Example 1

A company has two plants to manufacture hydraulic machines. Plant I manufactures 70% of the hydraulic machines, and Plant II manufactures 30%. At Plant I, 80% of hydraulic machines are rated standard quality; and at Plant II, 90% of hydraulic machines are rated standard quality. A machine is picked up at random and is found to be of standard quality. What is the chance that it has come from Plant I? **[Summer 2015]**

Solution

Let A_1 and A_2 be the events that the hydraulic machines are manufactured in Plant I and Plant II respectively. Let B be the event that the machine picked up is found to be of standard quality.

$$P(A_1) = \frac{70}{100} = 0.7$$

$$P(A_2) = \frac{30}{100} = 0.3$$

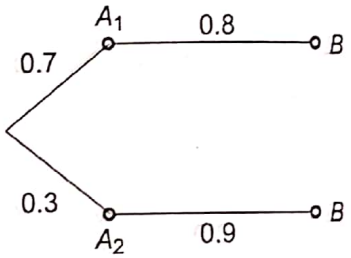

Fig. 1.3

Probability that the machine is of standard quality given that it is manufactured in Plant I

$$P(B/A_1) = \frac{80}{100} = 0.8$$

Probability that the machine is of standard quality given that it is manufactured in Plant II

$$P(B/A_2) = \frac{90}{100} = 0.9$$

Probability that a machine is manufactured in Plant I given that it is of standard quality

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$= \frac{0.7 \times 0.8}{0.7 \times 0.8 + 0.3 \times 0.9}$$

$$= 0.6747$$

Example 2

A bag A contains 2 white and 3 red balls, and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball is drawn from the bag B.

Solution

Let A_1 and A_2 be the events that the ball is drawn from bags A and B respectively. Let B be the event that the ball drawn is red.

$$P(A_1) = \frac{1}{2}$$

$$P(A_2) = \frac{1}{2}$$

Probability that the ball drawn is red given that it is drawn from the bag A

$$P(B/A_1) = \frac{3}{5}$$

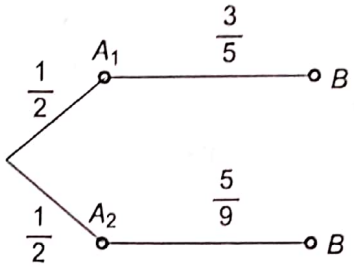

Fig. 1.4

Probability that the ball drawn is red given that it is drawn from the bag B

$$P(B/A_2) = \frac{5}{9}$$

Probability that the ball is drawn from the bag B given that it is red

$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$= \frac{\frac{1}{2}\times\frac{5}{9}}{\left(\frac{1}{2}\times\frac{3}{5}\right)+\left(\frac{1}{2}\times\frac{5}{9}\right)}$$

$$= \frac{25}{52}$$

Example 3

The chances that Doctor A will diagnose a disease X correctly is 60%. The chances that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of Doctor A, who had the disease X, died. What is the chance that his disease was diagnosed correctly?

Solution

Let A_1 be the event that the disease X is diagnosed correctly by Doctor A. Let A_2 be the event that the disease X is not diagnosed correctly by Doctor A. Let B be the event that a patient of Doctor A who has the disease X, dies.

$$P(A_1) = \frac{60}{100} = 0.6$$

$$P(A_2) = P(\bar{A}_1) = 1 - P(A_1) = 0.4$$

Probability that the patient of Doctor A who has the disease X dies given that the disease X is diagnosed correctly

$$P(B/A_1) = \frac{40}{100} = 0.4$$

Probability that the patient of Doctor A who has the disease X dies given that the disease X is not diagnosed correctly

$$P(B/A_2) = \frac{70}{100} = 0.7$$

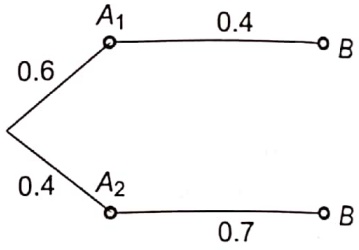

Fig. 1.5

Probability that the disease X is diagnosed correctly given that a patient of Doctor A who has the disease X dies

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$= \frac{0.6 \times 0.4}{(0.6 \times 0.4) + (0.4 \times 0.7)}$$

$$= \frac{6}{13}$$

Example 4

In a bolt factory, machines A, B, C manufacture 25%, 35%, and 40% of the total output and out of the total manufacturing, 5%, 4%, and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probabilities that it is manufactured from (i) Machine A, (ii) Machine B, and (iii) Machine C.

