!! Logarithm !!

→ Every ⊕ve real number N can be expressed in exponential form as.

$$a^x = N$$

$a \to$ ⊕ve real ~~no~~ number > 0 but $\neq 1$

$x \to$ Exponent

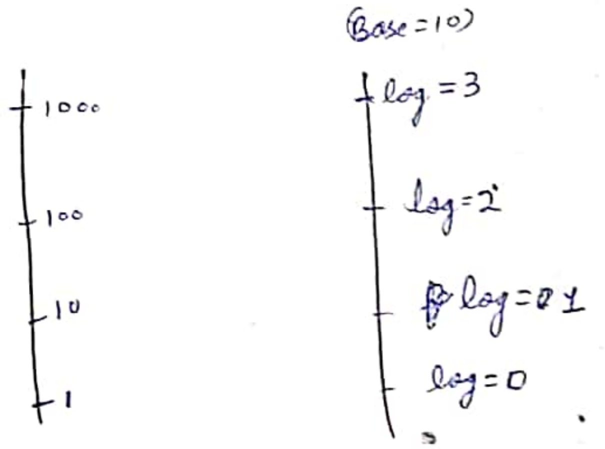
eg. $2^2 = 4$, $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ etc

Reason I

$2^{y} = 4$, $\left(\frac{1}{2}\right)^y = \frac{1}{16}$ $(8 \cdot 2)^y = 16 \cdot 32$

To find y.

Reason II



For the above reasons we introduced log. It is expressed as

$$\log_a N = x$$

$a \to$ Base
$x \to$ exponent

[a की Power को कितना Raise करें की N आ जाये.]

eg. $\log_2 8 = 3$ [2 की Power को कितने Raise करें की 8 aaye]

$\log_6 216 = 3$

$\log_2 \frac{1}{16} = -4$

$\log_{0.6}\left(\frac{25}{9}\right) =$

$0.6 = \frac{6}{10}$

$= \frac{3}{5}$

$= \left(\frac{3}{5}\right)^{-2}$

$= \left(\frac{5}{3}\right)^{2}$

$= \frac{25}{9}$

$$\boxed{\log_{0.6}\left(\frac{25}{9}\right) = -2}$$

$\log_{\frac{1}{2}}(1) = 0$

$\log_{\frac{1}{3}} 27 = -3$

$\log_1\left(\frac{1}{2}\right) =$ Not Defined

$\log_{(-2)}(16) = 4$ (Wrong)

$16^{1/4} = \sqrt{4} = |\pm 2| = 2 \neq -2$

Thus Base is $\oplus$ve

$\neq 1$

$\log_a N = x$

$a > 0$ $\quad$ $a \neq 1$ $\quad$ $N > 0$

Q Conversion of Logarithm form in exponential form.

① $\log_2 32 = 5 \longrightarrow 2^5 = 32$

② $\log_{36} 6 = \frac{1}{2} \longrightarrow 36^{1/2} = 6$

③ $\log_8 1 = 0 \longrightarrow 8^0 = 1$

④ $\log_{10}(0.001) = -3 \longrightarrow 10^{-3} = 0.001$

Q find the value of x if $\log_5 125 = x$

$5^x = 125$

$5^x = (5)^3$

$\boxed{x = 3}$

Q $\log_2 m = 1.5$

$2^{1.5} = m$

$2^{3/2} = m$

$\sqrt{2^3} = m$

$\sqrt{8} = m$

$\boxed{m = 2\sqrt{2}}$

Note - For same number different bases gives different answers.

Q Find log

(1) 32 (base $\frac{1}{2}$)

$\log_{1/2} 32 = x$

$\left(\frac{1}{2}\right)^x = 32$

$\frac{1}{2^x} = 32$

$\frac{1}{2^x} = 2^5$

$2^{-x} = 2^5$

$-x = 5$

$\boxed{x = -5}$

(2) 32 (base 2)

$\log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$\boxed{x = 5}$

(3) $3\sqrt{3}$ (base 3)

$\log_3 3\sqrt{3} = x$

$3^x = 3\sqrt{3}$

$3^x = \sqrt{27}$

$3^x = \cancel{27}[(3)^3]^{1/2}$

$3^x = 3^{3/2}$

$\boxed{x = 3/2}$

(4) $3\sqrt{3}$ (base $\frac{1}{3}$)

$\log_{1/3} 3\sqrt{3} = x$

$\left(\frac{1}{3}\right)^x = 3\sqrt{3}$

$3^{-x} = 3^{3/2}$

$-x = \frac{3}{2}$

$\boxed{x = -3/2}$

Note :-

① $\log_a 1 = 0 \quad (a > 0, a \neq 1)$

② $\log_N N = 1$

③ $\log_{\frac{1}{N}} N = -1$ or $\log_N \frac{1}{N} = -1 \quad (N \geq 0, N \neq 1)$

