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Saturday, January 8, 2022 12:54 PM

Zero lecture

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MTH302

Probability and Statistics

Lecture #0

Course details

- LTP – 3 0 0 [Three lectures/week]
- **Text Book**

FUNDAMENTALS OF MATHEMATICAL STATISTICS by S.C.GUPTA AND V.K.KAPOOR, SULTAN CHAND & SONS, 2nd Edition, (2007)

- **Reference Books**

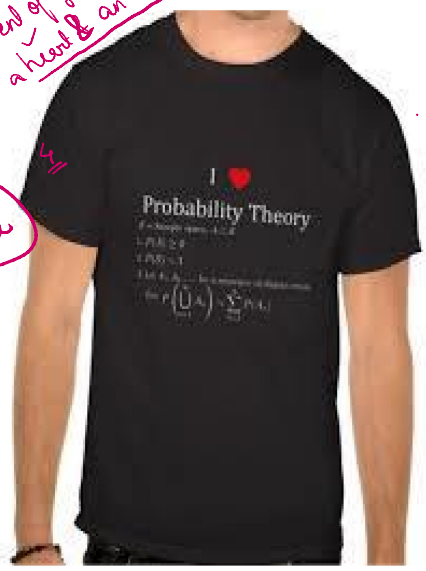
PROBABILITY STATISTICS AND RANDAM PROCESSES by T VEERARAJAN, MC GRAW HILL, 3rd Edition, (2011)

Course Assessment Model

- **Marks break up***
- Attendance — 5
- CA (Two best out of three tasks) — 25
- MTE — 20
- ETE — 50
- **Total** — **100**

Introduction

- A branch of mathematics concerned with the study of randomness and uncertainity.



52 cards

13 13 13 13

Spade hearts clubs Diamonds.

Ace 6
2 7 Jack
3 8 king
4 9 queen
5 10

1, 2, 3, 4, 5, 6

S = { ... 52 }

$E \subseteq S$

WHAT IS PROBABILITY?

- It is a measure of how often a particular event will happen if something is done repeatedly.
- If an event is certain to happen then its probability is 1.
- If an event is not certain to happen then its probability is 0.
- Probability is always between 0 and 1.

dice

E = 1 to 6

P(E) = 1

E = getting 7

P(E) = 0

R.E.

P(S) = 1

$0 \le P(E) \le 1$

$P \in [0, 1]$

null sure.

S = {H, T}

S = {HH, HT, TH, TT}

S = {1, 2, 3, 4, 5, 6}

Examples:-

- If you draw a card from a standard deck of cards, what is the probability of *not* drawing a spade?
- In a certain population, 10% of the people are rich, 5% are famous, and 3% are both rich and famous. A person is randomly selected from this population. What is the chance that the person is

52

13 $\frac{39}{52}$ $\frac{3}{4}$ 0.75

10 %

100 %

75 %

90 %.

n(R) = 10

n(F) = 5

$n(R \cap F) = 3$

famous, and 3% are both rich and famous. A person is randomly selected from this population. What is the chance that the person is

- not rich?
- rich but not famous?
- either rich or famous?

$n(R \cap F) = 3$

$n(R) - n(R \cap F)$

$10 - 3$

$= 7\%$

$10 + 5 - 3$

12

The course contents

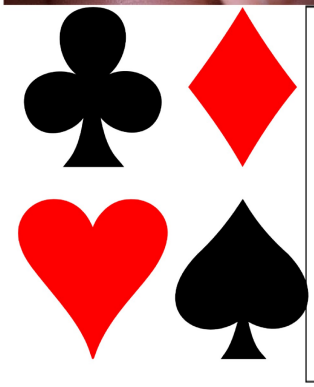
- Unit 1 : Basics of probability

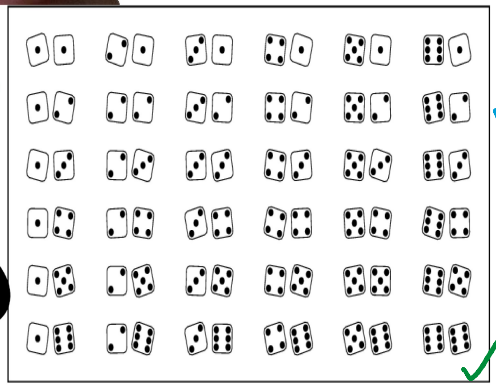
- Sample Space and Events
- Probability of an Event
- Rules of probability
- Independent events
- Conditional Probability

$P(A \setminus B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A)P(B)}{P(B)}$

$= P(A)$





$P(A \cap B) = P(A)\,P(B)$

$P(A \setminus B) = P(A)$

$P(B \setminus A) = P(B)$

2 die

5 (7)

35

12

$\Sigma = \{(2,6), (3,4), (4,3), (6,2)\}$

$P(\Sigma) = \frac{4}{36}$

$= \frac{1}{9}$

- Unit 2 : Random variable and its characterization

- Discrete and Continuous random variable
- Cumulative distribution function
- Expectations
- Mean
- Variance
- Moment
- Moment generating function

[ ]
( )

Discrete Coin $X : \{H, T\} \to \{0, 1, 2, \dots\}$
X = no. of heads appear.

$X : S \to \mathbb{R}$

Discrete $X : S \to \mathbb{Z}\{0, 1, 2, 3, \dots\}$

Cont. $X : S \to \mathbb{R}$

Random Variable	Possible Values	Random Events

$X =$ { 0 ←  ; 1 ←  }

Two coins $X : \{HH, HT, TH, TT\} \to \{0, 1, 2, 3 \dots\}$
X = no. of head appears.

