

## Artificial Intelligence Practical 4

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AIM: Implement the 8 puzzle problem using A\* Algorithm

8 puzzle problem is about reaching the final state from any intermediary state. Final State is defined as below:

1		2		3
-----				
4		5		6
-----				
7		8		

### Misplaced Tiles

This heuristic function returns integer representing the number of tiles no in the correct position

### Manhattan Distance

This heuristic function returns integer giving sum of manhattan distance of the each tile with its correct position

```
import numpy as np
```

```
class Node:
```

```
    def __init__(self, data, parent, act, g, h_ = 0):
        self.data = data
        self.parent = parent
        self.act = act
        self.g = g
        self.h_ = h_

    def __str__(self):
        return self.data.__repr__()
```

```
initial_node = Node(np.array([[1,2,3],[4,0,6],[7,5,8]]), None, None,
0, 0)
```

```
def whereEmpty(node):
    return np.argwhere(node.data == 0)[0]
```

```
whereEmpty(initial_node)
```

```
array([1, 1])
```

```
print(initial_node)
```

```

array([[1, 2, 3],
       [4, 0, 6],
       [7, 5, 8]])

def misplaced_tiles(data):
    return 9 - np.sum(data == np.array([[1,2,3],[4,5,6],[7,8,0]]))

def manhattan_distance(data):
    place = {
        0: (2,2),
        1: (0,0),
        2: (0,1),
        3: (0,2),
        4: (1,0),
        5: (1,1),
        6: (1,2),
        7: (2,0),
        8: (2,1)
    }
    md = 0
    for (x,y),d in np.ndenumerate(data):
        md += abs(place[d][0] - x) + abs(place[d][1] - y)
    return md

def canMoveLeft(node):
    return whereEmpty(node)[1] != 0

def moveLeft(node, h_function):
    pointer = whereEmpty(node)
    new_Node = Node(np.copy(node.data), node, 'left', node.g + 1)

    temp = new_Node.data[pointer[0], pointer[1] - 1]
    new_Node.data[pointer[0], pointer[1] - 1] =
new_Node.data[pointer[0], pointer[1]]
    new_Node.data[pointer[0], pointer[1]] = temp

    new_Node.h_ = h_function(new_Node.data)

    return new_Node

print(canMoveLeft(initial_node))
print(moveLeft(initial_node, misplaced_tiles))

True
array([[1, 2, 3],
       [0, 4, 6],
       [7, 5, 8]])

def canMoveRight(node):
    return whereEmpty(node)[1] != 2

```

```

def moveRight(node, h_function):
    pointer = whereEmpty(node)
    new_Node = Node(np.copy(node.data), node, 'right', node.g + 1)

    temp = new_Node.data[pointer[0], pointer[1] + 1]
    new_Node.data[pointer[0], pointer[1] + 1] =
new_Node.data[pointer[0], pointer[1]]
    new_Node.data[pointer[0], pointer[1]] = temp

    new_Node.h_ = h_function(new_Node.data)

    return new_Node

```

```

print(canMoveRight(initial_node))
print(moveRight(initial_node, misplaced_tiles))

```

```

True
array([[1, 2, 3],
       [4, 6, 0],
       [7, 5, 8]])

```

```

def canMoveUp(node):
    return whereEmpty(node)[0] != 0

```

```

def moveUp(node, h_function):
    pointer = whereEmpty(node)

    new_Node = Node(np.copy(node.data), node, 'up', node.g + 1)

    temp = new_Node.data[pointer[0] - 1, pointer[1]]
    new_Node.data[pointer[0] - 1, pointer[1]] =
new_Node.data[pointer[0], pointer[1]]
    new_Node.data[pointer[0], pointer[1]] = temp

    new_Node.h_ = h_function(new_Node.data)

    return new_Node

```

```

print(canMoveUp(initial_node))
print(moveUp(initial_node, misplaced_tiles))

```

```

True
array([[1, 0, 3],
       [4, 2, 6],
       [7, 5, 8]])

