PHY109 – ENGINEERING PHYSICS

Lecture 1

UNIT-1 ELECTROMAGNETIC THEORY

Lecture 1 Scalar and Vector field

Lecture 2 Concept of Gradient, Divergence and Curl

Lecture 3 Gauss theorem and Stokes theorem (qualitative)

Lecture 4 Poisson, Laplace Equations, Continuity Equation

Lecture 5 Ampere Circuital law, Maxwell's displacement current and corrections in Ampere Circuital Law, dielectric constant

Lecture 6 Maxwell's Electromagnetic Equations(Differential and integral forms)

Lecture 7 Electromagnetic waves, Physical significance of Maxwell Equations, electromagnetic spectrum

Physical quantities

- ❑ Any quantity that can be measured/determined and has a magnitude and unit.

Examples: Mass, weight, distance, length displacement, speed, velocity, pressure, temperature, force, acceleration, energy, current ..etc..

Scalar

Vector

- ❑ Physical quantity that has only magnitude and has no direction

Do you know about TENSOR?

If a tensor has only magnitude and no direction (i.e., rank 0 tensor), then it is called scalar.

If a tensor has magnitude and one direction (i.e., rank 1 tensor), then it is called vector.

If you didn't learn it so far, better learn it and will be useful as an Engineer ☺

What are Scalars and Vectors?

Scalar quantity

- It is enough to know its magnitude (numerical value and unit to express it)

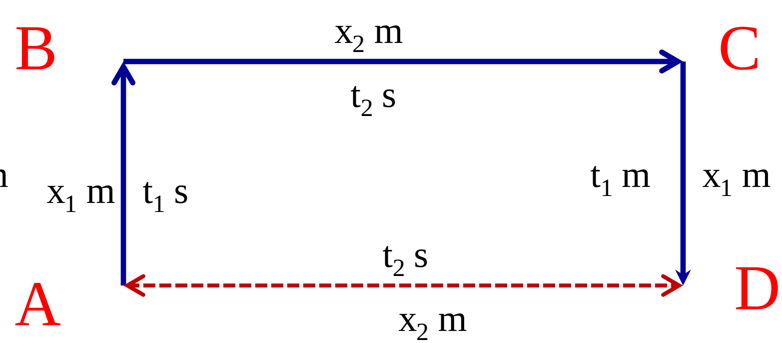
Examples:. Mass (kg), length (m), Current (A),time (s), speed (m/s), temperature (°C), Energy (J)

Vector Quantity

- It is necessary to know its magnitude (numerical value and unit) and also the direction

Examples: Displacement (m), velocity (m/s), acceleration (m/s^2), force (N), Weight (N)

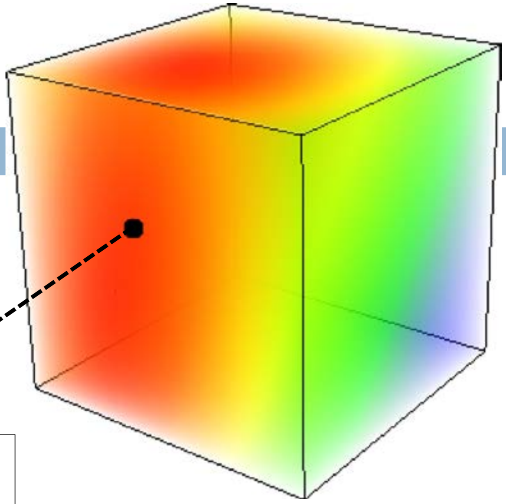
Distance=AB+BC+CD=$(2x_1+x_2)$m
Speed = $(2x_1+x_2)/(2t_1+t_2)$ m/s



Displacement= AD=x_2 m
Velocity=(x_2/t_2) m/s

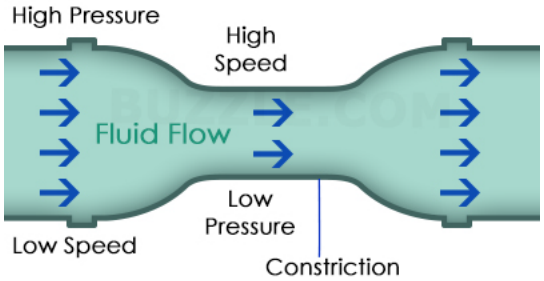
Scalar and Vector fields?

Eg. Temperature (°C)
Temperature distribution in the room with a heat source is a scalar field



Heat source

Eg. Velocity (m/s)
Velocity distribution of fluid flow through a tube of varying diameter is a vector field



Velocity field

Vector and scalar fields may depend also on time in addition to their dependencies on space.

Vector Calculus

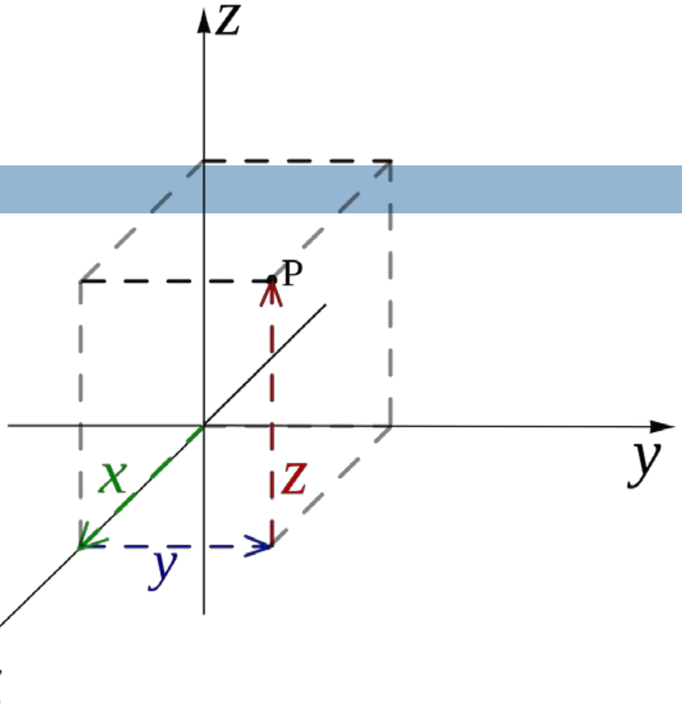
- Introducing mathematical operations in vector calculus for EM theory
- Study the rate of change of scalar and vector fields

Vector differential operator (∇)

-called Del or Nabla

- Rectangular co-ordinate system (Cartesian)
- Cylindrical coordinate system
- Spherical polar coordinate system

1. DEL OPERATOR



In the Rectangular coordinate system

$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

In the Cylindrical coordinate system

$$\vec{\nabla} = \hat{s}\frac{\partial}{\partial s} + \hat{\phi}\frac{1}{s}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$$

In the spherical polar coordinate system

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

Refer T1: Mailk and Singh section 10.3 figure 10.1 to see the definition of (x,y,z), (s,ϕ,z) and (r,θ,ϕ)…Also depicted in the next slide

Please learn about curvilinear co-ordinate systems!

- **Curvilinear coordinate system**
- **Coordinate transformation**
- **Partial differential calculus**

Refer
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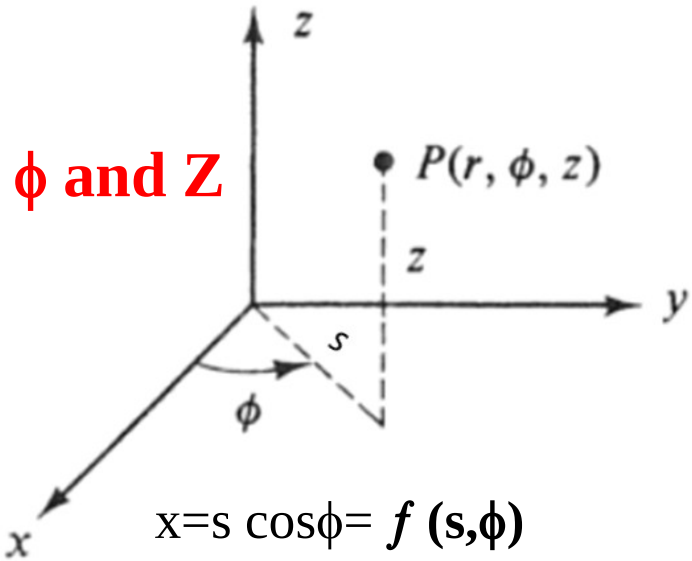
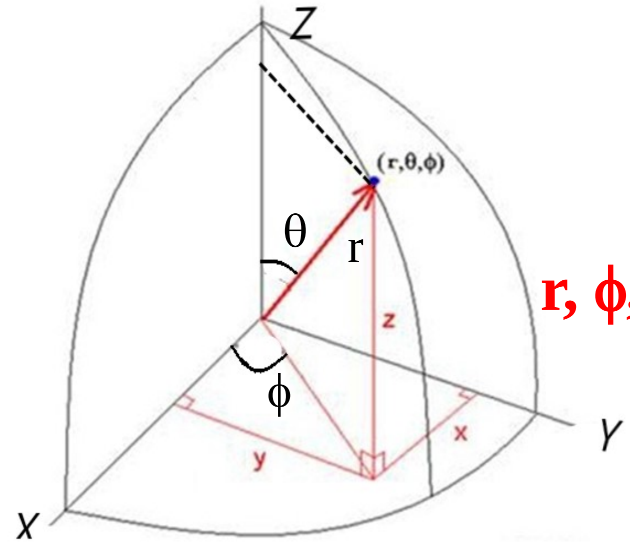
S, ϕ and Z



$x = s\cos\phi = f\,(\mathbf{s},\phi)$
$y = s\sin\phi = f\,(\mathbf{s},\phi)$
$z = z$



r, ϕ,θ

r-projection = $r\,\mathrm{Sin}\theta = f\,(\mathbf{r},\theta)$
x=r-projection. $\mathrm{Cos}\phi = r\,\mathrm{Sin}\theta\,\mathrm{Cos}\phi = f\,(\mathbf{r},\theta,\phi)$
y=r-projection.$\mathrm{Sin}\phi = r\,\mathrm{Sin}\theta\,\mathrm{Sin}\phi = f\,(\mathbf{r},\theta,\phi)$
$z = r\,\mathrm{Cos}\theta = f\,(\mathbf{r},\theta)$

2. GRADIENT

In the rectangular coordinate, the Gradient of a Scalar function F(x,y,z)

$$\text{Grad } F(x,y,z) = \vec{\nabla} F = \hat{\boldsymbol{\imath}}\frac{\partial F}{\partial x} + \hat{\boldsymbol{\jmath}}\frac{\partial F}{\partial y} + \widehat{\boldsymbol{k}}\frac{\partial F}{\partial z}$$

$$\vec{\nabla} F = \frac{\partial F}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial F}{\partial \phi}\hat{\phi} + \frac{\partial F}{\partial z}\hat{\mathbf{z}}$$

Cylindrical coordinate system

$$\vec{\nabla} F = \frac{\partial F}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial F}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\hat{\phi}$$

