

!! Projectile Motion !!

Projectile motion - A ~~for~~ object has ~~that~~ ~~got~~ ~~it~~ and it lands somewhere else.

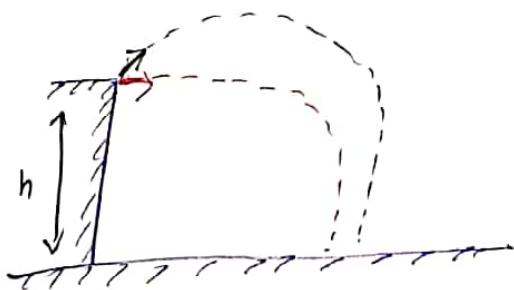
- the object always moves in a single plane
- The object moves in parabola path.
- The object moves only under gravity.  
(ignore earth's curvature, air resistance etc)
- A projectile is any body that is given an initial velocity and then follows a path determined ~~entirely~~ by effects of acceleration due to gravity.

Types of projectile Motion -

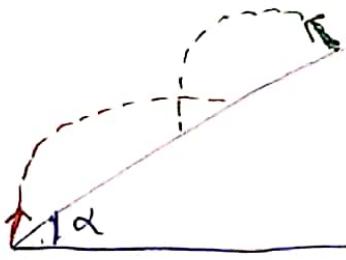
① Ground to ground (oblique) :-



② Projection from a height .



### ③ ~~projection~~ on a plane:-

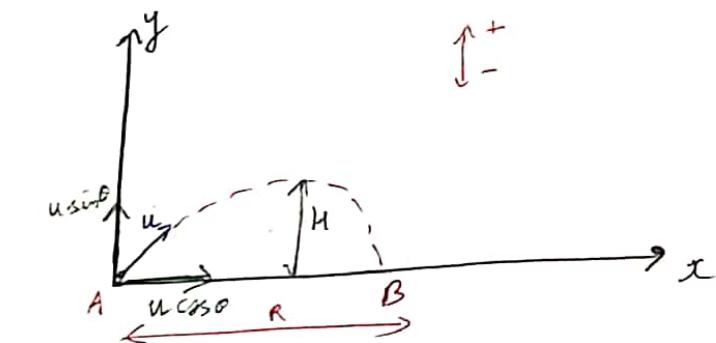


Projectile motion -

↓  
2 dimensional motion (2 d)  
(motion in a plane)

Horizontal ← independent of each other. → Vertical  
(uniform motion) (uniformly accelerated motion)

Oblique projection :- (ground to ground projection)



Horizontal  
(uniform)

$$v_x = u \cos \alpha$$

$$a_x = 0$$

Vertical  
(uniformly accelerated)

$$v_y = u \sin \alpha$$

$$a_y = -g$$

## Time of flight ( $T$ )

$$s_y = 0$$

Using  $s = ut + \frac{1}{2} at^2$

$$0 = usin\theta T + \frac{1}{2} (-g) T^2$$

$$0 = T \left( u\sin\theta - \frac{gT}{2} \right)$$

$T=0$  (at point of projection)

$$u\sin\theta = \frac{gT}{2}$$

$$\frac{2u\sin\theta}{g} = T \quad (\text{at } B)$$

$$T = \frac{2u\sin\theta}{g}$$

## Highest Point ( $H$ )

At highest point,  $v_y = 0$

Using  $v^2 = u^2 + 2as$

$$(v_y)^2 = (u_y)^2 + 2ay H$$

$$0 = (u\sin\theta)^2 + 2(-g)H$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

for fixed  $u$ ,  
maximum height  $H$  will be  
for  $\sin\theta = 1$

$$\theta = 90^\circ$$

## Range (R)

$$R = U_x T$$

$$R = \frac{U \cos \theta \times 2 U \sin \theta}{g}$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

For complimentary angle of projection

$$\theta \rightarrow (90 - \theta)$$

$$R' = \frac{U^2 \sin [2(90 - \theta)]}{g}$$

$$R' = \frac{U^2 \sin (180 - 2\theta)}{2}$$

$$R' = \frac{U^2 \sin 2\theta}{g}$$

Range is always same for complimentary angles.

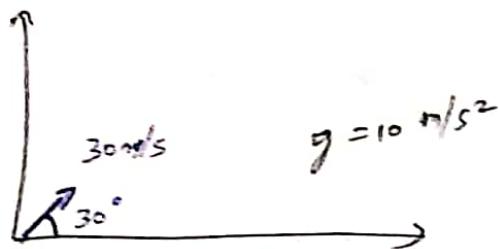
For max range

$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

Q



a) time at which ball reaches highest point

- ~~b)~~
- b) maximum height reached
- c) horizontal range
- d) time of flight

$$T = \frac{2u \sin \theta}{g}$$

$$= \frac{60 \times \frac{1}{2}}{9.8}$$

$$= \frac{30}{9.8}$$

$$\text{time to reach max height} = \frac{30}{9.8} \times \frac{1}{2} = \boxed{\frac{15}{9.8} \text{ s}}$$

$$= \frac{15}{10} \text{ s}$$

$$= 1.5 \text{ s} \checkmark$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$= \frac{15 \times 15 \times \frac{1}{2}}{2 \times 9.8}$$

~~$$= \frac{225}{19.6} \text{ m}$$~~

$$= \frac{225}{20}$$

$$= \frac{112.5}{10}$$

$$\boxed{= 11.25 \text{ m}} \checkmark$$

$$Q) R = \frac{u^2 \sin 2\theta}{g}$$

$$= 30 \times 30 \times \frac{\sqrt{3}}{2}$$

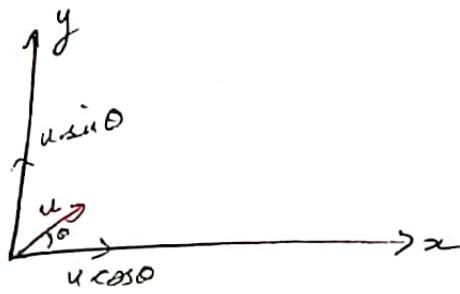
$$\boxed{= 45\sqrt{3} \text{ m}} \checkmark$$

$$T = 1.5 \times 2$$

$$\boxed{= 3 \text{ s}} \checkmark$$

Q 04

Velocity at ~~time~~ time 't' -



$$u_x = u \cos \theta$$

$$v_x = u \cos \theta$$

$$u_y = u \sin \theta$$

$$\bullet v_y = u_y + \alpha_y t$$

$$v_y = u \sin \theta - gt$$

$$\vec{v} = (u \cos \theta) \hat{i} + (u \sin \theta - gt) \hat{j}$$

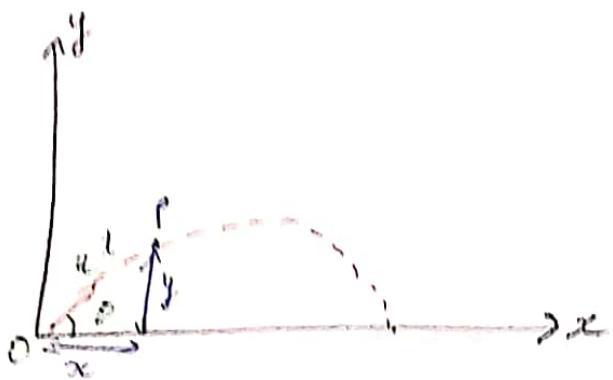
Q A ball is projected with velocity  $u$  at angle of projection  $\theta$ . After what time ball is moving at right angles to initial direction.  
initial direction =  $\theta = \frac{\pi}{2}$  =  $u \sin \theta \hat{j} + u \cos \theta \hat{i}$

$$\vec{v} = (u \cos \theta) \hat{i} + (u \sin \theta - gt) \hat{j}$$
$$\neq \frac{u^2 \cos^2 \theta + u \sin \theta (u \sin \theta - gt)}{u \sqrt{u^2 \cos^2 \theta + u^2 \sin^2 \theta + g^2 t^2 - 2 u \sin \theta g t}}$$
$$= u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta g t$$
$$= u^2 - u \sin \theta g t$$
$$\frac{u^2 + g^2 t^2 - 2 u \sin \theta g t}{u \sqrt{u^2 + g^2 t^2 - 2 u \sin \theta g t}}$$
$$= u - \sin \theta g$$
$$\vec{u}, \vec{v} = 0$$
$$u^2 \cos^2 \theta + u^2 \sin^2 \theta - u \sin \theta g t = 0$$

$$u(u - \sin \theta g t) = 0$$

$$\boxed{t = \frac{u}{\sin \theta g}}$$

## Equation of trajectory



$$\begin{aligned} x &= u_x t \\ x &= u \cos \theta t \\ t &= \frac{x}{u \cos \theta} \quad (1) \end{aligned} \quad \left. \begin{aligned} s_y &= u_y t + \frac{1}{2} g y t^2 \\ y &= u \sin \theta (t) + \frac{1}{2} g t^2 \end{aligned} \right\} \quad (2)$$

$$y = \frac{u \sin \theta (x)}{u \cos \theta} - \frac{1}{2} g \frac{\frac{x^2}{u^2 \cos^2 \theta}}{}$$

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}$$

$$\boxed{y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}}$$

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta} \quad \frac{\sin \theta}{\sin \theta}$$

$$y = x \tan \theta - \frac{x^2 \cancel{\sin \theta \cos \theta}}{\cancel{2 u^2 \sin \theta \cos \theta}} \quad \boxed{\text{Range formula}}$$

$$y = x \tan \theta - \frac{x^2 \tan \theta}{R}$$

$$\boxed{y = x \tan \theta \left[ 1 - \frac{x}{R} \right]}$$

Q A grasshopper can jump its height 'h', find the maximum distance through which it can jump along horizontal ground.

$$h = \frac{u^2 \sin^2 \theta}{2g}$$

$$h = \frac{u^2}{2g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

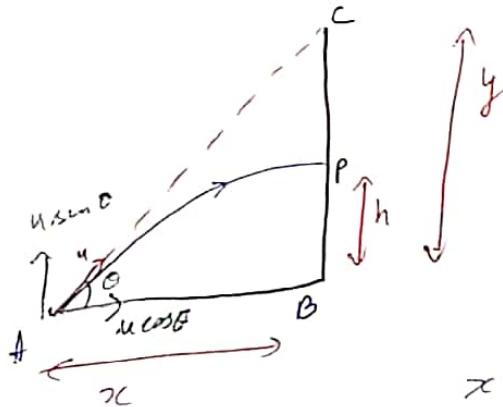
$$\sqrt{2gh} = u$$

$$R = \frac{2gh \times}{g}$$

~~$R = 2h$~~

$\boxed{R_{\text{max}} = 2h}$  ✓

Q A hunter aims his gun and fires a bullet directly on a monkey on a tree. At the instant bullet is fired, the monkey drops. Will the bullet hit the monkey?



$$\tan \theta = \frac{y}{x}$$

for bullet,

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$h = u \sin \theta t - \frac{1}{2} g t^2$$

$$x = u \cos \theta t \Rightarrow t = \frac{x}{u \cos \theta}$$

$$h = u \sin \theta \left( \frac{x}{u \cos \theta} \right) - \frac{1}{2} g t^2$$

$$h = x \tan \theta - \frac{1}{2} g t^2$$

$$h = y - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 = y - h$$

For monkey

$$z = 0 + \frac{1}{2} g t^2$$

$$z = gt^2$$

As, the height is same for monkey & bullet at some time,  
Yes the bullet will hit monkey.

Q The range of a projectile fired at an angle  $15^\circ$  is 40 m. if it is fired with the same speed at an angle of  $45^\circ$  find its range.  
 $g = 10 \text{ m/s}^2$

$$R = \frac{u^2 \sin 2\theta}{g}$$

R =

$$u_0 = \frac{u^2 \times \frac{1}{2}}{g}$$

$$400 \times 2 = u^2$$

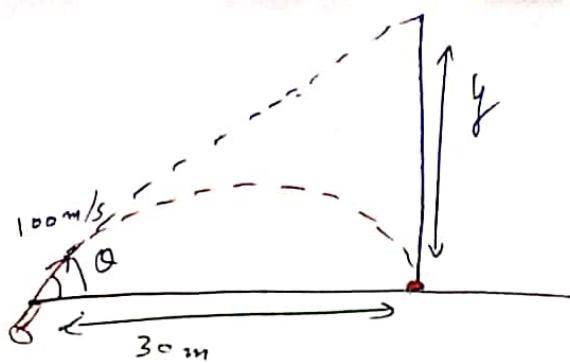
$$\underline{20\sqrt{2} = u}$$

$$R' = \frac{800 \times \sin 45^\circ}{g}$$

$$R' = \frac{800}{10}$$

b)  $\boxed{R' = 80 \text{ m}}$

Q



At how high above the target  
the gun must be aimed to hit  
the target on ground at 30 m  
from the gun?

Find  $y$

$$R = \frac{100 \times 10\phi \sin 2\theta}{10}$$

$$R = 1000 \sin 2\theta$$

$$\tan \theta = \frac{4}{3}$$

$$30 \frac{\sin \theta}{\cos \theta} = y$$

$$\tan 0.015 = \frac{y}{30}$$

$$30 \times 0.015 = y$$

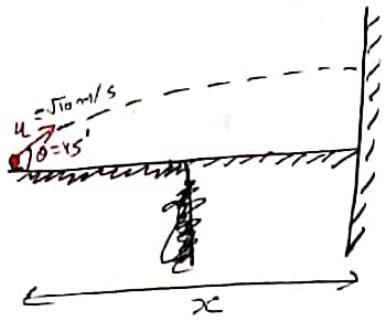
$$0.15 \times 3 = y$$

$$0.45 = y$$

$$y = 0.45 \text{ m}$$

$$\begin{aligned} \frac{3\phi}{100\phi} &= \sin 2\theta \\ \frac{3}{100} &= 2 \sin \theta \cos \theta \\ \frac{3}{100} &= \sin \theta \cos \theta \\ \sin 2\theta &= 0.03 \rightarrow (\text{approximation}) \\ 2\theta &= 0.03 \\ \theta &= 0.015 \end{aligned}$$

Q



$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{10 \times 1}{10}$$

$$R = 1 \text{ m}$$

- a)  $x = y_2 \text{ m}$  ✓ will hit wall  
 b)  $x = 2 \text{ m}$  X hit wall  
 c) where does it hit wall  
 d) max height

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$2g$$

$$H = 10 \times \frac{1}{2} = \frac{1}{2} \text{ m}$$

$$H = \frac{1}{4} \text{ m}$$

$$y = x \tan \theta \left[ 1 - \frac{x}{R} \right]$$

$$y = \frac{1}{2} \times 1 \left[ 1 - \frac{1}{2} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{2} \right]$$

$$= \frac{1}{4} \text{ m}$$

$$\boxed{y = \frac{1}{4} \text{ m}}$$

OR

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

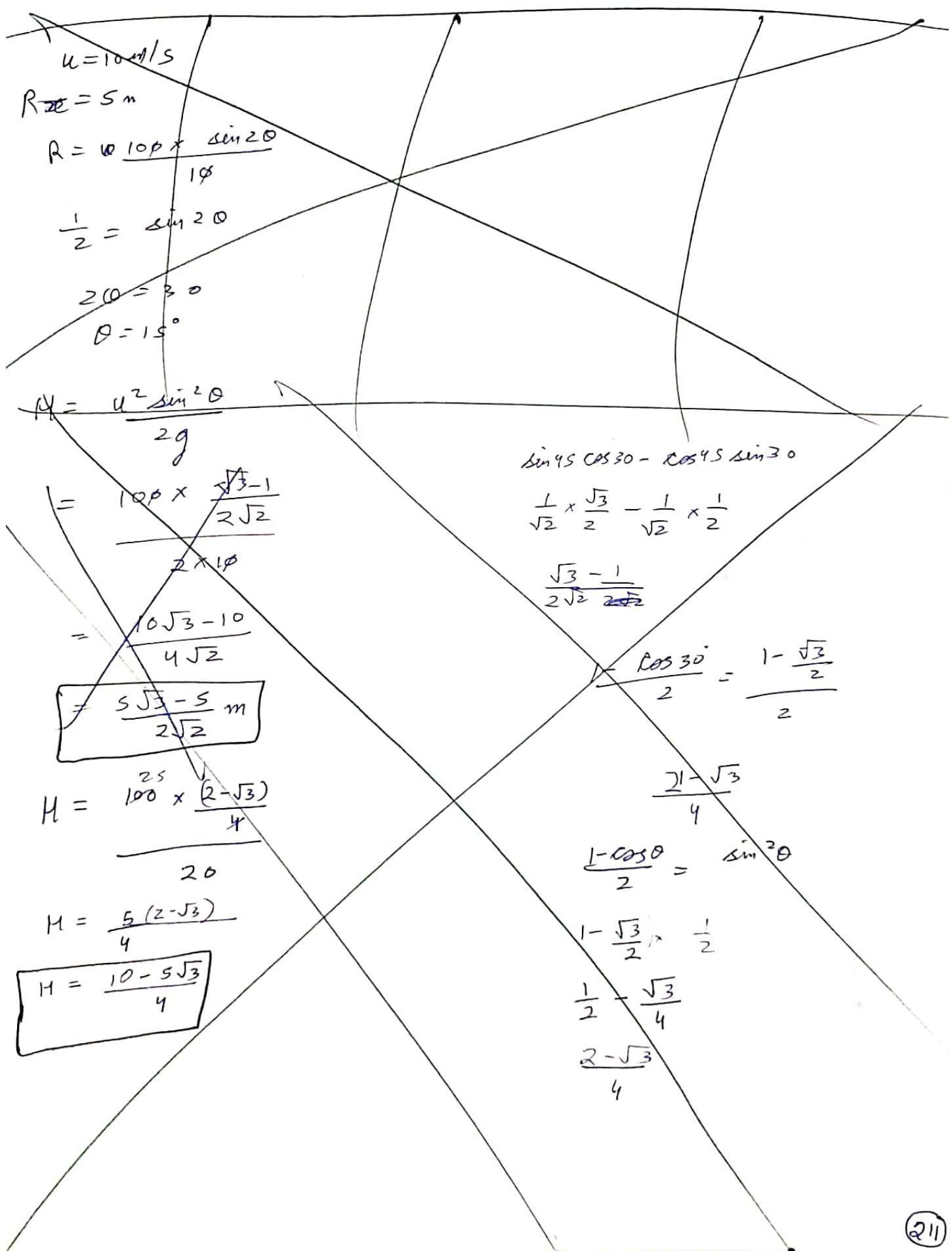
$$= \frac{1}{2} \times 1 - \frac{10 \times \frac{1}{4}}{2 \times 10 \times \frac{1}{2}}$$

