}{a_1} = \frac{+b_2}{a_2}$

$= \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\alpha\beta = \boxed{\frac{c_1}{c_2} = \frac{a_1}{a_2} = \frac{b_1}{b_2}}$

Q If $a, b, c \in R$ & eq $ax^2+bx+c=0$ & $x^2+2x+9=0$ have both roots as common, then find $a:b:c$

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{9}$$

~~$a:b:c$~~

~~$\frac{18a}{9}$~~

$$\frac{18a}{18} = \frac{9b}{18} = \frac{2c}{18}$$

$a:b:c$

$\boxed{18:9:2}$

$$\frac{1}{a} = \frac{2}{b} = \frac{9}{c}$$

$a:b:9$

$\boxed{1:2:9}$

Q. $2x^2+x+k=0$ & $x^2+\frac{x}{2}-1=0$ have 2 common roots find k

$$\frac{2}{1} = \frac{1}{1} \times 2 = \frac{k}{-1}$$

$$\frac{2}{1} = -\frac{k}{1}$$

$\boxed{-2 = k}$

Q. find k $x^2+2kx+1=0$ & $x^2+x+1=0$ have 1 common root

$D<0$, so roots are in pair as one is common, other will be common as well.

$$\frac{2k}{1} = 1$$

$2k = 1$

$\boxed{k = \frac{1}{2}}$

Case 3 At least one root common.

Q Possible values of 'a' for which $x^2+ax+1=0$ & $x^2+x+a=0$ have at least one common root

Case 1 $x^2-x^2+ax-x+1-a=0$

$x(a-1)-1(a-1)=0$

$(a-1)(x-1)=0 \rightarrow x-1=0$

$\boxed{a=1}$ $x=1$

$(1)^2+a(1)+1=0$

$a+2=0$

$\boxed{a=-2}$

Case 2 $\frac{1}{1}=\frac{a}{1}=\frac{1}{a}$

$\frac{1}{a}=1$ $a^2=1$

$\boxed{a=1}$ $a=\pm1$

$\boxed{a=-1}$ not satisfy

Case 1 ∪ case 2 $=$ $\boxed{a\in\{1,-2\}}$

Case 4 QnCu - One common root between a quadratic & cubic

Q $x^3-3x^2+(2k-1)x+3=0$ & $2kx+1-x^2=0$ have one common ~~same~~ root. find k.

$x^3-3x^2+(2k-1)x+3-2kx-1+x^2=0$

$x^3-2x^2+(2k-1-2k)x + 2=0$

$x^3-2x^2-x+2=0$

$\boxed{x=1}$

$x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$\boxed{x=2,-1,1}$

$2k+1-1=0$

$2k=0$

$k=0$

$-2k+1+1=0$

$-2k=-2$

$k=1$

$4k+1-4=0$

$k=\frac{3}{4}$

7

① H.W. 20-05-2024

DYS-4 {3, 4, 5, 6, 7}

O-1 {Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Q If the eq $ax^2+bx+c=0$ & $x^3+3x^2+3x+2=0$ have 2 common roots. then –

a) $a=b\neq c$

B) $a=-b=c$

c) $a\neq b\neq c$

✓D) $a+b+c=3$

$-1+3-3+2$

$-8+12-6+2$

$(x+2)=0$

x^2+x+1

$x=\frac{-1\pm\sqrt{1-4}}{2}$

x^2+x+1

ax^2+bx+c

$\frac{1}{a}=\frac{1}{b}=\frac{1}{c}$

$a=1=c$

$a=1$

$1+1+1=3$

Q If $x^3+1=0$ & $ax^2+bx+c=0$ $a,b,c\in R$. have 2 common roots. then $a+b=$?

$x^3+1=0$

$x^3=-1$

$\boxed{x=-1}$

~~$\alpha+\beta=-2$~~

~~$\alpha\beta=1$~~

~~x^2+2x+1~~

$(x+1)$ is a factor

~~$b=2$~~

~~$c=1$~~

~~$a=1$~~

~~$a+b=2+1$~~

~~$=3$~~

~~$\boxed{D}$~~

$x+1 \overline{)\,x^3+1\,}(x^2-x+1$

$-x^3-x^2$

$-x^2+1$

$+x^2+x$

$x+1$

x^2-x+1

~~$x=$~~ $D<0$

$x^2-x+1=ax^2+bx+c$

$a=1$

$b=-1$

$\boxed{a+b=0}$

$\boxed{B}$

Quadratic expression and its graphs.

$ax^2+bx+c \ (a, b, c \in R), \ a \neq 0$

① $a<0$ concave down

$a>0$ concave up

② $D>0$ Roots real & unequal → Graph cuts X axis at 2 diff points

$D=0$ Roots are equal → Graph touch x-axis

$D<0$ Graph does not cut x-axis.

③ Vertex : $(x, y) = \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$.

Proof:- $y = ax^2+bx+c$

$$y = a\left[x^2+\frac{b}{a}x+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}+\frac{c}{a}\right]$$

$$y = \left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2+4ac}{4a^2}\right]$$

$$y = a\left[\left(x+\frac{b}{2a}\right)^2-\left(\frac{-D}{4a^2}\right)\right]$$

$$y = a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a}$$

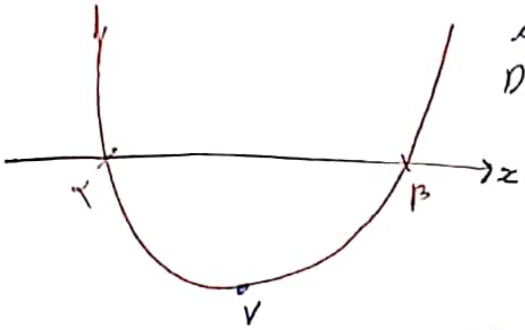
$$\left(y+\frac{D}{4a}\right) = a\left(x+\frac{b}{2a}\right)^2$$

$$\boxed{x = \frac{-b}{2a}}$$

$$\boxed{y = \frac{-D}{4a}}$$

(4) Graphs

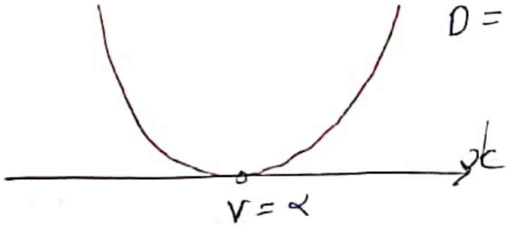
(1)



$a > 0$
$D > 0$

$y = \oplus, \ominus, 0$

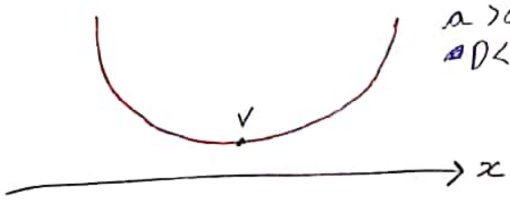
(2)



$a > 0$
$D = 0$

$y = \oplus, 0$

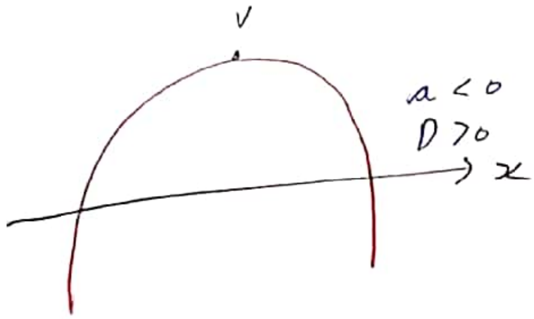
(3)



$a > 0$
$D < 0$

$y = \oplus$

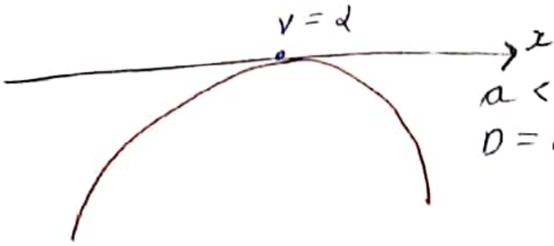
(4)



$a < 0$
$D > 0$

$y = \oplus, \ominus, 0$

(5)

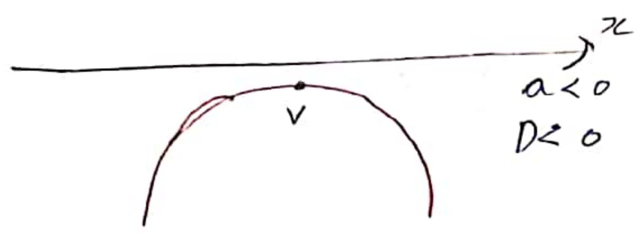


$a < 0$
$D = 0$

$y = \ominus, 0$

(e)

(6)



$y = \ominus$

$a, b, c \in R$

(1) $ax^2+bx+c>0$ ($a>0$, & $D<0$) Quadratic always $\oplus$ve

(2) $ax^2+bx+c<0$ ($a<0$ & $D<0$) Quadratic is always $\ominus$

(3) $ax^2+bx+c \geq 0$ ($a>0$, & $D \leq 0$) Quadratic is $\oplus$ or 0

Q Find 'a' for which $ax^2+3x+4>0$ $\quad x \in R$

$a>0, \; D<0$

$9-\cancel{4a}\,16a<0$

$16a>9$

$a>9/16 \quad \cup \; a>0$

$\boxed{a>\frac{9}{16}}$ $\quad a \in \left(\frac{9}{16}, \infty\right)$

Q. $ax^2+2ax+\frac{1}{2}<6$

$a<0, \; D<0$

$b^2-4ac<0$

$4a^2-2a<0$

$2a(2a-1)<0$

$a \in (0, 1/2) \cup a<0$

$\boxed{a \in \phi}$

(12)

Q $kx^2 + x + k > 0$

$a > 0$
$D < 0$

$1 - 4k^2 < 0$

$\cancel{4k^2 > 1}$
$\cancel{k^2 > \frac{1}{4}}$
$\cancel{k > \frac{1}{2}}$

$4k^2 - 1 > 0$

$+ \quad -\frac{1}{2} \quad - \quad \frac{1}{2} \quad +$

$k \in [(-\infty, -\tfrac{1}{2}) \cup (\tfrac{1}{2}, \infty)] \cap (0, \infty)$

$\boxed{k \in (\frac{1}{2}, \infty)}$ ✓

H.W. 21-05-2024

DYS-8 $[4, 11] - \{5\}$

O-1 $[11, 18] - \{15, 16, 17\}$

O-2 $[1, 6] - \{4\}$

Range of A Quadratic (values of y)

Range $\in [y_{min}, y_{max}]$

Type 1:- $x \in R$

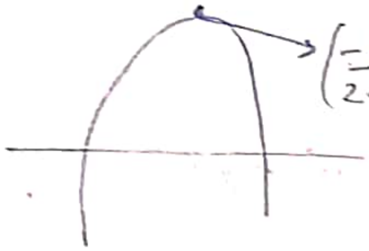
$a > 0$

Range $\in \left[\frac{-D}{4a}, \infty\right)$

$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$a < 0$

Range $\in \left(-\infty, \frac{-D}{4a}\right]$



$\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

Type-2:- x is restricted.

