PS - ASSIGNMENT - 2

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Q.1. A sample of 100 students is taken from a large population. The mean height of students is 65 inches and standard deviation is 4 inches. Can it be reasonably regarded that the population mean height is 66 inches?

$n = 100$

$\bar{x} = 65$ inches, $S = 4$ inches

i) Null Hypothesis H_0: The population mean height is 66 inches

$H_0 : \mu = 66$

Alternate Hypothesis:

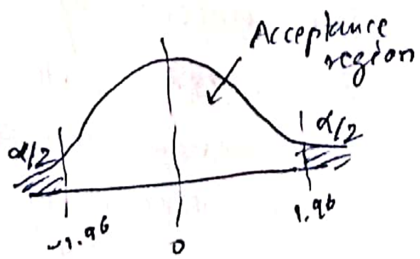
$H_1 : \mu \neq 66$

ii) Now the test statistics

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{65 - 66}{4/10} = -2.5$$

iii) Critical Region : (level of significance or p-value)

$\alpha = 5\% = 0.05$



Rejection region: $z < -1.96$ and $z > 1.96$

Since $z < -1.96$

We reject H_0

Thus population mean is μ is not 66 inches.

Q.2 A random sample of 200 tins of ground nut oil gave an average weight of 4.95 kg with std. of ~~0.41~~ 0.21 kg. Should we accept the hypothesis of net weight of 5 kg per tin at 1% significance ?

we have,

$n = 200, \quad \bar{x} = 4.95 \text{ kg}, \quad s = 0.21 \text{ kg}$

i) Let null hypothesis be,

$H_0 : \mu = 5 \text{ kg}$

So, Alternate hypothesis be

$H_1 : \mu \neq 5 \text{ kg}$

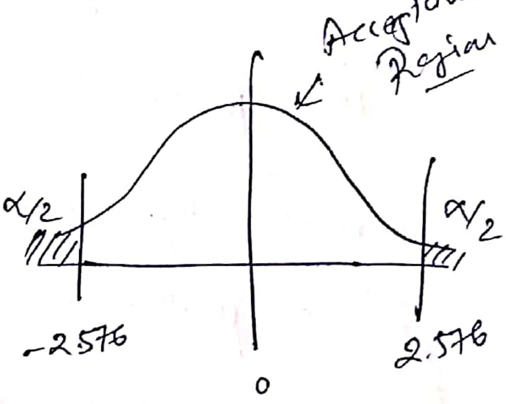
ii) Test statistics

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{4.95 - 5}{0.21 / \sqrt{200}} = -3.367$$

iii) From Critical Region

$\alpha = 01\% = 0.01\%$

Rejection Region: $z < -2.576$ & $z > 2.576$



Here,

$z < -2.576$ so Null Hypothesis H_0 is rejected.

We reject hypothesis of net wt of 5 kg.

Q.3 The heights of college students in a city are normally distributed with S.D. 6 cms. A sample of 1000 students has mean height 158 cms. Test the hypothesis that the mean height of college students in a city is 160 cms.

we have

$n = 1000$

$\bar{x} = 158 \text{ cm}$

$s = 6 \text{ cm}$

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i) Let null hypothesis be,
$H_0 : \mu = 160$ cm (i.e. mean height of college students in the city is 160 cm)

So, Alternate Hypothesis is,

$H_1 : \mu \neq 160$ cm

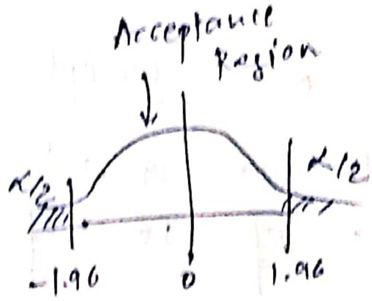
ii) <u>z statistics :</u>

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{158 - 160}{6/\sqrt{1000}} = \underline{-10.54}$$

iii) Critical Region

With $\alpha = 5\% = 0.05$

Rejection Region : $z < -1.96$ & $z > 1.96$



Here
$z < -1.96$ so <u>we reject null hypothesis</u> that the mean of college student in the city is not 160 cms.