```

```

def canMoveDown(node):
    return whereEmpty(node)[0] != 2

```

```

def moveDown(node, h_function):
    pointer = whereEmpty(node)
    new_Node = Node(np.copy(node.data), node, 'down', node.g + 1)

    temp = new_Node.data[pointer[0] + 1, pointer[1]]
    new_Node.data[pointer[0] + 1, pointer[1]] =
new_Node.data[pointer[0], pointer[1]]
    new_Node.data[pointer[0], pointer[1]] = temp

    new_Node.h_ = h_function(new_Node.data)

    return new_Node

print(canMoveDown(initial_node))
print(moveDown(initial_node, misplaced_tiles))

True
array([[1, 2, 3],
       [4, 5, 6],
       [7, 0, 8]])

def CheckIfFinal(node):
    return np.all(node.data == np.array([[1,2,3],[4,5,6],[7,8,0]]))

CheckIfFinal(initial_node)

False

# Python3 program to check if a given
# instance of 8 puzzle is solvable or not

# A utility function to count
# inversions in given array 'arr[]'

def getInvCount(arr):
    inv_count = 0
    empty_value = -1
    for i in range(0, 9):
        for j in range(i + 1, 9):
            if arr[j] != empty_value and arr[i] != empty_value and
arr[i] > arr[j]:
                inv_count += 1
    return inv_count

# This function returns true
# if given 8 puzzle is solvable.
def isSolvable(puzzle) :

    # Count inversions in given 8 puzzle
    inv_count = getInvCount([j for sub in puzzle for j in sub])

```

```

    # return true if inversion count is even.
    return (inv_count % 2 == 0)

    # Driver code
puzzle = [[4, 1, 0], [7, 2, 3], [5, 8, 6]]
if(isSolvable(puzzle)) :
    print("Solvable")
else :
    print("Not Solvable")

Solvable

import random

def generatePuzzle(dep):
    node = Node(np.array([[1,2,3],[4,5,6],[7,8,0]]), None, None, 0)

    while (node.g != dep):
        step = random.randint(1,4)

        if step == 1 and canMoveLeft(node) and node.act != "right":
            node = moveLeft(node, misplaced_tiles)
        elif step == 2 and canMoveRight(node) and node.act != "left":
            node = moveRight(node, misplaced_tiles)
        elif step == 3 and canMoveUp(node) and node.act != "down":
            node = moveUp(node, misplaced_tiles)
        elif step == 4 and canMoveDown(node) and node.act != "up":
            node = moveDown(node, misplaced_tiles)
        else:
            continue
    print(node)

generatePuzzle(50)

array([[1, 2, 3],
       [4, 5, 6],
       [7, 0, 8]])
array([[1, 2, 3],
       [4, 0, 6],
       [7, 5, 8]])
array([[1, 2, 3],
       [4, 6, 0],
       [7, 5, 8]])
array([[1, 2, 3],
       [4, 6, 8],
       [7, 5, 0]])
array([[1, 2, 3],
       [4, 6, 8],
       [7, 0, 5]])

