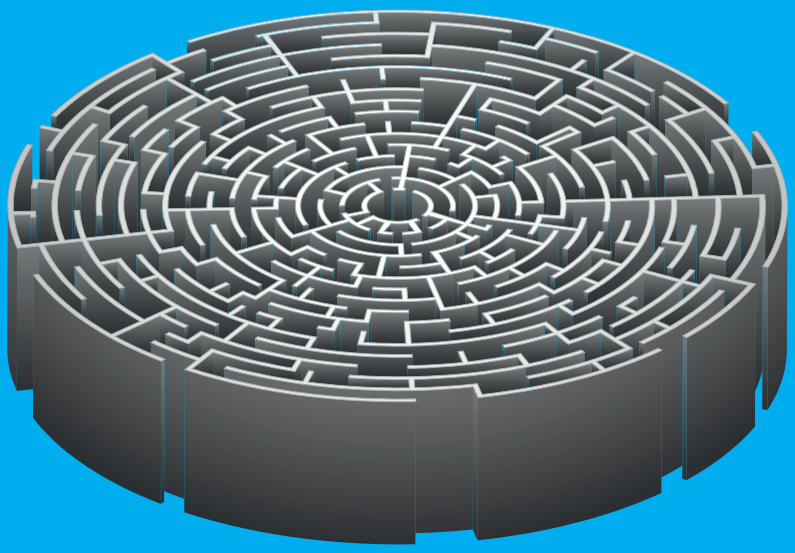
The Pearson Guide to

IIM Indore

IPM

(Integrated Programme in Management)

Entrance Examination



dueNorth Academics (An IIM Alumni Body)

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PEARSON

The Pearson Guide to IIM Indore–IPM

(Integrated Programme in Management)

Entrance Examination

dueNorth Academics

PEARSON

Delhi • Chennai

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Preface

The Integrated Programme in Management (IPM) has been started by IIM Indore in the year 2011. It is a boon for students aspiring for a management degree from IIM due to the following reasons:

(a) An entry into the prestigious management program of IIM at an undergraduate level (UG)
(b) An opportunity to interact with IIM professors and peer group for five years (instead of two years) at a world class infrastructure
(c) Better success ratio at UG level (for IPM program) than that of at CAT (for PGDM program)

In the last three years, several of our students have qualified into IIM Indore–IPM. This provided us an opportunity to present this book to undergraduate students.

The book is designed keeping in mind the 3Cs:

(i) **Concise:** We understand that students at UG level are already occupied with their X+2 syllabus. This book is prepared keeping in mind that students are learning 'to-the-point' concepts and practising quality questions.
(ii) **Comprehensive:** While preparing the manuscript, the last three years' question papers were analyzed, the scope of Aptitude Tests discussed with trainers across the country and future projections made accordingly. After deliberations, we developed the table of contents and related concepts.

The book also consists of two Section Tests and three Full Length Test Papers to help students assess their preparation level.

(iii) **Clarity:** All concepts are explained with the help of illustration and worked-out examples. Moreover, five practice exercises are provided to supplement the learning.

We are sure this book will help students immensely in cracking this examination.

Although we have taken utmost care to prepar the manuscript and checking subsequent proofs, there may be a possibility of some errors creeping inside the book. Please mail us your suggestions on: contentdev.duenorth@gmail.com

Happy Learning !!!

Nishit K. Sinha
(Alumnus, IIM Lucknow)
Founder Director, dueNorth Academics

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About IIM Indore–IPM (Integrated Programme in Management)

The Five-year Integrated Programme in Management (IPM) is aimed at students who have passed out class XII/Higher Secondary or equivalent from various schools in India. Indian Institute of Management (IIM) Indore has started Five-year Integrated Programme in Management (IPM) in 2011. Classes for the 1st batch was started in October, 2011.

Degree Awarded after successful completion of course is—Integrated Diploma in Management (IDM)

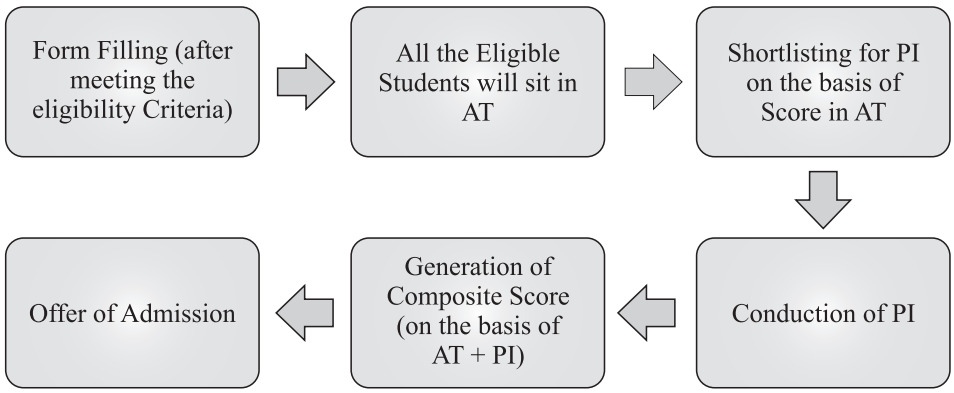
Eligibility Criteria

- 60% aggregate marks at Secondary/X Standard/Equivalent and at Higher Secondary/XII/+2 level/ Equivalent for General Candidates and NC-OBC category candidates.
- 55% aggregate marks at Secondary/X Standard/Equivalent and at Higher Secondary/XII/+2 level/Equivalent for SC, ST and PWD/DA candidates.
- All candidates meeting the minimum eligibility mentioned above will be called for the Aptitude Test.
- Government of India reservation norms would be followed while offering the admission.

Age Limit for 2014–2019 Programme

- Not more than 20 years as on July 31st, 2014 for General/NC-OBC Candidates.
- Not more than 22 years as on July 31st, 2014 for SC/ST/PWD/DA Candidates.

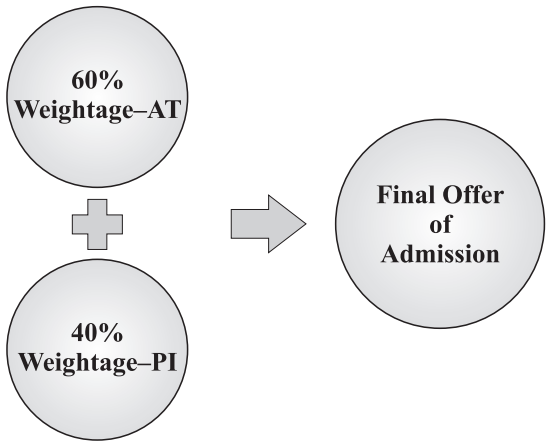
Selection Process



AT = Aptitude Test PI = Personal Interview

Candidates will be called in the ratio of 1:10 in each category, *i.e.*, for every seat, 10 students will be called for PI. Total number of seats will be 121 (for academic calendar 2014–2019).

Weightage for Final Offer of Admission



Timeline for Academic Calendar 2014–2019

Sl. No.	Activity Description	Date (Day)
1.	Release of admission notification in newspapers	February 3rd, 2014 (Monday)
2.	Last date for receipt of ONLINE/OFFLINE applications along with application fee DD/Banker's Cheque (BC)	April 4th, 2014 (Friday)
3.	Announcement of Aptitude Test (AT)	April 14th, 2014 (Monday)
4.	Conduct of Aptitude Test (AT)	May 15th, 2014 (Thursday)
5.	Announcement of shortlist for Personal Interview (PI)	May 26th, 2014 (Monday)
6.	Conduct of Personal Interview (PI)	June 3rd–6th, 2014 (Tuesday–Friday)
7.	Announcement of final admission offers	June 16th, 2014 (Monday)
8.	Last date for acceptance of Admission offer and payment of Commitment fee	June 30th, 2014 (Monday)
9.	Programme Registration	August 1st, 2014 (Friday)
10.	Commencement of classes	August 4th, 2014 (Monday)

Note: The above timelines are tentative and subject to change as required.

Tuition Fee (for Academic Calendar 2014–2019)

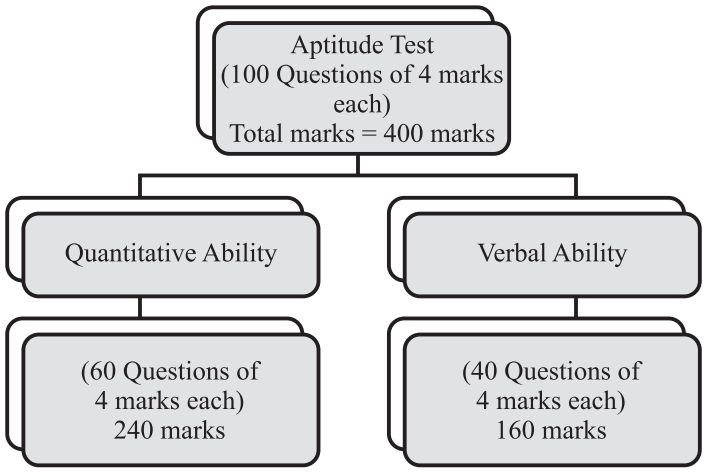
	1st Year	2nd Year	3rd Year	4th Year	5th Year	Total
Per Annum Tuition Fee	3 lacs	3 lacs	3 lacs	5 lacs	5 lacs	19 lacs

Boarding and Lodging fee will be in addition to the above mentioned Tuition Fee.
Should you have any query, you can mail us at email id: contentdev.duenorth@gmail.com

Aptitude Test Paper Pattern over the year

Paper pattern of IIM–IPM Aptitude Test (AT) has been fairly constant across the years so far—60 questions in Quantitative Ability (4 marks each, totaling 240 marks) and 40 questions in Verbal Ability (4 marks each, totaling 160 marks). There is 25% negative marking.

SNAPSHOT of the Paper Pattern



Negative Marking—Yes. 25% negative marking is there.

Quantitative Ability

Questions are asked from a range of topics like Percentage, Profit and Loss, Simple Interest (SI) & Compound Interest (CI), Number System, Average, Allegation, Mixture and Solution, Ratio and Proportion, Time, Speed and Distance, Time and Work, Progressions, Geometry and Mensuration, Inequalities, Logarithms and Graphs. AT 2012 had questions from Data Interpretation (DI) and Logical Reasoning (LR). Though in AT 2013, DI and LR questions were not asked.

Verbal Ability

This section consists of—Reading Comprehension (2–3 passages with 5 questions each), Vocabulary (Around 10–12 questions on Antonyms and Synonyms), Parajumble, Para Completion, Word Usage, Fill in the Blanks (6–8 questions) and Proper Usage of words. The passages were asked from economy, culture and developmental issues.

Should you have any query, you can mail us at email id: contentdev.duenorth@gmail.com

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SECTION 1

PART A

Quantitative Aptitude

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1 Number System

Number is "a numeral or group of numerals". In other words it is a word or symbol, or a combination of words or symbols, used in counting several things.

TYPE OF NUMBER

Broadly number can be defined to be of two types—Real Number and Imaginary Number.

Real Number

Real number is a number that can be expressed on the number line. In other words, all the real numbers can be felt or experienced in the real world.

Imaginary Number

In contrast to Real numbers, Imaginary numbers cannot be plotted on the number line, or, they cannot be felt of experienced in the real world.

Now we will see the types of Real Number: All the real numbers are either a rational number or an irrational number. No real number can be both a rational number and an irrational number.

Rational Number

A rational number is a number that can be expressed as a fraction (p/q) where p and q are integers and $q \neq 0$. The term 'rational' comes from other word **Ratio**, because the rational numbers are the ones that can be written in the format of a ratio.

For example, 3/5, 4/8, 22/7, etc. are rational numbers. Rational number can be positive as well as negative.

In a rational number p/q, p is known as numerator and q is known as denominator.

We also understand that:

- If numerator is more than denominator, then ratio is known as improper ratio. Example: 5/3, 7/4.
- If numerator is less than denominator, then ratio is known as proper ratio. Example: 3/7, 7/18, etc.

So what kind of numbers are Rational Numbers?

1. All the integers–positive or negative or zero—are rational numbers. For example: (–5), (–10), 10, 20, etc. are rational numbers.
2. Is 0.7777 ... (7 repeating till infinity) a rational number?
   Yes, it is. Because 0.777 ... (7 repeating till infinity) = 7/9.
   All the repeating decimal numbers with a fixed period are rational.
   For example, 0.315315315 ... (315 repeating till infinity).
3. All finite decimals like 0.48 are rational. In other words all the terminating decimals are rational numbers.
   For example, 0.5678345678 is a rational number because it can be written in the form of p/q.

Method to Convert Decimals into Fractions

1. If a number is finite decimal like 0.76

$$0.76 = \frac{76}{100}$$

$$2.32 = \frac{232}{100}$$

$$0.7 = \frac{7}{10} = \frac{70}{100}$$

2. If there is one digit repeating, then the denominator will be 9.
   For example, consider the number
   $0.4444 \ldots = 0.\overline{4}$
   (4 repeating till infinity)
   $0.4444 \ldots = 0.\overline{4} = \frac{4}{9}$
   If there are two digits repeating, then the denominator will be 99.
   For example, consider the number
   $0.626262 \ldots = 0.\overline{62}$
   (62 repeating till infinity)
   $0.626262 \ldots = 0.\overline{62} = \frac{62}{99}$
   If there are three digits repeating, then the denominator will be 999.
   For example, consider the number
   $0.678678678678 \ldots = 0.\overline{678}$
   $0.678678678678 \ldots = 0.\overline{678} = \frac{678}{999}$
   Similarly, if the number has an integer as quotient:
   For example, consider the number
   $1.23232323 \ldots = 1.\overline{23}$
   To solve such questions, break this number down in two parts such that 1st part is constant and second part is repeating:
   $1.23232323 \ldots = 1.\overline{23}$
   $= 1 + 0.232323 \ldots$
   Now convert the decimal recurring part into a ratio using the concept as given above:
   $1 + 0.232323 \ldots = 1 + \frac{23}{99} = \frac{122}{99}$

Summarizing the whole discussion: Dividing by 9 repeats one digit, dividing by 99 repeats two digits, dividing by 999 repeats three digits and so on. In other words, number of 9's in the denominator will be equal to the number of repeating digits in the number (Table 1.1).

Example 1

Write down $4.321321321 \ldots = 4.\overline{321}$ in the ratio format.

Solution

At first, Let us break this number down in two parts such that one part is fixed and other part is repeating

$4.321321321 \ldots$
$= 4 + 0.321321321 \ldots$
$= 4 + 0.\overline{321}$

Since three digits are repeating, denominator will be 999

$$4 + 0.\overline{321} = 4 + \frac{321}{999} = \frac{4317}{999}$$

Example 2

Convert 3.1232323 ... into rational form.

Solution

Digits are not repeating after the decimal, rather digits are repeating after one digit from decimal.

So we can write 3.1232323 ...
$= 3.1 + 0.0232323 \ldots$

We just have to convert 0.0232323 ... into rational form. Remaining part = 3.1 can be converted to rational part very easily

$0.02323232 \ldots$
$= 0.1 \times 0.232323 \ldots$
$= \frac{1}{10} \cdot \frac{23}{99} = \frac{23}{990}$

Table 1.1

1/9	2/9	3/9	4/9	5/9	7/9	25/99	214/99
0.1111 ...	0.2222 ...	0.3333 ...	0.444 ...	0.5555 ...	0.777 ...	0.252525 ...	2.151515 ...

So, 3.1232323 ...

$$= \frac{31}{10} + \frac{23}{990} = \frac{3092}{990}$$

Alternatively, we can do this question in the following way too:

Let $P = 3.1232323 \ldots$

$\Rightarrow$ $10P = 31.232323 \ldots$

$$= 31\frac{23}{99} = \frac{3092}{99}$$

Hence $P = \frac{3092}{990}$

Example 3

Convert 3.15474747 ... into rational form.

Solution

Assume $P = 3.15474747 \ldots$

We can see that digits repeat two digits after decimal

$100P = 315.474747 \ldots$

$$= 315\frac{47}{99} = \frac{31232}{99}$$

Hence $P = \frac{31232}{9900}$

Irrational Numbers

Any real number that is not rational is irrational number. An irrational number is a number that cannot be expressed as a fraction p/q for any integer p and q. Irrational numbers have decimal expansion that neither terminate nor become periodic.

A Dilemma

$$\pi = \frac{22}{7}$$

π is an irrational number, but $\frac{22}{7}$ is a rational number. How is this possible that a rational number is equal to an irrational number?

Explanation: π is approximately equal to $\frac{22}{7}$. It is not exactly equal to $\frac{22}{7}$. This is how it is possible: $\frac{22}{7}$ is an irrational number whereas π is an irrational number.

π is an Irrational Number.

Most famous irrational number is $\sqrt{2}$, also known as Pythagoras constant.

A Practical Approach to Understand Rational and Irrational Number:

What kind of person would you identify as Rational person and what kind of person is an irrational person? Answer lies in—a person whose behavior can be predicted is a rational person, and a person whose behavior cannot be predicted is an irrational person. You may use the same criteria of "Predictability" to identify the Rational and Irrational number. So, **if you can predict the next digit (after decimal) in the number, then the number is Rational, and if cannot predict the next digit (after the decimal), then it is an Irrational number.**

For example,

1. What is the value of A in $2.131313A \ldots$ (Number is $2.\overline{13}$) ?
   Answer = 1. Hence, this number is a rational number.
2. What is the next digit that will come in 2.13?
   Answer = 0. Hence, this number is a rational number.
3. What is the next digit that will come in 12?
   Answer = 0 (As in 12.0000). Hence, this number is a rational number.

Now try answering this question—what is the next digit that will come in 4.783756395835144967 ...? Can we answer this? Answer is NO. it is simply because we do not have any pattern of the digits. Hence, this number is an Irrational number.

Test Your Learning (Intro to Numbers)

1. Identify which of the following are rational numbers?

(a) $\frac{7}{9}$

(b) 0.44444 ... (4 repeating till infinity)

(c) $\sqrt{3}$

(d) $\frac{22}{7}$

(e) π

(f) 0.173173173 ... (173 repeating till infinity)

(g) 5.63796778472867346521 (till infinity)

(h) $\sqrt{7}^{100}$

(i) $\sqrt{7}^{100} + \frac{1}{\sqrt{5}^{100}}$

Answer and Solution

(a) Rational

(b) **Rational:** 0.444 ... (4 repeating till infinity) $= \frac{4}{9} =$ Rational number.

(c) **Irrational:** $\sqrt{3}$ cannot be written in the form of $\frac{p}{q}$ where p and q are integers ($q \neq 0$). Hence, irrational number.

(d) $\frac{22}{7}$ is in the form of $\frac{p}{q}$. Hence, Rational number.

(e) **Irrational number:** Exact value of π is not known. $\pi \left(\frac{22}{7}\right)$ is only an approximate value, and not the exact value. Hence, next digit of π cannot be predicted. So, it is an Irrational number.

(f) 0.173173 … (173 repeating till infinity) $= \frac{173}{999} =$ Rational number.

(g) **Irrational number:** Despite 5.63796778472867346521 (till infinity) goes till infinity, but it does not show any pattern (unlike previous questions), hence cannot be presented as a ratio = p/q.

(h) $\sqrt{7}^{100} = 7^{50} =$ rational number.

(i) $\sqrt{7}^{100} + \frac{1}{\sqrt{5}^{100}}$ rational number + another rational number = Rational number.

A dilemma—is 0.9999999 ... equal to 1?

Obviously when we write this down mathematically,

$$0.9999999 \ldots = 0.\overline{9} = \frac{9}{9} = 1$$

Although a more logical question arises that any number of times we write 9 after the decimal as in 0.99999 ..., this should not be equal to 1.

An explanation to this is given in the following manner:

Don't look at 0.99999 ... as if we are approaching towards 1, see this number as if we are moving away from 1 and look at the distance between 1 and this number.

So, distance between 1 and 0.9 = 0.1

Distance between 1 and 0.99 = 0.01

Distance between 1 and 0.999 = 0.001

Distance between 1 and 0.9999 = 0.0001

We can see that the distance is very slowly getting smaller, and is tending towards zero. So after writing a number of 9's as in 0.999999 ..., distance will become equal to zero, and number = 1.

INTEGERS

Integers can be either positive or negative or zero. Integers are also classified as odd or even integers.

Even Integers

Any integer that can be written in the format of $2N$ is an even integer, where N is an integer. In other words, any number that is divisible by 2 is an even integer. For example: 2, 4, 100, etc.

Is (–10) an even integer → Yes (–10) is an even integer.

Is zero an even integer → Yes, 0 is an even integer.

Odd Integers

Any integer that can be written in the format of $2N + 1$ is an odd integer, where N is an integer. For example 1, 5, 101, –23 are odd integers.

Odd ± Odd = Even
Even ± Even = Even
Odd ± Even = Odd
Even ± Odd = Odd
(Odd) Even = Odd
(Even) Odd = Even
Odd × Even = Even
Odd × Odd = Odd
Even × Even = Even

Well, the idea is not asking you to mug it up. Rather develop an understanding and whenever required, you should be able to verify it on your own.

REAL NUMBER LINE

Real number line (in short also known as number line) consists of the union of the rational and irrational numbers.

Every real number can be associated with a single point on the **real number line** (Fig. 1.1)

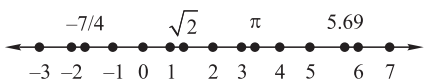


Fig. 1.1

- On the left hand side, it extends to – infinity (–∞) to + infinity (+∞) on the right hand side.
- Numbers on the right hand side are always larger than the numbers on the left hand side.
- When we do addition operation, we move on to the right hand side of numbers line.
  - For example, when we add 4 units to 3 (= 3 + 4), we are required to move 4 units right hand side to 3, landing up at 7.
- When we do subtraction operation, we move on to the left hand side of numbers line.
  - For example, when we subtract 3 units from 4 (= 4 – 3), we are required to move 3 units left hand side to 4, landing up at 1 (Figs. 1.2 and 1.3).

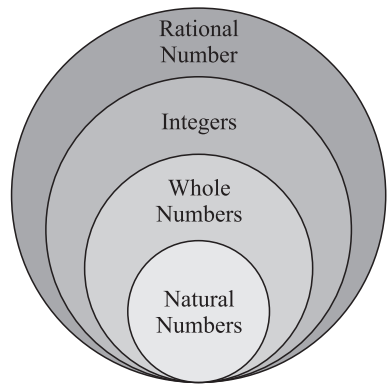


Fig. 1.3

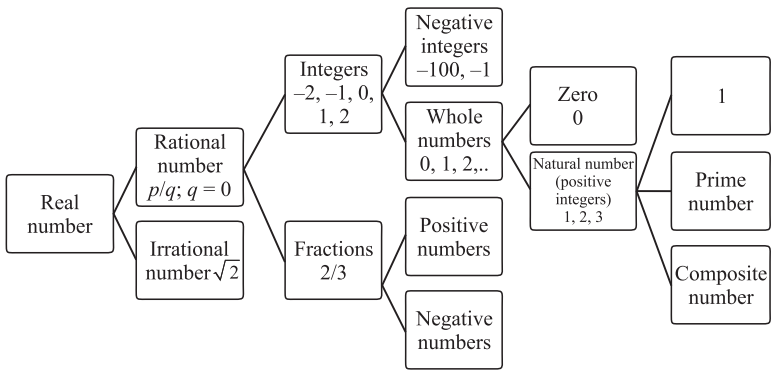


Fig. 1.2

NATURAL NUMBER

Now we will see the types of natural numbers. All the natural numbers greater than 1 has at least two factors—and on the basis of number of factors we can categorize natural numbers greater than 1 either as Prime number or Composite number.

Prime Number

Any number that has exactly two distinct factors is known as prime number. In other words, any number which is divisible by 1 and itself only is a prime number. For example, 2 is a prime number, because 2 is divisible by 1 and 2 only (two distinct factors only). Similarly 103 is a prime number because it is divisible by 1 and 103 only. 2 is the only even prime number, and all the other prime numbers are odd.

In that way prime numbers are building blocks of numbers *i.e.*, Prime numbers are used to build the other numbers—known as composite numbers.

Example 4

A, B, C, D and E are five prime numbers, not necessarily consecutive. Sum of these five prime numbers = 180. It is also given that $A < B < C < D < E$. What is the value of A^4?

Solution

You should not try to solve this question by trying to find out the actual values of prime numbers. Rather think of the logic behind the question.

Summation of five prime numbers
= 180
= Even number

We know that except 2, all the other prime numbers are odd. Since summation of five prime numbers is even, hence four of these prime numbers are odd and one is even.

Since A is the lowest prime number, hence
A = Even prime number = 2

So $A^4 = 24$
$= 16.$

Properties of Prime Numbers

1. There are infinite prime numbers.
2. There are infinite set of prime numbers such that difference between them = 2 (represented as N and $N + 2$, where N is a prime number). For example, 3 and 5, or, 5 and 7, or, 11 and 13, etc. Set of these two prime numbers are known as "Twin Primes."
3. There is exactly one set of prime number triplet N, $N + 2$, $N + 4$, where N is a prime number (3, 5, 7).
4. All the natural numbers greater than 1 has atleast one prime factor.
5. All the prime numbers greater than 3 are of the format $6N \pm 1$. It means that all the prime numbers will give either a remainder of +1 or –1 when divided by 6. Understand that this is only a sufficient condition and not necessary one. In other words, if a number is a prime number, it will be of the format $6N \pm 1$, but it does not mean that if any number is of the format $6N \pm 1$, then its going to be a prime number. For example, $25 = 6N + 1$, but 25 is not a prime number.

Composite Number

If we subtract the prime numbers and 1 from the list of natural numbers, we obtain composite numbers.

Composite Number
= All the natural numbers
– all the prime numbers – 1

In other words, composite numbers will have atleast one more factor other than 1 and itself. Hence, composite numbers will have atleast 3 factors.

Lowest composite number = 4

List of first few composite factors (refer Table 1.2).

Properties of Composite Number

1. A composite number has atleast three factors.

Table 1.2

Number	4	6	8	9	10	12	14	15
Prime Factorization	2^2	$2^1 \times 3^1$	2^3	3^2	$2^1 \times 5^1$	$2^2 \times 3^1$	$2^1 \times 7^1$	$3^1 \times 5^1$
Number of factors	3	4	4	3	4	6	4	4

2. All the natural numbers other than prime numbers and 1 are composite numbers.

Out of any *N* consecutive natural numbers:

1. Out of any 2 consecutive natural numbers, exactly one number is divisible by 2.
2. Out of any 3 consecutive natural numbers, exactly one number is divisible by 3.
3. Out of any 4 consecutive natural numbers, exactly one number is divisible by 4.
4. Out of any 5 consecutive natural numbers, exactly one number is divisible by 5.

Example 5

Mark true or false:

(a) Out of any 10 consecutive natural numbers, exactly 5 natural numbers are divisible by 2. (True/False)

(b) Out of any 11 consecutive natural numbers, exactly 4 natural numbers are divisible by 3. (True/False)

(c) Out of any 100 consecutive natural numbers, exactly 1 natural number is divisible by 100. (True/False)

(d) Out of any 101 consecutive natural numbers, exactly 1 natural number is divisible by 100. (True/False)

(e) Out of any 99 consecutive natural numbers, exactly 1 natural number is divisible by 100. (True/False)

Solution

(a) True. 10/5 = 2

(b) It may be 3 or 4 numbers depending upon which is the 1st number. For example, if we start from 3, then there are 4 numbers divisible by 3. However if we start from 4, then there are three numbers only.

(c) True. 100/100 = 1

(d) False. There may be one or two numbers divisible by 100.

(e) False. There may be zero or one number divisible by 100.

Counting of Numbers in any Range

If N is a natural number, number of natural numbers in the range (A and B are natural numbers):

$A \le N \le B = (B - A) + 1$ [Both ends included]

$A \le N < B = (B - A)$ [Only one of the ends included]

$A < N \le B = (B - A)$ [Only one of the ends included]

$A < N < B = (B - A) - 1$ [None of the ends included]

For example, number of natural numbers in the following range is as given in Fig. 1.4.

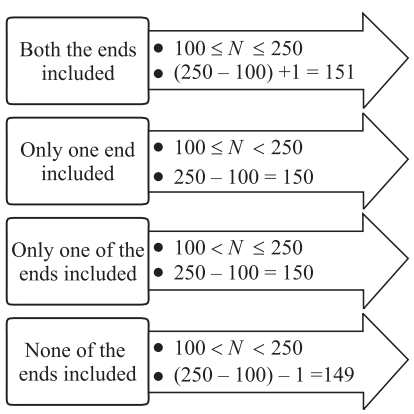


Fig. 1.4

Alternatively we can generate a pattern to count the number of numbers. This can be seen through the following example.

Example 6

How many natural numbers from 200 to 500 (including both the limits) will be divisible by 3?

Solution

Method 1: Pattern method:

1st number = 201 and next numbers are 204, 207, ..., 498.

$201 = 67 \times 3, ..., 498 = 166 \times 3$

So we have to count till 166th multiple of 3 leaving 1st 66 multiple.

Hence, number of numbers

$= 166 - 66 = 100$

Method 2: Counting Method:

1st number = 201 and next numbers are 204, 207, ..., 498.

So number of numbers

$$= \frac{498 - 201}{3} + 1 = \frac{297}{3} + 1$$

$= 99 + 1 = 100.$

Some General Properties

1. If we reverse any two digit natural number, the difference between the original and resulting number will be equal to 9 times the difference between tens place digit and units place digit.

   To understand this more, let us see it in details:

   If xy is a two digit number,

   with x = Tens place digit,

   and y = Units place digit,

   then $xy - yx = 9(x - y)$

   For example,

   $91 - 19 = 9(9 - 1) = 72$

   $74 - 47 = 9(7 - 4) = 27$

   $85 - 58 = 9(8 - 5) = 27$ and so on.

   It is actually very simple to see that why this happens:

   $xy = 10x + y$

   and $yx = 10y + x$

   So, $xy - yx = 10x + y - (10y + x)$

   $= 9(x - y)$

   Similarly if we add the numbers, let us see what happens, then:

   $xy + yx = 10x + y + 10y + x$

   $= 11(x + y)$

   $\Rightarrow$ 11 times sum of the digits at unit and tens place.

Example 7

When a two digit number is reversed, another two digit number is obtained. Difference between original number and resulting number = 48. How many such two digit natural numbers are possible?

Solution

As we have seen that difference between original number and resulting number should be a multiple of 9, and 48 is not a multiple of 9, hence no such number exists.

Example 8

When a two digit number is reversed, another two digit number is obtained. Difference between original number and resulting number = 45. How many such two digit natural numbers are possible?

Solution

Taking a clue from the above discussions,

$9(x - y) = 45$

$\Rightarrow \quad x - y = 5$

$\Rightarrow$ Sets of (x, y) = (5, 0), (6, 1), (7, 2), (8, 3), (9, 4).

However we cannot take set (5, 0) as interchanging this will lead to '05' which is not a two digit number. Hence, number of sets possible = 4.

Example 9

Digits of a two digit number are interchanged. Sum of new number obtained and original number is a perfect square. If tens place digit is larger than units place digit, how many such two digit numbers exist?

Solution

Assume xy is the two digit number. After interchanging the digits, new number obtained $= yx$.

According to the question,

$$xy + yx = 10x + y + 10y + x$$
$$= 11\,(x + y)$$

$11\,(x + y)$ is perfect square $\Rightarrow$ Since 11 is a prime number, $(x + y)$ should be equal to 11. Now we will see different possibilities so that

$$x + y = 11.$$

Set of $(x, y) = (9, 2), (8, 3), (7, 4), (6, 5)$ [Since tens place digit is larger than units place digit].

Hence, 4 sets are possible.

2. If we reverse any three digit natural number, the difference between the original and resulting number will be equal to 99 times the difference between hundreds place digit and units place digit.

To understand this more precisely, let us see it in details:

If xyz is a three digit number,

with x = Units place digit,

and y = Tens place digit,

z = Units digit

then $xyz - zyx$

$$= 100x + 10y + z - (100z + 10y + x)$$
$$= 99\,(x - z)$$

UNIT DIGIT

To calculate the unit digit, we should focus only upon unit digit. It also means that we should not focus upon the tens digits and other digits on the left hand side.

For example, unit digit of $AB2 \times CDE4 = 8$. It is independent of the values of A, B, C, D and E.

Unit digit of 252 + 6784 = 6. It is independent of the values of the digits on the tens place or hundreds place and so on.

Calculating the Unit Digit

Let us start with calculating the unit digit of 2^5. We can simply right down $2^5 = 32$, hence unit digit of $2^5 = 2$.

However, had it been a larger number, it would not have so easy to calculate the exact value of the number, and hence not so easy to calculate the unit digit. Here comes the need to define an alternative method to find out the unit digit, without actually finding the exact value.

Step Solution to all the questions related to finding the unit digit.

Example 10

Find out the unit digit of 789^{7152}

Solution

Step 1: Unit digit of 789^{7152}

$$= \text{Unit digit of } 9^{52}$$
$$= (\text{Unit digit of the number})^{\text{Last two digits of the power}}$$

So, we will consider only the unit digit of the number and last two digits of the power.

Step 2: There are two rules—one rule for even digits and other rule for odd digits:

Rule (i): $(\text{Any odd digit at the unit place})^{\text{any natural number divisible by 4}}$ will give 1 at the unit place.

Exception: If the odd digit at the unit place = 5, then the unit digit obtained for

$$5^{\text{any natural number}} = 5$$

Rule (ii): $(\text{Any even digit at the unit place})^{\text{any natural number divisible by 4}}$ will give 6 at the unit place.

Exception: If digit = 0, then the unit digit obtained for $0^{\text{any natural number}} = 0$. For example, if we have to calculate the unit digit of $(210)^{40}$, though we can see that number is even and power is divisible by 4, unit digit will be = 0, and not 6.

Getting back to the question:

Unit digit of 789^{7152}

$$= \text{Unit digit of } 9^{52}$$
$$= (\text{Odd digit at the unit place})^{\text{any natural number divisible by 4}}$$

So unit digit obtained

$$= 1 \quad \text{[Using rule (i) as given above]}$$

Example 11

Find out the unit digit of 5984^{1932}

Solution

Step 1: Unit digit of 5984^{1932}
= Unit digit of 432

Step 2: Since the number is even, it will follow rule (ii).

(Even digit at the unit place)$^{\text{any natural number divisible by 4}}$
= 6

If the Power is not Divisible by 4

Example 12

Find out the unit digit of 598^{193}

Solution

Step 1: will remain exactly the same:
Unit digit of 598^{193}
= Unit digit of 8^{93}

Step 2: Now, since the power is not divisible by 4, we cannot use the rules directly as given in the previous two worked out examples. We will break down the power in two parts—1st part divisible by 4 and other part is the remainder when the power is divided by 4.

Unit digit of 8^{93}
= Unit digit of $(8^{92} \times 8^1)$
[93 divided by 4 gives 1 as the remainder]

Now, Unit digit of 8^{92}
= 6 [Using rule (ii)]

and Unit digit of 8^1
= 8 [Using actual calculation]

Unit digit of 8^{93}
= Unit digit of $(8^{92} \times 8^1)$
$= 6 \times 1 = 6$

Some More Facts

1. If the unit digit of any number is 0, 1, 5, 6 ⇒ It will give 0, 1, 5, 6 as the unit digit respectively irrespective of the power.
   For example:
   20^{457} will give 0 as the unit digit
   21^{457} will give 1 as the unit digit
   25^{457} will give 5 as the unit digit
   26^{457} will give 6 as the unit digit.
2. (Any digit)$^{4n+1}$ = Same digit [where n is any whole number].
   For example:
   $3^{45} = 3$ [since power = 4^5 is of the format $4n + 1$]

Test Your Learning (Unit Digit)

1. Find the unit digit of the following expressions:
   (a) 3984^{345}
   (b) 78563^{87564}
   (c) $875^{603} \times 48^{76}$
   (d) $74^{998} + 874^{3547} + 8^{762} \times 36^{765}$
   (e) $985^{445} - 87^{48} + 587^{33}$

Answer and Solution

(a) Unit digit of 3984^{345}
= Unit digit of 4^{45}
= Unit digit of $4^{44} \times 4^1 = \ldots 6 \times 4 = 4$

(b) Unit digit of 78563^{87564}
= Unit digit of $3^{64} = 1$
[since power is divisible by 4]

(c) Unit digit of 875^{603}
= Unit digit of 5^{03}
$= 5$ ($5^{\text{any number}} = 5$)

Unit digit of 48^{76}
= Unit digit of $8^{76} = 6$
(since power is divisible by 4)

Hence, Unit digit of $875^{603} \times 48^{76}$
$= 5 \times 6 = 30 \Rightarrow$ Unit digit = 0

Short Cut: Unit digit of any number with 5 as the unit digit will provide 5 at the units place. Next, 48^{76}.
Is an even number, hence the unit digit will be even. Now unit digit 5 multiplied to any even number will give 0 (zero) at the unit place. Hence, Final Unit digit = 0.

(d) Unit digit of 74^{998}
= Unit digit of 4^{98}
= Unit digit of $4^{96} \times$ Unit digit of 42
$= \ldots 6 \times 6 = \ldots 6$

Unit digit of 874^{3547}
= Unit digit of 4^{47}

= Unit digit of 4^{44} × Unit digit of 43
= ... 6 × ... 4 = ... 4

Unit digit of 8^{762}
= Unit digit of 8^{62}
= Unit digit of 8^{60} × Unit digit of 82
= ... 6 × ... 4 = 4

Unit digit of 36^{765}
= 6 (if the unit digit of the number = 6, it will provide 6 at the unit place irrespective of the power)

So, Unit digit of
$74^{998} + 874^{3547} + 8^{762} \times 36^{765}$
= ... 6 + ... 4 + (... 4 × ...6) = ... 4.

Hence, unit digit = 4.

(e) Unit digit of $985^{445} - 87^{48} + 587^{33}$
= ... 5 – ... 1 + ... 7 = ... 1

Hence, unit digit obtained = 1.

INDICES AND SURDS

Indices

Exponents or power is a way to show that how many times a number is multiplied to itself. For example, when we write y^2, we mean to say that y is multiplied to itself.

y^3 = y is multiplied to itself twice more, or a total of three times
$= y \times y \times y$.

In the example given above, y is known as base, and 3 is known as exponent or power.

Rules of Exponents

1. $a^m \times a^n = a^{m+n}$
2. $a^m/a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^m \,.\, b^m = (ab)^m$
5. $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$
6. $a^0 = 1$
7. $a^{-b} = \dfrac{1}{a^b}$
8. $a^{m/n} = \sqrt[n]{a^m}$
9. $a^{b^c} = a^{[b^c]}$

   For example, $2^{3^4} = 2^{81}$ (and not 2^{12})

   By convention, we always calculate powers from 'top to bottom'. So if we are given with $2^{3^{4^5}}$, first we will calculate 4^5 (= 1024), then 3^{1024} and finally $2^{3^{1024}}$.
10. $a^x = a^y, \Rightarrow x = y$ [a is not equal to –1 or 0 or 1].

Surds

Discussion on surds has been occurring for a long time, to be more precise, since the development of the Pythagorean Theorem. Look at the right-angled triangle with perpendicular and base = 1 and respectively.

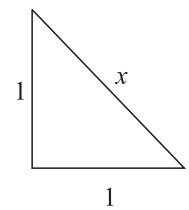


Fig. 1.5

Using Pythagorean theorem,

$x^2 = 1 + 1 = 2 \Rightarrow x = \sqrt{2}$

We have already seen that:

- A number that can be expressed as a fraction of integers (or in the format p/q, where p and q are integers and $q \neq 0$) is called a rational number. Examples of rational numbers are $\dfrac{5}{7}, -\dfrac{3}{8}$, 4.
- A number which cannot be expressed as a fraction of two integers is called an irrational number. Examples of irrational numbers are p, $\sqrt{2}$, $\sqrt[3]{11}$, etc.

Now we will define 'surd':

An irrational number involving a root is called a surd. Examples of surds are $4\sqrt[3]{5}$, $\sqrt{6}$, etc.

Rules of Surds

1. $a^{1/b} = \sqrt[b]{a}$
2. Two basic laws of surds are (for all positive real numbers a and b) as follows.

$$\sqrt{a}.\sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

3. Conjugate surds are specially related surds such as: $\sqrt{a} + \sqrt{b}$, $\sqrt{a} - \sqrt{b}$
   Their product is a rational number, as
   $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$
   $= (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
   This idea of conjugate surds is used for rationalizing the denominator.
4. Surds are further categorized as Pure Surd or Mixed Surd. A pure surd is a number without any visible natural number component added to it. For example, $\sqrt[4]{48}$ is a pure surd. A mixed surd is a number which has a visible natural number component attached to it. For example $2\sqrt[4]{3}$ is a mixed surd.

LCM AND HCF

LCM

LCM stands for Least Common Multiple. In other words, LCM is the lowest number divisible by all the numbers taken. This is also the 1st number that comes common in the list of multiples (or table) of all the numbers taken. To understand this better, let us see the following example:

1. LCM of 10, 20 and 25

1st few multiples of 10

= 10, 20, 30, 40, 50, 60, 70, 80, 90, (100), 110, 120, 130, ...

1st few multiples of 20

= 20, 40, 60, 80, (100), 120 ...

1st few multiples of 25

= 25, 50, 75, (100), 125 ...

1st number common in the list of multiples of 10, 20 and 25 = 100. Hence, 100 is the LCM of 10, 20 and 25.

Meaning of LCM

Taking a cue from above example, we can also say that 100 is the lowest number divisible by 10, 20 and 25. Giving a second thought, is 100 actually the lowest number divisible by 10, 20 and 25?

Some numbers that are less than 100 and divisible by 10, 20 and 25 are = 0, –100, –200, etc. Despite these numbers being lower than 100, why these numbers are not the LCM of taken numbers?

Reason: 0 is not a multiple of 10 or 20 or 25. At the same time, –100 or –200 are also not the multiples of 10 or 20 or 25. Since these are not the multiples, hence not the LCM.

Example 13

LCM of 10, –20 and –25

LCM is defined only for positive numbers. In other words, LCM is defined only for positive fractions and natural numbers—positive numbers.

Important: In other words, LCM for negative numbers as well as zero is not defined. Hence, no LCM of 10, –20 and –25 is possible.

Process to find out LCM

Standard method: Standard method of finding LCM is through factorization of numbers. Let us understand this 3-step process:

Step 1: Factorize the numbers into prime numbers.

Step 2: Raise each prime factor to its maximum available power and multiply these.

Finding LCM of 10, 20 and 25

Step 1: Prime numbers present are 2 and 5.

Step 2: Maximum power of 2 occurs in

20 = 22

and maximum power of 5 occurs in

25 = 52

Hence, LCM = 22 × 52

This method leads us to Straight Line method of finding LCM—Straight Line method of finding LCM involves taking numbers in succession one by one and then observing which new prime factors are occurring.

Example 14

Find LCM of 24, 30, 52, 76.

Solution

Step 1: Start observing the prime factors in all these numbers:

24 has 2 and 3 as the prime factors.
30 has 2, 3 and 5 as the prime factors.
52 has 2 and 13 as the prime factors.
76 has 2 and 19 as the prime factors.
So, the prime factors present
= 2, 3, 5, 13, 19

Step 2: Maximum powers of all these prime numbers are:

Maximum power of
$2 = 2^3$ (occurs in 24)
Maximum power of
$5 = 5^1$ (occurs in 30)
Maximum power of
$3 = 3^1$ (occurs in 24 and 30 both)
Maximum power of
$13 = 13^1$ (occurs in 52)
Maximum power of
$19 = 19^1$ (occurs in 76)
Hence, LCM of 24, 30, 52, 76
$= 2^3 \times 3^1 \times 5^1 \times 13^1 \times 19^1 = 29640$

Now we'll see the Straight Line method of finding LCM for the same numbers:

Let us start with 24. Take next number 30. And think what number should be multiplied to 24 so that 30 divides 24 × N (where N is the number multiplied to 24). **It is going to be the factors of 30 which are not present in 24**. Now $30 = 2^1 \times 3^1 \times 5^1$ and $24 = 2^3 \times 3$. Factors of 30 which are missing in $24 = 5^1$. Hence, LCM of 24 and 30 = 24 × 5.

[Alternatively it we start with 30, and then we will be required to think that which number should be multiplied to 30 so that (30 × K) is divisible by each of 30 and 24.

Now $30 = 2^1 \times 3^1 \times 5^1$
and $24 = 2^3 \times 3$

Factors of 24 that is not present in $30 = 2^2$. Hence, LCM of 30 and 24 = 30 × 4]

Next number is $52 = 13^1 \times 2^2$. 2^2 is already present in the LCM (that we have calculated till now) of 24 and 30, hence we will multiply it by 13^1 (which is not present in 24 × 5). So, LCM of 24, 30 and 52 = 24 × 5 × 13.

Next number is $76 = 19^1 \times 2^2$. It can be seen that 2^2 is already present in the LCM (that we have calculated till now) of 24, 30 and 52. Hence, we will multiply this by 19^1.

Hence, LCM of 24, 30, 52, 76
= 24 × 5 × 13 × 19 = 29640

Straight line approach can also be shown by the following expression:

LCM (A, B, C)
= LCM (LCM (A, B), C)
= LCM (A, LCM (B, C))
= LCM (B, LCM (A, C))

Similarly, LCM (A, B, C, D)
= LCM (LCM (A, B), LCM (C, D))
and so on.

Important: In a nutshell, LCM is Commutative, Associative and Distributive.

HCF

HCF stands for Highest Common Factor. It is also known as GCD (Greatest Common Divisor). This is the largest number that can divide the given set of numbers.

A number is a common factor of two or more than two numbers if it divides all the numbers.

For example, let us see the factors of some of the numbers:

Factors of 20 = 1, 2, 4, 5, 10, 20
Factors of 40 = 1, 2, 4, 5, 8, 10, 20, 40
Factors of 45 = 1, 3, 5, 9, 15, 45

Common factors of 20, 40 and 45 = 1 and 5 (these two factors will divide 20, 40 and 45).

The Highest Common Factor of the numbers is, as the name suggests, the largest factor common across all the given numbers.

In the above example, 5 is the Highest Common Factor.

In other words, in the above example, 5 is the largest number which will divide each of 20, 40 and 45.

Properties of HCF

1. HCF $(a, a) = a$
2. HCF (a, b) = HCF (b, a)
3. If $a > b > 0$, and $a = bq + r$, then HCF (a, b) = HCF (b, r).
   Let us understand this with the help of an example:
   Compute HCF (803, 154).
   HCF (803, 154) = HCF (154, 33)
   since $803 = 154 \cdot 5 + 33$
   HCF (154, 33) = HCF (33, 22)
   since $154 = 33 \cdot 4 + 22$
   HCF (33, 22) = HCF (22, 11)
   since $33 = 22 \cdot 1 + 11$
   HCF (22, 11) = 11
   since $22 = 11 \cdot 2 + 0$
   Hence,
   HCF (803, 154) = 11.
   This is also known as Euclidean Algorithm for finding HCF.
4. If HCF of $(a, b) = N$ (where a and b are natural numbers and $a > b$), then HCF of $[b, (a + b)]$
   $= [b, (a - b)] = N$
   For example, HCF of (28, 20) = 4
   Hence, HCF of [20, (28 + 20)]
   = HCF of [20, (28 – 20)] = 4
5. For all a, b, n (non-zero values),
   HCF $(na, nb) = n$ HCF (a, b)
   Same rule is applicable for LCM too.
   LCM $(na, nb) = n$ LCM (a, b)
   For example,
   HCF (15, 25) = 5,
   hence HCF (15 × 10, 25 × 10)
   = 5×10 = 50

Similarly,
LCM (15, 25) = 25 × 3,
hence, LCM (15 × 10, 25 × 10)

Summarizing LCM and HCF

It is very essential to understand the mechanism to find out LCM and HCF. We can simply understand the mechanism to find out lowest common multiple and highest common factor through this example:

Example 15

Find out LCM and HCF of 16, 12, 24.

Solution

Number	Multiples	Factors
16	16, 32, 48, 64, 80, 96, 112, 128, ...	1, 2, 4, 8, 16
12	12, 24, 36, 48, 60, 72, 84, 96, 108, ...	1, 2, 3, 4, 6, 12
24	24, 48, 72, 96, 120, 144, 168, 192, ...	1, 2, 3, 4, 6, 8, 12, 24
	Common Multiple	Common Factor
	48	1, 2, 4, ...
	Least common multiple	Highest common factor
	48	4

Finally analogy between Union of the sets and Intersection can be understood with the help of LCM and HCF of sets. Consider this for the following example:

Finding LCM and HCF of 20 and 30.

$20 = 2^2 \times 5^1$

$30 = 2^1 \times 3^1 \times 5^1$

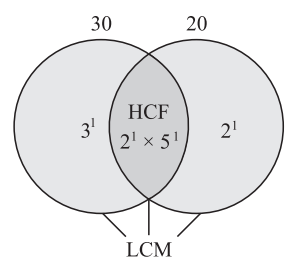


Fig. 1.6

LCM $= 2^2 \times 3^1 \times 5^1$
and HCF $= 2^1 \times 5^1$

It can be seen that HCF is the intersection of the sets, and LCM is the union of the sets.

Application of LCM/HCF

Case 1: Time and work: In the questions involving individual efficiency, we use the concept of LCM to assume the total units of work instead of going through the actual method of solving the question.

Example 16

Tatto can do a work in 10 days and Tappo can do same work in 12 days. How many days will it take if both of them start working together?

Solution

Normal Method:

Work done by Tatto in one day
= 1/10 of the total work

Work done by Tappo in one day
= 1/12 of the total work

Total work done by Tatto and Tappo in one day

$$= \left[\frac{1}{10} + \frac{1}{12}\right] = \frac{11}{60} \text{ of the work}$$

Hence, total time taken by Tatto and Tappo to finish the work

$$= \frac{60}{11} = 5\frac{5}{11} \text{ days}$$

LCM Method:

Let us assume total work
= LCM of (10, 12) units = 60 units

Now since 60 units of work is being done by Tatto in 10 days, so Tatto is doing 6 units of work per day and similarly, Tappo is doing 5 units of work per day. Hence, they are doing 11 units of work in one day together.

So, time taken to finish the work by Tatto and Tappo

$$= \frac{\text{Total work}}{\text{Work done in one day}}$$

$$= \frac{60}{11} = 5\frac{5}{11} \text{ day.}$$

Case 2: Number system: Tolling the bell

Example 17

There are two bells in a temple. Both the bells toll at a regular interval of 66.66 sec and 50 sec respectively. After how much time will they toll together for the first time?

Solution

Time taken by 1st bell to toll = 66.66 sec

Time taken by 2nd bell to toll = 50 sec

LCM (66.66, 50) = 200 sec

It can be observed here that Mathematical interpretation of both the questions are same, just the language has been changed.

Case 3: Number system: Number of rows

Example 18

There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains same number of fruits of one type. What is the minimum number of rows required for this to happen?

Solution

We can put one fruit in one row, and in (24 + 36 + 60) 120 rows, we can arrange all the fruits. Or, we may put two fruits in one row and can arrange all the fruits in 60 rows. But for the rows to be minimum, number of fruits should be maximum in one row.

HCF of 24, 36, 60 = 12,

so 12 fruits should be there in one row.

Hence, number of rows
= 10

Case 4: Number system: Finding remainder

Example 19

There is a natural number which when divided by 4 and 5 gives 3 as the remainder. What is the lowest three digit number which satisfy this condition?

Solution

Let us assume that there is no remainder. So, number has to be a multiple of LCM of 4 and 5. Now, LCM (4, 5) = 20

But there is a remainder of 3 when divided by 4 and 5. So, the number will be 3 more than 20 = 23, and so on 43, 63, etc. in general, the number will be of the form $(20N + 3)$.

Hence, numbers are 23, 43, 63, 83,103 and so on ...

So, 103 is the answer.

Standard Formula

1. LCM × HCF = Product of two numbers. This formula can be applied only in case of two numbers.
2. LCM of fractions
$$= \frac{\text{LCM of numerator of all the fractions}}{\text{HCF of numerator of all the fractions}}$$
It the fractions are $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$, then LCM of fractions
$$= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$
$$= \frac{\text{LCM of } (x, a, m)}{\text{HCF of } (y, b, n)}$$
LCM of decimals can be calculated by converting decimals into fractions first, and then using above method.
3. HCF of fractions
$$= \frac{\text{HCF of numerator of all the fractions}}{\text{LCM of numerator of all the fractions}}$$
If the fractions are $\frac{x}{y}, \frac{a}{b}$ and $\frac{m}{n}$, then HCF of fractions
$$= \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$
$$= \frac{\text{LCF of } (x, a, m)}{\text{LCM of } (y, b, n)}$$
HCF of decimals can be calculated by converting decimals into fractions first, and then using above method (Fig. 1.7).

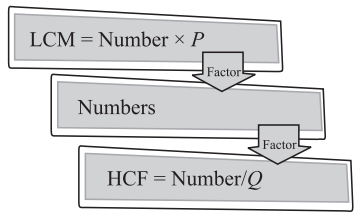


Fig. 1.7

4. LCM for a given set of numbers has to be greater than or equal to the HCF of the same set of numbers.
Whatever numbers do we take, LCM is a multiple of these numbers.
So LCM ≥ Numbers
At the same time, HCF is a factor of Numbers or Numbers taken are multiple of HCF.
Hence, number ≥ HCF
Combining these two, we get
LCM ≥ Numbers ≥ HCF

Corollary 1: LCM can be equal to HCF only when all the numbers taken are equal. HCF obtained in this case is also going to be largest possible HCF.

Corollary 2: It also gives that LCM will be always divisible by HCF.

Test Your Learning (LCM and HCF)

1. Find the LCM of:
(i) 36, 46, 56, 76
(ii) 22, 32, 42, 52
(iii) 38, 56, 78, 82

2. LCM of 1st 10 natural numbers is *N*. What is the LCM of 1st 15 natural numbers?

3. What is the ratio of the LCM of first 20 natural numbers and first 10 natural numbers?
(i) 11 × 13 × 17 × 19
(ii) 2 × 11 × 13 × 17 × 19
(iii) 11 × 13 × 17
(iv) 2 × 11 × 13 × 17 × 19
(v) None of these

4. What is the HCF of:
   (i) 12, 15, 18, 21
   (ii) 120, 150, 180, 210?
5. What is the HCF of 72, 47, 78, 46?
6. What is the HCF of x^2, x^4, x^5?
7. What is the HCF of $(x^{24} - y^{24})$, $(x^{16} - y^{16})$, $(x^{12} - y^{12})$?
8. What is the HCF of any two consecutive natural numbers?
9. What is the HCF of any two consecutive even natural numbers?
10. What is the HCF of any two consecutive odd natural numbers?

Test Your Learning (LCM)

1. (i) $36 \times 23 \times 7 \times 2 \times 19$
   (ii) $22 \times 16 \times 21 \times 13$
   (iii) $19 \times 7 \times 8 \times 13 \times 3 \times 41$
2. We'll have to see which new prime numbers are occurring after 10 till 15. Those are 11 and 13.
   Hence, new LCM = $N \times 11 \times 13$.
3. To solve this question, we just have to understand that which factors (of numbers exceeding 10) will be multiplied to the LCM of first 10 natural numbers.
   So what numbers should be multiplied? Did you get $11 \times 13 \times 17 \times 19$?
   Observe once again—LCM of first 10 natural numbers will have 2^3 only, whereas LCM of first 20 natural numbers should have 2^4 (because of presence of 16).
   Hence, ratio of (1st 20 natural numbers/LCM of 1st 10 natural numbers)
   $= 11 \times 13 \times 17 \times 19 \times 2$.
4. (i) Step 1 goes like →
   $12 = 2^2 \times 3^1$; $15 = 3^1 \times 5^1$;
   $18 = 2^1 \times 3^2$; $21 = 3^1 \times 7^1$
   Step 2 goes like → 3 (only prime factor that is common across all the numbers).
   Step 3 goes like → 3^1 (minimum power of 3 = 1). Hence, HCF = 3.
   (ii) 30
5. Answer is 1. There is no number other than 1 common in all the numbers. In other words, 1 is the largest number that will divide all the numbers.
6. We can take out only 'x' common out of the given numbers. Hence, HCF = x.
7. Answer is: $x^{\text{HCF of 24, 16 and 12}} - y^{\text{HCF of 24, 16 and 12}} = x^4 - y^4$
   Note: You may use the solution given above as a rule also.
   Rule: HCF of $(x^n - y^n)$ and $(x^m - y^m)$
   $= x^{\text{HCF of } n \text{ and } m} - y^{\text{HCF of } n \text{ and } m}$.
8. HCF of any two consecutive natural numbers = 1.
9. HCF of any two consecutive even natural numbers = 2.
10. HCF of any two consecutive odd natural numbers = 1.

DIVISIBILITY RULES

For 2: If unit digit of any number is 0, 2, 4, 6 or 8, then the number will be divisible by 2.

For example: 1240, 1352, 1088, etc.

For 3: If sum of digits of any number is divisible by 3, then the number will be divisible by 3.

For example: 1323, 3456

For 4: If last two digits of a number is divisible by 4, then that number will be divisible by 4.

For example: 4236, 2488, 256, etc.

For 5: If last digit of the number is 5 or 0, then the number will be divisible by 5.

For 6: If last digit of the number is divisible by two and sum of all the digits of number is divisible by 3, then that number will be divisible by 6.

For example: 3816, 4524, etc.

For 7: The integers is divisible by 7 if and only if the difference of the number of its thousands and the remainder of its divisible by 1,000 is divisible by 7.

For 8: If last 3 digits of a number is divisible by 8, then the number itself will be divisible by 8.

For 9: If sum of digits of the number is divisible by 9, then the number will be divisible by 9.

For 10: If the unit digit of the number = 0, then the number will be divisible by 10.

For 11: A number is divisible by 11, if the difference between the sum of the digits at the even places and the sum of the digits at the odd places is divisible by 11 (zero is divisible by 11).

For example: 6595149 is divisible by 11 as the difference of 6 + 9 + 1 + 9 = 25 and 5 + 5 + 4 = 14 is 11.

For 12: If the number is divisible by 3 and 4, then the number will be divisible by 12.

For example: 144, 348.

For 16: A number is divisible by 16, if the number formed by the last 4 digits of the given number is divisible by 16.

PRACTICE EXERCISE

EXERCISE 1

Direction for questions 1–20: ***Solve these questions independently.***

1. If $N = (n - 1)(n + 1)$ is divisible by 8, then n is certainly:
(a) An even natural number
(b) An odd natural number
(c) Any natural number
(d) Any rational number

2. The square of any natural number cannot be in the form of:
(a) $5m$ (b) $5m + 1$
(c) $5m + 3$ (d) $5m + 4$

3. If the number 13915 can be factorized as $xy^2 z$, where x, y and z are prime numbers, then what is the value of $x + y + z$?
(a) 28 (b) 39
(c) 49 (d) None of these

4. Let x, y and z be distinct integers, that are odd and positive. Which of the following statements is not true?
(a) xyz^2 is odd
(b) $(x - y)^2 z$ is even
(c) $(x + y - z)(x + y)$ is even
(d) $(y + z)(x + y - z)$ is odd

5. Anil had to do a multiplication. Instead of taking 35 as one of the multipliers, he took 53. As a result, the product went up by 540. What is the new product?
(a) 1050 (b) 540
(c) 1440 (d) 1590

6. When you reverse the digits of the number 13, the number increases by 18. How many total two-digit numbers increase by 18 when their digits are reversed?
(a) 5 (b) 6
(c) 7 (d) 8

7. How many natural numbers up to 200 are divisible by 7?
(a) 28 (b) 29
(c) 27 (d) 100

8. How many natural numbers up to 500 are divisible by 23?
(a) 23 (b) 27
(c) 21 (d) 19

9. How many natural numbers up to 200 are divisible 2 and 3 both?
(a) 35 (b) 33
(c) 29 (d) 27

10. How many natural numbers between 100 and 300 are divisible by 11?
(a) 11 (b) 10
(c) 12 (d) 18

11. $N = 2^{x+1} \times 3^{x+4} \times 5^{x+4} \times 7^{x+7}$. If N ends with six zeroes, then what is the minimum value of x?

(a) 5 (b) 6
(c) 4 (d) None of these

Direction for questions 12–13: ***Go through the passage given below and solve the questions based on it.***

Sweetu has 60 apples, 156 oranges and 204 guavas. She distributes the fruits equally among a group of kids such that each kid gets maximum number of fruits, and all the kids get exactly one type of fruit.

12. Find the number of fruits that each child will get:
(a) 12 (b) 24
(c) 18 (d) 6

13. Find the number of children.
(a) 12 (b) 24
(c) 18 (d) 35

14. Determine the HCF of $25ab^2c^3$, $80ab^3c^2$ and $12a^2b^2c$.
(a) $10a^2b^2c^3$
(b) $5ab^2c$
(c) ab^2c
(d) $10ab^2c^2$

15. What is the least number of soldiers in a regiment, such that they stand in rows of 18, 15 and 25 and also form a solid square?
(a) 1800 (b) 225
(c) 900 (d) None of these

16. Find the number of pairs of two natural numbers having product = 3600 and HCF = 30.
(a) 0 (b) 1
(c) 2 (d) 5

17. Four bells toll at regular interval of 24 sec, 36 sec, 40 sec and 48 sec respectively. How many times they next toll together in 1 hour (not counting $T = 0$ situation)?
(a) 10 (b) 4
(c) 5 (d) 6

18. How many zeroes will be there at the end of the number: $11^{(5!)!} - 1$?
(a) 0 (b) 1
(c) 11 (d) 49

19. A and B are two natural numbers, each divisible by 5. Which of the following is not necessarily true?
(a) $A + B$ is divisibly by 5
(b) $A + B$ is divisible by 10
(c) $A - B$ is divisible by 5
(d) All of these are definitely true

20. I have to pack some apples into some boxes. If I pack 3, or 4 or 5 or 6 apples in a box, I am left with one apple in each of the cases. However if I pack 7 apples in a box, I am left with none. What is the minimum number of apples that I have?
(a) 301 (b) 601
(c) 61 (d) 436

ANSWER KEY

Exercise 1

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (c)
9. (b)	**10.** (d)	**11.** (a)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (b)
17. (c)	**18.** (b)	**19.** (b)	**20.** (a)				

HINTS AND EXPLANATIONS

Exercise 1

1. **Method 1:**
   (n – 1) and (n + 1) are either two consecutive odd natural numbers or two consecutive even natural numbers. For N to be divisible by 8, (n – 1) and (n + 1) both should be even number. We also understand that one of the two consecutive even natural number will be divisible by 2 and other would be divisible by 4. Hence, product of (n – 1) and (n + 1) would be divisible by 8.
   Now, for (n – 1) and (n + 1) to be even numbers, n should be odd natural number. Hence, option (b) is the answer.
   Method 2:
   Let us go through the options:
   Option (a) An even natural number
   For example, take $n = 4$.
   So $N = (n - 1)(n + 1) = 3 \times 5 = 15$,
   which is not divisible by 8. Hence, option (a) is not the answer.
   Option (b) An Odd natural number.
   For example, take $n = 5$.
   So $N = (5 - 1)(5 + 1) = 4 \times 6 = 24$,
   which is divisible by 8. Hence, option (b) is the answer.
   Option (c) Any natural number—Since option (a) is not true, hence option (c) also will not be true.
   Option (d) Any rational number—Since option (a)is not true, hence option (d) also will not be true.
   Hence, option (b) is the answer.

2. Unit digit of any square number can be the following: 0, 1, 4, 5, 6, 9. It cannot be 2, 3, 7 or 8. Using this, we can say option (c) is the answer as 3 or 8 cannot be the unit digit of any perfect square.

3. 13915 is divisible by 5 = 2783 × 5. Now we are required to factorize the number
   $2783 = 11^2 \times 23$
   Hence, $13915 = 5 \times 11^2 \times 23$
   $\Rightarrow x = 5, y = 11$ and $z = 23$.
   So, $x + y + z = 39$.
   Hence, option (b) is the answer.

4. Assume any three odd numbers 1, 3 and 5. Now let us check the options one by one:
   Option (a) Product of any number of odd numbers is odd. Hence, option (a) is true.
   Option (b) $(x - y) = (1 - 3) =$ Even
   $\Rightarrow$ Product will be even. Hence, option (b) is true.
   Option (c) $(x + y) = (1 + 3) = 4 =$ Even
   $\Rightarrow$ Product will be even. Hence, option (c) is true.
   Option (d) $(y + z) = (3 + 5) = 8 =$ Even
   $\Rightarrow$ Product will be even. Hence, option (d) is false and it is the answer.

5. Let the number be x.
   Increase in product $= 53x - 35x$
   $= 18x = 18x = 540$
   $\Rightarrow x = 30$
   Hence, new product $= 53 \times 30 = 1590$.
   Hence, option (d) is the answer.

6. We know that $xy - yx = 9(x - y)$, where xy is a two digit number and $xy \geq yx$.
   So, $x - y = 2$
   The possible pairs of (x, y) is (3, 1) (4, 2) (5, 3) (6, 4), (7, 5) (8, 6) (9, 7)
   So, there are 7 possible numbers are viz. 13, 24, 35, 46, 57, 68, 79.
   Hence, option (c) is the answer.

7. Starting from 7, every 7th number will be divisible by 7 (*e.g.*, 14, 21, etc). This is same as Quotient obtained when 200 is divided by 7 is 28. Hence, option (a) is the answer.

8. Using the concept given in the previous question, quotient obtained when 500 is

divided by 23 is 21. Hence, option (c) is the answer.

9. A number divisible by 2 and 3 both will be divisible by the LCM of 2 and 3 = 6. Quotient obtained when 200 is divided by the LCM of 2 and 3, *i.e.*, 6 is 33. Hence, option (b) is the answer.

10. Quotients obtained when 300 and 100 are divided by 11 are 27 and 9. Hence, between 300 and 100, there are 27 – 9, *i.e.*, 18 numbers. Hence, option (d) is the answer.

11. In this question, going through the options is the best way of solving this question. Using the options, option (a) is the answer.

12. HCF of (60, 156, 204) = 12. Hence, each child will get 12 fruits. Hence, option (a) is the answer.

13. Number of children

$$= \frac{\text{Total number of fruits}}{\text{Number of children given to each child}}$$

$$= \frac{60 + 156 + 204}{12} = 35$$

Hence, option (d) is the answer.

14. $25ab^2c^3 = 5^2 \times a \times b^2 \times c^3$
$80ab^3c^2 = 5 \times 24 \times a \times b^3 \times c^2$
$12a^2b^2c = 3 \times 2^2 \times a^2 \times b^2 \times c$
Hence, required HCF = $a \times b^2 \times c$. Hence, option (c) is the answer.

15. Number of soldiers should be a multiple of 18, 15 and 25 ⇒ Number should be a multiple of LCM (18, 15, 25) = 450. Now lowest multiple of 450 that is a perfect square = 900. Hence, option (c) is the answer.

16. Since HCF = 30, numbers can be assumed to be $30a$ and $30b$.
Hence, $30a \times 30b = 3600 \Rightarrow ab = 4$
⇒ There is only one set possible $a = 4$, $b = 1$ [we cannot take $a = b = 2$, as in that case HCF will become = 60].
Hence, numbers are 30, 120. Hence, option (b) is the answer.
[Understand that we are considering 'un-ordered pairs'. Hence, (4, 1) and (1, 4) are the same] sets.

17. Time taken to toll together = LCM of 24, 36, 40 and 48 = 720 seconds = 12 minutes. So they will toll together 60/12 = 5 times in 1 hour.
Hence, option (c) is the answer.

18. Unit digit of the $11^{(5!)!} = 1$. Hence, unit digit of $11^{(5!)!} - 1 = 0$. This is the only zero at the end (None of the powers of 11 ends with 01). Hence, option (b) is the answer.

19. If A and B are divisible by 5, then $(A + B)$ is also divisible by 5. It may or may not be divisible by 10. For example, for $A = 10$, $B = 15$, $A + B$ is not divisible by 10. Hence, option (b) is not necessarily true.

20. **Method 1:** LCM of 3, 4, 5, 6 = 60. So the number should be of the format $60K + 1$. 1st number of the format $60K + 1$ which is divisible by 7 = 301.
Method 2: Go through the options.
Option (a) satisfies the given conditions. Hence, option (a) is the answer.

2 Percentage

WHAT IS PERCENTAGE?

Percentage is a way to express a number or quantity as a fraction of 100 (*per cent* meaning 'per hundred').

It is denoted using the sign "%". For example, 45% (read as "forty-five per cent") is equal to $\frac{45}{100} = 0.45$.

Percentage can be also seen as a common platform. It can be further understood with the help of the Table 2.1 given below, which gives us the marks obtained by different students in their class-10 exams.

A student in America gets 100 marks out of 1,000 marks. If we convert the marks obtained on a base of 100, then it becomes 10. According to the definition of percentage, this student has obtained 10% marks.

In fact, we use percentage as a common platform of 100 to compare the given values.

Using only 'Marks Obtained' we cannot say, the student from which country has performed best. We need to know 'Total Marks' too.

Needless to say, if we know the percentage marks obtained only (as given in rightmost column), we can find out the best performer. So, this way, percentage provides a common platform for comparing similar quantities.

Before we move ahead, it is important to understand the basic statements used in percentage.

Basic Statement 1: What is x % of y?

$\Rightarrow \quad \frac{x \times y}{100}$

It can also be seen that x % of $y = y$ % of x.

Example, 4.5% of 20,000 = 20,000% of 4.5.

This one simple fact can be used to divide or multiply any number by 50 or 25 or so. Let us see this with the help of an example—We are trying to find out the value of 25 × 32, which is nothing but $32 \cdot \frac{100}{4} = 800$. Similarly, if we have to divide any number by 50, we should be multiplying the number by 100 and dividing it by 2 finally.

Using this, we can see that if we have to calculate 24% of 25 (or any other calculation of similar nature), it is better to find out 25% of 24 $\left(24 \times \frac{1}{4}\right) = 6 \left\{25\% = \frac{25}{100} = \frac{1}{4}\right\}$.

1. What is 20% of 50% of 40% of 20?

Solution

%age means 'per hundred'

Table 2.1

Student in country	Marks obtained	Total Marks	Marks obtained/100 marks
America	100	1,000	10%
India	25	25	100%
China	45	300	15%
France	50	100	50%

So, 20% of 50% of 40% of

$$20 = \frac{20}{100} \cdot \frac{50}{100} \cdot \frac{40}{100} \cdot 20 = 0.8$$

What we can observe here is that even if we change the order of values here, the final result will be same. So, 20% of 50% of 40% of 20 = 50% of 40% of 20% of 20.

Basic Statement 2: What %age of x is y?

$$= \frac{y \cdot 100}{x}$$

Basic Statement 3: Change

(*a*) **Percentage change**

$$= \frac{\textbf{Change}}{\textbf{Initial value}} \cdot \mathbf{100}$$

(*b*) **Percentage point change: It is the numerical difference between the values for which we have to calculate change.**

Let us assume some values to understand the above written concept (Table 2.2):

Table 2.2

Market share	2008–09	2009–10
Maruti	40%	48%
Honda	30%	26%

Percentage change in the market share of Maruti over the years $= \frac{48 - 40}{40} \times 100 = 20\%$

Percentage point change in the market share of Maruti over the years = 48% – 40% = 8%

Similarly, if we have to increase any quantity N by S%, then final quantity $= N\left(1 + \frac{S}{100}\right)$ and when the same quantity N is to be decreased by S%, then final quantity $= N\left(1 - \frac{S}{100}\right)$.

Observations

(i) An increase of 100% is equal to the final amount becoming 200% of initial value or twice the initial value.

(ii) An increase of 500% is equal to the final amount becoming 600% of initial value or six times the initial value.

(iii) A decrease of 100% is equal to the final amount becoming zero. Hence, 0% of initial value.

(iv) Concept of Multiplier: Multiplier is the factor which provides the final value after getting multiplied with initial value.

$$100 \rightarrow 20\% \uparrow \rightarrow 100 \times 1.2 = 120$$
$$100 \rightarrow 20\% \downarrow \rightarrow 100 \times 0.8 = 80$$

In the above examples, 1.2 and 0.8 are the multipliers obtained as a result of increasing by 20% and decreasing by 20% respectively.

$$150 \rightarrow 30\% \uparrow \rightarrow 150 \times 1.3 = 195$$
$$150 \rightarrow 30\% \downarrow \rightarrow 150 \times 0.7 = 105$$

In the above examples, 1.3 and 0.7 are the multipliers obtained as a result of increasing by 30% and decreasing by 30% respectively.

$$210 \rightarrow 27\% \uparrow \rightarrow 210 \times 1.27 = 266.7$$
$$210 \rightarrow 27\% \downarrow \rightarrow 210 \times 0.73 = 153.3$$

In the above examples, 1.27 and 0.73 are the multipliers obtained as a result of increasing by 27% and decreasing by 27% respectively.

So, if final value and %age increase or %age decrease is given and we have to find out the initial value, then it can be done in the similar way.

Using $S \rightarrow 30\% \uparrow \rightarrow S \times 1.3 = 195$

So, if final value 195 and 30%↑ is given, then initial value of

$$S = \frac{195}{1.3} = 150.$$

Some More Type of Questions

Type 1: Questions based on Simple Calculations in Percentage

2. Anoop got 273 marks in his graduation exam and got 5% more than the pass percentage. If Siddharth got 312 marks, then by what percentage above the pass marks did he pass the exam?

(a) 9% (b) 12.5%

(c) 20% (d) 25%

Solution

Let us assume that pass marks = N marks. Since Anoop got 5% more than the pass percentage, we can say the multiplier here is 1.05. So, $1.05 \times N = 273 \Rightarrow N = 260$. Siddharth got 312 marks. $\Rightarrow$ 52 marks more than the pass marks. So he obtained $\frac{52}{260} \cdot 100 = 20\%$ more marks than the pass marks. Hence, option (c) is the answer.

3. Express 5:2 as percentage.
(a) 12.5% (b) 40%
(c) 80% (d) 250%

Solution

$5:2 = \frac{5}{2} \cdot$ To convert this in percentage, we need to multiply the ratio by

$$100 = \frac{5}{2} \cdot 100\% = 250\%.$$

Hence, option (d) is the answer.

4. A's income is 60% of B's income, and A's expenditure is 70% of B's expenditure. If A's income is 75% of B's expenditure, find the ratio of A's saving to B's saving.
(a) 5:1 (b) 1:5
(c) 7:2 (d) 2:7

Solution

Let us assume that income of B = ₹ 100, then income of A = ₹ 60. Now using "A's income is 75% of B's expenditure", B's expenditure = 60/0.75 = ₹ 80. So, B's savings = ₹ 20.

Expenditure of B = ₹ 80 $\Rightarrow$ expenditure of A = 70% of 80 = ₹ 56.

So savings of A = income of A – expenditure of A = ₹ 60 – ₹ 56 = ₹ 4.

So savings of A : savings of B = ₹ 4 : ₹ 20 = 1:5. Hence, option (b) is the answer.

5. Salary of Anil and Vinit is 20% and 30%, respectively less than salary of Dheeraj. By what percentage is the salary of Anil more than salary of Vinit?
(a) 33.33% (b) 50%
(c) 10% (d) 14.28%

Solution

Assume that the salary of Dheeraj = ₹ 100 $\Rightarrow$ salary of Anil = ₹ 80 and salary of Vinit = ₹ 70.

To calculate "by what percentage is the salary of Anil more than salary of Vinit", we would 1st calculate salary of Anil is how much more than the salary of Vinit = ₹ 10. Since "than the salary of Vinit" is to be calculated, salary of Vinit will be the base (denominator). Percentage increase $= \frac{10}{70} \cdot 100 = 14.28\%$. Hence, option (d) is the answer.

6. If 120 is 20% of a number, then 120% of that number will be:
(a) 20 (b) 120
(c) 360 (d) 720

Solution

Let the number be x.

Then, 20% of $x = 120 \leftrightarrow \left(\frac{20}{100} \times x\right)$

$$= 120 \leftrightarrow x = \left(\frac{120 \times 100}{20}\right) = 600$$

Hence, 120% of $x = \left(\frac{120}{100} \times 600\right) = 720.$

Hence, option (d) is the answer.

Alternatively, if 20% = 120, so 120% will be six times of 120 = 720.

7. 30% of 28% of 480 is the same as:
(a) 15% of 56% of 240
(b) 60% of 28% of 240
(c) 60% of 56% of 240
(d) None of these

Solution

Clearly, 60% of 28% of 240

$$= \left(\frac{60}{100} \times \frac{28}{100} \times 240\right)$$

$$= \left(\frac{30}{100} \times \frac{28}{100} \times 2 \times 240\right)$$

$$= \left(\frac{30}{100} \times \frac{28}{100} \times 480\right)$$

= 30% of 28% of 480.

Hence, option (b) is the answer.

8. When 35 is subtracted from a number, it reduces to its 80%. What is four-fifth of that number?

(a) 70 (b) 90
(c) 120 (d) 140

Solution

Let the number be x.

Analyse the statement and look at the preposition "to"—it reduces to its 80%—it means a loss of 20% = 35 subtracted from the number. Hence, 100% = 35 × 5 = 175.

Four-fifth of number $= 175 \cdot \frac{4}{5} = 140$

Hence, option (d) is the answer.

9. 45% of 150 + 35% of 170 = ? % of 317.5

(a) 30 (b) 35
(c) 45 (d) None of these

Solution

Let N% of 317.5

= 45% of 150 + 35% of 170

Then, $N \times \frac{317.5}{100}$

$$= \left(45 \times \frac{150}{100} + 35 \times \frac{170}{100}\right)$$

$$= 67.5 + 59.5 = 127$$

So, $N = \left(\frac{127 \times 100}{317.5}\right) = 40$

Hence, option (d) is the answer.

Alternatively, this question can be done with the help of options too.

Type 2: Questions based on Percentage Change

10. Tatto's working hours per day were increased by 25% and her wages per hour were increased by 20%. By how much per cent were her daily earnings increased?

(a) 20% (b) 25%
(c) 50% (d) 45%

Solution

Assume that she works for 10 hours daily at the rate of ₹ 10/hour.

Hence, her old earning = 10 × 10 = 100

New earning = 12.5 × 12 = 150. Hence, net percentage increase = 50%. Hence, option (c) is the answer.

11. If the price of an article rose by 25% every odd year and fell by 20% every even year, what would be the percentage change after 180 years?

(a) 10% increase (b) 25% increase
(c) No change (d) 20% decrease

Solution

There are 90 odd years and 90 even years. Or in other words, 90 pairs of (25% rise and 20% fall) are there. Net result of rise of 25% and fall of 20% = 0% change. Hence, percentage change after 180 years = 0%. Hence, option (c) is the answer.

12. Vijyendra Bisht saves 10% of his total salary. Next year he increases his expenses by 20% but his percentage savings remain the same. What is the percentage increase in his salary next year?

(a) 10% (b) 20%
(c) 40% (d) 16.66%

Solution

Let us assume salary = ₹ 100. So savings = ₹ 10 and expenses = ₹ 90.

New expenses = ₹ 108

Though percentage saving remains the same = 10%. So old expenses percentage = new expenses percentage = 90%.

So, ₹ 108 = 90% of new salary ⇒ new salary = ₹ 120. Hence, percentage increase in the salary = 20%. Hence, option (c) is the answer.

13. The length and breadth of a rectangular plot are increased by 20% and 50%, respectively. Then the new area is how many times the original area?

(a) 5/8 (b) 9/5
(c) 3/7 (d) 7/3

Solution

Let us assume that the length of rectangle = 10 units and breadth of the rectangle = 10 units.

Hence, area = length × breadth = 10 × 10
= 100

New length = 1.2 × 10 = 12 units

New breadth = 1.5 × 10 = 15 units

Hence, new area = 12 × 15 = 180 units

Hence, new area/old area $= \frac{180}{100} = \frac{9}{5}$

Hence, option (b) is the answer.

14. The entry fee in an exhibition was ₹ 10. Later this was reduced by 25%, which increased sales revenue by 20%. Find the percentage increase in the number of visitors.

(a) 54 (b) 57
(c) 60 (d) 66

Solution

Assume that earlier there were N visitors.

Hence, initial revenue
= Number of tickets sold × entry fee (price/ticket)
= ₹ 10 × N = 10N

New entry fee
= 0.75 × ₹ 10 = ₹ 7.5

New sales revenue
= 1.2 × 10N = 12N

So, new number of visitors
$= \frac{12N}{7.5} = 1.6N$

Increase in number of visitors
= 1.6N – N
= 0.6N

Hence, percentage increase in the number of visitors = 60%

Hence, option (c) is the answer.

Type 3: Questions based on Sets

15. In an examination, a total of 6,00,000 students appeared. 40% of them were females while the rest were males. Pass percentage among males is 75% and the overall pass percentage is 70%. What is the pass percentage for females?

(a) 37.5% (b) 50%
(c) 62.5% (d) 4.70%

Solution

Look at the tree (Fig. 2.1).

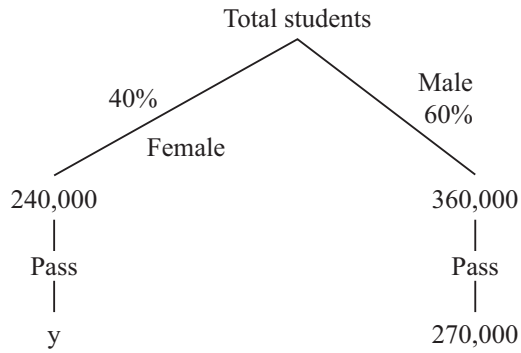


Fig. 2.1

It is given that a total of 70% students pass the examination, So total number of students passed = 4,20,000. So total number of females passed = 4,20,000 – 2,70,000 = 1,50,000. Hence, pass percentage for females

$$= \frac{150000}{240000} \cdot 100 = 62.5\%$$

Hence, option (c) is the answer.

16. In an examination, 80% students passed in Philosophy and 70% students passed in Maths. At the same time, 15% failed in both the subjects. If 390 students passed in both the subjects, then how many students appeared in the examination?

(a) 500 (b) 400
(c) 800 (d) 600

Solution

First of all, understand all the possibilities and statements (Table 2.3):

Table 2.3

Philosophy	Pass	Fail	Pass	Fail
Maths	Pass	Pass	Fail	Fail

15% failed in both the subjects does not mean that 85% passed in both the subjects. It means summation of following three possibilties:

Table 2.4

Philosophy	Pass	Fail	Pass
Maths	Pass	Pass	Fail

We have the information regarding the Philosophy pass percentage (and obviously fail percentage too can be calculated from this data), and Maths pass percentage (and obviously fail percentage too can be calculated from this data).

Using Set Theory will provide a better view of the whole scenario (Fig. 2.2).

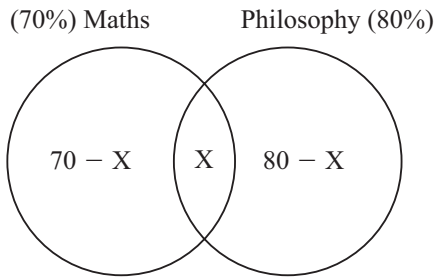


Fig. 2.2

According to the question:

$85\% = (70\% - x) + x + (80\% - x)$

$\Rightarrow \quad x = 65\%$

= students who passed in both the subjects = 390

$\Rightarrow \quad 100\% = \frac{390}{65} \cdot 100 = 600$

Hence, option (d) is the answer.

Expression for successive percentage change:

If there are two sucessive percentage changes of a% and b% then net % change

$$= \left(a + b + \frac{ab}{100}\right)\%$$

For example:

(a) If population of a town increases by 20% in 1st year and by 30% in 2nd year, then net % change

$$= \left(20 + 30 + \frac{20 \times 30}{100}\right)\%$$

$= (50 + 6)\%$

$= 56\%$

(b) If population of a town increases by 20% in 1st year and decreases by 30% in 2nd year, then net % change

$$= \left(20 - 30 - \frac{20 \times 30}{100}\right)\%$$

$= (-10-6)\%$

$= -16\%$

= Decrease of 16%

PRACTICE EXERCISES

EXERCISE 1

1. Difference between a number and its two-fifth is 510. What is 10% of that number?

(a) 12.75% (b) 85

(c) 204 (d) None of these

2. If 35% of a number is 12 less than 50% of that number, then the number is:

(a) 40 (b) 50

(c) 60 (d) 80

3. Three candidates contested an election and received 1,136, 7,636 and 11,628 votes respectively. If these three were the only candidates contesting the election, what percentage of the total valid votes did the winning candidate get?

(a) 57% (b) 60%

(c) 65% (d) 90%

4. In an election, only two candidates are contesting. Winner of the election gets 84% of the valid votes and is elected by

a majority of 476 votes. What is the total number of valid votes?
(a) 672 (b) 700
(c) 749 (d) 848

5. Two labourers A and B are paid a total of ₹ 550 per week. If A is paid 120 percentage of the sum paid to B, how much is B paid per week?
(a) ₹ 200 (b) ₹ 250
(c) ₹ 300 (d) None of these

6. 1,100 boys and 700 girls are examined in a test. 42% of the boys and 30% of the girls pass. The percentage of the total who failed is:
(a) 58% (b) 62⅔%
(c) 64% (d) 78%

7. If x% of x is 49, then x is equal to:
(a) 7 (b) 70
(c) 700 (d) 4,900

8. Subtracting 6% of x from x is equivalent to multiplying x by how much?
(a) 0.094 (b) 0.94
(c) 9.4 (d) 9.4

9. Dilip spends 30% of his monthly income on food articles, 40% of the remaining on conveyance and clothes and saves 50% of the remaining. If his monthly salary is ₹ 1,840, how much money does he save every month?
(a) ₹ 362.4 (b) ₹ 386.4
(c) ₹ 426.4 (d) ₹ 588.8

10. If 8% of x = 4% of y, then 20% of x is:
(a) 10% of y (b) 16% of y
(c) 80% of y (d) None of these

11. If $A = x$% of y and $B = y$% of x, then which of the following is true?
(a) A is smaller than B
(b) A is greater than B
(c) Relationship between A and B cannot be determined
(d) $A = B$

12. In an examination, there are three papers and a candidate has to get 35% of the total to pass. In one paper, he gets 62 out of 150 and in the second 35 out of 150. How much must he get, out of 180, in the third paper to pass the examination?
(a) 60.5 (b) 68
(c) 70 (d) 71

13. A number is decreased by 10% and then increased by 10%. The number so obtained is 10 less than the original number. What was the original number?
(a) 1,000 (b) 1,050
(c) 1,500 (d) 2,000

14. Entry fee in an exhibition was ₹ 10. Later, the entry fee was reduced by 25% which increased the sales by 20%. The percentage increase in the number of visitors is:
(a) 54 (b) 57
(c) 60 (d) 66

15. Price of an article increases at the rate of 8% p.a. What will be the new price of a ₹ 20 article two years later?
(a) Between ₹ 24 and ₹ 25
(b) Between ₹ 21 and ₹ 22
(c) Between ₹ 22 and ₹ 23
(d) Between ₹ 23 and ₹ 24

16. The present population of a country estimated to be 10 crores is expected to increase to 13.31 crores during the next three years. The uniform rate of growth per annum is:
(a) 8% (b) 10%
(c) 12.7% (d) 15%

17. Amar's salary is 50% more than Barun's. How much per cent is Barun's salary less than Amar's?
(a) 33% (b) 33¼%
(c) 33⅓% (d) 33½%

18. 5% of income of A is equal to 15% of income of B and 10% of income of B is equal to 20% of income of C. If C's income is ₹ 2,000, then the total income of A, B and C is:
(a) ₹ 6,000 (b) ₹ 14,000
(c) ₹ 18,000 (d) ₹ 20,000

19. A milk mixture contains 5% water. What quantity of pure milk should be added to 10 litres of this mixture to reduce the water concentration to 2%?
(a) 5 litres
(b) 7 litres
(c) 15 litres
(d) Cannot be determined

20. 405 chocolates were distributed equally among students in such a way that the number of chocolates received by each student is 20% of the total number of students. How many chocolates did each student receive?
(a) 9 (b) 15
(c) 18 (d) 45

EXERCISE 2

1. Rate of inflation is 1,000% per annum. What is the value of an article two years from now if it costs ₹ 6 today?
(a) ₹ 66 (b) ₹ 126
(c) ₹ 660 (d) ₹ 726

2. Dataman Infosys systems is a Lucknow-based software company which is growing at a furious pace. From a turnover of ₹ 50 crore in 1997 it doubled its turnover in 1998. Then it tripled its turnover in 1999 and grew by 50% in 2000. What is the turnover at the end of 2,000?
(a) ₹ 250 cr (b) ₹ 450 cr
(c) ₹ 300 cr (d) ₹ 600 cr

3. In the recently concluded CBSE Boards examination, a total of 6,00,000 students appeared. 40% of them were females while the rest were males. Pass percentage among males is 75% and the overall pass percentage is 70%. What is the pass percentage for females?
(a) 37.5 % (b) 50%
(c) 62.5% (d) 70%

4. Because of the budget presented by Yashwant Sinha, the price of sugar increased by 40%. The Verma family reduced its consumption so that expenditure on sugar is up by 12%. If the total consumption of sugar before the rise in price was 50 kg, what is the present consumption of sugar (in kg)?
(a) 48 kg (b) 40 kg
(c) 36 kg (d) 32 kg

5. Tatto's working hours per day were increased by 25% and his wages per hour were increased by 20%. By how much per cent were his daily earnings increased?
(a) 20% (b) 25%
(c) 50% (d) 45%

6. *A*'s income is reduced from ₹ 75,000 to ₹ 60,000, while *B*'s income for the same period is increased from ₹ 60,000 to ₹ 75,000. What percentage of decrease in *A*'s income is the increase in *B*'s income?
(a) 125% (b) 75%
(c) 133% (d) 100%

7. 18% of A plus 15% of B plus 19% of C is equal to 17% of the sum of A, B, and C. If $A - B = 500$ and $A - C = 3{,}400$. What is the value of $A + B + C$?
(a) 12,400 (b) 11,600
(c) 13,500 (d) None of these

8. Out of 51,600 candidates who sat at an examination, 35% failed, while 15% passed with honours. What is the number of candidates who passed the examination, but failed to obtain honours, assuming no failed student can obtain honours?
(a) 25,000 (b) 30,000
(c) 27,309 (d) 28,509

9. *A* and *B* have some guavas divided among themselves. *A* says to *B*, "If I give you 25% of the guavas I have, I will still have 2 more guavas than you." To this, *B* says, "If you give me guavas equal to 70% of what I have now, I will have 4 more guavas than you." What is the total number of guavas that they have?
(a) 80 (b) 64
(c) 36 (d) 88

10. Raghupati Raghav was able to score a total of 600 in 12 tests. He scored less than or equal to 80% of his average score per test in four of these tests. If he did not score more than 60 in any of the tests, what is the minimum number of tests in which he should have scored more than 50?
(a) 8 (b) 4
(c) 3 (d) 2

11. A locomotive engine runs at a speed of 50 km/hr when no compartment is attached to it. For every new compartment that is attached to it, the speed of the train reduces by 10% of the earlier speed. At most how many compartments can be attached so that the train can cover a distance of 180 km in a maximum of 8 hours?
(a) 7 (b) 5
(c) 6 (d) 8

12. There was a fundraising effort for a party being organised by Babloo. 60% of the participants contributed 80% of the funds. The average contribution of all the people who attended is ₹ 50. What is the average contribution of the remaining 40% of the people?
(a) ₹ 100 (b) ₹ 150
(c) ₹ 250 (d) ₹ 25

13. If the price of an article rose by 25% every odd year and fell by 20% every even year, what would be the percentage change after 180 years?
(a) 10% increase (b) 25% increase
(c) No change (d) 20% decrease

14. Pranav saves 10% of his total salary. Next year he increases his expenses by 20% but his percentage savings remain the same. What is the percentage increase in his salary next year?
(a) 10% (b) 20%
(c) 40% (d) 16.66%

15. In an examination, 80% students passed in Philosophy and 70% students passed in Maths. At the same time, 15% failed in both the subjects. If 390 students passed in both the subjects, then how many students appeared in the examination?
(a) 500 (b) 400
(c) 800 (d) 600

16. Salary of Anil and Vinit is 20% and 30% respectively less than salary of Dheeraj. By what percentage is the salary of Anil more than salary of Vinit?
(a) 33.33% (b) 50%
(c) 10% (d) 14.28%

17. In the recent census report of Patna, it was found that 40% of total male adult population and 55% of total female adult population is married. What percentage of total adult population of Patna is married? (Polygamy is not in practice in Patna).
(a) 46.32% (b) 43.6%
(c) 49.81% (d) 40%

18. Sona and Ellaraputtu are very close friends. One day, they were figuring out about their respective incomes and expenses. It was found that income of Sona and Ellaraputtu is 'S' and 'E', respectively. Sona spends 12% of her income and Ellaraputtu also spends the same amount. What percentage of her income Ellaraputtu is spending?
(a) $E/(12S)$ (b) $12S/E$
(c) $12E/S$ (d) $S/(12E)$

19. In an examination, a candidate who gets 30% of total marks failed by 100 marks. Another candidate gets 20 more marks than necessary marks for passing and gets 36% marks of total marks. What is the passing marks percentage?
(a) 35% (b) 40%
(c) 38% (d) 36%

20. In an election, 10% of the voters on the voters' list did not cast their votes and 60 voters left their ballot papers blank. There were only two candidates. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. The number of voters on the list was:
(a) 3,600 (b) 6,200
(c) 4,575 (d) 6,028

ANSWER KEY

Exercise 1

1. (b)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (b)
9. (b)	**10.** (a)	**11.** (d)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (b)
17. (c)	**18.** (c)	**19.** (c)	**20.** (a)				

Exercise 2

1. (d)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (d)
9. (b)	**10.** (b)	**11.** (a)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (d)
17. (a)	**18.** (b)	**19.** (a)	**20.** (b)				

HINTS AND EXPLANATIONS

Exercise 1

1. Let the number be x then,

$$x - \frac{2}{5}x = 510 \Rightarrow \frac{3}{5}x = 510$$

$$\Rightarrow \quad x = \left(510 \times \frac{5}{3}\right) = 850$$

$\therefore$ 10% of 850

$= 85.$

2. Let the number be x. Then, 50% of x – 35% of

$$= 12 \Rightarrow \frac{50}{100}x - \frac{35}{100x}$$

$$= 12 \Rightarrow \frac{15}{100}x = 12$$

$$= 12 \Rightarrow x = \left(12 \times \frac{100}{15}\right) = 80.$$

3. Total number of votes polled

$= (1{,}136 + 7{,}636 + 11{,}628) = 20{,}400$

$\therefore$ Required percentage

$$= \left(\frac{11{,}628}{20{,}400} \times 100\right)\% = 57\%.$$

4. Let the total number of votes polled be x. Then, valid votes polled by other candidate

$= (100 - 84)$ % of $x = 16\%$ of x

$\therefore$ 84% of x – 16% of x

$= 476 \leftrightarrow 68/100x$

$= 476 \leftrightarrow x = (476 \times 100/68) = 700.$

5. Let the sum paid to B per week be ₹ P. Then, P + 120% of P = 550

$\Rightarrow P + 1.2P = 550 \Rightarrow 2.2P = 550$

$\Rightarrow \quad P =$ ₹ 250.

6. Total number of students

$= 1{,}100 + 700 = 1{,}800$

Number of students passed

$= (42\%$ of $1{,}100 + 30\%$ of $700)$

$= (462 + 210) = 672$

Number of failures

$= 1{,}800 - 672 = 1{,}128$

Hence, percentage failure

$= (1{,}128/1{,}800 \times 100)\%$

$= 62\frac{2}{3}\%.$

7. $x\%$ of $x = 49 \Rightarrow \left(\frac{x}{100} \times x\right) = 49$

$\Rightarrow \quad x^2 = 4{,}900 \Rightarrow x = 70.$

8. $x - 6\%$ of $x = 94\%$ of $x = 0.94x$. Hence, option (b) is the answer.

9. Saving = 50% of (100 – 40)% of (100 – 30)% of ₹ 1,840

$$= ₹\left(\frac{50}{100} \times \frac{60}{100} \times \frac{70}{100} \times 1840\right)$$

$$= ₹\ 3{,}86.4.$$

10. 8% of x = 4% of $y \rightarrow$

$$\frac{8}{100}x = \frac{4}{100}y \rightarrow x = \frac{y}{2}$$

Hence, 20% of x = 20% of 1/2 y = 10% of y.

11. A and B are equal. Hence, option (d) is the answer.

12. Let the marks required be x. Then

$(62 + 35 + x) = 35\%$ of $(150 + 150 + 180)$

$$\Rightarrow \quad 97 + x = \frac{35}{100} \cdot 480$$

$$\Rightarrow \quad x = 168 - 97 = 71.$$

13. Let the original number be x.

Final number obtained

$= 110\%$ of $(90\%$ of $x)$

$$= \left(\frac{110}{100} \times \frac{90}{100} \times x\right) = 99\% \text{ of } x$$

Hence, $x - 99\%$ of $x = 10$

$\Rightarrow$ 1% of $x = 10 \Rightarrow x = 1{,}000$.

14. Let the total original sale be ₹ 100. Then, original number of visitors = 10

New number of visitors = 120/7.5 = 16

Hence, increase % = 60%.

15. Cost after 2 years

$= ₹\ [20 \times (1 + 8/100)^2]$

$= ₹\ (20 \times 27/25 \times 27/25) = ₹\ 23.33.$

16. 10 crores $\times (1 + R/100)^3$ = 13.31 crores.

Or, $(1 + R/100)^3$

= 13.31 crores/10 crores

$= 13.31/10 = 1331/1000 = (11/10)^3$

So, $(1 + R/100)$

$= 11/10 \leftrightarrow (1 + R/100)$

$= (1 + 1/10) \leftrightarrow R/100 = 1/10$

$\leftrightarrow \quad R = 10.$

17. Barun's salary is less than Amar's by $[50/(100 + 50) \times 100]\%$

i.e., 100/3% = 33⅓%.

18. $5/100A = 15/100B$ and

$10/100B = 20/100C$

$\rightarrow \quad A = 3B$ and $B = 2C$

$= 2 \times 2{,}000 = 4{,}000$

Hence, $A = 3 \times 4{,}000 = 12{,}000$

Hence, $A + B + C$

$= (12{,}000 + 4{,}000 + 2{,}000) = 18{,}000.$

19. Quantity of water in 10 litres

= 5% of 10 litres = 0.5 litres

Let x litres of pure milk be added. Then,

$0.5/10 + x = 2/100$

$\Rightarrow \quad 2x = 30$

So $\quad x = 15.$

20. Let the total number of students be x

Then, $x \times (20\%$ of $x)$

$= 405,$

or, $\dfrac{x^2}{5} = 405,$

or, $x^2 = 2025$

Hence, $x = 45$

Hence, number of chocolates received by each student

= 20% of 45

= 9.

3 Simple Interest and Compound Interest

Interest is the cost of borrowing money. In other words, interest is also defined as "time value of money". **There are two types of interests:**

SIMPLE INTEREST

In case of Simple Interest, interest as well as principal remains fixed for every compounding period.

Simple interest is calculated for **original principal only**. Accumulated interest from previous periods is not used in calculations for the next period's interest.

Example:

If the rate of interest
= 10%,
and the principal
= ₹ 1,000,
then: Interest for 1st year
= 10% of ₹ 1,000
= ₹ 100
Interest for 2nd year
= 10% of ₹ 1,000
= ₹ 100
Interest for 3rd year
= 10% of ₹ 1,000
= ₹ 100

It can be seen that the Interest generated every year = ₹ 100 (Table 3.1).

Table 3.1

	Principal	Rate	Interest
1st Year	1,000	10%	100
2nd Year	1,000	10%	100
3rd Year	1,000	10%	100

COMPOUND INTEREST

In case of Compound Interest, interest as well as principal keeps on changing for every compounding period. Interest keeps on increasing every compounding period because principal increases every year. Understand this in the following way:

Principal of 1st year (initially)
$= P$
Principal of 2nd year
$= P$ + Interest of 1st year
Principal of 3rd year
$= P$ + Interest of 1st year
+ Interest of 2nd year

In case of Compound Interest, interest gets added to the principal and for next years, interest is accrued over (Principal + Interest accrued so far). So in that way, Compound interest is interest that is paid on both the principal and also on all interest from past years.

Example

If the rate of interest = 10%,
and the principal = ₹ 1,000,
then: Interest for 1st year = 10% of ₹ 1,000
= ₹ 100

Table 3.2

	Principal	Rate	Interest
1st Year	1,000	10%	100
2nd Year	1,000 + 100 = 1,100	10%	110
3rd Year	1,000 + 100 + 110 = 1,210	10%	121

Expression for Simple Interest (SI)/ Compound Interest (CI)

$$\text{SI} = \frac{\text{Principal} \cdot \text{Rate of interest} \cdot \text{Time}}{100}$$

$$\text{CI} = \text{Principal} \times \left[1 + \frac{R}{100}\right]^N - \text{Principal}$$

Principal = Sum invested or lent
R = Rate of interest per annum
N = Number of years

It should be noted that the unit of rate of interest and time should be same. So, if rate of interest is "per year", then time should also be in "year".

In case of Compound Interest, if the compounding is not done annually, then formula changes like the following:

Half-yearly compounding

It means interest is given after every six months. In this case, after every six months, interest will be added to the principal (Table 3.3).

$$\text{CI} = \text{Principal} \times \left[1 + \frac{\frac{R}{2}}{100}\right]^{2N} - \text{Principal}$$

Quarterly compounding

It means interest is given after every three months. In this case, after every three months, interest will be added to the principal (Table 3.4).

$$\text{CI} = \text{Principal} \times \left[1 + \frac{\frac{R}{4}}{100}\right]^{4N} - \text{Principal}$$

Understand that the expression

$$\textbf{Principal} \times \left[\mathbf{1} + \frac{\boldsymbol{R}}{\mathbf{100}}\right]^{\boldsymbol{N}}$$

in Compound Interest provides the amount = Principal + Interest. To calculate interest, we need to subtract principal from this.

1. Find simple interest for the following data:
Principal = ₹ 400,
Rate of interest = 20% per annum,
Time = 4 months

Solution

In this case, we can see that units of rate and time are not same. We can convert any one of the two to be in one single unit—either in months/year.

Time = 4 months

$$= \frac{4}{12} \text{ year} = \frac{1}{3} \text{ year}$$

$$\text{SI} = \frac{\text{Principal} \cdot \text{Rate of Interest} \cdot \text{Time}}{100}$$

$$= \frac{400 \cdot 20 \cdot \frac{1}{3}}{100} = \frac{400 \cdot 20 \cdot 1}{100 \cdot 3}$$

$$= ₹\ 26.66$$

2. Find compound interest for the data:
Principal = ₹ 400,
Rate of interest = 20% per annum,
Time = 12 months.
Interest is compounded half yearly.

Table 3.3

Rate of Interest	Compounding Period	Interest in 6 months (Half year)	Number of Compounding Period in a Year
R% per year	Half yearly	(R/2)%	2 (12 months/6 months)

Table 3.4

Rate of Interest	Compounding Period	Interest in 3 months (Quarter year)	Number of Compounding Period in a Year
R% per year	Quarterly	(R/4)%	4 (12 months/3 months)

Solution

Since interest is compounded half yearly, in 12 months, interest will be added (or compounded) twice.

Rate of interest for six months

$$= \frac{20}{2} = 10\%$$

$$\text{CI} = 400 \times \left[1 + \frac{10}{100}\right]^2 - 400$$

$$= 400\,(1.1)^2 - 400$$

$$= ₹\,484 - ₹\,400$$

$$= ₹\,84$$

Alternatively, it can be calculated through simple addition too:

Interest for 1st six months

= 10% of ₹ 400 = ₹ 40

Interest for next six months

= 10% of ₹ 40

(interest for the interest for 1st six months) + 10% of ₹ 400 = ₹ 44

Hence, total Interest

= ₹ 30 + ₹ 44

= ₹ 84.

3. A sum of money becomes 3 times in 5 years. Find in how many years will the same sum become 6 times at the same rate of SI.

Solution

Sum of money gets 3 times, it means 200% is being added up to the original sum (principal) in 5 years.

So, 500% will be added up in $12\frac{1}{2}$ years.

4. Difference between two years of Compound Interest and Simple Interest at 10% over ₹ X is ₹ 10. What is the value of X?

Solution

See Table 3.5.

So, 1% = ₹ 10

⇒ 100% = ₹ 1,000

Table 3.5

	SI	CI
At the end of 1st year	10%	10%
At the end of 2nd year	10%	10% + 10% of 10% = 11%
	= 20%	= 21%

Important Points

- If the rate of interest = R% per annum for both CI and SI, then the difference between CI and SI for 2 years will be equal to (R% of R)% of principal $= \frac{R^2}{100}$% of principal

  In the above case, R = 10%, so the difference between CI and SI for 2 years = 1% of Principal
- If a sum doubles itself in n years at simple interest, then rate of interest $= \frac{100}{n}$%

5. A sum of money doubles itself in 5 years at SI. What is the rate of interest?

Solution

$$\text{Rate of interest} = \frac{100}{5} = 20\%$$

6. A sum of money amounts to ₹ 2,600 in 3 years, and to ₹ 3,000 in next two years at simple interest. What is the rate of interest?

Solution

Increase in interest in two years

= ₹ 400 (See the statement—'next two years')

Increase in interest in one year

= ₹ 200

Now, amount at the end of 3 years

= ₹ 2,600

Principal = 2,600 – (3 × 200)

= ₹ 2,000

$$\text{Interest Rate} = \frac{200}{2000} \cdot 100 = 10\%$$

Comparison between Compound Interest and Simple Interest

Assume two different sums are getting double at their respective rates of SI and CI in 5 years. Table 3.6 gives us the mechanism of getting money *n* times in above situation.

Table 3.6

	After 5 years	After 10 years	After 15 years	After 20 years
At SI	2 times	3 times	4 times	5 times
At CI	2 times	4 times	8 times	16 times

PRACTICE EXERCISE

EXERCISE 1

1. A certain amount earns simple interest of ₹ 1,750 after 7 years. Had the interest been 2% more, how much more interest would it have earned?
(a) ₹ 35
(b) ₹ 245
(c) ₹ 350
(d) Cannot be determined

2. A sum of money amounts to twice the original sum in 5 years at SI. What is the rate of interest?
(a) 10% (b) 12.5%
(c) 20% (d) 25%

3. The simple interest on a certain sum of money at the rate of 5% p.a. for 8 years is ₹ 840. At what rate of interest the same amount of interest can be received on the same sum after 5 years?
(a) 6% (b) 8%
(c) 9% (d) 10%

4. A sum of money amounts to three times in 5 years at CI. In how many years will the same sum amount to nine times the original sum?
(a) 10 (b) 15
(c) 20 (d) 9

5. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years?
(a) 1:3 (b) 1:4
(c) 2:3 (d) Data inadequate

6. Ravi Shankar wishes to buy an AC with the money in the bank, which is currently earning interest at the rate of 15% p.a. compounded annually. But Tanzar, his friend, forecasts that the inflation rate applicable to AC is going to be 14%; 15% and 16% respectively for the next 3 consecutive years and advises Ravi Shankar to postpone the purchase by 3 years. Does Ravi Shankar gain monetarily, if he takes Tanzar's tip?
(a) Yes
(b) No
(c) He neither gains nor losses
(d) He gains only if the purchase is made in the second year.

7. The simple interest on ₹ 10 for 4 months at the rate of 3 paise per rupee per month is:
(a) ₹ 1.20 (b) ₹ 1.60
(c) ₹ 2.40 (d) ₹ 3.60

8. Likhit earns x% on the first ₹ 2,000 and y% on the rest of his income. If he earns ₹ 700 from ₹ 4,000 income and ₹ 900 from ₹ 5,000 income, find y%.
(a) 20% (b) 15%
(c) 25% (d) None of these

9. If a sum of money at simple interest doubles in 6 years, it will become 4 times in:
(a) 12 years (b) 14 years
(c) 16 years (d) 18 years

10. Likhit earns x% on the first ₹ 2,000 and y% on the rest of his income. If he earns ₹ 700 from ₹ 4,000 income and ₹ 900 from ₹ 5,000 income, find x%.
(a) 20% (b) 15%
(c) 25% (d) None of these

11. The difference between the simple interest received from two different sources on ₹ 1,500 for 3 years is ₹ 13.50. The difference between their rates of interest is:
(a) 0.1% (b) 0.2%
(c) 0.3% (d) 0.4%

12. How much time will it take for an amount of ₹ 450 to yield ₹ 81 as interest at 4.5% per annum of simple interest?
(a) 3.5 years (b) 4 years
(c) 4.5 years (d) 5 years

13. Simple interest on a certain amount is 9/16 of the principal. If the numbers representing the rate of interest in per cent and time in years be equal, then time for which the principal is lent out, is:
(a) 5½ years (b) 6½ years
(c) 7 years (d) 7½ years

14. The rate of inflation is 1,000%. What will be the cost of an article, which costs ₹ 6 today, two years from now?
(a) 66 (b) 660
(c) 36 (d) 726

15. The rates of simple interest in two banks A and B are in the ratio 5:4. A person wants to deposit his total savings in two banks in such a way that he receives equal half yearly interest from both. He should deposit the savings in banks A and B in the ratio:
(a) 2:5 (b) 4:5
(c) 5:2 (d) 5:4

16. The difference between SI and CI on a sum of money at the rate of 5% per annum for two years is ₹ 25. What is the principal?
(a) ₹ 1,000 (b) ₹ 10,000
(c) ₹ 5,000 (d) ₹ 2,500

17. What is the compound interest at the rate of 10% for 3 years on that principal which in 3 years at rate of 10% p.a. yields ₹ 300 simple interest?
(a) ₹ 310 (b) ₹ 331
(c) ₹ 330 (d) None of these

18. A sum of money placed at compound interest doubles itself in 3 years. In how many years will it amount to 8 times the original sum?
(a) 6 years (b) 8 years
(c) 9 years (d) 12 years

19. Vinit invests ₹ 3,000 at the rate of 5% per annum. How much more should he invest at the rate of 8% per annum so that he earns a total of 6% per annum (consider Simple Interest)?
(a) ₹ 1,000 (b) ₹ 1,200
(c) ₹ 1,500 (d) ₹ 2,000

20. How many years would it take for a sum of grow from ₹ 1,250 to ₹ 10,000 if it is invested at 12.5% p.a. at SI?
(a) 28 (b) 56
(c) 48 (d) 70

ANSWER KEY

Exercise 1

1. (d)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (a)
9. (d)	**10.** (b)	**11.** (c)	**12.** (b)	**3.** (d)	**14.** (d)	**15.** (b)	**16.** (b)
17. (b)	**18.** (c)	**19.** (c)	**20.** (b)				

HINTS AND EXPLANATIONS

Exercise 1

1. We need to know the SI principal and time to find the rate. Since the principal is not given, so data is inadequate. Hence, option (d) is the answer.

2. Being twice means an increase of 100%. Now, 100% is increased in 5 years.
$\Rightarrow$ Hence, per annum increase = 20%.
Hence, option (c) is the answer.

3. SI = ₹ 840, R = 5%, T = 8 years.
Principal = ₹ (100 × 840/5 × 8)
= ₹ 2,100
Now, P = ₹ 2,100,
SI = ₹ 840, T = 5 years.
∴ Rate = (100 × 840/2,100 × 5)%
= 8%.

4. If x becomes $3x$ in 5 years, then $3x$ will become $9x$ in 10 years. Hence, option (a) is the answer.

5. Let the principal be P and rate of interest be R%.
∴ Required ratio
$= [(P \times R \times 6/100)/(P \times R \times 9/100)]$
$= 6PR/9PR = 6/9 = 2:3$.

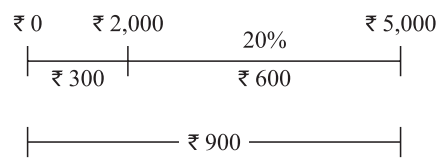
7. SI = ₹ (10 × 3/100 × 4) = ₹ 1.20.

8. Likhit earns ₹ 700 from ₹ 4,000 income and ₹ 900 from ₹ 5,000 income. It means he is earning ₹ 200 (₹ 900 – ₹ 700) from ₹ 1,000 (₹ 5,000 – ₹ 4,000).
Hence, $y = \dfrac{200}{1000} \cdot 100 = 20\%$
Hence, option (a) is the answer.

9. **Method 1:**
Let sum = x, then, SI = x.
∴ Rate = (100 × x/x × 6) % = 50/3%.
Now, sum = x, SI = $3x$, Rate = 50/3%.
∴ Time = 100 × $3x/x$ × 50/3 = 18 years.

Method 2:
Assume that sum invested initially = ₹ 100. It becomes ₹ 200 in 6 years. Hence, ₹ 100 interest is incurred in 6 years. For the sum to be 4 times, amount = ₹ 400. So interest required to incurred = ₹ 300.
Now ₹ 100 interest is incurred in 6 years, hence ₹ 300 interest would incur in 18 years.
Hence, option (d) is the answer.

10. Likhit earns ₹ 700 from ₹ 4,000 income and ₹ 900 from ₹ 5,000 income. It means he is earning ₹ 200 (= ₹ 900 – ₹ 700) from ₹ 1,000 (₹ 5,000 – ₹ 4,000).
Hence, $y = \dfrac{200}{1000} \cdot 100 = 20\%$
So income earned would be like:



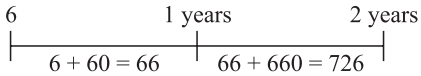
So he earns ₹ 300 from ₹ 2,000.
Hence, $x\% = \dfrac{300}{2000} \cdot 100 = 15\%$
Hence, option (b) is the answer.

11. $\dfrac{1500 \times R_1 \times 3}{100} - \dfrac{1500 \times R_2 \times 3}{100} = 13.5$
$\Rightarrow$ $4{,}500\,(R_1 - R_2) = 1{,}350$
So $R_1 - R_2 = 1{,}350/4{,}500$
$= 0.3\%$.

12. Time = (100 × 81/450 × 4.5) years
= 4 years.

13. Let sum = x, then, SI = $9/16x$.
Let rate = R% and time = R years.
∴ $(x \times R \times R/100)$
$= 9x/16 \leftrightarrow R^2 = 900/16$
$\leftrightarrow$ $R = 30/4 = 7\frac{1}{2}$
Hence, time = 7½ years.

14. 1,000% inflation means an increase of 10 times.



15. Let the savings be X and Y and the rates of simple interest be $5x$ and $4x$ respectively. Then, $X \times 5x \times 1/2 \times 1/100$

$$= Y \times 4x \times \frac{1}{2} \cdot \frac{1}{100}$$

or $X/Y = 4/5$,

i.e., $X:Y = 4:5$.

16. Difference between CI and SI for two years

$$= \frac{R^2}{100}\% \text{ of Principal}$$

$$= \frac{5^2}{100}\% \text{ of Principal}$$

$\Rightarrow$ 1/4% of Principal = ₹ 25

1% of Principal = ₹ 100

$\Rightarrow$ Principal = ₹ 10,000

18. X amounts to $2x$ in 3 years $\Rightarrow$ $2x$ will amount to $4x$ in another 3 years = total 6 years $\Rightarrow$ $4x$ will amount to $8x$ in another three years = total 9 years.

20. Interest to be added

= ₹ 8,750

So, $$8{,}750 = \frac{1250 \cdot 12.5 \cdot T}{100}$$

$\Rightarrow$ $T = 56$ years.

4 Profit, Loss and Discount

BASIC TERMINOLOGY

Cost Price (CP)

This is the price which a person pays to purchase something or cost incurred while manufacturing something.

Type of Cost

1. **Fixed cost:** As obvious from the name, it is that kind of cost which is fixed in all the cases.
2. **Variable cost:** Variable costs are those costs which vary according to the number of units produced.
3. **Semi-variable cost:** Semi variable costs are those costs which are fixed in one particular strata, but the costs varies among the different layers.

One good example of Fixed Cost, Variable Cost and Semi-variable cost is the bill we receive for the telephone connections at our home. A part of that bill, rental, is fixed cost; and the rest part of the bill is calculated on the basis of the number of calls made.

Selling Price (SP)

This is the price at which something is sold.

Now, there are three situations possible:

Case 1: Selling Price > Cost Price, then Profit occurs.

Profit $= SP - CP$

Profit percentage $= \frac{\text{Profit}}{CP} \cdot 100$

Case 2: Cost Price > Selling Price, then Loss occurs.

Loss $= CP - SP$

Loss percentage $= \frac{\text{Loss}}{CP} \cdot 100$

Case 3: Selling Price = Cost Price, then there is no profit no loss. We call it Break-Even case.

Marked Price or Mark-up Price (MP)

This is the price which the shopkeeper fixes in anticipation of some discount being asked by customer.

List price or Tag price: As obvious from the name, this is the price which is printed on the tag of the article.

For our calculations related to the concept of PLD, till the moment nothing is stated in the questions we won't see much difference between Marked Price and List price.

Type of Questions

Type 1

CP and SP are given, and profit percentage or loss percentage is to be calculated:

1. An article is bought for ₹ 600 and sold for ₹ 750. What is the profit percentage?

(a) 20% (b) 25%
(c) 30% (d) None of these

Solution

CP = ₹ 600 and SP = ₹ 750. Since SP is more than CP, there will be profit.

Profit $= SP - CP =$ ₹ 150

Profit percentage $= \frac{\text{Profit}}{CP} \cdot 100$

$= \frac{150}{600} \cdot 100 = 25\%$

Type 2

CP and profit percentage/loss percentage are given, and SP is to be calculated:

If one of CP or SP is given alongwith Profit percentage or Loss percentage, using the concept of multiplier makes the whole calculation simple. [To know about the concept of multiplier, read Percentage chapter]

In general,

1. CP × Multiplier = SP
2. $CP = \frac{SP}{\text{Multiplier}}$
3. If there is a profit, multiplier will be more than 1, and if there is a loss, multiplier will be less than 1. [Irrespective of the fact that we have to find out CP or SP].

