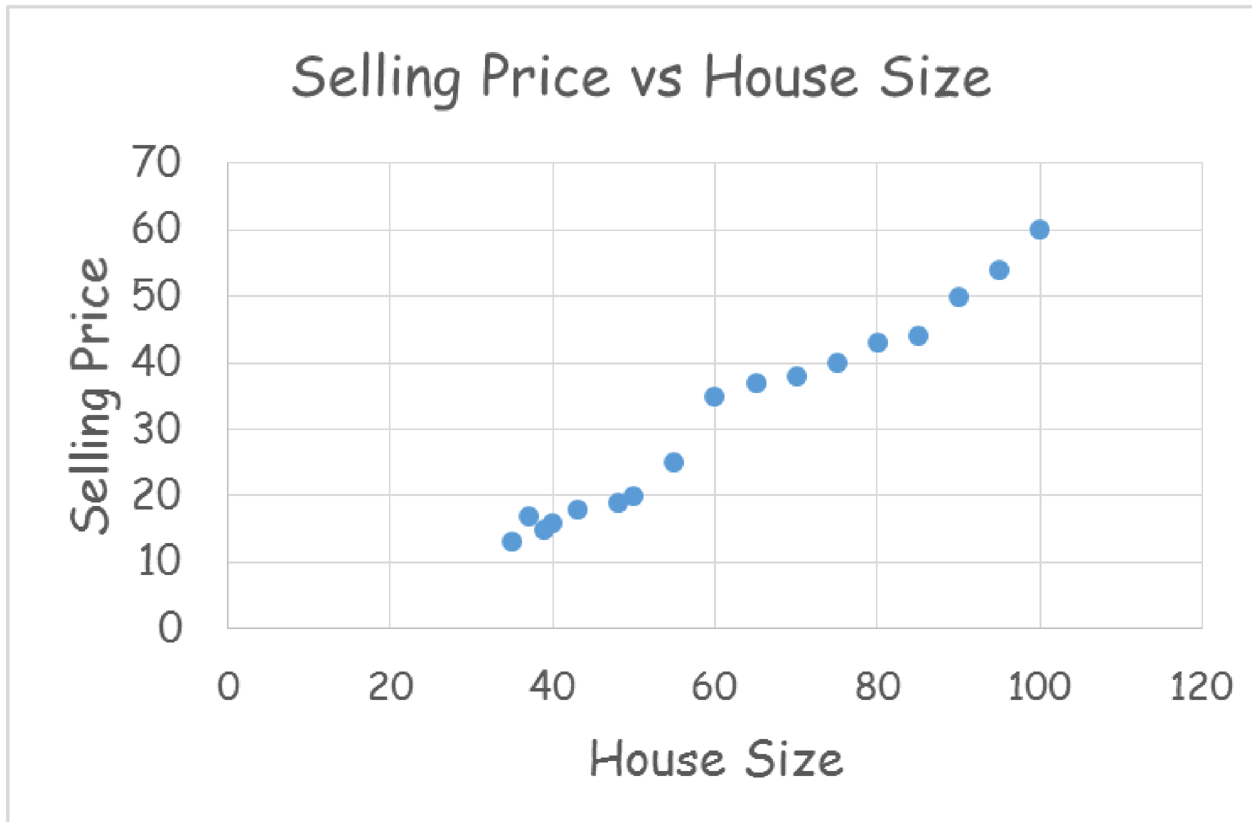
Simple Linear Regression

House Price Prediction

Area (SQM)	Selling Price (Lacs)
35	13
37	17
39	15
40	16
43	18
48	19
50	20
55	25
60	35
65	37
70	38
75	40
80	43
85	44
90	50
95	54
100	60



