1. Read N from user

(a) N = 5

$X1$	$X2$
0	1
1	1
1	0
0	0
1	1

$P(X_1=1) = \frac{3}{5}$ $\qquad P(X_1=1, X_2=1) = \frac{2}{5}$

$P(X_2=1) = \frac{3}{5}$

(b) $Z = X1 + X2$

$Z = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ $\qquad P(X1=x, Z=z) = P(X1=x) \cdot P(Z=z)$, $x \in \{0,1\}$, $z \in \{0,1,2\}$.

$P(X1=0, Z=0) = P(X1=0)\ P(Z=0)$

$P(X1=0, Z=1) =$

$P(X1=0, Z=2) =$

$P(X1=1, Z=0) =$

$P(X1=1, Z=1) =$

$P(X1=1, Z=2) =$

IC252 Lab 4

1. **Independence and conditional probability.** Let $X1$ and $X2$ be independent random variables, taking values from the set $\{0,1\}$, such that both outcomes are equally likely. Define $Z = X1 + X2$. Note that Z can take values $\{0,1,2\}$. Do the following.

   - (a) Demonstrate by counting, that the $X1$ and $X2$ are independent. Generate $X1$ and $X2$ a large number of times (call this N). Count the number of times $X1$ takes the value 1, $X2$ takes the value 1, and the number of times both $X1$ and $X2$ take the value 1. This should be approximately equal. Compute the probability by hand and compare with the result of the simulation. $X1$ and $X2$ can be stored in two arrays.
   - (b) Now generate Z, using the already computed values of $X1$ and $X2$. Is Z independent of $X1$? Determine using counting, and by hand.
   - (c) Now condition $X1$ and $X2$ on $Z = 1$. Is $X1$ conditioned on Z, independent of $X2$ conditioned on Z? In other words, is $P(X1 = 1, X2 = 1|Z = 1) = P(X1 = 1|Z = 1)P(X2 = 1|Z = 1)$? Demonstrate by counting and calculate by hand.

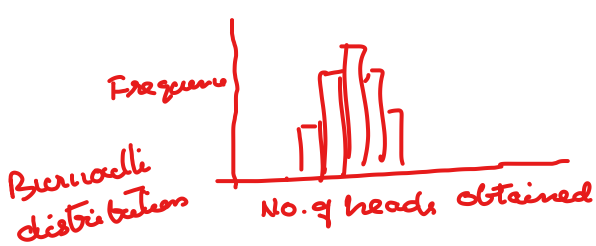
2. Simulate the number of heads obtained in N independent throws of a coin with $p(H) = p$. Accept N and p form the user. Run the experiment 10,000 times and plot the histograms of the number of heads obtained, for various values of N and p. Each N, p will need a separate histogram.

3. Count the number of times a message needs to be transmitted until it reaches correctly at the destination. The probability for a successful transmission is p. Repeat the experiment 10,000 times and plot the histograms of the count for various values of p. Each p will need a separate histogram. Read p from the user.
   Hint: Generate a random bit b with $P(b = 1) = p$. Keep adding to the count until you generate a 1. A successful transmission at the 4th try (ie. count = 4) corresponds to the binary string 0001.

2. N = no. of time coin tossed
p = P(H)

10000 Trials.

Trial 1: Two way to generate H & T coin tossing
(i) Generate N samples of 'H' (1) and 'T' (0) such that P(H) = p & P(T) = 1 − p
(ii) generate N random numbers bt 0 to 1 count the number of samples > p and consider it as number of heads.



Plot histogram from 10000 elements.
use default bin size

(a) Take N=1000 and p=0.1, 0.25, 0.5, 0.75, and 0.9. Generate histogram
Keep x-axis in the range 0 to 1000.
(b) Take differe N values different p.

3. Trial 1: generate randomly 0 & 1. such prob. of 1 is p.
001 → No. of message transmitted before success (1) is 3
Repeat for 10,000 Times. Each time store no. of messages transmitted before 1.

Frequency
Count of messages transmitted before success

Geometric distribution