Case 1

when $x = \frac{-b}{2a}$ lies in $[x_1, x_2]$

$f\left(\frac{-b}{2a}\right), f(x_1), f(x_2)$

Case 2

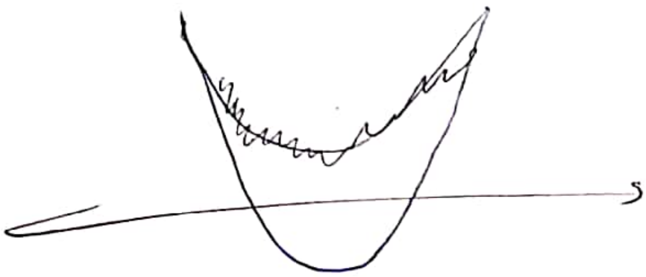
when $x = \frac{-b}{2a}$ dont lie in $[x_1, x_2]$

check :- $f(x_1)$, & $f(x_2)$

Q Draw the graph of $x^2 - 5x + 6 = 0$

① find Minimum value & point where min value occurs.

② Range of quadratic.



① min value $= \frac{-D}{4a}$

$= \frac{-1}{4}$

$\boxed{= -1/4}$

$x_{min} = \frac{5}{2}$

$\boxed{x_{min} = \frac{5}{2}}$

② Range $\in [y_{min}, y_{max}]$

$y_{min} = -1/4$

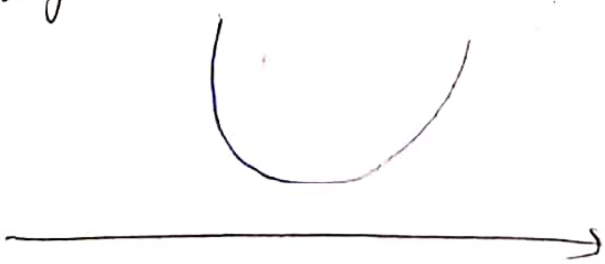
$y_{max} = \infty$

$\boxed{\text{Range } [-1/4, \infty)}$

Q Draw graph of $x^2 + x + 1 = 0$

① find min value & Point

② Range



① $y_{min} = \frac{-D}{4a}$

$= \frac{4-1}{4}$

$\boxed{= \frac{3}{4}}$

$x_{min} = \boxed{\frac{-1}{2}}$

$x_{min} = \frac{-b}{2a}$

② $y_{max} = \infty$

Range $\boxed{[\frac{3}{4}, \infty)}$

Q find the range of $-x^2+2x-1$

$y_{max} = -\frac{(4-4)}{2}$

$= 0$

$\boxed{(-\infty, 0]}$

Q $y = x^2 - 2x - 3$

① $x \in R$

$y_{min} = -\frac{(4+12)}{4}$

$= -4$

$\boxed{[4, \infty)}$

② $x \in [0, 3]$

$x \in [-8, \infty] \cap [0, 3]$

$\boxed{x \in [0, 3]}$

③ $x \in [-2, 0]$

$\boxed{x \in [-2, 0]}$

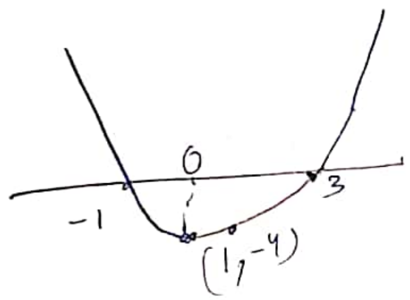
② $x \in [0,3]$

$y|_{x=0} = 0-0-3$
$= -3$

$y|_{x=3} = 9-6-3$
$= 0$

$\boxed{[-4,0]}$

$y|_{x=1} = 1-2-3$
$= -4$

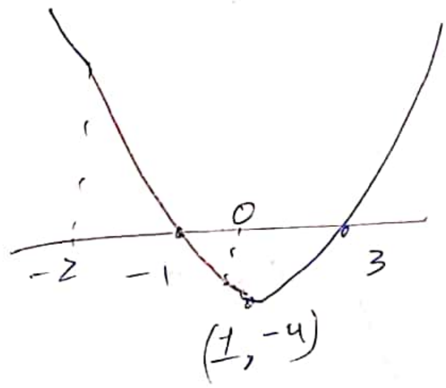


③ $x \in [-2,0]$

$f(-2) = 4+4-3$
$= 8-3$
$= 5$

$f(0) = -3$

$\boxed{[-3,5]}$



Q $y = f(x) = x^2-5x+6$

$y_{min} = -\frac{(25-24)}{4}$

$= \boxed{-\frac{1}{4}} = -0.25$

$x_{min} = \boxed{\frac{5}{2}} = 2.5$

① $[-3,0]\ x \in x$

$f(-3) = 9+15+6$
$= 30$

$f(0) = 6$

$\boxed{y \in [6,30]}$

② $x \in [1,5]$

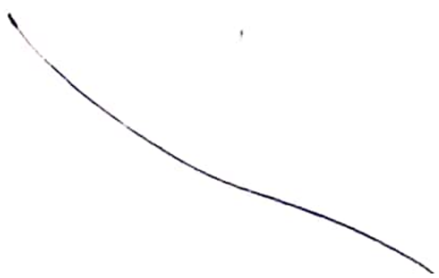
$f(1) = 2$
$f(5) = 6$
$f(2.5) = -1/4$

$\boxed{y \in [-1/4, 6]}$

③ $x \in [3,4]$

$f(3) = 0$
$f(4) = 16-20+6$
$= 2$

$\boxed{y \in [0,2]}$



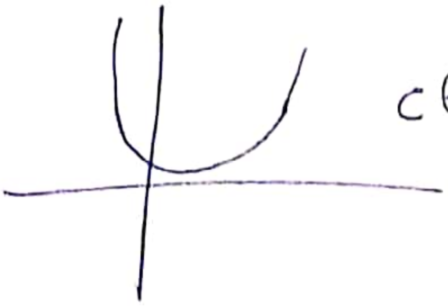
Commenting of *Signs of a, b, c

① $y = ax^2 + bx + c$

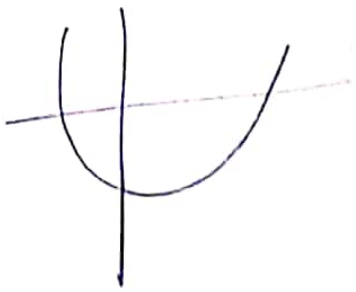
$a > 0$ concave up

$a < 0$ concave down

② $c \rightarrow$ cut on y-axis



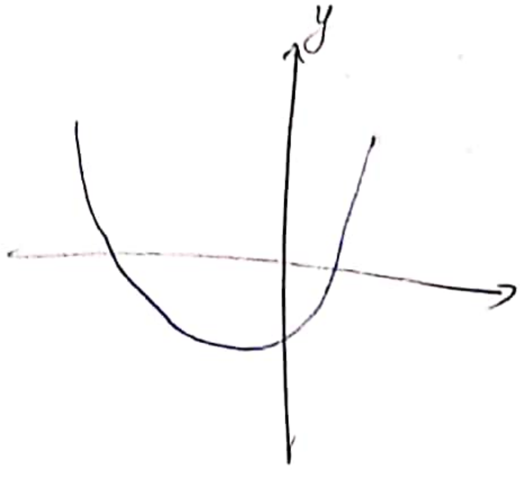
c ⊕ve



c ⊖ve

③ $b \rightarrow$ no fixed rule

Q Comment on the signs of a, b, c



a = ⊕ve → concave up

c = ⊖ve → cut y-axis below x-axis.

