Physics-1

Ch1. Units, Dimensions & Vectors

Ch2. Kinematics 1-D

Ch-3 Projectile Motion

Units & Dimentions

Physical Quantity - Can be measured eg. time, distance

Fundamental Quantity (7)

1 → Mass → kg
2 → Length - m
3 → Time - second
4 → Temperature - K
5 → Electric current - Ampere
6 → Luminous Intensity - Candela (cd)
7 → Amount of Substance - mole

Derived Quantity

eg. → speed = $\frac{\text{distance}}{\text{time}}$ (m/s)

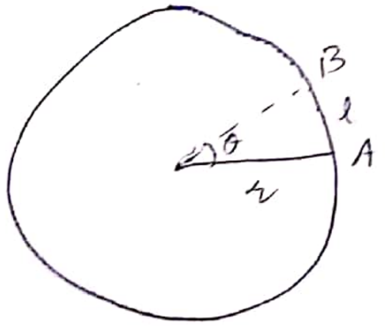
→ velocity
→ momentum
→ density (kg/m^3)

$1\ g/cc = 10^3\ kg/m^3$
$=$ water density ($4^\circ C$)

water
$0^\circ C \xrightarrow[\text{contract}]{} 4^\circ C \xrightarrow[\text{expand}]{}$

Supplementary units

1. Plane Angle
   SI → radian



$$\theta = \frac{l}{r}$$

2. Solid Angle

SI ⇒ Ste-radian

Practical Units

1. Light Year
2. Horse Power (HP)

   1 HP = 746 W
3. mile (1 mile = 1·6 km)

Improper Units

1. $kg_f = 1\ kg\text{-}wt = 9 \cdot 8\ N$

   eg. $60\ kg_f = 60 \times 10 = 600\ N$

②

Dimensions → Dimensions of a physical quantity are the powers to which fundamental quantities must be raised to represent a given physical quantity.

eg. Dimensions of ~~~~Mass - $[M^1]$

Length - $[L]$

Time - $[T]$

Temperature - $[\theta]$

Current - $[A]$

dimensions of speed $= \frac{dist}{time} = \frac{[L^1]}{[T^1]} = [LT^{-1}]$

~~density~~ energy =

work $= F \times S = [MLT^{-2}][L]$

$= [ML^2T^{-2}]$

Pressure $= \frac{F}{A} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

Force $= ma = M \times LT^{-2} = [MLT^{-2}]$

Applications of Dimensional Analysis

① To find dimensions of physical constraints

$F = \frac{GMm}{d^2}$

$N = \frac{G\,kg^2}{m^2}$

$G = N\,m^2\,kg^{-2}$

$G = [M^{-1}L^3T^{-2}]$

$E = h\nu \rightarrow$ frequency
$\downarrow$
Plank's constant

$ML^2T^{-2} = h\,T^{-1}$

$h = [ML^2T^{-1}]$

$\nu = \frac{1}{\cancel{f}\,\rho t}$

2. To check if a equation is dimensionally correct

eg. $F = ma^2$

$MLT^{-2} = M\,[LT^{-2}]^2$

$MLT^{-2} \neq ML^2T^{-4}$ ($F = ma^2$ is wrong as dimensions are diff)

$F = \frac{mv^2}{r}$

$= \frac{M\,[LT^{-1}]^2}{L}$

$= \frac{ML^{\cancel{2}}T^{-2}}{\cancel{L}}$

$MLT^{-2} = MLT^{-2}$ (correct equ)

$S = ut + \frac{1}{2}at^2$ (numbers are dimension less)

($[L]+[L] = [L]$)

$L = LT^{-1} \times T + \frac{1}{2}[LT^{-2}][T^2]$

$= L + \cancel{f}L$

$[L] = [L]$ ~~correct~~ dimensionally correct

(4)

3. Derive new equations

$E = mc^2$

$E = f(m, c)$

$E = \cancel{f}\ K\, m^x c^y$ (K is constant)

$ML^2T^{-2} = K\, M^x [LT^{-1}]^y$

$ML^2T^{-2} = K\, M^x L^y T^{-y}$

$x = 1$

$y = 2$

$E = K mc^2$

$K = 1$ (by experiments)

$\boxed{E = mc^2}$

Q If velocity, force & time are taken as funda. quan. find dimensions of mass & energy

~~vel = dis / time~~

~~force = mass × dis/time² × vel/time~~

~~$\frac{MLT^{-2} \times T}{LT^{-1}}$~~

~~M~~

$m = f(v, f, t)$

$m = K v^x F^y t^z$

$m = K [LT^{-1}]^x [MLT^{-2}]^y [T]^z$

$M^1 = K\, M^y L^{x+y} T^{-x-2y+z}$

$y = 1$

$x + y = 0$

$$E = f(v, f, t)$$

$$ML^2T^{-2} = M^y L^{x+y} T^{-x-2y+z}$$

$$\boxed{y = 1}$$

$$x + y = 2$$

$$\boxed{x = 1}$$

$$-x - 2y + z = -2$$

$$-1 \cancel{-2} + z = \cancel{-2}$$

$$\boxed{z = 1}$$

$$\boxed{E = K\, v^1 F^1 T^1}$$

Q if momentum, area & time are taken

$E = ?$

area $= L^2$

Time $= T^1$

momentum $= MLT^{-1}$

$$ML^2T^{-2} = K P^x A^y T^z$$

$$ML^2T^{-2} = K[MLT^{-1}]^x [L^2]^y [T]^z$$

$$= K M^x L^{x+2y} T^{-x+z}$$

$$\boxed{x = 1}$$

$$x + 2y = 2$$

$$2y = 1$$

$$\boxed{y = 1/2}$$

$$z = 1 + -2$$

$$\boxed{z = -1}$$

$$\boxed{E = K\, P A^{1/2} T^{-1}}$$

Q

$$P = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$D = \frac{M}{L^3} = ML^{-3}$$

$$E = ML^2T^{-2}$$

$$T' = K\left(ML^{-1}T^{-2}\right)^x \left[ML^{-3}\right]^y \left[ML^2T^{-2}\right]^z$$

$$T' = K \; M^{x+y+z} \; L^{-x-3y+2z} \; T^{-2x-2z}$$

$$-2x - 2z = 1$$

$$-2z = 2x$$

$$\boxed{z = -x}$$

$$z = \frac{1+2x}{2}$$

$$~~-x - 3y + 2x = 0~~$$

$$~~-3y = 3x~~$$

$$~~\boxed{x = -y}~~$$

$$~~-y + y + z = 0~~$$

$$~~\boxed{z = 0}~~$$

$$~~\boxed{x = 0}~~$$

$$~~\boxed{y = 0}~~$$

$$-x - 3y + 2\left(\frac{1+2x}{2}\right)$$

$$-x + 3y + 1 + 2x$$

$$x + 3y + 1 = 0$$

$$\boxed{x = -1 - 3y}$$

$$-1 - 3y + y + \frac{1 + 2(-1-3y)}{2}$$

$$-1 - 3y + y + \frac{1 - 2 - 6y}{2}$$

$$\frac{-2 - 4y + 1 - 2 - 6y}{2}$$

$$\frac{-3 - 10y}{2} = 0$$

$$y = \frac{3}{-10}$$

$$\boxed{y = \frac{-3}{10}}$$

$$x = -1 - 3\left(\frac{-3}{10}\right)$$

$$= -1 + \frac{9}{10}$$

$$= \frac{-10 + 9}{10}$$

$$\boxed{x = \frac{-1}{10}}$$

$$z = \frac{1 - \frac{2}{10}}{2}$$

$$z = \frac{\frac{8}{10}}{2}$$

$$\boxed{z = \frac{2}{5}}$$

7

Scanned with CamScanner

$T = K \rho^x d^y E^z$

$M^0 L^0 T^1 = [ML^{-1}T^{-2}]^x \; [ML^{-3}]^y [ML^2 T^{-2}]^z$

$M^0 L^0 T^1 = M^{x+y+z} \; L^{-x-3y+2z} \; T^{-2x-2z}$

$x + y + z = 0$

$-x - 3y + 2z = 0$

$-2x - 2z = 1$

$-x = \frac{1+2z}{2}$

$-x - 3y + 2z = 0$

$\frac{1+2z}{2} - 3y + 2z$

$1 + 2z - 6y + 4z = 0$

$1 - 6y + 6z = 0$

$y = \frac{6z+1}{6}$

$\frac{6z+1}{6} + \frac{-1-2z}{2} + z = 0$

$\frac{6z + 1 - 3 - \cancel{6z} + 6z}{6}$

$\frac{6z-2}{6} = 0$

$6z = 2$

$\boxed{z = \frac{1}{3}}$

$x = \frac{-1 - \frac{2}{3}}{2}$

$x = \frac{-3-2}{6}$

$\boxed{x = \frac{-5}{6}}$

$y = \frac{2+1}{6}$

$\boxed{y = \frac{1}{3}}$

$\boxed{t = K \rho^{-5/6} d^{1/3} E^{1/3}}$

System of units

FPS (Foot, Pound, Second) (British system)

CGS (Gausion system) (centimeter, gram, second)

MKS (Metric system) (Meter, kg, Second)

$1N = 10^5$ dyne (C.G.S)

$1J = 10^7$ erg (C.G.S)

Trignometry

Formulas

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
5. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
6. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
7. $\cos(A-B) = \cos A \cos B + \sin A \sin B$

8. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

9. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

10. $\sin 2\theta = 2 \sin\theta \cos\theta$

11. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= 1 - 2\sin^2\theta$$

$$= 2\cos^2\theta - 1$$

12. $1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right)$

13. $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

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eg. $\sin 150^\circ = \sin(90+60)$

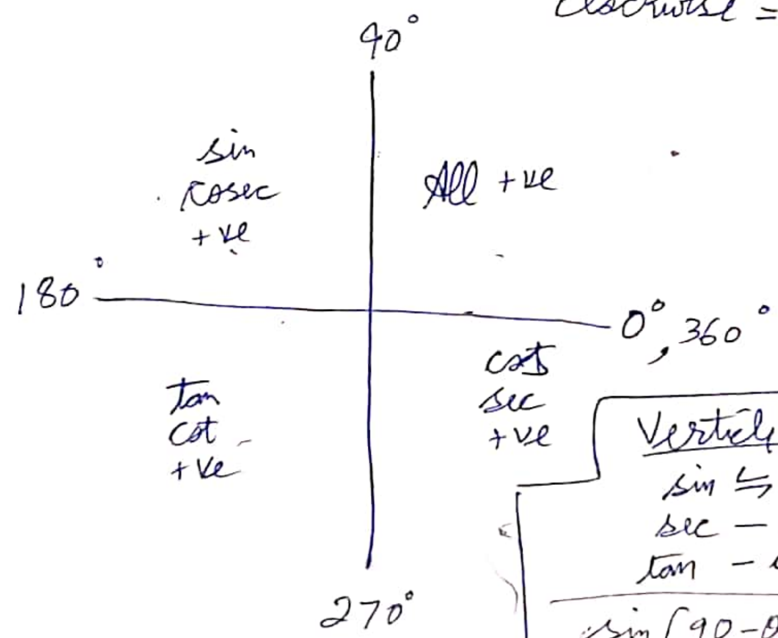
$\sin(A+B) = \sin 90 \cos 60 + \sin 60 \cos 90$

$$= 1 \times \frac{1}{2} + 0$$

$$= \frac{1}{2}$$

~~ATSC~~ rule :-
ASTC

anticlockwise = positive angle
clockwise = -ve angle



Vertical line
sin ⇆ cos
sec — cosec
tan — cot

$\sin(90-\theta) = +\cos\theta$
$\sin(90+\theta) = +\cos\theta$
$\cos(90+\theta) = -\sin\theta$
$\cos(90-\theta) = +\sin\theta$
$\tan(90-\theta) = \cot\theta$
$\tan(90+\theta) = -\cot\theta$

Horizontal Line (same sign & ratio)
$\sin(180-\theta) = +\sin\theta$
$\sin(180+\theta) = -\sin\theta$
$\cos\theta(180-\theta) = -\cos\theta$
$\tan\theta(180+\theta) = \tan\theta$

eg $\sin 150^\circ = \sin(90^\circ + 60^\circ)$
$= \cos 60^\circ$
$= \frac{1}{2}$

~~$\tan(270+\theta) = -\cot\theta$~~

eg. $\cos(120^\circ) = \cos(90+30)$
$= -\sin 30$
$\boxed{= -\frac{1}{2}}$

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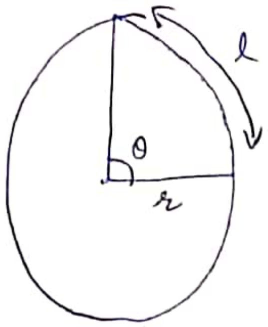
Angles

① Sexagesimal system :- Angle is measured in degrees (θ°)

$1^\circ = 60'$ (arc minutes)
$1' = 60''$ (arc seconds)

② Circular system :- Angle in radians (rad) (θ^c) (θ - no. superscript)



$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi^c = 360^\circ$

$1^\circ = \frac{\pi}{180}$ rad

$180^\circ = \pi^c$
$90^\circ = \frac{\pi}{2}^c$
$45^\circ = \frac{\pi}{4}^c$
$30^\circ = \frac{\pi}{6}^c$
$60^\circ = \frac{\pi}{3}^c$

* If θ is very small ($\theta < 5^\circ$)

$$\sin\theta \approx \theta$$
$$\cos\theta \approx 1 \quad \text{or} \quad 1 - \frac{\theta^2}{2}$$
$$\tan\theta \approx \theta.$$

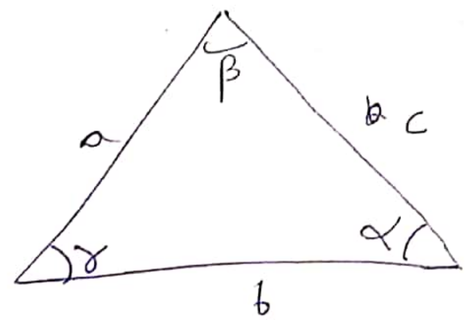
(θ must in radian)

$\sin 37^\circ = \frac{3}{5}$ $\quad$ $\sin 53^\circ = \frac{4}{5}$

$\cos 37^\circ = \frac{4}{5}$ $\quad$ $\cos 53^\circ = \frac{3}{5}$

$\tan 37^\circ = \frac{3}{4}$ $\quad$ $\tan 53^\circ = \frac{4}{3}$

Sine Rule



$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

Cosine Rule:-

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Binomiol Theorm (Expression with 2 terms)

Factorial (!)

$5! = 5 \times 4 \times 3 \times 2 \times 1$
$\quad = 120$

$4! = 4 \times 3 \times 2 \times 1 = \cancel{20}24$

$3! = 3 \times 2 \times 1 = 6$

$2! = 2 \times 1 = 2$

$\cancel{0}1! = 1 = 1$

$0! = 1$

$n!$ = Product of n natural numbers
$\quad = n \times (n-1) \times (n-2) \ldots \times 3 \times 2 \times 1$

$\underbrace{(x+y),\ (4x-3y),\ (a+b)^2}_{\text{Binomiol Expression :-}}$

Binomial Formula

$$(x+a)^{n} = {}^{n}C_0\, x^{n} a^{0} + {}^{n}C_1\, x^{(n-1)} a^{1} + {}^{n}C_2\, x^{(n+2)} a^{2} + {}^{n}C_n\, x^{0} a^{n}$$

$${}^{n}C_r = \frac{n!}{r!\,(n-r)!}$$ combination formula

eg. $(a+b)^2 = {}^2C_0\, a^2 b^0 + {}^2C_1\, a^1 b^1 + {}^2C_2\, a^0 b^2$

$= \frac{\cancel{2!}}{0!\,\cancel{2!}} a^2 + \frac{2!}{1!\,1!} ab + \frac{\cancel{2!}}{\cancel{2!}\,0!} b^2$

$= \frac{a^2}{1} + \frac{2}{1\times1} ab + \frac{b^2}{1}$

$= a^2 + 2ab + b^2$

eg 2. $(a+b)^3 = {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a^1 b^2 + {}^3C_3\, a^0 b^3$

$= \frac{\cancel{3!}}{0!\,\cancel{3!}} a^3 + \frac{3!}{1!\,2!} a^2 b + \frac{3!}{2!\,1!} ab^2 + \frac{\cancel{3!}}{\cancel{3!}\,0!} ab^3$

$= \frac{a^3}{1} + \frac{6}{2} a^2 b + \frac{6}{2} ab^2 + \frac{b^3}{1}$

$= a^3 + 3a^2 b + 3ab^2 + b^3$

eg. 3. $(1+x)^6 = {}^6C_0\, 1^6 x^0 + {}^6C_1\, 1^5 x^1 + {}^6C_2\, 1^4 x^2 + {}^6C_3\, 1^3 x^3 +$
${}^6C_4\, 1^2 x^4 + {}^6C_5\, 1^1 x^5 + {}^6C_6\, 1^0 x^6$

$= \frac{6!}{0!\,6!} \times 1 + \frac{6!}{1!\,5!} x + \frac{6!}{2!\,4!} x^2 + \frac{6!}{3!\,3!} x^3 + \frac{6!}{4!\,2!} x^4 + \frac{6!}{5!\,1!} x^5 + x^6$

$360\ 180\ 90\ 45\ 15$ $\quad 360\ 180\ 60\ 20$

$= 1 + \frac{\overset{6}{\cancel{720}}}{120} x + \frac{720}{\cancel{24\times2}} x^2 + \frac{720}{6\times6} x^3 + \frac{720}{240} x^4 + \frac{720}{120} x^5 + x^6$

$12 \quad 2 \quad 3 \quad 3 \quad 3$

$$\boxed{= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6}$$

Binomial approximation:-

$$\boxed{(1+x)^n \approx 1+nx \quad x<<1}$$

$(1+x)^n$.

$x<<1$ (<< means very small)

eg. $(1+x)^2 = 1+x^2+2x \approx 1+2x$

$(1+x)^3 = 1+x^3+3x^2+3x \approx 1+3x$

$x<<1$

$x^2, x^3 \to$ higher orders of $x \to$ even smaller

eg. $g_h = g\left[\frac{R}{(R+h)}\right]^2$

$R = 6400\,km$

$g = 9.8$

~~h = height above~~

if $h<<R$

$$g_h = g\left[\frac{\cancel{R}}{\cancel{R}(1+\frac{h}{R})}\right]^2$$

$$= g\left[\frac{1}{1+\frac{h}{R}}\right]^2$$

$$= g\left(1+\frac{h}{R}\right)^{-2}$$

~~$= g(1-$~~

$h<<R$

$\frac{h}{R}<<1$

$$g_h = g\left(1-\frac{2h}{R}\right)$$

Vectors

Physical Quantity

Scalars

Only magnitude
no. direction

Vectors

1) magnitude
2) ~~direction~~ specified direction
3) obey laws of vector addition

Ratio

To compare two Similar Quantities
(no unit)

Tensors

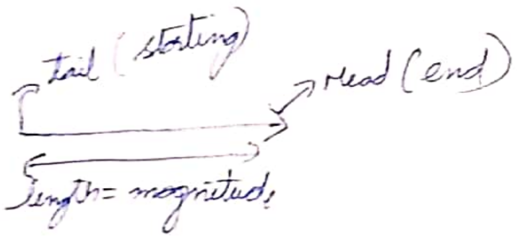
~~1) Magnitude must change with direction~~

1) direction don't effect value

eg. Movement of Inertia

Representation of a vector :-

$\vec{F}$ $\vec{V}$

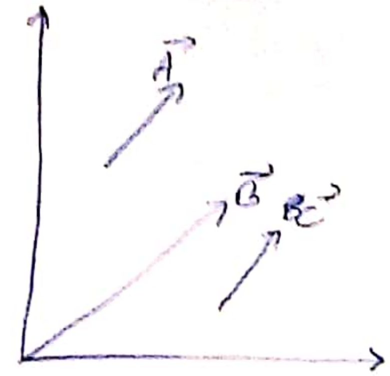


~~$[\vec{A}]$ (taking~~

→ For two vectors to be same, their magnitude & direction both should be equal.

$|\vec{A}|$ (taking only magnitude of the vector)

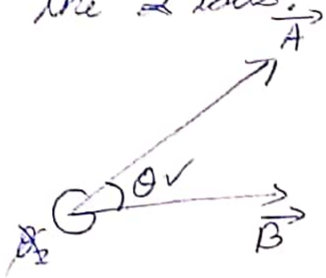
→ we can displace a vector parallel to itself without affecting it.



$\vec{A} = \vec{C}$

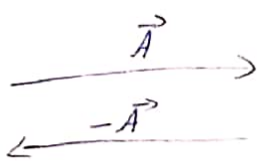
Angle between 2 vectors.

→ Join the 2 vectors by tail.
Angle between 2 vectors is the smaller vector between the 2 tails.



Negative of a vector

→ A vector of same magnitude but opposite in direction.



Multiply

① with Positive number

multiplying vector $\vec{A}$ with α will give new vector $\vec{B}$ with magnitude α times & same direction.

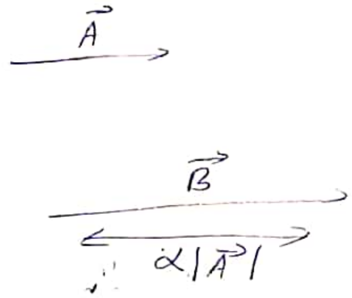
② with negative number

Opp direction & magnitude gets multiplied.

$\vec{A} \times \alpha$

$\downarrow$

$\vec{B} = \alpha \vec{A}$



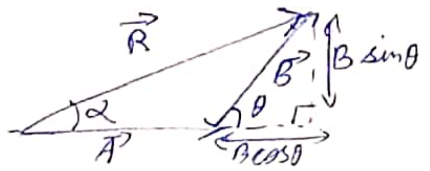
Null Vector - Vector with 0 magnitude & undetermined direction.

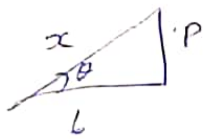
~~Also~~ Also known as 'zero vector'

Addition of Vectors

(I) Triangle law of vector addition -

$\vec{A} + \vec{B} = \vec{R}$





$\sin\theta = \frac{P}{x}$

$P = x\sin\theta$

$\cos\theta = \frac{b}{x}$

$b = x\cos\theta$

PGT

$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$

$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$

$$= B^2(\sin^2\theta + \cos^2\theta) + A^2 + 2AB\cos\theta$$

$$\boxed{R = \sqrt{B^2 + A^2 + 2AB\cos\theta}} \rightarrow \text{Magnitude of resultant vector}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

for $\boxed{\begin{array}{l}\theta = 0^\circ \\ R = A + B\end{array}}$ $\boxed{\begin{array}{l}\theta = 180^\circ \\ R = A - B\end{array}}$

$\boxed{\begin{array}{l}\theta = 90^\circ \\ R = \sqrt{A^2 + B^2}\end{array}} \rightarrow \tan\alpha = \dfrac{B}{A}$ $\quad |A - B| \le R \le |A + B|$

eg. ✱ Only the vectors of the same type can be added & subtracted & result is also same type.

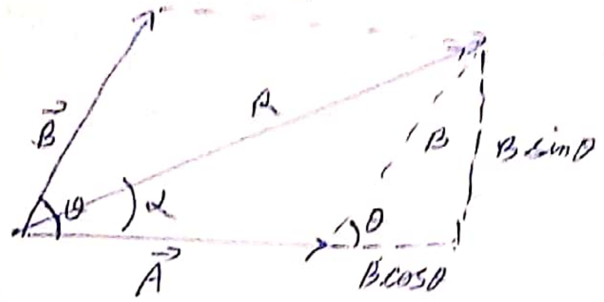
✱ Vector addition is commutative i.e

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

✱ Vector addition is associative i.e

$$(\vec{P} + \vec{Q}) + \vec{R} = \vec{P} + (\vec{Q} + \vec{R}) = \vec{Q} + (\vec{P} + \vec{R})$$

II Parallelogram Law of Vector addition -



$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

* If two non-zero vectors are represented by two adjacent sides of a parallelogram, then resultant vector is represented by its diagonals passing through point of intersection of 2 vectors.

O-1 09-04-2024

Q1 - D	Q2 - C	Q3 :- A	Q4 - A
Q5 - B	Q6 - A	Q7 - C	Q8 - D
Q9 - D	Q10 - A	Q11 - C	Q12 - A
Q13 - D	Q14 - B	Q15 - D	Q16 - B
Q17 - C	Q18 - B		

S-1

Q1 - $k = [ML^2T^{-2}]$ $a = [L^{-1}]$ Q2 - $R_B = \frac{R_g m^2}{Ks^2}$

Q3 - $t = a\sqrt{\frac{m}{R}}$ Q4 - $\omega = K\sqrt{\frac{mG}{r^3}}$ Q5 - $k = 6$

Q6 - $[MLT^{-1}]$ Q7 i) $24/25$ ii) $-7/25$ iii) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ iv) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

Q8 - $l = \frac{2\pi}{3} \times 6.4 \times 10^6\ m$

$PK = \sqrt{3}\ 6.4 \times 10^6\ m$

Q9 - a - 9.95

b - 9.99

Q10. $\frac{\pi}{100}\ m$ QIT.

JEE - Mains

Q1-3) Q2-4) Q3-~~2~~4) Q4-2) Q5-2) ~~Q10~~

Q10-3) Q11-4) Q16-1) Q21-C) Q22-C) Q23-B)

Q7 i) $\sin 74^\circ$

$\underline{\sin 2(37)}$

$\sin 2\theta = 2 \sin\theta \cos\theta$

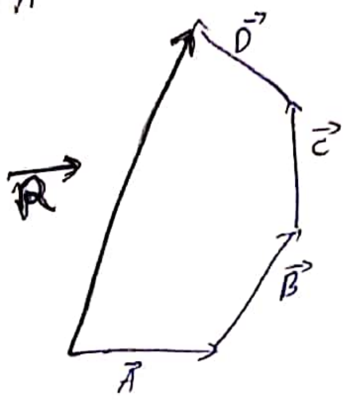
$\sin 2(37^\circ) = 2 \sin 37^\circ \cos 37^\circ$

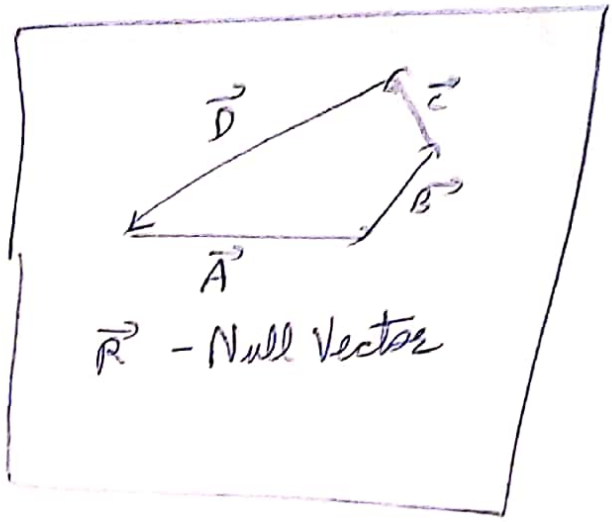
$\sin 74^\circ = 2 \times \frac{3}{5} \times \frac{4}{5}$

$\sin 74^\circ = \frac{24}{25}$

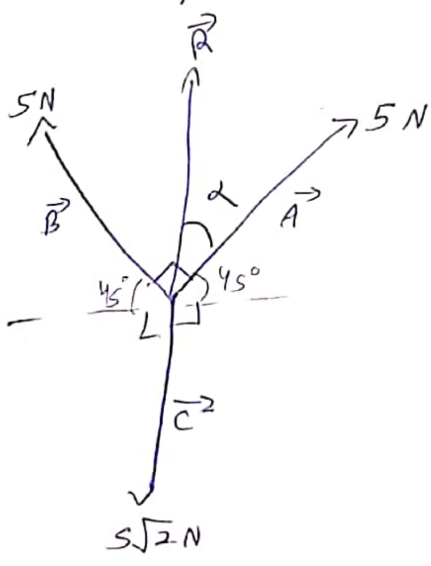
Polygon law of vector addition:

$\vec{A} + \vec{B} + \vec{C} + \vec{D}$





- ☆ If All the vectors are arranged to form sides of a polygon in one order then their resultant is represented by closing side of polygon in opposite order
- ☆ If n vectors are represented by n sides of a polygon in cyclic manner, the resultant of these vectors is a null vector.
- ☆ Resultant of two unequal vectors cannot be zero.
- ☆ Resultant of 3-co-planer vectors may or may not be zero.



$$\vec{A}+\vec{B}+\vec{C}=(\vec{A}+\vec{B})+\vec{C}$$

$$\cancel{5\sqrt{2}+5\sqrt{2}=0}$$

$$\cancel{\alpha=45^\circ}$$

$$\vec{A}+\vec{B}=\vec{R}$$

$$\vec{R}=\sqrt{25+25}$$

$$\vec{R}=5\sqrt{2}.$$

$$\tan\alpha=\frac{B}{A}$$

$$\tan\alpha=1$$

$$\tan 45=1$$

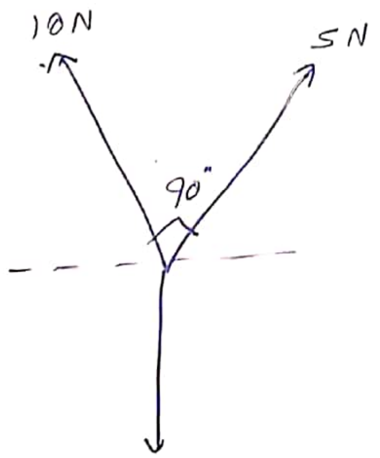
$$\alpha=45$$

$$\vec{R}+\vec{C}=\sqrt{(5\sqrt{2})^2-(5\sqrt{2})^2}$$

$$=0$$

---

eg 2.



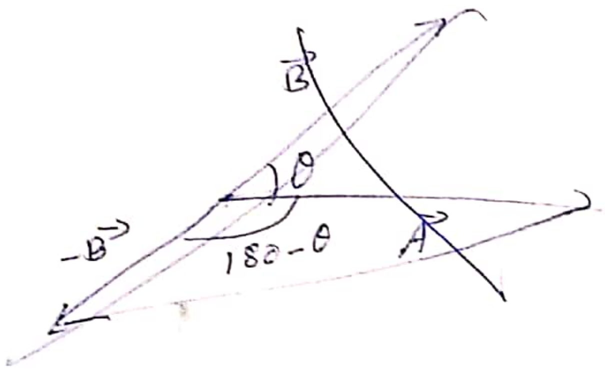
$$\vec{A}+\vec{B}+\vec{C}\neq 0$$

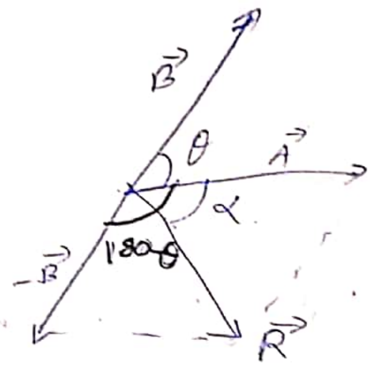
★ The result of non-co-planer vectors cannot be zero

Subtraction of vectors

$\vec{A} - \vec{B}$

$\vec{A} + (-\vec{B})$





$$\vec{A} - \vec{B} = \vec{R}$$

$$\vec{R} = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$$

$$\vec{R} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

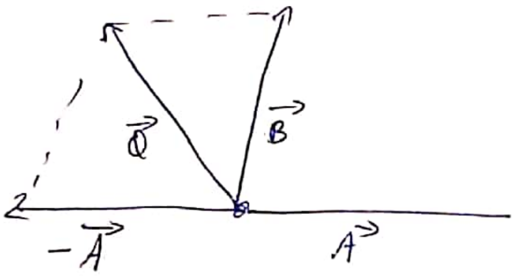
$$\tan\alpha = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)}$$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$ ★ Non-commutative

$\vec{B} - \vec{A} = \vec{Q}$

$\vec{B} + (-\vec{A})$



$$\boxed{\vec{A} - \vec{B} \neq \vec{B} - \vec{A}}$$

Q Given that $\vec{C} = \vec{A} + \vec{B}$.

Also $|\vec{A}| = 12$ $|\vec{B}| = 5$ $|\vec{C}| = 13$ units

Find angle btw $\vec{A}$ & $\vec{B}$

$|\vec{A}| = 13$ $\quad A = 13$ units

$|\vec{B}$ $\quad B = 13/12$ units

$C = 13/5$ units

$$\frac{13}{5} = \sqrt{(13)^2 + \left(\frac{13}{12}\right)^2 + 2 \times \frac{13}{12} \times 13 \times \cos\theta}$$

$$\frac{169}{25} = 169 + \frac{169}{144} + \frac{169}{6} \times \cos\theta$$

Ans.

$$C = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$13 = \sqrt{(12)^2+\cancel{B}(5)^2+(12)(5)\cos\theta}$$

$$169 = 144+25+120\cos\theta$$

$$\cos\theta = 0$$

$$\theta = 90° = \frac{\pi}{2}^c$$

A & B are Perpendicular / orthogonal

Q2. The greatest & least resultant of two forces acting at a point is 10N & 6N. If each is increased by 3N, find the resultant of two forces when acting at a point at an angle of 90° with each other.

$$A+B=10$$
$$A-B=6$$
$$A=6+B$$

$$6+B+B=10$$
$$2B=4$$
$$B=2$$

$$A=8$$

$$A=8+3=11$$
$$B=2+3=5$$

$$\theta=90°$$

$$\vec{R} = \sqrt{121+25}$$

$$\boxed{\vec{R} = \sqrt{146}\ N}$$

$$\tan\alpha = \frac{5}{11}$$

Q. The maximum & minimum of two forces are in ratio 7:3 find force ratio.

$A+B=7x$

$A-B=3x$

$A=3x+B$

$3x+2B=7x$

$2B=4x$

$\underline{\underline{B=2x}}$

$\frac{\cancel{A}}{B} = \frac{3x+B}{B}$

$\underline{\underline{A=5x}}$

$\frac{A}{B} = \frac{5x}{2x} = \frac{5}{2}$

$\boxed{A:B = 5:2}$ ✓

Q Two equal forces have their resultant equal to either. find angle.

$C = \sqrt{2x^2+2x^2\cos\theta}$

$C = \cancel{x\sqrt{\ldots}}$

$x^2 = 2x^2+2x^2\cos\theta$

$-x^2 = 2x^2\cos\theta$

$\frac{-1}{2} = \cos\theta$

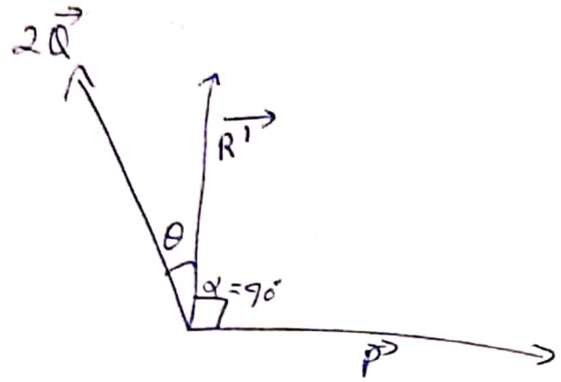
$\cancel{\boxed{\theta = 150}}$

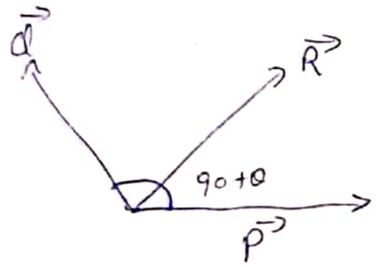
$\boxed{\theta=120^\circ}$ ✓

$\cancel{\vec{P} + \vec{Q} = \vec{R}}$

$\cancel{\tan 90 = \frac{2Q\sin\theta}{P+}}$

Q The resultant of two vectors $\vec{P}$ & $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new vector is $\perp$ to $\vec{P}$. find $\vec{R}$





$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\tan 90 = \frac{2Q\sin\theta(90+\theta)}{P+2Q\cos(90+\theta)}$$

0 (as $\tan 90°$ is undefined)

$P + 2Q\cos(90+\theta)$

$$\cos(90+\theta) = \frac{-P}{2Q}$$

$$R = \sqrt{P^2+Q^2+2PQ\cos(90+\theta)}$$

$$R = \sqrt{P^2+Q^2+2PQ \times \frac{-P}{2Q}}$$

$$\boxed{R = Q}$$

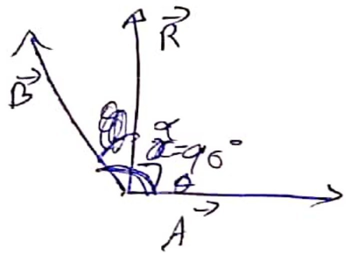
Q.6 The ~~sin~~ sum of magnitude of two forces at a point is 18 N and magnitude of resultant is 12 N. If the resultant makes an angle of 90° with the force of smaller magnitude, what are the magnitude of two forces.

