Physics - 2

Ch-3 Projectile Motion (continue)

Ch-4 Newton's Laws of Motion & Friction

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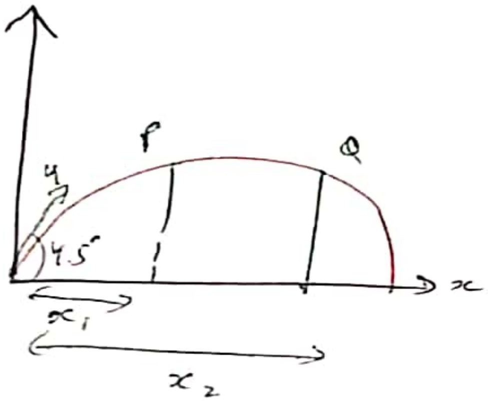
Q A particle fired with a velocity 20 m/s from a gun adjusted for maximum range. It passes through P & Q whose heights above horizontal are 5 m each. Find separation between P & Q?



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$5 = x - \frac{10 \times x^2}{2 \times 400 \times \frac{1}{2}}$$

$$200 = 40x - x^2$$

$$x^2 - 40x + 200 = 0$$

$$x = \frac{40 \pm \sqrt{1600 - 800}}{2}$$

$$x = \frac{40 \pm \sqrt{800}}{2}$$

$$x = 20 \pm \sqrt{200}$$

separation :- $20 + \sqrt{200} - 20 + \sqrt{200}$

$$\boxed{= 20\sqrt{2}\ m}$$

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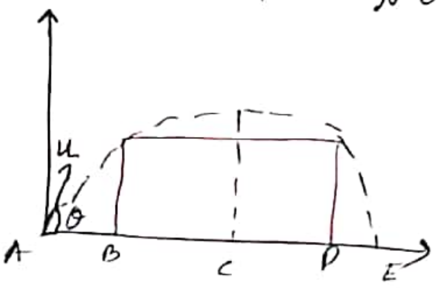
Q bird height — h
max height — 2h
find ratio of ~~speed~~ velocity of bird & horizontal velocity of the stone thrown



$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$h = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{u^2\sin^2\theta}{4g} = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\frac{gx^2}{2u^2\cos^2\theta} - x\tan\theta + \frac{u^2\sin^2\theta}{4g} = 0$$

$$x = \frac{\tan\theta \pm \sqrt{\tan^2\theta - \frac{gu^2\sin^2\theta}{2u^2\cos^2\theta\, g}}}{\frac{g}{u^2\cos^2\theta}}$$

$$x = \frac{\tan\theta \pm \tan\theta\sqrt{\frac{1}{2}}}{\frac{g}{u^2\cos^2\theta}}$$

$$\frac{\tan\theta + \frac{\tan\theta}{\sqrt{2}}}{\frac{g}{u^2\cos^2\theta}} - \frac{\tan\theta - \frac{\tan\theta}{\sqrt{2}}}{\frac{g}{u^2\cos^2\theta}}$$

$$\frac{\tan\theta\, u^2\cos^2\theta}{\sqrt{2}g} + \frac{\tan\theta\, u^2\cos^2\theta}{\sqrt{2}g}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

$$h = \frac{u^2\sin^2\theta}{4g}$$

$$u\sin\theta = 2\sqrt{gh}$$

$$\frac{\sqrt{2}\tan\theta\, u^2\cos^2\theta}{g}$$

(2)

$h = ut + \frac{1}{2} gt^2$

$gt^2 + 2ut - 2h = 0$

$gt^2 + 2ut - \frac{u^2 \sin^2\theta}{2g} = 0$

$t = \frac{-2u \pm \sqrt{4u^2 + 2gu^2\sin^2\theta}}{2g}$

$\frac{-2u + \sqrt{4u^2 + 2gu^2\sin^2\theta} + 2u + \sqrt{4u^2 + 2gu^2\sin^2\theta}}{2g}$

$\frac{\sqrt{4u^2 + 2gu^2\sin^2\theta}}{g}$

Speed = $\dfrac{\dfrac{\sqrt{2}\tan\theta \, u^2\cos^2\theta}{g}}{\dfrac{\sqrt{4u^2 + 2gu^2\sin^2\theta}}{g}}$

$= \frac{\sqrt{2}\tan\theta \, u^2\cos^2\theta}{\sqrt{4u^2 + 2g \times 4gh}}$

$= \frac{\sqrt{2}\tan\theta \, u^2\cos^2\theta}{\sqrt{4u^2_{\sin^2\theta} + 8g^2h}}$ (vertical velocity

$= \frac{\sqrt{2}\tan\theta \, u^2\cos^2\theta}{\sqrt{8gh + 8g^2h}}$

$= \frac{\tan\theta \, u^2\cos^2\theta}{2\sqrt{gh}}$

Speed

$u\cos\theta$

$\boxed{\frac{\tan\theta \, u\cos\theta}{2\sqrt{gh}}}$

$\frac{u\sin\theta}{2\sqrt{gh}}$

$\frac{2\sqrt{gh}}{2\sqrt{gh}} = \boxed{\sqrt{1}\sqrt{h}}$

$\boxed{= \frac{2\sqrt{2}}{2+\sqrt{2}}}$

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Q2

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$2h = \frac{u^2 \sin^2\theta}{2g}$$

$$u^2 \sin^2 g = 4gh$$

$$\boxed{u\sin\theta = 2\sqrt{gh}}$$

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$h = u\sin\theta\, t - \frac{1}{2} g t^2$$

$$h = 2\sqrt{gh}\, t - \frac{gt^2}{2}$$

$$gt^2 - 4\sqrt{gh}\, t + 2h = 0$$

$$t = \frac{4\sqrt{gh} \pm \sqrt{16gh - 8gh}}{2g}$$

$$t_1 = \frac{4\sqrt{gh} - 2\sqrt{2gh}}{2g}$$

$$t_2 = \frac{4\sqrt{gh} + 2\sqrt{gh}}{2g}$$

$$AD = AB + BD$$

$$u\cos\theta\, t_2 = u\cos\theta\, t_1 + V_B t_2$$

$$u\cos\theta (t_2 - t_1) = V_B t_2$$

$$\frac{V_B}{u\cos\theta} = \frac{t_2 - t_1}{t_2}$$

$$= \frac{4\sqrt{2gh}}{4\sqrt{gh} + 2\sqrt{2gh}}$$

$$= \frac{4\sqrt{2}}{4 + 2\sqrt{2}}$$

$$\boxed{= \frac{2\sqrt{2}}{2 + \sqrt{2}}}$$

④

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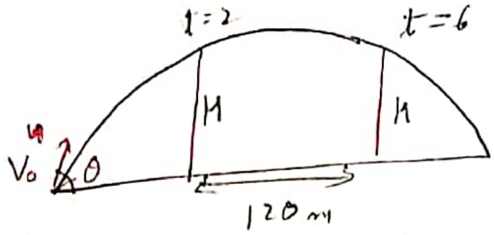
Q



A projectile crosses 2 walls of height H. find

a) time of flight

b) $H = ?$

c) Max height attained $= ?$

d) Range, $R = ?$

e) $\theta = ?$

f) $V_0 = ?$

$g = 10$

$u = V_0$

$u\cos\theta = \frac{\cancel{120}}{\cancel{4}}\ 30$

$\boxed{u\cos\theta = 30}$

$S = ut + \frac{1}{2}at^2$

$H = u\sin\theta(t) + 5t^2$

$\cancel{5t^2 + u\sin\theta t - H = 0}$

$\cancel{t = \frac{-u\sin\theta + \sqrt{u^2\sin^2\theta + 20H}}{10}}$

$u\sin(2) + 5(2)^2 = u\sin(6) + 5(6)^2$

$2u\sin\theta - 20 = 6u\sin\theta - 180$

$0 = 4u\sin\theta + 200$

$0 = u\sin\theta + 50$

$\boxed{u\sin\theta = +40}$

$\boxed{u = \frac{40}{\sin\theta}}$

$T = \frac{2u\sin^2\theta}{2\times g}$

$= \frac{2\times 40}{1\times 10}$

$\cancel{T = 40s}$

$\boxed{= 8s}$ a)

$H = \frac{u^2\sin^2\theta}{2g}$

$H = \frac{1600}{2\times 10}$

$\boxed{H = 80\,m}$ c) max height

$H = 40(2) - 5(4)$

$H = 80 - 20$

$\boxed{H = 60}$ b)

$\frac{40}{\sin\theta} \times \cos\theta = 30$

$\frac{40}{30} = \tan\theta$

$\tan\theta = \frac{4}{3}$

$\boxed{\theta = 60^\circ}$ e)

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$u \cos\theta = 30$

$u \times \frac{3}{5} = 30$

$u = \frac{30 \times 5}{3}$

$\boxed{u = 50\ m/s}$ f)

$R = \frac{u^2 \sin 2\theta}{g}$

$R = \frac{50 \times 50 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10}$

$R = 10 \times 24$

$\boxed{R = 240m}$ d)

Horizontal ~~Hori~~ Projection From A Height

Horizontal

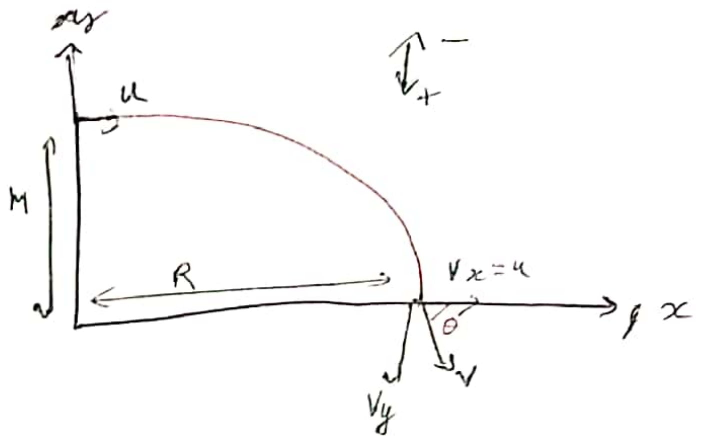
$U_x = u$

$a_x = 0$

Verticle

$U_y = 0$

$a_y = g$



Time of flight (T):-

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$H = \frac{1}{2} g T^2$$

$$\boxed{T = \sqrt{\frac{2H}{g}}}$$

Range $(R) = U_x T$

$$\boxed{R = u\sqrt{\frac{2H}{g}}}$$

$$V_y^2 = u_y^2 + 2 g S_y$$

$$V_y^2 = 0 + 2gH$$

$V_y = \sqrt{2gh}$ (Verticle component of velocity on hitting ground)

$$\boxed{V_y = \sqrt{2gh}}$$

$\cancel{\theta = \tan}$

$$\boxed{\theta = \tan^{-1}\frac{\sqrt{2gH}}{u}}$$

$$\boxed{\vec{V} = u\hat{i} + \sqrt{2gh}\,\hat{j}}$$

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Q A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find

i) time to reach ground

ii) The horizontal distance from foot of hill to ground

iii) The velocity with which it hits the ground. $(g = 9.8)$

$$T = \sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2 \times 490}{9.8}}$$

$$T = \sqrt{2 \times 50}$$

$$T = \sqrt{2 \times 5 \times 5 \times 2}$$

$$T = 2 \times 5$$

$$\boxed{= 10 s}$$ i)

$$R = u \times T$$

$$= 98 \times 10$$

$$\boxed{= 980 m}$$ ii)

$$\vec{v} = 98\hat{i} + \sqrt{2 \times 9.8 \times 490}\ \hat{j}$$

$$v = \sqrt{98 \times 98 + 2 \times 98 \times 49}$$

$$v = \sqrt{98(98 + 98)}$$

$$v = \sqrt{98 \times 98 \times 2}$$

$$\boxed{v = 98\sqrt{2} \text{ m/s}}$$ iii)

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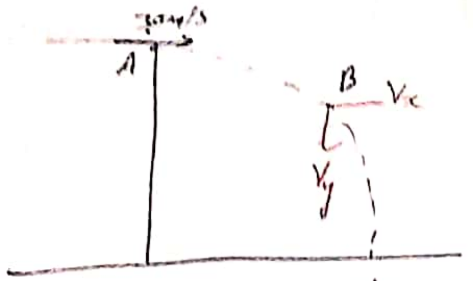
Q.



$|V_x| = |V_y|$

find time taken to reach point ~~A~~ B.

$$T = \sqrt{\frac{2H}{g}}$$

$$V_y = \sqrt{2gH}$$

$$30 = \sqrt{2 \times 10 \times H}$$

$$900 = 20H$$

$$H = \frac{900}{20}$$

$$\boxed{H = 45\,m}$$

$$T = \sqrt{\frac{2 \times 45}{10}}$$

$$T = \sqrt{\frac{45}{5}}$$

$$T = \sqrt{9}$$

$$\boxed{T = 3\,s}$$

(OR)

$$v = u + at$$

$$30 = 0 + (+10)\,t$$

$$30 = 10t$$

$$t = \frac{30}{10}$$

$$\boxed{t = 3\,s}$$

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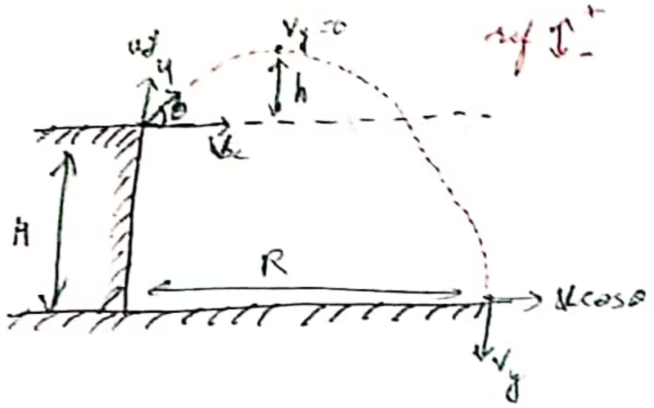
Projection at an Angle from a height



Time of flight:

$s = ut + \frac{1}{2}gt^2$

$-H = u\sin\theta\, T - \frac{1}{2} g t^2$

$T = ?$

Range $(R) = u\cos\theta\, T$

$h = \frac{u^2 \sin^2\theta}{2g}$

$v_y^2 = (u\sin\theta)^2 + 2gH$

$v_y = \sqrt{u^2\sin^2\theta + 2gH}$

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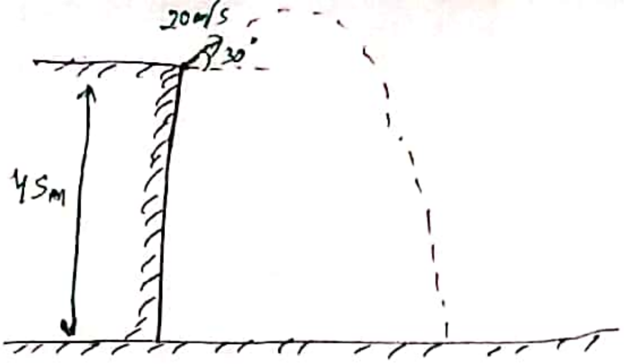
Q



find a) T
b) R
c) speed (V)

$s = ut + \frac{1}{2} a t^2$

$-45 = u\sin\theta\, t - 5t^2$

$5t^2 - 20 \times \frac{1}{2} t - 45 = 0$

$t^2 - 2t - 9 = 0$

$t = \frac{2 \pm \sqrt{4 + 36}}{2}$

$t = \frac{2 \pm \sqrt{40}}{2}$

$t = 1 \pm \sqrt{10}$

$\boxed{t = 1 + \sqrt{10}\, s}$

$R = u\cos\theta\, t$

$R = 20 \times \frac{\sqrt{3}}{2} \times (1 + \sqrt{10})$

$R = 10\sqrt{3} + 10\sqrt{30}$

$\boxed{R = 10(\sqrt{3} + \sqrt{30})\ m}$

$v = (u^2\sin^2\theta + 2gH)^{1/2}$

$v = 400 \times \frac{1}{4} + 2 \times -10 \times -45$

$V = 100 + 900$

$V = \sqrt{1000}$

$V_y = 10\sqrt{10}$ m/s

$|\vec{V}| = \sqrt{(10\sqrt{10})^2 + (10\sqrt{2})^2}$

$\cancel{= \sqrt{1000 + 200}}$

$\cancel{= \sqrt{1200}}$

$\cancel{= 10\sqrt{12}}$

$\cancel{\boxed{= 20\sqrt{3}\ m/s}}$

$= \sqrt{1300}$

$\boxed{= 10\sqrt{13}\ m/s}$

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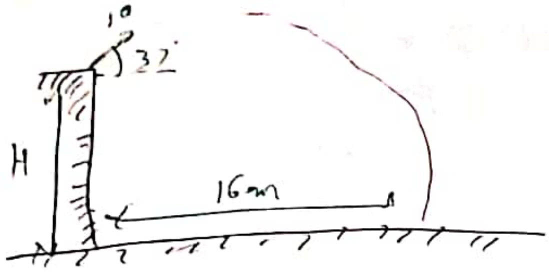
Q