$X = 0, 1.$
↓
$X = 0, 1, 2$
$P(X=0) = \frac{1}{4}$
$P(X=1) = \frac{2}{4} = \frac{1}{2}$
$P(X=2) = \frac{1}{4}$ //

Two Types of Random Variables

- A **discrete random variable** can assume a countable number of values.
  - Number of steps to the top of the Eiffel Tower*
- A **continuous random variable** can assume any value along a given interval of a number line.

- assume any value along a given interval of a number line
  - The time a tourist stays at the top once s/he gets there

*Believe it or not, the answer ranges from 1,652 to 1,789. See Great Buildings

Two Types of Random Variables

- **Discrete random variables**
  - Number of sales
  - Number of calls
  - Shares of stock
  - People in line
  - Mistakes per page

- **Continuous random variables**
  - Length
  - Depth
  - Volume
  - Time
  - Weight

• Unit 3: Probability distributions

- Binomial Distribution
- Poisson Distribution
- Normal Distribution

The Binomial Distribution

- A Binomial Random Variable
  - *n* identical trials → Flip a coin 3 times
  - Two outcomes: **S**uccess or **F**ailure → Outcomes are Heads or Tails
  - P(**S**) = *p*; P(**F**) = *q* = 1 – *p* → P(H) = .5; P(F) = 1-.5 = .5
  - Trials are independent → A head on flip *i* doesn't change P(H) of flip *i* + 1
  - *x* is the number of **S**'s in *n* trials

p

$q = 1 - p$

$p = 0.5$

$q = 1 - 0.5 = 0.5$

The Binomial Distribution

Results of 3 flips	Probability	Combined	Summary
HHH	$(p)(p)(p)$	p^3	$(1)p^3q^0$
HHT	$(p)(p)(q)$	p^2q	
HTH	$(p)(q)(p)$	p^2q	$(3)p^2q^1$
THH	$(q)(p)(p)$	p^2q	
HTT	$(p)(q)(q)$	pq^2	
THT	$(q)(p)(q)$	pq^2	$(3)p^1q^2$
TTH	$(q)(q)(p)$	pq^2	
TTT	$(q)(q)(q)$	q^3	$(1)p^0q^3$

p q

H T

$P(A \cap B) = P(A)\,P(B)$

$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

The Binomial Distribution

- Say 40% of the class is female.
- What is the probability that 6 of the first 10 students walking in will be female?

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

$$= \binom{10}{6} (.4^6)(.6^{10-6})$$

$$= 210(.004096)(.1296)$$

$$= .1115$$

$P(x) = {}^nC_x \, p^x q^{n-x}$

$= {}^{10}C_6 (0.4)(0.6)^4$

$p = 0.4$

$q = 0.6$

$= 0.1115$

${}^nC_x = \frac{n!}{x!(n-x)!}$

The Poisson Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- λ = mean number of occurrences in the given unit of time, area, volume, etc.
- e = 2.71828….
- μ = mean
- $\sigma^2 = \lambda$

The Poisson Distribution

- Say in a given stream there are an average of 3 striped trout per 100 yards. What is the probability of seeing 5 striped trout in the next 100 yards, assuming a Poisson distribution?

$$P(x=5)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{3^5 e^{-3}}{5!}=.1008$$

McClave, Statistics, 11th ed. Chapter 4: Discrete Random Variables

$\lambda = 3$

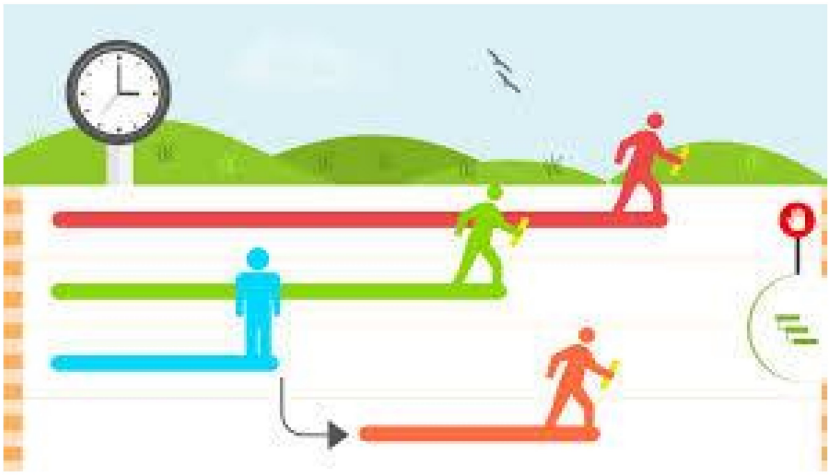
$e = 2.71$

$x = 5$

After Mid-term

- Unit 4: Point Estimation
  - Definition
  - Unbiased estimators
  - Consistent Estimators
  - Sufficient Estimator
  - MLE (Method of Maximum Likelihood)

Maximum likelihood estimator



- Null -
- Alternating -

- Unit 5: Testing of Hypothesis
  - Population and Sample
  - Types of Error
  - Test of Goodness of a Fit
  - Student t-test
  - Z-test
  - F-test

- Unit 6: Correlation and Regression
  - Bi-variate data
  - Scatter plots
  - Linear correlation
  - Regression
  - Fitting the line and curve
  - Properties of correlation coefficient.

The course outcomes...

The course MTH302: Probability and Statistics will...

- Recall the basic principles of probability and Bayes theorem.
- Visualize and use the concept of random variables to find the probability of an event.
- Use of some important distributions to find the probabilities.
- Discuss the method to find out the estimator and properties of estimator.
- Illustrate the concept of correlation, regression, hypothesis testing and their applications.

Thank You