Spherical coordinate system

2. GRADIENT

But $$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

dF is variation in F for a small change x, y and z

And is nothing but the dot product of ∇F with $\overrightarrow{dl} = \hat{\imath}\mathrm{dx} + \hat{\jmath}dy + \hat{\boldsymbol{k}}\boldsymbol{dz}$

i.e.….dF= $\vec{\nabla} F \bullet \overrightarrow{dl} = |\nabla F||dl|\cos\theta$

So maximum when θ=0; i.e when spatial change is in the direction of the vector ∇F

so…it divergence gives an idea about the direction along which maximum change in the scalar function (F) occurs- and that will be in the direction of the vector ∇F

3 DIVERGENCE

In the rectangular coordinate system, Divergence of the vector,

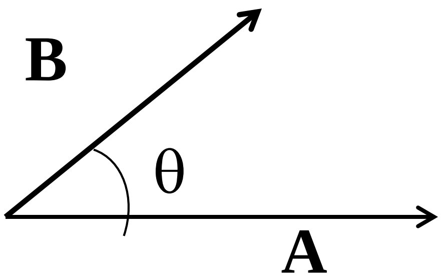
$$\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y) \text{ results in } \vec{\nabla}\bullet\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z})\bullet(\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$$

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1;\ \hat{k}\cdot\hat{\imath} = \hat{k}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{\imath} = 0 \ and\ \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = \hat{\imath}\cdot\hat{\jmath} = 0$$

Vectors **A** and **B** with an θ between them

A•B =AB cosθ



$$\vec{\nabla}.\vec{\mathrm{A}} = \frac{1}{s}\frac{\partial(sA_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

Cylindrical coordinate system

$$\vec{\nabla}.\vec{\mathrm{A}} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{\mathrm{r}\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Spherical coordinate system

Outward normal flux of vector field from a closed surface is Solenoidal if the divergence of the vector is zero..

What if the Divergence is ±x? Source ? Sink? Think about it…

LECTURE 2

What we learned in Lecture-1?

1. **Scalar and Vector quantities**
2. **Scalar and vector field**
3. **Del operator in Rectangular, cylindrical and spherical polar coordinate systems**
4. **Operation with del operator**
   - **Gradient of scalar function – maximum change of the function is along the direction of vector ∇F**
   - **Divergence of a Vector function- solenoidal or divergenceless when divergence of the vector is zero**

Curl of a vector $\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$

Rectangular coordinate system (x,y,z)

$$\vec{\nabla} \times \vec{A} = (\hat{\imath}\frac{\partial}{\partial x} + \hat{\boldsymbol{j}}\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial}{\partial z}) \times (\hat{\imath}A_x + \hat{\jmath}A_{y+}\hat{k}A_z)$$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0;\ \hat{\imath} \times \hat{\jmath} = k;\ \hat{\jmath} \times \hat{k} = \hat{\imath};\ \hat{\imath} \times \hat{k} = -\hat{\jmath};\ \hat{\imath} \times \hat{\jmath} = -\hat{k};\ \hat{k} \times \hat{\jmath} = -\hat{\imath}; \hat{\imath} \times \hat{k} = -\hat{\jmath}$$

$$\vec{\nabla} \times \vec{A} = \hat{\imath}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

- Curl of a vector is a vector quantity--- as it has both magnitude and direction, and
  - is a rotational vector
  - Its magnitude is the maximum circulation per unit are
  - Its direction is normal to the area that make circulation maximum.. *Right hand rule*
- It is not possible to have the curl of a scalar quantity

Curl of a vector

$$\vec{\nabla} \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (sA_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$$

Cylindrical coordinate system (s,ϕ,z)

Spherical polar coordinate system (r,θ,ϕ)

$$\vec{\nabla} \times \vec{A} = \frac{1}{r sin\theta}\left[\frac{\partial (sin\theta\ A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{sin\,\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (rA_\phi}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial (rA_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\phi}$$

If **∇ x A** =0, we can say that vector A is irrotational

If **∇ x A** ≠0, then vector A is rotational vector and cross product gives the vorticity of the vector field A

Theorems in VECTOR CALCULUS

- Fr Gradient of scalar function
  - Fundamental theorem for Gradient….. **Skip it**
- For Divergence of the vector function
  - Fundamental theorem for Divergence
- For Curl of the vector function
  - Fundamental theorem for Curl

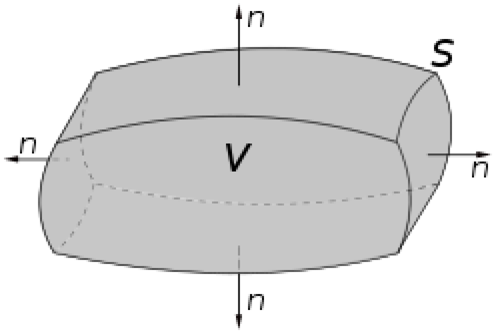
Learn about line integral, surface integral, and volume integral!!

Gauss theorem

Also called Gauss Divergence theorem; Which states volume integral of the divergence of a vector field over the volume is equal to the surface integral of that field enclosing the volume. Also known as **GREEN's Theorem**

i.e.

$$\oiiint_V \vec{\nabla} \cdot \vec{A}\, dV = \oiint_S \vec{A} \cdot \overrightarrow{dS}$$



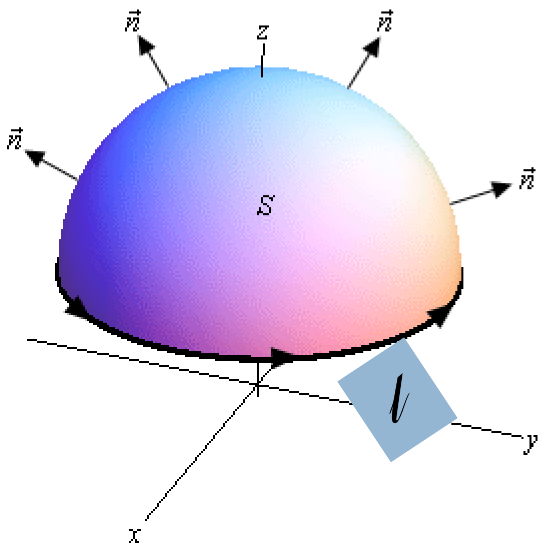
Where , $\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$ is the vector and V is the volume bounded by the closed surface S

❑ In short by this theorem volume integral can be converted to surface integral– useful when the volume integration is difficult to achieve the result.

Stokes' theorem

Stokes' theorem states that surface integral (over a patch of the surface, **S**) of the curl of a vector is equal to line integral of that vector over a closed curve ***(l)*** defining the boundary of that surface (**S**)

$$\iint (\vec{\nabla} \times \vec{A}) \cdot \overrightarrow{dS} = \oint \vec{A} \cdot \overrightarrow{dl}$$



Where , $\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$ is the vector and S is bounded by the closed path ***l***

- Convert surface integral into the line integral
- Curl of the vector relate to its line integration
- Right hand thump rule to know the direction of **dS**
- Not depend on the shape of the surface
- Depends on the boundary line

Lecture 3

What we learned so far?

1. **Scalar and Vector quantities**
   - It is *enough* to have a magnitude for **scalar** physical quantities where as it is *essential* to have both magnitude and direction for the **vector** physical quantities.
2. **Scalar and vector field**
   - **Region of space/domain** in which a function, f(x,y,z), signifies a physical quantity (Temperature, Velocity) is the **field**.
   - **Scalar field**: Each point in space is associated with a **scalar point function** (Temperature, potential) having magnitude.
   - **Vector field**: Each point space is associated with a **vector point function** (Electric field, Gravitational field) having magnitude and direction, both of which changes from point to point.
3. **Del operator (∇)**
   - It is a differential operator
   - It is not a vector by itself
   - It operate on scalar and vector functions and the resulting function may be a vector or scalar function depending on the type of operation.

Rectangular (x,y,z), cylindrical (s,ϕ,z) and spherical polar(r, ϕ,θ) coordinate systems

- **Curvilinear coordinate system**
- **Coordinate transformation**
- **Partial differential calculus**

→ **Tutorial 2**
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What we learned so far?

4. Operation with del (∇) operator

- Gradient of *<u>scalar function F</u>* – Directional derivative..maximum change of the scalar function is along the direction of vector ∇F, which nothing but the direction of outward surface normal vector; Advantage: A vector can be obtained from a scalar function which can be handled more easily than a vector.

- Divergence of a *<u>Vector function</u>* **A** - Gives the measure of the vector function's spread out at a point- is solenoidal or divergenceless when divergence of the vector is zero which means that flux of the such vector field entering into a region is equal to that leaving the region, a condition known as incompressibility; also gives an idea about source (∇.**A**>0) means vector diverge and sink(∇.**A**<0) means vector converge.

- Curl of a *<u>Vector function</u>* **A**– regarding the rotation of the vector and the vector function is irrotational when curl of the vector is zero, such fields are known as conservative fields.

5. Gauss Theorem: Conversion of volume integral to surface integral

- For the divergence of the vector

6. Stokes' Theorem: Conversion of surface integral to line integral

- For the curl of the vector

What to learn today?

Gauss law in Electrostatic
Poisson Equations
Laplace Equations
Continuity Equation

Gauss's law in Electrostatic (First law)

Electric flux (Φ_E) : The area integral of the Electric field (**E**) over any closed surface is the Φ_E or electric field is the flux per unit area

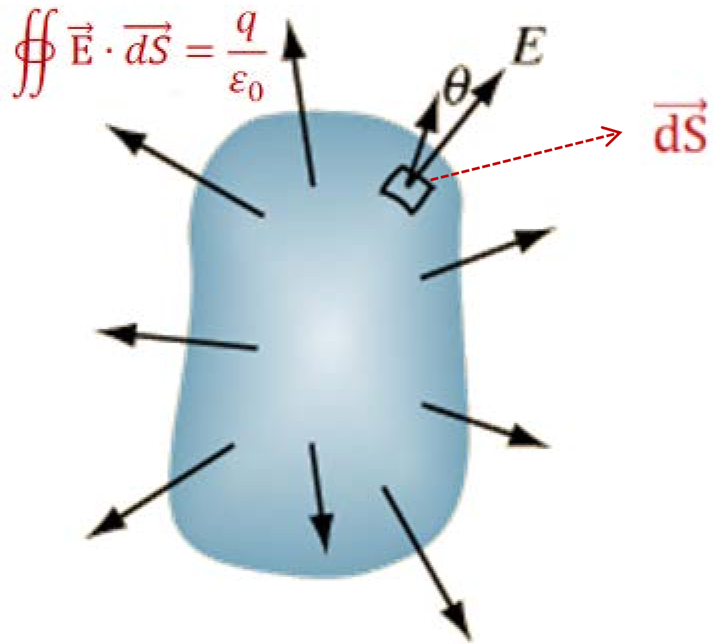
$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} \longrightarrow \text{Eq..1}$$

Gauss's law: Electric flux (Φ_E) from a closed surface (Gaussian surface) is equal to $1/\varepsilon_o$ times the charge (q) enclosed by the surface

$$\Phi_E = \frac{q}{\varepsilon_0} \longrightarrow \text{Eq..2}$$

ε_0 is the permittivity of free space

From Eq.1 and 2 $$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$



•Gauss's Law is a general law applying to any closed surface.