$A + B = 18 \rightarrow B = 18 - A$

$|R| = 12N$

$\alpha = 90°$

$$\tan 90 = \frac{A\sin\theta}{A + B\cos\theta} \rightarrow 0$$



$A + B\cos\theta = 0$

$$\cos\theta = \frac{-A}{B}$$

$\vec{R} = \vec{A} + \vec{B}$

$$12 = \sqrt{A^2+B^2+2AB \times \frac{-A}{B}}$$

$144 = A^2 + B^2 - 2A^2$

$144 = B^2 - A^2$

$144 = (18-A)^2 - A^2$

$144 = 324 + \cancel{A^2} - 36A - \cancel{A^2}$

$36A = 324 - 144$

$$A = \frac{180}{36}$$

$$\boxed{\begin{array}{l} A = 5\,N \\ B = 13\,N \end{array}}$$ ✓

18
18
144
180
324

Q 7. A vector $\vec{B}$ which has a magnitude 8. is added to $\vec{A}$ which lie along the x axis, the resultant lie along y-axis and has a magnitude that is twice of $\vec{A}$ magnitude of $\vec{A}$. The magnitude of $\vec{A}$ is ____.

$$\tan 90° = \frac{AB\sin\theta}{A+B\cos\theta}$$

$$\cos\theta = \frac{-A}{B}$$

$$(2x)^2 = A^2+B^2+2AB\cos\theta$$

$$4x^2 = x^2+64+2\times A\times B\times\frac{-A}{B}$$

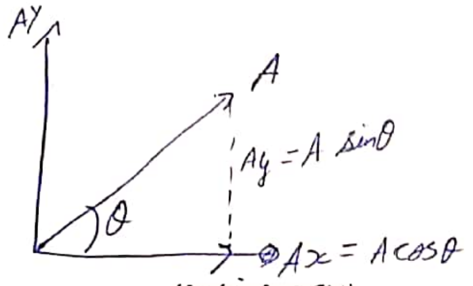
$$4x^2 = 64-x^2$$

$$5x^2 = 64$$

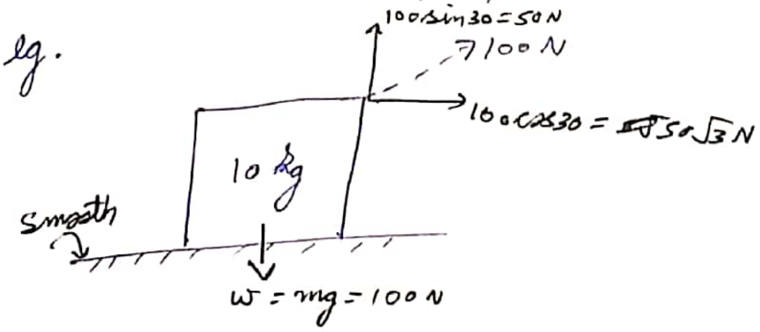
$$x^2 = \frac{64}{5}$$

$$\boxed{x = \frac{8}{\sqrt{5}}}$$

Components of Vectors



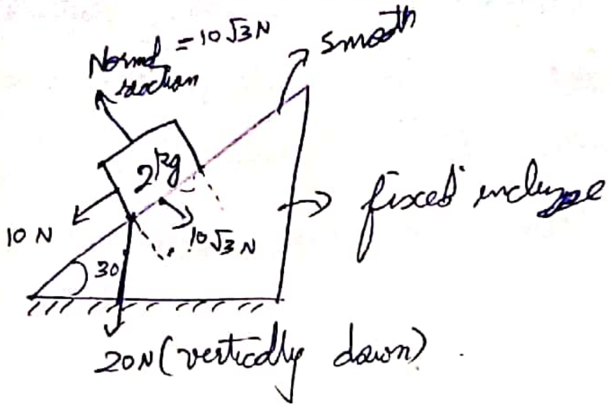
eg.

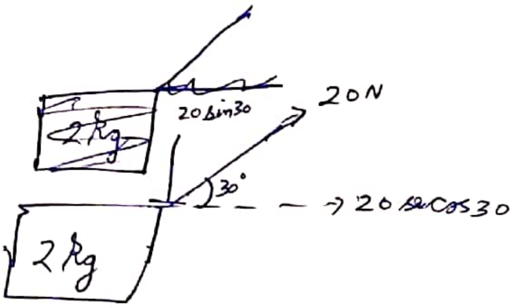


$F = ma$

$$\frac{50\sqrt{3}}{10} = a = 5\sqrt{3}\ m/s^2$$

Ag 2.



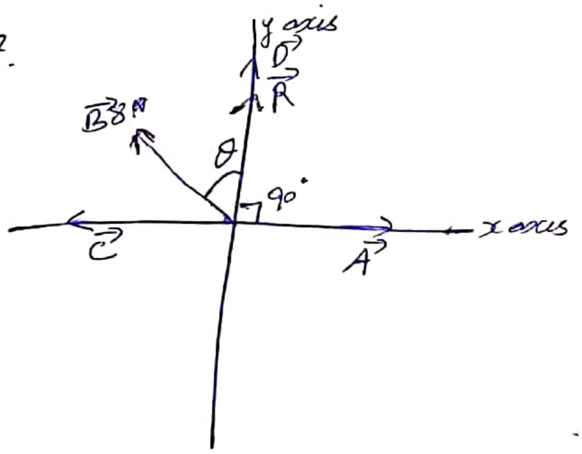


$20 \sin 30^\circ = 10 N$

$F = ma$

$\frac{10}{2} = a = \boxed{5 \ m/s^2}$

Ans 7.



$|B| = 8$

$|R| = 2|A|$

$\vec{C} = B \sin\theta$

$\vec{D} = ~~A~~B\cos\theta$

$\vec{A} + \vec{C} = 0$

$A = C$

$A = B \sin\theta$

$\vec{D} = \vec{R}$

~~$B\cos\theta = 2B\sin\theta$~~

$R = B\cos\theta$

$A^2 + R^2 = B^2 \sin^2\theta + B^2 \cos^2\theta$

$A^2 + R^2 = B^2$

$A^2 + R^2 = 64$

$A^2 + 4A^2 = 64$

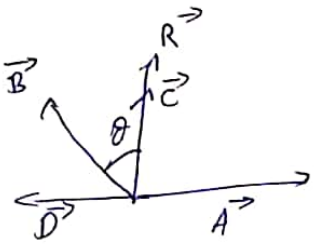
$A^2 = \frac{64}{5}$

$\boxed{A = \frac{8}{\sqrt{5}}}$

Ans6.

$|A| + |B| = 18$ $\qquad B = 18 - A$

$R = 12$



$C = B\cos\theta$
$D = B\sin\theta$

$D = A$ (as force is ⊥ & so they will cancle)

$A = B\sin\theta$

$R = C = B\cos\theta$

$A^2 + R^2 = B^2\sin^2\theta + B^2\cos^2\theta$

$A^2 + R^2 = B^2$

$R^2 = B^2 - A^2$

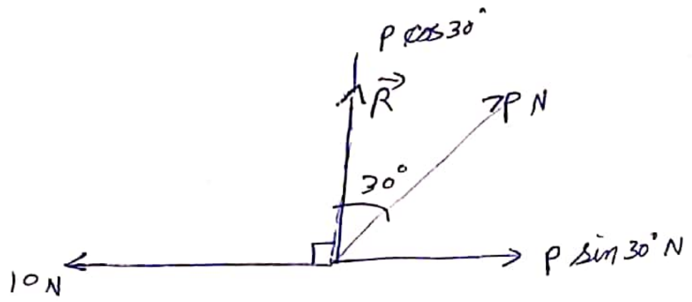
$144 = \cancel{18-A}(B-A)(B+A)$

$\dfrac{\cancel{144}^{72\ 24\ 8}}{\cancel{18}_{9\ 3}} = B - A$

$B - A = 8$
$B + A = 18$
$2B = 26$

$\boxed{\begin{matrix} B = 13 \\ A = 5 \end{matrix}}$

Q8. Two horizontal forces of 10N & PN act on a particle. The force of magnitude 10N acts west & the force of PN acts on a bearing of 30° east of north as shown in figure. The resultant of these two force acts due north. Find the magnitude of this resultant.



$$P\sin 30° = 10$$

$$\frac{P}{2} = 10$$

$$\boxed{P = 20}$$

$$P\cos 30 = 20 \times \frac{\sqrt{3}}{2} = \boxed{10\sqrt{3} N}$$

or

$$\tan\alpha = 90 = \frac{B\sin\theta}{A + B\cos\theta}$$

$$10 + P\cos\theta = 0$$

$$\cos\theta = \frac{-10}{P}$$

$$R^2 = (10)^2 + P^2 + 2 \times 10 \times P \times \frac{-10}{P}$$

$$R^2 = 100 + P^2 - 200$$

$$R^2 = P^2 - 200$$

$$R^2 = 400 - 200$$

$$R = \sqrt{200}$$

$$R \simeq 10\sqrt{2}$$

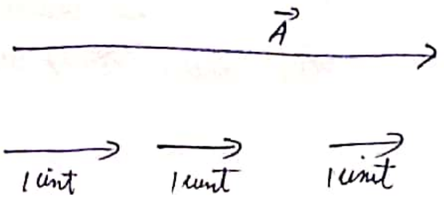
$$\tan 30 = \frac{10 \times \cancel{\sin 120} \; \sin(90+30)}{P + 10 \, \cancel{\sin 120} \; \cos(90+30)}$$

$$\frac{1}{\sqrt{3}} = \frac{5\sqrt{3}}{P - 5}$$

$$P - 5 = 15$$

$$\boxed{P = 20}$$

Unit Vector



$|\vec{A}| = 3$

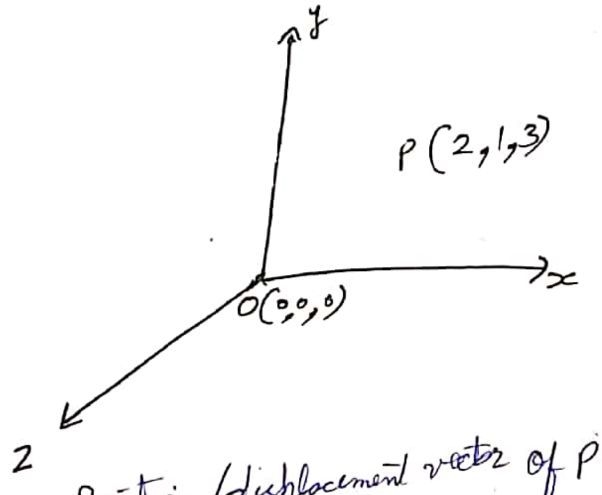
unit vector of $\vec{A} = \hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

dividing a vector by its magnitude, we get a vector of magnitude 1 & ~~as~~ same direction as $\vec{A}$.

→ It has no dimensions. ~~no~~ and no units

→ Cartesian form/rectangular form

unit vector of x-axis – $\hat{i}$
y-axis – $\hat{j}$
z-axis – $\hat{k}$



Position/displacement vector of $P = (2-0)\hat{i} + (1-0)\hat{j} + (3-0)\hat{k}$

$$\vec{OP} = 2\hat{i} + \hat{j} + 3\hat{k}$$

$$|\vec{OP}| = \sqrt{2^2 + 1^2 + 3^2}$$

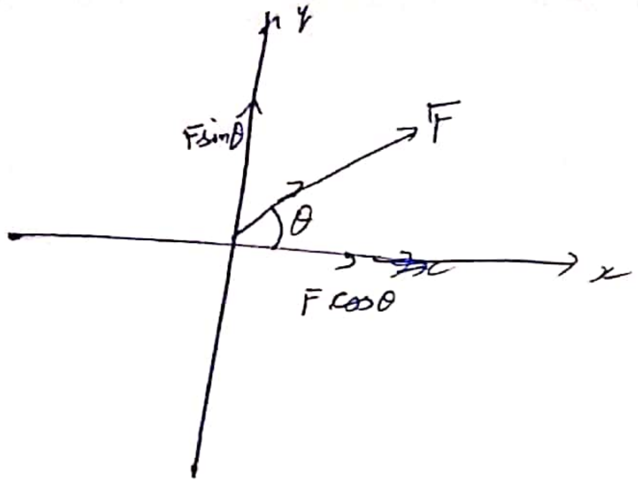
$$= \sqrt{4+1+9}$$

$$= \sqrt{14} \text{ units}$$

$$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$$

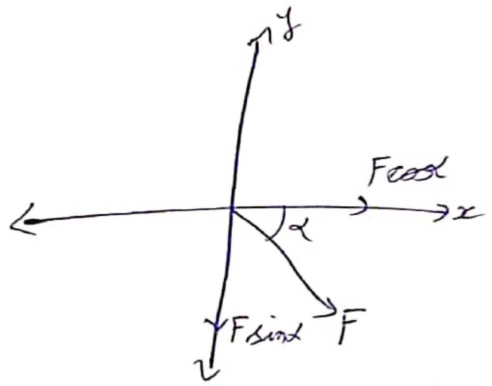
$$|\vec{A}| = \sqrt{a^2 + b^2 + c^2}$$

eg.



$$\boxed{\vec{F} = F\cos\theta\,\hat{i} + F\sin\theta\,\hat{j}}$$

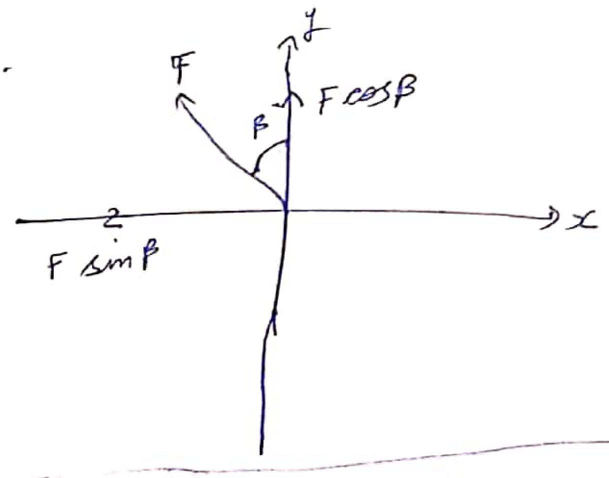
eg 2



$$\vec{F} = F\cos\alpha\,\hat{i} + F\sin\alpha\,(-\hat{j})$$

$$\boxed{= F\cos\alpha\,\hat{i} - F\sin\alpha\,\hat{j}}$$

eg 3.



$$\boxed{\vec{F} = \cancel{F\sin\beta}\ F\cos\beta\,\hat{j} - F\sin\beta\,\hat{i}}$$

Q9. find c if $0.3\hat{i} + 0.4\hat{j} + c\hat{k}$ is a unit vector.

~~$c = 0.3$~~

$$c = \sqrt{(0.3)^2 + (0.4)^2 + c^2}$$

$$1 = 0.09 + 0.16 + c^2$$

$$1 - 0.25 = c^2$$

$$0.75 = c^2$$

$$\frac{\sqrt{3}}{100} = c^2$$

$$\boxed{c = \frac{\sqrt{3}}{2}}$$

Q10. find unit vector of $\underbrace{2\hat{i} + \hat{j} - 2\hat{k}}_{\vec{A}}$.

$$|\vec{A}| = \sqrt{4+1+4}$$

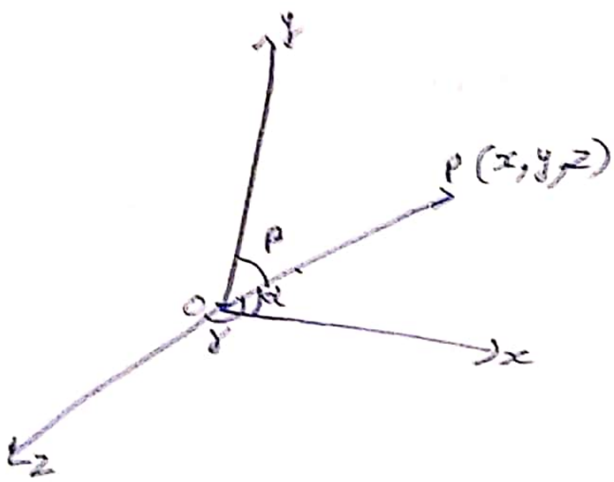
$$|\vec{A}| = \sqrt{9}$$

$$|\vec{A}| = 3$$

$$\hat{A} = \frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}$$

$$\boxed{\hat{A} = \tfrac{2}{3}\hat{i} + \tfrac{1}{3}\hat{j} - \tfrac{2}{3}\hat{k}}$$

Direction cosines :-

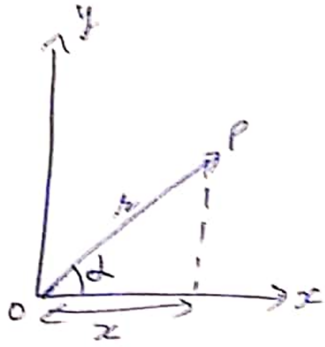


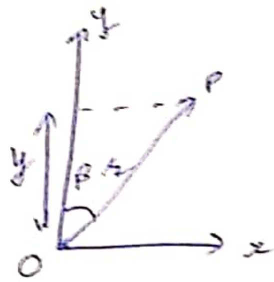
angle with x -axis = α
y = β
z = γ

α + β ≠ 090

$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{OP}| = \sqrt{x^2+y^2+z^2} = r$





$\cos\alpha = \frac{x}{r}$

$\cos\beta = \frac{y}{r}$

$\cos\gamma = \frac{z}{r}$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$$

$$= \frac{x^2+y^2+z^2}{r^2}$$

$$= \frac{r^2}{r^2}$$

$$= 1$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

The sum of square of direction cosines of a vector is always equal to one.

Q what are direction cosines of $\hat{i} + \hat{j} + \hat{k}$?

~~$\cos\alpha = \frac{x}{r}$~~ $x = 1$, $y = 1$, $z = 1$

$r = \sqrt{1^2 + 1^2 + 1^2}$

$r = \sqrt{3}$

$\cos\alpha = \frac{x}{r}$

$$\boxed{\cos\alpha = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos\beta = \frac{y}{r} = \frac{1}{\sqrt{3}}}$$

$$\boxed{\cos\gamma = \frac{z}{r} = \frac{1}{\sqrt{3}}}$$

Q2. if $\vec{P} = 4\hat{i} - 2\hat{j} + 6\hat{k}$, $\vec{Q} = \hat{i} - 2\hat{j} - 3\hat{k}$

find angle $(\vec{P} + \vec{Q})$ makes with x axis

$$\vec{P} + \vec{Q} = 5\hat{i} - 4\hat{j} + 3\hat{k}$$

$$r = \sqrt{25 + 16 + 9}$$

$$= \sqrt{50}$$

$$r = 5\sqrt{2}$$

$$\cos\alpha = \frac{\cancel{5}}{\cancel{5}\sqrt{2}}$$

$$\boxed{\cos\alpha = \frac{1}{\sqrt{2}}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\boxed{\alpha = 45^\circ} = \frac{\cancel{45} \times \pi}{\cancel{180}\,4} = \frac{\pi}{4}^{c}$$

$\boxed{\sin^{-1}\left(\frac{1}{2}\right) \text{ this angle ka like } \sin\frac{1}{2} \text{ hoga.}}$

Q3. A bird moves with velocity 20m/s in a direction making an angle of 60° with the eastern line & 60° with vertical upword. write velocity in rectangular form.

$$\alpha = 60^\circ$$

$$\gamma = 60^\circ$$

$$\cos 60^\circ = \frac{1}{2} \qquad \cos 60^\circ = \frac{1}{2}$$

$$\frac{x}{r} = \frac{1}{2} \qquad \frac{z}{r} = \frac{1}{2}$$

$$2x = r \qquad 2z = r$$

$$2x = 2z$$

$$x = z$$

$$\cancel{r = 20}$$

$$\cancel{x = 10}$$

$$\cancel{z = 10}$$

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{2x^2}$$

$$r^2 = 2x^2$$

$$\boxed{\cancel{\vec{V} = 10\hat{i} + 10\hat{k}}}$$

$|\vec{V}| = 20\ m/s$

$\alpha = 60$

$\gamma = 60^\circ$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\frac{1}{4} + \cos^2\beta + \frac{1}{4} = 1$

$\cos^2\beta = \frac{1}{2}$

$\cos\beta = \frac{1}{\sqrt{2}}$

$\cos\alpha = \frac{1}{2} = \frac{x}{20}$

$x = 10$

$z = 10$

$y = 10\sqrt{2}$

$\boxed{\vec{V} = 10\hat{i} + 10\sqrt{2}\hat{j} + 10\hat{k}}\ m/s$

Q4. find the vector which must be added to the sum of 2 vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ to get resultant of unit vector along z axis.

$\vec{V} = 2\hat{i} + 0\hat{j} + \hat{k}$

$\vec{V}$ - unit vector along z $= -\vec{R}$

$-\vec{R} = 2\hat{i} + \hat{k} - \hat{k}$

$\boxed{\vec{R} = -2\hat{i}}$ ✓

Q 5. A particle is moving towards east with 5 m/s in 10 seconds, its velocity becomes 5 m/s towards north. find average acceleration

$\vec{u} = 5\hat{i}$ m/s

$\vec{v} = 5\hat{j}$ m/s

$t = 10s$

$$\vec{a} = \frac{\vec{v} - \vec{u}}{10}$$

$$\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$$

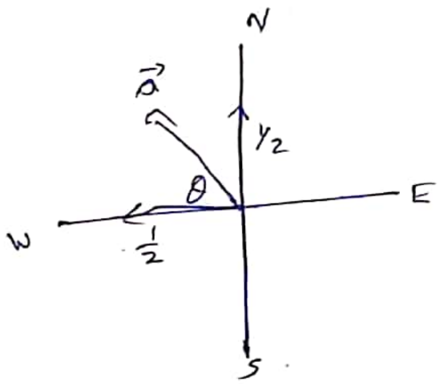
$$\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$$

$$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$r = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

$\boxed{|\vec{a}| = \frac{1}{\sqrt{2}} \text{ m/s}^2}$ (at an angle 45° west of north)

$$\vec{a} = \frac{-1}{2}\hat{i} + \frac{1}{2}\hat{j}$$



$a \cos\theta = \frac{1}{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\boxed{\theta = 45°}$

Q6. A car is moving towards North with speed of $\frac{36}{\cancel{60}}$ km/h. In 5s it turns towards left with its speed unchanged. find acc of car.

$\vec{u} = \frac{36}{\cancel{60}}\hat{j}$ m/s

$\vec{v} = -36\hat{i}$ m/s

$t = 5s$

$\vec{a} = \frac{-36\hat{j} - 36\hat{i}}{5}$

$\vec{a} = -\frac{(36\hat{i} + 36\hat{j})}{5}$

$\vec{a} = -12\frac{(3\hat{i} + 3\hat{j})}{5}$

$\cancel{|\vec{a}| = \sqrt{36^2 + 3}}$

$|\vec{a}| = \sqrt{9+9}$

$= \sqrt{18}$

$|\vec{a}| = 3\sqrt{2} \times \frac{-12}{5}$

$|\vec{a}| =$

$\vec{u} = 36 \times \frac{5}{18} = 10\hat{j}$

$\vec{v} = -10\hat{i}$

$t = 5s$

$\cancel{\vec{a} = -\frac{10}{2}\hat{i} - \frac{10}{2}\hat{j}}$

$\cancel{\vec{a} = -\left(\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}\right)}$

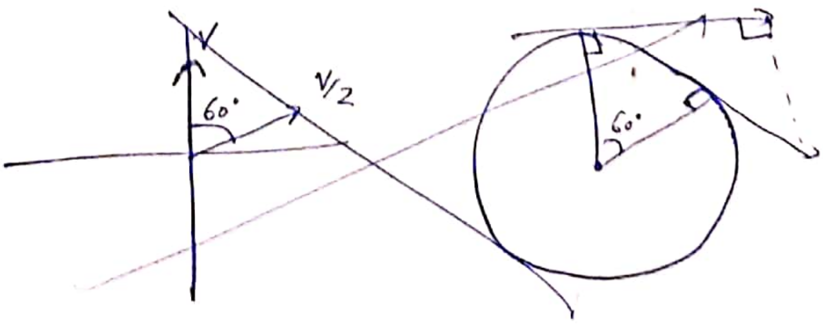
$\vec{a} = -2\hat{i} - 2\hat{j}$

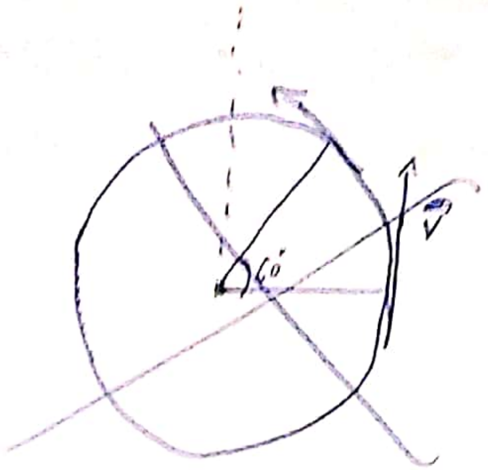
$|\vec{a}| = \sqrt{4+4}$

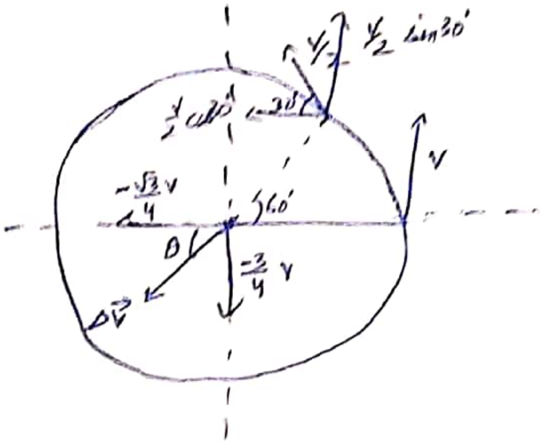
$= \sqrt{8}$

$\boxed{|\vec{a}| = 2\sqrt{2}\ m/s^2}$ (45° towards south of west)

Q7. A particle is moving on a circular path with speed v at a particular instant after some time, when it has described an angle 60° its speed becomes $\frac{v}{2}$ what is change in velocity.







$\vec{v} - \vec{u}$

$\vec{u} = v\hat{j}$

$\vec{v} = \frac{v}{2}\sin 30^\circ(\hat{j}) + \frac{v}{2}\cos 30^\circ(-\hat{i})$

$\vec{v} = \frac{v}{4}\hat{j} - \frac{\sqrt{3}}{4}v\hat{i}$

$\Delta\vec{v} = \vec{v} - \vec{u}$

$= -\frac{\sqrt{3}}{4}v\hat{i} + \frac{v}{4}\hat{j} - v\hat{j}$

$\boxed{\Delta\vec{V} = -\frac{\sqrt{3}}{4}v\hat{i} - \frac{3}{4}v\hat{j}}$

$|\Delta\vec{v}| = \sqrt{\left(\frac{-\sqrt{3}}{4}v\right)^2 + \left(-\frac{3}{4}v\right)^2}$

$\boxed{|\Delta\vec{v}| = \frac{\sqrt{3}v}{2}}$

$\tan\theta = \frac{\frac{3}{4}v}{\frac{\sqrt{3}}{4}v} = \sqrt{3}$

$\boxed{\theta = 60^\circ}$

Q 8. A particle whose speed is 50 m/s moves along a line from A(2,1) to B(9,25) find its velocity in rectangular form.

$u = 2\hat{i} + \hat{j}$

$v = 9\hat{i} + 25\hat{j}$

$v - u = 7\hat{i} + 24\hat{j}$

$\vec{v} = 7\hat{i} + 24\hat{j}$

$\therefore \hat{v} = \dfrac{7\hat{i} + 24\hat{j}}{\sqrt{(7)^2 + (24)^2}}$

$\hat{v} = \dfrac{7\hat{i} + 24\hat{j}}{25}$

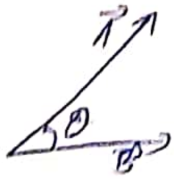
$\hat{v} \times 50 =$ velocity

$\boxed{\overrightarrow{\text{Velocity}} = 14\hat{i} + 48\hat{j} \text{ m/s}}$

Product of two vectors

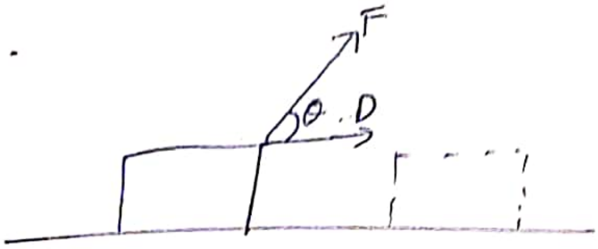
I) Scalar Product/dot product:-

$$\boxed{\begin{aligned}\vec{A}\cdot\vec{B} &= |\vec{A}||\vec{B}|\cos\theta \\ &= AB\cos\theta\end{aligned}}$$



→ The scalar product is a way to multiply two vectors to get a scalar result

eg.



$W = \vec{F}\cdot\vec{S}$

$W = F.S\cos\theta$

★ $\theta = 0$
$\cos 0^\circ = 1$
$\vec{A}\cdot\vec{B} = AB$ (max. value)

★ $\theta = 180$
$\cos\theta = -1$
$\vec{A}\cdot\vec{B} = -AB$ (negative non max value)

★ $\theta = 90$
$\cos\theta = 0$
$\vec{A}\cdot\vec{B} = 0$ (min value)

$\hat{i}\cdot\hat{i} = 1$	$\hat{i}\cdot\hat{j} = 0$
$\hat{j}\cdot\hat{j} = 1$	$\hat{i}\cdot\hat{k} = 0$
$\hat{k}\cdot\hat{k} = 1$	$\hat{j}\cdot\hat{k} = 0$

~~$\vec{A} = A_x \hat{i} + B_x \hat{i} +$~~

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k})(B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

* To find angle between two vectors

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\boxed{\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}}$$

* Commutative

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

* distributive

$\vec{A} (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

* $\vec{A} \cdot \vec{A} = (A)(A)\cos 0^\circ = A^2$

$|P+Q|^2 = (\vec{P} + \vec{Q})(\vec{P} + \vec{Q})$

$= \vec{P} \cdot \vec{P} + \vec{P} \cdot \vec{Q} + \vec{Q} \cdot \vec{P} + \vec{Q} \cdot \vec{Q}$

$= P^2 + Q^2 + 2PQ\cos\theta$

$|P+Q| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Q $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{B} = 5\hat{i} + 2\hat{j} - 7\hat{k}$

$\vec{A} \cdot \vec{B} = 2 \times 5 + (-3) \times 2 + -(7) \times 1$

$= 10 - 6 - 7$

$= 10 - 13$

$\boxed{= -3}$

Q2. $\vec{A} = \hat{i} + \hat{j} - \hat{k}$

$\vec{B} = \hat{i} - \hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = ?$

$\vec{A} \cdot \vec{B} = 1 - 1 - 1$

$= -2 + 1$

$\boxed{= -1}$

Q3. $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$

$\vec{B} = 3\hat{i} + 2\hat{j}$

$\vec{A} \cdot \vec{B} = 6 - 6 + 0$

$\boxed{= 0}$

Q Find value of n, such that $(2\hat{i} - 3\hat{j} + \hat{k})$ is perpendicular to $\hat{i} + 2\hat{j} + n\hat{k}$.

$$\cos\theta = \frac{2 - 6 + n}{\sqrt{4+9+1} \times \sqrt{1+4+n^2}}$$

$$\cos\theta = \frac{n-4}{\sqrt{14} \times \sqrt{5+n^2}}$$

$$\cos 90 = \frac{n-4}{\sqrt{70+14n^2}}$$

$n - 4 = 0$

$\boxed{n = 4}$

Q find angle between $(\hat{i}+\hat{j})$ & $(\hat{i}-\hat{j})$

$$\cos\theta = \frac{1-1}{AB}$$

$\boxed{\theta = 90^\circ}$ as $1-1=0$

Q find angle b/w $(\hat{i}+\hat{j})$ & $(\hat{j}+\hat{k})$

$$\cos\theta = \frac{0+1+0}{AB}$$

$$\cos\theta = \frac{1}{\sqrt{1+1}\times\sqrt{1+1}}$$

$$\cos\theta = \frac{1}{\sqrt{2}\times\sqrt{2}}$$

$$\cos\theta = \frac{1}{2}$$

$$\boxed{\theta = 60^\circ}$$

Q A force $\vec{F} = (10\hat{i} - 3\hat{j} + 6\hat{k})$ N acts on a body of mass 100 g & displaces it from $(6\hat{i} + 5\hat{j} - 3\hat{k})$ m to $(10\hat{i} - 2\hat{j} + 7\hat{k})$ m find work done.

displacement $= 4\hat{i} - 7\hat{j} + \overset{10}{\cancel{9}}\hat{k}$ m

work $= F.S$

$= (10\hat{i} - 3\hat{j} + 6\hat{k}).(4\hat{i} - 7\hat{j} + \overset{10}{\cancel{9}}\hat{k})$

$= 40 + 21 + \cancel{29}\,60$

$\boxed{\cancel{= 85\ J}}$

$\boxed{= 121\ J}$

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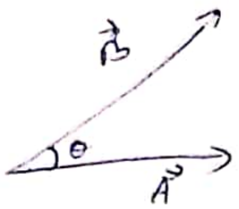
II Cross Product / Vector Product



$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta\ \hat{n}$ ← direction of $\vec{A} \times \vec{B}$ (Right hand thumb rule)

$\odot$ out of plane

$\otimes$ in plane

* not commutative

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = AB\sin\theta$$

* distributive

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

$\hat{i} \times \hat{i} = (1)(1)\sin 0° = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = (1)(1)\sin 90\ \hat{k}$

$\hat{i} \times \hat{j} = \hat{k}$

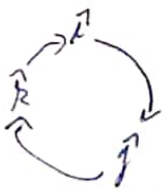
$\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$



clockwise – (+ve)

anticlockwise negative – (−ve)

* If cross product is 0, vectors are either equal or parallel
* If dot product is 0, vectors are ⊥ to each other

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$$\vec{A} \times \vec{B} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \end{array}$$ 3 by 3 determinant

$\cancel{\vec{A} \times \vec{B} = \hat{i}(A_y - B_z) - \hat{j}(A_x - B_z) + \hat{k}(A_x - B_y)}$

$$\boxed{\vec{A} \times \vec{B} = \hat{i}\,(A_y B_z - B_y A_z) - \hat{j}\,(A_x B_z - B_x A_z) + \hat{k}\,(A_x B_y - A_x A_y)}$$

Q $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$
$\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$\vec{A} \times \vec{B} = \hat{i}\,(4 - (-4)) - \hat{j}\,(12 - 4) + \hat{k}\,(-6 - 2)$