Solution

Let A_1, A_2 and A_3 be the events that bolts are manufactured by machines A, B, and C respectively. Let B be the event that the bolt drawn is defective.

$$P(A_1) = \frac{25}{100} = 0.25$$

$$P(A_2) = \frac{35}{100} = 0.35$$

$$P(A_3) = \frac{40}{100} = 0.4$$

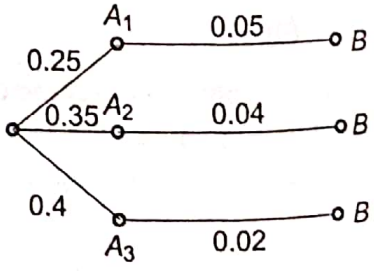

Fig. 1.6

Probability that the bolt drawn is defective given that it is manufactured from Machine A

$$P(B/A_1) = \frac{5}{100} = 0.05$$

Probability that the bolt drawn is defective given that it is manufactured from Machine B

$$P(B/A_2) = \frac{4}{100} = 0.04$$

Probability that the bolt drawn is defective given that it is manufactured from Machine C

$$P(B/A_3) = \frac{2}{100} = 0.02$$

(i) Probability that a bolt is manufactured from Machine A given that it is defective

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{0.25 \times 0.05}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 \times 0.02)}$$

$$= 0.3623$$

(ii) Probability that a bolt is manufactured from Machine B given that it is defective

$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{0.35 \times 0.04}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 \times 0.02)}$$

$$= 0.4058$$

(iii) Probability that a bolt is manufactured from Machine C given that it is defective

$$P(A_3/B) = \frac{P(A_3)\,P(B/A_3)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{0.4 \times 0.02}{(0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 \times 0.02)}$$

$$= 0.2319$$

Example 5

A businessman goes to hotels X, Y, Z for 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings. What is the probability that the businessman's room having faulty plumbing is assigned to Hotel Z?

Solution

Let A_1, A_2 and A_3 be the events that the businessman goes to hotels X, Y, Z respectively.
Let B be the event that the rooms have faulty plumbings.

$$P(A_1) = \frac{20}{100} = 0.2$$

$$P(A_2) = \frac{50}{100} = 0.5$$

$$P(A_3) = \frac{30}{100} = 0.3$$

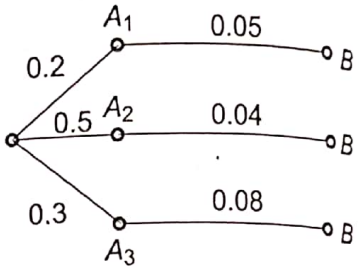

Fig. 1.7

Probability that rooms have faulty plumbings given that rooms belong to Hotel X

$$P(B/A_1) = \frac{5}{100} = 0.05$$

Probability that rooms have faulty plumbing given that rooms belong to Hotel Y

$$P(B/A_2) = \frac{4}{100} = 0.04$$

Probability that rooms have faulty plumbings given that rooms belong to Hotel Z

$$P(B/A_3) = \frac{8}{100} = 0.08$$

Probability that the businessman's room belongs to Hotel Z given that the room has faulty plumbing

$$P(A_3/B) = \frac{P(A_3)\,P(B/A_3)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{0.3 \times 0.08}{(0.2 \times 0.05) + (0.5 \times 0.04) + (0.3 \times 0.08)}$$

$$= \frac{4}{9}$$

Example 6

Of three persons the chances that a politician, a businessman, or an academician would be appointed the Vice Chancellor (VC) of a university are 0.5, 0.3, 0.2 *respectively. Probabilities that research is promoted by these persons if they are appointed as VC are* 0.3, 0.7, 0.8 *respectively.*

(i) Determine the probability that research is promoted.

