AVERAGE

Average is a very simple but effective way of representing an entire group of by a single value.

The term average is also referred to as 'Mean'. Basic formula to calculate the average is as follows

$$\text{Average} = \frac{\textit{Sum of Observations}}{\textit{Number of Observations}}$$

Average is used quite regular in our day to day life. For example to calculate the average marks of the students, Average height of a particular group etc.

Points to remember

(i)   If the value of each item is increased by the same value P, then the average of the group or items will also increase by p.

(ii)   If the value of each item is decreased by the same value p, then the average of the group or items will also decreased by p.

(iii)   If the value of each item is multiplied by the same value, then the average of the group or items will also be multiplied by p.

(iv)   If the value of each item is divided by the same value P ($P \neq 0$), then the average of the group or items will also be divided by p.

(v)   The average of a group of items will always lie between the smallest value in the group and largest value in the group – i.e., the average will be greater than the smallest value and less than the largest value in the group

Example 1: What is the average of First 10 Prime numbers?

Solution: First 10 Prime numbers are 2,3,5,7,11,13,17,19,23,29.

Hence, Average = {2+3+5+7+11+13+17+19+23+29} / 10

= 129 / 10 = 12.90

Example 2: Four children have 30, 40, 50 and 60 toffees with them. What is the average number of toffees with them?

Solution: Average = (30 + 40 + 50 + 60) / 4 = 45

Using the above point, the average is (40+50)/2 = 45

Deviation Method for Calculation of Average

Now take a set of values which are typically cumbersome to deal with.

Example 3: Find average of 213, 227, 233, 223 and 229.

Solution: In the above case take the arbitrary number to be 220.

Then the average can be found as {220 + (-7+7+13+3+9)}/5 = 225.

Example 4: Find the average of 1093, 1103, 1109, 1089 and 1096?

Solution: This can be done as (1093 + 1103 + 1109 + 1089 + 1096) / 5

Or we observe that the numbers are close to 1100. Let us say, the numbers are

1093 = 1100 – 7;   1103 = 1100 + 3;   1109 = 1100 + 9

1089 = 1100 – 11;   1096 = 1100 – 4

Sum = 1100 x 5 – 10

So, average = (1100 x 5 – 10) / 5 = (1100/5) – 2 = 1098

This can be written as

$$P = P + \frac{\Sigma(P_i - P)}{n}$$

Here it doesn't really make a difference whether we assume the mean as 1100 or 1095 etc.

This method would come in handy while dealing with weighted averages or in DI.

Weighted Average
When two groups of items are combined together, then we can talk of the average of the entire group. However, if we know only the average of the two groups individually, we cannot find out the average of the combined group of items.

For example, there are two sections A and B of a class where the average height of section A is 150 cm and that of section B is 160 cm. On the basis of this information alone, we cannot find the average of the entire class (of the two sections).

Important Formulae Related to Average of numbers

1. Average of first n natural number = (n+1)/2

2. Average of first n even number = (n+1)

3. Average of first n odd number = n

4. Average of consecutive number = (First number + Last number)/2

5. Average of 1 to n odd numbers = (Last odd number+1)/2

6. Average of 1 to n even numbers = (Last even number+2)/2

7. Average of squares of first n natural numbers = [(n+1) (2n+1)]/6

8. Average of the cubes of first n natural number = [n (n+1)^2]/4

9. Average of n multiples of any number = [Number*(n+1)]/2

Example 5: Find out the average of 2, 4, 6, 8, 10, 12 and 14.
Solution: As we know that average of 1st 'n' even numbers = $(n + 1)$
∴ Required average = (7 + 1) = 8

Example 6: Calculate the average of 1, 3, 5, 7, 9, 11, 13, 15 and 17.
Solution: As we know, average of 1st 'n' odd numbers = n
∴ Required average = 9

Example 7: What will be the average of 1, 2, 3, 4…………….………51, 52, 53?
Solution: As we know, average of consecutive numbers = $\frac{First\ Number + Last\ Number}{2}$

Where first number = 1 and last number = 53

∴ Required average = $\frac{1+53}{2} = \frac{54}{2} = 27$

Example 8: Calculate the average of the squares of natural numbers from 1 to 25.

