Fundamentals of Algebra

~~Indices / Powers~~

Indices & ~~Q~~ Surds (Powers & Exponents).

a^{m} $m \rightarrow$ Exponent/Power

$a \rightarrow$ Base

$m \rightarrow$ ~~Our real no.~~ Integers

$a \rightarrow$ non-zero real no. or complex number

Eg. 2^4, $(-4)^6$, $(\sqrt{3})^9$, $(\pi)^8$; $(2+3i)^{15}$ etc

Laws:- $(a \neq 0)$

(1) $a^{0} = 1$

(2) $a^{-m} = \frac{1}{a^m}$

(3) $a^m \times a^n = a^{m+n}$

(4) $\frac{a^m}{a^n} = a^{m-n}$

(5) $(a^m)^n = a^{m \times n}$

(6) $\sqrt[c]{a^b} = a^{b/c}$, $c \in N$, $c \geq 2$

Q1. find values

① $\left(\left(256^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right)^{2}$

$\frac{-1}{\cancel{2}} \times -\frac{1}{4} \times 2 = +\frac{1}{4}$

$\sqrt{\sqrt{256}} = \sqrt{16} = \boxed{4}$

② $\left(125^{\frac{1}{3}} + 64^{\frac{1}{3}}\right)^{3}$

$\left(\sqrt[3]{125} + \sqrt[3]{64}\right)^{3}$

$(5+4)^{3} = 9\cancel{8}^{3}\ \cancel{= 512} = \boxed{729}$

$\begin{array}{r} 81 \\ \times\ 9 \\ \hline 729 \end{array}$

③ $\left\{\sqrt[5]{\left(\frac{1}{a}\right)^{-15}}\right\}^{-\frac{4}{3}}$

$\left\{\sqrt[5]{a^{15}}\right\}^{-\frac{4}{3}}$

$\left(a^{\frac{15}{5}}\right)^{-\frac{4}{3}}$

$a^{\frac{15}{5} \times -\frac{4}{3}}$

$\boxed{a^{-4}}$

④ $\dfrac{\sqrt{x^3}\times\sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}}$

$\cancel{\dfrac{x^{\frac{1}{3}}\times x^{\frac{5}{3}}}{x^{\frac{3}{5}}} \times x^{\frac{77}{30}}}$

$\cancel{x^{\frac{1}{3}+\frac{5}{3}+\frac{77}{30}-\frac{3}{5}}}$

$\cancel{x^{\frac{10+50+77-24}{30}}}$

$\cancel{x^{\frac{63}{30}}}$

$\cancel{\boxed{\sqrt[10]{x^{21}}}}$ $\cancel{\boxed{x^{\frac{71}{10}}}}$

$\dfrac{x^{3/2}\times x^{5/3}}{x^{3/5}}\times x^{\frac{77}{30}}$

$x^{\frac{3}{2}+\frac{5}{3}+\frac{77}{30}+\frac{3}{5}}$

x^{y}

$y = \frac{3}{2}+\frac{5}{3}+\frac{77}{30}+\frac{3}{5}$

$y = \frac{45+50+77+18}{30}$

$y = \frac{190}{30}$

$\boxed{y = 19/3}$

$\boxed{x^{19/3}}$

$$\begin{array}{r} {}^{2}\\ 95\\ 77\\ 18\\ \hline 190 \end{array}$$

⑤ $\sqrt[5]{\sqrt[4]{\sqrt[3]{x}}} = x^{\frac{1}{30k}}$

$\left(\left(x^{1/3}\right)^{1/4}\right)^{1/3}$

$\cancel{x^{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}}$

$\cancel{x^{\frac{20+15+12}{60}}}$

$\cancel{x^{\frac{47}{60}} = x^{\frac{1}{30k}}}$

$\cancel{\frac{47}{60} = \frac{1}{30k}}$

$\cancel{\boxed{k = \frac{47}{2}}}$

$x^{\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}}$

$x^{\frac{1}{60}}$

$\frac{1}{60} = \frac{1}{30k}$

$\boxed{k = 2}$

Q2. $\left(\frac{1}{100}\right)^{1/2} \cdot (125)^{1/3} + (64)^{1/3} \cdot (4)^{-2} + \left((729)^{-1/9}\right)^{-3}$

$10000 \times \frac{1}{125} \times \frac{1}{125} \times \frac{1}{125} + 4 \times \frac{1}{16} + (729)^{1/3}$

$\frac{\overset{16}{\cancel{\overset{80}{\cancel{\overset{400}{\cancel{\overset{2000}{\cancel{10000}}}}}}}}}{\underset{25}{125} \times \underset{25}{\cancel{125}} \times \underset{5}{\cancel{\underset{25}{\cancel{125}}}}} + \frac{1}{4} + 9$

$\frac{16}{625 \times 5} + \frac{1}{4} + 9$

$10 \times \frac{1}{5} + \frac{1}{4} + 9$

$2 + \frac{1}{4} + 9$

$11 + \frac{1}{4}$

$\boxed{\frac{45}{4}}$

Q3. $\cancel{\dfrac{(2\cdot3)^{n+1} - (7\cdot3)^{n-1}}{3^{n+1} + 2\left(\frac{1}{3}\right)^{1-n}}}$

$$\frac{2\times(3)^{(n+1)} - (7\times3)^{(n-1)}}{3^{n+1} + 2\times 3^{(n-1)}}$$

$$\frac{2\times 3^{n}\times 3^{1} - 7\times 3^{n}\times 3^{-1}}{3^{n}\times 3^{1} + 2\times 3^{n}\times 3^{-1}}$$

$$\frac{\cancel{3^{n}}\left(6-\frac{7}{3}\right)}{\cancel{3^{n}}\left(3+\frac{2}{3}\right)}$$

$$\frac{6-\frac{7}{3}}{3+\frac{2}{3}}$$

$$\frac{\frac{18-7}{\cancel{3}}}{\frac{9+2}{\cancel{3}}} = \frac{18-7}{9+2} = \frac{11}{11} = \boxed{1}$$

Q4. $a^x = b$

$b^y = c$

$c^z = a$

prove $xyz = 1$

$a^{xy} = c^1$

$a = c^z$

$(c^z)^{xy} = c^1$

$c^{xyz} = c^1$

$\boxed{xyz = 1}$

Hence Proved

Q5. $\cancel{2^{4^{0.5^{0.7}}}} \ldots \infty \quad 2^{4^{0.5^{0.7^{0.9}}}} \ldots \infty$

$16^{0.5^{0.7^{0.9}}} \ldots \sim$

$2^{4^{0.5^{0.7^{0.9^{1.1^{1.3}}}}}} \ldots \infty \rightarrow 2^{4^{0.5^{0.7^{0.9^{1.1^{1.3^{1.5^{1.7}}}}}}}} \rightarrow 0.9$ to power $\infty = 0$

$2^{4^{0.5^{0.7^{0}}}} \rightarrow 2^{4^{0.5^{1}}} \rightarrow 16^{1/2} \boxed{\rightarrow 4}$

$(1)^{\infty} = 1$

$1^{0} = 1$

$(\text{value btw } 0 \,\&\, 1)^{\cancel{\infty}\, 0} = 1$

$(\text{value btw } 0 \,\&\, 1)^{\infty} = 0$

$(\text{value greater than } 1)^{0} = 1$

$(\text{value greater than } 1)^{\infty} = \infty$

Note:- ① If $x = y$ then $a^x = a^y$ $(0 \neq a)$ and
If $a^x = a^y$ then $x = y$ $(a \neq 0, 1, -1)$ is not always true.
& ~~not always~~

② Same base and different powers:

$a^x = a^y$

Case 1	Case 2	Case 3	Case 4
$a = 1$	$a = -1$	$a = 0$	$x = y$

$\Rightarrow$ verify in each case.

$\cancel{(2x^2-1)^{5x+2} = (2x^2-1)^{x^2+6}}$

$\cancel{x^2+6 = 5x+2}$

Q $\left(2x^2-1\right)^{5x+2} = \left(2x^2-1\right)^{x^2+6}$

Sol case-1 $\rightarrow a=1$

$2x^2-1=1$

$2x^2=2$

$x^2=1$

$x=\pm 1$

verify $\rightarrow x=+1 \rightarrow (2-1)^{5(1)+2} = (2-1)^{(1)^2+6}$

$1^7 = 1^7$ ✓

$x=-1 \rightarrow (2-1)^{-3} = (2-1)^7$

$1^{-3} = 1^7$

$\frac{1}{1^3} = 1^7$

$1=1$ ✓

case 2 $\rightarrow a=0$

$2x^2-1=0$

$2x^2=1$

$x^2=\frac{1}{2}$

$x=\pm\frac{1}{\sqrt{2}}$

verify $\rightarrow x = +\frac{1}{\sqrt{2}} \rightarrow \left(2\times\frac{1}{2}-1\right)^{\frac{5}{\sqrt{2}}+2} = \left(2\times\frac{1}{2}-1\right)^{\frac{1}{2}+6}$

$0^{\oplus ve} = 0^{\oplus ve}$ ✓

$x=\frac{-1}{\sqrt{2}} \rightarrow \left(2\times\frac{1}{2}-1\right)^{-\frac{5}{\sqrt{2}}+2} = \left(2\times\frac{1}{2}-1\right)^{\frac{1}{2}+6}$

$0^{\ominus ve} = 0^{+ve}$ ✗ (0^{-ve} is not defined) case-3 → $a = -1$

$2x^2 \cancel{-1} = \cancel{-1}$

$2x^2 = 0$

$x^2 = 0$

$x = 0$

verify:- $(0-1)^{0+2} = (0-1)^{0+6}$

$-1^2 = -1^6$

$1 = 1$ ✓

Case 4 → $a^x = y$

$x^2+6 = 5x+2$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$

$x(x-4) - 1(x-4) = 0$

$(x-1)(x-4) = 0$

$x = 1, x = 4$

verify :- $x = 1$ → verified already

$x = 4 \rightarrow (32-1)^{20+2} = (32-1)^{16+6}$

$\rightarrow 31^{22} = 31^{22}$ ✓

Final answer - $\boxed{x = 1, -1, 0, \frac{1}{\sqrt{2}}, 4 \text{ Answer}}$

Note:-

(3) Same Power and Different Base

$a^x = b^x$

Case-1	Case 2	Case 3
a & b = 1 or a = b	~~a & b = 0~~ Power = 0	a & b = -1 or a = -b

Q find x, $(x+2)^{(x-3)} = (2x-5)^{(x-3)}$

case 1 → $a = b$

$x + 2 = 2x - 5$

$7 = x$

verify

$(7+2)^{(7-3)} = (14-5)^{(7-3)}$

$9^4 = 9^4$ ✓

case 2 → $(x-3) = 0$

$x - 3 = 0$

$x = 3$

verify.. $(3+5)^{(3-3)} = (6-5)^{(3-3)}$

$8^0 = 1^0$

$1 = 1$ ✓

Case 3 → $a = -b$

$$x + 2 = 5 - 2x$$

$$3x = 3$$

$$\underline{x = 1}$$

verify :-

$$(1+2)^{(1-3)} = (2-5)^{(1-3)}$$

$$3^{-2} = -3^{-2}$$

$$\frac{1}{9} = \frac{1}{9} \checkmark$$

$\boxed{x = 1, 3, 7 \text{ Answer}}$

H.W. 23-04-2024

Q1. Find the values of x

(1) $(4-x)^{x^3-4x} = (4-x)^{(x^2-4)x(x-1)}$

(2) $(x^2-4)^{2x} = (x^2+2x)^{2x}$

Race I - 8

Surds

→ Any root of a rational number which cannot be exactly found as a whole or rational number.

$\sqrt[n]{a}$ (a is a rational number)

n → degree of surd

eg $\sqrt{2}$, $7^{3/7}$, $(+3)^{4/15}$ → 15 degree

$\sqrt{2}$ → 2 degree, $7^{3/7}$ → 7 degree

→ If a is not rational then $\sqrt[n]{a}$ is not a surd

eg $\sqrt{2+\sqrt{3}}$ is not a surd

→ 1 term → simple surd
2 term → Binomial surd
4 term → Bi Quadratic surd.

→ Two surds which differ ~~differ~~ only in sign which connects their terms are conjugate or complimentary to each other.

eg. $2\sqrt{7}+5\sqrt{3}$

↓

conjugate

↓

$-2\sqrt{7}+5\sqrt{3}$ Or $2\sqrt{7}-5\sqrt{3}$

→ Product of a surd and its conjugate is rational or irrational.

eg.1. $\sqrt{2}+\sqrt{3} \longrightarrow \sqrt{2}-\sqrt{3}$

$(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$

$(\sqrt{2})^2-(\sqrt{3})^2$

$2-3=1$ (rational) (if degree = 2, rational)

eg 2. $\sqrt[3]{2}+\sqrt[3]{3} \rightarrow \sqrt[3]{2}-\sqrt[3]{3}$

$(\sqrt[3]{2}+\sqrt[3]{3})(\sqrt[3]{2}-\sqrt[3]{3})$

$(\sqrt[3]{2})^2-(\sqrt[3]{3})^2$

$2^{2/3}-3^{2/3} \rightarrow$ (irrational)

Q. Arrange the following in assending order

$\sqrt[3]{9}, \sqrt[4]{11}, \sqrt[6]{17}$

$9^{1/3}, 11^{1/4}, 17^{1/6}$ (LCM of 3, 4 & 6 → 24)

$9^8, 11^6, 17^4$

$81^4, 11^6, 17^4$

$81^{4/24}, \cancel{12221}^{46/24}, 17^{4/24}$

1331

~~$\sqrt[4]{11} > \sqrt[3]{9} > \sqrt[6]{17}$~~

$\sqrt[3]{9} > \sqrt[4]{11} > \sqrt[6]{17}$

$$\begin{array}{r} 11 \\ \times 11 \\ \hline 11 \\ 11 \\ \hline 121 \\ \times 11 \\ \hline 121 \\ 121 \\ \hline 1221 \end{array}$$

12221

1331

Square Root

→ $\sqrt{x^2} = \cancel{\pm}|x|$ ($-x$ is not right) (x be + ve)

$(\sqrt{x})^2 = x$

→ It gives us only non-negative values
→ All quantities in underroot must be positive
→ Square Root of $a+\sqrt{b}$ ($a \geq 0, b \geq 0$)

$\sqrt{a+\sqrt{b}} = \sqrt{x}+\sqrt{y}$ ($x, y > 0$)

Squaring

$a+\sqrt{b} = x+y+2\sqrt{xy}$

$a+\sqrt{b} = c+\sqrt{D} \rightarrow a = c, b = d$

$\boxed{a = x+y} \rightarrow y = a-x$

$\sqrt{b} = 2\sqrt{xy}$

$\boxed{b = 4xy}$

$b = 4x(a-x)$

$b = 4ax - 4x^2$

$4x^2 - 4ax + b = 0$

$x = \dfrac{-(-4a) \pm \sqrt{16a^2 - 4(4)(b)}}{2(4)}$

$x = \dfrac{4a \pm \sqrt{16a^2 - 16b}}{8} = \dfrac{a \pm \sqrt{16a^2-16b}}{2}$ (+ ve because x is positive)

$y = \dfrac{a - \sqrt{16a^2-16b}}{2}$

$\sqrt{a \pm \sqrt{b}} = \sqrt{\dfrac{a+\sqrt{a^2-b}}{2}} \pm \sqrt{\dfrac{a-\sqrt{a^2-b}}{2}}$

H.W. 23-04-2024

Q1.
(1) $(4-x)^{x^3-4x} = (4-x)^{(x^2-4)(x-1)}$

Case 1:- $a=0$

$4-x=0$

$x=4$

verify

$(4-4)^{4^3-4\times4} = (4-4)^{(4^2-4)(4-1)}$

$0^{64-16} = 0^{(12)(3)}$

$0^{48} = 0^{36}$

$0=0$ ✓

case 2:- $a=-1$

$4-x=-1$

$4+1=x$

$x=5$

verify:- $(4-5)^{125-20} = (4-5)^{(25-4)(4)}$

$(-1)^{105} = (-1)^{84}$

$-1 = 1$ ✗

Case 3 :- $a = \cancel{0}\,1$

$$4 - x = \cancel{0}1$$

$$x = 3$$

verify :-

$$(4-3)^{27-12} = (4-3)^{(7-4)(3-1)}$$

$$1^{15} = 1^{5\times 2}$$

$$1^{15} = 1^{20}$$

$$1 = 1 \checkmark$$

Case 4 :- $x = y$

$$x^3 - 4x = (\cancel{0}x^2 - 4)(x-1)$$

$$\cancel{x^3} - 4x = \cancel{x^3} - x^2 - 4x + \cancel{0}4$$

$$\cancel{x^2 + 4 = 0}$$

$$\cancel{x^2 = -4}$$

$$\cancel{x = \pm 2}$$

$$x^2 = 4$$

$$x = \pm 2$$

verify $x = 2 \longrightarrow (4-2)^{8-8} = (4-2)^{\cancel{(5-4)}(4-4)(2-1)}$

$$2^0 = 2^0 \checkmark$$

$x = -2 \longrightarrow \cancel{(-2-2} \; (4+2)^{-8+8} = (4+2)^{(4-4)(-3)}$

$$6^0 = 6^0 \checkmark$$

$$\boxed{x = 2, -2, 3, 4,}$$

② $(x^2-4)^{2x} = (x^2+2x)^{2x}$

case 1 :- $a=b$

$\cancel{x^2}-4 = \cancel{x^2}+2x$

$x = \frac{-4}{2}$

$x = -2$

verify :-

$((-2)^2-4)^{-2\times 2} = ((-2)^2+2(-2))^{2(-2)}$

$(4-4)^{-4} = (4 + -4)^{-4}$

$0^{-4} = 0^{-4}$ ✗ (0 $\frac{1}{0^4}$ is not valid)

Case 2 :- $a=-b$

$x^2-4 = -x^2-2x$

$x^2+x^2 = -2x+4$

$2x^2+2x-4=0$

$x^2+x-2=0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$(x+2)(x-1)$

$x=-2$ (verified)

