Testing of Hypotheses.

(*) Two major areas of statistical inference are (i) estimation and (ii) testing of hypotheses.

→ In estimation we use a statistic (a function of sample observations only) to make a close guess about the unknown value of a population parameter.

→ In testing of hypotheses, we ascertain the truth of a statement about a population parameter by using a proper sample statistic.

In this chapter we shall discuss the problem of testing of hypotheses. In particular, we shall discuss t, F and chisquare tests.

(*) certain terms connected with the problem of testing of hypotheses.

1) Hypothesis : A hypothesis is a statement about a population parameter.

2) statistical hypothesis : A statistical hypothesis is some assumption or statement, which may or may not be true, about the probability distribution of a given population which we want to test on the basis of the information contained in a sample which is drawn from the given population

3) <u>Null hypothesis</u> : A statistical hypothesis which is formulated with a view of verifying its validity is called a null hypothesis. It is denoted by H_0

4) <u>Alternative hypothesis</u> : The negation or complement of the null hypothesis is called the alternative hypothesis. In other words, a hypothesis which is accepted in the event of H_0 being rejected is called the alternative hypothesis and it is denoted by H_1 or H_A.

5) <u>Test of hypothesis</u> : It is a procedure to decide whether to accept the null hypothesis H_0 or to reject it. A suitable test statistic is selected and on the basis of sampling distribution of this statistic, we fix up some criterion for accepting or rejecting H_0.

6) <u>Critical Region</u> : Using the sampling distribution of the test statistic T, we partition the whole range of the test stistic into two mutually exclusive subsets, say S and S^c.

If $T \in S$, we reject H_0

If $T \in S^c$, we accept H_0.

Here the region S is called the critical region of the test.

7) Types of errors in testing of a hypothesis:

There is every chance that the decision taken about the null hypothesis may be correct or may not be correct.

When a statistical hypothesis H_0 is tested, we have four possibilities.

(i) H_0 is true and is accepted by the test.
(ii) H_0 is false and it is rejected by the test.
(iii) H_0 is true but it is rejected by the test.
(iv) H_0 is false but it is accepted by the test.

The first two are correct decisions but the later two lead to errors. If a hypothesis is true and it is rejected by the test, we say that type I error has been committed. If a hypothesis is false and it is accepted by the test, we say that type II error has been committed.

Decision	H_0 true	H_0 false.
Accept H_0	Correct decision	Type II error
Reject H_0	Type I Error	Correct decision.

→ The probability of committing type - I error is denoted by α and it is called the level of significance.

→ The probability of committing type II error is denoted by β and $1-\beta$ is known as the power of the test.

8) Level of significance : The level of significance is the maximum probability of making type I error. It is denoted by α. The probability of making the correct decision is then $(1-\alpha)$. The best value for fixing the level of significance depends on the seriousness of the results of the two types of errors.

The commonly used values of the level of significance are $\alpha = 0.05$ or $\alpha = 0.01$ i.e 5% or 1%.

For example, when a decision is taken at 5% level of significance, then there is a chance that out of 100 such decisions, in 5 decisions we will reject a hypothesis when it is true.

9) Two - tailed and one - tailed tests :

+ The probability curve of the sampling distribution of the test statistic T is generally a normal curve. If we wish to test the null hypothesis $H_0 : \theta = \theta_0$ against an alternative hypothesis $H_1 : \theta \neq \theta_0$, we use a two tailed test in which we reject H_0, if the value of the test statistic T is higher than a value T_1 of T or lower than a value T_2 of T. Here the values T_1 and T_2 of T are determined by

$$P(T \geq T_1) = \frac{\alpha}{2} \quad \text{and} \quad P(T \leq T_2) = \frac{\alpha}{2}$$

We can find these values T_1 and T_2 using the area under the normal curve.