Q find value of

① $\log_{2+\sqrt{3}} (2-\sqrt{3}) = x$

$$(2+\sqrt{3})^x = 2-\sqrt{3}$$

$$(2+\sqrt{3})^x = \frac{(2-\sqrt{3})}{2+\sqrt{3}} \times (2+\sqrt{3})$$

$$(2+\sqrt{3})^x = \frac{\cancel{(}2^2 - \sqrt{3}\cancel{)}^2}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = \cancel{} \frac{\cancel{}\ 4-3}{2+\sqrt{3}}$$

$$(2+\sqrt{3})^x = (2+\sqrt{3})^{-1}$$

$$\boxed{x = -1}$$

② $\log_{(1+\sqrt{2})} (\cancel{a}\sqrt{3+2\sqrt{2}}) = x$

$$(1+\sqrt{2})^x = (\sqrt{2}+1)^1$$

$$\boxed{x = 1}$$

H.W. 03-06-2024

DYS-1 {Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Principle Properties of log $\{a>0, \neq 1 \,;\, m \,\&\, n > 0\}$

① $\log_a (mn) = \log_a m + \log_a n$ (can put more than two terms)

② $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

③ $\log_a (m^n) = n \log_a m$

Proof :-

① $\log_a (mn) = \log_a m + \log_a n$

let $\log_a m = x$, $\log_a n = y$

$\downarrow$ $\quad\quad$ $\downarrow$

$a^x = m$ $\quad\quad$ $a^y = n$

$\therefore mn = a^x \times a^y$

$= a^{x+y}$

$\log_a (mn) = x+y$

$= \log_a m + \log_a n$

③ $\log_a m^n = n \log_a m$

LHS

$\log_a (\underbrace{m \times m \times m \ldots}_{n \text{ times}})$

$= \log_a m + \log_a m + \ldots n \text{ times}$

$= n \log_a m = \text{RHS}$

Hence Proved

Note -

① $\log_{1/2} \log_4 2$

$\log_4 2 = \frac{1}{2}$

$\log_{1/2}\left(\frac{1}{2}\right) = x$

$\boxed{x = 1}$

② $\log_a N \cdot \log_b M$

$\boxed{\log_a N \times \log_b M}$

③ $\log_a N \cdot 3$

$\log_a N \times 3$

$3\log_a N$

$\boxed{\log_a N^3}$

~~Q log 3 + log (Base 1~~

Q Find value (Base 10)

① $\log 3 + \log 5$

$\log_{10}(3 \times 5)$

$\boxed{\log_{10} 15}$

② $\log 6 - \log 2$

$\log\left(\frac{6}{2}\right)$

$\log_{10} 3$

(3) $3\log 4$

$\log (4^3)$

$\cancel{\log (64)}$

$\log_{10} 64$

(4) $\log 2\ 36 - \log 1$

$\log_{10} 236$

(5) $\cancel{2\log}$ $2\log 3 - 3.\log 2$

$\log 3^2 - \cancel{3}\log 2^3$

$\log 9 - \log 8$

$\log_{10} \left(\frac{9}{8}\right)$

(6) $\log 2 + \log 3 + \log 4$

$\log (2 \times 3 \times 4)$

$- \log (12 \times 2)$

$\log_{10} (24)$

(7) $5\log 10 + 2\log 3 - \log 2$

$\log 10^5 + 2\log 3^2 - \log 2$

$\log 10^5 + \log 9 - \log 2$

$\log 10^5 + \log \frac{9}{2}$

$\log_{10} (4.5 \times 10^5)$

Properties

(4) fundamental log. Identity

$$\boxed{a^{(\log_a N)} = N}$$

→ in power

Proof :- $a^{(\log_a N)} = N$

Taking $\log_a$ both sides

$$\log_a \left(a^{\log_a N}\right) = \log_a N$$

$$\log_a N \cdot \log_a a = \log_a N$$

$$\cancel{\log_a N} = \cancel{\log_a N}$$

$$N = N$$

Hence, Proved

(5) Base Changing Theorem

$$\boxed{\frac{\log_a m}{\log_a n} = \log_n m}$$

Proof :-

$\log_n m = P \Leftrightarrow n^P = m$

$\log_a m = q \Leftrightarrow a^q = m$

$\log_a n = r \Leftrightarrow a^r = n$

$n^P = a^q$

$(a^r)^P = a^q$

$a^q = a^{rP}$

$q = rP$

$\frac{q}{r} = P$

$$\frac{\log_a m}{\log_a n} = \log_n m$$

⑥ DDF

$$\boxed{a^{\log_b c} = \cancel{a}\, c^{\log_b a}}$$

Proof

$$\log_b c = x \Leftrightarrow b^x = c$$

L.H.S

a^x

RHS

$c^{\log_b a}$

$\cancel{b}^{x \log_b a}$

$\cancel{b}^{\log_b a^x}$

a^x

LHS = RHS

Hence, Proved.