$x=1$

~~xxx~~

verify :-

$$\left((1)^2-4\right)^{2\times1} = \left((1)^2+2(1)\right)^{2\times1}$$

$$(1-4)^2 = (1+2)^2$$

$$(-3)^2 = (3)^2$$

$$9 = 9 \checkmark$$

Case 3 :- power = 0

$$2x = 0$$
$$x = \frac{0}{2}$$
$$\boxed{x = 0}$$

Verify :- $\left((0)^2-4\right)^{2\times0} = \left[(0)^2+2(0)\right]^{2\times0}$

$$(0-4)^0 = (0+0)^0$$

$$-4^0 = 0^0 \quad \times \quad (0^0 \text{ is not defined})$$

$$\boxed{x = 1}$$

Scanned with CamScanner

① find: $\sqrt{8+2\sqrt{15}}$

② $\sqrt{3+\sqrt{5}}$

① $\sqrt{8+2\sqrt{15}} =$

$$\sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

$$\sqrt{8+\sqrt{64-}}$$

① Method 1

$$(a+b)^2 = a^2+b^2+2ab$$

$$\sqrt{8+2\sqrt{15}}$$

$$\sqrt{8+2\sqrt{5}\sqrt{3}}$$

$$\sqrt{3+5+2\sqrt{5}\sqrt{3}}$$

$$\sqrt{(\sqrt{3})^2+(\sqrt{5})^2+2\sqrt{5}\sqrt{3}}$$

$$\sqrt{(\sqrt{3}+\sqrt{5})^2}$$

$$\underline{\underline{\sqrt{3}+\sqrt{5}}}$$

Method - 2

$$\sqrt{8+2\sqrt{15}}$$

$$\sqrt{8+\sqrt{4}\sqrt{15}}$$

$$\sqrt{8+\sqrt{60}}$$

$$\sqrt{\frac{8+\sqrt{64-60}}{2}} + \sqrt{\frac{8-\sqrt{64-60}}{2}}$$

$$\sqrt{\frac{8+\sqrt{4}}{2}} + \sqrt{\frac{8-\sqrt{4}}{2}}$$

$$\sqrt{\frac{8+2}{2}} + \sqrt{\frac{8-2}{2}}$$

$$\underline{\underline{\sqrt{5}+\sqrt{3}}}$$

~~3) $\sqrt{\ }$~~

② $\sqrt{3+\sqrt{5}}$

$$\sqrt{\frac{3+\sqrt{9-5}}{2}} + \sqrt{\frac{3\mp\sqrt{9-4}}{2}}$$

$$\sqrt{\frac{3+\sqrt{4}}{2}} + \sqrt{\frac{3-\sqrt{4}}{2}}$$

$$\boxed{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \quad ②}$$

$$\sqrt{3+\sqrt{5}}$$

$$\sqrt{\left(\sqrt{\tfrac{5}{2}}\right)^2+\left(\sqrt{\tfrac{1}{2}}\right)^2+2\,\frac{\sqrt{5}}{2}\,\frac{\sqrt{1}}{2}}$$

$$\sqrt{\left(\frac{\sqrt{5}}{2}+\frac{\sqrt{1}}{2}\right)^2}$$

$$\cancel{\frac{\sqrt{5}}{2}+\frac{\sqrt{1}}{2}}$$

$$\cancel{\sqrt{\left(\sqrt{\tfrac{5}{2}}+\sqrt{\tfrac{1}{2}}\right)}}$$

$$\boxed{\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}} \quad ②}$$

$$\frac{\sqrt{5}}{\sqrt{2}}+\frac{1}{\sqrt{2}}$$

$$\boxed{\frac{\sqrt{5}+1}{\sqrt{2}}}$$

(86)

3) $\sqrt{7-3\sqrt{5}}$

$\cancel{\frac{1}{\sqrt{2}}\sqrt{7-2\times 3\sqrt{5}\sqrt{9}}}$

$\sqrt{7-\sqrt{45}}$

$\sqrt{7-\frac{2\sqrt{45}}{2}}$

$\frac{1}{\sqrt{2}}\sqrt{14-2\sqrt{45}}$

$\frac{1}{\sqrt{2}}\sqrt{(\sqrt{9})^2+(\sqrt{5})^2-2\sqrt{45}}$

$\frac{1}{\sqrt{2}}\sqrt{(\sqrt{9}-\sqrt{5})^2}$

$\frac{1}{\sqrt{2}}(\sqrt{9}-\sqrt{5})$

$\frac{\sqrt{9}}{\sqrt{2}}-\frac{\sqrt{5}}{\sqrt{2}}$

$\boxed{\frac{\sqrt{9}-\sqrt{5}}{\sqrt{2}} \quad 3}$

④ $\sqrt{2\sqrt{3}-3}$

$\sqrt{-(3-2\sqrt{3})}$

$\sqrt{-(\sqrt{3})^2+}$

$\sqrt{-(}$

$\sqrt{\sqrt{2\sqrt{3}-\sqrt{9}}}$

H.W - 25-04-2024

① $\sqrt{\cancel{2\sqrt{3}-3}}$ $\sqrt{2\sqrt{2}-3}$

② $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}}$

③ $\sqrt[4]{17+12\sqrt{2}}$

~~④ $x=\sqrt{3+2\sqrt{3}}$ find x^3-x~~

④ $x=\dfrac{1}{2+\sqrt{3}}$ find $x^3-x^2-11x+4$

i) $x = \sqrt{3+2\sqrt{2}}$

$x^3 - x^2 - 11x + 4$

$x = \sqrt{3+\sqrt{9-12}}$

$x = \frac{1}{\sqrt{2}}\sqrt{4\cancel{2}\sqrt{3} + 2\sqrt{9}}$

$x = \sqrt{(\sqrt{2})^2 + (\sqrt{1})^2 + 2\sqrt{2}\sqrt{1}}$

$= (\sqrt{2}+\sqrt{1})$

$(\sqrt{2}+\sqrt{1})^3 - (\sqrt{2}+\sqrt{1})^2 - 11(\sqrt{2}+\sqrt{1}) + 4$

$(\sqrt{2}+\sqrt{1})^3 - 3\sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$

$(\sqrt{2})^3 + (\sqrt{1})^2 + 3\sqrt{2}(\sqrt{2}+\sqrt{1}) - 3 - \sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$

$(\sqrt{2})^3 + 1 + 6 + 1 - 3 - \sqrt{2} - 11\sqrt{2} - 11\sqrt{1} + 4$

$(\sqrt{2})^3 + 12 - 3 - 12\sqrt{2} - 11$

$(\sqrt{2})^3 - 2 - 12\sqrt{2}$

Q $\sqrt{3+2\sqrt{2}}$ find (1) $x^3-x^2-11x+4$
(2) $2x^3-x^2-8x+120$

(1)(2) $\sqrt{3+2\sqrt{2}}$

$\sqrt{(\sqrt{2})^2+(\sqrt{1})^2+2\sqrt{2}\sqrt{1}}$

$x=\sqrt{2}+1$

$(x-1)=\sqrt{2}$

$2=x^2+1-2x$

$x^2-1-2x=0$

$x^2-2x-1=0$

$2x^2-4x-2=0$

$2x^3-x^2-8x+120$

$x(2x^2-x-8)+120$

$x(x^2-2x-1+x^2+x-7)+120)=0$

$x(0x^2+x-7)+120=0$

$x^3+x^2-7x+120=0$

$x^3+x^2-2x-1+121-5x=0$

$x^3+0+121-5x=0$

$x^3+121-5x=0$

$x^3-5x+121=0$

$x(2x^2-4x-2-6+3x)+120=0$

$x(0-6+3x)+120=0$

$3x^2-6x+120=0$

$3(x^2-2x+40)=0$

$3(x^2-2x-1+41)=0$

$3(41)$

(10) $\boxed{123}$

① $x^3 - x^2 - 11x + 4$

$x(x^2 - x - 11) + 4$

$x(x^2 - 2x - 1 - 10 + x) + 4$

$x(x - 10) + 4$

$x^2 - 10x + 4$

$x^2 - 2x - 1 + 5 - 8x$

$0 + 5 - 8x$

$5 - 8x$

$5 - 8(\sqrt{2} + 1)$

$5 - 8\sqrt{2} - 8$

$\boxed{-8\sqrt{2} - 3}$

H.W. 25-04-2024

1. $\cancel{\sqrt{2\sqrt{2}-3}}$

$\cancel{\sqrt{-(3-2\sqrt{2})}}$

$\cancel{\sqrt{-((\sqrt{2})^2+(\sqrt{1})^2-2\sqrt{2}\sqrt{1})}}$

$\cancel{\sqrt{-(\sqrt{2}-\sqrt{1})^2}}$

$\cancel{\sqrt{2\sqrt{2}-3}}$

2. $\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{7+\sqrt{48}}}}$

$\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{3+4+2\sqrt{4}\sqrt{3}}}}$

$\sqrt{3+\sqrt{3}+\sqrt{2+\sqrt{3}+\sqrt{3}+\sqrt{4}}}$

$\sqrt{3+\sqrt{3}+\sqrt{4+2\sqrt{3}\sqrt{1}}}$

$\sqrt{3+\sqrt{3}+\sqrt{3+1\ 2\sqrt{3}\ \sqrt{1}}}$

$\sqrt{3+\sqrt{3}+\sqrt{(\sqrt{3})^2+1^2+2\sqrt{3}\sqrt{1}}}$

$\sqrt{3+\sqrt{3}+\sqrt{(\sqrt{3}+1)^2}}$

$\sqrt{3+\sqrt{3}+\sqrt{3}+1}$

$\sqrt{4+2\sqrt{3}}$

$\sqrt{(\sqrt{3}+1)^2}$

Q2 $\boxed{\sqrt{3}+1 \quad Q2}$

(3) $\sqrt[4]{17+12\sqrt{2}}$

$\sqrt[4]{17+2\sqrt{2}\sqrt{36}}$

$\sqrt[4]{17+2\sqrt{72}}$

$\sqrt[4]{9+8+2\sqrt{9}\sqrt{8}}$

$\sqrt[4]{(\sqrt{9})^2+(\sqrt{8})^2+2\sqrt{9}\sqrt{8}}$

$\sqrt[4]{(\sqrt{9}+\sqrt{8})^2}$

$\sqrt{\sqrt{9}+\sqrt{8}}$

$\sqrt{3+\sqrt{8}}$

$\sqrt{3+2\sqrt{2}}$

$\sqrt{(\sqrt{2})^2+1^2+2\sqrt{2}\sqrt{1}}$

$\sqrt{(\sqrt{2}+1)^2}$

$\boxed{\sqrt{2}+1 \quad (3)}$

(4) $x = \frac{1}{2+\sqrt{3}}$, find $x^3-x^2-11x+4$

$x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \frac{2-\sqrt{3}}{4-3}$

$x = 2-\sqrt{3}$

$x = 2 - \sqrt{3}$

$\sqrt{3} = 2 - x$

$(\sqrt{3})^2 = (2 - x)^2$

$3 = 4 + x^2 - 4x$

$x^2 - 4x + 1 = 0$ $\qquad$ $3x^2 - 12x + 3 = 0$

$x^3 - 2x^2 - 11x + 4$

$x(x^2 - x - 11) + 4$

$x(x^2 - 4x + 1 + 3x - 12) + 4$

$x(0 + 3x - 12) + 4$

$3x^2 - 12x + 4$

$3x^2 - 12x + 3 + 1 = 0$

$0 + 1 = 0$

$\boxed{= 1 \qquad (1)\ 4.}$

O-3

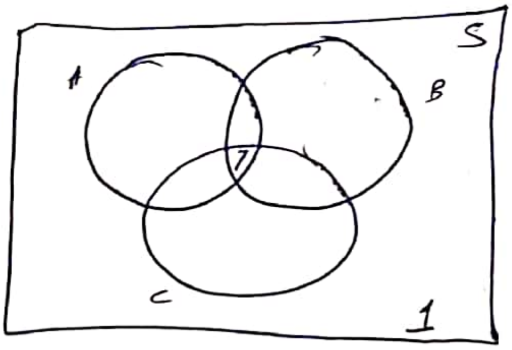
Q4. ✓A) $A = B = C$

✓B) $A = B = C$ (on solving)

✓C) $A - B = (A \cup B \cup C) - (A \cap B \cap C)$

$\phi = \phi$ for $(A \cap B \cap C)$ be max

✗D)

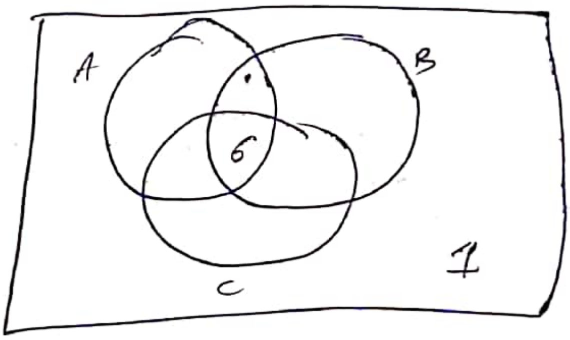


Q5. ✗A)

✓B)

✓C)

✓D)

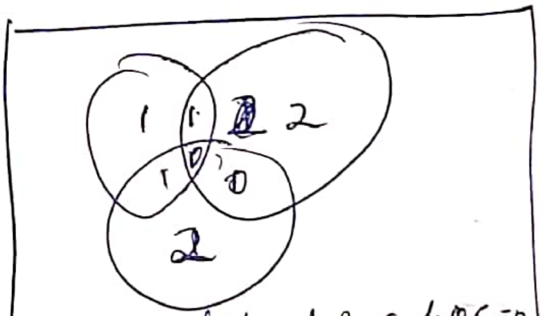


Q6. ✓A)

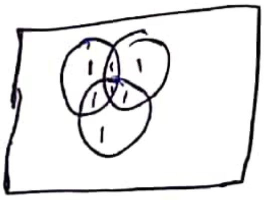
✓B)

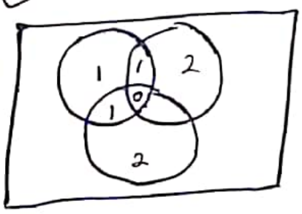
C)

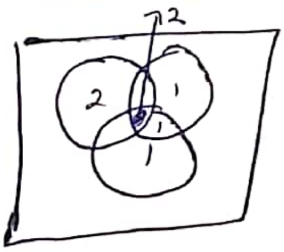
✓D)

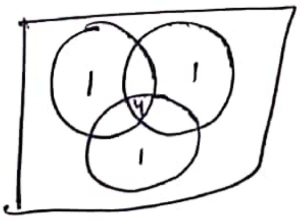


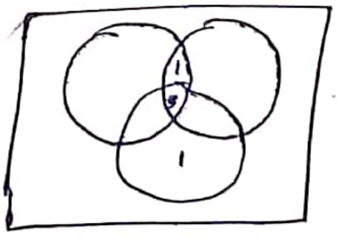
min val of $A \cap B \cap C$ for Q6 = 0

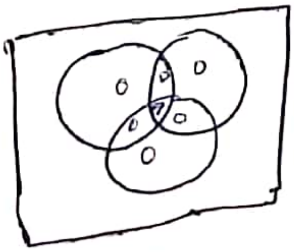


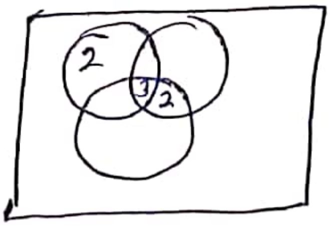












7 cases

Q $x = \sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{7\ldots\infty}}}}}}$

~~$x = 7^{1/2}$~~

$x = \sqrt{7x}$

$x^2 = 7x$

$\frac{x^2}{x} = 7$

$\boxed{x = 7}$, $\boxed{\cancel{x = 0}}$ $\sqrt{7}$ is greater than 0.

Q $x = \sqrt{6+\sqrt{6+\sqrt{6\ldots\infty}}}$

$x = \sqrt{6+x}$

$x^2 = 6 + x$

~~$x^2 - x = 6$~~

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$x - 3 = 0$

$x + 2 = 0$

$\boxed{x = 3}$

$\boxed{\cancel{x = -2}}$

Q $\sqrt{\frac{1}{\sqrt{10}+\sqrt{9}}+\frac{1}{\sqrt{11}+\sqrt{10}}+\frac{1}{\sqrt{12}+\sqrt{11}}} = \sqrt{a\sqrt{b}-b}$

$\cancel{\frac{\sqrt{11}+\sqrt{10}+\sqrt{10}+\sqrt{9}}{\sqrt{10}\sqrt{11}+10+\sqrt{9}+\sqrt{11}+\sqrt{9}+\sqrt{10}}}$

$\cancel{\frac{\sqrt{9}+2\sqrt{10}+\sqrt{11}}{10+\sqrt{110}+\sqrt{99}+\sqrt{90}}}$

$\cancel{\frac{3+2\sqrt{10}+11}{10+\sqrt{90}}}$

$\frac{\sqrt{10}-\sqrt{9}}{1} + \sqrt{11}-\sqrt{10} + \sqrt{12}-\sqrt{11}$

$\cancel{\sqrt{10}}-\sqrt{9}+\cancel{\sqrt{11}}-\cancel{\sqrt{10}}+\sqrt{12}-\cancel{\sqrt{11}}$

$\sqrt{12}-\sqrt{9}$

$\sqrt{12}-3$

$2\sqrt{3}-3 = a\sqrt{b}-b$

$b=3$

$a=2$

$a+b$

$=3+2$

$\boxed{=5}$

H.W. 27-07-2024

(1) $\sqrt{x}+\frac{1}{\sqrt{x}}=3$

$x^3+\frac{1}{x^3}=?$

(2) $\sqrt{8+2\sqrt{15}}+\sqrt{8-2\sqrt{15}}-\sqrt{20}$

(3) $\sqrt{2x+3}-\sqrt{3x-5}=1$

(4) $\sqrt{1988^{\sqrt{x}}}=(1988)^{\frac{1}{\sqrt{x}}}$

Class 9 & 10th polynomial, factorisation, formula (square, cube)

reminder Theorm

factor Theorm

find k using factor

SOS (Sum of Squares)

$x^2 \geq 0$

$(x^2 - a)^2 \geq 0$

$(x + a)^2 \geq 0$

$(x-a)^2 + (x+6)^2 + (x-c)^2 \cancel{+2(x+d)^2} \cancel{\geq 0} \geq 0$

$$\boxed{(x-a)^2 + (y-b)^2 + (z+c)^2 = 0 \text{ if } \begin{array}{l} x = a \\ y = b \\ z = -c \end{array}}$$

$\sqrt{x} \geq 0$

$\sqrt{x-a} \geq 0$

$\sqrt{x+a} \geq 0$

$$\boxed{\sqrt{x-a} + \sqrt{y-b} + \sqrt{z+c} = 0 \text{ if } \begin{array}{l} x = a \\ y = b \\ z = -c \end{array}}$$