• It is an important tool since it permits the assessment of the amount of enclosed charge by mapping the **electric field normal** to the surface outside the charge distribution

•Or can be used to calculate electric field

Poisson’s Equations

Eq.1 $$\oint\!\!\!\oint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

Gauss’s first law

Charge distributed over a volume with ρ is the volume charge density

Eq.2 $$\iiint \rho \, dV = q$$

Eq.3 $$\varepsilon_0 \oint\!\!\!\oint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = q$$

Applying divergence theorem to eq.3

$$\varepsilon_0 \iiint \vec{\nabla} \cdot \vec{E} \, dV = \iiint \rho \, dV \dashrightarrow \vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0}$$

Eq.4

Integrands must be equal for LHS and RHS

$$\vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0}$$ **Eq.4**

Electric field (**E**) and potential (V) are related as

But $$\vec{\mathrm{E}} = -grad\, V = -\vec{\nabla} V$$ **Eq.5**

$$\vec{\nabla} \cdot (-\vec{\nabla} V) = \frac{\rho}{\varepsilon_0}$$ **Eq. 5 in Eq.4**

By using the vector identity for $\vec{\mathrm{A}} \cdot \vec{\mathrm{A}} = \mathrm{A}^2$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

Eq.6 is the Poisson's Equation

Laplace Equations and Laplace Operator

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

For a charge free region i.e $\rho=0$, then the Poisson's Equation changes to

Eq.7
$$\nabla^2 V = 0$$

This, Eq.7 is known as Laplace's equation and ∇^2 is the Laplacian operator.

Laplace Equation

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Cartesian coordinate

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} = 0$$

Cylindrical coordinate

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{\mathrm{r}^2 \sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{\mathrm{r}^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

Spherical coordinate

LECTURE 4

What we learned in the last lecture

Gauss's law in Electrostatic

$$\oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

•Where E is the electric field vector, q is the charge and Φ is the electric flux

•Important tool since it permits the assessment of the amount of enclosed charge by mapping the **electric field normal** to the surface outside the charge distribution or vice versa

Poisson & Laplace Equations

$$\iiint \rho \, dV = q \qquad \vec{\nabla} \cdot \vec{\mathrm{E}} = \frac{\rho}{\varepsilon_0} \qquad \vec{\mathrm{E}} = -grad\, V = -\vec{\nabla} V$$

Where ρ is the electric charge density in the closed volume. And V is the electric potential

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad \nabla^2 V = 0$$

Poisson Equations
(a region with charge)

Laplace Equations
(a region free of charge)

What we learned in the last lecture

Continuity Equation

$$I = \frac{dq}{dt} \qquad q = \iiint \rho \, dV \quad \text{and} \quad I = \iint \vec{J} \cdot \overrightarrow{dS}$$

Where I is the current and J is the current density

$$\oiiint \vec{\nabla} \cdot \vec{J} \, dV = -\oiiint \frac{d\rho}{dt} \, dV$$

$$\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$$

Current density flowing out of the closed volume is equal to the rate of decrease of charge within that volume.

What to learn today?

- **Gauss law of magnetostatics**
- **Faraday's law of electromagnetic induction**
- **Ampere Circuital Law**
- **Maxwell displacement current**
- **Correction in Ampere Circuital Law**

Understand the relation between electricity and Magnetism

Gauss law of magnetostatics

Concept of magnetic flux (ϕ_B)

We can calculate the amount of electric field (**E**) that passes through a surface by a quantity called electric flux (Φ_E), which we have seen in the last lecture as

$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS}$$

Similarly, e can calculate the amount of magnetic field (**B**) that passes through a surface by a quantity called magnetic flux (Φ_B),

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS}$$

Gauss's laws of magnetostatics and Electrostatics

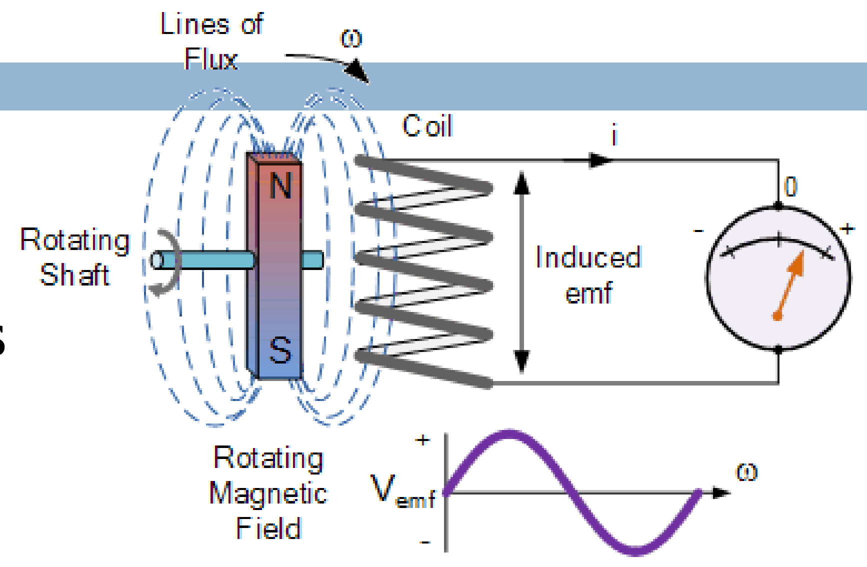
Gauss law of magnetostatic (Gauss's 2nd law) asserts that the net magnetic flux through any closed Gaussian surface is zero.

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS} = 0$$ Gauss's 2nd law for magnetic field

$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$ Gauss's 1st law for electric field

Magnetic monopoles do not exist, where as electric monopoles do exist

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION



Faraday's first law: Whenever the magnetic flux (ϕ_B) linked with a circuit changes and emf ($\mathbf{E}_{\mathbf{emf}}$) is induced in the circuit.

Faraday's second law: This induced emf, $\mathbf{E}_{\mathbf{emf}}$ is equal to the negative rate of change of magnetic flux (ϕ_B) with time linked with the circuit.

$$\mathrm{E}_{emf} = -\frac{\partial \phi_B}{\partial t}$$

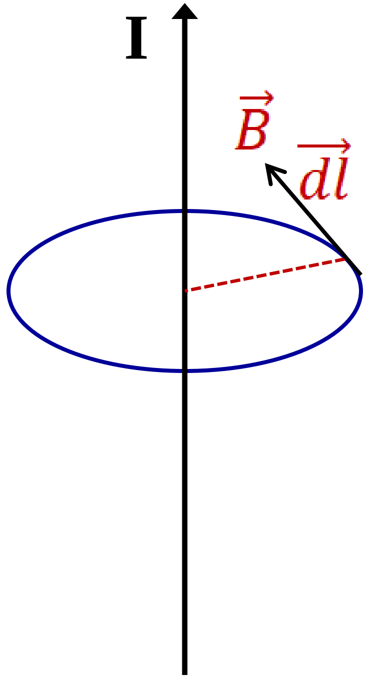
Negative sign indicating that induced emf ($\mathbf{E}_{\mathbf{emf}}$) always **opposes** the change in flux

Ampere Circuital Law in Magnetostatics

Ampere circuital law: The line integral of the magnetic field (**B**) around any closed loop is equal to **μ_0** (***permeability of the free space***) times the net current (**I**) flowing through the area enclosed by the loop.

Mathematically, this can be expressed as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$



Amperian loop

Proof of Ampere law $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

$$LHS = \oint \vec{B} \cdot \vec{dl}$$ Vector dot product

$$= \oint B\, dl \cos\theta = \oint B\, dl$$

θ is either 0 or 180° so

$$= B \oint dl = B\, 2\pi r$$

By **Biot-Savart law** the magnitude of the magnetic field **B** at a point r from the conductor carrying a current **I** is given by

$$B = \frac{\mu_0}{4\pi} \frac{2I}{r}$$

Substituting for B in the above equation

$$LHS = \frac{\mu_0}{4\pi} \frac{2I}{r} 2\pi r = \mu_0 I = RHS$$

Hence proved

Gauss's Electrostatic vs. Ampere Magnetostatics

$$\oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

Gauss's law of Electrostatic

Gauss's law charge constant (do not change with time)

$$\oiint \vec{\mathrm{B}} \cdot \overrightarrow{dS} = 0$$

Gauss's law of magnetostatic

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I$$

Ampere's law of Magnetostatic

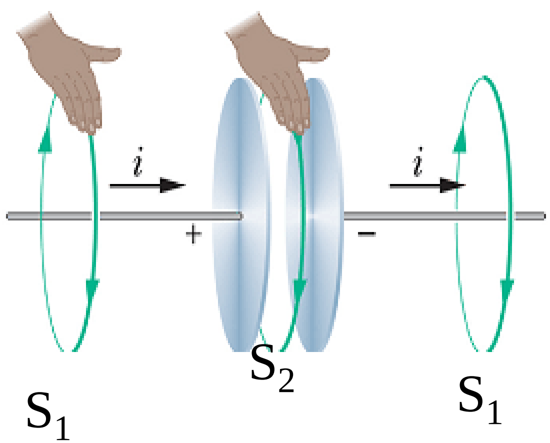
Ampere's Law all currents have to be steady (i.e. do not change with time).

Correction in Ampere Circuital Law

Concept of displacement current due to the charge/discharge of a capacitor leads to the correction/modification to the Ampere's law

$$\oint_{s_1} \vec{B} \cdot \vec{dl} = \mu_0 I$$ Eq.1

$$\oint_{s_2} \vec{B} \cdot \vec{dl} = 0$$ Eq.2



Because no current is enclosed by S_2

Eq.1 and Eq.2 are contradicting and Maxwell corrected Ampere's law by putting another 'current' term in equation 1

MAXWELL's LAW OF ELECTROMAGNETIC INDUCTION

Hence Maxwell (like Faraday's idea) introduced the idea of changing electric field as the source of magnetic field in the gap between the capacitor plate (during charging) and introduced displacement current I_d

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

Add this I_d into

$$\oint_{s_1} \vec{B} \cdot \overrightarrow{dl} = \mu_0 I$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt} \right) = \mu_0 (I + I_d)$$

Thus corrected Ampere's law to take care of the **continuity equation and** will be verified later..

