CS711: Introduction to Game Theory and Mechanism Design **Sep-Dec 2020**

Project Report: Game Theory in Product Differentiation

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Abstract

The below project report describes the analysis of the market of a product having high degree of possible differentiability. The report initially describes a model to refine the manufacturing strategy in an environment with some broad assumptions. Going further, it describes the pricing mechanism and equilibrium price notion considering and altering various factors like quality, customer income etc.

1 Introduction to the problem

In a differentiated product environment like mobile, laptops, automobiles there are many attributes associated with a product and so many variants are available for the same. In such a market, firms must take informed decisions to emerge among the competition and satisfy customer needs as well. Initially we take a look at a simple model that make broad assumptions to make decisions on which products a firm should manufacture. Further, the assumptions are considered in detail in real world context and to describe the pricing mechanism that should be employed by firms over their products. Further the notion of equilibrium is considered for such a market in duopolistic and oligopolistic setting.

1.1 Related work

There has been extensive work performed to study differential product market in various directions. Many papers focus on branding, market equilibrium etc. Also, there has been some work done in context of how firms should invest in product differentiation [1]. There has also been study on how the firms are employing product differentiation in practice by considering various attributes possible, especially in mobile market [2]. However, we have considered a model in the project with a much more general setting considers how a firm should act upon the problem of product selection & distribution in such a differentiated setting considering the limitations. However, the model is based on some very high level assumptions like strict preferences of customer being available. But there has also been some work done in this area as well that describes how a group of users were surveyed to rate a set of mobile devices [3]. A good collection of game theoretic models are discussed in [5] including static and dynamic pricing when products are differentiated.

1.2 Brief overview of the report

The Section 2 describes the formal models considered for the project. It starts initially by describing a simple model that considers user preferences and the principle of social preference ordering to work upon the problem of product selection and distribution among given feasible set of products. However this model is quiet general, in the sense it does not consider how different attributes impact the customer behaviour and what factor the price of the products play on customer. After that, we consider different pricing models which take into account different aspect of consumers. First one takes into consideration the quality of product as a measure of pricing. In next sub-section a pricing mechanism based on income since income plays a vital role in product differentiation is introduced. Finally, a pricing mechanism considering competitor's product price into account is described along with the dynamic variant of the same.

2 Formal model of the problem

2.1 Product Selection model

Initially, a model is described that works upon the problem of selecting a set of products among the feasible products, manufacturing which would be in the best interest of the firm and customers. Such a model can be extended to use when a firm is re-considering its manufacturing strategy or deciding to develop/manufacture a few set of products among given prototypes/blueprints with the provision of customer preferences as market knowledge. Formally defining this problem, consider a set of N mobiles M = $\{M_1, M_2 ... M_N\}$. Also, we have the preference of L customers P = $\{P_1, P_2 ... P_L\}$ where each P_i is some order of all the mobiles in M, which are assumed to be strict in order. Also, it is assumed that the actual market demand is also a near proportional approximation of these preferences. Initially consider that the firm can manufacture only one mobile, which will be extended later. Now consider the notation ${}_iC_j$ as the count of preferences in which Mobile i is more preferred over Mobile j. If firm considers to manufacture a product **z** such that if ${}_zC_j >={}_j C_z \quad \forall j \in M - \{z\}$, then manufacturing any product other than z is preferred by less or equal number of people than that by z.

To look into it in more detail, consider the given example in figure with products A,B,C,D. Here, B satisfies the criteria described above. To look into the intuition about what role the above criteria plays, Consider that the firm decides to manufacture A, then 4/6 people prefer manufacturing B over A. Similarly, if firm decides to manufacture C or D, then also 4/6 people prefer to manufacture B over C or D. That's why since B satisfies above criteria, the firm can safely manufacture B to among the given alternatives. The above criteria is a weak form of Condorcet winner wherein ties are also allowed. Now, since we know that Copeland Winner is also consistent with Condorcet Winner, Copeland Winner is used to get the model that should be manufactured. Here Copeland score of A=1.5, B=3, C=1, D=0.5, consistent with our analysis above. This is not just a coincident but Condorcet Winner assures that the winner has a chance of at least a draw in every pair-wise competition and so is safe to be manufactured. But Condorcet Winner does not exist in all cases, in those case, Copeland Winner gives the Mobile that wins maximum times in pairwise competition. However, there is no perfect solution in such cases since for every mobile, there always exists another mobile manufacturing which would satisfy preferences for more number of people.