⑦ Base - Power Theorm

$$\boxed{\log_{a^n} b^m = \frac{m}{n} \log_a b}$$

Q find value (Base 10)

1. $2^{\log_2 5}$

$\boxed{5}$

2. $\pi^{\log_\pi 60}$

$\boxed{60}$

3. $25^{\log_5 8}$

$5^{2\log_5 8}$

$5^{\log_5 8^2}$

8^2

$\boxed{64}$

4. $\left(\frac{1}{16}\right)^{\log_2 2}$

$2^{\log_2 \pi^{-4}}$

$\boxed{\pi^{-4}}$

5. $\cancel{\log_4 8}$ $\log_{64} 8$

$\log_{2^6} 2^3$

$\frac{3}{6}\log_2 2$

$\frac{3}{6}$

$\boxed{\frac{1}{2}}$

6. $\log_3 2 \times \log_4 3 \times \log_5 4$

$\log_4 3^{\log_3 2} \times \log_5 4$

$\log_5 4^{\log_4 3^{\log_3 2}}$

$\log_5 2$

7. $4^{\log_3 7} - 7^{\log_3 4}$

8. $2^{\log_3 5} + 3^{\log_7 6} - 5^{\log_3 2} - 6^{\log_7 3}$

9. $\log_2\left[\log_2\left\{\log_3\left(\log_3 27^3\right)\right\}\right]$

10. $\log(\tan 1^\circ) . \log(\tan 2^\circ) \ldots\ldots \log(\tan 89^\circ)$

11. $7^{\log_7 x^2} + x - 2 = 0$

12. $\log(\sin 1^\circ) . \log(\sin 2^\circ) \ldots \log(\sin 90^\circ)$

13. $\log_{10}\left\{\left(\sqrt{a^{-2} b}\right)\left(\sqrt[3]{a b^{-3}}\right)\right\}$

14. $\log_2 3 . \log_3 4 . \log_4 5 \ldots \log_n (n+1) = 5$

15. Prove: $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$

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Q7 $4^{\log_3 7} - 7^{\log_3 4}$

$4^{\log_3 7} = 7^{\log_3 4}$

$\boxed{0}$

Q8. $2^{\log_3 5} = 5^{\log_3 2}$

$3^{\log_7 6} = 6^{\log_7 3}$

$\boxed{0}$

Q9. $\log_3 27^3 = 9$

$\log_3 9 = 2$

$\log_2 2 = 1$

$\log_2 1 = 0$

$\boxed{0}$

Q11. $7^{\log_7 x^2} + x - 2 = 0$

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$\boxed{x = 1, -2}$

13) $\cancel{10^x = (\sqrt{a^{-2}.b})(\sqrt[3]{ab^{-3}})}$

$\cancel{10^x = a^{-1}.b^{1/2}.a^{1/3}.b^{-1}}$

$\cancel{10^x = a^{-2/3}.b^{-1/2}}$

13) $10^x = a^{-2/3}.b^{-1/2}$

$\log_{10}(a^{-2/3}) + \log_{10}(b^{-1/2})$

$\boxed{\frac{-2}{3}\log_{10} a - \frac{1}{2}\log_{10} b}$

10) It includes $\tan 45°$,

$\log(\tan 45°)$

$= \log(1)$

$\boxed{= 0}$

12) $\lg(\sin 90°)$

$\log(1)$

$\boxed{0}$

14) $\frac{\log_a 3}{\log_a 2} \times \frac{\log_a 4}{\log_a 3} \cdots$

$\frac{\log_a 4}{\log_a 2} \cdots$

$\frac{\log_a n+1}{\log_a 2} = \boxed{\log_2 (n+1)}$

~~Q15. LHS~~

~~$2^{\sqrt{\log \frac{3}{2}}}$~~

$$\log_2 (n+1) = 5$$

$$2^5 = n+1$$

$$3^2 = n+1$$

$$\boxed{n = 31}$$

~~Q15~~

~~LHS~~ ~~RHS~~

~~$2^{\sqrt{\log \frac{3}{2}}}$~~ ~~$3^{\sqrt{\log_3 2}}$~~

~~$3^{1/2}$~~ ~~$2^{1/2}$~~

~~$\sqrt{3}$~~ ~~$\sqrt{}$~~

H.W.

DYS-2 {1, 2, 3, 4}

DYS-3 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

O-1 {1, 2, 3, 4, 5, 6}

Q15. $\cancel{\log_2 a = \sqrt{\log_2 3}}$

$\cancel{a = 2^{\sqrt{\log_2 3}}}$

$\cancel{\log_2 2^{\sqrt{\log_2 3}}}$

$\cancel{\sqrt{\log_2 3} \times \log_2 2}$

Q15. $a = 2^{\sqrt{\log_2 3}}$ $\qquad$ $b = 3^{\sqrt{\log_3 2}}$

$= \log_2 b$

$= \log_2 3^{\sqrt{\log_3 2}}$

$= \sqrt{\log_3 2} \times \log_2 3$

$\cancel{= \dfrac{\sqrt{\log_2 2}}{\sqrt{\log_2 3}} = \log_2 3}$

$\log_3 2 = x$

$\dfrac{1}{\log_2 3} = \dfrac{1}{x}$

$3^x = 2$

$\cancel{3 = \sqrt{2}}$ $\quad 3 = 2^{1/x}$

$\dfrac{1}{\log_3 2} = \log_2 3$

$\dfrac{1}{\sqrt{\log_2 3}} \times \log_2 3$

$\sqrt{\log_2 3} = \log_2 b$

$2^{\sqrt{\log_2 3}} = b$

$\boxed{a = b}$

Hence, Proved

(88)