Q1. find x, y, z for

① $\sqrt{3x-2} + \sqrt{y-2} = 6$

$\boxed{x = \frac{2}{3} \quad y = 2}$

② $\sqrt{x+2} + (y-3)^2 + (z+4)^2 = 6$

$\boxed{x = -2, y = 3, z = -4}$

③ $(p-2)^2+(q-100)^2+(r-3)^2=0$

$$\boxed{\begin{aligned} p&=2\\ q&=100\\ r&=3\end{aligned}}$$

④ $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}\ ,\ x\in R$

$(x+1)+(x-1)-2\sqrt{x+1}\sqrt{x-1}$

$x+1+x-1-2\sqrt{(x+1)(x-1)}$

$2x-2\sqrt{x^2-1}=4x-1$

$2x-4x+1=2\sqrt{x^2-1}$

$\dfrac{-2x+1}{2}=\sqrt{x^2-1}$

$\dfrac{4x^2+1+4x}{4}=x^2-1$

$\cancel{4x^2}+1+4x=\cancel{4x^2}-4$

$4x=-5$

$$\boxed{x=\frac{-5}{4}}$$

100

$$(x+1)+(x-1) - 2\sqrt{x^2-1} = 4x-1$$

$$x+x\cancel{+1-1} - 2\sqrt{x^2-1} = 4x-1$$

$$\frac{-2x+1}{2} = \sqrt{x^2-1}$$

$$\frac{1-2x}{2} = \sqrt{x^2-1}$$

$$\frac{1+4x^2-4x}{4} = x^2-1$$

$$\cancel{4x^2}-4x+1 = \cancel{4x^2}-4$$

$$-4x = -4-1$$

$$-4x = -5$$

$$\boxed{x = \frac{5}{4}}$$ Rejected

no value of x

H.W. 27-04-2024

Q1. $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2} + 2 \times \sqrt{x} \times \frac{1}{\sqrt{x}}$

$(3)^2 = x + \frac{1}{x} + 2$

$(3)^2 = x + \frac{1}{x} + 2$

$9 - 2 = x + \frac{1}{x}$

$\boxed{x + \frac{1}{x} = 7}$

$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x+\frac{1}{x}\right)$

$(7)^3 = x^3 + \frac{1}{x^3} + 3(7)$

$343 = x^3 + \frac{1}{x^3} + 21$

$343 - 21 = x^3 + \frac{1}{x^3}$

$\boxed{x^3 + \frac{1}{x^3} = 322}$

② $\sqrt{8+2\sqrt{15}}+\sqrt{\cancel{3x-5}\,8-2\sqrt{15}}-\sqrt{20}$

$\sqrt{5+3+2\sqrt{15}}+\sqrt{8-5+3-2\sqrt{15}}-\sqrt{20}$

$\sqrt{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\sqrt{3}}+\sqrt{(\sqrt{5})^2+(\sqrt{3})^2-2\sqrt{5}\sqrt{3}}-\sqrt{20}$

$\sqrt{(\sqrt{5}+\sqrt{3})^2}+\sqrt{(\sqrt{5}-\sqrt{3})^2}-\sqrt{20}$

$(\sqrt{5}+\sqrt{3})+(\sqrt{5}-\sqrt{3})-\sqrt{4}\sqrt{5}$

$\sqrt{5}+\cancel{\sqrt{3}}+\sqrt{5}-\cancel{\sqrt{3}}-2\sqrt{5}$

$2\sqrt{5}-2\sqrt{5}$

$\boxed{=0}$

Q3. $\sqrt{2x+3}-\sqrt{3x-5}=1$

$\sqrt{2x+3}=1+\sqrt{3x-5}$

$2x+3=1+3x-5+2\sqrt{3x-5}$

$\frac{2x+3-1+5-3x}{2}=\sqrt{3x-5}$

$\frac{7-3x}{2}=\sqrt{3x-5}$

$$\frac{49 + 9x^2 - 42x}{4} = 3x - 5$$

$$49 + 9x^2 - 42x = 12x - 20$$

$$9x^2 - 42x - 12x + 49 + 20 = 0$$

$$9x^2 - 54x + 69 = 0$$

$$3x^2 - 18x + \cancel{3}23 = 0$$

$$\cancel{x = +324} \pm \sqrt{}$$

$$x = +6 \pm \sqrt{b^2 - 4ac}$$

$$x = \frac{18 \pm \sqrt{324 - 276}}{6}$$

$$= \frac{18 \pm \cancel{1}7}{6}$$

$$\boxed{x = \frac{\cancel{1}11}{6}}$$

$$\boxed{x = \frac{25}{6}}$$

$$\begin{array}{r} 18 \\ \times 18 \\ \hline 144 \\ 180 \\ \hline 324 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

$$\begin{array}{r} 4 \\ \times 12 \cdot 2x \\ 6 \\ \hline 72 \end{array}$$

$$\begin{array}{r} 13 \\ \times 5 \\ \hline 65 \end{array}$$

$$\begin{array}{r} 14 \\ \times 4 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 24 \\ + 24 \\ \hline 48 \end{array}$$

Q4.

$$\sqrt{1988^{\sqrt{x}}} = 1988^{\frac{1}{\sqrt{x}}}$$

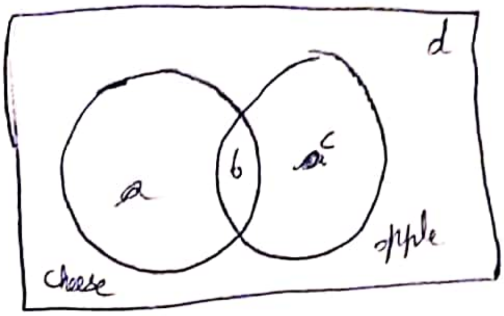
$$1988^{\frac{\sqrt{x}}{2}} = 1988^{\frac{1}{\sqrt{x}}}$$

$$\frac{\sqrt{x}}{2} = \frac{1}{\sqrt{x}}$$

$$\boxed{x = 2}$$

0-4

Q2.



$a+b=63$

$b+c=76$

$a+2b+c=139$

$a+b+c+d=100$

$\cancel{a+d=}$

$b=39+d$

$b=39$ (min)

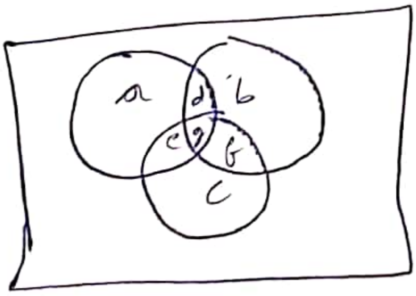
$b=63$ (max)

no. of values = $\cancel{63-39}$ $[39,64]$

$=64-39+1$

$\boxed{=25}$

Q3b.



$d+e+f=?$

$e+f+c=90$

$b+d+f=120$

$a+d+e=170$

1 + 2 + 3

$a+b+c+2(d+e+f)=380$ - 4

$a+b+c+d+e+f+g=300$ - 5

4 - 5

$\boxed{d+e+f=80}+30=110$

106

Q9. [1, 2000]

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 (6)

14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26 (6)

$\frac{2000}{13} = 153 \times 6$

$= 918$

$918 + 6 \rightarrow 924$

wrong in module

1990
1991
1992
1993
1994
1995
1996
1997
1998 (6)
1999
2000

$\frac{\cancel{2000}}{13}$ 5 x

16

$13 \overline{)1989}$ (153

-13

68

-65

39

$-$

13
26
39
42
55

$153 \times 5 = 765$

factorisation, factor theorem & cyclic expression

1. $a^2-b^2 = (a+b)(a-b)$
2. $(a+b)^2 = a^2+b^2+2ab$
3. $(a-b)^2 = a^2+b^2-2ab$
4. (2) − (3)

$$\boxed{(a+b)^2-(a-b)^2 = 2ab+2ab}$$

5. $(a+b)^3 = a^3+b^3+3ab(a+b)$
6. $(a-b)^3 = a^3-b^3-3ab(a-b)$
7. $a^3+b^3 = a+b(a^2+b^2-ab)$
8. $a^3-b^3 = (a-b)(a^2+b^2+ab)$
9. $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ac)$

$$\Downarrow$$

$$\boxed{= a^2+b^2+c^2+2abc\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)}$$

10. $a^2+b^2+c^2-ab-bc-ac$

$$=\frac{1}{2}\left[2a^2+2b^2+2c^2-2ab-2bc-2ac\right]$$

$$=\frac{1}{2}\left[a^2+a^2+b^2+b^2+c^2+c^2-2ab-2bc-2ac\right]$$

$$=\frac{1}{2}\left[a^2+b^2-2ab+a^2+c^2-2ac+b^2+c^2-2bc\right]$$

$$\boxed{= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (a-c)^2\right]}$$

Always greater or equal to zero.

(11) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

(12) $a^3+b^3+c^3-3abc=0$ if $(a+b+c)=0$

or $\left[\frac{1}{2}\left[(a-b)^2+(b-c)^2+(a-c)^2\right]\right]=0$

$\downarrow$

so, $a=b=c$

$$\boxed{\begin{gathered} a^3+b^3+c^3-3abc=0 \\ a=b=c \\ (a+b+c)=0 \end{gathered}}$$

(13) $a^4-b^4 = (a^2)^2-(b^2)^2$

$= (a^2-b^2)(a^2+b^2)$

$\boxed{= (a-b)(a+b)(a^2+b^2)}$

(14) $a^4+1+a^2 = a^4+2a^2+1+a^2-2a^2$

$= (a^2+1)^2-a^2$

$\boxed{= (a^2+1+a)(a^2+1-a)}$

H.W. 29-04-2024

Q find value of the following:-

~~(1) $(x-y)^2$.~~

(1) $$\frac{(x-y)^3+(y-z)^3+(z-x)^3}{3[(x-y)\cancel{(x-z)}(y-z)(z-x)}$$

(2) $$\frac{(2.75)^3-(2.25)^3}{(2.75)^2+2.75\times2.25+(2.25)^2}$$

(3) if $p=2-a$

$a^3+6ap+p^3-8$

(4) DYS-2
Q1, 2, 3, 6, 4, 5
DYS-1 (all)

① $A = x-y$

$B = y-z$

$C = z-x$

$A+B+C = (x-y)+(y-z)+(z-x)$

$= \cancel{x}-\cancel{y}+\cancel{y}-\cancel{z}+\cancel{z}-\cancel{x}$

$= 0$

$A^3+B^3+C^3 = 3ABC$

$(x-y)^3+(y-z)^3+(z-x)^3 = 3(x-y)(y-z)(z-x)$

Given Eqn

$$\frac{(x-y)^3+(y-z^3)+(z-x)^3}{3(x-y)(y-z)(z-x)}$$

$$\frac{\cancel{3(x-y)(y-z)(z-x)}}{\cancel{3(x-y)(y-z)(z-x)}}$$

$\boxed{=1}$

② let $A = 2.75$

$B = 2.25$

Given :-

$$\frac{A^3-B^3}{A^2+AB+B^2}$$

$$= \frac{(A-B)\cancel{(A^2+B^2+AB)}}{\cancel{A^2+B^2+AB}}$$

$= A - B$

$= 2.75 - 2.25$

$= 0.50$

$\boxed{= \frac{1}{2}}$

Q3. $p = 2 - a$

find $a^3 + 6ap + p^3 - 8$

$p + a = 2$

$(p+a)^3 = (2)^3$

$p^3 + a^3 + 3pa(p+a) = 8$

$p^3 + a^3 - 8 = -3pa(p+a)$

$-3pa(p+a) + 6pa$

$+ 3pa(2 - (p+a))$

$+ 3(2 - p - a)$

$\boxed{6 - 3p - 3a}$

① DYS-1

Q6, iii) $\dfrac{\sqrt{6+2\sqrt{3}+2\sqrt{2}+2\sqrt{6}}-1}{\sqrt{5+2\sqrt{6}}}$

$\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+(\sqrt{1})^2+2\sqrt{2}\sqrt{1}+2\sqrt{3}\sqrt{1}+2\sqrt{3}\sqrt{2}}$

$\dfrac{\sqrt{(\sqrt{3}+\sqrt{2}+\sqrt{1})^2}-1}{\sqrt{3}+\sqrt{2}}$

$\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$\boxed{=1}$

DYS-2

Q6 i) $8a^6+5a^3+1$

$(2a^2)^3+6a^3-a^3+1$

$(2a^2)^3+(-a)^3+(1)^3+6a^3$

$A^3+B^3+C^3-3ABC = (A+B+C)(a^2+b^2+c^2+ab+bc+ac)$

$\boxed{(2a^2-a+1)(4a^4+a^2+1-2a^3-a+2a^2)}$

Q If $4x^2 + 3y^2 + 16z^2 - 4x + 12y \cancel{24z} - 24z + 14 = 0$
$x, y, z = ?$

$\cancel{(2x)^2 + (\sqrt{3}y)^2 + (4z)^2 - (2)(x) +}$

$\cancel{(2x)^2 + (3y)^2 + (4z)^2 - 4x + 12y - 24z + 14 = 0}$

$\cancel{(2x)^2 + (3y)^2 + (4z)^2 - 4(x - 3y + 6y) + 14 = 0}$

$4x^2 - 4x + 9y^2 + 12y + 16z^2 - 24z + 14 = 0$

$(2x)^2 - (2)(2x) + (1)^2 + (3y)^2 + (2)(3y)(2) + (2)^2 + (4z)^2 - (2)(4z)(3) + (3)^2$

$(2x - 1)^2 + (3y + 2)^2 + (4z - 3)^2 = 0$

$2x - 1 = 0$	$3y + 2 = 0$	$4z - 3 = 0$
$2x = 1$	$3y = -2$	$4z = 3$
$\boxed{x = 1/2}$	$\boxed{y = -\frac{2}{3}}$	$\boxed{z = 3/4}$

Factorisation:

Method ~~Type~~ -1 :- Conversion into perfect square.

Q- $x^4 + 3x^2y^2 + 4y^4$

$(x^2)^2 + (2y)^2 + 4x^2y^2 - x^2y^2$

$(x^2 + 2y^2)^2 - x^2y^2$

$(x^2 + 2y^2 + xy)(x^2 + 2y^2 - xy)$

Q $4x^4 + 81$

$(2x^2)^2 + (9)^2 + 36x^2 - 36x^2$

$(2x^2+9)^2 - (6x)^2$

$\boxed{(2x^2+9-6x)(2x^2+9+6x)}$

Method -2 - Factor Theorm

$6x^3 - 5x^2 - 3x + 2$

$\cancel{x=1}$

$\cancel{6-5-3+1}$

$\cancel{-6-5+3+2}$

$\cancel{48-20-6+2}$

$6-5-3+2$

$-8+8=0$

$(x-1)$

$6x^2 + x - 2$

$6x^2 + 4x - 3x - 2$

$2(3x+2) - 1(3x+2)$

$(2x-1)(3x+2)$

$\cancel{\boxed{x=\frac{1}{2}}}$ $\cancel{\boxed{x=-\frac{2}{3}}}$

$(2x-1)(3x+2)(x-1)$

Method-3

$$6x^3 - 5x^2 - 3x + 2 = (x-1)(6x^2 + \overset{b}{\cancel{b}}x - 2)$$

$$\cancel{6x^2} \cdot 6x^2 - 6x^2 = -5x^2$$

$$6x^2 = \cancel{x}\ 6x^2 - 5x^2$$

$$6x^2 = x^2$$

$$b = 1$$

$$= (x-1)(6x^2 + x - 2)$$

Q $6x^3 + 11x^2 + 6x + 1$

$-6 + 11 - 6 + 1$

$(x+1)(6x^2 + 6x + 1)$

$6x + 6x^2 = 11x^2$

$6x^2 = 5x^2$

$b = 5$

$(x+1)(6x^2 + 5x + 1)$

$$\boxed{(x+1)(3x+1)(2x+1)}$$

Q Factorise

① $x^3 + y^3 + 8z^3 - 6xyz$

② $(x-y)^3 + (y-z)^3 + (z-x)^3$

(2) $\boxed{(3)(x-y)(y-z)(z-x)}$

(1) $\cancel{(x+y+z)(x^2+y^2+z^2+xy+yz+xz)}$

$$(x)^3+(y)^3+(2z)^3+3(2z\times x\times y)$$

$$A^3+B^3+C^3-3ABC=(A+B+C)(A^2+B^2+C^2-AB-BC-AC)$$

$$\boxed{(x+y+2z)(x^2+y^2+4z^2-xy-2xz-2yz)}$$

(3) Cyclic Expression & its factors

→ Expression will be unchanged when variables interchanged

eg. $x+y+z$ is cyclic

$\downarrow \ \downarrow \ \downarrow$

$y+z+x$

eg 2 $x-y+z$. not cyclic

$\downarrow$

$y-z+x$

eg 3. $x^2+y^2+z^2+xy+yz+zx$ is cyclic

eg 4. $x(y-z)+y(z-x)+z(x-y)$

$y(z-x)+z(x-y)+x(y-z)$ is cyclic

Note- In cyclic Expression if $(x-y)$ is a factor then $(y-z)$ and $(z-x)$ are also factors.

Q $x^2(y-z)+y^2(z-x)+z^2(x-y)$

$x^2(x-z)+x^2(z-x)+\cancel{z^2(x-x)}$

$x^2(x-z+z-x)=0$ so $(x-y)$ is a factor) and it is

cyclic so. $\boxed{(x-y)(y-z)(z-x)}$ are also factors.

Q $2xyz+x^2y+y^2z+z^2x+xy^2+yz^2+zx^2$

$\cancel{2xyz+x(}$

$\cancel{x(y}$

$\cancel{2xyz}+x^2(y+z)+y^2(z+x)+z^2(x+y)$ ~~cyclic~~

~~$x=y$~~

$x=-y$ Eqn is cyclic

$(x+y)$ is factor

$(y+z)$

$(z+x)$ is a factor as well

~~DYS-2~~ H.W- 30-04-2024

DYS-2

Q8 - i), ii), iii), iv), v), vi), vii), viii)

DYS-3

Q1, 2, 3, 4, 5, 7

Q $2xyz + x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2$

It is cyclic

$x = -y$, value $= 0$

$(x+y)$ is a factor

so $(y+z)$ are also factors
$(z+x)$

$E(x, y, z) = 2xyz + x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2$
$= A(x+y)(y+z)(z+x)$
$\downarrow$
find A.