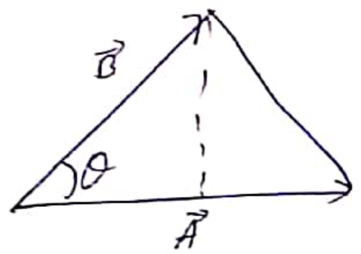
$$\boxed{= 8\hat{i} - 8\hat{j} - 8\hat{k}}$$

$|\vec{A} \times \vec{B}| = \sqrt{8^2 + (-8)^2 + (-8)^2}$

$$\boxed{= 8\sqrt{3}}$$

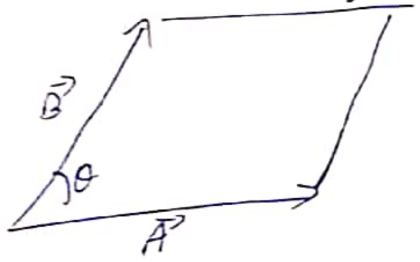
Uses of vector product

1) To Calculate area of Triangle :-



$$\text{area of triangle} = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times (A)(B \sin\theta)$$
$$= \frac{1}{2} |\vec{A} \times \vec{B}|$$

2) To calculate area of ∥gm :-



$$\text{area} = |\vec{A} \times \vec{B}|$$

Q1. The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{r} = 4\hat{j} - 3\hat{k}$ find $|\vec{v}|$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$$

$\cancel{\vec{v} = \hat{i}(6-8) - \hat{j}(-3 - 2) + \hat{k}(4+2)}$

$\cancel{= -2\hat{i} + 3\hat{j} + 6\hat{k}}$

$\cancel{|\vec{v}| = \sqrt{4 + 9 + 36}}$

$\cancel{|\vec{v}| = \sqrt{65}}$

$\cancel{= \sqrt{5 \times 13}}$

$\vec{v} = \hat{i}(6-8) - \hat{j}(-3-0) + \hat{k}(4)$

$= -2\hat{i} + 3\hat{j} + 4\hat{k}$

$|\vec{v}| = \sqrt{4+9+16}$

$\cancel{= \sqrt{29}}$

$\boxed{= \sqrt{29}}$

Q2. Calculate area of triangle determined by $\vec{A} = 3\hat{i}+4\hat{j}$
$\vec{B} = -3\hat{i}+7\hat{j}$

$$\cancel{B}\ \cancel{A}\ \vec{Ar} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -3 & 7 & 0 \end{vmatrix}$$

$$\vec{Ar} = \hat{i}(0) - \hat{j}(0) + \hat{k}(21+12)$$

$$= 33\hat{k}$$

$$|\vec{Ar}| = \sqrt{(33)^2}$$

$$= 33$$

$$\text{Area} = \frac{33}{2}$$

$$\boxed{= 16.5\ m^2}$$

Q3. $\vec{A} = 2\hat{i} + p\hat{j} + q\hat{k}$
$\vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

$$\sqrt{4+p^2+Q^2} = \sqrt{25+49+9}$$

~~$\cos\theta = 10\hat{i} + 7p\hat{j} + 3q\hat{k}$~~

$$p^2+Q^2+4 = 83$$

$$p^2+Q^2 = 79$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & p & Q \\ 5 & 7 & 3 \end{vmatrix}$$

$$= \hat{i}(3p-7Q) - \hat{j}(6-5Q) + \hat{k}(14-5p)$$

$6 - 5Q = 0$

$\boxed{Q = \frac{6}{5}}$

$14 - 5p = 0$

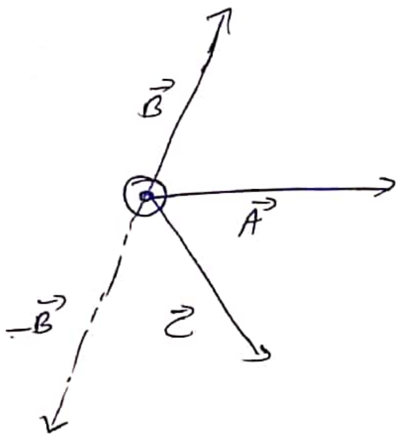
$\boxed{\cancel{p = B}}$

$\boxed{p = \frac{14}{5}}$

Q3. What is the angle between $\vec{A} - \vec{B}$ and $\vec{A} \times \vec{B}$

$$\cancel{\cos\theta = \frac{(\vec{A} \times \vec{B}) \cdot (\vec{A} - \vec{B})}{|\vec{A} - \vec{B}|\,|\vec{A} \times \vec{B}|}}$$

$$\cancel{= AB \sin\theta \, \hat{n} \cdot}$$



$\boxed{\theta = 90^\circ}$

Q4. $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

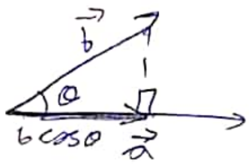
$$\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$$

$$4AB\cos\theta = 0$$
$$\theta = 90^\circ$$

Projection of vector over another:-

I) Scalar projection of $\vec{b}$ on $\vec{a}$ $= b\cos\theta\,\frac{a}{a} = \frac{ab\cos\theta}{a} = \frac{\vec{a}\cdot\vec{b}}{a}$

जिसपर projection ले रहे हैं।



II) vector projection $= \frac{\vec{a}\cdot\vec{b}}{a}\,\hat{a} = \frac{\vec{a}\cdot\vec{b}}{a}\cdot\frac{\vec{a}}{a} = \frac{\vec{a}}{a^2}(\vec{a}\cdot\vec{b})$

जिसपर projection लेना है।

Q find scalar & vector projection of $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$ on $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$.

$$\frac{\vec{a}\cdot\vec{b}}{b} = \frac{2+6+2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$$

vector $\frac{\vec{b}}{b}(\vec{a}\cdot\vec{b}) = \frac{5}{3}\hat{i}+\frac{10}{3}\hat{j}+\frac{5}{3}\hat{k}$

① find scalar ~~product~~ of $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on $\vec{B} = \hat{i} + \hat{j}$ projection and vector projection

$$\frac{\vec{A} \cdot \vec{B}}{b} = \frac{1+1+0}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}}$$

V.P

$$\frac{\vec{B}}{b} \cdot (\vec{a} \cdot \vec{B}) = \frac{\hat{i} + \hat{j}}{\cancel{2}} \times \cancel{2} = \boxed{\hat{i} + \hat{j}}$$

~~Q4.~~ $\vec{A} + \vec{B} = 2\hat{i}$

$\vec{A} - \vec{B} = 4\hat{j}$

$- A^2 + B^2 + 2AB\cos\theta = ~~A^2 + B^2 + 2AB\cos\theta~~ \; 2\hat{i}$

$A^2 + B^2 - 2AB\cos\theta = 4\hat{j}$

$-4AB\cos\theta = 2\hat{j}$

$\cos\theta = \frac{-1}{2AB}$

~~cos (90+~~

$\vec{A} + \vec{B} = 2\hat{i}$

$\vec{A} - \vec{B} = 4\hat{j}$

θ

$\cos\theta = 90°$

Q1. $\vec{A} + \vec{B} = 2\hat{i}$ A) $127°$
$\vec{A} - \vec{B} = 4\hat{j}$ B) $143°$
$\theta = ?$ C) $53°$
D) $37°$

$\vec{A} + \vec{B} = 2\hat{i}$
$\vec{A} - \vec{B} = 4\hat{j}$

$2\vec{A} = 2\hat{i} + 4\hat{j}$
$\vec{A} = \hat{i} + 2\hat{j}$
$\vec{B} = \hat{i} - 2\hat{j}$

$\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|AB|}$

$= \dfrac{1-4}{\sqrt{5} \times \sqrt{5}}$

$\cos\theta = \dfrac{-3}{5}$

$\cos(90+\theta) = -\sin\theta$

$-\sin\theta = -\dfrac{3}{5}$

$\sin 37° = \dfrac{3}{5}$

$90 + 37 = 127°$

$\boxed{\text{A) } 127°}$

56

Q2. two forces P & Q are in ratio 1:2 if their resultant is at an angle $\tan\alpha = \frac{\sqrt{3}}{2}$ to vector P Then angle between P & Q is.

A) $\tan^{-1}\left(\frac{1}{2}\right)$ B) 45° C) 30° D) 60°

$$\tan\alpha = \frac{\sqrt{3}}{2} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

$$\frac{P}{Q} = \frac{1}{2}$$

$$2P = Q$$

$$Q = 2P$$

$$\sqrt{3}P + \sqrt{3}Q\cos\theta = 2Q\sin\theta$$

$$\sqrt{3}P = 2Q\sin\theta - \sqrt{3}Q\cos\theta$$

$$\sqrt{3}(P+Q\cos\theta) = 2Q\sin\theta$$

$$\sqrt{3}(P+2P\cos\theta) = 4P\sin\theta$$

$$\sqrt{3}P(1+2\cos\theta) = 4P\sin\theta$$

$$\frac{1+2\cos\theta}{\sin\theta} = \frac{4\cancel{P}}{\sqrt{3}\cancel{P}}$$

$$\frac{1}{\sin\theta} + \frac{2\cos\theta}{\sin\theta} = \frac{4}{\sqrt{3}}$$

$$\frac{1}{\sin\theta} + 2\cot\theta = \frac{4}{\sqrt{3}}$$

$$\boxed{\sqrt{3} + 2\sqrt{3}\cos\theta = 4\sin\theta}$$

$$\sqrt{3} + \cancel{2}\sqrt{3} \times \frac{\sqrt{3}}{\cancel{2}} = \cancel{4}^{2} \times \frac{1}{\cancel{2}}$$

$$\sqrt{3} + 3 = 2 \times$$

$$\cancel{\sqrt{3}} + \cancel{2}\cancel{\sqrt{3}} \times \frac{1}{\cancel{2}} = \cancel{4}^{2} \times \frac{\cancel{\sqrt{3}}}{\cancel{2}}$$

$$1 + 1 = 2$$

LHS = RHS

$$\boxed{\theta = 60^\circ}$$

$$\boxed{D)\ 60}$$

Q3. $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$

find the angle between $(\vec{A} - \vec{B})$ & $\vec{C}$

A) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ B) $\cos\theta = \frac{\sqrt{3}}{2}$ C) $\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

D) none.

$\vec{A} - \vec{B} = -1\hat{i} + 5\hat{j} + \hat{k}$

$\vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$

$\frac{-p + 5p + 2p}{3\sqrt{3} \times p\sqrt{6}}$

$\frac{6p}{3p\sqrt{18}} \quad \frac{2}{\sqrt{18}} = \frac{2}{3\sqrt{2}}$

$\cos\theta = \frac{2}{3\sqrt{2}}$

$= \frac{\sqrt{2}}{3}$

$\boxed{C}$

Q4 Two forces $\hat{i} + \hat{j} + \hat{k}$ N & $\hat{i} + 2\hat{j} + 3\hat{k}$ N acts on a particle and displace it from $(2, 3, 4)$ to point $(5, 4, 3)$.

$\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{S} = 3\hat{i} + \hat{j} - 1\hat{k}$

A) 5 J

B) 4 J

C) 3 J

D) None

$W = 6 + 3 - 4$

$W = 5\ J$

$\boxed{A}$

Q5. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$

$\vec{D} = -4\hat{i} + 2\hat{j} + 3\hat{k}$

work $= 6$ J

A) 12 B) 0 C) 6 D) 1

Work $= -12 + 2c + 6$

$6 = -6 + 2c$

$12 = 2c$

$\boxed{c = 6}$

$\boxed{C}$

Q6. A force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$

$a = 1\ m/s^2$

find mass

$F = ma$

A) $6\hat{i} - 8\hat{j} + 10\hat{k}$

B) 100

C) $10\sqrt{2}$

D) 10

$6\hat{i} - 8\hat{j} + 10\hat{k} = m \times 1$

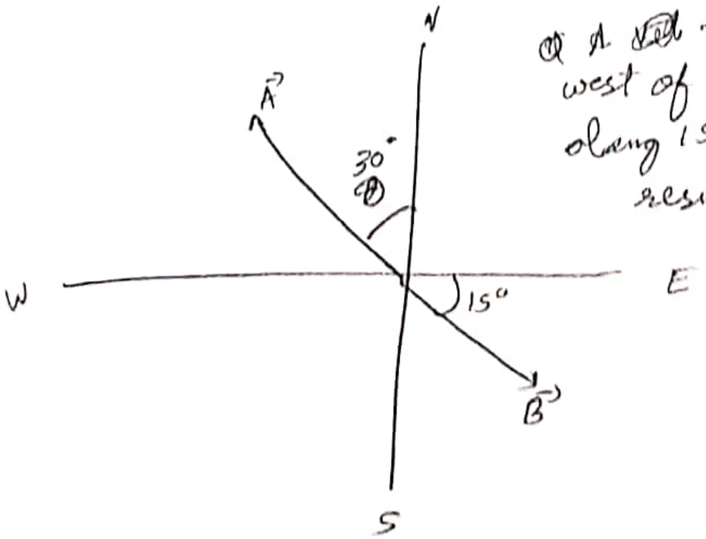
$m = \sqrt{36 + 64 + 100}$

$= \sqrt{200}$

$= 10\sqrt{2}$ kg

$\boxed{C}$

Q7.



Q A vector $\vec{A}$ is directed 30° west of north & another vector $\vec{B}$ along 15° south of east. There resultant cannot be in

$\boxed{\text{South}}$

Q 8. The sum of $\vec{F_1} = 100N$, $\vec{F_2} = 80N$ & $\vec{F_3} = 60N$ is 0, angle between F_1 & F_2 is

A) 53
B) 143
C) 37
D) 127

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

$$\cancel{\sqrt{10000 + 6400 + 3600}}$$

$$\sqrt{10000 + 6400 + 16000\cos\theta} = 0$$

$$= \cancel{\sqrt{32400 \cos}}$$

$$= \sqrt{16400 + 16000\cos\theta}$$

$$= 100\sqrt{164 + 160\cos\theta}$$

$$= 20\sqrt{41 + 40\cos\theta} = -60$$

$$= \sqrt{41 + 40\cos\theta} = -3$$

$$\cancel{900 \times (41 + 40\cos\theta) + 3600 +}$$

$$41 + 40\cos\theta = 9$$

$$\cos\theta = \frac{9 - 41}{40}$$

$$\cos\theta = \frac{-32}{40}$$

$$\cos\theta = \frac{-4}{5}$$

$$\cos(90 + 53^\circ) = \frac{-4}{5}$$

$$\theta = 90 + 53$$
$$= 143^\circ$$

$$\boxed{B}$$

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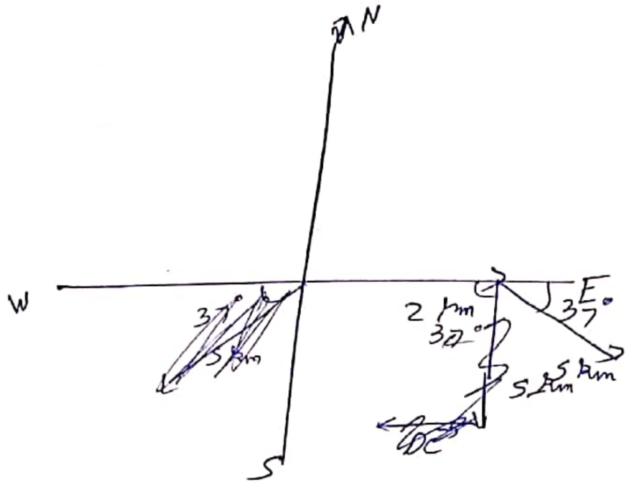
Q9. A sail boat sails 2 km east, 5 km 37° south of East and an unknown displacement. The final displacement is 6 km east, find unknown.



$\vec{A} = 2\hat{i}$

$\vec{B} = +5\times\frac{4}{5}\hat{i} - \frac{3}{5}\times 5\hat{j}$

$= -4\hat{i} - 5\hat{j}$

$\vec{C} = ?$

$\vec{A}+\vec{B}+\vec{C} = 6\hat{i}$

$\cancel{2\hat{i}+4\hat{i}} - 3\hat{j} = 6\hat{i}$

$= \cancel{6\hat{i}+2\hat{i}} + 3\hat{j}$

$\boxed{\cancel{\quad + 3\hat{j}}}$

$\boxed{= 3\hat{j}}$

Calculaus (Differentation & Integration)

Differentiation – Differential calculaus is used to study the nature (increase or decrease) and the amount of variation in a quantity when another quantity (on which first quantity depends) changes independently.

$\frac{dy}{dx}$ (y किस तरह change ho raha hai with respect o to x)

↳ is the instatanious rate of change of function y with respect to variable x.

$a = \frac{dv}{dt} = \frac{d}{dt}(v)$ $\qquad$ $\frac{dy}{dx} = y'$

$v = \frac{ds}{dt} = \frac{d}{dt}(s)$

~~formulas~~ derivatives of some imp functions.

① $\frac{d}{dx}(x^n) = nx^{n-1}$

② $\frac{d}{dx}(e^x) = e^x$

③ $\frac{d}{dx}(\text{constant}) = 0$ (does not change)

④ $\frac{d}{dx}(\sin x) = \cos x$

⑤ $\frac{d}{dx}(\cos x) = -\sin x$

⑥ $\frac{d}{dx}(\tan x) = \sec^2 x$

⑦ $\frac{d}{dx}(\sec x) = \sec x \tan x$

natural log

⑧ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

⑨ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

⑩ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

⑪ $\frac{d}{dx}(ax^n) = a\frac{d}{dx}(x^n) = anx^{n-1}$

⑫ $\frac{d}{dx}(a^x) = a^x \ln a$

↳ constant

Q1. Differentiate following w.r.t respect to x

① $y = 3x^2 - 5x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}(y) = \frac{d}{dx}(3x^2 - 5x + 1)$$

$$= \frac{d}{dx}(3x^2) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

$$= 3\frac{d}{dx}(x^2) - 5\frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$= 3 \times 2 \times x^{2-1} - 5 \times 1 \times x^{1-1} + 0$$

$$\boxed{= 6x - 5}$$

② . $y = ax^2 + bx + c$

$\frac{d}{dx}(y)$

$= \frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)$

$= a\frac{d}{dx}(x^2) + b\frac{d}{dx}(x) + 0$

$= a \times 2 \times x + b \times 1 + 0$

$\boxed{= 2ax + b}$

③ $y = {}^1\!/_2$

$\frac{d}{dx}(x^{1/2})$

$= {}^1\!/_2 \times x^{0}$

$= \frac{1}{2} \times \frac{1}{\sqrt{x}}$

$\boxed{= \frac{1}{2\sqrt{x}}}$

④ $y = \sqrt[3]{x}$

$\frac{d}{dx}(x^{1/3})$

$\frac{1}{3} \times \cancel{2} \times x^{-2/3}$

$\boxed{= \frac{1}{3\sqrt[3]{x^2}}}$

$1 - \frac{1}{3}$

$\frac{3-1}{3} = \frac{2}{3}$

$\frac{1}{3} - 1$

$\frac{1-3}{3} = \frac{-2}{3}$

⑤ $y = 2\sqrt{x} - \frac{1}{x} + \sqrt[4]{3}$

$\frac{d}{dx}(y) =$

$= \frac{d}{dx}(2\sqrt{x}) - \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(\sqrt[4]{3})$

$= 2\frac{d}{dx}(x^{+1/2}) - \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(3^{1/4})$

$= \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{x}} - (-1) \times \frac{1}{x^2} + \frac{1}{4} \times \frac{1}{3^{3/4}}$ $\qquad \frac{1}{4} - 4 = -\frac{3}{4}$

$= \frac{1}{\sqrt{x}} + \frac{1}{x^2} + \frac{1}{4\sqrt[4]{3^3}}$

$= \frac{d}{dx}(2\sqrt{x}) - \frac{d}{dx}(1/x) + \cancel{\frac{d}{dx}\sqrt[4]{3}}\ 0$ $\qquad \frac{1}{4} - 1 = -\frac{3}{4}$

$= \cancel{2} \times \frac{1}{\cancel{2}} \times \frac{1}{\sqrt{x}} - (-1) \times \frac{1}{x^2} + \cancel{\frac{1}{4} \times \frac{1}{\sqrt[4]{27}}}\ 0$

$= \frac{1}{\sqrt{x}} + \frac{1}{x^2} + 0$

$\boxed{= \frac{1}{\sqrt{x}} + \frac{1}{x^2}}$

Product Rule

$y = A \cdot B$

(A & B are functions of x)

$$\frac{dy}{dx} = A\frac{d}{dx}(B) + B\frac{d}{dx}(A)$$

eg. $y = 2\sin x \cos x$

$$\frac{d}{dx}(2\sin x \cos x$$

$$= 2\frac{d}{dx}(\sin x \cos x)$$

$$= 2\left[\sin x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(\sin x)\right]$$

$$= 2\left[\sin x(-\sin x) + \cos x \, e(\cos x)\right]$$

$$\boxed{= 2\left[\cos^2 x - \sin^2 x\right]}$$

Quotient Rule

$y = \frac{A}{B}$ (A & B are functions of x)

$$\frac{dy}{dx} = \frac{B \cdot \frac{d}{dx}(A) - A \cdot \frac{d}{dx}(B)}{(B)^2}$$

eg. $y = \frac{\sin x}{\cos x}$

$$\frac{d}{dx}(y) = \frac{\cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{(\cos x)^2}$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\boxed{= \sec^2 x}$$

Q6. $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$

$\frac{d}{dx}(y)$

$\frac{d}{dx}\left[(x^2 - 3x + 3)(x^2 + 2x - 1)\right]$

$(x^2 + 2x - 1)\frac{d}{dx}(x^2 - 3x + 3) + (x^2 - 3x + 3)\frac{d}{dx}(x^2 + 2x - 1)$

$(x^2 + 2x - 1)(2x - 3 + 0) + (x^2 - 3x + 3)(2x + 2 + 0)$

$(x^2 - 2x - 1)(2x - 3) + (x^2 - 3x + 3)(2x - 2)$

$(2x^3 - 2x^2 - 2x^2 + 6x - 2x + 3) + (2x^3 - 2x^2 - 6x^2 + 6x + 6x - 6)$

$(2x^3 - 4x^2 + 4x + 3) + (2x^3 - 8x^2 + 12x - 6)$

$\boxed{4x^3 - 12x^2 + 16x - 3}$

Q7. $y = (x^3 - 3x + 2)(x^4 + x^2 - 1)$

$\frac{d}{dx}\left[(x^2 - 3x + 2)\ \cancel{x^4 \quad 2x}\ (x^4 + x^2 - 1)\right]$

$(x^2 - 3x + 2)\frac{d}{dx}(x^4 + x^2 - 1) + (x^4 + x^2 - 1)\frac{d}{dx}(x^2 - 3x + 2)$

$(x^2 - 3x + 2)(4x^3 +$

Q6. $y = (x^2 - 3x + 3)(x^2 + 2x - 1)$

Q7. $y = (x^3 - 3x + 2)(x^4 + x^2 - 1)$

Q8. $y = (\sqrt{x} + 1)\left[\frac{1}{\sqrt{x}} - 1\right]$

Q9. $y = (x^2 - 1)(x^2 - 4)$

Q10. $y = \frac{x + 1}{x - 1}$

Q 6. $(x^2 - 3x + 3)(2x + 2) + (x^2 + 2x - 1)(2x - 3)$

Q 7. $(x^4 + x^2 - 1)(3x^2 - 3) + (x^3 - 3x + 2)(2x + 2)$

Q 8. $\frac{d}{dx}\left[(\sqrt{x} + 1)\left(\frac{1}{\sqrt{x}} - 1\right)\right]$

$$\left(\frac{1}{\sqrt{x}} - 1\right)\frac{d}{dx}(\sqrt{x} + 1) + (\sqrt{x} + 1)\frac{d}{dx}\left(\frac{1}{\sqrt{x}} - 1\right)$$

$$\left[\left(\frac{1}{\sqrt{x}} + 1\right)\frac{1}{2\sqrt{x}} + (\sqrt{x} \mp 1) \cdot \frac{1}{2\sqrt{x^3}}\right]$$

Q9. $\frac{d}{dx}\left((x^2 - 1)(x^2 - 4)\right)$

$= 2x(2x^2 - 5)$

Q10. $\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$

$$\frac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$\frac{(x-1)(1) - (x+1)(1)}{x^2+1-2x}$$

$$\boxed{\frac{(x-1)-(x+1)}{(x-1)^2}} = \cancel{\frac{-2}{(x-1)^2}} \quad \boxed{\frac{-2}{(x-1)^2}}$$

Q11. $y = \frac{x}{x^2+1}$

Q12. $y = \frac{3x^2+1}{x-1}$

Q13. $y = \frac{2x}{x^3-1}$

Q14. $y = x \sin x$

Q15. $y = \frac{x}{\sin x}$

Q16. $y = x^2 \tan x$

Q17. $y = \frac{x^2}{\sec x}$

Q18. $y = x \ln x$

Q19. $y = xe^x$

Q20. $y = \frac{x}{e^x}$

Q11. $\dfrac{dy}{dx} = \dfrac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$ -

$= \dfrac{x^2+1-2x^2}{\cancel{(x^2+1)^2}\,(x^2+1)^2}$

$= \boxed{\dfrac{1-x^2}{(x^2+1)^2}}$

Q12. $\dfrac{dy}{dx} = \boxed{\dfrac{(x-1)(6x) - (3x^2+1)(1)}{(x-1)^2}}$

Q13. $\dfrac{d}{dx}\left(\dfrac{2x}{x^3-1}\right)$

$= \boxed{\dfrac{(x^3-1)(2) - (2x)(3x^2)}{(x^3-1)^2}}$

Q14. $\dfrac{d}{dx}(x \sin x)$

$= \sin x\,(1) + (x)(\cos x)$

$\boxed{= \sin x + x \cos x}$

Q15. $\dfrac{d}{dx}\left(\dfrac{x}{\sin x}\right)$

$= \dfrac{\sin x\,(1) - (x)(\cos x)}{(\sin^2 x)^2}$

$\boxed{= \dfrac{\sin x - x\cos x}{\sin^2 x}}$

Q16 $\frac{d}{dx}(x^2 \tan x)$

$= \tan x(2x) + (x^2)(\sec^2 x)$

$\boxed{= 2x \tan x + x^2 \sec^2 x}$

Q17. $\frac{d}{dx}\left(\frac{x^2}{\sec x}\right)$

$= \frac{\sec x(2x) - (x^2)(\tan x \sec x)}{(\sec x)^2}$

$= \frac{2x \sec x - x^2 \sec x \tan x}{\sec^2 x}$

$= \frac{x \sec x(2 - x \tan x)}{\sec^2 x}$

$\boxed{= \frac{x(2 - x\tan x)}{\sec x}}$

Q 18. $\frac{d}{dx}(x \ln x)$

$= (\ln x)(1) + (x)\left(\frac{1}{x}\right)$

$\boxed{= \ln x + 1}$

Q19. $\frac{d}{dx}(xe^x)$

$= (e^x)(1) + (x)(e^x)$

$= e^x + xe^x$

$\boxed{= e^x(1+x)}$

Q20. $\frac{d}{dx}\left(\frac{x}{e^x}\right)$

$= \frac{e^x(1) - (x)(e^x)}{(e^x)^2}$

$= \frac{e^x - xe^x}{e^{2x}}$

$= \frac{e^x(1-x)}{e^{2x}}$

$\boxed{= \frac{1-x}{e^x}}$

Chain rule

eg 1. $y = \sin(x^2-4)$

let $t = x^2-4$

let $y = \sin t$

$$\frac{d\cancel{x}y}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dt} = \frac{d}{dt}(y) = \frac{d}{dt}(\sin t) = \cancel{\cot}\cos t$$

$$\frac{dt}{dx} = \frac{d}{dx}(t) = \frac{d}{dx}(x^2-4) = 2x$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \cancel{\frac{dt}{}}\frac{dt}{dx} = 2x\,\cancel{\cot}\cos t$$

$$\boxed{= 2x\cos[x^2-4]}$$

eg 2. $y = \sin^2 x = (\sin x)^2$

$t = \sin x$

$y = t^2$

$$\frac{dy}{dt} = 2t$$

$$\frac{dt}{dx} = \cos x$$

$$\frac{dy}{dx} = \cos x \times 2t$$

$$= \cos x \times 2\sin x$$

$$\boxed{= 2\sin x\cos x}$$

eg3. $y = \sin 2x$

$t = 2x$

$y = \sin t$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 2$$

$$\frac{dy}{dx} = 2 \cos t$$

$$\boxed{= 2 \times \cos(2x)}$$

$=$

eg4. $y = \sin x^3$

$t = x^3$

$y = \sin t$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 3x^2$$

$$\frac{dy}{dx} = 3x^2 \times \cot t$$

$$= 3x^2 \times \cos(x^3)$$

$$\boxed{= 3x^2 \cos(x^3)}$$

Q ~~0~~1. $y = \sin 3x$

Q 2. $y = 3 \sin(3x+5)$

Q 3. $y = \sin(\sin x)$

Q 4. $y = \ln^2 x = [\ln x]^2$

Q 5. $y = \frac{1}{3^x}$

Pg - 41, 42

Q1 - Q ~~29~~ 39

Q 1. $y = \sin 3x$

let $t = 3x$

$y = \cancel{t} \sin t$

$\frac{dy}{dt} = \frac{d}{dt}(\sin t) = \cos t$

$\frac{dt}{dx} = \frac{d}{dx}(3x) = 3$

$\frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dx} = \cancel{3\cos t}\ \boxed{3\cos t} = 3\cos(3x)$

Q 2. $y = 3\sin(3x+5)$

let $t = 3x+5$

$y = 3\sin t$

$\frac{dy}{dt} = 3\cos t$

$\frac{dt}{dx} = 3$

$\frac{dy}{dx} = 3 \times 3 \cos t$

$\boxed{= 9\cos t}$

$= 9\cos(3x+5)$

Q 3. $y = \sin(\sin x)$

$t = \sin x$

$y = \sin t$

$\frac{dy}{dt} = \cos t$

$\frac{dt}{dx} = \cos x$

$\frac{dy}{dx} = \cos x \cos t$

$\boxed{= \cos x \cos(\sin x)}$

Q4. $y = (\ln x)^2$

$t = \ln x$

$y = t^2$

$\frac{dy}{dt} = 2t$

$\frac{dt}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{1}{x} \times 2t$

$\boxed{\cancel{= \frac{(\ln x)^2}{x}}}$

$= \boxed{\frac{2 \ln x}{x}}$

Q 5 $y = \frac{1}{3^x} = 3^{-x}$

let $t = -x$

$y = 3^t$

$\frac{dy}{dt} = 3^t \ln 3$

$\frac{dt}{dx} = 1$

$\frac{dy}{dx} = 3^t \ln 3$

$\boxed{= -3^{-x} \ln 3}$

Q Differentiate the following

① $y = \frac{1}{4} \tan^4 x$

$\cancel{\frac{d}{dx} = \frac{1}{4} \times \sec^2}$

let $t = \tan x$

$y = \frac{1}{4} t^4$

$\frac{dy}{dt} = \frac{1}{4} \times 4t^3$

$= t^3$

$\frac{dt}{dx} = \sec^2 x$

$\frac{dy}{dx} = t^3 \sec^2 x$

$= \boxed{\tan^3 x \sec^2 x}$

=

(2) $y = \cot\sqrt{1+x^2}$

$t = \sqrt{1+x^2}$

$= (1+x^2)^{1/2}$

~~$s = 1+x^2$~~ ~~$t = 1 + x + 2x^2$~~

$S = 1 + x^2$

$t = s^2$

$y = \cot t$

$\frac{dy}{dt} = -\operatorname{cosec} t \cot t$

~~$\frac{ds}{dx}$~~

(3) $y = \sin^2 3x$

$t = 3x$

$y = \sin^2 t$

$\frac{dy}{dt} = \cancel{B}\ \sin t \cos t + \sin t \cos t$

$= 2 \sin t \cos t$

$\frac{dt}{dx} = 3$

$\frac{dy}{dx} = 6 \sin t\ \cancel{\cot}\ \cos t$

$\boxed{= 6 \sin 3x \cos 3x}$

Q 2) $y = \cot\sqrt{1+x^2}$

$$y = \cot[1+x^2]^{1/2}$$

$$t = [1+x^2]^{1/2}$$

$$y = \cot t$$

$$\frac{dy}{dt} = -\operatorname{cosec}^2 t$$

$$z = 1+x^2$$

$$t = z^{1/2}$$

$$\frac{dt}{dz} = \frac{1}{2} z^{-1/2}$$

$$\frac{dz}{dx} = 2x$$

$$\frac{dy}{dx} = \boxed{-\operatorname{cosec}^2(1+x^2)^{1/2} \times \frac{1}{\cancel{2}}(1+x^2)^{-1/2} \times \cancel{2}x}$$

4) ~~$y = \cos^3(4x)$~~

~~$t = 4x$~~

~~$y = (\cos^3 t)^3$~~

~~$z = \cos t$~~

~~$y = z^3$~~

~~$\frac{dy}{dz}$~~

(4) $y = \cos^3(4x)$

$$\frac{dy}{dx} = 3(\cos 4x)^2 \times -\sin 4x \times 4$$

$$\boxed{\cancel{= 3\cos^2 4x - \sin 4x \times 4}}$$

$$\boxed{= -12\cos^2 4x \sin 4x}$$

(5) $y = (1+\sin^2 x)^4$

$$= 4(1+\sin^2 x)^3 \times (2\sin x) \times (\cos x)$$

$$\boxed{= 8(1+\sin^2 x)^3 \sin x \cos x}$$

(6) $y = (3x + \tan^2 x)^2$

$$\frac{dy}{dx} = 2(3x + \tan^2 x) \times (3 + 2\tan x \times \sec^2 x)$$

$$= 2(3x+\tan^2 x) \times (3 + 2\tan x \sec^2 x)$$

$$\frac{dy}{dx} = 2(3x + \tan^2 x) \times \frac{d}{dx}(3x + \tan^2 x$$

$$= 2(3x + \tan^2 x) \times (3 + \frac{d}{dx}(\tan^2 x))$$

$$= 2(3x + \tan^2 x) \times (3 + 2\tan x \times \frac{d}{dx}(\tan x))$$

$$\boxed{= 2(3x + \tan^2 x) \times (3 + 2\tan x \sec^2 x)}$$

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Double differentiation

$$y = f(x)$$

$$\frac{dy}{dx} = y' \text{ (first derivative)}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = y'' \text{ (double differentiation)}$$

↳ (Leibniz notation)

eg $y = 3x^2+5$

$y' = 6x$

$y'' = 6$

Displacement (s)

velocity (v) = rate of change of position $= \frac{\Delta s}{\Delta t}$

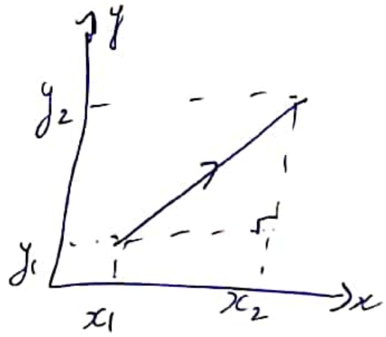
$$= \frac{ds}{dt}.$$

acceleration (a) = rate of change of velocity $= \frac{\Delta v}{\Delta t}$

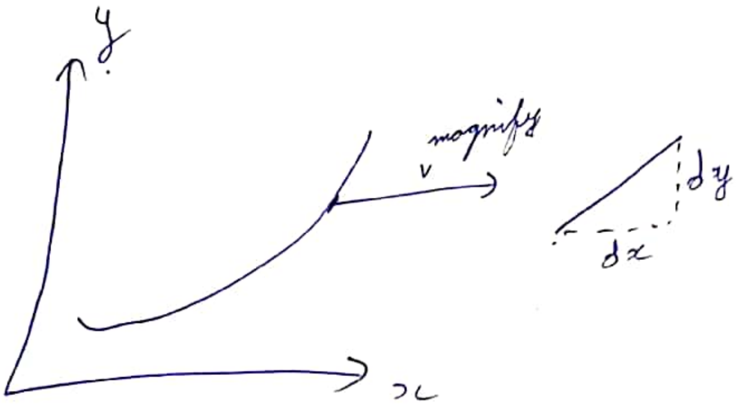
$$= \frac{d}{d\cancel{x}t}\left(\frac{ds}{dt}\right)$$

$$= \frac{d^2s}{\cancel{dt}\,dt^2}$$

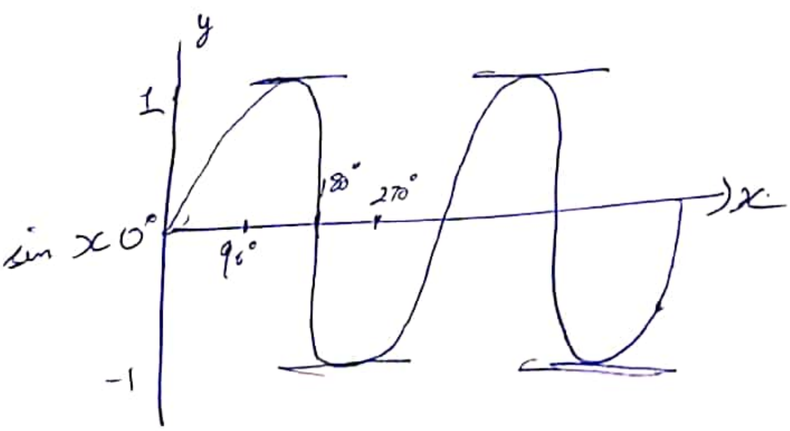
Slope



$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



$$\text{slope} = \frac{dy}{dx}$$



$y = \sin x$

at topmost (maxima) and bottommost (minima), slope = 0

$$\boxed{\frac{dy}{dx} = 0}$$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x$$

$\cos x = 0$

$x = 90°, 270°$

Curvature ⇒ Double Differentiation

maxima ⇒ slope decrease ⇒ $\frac{d^2y}{dx^2} < 0$

minima ⇒ slope increase ⇒ $\frac{d^2y}{dx^2} > 0$

For maxima, $\frac{dt}{dx} = 0$, $\frac{d^2y}{dx^2} < 0$

For minima, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} > 0$

Q $y = x^3 - 3x^2 + 6$

$$\frac{dy}{dx} = 3x^2 - \cancel{3x}\ 6x$$

$$\frac{dy}{dx} = 0$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$\boxed{\begin{matrix} x = 0 \\ x = 2 \end{matrix}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 6x)$$

$$= 6x - 6$$

at $x = 0$

$$\frac{d^2y}{dx^2} = 0 - 6$$
$$= -6$$

at $x = 2$

$$\frac{d^2y}{dx^2} = 12 - 6$$
$$= 6$$

$$\boxed{\begin{matrix} \text{Maxima} = a(x=0) \\ \text{Minim} = (x=2) \end{matrix}}$$

$= 6$

$= 8 - 12 + 6$

$= 2$

② $y = 3x^4 + 4x^3 - 12x^2 + 12$

find the points of maxima & minima.