$$= \frac{1}{2} - \frac{\frac{5}{2}}{10}$$

$$= \frac{1}{2} - \frac{5}{20} = \frac{5}{4}$$

$$\boxed{= \frac{1}{4} \text{ m}}$$

Q we have a hose pipe which disposes water at speed of  $10 \text{ m/s}$ .  
 The safe distance from a building on fire, on ground is  $5\text{m}$ . How  
 high can this water go. ( $g = 10 \text{ m/s}^2$ )



$$y = x \tan \theta - \frac{g x^2}{2 u^2 \sec^2 \theta}$$

$$y = s \tan \theta - \frac{10 \times 25}{2 \times 10 \times 16 \times \sec^2 \theta}$$

$$y = s \tan \theta - \frac{s \sec^2 \theta}{4}$$

~~first diff~~  
max height

$$\frac{dy}{d\theta} = 0$$

~~first change~~ for stationary

$$\frac{dy}{d\theta} = s \sec^2 \theta - \frac{s}{4} \times 2 \sec \theta \times \sec \theta \tan \theta$$

$$= s \sec^2 \theta - \frac{s}{2} \sec^2 \theta \tan \theta$$

$$\frac{s}{2} \sec^2 \theta = \cancel{s} \sec^2 \theta$$

$$\frac{\tan \theta}{2} = 0$$

$$\tan \theta = 0$$

$$\tan \theta = 2$$

$$p = 2$$

$$B = 1$$

$$H = \sqrt{5}$$

$$\sec \theta = \frac{H}{B}$$

$$= \sqrt{5}$$

$$y_{\text{max}} = \cancel{(10)} 5 \times 2 - \frac{s}{4} \times 5$$

$$= 10 - \frac{25}{4}$$

$$= \frac{15}{4} \text{ m}$$

Or

$$y = s \tan \theta - \frac{s}{4} \sec^2 \theta$$

$$y = s \tan \theta - \frac{s}{4} (1 - \tan^2 \theta)$$

$$y = s \tan \theta - \frac{s}{4} + \tan^2 \theta$$

$$\tan^2 \theta + s \tan \theta - \frac{5s+4y}{4}$$

for real roots

$$\tan^2 \theta - 0 \geq 0$$

$$25 + 5y \geq 0$$

$$5(5+y) \geq 0$$

$$5+y \geq 0$$

$$25 + 5+4y \geq 0$$

$$30+4y \geq 0$$

$$y \geq -\frac{30}{4}$$

$$y \leq \frac{15}{4}$$

Q A shot is fired with a velocity 20 m/s at a vertical wall whose distance from point of projection is 20m. Find the greatest height above the level of point of projection at which bullet can ~~not~~ hit the wall.

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$y = 20 \tan \theta - \frac{10 \times 20 \times 20}{2 \times 20 \times 20 \times \cos^2 \theta}$$

$$y = 20 \tan \theta - 5 \sec^2 \theta$$

$$\frac{dy}{d\theta} = 20 \sec^2 \theta - 10 \sec^2 \theta \tan \theta$$

$$0 = 2 \sec^2 \theta - \sec^2 \theta \tan \theta$$

$$\sec^2 \theta (2 - \tan \theta)$$

$$2 - \tan \theta = 0$$

$$\tan \theta = 2$$

$$P = 2$$

$$B = 1$$

$$u = \sqrt{5}$$

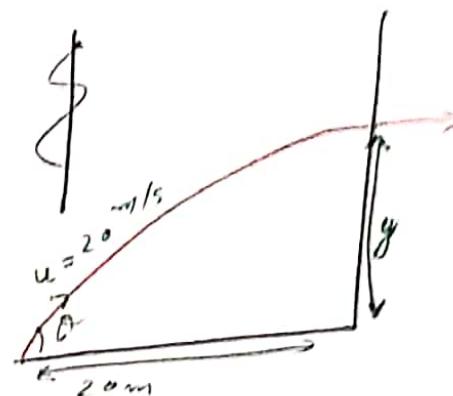
$$\sec \theta = \sqrt{5}$$

$$\sec^2 \theta = 5$$

$$y_{\text{max}} = 20 \times 2 - 5 \times 5$$

$$= 40 - 25$$

$$\boxed{= 15 \text{ m}}$$



(Q2)

$$y = 20 \tan \theta - 5 \sec^2 \theta$$

$$= 20 \tan \theta - 5(1 + \tan^2 \theta)$$

$$y = 20 \tan \theta - 5 - 5 \tan^2 \theta$$

$$5 \tan^2 \theta - 20 \tan \theta + 5 + 15 =$$

$$400 - (4)(5)(y+5) \geq 0$$

$$400 - 20y - 100 \geq 0$$

$$-20y + 300 \geq 0$$

$$-20y \geq -300$$

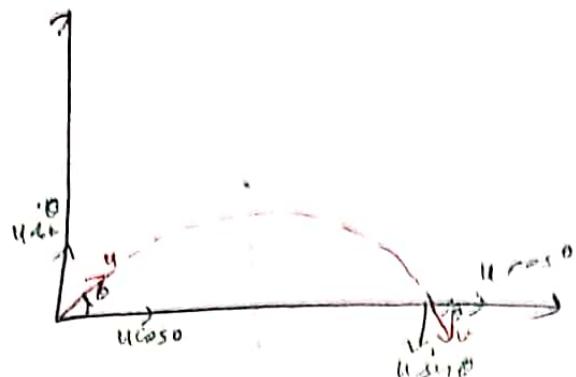
$$20y \leq 300$$

$$y \leq \frac{300}{20}$$

$$\boxed{y \leq 15}$$

Change in momentum between any two positions of projectile

I) Between point of projection & highest point :-



$$\vec{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$$

$$\vec{v} = u \cos \theta \hat{i}$$

$$\Delta \vec{p} = m \vec{v} - m \vec{u}$$

$$= m (\vec{v} - \vec{u})$$

$$\Delta \vec{p} = m (-u \sin \theta \hat{j})$$

$$\boxed{\Delta \vec{p} = m (-u \sin \theta \hat{j})}$$

$$\boxed{|\Delta \vec{p}| = m u \sin \theta}$$

2) Between initial & final points.

$$\vec{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$$

$$\vec{v} = u \cos \theta \hat{i} - u \sin \theta \hat{j}$$

$$\Delta \vec{p} = m (\vec{v} - \vec{u})$$

$$= m (-2 \sin \theta \hat{j})$$

$$\Delta \boxed{\Delta \vec{p} = -2 m u \sin \theta \hat{j}}$$

$$\boxed{|\Delta \vec{p}| = 2 m u \sin \theta}$$

Q A object of mass 0.5 kg is projected under gravity with a speed  $u$  of 98 m/s at an angle of  $60^\circ$ . find magnitude of change in momentum after 10s ( $g = 9.8 \text{ m/s}^2$ )

$$m = 0.5 \text{ kg}$$

$$u = 98 \text{ m/s}$$

$$\theta = 60^\circ$$

$|\Delta \vec{p}| =$

$$u = 98 \sin 60^\circ \uparrow$$

$$t = 10 \text{ s}$$

$$g = 9.8$$

$$v = 98 - 9.8 \cdot \frac{\sqrt{3}}{2} + 98$$

$$v = 49\sqrt{3} + 98$$

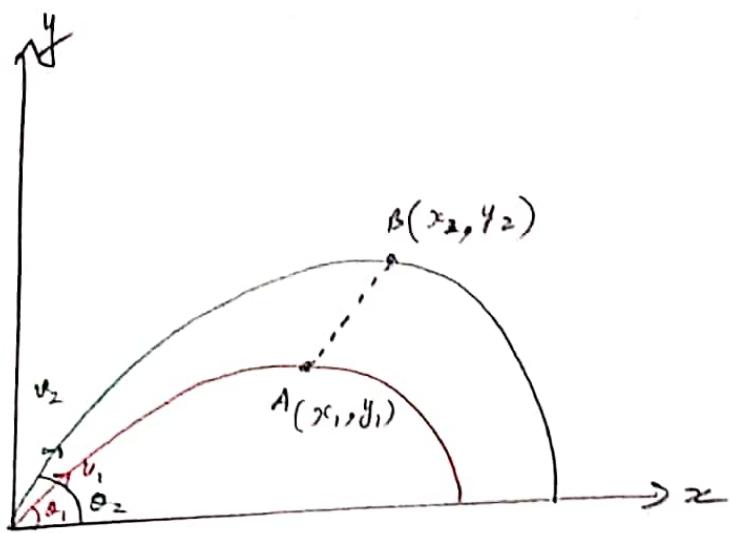
$$\vec{v} = 49\sqrt{3} + 98 \hat{j} + u \cos 60^\circ \hat{i}$$

$$|\Delta \vec{p}| = m (49\sqrt{3} + 98 - 49\sqrt{3})$$

$$= 98 \times 0.5$$

$$|\Delta \vec{p}| = 49 \text{ kg m/s} \quad \checkmark$$

~~Path of one projectile in reference of another projectile~~



$$\left. \begin{array}{l} x_1 = u_1 \cos \theta_1 t \\ x_2 = u_2 \cos \theta_2 t \end{array} \right\} x_2 - x_1 = (u_2 \cos \theta_2 - u_1 \cos \theta_1) t$$

$$\left. \begin{array}{l} y_1 = u_1 \sin \theta_1 t - \frac{1}{2} g t^2 \\ y_2 = u_2 \sin \theta_2 t - \frac{1}{2} g t^2 \end{array} \right\} = u_2 \sin \theta_2 - u_1 \sin \theta_1$$

$$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \frac{u_2 \sin \theta_2 - u_1 \sin \theta_1}{u_2 \cos \theta_2 - u_1 \cos \theta_1}}$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$\hookrightarrow$  constant

If  $u_1 \cos \theta_1 = u_2 \cos \theta_2$

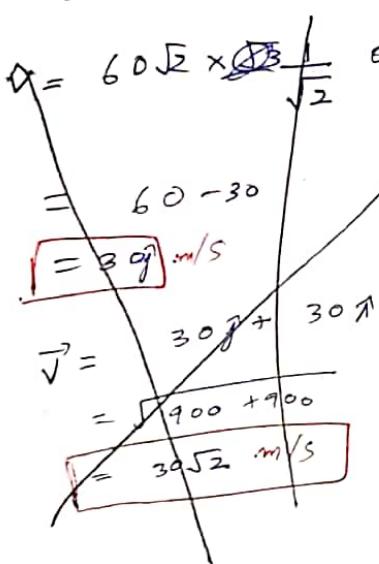
$$\frac{y_2 - y_1}{x_2 - x_1} = \infty \infty \quad (\text{vertical straight line})$$

Q.

$$\begin{array}{c} 60\sqrt{2} \\ \swarrow \\ 10\sqrt{5} \end{array}$$

find after 3s.

- velocity of ball
- angle made by ball with horizontal
- horizontal & vertical displacement.



$$\begin{aligned} \vec{v} &= (u \cos \theta)\hat{i} + (u \sin \theta - gt)\hat{j} \\ &= 60\sqrt{2} \times \frac{1}{\sqrt{2}}\hat{i} + 60\sqrt{2} \times \frac{1}{\sqrt{2}} - 30\hat{j} \\ &= 60\hat{i} + 30\hat{j} \end{aligned}$$

$$\begin{aligned} |\vec{v}| &= \sqrt{60 \times 60 + 30 \times 30} \\ &= \sqrt{3600 + 900} \\ &= \sqrt{4500} \\ &= 10\sqrt{45} \\ &= 30\sqrt{5} \text{ m/s} \checkmark \end{aligned}$$

c) horizontal - ~~180 m~~  $180 \text{ m}$

vertical  $S = ut + at^2$

$$S = 60 \times 3 - \frac{10}{2} \times 9$$

$$S = 180 - 45$$

$$S = 135 \text{ m}$$

$\cos \alpha = \frac{30\sqrt{5}}{60}$   $\theta = \tan^{-1}\frac{30}{60} = \frac{1}{2}$

$\alpha = \cos^{-1}\frac{\sqrt{5}}{2}$

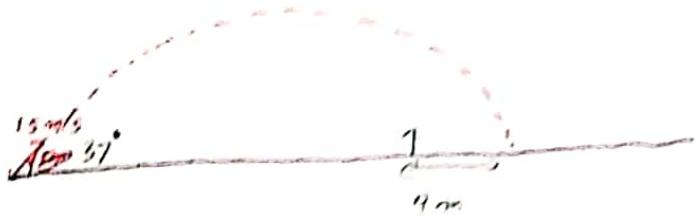
$\cos \theta = \frac{30}{60} = \frac{1}{2}$

$B = 1$

$\cos \alpha = \frac{60}{30\sqrt{5}}$

(21)

Q With what minimum velocity man should run to catch the ball?  
( $g = 10 \text{ m/s}^2$ )



$$2P = T = \frac{24 \sin \theta}{g}$$

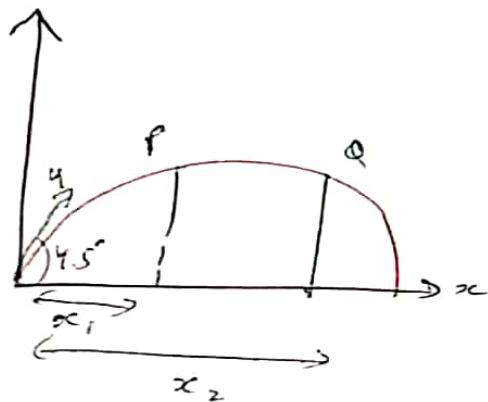
$$T = \frac{24 \times 1.5 \times \frac{3}{5}}{10}$$

$$\boxed{T = 1.86 \text{ s}}$$

$$\text{Speed} = \frac{9}{1.8}$$

$$\boxed{= 5 \text{ m/s}}$$

Q A particle fired with a velocity 20 m/s from a gun adjusted for maximum range. It passes through P & Q whose heights above horizontal are 5m each. find separation between P & Q?