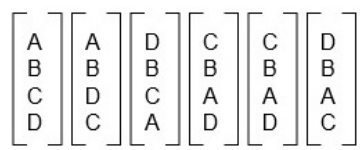

Figure 1: Preferences Example

Consider the given example in the figure 2, now consider if the firm decides to manufacture A, then 15/20 people prefer using D over A. Similarly, if the firm decides to manufacture B, then 16/20 people prefer using C over B. For C, we have 14/20 people preferring A over C and for D, there are 15/20 people preferring B over D. So, no product can be selected safely and Copeland Voting gives C that wins maximum pairwise competition i.e. over B and D but not A. However, if the firm has capacity to manufacture more than one products, it gives additional advantages. Let's take a look at that ahead.

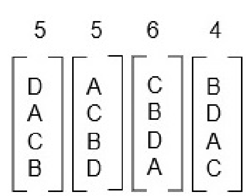

Figure 2: No Condorcet Winner

Now, consider the extension in problem, that now instead of selecting 1 product, we need to select K products. For that, we define the term **Alliance** as a set/bundle of K different products. Now the problem reduces to selecting the best alliance but for that we need to assimilate the notation into the customer preferences.

To compare any two alliance w.r.t a given preference, we introduce a notation. For a given preference P_i and alliance **X**, **top**(P_i,**X**) $= \{z : z \in X, zP_iy, \forall y \in X - \{z\}\}$. This indicates to the element of alliance that is preferred over any other element of the alliance w.r.t the given customer preference. So based on this for a given preference P_i and two alliances X and Y, X P_i Y if top(P_i,X) P_i top(P_i,Y), and vice versa for Y P_i X. Also if top(P_i,X) = top(P_i,Y), both X and Y are equally preferred. So, now the preferences are transformed among alliances and these preferences need not be strict since two bundles may have a common element that is most preferred by a customer. Also, this scheme is not random but based on the fact that a bundle X is more preferred over Y if it contains an element that is more preferred over every element in Y. So now, as we have established how to compare alliances. We take all combinations of alliance and create preference sets for each preference that now contain alliances instead of original models. Based on them, we output the alliance that is the Condorcet Winner. This alliance should be selected by the firm to be manufactured since it wins in pairwise competition over every other alliance. So, consider the example same as in figure 2 with K=2 and consider alliances $A1 = \{A, B\}, A2 = \{A, C\}, A3 = \{A, D\}, A4 = \{B, C\}, A5 = \{B, D\}, A6 = \{C, D\}$. The resultant preference set is as shown in Figure 3. Through Copeland Winner, alliance A6 was found to be the winner which was also found to be the Condorcet Winner. Please note that it may be possible that Condorcet Winner doesn't still exist. However, if K=N, then there is only one possible alliance which is also the solution.

After deciding the alliance say X, now the firm might need to decide the distribution to manufacture the products in X. This can be decided by using a distribution mapping $D : e \implies [0, 100] : \frac{(count(top(P_i,X)==e))*100}{|P|}$. This distribution assigns the element e the proportion of preferences in which e is more preferred than any other element in X.

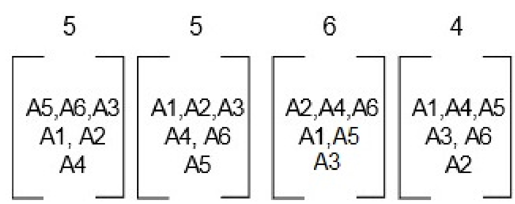

Figure 3: No Condorcet Winner

2.2 Quality Pricing model

The following model's objective is computing equilibrium prices of a product's substitutes produced in an oligopoly market by multiple firms of various qualities. We address the situation where though each person wants to buy the highest quality but is bounded by their willingness to pay, which is associated with the consumer's income. This separation in consumers in terms of valuation of willingness to pay will split the market into strict subsets for firms to serve.

Valuation and quality will go hand in hand. More the valuation of a person, the higher the quality he shall buy.

