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↳ Theory

★ Definitions

★ Examples

Unit-3 → Counting Principles & Relations

★ Sets :- collection of well-defined objects.
↳ represented in capitals.

Relations :- subset of cartesian product of 2 sets A & B
$R \subseteq (A \times B)$

$\rightarrow \boxed{n(R) = 2^{n(A \times B)}}$

Set :- A collection of elements which are well-defined

Example, students of KOCCF

Cartesian Product :-
$A = \{1, 2\}$
$B = \{3, 4, 5\}$

$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$

Relations :- Let A & B be the two sets, then the relation from the set A to the set B is a subset of $(A \times B)$.

$$R \subseteq A \times B$$

★ **Note** :- If set N is having n elements & the set M is having m elements then the number of elements in the set $N \times M$ is mn.

$$n(N \times M) = mn$$
$$n(R(N \rightarrow M)) = 2^{mn}$$

Types of Relations (Properties of Relations) :-

1. **Reflexive** :- A relation R on the set A is called reflexive if $(a, a) \in R \ \forall a \in A$ or
$aRa \ \forall a \in A$.

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Q. Is the divides relation on the set of positive integers reflexive?

S. $R = \{(a,b) ; a \text{ divides } b \ \forall\, a,b \in Z^+\}$

Yes, all the positive integers are divisible by itself.

Hence, Relation is reflexive.

★ matrix representation of reflexive relation $a_{ii} = 1$ always ($\Rightarrow$ diagnol elements are always 1).

Example,

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

2. Symmetric & Anti-symmetric :-

A relation R on a set A is called symmetric if $(a,b) \in R$ or aRb then $(b,a) \in R$ or bRa $\forall\, a,b \in A$.

Example, $A = \{1,2,3,4\}$

$R_1 = \{(1,2),(2,1),(3,4),(4,3)\}$

$R_2 = \{(1,1),(1,2),(2,1)\}$

A relation R on a set A is called antisymmetric if $(a,b) \in R$ then $(b,a) \in R$ 'or' $(a,b) \in R$ & $(b,a) \in R$ then $a=b$.

Example, $R_1 = \{(1,1),(1,2),(3,4)\}$

$R_2 = \{(1,1),(2,2),(3,3)\}$

★ If we have an ordered pair where the elements are related by itself then it is reflexive, symmetric & anti-symmetric. Eg, $A = \{1,2,3,4\}$

$R = \{(1,1),(2,2),(3,3),(4,4)\}$

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Q1. Is the divide relation on the set of positive integers symmetric, anti symmetric or neither?

↳ Anti-symmetric

2 divides 4 $(2,4) \in R$

4 divides 2 $(4,2) \notin R$

$(a,b) \in R$ then $(b,a) \notin R$

Hence, Anti-symmetric

3. **Transitive** :- A relation R on a set A is called transitive if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$.

$A = \{1,2,3,4\}$

Example, $R = \{(1,2),(2,3),(1,3),(1,4)\}$

Q1. Is the divide relation on the set of Z^+ transitive?

↳ Yes, it is transitive.

4. **Composite** :- Let R be a relation from a set A to the set B & S be the relation from set B to the set C. The composite of R & S is the relation consisting of the ordered pairs (a,c), where $a \in A$ & $c \in C$ & for which there exists an element $b \in B$ such that $(a,b) \in R$ & $(b,c) \in S$.

Represented as $S \circ R$.

Example,

Composite of relation R & S where R is defined from $R: \{\underbrace{1,2,3}_{A}\}$ to $\{\underbrace{1,2,3,4}_{B}\}$ &

$R = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$ &

$S: \{\underbrace{1,2,3,4}_{B}\}$ to $\{\underbrace{0,1,2}_{C}\}$ &

$S = \{(1,0),(2,0),(3,1),(3,2),(4,1)\}$

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G $S \circ R = \{(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)\}$

Q1 Suppose relation R on the set is represented as matrix

$$\begin{array}{c|c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

. Is the relation R reflexive, symmetric or antisymmetric?

