# Lecture 1: Introduction

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# Why study probability?

- Probability is the logic of uncertainty.
- Statistics is concerned with collection, organization, analysis, interpretation and presentation of data.

Probability is extremely useful in a wide variety of fields, such as

- Physics: Quantum physics, statistical mechanics
- Biology: study of random mutations of genes
- Computer science: Randomized algorithms
- Meteorology: weather forcast
- Finance and gambling: Modeling stock prices
- Political science: Analysis of public opinion, prediction
- Medicine: Randomized clinical trials

# Introduction to IC252

- IC252 - a foundation course on which you will build understanding for other advanced courses.

  Topics:
- Probability (6 lectures)
- Random variables (9 lectures)
- Measures of central tendency, dispersion and association (11 lectures)
- Statistics (14 lectures)
- Case study (2 lectures)

# Introduction to IC252

Reference books:

- Sheldon Ross, *Introduction to Probability and Statistics for Engineers*, 5/e (2014), Elsevier
- Morris H. DeGroot and Mark J. Schervish, *Probability and Statistics* (4/e)(2012), Addison- Wesley.
- Blitzstein and Hwang, *Introduction to Probability* (2015), CRC Press.
- William Feller, *An Introduction to Probability*, (3/e) (2008), Volume 1, Wiley.
- Freedman, Pisani, Purves, *Statistics* (4/e)(2014), W. W. Norton & Company.

# Introduction to IC252

approx.

Evaluation (Theory 70%, Lab 30 %):

- Theory (approx. %): Quiz-1 (20%), Quiz-2 (20%), Endsem exam (50%), Assignments (10%)
- Lab (approx. %): Weekly evaluations / exams

Lecture hours:

- Monday 1-1:50 PM, Tuesday 10-10:50 AM, Friday 10-10:50 AM

Labs:

- 2-4 PM
  - Tuesday
  - Wednesday
  - Thursday
  - Friday

# Introduction to IC252

**Other Information:**

- Lecture notes, assignments and other relevant material will be shared on Moodle
- Lecture videos will be shared by YouTube
- Email me your doubts/queries at satyajit@iitmandi.ac.in
- Two Google Docs will be maintained: IC252 Technical Queries, IC252 Announcements

**Instructions for theory assignments:**

- Write solutions in A4 sized blank sheets only and in the same order in which problems are given in the assignment.
- Upload the assignments on Moodle by the deadline. Late submissions may face marks penalty or may not be considered as submissions.

# Sets

- The mathematical framework for probability is built around sets.
- A set is a collection of objects (also called elements).
- The objects can be anything, e.g., numbers, names
- Set notation $A, B, C, \ldots$
- Examples $A = \{1, 2, 3\}, \quad B = \{\alpha, \beta, a, b\}$
- The empty set is the smallest set containing no elements.
- Denoted $\emptyset$ or $\{\}$. Note that, $\emptyset \neq \{\emptyset\}$

empty set

a set containing the empty set as an element.

- Subset relation: $A$ is a subset of $B$, denoted $A \subseteq B$, if every element of $A$ is also an element of $B$.

# Sets

- Union of sets $A, B$, denoted $A \cup B$, is the set of all objects that are in $A$ or $B$.
- Intersection of sets $A, B$, denoted $A \cap B$, is the set of all objects that are in both $A$ and $B$
- $A$ and $B$ are disjoint sets if $A \cap B = \emptyset$.
- In many applications, all the sets we're working with are subsets of some set $S$.
- Complement of a set $A$, denoted $A^c$, is the set of all objects in $S$ that are not in $A$.
- DeMorgan's law: $(A \cap B)^c = A^c \cup B^c$ and $(A \cup B)^c = A^c \cap B^c$

# Sets

- If $A$ is a finite set, we write $|A|$ for the number of elements in $A$, which is called its size or cardinality.
- Note that $|A \cup B| = |A| + |B| - |A \cap B|$ (Inclusion-Exclusion Formula for 2 sets).
- Cartesian product of sets $A, B$ is defined as

$$A \times B = \{\underbrace{(a, b)}_{} : a \in A, b \in B\}.$$

called tuple (or ordered pair).

- Example

$A = \{\alpha, \beta\}$

$B = \{1, 2, 3\}$

$\Rightarrow A \times B = \{(\alpha, 1), (\alpha, 2), (\alpha, 3), (\beta, 1), (\beta, 2), (\beta, 3)\}$

- The above operations results can be generalized for $n$ sets.

# Outcomes, sample spaces and events

- Imagine that an experiment is performed, resulting in one out of a set of possible outcomes.
- The sample space $S$ of an experiment is the set of all possible outcomes of the experiment.
- An event $A$ is a subset of the sample space $S$.
- We say that $A$ occurred if the actual outcome is in $A$.
- The sample space of an experiment (i.e., the cardinality of $S$) can be finite, countably infinite, or uncountably infinite (advance topic)
- Example: Experiment of tossing a coin.

Sample space : $S = \{H, T\}$

Outcomes : $H, T$

Events : $\phi, \{H\}, \{T\}, \{H, T\}$

# Outcomes, sample spaces and events

- Example (Coin flips). A coin is flipped 10 times. Writing Heads as $H$ and Tails as $T$, a possible outcome is $HHHTHHTTHT$, and the sample space is the set of all possible strings of length 10 of $H$'s and $T$'s. We can encode $H$ as 1 and $T$ as 0, so that an outcome is a sequence $(s_1, \ldots, s_{10})$ with $s_j \in \{0, 1\}$, and the sample space is the set of all such sequences. Now let's look at some events:
- Let $A_1$ be the event that the first flip is Heads.

$$A_1 = \{(1, s_2, \ldots, s_{10}) : s_j \in \{0,1\} \text{ for } 2 \le j \le 10\}$$

- Similarly, define $A_j$ as the event that $j$th flip is H for $j \in \{2, \ldots, 10\}$.

# Outcomes, sample spaces and events

- Let $B$ be the event that at least one flip was Heads. Write $B$ in terms of $A_j$'s:

$$B = \bigcup_{j=1}^{10} A_j$$

- Let $C$ be the event that all the flips were Heads. Write $C$ in terms of $A_j$'s:

$$C = \bigcap_{j=1}^{10} A_j$$

- Let $D$ be the event that there were at least two consecutive Heads. Write $D$ in terms of $A_j$'s:

$$D = \bigcup_{j=1}^{9} \left( A_j \cap A_{j+1} \right)$$

# Naive probability

- The earliest definition of the probability of an event was to count the number of ways the event could happen and divide by the total number of possible outcomes for the experiment.
- Let $A$ be an event for an experiment with a finite sample space $S$. The naive probability of $A$ is

$$P(A) = \frac{|A|}{|S|} = \frac{\text{number of outcomes favorable to } A}{\text{total number of outcomes in } S}.$$

- The naive definition requires $S$ to be finite, and the outcomes to be equally likely.
- Reading: Blitzstein and Hwang, *Introduction to Probability* (2015), CRC Press.