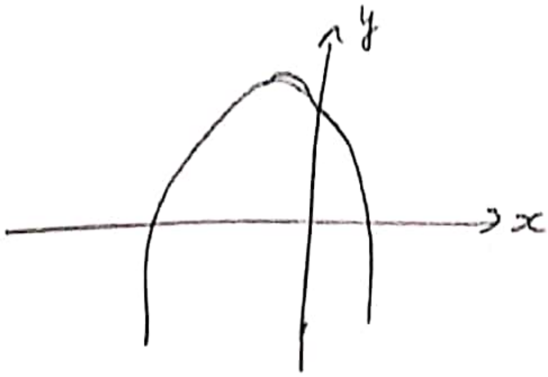
$\frac{-b}{2a} < 0$

⊕ve so $-b$ = ⊖ve

$\boxed{b = \oplus ve}$

Q Comment on a, b, c signs

①



$\boxed{a = \ominus ve}$

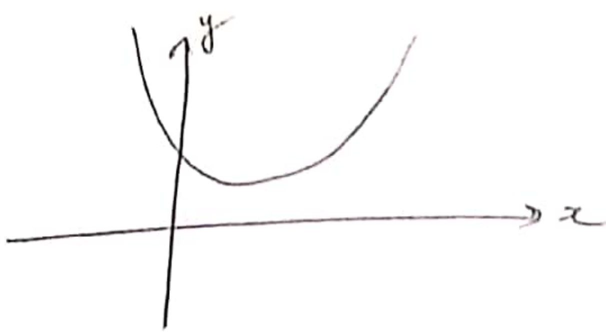
$\boxed{C = \oplus ve}$

$\frac{-b}{2a} = \ominus ve$

$2a \to a$ is $\ominus ve$

$\boxed{b = \ominus ve}$ $\boxed{b = \ominus ve}$

②

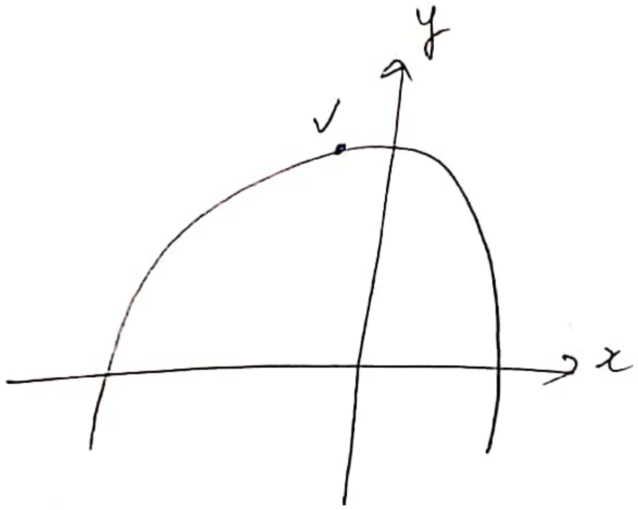


$a = \oplus ve$

$c = \oplus ve$

$b = \ominus ve$

Q



i) a ⊖ve

ii) b ~~⊕ve~~ ⊖ve

iii) c ⊕ ve

iv) bc ⊖ve

v) c−a ⊕ve

vi) ab^2 ⊖ve

vii) abc ⊕ ve

viii) $\frac{a+b}{c}$ ⊖ ~~⊕~~ ve

H.W. (23-5-2024)

~~O-1~~ DYS-8 (Q1, Q2, Q3)

O-1 (Q15, 22)

~~O-2 (Q9, 8)~~,

O-2 (Q8, 9, 11, 16, 17, 18, 19,)

J-M (Q2, 3, 4, 5, 6, 7, 14)

Q $y = ax^2 + bx + c$, & $c < 0$ does not have any real roots then comment on the signs of - $a < 0$, $c < 0$

(A) $c(a+b+c)$

(B) $c(a-b+c)$

(C) $4a+2b+c$

$\frac{-D}{4a} < 0$

(A) For $x = 1$

$y = a+b+c$

where $y < 0$

$a+b+c < 0$

$c < 0$

$c(a+b+c)$ $\boxed{\oplus ve}$

$\frac{-b}{a} > 0$

(B) for $x = -1$

$y = \ominus ve$

$c(\ominus ve)$

$\ominus(\ominus ve)$

$\boxed{\oplus ve}$

(C) $x = 2$

$\boxed{y = 4 \ominus ve}$

Q If $c < 0$ & $y = ax^2 + bx + c$ has no real roots find signs.

$c < 0$, $a < 0$

(A) $9a + 3b + c = y$

for $x = 3$

$\boxed{y = \ominus ve}$

(B) $a + 2b + 4c$

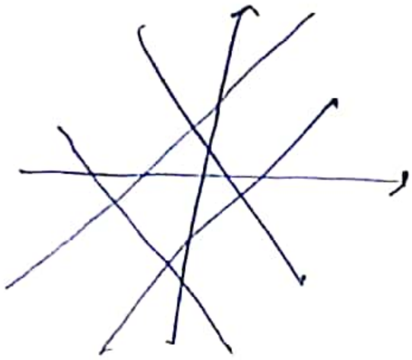
$\ominus ve$ for $x = 1/2$

Range of Linear, $\frac{\text{Linear}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Linear}}$ & $\frac{\text{Quad}}{\text{Quad}}$

① Linear –

$y = ax + b$

$y \in R$ always



eg. $y = 2x + 3$

Range – $(-\infty, \infty)$

$y = \sqrt{2}x - \frac{7}{2}$

Range $= (-\infty, \infty)$

② $\frac{\text{Linear}}{\text{Linear}}$ – $y = \frac{ax+b}{cx+d}$

Range $\rightarrow y \in R - \left\{\frac{a}{c}\right\}$

eg – $y = \frac{2x-3}{x+2}$

$y \in R - \left\{\frac{2}{1}\right\}$

$y \in R - \{2\}$

$y \in (-\infty, 2) \cup (2, \infty)$

Q find ranges

1) $y = \frac{3x+1}{2x-1}$

$y \in R - \{\frac{3}{2}\}$

2) $y = \frac{x}{2x-1}$ ~~$\{\frac{1}{2}\}$~~

$y \in R - \{\frac{1}{2}\}$

3) $y = \frac{5x-2}{7-4x}$

$y \in R - \{\frac{5}{-4}\}$

4) $y = \frac{1}{3x+4}$

$y \in R - \{0\}$

3) $\frac{\text{Quad}}{\text{Linear}}$, $\frac{1}{\text{Quad}}$, $\frac{\text{Quad}}{\text{Linear}}$, $\frac{\text{Quad}}{\text{Quad}}$

Process:- Do cross Multiply

case 1:- when leading coefficient $\neq 0$ then apply $D \geq 0$ & solve inequality in y.

case 2: when leading coefficient $= 0$, if any value of x is comming then no problem. otherwise exclude.

Q find range of $y = \frac{x^2-3x+4}{x^2+3x+4}$ $\quad x \in R$

$yx^2 + 3yx + 4y = x^2 - 3x + 4$

$(y-1)x^2 + (3y+3)x + (4y-4)$

Case 1 $\quad y-1 \neq 0$

$D \geq 0$

$(3y+3)^2 + (-4)(4y-4)(4y-1) \geq 0$

$9y^2 + 9 + 18y - 16(y^2+1-2y) \geq 0$

$9y^2 - 16y^2 + 18y + 32y + 9 - 16 \geq 0$

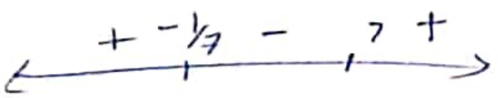
$-7y^2 + 50y - 7 \geq 0$

$7y^2 - 50y + 7 \leq 0$

$7y^2 - 49y - y + 7 \leq 0$

$7y(y-7) - 1(y-7) \leq 0$

$(7y-1)(y-7) \leq 0$



$y \in \left[\frac{1}{7}, 7\right]$

Case 2 :-

$y-1=0$

$y=1$

Put in Question

$\cancel{x^2}+3x+4-\cancel{x^2}+3x-4=0$

$6x=0$

$x=0$

$\therefore$ Value of x coming in Case 2 so we need to exclude $y=1$

Hence, $y\in\left[\frac{1}{7},7\right]$

~~Q. $y=\frac{8x-4}{x^2+2x-1}$~~

Q $y=\frac{8x-4}{x^2+2x-1}$ $\quad x\in R$

Case 1 :-

$yx^2+2yx-y=8x-4$

$yx^2+(2y-8)x-(y-4)=0$

$D\geq 0$

$(2y-8)^2+(+4)(y)(y-4)=0$

$4y^2+64-32y+4y^2-16y$

$8y^2-48y+64\geq 0$

$y^2-6y+8\geq 0$

$y^2-4y-2y+8\geq 0$

$$y(y-4)-2(y-4) \geq 0$$
$$(y-2)(y-4) \geq 0$$

$$y \in (-\infty, 2] \cup [4, \infty)$$

Case 2 $\quad y = 0$

$$8x - 4 = 0$$
$$8x = 4$$
$$x = \frac{4}{8}$$
$$x = \frac{1}{2}$$

$$\boxed{y \in (-\infty, 2] \cup [4, \infty)}$$

Q $\quad y = \dfrac{x^2+2x-3}{x^2+2x-8}$, Find range when $x \in R$

$$y = \frac{x^2+2x-3}{x^2+2x-8}$$

$$yx^2 + 2yx - 8y = x^2+2x-3$$
$$yx^2 - x^2 + 2yx - 2x - 8y + 3 = 0$$
$$(y-1)x^2 + (2y-2)x - (8y-3) = 0$$

Case 1 $\quad D \geq 0$

$b^2 - 4ac \geq 0$

$(2y-2)^2 + (+4)(y-1)(8y-3)$

$4y^2 + 4 - 8y + 4(8y^2 - 3y - 8y + 3)$

$4y^2 + 4 - 8y + 32y^2 - 44y + 12$

$36y^2 - 52y + 16 = 0$

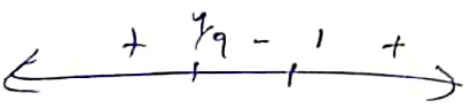
$18y^2 - 26y + 8 = 0$

$9y^2 - 13y + 4 = 0$

$9y^2 - 9y - 4y + 4 \geq 0$

$9y(y-1) - 4(y-1) \geq 0$

$(9y-4)(y-1) \geq 0$



$y \in (-\infty, 4/9] \cup [+1, \infty)$

Case 2 $\quad y - 1 = 0$

$\boxed{y = 1}$

$1 = \dfrac{x^2 + 2x - 3}{x^2 + 2x - 8}$

$\cancel{x^2} + \cancel{2x} - 8 = \cancel{x^2} + \cancel{2x} - 3$

$-8 \neq -3$

so exclude $\boxed{y = 1}$

$$\boxed{y \in (-\infty, 4/9] \cup (1, \infty)}$$

H.W. 24-05-2024

DYS-9 [All] ✓

O-1 {23}

~~O-2 {~~

J-M {1, 13}

J-A {3, 4}

O-4 {1} ✓

O-3 {7, 8, 1, 2, 3}

~~O-2 {~~

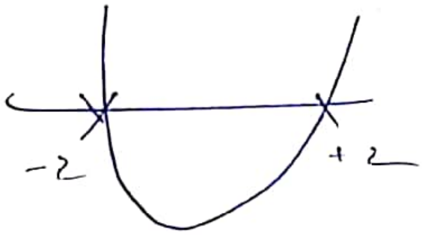
O-2 {7} ✓

Location of roots (Real nos.) (Type 1)

① Roots are equal in magnitude & opposite in sign.

$$\boxed{b=0 \ \& \ D>0}$$

eg.