$g = 10\, m/s^2$

u

$-H = 6t - 5t^2$

$\cancel{5t^2 - 6t - H = 0}$

$\cancel{t = \frac{6 \pm \sqrt{36 + 20H}}{2}}$

$\cancel{t = 3 \pm \sqrt{9+5H}}$

$16 = u\cos\theta\, t$

$16 = 8t$

$t = 2$

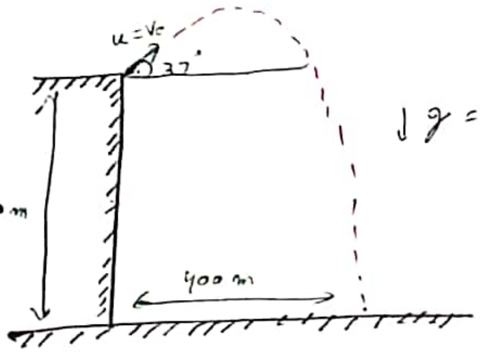
$\cancel{3 + \sqrt{9+5H} = 2}$

$\cancel{\sqrt{9+5H} =}$

$-H = 12 - 20$

$\boxed{H = 8m}$

Q



$\downarrow g = 10\, m/s$

$400 = u \times \frac{4}{5} \times t$

$500 = ut$

$\frac{500}{u} = t$

$-100 = u \times \frac{3}{5} \times \frac{500}{u} - \frac{1}{2} \times 10 \times \frac{500}{u} \times \frac{500}{u}$

$-100 = 300 - \frac{1250000}{u^2}$

$-100u^2 = 300u^2 - 12500$

$12500 = 400u^2$

$\frac{12500}{4} = u^2$

$\cancel{u = \frac{5\sqrt{5}}{\sqrt{2}}\, m/s}$

$\cancel{u^2 = \frac{6250}{2}}$

$u = \frac{25\sqrt{10}}{2}$

$u^2 = \frac{6250}{2}$

$u^2 = 3125$

$\boxed{u^2 = 25\sqrt{5}}$

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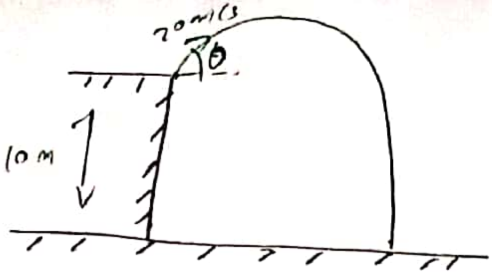
Q



$$-10 = 20\sin\theta\, t - 5t^2$$
$$-2 = 4\sin\theta\, t - t^2$$
$$t^2 - 4\sin\theta\, t - 2 = 0$$
$$t = \frac{4\sin\theta + \sqrt{16\sin^2\theta + 8}}{2}$$
$$t = 2\sin\theta + \sqrt{4\sin^2\theta + 2}$$
$$0 = 2\cos\theta + \frac{1}{2\sqrt{4\sin^2\theta + 2}}$$
$$0 = 4\cos\theta\sqrt{4\sin^2\theta + 2} + 1$$
$$0 =$$
$$1 = 16\cos^2\theta\,(4\sin^2\theta + 2)$$
$$h =$$
$$\sec^2\theta = 64\sin^2\theta + 2$$

$$\cancel{R = u\cos\theta\, t}$$
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$
$$-10 = x\tan\theta - \frac{10x^2}{2\times 400\times\cos^2\theta}$$
$$-10 = x\tan\theta\left[1 - \frac{x}{R}\right]$$
$$-10 = x\tan\theta - \frac{x^2}{80\cos^2\theta}$$
$$-800\cos^2\theta = 80x\sin\theta\cos\theta - x^2$$
$$x^2 - x\sin\theta\cos\theta - 800\cos^2\theta$$
$$\frac{dy}{dx} = 2x -$$
$$-10 = x\tan\theta - \frac{10x^2}{400\times 2\times\cos^2\theta}$$
$$-10 = \overset{80}{x}\tan\theta - \frac{x^2\sec^2\theta}{80}$$
$$-800 = 80x\tan\theta - x^2\sec^2\theta$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$
$$-10 = x\tan\theta - \frac{10x^2\sec^2\theta}{800}$$
$$-10 = x\tan\theta - \frac{x^2(1 - \tan^2\theta)}{80}$$
$$-800 = 80x\tan\theta - x^2 + x^2\tan^2\theta$$
$$x^2\tan^2\theta + 80x\tan\theta - (x^2 - 800) = 0$$
$$\tan\theta \quad b^2 - 4ac \geq 0$$

$$6400x^2 + 4x^2(x^2 - 800) \geq 0$$
$$6400x^2 + 4x^4 - 3200x^2 \geq 0$$
$$9600x^2 - 4x^4 \geq 0$$
$$2400 \leq R^2$$
$$\boxed{R \leq 20\sqrt{6}}$$

$$\cancel{2400\tan^2\theta}$$
$$\tan\theta = \frac{1600\sqrt{6} \pm \sqrt{9600x^2 - 4x^4}}{4800}$$

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$$\tan\theta = \frac{1600\sqrt{6} + \sqrt{9600 \times 2400 + 4 \times 2400}}{2400 \times 2}$$

$$\tan\theta = \frac{2/3\,\sqrt{6} + \sqrt{9600+4}}{2}$$

$$\tan\theta = \frac{1}{3}\sqrt{6} + \sqrt{2401}$$

$$\theta = \tan^{-}\left[\frac{\sqrt{6}}{3} + \sqrt{2401}\right]$$

$$x = 20\sqrt{6}$$
$$x^2 = 400 \times 6$$
$$= 2400$$

$$x^2\tan^2\theta + 80x\tan\theta - (x^2 - 800) = 0$$

$$2400\tan^2\theta + \cancel{8}1600\tan\theta - (2400 - 800)$$
$$- 1600$$

$$2400\tan^2\theta + 1600\sqrt{6}\tan\theta - 1600 = 0$$

$$\cancel{3\tan^2\theta + 2\tan\theta - 2 = 0}$$

$$\cancel{\tan\theta = \frac{-2 \pm \sqrt{4+24}}{2}}$$

$$\cancel{\tan\theta = \frac{-2 + \sqrt{30}}{2}}$$

$$3\tan^2\theta + 2\sqrt{6}\tan\theta - 2 = 0$$

$$\tan\theta = \frac{-2\sqrt{6} \pm \sqrt{24+24}}{2 \times 3}$$

$$\cancel{\tan\theta = \frac{-\sqrt{6} \pm 12\sqrt{2}}{3}}$$

$$\tan\theta = \frac{-\sqrt{6} \pm \sqrt{12}}{3}$$

$$\boxed{\tan\theta = \frac{-\sqrt{6} \pm 2\sqrt{3}}{3}}$$

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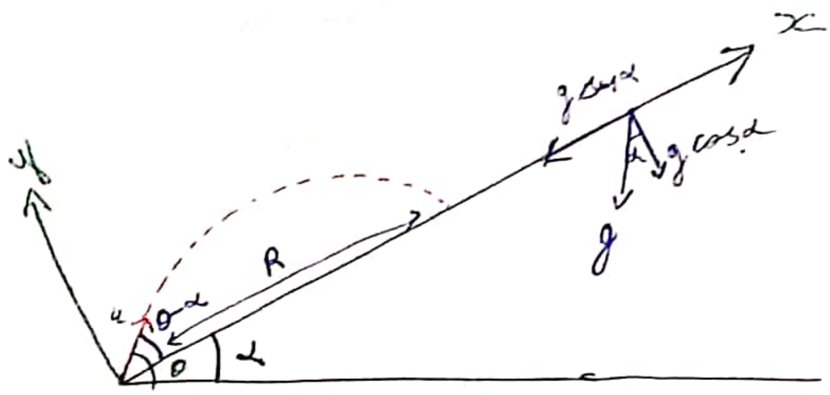
Projection up the inclined plane



Horizontal

$u_x = u\cos(\theta-\alpha)$

$a_x = -g\sin\alpha$

Vertical

$u_y = u\sin(\theta-\alpha)$

$a_y = -g\cos\alpha$

Time of Flight (T) =

$s = ut + \frac{1}{2}at^2$

$0 = u_y t + \frac{1}{2}a_y t^2$

$0 = u\sin(\theta-\alpha)\, t + \frac{1}{2}(-g\cos\alpha)\, t^2$

$\frac{g\cos\alpha}{2} t^2 = u\sin(\theta-\alpha)\, t$

$t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$

$$\boxed{t = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}}$$

Range :- $U_x T + \frac{1}{2} a_x T^2$

$R = u\cos(\theta-\alpha)\, T + \frac{1}{2}(\cancel{-g\cos\alpha})(-g\sin\alpha)\, T^2$

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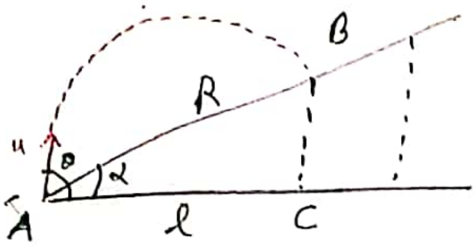

$l = u\cos\theta\, T$ & $\cos\alpha = \frac{l}{R}$

$$\cos\alpha = \frac{u\cos\theta T}{R}$$

$$R = \frac{u\cos\theta T}{\cos\alpha}$$

$$R = \frac{u\cos\theta}{\cos\alpha} \times \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$$

$$\boxed{R = \frac{2u^2\cos\theta\sin(\theta-\alpha)}{g\cos^2\alpha}}$$

$$\boxed{R_{max} = \frac{u^2}{g(1+\sin\alpha)}}$$

(16)

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JM6 (Ch - $\cancel{8}$)

Q8. $x^2 = at^2 + 2bt + c$

$$\frac{d}{dt}(x^2) = \frac{d}{dt}(at^2 + 2bt + c)$$

$$2x \times \frac{dx}{dt} = 2at + 2b$$

$$x\,v = at + b$$

$$v \times \frac{dx}{dt} + \frac{x\,dv}{dt} = a$$

$$v^2 + ax = a$$

$$A = \frac{a - v^2}{x}$$

$$A = \frac{a - \left(\frac{at+b}{x}\right)^2}{}$$

$$A = \frac{a - \frac{a^2t^2 + b^2 + 2abt}{x^2}}{x}$$

$$A = \frac{ax^2 - (a^2t^2 + b^2 + 2abt)}{x^3}$$

$$A = \frac{a^2t^2 + 2abt + ac - a^2t^2 - b^2 - 2abt}{x^3}$$

$$A = \frac{ac - b^2}{x^3}$$

$$A \propto x^{-3}$$

$$\boxed{n = 3}$$

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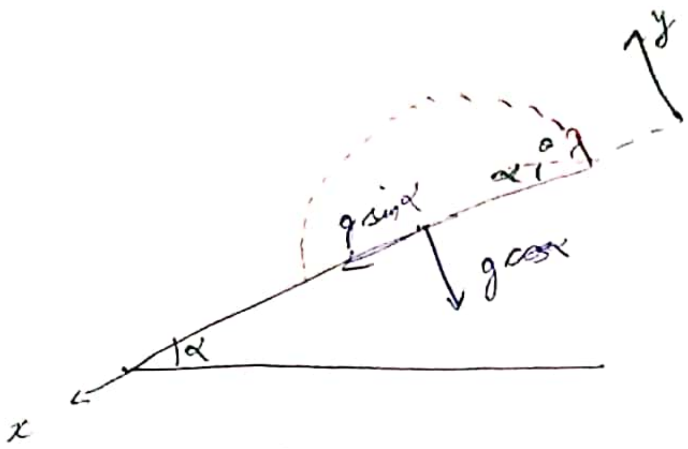
Projection down the inclined plane



Horizontal

$U_x = u\cos(\theta+\alpha)$

$a_x = g\sin\alpha$

Vertical

$U_y = u\sin(\theta+\alpha)$

$a_y = -g\cos\alpha$

Time of flight (T):-

$S_y = 0 = U_y T + \frac{1}{2} a_y T^2$

$0 = U\sin(\theta-\alpha)T + \frac{1}{2}(-g\cos\alpha)T^2$

$$\boxed{T = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}}$$

$R = U_x T + \frac{1}{2} a T^2$

$R = u\cos(\theta+\alpha)T + \frac{1}{2} g\sin\alpha T^2$

$$\boxed{R = u\cos(\theta+\alpha)T + \frac{1}{2} g\sin\alpha T^2}$$

$$\boxed{R = \frac{2u^2\sin(\theta+\alpha)\cos\theta}{g\cos^2\alpha}}$$

$$R_{max} = \frac{u^2}{g(1-\sin\alpha)}$$

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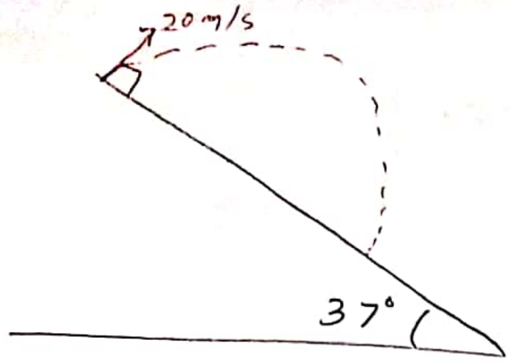
Q



$$R = \frac{2u^2 \sin(\theta \pm \alpha) \cos\theta}{g \cos^2\alpha}$$

$$R = \frac{2 \times 20 \times 20 \times \cos 37 \times 1}{10 \times (\cos 37)^2}$$

$$R = \frac{80 \times \frac{4}{5}}{\frac{16}{25}}$$

$$R = \frac{80 \times 4 \times 25}{16 \times 5}$$

$$R = 100$$

$$R = \frac{2u^2 \sin(\theta + \alpha) \cos\theta}{g \cos^2\alpha}$$

$$R = \frac{2 \times 20 \times 20 \times 1 \times \frac{3}{5}}{10 \times \frac{16}{25}}$$

$$R = \frac{80 \times 3 \times 25}{16 \times 5}$$

$$R = 25 \times 3$$

$$\boxed{R = 75\,m}$$

Or

$$T = \frac{2u \sin(\theta + \alpha)}{g \cos\alpha}$$

$$= \frac{2 \times 20 \times 1}{2 \times 10 \times \frac{4}{5}}$$

$$\boxed{T = 5s}$$

$$S = ut + \frac{1}{2} g t^2$$

$$R = u \cos(\theta + \alpha) T - \frac{10 \times \sin\alpha \times t^2}{2}$$

$$R = 20 \times 0 - 5 \times \frac{3}{5} \times 5 \times 5$$

$$\boxed{R = + 75\,m}$$

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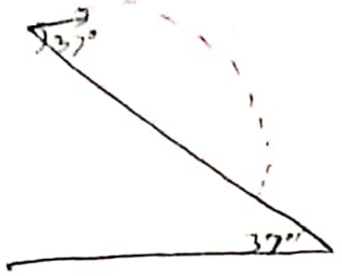
Q



$T = 5s$

$u = ?$

~~T = 5s find u~~

$$T = \frac{2u \sin(\theta + \alpha)}{g\cos\alpha}$$

$\cancel{T = 2u\, 2 \times u \times}$

$$5 = \frac{\cancel{2}\times u \times \frac{3}{5}}{\cancel{2}\cdot 10 \times \frac{4}{5}}$$

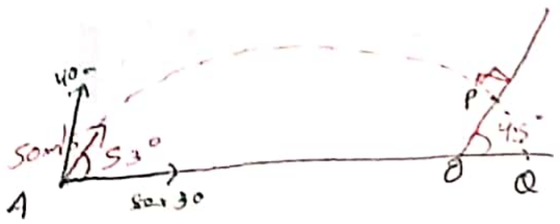
$$\frac{20 \times 5}{3} = u$$

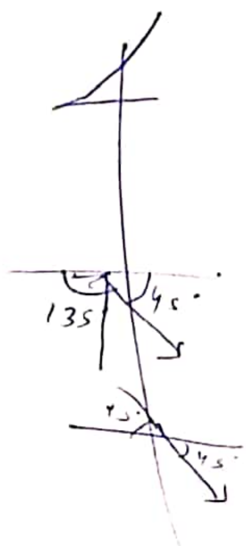
$$\boxed{\frac{100}{3} = u}$$

Q

$T = ?$

$AO = ?$





$\vec{v} = u\cos 45°\,\hat{i} + u\sin 45°\,\hat{j}$

$\vec{v} = u\cos 53°\,\hat{i} + u\sin 53 - gt\,\hat{j}$

$$\frac{50}{\sqrt{2}} = 50 \times \frac{40}{5} - 10t$$

$$10t = 40 - \frac{50}{\sqrt{2}}$$

$$\cancel{10}t = 4 - \frac{5}{\sqrt{2}}$$

$$t = 4\sqrt{2} - 5\,s$$

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Velocity at P $= \vec{v}$

$\vec{v} = \frac{v}{\sqrt{2}}\hat{i} - \frac{v}{\sqrt{2}}\hat{j}$

$\vec{u} = 30\hat{i} + 40\hat{j}$

$\frac{v}{\sqrt{2}} = 30$

$\underline{v = 30\sqrt{2}}$

$v(\text{in } y) = \frac{30\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}$

$= -30$

Using $v - u = at$

$\frac{-30-40}{-10} = t$

$\frac{-70}{-10} = t$

$\boxed{7 = t}$

$\boxed{t = 7s}$

$s = ut + \frac{1}{2}at^2$

$PQ = 40(7) + \frac{1}{2}(-10)(7)^2$

$= 280 - 245$

$= 35 = OQ$

$AQ = 30 \times 7 = 210$

$AO = AQ - PQ = 210 - 35$

$\boxed{= 175\ m}$

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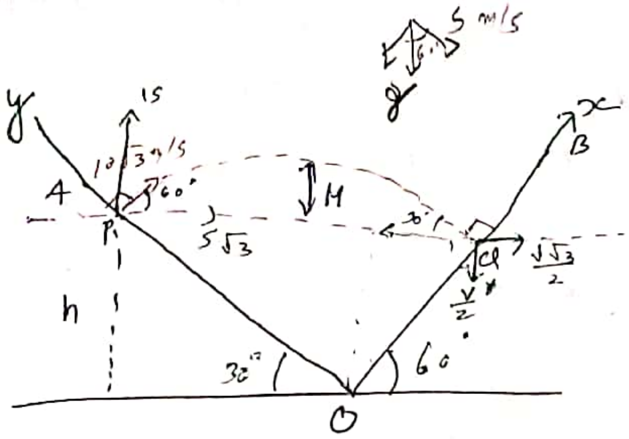
Q



a) Time of flight
b) velocity at OB.
c) $h = ?$
d) Max height attained by particle from O
e) distance PQ.