$(A = 1)$

DYS-3

Q6. $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$

$a^3 = 2+\sqrt{5} \qquad b^3 = 2-\sqrt{5}$

$(a^3)^{1/3} + (b^3)^{1/3} = a+b = t$

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$t^3 = 4+3(4-5)(t)$

$t^3 = 4-3t$

$t^3+3t-4=0$

$(t-1)$

$\cancel{t^2+t+4}$ $\qquad -t^2+3t-4$

$\cancel{t^2+2}.$ $\qquad -t$

$\qquad 2t-4$

t^2-t+2

$t^2+2t-t+2$

$t(t+2)-1(t+2)$

$(t-1)(t+2)$

$\boxed{t=1,-2}$

Q. $\sqrt{2024 \times 2022 \times 2020 \times 2018 + 16}$

$\cancel{4\sqrt{253 \times 1011 \times 2020 \times 2018 + 1}}$

let $x = 2021$

$\sqrt{(x+3)(x+1)(x-1)(x-3)+16}$

$\sqrt{(x^2-9)(x^2-1)+16}$

$\sqrt{x^4-10x^2+9+16}$

$\sqrt{x^4-10x^2+25}$

$\sqrt{(x^2-5)^2}$

x^2-5

2021^2-5

$\boxed{4462446}$

$$\begin{array}{r} 2021 \\ 2021 \\ \hline 2021 \\ 40420 \\ 4042000 \\ \hline 4441 \end{array}$$

4462

4462441

Q2. $(2+1)(2^2+1)(2^4+1)(2^8+1) - 2^{16}$

$\cancel{(3)(5)(17)(257)} - 2^{16}$

$1 \times (2-1) =$

$$\frac{(2-1)(2+1)(2^2+1)(2^4+1)(2^8+1) - 2^{16}}{\cancel{(2+1)}}$$

$(2^2-1)(2^2+1)$

$(2^4-1)(2^4+1)$

$(2^8-1)(2^8+1)$

$(\cancel{2^{16}}-1) - \cancel{2^{16}}$

$\boxed{-1}$

Polynomial

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots\dots a_2 x^2 + a_1 x + a_0$

$a_n \neq 0$ ($a_n \rightarrow$ leading coefficient)

$n \in$ whole no.

$a_{n-1}, a_{n-2} \dots a_0$

$a_n = 1$ (monic polynomial)

monic polynomial $\rightarrow$ coefficient is 1

monomial $\rightarrow$ one term.

Types of Polynomial

Name	Degree	Format
1. zero polynomial	Not-Defined	$f(x) = 0$ $f(x) = 0x^{3}$ $= 0x^2$ $= 0x^3$ 0
2. Non-zero polynominal	0	$f(x) = c \quad (c \neq 0)$
3. Linear	1	$f(x) = ax + b \quad (a \neq 0)$
4 Quadratic	2	$f(x) = ax^2 + bx + c \quad (a \neq 0)$
5 Cubic	3	$f(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$
6 Biquadratic	4	$f(x) = ax^4 + bx^3 + cx^2 + dx + e$ $f(x) = ax^4$ $(a \neq 0)$

* Adding two polynomials may or not result in change in degree

$(x^8+x^7) + (x^8-x^6) = 2x^8+x^7-x^6$

$(x^8+x^7) - (x^8-x^6) = x^7+x^6$ (change)

*

Division in polynomial

$$\underbrace{P(x)}_{\text{Divident}} = \underbrace{Q(x)}_{\text{Quotient}} \cdot \underbrace{d(x)}_{\text{Divisor}} + \underbrace{r(x)}_{\text{Remainder}}$$

① Degree of $d(x) >$ degree $r(x)$

② Degree of $d(x) \leq P(x)$

③ $Q(x)$ & $r(x)$ are unique

④ If $d(x)$ is divisor of $P(x)$ then $k \times d(x)$ also be the divisor of $P(x)$. $(K \neq 0)$

Remainder Theorm

$P(x) = Q(x)(ax-b) + R$

$ax - b = 0$

$x = b/a$

$\cancel{\boxed{P(b/a) = R}}$

$P(b/a) = Q(b/a)(a \times \frac{b}{a} - b) + R$

$\boxed{P(b/a) = 0 + R}$

Q1. find remainder if $P(x) = x^3 - 6x^2 + 11x - 6$ is divided by

1. x
2. $2x-4$
3. $x-4$
4. $x^2 - 6x + 11$

(1) $x = 0$

$P(0) = 0 - 0 + 0 - 6$

$\boxed{= -6}$

(2) $2x - 4 = 0$

$x = \frac{4}{2}$

$x = 2$

$P(2) = 2^3 - 6 \times 2^2 + 11(2) - 6$

$= 8 - 24 + 22 - 6$

$\boxed{= 0}$

(3) $x - 4 = 0$

$x = 4$

$64 - 96 + 44 - 6$

$108 - 102$

$\boxed{6}$

(4) $x^2 - 6x + 11 \neq 0$

$\cancel{6 \pm \sqrt{36 - 44}}$

$\cancel{12x^2 - 11x}$

$\cancel{x + 12 + 72x}$

$\cancel{61x}$

(4) $x^2-6x+11 \overline{)\, x^3-6x^2+11x-6 \,}$ x

$-x^3+6x^2-11x$

$\boxed{-6}$

OR

$x^2-6x+11=0$

$x^3-6x^2+11x-6$

$x(x^2-6x+11)-6$

$x(0)-6$

$0-6$

$\boxed{-6}$

Q2. find the constants. a, b & c such that
$(2x^2+3x+7)(ax^2+bx+c)=2x^4+11x^3+9x^2+13x-35$

$2x^2+3x+7 \overline{)\, 2x^4+11x^3+9x^2+13x-35 \,}$ ax^2+4x-5

$-2x^4-3x^3-7x^2$

$8x^3+2x^2+13x-35$

$-8x^3-12x^2-28x$

~~$14x^2+24x-50$~~

~~$14x^2+41x-25$~~

$-10x^2-15x-35$

$+10x^2+15x+35$

0

$a=1$

$b=+4$

$c=-5$

$2x^{1} = 2x^{2} \times ax^{2}$

$2x^{4} = 2ax^{4}$

$\therefore \boxed{a=1}$

$2b + 3a = 11$ (for x^3)

$2b + 3 = 11$

$2b = 8$

$\boxed{b=4}$

$\cancel{3c + 7b = 9}$ (for x^2)

$\cancel{3c + 28 = 9}$

$\cancel{3c = 9 - 28}$

$\cancel{c = -\frac{19}{3}}$

$2c + 7a + 3b = 9$

$2c + 7 + 12 = 9$

$c = \frac{9 - 19}{2}$

$c = -\frac{10}{2}$

$\boxed{c = -5}$

Q3. find remainder when $P(x) = x^5 - 3x^3 + 2x^2 + 3x + 1$ is divided by $x^2 - 1$

$-2x$

$x + 3$

$x^3 - 2x + 2$

$\boxed{R = x + 3}$

~~M II~~

$\cancel{x^5 - 3x^3 + 2x^2 + 3x + 1}$

$\cancel{x^5 - x}$

$x^2 \quad x^5$

$P(x) = Q(x)(x^2 - 1) + (ax + b)$

for $x = 1 \quad (x^2 - 1 = 0)$

$P(1) = 4$

$P(1) = 0 + (a + b$

$(a + b) = 4$ — I

$P(-1) = 0 + (-a + b)$

$2 = b - a$ — II

I + II

$2b = 6$

$b = 3$

$a = 4 - b$

$a = 4 - 3$

$a = 1$

$ax + b$

$\boxed{x + 3}$

Q4. $(x-2)$ is factor of x^5-4x^3+x+k

$x-2=0$
$x=2$

$32-32+2+k=0$

$\boxed{k=-2}$

Q5. ax^3+bx^2+cx-6 has $(x-1),(x-2)$ & $(x-3)$ as factors. find a b & c

$x=1$

$a+b+c-6=0$

$\boxed{a+b+c=6}$ $a = 6-b-c$

$x=2$
$8a+4b+2c-6=0$

$\boxed{4a+2b+c=3}$

$x=3$
$27a+9b+3c-6=0$

$\boxed{9a+3b+c=2}$

$4(6-b-c)+2b+c=3$

$24-4b-4c+2b+c=3$

$21=+2b+3c$

$\frac{21-2b}{3}=c$

$9(6-b-c)+3b+\frac{21-2b}{3}=c$

$54-9b-9c+3b+\frac{21-2b}{3}=3$

$-6b$

H.W.

DYS-4 (Q1, 2, 3, 4, 5, 6, 7, 8)

DYS-3 (Q8)

Q5. $-a + b + c = 6$

$4a + 2b + c = 3$

$3a + b = -3$

$-9a + 3b = +9$

$9a + 3b + c = 2$

$\boxed{c = 11}$

$a + b + 11 = 6$

$a + b = -5$

$a = -5 - b$

$-15 - 3b + b = -3$

$-15 - 2b = -3$

$-12 = 2b$

$\boxed{b = -6}$

$a = -5 - b$

$a = -5 + 6$

$\boxed{a = 1}$

Q5. <u>Method II</u>

$$ax^3+bx^2+cx-6 = (x-1)(x-2)(x-3)$$
$$= (x^2-2x-x+2)(x-3)$$
$$= (x^2-3x+2)(x-3)$$
$$= x^3-3x^2+2x-3x^2+9x-6x$$
$$= x^3-6x^2+11x-6$$

$$\boxed{\begin{array}{ll} a=1 & [x^3] \\ b=-6 & [x^2] \\ c=11 & [x] \end{array}}$$

Q If $f(x)$ is a 4 degree polynomial having leading coefficient 1 such that

$f(1)=1$
$f(2)=4$
$f(3)=9$
$f(4)=16$

then find the value of $f(5)=25$

$$f(x) = x^2$$

$$f(x)-x^2 = (x-1)(x-2)(x-3)(x-4)$$

$x=5$

$$f(5)-25 = 4\times3\times2\times1$$
$$= 24+25$$
$$f(5) = 49$$

DYS-4

Q10

$f(1)=1$

$f(2)=2$

$f(3)=3$

$f(4)=4$

$f(0)=1$

$f(x)-x = A(x-1)(x-2)(x-3)(x-4)$

$f(0)-0 = A(-1)(-2)(-3)(-4)$

$\frac{1}{A} = 24$

$\boxed{A = \frac{1}{24}}$

$f(5)-5 = \frac{1}{\cancel{24}}(4)(3)(2)(1)$

$\boxed{\cancel{f(5) = 5}}$ $\frac{1}{\cancel{24}} \times 24$ 1

$f(5)-5=1$

$f(5)=1+5$

$\boxed{f(5) = 6}$

Q9. $f(x) = (x-1)(x+2)\cancel{(x+1)}$

$f(x) \cancel{-4} = (x+1) + 4$

$f(x) \cancel{-28} = (x-2) + 28$

$(x^2+2x-x-2)(x+\cancel{a}5)$

$(x^2+x-2)(x+5)$

$x^3+5x^2+x^2+5x-2x-10$

$A(x^3+6x^2+3x-10) =$

$A(8+24+6-10) = 28$

$A(28) = 28$

$A = 1$

$x^3+6x^2+3x-10$

DYS-4

Q9. $f(x) = (x-1)(x+2)(ax+b)$

$x = -1$

$4 = (-2)(1)(b-a)$

$b - a = 2$

$28 = (1)(4)(2a+b)$

$7 = 2a+b$

$\cancel{2 = b - a}$

$+2 = -b + a$

$9\cancel{a} = 3a$

$\boxed{a=3}$

$\boxed{b=5}$

$(x-1)(x+2)(3x+5)$

$(x^2+x-2)(3x+5)$

Equations Reducible to quadratic

$a(\text{कुछ ई})^2 + b(\text{कुछ ई}) + c = 0$

$\text{कुछ ई} \rightarrow x^{1/5}, x^4, (\sqrt{5}+\sqrt{7})^x, x^2+\frac{1}{x^2}, (x+\frac{1}{x})^2, 2^x$ etc

eg.1. $x^{2/5} + x^{1/5} + 2 = 0$

$1(x^{1/5})^2 + 1(x)^{1/5} + 2$

$t = x^{1/5}$

$t^2 + t + 2$

$\cancel{t^2+2}$

eg 2. $4^x + (2 \cdot 2)^x - 7 = 0$ → multiply

eg 3. $(\sqrt{5}+\sqrt{3})^x + (\sqrt{5}-\sqrt{3})^x - 8 = 0$

Method → 1. Assume कुछ ई = t such that first term have square, second term is linear and 3rd term is constant.

2. Solve the quadratic in t

3. get values of small t & replace t by कुछ ई

Q1. $5^{2x} - 6 \times 5^{x+1} + 125 = 0$

$(5^{x})^2 - 30(5^{x}) + 125 = 0$

$t = 5^{x}$

$t^2 - 30t + 125$

$t^2 - 25t - 5t + 125$

$t(t-25) - 5(t-25)$

$t = 5$

$5 = 5^{x}$

$\boxed{x = 1}$

$t = 25$

$25 = 5^{x}$

$\boxed{x = 2}$

Q2. $x^{2/3} + x^{1/3} - 2 = 0$

$(\sqrt[3]{x})^2 + 2(\sqrt[3]{x}) - \sqrt[3]{x} - 2 = 0$

$\sqrt[3]{x}(\sqrt[3]{x} + 2) - 1(\sqrt[3]{x} + 2)$

$(\sqrt[3]{x} - 1)(\sqrt[3]{x} + 2)$

$\sqrt[3]{x} = 1$

$\boxed{x = 1}$

$\sqrt[3]{x} = -2$

$\boxed{x = -8}$

Q3. $4^x + 3 \cdot 2^{x+3} + 128 = 0$

$(2^x)^2 + 24(2^x) + 128 = 0$

$(2^x)^2 + 16(2^x) + 8(2^x) + 128 = 0$

$2^x(2^x+16) + 8(2^x+16)$

$2^x + 8 = 0$

$2^x = -8$

$\cancel{2^x = -(2^3)}$

$\cancel{\boxed{x = -3}}$

not possible

$2^x + 16 = 0$

$\cancel{\boxed{x = -4}}$

not possible

Q4. $4^x - 3 \cdot 2^{x+3} + 128$

$2^x = 8$

$\boxed{x = 3}$

$2^x = 16$

$\boxed{x = 4}$

H.W. 3-5-24

DYS-5

Q 2, 3, Q 5, Q 7, (Q 15-f)

DYS-4

Q 8, 12

Q $3 \cdot 4^x + 2 \cdot 9^x - 5 \cdot 6^x = 0$

$3 \cdot 2^{2x} + 2 \cdot 3^{2x} - 5 \cdot 2^x 3^x$

$\cancel{\frac{3 \cdot 2^{2x}}{3^x} + \frac{2 \cdot 3^{2x}}{3^x} - \frac{5 \cdot 2^x 3^x}{3^x}}$

$\cancel{\frac{3 \cdot 2^{2x}}{2^x} + \frac{2 \cdot 3^{2x}}{2^x} - \frac{5 \cdot 2^x 3^x}{2^x}}$

$\frac{3 \cdot 2^{2x}}{2^{2x}} + \frac{2 \cdot 3^{2x}}{2^{2x}} - \frac{5 \cdot 2^x 3^x}{2^{2x}}$

$3 + 2 \cdot \left(\frac{3}{2}\right)^{2x} - 5 \times \left(\frac{3}{2}\right)^x$

$t = \left(\frac{3}{2}\right)^x$

$2t^2 - 5t + 3$

$\frac{5 \pm \sqrt{24-24}}{4}$

$\frac{5+1}{4}$ $\frac{5-1}{4}$

$\boxed{t = \frac{3}{2}}$ $\boxed{t = 1}$

$\frac{3}{2} = \frac{3}{2}^x$

$\boxed{x = 1}$

$\left(\frac{3}{2}\right)^x = 1$

$\boxed{x = 0}$

Note -

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\therefore x+\frac{1}{x} = t$$

$$t^2 = x^2+\frac{1}{x^2}+2$$

$$t^2-2 = x^2+\frac{1}{x^2}$$

$$\boxed{\begin{array}{ll} x+\frac{1}{x} = t & x-\frac{1}{x} = t \\ x^2+\frac{1}{x^2} = t^2-2 & x^2+\frac{1}{x^2} = t^2+2 \end{array}}$$

Q1. $3\left(x^2+\frac{1}{x^2}\right) - 16\left(x+\frac{1}{x}\right) + 26 = 0$

$3(t^2-2) - 16(t) + 26 = 0$

$3t^2 - 6 - 16t + 26$

$3t^2 - 16t + 20$

$3t^2 - 6t - 10t + 20$

$3t(t-2) - 10(t-2)$

$(3t-10)(t-2)$

$\boxed{t=2}$ $\boxed{t = 10/3}$

$x+\frac{1}{x} = t$

$x^2+1 = 2x$

$x^2-2x+1=0$

$x+\frac{1}{x} = \frac{10}{3}$

$x^2+1 = \frac{10x}{3}$

$3x^2-10x+3 = 0$

$x^2 - x - x + 1$

$x(x-1) - 1(x-1)$

$\boxed{x = 1}$

$\dfrac{10 \pm \cancel{\sqrt{}}\ 8}{6}$

$\dfrac{2}{6} = \dfrac{1}{3}$

$\boxed{x = \dfrac{1}{3}}$ $\boxed{x = 3}$

DVS - ds

(6) $2\left(x^2 + \dfrac{1}{x^2}\right) - 3\left(x - \dfrac{1}{x}\right) - 4 = 0$

$2(t^2 + 2) - 3t - 4 = 0$

$2t^2 + 4 - 3t - 4 = 0$

$2t^2 - 3t = 0$

$t(2t - 3) = 0$

$t = 0$ $\boxed{t = 3/2}$ ✓

$t = 3/2$

$x - \dfrac{1}{x} = t$

$x - \dfrac{1}{x} = \dfrac{3}{2}$

$x^2 - 1 = \dfrac{3x}{2}$

$2x^2 - 3x - 2 = 0$

$\dfrac{3 \pm \sqrt{9 + 16}}{4}$

$\dfrac{3 + 5}{4}$ $\dfrac{3 - 5}{4}$

$\boxed{2}$ $\boxed{-\dfrac{1}{2}}$

$t = 0$

$x - \dfrac{1}{x} = 0$

$x^2 - 1 = 0$

$x^2 = +1$

$\boxed{x = \pm 1}$

Q $2^{2x+1} - 7\cdot 10^{x} + 5^{2x+1} = 0$

$2\cdot 2^{2x} - 7\cdot 2^{x}\cdot 5^{x} + 5\cdot 5^{2x}$

$\dfrac{2\cdot 2^{2x}}{5^{2x}} - \dfrac{7\cdot 2^{x}\cdot 5^{x}}{5^{2x}} + 5$

$2\cdot\left(\left(\frac{2}{5}\right)^{\cancel{2x}}\right)^{x^2} - 7\cdot\left(\frac{2}{5}\right)^{x} + 5$