Antilog

$$\boxed{\text{Antilog}_a x = a^x}$$

$\log_a N = x$

$\text{Antilog}_a (\log_a N) = \text{Antilog}_a x$

DYS - 3

Q13 $\text{Antilog}_{64}\left(\frac{5}{6}\right)$

$(64)^{5/6}$

$(2)^5$

$\boxed{32}$

~~Illustrati~~ Illustration - 8

$\left(\log_b a \cdot \log_c a - \log_a a\right) + \left(\log_a b \times \log_c b - \log_b b\right) + \left(\log_a c \cdot \log_b c - \log_c c\right) = 0$

$\frac{\log a}{\log b} \times \frac{\log a}{\log c} + \frac{\log b \log b}{\log a \log c} + \frac{\log c \log c}{\log a \log b} - 3 = 0$

$(\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a \log b \log c$

$\log a + \log b + \log c = 0$

$\log (abc) = 0$

$\boxed{abc = 1}$

Illustration 10

$\log_4 18 = x$

$4^x = 18$

$2^{2x} = 9^2$

thus irrational.

Q $2\log_2(\log_2 x) + \log_{1/2}\left(\frac{3}{2} + \log_2 x\right) = 1$

$\log_2 \left((\log_2 x)^2 \times \frac{1}{\log_4 x^3}\right) = 1$

$\log_2 x \times \cancel{\log_2 x} \times \frac{1}{\frac{3}{2}\cancel{\log_2 x}} = 2$

$\log_2 x = 2 \times \frac{3}{2}$

$\log_2 x = 3$

$2^3 = x$

$\boxed{8 = x}$

Q 3. $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}} \cdots \infty}}\right)$

$x = \sqrt{4 - \frac{x}{3\sqrt{2}}}$

$x^2 = 4 - \frac{x}{3\sqrt{2}}$

$3\sqrt{2}x^2 = 12\sqrt{2} - x$

$3\sqrt{2}x^2 + x - 12\sqrt{2} = 0$

$x = -1 \pm \sqrt{1 + 288}$

$x = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$x = \frac{16}{6\sqrt{2}}$

$6 + \log_{3/2} \frac{4}{9}$

$6 - 2$

$\boxed{4}$

$6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{16}{3\sqrt{2}}\right)$

$6 + \log_{3/2} \frac{8}{9}^{4} \times 2$

$\left(\frac{3}{2}\right)^x = \frac{8}{9}$

$\left(\frac{2}{3}\right)^{-x} = \frac{8}{9}$

$6 - 2 + \log_{1/2} 2$

$4 + \sqrt{8}$

$\boxed{4 + 2\sqrt{2}}$

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Scanned with CamScanner

Q.20

Q. $2\log_2(\log_2 x) + \log_{1/2}\left(\frac{3}{2} + \log_2 x\right) = 1$

$\log_2 t^2 + \log_2\left(\frac{1}{\frac{3}{2}+t}\right) = 1$

$$\frac{t^2}{\frac{3+2t}{2}} = 1$$

$$\frac{2t^2}{3+2t} = 1$$

$2t^2 = 3 + 2t$

$2t^2 - 2t - 3 = 0$

~~$t = 2 \pm \sqrt{4+24}$~~

$t = 3, -1$

$\log_2 x = 3$ $\qquad$ $\log_2 x = -1$ ✗

$\boxed{x=8}$

$\boxed{x=8}$

H.W

J.A - {5, 6}

DYS-4 [4, 10]