Solution: According to the formula, average of squares of first 'n' natural numbers = $\frac{(n+1)(2n+1)}{6}$

Where n = 25.

$$\therefore \quad \text{Required average} = \frac{(25+1)(2\times25+1)}{6} = \frac{26\times51}{6} = \frac{1326}{6} = 221$$

Example 9: The average weight of a group of 15 friends increases by 1 kg, when a person joins the group. Find the weight of the person who joins the group, if the initial average weight of the group is 48 kg.

Solution: Let the weight of the person joining the group be x kg.

   Given, $(15 \times 48 + x) / 16 = 49$

   $\Rightarrow x = 16 \times 49 - 15 \times 48 = 784 - 720 = 64$

   Hence the weight of the new person is 64 kg.

Alternate Method:

When the new person joins group as the average weight of the group increases by 1 kg, we can understand that this person is bringing 1 kg additionally not only for himself but even for others. Hence his weight should be $48 + 1 (16)$ i.e., 64 kg

Example 10: In a class there are two sections –> A and B. Section A contains 50 students with an average of 30 marks in Maths. Section B contains 75 students with an average of 40 marks in Maths. What is the average mark of the whole class?

Solution: For finding out the average mark of the whole class we will need the sum of marks of the whole class.
   The sum of marks of all students in section A is 50×30 = 1500
   The sum of marks of all students in section B is 75×40 = 3000
   Thus total sum of marks for the class is (1500+3000) = 4500
   So the average mark of the class is 4500 / (50+75) = 36

Example 11: The average age of a group of friends is 34 years. If five new friends with an average age of 30 years join the group, the average of the entire group becomes 32 years. How many people were there in the group initially?

Solution: Let there be 'n' people initially in the group. Then the total age of the group after the five new friends joined the group is 34n + (5 x 30)

   But this is also equal to 32 (n + 5)
   $\therefore 32(n+5) = 34n + 150 \Rightarrow n = 5$
   So, there were 5 friends in the group initially.

Example 12: A batsman scored an average of 55 runs in the first 6 tests. If the first test is not counted and the seventh is counted then his average score goes, up to 57. If the score in the first test as 50 runs. Find his score in the seventh test.

Solution: Total score in the first sic tests = 6 x 55 = 330
   Total score in the last six tests = 6 x 57 = 342
   Score in all seven tests = Total score in the last
   Six tests + score in the 1st test = 342 + 50 = 392
   $\therefore$ Score in the 7th test = Total score in all 7 test – total score in the first 6 tests
   = 392-330 = 62

Example 13: Twelve years ago, the average of the ages of the members of a joint family having ten members was 25 years. Four years later a member aged 50 years died and a child was born in the family that year. Four years after that, another member aged 50 years died and another child was born. Find the present average age of the members of the family (in years).

Solution: Had there been no alterations, the current average would have been 25 + 12 = 37.
The first person who goes out takes with him 50 years, Similarly, the second person who goes out takes with him another 50 years.
Present average = 37 - (50 + 50)/10 = 37 – 10
= 27 years.

Example 14: The average of five positive numbers is 213. The average of the first two numbers is 233.5 and the average of last two numbers is 271. What is the third number?
Solution: The sum of the five numbers = 5 × 213 =1065
The sum of the first two numbers = 2 × 233.5 = 467
The sum of the last two numbers = 542
Then the sum of the four numbers = 467 + 542 =1009
So, the third number will be = 1065 – 1009 = 56.

Example 15: The average marks of 65 students in a class were calculated as 150. It was later realized that the marks of one of the students was calculated as 142, whereas his actual average marks were 152. What is the actual average mark of the group of 65 students? (Rounded off to two digits after decimal)
Solution: Increase in total marks = 152 – 142 = 10
Therefore the New average = 150 + 10/65 = 150.15

Example 16: In a class there are 32 boys and 28 girls. The average age of the boys in the class is 14 years and the average age of the girls in the class is 13 years. What is the average age of the whole class? (Rounded off to two digits after decimal)
Solution: The sum of the ages of 32 boys = 32 × 14 = 448
The sum of the ages of 28 girls = 28 × 13 = 364
Therefore, the sum of the ages of the whole class of 60 students =812
The average age of the whole class of 60 students = 812/60 = 13.53