G

⇒ Reflexive ✓ → Diagnol elements are 1

$R = \{(a,a), \underline{(a,b)}, \underline{(b,a)}, (b,b), \underline{(b,c)}, \underline{(c,b)}, (c,c)\}$

⇒ Symmetric ✓

⇒ ~~No~~ ~~Anti-symmetric~~ X

⇒ ~~Transitive~~ X

5. **Equivalence** :- A relation R on a set A is called equivalence relation if it is reflexive, symmetric & transitive.

6. **Partial ordering** :- A relation R on a set S is called a partial ordering relation if it is reflexive, antisymmetric & transitive. A set S together with a partial ordering relation R is called partially ordered set (poset).

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Comparable & Incomparable :- The elements a & b of the Poset $(S, \leq)$ are called comparable if either $a \leq b$ or $b \leq a$.

When a & b are the elements of S such that neither $a \leq b$ nor $b \leq a$ then it is called as incomparable. Represented by $\boxed{a/b = \frac{a}{b}}$ & $\boxed{a|b = \frac{b}{a}}$.

Example, In the Poset $(Z^+, 1)$ are the integers

(i) 3 & 9 comparable? → Yes

(ii) 5 & 7 comparable? → No

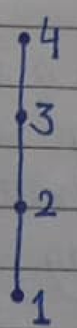
Totally ordered :- If $(S, \leq)$ (set, Relation) Poset & every 2 elements of S are comparable then S is called as totally ordered / linearly ordered set. A totally ordered set is also called a $\boxed{\text{chain}}$.

Most Impt. $\boxed{\text{for CA \& ETE}}$

★ # Hasse Diagram :- pictorial representation of Posets.

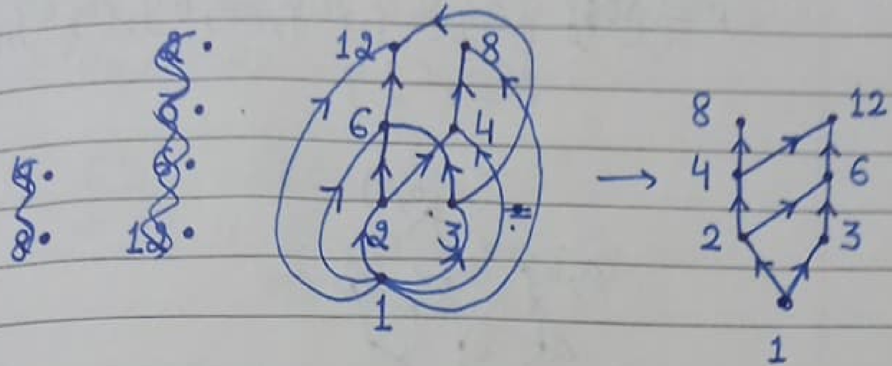
↳ always in upper direction (lowest to highest value)

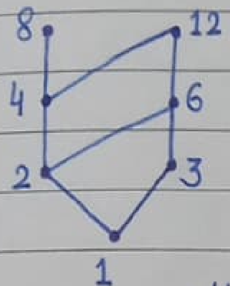
1. $(\{1,2,3,4\}, \leq)$



Hasse Diagram

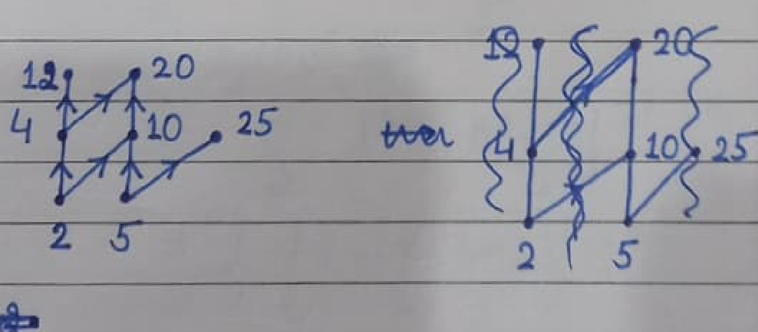
2. $(\{1, 2, 3, 4, 6, 8, 12\}, |)$

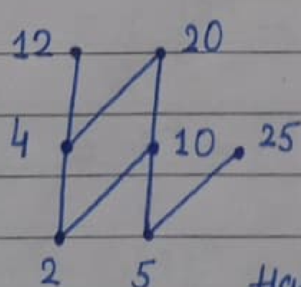




Hasse Diagram
(Final)