Q.4. An auto company decided to introduce a new six cylinder car whose mean petrol consumption is claimed to be lower than that of existing engine. It was found that the mean petrol consumption for 50 cars was 10 Km/litre with SD of 3.5 km/litre. Test for few significance of 5% whether the claim the new car petrol consumption is 9.5 km/litre on the average is acceptable.

We have $n = 50$, $\bar{x} = 10$ km/l, $s = 3.5$ km/l

i) Let null hypothesis, H_0 be $\mu = 9.5$

Alternate Hypothesis, $H_1 : \mu \neq 9.5$

ii) Now, test statistics

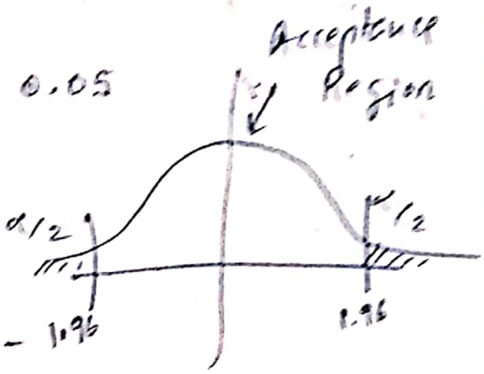
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{10 - 9.5}{3.5 / \sqrt{50}} = \underline{\underline{1.01}}$$

iii) Critical Region

with level of significance $\alpha = 5\% = 0.05$

From Z table, $z > -1.96$ & $z > 1.96$

since $-1.96 < 1.01 < 1.96$



We accept Null Hypothesis

The avg petrol consumption of new car is 9.5 km/l at 5% . level of significance

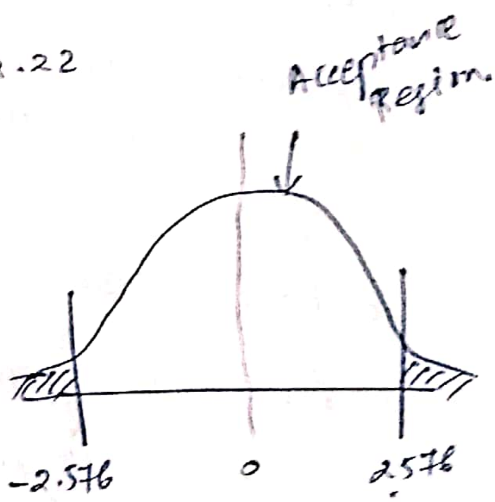
Q.5 It has previously been recorded that the average depth of ocean at perticular region is 67.4 fathoms. It there reason to believe this at 0.01 level of significance if reading at 40 random Locations in this perticular region showed a mean of 69.3 with S.D. of 5.4 fathoms.

i) Null Hypothesis : $H_0 : \mu = 67.4$

Alternate Hypothesis : $H_1 : \mu \neq 67.4$

ii) Now the test statistics.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{69.3 - 67.4}{5.4 / \sqrt{40}} = 2.22$$



iii) Critical Region

with ~~acc~~ significance level of 0.01

From Z table,

$z < -2.576$ & $z > 2.576$

As z is in Acceptance Region.

Null Hypothesis is Accepted

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It is reasonable to believe that the avg. depth of ocean in a perticular region ~~sho~~ is 69.3 fathoms.

Q.6. A sample of 64 students have mean weight of 70 kg. Can this be regarded as a sample from population with mean weight 65 kg and S.D. 25 kg.

we have,

$\bar{x} = 65$ kg $s = 25$ kg $n = 64$

$\mu = 70$ kg

i) Lets Null Hypo. be $H_0 : \mu = 70$

& Alternate Hypo. be $H_1 : \mu \neq 70$

ii) test statistics,

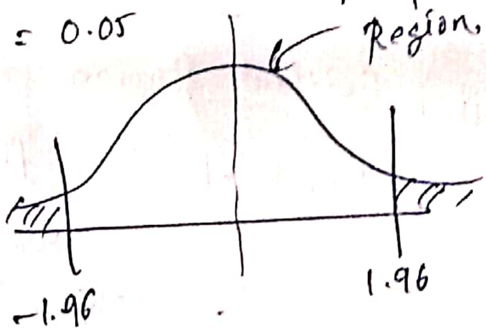
$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{65 - 70}{25/\sqrt{64}} = 1.92$$

iii) Critical Region

with level of significance $\alpha = 5\% = 0.05$

from z table,

$z < -1.96$ & $z > 1.96$



iv) As $z \in (-1.96, 1.96)$ ie in acceptance region

we accept Null Hypothesis.