$\frac{d}{dx} = 12x^3 + 12x^2 - 24x$

$\cancel{0 =}$

$24x = 12x^3 + 12x^2$

$2 = x^2 + x$

$\cancel{2 = x(x+1)}$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2$

$x(x+2) - 1(x+2)$

$(x-1)(x+2)$

$x = 1 \qquad x = 0$

$x = -2$

$\frac{d^2}{dx^2} = 36x^2 + 24x - 24$

$= 36 + 24 - 24$

$= 36$ (minima, $x = 1$)

$= 36(-2)^2 + 24(-2) - 24$

$= 36 \times 4 = 48 - 24$

$= 144 - 48 - 24$

$= 72$ (minima)

$= 0 + 0 - 24$

$= 0 - 24$ (maxima)

$y_{max} = 12$

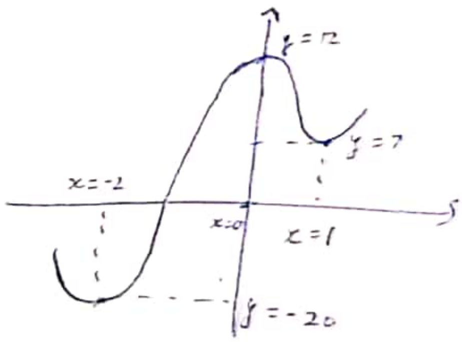
$y_{min} = 3 + 4 - 12 + 12$

$= 7$

$y_{min} = 16 \times 3 - 4 \times 8$

$= -20$

$\boxed{\begin{array}{l} \text{minimum} = -20 \\ \text{maximum} = 12 \end{array}}$



(3) find minimum value of $5x^2-2x+1$

$$\frac{dy}{dx} = 10x-2 = 0$$

$\cancel{0 = 10x}$

$2 = 10x$

$x = \frac{1}{5}$

$$\frac{d^2y}{dx^2} = 10 \text{ (minima)}$$

value $= 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right)+1$

$= \frac{\cancel{5}}{\cancel{25}\,5} - \frac{2}{5}+1$

$= \frac{1-2+5}{5}$

$\boxed{= \frac{4}{5} \quad (3)} = 0.8$

(4) find turning points of function $4x^3+12x^2+12x+10$

$\frac{dy}{dx} = 12x^2+24x+12$

$= 12x^2+24x+12$

$= x^2+2x+1$

$0 = x^2+x+x+1$

$= x(x+1)+1(x+1)$

$\boxed{= x=-1}$

$y = 12 - 24+12$

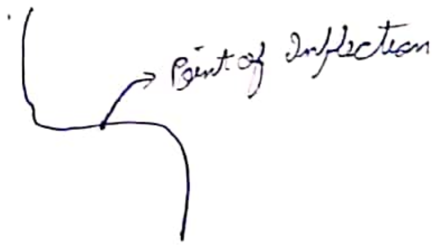
$= 0$

$(-1, 0)$

$\frac{d^2y}{dx^2} = 24x+24$

$= -24+24$

$= 0 \rightarrow$ Point of Inflection, no minima or maxima



Application of ~~Derivati~~ Derivatives

Q The radius of circle is increasing at the rate of 0.7 cm per second. What is the rate of Increase in its circumference.

$\frac{dr}{dt} = 0.7$ cm/s

$C = 2\pi r$

$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$

$= 2 \times 0.7 \times \pi$

$= 1.4\pi$ m/s

Q②. The side of square is increasing at the rate of 0.2 cm/s find rate of increase in its perimeter.

$P = 4s$

$\frac{ds}{dt} = 0.2$ cm/s

$\frac{dP}{dt} = 4 \times \frac{ds}{dt}$

$\boxed{= 0.8 \text{ cm/s}}$

Q③. The radius of a spherical balloon is decreasing at the rate of 10 cm/s. At what rate is the surface area of the balloon decreasing when its radius is 15 cm.

$$\frac{dr}{dt} = -10 \text{ cm/s}$$

area $= \cancel{R\pi}\ 4\pi r^2$

$$\cancel{\frac{da}{dt} = 4\pi(-10)^2}$$
$$\cancel{= 4\pi\ -100}$$
$$\cancel{= -400\pi \text{ cm}^2/s}$$

$$\frac{dA}{dt} = \frac{d}{dt}(4\pi r^2)$$

$$= \cancel{\frac{d}{d}}\ 4\pi \frac{d}{dt}(r^2)$$

$$= 4\pi\ 2r \times \cancel{\frac{d}{dt}}\left(\frac{dr}{dt}\right)$$

$$= 4\pi \times 2r \times -10$$

$$= -80\cancel{0}\ \pi r$$

$r = 15$ (given)

$$= -80\cancel{0} \times 15\pi$$

$$\boxed{= -1200\pi \text{ cm}^2/s}$$

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Q4. An Edge of a variable cube ~~Of~~ ~~a va~~ is increasing at a ~~ro~~ rate of 3 cm/s. How fast is its volume increasing when the edge is 10cm long.

$$\frac{de}{dt} = 3\ cm/s$$

$$v = e^3$$

$$\frac{dv}{dt} = \frac{d}{dt}(e^3) = 3e^2 \times 3$$

$$e = 10$$

$$= 3(10)^2 \times 3$$

$$= 9 \times 100$$

$$\boxed{= 900\ cm^3/s}$$

Q5. The volume of of a spherical balloon is increasing at the rate ~~at~~ of 25 cm³/s. Find the rate of change of its surface area at the ~~sa~~ instant when ~~k~~ its radius is 5 cm.

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 25$$

$$25 = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$$

$$\frac{\cancel{25}^{5} \times \cancel{3}}{4 \times \pi \times \cancel{3} \times \cancel{5} \times 5}$$

$$\frac{1}{4\pi} = \frac{dr}{dt}$$

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 4\pi(2r) \times \frac{dr}{dt}$$

$$= 4\pi \times 2 \times 5 \times \frac{1}{4\pi}$$

$$\boxed{= 10\ \cancel{\tfrac{50}{25}}\ cm^2/s}$$

Integration

$$\frac{d}{dx}(x^2)=2x$$

$$\frac{d}{dx}(x^2+1)=2x$$

$$\frac{d}{dx}(x^3+3)=2$$

$\int(2x)\,dx = x^2+C \rightarrow$ constant of Integration

Indefinite Integration

formulas:-

① $\int c\, f(x)\, dx = c\int f(x)\, dx$

② $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

③ $\int \frac{1}{x}\, dx = \ln x + c$

④ $\int e^x\, dx = e^x + c$

⑤ $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c \quad (n \neq -1)$

⑥ $\int e^{(ax+b)}\, dx = \frac{e^{(ax+b)}}{a} + c$

⑦ $\int \sin x \, dx = -\cos x + c$

⑧ $\int \cos x \, dx = \sin x + c$

⑨ $\int \sin(ax+b) \, dx = \dfrac{-\cos(ax+b)}{a} + c$

⑩ $\int \cos(ax+b) \, dx = \dfrac{\sin(ax+b)}{a} + c$

⑪ $\int \sec^2 x \, dx = \tan x + c$

⑫ $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

⑬ $\int \sec x \tan x \, dx = \sec x + c$

① $\int \sqrt{x} \, dx$

$$\int x^{1/2} \, dx$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + c$$

$$\boxed{= \frac{2\sqrt{x^3}}{3}}$$

(2) $\int \frac{dx}{x^2}$

$\int x^{-2} dx$

$\frac{x^{-1}}{-1} + c$

$\boxed{\frac{1}{-x} + c}$

(3) $\int (1-2x) dx$

$\cancel{\frac{(-2x+1)^{2}}{}}$

$\frac{(1-2x)^{1+1}}{(-2)(1+1)} + c$

$\boxed{\frac{(1-2x)^2}{-4} + c}$

(4) $\int \cos^2 x \, dx$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{1}{2} + \frac{\cos^2 x}{} dx$

$\frac{1}{2} \int 1 + \cos^2 x \, dx$

$\frac{1}{2} \int 1 \, dx + \int \cos 2x \, dx$

$\boxed{\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c}$

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⑤ $\int \sin^2 x \, dx$

⑥ $\int \frac{1}{2x-1} \, dx$

⑤ $\int \frac{1-\cos 2x}{2} \, dx$

$\frac{1}{2} \int 1-\cos 2x \, dx$

$\frac{1}{2} \int 1 \, dx - \int \cos 2x \, dx$

$$\boxed{\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)}$$

⑥ $\int \frac{1}{2x-1} \, dx$

$\int \frac{1}{2x} \, dx - x$

$\frac{1}{2} \int \frac{1}{x} \, dx - x$

$\frac{1}{2} \times \ln x - x + c$

$$\boxed{\frac{\ln x}{2} - x + c}$$

(7) $\int\left(x-\frac{1}{x}\right)dx$

$$\int x\,dx - \int \frac{1}{x}\,dx$$

$$\frac{x^2}{2} - \ln$$

$$\boxed{\frac{x^2-2\ln}{2}+c}$$

(8) $\int \cos(2x+5)\,dx$

$$\boxed{\frac{\sin(2x+5)}{2}+C}$$

(9) $\int\left(e^{2x}+\frac{1}{x^3}\right)dx$

$$e^{2x}+\int x^{-3}\,dx$$

$$e^{2x}+\frac{x^{-2}}{-2}$$

$$e^{2x}-\frac{1}{2x^2}$$

$$\boxed{\frac{e^{2x}}{2}-\frac{1}{2x^2}+c}$$

(10) $\int \cos(1-2x)\,dx$

$$\boxed{\frac{\sin(1-2x)}{-2}+C}$$

Substitution method

① $\int \frac{(2x-3)\,dx}{x^2-3x+8}$

let $t = x^2-3x+8$

$\frac{dt}{dx} = 2x-3$

$\therefore dt = (2x-3)\,dx$

$\int \frac{dt}{x^2-3x+8}$

$\int \frac{dt}{t}$

$\int \frac{1}{t}\,dt$

$\ln t + c$

$= \ln(x^2-3x+8) + c$

② $\int \frac{x^2\,dx}{x^3+1}$

$t = x^3-1$

$\frac{dt}{dx} = 3x^2$

$dt = 3x^2\,dx$

$\frac{dt}{3} = x^2\,dx$

$\int \frac{dt}{3} \cdot \frac{1}{t}$

$\frac{\ln t}{3} + c$

$\frac{\ln(x^3+1)}{3} + c$

③ $\int \frac{e^{2x}\,dx}{e^{2x}+a^2}$

let $t = e^{2x} + a^2$

$\frac{dt}{dx} = e^{2x} + \cancel{2a}$

~~$dt - 2a\,dx = e^{2x}\,dx$~~

~~$dt = e^{2x}\,dx$~~

$\frac{dt}{dx} = e^{2x} \times 2$

$dt = e^{2x}\,dx \times 2\,dx$

~~$dt - 2\,dx = e^{2x}\,dx$~~

$\frac{dt}{2} = e^{2x}\,dx$

~~$\int \frac{dt}{t}$~~

~~$= \ln t + c$~~

~~$= \ln(e^{2x}+a^2) + C$~~

~~④ $\int \frac{dx}{x \ln x}$~~

$\int \cancel{dt}\,\frac{dt}{2} \times \frac{1}{t}$

$\frac{\ln t}{2} + c$

$\frac{\ln(e^{2x}+a^2)}{2} + c$

④ $\int \frac{dx}{x \ln x}$

$t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt \times x = dx$

$\int \frac{dt\,x}{xt}$

$\cancel{x} \times \ln t$

$\boxed{\cancel{x} \ln(\ln x)} + c$

⑤ $\int e^{x}(\sin e^{x})\,dx$

let $t = e^{x}$

$\frac{dt}{dx} = e^{x}$

$dt = e^{x}\,dx$

$\int dt\,(\sin e^{x})$

$\int dt \times \cancel{\cos t}\ \sin t$

$-\cos t + c$

$\boxed{-\cos e^{x} + c}$

⑥ $\int \frac{x\,dx}{x^{2}+1}$

$t = x^{2}+1$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x\,dx$

$\int \frac{dt}{2} \times \frac{1}{\cancel{x^2}\,t}$

$\frac{\ln t}{2} + c$

$\boxed{\frac{\ln(x^{2}+1)}{2} + c}$

⑦ $\int \frac{(\ln x)^m}{x} dx$

$t = \ln x$

$\frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{dx}{x}$

$\int \cancel{(\ln x)^m} \, t^m \, dt$

$\frac{t^{m+1}}{m+1} + c$

$\boxed{\frac{(\ln x)^{m+1}}{m+1} + c}$

⑧ $\int e^{\sin x} \cos x \, dx$

let $t = \sin x$

$\frac{dt}{dx} = \cos x$

$dt = \cos x \, dx$

$\int e^t \, dt$

e^t

$\boxed{= e^{\sin x} + c}$

⑨ $\int e^{x^2} x\,dx$

$t = x^2$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x\,dx$

$\int e^{t} \frac{dt}{2}$

$\boxed{\cancel{\frac{e^{t}}{2} + c}} \rightarrow \boxed{\frac{e^{x^2}}{2} + c}$

⑩ $\int e^{-x^3} x^2\,dx$

$t = -x^3$

$\frac{dt}{dx} = -3x^2$

$\frac{dt}{-3} = x^2\,dx$

$\int e^{t} \times \frac{dt}{-3}$

$\boxed{\frac{e^{-x^3}}{-3} + c}$

(14) $\int \frac{x\,dx}{\sqrt{x^2+1}}$

$t^2 = x^2+1$

$2t\,dt = 2x\,dx$

$t\,dt = x\,dx$

$= \int \frac{\cancel{t}\,dt}{\cancel{t}}$

$= \int dt$

$= t$

$\boxed{= \sqrt{x^2+1} + C}$

$\frac{1}{t} \quad x^2+1$

(15) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

$t^3 = \sin x$

$3t^2\,dt = \cos x\,dx$

$\int \frac{3t^{\cancel{2}}\,dt}{\cancel{t}}$

$3\int t\,dt$

$3 \times \frac{t^2}{2}$

$\boxed{\frac{3}{2}(\sin x)^{2/3} + C}$

(16) $\int \frac{\sqrt{\ln x}}{x}\, dx$

(17) $\int x^2 \sqrt[5]{x^3+2}\ dx$

(16) $t^2 = \ln x$

$2t\,dt = \frac{1}{x}\,dx$

$\int t \times 2t\,dt$

$2\int t^2\,dt$

$2 \times \frac{t^3}{3}$

$$\boxed{\frac{2}{3}(\ln x)^{3/2} + c}$$ ✓

(17) $t^5 = x^3 + 2$

$5t^4\,dt = 3x^2\,dx$

$\frac{5}{3}t^4\,dt = x^2\,dx$

$\int t \times \frac{5}{3} t^4\,dt$

$\frac{5}{3}\int t^5\,dt$

$\frac{5}{3} \times \frac{t^6}{6}$

$\frac{5}{3} \times \frac{(x^3+2)^{6/5}}{6}$

$$\boxed{\frac{5(x^3+2)^{6/5}}{18} + c}$$ ✓

(14) $\int \frac{x\,dx}{\sqrt{x^2+1}}$

$t^2 = x^2+1$

$2t\,dt = 2x\,dx$

$t\,dt = x\,dx$

$= \int \frac{\cancel{t}\,dt}{\cancel{t}}$

$= \int dt$

$= t$

$\boxed{= \sqrt{x^2+1} + C}$

$\frac{1}{t} \quad \cancel{x^2+1 \quad 2x}$

(15) $\int \frac{\cos x}{\sqrt[3]{\sin x}}\,dx$

$t^3 = \sin x$

$3t^2\,dt = \cos x\,dx$

$\int \frac{3t^{\cancel{2}}\,dt}{\cancel{t}}$

$3\int t\,dt$

$3 \times \frac{t^2}{2}$

$\boxed{\frac{3}{2}(\sin x)^{2/3} + C}$

(16) $\int \frac{\sqrt{\ln x}}{x} \, dx$

(17) $\int x^2 \sqrt[5]{x^3+2} \, dx$

(16) $t^2 = \ln x$

$2t \, dt = \frac{1}{x} dx$

$\int t \times 2t \, dt$

$2 \int t^2 \, dt$

$2 \times \frac{t^3}{3}$

$\boxed{\frac{2}{3}(\ln x)^{3/2} + c}$ ✓

(17) $t^5 = x^3 + 2$

$5t^4 \, dt = 3x^2 \, dx$

$\frac{5}{3} t^4 \, dt = x^2 \, dx$

$\int t \times \frac{5}{3} t^4 \, dt$

$\frac{5}{3} \int t^5 \, dt$

$\frac{5}{3} \times \frac{t^6}{6}$

$\frac{5}{3} \times \frac{(x^3+2)^{6/5}}{6}$

$\boxed{\frac{5(x^3+2)^{6/5}}{18} + c}$ ✓

Definite Integral

~~करके~~ a से b

$$I = \int_a^b f(x)\,dx = \left[g(x)\right]_a^b = g(b) - g(a)$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$a < c < b$$

① $\int_1^3 x^2\,dx$

$$\left[\frac{x^3}{3}\right]_1^3$$

$$\frac{1}{3}\left[x^3\right]_1^3$$

$$\frac{1}{3}\left[(3)^3 - (1)^3\right]$$

$$\frac{1}{3}(27-1)$$

$$\boxed{\frac{26}{3}}$$

~~②~~

② $I = \int_0^2 (ax^2+bx+c)\,dx$

$$\left[\frac{a}{3}x^3 + \frac{b}{2}x^2 + cx\right]_0^2$$

$$\left(\frac{8a}{3} + \frac{4b}{2} + 2c\right) - \left(\cancel{\frac{a}{3}} + \cancel{\frac{b}{2}}\, 0 + 0 + 0\right)$$

$$\frac{8a + \overset{6}{\cancel{2}}b + \overset{6}{\cancel{2}}c}{3}$$

$$\boxed{\frac{2}{3}(4a+3b+3c)}$$ ✓

③ $I = \int_0^{\pi} \sin^2 x\,dx$

$$\frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$\frac{1}{2}\left[\left(\pi - \frac{\sin 2\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

$$\boxed{\frac{1}{2}[\pi - 0] = \frac{\pi}{2}}$$ ✓

(4) $\int_0^4 \sqrt{2x+1}\, dx$

Method - I

$I = \int_0^4 (2x+1)^{1/2}\, dx$

$= \left[\frac{(2x+1)^{\frac{3}{2}}}{2\left(\frac{1}{2}+1\right)} \right]_0^4$

$= \frac{1}{3}\left[9^{3/2} - 1^{3/2} \right]$

$= \frac{1}{3} \times 26$

$= \frac{26}{3}$

Method II

$I = \int_0^4 \cancel{(2x+1)} \sqrt{2x+1}\, dx$

$t^2 = 2x+1$

U.L. $\Rightarrow \sqrt{2(4)+1} \to 3$

L.L $\Rightarrow \sqrt{2(0)+1} \to 1$

$2t\,dt = 2\,dx$

$dx = t\,dt$

$\int_1^3 t^2\,dt$

$\left[\frac{t^3}{3}\right]_1^3$

$\frac{1}{3}\left[t^3\right]_1^3$

$\frac{1}{3}(27-1)$

$\frac{26}{3}$

(a) (5) $\int_0^1 x(x^2+1)^3 dx$

$t = x^2+1$

$\frac{dt}{dx} = 2x$

$\frac{dt}{2} = x\,dx$

$\int_1^2 \frac{t^3}{2}\, dt$

$\left[\frac{t^4}{2\times 4}\right]_1^2$

$\frac{1}{8}\left[t^4\right]_1^2$

$\frac{1}{8}(16-1)$

$\boxed{\frac{15}{8}}$ ✓

~~(7) (6) $\int_0^1 \frac{x^3}{(1+x^2)^4} dx$~~

~~$t = 1+x^2$~~

~~$1 + x^4 + 2x^2$~~

~~$2x^3$ $4x^3 + 4x$~~

(6) $$\int_0^1 \frac{x^3}{(x^2+1)^4}\,dx$$

$t = 1 + x^2$

$x^2 = t - 1$

$\frac{dt}{2} = x\,dx$

$$\int_1^2 \frac{(t-1)\,\frac{dt}{2}}{t^4}$$

$$\frac{1}{2}\int_1^2 t^{-3}dt \int_1^2 t^{-4}\,dt$$

$$\frac{1}{2}\left[\frac{t^{-2}}{-2} - \frac{t^{-3}}{-3}\right]_1^2$$

$$\frac{1}{2}\left[-\frac{1}{2t^2} + \frac{1}{3t^3}\right]_1^2$$

$$\frac{1}{2}\left[\frac{1}{3t^3} - \frac{1}{2t^2}\right]_1^2$$

$$\frac{1}{2}\left[\left(\frac{1}{24} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$

$$\frac{1}{2}\left[\left(\frac{-2}{24}\right) - \left(\frac{-1}{6}\right)\right]$$

$$\frac{1}{2}\left[-\frac{1}{12} + \frac{1}{6}\right]$$

$$\frac{1}{2}\left[\frac{2}{12} - \frac{1}{12}\right]$$

$$\frac{1}{2}\left(\frac{1}{12}\right) = \boxed{\frac{1}{24}}$$

Average value of a continuous function in an Interval:-

$y = f(x)$

Ans. → Average value of y in an interval $a \leq x \leq b$ is given by

$$\langle y \rangle \text{(average value of } y) = \frac{\int_a^b y\,dx}{\int_a^b dx}$$

Q find average value from $x = 0$ to $x = \pi$ for $\sin x$

$$\langle \sin x \rangle = \frac{\int_0^\pi \sin x\,dx}{\int_0^\pi dx}$$

$$= \frac{[-\cos x]_0^\pi}{[x]_0^\pi}$$

$$= \frac{-[\cos \pi - \cos 0]}{\pi - 0}$$

$$= \frac{-(-1-1)}{\pi} = \frac{2}{\pi}$$

Q Determine average value of $y = x+5$ in the interval $0 \leq x \leq 1$

$y = x+5$

$$\langle y \rangle = \frac{\int_0^1 x+5\,dx}{\int_0^1 dx}$$

$$= \frac{\left[\frac{x^2}{2} + 5x\right]_0^1}{[x]_0^1}$$

$$= \frac{\left(\frac{1}{2} + 5\right) - (0)}{1 - 0}$$

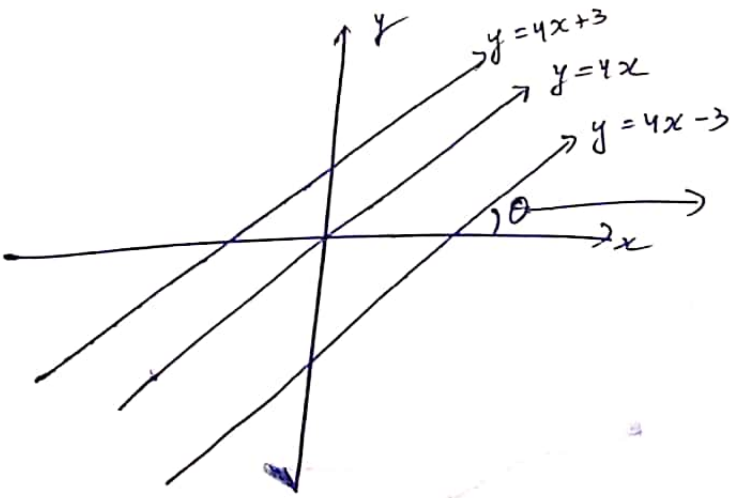
$$= \frac{\frac{11}{2}}{1}$$

$$\boxed{\langle y \rangle = \frac{11}{2}}$$

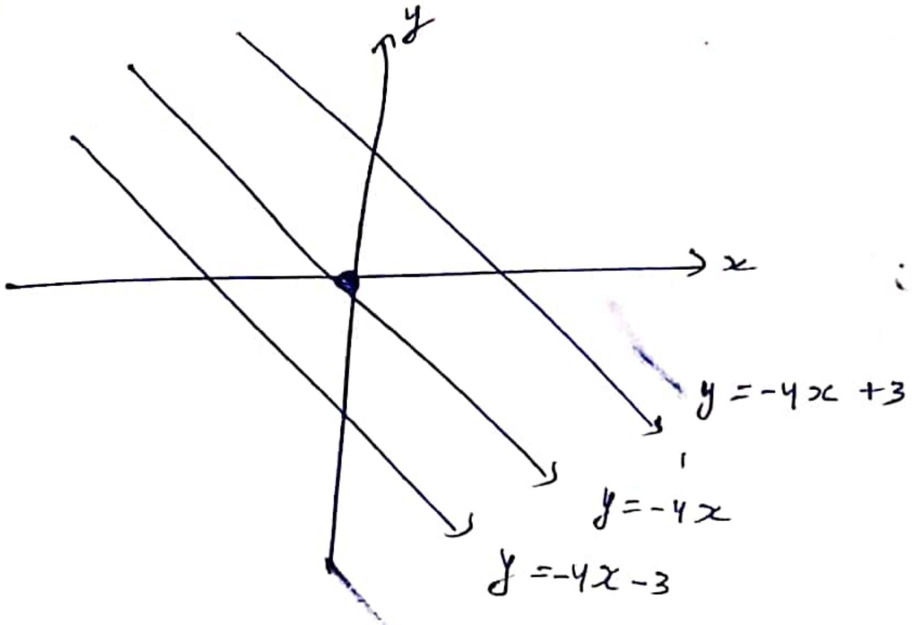
Graphs

1. Straight line

equ = $y = \underbrace{m}_{\text{slope}}x + \underbrace{c}_{\text{intercept on } y\text{-axis}}$

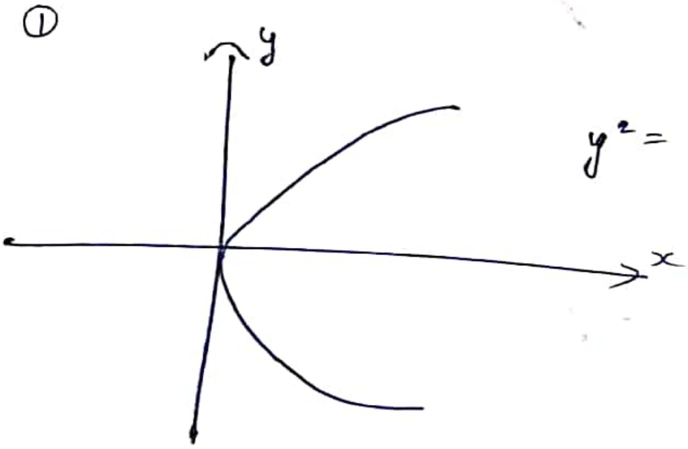


slope = $\tan\theta$



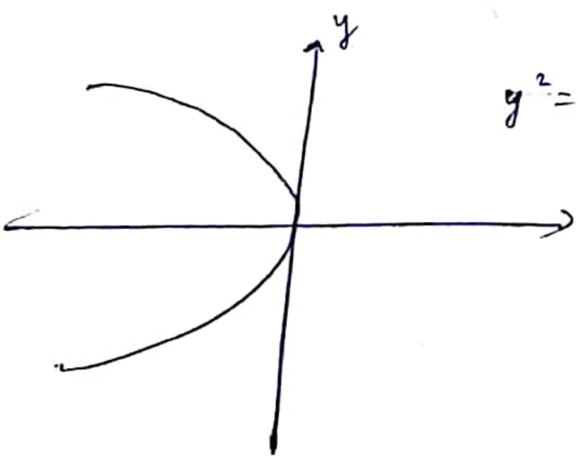
Quadratic ~~Eqn~~ Equation Graph

①



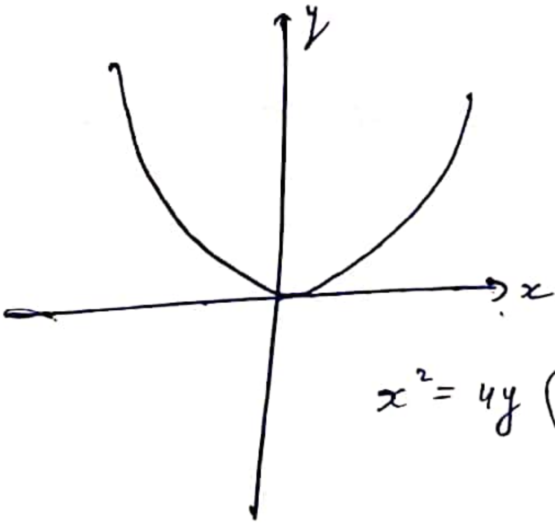
$y^2 = 4x$

②



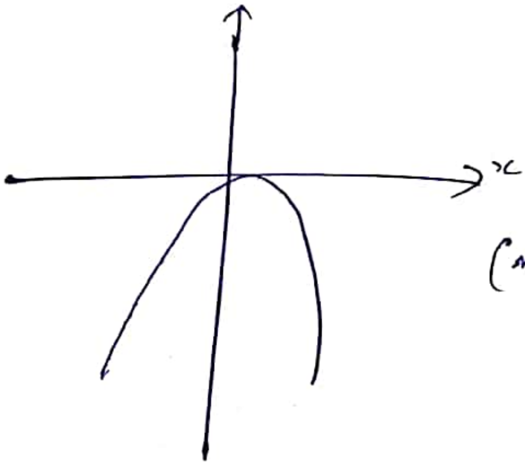
$y^2 = -4x$

③



$x^2 = 4y$ (mouth of parabola opening upwards

④



$x^2 = -4y$

(mouth of parabola opening downwards)

Kinematics (Motion in one Dimension)

Kinematics - The branch of theoretical mechanics which deals with study of motion of rigid bodies (shape cannot be changed) without taking into account the cause of motion (i.e. forces acting on them) is called **Kinematics**.

point object - A particle in its strict sense means an object without dimensions (no size , only a point)

Translatory motion - when their is no change in the orientation of a object while moving. (every point traveles same distance)

rectilinear (straight line)

curvilinear ~~curvilinear~~ (curved path)

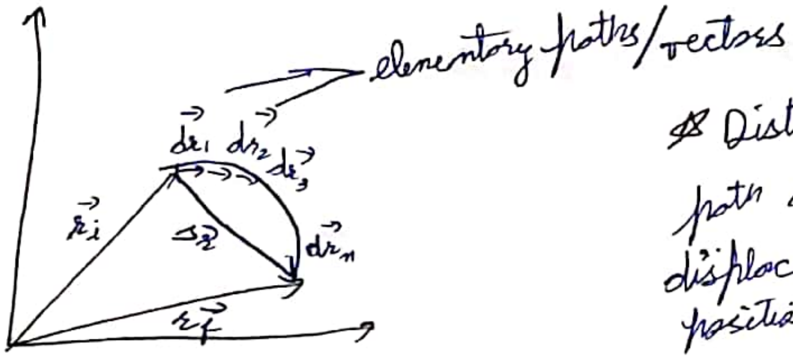
rectilinear

curvilinear

* Position of a particle in space is determined relative to some fixed point, position depends on position of observer.
  ↳ refrence point.

* The path followed by a point object during its motion is called its trajectory.

Distance & Displacement



Distance is the length of the path actually traversed ~~which~~ while displacement is the change in position vector.

Displacement; $\Delta\vec{r} = \vec{r_f} - \vec{r_i}$

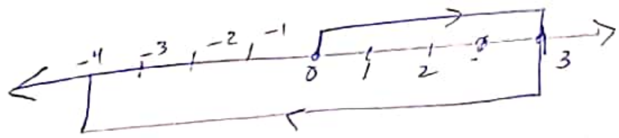
$$\boxed{\Delta\vec{r} = \int \vec{dr}}$$ adding all elementary vectors

$$\boxed{\text{Distance} = \int |\vec{dr}|}$$ adding of all magnitudes of elementary vectors.

→ Distance is multi ~~valued~~ valued function while displacement is a single valued function.

→ Distance ≥ Displacement → can be +ve, −ve or zero
↳ always Positive

eg.