Alternatively

If there is a profit of R%, then CP

$$= \frac{SP}{100+R} \cdot 100$$

If there is a loss of R%, then CP

$$= \frac{SP}{100-R} \times 100$$

2. Nitika buys a kinetic for ₹ 16,000. If she wants to gain 40%, how much should she charge for the kinetic?

Solution

CP = ₹ 16,000

Profit percentage = 40%

SP = CP + 40% of CP = 1.4 × CP

[Here the multiplier = 1.4]

= 1.4 × 16,000

= ₹ 22,400.

3. By selling a VCD player for ₹ 1,950, I got a profit of 30%. At what price should I have sold it in order to get a profit of 40%?

(a) ₹ 2,000 (b) ₹ 2,100

(c) ₹ 2,500 (d) None of these

Solution

SP = ₹ 1,950 and profit percentage = 30%.

Understand that if profit percentage = 30%, multiplier has to be equal to 1.3. Now only thing you have to think is: Shall I multiply SP by 1.3 or divide SP by 1.3? Understand this further that if there is profit, then SP > CP. So, multiplying SP by 1.3 will make it larger than SP, hence we conclude that SP should be divided by 1.3.

To calculate CP, we can use the following methods:

Method 1: $CP = \frac{SP}{\text{Multiplier}}$

$= \frac{1950}{1.3} = ₹1500$

Method 2: $CP = \frac{SP}{100+R} \cdot 100$

$= \frac{1950}{130} \cdot 100$

= ₹ 15 × 100

= ₹ 1,500

Now, I want to earn a profit of 40%. Hence, multiplier = 1.4

So, SP = CP × 1.4 = 1,500 × 1.4 = ₹ 2,100

4. By selling an article for ₹ 360, loss incurred is 10%. At what minimum price should the article be solve to avoid loss?

(a) ₹ 320 (b) ₹ 324

(c) ₹ 396 (d) ₹ 400

Solution

SP = ₹ 360

Loss percentage = 10%

Hence, multiplier = 0.9

Method 1: $CP = \frac{SP}{\text{Multiplier}}$

$= \frac{360}{0.9} = ₹400.$

Method 2: $CP = \frac{SP}{100-R} \times 100$

$$= \frac{360}{100-10} \times 100$$

$$= \frac{360}{90} \cdot 100$$

$$= ₹\ 4 \times 100$$

$$= ₹\ 400$$

Hence, to avoid any loss, SP should be atleast equal to the CP = ₹ 400.

Type 3

Questions involving Marked Price/Tag Price

5. A shopkeeper wants to earn a profit of 20% and at the same time, the minimum discount which he wants to offer is 25%. What should be the minimum percentage mark-up over CP?

(a) 50% (b) 42.5%
(b) 62.5% (d) 35%

Solution

Assume that Cost Price = ₹ 100
To earn a profit of 20%,
Multiplier = 1.2
Hence, SP = CP × 1.2
= 100 × 1.2
= ₹ 120 (i)

Now discount offered = 25%. Discount is always provided on Tag price or MarkUp price.

Multiplier related to 25% discount = 0.75.
Assume that MarkUp price = M
Hence, Selling price = 25% discount on MarkUp price = $0.75M$
Using (i), $0.75M$ = ₹ 120

$$\Rightarrow \quad M = \frac{120}{0.75} = ₹150$$

$$= ₹\ 150$$

Hence, percentage mark up

$$= \frac{\text{Mark Up}}{\text{CP}} \cdot 100$$

$$= \frac{(150-100)}{100} \times 100$$

$$= 50\%$$

Type 4

Questions involving number of articles sold and number of articles bought

6. Cost price of 40 apples is equal to selling price of 30 apples. What is the profit percentage?

Solution

Best way to solve these questions is by assuming a value (ideally LCM of 30 and 40).

CP of 40 apples = SP of 30 apples
= ₹ 120 (LCM of 30 and 40)
⇒ CP of one apple = ₹ 3
and SP of one apple = ₹ 4
Hence, profit = SP – CP = ₹ 1
Hence, profit percentage

$$= \frac{1}{3} \cdot 100 = 33.33\%$$

In general,

$$\boxed{\text{Profit Percentage or Loss Percentage} = \frac{\text{Goods Left or Goods Added}}{\text{Number of articles sold}} \cdot 100}$$

In this case, there are 10 apples left out (after selling 30 apples out of 40 apples, 10 are left out).

Hence, profit percentage

$$= \frac{10}{30} \cdot 100 = 33.33\% \text{ profit.}$$

7. Cost price of 40 apples is equal to selling price of 50 apples. What is the profit/loss percentage?

Solution

Since number of articles sold is more than number of articles bought, hence there is loss [shopkeeper is selling more number of articles than he has got].

$$\text{Loss percentage} = \frac{10}{50} \cdot 100 = 20\% \text{ loss}$$

Type 5

Questions based upon faulty balance.

In these questions, shopkeeper cheats his customers by selling less quantity than what he is professing.

If shopkeeper sells x gms instead of 1,000 gms (where $x < 1{,}000$),

$$\text{Profit percentage} = \frac{1000 - x}{x} \times 100\%$$

8. A shopkeeper professes to sell his goods at Cost Price. However he sells only 800 gms at the place of 1,000 gms. What is his profit percentage?

Solution

Profit will be obtained for 200 gms (1000 – 800).

$$\text{Profit percentage} = \frac{200}{800} \cdot 100\% = 25\%$$

9. A shopkeeper professes to sell his goods at cost price. However, he sells x gms at the place of 1,000 gms, and thus earns a profit of 20%. What is the value of x?

Solution

$$\frac{1000 - x}{x} \times 100 = 20$$

$$\Rightarrow \quad \frac{1000 - x}{x} = 1/5 = 1/5$$

$$\Rightarrow \quad x = 833.33 \text{ gms.}$$

Some Important Points

When SPs of two Articles are same

1. First article is sold at a profit of x% and second article is sold at a loss of x%.
   In this case, there will be always a loss.
   $$\text{Net Loss} = \frac{x^2}{100}\% \text{ of CP}$$
2. First article is sold at a profit of x% and second article is sold at a profit of y%.
   Ratio of $CP_1:CP_2$
   $= (100 + y):(100 + x)$
3. First article is sold at a profit of x% and second article is sold at a loss of y%.
   Ratio of $CP_1:CP_2$
   $= (100 - y):(100 + x)$
   In this case, we cannot find if there will be net profit or loss.
4. First article is sold at a loss of x% and second article is sold at a loss of y%.
   Ratio of $CP_1:CP_2$
   $= (100 - y):(100 - x)$
5. First article is sold at a loss of x% and second article is sold at a profit of y%.
   Ratio of $CP_1:CP_2$
   $= (100 + y):(100 - x)$
   In this case, we cannot find if there will be net profit or loss.

PRACTICE EXERCISES

EXERCISE 1

1. A sells an article which costs him ₹ 400 to B at a profit of 20%. B then sells it to C, making a profit of 10% on the price he paid to A. How much does C pay to B?
(a) ₹ 472 (b) ₹ 476
(c) ₹ 528 (d) ₹ 532

2. A fruit seller sells mangoes at the rate of ₹ 9 per kg and thereby loses 20%. At what price per kg, he should have sold them to make a profit of 5%?
(a) ₹ 11.81
(b) ₹ 12
(c) ₹ 12.25
(d) ₹ 12.31

3. A man gains 20% by selling an article for a certain price. If he sells it at double the price, the percentage of profit will be:
(a) 40 (b) 100
(c) 120 (d) 140

4. Profit earned by selling an article for ₹ 1,060 is 20% more than the loss incurred by selling the article for ₹ 950. At what price should the article be sold to earn 20% profit?
(a) ₹ 980 (b) ₹ 1,080
(c) ₹ 1,800 (d) None of these

5. If the selling price of 50 articles is equal to the cost price of 40 articles, then the loss or gain percent is:
(a) 20% loss (b) 20% gain
(c) 25% loss (d) 25% gain

6. On an order of 5 dozen boxes of a consumer product, a retailer receives an extra dozen free. This is equivalent to allowing him a discount of:
(a) 15% (b) 161/6%
(c) 16⅔% (d) 20%

7. A man bought some fruits at the rate of 16 for ₹ 24 and sold them at the rate of 8 for ₹ 18. What is the profit percent?
(a) 25% (b) 40%
(c) 50% (d) 60%
(e) None of these

8. By selling 12 toffees for a rupee, a man loses 20%. How many for a rupee should he sell to get a gain of 20%?
(a) 5 (b) 8
(c) 10 (d) 15

9. A dairyman pays ₹ 6.40 per litre of milk. He adds water and sells the mixture at ₹ 8 per litre, thereby making 37.5% profit. The proportion of water to milk received by the customers is:
(a) 1:10 (b) 1:12
(c) 1:15 (d) 1:20

10. A dishonest dealer uses a scale of 90 cm instead of a metre scale and claims to sell at cost price. His profit is:
(a) 9% (b) 10%
(c) 12% (d) None of these

11. A shopkeeper cheats to the extent of 10% while buying as well as selling, by using false weights. His total gain is:
(a) 10% (b) 11%
(c) 20% (d) $22\frac{2}{9}\%$

12. A man buys an article for 10% less than its list price and sells it for 10% more than its list price. His gain or loss percent is:
(a) no profit, no loss
(b) 20% loss
(c) less than 20% profit
(d) more than 20% profit

13. If a man reduces the selling price of a fan from ₹ 400 to ₹ 380, his loss increases by 2%. The cost price of the fan is:
(a) ₹ 480 (b) ₹ 500
(c) ₹ 600 (d) None of these

14. The difference between the cost price and sale price of an article is ₹ 240. If the profit is 20%, the selling price is:
(a) ₹ 1,240 (b) ₹ 1,400
(c) ₹ 1,600 (d) none of these

15. A house and a shop were sold for ₹ 1 lac each. In this transaction, the house sale resulted into 20% loss whereas the shop sale resulted into 20% profit. The entire transaction resulted in:
(a) no loss, no gain
(b) loss of ₹ 1/12 lakh
(c) loss of ₹ 1/18 lakh
(d) gain of ₹ 1/24 lakh

16. Piyush purchased 20 dozen notebooks at ₹ 48 per dozen. He sold 8 dozen at 10% profit and the remaining 12 dozen with 20% profit. What is his profit percentage in the transaction?
(a) 6.68 (b) 15
(c) 16 (d) 19.2

17. A vegetable vendor sold half of his stock at 20% profit, half of the remaining at 20% loss

and the rest was sold at the cost price. In the total transaction, his gain or loss will be:
(a) neither loss, nor gain
(b) 5% loss
(c) 5% gain
(d) 10% gain

18. Cost Price of two cameras taken together is ₹ 840. By selling one at a profit of 16% and the other at a loss of 12%, there is no loss or gain in the whole transaction. Find the CP of the two watches:
(a) ₹ 360, ₹ 480 (b) ₹ 480, ₹ 360
(c) ₹ 380, ₹ 460 (d) ₹ 400, ₹ 440

19. List price of an article at a showroom is ₹ 2,000. It is sold at successive discounts of 20% and 10%. Its net selling price will be:
(a) ₹ 1,400 (b) ₹ 1,440
(c) ₹ 1,500 (d) ₹ 1,700

20. A trader marked the price of his commodity so as to include a profit of 25%. He allowed discount of 16% on the marked price. His actual profit was:
(a) 5% (b) 9%
(c) 16% (d) 25%

21. Rahul bought a TV with 20% discount on the tag price. Had he bought it with 25% discount, he would have saved ₹ 500. At what price did he buy the TV?
(a) ₹ 5,000 (b) ₹ 10,000
(c) ₹ 12,500 (d) ₹ 15,000

22. Cost price of 40 apples is same as selling price of N apples. In the process, he obtains a profit of 25%. What is the value of N?
(a) 30 (b) 32
(c) 50 (d) 36

23. CP of an article is 40% of the SP. The percentage that the SP is of CP is:
(a) 250 (b) 240
(c) 60 (d) 40

24. A shopkeeper sells his articles at 10% profit. Besides, he sells only 900 gms at the place of 1,000 gms. What is his net profit percentage?
(a) 20 (b) 21
(c) 22.22 (d) 18.18

25. A shopkeeper sells 25 articles at ₹ 45 per article after giving 10% discount on marked price and earns 50% profit. Had there been no discount, profit percentage would have been:
(a) 60% (b) 60⅔%
(c) 66% (d) 66⅔%

EXERCISE 2

Direction for questions 1–3: ***Read the passage below and solve the questions based on it.***

Had the CP been 10% less and SP been 10% more, profit % would have been double than that of earlier case.

1. What is the actual CP?
(a) ₹ 70
(b) ₹ 100
(c) ₹ 140
(d) Cannot be determined

2. What is the profit % in original case?
(a) 20%
(b) 28.56%
(c) 14.28%
(d) Cannot be determined

3. How many values for actual SP is/are possible?
(a) 1 (b) 0
(c) 3 (d) Infinite

4. Sunny marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance also which reads 800 gms for 1,000 gms. What is his net profit/loss percentage?
(a) 8% (b) 57.2%
(c) 37.6% (d) None of these

5. A shopkeeper marks up his goods by 20% and then gives a discount of 20%. Besides he cheats both his supplier and customer by 100 gms, *i.e.*, he takes 1,100 gram from

his supplier and sells only 900 gms to his customer. What is his net profit percentage?
(a) 24.5% (b) 17.33%
(c) 25% (d) 32.5%

6. Some mangoes are purchased at the rate of 8 mangoes per rupee and some more mangoes at the rate of 6 mangoes per rupee, investment being equal in both the cases. Now, the whole quantity is sold at the rate of 7 mangoes per rupee. What is the net percentage of profit/loss?
(a) 0.6% profit (b) 0.6% loss
(c) 1.2% loss (d) no profit/no loss

7. Some olives are purchased at the rate of 8 olives per rupee and same number of olives at the rate of 6 olives per rupee. Now, the whole quantity is sold at the rate of 7 olives per rupee What is the net percentage of profit/loss?
(a) 0.6% profit
(b) 0.6% loss
(c) 1.2% loss
(d) no profit/no loss

Direction for questions 8–12: ***Read the passage below and solve the questions based on it.***

A shop is having following discount layers:

1st layer—Buy 1, get 2 free.
2nd layer—Buy 2, get 3 free.
3rd layer—Buy 3, get 4 free.

And so on there are infinite layers on the same pattern.

8. For how many of these discount layers, discount percentage offered is more than 50%?
(a) 4 (b) 27
(c) 99 (d) None of these

9. Now, shopkeeper wishes to lessen the percentage of discount by clubbing any two layers. Which two layers should be clubbed so that discount is minimized by maximum percentage?
(a) 1st + 2nd (b) 2nd + 3rd
(c) 3rd + 4th (d) None of these

10. A car is sold for ₹ 2,400 at a profit of 20%. What is the CP?
(a) ₹ 2,100 (b) ₹ 2,000
(c) ₹ 1,800 (d) ₹ 2,200

11. A car is sold for ₹ 2,400 at a profit of 20% over SP. What is the CP?
(a) ₹ 2,000 (b) ₹ 1,920
(c) ₹ 1,980 (d) ₹ 1,800

12. A car is sold for ₹ 2,400 at a profit of 20% over SP. What is the actual profit percentage?
(a) 16.66% (b) 25%
(c) 21.21% (d) 14.28%

13. A shopkeeper wants to earn a profit of 20% and at the same time, the minimum discount which he wants to offer is 25%. What should be the minimum percentage mark-up over CP?
(a) 50% (b) 42.5%
(c) 62.5% (d) 35%

14. 100 kgs of gold are purchased for ₹ 1,100. It is sold in such a way that after selling the whole quantity, the quantum of loss is equal to the amount obtained by selling 20 kgs of gold. What is the selling price?
(a) ₹ 9.16 (b) ₹ 18.32
(c) ₹ 11.11 (d) ₹ 25

15. Sum of CPs of two cows is ₹ 13,000. Both the cows are sold at a profit of 20% and 40% respectively with their SPs being the same. What is the difference of CPs of both the cows?
(a) ₹ 1,000 (b) ₹ 2,000
(c) ₹ 1,500 (d) ₹ 2,500

16. Buy three, get one free. What is the percentage discount being offered here?
(a) 33.33% (b) 25%
(c) 20% (d) 28.56%

17. CP of 40 articles is equal to the SP of 30 articles. What is the profit/loss percentage?
(a) 25% profit (b) 33.33% profit
(c) 25% loss (d) 33.33% loss

18. Due to a price hike of 20%, 4 kgs less tea is available for ₹ 120. What is the original price of tea?
(a) ₹ 4 per kg (b) ₹ 5 per kg
(c) ₹ 6 per kg (d) ₹ 4.5 per kg

19. A shopkeeper sells two cows in such a way that CP of first cow is equal to the SP of second cow and SP of first cow is equal to the CP of second cow. Which of the following is final result after the whole transaction?
(a) There is a loss
(b) There is a profit
(c) No profit, no loss
(d) Cannot be determined

20. A milkman professes to sell milk at its CP only. But still he is making a profit of 20% since he has mixed some amount of water in milk. What is the percentage of milk in the mixture?
(a) 80% (a) 83.33%
(c) 75% (d) 66.66%

ANSWER KEY

Exercise 1

1. (c)	**2.** (a)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (b)
9. (a)	**10.** (d)	**11.** (d)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (c)
17. (c)	**18.** (a)	**19.** (b)	**20.** (a)	**21.** (b)	**22.** (b)	**23.** (a)	**24.** (a)
25. (d)							

Exercise 2

1. (d)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (d)
9. (a)	**10.** (b)	**11.** (b)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (a)	**16.** (b)
17. (b)	**18.** (b)	**19.** (c)	**20.** (b)				

HINTS AND EXPLANATIONS

Exercise 1

1. CP for B = 120% of ₹ 400

$$= ₹\left(\frac{120}{100}\times 400\right) = ₹\,480$$

CP for C = 110% of ₹ 480

$$= ₹\left(\frac{110}{100}\times 480\right) = ₹\,528.$$

2. $$\text{CP} = \frac{9}{100-20}\times 100$$

$$= \frac{9}{80}\cdot 100 = 9\cdot\frac{5}{4}$$

$$\text{SP} = 9\cdot\frac{5}{4}\cdot\frac{105}{100} = ₹11.81$$

Hence, option (a) is the answer.

3. Let CP $= ₹\,x$

Then SP $= ₹\,(120\%\text{ of } x) = ₹\,\frac{6x}{5}$

New SP $= ₹\left(2\times\frac{6x}{5}\right) = ₹\,\frac{12x}{5}$

Profit $= ₹\left(\frac{12x}{5} \times x\right) = ₹\frac{7x}{5}$

Hence,

Profit % $= \left(\frac{7x}{5} \times \frac{1}{x} \times 100\right)\% = 140\%.$

4. Let CP be ₹ x

Then, $(1060 - x) = \frac{120}{100}(x - 950)$

$\Rightarrow \quad 106000 - 100x = 120x - 120 \times 950$

$\Rightarrow \quad 220x = 220000$

$\Rightarrow \quad x = 1000$

$\therefore$ Desired SP $= ₹\left(\frac{120}{100} \times 1000\right) = ₹1200.$

5. Let CP of each article be ₹ 1

Then, CP of 50 articles = ₹ 50

SP of 50 articles = ₹ 40

$\therefore$ Loss % $= \left(\frac{10}{50} \times 100\right)\% = 20\%.$

6. Clearly, the retailer gets 1 dozen out of 6 dozens free

$\therefore$ Equivalent discount

$= \left(\frac{1}{6} \times 100\right)\% = 16\frac{2}{3}\%.$

7. Suppose, number of fruits bought

= LCM of 16 and 8 = 16

CP of 16 fruits = ₹ 24.

SP of 16 fruits $= ₹\left(\frac{18}{8} \times 16\right) = ₹36$

$\therefore$ Profit % $= \left(\frac{12}{24} \times 100\right)\% = 50\%.$

8. Let SP of 12 toffees be ₹ x.

Then, 80:1 = 120:x or x

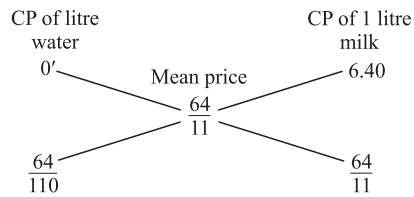
$= \left(\frac{120}{80}\right) = \frac{3}{2}$

For ₹ $\frac{3}{2}$, toffees sold = 12

For ₹ 1, toffees sold $= \left(12 \times \frac{2}{3}\right) = 8.$

9. Mean cost price $= ₹\left(\frac{100}{137.5} \times 8\right) = ₹\frac{64}{11}$

By the rule of alligation:



$\therefore$ Required ratio

$= \frac{64}{110} : \frac{64}{11} = 1:10.$

10. Gain % $= \left(\frac{10}{90} \times 100\right)\% = 11\frac{1}{9}\%.$

11. Shopkeeper gains goods worth ₹ 1,100 and sells worth ₹ 900

Percentage gain $= \frac{200}{900} \cdot 100 = 22\frac{2}{9}\%.$

12. Assume list price = ₹ 100

CP = ₹ 90 and SP = ₹ 110

Hence, profit percentage

$= \frac{20}{90} \cdot 100 = 22\frac{2}{9}\%.$

13. Let CP be ₹ x

Then, 2% of x = (200 – 380) = 20

$\Rightarrow \quad \frac{x}{50} = 20$

$\Rightarrow \quad x = 1000.$

14. Let the CP be ₹ x

Then, SP = 120% of ₹ x

$= ₹\left(x \times \frac{120}{100}\right) = ₹\frac{6x}{5}$

$\therefore \quad \frac{6x}{5} - x = 240$

$\Rightarrow \quad x = 1200$

Hence, SP $= ₹\left(\frac{6}{5} \times 1200\right)$

= ₹ 1,200.

15. Net Loss $= \frac{20^2}{100}\% = 4\%$

It means for every ₹ 100y invested, 96y comes back and ₹ 4y is loss

In this case, 96y = ₹ 2 lacs

Hence, Net Loss

$$= 4y = \frac{4y}{96y} \cdot 2 = \frac{1}{12} \text{ lac loss.}$$

16. Total CP = ₹ 48 × 20
= ₹ 960

Total SP = (₹ 48 × 1.1) × 8
+ (₹ 48 × 1.2) × 12
= ₹ 1,113.6

Profit percentage

$$= \frac{153.6}{960} \cdot 100 = 16\%$$

Hence, option (c) is the answer.

17. Let CP of whole be ₹ x.

CP of 1/2 stock = ₹ $\frac{x}{2}$,

CP of 1/4 stock = ₹ $\frac{x}{4}$

Total SP

$$= ₹\left[\left(120\% \text{ of } \frac{x}{2}\right) + \left(80\% \text{ of } \frac{x}{4}\right) + \frac{x}{4}\right]$$

$$= ₹\left(\frac{3x}{5} + \frac{x}{5} + \frac{x}{4}\right) = ₹\ \frac{21x}{20}$$

Gain $= ₹\left(\frac{21x}{20} - x\right) = ₹\ \frac{x}{20}$

$\therefore$ Gain % $= \left(\frac{x}{20} \times \frac{1}{x} \times 100\right)\% = 5\%.$

18. Going through the options:

Option (a) SP_1 = ₹ 417.6

and SP_2 = ₹ 422.4

$\Rightarrow$ Total SP = ₹ 840

Total CP = Total SP

Hence, option (a) is the answer.

19. Selling price = ₹ 2,000 × 0.8 × 0.9 = ₹ 1,440

Hence, option (b) is the answer.

20. Let CP be ₹ 100.

Then, marked price = ₹ 125

SP 84% of ₹ 125 $= ₹\left(\frac{84}{100} \times 125\right)$

= ₹ 105

Hence, Profit % = (105 – 100)%
= 5%.

21. Let the labeled price be ₹ x

Then, (80% of x) – (75% of x)

= 500 → 5% of x

= 500 → x

$$= \left(500 \times \frac{100}{5}\right)$$

= 10,000.

22. Profit percentage $= \frac{40 - N}{N} \times 100$

$= 25$

$\Rightarrow \quad \frac{40 - N}{N} = \frac{1}{4}$

$\Rightarrow \quad N = 32$

Hence, option (b) is the answer.

23. CP $= \frac{40}{100} \times \text{SP}$

$\rightarrow$ SP $= \frac{5}{2}$

CP $= \left(\frac{5}{2} \times 100\right)\%$ of CP

= 250% of CP.

24. Assume that his cost price

= ₹ 100 for 1,000 gms

So his selling price

= ₹ 110 for 900 gms

To find out the profit, we need to find out the cost price of 900 gms

Cost price = ₹ 90 for 900 gms

Hence, profit percentage

$$= \frac{100 - 90}{90} \times 100$$

$$= \frac{20}{90} \cdot 100 = 22.22\%.$$

25. Marked price $= \frac{45}{0.9} =$ ₹ 50

SP = ₹ 45 and profit percentage = 50%

CP $= \frac{45}{100+50} \cdot 100$

$= 45 \cdot \frac{100}{100+50} =$ ₹ 30

Had there been no discount, new selling price would have been

= ₹ 50

Hence, new profit percentage

$$= \frac{\text{Profit}}{\text{CP}} \cdot 100 = \frac{20}{30} \cdot 100$$

$$= 66\frac{2}{3}\%$$

Hence, option (d) is the answer.

5 Average

Average is calculated by dividing the sum of all the numbers by the number of numbers.

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of numbers}}$$

Let us find the average of four numbers 214, 215, 219, 224.

$$\text{Average} = \frac{214 + 215 + 219 + 224}{4}$$

$$= 218$$

Example 1

Ramesh obtained 76, 65, 82, 67 and 85 marks (out of 100) in English, Mathematics, Physics, Chemistry and Biology. What is his average marks in all the mentioned subjects?

Solution

$$\text{Average} = \frac{76 + 65 + 82 + 67 + 85}{5}$$

$$= \frac{375}{5} = 75$$

Example 2

Average of 10 numbers is zero. Of them, at the most, how many may be greater than zero?

Solution

Average of 10 numbers

$= 0 \Rightarrow$ Sum of 10 numbers

$= (0 \times 10) = 0$

It is quite possible that 9 of these numbers may be positive and if their sum is x, then 10th number is $(-x)$.

Hence, 9 is the answer.

CENTRAL VALUE MEANING OF AVERAGE

Average can also be seen as the central value of all the values given.

Applying this definition for the above written numbers; let us assume the central value of all the given numbers = 214

Now find deviations of all the numbers from 214

214 215 219 224

When assumed central value is (214)

0 +1 +5 +10

Now finding the average of deviation gives us

$$\frac{0 + 1 + 5 + 10}{4} = \frac{16}{4} = 4$$

So **Average = Assumed central value + average of deviations**

$= 214 + 4 = 218$

In the similar fashion, we can assume any value to be the assumed average and then finding the average of all the deviations will give us average. And when we are adding all the numbers and dividing it by number of numbers, then unintentionally we have assumed 0 to be the central value.

Example 3

Average age of A, B, and C is 84 years. When D joins them average of A, B, C and D becomes 80 years. Now a new person E, whose age is 4 years more than D, replaces A and average age of B, C, D and E becomes 78 years. What is the age of A?

Solution

Since average age of A, B and C is 84 years so we can very safely assume that age of each of A, B, and C is 84 years.

$A = 84$ years

$B = 84$ years

$C = 84$ years

After D has joined them, the whole situation can be seen as (Table 5.1):

Table 5.1

	Initially	Finally
A	84 yrs	80 yrs
B	84 yrs	80 yrs
C	84 yrs	80 yrs
D	…	80 yrs

Decrease in the age of A, B, and C can be attributed to the increase in the age of D. So now after getting 12 years in total (4 years each from A, B, and C) D is at 80 years. So original age of $D = 80 - 12 = 68$ years.

So age of E = 72 years

Now average age of A, B, C, and D = 80 years;

$A + B + C + D = 320$

And average of B, C, D, and E = 78 years;

$B + C + D + E = 312$

(Since the average difference between the ages of A and E is 2 years)

Difference $(A - E)$ = $2 \times 4 = 8$ years

Since $E = 72$ years,

so $A = 80$ years

Best part of using central value method of averages lies in the fact that with help of this method every question of average can be done by mental calculation only.

Example 4

Average of ten two-digits numbers is S. However, when we reverse one of the numbers AB as BA from the given 10 numbers, then the average becomes $(S + 1.8)$. What is the value of $B - A$?

Solution

1st Method: Average of 10 numbers is increasing by 1.8, so it can be assumed that 1.8 has been added up to all the numbers.

So, BA is $(1.8 \times 10 = 18)$ more than AB and $BA - AB = 18$

There are so many two-digit numbers which satisfy above condition. Using hit and trial, number can be 13, 24, 35, 46, 57, 68, 79. In every case, difference between the digits = 2

2nd Method: We can use the formula

$(BA - AB) = 9 \times (B - A)$

where BA and AB are two-digit numbers. So,

$18 = 9 \times (B - A)$

$\Rightarrow \quad B - A = 2.$

Example 5

Average score of Rahul Dravid after 25 innings is 46 runs per innings. If after 26th inning his average runs increased by 2 runs, then what is his score in the 26th inning?

Solution

Runs in 26th inning

= Runs total after 26 innings – Runs total after 25 innings

$= 26 \times 48 - 25 \times 46$

$= 98$

Alternatively, we can do this question by above given Central value meaning also. Since the average increases by 2 runs per innings, so we can assume that 2 runs have been added to his score in each of the first 25 innings. Now, total runs added in these innings have been contributed by runs scored in 26th inning which must be equal to $25 \times 2 = 50$ runs.

And after contributing 50 runs, his score in the 26th inning is 48 runs.

Hence, Runs scored in 26th inning

= New average + old innings × change in average

$= 48 + 25 \times 2 = 98.$

To have a mental mapping, we can see the whole situation as (Table 5.2).

Table 5.2

Number of innings	Average in 1st 25 innings	Average in 1st 26 innings	Addition
1	46	48	2
2	46	48	2
3	46	48	2
…	…	…	..
…	…	…	..
…	…	…	..
25	46	48	2
26		48	

Properties of Average

1. Average always lies in between maximum and minimum value. It can be equal to the maximum or minimum value only in case of all the numbers being equal.
   Example: $A1$, $A2$, $A3$, and $A4$ are four numbers given where $A1 > A2 > A3 > A4$. Average of these four numbers will always lies in between $A1$ and $A4$.
   However, if all the four numbers are equal ($A1 = A2 = A3 = A4$) then the average will be equal to each of these numbers.
   Average $= A1 = A2 = A3 = A4$.
2. Average is the resultant of net surplus and net deficit, as used in Central Tendency method.
3. When weights of different quantities are same, then we use simple method to find the average however when we take different weights of different quantities, then it is known as weighted average and we use the method of weighted average to find the average.
   Example: Assume per capita income of India is USD 500 and per capita income of US is USD 200. Now if we merge India and US into one country then it can be seen that per capita income of this new country will **not** be equal to $\frac{500 + 200}{2}$ = USD 350 unless we know the exact population or the ratio of the population of two countries.
4. If the value of the each quantity is increased or decreased by the same value S, then the average will also increase or decrease respectively by S.
5. If the value of each quantity is multiplied by the same value S. Then the average will also be multiplied by S.
6. If the value of each quantity is divided by the same value S ($S \neq 0$) then the average will also be divided by S.

Some Special Cases

1. **If the series is in arithmetic progression**

   $$\text{Average} = \frac{\text{1st number + Last Number}}{2}$$

   [Arithmetic Progression is a series in which difference between any two consecutive terms is same. To know more, go through the chapter – Sequence and Series in this book].

Example 6

Average of first five multiples of 3 is:

Solution

Method 1: Using general definition of Average = 3 (1 + 2 + 3 + 4 + 5)/5 = 45/5 = 9.

Method 2: Since consecutive multiples will be in Arithmetic Progression,

$$\text{Average} = \frac{\text{1st number + Last Number}}{2}$$

1st Number = 1st multiple of 3 = 3, and last number = 5th multiple of 3 = 15

$$\text{Hence, average} = \frac{3 + 15}{2} = 9$$

Example 7

Average of 5 observations p, $p + 2$, $p + 4$, $p + 6$ and $p + 8$ is 11. What is the average of last three observations?

Solution

Method 1: It can be seen that numbers are in AP.

Hence, average

= Middle Number

Hence, middle number

= Average = $p + 4 = 11 \Rightarrow p = 7$

Average of last three observations

= Middle Number

$= p + 6 = 7 + 6 = 13$

Method 2: Going through general approach:

$[p + (p + 2) + (p + 4) + (p + 6) + (p + 8)]/5$

$= 11$ or $5p + 20 = 55$ or $p = 7$.

So, the numbers are 7, 9, 11, 13, 15.

Hence, Required mean = $(11 + 13 + 15)/3$

$= 39/3 = 13$.

2. Average involving time, speed and distance

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Example 8

Lovely goes to Patna from New Delhi at a speed of 40 kmph and returns with a speed of 60 kmph. What is her average speed during the whole journey?

Solution

1st of all understand that, average speed in this case is NOT equal to $= \dfrac{40 + 60}{2} = 50$ kmph.

To solve this question, we need to have the distance and time known. We can assume the distance either a variable (x) or a constant (number). In my perception, it is always better, in these cases, to assume number.

Let us assume that the total distance between Patna and New Delhi is 120 km (LCM of 40 and 60).

So, total time taken (Patna – New Delhi and New Delhi – Patna) = 3 + 2 = 5 hours

So, Average Speed $= \dfrac{240}{5} = 48$ kmph.

Alternatively:

(a) If a person covers the same distance with different speeds A and B,

Average speed for the whole journey

$= \dfrac{2AB}{A + B} =$ Harmonic Mean of A and B.

(b) If a person covers some distance with different speeds A and B for equal time,

Average speed for the whole journey

$= \dfrac{A + B}{2} =$ Arithmetic Mean of A and B.

Example 9

Lovely goes to Patna from New Delhi at a speed of 40 kmph and returns with a speed of N kmph. Her average speed during the whole journey is 60 kmph. What is the value of N?

Solution

Since the distance covered is same, Average speed

$$= \frac{2AB}{A + B} = \frac{2 \cdot 40 \cdot N}{40 + N} = 60$$

$\Rightarrow \quad (40 + N) \times 60 = 80 \times N$

$\Rightarrow \quad 2400N + 60N = 80N \Rightarrow N = 120$

Example 10

Munu Shing is going to Patna from New Delhi. She covers 1st half of the distance with a speed of 40 kmph and 2nd half of the distance with a speed of 60 kmph. What is her average speed for the whole journey?

Solution

This question is exactly same as the question 8. Hence, answer remains the same

= 48 kmph.

Example 11

Ram Prasad is going to Delhi from Dehradun. For half the time, he travelled with a speed of 40 kmph and for other half time, with a speed of 60 kmph. What is his average speed during the whole journey?

Solution

Since time invested is same, average speed

$$= \frac{A+B}{2} = \frac{40+60}{2} = 50 \text{ kmph.}$$

3. **Average involving age**
   If Average of a group of n persons is given at any point of time and we are required to find out the average of same group consisting same n persons, then it is done in the Table 5.3.

Table 5.3

5 years back	10 years back	Now	10 years later	5 years later
$N-5$	$N-10$	N	$N+10$	$N+5$

Example 12

Average age of 5 members of a family is 20 years. The youngest member of the family is 4 years old. At the time of birth of this youngest member, average age of rest of the members of the family was N years. What is the average age of the family (in terms of N) excluding the youngest member?

Solution

Sum of ages of all the members of the family

$= 100$

Sum of ages of all the members of the family excluding the youngest number

$= 100 - 4 = 96$

So, average age of all the members of the family excluding the youngest number

$= 96/4 = 24$

$= N$

What is the average age of the family (in terms of N) excluding the youngest member

$= N + 4$

4. **Average involving Number system**
   (a) Average of 1st n consecutive natural numbers $= \frac{n+1}{2}$
   Example: Average of 1st five natural numbers = 3
   (b) Average of 1st n consecutive even natural numbers $= n + 1$
   Sum of 1st n consecutive even natural numbers $= n(n+1)$
   Example: Average of 1st five even natural numbers = 6
   (c) Average of 1st n consecutive odd natural numbers $= n$
   Sum of 1st n consecutive odd natural numbers $= n^2$
   Example: Average of 1st five odd natural numbers = 5

Weighted Average

We have seen that average can be used only if the weights of all the factors, for which average is to be calculated, is same. So, we can see Weighted Average as a more generalized form of average. This can be further understood with the following illustration (Table 5.4).

Table 5.4

	Class *A*	Class *B*
No. of students	10	10
Average age	12 years	16 years

Now if we combine both these classes, then average age of all the students

$$= \frac{12+16}{2} = \frac{28}{2} = 14 \text{ years.}$$

And this is one standard example of average.

Let us see another example (Table 5.5):

Table 5.5

	Class *A*	Class *B*
No. of students	12	16
Average age	10 years	14 years

Now, if we combine these two classes now, then average cannot be calculated in the above mentioned method, since this is an example where weights attached to different averages are different.

Let us see a general scenario for two groups (Table 5.6):

Table 5.6

Group No.	No. of members	Average age of group
G_1	N_1	A_1
G_2	N_2	A_2

If we combine both the groups, then average age of all the members

$$= \frac{(N_1 \cdot A_1 + N_2 \cdot A_2)}{N_1 + N_2}$$

$$= A_w = \text{Weighted Average}$$

We write this in the conventional "Criss-Cross" method as given in Fig. 5.1.

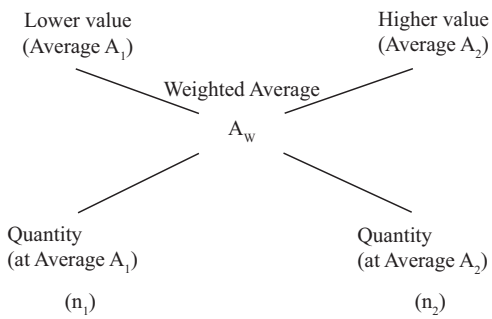


Fig. 5.1

And we write this as:

$$\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$$

i.e., $$\frac{\text{Quantity (Lower Priced)}}{\text{Quantity (Higher Priced)}}$$

$$= \frac{\text{Higher Price} - \text{Average Price}}{\text{Average Price} - \text{Lower Price}}$$

Example 13

Two mixtures of 70% milk solution and 30% milk solution are mixed to from 45% milk solution. In what ratio they have been mixed?

Solution

See Fig. 5.2.

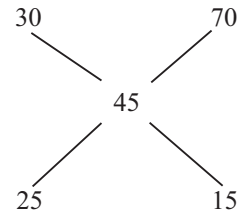


Fig. 5.2

Hence, quantity of 30% milk solution : 70% milk solution = 25:15 = 5:3.

Example 14

Two varieties of rice are mixed in the ratio 2:3. Price of mixture is ₹ 12 per kg and price of the variety having lower weight is ₹ 10 per kg. Find the price of other variety.

Solution

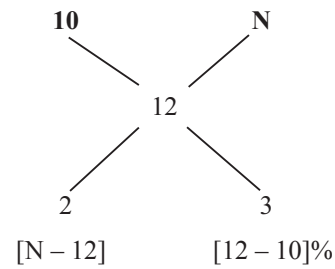


Fig. 5.3

Now, 2:3 $= [N - 12]/[12 - 10]$

$= [N - 12]/2$

So, $N =$ ₹ 13.33 per kg.

PRACTICE EXERCISES

EXERCISE 1

1. If a, b, c, d, e are five consecutive odd numbers, their average is:
(a) $3(a + 4)$
(b) $abcde/5$
(c) $5(a + b + c + d + e)$
(d) none of these

2. The average of 7 consecutive natural numbers is 20. The largest of these numbers is
(a) 20 (b) 22
(c) 23 (d) 24

3. The sum of three consecutive odd numbers is 38 more than the average of these numbers. What is the first of these numbers?
(a) 13 (b) 17
(c) 19 (d) 21

4. A library has an average of 510 visitors on Sundays and 240 on other days. The average number of visitors per day in a month of 30 days beginning with a Sunday is
(a) 282
(b) 285.6
(c) 285
(d) Cannot be determined

5. The average weight of 16 boys in a class is 50.25 kg and that of the remaining 8 boys is 45.15 kg. Find the average weight of all the boys in the class.
(a) 47.55 kg (b) 48 kg
(c) 48.55 kg (d) 49.25 kg

6. The average of six numbers is x and the average of three of these is y. If the average of the remaining three is z, then
(a) $x = y + z$ (b) $2x = y + z$
(c) $x = 2y + 2z$ (d) None of these

7. The average of 50 numbers is 30. If two numbers, 35 and 40 are discarded, then the average of the remaining numbers is nearly
(a) 28.32 (b) 28.78
(c) 29.27 (d) 29.68

8. The average price of three items of furniture is ₹ 15,000. If their prices are in the ratio 3:5:7, the price of the cheapest item is
(a) ₹ 9,000 (b) ₹ 15,000
(c) ₹ 18,000 (d) ₹ 21,000

9. Average of ten positive numbers is x. If each number is increased by 20%, then x
(a) remains unchanged
(b) decreases by 20%
(c) increases by 20%
(d) increases by 10%

10. Of the four numbers, whose average is 60, the first is one-fourth of the sum of the last three. The first number is
(a) 15 (b) 45
(c) 48 (d) 60.25

11. Arithmetic mean of 50 numbers is 35. If each number is increased by 5, then the average of new numbers is
(a) 30 (b) 40
(c) 70 (d) 90

12. The average monthly salary of 20 employees in an organization is ₹ 1,500. If the manager's salary is added, then the average salary increases by ₹ 100. What is the manager's monthly salary?
(a) ₹ 2,000 (b) ₹ 2,400
(c) ₹ 3,600 (d) ₹ 4,800

13. The average weight of 45 students in a class is 52 kg. Five of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class?

(a) 52⅓ (b) 52½
(c) 51⅔ (d) None of these

14. The average of five consecutive numbers is n. If the next two numbers are also included, the average will:
(a) remain the same (b) increase by 1
(c) increase by 1.4 (d) increase by 2

15. A motorist travels to a place 150 km away at an average speed of 50 km/h and returns at 30 km/h. His average speed for the whole journey in km/h is:
(a) 35 (b) 37
(c) 37.5 (d) 40

16. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of the husband is:
(a) 35 years (b) 40 years
(c) 50 years (d) None of these

17. Ramesh goes from A to B at an speed of 40 kmph and comes back with a speed of N kmph. During his whole journey, his average speed = 80 kpmh. What is the value of N?
(a) 120 (b) 140 years
(c) 250 years (d) None of these

18. The average salary of all the workers in a workshop is ₹ 8,000. The average salary of 7 technicians is ₹ 12,000 and the average salary of the rest is ₹ 6,000. The total number of workers in the workshop is:
(a) 20 (b) 21
(c) 22 (d) 23

19. The average of five numbers is 39.20 and the average of three of these numbers is 41. Find the average of the remaining two numbers.
(a) 35.5 (b) 36.5
(c) 37.5 (d) 38.5

20. In a class of 60 students, the average height of 30 students is x cm and that of the remaining students is y cm. Find the average height of the whole class.
(a) $(x + y)$ cm (b) 30 cm
(c) $\frac{x+y}{2}$ cm (d) None of these

EXERCISE 2

1. In a preparatory school, the average weight of 30 girls in a class of 50 students is 16 kg and that of the remaining students is 15.5 kg. What is the average weight of the all the students in the class (Class comprises of boys and girls only)?
(a) 15.2 kg (b) 15.8 kg
(c) 15.4 kg (d) None of these

2. The average sum with 10 girls is ₹ 45. When two more girls join, the average increases by ₹ 2. Find the average sum of the two new girls.
(a) ₹ 57 (b) ₹ 54
(c) ₹ 48 (d) None of these

3. In what ratio must 35% spirit solution be mixed with pure spirit to get a resultant solution of 56% spirit?
(a) 35:44 (b) 44:21
(c) 8:3 (d) 56:9

4. A beggar is having 50 coins with an amount of ₹ 15 in denominations of 25 paise and 50 paise. How many 25 paise coins are there with the beggar?
(a) 36 (b) 40
(c) 30 (d) 25

5. 2 litres of pure spirit is added to 6 litres of a spirit solution containing $16\frac{2}{3}$% spirit. What is the concentration of spirit in the resultant solution?
(a) 12.5% (b) 25%
(c) 37.5% (d) None of these

6. 3 litres of pure milk is added to 10 litres of a milk solution containing 9% milk. Find the concentration of the resultant solution.
(a) $33\frac{1}{3}$% (b) 40%
(c) 50% (d) 30%

7. In what ratio must a quality of sugar @ ₹ 15.50/kg be mixed with another quality of sugar @ ₹ 21.80/kg so that selling the mixture at ₹ 20.02/kg results in 10% profit?
(a) 5:4 (b) 3:2
(c) 2:1 (d) 4:3

8. Parul Jaiswal has ₹ 11,200 in ₹ 100 and ₹ 500 notes. What is the ratio of the number of notes of the two types, if she has a total of 40 notes with her?
(a) 3:2 (b) 7:3
(c) 8:7 (d) 11:9

9. A shrewd milkman mixes water and milk in the ratio 2:3. What part of this mixture should be removed and replaced with water so that the solution contains water and milk in the ratio 1:1?
(a) 1/6 (b) 1/4
(c) 1/3 (d) 1/2

10. A milkman sells his mixture of milk and water at the cost price of milk itself and thus he gains a profit of 20%. What is the ratio of milk and water in the mixture?
(a) 4:1 (b) 5:1
(c) 1:4 (d) None of these

11. I went to a hotel along with 12 friends of mine. I paid ₹ 145 and all the others paid an equal amount. In the end when we did some calculations, we found that the average sum paid by all of us was ₹ 5 more than what was originally paid by each of my friends. How much money did each of my friends pay?
(a) ₹ 120 (b) ₹ 100
(c) ₹ 80 (d) ₹ 70

12. Saketprakash married ten years ago at the age of 27 years. His wife was 23 years old then. Six years after their marriage the average age of Saketprakash, his wife and their son was 22 years. After how many years of Saketprakash' marriage, he got his son?
(a) 6 years (b) 3 years
(c) 2 years (d) 4 years

Direction for questions 13–14: ***Read the passage below and solve the questions based on it.***

The average age of the students in class was 35 years.

13. If a student whose age was 25 is absent from the class, the average of those present went up by 1. How many students are there in the class originally?
(a) 9 (b) 10
(c) 11 (d) 12

14. If 2 students whose ages were 30 years and 20 years left, by what value would the average of the remaining increase? (use the result from question 13)
(a) $\frac{20}{9}$ (b) 2
(c) $\frac{20}{11}$ (d) None of these

15. If the average of m numbers is a, and on adding x to the m numbers, the average of the $(1 + m)$ numbers is b. Then find the value of x.
(a) $m(b - a) + b$ (b) $m(b + a) + a$
(c) $m(a - b) + a$ (d) None of these

16. One out of five weights A, B, C, D and E is of different weight. A person makes a measurement and finds the following:

$$A + B = C + D + E$$

Which of the following you should know to find out which one is of different weight?
(a) $A + C > B + D + E$
(b) $A + D + E < B + C$
(b) $A + B + C > D + E$
(d) $A + C < B + D + E$

17. If the average mark of "a" students in a class is "c" and that of the remaining "b" students is "d", what is the average mark of the class?
(a) $\frac{ac + bd}{b + d}$ (b) $\frac{ab + cd}{a + d}$
(c) $\frac{ac + bd}{a + b}$ (d) $\frac{ad + cd}{b + d}$

18. Average of 10 two digit positive integer is Z. However one number AB is taken as BA, then the average increases to $Z + 2.7$. What is the value of $|B - A|$?
(a) 1 (b) 2
(c) 3 (d) 4

19. First ten multiples of 1, 2, ... 10 are taken. What is the average of all these 100 numbers?
(a) 25.5 (b) 50.5
(c) 75.5 (d) None of these

20. The average of 38, 84, 63, 45, 32, n lies between 50 and 60. If n is an integer greater than the average of the above numbers, then n lies between
(a) $53 < n < 97$ (b) $50 < n < 100$
(c) $52 < n < 98$ (d) None of these

ANSWER KEY

Exercise 1

1. (d)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (a)
9. (c)	**10.** (c)	**11.** (b)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (b)
17. (d)	**18.** (b)	**19.** (b)	**20.** (c)				

Exercise 2

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (d)
9. (a)	**10.** (b)	**11.** (c)	**12.** (c)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (d)
17. (c)	**18.** (c)	**19.** (b)	**20.** (a)				

HINTS AND EXPLANATIONS

Exercise 1

1. Assume numbers to be 1, 3, 5, 7, 9. Obviously, Average = Middle Number = 5 (Since the series is in AP). We will check which option = 5 for $a = 1$, $b = 3$, $c = 5$, $d = 7$ and $e = 9$.
None of these options given 5. Hence, option (d) is the answer.

2. We have taken the consecutive numbers, hence the series is in AP $\Rightarrow$ Average = Middle number of the Series = 4th number = 20.
Since 4th number = 20, 5th number = 21, 6th number = 22 and 7th number = 23. Hence, option (c) is the answer.

3. Let the numbers be x, $x + 2$ and $x + 4$.
Then, $(x + x + 2 + x + 4)$

$$\frac{(x + x + 2 + x + 4)}{3} = 38$$

or, $(3x + 6) - \left(\frac{3x + 6}{3}\right)$

$= 38$ or $2\,(3x + 6)$
$= 114$ or $6x$
$= 102$
or $x = 17$
So, first number
$= x$
$= 17.$

4. Since the month begins with a Sunday, so there will be five Sunday in the month.

Hence, required average

$$= \frac{(510 \cdot 5 + 240 \cdot 24)}{30} = 285$$

Hence, option (c) is the answer.

5. Required average
$= (50.25 \times 16 + 45.15 \times 8/16 + 8)$
$= (804 + 361.20/24) = 1165.20/24$
$= 48.55.$

6. We have: $x = (3y + 3z/6)$ or $2x = y + z$.
Hence, option (b) is the answer.

7. Sum of 50 numbers
$= 30 \times 50 = 1{,}500$
Sum of remaining 48 numbers
$= 1{,}500 - (35 + 40) = 1{,}425$
Required average
$= (1{,}425/48) = 475/16 = 29.68$
Hence, option (d) is the answer.

8. Let their prices be $3x$, $5x$ and $7x$.
Then, $3x + 5x + 7x = (15000 \times 3)$
or, $x = 3000$
Hence, Cost of cheapest item
$= 3x =$ ₹ 9,000.

9. If all the numbers increase by 20% (or change by x%), average will also increase by 20% (change by x%).

10. Let the first number be x.
Then, sum of the four numbers,
$= x + 4x = 5x$
So, $5x/4 = 60$ or $x = (60 \times 4/5) = 48$.

11. Same as 9. Hence, option (b) is the answer.

12. Manager's monthly salary
= ₹ $(1{,}600 \times 21 - 1{,}500 \times 20)$
= ₹ 3,600
Hence, option (c) is the answer.

13. Sum of the weights of the students after replacement
$= [(52 \times 45) - (48 \times 5) + (54 \times 5)]$ kg
= 2,370 kg
∴ New average
= (2,370/45) kg = 52⅔ kg
Hence, option (d) is the answer.

14. Let five consecutive numbers be x, $x + 1$, $x + 2$, $x + 3$ and $x + 4$.
Their average
$= 5x + 10/5 = (x + 2)$
Average of 7 numbers
$= (5x + 10) + (x + 5) + (x + 6)/7$
$= 7x + 21/7 = (x + 3)$
So, the average increased by 1. Hence, option (b) is the answer.
Alternatively, we could have assumed that values and solved this question faster.

15. Average speed
$= 2xy/x + y$ km/h
$= (2 \times 50 \times 30/50 + 30)$ km/h
= 37.5 km/h.

16. Sum of the present ages of husband, wife and child
$= (27 \times 3 + 3 \times 3)$ years = 90 years
Sum of the present ages of wife and child
$= (20 \times 2 + 5 \times 2)$ years = 50 years
∴ Husband's present age
$= (90 - 50)$ years = 40 years.

17. This is not possible. Average speed has to be less than twice the lower speed. In this case, lower speed = 40 kmph, hence average speed will be less than 80 kmph. Hence, option (d) is the answer.

18. Let the total number of workers be x.
Then, $8000x = (12000 \times 7) + 6000\,(x - 7)$
$\leftrightarrow \quad 2000x = 42000 \Rightarrow x = 21.$

20. Since the number of students is same, hence the average age
$= (x + y)/2$ years.

6 Time and Work

There is definite relationship among the amount of time taken, number of persons doing the work and the unit of work done. There are certain formulae that demonstrate the above-mentioned relationship and exhibit how each of these factors influence the other two. We shall now learn the working of these formulae and the concept of time and work.

BASIC TIME-WORK EQUIVALENCE

Number of days × Number of men
= Work done

$$D \times M = W$$

This gives us a very important concept of man-days.

Suppose there are 20 persons working for 10 days to complete a job, then total work done is equal to 200 man-days. Now if we change the number of days in which work is to be completed, then the other factor *i.e.*, number of persons will change accordingly so that product of the factors becomes equal to 200 man-days.

It can also be seen that if Work done is constant (W), then number of persons (M) and number of days (D) will be inversely proportional. So if number of persons is doubled, then number of days would be halved. So if W is constant (Table 6.1).

Table 6.1

Number of Persons	Number of Days
Multiplied by 2/3	Will be Multiplied by 3/2
Multiplied by 2	Will be Multiplied by 1/2
Multiplied by N	Will be Multiplied by $1/N$

Example 1

Instead of allocating initially planned 20 men, a contractor allocates only 15 men for a project. If initially planned schedule was of x days, then how many days would this project now take?

Solution

Since number of men has become 3/4 of the initial, hence number of days would become 4/3 of the original schedule.

Taking it further from $W = M \times D$. In general we have the following relation:

$$\frac{W_1}{W_2} = \frac{M_1}{M_2} \cdot \frac{D_1}{D_2}$$

W_1 = Work done in the 1st case
W_2 = Work done in the 2nd case
M_1 = Men in the 1st case
M_2 = Men in the 2nd case
D_1 = Time required in the 1st case
D_2 = Time required in the 2nd case

Example 2

20 men can cut 15 trees in 18 days. In how many days can 25 men cut 10 trees?

Solution

In this question,

$W_1 = 15$, $W_2 = 10$, $M_1 = 20$, $M_2 = 25$,
$D_1 = 18$, D_2 = To be calculated.

$$\frac{15}{10} = \frac{20}{25} \cdot \frac{18}{D_2}$$

$$D_2 = \frac{20 \cdot 18 \cdot 10}{15 \cdot 25} = 9.6 \text{ days.}$$

Example 3

Yadavjee contractor undertakes to do a work in 50 days by 50 labourers. After 40 days, he realizes that only 50% work is done. How many more men should be employed so that work is complete on time?

Solution

W =	M	×	D	
50%	50	40	—— —— ——	(i)
Rest 50%	$(50 + M)$	10	—— —— ——	(ii)

Since work is constant in both the cases, so, number of men and number of days will be reciprocal to each other. As number of days left in (ii) is 1/4 of (i) initial period, so, number of persons will become 4 times of the initial number of persons.

Hence, number of persons required

$= 50 \times 4 = 200$

So, Additional men required

$= M = 150$ men

Example 4

N men can do a piece of work in 8 days. The same work can be done by $(N + 4)$ men in 6 days. Find the value of N.

(a) 10 (b) 12
(c) 14 (d) None of these

Solution

According to the question,

$N \times 8 = (N + 4) \times 6$

$\Rightarrow \quad N = 12.$

Concept of Individual Work/ Individual Efficiency

If Amit can do a work in 10 days, then Amit will do 1/10th of the work in one day.

Example 5

Amit can do some work in 12 days and Vinit can do the same work in 15 days. In how many days will both of them do the work working together?

Solution

Assume total work = 1 unit

Work done by Amit in one day

= 1/12 unit

Work done by Vinit in one day

= 1/15 unit

Work done by both of them in one day working together

= (1/12) + (1/15) = 9/60 unit

Hence, they will do the whole work in

$\frac{60}{9}$ days $= 6\frac{2}{3}$ days

LCM Method of Solving Time and Work Questions:

Let us assume total work to be equal to the LCM of the days taken by Amit and Vinit (*i.e.*, of 10 and 15)

Assume work = 60 units

Work done by Amit in one day

= 5 units

Work done by Vinit in one day

= 4 units

Work done by both of them in one day working together

= 9 units

So number of days taken by both of them working together

$= \frac{60}{9} = 6.6$ days

Example 6

A, B and C can do a work individually in 8 days, 12 days and 15 days respectively. A and B start working, but A quits after working for 2 days. After this, C joins B till the completion of work. In how many days will the work be complete?

Solution

Let us assume that the work

= LCM (8, 12, 15)

= 120 units

So, work done by A in one day

= 15 units

Work done by B in one day
= 10 units
Work done by C in one day
= 8 units
Work done by A and B in two days
$= 2 \times 25 = 50$ units
Remaining work
= 70 units
Work done by C and B in one day
= 18 units
Time taken to complete the remaining work by C and B

$$= \frac{70}{18} = 3\frac{16}{18} \text{ days}$$

So, total number of days

$$= 5\frac{16}{18} \text{ days}$$

Example 7

A and B together can do a work in 12 days, B and C together can do the same work in 10 days and A and C together can do the same work in 8 days. In how many days will the work be complete if A, B and C are working together?

Solution

Let us assume work
= LCM of (12, 10, 8)
= 120 units

So, A and B are doing 10 units in one day, B and C are doing 12 units in a day and A and C are doing 15 units in a day.

Adding all these, 2 $(A + B + C)$ are doing 37 units in a day.

$\Rightarrow$ $(A + B + C)$ are doing

$$\frac{37}{2} = 18.5 \text{ units in a day}$$

So, Time taken to complete the work

$$= \frac{120}{18.5} \text{ days} = 6.48 \text{ days}$$

Example 8

A can do a work in 20 days and B can do the same work in 30 days. A and B complete a work, working together and they are paid ₹ 240. What is the share of A in the whole payment?

Solution

Since number of days taken by A and B are 20 days and 30 days respectively, ratio of their efficiency = 3:2. Payment done is in the same ratio as that of efficiency.

Hence, payment obtained by A

$$= \frac{3}{5} \cdot 240 = ₹\ 144.$$

Example 9

Working together, A and B can do a work in 10 days. A alone can do the same work working alone in 15 days. In how many days, B alone can do the whole work working alone?

Solution

Let us assume total work = 30 units.

So, $A + B$ will do

$$\frac{30}{10} = 3 \text{ units in a day,}$$

and Aalone will do

$$\frac{30}{15} = 2 \text{ units in a day.}$$

So, Number of units B will be doing in a day

$$= 3 - 2 = 1 \text{ unit per day.}$$

Hence, number of days taken by B working alone

$$= 30 \text{ days.}$$

Application of Concept of Individual Work/Individual efficiency

Pipes and Cisterns

Pipes and Cisterns is just another application of the concepts of Time and Work. While we see only +ve work being done in normal cases of Time and Work, in case of Pipes and Cisterns, –ve work is also possible.

Given that pipe A and B can fill a tank in 20 mins and 25 mins working individually $\Rightarrow$ this

statement is similar to "*A* can do a work in 20 mins and *B* can do the same work in 25 mins."

Again Given that pipe *C* can empty a tank in 40 mins ⇒ this statement is similar to "*C* can demolish a wall in 40 minutes (assuming that the work is building and demolishing the wall).

Let us understand this with the help of an example:

Example 10

A and *B* are two taps which can fill a tank individually in 10 min and 20 min respectively. However, there is a leakage at the bottom which can empty a filled tank in 40 min. If the tank is empty initially, how much time will both the taps will take to fill the tank?

Solution

Let us assume the units of work
= LCM of (10, 20, 40) = 40 units
Work done by Tap *A*/min
= 4 units/min (Positive work)
Work done by Tap *B*/min
= 2 units/min (Positive work)
Work done by leakage/min
= 1 unit/min (Negative work)
Net work done/min
= 4 + 2 – 1 = 5 units/min
Hence, Time taken
= 8 mins.

Example 11

Pipe *A* can fill a tank in 3 hours. But there is a leakage also, due to which it takes 3.5 hours for the tank to be filled. How much time will the leakage take in emptying the tank if the tank is filled initially?

Solution

Assume total units of work
= 10.5 units
Work done by Tap *A*/hour
= 3.5 units/hour (Positive work)
Work done by leakage/hour
= 3 units/hour (Negative work)
Net work done/hour
= 0.5 units/hour
So time taken

$$= \frac{10.5}{0.5} = 21 \text{ hour.}$$

General Expression Correlating Time taken and Efficiency:

If efficiency of *A* is *x*% more than efficiency of *B* and *B* takes '*B*' days to complete the work, then

A will take $\left(\frac{B}{100+x} \times 100\right)$ days to complete the same work.

If efficiency of *A* is *x*% less than efficiency of *B* and *B* takes '*B*' days to complete the work, then *A* will take $\left(\frac{B}{100-x} \times 100\right)$ days to complete the same work.

So, if *A* is 20% more efficient than *B* and *B* takes '*B*' days to complete the work, then *A* will take $\frac{B}{1.2}$ days to do the same work.

With this, it can also be observed that if work is constant then Time taken is inversely proportional to the efficiency.

Example 12

A is 25% more efficient than *B*. *B* takes 20 days to complete the job working alone. How many days will *A* take to finish the job working alone?

Solution

Number of days taken by *A*

$$= \frac{20}{1.25} = 16 \text{ days.}$$

Example 13

John is thrice as efficient as Abraham and hence completes a work in 60 days less than the number of days taken by Abraham. What will be the number of days taken by both of them working together?

Solution

Since John is thrice as efficient as Abraham, so number of days taken by him will be 1/3rd the number of days taken by Abraham. So, if John is taking x days, then Abraham will take $3x$ days to complete the same work.

Now, $3x - x = 2x = 60$ days

So, $x = 30$ days and $3x = 90$ days

Let us assume that total work

= 90 units (LCM of 30 and 90)

So, total work done by both of them in one day

= 3 + 1 = 4 units of work.

So, total number of days

$$= \frac{90}{4} \text{ days} = 22.5 \text{ days.}$$

Some More Worked Out Examples

Example 14

10 men or 22 boys can do a work in 18 days. In how many days, can 15 men and 20 boys finish the work working together?

Solution

Since the work force supplied contains 'and', we are required to convert efficiency of men into efficiency of boys or vice versa. In short, we will solve this question by converting 'men' into 'boys' or vice-versa.

Since 10 men = 22 boys,

hence 15 men = 33 boys.

So, 15 men + 20 boys = 22 boys + 20 boys

= 42 boys.

Using $$\frac{W_1}{W_2} = \frac{M_1}{M_2} \cdot \frac{D_1}{D_2},$$

since W_1 and W_2 are same, hence $M_1 D_1 = M_2 D_2$

or, $22 \times 18 = 42 \times D_2$

$\Rightarrow$ $x = 9.42$ days.

Example 15

Nandan is twice as good a workman as Divya and together they complete the work in 15 days. In how many days can Divya alone complete the work?

(a) 45 days (b) 40 days

(c) 35 days (d) None of these

Solution

Assume that Nandan works 2 units/day, and Divya works 1 unit/day.

So, number of units done per day

= 3 units/day

Hence, Work done

= 15 × 3

= 45 units

= Work Equivalent

Since Divya does 1 unit/day, she would take

$$\frac{45}{1} = 45 \text{ days.}$$

PRACTICE EXERCISES

EXERCISE 1

1. Aftab can do a piece of work in 12 days and Ravi can do the same work in 18 days. If Aftab starts the work and quits after working for three days, then in how many days will the remaining work be completed by Ravi alone?

(a) 13½ (b) 9

(c) 16½ (d) 12

2. Efficiency of Ravi and Mayank of doing the same work is 3:2. If they together can complete a work in 18 days, then in how many days Ravi can complete the work alone?

(a) 45 (b) 30

(c) 24 (d) 40

3. A tap can fill an empty tank in 12 hours and a leakage can empty half the tank in 10 hours. If tap and leakage are working when the tank is empty, then how long would it take for the tank to be filled to half its capacity?
(a) 60 hours (b) 30 hours
(c) 15 hours (d) 12 hours

4. Pipe *A* can fill an empty tank in eight hours and pipe *B* can empty the full tank in four more hours. If both the pipes are opened simultaneously, then how long would it take to fill an empty tank?
(a) 12 hours (b) 24 hours
(c) 18 hours (d) 30 hours

5. Sanjay can wash a truck in 45 minutes and he works for six hours a day. How many trucks can he wash in three days?
(a) 24 (b) 12
(c) 21 (d) 15

6. *N* men can do a piece of work in 8 days. The same work can be done by $(N+4)$ men in 6 days. Find the value of *N*.
(a) 10 (b) 12
(c) 14 (d) None of these

7. Harveen can do a piece of work in 18 days. He worked for 12 days and left. Deepak finished the remaining work in 8 days. In how many days can Deepak alone complete the work?
(a) 21 (b) 22
(c) 23 (d) 24

8. Munchun can do a work in 20 days and Chunmun alone can do it in 10 days. Both worked together and were paid ₹ 1,800. The share of Chunmun is
(a) ₹ 1,200 (b) ₹ 1,500
(c) ₹ 900 (d) None of these

9. Shahid Bhai can do 2/3rd of work in 18 days. In how many days can he finish 1/4th of the work?
(a) 6½ (b) 6.75
(c) 6 (d) 13.5

10. *A* can do half as much work as *B* in one day. *B* alone can do a certain work in 12 days. In how many days can *A* and *B* together finish that work?
(a) 6 days (b) 7 days
(c) 8 days (d) None of these

11. If 35 persons can do a piece of work in 6 days, in how many days can 15 persons do it?
(a) 12 (b) 14
(c) 16 (d) 18

12. *A* and *B* can complete a work in 15 days. *A* is 50% more efficient than *B*. How long would *A* take to complete the work working alone?
(a) 20 days (b) 22 days
(c) 24 days (d) None of these

13. 40 persons can finish a work in 60 days. If 40 persons start the work and at the end of every 10 days 40 more persons of the same efficiency join them, then how long will it take for the work to be completed?
(a) 30 days (b) 20 days
(c) 40 days (d) None of these

14. A group of 18 men and 12 women can do a work in 18 days. A woman takes twice as much time as a man to do the work. How many days will 8 men take to finish the same work?
(a) 45 (b) 48
(c) 54 (d) None of these

15. *A* and *B* can do a work in 8 days, *B* and *C* can do the same work in 12 days. *A*, *B* and *C* together can finish it in 6 days. In how many days *A* and *C* together will do the same work?
(a) 4 days (b) 6 days
(c) 8 days (d) 12 days

16. *A* is twice as good a workman as *B* and together they finish a piece of work in 14 days. The number of days taken by *A* alone to finish the work is:
(a) 11 (b) 21
(c) 28 (d) 42

17. A is 50% as efficient as B. C does half of the work done by A and B together. If C alone does the work in 40 days. Then A, B and C together can do the work in:
(a) 13⅓ days (b) 15 days
(c) 20 days (d) 30 days

18. A and B can complete a work in 15 days and 10 days respectively. They stared doing the work together but after 2 days B had to leave and A alone completed the remaining work. The whole work was completed in:
(a) 8 days (b) 10 days
(c) 12 days (d) 15 days

19. X and Y can do a piece of work in 20 days, and 12 days respectively. X started the work alone and then after 4 days, Y joined him till the completion of the work. In how many days the whole work is complete?
(a) 6 days (b) 10 days
(c) 15 days (d) 20 days

20. A does 4/5 of a work in 20 days. He then calls in B and they together finish the remaining work in 3 days. How long B alone would take to do the whole work?
(a) 23 days (b) 37 days
(c) 37½ days (d) 40 days

EXERCISE 2

1. $A + B$ can do a work in 10 days where as $B + C$ can do the same work in 8 days. Who is most efficient among the three?
(a) A
(b) B
(c) C
(d) Cannot be determined

2. A man works twice as fast as a woman. A woman works twice as fast as a child. If 16 men can complete a job in 12 days, how many days would be required for 32 women and 64 boys together to complete the same job?
(a) 2 days (b) 3 days
(c) 4 days (d) 6 days

3. A tank with capacity T litres is empty. If water flows into the tank from pipe X at the rate of X litres per minute and water is pumped out by pipe Y at the rate of Y litres per minute and $X > Y$, then in how many minutes will the tank be filled?
(a) $T/(Y - X)$ (b) $T/(X - Y)$
(c) $(T - X)\ Y$ (d) $(X - Y)/60T$

4. A can do a piece of work in 90 days, B in 40 days and C in 12 days. They work for a day each in turn *i.e.*, first day A does it alone, second day B alone and third day C alone. After that the cycle is repeated till the work is finished. They get ₹ 240 for this job. If the wages are divided in proportion to the work each had done, find the amount A will get:
(a) 14 (b) 24
(c) 34 (d) 36

5. There are 12 pipes connected to a tank. Some of them are fill pipes and the others are drain pipes. Each of the fill pipes can fill the tank in 8 hours and each of the drain pipes can drain completely in 6 hours. If all the pipes are kept open, an empty tank gets filled in 24 hours. How many of the 12 pipes are fill pipes?
(a) 5 (b) 6
(c) 7 (d) 8

Direction for questions 6 and 7: ***Read the passage below and solve the questions based on it.***

The tank at a water supply station is filled with water by several pumps. At first, three pumps of the same capacity are turned on 2.5 hour later, two more pumps (both the same) of a different capacity are set into operation. After 1 hour the additional pumps were set into operation; the tank was almost filled to its capacity (15 m^3 were still lacking); in another hour, the tank was full. One of the two additional pumps could have filled the tank in 40 hours.

6. What is the volume of the tank?
(a) 1.60 m^3 (b) 2.80 m^3
(c) 3.75 m^3 (d) 4.90 m^3

7. How much water does one of the first three pumps emit in an hour?
(a) 1.5 m^3 (b) 2.4 m^3
(c) 3.3 m^3 (d) 4.2 m^3

Direction for questions 8–9: *Read the information below and solve the questions based on it.*

Two products A and B can be made on either of the two extrusion machines M_1 or M_2. The time in minutes, taken by these products according to the machine used, are as given below. Assume a day has 8 working hours.

	M_1	M_2
A	8	10
B	6	6

8. If A and B are produced in the ratio 3:1, then what is the maximum total of products A and B that can be produced in a day?
(a) 120 (b) 160
(c) 116 (d) None of these

9. If A is produced only on Machine M_1 and B produced only on machine M_2, then what is the unutilised time on M_2 in a day, if A and B are produced in 1:1 ratio?
(a) 20 minutes (b) 60 minutes
(c) 150 minutes (d) None of these

Direction for questions 10–13: Read the information below solve the questions based on it.

There are infinite pipes attached to a very big tank. Pipes are numbered like P_1, P_2, P_3 ... and so on. Its also known that efficiency of every subsequent pipe is half the efficiency of earlier pipe *i.e.*, efficiency of P_2 pipe is half the efficiency of P_1 pipe and so on. However, in case of P_1 pipe, this rule is not true since there is no earlier pipe. When all the pipes are working together, tank gets filled in 2 hours.

10. What time will P_4 take to fill the tank working alone?
(a) 8
(b) 16
(c) 32
(d) Cannot be determined

11. What is the difference in the time taken by P_5 and P_6?
(a) 32
(b) 64
(c) 128
(d) Cannot be determined

12. How many pipes can fill the tank within 100 hours working alone?
(a) 4 (b) 5
(c) 6 (d) Infinite

13. Due to some technical problem, only four pipes P_1, P_2, P_3 and P_4 are in working condition. These pipes are now paired up to obtain two pairs of pipes. Now it is found that one pair of pipe is taking 2/3rd of the time taken by the other pair to fill the tank independently. Which of the following is one of the two pairs?
(a) P_1, P_2
(b) P_1, P_3
(c) P_1, P_4
(d) Cannot be determined

14. Ramesh and Suresh can finish the garden cleaning in 'p' days. They worked together for 'q' days; then Ramesh fell sick and remaining cleaning was completed by Suresh in 'r' days. In how many days could each of them clean the garden?
(a) Ramesh $= \dfrac{pr}{r - p + q}$ days;
Suresh $= \dfrac{pr}{p - q}$
(b) Ramesh $= \dfrac{p}{r - p + q}$ days;
Suresh $= \dfrac{p}{p - q}$ days
(c) Ramesh $= \dfrac{pr}{r + p - q}$ days;
Suresh $= \dfrac{pr}{q - p}$ days

(d) Ramesh $= \dfrac{pr^2}{r - p + q}$ days;

Suresh $= \dfrac{pr^2}{q - p}$ days

15. *A* can do a piece of work in 14 days which *B* can do in 21 days. They begin together but 3 days before the completion of the work, A leaves off. The total number of days to complete the work is:
(a) $6^3/_5$ (b) 8½
(c) 10.2 (d) 13.5

16. *A*, *B* and *C* can do a piece of work in 36, 54 and 72 days respectively. They started the work but *A* left 8 days before the completion of the work while *B* left 12 days before the completion. The number of days for which *C* worked is:
(a) 4 (b) 8
(c) 12 (d) 24

17. 12 men can complete a piece of work in 4 days, while 15 women can complete the same work in 4 days. 6 men start working on the job and after working for 2 days, all of them stopped working. How many women should be put on the job to complete the remaining work, if it is to be completed in 3 days?
(a) 15 (b) 18
(c) 22 (d) 25

18. If 12 men and 16 boys can do a piece of work in 5 days; 13 men and 24 boys can do it in 4 days, then the ratio of the daily work done by a man to that of a boy is:
(a) 2:1 (b) 3:1
(c) 3:2 (d) 5:4

19. 10 women can complete a work in 7 days and 10 children take 14 days to complete the work. How many days will 5 women and 10 children take to complete the work?
(a) 3 (b) 5
(c) 7 (d) None of these

20. *A* can work half as fast as *B* and *C* together. *A* and *B* together can work three times as fast as *C*. If all three of them complete a job in 15 days, how long would *C* take to complete the same work?
(a) 45 days (b) 54 days
(c) 60 days (d) 65 days

ANSWER KEY

Exercise 1

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (a)
9. (b)	**10.** (c)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (b)
17. (a)	**18.** (c)	**19.** (b)	**20.** (c)				

Exercise 2

1. (d)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)
9. (c)	**10.** (c)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (d)
17. (a)	**18.** (a)	**19.** (c)	**20.** (c)				

HINTS AND EXPLANATIONS

Exercise 1

1. Work done by Aftab in 3 days

$$= \frac{3}{12} = \frac{1}{4} \text{ of the total work}$$

Remaining work

$$= 1 - \frac{1}{4} = \frac{3}{4} \text{ of the total work}$$

Ravi will do the remaining work in

$$= \frac{3}{4} \cdot 18 = 13.5 \text{ days.}$$

2. Since efficiency of Ravi and Mayank of doing the same work is 3:2, assume that Ravi does 3 units of work in a day and Mayank does 2 units of work in a day.
So, work done by them (both together) in one day = 5 units. They complete the work in 18 days, working together.
So, total units of work

$$= 18 \times 5 = 90 \text{ units}$$

Hence, time taken by Ravi to complete the work working alone

$$= \frac{90}{3} \text{ days.}$$

3. Time taken by tap to fill the tank

= 12 hours

Time taken by leakage to empty the whole tank

= 20 hours

Assume total volume of tank

= LCM of 12 and 20 = 60 litres

So water filled by tap/hour

$$= \frac{60}{12} = 5 \text{ litres/hour}$$

and water removed by leakage/hour

$$= \frac{60}{20} = 3 \text{ litres/hour}$$

Hence, quantity of water filled per hour litres/hour – 3 litres/hour

= 2 litres/hour

So, time taken by the tap to fill half the tank

$$= \frac{30}{2} = 15 \text{ hours.}$$

4. Time taken by pipe A to fill the tank

= 8 hours

Time taken by pipe B to empty the tank

= 12 hours

Assume total volume of tank

= LCM of 8 and 12 = 24 litres

So water filled by pipe A/hour

$$= \frac{24}{8} = 3 \text{ litres/hour}$$

and water removed by pipe B/hour

$$= \frac{24}{12} = 2 \text{ litres/hour}$$

Hence, quantity of water filled per hour

= 3 litres/hour – 2 litres/hour

= 1 litre/hour

So, time taken by the tap to fill the tank

$$= \frac{24}{1} = 24 \text{ hours.}$$

5. Sanjay works everyday for

$6 \times 60 = 360$ minutes

He takes 45 minutes to wash a truck.
Hence, number of trucks he will wash in a day

$$= \frac{360}{45} = 8$$

Hence, number of trucks he will wash in 3 days

$$= 8 \times 3 = 24.$$

6. $N \times 8 = (N + 4) \times 6$

Or, $2N = 24$. Hence, $N = 12$.

7. Work done by Harveen = the whole work

So, remaining work $= 1 - \frac{2}{3} = \frac{1}{3}$ of the total = Done by Deepak in 8 days.

Hence, days taken by Deepak to finish the whole work = 8 × 3 = 24 days.

8. Ratio of efficiency is inversely proportional to the number of days taken. So, ratio of efficiency of Munchun to Chunmun = 1:2 (since ratio of time taken = 2:1).
We also understand the payment should be done in the ratio of Efficiency. Hence, ratio of payment done to Munchun to Chunmun = 1:2.
Hence, amount received by Chunmun $= \frac{2}{3}$ of the total = ₹ 1,200.

9. Work done is proportional to the number of days taken, and we can use 'Unitary Method' of solving questions in these situations.
Since Shahid Bhai takes 18 days to finish 2/3rd of the work, hence time taken by him to do the whole work

$$= 18 \cdot \frac{3}{2} = 27 \text{ days}$$

Hence, time taken by him to do 1/4th of work

$$= \frac{27}{4} = 6.75 \text{ days.}$$

10. A can do half as much work as B in one day ⇒ it means that if B does 2 units in a day, then A does 1 unit in a day. In other words, it also means that if B will take twice the time taken by A in doing the same quantity of work.
So, time taken by A
= Twice the time taken by B
= 2 × 12 = 24 days
Assume total units of work
= LCM of 12 and 24 = 24 units
Hence, work done by A in one day
= 1 unit and work done by B in one day
= 2 units
So, work done by both of them together in one day
= 3 units
Hence, total time taken by both of them to finish the work

$$= \frac{24}{3} = 8 \text{ days.}$$

11. Using $\frac{W_1}{W_2} = \frac{M_1}{M_2} \cdot \frac{D_1}{D_2}$

$M_1 \times D_1 = M_2 \times D_2 \Rightarrow 35 \times 6 = 15 \times D$
⇒ $D = 14$ days.

12. A is 50% more efficient than B ⇒ If B does 2 units in a day, then A does 3 units in a day.
So, work done by A and B together in a day
= 5 units
Hence, total work = 5 × 15 = 75 units
So, time taken by A to finish the work alone

$$= \frac{75}{3} = 25 \text{ days.}$$

13. Total man days required to finish the work
= 40 × 60 = 2,400 units
Now, for the first 10 days, 40 men were working. So total work done
= 40 × 10 = 400 units
For the next 10 days, 80 men were working. So total work done
= 80 × 10 = 800 units
For the next 10 days, 120 men were working. So total work done
= 120 × 10 = 1,200 units
Total work done
= 1,200 + 800 + 400 = 2,400 units
= Total work
So, work is finished in 30 days.

14. A woman takes twice as much time as a man to do the work
⇒ So, 2 women = 1 man
So, 12 women = 6 man
18 men and 12 women
= 18 men + 6 men = 24 men
So, 24 men can do the work in 18 days.
Using $M_1 \times D_1$
$= M_2 \times D_2 \Rightarrow 24 \times 18$
$= 8 \times D$
⇒ $D = 54$ days.

15. $(A + B)$ take 8 days, $(B + C)$ take 12 days.
So assume total units of work
$= 24$ units
Units of the work done by $(A + B)$
$= 3$ units/day (i)
Units of the work done by $(B + C)$
$= 2$ units/day (ii)
Units of the work done by $(A + B + C)$
$= 4$ units/day (iii)
Subtracting (iii)–(i) gives
$C = 1$ unit/day
and Subtracting (iii)–(ii) gives
$A = 2$ units/day
So, work done by $A + C$
$= 3$ units/day
Hence, time taken by A and C working together

$$= \frac{24}{3} = 8 \text{ days.}$$

16. Assume work done by A in one day
$= 2$ units
So work done by B
$= 1$ unit/day
So total work done by A and B in one day
$= 3$ units/day,
Hence, total work
$= 14 \times 3 = 42$ units,
Time taken by A alone to finish the work

$$= \frac{42}{2} = 21 \text{ days.}$$

7 Ratio, Proportion and Variation

Ratio is a quantity which expresses the relationship between two similar quantities. It expresses a magnitude by which one quantity is a part of another quantity or a multiple of another quantity.

If the value of A and B is 8 and 6 respectively, then we say that they are in the ratio 8:6 (read as 8 is to 6). Ratio can be also understood as the relationship which one quantity bears with the other of the same kind. Due to this reason, we cannot compare salary of one person with the percentage expenditure of other person.

Ratio of two quantities A and B is written as $A:B$. Here A is known as antecedent and B is known as consequent. It can be also seen that $A:B = kA:kB$, where k is any constant known as constant of proportionality, $k \neq 0$.

- If antecedent is more than consequent (or, numerator is more than denominator), then this kind of ratio is known as improper ratio.

  Example: 5/3, 9/5 etc.

- If antecedent is less than consequent (or, numerator is less than denominator), then this kind of ratio is known as proper ratio.

  Example: 3/7, 7/18 etc.

Example 1

Consider any ratio a/b. Now x is added to the numerator and denominator of this fraction. Which of the following is greater: $\frac{a+x}{b+x}$ or $\frac{a}{b}$?

Solution

It depends upon two factors:

(i) If the ratio is proper or improper

(ii) x is positive or negative.

If $\frac{a}{b} > 1$ and $x > 0$,

or, $\frac{a}{b} < 1$

and $x < 0$

$$\frac{a}{b} > \frac{a+x}{b+x}$$

and if $\frac{a}{b} > 1$ and $x < 0$,

or, $\frac{a}{b} < 1$ and $x > 0$

$$\frac{a}{b} < \frac{a+x}{b+x}.$$

We can understand ratio in two ways:

1. Ratio as a bridging element
2. Ratio as a multiplier

1. **Ratio as a bridging element:** Ratio as a bridging element helps us in establishing the relationship between more than two quantities. This can be further understood with the following example:

   Suppose conversion rate of our currency Rupee is given with respect to US dollar. Again, conversion rate of Rupee is given with respect to Pound Sterling also. Now, if we have to find out conversion ratio of US dollar with respect to Pound Sterling, we can find it out by making Ru-

pee as the bridge between US dollar and Pound sterling.

Example 2

Ratio of the age of A and B is 2:5 and ratio of age of B and C is 3:4. What is the ratio of age of A, B and C?

Solution

Since B is the common platform which associates A and C, so we will try to make B equal in both the cases.

Age of A:Age of B = [2:5] × 3

Age of B:Age of C = [3:4] × 5

or, Age of A:Age of B = 6:15 (i)

Age of B:Age of C = 15:20 (ii)

Since ratio of B is same in both the cases, hence age of A:age of B:age of C

= 6:15:20.

Example 3

Given that

Salary of A : salary of B = 1:2

Salary of B : salary of C = 3:4

Salary of C : salary of D = 5:6

Salary of D : salary of E = 7:8

Salary of E : salary of F = 9:10

What is ratio of salary of A : salary of B : salary of C : salary of D : salary of E : salary of F?

Solution

Salary of A : salary of B : salary of C : salary of D : salary of E : salary of F = (1 × 3 × 5 × 7 × 9):(2 × 3 × 5 × 7 × 9):(2 × 4 × 5 × 7 × 9):(2 × 4 × 6 × 7 × 9):(2 × 4 × 6 × 8 × 9):(2 × 4 × 6 × 8 × 10).

(Understand the above mechanism with the help of method given in Example 2. We can use this method as short-cut also to find out ratio in these cases in following way: For A–take all the leftmost digits, and now keep shifting towards right digits by quitting one by one all the leftmost digits. So, B = Right digit of 1st ratio and so on for C, D, E and F).

Example 4

If A:B = 3:4,

B:C = 5:7

C:D = 10:11

What is the ratio of A:D?

Solution

$A = 3 \times 5 \times 10$

and $D = 4 \times 7 \times 11$

So, ratio = 150:308

Alternatively, $(A/B) \times (B/C) \times (C/D)$

$= (3/4) \times (5/7) \times (10/11)$

$= (3 \times 5 \times 10)/(4 \times 7 \times 11)$

= 150:308

Example 5

A, B, C and D purchase a gift worth ₹ 60. A pays 1/2 of what others are paying, B pays 1/3rd of what others are paying and C pays 1/4th of what others are paying. What is the amount paid by D?

Solution

Since A is paying 1/2 of what others are paying, so A is paying 1/3rd of total amount. (To understand this, let us assume that B, C, D are paying ₹ $2x$. So A is paying ₹ x. So, total amount being paid by A, B, C and D = $3x$ = ₹ 60, hence amount paid by $A = x/3x$ = 1/3rd of total).

So, amount paid by A = 60/3 = ₹ 20

Similarly, B is paying 1/4th of total and C is paying 1/5th of total.

Hence, amount paid by B and C are ₹ 15 and ₹ 12 respectively.

So, amount paid by D = ₹ 13

2. **Ratio as a multiplier:** The moment we say that ratio of two numbers A and B is 5:1, what we mean to say is that A is 5 times of B.

   This also can be seen that A:B:C in A/2:B/3:C/4 = K is not same as A:B:C = 1/2:1/3:1/4 since multiplier of A, B and C are not same in both the cases.

Ratio of $A:B:C$ in $A/2:B/3:C/4 = K$ can be calculated in following way:

Since $A/2 = B/3 = C/4 = K$, so

$A = 2K$, $B = 3K$ and $C = 4K$

Hence, ratio of $A:B:C$

$= 2:3:4$.

While calculating the ratio of A, B and C in

$A:B:C = 1/2:1/3:1/4$,

we will multiply each of A, B and C by the LCM of denominator of all the ratios *i.e.*, 12. So,

$A:B:C = 6:4:3$

Example 6

10 persons can cut 8 trees in 12 days. How many days will 8 persons take to cut 6 trees?

Solution

Let us see this question in a bit changed perspective:

Suppose if the question is: 10 persons can cut 8 trees in 12 days. How many days will 10 persons take to cut 4 trees?

Answer to this question is: Since amount of work is getting halved, so number of days will also get halved.

Now, there are three factors namely, number of men, number of days and number of trees, which are responsible for the final answer.

Let us see how these factors are affecting the final result:

Since number of men is less now in the final situation, so more days will be required. Hence, multiplier = 10/8 (Had there been 12 persons, multiplier would have been 10/12).

Number of trees is less now in the final situation, so less days will be required. So, multiplier = 6/8

Hence, total number of days

$= 12 \times (10/8) \times (6/8) = 90/8 = 11.25$ days

Example 7

A train approaches a tunnel AB. Inside the tunnel, a cat is located at a point that is 3/8 of the distance AB measured from the entrance A. When the train whistles, the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat exactly at the exit. What is the ratio of speed of train and speed of cat?

Solution

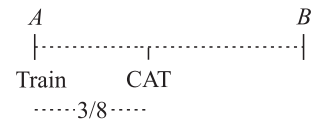


Fig. 7.1

Initially, this is the position of train and cat. Now, let us assume that cat is moving towards exit B. The moment cat covers 3/8 AB distance in the direction of exit B, train will be at the entrance A.

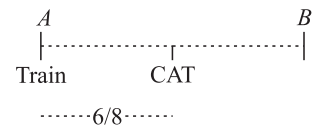


Fig. 7.2

Now, if the cat moves in the direction of exit B, train is catching up with the cat at the exit B. So, in the time cat covers 2/8 of AB, train is covering whole distance from A to B.

So, Ratio of the distance covered by train and the distance covered by cat = 4:1

So, Ratio of speed = 4:1.

Example 8

Pranesh can do a work in 15 days. In how many days will the work be completed by his brother Saket if efficiency of Saket is 60% more than that of Pranesh?

Solution

Ratio of efficiency of Pranesh and Saket

$= 100:160 = 5:8$

So, number of days taken by Pranesh and Saket will be in the ratio of 8:5.

Now, Pranesh takes 15 days to do this work, so Saket will take 15 × 5/8 = 9.37 days.

Some Important Results

1. Continued proportion a, b and c are said to be in continued proportion if

$$\frac{a}{b} = \frac{b}{c}$$

So, $b^2 = ac$

Here b is known as the mean proportion. Similarly, if a, b, c and d are in continued proportion, then we get

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$

2. Componendo:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$

3. Dividendo:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$

4. Componendo and Dividendo:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

5. Invertendo:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$

6. Alterando:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

7. $$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$

In general, if

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = K$$

Then, $$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = K$$

$$= \frac{a+c+e+\ldots}{b+d+f+\ldots}$$

= (any combination of numerator/any combination of corresponding denominator).

Example,

$$1/2 = 3/6 = 4/8 = \ldots = \frac{(1+3+4)}{(2+6+8)}$$

$$= (3+4)/(6+8)$$

8. If we multiply the numerator and denominator of a ratio by any number N ($N \neq 0$), then the ratio remains same.

$$A/B = NA/NB$$

9. If we divide the numerator and the denominator of a ratio by any number N ($N \neq 0$), then the ratio remains same.

$$A/B = (A/N)/(B/N)$$

10. If a/b, c/d, e/f ... etc., are all unequal ratios, then the value of $\frac{(a+c+e+\ldots)}{(b+d+f+\ldots)}$ lies in between minimum and maximum of all these ratios.

PRACTICE EXERCISES

EXERCISE 1

1. If $A:B = 5:7$ and $B:C = 6:11$, then $A:B:C$ is:
(a) 55:77:66 (b) 30:42:77
(c) 35:49:42 (d) None of these

2. If $A:B = 8:15$ and $C:D = 4:5$, then $A:D$ is equal to:
(a) 2:7 (b) 4:15
(c) 8:15 (d) 15:4

3. If $2A = 3B = 4C$, then $A:B:C$ is:
(a) 2:3:4 (b) 4:3:2
(c) 6:4:3 (d) 20:15:2

4. If $2A = 3B$ and $4B = 5C$, then $A:C$ is:
(a) 4:3 (b) 8:15
(c) 15:8 (d) 3:4

5. If $x:y = 5:2$, then $(8x + 9y)$: $(8x + 2y)$ is:
(a) 22:29 (b) 26:61
(c) 29:22 (d) 61:26

6. If $(x:y) = 2:1$. Then $(x^2 - y^2):(x^2 + y^2)$ is:
(a) 3:5 (b) 5:3
(c) 1:3 (d) 3:1

7. If $(a + b):(b + c):(c + a) = 6:7:8$ and $(a + b + c) = 14$, then the value of c is:
(a) 6 (b) 7
(c) 8 (d) 14

8. If ₹ 782 be divided into three parts, proportional to 1/2:2/3:3/4, then the first part is:
(a) ₹ 182 (b) ₹ 190
(c) ₹ 196 (d) ₹ 204

9. Two numbers are in the ratio 3:5. If 9 is subtracted from each, the new numbers are in the ratio 12:23. The smaller number is:
(a) 27 (b) 33
(c) 49 (d) 55

10. ₹ 1,210 is divided among A, B, C so that $A:B = 5:4$ and $B:C = 9:10$. Then, C gets:
(a) 340 (b) 400
(c) 450 (d) 475

11. ₹ 366 are divided amongst A, B and C so that A may get 1/2 as much as B and C together, B any get 2/3 as much as A and C together, then the share of A is:
(a) ₹ 122 (b) ₹ 129.60
(c) ₹ 146.60 (d) ₹ 183

12. A sum of money is to be distributed among Amar, Bidya, Charu, and Dhanraj in the proportion of 5:2:4:3. If Charu gets ₹ 1,000 more than Dhanraj, what is Bidya's share?
(a) ₹ 500 (b) ₹ 1,500
(c) ₹ 2,000 (d) None of these

13. Ratio of the earnings of A and B is 4:7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earnings becomes 8:7. What are A's earnings?
(a) ₹ 21,000
(b) ₹ 26,000
(c) ₹ 28,000
(d) Data inadequate

14. A sum of ₹ 53 is divided among A, B, C in such a way that A gets ₹ 7 more than what B gets and B gets ₹ 8 more than what C gets. The ratio of their shares is:
(a) 16:9:18 (b) 25:18:10
(c) 18:25:10 (d) 15:8:30

15. The ratio of the number of boys and girls in a school is 3:2. If 20% of the boys and 25% of the girls are scholarship holders, what percentage of the students does not get the scholarship?
(a) 56 (b) 70
(c) 78 (d) 80

16. 60 kg of an alloy A is mixed with 100 kg of alloy B. If alloy A has lead and iron in the ratio 3:2 and alloy B has iron and copper in the ratio 1:4, then the amount of iron in the new alloy is:
(a) 36 kg (b) 44 kg
(c) 53 kg (d) 80 kg

17. Number of students for Mathematics, Physics and Biology in a school are in the ratio 5:7:8. If number of these seats is increased by 40%, 50% and 75% respectively, what will be the ratio of students after the increase?
(a) 2:3:4 (b) 6:7:8
(c) 6:8:9 (d) None of these

18. 85 litres of a solution contains milk and water in the ratio 27:7. How much more water is to be added to get a new solution containing milk and water in the ratio 3:1?
(a) 5 litres (b) 6.5 litres
(c) 7.25 litres (d) 8 litres

19. Three jars have their volumes in the ratio 3:4:5. They are full of mixtures of milk and water. The mixtures contain milk and water in the ratio of (4:1), (3:1) and (5:2) respectively. The contents of all these three jars are poured into a fourth jar. The ratio of milk and water in the fourth jar is:

(a) 4:1 (b) 151:48
(c) 157:53 (d) 5:2

20. Ratio of income of *A* and *B* is 5:4 and the ratio of their expenditure is 3:2. At the end of the year, each saves ₹ 1,600. What is the income of *A*?
(a) ₹ 3,400 (b) ₹ 3,600
(c) ₹ 4,000 (d) ₹ 4,400

EXERCISE 2

1. Sum of four numbers is 253. The ratio of the first number to the second number is 2:3. The ratio of the second number to the third number is 5:6. The ratio of the third number to the fourth number is 8:9. What is the average of the second number and the third number?
(a) 72 (b) 132
(c) 60 (d) None of these

2. Population of the Balearic Island situated in Mediterranean Sea is 18,000. Balearic Island has three ports *A*, *B* and *C*. Every year the entire population of each port moves to the other two ports, half going to one of them and the remaining half going to the other. The current populations of *A*, *B* and *C* are 2,000, 6,000 and 10,000 respectively. Then four years from now, the population of port *A* will be
(a) 5,000 (b) 6,500
(c) 6,000 (d) 5,750

3. Gandhiji owned cows, some black and some white. He found that 4 black cows and 3 white cows give the same amount of milk in 5 days as 3 black cows and 5 white cows give in 4 days. What is the ratio of milk given by a black cow in a day to that given by a white cow in a day?
(a) 8:5 (b) 5:8
(c) 3:5 (d) 5:3

4. Two software developers Vinit and Harpal share a mainframe and the final cost is ₹ 74,750. Vinit uses 5 type I terminals for 7 weeks and 22 type II terminals for 4 weeks. Harpal uses 10 type I terminals for 6 weeks and 8 type II terminals for 6 weeks. If cost of operating each type I and type II terminal is in a ratio of 2:3, find the share of Vinit and Harpal in the final cost.
(a) ₹ 41,750, ₹ 33,000
(b) ₹ 40,000, ₹ 34,000
(c) ₹ 41,000, ₹ 3,80,000
(d) ₹ 50,000, ₹ 24,750

5. A metal trader buys 2 kinds of silver foils, the ratio of their prices being 1:4. He sells the alloy at ₹ 90 per kg so that he can make a profit of 20%. If ratio of their quantities present in a alloy is 6:1 respectively, find the purchase price of foil present in lesser quantity.
(a) ₹ 52.5 (b) ₹ 55
(c) ₹ 47.5 (d) ₹ 45

Direction for questions 6 and 7: ***Read the passage below and solve the questions based on it.***

A book having pages between 4,000 and 5,000 is divided into four parts, each part being divided into chapters. The total number of pages in each of the four parts is the same. The ratio of the chapters across all the parts is 6:5:10:14. Number of chapters in fourth part is 70.

6. What is the total number of pages in the book?
(a) 4,000 (b) 4,800
(c) 4,200 (d) 4,600

7. What is the total number of chapters in the book?
(a) 113 (b) 226
(c) 127 (d) None of these

8. A rat takes 5 leaps for every 4 leaps of a cat, but 3 leaps of the cat are equal to 4 leaps of the rat. What is the ratio of the speed of the rat to that of the cat?
(a) 11:15 (b) 15:11
(c) 16:15 (d) 15:16

9. Ratio of age of Sita and her mother is 2:3. N years from now, ratio of their ages will become 3:5. What is the value of N?
(a) Infinite
(b) 1
(c) More than 1 but finite
(d) Not possible

10. P works twice as fast as Q, whereas P and Q together can work three times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings?
(a) 2:1:1 (b) 4:2:1
(c) 4:3:2 (d) 4:2:3

11. Given that $x:y = a:b = 1:5$. Then the value of $\frac{x^2 + a^2}{y^2 + b^2}$ is:
(a) $\frac{1}{5}$ (b) $\frac{1}{50}$
(c) $\frac{1}{250}$ (d) None of these

12. A watermelon is cut into two pieces in the ratio 3:5 by weight. The bigger of the two is cut further in the ratio 5:7 by weight. Find the ratio of each of the three pieces.
(a) 3:5:7 (b) 36:25:35
(c) 15:25:56 (d) None of these

13. Two alloys of chromium have different percentage of chromium in them. First one weighs 6 kg and second one weighs 12 kg. One piece each of equal weight was cut off from both the alloys and the first piece was alloyed with the second alloy and the second piece alloyed with the first one. As a result, the percentage of chromium became the same in the resulting two new alloys. What was the weight of each piece cut-off?
(a) 4 kg (b) 2 kg
(c) 3 kg (d) 1 kg

14. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$ then each fraction is equal to:
(i) –1 (ii) 1/2
(iii) 1
(a) (i) and (ii) only
(b) (ii) and (iii) only
(c) (i), (ii) and (iii) only
(d) None of these

Direction for questions 15 and 16: ***Read the passage below and solve the questions based on it.***

There is a certain number of apples, guavas and oranges in a basket. Number of each variety is more than one. Ratio of the number of apples to the number of guavas is equal to the ratio of the number of guavas to the number of oranges.

15. If total number of fruits is 61, then find the number of guavas.
(a) 16
(b) 20
(c) 25
(d) Cannot be determined

16. If the number of guavas is 21, then which of the following can be the total number of all the in the basket?
(a) 63 (b) 89
(c) 101 (d) 117

17. Satish used 6 L of oil paint to paint a map of India of 6 m high. How many litres of paint would Santosh need for painting a proportionally scaled map of height 18 m?
(a) 54
(b) 18
(c) 30
(d) Cannot be determined

18. If $A:B = 3:4$. What will be the value of $(3A^2 + 4B)/(3A - 4B^2)$?
(a) 43:55
(b) –43:55
(c) either (a) or (b)
(d) Cannot be determined

19. The value of a diamond varies directly as the square of its weight. If a diamond worth ₹ 10,000 is divided into 2 pieces in the ratio of 4:6, what is the loss in value?
(a) 52% (b) 48%
(c) 36% (d) None of these

20. The monthly income of A and B are in the ratio of 3:2, and their expenditures are in the ratio of 5:3. If each of them saves ₹ 1,000 a month, then what is the income of A?

(a) ₹ 4,000 (b) ₹ 6,000
(c) ₹ 9,000 (d) None of these

ANSWER KEY

Exercise 1

1. (b)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (d)
9. (b)	**10.** (b)	**11.** (a)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (b)
17. (a)	**18.** (a)	**19.** (c)	**20.** (c)				

Exercise 2

1. (a)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (d)
9. (d)	**10.** (a)	**11.** (d)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (a)
17. (a)	**18.** (d)	**19.** (b)	**20.** (b)				

HINTS AND EXPLANATIONS

Exercise 1

1. $A:B = 5:7,\ B:C = 6:11$
$= (6 \times 7/6):(11 \times 7/6)$
$= 7:77/6$
$A:B:C = 5:7:77/6 = 30:42:77$
Hence, option (b) is the answer.

2. $A/B = 8/15,\ B/C = 5/8$ and
$C/D = 4/5 \rightarrow A/D$
$= (A/B \times B/C \times C/D)$
$= (8/15 \times 5/8 \times 4/5) = 4/15$
$\Rightarrow \quad A:D = 4:15$
Hence, option (b) is the answer.

3. Let $2A = 3B = 4C = k$,
then, $A = k/2,\ B = k/3$ and $C = k/4$
$\Rightarrow \quad A:B:C = k/2:k/3:k/4$
$= 6:4:3$
Hence, option (c) is the answer.

4. $2A = 3B$ and $4B = 5C \rightarrow A/B$
$= 3/2$ and $B/C = 5/4$
$\Rightarrow \quad A/C = (A/B \times B/C)$
$= (3/2 \times 5/4)$
$= 15/8 \rightarrow A:C$
$= 15:8$
Hence, option (c) is the answer.

5. Put $x = 5$
and $y = 2$
Then, $8x + 9y/8x + 2y$
$= (8 \times 5) + (9 \times 2)/(8 \times 5) + (2 \times 2)$
$= 58/44$
$= 29/22$
Hence, option (c) is the answer.

6. Simply put $x = 2$ and $y = 1$ in the expression.

7. Let $(a + b) = 6k$,
$(b + c) = 7k$
and $(c + a) = 8k$
Then, $2\,(a + b + c)$
$= 21k \leftrightarrow 2 \times 14 = 21k$

$\leftrightarrow \quad k = 28/21 = 4/3$

$\therefore \ (a + b) + 6 \cdot \dfrac{4}{3}$

$= 8 \rightarrow c$

$= (A + B + C) - (A + B)$

$= (14 - 8) = 6.$

8. Given ratio $= 1/2:2/3:3/4$

$= 6:8:9$

$\therefore$ 1st part $= \left(782 \times \dfrac{6}{23}\right) =$ ₹ 204.

9. Let the numbers be $3x$ and $5x$. Then,

$(3x - 9)/(5x - 9)$

$= 12/23 \leftrightarrow 23\ (3x - 9)$

$= 12\ (5x - 9)$

Hence, smaller number

$= (3 \times 11) = 33$

Alternatively, this question can also be done with the help of options.

11. $A:(B + C) = 1: 2 \rightarrow A$'s share

= ₹ $(366 \times 1/3)$

= ₹ 122.

12. Let the shares of them be ₹ $5x$, ₹ $2x$, ₹ $4x$ and ₹ $3x$ respectively.

Then, $4x - 3x = 1{,}000$

$\Rightarrow \quad x = 1{,}000$

Hence, Bidya's share = ₹ $2x$

= ₹ $(2 \times 1{,}000)$

= ₹ 2,000.

13. Let the original earnings of A and B be ₹ $4y$ and ₹ $7y$.

New earnings of A

= 150% of ₹ $4y$

= ₹ $(150/100 \times 4y)$ = ₹ $6y$

New earnings of B

= 75% of ₹ $7y$

= ₹ $(75/100 \times 7y)$ = ₹ $21y/4$

$\therefore \ 6x:21y/4 = 8:7 \leftrightarrow 6y \times 4/21y = 8/7$

This does not provide the value of y. Hence, answer cannot be determined.

14. Suppose C gets ₹ x, Then, B gets ₹ $(x + 8)$ and A gets ₹ $(x + 15)$.

Then, $x + (x + 8) + (x + 15)$

$= 53 \leftrightarrow x = 10$

Hence, A: B: C

$= (10 + 15):(10 + 8):10$

$= 25:18:10.$

15. Assume that number of boys = 30 and number of girls = 20

Number of those who get scholarship

= 20% of 30 + 25% of 20

$= 6 + 5 = 11$

Number of students who do not get scholarship $= 50 - 11 = 39$

Required percentage

$= \dfrac{39}{50} \cdot 100) = 78\%$

Hence, option (c) is the answer.

16. Quantity of iron in 60 kg of A

$= (60 \times 2/5)$ kg = 24 kg

Quantity of iron in 100 kg of B

$= (100 \times 1/5)$ kg = 20 kg

Quantity of iron in the new alloy

$= (24 + 20)$ kg = 44 kg.

17. Originally, let the number of students for Mathematics, Physics and Biology be 50, 70 and 80 respectively.

New number of students

= (140% of 50), (150% of 70) and (175% of 80)

Hence, required ratio

$= 70:105:140$

$= 2:3:4$

Hence, option (a) is the answer.

19. Let the three jars contain 30, 40 and 50 litres of mixtures respectively.

Milk in 1st mixture

$= (30 \times 4/5)$ litres

= 120/5 litres

Water in 1st mixture

$= (30 - 12x/5)$ litres

= 30/5 litres

Milk in 2nd mixture

$= (40 \times 3/4)$ litres = 30 litres

Water in 2nd mixture

= (40 – 30) litres = 10 litres

Milk in 3rd mixture

= (50 × 5/7) litres

= 250/7 litres

Water in 3rd mixture

= (50 – 25x/7) litres

= 100/7 litres

Total milk in final mixture

= (120/5 + 30 + 250/7) litres

= 3140/35 litres

Total water in final mixture

= (30/5 + 10 + 100/7) litres

= 1060/35 litres

Ratio of milk and water in 4th jar

= (3140/35):(1060/35)

= 157:53.

20. Let the incomes of A and B be ₹ 5x and ₹ 4x respectively and let their expenditures be ₹ 3y and ₹ 2y respectively.

Then, $5x - 3y = 1600$ (i)

and $4x - 2y = 1600$ (ii)

Solving (i) and (ii), we get:

$2x = 1600 \Rightarrow x = 800$

Hence, A's income = ₹ 5x = ₹ (5 × 800)

₹ 4,000.

8 Time, Speed and Distance

INTRODUCTION

Concept of Time, Speed and Distance is used very extensively in different situations to generate questions pertaining to different areas. In this chapter, we study the following:

1. Motion in a straight line
2. Boats and streams/Escalators
3. Races
4. Circular Motion
5. Trains
6. Clocks

TIME, SPEED AND DISTANCE

As we know, Distance = Speed × Time. It means that if a person is running at a speed of 20 kmph and he runs for 2 hours, he will cover a total distance of 40 km. Distance = 20 × 2 = 40 km.

CONVERSION FROM M/S TO KM/H AND VICE-VERSA

If speed is given in m/s and we are required to convert it into km/h, we multiply it by $= \frac{18}{5}$ and when speed is given in km/h and we have to convert it into m/s, we multiply it by $\frac{5}{18}$.

$$36 \text{ km/h} = 36 \cdot \frac{5}{18} = 10 \text{ m/s}$$

Similarly, $$20 \text{ m/s} = 20 \cdot \frac{18}{5} = 72 \text{ km/h}$$

1. **Motion in a Straight Line:** It is one of the primary areas of application of Time, Speed and Distance. Using the basic relationship among time, speed and distance, we can observe that three different cases are possible:

Case 1: When S (Distance) is constant

$$V \propto \frac{1}{T} \cdot \text{ So, } V_1/V_2 = T_2/T_1$$

It is read as $-V$ is inversely proportional to T.

It can be understood in terms of the reciprocal-ratio-multiplication relationship between V and T.

Example 1

A man cycles at a speed of 10 kmph and reaches his office at 1 pm. When he cycles at a speed of 15 kmph, he reaches his office at 11 am. At what speed should he cycle so that he reaches his office at 12 noon?

Solution

In other words, speed becomes $\frac{3}{2}$ of original. So time will become $\frac{2}{3}$ of original.

Speed is being increased by 50%, so time taken will reduce by 33.33%.

So, 33.33% of time = 2 hours. Hence, total time = 6 hours.

So, distance = 10 × 6 = 60 kms

This distance is to be covered in 5 hours (6 – 1) hours.

So, speed = 60/5 =12 kmph

Case 2: When T (Time) is constant

$$S \propto V \Rightarrow \text{So, } S_1/S_2 = V_1/V_2$$

The higher the speed, the more will be the distance covered and the slower the speed, the

lesser will be the distance covered. We'll see that T constant is a situation specific to meeting point cases.

Example 2

Distance between two points $AB = 110$ km. Manoj starts running from A at a speed of 60 kmph and Ravi starts running from B at a speed of 40 kmph at the same time. They meet at a point X, somewhere on the line AB. What is ratio of AX to BX?

A ———————— X ———————— B

Solution

Since both Manoj and Ravi are running for the same time, T is constant. Hence, ratio of distance covered by them will be same as ratio of their speed.

So, $$\frac{AX}{BX} = \frac{\text{Speed of Manoj}}{\text{Speed of Ravi}}$$

$$= 60/40 = 3:2$$

Some typical meeting point cases:

When two persons are running between the ends of a linear track for infinite time:

Example 3

Two robots Mango and Bango start from the opposite ends A and B of a linear track respectively and keep running between the ends for infinite time. They meet for the first time at a point 60 m from A. If $AB = 100$ m, which point is their point of 4th meeting?

Solution

1st meeting point:

A ———————————————— B

——— 80 m ———┘

Ratio of speed of Mango and Bango is 60:40 = 3:2. Now Mango is moving towards B and Bango is moving towards A.

For 2nd meeting, in the time Bango reaches A after covering 60 m, Mango must have covered 90 m. So, he is at the mid-point of A and B now. Now, the distance between Mango and Bango is 50 m. They will cover this distance of 50 m in the ratio 3:2. So, point of their 2nd meeting is:

A ———————————————— B

——— 20 m ———┤

It can be seen here that the sum of distance covered by both Mango and Bango till now from the starting is 300 m, and the difference between the distance covered between the 1st meeting and 2nd meeting is 200 m.

So, again they will meet for the 3rd time when they have covered a total distance of 200 m together. Mango and Bango will cover this distance of 200 m in the ratio 3:2.

Distance covered by Mango = 120 m
and distance covered by Bango = 80 m

So, 3rd meeting point is point B. To have a 4th meeting, they will again have to cover a total distance of 200 m. So, point of their 4th meeting is 20 m from A.

So, now we can generalise the above situation as:

Distance covered by both of them for:
1st meeting = 100 m
Distance covered by both of them for:
2nd meeting = 100 m + 200 m = 300 m
Distance covered by both of them for:
3rd meeting = 300 m + 200 m = 500 m
and so on ...

Ratio of speed of Ram and Mohan is 3:2, so distance covered by Ram = 420 m. Hence, they will meet at 20 m from A.

Limitation of above generalization: Ratio of speed should lie in between 1/2 and 2.

Case 3:

When V (Speed) is constant

$S \propto T$. So, $S_1/S_2 = T_1/T_2$

In layman terms, if a person is running at a speed of 20 kmph, then the ratio of distance covered in one hour to the distance covered in two hours will be 1:2.

2. **Boats and Streams/Escalator:** Boats and streams should be ideally seen as just a logical extension of Motion in a Straight Line, with distance being constant. As we know, if distance is constant, then $V \alpha 1/T$.

Basic Terminology

Downstream Movement

When the direction of the movement of a river and a boat is same, their collective movement is known as Downstream Movement. And the distance covered by boat is known as Downstream Distance.

If speed of river $= R$
and speed of boat $= B$,
then downstream speed $= B + R$

Upstream Movement

When direction of movement of river and boat is opposite, they are said to be in Upstream Movement. Distance covered in this case is known as Upstream Distance.

If speed of river $= R$ and speed of boat $= B$, then upstream speed $= B - R$ (conventionally we take speed of boat more than speed of river and if this would not have been the case, boat would not be able to go back).

Now, speed of boat

$= 1/2$ (Downstream Speed + Upstream Speed)

$= 1/2\ (B + R + B - R) = B$

And speed of river

$= 1/2$ (Downstream Speed – Upstream Speed)

$= 1/2\ (B + R - B + R) = R$

Hence, if downstream speed and upstream speed are given as 20 kmph and 10 kmph, then
speed of boat = 15 km/h
and speed of river = 5 km/h.

In most of the cases of boats and streams, distances covered downstream and upstream are same. In those cases, ratio of time taken becomes inversely proportional of the ratio of the speeds.

Time taken downstream: Time taken upstream = Upstream speed: Downstream speed.

General Expressions

1. $$\frac{\text{Time taken in Downstream movement}}{\text{Time taken in Upstream movement}} = \frac{\text{Upstream Speed}}{\text{Downstream Speed}}$$

   [Assuming same distance is travelled downstream and upstream]

2. If time taken in covering a certain distance downstream and upstream $= x$ hour and y hour respectively, then time taken in covering the same distance in still water

   $$= \frac{2xy}{x + y} \text{ hour}$$

Example 5

Speed of boat in still water is 6 km/h and the speed of river is 1.2 km/h. It takes a total of 10 hours to go to a place and come back. What is the total distance covered in the whole process?

Solution

Method 1: Let us assume D is the distance.

Upstream speed = 4.8 km/h
and downstream speed = 7.2 km/h

According to the question,

$(D/4.8) + (D/7.2) = 10$

So, $D = 28.8$ km
and hence, total distance = 57.6 km.

Method 2: (Using generalizsation 1 as given on previous page)

Ratio of downstream speed: upstream speed = 3:2 ⇒ Ratio of downstream time: upstream time = 2:3. So, time taken in downstream movement = 4 hours and time taken in upstream movement = 6 hours.

So, distance covered $= 4 \times 7.2$
$= 6 \times 4.8 = 28.8$ km

And hence,
total distance = 57.6 km

Example 6

Ramesh takes 6 hours in downstream to cover a certain distance. He can cover the same distance in 8 hours going upstream. What is the time taken by him to cover the same distance in still water?

Solution

Using generalization 2 as given on previous page,

Time taken in still water

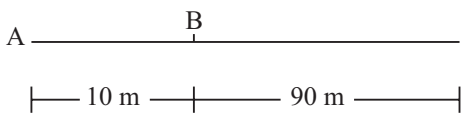
$$= \frac{2xy}{x+y} \text{ hours} = \frac{2 \cdot 6 \cdot 8}{6+8} \text{ hours}$$

$= 6.85$ hours

3. Races

Basic statements:

(i) A gives a start of 10 m to $B \rightarrow$ When B has already run 10 m, then A starts running.



Example 7

In a race of 100 m, A gives a start of 10 m to B. Despite this, A wins the race by 20 m. What is their ratio of speed?

Solution

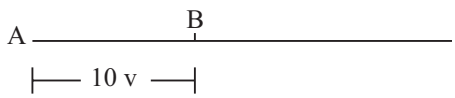
Obviously, time taken by A to cover 70 m

= Time taken by B to cover 100 m

Since distance is constant,

ratio of speed of A and B = 10:7

(ii) A gives a start of 10 seconds to $B \rightarrow B$ has already run for 10 seconds, now A starts running.



where v m/s is the speed of B.

Ratio of speeds = 100: (100 – 10V)

Example 8

In a 100 m race, Tom runs at a speed of 1.66 m/s. If Tom gives a start of 4 m to Jerry and still beats him by 12 seconds, what is the speed of Jerry?

Solution

Time taken by Tom to cover

100 m = 60 seconds

Now, since Tom beats Jerry by 12 seconds, time taken by Jerry = 72 seconds

And distance covered by Jerry

= 96 m

So, speed = 96/72 = 1.33 m/s

Example 9

Karan and Arjun run a 100 metre race, where Karan beats Arjun by 10 metres. To do a favour to Arjun, Karan starts 10 metres behind the starting line in the second 100 metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race?

(i) Karan and Arjun reach the finishing line simultaneously.

(ii) Arjun beats Karan by 1 metre.

(iii) Arjun beats Karan by 11 metres.

(iv) Karan beats Arjun by 1 metre.

Solution

Situation (I): In whatever time Karan covers a distance of 100 m, Arjun covers 90 m in the same time. Situation (II): Now Karan is 10 m behind the starting point. Once again to cover 100 m from this new point, Karan will be taking the same time as before. In this time Arjun will be covering 90 metres only. This means that now both of them will be at the same point, which will be 10 metres away from the finish point. Since both of them are required to cover the same distance of 10 m now and Karan has a higher speed, he will beat Arjun. No need for calculations as option (4) is the only such option.

Example 10

A can beat *B* by 20 metres in a 200 metres race. *B* can beat *C* by 10 metres in a 250 metres race. In a race of 100 m, by how many metres *A* can beat *C*?

Solution

1st of all, we are required to find the ratio of their speeds:

Speed of *A*:Speed of *B* = 10:9
Speed of *B*:Speed of *C* = 25:24
Hence, Speed of *A*:Speed of *B*:
Speed of *C* = 250:225:216
[See 'Ratio-Proportion' Chapter]
So when *A* covers 250 m, *C* covers 216 m
Hence, when *A* covers 100 m,

$$C \text{ covers } \frac{216}{250} \times 100 = 86.4 \text{ m}$$

Hence, *A* would beat *C* by
100 – 86.4 = 13.6 m.

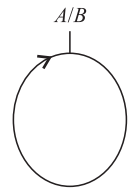
4. **Circular Motion:** In the case of races and motions in straight line, we have observed that if two bodies or persons are moving with different speeds in a straight line in one direction, they will never meet. It is all due to the fact that with the passage of time, distance between them is constantly increasing. Since it is an enclosed track (by virtue of it being circular), runners are bound to meet at some point or the other.
Now there is a possibility of so many different situations. Let us see some of those situations:

Case 1: When two or more than two persons are running around a circular track in same direction.

Example 9

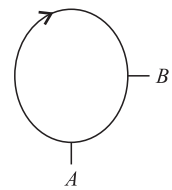
When will they meet for the first time anywhere on the track?

To understand the situation completely, let us assume that there are two persons *A* and *B*. Speed of *A* = 20 m/s, speed of *B* = 10 m/s, length of track is 1,000 m and they are running in same direction. It can be seen in Fig. 1 that initially both of them are at the same point, *i.e.*, the starting point.