$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\frac{1}{2} = x - \frac{10x}{2 \times 40 \times \frac{1}{2}}$$

$$200 = 20x - x^2$$

$$x^2 - 40x + 200 = 0$$

$$x = \frac{40 \pm \sqrt{1600 - 800}}{2}$$

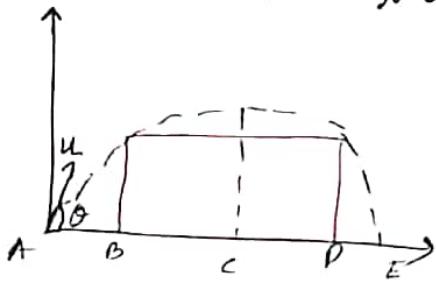
$$x = \frac{40 \pm \sqrt{800}}{2}$$

~~$$x = 20 \pm \sqrt{200}$$~~

$$\text{separation} = 20 + \sqrt{200} - 20 + \sqrt{200}$$

$$= 20\sqrt{2} \text{ m}$$

Q Bird height -  $h$   
 max height -  $2h$   
 find ratio of velocity of bird & horizontal velocity of the stone thrown



$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$h = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\frac{u^2 \sin^2 \theta}{4g} = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\frac{gx^2}{2u^2 \cos^2 \theta} - x \tan \theta + \frac{u^2 \sin^2 \theta}{4g} = 0$$

$$x = \frac{\tan \theta \pm \sqrt{\tan^2 \theta - \frac{g u^2 \sin^2 \theta}{2u^2 \cos^2 \theta}}}{\frac{g}{u^2 \cos^2 \theta}}$$

$$x \propto = \frac{\tan \theta \pm \tan \theta \sqrt{\frac{1}{2}}}{\frac{g}{u^2 \cos^2 \theta}}$$

$$\frac{\tan \theta + \frac{\tan \theta}{\sqrt{2}}}{\frac{g}{u^2 \cos^2 \theta}} - \frac{\tan \theta - \frac{\tan \theta}{\sqrt{2}}}{\frac{g}{u^2 \cos^2 \theta}}$$

$$\frac{\tan \theta u^2 \cos^2 \theta}{\sqrt{2} g} + \frac{\tan \theta u^2 \cos^2 \theta}{\sqrt{2} g}$$

$$\left. \begin{aligned} H &= \frac{u^2 \sin^2 \theta}{2g} \\ h &= \frac{u^2 \sin^2 \theta}{4g} \\ u \sin \theta &= 2\sqrt{gh} \end{aligned} \right\} \frac{\sqrt{2} \tan \theta u^2 \cos^2 \theta}{g}$$

$$h = ut + \frac{1}{2} gt^2$$

$$\cancel{gt^2} + 2ut - 2h = 0$$

$$gt^2 + 2ut - \frac{u^2 \sin^2 \theta}{2g} = 0$$

$$t = \frac{-2u \pm \sqrt{4u^2 + 2g u^2 \sin^2 \theta}}{2g}$$

$$\frac{-2u + \sqrt{4u^2 + 2g u^2 \sin^2 \theta}}{2g} + 2u + \sqrt{u^2 + 2g u^2 \sin^2 \theta}$$

$$\frac{\sqrt{4u^2 + 2g u^2 \sin^2 \theta}}{g}$$

$$\text{Speed} = \frac{\sqrt{2 \tan \theta u^2 \cos^2 \theta}}{g}$$

$$\frac{\sqrt{4u^2 + 2g u^2 \sin^2 \theta}}{g}$$

$$= \frac{\sqrt{2 \tan \theta u^2 \cos^2 \theta}}{\sqrt{4u^2 + 2g u^2 \sin^2 \theta}}$$

$$= \frac{\sqrt{2 \tan \theta u^2 \cos^2 \theta}}{\sqrt{4u^2 + 8g^2 h}} \quad \begin{matrix} \sin^2 \theta \\ \text{Vertical velocity} \end{matrix}$$

$$= \frac{\sqrt{2 \tan \theta u^2 \cos^2 \theta}}{\sqrt{8gh + 8g^2 h}}$$

$$= \frac{\tan \theta u^2 \cos^2 \theta}{2\sqrt{gh}}$$

Speed  
 $u \cos \theta$

$$\boxed{\cancel{\frac{\tan \theta u \cos \theta}{2\sqrt{gh}}}}$$

$$\frac{u \sin \theta}{2\sqrt{gh}}$$

$$\frac{2\sqrt{gh}}{2\sqrt{gh}} = \boxed{\sqrt{n}/\sqrt{n}}$$

$$\boxed{= \frac{2\sqrt{2}}{2+\sqrt{2}}}$$

Q2

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$2h = u^2 \frac{\sin^2 \theta}{2g}$$

$$u^2 \sin^2 \theta = 4gh$$

$$\boxed{u \sin \theta = 2\sqrt{gh}}$$

$$s_y = v_y t + \frac{1}{2} \circ_y t^2$$

$$h = u \sin \theta t - \frac{1}{2} g t^2$$

$$h = 2\sqrt{gh} t - \frac{gt^2}{2}$$

$$gt^2 - 4\sqrt{gh} t + 2h = 0$$

$$t = \frac{4\sqrt{gh} \pm \sqrt{16gh - 8gh}}{2g}$$

$$\boxed{t_1 = \frac{4\sqrt{gh} - 2\sqrt{2gh}}{2g}}$$

$$\boxed{t_2 = \frac{4\sqrt{gh} + 2\sqrt{2gh}}{2g}}$$

$$AD = AB + BD$$

$$u \cos \theta t_2 = u \cos \theta t_1 + v_B t_2$$

$$u \cos \theta (t_2 - t_1) = v_B t_2$$

$$\frac{v_B}{u \cos \theta} = \frac{t_2 - t_1}{t_2}$$

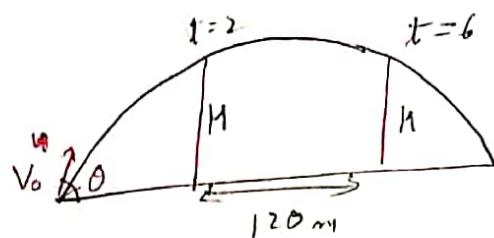
$$= \frac{4\sqrt{2h}}{4\sqrt{2h} + 2\sqrt{2gh}}$$

$$= \frac{4\sqrt{2}}{4 + 2\sqrt{2}}$$

$$\boxed{= \frac{2\sqrt{2}}{2 + \sqrt{2}}}$$

④

Q



$$g = 10$$

$$u = v_0$$

A projectile crosses walls of height  $H$ . find

- time of flight
- $H = ?$
- Max height attained = ?
- Range,  $R = ?$
- $\theta = ?$
- $v_0 = ?$

$$v \cos \theta = \frac{120}{t} \quad 30$$

$$\boxed{u \cos \theta = 30}$$

$$S = ut + \frac{1}{2} at^2$$

$$H = u \sin \theta t + \frac{1}{2} gt^2$$

$$5t^2 + u \sin \theta t - H = 0$$

$$t = \frac{-u \sin \theta + \sqrt{u^2 \sin^2 \theta + 20H}}{10}$$

$$u \sin^2 \theta + 5(2)^2 = u \sin^2 \theta + 5(6)^2$$

$$2u \sin \theta \cancel{20} = 6u \sin \theta \cancel{180}$$

$$0 = 4u \sin \theta + 200$$

$$0 = u \sin \theta + 50$$

$$\boxed{u \sin \theta = -50}$$

~~$$u \cos \theta = 30$$~~
~~$$u \cos \theta = 30$$~~
~~$$u \cos \theta = 30$$~~

$$\boxed{u = \frac{40}{\sin \theta}}$$

$$T = \frac{2u \sin \theta}{g}$$

~~$$= \frac{2 \times 40}{10} \times 10$$~~

~~$$= 8 \Delta$$~~

$$\boxed{= 8 \Delta} \quad a)$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = \frac{1600}{2 \times 10}$$

$$\boxed{H = 80 \text{ m}} \quad c) \text{ max height}$$

$$H = 40(2) + 5(4)$$

$$H = 80 - 20$$

$$\boxed{H = 60} \quad b)$$

~~$$\frac{40}{\sin \theta} \times \cos \theta = 30$$~~

$$\frac{40}{30} = \tan \theta$$

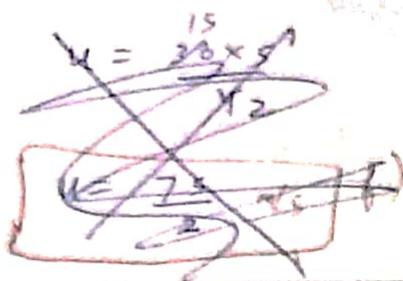
$$\tan \theta = \frac{4}{3}$$

$$\boxed{\theta = 60^\circ} \quad e)$$



$$u \cos \theta = 30$$

$$u \times \frac{15}{5} = 30$$



$$u = \frac{30 \times 5}{3}$$

$$\boxed{u = 50 \text{ m/s}} \quad d)$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

~~$$R = \frac{75 \times 75}{2 \times 2}$$~~

$$R = \frac{50 \times 50 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10}$$

$$R = 10 \times 24$$

$$\boxed{R = 240 \text{ m}} \quad d)$$

# Horizontal Projection From A Height

Horizontal

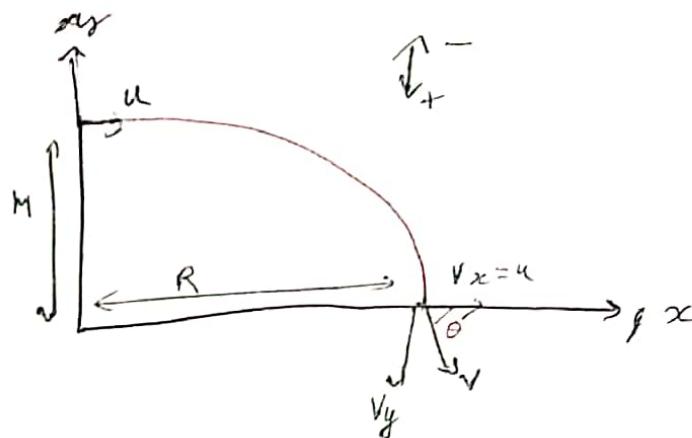
$$U_x = u$$

$$A_x = 0$$

Vertical

$$U_y = 0$$

$$g \cdot a_y = g$$



Time of flight  $\tau(T)$  :-

$$S_y = U_y t + \frac{1}{2} a_y t^2$$

$$H = \frac{1}{2} g T^2$$

$$\therefore T = \sqrt{\frac{2H}{g}}$$

Range ( $R$ ) =  $U_x T$

$$R = u \sqrt{\frac{2H}{g}}$$

$$V_g^2 = U_y^2 + 2 g S_y$$

$$V_y^2 = 0 + 2 g H$$

$V_y = \sqrt{2gh}$  (vertical component of velocity on hitting ground)

$$V_y = \sqrt{2gh}$$

$\theta = \tan^{-1} \frac{V_y}{U_x}$

$$\theta = \tan^{-1} \frac{\sqrt{2gh}}{u}$$

$$\vec{V} = u \hat{i} + \sqrt{2gh} \hat{j}$$

Q A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find

i) time to reach ground

ii) The horizontal distance from foot of hill to ground

iii) The velocity with which it hits the ground. ( $g = 9.8$ )

$$T = \sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2 \times 490}{9.8}}$$

$$T = \sqrt{2 \times 50}$$

$$T = \sqrt{2 \times 5 \times 5 \times 2}$$

$$T = 2 \text{ s}$$

$$= 10 \text{ s} \quad \boxed{i)$$

$$R = u \times T$$

$$= 98 \times 10$$

$$= 980 \text{ m} \quad \boxed{ii)}$$

$$v = 98t + \sqrt{2 \times 9.8 \times 490} \quad \vec{j}$$

$$v = \sqrt{98 \times 98 + 2 \times 98 \times 49}$$

$$v = \sqrt{98(98+98)}$$

$$v = \sqrt{98 \times 2 \times 98 \times 2}$$

$$\boxed{v = 98\sqrt{2} \text{ m/s}} \quad \boxed{iii)}$$

Q.



~~Given~~ find time taken to reach point B.

$$T = \sqrt{\frac{2H}{g}}$$

$$V_f = \sqrt{2gH}$$

$$30 = \sqrt{2 \times 10 \times H}$$

$$200 = 20H$$

$$H = \frac{200}{20}$$

$$\boxed{H = 45 \text{ m}}$$

$$T = \sqrt{\frac{2 \times 45}{10}}$$

$$T = \sqrt{\frac{45}{5}}$$

$$T = \sqrt{9}$$

$$\boxed{T = 3 \text{ s}}$$

Or

$$v = u + at$$

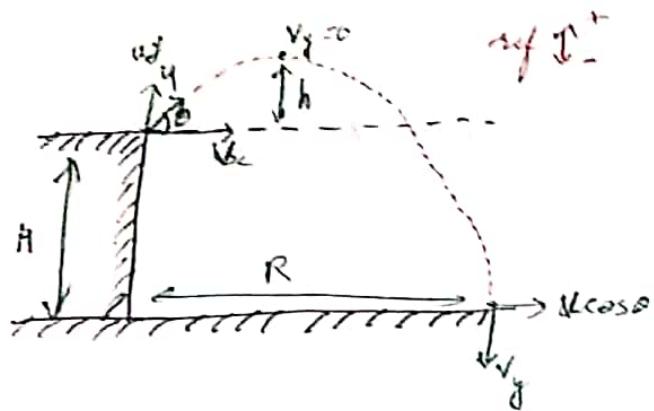
$$30 = 0 + (10)t$$

$$30 = 10t$$

$$t = \frac{30}{10}$$

$$\boxed{t = 3 \text{ s}}$$

## Projection at an angle from a height



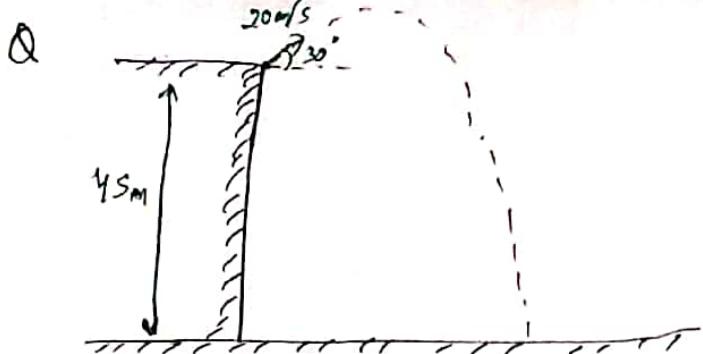
Time of flight:

$$s = ut + \frac{1}{2}gt^2$$

$$-H = u \sin \theta T - \frac{1}{2}gt^2$$

$$T = ?$$

$$\left. \begin{aligned} \text{Range } (R) &= u \cos \theta T \\ h &= \frac{u^2 \sin^2 \theta}{2g} \\ v_f^2 &= (u \sin \theta)^2 + 2gH \\ v_f &= \sqrt{\dots} \\ v_f &= \sqrt{u^2 \sin^2 \theta + 2gH} \end{aligned} \right\}$$



find  
 a) T  
 b) R  
 c) speed (v)

~~Q~~

$$S = ut + \frac{1}{2} a t^2$$

$$-45 = u \sin \theta t + \frac{1}{2} a t^2$$

$$\frac{1}{2} a t^2 - 20 \times \frac{1}{2} t - 45 = 0$$

$$t^2 - 2t - 90 = 0$$

$$t = \frac{2 \pm \sqrt{4 + 36}}{2}$$

$$t = \frac{2 \pm \sqrt{40}}{2}$$

$$t = 1 \pm \sqrt{10}$$

$$t = 1 + \sqrt{10} s$$

$$R = u \cos \theta t$$

$$R = 20 \times \frac{\sqrt{3}}{2} \times (1 + \sqrt{10})$$

$$R = 10\sqrt{3} + 10\sqrt{30}$$

$$R = 10(\sqrt{3} + \sqrt{30}) m$$

$$v = \sqrt{u \sin^2 \theta + 2 g h}$$

$$v = 400 \times \frac{1}{2} + 2 \times 10 \times -45$$

$$v = 100 + 900$$

$$v = \sqrt{1000}$$

$$v = 10\sqrt{10} m/s$$

$$|\vec{v}| = \sqrt{(10\sqrt{10})^2 + (10\sqrt{2})^2}$$

$$= \sqrt{1000 + 200}$$

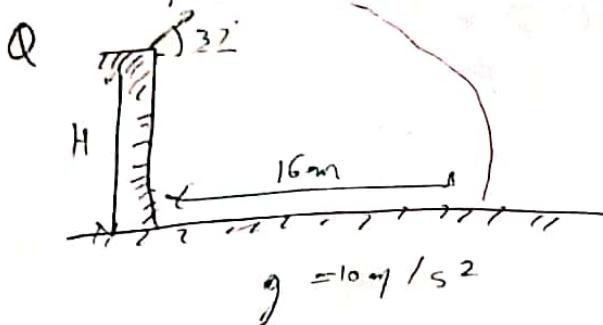
$$= \sqrt{1200}$$

$$= 10\sqrt{12}$$

$$= 20\sqrt{3} m/s$$

$$= \sqrt{1300}$$

$$= 10\sqrt{13} m/s$$



ii

$$-H = 6t - 5t^2$$

$$\Delta t^2 + 6t - H = 0$$

$$t = 6 \pm \sqrt{36 + 20H}$$

$$t = 3 \pm \sqrt{9 + 5H}$$

$$16 = u \cos \theta t$$

$$16 = 8t$$

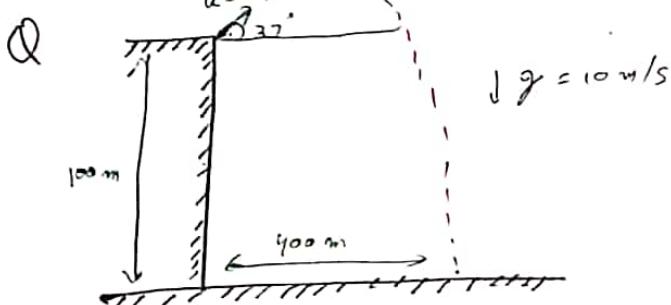
$$t = 2$$

$$3 + \sqrt{9 + 5H} = 2$$

$$\sqrt{9 + 5H} = -1$$

$$-H = 12 - 20$$

$$\boxed{H = 8 \text{ m}}$$



$$400 = u \times \frac{4}{5} \times t$$

$$500 = ut$$

$$\frac{500}{u} = t$$

$$-100 = u \times \frac{3}{5} \times \frac{500}{u} - \frac{1}{2} \times 10 \times \frac{500}{u} \times \frac{500}{u}$$