This section implements the paper [6].
The Utility of a consumer of valuation v willing to pay $p(q)$ for quality q is given by $U_v(q) = v \cdot q - p(q)$
To keep utility $U_v \geq 0$, a person can only buy products of quality 'q', if its valuation is :

$$v \geq \min_q [\frac{p(q)}{q}] \tag{1}$$

For the model, we assume valuation v of total mass 'L' as uniformly distributed in range [0 , v_{max}] : $v \sim U([0, v_{max}])$
Production Marginal Cost of a product of quality 'q': $c(q) = \psi q^\theta$, where $\psi > 0$ governs marginal production cost and $\theta > 1$ decides level of cost increase with increase in quality
In the market, Oligopoly firm indices: $nin\{0, -1, -2, ...\}$ such that $q_n < q_{n+1}$. Quality Ratio of any two neighbouring qualities is constant :$q_{n+1} = \gamma q_n$ where $\gamma \geq 1$ A person with valuation v can be indifferent between goods of quality q_n and q_{n+1}, if $v = (p_{n+1} - p_n)/(q_{n+1} - q_n)$
Firm 'n' can serve customers with valuations $v \in [\underline{v_n}, \bar{v_n}]$: such that:

$$\underline{v_n} = \frac{p_{n+1} - p_n}{q_{n+1} - q_n} \text{ and } \bar{v_n} = \begin{cases} v_{max}, & \text{if } n = 0 \\ \frac{p_{n+1}-p_n}{q_{n+1}-q_n}, & \text{if } n < 0 \end{cases} \tag{2}$$

The authors in the referred paper designed a model using above considerations to find an equilibrium in the market by finding prices such that the prices guarantee that all the firms stay in the market while maximizing their margin. The referred paper delivers equilibrium pricing as:

$$p_n = A\lambda^n + \alpha c_n \text{ where } \alpha = \frac{\gamma+1}{2(\gamma+1) - \gamma^{\theta} - \gamma^{1-\theta}},\ \lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1}$$
$$A = \frac{\lambda}{2\lambda - 1}(1 - \alpha(2 - \gamma^{-\theta}) + \frac{\gamma - 1}{\gamma} . \frac{q_0 v_{max}}{c_0})c_0 \tag{3}$$

The firms can only hold on in the market if the price p_n is more than Production cost c_n. This holds only if:

$$A/c_0 + \alpha \geq 1 \ \ if \ A < 0, \alpha \geq 1 \ \ if A \geq 0 \tag{4}$$

Implementation: See this for the same A.

2.3 Pricing Mechanism Based on Income[4]

In this sub-model, we will see how income is used as a parameter to decide the selling price of a mobile phone. We will consider that duopoly in the market - A is mobile phone brand of high quality and B is mobile phone brand of low quality(in comparison to A). We will also that income varies in linear fashion such that - R(t) = $R_1 + R_2(t)$ where R(t) is income of customer 't', R_2 is rate of change of income and R_1 is a constant. Customer will buy the product if following condition satisfies - $U(0, R) \leq U(A, R - p_A)$ where U(0,R) denotes utility if consumer has old or no mobile phone and $U(A, R - p_A)$ denotes the utility after purchasing mobile phone of brand A with price p_A.
Notations - Here, T is a set of all customers in order of increasing income. p_A and p_B is price of mobile A and B respectively. π_A and π_B denotes budget of consumers for mobile A and B. U(A, R(t)) can be defined as - $U(A, R(t)) = U_A.R(t)$ where U_A(resp. U_B) is some scalar showing preferences of consumers.
This model is broken into 3 cases as shown below -

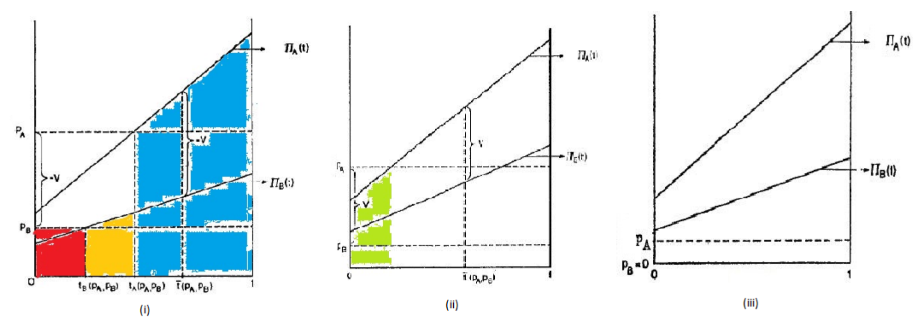

Figure 4: Scenarios under consideration

In case(i) of Figure 4, in red region π_A, π_B both are less than p_A, p_B, so no customer will be served in this region. In yellow region, Mobile B is under budget of some consumers, so they can purchase only mobile B in this region. Now, in blue region Both mobiles are in budget and consumer will buy the mobile with which his utility is more. So in this case, some of the potential customers will not be served(in red region). In case