(ii) If research is promoted, what is the probability that the VC is an academician?

Solution

Let A_1, A_2 and A_3 be the events that a politician, a businessman or an academician will be appointed as the VC respectively. Let B be the event that research is promoted by these persons if they are appointed as VC.

$$P(A_1) = 0.5$$
$$P(A_2) = 0.3$$
$$P(A_3) = 0.2$$

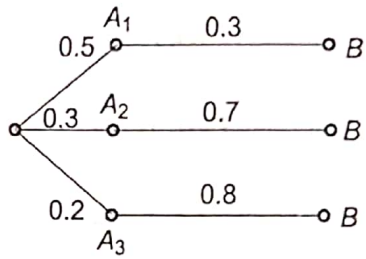

Fig. 1.8

Probability that research is promoted given that a politician is appointed as VC

$$P(B/A_1) = 0.3$$

Probability that research is promoted given that a businessman is promoted as VC

$$P(B/A_2) = 0.7$$

Probability that research is promoted given that an academician is appointed as VC

$$P(B/A_3) = 0.8$$

(i) Probability that research is promoted

$$P(B) = P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)$$
$$= (0.5 \times 0.3) + (0.3 \times 0.7) + (0.2 \times 0.8)$$
$$= 0.52$$

(ii) Probability that the VC is an academician given that research is promoted by him

$$P(A_3/B) = \frac{P(A_3)\,P(B/A_3)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$
$$= \frac{0.2 \times 0.8}{0.52}$$
$$= \frac{4}{13}$$

Example 7

The contents of urns I, II, and III are as follows:

1 *white,* 2 *red, and* 3 *black balls,*

2 *white,* 3 *red, and* 1 *black ball, and*

3 *white,* 1 *red, and* 2 *black balls.*

One urn is chosen at random and two balls are drawn. They happen to be white and red. Find the probability that they came from (i) Urn I, (ii) Urn II, and (iii) Urn III.

Solution

Let A_1, A_2, and A_3 be the events that urns I, II and III are chosen respectively. Let B be the event that 2 balls drawn are white and red.

$$P(A_1) = \frac{1}{3}$$

$$P(A_2) = \frac{1}{3}$$

$$P(A_3) = \frac{1}{3}$$

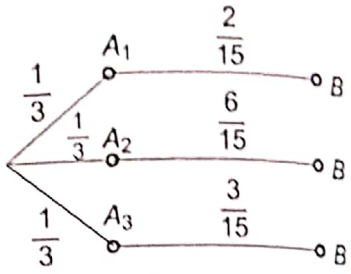

Fig. 1.9

Probability that 2 balls drawn are white and red given that they are chosen from the urn I

$$P(B/A_1) = \frac{{}^1C_1 \times {}^2C_1}{{}^6C_2} = \frac{1\times 2}{15} = \frac{2}{15}$$

Probability that 2 balls drawn are white and red given that they are chosen from the urn II

$$P(B/A_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^6C_2} = \frac{2\times 3}{15} = \frac{6}{15}$$

Probability that 2 balls drawn are white and red given that they are chosen from the urn III

$$P(B/A_3) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{3\times 1}{15} = \frac{3}{15}$$

(i) Probability that 2 balls came from the urn I given that they are white and red