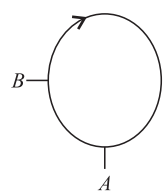
Starting

Fig. 1



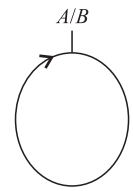
After 25 seconds

Fig. 2



After 75 seconds

Fig. 3



After 100 seconds

Fig. 4

They will be meeting for the first time only if the faster runner *A* has taken one more round of the track than the slower runner *B*. This can be interpreted as: *A* will have to cover 1,000 m more than *B*.

It can be understood with the help of the figures given above that the distance will keep on increasing between them with the passage of time. And the moment distance between them becomes equal to 1,000 m, they will be at the same point, *i.e.*, they will meet (refer Fig. 4).

So,
Time taken = Distance/Relative speed
= 1000/10
= 100 seconds.

Example 10

When will they meet for the first time at the starting point?

To calculate this, we will use the concept of LCM.

Find the time taken by each individual to take one round and then calculate LCM of these values.

Assume there are three persons A, B and C with a respective speed of 30 m/s, 20 m/s and 10 m/s running in same direction. Length of circular track is 1,000 m.

Time taken by A to take one round

$= t_1 = 1000/30 = 33.33$ seconds

Time taken by B to take one round

$= t_2 = 1000/20 = 50$ seconds

Time taken by C to take one round

$= t_3 = 1000/10 = 100$ seconds.

LCM of (t_1, t_2, t_3)

$= 100$ seconds.

Example 11

At how many different points of the track will they be meeting?

In general, Number of meeting points = Difference of ratio of speed of A and B in its simplest form.

Ratio of speed of A and B

$= 5:2$

So, number of different meeting points

$= 5 - 2 = 3$ points

Example 12

When will they meet for the first time anywhere on the track?

Since they are running in opposite direction, relative speed $= 10 + 20 = 30$ m/s

So,

Time taken = Distance/Relative speed

$= 1000/30 = 33.33$ seconds.

Example 13

When will they meet for the first time at the starting point?

Calculate the time taken by each individual to take one round and then calculate the LCM of those values.

Time taken by A to take one round

$= t_1 = 1000/20 = 50$ seconds

Time taken by B to take one round

$= t_2 = 1000/10 = 100$ seconds

LCM of (t_1, t_2) $= 100$ seconds.

Example 14

At how many different points of the track will they be meeting?

They are meeting after 33.33 seconds for the first time. Till now, A has covered 666.66 m and B has covered 333.33 m. So, obviously they are meeting at a distance of 666.66 m from starting point in the direction of A. Next point will be again 666.66 m ahead of this point. which will be starting point.

So, a total of 3 points will be there. In general, Number of distinct meeting points = Addition of ratio of speed of A and B in its simplest form. Ratio of speed of A and $B = 2:1$.

So, number of different meeting points

$= 2 + 1 = 3$ points

Generalisations

1. Time taken for the runners to meet for the 1st time at the starting point

   $= \text{LCM}\ (T_a, T_b, T_c, ..., \text{etc.})$

   where T_a, T_b and T_c are the time taken by A, B and C to take one complete lap. It can be seen that answer to this question is independent of direction of their movement, *i.e.*, irrespective of their direction of movement being clockwise or anti-clockwise, they will take same time in taking a complete lap.

2. **Time taken by two runners A and B to meet for the 1st time anywhere on the track**

   $$= \frac{\text{Length of Track}}{\text{Relative Speed}}$$

   If there are more than two runners: Let us assume that there are three runners

   $A = 15$ m/s, $B = 13$ m/s and $C = 7$ m/s

   They start from the same point at the same time in the same direction. We are required to calculate that after how much time they will meet for the 1st time anywhere on the track?

Find T_{AB} = Time taken by A and B to meet for the 1st time anywhere

$$= \frac{1000}{2} = 500 \text{ seconds}$$

[Relative Speed = 15 m/s – 13 m/s = 2 m/s]
Then find any one of T_{BC} or T_{AC} in the similar way.
T_{BC} will get us a fractional value. Hence, let us calculate

$$T_{AC} = \frac{1000}{(50 - 7 =)\ 8\frac{m}{s}} = 125 \text{ seconds}$$

Time taken by all of them to meet for the 1st time anywhere on the track
= LCM [T_{AC} and T_{AB}]
= LCM [500, 125] = 500 seconds.

3. **Total number of distinct meeting points on the track**

$$= \frac{\text{(Time taken for the runners to meet for the 1st time at the starting point)}}{\text{(Time taken by two number } A \text{ and } B \text{ to meet for the 1st time anywhere on the track)}}$$

5. **Trains:** We know that when the direction of movement of boat and river is same, relative speed is obtained by adding the speeds of both, the boat and river. But if two trains are moving in same direction, then what is the relative speed?
Let us see some cases:

(i) When two trains of length L_1 and L_2 and speed V_1 m/s and V_2 m/s respectively are crossing each other

(a) Direction of movement of both the trains is same:
Relative speed = $|V_1 - V_2|$ and total distance covered = $L_1 + L_2$

(b) Direction of movement of both the trains is opposite:
Relative speed = $|V_1 + V_2|$ and total distance covered = $L_1 + L_2$

(ii) When a train is crossing a stationary object

(a) When the train is crossing a pole or a stationary human being:
Relative speed = $V_1 + V_2$, since $V_2 = 0$, so, relative speed = Speed of train
Total distance covered = $L_1 + L_2$, since L_2 (width of pole) is negligible with respect to L_2 (Length of train), so we don't consider it while calculating the quantities. So, distance = L_1

(b) When the train is crossing a platform or a standing train:
Relative speed = V_1, where V_1 is the speed of moving train and V_2 is the speed of standing train or platform.

Since $V_2 = 0$, so,
relative speed $= V_1$ = Speed of Train
and total distance covered
$= L_1 + L_2$
where L_1 is the length of moving train and L_2 is the length of standing train or platform.

Note that distance will be always added up irrespective of the direction of the movement of the objects.

Example 15

A train takes 10 seconds to cross a pole and 20 seconds to cross a platform of length 200 m. What is the length of train?

Solution

Train takes 10 seconds to cross its own length and 20 seconds to cross its own length and length of platform.

So, it infers that train takes 10 seconds to cross platform and 10 seconds to cross its own length.

Since time taken to cross platform = time taken to cross its own length. So, length of platform = length of train = 200 m.

Example 16

Speed of a train is 36 kmph. It takes 25 seconds to cross a pole. What is the length of this train?

Solution

Speed of train = 10 m/s (36 × 5/18)
Distance covered = 10 × 25 = 250 m
So, length of train = 250 m.

Example 17

A man running with a speed of 18 kmph crosses a train of length 280 m running in the opposite direction in 20 seconds. What is the speed of train?

Solution

Total distance covered = 280 m,
and time taken = 20 seconds

Hence, relative speed = $\frac{280}{20}$ = 14 m/s

Speed of man = 18 kmph = 5 m/s
Hence, speed of train = 9 m/s.

Example 18

Two trains of length 200 m and 220 m are running on parallel tracks in the same direction. Speed of 1st train is 72 kmph and speed of second train is 54 kmph. What is the time taken by both the trains to cross each other?

Solution

Relative speed = 72 kmph – 54 kmph
= 18 kmph = 5 m/s

Total distance to be covered
= 200 + 220 = 420 m
[Distance will be always added]

So time taken = $\frac{420}{5}$ = 84 seconds.

Example 19

In the above question, what would have been the time taken had their direction of movement been the opposite direction?

Solution

Relative speed = 72 kmph + 54 kmph
= 126 kmph = 35 m/s

Total distance to be covered
= 200 + 220 = 420 m
[Distance will be always added]

So time taken = $\frac{420}{35}$ = 12 seconds.

6. **Clocks:** Clock is a typical example of circular motion where length of track is equal to 60 km (assume 1 min = 1 km). Now on this track, two runners *i.e.*, hour hand and minute hand are running with a speed of 5 km/h and 60 km/h respectively. Since direction of their movement is same, so relative speed = 55 km/h.

Example 20

When will the hour hand and the minute hand of a clock be together between 1 and 2?

Solution

Hands have to be together in between 1 O'clock to 2 O'clock.

At 2 O'clock, distance between hour and minute = 5 km and relative speed = 55 km/h

Time = 5/55 hour = 1/11 hour = 60/11 min
= 5 5/11 min = 5300/11 seconds
= 5:27.27 seconds

So, hour hand and minute hands will be together at 1:05:27.27 seconds.

Students can learn this value 5 min 27.27 seconds by heart as a standard result.

So, both the hands will meet at:

1:05:27.27—Between 1 O'clock and 2 O'clock

2:10:54.54—Between 2 O'clock and 3 O'clock

3:16:21.81—Between 3 O'clock and 4 O'clock ... and so on ...

Degree Concept of Clocks

Total angle subtended at the centre of a clock = 360°.

Angle made by hour hand at the centre per hour = 30° per hour, OR, 0.5° per minute

Angle made by minute hand at the centre per hour = 360° per hour, OR, 6° per minute.

Solving **Example 20** by this method, angle between hour hand and minute hand at 1 O'clock = 30°

Relative speed (in terms of angle)
= 5.5°/hour

So, time taken = 30°/5.5° = 60/11 min.

IMPORTANT DERIVATIONS

- Number of times hands of a clock are in a straight line (either at 0° or at 180°) in 24 hours = 44
- Number of times hands of a clock are at right angle (at 90°) in 24 hours = 44
- Both the hands of clock are together after every $65\frac{5}{11}$ min.

(So if both the hands of the clock are meeting after every 65 minutes or anything less than $65\frac{5}{11}$ mins, then clock is running fast and if both the hands of the clock are meeting after every 66 minutes or anything more, than $65\frac{5}{11}$ mins, then clock is running slow).

PRACTICE EXERCISES

EXERCISE 1

1. A man is running at the speed of 36 kmph, then how many metres does he cover in 40 seconds?
(a) 400 m (b) 300 m
(c) 2,400 m (d) 100 m

2. The speed of a train is 20 m/s. The time it takes to cross a telephone pole is 10 seconds. What is its length?
(a) 100 m (b) 1.5 km
(c) 150 m (d) 200 m

3. A train having a length of 400 m, travelling at a speed of 15 m/s, crosses a platform in one minute. What is the length of the platform?
(a) 2,000 m
(b) 500 m
(c) 400 m
(d) Cannot be determined

4. Speed of a train is 88 kmph. What time will it take to cross a platform 180 m long if the length of the train is 150 m?
(a) 11½ seconds (b) 10½ seconds
(c) 13½ seconds (d) 14½ seconds

5. A certain distance is covered at a speed V km/h. If half of the same distance is covered in double the time, then the ratio for the former speed to that of the latter is:
(a) 4:1 (b) 1:4
(c) 2:1 (d) 1:2

6. A person can row a distance of 1 km upstream in 10 minutes and downstream in 4 minutes. What is the speed of the stream?
(a) 4 km/h (b) 9 km/h
(c) 5.6 km/h (d) 4.5 km/h

7. Anil calculated that it will take 45 minutes to cover a distance of 60 km by his car. How long will it take to cover the same distance if the speed of his car is reduced by 15 kmph?
(a) 36 minutes (b) 60 minutes
(c) 48 minutes (d) None of these

8. A car travels from Patna to Jehanabad at the speed of 65 kmph in one hour. If the speed is reduced by 15 kmph then, how much more time will the car take to cover the same distance?
(a) 12 minutes (b) 16 minutes
(c) 18 minutes (d) 44 minutes

9. In a kilometer race, *A* beats *B* by 10 metres. In a 2 kilometres race, *A* beats *B* by:
(a) 10 m (b) 20 m
(c) 40 m (d) 25 m

10. A train, 225 m long, is running at 45 km/hr. In what time does it cross a man running at 18 km/hr in the same direction?
(a) 20 seconds (b) 30 seconds
(c) 25 seconds (d) 15 seconds

11. Two trains started from a station, at the same time in the same direction, at speeds of 70 kmph and 60 kmph. What will be the distance between the two trains after 6 hours?
(a) 42 km (b) 54 km
(c) 60 km (d) 84 km

12. A man can row 1/2 km against the current in 12 minutes and returns in 6 minutes. Find the speed of the current.
(a) 1.25 kmph (b) 1.5 kmph
(c) 2.5 kmph (d) 3 kmph

13. In a km race, Amit beats Bahadur by 100 m and Bahadur beats Chandra by 200 m. By how many metres does Amit beat Chandra in the same race?
(a) 100 m (b) 83⅓ m
(c) 68 m (d) 280 m

14. On a river, *Q* is the mid-point between two points *P* and *R* on the same bank of the river. A boat can go from *P* to *Q* and back in 12 hours, from *P* to *R* in 16 hours 40 minutes. How long would it take to go from *R* to *P*?
(a) 3.33 hours (b) 5 hours
(c) 6.66 hours (d) None of these

15. Two trains are running on parallel lines in the same direction at the speed of 50 kmph and 30 kmph respectively. Given that the faster train crosses a man in the slower train in 18 seconds, what is the length of the faster train?
(a) 170 m (b) 100 m
(c) 98 m (d) 85 m

16. A car driver travels from the plains to the hill station, which is 200 km apart at an average speed of 40 km/h. In the return trip; he covers the same distance at an average speed of 20 km/h. The average speed of the car over the entire distance of 400 km is:
(a) 25 km/h (b) 26.67 km/h
(c) 28.56 km/h (d) 30 km/h

17. The average speed of a train in the onward journey is 25% more than that in the return journey. The train halts for one hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours, covering a distance of 800 km. The speed of the train in the onward journey is:
(a) 45 km/h (b) 47.5 km/h
(c) 52 km/h (d) 56.25 km/h

18. Walking $6/7^{th}$ of his usual speed, a man is 12 minutes too late. The usual time taken by him to cover that distance is:
(a) 1 hour (b) 1 hr 12 min
(c) 1 hr 15 min (d) 1 hr 20 min

19. A car covers a distance of 715 km at a constant speed. If the speed of the car would have been 10 km/hr more, then it would have taken 2 hours less to cover the same distance. What is the original speed of the car?
(a) 45 km/h (b) 50 km/h
(c) 55 km/h (d) 65 km/h

20. A train *M* leaves Meerut at 5 am and reaches Delhi at 9 am. Another train leaves Delhi at 7 am and reaches Meerut at 10.30 am. At what time do the two trains cross each other?
(a) 7.36 am (b) 7.56 am
(c) 8 am (d) 8.26 am

EXERCISE 2

1. When speed of a train is increased by 20%, it takes 20 minutes less to cover the same distance. What is the time taken to cover the same distance with the actual speed?

(a) 2 hours (b) 1 hour
(c) 1.5 hour (d) 2.5 hour

2. Difference between time taken in covering a certain journey decreases by 45 minutes when speed is increased to 20 kmph from 10 kmph. What is the difference between the time taken when the same distance is covered at a speed of 40 kmph and 25 kmph?
(a) 12.5 mins (b) 13.5 mins
(c) 18 mins (d) None of these

3. Shramjeevi Express goes to Patna from New Delhi at a speed of 60 kmph. At what speed (in kmph) should Shramjeevi Express return from Patna to New Delhi so that average speed during the whole journey is 120 kmph?
(a) 180
(b) 240
(c) 120
(d) This is not possible

4. Sona travels 600 km to his home partly by train and partly by car. She takes 8 hours if she travels 120 km by train and the rest by car. She takes 20 minutes more if she travels 200 km by train and the rest by car. Find the speed of the train in kmph.
(a) 45 (b) 60
(c) 75 (d) None of these

5. Sama traveled half the distance from her house to school at a speed of 10 km/hr and for exactly half of the remaining time she traveled at 15 km/hr and the rest of the time at 20 km/hr. Find her average speed.
(a) 13 km/hr (b) $12^{8}/_{11}$ km/hr
(c) 15 km/hr (d) None of these

6. Shilu can go 30 km upstream and 44 km downstream in 10 hours. Also, she can go 40 km upstream and 55 km downstream in 13 hours. What is her speed in still water?
(a) 6 km/h (b) 12 km/h
(c) 8 km/h (d) 16 km/h

7. A train travels from a station *P* to another station *Q*, a distance of 192 km at an average speed of 80 km per hour. It then travels back to *P* and the total time taken by it for the return journey is 5 hours 36 minutes. Find the average speed of the train on the return trip to *P*.
(a) 72 kmph (b) 36 kmph
(c) 20 kmph (d) 60 kmph

8. Two persons, 27 km apart, setting out at the same time are together in 9 hours if they walk in the same direction, but in 3 hours if they walk in opposite directions. What are their speeds (in kmph)?
(a) 5, 4 (b) 4, 3
(c) 6, 3 (d) 8, 7

9. In a race of 300 m, Abhishek beats Bijay by 30 metres while Bijay beats Chandan by 50 metres. By what distance must Abhishek have beaten Chandan in the same 300 m race?
(a) 80 metres (b) 40 metres
(c) 160 metres (d) 75 metres

10. What is the speed of a train, which overtakes a cow walking at a speed of 5 kmph in 30 seconds, if the train is 274 metres long?
(a) 51.88 kmph (b) 27.88 kmph
(c) 37.88 kmph (d) 21.67 kmph

11. Shramjeevi Express leaves Patna for New Delhi at 10:10 am and reaches New Delhi at 3:30 pm. Magadh Express leaves New Delhi at 9:48 am and reaches Patna at 2:36 pm. If routes of both the trains are same, then at what time both the trains will meet each other?
(a) 12:52 pm (b) 1:12 pm
(c) 1:32 pm (d) None of these

12. At noon, ship Vikrant starts from a point *P* towards a point *Q* and at 1.00 pm, ship Viraat starts from *Q* towards *P*. If ship Vikrant is expected to complete the voyage in 6 hours and ship Viraat is moving at a speed 2/3 of that of ship Vikrant, at what time are the two ships expected to meet?
(a) 4 pm (b) 4.30 pm
(c) 3 pm (d) 2.30 pm

13. Amit Kumar starts driving from Patna to Quillon at 9 am and Bhartendu starts driving from Quillon to Patna at 10 am. Bhartendu is 50% faster than Amit Kumar. What is the time when they meet if Patna and Quillon are 300 km apart and Amit Kumar's speed is 50 kmph?
(a) 12.30 pm (b) 12 noon
(c) 11 am (d) 11.30 am

14. A ship 156 km away from the sea coast springs a leak, which admits 2.33 metric tons of water in 6.5 minutes, but the pumps throw out 15 metric tons of water in 1 hour. 70 metric tons would suffice to sink the ship. What is the average speed (in kmph) so that the ship may just reach the shore as ship begins to sink?
(a) 14.5 (b) 15
(c) 18 (d) 20

15. A man rows a boat upstream a certain distance and then returns to the same place. If the time taken by him in going upstream is twice the time taken in rowing downstream, find the ratio of the speed of the boat in still water and the speed of stream.
(a) 2:1 (b) 3:2
(c) 5:3 (d) 3:1

16. Excluding stoppages, the speed of a bus is 54 kmph and including stoppages it is 45 kmph. What is the stoppages time of bus (in minutes) per hour?
(a) 10 (b) 12
(c) 9 (d) 20

17. For covering a distance P km, a man takes t hour. If he moves 3 kmph faster, he takes 40 minutes less. But if he moves 2 kmph slower, he takes 40 minutes more. Then P is equal to:
(a) 22.5 km (b) 32.5 km
(c) 42.5 km (d) None of these

18. Two trains start at the same time from A and B and proceed towards B and A at 36 kmph and 42 kmph respectively. When they meet, it is found that one train has moved 48 km more than the other. Then the distance between A & B (in km) is:
(a) 624 (b) 636
(c) 544 (d) 460

19. Anand can beat Bidhan by 5 metres in a 100 m race and Bidhan can beat Chandan by 10 metres in a 200 m race. Then in a race of 400 m, Anand can beat Chandan by:
(a) 40 metres (b) 39 metres
(c) 15 metres (d) 10 metres

20. A man can swim with the stream at the rate of 3 kmph and against the stream at the rate of 2 kmph. How long will it take him to swim 7 km in still water?
(a) 3 hours (b) 2.8 hours
(c) 2.6 hours (d) 3.2 hours

21. A can give a lead 20 m to B in a race of 200 m and B can give a lead of 10 m to C in a race of 200 m. By how many metres can A beat C in the same race of 200 m?
(a) 21 m (b) 28 m
(c) 29 m (d) 31 m

22. A and B start at the same time from P and Q (55 km apart) to Q and P at 3 and 2½ kmph respectively. They meet at R, reach Q and P, return immediately and meet again at S. The distance from R to S is:
(a) 10 km (b) 12 km
(c) 15 km (d) 16 km

23. A runs 1.75 times as fast as B. If A gives B a start of 60 m, how far must be the winning post be so A and B reach it at the same time?
(a) 140 m (b) 70 m
(c) 210 m (d) 175 m

24. X, Y and Z walk @ 6, 12, and 18 kmph respectively. They start from Jhumritalaiya towards Delhi at 2, 5, and 7 pm respectively, when Y catches up with X, Y sends X back with a message to Z; when will Z get the message?
(a) 9:45 pm (b) 9 pm
(c) 8:45 pm (d) 9:15 pm

25. At his normal speed, Rishu can go 18 km downstream in a flowing river in 9 hours less than what he takes to go the same distance upstream. The downstream journey would take 1 hour less than what the upstream journey would take provided he doubles his rate of rowing. Find the speed of the river (in km/hr)?

(a) 6.66 (b) 8
(c) 9.5 (d) 12

26. Two guns were fired from the same place at an interval of 10 minutes and 30 seconds, but a person in the train approaching the place hears the second shot 10 minutes after the first. The speed of the train (in km/hr), supposing that speed travels at 330 metres per second, is:

(a) 19.8 (b) 57.6
(c) 59.4 (d) 111.80

ANSWER KEY

Exercise 1

1. (a)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (c)
9. (b)	**10.** (b)	**11.** (c)	**12.** (a)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (b)
17. (d)	**18.** (b)	**19.** (c)	**20.** (b)				

Exercise 2

1. (a)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (c)
9. (d)	**10.** (b)	**11.** (d)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (a)
17. (d)	**18.** (a)	**19.** (b)	**20.** (b)	**21.** (c)	**22.** (a)	**23.** (a)	**24.** (c)
25. (a)	**26.** (c)						

HINTS AND EXPLANATIONS

Exercise 1

1. Speed $= \frac{5}{18} \cdot 36$ m/s = 10 m/s

Distance = Speed × Time

= 10 × 40 = 400 m

Hence, option (a) is the answer.

2. Length of train = Distance travelled by it while crossing a pole (or any stationary object of negligible length)

= 20 × 10 = 200 m

Hence, option (d) is the answer.

3. Total distance travelled by the train while crossing a platform

= Length of Train + Length of platform

Total distance

= Speed × Time = 15 × 60 secs = 900 m

So, Length of Train + Length of platform

= 900 m ⇒ Length of platform

= 900 m – 400 m = 500 m.

Hence, option (b) is the answer.

4. Total distance to be travelled

= 180 m + 150 m = 330 m

Speed of train $= 88 \cdot \frac{5}{18}$ m/s

So, Time taken $= \frac{330 \cdot 18}{88 \cdot 5} = 13.5$ secs

Hence, option (c) is the answer.

5. Assume that the time taken is T hrs with V km/h.

So distance = Speed × Time = VT.

New Speed $= \frac{\text{New Distance}}{\text{New Time}}$

$$= \frac{\frac{VT}{2}}{2T} = \frac{V}{4}$$

So ratio of former speed:new speed

$$= V : \frac{V}{4} = 4:1$$

Hence, option (a) is the answer.

6. Upstream speed $= B - R = 6$ km/hour
[One km in 10 minutes, so 6 km in 60 minutes] (i)

Donwstream speed $= B + R$
$= 15$ km/hour
[One km in 4 minutes, so 15 km in 60 minutes] (ii)

Speed of Boat

$$= \frac{\text{Downstream speed + Upstream speed}}{2}$$

$$= \frac{15+6}{2} = \frac{21}{2} = 10.5$$

So, speed of stream
$= 15 - 10.5 = 4.5$ kmph.

Hence, option (d) is the answer.

7. Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{60}{3/4}$
$= 80$ kmph

New speed $= (80 - 20)$ kmph $= 60$ kmph

So time taken $= \frac{60}{60} = 1$ hr

Hence, option (b) is the answer.

8. Time taken to meet for the 1st time

$$= \frac{\text{Length of track}}{\text{Relative speed}} = \frac{1000}{40} = 25 \text{ secs}$$

So, they will keep on meeting after every 25 secs.

Number of times they will meet in 10 mins (600 secs)

$$= \frac{600}{25} = 24 \text{ times}$$

Hence, option (c) is the answer.

9. This is simple:

In a kilometer race, A beats B by 10 metres ⇒ when A travels 1,000 m, B travels 990 m ⇒ When A travels 2,000 m, B will travel 1,980 m. Hence, A will beat B by 20 m. Hence, option (b) is the answer.

10. Total distance to be travelled
= Length of train
= 225 m

Relative Speed = 45 km/hr – 18 km/hr
$= 27$ km/hr $= 27 \cdot \frac{5}{18}$
$= 7.5$ m/s

So, time taken $= \frac{225}{7.5} = 30$ secs

Hence, option (b) is the answer.

11. In one hour, they will be 10 kms apart. So, in six hour, they will be 60 kms apart. Hence, option (c) is the answer.

12. Upstream speed $= B - R = 2.5$ km/hour
[1/2 km in 12 minutes, so 2.5 km in 60 minutes] (i)

Donwstream speed $= B + R = 5$ km/hour
[1/2 km in 6 minutes, so 5 km in 60 minutes] (ii)

Speed of Boat

$$= \frac{\text{Downstream speed + Upstream speed}}{2}$$

$$= \frac{5+2.5}{2} = \frac{7.5}{2} = 3.75 \text{ kmph}$$

So, speed of current $= 5 - 3.75$
$= 1.25$ kmph

Hence, option (a) is the answer.

13. When Amit travels 1 km, Bahadur travels 900 m.

Hence, ratio of speed of Amit:Bahadur

$= 10:9$ (i)

When Bahadur travels 1 km, Chandra travels 800 m.

Hence, ratio of speed of Bahadur:Chandra

$= 10:8$ (ii)

So, ratio of speed of Amit:Bahadur: Chandra $= 100:90:72$

$\Rightarrow$ When Amit travels 100 m, Chandra travels 72 m

$\Rightarrow$ When Amit travels 1000 m, Chandra travels 720 m. Hence, Amit will beat Chandra by 280 m. Hence, option (d) is the answer.

14. See the diagram:

$$P \xrightarrow[\substack{12\text{ hr} - 8\text{ hr } 20\text{ min} \\ = 3\text{ hr } 40\text{ min}}]{16:40/2 = 8\text{ hr } 20\text{ min}} Q \xrightarrow[\substack{12\text{ hr} - 8\text{ hr } 20\text{ min} \\ = 3\text{ hr } 40\text{ min}}]{16:40/2 = 8\text{ hr } 20\text{ min}} R$$

Time taken $= 3$ hr 40 min + 3 hr 40 min

$= 7$ hr 20 mins.

15. Relative Speed = (50 – 30) kmph

$= 20$ kmph $= 20 \times$ m/s

$= 20 \cdot \frac{5}{18}$ m/s

$= 20 \cdot \frac{5}{18}$ m/s

Length of faster train = Distance travelled

$= 20 \cdot \frac{5}{18} \cdot 18 = 100$ m

Hence, option (b) is the answer.

18. Speed is inversely proportional to the time. So, when speed becomes 6/7 of original speed, time will become 7/6 of original time (assume T).

So, change in time taken $= \frac{7}{6}T - T$

$= \frac{1}{6}T = 12$ mins

$\Rightarrow$ $T = 72$ mins

Hence, option (b) is the answer.

19. **Method 1:** Assume Speed = V kmph.

So time taken $= \frac{715}{V}$

New Time taken $= \frac{715}{V+10}$

According to the question,

$$\frac{715}{V} - \frac{715}{V+10} = 2 \text{ hrs}$$

Solving it,

we get $V = 55$ kmph

Method 2: Go through the options.

20. Train M takes 4 hrs and train Q takes 3:30 hours. Assume distance = 140 kms

Speed of train M $= \frac{140}{4} = 35$ kmph

Speed of train Q $= \frac{140}{3.5} = 40$ kmph

At 7 am, train M has already travelled

$35 \times 2 = 70$ kms

Post 7 am, they are required to travel 70 kms together.

Distance $= 70$ kms,

relative speed $= 35 + 40 = 75$ km/h

Time taken $= \frac{70}{75} \cdot 60$ mins

$= 56$ mins

Hence, the trains will meet at 7:56 am. Hence, (b) is the answer.

9 Sequence and Series

Let us consider the following series:

(i) 1, 4, 9, 16, ...
(ii) 2, 6, 12, 20, ...

It can be observed here that each of these two series is sharing some common property:

Series (i) is $\rightarrow 1^2, 2^2, 3^2, 4^2$ ...

Series (ii) is $\rightarrow 1^2+1, 2^2+2, 3^2+3, 4^2+4$ ...

With this we can find out any term very easily, or in general t_n, for any of these two series.

For series (i), $t_{10} = 10^2$

For series (ii), $t_{10} = 10^2 + 10$.

If the terms of a sequence are written under some specific conditions, then the sequence is called a **Progression**.

We will confine ourselves only to following certain and standard progression:

1. Arithmetic Progression
2. Geometric Progression
3. Harmonic Progression

1. **Arithmetic Progression:** A succession of numbers is said to be in Arithmetic Progression (AP) if the difference between any term and the previous term is constant throughout. This constant difference is known as Common Difference and is denoted by 'd'.

   In other words, difference between any of the two consecutive terms should be same. This difference which is common between any two consecutive terms is known as Common Difference of this AP and is denoted by d.

   Example $\rightarrow$ Series (i) 1, 2, 3, ...

   Series (ii) $\rightarrow$ 2, 5, 7, 10, ...

   Series (iii) $\rightarrow a, a + d, a + 2d$, ...

   Common Difference (d) of series (i) = 1

   Common Difference (d) of series (ii) = 3

   Common Difference (d) of series (iii) = d

*n*th term of an Arithmetic Progression

First term $t_1 = a = a + (1 - 1)\,d$

Second term $t_2 = a + d = a + (2 - 1)\,d$

Third term $t_3 = (a + d) + d = a + 2d$
$= a + (3 - 1)\,d$

Fourth term $t_4 = a + 3d = a + (4 - 1)\,d$

*n*th term $t_n = a + (n - 1)\,d$

where a is the first term, d is the common difference and n is the number of terms.

Important Points

1. t_n is also known as general term of AP.
2. If in any question, some particular term is given (likewise t_4 or t_{10}), then we should assume those terms in the form of t_n. However if total number of terms are given, then we should assume the terms in following way:

   If three terms or odd number of terms are involved. then we should assume these terms as $a - d$, a, $a + d$ and so on.

   If four terms or even number of terms are involved, then we should assume these terms as $a - 3d$, $a - d$, $a + d$, $a + 3d$ and so on.

Example 1

Sum of three consecutive numbers in AP is 27 and sum of their squares is 293. Find the numbers.

Solution

Let the numbers be $a - d$, a, $a + d$

Given is $(a - d + a + a + d) = 27$

So, $a = 9$

Also, $(a - d)^2 + a^2 + (a + d)^2 = 293$

$\Rightarrow$ $d^2 = 25$

$\Rightarrow$ $d = \pm 5$

When $d = +5$, then the terms are 4, 9, 14

When $d = -5$, then the terms are 14, 9, 4

Alternatively, this question can be done very easily with the help of options (if provided).

Example 2

If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

(a) 0 (b) –1

(c) 1 (d) Not unique

Solution

Given

$t_1 + t_2 + ... + t_{11} = t_1 + t_2 + ... + t_{19}$ (for an AP)

$$\frac{11}{2}[2a + (11-1)d] = \frac{19}{2}[2a + (19-1)d]$$

$$22a + 110d = 38a + 342d$$

$$16a + 232d = 0$$

$$2a + 29d = 0$$

Now, $$S_{30} = \frac{30}{2}[2a + (30-1)d]$$

$$S_{30\text{ terms}} = 0.$$

Properties of AP

If a, b, c, d, ... are in AP, then

1. $a + k$, $b + k$, $c + k$, $d + k$, ... will be in AP, where k is any constant.
2. $a - k$, $b - k$, $c - k$, $d - k$, ... will be in AP, where k is any constant.
   In the above two cases, common difference will be same as earlier.
3. ak, bk, ck, dk, ... will be in AP, where k is any constant.
   In this case, new common difference will be k times the earlier common difference.
4. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \frac{d}{k}$ will be in AP, where $k \neq 0$.
   In this case, new common difference will be $\frac{1}{k}$ times the earlier common difference.

Sum of *n* terms of an Arithmetic Progression

$$S_n = \frac{n}{2}[2a + (n-1)d],$$

where, n = number of terms,

a = 1st term

and d = common difference.

Example 3

Find the sum of AP 3, 5, 7, ..., 50 terms.

Solution

Here, $n = 50$, $d = 2$ and $a = 3$

Using formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= 25 \times [2 \times 3 + (50 - 1)\, 2]$$

$$= 25 \times 104 = 2{,}600$$

However, we can find out the sum of any AP in a better way through Average also.

Last term of this series

$= 101$

so average $= \frac{3 + 101}{2} = 52$

So, sum = Average × Number of numbers

$= 52 \times 50 = 2{,}600$

Hence, $$S_n = \frac{F \cdot L}{2} \cdot n,$$

where F is the first term, L is the last term and n is the number of terms of that AP.

Example 4

What is the sum of all the two-digit numbers which when divided by 7 gives a remainder 3?

Solution

This series is like → 10, 17, 21, ..., 94

Here, $n = 13$, $d = 7$ and $a = 10$

Using the formula for sum

$$S_n = \frac{n}{2}[2a + (n-1)d],$$

sum = 676

Alternatively, using average method,

$$\text{average} = \frac{(\text{1st number} + \text{last number})}{2}$$

$$\text{Average} = \frac{10+94}{2} = 52$$

So, sum = Average × Number of numbers
= 52 × 13 = 676.

Example 5

Find the value of the expression:
1 – 4 + 2 – 5 + 3 – 6 + ... to 100 terms

(a) –250 (b) –500
(c) –450 (d) –300

Solution

We can write the given expression
(1 – 4 + 2 – 5 + 3 – 6 + ... to 100 terms) as:
(1 + 2 + 3 + ... to 50 terms) – (4 + 5 + 6 + ... to 50 terms)

Both these are APs with different values of 'a' and 'd'.

$a = 1$, $n = 50$ and $d = 1$ for 1st series and $a = 4$, $n = 50$ and $d = 1$ respectively.

Using the formula for sum of an

$$\text{AP} = \frac{n}{2}[2a + (n-1)d]$$

we get $\frac{50}{2}[2.1 + (50-1).1] - \frac{50}{2}[2.4 + (50-1).1]$

= 25 (2 + 49) – 25 (8 + 49)
= 25 (51 – 57) = –150

Alternatively, logically we can do this question a bit faster by assuming (1 – 4), (2 – 5) etc. as one unit.

1 – 4 = 2 – 5 = ... = –3. Thus the above series is equivalent to a series of fifty (–3)'s added to each other.

So, (1 – 4) + (2 – 5) + (3 – 6) + ... 50 terms = – 3 × 50 = –150.

2. **Geometric Progression:** A succession of numbers is said to be in Geometric Progression if the ratio of any term and the previous term is constant. This constant ratio which is common to any of the two terms is known as Common Ratio and is denoted by 'r'.

Example:

(i) 1, 2, 4, 8, ...
(ii) 20, 10, 5, ...
(iii) a, ar, ar^2, ...

Common ratio of series (i) is 2
Common ratio of series (ii) is 0.5
Common ratio of series (iii) is r.

*n*th term of a Geometric Progression

First term	$t_1 = a = ar^{1-1}$
Second term	$t_2 = ar = ar^{2-1}$
Third term	$t_3 = ar^2 = ar^{3-1}$
Fourth term	$t_4 = ar^3 = ar^{4-1}$
***n*th term**	$\boldsymbol{t_n = ar^{n-1}}$

where a is the first term, r is the common ratio and n is the number of terms.

Important Points

1. t_n is also known as general term of GP.
2. If in any question, some particular term is given (likewise t_4 or t_{10}), then we should assume those terms in the form of t_n. However if total number of terms are given then we should assume the terms in following way:

   If three terms or odd number of terms are involved, then we should assume these terms as $\frac{a}{r}$, a, ar and so on.

Example 6

Seventh term of a GP is 8 times the fourth term and 5th term of the same GP is 48. Find the 6th term of this GP.

Solution

Given $t_7 = 8 \times t_4$,
Or, $ar^6 = 8 \times ar^3$
$\Rightarrow$ $r^3 = 8$, or, $r = 2$

Now, $ar^4 = 48$, so, $a = 3$ (since $r = 2$)
So, 6th term $= ar^5 = 3 \times 2^5 = 96$.

Properties of GP

If $a, b, c, d, \ldots$ are in GP, then

1. $ak, bk, ck, dk, \ldots$ will be in GP, where k is any non-zero constant.
2. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \frac{d}{k}$ will be in GP, where k is any non-zero constant.
   In the above two cases, common ratio will be same as earlier.
3. If a GP of even number of terms is given, then its common ratio will be the same as ratio of sum of all the even terms and sum of all the odd terms.

Sum of *n* terms of a Geometric Progression

$$S_n = \frac{a(1-r^n)}{1-r} \text{ when } r \neq 1$$

$$S_n = na \text{ when } r = 1$$

where, n = number of terms,
a = 1st term
and d = common difference.

Sum of infinite GP

$$= \frac{a}{1-r}, \text{ when } -1 < r < 1.$$

Example 7

After striking the floor, a ball rebounds to 4/5th of the height from which it has fallen. What is the total distance that it travels before coming to rest if it is gently dropped from a height of 120 m?

Solution

Distance covered before first rebound
= 120 m (Fig. 9.1)

And then the ball bounces to a height of $120 \times 4/5$ and then falls from the same height.

Next time ball will go up by $120 \times \frac{4}{5} \times \frac{4}{5}$ and then will fall from the same height.

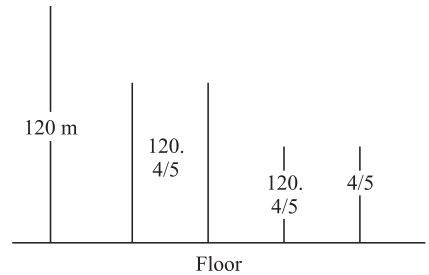


Fig. 9.1

So, total distance covered

$$= 120 + 2 \times 120 \times \frac{4}{5} + 2 \times 120 \times \frac{4}{5} \times \frac{4}{5} + \ldots \infty$$

$$= 120 + 2 \times 120\left(\frac{4}{5} + \frac{4}{5} \times \frac{4}{5} + \ldots \infty\right)$$

$$= 120 + 240 \frac{\frac{4}{5}}{1 - \frac{4}{5}}$$

= 1,080 m

> Alternatively, if the ball rebounds to $\frac{a^{\text{th}}}{b}$ of original height H, then total distance covered $= H \times \frac{a+b}{a-b}$.

In the above example, total distance covered

$$= 120 \times \frac{4+5}{5-4}$$

= 1,080 m.

3. **Harmonic Progression:** Unequal numbers $a, b, c, \ldots$ are said to be in HP if reciprocals of these terms *i.e.*, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \ldots$ are in AP. It is noteworthy here that no term of an HP can be equal to zero.

*n*th term of a Harmonic Progression

nth term of HP

$$= \frac{1}{(n\text{th term of the corresponding AP})}$$

Sum of *n* terms of a Harmonic Progression

There is no standard formula for finding out the sum of n terms of a HP.

MEANS

Arithmetic Mean

It two numbers a and b are in AP, then their AM

$$= \frac{a+b}{2}$$

In general, if $a, b, c, ..., n$ terms are in AP, then their AM

$$= \frac{a+b+c+...+n \text{ terms}}{n}$$

It's imperative here to mention that, in simple terms, AM of n terms is nothing but average of n terms.

Geometric Mean

If two numbers a and b are in GP then their GM

$$= \sqrt{ab}$$

In general, if $a, b, c, ..., n$ terms are in GP, then GM

$$= \sqrt[n]{a \cdot b \cdot c \cdot ... n \text{ terms}}$$

So, if three terms a, b and c are in GP, then their GM

$$= b = \sqrt[3]{a \cdot b \cdot c}$$

Harmonic Mean

If two numbers a and b are in HP, then their HM

$$= \frac{2ab}{a+b}$$

Relationship between AM, GM and HM

Now we know that for any given two numbers a and b,

$$\text{AM} = \frac{a+b}{2}$$

$$\text{GM} = \sqrt{ab}$$

$$\text{HM} = \frac{2ab}{a+b}$$

1. AM, GM and HM will be in GP.
   So, GM is the geometric mean of this series.
   And, $\text{GM} = \sqrt{\text{AM} \cdot \text{HM}}$
2. AM ≥ GM ≥ HM

Sum of *n* Terms of Some Special Series

In this part of Progression, we will see the sum of some other special sequences.

1. Sum of 1st n natural numbers:
   $$S_n = \frac{n(n+1)}{2}$$
   Hence, $\sum_{S=1}^{n} S = 1 + 2 + 3 + ... + n$
   $$= \frac{n(n+1)}{2}$$
2. Sum of the squares of 1st n natural numbers:
   $$S_n = \frac{n(n+1)(2n+1)}{6}$$
   Hence, $\sum_{S=1}^{n} S^2 = 1^2 + 2^2 + 3^2 + ... + n^2$
   $$= \frac{n(n+1)(2n+1)}{6}$$
3. Sum of the cubes of 1st n natural numbers:
   S_n = (Sum of 1st n natural numbers)^2
   $$= \frac{n(n+1)^2}{2}$$
   It can be seen that $(1 + 2 + 3)^2$
   $= 1^3 + 2^3 + 3^3$
   Hence, $\sum_{S=1}^{n} S^3 = 1^3 + 2^3 + 3^3 + ... + n^3$
   $$= \frac{n(n+1)^2}{2}$$
4. $2^2 + 4^2 + 6^2 + ... + n$ terms
   $$= \frac{n(n+1)(2n+1)}{6} \cdot \frac{1}{4}$$
5. $1 + 3 + 5 + 7 + ...+ n$ terms $= n^2$
6. $2 + 4 + 6 + 8 + ... + n$ terms $= n(n + 1)$

PRACTICE EXERCISE

EXERCISE 1

1. Find the sum of the first 15 terms of the series whose nth term is $(4n + 1)$.

(a) 485 (b) 495
(c) 505 (d) 630

2. Find the sum of all the numbers divisible by 6 in between 100 and 400.

(a) 12,550 (b) 12,450
(c) 11,450 (d) 11,550

3. In a GP first term is 7 and nth term is 448, and the sum of n terms is 889. What is the common ratio of this GP?

(a) 2 (b) 4
(c) 1 (d) 3

4. The sum of 15 terms of an AP is 600, and the common difference is 5. Find the first term?

(a) 4 (b) 5
(c) 3 (d) None of these

5. A man arranges to pay off a debt of ₹ 3,600 by 40 annual installments which form an AP. When 30 of the installments repaid, he dies leaving a third of the debt unpaid. What is the value of the first installment?

(a) 50 (b) 51
(c) 52 (d) 53

6. An AP has 23 terms, sum of the middle three terms is 144, the sum of the last three terms is 264. What is the 16th term?

(a) 102 (b) 64
(c) 82 (d) None of these

7. Find a, b, c between 2 and 18 such that (i) their sum is 25, (ii) 2, a, b are consecutive terms of an AP and (iii) b, c, 18 are the consecutive terms of a GP?

(a) 5, 8, 14 (b) 4, 8, 12
(c) 5, 8, 12 (d) None of these

8. If the sum of an AP is the same for p terms as for q terms, find the sum for $(p + q)$ terms?

(a) 2 (b) 0
(c) 4 (d) None of these

9. Sum of three numbers in GP is 70. If the two extreme terms are multiplied by 4, and the middle term by 5, the resultants are in AP. Find the numbers?

(a) 10, 25, 35 (b) 10, 20, 40
(c) 15, 30, 35 (d) 12, 24, 34

10. Find the first term of the infinite GP whose first two terms add up to 5, and whose each term is three times in the sum of all terms that follow it.

(a) 1 (b) 2
(c) 3 (d) 4

11. Sum of an infinite geometric series is 4 and the sum of the cubes of the terms of the same GP is 192. The common ratio of the original geometric series is:

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

12. The 10th common term between the series 3 + 7 + 11 + ... and 1 + 6 + 11 + ... is:

(a) 191 (b) 193
(c) 211 (d) None of these

13. In a GP of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the GP is:

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$
(c) 4 (d) None of these

14. The ratio of HM and GM for two positive numbers is 4:5. What is the ratio of the numbers?

(a) 4:1 (b) 3:2
(c) 3:4 (d) 2:3

15. The 288th term of the series $a, b, b, c, c, c, d, d, d, d, e, e, e, e, e, \ldots$ is:

(a) u (b) v
(c) w (d) x

16. The product of n positive numbers is 1. Their sum is:

(a) a positive integer
(b) divisible by n
(c) equal to $n + \frac{1}{n}$
(d) greater than or equal to n

17. The 5th term of an AP is 15 and the 9th term is 23. Find the 14th term.

(a) 31 (b) 33
(c) 35 (d) 37

18. If the 9 times the 9th term in AP is equal to 15 times the 15th term in the AP what is the 24th term?

(a) –1 (b) –3
(c) 0 (d) 1

19. Find the sum of an AP of 14 terms and whose first and last terms are 8 and 99 respectively.

(a) 707 (b) 749
(c) 789 (d) 747

20. In an infinite GP every term is equal to the sum of all the terms that follow. Find the common ratio.

(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/6

21. A GP consists of 1,000 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio of this GP.

(a) P_2/P_1 (b) P_1/P_2
(c) $(P_2 - P_1)/P_1$ (d) $(P_2 + P_1)/P_2$

22. If the terms of one geometric progression are multiplied by the corresponding terms of another geometric progression, the sequence obtained will be in:

(a) AP
(b) HP
(c) GP
(d) Cannot be determined

23. Consider the following pyramid of natural numbers:

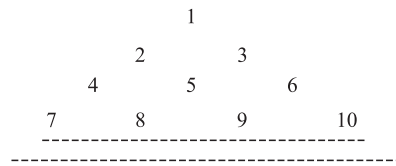


Fig. 9.2

As it can be seen that 1st row has only 1 number = 1, 2nd row has two numbers = 2, 3 and so on.

Find the 1st term of 50th row.

(a) 1,225 (b) 1,226
(c) 1,250 (d) 1,251

24. If m times the mth term of an AP is equal to p times the pth term, find the $(m + p)$th term.

(a) 0 (b) $a^2 - b^2$
(c) $a - b$ (d) 1

25. Mid-points of adjacent sides of a square are joined. Again the mid-points of adjacent sides of newly formed figure are connected and this process is repeated again and again. Calculate the sum of areas of all such figures given that the diagonal of outermost square is:

(a) 35 cm^2 (b) 44 cm^2
(c) 72 cm^2 (d) 58 cm^2

ANSWER KEY

Exercise 1

1. (b)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (b)
9. (b)	**10.** (d)	**11.** (b)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (d)
17. (b)	**18.** (c)	**19.** (b)	**20.** (a)	**21.** (a)	**22.** (c)	**23.** (b)	**24.** (a)
25. (c)							

HINTS AND EXPLANATIONS

Exercise 1

1. Series is 5, 9, 13, 17, ..., 61

Sum = Average × Number of terms

$$= \frac{\text{1st term + last term}}{2} \cdot \text{ Number of terms}$$

$$= \frac{5+61}{2} \cdot 15$$

$$= 33 \times 15 = 495$$

Hence, option (b) is the answer.

2. Series is 102, 108, 114, ..., 396

Sum = Average × Number of terms

$$= \frac{\text{1st term + last term}}{2} \cdot \text{ Number of terms}$$

$$= \frac{102+396}{2} \cdot 50$$

$$= 249 \times 50 = 12{,}450$$

Hence, option (b) is the answer.

3. $t_n = ar^{n-1}$

It is given that first term $= 7$

Hence, $t_n = 7 \times r^{n-1}$

$= 448$

$\Rightarrow \quad r^{n-1} = \frac{448}{7} = 64$

It can be either 4^3 or 2^6

If it is 4^3,

then $r = 4$ and $n = 4$ (i)

If it is 2^6,

then $r = 2$ and $n = 7$ (ii)

Sum of n terms $= a\,(r^n - 1)/(r - 1)$

$7\,(r^n - 1)/(r - 1) = 889$

$\Rightarrow \quad (r^n - 1)/(r - 1) = 127$

Now let us verify which of (i) or (ii) will satisfy this?

It can be seen that $r = 2$ and $n = 7$ will satisfy this. Hence, option (a) is the answer.

4. Average $=$ 8th term $= a + 7d$

$$= \frac{600}{15} = 40$$

$\Rightarrow$ Given that common difference $= 5$, hence, $a = 5$.

5. Total sum = ₹ 3,600

Number of terms = 40

Installments are increasing in an AP. When 30 installments have been paid, 2/3rd of the debt has been repaid.

$$S_{30} = \frac{30}{2}[2a + (30-1)d]$$

$$= \frac{2}{3} \cdot 3600 = 2400$$

$\Rightarrow 2a + 29d = 160$ (i)

$$S_{40} = \frac{40}{2}[2a + (40-1)d]$$

$$= 3600$$

$\Rightarrow 2a + 39d = 180$ (ii)

Solving (i) and (ii) gives: $d = 2$ and $a = 51$. Hence, 1st installment = ₹ 51. Hence, option (b) is the answer.

6. Middle three terms will be 11th, 12th, 13th term:

11th term = $a + 10d$
12th term = $a + 11d$
13th term = $a + 12d$

Given that $a + 10d + a + 11d + a + 12d$
$= 3a + 33d = 144$

$\Rightarrow \quad a + 11d = 48$ (i)

Similarly, 21st term + 22nd term + 23rd term
$= a + 20d + a + 21d + a + 22d$
$= 3a + 63d = 264$

$\Rightarrow \quad a + 21d = 88$ (ii)

Solving (i) and (ii), $d = 4$ and $a = 4$. Hence, 16th term = $a + 15d = 64$. Hence, option (b) is the answer.

7. Go through the options.

8. If sum of 1st p terms of an AP = sum of 1st q terms of the same AP $\Rightarrow$ Sum of $(p + q)$ terms = 0. Hence, option (b) is the answer.

9. Go through the options.

10. Series of common term = 11, 31, 51, ... and so on. It can be observed that 10th common term will be 191.

$$t_n = a + (n - 1)\,d$$
$$= 11 + 9 \times 20 = 191$$

Hence, option (a) is the answer.

11. Sum of Infinite GP

$$= \frac{a}{1-r} = 4 \quad \text{(i)}$$

If we cube all the terms of this GP, then Sum of Infinite GP

$$= \frac{a^3}{1-r^3} = 192 \quad \text{(ii)}$$

Cubing (i) and then dividing by (ii) gives: $\frac{1-r^3}{(1-r)^3} = \frac{1}{3}$. Now it is better that we put the options to check the answer.

It can be seen that option (b) $-\frac{1}{2}$ satisfies the equation obtained. Hence, option (b) is the answer.

12. Assume that series is $a, ar, ar^2, ar^3, \ldots$

Given that:

Equation (i) $\quad a + ar = 5$

$\Rightarrow \quad a(1 + r) = 5$

Equation (ii) $\quad a = 3(ar + ar^2 + ar^3 + \ldots)$

$$= 3 \times \frac{ar}{1-r}$$

$$\Rightarrow \quad 1 = 3 \times \frac{r}{1-r}$$

$$\Rightarrow \quad 1 - r = 3r$$

$$\Rightarrow \quad r = \frac{1}{4}$$

Now we will put the value of r in equation (i) to obtain the value of a.

$$a\left(1 + \frac{1}{4}\right) = 5$$

$$\Rightarrow \quad a\left(\frac{5}{4}\right) = 5$$

$$\Rightarrow \quad a = 4$$

Hence, option (d) is the answer.

13. Question gives a general situation "for even number of terms". So, let us assume there are four terms. And verify the options:

Option (a)

Assume the series is: 1, 2, 4, 8

Sum of all the terms $= (1 + 2 + 4 + 8)$
$= 15$

Sum of odd terms
= Sum of 1st term + 3rd term
$= 1 + 4 = 5$

Ratio of $\frac{\text{Sum of all the terms}}{\text{Sum of odd terms}} = \frac{15}{5} = 3$

This does not satisfy the condition given in the question. Hence, not the answer.

Option (b)

Assume the series is 1, 4, 16, 64

Sum of all the terms $= (1 + 4 + 16 + 64)$
$= 85$

Sum of odd terms
= Sum of 1st term + 3rd term
= 1 + 16 = 17

Ratio of $\dfrac{\text{Sum of all the terms}}{\text{Sum of odd terms}} = \dfrac{85}{17} = 5$

This satisfies the condition given in the question.

Hence, option (b) is the answer.

14. Let us verify the options:

Option (a) Assume the numbers are 4 and 1

$$\text{HM} = \frac{2ab}{a+b} = \frac{2 \cdot 4 \cdot 1}{4+1} = \frac{8}{5}$$

$$\text{GM} = \sqrt{ab} = \sqrt{4 \cdot 1} = 2$$

Ratio of HM and GM $= \dfrac{8}{5} : 2 = 4:5$

This satisfies the given condition in the question. Hence, option (a) is the answer.

15. a occurs once, b occurs twice, c occurs thrice, d occurs four times ... 10th letter will occur 10 times, ... 20th letter will occur 20 times and so on.

We will try to identify that which letter will occur immediately before where 288th letter starts.

1 + 2 + 3 + 4 + 5 + ... + 23 = 276

Hence, 24th letter will come afterwards for 24 times. So, 288th term will be 24th letter of the alphabet = x.

Hence, option (d) is the answer.

16. It is known that AM ≥ GM

$$\frac{\text{Sum}}{n} \geq \text{GM} \Rightarrow \frac{\text{Sum}}{n} \geq 1$$

Sum ≥ n

Hence, option (d) is the answer.

17. 5th term $= a + 4d = 15$
9th term $= a + 8d = 23$
$\Rightarrow \quad a = 7, d = 2$
14th term $= a + 13d$
$= 7 + 13 \times 2 = 33$

Hence, option (b) is the answer.

18. 9th term of AP $= a + 8d$ and
15th term $= a + 14d$
Given that: $9 \times (a + 8d) = 15\,(a + 14d)$
$\Rightarrow \quad 9a + 72d = 15a + 210d$
$\Rightarrow \quad 6a + 138d = 0$
$\Rightarrow \quad a + 23d = 0 =$ 24th term

Hence, option (c) is the answer.

19. $a = 8$ and $a + 13d = 99 \Rightarrow d = 7$.

Sum of all the 14 terms

$$= \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{14}{2}[2 \times 8 + (14-1)7] = 749$$

Hence, option (b) is the answer.

20. Assume the series is a, ar, ar^2 and so on

Given that $a = \dfrac{ar}{1-r}$

$$\Rightarrow \quad 1 = \frac{r}{1-r} \Rightarrow r = 1 - r$$

$$\Rightarrow \quad r = 1/2$$

Hence, option (a) is the answer.

21. Using property (iii) of GP, common ratio

$$= \frac{\text{Sum of all the even terms}}{\text{Sum of all the odd terms}} = \frac{P_2}{P_1}$$

Hence, option (a) is the answer.

22. Consider 1st GP = 1, 2, 4, 8, ...
and 2nd GP = 3, 9, 27, 81, ...
Multiplying these two gets
= 3, 18, 108, 648, ...

This is in GP with a common ratio
= Common ratio of 1st GP
× Common ratio of 2nd GP

Hence, option (c) is the answer.

23. Last term of every row = 1, 3, 6, 10, ...

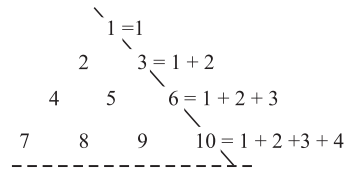


Fig. 9.3

So, last element of the 49th row

$= 1 + 2 + 3 + ... + 49 = 1{,}225$

Hence, 1st element of the 50th row

$= 1{,}226$

Hence, option (b) is the answer.

25. Diagonal of outermost square *ABCD*

$= 6\sqrt{2}$ cm

$\Rightarrow$ Side of outermost square *ABCD*

$= 6$ cm

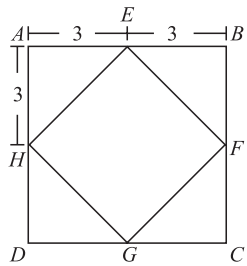


Fig. 9.4

When mid-points are joined, newly formed structure *EFGH* will also be square

$AE = AH = 3$ cm

Hence, $EH = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ cm

Area of outermost square

$ABCD = 36$ sq cm,

area of square

$EFGH = (3\sqrt{2})^2 = 18$ sq cm

and so on area of the next square

$= 9$ sq cm. And so on.

Sum of areas of all such squares formed $= 36 + 18 + 9 + ...$ infinite terms

It can be seen that the series formed 36, 18, 9, ... is a GP with infinite terms

Hence, summation

$$= \frac{a}{1-r} = \frac{36}{1-\frac{1}{2}} = \frac{36}{\frac{1}{2}}$$

$= 36 \times 2$

$= 72$ sq cm

Hence, option (c) is the answer.

10 Equations

DEFINITIONS

Polynomials: Assume that a_1, a_2, a_3, a_4, ... are real numbers and x is a real variable.

Then $f(x) = a_1 x^n + a_2 x^{n-1} + a_3 x^{n-2}\ 1 + ... + a_{n-1} x + a_n x$

is called a polynomial.

Example: $5x^5 + 3x^4 + ... + x$ is a polynomial in x, where x is a real variable.

Degree of a polynomial: Polynomial $f(x) = a_1 x^n + a_2 x^{n-1} + a_3 x^{n-2} + ... + a_{n-1} x + a_n x$ is a polynomial of degree n, where $a_1 \neq 0$.

Example: $5x^5 + 3x^4 + ... + x$ is a polynomial of degree 5.

$2x^4 + x^3 + 4x^2 + 2x + 10$ is a polynomial of degree 4.

$4x^3 + 4x^2 + 2x + 10$ is a polynomial of degree 3.

$4x^2 + 2x + 10$ is a polynomial of degree 2.

Remember that:

1. Degree of polynomial is defined for real as well as complex polynomials.
2. Degree of polynomial cannot be fractional.

QUADRATIC EQUATION

Any equation of degree 2 is known as Quadratic Equation.

$ax^2 + bx + c = 0$ is known to be the standard equation of quadratic equation. It can be seen that this equation will have two roots, real or imaginary.

Geometrical Meaning of Roots: For any given equation $y = f(x) = 0$, number of times graph of this equation cuts X-axis is equal to the distinct real roots of this equation.

Example: $y = (x-1)(x+2)(x-2) = 0$ will intersect X-axis at three distinct points, namely $x = 1, -2$ and 2.

For any quadratic equation $f(x) = (x-1)(x-3)$, graph of this equation (can be seen below) will intersect X-axis at two distinct points, namely $x = 1$ and $x = 3$.

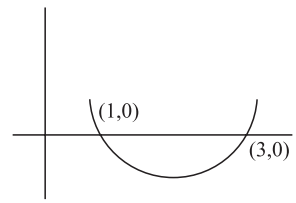


Fig. 10.1

So, the roots of the equation

$$f(x) = x^2 - 4x + 3 = 0$$

will have two roots, *i.e.*, two values of $x = 1$ and $x = 3$, which will satisfy this equation.

Properties of Roots

1. A polynomial equation of degree n will have n roots, real or imaginary.
2. Complex roots or surds always occur in pair.
   Example: If $2+\sqrt{5}$ is a root of any equation, then $2-\sqrt{5}$ will also be a root of that equation.
3. Any equation with sum of all of its coefficients equal to zero, will have 1 as its one of the roots.
   Example: $x^2 - 5x + 4 = 0$ will have 1 as one of its roots.
4. If all the terms of an equation are +*ve* and it does not involve any odd powers of x, then this equation will not have any real root.

Example: $f(x) = x^4 + x^2 + 1 = 0$ will not have any real root. Obviously $x^4 \geq 0$, $x^2 \geq 0$ and 1 is positive. And we know that sum of three positive numbers can never be equal to zero for any real value of x.

Example 1

How many real roots will be there of the quadratic equation

$$f(x) = x^2 + 5\,|x| + 6 = 0?$$

Solution

If $x > 0$

then, $f(x) = x^2 + 5x + 6 = 0$

Or,

$x^2 + 5x + 6 = (x + 2)(x + 3) = 0$

So, $x = 22$ and $x = 23$

But we have taken $x > 0$, so negative values of x are not admissible

Now, if $x < 0$

then, $f(x) = x^2 - 5x + 6 = 0$

Or,

$x^2 - 5x + 6 = (x - 2)(x - 3) = 0$

So, $x = 2$ and $x = 3$

But we have taken $x < 0$, so positive values of x are not admissible

And $x = 0$ is not possible

So, no real value of x can be obtained

Alternatively, it is worth observing that all the terms of $f(x)$, *viz.*, x^2 and $5|x|$ and 6, are positive. So, sum of these three terms cannot be equal to zero for any real value. Hence, no real value of x can be obtained.

Roots of a Quadratic Equation and their Nature

Given is the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

Assuming that α and β are the roots of this equation.

Then, $$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and $$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

It can be seen that nature of both of these two roots depends upon the value of $b^2 - 4ac$. $(b^2 - 4ac)$ is also known as **Discriminant (D)** of the quadratic equation $ax^2 + bx + c = 0$.

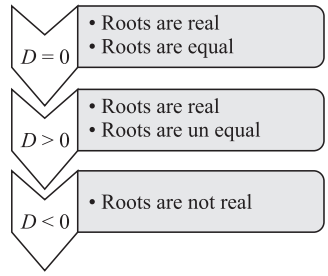


Fig. 10.2

If D is a perfect square (including $D = 0$) and a, b and c are rational, then roots will be rational.

Example 2

Given is the quadratic equation $ax^2 + bx + 1 = 0$, where a, b, ε (1, 2, 3, 4). For how many set of values of (a, b), quadratic equation $ax^2 + bx + 1 = 0$ will have real roots?

Solution

For roots to be real, $D \geq 0$.

$D = b^2 - 4a \geq 0$.

Forming the Table 10.1 for the above written condition:

Table 10.1

b	a
1	Not possible
2	1
3	1
3	2
4	1
4	2
4	3
4	4

So, there are seven sets of values for (a, b).

Example 3

Sum of a certain integer and its square is 90. What is the integer?

Solution

Let the number be x

$$x + x^2 = 90$$

$$\Rightarrow \quad x^2 + x - 90 = 0$$

$$\Rightarrow \quad (x + 10)(x - 9) = 0$$

Hence, $x = 9$ or -10.

ALGEBRAIC CALCULATIONS ON ROOTS

Quadratic Equation

$f(x) = ax^2 + bx + c = 0$

Assume that the roots are α and β.

$$\alpha + \beta = \left(\frac{\text{Co-efficient of } x^{n-1}}{\text{Co-efficient of } x^n}\right) \times (-1)^n$$

$$= \frac{b}{a}(-1)^1 = -\frac{b}{a}$$

$$\alpha\beta = \left(\frac{\text{Constant term}}{\text{Co-efficient of } x^n}\right) \times (-1)^n$$

$$= \frac{c}{a}(-1)^2 = \frac{c}{a}$$

Cubic Equation

$f(x) = ax^3 + bx^2 + cx + d = 0$

Assume that the roots are α, β and γ.

$\alpha + \beta + \gamma$

$$= \left(\frac{\text{Co-efficient of } x^{n-1}}{\text{Co-efficient of } x^n}\right) \times (-1)^n$$

$$= \frac{b}{a}(-1)^1 = -\frac{b}{a}$$

$\alpha + \beta + \gamma\ \alpha + \beta\gamma$

$$= \left(\frac{\text{Co-efficient of } x^{n-2}}{\text{Co-efficient of } x^n}\right) \times (-1)^n$$

$$= \frac{c}{a}(-1)^2 = \frac{c}{a}$$

$$\alpha\beta\gamma = \left(\frac{\text{Constant term}}{\text{Co-efficient of } x^n}\right) \times (-1)^n$$

$$= \frac{d}{a}(-1)^3 = \frac{-d}{a}$$

Example 4

If the polynomial $ax^4 + bx^3 + cx^2 + dx + e$ has the property that the product of all the roots, taken all at a time, is 1/3rd of the sum of the product of roots, taking two at a time. What is the relationship between e and c?

Solution

Product of all the roots, taking at a time

$$= \frac{e}{a}$$

Sum of the product of roots, taking two at a time

$$= \frac{c}{a}$$

Now, $\frac{e}{a} = 1/3\frac{c}{a}$

So, $c = 3 \times e$

Example 5

Find the quadratic equation with sum of its roots = 1, their product is 12.

(a) $x^2 - 12x + 1 = 0$

(b) $x^2 - 4x + 1 = 0$

(c) $x^2 - x - 12 = 0$

(d) $x^2 - 3x + 4 = 0$

Solution

Going through the options, $-b/a = 1$, which is true only in case of option (c). Hence, option (c) is the answer.

Example 6

If a, b and c are the roots of the equation $x^3 - 3x^2 + 2x + 1 = 0$, then what is the value of $= \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$?

Solution

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{ab+bc+ca}{abc}$$

$ab + bc + ca = 2$ and $abc = -1$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{ab+bc+ca}{abc}=-2$$

Formation of Quadratic Equation from its Roots

If α and β are the roots of the equation $ax^2 + bx + c = 0$, then we can write $ax^2 + bx + c = x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - (\text{sum of roots})x + \text{product of roots} = 0$, or, $ax^2 + bx + c = a(x - \alpha(x - \beta) = 0$.

Example 7

What will be the quadratic equation having the roots opposite in sign as that of the quadratic equation $x^2 + 5x + 6 = 0$?

Solution

Putting $(-x)$ at the place of x will give us the solution

Desired equation is $(-x)^2 + 5(-x) + 6 = 0$

So, equation is $x^2 - 5x + 6 = 0$

Alternatively, roots of the quadratic equation $x^2 + 5x + 6 = 0$ are 22 and 23

So, roots of the desired equation should be 2 and 3

So, the quadratic equation is $(x-2)(x-3)=0$, or, $x^2 - 5x + 6 = 0$.

Solution of Inequations

First case:

$$f(x) = x^2 - 5x + 6 \geq 0$$
$$f(x) = x^2 - 5x + 6$$
$$= (x-2)(x-3) = 0$$

Fig. 10.3

We have got three intervals of values here: $-\infty$ to 2, 2 to 3 and 3 to $+\infty$.

To check the values of x satisfying the above written inequation, we will take any one value of x from any of the intervals. If this value satisfies the inequation, then inequality will be satisfied in the alternate intervals.

Taking $x = 10$, $f(x) = (x-2)(x-3) = (10-2)(10-3) = +\text{ve}$

So, inequality is satisfied in this interval. Hence, it will not be satisfied in the interval $2 \leq x \leq 3$ and again will be satisfied in the interval $-\infty < x \leq 2$.

So, values of x satisfying the above written inequality lies outside 2 and 3, *i.e.*, $x \leq 2$ or $x \geq 3$.

Second case:

$$f(x) = x^2 - 5x + 6 \leq 0$$

2nd case $f(x) = x^2 - 5x + 6 \geq 0$

Fig. 10.4

It can similarly be observed that values of x are lying in between 2 and 3.

Hence, $2 \leq x \leq 3$.

In general, if $(x - a)(x - b) > 0$, then $a < x < b$ (we read this as x lies inside a and b) and if $(x - a)(x - b) < 0$ and $a < b$, then $x < a$ or $x > b$ (we read this as x lies outside a and b).

Example 8

What values of x will satisfy the quadratic inequation $f(x) = -x^2 + 3x + 4 > 0$?

Solution

$$f(x) = -x^2 + 3x + 4 > 0$$

or, $x^2 - 3x - 4 < 0$

or, $(x + 1)(x - 4) < 0$

So values of x satisfying

$$f(x) = -1 < x < 4.$$

Some More Worked Out Problems

Example 9

Find the real values of N for which the quadratic equation $2x^2 - (N^3 + 8N - 1)x + N^2 - 4N = 0$, will have roots of opposite sign.

Solution

The roots of the given equation will be of opposite signs only if they are real and the product of the roots is negative.

Or, $D \geq 0$, and product of roots < 0

Or, $(N^3 + 8N - 1)^2 - 8(N^2 - 4N) \geq 0$

and $\dfrac{N^2 - 4N}{2} < 0$

Or, $N^2 - 4N < 0$

Hence, $0 < N < 4$.

Example 10

For which value of k does the following pair of equators yield a unique solution of x such that the solution is positive?

$$x^2 - y^2 = 0$$

$$(x - k)^2 + y^2 = 1$$

(a) 2 (b) 0

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

Solution

(c) $y^2 = x^2$

$2x^2 - 2kx + k^2 - 1 = 0$

$D = 0$

$\Rightarrow \quad 4k^2 = 8k^2 - 8$

$\Rightarrow \quad 4k^2 = 8$

$k = \sqrt{2}$

Example 11

Let $x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \ldots}}}}$ to infinity. Then x equals

(a) 3 (b) $\dfrac{\sqrt{13} - 1}{2}$

(c) $\dfrac{\sqrt{13} + 1}{2}$ (d) $\sqrt{3}$

Solution

(c) $x = \sqrt{4 + \sqrt{4 - x}}$

$x^2 - 4 + \sqrt{4 - x}$

$(x^2 - 4) = \sqrt{4 - x}$

Now put the values from options.

Only third option satisfies the condition. Alternatively, value of x will be more than 2, which is given there in only option (a) and option (c). Since it is only slightly more than 3, option (b) will be the answer.

PRACTICE EXERCISE

EXERCISE 1

1. What is the value of k such that the equation $x^2 - (k + 6)\,x + 2\,(2k - 1) = 0$ has sum of the roots equal to half of their product?

(a) –2 (b) 7

(c) 9 (d) 12

2. If $x + y + z = 0$, then $x^3 + y^3 + z^3$ is equal to:

(a) 0

(b) $3xyz$

(c) $\dfrac{xy + yz + zx}{xyz}$

(d) $xyz\,(xy + yz + zx)$

3. The cost of 3 apples and 2 guavas is ₹ 23 and that of one apple and four guavas of the same type is ₹ 21. What is the cost of 5 guavas?

(a) ₹ 4 (b) ₹ 20

(c) ₹ 12 (d) ₹ 25

4. Roots of the quadratic equation $ax^2 + bx + c = 0$ are 1 and 2, for some certain value of b and c. If $a = 1$, what will be the value of $(a + b + c)$?

(a) 2

(b) –2

(c) –3

(d) Cannot be determined

5. Let $x = \sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-...}}}}$ to infinity. Then x equals

(a) 3 (b) $\left(\frac{\sqrt{13}-1}{2}\right)$

(c) $\left(\frac{\sqrt{13}+1}{2}\right)$ (d) $\sqrt{3}$

6. What is the sum of roots of the quadratic equation $3x^2 + 2x - 1 = 0$?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 3 (d) $\frac{-2}{3}$

7. What is the product of roots of the quadratic equation $3x^2 + 2x - 1 = 0$?

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 3 (d) $\frac{-2}{3}$

8. What is the sum of roots of the cubic equation $x^3 + 2x^2 + x - 1 = 0$?

(a) –2 (b) $\frac{-2}{3}$

(c) –1 (d) 2

9. Which of the following quadratic equations will not have real roots?

(a) $3x^2 + 2x - 1 = 0$
(b) $x^2 + 2x + 1 = 0$
(c) $3x^2 + 2x + 5 = 0$
(d) $2x^2 + 3x + 1 = 0$

10. Roots of the quadratic equation $ax^2 + bx + c = 0$ are 1 and 2, for some certain value of a, b and c. What will be sum of roots of the equation having the roots opposite in sign of this equation $ax^2 + bx + c = 0$?

(a) 3 (b) 6
(c) –3 (d) None of these

11. Present age of a father is eight times the present age of the son. After three years, the father's age will become five times his son's age. What is the present age of the father?

(a) 24 years (b) 32 years
(c) 40 years (d) 48 years

12. Which of the following satisfies the inequality $-x^2 + 6x - 8 > 0$?

(a) $3 < x < 5$ (b) $2 < x < 4$
(c) $x < 2$ and $x > 4$ (d) $x < 3$ and $x > 5$

13. For what value of K, the equations $2x + 3y = 5$ and $4x + Ky = A$ will not have any solution?

(a) 2
(b) –2
(c) 6
(d) More than one value is possible

14. Five burgers, six pizzas and seven cold drinks cost ₹ 178 and six burgers, four pizzas and two cold drinks cost ₹ 124. What is the cost of 3 (pizzas + burgers + cold drinks)?

(a) ₹ 60
(b) ₹ 62.5
(c) ₹ 90
(d) Cannot be determined

15. A and B are the roots of the equation $x^2 - 13x + N = 0$. If $A - B = 3$, what is the value of N?

(a) 20 (b) 27
(c) 36 (d) None of these

16. If $x < 0$, then what is the maximum value of $\frac{9}{x} + \frac{x}{9}$?

(a) 2 (b) –2
(c) $+\infty$ (d) None of these

17. If the co-efficient of x^2 and the constant term of a quadratic equation are interchanged, then which of the following will not get changed?

(a) Sum of the roots
(b) Product of the roots
(c) Roots of the equation
(d) None of these

18. If $(x + 2)^2 = 9$ and $(y + 3)^2 = 25$, then what is the maximum value of $\frac{x}{y}$?

(a) $\frac{5}{-8}$ (b) $\frac{1}{-8}$
(c) $\frac{1}{2}$ (d) $\frac{-5}{-8}$

19. If one of the roots of the quadratic equation $2x^2 - 7x + q = 0$ is 3, what is the other root of this equation?
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

20. What is the remainder when the polynomial $x^4 - 3x^2 + 7x - 10$ is divided by $x - 2$?
(a) 12 (b) 1
(c) –1 (d) 8

21. Given that $as^4 + by^3 + cy^2 + dy + e = 0$ is a bi-quadratic equation in y and $a \neq 0$. If y_1, y_2, y_3 and y_4 are the roots of the equation, what is the value of $(1 - y_1)(1 - y_2)(1 - y_3)(1 - y_4)$?
(a) 1
(b) $\frac{a+b+c+d+e}{a}$
(c) 0
(d) $a + b + c + d + e$

22. If $(x + 2)$ is a factor of $x^4 - 4x^2 + 2ax + 3 = 0$, then what is the value of a?
(a) 0 (b) 1
(c) $\frac{1}{2}$ (d) None of these

23. In writing a quadratic equation of the form $x^2 + bx + c = 0$, a student writes the coefficient of x incorrectly and finds the roots as 7 and 8. Another student makes a mistake in writing the constant term and gets the roots as 8 and –3. Find the correct quadratic equation.
(a) $x^2 - 5x + 56 = 0$
(b) $x^2 - 5x + 21 = 0$
(c) $x^2 - 21x + 56 = 0$
(d) $x^2 - 7x + 21 = 0$

24. $x^{3^n} + y^{3^n}$ is divisible by $x + y$ if:
(a) n is any integer ≥ 0
(b) n is an odd positive integer
(c) n is an even positive integer
(d) n is a rational number

25. Sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is ...:
(a) 0 (b) –4
(c) 4 (d) 2

ANSWER KEY

Exercise 1

1. (b)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (a)
9. (c)	**10.** (c)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (d)	**16.** (b)
17. (d)	**18.** (b)	**19.** (b)	**20.** (d)	**21.** (b)	**22.** (d)	**23.** (a)	**24.** (a)
25. (c)							

HINTS AND EXPLANATIONS

Exercise 1

1. Sum of the roots

$$= \frac{-b}{a} = \frac{(k+6)}{1} = k + 6$$

Product of the roots

$$= \frac{c}{a} = \frac{2(2k-1)}{1} = 4k - 2$$

Given that Sum of the roots $= \frac{1}{2}$ of product of roots.

$\Rightarrow \quad k + 6 = \frac{1}{2}(4k - 2)$

$\Rightarrow \quad 2k + 12 = 4k - 2$

$\Rightarrow \quad 2k = 10$

$\Rightarrow \quad k = 7$

Hence, (b) is the answer.

2. $x + y = -z$

$\Rightarrow \quad (x + y)^3 = (-z)^3$

$\Rightarrow \quad x^3 + y^3 + 3xy\,(x + y) + z^3 = 0$

So, $x^3 + y^3 + z^3 = 3xyz$.

3. Assume that price of an apple = ₹ X and price of a guava = ₹ Y

Given is $3X + 2Y = 23$

and $X + 4Y = 21$

Solving we get $Y = 4$

Hence, cost of 5 guavas = ₹ 20.

4. If the roots are 1 and 2, equation will be

$(x - 1)\,(x - 2) = 0$

$\Rightarrow \quad x^2 - 3x + 2 = 0$

Equating it with

$ax^2 + bx + c = 0$

gives $a = 1, b = -3$ and $c = 0$

Hence, option (b) is the answer.

5. Method 1:

$x = \sqrt{4 + \sqrt{4 - x}}$

$\Rightarrow \quad x^2 - 4 + \sqrt{4 - x}$

$(x^2 - 4) = \sqrt{4 - x}$

Now put the values from options.

Only 3rd option satisfies the condition.

Method 2: Alternatively, value of x will be more than 2, which is given there in only option 1 and option 3. Since it is only slightly more than 3, option (c) will be the answer.

6. Sum of the roots $= \frac{-b}{a} = \frac{-2}{3}$

Hence, option (d) is the answer.

7. Product of the roots $= \frac{c}{a} = \frac{-1}{3}$

Hence, option (a) is the answer.

8. Sum of the roots $= \frac{-b}{a} = \frac{-2}{1}$

Hence, option (a) is the answer.

9. Only option (c) $3x^2 + 2x + 5 = 0$ will have its discriminant less than zero. Hence, option (c) is the answer.

10. New equation will have the following roots = –1 and –2. Hence, sum of roots = –1 – 2 = –3. Hence, option (c) is the answer.

11. Assume father's present age = x years and son's present age = y

According to the question

$x = 8y$ and $(x + 3) = 5\,(y + 3)$

$\Rightarrow \quad x - 5y = 12$

Solving the equations give

$y = 4$ and $x = 32$

Hence, option (b) is the answer.

12. $-x^2 + 6x - 8 > 0$ can be written as

$x^2 - 6x + 8 < 0$

$\Rightarrow \quad (x - 2)\,(x - 4) < 0$

$\Rightarrow \quad 2 < x < 4$

Hence, option (b) is the answer.

13. Equations will not have a solution if ratio of coefficients of x = ratio of coefficients of y

$\Rightarrow \quad \frac{2}{4} = \frac{3}{k}$

$\Rightarrow \quad k = 6$

Hence, option (c) is the answer.

15. Sum of the roots

$$A + B = \frac{-b}{a} = 13 \qquad \text{(i)}$$

Product of the roots

$$AB = \frac{c}{a} = N$$

Given that $A - B = 3$ (ii)

Solving (i) and (ii) gives: $A = 8$, $B = 5$

Hence, $N = AB = 8 \times 5 = 40$. Hence, option (d) is the answer.

16. $\frac{9}{x}$ is the inverse of $\frac{x}{9}$. Hence, maximum value = –2. Hence, option (b) is the answer.

17. There is no necessity that any of the options will not change. It might change or might not change depending upon the values of a and c. Hence, answer is option (d) None of these.

18. $(x + 2)^2 = 9$

$\Rightarrow$ so $(x + 2)$ = either + 3 or –3

$\Rightarrow$ x = 1 or –5

Similarly,

$(y + 3)^2 = 25$

$\Rightarrow$ so $(y + 3)$ = either +5 or –5

$\Rightarrow$ $y = 2$ or –8

$$\frac{x}{y} = \text{either } \frac{1}{2} \text{ or } \frac{-5}{2} \text{ or } \frac{-5}{-8}$$

We'll have to choose maximum value out of the four given fractions above.

It can be seen that $\frac{-5}{-8} = \frac{5}{8}$ is the maximum among all these fractions. Hence, option (d) is the answer.

20. Putting $x = 2$ in the original equation will give the answer.

Remainder obtained

$$= 2^4 - 3 \times 2^2 + 7 \times 2 - 10$$
$$= 16 - 12 + 14 - 10 = 8$$

Hence, option (d) is the answer.

21. y_1, y_2, y_3 and y_4 are the roots of the equation. So equation will be

$$a(y - y_1)(y - y_2)(y - y_3)(y - y_4)$$
$$= ay_4 + by_3 + cy_2 + dy + e$$

(original equation)

Putting $y = 1$

$$a(1 - y_1)(1 - y_2)(1 - y_3)(1 - y_4)$$
$$= a + b + c + d + e$$

Hence, $(1 - y_1)(1 - y_2)(1 - y_3)(1 - y_4)$

$$= \frac{a + b + c + d + e}{a}$$

Hence, option (b) is the answer.

22. Putting $x = -2$ in the equation

$(-2)^4 - 4(-2)^2 + 2a(-2) + 3 = 0$

$\Rightarrow$ $-4a + 3 = 0$

$\Rightarrow$ $a = \frac{3}{4}$

Hence, option (d) is the answer.

23. Best way to solve this question is to look at the options and keep eliminating it:

Point (1) A student writes the coefficient of x incorrectly and finds the roots as 7 and 8 $\Rightarrow$ So his constant term was right. For him the equation would be $(x - 7)(x - 8) = 0 \Rightarrow$ Constant term = 56. So we can eliminate option (b) and option (d) and we are left out with option (a) and option (c).

Point (2) Another student makes a mistake in writing the constant term and gets the roots as 8 and –3. $\Rightarrow$ So his co-efficient of x was right. For him the equation would be $(x - 8)(x + 3) = 0 \Rightarrow$ Co-efficient of $x = -5$. So we can eliminate option (c). Hence option (a) is the answer.

24. $x^n + y^n$ is divisible by $(x + y)$ if n is an odd number. Hence option (a) is true.

Note: In this question, in fact option (b) and option (c) are also true, but option (a) provides the largest set. Hence, we will mark option (a) as the answer.

25. Assume $|x - 2| = z$

$|x - 2|^2 + |x - 2| - 2 = 0$

$\Rightarrow$ $z^2 + z - 2 = 0$

$\Rightarrow$ $(z - 1)(z + 2) = 0$

So, $z = 1$ or –2

Since z is a modulus value, it cannot be negative. Hence, $z = -2$ is not true. So the only value of $z = 1$

$|x - 2| = 1$

$\Rightarrow$ $x - 2 = +1$ or –1

$\Rightarrow$ $x = 3$ or 1

Hence, sum of real roots = 3 + 1 = 4. Hence, option (c) is the answer.

11 Logarithm

INTRODUCTION

If any number N is expressed in the form a^x then the index 'X' is called the logarithm of the number N to the base 'a'.

Thus, If $N = a^x \Rightarrow$ Then $X = \log_a N$. It is read as Log of N to the base a.

A simple way to recollect the formula (though not exactly mathematical):

$$N = a^x \qquad\qquad N = a^x$$

$$x = \log_a N \qquad\qquad x = \log_a N$$

1. Base of index will become the base of log.
2. Left hand number will go on the right hand side and vice-versa.

For example, $10^2 = 100$ will be written in logarithm format as follows:

$$2 = \log_{10} 100$$

1. (i) Write the following in the index format:

$$x = \log_5 6.$$

Solution

Base of log will become the log of index and left hand side number (x) will go the right hand side and vice-versa.

$$x = \log_5 6$$

$$\Rightarrow 6 = 5^x$$

(ii) Write the following in the logarithm format:

$$3^4 = 81.$$

Solution

Base of index will become the base of logarithm and left hand side number will go to right hand side number and vice-versa.

$$3^4 = 81$$

$$\Rightarrow \log_3 81 = 4$$

(iii) Write the following in the index format:

$$x = \log_y z$$

Solution

$$y^x = z$$

(iv) Write the following in the logarithm format:

$$a^b = c$$

Solution

$$b = log_a c$$

Generally logarithm of any number is calculated to the base 10. When base is not mentioned, it should be taken as 10.

Restrictions with Logarithm of any Number

For logarithm of any number to be defined, number should be greater than zero and base should be positive and not equal to 1.

$\Rightarrow$ $\log_a x$ to be defined, $x > 0$ and $a > 0$ and $a \neq 1$.

$\Rightarrow$ log of negative number is not defined. For example log (–10) is not defined.

$\Rightarrow$ log to the base any negative number or log to the base = 1 is not defined. For example, $\log_{(-5)} x$ is not defined. Similarly, $\log_1 x$ is not defined.

It can be seen with the help of graph of log x also (given alongside).

Following observations can be made from this graph:

1. Value of y can be negative for some value of x.
2. Value of x cannot be negative in any case.
3. For constant x, if base is lying in between 0 and 1, then log x becomes decreasing function. Otherwise it is an increasing function (Fig. 11.1).

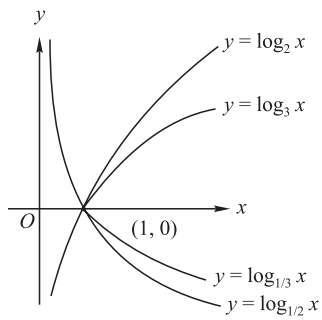


Fig. 11.1

Some Important Formulas

In case of all the following formulas given ahead, we will be using the standard restrictions on logarithm.

1. $\log_a (XY) = \log_a X + \log_a Y$

   If we don't use the restrictions given regarding log of any number, then we can see a good number of contradictions about numbers. One of the examples of similar nature can be seen here:

   $$\log (12) = \log (-4 \times -3)$$
   $$= \log (-4) + \log (-3)$$

   Now on the left hand side, we have a defined value, but on the right hand side we get a value which is not defined.

   It is all owing to the fact that

   $$\log_a (XY) = \log_a X + \log_a Y$$

   is possible only if $X > 0$ and $Y > 0$.
2. $\log_a (X/Y) = \log_a X - \log_a Y$
3. (a) $\log_a (X^k) = k \log_a X$,

   (b) $\log_a \sqrt[k]{X} = \frac{1}{k} \cdot \log_a x$
4. $\log_a 1 = 0$ (as $a^0 = 1$ provided $a \neq 1$)
5. $\log_x X = 1$
6. $\log_a X = \frac{1}{\log_x a}$
7. Base change rule

   $$\log_a x = \log_a b \times \log_b x$$
   $$= \log_b x \times \log_a b$$
8. $a^{(\log_a x)} = x$
9. If $a > 1$ and $x > a$, then $\log_a x > 0$
10. Increasing and Decreasing nature of logarithm function:

    (i) If $0 <$ base < 1

    If $x > y$, then $\log_a x < \log_a y$

    For example: $\log_{0.5} 10 < \log_{0.5} 5$

    (ii) If base (assume $= a$) > 1

    If $x > y$, then $\log_a x > \log_a y$

    For example: $\log_5 10 > \log_5 5$
11. $\log_a 1 = 0$
12. $\log_a b = \frac{\log b}{\log a}$

Characteristics and Mantissa

The integral part of logarithm is called Characteristic and its decimal part is called Mantissa. Logarithms to the base 10 are called Common logarithms. The Characteristic of common logarithm can be found out by visual inspection. The Characteristics of the logarithm (base 10) of a number greater than 1 is less by one than the number of digits in the integral part and is positive. However if a decimal fraction number is less than 1 but positive, its characteristic will greater by unity than the number of consecutives zeros immediately after the decimal point and is negative.

Example 1

What is the value of $\log_{125} 625$?

Solution

$$\log_{125} 625 = \log_{5^3} 5^4 = 4 \cdot \frac{1}{3}(\log_5 5)$$
$$= \frac{4}{3} \cdot 1 = \frac{4}{3}$$

Example 2

What is the value of
$\log_3 2 . \log_4 3 . \log_5 4 \ldots \log_{16} 15$?

Solution

$$\log_3 2 = \frac{\log 2}{\log 3};$$

$$\log_4 3 = \frac{\log 3}{\log 4}, \text{ and so on ...}$$

$$\log_3 2 . \log_4 3 . \log_5 4 \ldots \log_{16} 15$$
$$= \frac{\log 2}{\log 3} \cdot \frac{\log 3}{\log 4} \ldots$$
$$\frac{\log 15}{\log 16} = \frac{\log 2}{\log 16} = \log_{16} 2$$
$$\Rightarrow \quad \log_{16} 2 = \log_{2^4} 2 = \frac{1}{4}(\log_2 2)$$
$$= \frac{1}{4} \cdot 1 = \frac{1}{4}$$

Example 3

What is the value of x in the following expression?

$$x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$$

(a) 1 (b) 0
(c) –1 (d) 3

Solution

Ideally these questions should be solved using the options.

Checking option (a), putting $x = 1$

$$\text{LHS} = 1 + \log_{10}(1 + 2^1) = 1 + \log_{10}(3)$$
$$\text{RHS} = \log_{10} 5 + \log_{10} 6 = \log_{10}(6 \times 5)$$
$$= \log_{10} 30 = \log_{10} 10 + \log_{10} 3$$
$$= 1 + \log_{10}(3)$$
$$\text{LHS} = \text{RHS}$$

Hence, option (a) is the answer.

Note:

(i) If you get plus sign (+) in any logarithm question, you should try to use formula number (1) as mentioned previously and vice-versa
[$\log_a(XY) = \log_a X + \log_a Y$].

(ii) If you get minus sign (–) in any logarithm question, you should try to use formula number (2) as mentioned previously and vice-versa
[$\log_a(X/Y) = \log_a X - \log_a Y$].

Example 4

What is the number of digits in 2^{50} (Given that log 2 = 0.301)?

Solution

Taking log gives us 50 log 2 = 50 × 0.301 = 15.05

Number of digits will be immediate next integer if the result obtained is not an integer. In this case, we obtain, product = 15.05, hence, number of digits = 16.

Example 5

What is the number of digits in 3^{60} (Given that log 2 = 0.477)?

Solution

Taking log gives us 60 log 3 = 60 × 0.477 = 28.62

Number of digits will be immediate next integer if the result obtained is not an integer. In this case, we obtain, product = 28.62, hence, number of digits = 29.

Example 6

If $5 \log_{27}(y) + 2 \log_9(81y^2) = 20$, then y is equal to:

(a) 1/7 (b) 81
(c) 729 (d) 243

Solution

$$5 \log_{27}(y) + 2 \log_9(81y)$$
$$= \frac{5}{3}\log_3(y) + \frac{2}{2}\log_3(81y)$$
$$= \frac{5}{3}\log_3(y) + \log_3(81y)$$
$$= \frac{5}{3}\log_3(y) + \log_3(81) + \log_3(y)$$

$$= \left(\frac{5}{3} + 1\right)\log_3(y) + \log_3(81)$$

...[taking ($\log_3 y$) common]

$$= \frac{8}{3}\log_3 y + 4\log_3 3$$

$$= \frac{8}{3}\log_3 y + 4$$

Given that this value = 20.

So, $\frac{8}{3}\log_3 y + 4 = 20$

$\Rightarrow$ $\frac{8}{3}\log_3 y = 20 - 4 = 16$

$\Rightarrow$ $\log_3 y = 6 \Rightarrow y = 3^6 = 729.$

Hence, option (c) is the answer.

Example 7

If $x \geq y$ and $y > 1$, then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be equal to:

(a) –1 (b) –0.5
(c) 0 (d) 1

Solution

Assume $N = \log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$

$$= \log_x x - \log_x y + \log_y y - \log_y x$$
$$= 1 - \log_x y + 1 - \log_y x$$
$$= 2 - \log_x y - \log_y x$$

Assume that $\log_x y = t$

$$N = 2 - \frac{1}{t} - t = -\left(\frac{1}{t} + t - 2\right)$$

$$= -\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2$$

$$= -\text{(Perfect square)}$$

Minimum value of a perfect square = 0. Since negative of a perfect square cannot be equal to a positive value $\Rightarrow$ can never be equal to 1. Hence, option (d) is the answer.

Logarithmic Inequality

Case (1): If base (Assume = N) > 1

(i) If $x > y$, then $\log_N x > \log_N y$.
(ii) Vice-versa of above rule is also true. *i.e.*, if $\log_N x > \log_N y \Rightarrow x > y$.

Case (2): If base = N is $0 < N < 1$

(i) If $x > y$, then $\log_N x < \log_N y$.
(ii) Vice-versa of above rule is also true. *i.e.*, if $\log_N x > \log_N y \Rightarrow x < y$.

Note: For base > 1, log is an increasing function. For base between 0 and 1, log is a decreasing function.

Rules for Solving Questions Based upon Logarithmic Inequality

In addition to the rules written above, we need to take care of the following also:

For $\log_N x$ to be defined:

1. $N > 0$, and $N \neq 0$, $N \neq 1$.
2. $x > 0$.

Example 8

Solve the following equations for x:

$\log_{10} x + \log_{10}(x - 1) < \log_{10} 6.$

Solution

Base > 1, hence, given logarithmic arguments will be in increasing order.

As the base is greater than 1,

$\Rightarrow$ $x(x - 1) < 6$

or, $x^2 - x - 6 < 0$

$\Rightarrow$ $(x - 3)(x + 2) < 0$

Hence, range of value of x

$= -2 < x < 3$ (i)

Let us mark this on number line (Fig. 11.2):

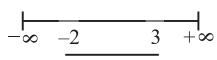


Fig. 11.2

Next, for log to be defined:

$x > 0$ and $x - 1 > 0$

$\Rightarrow$ $x > 1$ (ii)

Again mark this on number line (Fig. 11.3):

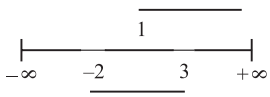


Fig. 11.3

It can be seen that the circled area (as shown in Fig. 11.4) is common and is the answer:

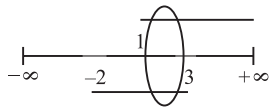


Fig. 11.4

Hence, following is the answer:

$1 < x < 3.$

PRACTICE EXERCISE

EXERCISE 1

1. What is the value of

$$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{40} n}?$$

(a) $\dfrac{1}{\log_{40!} n}$ (b) $\log_n 40!$

(c) 1 (d) 0

2. What is the number of digits in the expansion of 5^{20} given that $\log 5 = 0.698$?

(a) 12 (b) 13

(c) 14 (d) 15

3. What is the value of $\log_{3\sqrt{3}} 27$?

(a) 2 (b) 3

(c) 4 (d) 5

4. Evaluate $\log\ (36\sqrt{6})$ to the base 6.

(a) 1/2 (b) 5/2

(c) 3/2 (d) 7/2

5. Find the value of:

$$3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$$

(a) log 2 (b) log 3

(c) 1 (d) None of these

6. If $[N]$ = the greatest integer less than or equal to N, then $[\log_{10} 6730.4]$ is equal to:

(a) 6 (b) 4

(c) 3 (d) 7

7. What is the value of x in the following expression:

$\log_7 \log_5 [\sqrt{(x+5)} + \sqrt{x}] = 0$?

(a) 1 (b) 2

(c) 3 (d) 4

8. Arrange the following in ascending order:

$A = \log_7 2401$ $B = \log_{7\sqrt{7}} 343$

$C = \log_{\sqrt{6}} 216$ $D = \log_2 32$

(a) $ABCD$ (b) $BDCA$

(c) $BDAD$ (d) $BADC$

9. Given that x, y and z are three angles of a triangle. Which of the following will be one possible set of value for x, y and z such that $\log (x \times y \times z) = 3 \log X + 4 \log 2$ (x, y and z are integers)?

(a) 30°, 70°, 80° (b) 30°, 60°, 90°

(c) 20°, 80°, 80° (d) 30°, 50°, 100°

10. If $2\ [\log (x + y) - \log 5] = \log x + \log y$, then what is the value of $x^2 + y^2$?

(a) $20xy$ (b) $23xy$

(c) $25xy$ (d) $28xy$

11. What is the value of $\log_{32} 27 \times \log_{243} 8$?

(a) $\dfrac{\log 9}{\log 4}$ (b) $\dfrac{\log 3}{\log 2}$

(c) 1 (d) None of these

12. If $x > 1$, $y > 1$, $z > 1$ are three numbers in Geometric Progression then,

$$\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z} \text{ are in:}$$

(a) Arithmetic Progression
(b) Harmonic Progression
(c) Geometric Progression
(d) None of these

13. If $\log_k N = 6$, and $\log_{25k}(8N) = 3$, then k is:
(a) 12.5 (b) $(12.5)^2$
(c) $\frac{5^{\frac{2}{3}}}{2}$ (d) $(12.5)^{\frac{2}{3}}$

14. What is the value of x if:
$\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \frac{25}{4}$?
(a) 9 (b) 27
(c) 81 (d) None of these

15. What is the value of x in the following expression:
$\log_2(3 - x) + \log_2(1 - x) = 3$?
(a) 1 (b) 0
(c) –1 (d) Not possible

16. The logarithm of a number to a certain base is 9. The logarithm of 64 times the number to a base which is 11 times the original base is 6. Find the actual base.
(a) $\frac{11}{2}$
(b) $\frac{121}{2}$
(c) $\frac{121}{4}$
(d) Cannot be determined

17. If $\log N^2 - \log 2N = 3 \log 3 - \log 6$, then what is the value of N?
(a) 1
(b) 2
(c) 9
(d) More than one value

18. If $x = \log_a(bc)$, $y = \log_b(ca)$ and $z = \log_c(ab)$ then what is the value of:
$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}?$$
(a) 0 (b) 1
(c) xyz (d) –1

19. If $\log_4 5 = a$ and $\log_5 6 = b$ then what is the value of $\log_3 2$?
(a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$
(c) $2ab + 1$ (d) $\frac{1}{2ab-1}$

20. The number of solutions of
$\log_2(x + 5) = 6 - x$ is:
(a) 2 (b) 0
(c) 3 (d) 1

ANSWER KEY

Exercise 1

1. (a)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (d)
9. (a)	**10.** (c)	**11.** (b)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (c)
17. (c)	**18.** (b)	**19.** (d)	**20.** (d)				

HINTS AND EXPLANATIONS

Exercise 1

1. Given expression can be written as:

$\log_n 2 + \log_n 3 + \log_n 4 + ... + \log_n 40$
$= \log_n (2 \times 3 \times 4 \times ... 40)$
$= \log_n 40!$

It is equal to $\dfrac{1}{\log_{40!} n}$.

Hence, option (a) is the answer.

2. $5 \log 20 = 5 \times 0.698 = 13.96$. Hence, number of digits = 14. Hence, option (c) is the answer.

3. $\log_{3\sqrt{3}} 27 = \log_{3\sqrt{3}} (3\sqrt{3})^2$
$= 2 \cdot \log_{3\sqrt{3}} (3\sqrt{3}) = 2$

Hence, option (a) is the answer.

4. $\log_6 36\sqrt{6} = \log_6 6^{2.5}$
$= 2.5(\log_6 6) = 2.5$

Hence, option (b) is the answer.

5. $3\log\dfrac{81}{80} = \log\left[\dfrac{81}{80}\right]^3$;

$5\log\dfrac{25}{24} = \log\left[\dfrac{25}{24}\right]^5$;

$7\log\dfrac{16}{15} = \log\left[\dfrac{16}{15}\right]^7$

So $3\log\dfrac{81}{80} + 5\log\dfrac{25}{24} + 7\log\dfrac{16}{15}$

$= \log\left(\left[\dfrac{81}{80}\right]^3 \times \left[\dfrac{25}{24}\right]^5 \times \left[\dfrac{16}{15}\right]^7\right)$

$= \log 2$.

Hence, option (a) is the answer.

6. Assume that the value of $\log_{10} 6730.4 = z$

It can be seen that $1000 < 6730.4 < 10000$.

Hence, $\log_{10}(10^3) < \log_{10} 6730.4 < \log_{10}(10^4)$

Taking antilog, $3 < z < 4$. So value of z lies in between 3 and 4.

So, $[z]$ = Greatest Integer less than or equal to $z = 3$. Hence, option (c) is the answer.

7. In these questions, it is always advisable to go through the options.

$\log_7 \log_5 [\sqrt{(x+5)} + \sqrt{x}] = 0$

Since one of the components in the question is $\sqrt{(x+5)}$ and $\sqrt{x}$ it is advisable to start with an option that will give us the square root.

Let us start with option (d). Putting $x = 4$.

$\log_7 \log_5 [\sqrt{(4+5)} + \sqrt{4}]$

$= \log_7 \log_5 [\sqrt{(9} + \sqrt{4})]$

$= \log_7 \log_5 5 = \log_7 1 = 0$

Hence, option (d) is the answer.

8. $A = \log_7 2401 = \log_7 7^4$
$= A = 4 \log_7 7 = 4$
$B = \log_{7\sqrt{7}} 343$
$= \log_{7\sqrt{7}} 7\sqrt{7}^2 = 2$
$C = \log_{\sqrt{6}} 216 = 9$
$D = \log_2 32 = 5$

Hence, the Ascending order is: $BADC$. Hence, option (d) is the answer.

9. Method 1:

RHS $= 3 \log X + 4 \log 2 = \log (x^3 \times 2^4)$
$= \log (16x^3)$

Let us go through the options:

Checking option (a):

LHS $= \log (x \times y \times z)$
$= \log (30 \times 70 \times 80)$
$= \log (168 \times 1000)$

RHS $= \log (16x^3)$
$= \log (16 \times 30 \times 30 \times 30)$

It can be seen that LHS $\neq$ RHS

Similarly option (b) also will not satisfy.

Checking option (c):

LHS = log $(x \times y \times z)$
= log $(20 \times 80 \times 80)$
= log $(128 \times 1{,}000)$

RHS = log (16×20^3) = log $(128 \times 1{,}000)$

LHS = RHS.

Hence, option (c) is the answer.

Method 2: It can be seen that RHS has a term in x, but not in y and z. It means y and z can be used interchangeably $\Rightarrow y$ and z are equal. That is given only in option (c). Hence, option (c) is the answer,

10. $2\,[\log(x+y) - \log 5] = \log x + \log y$ can be written as:

$$[\log(x+y) - \log 5] = \frac{1}{2}(\log x + \log y)$$

or, $$\log\frac{x+y}{5} = \log\sqrt{xy}$$

Taking antilog both the sides, we obtain the following:

$$\frac{x+y}{5} = \sqrt{xy} \Rightarrow x + y = 5\sqrt{xy}$$

Squaring both the sides,

$$(x+y)^2 = 25xy \Rightarrow x^2 + y^2 + 2xy$$
$$= 25xy \Rightarrow x^2 + y^2 = 23xy.$$

Hence, option (b) is the answer.

11. $$\log_{32} 27 = \log_{2^5} 3^3 = \frac{3}{5}\log_2 3$$

$$\log_{243} 8 = \log_{3^5} 2^3 = \frac{3}{5}\log_3 2$$

Hence, $\log_{32} 27 \times \log_{243} 8$

$$= \frac{3}{5}\log_2 3 \cdot \frac{3}{5}\log_3 2 = \frac{9}{25}$$

Hence, option (d) is the answer.

12. In these questions, it is better to assume values and verify the options.

Assume $x = 10$, $y = 100$ and $z = 1000$.

$$\frac{1}{1+\log x} = \frac{1}{1+\log 10} = \frac{1}{1+1} = \frac{1}{2}$$

$$\frac{1}{1+\log y} = \frac{1}{1+\log 100} = \frac{1}{1+2} = \frac{1}{3}$$

$$\frac{1}{1+\log z} = \frac{1}{1+\log 1000} = \frac{1}{1+3} = \frac{1}{4}$$

Now, it can be clearly seen that 1/2, 1/3, 1/4 are in Harmonic Progression. Hence, option (b) is the answer.

13. $$\log_k N = 6 \Rightarrow N = k^6 \quad \text{(i)}$$

Similarly, $\log_{25k}(8N) = 3$

$$\Rightarrow \quad (8N) = (25k)^3 \quad \text{(ii)}$$

Dividing (i) by (ii):

$$\frac{N}{8N} = \frac{k^6}{(25k)^3}$$

$$k = \frac{25}{2} = 12.5$$

Hence, option (a) is the answer.

14. Go through the options.

15. Go through the options.

It can be seen that $x = -1$ satisfies the equations. Hence, option (c) is the answer.

16. Assume $\log_n y = 9 \Rightarrow y = n^9$ (i)

Given that $\log_{11n} 64y = 6 \Rightarrow 64y = (11n)^6$ (ii)

Putting the value of y from equation (i) in equation (ii):

$$64 \times n^9 = 11^6 \times n^6$$

$$\Rightarrow \quad n^3 = \frac{11^6}{2^6} \Rightarrow n = \frac{11^2}{2^2} = \frac{121}{4}$$

Hence, option (c) is the answer.

17. Again, in this question, we have to find out the value of N. It is better to go through the options.

$N = 9$ satisfies the equation. Hence, option (c) is the answer.

18. $$1 + x = \log_a a + \log_a (bc)$$
$$= \log_a (abc)$$

Similarly, $1 + y = \log_b (abc)$

and $1 + z = \log_c (abc)$

$$\frac{1}{1+x} = \log_{abc} a$$

$$\frac{1}{1+y} = \log_{abc} b$$

$$\frac{1}{1+z} = \log_{abc} c$$

$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc} abc = 1$$

Hence, option (c) is the answer.

19. $$ab = \log_4 5 \times \log_5 6 = \log_4 6$$

$$= \frac{1}{2}\log_2 6$$

$$= \frac{1}{2}(\log_2 2 + \log_2 3)$$

$$= \frac{1}{2}(1 + \log_2 3)$$

$$\Rightarrow \quad 2ab = (1 + \log_2 3)$$

$$\Rightarrow \quad \log_2 3 = 2ab - 1$$

$$\Rightarrow \quad \log_3 2 = \frac{1}{2ab - 1}$$

Hence, option (d) is the answer.

20. Graph of $(6 - x)$ will be straight line like this (Fig. 11.5):

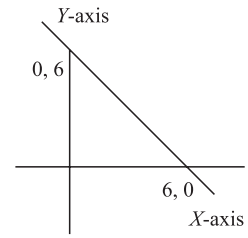


Fig. 11.5

Whereas graph of $\log_2 (x + 5)$ will be curve moving upwards.

Hence, they will intersect at only one point. Hence, option (d) is the answer.

12 Geometry

In Geometry, we will go through the following:

1. Line Angles and Polygons
2. Triangles
3. Circles
4. Quadrilaterals

LINES ANGLES AND POLYGONS

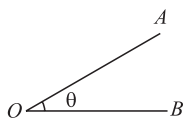
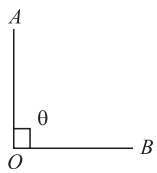
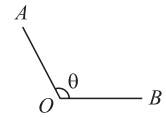
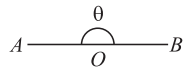
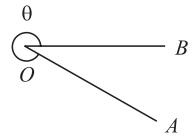
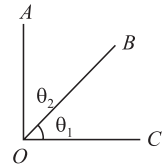
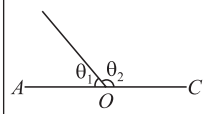
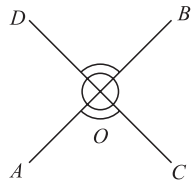
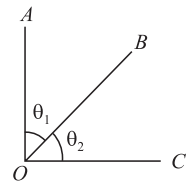
At the onset, let us get ourselves acquainted with the basic terms:

Different Angles and Pair of Angles

Table 12.1 Measurement and nomenclature

Value	Less than 90°	Equal to 90°	Greater than 90° but less than 180°	Greater than 180° but less than 360°
Name	Acute	Right	Obtuse	Reflex

Table 12.2 Types of Angles, Property and Diagram

Types of Angles	Property	Diagram
Acute Angle	$0° < \theta < 90°$ ($\angle AOB$ is an acute angle)	
Right Angle	$\theta = 90°$	
Obtuse Angle	$90° < \theta < 180°$	
Straight Line	$\theta = 180°$	
Reflex Angle	$180° < \theta < 360°$	
Complementary Angle	$\theta_1 + \theta_2 = 90°$ Two angles whose sum is 90°, are complementary to each other	
Supplementary Angle	$\theta_1 + \theta_2 = 180°$ Two angles, whose sum is 180°, are supplementary to each other.	
Vertically opposite Angle	$\angle DOA = \angle BOC$ and $\angle DOB = \angle AOC$	
Adjacent angles	$\angle AOB$ and $\angle BOC$ are adjacent angles Adjacent angles must have a common side (*e.g.*, OB)	

Angles Associated with Two or more Straight Lines

When two straight lines cross each other, $\angle d$ and $\angle b$ are pair of vertical angles. Similarly, $\angle a$ and $\angle c$ are pair of vertical angles. Vertical angles are always equal (Fig. 12.1).

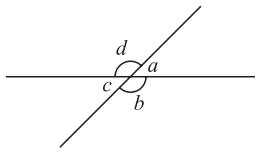


Fig. 12.1

Alternate angles and Corresponding angles

In Fig. 12.2 given below:

Corresponding angles are $\angle a$ and $\angle e$, $\angle b$ and $\angle f$, $\angle d$ and $\angle h$, $\angle c$ and $\angle g$.

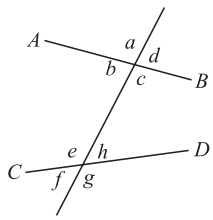


Fig. 12.2

Alternate angles are $\angle b$ and $\angle h$, $\angle c$ and $\angle e$ are some of the set of alternate angles.

Angles Associated with Parallel Lines

A line passing through two or more lines in a plane is called a transversal. When a transversal cuts two parallel lines, then set of all the corresponding angels will be equal and similarly, set of all alternate angles will be equal (Fig. 12.3).

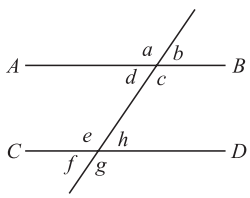


Fig. 12.3

In Fig. 12.3, corresponding $\angle a = \angle e$, corresponding $\angle b = \angle f$, corresponding $\angle d = \angle h$ and corresponding $\angle c = \angle g$.

Similarly, Alternate $\angle b = \angle h$ and alternate $\angle c = \angle e$.

Now , $\angle b + \angle c = 180^o$,

so $\angle b + \angle e = \angle h + \angle c = 180^o$

So we can conclude that sum of angles contained on one side of transversal and between parallel lines will be equal to 180°.

Converse of above theorem is also true: When a transversal cuts two lines, if corresponding angles are equal in size, or if alternate angles are equal in size, then two lines are parallel.

Example 1

In Fig. 12.4, find the value of $\angle b$ in terms of $\angle a$.

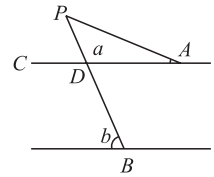


Fig. 12.4

Solution

In Fig. 12.4, $\angle b$ = Alternate $\angle PDC = 180^o - \angle PDA = 180^o - a$

POLYGONS AND THEIR PROPERTIES

Any closed plane figure with n sides is known as polygon. If all sides and angles of this polygon are equivalent, the polygon is called Regular Polygon. Polygons can be Convex or Concave. The word "polygon" derives from the Greek *poly*, meaning "many," and *gonia*, meaning "angle." The most familiar type of polygon is the Regular Polygon, which is a Convex Polygon with equal side lengths and angles.

Every triangle is strictly a convex polygon.

Following is an example of Convex Polygon (Fig. 12.5): If a simple polygon is not

convex, it is called concave. At least one internal angle of a concave polygon is larger than 180 degrees.

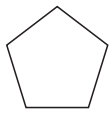


Fig. 12.5

Following is an example of Concave Polygon (Fig. 12.6): As we can see in the Fig. 12.6, one of the internal angles is more than 180°.

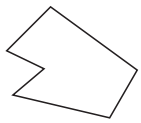


Fig. 12.6

Here onwards, whatever we are discussing vis-s-vis to polygons, it is about Regular Polygons only.

Polygons are named on the basis of the number of sides that they have. A list of some of the polygons is given in Table 12.3.

Table 12.3 List of Polygons

Number of sides	Name of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Area and Perimeter of a Regular Polygon

See Fig. 12.7.

Given: $A_1A_2A_3A_4 \ldots A_n$ is a 'n' sides regular polygon.

$$A_1A_2 = A_2A_3 = A_3A_4 = \ldots$$
$$= A_{(n-1)}A_n = \text{a units}$$

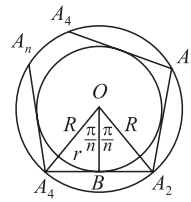


Fig. 12.7

OB (in radius) $= r$

and $OA_1 = OA_2$ (Circum radius) $= R$

1. Perimeter $(P) = na$
2. Area $= \dfrac{na}{2} \cdot r = \dfrac{p}{2} \cdot r$
3. Area $= \dfrac{na^2}{4} \times \cot\left(\dfrac{\pi}{n}\right)^{\circ}$

Other Properties of Polygon

1. Sum of all the interior angles of n sided polygon $= (2n - 4) \times 90^{\circ}$
2. Measurement of One interior angle of n sided regular polygon $= \dfrac{(2n-4) \times 90}{n}$

   A better way of finding out the value of each interior angle of any regular polygon is to subtract $[360^{\circ}/n]$ from 180°. The principle behind this is same—Sum of all the angles on any straight line on one side $= 180^{\circ}$
3. One Interior Angle + One Exterior Angle $= 180°$
4. Number of diagonals in an n-sided polygon $= \dfrac{n(n-3)}{2}$
5. Sum total of all the exteriors angles of any polygon $= 360^{\circ}$
6. Measure of each exterior angle of a regular polygon $= \dfrac{360°}{n}$
7. The ratio of sides of a polygon to the diagonals of a polygon is $2:(n-3)$
8. The ratio of interior angle of a regular polygon to its exterior angle is $(n-2):2$

Example 2

If $PQRSTUVW$ is a regular octagon, then what is the measure (in degrees) of $\angle WPQ - \angle PVQ$ (Fig. 12.8)?

(a) 135 (b) 22.5
(c) 112.5 (d) 37.5

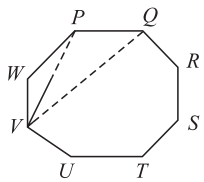


Fig. 12.8

Solution

$\angle WPQ$ = Interior angle
= 135°

In ΔWVP,

$\angle WVP = \angle WPV$

and $\angle VWP = 135°$

So, $\angle WVP = 45/2$
$= 22.5°$

So, $\angle PVQ = 45 - \angle WVP$
$= 22.5°$

Hence, $\angle WPQ - \angle PVQ = 112.5°$.

Example 3

The exterior angle of a regular polygon is 1/3rd of its interior angle. How many sides does the polygon have?

Solution

Assume that value of exterior angle = P, then value of interior angle = $3P$

Now $P + 3P = 180°$

So, $P = 45°$
= Exterior angle of Polygon

So, number of sides of polygon

$$= \frac{360}{45} = 8$$

Important Deductions

See Table 12.4.

Table 12.4

No. of sides of a regular polygon	No. of Diagonals	Each interior angle	Each exterior angle
3	0	60	120
4	2	90	90
5	5	108	72
6	9	120	60
7	14	$128\frac{4}{7}$	$51\frac{3}{7}$
8	20	135	45
9	27	140	40
10	35	144	36

Remember

1. For a fixed perimeter, area of a polygon with higher number of sides will always be more than the area of a polygon of lesser sides.

   So, if an Equilateral Triangle, Square, Regular Pentagon, Regular Hexagon have same perimeter, then following will be ascending order of area: Equilateral Triangle < Square < Regular Pentagon < Regular Hexagon.
2. However if circumference of a circle is same as perimeter of a regular polygon of n sides, then area of circle will be always more than the area of regular polygon of n sides, for any value of n.
3. For any fixed area, perimeter of a regular polygon with lesser number of sides will always be more than a regular polygon of higher number of sides.
4. For any fixed area, if perimeter of a regular polygon will be always more than circumference of a circle.

Example 4

The length of circumference of a circle equals the perimeter of an equilateral triangle and also the perimeter of a square.The areas covered by the circle, triangle and square are c, t and s, respectively, then:

(a) $s > t > c$ (b) $c < s < t$
(c) $c > s > t$ (d) $s > c > t$

Solution

Taking a cue from the points given above, order is $c > s > t$.

Example 5

Consider the Fig. 12.9:

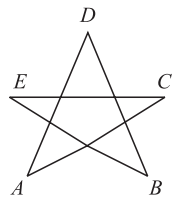


Fig. 12.9

Find the sum of $\angle A + \angle B + \angle C + \angle D + \angle E$.

Solution

In ΔABD (Fig. 12.10),

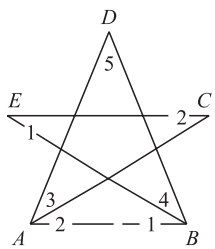


Fig. 12.10

$$1 + 2 + 3 + 4 + 5 = 180^{o}$$

Some Frequently Used Polygons

Apart from Triangles and Quadrilaterals, Regular Hexagon and regular Octagon are worth mentioning.

Regular Hexagon

In Fig. 12.11 *ABCDEF* is a regular hexagon with each side measuring a units point *O* inside the hexagon is the centre of the hexagon.

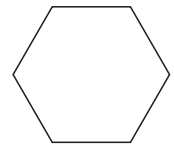


Fig. 12.11

Sum of interior angles = 720°
Each interior angle = 120°
Each exterior angle = 60°
Area $= \frac{3\sqrt{3}}{2} a^2$

Regular Octagon

In Fig. 12.12 alongwith *ABCDEFGH* is a regular octagon with each side measuring '*a*' units.

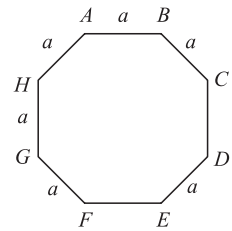


Fig. 12.12

Sum of interior angles = 1080°
Each interior angle = 135°
Each exterior angle = 45°
Area $= 2a^2(1 + \sqrt{2})$

TRIANGLE

A triangle is a figure enclosed by three sides. In Fig. 12.13, *ABC* is a triangle with sides *AB*, *BC* and *CA* measuring *c*, *a* and *b* units respectively. Line *AD* represents the height of the triangle corresponding to the side *BC* and is denoted by *h*.

