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The following is the description of the program written for Question 1:

- Programming Language Used : Python
- Execution Instructions : The file submitted is named **Assign2\_Problem1.ipynb**, so run the file to get the results. Appropriate comments have been added to understand what the program does.
- Description of the Program: The program contains 2 functions:
  1. **CRC\_Div** which does the modulo-2 division and returns the remainder which will be the frame check sequence (FCS) for the given channel.
  2. **encode\_frame** which takes the FCS of the data according to the given pattern and returns the encoded frame which is sent by the transmitter.

The output has been checked on the following 5 test cases:

Test Case No.	Message Sent	Error Frame	Message Received	Result
1	1100111011	000101110111111	110110011001010	Accepted
2	1111000110	111000110000010	000100101010011	Rejected
3	1110110000	111001110100001	000010110110100	Rejected
4	1110001101	101011011110110	010011101010101	Rejected
5	1100110011	101100111111000	011111110011011	Rejected

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The maximum window size that can be used in the Go-back-N ARQ mechanism that uses  $k$ -bit sequence numbers is  $2^k - 1$ .

In the Go-back-N ARQ mechanism, the sender has to retransmit the frames and all the other subsequent frames in case of any damage. Now, consider a situation where  $k=3$ , so we can have  $2^3 = 8$  different sequences possible, so let's call its frame numbers to be 0,1,2....7. Now, suppose the following series of events:

- Sender sends frame 0 and receives ACK RR1.
- Now, the sender sends frames 1,2...7,0 and receives RR1.
- It can be the case that all the 8 frames were acknowledged by the receiver, or all the 8 may even be damaged or lost in the transmission and receiving station is repeating its previous RR1.

So, to avoid this overlap, we keep the maximum window size for accepting the frames to 1 less than the total number of frames possible, i.e.  $2^k - 1$ .

The window size in Go-back-N ARQ is limited to  $2^k - 1$  and not  $2^k$  to ensure that the sequence numbers remain unique and to avoid overflow problems.

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The maximum window size that can be used in the Selective-Reject ARQ mechanism that uses  $k$ -bit sequence numbers is  $2^{k-1}$ .

In the Selective-Reject ARQ mechanism, only the rejected or timeout frames are retransmitted. Now, consider  $k=3$ . So, we can have  $2^3 = 8$  different sequences possible, so let's call its frame numbers to be 0,1,2,...7. Now, suppose the following series of events:

- Sender sends frames 0 to 6 to receiver.
- The receiver sends RR7 acknowledging the received frames but it gets lost during transmission.
- When timer of sender times out, its re-sends frame 0.
- Now, B had already advanced its window to accept frames to 7,0,...5, so on encountering 0, it feels that 7 is lost and accepts 0.

So, there's a clear overlap between the window of accepting frames of the sender and the receiver. So, to avoid this we consider the maximum frame size to be 4, so that no overlap can occur only the receiver will always have the window of  $4 = 2^{3-1}$  frames which is not in the window of the sender.

Now, suppose  $k$ -bit sequence numbers are used. The maximum number of frames that can be in transit at any given time is given by  $2^{k-1}$ . This is because the sequence numbers must be unique and range from 0 to  $2^k - 1$ . If the number of frames in transit exceeds this maximum value, the sequence numbers will wrap around and the receiver will no longer be able to distinguish between the old and new frames.

Hence, generalising this, we can say that for a  $k$ -bit sequence, we can choose a maximum window for accepting frames to be  $2^{k-1}$ .

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We are given that :

- Data Rate  $R = 40$  kbps
- Propagation delay  $T_p = 20$ ms
- Efficiency  $\eta \geq 0.5$

So, the efficiency of the stop-and-wait protocol is given by  $\eta = \frac{1}{1+2a}$  where  $a = \frac{T_p}{T_t}$  where  $T_t$  is the transmission delay.

Now, the transmission delay is given by

$$T_t = \frac{N}{R}$$

where  $N$  is the frame size to be transmitted.

$$\eta \geq 0.5 \implies a = \frac{T_p}{T_t} \leq 0.5$$

So, now we have that

$$\begin{aligned} T_t &\geq 2T_p \\ \frac{N}{R} &\geq 40ms \\ N &\geq 40ms * R \\ &\geq 40ms * 40kbps \\ &\geq 160 \text{ bits} \end{aligned}$$

So, the range of frame sizes( $N$ ) for which stop-and-wait gives an efficiency of at least 50% is  $N \geq 160$  bits.

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One frame consists of 4 bits. The probability of bit error is  $P_b = 10^{-3}$ .