3. $(\{2, 4, 5, 10, 12, 20, 25\}, |)$





Hasse Diagram

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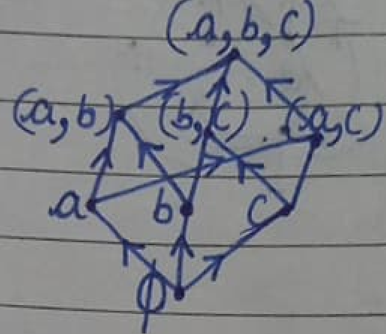
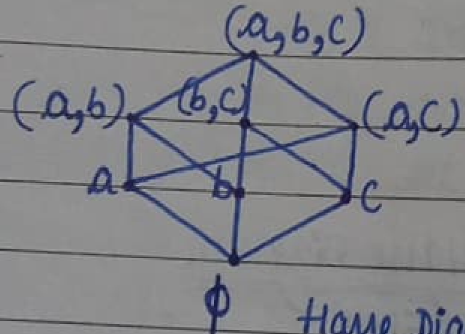
4. $S = \{a, b, c\}$

$R \rightarrow \subseteq$

$R = \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\},$
$\{a, b, c\}\}$

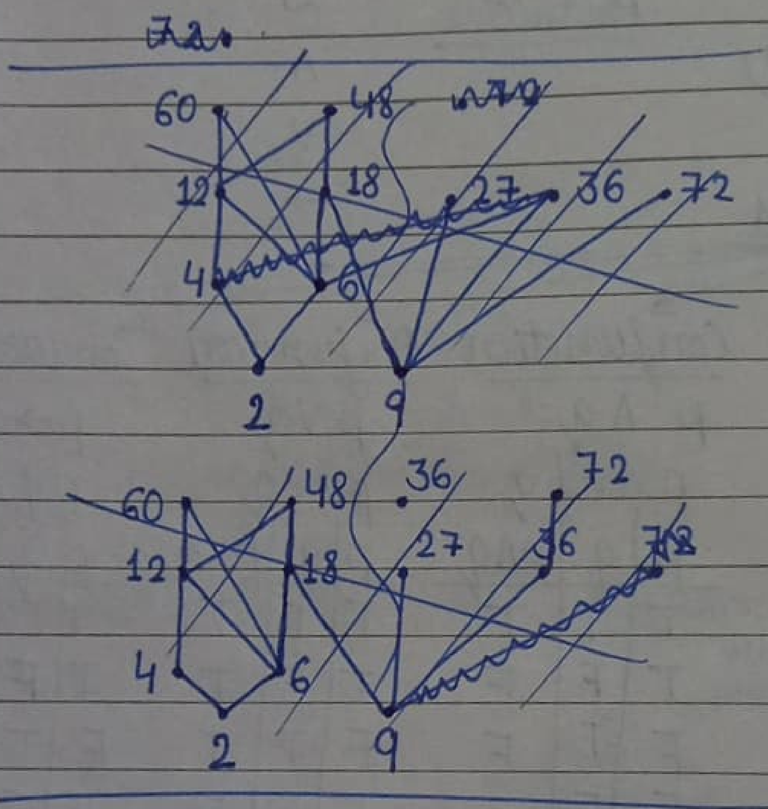


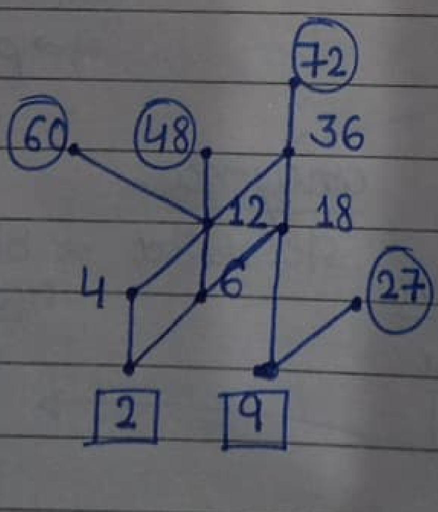


Hasse Diagram

⇒ Hasse Diagram

$(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$





O→ maximal elements

□→ minimal elements

Maximal & minimal elements

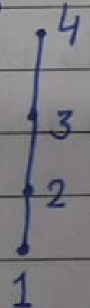
⇒ **Maximal elements** :- An element 'a' is called the maximal element of the Poset $(S, \leq)$ if there is no $b \in S$ such that $b \geq a$ or $b > a$. Example, 60, 48, 72 & 27 in the above hasse diagram