~~$\frac{V}{2} = 5\sqrt{3}$~~

~~$\boxed{V = 10\sqrt{3}}$~~

~~$\frac{V\sqrt{3}}{2} = \frac{10\sqrt{3}\times\sqrt{3}}{2} = \frac{30}{2} = 15$~~

$\frac{V\sqrt{3}}{2} = 5\sqrt{3}$

$\boxed{V = 10}$ b)

$\frac{-5-15}{-10} = t$

~~$\frac{-10}{-10} = t$~~ a)

~~$\boxed{t = 1s}$~~

$\frac{-20}{-10} = \boxed{2s}$ a)

$\tan 60° = \frac{PO}{PQ}$

$\sqrt{3} = \frac{20}{PQ}$ e)

$\boxed{PQ = \frac{20\sqrt{3}}{3}}$

~~$H = \frac{u^2 \sin^2\theta}{2g}$~~

~~$H = \frac{10\sqrt{3}\times 10\sqrt{3}\times\frac{3}{4}}{10\times 10}$~~

~~$H = \frac{9}{4}$~~

~~$T = \sqrt{\frac{2H}{g}}$~~

~~$2 = \sqrt{\frac{2\times PO}{10}}$~~

~~$4 = \frac{2PO}{10}$~~

~~$\frac{40}{2} = PO$~~

~~$\boxed{PO = 20}$~~

~~$\tan 30 = \frac{h}{PO}$~~

~~$\frac{1}{\sqrt{3}} = \frac{h}{20}$~~

~~$\boxed{h = \frac{20}{\sqrt{3}}}$~~ c)

~~$\text{height} = \frac{9}{4} + \frac{20}{\sqrt{3}}$~~

~~$\boxed{= \frac{9\sqrt{3}+80}{4\sqrt{3}}\ m}$~~ d)

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$$-OP = 0 + \frac{1}{2}(-5)(2)^2$$

$$S_y = V_y T + \frac{10}{2} y T^2$$

$$\boxed{OP = 10}$$

$$S_x = V_x T + \frac{1}{2} a_x T^2$$

$$OQ = 10\sqrt{3}(2) + \frac{1}{2}(-5\sqrt{3})(2)^2$$

$$= 20\sqrt{3} - 10\sqrt{3}$$

$$\boxed{OQ = 10\sqrt{3}\ m}$$

$$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{100 + 300}$$

$$\boxed{PQ = 20m}$$

$$\boxed{PQ = 20m}$$

$$\sin 30^\circ = \frac{h}{10}$$

$$\boxed{= 5m}$$

$$H = \frac{(10\sqrt{3})^2 \sin^2 \cancel{30} 60^\circ}{2(10)}$$

$$H = 15 \times \frac{3}{4} = \boxed{\frac{45}{4}}$$

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Relative Velocity

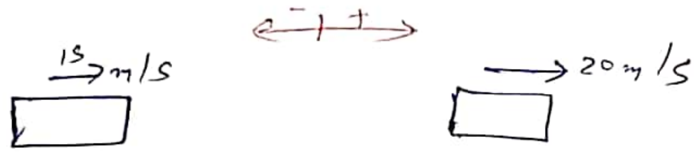
$$\boxed{\vec{V}_{obj,\,observer} = \vec{V}_{obj} - \vec{V}_{observer}}$$

→ Velocity of object with respect to observer.

→ If distance increases, velocity of separation else velocity of approach

eg.

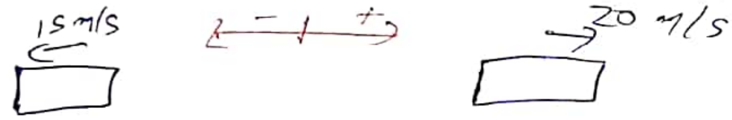
eg ①



$V_{AB} = V_A - V_B = 15 - 20 = -5$ m/s

$V_{BA} = V_B - V_A = 20 - 15 = 5$ m/s

eg ②



$V_{AB} = V_A - V_B = -15 - 20 = -35$ m/s

$V_{BA} = V_B - V_A = 20 - (-15) = 20 + 15 = 35$ m/s

eg ③.

$V_{AB} = V_A - V_B = v - v = 0$

④ -

$V_{AB} = V_A - V_B = v - (-v) = v + v = 2v$

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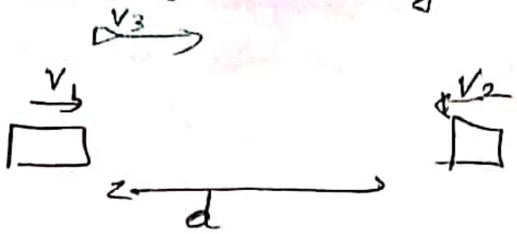
Q find distance travelled by bird till car meets.



$$V_{AB} = V_A - (-V_2) = V_1 + V_2$$

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

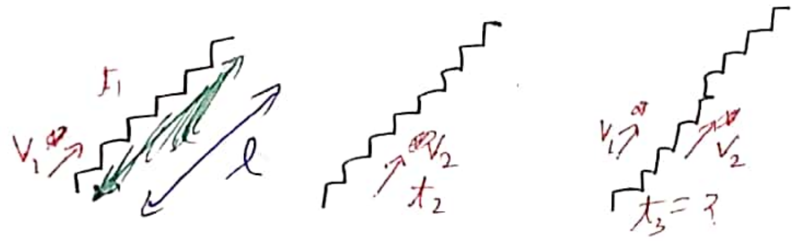
$$\boxed{\text{Time} = \frac{d}{V_1 + V_2}}$$

Distance by bird = speed × time

$$= V_3 \times \frac{d}{V_1 + V_2}$$

$$\boxed{= \frac{V_3 d}{V_1 + V_2}}$$

Q



$$V_1 = \frac{l}{t_1} \qquad V_2 = \frac{l}{t_2}$$

$$V_1 + V_2 = V_3$$

$$V_3 = \frac{l}{t_1} + \frac{l}{t_2}$$

$$\frac{l}{V_3} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$$

$$= \frac{t_2 l + t_1 l}{t_1 + t_2}$$

$$\frac{l}{V_3} = t_3 = \frac{l}{\frac{l(t_1 + t_2)}{t_1 + t_2}}$$

$$\boxed{= \frac{t_1 t_2}{t_1 + t_2}}$$

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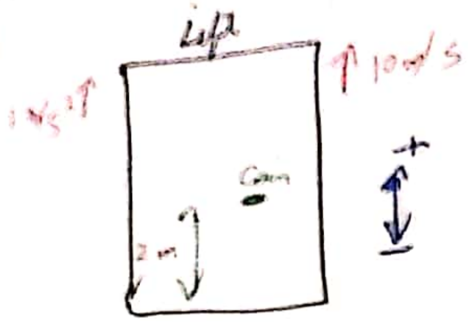
Q 

A coin is dropped inside lift. find Time it takes to reach the floor ($g = 10\,m/s^2$)

$$u_{relative} = u_{c,l} = u_c - u_l = 10 - 10 = 0\ m/s$$

$$a_{rel} = a_{c,l} = a_c - a_l = -10 - (+1) = -11\ m/s$$

$$S_{rel} = -2m$$

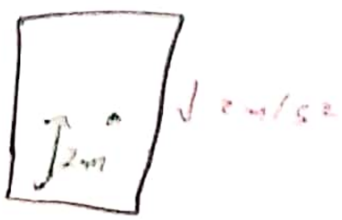
$$S = ut + \frac{1}{2}at^2$$

$$S_{rel} = u_{rel}t + \frac{1}{2}a_{rel}t^2$$

$$-2 = -\frac{11}{2}t^2$$

$$\frac{-4}{-11} = t^2$$

$$\boxed{t = \frac{2}{\sqrt{11}}\ s}$$

Q 

$$a_{rel} = -8$$
$$u_{rel} = 0$$
$$S_{rel} = -2$$

$$-2 = \frac{1}{2} \times -8t^2$$

$$\frac{4}{8} = \frac{1}{2} = t^2$$

$$\boxed{t = \frac{1}{\sqrt{2}}\ s}$$

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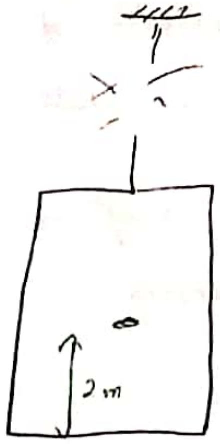
Q



~~ad~~ $V_{cl} = 0\, m/s$

$a_{c,l} = -10 - (-10) = -10 + 10 = 0$

$-2 = 0 + \frac{1}{2}(0) t^2$

$-2 = 0 t^2$

$\frac{-2}{0} = t^2$

$t^2 = \infty$

$\boxed{t = \infty\, s}$ The coin will never reach the floor of lift.

Q

$v_1 > v_2$ (initially)

~~If~~ Applies break for $a = -\alpha$ within distance is d, find min d to avoid collision. Speed of A must be equal to v_2 after relative distance d.

$v_3 = v_1 - v_2 = v_{rel}$

$a = -\alpha - 0 = -\alpha$

$s = d$

$\cancel{d = v_3 t + 0 - \frac{1}{2} \alpha t^2}$

$0 - v_3^2 = 2 \times (-\alpha) \times d$

$$\boxed{\frac{+(v_1 - v_2)^2}{+2\alpha} = d}$$

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Q A platform is moving upwards with a constant acceleration of $2 m/s^2$, At $t = 0$, a boy standing on platform throws a ball upwards with a relative speed of 8 m/s. At this instant, platform was at height of 4 m from the ground and was moving with a speed of 2 m/s. Find

a) when & where ball strikes the platform

b) Max height of ball from ground

c) Max height of ball from platform.

$a_{rel} = \cancel{8}\, -12 \, m/s^2$

$u_{rel} = 8 m/s$

$v = 0$

$\frac{-8}{-12} = t$

$t = \frac{2}{3}$ (to go up

~~t = 2s~~ $\boxed{t = 4/3}$ a)

~~S = H + 1/2 × 2 × 4/2~~

~~S = 8 m + 4~~

~~= 12 m~~ a)

~~S = 4 + 1/2 ×~~

$S = 2 \times \frac{4}{3} + \frac{1}{2} \times 2 \times \frac{4}{3} \times \frac{4}{3}$

$S = \frac{8 \times 3}{9} + \frac{16}{9}$

$S = \frac{40}{9} + 4$

~~S = 4.66 m~~ a)

$\boxed{S = 8.66 \, m}$ a)

~~s = 8 × 0 + 1/2~~

~~v + 64 = 2 × + 12 × H (from const) ;~~

~~H = 4m~~ c)

~~H = 8/3 m~~ c)

$v + 100 = 2 \times + 10 \times h$ (from ground) $+ 4$

$\frac{100}{20} + 4 = h$

$h = 5 + 4$

$\boxed{h = 9 m}$ b)

W.r.t platform, max ball height will be for velocity of ball = velocity of platform.

$U_{BP} = 8 m/s$

$V_{B \cdot P} = 0$

$a_B P = -12 m/s^2$

$s_B P = ?$

$v^2 - u^2 = 2as$

$0 + 64 = 2 \times + 12 \times h$

$\frac{64}{24} = h$

$\boxed{h = \frac{8}{3} m}$ c)

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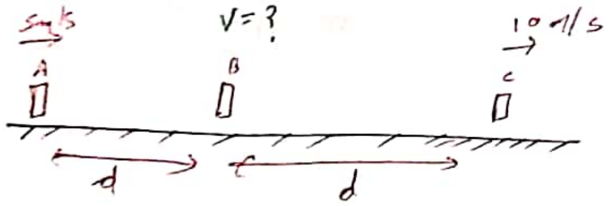
Q



~~When A catches B, sep~~ When B catches C, separation between A & C is 3d. find speed of B.

Time taken by B to catch C, $t_1 = \frac{d-0}{v-10}$

A & C

$10-5 = \frac{3d-2d}{T_1}$ ↗ change in separation

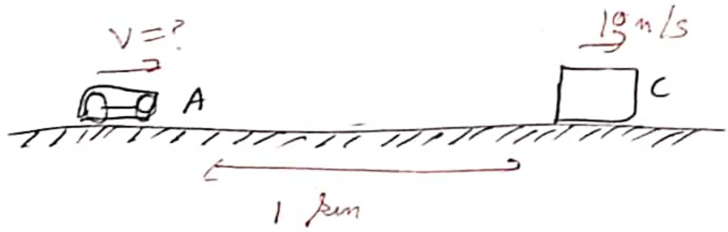
$10-5 = \frac{d}{\frac{d}{v-10}}$

$10-5 = v-10$

$5+10 = v$

$\boxed{v = 15\ m/s}$

Q



find the speed with which A should chase C to overtake it in 100 s.

$(v-10)\times 100 = 1000$

$\boxed{v = 20\ m/s}$

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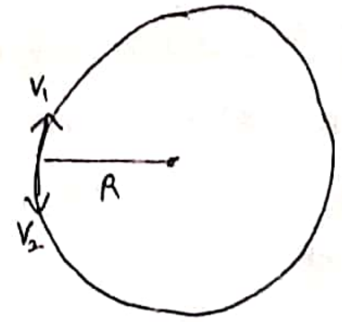
Q



After what time will they meet

$$\frac{2\pi R}{V_1 + V_2}$$

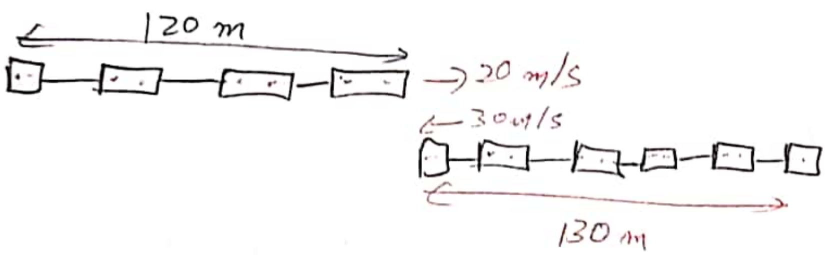
rel Velocity = ~~$V_1 - V_2$~~ $V_1 - (-V_2) = V_1 + V_2$

distance $= 2\pi R$ (circumference)

Time $= \frac{dis}{Time}$

$$\boxed{= \frac{2\pi R}{V_1 + V_2}}$$

Q



$V_1 = 20$

$V_2 = 30$

$V_{rel} = 30 + 20$

$= 50$

distance $= 120 + 130$

$= 250$

Time $= \frac{\text{distance}}{\text{~~Total~~ Speed}}$

$= \frac{250}{50}$

$\boxed{= 5s}$

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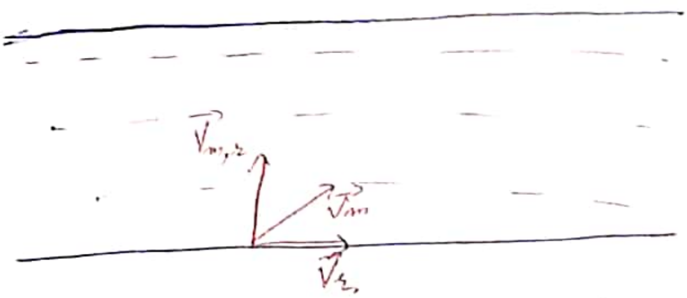
river man problems.

upstream : against the river flow
downstream : with the river flow

$t_{down} = \frac{d}{V_m + V_r}$ $(A \to B)$ $\quad T_{up} = \frac{d}{V_m - V_r}$ $(B \to A)$



$\vec{V}_{m,r}$ = Velocity of man w.r.t river (man generated v in still water)

$\vec{V}_{r}$ = Velocity ~~f~~ of flow of river

$\vec{V}_m$ = Net velocity of man

$\vec{V}_m = \vec{V}_{m,r} + \vec{V}_r$

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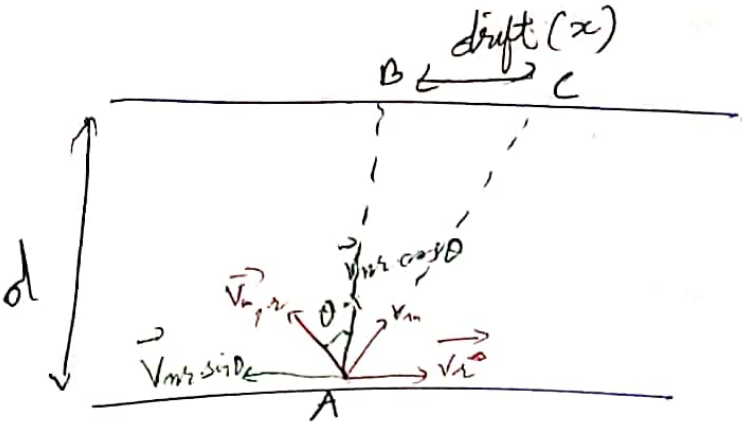


Time Taken = $\boxed{\dfrac{d}{V_{mr}\cos\theta}}$

drift $(x) = (V_r - V_{mr}\sin\theta) \times t$

$$\boxed{x = \frac{(V_r - V_{mr}\sin\theta)\, d}{V_{mr}\cos\theta}}$$

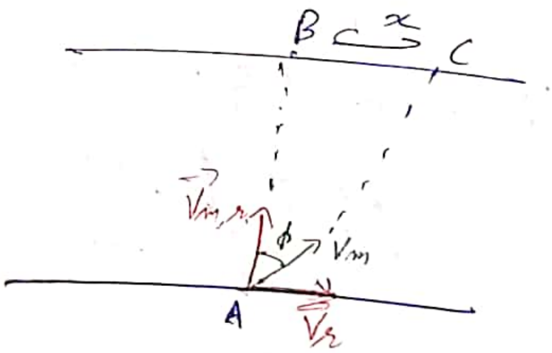
Case I :- For minimum time

$\cos\theta = 1$ (max)

$\theta = 0°$

$$\boxed{V_m = \sqrt{V_{mr}^2 + V_r^2}}$$

man should swim ⊥ to river flow



$x = V_r t$

$$\boxed{x = \frac{V_r d}{V_{mr}}}$$

$$\boxed{\tan\phi = \frac{V_r}{V_{mr}}}$$

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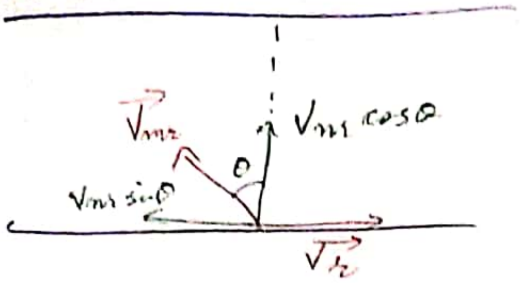
Case 2



II To cross river with zero drift.