(ii), all customers will be served. In case(iii), Price of mobile B will be such that it will be get out of the market. Now lets formalize all these cases -
Let $S_A = \{p_A | 0 \leq p_A \leq \pi_A(1)\}$ and $S_B = \{p_B | 0 \leq p_B \leq \pi_B(1)\}$ are strategy sets of A and B respectively. Here, market partition for A and B respectively is given as - $M_A(p_A, p_B) = \{t \in T$— t buys A at a price p_A $\}$, $M_B(p_A, p_B) = \{t \in T$— t buys B at a price $p_B\}$ and $(M_0(p_A, p_B))$ denotes customers who neither buys A nor B. Now, we can define the demand function for each product as a function of p_A and p_B -
$D_1 = \{(p_A, p_B) | \mu_A(p_A, p_B) + \mu_B(p_A, p_B) < 1; \mu_A(p_A, p_B) > 0, \mu_B(p_A, p_B) > 0\}$
$D_2 = \{(p_A, p_B) | \mu_A(p_A, p_B) + \mu_B(p_A, p_B) = 1; \mu_A(p_A, p_B) > 0, \mu_B(p_A, p_B) > 0\}$
$D_3 = \{(p_A, p_B) | \mu_B(p_A, p_B) = 0; 0 \leq \mu_A(p_A, p_B) \leq 1\}$
Here, D_1, D_2, D_3 respectively shows the demand function for case(i),(ii) and (iii) and $\mu_A(p_A, p_B) = \mu(M_A(p_A, p_B))$, $\mu_B(p_A, p_B) = \mu(M_B(p_A, p_B))$.
<u>Lemma - 1</u> $\rightarrow$If $(p_A, p_B) \in D_1$, then
$\mu_A(p_A, p_B) = 1 - \frac{U_A p_A - U_B p_B}{(U_A - U_B) R_2} + \frac{R_1}{R_2}$ and $\mu_B(p_A, p_B) = \frac{U_A p_A - U_B p_B}{(U_A - U_B) R_2} - \frac{U_B p_B}{(U_B - U_0) R_2}$
<u>Lemma - 2</u> $\rightarrow$If $(p_A, p_B) \in D_2$, then
$\mu_A(p_A, p_B) = 1 - \frac{U_A p_A - U_B p_B}{(U_A - U_B) R_2} + \frac{R_1}{R_2}$ and $\mu_B(p_A, p_B) = \frac{U_A p_A - U_B p_B}{(U_A - U_B) R_2} - \frac{R_1}{R_2}$
<u>Lemma - 3</u> $\rightarrow$If $(p_A, p_B) \in D_3$, then
$\mu_A(p_A, p_B) = Min\{1, 1 - \frac{U_A p_A}{(U_A - U_B) R_2} + \frac{R_1}{R_2}\}$ and $\mu_B(p_A, p_B) = 0$
Proof for $\mu_A(p_A, p_B)$ of Lemma - 1 is shown in appendices. Similarly, others can be proved. Now, equilibrium prices can be obtained by solving the first order conditions for maximization of p_A and p_B. We will get equilibrium prices and some conditions as follows -
For case(i) - $\frac{R_1}{R_2} < \frac{1}{3} \frac{U_A - U_B}{U_A - U_0}$ and (p_A^*, p_B^*) can be given as -
$p_A^* = \frac{2(U_A - U_0)(U_A - U_B)(R_1 + R_2)}{U_A(4U_A - U_B - 3U_0)}$ and $p_B^* = \frac{(U_B - U_0)(U_A - U_B)(R_1 + R_2)}{U_B(4U_A - U_B - 3U_0)}$
For case(ii) - $\frac{1}{3} \frac{U_A - U_B}{U_A - U_0} \leq \frac{R_1}{R_2} \leq 1$ and (p_A^*, p_B^*). Now, there are two sub-cases -
If $\frac{1}{3} \frac{U_A - U_B}{U_A - U_0} \leq \frac{R_1}{R_2} < \frac{U_A - U_B}{U_A + 2U_B - 3U_0}$, then
$p_A^* = \frac{(U_A - U_B) R_2 + (U_A - U_0) R_1}{2U_A}$ and $p_B^* = \frac{U_B - U_0}{U_B}$
If $\frac{U_A - U_B}{U_A + 2U_B - 3U_0} \leq \frac{R_1}{R_2} \leq 1$, then
$p_A^* = \frac{(U_A - U_B)(2R_2 + R_1)}{3U_A}$ and $p_B^* = \frac{(U_A - U_B)(R_2 - R_1)}{3U_B}$
For case(ii) - $\frac{R_1}{R_2} > 1$ and (p_A^*, p_B^*) can be given as -
$p_A^* = \pi_A(0) - \frac{U_B}{U_A} \pi_B(0)$ and $p_A^* = 0$
So, this is the equilibrium prices for each of the case. Now, we can see these equilibrium prices also helps us in maximizing our profit. We require two things to maximize our profit - one is the highest price, we can sell our mobile phone for without losing the market competition and number of units sold. Here, since our equilibrium prices are maximum without losing the market and since we are not losing the market then, number of units sold will also be maximized. So ultimately our profit will also be maximized. Lets formally define it.
The profit function for A on S_A and S_B can be defined as $P_A(p_A, p_B) = p_A.\mu_A(p_A, p_B)$ and similarly $P_B(p_A, p_B) = p_B.\mu_B(p_A, p_B)$. An equilibrium point is a pair of strategies (p_A^*, p_B^*) such that, $\forall p_A \in S_A, P_A(p_A, p_B^*) \leq P_A(p_A^*, p_B^*)$ and $\forall p_B \in S_B, P_B(p_A^*, p_B) \leq P_B(p_A^*, p_B^*)$ and $\mu_A(p_A^*, p_B^*) + \mu_B(p_A^*, p_B^*) \leq 1$.
Result is described in section - 3.