Example 17: Srikanth earned an average of Rs. 16,00 per month from January to June. Then, he earned Rs. 1500, Rs, 1800, Rs, 1900 and Rs. 2150 respectively during the months July, through October. During November he earned 5-% of what he earned in December. If his average earnings for the entire year is Rs. 1,600. Find his earnings in the month of November.
Solution: Let earnings for the month of November be 'x'.
Then total annual earnings will be
(1600 x 6) + 1500 + 1800 +1900 + 2150 + 3X =16950 +3x
Total earnings for the entire year
= 12 x 1600 (given)
= 19200 which is equal to 16950 + 3x
16950 + 3x = 19200 $\Rightarrow$ 3x = 250
∴ x = Rs 750
Hence, the earnings in the month of November is Rs 750.

Level - I

1. The average of first five prime numbers is:
   A] 4.5   B] 5   C] 5.6   D] 7.5

2. The average of first five multiples of 3 is:
   A] 3   B] 9   C] 12   D] 15

3. The average height of 30 boys out of a class of 50 is 160 cm. If the average height of the remaining boys is 165 cm, the average height of the whole class (in cm) is:
   A] 161   B] 162   C] 163   D] 164

4. The average of three numbers is 20. If the two numbers are 16 and 22, the third number is:
   A] 22   B] 20   C] 19   D] 18

5. The average of five results is 46 and that of the first four is 45. The fifth result is:
   A] 1   B] 10   C] 12.5   D] 50

6. The average of Radhika's marks in 7 subjects is 75. His average in six subjects excluding science is 72. How many marks did he get in Science?
   A] 72   B] 90   C] 93   D] None of these

7. The average of eight numbers is 14. The average of six of these numbers is 16. The average of the remaining two numbers is:
   A] 4   B] 8   C] 16   D] Data inadequate

8. Average of four consecutive odd numbers is 106. What is the third number in ascending order?
   A] 107   B] 111   C] 113   D] cannot be determined

9. Of the three numbers, the average of the first and the second is greater than the average of the second and the third by 15. What is the difference between the first and the third of the three numbers?
   A] 15   B] 45   C] 60   D] None of these

10. The ratio of roses and lilies in a garden is 3:2 respectively. The average number of roses and lilies is 180. What is the number of lilies in the garden?
    A] 144   B] 182   C] 216   D] 360

11. The average price of three items of furniture is Rs. 15000. If their prices are in the ratio 3:5:7, the price of the cheapest item is:
    A] R9000   B] R15000   C] R18000   D] R21000

12. Of three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, the largest number is:
    A] 24   B] 36   C] 72   D] 108

13. The average of ten numbers is 7. If each number is multiplied by 12, then the average of new set of numbers is:
    A] 7   B] 19   C] 82   D] 84

14. The average age of 30 students of a class is 12 years. The average age of a group of 5 of the students is 10 years and that of another group of 5 of them is 14 years. What is the average age of the remaining students?

A] 8 years   B] 10 years   C] 12 years   D] 14 years

15. The average of 50 numbers is 38. If two numbers 45 and 55 are discarded, the average of the remaining numbers is:

A] 36.5   B] 37   C] 37.5   D] 37.52

16. The mean of 100 observations was calculated as 40. It was found later on that one of the observations was misread as 83 instead of 53. The correct mean is:

A] 39   B] 39.7   C] 40.3   D] 42.7

17. The average of six numbers is 30. If the average of first four is 25 and that of last three is 35, the fourth number is:

A] 25   B] 30   C] 35   D] 40

18. The average of 11 observations is 60. If the average of first five observations is 58 and that of the last five is 56, the sixth observation is:

A] 90   B] 110   C] 85   D] 100

19. Harish has twice as much money as Rohan and Rohan has 50% more money than what Anita has. If the average money with them is Rs.110, then Harish has:

A] Rs.55   B] Rs. 60   C] Rs.90   D] Rs. 180

20. A motorist has travels to a place 150 km away at an average speed of 50 km per hour and returns at 30 km per hour. His average speed for whole journey in km per hour is

A] 35   B] 37   C] 37.5   D] 40

LEVEL – II

1. The average of 5 numbers is 7. When 3 new numbers are added, the average of the eight numbers is 8.5. The average of three new numbers is:

A] 11   B] 7.75   C] 8.5   D] 7

2. The average age of 30 students is 9 years. If the age of their teacher is included, it becomes 10 years. The age of the teacher (in years) is:

A] 27   B] 31   C] 35   D] 40

3. The average age of 24 boys and the teacher is 15 years. When the teacher's age is excluded, the average decreases by 1. What is the age of the teacher?