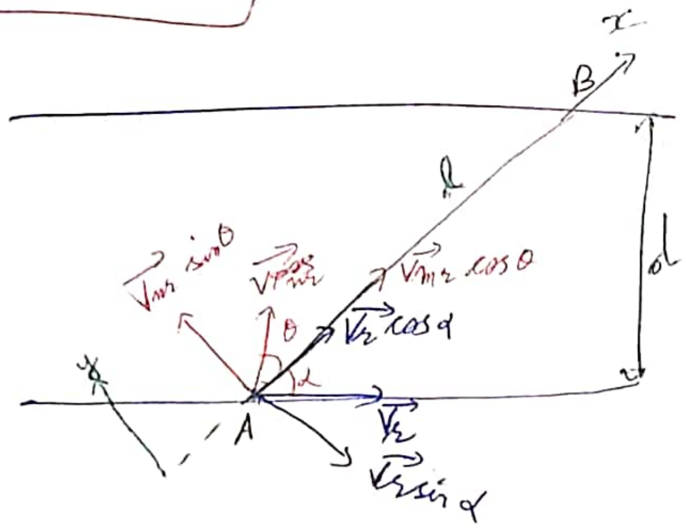
$$V_r = V_{Mr}\sin\theta$$

$$\sin\theta = \frac{V_r}{V_{mr}}$$ Only valid till $V_{mr} > V_r$.

$$\boxed{\theta = \sin^{-1}\left[\frac{V_r}{V_{mr}}\right]}$$

$$\boxed{t = \frac{d}{V_{mr}\cos\theta}}$$

Q



$$V_{mr}\sin\theta = V_r\sin\alpha$$

$$\boxed{\sin\theta = \frac{V_r}{V_{mr}}\sin\alpha}$$

↳ Angle with which boat will be steered.

$$\boxed{t = \frac{l}{V_r\cos\alpha + V_{mr}\cos\theta}}$$

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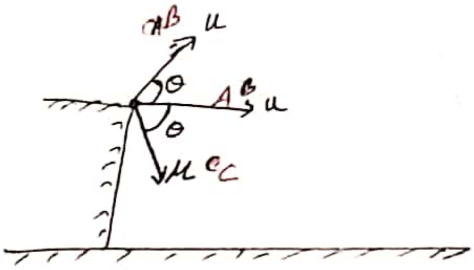
Extra Questions (Ch-1, 2, 3)

Q1.



A) $V_A > V_B > V_C$

B) $V_A = V_B = V_C$

C) $V_C > V_B > V_A$

D) cannot find.

$\cancel{V_{B_y} = \sqrt{2gh} = \sqrt{20h}}$

$\cancel{V_{A_y} = u\sin\theta + 10}$

$\cancel{V_C =}$

$\vec{V}_{BA} = \sqrt{u^2 + 2gh}$

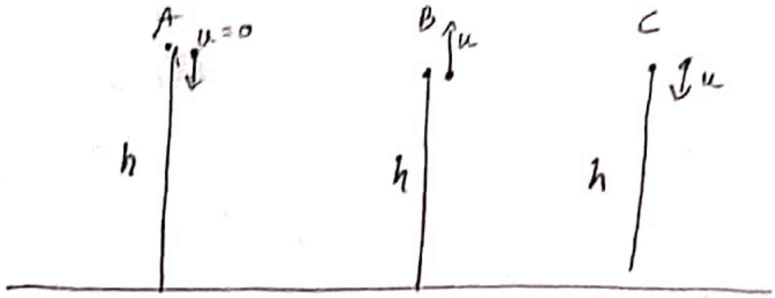
$V_B = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta + 2gH}$

$= \sqrt{u^2 + 2gH}$

$V_C = \sqrt{u^2 + 2gH}$

$\boxed{\text{B) } V_A = V_B = V_C}$

Q2.



a) velocity relation

$V_A^2 = 2gH$

$V_B^2 = 2gH + u^2$

$V_C^2 = 2gH + u^2$

A) $V_C > A_B > V_A$

B) $V_A < V_B < V_C$

C) $V_A < V_B = V_C$

d) Nota

Q3. If initial vel. of a particle projected from ground is $\vec{u} = 6i + 8j$ m/s find the horizontal range

$R = \frac{u^2 \sin 2\theta}{g} \cdot \frac{2 u\sin\theta \, u\cos\theta}{g} = \frac{2 \times 8 \times 6}{10} = \frac{96}{10} = 9.6$ m

Q4. In the situation shown a player kicks the ball at 45° with 20 m/s vel the distance of a 3 m high goal post is 25 m from the player. find weather there will be a goal or not

$= 25 - \frac{10 \times 625}{2 \times 20 \times 20 \times \frac{1}{2}}$

$= 25 - \frac{625}{40}$

$= \frac{1000 - 625}{40}$

$= \frac{375}{40}$

$= \frac{93.\text{something}}{10}$

$y = 9.3$ something

at 25 m, ball's height will be 9.3 so there will be no goal.

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Q5. A particle is thrown from ground at angle 30° with horizontal at 40 m/s find time for which it will be at the height more than 20 m

$$20 = \cancel{40}^{20} \times \frac{1}{\cancel{2}} t - 5t^2$$

$$\frac{\cancel{40}^{20} \times \cancel{40}^{10} \times \frac{1}{4}}{} = 2 \times 10$$

$$5t^2 - 20t + 20 = 0$$

$$t^2 - 4t + 4 = 0$$

$$t = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$\boxed{t = 2\,s}$$

$$\boxed{\cancel{t = 3\,s}}$$

Max heigh = 20
time duration for which it will stay there = 0 s.

$$t^2 - \cancel{2}^{4}t + 2 = 0$$

$$t = \frac{4 \pm 2\sqrt{2}}{2}$$

$$t = 2 \pm \sqrt{2}$$

$$2 + \sqrt{2} - 2 + \sqrt{2}$$

$$\boxed{2\sqrt{2}\,s}$$

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Q6. चोर is running away on a straight road at 9 m/s. A policeman chases him on a motorcycle at 10 m/s if the distance between them is 100 m how long will it take for police to catch the thief.

$V_t = 9$

$V_p = 10$

$V_{rel} = 10 - 9$

$= 1 \, m/s$

$S_{rel} = 100 \, m$

$$Time = \frac{Distance}{Re \; speed}$$

$$Time = \frac{100}{1}$$

$$\boxed{= 100 s}$$

Q7. A bus moving at 10 m/s on straight road. a bikes 80 m behind it moving at 5 m/s wishes to overtake the bus in 100 s find acc require.

$V_{rel} = 5 - 10$

$\boxed{= -5 \, m/s}$

$S_{rel} = 80 \, m$

$t = 100 s$

$8\cancel{0} = -5\cancel{00} + \frac{1}{\cancel{2}} a \times \overset{5}{\cancel{1000}}$

$8 + 50 = a \times 500$

$a = \frac{58}{500}$

$a = \frac{58}{5 \times 100}$

$a = 11.6$

$\boxed{a = 0.116 \, m/s^2}$

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H.W.

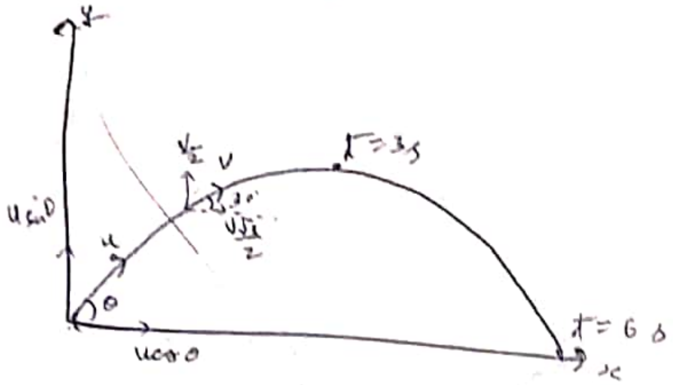
Q. A body is projected with velocity u at angle θ with horizontal. The body makes angle 30° with horizontal at t = 2s & after 1s reaches max height.

a) angle of projection

b) speed of projection.

$\cancel{v = u\sin\theta = 20}$

$\cancel{u\cos\theta = u\sin\theta \quad 20 \times \frac{\sqrt{3}}{2}}$



$$T = \frac{2u\sin\theta}{g}$$

$$\frac{6 \times 10}{2} = u\sin\theta$$

$$30 = u\sin\theta$$

$$\boxed{u = \frac{30}{\sin\theta}}$$

$$\frac{v}{2} = u\sin\theta - gt$$

$$= 30 - 20$$

$$\frac{v}{2} = 10$$

$$v = 20$$

$$\frac{20\sqrt{3}}{2} = u\cos\theta$$

$$\cancel{20\sqrt{3}}$$

$$10\sqrt{3} = 30\cot\theta$$

$$\frac{30}{10\sqrt{3}} = \tan\theta$$

$$\sqrt{3} = \tan\theta$$

$$\boxed{\theta = 60^\circ}$$

$$u = \frac{30}{\sin 60}$$

$$= \frac{30}{\sqrt{3}} \times 2$$

$$\boxed{u = 20\sqrt{3}\ m/s}$$

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Ch-3

Q1. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min. with a drift 120 m. If he crosses the river taking shortest route he takes 12.5 min. find velocity of boat w.r.t water.

$\theta = 0$

$600 = \frac{d}{V_{m/r}}$

$120 = 600 u_r$

$u_r = \frac{120}{600}$ m/s

$\boxed{u_r = \frac{1}{5} \text{ m/s}}$

~~$\sin\theta = \frac{V_{m/r}}{V_r}$~~

~~$\sin\theta = V_{m/r} \times 5$~~

~~$\sin\theta = 5 V_{m/r}$~~

~~$750 = \frac{d}{V_{m/r}\cos\theta}$~~

~~$750 = \frac{5d}{V_m}$~~

$750 = \frac{d}{V_{m/r}\cos\theta}$

$\frac{600}{750} = \cos\theta$

$\cos\theta = \frac{4}{5}$

~~$750 = \frac{d}{V_{m/r}} \times \frac{5}{4}$~~

$600 = \frac{d}{V_m}$

$\theta = 37^\circ$

$V_{m/r} \times \frac{3}{5} = \frac{1}{5}$

$V_{m/r} = \frac{1}{3}$ m/s

$= \frac{1}{3} \times 60$

$\boxed{= 20 \text{ m/min}}$

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Q2. A man can swim at the rate of 5km/hr in still water. A 1 km wide river flows at 3km/h. The man wishes to swim across the river directly opposite to starting point.

a) along what direction should the man swim?

b) What should be his resultant velocity?

c) Find time taken to cross the river.

$d = 1\ km$

$V_R = 3\ km/h$

$V_{mR} = 5\ km/hr$

$V_{mR} \sin\theta = V_R$

$\sin\theta = \frac{3}{5}$

$\boxed{\theta = 37°}$ a)

b) $\boxed{\vec{V} = 4\hat{j}\ km/hr}$ b)

$\boxed{Time = \frac{1}{4}\ hr}$ c)

Q3. A man swims ⊥ to river flow. His vel relative to water is 4m/s the river flow = 2 m/s. wid of river = 800 m

a) velocity relative to ground

b) time to cross river

c) drift.

$V_{mR} = 4$

$V_R = 2$

$d = 800$

$V_m = \sqrt{16 + 4}$

$\boxed{= \sqrt{20}\ m/s}$ a)

$Time = \frac{800}{4}$

$\boxed{= 200\ s}$ b)

drift = ~~400 m~~ $\frac{distance}{~~Time~~\ Speed}$

$= \frac{800}{\cancel{2}\ 2}$

$= \frac{800}{2}$

$\boxed{= 400\ m}$ c)

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Q4. A swimmer crosses a 200 m wide river & return 10 minutes at a point away from starting point (downstream). Find velocity of man with respect to ground if he heads towards the bank & at right angles all the times.

$d = 200\,m$

$T = 300\,s$

$x = 150\,m$

$\theta = 90$

$$V_{mr} = \frac{2\cancel{0}0}{3\cancel{0}0}$$

$$= \frac{2}{3}\,m/s$$

$$\cancel{\frac{150}{2} \times 3 = V_r}$$

$$\frac{150}{300} = \frac{1}{2} = V_r$$

$$V_r = \frac{1}{2}\,m/s$$

$$\sqrt{\frac{1}{4} + \frac{4}{9}}$$

$$\sqrt{\frac{9+16}{36}}$$

$$\sqrt{\frac{25}{36}}$$

$$\boxed{\frac{5}{6}\,m/s}$$

Q6.

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Q6. A boat moves relative to water with velocity v which is η times less than river flow u. At what angle river flow must the boat move to minimise drifting?

$V_{mr} = \frac{u}{\eta}$

$V_r = u$

Time taken to cross river, $t = \frac{d}{V_{mr}\cos\theta}$

Drift, $x = (V_r - V_{mr}\sin\theta) \times \frac{d}{V_{mr}\cos\theta}$

$$= \frac{d}{V_{mr}} \frac{(V_r - V_{mr}\sin\theta)}{\cos\theta}$$

$\frac{dx}{d\theta} = 0$

$\cos\theta(0 - V_{mr}\cos\theta) + (V_r - V_{mr}\sin\theta)(\sin\theta) = 0$

$-V_{mr}\cos^2\theta + V_r\sin\theta - V_{mr}\sin^2\theta = 0$

$V_r\sin\theta = V_{mr}(\sin^2\theta + \cos^2\theta)$

$\sin\theta = \frac{V_{mr}}{V_r}$

$\theta = \sin^{-1}\left(\frac{V_{mr}}{V_r}\right)$

$\theta = \sin^{-1}\left(\frac{u}{\eta u}\right) = \frac{1}{\eta}$

$$\boxed{\theta = \sin^{-1}\left(\frac{1}{\eta}\right)}$$

Angle with river flow $= \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{\eta}\right)$

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Q 7. Two swimmers leave point A on the bank of river to land B right cross other bank. One of them crosses river along AB and other swims at right angles to river and then walks the drift distance to get to point B. find velocity u of his walking if both arrive at B at same time. ($V_r = 2$ km/hr, $V_{mr} = 2.5$ km/h)

$\sin\theta = \frac{2}{2.5}$

$\theta = 53^\circ$

$T = \frac{d}{2.5 \times \frac{3}{5}}$

$T_1 = \frac{2d}{3}$

$T_2 = \frac{d}{2.5}$

$R = 2 \times \frac{d}{2.5}$

drift, $R = \frac{4d}{5}$

time to come to B for 2nd swimmer $= \cancel{\frac{d}{2.5} - \frac{2d}{3}}$

$= \cancel{\frac{3d - 5d}{7.5}}$

$= \frac{2d}{3} - \frac{d}{2.5}$

$= \frac{5d - 3d}{7.5}$

$= \frac{2d}{7.5}$

speed $= \frac{4d}{5} \times \frac{7.5}{2d}$

$= \frac{15}{5}$

$\boxed{V = 3 \text{ km/h}}$

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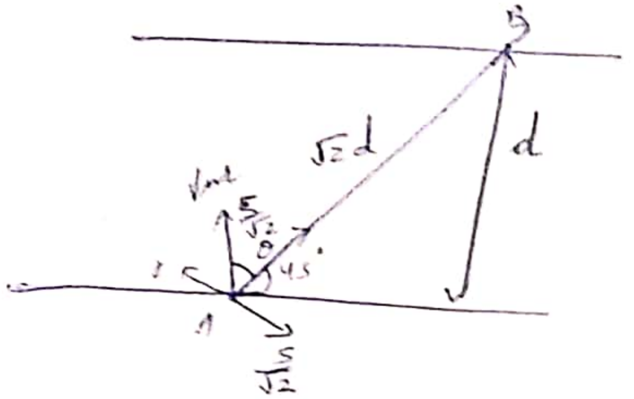
Q8. A swimmer crosses a river along the line making an angle of 45° with the direction of flow. Velocity of river water is 5m/s. Swimmer takes 6 s to cross the river of width 60m. Find velocity of swimmer w.r.t water.



$$\frac{\sqrt{2}d}{6} = \text{speed}$$

$$\boxed{\cancel{10\sqrt{2} = \text{speed}}}$$

$$10\sqrt{2} - \frac{5}{\sqrt{2}}$$

$$\frac{20-5}{\sqrt{2}} = \boxed{\frac{15}{\sqrt{2}}\ m/s} = V_{mr}\cos\theta$$

$$V_{mr}\sin\theta = \frac{5}{\sqrt{2}}$$

$$\cancel{\tan\theta = \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{15} = \frac{1}{3}}$$

$$V_{mr}^2 = \frac{25}{2} + \frac{225}{2}$$

$$V_{mr}^2 = \frac{250}{2}$$

$$V_{mr} = \frac{5\sqrt{10}}{\sqrt{2}}$$

$$\boxed{V_{mr} = 5\sqrt{5}\ m/s}$$

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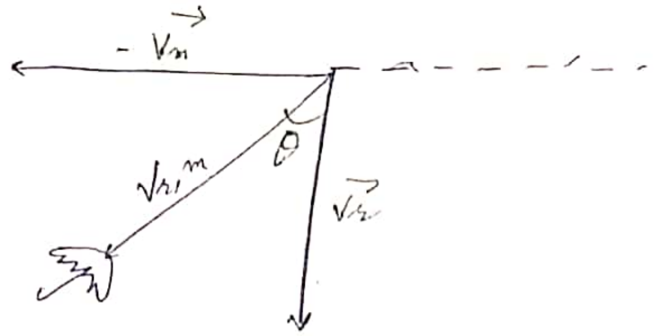
H. W.