```

```
array([[1, 2, 3],
       [4, 6, 8],
       [0, 7, 5]])
array([[1, 2, 3],
       [0, 6, 8],
       [4, 7, 5]])
array([[0, 2, 3],
       [1, 6, 8],
       [4, 7, 5]])
array([[2, 0, 3],
       [1, 6, 8],
       [4, 7, 5]])
array([[2, 3, 0],
       [1, 6, 8],
       [4, 7, 5]])
array([[2, 3, 8],
       [1, 6, 0],
       [4, 7, 5]])
array([[2, 3, 8],
       [1, 6, 5],
       [4, 7, 0]])
array([[2, 3, 8],
       [1, 6, 5],
       [4, 0, 7]])
array([[2, 3, 8],
       [1, 0, 5],
       [4, 6, 7]])
array([[2, 3, 8],
       [0, 1, 5],
       [4, 6, 7]])
array([[0, 3, 8],
       [2, 1, 5],
       [4, 6, 7]])
array([[3, 0, 8],
       [2, 1, 5],
       [4, 6, 7]])
array([[3, 8, 0],
       [2, 1, 5],
       [4, 6, 7]])
array([[3, 8, 5],
       [2, 1, 0],
       [4, 6, 7]])
array([[3, 8, 5],
       [2, 0, 1],
       [4, 6, 7]])
array([[3, 8, 5],
       [2, 6, 1],
       [4, 0, 7]])
array([[3, 8, 5],
       [2, 6, 1],
```

```
    [0, 4, 7]])
array([[3, 8, 5],
       [0, 6, 1],
       [2, 4, 7]])
array([[0, 8, 5],
       [3, 6, 1],
       [2, 4, 7]])
array([[8, 0, 5],
       [3, 6, 1],
       [2, 4, 7]])
array([[8, 5, 0],
       [3, 6, 1],
       [2, 4, 7]])
array([[8, 5, 1],
       [3, 6, 0],
       [2, 4, 7]])
array([[8, 5, 1],
       [3, 6, 7],
       [2, 4, 0]])
array([[8, 5, 1],
       [3, 6, 7],
       [2, 0, 4]])
array([[8, 5, 1],
       [3, 0, 7],
       [2, 6, 4]])
array([[8, 0, 1],
       [3, 5, 7],
       [2, 6, 4]])
array([[0, 8, 1],
       [3, 5, 7],
       [2, 6, 4]])
array([[3, 8, 1],
       [0, 5, 7],
       [2, 6, 4]])
array([[3, 8, 1],
       [2, 5, 7],
       [0, 6, 4]])
array([[3, 8, 1],
       [2, 5, 7],
       [6, 0, 4]])
array([[3, 8, 1],
       [2, 0, 7],
       [6, 5, 4]])
array([[3, 8, 1],
       [2, 7, 0],
       [6, 5, 4]])
array([[3, 8, 1],
       [2, 7, 4],
       [6, 5, 0]])
array([[3, 8, 1],
```

```

        [2, 7, 4],
        [6, 0, 5]])
array([[3, 8, 1],
       [2, 0, 4],
       [6, 7, 5]])
array([[3, 0, 1],
       [2, 8, 4],
       [6, 7, 5]])
array([[0, 3, 1],
       [2, 8, 4],
       [6, 7, 5]])
array([[2, 3, 1],
       [0, 8, 4],
       [6, 7, 5]])
array([[2, 3, 1],
       [6, 8, 4],
       [0, 7, 5]])
array([[2, 3, 1],
       [6, 8, 4],
       [7, 0, 5]])
array([[2, 3, 1],
       [6, 8, 4],
       [7, 5, 0]])
array([[2, 3, 1],
       [6, 8, 0],
       [7, 5, 4]])
array([[2, 3, 1],
       [6, 0, 8],
       [7, 5, 4]])
array([[2, 3, 1],
       [0, 6, 8],
       [7, 5, 4]])
array([[0, 3, 1],
       [2, 6, 8],
       [7, 5, 4]])

```

```

def A_star(initial_node, h_function):

    if(not isSolvable(initial_node.data)):
        print("Puzzle Not Solvable")
        return

    Closed = []
    Opened = [initial_node]

    min_node = None

    while True:

        if(len(Opened) == 0):

```



```

        return False

min_node = Opened[0]
for n in Opened:
    if ((n.g+n.h_) < (min_node.g+min_node.h_)):
        min_node = n

Opened.remove(min_node)
Closed.append(min_node)

if(CheckIfFinal(min_node)):
    print("Final State Obtained")
    break

if canMoveLeft(min_node) and min_node.act != "right":
    node_l = moveLeft(min_node, h_function)
    if(CheckIfFinal(node_l)):
        print("Final State Obtained")
        min_node = node_l
        break
    Opened.append(node_l)
if canMoveRight(min_node) and min_node.act != "left":
    node_r = moveRight(min_node, h_function)
    if(CheckIfFinal(node_r)):
        print("Final State Obtained")
        min_node = node_r
        break
    Opened.append(node_r)
if canMoveUp(min_node) and min_node.act != "down":
    node_u = moveUp(min_node, h_function)
    if(CheckIfFinal(node_u)):
        print("Final State Obtained")
        min_node = node_u
        break
    Opened.append(node_u)
if canMoveDown(min_node) and min_node.act != "up":
    node_d = moveDown(min_node, h_function)
    if(CheckIfFinal(node_d)):
        print("Final State Obtained")
        min_node = node_d
        break
    Opened.append(node_d)

moves = []
while (min_node != None):
    moves.insert(0, min_node.act)
    min_node = min_node.parent

print(f"Solution: {len(moves) - 1} moves")
print("=====")

```

```

print("Initial Node: ")

node = initial_node
for move in moves:
    print(move)
    if move == 'left':
        node = moveLeft(node, misplaced_tiles)
    elif move == 'right':
        node = moveRight(node, misplaced_tiles)
    elif move == 'up':
        node = moveUp(node, misplaced_tiles)
    elif move == 'down':
        node = moveDown(node, misplaced_tiles)
    print(node)

return True