Distance $= 3+3+4 = 10$
Displacement Δx $= x_f - x_i = -4-0 = -4$

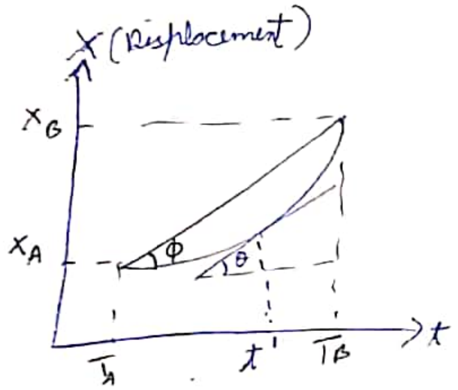
Speed & Velocity -

Average Speed - ~~Total~~ $\dfrac{\text{Total Distance}}{\text{Total Time}}$

Average Velocity $= \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{\Delta \vec{x}}{\Delta t}$

Instantaneous velocity - ~~velocity~~ Rate of change of position at given time instance.

$$\boxed{\text{Velocity} = \frac{dx}{dt}}$$

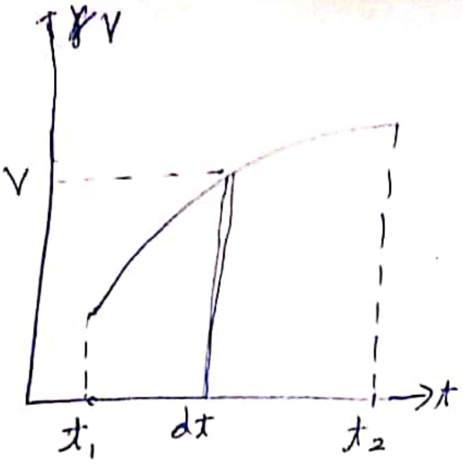


instantaneous velocity at t_1 slope

$$\boxed{v = \frac{dx}{dt} = \tan\theta}$$

Average velocity = slope of chord

$$\text{Average velocity} = \boxed{\frac{x_B - x_A}{t_B - t_A} = \tan\phi}$$



~~Velocity under~~
area under velocity graph = displacement

$V.dt$ = area under small time instance

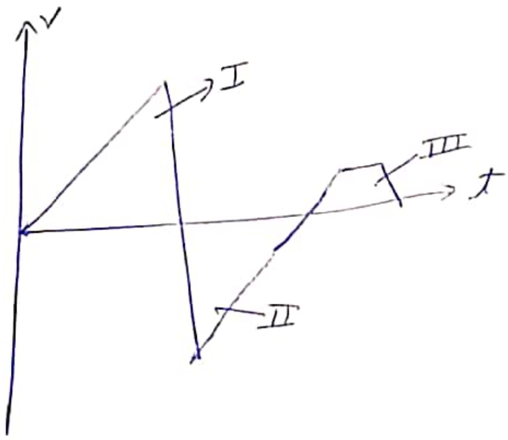
→ area under curve is Integration

$\int_{t_1}^{t_2} v\,dt$ = area under whole graph (by adding all small time instances)

$$\boxed{\begin{aligned} v &= \frac{dx}{dt} \\ x &= \int dx = \int_{t_1}^{t_2} v\,dt \end{aligned}}$$

x is displacement not distance

Ex.



Distance = I + II + III

Displacement = I − II + III

~~x =~~ .

eg 1 $x = 2t - 3t^2$, find velocity at $t = 2s$

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}(2t - 3t^2)$$

$$v = 2 - 6t$$

$$v\Big|_{t=2} = 2 - 6(2)$$

$$= 2 - 12$$

$$\boxed{= -10\ m/s}$$

eg 2. $v = (3 + 2t)\ m/s$

→ at $t = 0$, $x = 0$

find displacement at $t = 2$ sec

$$x = \int 3 + 2t \, dt$$

$$x = 3t + \frac{2t^2}{\cancel{2}} + c$$

$$x = 3t + t^2 + c$$

~~x~~

$$0 = 3(0) + (0)^2 + c$$

$$c = 0$$

$$x = 3(2) + (2)^2$$

$$x = 6 + 4$$

$$\boxed{x = 10\ m}$$

(122)

Note:- ① Slope of Displacement-time graph gives velocity.

$$v = \frac{dx}{dt}$$

② Area under v-t graph with proper sign gives displacement & without sign gives distance

$$x = \int v\,dt$$

Acceleration:-

Velocity
- Uniform Velocity
  - Constant velocity
  - Same Direction
- Non-uniform Velocity
  - Speed change
  - Direction change
  - Both change

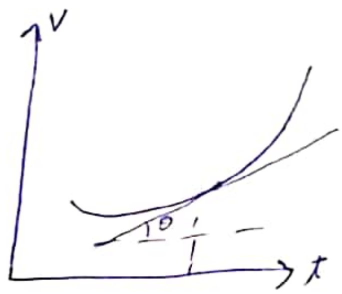
Constant velocity, Same Direction → acceleration = 0

u (initial velocity)
v (final velocity)

$$a = \frac{v-u}{t}$$

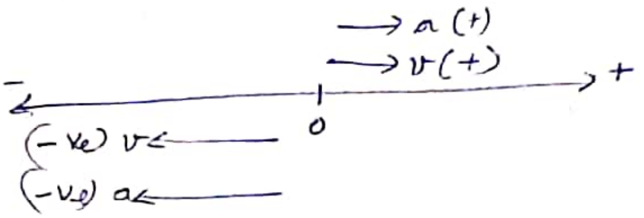
Acceleration - Rate of change of velocity

$$\boxed{a = \frac{dv}{dt}}$$



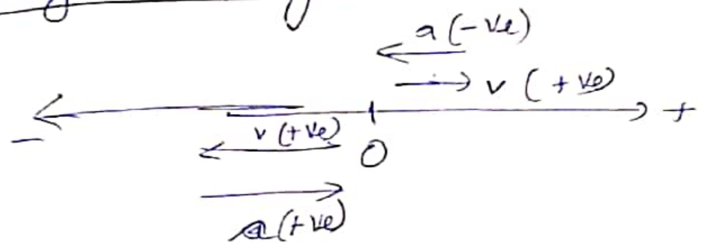
* slope of v-t graph gives acceleration.

→ Body is speeding up



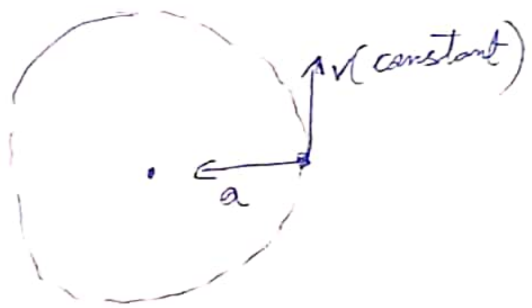
When velocity & acceleration are of same sign (either of them be ⊕ve or ⊖ve), speed of body will increase.

→ Body is slowing down (Retardation / deceleration)



when $\vec{v}$ & $\vec{a}$ are in opp sign, speed of body decreases.

→ Velocity ⊥ Acceleration.



speed = constant
direction = change
resulting path — circle

uniform circular motion

$$\boxed{a = \frac{dv}{dt}}$$

$$\because v = \frac{dx}{dt}$$

$$a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$a = \frac{d^2x}{dt^2}$$

$$\boxed{a = \frac{d^2x}{dt^2}}$$

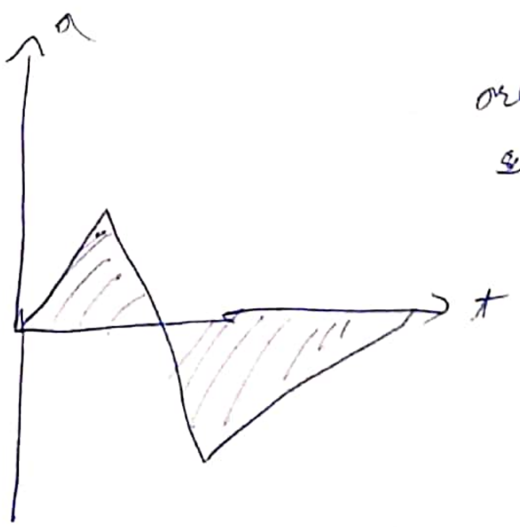
$$a = \frac{dv}{dx} \times \frac{dx}{dt}$$

$$\cancel{a = v\frac{dv}{dx}}$$

$$\boxed{a = v\frac{dv}{dx}}$$

$$a = \frac{dv}{dt}$$

$$\boxed{\Delta v = \int_u^v dv = \int a\, dt}$$



area under a-t graph = velocity

S-t Graph → Slope → v , $v = \frac{dx}{dt}$

V-t Graph → Slope → a , $a = \frac{dv}{dt}$, $a = v\frac{dv}{dx}$, $a = \frac{d^2x}{dt^2}$

→ area → Displacement , $x = \int v\,dt$

→ Distance

a-t Graph → area → Δv $\Delta v = \int a\,dt$

→ slope → Jerk

$\frac{d}{dt}|\vec{v}|$:- rate of change of speed

$\frac{d\vec{v}}{dt}$:- rate of change of velocity = acceleration, $\vec{a}$ (gives vector)

$\left|\frac{d\vec{v}}{dt}\right|$:- magnitude of acceleration

Q Can $\frac{d}{dt}|\vec{v}| = 0$ while $\left|\frac{d\vec{v}}{dt}\right| \neq 0$? Yes

Speed = constant

Its direction may change , acc might be non-zero

Q Can $\frac{d}{dt}|\vec{v}| \neq 0$ while $\left|\frac{d\vec{v}}{dt}\right| = 0$? No

Q1. A train travels a distance of 20 km with a uniform speed of 60 km/hr. It travels another distance of 40 km with a uniform speed of 80 km/hr. Calculate average speed of train.

Average speed $= \cancel{80+60}\ \cancel{\frac{V_1+V_2}{2}}$

$= \frac{V_1+V_2}{2}$

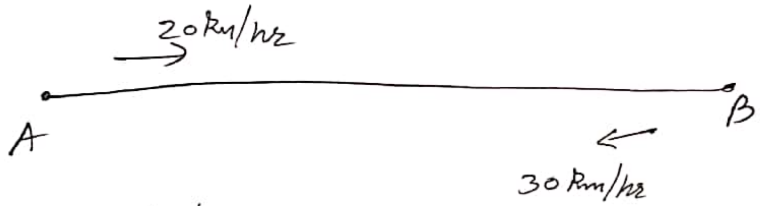
$= \cancel{80}$

$T_1 = \cancel{200}\ \frac{20}{60} = \frac{1}{3}$ hr

$T_2 = \frac{40}{80} = \frac{1}{2}$ hr

$\frac{\cancel{64}\cancel{8}0+20}{\frac{1}{2}+\frac{1}{3}} = \frac{\overset{60}{\cancel{100}}}{\frac{5}{6}\text{ hr}} = \frac{\overset{360}{\cancel{600}}}{5} = \cancel{300}\ \frac{360}{5} = \boxed{72\text{ km/hr}}$

Q2.



~~V = 20 km/hr~~

Average Velocity = 0

$\cancel{\frac{2V_1V_2}{V_1+V_2} = \frac{2\times20\times30}{20+50} = \frac{1200}{70} = 17}$

dis = x

$t_1 = \frac{x}{20}$

$T_2 = \frac{x}{30}$

A. Speed $= \frac{x+x}{\frac{x}{20}+\frac{x}{30}}$

$= \frac{2x}{\frac{50x}{600}}$

$= \frac{120\cancel{0}\cancel{x}}{5\cancel{0}\cancel{x}}$

$\boxed{= 60\text{ km/hr}}$

Average speed $= \frac{2x}{\frac{x}{20}+\frac{x}{30}}$

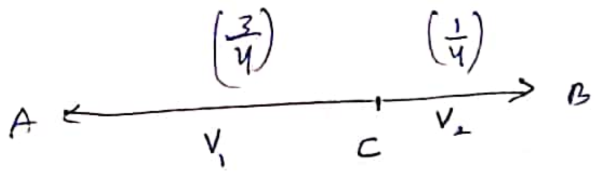
$= \frac{2\cancel{x}}{\cancel{x}\left(\frac{1}{20}+\frac{1}{30}\right)}$

$= \frac{2}{\frac{3+2}{60}}$

$= \frac{120}{5}$

$\boxed{= 24\text{ km/hr}}$

Q3.



$$\text{Average velocity} = \frac{x}{\frac{\frac{3x}{4}}{V_1} + \frac{\frac{x}{4}}{V_2}}$$

$$= \frac{x}{\frac{3x}{4V_1} + \frac{x}{4V_2}}$$

$$= \frac{1}{\frac{3}{4V_1} + \frac{1}{4V_2}}$$

$$= \frac{4}{\frac{3}{V_1} + \frac{1}{V_2}}$$

$$\boxed{= \frac{4\,V_2 V_1}{(3V_2 + V_1)}}$$

Q4. $x = (5t^2 + 4t + 3)$ m

find velocity and acceleration at $t = 2s$

$$v = \frac{dx}{dt} = \frac{d}{dt}(5t^2+4t+3)$$

$$= 10t + 4$$

$$= 20+4$$

$$\boxed{= 24 \text{ m/s}}$$

$$a = \frac{d}{dt}(10t+4)$$

$$\boxed{= 10 \text{ m/s}^2} \rightarrow \text{uniform acceleration}$$

Q5. $x = At^3 + Bt^2 + Ct + D$

$A = 1$

$B = 4$ (all SI units are used)

$C = -2$

$D = 5$

a) find dimensions of A B C & D

b) Find velocity at $T = 4s$

c) find Average velocity from $t=0$ to $t=4s$

d) find ~~Avg~~ Acceleration of particle at $t=4s$

e) find acc Avg from $t=0$ to $t=4s$

All At^3, Bt^2, Ct & D are with dimensions $[L]$

$[D] - [L]$

$[C] - [LT^{-1}]$

$[B] - [LT^{-2}]$

$[A] - [LT^{-3}]$

b) $v = \frac{d}{dt}(t^3 + 4t^2 - 2t + 5)$

$= (2t^2 + 4t - 2)$ $\quad = 3t^2 + 8t - 2$

$= 2 \times 16 + 4 \times 4 - 2$ $\quad = 48 + 32 - 2$

$= 32 + 16 - 2$ $\quad \boxed{= 78 \text{ m/s}}$ ✓

$= 48 - 2$

$\boxed{= 46 \text{ m/s}}$

c) $t = 0$ $\quad t = 4s$

$x = 5$ $\quad 64 + 64 - 8 + 5$

$x = 125$

$\frac{125 - 5}{4} = \frac{120}{4} = \boxed{30 \text{ m/s}}$ ✓

d) $\frac{d^2x}{dt^2} = \frac{d}{dt}(3t^2 + 8t - 2)$

$= 6t + 8$

$= 6(4) + 8$

$= 24 + 8$

$\boxed{= 32 \text{ m/s}^2}$ ✓

e) $\frac{78 + 2}{4} = \frac{80}{4} = \boxed{20 \text{ m/s}^2}$ ✓

Scanned with CamScanner

Q6. $x = t^3 - 6t^2 + 3t + 4$ in meters

find velocity of particle at instant when $a = 0$

$\frac{dv}{dt} = 0$

$v =$ constant

$\frac{dx}{dt} = 3t^2 - \cancel{6t+3}\ 12t + 3$

$\cancel{\frac{d^2x}{dt^2} = 6t - 6}$

$\cancel{6t - 6 = 0}$

$\cancel{6t = 6}$

$\cancel{t = 1}$

$\cancel{v = 3(1)^3 - 6(1) + 3}$

$\cancel{= 3 - 6 + 3}$

$\frac{d^2x}{dt^2} = 6t - 12$

$6t - 12 = 0$

$t = 2$

$v = 3(2)^2 - 12(2) + 3$

$= 3 \times 4 - 12(2) + 3$

$= 12 - 24 + 3$

$= -12 + 3$

$\boxed{= -9 \text{ m/s}}$

Q7. $a = 3t^2 + 2t + 2$

$t = 0,\ v = 2$ m/s

find v at $t = 2$ s

$\Delta v = \int 3t^2 + 2t + 2 \; dt$

$2 = \frac{\cancel{3}t^3}{\cancel{3}} + \frac{\cancel{2}t^2}{\cancel{2}} + 2t + C$

$\cancel{6 \times 2 = 6 3t^3 + 6t^2 + 12t}$

$2 = t^3 + t^2 + 2t + C$

$2 = 0 + 0 + 0 + C$

$C = 2$

$v = 8 + 4 + 4 + 2$

$\boxed{v = 18 \text{ m/s}}$

OR

$\int_{2}^{x} dv = \int_{0}^{2} 3t^2 + 2t + 2$

जब $t=2, x=?$

जब $t = 0$, $v = 2$

$[v]_2^x = [t^3 + t^2 + 2t]_0^2$

$x - 2 = 8 + 4 + 4$

$\boxed{x = 18 \text{ m/s}}$

Q7. $a = 6t + 6 \; m/s^2$

$t = 0, \; x = 0, \; v = 2 \; m/s$

velocity & displacement as a function of time.

$$\frac{dv}{dt} = a$$

$$dv = a\,dt$$

$$\int dv = \int a\,dt$$

$$v = \int 6t + 6 \, dt$$

$$v = \frac{6t^2}{2} + 6t$$

$$\boxed{v = 3t^2 + 6t + 2}$$

$$\frac{dx}{dt} = v$$

$$x = \int v\,dt$$

$$x = \int 3t^2 + 6t + 2 \, dt$$

$$x = \frac{3t^3}{3} + \frac{6t^2}{2} + 2t + c$$

$$x = t^3 + 3t^2 + 2t + c$$

$$\boxed{x = t^3 + 3t^2 + 2t}$$

OR

$$\frac{dv}{dt} = 6t + 6$$

$$\int_2^v dv = \int_0^t (6t + 6)\,dt$$

$$[v]_2^v = [3t^2 + 6t]_0^t$$

$$v - 2 = 3t^2 + 6t$$

$$\boxed{v = 3t^2 + 6t + 2}$$

$$\frac{dx}{dt} = 3t^2 + 6t + 2$$

$$[x]_0^x = [t^3 + 3t^2 + 2t]_0^t$$

$$\boxed{x = t^3 + 3t^2 + 2t}$$

Q8. $x = \left(-\frac{2}{3}t^2 + 16t + 2\right) m$

at $t = 0$, $x = 0$

a) u (initial velocity

$\cancel{\frac{dv}{dt}}$

$\cancel{\int dv = \int -\frac{2}{3}t^2 + 16t + 2 \, dt}$

$$\frac{dx}{dt} = -\frac{4}{3}t + 16$$

$$= -\frac{4}{3}(0) + 16$$

$$\boxed{= 16 \ m/s}$$ ✓

b) how long to come to rest

$$-\frac{4}{3}t + 16 = 0$$

$$t = \frac{-16 \times 3}{-4}$$

$$t = 4 \times 3$$

$$\boxed{t = 12 s}$$ ✓

c) acc, $t = 12$

$$\boxed{a = -\frac{4}{3} \ m/s^2}$$ ✓

Q9:- The position of particle is given by $x = 10 + 20t - 5t^2$ in metre,
find a) displacement in 1s
b) distance in 3s
c) initial acceleration
d) velocity at 4s

$\cancel{\frac{d}{dt}(10+20t-5t^2)}$

$\cancel{= -10t + 20}$

$\cancel{x\Big|_{t=1} = 20-10(1)}$

$\cancel{= 10\,m}$

a) $\Delta x = x_f - x_i$

$= (10+20-5) - 10$

$\boxed{= 15\,m}$ ✓

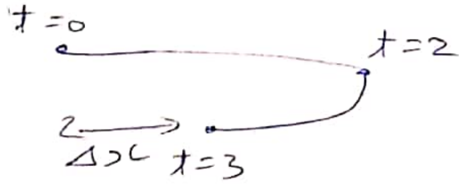
b) $v = \frac{dx}{dt} = 0 + 20 - 10t$

$v = 20 - 10t$

for $v = 0$

$20 - 10t = 0$

$t = 2s$



$x = \int v\,dt$

~~x~~ Distance = 0/

$$\text{Distance} = \left| \int_0^2 (20-10t)\,dt \right| + \left| \int_2^3 (20-10t)\,dt \right|$$

$$\left| \left[20t - 5t^2 \right]_0^2 \right| + \left| \left[20t - 5t^2 \right]_2^3 \right|$$

$$\left| \left[\cancel{20(2)}\ 20 - 0 \right] \right| + \left| \left[15 - 20 \right] \right|$$

$$|20| + |-5|$$

$$20 + 5$$

$$\boxed{= 25\ m}$$

c) $a = \dfrac{d^2x}{dt^2}$

$a = \cancel{20-10t}\ \dfrac{d}{dt}(20 - 10t)$

$$\boxed{a = -10\ m/s^2}$$

d) $v = 20 - 10t$

$= 20 - \cancel{4}\ 10(4)$

$= 20 - 40$

$$\boxed{= -20\ m/s}$$

Q10. The acc of body is given by $(4-2t)\,m/s^2$ at $t=0$, $v = 5\,m/s$. find distance at 12 s.

$$v = \int a\,dt$$

$$v = \int 4-2t\,dt$$

$$v = 4t - t^2 + c$$

$$5 = 0 - 0 + c$$

$$c = 5$$

$$v = 4t - t^2 + 5$$

~~distance =~~

$$0 = 4t - t^2 + 5$$

$$t^2 - 4t - 5 = 0$$

$$t^2 - 5t + t - 5$$

$$t(t-5) + 1(t-5)$$

$$t = 5$$

$$t = -1 \times$$

$$v = 0,\ t = 5$$

$$\text{Displacement} = \int_0^{12} v\,dt$$

$$= \left[2t^2 - \frac{t^3}{3} + 5t\right]_0^{12}$$

$$= 288 - \frac{1728}{3} + 60$$

$$\boxed{= -228\,m}$$

$$\text{dis} = \left|\int_0^5 4t - t^2 + 5\,dt\right| + \left|\int_5^{12} 4t - t^2 + 5\right|$$

$$\text{dis} = \left|\left[2t^2 - \frac{t^3}{3} + 5t\right]_0^5\right| + \left|\left[2t^2 - \frac{t^3}{3} + 5t\right]_5^{12}\right|$$

$$= \left|\left[50 - \frac{125}{3} + 25 - 0\right]\right| + \left|\left[288 - \frac{1728}{3} + 60 - 50 + \frac{125}{3} - 25\right]\right|$$

$$= \left|75 - \frac{125}{3}\right| + \left|-228 - 75 + \frac{125}{3}\right|$$

$$= \left|75 - \frac{125}{3}\right| + \left|-303 + \frac{125}{3}\right|$$

$$= \frac{225-125}{3} + \frac{784}{3}$$

~~$= \frac{100+178}{3}$~~

~~$= \frac{278}{3}$~~

$$\boxed{= \frac{884}{3}\,m}$$

744
12
288
144
1728

136

Q11. $a = -\cos t$

$t=0$; $v=0$, $x=1$

find

a) position at $t = \pi$

b) distance from $t = 0$ to $t = 2\pi$

a) $v = \int -\cos t \, dt$

$v = -\sin t + c$

$0 = c$

$v = -\sin t$

~~$x = \int v\,dt$~~

~~$x = \int -\sin\, dt$~~

~~$x = \cos t + c$~~

~~$0\ 1 = 90 + c$~~ ~~$1 = 1 + c$~~

~~$c = -89$~~

~~$x = \cos t - 89$~~

~~$x = \cos(180^\circ) - 89$~~

~~$\boxed{x = -90 \text{ m}}$~~

~~b) $v = 0$; for $t = 90, 180$, $\frac{\pi}{2}$, π, $\frac{\pi}{2}$, $\frac{2}{3}\pi$, 2π~~

$x = \int v\,dt$

$x = \cos t + c$

$1 = \cos 0 + c$

$1 = 1 + c$

$c = 0$

$x = \cos t$

$x = \cos(180^\circ)$

$\boxed{x = -1 \text{ m}}$

6) ~~cost~~ $v = 0$

$-\sin t = 0$

$\sin t = 0 \rightarrow t = 0$, $t = \pi$

$D = \left| \int v \, dt \right|$

$D = \left| \int_0^{\pi} -\sin t \, dt \right| + \left| \int_{\pi}^{2\pi} -\sin t \, dt \right|$

$D = \left| [\cos t]_0^{\pi} \right| + \left| [\cos t]_{\pi}^{2\pi} \right|$

$D = \left| \cos\pi - \cos 0 \right| + \left| \cos 2\pi - \cos\pi \right|$

$D = \left| -1 - 1 \right| + \left| 1 - (-1) \right|$

$= |-2| + |2|$

$\boxed{= 4 \ m}$ ✓

Q12. $V = 6t - 3t^2$ m/s

find average speed and average velocity for first 4 seconds.

~~$\cancel{V. \ x}$~~

$V = 0$

$0 = 6t - 3t^2$

$3t^2 - 6t = 0$

$\cancel{3t(3-t)}$

$3t(t-2) = 0$

$t = 2 \quad | \quad t = 0$

$$x_4 = \left| \int_0^2 6t - 3t^2 \, dt \right| + \left| \int_2^4 6t - 3t^2 \, dt \right|$$

$$= \left| \left[3t^2 - t^3\right]_0^2 \right| + \left| \left[3t^2 - t^3\right]_2^4 \right|$$

$$= |4| + |\cancel{0}' - 16 - 4|$$

$$= 4 + 20$$

$$= 24 \text{ m}$$

$$\frac{24}{4} = \boxed{6 \text{ m/s}}$$ Average speed

$$x = \int v\,dt$$

$$x\Big|_{t=4} = \int 3t^2 - t^3$$

$$\boxed{x = -16\ m}$$

$$\frac{-16}{4} = \boxed{-4\ m/s}$$ Average Velocity

Q 13. $v = (t-2)\ m/s$

calculate

distance & displacement from $t=0$ to $t=4$

$$x = \int t-2\ dt$$

$$x = \frac{t^2}{2} - 2t$$

$$x\Big|_{t=4} = \frac{16}{2} - 8$$

$$\boxed{= 0\ m}$$

$0 = t-2$

$t = 2$

distance $= \left|\int_0^2 t-2\ dt\right| + \left|\int_2^4 t-2\ dt\right|$

$$= \left|\left[\frac{t^2}{2} - 2t\right]_0^2\right| + \left|\left[\frac{t^2}{2} - 2t\right]_2^4\right|$$

$$= |-2| + |2|$$

$$= 2+2$$

$$\boxed{= 4\ m}$$

Q14. A Particle starts moving along straight line such that

$v = t^2 - t$ m/s

find the interval for which particle retards.

$a = \int t^2 - t \, dt$

$a = \frac{t^3}{3} - \frac{t^2}{2}$ m/s^2

$av < 0$

$\frac{t^3}{3} - \frac{t^2}{2} \left(t^2 - t\right) < 0$

$\frac{2t^3 - 3t^2 (2t^2 - t)}{6} < 0$

$\cancel{2}4t^5 - 2t^4 - 6t^4 + 3t^3 < 0$

$t^3 (4t^2 - 2t - 6t + 3) < 0$

$4t^2 - 2t - 6t + 3 < 0$

$\cancel{t(4t-2}$

$4t^2 - 8t + 3 < 0$

$4t^2 - 6t - 2t + 3 < 0$

$2t(2t-3) - \cancel{2}1(2t-3) < 0$

$(2t-1)(2t-3) < 0$

$t < \frac{1}{2}$ $\quad$ $t < \frac{3}{2}$ ×

Q14. A particle starts moving along straight line such that

$v = t^2 - t$ m/s

find interval when particle retards

$v = t^2 - t$

$a = \frac{d}{dx}(t^2 - t) = 2t - 1$

$av < 0$

$(2t-1)(t^2-t)$

$t(t-1)(2t-1) < 0$

$t - 1 < 0$ | $2t - 1 > 0$

$t < 1$ | $t > \frac{1}{2}$

or

$t - 1 > 0$ | $2t - 1 < 0$

$t > 1$ ✗ | $t < \frac{1}{2}$

Q15. Velocity of a particle moving in the (+)ve x-axis varies as $V = a\sqrt{x}$ where a is a constant & x is displacement

at $t=0$, $x=0$

find time dependence of velocity.

$\cancel{x = \int v\,dt}$

$\cancel{\frac{dx}{dt}}$

$\cancel{\frac{dv}{dt} \times \frac{dt}{dx}}$

$V = \frac{dx}{dt} = a\sqrt{x}$

$\int dx = \int a\sqrt{x}\,dt$

$\int \frac{dx}{a\sqrt{x}} = \int dt$

$\int \frac{1}{a\sqrt{x}}\,dx = t$

$\frac{1}{a}\frac{x^{1/2}}{\frac{1}{2}}$

$\frac{2\sqrt{x}}{a} = t$

$\sqrt{x} = \frac{at}{2}$

$\cancel{x = \frac{a^2t^2}{2}}$

$\sqrt{x} + C = t$

$C = 0$

$\boxed{x = \frac{a^2t^2}{4}}$

$\frac{dx}{dt} = V$

$\frac{d}{dt}\left(\frac{a^2t^2}{4}\right) = V$

$\boxed{\frac{a^2t}{2} = V}$

$\cancel{\frac{dv}{dt} = \frac{d}{dt}\left(\frac{a^2t}{2}\right) = \frac{a^2}{2}}$

Q16. $\vec{v} = 4t\hat{i} + 3\hat{j}$

at $t = 0$

$\vec{x} = 2\hat{j}$

a) find position vector at $t = 2s$

b) find average acc for $t = 0s$ से $t = 2s$.

$\cancel{\vec{v} = 4t\hat{i} + 3\hat{j}}$

$\cancel{|\vec{v}| = \sqrt{16t^2+9}}$

$\cancel{x = \int \sqrt{16t^2+9}\, dt}$

$\cancel{x = \int 4t\hat{i} + 3\hat{j}\, dt}$

$\cancel{x = 2t^2\hat{i} + 3t\hat{j}}$

$\cancel{x|_{t=2} = 8\hat{i} + 6\hat{j}}$

$\cancel{\int_0^t x = \int}$

$\vec{v} = 4t\hat{i} + 3\hat{j}$

$$\int_{2\hat{j}}^{x} x\,dt = \int_0^t 4t\hat{i} + 3\hat{j}\, dt$$

$$x - 2\hat{j} = \left[2t^2 + 3t\right]_0^t$$

$$x - 2\hat{j} = 2t^2\hat{i} + 3t\hat{j}$$

$$x|_{t=0} = 2(2)^2\hat{i} + 3(2)\hat{j} + 2\hat{j}$$

$$\boxed{r = 8\hat{i} + 8\hat{j}}$$

$\cancel{\int a\, dt}$

$$\cancel{avg\ a = \int_0^2 4t\hat{i} + 3\hat{j}\ dt}$$

=

$$a_{avg} = \frac{\vec{v} - \vec{u}}{T}$$

$$= \frac{8\hat{i} + 3\hat{j} - 3\hat{j}}{2}$$

$$\boxed{= 4\hat{i}\ m/s^2}$$

Q17. $\vec{a} = 2\hat{i} - \hat{j}$

$t = 0,\ u = 0,\ \vec{x} = 3\hat{i} + \hat{j}$

find $\vec{x}$ at time t

$$\int_0^v dv = \int_0^t 2\hat{i} - \hat{j}\ dt$$

$$v = 2t\hat{i} - t\hat{j}$$

$$\int_{3\hat{i}+\hat{j}}^{x} d\vec{x} = \int_0^t 2t\hat{i} - t\hat{j}$$

$$x - 3\hat{i} - \hat{j} = t^2\hat{i} - \frac{t^2}{2}\hat{j}$$

$$x = t^2\hat{i} + 3\hat{i} - \frac{t^2}{2}\hat{j} + \hat{j}$$

$$\boxed{x = (t^2+3)\hat{i} - \left(\frac{t^2-2}{2}\right)\hat{j}}$$

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Q18. A particle with velocity v_0 at $t=0$ is decelerated at rate $|a| = \alpha\sqrt{v}$ (α is a +ve constant)

a) find time after which it comes to rest.
b) distance.

$\cancel{a = \alpha\sqrt{v}}$

$\cancel{\frac{dv}{dt} = \alpha\sqrt{v}}$

$\cancel{dv = \alpha\sqrt{v}\,dt}$

$\cancel{\int_0^v dv = \int_0 \alpha\sqrt{v}\,dt}$

$\cancel{v =}$

$\cancel{\int \frac{da}{dv} = \alpha\sqrt{v}}$

$\cancel{da = \alpha\sqrt{v}\,dv}$

$\cancel{a = \frac{2\alpha\sqrt{v^3}}{3}}$

$\cancel{\alpha\sqrt{v} = \frac{2\alpha\sqrt{v^3}}{3}}$

$\cancel{v = \frac{4v^3}{9}}$

$\cancel{\frac{9}{4} = v^2}$

$\cancel{v = \frac{3}{2}}$

$a = -\alpha\sqrt{v}$

$\frac{dv}{dt} = -\alpha\sqrt{v}$

$\frac{dv}{\sqrt{v}} = -\alpha\,dt$

$\left[2\sqrt{v}\right]_{v_0}^{v} = -\alpha t$

$2\sqrt{v} - 2\sqrt{v_0} = -\alpha t$

$\boxed{2\sqrt{v} = 2\sqrt{v_0} - \alpha t}$

a) v = 0

$0 = 2\sqrt{v_0} - \alpha t$

$\alpha t = 2\sqrt{v_0}$

$\boxed{t = \frac{2\sqrt{v_0}}{\alpha}}$

b) $\sqrt{v} = \sqrt{v_0} - \frac{\alpha t}{2}$

$v = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$

$\frac{dx}{dt} = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$

$\int_0^x dx = \int_0^{\frac{2\sqrt{v_0}}{\alpha}} \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2 dt$

$x = \left[\frac{\left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^3}{3\left(\frac{-\alpha}{2}\right)}\right]_0^{\frac{2\sqrt{v_0}}{\alpha}}$

$$x = \frac{-2}{3\alpha}\left[\left(\sqrt{V_0} - \frac{\alpha \times 2\sqrt{V_0}}{2\alpha}\right)^3 - \left(\sqrt{V_0}\right)^3\right]$$

$$x = \frac{-2}{3\alpha}\left[\sqrt{V_0} - \sqrt{V_0}\right]^3 - \left(\sqrt{V_0}\right)^3$$

$$x = \frac{-2}{3\alpha} \times -\left(\sqrt{V_0}\right)^3$$

$$\boxed{x = \frac{2\left(\sqrt{V_0}\right)^3}{3\alpha}}$$

Equations of Motion :-

① Velocity Time relation ($v = u + at$)

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$\int_u^v dv = \int_0^t a\,dt$$

$$[v]_u^v = [at]_0^t$$

$$v - u = at$$

$$\boxed{a = \frac{v-u}{t}}$$

② Displacement time relation ($s = ut + \frac{1}{2}at^2$)

$$v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$v = u + at$$

$$dx = u + at\ dt$$

$$\int_0^s dx = \int_0^t u + at\ dt$$

$$s = \left[ut + \frac{at^2}{2}\right]_0^t$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

(3) Velocity - Displacement relation ($v^2 = u^2 + 2as$)

$$a = v\frac{dv}{dx}$$

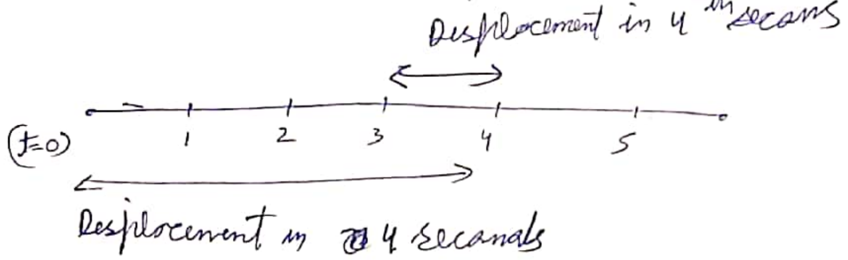
$$\int_u^v v\,dv = \int_0^s a\,dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a\left[x\right]_0^{x=s}$$

$$\frac{v^2}{2} - \frac{u^2}{2} = as$$

$$\boxed{v^2 - u^2 = 2as}$$

(4) Displacement in n^{th} second :-



$$S_{n^{th}} = S_n - S_{(n-1)}$$

$$= un - \frac{1}{2}an^2 - \left[u(n-1) - \frac{1}{2}a(n-1)^2\right]$$

$$= un - \frac{an^2}{2} - \left[un - u - \frac{an^2}{2} - \frac{a}{2} + \frac{2an}{2}\right]$$

$$= un - un - \frac{an^2}{2} + \frac{an^2}{2} + u + \frac{a}{2} - an$$

$$= u + an - \frac{a}{2}$$

$$\boxed{= u + \frac{a}{2}(2n-1)} \qquad \boxed{S_n^{th} = u + \frac{a}{2}(2n-1)}$$

Q A Train is travelling at speed 90 km/hr breaks are applied so as to produce a uniform retardation of $0.5 m/s^2$. find how far the train goes before comming to rest.

$$u = 90 \times \frac{5}{18} = 5 \times 5 = 25 m/s$$

$$a = 0.5$$

$$v^2 - u^2 = 2as$$

$$\frac{+625}{2 \times 0.5} = s$$

$$\boxed{s = 625 m}$$

Q2. A particle moving with constant acceleration from A to B in a straight line AB has velocities u & v at A & B. find velocity while passing through if C is the mid of AB, C.

$$s = \frac{AB}{2}$$

$$u = u$$

$$V_c = ?$$

$$a = a$$

$$V_c^2 = u^2 + 2a\frac{s}{2}$$

$$V_c^2 = u^2 + \cancel{2}as$$

$$\boxed{V_c = \sqrt{u^2 + a\cancel{B}AB}}$$

$\cancel{a = \frac{v-u}{t}}$

$\cancel{V_c^2 = 2a \times \frac{AB}{2} - v^2}$

$$V_c^2 = v^2 - a AB$$

$$\sqrt{v^2 - aAB} = V_c$$

$$2V_c^2 = v^2 + v^2$$

$$x = \boxed{\frac{v^2 + u^2}{2}}$$

Q3. A particle starts from rest with uniform ~~velocity~~ acceleration a. Its velocity after n seconds is v. Find the displacement of body in last two seconds

$u = 0$

$v = v$

$t = n$

$$S^{th}_{(n-2)} = u + \frac{a}{2}(2n-1)$$

$$= u + \frac{a}{2}(2(n-2)-1)$$

$$= u + \frac{a}{2}(2n-4-1)$$

$$= u + \frac{a}{2}(2n-5)$$

$$= u + an - \frac{5a}{2}$$

$$S^{th}_{n} + S^{th}_{(n-1)} = u + \frac{a}{2}(2n-1) + u + \frac{a}{2}(2n-3)$$

$$= 2u + an - \frac{a}{2} + an - \frac{3a}{2}$$

$$= 2u + 2an - 2a$$

$$= 2(u + a + an)$$

$$= 2[u + a(1+n)]$$

or

$$= 2[a(n-1)]$$

$u = 0$ or

$v = an \ldots (1) \ldots$

Disp in last 2 sec $= S_n - S_{(n-2)}$

$$= \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= \frac{a}{2}(n^2 - n^2 + 4 - 2n)$$

$$= \frac{a}{2}(4 - 2n) = 2a - an$$

$$= 2a(n-1)$$

Q4. A body starting from rest moves with constant acceleration. Find the ratio of distance covered during fifth to that covered in five seconds.

$$= \frac{\frac{a}{2}(2n-1)}{\frac{1}{2}an^2}$$

$$= \frac{a(10-1)}{2 \times 5^2}$$

$$= \frac{9}{25}$$

$\boxed{9:25}$

Q5. A particle moves with constant acceleration for 6 sec. after starting from rest. Find the ratio of distance travelled it during the intervals of two seconds each.