Pg 72 Q 29-45

Rain Man Problems

If rain is falling vertically with velocity $\vec{V_r}$ and ~~the~~ observer is moving horizontally with velocity $\vec{V_m}$, then velocity of rain w.r.t. man, $\vec{V_{r,m}} = \vec{V_r} - \vec{V_m}$

$$\vec{V_{r,m}} = \vec{V_r} - \vec{V_m} = \vec{V_r} + (-\vec{V_m})$$



Angle at which umbrella has to be held.

$$\boxed{\tan\theta = \frac{V_m}{V_r}}$$

$V_{r,m}$ ~~V_{rm}~~ $= \sqrt{V_m^2 + V_r^2}$

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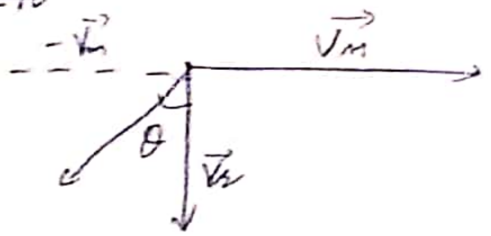
Q Rain falls vertically at speed of 10 m/s. A man walks with a speed of 6 m/s on a horizontal road. find angle at which man should hold his umbrella to avoid getting wet?

$\cancel{V_r = 10}$



$|\vec{V_r}| = 10\ m/s$

$\vec{V_r} = -10\hat{j}\ m/s$

$|\vec{V_m}| = 6\ m/s$

$\vec{V_m} = 6\hat{i}\ m/s$

$\vec{V_{r,m}} = \vec{V_r} - \vec{V_m}$

$= -10\hat{j} - 6\hat{i}$

$\tan\theta = \frac{6}{10} = \frac{3}{5}$

$$\boxed{\theta = \tan^{-1}\left(\frac{3}{5}\right)}$$

Q A man moving with 6 m/s observes rain falling at 12 m/s vertically. find direction & speed of rain w.r.t ground.

$V_m = 6\ m/s$

$V_{r,m} = 12$

$\vec{V_{r,m}} = -12\hat{j}$

$\vec{V_m} = 6\hat{i}$

$\vec{V_r} = \vec{V_m} + \vec{V_{r,m}}$

$= 6\hat{i} - 12\hat{j}$

$|\vec{V_r}| = \sqrt{36 + 144}$

$= \sqrt{180}$

$= \sqrt{3 \times 3 \times 2 \times 10}$

$= 3\sqrt{20}$

$\boxed{= 6\sqrt{5}\ m/s}$

$\cos\beta = \frac{+12}{6\sqrt{5}}$

$= +\frac{2}{\sqrt{5}}$

$\theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\tan\theta = \frac{6}{12}$

$\tan\theta = \frac{1}{2}$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$$

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Q3. Rain is falling with velocity of 20 m/s at an angle 30° with vertical. How fast should he move so that rain appears ⊥ to vertical to him?

$\vec{V_r} = 20\ m/s$

$\vec{V_r} = 20 \sin 30° \,\hat{i} - 20 \cos 30° \,\hat{j}$

$= 20 \times \frac{1}{2} \,\hat{i} - 20 \times \frac{\sqrt{3}}{2} \,\hat{j}$

$= 10\hat{i} - 10\sqrt{3}\,\hat{j}$

If rain is vertical,

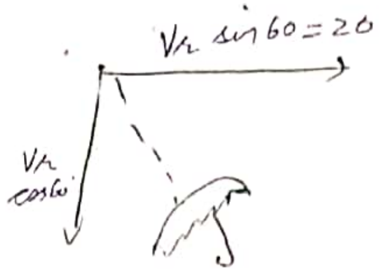
$10 = V_m$

$\boxed{V_m = 10\ m/s}$

Q4. Nirjhar is standing on road has to hold his umbrella at 60° with vertical. He throws his umbrella and starts moving at 20 m/s. He finds that rain drops are hitting his head vertically. Find speed of rain drops w.r.t (a) road, (b) Nirjhar

$\vec{V_r} \sin 60 = \frac{V_r \sqrt{3}}{2} = 20$

$\boxed{\vec{V_r} = \frac{40}{\sqrt{3}}}$ a)



$V_r \cos 60 = \frac{40}{\sqrt{3}} \times \frac{1}{2} = \boxed{\frac{20}{\sqrt{3}}}$ b)

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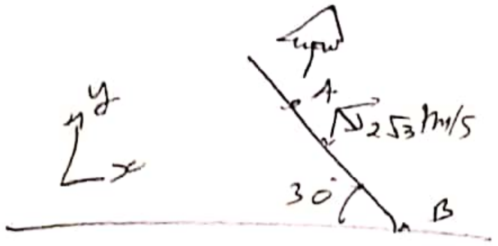
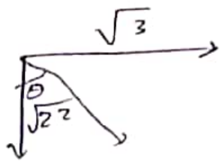
Q The man is standing on a ramp with speed $2\sqrt{3}$ m/s and keeps umbrella vertical. Actual speed of rain is 5 m/s. At what angle should he keep his umbrella with vertical when he reaches & stops at B





$\vec{A} = \frac{2\sqrt{3}}{2}\hat{i} - 2\sqrt{3} \times \frac{\sqrt{3}}{2}\hat{j}$

$= \sqrt{3}\hat{i} - 3\hat{j}$

$\vec{R} = \sqrt{3}\hat{i} - \sqrt{22}\hat{j}$

$\frac{\sqrt{22}}{\sqrt{3}} = \cancel{\tan\theta}$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{22}}$

$\vec{A} = 3\hat{i} - \sqrt{3}\hat{j}$

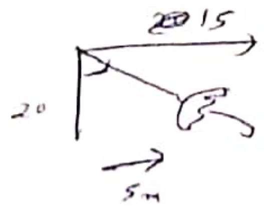
$\vec{R} = 3\hat{i} - 4\hat{j}$

$\tan\theta = \frac{3}{4}$

$\boxed{\cancel{\theta = 30°}}$ $\boxed{\cancel{\theta = 37°}}$

$\boxed{\theta = 37°}$

Q Rain is falling with a speed 30 m/s. A Person running in rain with a velocity of 5 m/s & wind is also blowing with a speed of 15 m/s (both from west). The angle with the vertical at which the person should hold his umbrella so that he may not get drenched is.



$V_{rain/man} = -20\hat{j} + 15 - 5\hat{i}$

$= -20\hat{j} + 10\hat{i}$

$\tan\theta = \frac{10}{20}$

$= \frac{1}{2}$

$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$

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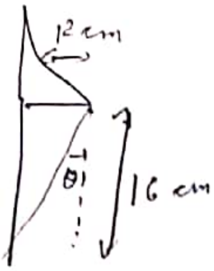
Q A wearing a hat of extended length 12 cm is running in rain falling vertically with 10 m/s. find max speed with which man can run so that raindrops does not fall on his face. (length of face is 16 cm)



V_r (vertical) $= 10$ m/s

$= 1000$ cm/s

$\tan\theta = \frac{12}{16}$

$\tan\theta = \frac{3}{4}$

$\theta = 37^\circ$

$\frac{V_m}{V_r(\text{vertical})} = \frac{3}{4}$

$\frac{V_m}{1000} = \frac{3}{4}$

$V_m = \frac{\cancel{3000}^{\cancel{1500}\ 750}}{\cancel{4}_{2}}$ cm/s

$\boxed{V_m = 7.5 \text{ m/s}}$

Q

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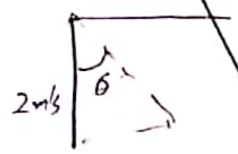
a



$\tan\theta = \frac{\text{velocity}}{2}$

~~$\theta = \tan$~~

$\frac{d\theta}{dt}$

~~$\frac{d\theta}{dv} = \tan^{-1}(\frac{v}{2})$~~

~~$d\theta =$~~

$\frac{dv}{dt} = 2\,m/s^2, \quad v = 2t$

$\frac{d\theta}{dv} \times \frac{dv}{dt} = \frac{d\theta}{dt}$

$= 2 \times \tan^{-1}\left(\frac{v}{2}\right)$

$\theta =$

$\frac{d\theta}{dt} = \frac{d\theta}{dv} \times \frac{dv}{dt}$

$= 2 \times \frac{\sec^{-2}(\frac{v}{2})}{2}$

$\boxed{\frac{d\theta}{dt} = [\sec^{-1}(t)]^2}$

$\theta = \tan^{-1}\left(\frac{v}{2}\right)$

$\frac{d\theta}{dv} = \sec^{-2}\left(\frac{v}{2}\right) \times \frac{1}{2}$

$= \frac{\sec^{-2}(\frac{v}{2})}{2}$

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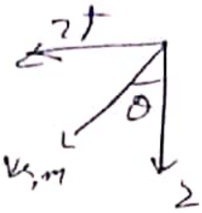
Q rain falling vertically with 2 m/s. A boy at rest accelerates at $2 m/s^2$ along straight road. find rate of change of angle of umbrella?

$V_r = -2\hat{j}$

at t $V_m = 2t\hat{i}$

$V_{rm} = V_r - V_m$

$V_{rm} = -2\hat{j} - 2t\hat{i}$



$\tan\theta = \frac{2t}{2}$

$\tan\theta = t \longrightarrow$

$\sec^2\theta \, d\theta = dt$

$\frac{d\theta}{dt} = \frac{1}{\sec^2\theta}$

$\frac{d\theta}{dt} = \cos^2\theta$

$\frac{d\theta}{dt} = \frac{1}{\sqrt{1+t^2}}$

$$\boxed{\frac{d\theta}{dt} = \frac{1}{1+t^2}}$$

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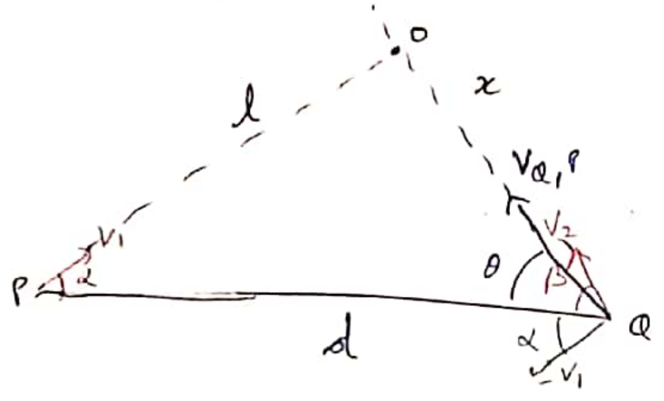
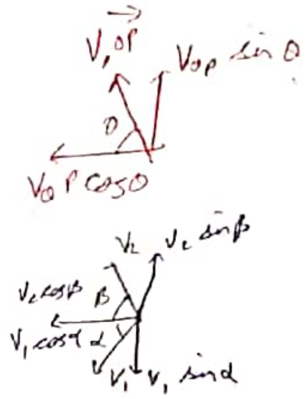
Shortest Distance Between two moving particles

(Distance of closest approach)





$$V_{Q,P} = \vec{V_Q} - \vec{V_P}$$

$$\vec{V_{QP}} = \vec{V_2} - \vec{V_1} = \vec{V_2} + (-\vec{V_1})$$

$$V_{QP}\cos\theta = V_1\cos\alpha + V_2\cos\beta \quad \text{--- (1)}$$

$$V_{QP}\sin\theta = V_2\sin\beta + V_1\sin\alpha \quad \text{--- (2)}$$

$(1)^2 + (2)^2$

$$(V_{QP})^2 = (V_1\cos\alpha + V_2\cos\beta)^2 + (V_2\sin\beta + V_1\sin\alpha)^2$$

$$V_{QP} = ?$$

$(2) \div (1)$

$$\tan\theta = \frac{V_2\sin\beta + V_1\sin\alpha}{V_1\cos\alpha + V_2\cos\beta}$$

$$\theta = ?$$

$$\sin\theta = \frac{l}{d}$$

$$\boxed{l = d\sin\theta}$$ (for particles to collide, $d=0$)

$$\cos\theta = \frac{x}{d}$$

$$x = d\cos\theta$$

$$\text{time} = \frac{x}{V_{QP}}$$

$$\boxed{= \frac{d\cos\theta}{V_{QP}}}$$

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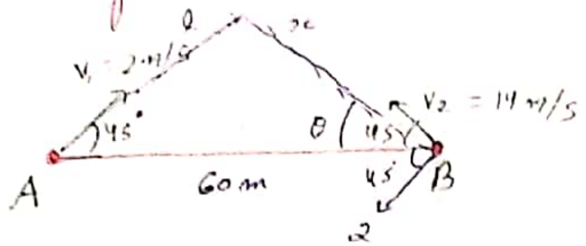
Q1. Motion of A & B takes place in horizontal plane



a) find distance of closest approach
b) At what time is the separation b/t them minimum.