$$-100 = 300 - \frac{1250000}{u^2}$$

$$-100u^2 = 300u^2 - 125000$$

$$125000 = 300u^2$$

$$\frac{125000}{300} = u^2$$

$$u = \frac{5\sqrt{5}}{\sqrt{2}} \text{ m/s}$$

~~$u^2 = 6250$~~ 

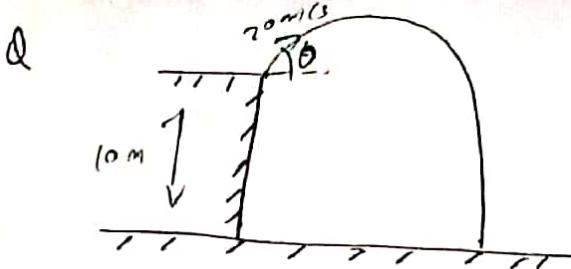
$$u = \frac{25\sqrt{10}}{2}$$

$$u^2 = \frac{6250}{2}$$

$$\boxed{u^2 = 3125}$$

$$\boxed{u^2 = 25\sqrt{5}}$$

(1B2)



$$-10 = 20 \sin \theta t - 5t^2$$

$$-2 = 4 \sin \theta t - t^2$$

$$t^2 - 4 \sin \theta t - 2 = 0$$

$$t = \frac{4 \sin \theta + \sqrt{16 \sin^2 \theta + 8}}{2}$$

$$t = \frac{4 \sin \theta + \sqrt{4 \sin^2 \theta + 2}}{2}$$

$$\theta = 2 \cos \theta + \frac{1}{2 \sqrt{4 \sin^2 \theta + 2}}$$

$$\theta = 4 \cos \theta \sqrt{4 \sin^2 \theta + 2} + 1$$

$$\theta = 100 \cos^2 \theta (4 \sin^2 \theta + 2)$$

$$h =$$

$$\sec^2 \theta = 64 \sin^2 \theta + 2$$

$$y = x \tan \theta - \frac{gx^2}{24^2 \cos^2 \theta}$$

$$-10 = x \tan \theta - \frac{10x^2 \sec^2 \theta}{24^2}$$

$$-10 = x \tan \theta - \frac{x^2 (1 - \tan^2 \theta)}{80}$$

$$-800 = 80x \tan \theta - x^2 + x^2 \tan^2 \theta$$

$$x^2 \tan^2 \theta + 80x \tan \theta - (x^2 - 800) = 0$$

$$\tan^2 \theta - 4ac \geq 0$$

~~$$R = \text{constant}$$~~

$$y = x \tan \theta - \frac{gx^2}{24^2 \cos^2 \theta}$$

~~$$-10 = x \tan \theta - \frac{10x^2}{24^2 \cos^2 \theta}$$~~

~~$$-10 = x \tan \theta + \left[ 1 - \frac{x^2}{R} \right]$$~~

~~$$-10 = x \tan \theta - \frac{x^2}{80 \cos^2 \theta}$$~~

~~$$-800 \cos^2 \theta = x^2 \sin^2 \theta \cos^2 \theta - x^2$$~~

~~$$x^2 \sec^2 \theta \sin^2 \theta \cos^2 \theta - 800 \cos^2 \theta$$~~

~~$$\frac{dy}{dx} = 2x -$$~~

~~$$-10 = x \tan \theta + \frac{10x^2}{480 \times 2 \times \cos^2 \theta}$$~~

~~$$-10 = \frac{80}{x} \tan \theta - \frac{x^2 \sec^2 \theta}{80}$$~~

~~$$-800 = 8x \tan \theta - x^2 \sec^2 \theta$$~~

$$6400x^2 + 4x^2(x^2 - 800) \geq 0 \geq 0$$

$$6400x^2 + 4x^4 - 3200x^2 \geq 0 \geq 0$$

$$9600x^2 \geq 3200x^2 \Rightarrow 4x^4 \geq 0 \geq 0$$

$$2400 \leq R^2$$

$$R \leq 20\sqrt{6}$$

$$2400 = \tan^2 \theta$$

$$\tan \theta = \frac{1000\sqrt{6}}{4800} = \sqrt{\frac{9600x^2 - 4x^4}{4800}}$$

$$\tan \theta = \frac{1600\sqrt{6} + \sqrt{9600 \times 2400 + 4 \times 2400}}{2400 \times 2}$$

$$\tan \theta = \frac{\frac{2}{3}\sqrt{6} + \sqrt{9600+4}}{2}$$

$$\tan \theta = \frac{1}{3}\sqrt{6} + \sqrt{2401}$$

$$\theta = \tan^{-1} \left[ \frac{\sqrt{6}}{3} + \sqrt{2401} \right]$$

$$x^2 \tan^2 \theta + 80x \tan \theta - (x^2 - 800) = 0$$

$$x = 20 \sqrt{6}$$

$$x^2 = 400 \times 6$$

$$= 2400$$

$$2400 \tan^2 \theta + 1600 \tan \theta - (2400 - 800)$$

$$- 1600$$

$$2400 \tan^2 \theta + 1600 \frac{\sqrt{6}}{2} \tan \theta - 1600 = 0$$

~~$$3 \tan^2 \theta + 2 \tan \theta - 2 = 0$$~~

~~$$\tan \theta = \frac{-2 \pm \sqrt{4 + 24}}{2}$$~~

~~$$\tan \theta = \frac{-2 + \sqrt{30}}{2}$$~~

$$3 \tan^2 \theta + 2\sqrt{6} \tan \theta - 2 = 0$$

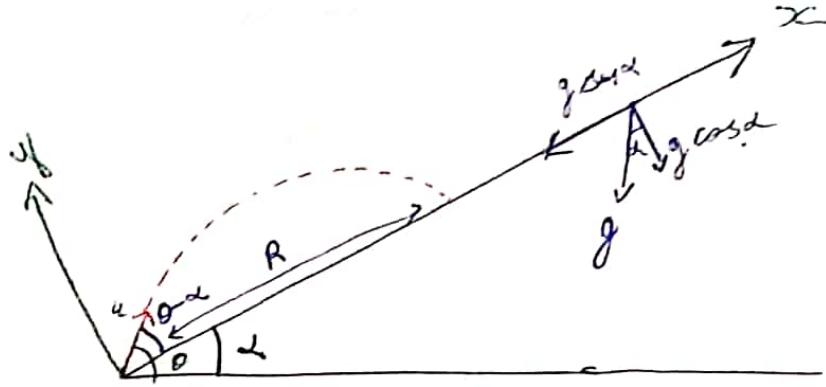
$$\tan \theta = \frac{-2\sqrt{6} \pm \sqrt{24 + 24}}{2 \times 3}$$

~~$$\tan \theta = \frac{-\sqrt{6} \pm \sqrt{12}}{3}$$~~

~~$$\tan \theta = \frac{-\sqrt{6} \pm \sqrt{12}}{3}$$~~

$$\tan \theta = \frac{-\sqrt{6} \pm 2\sqrt{3}}{3}$$

## Projection up the inclined plane



Horizontal

$$u_{x_2} = u \cos(\theta - \alpha)$$

$$a_x = -g \sin \alpha$$

Vertical

$$u_y = u \sin(\theta - \alpha)$$

$$a_y = -g \cos \alpha$$

Time of flight ( $T$ ) =

$$S = uT + \frac{1}{2} aT^2$$

$$0 = u_y T + \frac{1}{2} a_y T^2$$

$$0 = u \sin(\theta - \alpha) T + \frac{1}{2} (-g \cos \alpha) T^2$$

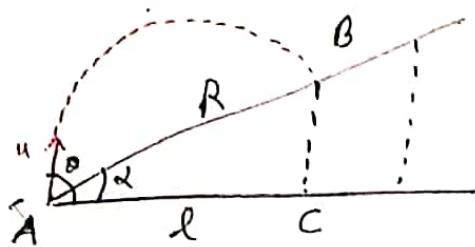
$$\frac{g \cos \alpha}{2} T^2 = u \sin(\theta - \alpha) T$$

$$T = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$$

$$T = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}$$

$$\text{Range: } u_x T + \frac{1}{2} a_x T^2$$

$$R = u \cos(\theta - \alpha) T + \frac{1}{2} \frac{(-g \cos \alpha)}{(-g \sin \alpha)} T^2$$



$$l = u \cos \alpha T \quad l \cos \alpha = \frac{l}{R}$$

$$\cos \alpha = \frac{u \cos \alpha T}{R}$$

$$R = \frac{u \cos \alpha T}{\cos \alpha}$$

$$R = \frac{u \cos \theta}{\cos \alpha} \times \frac{2u \sin (\theta - \alpha)}{g \cos \alpha}$$

$$R = \frac{2u^2 \cos \theta \sin (\theta - \alpha)}{g \cos^2 \alpha}$$

$$R_{max} = \frac{u^2}{g(1 + \tan \alpha)}$$

JMG Ch-8

Q8.  $x^2 = at^2 + bt + c$

$$\frac{d}{dt}(x^2) = \frac{d}{dt}(at^2 + bt + c)$$

$$2x \cdot \frac{dx}{dt} = 2at + 2b$$

$$\Rightarrow x \cdot v = at + b$$

$$v \cdot \frac{dx}{dt} + x \cdot \frac{dv}{dt} = a$$

$$v^2 + ax = a$$

$$A = \frac{a - v^2}{x}$$

$$A = A - \left( \frac{at + b}{x} \right)^2$$

$$A = a - \frac{a^2 t^2 + b^2 + 2abt}{x^2}$$

$$A = \frac{ax^2 - (a^2 t^2 + b^2 + 2abt)}{x^3}$$

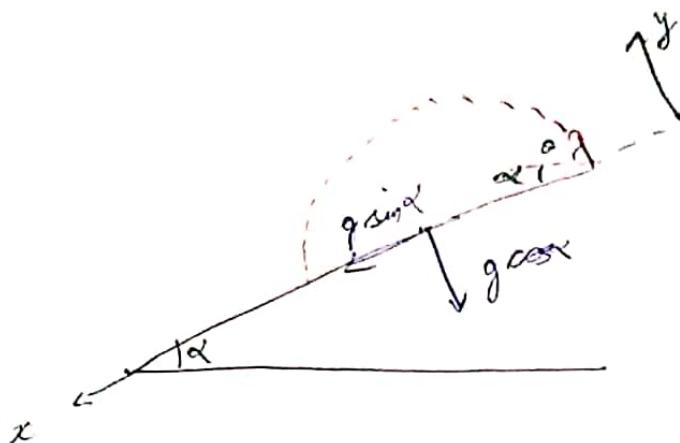
$$A = \frac{a^2 t^2 + 2abt + ac - a^2 t^2 - b^2 - 2abt}{x^3}$$

$$A = \frac{ac - b^2}{x^3}$$

$$A \propto x^{-3}$$

$$\boxed{n=3}$$

~~For~~ Projection down the inclined plane



Horizontal

$$v_x = u \cos(\theta + \alpha)$$

$$\alpha_x = -g \sin \alpha$$

Vertical

$$v_y = u \sin(\theta + \alpha)$$

$$\alpha_y = -g \cos \alpha$$

Time of flight ( $T$ ):-

$$S_y = 0 = v_y T + \frac{1}{2} \alpha_y T^2$$

$$0 = u \sin(\theta - \alpha) T + \frac{1}{2} (-g \cos \alpha) T^2$$

$$T = \frac{2 u \sin(\theta - \alpha)}{g \cos \alpha}$$

$$R = v_x T + \frac{1}{2} \alpha T^2$$

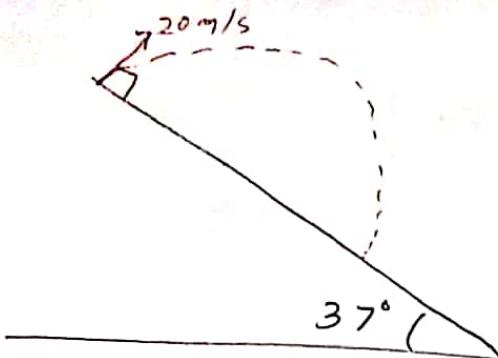
$$R = u \cos(\theta + \alpha) T + \frac{1}{2} g \sin \alpha T^2$$

$$R = u \cos(\theta + \alpha) T + \frac{1}{2} g \sin \alpha T^2$$

$$R = \frac{2 u^2 \sin(\theta + \alpha) \cos \alpha}{g \cos^2 \alpha}$$

$$R_{max} = \frac{u^2}{g(1 - \sin \alpha)}$$

Q



$$R = \frac{2u^2 \sin(\theta + \alpha) \cos \theta}{g \cos^2 \alpha}$$

$$R = \frac{2 \times 20 \times 20 \times \sin(37 + 1) \cos 37 \times 1}{10 \times (\cos 37)^2}$$

$$R = \frac{80 \times 1 \times \frac{4}{3}}{\frac{16}{25}}$$

$$R = \frac{80 \times 1 \times 25}{4 \times 16 \times 5}$$

$$R = 100$$

$$R = \frac{2u^2 \sin(\theta + \alpha) \cos \theta}{g \cos^2 \alpha}$$

$$R = \frac{2 \times 20 \times 20 \times 1 \times \frac{3}{5}}{10 \times \frac{16}{25}}$$

$$R = \frac{80 \times 3 \times 25}{16 \times 5} \times \frac{25}{4}$$

$$R = 25 \times 3$$

$$\boxed{R = 75 \text{ m}}$$

Q2

$$T = \frac{2u \sin(\theta + \alpha)}{g \cos \alpha}$$

$$= \frac{2 \times 20 \times 1}{10 \times \frac{4}{5}}$$

$$\boxed{T = 5 \text{ s}}$$

$$S = ut + \frac{1}{2} g t^2$$

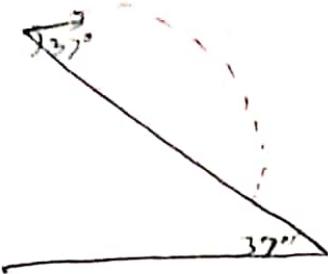
$$PR = u \cos(\theta + \alpha) T - \frac{1}{2} g L \cos \alpha \sin^2 \alpha \times T^2$$

$$R = 20 \times 1 - \frac{5 \times 3}{5} \times 5 \times 5$$

$$\boxed{R = 75 \text{ m}}$$

(19)

Q



$$T = 5 \text{ s}$$

$$u = ?$$

~~T = 5 s~~  
find u

$$T = \frac{2u \sin(\theta + d)}{g \cos \alpha}$$

~~$T = 2 \times u \times x$~~

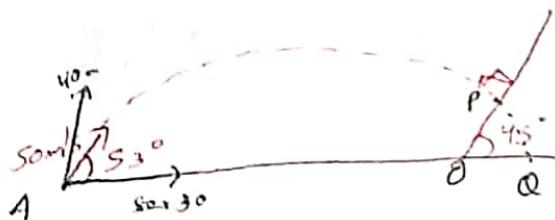
$$5 = 2 \times u \times \frac{3}{5}$$

$$\overline{2 \cdot 10 \times \frac{u}{5}}$$

$$\frac{20 \times 5}{3} = u$$

$$\boxed{\frac{100}{3} = u}$$

Q



$$T = ?$$

$$AO = ?$$

$$\vec{v} = u \cos 45^\circ \hat{i} + u \sin 45^\circ \hat{j}$$

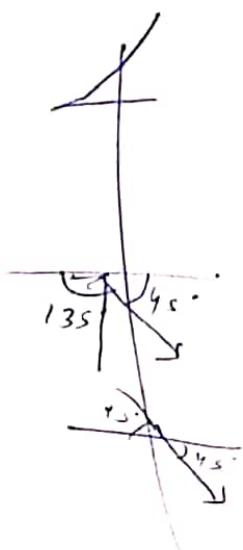
$$\vec{v} = u \cos 53^\circ \hat{i} + u \sin 53^\circ \hat{i} - gt \hat{j}$$

$$x = \frac{50}{\sqrt{2}} = \frac{50 \times 40}{5} - 10t$$

$$10t = 40 - \frac{50}{\sqrt{2}}$$

$$20t = 4 - \frac{5}{\sqrt{2}}$$

$$t = 4 \sqrt{2} - 5 \text{ s}$$



(20)

Velocity at P =  $\vec{v}$

$$\vec{v} = \frac{v}{\sqrt{2}} \hat{i} - \frac{v}{\sqrt{2}} \hat{j}$$

$$\vec{u} = 30 \hat{i} + 10 \hat{j}$$

$$\frac{v}{\sqrt{2}} = 30$$

$$v = 30\sqrt{2}$$

$$V(\text{in } \text{m/s}) = \frac{30\sqrt{2}}{\sqrt{2}}$$

$$= 30$$

using  $v - u = at$

$$\frac{-30 - 40}{-10} = t$$

$$\frac{-70}{-110} = t$$

$$\boxed{\cancel{t} = 7}$$

$$\boxed{\cancel{t} = 7 \text{ s}}$$

BB

$$s = ut + \frac{1}{2} at^2$$

$$PQ = u_0 t + \frac{1}{2} (-10) t^2$$

$$= 280 - 24 s$$

$$= 35 = 0$$

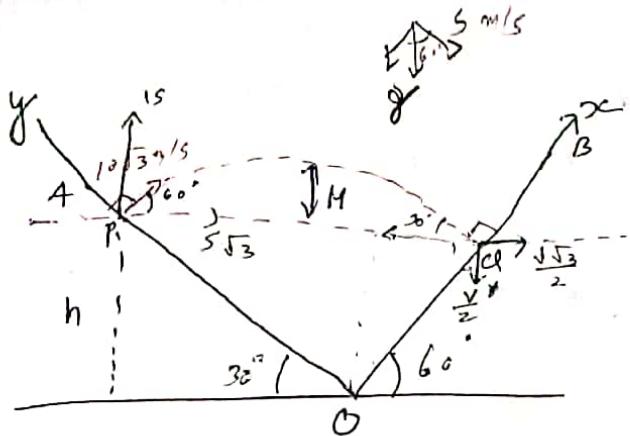
$$AD = 30 \times 7 = 210$$

$$AO = AD - PQ = 210 - 35$$

$$\boxed{= 175 \text{ m}}$$

(21)