```

A\_star(initial\_node, misplaced\_tiles)

Final State Obtained

Solution: 2 moves

=====

Initial Node:

None

```

array([[1, 2, 3],
       [4, 0, 6],
       [7, 5, 8]])

```

down

```

array([[1, 2, 3],
       [4, 5, 6],
       [7, 0, 8]])

```

right

```

array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])

```

True

%%time

```

initial_node = Node(np.array([[0,1,3],[4,2,6],[7,5,8]]), None, None, 0)

```

A\_star(initial\_node, misplaced\_tiles)

Final State Obtained

Solution: 4 moves

=====

Initial Node:

None

```

array([[0, 1, 3],
       [4, 2, 6],
       [7, 5, 8]])

```

```

right
array([[1, 0, 3],
       [4, 2, 6],
       [7, 5, 8]])
down
array([[1, 2, 3],
       [4, 0, 6],
       [7, 5, 8]])
down
array([[1, 2, 3],
       [4, 5, 6],
       [7, 0, 8]])
right
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
CPU times: user 4.7 ms, sys: 149 µs, total: 4.85 ms
Wall time: 5.48 ms

```

True

```

%%time
initial_node = Node(np.array([[4, 3, 6], [2, 0, 1], [7, 5, 8]]), None,
None, 0)
A_star(initial_node, misplaced_tiles)

```

Final State Obtained

Solution: 10 moves

=====

Initial Node:

None

```

array([[4, 3, 6],
       [2, 0, 1],
       [7, 5, 8]])

```

right

```

array([[4, 3, 6],
       [2, 1, 0],
       [7, 5, 8]])

```

up

```

array([[4, 3, 0],
       [2, 1, 6],
       [7, 5, 8]])

```

left

```

array([[4, 0, 3],
       [2, 1, 6],
       [7, 5, 8]])

```

down

```

array([[4, 1, 3],
       [2, 0, 6],
       [7, 5, 8]])

```

left

```

array([[4, 1, 3],
       [0, 2, 6],
       [7, 5, 8]])
up
array([[0, 1, 3],
       [4, 2, 6],
       [7, 5, 8]])
right
array([[1, 0, 3],
       [4, 2, 6],
       [7, 5, 8]])
down
array([[1, 2, 3],
       [4, 0, 6],
       [7, 5, 8]])
down
array([[1, 2, 3],
       [4, 5, 6],
       [7, 0, 8]])
right
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
CPU times: user 6.82 ms, sys: 7.84 ms, total: 14.7 ms
Wall time: 15.6 ms

```

True

```

%%time
initial_node = Node(np.array([[0,1,2],[4,5,3],[7,8,6]]), None, None,
0)
A_star(initial_node, misplaced_tiles)

```

Final State Obtained

Solution: 4 moves

=====

Initial Node:

None

```

array([[0, 1, 2],
       [4, 5, 3],
       [7, 8, 6]])

```

right

```

array([[1, 0, 2],
       [4, 5, 3],
       [7, 8, 6]])

```

right

```

array([[1, 2, 0],
       [4, 5, 3],
       [7, 8, 6]])

```

down

```

array([[1, 2, 3],

```

```
        [4, 5, 0],
        [7, 8, 6]])
down
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
CPU times: user 2.29 ms, sys: 3.64 ms, total: 5.93 ms
Wall time: 5.57 ms
```

True

```
%%time
initial_node = Node(np.array([[1, 2, 3], [4, 0, 5], [7, 8, 6]]), None,
None, 0)
A_star(initial_node, misplaced_tiles)
```

Final State Obtained

Solution: 2 moves

=====

Initial Node:

None

```
array([[1, 2, 3],
       [4, 0, 5],
       [7, 8, 6]])
```

right

```
array([[1, 2, 3],
       [4, 5, 0],
       [7, 8, 6]])
```

down

```
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
```

CPU times: user 3.28 ms, sys: 0 ns, total: 3.28 ms

Wall time: 3.43 ms

True

```
%%time
initial_node = Node(np.array([[1, 8, 2], [4, 6, 3], [0, 7, 5]]), None,
None, 0)
A_star(initial_node, misplaced_tiles)
```

Final State Obtained

Solution: 10 moves

=====

Initial Node:

