$J(\theta) \in R^{6\times n}$ $\quad$ $Rank(J) \le min(6, n)$

Full Rank: $Rank(J) = min(6, n)$

if $Rank(J) < min(6, n)$, robot is at singularity

① $n < 6$: Jacobian is tall, kinematically deficient

② $n = 6$: Jacobian is square

③ $n > 6$: Jacobian is fat, redundant

# Manipulability Ellipsoids

For manipulability ellipsoid, it assume $\|\dot{\theta}\| = 1$

$$\dot{\theta}^T \dot{\theta} = 1$$

$$(J^{-1} V_{tip})^T (J^{-1} V_{tip}) = 1$$

$$V_{tip}^T J^{-T} J^{-1} V_{tip} = 1$$

$$V_{tip}^T (J J^T)^{-1} V_{tip} = 1$$

$$V_{tip}^T A^{-1} V_{tip} = 1, \quad A = J J^T$$

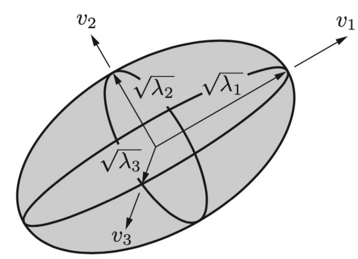


**Figure 5.13:** An ellipsoid visualization of $\dot{q}^T A^{-1} \dot{q} = 1$ in the $\dot{q}$ space $\mathbb{R}^3$, where th principal semi-axis lengths are the square roots of the eigenvalues $\lambda_i$ of $A$ and th directions of the principal semi-axes are the eigenvectors $v_i$.

As stuff in Linalg, $\dot{q}^T A^{-1} \dot{q} = 1$ is an ellipsoid in dimension of $A$

Let $v_i$ and $\lambda_i$ be the eigenvectors and eigenvalue of the ellipsoid. Direction and length of Principle are $v_i$ and $\sqrt{\lambda_i}$, respectively

$J(\theta) = \begin{bmatrix} J_\omega(\theta) \end{bmatrix}$ make two seperate ellipsoid with $A=J_\omega J_\omega^T$ and $A=J_v J_v^T$

Three measures to dicide whether a robot is easy to move at current config. (Away from singularity):

① Ratio of longest to shortest semi-axis

$$\mu_1(A)=\frac{\sqrt{\lambda_{max}(A)}}{\sqrt{\lambda_{min}(A)}}\geq 1$$

the more $\mu_1(A)$ close to 1, the better, isotropic

② Condition Number

$$\mu_2(A)=\frac{\lambda_{max}(A)}{\lambda_{min}(A)}\geq 1$$

Same as ①. better close to "1"

③ Volume Proportion

$$\mu_3(A)=\sqrt{\lambda_1\lambda_2\cdots}=\sqrt{\det(A)}$$

The larger means closer to a square or circle which is better

The force ellipsoid is just like manipulability ellipsoi

$$f^T J J^T f=f^T B^{-1} f=1$$

$$B=(JJ^T)^{-1}=A^{-1}$$

The length of force ellipsoid semi-axis is $\frac{1}{\sqrt{\lambda_i}}$

volume to be $\sqrt{\lambda_1\lambda_2\cdots}$

Therefore when $\mu_3(A)\left(\sqrt{\lambda_1\lambda_2\cdots}\right)$ to be infinity small, $\mu_3(B)\left(\frac{1}{\sqrt{\lambda_1\lambda_2\cdots}}\right)$ would be infinitely large.