Incomplete Ampere's law

According to the continuity equation $\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$, the rate of change of charge give rise to current density

$$\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$$

And will be zero only when there is no change in the charge density within a closed volume

That is $\vec{\nabla} \cdot \vec{J} = 0$ is zero only when $\frac{d\rho}{dt} = 0$

Let us consider Ampere's law and the relation between current density (**J**) and current (**I**)

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I \qquad I = \iint \vec{J} \cdot \vec{dS}$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \iint \vec{J} \cdot \overrightarrow{dS}$$

Now apply Stoke's theorem to the LHS

$$\iint (\vec{\nabla} \times \vec{B}) \cdot \overrightarrow{dS} = \mu_0 \iint \vec{J} \cdot \overrightarrow{dS} \quad \Rightarrow \quad \iint \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) \cdot \overrightarrow{dS} = \iint \vec{J} \cdot \overrightarrow{dS}$$

Integrands of the LHS and RHS must be equal, so we have

$$\frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) = \vec{J}$$

Take the divergence of both LHS and RHS we will get

$$\vec{\nabla} \cdot \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot \vec{J} \quad \Rightarrow \quad \frac{1}{\mu_0} \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot \vec{J}$$

But the divergence of the curl of *any* vector field **A** is always zero

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$$

That means in the last equation $\frac{1}{\mu_0} \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot \vec{J}$

the term $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B})$ is ZERO and we will end up with

$$0 = \vec{\nabla} \cdot \vec{J}$$

But we know that there is current and hence there is a rate of change of charge density… $\frac{d\rho}{dt}$ *is not zero*

So Ampere's law conflicts with the continuity equation $\vec{\nabla} \cdot \vec{J} = -\frac{d\rho}{dt}$
And hence the correction by Maxwell is justified.

Maxwell displacement current

- **Current** in a conductor produces magnetic field – **Ampere's circuital law**
- However, a changing electric field produces a **magnetic field** in vacuum or in a dielectric
- That means **a changing electric field** is equivalent to a **current** and is called the **DISPLACEMENT CURRENT**
- **Displacement current** produces the same effect as a conventional current in a metallic wire/conductor

Refer R-3 FUNDAMENTALS OF PHYSICS HALLIDAY. RESNICK, WALKER Chapter 8 278-281

LECTURE 5

what expected to learn! A step back as you were lost in ....(impedance mismatch)☺

1. **Scalar and Vector physical quantities**
   - It is *enough* to have a magnitude for **scalar** physical quantities where as it is *essential* to have both magnitude and direction for the **vector** physical quantities.
2. **Scalar and vector field**
   - **Region of space/domain** in which a function, f(x,y,z), signifies a physical quantity (Temperature, Velocity) is the **field**.
   - **Scalar field**: Each point in space is associated with a **scalar point function** (Temperature, potential) having magnitude and changes from point to point.
   - **Vector field**: Each point space is associated with a **vector point function** (Electric field, Gravitational field) having magnitude and direction, both of which changes from point to point.

In Lecture 1 we learned!

1. **Del operator (∇)**

$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

- It is a differential operator
- It is not a vector by itself
- It operate on scalar and vector functions and the resulting function may be a vector or scalar function depending on the type of operation.
- Where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are unit vectors (magnitude 1) along **X, Y and Z** directions. Similarly $\hat{s}, \hat{\phi}$ and $\mathbf{z}$ are the unit vectors for the cylindrical co-ordinate system along **S, ϕ and Z** directions;
  And $\hat{r}, \hat{\theta}$ and $\hat{\phi}$ are the unit vectors in the spherical coordinate system along **r, ϕ,θ** directions.

Rectangular (x,y,z), **cylindrical (s,ϕ,z) and** **spherical polar(r, ϕ,θ) coordinate systems**

$$\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla} = \hat{s}\frac{\partial}{\partial s} + \hat{\phi}\frac{1}{s}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$$

x=s cosϕ

y=s sinϕ

z= z

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

x = r Sinθ Cosϕ

y= r Sinθ Sinϕ

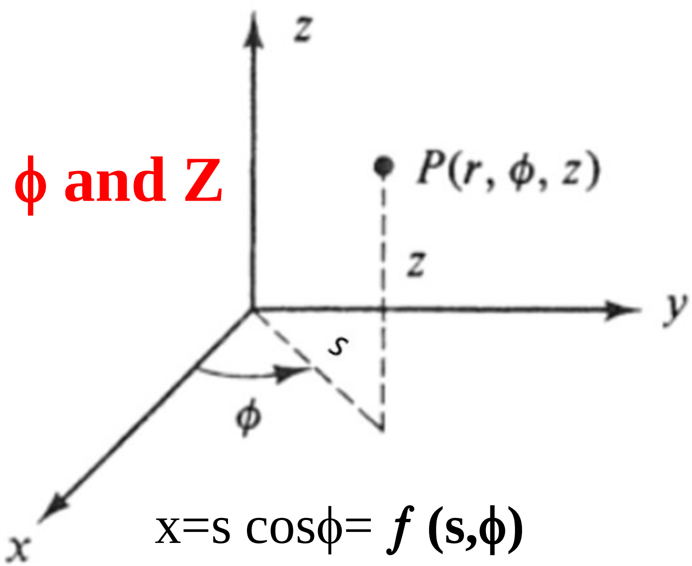
z= r Cosθ

In lecture 1&2 we learned!

- **Curvilinear coordinate system**
- **Coordinate transformation**
- **Partial differential calculus**

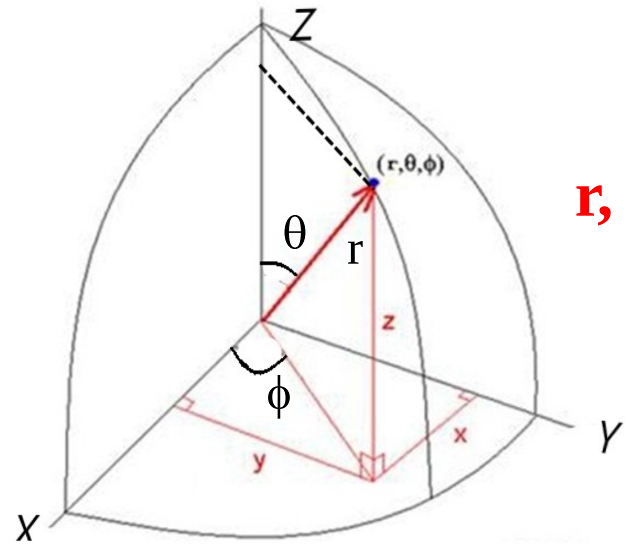
Refer
Advanced Engineering Mathematics
By ERWIN KREYSZIG

S, ϕ and Z



$x = s\cos\phi = \boldsymbol{f}\,(\mathbf{s},\boldsymbol{\phi})$
$y = s\sin\phi = \boldsymbol{f}\,(\mathbf{s},\boldsymbol{\phi})$
$z = z$

r, ϕ,θ



r-projection $= r\,\mathrm{Sin}\theta = \boldsymbol{f}\,(\mathbf{r},\boldsymbol{\theta})$
x=r-projection. $\mathrm{Cos}\phi = r\,\mathrm{Sin}\theta\,\mathrm{Cos}\phi = \boldsymbol{f}\,(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\phi})$
y=r-projection.$\mathrm{Sin}\phi = r\,\mathrm{Sin}\theta\,\mathrm{Sin}\phi = \boldsymbol{f}\,(\mathbf{r},\boldsymbol{\theta},\boldsymbol{\phi})$
$z = r\,\mathrm{Cos}\theta = \boldsymbol{f}\,(\mathbf{r},\boldsymbol{\theta})$

In lecture 2&3 we learned!

4. Operation with del (∇) operator

4.1 Gradient of *scalar function F*

$$\vec{\nabla}F = \hat{\boldsymbol{i}}\frac{\partial F}{\partial x} + \hat{\boldsymbol{j}}\frac{\partial F}{\partial y} + \widehat{\boldsymbol{k}}\frac{\partial F}{\partial z} \qquad \mathrm{F} = f(\mathrm{x,y,z})$$

Directional derivative..maximum change of the scalar function is along the direction of vector ∇F, which nothing but the direction of outward surface normal vector; Advantage: A vector can be obtained from a scalar function which can be handled more easily than a vector.

4.2 Divergence of a *Vector function* **A**

$$\vec{\nabla}\bullet\vec{\mathrm{A}} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad \vec{\mathrm{A}} = (\hat{\imath}\mathrm{A_x} + \hat{\jmath}\mathrm{A_y} + \hat{\mathrm{k}}\mathrm{A_y})$$

Gives the measure of the vector function's spread out at a point- is solenoidal or divergenceless when divergence of the vector is zero which means that flux of the such vector field entering into a region is equal to that leaving the region, a condition known as incompressibility; also gives an idea about source (∇.**A**>0) means vector diverge and sink(∇.**A**<0) means vector converge.

4.3 Curl of a *Vector function* **A**

$$\vec{\nabla}\times\vec{A} = \hat{\imath}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

Regarding the rotation of the vector and the vector function is irrotational when curl of the vector is zero, such fields are known as conservative fields.

In lecture 3 we learned!

5. Theorems in vector calculus

5.1 Gauss Theorem: Conversion of volume integral to surface integral

- For the divergence of the vector

$$\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y) \qquad \overrightarrow{dS} = dS\,\hat{n} \qquad \hat{n} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\oiiint_V \vec{\nabla}\cdot\vec{A}\,dV = \oiint_S \vec{A}\cdot\overrightarrow{dS}$$

5.2 Stokes' Theorem: Conversion of surface integral to line integral

- For the curl of the vector

$$\iint (\vec{\nabla}\times\vec{A})\cdot\overrightarrow{dS} = \oint \vec{A}\cdot\overrightarrow{dl} \qquad \vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$$

$$\overrightarrow{dS} = dS\,\hat{n} \qquad \overrightarrow{dl} = \hat{\boldsymbol{\imath}}dx + \hat{\boldsymbol{\jmath}}dy + \hat{\boldsymbol{k}}\boldsymbol{dz}$$

Lecture 3 we learned!

6. Gauss's law in Electrostatic

$$\oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0} = \Phi_E$$

- Where E is the electric field vector, q is the charge, ε_0 is the permittivity of vacuum and Φ_E is the electric flux
- Important tool since it permits the assessment of the amount of enclosed charge by mapping the **electric field normal** to the surface outside the charge distribution or vice versa

7. Poisson & Laplace Equations

$$\iiint \rho \, dV = q \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{E} = -grad\, V = -\vec{\nabla} V$$

Where ρ is the electric charge density in the closed volume. And V is the electric potential

Poisson Equations

(a region with charge)

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

Laplace Equations

(a region free of charge)

$$\nabla^2 V = 0$$

Lecture 3 we learned!

8. Continuity Equation

$$I = \frac{dq}{dt} \qquad q = \iiint \rho \, dV \quad \text{and} \quad I = \iint \vec{J} \cdot \overrightarrow{dS}$$

Where I is the current and J is the current density

$$\oiiint \vec{\nabla} \cdot \vec{J} \, dV = - \oiiint \frac{d\rho}{dt} \, dV$$

$$\vec{\nabla} \cdot \vec{J} + \frac{d\rho}{dt} = 0$$

Current density flowing out of the closed volume is equal to the rate of decrease of charge within that volume.

In lecture 4 we learned!

9. Gauss's law of magnetostatic (Gauss's 2nd law)

asserts that the net magnetic flux through any closed Gaussian surface is zero.

