# Exponential and Gamma distribution

# Gamma Distribution

**Gamma Distribution** The continuous random variable $X$ has a **gamma distribution**, with parameters $\alpha$ and $\beta$, if its density function is given by

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$.

The **gamma function** is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx, \qquad \text{for } \alpha > 0.$$

$\Gamma(n) = (n-1)!$ for a positive integer $n$.

$\Gamma(1) = 1$. $\qquad \Gamma(1/2) = \sqrt{\pi}$.

# Mean, Variance and mgf of Gamma Distribution

The mean and variance of the gamma distribution are

$$\mu = \alpha\beta \text{ and } \sigma^2 = \alpha\beta^2.$$

The moment generating function of gamma distribution is

$M_X(t) = (1 - t\beta)^{-\alpha}$

# Exponential distribution

**Exponential Distribution** The continuous random variable $X$ has an **exponential distribution**, with parameter $\beta$, if its density function is given by

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0, \\ 0, & \text{elsewhere}, \end{cases}$$

where $\beta > 0$.

# Mean, variance and mgf of exponential distribution

The mean and variance of the exponential distribution are

$$\mu = \beta \text{ and } \sigma^2 = \beta^2.$$

The moment generating function of exponential distribution is

$M_X(t) = (1 - t\beta)^{-1}$

Q: Suppose that telephone calls arriving at a particular switchboard follow a Poisson process with an average of 5 calls coming per minute. What is the probability that up to a minute will elapse by the time (a) 1 call have come in to the switchboard? (b) 3 calls have come in to the switchboard?

**6.41** If a random variable $X$ has the gamma distribution with $\alpha = 2$ and $\beta = 1$, find $P(1.8 < X < 2.4)$.

**6.42** Suppose that the time, in hours, required to repair a heat pump is a random variable $X$ having a gamma distribution with parameters $\alpha = 2$ and $\beta = 1/2$. What is the probability that on the next service call

(a) at most 1 hour will be required to repair the heat pump?

(b) at least 2 hours will be required to repair the heat pump?

**6.45** The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

**6.44** In a certain city, the daily consumption of electric power, in millions of kilowatt-hours, is a random variable *X* having a gamma distribution with mean $\mu$ = 6 and variance $\sigma^2$= 12.

(a) Find the values of $\alpha$ and $\beta$.

(b) Find the probability that on any given day the daily power consumption will exceed 12 million kilowatthours.

# Central limit theorem

**Central Limit Theorem:** If $X$ is the mean of a random sample of size $n$ taken from a population with mean $\mu$ and finite variance $\sigma^2$, then the limiting form of the distribution of

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}},$$

as $n \to \infty$, is the standard normal distribution $n(z; 0, 1)$.

Q1. The lifetime of certain brand of an electric bulb may be considered a RV with mean 1200 h and standard deviation 250 h. Find the probability using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250 h.

Q2. If $X_1, X_2, \dots, X_n$ are independent Poisson variates with parameter 2, if $\mathrm{S_n} = X_1 + X_2 + \cdots + X_n$ and $n$ = 75 then Use Central Limit Theorem to estimate $P(120 \le S_n \le 160)$.

Q3. The guaranteed average life of a certain type of electric light bulb is 1000 h with a standard deviation of 125 h. It is decided to sample the output so as to ensure that 90% of the bulbs do not fall short of guaranteed average by more than 2.5%. Use CLT to find the minimum sample size.