2.4 Static and Dynamic Pricing

We assume here that some consumers receive different utilities(since products are differentiated) from using products and are hence loyal, means, they will purchase from firm(A or B), even when the other firm is offering it at lower price.

2.4.1 Best Response(Static) Pricing

Let's assume that there are two firms, A and B in the market and sell product at price p_i(i= A or B). Our main goal is to estimate the price p_i in such a way that the profit generated is maximum. Let's define demand function for product i when the competitors i and j charge the price p_i and p_j as-

$$d_i(p_i, p_j) = a - b \cdot p_i + c \cdot p_j$$

a, b and c are constants. Whereas a may be treated as (expected)consumer base, b defines how the demand of a product depends on its own price, c defines the demand dependence of a product on the price of its competitor's product. Depending on the value of c, there are three cases that arise:

- If $c > 0$, then on increasing product price of firm j increases the demand of i firm's product.
- If $c < 0$, then on decreasing the price of one firm's product raises demand for products of both the firms.
- If $c = 0$, in this case the products of both the firms are independent and there is no effect of price change of one product to the demand of the other product.

Now, suppose the constant marginal cost(one unit production cost) of producing either of the products be m, total profit for firm i would be given by-

$$(p_i - m) \cdot d_i(p_i, p_j)$$

For maximizing the above, setting its first derivative w.r.t. p_i to zero gives,

$$\frac{d}{dp_i}\left((p_i - m) \cdot d_i(p_i, p_j)\right) = 0 \qquad \Rightarrow p_i^* = \frac{1}{2b} \cdot (a + bm + cp_j)$$

where p_i^* is the best optimal choice of price for firm i when the competitor's price is p_j. Similarly, we have,

$$p_j^* = \frac{1}{2b} \cdot (a + bm + cp_i)$$

Solving the just above two equations gives us,

$$p_i^* = p_j^* = \frac{a + bm}{2b - c} \tag{5}$$

Implementation: See A.

2.4.2 Dynamic Pricing

In the earlier section, we saw how to attain equilibrium with respect to the competitor's price. In this section we will see what happens when a competitor changes its product price, then how should we react to it in different periods of time. In fact it is very similar to repeated games, however here the firms strive for best prices that maximizes their over all profit.
Let's define the followings as follows-

$$r_{i,t} = \alpha \cdot r_{i,t-1} + (1 - \alpha) \cdot p_{i,t-1} \qquad 0 \leq \alpha < 1 \tag{6}$$