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS} = 0$$

Magnetic monopoles do not exist

10. Faraday's law of induction:

Whenever the magnetic flux (ϕ_B) linked with a circuit changes an emf (**E_{emf}**) is induced in the circuit. The induced emf, **E_{emf}** is equal to the negative rate of change of magnetic flux (ϕ_B) with time linked with the circuit.

$$E_{emf} = -\frac{\partial \phi_B}{\partial t} \quad \textit{or} \quad \oint \vec{E} \cdot \overrightarrow{dl} = -\frac{\partial \phi_B}{\partial t}$$

In lecture 4 we learned!

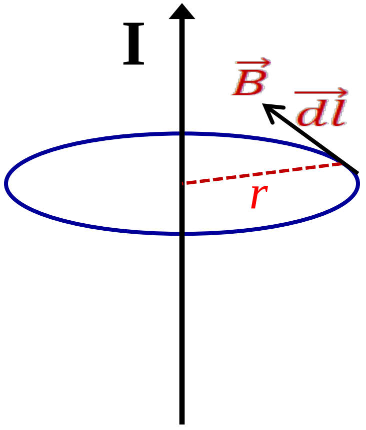
11. Ampere circuital law: The line integral of the magnetic field (**B**) around any closed loop is equal to $\boldsymbol{\mu_0}$ (***permeability of the free space***) times the net current (**I**) flowing through the area enclosed by the loop.

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I$$

12. Proved Ampere circuital law: By using Biot-Savart law

By **Biot-Savart law** the magnitude of the magnetic field **B** at a point r from the conductor carrying a current **I** is given by

$$B = \frac{\mu_0}{4\pi} \frac{2I}{r}$$



In lecture 4 we learned!

13. Maxwell's law of induction

Concept of **displacement current** due to the change/discharge of a capacitor leads to the correction/modification to the Ampere's law.

$$\oint \vec{\mathrm{B}} \cdot \overrightarrow{dl} = \mu_0 \varepsilon_0 \frac{\partial \phi_{\mathrm{E}}}{\partial t}$$

14. Correction to Ampere circuital law

Concept of **displacement current** due to the charge/discharge of a capacitor leads to the correction/modification to the Ampere's law.

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} \right) = \mu_0 (I + I_d)$$

Ampere-Maxwell law

Refer R-3 FUNDAMENTALS OF PHYSICS HALLIDAY. RESNICK, WALKER Chapter 8 278-281

Maxwell's equations of electromagnetism

- All relationships between electric & magnetic fields & their sources summarized by four equations.

- James Clerk Maxwell

13 June 1831 – 5 November 1879

Scottish scientist in the field of mathematical physics

LECTURE 5 Basic laws of ELECTROMAGNETISM-
MAXWELL'S EQUATIONS

Four Maxwell's equations in differential form in S.I. units are

$$\vec{\nabla} \cdot \vec{D} = \rho \quad \text{------------------------>} \; \textbf{Eq.1}$$

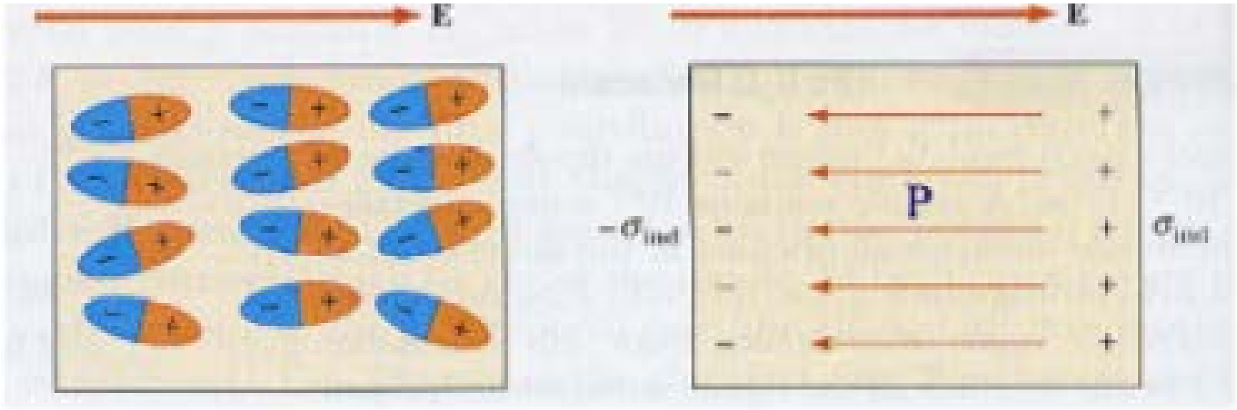
$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{------------------------>} \; \textbf{Eq.2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{------------------------>} \; \textbf{Eq.3}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{------------------------>} \; \textbf{Eq.4}$$

We knew the meaning of **E** and **B**, which are related to the Electric flux (Φ_E) and magnetic flux (Φ_B) , respectively. And now we will know how **D** and **H** are respectively, related to **E** and **B**.

LECTURE 5 THE FIELDS $\vec{E}$ and $\vec{P}$



Dielectric materials, which have no free electrons, can be polarized (internal dipole moments are induced) by an external electric field, **E** resulting in a polarization field, **P** (the sum of all the induced dipoles). At internal locations, the positive and negative charges cancel leaving a net positive surface charge on the right and net negative surface charge on the left. The resultant polarization field, which points in the opposite direction to **E**, is normalized by the volume of the material so that **P** has the units of electric dipole-moment per unit volume

LECTURE 5 THE FIELDS $\vec{E}$ and $\vec{P}$

When the applied electric field, $\vec{E}$ is weak and the medium is *isotropic*, $\vec{P}$ depends linearly on $\vec{E}$ and are parallel. Thus, we can write

$$\vec{P} = \chi_E(\vec{E})\vec{E}$$

Where, $\chi_E(\vec{E})$ is a scalar, in this case, and is called the electric susceptibility

Then total field is given by adding $\vec{P}$ to the applied field $\vec{E}$. However, we cannot simply $\vec{E}$ and $\vec{P}$ because they do not have the same units in the SI system of units.

LECTURE 5 THE FIELDS $\vec{E}$, $\vec{P}$, $\vec{D}$ and ε_r

The polarization has units of C-m/m$^3 \equiv$ C/m^2, which means χ_E must have units of C^2/N-m^2. Since ε_0, the vacuum permittivity, also has units of C^2/N-m^2, we multiply **E** by ε_0 and add the result to **P** to give a **new quantity** that bears the name the **electric displacement**,

$$\vec{D} = \varepsilon_o\vec{E} + \vec{P} = \varepsilon_o\vec{E} + \chi_E\vec{E} = (\varepsilon_o + \chi_E)\vec{E} = \varepsilon\vec{E}$$

Where ε is the permittivity of the dielectric medium and is defined as

$$\varepsilon = \varepsilon_0\varepsilon_r$$

This ε_r is the relative permittivity or the **Dielectric constant** of the dielectric medium

Lecture 5 THE FIELDS $\vec{B}$ AND $\vec{H}$

As in the electric case, we have two fields in the magnetic case $\vec{B}$ and $\vec{H}$. The quantity $\vec{H}$ plays the role of $\vec{D}$ for the magnetic case and is related to $\vec{B}$ through the relation

$$\vec{H} = \frac{\vec{B}}{\mu}$$

Where μ is the permeability of the medium

TABLE I: The names and units of the six electromagnetic fields: $\vec{E}$, $\vec{D}$, $\vec{P}$, $\vec{B}$ $\vec{H}$ and $\vec{M}$.

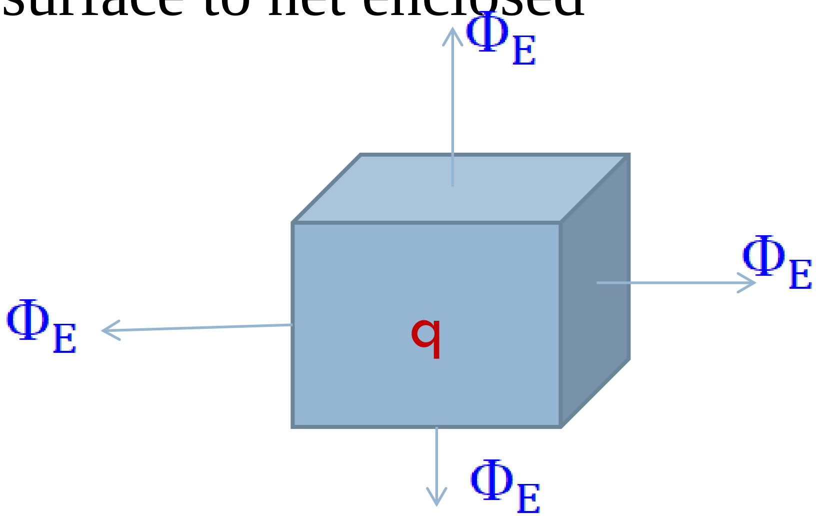
Symbol	Name	Units
$\vec{E}$	Electric Field	$\mathrm{V/m = N/C}$
$\vec{P}$	Polarization	C/m^2
$\vec{D}$	Electric Displacement	$\mathrm{C/m^2}$
$\vec{B}$	Magnetic Induction	N/A-m
$\vec{M}$	Magnetization	A/m
$\vec{H}$	Magnetic Intensity	A/m

LECTURE 5 ELECTROMAGNETISM–MAXWELL'S EQUATIONS

Now we have the necessary background for deriving the Maxwell's equations ☺

1. Derivation of Maxwell's First Equation

Let us consider the Gauss's law for Electrostatics, which relate the net electric flux Φ_E through a Gaussian surface to net enclosed electric charge;