Q



- Time of flight
- velocity at QB.
- $h = ?$
- Max height attained by particle from O
- distance PQ.

$$\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2} \quad \frac{V\sqrt{3}}{2} = \frac{10\sqrt{3} \times \sqrt{3}}{2} = \frac{30}{2} = 15$$

$V = 10\sqrt{3}$

$$\frac{V\sqrt{3}}{2} = 5\sqrt{3}$$

(b)

$$V = 10$$

$$\frac{-5 - 15}{-10} = t$$

$$\frac{10}{10} = t$$

a)

t = 1s

$$\frac{-20}{-10} = \sqrt{2} s$$

b)

$$\tan 60^\circ = \frac{PO}{PQ}$$

$$\sqrt{3} = \frac{20}{PQ}$$

c)

$$PQ = \frac{20\sqrt{3}}{3}$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$H = \frac{10\sqrt{3} \times 10\sqrt{3} \times \frac{3}{4}}{10 \times 10}$$

$$H = \frac{9}{4}$$

(22)

$$T = \sqrt{\frac{2H}{g}}$$

$$2 = \sqrt{\frac{2 \times PO}{g}}$$

$$4 = \frac{2 PO}{10}$$

$$\frac{40}{2} = PO$$

PO = 20

$$\approx \tan 30 = \frac{h}{PO}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20}$$

c)

$$h = \frac{10\sqrt{3}}{\sqrt{3}}$$

$$\text{height} = \frac{9}{4} + \frac{20}{\sqrt{3}}$$

$$= \frac{9\sqrt{3} + 80}{4\sqrt{3}} \text{ m}$$

d)

$$OP = 0 + \frac{1}{2} (-s) (z)^2$$

$$S_y = v_y T + \frac{10}{2} g T^2$$

$$\boxed{OP = 10}$$

$$S_x = v_x T + \frac{1}{2} a_x T^2$$

$$OP = 10 \sqrt{3} (z) + \frac{1}{2} (-s \sqrt{3}) (z)^2$$

$$= 20\sqrt{3} - 10\sqrt{3}$$

$$OP = 10\sqrt{3} \text{ m}$$

$$PQ = \sqrt{OP^2 + OQ^2} = \sqrt{100 + 300}$$

$$\boxed{PQ = 20 \text{ m}}$$

$$\boxed{PQ = 20 \text{ m}}$$

$$\sin 30^\circ = \frac{h}{10}$$

$$\boxed{= 5 \text{ m}}$$

$$M = \frac{(10\sqrt{3})^2 \sin^2 60^\circ}{2(10)}$$

$$M = 15 \times \frac{3}{4} = \boxed{\frac{45}{4}}$$

## Relative Velocity

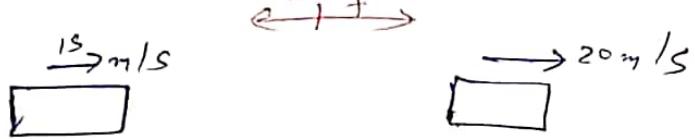
$$\vec{V}_{\text{obj, observe}} = \vec{V}_{\text{obj}} - \vec{V}_{\text{observe}}$$

→ Velocity of object with respect to observer.

→ If distance increases, velocity of separation else velocity of approach

Q.

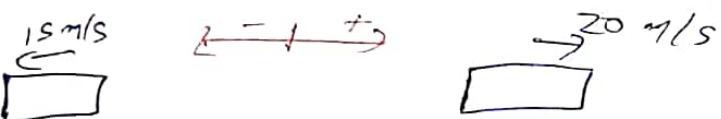
eg (1)



$$V_{AB} = V_A - V_B = 15 - 20 = -5 \text{ m/s}$$

$$V_{BA} = V_B - V_A = 20 - 15 = 5 \text{ m/s}$$

eg (2)



$$V_{AB} = V_A - V_B = -15 - 20 = -35 \text{ m/s}$$

$$V_{BA} = V_B - V_A = 20 - (-15) = 20 + 15 = 35 \text{ m/s}$$

eg (3).



$$V_{AB} = V_A - V_B = v - v = 0$$

(4) -



$$V_{AB} = V_A - V_B = v - (-v) = v + v = 2v$$

Q find distance travelled by bird till car meets.



$$v_{AB} = v_1 - (-v_2) = v_1 + v_2$$

$$\text{Time} = \frac{\text{distance}}{\text{Speed}}$$

$$\boxed{\text{Time} = \frac{d}{v_1 + v_2}}$$

$$\text{Distance by bird} = \text{Speed} \times \text{Time}$$

$$= v_3 \times \frac{d}{v_1 + v_2}$$

$$= \boxed{\frac{v_3 d}{v_1 + v_2}} \checkmark$$

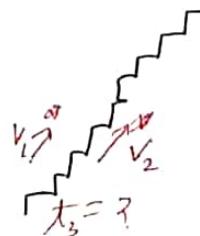
Q



$$v_1 = \frac{l}{t_1}$$



$$v_2 = \frac{l}{t_2}$$



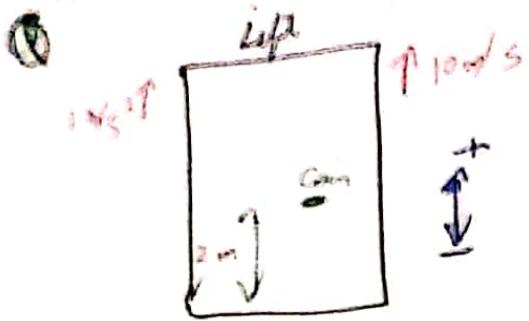
$$v_1 + v_2 = v_3$$

$$t_3 = t_1 + t_2 = \frac{l}{v_3}$$

$$\boxed{t_3 = \frac{t_1 + t_2}{v_3}} \checkmark$$

$$v_3 = \frac{l}{t_1} + \frac{l}{t_2}$$

$$\begin{aligned} \frac{l}{v_3} &= \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}} \\ &= \frac{t_1 t_2}{t_2 l + t_1 l} \end{aligned}$$



A coin is dropped inside lift f.  
find time it takes to reach the  
floor ( $g = 10 \text{ m/s}^2$ )

$$U_{\text{relative}} = U_{c,e} = V_c - V_e = 10 - 10 = 0 \text{ m/s}$$

$$\alpha_{\text{rel}} = \alpha_{c,e} = \alpha_c - \alpha_e = -10 - (+1) = -11 \text{ m/s}^2$$

$$S_{\text{rel}} = -2 \text{ m}$$

$$S_{\text{rel}} = ut + \frac{1}{2} \alpha_{\text{rel}} t^2$$

$$-2 = ut + \frac{1}{2} \alpha_{\text{rel}} t^2$$

$$-2 = -11t^2$$

$$\frac{-4}{11} = t^2$$

$$t = \frac{2}{\sqrt{11}} \text{ s}$$



$$\alpha_{\text{rel}} = -8 \text{ m/s}^2$$

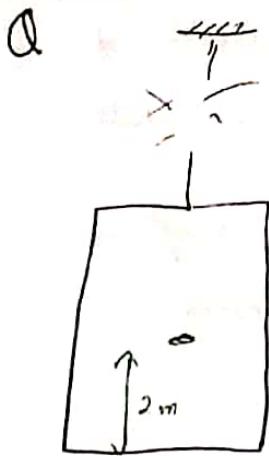
$$u_{\text{rel}} = 0$$

$$S_{\text{rel}} = -2 \text{ m}$$

$$-2 = \frac{1}{2} \alpha_{\text{rel}} t^2$$

$$\frac{4}{8} = \frac{1}{2} = t^2$$

$$t = \frac{1}{\sqrt{2}} \text{ s}$$



$$\cancel{v_0} = v_{ce} = 0 \text{ m/s}$$

$$a_{ce} = -10 - (-10) = -10 + 10 = 0$$

$$-2 = 0 + \frac{1}{2}(0)t^2$$

$$-2 = 0t^2$$

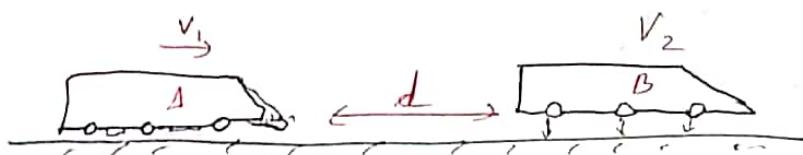
$$\frac{-2}{0} = t^2$$

$$t^2 = \infty$$

$$\boxed{t = \infty \text{ s}}$$

The coin will never reach the floor of lift.

Q



$$v_1 > v_2 \text{ (initially)}$$

If applies break for  $a = -\alpha$  within distance is  $d$ , find time  $t$  to avoid collision. Speed of A relative equal to  $v_2$  after relative distance  $d$ .

$$v_3 = v_1 - v_2 = v_{rel}$$

$$a = -\alpha - 0 = -\alpha$$

$$s = d$$

$$\left| \begin{array}{l} \cancel{d = v_3 t + \frac{1}{2} \alpha t^2} \\ 0 - v_3^2 = 2 \times (-\alpha) \times d \\ \frac{-(v_1 - v_2)^2}{2\alpha} = d \end{array} \right.$$

(27)

- Q A platform is moving upwards with a constant acceleration of  $2 \text{ m/s}^2$ . At  $t = 0$ , a boy standing on platform throws a ball upwards with a relative speed of  $8 \text{ m/s}$ . At this instant, platform was at height of  $4 \text{ m}$  from the ground and was moving with a speed of  $2 \text{ m/s}$ . Find
- when & where ball strikes the platform
  - Max height of ball from ground
  - Max height of ball from platform.

$$a_{\text{rel}} = \frac{-12}{2} \text{ m/s}^2$$

$$u_{\text{rel}} = 8 \text{ m/s}$$

$$v = 0$$

$$\frac{-8}{-12} = t$$

$$t = \frac{2}{3} \text{ s} \quad (\text{to g up})$$

$$\boxed{t = \frac{4}{3} \text{ s}} \quad (2)$$

$$\begin{aligned} S &= H + \frac{1}{2} a t^2 \\ S &= 4 + \frac{1}{2} \times 2 \times \left(\frac{4}{3}\right)^2 \\ S &= 12 \text{ m} \end{aligned}$$

$$S = \frac{4}{3} + \frac{1}{2} x^2$$

$$S = 2 \times \frac{4}{3} + \frac{1}{2} x^2 \times \frac{4}{3} \times \frac{4}{3}$$

$$S = \frac{8x3}{9} + \frac{16}{9}$$

$$S = \frac{40}{9} + 4$$

$$\boxed{S = 4.66 \text{ m}} \quad (2)$$

$$\boxed{S = 8.66 \text{ m}} \quad (a)$$

$$S = \frac{8x0 + 1}{2}$$

$$\begin{aligned} v + 64 &= 2x + 8 \times \frac{12}{2} \text{ m} \quad (\text{from const}) \\ v &= 8 \text{ m/s} \\ H &= 4 \text{ m} \\ H &= \frac{8}{3} \text{ m} \end{aligned}$$

$$v + 16 = 2x + 10 \times h \quad (\text{from ground}) + 4$$

$$\frac{100}{20} + 4 = h$$

$$h = 5 + 4$$

$$\boxed{h = 9 \text{ m}} \quad (b)$$

w.r.t platform, max ball height will be  
for velocity of ball = velocity of platform.

$$V_{BP} = 8 \text{ m/s}$$

$$V_{B.P} = 0$$

$$a_{B.P} = -12 \text{ m/s}^2$$

$$S_{B.P} = ?$$

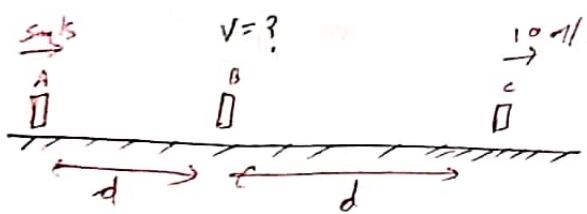
$$v^2 - u^2 = 2as$$

$$0 - 64 = 2x + 12 \times 4$$

$$\frac{64}{24} = h$$

$$\boxed{h = \frac{8}{3} \text{ m}} \quad (c)$$

Q



When A catches B, sep  
when B catches C, separation  
between A & C is 3d.  
find speed of B.

Time taken by B to catch C,  $t_1 = \frac{d}{v-10}$

A & C

$$10 - s = \frac{3d - 2d}{T_1} \rightarrow \text{change in separation}$$

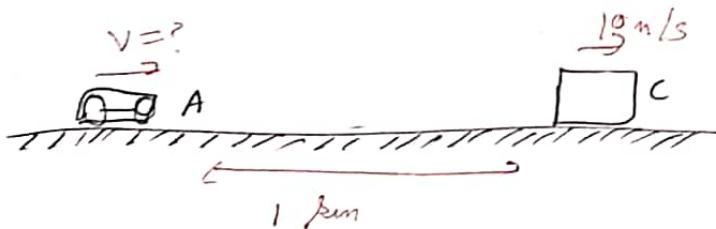
$$10 - s = \frac{d}{\frac{d}{v-10}}$$

$$10 - s = v - 10$$

$$s + 10 = v$$

$$\boxed{v = 15 \text{ m/s}}$$

Q

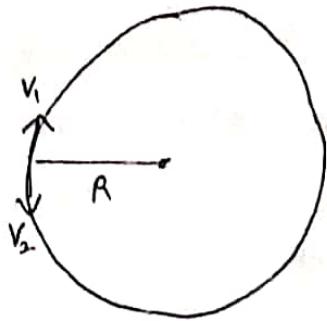


find the speed with which  
A should chase  
C to catch it in 100s.

$$(v - 10) \times 100 = 10^4$$

$$\boxed{v = 20 \text{ m/s}}$$

Q



after what time will they meet

$$\frac{2\pi R}{v_1 + v_2}$$

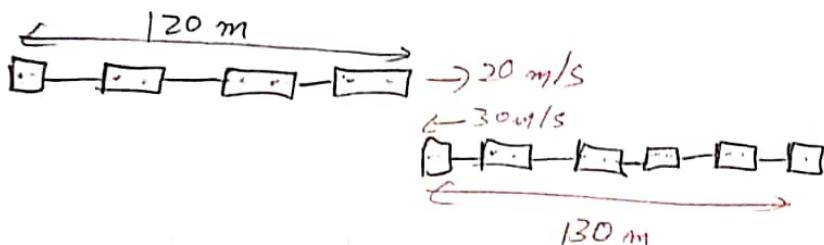
rel Velocity = ~~v1 + v2~~  $v_1 - (-v_2) = v_1 + v_2$

distance =  $2\pi R$  (circumference)

Time =  $\frac{\text{dis}}{\text{Time}}$

$$= \frac{2\pi R}{v_1 + v_2}$$

Q



$$v_1 = 20$$

$$v_2 = 30$$

$$\begin{aligned} \text{rel} &= 30 + 20 \\ &= 50 \end{aligned}$$

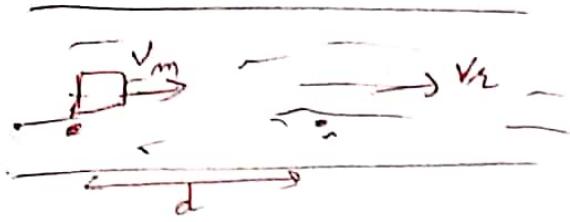
$$\begin{aligned} \text{distance} &= 120 + 130 \\ &= 250 \end{aligned}$$

$$\begin{aligned} \text{Time} &= \frac{\text{distance}}{\text{rel Speed}} \\ &= \frac{250}{50} \\ &= 5 \text{ s} \end{aligned}$$

(30)

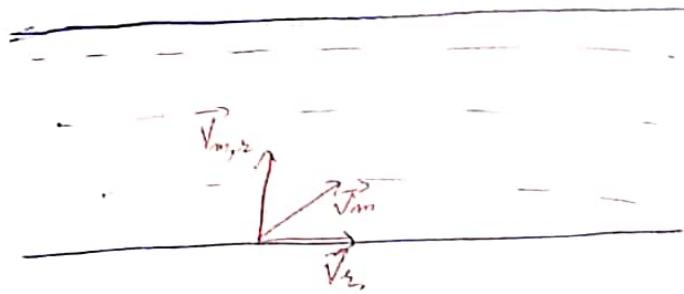
~~#~~ river man problem.