$(x-2)(x+2)=0$

$x^2-4=0$

↳ Hence $b=0$

② Only 1 root is always zero

$$\boxed{c=0}$$

let roots $\alpha, 0$

~~③ Roots are~~

$(x-\alpha)(x-0)=0$

$x^2-\alpha x=0$

↳ Hence $c=0$

③ Roots are reciprocal to each other.

$$\boxed{a=c \ \& \ D\geq 0}$$

$(x-\alpha)\left(x-\frac{1}{\alpha}\right)=0$

$x^2-x\left(\alpha+\frac{1}{\alpha}\right)+1=0$

$$\boxed{a=c}$$

(4) Roots are of opposite sign

$\boxed{POR < 0 \text{ \& } D > 0}$

$(x - \alpha)(x + \beta) = 0$

SOR ⇒ $\alpha - \beta$ (any can be bigger)

POR ⇒ $\alpha(-\beta) = \underbrace{-\alpha\beta}_{\text{always } \ominus\text{ve}}$

(5) Both roots are ⊖ve

$\boxed{SOR < 0 \text{ \& } POR > 0 \text{ \& } D \geq 0}$

$(x + \beta)(x + \alpha) = 0$

$-\alpha - \beta = \ominus$ ve
$-\alpha \cdot -\beta = \oplus$ ve
always

(6) Both roots are ⊕ve

$\boxed{SOR > 0 \text{ \& } POR > 0 \text{ \& } D \geq 0}$

$(x - \beta)(x - \alpha) = 0$

$\beta + \alpha = \oplus$ve
$\beta\alpha = \oplus$ve
always

leading coefficient must be +

Q $f(x) = x^2 + 2(a-1)x + (a+5)$ find 'a'.

a) Roots are of opposite sign

$a+5 < 0$

$\boxed{a < -5}$

$a \in (-\infty, -5)$

intersection

$4(a-1)^2 - 4(a+5) > 0$

$a^2 + 1 - 2a - a - 5 > 0$

$a^2 - 3a \cancel{+6} - 4 > 0$

$\cancel{a^2 - 2a - a + 6 > 0}$

$\cancel{a(a-2) + (a}$

$\cancel{a = \frac{3 \pm \sqrt{9-24}}{2}}$

$a^2 - 4a + a - 4 > 0$

$a(a-4) + 1(a-4) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$a \in (-\infty, -1) \cup (4, \infty)$

$\boxed{a \in (-\infty, -5)}$

b) Roots equal in magnitude but opposite in sign

$a - 1 = 0$

$a = 1$

$a \in (-\infty, -1) \cup (4, \infty)$

$\boxed{a \in \phi}$

c) Both roots ⊕ve

$SOR > 0$

$POR > 0$

$D \geq 0$

SOR $2(1-a) > 0$

$1-a > 0$

$\boxed{a < 1}$

$a+5 > 0$

$\boxed{a > -5}$

$a \in (-\infty, -1] \cup [4, \infty)$

$\boxed{a \in (-5, -1]}$

d) Both roots ⊖ve

$SOR < 0$

$a-1 > 0$

$\boxed{a > 1}$

$\boxed{a \in [4, \infty)}$

Q $f(x) = x^2 - (m-3)x + m$. find 'm'

a) Roots are of opposite sign
b) roots equal magnitude but opposite sign
c) Both roots are ⊕ve
d) Both roots are ⊖ve

$x^2-(m-3)x+m$

$(m-3)^2-4m=D$

a) $POR<0$

$D>0$

$m^2+9-6m-4m>0$

$m^2+9-10m>0$

$m^2-9m-m+9>0$

$m(m-9)-1(m-9)>0$

$\overset{+}{\longleftarrow} 1 \; - \; 9 \overset{+}{\longrightarrow}$

$(-\infty,1)\cup(9,\infty)$

$m<0$

$\boxed{m\in(-\infty,0)}$

b) $b=0 \quad D>0$

$-(m-3)=0$

$m=3$

$\boxed{m\in\phi}$

c) $SOR>0$

$POR>0$

$D\geq 0$

$m>0$

$m-3>0$

$m>3$

$\boxed{m\in[9,\infty)}$

d) $m>0$

$m-3<0$

$m<3$

$\boxed{m\in(0,1]}$

Q. If $f(x) = 3x^2 - 5x + p$ & $f(0)$ & $f(1)$ are of opposite signs find P.

$f(0) \cdot f(1) < 0$

$p(3-5+p) < 0$

$p(p-2) < 0$

$p \in (0, 2)$

<u>Location of Roots</u> <u>Type-2</u>

$f(x) = ax^2 + bx + c$

⇒ Leading coeff must be 1.

(1) Both ~~the~~ roots of a quad are greater than a number 'd'.

$$\boxed{\begin{array}{l} D \geq 0 \\ f(d) > 0 \\ d < \frac{-b}{2a} \end{array}}$$

Q① find "λ" for both roots of quadratic $x^2 - 6\lambda x + 9\lambda^2 - 2\lambda + 2 = 0$ are greater than 3

$$36\lambda^2 - 36\lambda^2 + 8\lambda - 8 \geq 0$$

$$8(\lambda - 1) \geq 0$$

$$\lambda - 1 \geq 0$$

$$\boxed{\lambda \geq 1}$$

$$9 - 18\lambda + 9\lambda^2 - 2\lambda + 2 > 0$$

$$9\lambda^2 - 20\lambda + 11 > 0$$

$$9\lambda^2 - 11\lambda - 9\lambda + 11 > 0$$

$$\lambda(9\lambda - 11) - 1(9\lambda - 11) > 0$$

$$(9\lambda - 11)(\lambda - 1) > 0$$

intersection

$+ \quad 1 \quad - \quad 11/9 \quad +$

$$\boxed{\lambda \in (-\infty, 1) \cup (11/9, \infty)}$$

$$3 + \frac{-6\lambda}{2} < 0$$

$$6 - 6\lambda < 0$$

$$(1 - \lambda) < 0$$

$$\lambda - 1 > 0$$

$$\boxed{\lambda > 1}$$

$$\boxed{\lambda \in (11/9, \infty)}$$

Q find 'k' for both the roots of the quadratic $(k+1)x^2 - 3kx + 4k = 0$ are greater than 1

$$x^2 - \frac{3k}{k+1}x + \frac{4k}{k+1} = 0$$

$D \geq 0$

$$\cancel{\frac{9k^2}{k^2+1+2k}}$$

$$\left(\frac{3k}{k+1}\right)^2 - 4\left(\frac{4k}{k+1}\right) \geq 0$$

$$\frac{9k^2 - 16k^2 - 16k}{k^2+1+2k} \geq 0$$

$$\frac{7k^2+16k}{k^2+2k+1} \leq 0$$

$$\frac{k(7k+16)}{(k+1)^2} \leq 0$$

$-\frac{16}{7}$ $\quad -1 \quad$ 0

$+ \quad - \quad - \quad +$

$$\boxed{\left[-\frac{16}{7}, -1\right) \cup (-1, 0]}$$

$$\boxed{k \neq -1}$$

$(k+1) - 3k + 4 = 0$

$k + 1 - 3k + 4 = 0$

$\cancel{-2k + 5 = 0}$ $\quad 2k < 1$

$\cancel{2k - 5 < 0}$

$\cancel{\boxed{k < \frac{5}{2}}}$

$$\boxed{k < \frac{1}{2}}$$

$\cancel{1 + \left(\frac{-3k}{k+1}\right) \times \frac{1}{2} \times \frac{k+1}{4k} \not< 0}$

$\cancel{1 + \frac{(-3k^2 - 3k)}{8k^2 + 8} < 0}$

$\cancel{\frac{8k^2 + 8 - 3k^2 - 3k}{8(k^2+1)} < 0}$

$\cancel{\frac{5k^2 - 3k + 8}{k^2+1} < 0}$

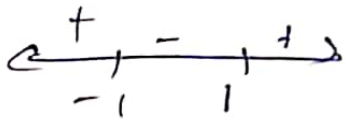
$\cancel{5k^2 - 8k -}$

(36)

$$\frac{-\left(\frac{-3k}{k+1}\right)}{2} > 1$$

$$\frac{3k}{2(k+1)} - 1 > 0$$

$$\frac{k-1}{k+1} > 0$$



$$(-\infty, -1) \cup (2, \infty)$$

$$\boxed{\left[-\frac{16}{7}, -1\right) \cup}$$

H.W. 25-05-2024

Type (2)

(2) Both roots are less than any specific number 'd'.