$a = a$

$t = 6$

$u = 0$

$$\frac{S_4 - S_2}{S_2 - S_0}$$

$$\frac{\frac{a}{2} \times 4^2 - \frac{a}{2} 2^2}{\frac{a}{2} \times 2^2 - \frac{a}{2} \times 0^2}$$

$$\frac{8a - 2a}{2a}$$

$$= \frac{6a}{2a}$$

$\boxed{= 3:1}$

$$\cancel{36-8}\ \frac{18-8}{8-2}$$

$$= \frac{10}{6}$$

$$= \frac{5}{3}$$

$\cancel{\boxed{5:3:1}}$

$\boxed{1:3:5}$

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Q6. A driver driving a truck at a constant speed of 20 m/s suddenly saw a parked car ahead of him by 95 m. He could apply the brake after some time to produce retardation of 2.5 m/s². If an accident was ~~just~~ just avoided, find his reaction time.

$a = -2.5 m/s^2$

$u = 20 m/s$

$v = 0$

$S = 95 m$

$$\frac{0 - 20}{-2.5} = t$$

$8s = t$

$S = 20(8) + \frac{1}{2}(-2.5)(64)$

$S = 160 - 80$

$S = 80 m$

$t = \frac{15}{20}$ $\boxed{\frac{3}{4} \text{ sec}}$

Q 7. A body starts its motion with initial velocity of 9 m/s towards ~~we~~ east and ~~its~~ its acceleration is $2 m/s^2$ west. find distance covered in fifth second of its motion.

$\cancel{u = 9 m/s}$

$\cancel{a = -2 m/s^2}$

$\cancel{S_{5^{th}} = 9 + \frac{-2}{2}(10-2)}$

$\cancel{= 9 - 8}$

$\cancel{\boxed{= 1 \text{ meter}}}$

$\cancel{S = 9 \times 4.5 - \frac{1}{2} \times 2 (4.5)^2}$

$\cancel{= 4.5 (9 - 4.5)}$

$\cancel{= (4.5)^2}$

$\cancel{S = 9 \times 0.5 + \frac{1}{2} \times 2 (0.5)^2}$

$\cancel{= 0.5(9 - 0.5)}$

$\cancel{= 8.5 \times 0.5}$

$\cancel{= 4.25 \text{ cm} \times 2}$

$\cancel{\boxed{= 8.5 \text{ meter}}}$

$$
\begin{array}{r}
45 \\
45 \\
\hline
225 \\
1800 \\
\hline
2025
\end{array}
$$

$S = AC - AB$

$= ut + \frac{1}{2} at^2 - ut + \frac{1}{2} at^2$

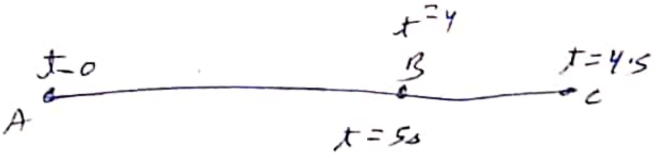
$= 9 \times 4.5 - \frac{1}{2} \times 2 (4.5)^2 - 9 \times 4 - \frac{1}{2} \times 2 (4)^2$

$= 4.5 \times 4.5 - 20$

$= 20.25 - 20$

$= \frac{1}{4} \times 2$

$\boxed{= \frac{1}{2} \text{ meter}}$



Q 8. A Particle starts with velocity 10 m/s & deceleration 5 m/s^2 find displacement & distance covered in 6 s.

$S = \frac{1}{2} \times 5 \times \cancel{36}\ 18$

$\cancel{\boxed{S = 90\ m}}$

$S = 60 - 90$

$\boxed{S = -30\ m}$

$S_1 = 20 - \frac{1}{2} \times 5 (4) 2$

$S_1 = 10\ m$

$S_2 = \frac{1}{2} \times 5 (4) \times 4^2$

$S_2 = 40\ m$

$\boxed{S_1 + S_2 = 50\ m}$

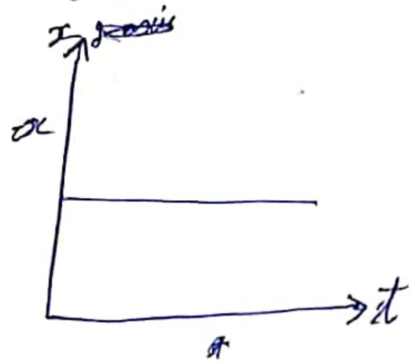
H. W. 06-05-2024

Pg 47 → Q1-25

Graphs

1) Position - Time Graph (x-t graph)

a) At rest



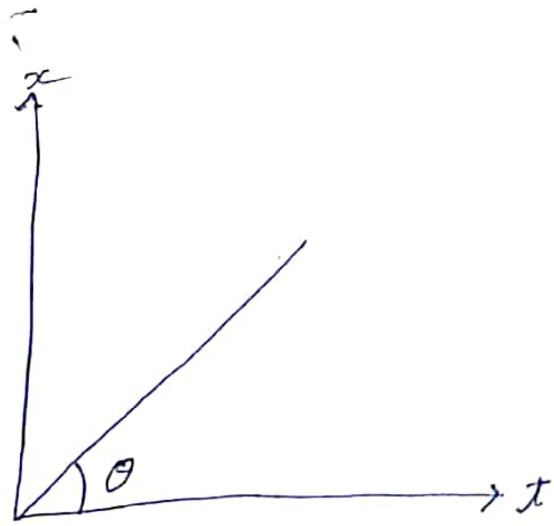
→ straight line parallel to ~~x~~ time axis

→ slope $= \frac{dx}{dt} = v = 0$

$= \tan\theta = \tan 0^\circ = 0$

↳ angle with time axis.

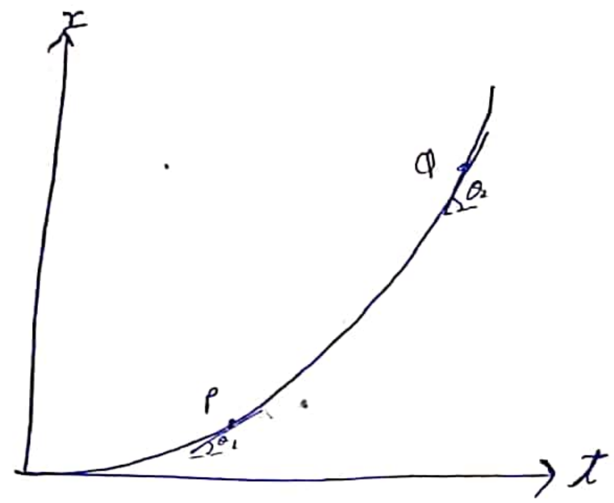
b) Uniform velocity



slope $= \tan\theta =$ constant

$= \frac{dx}{dt} = v =$ constant

iii) Non-Uniform Velocity (increasing with time) :-



Smiling Curve
(a is > 0)

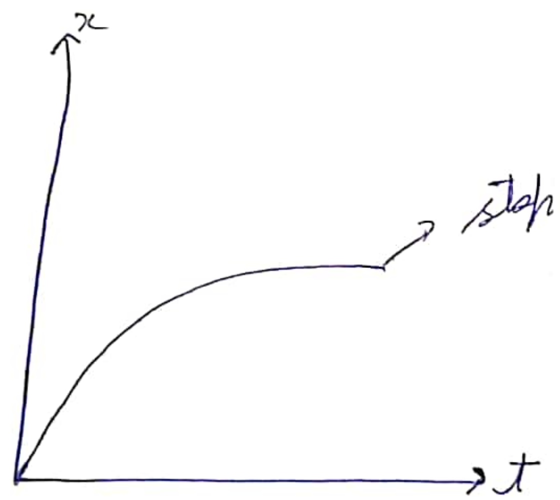
$\theta_1 < \theta_2$

$\tan\theta_1 < \tan\theta_2$

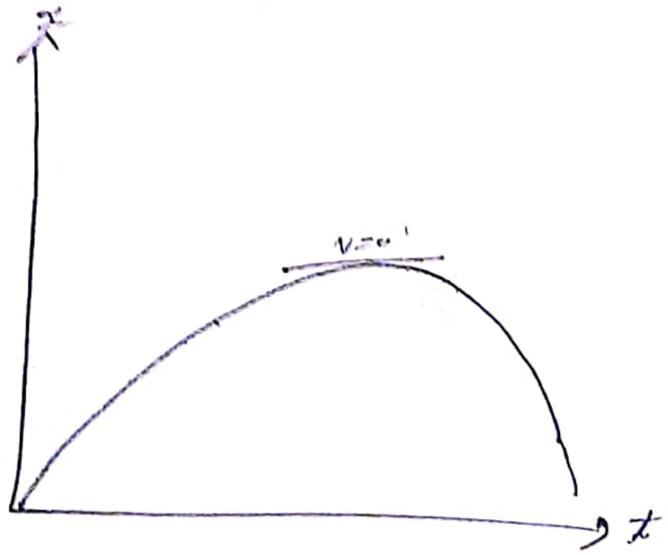
$v_1 < v_2$

so velocity is increasing.

iv) Non-Uniform Velocity (decreasing with time)



body finally stops

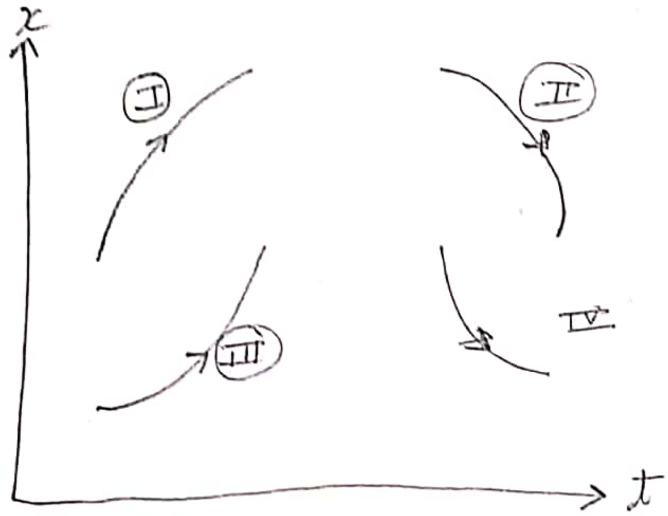


Crying curve
(a < 0)

$\frac{d^2x}{dt^2} < 0$

body reverse direction

Q1.



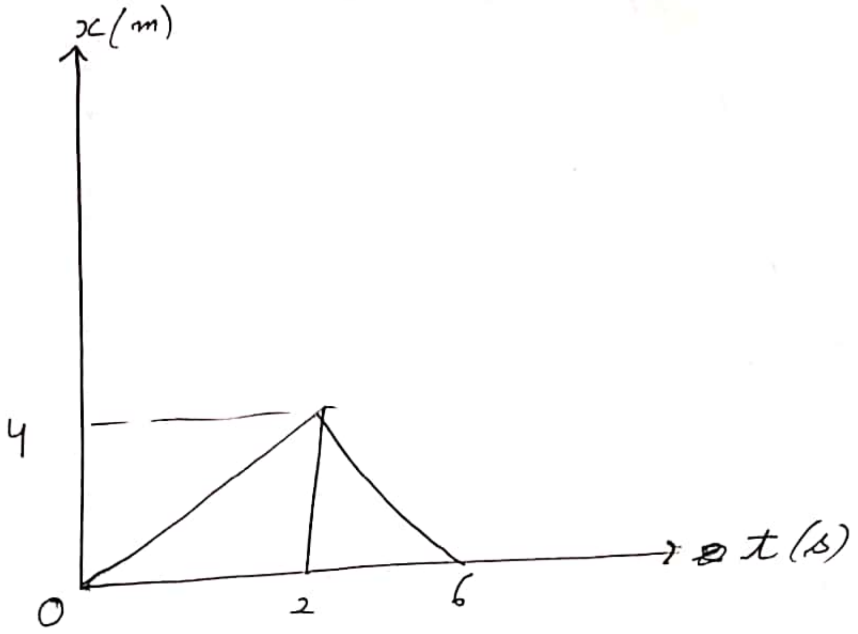
I - velocity > 0, acceleration < 0

II - V < 0, a < 0

III - V > 0, a > 0

IV - V < 0, a > 0

Q2.



find velocity

a) 0-2s

b) 2-6s

a) $\frac{dx}{dt}$ = velocity

$\cancel{v = \frac{d}{dt}}$ $v = \frac{d}{t}$

$v = \frac{4}{2}$

$\boxed{v = 2\,m/s}$

b) $v = \frac{d}{t}$

$= \frac{-4}{4}$

$\boxed{v = -1\,m/s}$

2) Velocity - Time Graph (v-t graph)

a) <u>Uniform Velocity</u> -

$v =$ constant

$a = 0$

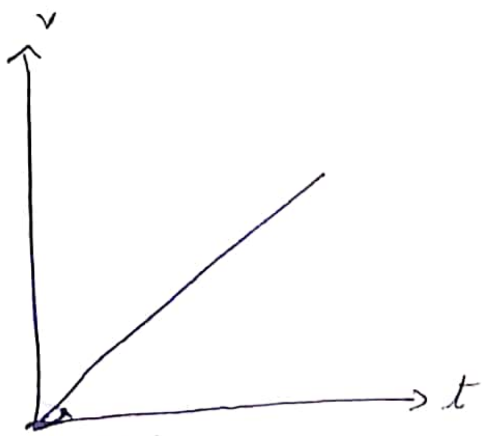
$\frac{dv}{dt} = 0$

slope $= \frac{dv}{dt} = 0$

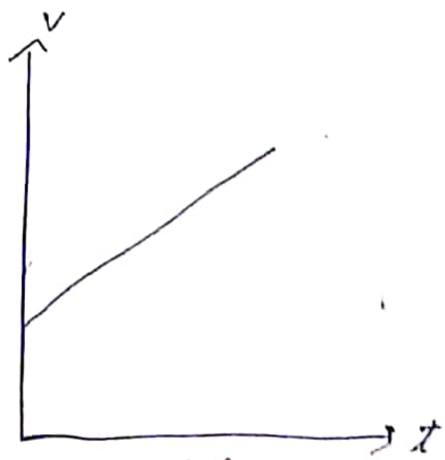
6) <u>Uniform Velocity</u>

$a =$ const

$\frac{dv}{dt} =$ constant $=$ slope

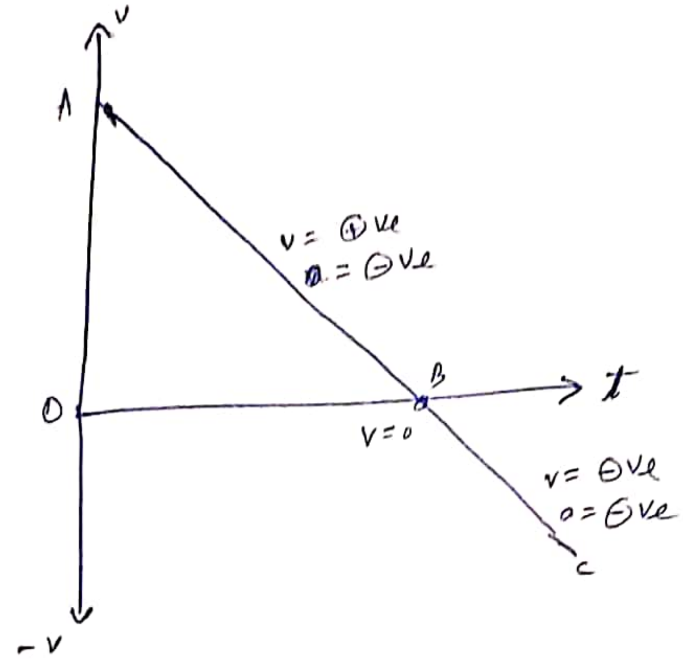


$u = 0$



$u \neq 0$

c) Uniform ~~Acceleration~~. Retardation

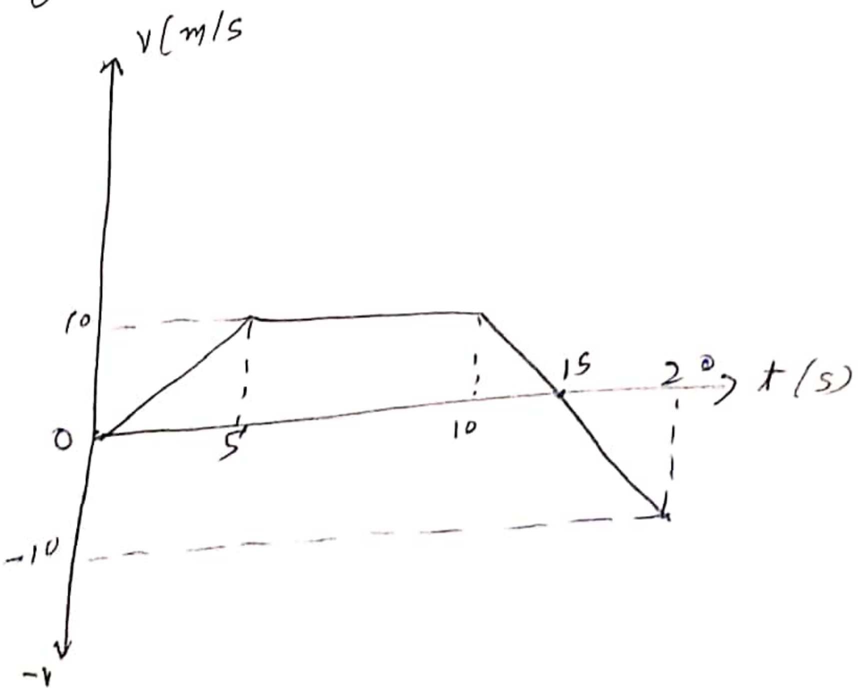


A → B
v = +ve & decreasing

B →
v = 0 & after that body reversed direction

B → c body is increasing speed in ⊖ve direction.

Q Calculate distance travelled & ~~a~~ & displacement of body. find acceleration from 0 – 5 s & 10 – 15 s.



~~Ans.~~

$~~S(\text{displacement}) = \frac{1}{2} \times \frac{10}{5} \times 20 \times 20~~$

$~~= 200 \times 2~~$

$~~= 400~~$

$~~S_1 = \frac{1}{2} \times \frac{10}{5} \times 15 \times 15~~$

Distance $= \frac{1}{\cancel{2}} \times \cancel{10}^{5} \times 5\cancel{+}15 + 10 \times 5 \times \frac{1}{2}$

$= 100 + 25$

$\boxed{= 125\ m}$

Displacement $= 100 - 25$

$\boxed{= 75\ m}$

$a_{0-5} \rightarrow \frac{10}{5}$

$\boxed{= 2\ m/s^2}$

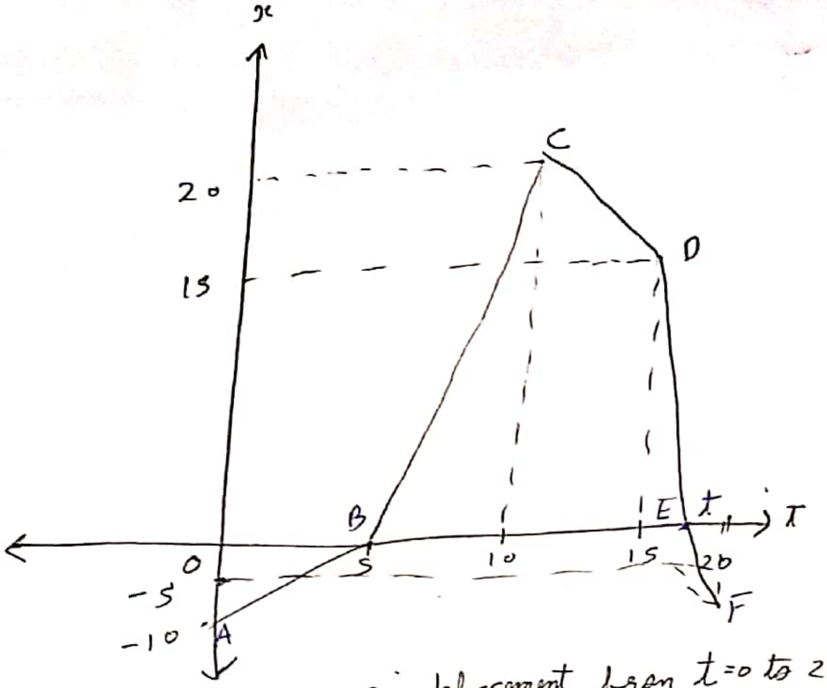
$a_{10-15} \rightarrow -\frac{10}{5}$

$\boxed{= -2\ m/s^2}$

H.W. 07-05-2024

Pg 55 - Q1-15

Q 1.



a) find distance covered & Displacement from $t=0$ to $20s$.

b) find average speed & velocity from $t=0$ to ~~15~~ 15 sec.

c) find velocity of particle in $t=5$ to $10s$

d) find velocity of particle in $t=10$ to $15s$

a) Distance = $10+20+5+15+5$

$\boxed{= 55\,m}$ ✓

Displacement $\boxed{= +5\,m}$ ✓

e) find t

f) How many time particle has taken turn during $t=0$ to $20s$

b) average speed $= \frac{35}{120}$ $= \boxed{\frac{7}{3}\,m/s}$

~~= 2.75 m/s~~

average velocity $= \cancel{\frac{+5}{20}}$ $\frac{25}{15} = \boxed{\frac{5}{3}\,m/s}$

~~= +0.25 m/s~~

c) ~~a~~ velocity $\Big|_{t=5 \text{ to } 10s}$ $= \frac{\text{distance}}{\text{Time}}$

$= \frac{20}{5}$

$\boxed{= 4\ m/s}$ ✓

d) velocity $\Big|_{t=10-15s}$ $= \frac{-5}{5}$

$\boxed{= -1\ m/s}$ ✓

e) ~~speed~~velocity $= \frac{20}{5} = 4$

dis $= 15$

$\boxed{\text{time} = \frac{15}{4}\ s}$

$t = 15 + \frac{15}{4}\ s$

$= 15 + 3.75$

$\boxed{= 18.75\ s}$ ✓

or

DEF is a straight line

$\text{slope}_{DE} = \text{slope}_{EF}$

$\frac{0-15}{t-15} = \frac{-5-0}{20-t}$

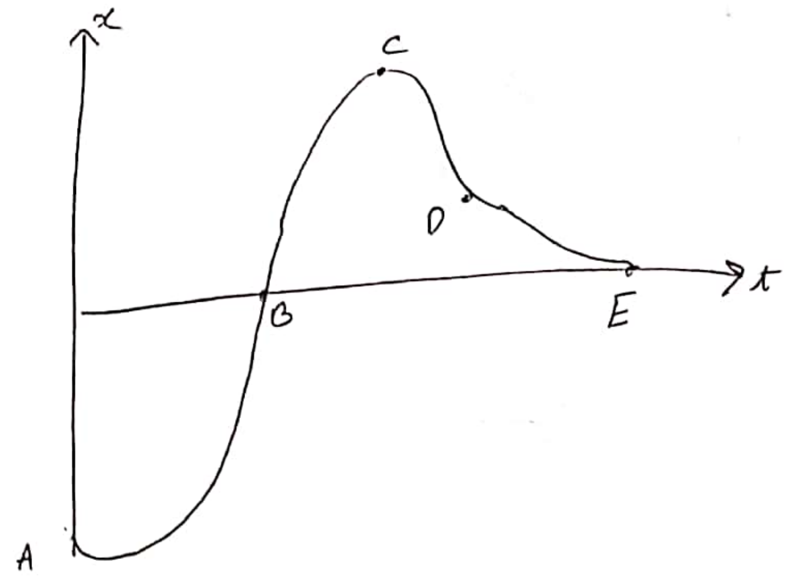
$-300 + 15t = -5t + 75$

$20t = 375$

$\boxed{t = \frac{375}{20}\ s}$ ✓

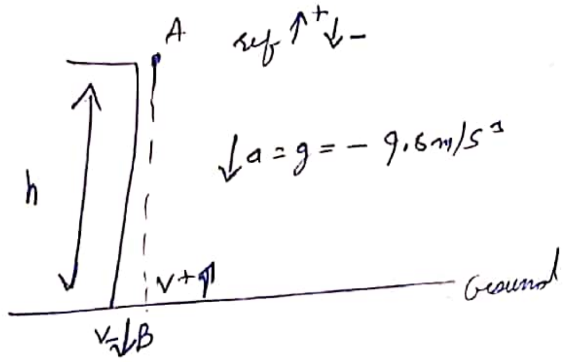
f) $\boxed{1 \text{ time}}$ at Point C ✓

Q2.

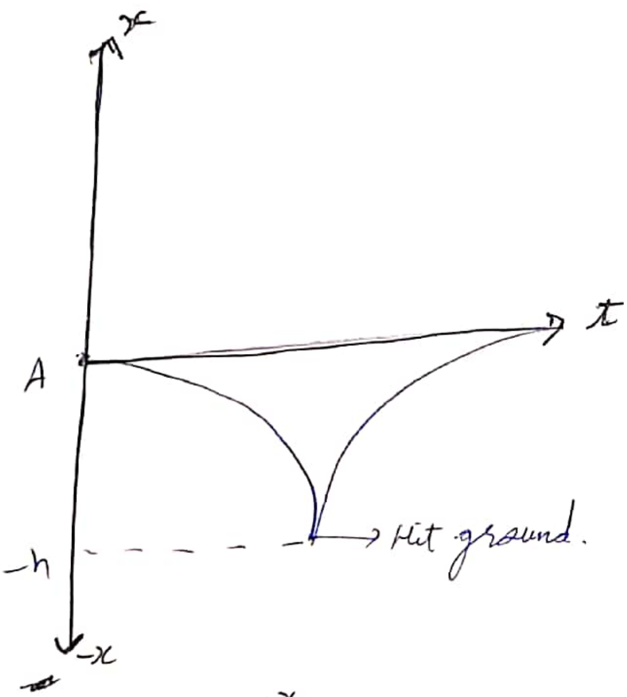


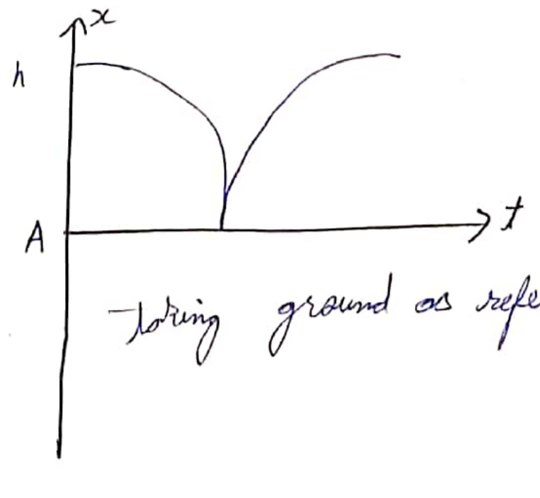
	v	a
A	0	+
B	+	0
C	0	−
D	−	+
E	0	+

Q3. A body is dropped from a height, It strikes the ground elastically. draw all the 3 ~~[illegible]~~ kinematic graphs taking upward direction as +ve.

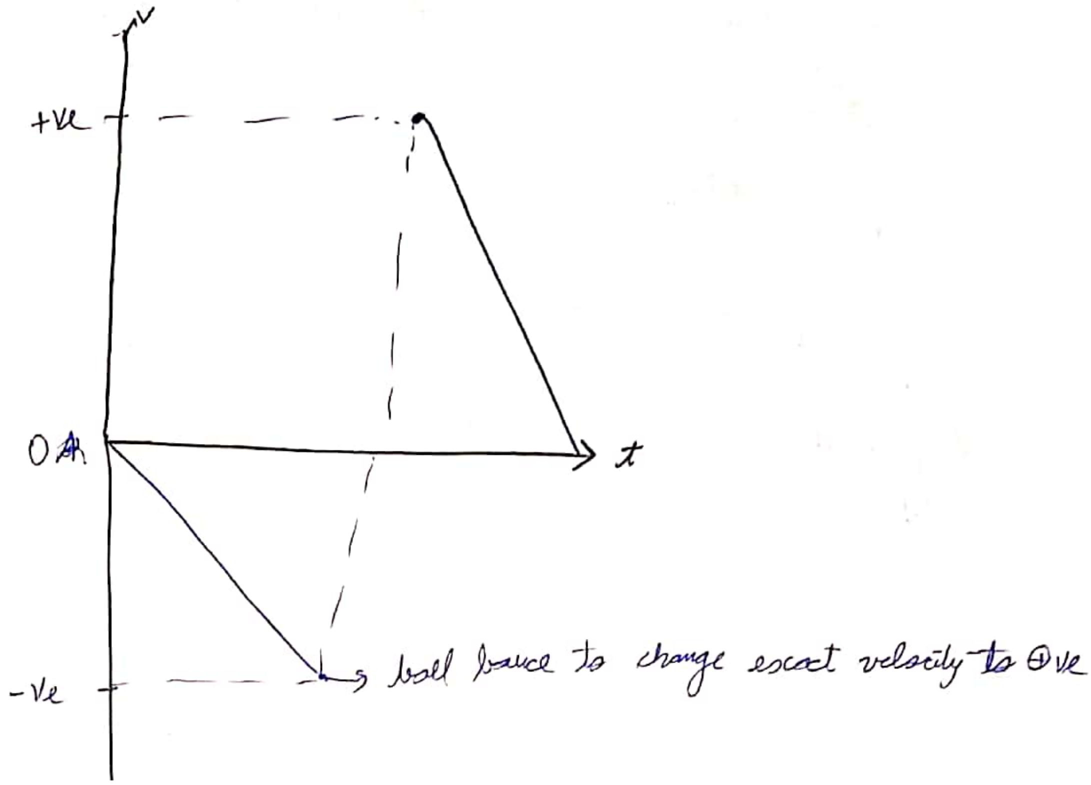


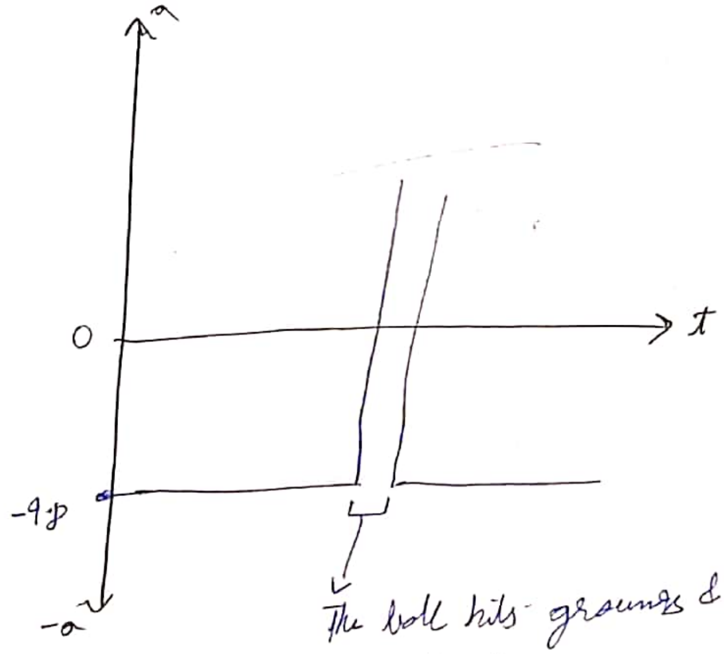
A → B speeding up
B → A slowing down





Taking ground as reference



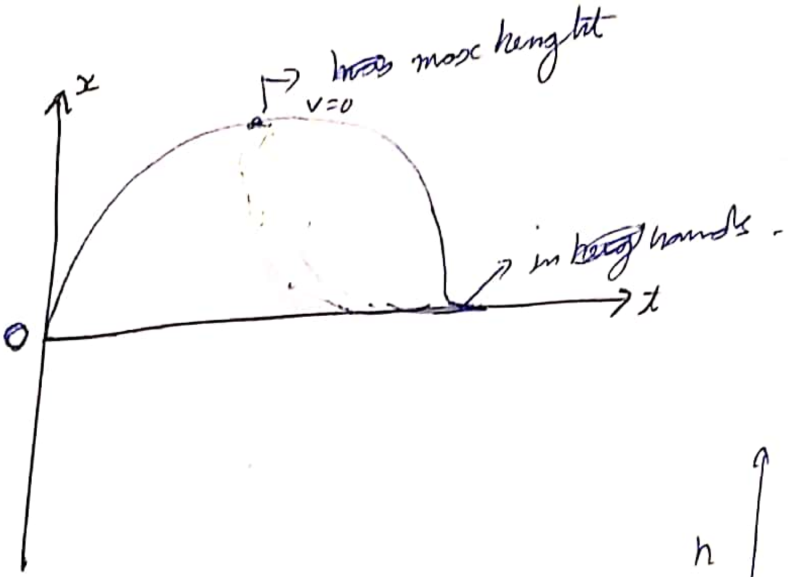


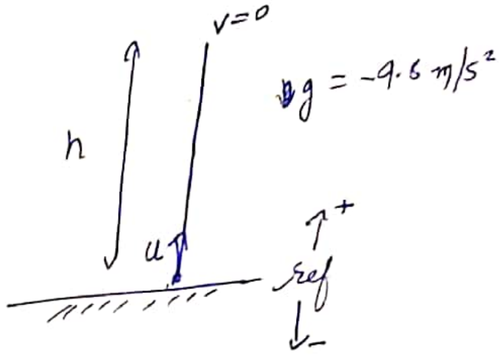
The ball hits grounds & changes vel from
$-v$ to $+v$