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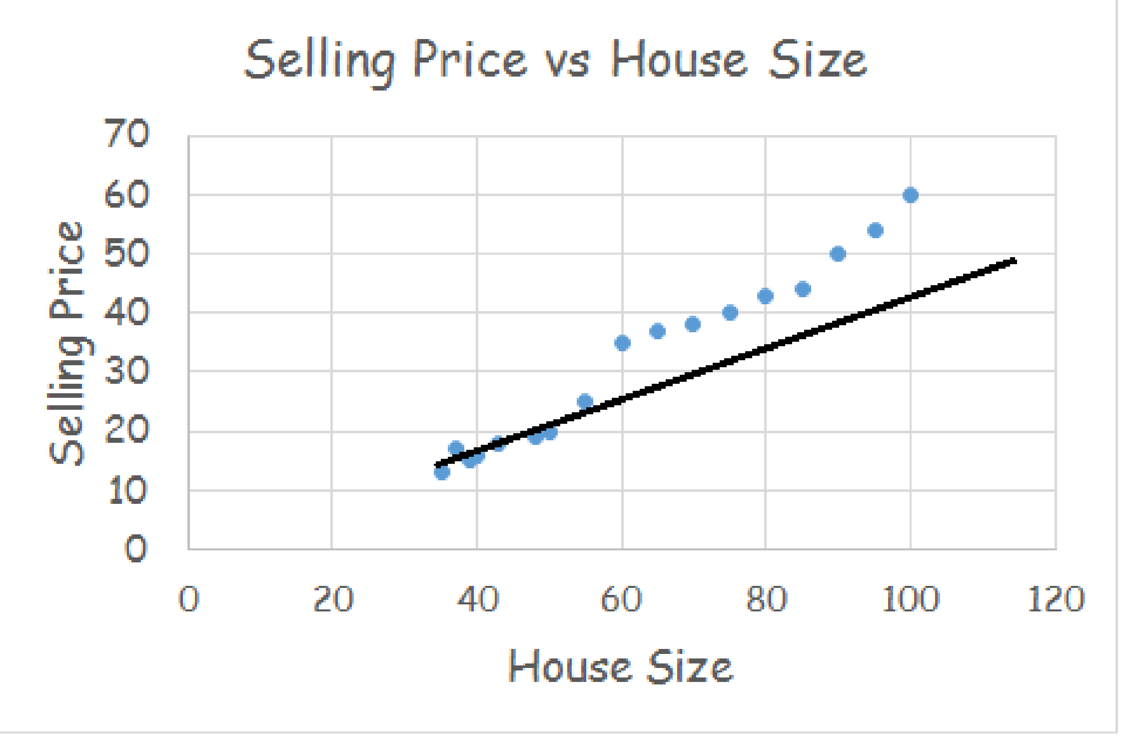