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{15}}{\left(\frac{1}{3}\times\frac{2}{15}\right)+\left(\frac{1}{3}\times\frac{6}{15}\right)+\left(\frac{1}{3}\times\frac{3}{15}\right)}$$

$$= \frac{2}{11}$$

(ii) Probability that 2 balls came from the urn II given that they are white and red

$$P(A_2/B) = \frac{P(A_2)\,P(B/A_2)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{\frac{1}{3}\times\frac{6}{15}}{\left(\frac{1}{3}\times\frac{2}{15}\right)+\left(\frac{1}{3}\times\frac{6}{15}\right)+\left(\frac{1}{3}\times\frac{3}{15}\right)}$$

$$= \frac{6}{11}$$

(iii) Probability that 2 balls came from the urn III given that they are white and red

$$P(A_3/B) = \frac{P(A_3)\,P(B/A_3)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2) + P(A_3)\,P(B/A_3)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{15}}{\left(\frac{1}{3} \times \frac{2}{15}\right) + \left(\frac{1}{3} \times \frac{6}{15}\right) + \left(\frac{1}{3} \times \frac{3}{15}\right)}$$

$$= \frac{3}{11}$$

EXERCISE 1.4

1. There are 4 boys and 2 girls in Room *A* and 5 boys and 3 girls in Room *B*. A girl from one of the two rooms laughed loudly. What is the probability the girl who laughed was from Room *B*?

[Ans.: $\frac{9}{17}$]

2. The probability of *X*, *Y*, and *Z* becoming managers are $\frac{4}{9}$, $\frac{2}{9}$, and $\frac{1}{3}$ respectively. The probabilities that the bonus scheme will be introduced if *X*, *Y*, and *Z* become managers are $\frac{3}{10}$, $\frac{1}{2}$, and $\frac{4}{5}$ respectively. (i) What is the probability that the bonus scheme will be introduced? (ii) If the bonus scheme has been introduced, what is the probability that the manager appointed was *X*?

[Ans.: (i) $\frac{23}{45}$ (ii) $\frac{6}{23}$]

3. A factory has two machines, *A* and *B*. Past records show that the machine *A* produces 30% of the total output and the machine *B*, the remaining 70%. Machine *A* produces 5% defective articles and Machine *B* produces 1% defective items. An item is drawn at random and found to be defective. What is the probability that it was produced (i) by the machine *A*, and (ii) by the Machine *B*?

[Ans.: (i) 0.682 (ii) 0.318]

4. A company has two plants to manufacture scooters. Plant I manufactures 80% of the scooters, and Plant II manufactures 20%. At Plant I, 85 out of 100 scooters are rated standard quality or better. At Plant II, only 65 out of 100 scooters are rated standard quality or better. What is the probability that a scooter selected at random came from (i) Plant I, and (ii) Plant II if it is known that the scooter is of standard quality?

[Ans.: (i) 0.84 (ii) 0.16]

5. A new pregnancy test was given to 100 pregnant women and 100 non-pregnant women. The test indicated pregnancy in 92 of the 100 pregnant women and in 12 of the 100 non-pregnant women. If a randomly selected woman takes this test and the test indicates she is pregnant, what is the probability she was not pregnant?

$\left[\text{Ans.: } \frac{3}{26}\right]$

6. An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probability of an accident is 0.01, 0.03, and 0.15 in the respective category. One of the insured drivers meets with an accident. What is the probability that he is a scooter driver?

$\left[\text{Ans.: } \frac{1}{52}\right]$

7. Consider a population of consumers consisting of two types. The upper-income class of consumers comprise 35% of the population and each member has a probability of 0.8 of purchasing Brand *A* of a product. Each member of the rest of the population has a probability of 0.3 of purchasing Brand *A* of the product. A consumer, chosen at random, is found to be the buyer of Brand *A*. What is the probability that the buyer belongs to the middle-income and lower-income classes of consumers?

$\left[\text{Ans.: } \frac{39}{95}\right]$

8. There are two boxes of identical appearance, each containing 4 spark plugs. It is known that the box I contains only one defective spark plug, while all the four spark plugs of the box II are non-defective. A spark plug drawn at random from a box, selected at random, is found to be non-defective. What is the probability that it came from the box I?

$\left[\text{Ans.: } \frac{3}{7}\right]$