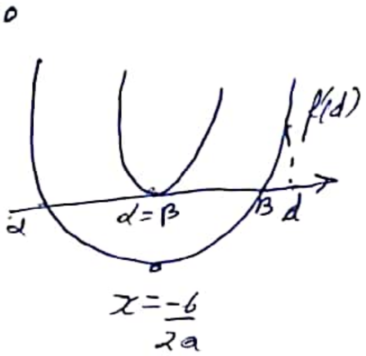
$D \geq 0$

$\frac{-b}{2a} < d$

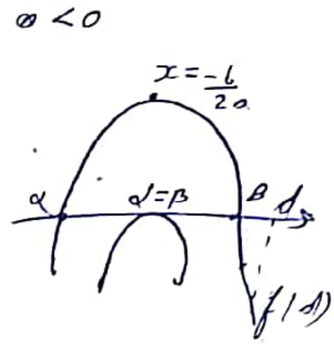
$a f(d) > 0$

intersection

$a > 0$



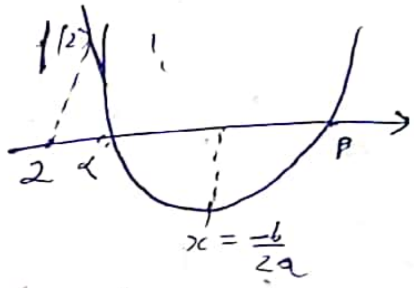
$a < 0$



Q let $x^2 - (m-3)x + m = 0$ $(m \in R)$ be a quadratic equation find the value of m for which.

(1) both roots are greater than 2.

(2) both the roots are smaller than 2.



(2) $x^2 - (m-3)x + m = 0$

(1) $D = (m-3)^2 - 4m$

$= m^2 + 9 - 6m - 4m$

$= m^2 + 9 - 10m$

~~$= m^2 + 10 -$~~

$= m^2 - 9m - m + 9$

$= m(m-9) - 1(m-9)$

$= (m-1)(m-9) \geq 0$

$+ \quad 1 \quad - \quad 9 \quad +$

$\boxed{m \in (-\infty, 1] \cup [9, \infty)}$ ... (1)

$\frac{m-3}{2} < 2$

$m - 3 < 4$ ... (2)

$\boxed{m < 7}$

$4 - (m-3) \times 2 + m > 0$

$4 - 2m + 6 + m > 0$

$10 - m > 0$

$\boxed{m < 10}$ ... (3)

(1) ∩ (2) ∩ (3)

$\boxed{m \in (-\infty, 1]}$

(2) $\frac{m-3}{2} > 2$

$m - 3 > 4$ ... (1)

$\boxed{m > 7}$

$\boxed{m < 10}$

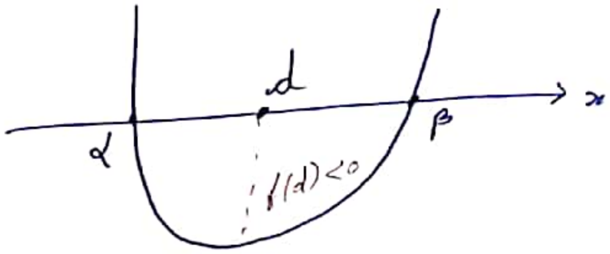
(1) ∩ (2) ∩ (3)

$\boxed{m \in [9, 10)}$

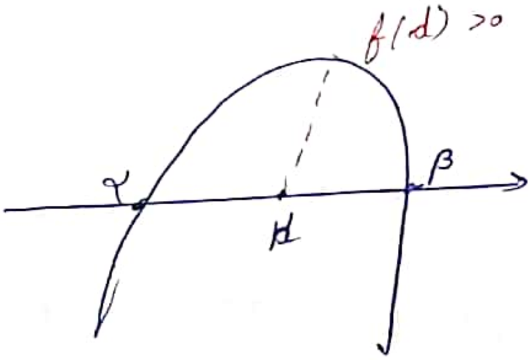
Type 3

Both roots lie on either side of specific no. d
→ One root is greater than D & one is less
→ Specific no. d lies between the roots

$a > 0$



$a < 0$



$\boxed{a \cdot f(d) < 0}$

$\left.\begin{array}{l} a > 0,\ f(d) < 0 \\ a < 0,\ f(d) > 0 \end{array}\right]$ so $\underline{a f(d) < 0}$

Q find all possible values of a for which exactly one root of $x^2-(a+1)x+2a=0$ lie in interval $(0, 3)$.

$f(0) = 2a$

$f(3) = 9 - 3a + 3 + 2a$

$= 6 - a$

$f(0) \cdot f(3) < 0$

$a(6-a) < 0$

$- \quad 0 \quad + \quad 6 \quad -$

$\underline{(-\infty, 0) \cup (6, \infty)}$

Check at the points of interval

put $a = 0$

$x^2 - x = 0$

$x = 1, 0 \rightarrow$ 1 lie ~~between~~ in interval

so include 0 in answer

put $a = 3$

$D < 0$ so no real roots

$\boxed{a \in (-\infty, 0] \cup (6, \infty)}$

Q find 'K' for which one root of equation $x^2 - (K+1)x + K^2 + K - 8 = 0$ is greater than 2 & other is less than 2.

$d = 2$

$f(2) = 4 - (K+1)2 + K^2 + K - 8 = 0$

$= 4 - 2K - 2 + K^2 + K - 8$

$= K^2 - K - 6 < 0$

$K^2 - 3K + 2K - 6 < 0$

$K(K-3) + 2(K-3) < 0$

$(K+2)(K-3) < 0$

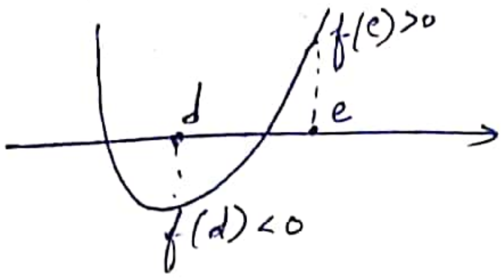
$+ \quad -2 \quad - \quad 3 \quad +$

$\boxed{K \in (-2, 3)}$

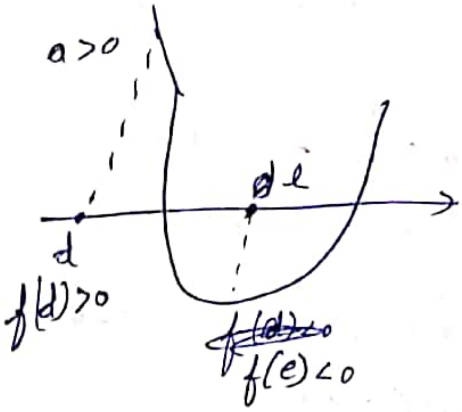
② Exactly one root lies in ($d<e$) the interval (d,e)

Type-4

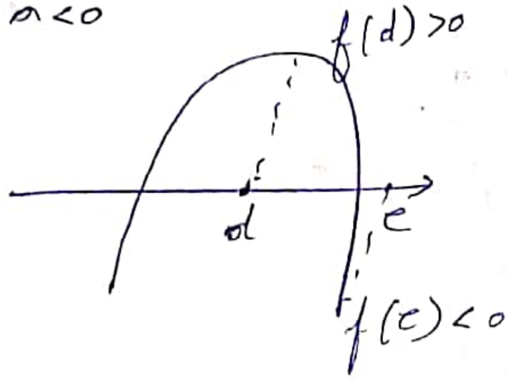
$a>0$



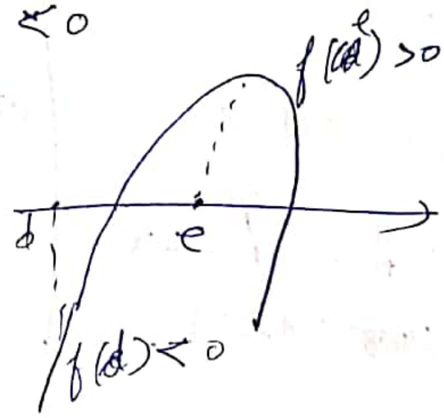
$a>0$



$a<0$



$a<0$



So $f(d)\cdot f(e)<0$

$$\boxed{f(d)\cdot f(e)<0}$$

Note:- Check at the extreme points of Interval.

Pg 158

Q8. $(c-5)x^2-2cx+(c-4)=0$

$-2x^2-6x-1$

$2x^2+6x+1$

$f(0) = c-4$

$f(2) = 4(c-5)-2c+c-4=0$

$f(2) = 4c-20-\cancel{4c}+c-4$

$= \cancel{5}c-24$

$f(3) = \cancel{9}c-45-\cancel{6}c+c-4$

$= 4\cancel{7}c-49$

$f(0)\cdot f(2)<0$

$(c-4)(3c-24)<0$

$+ \quad 4 \quad - \quad \frac{24}{3} \quad +$

$\cancel{(\;)}\ (4, 24)$

$f(2)\cdot f(3)<0$

$(\overset{4}{\cancel{7}}c-49)(3c-24)<0$

$+ \quad \frac{49}{4} \quad - \quad \frac{24}{3} \quad +$

$\cancel{(\;)}$

$\left(\frac{49}{4}, 24\right)$

intersection

$(1\overset{23}{\cancel{3}}, 24)$

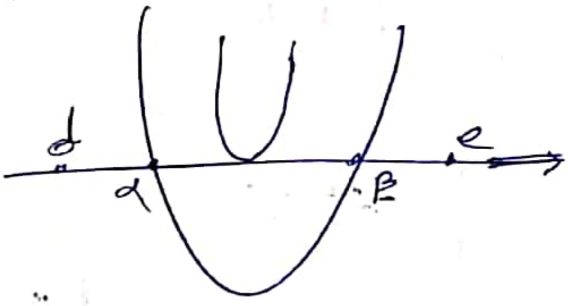
$24-1\frac{2}{3}-1$

$\cancel{2}11$

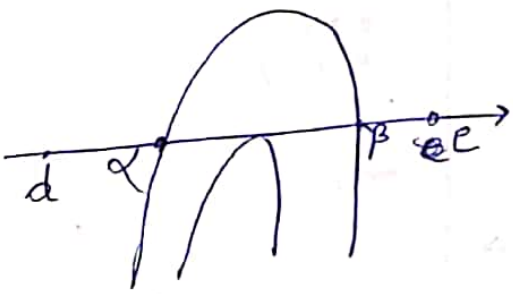
$\boxed{31}$

Type - 5 Both the roots lie between number d & e (d<e)

$a > 0$



$a < 0$



Union of ① & ③

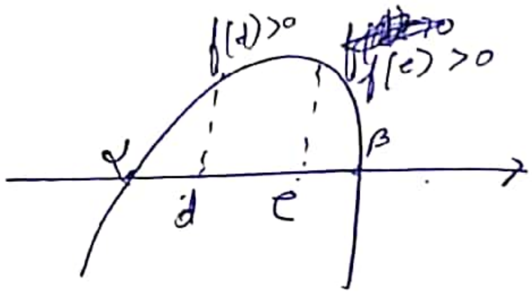
① $D \geq 0$

② $d < \frac{-b}{2a} < e$

③ $a\,f(d) > 0$

④ $a\,f(e) > 0$

Type - ⑥ One root is greater than e & one is less than d



$a\,f(d) < 0$

$a\,f(e) < 0$

Q If α, β are the roots of $x^2+2(K-3)x+9=0$ if α, β belongs to $(-6, 1)$ find K.