$$d_{i,t} = a_i - b_i \cdot p_{i,t} + c_i \cdot p_{j,t} + g_i(r_{i,t} - p_{i,t}) \text{ where, } g_i = \begin{cases} \delta & \text{if } r_{i,t} > p_{i,t} \\ \gamma & \text{Otherwise} \end{cases} \tag{7}$$

where $r_{i,t}$ is the reference price in time period t for firm i's product. It defines anchoring level of prices formed by consumers, based on pricing environment. $p_{i,t}$ is the price of product by firm i. $p_{i,t-1}$ and $r_{i,t-1}$ are similarly defined. α controls importance given to $r_{i,t}$ and $p_{i,t}$. $d_{i,t}$ represents demand of firm i's product in time period t. b_i and c_i are defined as previously(in case of static pricing). The control variables δ and γ define impact of perceived gain and perceived loss by a consumer respectively.

Now, the objective of each firm i is to maximize its profit over long period of time,

$$\max_{p_{i,t}} \sum_{t=1}^{\infty} \beta^{(t)}(p_{i,t} - c) \cdot d_{i,t} \qquad 0 < \beta^{(t)} \le 1 \tag{8}$$

$$\sum_{t=1}^{\infty} \max_{p_{i,t}} \left(\beta^{(t)}(p_{i,t} - c) \cdot d_{i,t}\right) \qquad 0 < \beta^{(t)} \le 1 \tag{9}$$

where $\beta^{(t)}$ represents extent of discount given in time period t and c is the marginal cost of the product of firm i.

For maximizing the profit, we would set differential of profit w.r.t. $p_{i,t}$ equal to zero,

$$\frac{d}{dp_{i,t}} \cdot \left(\beta^{(0)}(p_{i,t} - c)d_{i,t}\right) = 0 \Rightarrow p_{i,t} = \frac{a_i + b_i \cdot c + c_i \cdot p_{j,t} + g_i(r_{i,t} + c)}{2b_i + 2g_i}$$

g_i is based on $r_{i,t-1}$ and $p_{i,t-1}$.
Implementation: Have a look at A.

3 Main results/findings

So, as described in the previous section for the initial framework trying to model problem of selection of K out of N products, we are employing Copeland Division to find Condorcet Winner. However, as we know that since Condorcet Winner does not always exist, there may be cases where an alliance cannot win in pairwise competition against all other alliances. One such example was constructed as similar to the one shown in figure 2 but with the preference [A C D B] having count 7 instead of 5 due to which A3 had slightly high score and was able to triumph over A6, due to which no Condorcet Winner emerged. The implementation and simulation for the same has been done explained shortly in Section 4. But despite non-existence of Condorcet Winner, Copeland Voting still give winner. This winner is an indication of the alliance that would possess maximum win against other alliances in pairwise competition. Besides Copeland, there are other methods as well that which are Condorcet consistent i.e. the output will be a Condorcet, if exists. One among them is Kemeny-Young, which has also been simulated as well. It selects the alliance not only considering the pairwise competition but also considering how many user preferences are diverted to that winner as compared to others.
The observation from the equilibrium pricing model discussed in section 2.2 is discussed ahead. In the equilibrium pricing equation, α denotes constant markup over marginal production costs of products i.e. c_n and A denotes distortion in demand elasticity of top quality firms, since top quality firm has a single competitor that is n=-1, hence if we take v_{max} as large enough $A > 0$, more number of people will be able to pay for high quality firm and hence demand will increase for top quality firms, so top quality firm will keep unusually higher or asymmetrical price in the market. This distortion will also be reflected in subsequent minor quality firms since their competitor is showing asymmetrical behaviour.
The distortion will fade away with lower quality firms. This is controlled by the term λ^n hence the complete distortion term $A\lambda^n$ will control the asymmetry in prices as per quality of the firm. Conversely, if v_{max} is smaller then $A < 0$ which will cause demand of top quality firms to go down and hence will charge

asymmetrically lower prices in the market.
In section - 2.3, we can see, if any mobile brand doesn't follow equilibrium prices, then it can not maximize its profit and also whole market may not be covered so, it will lead to less number of units sold.
In section - 2.4, we saw how pricing can be done in static as well as dynamic setting. However, this method requires a number of control variables to be set and hence an extensive market research is necessary.