Q

① $\cancel{2\log_2}\ 2^{\log_2 x^2} - 3x - 4 = 0$

$2^{\log_2 x^2} = 3x + 4$

$\log_2 3x+4 = \log_2 x^2$

$3x + 4 = x^2$

$x^2 - 3x - 4 = 0$

$x = 4, -1$

$\boxed{x = 4, -1}$

② $2^{2\log_2 x} - 3x - 4 = 0$

$x = 4, -1$

-1 is rejected as ⊖ve not possible in log

③ $\log_2 (x^2 - 1) = 3$

$8 = x^2 - 1$

$9 = x^2$

$x = \pm 3$

④ $\log_2 (x+1) + \cancel{2}\ \log_2 (x-1) = 3$

$\log_2 (x+1)(x-1) = 3$

$8 = x^2 - 1$

$x^2 + 9$

$x = \pm 3$

-3 is reject

$\boxed{x = 3}$

Q 5 $x^2+7^{\log_7 x}-2=0$

$x^2+x-2=0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$x=1,-2$

-2 reject

$\boxed{x=1}$

Q 7. $5^{(\log_5 x)^2}+x^{\log_5 x}$

$5^{y^2}+x^y=1250$

$\log_5 x=y$

$5^y=x$

$5^{y^2}+5^{y^2}=1250$

$5^{y^2}=625$

$5^{y^2}=5^4$

$y^2=4$ $\quad y=\pm 2$

$\boxed{x=25,\frac{1}{25}}$

Q 6. $\log_4(2\log_3(1+\log_2(1+3\log_2 x)))=\frac{1}{2}$

$4^{1/2}=2\log_3(1+\log_2(1+3\log_3 2))$

$2=\log_2(1+3\log_2 x)$

$1=\log_2 x$

$1=\log_2 x$

$\boxed{x=2}$

Q 8. $\log_2(9-2^x)=10^{\log_{10}(3-x)}$

$2^{3-x}=9-2^x$

$2^3\times 2^{x}=9-2^x$

$8y=9-y$

$y=8,1$

$x=3,0$ $\quad \boxed{x=0}$

3 reject

Q9. $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$

$\sqrt{x+5} + \sqrt{x} = 5$

$5 = \sqrt{x+5} + \sqrt{x}$

$25 = x+5+x+2\sqrt{(x+5)(x)}$

$\cancel{25} \; 10 = x + \sqrt{x^2+5x}$

$10 - x = \sqrt{x^2+5x}$

$100 = 25x$

$\boxed{x = 4}$

Q10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$

$x+1 = y$

$y^{\log_{10} y} = 100y$

$\log_{10} y = \log_y 100y$ (take $\log_{10}$) both sides

$\log_{10} y = \log_y y + \log_y 100$

$\log_{10} y = 1 + 2\log_y 10$

$z = 1 + \frac{2}{z}$

$z = 2, -1$

$\log_{10} y = -1$

$y = \frac{1}{10}$

$x+1 = \frac{1}{10}$

$\boxed{x = \frac{-9}{10}}$

$\log_{10} y = 2$

$y = 100$

$x+1 = 100$

$\boxed{x = 99}$

Q (11) $\log_{x-1}(4) = 1 + \log_2(x-1)$

$\log_y 4 = 1 + \log_2 y$

$\cancel{y^{1+\log_2 y}}$

$2\log_y 2 = 1 + \log_2 y$

$\log_2 y = z$

$\frac{2}{z} = 1 + z$

$2 = z + z^2$

$z^2 + z - 2 = 0$

$z = \frac{-1 \pm \sqrt{1+8}}{2}$

$z = \frac{-1 \pm 3}{2}$

$z = -2, 1$

$\log_2 y = -2$

$\frac{1}{4} = y$

$x - 1 = y$

$x - 1 = \frac{1}{4}$

$x = \frac{1}{4} + \frac{4}{4}$

$x = \frac{5}{4}$

$\log_2 y = 1$

$y = 2$

$x - 1 = 2$

$x = 3$

$\boxed{x = 3, 5/4}$

(12) sum of values of x A) 1, B) 4 C) 0 D) 3

$\log_{2x-1}(x^3+3x^2-13x+10) = 2$

$(2x-1)^2 = x^3+3x^2-13x+10$

$4x^2+1-4x = x^3+3x^2-13x+10$

$x^3 - x^2 - 9x + 9 = 0$

$x = 1$

$(x-1)$

$x^2 - 9 = 0$

$x = \pm 3$

$x = 1, 3, -3$ but, -3 is reject as $2x-1$ will be $\ominus$ve

$\cancel{3 + -3 + 1}$

$\cancel{1}$

$\cancel{\boxed{A}}$

$\cancel{\boxed{A}}$

1 reject as $2x-1$ would be 1

$x = 3$

$\boxed{D)}$

(13)

$$5^{1+\log_4 x} + 5^{(\log_{1/4} x)-1} = \frac{26}{5}$$

$$5^{1+\log_4 x} + 5^{-(\log_4 x+1)} = \frac{26}{5}$$

$$\cancel{5^y + \frac{1}{5^y} = \frac{26}{5}}$$

$$\cancel{5^{2y}+1 = \frac{26\times 5^y}{5}}$$

$$\cancel{5 \times 5^{2y}+1 = 26 \times 5^y}$$

$$\cancel{5\times 5^{x^2}+1 = 26}$$

$$\cancel{5\times z^2+1 = 26z}$$

$$\cancel{5z^2-26z+1=0}$$

$$\cancel{z = \frac{26 \pm \sqrt{676-20}}{2}}$$

$$\cancel{z = \frac{26 \pm \sqrt{656}}{2}}$$

$$\cancel{t + \frac{1}{t} = \frac{26}{5}}$$

$$\cancel{t^2+1 = \frac{26t}{5}}$$

$$\cancel{5t^2-26t+1=0}$$

$$5 t + \frac{1}{5t} = \frac{26}{5}$$

$$25t^2+1 = \frac{26\times 5t}{5}$$

$$25t^2-26t+1$$

$$25t^2-25t-t+1$$

$$25t(t-1)-1(t-1)$$

$$t = 1, \ 1/25$$

$$5^{\log_4 x} = 1$$

$$\cancel{\log_5 1 = \log_4 x}$$

$$\log_4 x = 0$$

$$4^0 = x$$

$$\boxed{x=1}$$

$$5^{\log_4 x} = \frac{1}{25}$$

$$\log_4 x = -2$$

$$\boxed{\frac{1}{16} = x}$$

$$\boxed{x = 1, \ 1/16}$$

$$\begin{array}{r} 26 \\ \times 26 \\ \hline 156 \\ 520 \\ \hline 676 \end{array}$$