$t = \left(\frac{2}{5}\right)^{x}$

$2t^{2} - 7t + 5$

$\dfrac{7 \pm \sqrt{9}}{4}$

$\dfrac{10}{4}$ | $\dfrac{4}{4}$

$t = \dfrac{5}{2}$

$\left(\frac{2}{5}\right)^{x} = \frac{5}{2}$

$\boxed{x = -1}$ ✓

$t = 1$

$\left(\frac{2}{5}\right)^{2} = 1$

$\left(\frac{2}{5}\right)^{x} = \left(\frac{2}{5}\right)^{0}$

$\boxed{x = 0}$ ✓

Q15. F $(5^{2x}-7^{x})-35(5^{2x}-7^{x})=0$

$$(5^{2x}-7^{x})(-34)=0$$

$$5^{2x}=7^{x}$$

$$25^{x}=7^{x}$$

$$A \neq B$$

$$A \neq B$$

Power $= \boxed{x=0}$

Q $(5+2\sqrt{6})^{x/2}+(5-2\sqrt{6})^{x/2}=10$

$$5-2\sqrt{6}=5-2\sqrt{6}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}=\frac{25-24}{5+2\sqrt{6}}=\frac{1}{5+2\sqrt{6}}$$

$$(5+2\sqrt{6})^{x/2}+\frac{1}{5+2\sqrt{6}}^{x/2}=10$$

$$t=(5+2\sqrt{6})^{x/2}$$

$$t+\frac{1}{t}=10$$

$$t^2+1=10t$$

$$t^2-10t+1$$

$$t=\frac{10\pm\sqrt{100-4}}{2}$$

$$=5\pm\sqrt{24}$$

$$t=5\pm2\sqrt{6}$$

$t=5+2\sqrt{6}$

$(5+2\sqrt{6})^{x/2}=(5+2\sqrt{6})^{1}$

$\frac{x}{2}=1$

$\boxed{x=2}$

$t=5-2\sqrt{6}$

$(5+2\sqrt{6})^{x/2}=\frac{1}{5+2\sqrt{6}}$

$\frac{x}{2}=-1$

$\boxed{x=-2}$

~~Q2.~~ ~~DYS-3~~ H.W
~~Q9 (~~

DYS-5 (Q180), Q12, Q13, Q 8
DYS-2 (Q7)

DIS - 5

Q9 $(\sqrt{3}+\sqrt{2})^{x}+(\sqrt{3}-\sqrt{2})^{x}-2\sqrt{3}=0$

$$\frac{\sqrt{3}-\sqrt{2}\times\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{3-2}{\sqrt{3}+\sqrt{2}}=\frac{1}{\sqrt{3}+\sqrt{2}}$$

$t=(\sqrt{3}+\sqrt{2})^{x}$

$t+\frac{1}{t}-2\sqrt{3}=0$

$t^{2}+1=2\sqrt{3}t$

$t^{2}-2\sqrt{3}t+1$

$$t=\frac{2\sqrt{3}\pm\sqrt{12-4}}{2}$$

$$=\frac{2\sqrt{3}\pm2\sqrt{2}}{2}$$

$=\sqrt{3}\pm\sqrt{2}$

$\sqrt{3}+\sqrt{2}^{x}=\sqrt{3}+\sqrt{2}$

$x=1$

$(\sqrt{3}+\sqrt{2})^{x}=\sqrt{3}-\sqrt{2}=\frac{1}{\sqrt{3}+\sqrt{2}}$

$\boxed{x=-1}$

$\boxed{x=\pm 1}$

Q $x(x+1)(x+2)(x+3)-8=0$

$x(x+3)(x+1)(x+2)=8$

$(x^2+3x)(x^2+3x+2)=8$

$x^2+3x=t$

$t(t+2)=8$

$t^2+2t-8=0$

$D=\frac{-2\pm\sqrt{4+32}}{2}$

$=\frac{-2\pm 6}{2}$

$=\frac{-8}{2},\ \frac{4}{2}$

$t=-4, 2$

$x^2+3x=-4$

$x^2+3x+4=0$

$x=-3\pm\sqrt{9-16}$

Not Possible as $\sqrt{}$ is negative.

x^2+3x-2

$x=\frac{-3\pm\sqrt{9+8}}{2}$

$=\frac{-3\pm\sqrt{17}}{2}$

$x=\boxed{\frac{-3\pm\sqrt{17}}{2}}$

DYS-5

Q 11 $x(x+1)(x+2)(x+3)=24$

$t=x^2+3x$

$t(t+2)=24$

$t^2+2t-24=0$

$t=\frac{-2\pm\sqrt{4+96}}{2}$

$=\frac{-2\pm 10}{2}$

$=-6, 4$

$t=-6$

$x^2+3x+6=0$

$x=-3\pm\sqrt{9-24}$

$\times$

$x^2+3x-4=0$

$x=\frac{-3\pm\sqrt{9+16}}{2}$

$x=\frac{-3\pm 5}{2}$

$\boxed{x=-4, 1}$

Q $(x+1)(x+2)(x+3)(x+6)=3x^2$

$\cancel{(x^2+4x+3)(x^2+8x+12)}$

$(x^2+7x+6)(x^2+5x+6)=3x^2$

$\cancel{t=x^2+6}$

$\cancel{(t+7x)(t+5x)=3x^2}$

$$\left(\frac{x^2+7x+6}{x}\right)\left(\frac{x^2+5x+6}{x}\right)=3$$

$$\cancel{\frac{x^2+6}{x} = 7}$$

$$\left(x+7+\frac{6}{x}\right)\left(x+5+\frac{6}{5}\right)=3$$

$$t = x+\frac{6}{x}$$

$$(t+7)(t+5)=3$$

$$t^2+12x+35-3=0$$

$$t^2+12x+32=0$$

$$t = \frac{-12\pm\sqrt{144-128}}{2}$$

$$= \frac{-12\pm 4}{2}$$

$$= -8, -4$$

$$-8 = x+\frac{6}{x}$$

$$-8x = x^2+6$$

$$x^2+8x+6=0$$

$$x = \frac{-8\pm\sqrt{64-24}}{2}$$

$$= -8\pm\sqrt{40}$$

$$\boxed{x = -4\pm\sqrt{10}}$$

$$-4x = x^2+6$$

$$x^2+4x+6$$

$$x = \frac{-4\pm\sqrt{16-24}}{2}$$

✗

147

Q $(x+2)(x+3)(x+8)(x+12)=4x^2$

$$(x^2+14x+24)(x^2+11x+24)=4x^2$$

$$\left(x+14+\frac{24}{x}\right)\left(x+11+\frac{24}{x}\right)$$

$$t = x+\frac{24}{x}$$

$$(t+14)(t+11)=4$$

$$t^2+25t+\cancel{1}154=4$$

$$t^2+25t+150=0$$

$$t=\frac{-25\pm\sqrt{625-600}}{2}$$

$$=\frac{-25\pm5}{2}$$

$$=-15,-10$$

$x^2+24=-15x$

$x^2+15x+24=0$

$$x=\frac{-15\pm\sqrt{225-96}}{2}$$

$$\boxed{x=\frac{-15\pm\sqrt{129}}{2}}$$

$x^2+\cancel{20}\,24=-10x$

$x^2+10x+24$

$$x=\frac{-10\pm\sqrt{100-96}}{2}$$

$$=\frac{-10\pm2}{2}$$

$$\boxed{=-4,-6}$$

Q $x^4-2x^3+3x^2-2x=0$

$x(x^3-2x^2+3x-2)=0$

$\boxed{x=0}$

~~Q $x^4-2x^3+3x^2-2x+1=0$~~

Q $x^4-2x^3+3x^2-2x+1=0$

$x^4+1-2(x^3+x)+3x^2=0$

$\frac{x^4+1}{x^2}-2\left(\frac{x^3+x}{x^2}\right)+\frac{3\cancel{x^2}}{\cancel{x^2}}=0$

$x^2+\frac{1}{x^2}-2\left(x+\frac{1}{x}\right)+3=0$

$x+\frac{1}{x}=t$

$(t^2-2)-2t+3=0$

$t^2-2t+1=0$

$t=\frac{2\pm\sqrt{4-0}}{2}$

$=1$

$x^2+1=x$

$x^2-x+1=0$

$x=\frac{1\pm\sqrt{1-4}}{2}$

Not Possible

DYS-5

Q14 $x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

$$\frac{x^4+1}{x^2} - 2\left(\frac{x^3 - x}{x^2}\right) - \frac{2x^2}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2\left(x - \frac{1}{x}\right) - 2 = 0$$

$$(t^2+2) - 2t - 2 = 0$$

$$t^2 - 2t = 0$$

$$t(t-2) = 0$$

$t = 2$

$$x^2 - 1 = 2x$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$\boxed{= 1 \pm \sqrt{2}}$$

$t = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\boxed{x = \pm 1}$$

Q $(x-a)^4 + (x-b)^4 = c$

$$t = \frac{x - a + x - b}{2}$$

$$2t = 2x - (a+b)$$

$$2t + (a+c) = 2x$$

$$\boxed{x = t + \frac{(a+b)}{2}}$$

$\cancel{x}$

Q $(x-1)^4+(x-7)^4=272$

$$\left(t+\frac{8}{2}-1\right)^4+\left(t+\frac{8}{2}-7\right)^4=272$$

$$(t+4-1)^4+(t+4-7)^4=272$$

$$(t+3)^4+(t-3)^4=272$$

$$(\underbrace{t^2+9+6t}_{a^2})^2+(\underbrace{t^2+9-6t}_{b^2})^2=272$$

$$(a^2+b^2)^2=a^4+b^4+2a^2b^2$$

$$(t^2+9+6t+t^2+9-6t)^2=272+2(t^2-9)^2$$

$$(2t^2+18)^2=272+2(t^4+81-18t^2)$$

$$4t^4+324+72t^2+2t^4+162-36t^2$$

$$\cancel{6}^{3}t^4+\cancel{486}^{243}+\cancel{36}^{18}t^2=\cancel{272}^{136}$$

$$3t^4+18t^2-136+243=0$$

$$3t^4+18t^2+107=0$$

$$t^2=\cancel{324}\;-18\pm\sqrt{324-}$$

Scanned with CamScanner

$$4t^4+324+72t^2-2t^4-162+36t^2=272$$

$$2t^4+\overset{81}{\cancel{162}}+\overset{54}{\cancel{108}}t^2=\overset{136}{\cancel{272}}$$

$$t^4+54t^2-55=0$$
$$t^4+55t^2-t^2-55=0$$
$$t^2(t^2+54)-55=0$$

$$(t^2-1)(t^2+55)=0$$

$$t^2-1=0$$
$$t^2=1$$
$$\boxed{t=\pm 1}$$

P Minor Test -1

Q18. $$\underbrace{\left(\frac{x-1}{x}\right)^{1/2}}_{a^2}+\underbrace{\left(1-\frac{1}{x}\right)^{1/2}}_{b^2}=x$$

$$a+b=x$$

$$a^2-b^2=\frac{x-1}{x}-\frac{1+1}{x}$$

$$(a+b)(a-b)=(x-1)$$

$$(a-b)x=(x-1)$$

$$a-b=\frac{x-1}{x}$$

$$2a=\frac{x-1}{x}+x$$

$$2a=\frac{x^2+x-1}{2x}$$

$D=$ ove

So irrational

(152)

Integer type

Q5.

$$x^{x^8} = 8$$

$$x^8 = t$$

$$x^t = 8 \Rightarrow \boxed{x = t^{1/8}}$$

$$\left(t^{1/8}\right)^t = 8$$

$$t^{t/8} = 8$$

$$t^t = 8^8$$

$$\boxed{t = 8}$$

DYS - 5

Q8. $(x-1)^4+(x-5)^4 = 82$

$t = x-3$

$x = t+3$

$(t+2)^4 + (t-2)^4 = 82$

$\cancel{t^2+4+4t+t^2+4-4t = 82}$

$\cancel{(2t^2+8t)^2 =}$

$(a^2+b^2)^2 - 2a^2b^2 = a^4+b^4$

$(2t^2+8)^2 - 2(t^2+4-2t)(t^2+4+2t) = 82$

$4t^4+64+32t^2 - 2(t^4+16+8t^2-4t^2) = 82$

$4t^4+64+32t^2 - 2t^4 - 32 - 8t^2 = 82$

$2t^4+24t^2+32 = 82$

$t^4+12t^2-28 = 0$

$t^4+14t^2-2t^2-28 = 0$

$t^2(t^2+14) - 2(t^2+14) = 0$

$t^2 = 2$

$t = \pm\sqrt{2}$

$\cancel{t^2 = 14}$

$\cancel{t = \pm 2}$

$\cancel{\boxed{x = 5, 1}}$

154

System of Equation

→ It comprises two or more equations which are satisfied ~~by~~ by the same set of values of variable

Q $x^2 - y^2 = 16$

$x + y = 8$

$x = 8 - y$

$(8-y)^2 - y^2 = 16$

$\cancel{y^2} + 64 - 16y - \cancel{y^2} = 16$

$16y = 64 - 16$

$y = \frac{48}{16}$

$\boxed{y = 3}$

$\boxed{x = 5}$

Q4. $x - y = 1$

$x^3 - y^3 = 7$

$(1+y)^3 - y^3 = 7$

$1 + \cancel{y^3} + 3y(y+1) - \cancel{y^3} = 7$

$3y(y+1) + 1 = 7$

$3y^2 + 3y = 86$

$y^2 + y - 2 = 0$

$y^2 + 2y - y - 2 = 0$

$y(y+2) - 1(y+2)$

$(y-1)(y+2)$

$y = 1$ $x = 2$	$y = -2$ $x = -1$

H.W.

D YS-6

QS.

O-1 (Q1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18)

Q.6. $x^2+y^2+6x+2y=0$

$x+y+8=0$

$x=-(y+8)$

$(-y+8)^2+y^2-6y-48+2y=0$

$y^2+64+16y+y^2-6y-48+2y$

$12y+2y^2+16=0$

$y^2+6y+8=0$

$y^2+4y+2y+8=0$

$y(y+4)+2(y+4)=0$

$(y+2)(y+4)=0$

$\boxed{\begin{array}{l} y=-2 \\ x=-6 \end{array}}$ $\boxed{\begin{array}{l} y=-4 \\ x=-4 \end{array}}$

~~Method 2~~

Q7. $\frac{x}{y}-\frac{y}{x}=\frac{5}{6}$

$x^2-y^2=5$

$\cancel{(x-y)^2=x^2+y^2-12}$

$\frac{36}{y^2}-y^2=5$

$36-y^4=5y^2$

$x^2-y^2=\frac{5xy}{6}$

$5=\frac{5}{6}xy$

$xy=\frac{5}{\frac{5}{6}}$

$\frac{30}{5}=6=xy$

$\frac{6}{y}=x$

$y^4 + 5y^2 - 36 = 0$

$y^4 + 9y^2 - 4y^2 - 36$

$y^2(y^2+9) - 4(y^2+9)$

$(y^2-4) \quad (y^2+9)$

$\boxed{y = \pm 2}$ $\quad$ $y^2 = -3$ ✗

$\cancel{x = \frac{6}{\pm 4}}$

$\cancel{\boxed{x = \pm \frac{3}{2}}}$

$\boxed{x = \pm 3}$

$\cancel{\frac{3}{8} - \frac{4}{3}}$

$\cancel{\frac{9 - 64}{24}}$

$\cancel{-\frac{55}{24}}$

Q8. $\frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{13}{6}$

$xy=5$

$$\frac{(x+y)^2 + (x-y)^2}{x^2-y^2} = \frac{13}{6}$$

$$x^2+y^2+\cancel{2xy}+x^2+y^2-\cancel{2xy} = \frac{13x^2-13y^2}{6}$$

$$12x^2+12y^2 = 13x^2-13y^2$$

$$25y^2 = x^2$$

$$(5y)^2 = x^2$$

$$x=5y$$

$$\cancel{5}y^2=\cancel{5}$$

$$y^2=1$$

$$\boxed{y=\pm 1}$$

$$\boxed{x=\pm 5}$$

$$\frac{5+1}{5-1} + \frac{5-1}{5+1} =$$

$$\frac{6}{4}+\frac{4}{6}$$

$$\frac{36+16}{24}$$

$$\frac{\cancel{52}\ \cancel{26}\ 13}{\cancel{24}\ \cancel{12}\ 6}$$

159

Q9 $\dfrac{1}{x+1} + \dfrac{1}{y} = \dfrac{1}{3}$ $\dfrac{1}{(x+1)^2} - \dfrac{1}{y^2} = \dfrac{1}{4}$

$y + x + 1 = \dfrac{xy + y}{3}$

$3y + 3x + 3 = xy + y$

$2y + 3x + 3 = xy$

$x + 1 = a$
$y = b$

$a + b = \dfrac{1}{3}$

$b = \dfrac{1}{3} - a$

$a^2 - b^2 = \dfrac{1}{4}$

$a^2 - \left(\dfrac{1}{3} - a\right)^2 = \dfrac{1}{4}$

$a^2 - \dfrac{1}{9} + a^2 - \dfrac{2a}{3} = \dfrac{1}{4}$

$a^2 - \dfrac{1-6}{9} = \dfrac{1}{4}$

$a^2 - \dfrac{7}{9} = \dfrac{1}{4}$

$a^2 = \dfrac{1}{4} + \dfrac{7}{9}$

$a^2 = \dfrac{9+28}{36}$

$a^2 = \dfrac{37}{36}$

$\boxed{a = \dfrac{\sqrt{37}}{6}}$

$b = \dfrac{1}{3} - \dfrac{\sqrt{37}}{6}$

$\boxed{b = \dfrac{2 - \sqrt{37}}{6}}$

160

Scanned with CamScanner

~~Q10~~. ~~Q11~~ Q11. $\frac{1}{y-1} - \frac{1}{y+1} = \frac{1}{x}$

$y+1-y+1 = \frac{y^2-1}{x}$

$2x = y^2-1$

$y^2 = 2x+1$

$y^2 = 8+1$

$y^2 = 9$

$y = \pm 3$

$\boxed{(4,3)\ (4,-3)}$

$y^2 - x - 5 = 0$

$2x+1-x-5 = 0$

$x = 4$

Q10. $x^2+y^2 = 25-2xy$

$(x+y)^2 = 5^2$

~~$x+y=5$~~

~~$x = 5-y$~~

~~$x=3$~~

~~$(3,2)\ (-3,-2)$~~

$x+y = \pm 5$

$\boxed{(3,2)\ (-3,-2)}$

$y(x+y) = 10$

~~$5y = 10$~~

~~$y = 2$~~

$xy + y^2 = 10$

$(5-y)y + y^2 = 10$

~~$5y - y^2 + y^2 = 10$~~

~~$5y = 10$~~

~~$y = 2$~~

$y(\pm 5) = 10$

$\boxed{y = \pm 2}$

$\boxed{x = \pm 3}$

Method - 2

$$y(x+y) = 10$$

$$y^2(x+y)^2 = 100$$

$$(x+y)^2 = 25$$

$$y^2(25) = 100$$

$$y^2 = 4$$

$$y = \pm 2$$

$$x = \pm 3$$

~~Q13.~~ $\cancel{2xy}$

Q12. $2xy + y^2 - 4x - 3y + 2 = 0$

$\cancel{-4x + y(2x + y - 3) + 2 = 0}$

$\cancel{xy - 2y^2 - 2x + 11y - 14 = 0}$

$$-5y^2 + 25y - 30 = 0$$

$$-y^2 + 5y - 6 = 0$$

$$y^2 - 5y + 6 = 0$$

$\cancel{y^2 - 6y + y + 6 = 0}$

$\cancel{y(y-6) + 1(y-6)}$

$\cancel{2x + 4312 - 2x}$

$\cancel{3x + 18 - 2x - 42y + 16}$

$\cancel{-x + 8 = 0}$

$2xy + 6y^2 - 2x - 28y + 32 = 0$

$$y^2 - 3y - 2y - 6 = 0$$

$$y(y-3) - 2(y-3)$$

$$(y-2) \quad (y-3)$$

$y = 2$ $y = 3$

×

$\cancel{-x = 8}$

$3x + 27 - 2x - 42 + 16$

$$\boxed{x = -1}$$

(162)

Scanned with CamScanner

H.W.