4 Experiments/Simulations

For the initial model discussed to cover the problem of Product Selection and Distribution was also implemented. It was also executed as simulation to work upon data described in Figure 2 and the one with few alteration over it as described in Section 3. For the first simulation, the alliance with Condorcet Winner C,D was outputted. For the other input, there was no Condorcet Winner as thee didn't exist one and two alliances were reported as Copeland Winner: [A,B] and [C,D]. This is because each alliance won against three alliances and tied against one. So, Copeland Winner prefers such Winner in the case of 'No Condorcet Winner' case that would win in maximum case of pairwise competition. The Kemeny Young Simulations, on the other hand outputted only [A,B] in the latter case while giving same output on the former one. [A,B] was selected because although [A,B] and [C,D] had same number of pairwise wins but [A,B] had higher number of user preferences w.r.t other alliances as compared to [C,D]. However, the simulations could not be extended to formal validation experiments due to lack of availability of appropriate dataset.

The plots obtained from the rest of the models have been displayed in A

5 Summary and Discussions

So, to summarize the discussion of the project, it is shown that how in a differentiated product market, a firm can decide upon the strategy on how to select the products to manufacture provided it knows about the preferences of customers in the market. This was a general model and did not delved on how customers would react if prices were changed or if they had equal preferences or no preferences for some products at all. It is also discussed that how firms should set up equilibrium prices by modelling consumer behaviour in a duopoly and oligopoly market in a static and dynamic sense. Moreover, it is also discussed how the income level of people should affect the price of the products in market.

References

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Appendices

Proof for $\mu_A(p_A, p_B)$ of Lemma - 1
To Prove - $\mu_A(p_A, p_B) = 1 - \frac{U_A p_A - U_B p_B}{(U_A - U_B)R_2} + \frac{R_1}{R_2}$
Proof - (Refer Figure 4) mobile A will be in the market for blue region and customer will buy that product if following condition is satisfied -
$U(A, R(t) - p_A) \geq U(B, R(t) - p_B)$
$\implies U_A(R_1 + R_2(t) - p_A) = U_B(R_1 + R_2(t) - p_B)$
$\implies U_A R_1 + U_A R_2 - U_A p_A = U_B R_1 + U_B R_2 - U_B p_B$
$\implies R_1(U_A - U_B) + R_2(U_A - U_B) = U_A p_A - U_B p_B$
$\implies R_1 + R_2 = \frac{U_A p_A - U_B p_B}{U_A - U_B} \implies \frac{R_1}{R_2} + 1 = \frac{U_A p_A - U_B p_B}{(U_A - U_B)R_2}$
$\implies \mu_A(p_A, p_B) = 1 - \frac{U_A p_A - U_B p_B}{(U_A - U_B)R_2} + \frac{R_1}{R_2}$

A Additional Experiments

Quality Pricing model:
Following parameters were used while implementing:
$\psi = 4$, $\theta = 1.2$, $\gamma = 1.2$, n=4 (Four firms), $q = [7, 5, 3, 1]$ (qualities offered by each firm)

```
c= [41.32164852464746, 27.594593229224294, 14.948771275386207, 4.0]
alpha =1.003996476981579
lamda= 4.107878402833892
A=8.548374895034446
Equilibrium prices for firms:

Firm 1: 50.03516443685156

Firm 2: 29.785845158807696

Firm 3: 15.515094117227312

Firm 4: 4.139305142786496
```

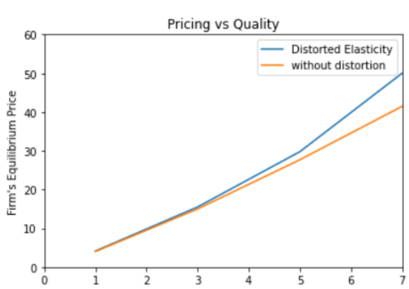

Figure 5: $v_{max} = 20$

```
c= [41.32164852464746, 27.594593229224294, 14.948771275386207, 4.0]
alpha =1.003996476981579
lamda= 4.107878402833892
A=1.9066255506804248
Equilibrium prices for firms:

Firm 1: 43.393415092497534

Firm 2: 28.16901313676632

Firm 3: 15.12150115670395

Firm 4: 4.0434909725686445
```