$4(K-3)^2 - 36 \geq 0$

$4K^2 + 36 - 24K - 36 \geq 0$

$K^2 - 6K \geq 0$

$K(K-6) \geq 0$

~~$K = 6,$~~

$+ \quad - \quad +$

$0 \quad 6$

$K \in (-\infty, 0) \cup (6, \infty)$

$\frac{-2K+6}{2}$

$3 - K$

$-6 < 3 - K$

$K < 9$

$3 - K > 1$

$K > 2$

$f(-6) = 36 - 12K + 36 + 9 = 0$

$= 81 - 12K > 0$

$f(1) = 1 + 2K - 6 + 9$

$= 2K + 4 > 0$

$f(-6)$ $81 - 12K > 0$

$\boxed{K < \frac{27}{4}}$

$f(1) = K + 2 > 0$

$\boxed{K > -2}$

$\boxed{\left(6, \frac{27}{4}\right)}$

Summary (Location of Roots)

① Both roots greater than d. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} > d\right)$

② Both roots less than d. $(D \geq 0)$ $(af(d) > 0)$ $\left(\frac{-b}{2a} < d\right)$

③ 'd' lies between the roots. $(af(d) < 0)$

④ exactly one root lies in (d, e) $(f(d) \cdot f(e) < 0)$

⑤ Both roots lie between d and e $(D \geq 0)$ $(af(d) > 0)$ $(af(e) > 0)$

$\left(d < \frac{-b}{2a} < e\right)$

~~⑥ Both roots lie between~~

⑥ Both points lies between 2 roots. $(af(d) < 0)$ $(af(e) < 0)$

Irrational Inequality

① Inequalities having $\sqrt{\ }$ sign.

② Direct Squaring is not allowed without checking

Q $\sqrt{2x-5} < 3$

$2x-5 \geq 0$ [under root quantity is always ⊕)

$2x-5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$x \in \left[\frac{5}{2}, \infty\right)$ — ①

$\underbrace{\sqrt{2x-5}}_{\oplus ve} < \underbrace{3}_{\oplus ve}$

Square both sides so we can square

$2x-5 < 9$

$2x < 14$

$x < 7$ — ②

$x \in (-\infty, 7)$

① ∩ ②

$\boxed{\left[\frac{5}{2}, 7\right)}$

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Q $\sqrt{x+6} < x-6$

$x+6 \geq 0$

$x \geq -6$

$x \in [-6, \infty)$

Case ① $x-6 < 0$

$x < 6$

$\oplus < \ominus$ not possible

$x \in \phi$

case ② $x-6 \geq 0$

$x \geq 0$

$x+6 < x^2+36-12x$

$\cancel{x^2+13}$ -

$x^2 - 13x + 30 > 0$

$x^2 - 10x - 3x + 30 \geq 0$

$x(x-10) - 3(x-10) \geq 0$

$(x-3)(x-10) \geq 0$

$+ \quad 3 \quad - \quad 10 \quad +$

$(-\infty, 3) \cup (10, \infty)$ but $x \in (6, \infty)$

$x \in (10, \infty)$

case ① $\cup$ case ② $\Rightarrow \cancel{(-\infty, 3) \cup}\ (10, \infty)$

intersection with $x \in [-6, \infty)$

$$\boxed{x \in (10, \infty)}$$

Q $x+1 \geq \sqrt{5-x}$

$\sqrt{5-x} \leq x+1$

$5-x \geq 0$

$x-5 \leq 0$

$\boxed{x \leq 5}$ - (1)

Case (1) $x+1 < 0$

$x < -1$

$(+) \leq 0$ ×

$\boxed{x \in \phi}$ - (2)

Case (2) $x+1 \geq 0$

$x \geq -1$

$5-x \leq x^2+1+2x$

$x^2+3x-4 \geq 0$

$x^2+4x-x-4 \geq 0$

$x(x+4)-1(x+4) \geq 0$

$(x-1)(x+4) \geq 0$

$+ \quad -4 \quad - \quad 1 \quad +$

$(-\infty,-4] \cup [1,\infty)$

$\boxed{x \in [1,\infty)}$ - (3) Case 1 $\cup$ Case 2

(2) $\cup$ (3) $= x \in [1,\infty)$ - (4)

$\boxed{[1,5]}$

Q $\sqrt{x+18} < 2-x$

$x+18 \geq 0$

$\boxed{x \geq -18}$

Case(1) $2-x < 0$

$-x < -2$

$x > 2$

$(+) < (-)$ ×

$x \in \phi$

Case (2) $2-x > 0$

$x < 2$

$x+18 < x^2+4-4x$

$x^2 - 5x - 14 > 0$

$x^2 - 7x + 2x - 14 > 0$

$x(x-7) + 2(x-7) > 0$

$(x+2)(x-7) > 0$

$+ \quad -2 \quad - \quad 7 \quad +$

$(-\infty, -2) \cup (7, \infty)$

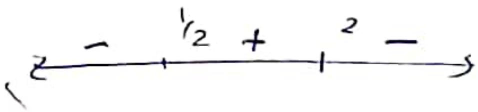
$(-\infty, -2) \cap [-18, \infty)$

$\boxed{x \in [-18, -2)}$

d

$$\sqrt{\frac{x-2}{1-2x}} > -1$$

$$\frac{x-2}{1-2x} \geqslant 0$$



$$\left[\frac{1}{2}, 2\right]$$

$\cancel{x \in \phi}$

$x \in R - \left\{\frac{1}{2}\right\}$

$$\boxed{\left[\frac{1}{2}, 2\right]}$$

H.W. 28-05-2024

DYS-6 (full)

DYS-5 (full)

DYS-8 (Q5)

DYS-10 (full)

O-1 {16, 17, 19, 20, 21, 24, 25, 26, 27}

O-2 {10, 14, 15, 20 ..., ∞} − {12, 13}

~~O-3~~

Modulous Equality

(1) If $a = +ve$ ($a \to$ constant)

$|x| \le a \quad x \in [-a, a]$

$|x| < a \quad x \in (-a, a)$

$|x| \ge a \quad x \in (-\infty, -a] \cup [a, \infty)$

$|x| > a \quad x \in (-\infty, -a) \cup (0, \infty)$

(2) If $a = -ve$ ($a \to$ constant)

$|x| \le a \quad x \in \phi$

$|x| < a \quad x \in \phi$

$|x| \ge a \quad x \in R$

$|x| > a \quad x \in R$

(3) $|x|^2 = |x^2|$

(4) $|x||y| = |xy|$

(5) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

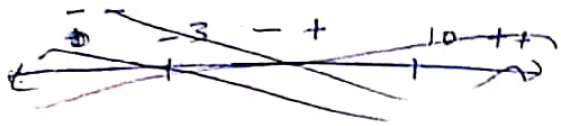
(6) $\sqrt{x^2} = |x|$

(7) $\big||x| - |y|\big| \le |x+y| \le |x| + |y|$

(8) $|x+y| = |x| + |y| \Rightarrow xy \ge 0$

$|x-y| = |x| + |y| \Rightarrow xy \le 0$

Q $|2x-7| = |x+3| + |x-10|$



$|x+3+x-10| = |x+3| + |x-10|$

$(x+3)(x-10) \geq 0$

x: + -3 − 10 +

$(-\infty, -3] \cup [10, \infty)$

Q $|x-2| + |x-7| = 5$

$(x-2)(x-7) \leq 0$

+ 2 − 7 +

$\boxed{[2, 7]}$

Q $|x^2+6x+6| = |x^2+4x+9| + |2x-3|$

$\cancel{x^2+4x}$

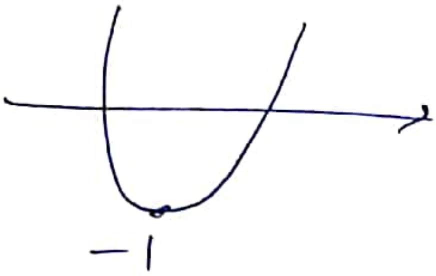
$(x^2+4x+9)(2x-3) \geq 0$

+ − $3/2$ +

$\boxed{\left(\frac{3}{2}, \infty\right)}$

O-1

Q27.



as it is biggest -ve integer

$\frac{-p}{4} = -1$

$-p = -4$

$p = 4$

$2x^2 + 4x + 1$

$x = \frac{-4 \pm \sqrt{16 - 8}}{4}$

$= \frac{-4 \pm 2\sqrt{2}}{4}$

$= \frac{-2 \pm \sqrt{2}}{2}$

$x = \frac{-2 + \sqrt{2}}{2} \times x = \frac{-2 - \sqrt{2}}{2}$

~~$x = -p$~~

$x = \frac{(\sqrt{2} - 2)}{2} \times \frac{-(\sqrt{2} + 2)}{2}$

$2x^2 + px + 1 = -1$

$2x^2 + px + 2 = 0$

$p^2 - 16 = 0$

$p = \pm 4$

$+4 \times -4$

$\boxed{= -16}$

$\boxed{B}$

Q find 'x'