Q1. $\frac{1}{x+1} + \frac{1}{y} = \frac{1}{3}$ $\qquad$ $\frac{1}{(x+1)^2} - \frac{1}{y^2} = \frac{1}{4}$

$$a + b = \frac{1}{3}$$

$$3a + 3b = 1$$

$$3a = 1 - 3b$$

$$\cancel{3}a = \frac{1-3b}{3}$$

$$a = \frac{1 + \frac{15}{24}}{3}$$

$$a = \frac{24+15}{24 \times 3}$$

$$= \frac{\cancel{39}^{13}}{3 \times 24}$$

$$= \frac{13}{24}$$

$$x + 1 = \frac{13}{24}$$

$$24x + 24 = 13$$

$$x = \frac{13-24}{24}$$

$$\boxed{x = \frac{-11}{24}}$$

$$\boxed{y = \frac{-5}{24}}$$

$$a^2 - b^2 = \frac{1}{4}$$

$$\left(\frac{1-3b}{3}\right)^2 - b^2 = \frac{1}{4}$$

$$\left(\frac{1-3b}{3} + b\right)\left(\frac{1-3b}{3} - b\right) = \frac{1}{4}$$

$$\left(\frac{1-\cancel{3b}+\cancel{3b}}{3}\right)\left(\frac{1-3b-3b}{3}\right) = \frac{1}{4}$$

$$\frac{1}{3}\left(\frac{1-6b}{3}\right) = \frac{1}{4}$$

$$\frac{1-6b}{9} = \frac{1}{4}$$

$$1 - 6b = \frac{9}{4}$$

$$-6b = \frac{9-4}{4}$$

$$6b = \frac{-5}{4 \times 6}$$

$$\boxed{b = \frac{-5}{24}}$$

$0-1$

Q6. $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$

$\frac{1}{(a-b)(a-c)} - \frac{1}{(b-c)(a-b)} + \frac{1}{(a-c)(b-c)}$

$\cancel{(a-b)(a-c)(a-}$

$b-c-a+c+a-b=0$

$x^0 = 1$

Q11.

$(a^n)^m = a^{m^n}$

$mn = m^n$

$mn - m^n = 0$

$m\left(n - \frac{m^n}{m}\right) = 0$

$m(n - m^{n-1}) = 0$

$m = 0$ (rejected $m>0$ given)

$n - m^{n-1} = 0$

$m^{n-1} = n$

$m^{\frac{n-1}{n-1}} = n^{\frac{1}{n-1}}$

$m = n^{\frac{1}{n-1}}$

$\boxed{A}$

H.W. 07-05-2024

DYS-6 Q 1 & 3

DYS-7 (Q1, 2, 5)

O-1 (Q 22, 23, 24, 25)

O-2 (Q1-5, 6, 8, 9, 10, 11, 12, 13, 14, 15)

DYS-7

Q5.

$5 > 3$

$\sqrt{14} > \sqrt{3}$

$\sqrt{14} - 5 > \sqrt{3} - 3$

DPS - 6

Q 14.

$x^4 + y^4 = 82$

$xy = 3$

$y = \frac{3}{x}$

$y^4 = \left(\left(\frac{3}{x}\right)^2\right)^2$

$= \left(\frac{9}{x^2}\right)^2$

$= \frac{81}{x^4}$

$x^4 + \frac{81}{x^4} = 82$

$x^8 + 81 = 82x^4$

$x^4 = a$

$a^2 - 82a + 81 = 0$

$a^2 - 81a - 1 + 81 = 0$

$a(a - 81) - 1(a - 81)$

$a = 1$	$a = 81$
$x^4 = 1$	$x^4 = 81$
$x = \pm 1$	$x = \pm 3$
$y = \pm 3$	$y = \pm 1$
$(1, 3)\ (3, 1)$	$(-1, -3),\ (-3, -1)$

Method - 2

$(x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$

$(x^2 + y^2)^2 = 82 + 2 \times (3)^2$

$x^2 + y^2 = \pm 10$

put $y = \frac{3}{x}$ & solve

3 Variables

Q1.
$a+b=10$
$b+c=15$
$a+c=25$
Find a, b, c

$b=10-a$

$10-a+c=15$

$c-a=5$
$a+c=25$

$2c=30$
$\boxed{c=15}$

~~a=10~~ $a=15-5$
$\boxed{a=10}$

$b=10-10$
$\boxed{b=0}$

Q2. $2x+3y+z=1$

$\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z}{6}$

$-2x+2=y+1$

$-(2x+y=1)$
$2x+3y+z=1$

$-(2y+z=0)\times 2$
$6y+2z=-6$

$2y=-6$
$\boxed{y=-3}$

$6y+6=-2z$

$6y+2z=-6$

$-18+2z=-6$

$2z=-6+18$
$\boxed{z=6}$

$2x-9+6=1$
$2x=1+3$
$\boxed{x=2}$

Method-2
let $\frac{x-1}{1}=\frac{y+1}{-2}=\frac{z}{6}=k$

put k in eq 1

Q13.

$w + x + y = -2$

$w + x + z = 4$

$w + y + z = 19$

$x + y + z = 12$

$w + y + z = 19$

$-w - x - y = 2$

$z - x = 21$

$\underline{z = 21 + x} \rightarrow x = z - 21$

$6 + y = 21 + x$

$z - y = 6$

$\underline{z = 6 + y} \rightarrow y = z - 6$

$y - x = 21 - 6$

$y - x = 15$

~~$x - y = -15$~~

$z - 21 + z - 6 + z = 12$

$3z - 27 = 12$

$3z = 39$

$\boxed{z = 13}$

$\boxed{x = -8}$

$\boxed{y = 7}$

Q4. $x+y+z=4$

$x^2+y^2+z^2=6$

$x^3+y^3+z^3=8$

~~i) xyz~~

ii) $xy+yz+xz=?$

$(4)^2=6+2(xy+yz+xz)$

$\frac{16-6}{2}=\frac{10}{2}=5$

$\boxed{xy+yz+xz=5}$

~~i) xyz~~

~~$64=8+3xyz(4)$~~

~~$\frac{56}{12}=xyz$~~

~~$\boxed{xyz=\frac{14}{3}}$~~

~~iii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$~~

~~$\frac{4\times3}{14}=\frac{12}{14}=\boxed{\frac{6}{7}}$~~

i) xyz

$8-3abc=(4)(6-5)$

$8-3abc=4$

$3abc=4$

$\boxed{abc=\frac{4}{3}}$

~~iii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$~~

~~$\frac{4}{4}\times3=\boxed{3}$~~

~~$\frac{x+y+z}{xyz}=\frac{4}{\frac{4}{3}}=\boxed{3}$~~

Scanned with CamScanner

iii) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

$$\frac{xy+yz+xz}{xyz} = \frac{5}{\frac{4}{3}} = \boxed{\frac{15}{4}}$$ ✓

Inequalities

→ ~~Comparing~~ Comparibility in real numbers.

→ $\underbrace{>, <,}_{\text{Strict Inequality}}$ $\underbrace{\geq, \leq}_{\text{slack inequality}}$

eg. $4 > -2$ $(x \in R)$

$3 \geq 3$ $(x \in R)$

$4 > 3$ $(x \in R)$

$3 > 4$ $(x \in \phi)$

→ Properties –

① Addition, Subtraction, Multiplication of any constant.

$5 > 2$

$+$

$5 > 2$

$5+2 > 2+2$

$7 > 4$

$-$

$5 > 2$

$5-2 > 2-2$

$3 > 0$

$$5 > 2$$

Multiply ~~the~~ | Division

$(3)\ 5 > 2\ (3)$

$15 > 6$

$\frac{5}{1} > \frac{2}{1}$

$5 > 1$

$\ominus$ve

$(-3)\ 5 > 2\ (-3)$

$-15 > -2$ (not valid)

$\ominus$ve

$\frac{5}{-1} > \frac{2}{-1}$

$-5 > -2$ (not valid)

→ If we multiply ~~of~~ or divide any negative constant, then we have to <u>reverse the sign</u> of enequality.

→ Never multiply or divide any thing whose ~~s~~ sign we dont know.

eg — $9 > 2$

$9x > 2x$ (not valid)

$x = 3$

$9(3) > 2(3)$

$27 > 6$

$x = -3$

$9(-3) > 2(-3)$

$-27 > -6$.

(wrong)

~~a~~ eg ② — $9 > 2$

$9(x^2+2) > 2(x^2+2)$ (we know (x^2+2) is always $\oplus$ve)

→ Inequalities sign reverse when we take <u>resiprocal both sides</u>

eg $3 > 2$

$\frac{1}{3} < \frac{1}{2}$

→ Cross Multiplication is not valid until unless we dont know the sign.

eg. $\frac{2}{x} > 1$

$2 > x$ (wrong)

$\frac{2}{x} - 1 > 0$

$\frac{2-x}{x} > 0$ (right)

→ If we cancle anything (variables) in inequality then mension it

eg 2. $9 > \frac{\cancel{2-x}}{\cancel{2-x}}$ $\quad x \neq 2$

$9 > 1$ (for $x \in R, x \neq 2$) right

$9 > \frac{\cancel{(2-x)}}{\cancel{(2-x)}}$ wrong as for $x = 2$

$\forall x \in R$ $\quad 9 > \frac{0}{0}$ (not defined)

$9 > 1$

→ we can concel out anything (constant & variables) which are in addition of subtraction in inequality.

eg. $\cancel{x}+2 > \cancel{x}-1$

$2 > -1$ (right)

$2\cancel{x} > \cancel{x}(-1)$

$2 > -1$ (wrong)

→

$AB > AC$ ∴

⇓

$B > C$ (wrong)

$AB - AC > 0$

$A\cdot(B-C) > 0$ (right)

→ 3. Addition of two inequalities is valid with same side sign, but subtraction, multiplication & division is not valid.

eg. $2>1$
$+\ 3>2$
$5>3$ (right

$2>1$
$+\ 3<5$
$5 \cdot 6$ (wrong)

eg (2). ~~$-3<1$~~
~~$10<11$~~

$-3<1$

$10<11$

$183<10$ (wrong)

Wavy Curvy Method

Q $x-3>0$

Factors ⇒ $x-3=0$
$x=3$

$-\infty$ ⊖ 3 ⊕ $+\infty$

two sets $(3,\infty)$ & $(-\infty,3)$

↓

take random val

$x=1000$

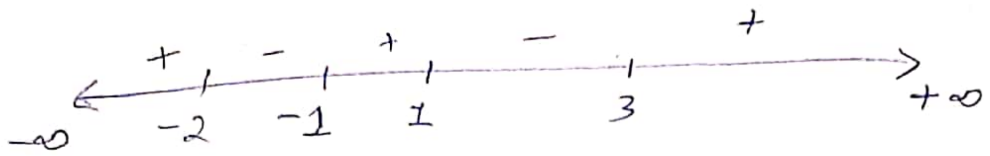
$1000-3 = \oplus ve$

so $(x-3)$ ⊕ve for x in $(3,\infty)$

Q2. $(x-1)(x+1)(x+2)(x-3) \le 0$

$x-1=0$

$x = 1, -1, -2, 3$



~~Intervals:~~ let $x =$ 4 ~~1000~~ [in $(3, \infty)$]

$(4-1)(4+1)(4+2)(4-3)$

$3 \times 5 \times 6 \times 1 = +ve$

x must be $-ve$

$$\boxed{[-2, -1] \cup [1, 3]}$$

Q3. $(x+4)(x-1) < 0$

$x = 1, -4$

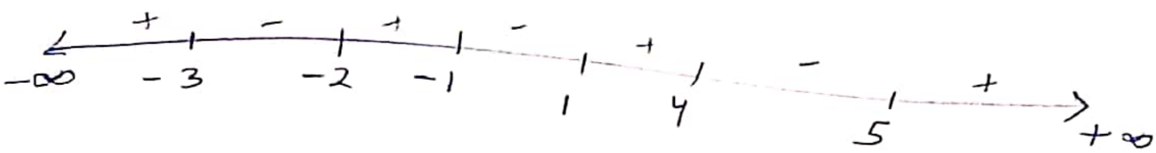
$x = 1000$

$(1004)(999) = +ve$

$$\boxed{(-4, 1)}$$

Q4. $(x-1)(x+3)(x+1)(x+2)(x-4)(x-5) \geq 0$

$x = 1, -3, -1, -2, 4, 5$



let $x = 6$

$5 \times 9 \times 7 \times 8 \times 2 \times 1 = \oplus ve$

$$\boxed{x \in [-\infty, -3] \cup [-2, -1] \cup [1, 4] \cup [5, \infty)}$$

Note:-

① $(x-a)(x-b) \leq 0 \rightarrow x \in [a, b]$

② $(x-a)(x-b) < 0 \rightarrow x \in (a, b)$

③ $(x-a)(x-b) \geq 0 \rightarrow x \in (-\infty, a] \cup [b, \infty)$

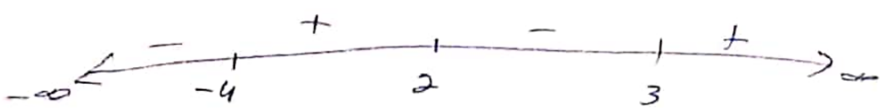
④ $(x-a)(x-b) > 0 \rightarrow x \in (-\infty, a) \cup (b, \infty)$

Q5. $(2-x)(4+x)(x-3) < 0$

$-(x-2)(4+x)(x-3) < 0$

(−1 multiply)

$(x-2)(x+4)(x-3) > 0$



$x \in (-4, 2) \cup (3, \infty)$

Q6. $(4-x)(x+4) \leq 0$

$$\boxed{x \in [4, \infty) \cup (-\infty, -4]}$$

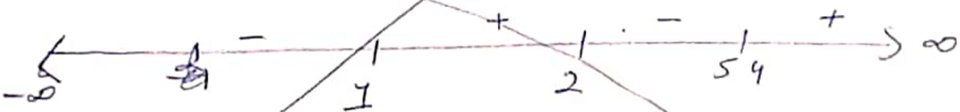
Q7. $(x-1)(x+2)(5-x)(1000-x) > 0$

$$\boxed{x \in (1000, \infty) \cup (1, 5) \cup (-\infty, -2)}$$

Q8. $(x^2-3x+2)(x^2-55x+54) > 0$

$(x^2-2x-x+2)\ (x^2-54x-x+54)$

$x(x-2)-1(x-2)\ \ x(x-54)-1(x-54)$



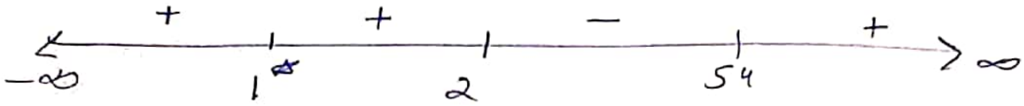
$$\boxed{x \in (54, \infty) \cup (1, 2)}$$

Note:- ① Quantities which have whole even powers does not change sign and for whole odd powers we can change sign

Q8. $(x^2-3x+2)(x^2-55x+54)>0$

$(x-2)(x-1)\ (x-1)(x-54)>0$

$(x-2)\ (x-1)^2\ (x-54)$



$\boxed{x \in (-\infty,1)\cup(1,2)\cup(54,\infty)}$

Q9. $(x-1)^2\ (x+1)^3\ (x-4)<0$

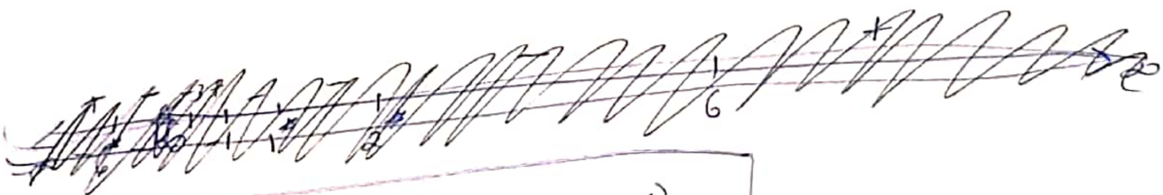
$(-1,1)\cup(1,4)$

$\boxed{x \in (-1,1)\cup(1,4)}$

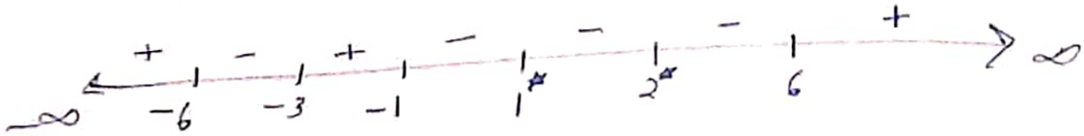
Q10. $(x-1)^2(x+1)^3(x-2)^4(x+3)^5(x^2-36)>0$

$(x-1)^2(x+1)^3(x-2)^4(x+3)^5(x+6)(x-6)>0$

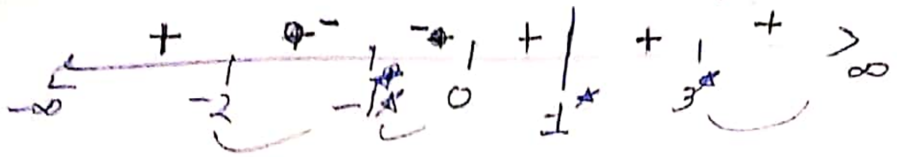
$(x-1)^2\ (x+1)^3\ (x-2)^4$



$\boxed{x \in (-\infty,-6)\cup(-3,-1)\cup(6,\infty)}$



Q11. $(x-1)^2 (x+1)^4 (x) (x-3)^6 (x+2)^7 \geq 0$



~~$[-2,-1] \cup [-1,0] \cup [3,\infty)$~~

~~$x \in [-2,0] \cup [3,\infty)$~~

$\boxed{x \in (-\infty,-2] \cup [0,\infty) \cup \{-1\}}$

Note :-

① whenever we solve questions of odd & even whole powers, we will always check at the end points or factors values.