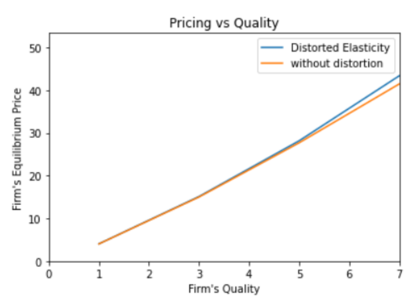

Figure 6: $v_{max} = 10$.

```
c= [41.32164852464746, 27.594593229224294, 14.948771275386207, 4.0]
alpha =1.003996476981579
lamda= 4.107878402833892
A=-2.74259899036739
Equilibrium prices for firms:

Firm 1: 38.74419055144972

Firm 2: 27.03723072133736

Firm 3: 14.845986084337595

Firm 4: 3.976421053416148
```

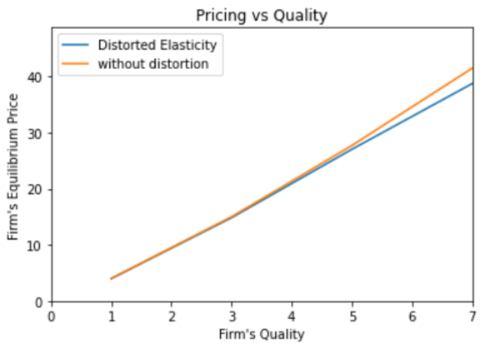

Figure 7: $v_{max} = 3$

Static Pricing:

```
Competitor's price 460
Best response price for the company is 400.0
Equilibrium price for both companies is 380.0
```

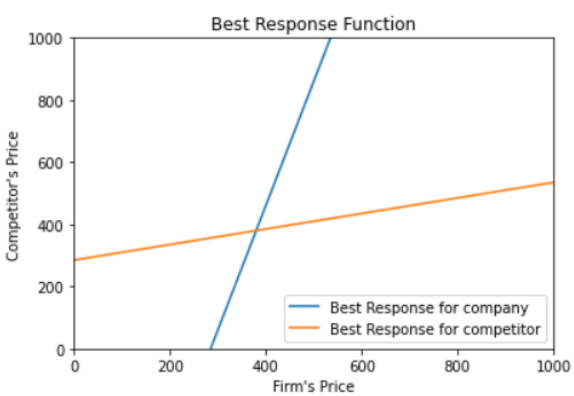

Figure 8: $a = 1000$, $b = 4$, $c = 2$

```
Competitor's price 40
Best response price for the company is 275.0
```

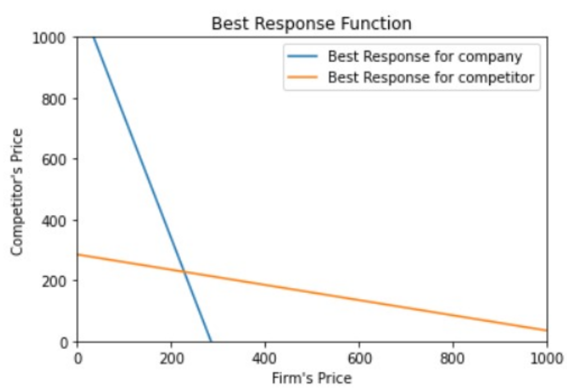

Figure 9: $a = 1000$, $b = 4$,$c = -2$

Dynamic Pricing:
Control variables are initialized as follows-
$\alpha_i = 0.6, a_i = 20, c_i = 0.5, r_{i,0} = 140$
for both i,j $c = 125, \delta = \gamma = 0,$
$\alpha_j = 0.4, a_j = 20, c_j = 0.6, p_{j,0} = 170, r_{j,0} = 150$

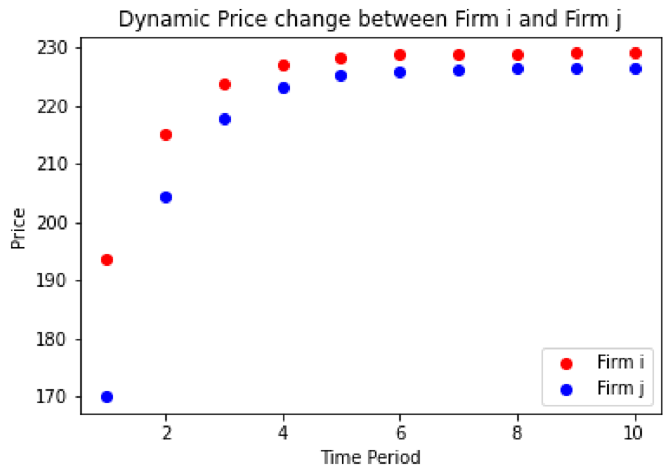

Figure 10: Dynamic Pricing at play