MA-203
NUMERICAL METHODS

Mathematical Modelling of Damped Vibrations with One Degree of Freedom

GROUP 32:
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ABSTRACT:

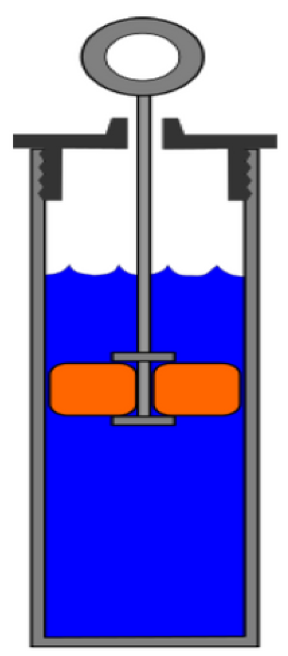
Studying viscous damping's impact on dynamic mechanical systems is vital in engineering. This research assesses damping's influence by analyzing velocity and acceleration patterns with varying coefficients and initial conditions. These insights will enhance vibration control in applications like shock absorbers, landing gear, and seismic dampers. The project involves developing a theoretical model, deriving equations, and using numerical methods to determine motion profiles, contributing to better engineering solutions.

THEORY & EQUATIONS:

On performing required calculations, we come up with this expression for critical damping:

$$\zeta = 2mw_n$$

$$\frac{d^2x}{dt^2} + 2\zeta w_n \frac{dx}{dt} + w_n^2 x = 0$$



(Viscous Damper)

NUMERICAL APPROACH:

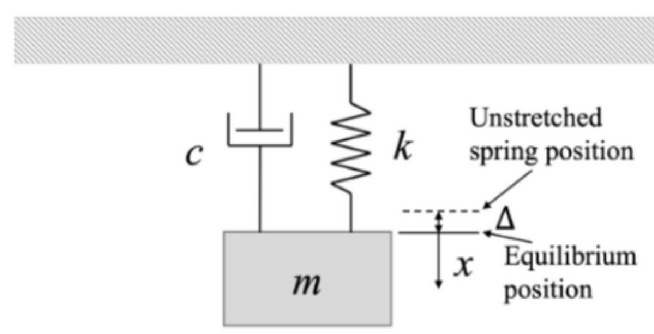
Here, we shall be using two numerical methods to solve the equations associated with viscous damping vibrations:

1. **Euler's Method:**

Euler's method, also known as the point-slope method, is a numerical algorithm used to approximate solutions to differential equations. It directly estimates a function's behavior within a small interval by approximating the slope at a specific point using the function's first derivative.

Using this approximation, Euler's method predicts a new value yi+1 at the next point xi+1 using the formula:

$$y_{i+1} = y_i + f(x_i, y_i).h$$



$\vec{F}_c = c\dot{\vec{x}}$

(FBD of a damped spring system as an initial step towards numerical approach)

2. **Runge- Kutta Method:**

The Runge-Kutta method is an iterative numerical technique for solving ordinary differential equations. Solution accuracy depends on step size and the method's convergence. RK methods provide high accuracy similar to a Taylor series approach without the need to calculate higher-order derivatives

We will use the fourth-order Runge-Kutta method:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \cdot h$$

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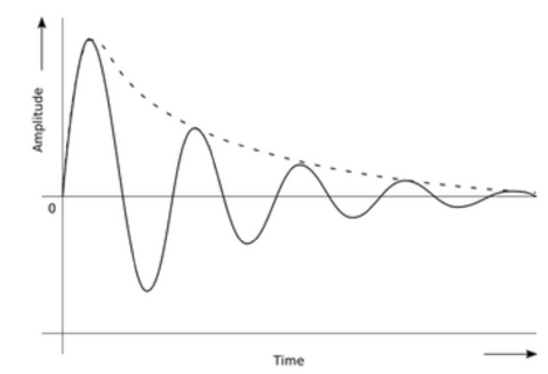
CASES OF DAMPING:

1. Underdamped
2. Overdamped
3. Critically
4. Undamped



RESULTS:

Plotting position and velocity graphs reveals nearly identical results due to very small step sizes and numerous iterations. The plots in the following pages show decreasing vibration amplitudes.