IEOR 4500 Application Programming for FE

Assignment 2

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This file includes te write-up and explanations for any of the problems in the aforementioned assignment

Problem 2

In the second part we transform the Q matrix as follows:-

$$Q = V^T F V \tag{1}$$

F is the diagonal matrix of the eigen values. V is 2x2 matrix because the first two assets with highest eigenvalues explain over 95% variability of the portfolio. (Results from a PCA analysis) The optimization problem that we are trying to solve is:-

$$F(x) = min.\ \lambda x^T Q x - \mu^T x \tag{2}$$

where the constraints are:-

$$1^T x = 1 \tag{3}$$

$$Ix \geqslant L \tag{4}$$

$$(-I)x \geqslant -U \tag{5}$$

where L are the lower bounds vector and U is the upper bound vector. Substituting Q as shown in equation 1, the minimization problem changes to:

$$F(x) = min.\ \lambda x^T V^T F V x - \mu^T x \tag{6}$$

where the constraints are:-

$$1^T x = 1 \tag{7}$$

$$Ix \geqslant L \tag{8}$$

$$(-I)x \geqslant -U \tag{9}$$

Taking the substitution:-

$$Vx = y \tag{10}$$

We do this substitution because our current function for solving the quadratic program accepts input in terms of $A^T BA$ and using the property of V:

$$V^T = V^{-1} \tag{11}$$

Hence the optimization problem changes to:-

$$F(x) = min.\ \lambda y^T Fy - \mu^T V^T y \tag{12}$$

where the constraints are:-

$$1^T V^T y = 1 \tag{13}$$

$$V^T y \geqslant L \tag{14}$$

$$-V^T y \geqslant -U \tag{15}$$

Again for verifying the constraint:

$$1^T V^T y = 1 \tag{16}$$

Our code accepts a vector of portfolio weights. Hence we introduce a matrix D such that:

$$Dy = z \tag{17}$$

or

$$y = D^{-1} z \tag{18}$$

For example in our case the constraint transforms to:-

$$1 = \begin{bmatrix} -0.999 & 0.8523 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$z_1 = -0.999 y_1 \tag{19}$$

$$z_2 = -0.8523 y_2 \tag{20}$$

And hence we get the vector of z such that:-

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -0.999 & 0 \\ 0 & 0.8523 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Hence the D matrix in our case is:

$$D = \begin{bmatrix} -0.999 & 0 \\ 0 & 0.8523 \end{bmatrix}$$

Hence the optimization problem finally changes to:-

$$F(x) = min.\ \lambda z^T D^{-1} F D^{-1} z - \mu^T V^T D_{-1} z \tag{21}$$

where the constraints are:-

$$1^Z = 1 \tag{22}$$

$$Z \geqslant DVL \tag{23}$$

$$-Z \geqslant -DVU \tag{24}$$