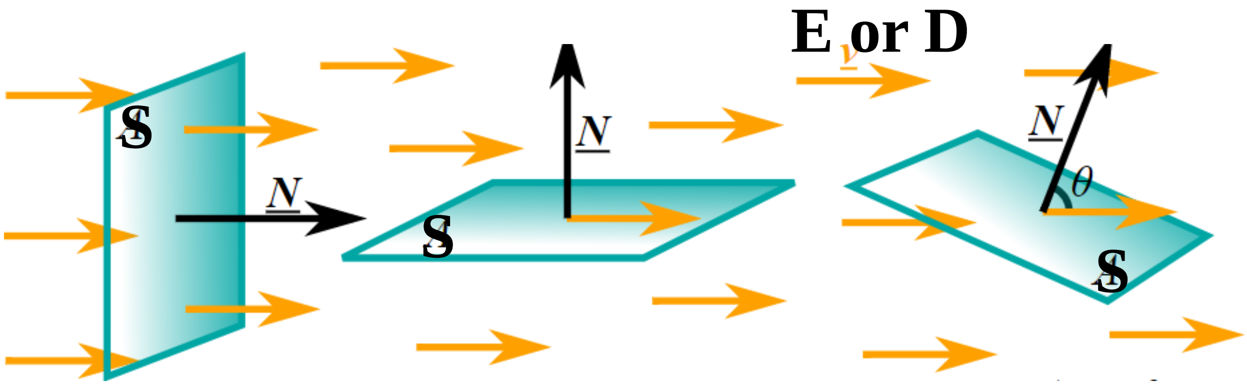
$$\Phi_E = \oiint \vec{E} \cdot \overrightarrow{dS} = \frac{q}{\varepsilon_0}$$

$$\oiint \varepsilon_0 \vec{E} \cdot \overrightarrow{dS} = q$$

But and $\vec{D} = \varepsilon \vec{E}$ and $q = \iiint \rho \, dV$

ELECTRIC FLUX

$$\Phi_E = \vec{E} \cdot \vec{S} = ES\cos\theta$$

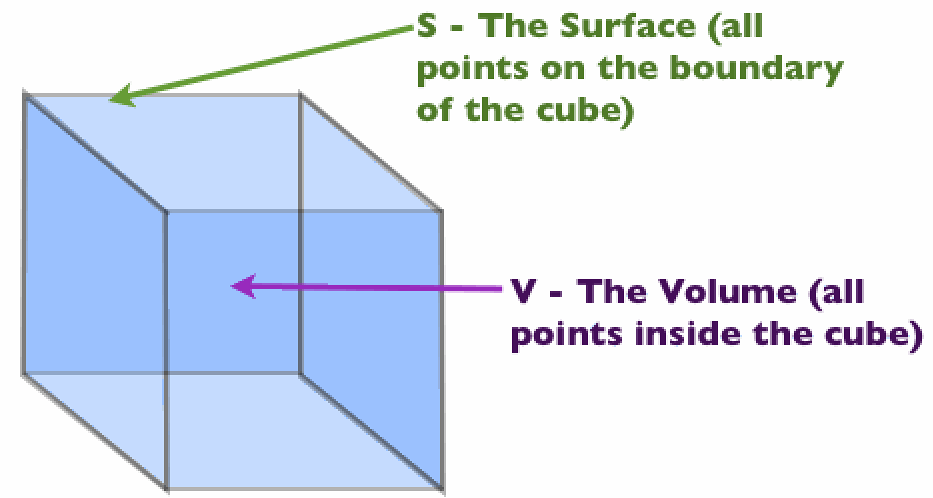


$\Phi_E = ES$ $\Phi_E = 0$ $\Phi_E = ES\cos\theta$

That was just to have the feeling about the ‘planes’ which I was talking about!

LECTURE 5 ELECTROMAGNETISM–MAXWELL'S EQUATIONS

$$\oiint \vec{D} \cdot \overrightarrow{dS} = \iiint \rho \, dV$$



Now apply Gauss's divergence theorem on the LHS..

$$\iiint \vec{\nabla} \cdot \vec{D} \, dV = \iiint \rho \, dV$$

This equation hold true for any arbitrary volume and for that, the integrands must be same. So we have now

$$\vec{\nabla} \cdot \vec{D} = \rho$$

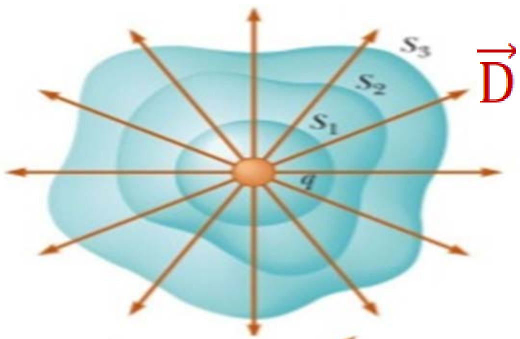
This is the Maxwell's FIRST EQUATION

Or

$$\text{div}\, \vec{D} = \rho$$

LECTURE 5 ELECTROMAGNETISM-MAXWELL'S EQUATIONS

Charge enclosed by a region



$\vec{D}$

$\vec{\nabla} \cdot \vec{D} = \rho$

ρ Positive means? **D** diverge (source)

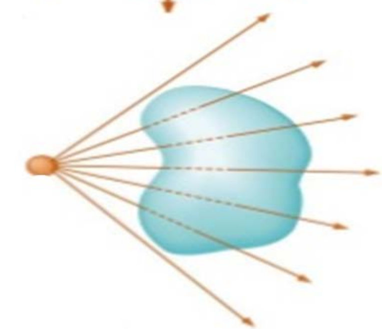
ρ Negative means? **D** Converge (sink)

ρ=0, **D** called solenoidal

1 $\vec{\nabla} \cdot \vec{D} = \rho$

Electric field have a source or sink and can also be solenoidal vector field.

q



$\vec{D}$

$\vec{\nabla} \cdot \vec{D} = 0$

Charge free region

Electric monopoles exist

LECTURE 6

LECTURE 6

$$F = f(x, y, z)$$

$$\vec{A} = (\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_y)$$

$$\vec{\nabla}F \qquad \vec{\nabla}\bullet\vec{A} \qquad \vec{\nabla}\times\vec{A}$$

$$\underset{V}{\oiiint} \vec{\nabla}\cdot\vec{A}\,dV = \underset{S}{\oiint} \vec{A}\cdot\overrightarrow{dS}$$

$$\iint (\vec{\nabla}\times\vec{A})\cdot\overrightarrow{dS} = \oint \vec{A}\cdot\overrightarrow{dl}$$

$$\oiint \vec{E}\cdot\overrightarrow{dS} = \frac{q}{\varepsilon_0} = \Phi_E$$

$$\iiint \rho\,dV = q$$

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\nabla^2 V = 0$$

$$\vec{\nabla}\cdot\vec{J} + \frac{d\rho}{dt} = 0$$

$$\Phi_B = \oiint \vec{B}\cdot\overrightarrow{dS} = 0$$

$$\oint \vec{E}\cdot\overrightarrow{dl} = -\frac{\partial\phi_B}{\partial t}$$

$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0 I$$

$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0\varepsilon_0\frac{\partial\phi_E}{\partial t}$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\varepsilon = \varepsilon_0\varepsilon_r$$

$$\oint \vec{B}\cdot\overrightarrow{dl} = \mu_0\left(I + \epsilon_0\frac{d\Phi_E}{dt}\right) = \mu_0(I + I_d)$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\vec{\nabla}\cdot\vec{D} = \rho$$

2. Derivation of Maxwell's 2nd Equation

Like electric flux $\mathbf{\Phi_E}$, magnetic flux $\mathbf{\Phi_B}$ is defined as

$$\Phi_B = \oiint \vec{B} \cdot \overrightarrow{dS}$$

But Gauss's law for Magnetostatics say $\mathbf{\Phi_B}$ =0, flux of magnetic field B across any closed **surface** is zero

So, we have $\oiint \vec{B} \cdot \overrightarrow{dS} = 0$

Now apply Gauss's divergence theorem on the LHS..

$$\iiint \vec{\nabla} \cdot \vec{B} \, dV = 0$$

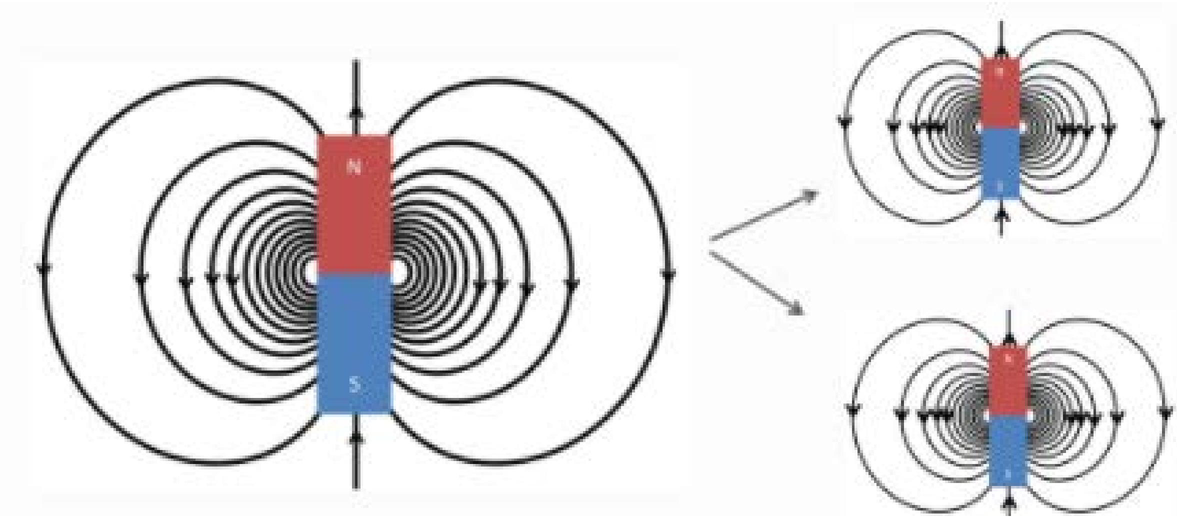
LECTURE 6 ELECTROMAGNETISM-MAXWELL'S EQUATIONS

above equation hold true for any arbitrary volume and for that, the integrands must be zero. So we have now

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \textit{or} \qquad \operatorname{div} \vec{B} = 0$$

This is the Maxwell's SECOND EQUATION

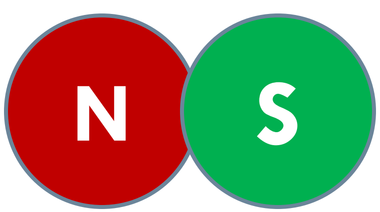
The **magnetic line of force** are either closed or go off to infinity, the number of magnetic lines entering any **volume** is exactly equal to the number of lines leaving volume.



2

$$\vec{\nabla} \cdot \vec{B} = 0$$

NON-EXISTANCE OF MAGETIC MONOPOLE. **Lowest unit is 'dipole'.** Magnetic fields have no source or sink but it is always a **solenoidal vector field**.



Magnetic dipoles exist but monopoles do not exist

3. Derivation of Maxwell's 3rd Equation

Let us consider the Faraday's law for induction, which relates the induced electric field to changing magnetic flux;

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \phi_B}{\partial t}$$

But $\phi_B = \oiint \vec{B} \cdot \vec{ds}$

then RHS can be written as

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \oiint \vec{B} \cdot \vec{ds}$$

Upon re-arranging

LECTURE 6 ELECTROMAGNETISM-MAXWELL'S EQUATIONS

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\oiint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

Now we can apply Stokes' theorem to LHS, and then line integral changes to surface integral as

$$\oiint (\vec{\nabla} \times \vec{E}) \cdot \overrightarrow{dS} = -\oiint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

Re-arranging

$$\oiint (\vec{\nabla} \times \vec{E}) \cdot \overrightarrow{dS} = \oiint \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot \overrightarrow{dS}$$

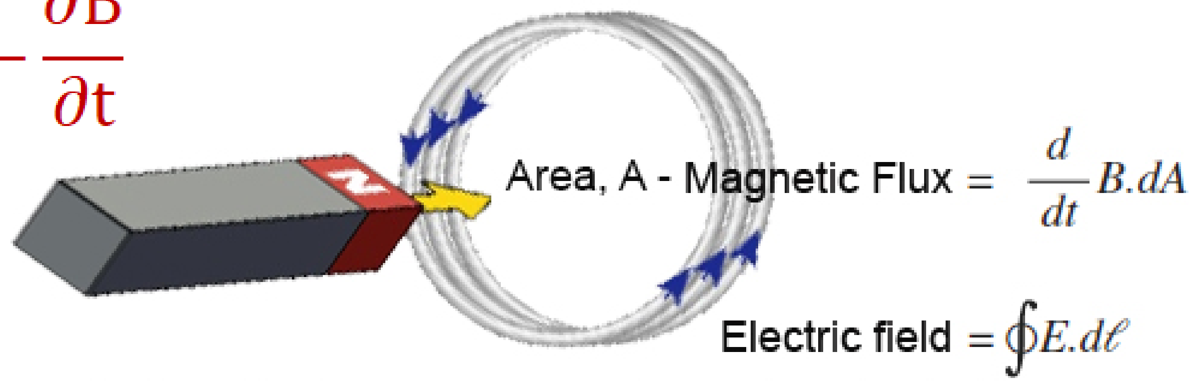
i.e. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ or $\text{curl}\, \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

This is the Maxwell's THIRD EQUATION

LECTURE 6 ELECTROMAGNETISM–MAXWELL'S EQUATIONS

3

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



This relation may look odd. But as many people point out that - this is the formula that runs the entire economy. Because this is the basic formula that describe how power station works. When steam drive a turbine, the turbine that has effectively coil of wire will spin in a magnetic field. It doesnt matter whether the magnet is moving or the coil wire is moving.