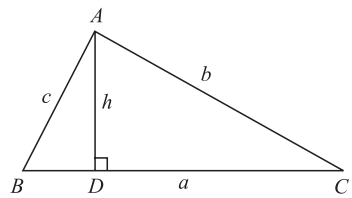


Fig. 12.13

In any triangle ABC

Area = 1/2 × base × corresponding height

$$= \frac{1}{2} \cdot BC \cdot AD = \frac{1}{2} a \cdot h.$$

Important Points

1. Sum of all the angles of a triangle = 180°
2. Sum of lengths of two sides > Length of third side
3. Difference of any two sides of any triangle < Length of the third side
4. Area of any triangle can be found by several methods:

   (i) Area of any triangle $= \frac{1}{2} \times$ base × perpendicular to base from opposite vertex.

   (ii) Area of any triangle $= \sqrt{S(S-a)(S-b)(S-c)}$, where S is the semi-perimeter of the triangle and a, b and c are sides of the triangle. This is known as Heron' Formula (more commonly hero's formula).

   (iii) Area of any triangle
   $= (1/2) \times bc \sin A$
   $= (1/2) \times ab \sin C$
   $= (1/2) \times ac \sin B$

Besides, there are some formulas which we use exclusively in some particular cases. We will see the same at appropriate places through this chapter.

Example 6

What is the number of distinct triangles with integral valued sides and perimeter as 14?

(a) 6 (b) 5
(c) 4 (d) 3

Solution

Sum of lengths of two sides > Length of third side

So, maximum length of any particular side can be 6 units.

Now if $a = 6$, then $b + c = 8$, so the possible sets are (6, 6, 2), (6, 5, 3) and (6, 4, 4).

If $a = 5$, then $b + c = 9$, so the possible set is (5, 5, 4).

So, Number of distinct triangles = 4.

Example 7

Two sides of a triangle are 100 units and 200 units respectively. Find the range of length of third side.

Solution

Third side will be greater than the difference of two sides and smaller than the sum of two sides. If length of third side = x, then

$(200 - 100) < x < (200 + 100)$

$\Rightarrow \quad 100 < x < 300.$

Classification of Triangles

Table 12.5 Based upon sides

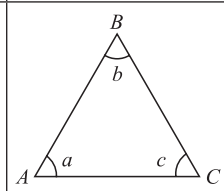
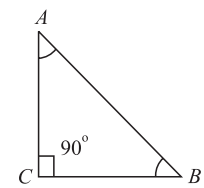
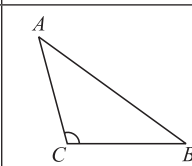
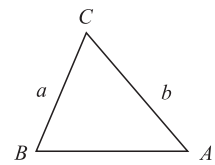
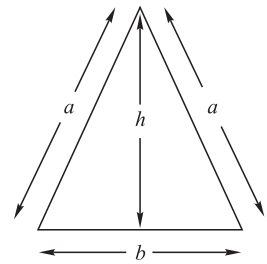
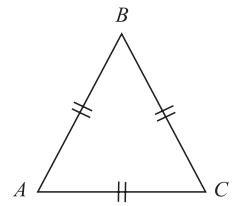
Types of Triangles	Property/ Definition	Diagram
Acute angle triangle	Each of the angle of a triangle is less than 90° *i.e.*, $a < 90°$, $b < 90°$, $c < 90°$	 {Angle a, Angle b, Angle c} < 90°
Right angled triangle	Exactly One of the angles is equal to 90°, then it is called as right angled triangle. Rest two angles are complementary to each other.	 Angle $C = 90°$
Obtuse angle triangle	Exactly One of the angles is obtuse (*i.e.*, greater than 90°), then it is called as Obtuse angle triangle.	 Angle $C > 90°$

Table 12.6 According to the length of sides

Types of Triangles	Property/Definition	Diagram
Scalene triangle	A triangle in which none of the three sides is equal is called a scalene triangle (all the three angles are also different). Area $= \sqrt{S(S-a)(S-b)(S-c)}$, S = Semi-perimeter a, b, c are the sides.	 $a \neq b \neq c$
Isosceles triangle	A triangle in which two sides are equal is called as isosceles triangle. In this triangle, the angles opposite to the congruent sides are also equal. $AB = BC = a$ and $AC = b$ Height $= \sqrt{a^2 - \frac{b^2}{4}}$ Area $= \frac{b}{4}\sqrt{4a^2 - b^2}$	 $AB = BC$ $\angle A = \angle C$
Equilateral triangle	A triangle in which all the three sides are equal is called an equilateral triangle. In this triangle each angle is congruent and equal to 60°. Height $= \frac{\sqrt{3}}{2}$ (side) $= \frac{\sqrt{3}}{2}a$ Area $= \frac{\sqrt{3}}{4}$ (side)$^2 = \frac{\sqrt{3}}{4}a^2$	 $AB = BC = AC$ $\angle A = \angle B = \angle C = 60°$

Example 8

What is the area of the triangle with side lengths 4 units, 5 units and 10 units?

Solution

This triangle is not possible (Since Sum of lengths of two sides > Length of third side).

Example 9

What is the area of the triangle with side lengths 7 units, 5 units and 6 units?

Solution

$$S = (7 + 5 + 6)/2 = 9 \text{ units}$$

$$\text{Area} = \sqrt{9(9-7)(9-5)(9-6)}$$
$$= \sqrt{9 \cdot 2 \cdot 4 \cdot 3} = 6\sqrt{6} \text{ sq units}$$

Example 10

Find the area of a triangle whose base is 14 cm and height is 10 cm.

Solution

Base = 14 cm and height = 10 cm
Hence area $= 1/2 \cdot 14 \cdot 10 = 70 \text{ cm}^2$

Pythagoras Theorem

Pythagoras Theorem is applicable in case of right-angled triangles. It says that, the square of

hypotenuse is equal to the sum of the squares of the other two sides.

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$
$$a^2 + b^2 = c^2.$$

The smallest integral example is $a = 3$, $b = 4$, and $c = 5$. You can check that

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2.$$

Sometimes we use the notation (a, b, c) to denote such a triple.

Notice that the greatest common divisor of the three numbers 3, 4, and 5 is 1. Pythagorean triplets with this property are called *primitive*.

Pythagorean Triplets

A *Pythagorean triplet* is a set of three positive whole numbers a, b, and c that are the lengths of the sides of a right triangle.

$$a^2 + b^2 = c^2.$$

Some Pythagoras triplets are:

3	4	5	$(3^2 + 4^2 = 5^2)$
5	12	13	$(5^2 + 12^2 = 13^2)$
7	24	25	$(7^2 = 24^2 = 25^2)$
8	15	17	$(8^2 + 15^2 = 17^2)$

Note: If each term of any Pythagorean triplet is multiplied or divided by a constant (say P, $P > 0$) then the triplet so obtained will also be a Pythagorean triplet. This is because if,

$a^2 + b^2 = c^2$, then $(Pa)^2 + (Pb)^2 = (Pc)^2$, where $P > 0$.

For example,

3×2	4×2	5×2	Gives
6	8	10	$(6^2 + 8^2 = 10^2)$

Example 11

Two sides of a plot measure 32 m and 24 m and the angle between them is a right angle. The other two sides measure 25 m each and the other three angles are not right angles (Fig. 12.14).

What is the area of the plot (In M^2)?

(a) 768 (b) 534
(c) 696 (d) 684

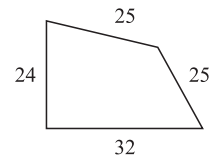


Fig. 12.14

Solution

Figure 12.14 can be seen like:

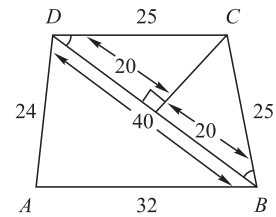


Fig. 12.15

Since ABD is a right angled triangle, so it will satisfy Pythagoras theorem. And the triplet used here is: 3 (× 8), 4 (× 8) and 5 (× 8). And similarly the other part of figure can also be bifurcated by drawing a perpendicular from C on BD.

So, area of plot is:

Area (ΔABD) + Area (ΔCBD)
$= 1/2 \times 24 \times 32 + 2 \times (1/2 \times 20 \times 15)$
$= 684\ m^2$

Two important Points

Side opposite to the largest angle will be largest.

Side opposite to the smallest angle will be smallest.

Example 12

In Fig. 12.16, $AD = CD = BC$. What is the value of $\angle CDB$?

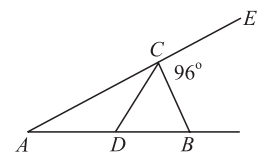


Fig. 12.16

Solution

$$x + y = 180 - 96 = 84^o \quad \text{(i)}$$

In ΔCDB,

$$4x + y = 180 \quad \text{(ii)}$$

Solving (i) and (ii),

$$x = 32^o$$

So, $2x = 64^o$

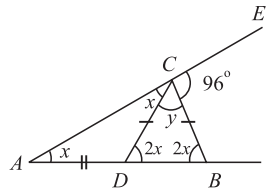
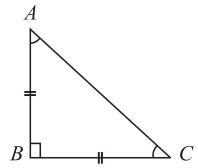
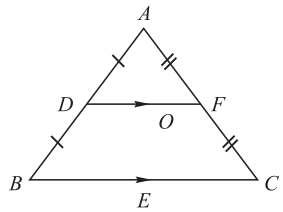
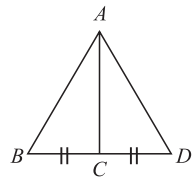
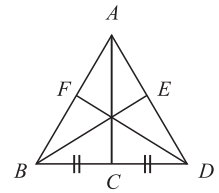
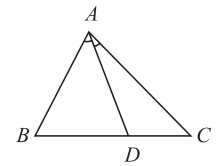
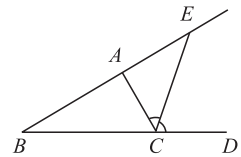


Fig. 12.17

Table 12.7 Important Theorems Related to Triangle

Theorem	Statement/Explanation	Diagram
45° – 45° – 90° triangle theorem	If the angles of a triangle are 45°, 45° and 90°, then the hypotenuse (*i.e.*, longest side) is $\sqrt{2}$ times of any smaller side. Excluding hypotenuse rest two sides are equal. *i.e.*, $AB = BC$ and $AC = \sqrt{2}\,AB = \sqrt{2}\,BC$ Ratio of sides 45° – 45° – 90° = $1:1:\sqrt{2}$	 $\angle A = 45°$ $\angle B = 90°$ $\angle C = 45°$
30° – 60° – 90° triangle theorem	If the angles of a triangle are 30°, 60° and 90°, then the sides opposite to 30° angle is half of the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse, *e.g.*, $AB = \frac{AC}{2}$ and $BC = \frac{\sqrt{3}}{2}AC$ $AB:BC:AC = 1:\sqrt{3}:2$	
Basic proportionality theorem (BPT)	Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally, *i.e.*, $\frac{AD}{DB} = \frac{AF}{FC}$ or $\frac{AD}{AB} = \frac{AF}{AC}$	
Mid-point theorem	Any line parallel to one side of a triangle of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side, *i.e.*, if $AD = BD$ and $AE = CE$ then $DE \parallel BC$	
Apollonius' theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side, *i.e.*, $AB^2 + AD^2 = 2\,(AC^2 + BC^2)$	

Theorem	Statement/Explanation	Diagram
Extension of Apollonius' theorem	In the given ΔABC, AC, BE and DF are medians. 3 (Sum of squares of sides) = 4 (Sum of squares of medians) $3\,(AB^2 + AD^2 + DB^2) = 4\,(AC^2 + EB^2 + FD^2)$	
Interior angle bisector theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. *i.e.*, $\frac{BD}{CD} = \frac{AB}{AC}$	
Exterior angle bisector theorem	In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides *i.e.*, $\frac{BE}{AE} = \frac{BC}{AC}$	

Congruency of Triangles

Two figures are said to be congruent if when placed one over the other, they completely overlap each other. They would have the same shape and the same area and will be identical in all respects.

Rules for two Triangles to be Congruent

1. $S - S - S$
   If in two triangles each side of one triangle is equal to a side of the other triangle, the two triangles are congruent. This rule is $S - S - S$ rule (Fig. 12.18).

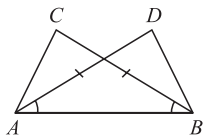


Fig. 12.18

2. $S - A - S$
   In ΔABC and ΔABD
   $AB = AB$ (common side)
   $\angle ABC = \angle BAD$ (given)
   $BC = AD$ (given)
   Thus by rule $S - A - S$ the two triangles are congruent.
   Remember that for this rule to hold true, the angles that are equal has to be the included between two equal sides (*i.e.*, the angle should be formed the two sides that are equal).
3. $A - A - S$ (Fig. 12.19)

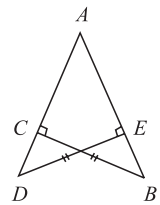


Fig. 12.19

In ΔABC and ΔADE,
$\angle ACB = \angle AED$ (given)
$\angle BAC = \angle DAE$ (common angle)
$BC = DE$ (given)
Thus by rule $A - A - S$ the two triangles are congruent.
For this rule, the side NEED NOT be the included side.

4. $R - H - S$
   This is applicable only for Right-angled triangles.
   If two right-angled triangles have their hypotenuse and one of the sides same, then the triangles will be congruent.

Similarity of the Triangles

If we take two maps of India of different sizes (breadths and lengths), then the map of all the 28 states of India will cover proportionally the same percentage area in both the maps.

Let us see this in geometry:

Criteria for Similarity of Two Triangles

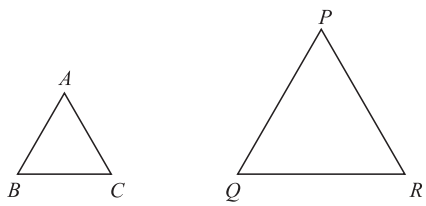


Fig. 12.20

Two triangles are similar if (i) their corresponding angles are equal And/Or (ii) their corresponding sides are in the same ratio. That is, if in two triangles ABC and PQR, (a) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, And/Or (b) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$, the two triangles are similar.

All regular polygons of same number of side such as equilateral triangles, squares, etc are similar. In particular, all circles are also similar.

So, we can say that all congruent triangles are Similar triangles, but vice-versa is not true always.

Theorems for Similarity

Besides the criteria for the Congruency, following are worth noting:

1. If in two triangles, the corresponding angles are equal, then their corresponding sides will be proportional (*i.e.*, in the same ratio). Thus the two triangles are similar.
   This property is referred to as the AAA similarity criterion for two triangles.
   Corollary: If two angles of a triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This is referred to as the AA similarity criterion for two triangles. It is true due to the fact that if two angles of one triangle are equal to the two angles of another triangle, then third angle of both the triangles will be automatically same.
2. The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
3. If a perpendicular is drawn from the vertex of the right angled of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

Example 13

ΔABC is a right angled triangle $BC \perp AC$. If $AD = 8$ cm and $DC = 2$ cm, then $BD = ?$

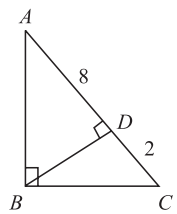


Fig. 12.21

(a) 4 cm
(b) 4.5 cm
(c) 5 cm
(d) Cannot be determined

$\Delta ADB \sim \Delta BDC$

$\therefore \quad \frac{AD}{BD} = \frac{BD}{DC}$

$\therefore \quad BD^2 = AD \times DC = 8 \times 2$

$\therefore \quad BD^2 = 16$

$\Rightarrow \quad BD = 4$ cm

This expression is used as a formula too:

$BD^2 = AD \times DC$

CIRCLE AND ITS PROPERTIES

A circle is the path travelled by a point which moves in such a way that its distance from a fixed point remains constant. The fixed point is known as centre and the fixed distance is called the radius.

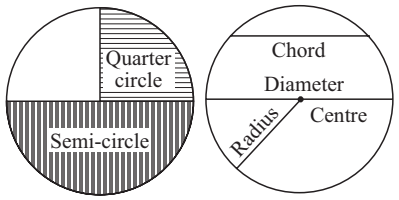
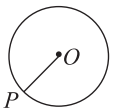
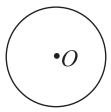
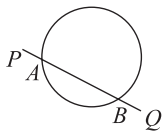
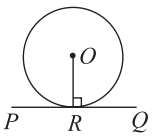
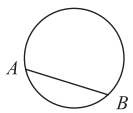
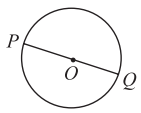
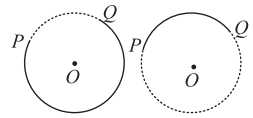
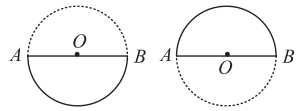
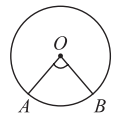
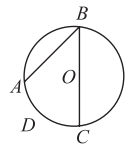
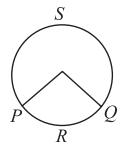
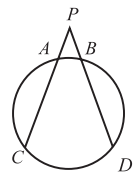
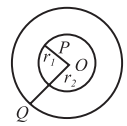
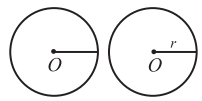
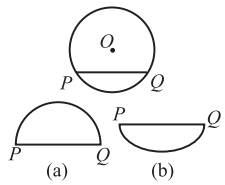


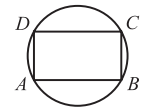
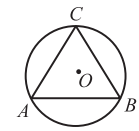
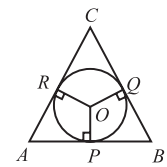
Fig. 12.22

Before we move ahead, let us understand the terms used in case of circle:

Table 12.8

Nomenclature	Definition	Diagram
Centre	The fixed point is called the centre. In the given diagram 'O' is the centre of the circle.	
Radius	The fixed distance is called a radius. In the given diagram, OP is the radius of the circle (point P lies on the circumference).	
Circumference	The circumference of a circle is the distance around a circle, which is equal to $2\pi r$. (r = radius of the circle)	
Secant	A line segment which intersects the circle in two distinct points, is called secant. In the given diagram, secant PQ intersects circle at two points at A and B.	
Tangent	A line segment which has one common point with the circumference of a circle, *i.e.*, it touches only at only one point is called tangent of circle. The common point is called point of contact. In the given diagram, PQ is a tangent which touches the circle at a Point R.	 (R is the point of contract) **Note:** Radius is always perpendicular to tangent.
Chord	A line segment whose end points lie on the circle. In the given diagram, AB is a chord.	
Diameter	A chord which passes through the centre of the circle is called the diameter of the circle. The length of the diameter is twice the length of the radius. In the given diagram, PQ is the diameter of the circle (O = is the centre of the circle). Diameter is also known as the longest chord of the circle.	

Nomenclature	Definition	Diagram
Arc	Any two points on the circle divide the circle into two parts. The smaller part is called minor arc and the larger part is called major arc. In the given diagram, PQ is arc.	
Semicircle	A diameter of the circle divides the circle into two equal parts. Each part is called semicircle.	
Central angle	An angle formed at the centre of the circle, is called the central angle. In the given diagram, $\angle AOB$ in the central angle.	
Inscribed angle	When two chords have one common end point, then the angle included between these two chords at the common point is called the inscribed angle. $\angle ABC$ is the inscribed angle by the arc ADC	
Measure of an arc	Basically, it is the central angle formed by an arc. *e.g.*, (a) measure of a circle = 360° (b) measure of a semicircle = 180° (c) measure of a minor arc = $\angle POQ$ (d) measure of a major arc = 360 – $\angle POQ$	 m (arc $PRQ = m\ \angle POQ$ m (arc PSQ) = 360° – m (arc PRQ)
Intercepted arc	In the given diagram, AB and CD are the two intercepted arcs, intercepted by $\angle CPD$. The end points of the arc must touch the arms of $\angle CPD$ *i.e.*, CP and DP.	
Concentric circles	Circles having the same centre in a plane are called the concentric circles. In the given diagram, there are two circles with radii r_1 and r_2 having the common (or same) centre. These are called the concentric circles.	
Congruent circles	Circles with equal radii are called the congruent circles.	
Segment of a circle	A chord divides a circle into two regions. These two regions are called the segments of a circle. (a) major segment (b) minor segment	

Nomenclature	Definition	Diagram
Cyclic quadrilateral	A quadrilateral whose all the four vertices lie on the circle.	
Circum-circle	A circle which passes through all the three vertices of a triangle. Thus, the circumcentre is always equidistant from the vertices of the triangle. $OA = OB = OC$ (circumradius)	
In circle	A circle which touches all the three sides of a triangle *i.e.*, all the three sides of a triangle are tangents to the circle and called an in-circle. Incircle is always equidistant from the sides of a triangle.	

Area of circle $= \pi r^2$

Circumference of circle $= 2\pi r$

Area of sector $= \pi r^2 \dfrac{\theta}{360°}$

Perimeter of sector $= 2r \dfrac{\pi\theta}{360} + 1$

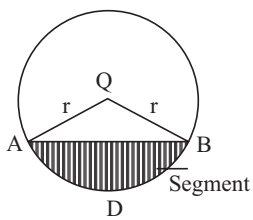


Fig. 12.23

Cyclic Quadrilateral

Consider the Fig. 12.24.

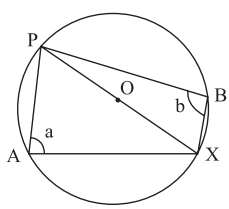


Fig. 12.24

If we have $a + b = 180°$ and quadrilateral $AXBP$ has all its vertices on a circle, then such a quadrilateral is called a cyclic quadrilateral.

For a cyclic quadrilateral, sum of opposite angles of a quadrilateral in a circle is 180°.

It can also be seen that exterior

$\angle CBE$ = internal

$\angle ADC = 180° - \angle ABC$.

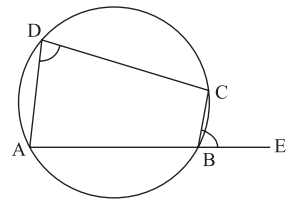


Fig. 12.25

If length of sides of the cyclic quadrilateral are a, b, c and d, then:

Area of a cyclic quadrilateral

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

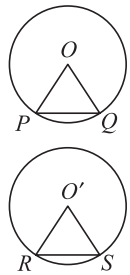
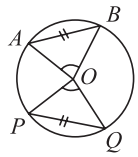
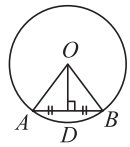
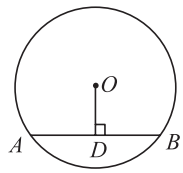
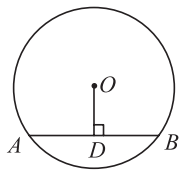
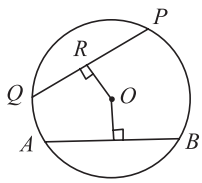
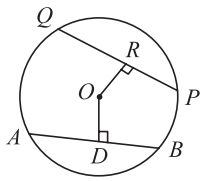
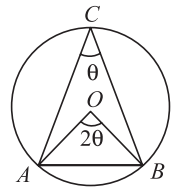
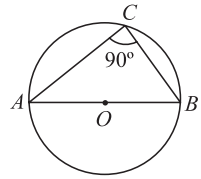
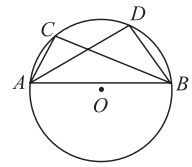
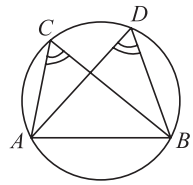
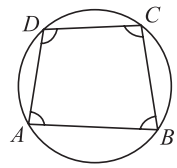
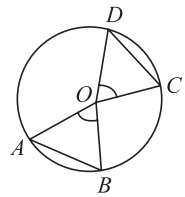
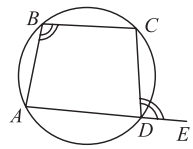
where S = Semi perimeter

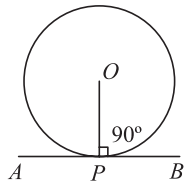
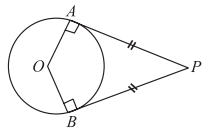
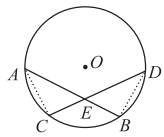
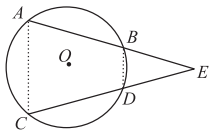
$$= \frac{a+b+c+d}{2}.$$

Theorems Related to Circle

See Table 12.9.

Table 12.9

S. No.	Theorem	Diagram
1.	In a circle (or of congruent circles), equal chords make equal arcs. $\{OP = OQ\} = \{O'R = O'S\}$ $PQ = RS$	
2.	Equal arcs (or chords) subtend equal angles at the centre $PQ = AB$ (or $PQ = AB$) $\angle POQ = \angle AOB$	
3.	The perpendicular from the centre of a circle to a chord bisects the chord, *i.e.*, if $OD \perp AB$, then $AD = DB$	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. $AD = DB$ $OD \perp AB$	
5.	Perpendicular bisector of a chord passes through the centre, *i.e.*, $OD \perp AB$ and $AD = DB$ $\therefore$ O is the centre of the circle	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre If $AB = PQ$, then $OD = OR$	
7.	Chords of a circle (or of congruent circles) which are equidistant from the centre are equal. If $OD = OR$, then $AB = PQ$	
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. $\angle AOB = 2\angle ACB$.	
9.	Angle in a semicircle is a right angle.	
10.	Angles in the same segment of a circle are equal, *i.e.*, $\angle ACB = \angle ADB$	
11.	If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle ACB = \angle ADB$ $\therefore$ Points A, C, D, B are concyclic, *i.e.*, lie on the circle	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180°. $\angle DAB + \angle BCD = 180°$ and $\angle ABC + \angle CDA = 180°$ (Inverse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centre. If $AB = CD$, then $\angle AOB = \angle COD$ (Inverse of this theorem is also true)	
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. $m\angle CDE = m\angle ABC$	

S. No.	Theorem	Diagram
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (Inverse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal, *i.e.*, $AP = BP$	
17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then $AE \times BE = CE \times DE$	 

QUADRILATERALS

A quadrilateral is a figure bounded by four sides. In Fig. 12.26, $ABCD$ is a quadrilateral. Line AC is the diagonal of the quadrilateral (denoted by d) and BE and DF are the heights of the triangles ABC and ADC, respectively (denoted by h_1 and h_2).

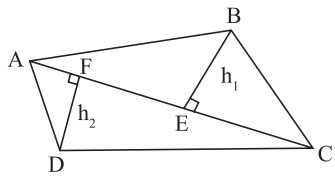


Fig. 12.26

$AC = d$, $BE = h_1$ and $DF = h_2$

1. Area $= \frac{1}{2} \cdot$ one diagonal × (sum of perpendiculars to the diagonal from opposite verticals) $= \frac{1}{2} d\,(h_1 + h_2)$
2. Area $= \frac{1}{2} \cdot$ product of diagonals × sine of angle between them
3. Area of cyclic quadrilateral

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where a, b, c and d are the sides of quadrilateral and s = semi-perimeter

$$= \frac{a+b+c+d}{2}$$

Now we will Discuss Types of Quadrilaterals one by one

Parallelogram

A parallelogram is a quadrilateral whose opposite sides are equal and parallel. Diagonals of a parallelogram bisect each other (Fig. 12.27).

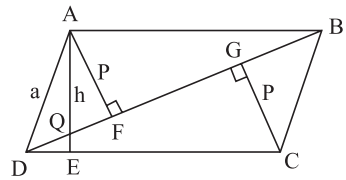


Fig. 12.27

In parallelogram $ABCD$ given above:

(i) $\angle A = \angle C$ and $\angle B = \angle D \Rightarrow$ Opposite angles are equal.

(ii) $\angle A + \angle D = \angle A + \angle B = \angle C + \angle D = \angle B + \angle C$ = Sum of any two adjacent angles = 180°

Given: $AD = BC = a$ and $AB = DC = b$

$BD = d$

AF = (Height of ΔABD)

$= CG$ (Height of ΔCBD) and

AE = height of parallelogram = h

$\angle ADC = \theta$

(i) Area = base × height

(ii) Area = (any diagonal) × (perpendicular distance to the diagonal from opposite vertex)

(iii) Area = (product of adjacent sides) × (sine of the angle between them)

Area = $ab \sin \theta$, where a and b are the side lengths and θ is the angle between them.

(iv) Area = $2\sqrt{s(s-a)(s-b)(s-d)}$, where a and b are the adjacent sides and d is any one of diagonals.

Rectangle

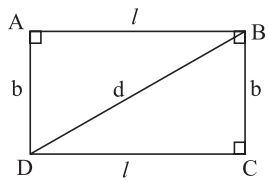


Fig. 12.28

A rectangle is a quadrilateral whose opposite sides are equal and each internal angle equals 90°. Diagonals of a rectangle are equal and bisect each other.

Given: $AD = BC = b$ and $AB = DC = l$, $BD = d$

(i) Area = length × breadth
Area = lb

(ii) Perimeter = 2 (length + breadth)
Perimeter = 2 $(l + b)$

(iii) Diagonal^2 = length^2 + breadth^2 (Pythagoras Theorem)

$$d^2 = l^2 + b^2$$

$$d = \sqrt{l^2 + b^2}$$

Square

A square is a quadrilateral whose all sides are equal and each internal angle is 90°. The diagonals of a square bisect each other at right angles (90°).

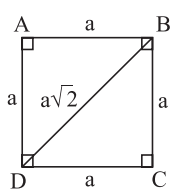


Fig. 12.29

Given: $AB = BC = CD = DA = a$

BD (diagonal) $= a\sqrt{2}$

(i) Area $= (\text{side})^2$

$$= \frac{(\text{diagonal})^2}{2} = \frac{(\text{perimeter})^2}{16}$$

$$\text{Area} = a^2 = \frac{d^2}{2} = \frac{P^2}{16}$$

(ii) Perimeter = 4 (side)
Perimeter = $4a$

Rhombus

A rhombus is a quadrilateral whose all sides are equal. The diagonals of a rhombus bisect each other at right angles (90°).

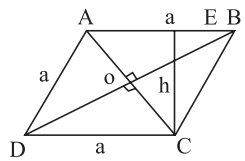


Fig. 12.30

Given $= AB = BC = CD = DA = a$

$\angle AOB = \angle BOC = \angle COD$
$= \angle DOA = 90°$

$AC = d_1$ $(AO = OC)$

and $BD = d_2$ $(BO = OD)$

CE (height) $= h$

(i) Area $= \frac{1}{2} \cdot$ (product of the diagonals)

$$\text{Area} = \frac{1}{2} d_1 d_2$$

Remember, the sum of the square of the diagonals is equal to four time the square of the side, *i.e.*, $d_1^2 + d_2^2 = 4a^2$.

Trapezium

A trapezium is a quadrilateral in which only one pair of opposite sides is parallel:

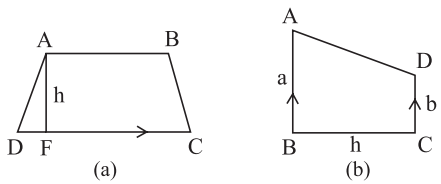


Fig. 12.31

Given: $AB = a$ and $CD = b$

In Fig. 12.31 (a), AF (height) = h and in Fig. 12.31 (b), BC (height) = h

(i) Area $= \frac{1}{2} \cdot$ (sum of parallel sides) × (distance between the parallel sides)

Area $= \frac{1}{2}(a + b)h$

Kite

Kite is a quadrilateral in which both the pairs of **adjacent sides** are equal. Diagonals of kite bisect each other at right angles (90°). Larger diagonal bisects the shorter diagonal. In Fig. 12.32, $DO = OB$ and AC is perpendicular to BD.

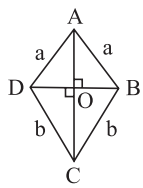


Fig. 12.32

Given: $AB = AD = a$ and $BC = DC = b$

$AC = d_1$

and $BD = d_2$ ($BO = OD$)

$\angle AOB = \angle BOC = \angle COD$

$= \angle DOA = 90°$

(i) Area of kite $= \frac{1}{2} \cdot$ (Product of the diagonals)

Area $= \frac{1}{2} d_1 d_2$

Summary Chart Regarding Diagonals of the Quadrilaterals

Properties of Diagonals

See Table 12.10.

Structure Formed by Joining the Mid-points of Different Quadrilaterals

Table 12.11

Initial		Final
Square	Structure formed by joining the mid-points of sides	Square
Rectangle		Rhombus
Parallelogram		Parallelogram
Rhombus		Rectangle
Trapezium		Parallelogram

Important Point

1. Triangles on the same base and between the same parallel lines are equal in area.

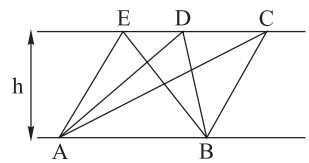


Fig. 12.33

Area (ΔABC) = Area (ΔABD)

= Area (ΔABE)

Table 12.10

S.No.		Types of quadrilaterals				
		Square	Rectangle	Parallelogram	Rhombus	Trapezium
1.	Diagonals are equal	Y	Y	N	N	N
2.	Diagonals bisect each other	Y	Y	Y	Y	N
3.	Diagonals bisect vertex angles	Y	N	N	Y	N
4.	Diagonals are at right angle	Y	N	N	Y	N
5.	Diagonals make 4 congruent triangles	Y	N	N	Y	N

$$= \frac{1}{2}(AB) \cdot h$$

(Base AB is same and height 'h' is same).

2. If a parallelogram and a triangle are drawn on the same base and between same parallel lines, area of parallelogram is twice the area of the triangle.

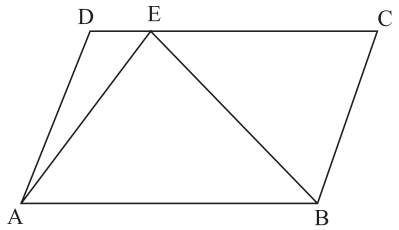


Fig. 12.34

Area of $ABCD = 2 \times$ (Area of ΔABE)

PRACTICE EXERCISES

EXERCISE 1

1. In the Fig. 12.35, $AB \parallel CD$, $\angle ABE = 45^o$, and $\angle CED = 110^o$. Find $\angle FDC$.

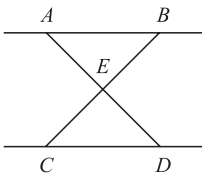


Fig. 12.35

2. In Fig. 12.36, $AB \parallel CD$, and AD is a transversal. Further, $\angle BAE = 45^o$, $\angle DCE = 50^o$. Find $\angle DEC$.

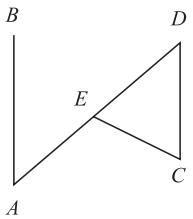


Fig. 12.36

3. In Fig. 12.37, $AB \parallel CD$, $\angle EFC = 30^o$ and $\angle ECF = 100^o$. Find the value of $\angle BAF$.

4. In Fig. 12.38, AB and CD are two parallel lines and transversal PQ intersects them at E and F respectively. Given that $\angle PEA = 115^o$. Find $\angle EFD$.

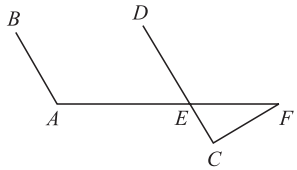


Fig. 12.37

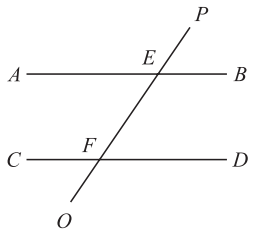


Fig. 12.38

5. Given that $AB \parallel CD$. PQ cuts AB and CD at E and F respectively. EL is the bisector of $\angle FEB$. If $\angle LEB = 35^o$, then find the value of $\angle QEA$.

6. In Fig. 12.39, $AB \parallel CD$ and PQ is the transversal. Find the value of y.

7. In Fig. 12.40, MN and KL are parallel lines. If $\angle LKO = 70^o$, $\angle KON = 100^o$. Find $\angle MNO$.

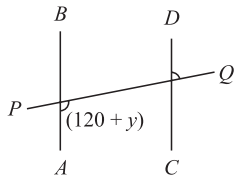


Fig. 12.39

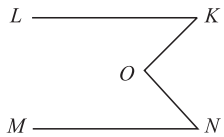


Fig. 12.40

8. In Fig. 12.41, $L1$ and $L2$ are parallel lines. $\angle ABC = 80°$. If the lines BD and CF are parallel and $\angle DBC = 30°$, find $\angle FCE$.

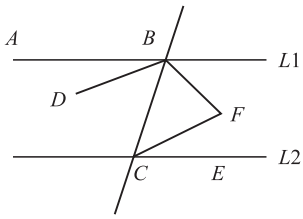


Fig. 12.41

Direction for questions 9–12: ***Refer to the Fig. 12.42 and solve the questions based on it.***

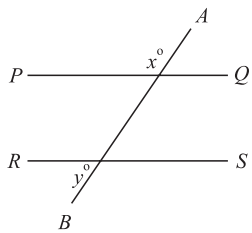


Fig. 12.42

PQ and RS are parallel lines and AB is a transversal line.

9. If $x - y = 10$, find the value of x.

10. If the ratio of $x:y = 3:2$, find the value of y.

11. If $y + (x + y) = 320$, what is x?

12. If $\dfrac{x}{x - y} = 3$, what is the value of x?

Direction for questions 13–16: ***Refer to Fig. 12.43 and solve the questions based on it.***

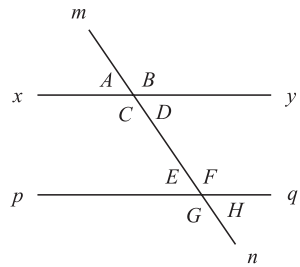


Fig. 12.43

Line XY is parallel to the line QU. Line MN is transversal.

13. If $A + E = 150$, find F.

14. If $A = m$, $G = 3m + 20$ and $F = 2x$, find x.

15. If $G = 11n$; $A = 4m - n$ and $D = 5n + 2m - 20$, find the value of H.

16. If $B = 4m$ and $E = m + 2n$; $D = 3n + 8$, find the value of H.

17. How many degrees are there in an angle, which equals one fifth of its supplement?

18. Two straight lines AB and CD cut each other at O. If Angle $BOD = 63°$, find angle BOC.

19. Ratio between one internal angle and one external angle of a regular polygon is 2:1. Which polygon is this?

20. Ratio between one internal angle and one external angle of a regular polygon is 1:2. Which polygon is this?

21. What is the number of diagonals in a pentagon?

22. It is given that perimeter of a square = Perimeter of a regular Octagon = Perimeter of a regular Decagon. Area of Square = S, Area of Octagon = O and Area of Decagon

= D. What the order for the areas in ascending order?

23. There are two regular polygons with number of sides in the ratio 4:5 and the interior angles in the ratio 25:26. Number of sides in the first polygon is?

24. The sides of a regular octagon are extended of form a star. Find the measure of the internal angle at each point of the star.

25. Under the usual 2-dimensional coordinate system, which structure is represented by the equation $|x| + |y| = 1$, where x and y are real numbers?

EXERCISE 2

1. In ΔACD, $AD = AC$ and $\angle C = 2\angle E$. The distance between parallel line AB and CD is h (Fig. 12.44).

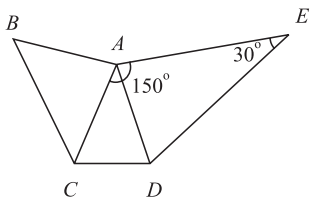


Fig. 12.44

I. Area of parallelogram $ABCD$
II. Area of ΔADE

(a) I > II
(b) I < II
(c) I = II
(d) Nothing can be said

2. Which one of the following cannot be the ratio of angles in a right angled triangle?
(a) 1:2:3 (b) 1:1:2
(c) 1:3:6 (d) None of these

3. In the adjoining Fig. 12.45, $AC + AB = 5AD$ and $AC - AD = 8$. Then the area of the rectangle $ABCD$ is:
(a) 36 (b) 50
(c) 60
(d) Cannot be answered

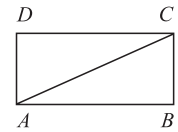


Fig. 12.45

4. In ΔABC, $\angle ACD = 100°$ and $\angle ABC = 40°$. Find $\angle BAC$:

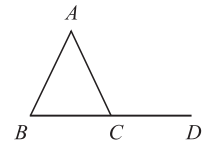


Fig. 12.46

(a) 40° (b) 100°
(c) 80° (d) 60°

5. The length of a ladder is exactly equal to the height of the wall it is resting against. If lower end of the ladder is kept on a stool of height 3 m and the stool is kept 9 m away from the wall the upper end of the ladder coincides with the tip of the wall. Then, the height of the wall is:
(a) 12 m (b) 15 m
(c) 18 m (d) 11 m

6. In triangle ABC, angle B is a right angle. If AC is 6 cm and D is the mid-point of side AC. The length of BD is:

(a) 4 cm (b) $\sqrt{6}$ cm
(c) 3 cm (d) 3.5 cm

7. In Fig. 12.47, $PQ \times PS = PT \times PR$. If $\angle PQR = \angle PST + 30°$ and $\angle PTS = 100°$ then find $\angle PRQ$.

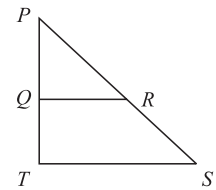


Fig. 12.47

(a) 100° (b) 70°
(c) 130° (d) 50°

8. To find the area of a triangle which of the following statements is/are sufficient?

A. All the length of the sides of the triangles.

B. Any side and the corresponding altitude of the triangle.

C. Two sides and the angle between them.

(a) A alone is sufficient
(b) B alone is sufficient
(c) Either A or B alone is sufficient
(d) Any one of the three is sufficient

Direction for questions 9 to 10: *Read the information given below and answer the question that follows*:

A cow is tethered at A by a rope. Neither the rope nor the cow is allowed to enter the triangle ABC (Fig. 12.48).

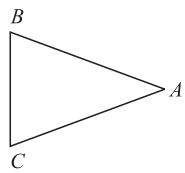


Fig. 12.48

$m\angle A = 30^\circ$,
$\ell(AB) = \ell(AC) = 10$ m,
$\ell(BC) = 6$ m.

9. What is the area that can be grazed by the cow if the length of the rope is 8 m?

(a) $133\frac{1}{6}\pi$ sq m (b) 121π sq m
(c) 132π sq m (d) $\frac{176}{3}\pi$ sq m

10. What is the area that can be grazed by the cow if the length of the rope is 12 m?

(a) $133\frac{1}{6}\pi$ sq m (b) $134\frac{1}{3}\pi$ sq m
(c) 132π sq m (d) $\frac{176}{3}\pi$ sq m

11. ABC and DBC are right triangles with common hypotenuse BC. The side AC and BD are extended to intersect at P. Then $\frac{AP \cdot PC}{DP \cdot PB} = ?$

(a) 2 (b) 1/3
(c) 1 (d) None of these

12. In ΔABC, points P, Q and R are the midpoints of side AB, BC and CA respectively. If area of Δabs is 20 sq units then find the area of ΔPQR.

(a) 10 sq units (b) $5\sqrt{3}$
(c) 5 sq units (d) None of these

13. If a, b, c are side of the Δ, and $a^2 + b^2 + c^2 = ab + bc + ca$ then the triangle is:

(a) Isosceles
(b) Scalene
(c) Equilateral
(d) Cannot be determined

14. In a triangle ABC, $AB = 6$, $BC = 8$ and $AC = 10$. A perpendicular dropped from B, meets in side AC at D. A circle of radius BD (with center B) is drawn. If the circle cuts AB and BC at P and Q respectively, then AP:QC is equal to:

(a) 1:1 (b) 3:2
(c) 4:1 (d) 3:8

15. The sides of a triangle are 5 cm, 7 cm and 10 cm, find the length of the median to the longest side.

(a) 4.5 cm (b) 3.1 cm
(c) 3.5 cm (d) 4.2 cm

16. A ladder 50 cm long is placed so as to reach a window 48 cm high and on turning the ladder over to other side of the street, it reaches a point on the opposite wall 14 cm high. Find the breadth of the street.

(a) 56 m (b) 60 cm
(c) 64 cm (d) 62 cm

17. Two sides of a plot in the form of a quadrilateral measure 32 and 24 meters and the angle between them is a right angle. The other two sides measure 25 m each. Find the area of the plot (Fig. 12.49):

(a) 684 (b) 342
(c) 748 (d) None of these

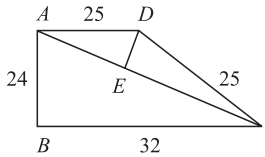


Fig. 12.49

18. In Fig. 12.50, *ABCD* is a rectangle with *AE* = *EF* = *FB*. What is the ratio of the area of the triangle *CEF* and that of the rectangle?

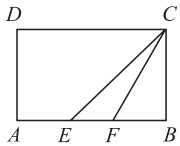


Fig. 12.50

(a) 1:4 (b) 1:6
(c) 2:5 (d) 2:3

19. Four villages lie at the vertices of a squares of side 1 km, what is the smallest length of road needed to link all village together?
(a) 3 km (b) 2.73 km
(c) 2.88 km (d) 4 km

20. What is the area (in cm^2 upto two decimal points) of a triangle whose sides and the angle between them are given as:
$a = 5$ cm and $b = 7$ cm and angle $C = 45^o$?
(a) 12
(b) 12.37
(c) 2.88 km
(d) Cannot be determined

EXERCISE 3

1. In Fig. 12.51, *AB* and *CD* are two chords of a circle intersecting at *O*. If *AO* = 4 cm, *OB* = 6 cm and *OC* = 3 cm, find *OD*.

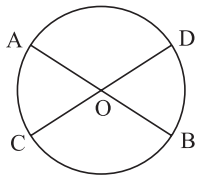


Fig. 12.51

(a) 4 cm (b) 6 cm
(c) 8 cm (d) 10 cm

2. In Fig. 12.52, if 2 $\angle POR$ = 3 $\angle ROQ$ and $\angle POQ = 100°$, then find $\angle POR$.

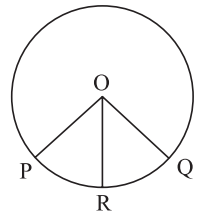


Fig. 12.52

(a) 20° (b) 30°
(c) 45° (d) 60°

3. In Fig. 12.53, *O* is the centre of the circle and $\angle AOB = 120^o$. Find $\angle ACB$.

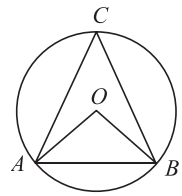


Fig. 12.53

(a) 60° (b) 30°
(c) 45° (d) 50°

4. In Fig. 23.54, *O* is the centre of the circle, $\angle AOC = 160°$. If *AB* = *BC*, then find $\angle BAC$.

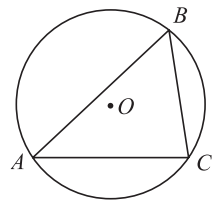


Fig. 12.54

(a) 45° (b) 50°
(c) 60° (d) 75°

5. In Fig. 12.55, *O* is the centre of the circle and $\angle OQP + \angle ORP = 70^o$. Find $\angle ORQ$.

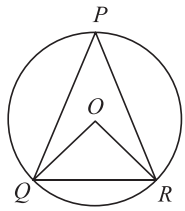


Fig. 12.55

(a) 20° (b) 30°
(c) 40° (d) 50°

6. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle?
(a) 88 cm^2 (b) 154 cm^2
(c) 216 cm^2 (d) None of these

7. If the circumference and the area of a circle are numerically equal, then the diameter is equal to:
(a) $\pi/2$ (b) 2π
(c) 2 (d) 4

8. The sum of areas of two circles *A* and *B* is equal to the area of a third circle *C*, whose diameter is 30 cm. If the diameter of circle *A* is 18 cm, then the radius of circle *B* is:
(a) 10 cm (b) 12 cm
(c) 15 cm (d) 18 cm

9. If the ratio of areas of two circles is 4:9, then the ratio of their circumferences will be:
(a) 2:3 (b) 3:2
(c) 4:9 (d) 9:4

10. *AB* and *CD* are two parallel chords of a circle with centre *O* such that $AB = 6$ cm and $CD = 12$ cm. The chords are on the same side of the centre and the distance between them is 3 cm. What is the radius of the circle?
(a) 3 cm (b) $3\sqrt{5}$
(c) $7\sqrt{5}$ (d) 5 cm

11. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, find the radius of the circle.
(a) 30 cm (b) $3\sqrt{50}$ cm
(c) 70 cm (d) 20 cm

12. Two circles touch externally. The sum of their areas is 130π sq cm and the distance between their centres is 14 cm. Find the radius of the larger circle.
(a) 10 cm (b) 22 cm
(c) 7 cm (d) 11 cm

13. Two circles touch internally. The sum of their areas is 116π sq cm and the distance between their centres is 6 cm. Find the radius of the larger circle.
(a) 10 cm (b) 22 cm
(c) 7 cm (d) 11 cm

14. A chord of length 14 cm is at a distance 6 cm from the centre of a circle. Find the length of another chord at a distance of 2 cm from the centre.
(a) 18 cm (b) 24 cm
(c) 21 cm (d) 12 cm

15. What is the length of the tangent to a circle of radius 5 cm from a point at a distance of 13 cm from the centre of the circle?
(a) 8 cm (b) 12 cm
(c) 18 cm (d) 24 cm

16. Two circles of radius 10 cm, and 8, cm respectively, touch each other internally. What is the distance between their centres?
(a) 8 cm (b) 2 cm
(c) 18 cm (d) None of these

17. What is the length of the common chord of the two circles of radii 6 cm and 8 cm with their centres 10 cm apart?
(a) 8.4 cm (b) 9.6 cm
(c) 10 cm (d) 12 cm

18. A horizontal pipe for carrying water has diameter 1 m when water in it is 10 cm deep. What is the breadth of the water surface?
(a) 80 cm (b) 120 cm
(c) 60 cm (d) 24 cm

19. In ΔABC, base side BC is parallel to Side DE (D point is on side AB and E point on AC). $AC = 5.6$ cm, $AD = 3$ cm and $BD = 5$ cm. Find the length of AE.
(a) 2.1 cm (b) 1.2 cm
(c) 5.60 cm (d) 2.4 cm

20. Two identical circles intersect so that their centres, and the points at which they intersect, form a square of side 1 cm. The area in sq cm of the portion that is common to the two circles is:
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2} - 1$
(c) $\frac{\pi}{5}$ (d) $\sqrt{2} - 1$

21. A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, A and B, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. A jogs along the rectangular track, while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting point?
(a) 3.88% (b) 4.22%
(c) 4.44% (d) 4.72%

22. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?
(a) 1 or 7 (b) 2 or 14
(c) 3 or 21 (d) 4 or 28

23. Four points A, B, C and D lie on a straight line in the X-Y plane, such that $AB = BC = CD$, and the length of AB is 1 metre. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one metre of any insect repellent. The minimum distance in metres the ant must traverse to reach the sugar particle is:
(a) $3\sqrt{2}$ (b) $1 + \pi$
(c) $\frac{4\pi}{3}$ (d) 5

24. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral $PQSR$?
(a) $2r(1 + \sqrt{3})$ (b) $2r(2 + \sqrt{3})$
(c) $r(1 + \sqrt{5})$ (d) $2r + \sqrt{3}$

25. The ratio of the area of a square inscribed in a semi-circle to that of the area of a square inscribed in the circle of the same radius is
(a) 2:1 (b) 2:3
(c) 2:5 (d) 2:7

EXERCISE 4

Note from Author: In this exercise, there are some questions that do not contain options. I have deliberately removed the options from those questions so that students don't get biased by looking at the options.

1. One side of a parallelogram is 18 cm and its distance from the opposite side is 8 cm. The area of the parallelogram is:
(a) 48 cm^2 (b) 72 cm^2
(c) 100 cm^2 (d) 144 cm^2

2. A parallelogram has sides 30 m and 14 m and one of its diagonals is 40 m long. Then, its area is:
(a) 168 m^2 (b) 336 m^2
(c) 372 m^2 (d) 480 m^2

3. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is:
(a) $10\sqrt{2}$ m (b) 100
(c) $100\sqrt{2}$ m (d) 200 m

4. If a parallelogram with area P, a rectangle with area R and a triangle with area T are

all constructed on the same base and all have the same altitude, then which of the following statements is false?
(a) $P = R$ (b) $P + T = 2R$
(c) $P = 2T$ (d) $T = (1/2)\,R$

5. The area of a rhombus is 150 cm^2. The length of one of its diagonals is 10 cm. The length of the other diagonal is:
(a) 25 cm (b) 30 cm
(c) 35 cm (d) 40 cm

6. One of the diagonals of a rhombus is double the other diagonal. Its area is 25 sq cm. The sum of the diagonals is:
(a) 10 cm (b) 12 cm
(c) 15 cm (d) 16 cm

7. The perimeter of a rhombus is 56 m and its height is 5 m. Its area is:
(a) 64 m^2 (b) 70 m^2
(c) 78 m^2 (d) 84 m^2

8. Each side of a rhombus is 26 cm and one of its diagonals is 48 cm long. The area of the rhombus is:
(a) 240 cm^2 (b) 300 cm^2
(c) 360 cm^2 (d) 480 cm^2

9. The length of one diagonal of a rhombus is 80% of the other diagonal. The area of the rhombus is how many times the square of the length of the other diagonal?
(a) 4/5 (b) 2/5
(c) 3/4 (d) 1/4

10. If a square and a rhombus stand on the same base, then the ratio of the areas of the square and the rhombus is:
(a) Greater than 1 (b) equal to 1
(c) equal to 1/2 (d) equal to 1/4

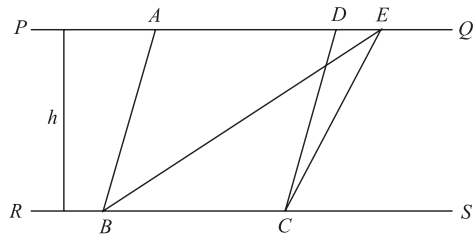
11. The two parallel sides of a trapezium are 1.5 m and 2.5 m, respectively. If the perpendicular distance between them is 6.5 metres, the area of the trapezium is:
(a) 10 m^2 (b) 13 m^2
(c) 20 m^2 (d) 26 m^2

12. The area of a field in the shape of a trapezium measures 1,440 m^2. The perpendicular distance between its parallel sides is 24 m. If the ratio of the parallel sides is 5:3, the length of the longer parallel side is:
(a) 45 m (b) 60 m
(c) 75 m (d) 120 m

13. If sides of a rectangle are 10 cm and 8 cm, then find:
(a) Length of diagonal
(b) Perimeter
(c) Semi perimeter
(d) Area

14. Find all the angles of parallelogram *ABCD* in which $\angle B$ is 60° (points are in order).

15. In Fig. 12.56, find the area of the parallelogram *ABCD* (in square units) if area of ΔEBC is 15 cm^2.



(a) 45 (b) 60
(c) 30 (d) 120

16. If two adjacent sides of a parallelogram are 6 cm and 8 cm and angle between them is 60°, then find:
(a) Perimeter (b) Area

17. Find the area of a trapezium whose shorter and longer parallel sides have the length 10 cm and 14 cm and are 8 cm apart from each other.

18. In parallelogram *KLMN*, if diagonal *KM* measures 30 inches, then:
(a) *KL* measures 18 inches
(b) *LM* measures 24 inches
(c) Diagonal *LN* is perpendicular to diagonal *KM*
(d) None of these

19. Length of diagonals of a rhombus are 16 cm and 12 cm. What is the side of the rhombus?

20. In a quadrilateral $ABCD$, if AO and BO be the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 70°$ and $\angle D = 30°$, then find $\angle AOB$.

21. The bisectors of any two adjacent angles of a parallelogram intersect at what angle?

22. If an angle of a parallelogram is two-third of its adjacent angle, then find smallest angle of the parallelogram.

23. Two parallelograms stand on equal bases and between the same parallel lines. What is the ratio between their areas?

24. $ABCD$ is a parallelogram. M is the mid-point of BD and BM bisects $\angle B$. Find the value of $\angle AMB$.

25. $ABCD$ is a square of side 13 cm. K is a point on CD such that $CK:KD = 1:2$. CP is perpendicular to AK produced. Find the length of CP.

26. Three sides of a parallelogram are 17 cm, 15 cm and 10 cm. Length of fourth side must be less than:
(a) 32 cm (b) 42 cm
(c) 27 cm (d) 25 cm

27. Find the area of a cyclic quadrilateral whose sides are 10 cm, 12 cm, 16 cm and 20 cm.

28. Sides of a board consecutively measure 9 feet, 5 feet, 9 feet and 5 feet. Without any further information, you can determine that board is a:
(a) rectangle (b) rhombus
(c) parallelogram (d) square

29. Mid-points of an irregular quadrilateral are joined to form a quadrilateral (irregular quadrilateral cannot have all four sides of the same length). Name the quadrilateral formed as a result.
(a) Trapezium
(b) Rhombus
(c) Parallelogram
(d) Square

30. Mid-points of a rectangle are joined to form a quadrilateral. Name the quadrilateral formed as a result.
(a) Rectangle (b) Rhombus
(c) Parallelogram (d) Square

ANSWER KEY

Exercise 1

1. 25° **2.** 85° **3.** 130° **4.** 65° **5.** 110° **6.** 30° **7.** 30° **8.** 50°
9. 95° **10.** 72° **11.** 140° **12.** 120° **13.** 105° **14.** 70° **15.** 70° **16.** 68°
17. 30° **18.** 117° **19.** Hexagon **20.** Equilateral Triangle **21.** 5
22. $S < O < D$ **23.** 12 **24.** 90°
25. A square whose sides are not parallel to the coordinate axes

Exercise 2

1. (c) **2.** (c) **3.** (c) **4.** (d) **5.** (b) **6.** (c) **7.** (b) **8.** (d)
9. (d) **10.** (b) **11.** (c) **12.** (c) **13.** (c) **14.** (d) **15.** (c) **16.** (d)
17. (a) **18.** (b) **19.** (b) **20.** (b)

Exercise 3

1. (c)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (b)
9. (a)	**10.** (b)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (b)
17. (b)	**18.** (c)	**19.** (a)	**20.** (b)	**21.** (d)	**22.** (d)	**23** (b)	**24.** (a)
25. (c)							

Exercise 4

1. (d)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (d)
9. (b)	**10.** (b)	**11.** (b)	**12.** (c)	**13.** *	**14.** *	**15.** (c)	**16.** *
17. 96 cm^2	**18.** (d)	**19.** 10 cm	**20.** 50	**21.** 90	**22.** 72	**23.** 1:1	**24.** 90
25. $\sqrt{13}$	**26.** 2	**27.** *	**28.** (c)	**29.** (c)	**30.** (b)		

HINTS AND EXPLANATIONS

***Answer Given Ahead.**

13. Sides of a rectangle are 6 cm and 8 cm

(a) Length of diagonal $= \sqrt{6^2 + 8^2} = 10$ cm
(b) Perimeter = 2 (6 cm + 8 cm) = 28 cm
(c) Semi perimeter = (6 cm + 8 cm) = 14 cm
(d) Area = 6 cm × 8 cm = 48 cm^2

14. $\angle B = 60° \Rightarrow \angle D = 60°$
$\angle A = \angle C = 120°$

16. If two adjacent sides of a parallelogram are 6 cm and 8 cm and angle between them is 60°, then find:

(a) Perimeter = 2 × (6 + 8) cm = 28 cm
(b) Area $sb \sin\theta = 6 \cdot 8 \cdot \sin 60° = 48 \cdot \dfrac{\sqrt{3}}{2} = 24\sqrt{3}$ cm^2

17. Area of trapezium = 1/2 × (10 + 14) × 8 cm^2 = 96 cm^2

27. Area of Cylic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

where $s = \dfrac{10 + 12 + 16 + 20}{2} = 29$

13 Mensuration

Solids

A solid is a figure bounded by one or more surfaces. A solid has three dimensions namely length, breadth or width and height or thickness. The plane surfaces that bind it are called its faces and the solid so generated is known as a polyhedron.

The volume of any solid figure is the amount of space enclosed within its bounding faces. A solid has edges, vertices and faces which are shown in Fig. 13.1.

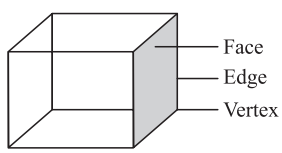


Fig. 13.1

A solid has two types of surface areas:

(i) Lateral Surface Area (LSA)—LSA of a solid is the sum of the areas of all the surfaces it has except the top and the base. In case of structures like cylinder or cone (which have curvature instead of plane surface), Lateral Surface Area is known as Curved Surface Area.

(ii) Total Surface Area (TSA)—TSA of a solid is the sum of the lateral surface area and the areas of the base and the top.

Note: In case of solids like cube and cuboid, the Lateral Surface Area consists of plane surface areas (*i.e.*, area of all surfaces except the top and base) whereas in case of solids like cone and cylinder, it consists of curved surface areas (CSA). Thus, in case of such solids, the LSA is also called CSA.

Cuboid (Rectangular Box)

A cuboid is a rectangular solid having 6 rectangular faces. The opposite faces of a cuboid are equal rectangles. A cuboid has a length (l), breadth (b) and height (h).

In Fig. 13.2 (b), ED is the diagonal of the cuboid. Moreover the area of the surface $GDCH$ is X, the area of the surface $HEBC$ is Y and the area of the surface $GFEH$ is Z.

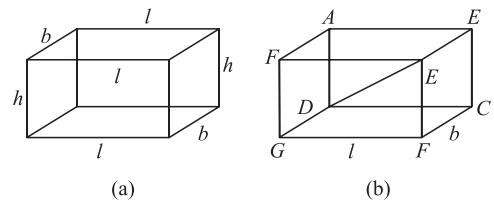


Fig. 13.2

(i) Volume = Area of base × Height = lbh

(ii) Volume $= \sqrt{xyz}$

(iii) Volume $= xh = yl = zb$

(iv) Lateral Surface Area (LSA) of a room or area of 4 walls = $2(l + b)h$ [lb will not be counted]

(v) Total Surface Area (TSA) = $2(x + y + z)$ $= 2(lb + bh + lh)$

(vi) Diagonal $= \sqrt{l^2 + b^2 + h^2}$

Cube

A cube is a six faced solid Fig. 13.3. All the faces of a cube are equal squares (let's say of side 'a'). Thus, the length, breadth and height of a cube are equal.

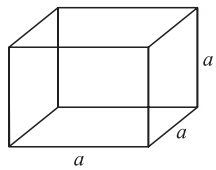


Fig. 13.3

(i) Volume = a^3

(ii) Lateral Surface Area (LSA) or area of 4 walls = $4a^2$

(iii) Total Surface Area (TSA) = $6a^2$

(iv) Diagonal $= a\sqrt{3}$

Right Circular Cylinder

In Fig. 13.4, r is the radius of the base and h is the height of a right circular cylinder. A cylinder is generated by rotating a rectangle or a square by fixing one of its sides.

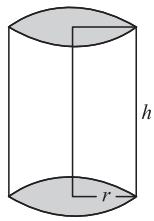


Fig. 13.4

(i) Volume = Area of base × Height

Volume = $\pi r^2 h$

(ii) Curved Surface Area (CSA) = Perimeter of base × Height

LSA = $2\pi rh$

(iii) Total Surface Area (TSA) = LSA + Area of top + Area of base

TSA = $2\pi rh + \pi r^2 + \pi r^2$

TSA = $2\pi r\,(r + h)$

Right Circular Cone

In Fig. 13.5, 'r' is the radius of the base, h is the height and λ is the slant height of right circular cone.

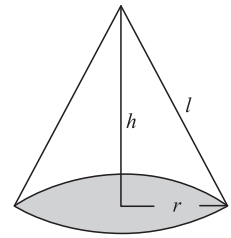


Fig. 13.5

(i) Volume $= \frac{1}{3} \times$ area of base × height

Volume $= \frac{1}{3}\pi r^2 h$

It can also be seen that volume of cone is 1/3rd the volume of corresponding cylinder (with same radius and height).

(ii) Slant height = $l = \sqrt{r^2 + h^2}$

(iii) Curved Surface Area (CSA) = πrl

(iv) Total Surface Area (TSA) = (CSA + Area of base)

TSA = $\pi rl + \pi r^2$

Important Deductions

Frustum of Cone

A cone whose top portion is sliced off by a plane which is parallel to the base is called frustum of cone.

Formation of Frustum:

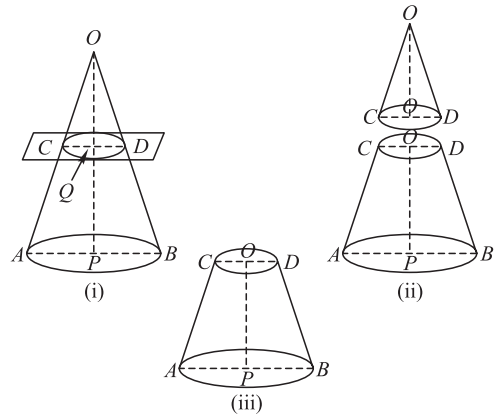


Fig. 13.6

However, for the sake of representing the formula, we will use another form of Frustum right now as given below:

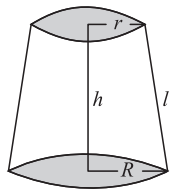


Fig. 13.7

(i) Volume of Frustum = Volume of original cone – Volume of cone removed
(ii) Total Surface Area (TSA) = CSA + Area of top + Area of base
(iii) To find the height (H) of original cone.

$$H = \frac{Rh}{R - r}$$

Pyramid

A pyramid is a solid having an n-sided polygon at its base. The side faces of a pyramid are triangular with the top as a point.

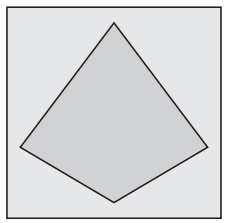


Fig. 13.8

In Fig. 13.9, OM is the height of the pyramid.

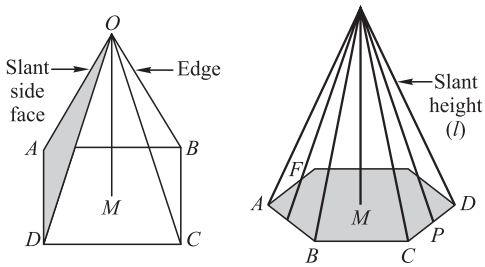


Fig. 13.9

(i) Volume $= \frac{1}{3} \times$ Area of base × Height
(ii) Lateral Surface Area $= \frac{1}{2} \times$ (Perimeter of base) × Slant height
(iii) Total Surface Area (TSA) = LSA + Area of base

Frustum of Pyramid

A pyramid whose top portion is sliced off by a plane which is parallel to the base is called frustum of pyramid.

In Fig. 13.10, A_1 is the area of top face of the frustum, A_2 is the area of the bottom face of the frustum, h is the height of the frustum and λ is the slant height of the frustum.

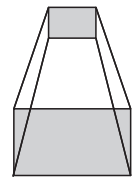


Fig. 13.10

Sphere

In Fig. 13.11, r is the radius of sphere.

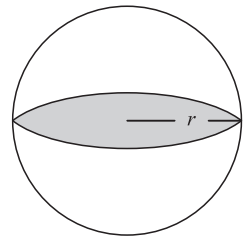


Fig. 13.11

(i) Volume $= \frac{4}{3}\pi r^3$
(ii) Surface area $= 4\pi r^2$

In case of Sphere, Surface Area or Total Surface Area of Curved Surface Area means the same thing.

Hemisphere

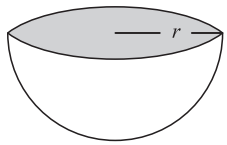


Fig. 13.12

(i) Volume $= \frac{2}{3}\pi r^3$

(ii) Curved surface area (CSA) = $2\pi r^2$

(iii) Total surface area (TSA) = LSA + Area of top face (read circle)
TSA = $2\pi r^2 + \pi r^2$
TSA = $3\pi r^2$

Example 1

A cylindrical container of 32 cm height and 18 cm radius is filled with sand. Now all this sand is used to form a conical heap of sand. If the height of the conical heap is 24 cm, what is the radius of its base?

Solution

Volume of cylinder $= \pi \times 18 \times 18 \times 32$
Volume of cone $= 1/3\pi R^2 H$
(where R and H are the radius and height of cone, respectively)
Now, $1/3\pi R^2 H = \pi \times 18 \times 18 \times 32$
Putting the value of $H = 24$ cm, we get
$R^2 = 1{,}296$
Hence, $R = 36$ cm

Example 2

The diameter of a metallic sphere is 6 cm. the sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36 m, find its radius.

Solution

The diameter of the metallic sphere is 6 cm. Hence radius of the sphere is 3 cm. Now, let the radius of the cross-section of wire be r cm. And as we know, metallic sphere is converted into cylindrical shaped wire, their volumes will be equal.

So, $\frac{4}{3} \times \pi \times 3^3 = \pi \times r^2 \times 3600$

or $4 \times 9\pi \times 3^3 = 3{,}600\pi r^2$

This gives $r^2 = 0.01$

i.e., $r = 0.1$

Example 3

Sardar Sarovar Dam, which is rectangular in shape, can produce electricity only if the height of water level in it is at least 7 cm. Now water is pumped in at the rate of 5 km per hour through a pipe of diameter 14 cm into the dam area of dimension 50 m × 44 m. In what time the dam will be able to produce electricity?

Solution

Volume of water flowing through the cylindrical pipe in one hour at the rate of 5 km (5,000 m) per hour

$$= \frac{22}{7} \cdot \frac{7}{100} \cdot \frac{7}{100} \cdot 5000 \text{ m}^3$$

$$\left[\text{Since radius} = 7 \text{ cm} = \frac{7}{100} \text{ m}\right]$$

$= 77 \text{ m}^3$

Thus, 77 m^3 of water will fall into the tank in 1 hour. Since the level of water is required to be raised by 7 cm, *i.e.*, $\frac{7}{100}$ m, so volume of required quantity of water $= 50 \cdot 44 \cdot \frac{7}{100} \text{ m}^3 = 154 \text{ m}^3$

Since, 77 m^3 of water falls into the tank in 1 hour, therefore, 154 m^3 of water will fall into the dam in $\frac{154}{77}$ hours, *i.e.*, 2 hours.

So, the level of water will rise by 7 cm in 2 hours.

Example 4

How many metres of cloth 10 m wide will be required to make a conical tent with base radius of 14 m and height 48 m?

Solution

Area of cloth $= \pi rl = \pi \times 14 \times 50$
$= 2{,}200 \text{ m}^2$

Hence length of cloth $= 2{,}200/10 = 220$ m.

Example 5

The ratio of heights of two cylinders is 3:2 and the ratio of their radii is 6:7. What is the ratio of their curved surface areas?

Solution

Ratio of curved surface areas
$= 2\pi r_1 h_1 / 2\pi r_2 h_2 = 9/7$

Example 6

A solid metal ball of diameter 16 cm is melted and cast into smaller balls, each of radius 1 cm. What is the number of such balls formed?

Solution

Volume of metal ball $= 4/3\ (\pi \times 8^3)$

Volume of smaller ball
$= 4/3\ (\pi \times 1^3)$

Hence, number of balls formed

$$= \frac{4/3\,(\pi \times 8^3)}{4/3\,(\pi \times 1^3)} = 512.$$

Example 7

A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15 cm and 12 cm respectively, what is the rise in the water level?

Solution

It can be seen that

Base area of vessel × Rise in water level = Volume of cube

Or, $15 \times 12 \times h = 11^3$

where, h = Rise in water level

Hence, $h = 7.4$ m (approx.)

PRACTICE EXERCISES

EXERCISE 1

1. If the side of a cube is increased by 100%, find by what percentage the surface area of the cube is increased?
(a) 150% (b) 200%
(c) 300% (d) 350%

2. A banquet hall has the dimensions 30 m × 12 m × 6 m. Each person should get 8 m^3 of space. Find the number of persons who can be accommodated in this hall.
(a) 240 (b) 250
(c) 270 (d) 300

3. Find the height of the cylinder whose volume is 511 cm^3 and the area of the base 36.5 cm^2.
(a) 3.5 cm (b) 10.5 cm
(c) 14 cm (d) None of these

4. The ratio between curved surface area and total surface area is 2:3 and the total surface is 924 cm^2. What is the volume of the cylinder?
(a) 2,156 cm^3 (b) 2,183 cm^3
(c) 2,492 cm^3 (d) None of these

5. If the curved surface area of a cylinder is 1,320 cm^2 and its base radius is 21 cm, then what is its total surface area?
(a) 4,092 cm^2 (b) 2,084 cm^2
(c) 5,104 cm^2 (d) None of these

6. The radius of an iron rod is decreased to one-fourth of its actual radius. If its volume remains constant, the length will become:
(a) 2 times (b) 12 times
(c) 8 times (d) 16 times

7. A reservoir is in shape of a frustum of a right circular cone. It is 8 m wide at the

top and 4 m wide at the bottom. If it is 6 m deep, then what is its volume?
(a) 224 m^3 (b) 176 m^3
(c) 204 m^3 (d) None of these

8. A spherical metal ball of 6 cm radius is melted and recast into three spherical balls. The radii of two of these balls are 3 cm and 4 cm. What is the radius of the third ball?
(a) 4.5 cm (b) 5 cm
(c) 6 cm (d) 7 cm

9. If the diagonals of a rhombus are 18 cm and 24 cm, respectively, then find its perimeter.
(a) 15 cm (b) 42 cm
(c) 60 cm (d) 68 cm

10. In the adjoining Fig. 13.13, *PQRS* is a rectangle of the dimension 8 cm × 6 cm and is inscribed in a circle. Find the area of the shaded portion.

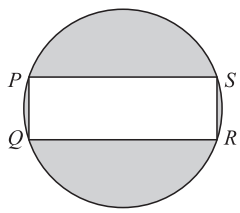


Fig. 13.13

(a) 44 cm^2 (b) 34.25 cm^2
(c) 32.50 cm^2 (d) None of these

11. The radius of base and the volume of right circular cone are doubled. What is the ratio of the length of the larger cone to that of the smaller cone?
(a) 1:4 (b) 1:2
(c) 1:3 (d) 4:1

12. A cone and a hemisphere have equal base radius and equal volumes. The ratio of their heights is:
(a) 3:1 (b) 2:1
(c) 4:1 (d) None of these

13. If the right circular cone is cut into three solids of volumes V_1, V_2 and V_3 by two cuts which are parallel to the base and trisect the altitude, then $V_1:V_2:V_3$ is:
(a) 1:2:3 (b) 1:4:6
(c) 1:6:9 (d) None of these

14. Water flows at the rate of 10 m per minute from a cylindrical pipe of radius 2.5 mm. A conical vessel whose diameter is 40 cm and depth 24 cm is filled with water flowing from this pipe. The time taken to fill the conical vessel is:
(a) Less than 30 mins
(b) Less than 50 minutes but more than 30 minutes
(c) More than 50 mins but less than 75 minutes
(d) More than 75 minutes

15. If a regular square pyramid has a base of side 8 cm and height of 30 cm, then what is its volume (in cc)?
(a) 360 (b) 480
(c) 640 (d) 800

16. A cylinder circumscribes a sphere. The ratio of their volumes is:
(a) 2:1 (b) 3:2
(c) 4:3 (d) 6:5

17. Find the area of the shaded region in the given Fig. 13.14 of square *ABCD*:

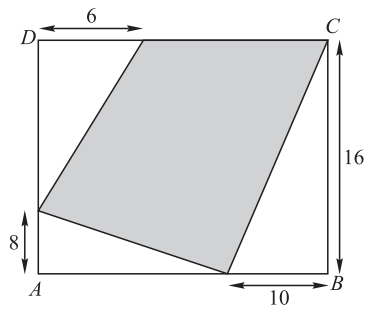


Fig. 13.14

(a) 128 cm^2 (b) 184 cm^2
(c) 154 cm^2 (d) 168 cm^2

18. An open box is made of wood 2 cm thick. Its internal dimension is 86 cm × 46 cm

× 38 cm. What is the cost of painting the outer surface of this box @ ₹ 10 per m^2?
(a) ₹ 12.35 (b) ₹ 8.85
(c) ₹ 15.70 (d) ₹ 16.50

19. A spherical ball was painted black. After getting painted, it was cut into 4 similar pieces. What is ratio of the painted area to the non-painted area?
(a) 1:1 (b) 1:2
(c) 3:1 (d) 3:2

20. Ratio of diagonals of two cubes is 3:2. What is the ratio of the surface areas of these two cubes, respectively?
(a) 27:8 (b) 3:2
(c) 9:4 (d) 16:9

EXERCISE 2

1. What is the radius of a spherical ball in inches which is formed by melting a cylinder of base diameter 8 inches and height 160 inches, if the conversion wastage results in a 10% weight loss?
(a) 6 (b) 8
(c) 12 (d) 16

2. A solid wooden toy is in the shape of a right circular cone mounted on base of a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wood needed to construct another such toy.
(a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^3

3. A well is dug 20 feet deep and the mud which came out is used to build a wall of width 1 feet around the well on the earth. If the height of the wall around the well is 5 feet, then what is the radius of the well?
(a) $\frac{\sqrt{5}+1}{4}$ (b) $\sqrt{5}$
(c) 1 (d) $\frac{1}{4}$

4. The foot of a ladder 15 m long is 9 m from a house and its top reaches the upper part of a circular window. When the foot is drawn away to a distance of 3 m remote from the house, the top reaches the lower edge of the window. The area of the window is:
(a) $5\frac{2}{3}$ m^2 (b) 6 m^2
(c) $7\frac{1}{14}$ m^2 (d) $8\frac{2}{3}$ m^2

5. A brick wall is to be built 20 m long, 3 m high and 40 cm thick. It has a door 3 m by 2 m. Supposing each brick is 15 cm long, $7\frac{1}{2}$ cm broad and 5 cm thick, how much will be the cost of bricks at the rate of ₹ 800 per thousand of bricks?
(a) ₹ 55,270 (b) ₹ 66,230
(c) ₹ 30,720 (d) ₹ 15,750

6. A rectangular field is of the dimension 15.4 m × 12.1 m. A circular well of 0.7 m radius and of 3 m depth is dug in the field. The mud dug out from the well is spread in the field. By how much would the level of the field rise?
(a) 1 cm (b) 2.5 cm
(c) 3.5 cm (d) 4 cm

7. Dimension of a room is thrice as long as it is high, but only twice as long as it is wide. Total cost of painting its walls @ ₹ 2.50 m^2 is ₹ 360. What is the cost of laying carpet on its floor @ ₹ 3/m^2?
(a) ₹ 81 (b) ₹ 125
(c) ₹ 216 (d) ₹ 260

8. A man by walking diametrically across a circular grass plot, finds that it has taken 45 seconds less than if he had kept to the path round the outside, if he walks at the rate of 80 m per minute. The diameter of the grass plot is:
(a) 35 m (b) 65 m
(c) 105 m (d) 145 m

9. Within a rectangular courtyard of length 60 feet, a gravelled path 3 feet wide is laid down along all the four sides. The cost of gravelling the path is ₹ 2 per $feet^2$. If the

path had been twice as wide, the gravel would have cost ₹ 984 more. The width of the courtyard is:

(a) 24 feet (b) 40 feet
(c) 45 feet (d) 54 feet

10. A vessel 2 m long, 1 m wide and 1.5 m deep contains 2 m^3 water. How many bricks 20 cm by 10 cm by 7.5 cm can be put in it so that water does not overflow provided that a brick is supposed to absorb 1/7 of its own volume of water?

(a) 666 (b) 111
(c) 555 (d) None of these

11. There is a playground inside Snehans Apartment of the dimension $7\frac{1}{2}$ m by 7 m. In the centre of the ground, there is a flower-bed cutout of the dimension 3 m by $2\frac{1}{2}$ m. What fraction of the whole ground is occupied by the flower-bed?

(a) $\frac{3}{7}$ (b) $\frac{3}{5}$
(c) $\frac{1}{7}$ (d) $\frac{1}{11}$

12. The height of a room is $\frac{1}{5}$ of the sum of its length and breadth. Cost of preparing its wall @ ₹ 4 per m^2 is ₹ 640. What is the height of the room?

(a) 4 m (b) 5 m
(c) 6 m (d) 7 m

13. A cylindrical container of 32 cm height and 18 cm radius is filled with sand. Now all this sand is used to form a conical heap of sand. If the height of the conical heap is 24 cm, the what is the radius of its base?

(a) 12 cm (b) 24 cm
(c) 36 cm (d) 48 cm

14. Total area of four walls of a room is 150 m^2. If the area of the floor is 50 m^2 and the width of the floor is 3 m, then find the height of the room.

(a) 2.6 m (b) 3.8 m
(c) 5.42 m (d) 7.32 m

15. Three equal cubes of unit side length are placed adjacent to each other in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of all the three cubes.

(a) 3:5 (b) 4:5
(c) 6:7 (d) 7:9

16. A classroom is to be built to accommodate 70 students. It should be done in such a way that for every student there is 2.2 m^2 of floor and 11 m^3 of space. If the length of the room is 14 m, then find the breadth and height of the room.

(a) 12 metres, 5.5 metres
(b) 11 metres, 5 metres
(c) 13 metres, 6 metres
(d) 11 metres, 4 metres

17. A cone and a cylinder have their height in the ratio 3:2 and the radii of their bases in the ratio 4:3. Find the ratio of their volumes.

(a) 9:1 (b) 9:2
(c) 8:9 (d) 3:1

18. A cylindrical structure standing on its base with radius 1.5 m and height 5 m is cut with a saw in such a way that the cutting planes go through all the points at a distance of 0.625 m from the base. Find volume of the remaining piece:

(a) 5.62π (b) 9.24π
(c) 9.04π (d) None of these

19. The height of a right circular cylinder is 6 m. Three times the sum of the areas of its two circular faces is twice the area of its curved surface. The radius of the base is:

(a) 4 m (b) 2 m
(c) 6 m (d) 1.5 m

20. A solid cone kept on its base is cut at 2/3rd of its height along a plane parallel to its circular base. The base radius and the slant height are 14 cm and 50 cm, respectively.

What is the ratio of the portion cut-out from the solid to the volume of remaining solid?
(a) 1:20 (b) 1:25
(c) 1:36 (d) None of these

21. A rectangular tank is of dimension 30 m × 20 m. Water is being flown into it through a square pipe of side length 5 cm. Find the speed of water if the level of water in the tank rises by 1 m in 8 hours?
(a) 30 km/hr (b) 36 km/hr
(c) km/hr (d) None of these

22. A cube of side length 3 cm weighs 12 kg. What is the weight of similar cube of same material whose side length is 12 cm?
(a) 768 kg (b) 678 kg
(c) 964 kg (d) 864 kg

23. A cube of side length 4 cm is cut into cubes of side 1 cm. Find the ratio of sum of surface area of all the small cubes to that of the large cube.
(a) 1:16 (b) 2:3
(c) 4:1 (d) 6:1

24. A cylindrical well of depth 12 m with internal ratio 1.75 m is dug up. The mud that came out from it is spread evenly to form a platform 10.5 m × 8.8 m. What is the height of the platform?
(a) 2.25 m (b) 3.25 m
(c) 1.25 m (d) 4.25 m

25. Diameter of a cylindrical vessel is made twice as large. How should the height be changed so that it still contains the same quantity of water?
(a) $\frac{1}{2}$ × height of two cylinder
(b) $\frac{1}{4}$ × height of original cylinder
(c) Let it be same
(d) None of these

ANSWER KEY

Exercise 1

1. (c)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (b)
9. (c)	**10.** (d)	**11.** (b)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (b)
17. (a)	**18.** (c)	**19.** (a)	**20.** (c)				

Exercise 2

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (c)	**8.** (c)
9. (b)	**10.** (d)	**11.** (c)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (b)
17. (c)	**18.** (d)	**19.** (a)	**20.** (d)	**21.** (a)	**22.** (a)	**23.** (c)	**24.** (c)
25. (b)							

14 Permutations and Combinations

PERMUTATION AND COMBINATION

If we go by the dictionary meaning of the words Permutation and Combination, then Permutation is the number of ways in which a set or number of things can be ordered or arranged and Combination means the number of ways in which a group of things can be chosen from a larger group without giving regard to their arrangement.

Let us go through an example:

Suppose there are four different batsmen *A*, *B*, *C* and *D*. And we have to select a group of three batsmen out of these four batsmen. Now, we can select any combination of three batsmen so that no set of batsmen has all the three batsmen same. These set of batsmen will be *ABC*, *BCD*, *ABD* and *ACD*. This is a case of combination as for every set of selection of three batsmen, order of selection does not play any role (*i.e.*, we can select anybody first or second or third and it creates no difference in the final selection as well as the total no. of selection).

Now, if we try to define their batting order also, *i.e.*, who bats first and 2nd and so on, then corresponding to every selection of a set of three batsmen, we will have six different arrangements of their batting. It can be seen below that corresponding to selection of *ABC* as a team, following is the list of different batting orders:

ABC, *ACB*, *BAC*, *BCA*, *CAB* and *CBA*

Now this is a case of permutation since order of occurrence has become important.

Since there are four different ways of selecting a group of three batsmen and every selection can be arranged in 6 different ways, so total number of ways of arranging 3 batsmen (or distinct things) out of 4 batsmen (or distinct things) = $4 \times 6 = 24$ ways.

Permutation and Combination can be better understood through examples of Hand-shake and Gifts exchange also. Assume that there are 20 persons in a party and everybody shakes hand with each other and presents a gift also. Now if we take a case of two persons *A* and *B*, then the event of shaking hand between them is a case of Combination because when *A* shakes hand with *B* or *B* shakes hand with *A*, the number of hand shake is just one. So, there is no order as such and hence, it is a case of Combination.

Similarly, event of presenting the gift is a case of Permutation because gift given to *B* by *A* and gift given to *A* by *B* are two different gifts. So, the order of case plays a role here and hence, it is a case of Permutation.

Standard Meanings and Definitions

(a) $n! =$ Product of all the natural numbers from n to $1 = n(n-1)(n-2)(n-3) \ldots \times 3 \times 2 \times 1$

(b) $0! = 1$

Note: Factorials are defined only for whole numbers, and not for negative numbers or fractions.

Fundamental Principle of Counting: Two Basic Theorem

Multiplication theorem

If there are two jobs in such a way that one of them can be done in m ways and when it is

completed in any of the ways out of m ways, second job can be completed in n ways, then the whole job can be done in $m \times n$.

Addition theorem

If there are two jobs in such a way that one of them can be done in m ways and second one can be done in n ways independently, then either of the jobs can be done in $(m + n)$ ways.

Basically, there's one point where these two theorems differ. in multiplication the job does not get completed while in addition it gets completed.

In layman's language, we multiply the no. of ways when job has not been completed and we add the no. of ways when jobs have been completed.

Permutations

As we have seen, each of the arrangements that can be made by taking some or all elements out of a number of things is called a Permutation. Permutation implies "arrangement" or that "order of the things" is important.

The permutations of three things a, b and c taken two at a time are ab, ba, ac, ca, cb and bc. Since the order in which the things are taken is important, ab and ba are counted as two different arrangements.

The number of permutations of n things taking r at a time is denoted by ${}^nP_r$.

Combinations

As we have seen, each of the groups or selections which can be made by taking some or all out of a number of things is called a Combination. In combinations, the order in which the things are taken is not important.

The combination of three things a, b and c taken two at a time are ab, bc and ca. Here, ab and ba are same because the order in which a and b are taken is not important. What is required is only a combination including a and b. The words "combination" and "selection" can be used without any differentiation.

The number of combinations of n things taking r at a time is denoted by ${}^nC_r$.

Meaning and derivation of ${}^nP_r$ and ${}^nC_r$

Number of Permutations of n different things taken r at a time = ${}^nP_r$

In this statement, we are doing following two assumptions:

- All the n things are distinct or distinct (or no two things are of the same type).
- Each thing is used at most once (*i.e.*, no thing being repeated in any arrangement).

The number of arrangements of n distinct things taken r things at a time is

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

If we take n things at a time, then we get ${}^nP_n$. From a discussion similar to that we had for filling the r boxes above. We can find that ${}^nP_n$ is equal to $n!$ The first box can be filled in n ways, the second one in $(n - 1)$ ways, the third one in $(n - 2)$ ways and so on, then the nth box in 1 way; hence, all the n boxes can be filled in ${}^nP_n = n!$

But if we substitute $r = n$ in the formula for ${}^nP_n$ then we get ${}^nP_n = \frac{n!}{O!}$; since we already found that ${}^nP_n = n!$

We can conclude that $0! = 1$

Number of combinations of n distinct things taken r at a time = ${}^nC_r$ and ${}^nC_r = \frac{n!}{(n-r)!} \times \frac{1}{r!}$

It can also be deduced from here that the number of selections of n distinct things taken all at a time will be equal to 1 (Since, there is only one way in which all the articles can be selected).

Alternatively $\quad {}^nC_n = \frac{n!}{0! \cdot n!} = 1$

Out of n things kept in a bag, if we select r things and remove them from the bag, we are left with $(n - 1)$ things inside the bag—that is, whenever r things are selected out of n things, we automatically have another selection of the $(n - 1)$ things. Hence, the number of ways of making combinations taking r out of n things is the same as selecting $(n - r)$ things out of n given things, *i.e.*, ${}^nC_r = {}^nC_{n-r}$.

In other words, if we carve out r things out of n things, it means $(n - r)$ things have been also partitioned in second part. So if we select r things, remaining $(n - r)$ have been selected automatically and vice versa. For example, if I select 3 persons out of 8 persons to be put in a room, I would've automatically selected the remaining (8 – 3) 5 persons to be put outside the room (in a group). Combination does not differentiates between who goes inside the room and who remains outside the room (Fig. 14.1):

Fig. 14.1

Before we move ahead, let's make it clear once again that whenever we are using ${}^nC_r$ and ${}^nP_r$, our assumption is that all the things are distinct *i.e.*, no two of them are same.

Some calculation tips

$${}^nC_0 = {}^nC_n = 1$$

For example,

$${}^{10}C_0 = {}^{10}C_{10} = 1$$

$${}^nC_1 = {}^nC_{n-1} = n$$

For example,

$${}^{10}C_1 = {}^{10}C_9 = 10$$

$${}^nC_2 = {}^nC_{n-2} = \frac{n(n-1)}{2}$$

For example,

$${}^{10}C_2 = {}^{10}C_8 = \frac{10(10-1)}{2}$$

$$= \frac{10 \cdot 9}{2} = 45$$

In fact, ${}^nC_2$ is also equal to:

1. Number of handshakes in a party among n persons where everybody shakes hand with everybody else = ${}^nC_2$.
   For example, if there are 20 persons in a party and everybody shakes hand with every other person, then total number of handshakes
   $$= {}^{20}C_2 = \frac{20(20-1)}{2} = \frac{20 \cdot 19}{2} = 190.$$
2. Maximum total number of points of intersection among n straight lines in the same plane = ${}^nC_2$.
   Note 1: Question will ask you the 'maximum' number of points of intersection, as it is not a fixed number of points of intersection among n straight lines.
   Note 2: Maximum total number of points of intersection among n circles in the same plane = ${}^nP_2$.
3. Total number of matches in a round robin league tournament among n teams where each team plays a match with every other team = ${}^nC_2$.
   Note 1: Total number of matches to declare the champion in a knockout tournament (where losing one match will oust you from the tournament) = $(n - 1)$.

Example 1

In a party where everybody shakes hand with everybody else, total number of handshake = 91. How many persons are present in that party?

Solution

Total number of handshake in this situation

$${}^nC_2 = \frac{n(n-1)}{2} = 91$$

$\Rightarrow n(n-1) = 91 \times 2 = 182 = 14 \times 13$

$\Rightarrow$ n = total number of persons = 14

Example 2

In the recently concluded IPL champions league, there were 8 teams in one group. Every team plays two matches with every other team in the group. How many matches have been played in this round in this group?

Solution

Total number of matches in this case if there is one match between any two teams

$${}^8C_2 = \frac{8 \cdot 7}{2} = 28$$

Total number of matches played because there are two matches between any two teams

$= 2 \times 28 = 56$ matches

Examples on Combination

Example 3

Mun has 10 children. She takes 3 of them to the zoo at a time as often as she can, but she does not take the same three children to the zero more than once. How many time Mun will be required to go to zoo?

(a) 120 (b) 45
(c) 90 (d) 180

Solution

Number of times (read ways) 3 children (read distinct things) can be selected from 10 children (read distinct things) = ${}^{10}C_3$. So, she will be required to go to zoo ${}^{10}C_3$ times. So, option (a) is the answer.

Example 4

In the above question, how many times a particular child will go?

(a) 72 (b) 45
(c) 90 (d) 36

Solution

Consider the case for any particular child C_1: Since C_1 has already been selected, so out of the rest 9 children Munchun will be required to select 2 more children. This can be done on ${}^{9}C_2$ ways.

So, option (d) is the answer.

Example 5

In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is:

(a) 200 (b) 216
(c) 235 (d) 256

Solution

Let there be m boys and n girls.

$${}^{n}C_2 = \frac{n(n-1)}{2}$$

$\Rightarrow \quad n(n-1) = 90$

$\Rightarrow \quad n = 10$

$${}^{m}C_2 = 190$$

$\Rightarrow \quad \dfrac{m(m-1)}{2} = 190$

$\Rightarrow \quad m(m-1) = 380$

$\Rightarrow \quad m = 20$

Number of games between one boy and one girl = $10 \times 20 = 200$. Hence, option (a).

Example 6

In how many ways can three persons be seated on five chairs?

Solution

This question is a very fundamental problem of arrangements without repetition. The first person can seat in 5 ways (into any of the five chairs), the second person can take place in 4 ways (into any of the remaining 4 chairs) and the third person can seat in 3 ways.

So, total number of ways in which these 3 persons can arrange themselves on 5 chairs is $5 \times 4 \times 3 = 60$.

Circular permutation

When n distinct things are to be arranged in a straight line, we can do this in $n!$ ways. However, if these n things are arranged in a circular manner, then the number of arrangements will not be $n!$.

Let us understand this:

Number of ways A, B and C can be arranged in a straight line = $3! = 6$.

The possible arrangements are ABC, ACB, BAC, BCA, CAB, CBA.

Now arrange these three people A, B and C in a circle (Fig. 14.2):

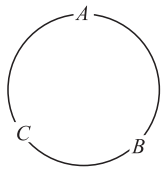


Fig. 14.2

What we can observe here is that arrangements *ABC*, *BCA* and *CAB* are same. And similarly arrangements *ACB*, *CBA* and *BAC* are same. So, there are only two permutations in this case of circular permutation. To derive the formula for Circular Permutation, we first fix the position of one thing. Then the remaining $(n-1)$ things can be arranged in $(n-1)!$ ways.

Hence, the number of ways in which *n* distinct things can be arranged in a circular arrangement is $(n-1)!$

1. Fundamental Principle of counting:
   (a) Multiplication rule: If a work is done only when all of a number of works are done, then number of ways of doing that work is equal to the product of number of ways of doing separate works.
   (b) Addition rule: If a work is done only when any one of a number of works is done, then number of ways of doing that work is equal to the sum of number of ways of doing separate works.
   Thus if a work is done when exactly one of a member of works is done, then number of ways of doing this work = sum of number of ways of doing all the works.
2. If $\quad {}^nC_x = {}^nC_y$
   then either $x = y$ or $x + y = n$
3. $= 1 \times 2 \times 3 \times 4 \times \ldots n$
4. (a) Number of permutation of n different articles taken r at a time is denoted by ${}^nP_r$ and ${}^nP_r = \dfrac{\lfloor n}{\lfloor n-r}$.
   (b) Number of permutations of n different articles taking all at a time
   (c) Number of permutations of N articles, out of which P are alike and are of one type, Q are alike and are of second type and rest are all different $= \dfrac{N!}{P!Q!}$.
5. Number of permutations (arrangements) of n different articles taking r at a time when articles can be repeated any number of times $= n \times n \times \ldots r$ times $= n^r$.
6. Circular permutations:
   (a) Number of circular permutations (arrangements) of n different articles $\lfloor n-1$.
   (b) Number of circular arrangements of n different articles when clockwise and anticlockwise arrangements are not different, *i.e.*, when observation can be made from both sides $= \dfrac{\lfloor n-1}{2}$.
7. Number or combinations of n different articles taken r at a time is denoted by ${}^nC_r$ and ${}^nC_r = \dfrac{\lfloor n}{\lfloor r \lfloor n-r}$.
8. Number of selections of r articles $(r \le n)$ out of n identical articles is 1.
9. Total number of selections of zero or more articles from n distinct articles $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.
10. Total number of selections of zero or more articles from n identical articles $= 1 + 1 + 1 + \ldots$ to $(n+1)$ terms $= n + 1$.
11. Number of ways of distributing n identical articles among r persons when each person may get any number of articles $= {}^{n+r-1}C_{r-1}$.
12. Number of ways of dividing $m + n$ different articles in two groups containing

m and n articles respectively ($m \neq n$) $= {}^{m+n}C_n \times {}^{m}C_m = \frac{\lfloor m+n}{\lfloor m \lfloor n}$.

13. Number of ways of dividing $2m$ different articles each containing m articles $= \frac{\lfloor 2m}{\lfloor m \lfloor m \lfloor 2}$.

14. Number of ways of selecting n distinct articles taken r at a time when p particular articles are always included $= {}^{n-p}C_{r-p}$.

15. ${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r$.

16. Number of diagonals in an n-sided regular polygon $= {}^{n}C_2 - n = n(n-3)/2$.

17. Maximum number of point of intersection among n lines $= {}^{n}C_2$.

Some Standard Formats of Questions

Word formation

As we know that order of occurrence of letters decide the formation of words, so Word Formation is one standard example of Permutation.

Let us understand with the help of some examples:

Example 7

How many words can be formed with the word 'LUCKNOW' which are having

(i) No restriction is there.
(ii) L is the 1st letter of the word.
(iii) L and W are the terminal letters.
(iv) All the vowels are together.
(v) L always occurs before U.
(vi) L always occurs before U and U always occurs before W.

Solution

(i) Total number of distinct letters = 7 (L, U, C, K, N, O, W).
So, total number of words that can be formed = 7!

(ii) Now we can arrange only 6 letters (As place of L is restricted).
So, total number of words that can be formed = 6!

(iii) Now we can arrange only 5 letters (As place of L and W are restricted).
So, Number of arrangements = 5!
But place of L and W can be interchanged between themselves.
So, total number of words that can be formed = 5! × 2!

(iv) U and O should be together, so we will assume these two letters to be tied up with each other.
Now we have 6 distinct things to be arranged (L, UO, C, K, N, W).
So, number of arrangements = 6!
But place of U and O can be interchanged between themselves.
So, total number of words that can be formed = 5! × 2!

(v) Occurrence of all the letters in the word are having equal likelihood, so in half of the cases L will occur before U and in the remaining half, U will occur before O.
So, total number of words that can be formed = 7!/2.

(vi) There are six possible arrangements (3!) corresponding to L, U and W. However only one out of these six will be in the prescribed order: L always occurs before U and U always occurs before W.
So, corresponding to 7! arrangements, no. of ways in which the condition will be satisfied = 7!/3! ways.

Example 8

How many new words can be formed with the word 'PATNA'?

Solution

Total number of letters are P, T, N are occurring once while A is occurring twice.

So, total number of words that can be formed = 5!/2! = 60

Total number of new words = 60 –1 = 59.

Example 9

How many words can be formed with the word 'ALLAHABAD'?

Solution

Letters are A—Four times

L—Twice

H, B and D are occurring once.

So, total number of words $= \frac{9!}{4!2!}$.

Number formation

Number Formation is another standard example of Permutation. Here we will introduce BOX Diagram method of solving the questions.

If a three digit number is to be constructed, then we will use

Hundred's place	Ten's place	Unit's place

If a four digit number is to be constructed, then we will use

Thousand's place	Hundred's place	Ten's place	Unit's place

And so on.

While solving questions related to **Number Formation**, we should have two things very clearly in our mind:

1. While using box diagram, we should start with the digit which is having any restriction, *i.e.*, some condition is imposed on that digit.
2. When nothing about repetition of digits is mentioned in the question, we will have to assume that repetition is allowed.

Example 10

How many different 3 digit numbers can be formed using the digits 1, 2, 3, 4 and 5:

(i) When repetition is not allowed

(ii) When repetition is allowed

Solution

Below given Box represents the respective positioning of digits in a three digit number.

Hundred's place	Ten's place	Unit's place

(i) Since repetition of the digits is not allowed, we can fill Unit's place in 5 ways, Ten's place in 4 ways and Hundred's place in 2 ways.

3	4	5

Using Multiplication Theorem, Total number of numbers which can be formed = 5 × 4 × 3 = 60.

Alternatively, 3 digits can be selected out of 5 digits in ${}^5P_3 = 60$.

(ii) Since repetition of the digits is allowed here, we can fill each of the hundred's, ten's and Unit's place in 5 ways.

5	5	5

Using Multiplication Theorem, Total number of numbers which can be formed = 5 × 5 × 5 = 125.

Example 11

How many 4 lettered numbers divisible by 4 can be formed from the digits 0, 1, 2, 3, 4, 5?

Solution

Any number divisible by 4 will have number formed by its last two digits divisible by 4.

In this case, last two digits of the number can be 00, 04, 12, 20, 24, 32, 40, 44, 52.

Corresponding to any one of 00, 04, 12, 20, 24, 32, 40, 44, 52, we can have the following digits at its Hundred's and Thousand's place:

5	6	1	1

Thousand's place cannot be filled by 0, so it can be filled in 5 ways.

Hundred's place can be filled by any of the 0, 1, 2, 3, 4, 5; hence, 6 ways.

So, corresponding to any one of 00, 04, 12, 20, 24, 32, 40, 44, 52, total number of ways = 5 × 6 = 30.

So, total number of numbers which can be formed = 30 × 9 = 270.

Example 12

In the above question, how many numbers can be formed if repetition of the digits is not allowed?

Solution

Last two digits of this number can be 04, 12, 20, 24, 32, 40, 52.

At this point now, we will have to bifurcate the process of solving this question—one part will be having those numbers which are containing '0' as one of its last two digits *viz.*, 04, 20, 40 and other part will be having remaining numbers *viz.*, 12, 24, 32, 52.

1st part—Last two digits are 04, 20, 40.

4	3	1	1

= 4 × 3 = 12 ways

Hence, total number of numbers which can be formed = 12 × 3 = 36.

2nd part—Last two digits are 12, 24, 32, 52.

'0' cannot occur at thousand's place.

3	3	1	1

= 3 × 3 = 9 ways

Hence, total number of numbers which can be formed = 9 × 4 = 36.

Total numbers = 36 + 36 = 72.

Example 13

How many odd integers from 1,000 to 8,000 have none of its digits repeated?

Solution

There are two restrictions in this question:

(i) For a number to be odd, unit digit should be either 1 or 3 or 5 or 7 or 9.

(ii) Thousand's place cannot be filled with 8 or 9.

For unit's digit—When it is filled with 9, thousand's place can be filled in 7 ways namely any digit from 1 to 7, and remaining two places can be filled in 8 × 7 = 56 ways.

So, total number of numbers formed in this way = 56 × 7 = 392.

Now, if unit's place is filled with any of the four digits 1, 3, 5 or 7, the thousand's place can be filled in 6 ways (0 will be excluded), and remaining two places can be filled in 8 × 7 = 56 ways.

So, total number of numbers formed in this way = 56 × 6 × 4 = 1,344.

So, total number of numbers = 392 + 1344 = 1,736.

Example 14

How many integers from 6,000 to 6,999 have atleast one of its digits repeated?

Solution

Total number of numbers = None of its digits repeated numbers + atleast one of its digits repeated number (*i.e.*, either digits will be repeated or not repeated).

Total numbers with none of its digits repeated = 1 × 9 × 8 × 7 = 504.

So, numbers having atleast one of its digits repeated = 1000 – 504 = 496.

Example 15

How many natural numbers less than a million can be formed using the digits 0, 7 and 8?

Solution

Numbers formed would be of Single digit, Two digits, Three digits, Four digits, Five digits and Six digits.

Single digit numbers = 7 and 8.

For two digits numbers,

2	3

= 2 × 3 = 6 numbers

For three digits numbers,

2	3	3

= 2 × 3 × 3 = 18 numbers

For four digits numbers,

2	3	3	3

= 2 × 3 × 3 × 3 = 54 numbers

For five digits numbers,

2	3	3	3	3

$= 2 \times 3 \times 3 \times 3 \times 3$
$= 162$ numbers

For six digits numbers,

2	3	3	3	3	3

$= 2 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 486$ numbers

So, total number of numbers = 728.

Example 16

How many diagonals will be there in an octagon (number of sides = 8)?

Solution

Number of diagonals in any polygon with n sides

$$= \frac{n(n-3)}{2} = \frac{8(8-3)}{2}$$

$$= \frac{8 \cdot 5}{2} = 20$$

Note: Result of this question can be used as a general result to find out the total number of diagonals.

PRACTICE EXERCISES

EXERCISE 1

1. How many numbers between 100 and 1,000 can be formed using the digits 0, 2, 4, 6, 8, 5, if repetition of digits in a number is not allowed?
(a) 99 (b) 82
(c) 100 (d) 120

2. How many numbers between 100 and 1,000 can be formed using the digits 0, 2, 4, 6, 8, 5, if repetition of digits in a number is allowed?
(a) 164 (b) 180
(c) 192 (d) 100

3. Two out of six papers set for an examination are of mathematics. What is the number of ways in which the papers can be set so that the two mathematics papers are not together?
(a) 480 (a) 2.520
(c) 492 (d) 512

4. In a letter lock, each of three rings is marked with 15 letters. What is the maximum number of unsuccessful attempts that one has to make before the lock is opened?
(a) 3,374 (b) 5,284
(c) 3^{15} (d) 3,375

5. A certain code consists of 5 variables, with each variable having 4 different constant values possible. What is the total number of coded messages that can be sent with 5 constants one from each variable?
(a) $1024 \times 5!$ (b) $1024 \times 4!$
(c) $1024 \times 3!$ (d) None of these

6. From 3 different soft-drinks, 4 different Chinese dishes and 2 different ice-creams, how many different menus can be planned if at least one of each of the three items is to be included?
(a) 315 (b) 282
(c) 864 (d) 345

7. The Governing Council of an institute has fifteen members and wants to hold its annual meeting. In how many ways can the council be seated around a round table if the Chairman and the Vice-Chairman of the council are always seated together?
(a) $10 \times 12!$ (b) $14 \times 10!$
(c) 13! (c) None of these

8. Akshay is planning to give a birthday party at his place. In how many ways can he invite one or more of five friends and seat them at a circular table?
(a) 84 (b) 89
(c) 78 (d) 81

9. In the above question, in how many ways can he invite one or more of five friends and seat them at a circular table with him?

(a) 325 (b) 205
(c) 265 (d) 320

10. A cricket team of eleven is to be chosen from among 8 batsmen, 6 bowlers and 2 wicket-keepers. In how many ways can the team be chosen if there must be at least four batsmen, at least four bowlers and exactly one wicket-keeper?
(a) 1,681 (b) 5,304
(c) 1,652 (d) None of these

11. How many words can be formed using the letters of the word "CORRESPONDENCE"?
(a) $\frac{8!}{(2!)^4(3!)^1}$ (b) 14!
(c) $\frac{14!}{(2!)^4(3!)^1}$ (d) $\frac{14!}{(2!)^3(3!)^1}$

12. How many words can be formed using the letters of the word "CORRESPONDENCE" if the consonants are always written together?
(a) 182 (b) 184
(c) 216 × 9! (d) None of these

13. If six persons are selected out of ten, in how many ways will a particular person be found among those six?
(a) 124 (b) 126
(c) 144 (d) 84

14. A committee of five is to be chosen from among six men and four ladies. In how many ways can this be done in order to include at least one lady?
(a) 252 (b) 246
(c) 244 (d) 152

15. How many ways are there to pick two different cards from a deck of 52 cards such that the first card is an Ace and the second is not a Queen?
(a) 188 (b) 198
(c) 164 (d) None of these

16. In the above question, what is the number of ways such that the first card is a spade and the second is not a Queen?
(a) 611 (b) 612
(c) 164 (d) None of these

17. How many different license plates of six entities involving three letters and three digits are there if three letters appear together, either at the beginning or at the end of the license?
(a) $2 \times 26^3 \times 10^3$ (b) 54,102
(c) $4 \times 25^2 \times 10^4$ (d) None of these

18. A joint student-teacher committee of 5 members is to be formed from among 4 teachers, 3 male students and 5 female students. How many different committees can be formed if the committee must consist at least 2 teachers, 1 male student and 2 female students?
(a) 170 (b) 152
(c) 180 (d) 104

19. There are 8 different locks, with exactly one key for each lock. All the keys have been mixed up. What is the maximum number of trials required in order to determine which key belongs to which lock?
(a) 44 (b) 28
(c) 24 (d) 32

20. Hoppers' Stop stocks four styles of trousers. For each style, there are ten different possible waist sizes, six different trousers lengths and four colour choices. How many different types of trousers could the store have?
(a) 1,024 (b) 960
(c) 921 (d) 924

EXERCISE 2

1. In a group of boys, the number of arrangements of 4 boys is 12 times the number of arrangements of 2 boys. The number of boys in the group is:
(a) 10 (b) 8
(c) 6 (d) None of these

2. From a group of persons the number of ways of selecting 5 persons is equal to that of 8 persons. The number of persons in the group is

(a) 13 (b) 40
(c) 3.18 (d) 21

3. For the BCCI, a selection committee is to be chosen consisting of 5 ex-cricketers. Now there are 10 representatives from four zones. It has further been decided that if Kapil Dev is selected, Sunil Gavaskar will not be selected and vice versa. In how many ways can this be done?
(a) 140 (b) 112
(c) 196 (d) 56

4. At a get-together, it was found that a total of 66 handshakes took place at the conclusion of the party. If each guest shook hand only once will with all the others, how many people were present.
(a) 33 (b) 22
(c) 12 (d) 13

5. A committee is to be formed comprising of 7 members such that there is a majority of men and at least 1 woman in every committee. The shortlist consists of 9 men and 6 women. In how many ways can this be done?
(a) 3,724 (b) 3,630
(c) 3,526 (d) ,4914

6. The number of 6 digit numbers that can be made with the digits 0, 1, 2, 3, 4 and 5 so that even digits occupy odd places, is:
(a) 24 (b) 36
(c) 48 (d) None of these

7. The total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together is:
(a) $\frac{7!}{3!}$ (b) $6! \times \frac{7!}{3!}$
(c) 3. 35 (d) None of these

8. Let A be the set of 4 digit numbers $a_1 a_2 a_3 a_4$ where $a_1 > a_2 > a_3 > a_4$ then how many values of A are possible?
(a) 126 (b) 84
(c) 210 (d) None of these

9. The total number of selections of at most n things from $(2n + 1)$ different things is 63. Then the value of n is:
(a) 3 (b) 2
(c) 4 (d) None of these

10. From 4 gentlemen and 6 ladies a committee of five is to be formed. Then number of ways in which the committee can be formed so that gentlemen are in majority is:
(a) 66 (b) 156
(c) 60 (d) None of these

11. The total number of 9 digit numbers of different digits is:
(a) 10 (9!) (b) 8 (9!)
(c) 9 (9!) (d) None of these

12. The number of all four digit numbers which are divisible by 4 that can be formed from the digits 1, 2, 3, 4 and 5 is:
(a) 125 (b) 30
(c) 95 (d) None of these

13. The total number of integral solutions for (x, y, z) such that $xyz = 24$ is:
(a) 36 (b) 90
(c) 120 (d) None of these

14. The greatest possible number of points of intersection of 8 straight lines and 4 circles is:
(a) 32 (b) 64
(c) 76 (d) 104

15. The number of positive integral solutions of $x + y + z = n$, $n \in N$, $n > 3$, is:
(a) ${}^{n-1}C_2$ (b) ${}^{n-1}P_2$
(c) $n(n - 1)$ (d) None of these

16. If ${}^mC_4$, ${}^mC_5$ and ${}^mC_6$ are in AP then m is:
(a) 8 (b) 9
(c) 14 (d) 9

17. There are 20 questions in a question paper. If no two students solve the same combination of questions but solve equal number of questions then the maximum number of students who appeared in the examination is:

(a) ${}^{20}C_9$ (b) ${}^{20}C_{11}$
(c) ${}^{20}C_{10}$ (d) None of these

18. The number of ways to distribute 32 different things equally among 4 persons is:
(a) $\frac{32!}{(8!)^3}$ (b) $\frac{32!}{(8!)^4}$
(c) $\frac{1}{4}(32!)$ (d) None of these

19. In a dinner party there are 10 Indians, 5 Americans and 5 Englishmen. In how many ways can they be arranged in a row so that all persons of the same nationality sit together?
(a) $10! \times 5! \times 5!$
(b) $20!$
(c) $3! \times 10! \times 5! \times 5!$
(d) $20!\ 3!$

20. In how many ways can 15 I.Sc. and 13 B.Sc. candidates be arranged in a line so that no two B.Sc. candidates may occupy consecutive positions?
(a) $15! \times 13!$
(b) $15! \times {}^{16}P_{12}$
(c) $13! \times {}^{16}P_{12}$
(d) $2! \times 15! \times 13!$

21. How many new words can be formed from the letters of the word 'circle' taken all together?
(a) 720 (b) 719
(c) 360 (d) 359

22. How many different words can be formed with the letters of the word 'Vice-Chancellor' so that the vowels are together?
(a) $10 \times 5!$ (b) $10! \times 5!$
(c) $5 \times 10!/2$ (d) $5 \times 10!$

23. In how many ways can the letters of the word 'Lucknow' be arranged so that the two vowels do not come together?
(a) 720 (b) 1,440
(c) 3,600 (d) None of these

24. How many words can be formed out of the letters of the word 'Article' so that the vowels occupy the even places?
(a) 72 (b) 144
(c) 288 (d) 36

25. How many numbers greater than four million (40,00,000) can be formed with the digits 2, 3, 0, 3, 4, 2, 5, ?
(a) 280 (b) 380
(c) 360 (d) None of these

ANSWER KEY

Exercise 1

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (d)	**7.** (4)	**8.** (2)
9. (a)	**10.** (c)	**11.** (c)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (a)
17. (a)	**18.** (c)	**19.** (b)	**20.** (b)				

Exercise 2

1. (c)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (c)
9. (a)	**10.** (a)	**11.** (c)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (c)
17. (c)	**18.** (b)	**19.** (c)	**20.** (b)	**21.** (d)	**22.** (d)	**23.** (c)	**24.** (b)
25. (c)							

HINTS AND EXPLANATIONS

Exercise 1

1. Number has to be of three digits. Zero cannot come at the 1st place.

Ways	5	5	4
	100's Place	Tens Place	Unit Place

Hence, number of numbers formed

$= 5 \times 5 \times 4 = 100$

Hence, option (c) is the answer.

2. Number has to be of three digits.

Ways	5	6	6
	100's Place	Tens Place	Unit Place

Hence, number of numbers formed

$= 5 \times 6 \times 6 = 180$

Hence, option (b) is the answer.

3. Total number of ways papers can be set without any restriction = 6!

Total number of ways in which both the Maths papers will be together = $5! \times 2$

Hence, number of ways the two Mathematics papers are not together

$= 6! - 5! \times 2 = 720 - 240$

$= 480$

Hence, option (a) is the answer.

4. Total number of attempts

$= 15 \times 15 \times 15 = 3375$

Maximum number of unsuccessful attempts = One less than the maximum total attempts = 3374. Hence, option (a) is the answer.

6. Soft drink can be chosen = either 1 or 2 or 3

Chinese dishes can be chosen = either 1 or 2 or 3 or 4

Ice Cream can be chosen = either 1 or 2

Soft Drink	1	2	3		Total Ways
Ways it can be done	${}^3C_1$	${}^3C_2$	${}^3C_3$		${}^3C_1 + {}^3C_2 + {}^3C_3 = 2 \wedge 3 - 1 = 7$
Chinese Dishes	1	2	3	4	
Ways it can be done	${}^4C_1$	${}^4C_2$	${}^4C_3$	${}^4C_4$	${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2 \wedge 4 - 1 = 15$
Ice Cream	1	2			
Ways it can be done	${}^2C_1$	${}^2C_2$			${}^2C_1 + {}^2C_2 = 2 \wedge 2 - 1 = 3$

Total number of ways menu can be chose $= 7 \times 15 \times 3 = 345$. Hence, option (d) is the answer.

7. Total number of members to be arranged = 13 members + (one pair of Chairman + Vice Chairman) = 14

Number of ways circular arrangement can be done $= (14 - 1)! \times 2! = 13! \times 2!$. Hence, option (d) is the answer.

8. He can invite either one or two or three or four or all five friends. Question is asking "for the friends (seat them) arrangements:

(a) Number of ways One friend can be chosen $= {}^5C_1 = 5$

Circular arrangements possible if there is only one friend $= (1 - 1)! = 0! = 1$

Hence, total number of arrangements $= 5 \times 1 = 5$ (i)

(b) Number of ways two friends can be chosen $= {}^5C_2 = 10$

Circular arrangements possible if there are two friends $= (2 - 1)! = 1! = 1$

Hence, total number of arrangements $= 10 \times 1 = 10$ (ii)

(c) Number of ways three friends can be chosen $= {}^5C_3 = 10$

Circular arrangements possible if there are three friends $= (3 - 1)! = 2! = 2$

Hence, total number of arrangements $= 10 \times 2 = 20$ (iii)

(d) Number of ways four friends can be chosen = ${}^5C_4 = 5$
Circular arrangements possible if there are four friends = (4 – 1)! = 3! = 6
Hence, total number of arrangements = 5 × 6 = 30 (iv)

(e) Number of ways five friends can be chosen = ${}^5C_5 = 1$
Circular arrangements possible if there are five friends = (5 – 1)! = 4! = 24
Hence, total number of arrangements = 1 × 24 = 24 (v)
Total number of ways = Sum of all these five = 5 + 10 + 20 + 30 + 24 = 89. Hence, option (b) is the answer.

9. He can invite either one or two or three or four or all five friends. In this question, Question is asking for arrangements alongwith him.