(12) $(x-2)^{\log^2(x-2)+\log(x-2)^5-12} = 10^{2\log(x-2)}$

$\cancel{(x-2)^{y^2}}$

$\cancel{x-2=y}$

$y^{\cancel{(\log y)^2(\log y)}+\log y^5-12} = \cancel{10}^{2\log y}$

$\cancel{\log y=2}$

$\cancel{(x-2)^{2y^2}}$

$x-2=y$

$y^{(\log y)(\log y)+5\log y-12} = 10^{2\log y}$

$y^{\log y(\log y+5)-12} = 10^{2\log y^2}$

$2\log y^2 \log_y 10 = \log_y(\log_y+5)-12$

$2\log_y 10 = \log y + 5 - \dfrac{12}{\log y}$

$2\log_y 100 = \dfrac{(\log y)^2+(\log y)-12}{\log_{10} y}$

Scanned with CamScanner

Q 12. $(x-2)^{\log(x-2)(\log(x-2)+\log(x-2)^5-12} = 10^{2\log(x-2)}$

Note – can assume same base on both sides.

$y^{\log_{10} y \times \log_{10} y + 5\log_{10} y^{0} - 12} = 10^{\log_{10} y^2}$

$y^{\log_{10} y(\log_{10} y + 5) - 12} = y^2$

$\log_{10} y(\log_{10} y + 5) - 12 = 2$

$100 = y(\log_{10} y + 5) - 12$

~~$100 = 5y + y\log_{10} y$~~

~~$100 =$~~

$2 = (\log_{10} y)^2 + 5\log_{10} y - 12$

$14 = z^2 + 5z$

$z^2 + 5z - 14 = 0$

$z = \frac{-5 \pm \sqrt{25 + 56}}{2}$

$z = \frac{-5 \pm 9}{2}$

$z = -7, 2$

$\log_{10} y = 2$

$y = 100$

$x - 2 = 100$

$\boxed{x = 102}$

$\log_{10} y = -7$

$\frac{1}{10^7} = y$

$x = \frac{1}{10^7} + 2$

$x = \frac{1 + 20000000}{10000000}$

$x = \frac{20000001}{10000000}$

$x = 2.0000001$

$\boxed{x = 102, 2.0000001}$

H.W. 07-06-2024

DYS-4 [10:]

~~Logarithmic~~

(100)

Logarithmic Inequalities

Constant Base

$\in (0,1)$ sign change

$(1, \infty)$ no sign change

Variable Base

case 1 Base $\in (0,1)$

case 2 $\in (1, \infty)$

union

eg. $\log_{1/2} x > \log_{1/2}(2x-1)$

$x < 2x-1$

$1 < x$

$\boxed{x > 1}$

eg. $\log_5 x > \log_5 (2x-1)$

$x > 2x-1$

$1 > x$

$\boxed{x < 1}$

Q $\log_{0.5}(x-3) > \log_{0.5}(2x)$

$x - 3 < 2x$

$-3 < x$

$x > -3$

$\therefore x \in (-3, \infty)$ — (1)

For $\log_{0.5}(x-3)$ — $x-3>0$

$x > 3$ — (2)

$\log_{0.5}(2x)$ — $2x > 0$

$x > 0$ — (3)

$\Rightarrow$ (1) $\cap$ (2) $\cap$ (3)

$\boxed{x \in (3, \infty)}$

Q2. $\log_7(x^2-3x) \geq \log_7(2x-6)$

$x^2 - 3x \geq 2x - 6$

$x^2 - 5x + 6 \geq 0$

$(x-2)(x-3) \geq 0$

$+ \quad - \quad +$

$2 \quad 3$

$x \in (-\infty, 2] \cup [3, \infty)$ — (1)

$\log_7(x^2-3x)$ — $x^2 - 3x > 0$

$x(x-3) > 0$

$0 \quad 3$

$+ \quad - \quad +$

$x \in (-\infty, 0) \cup (3, \infty)$ — (2)

$\log_7(2x-6)$ — $2x - 6 > 0$

$2x > 6$

$x > 3$

$x \in (3, \infty)$ — (3)

(1) ∩ (2) ∩ (3)