upstream: against the river flow  
downstream: with the river flow



$$T_{\text{down}} = \frac{d}{V_m + V_r} \quad (A \rightarrow B)$$

$$T_{\text{up}} = \frac{d}{V_m - V_r} \quad (B \rightarrow A)$$

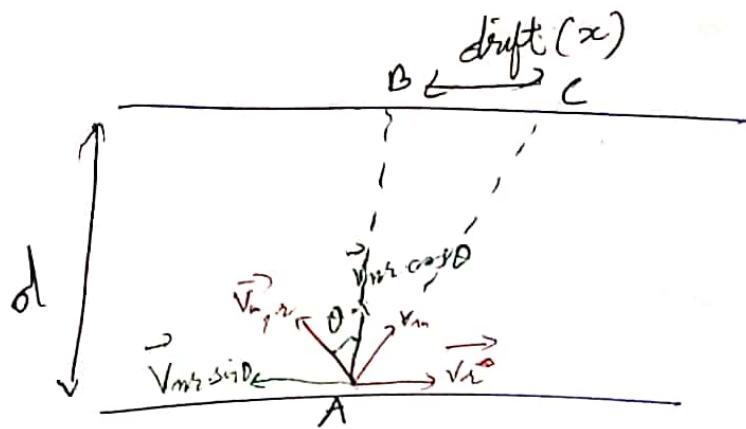


$\vec{V}_{m,r}$  = Velocity of man w.r.t river (man generates  $V$  in still water)

$\vec{V}_{r,r}$  = Velocity of flow of river

$\vec{V}_m$  = Net velocity of man

$$\vec{V}_m = \vec{V}_{m,r} + \vec{V}_r$$



$$\text{Time taken} = \boxed{\frac{d}{V_m \cos \theta}}$$

$$\text{drift}(x) = (V_x - V_m \sin \theta) \times t$$

$$x = \boxed{\frac{(V_x - V_m \sin \theta) d}{V_m \cos \theta}}$$

Case I :- For minimum time

$$\cos \theta = 1 \quad (\text{max})$$

$$\theta = 0^\circ$$

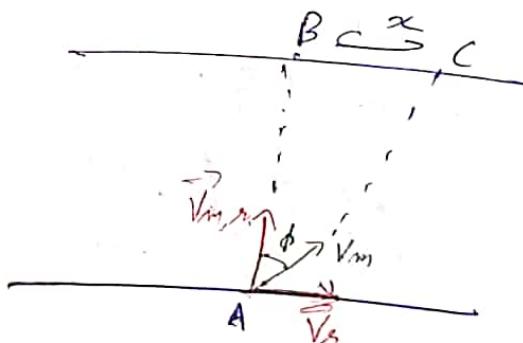
$$V_m = \sqrt{V_{m2}^2 + V_x^2}$$

now should switch to give floor

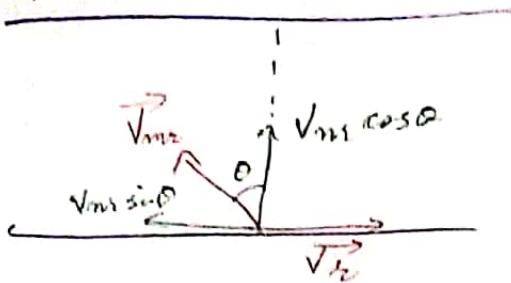
$$x = V_x t$$

$$x = \boxed{\frac{V_x d}{V_{m2}}}$$

$$\tan \phi = \boxed{\frac{V_x}{V_{m2}}}$$



case 2.



To cross river with zero drift.

$$V_n = V_m \sin \theta$$

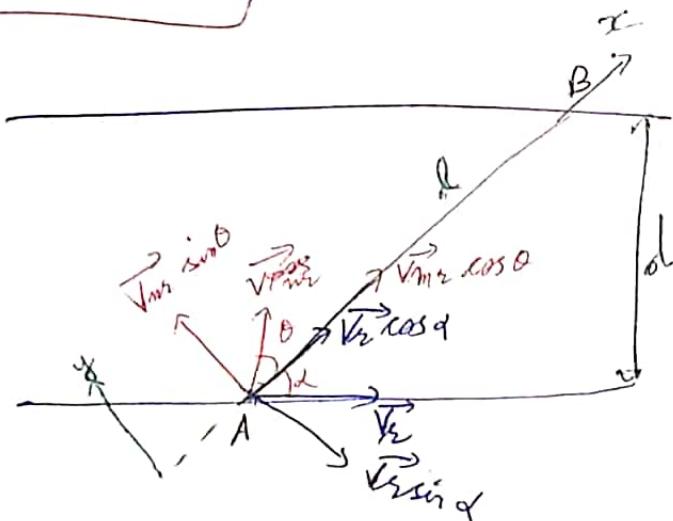
$$\sin \theta = \frac{V_n}{V_m}$$

Only valid till  $V_m > V_r$ .

$$\theta = \sin^{-1} \left[ \frac{V_n}{V_m} \right]$$

$$t = \frac{l}{V_m \cos \theta}$$

Q



$$V_m \sin \theta = V_r \sin \alpha$$

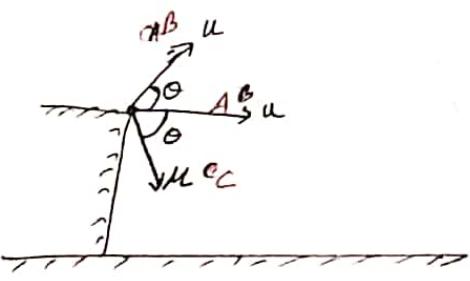
$$\sin \theta = \frac{V_r \sin \alpha}{V_m}$$

Angle with which boat will be steered.

$$t = \frac{l}{V_r \cos \alpha + V_m \cos \theta}$$

## Extra Questions (Ch-1, 2, 3)

Q1.



A)  $V_A > V_B > V_C$

B)  $V_A = V_B = V_C$

C)  $V_C > V_B > V_A$

D) cannot find.

$$V_{By} = \sqrt{2gh} = \sqrt{20h}$$

$$V_{Ay} = u \sin \theta + 10$$

$$V_C =$$

$$\overrightarrow{V}_{BA} = \sqrt{u^2 + \cancel{2gh}} = \sqrt{2gh}$$

$$V_B = \sqrt{u^2 \cos^2 \theta + u^2 \sin^2 \theta + 2gH} \\ = \sqrt{u^2 + 2gH}$$

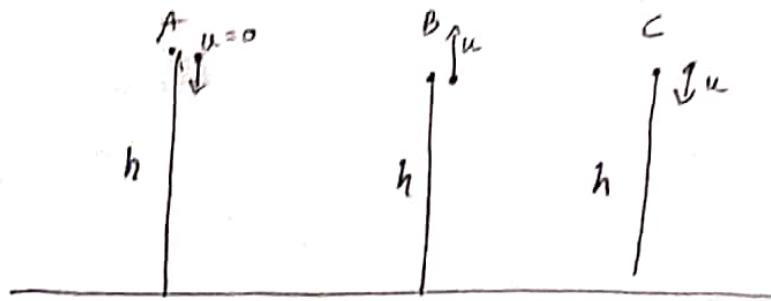
$$V_C = \sqrt{u^2 + \cancel{2gH}}$$

B)  $V_A = V_B = V_C$

(A)  ~~$V_B$~~

(34)

Q2.



a) velocity relation

$$v_A^2 = \cancel{u^2} - 2gH$$

$$v_B^2 = 2gH + u^2$$

$$v_C^2 = 2gH + u^2$$

A)  $v_C > v_B > v_A$

B)  $v_A < v_B < v_C$

C)  $v_A < v_B = v_C$

d) Note

Q3. If ~~zero~~ initial vel. of a particle projected from ground is  $\vec{u} = 6i + 8j$  m/s find the horizontal range

$$R = \frac{u \sin \theta}{g} \cdot \frac{2 u \cos \theta}{g} = \frac{2 \times 8 \times 6}{10} = \frac{96}{10} = 9.6 \text{ m}$$

~~Q4.~~ In the situation shown a player kicks the ball at  $45^\circ$  with  $20 \text{ m/s}$  vel. The distance of a  $3 \text{ m}$  high goal post is  $25 \text{ m}$  from the player. Find whether there will be a goal or not.

$$= 25 - \frac{\frac{100}{2} \times 62.5}{\frac{2 \times 20 \times 20 \times 1}{2}} =$$

at  $25 \text{ m}$ , ball's height will be  $9.3$  so there will be no goal.

$$= 25 - \frac{625}{40}$$

$$= \frac{1000 - 625}{40}$$

$$= \frac{375}{40}$$

$$= 9.3 \text{ something}$$

$$y = 9.3 \text{ something}$$

Q5. A particle is thrown from ground at angle  $30^\circ$  with horizontal at  $70 \text{ m/s}$  find time for which it will be at the height more than  $20 \text{ m}$

$$20 = 40 \times \frac{1}{2} t^2 - 5t^2$$

$$5t^2 - 20t + 20 = 0$$

$$t = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$\boxed{t = 2\Delta}$$

$$\boxed{\cancel{t = 0.13}}$$

$$\begin{array}{r} 20 \\ 40 \times 4 \\ \hline 2 \end{array} \times 10$$

Max height =  $20$   
time duration for which it will stay there =  $0.8$ .

$$0t^2 - 2t + 2 = 0$$

$$t = \frac{4 \pm 2\sqrt{2}}{2}$$

$$t = 2 \pm \sqrt{2}$$

$$2 + \sqrt{2} - 2 + \sqrt{2}$$

$$\boxed{2\sqrt{2}\Delta}$$

Q6. A thief is running away on a straight road at 9 m/s. A police man chases him on a motorcycle at 10 m/s if the distance between them is 100 m how long will it take for police to catch the thief.

$$V_t = 9$$

$$V_p = 10$$

$$\textcircled{Q} V_{rel} = \cancel{10 - 9} = 1 \text{ m/s}$$

$$\text{if } S_{rel} = 100 \text{ m}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Rel Speed}}$$

$$\text{Time} = \frac{100}{1}$$

$$\sqrt{100} \text{ s}$$

Q7. A bus moving at 10 m/s on straight road. A car lies 80 m behind it moving at 5 m/s wishes to overtake the bus in 100 s find acc required.

~~Distance~~

$$80 = -5 \text{ } \cancel{0} + \frac{1}{2} a \times 100^2$$

$$8 + 50 = a \times 500$$

$$a = \frac{58}{500}$$

$$a = \frac{58}{5 \times 100}$$

$$a = 11.6$$

$$a = 0.116 \text{ m/s}^2$$

$$\left\{ \begin{array}{l} V_{rel} = 5 - 10 \\ = -5 \text{ m/s} \end{array} \right.$$

$$S_{rel} = 80 \text{ m}$$

$$t = 100 \text{ s}$$

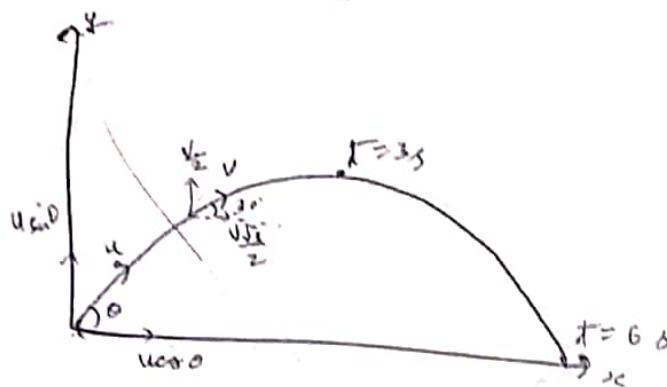
~~Distance~~

H.W.

- Q. A body is projected with velocity  $u$  at angle  $\theta$  from horizontal. The body makes angle  $30^\circ$  with horizontal at  $t = 2s$  & after 1s reaches max height.
- angle of projection
  - speed of projection.

$$v = u \sin \theta = 20$$

$$u \cos \theta = u \sin \theta \cdot \frac{\sqrt{3}}{2}$$



$$T = \frac{2u \sin \theta}{g}$$

$$\frac{6 \times 10}{2} = u \sin \theta$$

$$30 = u \sin \theta$$

$$\frac{y}{2} = u \sin \theta - gt$$

$$= 30 - 20$$

$$\frac{y}{2} = 10$$

$$y = 20$$

$$\frac{20\sqrt{3}}{2} = u \cos \theta$$

$$\frac{20\sqrt{3}}{10\sqrt{3}} = 30 \cot \theta$$

$$\frac{30}{10\sqrt{3}} = \tan \theta$$

$$\sqrt{3} = \tan \theta$$

$$\theta = 60^\circ$$

$$u = \frac{30}{\sin 60^\circ}$$

$$= \frac{30}{\sqrt{3}} \times 2$$

$$u = 20\sqrt{3} \text{ m/s}$$

Q1. A man wishes to cross a river in a boat. If he crosses the river in minimum time, he takes 10 min. with a drift 120 m. If he crosses the river taking shortest route he takes 12.5 min. find velocity of boat w.r.t water.

$$\theta = 0^\circ$$

$$d_{600} = \frac{d}{v_m \cos \theta}$$

$$120 = 600 u_r$$

$$u_r = \frac{120}{600} \text{ m/s}$$

$$u_r = \frac{1}{5} \text{ m/s}$$

~~$\sin \theta = \frac{v_m}{v_r}$~~

~~$\sin \theta = v_m / 5$~~

~~$\sin \theta = 5 v_m$~~

~~$750 = \frac{d}{v_m \cos \theta}$~~

~~$750 = \frac{13d}{\sqrt{2}}$~~

$$750 = \frac{d}{v_m \cos \theta}$$

~~$\frac{750}{600} = \frac{12.5}{v_m \cos \theta}$~~

$$\cos \theta = \frac{4}{5}$$

~~$750 = \frac{d}{v_m \cos \theta}$~~

~~$600 = \frac{d}{v_m}$~~

$$\theta = 37^\circ$$

$$v_m \times \frac{3}{5} = \frac{1}{5}$$

$$v_m = \frac{1}{3} \text{ m/s}$$

$$= \frac{1}{3} \times 60$$

$$= 20 \text{ m/min}$$

Q2. A man can swim at the rate of  $5 \text{ km/hr}$  in still water. A  $1 \text{ km}$  wide river flows at  $3 \text{ km/hr}$ . The man wishes to swim across the river directly opposite to starting point.

- along what direction should the man swim?
- What should be his resultant velocity?
- Find time taken to cross the river.

$$d = 1 \text{ km}$$

$$V_r = 3 \text{ km/hr}$$

$$V_{mr} = 5 \text{ km/hr}$$

$$\sqrt{V_{mr}^2 - V_r^2} = \sqrt{2}$$

$$\sin \theta = \frac{3}{5}$$

$$\theta = 37^\circ \quad \text{a)}$$

$$\boxed{V = 4 \text{ km/hr}} \quad \text{b)}$$

$$\boxed{\text{Time} = \frac{1}{4} \text{ hr}} \quad \text{c)}$$

Q3. A man swims  $\perp$  to river flow. His vel relative to water is  $4 \text{ m/s}$ , the river flow  $= 2 \text{ m/s}$ . width of river  $= 800 \text{ m}$

a) velocity relative to ground

b) time to cross river

c) drift.

$$V_{m_A} = 4$$

$$V_r = 2$$

$$d = 800$$

$$V_m = \sqrt{16 + 4} \\ = \sqrt{20} \text{ m/s} \quad \text{c)}$$

$$\text{Time} = \frac{800}{4}$$

$$= 200 \text{ s} \quad \text{b)}$$

$$\text{drift} = \frac{\text{distance}}{\text{Flow Speed}}$$

$$= \frac{800}{2} \text{ m}$$

$$= 400 \text{ m}$$

(40)

Q 4. A swimmer crosses a 200 m wide river & return 10 minutes at a point away from starting point (downstream). Find velocity of man with respect to ground if he heads towards the bank & at right angles all the times.

$$d = 200 \text{ m}$$

$$T = 300 \text{ s}$$

$$x = 150 \text{ m}$$

$$\theta = 90^\circ$$

$$V_{mr} = \frac{200}{300}$$

$$= \frac{2}{3} \text{ m/s}$$



$$\frac{225}{300} \frac{150}{300} = \frac{1}{2} = V_z$$

$$V_z = \frac{1}{2} \text{ m/s}$$

$$\sqrt{\frac{1}{4} + \frac{4}{9}}$$

$$\sqrt{\frac{9+16}{36}}$$

$$\sqrt{\frac{25}{36}}$$

$$\boxed{\frac{5}{6} \text{ m/s}}$$

Q 6.