A] 38 years   B] 39 years   C] 40 years   D] Data inadequate

4. The average salary per month of 30 employees in a company is Rs. 4000. If the manager's salary is added, the average salary increases to Rs. 4300, what is the salary of the manager?

A] Rs. 10000   B] Rs. 13000   C] Rs. 12000   D] Rs. 13300

5.	The average age of 40 students of a class is 15 years. When 10 new students are admitted, the average is increased by 0.2 years. The average age of new students is:
A] 15.2 years	B] 16 years	C] 16.2 years	D] 16.4 years

6.	The average weight of 8 men is increased by 1.5 kg when one of the men who weigh 65 kg is replaced by a new man. The weight of the new man is:
A] 76 kg	B] 76.5 kg	C] 76.7 kg	D] 77 kg

7.	The average weight of 6 men decreases by 3 kg when one of them weighing 80 kg is replaced by a new man. The weight of the new man is:
A] 56 kg	B] 58 kg	C] 62 kg	D] 76 kg

8.	The average age of a committee of eight members is 40 years. A member aged 55 years retired and his place was taken by another member aged 39 years. The average age of the present committee is:
A] 39 years	B] 38 years	C] 36 years	D] 35 years

9.	A cricketer has a certain average for 9 innings. In the tenth innings, the score is 100 runs, thereby increasing his average by 8 runs. His new average is:
A] 20 runs	B] 24 runs	C] 28 runs	D] 32 runs

10.	A man whose bowling average is 12.4 takes 5 wickets for 26 runs and thereby decreases his average by 0.4. The number of wickets, taken by him before his last match is:
A] 85	B] 78	C] 72	D] 64

11.	The mean temperature of Monday to Wednesday was 37⁰C and of Tuesday to Thursday was 34⁰C. If the temperature on Thursday was 4/5 that of Monday, the temperature on Thursday was:
A] 36.5⁰C	B] 36⁰C	C] 35.5⁰C	D] 34⁰C

12.	Three years ago, the average age of X, Y and Z was 27 years and that of Y and Z, 5 years ago was 20 years. X's present age is:
A] 30 years	B] 35 years	C] 40 years	D] 48 years

13.	Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is:
A] 2 years	B] 2.4 years	C] 3 years	D] 1.5 years

14.	The average monthly income of a family of four earning members was Rs.15130. One of the daughters in the family got married and left home, so the average monthly income of the family came down to Rs.14660. What is the monthly income of the married daughter?
A] Rs.15350	B] Rs.12000	C] Rs.16540	D] Cannot be determined

15.	The average of four positive integers is 73.5. The highest integer is 108 and the lowest integer is 29. The difference between the remaining two integers is 15. Which of the following is the smaller of the remaining two integers?
A] 80	B] 86	C] 73	D] None of these

16. The average age of a woman and her daughter is 46 years. The ratio of their ages is 15:8 respectively. What will be the respective ratio of their ages after 8 years?
    A] 8 : 5   B] 10 : 17   C] 17 : 10   D] 5 : 8

17. There are 50 boys in a class. One boy weighing 40 kg goes away and at the same time another boy joins the class. If the average weight of the class is thus decreased by 100 g, find the weight of the new boy.
    A] 35 kg   B] 43 kg   C] 36 kg   D] 30 kg

18. Kamlesh bought 65 books for Rs.1050 from one shop and 50 books for Rs.1020 from another. What is the average price he paid per book?
    A] Rs. 36.40   B] Rs.18.20   C] Rs. 24   D] Rs.18

19. A car covers the first 39 kms of its journey in 45 minutes and covers the remaining 25 kms in 35 minutes. What is the average speed of the car?
    A] 40 kms/hr   B] 64 kms/hr   C] 49 kms/hr   D] 48 kms/hr

20. The average marks in Science subject of a class of 20 students is 68. If the marks of two students were misread as 48 and 65 of the actual marks 72 and 61 respectively, then what would be the correct average?
    A] 68.5   B] 69   C] 69.5   D] 70