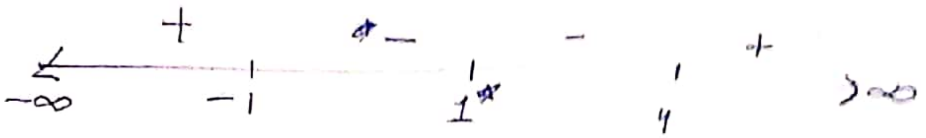
H.W. 09-05-2024

~~Q-3 (Q1 to Q7)~~

Q-3 $[1,7] \cup \{8\}$ (1 to 7)

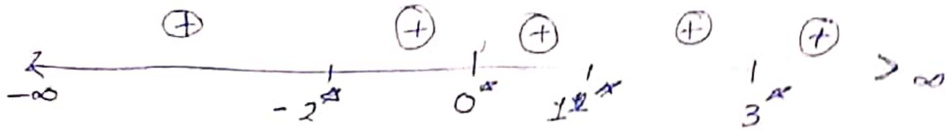
Q-4 $\{1, 2, 3, 4\}$

Q. $(x-1)^{12}(x+1)^{13}(x-4)^{7}<0$



$$\boxed{x \in (-1,1) \cup (1,4)}$$

Q $x^{4}(x-1)^{2}(x+2)^{8}(x-3)^{10} \leq 0$

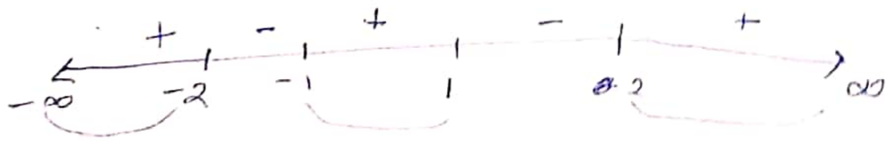


~~$(-\infty,-2] \cup \{0,2,3\}, -2\}$~~

$$\boxed{x \in \{0,1,3,-2\}}$$

Denominator based questions

Q1. $\dfrac{(x-1)(x-2)}{(x+1)(x+2)} \geq 0$



$x \in (-\infty,-2] \cup [-1,1] \cup [2,\infty)$

but $x \notin \{-2,-1\}$

$$\boxed{x \in (-\infty,-2) \cup (-1,1] \cup [2,\infty)}$$

DYS-10

Q2. $\dfrac{6x-5}{4x+1} < 0$

$$-\infty \quad + \quad -\tfrac{1}{4} \quad - \quad \tfrac{5}{6} \quad + \quad +\infty$$

$x \notin \{-1/4\}$

$\boxed{x \in (-1/4, 5/6)}$

Q3. $\dfrac{x(x-2)(x-5)}{(x+2)(x+1)} > 0$

$$-\infty \quad - \quad -2 \quad + \quad -1 \quad - \quad 0 \quad + \quad 2 \quad - \quad 5 \quad + \quad +\infty$$

$(-2,-1) \cup \cancel{(2,5)}\ (0,2) \cup (5,\infty)$

$x \notin \{-2,-1\}$

$\boxed{x \in (-2,-1) \cup (0,2) \cup (5,\infty)}$

Q $\dfrac{(x^2-4)(x+3)}{(x-2)} \geq 0$

$-\infty \quad + \quad -3 \quad - \quad -2 \quad + \quad 2^{*} \quad + \quad \infty$

$\boxed{x \in [-3,-2] \cup (2,\infty)}$

$x \in (-\infty,-3] \cup [-2,\infty) - \{2\}$

Q $\dfrac{(2x-1)(x-1)^2(x-2)^3}{(x-4)^4} > 0$

$-\infty \quad + \quad \tfrac{1}{2} \quad - \quad 1^{*} \quad - \quad 2 \quad + \quad 4^{*} \quad + \quad \infty$

$\boxed{x \in (-\infty, \tfrac{1}{2}) \cup (2,4) \cup (4,\infty)}$

Q $\dfrac{(x^2+4x+4)}{(2x^2-x-1)} < 0$

$\dfrac{(x+2)(x+2)}{(2x+1)(x-1)} < 0$

$2x^2-x-1$

$2x^2-2x+x-1$

$2x(x-1)+1(x-1)$

$-\infty \quad + \quad -2^{*} \quad + \quad -\tfrac{1}{2} \quad - \quad 1 \quad + \quad \infty$

$\boxed{x \in (-\tfrac{1}{2}, 1)}$

$x \notin \{-\tfrac{1}{2}, 1\}$

Q $\dfrac{(x^2-4)(x+3)}{(x-2)} \geq 0$

$\dfrac{(x+2)\cancel{(x-2)}(x+3)}{\cancel{(x-2)}} \geq 0$

$(x+2)(x+3) \geq 0$

$x \neq 2$

$\boxed{x \in (-\infty,-3] \cup (-2,\infty) - \{2\}}$

Note - ① we can cancle out factors from numerator & denominator but we have to mention their also that

② ~~we can~~ following quantities are always positive.

1. quadratic equations with D ⊖ve & a ⊕ve

$D < 0$ & $a > 0$

2. Quantities with whole even powers.

3. Modulus.

Q $\dfrac{(x-1)(x+2)}{(x^2+1)(x^2+x+1)} < 0$

$D = 1-4(1)(1) < 0$

$A = \oplus ve$

$D<0, a>0$

Positive

$(x-1)(x+2) < 0\,(x^2+1)(x^2+x+1)$

$(x-1)(x+2) < 0$

$x \in (-2, 1)$

① $\frac{3x-1}{(4x+1)(x^2)} \le 0$

$3x-1 \le 0$

$D = 0 - 4 \times 1 \times 4$

$x \notin \{-1/4, 0\}$

$-\infty \quad \frac{1}{3} \quad +\infty$

$3x - 1 \le 0$

$3x \le 1$

$x \le \frac{1}{3}$

~~$x \in \{1/3\}$~~

$x \in (-\infty, \frac{1}{3}] - \{-1/4, 0\}$

$\frac{3x-1}{4x+1} \le 0$

$x \notin \{0, -1/4\}$

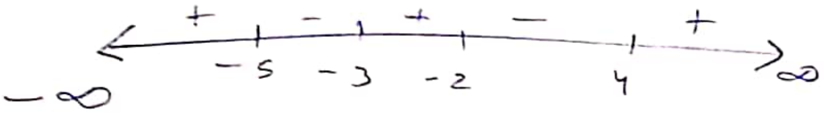
$+ \quad - \quad +$

$-\infty \quad -1/4 \quad 1/3 \quad \infty$

$\boxed{x \in [-1/4, \frac{1}{3}] - \{0\}}$

Q2. $\frac{(x+2)(x+3)(x+5)}{(x-4)^3 (x-6)^8} > 0$

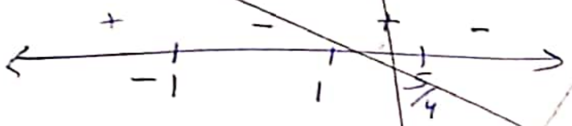
$x \notin \{6, 4\}$



$\boxed{x \in (-\infty, -5) \cup (-3, -2) \cup (4, \infty) - \{6\}}$

नये प्रकार के प्रश्न

Q1. $\frac{x^2+1}{4x-3} > 2$



$(-\infty, -1) \cup (1, 5/4)$

$x^2+1=2$

$x^2=1$

$x=\pm 1$

$4x-3=2$

$4x=5$

$x=5/4$

$\frac{41}{16}$

$\cancel{5-3}=2$

$\frac{41}{32}$

Q1. $\dfrac{x^2+1}{4x-3} > 2$

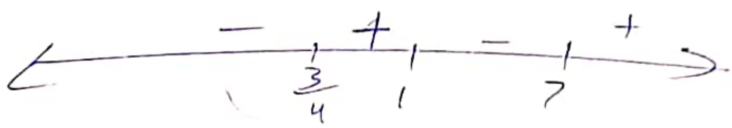
$$\frac{x^2+1-2(4x-3)}{4x-3} > 0$$

$$\frac{x^2-8x+7}{4x-3} > 0$$

$$\frac{x^2-7x-x+7}{4x-3} > 0$$

$$\frac{x(x-7)-1(x-7)}{4x-3} > 0$$

$$\frac{(x-1)(x-7)}{4x-3} > 0$$



$$\boxed{x \in \left(\frac{3}{4}, 1\right) \cup (7, \infty)}$$

(2) $$\frac{4x-3}{x^2+1} > 2$$

$$\frac{4x-3-2x^2-2}{x^2+1} > 0$$

$$4x-2x^2-5 > 0$$

$$-2x^2+4x-5 > 0$$

$$x = \frac{-4 \pm \sqrt{16-40}}{2}$$

$\cancel{x = -4 \pm}$

$D = b^2 - 4ac$

$D = 0 - 4(1)(1)$

D

$D < 0, a > 0$

always positive, never < 0

$$\boxed{x \in \phi}$$

(3) $$\frac{x}{x+1} > 2$$

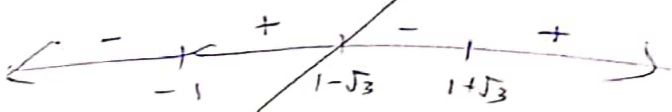
$$\frac{x-2x-2}{x+1} > 0$$

$$x = \frac{2 \pm \sqrt{4+8}}{2}$$

$x = 1 \pm \sqrt{3}$

$x = 1+\sqrt{3}, \quad 1-\sqrt{3}, -1$

$x \neq -1$



$$\boxed{x \in (-1, 1-\sqrt{3}) \cup (1+\sqrt{3}, \infty)}$$

Q4. $\dfrac{x+1}{(x-1)^2} < 1$

$\dfrac{x+1}{x^2+1-2x} < 1$

$\dfrac{x+1-(x-1)^2}{(x-1)^2} < 0$

$x+1-x^2-1+2x$

$\cancel{3x-3x}$

$\dfrac{3x-x^2}{(x-1)^2} < 0$

$\dfrac{x(3-x)}{(x-1)^2} < 0$

$-\quad+\quad+\quad-$

$-\infty \quad 0 \quad 1 \quad 3 \quad \infty$

$\cancel{x \in (0,3) - \{1\}}$

$\boxed{x \in (-\infty, 0) \cup (3, \infty)}$

Method-2

$x+1 < (x-1)^2$

$x+1 < x^2-2x+1$

$x < x^2-2x$

$\cancel{0 < x - x}$

$\cancel{x-2 > 0}$

$0 < x^2-3x$

$x^2-3x > 0$

$+\quad-\quad+$

$-\infty \quad 0 \quad 3 \quad \infty$

$\boxed{(-\infty, 0) \cup (3, \infty) \in x}$

Q 3. $\dfrac{x}{x+1} > 2$

$$\frac{x}{x+1} - \frac{2}{1} > 0$$

$$\frac{x-2(x+1)}{x+1} > 0$$

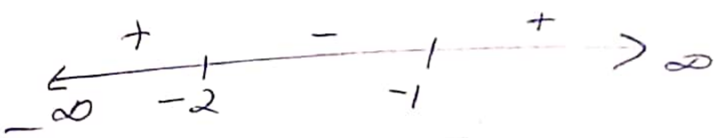
$$\frac{x-2x-2}{x+1} > 0$$

$$\frac{-x-2}{x+1} > 0$$

$$\frac{-(x+2)}{x+1} > 0$$

$$\frac{x+2}{x+1} < 0$$

$x \neq -1$



$$\boxed{x \in (-2,-1)}$$

HW· 10-05-2024

DYS-10

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16\}$

$[1, 16] - \{13\}$

Q13. $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$

$\frac{x-1+x-2}{(x-2)(x-1)} > \frac{1}{x}$

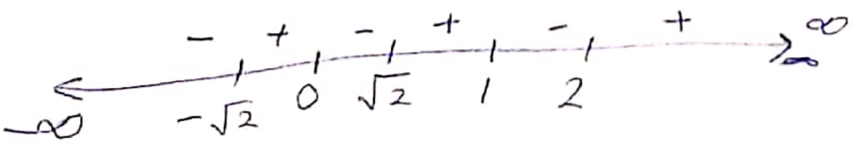
$\frac{2x-3}{(x-2)(x-1)} > \frac{1}{x}$

$\frac{x(2x-3) - (x-2)(x-1)}{(x-2)(x-1)x} > 0$

$\frac{2x^2-3x-x^2+3x-2}{(x-2)(x-1)x} > 0$

$\frac{x^2-2}{(x-2)(x-1)x} > 0$

$\frac{(x-\sqrt{2})(x+\sqrt{2})}{(x-2)(x-1)x} > 0$



$(-\sqrt{2}, 0) \cup (\sqrt{2}, 1) \cup$
$(2, \infty)$

$\boxed{(-\sqrt{2}, 0) \cup (\sqrt{2}, 1) \cup (2, \infty)}$

H.W. 21-05-2024

PYS-10 Q17

O-1 Q19-20-21

O-4 Q5, 6

Mean

→ For Any two positive real numbers, x & y $(y \geq x)$

$$AM \text{ (Airthmetic mean)} = \frac{x+y}{2}$$

$$\boxed{GM \text{ (Geometric mean)} = \sqrt{xy}}$$

$$\boxed{y \geq AM \geq GM \geq x}$$

Ex ①.

$x = 4$
$y = 16$

$$AM = \frac{16+4}{2} = \frac{20}{2} = \boxed{10}$$

$$GM = \sqrt{4 \times 16} = \sqrt{64} = \boxed{8}$$

$$y \geq AM \geq GM \geq x$$

②.

$x = 10$
$y = 90$

$$AM = \frac{10+90}{2} = \boxed{50}$$

$$GM = \sqrt{10 \times 90} = \sqrt{900} = \boxed{30}$$

③.

$x = 10$
$y = 10$

$$AM = \frac{10+10}{2} = \boxed{10}$$

$$GM = \sqrt{10 \times 10} = \sqrt{100} = \boxed{10}$$

Note:- AM = GM when $x = y$

→ In AM & GM, equality holds when $x = y$

→ For 3 ⊕ve quantities x, y, z

$$AM = \frac{x+y+z}{3}$$

$$\boxed{GM = \sqrt[3]{xyz}}$$

→ For n quantities a_1, a_2, a_3

$$AM = \frac{a_1 + a_2 + a_3 + a_4 \dots a_n}{n}$$

$$\boxed{GM = \sqrt[n]{a_1 \times a_2 \times a_3 \dots a_n}}$$

Hints

→ Maximum/Minimum or largest/smallest is in question

→ Given all quantities are ⊕ve

→ Multiplication of quantities will be a constant

Q Find the ~~maximum~~ minimum value of (1) $x+\frac{1}{x}$ (x is +ve)

(2) $x^2+\frac{4}{x^2}$

(1) $GM = \sqrt{x \times \frac{1}{x}}$

$= \sqrt{1}$

$\boxed{=1}$

$AM = \dfrac{x+\frac{1}{x}}{2}$

$= \dfrac{x^2+1}{2x}$

$1 \leq \dfrac{x+1}{x}$

$\boxed{\text{minimum value} = 2}$

(2) $GM = \sqrt{x^2 \times \frac{4}{x^2}}$

$\boxed{=2}$

$AM = \dfrac{x^2+\frac{4}{x^2}}{2}$

$x^2+\frac{4}{x^2} \geq 4$

$\boxed{\text{minimum} = 4}$

Q 2. find min value. $(x>0)$

① $x+\frac{1}{x}+3$

$$\cancel{AM = \frac{x+\frac{1}{x}+3}{2}}$$

$$\cancel{GM = \sqrt[2]{x\left(\frac{1}{x}+3\right)}}$$

$$\cancel{GM = \sqrt[2]{3}}$$

$$\cancel{GM = \sqrt{3}}$$

$$\cancel{2\sqrt{3} \le x+\frac{1}{x}+3}$$

~~min $2\sqrt{3}$~~

$\cancel{2 = x}$

$\cancel{1 = \frac{1}{x}+3}$

$x=\frac{1}{x}=3$ (not possible for any value of x)

So cannot use AM & GM.