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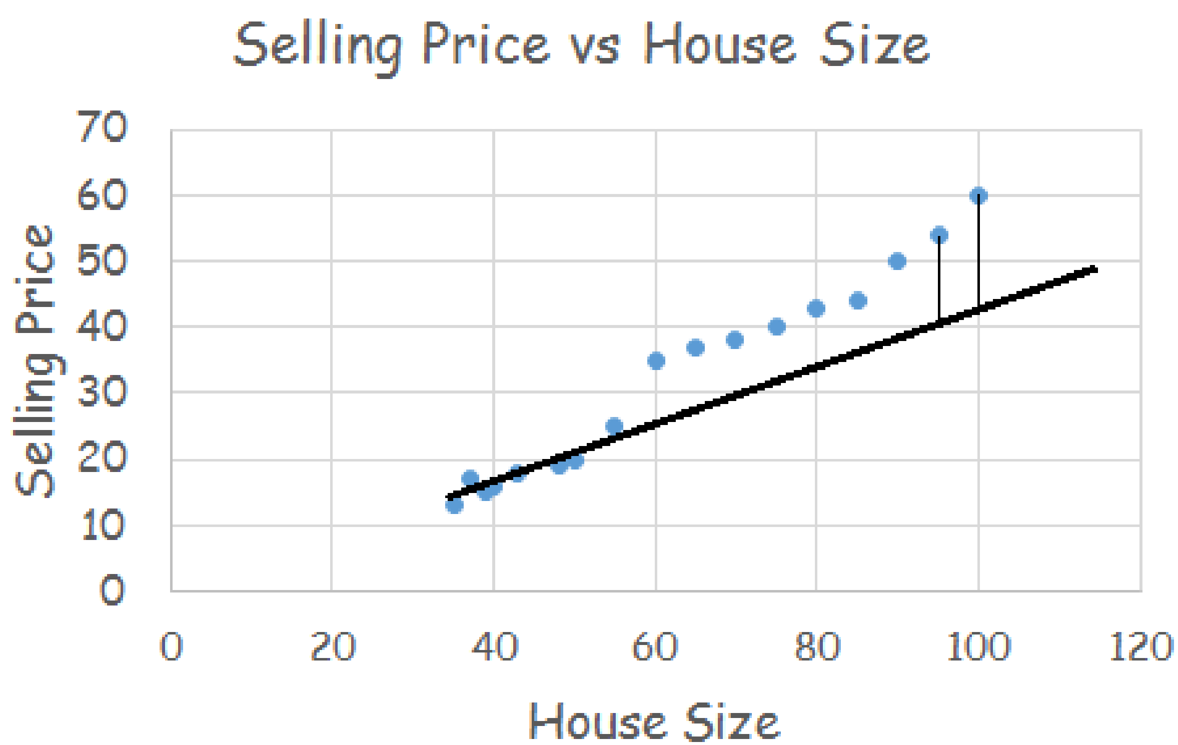

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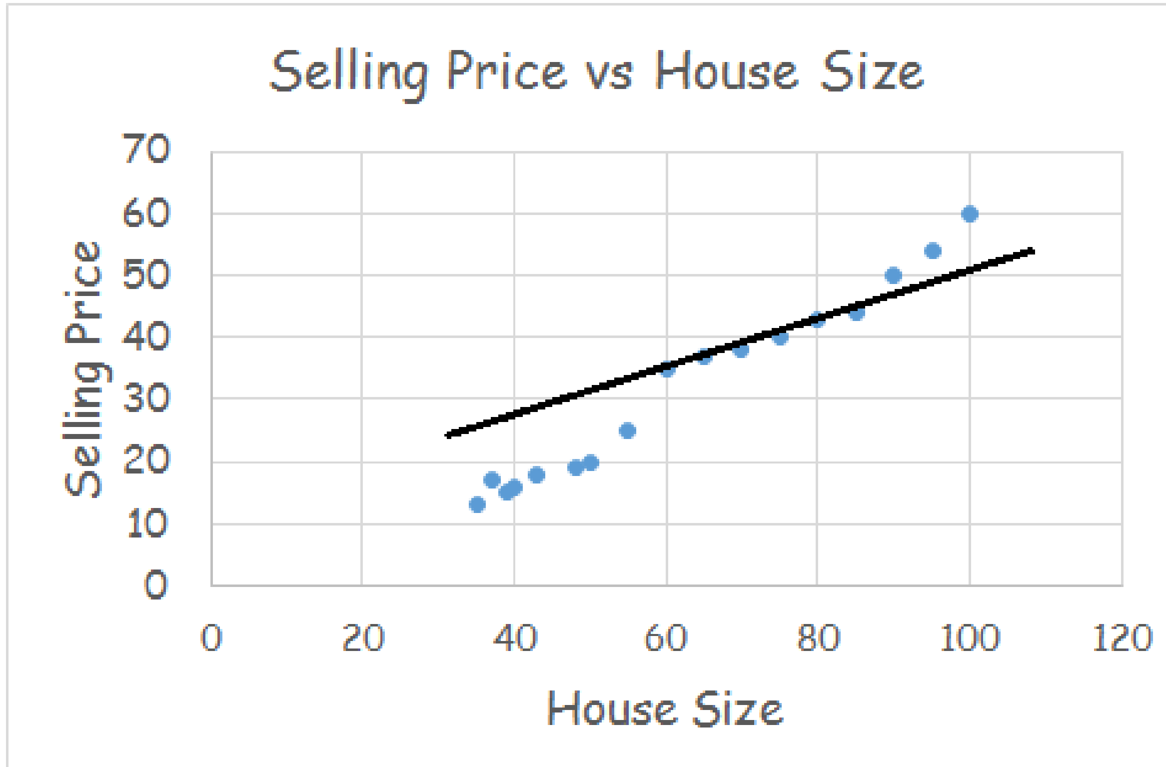