A moving coil in the presence of stationary magnetic field or the other way around will cause a current to flow. And that is how the electricity is generated & the formula that we just derived here is the one that governs the generating of electricity. ----- That is Maxwell 3rd Law!

4. Derivation of Maxwell's 4th Equation

Let us consider the Ampere-Maxwell law, which relates the induced magnetic field to changing electric flux and current

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} \right)$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt}$$

But…

$$I = \oiint \vec{J} \cdot \overrightarrow{dS}$$

$$\Phi_{\mathrm{E}} = \oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS}$$

Substituting for I and Φ_{E}

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \oiint \vec{J} \cdot \overrightarrow{dS} + \mu_0 \epsilon_0 \frac{d}{dt} \oiint \vec{\mathrm{E}} \cdot \overrightarrow{dS}$$

LECTURE 6 ELECTROMAGNETISM-MAXWELL'S EQUATIONS

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \oiint \vec{J} \cdot \overrightarrow{dS} + \mu_0 \frac{d}{dt} \oiint \varepsilon_0 \vec{\mathrm{E}} \cdot \overrightarrow{dS}$$

$$\vec{\mathrm{D}} = \varepsilon \vec{\mathrm{E}}$$

$$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \oiint \vec{J} \cdot \overrightarrow{dS} + \mu_0 \frac{d}{dt} \oiint \vec{D} \cdot \overrightarrow{dS}$$

$$\oint \frac{\vec{B}}{\mu_0} \cdot \overrightarrow{dl} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

$$\vec{\mathrm{H}} = \frac{\vec{\mathrm{B}}}{\mu}$$

$$\oint \vec{\mathrm{H}} \cdot \overrightarrow{\mathrm{dl}} = \oiint (\vec{\mathrm{J}} + \frac{\mathrm{d}\vec{\mathrm{D}}}{\mathrm{dt}}) \cdot \overrightarrow{\mathrm{dS}}$$

Now we can apply Stokes' theorem to LHS, and then line integral changes to surface integral as

$$\oiint (\nabla \times \vec{H}) \cdot \overrightarrow{dS} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

or

$$\text{Curl}\ \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$$

This is the Maxwell's **FOURTH EQUATION**

Second term on the RHS is called Maxwell's correction and is known as displacement current density, this along with Ampere's law is responsible for the EM fields..

MAXWELL'S EQUATIONS : In integral forms from differential Equations

	Differential forms	Integral forms
Eq.1	$\vec{\nabla} \cdot \vec{D} = \rho$	$\oiint \vec{D} \cdot \overrightarrow{dS} = q$
Eq.2	$\vec{\nabla} \cdot \vec{B} = 0$	$\oiint \vec{B} \cdot \overrightarrow{dS} = 0$
Eq.3	$\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot \overrightarrow{dl} = -\oiint \dfrac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$
Eq.4	$\vec{\nabla} \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$	$\oint \vec{H} \cdot \overrightarrow{dl} = \oiint (\vec{J} + \dfrac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$

MAXWELL'S 1st INTEGRAL EQUATION from differential Equation

Let us start with the first Maxwell's differential Equation $\vec{\nabla} \cdot \vec{D} = \rho$

Now integrate this equation over the volume V,

$$\iiint \vec{\nabla} \cdot \vec{D}\, dV = \iiint \rho\, dV$$

Now use divergence theorem on LHS and use the relation between charge density **ρ** and charge **q** on RHS $\iiint \rho\, dV = q$

1 $$\iiint \vec{\nabla} \cdot \vec{D}\, dV = q$$

Total electric displacement, **D** through the surface, **S** which define the volume, **V** is equal to the total charge contained in the volume

MAXWELL'S 2nd INTEGRAL EQUATION from differential Equation

Differential form of Maxwell's 2nd equation is $\vec{\nabla} \cdot \vec{B} = 0$

Now integrate this equation over the volume V, like before and we get

$$\iiint \vec{\nabla} \cdot \vec{B} \, dV = 0$$

Now use divergence theorem on LHS and the above equation becomes

2

$$\oiint \vec{B} \cdot \overrightarrow{dS} = 0$$

Total magnetic field, **B** through the surface, **S** which define the volume, **V** is equal to zero

MAXWELL'S 3rd INTEGRAL EQUATION from differential Equation

Differential form of Maxwell's 3rd equation is $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Here integrate over the surface, **S** bounded by the closed path

$$\oiint (\vec{\nabla} \times \vec{E}) \cdot \overrightarrow{dS} = -\oiint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{dS}$$

Now use Stoke's theorem on LHS and the above equation becomes

3

$$\oint \vec{E} \cdot \overrightarrow{dl} = -\frac{\partial}{\partial t} \oiint \vec{B} \cdot \overrightarrow{dS}$$

This means the **electromotive force** around a closed path is equal to the time derivative of the magnetic field through any closed surface bounded by that path.

MAXWELL'S 4th INTEGRAL EQUATION from differential Equation

Differential form of Maxwell's 4th equation is $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Here integrate over the surface, **S** bounded by the closed path

$$\oiint (\nabla \times \vec{H}) \cdot \overrightarrow{dS} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

Now use Stoke's theorem on LHS and the above equation becomes

4

$$\oint \vec{H} \cdot \overrightarrow{dl} = \oiint (\vec{J} + \frac{d\vec{D}}{dt}) \cdot \overrightarrow{dS}$$

Which says that **magnetomotive force** around a closed path is equal to the conventional conduction current plus displacement current through any surface bounded by that path.

LECTURE 7

For today… last one on EM theory

- *Wave equation and electromagnetic waves*
- *Physical significance of Maxwell Equations*
- *Electromagnetic spectrum*

WAVE EQUATION

The function u(x, t) is a solution to the classical one dimensional wave equation if it satisfies the Partial Differential Equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \qquad \text{Equation 1}$$

The wave function **u** is the amplitude of the wave as a function of time and position. The constant **v** is the wave's velocity in the x direction.

Let us now see whether we can get an equation like above for Electric field (E) and magnetic field (H)!!

ELECTROMAGNETIC WAVES

Maxwell's equations

In a medium with source

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

In vacuum without source

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{d\vec{D}}{dt}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$$

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{H} = \frac{\vec{B}}{\mu}$$

ELECTROMAGNETIC WAVES

Maxwell's third equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Take the curl of the above eqn. we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(-\frac{\partial \vec{B}}{\partial t})$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

Using this identity for triple cross product, we get

$$(\vec{\nabla} \cdot \vec{E})\vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla})\vec{E} = \vec{\nabla} \times (-\frac{\partial \vec{B}}{\partial t})$$

$$0 - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} \vec{\nabla} \times \vec{B}$$

$$\nabla^2 \vec{E} = \frac{\partial}{\partial t}\left[\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right] \Rightarrow \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Equation 2

ELECTROMAGNETIC WAVES

Maxwell's fourth equation $\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$

Take the curl of the above eqn. we get $\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times (\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

Using this identity for triple cross product, we get

$$(\vec{\nabla}.\vec{B})\vec{\nabla} - (\vec{\nabla}.\vec{\nabla})\vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}$$

$$-\nabla^2\vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(-\frac{\partial B}{\partial t})$$

$$\nabla^2\vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$ Equation 3

ELECTROMAGNETIC WAVE Equations

Eq.1

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

Eq.2

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Eq.3

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

Equation 2 and 3 are same as the wave equation 1..

The waves sent out by the oscillating charges are fluctuating electric and magnetic fields, so they are called electromagnetic waves

Meaning electric and magnetic field vectors are traveling with speed $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\frac{1}{v^2} = \mu_0 \varepsilon_0$$

Speed of the electromagnetic waves

Now we know the relationship for the speed of electromagnetic waves

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Where ε_0 and μ_0 are the permittivity and permeability of vacuum

But ε_0 = 8.85 x 10^{-12}F/m and μ_0 =1.26 x 10^{-6} H/m, also H=s^2/F

i.e v =1/(8.85 x 10^{-12}F/m x 1.26 x 10^{-6} s^2/F m)$^{1/2}$

= 3x10^8 m/s= c (speed of light)

Hence electromagnetic waves propagate in free space with the speed of light. This led scientists to conclude (correctly) that light is an electromagnetic wave.

Physical significance of Maxwell Equations

1) Maxwell's first equation $\vec{\nabla} \cdot \vec{D} = \rho$ or $\iiint \vec{\nabla} \cdot \vec{D}\, dV = q$ states electric displacement flux through any closed surface is equal to the total charge enclosed by the surface.

Electric field lines originate on positive charges and terminate on negative charges

2) Maxwell's second equation $\vec{\nabla} \cdot \vec{B} = 0$ or $\oiint \vec{B} \cdot \vec{dS} = 0$ states net magnetic flux through any closed surface is zero. Since a magnetic monopole does not exist, any closed volume always contains equal and opposite magnetic poles (north and south poles), resulting in the zero net magnetic pole strength. It also signifies that magnetic line of flux are continuous i.e., the number of magnetic lines of flux entering into a region is equal to the lines of flux leaving it.

Magnetic field lines always form closed loops—they don't begin or end anywhere.

