Parameter Estimation Assignment

Q1 Let ($x_1, x_2, \ldots$) be a random sample of size n taken from a Normal population with parameters : mean = θ_1 and variance = θ_2. Find the Maximum Likelihood ~~parameters~~ Estimates of these parameters

A1 Given :

Sample ($x_1, x_2 \ldots x_n$) from a normal distribution

Mean $\mu = \theta_1$, Variance $\sigma^2 = \theta_2$.

Likelihood Function for normal distribution:

$$L(\theta_1, \theta_2 | x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log on both sides

$$\ln(L) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

MLE for θ_1 : $\frac{\partial \ln(L)}{\partial \theta_1}$

$$\frac{\partial \ln(L)}{\partial \theta_1} = -0 - 0 - \frac{1}{2\theta_2}(2)(-1)\sum_{i=1}^{n}(x_i-\theta_1)$$

$$\frac{\partial \ln(L)}{\partial \theta_1} = \sum_{i=1}^{n}(x_i-\theta_1)$$

· To set this to max , $\frac{\partial \ln(L)}{\partial \theta_1} = 0$

$$\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\sum_{i=1}^{n} x_i - n\theta_1 = 0$$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}}$$

Scanned with CamScanner

M T W T F S S
Page No.:
Date:
YOUVA

MLE for θ_2 : $\frac{\partial \ln(L)}{\partial \theta_2}$

$$\frac{\partial \ln(L)}{\partial \theta_2} = \cancel{-n} - 0 - \frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

To maximize, set the derivative to 0

$$-\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = \frac{n}{2\theta_2}.$$

$$\sum (x_i - \theta_1)^2 = n\theta_2$$

$$\sum (x_i - \bar{x})^2 = n\theta_2$$

$$\cancel{\theta_2 = \frac{1}{n} \sum (x_i - \bar{x})^2}$$

$$\boxed{\theta_2 = \frac{1}{n} \sum (x_i - \bar{x})^2}$$

Scanned with CamScanner

Q2 Let $(X_1, X_2, \ldots X_n)$ be a random sample from $B(m, \theta)$ distribution where $\theta \in (0,1)$ is unknown & m is a known positive integer. Compute value of θ using the MLE

A2 Given :

① Sample $(X_1, X_2 \ldots X_n)$ from a Binomial distribution $B(m, \theta)$

② m is a known positive integer

③ θ is unknown & lies between $(0,1)$

Likelihood function for Binomial distribution

$$L(\theta|x) = \theta^{\sum_{i=1}^{n} x_i} (1-\theta)^{n.m - \sum_{i=1}^{n} x_i}$$

Taking log $\rightarrow$ $\ln(L(\theta|x)) = \left(\sum_{i=1}^{n} x_i\right) \ln(\theta) + \left(nm - \sum_{i=1}^{n} x_i\right) \ln(1-\theta)$

$$\frac{\partial \ln(L)}{\partial \theta} = \frac{\sum_{i=1}^{n} x_i}{\theta} - \frac{nm - \sum_{i=1}^{n} x_i}{1-\theta}$$

Solving for θ

$$\boxed{\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{nm}}$$

Scanned with CamScanner