Q6. A boat moves relative to water with velocity  $v$  which is  $n$  times less than river flow  $u$ . At what angle river flow must the boat move to minimize drifting?

$$V_{mr} = \frac{u}{n}$$

$$V_r = u$$

Time taken to cross river,  $t = \frac{d}{V_{mr} \cos \theta}$

$$\text{Drift, } x = (V_r - V_{mr} \sin \theta) t = \frac{d}{V_{mr} \cos \theta} (V_r - V_{mr} \sin \theta)$$

$$= \frac{d}{V_{mr}} \frac{(V_r - V_{mr} \sin \theta)}{\cos \theta}$$

$$\frac{dx}{d\theta} = 0$$

$$\cos \theta (0 - V_{mr} \frac{\cos \theta}{\cos \theta}) + (V_r - V_{mr} \sin \theta) (-\sin \theta) = 0$$

$$-V_{mr} \cos^2 \theta + V_r \sin \theta - V_{mr} \sin \theta \cos 2\theta = 0$$

$$V_r \sin \theta = V_{mr} (\sin^2 \theta + \cos^2 \theta)$$

$$\sin \theta = \frac{V_{mr}}{V_r}$$

$$\theta = \sin^{-1} \left( \frac{V_{mr}}{V_r} \right)$$

$$\theta = \sin^{-1} \left( \frac{u}{n u} \right) = \frac{1}{n}$$

$\theta = \sin^{-1} \left( \frac{1}{n} \right)$

$$\text{Angle with river flow} = \frac{\pi}{2} + \sin^{-1} \left( \frac{1}{n} \right)$$

Q7. Two swimmers leave point A on the bank of river to land B right across other bank. One of them crosses river along AB and other swims at right angles to river and then walks the drift distance to get to point B. find velocity  $u$  of his walking if both arrive at B at same time. ( $V_r = 2 \text{ km/h}$ ,  $V_{nr} = 2.5 \text{ km/h}$ )

$$\sin \theta = \frac{2.5}{2.5} = 1$$

$$\theta = 53^\circ$$

$$T = \frac{d}{2.5 \times \frac{3}{5}} = \frac{d}{2.5}$$

$$T_1 = \frac{2d}{3}$$

$$T_2 = \frac{d}{2.5}$$

$$R = 2 \times \frac{d}{2.5}$$

$$\text{drift} R = \frac{4d}{5}$$

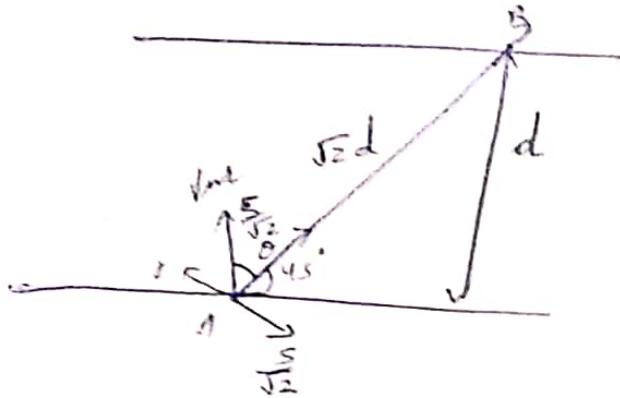
$$\begin{aligned}
 \text{time to come to B for 2nd swimmer} &= \cancel{\frac{d}{2.5}} - \cancel{\frac{2d}{3}} \\
 &= \cancel{\frac{3d}{7.5}} - \cancel{\frac{5d}{7.5}} \\
 &= \frac{2d}{3} - \frac{d}{2.5} \\
 &= \frac{5d - 3d}{7.5} \\
 &= \frac{2d}{7.5}
 \end{aligned}$$

$$\text{speed} = \frac{4d}{5} \times \frac{7.5}{2d}$$

$$= 1.5$$

$$\boxed{s = 3 \text{ km/h}}$$

Q8. A swimmer crosses a river along the line making an angle of  $45^\circ$  with the direction of flow. Velocity of river water is  $5 \text{ m/s}$ . Swimmer takes 6 s to cross the river of width 60 m. Find velocity of swimmer w.r.t. water.



$$\frac{\sqrt{2}d}{6} = \text{Speed}$$

~~Opposite speed~~

$$10\sqrt{2} - \frac{5}{\sqrt{2}}$$

$$\frac{20 - 5}{\sqrt{2}} = \left[ \frac{15}{\sqrt{2}} \text{ m/s} \right] = \text{Var 650}$$

$$V_{nr} \sin \theta = \frac{5}{\sqrt{2}}$$



$$V_{nr}^2 = \frac{25}{2} + \frac{225}{2}$$

$$V_{nr}^2 = 250$$

$$V_{nr} = \frac{5\sqrt{10}}{\sqrt{2}}$$

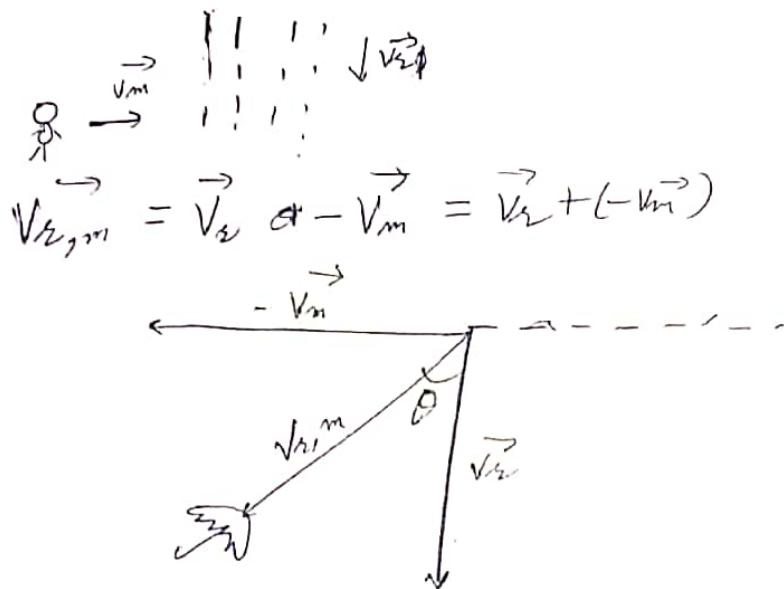
$V_{nr} = 5\sqrt{5} \text{ m/s}$

H. W.

Fig 72 Q 29-45

### Rain Man Problem

If rain is falling vertically with velocity  $\vec{V}_r$  and ~~the~~ observer is moving horizontally with velocity  $\vec{V}_m$ , then velocity of rain w.r.t. man,  $\vec{V}_{r,m} = \vec{V}_r - \vec{V}_m$

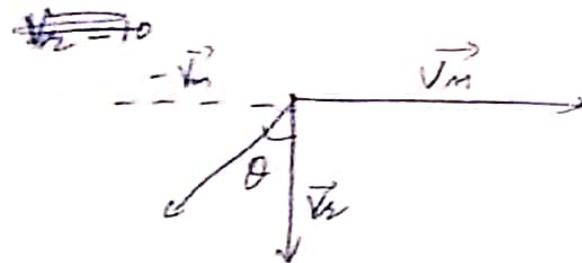


Angle at which umbrella has to be held.

$$\boxed{\tan \theta = \frac{V_m}{V_r}}$$

$$V_{r,m} = \sqrt{V_m^2 + V_r^2}$$

Q Rain falls vertically at speed of 10 m/s. A man walks with a speed of 6 m/s on a horizontal road. find angle at which man should hold his umbrella to avoid getting wet?



$$|\vec{V}_r| = 10 \text{ m/s}$$

$$\vec{V}_r = -10 \hat{j} \text{ m/s}$$

$$|\vec{V}_m| = 6 \text{ m/s}$$

$$\vec{V}_m = 6 \hat{i} \text{ m/s}$$

$$\vec{V}_{r,m} = \vec{V}_r - \vec{V}_m$$

$$= -10 \hat{j} - 6 \hat{i}$$

$$\tan \theta = \frac{6}{10} = \frac{3}{5}$$

$$\boxed{\theta = \tan^{-1}(\frac{3}{5})}$$

Q A man moving with 6 m/s observes rain falling at 12 m/s vertically. find direction & speed of rain w.r.t ground.

$$V_m = 6 \text{ m/s}$$

$$V_{r,m} = 12$$

$$\vec{V}_{r,m} = -12 \hat{j}$$

$$\vec{V}_m = 6 \hat{i}$$

$$\begin{aligned}\vec{V}_r &= \vec{V}_m + \vec{V}_{r,m} \\ &= 6 \hat{i} - 12 \hat{j}\end{aligned}$$

$$|\vec{V}_r| = \sqrt{36 + 144}$$

$$= \sqrt{144}$$

$$= 3\sqrt{20}$$

$$= 6\sqrt{5} \text{ m/s}$$

$$\cos \beta = \frac{-12}{6\sqrt{5}} \quad ; \quad \tan \beta = \frac{6}{12}$$

$$= \frac{-2}{\sqrt{5}} \quad ; \quad \tan \theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$$

Q3. Rain is falling with velocity of  $20 \text{ m/s}$  at an angle  $30^\circ$  with vertical. How fast should he move so that rain appears  $\perp$  to vertical to him?

$$\vec{V}_r = 20 \text{ m/s}$$

$$\vec{V}_r = 20 \sin 30^\circ \hat{i} - 20 \cos 30^\circ \hat{j}$$

$$= 20 \times \frac{1}{2} \hat{i} + 20 \times \frac{\sqrt{3}}{2} \hat{j}$$

$$= 10 \hat{i} + 10\sqrt{3} \hat{j}$$

If rain is vertical,

$$10 = V_m$$

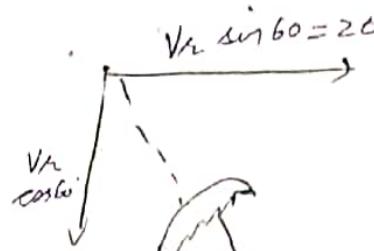
$$\boxed{V_m = 10 \text{ m/s}}$$

Q4. Nizhar is standing on road has to hold his umbrella at  $60^\circ$  with vertical. He throws his umbrella and starts moving at  $20 \text{ m/s}$ . He finds that rain drops are hitting his head vertically. Find speed of rain drops w.r.t (a) road, (b) Nizhar

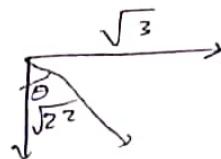
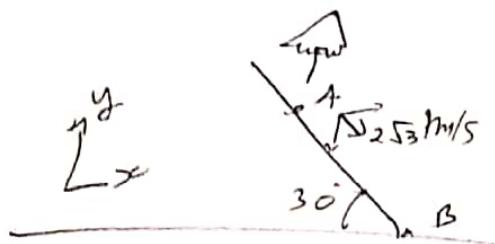
$$\vec{V}_r \sin 60 = \frac{V_r \sqrt{3}}{2} = 20$$

$$\boxed{\vec{V}_r = \frac{40}{\sqrt{3}} \hat{i}} \quad a)$$

$$V_r \cos 60 = \frac{40}{\sqrt{3}} \times \frac{1}{2} = \boxed{\frac{20}{\sqrt{3}}} \quad b)$$



Q The man is standing on a road with speed  $2\sqrt{3}$  m/s and keeps umbrella vertical. Actual speed of rain is 5 m/s. At what angle should he keep his umbrella with vertical when he reaches & stops at B



$$\begin{aligned}\vec{A} &= \frac{2\sqrt{3}}{2} \hat{i} - 2\sqrt{3} \times \frac{\sqrt{3}}{2} \hat{j} \\ &= \sqrt{3} \hat{i} - 3 \hat{j} \\ \vec{R} &= \sqrt{3} \hat{i} - \sqrt{2^2} \hat{j} \\ \tan \theta &= \frac{\sqrt{3}}{\sqrt{2^2}}\end{aligned}$$

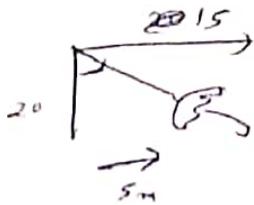
$$\vec{A} = 3 \hat{i} - \sqrt{3} \hat{j}$$

$$\vec{R} = 3 \hat{i} - 4 \hat{j}$$

$$\tan \theta = \frac{3}{4}$$

$$\begin{array}{c} \cancel{\theta = 30^\circ} \\ \cancel{\theta = 37^\circ} \\ \boxed{\theta = 37^\circ} \end{array}$$

Q Rain is falling with a speed 30 m/s. A Person running in rain with a velocity of 5 m/s & wind is also blowing with a speed of 10 m/s (both from west). The angle with the vertical at which the person should hold his umbrella so that he may not get drenched is.



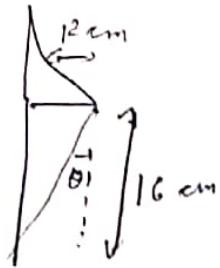
$$\begin{aligned}\vec{v}_{\text{rain}} &= -20 \hat{j} + 10 \hat{i} - 5 \hat{i} \\ &= -20 \hat{j} + 10 \hat{i}\end{aligned}$$

$$\tan \theta = \frac{10}{20}$$

$$= \frac{1}{2}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$$

Q A wearing a hat of extended length 12 cm is running in rain falling vertically with 10 m/s. find max speed with which man can run so that raindrops does not fall on his face. (length of face is 16 cm)



$$V_m (\text{vertical}) = 10 \text{ m/s} \\ = 1000 \text{ cm/s}$$

$$\tan \theta = \frac{12}{16}$$

$$\tan \theta = \frac{3}{4}$$

$$\theta = 37^\circ$$

$$\frac{V_m}{\sqrt{v_{\text{vertical}}}} = \frac{2}{4}$$

$$\frac{V_m}{1000} = \frac{3}{4} \\ V_m = \frac{1800}{4} \text{ cm/s}$$

$$\boxed{V_m = 7.5 \text{ m/s}}$$



Q

a



$$\tan \theta = \frac{\text{velocity}}{2}$$

~~$\theta = \tan^{-1}$~~

$$\frac{d\theta}{dt}$$

$$\frac{dv}{dt} = 2 \text{ m/s}^2, v = 2t$$

$$\frac{d\theta}{dv} \times \frac{dv}{dt} = \frac{d\theta}{dt}$$
$$= 2 \times \tan^{-1}\left(\frac{v}{2}\right)$$

$$\theta = \tan^{-1}\left(\frac{v}{2}\right)$$

$$\begin{aligned}\frac{d\theta}{dv} &= \sec^{-2}\left(\frac{v}{2}\right) \times \frac{1}{2} \\ &= \frac{\sec^{-2}\left(\frac{v}{2}\right)}{2}\end{aligned}$$

$$\begin{aligned}\frac{d\theta}{dt} &= \frac{d\theta}{dv} \times \frac{dv}{dt} \\ &= 2 \times \frac{\sec^{-2}\left(\frac{v}{2}\right)}{2}\end{aligned}$$

$$\boxed{\frac{d\theta}{dt} = \left[\sec^{-1}(t)\right]^2}$$

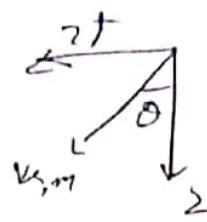
Q) rain falling vertically with  $2 \text{ m/s}$ . A boy starts accelerating at  $2 \text{ m/s}^2$  along straight road. Find rate of change of angle of umbrella?

$$V_R = -2 \text{ m/s}$$

$$\text{at } t, V_R = 2t \text{ m/s}$$

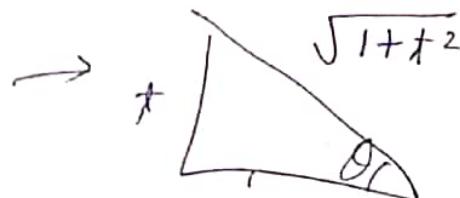
$$V_{RM} = V_R - V_m$$

$$V_{RM} = -2 + 2t \text{ m/s}$$



$$\tan \theta = \frac{2t}{2}$$

$$\tan \theta = t$$



$$\sec^2 \theta d\theta = dt$$

$$\frac{d\theta}{dt} = \frac{1}{\sec^2 \theta}$$

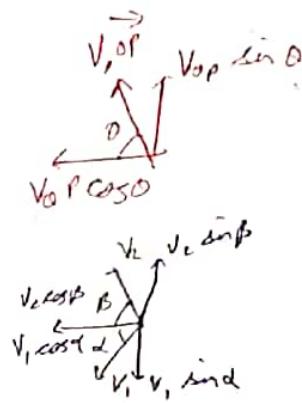
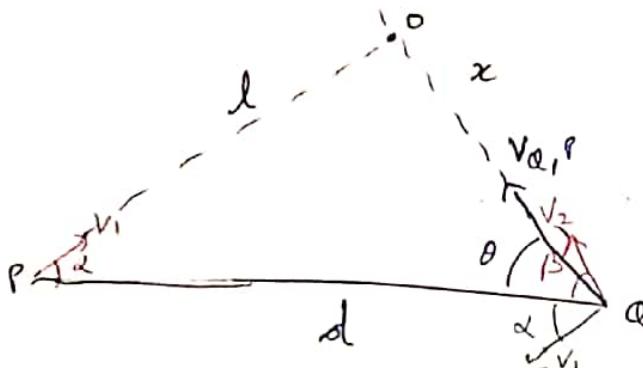
$$\frac{d\theta}{dt} = \cos^2 \theta$$

$$\frac{d\theta}{dt} = \frac{1}{\sqrt{1+t^2}}$$

$$\boxed{\frac{d\theta}{dt} = \frac{1}{1+t^2}}$$

## Shortest Distance Between two moving particles

(Distance of closest approach)



$$v_{0,p} = \vec{v}_0 - \vec{v}_p$$

$$\therefore \vec{v}_{0,p} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$$

$$v_{0,p} \cos \theta = v_1 \cos \alpha + v_2 \cos \beta \quad \dots \textcircled{1}$$

$$v_{0,p} \sin \theta = v_2 \sin \beta + v_1 \sin \alpha \quad \dots \textcircled{2}$$

$$\textcircled{1}^2 + \textcircled{2}^2$$

$$(v_{0,p})^2 = (v_1 \cos \alpha + v_2 \cos \beta)^2 + (v_2 \sin \beta + v_1 \sin \alpha)^2$$

$$v_{0,p} = ?$$

$$\textcircled{2} \div \textcircled{1}$$

$$\tan \theta = \frac{v_2 \sin \beta + v_1 \sin \alpha}{v_1 \cos \alpha + v_2 \cos \beta}$$

$$\theta = ?$$

$$\sin \theta = \frac{l}{d}$$

$$l = d \sin \theta \quad (\text{for particles to collide, } d=0)$$

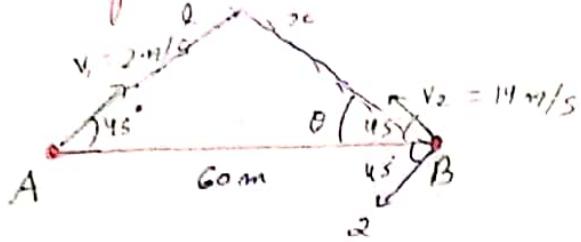
$$\cos \theta = \frac{x}{d}$$

$$x = d \cos \theta$$

$$\text{time} = \frac{dc}{v_{0,p}}$$

$$\boxed{\text{time} = \frac{d \cos \theta}{v_{0,p}}}$$

Q1. Motion of A & B takes place in horizontal plane



- a) find distance of closest approach  
b) At what time is the separation  
of 12 m minimum.