$V\cos\theta = \cos 45\times 2 + \cos 45 \times 14$

$= \frac{2}{\sqrt{2}} + \frac{14}{\sqrt{2}}$

$= \frac{16}{\sqrt{2}}$

$= 8\sqrt{2}$

$V\sin\theta = -\sin 45\times 2 + \sin 45\times 14$

$= \cancel{8\sqrt{2}}\ 6\sqrt{2}$

$\tan\theta = \frac{6\sqrt{2}}{8\sqrt{2}}$

$= \frac{3}{4}$

$\boxed{\cancel{\theta = 45^\circ}}$ $\boxed{\theta = 37^\circ}$

$\cancel{\sin\theta = \frac{l}{60}}$ $\sin 37 = \frac{l}{60}$

$\cancel{\frac{1}{\sqrt{2}} = \frac{l}{60}}$ $\frac{3}{5}\times 60 = l$

$\cancel{\boxed{30\sqrt{2} = l}}$ a) $\boxed{36 = l}$ a)

$\cancel{\frac{V}{\sqrt{2}} = 8\sqrt{2}}$

$\cancel{V = 8\times 2}$

$\cancel{\boxed{V = 16\,m/s}}$

$\cancel{x\cos 45 = \frac{x}{60}}$

$\cancel{\boxed{30\sqrt{2} = x}}$

$\cancel{\text{Time} = \frac{\text{Distance}}{\text{Speed}}}$

$\cancel{= \frac{30\sqrt{2}}{16}}$

$\frac{4V}{5} = 8\sqrt{2}$

$V = \frac{5\times 8\sqrt{2}}{4}$

$\boxed{V = 10\sqrt{2}}$

$\cos 37^\circ = \frac{x}{60}$

$\frac{4\times 60}{5} = x$

$\boxed{x = 48}$

$\text{Time} = \frac{48}{10\sqrt{2}}$

$= \frac{48\sqrt{2}}{20}$

$= \frac{24\sqrt{2}}{10}$

$\boxed{= \frac{12\sqrt{2}}{5}}$ b)

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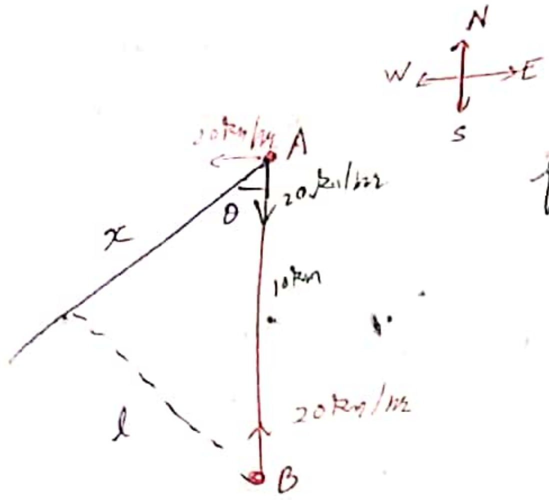
Q 2.



find distance of closest approach & time

$\theta = 45$

$V = \sqrt{400 + 400}$

$= 20\sqrt{2}$

$\sin 45 = \frac{l}{10}$

$\frac{1}{\sqrt{2}} = \frac{l}{10}$

$\boxed{5\sqrt{2} = l}$ a)

$\cos 45 = \frac{l}{10}$

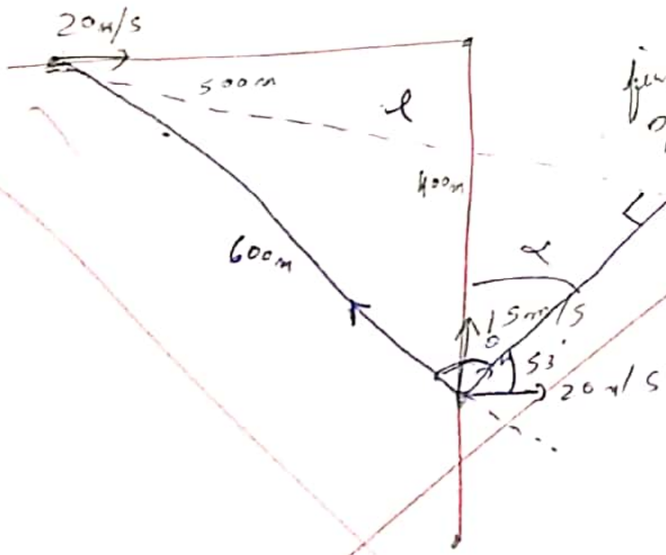
$\boxed{5\sqrt{2} = x}$

time $= \frac{5\sqrt{2}}{20\sqrt{2}}$

$= \frac{1}{4}$ hr

$\boxed{= 15 \text{ min}}$ b)

Q 3.



find distance of closest approach

$V = \sqrt{400 + 225}$

$= \sqrt{625}$

$\boxed{= 25 \text{ m/s}}$

θ

$\alpha = 37^\circ$

$15 \sin 37 = \sin 53 \times 20$

$15 \times \frac{3}{5} = \frac{4}{5} \times 20$

$9 =$

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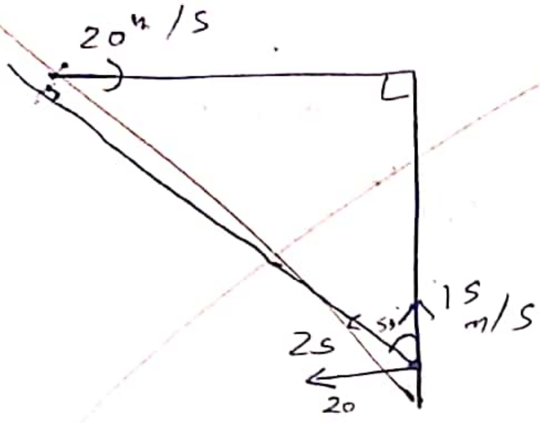
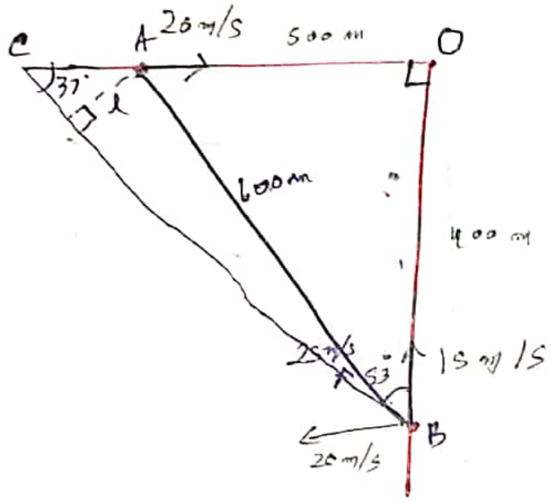
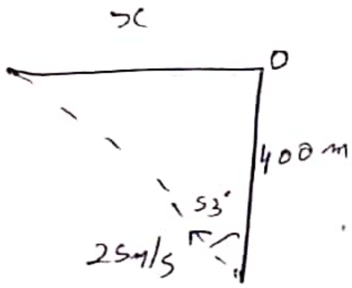


Q3.



find distance of closest approach



$\tan 53^\circ = \frac{x}{400}$

$\frac{4}{3} = \frac{x}{400}$

$\boxed{\frac{1600}{3} = x}$

$CA = \frac{1600 - 1500}{3}$

$= \frac{100}{3}$

$\sin 37 = l \times \frac{3}{100}$

$\frac{100}{3} \times \frac{3}{5} = \boxed{l = 20 m}$

$\frac{x}{400} = \frac{4}{3}$

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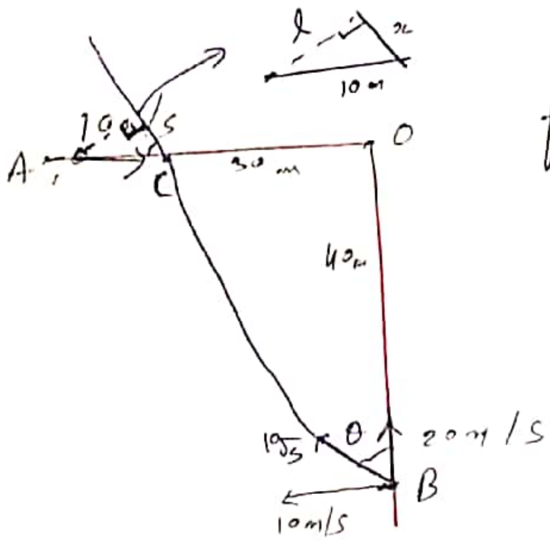
Q



find distance of closest approach

$$V_A = \sqrt{400+100}$$
$$= \sqrt{500}$$
$$= 10\sqrt{5}\ m/s$$

$$\tan\theta = \frac{10}{20}$$
$$= \frac{1}{2}$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$

$$\frac{x}{40} = \frac{1}{2}$$

$$x = 20\ m$$

$$AC = 10\ m$$

$$\frac{x}{l} = \frac{1}{2}$$

$$x = \frac{l}{2}$$

$$2x = l$$

PGT

$$100 = \sqrt{x^2+2x^2}$$

$$\cancel{100}$$
$$10 = 3x^2$$

$$x^2 = \frac{10}{3}$$

$$x = \frac{\sqrt{10}}{\sqrt{3}}$$

$$l = 2 \times \frac{\sqrt{10}}{\sqrt{3}}$$

$$=$$

$$\frac{2}{\sqrt{5}} = \frac{l}{10}$$

$$\cancel{2\sqrt{5} = l}$$

$$\frac{10 \times 2}{\sqrt{5}} = l$$

$$\boxed{l = 4\sqrt{5}}$$

60

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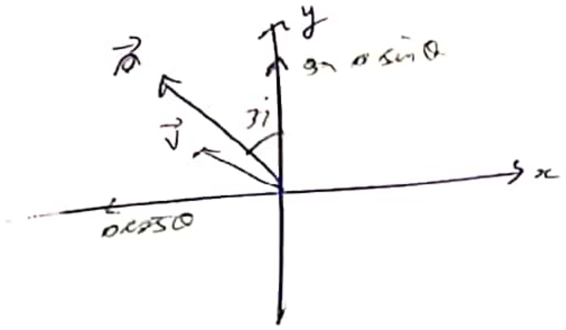
General 2-D motion Questions

Q. A Particle with initial velocity $\vec{V_0} = (-2\hat{i} + 4\hat{j})$ undergoes constant acceleration of 3 m/s² at $\theta = 127°$ from +ve x-axis, find $\vec{v}$ at $t = 5s$



$\vec{a} = 3 \times \frac{3}{5}\hat{i} + 3 \times \frac{4}{5}\hat{j}$

$= \frac{-9}{5}\hat{i} + \frac{12}{5}\hat{j}$

$\vec{v} = -2\hat{i} + 4\hat{j}$

in x axis,

$v = -2$

$a = -\frac{9}{5}$

$t = 5$

$v = -2 + \frac{-9}{5} \times 5$

$= -2 - 9$

$\boxed{v = -11 \text{ m/s}}$

in y axis

$v = 4$

$a = \frac{12}{5}$

$v = 4 + \frac{12}{5} \times 5$

$v = 12 + 4$

$\boxed{= 16 \text{ m/s}}$

final $\boxed{\vec{v} = -11\hat{i} + 16\hat{j}}$

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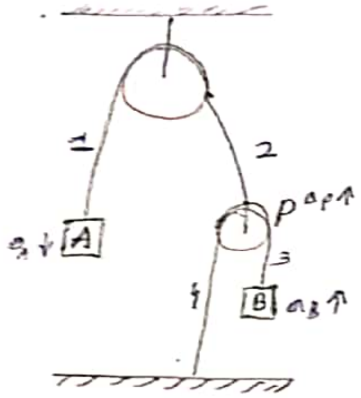
!! Chapter - 4 !!

Newton's Laws of Motion & Friction

Constraint Motions :-

→ The equations showing the relation of motion of bodies in which motion of one body is ~~con~~ constrained by the other are called constraint relations.

1. Pulley Constraint



Relation between acceleration of blocks

Δl_1 (change in length of rope 1) $= +x_A$
(जितना Block A निचे जाएगा)

$\Delta l_2 = -x_P$
(जितना pulley upar जाएगा)

$\Delta l_1 + \Delta l_2 = 0$
(rope length remains same)

$+x_A - x_P = 0$

$$\boxed{x_A = x_P}$$

$\Delta l_3 = +x_P - x_B$
(जितना Pulley upar जाएगा — जितना Block नीचे जाएगा)

$\Delta l_4 = x_P$

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$$\Delta l_3 + \Delta l_4 = 0$$

$$x_p + x_p - x_B = 0$$

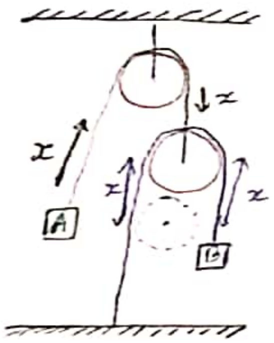
$$2x_p = x_p$$

$$2x_A = x_B$$

$$2V_A = \cancel{x_B}\, V_B$$

$$\boxed{2a_A = a_B}$$

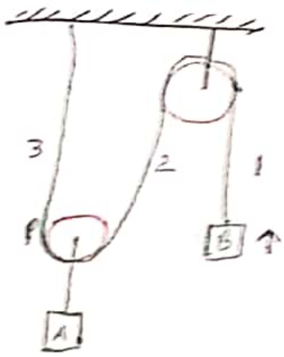
Method II



Displacement of the block B will be $2x$
A will be x.

$$\boxed{2a_A = a_B}$$

Q Find relation between movement of blocks



$\cancel{\Delta l_1 = x_B}$

$\cancel{\Delta l_2 = x_1}$

$$\Delta l_3 = +x_A$$
$$\Delta l_2 = +x_A$$
$$\Delta l_1 = -x_B$$

$$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$$

$$2x_A - x_B = 0$$

$$2x_A = x_B$$

$$2V_A = V_B$$

$$\boxed{2a_A = a_B}$$

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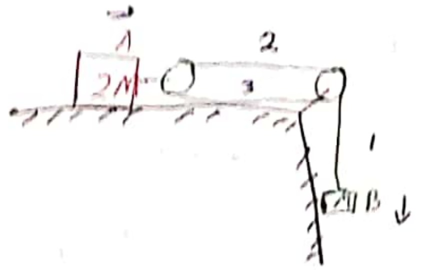
Q find constraint relation between acceleration of A & B

$\Delta l_1 = x_A$
$\Delta l_2 = -x_A$

$\cancel{\Delta l_1 + \Delta l_2 = 0}$
$\cancel{x_B = x_A}$

$\Delta l_3 = -x_A$



$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$

$x_B - 2x_A = 0$

$\boxed{2x_A = x_B}$

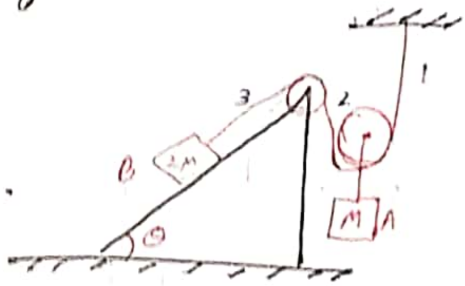
$\boxed{2a_A = a_B}$

Q find constraint relation between acceleration of A and B.

$\cancel{\Delta l_1} = \Delta l_3 = x_B$

$\Delta l_2 = -x_A$

$\Delta l_5 = -x_A$



$\Delta l_1 + \Delta l_2 + \Delta l_3 = 0$

$x_B - x_A - x_A = 0$

$x_B = 2x_A$

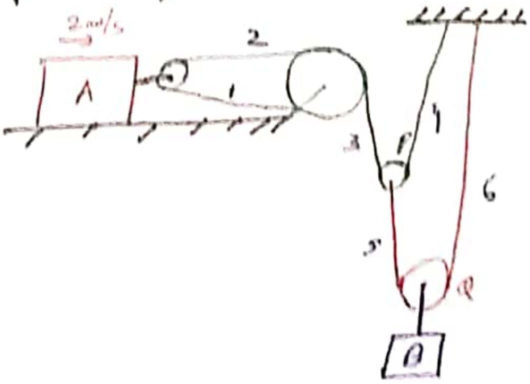
$2x_A = x_B$

$2V_A = V_B$

$\boxed{2a_A = a_B}$

Q find acceleration of block B, Pulley P & Q. If acceleration of A is given

$\Delta l_1 = -x_A$
$\Delta l_2 = -x_A$
$\Delta l_3 = x_P$
$\Delta l_4 = x_P$



$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 = 0$

$-2x_A + 2x_P = 0$

$\boxed{x_P = x_A}$

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$\Delta l_5 = +x_B \mp x_p$

$\Delta l_6 = +x_B$

$\Delta l_5 + \Delta l_6 = 0$

$+2x_B \mp x_p = 0$

$+x_p = +2x_B$

$x_A = 2x_p$

$V_A = 2V_B$

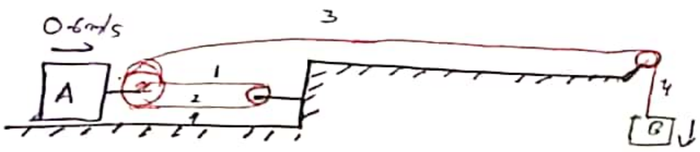
$\boxed{a_A = 2a_B}$

$a_A = 2 m/s$

$2 = 2a_B$

$\boxed{a_B = 1 \ m/s^2}$

Q Block A vel = 0.6 m/s to right, find V_B.



$\Delta l_1 = -x_A$

$\Delta l_2 = -x_{BA}$

$\Delta l_3 = -x_A$

$\Delta l_4 = x_B$

$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_3 = 0$

$-3x_A + x_B = 0$

$x_B = 3x_A$

$V_B = 3V_A$

$V_B = 3(0.6)$

$\boxed{V_B = 1.8 \ m/s}$

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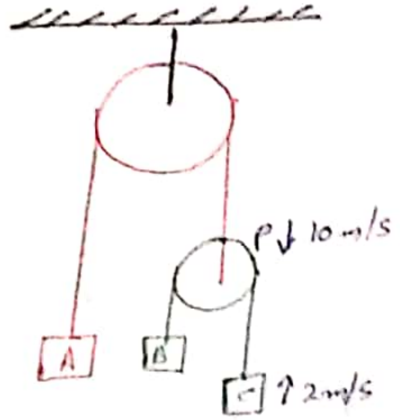
Q find velocities of A & B if velocity of P is 10 m/s downwards and velocity of C is 2 m/s upwards.



$V_A = -V_P$

$V_P = -10\hat{j}$

$\boxed{V_A = +10\hat{j}}$

$V_C = 2\hat{j}$

$V_B = ?$

$$\boxed{\vec{V}_{B,P} = -\vec{V}_{C,P}}$$

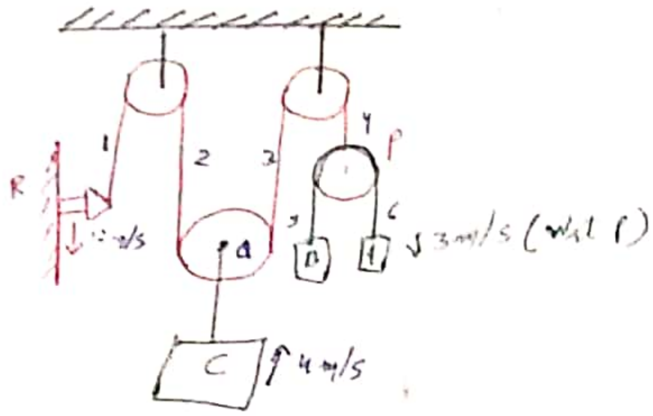
$$\vec{V}_B - \vec{V}_P = -(\vec{V}_C - \vec{V}_P)$$

$$2\vec{V}_P = \vec{V}_B + \vec{V}_C$$

$$2(-10\hat{j}) = \vec{V}_B + 2\hat{j}$$

$$\boxed{-22\hat{j} = \vec{V}_B}$$

Q ~~At an instant~~ Determine motion of B with ground



$\Delta l_1 = x_R$

$\Delta l_2 = -x_c$

$\Delta l_3 = -x_c$

$\Delta l_4 = x_P$

$x_R + x_P = 2x_c$

$x_P = 2(4) \mp 12$

$= 8 \mp 12$

$$\boxed{x_P = \cancel{20}\,4 \text{ m/s}}$$

$x_{A,P} = x_A - \cancel{x_P}$

$-3 - 20$

$= -23$

$\cancel{x_{B,P} =}$

$x_{A,P} = -3$

$x_{B,P} = x_B - x_P = (-3)$

$\cancel{x_B = +3 + 20}$

$\boxed{\cancel{x_B = 23 \text{ m/s}}}$

$-x_B = -3 - 4$

$\boxed{x_B = 7 \text{ m/s}}$

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Q5) Method II (P mikolne Re based)

$\cancel{\vec{V}_R = -12\hat{j}}$

$\cancel{\vec{V}_C = 4\hat{j}}$

$\vec{V}_{A,P} = -3\hat{j}$

$\vec{V}_{B,P} = -\vec{V}_{A,P}$

$\vec{V}_B - \vec{V}_P = 3\hat{j}$

$\vec{V}_B = 3\hat{j} + \vec{V}_P$

$\vec{V}_B = 3\hat{j} + 20\hat{j}$

$\boxed{= 23\hat{j} \text{ m/s}}$

Q find B acc.

