

IEOR 4500 Application Programming for FE

Assignment 2

Anurag Dutt, Shrey Goel, Jatindeep Singh, Vinayak Shinde, Aditya Zalte

October 15, 2018

This file includes the write-up and explanations for any of the problems in the aforementioned assignment

Problem 2

In the second part we transform the Q matrix as follows:-

$$Q = V^T F V \quad (1)$$

F is the diagonal matrix of the eigen values. V is 2x2 matrix because the first two assets with highest eigenvalues explain over 95% variability of the portfolio. (Results from a PCA analysis) The optimization problem that we are trying to solve is:-

$$F(x) = \min. \lambda x^T Q x - \mu^T x \quad (2)$$

where the constraints are:-

$$1^T x = 1 \quad (3)$$

$$I x \geq L \quad (4)$$

$$(-I)x \geq -U \quad (5)$$

where L are the lower bounds vector and U is the upper bound vector. Substituting Q as shown in equation 1, the minimization problem changes to:

$$F(x) = \min. \lambda x^T V^T F V x - \mu^T x \quad (6)$$

where the constraints are:-

$$1^T x = 1 \quad (7)$$

$$I x \geq L \quad (8)$$

$$(-I)x \geq -U \quad (9)$$

Taking the substitution:-

$$Vx = y \quad (10)$$

We do this substitution because our current function for solving the quadratic program accepts input in terms of A^TBA and using the property of V:

$$V^T = V^{-1} \quad (11)$$

Hence the optimization problem changes to:-

$$F(x) = \min. \lambda y^T Fy - \mu^T V^T y \quad (12)$$

where the constraints are:-

$$1^T V^T y = 1 \quad (13)$$

$$V^T y \geq L \quad (14)$$

$$-V^T y \geq -U \quad (15)$$

Again for verifying the constraint:

$$1^T V^T y = 1 \quad (16)$$

Our code accepts a vector of portfolio weights. Hence we introduce a matrix D such that:

$$Dy = z \quad (17)$$

or

$$y = D^{-1}z \quad (18)$$

For example in our case the constraint transforms to:-

$$1 = \begin{bmatrix} -0.999 & 0.8523 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$z_1 = -0.999y_1 \quad (19)$$

$$z_2 = -0.8523y_2 \quad (20)$$

And hence we get the vector of z such that:-

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -0.999 & 0 \\ 0 & 0.8523 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Hence the D matrix in our case is:

$$D = \begin{bmatrix} -0.999 & 0 \\ 0 & 0.8523 \end{bmatrix}$$

Hence the optimization problem finally changes to:-

$$F(x) = \min. \lambda z^T D^{-1} F D^{-1} z - \mu^T V^T D^{-1} z \quad (21)$$

where the constraints are:-

$$1^Z = 1 \quad (22)$$

$$Z \geq DVL \quad (23)$$

$$-Z \geq -DVU \quad (24)$$