$$\begin{aligned} V \cos \theta &= \cos 45 \times 2 + \cos 45 \times 14 \\ &= \frac{2}{\sqrt{2}} + \frac{14}{\sqrt{2}} \\ &= \frac{16}{\sqrt{2}} \\ &= 8\sqrt{2} \end{aligned}$$

$$V \sin \theta = \sin 45 \times 2 + \sin 45 \times 14$$

$$= 6\sqrt{2}$$

$$\tan \theta = \frac{6\sqrt{2}}{8\sqrt{2}} = \frac{3}{4}$$

$$\boxed{\theta = 45^\circ} \quad \boxed{\theta = 37^\circ}$$

$$\text{Q } \sin \theta = \frac{l}{60} \quad \sin 37 = \frac{l}{60}$$

$$\frac{1}{\sqrt{2}} \times \frac{l}{60} = l$$

$$30\sqrt{2} = l$$

$$\frac{3}{5} \times \frac{60}{\sin 37} = l$$

$$36 = l$$

$$\begin{aligned} \frac{V}{\sqrt{2}} &= 8\sqrt{2} \\ V &= 8 \times 2 \\ \boxed{V = 16 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} \frac{4V}{5} &= 8\sqrt{2} \\ V &= \frac{5 \times 8\sqrt{2}}{4} \\ \boxed{V = 10\sqrt{2}} \end{aligned}$$

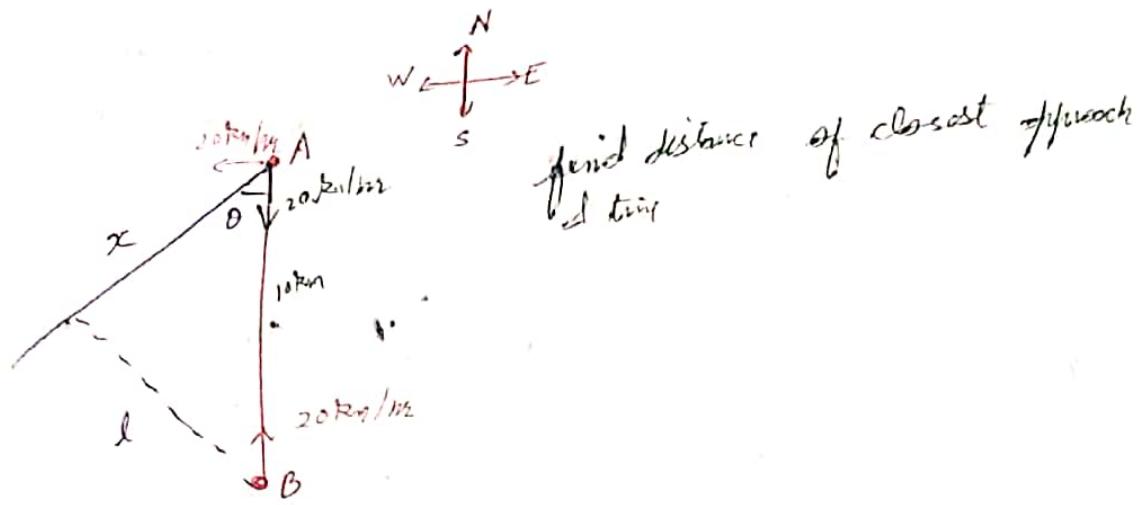
$$\cos 37 = \frac{x}{60}$$

$$30\sqrt{2} = x$$

$$\frac{4 \times 60}{5} = x$$

$$\begin{aligned} \text{Time} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{30\sqrt{2}}{16} \\ \text{Time} &= \frac{48}{10\sqrt{2}} \\ &= \frac{48\sqrt{2}}{20} \\ &= \frac{24\sqrt{2}}{10} \\ \boxed{= \frac{12\sqrt{2}}{5}} \end{aligned}$$

Q 2.



$$\theta = 45^\circ$$

$$V = \sqrt{400 + 400} = 20\sqrt{2}$$

$$\sin 45^\circ \approx \frac{l}{10}$$

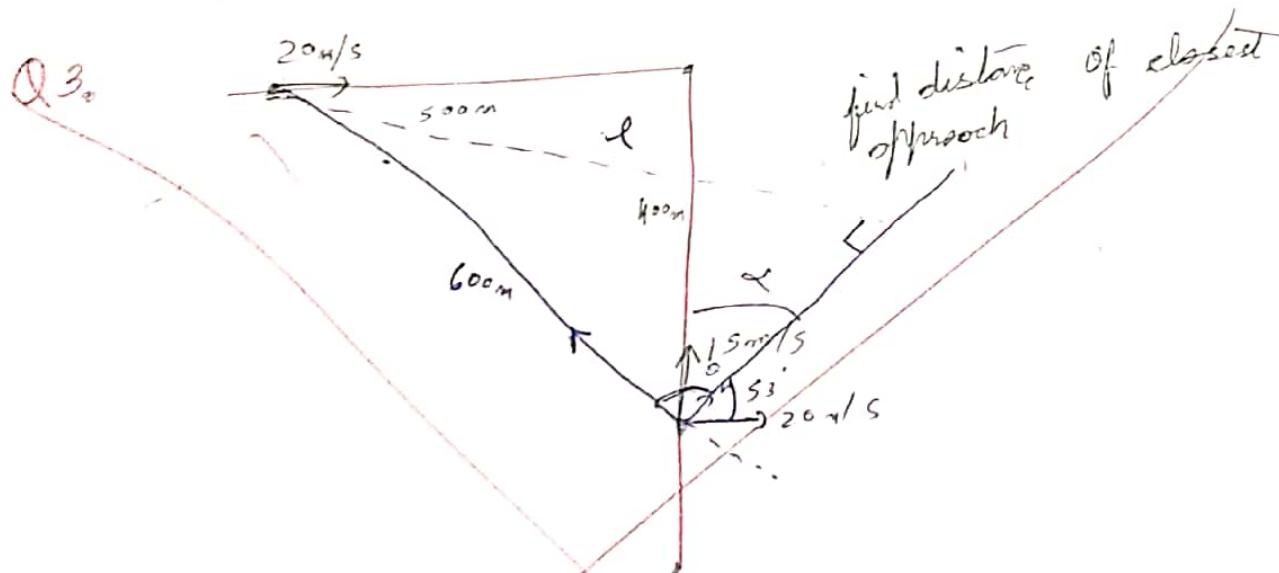
$$\frac{1}{\sqrt{2}} = \frac{l}{10} \Rightarrow l = 5\sqrt{2}$$

$$\cos 45^\circ = \frac{l}{10}$$

$$\sqrt{552} = x$$

$$\text{time} = \frac{s\sqrt{2}}{20\sqrt{2}} = \frac{1}{4} \text{ hr}$$

$$= 15 \text{ min}$$



$$V = \sqrt{400 + 225} = \sqrt{625} = 25 \text{ m/s}$$

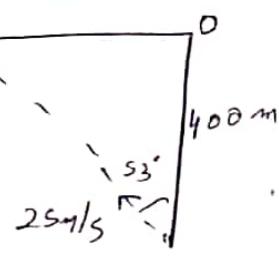
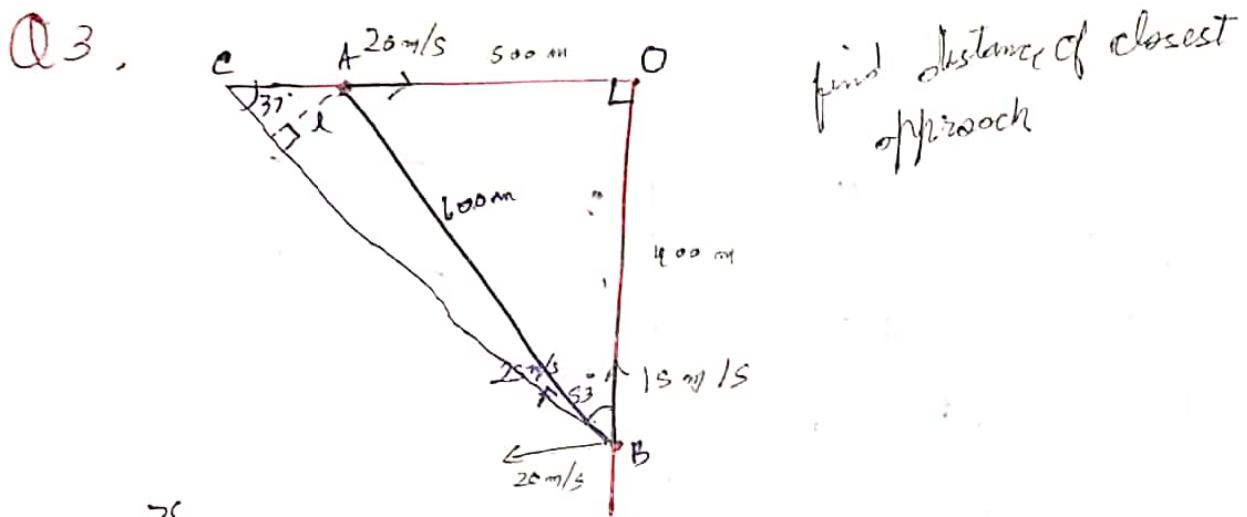
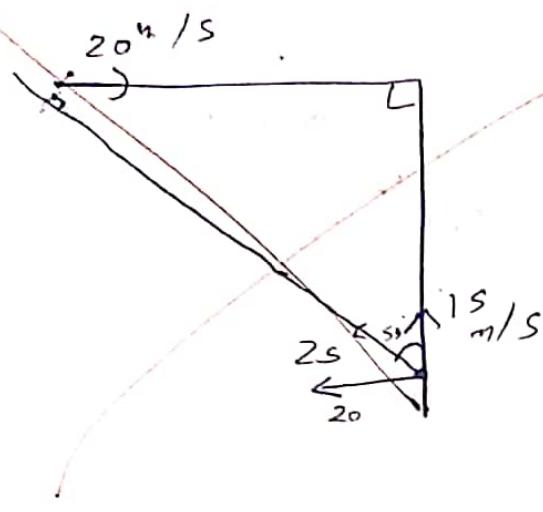
~~$$\theta = 37^\circ$$~~

$$15 \sin 37^\circ = 18 \sin 53^\circ$$

$$18 \sin 53^\circ = \frac{4}{5} \times 20$$

$$q =$$

(54)



$$\tan 53^\circ = \frac{x}{400}$$

$$\frac{4}{3} = \frac{x}{400}$$

$$\boxed{\frac{1600}{3} = x}$$

$$CA = \frac{1600 - 1500}{3} \\ = \frac{100}{3}$$

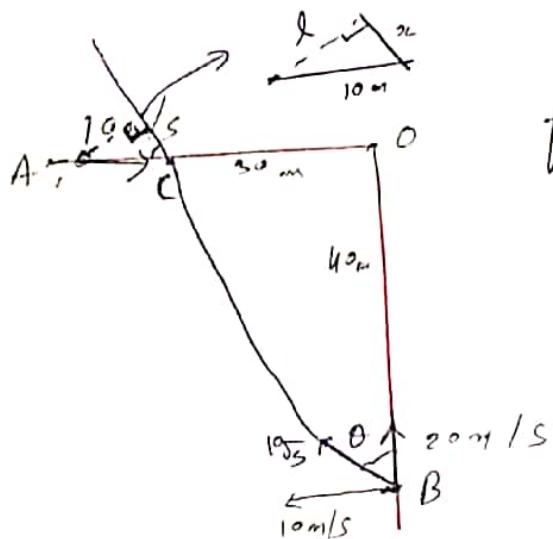
$$\sin 37^\circ = \frac{100}{l} \times \frac{3}{100}$$

$$\frac{100}{3} \times \frac{3}{5} = l = 20 \text{ m}$$



$$\frac{4}{5} = \frac{x}{400} \Rightarrow x = \frac{4}{5} \times 400$$

Q

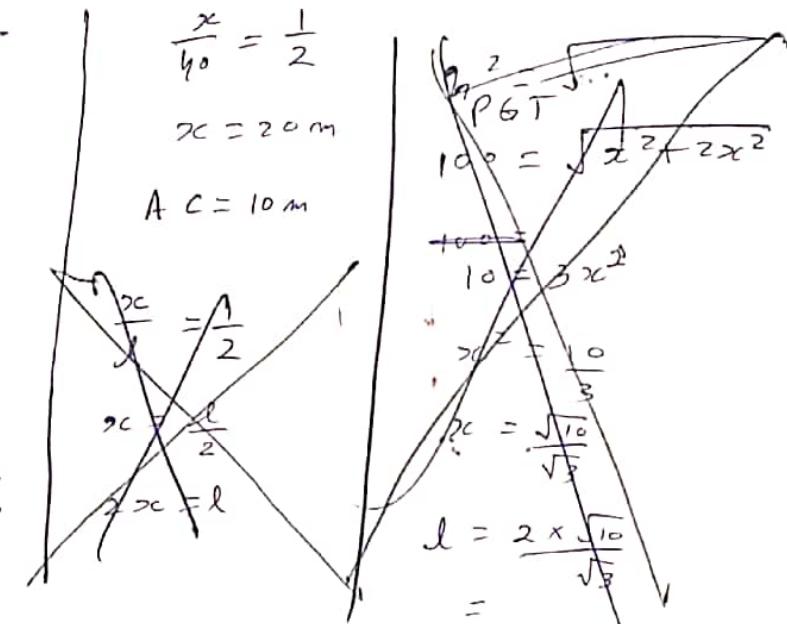


find distance of closest approach

$$\begin{aligned} V_A &= \sqrt{400 + 100} \\ &= \sqrt{500} \\ &= 10\sqrt{5} \text{ m/s} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{10}{20} \\ &= \frac{1}{2} \\ \cos \theta &= \frac{2}{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \frac{x}{40} &= \frac{1}{2} \\ x &= 20 \text{ m} \\ AC &= 10 \text{ m} \end{aligned}$$



$$\frac{1}{\sqrt{5}} = \frac{l}{10}$$

$$2\sqrt{5} = l$$

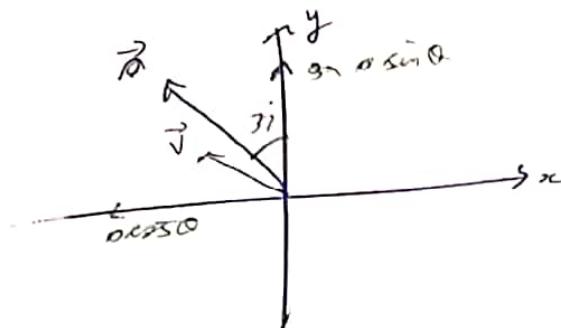
$$\frac{10 \times 2}{\sqrt{5}} = l$$

$$l = 4\sqrt{5}$$

60

## # General 2-D motion directions

Q. A Particle with initial velocity  $\vec{V}_0 = (-2\hat{i} + 4\hat{j})$  undergoes constant acceleration of  $3 \text{ m/s}^2$  at  $\theta = 12^\circ$  forward  $x$ - axis, find  $\vec{v}$  at  $t = 5\text{s}$



$$\vec{v} = 3 \times \frac{3}{5} \hat{x} + 3 \times \frac{4}{5} \hat{y}$$

$$= \frac{9}{5} \hat{x} + \frac{12}{5} \hat{y}$$

$$\vec{v} = -2\hat{x} + 4\hat{y}$$

in  $x$  axis,

$$v = -2$$

$$a = -\frac{9}{5}$$

$$t = 5$$

$$v = -2 + -\frac{9}{5} \times 5$$

$$= -2 - 9$$

$$\boxed{v = -11 \text{ m/s}}$$

in  $y$  axis

$$v = 4$$

$$a = \frac{12}{5}$$

$$v = 4 + \frac{12}{5} \times 5$$

$$v = 12 + 4$$

$$\boxed{v = 16 \text{ m/s}}$$

find  $\boxed{\vec{v} = -11\hat{x} + 16\hat{y}}$