$\boxed{x \in (3, \infty)}$

Q $\log_x (2x) > 2$

$x \in (0,1)$

Case 1 $2x < x^2$

$0 < x^2 - 2x$

$x(x-2) > 0$

$+ \quad - \quad +$

$0 \quad 2$

$x \in (-\infty, 0) \cup (2, \infty)$

$\xrightarrow{\cap} x \in \phi$

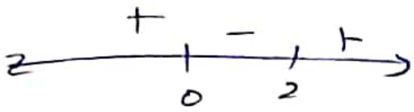
Case 2:-

$x \in (1, \infty)$

$2x > x^2$

$0 > x^2 - 2x$

$x(x-2) < 0$



$x \in (0, 2)$

$\xrightarrow{\cap} x \in (1, 2)$

Case 1 ∪ Case 2: $(x \in \phi) \cup (x \in (1,2))$

$x \in (1, 2)$ — ①

$\log_x 2x$

$2x > 0$

$x > 0$

$x > 0$,

$x \neq 1$

$\xrightarrow{\cap} x \in (0,1) \cup (1, \infty)$ — ②

① ∩ ②

$(x \in (1,2)) \cap (x \in (0,1) \cup (1,\infty))$

$\boxed{x \in (1,2)}$

Q ① $\log_{1/3}\left(\frac{1-2x}{x}\right) \leq 0$

$\cancel{\left(\frac{1}{3}\right)^0 \leq \frac{1-2x}{3}}$

$\cancel{\frac{1-2x}{3} \leq 1}$

$\cancel{1-2x \leq 3}$

$\cancel{-2x \leq 3-1}$

$\cancel{-2x = -2}$

$\cancel{2x = 2}$

$\cancel{x = \frac{2}{2}}$

$\cancel{x = 1}$

$\cancel{1-2x \leq 3}$

$\cancel{-2x \leq -2}$

$\cancel{2x \geq 2}$

$\cancel{x \geq 1}$

$\cancel{\log_{1/3}\left(\frac{1-2x}{3}\right) \longrightarrow \frac{1-2x}{3} > 0}$

$\cancel{1 > \frac{2x}{3}}$

$\cancel{3 > 2x}$

$\cancel{x < \frac{3}{2}}$

$\cancel{\boxed{x \in [1, 1.5)}}$

$\frac{1-2x}{x} \geq 1$

$\frac{1-2x-x}{x} \geq 0$

$\frac{1-3x}{x} \geq 0$

$-\quad 0 \quad + \quad 1/3 \quad -$

$\cancel{(-\infty, 0) \cup (1/3, \infty)}$

$x \in (0, 1/3]$ — ①

$\log_{1/3}\left(\frac{1-2x}{x}\right) \longrightarrow \frac{1-2x}{x} > 0$

$0 \quad 1/2$

$- \quad + \quad -$

$\cancel{x \in (-\infty, 0) \cup (1/2, \infty)}$

$x \in (0, 1/2)$ — ②

① $\cap$ ②

$\boxed{(0, 1/3]}$

(2) $\log_{2x}(x^2-5x+6) < 1$

Case 1 $\quad 2x \in (0,1)$

$x \in (0, 1/2)$

$x^2-5x+6 > 2x$

$x^2-7x+6>0$

$x^2-6x-x+6>0$

$(x-6)(x-1)>0$

$+ \quad 1 \quad - \quad 6 \quad +$

$x \in (-\infty,1) \cup (6,\infty)$

$x \in (0, 1/2)$ —— (1)

Case 2 $\quad 2x \in (1,\infty)$

$x \in (1/2, \infty)$

$(x-6)(x-1) < 0$

$+ \quad 1 \quad - \quad 6 \quad +$

$x \in (1,6)$

$x \in (1,6)$ — (2)

(1) $\cup$ (2)

$x \in (0,1/2) \cup (1,6)$ — (3)

$x^2-5x+6>0$

$x^2-3x-2x+6>0$

$x(x-3)-2(x-3)>0$

$(x-2)(x-3)>0$

$+ \quad 2 \quad - \quad 3 \quad +$

$x \in (-\infty,2) \cup (3,\infty)$

$2x>0$

$x>0, \neq \frac{1}{2}$

$x \in (0, 1/2) \cup (\frac{1}{2},\infty)$

$\cap$

$x \in (0,1/2) \cup (1/2,2) \cup (3,\infty)$ (4)

(3) $\cap$ (4)

$\boxed{(0,1/2) \cup (1,2) \cup (3,6)}$

H.W. 08-06-2024

DYS-5 [1, 2, 3, 5, 7, 8, 9, 10]

O-1 [7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]

