Just like $R = \hat{w}\theta$, T can be exponentially represented by $S\theta$, where S is screw axis

$\log: SE(3) \to se(3)$

$\exp: se(3) \to SE(3)$

Same procedure doing exp and log for R, we have

$$e^{[S]\theta} = I + [S]\theta + [S]^2\frac{\theta^2}{2!} + [S]^3\frac{\theta^3}{3!} + \cdots$$

$$= \begin{bmatrix} e^{[w]\theta} & G(\theta)v \\ 0 & 1 \end{bmatrix} \quad G(\theta) = I\theta + [w]\frac{\theta^2}{2} + [w]^2\frac{\theta^3}{3!} + \cdots$$

$$G(\theta) = I\theta + \left(\frac{\theta^2}{2!} - \frac{\theta^4}{4!}\cdots\right)[w] + \left(\frac{\theta^3}{3!} - \frac{\theta^5}{5!} + \cdots\right)[w]^2$$

$$= I\theta + (1-\cos\theta)[w] + (\theta - \sin\theta)[w]^2$$

If there's rotational part ($\|w\| = 1$), finally we have:

$$e^{[S]\theta} = \begin{bmatrix} e^{[w]\theta} & (I\theta + (1-\cos\theta)[w] + (\theta-\sin\theta)[w]^2)v \\ 0 & 1 \end{bmatrix}$$

If there's only translational part, we have:

$$e^{[S]\theta} = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$$

Algorithm: Given (R, p) written as $T \in SE(3)$, find a $\theta \in [0, \pi]$ and a screw axis $\mathcal{S} = (\omega, v) \in \mathbb{R}^6$ (where at least one of $\|\omega\|$ and $\|v\|$ is unity) such that $e^{[\mathcal{S}]\theta} = T$. The vector $\mathcal{S}\theta \in \mathbb{R}^6$ comprises the exponential coordinates for T and the matrix $[\mathcal{S}]\theta \in se(3)$ is the matrix logarithm of T.

(a) If $R = I$ then set $\omega = 0$, $v = p/\|p\|$, and $\theta = \|p\|$.

(b) Otherwise, use the matrix logarithm on $SO(3)$ to determine ω (written as $\hat{\omega}$ in the $SO(3)$ algorithm) and θ for R. Then v is calculated as

$$v = G^{-1}(\theta)p \tag{3.91}$$

where

$$G^{-1}(\theta) = \frac{1}{\theta}I - \frac{1}{2}[\omega] + \left(\frac{1}{\theta} - \frac{1}{2}\cot\frac{\theta}{2}\right)[\omega]^2. \tag{3.92}$$

Wrenches

For a point r represented in frame $\{a\}$, we have:

its $\begin{cases} \text{position}, r_a \\ \text{force}, f_a \end{cases}$, its moment (torque), $m_a = r_a \times f_a$

wrench of this point can be written as:

$$\mathcal{F}_a = \begin{bmatrix} m_a \\ f_a \end{bmatrix}$$

If there're more than 1 wrenches acting at same point, wrench of this point is just the sum of them.

(make sure they're represented in same frame)

Because the power generated by f_r and V_r is its property, which doesn't depend on the choice of frame.

$$V_a^T \mathcal{F}_a = V_b^T \mathcal{F}_b$$

$$V_a^T \mathcal{F}_a = ([Ad_{T_{ba}}] V_a)^T \mathcal{F}_b$$

$$V_a^T \mathcal{F}_a = V_a^T [Ad_{T_{ba}}]^T \mathcal{F}_b$$

$$\mathcal{F}_a = (Ad_{T_{ba}})^T \mathcal{F}_b \Leftrightarrow \mathcal{F}_b = (Ad_{T_{ab}})^T \mathcal{F}_a$$