(a) Number of ways One friend can be chosen = ${}^5C_1 = 5$
Circular arrangements possible if there is only one friend and Akshay = (2 – 1)! = 1! = 1
Hence, total number of arrangements = 5 × 1 = 5 (i)

(b) Number of ways two friends can be chosen = ${}^5C_2 = 10$
Circular arrangements possible if there are two friends and Akshay = (3 – 1)! = 2! = 2
Hence, total number of arrangements = 10 × 2 = 20 (ii)

(c) Number of ways three friends can be chosen = ${}^5C_3 = 10$
Circular arrangements possible if there are three friends and Akshay = (4 – 1)! = 3! = 6
Hence, total number of arrangements = 10 × 6 = 60 (iii)

(d) Number of ways four friends can be chosen = ${}^5C_4 = 5$
Circular arrangements possible if there are four friends and Akshay = (5 – 1)! = 4! = 24
Hence, total number of arrangements = 5 × 24 = 120 (iv)

(e) Number of ways five friends can be chosen = ${}^5C_5 = 1$
Circular arrangements possible if there are five friends and Akshay = (6 – 1)! = 5! = 120
Hence, total number of arrangements = 1 × 120 = 120 (v)
Total number of ways = Sum of all these five = 5 + 20 + 60 + 120 + 120 = 325. Hence, option (a) is the answer.

10. Following cases are possible:

	Batsman	Bowler	Wicketkeeper	Total
Case 1	6	4	1	11
Case2	5	5	1	11
Case 3	4	6	1	11
Case 4	6	4	1	11
Case 5	5	5	1	11
Case 6	4	6	1	11

11. In this word:

Letter	Number of times occurs
R	2
O	2
E	3
C	2
S	1
P	1
N	2
D	1

So total number of words formed

$$= \frac{14!}{(2!)^4(3!)^1}$$

Hence, option (c) is the answer.

12. Vowels are *O*, *E*
Consonants are *R*, *C*, *S*, *P*, *N*, *D*. Consonants will be in one group.
So, total entities to be arranged = *O*, *E*, (Group of consonants) = 3
Total ways of arrangements

$$= \frac{3! \cdot 9! \cdot 5!}{(2!)^4 (3!)^1}$$

This is option (d). Hence, option (d) is the answer.

13. If a particular person is to be selected always ⇒ It means we have to select five person out of nine persons remaining.
Number of ways this can be done = ${}^9C_5$ = 126. Hence, option (b) is the answer.

15. 1st card is an ace = ${}^4C_1 = 4$
Second is not a queen = ${}^{47}C_1 = 47$ (Total 52 – One Ace selected earlier – 4 queens = 47 cards remaining).
Total number of ways it can be done = 4 × 47 = 188. Hence, option (a) is the answer.

16. 1st card is an spade = ${}^{13}C_1 = 13$
Second is not a queen = ${}^{47}C_1 = 47$ (Total 52 – One spade selected earlier – 4 queens = 47 cards remaining)
Total number of ways it can be done = 13 × 47 = 611. Hence, option (a) is the answer.

17. There are total 26 letters and 10 digits.
Number of ways letters can be chosen

$$= \underset{1\text{st}}{26}\ \underset{2\text{nd}}{26}\ \underset{3\text{rd}}{26} = 26^3$$

These can be put at the end or starting.
Hence, number of ways = 2×26^3
Number of ways digits can be chosen = $10 \times 10 \times 10 = 10^3$
Total number of ways = $2 \times 26^3 \times 10^3$.
Hence, option (a) is the answer.

19. Maximum total number of trials for the key to be identified of 1st lock = 7 (once you have used all the seven keys and lock has not opened, it means 8th key is the right key).
And so on.
Maximum total number of trials = 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28. Hence, option (b) is the answer.

20. Total number of ways

$$= {}^4C_1 \times {}^{10}C_1 \times {}^6C_1 \times {}^4C_1$$
$$= 4 \times 10 \times 6 \times 4 = 960$$

Hence, option (b) is the answer.

15 Probability

Probability is "Possibility quantified". In other words, we want to put accuracy to the chance of occurrence of an event. For example, when we say "it may rain tomorrow", exact meaning of 'may' is not clear. Though if we say that "it is 80% probable that it will rain tomorrow", 80% has a clear interpretation about how much probable this event (rain) is:

Probability of an event E, written as $P(E)$, is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes (also known as sample space)}}$$

Points to Remember

1. $0 \le P(E) \le 1$ [Probability of any event lies in between 0 and 1].
   If probability of an event = 0, then we call it Impossible Event (0% chance of occurrence).
   If probability of an event = 1, then we call it Certain Event (100% chance of occurrence).
2. $P(E) + P'(E) = 1$ [Occurrence of an event + non-occurrence of the same event = 1].

Example 1

In a single throw of two die, what is the probability that the sum on the top face of both the dice will be at least 9?

Solution

When two die are thrown, sum of the numbers appearing on the faces can be anything from 2 to 12. To find out the number of favourable cases, we will be required to find out the cases in which sum is more than 9.

Following are the cases: (6, 3), (3, 6), (5, 4), (4, 5), (5, 5), (6, 4), (4, 6), (6, 5), (5, 6), (6, 6)

So, total number of favourable cases = 10

Total number of possible outcomes

$$= 6 \times 6 = 36$$

Hence, probability

$$= \frac{10}{36} = \frac{5}{18}$$

Example 2

Six dices are thrown simultaneously. Find the probability that all of them show the same face.

Solution

Sample space of throwing six dices

$$= 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6$$

All dices are showing the same face implies that we are getting same number on the entire six dices. The number of ways for which is ${}^6C_1$.

Hence, required probability

$$= \frac{{}^6C_1}{6^6} = \frac{1}{6^5}$$

Example 3

In the above question, find the probability that all of them show different face.

Solution

Total number of ways in which all dices show different numbers on their top-faces is same as the number of arrangement of 6 numbers 1, 2, 3, 4, 5, 6 by taking all at a time.

So, number of favourable cases = 6!

Hence, required probability

$$= \frac{6!}{6^6}$$

Example 4

Five persons entered the lift on the ground floor of an 8-floor apartment. Assuming that each one of them independently and with equal probability can leave the lift at any floor beginning with the first, what is the probability that all five persons are leaving the lift at different floors?

Solution

Apart from ground floor, there are 7 floors.

A person can leave the lift at any of the seven floors. Hence, total number of ways in which each of the five persons can leave the lift at any of the 7 floors = 7^5.

So, sample space = 7^5.

Five persons can leave the lift at five different floor ${}^7P_5$ ways.

So, favourable number of ways = ${}^7P_5$.

Hence, the required probability

$$= \frac{{}^6P_5}{7^5}$$

Example 5

If you have 3 tickets to a lottery for which 10 tickets were sold and 5 prizes are to be given, the probability that you will win at least one prize is:

(a) $\frac{7}{12}$ (b) $\frac{9}{12}$

(c) $\frac{1}{12}$ (d) $\frac{11}{12}$

Solution

Probability that you will win at least one prize = 1, probability that you will not win any prize.

$$= 1 - \frac{5C_3}{10C_3} = \frac{11}{12}$$

Example 6

Two coins are tossed at the same time. What is the probability that at least one of them shows a head?

Solution

Total different outcomes

$= (H, H), (H, T), (T, H), (T, T)$

$= 4$

Total favourable cases

$= (H, H), (H, T), (T, H) = 3$

Hence, probability $= \frac{3}{4}$

ODDS in FAVOUR and ODDS in AGAINST

Odds in Favour

$$= \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

Odds in Agaist

$$= \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

Understanding AND/OR

To understand the role played by And/Or in our calculation, let us take the example of throwing an unbiased dice. Let A and B be two events associated with it such that

A = getting an even number, B = getting a multiple of 3.

Then $A = \{2, 4, 6\}$ and $B = \{3, 6\}$.

We now define a new event "A or B" which occurs if A or B or both occur, *i.e.*, at least one of A, B occurs. Clearly the event "A or B" occurs if the outcome is any one of the outcomes {2, 3, 4, 6}. Thus, the event "A or B" is represented by the subset $A \cup B$.

Similarly, "A and B" means occurrence of both A and B which is possible if the outcome is {6}.

Hence, it is represented by the subset $A \cap B$.

Addition Theorem

If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary: If the events are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B)$$

Addition theorem can be extended for any number of events.

Example 7

A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes out 2 at random, what is the probability that either both are apples or both are good fruits?

Solution

Out of 30 items, two can be selected in ${}^{30}C_2$ ways. So, exhausted number of cases = ${}^{30}C_2$.

Consider the following events:

A = getting two apples

B = getting two good fruits

Required probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

There are 20 apples, out of which two can be drawn in ${}^{20}C_2$ ways.

$$\therefore \quad P(A) = \frac{{}^{20}C_2}{{}^{30}C_2}$$

There are 8 defective pieces and the remaining 22 are good. Out of 22 good pieces, two can be selected in ${}^{22}C_2$ ways.

$$\therefore \quad P(B) = \frac{{}^{20}C_2}{{}^{30}C_2}$$

Since there are 15 pieces which are good apples about of which 2 can be selected in ${}^{15}C_2$ ways, therefore $P(A \cap B)$ = Probability of getting 2 pieces which are good apples $= \dfrac{{}^{15}C_2}{{}^{30}C_2}$

From (i), required probability

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$= \frac{316}{435}$$

Conditional Probability

Let A and B be two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and $P(B) \neq 0$, is called the conditional probability and it is denoted by $P(A/B)$.

Thus, $P(A/B)$ = Probability of occurrence of A given that B has already happened.

Similarly, $P(B/A)$ = Probability of occurrence of B given that A has already happened.

Sometimes, $P(A/B)$ is also used to denote the probability occurrence of A when B occurs. Similarly, $P(B/A)$ is used to denote the probability of occurrence of B when A occurs.

Following examples illustrate various meanings of these notations.

Example 8

A bag contains 5 white and 4 red balls. Two balls are drawn from the bag one after the other without replacement. Consider the following events.

A = drawing a white ball in the first draw,

B = drawing a red ball in the second draw.

Now, $P(B/A)$ = Probability of drawing a red ball in second draw given that a white ball has already been drawn in the first draw.

Since 8 balls are left after drawing a white ball in first draw and out of these 8 balls, 4 balls are red, therefore

$$P(B/A) = \frac{4}{8} = \frac{1}{2}$$

Note that $P(A/B)$ is not meaningful in this experiment because A cannot occur after the occurrence of B.

Some More Examples

Example 9

One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is

1. A king
2. Either red or king
3. Red and a king

Solution

Out of 52 cards, one card can be drawn in ${}^{52}C_1$ ways. Therefore, exhaustive number of cases

$$= {}^{52}C_1 = 52$$

1. There are 4 kings in a pack of cards, out of which one can be drawn in ${}^4C_1$. Therefore, favourable number of cases = ${}^4C_1 = 4$. So the required probability $= \frac{4}{52} = \frac{1}{13}$.
2. There are 28 cards in a pack of cards which are either a red or a king. Therefore, one can be drawn in ${}^{28}C_1$ ways. Therefore, favourable number of cases = ${}^{28}C_1 = 28$. So the required probability $= \frac{28}{52} = \frac{7}{13}$.
3. There are 2 cards which are red and king, *i.e.*, red kings. Therefore, favourable number of cases = ${}^2C_1 = 2$. So the required probability $= \frac{2}{52} = \frac{1}{26}$.

Example 10

An urn contains 9 blue, 7 white and 4 black balls. If 2 balls are drawn at random, find the probability that:

1. Both the balls are blue
2. One ball is white

Solution

There are 20 balls in the bag out of which 2 balls can be drawn in ${}^{20}C_2$ ways. So the total number of cases (sample space) = ${}^{20}C_2 = 190$.

1. There are 9 blue balls out of which 2 balls can be drawn in ${}^9C_2$ ways. Therefore, favourable the required probability $= \frac{36}{190}$ $= \frac{18}{95}$.
2. There are 7 white balls out of which one white can be drawn in ${}^7C_1$ ways. One ball from the remaining 13 balls can be drawn in ${}^{13}C_1$ ways. Therefore, one white and one other colour ball can be drawn in ${}^7C_1 \times {}^{13}C_1$ ways. So the favourable number of cases = ${}^7C_1 \times {}^{13}C_1 = 91$. So the required probability $= \frac{91}{190}$.

Example 11

Three persons A, B and C are to speak at a function along with five others. If they all speak in random order, the probability that A speaks before B and B speaks before C is:

(a) 3/8 (b) 1/6
(c) 3/5 (d) None of these

Solution

Total number of ways in which 8 persons can speak is ${}^8P_8 = 8!$ = Total number of cases

Favourable number of cases = Number of ways in which A, B and C can be arranged in the specified speaking order $= \frac{8!}{3!}$.

Hence, required probability

$$= \frac{8!/3!}{8!} = \frac{8!}{3!/8!} = \frac{1}{6}.$$

Example 12

There is a point inside a circle. What is the probability that this point is closer to circumference than to the centre?

Solution

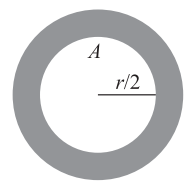


Fig. 15.1

Assume that the radius of the bigger circle is r, and the radius of the inner circle is $r/2$. Point will be closer to circumference than to the centre if point is lying in the segment B.

Area of segment B $= \frac{3}{4}\pi r^2$

So, probability of point being closer to the circumference $= \frac{\frac{3}{4}\pi r^2}{\pi r^2} = \frac{3}{4}$

PRACTICE EXERCISE

EXERCISE 1

1. Five boys and three girls are seated at random in a row. The probability that no boy sits between two girls is

(a) $\frac{1}{56}$ (b) $\frac{1}{8}$

(c) $\frac{3}{28}$ (d) None of these

2. Using the digits 0, 1, 2, 3 and 4, four-digit numbers are made without repetition. What is the probability that the number formed is even?

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$

(c) $\frac{5}{8}$ (d) $\frac{1}{2}$

3. Two cards are drawn at random from a pack of 52 cards. The probability of getting at least a spade and an ace is

(a) $\frac{1}{34}$ (b) $\frac{8}{221}$

(c) $\frac{1}{26}$ (d) $\frac{2}{51}$

4. If the letters of the word 'ATTEMPT' are written down at random, the chance that all Ts are consecutive is

(a) $\frac{1}{42}$ (b) $\frac{6}{71}$

(c) $\frac{1}{7}$ (d) None of these

5. 7 white balls and 3 black balls are placed in a row at random. The probability that no two black balls are adjacent is

(a) $\frac{1}{3}$ (b) $\frac{7}{15}$

(c) $\frac{2}{15}$ (d) $\frac{2}{3}$

6. 4 gentlemen and 4 ladies take seats at random round a table. The probability that they are sitting alternately is

(a) $\frac{4}{35}$ (b) $\frac{1}{70}$

(c) $\frac{2}{35}$ (d) $\frac{1}{35}$

7. Let $x = 33^n$. The index n is given a positive integral value at random. The probability that the value of x will have 3 in the Unit's place is:

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) None of these

8. Three dice are thrown simultaneously. The probability of getting a sum of 15 is:

(a) $\frac{1}{72}$ (b) $\frac{5}{36}$

(c) $\frac{5}{72}$ (d) None of these

9. Three dice are thrown. The probability of getting a sum which is a perfect square is:

(a) $\frac{2}{5}$ (b) $\frac{9}{20}$

(c) $\frac{1}{4}$ (d) None of these

10. A box contains 6 red balls, 7 green balls and 5 blue balls. Each ball is of a different size. The probability that the red ball being selected is the smallest red ball, is:

(a) $\frac{1}{18}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

11. Two distinct numbers are selected at random from the first twelve natural

numbers. The probability that the sum will be divisible by 3 is:

(a) $\frac{1}{3}$ (b) $\frac{23}{66}$

(c) $\frac{1}{2}$ (d) None of these

12. Three different numbers are selected at random from the set $A = \{1, 2, 3, ..., 10\}$. The probability that the product of two of the numbers is equal to the third is:

(a) $\frac{3}{4}$ (b) $\frac{1}{40}$

(c) $\frac{1}{8}$ (d) None of these

13. There are 7 seats in a row. Three persons take seats at random. The probability that the middle seat is always occupied and no two persons are consecutive is:

(a) $\frac{9}{70}$ (b) $\frac{9}{35}$

(c) $\frac{4}{35}$ (d) None of these

14. From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) None of these

Direction for questions 15 and 16: ***Read the passage below and solve the questions based on it.***

A and *B* are two possible events of an experiment such that $P(A \cup B) = 0.7$ and $P(A) = 0.4$, then find $P(B)$ given that:

15. *A* and *B* are mutually exclusive events:

(a) 0.6 (b) 0.3

(c) 0.2 (d) 0.5

16. *A* and *B* are independent events.

(a) 0.5 (b) 0.3

(c) 0.6 (d) 0.2

17. The odds against an event are 3 to 4 and the odds in favour of another independent event are 2 to 5. The probability that exactly one of them occurs is

(a) $\frac{23}{49}$ (b) $\frac{26}{49}$

(c) $\frac{6}{49}$ (d) $\frac{8}{49}$

18. In the above question, what is the probability that at least one of them occurs is:

(a) $\frac{34}{49}$ (b) $\frac{15}{49}$

(c) $\frac{23}{49}$ (d) $\frac{26}{49}$

19. Chunmun picks a letter from the set of English alphabet and finds it to be a vowel. What is the probability that the letter is *E*?

(a) 1/26 (b) 5/6

(c) 1/5 (d) None of these

20. The probability of a problem being solved by *A* is 1/4 and *B* solving it is 1/5. If they try independently, then what is the probability that the problem is not solved?

(a) 2/5 (b) 4/9

(c) 3/5 (d) 2/9

21. Ten horses are running in a race, the chances that *A* will win are 30%, that *B* will win are 20% and *C* will win are 10%. What is the probability that one of them will win?

(a) 0.689 (b) 0.598

(c) 0.498 (d) 0.398

22. If a year has 360 days and all months with 30 days. What is the probability that your birthday falls on a Monday and that is an even day of an even month, given that January 1 is a Monday?

(a) $\frac{1}{10}$ (b) $\frac{13}{360}$

(c) $\frac{1}{28}$ (d) $\frac{11}{360}$

23. If 4 whole numbers are taken at random, and multiplied together, then the probability that the last digit in the product is 1, 3, 7 or 9 will be:
(a) 13/976 (b) 17/529
(c) 16/625 (d) 13/625

24. There are 500 students in an examination. 150 students passed the first paper, 350 students passed the second paper and 50 students passed both the papers. Find the probability that a student selected at random has failed in both the papers.
(a) 1/5 (b) 1/10
(c) 3/10 (d) 3/5

25. A person is asked to randomly pick two balls from a bag which has 15 yellow and 5 red balls. The probability that the person picks two balls of the same colour is:
(a) 33/38 (b) 23/38
(c) 38/43 (d) 15/38

26. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is:
(a) $\frac{1}{5}$ (b) $\frac{1}{7}$
(c) $\frac{1}{4}$ (d) $\frac{1}{49}$

27. If the probability of *A* failing in an examination is and that of *B* failing the same examination is then probability that either *A* or *B* fails is:
(a) $\frac{1}{2}$ (b) $\frac{11}{25}$
(c) $\frac{19}{50}$ (d) None of these

28. A bag contains 5 green apples and 7 red apples. If two apples are drawn from the bag, what is the probability that one is red and other is green?
(a) 12/66 (b) 35/66
(c) 2/12 (d) 2/35

ANSWER KEY

Exercise 1

1. (c)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (d)
9. (d)	**10.** (c)	**11.** (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (b)	**16.** (a)
17. (b)	**18.** (a)	**19.** (c)	**20.** (c)	**21.** (d)	**22.** (b)	**23.** (c)	**24.** (b)
25. (b)	**26.** (c)	**27.** (b)	**28.** (b)				

SECTION 1

PART B

Data Interpretation

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1 Basics of Data Interpretation

INTRODUCTION

Data Interpretation is the act of transforming data with the objective of extracting useful information and facilitating conclusions on the basis of the given data. Depending upon the type of data and the question, we might be required to include the application of certain statistical tools with various methods to represent the given data. Before we move ahead to see and learn various methods of representing the data, let us understand some basics pertaining to Data.

MEANING OF DATA

Data is a means to represent facts, concepts, or instructions in a formalized manner suitable for communication, interpretation, or processing by humans or by automatic means.

In other words, a large class of practically important propositions are measurements or observations of a variable. Such propositions may comprise numbers, words, or images. And with the help of data, we try to quantify these propositions.

Data can be any numerical or non-numerical fact and figure related to a particular Social event or Economic event or Political event etc.

Let us see some examples of data:

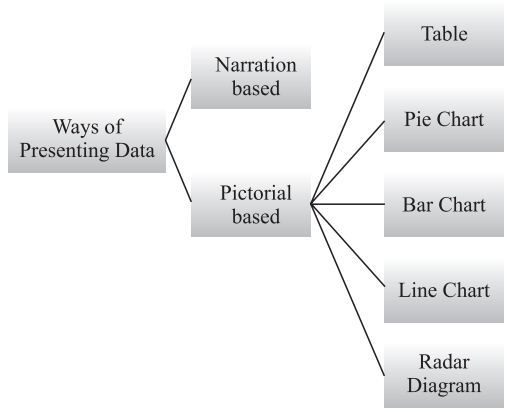
1. Yesterday sensex closed at 21762.
2. A total of 25% population of India is Below Poverty Line (BPL).

If the data given is distinct and separate, *i.e.*, They can be counted (1, 2, 3, ...), then it is known as discrete data. For example, Population of a country, production of cars in at a manufacturing plant.

However, if the data can take any value within a finite or infinite interval, then it is known as continuous variable. For example, percentage of sugar in mango or weight/height etc. are continuous variables.

DIFFERENT WAYS OF REPRESENTING DATA

Data representation in the DI section is primarily of two types:



Based

Also known as caselets, these questions often involve stories that define a situation and give details of various parameters involved, including their interrelationships.

Example: Mittal has recently acquired four companies viz., Bank of Bozoland (BOB), My Own Bank (MOB), Zany Obliterated Bank (ZOB), and Dogmatically Obscure Bank (DOB). He noticed that the sales of DOB are half that of BOB whereas the profits of DOB are double that

of BOB. The expenses of ZOB are ₹ 3 crores less than that of DOB whereas the profits of MOB is ₹ 1 Crore less than that of ZOB. The expenses of BOB are three times that of DOB. It is also known that the sales of ZOB are ₹ 15 crore or one-fourth that of MOB. All figures are for 1992–93. An insider further informs Mittal that the sales of DOS are ₹ 10 crores more than that of ZOB and the expenses of BOB are 90% of its own sales. Sales – Expenses = Profit.

1. The Total sales of all the four companies is (₹ crores):
   (a) 200 (b) 150
   (c) 125 (d) 160

Pictorial Based

This is the most common form of data representation. In such problems, data is presented in various pictorial forms such as—line graphs, bar diagrams, line charts, etc.

The important point to remember pertaining to all these questions is the fact that each and every question asked is based on some logic and reasoning and it is meant to check your aptitude. Few questions that involve numbers may also require a basic level of calculation skills.

Let us see the main types of pictorial based presentation of the data:

Table

Tabular method is the most fundamental way of representing the data. In fact, most of the different kinds of data presentation format—Bar chart, Line chart, etc.—originate from table only. In other words, presenting the data in a tabular format is the first step of forming other types of data presentation formats.

Example: Table 1.1 shows the break-up of the percentage of people of different age groups frequenting bars in 4 different metro cities viz. Delhi, Hyderabad, Bangalore and Patna in the year 2002.

Tabular format is considered to be the most versatile data presentation method. No such data exists which can be expressed in any other format but cannot be expressed in the format of table. On the other hand, it is quite possible that many a different kinds of data that can be presented in tabular format, but cannot be presented in a particular format—like pie chart—of data presentation.

Pie Chart

Pie charts are a typical type of data representation where every data is represented as a part of a circle. The circle represents the total value (or 100%), and different parts representing their own proportions (or percentage). We divide the circle into several parts and each part represents a certain proportion or percentage of the total. It can also be observed that in a pie chart, the arc length of each sector (and in turn its central angle and area), is proportional to the part it represents.

Origin of pie chart is traced with Florence Nightingale in 1858. This was the year when she presented a paper on the causes of death of army in the eastern part of the world.

There are two approaching of constructing a pie chart from the given data:

1. **Degree Approach:** The central angle on a circle represents 360°, so any part or

Table 1.1

Cities	Percentage break-up for age groups (Years) in 2006						
	Up to 15	15–20	20–25	25–30	30–35	35–40	Above 40
Delhi	8	13	24	21	11	17	6
Hyderabad	3	8	35	23	10	16	5
Bangalore	4	21	27	11	8	14	15
Patna	1	7	43	32	9	5	3

segment in a pie chart is calculated as a proportion of 360°.

2. **Percentage Approach:** In this case, any part or segment in a pie chart is calculated as a part of 100%.

For example, sales value of company *XYZ* in the year 2006.

Sales value of company *XYZ* (Fig. 1.1).

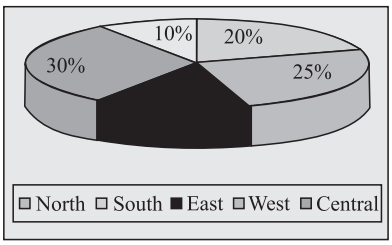


Fig. 1.1

If we convert the same pie chart into degrees format, we will be required to do following conversions:

Total = 100% = 360°
Hence, 1% = 3.6°
Central = 10% = 36°
North = 20% = 72°
South = 25% = 90°
East = 15% = 54°
West = 30% = 108°

Limitation of Pie chart

Despite pie chart being one of the most important ways to represent data, it is marred by limitations of its own:

1. Pie charts can be used only when the sum of all categories is given, for example if they represent proportions or percentage of total.
2. A single pie chart can represent only one continuous variable.

Significance of Pie chart

Pie chart has gained prominence due to the following reasons:

1. In a pie chart, we get a clear picture regarding contribution of different sectors to build up the total.
2. For the sake of comparison, comparing two pie charts is easier than comparing two bar charts or any other format of data representation.

For example, let us see the following data (Figs. 1.2 and 1.3):

Sales value in 2005 = ₹ 180 crores
Sales value in 2006 = ₹ 204 crores

Sales value of company *XYZ* in 2005

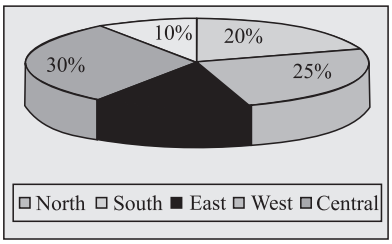


Fig. 1.2

Sales value of company *XYZ* in 2006

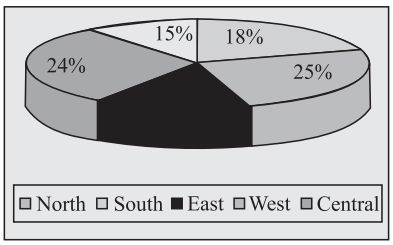


Fig. 1.3

1. What is the percentage increase in the sales value of east zone?

Solution: There are two percentage increase—(i) total sales value of company *XYZ* is increasing, (ii) percentage contribution of east zone is increasing.

Percentage increase in total sales value of company *XYZ* = 30%

Percentage increase in percentage contribution of east zone = 20%

Hence, net percentage increase = 56%.

Types of pie chart

There are two types of pie charts:

1. **Normal pie chart:** This displays the contribution of each of the components of the pie (Fig. 1.4).
   Sales value of company *XYZ* in 2006

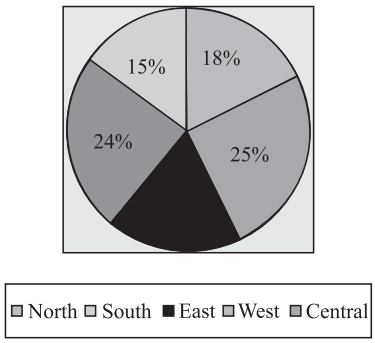


Fig. 1.4

2. **Exploded pie chart:** This pie chart has all the characteristics of a normal pie chart. The only addition is—Contribution of individual segments is highlighted (Fig. 1.5).
   Sales value of company *XYZ*

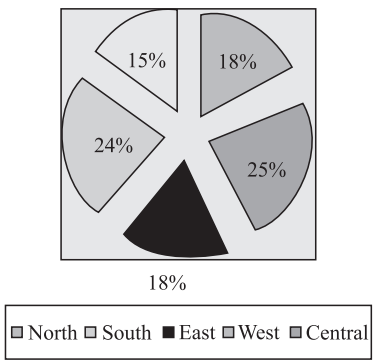


Fig. 1.5

Bar Chart/Bar Graph

While going through pie chart, we learnt the limitations of pie chart. Bar chart, in comparison, is more versatile in representing the data. It has been proved also that lengths as in case of bar charts are a better indicator of data with respect to area as in pie chart.

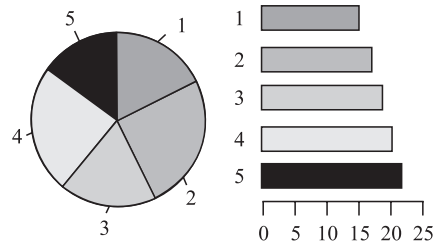


Fig. 1.6

In Fig. 1.6, the same data has been represented length wise in bar chart and area wise in pie chart. Obviously, it is easier to see the contribution of various segments in case of bar chart than to in case of pie chart.

A **bar chart** is a chart with rectangular bars of lengths proportional to the values which they represent. Usually, the notations 'bar chart' and 'bar graph' are used interchangeably. **It should also be noted that in a bar chart, what matters is the length of the bar and not the width of the bars.**

Example: Fig. 1.7 represents the number of sets won by different parties in the last general election:

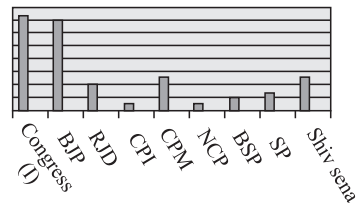


Fig. 1.7

Significance of bar chart

Following are the major specialties of bar chart:

1. Unlike pie chart, a single bar chart can be used to compare two or more than two continuous variables.

   Example: Figure 1.8 represents the number of sets won by different parties in the last two general elections.

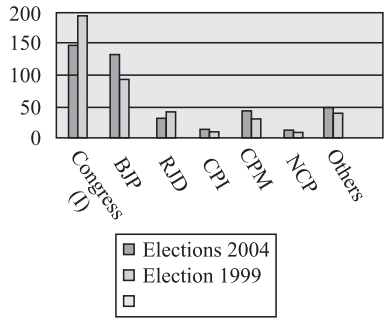


Fig. 1.8

2. As we have discussed earlier, since the length parameter is easier to observe than the area parameter, bar chart gives a quick understanding regarding the ranks of various parts. And hence, the time taken to have a mapping of the data becomes considerably lower in case of bar chart with respect to other types of data presentation format.

Types of bar chart

There are three types of bar chart:

1. **Normal Bar chart:** Normal bar chart is a simple bar chart with the values of different segments represented in the format of bars. Understand that the bars could be horizontal or vertical both.

   Example: Following bar chart represents the crime cases reported in Delhi in the year 2007 (Fig. 1.9):

   (i) Bar chart with Vertical bars

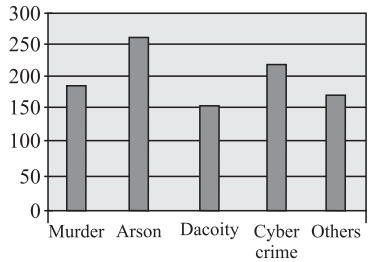


Fig. 1.9

   (ii) Bar chart with Horizontal bars

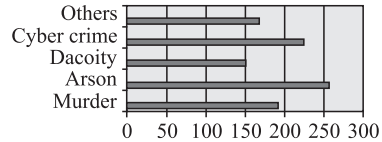


Fig. 1.10

Sometimes bar charts are shown with 3D effects too (Fig. 1.11).

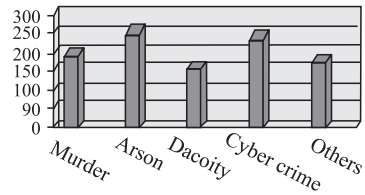


Fig. 1.11

2. **Stacked bar chart (value wise):** When the same variable is to be represented on more than one parameters (like—year, etc.), then we can have stacked bar chart.
3. **Stacked bar chart (percentage wise):** The only difference between value wise and percentage wise is that in the former we apply values to stack the bars and in the latter we apply percentage.

Stacked Bar Charts are also known as Cumulative Bar Charts.

Let us see an example:

We will see the same data in normal bar chart, stacked bar chart (value wise) and stacked bar chart (Percentage wise).

Normal bar chart:

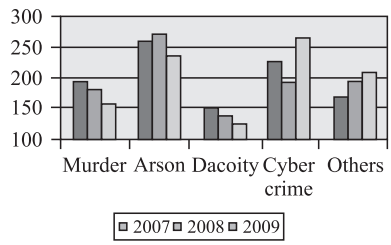


Fig. 1.12

Following things can be observed in the above bar chart (Fig. 1.12):

1. Bars of different crimes along the different years are proportional to the crimes reported.
2. Above bar chart takes the minimum value as 100, the same bar chart could have taken the minimum value as 0 or 50 or anything else too. The question lies here—what will happen if we take the minimum value as 150?

Since some of the values are less than 150, hence, what will happen to those? Find it yourself without using computer.

Stacked bar chart (value wise):

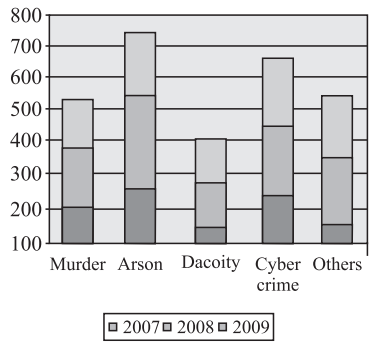


Fig. 1.13

Following things can be observed in the above stacked bar chart:

1. Total number of murders have been added up in one bar, and different years are shown as a part of that total.
2. The same data could have been represented by taking the sum of all the crimes reported in a particular year as a total, and then individual crimes being the part of that.

Stacked bar chart (Percentage wise) (Fig. 1.14):

Following things can be observed in the stacked bar chart as shown in Fig. 1.14:

1. Total number of murders have been added up and it is equal to 100%, and murders reported in different years have been taken as a percentage of that total.

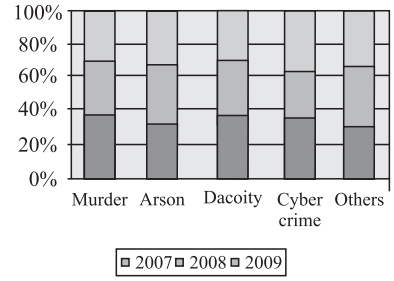


Fig. 1.14

2. The same data could have been represented by taking the sum of all the crimes reported in a particular year as a total, and then individual crimes being the part of that.

Line Chart/X–Y Chart

In layman terms, line charts can be seen as a simplified form of normal bar chart.

It can be seen through the following example:

The bar chart as shown in Fig. 1.15 represents the highest value of sensex in the given years.

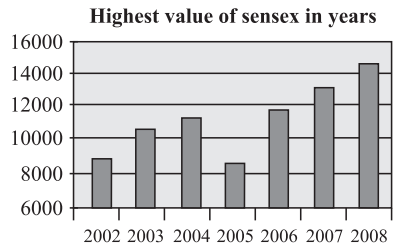


Fig. 1.15

If we convert the same data into a line chart, it will look like Fig. 1.16.

Sometimes in case of line chart, lines are not given and the only thing present in the whole graph is the dots. Let us see an example with the same data (Fig. 1.17).

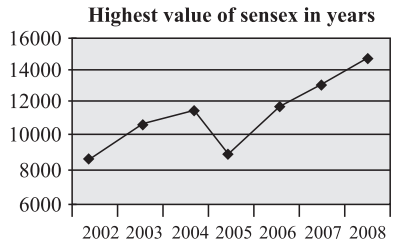


Fig. 1.16

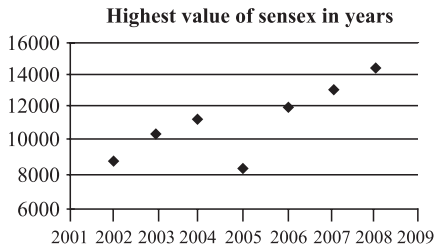


Fig. 1.17

Significance of Line Chart

1. Generally line charts are used in case of time being one of variables in the data. 'Time' variable can be hour, or day, or months or year or anything that represents chronological order of happening of events.
2. It is easier to calculate the percentage changes in the data in case of line chart with respect to any other chart that in turn facilitates understanding of the trends of data in a better way.
3. Line chart becomes very handy in case of data with two different scales.

Let us see an example:

Figure 1.18 represents production of number of soaps in a company over a period of five years and the inflation for the same period.

1. Two or more than two variables can be represented on line chart very easily. Besides, we see the movement of data very easily in case of line chart.

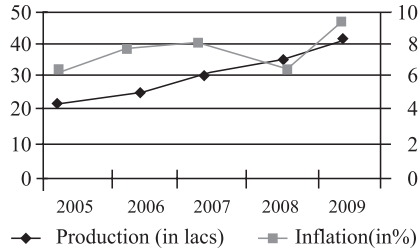


Fig. 1.18

Let us see an example:

Figure 1.19 represents the movements of highest value of two indexes over the given period.

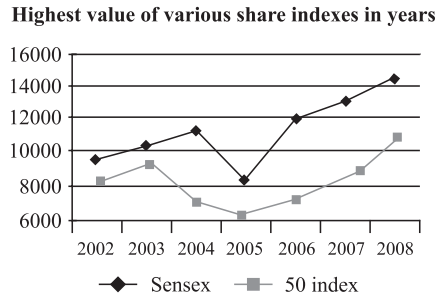


Fig. 1.19

The same data can be represented using dots only also (Fig. 1.20):

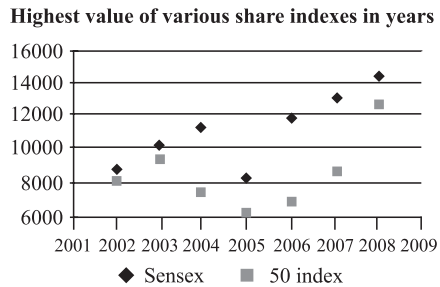


Fig. 1.20

Types of Line Chart

Broadly, line charts are of three types:

1. **Normal line chart:** A normal line chart is a simple line chart representing two or more than two variables.

In Fig. 1.21, total number of enrolments for four different years for four coaching institutes are represented:

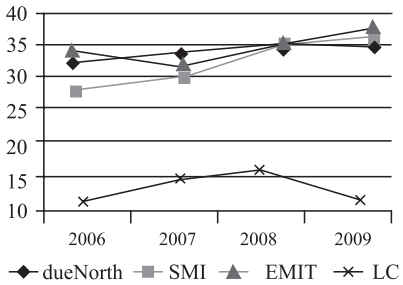


Fig. 1.21

2. **Stacked line chart:** In case of stacked line chart, we keep on adding the values to obtain the next value. Let us see this with an example (Fig. 1.22):

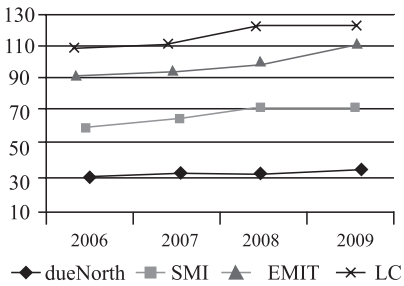


Fig. 1.22

Here, the bottom line (of *LC*) gets added with the values of SMI to represent the value of SMI. Now this represented value of SMI is added to the actual value of EMIT to give the value of EMIT on the chart. And finally the same is done with the value of *FC*.

Hence, in case of stacked line chart, to obtain the value of different constituents/segments, either start with the top line or the bottom line and then keep on subtracting the values to obtain the next value.

3. **Time-Speed-Distance line chart:** It is a typical type of line chart used in case of tables of data collected from experiments on physical processes (Fig. 1.23).

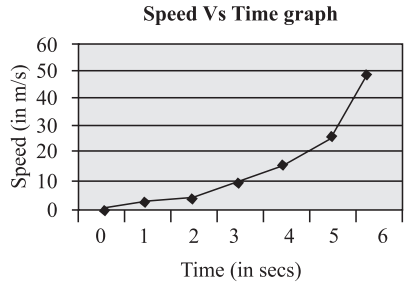


Fig. 1.23

Radar Diagram

It is a type of diagram where every value is represented with respect to a centre point. All the changes in the values too are expressed in the form of distance from this center point (Fig. 1.24).

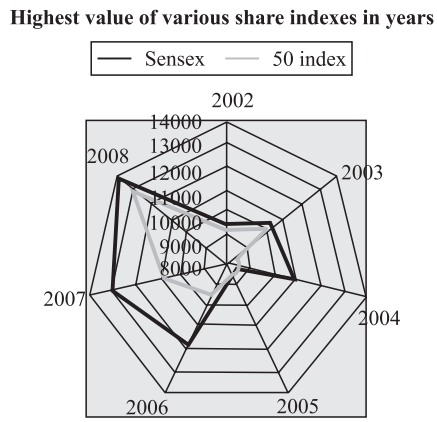


Fig. 1.24

It can be seen that the centre value = 8000. With every passing circle, value increases by 2000. Since there are seven values, hence, the diagram takes the shapes of heptagon. Had there been only six years, it would have taken the shape of hexagon. It can be seen in Fig. 1.25.

Similarly, had there been only four years, the diagram would have been like a square.

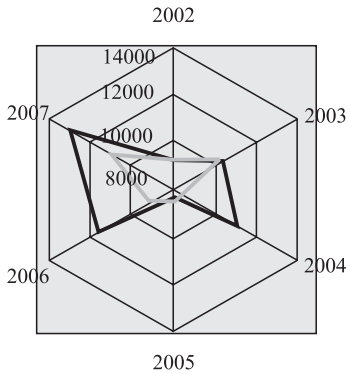


Fig. 1.25

Let us understand how questions are created:

For every DI set, there is a central idea (this happens in case of Reading Comprehension too) that question creator has in his/her mind. Ideally, in a set of 4–5 questions, 1–2 questions will be based upon central idea, and those questions will be of a slightly higher level of difficulty than the other questions. Although this may not be always true, but provides us the following "thumb rules" of solving Data Interpretation.

Thumb Rule 1: Out of a set of 4–5 questions, 1–2 questions will be slightly difficult. While attempting the set, 1st one should attempt the easy questions, and then only proceed for those central idea questions.

Thumb Rule 2: Before attempting the questions, read all the questions and try to identify which question is most difficult, and which question is easiest and should be attempted 1st.

Thumb Rule 3: Don't get impressed with the 'data'. Stay away from the technicalities. Keep focused that for solving a particular question, what data may be required. We call this 'Reverse Contemplating' of the solution.

Let us understand the above thumb rules the help of following example:

Direction for questions 1 to 3: ***Go through the data given below and solve the questions based on it.***

Following bar chart gives the distribution of green land in the following countries.

Total green land area = 13,754 sq km

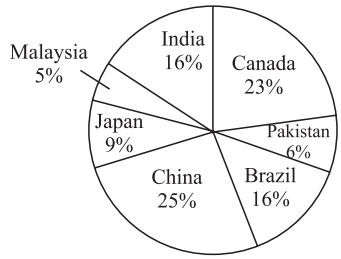


Fig. 1.26

Some of these green lands are protected and remaining are unprotected. Figure 1.27 gives that distribution.

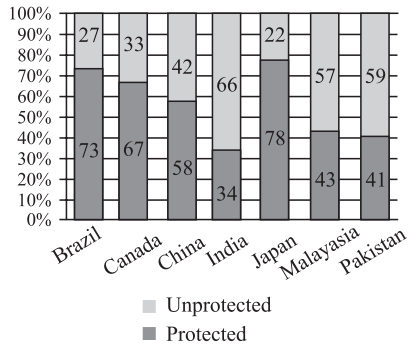


Fig. 1.27

1. What is the difference of protected and unprotected green land (approx.)?
   (a) 3015 (b) 2382
   (c) 2845 (d) 2654
2. Which country has the maximum protected green land?
   (a) China (b) Japan
   (c) Brazil (d) Canada
3. Protected green land of India is what percentage of the protected green land of Pakistan?
   (a) 48% (b) 65%
   (c) 160% (d) 220%

Let us 1st decide that what should be the order of attempting questions:

In my perception, Q2 should be attempted 1st, then Q3 and then Q1 at the end. Rather you may choose to not attempt Q1 as it will take atleast 3–4 minutes to calculate.

What all is required to solve which question:

1. Pure Calculation
2. Observation
3. Interpretation and calculation technique

How to solve:

2. Let us 1st have a look at all the options:
   (a) China (b) Japan
   (c) Brazil (d) Canada

Japan seems to be very unlikely, as its percentage share of green land is 9% only. Out of remaining three options, China and Canada are having very close shares at 25% and 23% but percentage of protected land in Canada is far higher than China (Canada = 67%, China = 58%), so quantity of protected land in Canada will be higher than that of China.

Next we have to compare Canada's protected green land and Brazil's protected green land.

Look at the following data:

Country	%age	Protected
Canada	23%	67%
Brazil	16%	73%

It can be seen that 23% of 67% of X ($0.23 \times 0.67 \times X$) is larger than 16% of 73% of X ($0.16 \times 0.73 \times X$). It can be calculated as follows:

For Canada:

67% = 2/3 (approx.), so 67% of 23% = approx 2/3 of 23% = 46%/3 = 15.33%

For Brazil:

16% = 1/6 (approx.), so 16% of 73% = approx 1/6 of 73% = 73%/6 = 12% approx.

Hence, Canada's protected green land is maximum.

Hence, option (d) is the answer.

Authors' Note: Please keep in your mind that all the calculations that you are seeing above should be done through mental calculation.

Now we would look at Q3:

3. Let us look at the data pertaining to India and Pakistan:

Country	%age	Protected
India	16%	34%
Pakistan	6%	41%

Question is asking for:

Protected green land of India is what percentage of the protected green land of Pakistan?

Protected green land of India = 16% of 34% of X

Protected green land of Pakistan = 6% of 41% of X

We have to just calculate ratio or percentage value, hence, don't need to calculate this of total green land, hence, we can take it X or any value.

$$\text{Answer} = \frac{0.16 \cdot 0.34}{0.06 \cdot 0.41} \cdot 100$$

Now before you actually calculate the final answer, look at the options. Our actually answer is clearly more than 200%, and there is just one option having a value more than 200%. Hence, option (d) is the answer.

Now we would look at Q1:

1. Ideally, in an examination condition, one should not attempt this question.
   Following calculation solves and answers this question (Table 1.2):

Table 1.2

Country	%age	Protected	Unprotected	Protected Area	Unprotected Area
India	0.16	0.34	0.66	748.2176	1452.4224
Canada	0.23	0.67	0.33	2119.4914	1043.9286
Pakistan	0.06	0.41	0.59	338.3484	486.8916
Brazil	0.16	0.73	0.27	1606.4672	594.1728
China	0.25	0.58	0.42	1994.33	1444.17
Japan	0.09	0.78	0.22	965.5308	272.3292
Malaysia	0.05	0.43	0.57	295.711	391.989
			Total	8068.0964	5685.9036

Difference of protected area and unprotected area = 2382

Hence, option (b) is the answer.

DIFFERENT CALCULATION TECHNIQUES

We will discuss this under two headings:

1. Approximation
2. Specific calculation techniques pertaining to the specific data set.

Approximation

Approximation is the art of doing mistake smartly. In other words, when somebody expects to approximate the value, it means we need to find out an answer without doing the actual calculation which is close to the actual value.

Thumb Rules for Approximation

1. Look at the options 1st—Before you start calculating the final answer, look at the options. They should not be too close as in that case approximation might lead us to choose the wrong option as the answer. A minimum gap of atleast 5% is desirable among options.
2. An approximation is good if it is in the range of ±2%.
3. At the end of approximation, one should know the direction of approximation. It means that one should know that approximated value is more than or less than the actual value.

How do we approximate in the calculation:

Let we divide $\frac{13}{79}$; it is approximately equal to $\frac{13}{67} = \frac{1}{6} = 0.1667$

It is so easy because of denominator is very-very close to the multiple of numerator. In reverse case, if numerator is very-very close to the multiple of denominator, then $\frac{79}{13}$ is approximately equal to $\frac{78}{13} = 6$, but if we consider 79 instead of 78, then we have another more approximation 6.07 = 6.1.

Let us do an exercise to understand it more clearly.

Exercise

1. Find the approximate value of the following:

(a) $\frac{95}{16}$ (b) $\frac{999}{110}$

(c) $\frac{456}{199}$ (d) $\frac{479}{159}$

(e) $\frac{111}{599}$

Explanations:

(a) Approx. $= \frac{96}{16} = 6$, but actual is something less than 6; very close to 5.9

(b) Approx. $= \frac{999}{111} = 9$, but actual is something more than 9; very close to 9.1

(c) Approx. $= \frac{456}{200} = \frac{4.56}{2} = 2.28$

(d) Approx. $= \frac{480}{160} = 3$

(e) Approx. $= \frac{111}{600} = \frac{37}{200} = \frac{.37}{2} = .185$

Specific Calculation Techniques Pertaining to the Specific Data Set

There are some certain techniques specific to particular type of data presentation formats:

Pie Chart

Figure 1.28 shows the sales figure of *due North Inc.* for the year 2007–08 and 2008–09. It is also given that share of North India in total sales figure is 20% and 24% in respective years.

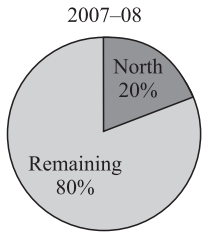


Fig. 1.28

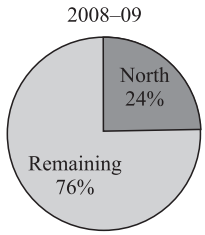


Fig. 1.29

Total sales for Year 2007–08 is ₹ 1,202 crores and for Year 2008–09, it is ₹ 1381 crores.

Now the question is: What is the percentage growth in the sales value of North India in 2008–09 over the sales value of 2007–08?

Normal way of doing this:

Sales value of North India in 2007–08 = 20% of ₹ 1,202 crores = ₹ 240.4 crores

Sales value of North India in 2008–09 = 24% of ₹ 1,381 crores = ₹ 331.4 crores

So, Growth in sales value = ₹ 91 crore

So, Percentage growth

$$= \frac{\text{Growth}}{\text{Initial value}} \cdot 100$$

$$= \frac{91}{240.4} \cdot 100 = 37.93\%$$

Alternatively, there are percentage increases in two factors in the pie-charts given viz. share is increasing from 20% to 24% and then total sales value is increasing from ₹ 1,202 crores to ₹ 1,381 crores.

Rather calculating the sales value of individual years, we can directly calculate the net percentage change by calculating percentage changes independently in above mentioned two factors.

Percentage growth in share = 20% and remember that it is not 4%)

$$= \left[\frac{\text{Growth} \times 100}{\text{Initial . value}}\right]$$

Percentage growth in total value ≅ Approx. 15%

So, $100 \rightarrow 20\%\uparrow \rightarrow 120 \rightarrow 15\%\uparrow \rightarrow 138$,

or $100 \rightarrow 15\%\uparrow \rightarrow 115 \rightarrow 20\%\uparrow \rightarrow 138$

So, net percentage growth = 38%

[Logic used here is somewhat similar to the fact as if length of a rectangle is increased by 20% and breadth of the rectangle is increased by 15%, then the corresponding percentage increase in the area =Product of multipliers = 1.15 × 1.2 = 1.38. Hence, net percentage increase = 38%]

Line Chart/Bar Chart

In case of line chart/bar chart, understanding the process of slope calculation between two consecutive periods/constituents/segments can tell us the highest or lowest percentage change in the whole chart.

Let us see an example (Fig. 1.30):

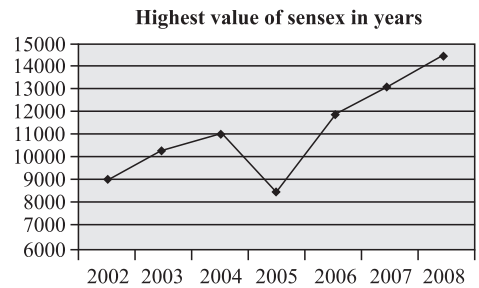


Fig. 1.30

1. When is the highest percentage increase occurring between any two years during the whole period?

   Solution: Let us start going through the slope of different lines joining two values. [Slope is defined as the tangent of the angle formed by the line in the anti-clock wise direction with the *X*-axis.]

However if we find out the angle only at the place of slope, it will serve the purpose.

It can be seen in the above line chart that the angle between 2005 and 2006 is highest. Hence, maximum percentage increase occurs during 2005–06.

2. What is to be done if there is almost same angle formed between two different periods? Let us see this through the final example (Fig. 1.31).

In the line chart given in Fig. 1.31, the line between 2005–2006–2007 is almost straight line; hence, same angle is being formed. But as we can see that the base (or, denominator) is lower in 2005 than to in 2006, hence, the percentage increase in 2005–06 will be higher than 2006–07.

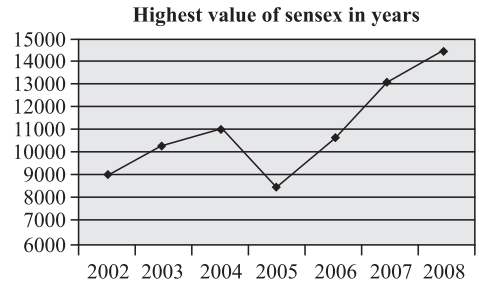


Fig. 1.31

Method discussed above for line chart is true for bar chart too.

2 Data Sufficiency

Before we move ahead on solving the data sufficiency questions, we should have a clear understanding of the different situations that might come with the questions. Besides, we should be clear with the format of the possible questions and various do's and don'ts related to the above situation and solution.

MEANING OF DATA SUFFICIENCY

Data sufficiency questions involve the typical situation where on the basis of the given statements, it will be required to find out if the question can be answered uniquely or not. Remember in Data Sufficiency questions, one is not required to find out the actual value or the answer, but only has to ascertain the possibility that if the answer can be found out or not on the basis of the statements given?

In most of the cases, a data sufficiency question will have the following structure:

1. A question will be given in below format:
   Statement 1: Some information will be given.
   Statement 2: Some information will be given.

On the basis of the statements given, four or five different options will be given. And one of those options will be the most probable choice and hence, the answer.

SIGNIFICANCE OF DATA SUFFICIENCY (DS)

A bureaucrat is virtually burdened with the weight of the data—sometimes organized and sometimes un-organized. DS questions test the ability of future civil servants to prune out the data to make decisions. Apart from testing the decision making ability of the individual, DS questions also try to evaluate the ability of the student to see into the future (read vision) that what can happen in the next 2–3 minutes. Understand it—we might be having the answer to the question given in 2–3 minutes time with the help of the statements given, but can we find the answer right now without actually investing those 2–3 minutes.

So the question now is—can we predict correctly that what is going to happen in the next 2–3 minutes without actually living those 2–3 minutes?

A data sufficiency question is designed—by virtue of its design and content—in such a way that it essentially checks students' ability to reason out. In most of the cases of DS questions, basics concepts of Quantitative Aptitude will be required.

DO'S AND DON'TS OF DATA SUFFICIENCY

Process to solve the DS questions is a bit different than the process to solve the QA questions or LR/DI questions. There are certain key things that we should keep in our mind before solving the DS questions. Let us see it one by one:

Understanding the Options

This is the first step of solving the DS questions—we must go through the options and understand it clearly. Sometimes, the options given in data set themselves can become a perennial source of problem.

Let us understand the statements given in the options:

If the statement reads like—"If the question can be solved by using only one of the statements alone, but not by the other statement alone".

It means—Out of the given two (or, more) statements, only one statement can answer the question given, and the other statement cannot give the answer to the question given. This is the case of either 1st statement will give the answer or the second one, but not both simultaneously, *i.e.*, if 1st statement can give the answer, then 2nd statement cannot give the answer and vice-versa.

If the statement reads like—"If the question can be solved by using either of the statements alone.

It means—Out of the given two (or, more) statements, the question can be solved by using any one of the statements alone. This is the case of either 1st statement will give the answer or the second one used individually and independently.

If the statement reads like—"If the question can be solved by using both (or all) the statements together, but not by either of the statements alone.

It means—Out of the given two (or, more) statements, the question can be solved only if the information given in both (or, all) the statements are used together. And using only one of the statements alone cannot solve the given question.

If the statement reads like—"If the question cannot be solved by even using both the statements together.

It means—Out of the given two (or, more) statements, the question cannot be solved even by using the information given in both (or, all) the statements together.

Once you have understood the statements given in the options, must not forget to see that which statement corresponds to which option. Examiners are known to introduce the surprise element, and this also can be one of those surprises which might catch you unaware if you are not careful of this basic fact.

General Awareness/Assumptions are not Allowed

As an un-proclaimed rule, the element of general awareness cannot be used to solve the DS questions. Let us understand this with an example:

Example 1

Is New Delhi the most polluted town in India?

Statement 1: Capital of India is the most polluted town in India.

Statement 2: New Delhi is the capital.

Solution

Well, we can understand that statement 1 alone cannot answer the question, as it is not given that New Delhi is the capital of India.

If your answer is "Both the statements together can answer the question", think once again. It is not given in statement 2 that New Delhi is the capital of India.

So, answer to this question is "Cannot be determined."

However, we should be having a clear idea regarding the differences between a universal fact like "Pythagoras theorem" and general awareness like "New Delhi is the capital of India".

Universal facts like mathematical principles are not the general awareness stuff, and hence, can be used.

Example 2

In how many days A and B working together can finish a work?

Statement 1: A can finish the same work alone in 10 days.

Statement 2: B can finish the same work alone in 15 days.

Solution

Understandably, we cannot have a unique answer to this question by using the statements alone. However, in this case we cannot solve this question even by using both the statement

together also because it is not given that A and B start working together and kept on working together till the work is finished.

And we cannot make assumptions that they started working together and kept on working till the work is finished.

Answer should be Unique

This is one aspect that makes the process of solving the DS questions different to the process of solving QA or DI/LR questions. While in case of QA, we can have multiple values of a variable; in DS there should be only one value of any variable or constant given.

Example 3

What is the value of x?

Statement 1: $x^2 - 5x + 6 = 0$

Statement 2: $x^2 + 2x - 15 = 0$

Solution

Using statement 1, value of $x = 2, 3$

Since we are not getting unique value of x using statement 1 alone, we will say that statement 1 alone is not sufficient to give the answer.

Using statement 2, value of $x = -5, 3$

Since we are not getting unique value of x using statement 2 alone, we will say that statement 2 alone is not sufficient to give the answer.

Till now we have seen that using the statements alone is not sufficient to find the answer. Hence, we will move on to using both the statements together.

Using statement 1 and 2 simultaneously, we get a common unique value of $x = 3$.

Hence, "both the statements together are needed to answer the question."

Example 4

What is the value of x, where x is the length of a rectangle?

Statement 1: $x^2 - 5x + 6 = 0$

Statement 2: $x^2 + 2x - 15 = 0$

Solution

Using statement 1, value of $x = 2, 3$

Since we are not getting unique value of x using statement 1 alone, we will say that statement 1 alone is not sufficient to give the answer.

Using statement 2, value of $x = -5, 3$

x being the length of the rectangle, it cannot be negative. Hence, $x = 3$ is the unique answer that we are getting using statement 2 alone.

Concern for the Answer, and not the Technicalities

Quite a few times I have observed that students, rather solving the questions, tend to get into the technicalities of the data. This furthers students' 'emotional attachment' with the question ending up in the wastage of time and efforts.

Example 5

Can we fill up a big drum of capacity 2,000 litres with exactly 88 litres of water (there is unlimited supply of water)?

Statement 1: There is a bucket of capacity 20 litres.

Statement 2: There is a bucket of capacity 50 litres and a mug of capacity 3 litres.

Solution

Using statement 1, we can fill anything in the multiples of 20 litres only. And it can never be ascertained that the drum is having exactly 88 litres of water. So, answer to the question is—No, we might be or might not be able to fill up a big drum with exactly 88 litres of water. And whatever is the quantity of the water in the drum is, we will not be sure if this is exactly 88 litres of water.

Using statement 2, we can fill the drum exactly with 88 litres of water (50 + 50 – 3 – 3 – 3 – 3). Hence, answer to the question is—Yes, we can fill up a big drum with exactly 88 litres of water.

Hence, using statement 2 alone we can find the answer.

Example 6

Is $y > x$?

Statement 1: $x^2 > y^2$

Statement 2: $x^3 > y^3$

Solution

Using statement 1, assuming some of the values of x and y can tell us that in some cases $x > y$ and in some of the cases $y > x$.

x	y	Remarks
5	4	$x > y$
–5	–4	$y > x$

While assuming the values, we can take only those values of x and y that satisfy the statement given.

Using statement 2,

$$x^3 > y^3$$

$$\Rightarrow \quad x > y$$

Now look at the question. Obviously we are getting an answer that y is not greater than x.

Hence, statement 2 alone is sufficient to answer the question.

Look for the Obvious, but don't miss the Hidden

Looking at any of the statements gives us an idea of the facts given, and sometimes we get so much enamored with these obvious facts only that we fail to see something hidden. Evidently, there is no point saying that, "You should not commit silly mistakes" (because nobody does it intentionally either, however we can reduce the incidences of these silly mistakes or missing some hidden information.

Some of the precautions that must be taken while tackling any DS question:

While assuming the values, take different set of values (like positive, negative, fractions between 0 and 1, etc.) to make an informed decision.

Any geometrical figure should not be taken as accurate unless it is given. For example, the below given triangle ABC cannot be taken as a right-angled triangle despite it appears to be (Fig. 2.1).

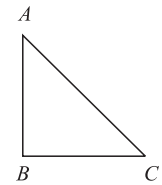


Fig. 2.1

Hence, no conclusion should be drawn by finding proportionate value of length or the angle in any geometrical figure unless stated otherwise.

Example 7

Is Arvind the father of Puja?

Statement 1: Puja is the daughter of Arvind.

Statement 2: Arvind is the aunt of Abhishek.

Solution

Using statement 1, despite Puja is the daughter of Arvind, we cannot conclude that Arvind is the father of Puja. Arvind can be mother too.

Using statement 2, in first impression statement 2 seems to be irrelevant, but looking closely at this statement gives us an idea that Arvind is a female (only females can be aunt of anybody). And hence, we can conclude that Arvind is not the father of Puja.

Hence, statement 2 alone is sufficient to answer the question.

PRACTICE EXERCISES

EXERCISE 1

Direction for questions 1–20: ***Each item is followed by two statements, A and B. Answer each question using the following instructions.***

Choose 1: If the question can be answered by one of the statements alone and not by the other.

Choose 2: If the question can be answered by using either statement alone.

Choose 3: If the question can be answered by using both the statements together, but cannot be using either statement alone.

Choose 4: If the question cannot be answered even by using both the statements together.

1. Is the point P on the circle with center O?
(a) Q is a point on the circle and the distance from P to Q is equal to the distance from O to Q.
(b) Q is on the circle and PQO is an equilateral triangle.

2. Is Arvind the father of Puja?
(a) Puja is the daughter of Arvind.
(b) Arvind is the brother of Abhishek.

3. If the ratio of boys to girls attending school in 2000 was 1/3, what was the ratio of boys to girls attending school in 2001?
(a) 100 more boys were attending school in 2001 than in 2000.
(b) 150 more girls were attending school in 2001 than in 2000.

4. If a and b are positive numbers, is $b > a$?
(a) $a^2 > b$ (b) $a^2 > b^2$

5. What is the area of the unshaded region in the Fig. 2.2 (Points E, F are on the line BC and point G is on the line AD)?
(a) $ABCD$ is a parallelogram.
(b) Area of $ABCD$ is 50 square units.

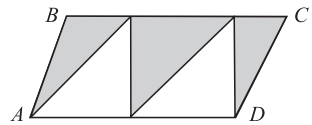


Fig. 2.2

6. If the integer P divisible by 15?
(a) Sum of the digits of P equals to 15.
(b) Unit digit of P is 6.

7. Is Q a positive number?
(a) $4Q + 24 > 0$
(b) $4Q - 24 < 0$

8. How many chocolate bars 2 inches wide and 4 inches long can be packet into carton C?
(a) The inside dimension of carton C are 8 cms by 12 cms.
(b) The width of carton C is equal to the height and 3/4th of the length.

9. What are the values of x and y?
(a) $3x + 2y = 45$
(b) $y = 22.5 - 1.5x$

10. Is x an even number?
(a) $4x + 3y$ is an even number
(b) $3x + 4y$ is an even number

11. What is the ratio of the number of boys and girls in a school (Number of boys and girls are natural numbers)?
(a) Number of boys is 40 more than girls
(b) Number of girls is 80% of the number of boys

12. What is the difference between two numbers?
(a) First number is 60% of the other number.
(b) 50% of the sum of first and second number is 24.

13. How many marks did Arpit Sinha obtain in mathematics?

(a) Arpit Sinha secured on an average 55% marks in mathematics, physics, and chemistry together.
(b) Arpit Sinha secured 10% marks more in mathematics than the average of mathematics, physics and chemistry.

14. Is $x > y$?
(a) $[(x^4 - y^4)/(x^3 + y^3)] > 0$
(b) $[(x^3 - y^3)/(x^4 + y^4)] > 0$

15. What is a two digits number?
(a) The number obtained by interchanging the digits is smaller than the original number by 63.
(b) Sum of the digits is 11.

16. X, Y, and Z are integers, is X an odd number?
(a) An odd number is obtained when X is divided by 5.
(b) $(X + Y)$ is an odd number.

17. What is the number X?
(a) The LCM of X and 18 is 36.
(b) The HCF of X and 18 is 2.

18. Is y greater than x?
(a) $x + y = 2$ (b) $x/y = 2$

19. Which of the four numbers w, x, y, and z is the largest?
(a) The average of w, x, y and z is 25.
(b) The numbers w, x, and y are each less than 24.

20. P, Q, R, and S are four consecutive even integers. What is the value of the largest integer among these?
(a) The average of the four numbers is the first prime number greater than 10.
(b) The ratio between the largest and the smallest of the number is less than 10.

21. Is A greater than B?
(a) $(A + 3)$ is greater than $(B + 2)$.
(b) Fourth power of A is greater than third power of B.

22. Which of the following is greater: $\dfrac{a+x}{b+x}$ or $\dfrac{a}{b}$?
(a) $\dfrac{a}{b} > 1$
(b) x is positive.

23. How many boys and girls are there in the family of 7 children?
(a) Each boy in the family has as many sisters as brothers.
(b) Each girl in the family has as many brothers as sisters.

24. In how many days does Binod finish a certain work?
(a) Binod is twice as efficient as Amar.
(b) Akshay is twice as efficient as Binod.

25. What is the equation of the straight line AB?
(a) Straight line AB is perpendicular to another straight line $3x + 5y = 10$
(b) Straight line AB passes through the point (1, 2).

EXERCISE 2

Direction for questions 1–15: ***Each question is followed by two statements.***

Mark 1: If the question can be answered by one of the statements alone and not by the other.

Mark 2: If the question can be answered by using either statement alone.

Mark 3: If the question can be answered by using both the statements together, but cannot be using either statement alone.

Mark 4: If the question cannot be answered even by using both the statements together.

1. Three football teams France, Brazil and England played one game with each of the other teams. What is the result of the game between Brazil and England?
Statement 1: France won both its matches.
Statement 2: England does not lose either of its games.

2. Who is heaviest among A, B and C?
Statement 1: A weighs equal to the sum of B and C.

Statement 2: Following is valid for their weights: $A + B < B + C < A + C$

3. What is the dimension of the rectangle?
Statement 1: Perimeter of the rectangle = 98 m.
Statement 2: Diagonal of the rectangle = 41 m.

4. In a series of five consecutive even numbers (in increasing order), find the fourth from bottom.
Statement 1: Sum of last two numbers is 34.
Statement 2: Sum of 1st two numbers is 22.

5. How much did Amit earn in 2006?
Statement 1: Amit earned ₹ 12,000 in 2007 which is 12% more than what he earned in 2006.
Statement 2: Amit and his wife earned ₹ 24,000 in 2007. Amit's wife earned 20% more in 2007 than what she earned in 2006.

6. What is the average marks of the class?
Statement 1: Boys are more than twice in number than the girls and their average marks is 78%.
Statement 2: Average marks of the girls, who are 20 less in number than the boys, is 84%.

7. What is the seating capacity of the coach *M*?
Statement 1: Not more than 45 passengers have ever traveled at one time in coach *M*.
Statement 2: There were 80 passengers traveling by another coach, which has twice the seating capacity of coach *M*.

8. Is Ram's age equal to one-third of his father Shyam's age at present?
Statement 1: Shayam's age is 50 at present.
Statement 2: 4 years back their ages were in the ratio of 1:2.

9. Is Sharat's age equal to half the age of his father Binod at present?
Statement 1: Binod's age is 60 at present.
Statement 2: 6 years back the ages of Sharat and Binod were in the ratio of 1:3.

10. $P > Q$. Is $P > (-Q)$?
Statement 1: P is negative.
Statement 2: Q is negative.

11. Is $X + Y + Z$ is positive?
Statement 1: $X + Y$ is positive.
Statement 2: $Y + Z$ is positive.

12. What is the ratio of the number of boys and girls in a school (Number of boys and girls are natural numbers)?
Statement 1: Number of boys is 40 more than girls.
Statement 2: Number of girls is 80% of the number of boys.

13. What is the difference between two numbers?
Statement 1: First number is 60% of the other number.
Statement 2: 50% of the sum of first and second number is 24.

14. What is the value of X?
Statement 1: X and Y are unequal even integers, less than 10, and X/Y is an odd integers.
Statement 2: X and Y are even integers, each less than 10, and product of X and Y is 12.

15. What are the ages of two individuals, X and Y?
Statement 1: The age difference between them is 6 years.
Statement 2: The product of their ages is divisible by 6.

Direction for questions 16–20: ***Each item is followed by two statements, A and B. Answer each question using the following instructions.***

Mark 1: If the question can be answered by first statement (statement *A*) alone but not by the second statement (statement *B*) alone.

Mark 2: If the question can be answered by second statement (statement *B*) alone but not by the first statement (statement *A*) alone.

Mark 3: If the question can be answered by using either statement alone.

Mark 4: If the question can be answered by using both the statements together, but cannot be using either statement alone.

Mark 5: If the question cannot be answered even by using both the statements together.

16. In an examination of five papers, marks obtained by Pranav are in the ratio 4:5:6:7:8 across the five papers. Each paper carries the same full marks. In how many papers did Pranav obtained more than 50% of the full marks?

A. Total marks obtained by Pranav in all the papers together is 300 marks.

B. Lowest percentage marks obtained by Pranav in any of the papers of this examination is 30%.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

17. *N* is a natural number. Is *N* a perfect square?

A. When *N* is divided by 100, remainder obtained is 2.

B. Total number of factors of *N* is odd.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

18. What are the respective speeds of two trains of lengths 70 m and 75 m?

A. They take 35 seconds to cross each other when they are running in the same direction.

B. They take 10 seconds to cross each other when they are running in the opposite direction.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

19. Vijay and Rupesh are two friends working in the same company. Sum of their monthly salaries is more than ₹ 1 lac per annum. Who saves more per month?

A. Ratio of monthly income of Vijay and Rupesh is 5:3.

B. Ratio of monthly expenses of Vijay and Rupesh is 3:1.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

20. Vijay and Rupesh are two friends working in the same company. Sum of their monthly salaries is more than ₹ 1 lac per annum. Who saves more per month?

A. Ratio of monthly income of Vijay and Rupesh is 5:3.

B. Ratio of monthly expenses of Vijay and Rupesh is 3:2.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

ANSWER KEY

Exercise 1

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (d)	**8.** (c)
9. (d)	**10.** (d)	**11.** (a)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (a)
17. (d)	**18.** (c)	**19.** (a)	**20.** (a)	**21.** (d)	**22.** (c)	**23.** (b)	**24.** (d)
25. (c)							

Exercise 2

1. (d)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (d)	**8.** (a)
9. (c)	**10.** (a)	**11.** (d)	**12.** (a)	**3.** (c)	**14.** (d)	**15.** (d)	**16.** (c)
17. (b)	**18.** (e)	**19.** (e)	**20.** (d)				

HINTS AND EXPLANATIONS

Exercise 1

1. (a) Using statement A alone, since it is given that the centre of the circle is O, hence, OQ is the radius of the circle. But point Q can be inside the circle or on the circle or outside the circle and in all these cases $OQ = PQ$. Hence, statement A alone does not give the answer.
Using statement B alone, $PQ = QO = PO$ = Radius of the circle.
Hence, point P is on the circle and option (a) is the answer.

2. Using statement A alone, Arvind can be father or mother of Puja.
Using statement B alone, obviously it cannot be solved.
Using both the statements together, since Arvind is the brother of Abhishek, hence, Arvind is a male. Hence, Arvind is the father of Puja and option (c) is the answer.

3. Using statement A alone, nothing can be concluded.
Using statement B alone, nothing can be concluded.
Using both the statements together, if the ratio of boys to girls in 2000 $= \dfrac{x}{3x}$, then the new ratio of boys to girls in 2001 $= \dfrac{x+100}{3x+150}$.
Now everything depends upon the value of x. Hence, cannot be determined uniquely and option (d) is the answer.

4. Using statement A alone, depending upon the values of A and B, either can be greater ($A = 10$, $B = 1$ or $A = 5$, $B = 10$) according to the condition given in the statement.
Using statement B alone, since both the numbers a and b are positive, and $a^2 > b^2$, hence, $a > b$. So, the answer to the question is—No, b is not greater than a.
Hence, option (a) is the answer.

5. Using both the statements together, since the base and height of $ABCD$ and unshaded region are same, hence, the area of unshaded region is half of area ($ABCD$). Hence, option (c) is the answer.

6. Using statement A alone, nothing can be concluded. Numbers can be 195 or 159, etc. Otherwise also, we know that if any number is divisible by 5 and 3 both, then the number will be divisible by 15. If the sum of the digits is 15, then the number is divisible by 3 but nothing can be said about the divisibility of this number by 5.
Using statement B alone, since the unit digit of P is 6, hence, this number is not divisible by 5, and hence, not divisible by 15.
Hence, option (a) is the answer.

7. A. $4Q + 24 > 0$
or, $4Q > -24$, or, $Q > -6$
Hence, Q can be negative or positive or zero as well.
B. $4Q - 24 < 0$
or, $4Q < 24$, or, $Q < 6$

Hence, Q can be negative or positive or zero as well.
Hence, option (d) is the answer.

8. (c)

9. Using statement A alone, since the equation is in two variables and there is one equation nothing can be concluded.
Using statement B alone, since the equation is in two variables and there is one equation nothing can be concluded.
Using both the equations together, both the equations are same. Hence, cannot be determined and option (d) is the answer.

10. Using statement A alone, if $4x + 3y$ is an even number then both $4x$ and $3y$ should be either odd or both should be even. Both $4x$ and $3y$ cannot be odd (as $4x$ is even) and hence, both of $4x$ and $3y$ are even.
Now even if $4x$ is even, then we cannot conclude that x is even or odd.
Using statement B alone, if $3x + 4y$ is an even number then both $3x$ and $4y$ should be either odd or both should be even.
If each of $3x$ and $4y$ is even, then x can take values like $x = 4/3$, which is neither odd nor even.
If each of $3x$ and $4y$ is odd, then $3x$ is odd. But using this we cannot conclude that x is also odd. X may be a fraction also like $x = 1/3$ or even it can be an odd number.
Hence, option (d) is the answer.

11. Using statement A alone, we cannot determine the ratio of boys and girls.
Using statement B alone, Number of girls = 4/5 Number of boys, now we can find out the ratio of boys and girls in the school.
Hence, option (a) is the answer.

12. Using statement A, we get the ratio between the two numbers but not the difference between the numbers. Statement B gives the sum of the numbers but not the difference. Using both the statements together gives the difference between the numbers.
Hence, option (a) is the answer.

13. In none of the statements given marks have been given in terms of numbers, hence, we cannot find the marks obtained in mathematics.
Hence, option (d) is the answer.

14. Using statement A, we cannot determine if $(x^4 - y^4)$ is greater than zero or less than zero since we do not have the sign of $(x^3 + y^3)$. Hence, we cannot determine anything.
Using statement B, we cannot determine if $(x^3 - y^3) > 0$ since $(x^4 + y^4) > 0$.
$(x^3 - y^3) > 0$, so, $x > y$.
Hence, option (a) is the answer.

15. Using statement A alone gives that the difference between the digits of the number is 7. This gives the possibility of many numbers like 81, 92, etc.
Using statement B alone also gives the possibility of many numbers.
Using both the statements together, we get a unique number.
Hence, option (c) is the answer.

16. Using statement A alone, we get that the quotient obtained when x is divided by an odd number is odd. Hence, x has to be odd.
Using statement B alone, one of x and y will be odd and other one will be even. So we cannot determine that which of x or y is even?
Hence, option (a) is the answer.

17. Using statement A, value of X can be $2^2 \times 3^{0-1}$. Hence, two values of X are possible.
Using statement B, value of X can be $2^1 \times 3^0 \times 5^N \times 7^N \times ...$, where value of N can be any real number.
Using both the statements together, we can see that no unique value of X is possible.
Hence, option (d) is the answer.

18. Using statement A alone cannot give us any result.

Using statement B alone cannot give us any result since the values of x and y can be positive as well as negative too.

Using both the statements together will give us the result.

Hence, option (c) is the answer.

19. Using statement A alone cannot give us any result.

Using statement B alone gives that value of three of the given numbers is less than the average. Hence, fourth number will be greater than the average. So, this fourth number is largest.

Hence, option (a) is the answer.

20. Using statement A alone, average of the given four numbers = 11, hence, their sum = 44, hence, the numbers will be 8, 10, 12, 14. Hence, the value of largest integer, whichever is that, is 14.

Using statement B alone is not going to give us the result.

Hence, option (a) is the answer.

3 Practice Exercise on Data Interpretation

PIE CHART

Direction for questions 1–5: ***Go through the pie chart given below and solve the questions based on it.***

Following is the cost analysis of a book "Pearson's Guide to Quantitative Aptitude for CAT".

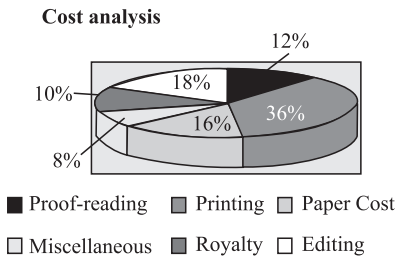


Fig. 3.1

Study the pie chart as shown in Fig. 3.1 and answer the questions:

1. What is the central angle showing the cost of paper?
(a) 42.8° (b) 32.6°
(c) 36.8° (d) 57.6°

2. If the cost of printing is ₹ 23,400, the royalty is:
(a) ₹ 6,500 (b) ₹ 2,340
(c) ₹ 4,680 (d) ₹ 7,840

3. If miscellaneous expenditure amounts to ₹ 18,000, the expenditure on editing will be:
(a) ₹ 8,000
(b) ₹ 14,400
(c) ₹ 46,800
(d) None of these

4. Royalty on the book is less than editing expenditure by:
(a) 8%
(b) 80%
(c) 44.44%
(d) None of these

5. If 5,500 copies are published and miscellaneous expenditure amount to ₹ 36,960 and the market price is 40% above cost price, then the market price of each copy is:
(a) ₹ 122.50 (b) ₹ 117.60
(c) ₹ 126.40 (d) ₹ 92.40

Direction for questions 6–10: ***Go through the pie charts as shown in Fig. 3.2 and solve the questions based on it.***

Pie charts given below represents the market share of different players in electronics market in 1995 and in the year 2000. A new player 'F' has entered the market in the given period (Fig. 3.2).

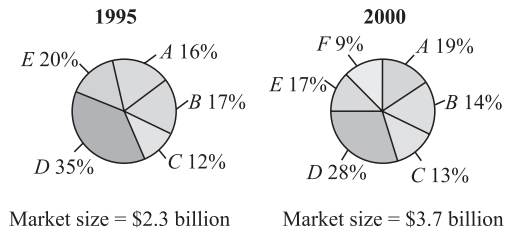


Fig. 3.2

6. What is the percentage increase in the sales of *A* in 2000 over 1995?
(a) 70% (b) 60%
(c) 80% (d) 50%

7. Find the CAGR with which the market has grown for the period 1995–2000?
(a) 18% (b) 10%
(c) 5% (d) 12%

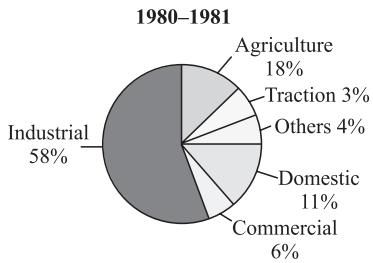
8. If the new entrant '*F*' has grown at the expense of '*B*', '*C*', and '*E*', what would be the change in the net value of sales of '*E*' in 2000 had '*F*' not entered the market?
(a) $111 million
(b) $340 million
(c) $247 million
(d) Cannot be determined

9. What is the increase in sales (in $ million) of *A*, *B* and *C* put together over the 5 years?
(a) 103 (b) 1202
(c) 668 (d) 854

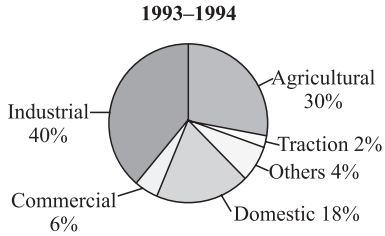
10. If the market grows at 10% p.a. over the next 2 years and '*D*' captures 1/3rd of the new market, what would be its market share in 2002?
(a) 29% (b) 27%
(c) 25% (d) 32%

Direction for questions 11–17: ***Go through the pie charts given and solve the questions based on it* [*Figs. 3.3* (*a*) *and* (*b*)].**

Electricity Consumption by Sector:



(a)



(b)

Fig. 3.3

11. By what percentage is the consumption of agriculture increased in 1993–94 over 1980–81?
(a) 66%
(b) 33%
(c) 133%
(d) Cannot be determined

12. Electricity consumption of how many sectors have definitely increased over the given period?
(a) 2 (b) 3
(c) 4
(d) Cannot be determined

13. If the total electricity consumption in 1993–94 has become 1.2 times of the total electricity consumption in 1980–81, then how many sectors have definitely increased by more than 50% during the same period?
(a) 1 (b) 2
(c) 3 (d) 4

14. If the total electricity consumption in 1993–94 has become 1.5 times of the total electricity consumption in 1980–81, then what is the percentage increase in the electricity consumption of agricultural over the given period?
(a) 50% (b) 100%
(c) 150% (d) None of these

15. What is the minimum number of sectors required to be added up to be more than 50% of the consumption in 1993–94?
(a) 1 (b) 2
(c) 3 (d) 4

16. Agricultural consumption of electricity doubled from 1980–81 to 1993–94. By how much percentage is the total electricity consumption grow from 1980–81 to 1993–94?
(a) 20% (b) 25%
(c) 50%
(d) Cannot be determined

17. If the electricity consumption of 'others' category has remained constant over the period, then what is the percentage increase in the electricity consumption of 'domestic' category?
(a) 63% (b) 38%
(c) 58%
(d) Cannot be determined

Direction for question 18–23: ***Go through the pie charts given below and solve the questions based on it.***

The two pie charts (Figs. 3.4 and 3.5) provide the expenses of two families:

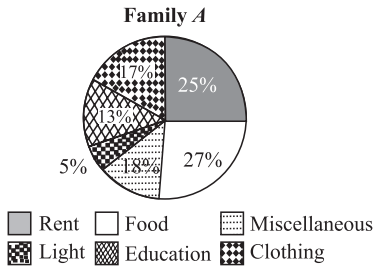


Fig. 3.4

Total expenses = ₹ 48,000.

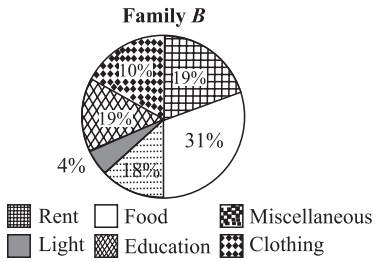


Fig. 3.5

Total expenses = ₹ 72,000.

18. Both the families decide to double the total expenditure keeping the patter of spending same as given above respectively. What will be the new ratio of expenditure on food by family *A* to family *B*?
(a) 27:31 (b) 31:27
(c) 2:3 (d) 3:2

19. If the total expenses of family *B* increases three fold keeping the expenses on education same as given above, the expenses on education would be:
(a) 6.33%
(b) 57%
(c) 19%
(d) None of these

20. The expenses on light by family *A* as a percentage of expenses on light of family *B* is equal to:
(a) 120% (b) 83.33%
(c) 62.5% (d) 66.66%

21. If family *A* and family *B* decide to combine their expenses, then which of the following heading will be responsible for the highest expenses?
(a) Rent (b) Miscellaneous
(c) Food (d) Education

22. In the above question, how many headings will have less percentage share in the combined total expenses of both the families than the percentage share of family *B* under the same heading?
(a) 1 (b) 2
(c) 3 (d) 4

23. Under how many headings, expenses of family *B* is more than the expenses of family *A*? [Headings like Food, Rent, etc.]
(a) Less than 3
(b) More than 3
(c) Equal to 3
(d) Cannot be determined

LINE CHART

Direction for questions 1–7: ***Go through the line chart given below and solve the questions based on it.***

Below given line chart represents the salary and expenditure (in ₹) of Guru Gulab Khatri for the given period (Fig. 3.6).

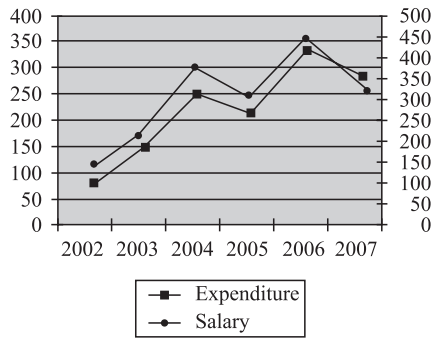


Fig. 3.6

It is given that salary is greater than expenditure atleast for four years.

1. By what percentage has the salary of Guru Gulab Khatri increased in 2004 over 2002?
   (a) 210% (b) 150%
   (c) 100% (d) 250%
2. By what percentage has the expenditure of Guru Gulab Khatri increased in 2004 over 2003?
   (a) 66.66% (b) 133.33%
   (c) 150% (d) None of these
3. How many years salary is more than expenditure?
   (a) 4
   (b) 5
   (c) 6
   (d) Cannot be determined
4. When is the percentage increase in the salary of Guru Gulab Khatri maximum?
   (a) 2003 (b) 2004
   (c) 2006 (d) 2003 or 2006
5. A year is said to be comfortable if Guru Gulab Khatri is able to save atleast 30% of his salary. How many years are comfortable during the given period?
   (a) 2 (b) 3
   (c) 4 (d) 5
6. Which of the following is the maximum difference (in ₹) between the salary and the expenditure for any year?
   (a) 70 (b) 75
   (c) 125 (d) 100
7. For how many years is the difference between salary and expenditure more than ₹ 80?
   (a) 2 (b) 3
   (c) 4 (d) 5

Direction for questions 8–12: ***Go through the line chart given below and solve the questions based on it (Fig. 3.7).***

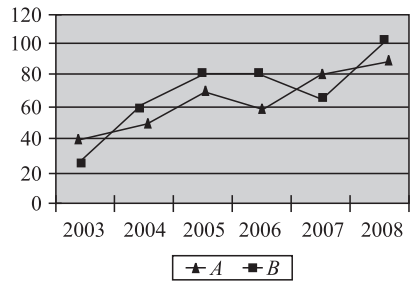


Fig. 3.7

Percentage profit earned by the two companies *A* and *B* over the years.

8. If the income for company *A* in the year 2004 was ₹ 35 lacs. What was the expenditure for company *B* in the same year?
   (a) ₹ 123.5 lacs
   (b) ₹ 128 lacs
   (c) ₹ 132 lacs
   (d) Cannot be determined
9. The income of company *A* in 2006 and the income of company *B* in 2007 are equal. What will be the ratio of expenditure of

company *A* in 2006 to the expenditure of company *B* in 2007?
(a) 26:7 (b) 17:6
(c) 15:170 (d) None of these

10. During which of the following years the ratio of percentage profit earned by company *A* to that of company *B* was the maximum?
(a) 2003 and 2006 both
(b) 2005 and 2007 both
(c) 2003 only
(d) 2008 only

11. If the expenditure of company *B* increased by 20% from 2005 to 2006, the income in 2006 will be how many times the income in 2005?
(a) 2.16 times (b) 1.2 times
(c) 1.8 times (d) None of these

12. If the income of company *A* in 2006 was ₹ 36 lakhs, what was the expenditure of company *A* in 2006?
(a) ₹ 12.5 lacs (b) ₹ 18.8 lacs
(c) ₹ 20 lacs (d) None of these

Direction for questions 13–17: ***Go through the line chart given below and solve the questions based on it (Fig. 3.8).***

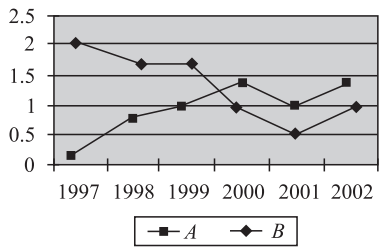


Fig. 3.8

Following line chart represents the ratio of exports (in ₹) to imports (in ₹) of two countries *A* and *B* over the given period:

13. For how many years both the countries have their respective exports more than their imports?
(a) 1 (b) 2
(c) 4 (d) None of these

14. Under some compulsions, one year during the period given these two countries have to do business between themselves only and they are not allowed to export to or import from any other countries. Which year is possibly that year?
(a) 1997 (b) 1998
(c) 1999 (d) 2001

15. What can be said regarding the imports/exports of the two countries?
(a) Value of exports of the country *A* are constantly on decline from 1997 to 2001.
(b) Value of imports of the country *B* are constantly on rise from 1997 to 2000.
(c) Value of imports of country *A* is equal to the value of import in 2002.
(d) None of these

16. For how many years, the value of exports is constantly on a rise over the previous year for country *B*?
(a) 2
(b) 3
(c) 4
(d) Cannot be determined

17. For how many years, the value of imports is constantly on a rise over the previous year for country *A*?
(a) 2
(b) 3
(c) 4
(d) Cannot be determined

TABLE

Direction for questions 1–4: ***Go through the Table 3.1 and solve the questions based on it.***

Table 3.1 gives the exports of top six industries (in value terms) from India in 2007.

Table 3.1

Industry	Exports (₹ crore)	Growth (in %)
Glass	1,093	11.7
Coal	1,080	7
Ruby	913	–8.7
Nickel	663	7.1
Softwares	605	–12.3
Diamond	505	–1.5

Overall export from India increased by 20% to ₹ 19,500 crore in 2007.

1. What is the share of Glass industry in the total exports in 2007?
(a) 2.5% (b) 4%
(c) 5.6% (d) 7%

2. For how many of the given six industries has the share in exports increased in 2007?
(a) 0 (b) 1
(c) 2 (d) 3

3. Which of the following has the maximum share among the given industries in 2006?
(a) Glass (b) Ruby
(c) Softwares (d) Coal

4. What is the percentage increase in the exports of the given six industries?
(a) 0.1% (b) 0.5%
(c) 1.05% (d) 1.75%

Direction for questions 5–9: ***Go through the Table 3.2 given below and solve the questions based on it.***

Table 3.2 gives the number of vehicles (in thousands) of different models and their colors sold in two cities—Patna and Lucknow—in a year.

5. The difference between the White-colored vehicles sold in the two metros of which of the following models is the minimum?
(a) *A* (b) *C*
(c) *D* (d) None of these

6. The total number of Blue-colored vehicles of model *E* and *D* sold in Metro *H* is exactly equal to the number of White-colored vehicles of which model in Metro *M*?
(a) *B* (b) *F*
(c) *C* (d) *A*

7. What is the difference between the number of Blue-colored vehicles of model '*C*' sold in Metro *M* and number of Red-colored vehicles of model '*F*' sold in Metro *H*?
(a) 8,000 (b) 10,000
(c) 12,000 (d) None of these

8. The total number of Silver-colored vehicles sold in Metro *H* is approximately what percentage of that in Metro *M*?
(a) 130 (b) 140
(c) 90 (d) 100

9. In Metro *M*, the number of vehicles sold was maximum for which of the color-model combination?
(a) White-*C* (b) Blue-*B*
(c) Silver-*B* (d) White-*D*

Table 3.2

	Patna					Lucknow				
Type	Colour					Colour				
	Black	Red	Blue	White	Silver	Black	Red	Blue	White	Silver
A	40	25	55	75	15	45	32	40	60	20
B	20	35	60	80	20	30	37	39	81	35
C	35	30	50	90	35	40	42	41	6	37
D	45	40	45	85	40	35	39	37	90	42
E	50	35	35	60	30	50	44	43	77	22
F	55	42	40	65	52	47	34	45	87	17

Direction for questions 10–14: ***Go through the Table 3.3 and solve the questions based on it.***

Table 3.3 gives the marks obtained by six students in six subjects:

Table 3.3

Student	History (150)	Science (200)	Geography (150)	English (200)	Hindi (100)	Math (200)
A	75	110	90	140	75	170
B	105	130	75	130	80	140
C	95	105	80	150	90	160
D	85	115	95	125	65	135
E	115	135	110	145	70	125
F	120	160	96	110	55	145

10. What was the difference between the percentage of marks obtained by the student *B* in History and the percentage of marks obtained by student *C* in Hindi?
(a) 25 (b) 20
(c) 35 (d) 30

11. The marks obtained by student *A* in Maths was how many times the percentage of marks obtained by student *F* in Science?
(a) 2.5 (b) 4.125
(c) 1.125 (d) 2.125

12. What was the difference between the percentage of marks obtained by student *C* in English and average percentage of marks of all the six subjects?
(a) 82 (b) 38
(c) 7 (d) 14

13. In how many of the given subjects did student *D* get marks more than seventy percent?
(a) None (b) One
(c) Two (d) Three

14. Approximately, what was the average percentage of marks obtained by the six students in English?
(a) 67
(b) 72
(c) 80
(d) Data inadequate

Direction for questions 15–18: ***Go through the data set given below and solve the questions based on it.***

Table 3.4 gives the break-up of the revenue earned by Phistream Private Limited over a period of five years. All figures in the table are in ₹ crores.

Table 3.4

	1994-95	1995-96	1996-97	1997-98	1998-99
Hardware:					
(a) Domestic	68	54	52	66	35
(b) Exports	540	600	730	1,770	1,827
Software:					
(a) Domestic	142	127	150	320	422
(b) Exports	1,100	1,200	1,339	2,500	2,215
Peripherals	25	10	25	20	37
Training	140	106	160	240	161
Maintenance	21	19	25	92	126
Others	12	10	19	40	24
Total	2,048	2,126	2,500	5,048	4,847

15. In which years the Hardware Exports lie between 35–40% of the total revenues for Phistream?
(a) 1997–98, 1998–99
(b) 1998–99, 1995–96
(c) 1997–98, 1994–95
(d) 1995–96, 1996–97

16. If in 1999–2000, the total revenue earned by Phistream decreases by five percent from the previous year but the revenue from Software increases to ₹ 2,437 crores, then what is the approximate percentage decrease in the combined revenue from the rest of the heads (excepting software) in 1999–2000?
(a) 1.0% (b) 1.8%
(c) 2.1% (d) 3.3%

17. In how many years during the given period, was the total revenue from Training and Maintenance less than ten percent of the total revenue for the corresponding years?

Table 3.5

Subject	Max. Marks	P		Q		R		S		T	
		B	G	B	G	B	G	B	G	B	G
English	200	85	90	80	75	100	110	65	60	105	110
History	100	40	55	45	50	50	55	40	45	65	60
Geography	100	50	40	40	45	60	55	50	55	60	65
Math	200	120	110	95	85	135	130	75	80	130	135
Science	200	105	125	110	120	125	115	85	90	140	135

(a) 2 (b) 3
(c) 4 (d) 5

18. In which of the following years was the revenue from Hardware business more than 50% of that from the Software business done by Phistream?
(a) 1994–95
(b) 1995–96, 1996–97
(c) 1997–98, 1994–95
(d) None of these

Direction for questions 19–21: ***Go through the Table 3.5 and solve the questions based on it.***

Table 3.5 gives the average marks obtained by 20 boys and 20 girls in five subjects from five different schools *P*, *Q*, *R*, *S* and *T*:

In Table 3.5, *B* = Boys and *G* = Girls.

19. What was the total marks obtained by the boys as given in Table 3.5 in History from school *Q*?
(a) 900 (b) 1,000
(c) 800 (d) 1,300

20. In Table 3.5, in which of the following subjects did the girls have highest average percentage of marks in each of the schools?
(a) Science (b) Geography
(c) English (d) History

21. What is the average marks obtained by all the students from school *P* taken all the subjects together?
(a) 82
(b) 84
(c) 80
(d) Cannot be determined

Direction for questions 22–28: ***Go through the Table 3.6 and solve the questions based on it.***

Table 3.6 shows the domestic sales of car by five manufacturers from 1995 to 2000 (All the figures are in thousands).

Table 3.6

Manufacturer	1995	1996	1997	1998	1999	2000
A	440	480	470	500	520	510
B	400	410	415	415	420	430
C	380	390	390	400	420	495
D	360	380	400	415	440	500
E	480	440	440	420	425	435

22. Considering the period given, the domestic sales of which manufacturer is the highest?
(a) *A* (b) *B*
(c) *C* (d) *D*

23. What is the approximate share of domestic sales of cars of the manufacturer *B* during 1996?
(a) 10.5% (b) 25.5%
(c) 15.5% (d) 19.5%

24. During 2000, the sales of which manufacturer has shown the maximum percentage increase over the previous year?
(a) *A* (b) *B*
(c) *C* (d) *D*

25. In respect of which of the following combinations, is the sales of cars highest over the period shown?
(a) *D*—2000 (b) *A*—1998
(c) *A*—2000 (d) *A*—1999

26. For which of the following manufacturer, there is a consistent increase across the years given?
(a) *A* (b) *B*
(c) *C* (d) *D*

27. How many of the manufacturers have never shown a decline in the number of cars sold in any particular year over the previous year?
(a) 1 (b) 2
(c) 3 (d) 4

28. How many times the number of cars sold is more than 450 for any manufacturer any year? (A manufacturer can have more than once achieved this fect.)
(a) 6 (b) 7
(c) 8 (d) 9

Direction for questions 29 and 30: ***Table 3.7 captured the comparative performance in the last four years of LIC and HDFC in the housing sector. All figures are (₹ in crores).***

29. For how many years are LIC loan sanctions 70% or more of HDFC loan sanctions?
(a) 0 (b) 1
(c) 2 (d) 3

30. The ratio of total revenue to the loan sanctions is least for LIC in:
(a) 1992–93 (b) 1993–94
(c) 1994–95 (d) 1991–92

Table 3.7

	1994–95		1993–94		1992–93		1991–92	
Attribute	LIC	HDFC	LIC	HDFC	LIC	HDFC	LIC	HDFC
Loan sanctions	618	1,495	564	1,025	607	859	511	758
Dues	24.2	29.6	20	27.8	16.1	25.17	5.32	19.57
Total loans	1,672	3,747	1,283	3,071	852	2,561	401	2,125
Default percentage	1.45	0.79	1.56	0.91	1.89	0.98	1.32	0.93
Total revenue	251	780	175	608	109	473	44.4	371
Net profit	41.4	146.1	21.53	105	13.5	55.55	6.71	45.6

ANSWER KEY

Pie Chart

1. (d) **2.** (a) **3.** (d) **4.** (c) **5.** (b) **6.** (a) **7.** (b) **8.** (d)
9. (c) **10.** (a) **11.** (d) **12.** (d) **13.** (b) **14.** (c) **15.** (b) **16.** (a)
17. (a) **18.** (a) **19.** (a) **20.** (b) **21.** (c) **22.** (b) **23.** (b)

Line Chart

1. (b) **2.** (a) **3.** (c) **4.** (b) **5.** (b) **6.** (c) **7.** (a) **8.** (d)
9. (d) **10.** (c) **11.** (b) **12.** (d) **13.** (b) **14.** (b) **15.** (d) **16.** (d)
17. (d)

Table

1. (c) **2.** (a) **3.** (d) **4.** (c) **5.** (d) **6.** (b) **7.** (d) **8.** (c)
9. (a) **10.** (b) **11.** (d) **12.** (c) **13.** (a) **14.** (a) **15.** (a) **16.** (c)
17. (d) **18.** (d) **19.** (a) **20.** (a) **21.** (d) **22.** (a) **23.** (d) **24.** (c)
25. (d) **26.** (d) **27.** (c) **28.** (c) **29.** (b) **30.** (d)

HINTS AND EXPLANATIONS

PIE CHART

1. Required angle $= \left(\frac{16}{100} \times 360\right) = 57.6°$

   Hence, option (d) is the answer.

2. Option (a) is the answer.

3. Option (d) is the answer.

4. If the editing charges are ₹ 18, royalty is ₹ 10. On ₹ 18, it is less by 8. On ₹ 100, it is less by (8/18 × 100)% = 44.44%.

   Hence, option (c) is the answer.

5. Let the total expenditure be ₹ x. Then, 8:100 = 36960:x. So x = ₹ 4,62,000. Therefore, Cost Price of 5,500 copies of the book = ₹ 4,62,000. Cost Price of each copy = ₹ (4,62,000/500) = ₹ 84. So, Market Price = 140% of ₹ 84 = ₹ 117.60.

   Hence, option (b) is the answer.

6. There are two successive increases—First is total increase of 60% approx ($3.7 billion in 2000 from $2.3 billion in 1995) and Second is percentage share of A by 20% approx. Hence, total percentage increase = 72% and option (a) is the answer.

7. Market size in 2000 = $3.7 billion

   Market size in 1995 = $2.3 billion

   Therefore, growth rate

   $$= \left[\left(\frac{3.7}{2.3}\right)^{1/5} - 1\right] \times 100 = 10\%$$

   [3.7/2.3 = 1.6, now to find $(1.6)^{1/5}$, 1.1^3 = 1.331 and 1.1^2 = 1.21 and 1.3 × 1.2 = 1.6, thus, $1.6^{1/5}$ = 1.1].

   Hence, option (b) is the answer.

8. Since we do not know that what part of market share of B. C and E has been eaten up by F, so cannot be determined.

   Hence, option (d) is the answer.

9. Sales value for $A + B + C$ in 2000,

   = (13 + 14 + 19)% of 3.7 billion

   = 46% × 3.7 = 1.702 billion

   Sales value for $A + B + C$ in 1995,

   = (16 + 17 + 12)% of 2.3 billion

   = 1.035 billion

   Hence, increase in sales value

   = 1703 – 1035 = $668 million

   and option (c) is the answer.

11. Since the total consumption for the given years is not known, we cannot determine it. Hence, option (d) is the answer.

12. Since the total consumption for the given years is not known, we cannot determine it. Hence, option (d) is the answer.

13. Total consumption has increased by 20%. So, any sector that shows an increase of even 25% will give us a net increase of 50%. There are only two sectors—Agriculture and Domestic. Hence, option (b) is the answer.

14. There are two increases—one is the increase in the share of Agriculture (66.66%) and the other increase is of 50% in total. Net increase = 150%. Hence, option (c) is the answer.

15. Option (a) is the answer. Also, see Fig. 3.3 for visual inspection.

16. There is an increase in the share of Agriculture (66.66%), to make the agricultural consumption double, total consumption is needed to be increased by 20%. Hence, option (a) is the answer.

17. The electricity consumption of 'others' category has remained constant over the period—It means that the total consumption of 1980–81 = Total consumption of 1993–94.

    So, the only increase $= \left[\frac{(18-11)}{11}\right] \times 100.$

    Hence, option (a) is the answer.

18. If there is same percentage increase in the expenditure of both the families, then the ratio will be the same. Hence, option (a) is the answer.

19. Total consumption has become 3 times keeping the expenses on education same. Hence, percentage consumption on education will become 1/3rd of the present. Option (a) is the answer.

21. Option (a) is the answer. Also, see Figs. 3.4 and 3.5 for visual inspection.

22. It should be understood that the final percentage of expenditure will always be in between the percentage of family *A* and family *B* (It is true of any mixture that the percentage composition of mixture will be always in between the percentage compositions of the components). Hence, option (b) is the answer.

23. Option (b) is the answer. Also, see Figs. 3.4 and 3.5 for visual inspection.

LINE CHART

Solution questions 1–7: For the sake of convenience, use the following chart (Fig. 3.9) (with the same data) to find out the solution:

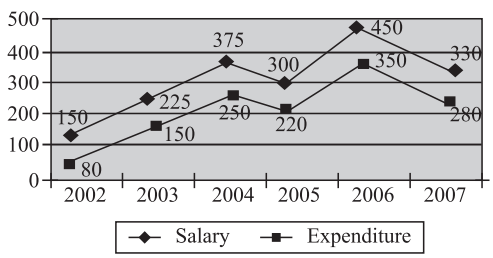


Fig. 3.9

Now all the questions can be done with ease.

8. Income-expenditures of company *A* and *B* cannot be correlated. Hence, cannot be determined.
Option (d) is the answer.

9. Expenditure of company *A* in 2006
= Income in 1996 of (*A*) × (100/160)
= Income in 1996 of (*A*) × 5/8.
Expenditure of company *B* in 2007
= Income in 1997 of (*B*) (100/170)
= 10/17 of income in 1997 of (*B*).

Now $\left[\dfrac{\text{Expenditure in 1996 of }(A)}{\text{Income in 1997 of }(B)}\right]$

$$= \left(\frac{5}{8}\right) \div \left(\frac{10}{17}\right)$$

[Since income in 1996 of (*A*)
= Income in 1997 of (*B*)]

$$= \frac{5}{8} \cdot \frac{17}{10} = \frac{17}{16} = 17:16$$

Hence, option (b) is the answer.

10. Ratio *A*:*B* is greater than 1 in only 2003 and 2007. It is 1.33 in 2003 and 1.1 in 2007. Hence, option (c) is the answer.

11. Percentage profits are the same for two years, hence, if expenditure increases by 20% the income should also increase by 20%. Hence, the required ratio = 120/100 = 1.2 and option (b) is the answer.

12. Option (d) is the answer.

13. It happens only in two years—1999, 2000. Understand that if export is more than import, then the ratio will be more than 1. If export is equal to import, then the ratio will be equal to 1 and if export is less than import, then the ratio will be less than 1. Hence, option (b) is the answer.

14. Ratio of one should be reciprocal of other. Hence, option (b) is the answer.

15. Despite the ratio is on a decline for country *A*, we cannot comment about the value of exports. IT might be a possibility that imports might have increased keeping exports constant. Similarly both the other options (b) and (c) can be refuted on the same grounds. Hence, option (d) is the answer.

16. Since only the ratio is given, we cannot comment on the values. Hence, option (d) is the answer.

TABLE

Solution questions 1–4 (Table 3.8):

Table 3.8

Exports (₹ Crore)		
Industry	**2006**	**2007**
Glass	978	1,093
Coal	1,009	1,080
Ruby	1,000	913
Nickel	619	663
Softwares	690	605
Diamond	513	505
Total	16,250	19,500

1. Share of Glass exports in 2007

$$= \frac{1093}{19500} \cdot 100 = 5.6\%$$

Hence, option (c) is the answer.

2. Overall exports has increased by 20% while none of the given six industries has grown by 20%. So we can conclude that market shares for all the six industries will decrease in 2003. Hence, option (a) is the answer.

3. Coal has the maximum exports in 2006. Hence, its market share is maximum and option (d) is the answer.

4. Exports for given six industries in 2006

= 4,809 crore

Exports for given six industries in 2007

= 4,859 crore

Therefore, percentage increase in the exports

$$= \left[\frac{(4859 - 4809)}{4809}\right] \times 100$$

$$= \frac{5000}{4809} \approx 1.05\%.$$

Hence, option (c) is the answer.

5. The difference between the White-coloured cars sold is the minimum in *B* type model. Hence, option (d) is the answer.

6. Blue ($E + D$) = 37 + 43 = 80 = White (B). Hence, option (a) is the answer.

7. Required difference

= (50 – 34) × 1,000 = 16,000.

Hence, option (d) is the answer.

8. Required percentage

= 173/192 × 100 = 90%.

Hence, option (c) is the answer.

9. Colour-model combinations of car in metro *M*.

White-C	Blue-B	Silver-B	White-D
90	60	20	85

Hence, option (a) is the answer.

10. Difference in percentage marks of *B* (History) and *C* (Hindi) = 90 – 70 = 20. Hence, option (b) is the answer.

11. Percentage of marks obtained by *F* in Science = 160/200 × 100 = 80%

Required value = 170/80 = 2.125.

Hence, option (d) is the answer.

12. Average percentage of marks of *C*

$$= \frac{680}{1000} \cdot 100 = 68\%$$

Percentage of marks of *C* in English

$$= \frac{150}{200} \cdot 100 = 75\%$$

Required difference = 75 – 68 = 7.

Hence, option (a) is the answer.

14. Average percentage of marks in English

$$= \frac{(140 + 130 + 150 + 125 + 145 + 110)}{6} \cdot \frac{100}{200}$$

= 67%

Hence, option (a) is the answer.

15. Hardware exports as percentage of total revenues for corresponding years are:

1994–95: $\frac{540}{2048} \cdot 100 < 33\%$

1995–96: $\frac{600}{2126} \cdot 100 < 33\%$

1996–97: $\frac{730}{2500} \cdot 100 < 33\%$

1997–98: $\frac{1770}{5048} \cdot 100 = 35.06\%$

1998–99: $\frac{1827}{4847} \cdot 100 = 37.7\%$

So, Hardware Exports lie between 35–40% of total revenue in the last 2 years. Hence, option (a) is the answer.

16. In 1999–2000:

Total revenue $= 0.95 \times 4847$
$= 4600$

Software revenue $= 2437$

Remaining revenue $= 4600 - 2437$
$= 2163$

In 1998–99: Revenue from heads other than Software
$= 4847 - 2637$
$= 2210$

$$\% \text{ Decrease} = \frac{2210 - 2163}{2210} \times 100$$
$= 2.12\%.$

Hence, option (c) is the answer.

17. The combined revenue from Training and Maintenance is never greater than 10% of total revenue for the corresponding year throughout the given period. Hence, option (d) is the answer.

18. See Table 3.9.

Table 3.9

Year	50% of Software Revenue	Hardware Revenue
1994–95	1242 × 0.5 = 621	608
1995–96	1327 × 0.5 = 663	654
1996–97	1484 × 0.5 = 744	782
1997–98	2820 × 0.5 = 1,410	1,836
1998–99	2637 × 0.5 = 1,318	1,862

Hardware business provides revenues > 50% of the revenue from software business in 1996–97, 1997–98 and 1998–99.

Hence, option (d) is the answer.

19. Average marks obtained by 20 boys in History from school $Q = 45$

Therefore, Total marks
$= 20 \times 45 = 900.$

Hence, option (a) is the answer.

20. From visual inspection (Table 3.5) it is clear that Science is the desired subject.

21. Table 3.5 gives the data pertaining to only 40 students of the school and it is not given if this is the total number of students in the school. Hence, option (d) is the answer.

22. It is simple calculation. Alternatively, it can be observed that A has always been more than the others but one year. Hence, option (a) is the answer.

23. Required share

$$= \left(\frac{410}{480} + 410 + 390 + 380 + 440\right) \times 100$$
$= 19.5\%.$

Hence, option (d) is the answer.

24. It can be observed that the difference is the highest in case of manufacturer C. We can further observe that largest percentage growth would for C, as the base is the smallest.

Hence, option (c) is the answer.

26. Option (d) is the answer. Also, see Table 3.6 for visual inspection.

27. Option (c) is the answer. Also, see Table 3.6 for visual inspection.

28. Option (d) is the answer. Also, see Table 3.6 for visual inspection.

PRACTICE EXERCISES

EXERCISE 1

Direction to questions 1–5: ***Go through the data set given below and solve the questions based on it.***

Pie charts as shown in Figs. 3.10 (a) and (b), given below present the percentage of students in various courses (A, B, C, D, E, F) and percentage of girls out of these courses.

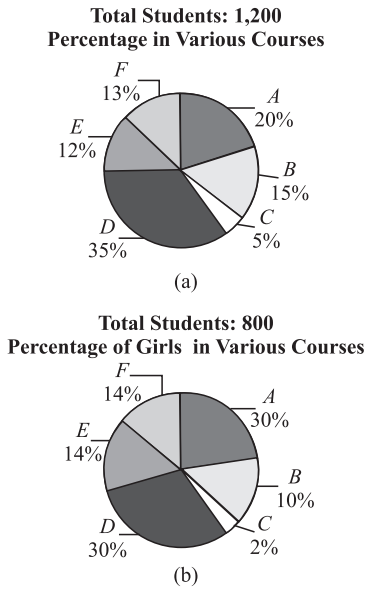


Fig. 3.10

1. For which course is the number of boys the minimum?
   (a) E (b) F
   (c) C (d) A
2. How many girls are there in course C?
   (a) 44 (b) 16
   (c) 40 (d) 160
3. For course D what is the respective ratio of boys and girls?
   (a) 3:4 (b) 4:5
   (c) 3:5 (d) 5:6
4. For which pair of courses is the number of boys the same?
   (a) E and F (b) A and D
   (c) C and F (d) B and D
5. For course E, the number of girls is how much per cent more than the boys for course E?
   (a) 250 (b) 350
   (c) 150 (d) 80

Direction to questions 6–10: ***Go through the data set given below and solve the questions based on it.***

Number of computers manufactured and sold by various companies in a year (number in lakhs) (Fig. 3.11).

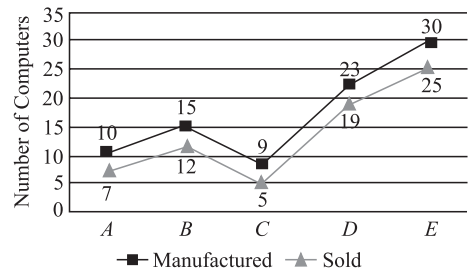


Fig. 3.11

6. What is the respective ratio of the number of computers manufactured by companies A and C together to the number of computers sold by companies A and C together?
   (a) 4:5 (b) 14:11
   (c) 8:9 (d) 7:5
7. What is the difference between the average number of computers manufactured by all the companies together and the average number of computers sold by all the companies together?

(a) 3,500 (b) 35,000
(c) 3,50,000 (d) 35,00,000

8. The number of computers sold by company *B* are what per cent of the number of computers manufactured by company *B*? (rounded off to two digits after decimal)
(a) 83.33 (b) 120
(c) 78.83 (d) 106.54

9. The number of computers manufactured by company *D* are what per cent of the number of computers manufactured by company *E*?
(a) 125 (b) 112.5
(c) 85 (d) 75

10. The number of computers manufactured by company *B* are approximately what per cent of the number of computers manufactured by all the companies together?
(a) 22 (b) 18
(c) 14 (d) 26

Direction to questions 11–15: ***Go through the data set given below and solve the questions based on it.***

Following pie chart as shown in Fig. 3.12 gives the degree wise Break-up of expenditure of a family in a particular month.

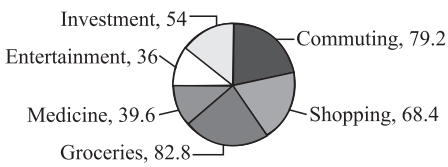


Fig. 3.12

Total amount spent in this month
= ₹ 45,800.

11. What is the amount spent by the family on commuting?
(a) ₹ 10,076 (b) ₹ 10,534
(c) ₹ 6,870 (d) ₹ 8,702

12. What is the respective ratio of amount spent by family on Medicine to the amount spent on Groceries?
(a) 1:2 (b) 13:21
(c) 3:5 (d) 11:23

13. What is the total amount spent by the family on Entertainment and Shopping together?
(a) ₹ 9,618 (b) ₹ 13,282
(c) ₹ 13,740 (d) ₹ 11,908

14. Total amount spent by the family on Groceries, Entertainment and Investments together forms approximately what per cent of amount spent on Commuting?
(a) 209 (b) 76
(c) 154 (d) 218

15. Amount spent by the family on Medicine forms what per cent of amount spent on Shopping? (rounded off to two digits after decimal)
(a) 43.67 (b) 49.52
(c) 57.89 (d) 61.89

Direction to questions 16–20: ***Go through the data set given below and solve the questions based on it.***

Note: All the values are in the multiples of 25 (Fig. 3.13).

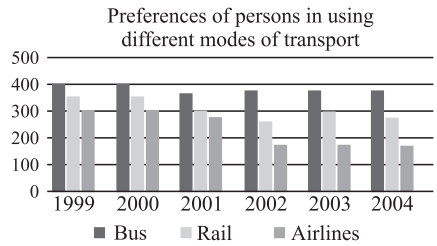


Fig. 3.13

16. From 1999 to 2004, the total number of people who preferred to travel by rail, was approximately how many millions?
(a) 1,300 (b) 1,500
(c) 1,600 (d) 1,800

17. The number of people preferring to travel by rail in 2004, was how many millions fewer than the number of people preferring to travel by rail in 1999?

(a) 70 (b) 45
(c) 75 (d) 50

18. In 2001, the people preferring to travel by bus represented approximately what per cent of the people preferring to travel by buses, rail and airlines together in that year?
(a) 65 per cent (b) 35 per cent
(c) 55 per cent (d) 45 per cent

19. What is the respective ratio of the number of people preferring to travel by bus to the number of people preferring to travel by rail in the year 2002?
(a) 15:11 (b) 9:7
(c) 7:9 (d) 11:15

20. In the year 2003, if all the airlines reduced their rates by 50% and 50% of the people who preferred to travel by rail, now preferred airlines, approximately how many millions prefer to travel by air?
(a) 290 (b) 313
(c) 330 (d) 325

EXERCISE 2

Direction to questions 1–5: ***Go through the data set given below and solve the questions based on it.***

Following pie chart as shown in Fig. 3.14 gives the percentage of people working in night shift from various industries. Table given ahead presents the percentage of females out of number of people working in night shift in different industries.