& ~~we can~~ The population mean weight of students is 70 kg.

Q.7. The mean breaking strength of cable supplied by manuf. is 1800 with a SD 100. By a new technique in manuf. process it is claimed that breaking strength of cable is increased. In order to test this claim a sample of 50 cables is tested. It is found that the mean breaking strength is 1850. Can we support the claim at 0.01 % level of significance?

If we have,

$n = 50$

$\bar{x} = 1850$

$\sigma = 100$

Let

~~i) Null Hypothesis be, $H_0 : \mu = 1800$~~

~~Alternate Hypothesis $H_1 : \mu = 1800$~~

i) Null Hypothesis be,

H_0: breaking strength of the cable does not increased

$\mu = 1800$

Alternate Hypothesis

H_1 : breaking strength of the cable increased.

$\mu > 1800$

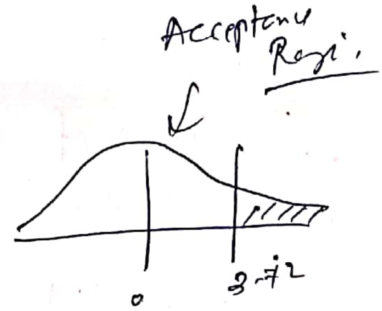
ii) Now test statistics

6 $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{1850 - 1800}{100/\sqrt{50}} = 3.53$

iii) Critical Region

Level of Significance $\alpha = 0.01\%$

$= 0.0001$



$z < 3.72$

Concludes, we will accept null Hypotheses

→ Breaking strength of cable has not increased.

Q.8 A normal population has a mean of 6.8 and S.D. of 1.5. A sample of 400 member gave a mean of 6.75. is the difference significant?

we,

$\mu = 6.8$ $n = 400$

$\bar{x} = 6.75$

$\sigma = 1.5$

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i) Null Hypothesis, and Alternate Hypothesis

$H_0 : \mu = 6.8$ $\quad H_1 : \cancel{\mu = 6.75}\ \mu \neq 6.8$

ii) Now, test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{6.75 - 6.8}{1.5/\sqrt{400}} = -0.66$$

iii) Critical Region

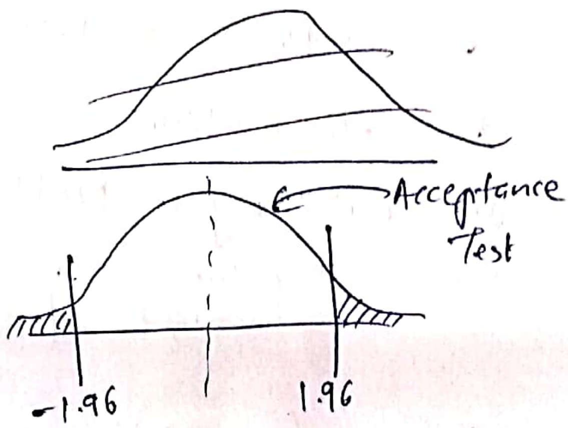
with 5% level of significant, (i.e. $\alpha = 0.05$)

from z table

Rejection: $z < -1.96$ & $z > 1.96$

As $-0.66 > -1.96$ & $-0.66 < 1.96$

we accept Null Hypothesis.



The population mean is $\mu = 6.8$.
Hence the difference is insignificant.

Q.9 The mean weight obtained from random sample of size 100 is 64 gms. The S.D. of wt. distribution of popul^n is 3 gms. Test the statement that the mean weight of population is 67 gm at 5% level of significance. Also set up 99% confidence limits of mean weight of population.

we have,

$n = 100$, $\mu = 67$

$\bar{x} = 64$ gm

$\sigma = 3$ gm

i) Let Null Hypothesis be.

