

Discrete distribution

Bernoulli trial

Bernoulli trial or distribution must possess the following properties:

1. The experiment consists of single trial.
2. trial result in two outcome as a success or a failure.
3. The probability of success, denoted by p , remains constant.

Probability mass function of Bernoulli distribution is $f(x) = q^{1-x}p^x, x = 0,1$

Bernoulli process

the Bernoulli process must possess the following properties:

1. The experiment consists of repeated trials.
2. Each trial results in an outcome that may be classified as a success or a failure.
3. The probability of success, denoted by p , remains constant from trial to trial.
4. The repeated trials are independent.

Binomial distribution

The number X of successes in n Bernoulli trials is called a **binomial random variable**. The probability distribution of this discrete random variable is called the **binomial distribution**, and its values will be denoted by $b(x; n, p)$ since they depend on the number of trials and the probability of a success on a given trial.

Binomial Distribution A Bernoulli trial can result in a success with probability p and a failure with probability $q = 1 - p$. Then the probability distribution of the binomial random variable X , the number of successes in n independent trials, is

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

Mean, Variance and Moment generating function

The mean and variance of the binomial distribution $b(x; n, p)$ are
 $\mu = np$ and $\sigma^2 = npq$.

Moment generating function of Binomial distribution is $M_X(t) = (q + pe^t)^n$

5.15 It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that

- (a) none contracts the disease;
- (b) fewer than 2 contract the disease;
- (c) more than 3 contract the disease.

5.11 The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation survive?

5.13 A national study that examined attitudes about antidepressants revealed that approximately 70% of respondents believe “antidepressants do not really cure anything, they just cover up the real trouble.” According to this study, what is the probability that at least 3 of the next 5 people selected at random will hold this opinion? find the mean and variance of X when 5 people are selected at random.

Negative Binomial Distribution

Negative Binomial Distribution If repeated independent trials can result in a success with probability p and a failure with probability $q = 1 - p$, then the probability distribution of the random variable X , the number of the trial on which the k th success occurs, is

$$b^*(x; k, p) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2, \dots$$

Geometric distribution

Geometric Distribution If repeated independent trials can result in a success with probability p and a failure with probability $q = 1 - p$, then the probability distribution of the random variable X , the number of the trial on which the first success occurs, is

$$g(x; p) = pq^{x-1}, \quad x = 1, 2, 3, \dots$$

The mean and variance of a random variable following the geometric distribution are

$$\mu = \frac{1}{p} \text{ and } \sigma^2 = \frac{1-p}{p^2}.$$

5.55 The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that a given student will pass the test

(a) on the third try;

(b) before the fourth try.

5.50 Find the probability that a person flipping a coin gets

- (a) the third head on the seventh flip;
- (b) the first head on the fourth flip.

5.51 Three people toss a fair coin and the odd one pays for coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than 4 tosses are needed .

5.52 A scientist inoculates mice, one at a time, with a disease germ until he finds 2 that have contracted the disease. If the probability of contracting the disease is $1/6$, what is the probability that 8 mice are required?

Poisson Distribution The probability distribution of the Poisson random variable X , representing the number of outcomes occurring in a given time interval or specified region denoted by t , is

$$p(x; \lambda t) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}, \quad x = 0, 1, 2, \dots,$$

where λ is the average number of outcomes per unit time, distance, area, or volume and $e = 2.71828 \dots$

Mean and Variance of Poisson Distribution

mean and the variance of the Poisson distribution $p(x; \lambda t)$ are λt .

Let X be a binomial random variable with probability distribution $b(x; n, p)$. When $n \rightarrow \infty$, $p \rightarrow 0$, and $np \xrightarrow{n \rightarrow \infty} \mu$ remains constant,

$$b(x; n, p) \xrightarrow{n \rightarrow \infty} p(x; \mu).$$

5.56 On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

- (a) exactly 5 accidents will occur?
- (b) fewer than 3 accidents will occur?
- (c) at least 2 accidents will occur?

5.58 A certain area of the eastern United States is, on average, hit by 6 hurricanes a year. Find the probability that in a given year that area will be hit by

- (a) fewer than 4 hurricanes;
- (b) anywhere from 6 to 8 hurricanes

5.67 The number of customers arriving per hour at a certain automobile service facility is assumed to follow a Poisson distribution with mean $\lambda = 7$.

(a) Compute the probability that more than 10 customers will arrive in a 2-hour period.

(b) What is the mean number of arrivals during a 2-hour period?

5.61 Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. If 10,000 returns are selected at random and examined, find the probability that 6, 7, or 8 of them contain an error.