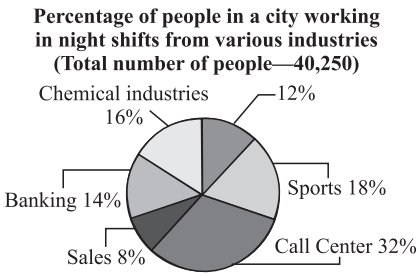


Fig. 3.14

Percentage of Females from various industries working in night shifts (Table 3.10):

Table 3.10

Industries	Females
IT	20%
Sports	20%
Call Centre	45%
Sales	60%
Banking	40%
Chemical Industries	15%

1. What is the respective ratio of men to the women working in night shifts from the Call Centre industry?
(a) 9:11 (b) 7:5
(c) 8:13 (d) None of these

2. What is the approximate average number of females working in night shifts from all the industries together?
(a) 2,227 (b) 4,481
(c) 3,326 (d) 2,823

3. What is the total number of men working in night shifts from al the industries together?
(a) 28,297 (b) 25,788
(c) 28,678 (d) 26,887

4. The number of women from the sports industry are what per cent of the total number of people working in the night shifts from all the industries together?
(a) 5.6 (b) 3.6
(c) 3.2 (d) 4.4

5. What is the difference between the total number of men and the total number of women working in night shifts from all the industries together?
(a) 13,254 (b) 13,542
(c) 13,524 (d) 13,363

Direction to questions 6–10: ***Go through the data set given below and solve the questions based on it.***

Following bar chart as shown in Fig. 3.15 presents the total number of boys and girls in five different departments in a college:

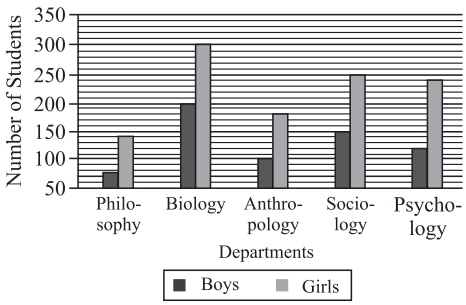


Fig. 3.15

6. The number of girls from Biology department is approximately what per cent of the total number of girls from all the departments together?
(a) 32 (b) 21
(c) 37 (d) 27

7. What is the difference between the total number of boys and the total number of girls from all the departments together?
(a) 440 (b) 520
(c) 580 (d) 460

8. What is the average number of boys from all the departments together?
(a) 122 (b) 126
(c) 130 (d) 134

9. The number of boys from Anthropology department is approximately what per cent of the total number of boys from all the departments together?
(a) 15 (b) 23
(c) 31 (d) 44

10. What is the respective ratio of number of girls from Philosophy department to the number of girls from Psychology department?
(a) 1:2 (b) 7:12
(c) 5:12 (d) 3:4

Direction to questions 11–15: ***Go through the data set given below and solve the questions based on it.***

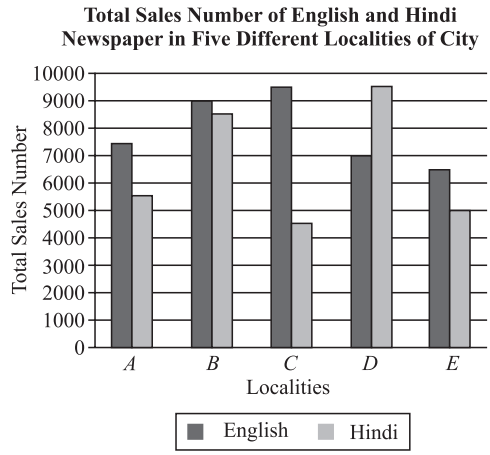


Fig. 3.16

11. What is the difference between the total sale of English Newspapers and the total sale of Hindi Newspapers in all the localities together?
(a) 6,000 (b) 6,500
(c) 7,000 (d) 7,500

12. The sale of English Newspaper in locality *A* is approximately what percent of the total sale of English Newspapers in all the Localities together?
(a) 527 (b) 25
(c) 111 (d) 19

13. What is the respective ratio of the sale of Hindi Newspapers in locality *A* to the sale of Hindi Newspapers in locality *D*?
(a) 11:19 (b) 6:5
(c) 5:6 (d) 19:11

14. The sale of English Newspaper in localities *B* and *D* together is approximately what per cent of the sale of English Newspaper in localities *A*, *C* and *E* together?
(a) 162 (b) 84
(c) 68 (d) 121

15. What is the average sale of Hindi Newspapers in all the localities together?
(a) 6,600 (b) 8,250
(c) 5,500 (d) 4,715

Direction to questions 16–20: ***Go through the data set given below and solve the questions based on it.***

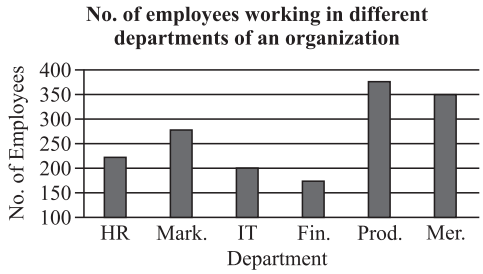


Fig. 3.17

All the data values given above are in the multiples of 25.

Table 3.11 presents the ratio of male and female in the organization:

Table 3.11

Department	Males	Females
HR	9	16
Marketing	3	2
IT	9	31
Finance	2	3
Production	11	4
Merchandising	4	3

Note: Mark.—Marketing; Fin.—Finance; Prod.—Production; Mer.—Merchandising

16. What is the total number of males working in all departments together?
(a) 755 (b) 925
(c) 836 (d) 784

17. What is the number of females working in the HR department?
(a) 158 (b) 128
(c) 136 (d) 144

18. What is the respective ratio of total number of employees working in the production department to those working in the Merchandising department?
(a) 15:14 (b) 8:7
(c) 14:15 (d) 7:8

19. Which is the department with lowest number of females working?
(a) Marketing (b) Production
(c) HR (d) Finance

20. What is the total number of employees from all departments together in the organization?
(a) 1,500 (b) 1,575
(c) 1,525 (d) 1,600

EXERCISE 3

Direction to questions 1–5: ***Go through the data set given below and solve the questions based on it.***

Study the following profile of parliament carefully and answer the questions given (Table 3.12).

Profile of Parliament (comprising of Lok Sabha and Rajya Sabha) in year 2014.

Total Members in Parliament = 640 (490 from Lok Sabha and 150 from Rajya Sabha).

Table 3.12

Lok Sabha (No. of Members)	Party	Rajya Sabha (No. of Members)
280	*A*	90
180	*B*	45
30	Others	15
490	Total	150
	Sex	
435	Males	120
55	Females	30
	Religion	
348	Hindus	85
42	Muslims	20
75	Sikhs	35
25	Christians	10
	Profession	
300	Graduates	50
45	Businessmen	19
60	Educators	11
85	Unknown	70

1. What is the approximate percentage of the Muslim members in Lok Sabha?
   (a) 9% (b) 11%
   (c) 13% (d) 14%

2. In Rajya Sabha if 30 male members were replaced by 30 female members, then what is the ratio of male members to female members respectively?
   (a) 3:1 (b) 3:2
   (c) 1:3 (d) 2:3

3. What percentage of members in Parliament are Businessmen?
   (a) 8% (b) 20%
   (c) 30% (d) 10%

4. If all the 'others' party members of Lok Sabha join the party '*B*' then what would be the ratio between members of party '*A*' to the members of party '*B*' respectively.
   (a) 3:2 (b) 6:5
   (c) 4:3 (d) 7:6

5. Out of total members of party '*B*' in parliament, what percentage of the members belongs to Rajya Sabha?
   (a) 30% (b) 35%
   (c) 25% (d) 20%

Direction to questions 6–10: ***Go through the data set given below and solve the questions based on it.***

Various food-grains sold by farmers at prices (Price per kg) mentioned (Table 3.13):

Table 3.13

Farmers	Food grains				
	Rice	Corn	Bajra	Paddy	Jowar
A	30	22.5	22	24	18
B	36	28	24.5	25	24
C	40	24	21	26	20.5
D	34.5	27.5	28	25	25
E	36	32	30	28.5	27

6. If farmer *A* sell 350 kgs of Rice, 150 kgs of Corn and 250 kgs of Jowar, how much would he earn?
   (a) ₹ 19,425 (b) ₹ 18,500
   (c) ₹ 15,585 (d) ₹ 18,375

7. What is the average price per kg. of Bajra sold by all the farmers together?
   (a) ₹ 25.10 (b) ₹ 24.50
   (c) ₹ 25 (d) ₹ 23.40

8. Farmer *D* and farmer *E*, both sell 240 kgs of Bajra each. What would be the respective ratio of their earnings?
   (a) 15:14 (b) 11:13
   (c) 14:15 (d) 13:15

9. Farmer *C* sells 180 kgs each of Corn, Paddy and Jowar grains. How much does he earn?
   (a) ₹ 13,540 (b) ₹ 12,550
   (c) ₹ 13,690 (d) ₹ 12,690

10. Earnings on 150 kgs of Paddy sold by farmer *B* is approximately what percentage of the earnings on the same quantity of Rice sold by the same farmer?
    (a) 65 (b) 69
    (c) 73 (d) 60

Direction to questions 11–15: ***Go through the data set given below and solve the questions based on it.***

Following bar chart as shown in Fig. 3.18 provides the percentage of Adult Males, Adult Females and Children out of total population in five colonies *A*, *B*, *C*, *D* and *E*:

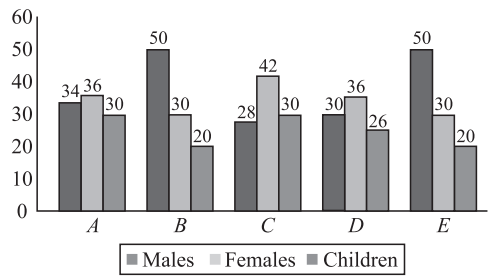


Fig. 3.18

Total number of residents in these colonies (Table 3.14):

Table 3.14

Colonies	Residents
A	1,250
B	2,050
C	1,800
D	1,150
E	1,620

11. What is the total number of adult females in colonies *A*, *B* and *C* together?
(a) 1,785
(b) 1,821
(c) 1,479
(d) 1,692

12. The number of children in colony *A* are approximately what per cent of the number of children in colony *E*?
(a) 121 (b) 116
(c) 75 (d) 101

13. What is the respective ratio of the number of adult males to the number of adult females in colony *B*?
(a) 3:5 (b) 7:5
(c) 8:7 (d) None of these

14. What is the average number of residents from all the colonies together?
(a) 1,654 (b) 1,600
(c) 1,580 (d) 1,574

15. What is the difference between the number of adult males and the number of children in colony *D*?
(a) 138 (b) 126
(c) 136 (d) 135

Direction to questions 16–20: ***Go through the data set given below and solve the questions based on it.***

Following pie chart as shown in Fig. 3.19 presents the percentage of employees working in different departments in an organization. Total number of employees = 2,500.

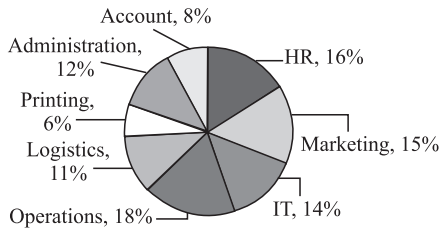


Fig. 3.19

Table 3.15 is the ratio of male to female in different departments in the same organization:

Table 3.15 Male : Female Ratio

Department	Male	:	Female
Administration	7	:	5
Accounts	2	:	3
HR	5	:	3
Marketing	7	:	8
IT	3	:	4
Operations	5	:	4
Logistics	6	:	5
Printing	2	:	1

16. What is the ratio between male employees in Administration and Printing departments respectively?
(a) 7:4 (b) 4:7
(c) 3:4 (d) 7:3

17. What is the difference between total number of employees in IT and Operations departments?
(a) 75 (b) 150
(c) 100 (d) 50

18. What is the ratio between total number of males in HR and Marketing and the total number of females in these two departments respectively?
(a) 13:15 (b) 15:13
(c) 13:17 (d) 17:14

19. How many female employees are there in HR department?
(a) 250 (b) 120
(c) 125 (d) 150

20. What is the difference between the male and female employees in Logistics department?
(a) 50 (b) 25
(c) 75 (d) 100

EXERCISE 4

Direction to questions 1–5: ***Go through the data set given below and solve the questions based on it.***

Following pie chart as shown in Fig. 3.20 gives the breakup of employees working in various departments of an organization. Table provides the ratio of men to women. It is given that Total Number of Employees = 1,800.

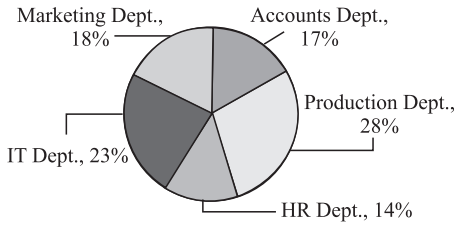


Fig. 3.20

Table 3.16 Ratio of Men to Women

Department	Men	Women
Production	11	1
HR	1	3
IT	5	4
Marketing	7	5
Accounts	2	7

1. What is the number of men working in the Marketing department?
(a) 132 (b) 174
(c) 126 (d) 189

2. The number of women working in the IT department of the organization forms approximately what per cent of the total number of employees in the organization from all departments together?
(a) 7 (b) 5
(c) 19 (d) 10

3. What is the respective ratio of the number of women working in the HR department of the organization and the total number of employees in that department?
(a) 3:4 (b) 2:5
(c) 2:9 (d) 3:7

4. What is the respective ratio of the number of men working in the Accounts department to the total number of employees working in that department?
(a) 9:2 (b) 7:6
(c) 2:9 (d) 6:7

5. The number of men working in the Production department of the organization forms what percentage of total number of employees working in that department? (rounded off to two digits after decimal)
(a) 89.76 (b) 91.67
(c) 88.56 (d) 94.29

Direction to questions 6–10: ***Go through the data set given below and solve the questions based on it.***

Table 3.17 Percent profit earned by six companies over the years

Company/ Year	*P*	*Q*	*R*	*S*	*T*	*U*
2004	11	12	3	7	10	6
2005	9	10	5	8	12	6
2006	4	5	7	13	12	5
2007	7	6	8	14	14	7
2008	12	8	9	15	13	5
2009	14	12	11	15	14	8

6. If the profit earned by Company *R* in the year 2008 was ₹ 18.9 lakhs, what was the income in that year?
(a) ₹ 303.7 lakhs (b) ₹ 264.5 lakhs
(c) ₹ 329.4 lakhs (d) ₹ 228.9 lakhs

7. What is the percentage rise in percent profit of Company *T* in year 2009 from the year 2004?

(a) 42 (b) 35
(c) 26 (d) None of these

8. If the profit earned by Company *P* in the year 2007 was ₹ 21 lakhs, what was its expenditure in that year?
(a) ₹ 30 lakhs (b) ₹ 15 lakhs
(c) ₹ 23 lakhs (d) ₹ 27 lakhs

9. What was the average percent profit of Company *S* over all the years together?
(a) 13.5 (b) 11
(c) 12 (d) 14

10. What is the difference between the percent profit earned by Company. *Q* in the year 2005 and the average of the percent profits earned by the remaining Companies together in that year?
(a) 4 (b) 2
(c) 1 (d) 3

Direction to questions 11–15: ***Go through the data set given below and solve the questions based on it.***

Table 3.18 provides the data regarding the number of tickets sold (number in thousands) in a week at multiplexes of six cities. Data has been given corresponding to five movies—*A*, *B*, *C*, *D* and *E*.

Table 3.18

Movie → / Cities ↓	*A*	*B*	*C*	*D*	*E*
Mumbai	20	15	35	26	18
Delhi	17	19	21	25	28
Kolkata	32	24	19	21	17
Chennai	18	21	32	28	34
Hyderabad	16	34	26	29	22
Lucknow	15	27	20	35	26

11. The number of tickets to movie *B* sold in Hyderabad was approximately. What percentage of the total number of tickets of the same movie sold in all the cities together?
(a) 15 (b) 18
(c) 12 (d) 24

12. What is the difference between the number of tickets of movie *D* sold in Kolkata and the number of tickets of movie *B* sold in Lucknow?
(a) 700 (b) 7,000
(c) 14,000 (d) None of these

13. What is the average number of tickets of movie *C* sold in all the six cities?
(a) 15,500 (b) 2,550
(c) 24,000 (d) 25,500

14. The number of tickets of movie *E* sold in Chennai is what percentage of number of tickets of movie *A* sold in Mumbai?
(a) 170 (b) 70
(c) 30 (d) 130

15. In which city was the total number of tickets of all the five movies together sold the minimum?
(a) Delhi (b) Chennai
(c) Lucknow (d) Kolkata

Direction to questions 16–20: ***Go through the data set given below and solve the questions based on it.***

Number of entertainment shows held in various cities in a year (number in hundreds) (Table 3.19).

Table 3.19

Cities	Shows				
	Dance	Music	Drama	Standup Comedy	Mimicry
M	15	21	24	0.8	0.9
N	12.4	13	26	2	0.5
O	5.7	8	12	0.3	0.2
P	11.3	6	18	1	1.5
Q	17	12.4	11	3	0.4
R	14	10.5	9.8	0.7	0.1

16. The mimicry shows held in city *M* are what per cent of the drama shows held in city *O*?

(a) 7 (b) 8.5
(c) 6.5 (d) None of these

17. What is the average number of entertainment shows held in city *P*?
(a) 756 (b) 678
(c) 786 (d) 698

18. If the number of music shows in cities *N* and *Q* is increased by 5%, what will be the total number of music shows in both the cities together?
(a) 2.602 (b) 2.667
(c) 2.540 (d) 2.605

19. What is the respective ratio of the number of dance shows held in city *N* to the number of drama shows held in city *R*?
(a) 49:62 (b) 49:51
(c) 62:45 (d) 62:49

20. What is the total number of standup comedy shows held in all the cities together?
(a) 820 (b) 740
(c) 780 (d) 810

EXERCISE 5

Direction to questions 1–5: ***Go through the data set given below and solve the questions based on it.***

Following pie chart as shown in Fig. 3.21 gives the Percentage wise breakup of total players who play five different sports. Given that total number of players = 4,200.

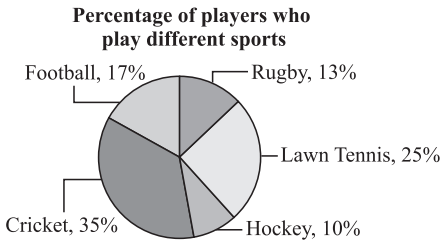


Fig. 3.21

Out of total 4,200 players, number of female Players is equal to 2,000. Following pie chart as shown in Fig. 3.22 presents the breakup of female players playing these five sports.

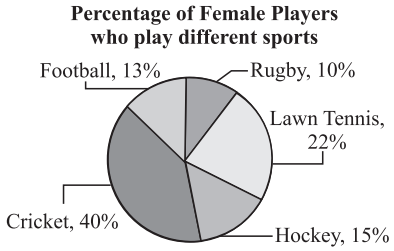


Fig. 3.22

1. What is the number of total players who play Football and Rugby together?
(a) 620 (b) 357
(c) 630 (d) 1,260

2. What is the difference between the number of the female players who play Lawn Tennis and the number of male players who play Rugby?
(a) 94 (b) 84
(c) 220 (d) 240

3. What is the respective ratio of the number of female players who play Cricket and number of male players who play Hockey?
(a) 20:7 (b) 4:21
(c) 20:3 (d) 3:20

4. What is the total number of male players who play Football, Cricket and Lawn Tennis together?
(a) 1,724 (b) 1,734
(c) 1,824 (d) 1,964

5. Number of male players who play Rugby is approximately what percentage of the total number of players who play Lawn Tennis?
(a) 33 (b) 39
(c) 26 (d) 21

Direction to questions 6–10: ***Go through the data set given below and solve the questions based on it.***

Number of articles (in thousands) Manufactured (M) and Defective (D) by 5 units of a company over the Years (Table 3.20).

Table 3.20

Year	UNIT									
	I		II		III		IV		V	
	M	D	M	D	M	D	M	D	M	D
1996	53	21	45	12	76	38	56	21	46	18
1997	49	18	32	10	45	24	63	24	36	14
1998	50	18	48	18	55	16	68	30	34	15
1999	65	20	68	15	57	20	54	19	48	12
2000	70	31	72	13	82	22	48	27	58	10
2001	44	15	56	22	38	32	40	15	60	11

6. What is the ratio between total number of articles manufactured by Unit III to that by Unit *V* for all the years together?
(a) 353:282 (b) 282:353
(c) 457:215 (d) 215:457

7. What is the average number of defective items from Unit II for the given years?
(a) 21,500 (b) 17,000
(c) 12,500 (d) 15,000

8. During which year the largest percentage of articles were defective out of the articles manufactured by unit IV?
(a) 1996 (b) 1997
(c) 1998 (d) 1999

9. What was the percentage (rounded off to nearest integer) of defective articles over the number of articles manufactured by all nits together in the year 2001?
(a) 42 (b) 40
(c) 37 (d) 33

10. During which year was the percentage increase/decrease in manufacture from the previous year the highest for Unit I?
(a) 1998 (b) 2001
(c) 1999 (d) 1997

Direction to questions 11–15: ***Go through the data set given below and solve the questions based on it.***

Table 3.21

Number of Males and Females staying in various Societies

Societies	Males	Females
A	250	350
B	400	150
C	300	275
D	280	300
E	180	250
F	325	300

Percentage of Male and Female Children in the Societies (Table 3.22):

Table 3.22

Societies	Children (out of total population)	Male Children (out of total children)	Female Children (out of total children)
A	25%	40%	60%
B	40%	75%	25%
C	16%	25%	75%
D	25%	80%	20%
E	40%	50%	50%
F	24%	46%	54%

11. What is the respective ratio of the number of adult females to the total number of female children staying in all the societies together?
(a) 243:82 (b) 112:71
(c) 82:243 (d) 71:112

12. What is the total number of female children staying in all the societies together?
(a) 314 (b) 433
(c) 410 (d) 343

13. What is the respective ratio of the total number of adult males in societies *A* and *B* together to the total number of adult males in societies *E* and *F* together?
(a) 3,520 (b) 3,360
(c) 4,100 (d) 3,000

14. What is the total number of members staying in all the societies together?

(a) 3,520 (b) 3,360
(c) 4,100 (d) 3,000

15. What is the difference between the number of male children in society *B* and the number of male children in society *F*?
(a) 84 (b) 14
(c) 96 (d) 26

Direction to questions 16–20: ***Go through the data set given below and solve the questions based on it.***

Following chart as shown in Fig. 3.23 gives the population of 2 states from year 2001 to year 2007:

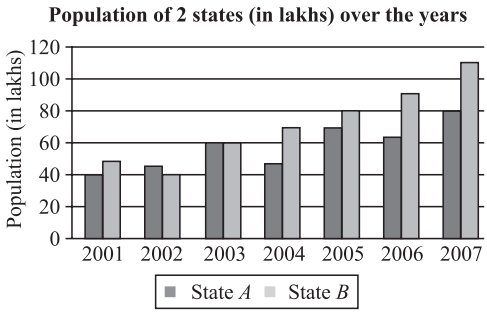


Fig. 3.23

16. Population of State *B* in 2002 is what per cent of the total population of State *B* in all the years together? (Rounded off to two digits after decimal)
(a) 8.56 (b) 7.26
(c) 8.32 (d) 8.16

17. What is the ratio between the total population of State *A* for the years 2001, 2002 and 2003 together and the total population of State *B* for the years 2005, 2006 and 2007 together respectively?
(a) 27:53 (b) 54:29
(c) 29:54 (d) 53:27

18. For which State and in which year the per cent rise in population from the previous year was the highest?
(a) State *B*—2003 (b) State *B*—2002
(c) State *A*—2004 (d) State *A*—2005

19. What is the per cent rise in population of State *B* from 2003 to 2004?
(a) $16\frac{1}{3}$ (b) $16\frac{2}{3}$
(c) $18\frac{2}{3}$ (d) $18\frac{1}{3}$

20. **Approximately** what is the average population of State *A* for all the given years?
(a) 65 lakhs (b) 50 lakhs
(c) 48 lakhs (d) 58 lakhs

ANSWER KEY

Exercise 1

1. (d)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (a)
9. (d)	**10.** (b)	**11.** (a)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (d)
17. (c)	**18.** (d)	**19.** (a)	**20.** (d)				

Exercise 2

1. (d)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (c)
9. (a)	**10.** (b)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (a)	**16.** (c)
17. (d)	**18.** (a)	**19.** (b)	**20.** (d)				

Exercise 3

1. (a)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (c)
9. (d)	**10.** (b)	**11.** (b)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (a)	**16.** (a)
17. (c)	**18.** (d)	**19.** (d)	**20.** (b)				

Exercise 4

1. (d)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (a)
9. (c)	**10.** (b)	**11.** (d)	**12.** (d)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (d)
17. (a)	**18.** (b)	**19.** (d)	**20.** (c)				

Exercise 5

1. (d)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (d)
9. (b)	**10.** (b)	**11.** (a)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (d)
17. (c)	**18.** (a)	**19.** (b)	**20.** (d)				

HINTS AND EXPLANATIONS

Exercise 1

1. Number of boys in course A is minimum, *i.e.*, zero. Hence, option (d) is the answer.

2. Number of girls in course $C = 16$. Hence, option (b) is the answer.

3. Required ratio = 180:240 = 3:4. Hence, option (a) is the answer.

4. Number of boys in each of courses C and $F = 44$. Hence, option (c) is the answer.

5. Required percentage = (112 – 32)/32 × 100 = 80/32 × 100 = 250. Hence, option (a) is the answer.

6. Number of computers manufactured by companies A and C together
= (10 + 7.5) lakh
= 17.5 lakhs
Number of computers sold by companies A and C together
= (7.5 + 5) lakh = 12.5 lakhs
Hence, required ratio
= 17.5:12.5 = 7:5
Hence, option (d) is the answer.

7. Average number of computers manufactured by all the companies together

$$= \left(\frac{10 + 15 + 7.5 + 22.5 + 30}{5}\right) \text{ lakhs}$$

$$= \left(\frac{85}{5}\right) \text{ lakhs} = 17 \text{ lakhs}$$

Average number of computers sold by all the companies together

$$= \left(\frac{7.5 + 12.5 + 5 + 17.5 + 25}{5}\right) \text{ lakhs}$$

= (67.5/5) lakhs = 13.5 lakhs
Hence, required difference
= (17 – 13.5) lakhs = 3.5 lakhs = 3,50,000.
Hence, option (c) is the answer.

8. Required percentage
= 12.5/15 × 100 = 38.33%.
Hence, option (a) is the answer.

9. Required percentage
= 22.5/30 × 100 = 75%
Hence, option (d) is the answer.

10. Number of computers manufactured by all the companies together = 85 lakhs

Number of computers manufactured by company B = 15 lakhs
Hence, required percentage
= 15/85 × 100 = 17.65 = 18
Hence, option (b) is the answer.

11. Angle for commuting
= 79.2° ⇒ 360° = 45,800
⇒ 79.2° = 45,800/360 × 79.2
= ₹ 10,076.
Hence, option (a) is the answer.

16. Total number of people travelling by rail
= (350 + 300 + 300 + 275 + 300 + 275) millions = 1,800 million
Hence, option (d) is the answer.

17. Required difference
= 350 – 275 = 75 millions
Hence, option (c) is the answer.

18. Total number of people travelling by buses, rail and airlines in 2001
= 375 + 300 + 175 = 850 millions
Number of people travelling by buses
= 375 millions.
Hence, required percentage
= 375/850 × 100 = 45.
Hence, option (e) is the answer.

19. Required ratio = 375:275 = 15:11.
Hence, option (a) is the answer.

20. In the year 2003:
Number of people travelling by rail
= 300 millions
Number of people travelling by air
= 175 millions
Now, 50% of people travelling by rail shift to air.
Hence, required number of people
= 175 + 150 = 325 million
Hence, option (d) is the answer.

Exercise 2

Solution to questions 1–5:

Number of people working in night shifts:
IT industry → 40,250 × 12/100 = 4,830
Number of women = 4,830 × 20/100 = 966 and number of men = 4,830 – 966 = 3,864
Sports industry → 40,250 × 18/100 = 7,245
Number of women = 7,245 × 20/100 = 1,449 and number of men = 7,245 – 1,449 = 5,796
Call centre → 40,250 × 32/100 = 12,880
Number of women = 12,880 × 45/100 = 5,796 and number of men = 12,880 – 5,796 = 7,084
Sales → 40,250 × 8/100 = 3,220
Number of women = 3,220 × 60/100 = 1,932 and number of men = 3,220 – 1,932 = 1,288
Banking → 40,250 × 14/100 = 5,635
Number of women = 5,635 × 40/100 = 2,254 and number of men = 5,635 – 2,254 = 3,381
Chemical industry → 40,250 × 16/100 = 6,440
Number of women = 6,440 × 15/100 = 966 and number of men = 6,440 – 966 = 5,474

1. Required ratio = 7,084:5,796 = 11:9.
Hence, option (d) is the answer.

2. Number of women:
= 966 + 1,449 + 5,796 + 1,932 + 2,254 + 966
= 13,363
Hence, required average
= 13,363/6 = 2,227.
Hence, option (a) is the answer.

3. Number of women:
= 3,864 + 5,796 + 7,084 + 1,288 + 3,381 + 5,474
= 26,887
Hence, option (d) is the answer.

4. Total number of working people = 40,250
Number of women in sports industry
= 1,449
Hence, required percentage
= 1,449/40,250 × 100 = 3.6
Hence, option (b) is the answer.

5. Required difference
= 26,887 – 13,363 = 13,524
Hence, option (c) is the answer.

6. Number of girls in all departments
= 140 + 300 + 180 + 250 + 240 = 1,110

Hence, required percentage
= 300/100 × 100 = 27
Hence, option (d) is the answer.

7. Total number of boys in all the departments
= 80 + 200 + 100 + 150 + 120 = 650
Number of girls = 1,110
Hence, required difference
= 1,110 – 650 = 460
Hence, option (d) is the answer.

8. Average number of boys = 650/5 = 130
Hence, option (c) is the answer.

9. Number of all boys = 650
Number of boys from Anthropology department = 100
Hence, required percentage
= 100/650 × 100 = 15.38 = 15
Hence, option (a) is the answer.

10. Number of girls from Philosophy department = 140
Number of girls from Psychology department = 240
Required ratio = 140:240 = 7:12
Hence, option (b) is the answer.

11. Total sale of Hindi Newspapers
= 5,500 + 8,500 + 4,500 + 9,500 + 5,000
= 33,000
Total sale of English Newspapers
= 7,500 + 9,000 + 9,500 + 7,000 + 6,500
= 39,500
Required difference
= 39,500 – 33,000 = 6,500
Hence, option (b) is the answer.

12. Required percentage
= 7,500/39,500 × 100 = 19
Hence, option (d) is the answer.

13. Required ratio = 5,500:9,500 = 11:19
Hence, option (a) is the answer.

14. Total sale of English Newspaper in localities *B* and *D*
= 9,000 + 7,000 = 16,000
Total sale of English Newspaper in localities *A*, *C* and *E*
= 7,500 + 9,500 + 6,500 = 23,500
Hence, required percentage
= 16,000/23,500 × 100 = 68%
Hence, option (c) is the answer.

15. Average sale of Hindi Newspaper
= 33,000/5 = 6,600
Hence, option (a) is the answer.

16. Number of males:
HR department = 225 × 9/25 = 81
Marketing department = 275 × 3/5 = 165
IT department = 200 × 9/40 = 45
Finance department = 175 × 2/5 = 70
Production department = 375 × 11/15 = 275
Merchandising department
= 350 × 4/7 = 200
Therefore Total number of males
= 81 + 165 + 45 + 70 + 275 + 200 = 836
Hence, option (c) is the answer.

17. Number of females working in the HR department = 225 × 16/25 = 144
Hence, option (d) is the answer.

18. Required ratio = 375:350 = 15:14
Hence, option (a) is the answer.

19. Number of females in different departments:
HR department = 225 – 81 = 144
Marketing department = 275 – 165 = 110
IT department = 200 – 45 = 155
Finance department = 175 – 70 = 105
Production department = 375 – 275 = 100
Merchandising department
= 350 – 200 = 150
Hence, option (b) is the answer.

20. Total number of employees
= 225 + 275 + 200 + 175 + 375 + 350
= 1,600
Hence, option (d) is the answer.

Exercise 3

1. Required percentage = 42/490 × 100 = 9%
Hence, option (a) is the answer.

2. Required ratio = (120 – 30):(30 + 30)
= 90:60 = 3:2
Hence, option (b) is the answer.

3. Number of businessmen = 45 + 19 = 64
Hence, required percentage
= 64/100 × 100 = 10%
Hence, option (d) is the answer.

4. Required ratio = 280:(180 + 30)
= 280:210 = 4:3
Hence, option (c) is the answer.

5. Required percentage

$$= \frac{45}{(180+45)} \cdot 100 = \frac{45}{225} \cdot 100 = 20\%$$

Hence, option (d) is the answer.

6. Farmer *A*'s earnings
= ₹ (350 × 30 + 150 × 22.5 + 250 × 18)
= ₹ (10,500 + 3,375 + 4,500) = ₹ 18,375
Hence, option (d) is the answer.

7. Average price of Bajra

$$= ₹\left(\frac{22 + 24.5 + 21 + 28 + 30}{5}\right)$$

= ₹ 25.10 per kg
Hence, option (a) is the answer.

8. Required ratio = 240 × 28:240 × 30 = 14:15
Hence, option (c) is the answer.

9. Farmer *C*'s earnings
= ₹ (180 × 24 + 180 × 26 + 180 × 20.5)
= ₹ 180 (24 + 26 + 20.5) = ₹ (180 × 70.5)
= ₹ 12,690
Hence, option (d) is the answer.

10. Required percentage = 25/36 × 100 = 69
Hence, option (b) is the answer.

11. Total number of adult females in colonies *A*, *B* and *C* together

$$= \left(\frac{1250 \times 36}{100} + \frac{2050 \times 30}{100} + \frac{1800 \times 42}{100}\right)$$

= (450 + 615 + 756) = 1,821
Hence, option (b) is the answer.

12. Number of children in colony *A*
= (1,250 × 30)/100 = 375
Number of children in colony *E*
= (1,620 × 30)/100 = 324
Required percentage
= 375/324 × 100 = 116
Hence, option (b) is the answer.

13. Required ratio = 50:30 = 5:3
Hence, option (d) is the answer.

14. Average number of residents from all the colonies together
= (1,250 + 2,050 + 1,800 + 1,150 + 1,620)/5
= 7,870/5 = 1,574
Hence, option (d) is the answer.

15. Required difference = (38 – 26)% of 1,150
= (12 × 1,150)/100 = 138
Hence, option (a) is the answer.

16. Total number of employees in administration department
= (2,500 × 12)/100 = 300
Number of male employees
= 7/12 × 300 = 175
Total number of employees in printing department
= (2,500 × 6)/100 = 150
Number of male employees
= 2/3 × 150 = 100
Therefore Required ratio
= 175:100 = 7:4
Hence, option (a) is the answer.

17. Required difference = 2,500 × (18 – 14)%

$$= \frac{2500 \cdot 16}{100} = 400$$

Hence, option (c) is the answer.

18. Total number of employees in HR department
= (2,500 × 16)/100 = 400
Hence, number of males
= 5/8 × 400 = 250
and number of females
= 400 – 250 = 150
Number of employees in marketing department
= (2,500 × 15)/100 = 375

Number of males
= 7/15 × 375 = 175
Number of females
= 375 – 175 = 200
Therefore, required ratio
= (250 + 175):(150+200)
= 425:350 = 17:14
Hence, option (d) is the answer.

20. Total number of employees in logistics department

$$= \frac{2500 \cdot 11}{100} = 275$$

= 275
Number of males = 6/11 × 275 = 150
Number of females = 275 – 150 = 125
Hence, required difference
= 150 – 125 = 25
Hence, option (b) is the answer.

Exercise 4

1. Number of men working in the Marketing department = 1,800 × 18/100 × 7/12 = 189
Hence, option (d) is the answer.

2. Number of women working in IT department
= 1,800 × 23/100 × 4/9 = 184
Hence, required percentage
= 184/1,800 × 100 = 10
Hence, option (d) is the answer.

3. Required ratio = 3:4. Hence, option (a) is the answer.

4. Required ratio = 2:9. Hence, option (c) is the answer.

5. Required percentage = 11/12 × 100 = 91.67.
Hence, option (b) is the answer.

6. If the expenditure be ₹ X lacs, then

$$18.9/x \times 100 = 9 \Rightarrow x = \frac{18.9 \cdot 100}{9}$$

= ₹ 210 lacs
Hence, income = ₹ (210 + 18.9) lacs
= ₹ 228.9 lacs
Hence, option (d) is the answer.

7. Percentage increase = 4/10 × 100 = 40
Hence, option (d) is the answer.

8. Required expenditure of the company

$$= \frac{2.1 \cdot 100}{7} = ₹\ 30 \text{ lacs}$$

Hence, option (a) is the answer.

9. Average per cent profit

$$= \frac{7+8+13+14+15+15}{6} = 12\%$$

Hence, option (c) is the answer.

10. Required difference

$$= 10 - \left(\frac{9+5+8+12+6}{5}\right)$$

= 10% – 8% = 2%
Hence, option (b) is the answer.

11. Total number of tickets sold of movie B
= (15 + 19 + 24 + 21 + 34 + 27) thousands
= 140 thousands
Required percentage = 34/140 × 100 = 24
Hence, option (d) is the answer.

12. Required difference = (27,000 – 21,000) = 6,000. Hence, option (d) is the answer.

13. Average number of tickets of movie C sold in all the cities

$$= \left(\frac{35+21+19+32+26+20}{6}\right) \text{thousand}$$

= 153/6 thousand = 25,500.
Hence, option (d) is the answer.

14. Required percentage = 34/20 × 100 = 170.
Hence, option (a) is the answer.

15. Total number of tickets sold of all the movies together:
Mumbai → (20 + 15 + 35 + 26 + 18) thousand
= 114 thousand
Delhi → (17 + 19 + 21 + 25 + 28) thousand
= 110 thousand
Kolkata → (32 + 24 + 19 + 21 + 17) thousand
= 113 thousand

Chennai → (18 + 21 + 32 + 28 + 34) thousand = 133 thousand
Hyderabad → (16 + 34 + 26 + 29 + 22) thousand = 127 thousand
Lucknow → (15 + 27 + 20 + 35 + 26) thousand = 123 thousand
Hence, option (b) is the answer.

16. Required percentage = 0.9/12 × 100 = 7.5.
Hence, option (d) is the answer.

17. Required average

$$= \frac{11.3+6+18+1+1.5}{5} \cdot 100$$

= 3780/5 = 756.
Hence, option (a) is the answer.

18. Number of music shows in cities *N* and *Q*
= 105% of (13 + 12.4) = 25.4 × 105/100
= 2,667.
Hence, option (b) is the answer.

19. Required ratio = 12.4:9.8 = 124:98 = 62:49.
Hence, option (d) is the answer.

20. Total number of standup comedy shows
= (0.8 + 2 + 0.3 + 1 + 3 + 0.7) × 100
= 780
Hence, option (c) is the answer.

Exercise 5

1. Average number of players who play Football and Rugby
= [(17 + 13)% of 4,200]
= 4,200 × 30/100 = 1,260
Hence, option (d) is the answer.

2. Number of players who play Rugby
= 4,200 × 13/100 = 546
Number of female players who play Rugby
= 2,000 × 10/100 = 200
Hence, number of male players who play Rugby
= 546 – 200 = 346
Number of female players who play Lawn Tennis
= 2,000 × 22/100 = 440
Hence, required difference
= 440 – 346 = 94.
Hence, option (a) is the answer.

3. Number of female cricketers
= 2,000 × 40/100 = 800
Number of male Hockey players

$$= \frac{4200\times 10}{100} - \frac{2000\times 15}{100}$$

= 420 – 300 = 120
Hence, required ratio
= 800:120 = 20:3
Hence, option (c) is the answer.

4. Number of male players who play Football, Cricket and Lawn Tennis
= (17 + 35 + 25)% of 4,200
– (13 + 40 + 22)% of 2,000
= 4,200 × 77/100 – 2,000 × 75/100
= 3,234 – 1,500 = 1,734
Hence, option (b) is the answer.

5. Number of male players who play Rugby
= 4,200 × 13/100 – 200 = 346
Number of players who play Lawn Tennis
= 4,200 × 25/100 = 1,050
Hence, required percentage
= 346/1,050 × 100 = 33
Hence, option (a) is the answer.

6. Number of articles manufactured by:
Unit III → (76 + 45 + 55 + 57 + 82 + 38) thousands = 353 thousands
Units V → (46 + 36 + 34 + 48 + 58 + 60) thousands = 282 thousands
Hence, required ratio = 353:282
Hence, option (a) is the answer.

7. Required average

$$= \frac{(12+10+18+15+13+22)}{6} \text{ thousands}$$

= 90/6 = 15 thousands
Hence, option (d) is the answer.

8. It was in the year 2000. Hence, option (d) is the answer.

9. Number of articles manufactured in 2001
= (44 + 56 + 38 + 40 + 60) thousands

= 238 thousands
Number of defective items
= (15 + 22 + 32 + 15 + 11) thousands
= 95 thousands
Hence, required percentage
= 95/238 ×100 = 39.9 = 40
Hence, option (b) is the answer.

10. Percentage increase in 1999
= 15/50 × 100 = 30%
Percentage decrease in 2001
= 26/70 × 100 = 37.14%
Hence, percentage decrease was highest in 2001.
Hence, option (b) is the answer.

11. Number of children:
Society $A \rightarrow 600 \times 25/100 = 15$
Number of female children
= 150 × 60/100 = 90
Number of male children
= 150 – 90 = 60
Society $B \rightarrow 550 \times 40/100 = 220$
Number of female children
= 220 × 25/100 = 55
Number of male children
= 220 – 55 = 165
Society $C \rightarrow 575 \times 16/100 = 92$
Number of female children
= 92 × 75/100 = 69
Number of male children
= 92 – 69 = 23
Society $D \rightarrow 580 \times 25/100 = 145$
Number of female children
= 145 × 20/100 = 29
Number of male children
= 145 – 29 = 116
Society $E \rightarrow 340 \times 40/100 = 172$
Number of female children
= 172 × 50/100 = 86
Number of male children
= 172 – 86 = 86
Society $F \rightarrow 625 \times 24/100 = 150$
Number of female children
= 150 × 54/100 = 81
Number of male children
= 150 – 81 = 69
Number of adult females
= (350 + 150 + 275 + 300 + 250 + 300) – 410
= 1,625 – 410 = 1,215
Number of female children
= 90 + 55 + 69 + 29 + 86 + 81 = 410
Hence, required ratio
= 1,215:410 = 243:82
Hence, option (a) is the answer.

12. Using the data obtained in the previous question:
Number of all female children = 410.
Hence, option (c) is the answer.

13. Number of adult males in societies *A* and *B*
= (250 – 60) + (400 – 165)
= 190 + 235 = 425
Number of adult males in societies *E* and *F*
= (180 – 86) + (325 – 69) = 94 + 256 = 350
Required ratio = 425:350 = 17:14
Hence, option (d) is the answer.

14. Number of all members
= (250 + 350) + (400 + 150) + (300 + 275) + (280 + 300) + (180 + 250) + (325 + 300) + 150 + 220 + 92 + 145 + 172 + 150 = 4,289
Hence, option (d) is the answer.

15. Required difference = 165 – 69 = 96
Hence, option (c) is the answer.

16. Total population of state *B* in all the years
= (50 + 40 + 60 + 70 + 80 + 90 + 100) lakhs
= 490 lakhs
Population of state *B* in 2002 = 40 lakhs
Hence, required percentage
= 40/490 × 100 = 8.16%
Hence, option (d) is the answer.

17. Required ratio
= (40 + 45 + 60):(80 + 90 + 100)
= 145:270 = 29:54
Hence, option (c) is the answer.

18. It is obvious from the graph.
Hence, option (a) is the answer.

19. Percentage increase
= (70 – 60)/60 × 100
= 50/3 = 162/3
Hence, option (b) is the answer.

20. Average population of the state *A*

$$= \left(\frac{40+45+60+50+70+65+80}{7}\right) \text{lakhs}$$

= 410/7 lakhs
= 58 lakhs
Hence, option (d) is the answer.

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SECTION 1

PART C

Logical Reasoning

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1 Sequencing and Arrangement

In the IIM Indore-IPM and other major B-School exams, sequencing and arrangement questions constitute a major portion of the paper. In this chapter, we will discuss sequencing and arrangement both as a skill and as a type of question.

However, throughout this book, other than this chapter, we will be using the word sequencing and arrangement more as a skill, rather than as a type of question.

WHAT IS SEQUENCING AND ARRANGEMENT?

Sequencing is the act of putting things in a specific sequence, order or a list. In our day-to-day life too, we go through many examples of sequencing or arranging things and events such as:

While writing appointments on a calendar or a mobile,

While placing an order in a restaurant—Talumein soup, kebabs, ice cream and finally mocha,

While taking a test and deciding which section is to be attempted first, second and so on.

The only difference between sequencing in our practical life and the LR set will be the fact that the LR set given in the questions will have a prior order of variables. As a student your job will be to find out that order or sequence.

TYPES OF SEQUENCING AND ARRANGEMENT

While a good number of times the statements/conditions given in the LR set will specify the exact positioning of the entities given. However, many a times only the relative positioning of the variables will be given.

Let us see some examples of such statements:

Context	Statement reads as
Which variables are exactly placed in the sequence?	M is fourth.
Which variables are forbidden from a specific position in the sequence?	M is not fifth.
Which variables are next to, before or immediately preceding or following one another?	M and N are consecutive. A is next to B. No variable comes between A and B. A and B are consecutively in the sequence.
Which variables cannot be next to, before or immediately preceding or following one another?	A does not immediately precede or follow B. A is not immediately before or after B. M and N are not consecutive in the sequence.
How far apart in the sequence are two particular variables?	Exactly two people come between A and B.
What is the relative position of A and B in the sequence?	A comes before B in the queue. M comes after N in the queue.

In the above given examples, we observe two types of sequencing:

Strict Sequencing and Arrangement

In these kinds of sequencing, relative positioning of the variables will be almost fixed. These are the rules which give us the exact position of the variables.

Example

Direction for questions 1 to 3: ***Read the following passage below and solve the questions based on it.***

During the summer placements at MU, which will run for a single week in June starting from Monday through Friday; Due North Inc. will interview three applicants Omkara, Tyagi, and Keshu. Exactly five interviews will be conducted in this week long schedule with strictly one interview per day.

Further information about the schedule of the interviews is as follows:

- Each applicant is interviewed at least once.
- Omkara is interviewed twice on two consecutive days.
- Tyagi has to meet his girl friend Billo on Wednesday, therefore he cannot be interviewed on Wednesday.
- Keshu is not interviewed on Friday as he has to go to meet Dolly who is Omkara's wife.
- If Tyagi is interviewed on Monday then Keshu must be interviewed on Tuesday.

1. Which out of the following week's schedule could be true?
(a) The person who is interviewed on Tuesday is also interviewed on Thursday.
(b) The person who is interviewed on Wednesday is also interviewed on Friday.
(c) The person who is interviewed on Monday is also interviewed on Tuesday.
(d) Tyagi is interviewed on Monday and Thursday.

2. If in addition to Omkara, one more person is interviewed on two consecutive days, then which of the following could be true?
(a) Tyagi is interviewed on Monday and Omkara is interviewed on Wednesday.
(b) Tyagi is interviewed on Tuesday and Friday.
(c) Tyagi is interviewed on Tuesday and Omkara is interviewed on Friday.
(d) Keshu is interviewed on Tuesday and Omkara is interviewed on Thursday

3. If the week's interview schedule has the same applicant scheduled for Monday and Thursday then on which day must Tyagi be interviewed?
(a) Monday (b) Tuesday
(c) Wednesday (d) Friday

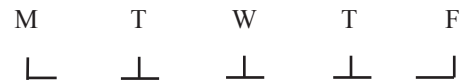
Solution

Scenario

There are three applicants—Omkara (O), Tyagi (T) and Keshu (K) and they are to interviewed for five interviews, Monday through Friday. Looking at the given limitations of exactly one interview per day and with only three applicants for five interviews, some of the applicants must be interviewed more than once.

Rules

The interviews are conducted one per day starting on Monday through Friday. Hence sketch a weekly schedule with five spaces first.



The first condition says each applicant will be interviewed at least once. The next condition says Omkara is interviewed on two consecutive days–lets have an OO block for two consecutive days. The next two conditions tell about the days on which Tyagi and Keshu cannot be interviewed; mark this information in your diagram. The last condition says that if Tyagi is interviewed

on Monday, Keshu must be interviewed on Tuesday.

T (MON) then – **K** (TUE)

The OO block for Omkara is the only big restricting factor present here. There is nothing that stops its placement on any particular pair of days in the schedule. But since Omkara is interviewed on exactly two days and Tyagi and Keshu are each to be interviewed at least once, you have four out of the five entities. The fifth interview will go to either P or S.

If Tyagi is interviewed on Monday, Keshu is interviewed on Tuesday, then Omkara will be interviewed either on Wednesday-Thursday or Thursday-Friday. Hence, one thing is sure that Omkara is definitely interviewed on Thursday. And if any of this does not happen, then it means that Tyagi won't be interviewed on Monday.

T (MON) then – **K** (TUE)

Mon	Tue	Wed	Thu	Fri
T		K	O	O
---	---	---	---	---
				O O

1. (c) To find out the 'could be true' statements, we should eliminate the options that must be false.

Option (a) does not allow to place the pair of days for Omkara anywhere, so it can't be true.

Option (b) cannot be true either: Tyagi is not interviewed on Wednesday and Keshu is not interviewed on Friday, the only applicant who can be interviewed on both Wednesday and Friday is Omkara, but his two interviews must happen on consecutive days.

Option (d) also can't be true; if Tyagi is interviewed on Monday, we have already concluded that Omkara will be interviewed on Thursday.

Only option (c) could be true.

2. (d) According to the conditions given in the question, besides an OO block, we will have either a TT block or a KK block.

Going through the options:

Option (a) suggests that if Tyagi is on Monday, then Keshu has to be on Tuesday. But with Keshu on Tuesday and Omkara on Wednesday and Thursday, we don't have the consecutive days block left either for PP or SS, so option (a) is ruled out.

Option (b) cannot be the answer because here Tyagi is interviewed on two non-consecutive days, and that stops every body besides Omkara to go on two consecutive days.

Option (c) Omkara is on Friday, so he is interviewed on Thursday also. Since both Keshu and Omkara can't go on Wednesday (using condition 3) Keshu must be interviewed on Wednesday. Taking Tyagi on Tuesday, he must be the second person to be interviewed on two consecutive day—Monday and Tuesday. But if we put Tyagi on Monday then Keshu should be interviewed on Tuesday.

So option (c) cannot be the answer.

Option (d) is the only option left out, so this has to be the answer.

If Keshu is on Tuesday and Omkara is on Thursday, then Keshu can be on Monday and Omkara can be on Wednesday. Now Tyagi can be interviewed on Friday.

M	T	W	T	F
Keshu	Keshu	Omkara	Omkara	Tyagi

3. (d) If the same applicant is to be scheduled for both Monday and Thursday, then it can't be Omkara, who must have the interview on two consecutive days. It can't be Tyagi too, because Tyagi on Monday and Omkara on Thursday is not an acceptable set. So it must be Keshu. Now, Monday

and Thursday are taken up by Keshu, then Omkara must be interviewed on Tuesday and Wednesday (the only consecutive days left). Hence Tyagi must be on Friday. Option (d) is correct.

Loose Sequencing and Arrangement

In this kind of sequencing, the data given will tell us the relative positioning of one variable with respect to the other variable; without giving the exact position.

Example

Direction for questions 4 and 5: ***Read the following passage and solve the questions based on it.***

Eight parties—Forward Block, Hindustan Party, Janta Party, Lokmat, New Socialist party, Praja party, Socialist party and TDP, contested in the last general election. The number of seats won by these parties vary largely. While announcing the final results, The election commission expressed the following observation regarding the number of seats won by these parties in terms of their popularity (the more the popularity, the more is the number of seats won by the party and vice-versa.

(While doing the ranking, take the first to be being the party with the maximum seats won and the eighth to be the party with minimum seats won.)

(i) No two parties won the same number of seats.
(ii) The Socialist party and the Forward block are each less popular than the TDP.
(iii) The Socialist party is more popular than the Janta party.
(iv) The Lokmat and the New Socialist party are each less popular than the Forward block.
(v) The Hindustan party and the Lokmat are each less popular than the Janta party.
(vi) The Praja party is less popular than the Lokmat.
(vii) The Praja party is not the eighth in rank.

Scenario

When planning the diagram for the sequencing first, decide if it should be drawn horizontally or vertically. To draw the diagram here, we will think of the rankings in terms of top to bottom.

Unlike the problems done in the strict sequencing section, writing 1 to 8 or creating eight spaces will not serve the purpose owing to the fact that no exact information regarding the positioning of variables, *e.g.* (The Praja party is not eighth) is given.

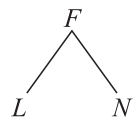
Rules

Start visualizing. Let us name each party with its first letter as in the Forward Block is *F*, etc.

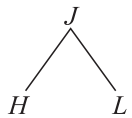
The first condition states that *S* and *F* are both less popular than *T*. Here we don't know anything about the space between these two entities, just the order. Draw *T* above both *S* and *F* by keeping in mind that nothing is said about the relationship between *S* and *F*.

The second condition: *S* is more popular than *J*

Next condition: Both *L* and *N* are less popular than *F*



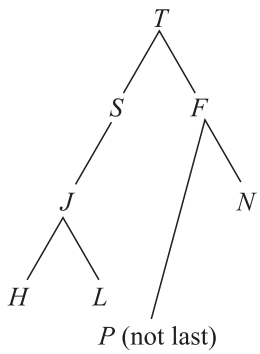
Next condition: Both *H* and *L* are below *J*



Next condition: P is less popular than L

Last condition: P is not the eighth. "$P \neq 8$" or "P is not the last"

Let us now start joining the information. If we connect S from condition 2 to condition 1, this is exactly what we get:



After joining the rules, we have the relationship between more parties.

Which is higher, F or P? Of course F is higher, and we know we can determine the relationship between them because we can travel from one to the other in one direction.

FLAW DETECTOR—Now have a look at F and H. Can we say that F is higher then H? The answer is No. It is all because of the fact that to go from F to H, we would have to "travel" along the tree in more than one direction (first up then down, or first down then up). By doing this there is no definite relationship. F could be ranked higher than H or H could be higher than F. Which party is ranked last? Either H or N.

Lets now consider the questions.

4. Which of the following could be the order of the parties listed from winning the maximum number of seats to the minimum number of seats?

(a) T, F, S, L, J, N, P, H
(b) S, T, F, N, J, L, P, H
(c) T, S, J, H, F, P, L, N
(d) T, S, J, F, L, N, P, H

(All the parties are being denoted by their first letter).

Ans. (d) To start with, this could be seen as the complete and accurate list, since all parties are taken into consideration in each answer choice.

Now, start going through the options.

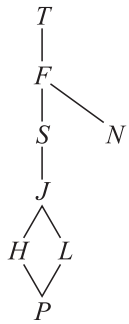
Apply each condition to the given options. Option (b) violates condition (i) by making S more popular than T, therefore option (b) must be incorrect. Condition (ii) and (iii) don't help eliminate any options, but (a) violates condition (iv) by making L more popular than J, and is hence incorrect. Option (c) gives P a higher popularity than L, a violation of condition (v). We have eliminated three options. So the remaining option D has to be the correct answer.

5. If the Forward block is more popular than the Socialist Party and the Hindustan party is more popular than the Praja Party, then which of the following must be true of the number of seats won by different parties in the election?

(a) The Hindustan Party is sixth
(b) The Lokmat is sixth
(c) The Lokmat is seventh
(d) The New Socialist Party is eighth

Ans. (d) Let us again use the first letter of the parties as their symbol.

This "if" question places two additional conditions on the order of popularity. The first condition places F above S. The second condition places H above P. Redraw your sketch to incorporate these new conditions:



Going through the options against the backdrop of the introduction of new facts like *F* is assigned to position 2 and the elimination of *H* as a possibility for the last position (leaving only *N* available for that slot) in the new sketch, option (d) is correct. Since both *H* and *L* could be in the sixth position, (a) and (b) don't have to be true, although these are probably true. Also, if *L* is placed at the seventh position, then *P* would have to be in the last position. And this violates the last condition, making option (c) incorrect.

Circular Arrangement

The Circular Arrangement questions are not necessarily the questions regarding a situation where in we need to make the seating arrangement around a circular table. It can be a rectangular table or even a hexagonal table. We should understand Circular Arrangement as an arrangement having a closed loop.

This can be understood with the following diagrams also:

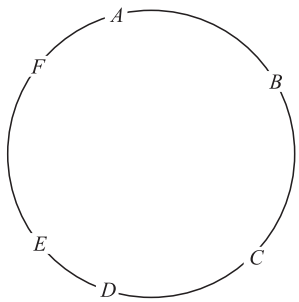


Fig. 1

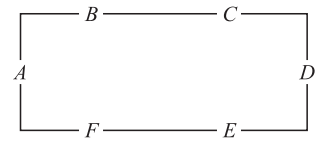


Fig. 2

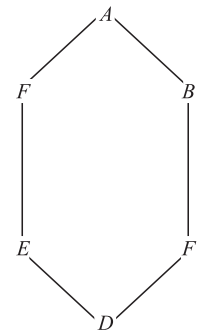


Fig. 3

The above given diagrams might appear to us as three different structures/arrangements. However, there would be a minimal difference in the descriptions given for the above situations. In all the diagrams, we find the following things common:

- *A* is sitting opposite to *D*.
- *B* is sitting on the immediate left of *A*.
- *E* is sitting in between *F* and *D*.

Now if we go on to making the diagrams pertaining to the details given, we will be clueless about the kind the diagram which is going to be formed.

If we form the question as—*A*, *B*, *C*, *D*, *E* and *F* are sitting around a circular table ... → We would be using Fig. 1.

If we form the question as—*A*, *B*, *C*, *D*, *E* and *F* are sitting around a rectangular table ... → We would be using Fig. 2.

If we form the question as—*A*, *B*, *C*, *D*, *E* and *F* are sitting around a hexagonal table ... → We would be using Fig. 3.

Similarly, we might be using a totally different arrangement in accordance with the given statements.

However, one thing that should be very clear in our mind is that, in any circular arrangement, what ever be the structure, the loop should be closed.

Example

Direction for questions 6 and 7: ***Read the following passage and solve the questions based on it.***

Five leaders from undivided India—Pt. Nehru (P), M.K. Gandhi (M), Rajendra Prasad (R), Subhash Chandra Bose (S) and Tyagi (T) participated during the 2nd Round Table Conference at London in 1930.

It was noted that Pt. Nehru sat two seats to the left of Rajendra Prasad and M.K. Gandhi sat two seats to the right of Rajendra Prasad.

6. If Subhash Chandra Bose sat between M.K. Gandhi and Rajendra Prasad, then who sat to the immediate right of Pt. Nehru?
(a) Tyagi
(b) Subhash Chandra Bose
(c) M.K. Gandhi
(d) Rajendra Prasad

7. If Subhash Chandra Bose did not sit next to M.K. Gandhi, then who sat between M.K. Gandhi and Subhash Chandra Bose?
(a) Rajendra Prasad
(b) Pt. Nehru
(c) Tyagi
(d) Cannot be determined

Solutions

Scenario

There are five leaders and five sitting positions.

Rules

Pt. Nehru sat two seats to the left of Rajendra Prasad and M.K. Gandhi sat two seats to the right of Rajendra Prasad. Let us see this with the help of a diagram:

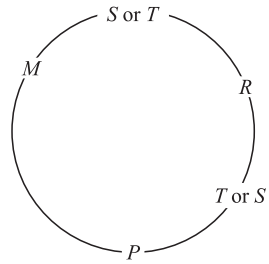


Fig. 4

6. If Subhash Chandra Bose sat between M.K. Gandhi and Rajendra Prasad, then the arrangement would be as follows:

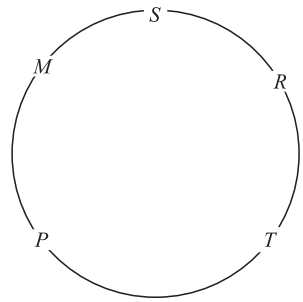


Fig. 5

It is obvious from the above given diagram that Tyagi sat to the immediate right of Pt. Nehru.
Hence, option (a) is the answer.

7. If Subhash Chandra Bose did not sat next to M.K. Gandhi, then the seating arrangement would be as follows:

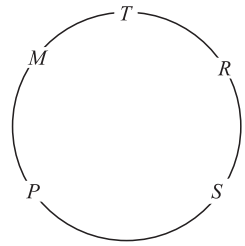


Fig. 6

Hence, option (b) is the answer.

PRACTICE EXERCISES

EXERCISE 1

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

(i) A resort has 6 rooms—*A*, *B*, *C*, *D*, *E* and *F*. Rooms '*A*' and '*C*' can accommodate two persons each; the rest can accommodate only one person each.
(ii) Eight persons *P*, *Q*, *R*, *S*, *T*, *U*, *W* and *X* are to live in these rooms. *Q*, *T*, and *X* are females while the rest are males. Persons of different genders can't be put together in the same room.
(iii) No man is willing to live in room *C* or *F*.
(iv) P wants to live alone but doesn't want to live in room *B* or *D*.
(v) *S* needs a partner but is not ready to live with either *U* or *W*.
(vi) *X* does not want to share her room.

1. Who among the following will live in room *E*?
(a) *U*
(b) *W*
(c) *P*
(d) Cannot be determined

2. Which of the given statements is not needed to complete the living arrangement so far?
(a) (ii)
(b) (iv)
(c) (v)
(d) All are necessary

3. In which of the following rooms will *U* live?
(a) *B* (b) *D*
(c) *A* (d) *B* or *D*

4. *X* will live in which of the following rooms?
(a) *C*
(b) *F*
(c) *B*
(d) Cannot be determined

Direction for questions 5–8: ***Read the passage and solve the questions based on it.***

(i) There are six different DVDs of different movies *P*, *Q*, *R*, *S*, *T* and *U*. These DVDs are kept one above the other on a shelf and belong to six different people—*A*, *B*, *C*, *D*, *E* and *F*. It is not necessary that the orders of these DVDs and persons are the same.
(ii) The DVD of movie *Q* is kept between the DVD of movie *P* and *T* and the DVD of movie *S* is kept between the DVD of movie *P* and *U*. The DVD of movie *R* is immediately above the DVD of movie *T*.
(iii) *C*'s DVD is kept on the top. *A* does not have the DVDs of movies *T* and *S*. The DVD of movie *P* belongs to *F*. The DVD of movie *U* belongs neither to *B* nor to *A*. *D*'s DVD is kept at the bottom.

5. Which movie DVD belongs to *A*?
(a) *Q* (b) *S*
(c) *P* (d) *T*

6. Who owns the DVD of movie *T*?
(a) *B* (b) *E*
(c) *C* or *E* (d) *B* or *E*

7. Who owns the DVD of movie *U*?
(a) *B* (b) *E*
(c) *D* (d) *C*

8. The DVD of which of the following movies is kept on the top?
(a) *T*
(b) *R*
(c) *U*
(d) Cannot be determined

Direction for questions 9–13: ***Read the following passage and solve the questions based on it.***

A, *B*, *C*, *D*, *E*, *F* and *G* are seven persons who travel to office everyday by a particular train

which stops at five stations 1, 2, 3, 4 and 5 respectively after leaving its base station.

(i) Three among them get on the train at the base station.
(ii) *D* gets down at the next station at which *F* gets down.
(iii) *B* does not get down either with *A* or *F*.
(iv) *G* alone gets on at station 3 and gets down with *C* after having passed one station.
(v) *A* travels between only two stations and gets down at station 5.
(vi) None of them gets on at station 2.
(vii) *C* gets on with *F* but does not get on with either *B* or *D*.
(viii) *E* gets on with two others and gets down alone after *D*.
(ix) *B* and *D* work in the same office and they get down together at station 3.
(x) None of them got down at station 1.

9. At which station does *E* get down?
(a) 2
(b) 3
(c) 4
(d) Cannot be determined

10. At which station do both *C* and *F* get on?
(a) 1 (b) 2
(c) 4 (d) None of these

11. At which of the following stations do *B* and *D* get on?
(a) 1
(b) 2
(c) 3
(d) Cannot be determined

12. After how many stations does *E* get down?
(a) 1 (b) 2
(c) 3 (d) 4

13. At which station, maximum people get down?
(a) 2 (b) 5
(c) 3 (d) 4

Direction for questions 14–17: ***Read the following passage and solve the questions based on it.***

Three men (Tom, Peter and Jack) and three women (Eliza, Anne and Karen) are spending a few months at a hillside. They are to stay in a row of nine houses, which are facing north, each one living in his or her own house. There are no others staying in the same row of houses. Following are the details given regarding each of their houses:

(i) Anne, Tom and Jack do not want to stay in any house, which is at the end of the row.
(ii) Eliza and Anne are unwilling to stay beside any occupied house.
(iii) When Karen, Peter and Jack stand facing north, Karen finds that houses of both Peter and Jack are on her left-hand side.
(iv) Between Anne and Jack's house there is just one vacant house.
(v) None of the girls occupy adjacent houses.
(vi) The house occupied by Tom is next to the house at the end.
(vii) House of *P* is at one of the ends.
(viii) Tom is beside Peter.

14. Which of the above statements can be said to be redundant?
(a) (i) (b) (ii)
(c) (iii) (d) (v)

15. How many of them occupy houses beside to a vacant house?
(a) (ii) (b) (iii)
(c) (iv) (d) (v)

16. Which among these statement(s) is/are definitely true?
I. Anne is between Eliza and Jack.
II. At the most four persons can have occupy houses on either side of them.
III. Tom stays beside Peter.
(a) I (b) II
(c) I and III (d) II and III

17. If we number all the houses from 1 through 9 and assign the house of *P* as number 1, then house number/s of how many people can NOT be definitely ascertained?
(a) None (b) (i)
(c) (ii) (d) (iii)

Direction for questions 18–20: ***Read the following passage and solve the questions based on it.***

Mr Sinha has six children—Pankaj, Ravi, Santosh, Tipu, Vikash and Prakash. They all were born on 13th November, but each was born in a different year, during six consecutive years. It is also known that:

(i) Pankaj is elder to Santosh.
(ii) Ravi is elder to both Tipu and Vikash.
(iii) Prakash is two years older than Tipu.
(iv) Pankaj was born either in the year 2002 or 2003.
(v) The oldest member of the group was born in the year 2000.

18. Which of the following could be a correct list of the group, from the youngest to the oldest?
(a) Santosh, Pankaj, Ravi, Tipu, Vikash, Prakas
(b) Santosh, Vikash, Pankaj, Tipu, Prakash, Ravi
(c) Santosh, Vikash, Tipu, Prakash, Pankaj, Ravi
(d) Santosh, Vikash, Tipu, Pankaj, Prakash, Ravi

19. If Pankaj was born in 2002, then which of the following is definitely true?
(a) Ravi was born in 2000
(b) Prakash was born in 2001
(c) Santosh was born in 2003
(d) Vikash was born in 2003

20. If Prakash is the eldest child, then which of the following is definitely true?
(a) Ravi was born in 2001
(b) Tipu was born in 2001
(c) Santosh was born in 2004
(d) Pankaj was born in 2004

EXERCISE 2

Direction for questions 1–3: ***Read the following passage and solve the questions based on it.***

In an obsession with the letter P, a person named all his sons starting with P.

(i) These are the six names—Pailashanand, Pamleshanand, Punalanand, Pedaranand, Parananand, Pamalanand.
(ii) Pailashanand is not the heaviest while Pedaranand is not the most handsome.
(iii) The lightest of the group is the most handsome of the group.
(iv) Pamleshanand is more handsome then Pamalanand, who is more handsome than Parananand.
(v) Pailashanand is less handsome than Pamalanand but is heavier than Pamalanand.
(vi) Pamleshanand is lighter than Parananand but heavier than Punalanand.
(vii) Parananand is more handsome than Pailashanand while Pamalanand is heavier than Parananand.

1. Who is the heaviest of the group?
(a) Pailashanand (b) Pedaranand
(c) Pamleshanand (d) Pamalanand

2. What is the rank of Pamalanand in the decreasing order of weight?
(a) Third (b) Fourth
(c) Fifth (d) Second

3. Who is most handsome?
(a) Pamleshanand (b) Pedaranand
(c) Pamalanand (d) Punalanand

Direction for questions 4–7: ***Read the following passage and solve the questions based on it.***

In the recent fashion show *LFW*, seven fashion designers presented their clothes to the viewers. Names of the fashion designers have been withheld due to security reasons, however to identify their clothes it has been decided that the first letter of their names will be used for their outfits. The names of the different fashion designers:

C, *L*, *W*, *G*, *D*, *J*, and *S*. Outfits made by these designers have been placed in a row in the following order:

(i) *S* is placed on the immediately left of *C*.
(ii) *C* is fourth to the left of *D*.
(iii) *L* is between *G* and *W*.

(iv) *D*, which is the third to the right of *G*, is at one of the ends.

4. How many outfits have been placed between *J* and *G*?
(a) 4 (b) 3
(c) 2 (d) 1

5. What is the position of *C*?
(a) Second to the left of *L*
(b) Second to the left of *W*
(c) Third to the left of *G*
(d) Between *S* and *J*

6. Which two outfits are at the two ends?
(a) *S* and *D* (b) *S* and *W*
(c) *J* and *W* (d) *J* and *D*

7. Which of the following is not true?
(a) *S* and *C* are consecutively placed
(b) *J* is at one of the ends
(c) There are two outfits between *C* and W
(d) Positions of *J* and *W* are interchangeable

Direction for questions 8–11: ***Read the following passage and solve the questions based on it.***

(i) *A*, *B*, *C*, *D*, *E*, *F* and *G* are seven members of a family standing in a row (not necessarily in the same order) facing a particular direction.

(ii) *C* and *B* have as many members between them as *G* and *C* have between them.

(iii) *D*, who is 3rd from the extreme left, is 3rd to the left of *E*.

(iv) *A* and *D* are neighbours and *F* and *C* are neighbours.

8. Which of the following statements may be false?
(a) *A* is 3rd to the left of *F*
(c) *D* is 3rd to the left of *E*
(b) *F* is 3rd to the right of *A*
(d) *B* is 3rd to the left of *C*

9. Which of the following statements is true?
(a) *C* and *E* are neighbours
(b) *E* is to the immediate left of F
(c) *C* is to the immediate left of *D*
(d) *A* is to the immediate left of *D*

10. Who is at the extreme right?
(a) *G* (b) *B*
(c) *E* (d) Data inadequate

11. Which of the following gives two pairs of neighbours?
(a) *A*, *C* and *D*, *C* (b) *A*, *B* and *E*, *G*
(c) *D*, *C* and *E*, *F* (d) *C*, *F* and *C*, *E*

Direction for questions 12–16: ***Read the following passage and solve the questions based on it.***

Five students—Pankaj, Jatin, Robin, Dinkar and Rahul went for the Group Discussion (GD) and the (Personal Interview) (PI). The panel judged these five students and gave them rankings for the GD and the PI in a descending order. Rahul, who was ranked first in the GD, was last in the PI. Robin had the same ranking in both and was just above Rahul in the PI. In the GD, Pankaj was just above Dinkar but in the PI he was in the middle, after Jatin.

12. Who ranked first in the PI?
(a) Jatin (b) Rahul
(c) Robin (d) None of these

13. Who ranked fifth in the GD?
(a) Dinkar (b) Jatin
(c) Robin (d) Data inadequate

14. Who among the following has the same rank in both the GD and the PI?
(a) Pankaj
(b) Robin
(c) Dinkar
(d) None of the five students

15. To get the final selection list, the ranks of the students in the GD and the PI are added up. The lower the sum of the ranks, the better the performance. Who among the following has the maximum chances of getting selected. (The better the performance, the better are the chances of getting selected).
(a) Pankaj (b) Jatin
(c) Robin (d) Dinkar

16. In the last question, who among the following has the least chance of getting selected?
(a) Pankaj (b) Jatin
(c) Robin (d) Dinkar

Direction for questions 17–20: ***Read the following passage and solve the questions based on it.***

(i) *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H* are standing in a row facing north.
(ii) *B* is not a neighbour of *G*.
(iii) *F* is at the immediate right of *G* and neighbour of *E*.
(iv) *G* is not at the extreme end.
(v) *A* is sixth to the left of *E*.
(vi) *H* is sixth to the right of *C*.

17. Who among the following are neighbours?
(a) *AB* (b) *CG*
(c) *FH* (d) *CA*

18. Which one among the following defines the position of *D*?
(a) Fourth to the right of *H*
(b) Third to the right of *A*
(c) Neighbour of *B* and *F*
(d) To the immediate left of *B*

19. Which of the following is true?
(a) *C* is to the immediate left of *A*
(b) *D* is a neighbour of *B* and *F*
(c) *G* is to the immediate right of *D*
(d) *A* and *E* are at the extreme ends

20. After making the linear arrangement, we join them to form a circular arrangement by joining *A* and *H*. Which of the following is the odd one out?
(a) *B–H* (b) *E–D*
(c) *B–F* (d) *H–G*

ANSWER KEY

Exercise 1

1. (c)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (b)
9. (c)	**10.** (d)	**11.** (d)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (c)	**16.** (c)
17. (a)	**18.** (d)	**19.** (a)	**20.** (a)				

Exercise 2

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (d)
9. (d)	**10.** (d)	**11.** (c)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (c)
17. (d)	**18.** (b)	**19.** (c)	**20.** (c)				

HINTS AND EXPLANATIONS

Exercise 1

Solution to questions 1–4

Room	Person
A	—
B	—
C	—
D	—
E	—
F	—

From (iii) and the other given facts, two women will live in room *C*.

From (iii) and (iv), *P* will live in room *E*.

From (v) and (iii), *S* will live in room *A* with *R* (because *P* will live alone). From (vi) and (iii), *X* will live in room *F*.

Hence, *Q* and *T* will live in room *C*.

Now the whole scene looks like this:

Room	Person
A	*S*, *R*
B	*U*/*W*
C	*Q*, *T*
D	*W*/*U*
E	*P*
F	*X*

Solution to questions 5–8

From (ii)

P	*T*
Q	*Q*
T	*P*
P	*U*
S	*S*
U	*P*

From the last sentence of (ii), only one possibility remains

R
T
Q
P
S
U

Now, using (iii) and the above derived result:

R	*C*
T	*E/B*
Q	*A*
P	*F*
S	*B/E*
U	*D*

Solution to questions 9–13

Persons traveling are: *A*, *B*, *C*, *D*, *E*, *F* and *G*.

Stations are base station, station 1, station 2, station 3, station 4, and station 5.

Using information (i), (iv), (v), (vi), (vii), (viii) and (x)

Station	Get on	Get down
Base station	- - -	× × ×
1		× × ×
2	× × ×	
3	Only *G*	*B*, *D*
4	*A*	Only *E*
5	× × ×	*A*, *G*, *C*

Using (ii) *F* gets down at station 2 and he got on either at the base station or at station 1.

Now, since *F* got down at station 2 and he had got on with *C*, it means that both *C* and *F* got on either at the base station or at station 1.

Again, since *B* and *D* get down at station 3, this means they too got on either at the base station or at station 1.

It is given that *E* got on with two other persons, *i.e.*, in a group of three persons. Obviously, *E* got on at the base station.

Summarizing the whole information:

Station	Get on	Get down
Base Station	*E* and (*C*, *F*) or (*B*, *D*)	× × ×
1	(*C*, *F*) or (*B*, *D*)	× × ×
2	× × ×	Only *F*
3	Only *G*	*B*, *D*
4	*A*	Only *E*
5	× × ×	*A G C*

Solution to questions 14–17

The first thing that we should do is make nine houses (symbols) in a row:

- - - - - - - - -

Now try to find out the information that we are sure about:

1. *T* is in either house 2 or house 8.
2. *A*/*T*/*J* cannot be in house 1 or 9.
3. Houses beside *E* and *A* have to be unoccupied.
4. House numbers of *P* and *J* have to be less than the house number of *K*, *i.e.*, the order

should be *PJK* or *JPK*, not necessarily together.

5. There are six people and 9 houses to be occupied by them.

Using point 5, 3 houses have to be vacant. It is possible only if neither *E* nor *A* are at the ends or else *E* is at one of the ends and the difference in the houses of *E* and *A* is more than 1.

Understand this—

If both of them are at the ends—

E	Vacant	3	4	5	6	7	Vacant	A
-	-	-	-	-	-	-	-	-

Well, this is otherwise also not possible because it violates condition (i) given in the question. In this case, only two houses are vacant.

If one of them is at the ends—

E	Vacant	3	4	5	6	7	8	9
-	-	-	-	-	-	-	-	-

If *E* is at 1 and *A* is anywhere from 4 to 8, then the total number of vacant houses = 3, but if *E* is at 1 and *A* is at 3, then only two houses will be vacant.

Now start taking the positions:

P	*T*	Vacant	*E*	Vacant	*A*	Vacant	*J*	*K*
-	-	-	-	-	-	-	-	-

14. We have not referred to statement (v) till now, therefore it is the redundant statement.

15. Obviously, four people are living beside a vacant house.

Solution to questions 18–20

18. This question is a typical example of the kind of questions which can be solved without actually making the whole diagram. Let us see:
Using the second statement (Ravi is elder to both Tipu and Vikash), we can eliminate option (a) Using the third statement (Prakash is two years older than Tipu), we eliminate (b) and (c). We are left with (d), which must be correct.

19. This question can also be solved without using a diagram, however for the sake of discussing the solution, let us consult a diagram.

2000	2001	2002	2003	2004	2005
	Prakash	Pankaj	Tipu		
		Pankaj	Prakash	Tipu	

Using statement (iii), Prakash is two years older than Tipu. Using the diagram, the only possible places which Prakash and Tipu can have are either 2003 and 2005 or 2001 and 2003.
Using statement (i), Pankaj is elder to Santosh, so the only possible place left for Santosh is 2004.
Using Statement (ii), Ravi is elder to Tipu and Vikash. Hence, the only place left for Ravi is 2000.

20. Using the diagram once again

2000	2001	2002	2003	2004	2005
Prakash		Tipu	Pankaj		

Assuming Prakash to be the eldest in the group, he must have been born in 2000, that means Tipu was born in the year 2002. Hence, we can conclude that Pankaj was born in 2003. Now using statement (ii), Ravi was elder to both Tipu and Vikash. Hence Ravi must have been born in 2001 and Vikash in either 2004 or 2005.

Exercise 2

Solution to questions 1–3

In this set, we will need to make two series-one for comparing beauty and the other for comparing weights. From (iv) and (vii), we can deduce the series for comparing beauty as

Pamleshanand > Pamalanand > Parananand > Pailashanand

From (v), (vi) and (vii), we deduce the series for comparing as

Pailashanand > Pamalanand > Parananand > Pamleshanand > Punalanand

Further, since Pailashanand is not the heaviest in the group (ii), Pedaranand has to be heavier than Pailashanand. Thus, the series

for comparing weights will be Pedaranand > Pailashanand > Pamalanand > Parananand > Pamaleshanand > Punalanand

⇒ Punalanand is lightest in the group and hence, the most handsome (iii)

Therefore the series for comparing beauty will be

Punalanand > Pamaleshanand > Pamalanand > Parananand > Pailashanand

From (ii), we can say that Pedaranand is less handsome than Punalanand.

Solution to questions 4–7

Using the first condition—*S* is on the immediate left of *C*, *i.e.*, *S C*

Using the second condition—There are three outfits between *C* and *D*. *C* _ _ _ *D*

Using the third condition—

G L W or *W L G*

Using the fourth condition—There are 2 outfits between *G* and *D*, these should be *L* and *W* and this arrangement satisfies the second condition too.

C G L W D

Using the result from the first condition:

S C G L W D

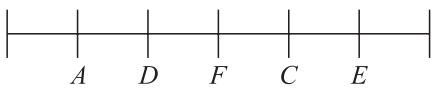
As *D* is at one corner, *J* must be at the second corner as there is no empty space anywhere else. So the complete arrangement is:

J S C G L W D

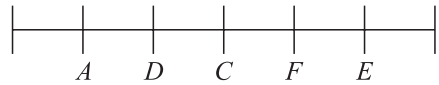
Solution to questions 8–11

From (iii)

Combining (iii) and (iv), we get the following arrangements:



or



or

So, the possible arrangements are:

GADCFEB

or

BADCFEG

8. Because position of *B* is not fixed.

11. As the position of *B* and *G* are not fixed, option b is not definitely true.

Solution to questions 12–16

	GD	PI
Pankaj	2	3
Jatin	5	2
Robin	4	4
Dinkar	3	1
Rahul	1	5

Solution to questions 17–20

Using the information given,

GFE (iii)

Now statements (v) and (vi) may be combined as:

AC - - - - EH (vii)

Or *CA - - - - HE* (viii)

But (viii) is not possible because it violates statement (iv)

Combining (iii) and (vii), we get

AC - - GFEH (viii)

Now, from statement (ii) and deduction (viii), we get

ACB – GFEH (ix)

This blank can be filled by '*D*', hence the arrangement is *ACBDGFEH*.

20. Two persons are sitting between all the four pairs, but in three of them it is clockwise and one of them is having it in anti-clockwise.

2 Team Selection

Questions belonging to this genre will provide a set of objects or people etc. and then a set of conditions. On the basis of these conditions, one is expected to make choices regarding the formation of the team. It is advisable to jot down the conditions and items to be formed into teams carefully.

In most of these set of questions, you will find that the parent data provides the basic guidelines and subsequently every question introduces one new condition.

> It is imperative here to understand that while parent data is applicable to all the questions in the set, data supplied in any question is applicable to that particular question only, and not to the other questions unless specified otherwise.

Understanding the Keywords and Types of Statements

These questions will throw a gauntlet of simple but contextually confusing words at the students. Some of these words/types of statements have been listed below:

1. At least one element—It means one or more—Symbolically, we can represent this as: 1+.
2. At most one element—It means zero or one—Symbolically, we can represent this as: 0/1.
3. At most two elements—It means zero or one or two—Symbolically, we can represent this as: 0/1/ 2.
4. Conditional clauses:
   If *A* is selected, then *B* will also be selected.
   Following inferences can be drawn:
   (a) If *A* is selected, *B* has to be selected.
   (b) There cannot be a case where *A* is selected but *B* is not selected.
   (c) It is possible that *B* has been selected but *A* may be/may not be selected.
   (d) If *B* has not been selected, then we can conclude that *A* also has not been selected.
   (e) It is not possible that *B* has not been selected, but *A* has been selected [rephrasing of inference (b) above].

In a nutshell, *A* alone cannot be selected though *B* alone can be selected.

Go through the following problem sets to understand the type of problems and techniques to solve the same. Do not look at the solution till you are convinced that you have attempted it at your best level.

Direction for questions 1–6: *Read the information given below and solve the questions based on it.*

Three adult women (*R*, *S*, and *T*), two adult men (*U* and *V*), and four children (*W*, *X*, *Y*, and *Z*) are going to a watch a movie. Though, during the online booking of the tickets, they realized that the nine seats available for the show are in three different classes—Silver Class, Gold Class and Lounge. The layout also showed that in each class, three adjacent seats are available.

To watch the movie, they decide to have the three groups of three members each as per the following conditions:

- No adults of the same gender can be together in ONE group.
- *W* cannot be in *R*'s group.
- *X* must be in a group with *S* or *U* or both.

PRACTICE EXERCISES

EXERCISE 1

1. If *R* is the only adult in one group, the other members of her group must be:
(a) *W* and *Y* (b) *X* and *Y*
(c) *X* and *Z* (d) *Y* and *Z*

2. *R* and *U* share a good rapport, and hence decide to be in the same group. Who can be in the second and third groups, respectively?
(a) *S*, *T*, *W*; *V*, *Y*, *Z* (b) *S*, *X*, *Y*; *T*, *W*, *Z*
(c) *T*, *V*, *W*; *S*, *Y*, *Z* (d) *W*, *X*, *Y*; *S*, *V*, *Z*

3. Which of the following pairs of people can be in the same group as *W*?
(a) *R* and *Y* (b) *S* and *U*
(c) *S* and *V* (d) *U* and *V*

4. Which of the following must be true?
(a) One of the women is in a group with two children.
(b) One of the two men is in a group with *W*.
(c) *R* is in a group with a man.
(d) One of the groups includes no children.

5. Any of the following pairs of people could be in a group with X EXCEPT:
(a) *R* and *U* (b) *S* and *T*
(c) *S* and *U* (d) *S* and *W*

6. Given that *T*, *Y*, and *Z* form one group. Which of the following must be together in one of the other groups?
(a) *R*, *S*, *V* (b) *R*, *U*, *W*
(c) *S*, *U*, *W* (d) *S*, *V*, *W*

Direction for questions 7–10: ***Read the information given below and solve the questions based on it.***

dueNorth presently employs three Marketing Managers (MM)—*A*, *B* and *C* and five faculty members—*D*, *E*, *F*, *G* and *H*. Company is planning to open a new office. It is planning to relocate two of the three marketing managers and three of the five faculty members to the new office. Management wants to ensure that the individuals who do not function well together should not be sent as a part of the team.

Following information was available to the HR department of due North:

Marketing Managers *A* and *C* cannot be sent as a team to the new office.

C and *E* are excellent performers, though, they do not share good rapport, and hence should not be sent together.

If *D* is sent, then *G* cannot be sent, and vice versa.

D and *F* should not be together in a team.

7. If *D* goes to the new office which of the following is (are) true?
A. *C* cannot go B. *A* cannot go
C. *H* must also go
(a) A only (b) B and C only
(c) A and C only (d) A, B and C

8. If A is to be moved as one of the Marketing Managers, which of the following cannot be a possible working unit?
(a) *ABDEH* (b) *ABFGH*
(c) *ABEGH* (d) *ABDGH*

9. If *C* and *F* are moved to the new office, how many combinations are possible?
(a) 0 (b) 1
(c) 2 (d) 3

10. Who among the Marketing Managers and the faculty members is sure to find a berth in the new office?
(a) *D* (b) *H*
(c) *G* (d) *B*

Direction for questions 11–13: ***Read the information given below and solve the questions based on it.***

Seven students at a B-School who live in a dormitory are being formed into groups that

consist of two or three or four members at a time. The groups may change but at any time, each person can be a part of one and only one group. The following guidelines have to be adhered to while forming the groups:

- The students are Nitin, Priyanka, Rahul, Sumit, Tanay, Urmila and Vinay.
- Nitin cannot be in the same group as Sumit.
- Tanay must be in a group that includes either Sumit or Vinay but not both.
- Vinay has to be in a group that does not contain even number of members.

11. Which of the following is a possible list of three groups that can coexist?

1	Nitin, Vinay	Tanay, Rahul, Urmila	Sumit, Priyanka
2	Nitin, Tanay	Vinay, Rahul, Sumit	Priyanka, Urmila
3	Nitin, Urmila	Rahul, Sumit	Vinay, Tanay, Priyanka
4	Urmila, Sumit	Vinay, Tanay	Priyanka, Nitin, Rahul

12. If a group of 3 members and another of 4 members is formed and one of these two groups has Vinay and Urmila, who else is likely to be in the same group?
(a) Nitin (b) Sumit
(c) Priyanka (d) Rahul

13. If a group of three is formed with Rahul, Tanay and Vinay, then how many groups will be there amongst the seven students?
(a) 2
(b) 3
(c) either 2 or 3
(d) Insufficient Data

Direction for questions 14 and 15: ***Read the passage given below and solve the questions based on it.***

Digviza and three other members of her yoga club dined out together. Each woman ordered either salad or soup to start with, and one of the three entrees (a dish served before the main course)—cabbage rolls, eggs benedict, or lasagna. Refer to the clues given below and answer the following questions:

- Every order was different. Each dish mentioned above was ordered at least once.
- Kejri and the lady who ordered the salad both ordered lasagna as the entree.
- Rajdev and Sonila both ordered soup as the first course.
- Sonila did not order the eggs benedict.

14. How many women ordered salad?
(a) 1 (b) 2
(c) 3 (d) 4

15. Who ordered salad and cabbage rolls?
(a) Digviza (b) Rajdev
(c) Sonila (d) None

Direction for questions 16–20: ***Read the information given below and solve the questions based on it.***

K, *L*, *M*, *N*, *P*, *Q*, *R*, *S*, *U* and *W* are the only ten members in a department. There is a proposal to form a team from within the members of the department, subject to the following conditions:

- A team must include exactly one among *P*, *R*, and *S*.
- A team must include either *M* or *Q*, but not both.
- If a team includes *K*, then it must also include *L*, and vice versa.
- If a team includes one among *S*, *U*, and *W*, then it must also include the other two.
- *L* and *N* cannot be members of the same team.
- *L* and *U* cannot be members of the same team.
- The size of a team is defined as the number of members in the team.

16. Who cannot be a member of a team of size 3?
(a) *L* (b) *M*
(c) *N* (d) *P*
(e) *Q*

17. Who can be a member of a team of size 5?
(a) *K* (b) *L*

(c) *M* (d) *P*
(e) *R*

18. What would be the size of the largest possible team?
(a) 8 (b) 7
(c) 6 (d) 5
(e) Cannot be determined

19. What could be the size of a team that includes *K*?
(a) 2 or 3 (b) 2 or 4
(c) 3 or 4 (d) Only 2
(e) Only 4

20. In how many ways a team can be constituted so that the team includes *N*?
(a) 2 (b) 3
(c) 4 (d) 5
(e) 6

Direction for questions 21–25: ***Read the information given below and solve the questions based on it.***

There are three projects—P1, P2 and P3. A student can select either one project or two projects or all the three projects subject to the conditions given below:

Condition 1: Both P1 and P2 have to be selected.

Condition 2: Either P1 or P3, but not both, has to be selected.

Condition 3: P2 can be selected only if P3 has been selected.

Condition 4: P1 can be selected only if P3 is selected.

21. How many different selections can be made if no conditions are imposed?
(a) 5 (b) 6
(c) 7 (d) 8
(e) 4

22. How many selections can be made to meet condition 1 as given above?
(a) 1 (b) 2
(c) 3 (d) 4
(e) 0

23. How many selections can be made to meet condition 2 as given above?
(a) 2 (b) 3
(c) 4 (d) 5
(e) 1

24. How many selections can be made meeting the conditions 2 and 3 as given above?
(a) 0 (b) 1
(c) 2 (d) 0 or 1
(e) None of these

25. How many selections can be made meeting the conditions 1, 2 and 3 as given above?
(a) 0 (b) 1
(c) 2 (d) 3
(e) None of these

EXERCISE 2

Direction for questions 1–4: ***Read the information given below and solve the questions based on it.***

Hosting Filmware award ceremony is a big affair. There are lot of things to be done—anchoring, managing the filler entertainment on-stage performance, crowd management, security etc. To conduct the filler entertainment on-stage performances, name of two male actors—Abhi and Riteish—and three female actors—Rani, Shreya and Alia—have been finalized. These five actors will give a performance of a satire play named "Angrezo Bharat Aao" that has exactly eight roles.

Following conditions are to be kept in mind while allocating roles:

Roles 1, 2 and 3 must be played by male actors.

Roles 4, 5 and 6 must be played by female actors.

Roles 7 and 8 can be played by either male or female actors.

Each actor must play at least one role.

The pairs of roles below are the only pairs that do NOT require the actors playing the roles to be on stage at the same time:

Roles 1 and 2 Roles 3 and 6

Roles 3 and 7 Roles 4 and 5
Roles 4 and 8 Roles 5 and 8

Appearances in these roles are spaced far enough apart to allow time for costume changes for actors playing more than one role.

Rani and Shreya cannot play roles that require them to be on stage together at the same time.

1. Each of the following pairs of roles could be played by the same actor EXCEPT:
(a) roles 1 and 2 (b) roles 3 and 6
(c) roles 3 and 7 (d) roles 4 and 8

2. If Riteish plays role 1, which of the following must be true?
(a) Abhi plays role 2
(b) Abhi plays role 7
(c) Shreya plays role 4
(d) Rani plays role 4

3. Which of the following is an acceptable assignment of roles 4, 5 and 6 respectively?
(a) Rani, Rani, Rani
(b) Rani, Alia, Rani
(c) Rani, Shreya, Alia
(d) Shreya, Alia, Rani

4. Abhi could play any of the following roles EXCEPT?
(a) 1 (b) 2
(c) 3 (d) 8

Direction for questions 5–8: ***Read the information given below and solve the questions based on it.***

On a Sunday, five friends—Ohn, Karan, Ranbir, Mithun and Onir—have gathered to play a game called Trios consisting of three rounds. In each round of the game, exactly three of these friends will play. Consider following rules regarding the participation:

No person can play in three consecutive rounds.

No person can sit out two consecutive rounds.

In any game, each of the five persons must play in exactly three rounds.

5. If John, Karan and Ranbir play in a first round, which of the following could be playing in that game's second round?
(a) John, Karan, Mithun
(b) John, Karan, Onir
(c) John, Ranbir, Mithun
(d) Karan, Mithun, Onir

6. In an individual game, Karan, Ranbir and Mithun Play in the first round and John, Ranbir and Mithun play in the third round. Then the players in the second round must be:
(a) John, Karan, Ranbir
(b) John, Karan, Mithun
(c) John, Karan, Onir
(d) Karan, Ranbir, Onir

7. In an individual game, Ranbir and Onir do not play in the first round. Which of the following must be true?
(a) Ranbir plays in rounds three and four
(b) Onir plays in rounds three and five
(c) Ranbir and Onir both play in round four
(d) Ranbir and Onir both play in round five

8. In an individual game, John, Ranbir and Mithun play in the first round, and Karan, Mithun and Onir play in the second round, which of the following must play in the fourth round?
(a) John (b) Karan
(c) Ranbir (d) Mithun

Direction for questions 9–11: ***Read the information given below and solve the questions based on it.***

During the Chemistry practical examination, a naughty student removed the labels pasted on the four bottles—that contain colorless liquids. Incharge of the lab knows that there are only six possibilities of the liquids present in the bottles—pure *X* liquid, pure *Y* liquid, pure *Z* liquid, or any mixture of these. No two bottles are filled with the same liquid or the same combination of any

of these. The only feasible way of testing for the identity of the liquids is to use strips of Mitlus paper—that turns red, black, or yellow depending on which of certain liquids it is dipped in. The full table of colour changes when Mitlus paper is dipped inside is given below:

Liquid	Turns to the colour when Mitlus paper is dipped inside
Pure *X*	Black
Pure *Y*	Red
Pure *Z*	Black
X and *Y*	Yellow
X and *Z*	Yellow
Y and *Z*	Red
X and *Y* and *Z*	Black

9. If none of the four liquids turns the Mitlus paper yellow, each of the following must be the contents of one of the bottles EXCEPT
(a) Pure *X*
(b) Pure *Y*
(c) *Y* mixed with *Z*
(d) *X*, *Y*, and *Z* mixed together

10. If the liquid in the first bottle tested turns the Mitlus paper red, and if the liquid in the second bottle tested turns the paper yellow, then a mix of some of the liquid from each of the first two bottles tested will turn the Mitlus paper
(a) Yellow
(b) Either red or black
(c) Either red or yellow
(d) Either black or yellow

11. If the liquid in the first bottle tested turns the Mitlus paper red and the liquid in the second bottle tested turns it black, and if a mix of some of the liquids from each of the first two bottles tested turns it red, then which of the following must be true?
(a) The first bottle tested contains pure *Y*
(b) The first bottle tested contains *Y* mixed with *Z*
(c) The second bottle tested contains pure *X*
(d) The second bottle tested contains pure *Z*

Direction for questions 12–15: ***Read the information given below and solve the questions based on it.***

Fargo, Goodday and HomeTrade are three retail companies, and *Q*, *R*, *S*, and *T* are four research associates. Each associate works for at least one of the retail companies.

Q always works for Fargo and at least one of the other companies.

Some of the time Goodday employs only one of these associates, the rest of the time it employs exactly two of them.

Fargo and HomeTrade each employ exactly two of these detectives all the time.

Answer all the questions on the basis of the information given above.

12. If *R* works for HomeTrade only, and if *S* works for Goodday and HomeTrade only, then *T* works for:
(a) Fargo only
(b) Goodday only
(c) HomeTrade only
(d) Both Fargo and Goodday

13. If *Q* and *R* both work for the same two retail companies, *T* must work for:
(a) Both Fargo and Goodday
(b) Both Fargo and HomeTrade
(c) Either Fargo or Goodday but not both
(d) Either Goodday or HomeTrade but not both

14. If only *S* works for Goodday, which of the following must be true?
(a) *R* works for either Fargo or Goodday but not both
(b) *T* works for either Goodday or HomeTrade but not both
(c) *R* and *T* cannot work for the same company
(d) *Q* and *R* cannot work for the same company

15. If *G* employs only one detective, which of the following must be true?
I. *R* works for two companies
II. *T* works for Goodday
III. *S* works for only one company
(a) I only (b) II only
(c) III only (d) I and II only

Direction for questions 16–19: ***Read the information given below and solve the questions based on it.***

When I went to buy an aquarium for my home, I was given a choice of seven fish species—*F*, *G*, *H*, *I*, *J*, *K*, and *L*—to be chosen from. I decided to put exactly six fishes (of same or different species) to be put in the aquarium. Though I was told by the owner of the shop that some of these fishes are quite violent towards other species of the fishes and if I put those fishes in the same aquarium tank, they will fight, and I obviously want to avoid that situation.

Consider the following Restrictions

Fish of species *F* will fight with fish of species *H*, *J*, and *K*.

Fish of species *I* will fight with fish of species *G* and *K*.

If three or more fish of species *I* are in one aquarium tank, they will fight with each other.

Fish of species *J* will fight with fish of species *I*.

If a fish of species *G* is to be in a aquarium tank, at least one fish of species *K* must also be in the aquarium tank.

16. If an aquarium tank is to contain fish of exactly three different species, which of the following could be these species?
(a) *F*, *G* and *I* (b) *F*, *I* and *K*
(c) *G*, *H*, and *I* (d) *H*, *I*, and *J*

17. Only two species of the fishes are to be put in the aquarium tank with the condition that three fishes of species *J* have to be out of six fishes to be put in the aquarium tank. Other three fishes in that aquarium tank could be from which of the following species?
(a) *F* (b) *G*
(c) *H* (d) *I*

18. If aquarium tank is to contain fish of exactly four different species, which of the following species of the fish cannot be put?
(a) *F* (b) *G*
(c) *H* (d) *J*

19. Which of the following species of the fishes can be put into aquarium tank with fish of species *G*?
(a) *F* and *I* (b) *F* and *J*
(c) *H* and *I* (d) *H* and *K*

Direction for questions 20–25: ***Read the information given below and solve the questions based on it.***

Members of the upper house and lower house are to be selected from exactly six qualified candidates. The six candidates are *U*, *V*, *W*, *X*, *Y* and *Z*.

Consider the following:

Each house must have exactly three members.

The two houses must have at least one member in common.

If *X* is selected for a house, *Y* must also be selected for that house.

20. If the members of the Upper House are selected first, which of the following could be those selected?
(a) *U*, *V* and *X* (b) *U*, *X* and *Z*
(c) *V*, *W* and *X* (d) *V*, *X* and *Y*

21. If the two houses have parallel terms of office, which of the following could be selected as the members of the Upper House and as the members of the Lower House, respectively, for one such term of office?
(a) *U*, *V* and *W*; *X*, *Y* and *Z*
(b) *U*, *W* and *Y*; *V*, *X* and *Z*
(c) *U*, *X* and *Y*; *U*, *X* and *Z*
(d) *W*, *X* and *Y*; *X*, *Y* and *Z*

22. If the members of the Upper House are *V*, *W*, and *Z*, and if the Lower House is to have as many members in common with the

Upper House as the rules allow, the Lower House must consist of:
(a) *U*, *V* and *W* (b) *V*, *W* and *Z*
(c) *W*, *Y* and *Z* (d) *X*, *Y* and *Z*

23. Assume that *U*, *V* and *W* make up the Upper House, and *W*, *Y*, and *Z* make up the Lower House. Which of these house members could yield his or her place on a house to *X* without causing any other changes?
(a) *U* (b) *W*
(c) *Y* (d) *Z*

24. If *U* and *X* are each selected for a house and only *Z* is selected for both house, which of the following must be true?
(a) *V* is selected for the same house as *W*
(b) *W* is selected for the same house as *Y*
(c) *U* is selected for a different house than *Y*
(d) *X* is selected for a different house than *Y*

25. If *X* and *Z* are both selected for the Upper House, and if *U* is selected for the Lower House, each of the following pairs of people could be the other two members of the Lower House EXCEPT:
(a) *V* and *W* (b) *V* and *Z*
(c) *W* and *Y* (d) *Y* and *Z*

ANSWER KEY

Exercise 1

1. (d)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (d)
9. (b)	**10.** (d)	**11.** (c)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (a)
17. (c)	**18.** (d)	**19.** (e)	**20.** (e)	**21.** (c)	**22.** (b)	**23.** (c)	**24.** (e)
25. (a)							

Exercise 2

1. (b)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (d)
9. (d)	**10.** (d)	**11.** (d)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (d)
17. (c)	**18.** (a)	**19.** (d)	**20.** (d)	**21.** (d)	**22.** (b)	**23.** (d)	**24.** (c)
25. (a)							

HINTS AND EXPLANATIONS

Exercise 1

Solution to questions 1–6

1. Since *W* cannot be in *R*'s group, we can eliminate 1st option. *X* can only be in a group with *S* or *U* or both. Since *R* is the only adult, neither *S* nor *U* can be in the group. So, 2nd option and 3rd option can be eliminated as both contain *X*. 4th option is the correct answer because the only two children remaining to fill out the group with *R* are *Y* and *Z*.
Hence, option (d) is the answer.

2. 1st option contains two women *S* and *T* in the second group. Hence, 1st option is eliminated. Similarly, in the 2nd option, first group contains *U* and *V*, both men; hence, 2nd can be eliminated.

Since *X* must be in a group with *S* or *U*, or both, the second option and 4th option can be eliminated.
3rd option, consisting of groups *R*, *U*, *X*; *T*, *V*, *W*; and *S*, *Y*, *Z*, meets all of the restrictions and is the correct answer.
Hence, option (c) is the answer.

3. *S*, *V*, and *W* could form a group if the other two groups were *R*, *Y*, *Z*, and *T*, *U*, *X* or *R*, *U*, *X* and *T*, *Y*, *Z*. Thus, the third option is the correct answer.
Option (a) is incorrect. *W* cannot be in *R*'s group.
Option (b) is incorrect. If *S* and *U* are in the same group, *X* must fill the remaining seat. But option shows that *W* will take the seat. Therefore, it is not possible.
D is not correct. *U* and *V* are both men and cannot be in the same group.
Hence, option (c) is the answer.

4. Now this is a sitter:
Option (a) is obviously correct, because there are three women and only one woman can be in ONE group, so each of the three groups must contain a woman. Further, there are more children than groups, hence at least one group must have two children. That group will include a woman along with the two children.
Option (b) is incorrect. *S*, *W*, *X*; *R*, *U*, *Y*; and *T*, *V*, *Z* is one of several possible sets of seating groups in which *W* is not in the same group as one of the two men.
Option (c) is incorrect. Using option (a) we can say that that woman can be *R*. Hence at best, this option is probably true, and not must be true.
Option (d) is also incorrect. Every group must include one child, because a group of three with no children would have to include two adults of the same sex, which is not possible as per the 1st condition.
Hence, option (a) is the answer.

5. Option a is possible and hence not the answer. *R*, *U*, *X*; *S*, *V*, *W*; and *T*, *Y*, *Z* is one of several possible sets of seating groups in which *R* and *U* are in *X*'s group.
S and *T* are both women and cannot be in the same group. Therefore, *B* is the correct answer.
Option (c) is also possible and hence cannot be the answer. *S*, *U*, *X*; *R*, *Z*, *Y*; and *T*, *V*, *W* is one of several possible sets of seating groups in which *S* and *U* are in *X*'s group.
Option (d) is also incorrect. *S*, *W*, *X*; *R*, *V*, *Y*; and *T*, *U*, *Z* is one of several possible sets of seating groups in which *S* and *W* are in *X*'s group.
Hence, option (b) is the answer.

6. If *T*, *Y*, and *Z* form one group, the remaining two groups must be chosen from among *R*, *S*, *U*, *V*, *W*, and *X*.
Now, out of these, *R* and *S* are females and *U* and *V* are males.
R and *S* must be in different groups as well as *U* and *V* must be in different groups (hence, 1st option is wrong). *W* must be in the group with *S* because *W* cannot be in the group with *R* (condition 2). *X* must thus be in the group with *R*, because both *U* and *V* are men and cannot both be in *R*'s group. Since *X* must be in a group with *U* or *S*, and *S* is already in another group, the remaining slot with *R* and *X* must be taken by *U*. It means *V* will fill the remaining space in *S*'s group.
Hence, option (d) is the answer.

Solution to questions 7–10

7. *D* and *G* as well as *D* and *F* cannot work together. New office requires 3 faculty members. Therefore, *E*, *H* should go. *C* and *E* cannot function together as a team. As *E* has to go, *C* cannot go. Therefore, *A* and *B* have to be the marketing managers if *D* is posted to San Jose. Hence, option (c) is the answer.

8. ABDGH is not a working unit as *D* and *G* cannot work together. Hence, option (d) is the answer.

9. If C and F are moved to the new office, then B should be the only other marketing manager who can go. Therefore, the managers are BC. Of the faculty members, E cannot go. Therefore, one has to choose 3 faculty members from D, F, G, H. D cannot be sent because if he goes, then F and G cannot go. Therefore, FGH are the faculty members and B, C are the marketing managers. Only one option available.
Hence, option (b) is the answer.

10. B is sure to find a berth in the group. Hence, option (d) is the answer.

Solution to questions 11–13

11. Only in option c, all the conditions are satisfied. Vinay is in a group of 3. Tanay is in a group that has Vinay or Sumit, but not both and Nitin is not in the same group as Sumit. Hence, option (c) is the answer.

12. Vinay has to be in a group that has 3 members. So, only one more member can be in that group. If Sumit is in that group, then Tanay cannot be a part of the other group as Tanay has to be in a group in which either Vinay or Sumit is there. So, Sumit has to be a part of the group that has 4 people. In which case, Nitin cannot be in the group that has 4 people and has to be a part of the group that has Vinay and Urmila. Hence, option (a) is the answer.

13. Nitin cannot be in the same group as Sumit. Therefore, we need to have two more groups in addition to the one formed with the three people. So, there will be 3 groups amongst the seven students. Hence, option (b) is the answer.

Solution to questions 14 and 15

Digviza ordered the salad and lasagna.
Rajdev ordered the soup and eggs benedict.
Sonila ordered the soup and cabbage rolls.
Kejri ordered the soup and lasagna.

14. Option (a) is the answer.

15. Option (d) is the answer.

Solution to questions 16–20

16. In the team of size 3, one member will be from P, R and S and one member will be from M and Q.
Now only one member has to be determined and we know that K and L will be always in team and since there is place for only 1 member, hence, K and L will not be in team of 3 members.
Hence, option (a) is the answer.

17. If we take K and L we cannot take N and U so the size of team will be 4 only. Now we will take S from first statement so U and W will also be there. Now as U so there L cannot be there and hence K will also not be there. As L is not in team so we can take N so 4 members of team are S, U, W and N and because of second statement any one of M or Q will be there and only M is given in the options.
Hence, option (c) is the answer.

18. Using first statement, if we take P or R, U and W cannot be in team. To get maximum members in team we should take S and from 4th statement U and W will also be in the team. Now either M or Q will be included in the team, and since U is there L can't be included in the team hence K will also not be there.
Now as L is not in the team, N can be included in the team. So team with maximum numbers of members will be S, W, U, N and any one from M and Q.
Hence, maximum possible size of team of team is 5.
Alternatively,
Start with all the 10 members and keep removing the member who cannot be in the team.
$K \quad L \quad M \quad N \quad P \quad Q \quad R \quad S \quad U \quad W$
Using condition 1, only one out of P, R and S can come. At the same time, S, U

and W will come together. Hence, we should select S out of P, R and S.

$K \quad L \quad M \quad N \quad \cancel{P} \quad Q \quad \cancel{R} \quad S \quad U \quad W$

Next, since U has come, L cannot come. So we remove L.

$K \quad \cancel{L} \quad M \quad N \quad \cancel{P} \quad Q \quad \cancel{R} \quad S \quad U \quad W$

Using point (2), only one out of M or Q can come. So eliminate M and retain Q (we could have done otherwise also).

$K \quad \cancel{L} \quad \cancel{M} \quad N \quad \cancel{P} \quad Q \quad \cancel{R} \quad S \quad U \quad W$

Using point (3), if L is removed, K will also be removed.

$\cancel{K} \quad \cancel{L} \quad \cancel{M} \quad N \quad P \quad Q \quad \cancel{R} \quad S \quad U \quad W$

Remaining members can be part of ONE team.

Maximum team size = 5.

Hence, option (d) is the answer.

19. If we have to take K, L will always be there so U and N will not be in the team. As U is not in the team so S and W will also not be included in the team.

As anyone from P and R will always be there in team and same in the case with M and Q, so the size of team that include K will always be 4.

Hence, option (e) is the answer.

20. Following are the ways in which a team can be constituted:

(1) P, M, N
(2) P, Q, N
(3) R, M, N
(4) R, Q, N
(5) S, U, W, N, M
(6) S, U, W, N, Q

Hence, option (e) is the answer.

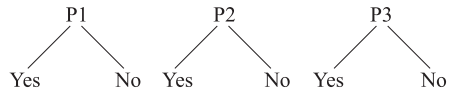
Solution to questions 21–25

21. If no conditions are imposed, we have following choices to select either one project or two projects and three projects [This is the part of parent statement, and conditions start afterwards]:

P1; P2; P3; P1 and P2; P1 and P3; P2 and P3; P1, P2 and P3 ⇒ 7 choices.

Alternatively:

For every project, we have two choices—either to select this or not to select this.



Total number of choices = 2 × 2 × 2 = 8

Out of this we will remove one choice that has—No, No, No.

Hence, option (c) is the answer.

22. We can use the solution to Q. 21 as our base. We will keep on eliminating the options that are not applicable:

P1; P2; P3; P1 and P2; P1 and P3; P2 and P3; P1, P2 and P3.

Condition 1: Both P1 and P2 have to be selected.

Hence, only possible selections = P1 and P2; P1, P2 and P3 ⇒ 2 choices.

Hence, option (b) is the answer.

23. Exhaustive sets = P1; P2; P3; P1 and P2; P1 and P3; P2 and P3; P1, P2 and P3.

Condition 2: Either P1 or P3, but not both, has to be selected.

Hence, only possible selections = P1; P3; P1 and P2; P2 and P3 ⇒ 4 choices.

Hence, option (c) is the answer.

24. Possible selections as per condition 2 = P1; P3; P1 and P2; P2 and P3.

Condition 3: P2 can be selected only if P3 has been selected ⇒ P3 can be selected alone, but P2 cannot be selected without the selection of P3.

Hence, only possible selections = P1; P3; P2 and P3 ⇒ 3 choices.

Hence, option (e) is the answer.

25. Possible selections as per condition 2 and condition 3 = P1; P3; P2 and P3.

Condition 1: Both P1 and P2 have to be selected.

Hence, only possible selections = None.

Hence, option (a) is the answer.

3 Practising Logical Reasoning

FOUNDATION EXERCISES

EXERCISE 1

Direction for questions 1–3: ***Read the following passage and solve the questions based on it.***

Ten coins are distributed among four people *P*, *Q*, *R* and *S* such that one of them gets one coin, another gets two coins, the third gets three coins and the fourth gets four coins. It is known that *Q* gets more coins than *P*, and *S* gets fewer coins than *R*.

1. If the number of coins distributed to *Q* is twice the number distributed to *P*, then which one of the following is necessarily true?
(a) *R* gets an even number of coins
(b) *R* gets an odd number of coins
(c) *S* gets an even number of coins
(d) *S* gets an odd number of coins

2. If *R* gets at least two more coins than *S*, then which one of the following is necessarily true?
(a) *Q* gets at least two more coins
(b) *Q* gets more coins than *S*
(c) *P* gets more coins than *S*
(d) *P* and *Q* together get at least five coins

3. If *Q* gets fewer coins than *R*, then which one of the following is not necessarily true?
(a) *P* and *Q* together get at least four coins
(b) *Q* and *S* together get at least four coins
(c) *R* and *S* together get at least five coins
(d) *P* and *R* together get at least five coins

Direction for questions 4–7: ***Read the following passage and solve the questions based on it.***

To make the non-technical background new joiners understand the process of manufacturing colour TVs better, LG has hired the services of Due North Inc. consultants. Due North is a consultancy firm which provides technical training of all the household equipment to the non-tech background new joiners at LG. To facilitate the training process, it has been decided that there will be six groups of new joiners namely *A*, *B*, *C*, *D*, *E* and *F* and each of the groups is scheduled at least once a week. All the groups will start their training on the same day and will also end their training on the same day.

Following points are to be taken into consideration while making the training schedule:

(i) Sunday is a holiday.
(ii) *B* group is scheduled all days except Friday and Saturday.
(iii) *C* group meets four days in succession.
(iv) *F* group meets only from Monday to Thursday.
(v) *E* group is scheduled everyday, but not on Thursday and Saturday.
(vi) *A* group is scheduled on alternate days.
(vii) *C* group does not meet on Monday and Tuesday only.
(viii) *A* and *D* groups never meet on the same days.
(ix) *D* group is scheduled only once a week on either Wednesday or Friday.

4. Which groups are scheduled for the same number of classes during the week?
   (a) *B*, *A*, *F* only (b) *E*, *B*, *C* only
   (c) *E*, *F*, *A* only (d) None of these
5. If a certain class of *D* is scheduled on the same day as that of the *B* group, then how many groups are scheduled on Friday?
   (a) 2 (b) 3
   (c) 4 (d) 5
6. For how many groups do we have a definite training schedule, for the whole week?
   (a) 4 (b) 2
   (c) 3 (d) 5
7. Which two groups can never be scheduled on the same day?
   (a) *C* and *D* (b) *C* and *E*
   (c) *A* and *D* (d) None of these

Direction for questions 8–11: ***Read the following passage below and solve the questions based on it.***

There are five identical looking boxes containing different objects in each of them and every box has a label indicating its contents. Following is the correct description of the contents and the label of each box (Table 3.1):

Table 3.1

Contents	Label
Two Pins	*PP*
Two Balls	*BB*
Two Clips	*CC*
One Pin and one Clip	*PC*
One Ball and one Clip	*BC*

Somebody has mischievously interchanged these labels in such a way that no box carries the label describing its contents correctly.

8. The first box which was opened contained the label *PP* and the second box opened contained the label *PC*. It is also known that out of the four items in the two boxes, one item was definitely a ball. Then which of the following has to be true?
   (a) Other three items will not contain two balls
   (b) Other three items will not contain any clip
   (c) Other three items will contain at least one clip
   (d) None of these
9. Box *PP* contained two clips, box *CC* contained two pins and the box *BB* contained one ball. Then which of the following will definitely be false?
   (a) The box *BC* contains one pin and one clip
   (b) The box *BB* contains one ball and one clip
   (c) The box *BC* contains two balls
   (d) The box *PC* contains two balls
10. If the first box containing the label *BC* was opened and it was found that one item is a ball, then which of the following is definitely true?
   (a) The other item may be either a ball or a clip
   (b) The other box with the *BB* label contains one ball and one clip
   (c) The other item will not be a ball
   (d) The other item will also be a ball
11. If the information is available that the box *PC* does not contain either any pin or any clip, box *PP* does not contain any pin and box and *CC* contains one clip and one ball. Which of the following will definitely be true if only one of the remaining boxes is opened?
   (a) The box will have one pin and one clip
   (b) The box will have at least one clip
   (c) The box will have at least one pin
   (d) None of these

Direction for questions 12–16: ***Read the following passage and solve the questions based on it.***

There are various rides available at the annual *Lucknow Mahotsav*. One of the rides that is

available is the roller coaster ride, comprising five cars, numbered 1 through 5 from the front to back. Each car can accommodate up to two riders, seated side by side. Six persons—*T*, *G*, *L*, *M*, *P* and *J*, are taking the ride simultaneously. The Following information is available regarding their seating pattern:

(i) *L* is sharing a car.
(ii) *M* is not sharing a car and is seated immediately behind an empty car.
(iii) *T* is not sharing a car with either *G* or *P*.
(iv) *G* is in either the third or the fourth car.

12. Which of the following people definitely occupy the second car?
(a) *L* only (b) *T* and *G*
(c) *L* and *M* (d) None of these

13. If *G* is immediately behind *L*'s car and immediately ahead of *T*'s car, all of the following must be true except:
(a) *G* is in the fourth car
(b) *P* is in the third car
(c) *T* is in the fifth car
(d) *L* is in the third car

14. Which one of the following statements cannot be true?
(a) Neither *T* nor *G* is sharing a car with anybody else.
(b) Neither *M* nor *J* is sharing a car with anybody else.
(c) *T* is sharing a car and *J* is sharing a car.
(d) *G* is sharing a car and *P* is sharing a car.