$\Delta l_1 + \Delta l_2 = 0$

$\cancel{x_A - x_Q = 0}$ $\quad x_A - x_Q - x_Q = 0$

$\cancel{x_Q = 12 \text{ m/s}^2}$ $\quad x_A = 2x_Q$

$\boxed{6 = x_Q}$

$\Delta l_3 + \Delta l_4 + \Delta l_5 + \Delta l_6 = 0$

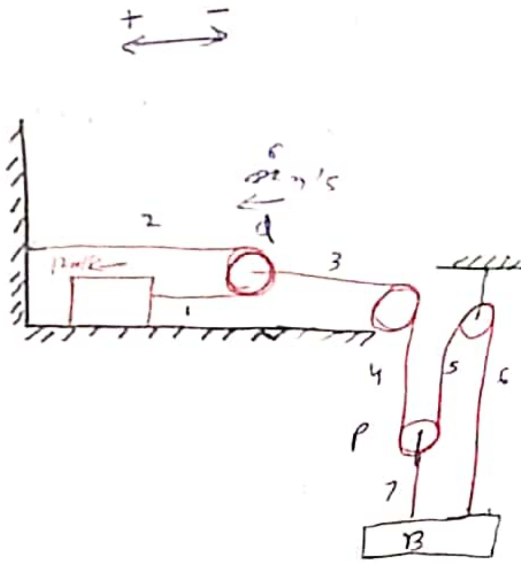
$x_Q - x_P - x_P - x_P = 0$

$x_Q = 3x_P$

$6 = 3x_P$

$x_P = \frac{6}{3}$

$\boxed{x_P = 2 \text{ m/s}^2}$



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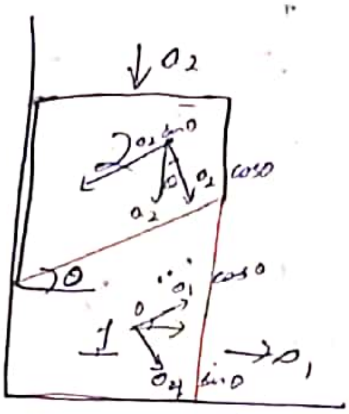
HW 01-28 (B) 04-06-2024

Wedge Constraint

→ Contact between the wedges is intact

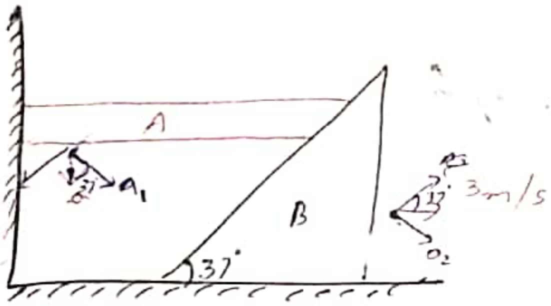
→ component of acceleration perpendicular to surface in contact is same for both.



$a_1 \sin\theta = a_2 \cos\theta$

$$\boxed{a_2 = a_1 \tan\theta}$$

Q find acceleration of A.? ($\theta = 37°$)



~~$a_A \tan\theta = a_B$~~

~~$a_A \times \frac{3}{4} = 3$~~

~~$a_A = 3 \times \frac{4}{3}$~~

~~$\boxed{a_A = 4\ m/s^2}$~~

Or

$a_1 = \cos 37° \times a$

$a_2 = \sin 37° \times 3$

$= \cancel{\ } \times 3 \times \frac{3}{5}$

$= \cancel{\ } = \frac{9}{5}$

$a_1 = a_2$

$a \times \frac{4}{5} = \frac{9}{5}$

$$\boxed{a = \frac{9}{4}\ m/s^2}$$

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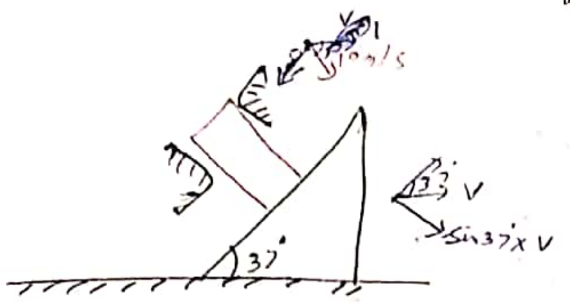
Q A rod is moving with speed 10m/s . find V



$V\theta_1 = 10 \cos 37$

$= 10 \times \frac{4}{5}$

$= 8\ m/s$

$V_2 = V \times \frac{3}{5}$

$V_2 = \frac{3V}{5}$

$V_1 = V_2$

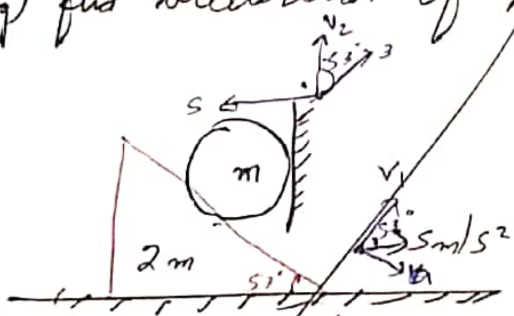
$8 = \frac{3V}{5}$

$\therefore V \sin 37^\circ = 10$

$V \times \frac{3}{5} = 10$

$\boxed{V = \frac{50}{3}\ m/s}$

Q (m=1kg) find acceleration of sphere



$V_1 = 5 \times \sin 53 = 5 \times \frac{4}{5} = 4\ m/s$

$V_1 = \cos 53^\circ \times 5 = 3\ m/s^2$

$V = \sqrt{9 + 25 - 30 \times \frac{4}{5}}$

$V = \sqrt{34 - 24}$

$V = \sqrt{10}$

$V_2 = 3 \cos 53^\circ$

$V_2 = 3 \times \frac{4}{5}$

$V_2 = \frac{12}{5}$

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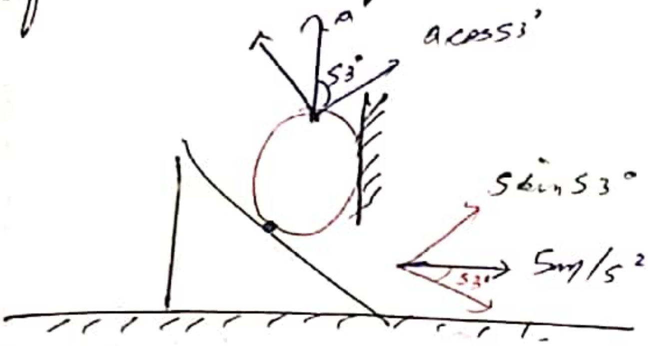
Q (find acceleration of sphere



$a \cos 53^\circ = 5 \sin 53^\circ$

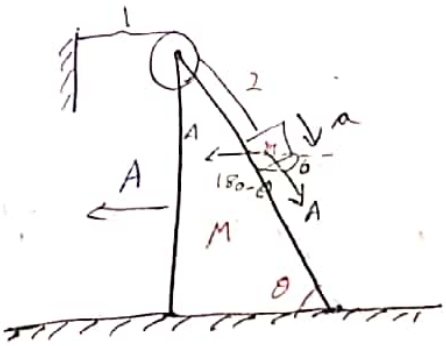
$a \times \frac{3}{5} = 5 \times \frac{4}{5}$

$a = \frac{4 \times 5}{3}$

$\boxed{a = \frac{20}{3}\ m/s^2}$

Pulley & Wedge constraint

Q Determine block acceleration w.r.t wedge.



$\Delta l_1 = -x_m$

$\Delta l_2 = x_m$

$x_M = x_m$

$a_M = a_A$

$\boxed{a_m = A \text{ (wrt wedge)}}$

$A_{\text{wrt ground}} = \sqrt{A^2 + A^2 + 2A^2 \cos(180-\theta)}$

$= \sqrt{2A^2 - 2A^2 \cos\theta}$

$= \sqrt{2A^2(1-\cos\theta)}$

$= \sqrt{2A^2 \times 2\sin^2\frac{\theta}{2}}$

$\boxed{= 2A \sin\frac{\theta}{2}}$

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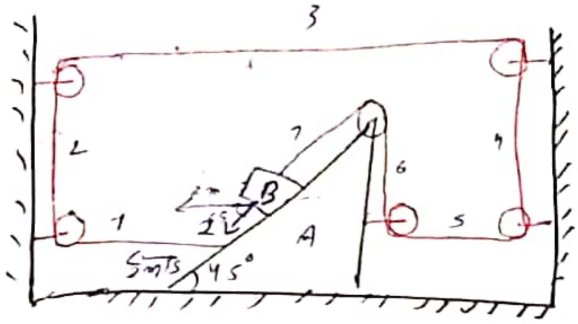
Q. vel B w.r.t. ground



~~$x_A = x_B$~~

~~$V_A = V_B$~~

~~$V_{B \text{ wrt } (A)} = 2m/s$~~

~~$V = \sqrt{8 V^2 + V^2 + 2V^2 \cos 45^\circ}$~~

~~$= \sqrt{8 + 8 \times \frac{1}{\sqrt{2}}}$~~

~~$= \sqrt{8 + 4\sqrt{2}}$~~

$\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4 + \Delta l_5 + \Delta l_6 + \Delta l_7$

$-x_A + x_A + x_B = 0$

$x_B = 0$

v_B (wrt wedge) $= 0$

wrt ground

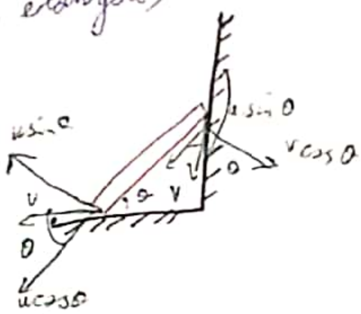
$\boxed{v_B = 2 m/s}$

General Constraint

Q Find vel of end B when rod makes an angle θ with horizontal

$v \sin\theta = u\cos\theta$ (so rod dont compress or elongate)

$\boxed{v = u\cot\theta}$



→ Component of velocity along the rod or string is equal for both ends.

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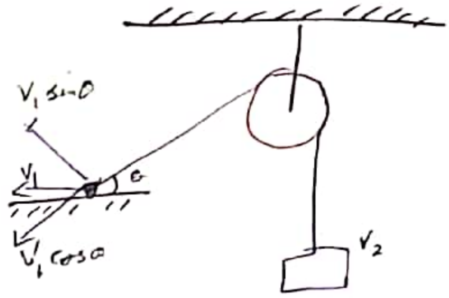
Q find relation b/w V_1 & V_2

(the component along string to remain same.

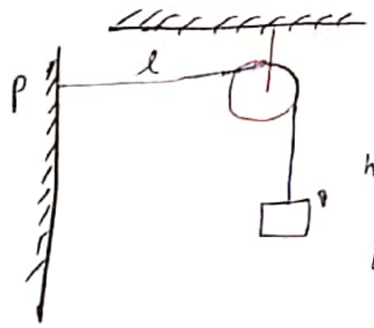
$$\boxed{V_1 \cos\theta = V_2}$$

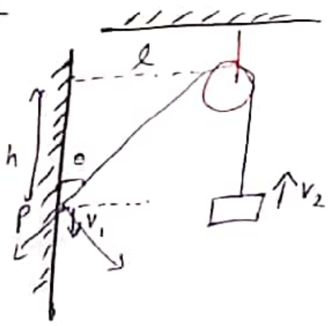


Q2. find relation b/w V_1 & V_2 if distance moved by P is h.

$V_1 \cos\theta = V_2$

$$\boxed{\frac{V_1 h}{\sqrt{h^2+l^2}} = V_2}$$





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Newton's Laws of Motion

→ cause of motion :- forces → Dynamics

Balanced Forces – Net Force = 0

- If body at rest → Remains at rest
- Body is in motion → Continues in motion with same speed & direction.

→ Balanced Forces may lead to change in size or shape of the object.

Unbalanced Forces – Net Force ≠ 0

- Body is at rest → Body will start moving in the direction of resultant force
- Body is in motion
  - Direction of net force is same as motion of the body → Body will speed up
  - Direction of net force opp. to motion of the body → Body will slow down & finally stop

~~Newton~~ Newton's First Law & Inertial Frame & non Inertial Frame.

→ Newton's First law / law of Inertia – defines a set of reference frames called inertial frames.

→ ~~Iner~~ Inertial Frame – Frames which do not have any acceleration.

→ Either the ~~Bo~~ frame is at rest or moving with a uniform velocity.

→ Newton's laws can be directly applied in such frames and dynamic equations can be written for objects in this frame. $\Sigma \vec{F} = m\vec{a}$

→ First Law – In the absence of external forces, when viewed from an inertial reference frame, every object continues to be in its state of rest or uniform motion.

Friction does not oppose the motion, It opposes the relative motion between two surfaces.

→ Inertial frames are also called as ~~call~~ 'Galilean Frames'.

→ Any reference frame that moves with a relative constant velocity to an inertial frame is itself an inertial frame.

→ First law is a qualitative law. (does not talk about the quantity of forces)

→ Non-Inertial Frame – A Frame of reference which is a in accelerated motion with respect to a Inertial frame.

→ Newton's Laws cannot be ~~dired~~ directly ~~applied~~, ~~same on~~ applicable

→ Tendency of an object to resist any attempt to change its velocity is called Inertia.

→ Depends on mass, more mass ↑ more Inertia ↑

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Linear Momentum & Newton's second Law.

Linear Momentum (P) – The quantity of motion contained in the body.

$$\boxed{\vec{P} = m\vec{v}}$$

SI unit :- kg m/s or Ns

It is a vector quantity

Q Two identical bodies are allowed to fall from two different heights h_1 & h_2. find the ratio of momentum just before striking the ground.

$V^2 = u^2 + 2as$

$V_1^2 = 2gh_1$

$V_1 = \sqrt{2gh_1}$

$V_2 = \sqrt{2gh_2}$

$P_1 = mV_1$
$P_2 = mV_2$ [same mass]

$\frac{P_1}{P_2} = \frac{\cancel{m}V_1}{\cancel{m}V_2}$

$= \frac{\sqrt{2gh_1}}{\sqrt{2gh_2}}$

$= \frac{\sqrt{gh_1}}{\sqrt{gh_2}}$

$= \sqrt{\frac{h_1}{h_2}}$

$P_1 : P_2$

$\sqrt{h_1} : \sqrt{h_2}$

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Q A ball of mass m is dropped from a height h on a smooth elastic floor, ~~such~~ such that it rebounds with same speed what is the change in momentum of ball before & after striking the floor is : (take vertically downward as positive)

$v^2 = u^2 + 2as$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$P_1 = m\sqrt{2gh}$

$v_2 = (-\sqrt{2gh})$

$P_2 = -m\sqrt{2gh}$

$|P_1| = m\sqrt{2gh}$

$|P_2| = m\sqrt{2gh}$

$$\boxed{|P_2| - |P_1| = 0}$$

b) find magnitude of change in momentum

$P_2 - P_1$

$-m\sqrt{2gh} - m\sqrt{2gh}$

$-2m\sqrt{2gh}$

$$\boxed{|P_2 - P_1| = 2m\sqrt{2gh}}$$

Newton's Second Law

→ When viewed from an inertial frame of reference, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$a \propto \frac{1}{m}$ $\qquad a \propto F$

→ Rate of change of momentum is directly proportional to net unbalanced force acting on it.

$P_i = mu$

$P_f = mv$

$\Delta P = mv - mu$

$$\boxed{\frac{\Delta P}{t} \propto F}$$

$F \propto \frac{m(v-u)}{t}$

$F \propto ma$

$F = k\,ma \quad (k = 1)$

$$\boxed{F = ma}$$

$$\boxed{\vec{F} = \frac{dP}{dt}}$$ → Slope of P - t graph

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$$\int_{p_i}^{p_f} dp = \int_0^t F\,dt$$

Change in momentum

↳ area under F-t graph

<u>Impulse</u> (J) :- It is the <u>change in momentum</u> of a body.

$J = \Delta P = mv - mu = Ft$

SI unit :- Ns or kg m/s

Q A machine gun has mass 5 kg. It fires 50 g bullets at the rate of 30 bullets per minute at a speed 400 m/s. What force is required to keep gun in position.