$|x| \le 2$

$|x| \le a$

$x \in [-a, a]$

$\boxed{x \in [-2, 2]}$

Q $|x-3| \le 2$

$x - 3 \in [-2, 2]$

$\boxed{x \in [1, 5]}$

Q $|x| \geq 9$

$\boxed{x \in (-\infty, -9] \cup [9, \infty)}$

Q $|x| < \sqrt{3}$

$\boxed{x \in (-\sqrt{3}, \sqrt{3})}$

Q $|2x| - 5 > 0$

$|2x| > 5$

$2x \in (-\infty, 5) \cup (5, \infty)$

$\boxed{x \in (-\infty, 5/2) \cup (5/2, \infty)}$

Q $|2-7x| < 8$

$2-7x \in (-8, 8)$

$-7x \in (-10, 6)$

$\boxed{\cancel{x \in (\frac{10}{7}, \frac{-6}{7})}}$ $\boxed{x \in (-\frac{6}{7}, \frac{10}{7})}$

Q no. of integral values of x such that $4 \le |x-4| \le 10$

$|x-4| \le 10$

$x-4 \in [-10, 10]$

$x \in [-6, 14]$

$|x-4| \ge 4$

$x-4 \in (-\infty, -4] \cup [4, \infty)$

$x \in (-\infty, 0] \cup [8, \infty)$

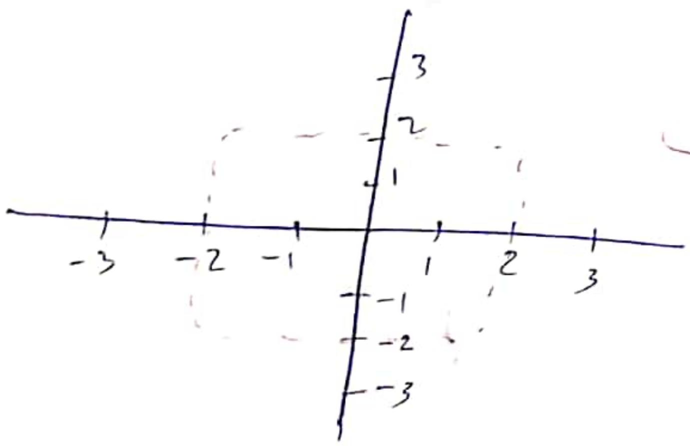
$\boxed{x \in [-6, 0] \cup [8, 14]}$

no. of integral values $= 7+7$

$\boxed{= 14}$

Q $|x| \le 2$ & $|y| \le 2$

$x \in [-2, 2]$ $y \in [-2, 2]$



area $= 4 \times 4$

$\boxed{\text{Area} = 16}$

Q $||x-2|-1| \le 5$

$|x-2|-1 \in [-5, 5]$

$|x-2| \in [-4, 6]$

$-4 \le |x-2| \le 6$

$|x-2| \geq -4$

$x \in R$

$|x-2| \le 6$

$x-2 \in [-6, 6]$

$\boxed{x \in [-4, 8]}$

Q $|x-3| < x-3$

$|x-3| < x-3$

$x-3 \in (3-x, x-3)$

$x \in (6-x, x)$

$6-x \le x \le x$

$6-x < x$

$6 < 2x$

$\boxed{3 < x}$

$x < x$

$0 < 2x$

$x \in R$

$x-3 > 0$

$\boxed{x > 3}$

$|x-3| < x-3$

$\cancel{x-3 < x-3}$

$x-3 \in (3-x, x-3)$

$x \in (6-x, x)$

$6-x < \boxed{x < x}$

$\boxed{x \in \phi}$

Theory of Equation

→ Deriving results for polynomials with degree 3 or more.

Let $a \neq 0$

(1) $ax^2+bx+c=0 \quad \alpha \quad \beta$

$$\alpha+\beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

(2) $ax^3+bx^2+cx+d=0 \quad \alpha \quad \beta \quad \gamma$

$$\alpha+\beta+\gamma = \frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

(3) $ax^4+bx^3+cx^2+dx+c \quad \alpha \quad \beta \quad \gamma \quad \delta$

$$\alpha+\beta+\gamma+\delta = \frac{-b}{a} \quad \text{(sum)}$$

$$\alpha\beta+\beta\gamma+\gamma\delta+\alpha\delta = \frac{c}{a} \quad \text{(2-2 sum)}$$

$$\alpha\beta\gamma+\beta\gamma\delta+\alpha\beta\delta+\alpha\gamma\delta = -\frac{d}{a} \quad \text{(3-3 sum)}$$

$$\alpha\beta\gamma\delta = \frac{e}{a}$$

general

$$f(x) = \overset{+}{a_0 x^n} + \overset{-}{a_1 x^{n-1}} + \overset{+}{a_2 x^{n-2}} + \overset{-}{a_3 x^{n-3}} \ldots + a_n$$

$\ominus$ for odd n $\oplus$ for even n

$$\boxed{\text{Sum of Roots} = \frac{-a_1}{a_0}}$$

$$2-2 \text{ sum} = \frac{a_2}{a_0}$$

$$3-3 \text{ sum} = \frac{-a_3}{a_0}$$

$$4-4 \text{ sum} = \frac{a_4}{a_0}$$

$$\boxed{\text{product} = \frac{(-1)^n a_n}{a_0}}$$

Q $2x^3 - 5x^2 + 4x - 1 = 0$ have roots $\alpha, \beta \,\&\, \gamma$

find (1) $\alpha^2+\beta^2+\gamma^2$

(2) $\alpha^3+\beta^3+\gamma^3$

$$(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2 + 2(\alpha\beta+\beta\gamma+\alpha\gamma)$$

$$\left(\frac{5}{2}\right)^2 = \alpha^2+\beta^2+\gamma^2 + 2(2)$$

$$\frac{25-16}{4} = \alpha^2+\beta^2+\gamma^2$$

$$\boxed{\frac{9}{4} = \alpha^2+\beta^2+\gamma^2}$$

(2) $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma = (\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2+\alpha\beta+\beta\gamma+\alpha\gamma)$

$$\alpha^3+\beta^3+\gamma^3 - \frac{1}{2} = \left(\frac{5}{2}\right)\left(\frac{9}{4}+2\right)$$

$$\cancel{\alpha^3+\beta^3+\gamma^3 = \frac{85}{8}+\frac{1}{2}}$$

$$\cancel{= \frac{85+4}{8}}$$

$$\cancel{= \frac{89}{8}}$$

$$\alpha^3+\beta^3+\gamma^3 = \frac{5}{2}\times\frac{1}{4}+\frac{1}{2}$$

$$\cancel{= \frac{5+4}{8}}$$

$$\cancel{= \frac{9}{8}}$$

$$= \frac{5}{8}+\frac{12}{8}$$

$$\boxed{= \frac{17}{8}}$$

Q $x^3-6x^2+10x-3=0$

$$\left(\alpha-\frac{1}{\beta\gamma}\right)\left(\beta-\frac{1}{\alpha\gamma}\right)\left(\gamma-\frac{1}{\alpha\beta}\right)$$

$$\frac{(\alpha\beta\gamma-1)}{\beta\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\gamma}\times\frac{(\alpha\beta\gamma-1)}{\alpha\beta}$$

$$\frac{(3-1)^3}{(3)^2} = \boxed{\frac{8}{9}}$$

Q $x^3 - 3x^2 + 2x + 1 = 0$

(i) $(\alpha-2)(\beta-2)(\gamma-2)$

$\alpha+\beta+\gamma-6$

$\boxed{= -3}$

$\alpha\beta - 2\alpha - 2\beta + 4 + \beta\gamma - 2\beta - 2\gamma + 4 + \alpha\gamma - 2\alpha - 2\gamma + 4$

$\alpha\beta + \alpha\gamma + \beta\gamma + 12 - 4(\alpha+\beta+\gamma)$

$-2 + 12 - 4(+3)$

$\cancel{-2+12+12}$

$-2 + 12 + 12$

$\boxed{= 22}$

$\alpha\beta\gamma - 2\alpha\gamma - 2\beta\gamma + 4\gamma - 2\alpha\beta + 4\alpha + 4\beta - 8$

$\alpha\beta\gamma - 2(\alpha\beta + \beta\gamma + \alpha\gamma) + 4(\alpha+\beta+\gamma) - 8$

$-1 - 4 + 12 - 8$

$\boxed{\cancel{+25}}$ $\boxed{-1}$

$\cancel{x^3 + 3 + 22 + 2}$

$\boxed{x^3 + 3x^2 + 22x + 1}$

60

② cubic with roots $\frac{\alpha+1}{2}, \frac{\beta+1}{2}, \frac{\gamma+1}{2}$

$$\frac{x+1}{2} = t$$

$$x+1 = 2t$$

$$x = 2t-1$$

$$(2t-1)^3 - 3(2t-1)^2 + 2(2t-1) + 1 = 0$$

$$8t^3 - 3 - 4t^2 + 2t - 12t^2 - 3 + 6t + 4t - 2 + 1 = 0$$

$$\boxed{8t^3 - 16t^2 + 12t - 7 = 0}$$

Q $x^3 - x + 1 = 0$ have roots α, β & γ

find

① Eq with roots are $\frac{\alpha+\beta}{\gamma^2}, \frac{\beta+\gamma}{\alpha^2}, \frac{\alpha+\gamma}{\beta^2}$

$$\alpha+\beta+\gamma = 0$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = -1$$

$$\alpha\beta\gamma = -1$$

~~$\alpha^2\beta^2(\alpha+\beta) + \beta^2\gamma^2(\beta+\gamma) + \alpha^2\gamma^2(\alpha+\gamma)$~~