$H_0 : \mu = 67$

& Alternate Hypo. be $H_1 : \mu \neq 67$

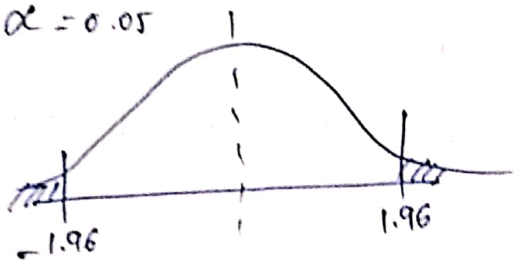
ii) test statistics

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{64 - 67}{3/\sqrt{100}} = \frac{-3}{3/10} = -10$$

ii) ~~zan~~ Critical Region

With 5% Significance level, $\alpha = 0.05$

From z table



$z < -1.96$ & $z > 1.96$

As $-10 < -1.96$

we will reject Null Hypothesis

Mean ~~Population~~ weight of population is not 67 gms.

⇒ Now for 99% confidence interval we get

$$1 - \alpha = 99\%$$
$$\alpha = 1\%$$
$$\alpha = 0.01$$

$$\mu = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 64 \pm 3.27\left(\frac{3}{\sqrt{100}}\right)$$

$$= 64 \pm 0.981$$

$$\Rightarrow 63.019 \le \mu \le 64.981$$

Q.10 A random sample of 200 measurement from a large population gave a mean value of 50 & S.D. of 9. Determine 95% confidence weight interval for mean of population

we have, $n = 200$

$\bar{x} = 50$

$s = 9$

$$\mu = \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$= 50 \pm 1.96\left(\frac{9}{\sqrt{200}}\right)$$

$$= 50 \pm 1.25$$

$$\Rightarrow \boxed{48.75 \leq \mu \leq 51.25}$$

Confidence interval for the mean of population.

Q.11 The mean of two single Large samples of 1000 & 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D. 2.5 inches ?

we have,

$n_1 = 1000$ $n_2 = 2000$

$\bar{x}_1 = 67.5$ inch $\bar{x}_2 = 68$ inch

$\sigma = 2.5$ inch

i) Let null hypothesis be,

$H_0 : \bar{x}_1 = \bar{x}_2$

Alternate hypothesis be

$H_1 : \bar{x}_1 \neq \bar{x}_2$

2) Now the test statistics

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}}$$

$$= \frac{-0.5}{0.097}$$

$$= -5.15$$

3) Critical Region
with level of significance $\alpha = 5\% = 0.05$

Acceptance Region $-1.96 < z < 1.96$

As $-5.15 \notin (-1.96, 1.96)$ we will reject null hypothesis.

The two sample does not belong to same population.

Q.12 An examination was given to 50 students of Hindu college and 60 students at Hans Raj College of Delhi University. The Hindu college has mean score 75 of with S.D. 9 and H.R.C. has mean score 79 with S.D. 7. Is there a significant difference b/w the mean score of two colleges?

1) Let the null hypothesis H_0,

H_0: There is no significant differece b/w the mean score of 2 colleges.

i.e. $H_0: \mu_1 = \mu_2$

So, Alternate Hypothesis, $H_1 = \mu_1 \neq \mu_2$

2) test Statistics

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{75 - 79}{\sqrt{\frac{9^2}{50} + \frac{7^2}{60}}} = \frac{-2}{1.56} = -1.282$$

3) With level of significance $\alpha = 5\% = 0.05$
From z table, $-1.96 \leq z \leq 1.96$

Since, $-1.282 > -1.96$, we accept the null hypothesis.

$\therefore$ There is no significance diff b/w mean scores of 2 colleges.

Q.13 If 60 new entrants in a given university are found to have a mean height of 68.6 inches an 50 seniors a mean height of 69.51 inches, is the evidence conclusive that men height of seniors is greater than that of new entrants? Assume the SD of height to be 2.48 inches.

$n_1 = 60$ $\quad n_2 = 50$

$\bar{x}_1 = 68.6$ inches $\quad \bar{x}_2 = 69.51$ inches

$\sigma_1 = \sigma_2 = \sigma = 2.48$ inches

1) Let the null hypothesis

Ho : The mean height of seniors is equal to that of juniors.

$H_0 : \mu_1 = \mu_2$

Alternate Hypothesis

$H_1 : \mu_1 \neq \mu_2$

2) Test Statistics

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\cancel{68.6 - 69.51}\; 69.51 - 68.6}{2.48\sqrt{\frac{1}{60} + \frac{1}{50}}}$$

$$= \frac{0.91 \cancel{\times 6.360}}{2.48 \times 0.191} = \boxed{1.916}$$

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3) <u>Critical Region</u>

level of significance $\alpha = 5\% = 0.05$.