⇒ **minimal elements** :- An element 'a' is called the minimal element of the Poset $(S, \leq)$ if there exists no $b \in S$ such that $b \leq a$ or $b < a$. Example, 2 & 9 in the above hasse diagram

Greatest & Least elements

If the Poset is having a unique maximal element & minimal element, then it is called as greatest & least element if it exists.

Example,



4 → greatest element (maximal)
1 → least element (minimal)

* for $(Z^+, |) \rightarrow$ least element $= \{1\}$ & greatest element $= \phi$.

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Upper bound & Lower bound

⇒ If 'u' is an element of S such that $a \leq u$ $\forall a \in A$, then u is called an upper bound of the set A.

⇒ If 'l' is an element of S such that $l \leq a$ $\forall a \in A$, then l is called lower bound of the set A.

Least upper bound (lub)

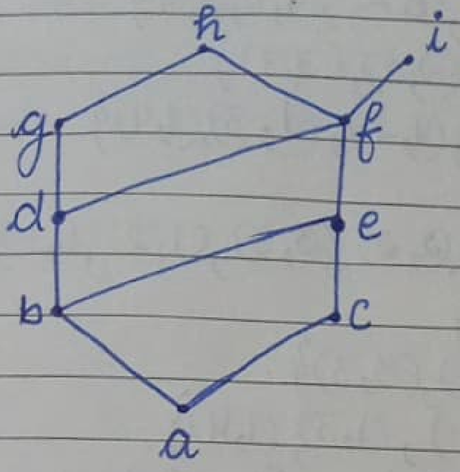
The element x is called the lub of the subset A if x is an upper bound which is smallest of all the upper bounds.

Greatest lower bound (glb)

The element y is called the glb of the subset A if it is a lower bound which is greatest of all the lower bounds.

Lattice

A poset in which every pair of elements have glb & lub is called a lattice.



$a \rightarrow$ minimal element
$h, i \rightarrow$ maximal elements
$\phi \rightarrow$ greatest element
$a \rightarrow$ ~~lower~~ least element

[★ elements should be related to a, b & c]

upper bound of $\{a, b, c\} = \{e, f, h, i\}$
lub $= \{e\}$
lower bound of $\{a, b, c\} = \{a\}$
glb $= \{a\}$

upper bound of $\{h, i\} = \phi$
lub $= \phi$
lower bound of $\{h, i\} = \{f, e, d, b, c, a\}$
glb $= \{f\}$

upper bound of $\{d, f, a, c\} = \{h, i, f\}$
lub $= \{f\}$
lb $= \{a\}$
glb $= \{a\}$

Combining Relation

Q1. $A = \{1,2,3\}$, $B = \{1,2,3,4\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (1,2), (1,3), (1,4)\}$

→ $R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (1,2), (1,3), (1,4)\}$

$R_1 \cap R_2 = \{(1,1)\}$

$R_1 - R_2 = \{(2,2), (3,3)\}$

$R_2 - R_1 = \{(1,2), (1,3), (1,4)\}$

$R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$

$= \{(2,2), (3,3), (1,2), (1,3), (1,4)\}$

Q2. no. of students of CSE = 25,
no. of students in MTH = 13,
no. of students in both = 8.
How many students are there in class?

→ $n(CSE) = 25$

$n(MTH) = 13$

$n(both) = n(CSE \cap MTH) = 8$

$n(CSE \cup MTH) = n(CSE) + n(MTH) - n(both)$

$= 25 + 13 - 8$

$= \boxed{30}$

Formula

☆ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.