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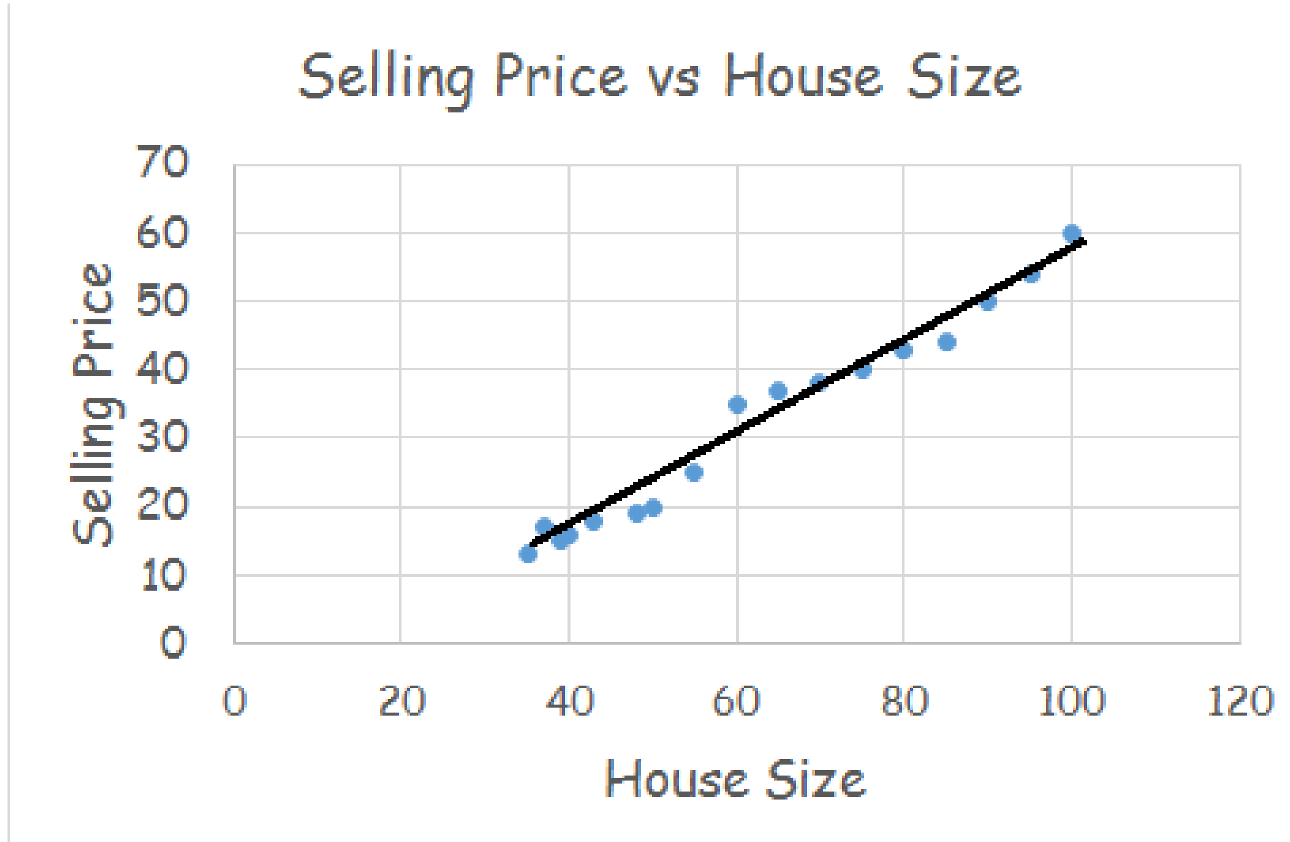