None

```
array([[1, 8, 2],
       [4, 6, 3],
       [0, 7, 5]])
```

right

```

array([[1, 8, 2],
       [4, 6, 3],
       [7, 0, 5]])
up
array([[1, 8, 2],
       [4, 0, 3],
       [7, 6, 5]])
up
array([[1, 0, 2],
       [4, 8, 3],
       [7, 6, 5]])
right
array([[1, 2, 0],
       [4, 8, 3],
       [7, 6, 5]])
down
array([[1, 2, 3],
       [4, 8, 0],
       [7, 6, 5]])
down
array([[1, 2, 3],
       [4, 8, 5],
       [7, 6, 0]])
left
array([[1, 2, 3],
       [4, 8, 5],
       [7, 0, 6]])
up
array([[1, 2, 3],
       [4, 0, 5],
       [7, 8, 6]])
right
array([[1, 2, 3],
       [4, 5, 0],
       [7, 8, 6]])
down
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
CPU times: user 13.2 ms, sys: 2.76 ms, total: 16 ms
Wall time: 18.6 ms

```

True

```

%%time
initial_node = Node(np.array([[4, 1, 8],[7, 2, 6],[0, 3, 5]]), None,
None, 0)
A_star(initial_node, manhattan_distance)

```

Final State Obtained  
Solution: 18 moves

=====

Initial Node:

None

array([[4, 1, 8],  
 [7, 2, 6],  
 [0, 3, 5]])

up

array([[4, 1, 8],  
 [0, 2, 6],  
 [7, 3, 5]])

up

array([[0, 1, 8],  
 [4, 2, 6],  
 [7, 3, 5]])

right

array([[1, 0, 8],  
 [4, 2, 6],  
 [7, 3, 5]])

down

array([[1, 2, 8],  
 [4, 0, 6],  
 [7, 3, 5]])

right

array([[1, 2, 8],  
 [4, 6, 0],  
 [7, 3, 5]])

up

array([[1, 2, 0],  
 [4, 6, 8],  
 [7, 3, 5]])

left

array([[1, 0, 2],  
 [4, 6, 8],  
 [7, 3, 5]])

down

array([[1, 6, 2],  
 [4, 0, 8],  
 [7, 3, 5]])

down

array([[1, 6, 2],  
 [4, 3, 8],  
 [7, 0, 5]])

right

array([[1, 6, 2],  
 [4, 3, 8],  
 [7, 5, 0]])

up

array([[1, 6, 2],  
 [4, 3, 0],  
 [7, 5, 8]])

```

left
array([[1, 6, 2],
       [4, 0, 3],
       [7, 5, 8]])
up
array([[1, 0, 2],
       [4, 6, 3],
       [7, 5, 8]])
right
array([[1, 2, 0],
       [4, 6, 3],
       [7, 5, 8]])
down
array([[1, 2, 3],
       [4, 6, 0],
       [7, 5, 8]])
left
array([[1, 2, 3],
       [4, 0, 6],
       [7, 5, 8]])
down
array([[1, 2, 3],
       [4, 5, 6],
       [7, 0, 8]])
right
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
CPU times: user 49.7 ms, sys: 186 µs, total: 49.9 ms
Wall time: 51.8 ms

```

True

```

%%time
initial_node = Node(np.array([[3, 5, 0],[8, 7, 2],[6, 4, 1]]), None,
None, 0)
A_star(initial_node, misplaced_tiles)

```

Final State Obtained

Solution: 24 moves

=====

Initial Node:

None

```

array([[3, 5, 0],
       [8, 7, 2],
       [6, 4, 1]])

```

down

```

array([[3, 5, 2],
       [8, 7, 0],
       [6, 4, 1]])