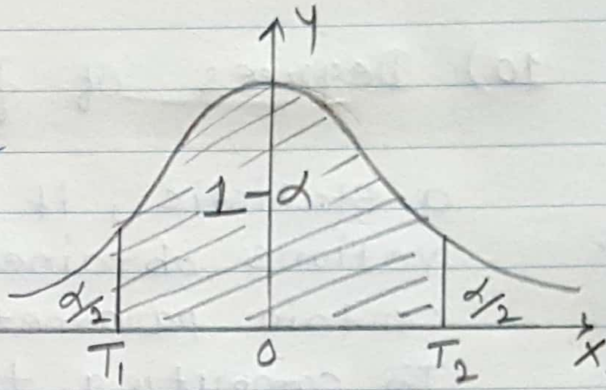

fig - 1

* If the null hypothesis is of the form $H_0 = \theta \leq \theta_0$ or $H_0 = \theta \geq \theta_0$, we use a one tailed test.

Suppose $H_0 : \theta \leq \theta_0$ is to be tested. Then we use a one tailed test in which we reject H_0, if the value of the test statistic T is higher than a value T' of T. Here T' is determined by $P(T \geq T') = \alpha$. (fig - 2)

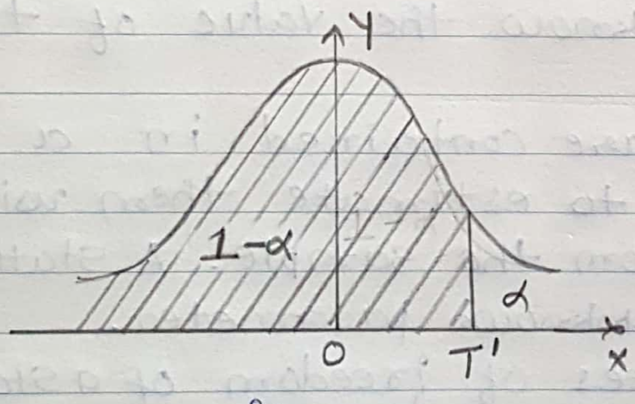

fig - 2

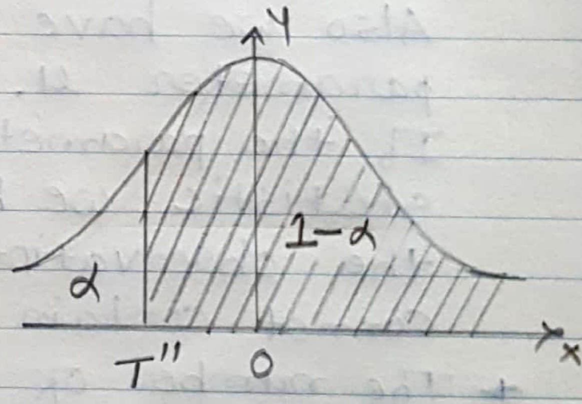

fig - 3.

If $H_0 : \theta \geq \theta_0$, then we use a one-tailed test in which we reject H_0; if the value of the test statistic T is lower than a value T'' of T. Here T'' is determined by $P(T \leq T'') = \alpha$ (fig - 3)

10) Degrees of freedom :

In order to compute a statistic, it is necessary to use observations obtained from a sample as well as certain parametric values

In computing the statistic.

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \quad \text{——— (1)}$$

We are required to know the values $x_1, x_2, \ldots x_n$ to compute $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ and.

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}.$$

Also we have to know the value of the parameter μ.

If the parameters are contained in a statistic, we have to estimate them using the observations from the sample. A statistic cannot contain an unknown parameter.

* The number of degrees of freedom of a statistic, generally denoted by ν, is defined as the number of n independent observations in the sample minus the number k of population parameters, which must be estimated from sample observations. Thus,

Degree of freedom $\nu = n - k$.

(n = sample size, k = no. of parameters estimated)

No. of independent observations in the sample is n. We take $\sigma = S \Rightarrow k = 1 \Rightarrow \nu = n - 1$.