From z table, $z < 1.645$

Since, $1.916 > 1.645$, we accept null hypothesis.

The mean of Seniors is same as mean of Juniors.

<u>Q.14.</u> A man buys 50 electric bulbs of 'Wipro' and 50 electric bulbs of 'Philips'. He finds that 'Wipro' bulbs gave an average life of 1500 hours with S.D. of 60 hours and 'Philips' bulb gave an average life of 1512 hours with ~~stand~~ S.D. of 80 hours. Is there a significant difference in the mean life of two makes of bulbs?

we have. $n_1 = 50 = n_2$

$\bar{x}_1 = 1500$ hrs $\quad s_1 = 60$ hours

$\bar{x}_2 = 1512$ hrs $\quad s_2 = 80$ hours

i) Let <u>null hypothesis</u>,

H_0 : there is no significant diff. in the mean life 2 makes of bulb.

$H_0 : \mu_1 = \mu_2$

<u>Alternate Hypothesis</u> : $H_1 : \mu_1 \neq \mu_2$

ii) <u>test statistics</u>

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} : \frac{1500 - 1512}{\sqrt{\frac{60^2}{50} + \frac{80^2}{50}}} = \underline{\underline{0.848}}$$

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iii) Critical Region

level of significance, $\alpha = 5\%$

$= 0.05$

From z table,

$z > -1.96$

$z < 1.96$

Since $0.848 < 1.96$, we accept the null hypothesis

There is no ~~significan~~ significance diff. in the mean life of two makers of bulbs.

Q.15 A sample of heights of 6400 soldier has a mean of 67.85 inches and a S.D of 2.56 inches. While another sample of heights of 1600 sailors has a mean of 68.55 inches with S.D. of 2.52 inches. Do the data indicate that the sailors are on the average taller than soldier?

we have,

$n_1 = 6400$ $\quad$ $n_2 = 1600$

$\bar{x}_1 = 67.85$ inches $\quad$ $\bar{x}_2 = 68.55$ Inches

$s_1 = 2.56$ inches $\quad$ $s_2 = 2.52$ inches

i) Null Hypothesis.

The height of soldier is same as that of sailor.

$H_0 : \mu_1 = \mu_2$

Alternate Hypothesis

$H_1 : \mu_1 \neq \mu_2$

ii) test statistics

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{68.55 - 67.85}{\sqrt{\frac{2.56^2}{6400} + \frac{2.52^2}{1600}}} = \frac{0.7 \times 80}{5.652} = 9.907$$

3) Critical Region:

Level of ~~Region~~, Significance

$\alpha = 5\% = 0.05$

From z table

$z < 1.645$

Since $9.907 > 1.645$, we will reject null hypothesis

So mean height of sailors is greater than mean height of soldiers.

Q.16 In a survey of buying habits, 400 women shoppers are chosen at random in supermarket A. Their average weekly food expenditure is Rs. 250 with a S.D. of Rs. 40. For 500 women shoppers chosen at supermarket B, the average weekly food expenditure is Rs. 220 with a S.D. of Rs. 45 test at 1% level of significance whether the average food expenditure of the two groups are equal.

We have,

$n_1 = 400$ $\quad$ $n_2 = 500$

$\bar{x}_1 = 250$ ₹ $\quad$ $\bar{x}_2 = 220$ ₹

$S_1 = 40$ ₹ $\quad$ $S_2 = 45$ ₹

1) Let null hypothesis H_0 be.

The average expenditure of 2 groups are same

$H_0 : \mu_1 = \mu_2$

Alternate Hypothesis be,

$H_1 : \mu_1 \neq \mu_2$

2) test statistics

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{250 - 220}{\sqrt{\frac{40^2}{400} + \frac{45^2}{500}}}$$

$$= \frac{30}{\sqrt{4 + 4.05}} = \underline{10.574}$$

3) Critical Region

level of Significance,

$\alpha = 1\% = 0.01$

From Z table, we have,

$-2.575 < z < 2.575$ as Acceptance Region

Since $10.574 > 2.575$, we will Reject Null Hypothesis

∴ The average food expenditure of two groups are not same