$x+\frac{1}{x}$, min $= 2$

$2+3=5$

$$\boxed{\text{min} = 5}$$

---

② $x^2+\frac{1}{x^2}+7$

$\cancel{x^2+\frac{1}{x^2}, \text{ min} = 4}$

$$\cancel{\boxed{4+7=11}}$$

AM =

$$\frac{x^2+\frac{1}{x^2}}{2} \geq 1$$

$$x^2+\frac{1}{x^2} \geq 2$$

$$\boxed{2+7=9}$$

Q3. $f(x) = \dfrac{x^2+1+8x}{x}$

find min value of x is (+)ve

$$\underbrace{x + \frac{1}{x}}_{min = 2} + 8$$

$2 + 8$

$\boxed{min = 10}$

Q4. mini value of $f(x) = (P+Q)\left(\frac{1}{P}+\frac{1}{Q}\right)$ P & Q (+)ve

$\frac{P}{Q} + \frac{Q}{P} + 2$

$$\frac{\frac{P}{Q}+\frac{Q}{P}}{2} \geq 1$$

$\frac{P}{Q} + \frac{Q}{P} \geq 2$

$2 + 4$

$\boxed{4 = min}$

Ratio & ~~Prat~~proportion

Ratio	Proportion
→ Comparision of quantities by the method of division.	→ It is a equation which defines that two given ratios are equivalent to each other.
→ It says how many times one quantity is equal to the another quantity.	→ $::$ or $=$
→ two numbers in ratio are compared only when they have the same unit.	→ 3 Types ① Direct proportion $(a \propto b)$ ② Inverse proportion $(a \propto \frac{1}{b})$ ③ Continued proportion $\frac{a}{b} = \frac{c}{d}$ or $a:b::c:d$
→ a to b $a:b$ a/b a → Antecedent b → Consequent	

Properties of ratio

① $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ ~~then~~

then:- $\frac{a+c+e}{b+d+f} = \frac{a}{b}$

eg. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

$\frac{1+2+3}{2+4+6} = \frac{6}{12} = \frac{1}{2}$

(2) ratio $a:b$

Duplicate → $a^2:b^2$

Triplicate → $a^3:b^3$

Sub Duplicate → $\sqrt{a}:\sqrt{b}$

Sub ~~d~~ triplicate → $\sqrt[3]{a}:\sqrt[3]{b}$

Q1. ~~Q~~ are the 2 ratios ~~10:8~~ 8:10 & 7:10 in proportion or not

$$\frac{8}{10} \neq \frac{7}{10}$$

not in proportion

~~Q~~. Properties of proportion -

(1) Componendo - $\frac{a}{b} = \frac{c}{d}$

componendo is $\frac{a+b}{b} = \frac{c+d}{d}$

(2) dividendo of $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{a-b} = \frac{c}{c-d}$

(3) Componendo - dividendo → $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

(4) Alternendo → $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \frac{a}{c} = \frac{b}{d}$$

(5) Invertendo → $\frac{a}{b} = \frac{c}{d}$

$$\Rightarrow \frac{b}{a} = \frac{d}{c}$$

(6) reverse componendo - dividendo - $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

$$\Rightarrow \frac{a}{b} = \frac{c}{d}$$

Q If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ show that $\frac{a^3b + 2c^2e - 3ae^2f}{b^4 + 2d^2f - 3bf^3} = \frac{ace}{bdf}$

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = t$

$a = bt,\ c = dt,\ e = ft$

$$\frac{(bt)^3 b + 2(dt)^2(ft) - 3(bt)(ft)^2(f)}{b^4 + 2d^2f - 3bf^3}$$

$$\frac{b^4t^3 + 2d^2ft^3 - 3bf^3t^3}{b^4 + 2d^2f - 3bf^3}$$

$$\frac{t^3\cancel{(b^4 + 2d^2f - 3bf^3)}}{\cancel{b^4 + 2d^2f - 3bf^3}}$$

$$t^3 = \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$$

$$= \frac{a \times c \times e}{b \times d \times f}$$

$$= \frac{ace}{bdf}$$

Q if $a:b = 1:2$ $\quad \frac{b-8a}{b-6a}$

$\frac{a}{b} = \frac{1}{2}$

$2a = b$

$\frac{2a-8a}{2a-6a} = \frac{-6\cancel{a}}{-4\cancel{a}}$

$\boxed{= \frac{3}{2}}$

Q2. $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ show $(a^2+b^2+c^2)(x^2+y^2+z^2) = (ax+by+cz)^2$

$x = at,\ y = bt,\ z = ct$

$(a^2+b^2+c^2)(a^2t^2+b^2t^2+c^2t^2)$

$a^4t^2+b^2a^2t^2+a^2c^2t^2+a^2b^2t^2+b^4t^2+b^2c^2t^2+a^2c^2t^2+b^2c^2t^2$
$+c^4t^2$

$a^4t^2+b^4t^2+c^4t^2+2a^2c^2t^2+2a^2b^2t^2+2b^2c^2t^2$

$t^2(a^2+b^2+c^2)^2 = LHS.$

$(a(at)+b(bt)+c(ct))^2$

$(a^2t+b^2t+c^2t)^2$

$t^2(a^2+b^2+c^2)^2 = RHS.$

$\therefore LHS = RHS$

Q 4. $\dfrac{3x^4+x^2-2x-3}{3x^4-x^2+2x+3} = \dfrac{5x^4+2x^2-7x+3}{5x^4-2x^2+7x-3}$ find x.

$$\frac{3x^4+(x^2-2x-3)}{3x^4-(x^2-2x-3)} = \frac{5x^4+(2x^2-7x+3)}{5x^4-(2x^2-7x+3)}$$

$$\frac{3x^4}{x^2-2x-3} = \frac{5x^4}{2x^2-7x+3}$$

$$6x^6-21x^5+9x^4 = 5x^6-10x^5-15x^4$$

$$x^6+24x^4 = 11x^5$$

$$x^6-11x^5+24x^4$$

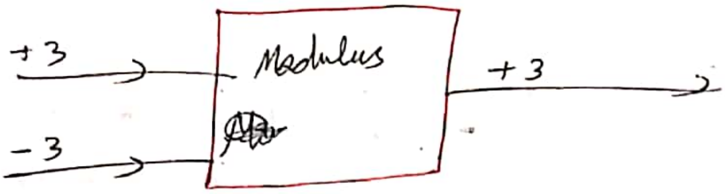
$$x^4(x^2-20x+24) = 0$$

$$x = \frac{11 \pm \sqrt{121-96}}{2}$$

$$x = \frac{11 \pm 5}{2}$$

$$\boxed{x = 8, 3, 0}$$

Modulus :-



→ Also called absolute value
→ $|x|$ (denotion)

$$f(x) = |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

$$f(x) = |x-2| = \begin{cases} (x-2) & x \geq 2 \\ -(x-2) & x < 2 \end{cases}$$

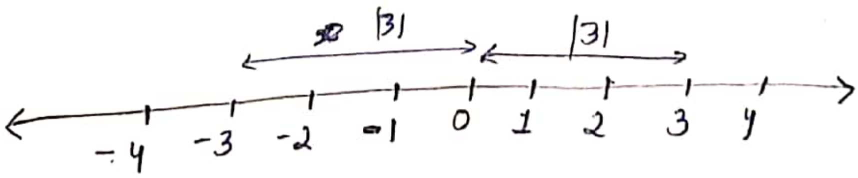
कहाँ पर zero होगा?

$$f(x) = |x+3| = \begin{cases} (x+3) & x \geq -3 \\ -(x+3) & x < -3 \end{cases}$$

Geometrical Representation -

$|x| = 3$ → Distance 3 from origin.

$x = 3, -3$



Scanned with CamScanner

- $|x| = 5 \Rightarrow$ Distance of x from origin is 5

  $x = \pm 5$

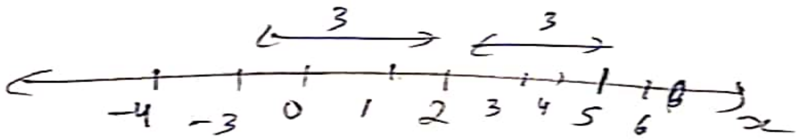
- $|x| = -2 \longrightarrow$ Not Possible

  ↳ Distance of any point from origin is -2. not possible.

  So, x is empty set.

- $|x-2| = 3$

  ↳ Distance of any point from 2 is 3

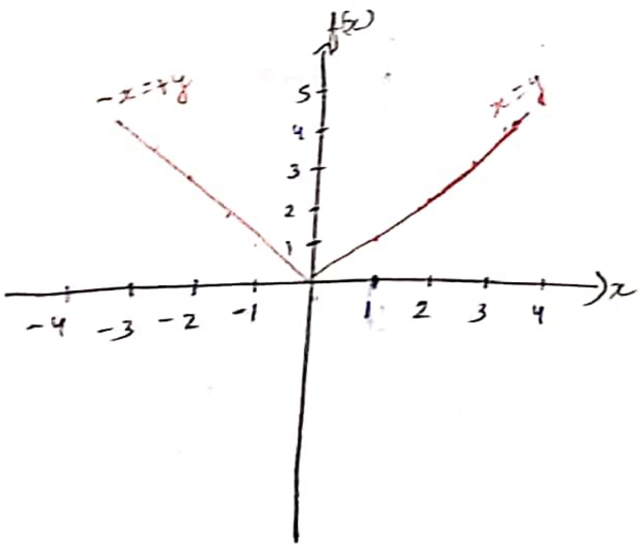


- $|x+5| = 7$

  ↳ Distance 7 from (-5).

Graphical Representation

$$f(x) = |x| \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



x	0	1	2	3
$f(x)$	0	1	2	3

$-x$	-1	-2	-3
$f(x)$	1	2	3

Note:-

① $|x| = |-x|$

$|x-2| = |2-x|$

$|x+3| = |-x-3|$

$|x| \neq -|x|$

② $\sqrt{x^2} = |x|$

③ value of modulus cannot be negative

Q Find the value of ~~your~~ x.

① $|3x-2| = 2$

$3x-2 = 2$

$3x = 4$

$\boxed{x = 4/3}$ ✓

$-3x+2 = 2$

$-3x = 0$

$\boxed{x = 0}$ ✓

② $|8x+1| = 7$

③ $|x-7| = 0$

④ $|2x-3| = -3$ - not possible

⑤ $\left|\frac{5x-10}{3}\right| = 4$

② $8x+1 = 7$

$8x = 6$

$\boxed{x = 3/4}$ ✓

$-8x-1 = 7$

$-8x = 8$

$\boxed{x = -1}$ ✓

③ $x-7 = 0$

$\boxed{x = 7}$ ✓

$-x+7 = 0$

$\boxed{x = 7}$ ✓

④ $x \in \phi$

⑤ $\frac{5x-10}{3} = 4$

$5x-10 = 12$

$5x = 22$

$\boxed{x = \frac{22}{5}}$ ✓

$\frac{5x-10}{3} = -4$

$5x-10 = -12$

$5x = -2$

$\boxed{x = \frac{-2}{5}}$ ✓

M.W. 13-05-2023

DYBS-8 (full) = $\{x : x \in \text{Left}\}$

DYS-9 (full)

DYS-10 = ϕ

DYS-11 = $\{1, 2, 3, 4, 5, 6\}$

O-4 = $\{7, 8, 9, 10\}$

J-M, J-A full

& others left

DYS-11 & [7, 21] - {13, 7, 12

Q

Hint:- with single modulus if constant is given in RHS or some positive quantity then we will solve modulus directly.

Q

① $|x^2-3x+2| = 5$

$x^2-3x+2=5$

$x^2-3x-3=0$

$\boxed{x = \frac{3 \pm \sqrt{9+12}}{2}}$

$\boxed{x = \frac{3 \pm \sqrt{21}}{2}}$ ✓

$x^2-3x+2=-5$

$x^2-3x+7=0$

$x = \frac{3 \pm \sqrt{9-28}}{2}$

× ✓

② $|5x-4| = |2x-3|$

$5x-4 = 2x-3$

$3x = 1$

$\boxed{x = 1/3}$ ✓

$5x-4 = -2x+3$

$7x = 7$

$\boxed{x = 1}$ ✓

③ $|8x-1| = x^2+1$

$8x-1 = x^2+1$

$x^2-8x+2=0$

$x = \frac{8 \pm \sqrt{64-8}}{2}$

$x = \frac{8 \pm \sqrt{56}}{2}$

$\boxed{x = 4 \pm \sqrt{14}}$

$8x-1 = -x^2-1$

$x^2+8x=0$

$x(x+8)=0$

$\boxed{x=0}$

$\boxed{x=-8}$

Q $2|x+3| = 3|x-4|$

$2x+6 = 3x-12$

$\boxed{18 = x}$

$2x+6 = -3x+12$

$5x = 6$

$\boxed{x = 6/5}$

Q $|x^2+x+1| = |x^2+x+2|$

~~$x^2+x+1 = x^2+x+2$~~

$\cancel{x^2}+\cancel{x}+1 = \cancel{x^2}+\cancel{x}+2$

$\times$

No solution

$x^2+x+1 = -x^2-x-2$

$2x^2+2x+3=0$

$x = \dfrac{-2 \pm \sqrt{4-24}}{4}$

$\times$

Q $|x| = 3x$

Case -1 $x \geq 0$

$x = 3x$

$x-3x=0$

$-2x=0$

$\boxed{x=0}$ ✓

Case - 2 $x<0$

$|x| = -x$

$x = -3x$

$x+3x=0$

$4x=0$

$\boxed{x=0}$ $\times$

Q ~~$|x^2+3x+2|$~~

Q $|x-1| = x+1$

case 1 $x-1 \geq 0$

$x \geq 1$

$\cancel{x}-1 = \cancel{x}+1$

$-1 = +1$

$x \in \phi$

Case 2 ~~$x < 0$~~

$x-1 < 0$

$x < 1$

$-(x-1) = x+1$

$-x+1 = x+1$

$2x = 0$

$x = 0$

$\boxed{x = 0}$

Q $|x^2+3x+2| = -(x+1)$

$x^2+3x+2 \geq 0$

$(x+2)(x+1) \geq 0$

$x^2+3x+2 < 0$

$(x^2+2)(x+1) < 0$

Case 1 $\quad x < -2$

$|(x+2)(x+1)| = (x+2)(x+1) \quad (\because$ take $|x| = x$, as $x \oplus)$

$(x+2)(x+1) = -(x+1)$

$(x+2)(x+1) - (x+1)$

$(x+1)(x \cancel{+} 2 \,\overset{+}{\cancel{-}}\, 1)$

$(x+1)(x+3)$

$x = -1, \quad x = -3$

$\downarrow$

satisfy

$\boxed{x = -3}$

Case 2 - $\quad -2 \leq x \leq -1$

$|(x+2)(x+1)| = -(x+2)(x+1) \quad ($take $|x| = -x$ as $x \ominus)$

$(x+2)(x+1) = -[-(x+1)]$

$(x+2)(x+1) - (x+1) = 0$

$(x+1)(x+2-1) = 0$

$(x+1)(x+1) = 0$

$\boxed{x = -1}$ (satisfy)

Case-3 $x > -1$

$(x+2)(x+1) = -(x+1)$

$x = -1$, $x = -3$
(not satisfy)

$$\boxed{x \in \{-3, -1\}}$$

combining answer from diff cases -

Q $|x| = x$

Case 1 $x \geq 0$

$x = x$
$x \in R^{+}$

$x \in R^{+} \cup \{0\}$

Case 2 $x \leq 0$

$x = -x$

$2x = 0$
$x = 0$ (satisfy)

$$\boxed{x \geq 0}$$

Q $|x+2| = -(x+1)$

Case 1 $\cancel{x+2}$ $x \geq 0$

$x+2 \geq 0$

$x \geq -2$

$x+2 = -(x+1)$

$x+2 = -x-1$

$2x = -3$

$\boxed{x = -3/2}$ (~~not~~ satisfy)

Case 2 $x \leq 0$

$x+2 = x+1$

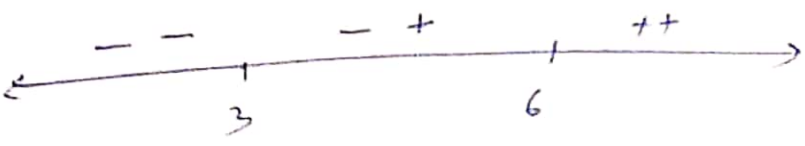
$x+1 = x$

$+1 = x - x$

$x \in \phi$

$\boxed{x = -3/2}$

Q $\underset{x=6}{|x-6|} + \underset{x=3}{|x-3|} = 1$



Case $x < 3$

$-(x-6) - (x-3) = 1$

$-x+6-x+3 = 1$

$-2x+9 = 1$

$9-1 = 2x$

$8 = 2x$

$x = 4$ (not satisfy)

Case 2 $\quad 3 \le x \le 6$

$$-(x-6)+(x-3)=1$$

$$-\cancel{x}+6+\cancel{x}-3=1$$

$$3=1$$

$$x \in \phi$$

Case 3 $\quad x > 6$

$$(x-6)+(x-3)=1$$

$$x-6+x-3=1$$

$$2x-9=1$$

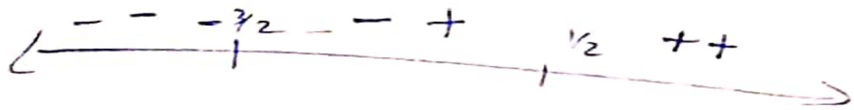
$$2x=10$$

$x=5$ (not satisfy)

$$\boxed{x \in \phi}$$

Q $|2x-1|+|2x+3|=6$

$x=1/2 \qquad x=-3/2$



Case 1 $\quad x < -3/2$

$$-(2x-1)-(2x+3)=6$$

$$-2x+1-2x-3=6$$

$$-4x-2=6$$

$$-4x=8$$

$$\boxed{x=-2}$$

Case 2 $\quad -3/2 \le x \le 1/2$

$$-(2x-1) + (2x+3) = 6$$

$$-\cancel{2x}+1 + \cancel{2x}+3 = 6$$

$$4 = 6$$

$$x \in \phi$$

Case 3 $\quad x \ge 1/2$

$$(2x-1) + (2x+3) = 6$$

$$4x + 2 = 6$$

$$4x = 4$$

$$\boxed{x = 1} \text{ (satisfy)}$$

$$\boxed{x \in \{-2, 1\}}$$

Double Inequality

① $-1 \le \underbrace{8x-3 \le 5}$

$-1 \le 8x-3$ $\qquad\qquad$ $8x-3 \le 5$

$2 \le 8x$ $\qquad\qquad$ $8x \le 8$

$\frac{1}{4} \le x$ $\qquad\qquad$ $x \le 1$

$x \in [\frac{1}{4}, \infty) \xleftrightarrow{\cap \text{ intersection}} x \in (-\infty, 1]$

Q $\quad 1 \leq \dfrac{x^2-5x-15}{x^2+x+1} \leq 2$

↳ $0 \leq 0$

$\cancel{x^2+x+1 \leq x^2-5x-15}$

$\cancel{0 \leq -6x-16}$

$\cancel{16+6x \leq 0}$

$\cancel{0 \leq x^2-5x+16}$

$\cancel{x^2-5}$

$\dfrac{x^2-5x-15}{x^2}$

$x^2+x+1 \leq x^2-5x-15$

$6x+16 \leq 0$

$x \in \left[\frac{-16}{6}, \infty\right)$

$x^2-5x-15 \leq 2x^2-10x-30$

$0 \leq x^2-5x-15$

$0 \leq$

Q $1 \leq \dfrac{x^2-5x-15}{x^2+x+1} \leq 2$

$a>0,\ D<0$

$\oplus$ ve

$x^2+x+1 \leq x^2-5x-15 \leq 2(x^2+x+1)$

$x^2+x+1 \leq x^2-5x-15$

$6x \leq -16$

$x \leq -\frac{8}{3}$

$x \in \left(-\infty, -\frac{8}{3}\right]$

$x^2-5x-15 \leq 2x^2+2x+2$

$0 \leq x^2+7x+17$

$x^2+7x+17 \geq 0$

$a>0,\ D<0$

Always $\oplus$ ve

$x \in R$

or $\cap$

~~Union~~ Intersection

$$\boxed{x \in \left(-\infty, -\frac{8}{3}\right]}$$