Q $\log_{0.5}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$

$\cancel{\log_{0.5}(\ldots)}$

$\log_6\left(\frac{x^2+x}{x+4}\right) > 1$

$\frac{x^2+x}{x+4} > 6$

$\frac{x^2+x}{x+4} - 6 > 0$

$\frac{x^2+x-6x-24}{x+4} > 0$

$\frac{x^2-5x-24}{x+4} > 0$

$\frac{x^2-8x+3x-24}{x+4} > 0$

$\frac{(x-8)(x+3)}{x+4} > 0$

$-4 \quad -3 \quad 8$

$- \quad + \quad - \quad +$

$x \in (-4,-3) \cup (8,\infty)$

$\frac{x^2+x}{x+4} > 0$

$\frac{x(x+1)}{x+4} > 0$

$-4 \quad -1 \quad 0$

$- \quad + \quad - \quad +$

$(-4,-1) \cup (0,\infty)$

$\frac{x^2+x}{x+4} > 1$

$\frac{x^2+x-x-4}{x+4} > 0$

$\frac{(x+2)(x-2)}{x+4} > 0$

$-4 \quad -2 \quad 2$

$- \quad + \quad - \quad +$

$x \in (-4,-2) \cup (2,\infty)$ $\cancel{x(x-8)+3(x-8)}$

$x \in (-4,-2) \cup (2,\infty)$

$x \in (-4,-3) \cup (8,\infty)$

$\boxed{x \in (-4,-3) \cup (8,\infty)}$

Q $\log_2 \log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$

$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 1$

$\log_7 \log_{1/3} (2x-3) < \frac{1}{2}$

$\log_{1/3} (2x-3) < \sqrt{7}$

$2x-3 > \frac{1}{3^{\sqrt{7}}}$

$2x-3 > 0$

$\log_{1/3} (2x-3) > 0$

$\log_7 \log_{1/3} (2x-3) > 0$

$\log_{1/2} \log_7 \log_{1/3} (2x-3) > 0$

Exponential Inequalities

→ Make the base same

→

Base

Base $\in (0,1)$
Sign Change

Base $\in (1,\infty)$
no sign change

Q1. $2^{x+2} > \left(\frac{1}{4}\right)^{\frac{1}{x}}$

$2^{x+2} > 2^{-2/x}$

$x+2 > -\frac{2}{x}$

$x+2+\frac{2}{x} > 0$

$\frac{x^2+2x+2}{x} > 0$

$\frac{1}{x} > 0$

0

$\boxed{x \in (0,\infty)}$

Q 2. $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

Q 3. $\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \frac{25}{4}$

$\left(\frac{2}{5}\right)^{\frac{6-5x}{2+5x}} < \left(\frac{2}{5}\right)^{-2}$

$\frac{5x-6}{5x-2} < 2$

$\frac{5x-6-10x-4}{5x-2} < 0$

$\frac{5x+2}{5x-2} > 0$

$-2/5 \quad 2/5$

$+ \quad - \quad +$

$(-\infty, -2/5) \cup (2/5, \infty)$

$\frac{2}{5} > \left(\frac{2}{5}\right)^{1/2\left(\frac{6-5x}{2+5x}\right)}$

$1 < \frac{1}{2}\left(\frac{6-5x}{2+5x}\right)$

$\frac{6-5x-2-5x}{2+5x} > 0$

$\frac{4-10x}{2+5x} > 0$

$\frac{10x-4}{5x+2} < 0$

PQ

$1 + \frac{12-5x}{2+5x} < 0$

$\frac{2x+5x+12-5x}{2+5x} < 0$

$\frac{6+x}{5x+2} < 0$

-6

$\frac{7x-10x+12}{5x+2} < 0$

$\frac{3x-12}{5x+2} > 0$

$-2/5 \quad 4$

$(-\infty, -2/5) \cup (4, \infty)$

(2) $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

$$\left(\frac{125}{100}\right)^{1-x} < \left(\frac{64}{100}\right)^{2(1+\sqrt{x})}$$

$$\left(\frac{5}{4}\right)^{1-x} < \left(\frac{5}{4}\right)^{-4(1+\sqrt{x})}$$

$$1-x < -4(1+\sqrt{x})$$

$$4(1+\sqrt{x}) < x-1$$

$$4+4t < t^2-1$$

$$t^2-4t-5>0$$

$$t-5t+t-5>0$$

$$t(t-5)+1(t-5)>0$$

$$(t+1)(t-5)>0$$

$+ \quad -1 \quad - \quad 5 \quad +$

$$t \in (-\infty,-1) \cup (5,\infty)$$

~~$x \in (-\infty$~~

$$\boxed{x \in (5,\infty)}$$

(3) $\left(\frac{5}{2}\right)^{\frac{5x-6}{5x+2}} < \left(\frac{5}{2}\right)^2$

$$\frac{5x-6}{5x+2} < 2$$

$$\frac{5x-10}{5x+2} < 0$$

$$\frac{x+2}{5x+2} > 0$$

$$x \in (-\infty,-2) \cup (-2/5,\infty)$$

DYS-6

(8) $\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > 1$

$$\left(\frac{2}{3}\right)^{\frac{|x|-1}{|x|+1}} > \left(\frac{2}{3}\right)^{0}$$

$$\frac{|x|-1}{|x|+1} < 0$$

$$|x|-1 < 0$$

$$|x| < 1$$

$$\boxed{x \in (-1, 1)}$$

H.W. ~~(x)~~
DYS-6-(Full) O-1(21, 22, 23, 24, 25, 26, 27, 28, 29)
O-2(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)

$e = 2.7$

DYS-4 S

(4) $(\log_{10} 100x)^2 + (\log_{10} 10x)^2 + \log_{10} x < 14$

$$\cancel{\frac{(\log 100x)^2}{(\log 10)^2} + \frac{(\log 10x)^2}{(\log 10)^2}} + \frac{\log x}{\log 10}$$

$$\cancel{\frac{(\log 100)^2 + (\log x)^2 + (\log 10)^2 + (\log x)^2 + \cdot}{\log 10 \log 10}}$$

$$(2 + \log_{10} x)^2 + (1 + \log_{10} x)^2 + \log_{10} x < 14$$

$$4 + y^2 + 4y + 1 + y^2 + 2y + y < 14$$

$$2y^2 + 7y - 9 < 0 \qquad 2y^2 + 9y - 2y - 9 < 0$$

$-9/2$ 1

$$\log_{10} x \in (-9/2, 1)$$

$$\boxed{x \in (10^{-9/2}, 10)}$$ ✓

(110)