Lecture 33:
Hypothesis Testing - Part II

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Hypothesis testing

- Any hypothesis we wish to test is called null hypothesis and is denoted by H_0.
- The rejection of H_0 leads to the acceptance of an alternative hypothesis, denoted by H_1.
- The null hypothesis H_0 nullifies or opposes H_1 and is often the logical complement to H_1.
- After analysis of a sample, the analyst arrives at one of the following two conclusions:
- (1) reject H_0 in favor of H_1 because of sufficient evidence in the data or
- (2) fail to reject H_0 because of insufficient evidence in the data.

Hypothesis testing

- How to test a hypothesis based on a sample?
- Let's understand this by example: A certain type of flu vaccine is known to be only 25% effective after a period of 2 years. To determine if a new and somewhat more expensive vaccine is superior in providing protection against the same virus for a longer period of time, suppose that 20 people are chosen at random and administered the vaccine dose.
- If more than 8 of those receiving the new vaccine surpass the 2-year period without contracting the virus, the new vaccine will be considered superior to the one presently in use.
- Let $X \sim \text{Bin}(20, p)$ be the binomial r.v.: "number of people who receive protection for a period of at least 2 years'.'

Hypothesis testing

- We are essentially testing the null hypothesis that the new vaccine is equally effective after a period of 2 years as the one now commonly used.
- The alternative hypothesis is that the new vaccine is in fact superior.
- This is equivalent to testing the hypothesis that the binomial parameter for the probability of a success on a given trial is $p = 1/4$ against the alternative that $p > 1/4$.
- Then the null and alternative hypothesis are

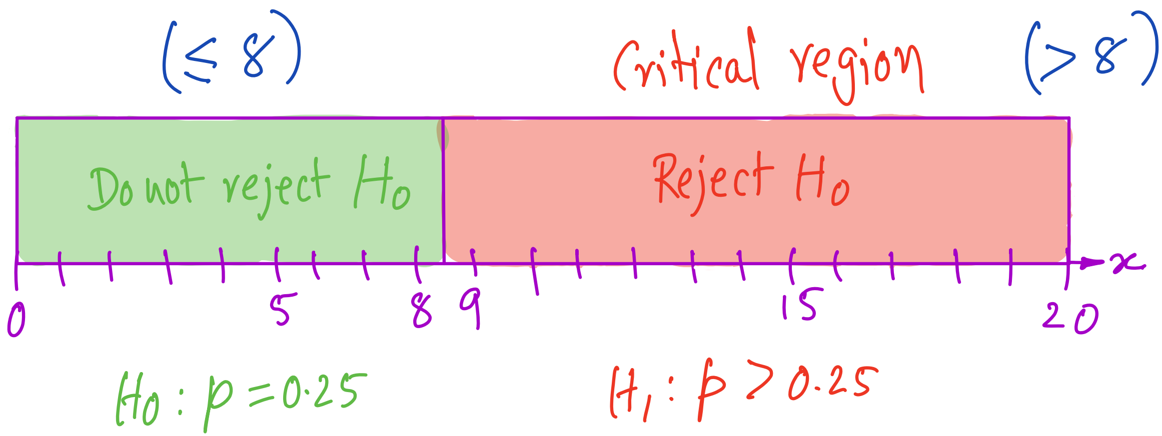
$$H_0 : p = 0.25,$$
$$H_1 : p > 0.25.$$

- The possible values of X are from 0 to 20.

Hypothesis testing

defined by the data analyst, it may be defined as 7 or 9 too.

- If the score is greater than 8 then we reject the null hypothesis H_0. This is referred to as the critical region or rejection region.
- The last value before passing into the critical region is called the critical value
- If the score is less than or equal to 8 then we fail to reject the null hypothesis H_0.



Hypothesis testing

- Null hypothesis is usually stated as an equality.
- A test of any statistical hypothesis where the alternative is one sided, such as

$$H_0 : \theta = \theta_0,$$
$$H_1 : \theta > (\text{or} <)\theta_0$$

  is called a one-tailed test.
- A test of any statistical hypothesis where the alternative is two sided, such as

$$H_0 : \theta = \theta_0,$$
$$H_1 : \theta \neq \theta_0$$

  is called a two-tailed test.

Hypothesis testing

- Example: A manufacturer of a certain brand of snack claims that the average saturated fat content does not exceed 1.5 grams per serving. State the null and alternative hypotheses to be used in testing this claim and determine the critical region.

- Let X be "the saturated fat content in a serving."

$$\left.\begin{aligned} H_0 &: \mu_X = 1.5 \\ H_1 &: \mu_X > 1.5 \end{aligned}\right\} \text{One-tailed test}$$

- Assume that the null hypothesis is rejected if $\mu_X \geq 1.7$.

- Then, this is the one tailed test and the critical region is the interval from 1.7 to $+\infty$. That is, if $\underbrace{\bar{X}}_{\text{sample mean}} \in [1.7, \infty)$ then we reject H_0.

Hypothesis testing

- Example: A real estate agent claims that 60% of all residences being built today are 3-bedroom homes. To test this claim, a large sample of new residences is inspected; the proportion of these homes with 3 bedrooms is recorded and used as the test statistic. State the null and alternative hypotheses to be used in this test and determine the critical region.

– Let $X = 1$ if a home built today is 3-bedroom and 0 otherwise. Then,

$$\left.\begin{array}{l} H_0 : \mu_X = 0.6 \\ H_1 : \mu_X \neq 0.6 \end{array}\right\} \text{two-tailed test}$$

– Assume that the sample size is 100. Then the critical region can be defined as $[0, 55]$ and $[65, 100]$ where we reject H_0 if $X_1 + \dots + X_{100}$ is in the critical region.

Hypothesis testing

- Now, note that we reject or fail to reject H_0 only based on the sample and hence, there may be an error in testing a hypothesis.
- Rejection of the null hypothesis H_0 when it is true is called a type I error.
- Nonrejection of H_0 when it is false is called a type II error.

	H_0 is true	H_0 is false
Do not reject H_0	Correct decision	Type II error
Reject H_0	Type I error	Correct decision

Hypothesis testing

- The probability of committing a type I error, also called the level of significance, is denoted by α.
- Type I error: Rejection of the null hypothesis when it is true.
- For our example, the type I error occurs if $X > 8$ but $p = 1/4$.
- Let's find the probability of the type I error:

Level of significance $\alpha = P\{\text{type I error}\}$

$$= P(X > 8 \text{ when } p = 1/4)$$

$$= \sum_{k=9}^{20} \binom{20}{k} (1/4)^k (3/4)^{20-k}$$

$$= 0.04093$$

Hypothesis testing

- The power of a test is the probability of rejecting H_0 given that a specific alternative is true, i.e., $1 - \beta$.
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Hypothesis testing

- In general, one wants to control the probability of committing type I or type II error or both.
- It is customary to choose $\alpha = .05$, or in some tests, $\alpha = .01$.
- p-value is an important parameter to decide whether to reject the null hypothesis.

- p-values are the probability of obtaining an effect at least as extreme as the one in your sample data, assuming the truth of the null hypothesis.

- Question: If the p-value for a given sample is very low, shall we accept or reject H_0?