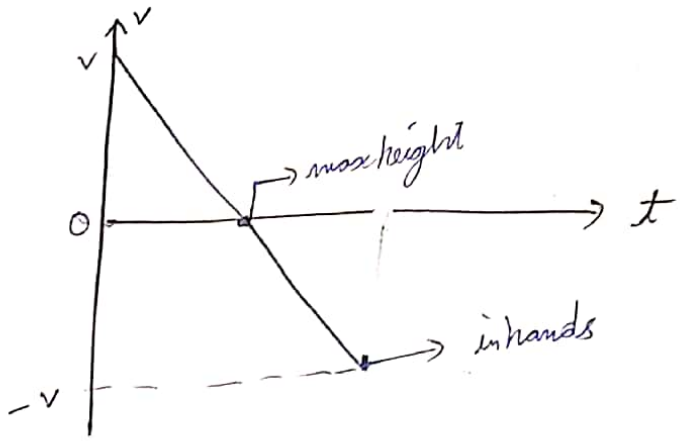
$a = \frac{\Delta v}{\Delta t \to 0}$

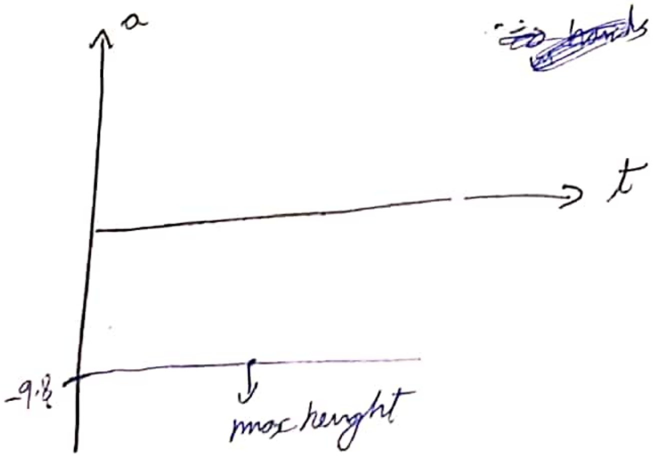
so $a =$ infinite in +ve direction

Q:- A body thrown upwards with some velocity. It reaches max height and again reaches hands of thrower. Draw all 3 kinematic graphs (take upward as +ve)

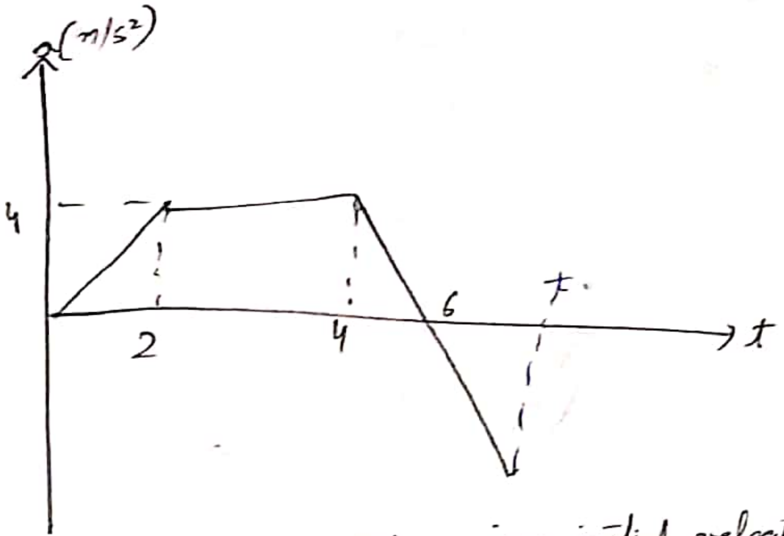








Q 5.



a) find time at which particle aquires initial velocity.

initial velocity (u) = ar (+ve)

$= \frac{1}{2} \times (2+6) \times 4$

$\boxed{= 16 \text{ m/s}}$

$v = 16$ m/s

$\frac{1}{2} \times t \times a = 16$

$t \times a = 32$

$\frac{a}{t} = 2$

$a = 2t$

$2t \times t = 32$

$t^2 = 16$

$t = 4$, $\boxed{-4 \times}$

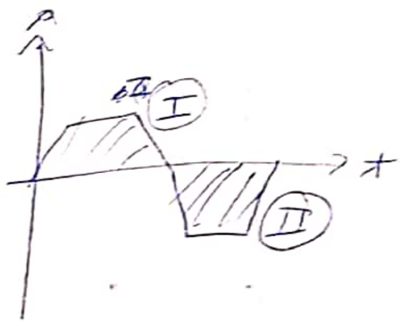
$4 + 6 = t$

~~10 = t~~

$\boxed{t = 10 \text{ s}}$

theory

area = change in velocity



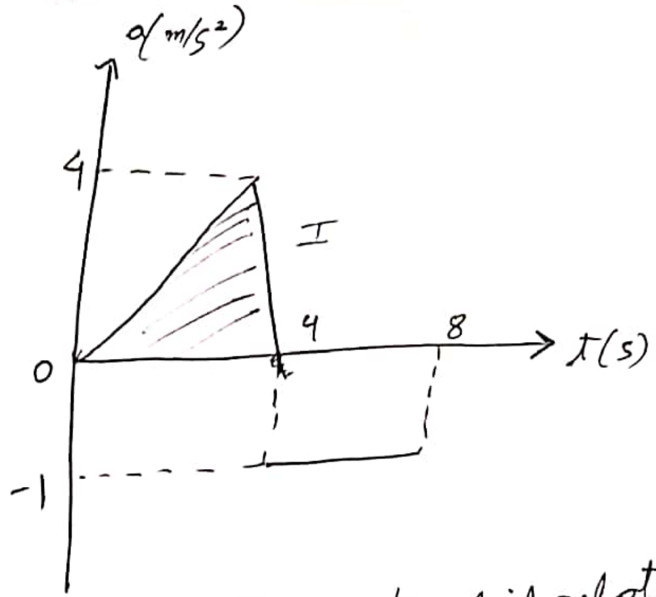
$\Delta v = I - II$

$\Delta = 0$

$I - II = 0$

$I = II$

Q6.



b) a) If initial velocity = 3 m/s find vel at t = 8 s

$I = \frac{1}{\cancel{2}} \times 4 \times \cancel{4}^{2}$

$= 8$

$\Delta v = 8 \text{ m/s}$

$u = 3$

$v = 11 \text{ m/s}$ at $t = 4$

$a = (-1) \text{ m/s}^2$

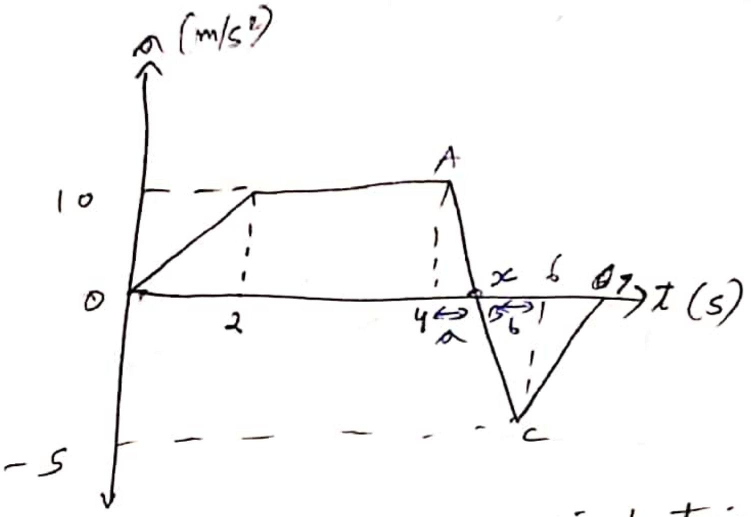
$\frac{v-u}{t} = a$

$v = (-1)(4) + (11)$, where $4 = (8-4)$

$v = -4 + 11$

$\boxed{v = 7 \text{ m/s}}$ ✓

Q 7.



If body starts from rest find time at which velocity is maximum.

max v, a = 0

v = x

$slope_{AB} = slope_{BC}$

$\frac{10}{a} = \frac{5}{b}$

$10b = 5a$

$\underline{2b = a}$

$a + b = 2$	$a = 2 - \frac{2}{3}$
$3b = 2$	$\underline{a = \frac{4}{3}}$
$\underline{b = 2/3}$	

$4 + \frac{4}{3}$

$\frac{12+4}{3}$

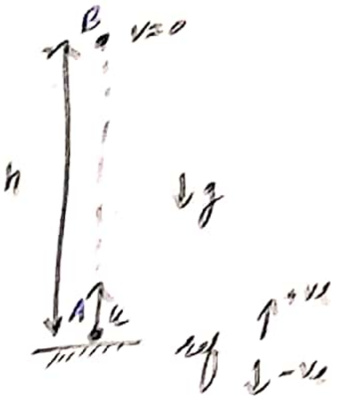
$\boxed{\frac{16}{3}\,s}$

$\boxed{5.33\,s}$

$3\overline{)16}(5.3$
-15
10
-9
10

Motion Under Gravity

① Case 1 →



→ Take point of projection as reference.

$$0-(u)^2 = 2(-g)(+h)$$

$$\boxed{h = \frac{u^2}{2g}} \text{ (maximum hight)}$$

→ Time of flight = time of ascent + time of descent

$v = u + at$

$A \to B$

$0 = u + (-g)t$

$$\boxed{t_1 = \frac{u}{g}}$$

time of ascent = time of descent

$$\boxed{\text{time of flight} = \frac{2u}{g}}$$

$B \to A$

$s = ut + \frac{1}{2}at^2$

$-h = 0 + \frac{1}{2}(-g)t^2$

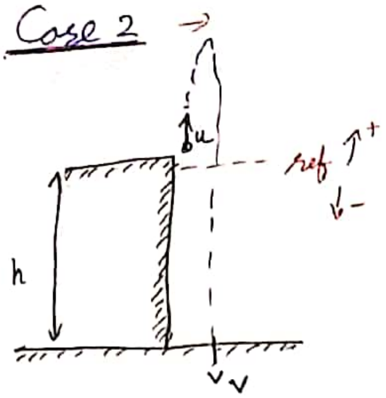
$\frac{u^2}{2g} = \frac{1}{2}gt^2$

$\frac{u^2}{2g} \times \frac{2}{g} = t^2$

$\frac{u^2}{g^2} = t^2$

$$\boxed{t_2 = \frac{u}{g}}$$

② Case 2 →



$$v^2 = u^2 + 2as$$

$$(-v)^2 = u^2 + 2(-g(-h))$$

$$v^2 = u^2 + \cancel{g}\,2gh$$

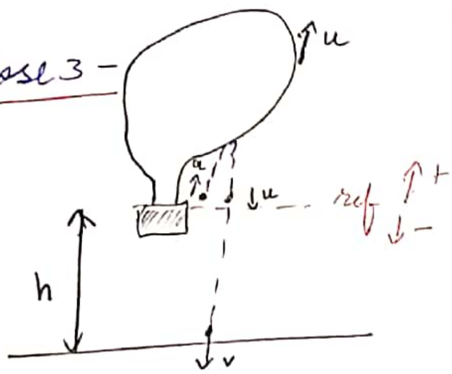
$$\boxed{v = \sqrt{u^2 + 2gh}}$$

→ To calculate time of flight

$$s = ut + \frac{1}{2} a t^2$$

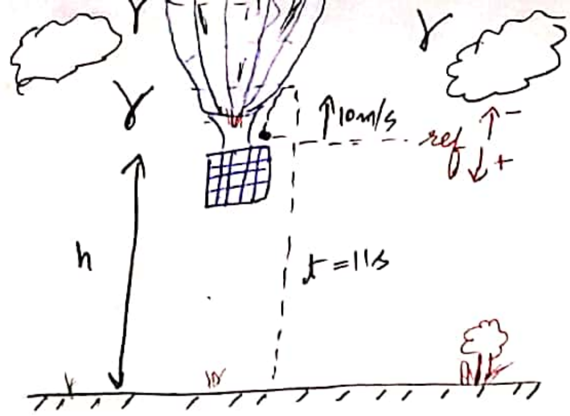
$$\boxed{-h = ut + \frac{1}{2}(-g) t^2}$$

③ Case 3 –



u = velocity of balloon.

Q



A packet is released,
$g = 10\ m/s^2$
$h = ?$

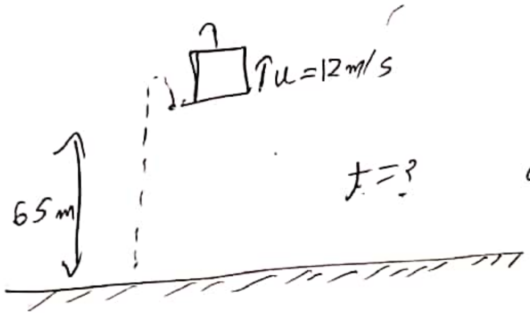
$s = ut + \frac{1}{2}at^2$

$h = (-10)(11) + \frac{1}{2} \times 10 \times 121$

$h = -110 + 605$

$\boxed{h = 495m}$

Q2.



$g = 10m/s$

$65 = -12t + \frac{1}{2} \times 10t^2$

$65 = -12t + 5t^2$

$5t^2 - 12t - 65$

$t = \frac{+12 \pm \sqrt{144 + 1300}}{10}$

$t = \frac{+12 \pm \sqrt{1444}}{10}$

$t = \frac{+12 + 38}{10}$

$t = \frac{+50}{10}$

$\boxed{t = 5s}$

$3 \overline{)1444}$ 268, 9, 544

2, 65, 5, 325, 4, 67, 7, 469, 6, 69, 9, 544

Q3. A balloon starts accending from rest at constant ~~a~~ $a = 2\,m/s^2$ when it was at 100 m height a pocket was released from it. after ~~t~~ how much time and with what velocity will it hit ground.

~~$u = 0$~~

$v^2 - 0 = 2 \times 2 \times 100$

$v^2 = 400$

$\boxed{v = 20\,m/s}$

for pocket

$u = -20\,m/s$

$100 = -20t + \frac{1}{2} \times 10 (t^2)$

$100 = -20t + 5t^2$

$20 = -4t + t^2$

$t^2 - 4t - 20 = 0$

$t = \frac{+4 \pm \sqrt{16 + 80}}{2}$

$t = \frac{4 \pm \sqrt{96}}{2}$

$t = 2 \pm \sqrt{24}$

$\boxed{t = 2 \pm 2\sqrt{6}\,s}$

$\boxed{t = 2 + 2\sqrt{6}\,s}$

$v^2 = u^2 + 2as$

$= 400 + 2 \times 10 \times 100$

$= 400 + 2000$

$v^2 = 2400$

$v = \sqrt{2400}$

$v = 10\sqrt{24}$

$\boxed{v = 20\sqrt{6}\,m/s}$

$\boxed{v = 20\sqrt{6}\,m/s}$

Q4. A parachutists bails out from an aeroplane and after 40 m, he ~~S = 40 m~~ opens parachute and decelerates at 2 m/s^2. If he reaches ground with speed 2 m/s, How long was he in air. At what height he jumped off the plane? ($g = 10\ m/s^2$)

~~$v = 2\ m/s$~~

$S = 40\ m$

$u = 0$

$g = 10\ m/s$

$v^2 = 2 \times 10 \times 40$

$v^2 = 800$

$\boxed{v = 20\sqrt{2}}$ ✓

$40 = \frac{1}{2} \times 10 \times t^2$

$80 = t^2$

$t^2 = 8$

$\boxed{t_1 = 2\sqrt{2}}$ ✓

After Parachute

$u = 20\sqrt{2}$

$a = -2\ m/s$

$v = 2\ m/s$

$v^2 - u^2 = 2as$

$4 - 800 = 2 \times -2 \times s$

$\frac{796}{4} = s$

$\boxed{S = 199\ m}$ ✓

$\frac{2 - 20\sqrt{2}}{-2} = t$

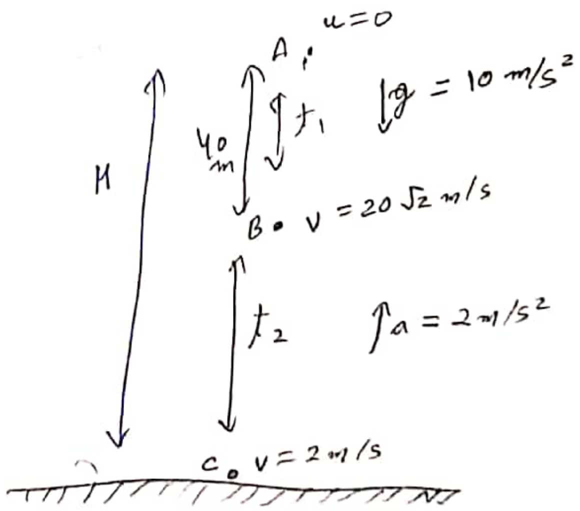
$-1 + 10\sqrt{2} = t$

$\boxed{t = 10\sqrt{2} - 1}$ ✓

Total height $= 199 + 40 = \boxed{239\ m}$ ✓

Total time $= 10\sqrt{2} - 1 + 2\sqrt{2}$

$\boxed{= 12\sqrt{2} - 1\ s}$ ✓



Q 5. A body throw from top of a cliff vertically up with velocity v, takes time t_1, the same body when thrown vertically down with same velocity takes time t_2. If dropped freely takes time t, find relation among three times.

$S = vt_1 + \frac{1}{2} at_1$

$S = \frac{1}{2} at^2$

$S = -vt + \frac{1}{2} a t_2^2$

$2S = -2vt_1 + gt_1^2$

$= gt^2$

$= -(2vt + at_2^2)$

$= 2vt_2 - at_2^2$

$2vt_1 + gt_1^2 = -2vt_2 - gt_2^2$

$-h = ut_1 + \frac{1}{2}(-g)(t_1)^2$

$-h = ut_1 - \frac{1}{2} gt_1^2$ --- (1) $\times t_2$

$-h = -ut_2 - \frac{1}{2} g t_2^2$ --- (2) $\times t_1$

$-ht_2 = ut_1 t_2 - \frac{1}{2} g t_1^2 t_2$

$-ht_1 = -ut_1 t_2 - \frac{1}{2} g t_2^2 t_1$

$+h(t_1 + t_2) = +\frac{1}{2} g t_1 t_2 (t_1 + t_2)$

$+h = \frac{t_1 t_2 g}{2}$ --- (4)

$+h = +\frac{1}{2} gt^2$ --- (3)

$\frac{t_1 t_2 g}{2} = \frac{1}{2} gt^2$ (4)

$t_1 t_2 = t^2$

$\boxed{t = \sqrt{t_1 t_2}}$

Q1. A particle is projected vertically up such that it posses a fixed point P after time t_1 and t_2 respectively. Find

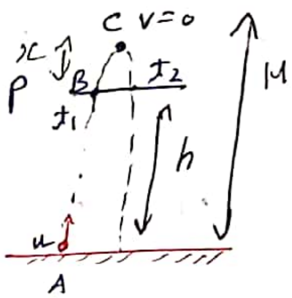
a) Height at which point is locate w.r.t point of projection.

b) speed of projection of ball

c) velocity at point P

d) Max height reached by ball w.r.t point of projection

e) Max height reached by ball w.r.t P.



a) $S = ut_1 - 4.9(t_1)^2$

$$h = ut - \frac{gt^2}{2}$$

$$2h = 2ut - gt^2$$

$$gt^2 - 2ut + 2h = 0$$

$$t = \frac{2u \pm \sqrt{4u^2 - (4)(2h)(g)}}{2g}$$

$$t = \frac{2u \pm \sqrt{4(u^2 - 2gh)}}{2g}$$

$$t = \frac{u \pm \sqrt{u^2 - 2gh}}{g}$$

$$t_1 = \frac{u - \sqrt{u^2 - 2gh}}{g}$$

$$t_2 = \frac{u + \sqrt{u^2 - 2gh}}{g}$$

$$t_1 + t_2 = \frac{2u}{g}$$

$$\boxed{u = \frac{g(t_1 + t_2)}{2}} \quad b)$$

A - B

$$v = u + at$$

$$v = g\frac{(t_1 + t_2)}{2} - g t_1$$

$$v = \frac{gt_1 + gt_2 - 2gt_1}{2}$$

$$v = \frac{gt_2 - gt_1}{2}$$

$$v = \frac{g}{2}(t_2 - t_1)$$

$$\boxed{v = \frac{g}{2}(t_2 - t_1)} \quad c)$$

from $A \rightarrow C$

$$v = 0$$

$$v^2 = u^2 + 2as$$

$$v^2 = \left(\frac{g}{2}(t_1 + t_2)\right)^2 - 2 \times g \times H$$

$$0 = \frac{g^2}{4}(t_1 + t_2)^2 - 2gH$$

$$2gH = \frac{g^2}{4}(t_1 + t_2)^2$$

$$H = \frac{g}{8}(t_1 + t_2)^2$$

$$\boxed{H = \frac{g}{8}(t_1 + t_2)^2 \quad d} \quad d)$$

from $B \rightarrow C$

$$v = 0$$

$$u = \frac{g(t_2 - t_1)}{2}$$

$$v^2 - u^2 = 2as$$

$$\pm \frac{g^2}{4}(t_2 - t_1) = + 2 \times g \times x$$

$$\boxed{\frac{g}{8}(t_2 - t_1)^2 = x} \quad e)$$

$H - x =$ hight of P

$$\frac{g}{8}(t_1+t_2)^2 - \frac{g}{8}(t_2-t_1)^2$$

$$\frac{g}{8}\left(\cancel{(t_1)^2}+\cancel{(t_2)^2}+2t_1t_2-\cancel{(t_2)^2}-\cancel{(t_1)^2}+2t_1t_2\right)$$

$$\frac{g}{8}(4t_1t_2)$$

$$\boxed{\frac{g(t_1t_2)}{2}}$$ a)

Q2. Drops of water fall from the roof of a building 9 m high at regular intervals of time, the first drop reaching the ground at the same instant fourth drop starts to fall. What are the distances of the second and third drop from the roof ($g = 10\,m/s^2$)

$u = 0$
$h = 9\,m$
$g = 10\,m$

$9 = \frac{1}{2} \times 10 \times t^2$

$\frac{18}{10} = t^2$

$t^2 = 1.8$

$t = \sqrt{1.8}$

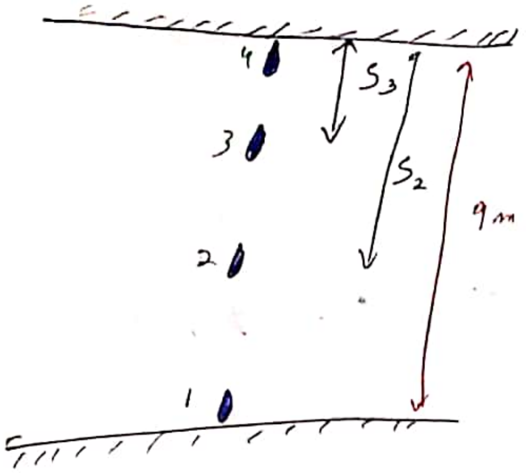
$\frac{3\times\sqrt{2}}{\sqrt{10}} \times \frac{1}{3} = \Delta t$

$S = \frac{1}{2}\times 10 \times \left(\frac{3\times\sqrt{2}}{\sqrt{10}}\times\frac{1}{4}\right)^2$

$S = 5 \times \left(\frac{9\times \cancel{2}}{10} \times \frac{1}{168}\right)$

$S = \cancel{8} \times \frac{9}{80}$ 16

$$\boxed{S = \frac{9}{16}\,m}$$



$$\frac{9\sqrt{2}}{3\sqrt{10}} - \frac{3\sqrt{2}}{4\sqrt{10}} = t_2$$

$$S_2 = \frac{1}{2} \times 10 \times \left(\frac{2\sqrt{2}}{\sqrt{10}}\right)^2$$

$$= 5 \times \frac{162}{160}$$

$$= 5 \times \frac{8}{10}$$

$$\boxed{S_2 = 4\ m}$$

$$S_3 = \frac{1}{2} \times 10 \times \left(\frac{2\sqrt{2}}{10}\right)^2$$

$$= 5 \times \frac{72}{160} = 5 \times \frac{2}{10}$$

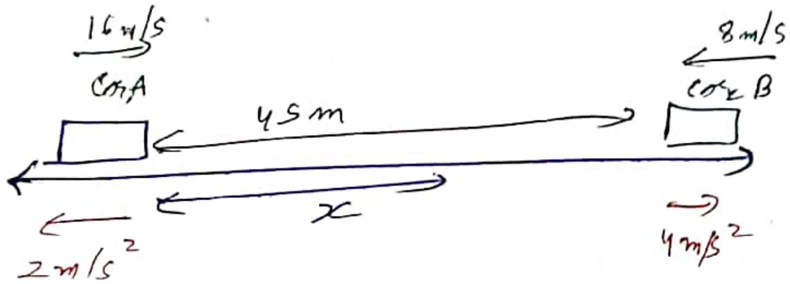
$$= \cancel{\frac{72}{32}} \quad \boxed{= 1\ m}$$

$$\boxed{\begin{array}{l} S_2 = 4\ m \\ S_3 = 1\ m \end{array}}$$

$$\cancel{S_2 - S_3}$$

$$\cancel{\frac{81}{16} - \frac{36}{16} = \boxed{\frac{45}{16}\ \text{meter}}}$$

Q.3. Two cars approach each other on a straight road. Car A moves at 16 m/s and car B moves at 8 m/s. When they are 45 m apart, both drivers apply their brakes. Car A slows down at 2 m/s², while car B slows down at 4 m/s². Where and when do they collide?



$x = 16t - \frac{1}{2} \times 2 (t)^2$

$45 - x = 8t - \frac{1}{2} 4 t^2$

$x = 16t - t^2$

$45 = 24t - 3t^2$

$3t^2 - 24t - 45 = 0$

$t^2 - 8t - 15 = 0$

$t = \frac{8 \pm \sqrt{64 + 60}}{2}$

$t = \frac{8 \pm \sqrt{124}}{2}$

$t = 4 \pm \sqrt{31}$

$t = 4 + \sqrt{31}$

$x = 56 + 16\sqrt{31} - 16 - 31$

$x = 56 - 47 + 16\sqrt{31}$

$x = 9 + 16\sqrt{31}$

Car - B

$u = 8$

$a = -4$

$v = 0$

$t = 2s$

$S = (8)(2) - \frac{1}{2} \times 4 \cdot (4)$

$S = 16 - 8$

$\boxed{S = 8m}$

Car - A

$u = 16$

$a = -2$

$V = 0$

$t = 8s$

$S = (8)(16) - \frac{1}{2} \times 2(64)$

$S = 128 - 64$

$\boxed{S = 64m}$

collision ~~when~~ 8m from B.

$45 - 8 = 37$

$37 = 16t - t^2$

$t^2 - 16t + 37 = 0$

$t = \frac{16 \pm \sqrt{256 - 148}}{2}$

$t = \frac{16 \pm \sqrt{108}}{2}$

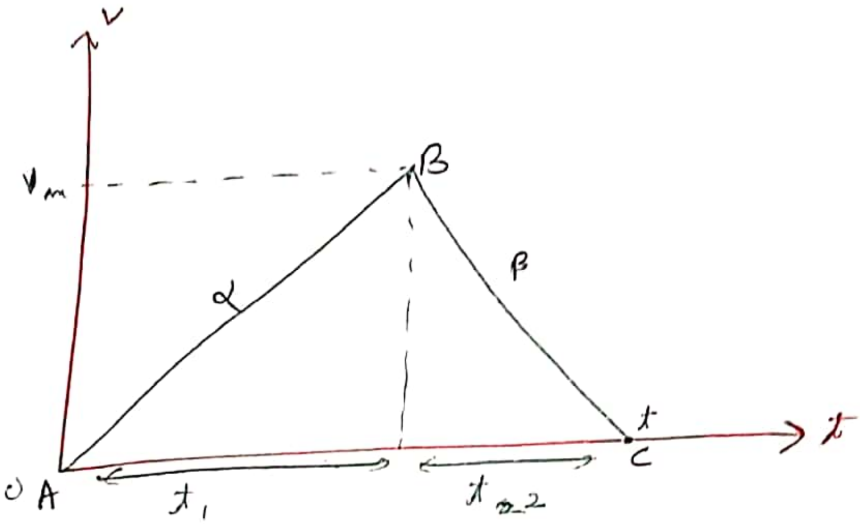
$t = 8 \pm \sqrt{27}$

$\boxed{t = 8 + \sqrt{27}\ s}$

$\boxed{t = 8 - \sqrt{27}\ s}$

Q A car accelerates from rest at a constant rate α for some time and then decelerates at constant rate β to come to rest, find Total time elapsed is t then, Calculate

a) Maximum velocity attained
b) Total distance travelled.



Slope $AB = \dfrac{V_m - 0}{t_1}$

$$\alpha = \frac{V_m}{t_1}$$

$$\boxed{t_1 = \frac{V_m}{\alpha}}$$

Slope $BC = \dfrac{V_m}{t_2}$

$$\boxed{t_2 = \frac{V_m}{\beta}}$$

$$t_1 + t_2 = t$$

$$\frac{V_m}{\alpha} + \frac{V_m}{\beta} = t$$

$$\boxed{V_m = \frac{t\alpha\beta}{\alpha+\beta}}$$

$$\boxed{V_m = \frac{t\alpha\beta}{\alpha+\beta}}$$

Distance $= \dfrac{1}{2} \times V_m \times t$

$$= \frac{1}{2} \times t \times \frac{t\alpha\beta}{\alpha+\beta}$$

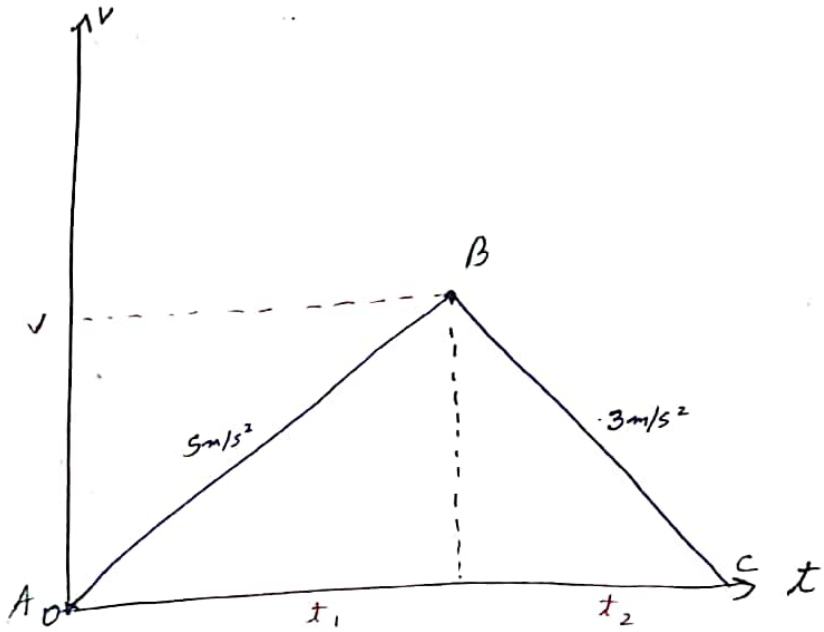
$$= \frac{\alpha\beta t^2}{2(\alpha+\beta)}$$

$$\boxed{\text{Dis} = \cancel{2}\,\frac{\alpha\beta t^2}{2(\alpha+\beta)}}$$

H.W. 11-05-2024

Q-7 (full)

Q A car can travel at maximum speed of 180 km/hr and can have maximum acceleration 5 m/s² and retardation 3 m/s². How fast can it start from rest and come to rest in travelling 500 m?



max speed = 180 km/hr

$= 180 \times \frac{5}{18} = 50 m/s$

area under graph = 500 m

A → B

$\frac{v}{t_1} = 5$

$\boxed{t_1 = \frac{v}{5}}$ --- (1)

B → C

$-\frac{v}{t_2} = -3$

$\frac{v}{t_2} = 3$

$\boxed{t_2 = \frac{v}{3}}$ --- (2)

area $= \frac{1}{2} \times b \times h$

$500 = \frac{1}{2} \times (t_1 + t_2) \times v$

$1000 = \left(\frac{v}{5} + \frac{v}{3}\right) v$

$1000 = \frac{3v + 5v}{15} \times v$

$15000 = 8v^2$

$v^2 = \frac{15000}{8}$

$v^2 = 1875$

$v = \sqrt{1875}$

$\boxed{v = 43.3 m/s}$

$\therefore v < 50 m/s$ (max speed)

$$t = t_1 + t_2$$

$$= \frac{v}{5} + \frac{v}{3}$$

$$= \frac{3v + 5v}{15}$$

$$= \frac{8v}{15}$$

$$= \frac{8}{15} \times 43.3$$

$$= 0.53 \times 43.3$$

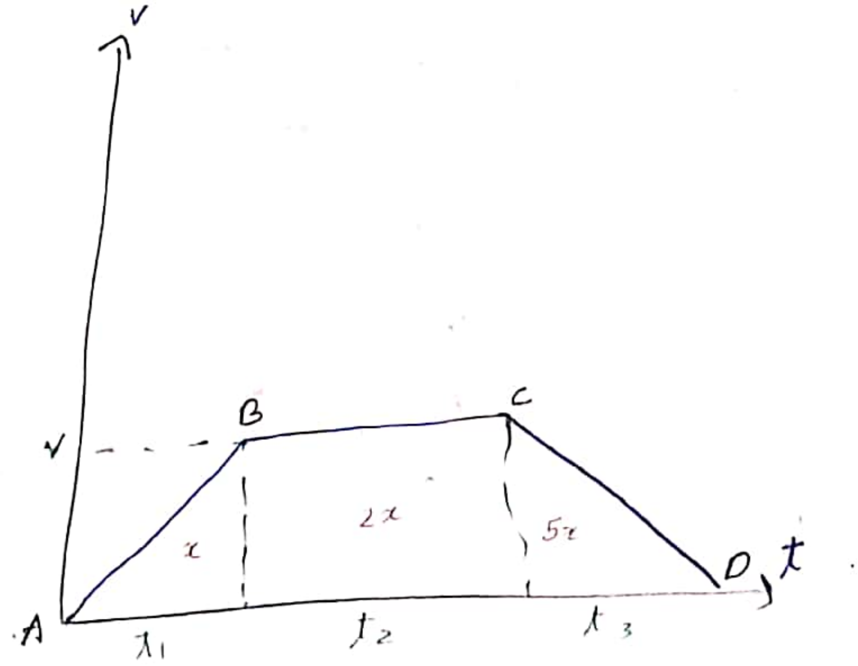
$$= 22.949\ s$$

$$\boxed{t = 22.949\ s}$$

$$\begin{array}{r} 43.3 \\ \times\ 53 \\ \hline 1299 \\ 21650 \\ \hline 22949 \end{array}$$

$$\begin{array}{l} 15)\ 80\ (0.53 \\ \ \ \ \ \ \ 75 \\ \hline \ \ \ \ \ \ \ 50 \\ \ \ \ \ -45 \\ \hline \ \ \ \ \ \ \ \ \ 5 \end{array}$$

Q A particle starts from rest and travels a distance x with uniform acceleration, then moves uniformly a distance $2x$ and finally comes to rest after moving further distance ~~2x so~~ $5x$ with uniform ~~acceleration~~ retardation. If the ratio of average speed to maximum speed is $k/7$ then find value of k.



average speed $= \dfrac{x+2x+5x}{t}$

$$\boxed{= \frac{8x}{t}}$$

~~max speed =~~

$\frac{1}{2} \times v \times t_1 = x$

$\dfrac{v}{2x} = \dfrac{1}{t}$

$t_1 = \dfrac{2x}{v}$

$\cancel{t_2 = 4}$

$t_2 \times v = 2x$

$t_2 = \dfrac{2x}{v}$

$t_3 \times \frac{1}{2} \times v = 5x$

$t_3 = \dfrac{10x}{v}$

$t = \dfrac{2x+2x+10x}{v}$

$t = \dfrac{14x}{v}$

$$\boxed{v = \frac{14x}{t}}$$

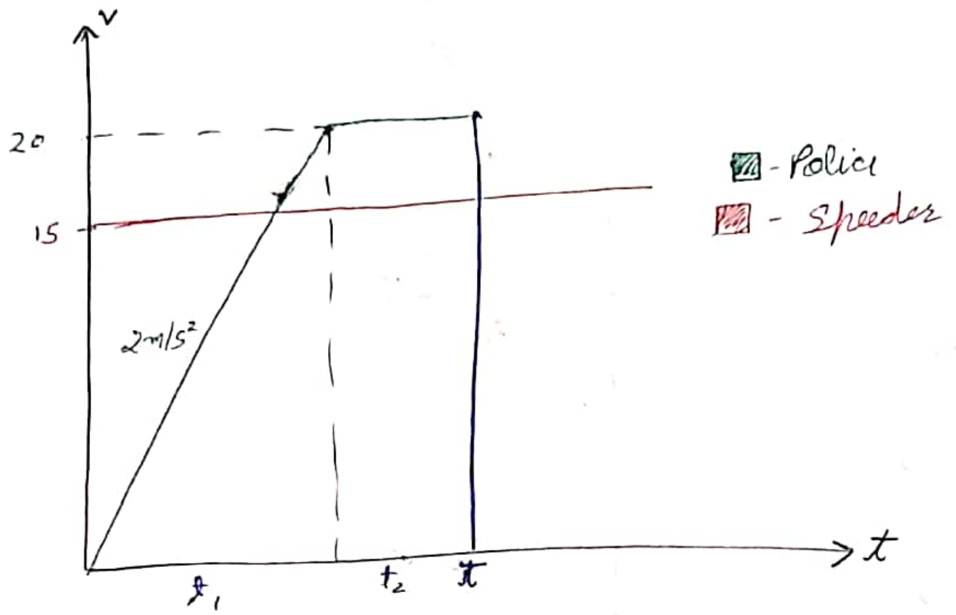
max speed $= \dfrac{14x}{t}$

$$\frac{\frac{8x}{t}}{\frac{14x}{t}} = \frac{k}{7}$$

$\dfrac{8 \times 7}{14} = k$

$$\boxed{k = 4}$$

Q A speeder moves at a constant speed 15 m/s in a school zone. A police car starts from rest just as the speeder passes it. The police car accelerates at 2 m/s2 until it reaches its maximum velocity of 20 m/s. when and where does the speeder get caught?