House Price Prediction

- Mathematically:
  - $Minimize \sum_{i=1}^{m}(y_i - \hat{y}_i)^2$ $\hat{y}$ = $h_\theta(x) = \theta_0 + \theta_1 x$

    OR

  - $Minimize(\frac{1}{2m})\sum_{i=1}^{m}(y_i - \hat{y}_i)^2$

    OR

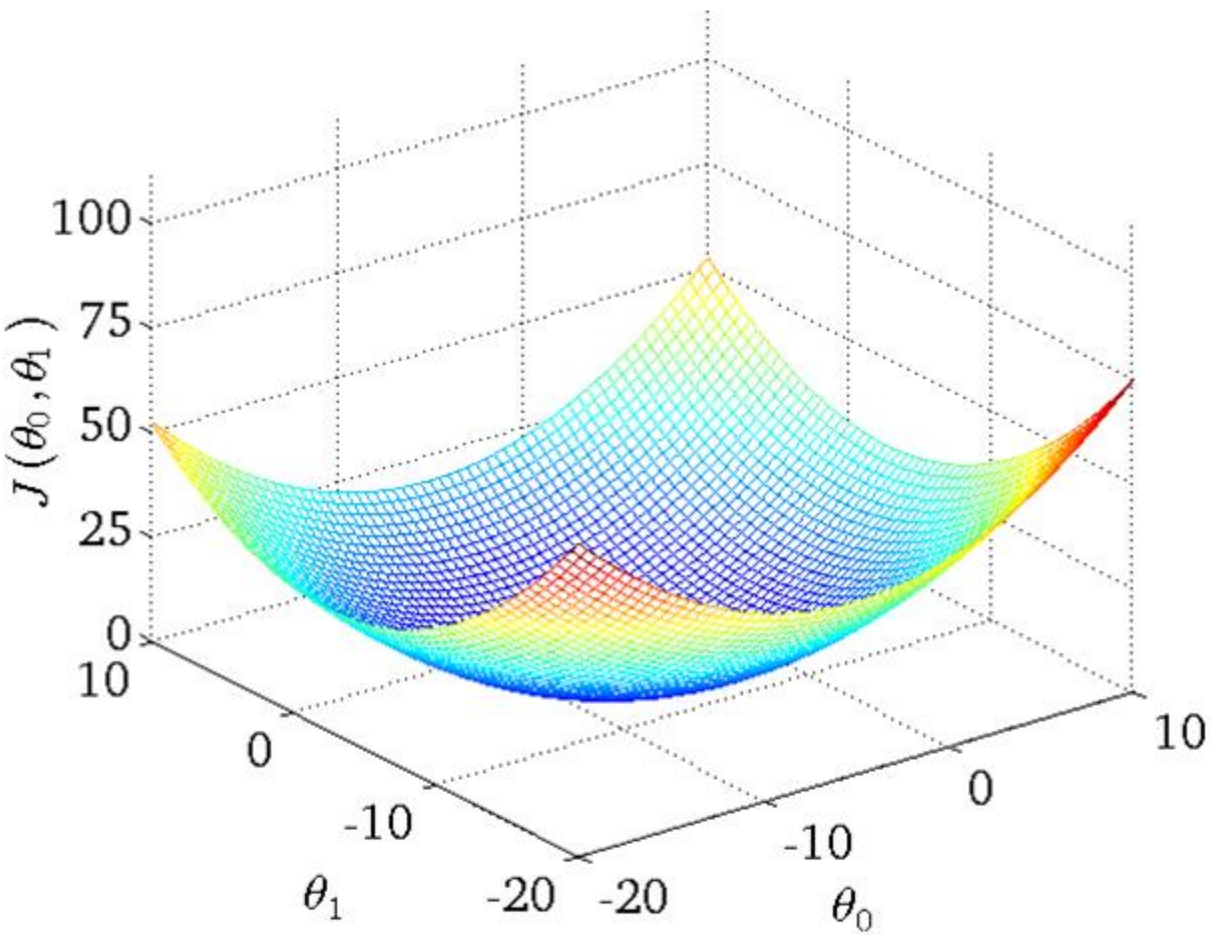
  - $J(\theta_0, \theta_1) = (\frac{1}{2m})\sum_{i=1}^{m}(y_i - \hat{y}_i)^2$