15. If *P* is in the second car, how many different combinations of riders are possible for the third car?
(a) 1 (b) 2
(c) 4 (d) None of these

16. Assume that a 7th person is in with *J* in the first car and all the other conditions remain the same. Which of the following is a complete list of persons who might be in the fifth car?
(a) *M* (b) *G*, *P*
(c) *T*, *L*, *P* (d) *T*, *L*, *M*

Direction for questions 17–20: ***Read the following passage and solve the questions based on it.***

'*Vaastu-shastra*' says that the dining table should not be rectangular, rather it should be hexagonal, as it helps in reducing the conflict. Accordingly, Mr Verma bought a hexagonal dining table for his six office employees *A*, *B*, *C*, *D*, *E* and *F*.

One day while taking lunch they were sitting along the sides of the hexagonal table. The following information is given regarding their seating postions:

(i) *F*, who is sitting exactly opposite *A*, is to the immediate right of *B*.
(ii) *D* is between *A* and *B* and is exactly opposite *C*.

17. *A* is sitting between which of the following pairs of persons?
(a) *D* and *E* (b) *B* and *E*
(c) *B* and *C* (d) *E* and *C*

18. Four of the following pairs are alike on the basis of sitting positions and so form a group. Which is the one that does not belong to the group?
(a) *A*, *D* (b) *B*, *C*
(c) *B*, *F* (d) *C*, *E*

19. Who is sitting opposite *B*?
(a) *C*
(b) *A*
(c) *E*
(d) Cannot be determined

20. Who is sitting to the left of *F*?
(a) *B* (b) *C*
(c) *A* (d) None of these

Direction for questions 21–23: ***Read the following passage and solve the questions based on it.***

There are nine judges—*G*, *H*, *I*, *K*, *L*, *M*, *N* and *O*, who have to appear on a series of three benches. Each bench will consist of three judges and each judge will appear in exactly one bench. The benches must be arranged according to the following conditions:

(i) *I* and *N* must be on the same bench.
(ii) *K* and *L* must be on the same bench.
(iii) *O* and *J* cannot be on the same bench.
(iv) *M* must appear on the second bench.
(v) Either *J* or *M* or both must appear on the bench with *H*.

21. Which of the following judges could appear on a bench together?
(a) *G*, *L*, *O* (b) *G*, *J*, *M*
(c) *K*, *I*, *M* (d) *N*, *I*, *J*

22. Which of the following cannot be true?
(a) *I* appears on the second bench
(b) *H* appears on the third bench
(c) *O* appears on the third bench
(d) *J* appears on the first bench, and *H* appears on the third

23. The third bench could consist of all of the following except:
(a) *K*, *L*, *O* (b) *K*, *I*, *J*
(c) *G*, *H*, *J* (d) *G*, *I*, *N*

EXERCISE 2

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

Under a special arrangement at FBI, telephone numbers of the employees are to be coded in the following way:

Digit	**7**	**3**	**5**	**0**	**2**	**1**	**6**	**4**	**9**	**8**
Code	N	H	L	T	F	D	R	Q	G	P

Following conditions are to be maintained:

(i) If the first digit is even and the last digit is odd, then they are to be coded as $ and @ respectively.
(ii) If the first digit is odd and the last digit is even, then they are to be coded as # and γ respectively.
(iii) If 0 is preceded as well as followed by an odd digit, then 0 is to be coded as *.
(iv) If 0 is preceded as well as followed by an even digit, then 0 is to be coded as ↑.
(v) Zero (0) is considered neither even nor odd.

1. What is the code for 1375490?
(a) DHNLQGT (b) #HNLQGE
(c) DHNLQG* (d) γHNLQG#
(e) None of these

2. Which of the following numbers will be coded as $ Q R L * H @?
(a) 8456037
(b) 8465032
(c) 6475031
(d) Cannot be determined

3. Which of the following numbers will be coded like—Q L P ↑ R N T?
(a) 4570680 (b) 4780650
(c) 6580470 (d) None of these

4. What will be the code for 36250084?
(a) #RFL**Gγ (b) #RFLG**γ
(c) #RF**LG* (d) None of these

Direction for questions 5–7: ***Read the following passage and solve the questions based on it.***

"Lets be God" is an organization imparting training to people to touch the different dimensions of life and be like God. It was planning to organize series of eight lectures *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H* (not necessarily in the same order) for three subjects 'Purpose of God', 'Alignment of Purpose' and 'Touching Lives' on three successive days.

For the sake of convenience, these three subjects are coded as *X*, *Y* and *Z* respectively. Subject *X* was to be covered first in three lecturers followed by *Z* and then subject *Y* in two lectures.

(i) Lectures *A*, *C* and *D* have to be on separate days.
(ii) Lectures *B* and *F* have to be kept on separate days. Lecture *B* cannot be clubbed with *A* or *G* or *D*.
(iii) Lectures *G* and *H* should happen on one day.

5. Which of the following pairs of lectures can go along with lecture *A* on subject *X*?
(a) *B*, *C* (b) *G*, *H*
(c) *D*, *E* (d) Data inadequate

6. Which combination of lecturers was arranged on the second day of the series?
(a) *C*, *G*, *H* (b) *B*, *D*, *E*
(c) *C*, *A*, *G* (d) Data inadequate

7. Which of the following lectures were for subject *Y*?
(a) *D*, *F* (b) *G*, *H*
(c) *B*, *C* (d) Data inadequate

Direction for questions 8–12: ***Read the following passage and solve the questions based on it.***

Pankaj, Qureshi, Rajesh and Sudhir live together in a house.
(i) Pankaj lives with his (or her) parents.
(ii) Qureshi lives with at least three persons younger than him (or her).
(iii) Sudhir lives with his mother, and is older than at least two persons living with him.
(iv) Rajesh lives with his (or her) son and is not older than Sudhir.

8. The total number of persons in that house is _____.
(a) 3 (b) 4
(c) 5 (d) 6

9. Qureshi is Pankaj's _____.
(a) father (b) mother
(c) son (d) grandmother

10. Sudhir is Pankaj's _____.
(a) brother (b) father
(c) mother (d) sister

11. Rajesh is Qureshi's _____.
(a) daughter (b) son
(c) grandson (d) daughter-in-law

12. Rajesh is Sudhir's _____.
(a) wife (b) husband
(c) son (d) daughter

Direction for questions 14–17: ***Read the following passage and solve the questions based on it.***

In Fig, 3.1; consists of three intersecting circles which represent the applicants for the post of *PA* who are computer literate; who have knowledge of office rules and regulations; and who have a bachelor's degree.

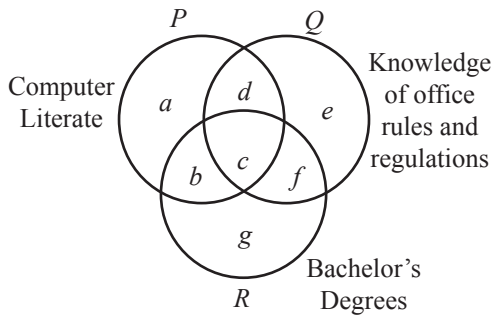


Fig. 3.1

13. Find the letter that represents all the applicants who are computer literate and have knowledge of office, rules and regulations but do not have a bachelor's degree.
(a) *B* (b) *C*
(c) *D* (d) *F*

14. Find the letter that represents all the applicants who are computer literate but do not have a bachelor's degree or any knowledge of office, rules and regulations.
(a) *A* (b) *B*
(c) *C* (d) *D*

15. Find the letter that represents the applicants who are computer literate and have a bachelor's degree but do not have knowledge of office, rules and regulations.
(a) *G* (b) *E*
(c) *C* (d) *B*

16. Find the letter that represents the applicants who are computer literate, know office, rules and regulations and have bachelor's degree.
(a) *B* (b) *C*
(c) *F* (d) *G*

17. Find the letter that represents the maximum number of applicants?
(a) *B*
(b) *C*
(c) *F*
(d) Cannot be determined

Direction for questions 18–22: ***Read the following passage and solve the questions based on it.***

Mr Manoj is a medical representative and he is supposed to visit six doctors—*M*, *N*, *P*, *Q*, *R* and *S*, exactly once every week. To visit the doctors, Mr Manoj has set up a schedule to visit each of the six doctors during the course of one week according to the following conditions:

(i) He must visit Dr *M* before Dr *N* and Dr *R*.
(ii) He must visit Dr *N* before Dr *Q*.
(iii) The third doctor he visits must be Dr *P*.

18. Which of the following must be true of Mr Manoj's schedule?
(a) He visits Dr *M* before Dr *Q*
(b) He visits Dr *N* before Dr *R*
(c) He visits Dr *P* before Dr *M*
(d) He visits Dr *P* before Dr *S*

19. If he visits Dr *S* first, which doctor must he visit second?
(a) Dr *M* (b) Dr *N*
(c) Dr *P* (d) Dr *Q*

20. Mr Manoj could visit any of the following doctors immediately after Dr *P* except:
(a) Dr *S* (b) Dr *R*
(c) Dr *Q* (d) Dr *M*

21. If he visits Dr *Q* immediately before Dr *R* and immediately after Dr *S* he must visit Dr *Q*:
(a) First (b) Second
(c) Fourth (d) Fifth

22. Which of the following could be the order in which he visits the six doctors?
(a) Dr *M*, Dr *S*, Dr *P*, Dr *N*, Dr *R*, Dr *Q*
(b) Dr *Q*, Dr *N*, Dr *P*, Dr *R*, Dr *S*, Dr *M*
(c) Dr *M*, Dr *R*, Dr *N*, Dr *Q*, Dr *P*, Dr *S*
(d) Dr *P*, Dr *S*, Dr *M*, Dr *R*, Dr *Q*, Dr *N*

Direction for questions 23 and 24: ***Read the following passage and solve the questions based on it.***

In the famous movie—*Hum Saat Aath Hain* there were six persons *A*, *B*, *C*, *D*, *E* and *F* in the family. *C* is the sister of *F*. *B* is the brother of *E*'s husband. *D* is the father of *A* and grandfather of *F*. There are 2 fathers, 3 brothers and a mother in the family and only these six people were the members of the family.

23. What is the name of husband of *E*?
(a) *B* (b) *C*
(c) *D* (d) None of these

24. Who is the mother?
(a) *A* (b) *B*
(c) *E* (d) *D*

EXERCISE 3

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

Five friends—Umesh, Vishnu, Xinhua, Yogesh and Zeta collected pebbles on the sea shore. They collected a total of 100 pebbles.

None of them collected less than 10 pebbles each. No two among them collected the same number of pebbles. Following information is given regarding the number of pebbles with each one of them:

(i) Umesh collected the same number of pebbles as Vishnu and Xinhua put together.
(ii) Xinhua collected 3 more pebbles than the cube of an integer.
(iii) The number of pebbles collected by Umesh was the square of an integer.
(iv) The number of pebbles collected by Vishnu was either the square or the cube of an integer.
(v) The number of pebbles collected by Yogesh and Zeta are in the ratio 4:3.

1. What was the number of pebbles collected by Umesh?
(a) 19 (b) 36
(c) 52 (d) 64

2. What was the number of pebbles collected by Vishnu?
(a) 16 (b) 25
(c) 46 (d) 64

3. What was the difference in the number of pebbles collected by Xinhua and Yogesh?
(a) 5 (b) 7
(c) 9 (d) 11

4. How many of the individual collection(s) of pebbles was/were prime numbers?
(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 5–9: ***Read the following passage and solve the questions based on it.***

Each of the alphabets given below represents a digit (from 1–9). No digit is represented by more than one alphabet and vice versa.

$$\begin{array}{rccccc} & & A & B & C & D \\ + & C & A & B & E & B \\ \hline & B & F & D & F & C \end{array}$$

5. Find the value of $B + C + D$.
(a) 12 (b) 14
(c) 16 (d) 19

6. What is the value of $A \times B \times C$?
(a) 72 (b) 108
(c) 24 (d) 36

7. What is the value of $(BF) \times (FC)$?
(a) 966 (b) 865
(c) 826 (d) 735

8. How many digits from A to F is/are prime numbers?
(a) 1 (b) 2
(c) 3 (d) 4

9. What is the sum of the three largest digits out of the given digits from A to F?
(a) 24 (b) 23
(c) 21 (d) 22

Direction for questions 10–13: ***Read the following passage and solve the questions based on it.***

(i) A group of five boys A, B, C, D and E and a group of five girls P, Q, R, S and T are standing in a row facing each other (not necessarily in the same order). The group of girls is facing north.

(ii) E is not at any of the ends. C is to the immediate right of B and D is to the immediate left of A, who is facing P. There are as many girls between P and Q as between R and S. A is second to the left of B, S and R are not facing either B or D.

10. Which of the following indicates the pair of students standing at the ends of the row?
(a) CB
(b) DB
(c) CD
(d) Cannot be determined

11. Which of the following is definitely true on the basis of the given information?
(a) C is second to the right of D
(b) P is third to the right of Q
(c) S is to the immediate right of P
(d) None of these

12. Who is standing to the immediate right of A?
(a) E
(b) C
(c) B
(d) Cannot be determined

13. Who is facing A?
(a) R (b) S
(c) P (d) None of these

Direction for questions 14–17: ***Read the following passage and solve the questions based on it.***

There are six students—A, B, C, D, E and F, participating in an evaluation test for Language and Science.

(i) A's total marks in language were just above C and in Science just above F.

(ii) B was just above C in Science but scored less than D in Language.

(iii) F got more marks than D and E in science, but didn't perform as well as D in Language.

(iv) No one scored in between *C* and *D* in Language and *C* and *A* in Science.

14. Who got the highest marks in Science?
(a) *A*
(b) *B*
(c) *C*
(d) Cannot be determined

15. Which of the following students has scored the least in Science?
(a) Only *D* (b) Only *E*
(c) Either *D* or *E* (d) Only *A*

16. Who scored just below *D* in Language?
(a) *B*
(b) *F*
(c) *C*
(d) Cannot be determined

17. Which of the given statements is not necessary to answer the questions?
(a) (ii)
(b) (iii)
(c) (iv)
(d) All are necessary

Direction for questions 18–22: ***Read the following passage and solve the questions based on it.***

In the annual performance appraisal, all employees were placed in three categories—average, good and excellent. To execute the appraisal plan more effectively, a software called 'Appraise Well' is being used. But somehow the computer got infected with a virus and some information was lost.

While trying to recover the data, only the following could be recovered (Table 3.2):

Table 3.2

	Average	Good	Excellent	Total
Male			12	
Female				36
Total		33		

(i) An employee can be placed in only one category.
(ii) Half the employees are either excellent or good.
(iii) 40% of the employees were females.
(iv) One-third of the males were average.

18. How many students are both females and excellent?
(a) 2 (b) 4
(c) 8 (d) 0

19. What proportion of the good employees are males?
(a) 0.4 (b) 0.5
(c) 0.6 (d) None of these

20. What proportion of the good employees are females?
(a) 0.4 (b) 0.5
(c) 0.6 (d) 0.27

21. What proportion of the females are good employees?
(a) 0.4 (b) 0.5
(c) 0.6 (d) None of these

22. How many employees are neither excellent nor males?
(a) 36 (b) 12
(c) 24 (d) 0

Direction for questions 23–25: ***Read the following passage and solve the questions based on it.***

A quiz has three rounds of two questions each. However, the scoring scheme is different in all the three rounds as per the following:

In the first round, each correct answer carries 20 points and each incorrect answer carries a penalty of 10 points. If both the questions in the 1st round are answered correctly a bonus of 10 points is awarded.

In the second round, each correct and incorrect answer gets the same points as in the first round. However, an additional penalty of 10 points is awarded if both the questions are answered incorrectly.

In the third round, each right answer fetches 40 points and an incorrect one fetches a penalty of 20 points.

23. In how many ways can a score of 40 be achieved?
(a) 0 (b) 1
(c) 2 (d) 3

24. If only two answers are incorrect in the whole quiz, what is the minimum possible score that a contestant can achieve?
(a) 20 (b) 30
(c) 40 (d) None of these

25. If only two answers are incorrect in the whole quiz, what is the maximum possible score that a contestant can achieve?
(a) 100 (b) 130
(c) 120 (d) None of these

EXERCISE 4

Direction for questions 1–3: ***Read the following passage and solve the questions based on it.***

A, *B*, *C*, *D* and *E* are five cities out of which two are hill stations and the rest are in the plains. Two cities, which are in the plains, are ports. Four cities out of five are state capitals and two are industrial cities. The Population of two cities is less than 5 lacs. The population of one city is 20 lacs and the more than 50 lacs of. Two cities are situated on the same latitudes and other two are on same longitude and the other two are on the same longitude. Latitudes and longitudes of both ports are different and out of these one is an industrial town. The population of both the industrial cities is more than 50 lacs. The longitude of one hill station and one of the industrial cities is the same. The latitudes and longitudes of the other hill station and the other port are different. One industrial town is neither a hill station nor a port. None of the hill stations is an industrial town. The hill station has the which same longitude as the port, is a capital. *B* is a hill station while the longitudes of *A* and *E* are the same. *E* is a port. The latitudes of *D* and *C* are the same and the population of *D* is 20 lacs. Both the ports are capitals and one of them is an industrial town.

1. Which of the following two cities have a population less than 5 lacs?
(a) *B* and *C* (b) *A* and *B*
(c) *A* and *D* (d) *D* and *B*

2. Which of the following cities is not a capital?
(a) *A* (b) *C*
(c) *B* (d) *E*

3. Which one of the following cities is a hill station as well as a capital?
(a) *A* (b) *B*
(c) *C* (d) *D*

Direction for questions 4 and 5: ***Read the following passage and solve the questions based on it.***

To smoothen the procedure of grievances reprisal, a team of five members is being constituted by the top management. As per the procedures established, it has to provide representation to both the workers and the middle management. The team must have two representatives of the middle management, two representatives of the workers and one representative of the top management.

As per the availability of persons, we know that:

(i) The middle management's representatives must be chosen from *X*, *Y* and *Z*.
(ii) The workers' representatives must be chosen from *A*, *B* and *C*.
(iii) The top management's representative must be chosen from either *J* or *K*.
Owing to some geographical constraints, we know that.
(iv) *A* cannot serve with *C*.
(v) *Z* cannot serve together with *C*.
(vi) *Y* cannot serve unless *K* is also in the team.

4. Which of the following persons must be chosen?
(a) *J* (b) *X*
(c) *Y* (d) *B*

5. If *A* and *B* are chosen to be the workers representatives, then which of the following statement(s) is/are true?

I. K is chosen
II. Both X and Y are chosen to represent teachers
III. J is chosen

(a) I only (b) II only
(c) III only (d) Both I and II

Direction for questions 6–9: ***Read the following passage and solve the questions based on it.***

There are five islands *A*, *B*, *C*, *D* and *E* in Nicobar. Two of these have post offices, three have schools and three are accessible by bridge. Two have a population of more than 5,000 each, two have a population between 2,000 and 5,000 each, and one has a population of less than 2,000. Two of these islands have electricity in addition to certain other facilities such as a school and accessibility by bridge. The island with a population of less than 2,000 has a school but does not have a post office nor is it accessible by bridge; while each of the islands with a population of more than 5,000 has a school. Of the two islands having a population between 2,000 and 5,000, only one has a post office and is accessible by bridge. Island *A* is accessible by bridge. Island *B* has a population of more than 5,000, island *D* has a school and is accessible by bridge but does not have a post office, while island *E* has a school but is not accessible by bridge.

6. Which island has a school and a post office?
(a) *A* (b) *B*
(c) *C* (d) *D*

7. Which island does not have any of the facilities available to other islands?
(a) *A* (b) *B*
(c) *C* (d) *D*

8. Which two islands have electricity?
(a) *A*, *B* (b) *B*, *C*
(c) *B*, *D* (d) *C*, *D*

9. Which three islands can be accessed by bridge?
(a) *A*, *B*, *D* (b) *A*, *B*, *E*
(c) *A*, *D*, *E* (d) *B*, *D*, *E*

Direction for questions 10–13: ***Read the following passage and solve the questions based on it.***

P, *Q*, *R*, *S*, *T* and *U* are six lecturers in a college each teaching a different subject Mathematics, Physics, Sociology, Biology, Geography and History not necessarily in the same order. There are only four female lecturers. Following is the information regarding who teaches what:

(i) *Q* teaches History.
(ii) No female lecturer teaches Biology or Mathematics.
(iii) *S*, who teaches Sociology, is the oldest.
(iv) *P* is older than *R* and teaches Physics.
(v) The youngest teaches Biology and *U* is not the youngest.
(vi) T teaches Mathematics and is older than *P* but younger than *Q* and *U*.

10. Which of the following group comprises only of females?
(a) *RPSU* (b) *RSTQ*
(c) *PQTU* (d) *PQSU*

11. Which subject does *U* teach?
(a) Geography
(b) Sociology
(c) Geography or Sociology
(d) Cannot be determined

12. Who teaches Biology?
(a) *P*
(b) *S*
(c) *R*
(d) None of these

13. If all the lecturers are arranged in descending order of their ages, what will be the position of the two male lecturers?
(a) 2nd and 6th
(b) 4th and 6th
(c) 3rd and 6th
(d) Cannot be determined

14. Who is the youngest out of the six lecturers?
(a) *Q*
(b) *T*
(c) *P*
(d) None of these

Direction for questions 15–19: ***Read the following passage and solve the questions based on it.***

(i) Five students *S*, *R*, *N*, *M* and *V* have a total of five books on various subjects like—Physics, Chemistry, Maths, Biology and English written by authors Gupta, Khanna, Harish, D'Souza and Edgar. Each student has only one book on any one of the five subjects.
(ii) Gupta is the author of the physics book, which is not owned by *V* or *S*.
(iii) *M* owns the book written by Edgar.
(iv) *N* owns the Maths book. *V* has the English book which is not written by Khanna. The Biology book is written by D'Souza.

15. Which of the following is the correct combination of subject, student and author?
(a) Maths – *N* – Harish
(b) Physics – *M* – Gupta
(c) English – *V* – Edgar
(d) Biology – *S* – D'Souza

16. Who is the author of the Chemistry book?
(a) Harish
(b) Edgar
(c) Khanna or Harish
(d) Edgar or Khanna

17. Who is the owner of the book written by Harish?
(a) *V*
(b) *S*
(c) *R*
(d) Cannot be determined

18. For how many authors, is it possible to uniquely determine the subject on which they have written a book?
(a) 2 (b) 3
(c) 4 (d) 5

19. For how many students is it possible to uniquely determine the books owned by them?
(a) 2 (b) 3
(c) 4 (d) 5

Direction for questions 20–24: ***Read the following passage and solve the questions based on it.***

There are seven students—*A*, *B*, *C*, *D*, *E*, *F* and *G* in a batch at Prep-Well Coaching institute. All these students sit on three benches 1st, 2nd and 3rd in such way that:
(i) There are at least two students sitting on each bench.
(ii) There is at least one girl on each bench.
(iii) *C*, a girl student, does not sit with *A*, *E* and *D*.
(iv) *F*, a boy student sits with only *B*. *A* sits with his best friend on bench *I*.
(v) *G* sits on the 3rd bench. *E* is the brother of *C*.

20. How many girl students are there?
(a) 3 (b) 4
(c) 2 (d) Data inadequate

21. Who sits with *C*?
(a) *B* (b) *G*
(c) *D* (d) *E*

22. Which of the following is a group of girls?
(a) *BAC* (b) *BFC*
(c) *CDF* (d) *BCD*

23. On which bench do three students sit?
(a) 2nd (b) 3rd
(c) 1st (d) 1st or 2nd

24. The sex of how many students is known?
(a) 4 (b) 5
(c) 6 (d) 7

EXERCISE 5

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

In a state, there are seven cities—*A*, *B*, *C*, *D*, *E*, *F* and *G*. The following cities have a two way connection between them: *D* and *F*, *E* and *H*, *A* and *B*, *B* and *C*, *G* and *E*, *C* and *E* and *G* and *F*. There is a one-way connection from *F* to *B*. (Two way connection means people can move in both the Directions whereas in a one way connection, people can move only in one way.)

1. If the connection between *E* and *C* is cut, then which route is not possible?
   (a) *B* to *E* (b) *E* to *C*
   (c) *F* to *B* (d) *D* to *G*
2. If a person wishes to travel from *D* to *E*, then what is the minimum number of cities that he needs to cross enroute?
   (a) 2 (b) 4
   (c) 3 (d) 5
3. If a person wants to go to city *F* from city *A*, how many cities will he be required to cross (excluding *A* and *F*)?
   (a) 2 (b) 4
   (c) 3 (d) 5
4. In the above question, which of the following will minimize the number of cities?
   (a) Make *F*—*B* a two way connection
   (b) Connect *F*—*C* with a two way connection
   (c) Connect *A*—*D* with a one way connection from *D* to *A*
   (d) Connect *C*—*G* with a two-way connection

Direction for questions 5–7: ***Read the following passage and solve the questions based on it.***

There are two groups namely Left and Right. Persons *A*, *B*, *C* and *D* are in the Left group and *E*, *F* and *G* are in the Right group. *A* committee is to be formed of 3 persons, at least one from each group. *B* and *C* will never be together. If *G* is there in the committee then *E* will also be there. *F* and *C* will be always together. The co-ordinator has to be from the minority group.

5. Which one of them cannot be a valid committee?
   (a) *BDE* (b) *FGB*
   (c) *AGC* (d) *EFG*
6. Which one of them cannot be a coordinator?
   (a) *F* (b) *B*
   (c) *G* (d) *D*
7. Which one is a false statement?
   (a) *FBA* is a possible group
   (b) *FCD* is a possible group
   (c) *FCE* is a possible group
   (d) *GEA* is a possible group

Direction for questions 8–12: ***Read the following passage and solve the questions based on it.***

P, *Q*, *R*, *S*, *T*, *V* and *W* are seven friends who left for seven different places—Delhi, Chennai, Hyderabad, Bangalore, Kolkata, Chandigarh and Patna—each one on a different day of the week. *R* left for Patna on Monday. On the last day of the week one person left for Bangalore. *T* left the next day after *P*, who left for Chandigarh and a day previous to *W*'s departure. *S* left for Kolkata on Friday. *Q* did not leave for either Hyderabad or Bangalore and *W* left for Delhi.

It is also given that the week starts on Sunday and ends on Saturday.

8. On which day of the week did *Q* leave?
   (a) Sunday
   (b) Saturday
   (c) Wednesday
   (d) Cannot be determined
9. Who left for Bangalore?
   (a) *T*
   (b) *P*
   (c) *V*
   (d) Cannot be determined
10. On which day of the week did *T* leave?
    (a) Tuesday (b) Thursday
    (c) Sunday (d) Wednesday
11. Which of the following combinations of person-place is not correct?
    (a) *R*—Patna (b) *P*—Chandigarh
    (c) *T*—Hyderabad (d) All are correct
12. Who left on Tuesday?
    (a) *P* (b) *W*
    (c) *Q* (d) *V*

Direction for questions 13–16: ***Read the following passage and solve the questions based on it.***

Manoj is a medical representative (*M R*) with Nicholas Piramal. He has to meet seven doctors

M, *N*, *O*, *P*, *Q*, *R* and *S* on a particular day between 9 a.m. and 4 p.m. Following details are available regarding his schedule:

(i) He takes 30 minutes with each doctor and keeps a gap of 25 minutes between two appointments for travel and preparation except after the 5th visit, when he takes a 50 minutes break for lunch, travel and preparation.

(ii) He has to meet doctor *M* immediately before *Q* but immediately after *R*, who is the third doctor to be met.

(iii) Dr *S* does not give any time for a meeting before 9.30 a.m. and after 10.30 a.m. Dr *P* and Dr *N* meet Manoj between 2 p.m. and 3.30 p.m. only.

13. Who among the following is likely to be the first doctor that day?
(a) *S* (b) *O*
(c) *R* (d) *Q*

14. Who will be the last doctor to be visited by him that day?
(a) *N*
(b) *P*
(c) *R*
(d) Cannot be determined

15. At what time is he likely to meet Dr *S*?
(a) 9.55 a.m. (b) 9:30 a.m.
(c) 9 a.m. (d) None of these

16. If Dr *R* asks him to visit again 25 minutes after his scheduled seventh visit is over, then at what time is Dr *R* asking him to meet?
(a) 3.55 p.m. (b) 3.40 p.m.
(c) 3.25 p.m. (d) None of these

Direction for questions 17–25: ***Read the following passage and solve the questions based on it.***

B, *C*, *D*, *F*, *G*, *H* and *J* are seven students studying in three colleges *P*, *Q* and *R*. Among them are three boys and four girls. There is at least one boy and one girl in each college. Three of them are in the Commerce discipline and two each in Arts and Science. *B* and her sister *G* are in the Science discipline but in different colleges. *F* studies Arts in college *Q* and he does not study with either *J* or *C*. *D* is not in the Commerce discipline and he studies in college *R*, only with *B*. All the three from the Commerce discipline do not study in the same college. *H* studies in the same college with her friend *G*.

17. In which college do only the Commerce students (from the given seven) study?
(a) *P* (b) *Q*
(c) *R* (d) *P* or *Q*

18. If *B* and *C* can interchange their colleges satisfying all other conditions, which of the following will definitely represent the girls?
(a) *BCHD* (b) *BCHG*
(c) *BFHG* (d) *DCHF*

19. Which of the following represents the three students in the Commerce discipline?
(a) *HJC*
(b) *HDC*
(c) *HFG*
(d) Cannot be determined

20. In which college do three of the students study?
(a) *P* (b) *R*
(c) *Q* (d) None of these

21. Which of the following represents the three boys?
(a) *DFJ*
(b) *DFC*
(c) *JFC*
(d) Cannot be determined

22. Pointing towards a man in the photograph, a woman said, "He is the father of the brother of my father". How is the man related to the woman?
(a) Father (b) Uncle
(c) Grandfather (d) Uncle or Father

23. To number the pages of a large book, the printer used a total of 1890 digits. How many pages are there in the book?

(a) 655 (b) 667
(c) 666 (d) 668

24. Five men *A*, *B*, *C*, *D* and *E* read a novel. The one who reads it first gives it to *C*. The one who reads it last had taken it from *A*. *E* was not the first or the last to read the novel. Also, it is known that there were two readers between *B* and *A*. Who read the novel last?
(a) *E* (b) *B*
(c) *D* (d) *C*

25. In the above question, who read the novel in the middle?
(a) *E* (b) *B*
(c) *D* (d) *C*

EXERCISE 6

Direction for questions 1–4: ***Read the passage given below and solve the questions based on it.***

Five persons—Yamini, Nitika, Monica, Rehana and Pragya are students of five different areas Medical, Engineering, Architecture, Arts and Management in no particular order. Each of them plays a different musical instrument from Sitar, Tabla, Sarod, Guitar and Violin, not given respectively.

Monica, a medical student, does not play Sarod or Sitar or Guitar.

Pragya is neither a student of Engineering nor of Management.

Rehana, who plays tabla, is an Arts student.

Neither Pragya nor Yamini plays Sarod.

1. Who among the following plays Sarod?
(a) Yamini (b) Nitika
(c) Pragya (d) Data inadequate
(e) None of these

2. The guitarist is a student of which of the following disciplines?
(a) Engineering
(b) Either Engineering or Management
(c) Architecture
(d) Data inadequate
(e) None of these

3. Who among the following plays Sitar?
(a) Yamini (b) Nitika
(c) Pragya (d) Data inadequate
(e) None of these

4. For how many of the given five students is it possible to find the exact discipline and the musical instrument the person plays?
(a) 0 (b) 1
(c) 2 (d) 3
(e) 4 (f) None of these

Direction for questions 5–10: ***Read the passage given below and solve the questions based on it.***

During their stint at IIM Shillong, ten students have opted for various electives named from *A* to *F*. In these electives, students are given the points on a scale of 1 to 5 points. Points obtained by the students can be integral points only.

It is also known that not all the electives are taken by all the students and not all the students are taking at least an elective.

The range of scores indicates the maximum and minimum scores in that elective by the students who have chosen that elective. However, if the range of the scores is 1–4, then at least one of students must have got 1 point and at least one student must have got 4 points in that elective.

The number of elective takers out of the 10 students is given in the last column (Table 3.3).

Table 3.3

Elective	Range of scores of all the elective takers (Minimum and maximum scored)	Average score of the elective takers	Number of elective takers
A	1–4	3.5	6
B	2–4	3	3
C	1–5	4	7
D	1–2	4/3	3
E	2–5	4	4
F	3–5	11/3	6

5. How many students have scored more than 4 points in at least 2 electives?
(a) 4 (b) 7
(c) 2 (d) 3
(e) Cannot be determined

6. What is the minimum number of students who must have scored less than 2 points in at least one elective?
(a) 3 (b) 4
(c) 2 (d) 3
(e) Cannot be determined

7. What is the maximum number of students who have scored more than 3 points in at least one elective?
(a) 5 2 6
(c) 7 (d) 3
(e) Cannot be determined

8. What is the minimum number of students who have scored more than 3 points in at least one elective?
(a) 5 (b) 6
(c) 7 (d) 0
(e) None of these

9. Elective *A* and elective *B* are merged to form a new elective *H*. This new elective *H* will be having all those students who have opted elective *A* and elective *B* and the scores of each of these electives have been taken into consideration while finding the average of elective *H*. If none of the students of elective *A* and elective *B* are common, then what will be the average score of elective *H*?
(a) 10/3 (b) 3
(c) 3.5 (d) None of these
(e) Cannot be determined

10. Elective *C* and elective *E* are merged to form a new elective *N*. This new elective *N* will be having all those students who have opted elective *C* and elective *E* and the scores of each of these electives have been taken into consideration while finding the average of elective *N*. What is the average score of elective *N*?
(a) 4 (b) 3
(c) 3.5 (d) 2
(e) Cannot be determined

Direction for questions 11–13: ***Read the passage given below and solve the questions based on it.***

There are six movies—*A*, *B*, *C*, *D*, *E* and *F* to be showed in a film festival. *B*, *C* and *E* are art movies and others are commercial movies. Only *F* and *D* are Hindi movies and the remaining movies are English movies. Movies *A*, *C* and *D* are made by Jagmohan Mundhra and movies *B*, *E* and *F* are made by Steven Spielberg.

11. Which English movie is a commercial movie?
(a) *A* (b) *B*
(c) *C* (d) *D*
(e) None of these

12. Which Hindi movie is made by Jagmohan Mundhra?
(a) *A* (b) *B*
(c) *C* (d) *D*
(e) None of these

13. Which of the English movies is/are art movies made by Steven Spielberg?
(a) *B* (b) *B* and *C*
(c) *C* (d) *B* and *E*
(e) None of these

Direction for questions 14–18: ***Read the passage given below and solve the questions based on it.***

Bus route number 761 has exactly six stops on its route. Any bus plying on this route starts from the initial position, then stops first at stop one and then at stops two, three, four, five, and six respectively. After the bus reaches stop six, the bus turns and returns to its initial position and repeats the cycle. Buses are not allowed to carry people on its return journey.

Following are the six stops—*L*, *M*, *N*, *O*, *P*, and *Q* in no particular order. Further, following observations have been made regarding the stops on this route:

Observation 1—*P* is the third stop.

Observation 2—*M* is the sixth stop.

Observation 3—The stop *Q* is the stop immediately after *O*.

Observation 4—*N* is the stop immediately before *L*.

14. If *N* is the fourth stop on this route, which among the following must be the stop immediately before stop *P*?
(a) *N* (b) *Q*
(c) *O* (d) *M*
(e) *L*

15. If *L* is the second stop on this route, which among the following must be the stop immediately before *M*?
(a) *N* (b) *Q*
(c) *O* (d) *P*
(e) *L*

16. A passenger boards a bus on this route at *O*, rides past one of the stops, and alights at *P*. Which of the following must be true?
(a) *O* is stop one (b) *P* is stop four
(c) *N* is stop five (d) *L* is stop six
(e) None of these

17. In how many different ways the stops can be fixed on this route?
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these

18. If we relax the observation 3, then in how many different ways the stops can be fixed on this route?
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these

Direction for questions 19–20: ***Read the passage given below and solve the questions based on it.***

During the Indian film festival at Goa, movies from five countries—Austria, Bhutan, China, Denmark, and England are to be shown. Movies from these countries have to follow a particular order as given follows:

Movie from Austria must be shown before the movie from China.

Movie from England should be the fifth movie to be shown.

Movie from Bhutan must be shown before the movie from Denmark.

19. Which of the following is the correct order for showing all the movies?
(a) Austria, China, Bhutan, Denmark, England
(b) Austria, China, Denmark, England, Bhutan
(c) Bhutan, Denmark, China, Austria, England
(d) Bhutan, Denmark, England, Austria, China
(e) England, Bhutan, China, Austria, Denmark

20. Movies from Denmark and England are shown at the farthest gap possible. Which among the following would be true?
(a) Movie from Austria is shown earlier than Movie from Bhutan
(b) Movie from England is shown earlier than Movie from China
(c) Movie from Denmark is shown earlier than Movie from Austria
(d) Movie from England is shown earlier than Movie from Bhutan
(e) Movie from China is shown earlier than Movie from Denmark

EXERCISE 7

Direction for questions 1–3: ***Read the passage given below and solve the questions based on it.***

Six sides of a cube are painted by six different colours—Black, blue, brown, green, red and white—one colour on one side. Following observations are made regarding the sides and the colour on it:

(i) The side coloured red is opposite the side coloured black.

(ii) The green side is between the red side and the black side.
(iii) The blue side is adjacent to the white side.
(iv) The brown side is adjacent to the blue side.
(v) The red side is the bottom face.

1. The four colours adjacent to the green side are:
(a) Black, blue, brown, red
(b) Black, blue, brown, white
(c) Black, blue, red, white
(d) Black, white, brown, red

2. Which of the following can be deduced using the observation 1 and observation 5?
(a) Black is on the top
(b) Brown is on the top
(c) Blue is on the top
(d) Brown is opposite to black

3. If the red side is exchanged for the green side and the blue is swapped for black, then which of the following is false?
(a) Red side is opposite to black side
(b) White side is adjacent to the brown side
(c) Green side is opposite to the blue side
(d) White side is adjacent to the blue side

Direction for questions 4–8: ***Read the passage given below and solve the questions based on it.***

In the latest *Rajshri* films *Hum aapke hain saath saath*, there are six generations in a family however total members in this family is 5. Names of the members of this family are Atul, Binod, Charu, Deepak and Faisal.

Following observations have been made regarding them:
(i) Charu is the only unmarried child whose grand parents have passed away.
(ii) Deepak, a widow, is the daughter of Faisal who is a widower.
(iii) None of the family members is an adopted son or an adopted daughter.
(iv) One generation has at most one family member.
(v) Every couple has only one child and the first generation did not have any siblings.

4. If Atul is the grandson of Binod, then which member is in the second last generation?
(a) A male (b) Atul
(c) Either (a) or (b) (d) Both (a) and (b)

5. If Binod is the eldest member in the family, then how is Deepak related to Atul?
(a) Grandmother (b) Grandfather
(c) Either (a) or (b) (d) None of these

6. If Binod is the eldest member in the family, then how is Atul related to Charu?
(a) Father
(b) Mother
(c) Uncle
(d) Cannot be determined

7. If Atul is Charu's father and Binod's grandson, then how many possible combinations of the family exist?
(a) 1 (b) 2
(c) 3 (d) 4

8. If grandfather of Charu were alive, he would have definitely belonged to which generation?
(a) 4th last (b) 3rd
(c) 2nd last (d) 3rd last

Direction for questions 9–12: ***Read the passage given below and solve the questions based on it.***

There are six classes to be held everyday by six different teachers—*A*, *B*, *C*, *D*, *E*, and *F* in a school. Three of these classes will take place in the morning session before the lunch break whereas the other three classes will be held in the afternoon session after the lunch break.

The scheme of the classes should follow the following restrictions:

Class by *B* should be immediately before the class by *C* and their presentations cannot be separated by the lunch break.

Class by *D* must be either the first or the last class.

9. If class by *C* is the fifth class of the day, then class by *B* is the ..... class of the day.
(a) First (b) Second
(c) Third (d) Fourth

10. Class by *B* can be placed at any of the following places in the order of presenters EXCEPT:
(a) First (b) Second
(c) Third (d) Fifth

11. Class by *F* is to be scheduled immediately after the class by *D*, then class by *C* could be slated for which class of the day?
(a) First (b) Third
(c) Fourth (d) Fifth

12. In case class by *F* and class by *E* are the fifth and sixth classes of the day respectively, then which of the following must be true?
(a) Class by *A* is first class of the day
(b) Class by *A* is third class of the day
(c) Class by *A* is fourth class of the day
(d) None of these

Direction for questions 13–20: *Read the passage given below and solve the questions based on it.*

There are three projects—P1, P2 and P3. A student can select either one project or two projects or all the three projects subject to the conditions given below:

Condition 1: Both P1 and P2 have to be selected.
Condition 2: Either P1 or P3, but not both, has to be selected.
Condition 3: P2 can be selected only if P3 has been selected.
Condition 4: P1 is selected only if P3 is selected.

13. How many different selections can be made if no conditions are imposed?
(a) 5 (b) 6
(c) 7 (d) 8

14. How many selections can be made to meet condition 1 as given above?
(a) 1 (b) 2
(c) 3 (d) 4

15. How many selections can be made to meet condition 2 as given above?
(a) 2 (b) 3
(c) 4 (d) 5

16. How many selections can be made to meet condition 3 as given above?
(a) 5 (b) 6
(c) 7 (d) 8

17. How many selections can be made to meet condition 4 as given above?
(a) 5 (b) 6
(c) 7 (d) 8

18. How many selections can be made meeting both the conditions 1 and 2 as given above?
(a) 0 (b) 1
(c) 2 (d) None of these

19. How many selections can be made meeting both the conditions 2 and 3 as given above?
(a) 0 (b) 1
(c) 2 (d) None of these

20. How many selections can be made meeting the conditions 1, 2 and 3 as given above?
(a) 0 (b) 1
(c) 2 (d) None of these

Direction for questions 21–26: *Read the passage given below and solve the questions based on it.*

Five universities—*A*, *B*, *C*, *L* and *K* participated in the last year inter-university athlete event. Each of the universities were represented by three members. According to the rules, the race-track maker gets wild card entry making the total number of athletes participating in the race equal to 16. The rank obtained by the individual will be the points obtained. And similarly the points obtained by an individual university will be equal to the sum of the ranks of the member athletes of the team. Lower the points, better the team and the best team wins the trophy.

The five universities tied for the trophy, their sum being equal, however no two athletes tied for the same position. In order to determine the order in which the universities will hold the trophy (they agreed to hold it for 73 days each), they multiplied the athletes positions together in each university. The university with the smallest product *K* will hold the trophy first and so on the university with the largest product *A* will hold the cup last. Unfortunately, universities *B* and *C* were still tied and had to be separated by a toss of coin.

It was also found that the no two athletes of a university finish at the consecutive positions. Had the race-track maker did not come in between two athletes of university *B*, university *B* would have achieved this feat of two athletes being at consecutive positions.

21. Which of the following is the position of the race-track maker?
(a) 6
(b) 11
(c) Either 6 or 11
(d) Neither 6 nor 11
(e) Cannot be determined

22. What is the product of the university *A*?
(a) 360 (b) 455
(c) 450 (d) 390
(e) None of these

23. What is the smallest point obtained by an individual of university *L*?
(a) 1 (b) 2
(c) 3 (d) 4
(e) 5

24. What is the product of the ranks obtained by all the athletes of university *K*?
(a) 120 (b) 150
(c) 128 (d) 144
(e) None of these

25. Which of the following is not a rank obtained by the athletes of university *C*?
(a) 15 (b) 6
(c) 4 (d) 2
(e) 4 and 2

26. Which of the following is a rank obtained by the athletes of university *B*?
(a) 14 (b) 6
(c) 4 (d) 5
(e) 3

ANSWER KEY

Exercise 1

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (d).
9. (c).	**10.** (d)	**11.** (c)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (d)
17. (a)	**18.** (b)	**19.** (c)	**20.** (a)	**21.** (d)	**22.** (d)	**23.** (b)	

Exercise 2

1. (a)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (b)
9. (b)	**10.** (d)	**11.** (d)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (b)
17. (d)	**18.** (a)	**19.** (a)	**20.** (d)	**21.** (d)	**22.** (a)	**23.** (d)	**24.** (c)

Exercise 3

1. (b)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (a)	**8.** (b)
9. (b)	**10.** (c)	**11.** (d)	**12.** (a)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (d)

17. (d) **18.** (d) **19.** (d) **20.** (d) **21.** (d) **22.** (a) **23.** (c) **24.** (c)
25. (c)

Exercise 4

1. (b) **2.** (c) **3.** (a) **4.** (d) **5.** (d) **6.** (b) **7.** (c) **8.** (c)
9. (a) **10.** (d) **11.** (a) **12.** (c) **13.** (b) **14.** (d) **15.** (d) **16.** (b)
17. (a) **18.** (d) **19.** (d) **20.** (d) **21.** (b) **22.** (d) **23.** (c) **24.** (c)

Exercise 5

1. (a) **2.** (a) **3.** (b) **4.** (a) **5.** (a) **6.** (a) **7.** (a) **8.** (a)
9. (c) **10.** (d) **11.** (d) **12.** (a) **13.** (b) **14.** (d) **15.** (a) **16.** (d)
17. (a) **18.** (b) **19.** (d) **20.** (c) **21.** (d) **22.** (c) **23.** (c) **24.** (c)
25. (a)

Exercise 6

1. (b) **2**. (d) **3**. (d) **4**. (c) **5**. (e) **6**. (c) **7**. (e) **8**. (e)
9. (a) **10**. (e) **11**. (a) **12**. (d) **13**. (d) **14**. (b) **15**. (b) **16**. (a)
17. (b) **18**. (d) **19**. (a) **20**. (c)

Exercise 7

1. (d) **2.** (a) **3.** (b) **4.** (d) **5.** (a) **6.** (d) **7.** (d) **8.** (d)
9. (d) **10.** (c) **11.** (d) **12.** (c) **13.** (c) **14.** (b) **15.** (c) **16.** (a)
17. (a) **18.** (b) **19.** (d) **20.** (a) **21.** (b) **22.** (b) **23.** (b) **24.** (c)
25. (d) **26.** (e)

HINTS AND EXPLANATIONS

Exercise 1

Solution to questions 1–3

1. It is given that $Q > P$ and $R > S$.

Q	P	R	S
4	2	3	1
2	1	4	3

The distribution of coins can be of two types; in both the cases, S gets an odd number.

2. is the answer as it is one of the conditions mentioned in the question itself.

3.

R	S	Q	P
4	2	3	1
4	1	3	2
4	3	2	1

Looking at the possible distribution of coins, we find that option (a) is not always true.

Solution to questions 4–7

Using the statements given above, we have Table 3.4 for the training schedule:

Table 3.4

Group	Mon	Tue	Wed	Thu	Fri	Sat
A	×	√	×	√	×	√
B	√	√	√	√	×	×

Group	Mon	Tue	Wed	Thu	Fri	Sat
C	×	×	√	√	√	√
D	×	×		×		×
E	√	√	√	×	√	×
F	√	√	√	√	×	×

Here, group *D* meets either on Wednesday or on Friday.

Solution to questions 8–11

8. Content combinations *PP* in 1st box and *PC* in the 2nd box are eliminated as no label indicates the contents of the boxes correctly. Now if one of the four items is a ball then one of the combinations is either *BC* or *BB*.

Option (a) is wrong when the combinations are *BC* and *BB*.

Option (b) is wrong when the combinations are (i) *BC* along with *PC* or *CC*, (ii) BB along with *CC*, *PC* or *BC*.

Option (c) is wrong because the combination in box *PP* is *BB* and in box *PC* is *PP*.

Option (d) is obviously the answer.

9. Label *PP* – *CC*

Label *CC* – *PP*

Label *PP* – *BC*

Now if the box with the label *BC* has contents *BB*, then the box with the label *PC* will have the contents *PC* and this is not allowed. Hence, option (c) is definitely wrong.

10. The box containing the label *BC* has one ball and one clip. After the interchange, it will not contain the same combination. Now, there are only two boxes which have the ball as one of the items, so the other item will also be a ball.

11. The box with the label *PC* contains two balls; the box with the label *PP* contains two clips and the box with the label *CC* contains one ball and one clip. The remaining two boxes contain two pins and one pin and one clip. Hence if only one of the remaining boxes is opened, it will have at least one pin.

Solution to questions 12–16

13. Let us take two possible scenarios:

Scenario 1: *L*, *G* and *T* would occupy the second, third and fourth cars, respectively. But this arrangement would not accommodate *M*, seated alone, immediately behind an empty car. Hence this scenario is eliminated.

Scenario 2: Given the additional information, *L* must occupy the third car while *T* occupies the fifth car. Accordingly, *M* must occupy the second car and the first car must be empty.

However, *P* may occupy either the third or the fourth car. Thus, statement (b) is not necessarily true.

Again, consider two basic scenarios.

In Scenario 1: *G* is in the 3rd car, Given that *J* and the 7th person occupy the first car, *M* must occupy the fifth car alone.

Scenario 2: *G* is in the 4th car. It is given than *J* and the 7th person occupy the 1st car, *M* must occupy the 3rd car (while the 2nd car is empty). Since *T* cannot share a car with *G*, *T* must occupy the 5th car. Again *T* cannot share a car with *P*, hence *P* must share the 4th car with *G*.

Accordingly, *L* should share the 5th car with *T*.

14. Total number of persons = 6

Total number of seats = 10

Number of vacant seats = 4

Out of these 4 seats, 3 seats are vacant because of *M*. So, only one out of remaining people can sit alone.

Solution to questions 17–20

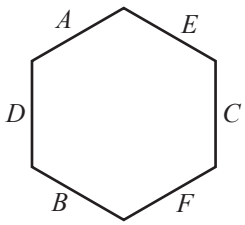


Fig. 3.2

18. The rest are sitting adjacent to each other

Solution to questions 21–23

21. Eliminating the options. *I* must appear with *N* so, we can eliminate option (c). Also *K* must appear with *L* so, we can eliminate option (a). Finally, *J* or *M* must be with *H*, so we can eliminate option (b).
Hence, option (d) is the answer.

22. The last condition requires that either *J* or *M* or both appear with *H*. *M* is on bench two. If *J* is assigned to bench one and *H* to bench three, then neither *J* nor *M* appears with *H*.

23. *I* cannot be assigned to a bench without *N* so, option (b) violates one of the initial conditions.

Exercise 2

Solution to questions 1–4

1.

1	3	7	5	4	9	0
D	H	N	L	Q	G	T

2. From (i) $ and @ can be replaced by the combination of 2, 4, 6, 8 and 1, 3, 5, 7, 9 respectively.

3. Using (iv)

Q	L	P	↑	R	N	T
4	5	8	0	6	7	0

4. Obviously, the answer is (d): None of these.

Solution to questions 5–7

Day 1: *X*—*AGH*
Day 2: *Z*—*BCE* or *DEF*
Day 3: *Y*—*DF* or *BC*

Solution to questions 9–13

The given information can be analysed as follows:

Qureshi—Mother of Sudhir	(ii) and (iii)
Sudhir—Son of Qureshi	(ii) and (iii)
Rajesh—Wife of Sudhir	(iv)
Pankaj—Son of Rajesh and Sudhir	(i)

Solution to questions 19–23

18. He visits Dr *M* before Dr *N* and Dr *N* before Dr *Q*. Hence, he must visit Dr *M* before Dr *Q*.

19. Out of the six doctors if Dr *S* is first, Dr *P* is third and the order Dr *M*, Dr *N*, Dr *Q* and Dr *M*, Dr *R* is followed. Hence, Dr *M* must be visited second.

20. Since Dr *P* is at the third place and order Dr *M*, Dr *N*, Dr *Q* and Dr *M*, Dr *R* is to be followed, so, immediately after Dr *P* he can visit any doctor except Dr *M* and which may occupy first or second place because Dr *Q*, Dr *R* and Dr *N* cannot precede him/her.

22. According to the given conditions, Dr *P* must be in third place and the order Dr *M*, Dr *N*, Dr *Q* must not be violated.

Solution to questions 23 and 24

23. *A* is *E*'s husband.

24. *E* is the mother of *F* and *C*.

Exercise 3

Solution to questions 1–4

U—Umesh *V*—Vishnu
X—Xinhua *Y*—Yogesh
Z—Zeta
We have

$$U + V + X + Y + Z = 100$$

and each one of $U, V, W, X, Z \leq 10$

$$U = V + X \text{ and } 3Y = 4Z$$

Now,

$$X = 8 + 3 = 11$$

or $X = 27 + 3 = 30$ and

$U = 16, 25, 36, 49$ and

$V = 16, 25, 27, 36$

Observing the values of U, V and X, we can easily get that

$$X(11) + V(25) = U(36)$$

So, $X = 11$, $V = 25$

and $U = 36$

Now, $Y + Z = 100 - (U + V + X)$

$= 100 - (11 + 36 + 25)$

$= 28$

now put $Y = 4/3\ Z$ in the above equation to get $Z = 12$ and then $Y = 16$.

Solution to questions 5–9

```
        A   B   C   D
+   C   A   B   E   B
-----------------------------------
    B   F   D   F   C
```

Start with the last column. We can infer that there has to be a carry over in the previous column so that $C + 1 = B$, or $B - C = 1$.

From the first column, $D + B = C$ or $D + B = C + 10$; $D + B = B - 1$ or $D + B = B + 9$. Hence, D is equal to either -1 or 9. Since -1 is not possible, so $D = 9$. Now there is a carry over to the next column, *i.e.*, $C + E + 1 = F$ or $C + E + 1 = F + 10$.

Moreover, $B + B = D$ is not possible as $B + B = 2B$ which is always an even number. Thus, $B + B + 1 = D$ or $B + B + 1 = D + 10 = 19$, *i.e.*, $B = 4$ or $B = 9$. But B cannot be equal to 9 as $D = 9$. Hence, $B = 4$,

$$C = B - 1, \text{ so, } C = 3$$

$$A + A = \text{F} + 10$$

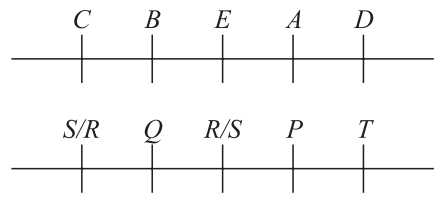
and $C + E + 1 = \text{F} + 10$

A	F	E	
6	2	8	possible
7	4		not possible as $B = 4$
8	6	12	not possible

Finally,

A	***B***	***C***	***D***	***E***	***F***
6	4	3	9	8	2

Solution to questions 8–13



Solution to questions 14–17

Order in Language: *EACDFB* or *EACDBF* or *ACDBFE* or *ACDBEF* or *ACDEBF* or *ACDEFB* or *ACDFEB* or *ACDFBE*

Order in Science: *BCAFDE* or *BCAFED*

Solution to questions 18–22

Table 3.5

	Average	Good	Excellent	Total
Male	18	24	12	54
Female	27	9	0	36
Total	45	33	12	90

Exercise 4

Solution to questions 4 and 5

The possible groups are *XYABK*, *XYBCK*, *XZBCJ* and *YZBCK*.

4. B must be chosen.

5. A and B are chosen, then Z cannot be in the team. So, X and Y will be in the team and therefore so will K. Both I and II only are true.

Solution to questions 6–9

Table 3.6

Villages	Post office	School	Accessibility By bridge	Electricity	Population >5000	Population 2000 to 5000	Population <2000
A	√		√			√	
B	√	√	√	√	√		

Villages	Post office	School	Accessibility By bridge	Electricity	Population		
					>5000	2000 to 5000	<2000
C						√	
D		√	√	√	√		
E		√					√
Total		√	√	2	2	2	1

Solution to questions 10–14

Table 3.7

Lecturer	Sex	Subject	Relative Age
Q	F	**History**	4/5
R	M	**Biology**	1
T	M	**Mathematics**	3
S	F	**Sociology**	6
P	F	**Physics**	2
U	F	**Geography**	5/4

The smaller the number, the lesser the age.

Solution to questions 15–19

Table 3.8

Student	Subject	Author
R	Physics	Gupta
M	Chemistry	Edgar
N	Maths	Khanna
V	English	Harish
S	Biology	D'souza

Solution to questions 20–24

It is given that *A* and *G* sit on the 1st and the 3rd bench respectively.

Now, since *F* is a boy who sits with only *B*, *B* has to be a female. *F* and *B* sit on the 2nd bench.

On the basis of the above information, we can summarize the details as follows:

1st Bench	*A*	—
2nd Bench	*F*(+)	*B*(–)
3rd Bench	*G*	—

(+) Indicates male; (–) indicates female

Now, since *C* (a girl) does not sit with *A*, *E* and *D*, it means that *C* sits on the 3rd bench (on the 2nd bench only two students sit). This by eliminating, *E* and *D* sit on the 1st bench.

Now, using the statement, "*A* sits with his best friend", hence *A* is a male. Again, *E* is a male as he is the brother of *C*. Eliminating the choices, *D* is a female but the sex of *G* is not known.

Exercise 5

Solution to questions 1–4

Let us first make the network of connections:

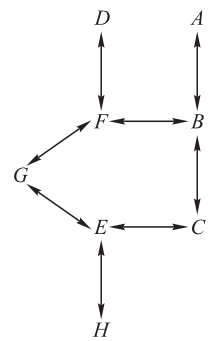


Fig. 3.3

1. Obviously, $B - E$ is not possible.
2. The path with the minimum possible cities in between will be $D - F - G - E$.
3. The route will be $A - B - C - E - F - G$.
4. Making $F - B$ a two way connection will give us the route $A - B - F$. In this case, only one city will need to be crossed.

Solution to questions 8–12

Let us prepare the scenario first:

Seven friends are *P*, *Q*, *R*, *S*, *T*, *V* and *W*. Seven destinations: Delhi, Chennai, Hyderabad, Bangalore, Kolkata, Chandigarh and Patna.

Let us proceed with the following concrete information:

R: Patna—Monday; Bangalore—Saturday

P: Chandigarh; *S*: Kolkata—Friday, *W*: Delhi

With the help of the above information we get the following Table 3.9:

Table 3.9

No.	Person	Destination	Day
1.	*R*	Patna	Monday
2.		Banglore	Saturday
3.	*P*	Chandigarh	
4.	*S*	Kolkata	Friday
5.	*W*	Delhi	

Using the indirect information—Since *Q* did not leave for either Hyderabad or Bangalore, the above table helps us conclude that *Q* left for Chennai.

Now, T left the next day of *P*'s departure and a day previous to *W*'s departure. Using Table 3.9, *P* and W did not depart on Monday, Saturday or Friday. *T* did not depart on Monday or Friday.

Now *P* can't depart on Sunday or Thursday because *T* did not depart on Monday or Friday respectively. *P* can't depart on Wednesday because he did not depart on Friday. So, by elimination, *P* left for Chandigarh on Tuesday.

Let us tabulate the information received till now (Table 3.10):

Table 3.10

No.	Person	Designation	Day
1.	*R*	Patna	Monday
2.		Bangalore	Saturday
3.	*P*	Chandigarh	Tuesday
4.	*S*	Kolkata	Friday
5.	*W*	Delhi	Thursday
6.		Hyderabad	
7.	*Q*	Chennai	

Finally, since *T* left for his destination on Wednesday, it is obvious that *T*'s destination is Hyderabad.

Hence, *V* is going to Bangalore and *Q* left for Chennai on Sunday.

So, the final table will look like (Table 3.11):

Table 3.11

No.	Person	Designation	Day
1.	*R*	Patna	Monday
2.	*V*	Bangalore	Saturday
3.	*P*	Chandigarh	Tuesday
4.	*S*	Kolkata	Friday
5.	*W*	Delhi	Thursday
6.	*T*	Hyderabad	Wednesday
7.	*Q*	Chennai	Sunday

Solution to questions 13–16

Table 3.12

Order	Doctor	Duration of meeting	Gap (in minutes)
1.	O	9 a.m. – 9.30 a.m.	25
2.	S	9.55 a.m. – 10.25 a.m.	25
3.	R	10.50 a.m. – 11.20 p.m.	25
4.	M	11.45 a.m. – 12.15 p.m.	25
5.	Q	12.40 p.m. – 1.10 p.m.	50
6.	P/N	2 p.m. – 2.30 p.m.	25
7.	N/P	2.55 p.m. – 3.25 p.m.	–

Solution to questions 17–21

Let us first find out the scenario—There are three colleges, three disciplines and seven students. Further, we know that three of them are in the Commerce discipline and two each in Arts and Science.

We have been given that *B* and *G* are in Science, *F* in Arts and *D* is not in Commerce. This implies that *D* is in Arts. By elimination, the remaining (*C*, *J* and *H*) are in Commerce.

Thus we get the Table 3.13:

Table 3.13

Discipline	Students
Science	*B* and *G*
Arts	*F* and *D*
Commerce	*C*, *J* and *H*

Now, let us proceed to make a table which correctly matches the colleges and the students/discipline.

It is given that D (a male student) studies in college R only with B. This implies that only two students study in college R and B is a female. (Note that there are at least one boy and one girl in each college).

Further, it is known that B's sister G, and H (a female student) study in the same college. Now, this college can't be P because in that case there will be only a single student (among the given students) in college Q. Hence, H and G study in college Q.

Still we do not know about the sex of J and C. Hence, the above information can be tabulated as follows (Table 3.14):

Table 3.14

College	Students
P	J and C
Q	F (male), H (female) and G (female)
R	D (male) and B (female)

22. 'Father of the brother of my father' is also the father of my father and hence is my (woman's) grandfather.

Solution to questions 24 and 25

Order of novel reading is as follows:

$B - C - E - A - D$.

Exercise 6

Solution to questions 1–4

2. Engineering or architecture or Management.

Solution to questions 5–10

5. We cannot find a definitive answer to this question because the solution give rise to multiple over-lapping.

6. Looking at the elective D, total number of points scored = 4 points and the number of students = 3. Since the range of the points obtained is 1–2, hence the maximum 2 points can be obtained by only one student and remaining two students are getting one mark each.

7. Same as that of Q1. Hence, cannot be determined.

8. To find the minimum number of students with more than 3 points, we should try to accommodate as much students as possible at 3 points each. And after we have accommodated enough students at 3 points each, remaining students will be accommodated at more than 3 points.

In case of elective A, maximum number of students who can get 3 points = 3. Hence, remaining 3 students are getting a total of 12 points. And in no way these 3 students are getting 3 points or less than 3 points to satisfy the conditions given.

In case of elective C, the minimum number of students that can be accommodated at 3 points each = 3. Hence, total points = 9. Now remaining 19 points are to accommodated among 4 students and none of these five students can get 3 points or less than 3 points [Otherwise then 16 points will be required to be accommodated among 3 students and in that case at least one student will get more than 5 points and that is a contradiction].

We can further see that each of these students will get more than 3 points now to satisfy the given conditions.

Hence, minimum number of the students who have scored more than 3 points in at least one elective = 4.

9. Total points obtained by the students of elective $A = 21$

Total points obtained by the students of elective $B = 9$

Total points obtained by the students of elective $A + B = 30$

Total number of students = 9

Hence, average = 30/9

10. In this question, at least one student is going to be overlapped. And without knowing the score of this student (or other overlapped students if any), we cannot find the average score of the elective N. Hence cannot be determined.

Solution to questions 14–18

Following is the structure of the stops on this route:

N	L	P	O	Q	M
Step 1	Step 2	Step 3	Step 4	Step 5	Step 6

or

O	Q	P	N	L	M
Step 1	Step 2	Step 3	Step 4	Step 5	Step 6

Now all the questions can be answered.

Exercise 7

Solution to questions 4–8

We get the following information using the statements given:

Statement 1:

1st Generation	
2nd Generation	
3rd Generation	
4th Generation	XX
5th Generation	
6th Generation	Charu

Statement 2:

Faisal (Female)

↓

Deepak (Male)

Faisal and Deepak will be in consecutive generations.

4. If Atul is the grandson of Binod, then the only possibility for this family is:

1st Generation	Faisal
2nd Generation	Deepak
3rd Generation	Binod
4th Generation	XX
5th Generation	Atul
6th Generation	Charu

Hence, Atul is in the second last generation. Since Atul is the grandson, so Atul is a male.

5. If Binod is the eldest member in the family, then following is the arrangement of the people in this family:

1st Generation	Binod
2nd Generation	Daisal
3rd Generation	Deepak (F)
4th Generation	XX
5th Generation	Atul
6th Generation	Charu

Hence, Deepak is a grandmother of Atul.

6. Using the arrangement given in Q5, we can see that Atul can be mother or father of Charu. Hence, cannot be determined.

8. Grandfather of Charu must have been in the fourth generation = 3rd last generation.

Solution to questions 13–20

13. Total number of selections = $2^3 = 8$
Total selections possible = 8 – 1 = 7

14. (P1, P2), (P1, P2, P3).

15. P1, P3, (P1, P2), (P2, P3).

16. P1, P3, (P1, P3), (P2, P3), (P1, P2, P3).

18. (P1, P2).

19. P1, P3, (P3, P2).

20. None of the selections can be done.

Solution to questions 21–26

Sum of ranks of all the 16 players = 136.

Sum of the ranks of all the universities is same, hence, their sum should be divisible by 5. So, their sum should be either 135, or 130, or 125 or 120. In turn, the rank of the race-track maker will be either 1, or 6, or 11 or 16 respectively.

Since race-track maker comes in between the two athletes of university *B*, hence rank cannot be 1 or 16.

Case 1: If the rank of race-track maker = 6.

Hence, the sum of the ranks obtained of all the universities = 130, and so the sum of ranks of all the athletes of each of the universities = 26.

Since the race track maker comes in between two athletes of university *B*, hence the ranks obtained by two of the athletes of university *B* are 5 and 7. Hence ,the rank of the third athlete of university *B* = 26 – (5 + 7) = 14.

Now the product of ranks of all the athletes of university *B* = Product of ranks of all the athletes of university *C*.

Product of ranks of university *B* contains 72 inter alia, and no other exponent of 7 is possible in the whole series of 1 to 16 for university *C*.

Hence, we can conclude that rank of race-track maker cannot be 6.

Now the only possible rank of race-track maker = 11.

Case 2: If the rank of race-track maker = 11.

Hence, the sum of the ranks obtained of all the universities = 125, and so the sum of ranks of all the athletes of each of the universities = 25.

Since the race track maker comes in between two athletes of university *B*, hence, the ranks obtained by two of the athletes of university *B* are 10 and 12. Hence the rank of the third athlete of university *B* = 25 – (10 + 12) = 3.

Hence, the ranks of athletes of university *B* = 3, 10, 12.

Now the product of ranks of all the athletes of university *B* = Product of ranks of all the athletes of university *C*.

Hence, the ranks of athletes of university *C* = 4, 6, 15.

We can observe that the product and sum of the ranks of athletes of university *B* and *C* are same.

From the remaining ranks now, for the smallest product of the ranks of athletes such that their sum is equal to 25, the ranks should be 1, 8, 16 (University *K*).

Similarly, for the maximum product of the ranks of the athletes for the university *A*, ranks should be 13, 7, 5.

Hence, the ranks for the athletes of the university *L* = 2, 9,14.

A	*B*	*C*	*L*	*K*
13 7 5	3 12 10	4 6 15	2 9 14	1 8 16

MODERATE EXERCISES

EXERCISE 1

Direction for questions 1–4: ***Read the following passage and solve the questions based on it. Some of these questions use Initial instead of Full Name.***

Three men (Tom, Peter and Jack) and three women (Eliza, Anne and Karen) are spending a few months at a hillside. They are to stay in a row of nine cottages, which are facing north; each one living in his or her own cottage. There are no others staying in the same row of houses.

(i) Anne, Tom and Jack do not want to stay in any cottage which is at the end of the row.
(ii) Eliza and Anne are unwilling to stay beside any occupied cottage.
(iii) When Karen, Peter and Jack stand facing north, Karen finds that the cottage of both Peter and Jack are on her left-hand side.
(iv) Between Anne and Jack's cottage there is just one vacant cottage.
(v) None of the girls occupy adjacent cottages.
(vi) The cottage occupied by Tom is next to an end cottage.
(vii) Tom is beside Peter.

1. In how many different ways can their living arrangements be made?
(a) 1 (b) 2
(3) 3 (d) 4

2. Consider the given statements:
Statement 1: The cottages of *P* and *J* are adjacent to each other.
Statement 2: The cottages of *T* and *K* are adjacent to each other.
Statement 3: None of the cottages between *E*'s and *A*'s cottages are occupied.
Statement 4: At least one of the cottages between *K*'s and *T*'s is occupied.
If statement 1 is true, then how many of the statement/s from statement 2 to statement 4 is/are definitely true?
(a) 0 (b) 1
(c) 2 (d) 3

3. In the above question, if statement 1 is true, then how many of the statement/s from statement 2 to statement 4 can be true?
(a) 0 (b) 1
(c) 2 (d) 3

4. In the second question, if statement 4 is true, then how many of the statement/s from statement 2 to statement 4 is/are definitely true?
(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 5–9: ***Read the following passage and solve the questions based on it.***

Amit, Bharat, Chandan, Dinesh, Eeshwar and Ferguson are cousins. None of them are of the same age, but all of them have birthdays on the same date. The youngest of them is 17 years old and Eeshwar, who is the eldest, is 22 years old. Ferguson is somewhere between Bharat and Dinesh in age. Amit is elder to Bharat and Chandan is older than Dinesh.

5. Which of the following is not possible?
(a) Dinesh is 20 years old
(b) Ferguson is 18 years old
(c) Ferguson is 19 years old
(d) Ferguson is 20 years old

6. If Bharat is 17 years old, then which of the following could be the ages of Dinesh and Chandan respectively?
(a) 18 and 19 (b) 19 and 21
(c) 18 and 20 (d) 18 and 21

7. If two of the cousins are between Chandan and Ferguson in age, then which of the following must be true?
(a) Amit is between Ferguson and Dinesh in age
(b) Bharat is 17 years old
(c) Bharat is younger than Dinesh
(d) Ferguson is 18 years old

8. If Amit is one year elder to Chandan, the number of logically possible orders of all six cousins by increasing age is:
(a) 2 (b) 3
(c) 4 (d) 5

9. If Chandan is 19 years old, which of the following must be true?
(a) Amit is 20 years old and Dinesh is 21 years old
(b) Bharat is 18 years old and Amit is 20 years old
(c) Bharat is 20 years old and Amit is 21 years old
(d) Dinesh is 17 years old and Bharat is 21 years old

Direction for questions 10 and 11: ***Read the following passage and solve the questions based on it.***

There are five rooms in my house in Patna—a hall, kitchen, study room, bedroom and dining room. There is one light in each room. The following clever arrangements are planned in the house:

(i) Whenever the kitchen light is 'on', the study room light is 'off'.
(ii) Whenever the dining room light is 'on', the kitchen light is also 'on'.
(iii) Whenever the study room light is 'on', the hall light has to be 'on'.

10. Which one of the following combinations cannot be 'on' at the same time?
(a) dining room and kitchen
(b) hall and bedroom
(c) dining room and study room
(d) kitchen and hall

11. How many lights, at the most, can be 'on' simultaneously?
(a) 5 (b) 4
(c) 3 (d) 2

Direction for questions 12–15: ***Read the following passage and solve the questions based on it.***

Guru Nanak Institute of Management will organize six once-a-month lecture series for young entrepreneurs as per the following schedule, with no dates conflicting for any two different programmes:

(i) Marketing—August through January
(ii) Entrepreneurship—April through October
(iii) Law—January through September
(iv) Finance—March through June
(v) Accounts—October through April
(vi) HR—October through December

12. During which month are the fewest lectures scheduled?
(a) January (b) February
(c) June (d) September

13. What is the maximum number of lectures that can be attended in a single month?
(a) 7 (b) 6
(c) 5 (d) 4

14. How many months of the year must a student attend to hear all the lectures on Marketing, Entrepreneurship and Finance?
(a) 11 (b) 10
(c) 9 (d) 8

15. How many different lectures can be attended during the months of January, February and March?
(a) 11 (b) 10
(c) 8 (d) 6

Direction for questions 16–19: ***Read the following passage and solve the questions based on it.***

Two ants start climbing a slippery wall together, from the bottom of the wall. Ant *A* climbs at the rate of 3 inches per minute. Ant *B* climbs at the rate of 4 inches per minute. However, owing to the fact that the wall is slippery, ant *A* slips back 1 inch for every 2 inches climbed and ant *B* slips back 1.5 inches for every 2 inches climbed. Besides this, ant *A* takes a rest of 1 minute after every 2 minutes and ant *B* takes a rest of 1 minute after every 3 minutes. (Assume that both ant *A* and ant *B* slip continuously while climbing.)

16. At what height on the wall do the two ants meet each other?
(a) Never (b) 3 inches
(c) 5 inches (d) 12 inches

17. If the widest gap achieved between the two ants, within the first 10 minutes, is *N* inches, then find the value of *N*?
(a) 1 inch (b) 3 inches
(c) 2.5 inches (d) 1.5 inches

18. If ant *B* does not have any periods of rest, then how many times do the ants meet in the first 10 minutes?
(a) 0 (b) 1
(c) 2 (d) 3

19. When ant *A* reaches a height of 12 inches on the wall, how far is ant *B* behind ant *A*?
(a) 3.5 inches (b) 2 inches
(c) 2.5 inches (d) 3 inches

Direction for questions 20–22: ***Read the following passage and solve the questions based on it.***

KK, an aspiring entrepreneur wanted to set up a pen drive manufacturing unit. Since technology was changing very fast, he wanted to carefully gauge the demand and the likely profits before investing. Market survey indicated that he would be able to sell 1 lac units before customers shifted to different gadgets. *KK* realized that he had to

incur two kinds of costs – fixed costs (the costs which do not change, irrespective of numbers of units of pen drives produced) and variable costs (= variable cost per unit multiplied by number of units). *KK* expected fixed cost to be ₹ 40 lacs and variable cost to be ₹ 100 per unit. He expected each pen drive to be sold at ₹ 200.

20. What would be the break-even point (defined as no profit, no loss situation) for *KK*'s factory, in term of sales?
(a) ₹ 80 lacs (b) ₹ 100 lacs
(c) ₹ 120 lacs (d) ₹ 140 lacs

21. *KK* was doubtful that per unit variable might increase by 10% though the demand might remain same. What will be the expected changes in profit in such a case?
(a) Profit would decrease by 10.33%
(b) Profit will increase will by 15.75%
(c) Profit would decrease by 15.75%
(d) Profit will decrease by 16.67%

22. He discussed his business with a chartered accountant. *KK* informed that he was contemplating a loan of ₹ 20 lacs at simple interest of 10% per annum for starting the business. The chartered accountant informed him that in such a case *KK* has to pay interest, followed by 30% tax.
By how much does *KK*'s earnings change with 20% growth in sales vis-à-vis the original sales volume, in both cases considering same tax and interest on loan?
(a) 20% (b) 16.7%
(c) 25.6% (d) 34.5%

EXERCISE 2

Direction for questions 1 and 2: *Read the following statement and solve the questions based on them.*

(i) Seema is Raju's grandmother
(ii) Ganesh is Raju's father
(iii) Arun is Seema's sister
(iv) Pratibha is Ganesh's wife
(v) Arun is not Ganesh's aunt

1. What is Seema's daughter Devaki to Raju?
(a) Mother (b) Paternal aunt
(c) Maternal aunt (d) Father's aunt

2. Pratibha is Arun's
(a) Sister (b) Sister-in-law
(c) Maternal uncle (d) Niece

Direction for questions 3–7: *Read the following passage and solve the questions based on it.*

There are six persons in a family, *viz.*, *A*, *B*, *C*, *D*, *E* and *F*—three males and three females, not necessarily in the same order. Out of these, there are two married couples and two persons who are unmarried. Each one of them likes a different newspaper, *viz.*, Times, Express, Hindustan Times, Financial Times, Bharat Times and Business Standard.

E, who likes the Express, is *A*'s mother-in-law and *A* is *C*'s wife. *D* is *F*'s father and he does not like the Times or the Business Standard. *B* likes the Bharat Times and is *F*'s sister, who likes the Hindustan Times. *C* does not read the Business Standard.

3. Who among the following likes the Times?
(a) *C*
(b) *D*
(c) *A*
(d) Cannot be determined

4. How is *F* related to *E*?
(a) Daughter (b) Brother
(c) Son (d) None of these

5. Which of the following is one of the married couples?
(a) *D–B* (b) *D–E*
(c) *B–F* (d) *E–F*

6. Which of the following newspapers is read by *A*?
(a) Times
(c) Bharat Times
(b) Business Standard
(d) Data Inadequate

7. How many sons does *E* have?
(a) Four (b) Three
(c) Two (d) One

Direction for questions 8–11: ***Read the following passage and solve the questions based on it.***

(i) There are five types of cards, *viz.*, *A*, *B*, *C*, *D* and *E*, and three cards of each type. These are to be kept in envelopes of three colours—violet, yellow and black and there are five envelopes of each colour.

(ii) *B*, *D* and *E* type cards are to be kept in the violet envelopes. *A*, *B* and *C* type cards are to be kept in the yellow envelopes; and *C*, *D* and *E* type cards are to be kept in the black envelopes.

(iii) Two cards each of *B* and *D* type are to be kept in the violet envelopes.

8. How many *E* type cards are kept in the black envelopes?
(a) Nil (b) One
(c) Two (d) Three

9. Which of the following combinations of regarding the type of cards and the number of cards is definitely correct with respect to the yellow coloured envelopes?
(a) *A*–2, *B*–1, *C*–2
(b) *B*–1, *C*–2, *D*–2
(c) *A*–2, *E*–1, *D*–2
(d) None of these

10. Which of the following combinations regarding the type of cards, the number of cards and the colour of envelopes is definitely correct?
(a) *C*–2, *D*–1, *E*–2, black
(b) *C*–1, *D*–2, *E*–2, black
(c) *B*–2, *D*–2, *A*–1, violet
(d) None of these

11. Which of the following combinations regarding the colour of envelopes and the number of cards is definitely correct with respect to the *E* type cards?
(a) Violet—2, black—1
(b) Violet—1, yellow—2
(c) Violet—2, yellow—1
(d) None of these

Direction for questions 12–15: ***Read the following passage and solve the questions based on it.***

(i) There are six picture cards—*A*, *B*, *C*, *D*, *E* and *F*, each bearing the picture of a king, a priest, a queen, a palace, a joker and a prince. These are printed in six different coloured inks—blue, red, green, grey, yellow and black and are arranged from left to right (not necessarily in the same order and colour).

(ii) The picture of the palace was in blue colour but it was not printed on card *D*.

(iii) Card *A*, which had the queen's picture printed in black ink, was at the extreme right.

(iv) The picture of the priest was neither on card *D* nor on card *E* and was not printed in either green or yellow coloured ink. Card *C* had a picture of the king on it in grey coloured ink. It was fifth from the right and next to card *B* which had the picture of the prince.

12. If the priest's card is between the cards of the palace and the prince, then at what number from the left is the joker's card placed?
(a) First (b) Fourth
(c) Fifth (d) Second

13. Which of the following combinations of card and colour is true for the picture of the priest?
(a) *E*—yellow
(b) *F*—red
(c) *B*—green
(d) Data inadequate

14. In which colour was the picture of the joker printed?
(a) Yellow (b) Red
(c) Green (d) Data inadequate

15. The picture of the palace was printed on which of the following cards?
(a) *E* (b) *F*
(c) *D* (d) Either *D* or *E*

Direction for questions 16–18: ***Read the following passage and solve the questions based on it.***

To smoothen the process of issuing of passports, MEA has devised a 'Single office—Three windows' system. Under this system, the windows are named W1, W2 and W3. To ease the pressure upon the employees, the windows operate at different time slots on different days.

(i) W1 is open between 10 a.m. and 2 p.m. on Monday, Wednesday and Thursday and between 2.30 p.m. and 4.30 p.m. on Tuesday, Friday and Saturday.

(ii) W2 is open between 9 a.m. and 12 noon on Tuesday, Thursday and Friday and between 1 p.m. and 3 p.m. on Monday, Wednesday and Saturday.

(iii) W3 is open between 8 a.m. and 11 a.m. on Monday, Tuesday, Thursday and Friday and between 1 p.m. and 3 p.m. on Monday, Wednesday and Saturday.

16. On which of the following days are all the three windows simultaneously open any time before noon?
(a) Saturday (b) Thursday
(c) Wednesday (d) Monday

17. On which day not more than one window is open simultaneously at any given time?
(a) Tuesday (b) Wednesday
(c) Friday (d) Saturday

18. On which day's afternoon shots are W1 and W2 open simultaneously for some time?
(a) Saturday (b) Thursday
(c) Monday (d) Tuesday

Direction for questions 19–23: ***Read the following passage and solve the questions based on it.***

(i) Two wooden cubes *A* and *B* are placed adjacent to each other in front of you in such a way that *A* is to your left and *B* is to your right.

(ii) One pair of the opposite faces of cube *A* is painted the same colour, *i.e.*, red colour. Another pair of opposite faces is painted blue and the other two remaining pairs of faces are painted Yellow and violet.

(iii) Only two opposite faces of cube *B* are painted blue in colour. The remaining pairs of opposite faces are painted in such a way that brown opposite to is green and one of the other two opposite faces is black and the other is white.

19. If the red surface of cube *A* and the blue surface of cube *B* are touching the table; and the yellow surface of cube *A* and the black surface of cube *B* are facing you; then which coloured side of cube *B* is facing the blue side of cube *A*?
(a) Brown
(b) Green
(c) White
(d) Either brown or green

20. If the black surface of cube *B* is kept on top of the red surface of cube *A*, which coloured side of cube *B* will face the sky?
(a) White (b) Blue
(c) Brown (d) Data inadequate

21. If the cubes are re-arranged one above the other in such a way that the white face of cube *B* is facing the sky and the yellow face of cube *A* is kept above it; then which coloured surface of cube *A* will be facing you?
(a) Violet
(b) Blue
(c) Either blue or red
(d) Either blue or violet

22. If cube *B* is kept to your left with the green coloured surface facing you and cube *A* kept to your right with the blue surface facing you; then which of the following pairs of colours of cube *A* and cube *B* will be facing each other?
(a) Yellow-black
(b) Yellow-white
(c) Black-violet
(d) Data inadequate

23. If cube *B* is kept behind cube *A* in such a way that the brown coloured surface of cube *B* is facing the yellow coloured face of cube *A* then which colour of cube *B* will be to your right?
(a) Blue
(b) Black
(c) Brown
(d) Data inadequate

Direction for questions 24 and 25: ***Read the following passage and solve the questions based on it.***

There were eight friends, *viz.*, *P*, *W*, *T*, *S*, *U*, *Q*, *V* and *R*—attending a New Year party. However, not all of them reached at the same time and not all of them guzzled the same quantity of beer at the party. Following is the data pertaining to their arrival and the drinks consumed by them at the party:

(i) *T* drank more than *P* but less than *V* and left later than *P* but earlier than *S*.
(ii) *P* drank more than *W* at the party and left later than *V*.
(iii) *U* drank less than *Q* but more than *V* and left later than *S*.
(iv) *W* drank more than *R* and left earlier than *V*.

24. Who left the earliest?
(a) *P* (b) *S*
(c) *T* (d) *W*

25. Who among the following drank the least?
(a) *P* (b) *R*
(c) *T* (d) *U*

EXERCISE 3

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

Six people—*P*, *Q*, *R*, *S*, *T* and *U* are *X*'s husband, sister, son, daughter, mother and aunt, though not necessarily in that order. They satisfy the following conditions:

(i) *R* and *S* are either both males or both females.
(ii) Either *P* or *Q*, or both are male.
(iii) Either *T* or *U*, or both are female.

1. Any of the following could be *X*'s son except:
(a) *P* (b) *Q*
(c) *S* (d) *T*

2. Which of the following statements must be false?
(a) *P* is *S*'s grandson
(b) *S* is *P*'s niece
(c) *P* is *Q*'s nephew
(d) *U* is *T*'s son

3. If *Q* is *U*'s daughter, then which of the following statements could be true?
(a) *P* is *X*'s sister (b) *U* is *R*'s aunt
(c) *T* is *S*'s nephew (d) *Q* is *S*'s niece

4. If *P* is *T*'s sister, then *U* could be any of the following except:
(a) *X*'s sister (b) *X*'s daughter
(c) *X*'s aunt (d) *X*'s husband

Direction for questions 5–9: ***Read the following passage and solve the questions based on it.***

A business school with six professors, *viz.*, *L*, *M*, *N*, *O*, *P* and *Q* has decided to implement a new scheme of course management. Each professor has to coordinate one course and support another course. This semester, *O*'s support course is Finance, while three other professors are its coordinator's. *P* and *Q* have Marketing as one of their subjects. *Q* coordinates Operations, which is a support course for both *N* and *P*. Finance and IT are *L*'s subjects. Both *L* and *O* have the same subjects. Strategy is a support course for only one of the professors.

5. Who coordinates the Strategy course?
(a) *M*
(b) *N*
(c) *O*
(d) None of the professors

6. Which course is supported by *M*?
(a) Finance (b) Strategy
(c) IT (d) Operations

7. Who coordinates the IT course?
(a) *L*
(b) *N*
(c) *O*
(d) None of the professors

8. Who all are coordinating the Finance course?
(a) *L*, *M* and *N* (b) *M*, *N* and *O*
(c) *N* and *O* (d) *L* and *N*

9. Which course has only one coordinator and only one support professor?
(a) Marketing (b) Operations
(c) Finance (d) Strategy

Direction for questions 10–12: ***Read the following passage and solve the questions based on it.***

During the Independence day celebrations at AR Academy last year, six different items, *viz.*, drama, singing, mimicry, speech, story-telling and dance, were performed by six children, *viz.*, Abha, Binita, Sophiya, Devika, Esha and Ferguson, not necessarily in the same order. The programme began with the song which was not sung by Binita and ended with the dance item. Sophiya performed the mimicry item immediately after the speech. Esha performed the drama just before the dance sequence. Devika or Ferguson were not available for the last performance. The speech was not given by Abha and an interval of 30 minutes was given immediately after the mimicry item with three more items remaining to be performed. Devika performed immediately after the interval.

10. Which item was performed by Ferguson?
(a) Drama (b) Song
(c) Speech (d) None of these

11. Who performed the dance item?
(a) Abha
(b) Binita
(c) Ferguson
(d) Cannot be determined

12. Who was the first perfomer?
(a) Abha
(b) Binita
(c) Sophiya
(d) Cannot be determined

Direction for questions 13–15: ***Read the following passage and solve the questions based on it.***

Next year in the Auto-Expo at Pragati Maidan, six cars, namely Uno, Verna, Wind, X-tra, Yale and Zen, are to be displayed. There will be six display stalls numbered 1, 2, 3, 4, 5 and 6 from the left to the right in such a way that only one car is showcased in each stall. However, the car Uno cannot be placed adjacent to the Verna and the Car Wind must be showcased to the left of X-tra. The Zen cannot be displayed in stall number 6.

Note: The direction (left or right) should be determined with respect to the observer/shopper.

13. Which of the following cars cannot be placed in stall number 1?
(a) Uno (b) Verna
(c) Wind (d) X-tra

14. If the X-tra is placed in stall number 3, in which stall can the Wind be placed?
(a) 1 (b) 2
(c) 4 (d) 5

15. If the Uno is placed in stall number 5, which of the following cars must be placed in stall number 6?
(a) Verna (b) Wind
(c) Yale (d) X-tra

Direction for questions 16 and 17: ***Read the following passage and solve the questions based on it.***

(i) Six men, *viz.*, Rajesh, Dinesh, Lokesh, Nilesh, Shailesh and Himesh work for different companies, namely *P*, *Q*, *R*, *S*, *T* and *U*. Each one wears a different coloured company-sponsored tie, *i.e.*, brown, green, pink, yellow, purple and red, though not necessarily in the same order.
(ii) The one wearing the brown tie works for the company *S* and the one wearing the green tie works for the company *P*.

(iii) Himesh does not work for either *R* nor *T*.
(iv) Rajesh wears a pink tie and works for the company *Q*.
(v) Nilesh does not work for the company *T* and a purple coloured tie is not given by the company *R*.
(vi) Shailesh works for the company *U* and neither Nilesh nor Dinesh work for the company *S*.
(vii) The company *T* does not sponsor a purple or a yellow coloured tie and Lokesh works for the company *P*.

16. Which of the following is true?
(a) The company *U* sponsors a green coloured tie
(b) Shailesh wears a red coloured tie
(c) Nilesh works for the company *T*
(d) The red coloured tie is sponsored by the company *T*

17. Which of the following sequence of companies represent Rajesh, Dinesh, Lokesh, Nilesh, Shailesh and Himesh in the same order?
(a) *Q*, *P*, *T*, *R*, *U*, *S* (b) *Q*, *T*, *P*, *R*, *U*, *S*
(c) *Q*, *P*, *T*, *S*, *U*, *R* (d) *Q*, *T*, *S*, *U*, *R*, *P*

Direction for questions 18–21: ***Read the following passage and solve the questions based on it.***

(i) The length, breadth and height of a rectangular piece of wood are 4 cm, 3 cm and 5 cm respectively.
(ii) Opposite sides of the 5 cm × 4 cm piece are coloured red.
(iii) Opposite sides of the 4 cm × 3 cm piece are coloured blue.
(iv) Rest of the sides of size 5 cm × 3 cm are coloured green on both sides.
(v) Now the rectangular piece is cut in such a way so that cubes of size 1 cm × 1 cm × 1 cm can be made.

18. How many cubes will have all the three colours?
(a) 8 (b) 10
(c) 12 (d) 14

19. How many cubes will not have any colour?
(a) 0 (b) 2
(c) 4 (d) 6

20. How many cubes will have only two colours (red and green) on their two sides?
(a) 8 (b) 12
(c) 16 (d) 20

21. How many cubes will have only one colour?
(a) 8 (b) 12
(c) 16 (d) 22

Direction for questions 22–26: ***Read the following passage and solve the questions based on it.***

Out of a stock of five essences, *viz.*, *L*, *M*, *N*, *O* and *P*, two or more essences are used by a manufacturer in making all the perfumes. He has learned that for a blend of essences to be agreeable, they should comply with all the rules listed below:
(i) A perfume containing the essence *L* should also contain the essence *N* and the quantity of *N* should be twice as that of *L*.
(ii) A perfume containing the essence *M* must also have the essence *O* as one of its components and they should both be in equal proportions.
(iii) No perfume should contain the essence *N* as well as the essence *O*.
(iv) The essence *O* and *P* the essence should not be used together.
(v) A perfume containing the essence *P* should contain it in such a proportion that the total amount of the essence *P* present should be greater than the total amount of the other essence or essences used.

22. Which is an agreeable formula for a perfume from among the following?
(a) one part *L*, one part *P*
(b) two parts *M*, two parts *L*
(c) three parts *N*, three parts *L*
(d) four parts *O*, four parts *M*

23. Adding more amounts of the essence *N*, which of the following perfumes will become agreeable?

(a) one part *L*, one part *N*, five parts *P*
(b) two parts *M*, two parts *N*, two parts *P*
(c) one part *M*, one part *N*, one part *P*
(d) two parts *M*, one part *N*, four parts *P*

24. The addition of which combination among the following would make an unagreeable perfume containing two parts *N* and one part *P* agreeable?
(a) one part *L* (b) one part *M*
(c) two parts *N* (d) two parts *P*

25. Which among the following combinations cannot be used together in an agreeable perfume containing two or more essences?
(a) *L* and *M* (b) *L* and *N*
(c) *L* and *P* (d) *P* and *N*

26. Among the below mentioned formulae, which can be made agreeable by eliminating some or all of one essence?
(a) one part *L*, one part *M*, one part *N*, four parts *P*
(b) one part *L*, two parts *N*, one part *O*, four parts *P*
(c) one part *L*, one part *M*, one part *O*, one part *P*
(d) two parts *M*, one part *N*, two parts *O*, three parts *P*

EXERCISE 4

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

A total of six teams participated in the PHL held at Chennai last year. The schedule of the tournament was such that each team had to play against every other team six times. Points were awarded for wins (*W*), losses (*L*) and draws (*D*) Table 3.15 gives the details of the various teams:

Table 3.15

	Sikkim	Goa	Gujarat	Bihar	MP	Chandigarh
W	13	7	12	*X*	10	9
L	15	18	15	*Y*	16	15
D	2	5	3	*Z*	4	6

1. How many *W*, *L* and *D* combinations were possible for the Bihar team?
(a) 1 (b) 2
(c) 3 (d) None of these

2. If we gave 5 points for a win, –3 points for a loss and 2 points for a draw, then what could be the maximum possible difference between the points of two teams?
(a) 123 (b) 153
(c) 163 (d) None of these

3. If Gujarat lost all the matches to only three teams with a point distribution of 5, 5 and 5 and Bihar scored the first rank with the maximum possible wins; then which team did Bihar lose its match against?
(a) Sikkim
(b) Gujarat
(c) MP
(d) Cannot be determined

4. The team which got the maximum number of points won the PHL. Which team was that?
(a) Sikkim
(b) Gujarat
(c) Bihar
(d) Cannot be determined

Direction for questions 5–9: ***Read the following passage and solve the questions based on it.***

P, *Q*, *R*, *S*, *T*, *U* and *V* are sitting in a circle facing the centre. Following are the details of their order of sitting:

(i) *S*, who is second to the right hand side of *R*, is not to the immediate right of *V*.
(ii) *U* is not between *V* and *T*.
(iii) *P* is between *R* and *Q*.

5. Which of the following statement is wrong?
(i) *T* is to the immediate left of *R*.
(ii) *Q* is to the immediate left of *U*.
(iii) *U*, *S* and *T* are in a sequence, one after the other.
(a) Only (i) (b) Only (ii)
(c) Only (iii) (d) (i) and (ii)

6. Which of the following are the two pairs of adjacent members?
(a) *VS* and *TR* (b) *SU* and *PQ*
(c) *PR* and *TQ* (d) None of these

7. What is the position of *T*?
(a) To the immediate left of *R*
(b) Second to the left of *P*
(c) Fourth to the left of *U*
(d) None of these

8. Which of the following statement is correct?
(i) *V* is third to the left of *R*
(ii) *U* is between *S* and *V*
(iii) *Q* is to the immediate left of *P*
(a) Only (i) (b) Only (ii)
(c) Only (iii) (d) (ii) and (iii)

9. If *Q* and *R* interchange places so as to be like *T* and *V*, then which of the following is true?
(a) *S* is third to the right of *R*
(b) *T* is second to the left of *R*
(c) *Q* is fourth to the right of *T*
(d) None of these

Direction for questions 10 and 11: ***Read the following passage and solve the questions based on it.***

(i) Five boys are standing in a line facing the wall. Each boy is either wearing red, green, yellow, white or blue dress.
(ii) The boy dressed in yellow is not standing at any end of the line.
(iii) The boy dressed in red is not standing at any end of the line.

10. What colour dress is the boy in the middle wearing?
(a) Green
(b) Blue
(c) Red
(d) Cannot be determined

11. What colour dress has been worn by the boy who is standing to the right side of the boy dressed in yellow?
(a) White
(b) Green
(c) Blue
(d) Cannot be determined

Direction for question 12–17: ***Read the following passage and solve the questions based on it.***

There are five specializations—Marketing, Finance, Production, Personnel and Systems—available at the IIM. It is necessary for all students to take up a combination of at least two of the above specializations.

Based on common preferences, pre-set combinations called modules are offered, which are coded S001, S002, S003, S004 and S005. Each of these modules has two specializations, so that every specialization is assigned to two modules. Further, to allow for uncommon preferences a student is allowed to take up more than one combination in such a way that the specialization common to the two modules is dropped.

However, a student can specialize in more than two subjects by manipulating the combination of modules.

Additional information is as follows:

S001 has not been assigned to Production. S003 has not been assigned to Marketing. S004 has not been assigned to Finance. By choosing S001 and S003, one may specialize in Marketing and Systems. By choosing S001 and S002 one may specialize in Marketing, Production, Finance and Personnel. By choosing S001 and S004 one may specialize in Finance and Personnel.

12. What specializations are possible by choosing S001, S002 and S003?
(a) Marketing, Finance and Personnel
(b) Marketing, Production and Personnel
(c) Finance, Personnel and Production
(d) Marketing, Production, Personnel and Systems

13. Which of the following number of modules give the maximum number of specializations?

(a) S003 and S004
(b) S002 and S004
(c) S002 and S003
(d) S004 and S005

14. What is the maximum number of modules that a student can take to specialize in at least two areas?
(a) 2 (b) 3
(c) 4 (d) 5

15. Which of the following number of specializations is impossible for any student to take?
(a) 2 (b) 3
(c) 4 (d) None of these

16. Personnel is available in modules:
(a) S002 and S003 (b) S001 and S003
(c) S002 and S004 (d) S003 and S005

17. Finance is available in modules:
(a) S001 and S003 (b) S003 and S004
(c) S002 and S003 (d) S001 and S004

Direction for questions 18–22: ***Read the following passage and solve the questions based on it.***

For a motorist, there are three ways of going from city *A* to city *C* by a 20 km long, toll highway the toll for which is ₹ 25 per entry; a 10 km long tunnel between the two cities, the toll for which is ₹ 40 per entry; and a two-lane, toll-free highway which goes 30 km east to city *B* and then 20 km north-west to city *C*.

The running cost of the motorist = ₹ 1/km.

18. Which of the following is the costliest way of going from city *A* to city *C*?
(a) Tunnel
(b) Toll highway
(c) Toll-free highway
(d) Tunnel or Toll-free highway

19. To connect the cities better, a master plan has been devised. According to this plan, a new toll-highway is to be built from the mid point of the highway connecting cities *A* and *B* to city *C*. The toll for this new highway is ₹ 30 per entry. Which of the following is the cheapest route between city *A* to city *C*?
(a) Tunnel
(b) Toll highway
(c) Toll-free highway
(d) New proposed highway

20. Due to CNG, the cost of the motorist has reduced to ₹ 0.5/km. Which of the following is the cheapest way to go from city *A* to city *C*?
(a) Tunnel
(b) Toll highway
(c) Toll-free highway
(d) Tunnel or Toll-free highway

21. If we combine the data of question 2 and question 3, then which of the following is the cheapest way to go to city *C* from city *A*?
(a) Tunnel
(b) Toll highway
(c) Toll-free highway
(d) New proposed highway

22. According to a new system, motorists are supposed to pay a bribe at the starting of their journey. This bribe has to be a one-time bribe only and no other bribe is to be paid during the earlier journey. What should be the bribe amount at the toll highway such that the total expenses on the toll highway become equal to the total expenses on the toll-free highway?
(a) ₹ 5 (b) ₹ 10
(c) ₹ 15 (d) None of these

Direction for questions 23–25: ***Read the following passage and solve the questions based on it.***

Two out of Anil, Balraj and Chandan are fighting each other. The given statements are:

(i) The shorter one out of Anil and Balraj is the older of the two fighters.
(ii) The younger one out of Balraj and Chandan is the shorter of the two fighters.

(iii) The taller one out of Anil and Chandan is the younger of the two fighters.

23. Who is not fighting?
(a) Anil
(b) Balraj
(c) Chandan
(d) Data inconsistent

24. Who is the tallest?
(a) Anil
(b) Balraj
(c) Chandan
(d) Data inconsistent

25. Who is the youngest?
(a) Anil
(b) Balraj
(c) Chandan
(d) Data inconsistent

EXERCISE 5

Direction for questions 1–4: ***Read the following passage and solve the questions based on it.***

A chess tournament is taking place at the college club and the players on all the four tables are engaged in their fourth game against their respective opponents. The players with the white pieces are: Sharukh, Sanjay, Saif and Shakti. The players with the black pieces are: Salman, Sunny, Sunil and Sohail. The scores are 3:0, 2.5:0.5, 2:1 and 1.5:1.5 (**Note:** Tied games result in a score of 0.5 for each player).

(i) The player using the white pieces at table 4 is Shakti; however, the current score at the table is not 2:1.
(ii) Saif is playing at the table on the right hand side of Sohail, who has lost all his games uptil now.
(iii) Sunil, who is not in the lead against his opponent, has not been in a tied game.
(iv) Salman is leading his match after his last three games.
(v) Sanjay is playing against Sunny.
(One win gets point for the winner whereas a player gets no point for losing the game).

1. What table is Sohail playing at, and what is the score at that table?
(a) Table 1:2.5–1.5 (b) Table 2:3–0
(c) Table 2:2.5–1.5 (d) Table 3:2–1

2. Which player has the higher score?
(a) Salman (b) Saif
(c) Sunny (d) Sunil

3. Which player had the black pieces alongwith and the lowest score?
(a) Salman (b) Sunny
(c) Sunil (d) Sohail

4. Who is the winning player at table 4?
(a) Salman (b) Shakti
(c) Sharukh (d) Sanjay

Direction for questions 5 and 6: ***Read the following passage and solve the questions based on it.***

If the sum of the rows, columns and the diagonals are equal in Table 3.16, then:

Table 3.16

y	x	$y-2$
$y-1$	$y+1$	—
—	—	—

5. If $x = 10$, then what is the value of y?
(a) 5 (b) 10
(c) 6 (d) 15

6. If the sum of any of the rows, columns or diagonals is 21, then what is the value of x?
(a) 5
(b) 11
(c) 15
(d) Cannot be determined

Direction for questions 7 and 8: ***Read the following passage and solve the questions based on it.***

In a multiplex, nine shops are connected by corridors. Anyone visiting these shops must begin at the reception which is a part of shop number 1.

From there, the other shops may be visited via the different corridors connecting them. The details of the corridors connecting the various shops are as under:

(i) Corridors connect the reception to shops number 2, 3 and 6.
(ii) A corridor connects the reception to shop number 2 and then to shop number 4.
(iii) Corridors connect the shops number 2 and 3 to shop number 7.
(iv) Shop number 6 is connected to shops number 1, 5 and 9.
(v) Shop number 7 is connected to shop number 9.
(vi) Shops number 7 and 9 are connected to shop number 8.

7. Without visiting any shop more than once, what is the maximum number of shops that a person can visit?
(a) 5 (b) 6
(c) 7 (d) 8

8. If a visitor wants to travel to shops number 4 and 5, which one of the following options must be False?
(a) The visitor will go to every shop except shop 3
(b) The visitor will go to shop 2 only once
(c) The visitor will not visit any shop more than once
(d) The visitor will visit at least six different shops

Direction for questions 9–11: ***Read the following passage and solve the questions based on it.***

(Any additional information provided with a particular question pertains to that individual question only.)

The placement office of a management school has to schedule seven companies, *viz.*, *P*, *Q*, *R*, *S*, *T*, *U* and *V* for the placement week (the week starts on a Sunday) but not necessarily in that order. This group of seven companies is divided into three categories, *viz.*, finance, manufacturing and sales. There are three companies in the final category and two each in one manufacturing and sales categories. The placement schedule is such that the same category companies have to be scheduled consecutively. It has also been decided that:

(i) Company *S*, which is a manufacturing company, cannot be scheduled on the sixth day of the week.
(ii) Companies *Q* and *U* are scheduled consecutively but not necessarily in that order.
(iii) Company *T* can be scheduled on any day of the week except on a Saturday.
(iv) Company *R*, which is in the same category as the companies *Q* and *U*, has to be scheduled on the third day of the week.

9. If *P* is scheduled on Wednesday, which one of the following options has to be True?
(a) *U* has to be scheduled on Thursday
(b) *V* has to be scheduled either on Friday or on Saturday
(c) *T* has to be scheduled on either Thursday or Friday
(d) *S* is always scheduled before *Q*

10. If *S* is scheduled on Monday, which one of the following options is definitely False?
(a) *P* and *T* are sales companies
(b) The maximum number of days in between the schedules of *T* and *U* is three
(c) *R* is always scheduled before *Q*
(d) There are three schedules for *P* being slotted on a Sunday

11. If *T* is a sales company and *S* is scheduled for Saturday, then which of the following schedules are possible?

Schedule I	*U–Q–R–T–P–V–S*
Schedule II	*P–T–R–U–Q–V–S*
Schedule III	*T–V–R–Q–U–P–S*
Schedule IV	*Q–U–R–P–V–T–S*

(a) I, III and IV
(b) II, III and IV
(c) I, II and IV
(d) I, II and III

Direction for question 12: ***Read the following passage and solve the questions based on it.***

There are five boxes—*T*, *U*, *W*, *X* and *Z*—that are to be delivered on five consecutive days, Monday through Friday, one box per day. The following conditions are to be kept in mind while formulating the delivery schedule of the boxes:

(i) Box *X* is not to be delivered on Monday.
(ii) If box *T* is delivered on Monday, then box *X* must be delivered on Friday.
(iii) If box *X* is delivered on Tuesday, box *U* is delivered on Monday.
(iv) Box *W* is delivered the next day following the delivery of box *Z*.

12. If box *W* is delivered on Friday, which of the following must be false?
(a) Box *Z* is delivered on Thursday
(b) Box *X* is delivered on Wednesday
(c) Box *T* is delivered on Tuesday
(d) Box *U* is delivered on Tuesday

Direction for questions 13–17: ***Read the following passage and solve the questions based on it.***

Tetraicosa is a game which is played by rolling six dices simultaneously. While playing the game—

If rolling six dices were rolled, then the following was observed:

(i) Three of the dices showed the same number. The rest showed different numbers.
(ii) Only one dice showed 6.
(iii) Not more than three dice showed 4 or more.

13. Find the minimum possible total of numbers on the faces of all the six dice if three dice show the same number 2.
(a) 14 (b) 21
(c) 18 (d) None of these

14. Find the maximum total if four of the dice show less than 4.
(a) 29 (b) 32
(c) 22 (d) None of these

15. If the third observation is waived, then what would be the maximum total if three dice were faulty and had only 5 on all the faces?
(a) 31 (b) 28
(c) 34 (d) None of these

16. If only one dice shows 1, what is the maximum number of dice with numbers greater than 4?
(a) 3 (b) 1
(c) 2 (d) 4

17. What is the maximum number that can be on the faces of the three dices which show the same number?
(a) 2 (b) 4
(c) 3 (d) 5

Direction for questions 18–21: ***Read the following passage and solve the questions based on it.***

Seven persons *A*, *B*, *C*, *D*, *E*, *F* and *G* contested in a game show that had a total prize money of ₹ 14 lakhs. Every contestant won some prize money and the highest prize money was ₹ 3.5 lakhs. No two contestants won the same amount of prize money. For every person the difference with the next highest and the next lowest contestant was the same.

(i) *E* won ₹ 2 lakhs.
(ii) *B* won more money than *A*.
(iii) The difference in prize money between *B* and *A* was the least.
(iv) The difference in prize money between *D* and *F* was not the least.
(v) There was at least one person whose prize money was between that of *E* and *G*.

18. Which of the following is a proper list of persons in an increasing order of prize money won?
(a) *G*, *C*, *F*, *B*, *E*, *D*, *A*
(b) *D*, *F*, *C*, *E*, *A*, *B*, *G*
(c) *F*, *C*, *D*, *E*, *A*, *B*, *G*
(d) *A*, *B*, *G*, *C*, *F*, *E*, *D*

19. If *D* won more than *E*, and *B* and *G* together won ₹ 3.5 lakhs, which of the following must be true?
(a) *D* won ₹ 3.5 lakhs
(b) *A* won ₹ 1.5 lakhs
(c) *B* won ₹ 1.5 lakhs
(d) *C* won ₹ 50,000

20. If the difference in prize money between *A* and *C* is the least; which of the following pairs must not have won prize money that differs by the minimal amount?
(a) *B* and *E* (b) *C* and *G*
(c) *D* and *G* (d) *A* and *E*

21. If the total money won by *A* and *D* is equal to that of *G* and the difference between *E* and *D* is at least 1 lakh, then which of the following must be True?
(a) *A* and *B* together won ₹ 3 lakhs
(b) *B* and *F* together won ₹ 3.5 lakhs
(c) *C* and *E* together won ₹ 3 lakhs
(d) *B* and *C* together won ₹ 3.5 lakhs

Direction for questions 22–25: ***Read the following passage and solve the questions based on it.***

Abhishek, Bhushan, Chandan, Dayal, Eklavya and Franklin are friends married to Rekha, Stuti, Urmila, Varsha, Trupti and Wanda, not necessarily in the same order.

Following facts are also given:
(i) Rekha and Stuti are Abhishek's sisters.
(ii) Neither Rekha nor Trupti is Chandan's wife.
(iii) Wanda is Eklavya's wife and Varsha is Bhushan's wife.
(iv) Dayal is not married to Rekha, Stuti or Trupti.

Nobody can marry his sister and vice-versa. Each person mentioned above practices monogamy.

22. Who is Abhishek's wife?
(a) Rekha
(b) Urmila
(c) Trupti
(d) Cannot be determined

23. Who is Rekha's husband?
(a) Chandan (b) Franklin
(c) Bhushan (d) Dayal

24. Who is Dayal married to?
(a) Urmila (b) Varsha
(c) Trupti (d) Rekha

25. Who is Stuti married to?
(a) Abhishek (b) Dayal
(c) Franklin (d) Chandan

ANSWER KEY

Exercise 1

1. (a)	**2.** (c)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (a)
9. (c)	**10.** (c)	**11.** (b)	**12.** (b)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (b)
17. (b)	**18.** (b)	**19.** (d)	**20.** (a)	**21.** (d)	**22.** (d)		

Exercise 2

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (c)
9. (d)	**10.** (a)	**11.** (d)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (b)
17. (c)	**18.** (c)	**19.** (d)	**20.** (a)	**21.** (c)	**22.** (d)	**23.** (d)	**24.** (d)
25. (b)							

Exercise 3

1. (a)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (a)
9. (a)	**10.** (d)	**11.** (d)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (d)
17. (a)	**18.** (a)	**19.** (d)	**20.** (b)	**21.** (d)	**22.** (d)	**23.** (a)	**24.** (d)
25. (a)	**26.** (b)						

Exercise 4

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (d)	**6.** (d)	**7.** (d)	**8.** (c)
9. (d)	**10.** (d)	**11.** (d)	**12.** (d)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (c)
17. (a)	**18.** (d)	**19.** (b)	**20.** (c)	**21.** (c)	**22.** (a)	**23.** (c)	**24.** (a)
25. (a)							

Exercise 5

1. (b)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (c)
9. (c)	**10.** (d)	**11.** (d)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (c)
17. (c)	**18.** (c)	**19.** (a)	**20.** (d)	**21.** (b)	**22.** (c)	**23.** (b)	**24.** (a)
25. (d)							

HINTS AND EXPLANATIONS

Exercise 1

Solution to questions 5–9

The given information can be sequenced as:

Eeshwar is the eldest;

Amit > Bharat;

Chandan > Dinesh and

Dinesh < Ferguson < Bharat or

Bharat < Ferguson < Dinesh

Now, the following possible arrangement can be attained:

	22	21	20	19	18	17
1.	Eeshwar	Amit	Bharat	Ferguson	Chandan	Dinesh
2.	Eeshwar	Amit	Bharat	Chandan	Ferguson	Dinesh
3.	Eeshwar	Amit	Chandan	Bharat	Ferguson	Dinesh
4.	Eeshwar	Chandan	Amit	Bharat	Ferguson	Dinesh
5.	Eeshwar	Chandan	Dinesh	Ferguson	Amit	Bharat
6.	Eeshwar	Chandan	Dinesh	Amit	Ferguson	Bharat
7.	Eeshwar	Chandan	Amit	Dinesh	Ferguson	Bharat
8.	Eeshwar	Amit	Chandan	Dinesh	Ferguson	Bharat

5. From the above arrangement, we see that Dinesh can be 20 years of age [5, 6], Ferguson can be 18 years old [2, 3, 4] and Ferguson can also be 19 years old [1, 5]. However, Ferguson can never be 20 years old.

6. If Bharat is 17 years old, the possible arrangements are 5, 6, 7 and 8.
From the options, we can see that the only possible answer is (b).

7. There are two cousins between Chandan and Ferguson in age reference [4, 6 and 7]. In all the cases, we have Ferguson's age as 18 years.

8. Amit is one year older than Chandan in only two arrangements 3 and 8.

9. If Chandan is 19 years old, the only possible arrangement is 2 Clearly, Amit is 21 years old and Bharat is 20 years old.

Solution to questions 10 and 11

10. Let us go through the various options to:

Option (a): The dinning room and kitchen lights can certainly be 'on' at the same time [reference (ii)].

Option (b): There is nothing given in the data to indicate that the hall and bedroom lights cannot be 'on' at the same time.

Option (c): Whenever the dining room light is 'on', the kitchen light is 'on' reference [(ii)] and whenever the latter is 'on', the study room is 'off' [reference (i)]. Hence, the dining room and the study room lights cannot be 'on' at the same time. Hence, the answer is (c).

Regarding option (d), note that the hall and kitchen lights can be 'on' at the same time. When the kitchen light is 'on', the study room light is 'off' [reference (i)]. Whenever the study room light is 'on' the hall light is also 'on' [reference (iii)]. But this does not mean that the hall light is 'off' when the study room light is 'off'. That would be true, only if statement (iii) says, "The hall light is 'on' only if the study room light is 'on'," but that is not so. Hence there is no restriction on the hall light being 'on' when the kitchen light is 'on'.

11. The bedroom, kitchen, dining room and the hall lights can all be 'on' simultaneously. Hence, the maximum number of lights that can be on simultaneously is 4.

Solution to questions 12–15

Fig. 3.4 can be made:

	Marketing	Ent	Law	Finance	Acc	HR
Jan						
Feb						
Mar						
Apr						
May						
June						
July						
Aug						
Sep						
Oct						
Nov						
Dec						

Fig. 3.4

Now all the questions can be answered.

Solution to questions 16–19

16. As per the question the following data is available to us:

Ant *A* climbs 3 inches per minute; ant *B* climbs 4 inches per minute; ant *A* slips back 1 inch for every 2 inches climbed; ant *B* slips back 1.5 inches for every 2 inches climbed.

Ant *A* takes a rest break of 1 minute after every 2 minutes.

Ant *B* takes a rest break of 1 minute after every 3 minutes.

After minutes			
1	2	3	4
5	6	7	8
9	10	11	12
Ant *A*			
2"	3"	3"	5"
6"	6"	8"	9"
9"	11"	12"	12"
Ant *B*			
1"	2"	3"	3"
4"	5"	6"	6"
7"	8"	9"	9"

After minutes			
1	2	3	4
5	6	7	8
9	10	11	12
Ant *A*			
2”	3”	3”	5”
6”	6”	8”	9”
9”	11”	12”	12”
Ant *B*			
1”	2”	3”	4”
5”	6”	7”	8”
9”	10”	11”	12”

Solution to questions 20–22

20. Expected profit margin (known as contribution) per unit

= ₹ 200 – ₹ 100 = ₹ 100

Hence break even sales units

$$= \frac{\text{Fixed cost}}{\text{Profit margin per unit}}$$

Hence net sales

= 40,000 × 200 = ₹ 80 lacs.

21. Original profit = Total sales – Total cost (fixed cost + variable cost) = (₹ 200 × 1 lac) – (40 lacs + ₹ 100 × 1 lac) = ₹ 60 lacs.
Now, new variable cost = ₹ 110/unit, and everything else remains same.
So new profit = (₹ 200 × 1 lac) – (40 lacs + ₹ 110 × 1 lac) = ₹ 50 lacs.
So, percentage decrease in profit

$$= \frac{₹10 \text{ lacs}}{₹60 \text{ lacs}} \cdot 100 = 16.67.$$

22. Original profit = ₹ 60 lacs (see above question)
Interest payment = ₹ 2 lacs
Profit after interest = ₹ 58 lacs
Profit after interest after tax = ₹ 58 lacs – 30% of ₹ 58 lacs = ₹ 40.6 lacs
In the new case, there is 20% growth in sales.
So, new profit = (₹ 200 × 1.2 lac) – (40 lacs + ₹ 100 × 1.2 lac) = ₹ 80 lacs
Profit after interest payment = ₹ 78 lacs
Profit after interest after tax = ₹ 78 lacs – 30% of ₹ 78 lacs = ₹ 54.6 lacs
Percentage increase in earning

$$= \frac{54.6 - 40.6}{40.6} \times 100$$

= 34.5% increase.

Exercise 2

Solution to questions 1 and 2

Seema is the mother of either Raju’s father (Ganesh) or his mother (Pratibha). As Arun is Seema’s sister, she is the maternal aunt of either Ganesh or Pratibha.

1. To solve the first question of the set; as Arun is not Ganesh’s aunt, Seema is not Ganesh’s mother (if she were, Arun would have been Ganesh’s aunt).
   Therefore, she is Pratibha’s mother.
   So, Seema’s daughter, Devaki, is Pratibha’s sister. Devaki is Raju’s maternal aunt.
2. It is given that Arun is not Ganesh’s aunt. So, she is Pratibha’s aunt. This means that Pratibha is Arun’s niece, and so the answer to the second question is (d).

Solution to questions 3–7

Table 3.17 and family tree can be easily made:

Table 3.17

Person	Newspaper	Sex
E	**Express**	Female
A	**Business Standard**	Female
C	**Times**	Male
D	**Financial Times**	Male
B	**Bharat Times**	Female
F	**Hindustan Times**	Male

Relation (–)*E* ↔ D(+)
|
(–)*B* – (+) *F* – (+) *C* ↔ A(–)

Solution to questions 8–11

From statement (ii) we can see that out of the fifteen cards, nine cards can be kept easily.

Table 3.18

Violet Envelope	Yellow Envelope	Black Envelope
B	*A*	*C*
D	*B*	*D*
E	*C*	*E*

From (iii) and using Table 3.18, we get

Table 3.19

Violet Envelope	Yellow Envelope	Black Envelope
B(2)	*A*	*C*
D(2)	*B*(1)	*D*(1)
E(1)	*C*	*E*(2)

Digits given in the brackets show the number of cards. From statement (i), it is clear that each coloured envelope contains five cards. So, there are two *C*-type cards in the black envelope. Therefore, the remaining one *C*-type card is in the yellow envelope. So, all the three *A*-type cards are in the yellow envelope.

Solution to questions 12–15

Starting with the definite information in statements (iii) and (iv) we get Table 3.20 (say Table 3.20);

Table 3.20

Card	Colour	Picture	Position
A	Black	Queen	6
B		Prince	
C	Grey	King	2
D			
E			
F			

The cards have been assigned the positions 1 to 6 from left to right.