~~area~~ displacement for speeder $= 15t$

displacement for police $= \frac{1}{2} \times 20 \times 10 + 20 \times (t - 10)$

$= 100 + 20t - 200$

$= 20t - 100$

Slope $= 2$

$2 = \frac{20}{t_1}$

$\boxed{t_1 = 10}$

$15t = 20t - 100$

$100 = 5t$

$\boxed{t = 20 s}$ ✓

$S = 15t$

$= 15 \times 20$

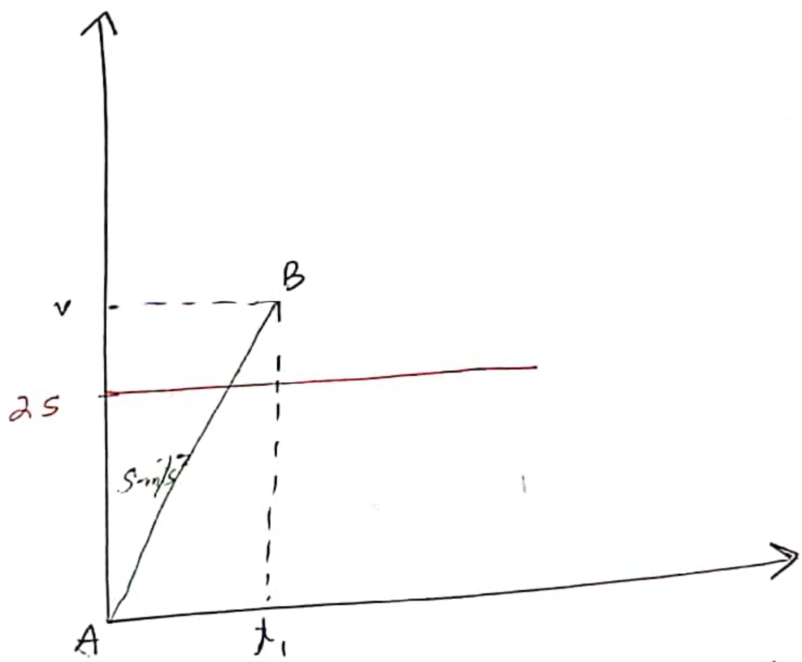
$\boxed{= 300 m}$ ✓

Q A car is speeding at 25 m/s in a low speed zone. A police car starts from rest as the speedster passes and accelerates at a constant rate of 5 m/s^2.

a) When does the police car catch the speeding car?

b) How fast is the police car at the instant when it catches up with the speedster?

c) How far have the cars travelled when the police car catches the speedster?



Displacement police $= \frac{1}{2} \times t_1 \times v$

Displacement speeder $= t_1 \times 25$

$\frac{1}{2} \cancel{t_1} v = 25 \cancel{t_1}$

$\boxed{V = 50 \text{ m/s}}$ b)

slope$_{AB}$ $= \frac{V}{t_1} = 5$

$50 = 5 t_1$

$\boxed{t_1 = 10 s}$ a)

$s = \frac{1}{2} \times 10 \times 50$

$\boxed{= 250 \text{ m}}$ c)

Q A railway track runs parallel to a road until a turn brings the road to railway crossing. A cyclist rides along the road everyday at a constant speed of 20 km/h. He normally meets a Train that Travels in same direction at the crossing. One day he was late by 25 minutes and met the train 10 km before crossing. find train speed.

~~S = 10~~

~~d = 10000 m~~

~~t = 25 × 60 s~~

~~$S = \frac{10000}{60 \times 25}$~~

~~$S = \frac{4020}{6_3}$~~

~~$S = \frac{20}{3}$ m/s~~

~~$S = \frac{10}{0.5}$~~

~~S =~~

~~0.5 hr~~

9:55 AM

?25 min late so

9:30 Am (cyclist everyday)

20 km/hr

X let 10:00 Am

10 km

Time = 5 min

$= \frac{5}{60}$ hr

$\text{speed} = \frac{10}{5} \times 60$

$\boxed{= 120 \text{ km/hr}}$

Scanned with CamScanner

Q A body starts with an initial velocity of 10 m/s and moves along a straight line with constant acceleration is reversed in direction $(at\ v = 0\ m/s)$. Find the velocity of the particle when it reaches the starting point.

$S = \frac{50^2 - 10^2}{2a}$

$S = \frac{2500 - 100}{2a}$

$S = \frac{2400}{2a}$

$S = \frac{1200}{a}$

$u = 60$

$S = \frac{1200}{a}$

$\frac{1200}{a} = 50t - \frac{1}{2}at^2$

$\frac{-40}{a} = t$ $\quad a = \frac{-40}{t}$

$\frac{1200}{a} = 50 \times \frac{-40}{a} - \frac{1}{2} \times a \times \frac{1600}{a^2}$ $\quad 800$

$\frac{1200}{a} = \frac{-2000}{a} - \frac{800}{a}$

$\frac{-1200t}{40} = 50t - \frac{1}{2}\left(\frac{-40}{t}\right) \times t^2$

$\frac{-1200t}{400} = 50t + 20t$

$-1200t = 2800t$

$2800t + 1200t = 0$

$t = 0s$

$50 - 10 = at$

$\frac{40}{a} = t$

$S = \frac{1200}{a}$

$u = 50$

$v = ?$

$2 \times a \times \frac{1200}{a} = v^2 - 2500$

$2500 + 2400 = v^2$

$4900 = v^2$

$\boxed{v = 70\ m/s}$

Ch-3

!! Projectile Motion !!

<u>Projectile motion</u> - एक चीज object को फेंका जाएगा and it lands somewhere else.

→ the object always moves in a single plane

→ The object moves in parabola path.

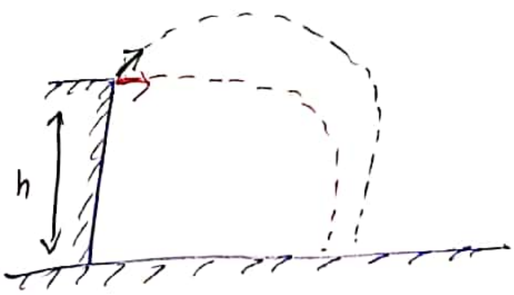
→ The object moves only under gravity. (ignore earth's curvature, air resistance etc)

→ A projectile is any body that is given an <u>initial velocity</u> and then follows a path determined ~~could~~ entirely by <u>effects of acceleration</u> due to <u>gravity</u>.

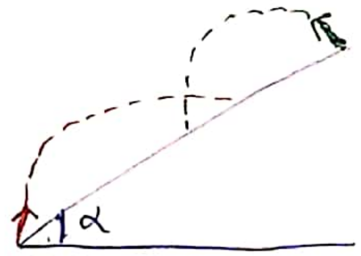
<u>Types of projectial Motion</u> -

① Ground to Ground (oblique) :-

② Projection from a height.



③ Projection on a plane :-



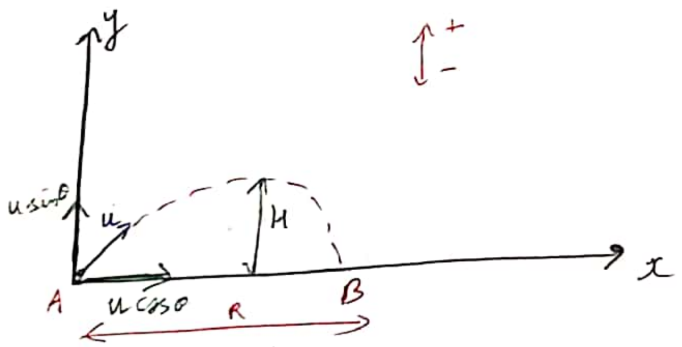
Projectile motion -

↓

2 dimentional motion (2 d)
(motion in a plane)

↓ Horizontal (uniform motion) ⟵ independent of each other. ⟶ ↓ Verticle (uniformly accelerated motion)

Oblique projection :- (ground to ground projection)



Horizontal (uniform)	Verticle (uniformly accelerated)
$V_x = u\cos\theta$	$V_y = u\sin\theta$
$a_x = 0$	$a_y = -g$

Time of flight (T)

$S_y = 0$

Using $s = ut + \frac{1}{2} at^2$

$0 = u\sin\theta\,(T) + \frac{1}{2}(-g)T^2$

$0 = T\left(u\sin\theta - \frac{gT}{2}\right)$

$T = 0$ (at point of projection)

$u\sin\theta = \frac{gT}{2}$

$\frac{2u\sin\theta}{g} = T$ (at B)

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

Highest Point (H)

at highest point, $V_y = 0$

Using $v^2 = u^2 + 2as$

$(V_y)^2 = (u_y)^2 + 2a_y H$

$0 = (u\sin\theta)^2 - 2gH$

$H = \frac{u^2\sin^2\theta}{2g}$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

for fixed u, maximum height H will be for $\sin\theta = 1$

$\boxed{\theta = 90^\circ}$

Range(R)

$R = V_x T$

$R = u\cos\theta \times \frac{2u\sin\theta}{g}$

$R = \cancel{2} \frac{u^2 \sin 2\theta}{g}$

$$\boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

For complimentry angle of projection

$\theta \rightarrow (90-\theta)$

$R' = \frac{u^2 \sin[2(90-\theta)]}{g}$

$R' = \frac{u^2 \sin(180-2\theta)}{2}$

$$\boxed{R' = \frac{u^2 \sin 2\theta}{g}}$$

Range is alway same for complimentry angles.

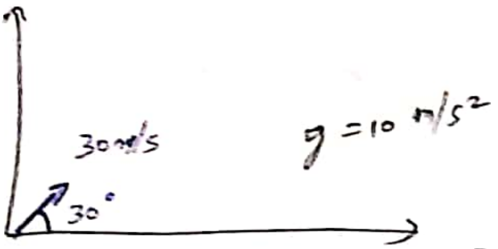
For max range

$\sin 2\theta = 1$

$2\theta = 90$

$$\boxed{\theta = 45^\circ}$$

Q



$g = 10\ m/s^2$

a) time at which ball reaches highest point

b) maximum height reached

c) horizontal range

d) time of flight

$T = \frac{2u \sin\theta}{g}$

$= \frac{60 \times \frac{1}{2}}{9.8}$

$= \frac{30}{9.8}$

time to reach max height $= \frac{30}{9.8} \times \frac{1}{2} = \boxed{\frac{15}{9.8}\ s}$ a)

$= \frac{15}{10}\ s$

$\boxed{= 1.5\ s}$

$H = \frac{u^2 \sin^2\theta}{2g}$

$= \frac{\cancel{30}^{15} \times \cancel{30}^{15} \times \frac{1}{\cancel{4}\,2}}{2 \times 9.8}$

$\boxed{\cancel{= \frac{225}{19.6}\ m}}$ b)

$= \frac{225}{20}$

$= \frac{112.5}{10}$

$\boxed{= 11.25\ m}$ b)

c) $R = \frac{u^2 \sin 2\theta}{g}$

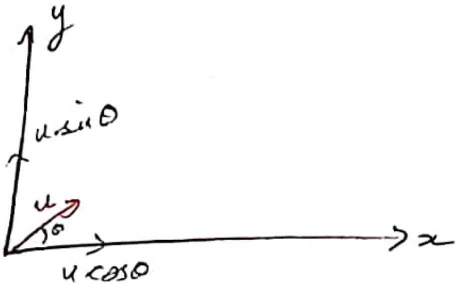
$= \frac{30 \times \cancel{30}^{3} \times \frac{\sqrt{3}}{2}}{\cancel{10}}$

$\boxed{= 45\sqrt{3}\ m}$ c)

$T = 1.5 \times 2$

$\boxed{= 3\ s}$ d)

Velocity at ~~time~~ time 't' —



$u_x = u\cos\theta$

$V_x = u\cos\theta$

$u_y = u\sin\theta$

$V_y = u_y + a_y t$

$\underline{V_y = u\sin\theta - gt}$

$$\boxed{\vec{V} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}}$$

Q A ball is projected with velocity u at angle of projection θ. After what time ball is moving at right angles to initial direction.

initial direction $= \dfrac{\vec{u}}{u} = u\sin\theta\,\hat{j} + u\cos\theta\,\hat{i}$

$\vec{V} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$$= \frac{u^2\cos^2\theta + u\sin\theta(u\sin\theta - gt)}{u\sqrt{u^2\cos^2\theta + u^2\sin^2\theta + g^2t^2 - 2u\sin\theta\, gt}}$$

$= u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta gt$

$$= \frac{u^2 - u\sin\theta\, gt}{u\sqrt{u^2 + g^2t^2 - 2u\sin\theta\, gt}}$$

$= u - \sin\theta g$

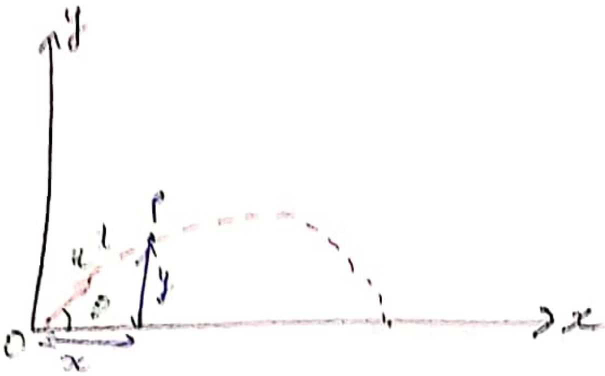
$\vec{u}\cdot\vec{V} = 0$

$u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta gt = 0$

$u(u - \sin\theta gt) = 0$

$$\boxed{t = \frac{u}{\sin\theta\, g}}$$

Equation of trajectory



$x = u_x t$

$x = u\cos\theta\, t$

$t = \frac{x}{u\cos\theta}$ (1)

$s_y = u_y t + \frac{1}{2} a_y t^2$

$y = u\sin\theta(t) - \frac{1}{2} g t^2$ ---(2)

$$y = \frac{u\sin\theta\,(x)}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}$$

$$\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}}$$

$$y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta} \cdot \frac{\sin\theta}{\sin\theta}$$

$$y = x\tan\theta - \frac{x^2 \tan\theta}{\left[\frac{2u^2\sin\theta\cos\theta}{g}\right]}$$ Range formula

$$y = x\tan\theta - \frac{x^2\tan\theta}{R}$$

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Q A grasshoper ~~can~~ jump into height 'h', find the maximum distance through which it can jump along horizontal ground.

$$h = \frac{u^2 \sin^2\theta}{2g} \qquad h = \frac{u^2}{2g}$$

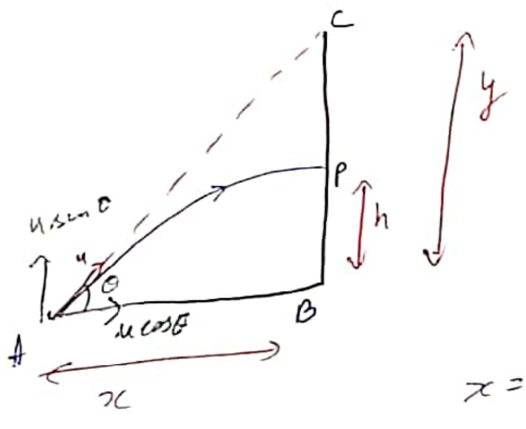
$$\sqrt{2gh} = u$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{2gh \times}{g}$$

$$\boxed{R_{max} = 2h}$$

Q A hunter aims his gun and fires a bullet directly on a monkey on a tree. At the instant bullet is fired, the monkey drops. Will the bullet hit the monkey?



$$\tan\theta = \frac{y}{x}$$

for bullet,

$$S_y = u_y t + \frac{1}{2} a_y t^2$$

$$h = u\sin\theta\, t - \frac{1}{2} g t^2$$

$$x = u\cos\theta\, t \Rightarrow t = \frac{x}{u\cos\theta}$$

$$h = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g t^2$$

$$h = x\tan\theta - \frac{1}{2} g t^2$$

$$h = y - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 = y - h$$

For Monkey

$$z = 0 + \frac{1}{2} g t^2$$

$$z = y - h$$

As, the height is same for monkey & bullet at same time,

Yes the bullet will hit monkey.

Q The range of a projectile fired at an angle 15° is 40 m. if it is fired with the same speed at an angle of 45° find its range.

$g = 10 m/s$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$40 = \frac{u^2 \times \frac{1}{2}}{g_0}$$

$$400 \times 2 = u^2$$

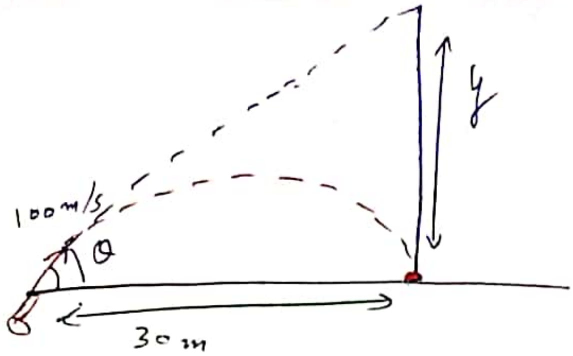
$$20\sqrt{2} = u$$

$$R' = \frac{800 \times 1}{g}$$

$$R' = \frac{800}{10}$$

$$\boxed{R' = 80 m}$$

Q



How high above the target the gun must be aimed to hit the target on ground at 30 m from the gun?

find y

$$R = \frac{100 \times 10\cancel{0} \quad \sin 2\theta}{1\cancel{0}}$$

$R = 1000 \sin 2\theta$

$\frac{3\cancel{0}}{100\cancel{0}} = \sin 2\theta$

$\frac{3}{100} = 2 \sin\theta \cos\theta$

$\frac{3}{100} = \sin\theta \cos\theta$

$\sin 2\theta = 0.03 \rightarrow$ (approximation)

$2\theta = 0.03$

$\theta = 0.015$

$\tan\theta = \frac{y}{30}$

$30 \frac{\sin\theta}{\cos\theta} = y$

$\tan 0.015 = \frac{y}{30}$

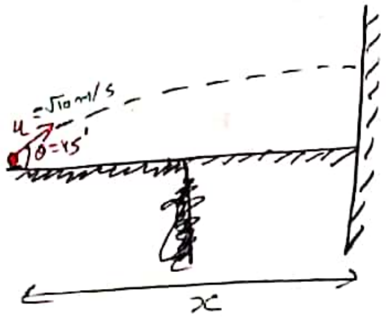
$30 \times 0.015 = y$

$0.15 \times 3 = y$

$0.45 = y$

$\boxed{y = 0.45 \text{ m}}$

Q



$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{10 \times 1}{10}$$

$$R = 1\,m$$

a) $x = 1/2$ m ✓
b) $x = 2$ m ✗

will water hit wall

c) where does it hit wall
d) max height

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = \frac{10 \times \frac{1}{2}}{2 \times 10}$$

$$\boxed{H = \frac{1}{4}\,m}$$

$$y = x \tan\theta \left[1 - \frac{x}{R}\right]$$

$$y = \frac{1}{2} \times 1 \left[1 - \frac{1/2}{1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{2}\right]$$

$$= \frac{1}{4}\,m$$

$$\boxed{y = \frac{1}{4}\,m}$$

OR

$$y = x \tan\theta - \frac{g x^2}{\cancel{2u^2 \sin^2\theta}\ 2u^2 \cos^2\theta}$$

$$= \frac{1}{2} \times 1 - \frac{10 \times \frac{1}{4}}{2 \times 10 \times \frac{1}{2}}$$

$$= \frac{1}{2} - \frac{\frac{5}{2}}{10}$$

$$= \frac{1}{2} - \frac{5}{20} \quad \frac{1}{4}$$

$$\boxed{= \frac{1}{4}\,m}$$

Q we have a hose pipe which disposes water at speed of 10 m/s. The safe distance from a building on fire, on ground is 5m. How high can the water go. ($g = 10 m/s^2$)

$u = 10 m/s$

$R = 5 m$

$R = \frac{100 \times \sin 2\theta}{10}$

$\frac{1}{2} = \sin 2\theta$

$2\theta = 30$

$\theta = 15°$

$H = \frac{u^2 \sin^2\theta}{2g}$

$= \frac{100 \times \frac{\sqrt{3}-1}{2\sqrt{2}}}{2 \times 10}$

$= \frac{10\sqrt{3}-10}{4\sqrt{2}}$

$\boxed{= \frac{5\sqrt{3}-5}{2\sqrt{2}} m}$

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\frac{\sqrt{3}-1}{2\sqrt{2}}$

$\frac{1-\cos 30}{2} = \frac{1-\frac{\sqrt{3}}{2}}{2}$

$\frac{2-\sqrt{3}}{4}$

$H = \frac{100 \times \frac{(2-\sqrt{3})}{4}}{20}$

$H = \frac{5(2-\sqrt{3})}{4}$

$\boxed{H = \frac{10-5\sqrt{3}}{4}}$

$\frac{1-\cos\theta}{2} = \sin^2\theta$

$1 - \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$\frac{1}{2} - \frac{\sqrt{3}}{4}$

$\frac{2-\sqrt{3}}{4}$

$$y = x\tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$$

$$y = 5\tan\theta - \frac{10 \times 25}{2 \times 10 \times 10 \times \cos^2\theta}$$

$$y = 5\tan\theta - \frac{5}{4}\sec^2\theta$$

जिसको max करना है

$$\frac{dy}{d\theta} = 0$$

जिससे change for settle hai

$$\frac{dy}{d\theta} = 5\sec^2\theta - \frac{5}{4} \times 2\sec\theta \times \sec\theta\tan\theta$$

$$= 5\sec^2\theta - \frac{5}{2}\sec^2\theta\tan\theta$$

$$\frac{5}{2}\cancel{\sec^2\theta}\tan\theta = \cancel{5\sec^2\theta}$$

$$\frac{\tan\theta}{2} = 1$$

$$\cancel{\tan\theta = 0}$$

$$\tan\theta = 2$$

$P = 2$

$B = 1$

$H = \sqrt{5}$

$$\sec\theta = \frac{H}{B} = \sqrt{5}$$

$$y_{max} = \cancel{5 \times (\sqrt{5})}\ 5 \times 2 - \frac{5}{4} \times 5$$

$$= 10 - \frac{25}{4}$$

$$\boxed{= \frac{15}{4}\ m}$$

Or

$$y = 5\tan\theta - \frac{5}{4}\sec^2\theta$$

$$y = 5\tan\theta - \frac{5}{4}(1 - \tan^2\theta)$$

$$y = 5\tan\theta - \frac{5}{4} + \tan^2\theta$$

$$\tan^2\theta + 5\tan\theta - \frac{5 + 4y}{4}$$

for real roots

$$D \geq 0$$

$$\cancel{25 + 5y \geq 0}$$

$$\cancel{5(5 + y) \geq 0}$$

$$\cancel{5 + y \geq 0}$$

$$25 + 5 + 4y \geq 0$$

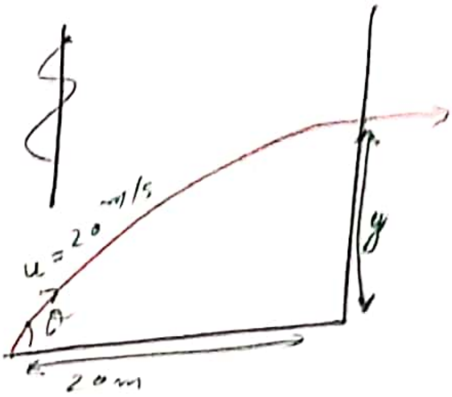
$$30 + 4y \geq 0$$

$$y \geq \frac{-30}{4}$$

$$\boxed{y \leq \frac{15}{4}}$$

(212)

Q A shot is fired with a velocity 20m/s at a vertical wall whose distance from point of projection is 20m. Find the greatest height above the level of point of projection at which bullet can ~~hit~~ hit the wall.



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = 20\tan\theta - \frac{10\times 20\times 20}{2\times 20\times 20\times\cos^2\theta}$$

$$y = 20\tan\theta - 5\sec^2\theta$$

$$\frac{dy}{d\theta} = 20\sec^2\theta - 10\sec^2\theta\tan\theta$$

$$0 = 2\sec^2\theta - \sec^2\theta\tan\theta$$

$$0 = \sec^2\theta(2-\tan\theta)$$

$$2-\tan\theta = 0$$

$$\tan\theta = 2$$

$P = 2$
$B = 1$
$H = \sqrt{5}$

$$\sec\theta = \sqrt{5}$$

$$\sec^2\theta = 5$$

$$y_{max} = 20\times 2 - 5\times 5$$

$$= 40-25$$

$$\boxed{= 15m}$$

OR

$$y = 20\tan\theta - 5\sec^2\theta$$

$$= 20\tan\theta - 5(1+\tan^2\theta)$$

$$y = 20\tan\theta - 5 - 5\tan^2\theta$$

$$5\tan^2\theta - 20\tan\theta + (5+y) =$$

$$400 - (4)(5)(y+5) \geq 0$$

$$400 - 20y - 100 \geq 0$$

$$-20y + 300 \geq 0$$

$$-20y \geq -300$$

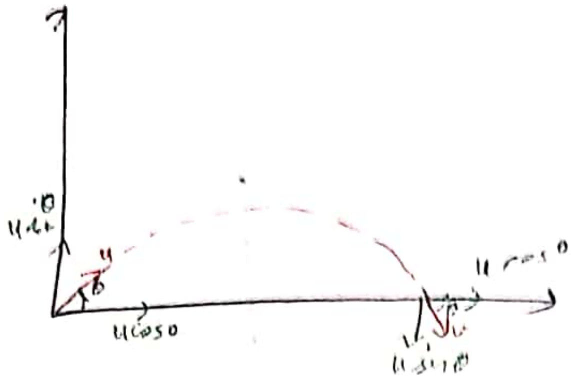
$$20y \leq 300$$

$$y \leq \frac{300}{20}$$

$$\boxed{y \leq 15}$$

Change in momentum between any two positions of projectile

I) Between point of projection & highest Point :-



$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v} = u\cos\theta\,\hat{i}$

$\Delta\vec{P} = m\vec{v} - m\vec{u}$

$= m(\vec{v} - \vec{u})$

$\Delta\vec{P} = m(-\sin\theta\,\hat{j})$

$\boxed{\Delta\vec{P} = m(-u\sin\theta\,\hat{j})}$

$\boxed{|\Delta\vec{P}| = mu\sin\theta}$

2) Between initial & final points.

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{v} = u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j}$

$\Delta\vec{P} = m(\vec{v} - \vec{u})$

$= m(-2\sin\theta\,\hat{j})$

$\boxed{\Delta\vec{P} = -2mu\sin\theta\,\hat{j}}$

$\boxed{|\Delta\vec{P}| = 2mu\sin\theta}$

Q A object of mass 0.5 kg is projected under gravity with a speed of 98 m/s at an angle of 60°. find magnitude of change in momentum after 10s ($g = 9.8\ m/s^2$)

$m = \cancel{10}\ 0.5\ kg$

$u = 98\ m/s$

$\theta = 60°$

$\cancel{|\Delta \vec{P}| =}$

$u = 98 \sin\theta\ \hat{i}$

$t = 10\ s$

$g = 9.8$

$v = \cancel{98 + 98}\ 98 \times \frac{\sqrt{3}}{2} + 98$

$v = 49\sqrt{3} + 98$

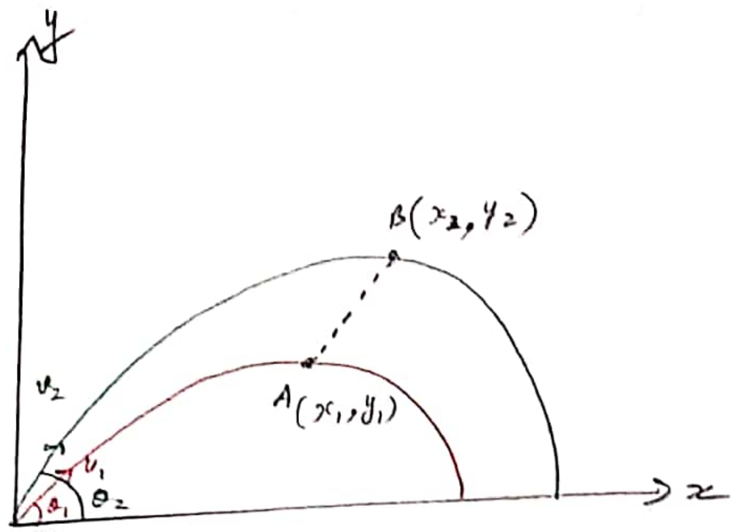
$\vec{v} = 49\sqrt{3} + 98\ \hat{j} + u\cos\theta\ \hat{i}$

$|\Delta \vec{P}| = m\,(49\sqrt{3} + 98 - 49\sqrt{3})$

$= 98 \times 0.5$

$\boxed{|\Delta \vec{P}| = 49\ kg\,m/s}$ ✓

path of one projectile in reference of another projectile :-



$$\left.\begin{aligned} x_1 &= u_1 \cos_1 t \\ x_2 &= u_2 \cos_2 t \end{aligned}\right] \quad x_2 - x_1 = (u_2 \cos_2\theta - u_1 \cos_1\theta)\, t$$

$$\left.\begin{aligned} y_1 &= u_1 \sin\theta_1 t - \tfrac{1}{2} g t^2 \\ y_2 &= u_2 \sin\theta_2 t - \tfrac{1}{2} g t^2 \end{aligned}\right] = u_2 \sin\theta_2 - u_1 \sin\theta_1$$

$$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \frac{u_2 \sin\theta_2 - u_1 \sin\theta_1}{u_2 \cos\theta_2 - u_1 \cos\theta_1}}$$

$$y_2 - y_1 = \underbrace{m}_{\text{constant}} (x_2 - x_1)$$

If $u_1 \cos\theta_1 = u_2 \cos\theta_2$

$$\frac{y_2 - y_1}{x_2 - x_1} = \infty \quad \text{(vertical straight line)}$$

Q.

find after 3s.

a) velocity of ball
b) angle made by ball with horizontal
c) horizontal & vertical displacement.

$u = 60\sqrt{2} \times \frac{1}{\sqrt{2}} \quad \theta - 10 \times 3$

$= 60 - 30$

$\boxed{= 30\hat{j}}$ m/s

$\vec{v} = 30\hat{j} + 30\hat{i}$

$= \sqrt{900 + 900}$

$\boxed{= 30\sqrt{2} \text{ m/s}}$

a) $\vec{v} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$= 60\sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + 60\sqrt{2} \times \frac{1}{\sqrt{2}} - 30\hat{j}$

$= 60\hat{i} + 30\hat{j}$

$|\vec{v}| = \sqrt{60 \times 60 + 30 \times 30}$

$= \sqrt{3600 + 900}$

$= \sqrt{4500}$

$= 10\sqrt{45}$

$\boxed{= 30\sqrt{5} \text{ m/s}}$

b) $\cos\alpha = \frac{30\sqrt{5}}{60}$

$= \frac{\sqrt{5}}{2}$

$\boxed{\alpha = \cos^{-1}\frac{\sqrt{5}}{2}}$

$\theta = \tan\theta = \frac{30}{60} = \frac{1}{2}$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

$\cos\theta = \frac{2}{\sqrt{5}} = \frac{B}{H}$

$B = 1$

$\cos\alpha = \frac{60}{30\sqrt{5}}$

c) horizontal — $\boxed{180 \text{ m}}$

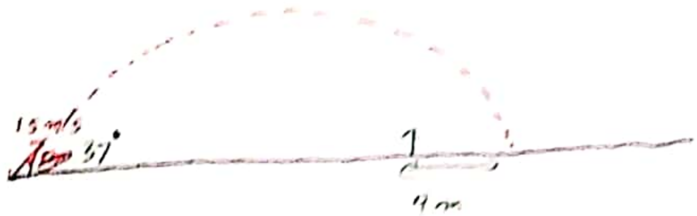
vertical $s = ut + at^2$

$s = 60 \times 3 - \frac{10}{2} \times 9$

$s = 180 - 45$

$\boxed{s = 135 \text{ m}}$

Q with what minimum velocity man should run to catch the ball? $(g = 10\, m/s^2)$



$$T = \frac{2u\sin\theta}{g}$$

$$T = \frac{2 \times 15 \times \frac{3}{5}}{10}$$

$$\boxed{T = 1.8\,s}$$

$$\text{speed} = \frac{9}{1.8}$$

$$\boxed{= 5\, m/s}$$