```

down



```
array([[3, 5, 2],
       [8, 7, 1],
       [6, 4, 0]])
left
array([[3, 5, 2],
       [8, 7, 1],
       [6, 0, 4]])
left
array([[3, 5, 2],
       [8, 7, 1],
       [0, 6, 4]])
up
array([[3, 5, 2],
       [0, 7, 1],
       [8, 6, 4]])
right
array([[3, 5, 2],
       [7, 0, 1],
       [8, 6, 4]])
right
array([[3, 5, 2],
       [7, 1, 0],
       [8, 6, 4]])
down
array([[3, 5, 2],
       [7, 1, 4],
       [8, 6, 0]])
left
array([[3, 5, 2],
       [7, 1, 4],
       [8, 0, 6]])
left
array([[3, 5, 2],
       [7, 1, 4],
       [0, 8, 6]])
up
array([[3, 5, 2],
       [0, 1, 4],
       [7, 8, 6]])
right
array([[3, 5, 2],
       [1, 0, 4],
       [7, 8, 6]])
right
array([[3, 5, 2],
       [1, 4, 0],
       [7, 8, 6]])
up
array([[3, 5, 0],
       [1, 4, 2],
```

```
        [7, 8, 6]])
```

```
left
```

```
array([[3, 0, 5],  
       [1, 4, 2],  
       [7, 8, 6]])
```

```
left
```

```
array([[0, 3, 5],  
       [1, 4, 2],  
       [7, 8, 6]])
```

```
down
```

```
array([[1, 3, 5],  
       [0, 4, 2],  
       [7, 8, 6]])
```

```
right
```

```
array([[1, 3, 5],  
       [4, 0, 2],  
       [7, 8, 6]])
```

```
right
```

```
array([[1, 3, 5],  
       [4, 2, 0],  
       [7, 8, 6]])
```

```
up
```

```
array([[1, 3, 0],  
       [4, 2, 5],  
       [7, 8, 6]])
```

```
left
```

```
array([[1, 0, 3],  
       [4, 2, 5],  
       [7, 8, 6]])
```

```
down
```

```
array([[1, 2, 3],  
       [4, 0, 5],  
       [7, 8, 6]])
```

```
right
```

```
array([[1, 2, 3],  
       [4, 5, 0],  
       [7, 8, 6]])
```

```
down
```

```
array([[1, 2, 3],  
       [4, 5, 6],  
       [7, 8, 0]])
```

```
CPU times: user 4min 52s, sys: 1.44 s, total: 4min 54s
```

```
Wall time: 4min 58s
```

```
True
```

```
%%time
```

```
initial_node = Node(np.array([[3, 5, 0],[8, 7, 2],[6, 4, 1]]), None,  
                    None, 0)
```

```
A_star(initial_node, manhattan_distance)
```

Final State Obtained

Solution: 24 moves

=====

Initial Node:

None

array([[3, 5, 0],  
[8, 7, 2],  
[6, 4, 1]])

down

array([[3, 5, 2],  
[8, 7, 0],  
[6, 4, 1]])

down

array([[3, 5, 2],  
[8, 7, 1],  
[6, 4, 0]])

left

array([[3, 5, 2],  
[8, 7, 1],  
[6, 0, 4]])

left

array([[3, 5, 2],  
[8, 7, 1],  
[0, 6, 4]])

up

array([[3, 5, 2],  
[0, 7, 1],  
[8, 6, 4]])

right

array([[3, 5, 2],  
[7, 0, 1],  
[8, 6, 4]])

right

array([[3, 5, 2],  
[7, 1, 0],  
[8, 6, 4]])

down

array([[3, 5, 2],  
[7, 1, 4],  
[8, 6, 0]])

left

array([[3, 5, 2],  
[7, 1, 4],  
[8, 0, 6]])

left

array([[3, 5, 2],  
[7, 1, 4],  
[0, 8, 6]])

up

array([[3, 5, 2],

```
        [0, 1, 4],
        [7, 8, 6]])
right
array([[3, 5, 2],
       [1, 0, 4],
       [7, 8, 6]])
right
array([[3, 5, 2],
       [1, 4, 0],
       [7, 8, 6]])
up
array([[3, 5, 0],
       [1, 4, 2],
       [7, 8, 6]])
left
array([[3, 0, 5],
       [1, 4, 2],
       [7, 8, 6]])
left
array([[0, 3, 5],
       [1, 4, 2],
       [7, 8, 6]])
down
array([[1, 3, 5],
       [0, 4, 2],
       [7, 8, 6]])
right
array([[1, 3, 5],
       [4, 0, 2],
       [7, 8, 6]])
right
array([[1, 3, 5],
       [4, 2, 0],
       [7, 8, 6]])
up
array([[1, 3, 0],
       [4, 2, 5],
       [7, 8, 6]])
left
array([[1, 0, 3],
       [4, 2, 5],
       [7, 8, 6]])
down
array([[1, 2, 3],
       [4, 0, 5],
       [7, 8, 6]])
right
array([[1, 2, 3],
       [4, 5, 0],
       [7, 8, 6]])
```

```
down
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 0]])
CPU times: user 623 ms, sys: 3.97 ms, total: 627 ms
Wall time: 639 ms

True
```