~~$\alpha^2\beta^2\gamma^2$~~

~~$\alpha^3\beta^2 + \beta\alpha^2\beta^3 + \beta^3\gamma^2 + \beta^2\gamma^3 + \alpha^3\gamma^2 + \alpha^2\gamma^3$~~

~~$(\alpha\beta\gamma)^2$~~

Scanned with CamScanner

$$\frac{\alpha+\beta}{\gamma^2}$$

$$\alpha+\beta+\gamma=0$$
$$\alpha+\beta=-\gamma$$

$$\frac{-\gamma}{\gamma^2} = -\frac{1}{\gamma}, -\frac{1}{\alpha}, -\frac{1}{\beta}$$ are roots.

$$-\frac{1}{x}=t$$

$$x=-\frac{1}{t}$$

$$\left(-\frac{1}{t}\right)^3 - \left(-\frac{1}{t}\right)+1=0$$

$$\frac{-1}{t^3}+\frac{1}{t}+1$$

$$\boxed{t^3+t^2-1=0}$$

(2) Value of $\frac{\beta\gamma}{(1-\beta)(1-\gamma)}+\frac{\alpha\gamma}{(1-\alpha)(1-\gamma)}+\frac{\alpha\beta}{(1-\alpha)(1-\beta)}$

$$\frac{\beta\gamma+1+\alpha\gamma+1+\alpha\beta+1}{(1-\beta)(1-\gamma)(1-\alpha)}$$

α, β, γ are roots

$\alpha^3-\alpha+1=0$

$1-\alpha=-\alpha^3$

$1-\beta=-\beta^3$

$1-\gamma=-\gamma^3$

$$=\frac{2}{(1-\beta)(1-\gamma)(1-\alpha)}$$

$$=\frac{2}{-(\alpha\beta\gamma)^3}$$

$$=\frac{2}{1}$$

$\boxed{=2}$ $\boxed{=2}$

(62)

$$\boxed{\begin{aligned}&\sum\alpha = \alpha+\beta+\gamma\\ &\sum\alpha^2=\alpha^2+\beta^2+\gamma^2\end{aligned}}$$

H.W. 30-5-2024

0-1 {28, 29, 30} *

0-3 {~~5-6~~ 4-6, 9, 10}

J-M {8, 9, 10, 11, 12}

General quadratic equation in two variable.

$$f(x, y) = ax^2 + by^2 + 2hxy + 2gx + 2fy + c$$

↳ two linear factors when

$$\boxed{abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$$

Proof:- consider quadratic in x

$$\underbrace{ax^2}_{x^2} + \underbrace{(2hy + 2g)\,x}_{x} + \underbrace{by^2 + 2fy + c}_{\text{constant}} = 0$$

$$x = \frac{-(2hy + 2g) \pm \sqrt{(2hy + 2g)^2 - 4(a)(by^2 + 2fy + c)}}{2a}$$

~~$x = -2hy - 2g$~~

Q find weather $x^2 + 2xy + 2x + 6y - 3 = 0$ have two linear factors or not.

$2 \times 3 - 9 + 3 = 0$

So it ~~can't~~ resolved in 2 linear factors.

Q $x^2 + 2xy + 2x + ky^2 + k = 0$ find k if the above equation has two linear factors

$a = 1$
$b = k$
$c = k$
$h = 1$
$g = 1$
$f = 0$

$k^2 - k - k = 0$
$k^2 - 2k = 0$

$$\boxed{k = 0, 2}$$

Type-2 – when two homogeneous equation have ~~common~~ common linear factors.

Homogeneity → when degree of all terms is same (xy degree $= 2$)

$a_1x^2+2h_1xy+b_1y^2=0$ $\quad\quad$ $a_2x^2+2h_2xy+b_2y^2=0$

Assume $y-mx=0$ is a common factor

$x=0$ is a factor

put $y=mx$ in equations

$a_1x^2+2h_1x(mx)+b_1m^2x^2=0$

$a_1x^2+2h_1mx^2+b_1m^2x^2=0$

$x^2(a_1+2h_1m+m^2b_1)=0$

we know $x=0$ is a common factor so can cut $x^2=0$

~~a_1+2h_1~~

or $b_1m^2+2h_1m+a_1=0$ $\quad\vert\quad$ $b_2m^2+2h_2m+a_2=0$

both have a common root.

Method 2:-

$m^2b_1+2h_1m+a_1=0$

$m^2b_2+2h_2m+a_2=0$

$$\frac{m^2}{\begin{matrix}2h_1 & a_1\\ 2h_2 & a_2\end{matrix}}=\frac{-m}{\begin{matrix}b_1 & a_1\\ b_2 & a_2\end{matrix}}=\frac{1}{\begin{matrix}b_1 & 2h_1\\ b_2 & 2h_2\end{matrix}}$$

Cross multiply

$$\frac{m^2}{2h_1 a_2 - 2h_2 a_1} = \frac{-m}{b_1 a_2 - b_2 a_1} = \frac{1}{2b_1 h_2 - 2h_1 b_2}$$

$$\cancel{m = \frac{b_1 a_2 - b_2 a_1}{}}$$

$$m = \frac{b_2 a_1 - b_1 a_2}{2b_1 h_2 - 2h_1 b_2}$$

Now -

$$\frac{m^2}{2b_1 a_2 - 2b_2 a_1} = \frac{1}{2b_1 h_2 - 2h_1 b_2}$$

Put value of m

$$\frac{\left(\frac{b_2 a_1 - b a_2}{2b_1 h_2 - 2h_1 b_2}\right)^2}{2h_1 a_2 - 2b_2 a_1} = \frac{1}{2b_1 b_2 - 2h_1 b_2}$$

$$\frac{(b_2 a_1 - b_1 a_2)^2}{(2b_1 h_2 - 2h_1 b_2)^2 (2h_1 a_2 - 2h_2 a_1)} = \frac{1}{(2b_1 h_2 - 2h_1 b_2)}$$

$$\frac{(b_2 a_1 - b_1 a_2)^2}{(2b_1 h_2 - 2h_1 b_2)(2h_1 a_2 - 2h_2 a_1)} = 1$$

$$(b_2 a_1 - b_1 a_2)^2 = (2b_1 h_2 - 2h_1 b_2)(2h_1 a_2 - 2h_2 a_1)$$

(48) (66)

DYS-11 - Complete H.W.
~~J~~ J-A - Complete

J-M

Q16.

$$\frac{|x+3|-1}{|x|-2} \geq 0$$

$--$ $+-$ $++$ at -3, 0

Case ①

$x \in (-\infty, -3)$

$$\frac{-(x+3)-1}{-x-2} \geq 0$$

$$\frac{[x+3+1]}{[x+2]} \geq 0$$

$$\frac{x+4}{x+2} \geq 0$$

-4, -2

$x \in (-\infty, -4] \cup (-2, \infty)$

$$\boxed{x \in (-\infty, -4]}$$

Case ②

$x \in [-3, 0]$

$$\frac{x+3-1}{-(x+2)} \geq 0$$

$$\frac{\cancel{x+2}}{-(\cancel{x+2})} \geq 0$$

$\ominus \geq 0$

$x \in \phi$

Case ③

$x \in (0, \infty)$

$$\frac{x+3-1}{x-2} \geq 0$$

$$\frac{x+2}{x-2} \geq 0$$

$+$ -2 $-$ 2 $+$

$x \in (-\infty, -2] \cup (2, \infty)$

$$\boxed{x \in (2, \infty)}$$

$x \in [-6, -2) \cup (-2, 2) \cup (2, 3]$

$$\boxed{x \in [-6, -4] \cup (2, 3]}$$

Part 2 - $x^2 - 7|x| + 9 \le 0$

$x^2 = |x|^2$

$|x|^2 - 7|x| + 9 \le 0$

$\cancel{t^2 - 7t + 9 \le 0}$

$\cancel{t = 7}$

$\cancel{+7|x| = x^2 + 9}$

$\cancel{|x| = \frac{x^2+9}{7}}$

$-7|x| \le \underline{x^2 + 9}$

$|x| \le \frac{x^2+9}{7}$

$x \in \left(-\frac{x^2-9}{7}, \frac{x^2+9}{7}\right)$

$-\frac{x^2-9}{7} \le x \le \frac{x^2+9}{7}$

$-x^2 - 9 \le 7x$ $\qquad$ $x^2 + 9 \ge 7x$

$\underbrace{x^2 + 7x + 9 \ge 0}$, $\qquad$ $\underbrace{x^2 - 7x + 9 \ge 0}$

$x \in (-\infty, -1.\text{sau}] \cup [1.\text{sau}, \infty) \cap x \in [-5.\text{sa}, -1.\text{sau}] \cup [1.\text{sa}, 5.\text{sau}]$

$x \in [-5.\text{sau}, -1.\text{sau}] \cup [1.\text{sau}, 5.\text{sau}]$

Q 10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$x+1 = y$

$y^{\log_{10} y} = 100y$

take $\log_y$ both sides

$\log_{10} y = \log_y 100y$

$1 = \log_y (100y + 10)$

$y = 100y + 10$

$99y + 10 = 0$

$y = -\frac{10}{99}$

$x+1 = -\frac{10}{99}$

$x = 1 - \frac{10}{99}$

$x = \frac{59}{99}$

$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$\log_{10} y = 1 + \frac{2}{\log_{10} y}$

$z = 1 + \frac{2}{z}$

$z^2 = z + 2$

$z^2 - z - 2 = 0$

$z = 2, -1$

$\log_{10} y = -1$

$\frac{1}{10} = y$

$x + 1 = \frac{1}{10}$

$x = \frac{1}{10} - 1$

$x = \frac{1-10}{10}$

$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$100 = y$

$x + 1 = 100$

$\boxed{x = 99}$