Physical significance of Maxwell Equations

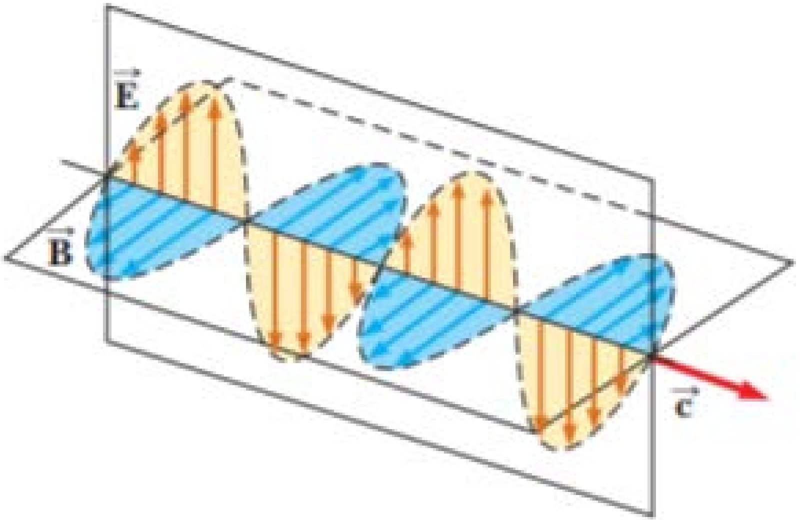
3) Maxwell's third equations $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ or $\oint \vec{E}\cdot\vec{dl} = -\oiint \frac{\partial\vec{B}}{\partial t}\cdot\vec{dS}$ states that the induced electromotive force around any closed surface is equal to the negative time rate of change of magnetic flux through the path enclosing the surface. This signifies that an electric field can also be produced by a changing magnetic flux.

4) Maxwell's fourth equation $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$ or $\oint \vec{H}\cdot\vec{dl} = \oiint (\vec{J} + \frac{d\vec{D}}{dt})\cdot\vec{dS}$ states that magnetomotive force around any closed path is equal to the sum of conduction current and the displacement current through the surface bounded by that path. This signifies that a conduction current or a changing electric flux produces a magnetic field.

5) In addition to unifying the formerly separate fields of electricity and magnetism, Maxwell's electromagnetic theory predicted that electric and magnetic fields can move through space as waves with the speed of light. So light is nothing but an electromagnetic wave.

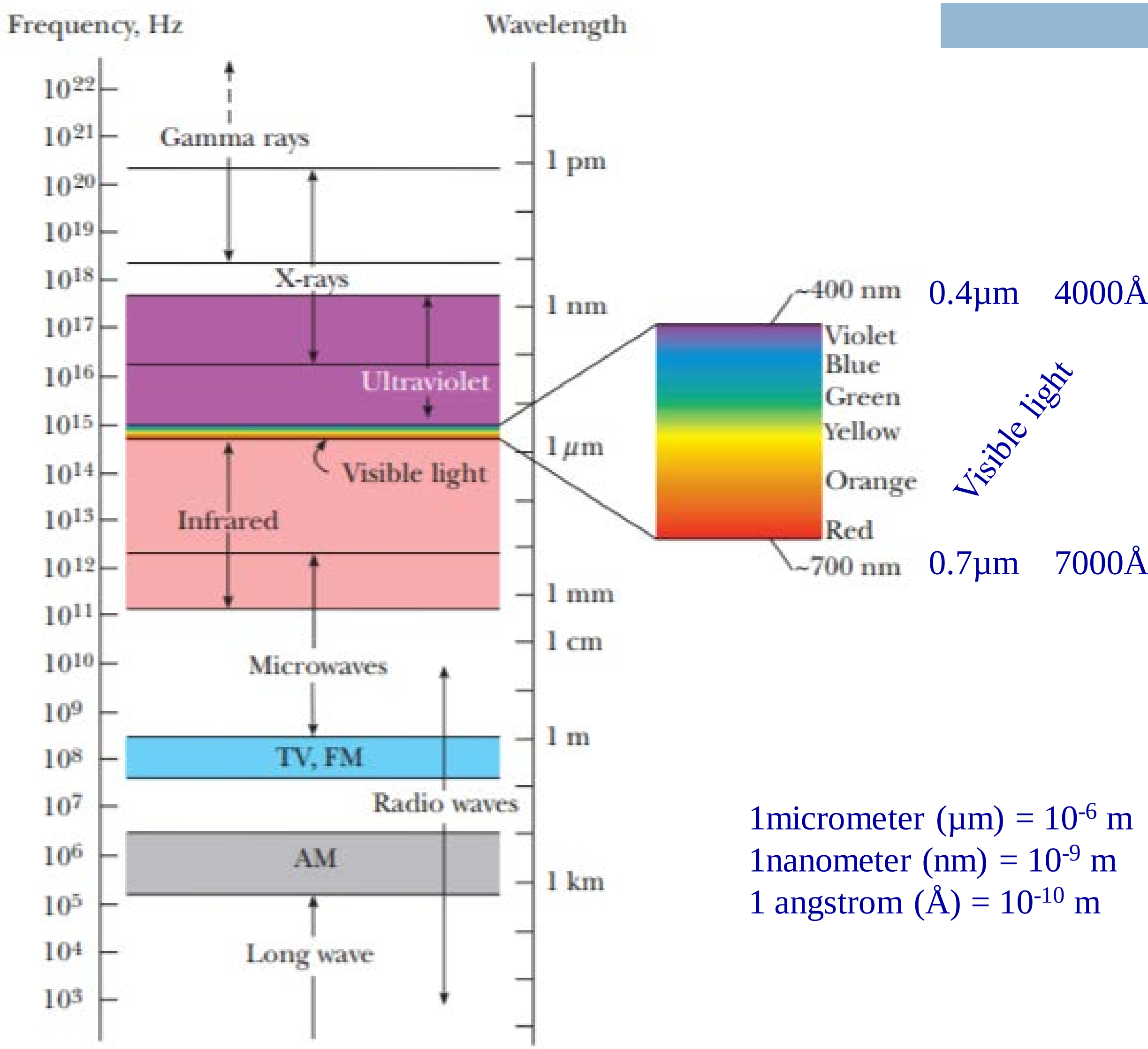
Physical significance of Maxwell Equations

Maxwell’s equations and the prediction of electromagnetic waves were truly one of the greatest discoveries of science, on a par with Newton’s discovery of the laws of motion. Like Newton’s laws, it had a profound influence on later scientific developments.



Electromagnetic Spectrum: An orderly distribution of electromagnetic waves according to their frequency (ν) or wavelength(λ)

$$c = \nu \lambda$$



1micrometer (μm) = 10^{-6} m
1nanometer (nm) = 10^{-9} m
1 angstrom (Å) = 10^{-10} m

Electromagnetic Spectrum

All forms of electromagnetic radiation are produced by accelerating charges.

Radio waves: are the result of charges accelerating through conducting wires. They are, of course, used in radio and television communication systems.

Microwaves (short-wavelength radio waves): have wavelengths ranging between about **1 mm and 30 cm** and are generated by electronic devices like klystron, magnetron etc. Their short wavelengths make them well suited for the radar systems used in aircraft navigation and for the study of atomic and molecular properties of matter. Microwave ovens are an interesting domestic application of these waves.

Infrared waves (Heat waves): produced by hot objects and molecules, have wavelengths ranging from about 1 mm to 0.7μm. Infrared radiation has many practical and scientific applications, including physical therapy, infrared photography, and the study of the vibrations of atoms.

Electromagnetic Spectrum

Visible light: the most familiar form of electromagnetic waves, may be defined as the part of the spectrum that is detected by the human eye. Light is produced by the rearrangement of electrons in atoms and molecules. The wavelengths of visible light are classified as colors ranging from **violet (0.4 µm) to red (0.7µ m**). The eye's sensitivity is a function of wavelength and is greatest at a wavelength of about 0.56 µm (yellow green).

Ultraviolet (UV) light: covers wavelengths ranging from about **400 nm to 0.6 nm**. The Sun is an important source of ultraviolet light. Most of the ultraviolet light from the Sun is absorbed by atoms in the upper atmosphere, or stratosphere. This is fortunate, because UV light in large quantities has harmful effects on humans. One important constituent of the stratosphere is ozone (O3), produced from reactions of oxygen with ultraviolet radiation. The resulting ozone shield causes lethal high energy ultraviolet radiation to warm the stratosphere

Electromagnetic Spectrum

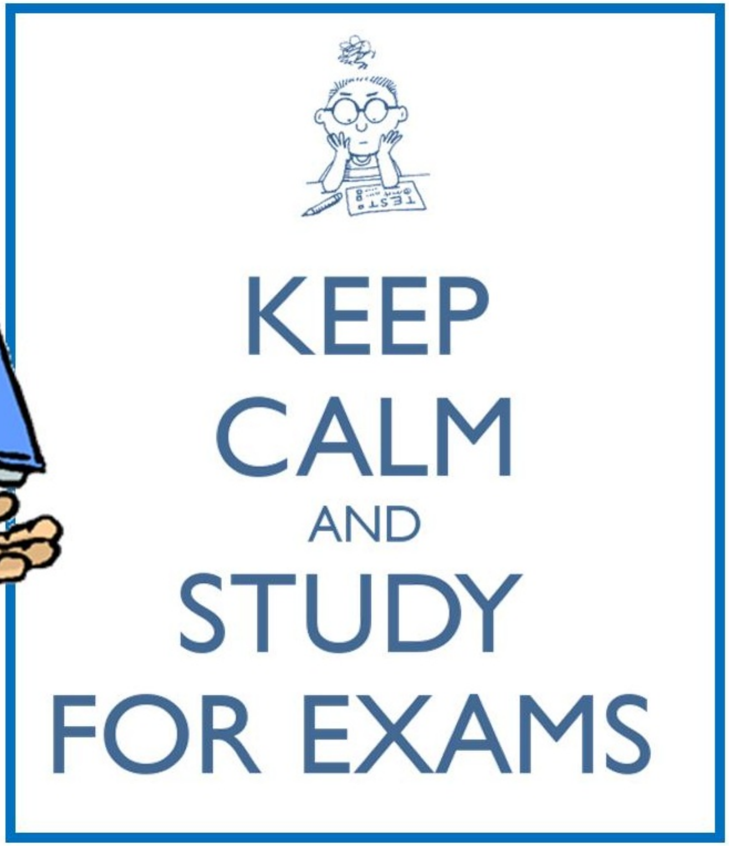
X-rays: are electromagnetic waves with wavelengths from about **10 nm to 10^{-4} nm**. The most common source of x-rays is the acceleration of high-energy electrons bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays easily penetrate and damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure and overexposure.

Gamma rays: electromagnetic waves emitted by radioactive nuclei—have wavelengths ranging from about **10^{-10} m to less than 10^{-14} m**. They are highly penetrating and cause serious damage when absorbed by living tissues. Accordingly, those working near such radiation must be protected by garments containing heavily absorbing materials, such as layers of lead.

UNIT-1 Electromagnetic theory- SYLLABUS

scalar and vectors fields, concept of gradient, divergence and curl, dielectric constant, Gauss theorem and Stokes theorem (qualitative), Poisson and Laplace equations, continuity equation, Maxwell electromagnetic equations (differential and integral forms), physical significance of Maxwell equations, Ampere Circuital Law, Maxwell displacement current and correction in Ampere Circuital Law

"Hope we overcame the impedance mismatch and going to have an efficient knowledge transfer here after"



Also hope that I "**uncovered**" the syllabus on EM theory for PHY109☺ and **THANK YOU FOR LISTENING ME!**

PHY109 – ENGINEERING PHYSICS

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