$F = m\dfrac{(v-u)}{t}$

$= \dfrac{50}{1000}\left[\dfrac{400-0}{2}\right]$

$= \dfrac{5}{100} \times 200$

$= 5 \times 2$

$\boxed{= 10\,N}$

Q A dish of mass 10 g is kept horizontally in air by firing 5 g bullets 10/s. If bullets rebound with same speed, ~~what~~ with what speed are bullets fired ($g = 9.8/s^2$)

Force to keep dish in air $= \dfrac{10}{1000} \times 9.8$

$= \dfrac{98}{1000}$ N

$\cancel{F = m\dfrac{(v-u)}{t}}$

$\cancel{= 5 \times \dfrac{(-u)}{1} \times 10}$

$\cancel{\dfrac{98}{1000} = -50u}$

$F = m\dfrac{(v-u)}{t}$

$\dfrac{98}{1000} = \dfrac{5\,(v-(-v))}{1000} \times 10$

$\dfrac{98}{1000} = 50 \times 2v$

$\dfrac{98}{1000} = \dfrac{100v}{1000}$

$V = \frac{98}{100}$

$\boxed{V = 0.98\ m/s}$

Q A body of mass 4 kg moving on horizontal surface with initial velocity 6 m/s comes to rest after 3 s. If one wants to keep moving the body with same speed of 6 on same surface. find required force.

$u = 6$
$v = 0$
$t = 3s$

$a = \frac{v-u}{t}$

$= \frac{-6}{3}$

$= -2$

- to keep moving, a = 2 to be applied

$F = ma$
$F = 4 \times 2$
$\boxed{F = 8N}$

Newton's Third Law

→ To Every Action there is a equal & opposite reaction.

→ Action & Reaction are equal in magnitude, opposite in direction and acts on two different bodies

→ if two forces are acting on the same object, even if they are equal in magnitude and opposite in direction, cannot be an action - reaction pair.

Free Body Diagram

→ diagram of a body showing all the forces on it along with direction & magnitude.

→ consider only ~~the~~ the forces applied on that body & not the forces the body applies on any other body.

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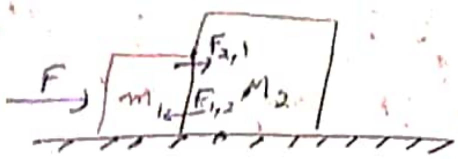
Types of Forces

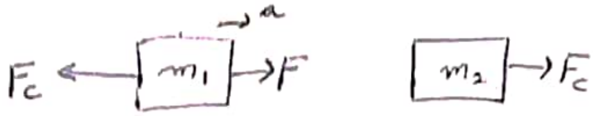
1. Contact forces - The force which acts between two bodies in contact are called contact forces.

$|F_{2,1}| = |F_{1,2}| = F_c$



$F = (m_1 + m_2)\, a$

$a = \frac{F}{(m_1 + m_2)}$



for m_2,

$F_c = m_2 a$

$$\boxed{F_c = \frac{m_2 F}{m_1 + m_2}}$$

for m_1,

$F - F_c = m_1 a$

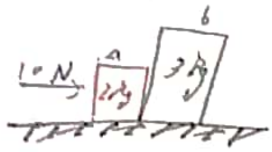
$F - F_c = \frac{m_1 F}{m_1 + m_2}$

$F_c = F\left[1 - \frac{m_1}{m_1 + m_2}\right]$

$$\boxed{F_c = \frac{m_2 F}{m_1 + m_2}}$$

$\boxed{\sum \vec{F} = m\vec{a}} \rightarrow$ Dynamic Equation.

Q find acceleration & contact force b/w A & B.



$F = ma$

$10 = 5 \times a$

$a = \frac{10}{5}$

$\boxed{a = 2\ m/s^2}$

for, b

$F_c = 3 \times 2$

$\boxed{F_c = 6\ N}$

for, a

$F - F_c = 2 \times 2$

$10 - F_c = 4$

$F_c = 10 - 4$

$\boxed{F_c = 6\ N}$

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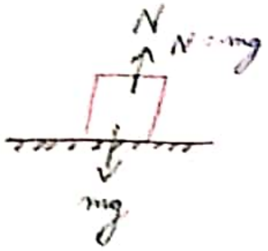
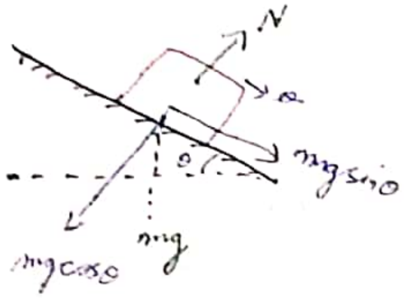
2. <u>Normal Force & Weight of body</u> - Normal force is a special type of contact force which always act $\perp$ to surface in contact





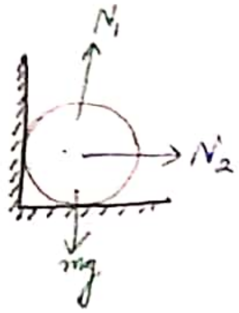
$N = mg\cos\theta$

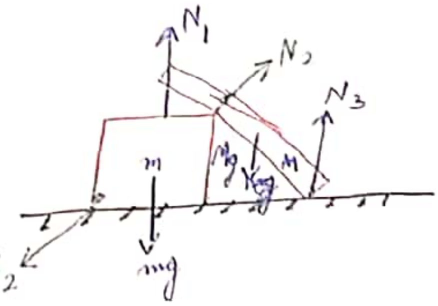
~~mgcos~~

$mg\sin\theta = ma$

$\boxed{g\sin\theta = a}$ (along incline)

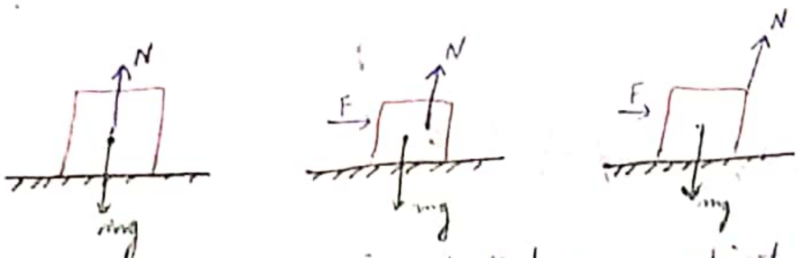
Eg





$N_1 = mg$

Note



→ when external force is applied on a object, the normal force shifts towards the direction of applied force.

→ on the verge of toppling, normal reaction passes through edge of the block.

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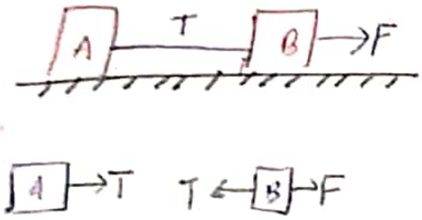
3. Tension Force – ~~The~~ The force with which elements of a string pull each other is called tension force.

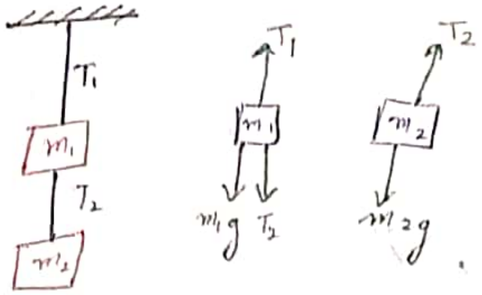
→ An Ideal string is considered to be massless, inextensible, pulls at any point on the string can pull but not push.

→ An Ideal pulley is assumed to be massless, frictionless. Action of pulley is to change the direction of force. Tension is same in the pulley on both sides of it.

→ Tension force is always directed away from point of contact.

Eg





$T_2 = m_2g$ (for body at rest)

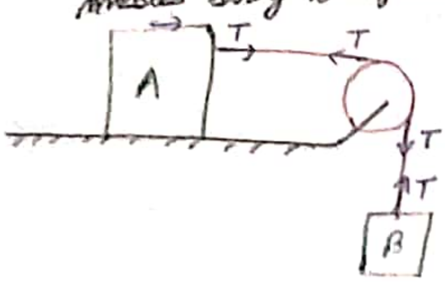
$T_2 = m_2g$

$T_1 = m_1g + T_2$

$T_1 = m_1g + m_2g$

$\boxed{T_1 = (m_1+m_2)g}$

massless string and frictionless pulley & String



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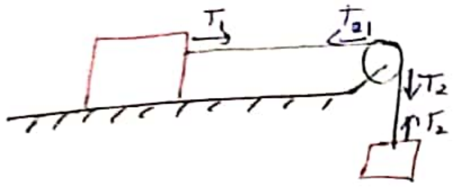
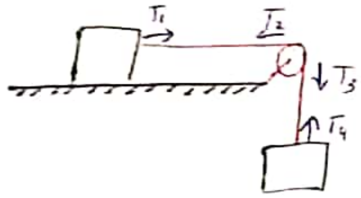
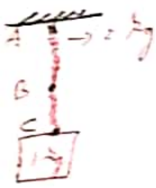
Massless String & pully is not frictionless



Massive String & Pulley is not frictionless



Eg



$g = 10\,m/s^2$

find ~~that~~ tension at A, B, C
↳ midpoint

at A, $T = mg$

$T = (2+1) \times 10$

$= 3 \times 10$

$\boxed{T_A = 30N}$

$T_B = mg$

$= (1+1) \times 10$

$\boxed{T_B = 20N}$

$T_c = mg$

$= 1 \times 10$

$\boxed{T_c = 10N}$

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Q A rope of uniform mass distribution of mass m & length l, find ~~bottom at~~ tension at distance x from bottom.

length till x = x

$$\text{mass} = \cancel{\frac{m(l-x)}{l}} = \frac{mx}{l}$$

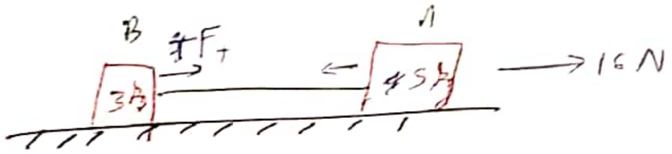
$$T = mg$$

$$= \cancel{\frac{m(l-x)}{l} g}$$

$$\boxed{T = \frac{mxg}{l}}$$

H.W.
Ch - 3
S - 1 (1-20)

Q Find acceleration of blocks & Tension in string connecting A.B.



$\cancel{F = ma}$

$\cancel{16 = 18 \times a}$

$\cancel{a = \frac{8}{9}}$

$F = ma$

$16 = 8a$

$a = \frac{16}{8}$

$\boxed{a = 2}\ m/s^2$

$F_T = 3 \times 2$

$\boxed{F_T = 6N}$

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Q with what min acceleration can a fireman slide down a rope whose breaking strength is of his 2/3 weight.

$W - F_t = ma$

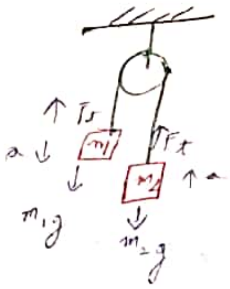
$W - \frac{2}{3}W = \frac{W}{g} \times a$

$1 - \frac{2}{3} = \frac{a}{g}$

$\frac{1}{3} = \frac{a}{g}$

$\boxed{\frac{g}{3} = a}$

Q Find acc & Tension $(m_1 > m_2)$



Atwood machine

~~acc = g~~

$m_1 g - F_t = m_1 a$ --- (1)

$F_t - m_2 g = m_2 a$ --- (2)

(1) + (2)

$m_1 g - m_2 g = m_1 a + m_2 a$

$$\boxed{\frac{(m_1 - m_2)g}{m_1 + m_2} = a}$$

$F_t = m_2 a + m_2 g$

$= m_2 [a + g]$

$= m_2 \left[\frac{(m_1 - m_2)g}{m_1 + m_2} + g\right]$

~~$= m_2 g$~~

$= m_2 g \left[\frac{m_1 - m_2 + m_1 + m_2}{m_1 + m_2}\right]$

$$\boxed{F_t = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$

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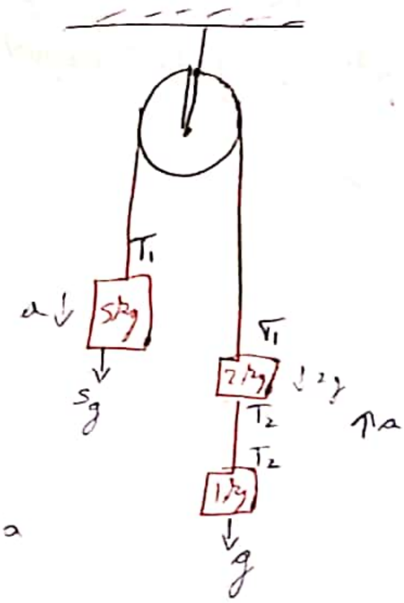
Q first acc & Tension

$5g - T_1 = 5a$ ---(1)

$T_1 - (2g + T_2) = 2a$ ---(2)

$T_2 - g = a$ ---(3)



(3) − (2)

$-g + 2g - T_1 = 0$

$\boxed{T_1 = g}$

$g + T_2 = a$

$T_2 - g = a$

$-2g + 2T_2 = 2a$

$5g - g = a$

$\boxed{a = 4g}$

$T_2 = a + g$

$= 4g + g$

$\boxed{T_2 = 5g}$

$T_2 = a + g$

$T_2 = 10 + 2.5$

$\boxed{T_2 = 12.5 N}$

(3) + (2)

$T_1 - 2g - g = 3a$

$T_1 - 3g = 3a$

$5g - T_1 = 5a$

$2g = 8a$

$a = \frac{g}{4}$

$a = \frac{10}{4}$

$\boxed{a = 2.5 m/s^2}$

$5g - T_1 = 5a$

$T_1 = 5g - 5a$

$T_1 = 5g - \frac{5g}{4}$

$T_1 = \frac{15g}{4}$

$T_1 = \frac{150}{4}$

$\boxed{T_1 = \frac{75}{2} N}$

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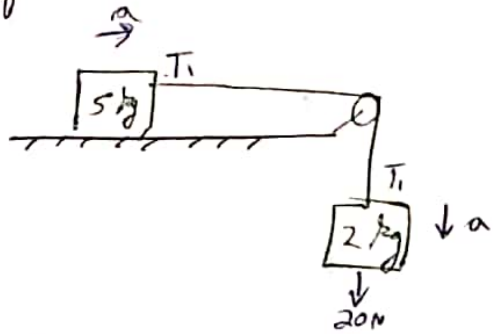
Q find acc & Tension.



$T_1 = 5a$ --- (1)

$\cancel{T_1 - 20 = 2a}$ --- (2)

$\cancel{20 = 3a}$

$\cancel{a = \frac{20}{3}\ m/s^2}$

$20 - T_1 = 2a$

$20 = 7a$

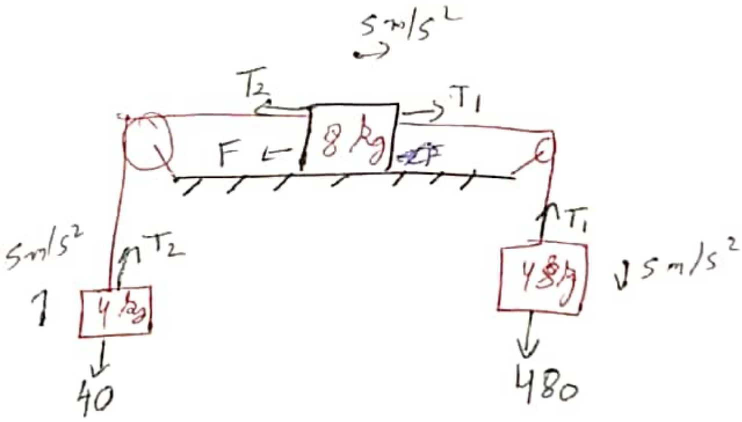
$\boxed{a = \frac{20}{7}\ m/s^2}$

$T = 5a$

$T = 5 \times \frac{20}{7}$

$\boxed{T = \frac{100}{7}\ N}$

Q. $a = 5\ m/s^2$
find friction of 8 kg block



for 48 kg

$480 - T_1 = 5 \times 48$

$480 - T_1 = 240$

$T_1 = 480 - 240$

$\boxed{T_1 = 240\ N}$

for 4 kg

$T_2 - 40 = 5 \times 4$

$\cancel{T_2 = 45\ N}$

$T_2 - 40 = 20$

$\boxed{T_2 = 60\ N}$

for 8 kg

$T_1 - T_2 - F = 8 \times 5$

$240 - 60 - F = 40$

$F = 240 - 100$

$\boxed{F = 140\ N}$

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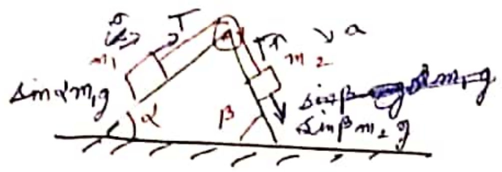
Q find acc.



$$\sin\beta\, m_2 g - T = m_2 a \quad \text{---} \; (1)$$

$$T - \sin\alpha\, m_1 g = m_1 a \quad \text{----} \; (2)$$

$$\sin\beta\, m_2 g - \sin\alpha\, m_1 g = m_2 a + m_1 a$$

$$\boxed{\frac{g\,(\sin\beta\, m_2 - \sin\alpha\, m_1)}{(m_2 + m_1)} = a}$$

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