- (a) The probability that the received frame contains no errors is the probability that every single bit of the frame transmitted by the sender doesn't contain any error. So, the probability that the received frame is error-free is :

$$P_1 = (\text{Probability that every bit is error free})^4 = (1 - 10^{-3})^4 = 0.996$$

- (b) The probability  $P_2$  that the received frame contains at least one error is 1 - (Probability that every bit is error free). So,

$$P_2 = 1 - (1 - 10^{-3})^4 = 0.004$$

- (c) One parity bit is added. Now, the errors can be there in 5 bits, the 4 bits earlier and the parity bit as well. So, if there are even number of errors in the received frame, then the error would not be detected. Also, we are asked about the probability of the received frames having errors which are not detected, so the frame has atleast one error. So, we have to take the cases where the errors in the received frame are either 2 or 4. Let  $P_3$  be the probability that the frame is received with errors that are not detected. So,

$$P_3 = \binom{5}{2}(10^{-3})^2(1 - 10^{-3})^3 + \binom{5}{4}(10^{-3})^4(1 - 10^{-3}) = 9.97 \times 10^{-6}$$

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## QUESTION

# 6

Cyclic Redundancy Check (CRC) is a method used to detect errors in digital data transmission. It works by appending a fixed number of bits to the original data, which forms the remainder when the data is divided by a specified divisor. Now, we are given that

- Message  $M = 11100011$
- Pattern  $P = 110011$

The Message is Multiplied by  $2^5$ , which gives  $T = 1110001100000$ . We want that  $2^{n-k}M + R$  should be divisible by  $P$ , where  $n$  is the length of the message to be sent,  $k$  is the size of the data block and  $R$  is the Frame Check Sequence, whose size is  $(n-k)$  bits. So, we divide  $T$  by  $P$  to get its remainder, which will be  $R$  for the given data  $M$  and pattern  $P$ . So, performing the calculations shown below, where  $n = 13$  and  $k = 5$ , we get

The image shows a handwritten long division of  $T = 1110001100000$  by  $P = 110011$ . The divisor  $P$  is written on the left. The dividend  $T$  is written as  $1110001100000$  with a label  $2^{n-k}M$  pointing to it. The quotient  $Q$  is written above the dividend as  $10110110$ . The remainder  $R$  is written at the bottom as  $11010$ . The division steps are shown with horizontal lines and subtractions.

$R = 11010$  which is the frame check sequence.

Hence the CRC for Message  $M = 11100011$  and Pattern  $P = 110011$  is  $FCS = R = 11010$ .

The encoded message  $T'$  transmitted by the sender is  $T' = T + R = 1110001111010$ .

- (a) In the CRC error-detecting scheme, we take  $P(X) = X^4 + X + 1$ , which is equivalent to 10011 in binary modulo 2 version of the CRC Method.

So, to encode the bits  $M = 10010011011$  using the given pattern  $P$ , we do the following calculations:

Since  $P$  is of 5 bits which is  $(n-k+1)$ , where  $k$  is the length of  $M$ , which is 11, we get  $n = 15$ .

In the calculations done in the figure below and all the following images have not shown the addition of 0's in the quotient in the cases where the dividend had leading zeroes.

The image shows a handwritten long division of the polynomial  $2^{n-k}M = 100100110110000$  by the divisor  $P = 10011$ . The quotient  $Q$  is 100010010100 and the remainder  $R$  is 1100.

$$\begin{array}{r}
 100010010100 \leftarrow Q \\
 100100110110000 \leftarrow 2^{n-k}M \\
 \underline{10011} \phantom{000000000000} \\
 10110 \phantom{000000000000} \\
 \underline{10011} \phantom{000000000000} \\
 10111 \phantom{000000000000} \\
 \underline{10011} \phantom{000000000000} \\
 10000 \phantom{000000000000} \\
 \underline{10011} \phantom{000000000000} \\
 1100 \leftarrow R
 \end{array}$$

So, we have that remainder  $FCS = 1100$  which is the frame check sequence or CRC of the given channel. The encoded message  $T$  transmitted by the sender is  $T = 2^{n-k}M + R = 100100110111100$ .

- (b) Suppose the channel introduces an error pattern  $E = 100010000000000$ , then we divide the new message obtained by the receiver  $M' = T \oplus E = 000110110111100$  by the pattern  $P$  and if the remainder comes out to be a string of only 0's, then the error will not be detected, else the error would be detected.

$$\begin{array}{r}
 \begin{array}{c} 10011 \\ \uparrow \\ P \end{array} \left| \begin{array}{r} 00011001110 \leftarrow Q \\ 000110110111100 \leftarrow M' \\ \underline{10011} \\ 10000 \\ \underline{10011} \\ 11111 \\ \underline{10011} \\ 11001 \\ \underline{10011} \\ 10100 \\ \underline{10011} \\ 1110 \leftarrow R \end{array}
 \end{array}$$

From the calculations above, we can see that the remainder obtained on dividing  $M'$  by  $P$  gives remainder  $R = 1110$  which is not equal to  $0000$ . So, the error is detected in this frame by the receiver.

- (c) Suppose the channel introduces an error pattern  $E = 100110000000000$ , then we divide the new message obtained by the receiver  $M' = T \oplus E = 000010110111100 = 1011011110$  by the pattern  $P$  and if the remainder comes out to be a string of only 0's, then the error will not be detected, else the error would be detected.

$$\begin{array}{r}
 \begin{array}{c} 10011 \\ \uparrow \\ P \end{array} \left| \begin{array}{r} 1010100 \leftarrow Q \\ 10110111100 \leftarrow M' \\ \underline{10011} \\ 10111 \\ \underline{10011} \\ 10011 \\ \underline{10011} \\ 0000 \leftarrow R \end{array}
 \end{array}$$

From the calculations above, we can see that the remainder obtained on dividing  $M'$  by  $P$  gives remainder  $R = 0000$  which is a string of only 0's. So, the error is not detected in this frame by the receiver.