    OR

  - $\underset{\theta_0, \theta_1}{Minimize}\, J(\theta_0, \theta_1)$

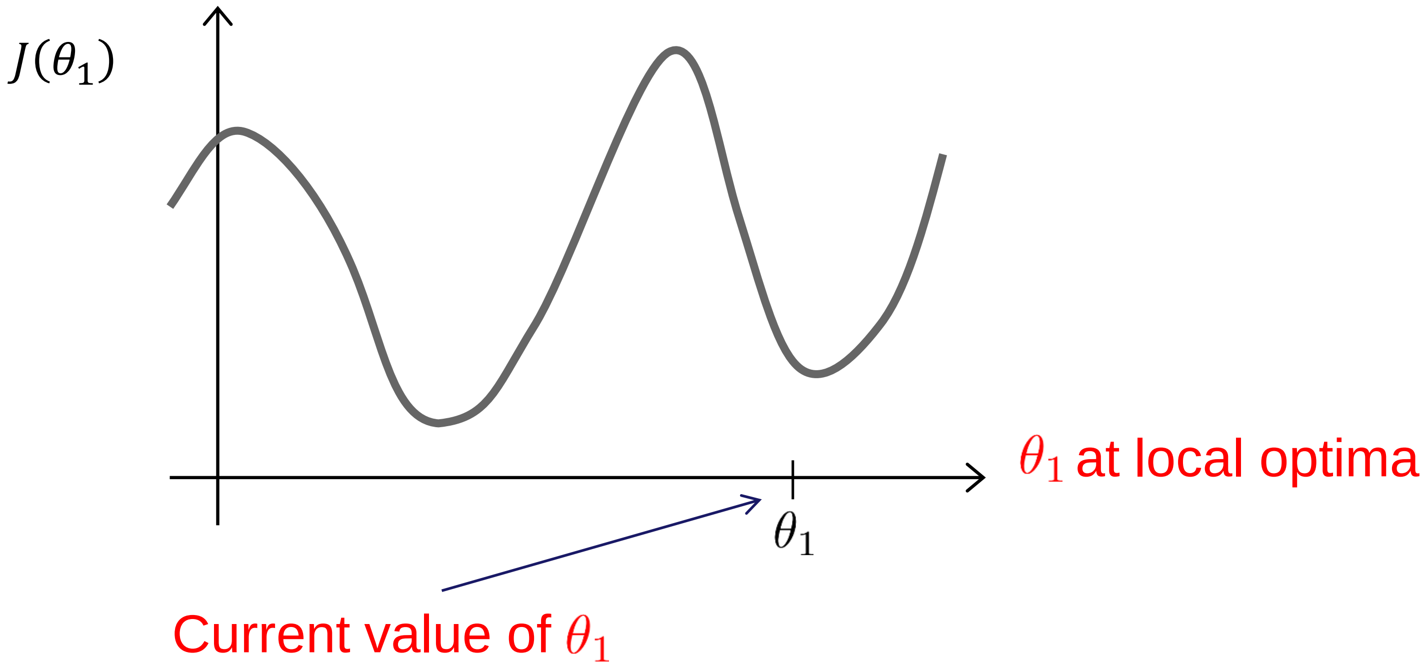
House Price Prediction

- Mathematically:



House Price Prediction

- Mathematically:



House Price Prediction

- Gradient Descent Algorithm

$$\text{repeat until convergence } \{$$
$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \qquad (\text{for } j = 0 \text{ and } j = 1)$$
$$\}$$

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

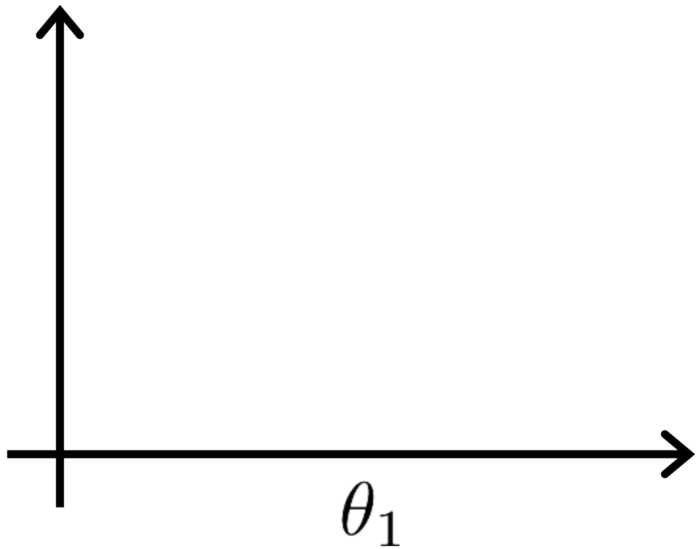
$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$

- $J(\theta_0, \theta_1) = (\frac{1}{2m}) \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$

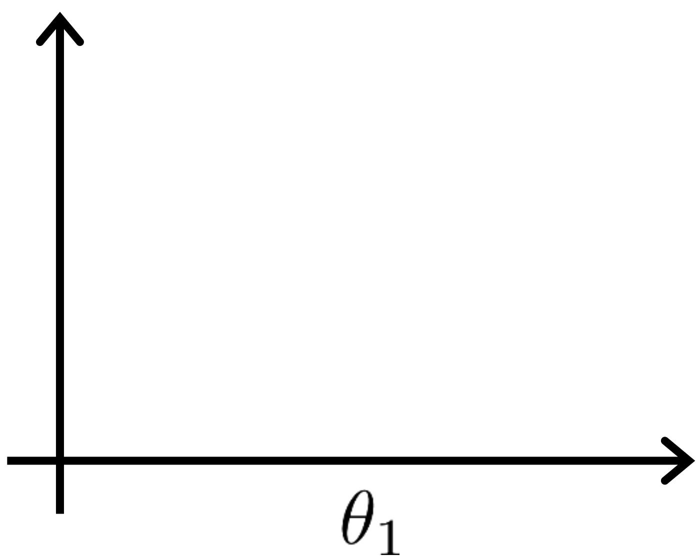
$$\hat{y} = h_\theta(x) = \theta_0 + \theta_1 x$$

$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



House Price Prediction

- Gradient Descent Algorithm

Gradient descent algorithm

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \\
&\qquad (\text{for } j = 1 \text{ and } j = 0) \\
&\}
\end{aligned}
$$

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

House Price Prediction

- Gradient Descent Algorithm

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) =$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) =$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) =$$

House Price Prediction

- Gradient Descent Algorithm

$$
\begin{aligned}
&\text{repeat until convergence } \{ \\
&\quad \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \\
&\quad \theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)} \\
&\}
\end{aligned}
$$

update θ_0 and θ_1 simultaneously

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$\hat{y} = h_\theta(x) = \theta_0 + \theta_1 x$

“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses all the training examples.

References

- Andrew Ng's slides on Linear Regression from his Machine Learning Course on Coursera.

Disclaimer

- Content of this presentation is not original and it has been prepared from various sources for teaching purpose.