Using the information given in the other statements:

Palace – blue – *D* (*x*) (i)
Prince – *D*, *E* (*x*) (ii)
Priest – green, yellow (*x*) (iii)
B – 2 or 4
From (i), (ii), (iii) and Table 3.20, we get
Prince – red (v)
From (ii), (v) and Table 3.20, we get
Prince – red – *F* (vi)
Using (i), (vi) and Table 3.20, we get
Palace – blue – *E* (vii)

Using (iv), (vi), (vii) and Table 3.20, and then filling up the remaining information, we get Table 3.21:

Table 3.21

Card	Colour	Picture	Position
A	Black	Queen	6
B	Green/Yellow	Prince	
C	Grey	King	2
D	Yellow/Green	Joker	
E	Blue	Palace	
F	Red	Priest	

12. The vacant positions are 1, 3, 4 and 5. The given conditions require that the three consecutive cards 3, 4 and 5 be assigned to these. So, the joker will be at the number 1 position from the left.

Solution to questions 16–18

	Mon	Tue	Wed	Thu	Fri	Sat
W1	10 a.m.–2 p.m.	2.30 p.m.–4.30 p.m.	10 a.m.–2 p.m.	2.30 p.m.–4.30 p.m.	2.30 p.m.–4.30 p.m.	2.30 p.m.–4.30 p.m.
W2	1 p.m.–3 p.m.	9 a.m.–12 noon	1 p.m.–3.p.m.	9 a.m.–12 noon	9 a.m.–12 noon	1 p.m.–3 p.m.
W3	8 a.m.–11 a.m.	8 a.m.–11 a.m.	12 noon–2 p.m.	8 a.m.–11 a.m.	12 noon–2 p.m.	12 noon–2 p.m.

Solution to questions 24 and 25

Drank More	Left Later
$P > W$	$P > V$
$T > P$	$T > P$
$V > T$	$S > T$
$Q > U$	$U > S$
$U > V$	$V > W$
$W > R$	

Hence, the order of decreasing drink
$Q - U - V - T - P - W - R$
Hence, the order of leaving earlier
$U - S - T - P - V - W$

24. Therefore, *W* left the earliest.

25. Therefore, *R* drank the least.

Exercise 3

Solution to questions 1–4

Let us first make Fig. 3.5 given facts:

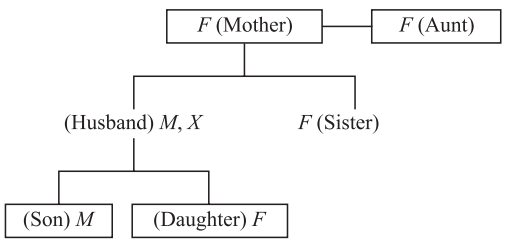


Fig. 3.5

Two (husband and son) of *X*'s six relatives are males and four are females. Since *R* is of the same sex as *S*, so both must be females, otherwise neither *P* nor *Q* could be males. The family relationship can be drawn around *X* like:

F—female and *M*—male

1. Since both *R* and *S* must both be females, so, *S* cannot be *X*'s son.
However, *P*, *Q*, *T* and *U* each could be *X*'s son because any of them might be male.

2. The only son among the six relatives is *X*'s son. If *U* were *T*'s son, then *U* would also be *X*'s son. Accordingly, *T* would be *X*'s husband. However, *T* and *U* cannot both be males. Thus statement (d) is false.

3. If *Q* is *U*'s daughter, then *Q* and *U* must either be *X*'s daughter and husband respectively or *X*'s sister and mother respectively. If *P* is *X*'s sister, *P* and *Q* would both be females, which is impossible so option (b) must be false. Since none of the six relatives could be *U*'s niece, so option (b) must be false. *T* and *S* would have to be *X*'s son and sister respectively. Accordingly, *U* and *Q* would have to be *X*'s husband and daughter but *T* and *U* cannot both be males. So, option (c) must be false. Option (d) could be true because *U* would be *X*'s mother, *S* would be *U*'s sister and *Q* would be *X*'s sister.

4. If *P* is *T*'s sister, then *P* and *T* must either be *X*'s daughter and son respectively or *X*'s mother and aunt (in either order). But *P*, *T* and *U* cannot all three be females, so *U* cannot be *X*'s daughter.

Solution to questions 5–9

O	Supports Finance
Q	Coordinates Operations
N	Supports Operations
P	Supports Operations

Professor	Coordinator	Support
L		
M		
N		Operations
O		Finance
P		Operations
Q	Operations	

P and *Q* have Marketing as one of their subjects

Professor	Coordinator	Support
L		
M		
N		Operations
O		Finance
P		Operations
Q	Operations	

P and *Q* have marketing as one of their subjects

Professor	Coordinator	Support
L		
M		
N		Operations
O	IT	Finance
P	Marketing	Operations
Q	Operations	Marketing

3 people coordinate Finance and

Professor	Coordinator	Support
L	Finance	
M	Finance	
N	Finance	Operations
O	IT	Finance
P	Marketing	Operations
Q	Operations	Marketing

IT is *L*'s subject

Strategy is supported by one of the professors.

Professor	Coordinator	Support
L	Finance	IT
M	Finance	Strategy
N	Finance	Operations
O	IT	Finance
P	Marketing	Operations
Q	Operations	Marketing

Solution to questions 10–12

For the sake of convenience, let us take the first letter of the names of all the children (Table 3.22).

Table 3.22

Item	Order	Performer	
Song	1	*B* (*x*)	… (i)
Dance	6		… (ii)
Mimicry	$x = 3$	*S*	… (iii)
Speech	$(x - 1) = 2$	*A* (*x*)	… (iv)
Drama	5	*E*	… (v)

(order can be determined with the help of Table 3.23)

Table 3.23

6	*D*/*F* (*x*)	…(vi)
4	*D*	…(vii)

'$x = 3$' is known by the second last line of the given information.

Table 3.24

Story telling	4	*D*
Drama	5	*E*
Mimicry	3	*S*
Speech	2	*B*/*F*
Song	1	*A*/*F*
Dance	6	*A*/*B*

Solution to questions 13–15

13. In any arrangement the Wind must be to the left of X-tra so, the X-tra cannot be placed in stall number 1.

14. If the X-tra is placed in stall number 3, then the Wind has to be immediately to the left of X-tra. So the Wind must be placed in stall number 2.

15. If the Uno is placed in stall number 5, neither the Wind nor the X-tra can be placed in stall number 6 as they must be placed next to each other. Also the Verna cannot be placed adjacent to the Uno. Hence, only the Yale can be placed in stall number 6.

Solution to questions 16 and 17

Tie	Company	Name
Brown	*S*	Himesh
Green	*P*	Lokesh
Pink	*Q*	Rajesh
Purple	*U*	Shailesh
Yellow	*R*	Nilesh
Red	*T*	Dinesh

Solution to questions 18–21

19. No surface coloured
$= (l - 2)(b - 2)(h - 2)$
$= 3 \times 1 \times 2 = 6.$

20. There are three cubes on each red – green interface (barring corner cubes). So, $4 \times 3 = 12$ cubes.

21. Single coloured cube
$=2(1 - 2)(b - 2) + 2(l - 2)(h - 2) + 2(b - 2)(h - 2) = 22$

Exercise 4

Solution to questions 1–4

1. Total number of matches $= 15 \times 6 = 90$
Number of matches played by each team = 30. The score table given in the question considers a total of 180 matches
Let total number of wins = x, so total loss = x
So, total number of draws on the score table
$D = 180 - 2x$
$D = 2(90 - x)$
We can conclude that the number of matches drawn must be a multiple of 2, *i.e.*, an even number and we know that the total number of matches for each team should be = 30
Or, $51 + P = 79 + Q$
Or, $P - Q = 28$
We know that
$P + Q \leq 30$ and
$P + Q + R = 30$
From the given equations, we can find the following possibilities:

$P = 29$	$Q = 1$	$R = 0$
$P = 28$	$Q = 0$	$R = 2$

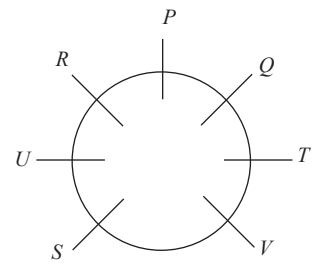
Therefore 2 *W*, *L* and *D* combinations were possible for the Bihar team.

2. The lowest score possible was for Goa
$= -9$
The highest score possible was for Bihar
$= 144$ (See question 1)
So, the maximum possible difference
$= 153$

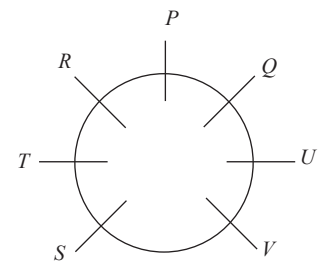
3. Bihar is maximum possible wins and losses were 29 and 1 respectively. It means Bihar has won all six matches against any 4 teams but it could win only 5 matches. Since Gujarat didn't lose any of its 6 matches against the other teams, it is definitely not the team which lost all its matches against Bihar. Hence, it must be the team which won the match against Bihar. Therefore, Bihar lost the match against Gujarat.

4. Since the scoring pattern for the wins (w), losses (l) and draws (d) of the match was not given, therefore it cannot be determined as to which team won the PHL.

Solution to questions 5–9



Case I



Case II

We cannot say that statement (iii) is wrong because case II (as shown above) makes the statement true. While case I and case II make the statement wrong. Since we are not certain about the positions of *U*, *V* and *T*, therefore, only statement (i) and (ii) are wrong.

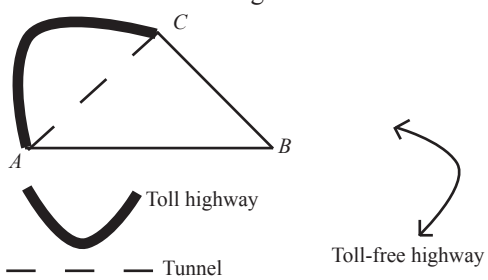
Solution to questions 12–17

Module S001 does not have Production and module S003 does not have Marketing. But their combination gives us Marketing and Systems. Hence, 5 module S001 must have Marketing. Since modules S001 and S002 have four different specializations, there must be no common specialization and module S002 will not have Marketing but will have Production. Module S003 will have Systems included because it is not a part of module S001. So, S001 will have Finance or Personnel and it will be common to module S003.

Now, since the S001 and S004 module combination does not have Marketing as a specialization, it means that they both have Marketing as common. Since they have Finance and Personnel common, therefore module S001 has Finance and module S004 has Personnel. Thus, module S002 also has Personnel and module S003 must also have Finance. This leaves module S005 with Production and Systems.

Solution to questions 18–22

Let us first see the arrangement of cities:



18. Total cost through the toll highway
= ₹ 20 + ₹ 25 = ₹ 45
Total cost through tunnel
= ₹ 10 + ₹ 40 = ₹ 50
Total cost through the toll-free highway
= ₹ 50
Therefore the costliest way of going from city *A* to city *C* is by the tunnel or the toll-free highway.

19. By using the data in the above answer we can see that the cheapest route between city *A* to city *C* is through the toll highway.

20. Total cost through the toll highway
= ₹ 10 + ₹ 25 = ₹ 35
Total cost through the tunnel
= ₹ 5 + ₹ 40 = ₹ 45
Total cost through the toll-free highway
= ₹ 25
Therefore the cheapest way to go from city *A* to city *C* using the CNG fuel option is through the toll-free highway.

21. By using the data in the above answer we can see that the cheapest way to go to city *C* from city *A* is through the toll-free highway.

22. Using the data from the solution of question 1, the difference = ₹ 5. Hence, if ₹ 5 is the bribe amount, then the costs will be equal.

Solution to questions 23–25

Using statement (i), Chandan is not the older fighter. From statement (iii): Balraj is not the younger fighter. So, either:

Case 1: Anil is the older fighter and Chandan is the younger fighter.

Case 2: Balraj is the older fighter and Anil is the younger fighter.

Case 3: Balraj is the older fighter and Chandan is the younger fighter.

From statement (iii):

Chandan is the taller fighter for case 1 and Chandan is taller than Anil for case 3.

From statement (i):

Balraj is the shorter fighter for case 2 and Balraj is shorter than Anil for case 3.

From statement (i) and (iii):

Chandan is taller than Balraj for Case 3.

Summarizing the whole discussion we get Table 3.25:

Table 3.25

	Older fighter	Younger fighter	Taller fighter	Shorter fighter
Case 1	Anil	Chandan	Chandan	Anil
Case 2	Balraj	Anil	Anil	Balraj
Case 3	Balraj	Chandan	Chandan	Balraj

From statement (ii):

Anil is not the shorter fighter; so case 1 is eliminated.

From statement (ii):

Balraj cannot be both older and shorter than Chandan, so case 3 is eliminated. Then case 2 is the correct one and Chandan is not fighting.

Then, from statement (ii): Balraj is younger than Chandan (so Chandan is the eldest out of the three and Anil is the youngest) and, from statement (iii): Anil is taller than Chandan (so Anil is the tallest out of the three).

Exercise 5

Solution to questions 1–4

1. Condition (ii) says Sohail lost all 3 games, hence his score should be 0–3.
2. The highest score could be that of Shahrukh or Salman. However, Salman is at the winning table 4, hence, option (a) is the correct answer.
5. As the sum of rows, columns and diagonals are equal, we can fill in the blanks in terms of "x and y" as below:

y	x	$y-2$
$y-1$	$y+1$	$x-2$
$x-1$	$y-3$	$y+2$

Further $(y+x+y-2)=y-2+x-2+y+2$

Hence, $x=y+5$

So, if $x=10$; $y=5$

6. From question 5; we have one equation

$$x=y+5$$

and as sum of a row (= sum of column = sum of diagonal) is given 21; then

$$x+2y=23$$

Solving these two equations we get,

$$y=6 \text{ and } x=11$$

Solution to questions 7 and 8

The diagram of the given arrangement can be shown in Figure 3.6:

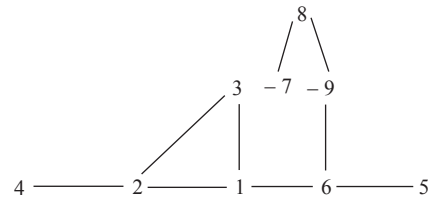


Fig. 3.6

7. By observing the diagram we can say that the maximum number of shops a person can visit without visiting any shop more than once is 8.
   Hence, the answer is option (d).
8. By observing the diagram we can say that option (c) must be false, which is the correct answer.

Solution to questions 9–11

The information that we have from the question is:

Q, U and R are finance companies

Now the 1st day is Sunday and the 7th day would be Saturday

S is a manufacturing company

R is scheduled on 3rd day of the week

S can't be scheduled on 6th day of the week

T can be scheduled on any day of the week except on a Saturday.

9. If P is scheduled on Wednesday and R is on the third day of the week then the possible arrangements are

1(Sun)	2(Mon)	3	4	5	6	7(Sat)
Q/U	*U/Q*	*R*	*P*	*S*	*T*	*V*

or,

1(Sun)	2(Mon)	3	4	5	6	7(Sat)
Q/U	*U/Q*	*R*	*P*	*T/V*	*V/T*	*S*

Now look at the answer options. The condition which has to be true is that T has to be scheduled on either Thursday or Friday. Hence, the answer is option (c).

10. If S is scheduled on Monday then possible arrangements are:

1(Sun)	2(Mon)	3	4	5	6 (xS)	7(Sat)(xT)
P/V	S	R	Q/U	Q/U	T	V/P
T	S	R	O/U	U/O	P/V	V/P

Now look at the answer options to be definitely false.

(a) P and T can be sales companies when V and S are manufacturing companies and Q and U and are finance companies. So it is not false.

(b) Maximum number of days between the schedules of T and U is 3. This is also possible when U is on the 5th day and T is on the 1st day. So this case is not definitely false.

(c) R is always scheduled before Q is definitely true as the arrangement can be either RUQ or RQU.

(d) This statement is false as only two cases are possible. The possible arrangements are:

1(Sun)	2	3	4	5	6(xS)	7(Sat)	(xT)
	P	S	R	O/U	U/O	T	V

i.e., 2 only.

11. Look at schedule IV. Since T is a sales company and S is a manufacturing company they cannot be together as the same category companies have to be scheduled consecutively. So except for schedule IV all the combinations are possible.
Hence, the answer is option (d).

12. Let us see the diagram:

Monday	Tuesday	Wednesday	Thursday	Friday
			Z	W

Now, there are three days and three boxes to be delivered.

Both X and T cannot be delivered on Monday [(condition (i) and condition (ii) given in the question)]. So, U has to be delivered on Monday.
Let us see the diagram now:

Monday	Tuesday	Wednesday	Thursday	Friday
U			Z	W

Box T and box X can interchangeably be delivered on either Tuesday or Wednesday. Go through the options now.

Answer is option (d).

Solution to questions 13–17

Based on observation (i) and (ii), we can say that the three dice with the same number either show 1 or 2 or 3.

13. $2 + 2 + 2 + 6 + 5 + 4 = 21$

14. $3 + 3 + 3 + 6 + 2 + 5 = 22$

15. $5 + 5 + 5 + 6 + 5 + 5 = 31$

16. 2 because one dice shows 6, one shows 1 and three are ≤ 3. Hence, only one more can be > 4.

17. One dice should show 6. But there cannot be more than three dice showing 4 or more than 4. The maximum number that the three dice can show is 3.

Solution to questions 18 to 21

18. Note that all the options except (c) violate some condition or the other.

19. If $B + G =$ ₹ 3.5 lacs, and $E =$ ₹ 2 lacs which leaves D to get the maximum.

20. If E is minimum and A and C are also minimum, then A and E are farthest apart.

21. $A + D = G$; and $E =$ ₹ 2 lacs, $D =$ ₹ 3 lacs, this leaves B and F together to win 3.5 lakhs.

MASTERING LOGICAL REASONING

EXERCISE 1

Direction for questions 1–6: ***Read the following passage and solve the questions based on it.***

My brother Vinit while was planning to go home last year was confused about the clothes that he should carry with him. He had the following types of clothing: belts, hats, jackets, ties, vests, T-shirts and handkerchiefs. It was also known that he had with him several pieces of each of the seven types of clothing. Finally, he set up an algorithm to decide the clothes that he would be taking along with him.

(i) If he packed ties, then he did not pack handkerchiefs.
(ii) If he packed belts, then he did not pack jackets.
(iii) If he packed belts, then he packed at least one handkerchief.
(iv) If he packed jackets, then he packed at least one vest.
(v) If he packed vests, then he packed at least one handkerchief.
(vi) If he packed handkerchiefs, then he packed at least two of them.

1. Which one of the following could be a complete list of the clothing that my brother packed?
(a) one belt, one T-shirt, one vest, two handkerchiefs
(b) one belt, one tie, one T-shirt, three handkerchiefs
(c) two belts, one ties, three T-shirts
(d) one jacket, one T-shirt, two vests and one handkerchief

2. If Vinit did not pack any handkerchiefs, what was the maximum number of the different types of clothing that he could pack?
(a) Two (b) Three
(c) Four (d) Five

3. Which one of the following statements must be false?
(a) Vinit packed exactly three items of clothing, one of which was a vest
(b) Vinit packed exactly four items of clothing, one of which was a hat
(c) Vinit packed exactly three items of clothing, one of which was handkerchief
(d) Vinit packed exactly three items of clothing, one of which was a belt

4. If Vinit packed as many different types of clothing as possible, then it must be true that he did not pack one of the following types of clothing.
(a) belts (b) hats
(c) ties (d) jackets

5. If Vinit packed at least one item, find out which one of the following are the minimum and the maximum numbers of the types of clothing that he could pack?
(a) 1, 4 (b) 1, 5
(c) 1, 6 (d) 2, 5

6. If Vinit did not pack any vests, then it could be true that he packed one or more:
(a) Ties and jackets
(b) T-shirts and belts
(c) Ties and belts
(d) T-shirts and ties

Direction for questions 7–9: ***Read the following passage and solve the questions based on it.***

The production, marketing, human resource, finance and management information system managers of a particular company meet for a round table meeting to discuss the strategy of the company. Out of the five, three are men and the other two are women. The following restrictions apply to their seating arrangement:

(i) The two women will not be seated next to each other.

(ii) The production manager, a man, will always be seated as far as possible from the marketing manager.
(iii) The finance manager will always be seated next to a woman.

7. If the human resource manager is a man, which of the following is definitely not true?
(a) The marketing manager is sitting in between the two women
(b) The marketing manager is sitting in between two men
(c) The finance manager is a man
(d) The marketing manager is a woman

8. If the finance manager always has a woman to his right, in how many different arrangements can the marketing manager be a woman?
(a) 2 (b) 3
(c) 4 (d) 5

9. If the management information system manager is sitting just to the right of the production manager, which of the following statement(s) is/are definitely True?
I. The marketing manager and the finance manager sit next to each other.
II. The human resource manager has to be a woman.
III. The management information system manager has to be a man.
(a) I only (b) I and II
(c) I and III (d) II and III

Direction for questions 10–14: ***Read the following passage and solve the questions based on it.***

The Snehans Apartment Welfare Association offers three activities to its members skating, soft ball and steam bath. To avail all these facilities the association has made separate activity centres, one each for all the three activities. Table 3.26 gives the details pertaining to the number of different types of members, the capacity of each type of activity centre and the time that must be invested in the individual activities (if used):

Table 3.26

	In Time	Number of Members	Skating	Soft ball	Steam Bath
Males	7.30 a.m.	200	90 minutes	75 minutes	20 minutes
Females	8.30 a.m.	160	60 minutes	40 minutes	15 minutes
Children	9.00 a.m.	220	60 minutes	90 minutes	N.A
Capacity			250	120	30

All members are divided into three categories based on their age and sex as males, females and children. All members report at the in-time and all of them do skating first. Members are entitled to use the specialties via. skating, softball or steam bath according to the first-come, first-served basis and the availability of space in that particular activity centre.

For example: All males (200) whose in-time is 7.30 a.m. are allowed to skate as the capacity of the skating centre is 250. When the females report at their in-time (8.30 a.m.) there are only 50 places available in the skating centre. So, the rest of the 110 females wait till it becomes available.

The additional information is given as below:

(i) No body can use the softball activity centre without going through the skating centre. If a member goes to the steam bath centre, he/she will have to go to the softball activity centre as well.
(ii) When members report to a particular centre, it is known as the reporting time for that centre; when they are allowed to enter the centre, it is known as their entry time for that centre. The difference between the reporting time and the entry time is known as the 'waiting time' for that centre.
(iii) 50% of the males who skate at a given time also play at the softball activity centre. Further, 50% of the males who go

to the softball centre at a given time also visit the steam bath activity centre.

(iv) 60% of the females who skate at a given time also play at the softball activity centre. Further, 50% of the females who go to the softball activity centre at a given time also we the steam bath faritity.

(v) 50% of the children who swim at a given time also play at the soft ball centre. No child takes a steam bath.

(vi) Members are served on the first-come, first-served basis. Further, if their reporting time at a particular centre is the same, then they are entertained on the basis of their in-time.

10. What is the number of children whose waiting time is 0, before entering the softball activity centre?

(a) 1.1 (b) 2.6
(c) 3.19 (d) 4.40

11. What percentage of men out of the total number of men, went through all the three centres in the minimum possible time?

(a) 2.5% (b) 5%
(c) 7.5% (d) 10%

12. What is the difference between the number of women who took the maximum time and those who took the minimum time, respectively, for going through all the three centres?

(a) 2 (b) 5
(c) 18 (d) 23

13. What is the maximum time taken by any member who went through all the three centres?

(a) 110 minutes (b) 130 minutes
(c) 150 minutes (d) 170 minutes

14. How many children are waiting at 9 a.m. for skating?

(a) 90 (b) 110
(c) 130 (d) 150

Direction for questions 15 and 16: ***Read the following passage and solve the questions based on it.***

There are five identical looking boxes containing different objects in each of them and every box has a label indicating its contents. Table 3.27 shours the correct description of the contents and the label on each box:

Table 3.27

Contents	Label
Two Pins	*PP*
Two Balls	*BB*
Two Clips	*CC*
One Pin and one Clip	*PC*
One Ball and one Clip	*BC*

15. Somebody has mischievously interchanged these labels in such a way that no box carries the correct label describing its contents. How many of the following statement/s is/are definitely true?

I. If two boxes are opened and it is found that there are three clips and one ball, then *CC* and *BC* are the actual labels on the boxes.

II. If two boxes are opened and it is found that there are a total of two clips and one pin among the four items inside the boxes, then *PC* and *BC* are the actual labels on the boxes.

III. If two boxes are opened and it is found that there are two pins among the four items inside the boxes, then *BC* has to be one of the actual labels on the boxes.

IV. If two boxes are opened and it is found that there is at least one clip and one ball among the four items inside the boxes, then at least one pin has to be there as well.

(a) 1 (b) 2
(c) 3 (d) None of these

16. Out of the five given boxes, three of the boxes have got their labels interchanged. It

is also known that the boxes with the right labels contain at least one pin or at least one ball inside it. In how many ways can the labels be put on the five boxes satisfying the above given conditions?

(a) 12 (b) 16
(c) 24 (d) None of these

Direction for questions 17–22: ***Read the following passage and solve the questions based on it.***

Transparency International is UN watch-dog which gives ranks to all the member countries of the UN in terms of the corruption level in the government machinery. There are 190 member countries of the UN since its establishment in 1945 and all of them are awarded the ranks which are valid for a particular year. The lower the rank, the less corruption infested the country is. And the higher the rank, the more corrupt the country is.

Table 3.28, the ranks of ten countries for three consecutive years are given. A positive change shows that the ranking has improved over the previous year and a negative change shows that the ranking has worsened over the previous year.

Table 3.28

Rank in 2006	Change over 2005	Country	Change over 2004
51	+3	Sri Lanka	–5
52	+4	Zambia	–3
53	+6	New Zealand	–7
54	–2	Tango	–3
55	+2	Austria	–3
56	–1	Hungary	+4
57	A	Australia	D
58	B	Kenya	E
59	C	Mozambique	F
60	Zero	UAE	NA

Somehow the data for Australia, Kenya and Mozambique are missing from the Table 3.28. NA indicated that the country was not in the list of the given ranks in that particular year.

17. What is the minimum value of $A + B + C$?
(a) 11 (b) 12
(c) 13 (d) 14

18. What is the maximum value of $D + E + F$?
(a) 14 (b) 15
(c) 16 (d) 17

19. What is the minimum value of $D + E + F$?
(a) 8 (b) 9
(c) 10 (d) 11

20. How many countries from the given countries in the table have definitely improved their ranking consistently every year during the given period?
(a) 1
(b) 2
(c) 3
(d) Cannot be determined

21. What can be the minimum change in the rank of any country in 2006 with respect to its rank in 2004?
(a) 0 (b) 1
(c) 2 (d) 3

22. For how many countries is it possible to have the same ranks in all the three years?
(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 23–25: ***Read the passage below and solve the questions based on it.***

Five friends—Amar, Binit, Charu, Deeksha, Eshwar, went to a fair. At a shooting stall there are three things to be shot–balloons, coins and needles. The number of balloons shot are 1, 4, 5, 6 and 8, while the number of coins shot are 0, 1, 2, 4 and 6. Following details are available:

(i) The number of coins shot by Amar is three times the number of coins shot by the person who shot 4 balloons.
(ii) Three persons including the one who shot four coins, did not shot any needle.
(iii) Binit did not shoot any needle.

(iv) The one who shot one balloon did not shoot any needle or coin. It is also known that he was not Charu.
(v) Deeksha shot balloons and coins but no needle.
(vi) Charu shot half as many coins as the person who shot twice as many balloons as he did.
(vii) Eshwar shot two more balloons than Amar, but Amar shot two more coins than Eshwar.

23. Which of the following is true?
(a) Charu shot 8 balloons and 4 coins but no needle
(b) The person who shot 5 balloons and one coin did not shoot any needle
(c) The person who shot an equal number of balloons and coins also shot needles
(d) The person who shot 4 balloons and 2 coins also shot needles

24. Who shot an equal number of coins and balloons?
(a) Amar (b) Binit
(c) Charu (d) Deeksha

25. Which of the following is true?
(a) Deeksha shot 5 balloons
(b) Amar shot 8 balloons
(c) Eshwar shot 1 balloon
(d) Eshwar shot 6 balloons

EXERCISE 2

Direction for questions 1–6: ***Read the following passage and solve the questions based on it.***

Looking upon the influx of high profile and sensitive cases in the Supreme Court. The CJI has decided that each case will be heard on a day to day basis by making a group of judges out of seven senior judges—Bindra, Goel, Kalyan, Mahajan, Deshbandhu, Raheja and Sinha. However all the judges need to serve on the panels collectively. They can serve either alone or in groups together, consistent with the following conditions:

(i) Bindra serves on every panel that Kalyan serves on.
(ii) Kalyan serves on every panel that Sinha serves on.
(iii) Mahajan serves on every panel that Raheja does not serve on.
(iv) If Bindra serves on a panel then neither Goel nor Mahajan serve on that panel.

1. Which one of the following could be a complete list of the judges who serve together on one panel?
(a) Goel, Kalyan, Raheja, Sinha
(b) Goel, Mahajan, Deshbandhu, Raheja
(c) Bindra, Kalyan, Mahajan, Deshbandhu
(d) Bindra, Deshbandhu, Raheja, Sinha
(e) Goel, Kalyan, Deshbandhu, Sinha

2. What is the maximum number of judges who could serve on a panel that Raheja does not serve on?
(a) Two (b) Three
(c) Four (d) Five
(e) Six

3. If exactly three judges serve together on a panel, then each of the following must be true except:
(a) Bindra and Kalyan both serve on the panel
(b) Raheja and Deshbandhu both serve on the panel
(c) Deshbandhu and Mahajan both serve on the panel
(d) Kalyan and Deshbandhu both serve on the panel
(e) Goel and Raheja both serve on the panel

4. If Sinha and Deshbandhu serve together on a panel, then exactly what would be the size of the panel?
(a) One (b) Two
(c) Three (d) Four
(e) Five

5. If Goel and Deshbandhu serve on a panel together, then which one of the following must be true?

(a) Sinha does not serve on the panel
(b) Bindra also serves on the panel
(c) Raheja also serves on the panel
(d) Kalyan also serves on the panel
(e) Mahajan does not serve on the panel

6. What is the largest possible size of the panel?
(a) 4 (b) 5
(c) 6 (d) 5 or 6
(e) Cannot be determined

Direction for questions 7–11: ***Read the following passage and solve the questions based on it.***

There are seven friends—*A*, *B*, *C*, *D*, *E*, *G* and *H* at a *B*-School. They specialize in exactly one of the two areas: Marketing or Systems. The students choose their specialization in accordance with the following:

(i) If *D* specializes in Systems then *B* specializes in Marketing.
(ii) If *E* specializes in Systems then *H* and *A* both specialize in Marketing.
(iii) *G* does not choose the same specialization as *D*.
(iv) If *C* specializes in Systems, then *B* also specializes in Systems.

7. Which one of the following could be a complete and accurate list of students who specialize in Systems?
(a) *D*, *E*, *C* (b) *H*, *A*, *G*
(c) *B*, *H*, *C* (d) *B*, *A*, *G*, *C*
(e) *D*, *B*, *H*, *G*

8. If *D* specializes in Marketing then which one of the following must be true?
(a) *E* specializes in Marketing
(b) *G* specializes in Marketing
(c) *B* specializes in Systems
(d) *C* specializes in Systems
(e) *H* specializes in Systems

9. If *H* and *G* both specialize in Systems, then which one of the following could be true?
(a) *E* and *A* both specialize in Marketing
(b) *C* and *A* both specialize in Systems
(c) *D* and *A* both specialize in Marketing
(d) *B* and *C* both specialize in Systems
(e) *D* and *E* both specialize in Systems

10. What is the maximum number of friends who could specialize in Systems?
(a) 2 (b) 3
(c) 4 (d) 5
(c) 6

11. Which one of the following is a pair of friends who cannot specialize in Marketing together?
(a) *H* and *G* (b) *D* and *B*
(c) *D* and *G* (d) *H* and *A*
(e) *B* and *G*

Direction for questions 12–15: ***Read the following passage and solve the questions based on it.***

King Amitabh of Bollysteel organized initials to decide the groom for his daughter in two steps—The preliminary stage and the final stage. The preliminary stage comprised of a written test whereas in the final stage there was an archery competition. The archery competition consisted of five rounds wherein the contestants could score from 2 to 9 points in each round. The prince who scored the maximum points in these five rounds would be considered the winner in the overall standing and would marry the princess.

When the results of the preliminary stage were announced, it was found that only five princes—Shikari, Bhikhari, Atyachari, Juari and Hazari, qualified the preliminary stage and hence these were the only contestants left to complete in the final stage.

During the final stage, however, the inilats who was tabulating the scores of the princes started taking sporadic short naps and so at the end, when King Amitabh asked for the score-card to find out the final winner, he was presented data as shown in Table 3.29.

Table 3.29

	1st Round	2nd Round	3rd Round	4th Round	5th Round
Shikari	9		8		3
Bhikhari		2	3		
Atyachari	2	9	4	8	
Juari	4	6		5	8
Hazari		3			9

However, the initals has made the following observations too:

(i) No two princes had scored equally in any round.
(ii) The difference between the total points scored by Prince Shikari and Prince juari was 6 points.
(iii) Prince Bhikhari's total points were always greater than that of Prince Atyachari's total points.
(iv) Prince Shikari scored an even number of points both in the 2nd and the 4th rounds.
(v) Each of the five princes scored greater than or equal to 24 points in the five rounds.
(vi) There was tie between any two princes in their overall points.

12. Prince Juari won the competition and married the Princess. Also, Prince Bhikhari scored 28 points from the five rounds. Then which of the following could be false?
(a) Prince Atyachari scores more points than Prince Hazari in round 4
(b) Prince Shikari scores more points than Prince Hazari in round 2
(c) Prince Bhikhari scores more points than Prince Hazari in round 1
(d) Prince Juari scores more than Prince Shikari in round 3
(e) None of these

13. If Prince Juari scores 5 points in the 3rd round and Prince Atyachari scores 4 points in the 5th round, then which of the following will definitely be false?
(a) Prince Shikari is 1st in the overall standing
(b) Prince Hazari is 2nd in the overall standing
(c) Prince Atyachari is 3rd in the overall standing
(d) Prince Juari is 4th in the overall standing
(e) None of these

14. If Prince Hazari scores 2 points in the 3rd round. Then, which of the following statements is sufficient to decide the winner?
I. Prince Atyachari scores the maximum possible points in the 5th round.
II. Prince Juari scores the maximum possible points in the 3rd round.
(a) Only I
(b) Only II
(c) Both I and II
(d) Even I and II together are not sufficient to decide the winner
(e) Using either I or II will be sufficient to decide the winner

15. If Prince Atyachari scores 5 points in the 5th Round but Prince Hazari becomes the winner with the least possible total points, then what is the least possible points scored by Prince Hazari in the 3rd round?
(a) 5 (b) 6
(c) 7 (d) 8
(e) Cannot be determined

Direction for questions 16 and 17: ***Read the following passage and solve the questions based on it.***

There are seven persons—*I*, *J*, *K*, *L*, *M*, *N* and *O* in a family. They have one set each of the following three games–cards, baseball and tag.

Following are the details pertaining to the games that they play among themselves:

(i) Each person must play exactly two out of the three games.
(ii) *I* must play cards.
(iii) *K* must play tag.
(iv) *N* must play baseball.
(v) *M* must play both games which *I* plays.

(vi) *O* must play both games which *L* plays.
(vi) Cards must be played by either 3 or 4 persons.
(vii) Baseball must be played by either 4 or 6 persons.
(ix) Tag can be played by any number of persons as long as there are a minimum of 2 persons.

16. If *K* plays the same two games as *N* plays, then which of the following is not true?
(a) *I* plays cards (b) *N* plays cards
(c) *K* plays baseball (d) *N* plays tag
(d) None of these

17. If *I* and *N* play tag and four persons play cards, then the persons playing cards besides *I* and *M* are necessarily one of the following options:
(a) *J* and *K* (b) *J* and *N*
(c) *K* and *N* (d) *K* and *O*
(e) None of these

Direction for questions 18–20: ***Read the following passage and solve the questions based on it.***

(i) In a newly constructed market, six shops on a floor in two rows facing North and South are allotted to *U*, *V*, *W*, *X*, *Y* and *Z*, not necessarily in that order. The number of shops in both the rows are equal in number.
(ii) *V* gets a North-facing shop and is not next to *X*.
(iii) *W*, which is next to *Z* gets a South-facing shop and *Y* gets a North-facing shop.

18. If *X* and *Z* get diagonally opposite shops and both these shops can never be at the extremes, then in how many ways can the be allotments of the shops done?
(a) 2 (b) 4
(c) 6 (d) 8
(e) None of these

19. In the above question, whose shop is between *V* and *X*?
(a) *Z* (b) *Y*
(c) *W* (d) *U*
(e) Cannot be determined

20. In Q.18, the shops of which other pairs, apart from *X* and *Z* are diagonally opposite to each other?
(a) *U* and *Y* (b) *V* and *U*
(c) *V* and *W* (d) *Y* and *X*
(e) None of these

Direction for questions 21–25: ***Read the following passage and solve the questions based on it.***

During the Doha Asian Games 2006, the following teams played in the soccer event—Afghanistan, Nepal, Pakistan, India, South Korea and Bangladesh. In the event, each team played with every other team exactly once.

The following was the scoring pattern of the event:

(i) Each win gave three points to the winning team.
(ii) Each draw gave one point to each of the teams.
(iii) There was no point for a loss.

Following was the table of the points at the end of all the matches in the soccer event. Table 3.30 showed the ranks of the teams in terms of points scored by them in the decreasing order of their total points.

Some information had been deliberately kept on hold due to certain reasons which were not to be made public. Even the results of the matches were not declared.

However, on public demand the result of only one match, which was played between India and Pakistan, was declared. It was announced that Pakistan had beaten India.

It was also given that no two teams finished with the same number of points.

For the sake of convenience, all the teams were denoted by their initials only.

21. Which of the following matches was a draw?

Table 3.30

Position	Team	Won	Drawn	Lost	Goals For	Goals Against	Total points
1	A				17	5	15
2	N				9	6	10
3	P					2	8
4	I				2	5	
5	S K				7	11	2
6	B				8	16	

(a) *I* vs *N* (b) *I* vs *S K*
(c) *B* vs *N* (d) *B* vs *S K*
(e) None of these

22. What was the total number of points scored by *I*?
(a) 3 (b) 5
(c) 6 (d) 7
(e) Cannot be determined

23. Find the total number of goals scored in the match between *N* and *P*.
(a) 3 (b) 2
(c) 0 (d) 4
(e) Cannot be determined

24. The number of goals scored by *A* against *I* could be at the most:
(a) 2 (b) 3
(c) 4 (d) 5
(e) 6

25. For how many teams could we determine the total points?
(a) 4 (b) 5
(c) 6 (d) 5 or 6
(e) Cannot be determined

EXERCISE 3

Direction for questions 1–6: ***Read the following passage and solve the questions based on it.***

(i) Six businessmen from six different nations are staying in six successive in a hotel.
(ii) Each of them owns a number of cars and has donated to a number of institutions last year.
(iii) The businessman in room number 102 owns twice as many cars as the businessman who donated to 8 institutions last year.
(iv) The businessman from Uruguay and the businessman in room number 106 together own a total of 40 cars.
(v) The businessman from Argentina owns 8 cars less than the businessman from England but donated to 10 more institutions last year.
(vi) Four times the number of cars owned by the businessman in room number 104 is lesser than the number of institutions to which he donated last year.
(vii) The businessman in room number103 owns 12 cars and donated to 8 institutions last year.
(viii) The businessman who owns 16 cars donated to 24 institutions last year.
(ix) The businessman in room number 105 owns 8 cars and donated to 2 institutions less than the businessman from Canada last year.
(x) The Brazilian businessman is staying two rooms ahead of the English businessman who is staying two rooms ahead of the Canadian businessman.

1. In which room is the Brazilian businessman staying?
(a) room number 102
(b) room number 103
(c) room number 104
(d) room number 105

2. How many institutions did the Argentinean businessman donate to last year?
(a) 8 (b) 3
(c) 18 (d) 24

3. The businessman of which country is staying in room number 106?
(a) Argentina (b) Canada
(c) Uruguay (d) Germany

4. The businessman of which country donated to 24 institutions last year?
(a) Argentina (b) Uruguay
(c) Canada (d) Germany

5. The businessman of which country owns the maximum cars?
(a) Argentina (b) Uruguay
(c) Germany (d) Brazil

6. How many cars does the English businessman own?
(a) 8 (b) 12
(c) 4 (d) 20

Direction for questions 7–11: ***Read the following passage and solve the questions based on it.***

Table 3.31 represents the average price of all the cars sold in a particular year from the year 2000–2004.

Table 3.31

Year	2000	2001	2002	2003	2004
Average price (in ₹ thousand)	30	40	P	30	50

Table 3.32

Years	Number of cars sold
Total number of cars sold in 2000 + 2001	*A*
Total number of cars sold in 2001 + 2002	*B*
Total number of cars sold in 2002 + 2003	*C*
Total number of cars sold in 2003 + 2004	*D*
Total number of cars sold in 2004 + 2000	*E*

7. I. If the value of $P = 40$, then only Table 3.31 is sufficient to find the average price of all the cars sold in 2002.
II. If the value of P is not given, then whatever is the value of A, B, C, D and E, the average price of all the cars sold in 2002 cannot be calculated.
(a) Only I is true
(b) Only II is true
(c) Both I and II are true
(d) None of the two is true

8. I. If the value of $P = 40$, then in 2002 no car has been sold.
II. If the value of P is given and it is not equal to 40, then the value of any three out of A, B, C, D and E is sufficient to find the value of the average price of all the cars sold in 2002.
(a) Only I is true
(b) Only II is true
(c) Both I and II are true
(d) None of the two is true

9. To find out the total number of cars sold in 2002 and 2003, we need the following:
(a) Only the value of P
(b) Value of P, A, B, C and D
(c) Value of P, A, B, C, D and E
(d) Cannot be determined

(Additional information for questions 10 and 11: Disregard Table 3.32 for both the questions).

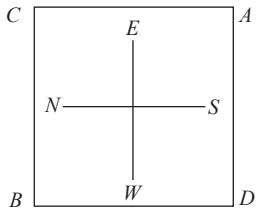
10. For what value of P, will the total number of cars sold in all the years be the least?
(a) $P = 30$
(b) $P > 40$
(c) $P = 40$
(d) Cannot be determined uniquely

11. It is given that in one particular year no car is sold. Which of the following is most likely to be that year?
(a) 2000 (b) 2001
(c) 2003 (d) 2004

Direction for questions 12–14: ***Read the following passage and solve the questions based on it.***

The following questions are based on the diagram as shown in Fig. 3.7, showing four persons—

A, *B*, *C* and *D* stationed at the four corners of a square piece of plot as shown:



E—East, *S*–South, *W*—West and *N*—North

Fig. 3.7

12. From the original position, *B* and *D* move one and a half lengths of sides clockwise and anti-clockwise respectively. Which one of the following statements is true?
(a) *B* and *D* are both at the mid-point between *A* and *C*
(b) *D* is at the mid-point between *A* and *C*; and *B* is at the corner originally occupied by *C*
(c) *B* is at the mid-point between *A* and *C*; and *D* is at the corner originally occupied by *A*
(d) *B* and *D* are both at the mid-point between *A* and *D*

13. From the positions in the original figure, *C* and *A* move diagonally to the opposite corners and then one side each clockwise and anti-clock wise respectively. Where is *A* now?
(a) At the north-west corner
(b) At the north-east corner
(c) At the south-east corner
(d) At the south-west corner

14. From the positions in the original figure, *C* and *A* move diagonally to the opposite corners and then one side each clockwise and anti-clockwise respectively. *B* and *D* move two sides each clockwise and anti-clockwise respectively. Who is now at the north-west corner?
(a) *A* (b) *B*
(c) *C* (d) *D*

Direction for questions 15–19: ***Read the following passage and solve the questions based on it.***

All the roads of a city are straight and perpendicular or parallel to each other. Roads *A*, *B*, *C*, *D* and *E* are parallel to one another. Roads *G*, *H*, *I*, *J*, *K*, *L* and *M* are parallel to one another.

(i) Road *A* is 1 km east of road *B*.
(ii) Road *B* is 0.5 km west of road *C*.
(iii) Road *D* is 1 km west of road *E*.
(iv) Road *G* is 0.5 km south of road *H*.
(v) Road *I* is 1 km north of road *J*.
(vi) Road *K* is 0.5 km north of road *L*.
(vii) Road *K* is 1 km south of road *M*.

15. Which is essentially true?
(a) *E* and *B* intersect
(b) *D* is 2 km west of *B*
(c) *D* is at least 2 km west of *A*
(d) *M* is 1.5 km north of *L*

16. If *E* is between *B* and *C*, then which of the following is false?
(a) *D* is 2 km west of *A*
(b) *C* is less than 1.5 km from *D*
(c) *E* is less than 1 km from *A*
(d) *D* is less than 1 km from *B*

17. If road *E* is between *B* and *C*, then the distance between *A* and *D* is:
(a) 0.5 km (b) 1 km
(c) 1.5 km (d) 2 km

18. Which of the following possibilities would make two roads coincide?
(a) *L* is 0.5 km north of *I*
(b) *C* is 1 km west of *D*
(c) *D* is 0.5 km east of *A*
(d) *E* and *B* are 0.5 km apart

19. If *K* is parallel to *I* and *K* is 0.5 km south of *J* and 1 km north of *G*, then which two roads would be 0.5 km apart?
(a) *I* and *K* (b) *J* and *K*
(c) *J* and *H* (d) *G* and *J*

Direction for questions 20–23: ***Read the following passage and solve the questions based on it.***

Mintu Babu lives in a smaller house than her brother. Mintu Babu lives in a larger house than her parents. Mintu Babu's children live with her. Mintu Babu has no other relatives.

20. If four females and two males live in a house smaller than Mintu Babu's brother, then, how many of Mintu Babu's children are boys and girls respectively?
(a) 1, 0 (b) 0, 1
(c) 2, 1 (d) 1, 2

21. If Mintu Babu's relative *U* lives in a larger house than her relative *S*. Then, all of the following may be true except:
(a) *S* is *U*'s father
(b) *S* is *U*'s mother
(c) *U* is younger to *S*
(d) *S* is younger to *U*

22. Out of all the relatives of Mintu Babu who could possibly be either older or younger than her, none are the same age or older, then, how many of Mintu Babu's relatives must be younger than her?
(a) less than 2 (b) 2
(c) 3 (d) more than 3

23. If the number of males related to Mintu Babu equals the number of females then which of the following can be true?
(a) Mintu Babu has 4 children
(b) Mintu Babu has 3 children
(c) Mintu Babu has 1 child
(d) Mintu Babu has 2 children

Direction for questions 24 and 25: ***Read the following passage and solve the questions based on it.***

Three committees are formed from eight people, *viz.*, *F*, *G*, *H*, *I*, *J*, *K*, *L* and *M*. Two of the committees have three members and one of the committees has only two members. Certain other conditions are as follows:

(i) *G* serves with *M*.
(ii) *L* serves with only one other person.
(iii) *F* does not serve with *M*.

24. If *K*, *J* and *I* serve on different committees, which one of the following must be true?
(a) *K* serves with *G*
(b) *I* serves on a committee of two
(c) *I* serves on a committee of three
(d) *H* serves on a committee of two

25. Which of the following is inconsistent with the given conditions?
(a) *K* serves with *H*
(b) *M* serves with *H*
(c) *M*, *H* and *I* serve together
(d) *F* does not serve with *G*

ANSWER KEY

Exercise 1

1. (a)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (b)
9. (a)	**10.** (d)	**11.** (d)	**12.** (a)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (d)
17. (b)	**18.** (c)	**19.** (b)	**20.** (d)	**21.** (a)	**22.** (c)	**23.** (c)	**24.** (a)
25. (a)							

Exercise 2

1. (b)	**2.** (b)	**3.** (d)	**4.** (e)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (a)
9. (a)	**10.** (c)	**11.** (e)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)
17. (a)	**18.** (b)	**19.** (b)	**20.** (a)	**21.** (d)	**22.** (c)	**23.** (c)	**24.** (b)
25. (c)							

Exercise 3

1. (d)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (d)
9. (c)	**10.** (d)	**11.** (c)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (d)	**16.** (a)
17. (d)	**18.** (d)	**19.** (b)	**20.** (d)	**21.** (d)	**22.** (c)	**23.** (b)	**24.** (a)
25. (d)							

HINTS AND EXPLANATIONS

Exercise 1

Solution to questions 7–9

Five categories of the managers are:

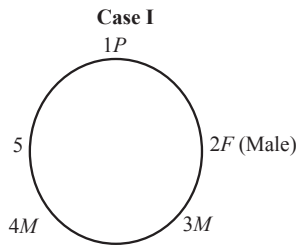
1. Production (*P*); 2. Marketing (*M*); 3. Human (*H*); 4. Finance (*F*) and 5. Management Information System (*MIS*).

Number of men = 3 and number of women = 2.

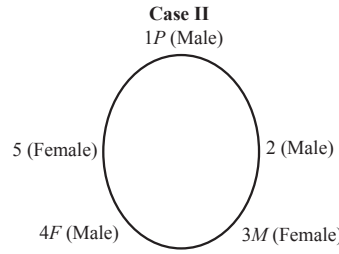
F will always sit next to a woman hence he is a male. The production manager is a male.

7. If *H* is a male:
Male—*H*, *F*, *P*
Female—*M*, *MIS*
As *M* herself is a woman so she cannot sit between the two women.
The statement in option (a) is definitely not true.
Hence, the answer is option (a).

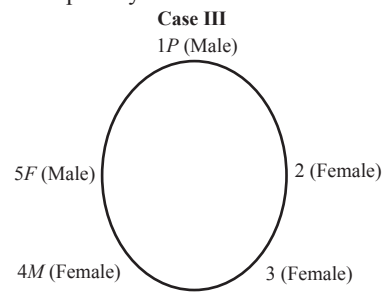
8.



Look at the above figure, finance (*F*) cannot be at the second position as he always has a woman to his right whereas *P*, *i.e.*, production manager is a man.

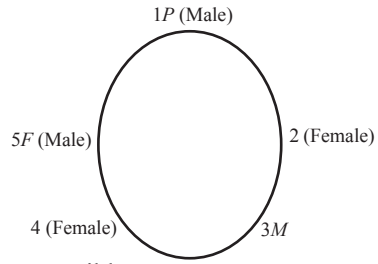


Assuming *F* is at the 4th position. Now *M* should be at the 3rd position because *P* should be seated as far as possible from *M*. Now *M* is a female as *M* is at the right of *F*. So 2nd position is occupied by a male as females cannot sit together. So the 5th position is occupied by a female. This is the 1st way.



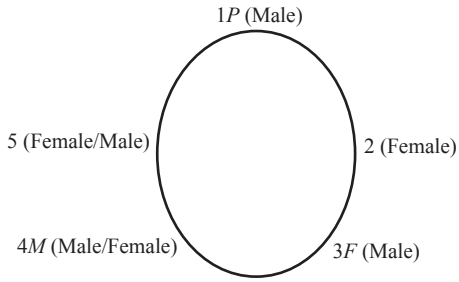
When F is at the 5th position and the 4th position is occupied by M who is female because she is on the right of F. Therefore, the 2nd position is occupied by a female and the 3rd is occupied by a male. This is the 2nd way.

Case IV



Not possible.

Case V



When F is at the 3rd position, the 2nd is occupied by a female. Now the 4th position is occupied by M who can be a male/female. But we can take a case from here in which M is a female. In that case the 5th position is occupied by a male.

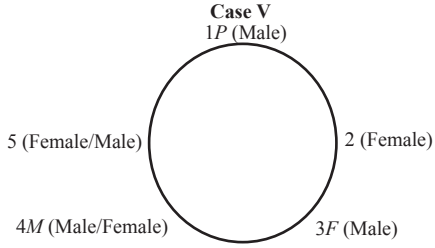
This is the 3rd way.

Hence, in all, three arrangements are possible.

So, the answer is option (b).

9. We can solve by negating the options given:

Case V



If MIS is a female and M is at the 4th position then the 3rd position can be occupied by F who is a male and the 2nd position by H who can be a female. So the condition III negated that H has to be a woman.

Now, let us see another case where H is a male. Now the 2nd option also gets negated that H has to be a female.

Therefore the answer is (I) only. Hence, the option (a) is the correct answer.

Solution to questions 10–14

See Table 3.33.

10. For 40 children (at the bottom of the table) the waiting time is zero. Hence, option (d) is correct.

11. A total of 20 males go through all the three centers in the minimum possible time. At 7.30 a.m. they enter the centre and at 10.35 a.m. they exit from the steam bath centre. Hence, 10% of the males took the minimum possible time. Therefore, option (d) is correct.

12. A total of 10 women took the minimum time (at 8.30 a.m. they enter and at 10.25 a.m. they exit from the steam bath centre). Similarly, 8 females took the maximum time (at 8.30 a.m. they entered whereas at 11.20 a.m. they exit from the steam bath centre). So, the difference between the maximum and the minimum is $10 - 8 = 2$. Hence, the option (a) is correct.

13. The maximum time taken by any male (5) is 135 minutes (from 7.30 a.m. to 10.45 a.m.). The maximum time taken by a female (8) is 170 minutes (8.30 a.m. to 11.20 a.m.) which is greater than the maximum time taken by any male. Hence, option (d) is correct.

14. At 9 a.m. all the males will be out of the skating centre and the only people inside the centers will be females. The total number of females inside the skating

Table 3.33

	N1	Skating			N2	Soft ball			N3	Steam Bath		
		In=R	Entry	Exit		R	Entry	Exit		R	Entry	Exit
Males	200	7.30	7.30	9.00	100	9.00	9.00	10.15	20	10.15	10.15	10.35
	50	8.30	8.30	9.30	20	9.30	9.30	10.10	5	10.15	10.25	10.45
					10	9.30	10.10	10.50	10	10.10	10.10	10.25
					10	10.10	10.10	10.50	5	10.50	10.50	11.05
Female	110	8.30	9.00	10.00					5	10.50	10.50	11.05
					56	10.00	10.15	10.55				
									20	10.55	10.55	11.10
	90	9.00	9.00	10.00	44	10.00	10.15	11.45	8	10.55	11.05	11.20
					1	10.00	10.50	12.20				
Kids	50	9.00	9.30	10.30	19	10.30	10.50	12.20				
					6	10.30	10.55	12.25				
	80	9.00	10.00	11.00	40	11.00	11.00	12.30				

activity centre at 9 a.m. = 160. Hence, 90 more members can be accommodated inside. So, (220 – 90) 130 children will be waiting.

Solution to questions 15 and 16

15. Only statements I and II are definitely true.

16. The correct ones should be either $PP + BB$, or, $PP + BC$, or $BC + PC$.

Solution to questions 17–22

17.

2006	2005	Country	2004
57	*A*	Australia	*D*
58	*B*	Kenya	*E*
59	*C*	Mozambique	*F*

The possible ranks for these three countries in 2005 = 51/53/58.

The minimum difference will occur if the signs of the change (+ or –) for these three countries are the same.

57	51	Australia
58	58	Kenya
59	53	Mozambique

Hence, net change = 12.

Alternatively, for minimum change = Sum of ranks in 2006 – sum of ranks in 2005 = (57 + 58 + 59) – (51 + 58 + 53) = 12.

18.

57	51	Australia	60
58	53	Kenya	58
59	58	Mozambique	56

Hence, the maximum value of $D + E + F$ = 9 + 5 + 2 = 16.

19.

57	51	Australia	56
58	53	Kenya	57
59	58	Mozawmbique	58

Min value of $D + E + F = 9$.

20. Cannot be determined because the ranks in 2005 and 2004 can be interchanged to give different answers.

21. The ranks available for 2004 = 56/57/58/60. Obviously the ranks 57 and 58 can be given to Australia and Kenya in 2006 as well as 2004. Hence, the difference = 0.

22. Since the question asks for possibility, we will have to see all the different options possible.

Ranks available for 2005 = 51/53/58 and

Ranks available for 2004 = 56/57/58/60.

We can see that rank 58 is available for both the years 2004 and 2005. The same rank can be given to Kenya during the whole given period.

Exercise 2

Solution to questions 7–11

Shikari–Juari = 6. Prince Shikari and Prince Juari will always have a difference of 6 points in their overall points. Though any one of them may have 6 points more than the other.

Considering the overall points, Bhikhari > Atyachari.

Also Prince Shikari scores 4/8 point in the 2nd round and 2/4/6 points in the 4th round.

Solution to questions 12–15

We can conclude the following:

12. If Prince Juari wins the competition and marries the princess and Prince Bhikhari scores 28 points from the given 5 rounds, then the Table 3.34 will be as given below:

Table 3.34

Prince	1st Round	2nd Round	3rd Round	4th Round	5th Round	Total
Shikari	9	4	8	2	3	26
Bhikhari	7	2	3	9	7	28
Atyachari	2	9	4	8		
Juari	4	6	9	5	8	32
Hazari		3			9	

If Shikari has scored points in the 2nd and the 4th rounds then the score of Juari should be 32 (*i.e.*, 6 more). And in this case Juari has to score 9 points in the 3rd round. Option (a), (b) and (d) will definitely be true for all cases. Only statement (c) could be false.

Hence, the correct option is (c).

13.

Table 3.35

Prince	1st Round	2nd Round	3rd Round	4th Round	5th Round	Total
Shikari	9	8	8	6	3	34
Bhikhari	7	2	3	9	7	29
Atyachari	2	9	4	8	4	27
Juari	4	6	5	5	8	28
Hazari		3			9	

Since, The total points scored by Juari are 28 so, Shikari will score a total of 34 points. So, he must have scored 8 points in the 2nd round and 6 points in the 4th round. Also, Prince Atyachari has scored a total of 27 points. Hence, Prince Bhikhari must score 29 points (as there are no ties) so Bhikhari scores 8, 9, 7 points respectively from the 1st, 4th and the 5th rounds. As it can be clearly seen from the Table 3.34 that Atyachari is fourth or fifth even though we do not know about Hazari's score.

So, option (c) is the correct answer.

14. According to statement I, Prince Atyachari can score a maximum of 5 points in the 5th round which makes his total points 28. As Prince Bhikhari can score a maximum of 29 points and Atyachari's score is always less than Bhikhari's score we get table as given below (Table 3.36):

Table 3.36

Prince	1st Round	2nd Round	3rd Round	4th Round	5th Round	Total
Shikari	9		8	6		
Bhikhari	8	2	3	9	7	29
Atyachari	2	9	4	8	5	28
Juari	4	6		5	8	
Hazari		3	2		9	

According to statement II, Juari scores the maximum possible point which is 9 in the 3rd round. Then his total will be 32. Also, Hazari can score a maximum of 30 points if he scores 7 points each in the 1st and the 4th

rounds. So, Juari will always be the winner with a total of 32 points (Table 3.37).

Table 3.37

Prince	1st Round	2nd Round	3rd Round	4th Round	5th Round	Total
Shikari	9	4	8		2	26
Bhikhari	8	2	3	9	7	29 (max)
Atyachari	2	9	4	8	5	28 (max)
Juari	4	6		5	8	32
Hazari		3		7	9	

Hence, statement II alone is sufficient to answer the question.

Therefore, option (b) is the answer.

15.

Table 3.38

Prince	1st Round	2nd Round	3rd Round	4th Round	5th Round	Total
Shikari	9		8		3	26
Bhikhari	8	2	3	9	7	29 (max)
Atyachari	2	9	4	8	5	28 (max)
Juari	4	6		5	8	32
Hazari	7	3		7	9	

The minimum score required by Hazari to win is 33 points. When Prince Atyachari scores 5 points in the 5th round, then the total points scored by him are 28. So, Prince Bhikhari must score more than 28 points. But taking into consideration the given Table 3.38, Prince Bhikhari can score a maximum of 29 points if he scores 8, 9, 7 points in the 1st, 4th and the 5th rounds respectively. But, one among Juari and Shikari will at least score 32. And Hazari can score a maximum (for the 3rd round to be minimum) of 7 points each in the 1st and the 4th rounds. So, the least possible points scored by Prince Hazari in the 3rd round = 33 – (7 + 3 + 7 + 9) = 7 which makes his total score as 33. Therefore, option (c) is the answer.

Solution to questions 16 and 17

On the basis of the given information, we can make the following table (Table 3.39):

Table 3.39

Cards	Tag	Baseball
I		
M	*K*	*N*

16. If *K* plays the same two games as *N* plays, this means that *K* and *N* will play tag and baseball. Therefore, options (a), (c) and (d) are true and option (b) is false.

17. *I* and *N* play tag and the fourth condition says that *N* must play baseball. Since no player can play more than two games, *N* cannot be one of the other two other persons playing cards. So options (b) and (c) are ruled out.

Option (d) cannot be the answer because *O* and *L* have to play together. Therefore, option (a) is the correct answer.

18–20

There are six shops *U*, *V*, *W*, *X*, *Y* and *Z* arranged in two rows such that the number of shops in both the rows are equal in number.

18. Using the given statements, we can conclude that:

North-facing shop: *X*, *Y*, *V*

South-facing shop: *W*, *Z*, *U*

Hence, there are four arrangements possible.

U	*Z*	*W*
V	*Y*	*X*

OR,

U	*Z*	*W*
X	*Y*	*V*

OR,

W	*Z*	*U*
V	*Y*	*X*

OR,

W	*Z*	*U*
X	*Y*	*V*

Hence, option (b) is the answer.

19. The shop between *V* and *X* is *Y*.
Hence, the answer is option (b).

20. It can be seen that diagonally opposite to each other are *U* and *Y*.
Hence, the answer is option (a).

Exercise 3

Solution to questions 1–6

From the given information:

Room	Country	Cars	Donations
101			
102			
103		12	8
104		(y)	$> 4y$
105		8	$x - 2$
106			

Here, the Brazil businessmen can stay in either room number 105 or room number 106. But from the options of the first question of the set, it must be 105. Let us fill the other data from it.

Room	Country	Cars	Donations
101	Canada	–	$y + 2$
102	Uruguay	24	
103	England	12	8
104	Argentina	4	18
105	Brazil	8	y
106	Germany	16	24

1. The Brazilian businessman is staying in room number 105.

2. The Argentinean businessman donated to 18 institutions.

3. The German businessman is staying in room number 106.

4. The German businessman donated to 24 institutions.

5. From the given options, the businessman from Uruguay owns the maximum cars.

6. The businessman from England has 12 cars.

Solution to questions 7–11

7. Using (I) if value of $P = 40$, then on two consecutive years 2001 and 2002, it has remained the same. This is only possible if the average in 2002 = 40.
Using (II) despite the value of *P* not given, we can find all the other things if *A*, *B*, *C*, *D* and *E* are given as equal. Hence, this statement is wrong and only I is true

8. Using I, if the value of $P = 40$, then there are two possibilities for 2002. Either all the cars have been sold for 40 itself or no car has been sold. Hence, this statement is incorrect. Using the given information in II we cannot find out. The average price of all the cars sold in 2002.
Hence, none of the two statements is true.

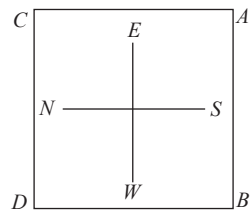
9. We need all the unknowns to find the value.

10. Using the solution given for Q. 8, if $P = 30$ or $P = 40$ is given, we can find that either no car has been sold or all the cars have been sold at the same price. Still we have the total freedom to increase the number of cars sold in 2000 or 2004, etc. We should also not forget that using the average we cannot find out the number of articles involved.
Hence, this cannot be determined uniquely.

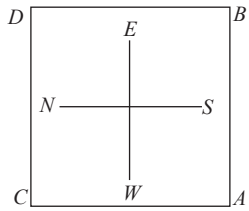
11. From the given options if $P = 30$, then in 2003 no car is sold.

Solution to questions 12–14

12. The position will be like this:



13. The position will be like this:



14. This is similar to the above question. Hence, at the north-west corner.

Solution to questions 20–23

Mintu Babu's parents (1 M, 1 F) live in the smallest house.

Mintu Babu stays with her children in the medium house.

So, Mintu Babu's brother stays in the largest house.

20. Given that 4 females and 2 males live in smaller house, 3 females and 1 male stay in Mintu Babu's house.

Hence, Mintu Babu has 3 children (1 boy and 2 girls).

21. The only relatives of Mintu Babu are:

U, staying in the larger house, is her brother.

S, staying in the smaller house, is her father/mother.

Hence, the statement 'd' cannot be true.

23. From Q. 20, it is clear that Mintu Babu has 3 children.

Solution to questions 24 and 25

24.

Committees	Two Members
K, G, M	*I, L*
J, F, H	*J, L*
I, F, H	

25. Use the list of the people given above.

Section Test 1

Quantitative Aptitude, Data Interpretation and Logical Reasoning

- There are sixty (60) questions in this test.
- Each question carries 4 marks.
- There is one-fourth negative marking.
- Total time allotted = 70 mins
- Qualifying marks for this test = 108 marks

1. Rahul mixes a solution '*A*' having water and oil in the ratio 3:4 with another solution '*B*' having oil and turpentine in the ratio 5:6. How much solution '*A*' and solution '*B*' should he mix so that the amount of oil in the new mixture is 9 litres (consider all values are integers)
(a) 11 litre and 7 litre
(b) 7 litre and 11 litre
(c) 9 litre and 9 litre
(d) 9 litre and 7 litre

2. Likhit earns x% on the first ₹ 2,000 and y% on the rest of his income. If he earns ₹ 700 from ₹ 4,000 income and ₹ 900 from ₹ 5,000 income, find x%.
(a) 20%
(b) 15%
(c) 25%
(d) None of these

3. Given $A = 2^{65}$ and
$B = (2^{64} + 2^{63} + 2^{62} + ... + 2^{0})$,
which of the following is true?
(a) $A = B$
(b) B is larger than A by 1
(c) A is larger than B by 1
(d) Cannot be determined

4. Six X's have to be placed in the squares of the Fig. 1 such that each row contains at least one X.

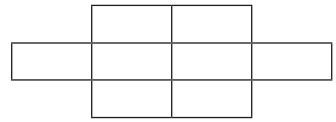


Fig. 1

(a) 28 (b) 26
(c) 36 (d) None of these

5. I have an unlimited supply of coins of the denominations ₹ 3, ₹ 6 and ₹ 20. How many amounts between ₹ 20 and ₹ 30, I cannot pay using these coins?
(a) 2 (b) 3
(c) 4 (d) None of these

6. When 98 is added to a perfect square, it gives another perfect square. How many such perfect squares are there?
(a) 1 (b) 2
(c) 3 (d) None of these

7. How many years would it take for a sum of grow from ₹ 1,250 to ₹ 10,000 if it is invested at 12.5 pcpa at SI?
(a) 28 (b) 56
(c) 48 (d) 70

8. Eleven oranges are bought for ₹ 10 and 10 oranges are sold for ₹ 11. What is the profit or loss percentage?
(a) 21% loss (b) 21% profit
(c) 10% profit
(d) No profit no loss

9. In a race of 100 m, A beats B by 20 m or 4 secs. What is the speed of A?
(a) 5 m/s (b) 10 m/s
(c) 6.25 m/s (d) None of these

10. Likhit purchases some items: buckets, mugs, glasses, pots and spoons. He bought at least five units of each of these. All the numbers bought are distinct and he does not buy anything else. Given that he bought the least number of buckets, the number of buckets he bought can be determined if the total number of items bought is at most:
(a) 37
(b) 39
(c) 41
(d) For all values > 41

11. Let N be an integer not divisible by 5. Which of the following numbers must leave 1 as the remainder when divided by 5?
(a) N^2 (b) N^4
(c) N^6 (d) N

12. In Fig. 2, $ABCD$ is a cyclic quadrilateral and $AB = 25$ cm, $BC = 39$ cm, $CD = 52$ cm and $AD = 60$ cm. What is the diameter of the circle?

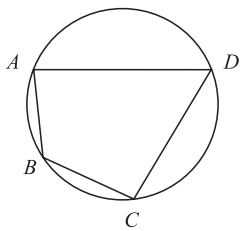


Fig. 2

(a) 60 cm (b) 65 cm
(c) 72 cm (d) 78 cm

13. How many natural numbers are there which give a remainder of 41 after dividing 1997?
(a) 2 (b) 4
(c) 6 (d) None of these

Direction for questions 14–17: ***Go through the chart given in Fig. 3 and solve the questions based on it.***

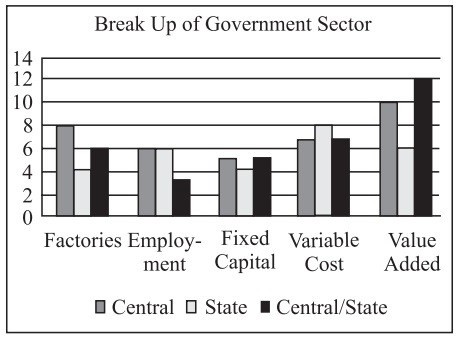


Fig. 3

Table 1 presents the percentage composition of five verticals under different sectors viz. Government, Public, Private, Joint.

For example, of all the factories, 18% are owned by Government.

Further, Government sector can be further broken down into three sub-sectors—Central, State, Central/State. Figure 3 gives a break-up of the Government sector data (as given in Table 1).

14. If the total work force was 76 million whereas the total value added was ₹ 225 million, then which of the following had the maximum value addition per worker?
(a) Central (b) State
(c) Central/State (d) Public

Table 1

Sector	Factories	Employment	Fixed Capital	Variable Cost	Value Added
Government	18	15	14	22	25
Public	12	8	6	19	8
Private	55	65	72	54	62
Joint	15	12	8	5	5
Total	100	100	100	100	100

15. Which of the following sectors has the maximum fixed capital invested per factory?
(a) Central (b) State
(c) Central/State (d) Public

16. If the variable is proportional to the number of employees and the production per employee, then for which of the following is the production highest?
(a) Government (b) Private
(c) Joint (d) Public

17. If the government has a fixed capital of \$200 million in the Iron & Steel industry, which corresponds to 20.012% of its total investment as fixed capital, then how much did the government invest (in ₹ million) in Maruti Udyog Ltd. which forms 25% of the investment in the joint sector? (1US \$ = ₹ 45)
(a) 7860 (b) 2500
(c) 143 (d) 145

18. Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone. 40 of them had both, a two-wheeler and a credit card, 30 had both, a credit card and a mobile phone and 60 had both, a two wheeler and mobile phone and 10 had all three. How many candidates had none of the three?
(a) 0 (b) 5
(c) 20 (d) 10

19. In a chess tournament every person played one game with every other person in the group. The total number of games that men played between themselves exceeded those played by men with women by 18. If there were 4 women in the tournament, totally how many games were played in the tournament?
(a) 112 (b) 120
(c) 162 (d) 190

20. The height of a tree varies as the square root of its age (between 5 and 17 years). When the age of a tree is 9 years, its height is 4 feet. What will be the height of the tree at the age of 16 (in inches)?
(a) 64 (b) 72
(c) 48 (d) 68

21. A can do a piece of work in 36 days, B in 54 days and C in 72 days. All three began the work together but A left 8 days and B 12 days before the completion of the work. For how many days in all did C work?
(a) 24 days (b) 28 days
(c) 32 days (d) 36 days

22. LCM of three consecutive natural numbers $(a, b, c) = N$. How many different value/s of HCF (a, b, c) is/are possible?
(a) 1
(b) 2
(c) 3
(d) More than one but finite

23. P works twice as fast as Q, whereas P and Q together can work three times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings?
(a) 2:1:1 (b) 4:2:1
(c) 4:3:2 (d) 4:2:3

24. There are fifty successive percentage discounts given in a series of 2%, 4%, 6%, 8% ... and so on. What is the net discount?
(a) 98% (b) 2550%
(c) 100% (d) Infinite

25. Everyone shakes hands with everyone else in a room. Total number of handshakes is 66. Find the number of persons.
(a) 10 (b) 11
(c) 12 (d) 13

Direction for questions 26–29: ***Go through the data set given below and solve the questions based on it.***

Table 2 gives the top 7 sectors attracting FDI equity inflow during the period 2001–02 to 2010–11 (as given in Cumulative Inflows which is the summation of all the equity inflows during the period 2001–02 to 2010–11 in that sector).

Table 2

Sectors attracting maximum FDI Equity Inflows (₹ Crores)					
Rank	**Sector**	**2008-09**	**2009-10**	**2010-11**	**Cumulative Inflows**
1	Services Sector	28,516	20,776	15,539	120,771
2	Computer	7,329	4,351	3,571	48,400
3	Telecom	11,727	12,338	7,546	48,220
4	Real Estate	12,621	13,586	5,149	43,192
5	Construction	8,792	13,516	5,077	40,770
6	Automobiles	5,212	5,754	6,008	26,831
7	Power	4,382	6,908	5,709	26,712

26. Which sector has attracted third highest FDI equity inflows from the year 2001–02 to 2007–08?
(a) Computer (b) Telecom
(c) Real Estate (d) Power

27. Telecom sector accounts for 8% of total cumulative inflows during the whole period. What percentage of total cumulative inflows is accounted for by Services sector during the whole period?
(a) 23%
(b) 20%
(c) 18%
(d) Cannot be determined

28. Out of the mentioned sector, how many sectors have seen a consistent rise or consistent decline across the years from 2008–09 to 2010–11?
(a) 2 (b) 3
(c) 4 (d) 1

29. For the year 2008–09, which sector accounts for the maximum percentage of the total FDI equity inflows of that particular sector for the whole period 2001–02 to 2010–11?
(a) Computer (b) Telecom
(c) Real Estate (d) Services Sector

30. There are three inlet taps whose diameters are 1 cm, 2 cm and 3 cm respectively. Rate of flow of water is directly proportional to the square of the diameter. It takes 9 mins for the smallest pipe to fill an empty tank. Find the time taken to fill an empty tank when all the three taps are opened.
(a) 9/14 (b) 14/9
(c) 3 (d) None of these

31. AB is the diameter of the given circle (Fig. 4), while points C and D lie on the circumference as shown. If AB is 15 cm, AC is 12 cm and BD is 9 cm, find the area of the quadrilateral $ABCD$.

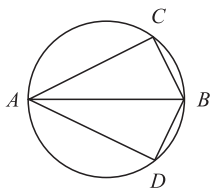


Fig. 4

(a) 54 sq cm (b) 216 sq cm
(c) 162 sq cm (d) None of these

32. How many triplets x, y and z can be chosen from the set {1, 2, 3, ..., 49, 50} such that $x \le y$.
(a) ${}^{50}C_2$ (b) ${}^{50}C_2 + {}^{50}C_3$
(c) ${}^{49}C_1\ {}^{50}C_1$ (d) None of these

33. The area of the square $ABCD$ is 64. Let E, F and G be mid-points of DC, AD and BC respectively. If P is any point inside the rectangle $ABGF$ and if X is the area of the triangle DPE, then which one of the following is true (Fig. 5)?
(a) $8 < X < 16$ (b) $8 < X < 32$
(c) $16 < X < 32$ (d) $16 < X < 64$

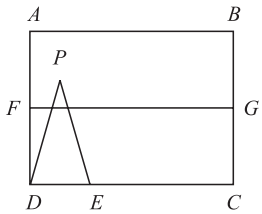


Fig. 5

34. *A* speaks truth in 75% of the cases and *B* in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

(a) 15 (b) 5
(c) 20 (d) 35

35. In a referendum about three proposals, 78% of the people were in against of at least one of the proposals. 50% of the people were against first proposal, 30% against second proposal and 20% against third proposal. If 5% of people were against all the three proposals, what percentage of people were against more than one of the three proposals?

(a) 10 (b) 17
(c) 27 (d) 22

36. A polygon has 44 diagonals. What is the number of sides in this polygon?

(a) 8 (b) 11
(c) 12 (d) 9

37. Surface area of cuboids is 24 cm^2. Find the maximum volume of the cuboids?

(a) 4 cm^3 (b) 6 cm^3
(c) 10 cm^3 (d) 8 cm^3

38. Find LCM of 1000! and (1001 × 1002 × 1003 × 1004 × 1005 × 1006)?

(a) 1001! (b) 1005!
(c) 1006! (d) 1000!

39. Likhit bought 100 kgs of rice for ₹ 1,100 and sold it at a loss of as much money as he received for 20 kgs of rice. At what price did he sell the rice (₹/kg)?

(a) ₹ 9.09 (b) ₹ 9.16
(c) ₹ 9.33 (d) ₹ 10

Direction for questions 40–43: ***Go through the chart given in Fig. 6 and solve the questions based on it.***

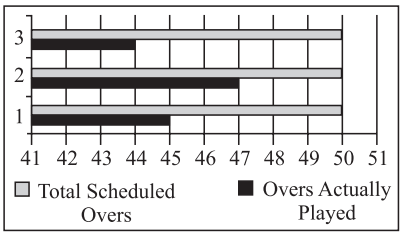


Fig. 6

In the game of cricket, there are 11 batsmen who bat during the match of 50 overs. First two batsmen come to bat first and when any one of the two gets out, next batsman come to bat alongwith the one of the two batsmen left out. And so on the next batsman comes to bat—in a fixed sequence for all the three matches. Only one batsman can get out for one ball. One over comprises of 6 balls—to be thrown one by one. Though if all the batsmen get out before scheduled 50 overs, match ends at that point (known as overs actually played).

Figure 6 shows total scheduled overs and overs actually played for 1st match (denoted by 1), 2nd match (denoted by 2) and 3rd match (denoted by 3).

Table 3 shows the runs scored in three matches series. Total runs scored = Runs scored by first Five players + Runs scored by remaining last six players + Extra runs.

Table 3

India vs Pakistan ODI Series—Runs Scored			
	1st Match	**2nd Match**	**3rd Match**
Runs scored by first five players	18	75	80
Runs scored by remaining last six players	187	72	70
Extra runs	22	18	17

$$\text{Average Runs per over} = \frac{\text{Total runs scored}}{\text{Overs actually played}}$$

40. In which of the matches, Indian Team's average runs per over is maximum?
(a) 1st match
(b) 2nd match
(c) 3rd match
(d) Either 1st or 2nd match

41. What would have the total runs scored in the 3rd match if Indian team would have played 50 overs with the same average which it maintained till the overs actually played?
(a) 180 (b) 190
(c) 200 (d) 170

42. Mr. Phani Ram, the team selector, has made following observations to improve the performance of the team:
(i) Performance of first five players have improved, whereas performance of last six players have gone down.
(ii) Had there been no extra runs, runs in 2nd match would have decreased by maximum percentage.
(iii) If team would have played for full 50 overs, it would have scored maximum in 1st match.
How many of these observations are true?
(a) Only (i) and (ii) but not (iii)
(b) Only (ii) and (iii) but not (i)
(c) Only (i) and (iii) but not (ii)
(d) All three

43. If extra runs are not considered, which match has the lowest average runs per over?
(a) 1st match
(b) 2nd match
(c) 3rd match
(d) Either 2nd or 3rd match

44. Forty percent of the employees of a certain company are men, and 75% of the men earn more than ₹ 25,000 per year. If 45% of the company's employees earn more than ₹ 25,000 per year, what fraction of the women employed by the company earn less than or equal to ₹ 25,000 per year?
(a) $\frac{2}{11}$ (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{3}{4}$

45. Consider the following data for a snacks shop 'Santushti' located at Mukherjee Nagar, Delhi:

	2009	2010
CP/Unit	A	B
SP/Unit	C	D

It is given that $B - A = D - C$ [All the values A, B, C and D are in natural numbers and ₹]
Which of the following is definitely true?
(a) Profit percentage per unit in 2010 is more than profit percentage per unit in 2009.
(b) Profit percentage per unit in 2009 is more than profit percentage per unit in 2010.
(c) Profit percentage per unit in 2010 is equal to the profit percentage per unit in 2009.
(d) Cannot be determined

46. A Geometric Progression consists of 1001 terms. Sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio of this GP.
(a) P_2/P_1
(b) P_1/P_2
(c) $(P_2 - P_1)/P_1$
(d) Cannot be determined

47. Let $N = \{x \mid x \text{ be a prime number and } x < 30\}$. The number of distinct rational numbers whose numerator and denominator belong to set N is:
(a) ${}^{10}C_2$ (b) ${}^{10}P_2$
(c) ${}^{10}C_2 + 1$ (d) ${}^{10}P_2 + 1$

48. In ΔDEF shown in Fig. 7, points A, B and C are taken on DE, DF and EF respectively

such that $EC = AC$ and $CF = BC$. If $\angle D = 40°$, then $\angle ACB =$

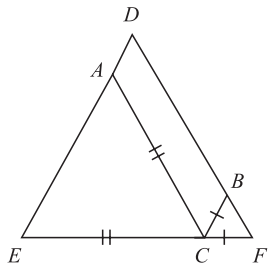


Fig. 7

(a) 140 (b) 70
(c) 100 (d) 50

49. $(17^{21} + 19^{21})$ is not divisible by:
(a) 36 (b) 8
(c) 9 (d) 18

50. If $x = 17 + 12\sqrt{2}$, then find the value of $x^3 + \frac{1}{x^3}$?
(a) 38202 (b) 39202
(c) 39302 (d) 38302

51. Consider the expression $y = x^2 - 2kx + k^2 + 1$. For which of the following values of K, y will be always greater than zero for any value of x?
(a) $K > 0$
(b) K is any real number
(c) $K > 2$
(d) $-2 < K < 7$

52. Inside a quadrilateral $ABCD$ (right angled at B), a circle is drawn in such a way that all the four sides of the quadrilateral are tangent to the circle. Circle touches side CD at P and side BC at R. Given that $PD = 9$ cm and $RC = 11$ cm. If radius of the circle = 5 cm, find the length of BC.
(a) 16 (b) 12
(c) 18 (d) 14

53. Using the digits 1, 2, 3 and 4, how many numbers of at most 4 digits can be formed (repetition allowed)?
(a) 340 (b) 256
(c) 64 (d) None of these

54. $x^2 + x + 1$ is a factor of $(x + 1)^n - x^n - 1$, whenever:
(a) n is odd
(b) n is odd and a multiple of 3
(c) n is an even multiple of 3
(d) n is odd and not a multiple of 3

55. Current sales tax is 8%. If sales tax had been only 5%, Junu would have paid ₹ 12 less sales tax. What was the total amount that Junu paid for the purchase including sales tax?
(a) 368 (b) 432
(c) 400 (d) 420

56. Consider the number
$= 11 \times 13 \times 17 \times 19 \times 23$
A decrease of 1 in which of the factors above would result in the greatest decrease in the product?
(a) 11 (b) 13
(c) 17 (d) 19

57. Of the positive integers that are multiples of 30 and are less than or equal to 360, what fraction are multiples of 12?
(a) 1/6 (b) 1/3
(c) 2/5 (d) 1/2

58. There are 1989 computers in all the schools of a state, which is approximately one computer for every 68.6 students. Of the following, which is the closest approximation, in thousands, of the number of students in that state?
(a) 132 (b) 137
(c) 142 (d) 144

59. What is greatest prime factor of $4^{17} - 2^{28}$?
(a) 2 (b) 3
(c) 7 (d) None of these

60. Define the function $f(x, y) = x + y + xy$. Natural numbers from 1 to 100 are written on the blackboard. A student comes and starts taking two numbers in order starting from the 1st pair as (1, 2), then

the resultant of $f(1, 2) = 1 + 2 + 1.2 = 5$ with 3 will be calculated as $f[f(1, 2), 3]$ and so on calculating the value of $f(x, y)$. What would be the final result at the end once all such values have been calculated?

(a) 100! – 1 (b) 101! – 1
(c) 101! + 1 (d) 100! + 1

MY PERFORMANCE SHEET IN SECTION TEST 1

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWER KEY

1. (b)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (b)	**6.** (d)	**7.** (b)	**8.** (b)
9. (c)	**10.** (b)	**11.** (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (b)
17. (a)	**18.** (d)	**19.** (b)	**20.** (a)	**21.** (a)	**22.** (a)	**23.** (a)	**24.** (c)
25. (c)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)	**31.** (d)	**32.** (b)
33. (a)	**34.** (d)	**35.** (b)	**36.** (b)	**37.** (d)	**38.** (d)	**39.** (b)	**40.** (a)
41. (b)	**42.** (c)	**43.** (b)	**44.** (d)	**45.** (d)	**46.** (d)	**47.** (a)	**48.** (c)
49. (b)	**50.** (b)	**51.** (b)	**52.** (a)	**53.** (a)	**54.** (d)	**55.** (b)	**56.** (a)
57. (d)	**58.** (b)	**59.** (c)	**60.** (b)				

HINTS AND EXPLANATIONS

1. Go through options. Volume of solution A should be a multiple of 7 and volume of solution B should be a multiple of 11.

Hence, option (b) is the answer.

2. He earns ₹ 700 from ₹ 4,000 income and ₹ 900 from ₹ 5,000 income

$\Rightarrow$ he earns ₹ 200 from ₹ 1,000

$\Rightarrow$ Rate of interest = 20% = y% (Fig. 8).

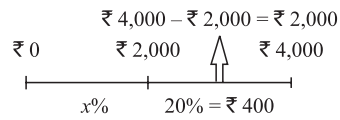


Fig. 8

So, Likhit earns ₹ 300 from first ₹ 2,000. Hence, rate of interest

$$= \frac{300}{2000} \cdot 100 = 15\% = x\%.$$

Hence, option (b) is the answer.

3. $B = (264 + 263 + 262 + ... + 20)$
$= (20 + 21 + 22 + ... + 264)$

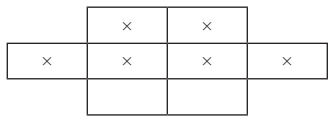
Using the summation formula of Geometric Progression:

$$B = (2^0)\frac{2^{65} - 1}{2 - 1} = 2^{65} - 1$$

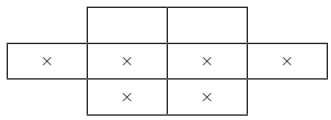
Hence, A is 1 more than B.

4. Total number of boxes = 8
Without any restriction, number of ways boxes can be filled = ${}^8C_6 = 28$
Out of which, following two cases are not to be counted.

Case 1:



Case 2:



Hence, total number of cases = 28 – 2 = 26

5. Following values cannot be paid:
= 22, 25, 28.
Hence, option (b) is the answer.

6. Difference between any two squares can be either odd, or if it is even, it has to be a multiple of 4. In other words, difference between two perfect squares cannot be an even number which is not a multiple of 4. Hence, no such value is possible.

7. Interest = ₹ 8,750, Principal = ₹ 1,250, Rate of interest = 12.5%

$$\text{Time} = \frac{\text{Interest} \cdot 100}{\text{Principal} \cdot \text{Rate}}$$

$$= \frac{8750 \cdot 100}{1250 \cdot 12.5} = 56 \text{ years}$$

Hence, option (b) is the answer.

8. Assume total 110 oranges are bought and 110 oranges are sold.

Total CP = ₹ 100 and Total SP = ₹ 121. Hence, profit percentage = 21%.

9. A beats B by 20 m or 4 secs $\Rightarrow$ it means B covers 20 m in 4 secs (and NOT A). Hence, speed of B = 5 m/s. So, time taken by

$$B = \frac{100}{5} = 20 \text{ secs.}$$

So, time taken by A = 4 secs less than B = 16 secs.

Hence, speed of $A = \frac{100}{16} = 6.25$ m/s.

Hence, option (c) is the answer.

10. Number of buckets is minimum = 5 (Assume), so minimum number of other items can be 6, 7, 8, 9. So, total number of items = 5 + 6 + 7 + 8 + 9 = 35.

For total number of items to be 36, 37, 38, 39, number of buckets has to be = 5 only. If total number of items becomes 40, number of items can be either 5 or 6. Hence, if there is maximum 39 items, number of buckets can be ascertained. But if there are 40 items, number of buckets cannot be ascertained. Hence, option (b) is the answer.

11. When we take $N = 3$, option (a), option (c) and option (d) can be eliminated. Hence, option (b) is the answer.

12. BC, CD and BD constitute a right-angled triangle (13 × 3, 13 × 4 and 13 × 5). If BD = 65, then it satisfies the other set of values of AB and AD also.

13. Let us assume that the quotient is Q and divisor is D.

Using the condition given in question, $1997 = QD + 41 \Rightarrow QD = 1956$. Now we will factorize 1956 in two parts such that D (divisor) is more than 41.

14. Value addition per worker

$$= \frac{\text{Value added}}{\text{number of employees}}$$

Total value added = ₹ 225 million and Total work force = 76 million.

We go by the options and calculate Value added per worker for all the given options:

For Central $= \dfrac{10\% \text{ of ₹ } 225 \text{ million}}{60\% \text{ of } 76 \text{ million}}$

For State $= \dfrac{3\% \text{ of ₹ } 225 \text{ million}}{60\% \text{ of } 76 \text{ million}}$

For Central/State

$$= \frac{12\% \text{ of ₹ } 225 \text{ million}}{3\% \text{ of } 76 \text{ million}}$$

For Public $= \dfrac{8\% \text{ of ₹ } 225 \text{ million}}{8\% \text{ of } 76 \text{ million}}$

Calculation Short Cut:

Inserted of calculating the real value in the above four options, what we can do is quick ratio comparison. We can observe that $\dfrac{\text{₹ } 225 \text{ million}}{76 \text{ million}}$ is present in all the ratios. Let us assume $\dfrac{\text{₹ } 225 \text{ million}}{76 \text{ million}} = N$.

So the four ratios become like the following:

For Central $= \dfrac{10\% \text{ of ₹ } 225 \text{ million}}{6\% \text{ of } 76 \text{ million}}$

$$= \frac{10\%}{6\%} N$$

For State $= \dfrac{3\% \text{ of ₹ } 225 \text{ million}}{6\% \text{ of } 76 \text{ million}}$

$$= \frac{3\%}{6\%} N$$

For Central/State

$$= \frac{12\% \text{ of ₹ } 225 \text{ million}}{3\% \text{ of } 76 \text{ million}}$$

$$= \frac{12\%}{3\%} N$$

For Public $= \dfrac{8\% \text{ of ₹ } 225 \text{ million}}{8\% \text{ of } 76 \text{ million}}$

$$= \frac{8\%}{8\%} N$$

It can be now clearly seen that the ratio is largest in case of Central/State. Hence, option (c) is the answer.

In such questions, we can proceed, in a nutshell, like following:

Value added per worker for various sectors will be proportional to following ratios:

Central	10/6
State	3/6
Central/state	12/3
Public	8/8

15. Using the method given in A1:

Fixed capital per factory will be proportional to the following ratios:

Central	5/8
State	4/4
Central/state	5/6
Public	6/12

Hence, option (b) is the answer.

16. If the variables are proportional to the number of employees and the production per employee, then we can say that the variable cost is directly proportional to the production. Since the variable cost is maximum for private sector, it will also have the maximum production.

Hence, option (b) is the answer.

17. Since options are not so close, we can take 20.012% = 20%

If 20% of govt. fixed capital is equal to \$200 million ⇒ Total govt. sector fixed capital = \$1,000 million which is equivalent to 14% of the total fixed capital. Now, 25% of Govt. investment in joint sector, *i.e.*, 25% of the 8% of the total = 2% of total fixed capital will be given by: $\dfrac{1000 \times 2}{14}$ = \$ 143 million

Value in ₹ is given by:

143 × 55 = ₹ 7,865 million

Hence, option (a) is the answer.

19. Assume there are m men. So number of matches where both the players are men

$$= {}^{m}C_2 = \frac{m(m-1)}{2}$$

Total number of matches where one player is male and other is a female

$$= {}^{m}C_1 \times {}^{4}C_1 = 4m$$

Given that $\frac{m(m-1)}{2} - 4m = 18$

$\Rightarrow \quad m = 12$

Hence, total number of matches

$$= {}^{m}C_2 + {}^{4}C_2 + 4m$$
$$= {}^{12}C_2 + {}^{4}C_2 + 4 \times 12$$
$$= 66 + 6 + 48 = 120.$$

Hence, option (b) is the answer.

22. HCF of any two consecutive natural numbers = 1. Hence, HCF of any three consecutive natural numbers = 1. So only one value is possible for HCF.

23. Assume that work done by Q in a day = 1 unit

So, work done by P in one day = 2 units

Work done by $P + Q$ in one day = 3 units = 3 times as fast as R. So work done by R = 1 unit.

So ratio of efficiency of P, Q and R = 2:1:1. Hence, they should be paid in the same ratio.

24. Last discount = 100%. Hence, net discount = 100%

25. Number of handshakes = ${}^{n}C_2$ [where n is the number of persons] $= \frac{n(n-1)}{2} = 66$

So, $n = 12$.

26. To solve this question, we need to add the FDI equity inflows from 2008–09 to 2010–11 and subtract this from Cumulative inflows. We are not required to do the actual calculation. We'll just take the Thousands value of all the years. We can also observe that answer has to be either Computer or Telecom (after doing preliminary observation of data).

Table 4

	2008–11	Remaining Years
Services Sector	63	57
Computer	14	32
Telecom	30	17
Real Estate	30	11
Construction	26	13
Automobiles	16	10
Power	16	10

Third highest is for Telecom. Hence, option (b) is the answer.

27. This is a simple calculation based question.

Given that 8% = 48,220

$\Rightarrow$ 1% = 6000 (Approx.)

So, 120,771 = 20%

Hence, (b) is the answer.

28. Following are the sectors:

Services Sector, Computer, Automobiles

Hence, option (b) is the answer.

29. We have to choose one of these four options. Since options do not mention 'none of these', we will focus upon these four options only.

Percentage to total FDI Equity in case of computer $= \frac{7329}{47700}$

Percentage to total FDI Equity in case of telecom $= \frac{11727}{48220}$

Percentage to total FDI Equity in case of Real Estate $= \frac{12621}{43192}$

Percentage to total FDI Equity in case of Services Sector $= \frac{28516}{120771}$

Let us first go through elimination:

In case of three options namely Computer, Telecom, Real Estate, Base (Denominator, *i.e.*, Cumulative Inflows) is approximately

same. Numerator of Real Estate is maximum, hence, out of these three options, Real estate is maximum.

So now we have to compare Real Estate with Services sector:

Real Estate $\frac{12621}{43192}$

Services Sector $\frac{28516}{120771}$

First of all, let us eliminate last two digits from the numerator and denominator from both the sectors:

Real Estate $\frac{126}{431}$ $\frac{285}{1207}$ Services Sector

IT can be seen that Numerator of Services sector is approximately 2.25 times of the numerator of Real Estate, but denominator of Services sector is more than 2.25 times of denominator of Real Estate (we just want to know if it is more than 2.25 or less than 2.25, we don't need the exact value) (Fig. 9).

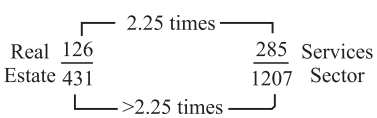


Fig. 9

Hence, Real Estate is the answer. Hence, option (c) is the answer.

31. Since AB is the diameter of the circle, $\angle ACB$ would be right angle. In this triangle, we know $AB = 15$ and $AC = 12$. So, we can find BC. Since 3–4–5 forms a triplet, 3 × (3–4–5) also forms a triplet.

So, 9–12–15 forms a triplet. Hence, $BC = 9$. Since $BC = BD$, $AD - AC$ (similar triangles).

Hence, area of ΔABC = Area of ΔABD $= 1/2 AC \times CB = 1/2 \times 12 \times 9 = 54$.

So, area of quadrilateral $ABCD = 2 \times 54 = 108$ sq cm. Hence, option (d) is the answer.

32. When the numbers are different, those can be chosen in ${}^{50}C_3$ ways. When $x = y$, then the numbers can be chosen in ${}^{50}C_2$ ways. Hence, answer = ${}^{50}C_3 + {}^{50}C_2$.

33. Given $DE = 4$ units. Let PQ be the altitude of the triangle DPE

$\therefore \quad 4 < PQ << 8 \rightarrow 1/2 \times 4 \times 4$

$<$ Area of triangle DPE

$< 1/2 \times 4 \times 8 \rightarrow 8 <$ Area < 16.

Hence, option (a) is the answer.

34. They will contradict if one of them is speaking the truth and other is speaking false

$= 0.75 \times 0.2 + 0.25 \times 0.8$

$= 0.15 + 0.2 = 0.35 = 35\%$ cases.

35. More than one proposal = Either two or three proposal = 12% + 5% = 17%

36. Number of diagonals $= \frac{n(n-3)}{2}$

Now use the option. Option (b) 11 satisfies this condition = 44 diagonals.

37. **Method 1:** Given that $2(ab + bc + ca) = 24$; if a, b and c are the sides of the cuboids.

$$\frac{ab + bc + ca}{3} \geq \sqrt[3]{ab.bc.ca}$$

$$abc \geq 8$$

Therefore, the maximum volume of the cuboids is 8 cm^3. Hence, option (d) is the answer.

Method 2: Maximum volume will occur when $ab = bc = ca = 8$ and $a = b = c = 2$.

38. $1001 = 13 \times 11 \times 7$

1002 is obviously a composite number when each prime factor will be less than 1000.

$1003 = 17 \times 59$

1004 is obviously a composite number when each prime factor will be less than 1000.

1005 is obviously a composite number when each prime factor will be less than 1000.

Hence, LCM will remain the same = 1000!

39. Assume SP/kg = x.
We know that $CP - SP$
$= \text{loss} \Rightarrow 1100 - 100x = 20x$
$\Rightarrow \quad 120x = 1100 \Rightarrow x = ₹\ 9.16/\text{kg}$
Hence, option (b) is the answer.

Solution to questions 40–43: ***Question provides two sets of data—Overs and Runs scored. While Total scheduled Overs is fixed at 50, overs actually played vary for all the three matches. Runs scored have been given in three parts—Runs Scored by 1st Five Players, Runs Scored by remaining Last Six Players and Extra Runs. Summation of these three components gives total runs. Further, Average runs per over has been defined.***

40. Clearly it can be seen that runs scored in the 1st match is much more than the total runs scored in either 2nd match or 3rd match. Though number of overs actually played is almost similar in all the three matches—45 overs in 1st match, 48 overs in 2nd match and 44 overs in 3rd match.
Since numerator (Total runs scored) is maximum in case of 1st match whereas denominator is almost similar, Average runs per over will be maximum for 1st match.
Hence, option (a) is the answer.

41. Total number of overs actually played in 3rd match = 44 overs
Total number of runs scored in 3rd match
$= 80 + 70 + 17 = 167$ runs
Method 1: Through actual calculation method:
Average runs per over in 3rd match
$$= \frac{167}{44} = 3.8 \text{ runs per over}$$
Total runs in 50 overs
$= 50 \times 3.8 = 190$ runs
Method 2: Through Approximation method:
$$44 \text{ overs} \xrightarrow{\text{Percentage increase of 12\%–13\%}} 50 \text{ overs}$$
Hence, runs scored will also increase by 12%–13%. Now we will not go through the actual calculation. A quick look at the options shows that options are not close, so we can have a good approximation.
Instead of increasing 167 runs (scored in 44 overs) by 12%–13%, we increase it by ONLY 10% to find a quick range of our answer.
$167 \xrightarrow{\text{Percentage increase of 10\%}} 167 + 16.7 \equiv 184$. Our actual answer will be a bit more than this. Option (b) is the closest one.
Hence, option (b) is the answer.

42. Evaluating the observations one by one:
Observation (i): Understand that you don't have to use your 'Cricket' knowledge to answer this question. Taking the data on its face value, it can be clearly seen that performance of first five players has improved whereas performance of last six players has gone down. Hence, observation I is correct.
Observation (ii): This is a close call. So let us go through the actual calculation.
Percentage decrease in case of Match 1
$$= \frac{22}{227} \times 100 = 9.69\%$$
Percentage decrease in case of Match 2
$$= \frac{18}{165} \cdot 100 = 10.9\%$$
Percentage decrease in case of Match 3
$$= \frac{17}{167} \cdot 100 = 10.17\%$$
So, maximum percentage decrease occurs in case of Match 3. Hence, Observation (ii) is wrong.
Observation (iii): Since the Average Runs per over is maximum in Match 1 (using answer to Q1), hence, runs scored in 50 overs would have been maximum in match 1. So observation (iii) is true.
So, only observations (i) and (iii) are correct.
Hence, option (c) is the answer.

43. This is simple calculation—similar to A1. Option (a) can be easily eliminated as this has the maximum runs scored. Though to decide between match 2 and match 3, we are required to calculate the actual value.

Average runs per over for Match 2 = 3.06 runs per over and average runs per over for match 3 = 3.40 runs per over. Hence, match 2 is the answer.

Hence, option (b) is the answer.

44. Men Women

40% 60%

Out of 40% men, 75% earn more than ₹ 25,000. Hence, 30% of the company (men) earn more than ₹ 25,000. But in all 45% of the employees earn more than ₹ 25,000. Hence, among women 15% earn more than ₹ 25,000 and the remaining (60–15)% earn less than or equal to ₹ 25,000.

Therefore, the fraction of women

$$= \frac{45}{60} = \frac{3}{4}$$

45. Profit percentage = Profit/CP

It can be seen that the profit amount remains the same the change in CP/unit = change in SP/unit. However we don't know that $A > B$ or $B > A$, so we do not have the information that if the base has increased or decreased. Hence, cannot be determined.

46. We can calculate this for equal number of odd terms and even terms.

For example, if the GP is = 1, 2, 4, 8, 16, 32

Common ratio = Sum of all the odd numbered terms/sum of all even numbered terms $= \frac{(1+4+16)}{(2+8+32)} = \frac{21}{42} = \frac{1}{2}$

Even if we take only first four terms, common ratio will remain same

$$\left[\frac{(1+4)}{(2+8)}\right] = \frac{1}{2}$$

However if the GP contains only five terms or seven terms, we cannot have unique value of common ratio.

47. $N = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$. A rational number will be made by taking any two elements of this set in any order.

Total number of elements = 10

Total number of ways in which two elements can be taken = ${}^{10}P_2$

Total number of distinct rational numbers = ${}^{10}P_2 + 1$ (including 1).

48. Option (c) is the answer (Fig. 10).

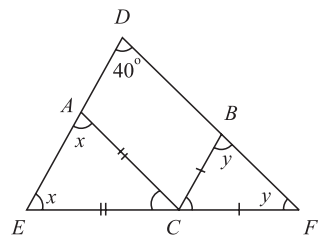


Fig. 10

Here $\angle ACE = 180 - 2x$,

$\angle BCF = 180 - 2y$

and $x + y + 40° = 180°$ (in ΔDEF)

So $x + y = 140°$

So $\angle ACB = 180° - \angle ACE - \angle BCF$

$= 180° - (180° - 2x) - (180° - 2y)$

$= 2(x + y) - 180°$

$= 2 \times 140 - 180 = 100°$.

49. This is divisible by 36. Hence, divisible by option (a), (c) and (d). Hence, option (b) is the answer.

50. Rationalize $\frac{1}{(17 + 12\sqrt{2})}$

$$\Rightarrow \quad \frac{1}{x} = 17 - 12\sqrt{2}$$

Hence, $x + \frac{1}{x} = 34$

Using the formula:

$$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

Hence, answer = $34^3 - 34 \times 3 = 39202$

Hence, option (b) is the answer.

51. $Y = (x - k)^2 + 1 > 0$. Hence, K can take any real value. Hence, option (b) is the answer.

52. $BC = BR + RC = 5 + 11 = 16$ cm. Hence, option (a) is the answer.

53. Total number of single digit = 4
Total number of two digits = 16
Total number of three digits = 64
Total number of four digits = 256
Total numbers = 4 + 16 + 64 + 256 = 340
Hence, option (a) is the answer.

54. Option (a) can be eliminated by assuming $n = 1$ and showing that it is not a factor.
Option (b) can be eliminated by assuming $n = 3$ and showing that it is not a factor.
Option (c) can be eliminated by assuming $n = 6$ and expanding it through binomial theorem to show that it is not a factor.
Option (d) should be the answer through elimination. Further, assume $n = 5$, and it can be seen that it is a factor. Hence, option (d) is the answer.

55. If x is the price without tax, then

$$1.08x - 1.05x = ₹\ 12 \Rightarrow 0.03x = ₹\ 12$$

Hence, $x = ₹\ 400$. So total amount paid by Junu = ₹ 432.
Hence, option (b) is the answer.

56. Go through the options.

57. Following are the multiples of 30 = 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360 = 12 numbers.
Out of these numbers, 60, 120, 180, 240, 300 and 360 are the multiples of 12 = 6 numbers.
Hence, $\frac{6}{12} = \frac{1}{2}\cdot$ Hence, option (d) is the answer.

58. Answer is $68.6 \times 1989 = 136445 = 137$
Thousands (approximately). Hence, option (b) is the answer.

59.
$$2^{34} - 2^{28} = 2^{28}(2^6 - 1) = 2^{28}(63)$$
$$= 2^{28} \times 7 \times 3^2$$

Hence, greatest prime factor = 7. Hence, option (c) is the answer.

60. This question can be solved by taking a small sample of (1, 2, 3, 4).

$$F(1, 2) = 5 = 3! - 1$$
$$F(5, 3) = 23 = 4! - 1$$
$$F(23, 4) = 119 = 5! - 1$$

Hence, in this case, the final answer would be = 101! – 1. Hence, option (b) is the answer.

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SECTION 2

Verbal Ability

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1 Reading Comprehension

WHAT IS READING COMPREHENSION?

Reading comprehension section in IIM Indore-IPM is advanced version of reading comprehension asked back school. Historically, the passages in IIM Indore-IPM have ranged from 350–1300 words, followed by a set of few questions. The passages are drawn from areas like pure sciences, social sciences, medicine, art, literature, technology etc. A student is supposed to answer the questions in light of the information given in the passage within the stipulated time. The ability of the student to make sense of the information and ideas presented in the passage is put to test through questions with multiple choice answers. More specifically, a person's ability to understand standard written English, analyse complex ideas, and ability to draw inferences from the written material is put to test.

The significance of Reading Comprehension, commonly known as RC can be gauged from the fact that almost half the questions in the Verbal Section are based on Reading Comprehension. Many students find RC to be tough and challenging, but with consistent practice, and patience, the art of Reading Comprehension can be mastered.

WHY IS READING COMPREHENSION ASKED?

RC is the most important and wide question type of the English section of IIM Indore-IPM and other B-School entrance test. RC section helps examiners in assessing the expertise of one in understanding the language. Besides, RC also checks how well a person understands someone else's (the author's) view point his preferences and prejudices.

1. Reading Comprehension is integral to success in IIM Indore-IPM.
   Reading is important not only for RC, but also for English Usage area (especially in the Critical Reasoning or FIJ or Sentence arrangement questions), and LR/DI section. Having a good reading habit will make more time available for solving the problem.
2. Reading is essential to do well in a B-School.
   Life in a B-School demands extensive reading and research for case studies, presentations, and business projects. Today's global manager has to keep pace with the latest happenings in the corporate world, political changes in the country and the world, changing preferences of the customer. Therefore, newspapers, journals and business magazines form the staple diet of a wannabe professional.
3. Reading helps in getting ahead in career.
   Recent researchers have tried to analyse the reading habits of adults working in different organizations at different levels of management hierarchy. One such research finds that people at the higher levels have more positive attitudes toward reading and spend more time in reading. It gives us an idea that reading helps probably because a well read person will have viewpoints and knowledge about diversified fields in his/her occupation, that may lead to faster growth.

Source: http://www.eric.ed.gov (sponsored by US Dept. of Education)

Let us now go through a sample RC passage:

Breaking the Ice with RC and English Usage

Read the passage carefully and answer the questions on the basis of the information supplied by the passage.

With Barack Obama taking oath yesterday as America's 44th and first African-American president, the United States turned a page and closed a chapter. Obama's spectacular success story is packed with poignant, and powerful, symbolism. If he accepted the Democratic nomination last August on the anniversary of Martin Luther King Jr's 'I Have a Dream' speech, his inauguration follows the American holiday in memory of King. It is the culmination of an extraordinary story and a new beginning. Obama rode on a ticket for change. A country left bitter, fearful and divided by eight years of George W Bush's presidency, welcomed him with relief and expectation. The world, which had viewed America with growing alarm during these years, tuned in to Obama as well. He represented hope that America would manage its own house responsibly and favour consensus and cooperation while dealing with the world. But as enormous as his moment in history are the challenges Obama will face from day one.

Undoubtedly, the gloomy economy will consume much of the new president's energies and he has so far shown signs of clear thinking on how to get America up on its feet again. Equally tough are the assortment of challenges that will present themselves on Obama's foreign policy plate. One war needs to be wound down responsibly while America's attention has to shift to the real battleground in Afghanistan and Pakistan. Obama cannot afford to engage Pakistan only to tackle al-Qaeda and the Taliban. To continue the world's war against terror, he will have to pursue the other extremist outfits like Lashkar-e-Taiba and its front organizations which export violence from that country. They have had a generally free run despite Pakistan's claims to the contrary. For the sake of the world's security, Obama must press Islamabad to clamp down on these groups and close down their bases, something that the Bush administration failed to do for most of its run. And then there is the Middle East mess. Trying to achieve a degree of resolution there will require fresh commitment and thinking from Washington. It is evident that Obama will have to hit the ground running. There are soaring expectations which cannot be all fulfilled. But he has a good base of credibility to start from. Opinion polls show he enjoys close to 80 per cent approval ratings as he picks up the keys to the White House and that the American people across political divides, are willing to give him a chance and their time. His commitment to consultative governance while being firmly in charge, and the A-list team he has picked, would hopefully serve America and the world well. Obama's inauguration party which has seen millions of Americans pour onto the streets to have a blast is a fine celebration of democratic ideals and values. Democracy's enabling promises are why Americans and those who share similar values elsewhere are raising a toast as they welcome President Barack Hussein Obama.

1. What does the author want to convey from the statement 'But as enormous as his moment in history are the challenges Obama will face from day one'?
   (a) His ascent heralds a significant change but it also poses many daunting tasks ahead.
   (b) The challenges in front of Obama are insignificant in comparison to his heroic stature.
   (c) This the biggest moment in the history of the US, but also marks the beginning of Osama's onerous journey.
   (d) Obama shall face the toughest challenges in the first phase of his presidency.

(e) Obama's historical win guarantees that he will be able to sail through the tough times easily.

2. What can be inferred about the policies of Obama's predecessor?
(a) His policies had less room for consensus and cooperation in world affairs.
(b) His policies lacked clear thinking on how to get America upon its feet.
(c) Pakistan was not engaged to tackle al-Qaeda and the Taliban.
(d) He showed lack of commitment to tackle the Middle East crisis.
(e) His policies were largely responsible for the looming financial crisis.

3. According to the passage, Obama is likely to face all the following major challenges except:
(a) To wind up the unresolved war.
(b) To improve the gloomy economic situation.
(c) To find a solution to the Middle East crisis.
(d) To put a curb on terrorism and terrorist organizations.
(e) To get the complete support of the White House and the American people push his plans ahead.

Answers and Explanation

1. The passage conveys that the change is remarkable but also throws new challenges ahead, hence option (a). The other options talk about something which is nowhere given or indicated in the passage.

2. It can be inferred from the second paragraph fourth line 'He represented hope that ... dealing with the world,' hence option (a).

3. The passage talks about 80 per cent approval ratings as he picks up the keys to the 'White House and that the American people across political divides', therefore 'to get the support of people' is not a challenge for him, so option (e).

How to Acquire Mastery over Reading Comprehension?

Knowing the weightage given to RC in the verbal section across the range of B-School entrance tests, a student is hardly left with any choice but to attempt at least a few passages.

In order to master the RC section, there are facts that a test-taker needs to know, prior to taking a test. These include:

Does Speed Matter?

Looking at the pattern of last few IIM Indore-IPM papers, average length of a passage have hovered around 900–1,000 words per passage including questions. Assuming that a student solves 3 passages in 25 minutes time, this leads to going through 3,000 words in 25 minutes ⇒ 120 words per minute. However, if a student goes through only two passages, then s/he will be going through only 2,000 words in 25 minutes ⇒ 80 words per minute.

Let us do a scenario analysis to understand it better (Table 1.1).

Table 1.1

	No. of passages	Words to be read	Reading Speed	No. of questions attempted	Accuracy	Net marks
Scenario 1	3	3000	120	15	80%	45
Scenario 2	2	2000	80	10	90%	35
Scenario 3	1	1000	40	5	100%	20

Obviously, this scenario analysis assumes many ifs and buts like there is no regression or there is 100 per cent comprehension, etc., however, it clearly drives home a basic point that with a speed of 80 words per minute and 90 per cent accuracy (at 80 words per minute, 90 per cent accuracy is not too high to expect), one can fetch 35 marks. Even if accuracy drops to 80 per cent, marks obtained in this section will be 30 marks under scenario 2.

Having said this, I strongly suggest pushing the limits and increasing the comprehension speed as much as possible, however not at the cost of accuracy.

Its more about Comprehension than Reading

There are two types of reading—reading with the eyes and reading with the brain.

Reading for RC is different from general reading where we read either to get some specific piece of information or we read at ease to relax or unwind ourselves after a long tiring day, and it may just be reading with the eyes ... But for the competitive exams we need to read with our brain because the purpose is different. And there comes the comprehension part. We read to find out the main idea of the passage and be accurate in answering the question, which even drains the physical energy. We read with sole objective to maximize our score, within a limited framework of time.

A practical suggestion can be given as to develop the habit of reading books of different genre, and developing a taste for intellectual debate and exploration. This can go a long way to develop the complete personality of the reader besides increasing the level of comfort in the RC section.

So, now what we focus upon is not reading speed but Comprehension speed.

Practice is the Key

Solving RC practice exercises regularly helps develop confidence and gives an exposure to the nuances of RC. An aspirant is suggested to find out the main-idea of the articles in the editorial section of the news-papers, so that identifying the main-idea comes naturally to a student as it is integral to answering questions in IIM Indore-IPM especially inferential ones.

And Finally, Analyse the Exercises Done

The work doesn't finish after solving the exercise. Analysing a test after solving is more important than students generally may think it to be. Analysis not only tells us where we went wrong and why we went wrong but also how those mistakes are to be avoided thence.

How to Increase your Reading Speed?

We have already discussed that with a decent speed of 80–100 words per minute only, RC can be managed for IIM Indore-IPM, although importance of improving the reading speed (or for that matter improving anything else too) cannot be denied.

Generally, we read slowly because we believe that if we read slowly we will understand the things better. On the other hand, reading slowly leads to loss of concentration as our mind gets easily distracted, which leads to loss of interest, which further lowers down the reading speed.

It is important to understand how we read. Generally we read one to two words at a glance, pause for a fraction of a second and move to the other chunk of words. If we can train our eyes to read more number of words in a glance, our reading speed can increase manifold. With consistent practice, it may be possible to read the lines of a newspaper column in a single glance.

While reading a piece of text, we have tendency to move back, cross check and re-read the things which we have already read. Generally, this happens because of lack of interest, complex vocabulary, or poor retention of the reader. This is known as regression or skip back. The habit of regression has to be minimized if not eliminated. Pausing once in a while to figure out the contextual meaning of words, however, is not regression.

Regression is a big time killer. It breaks the flow of thought, and leads to poor concentration, which leads to more regressions, which further lowers down the reading speed. Thus regression makes reading a slow and tiresome process. The best way to eliminate regression is to move a pen or pencil smoothly under the text and make your eyes follow the text. This is a great way to train oneself to read faster.

Some of us move our lips while reading. We tend to mumble the words audibly or inaudibly. The habit of sounding out words while reading is known as *vocalization*. The habit has its roots in our childhood when we were asked by our teachers in school to read aloud. Even at our homes, our mother would ask us to read aloud, when she used to be busy with the house chores. You could focus on the text (as there was no choice!), but how much of that could you understand? The problem with vocalization is it limits our reading speed, although it may help the reader to fix his mind on the subject matter.

Please note that as you experiment with these speed enhancement techniques, your comprehension will dip initially. Your mind may revolt, as many of us have been regressing and vocalizing throughout our school and college life. With a firm faith to improve reading effectiveness, coupled with consistent practice you can increase your reading speed manifold.

Instructions

Move your pen/pencil under the line of text uniformly and let your eyes follow the movement of the pacer (pen/pencil) and answer the following questions with Yes or No.

Is Muntazer al-Zaidi, the Iraqi who hurled his Number 10 footwear at George W Bush during a press conference in Baghdad, a hero or a heel, in more ways than one? Opinion is sharply divided on this. On the one hand or rather, foot there are those who feel that al-Zaidi's behaviour was totally unacceptable, breaching as it did the code of conduct for journalists who as opinion formers must stick to rules of parliamentary conduct in their professional lives. Conversely, the footloose, or shoe-loose, journo has been hailed as a champion by all those and there are many of them who feel that his was a robust and deserved retort-in-kind to Bush's jackbooted militarist policy in Afghanistan, Iraq itself, and elsewhere.

The Baghdad authorities have taken a dim view of the episode and al-Zaidi who has in a letter to the Iraqi prime minister pleading for clemency described his no-soles-barred attack as an "ugly act" could face two years imprisonment for his outburst. However, an enthralled Egyptian father has offered his daughter's hand in marriage to the feisty al-Zaidi, saying that he had nothing more valuable than his girl child to offer the barefoot guerrilla. Another admirer, a Saudi tycoon, had offered to buy the famous footwear for $10 million. Unfortunately, the sale could not take place as an embarrassed Iraqi officialdom had consigned the items in question to an incinerator.

While the jury remains out on whether al-Zaidi should eventually end up in the hall of fame or that of infamy, protocol officers and event managers of politicians and other public figures should put on their thinking caps to figure out ways and means to avoid or at least to minimize the impact of such incidents in future. All public rites and rituals marriages, funerals, birthday parties, press conferences are organized according to commonly accepted codes of behaviour. It is high time that the ritual of public protest so vital to democracy and the concept of a free society should have its own guide book of dos and don'ts; we need an etiquette of demonstrative dissent.

In this exercise, we might like to take a tip from the would-be novelist who, before a public reading of his latest work, went to the market and bought quantities of eggs and tomatoes which he distributed among the audience, explaining that if he were to be pelted for deficiencies in his prose he would prefer the missiles to be fresh and not old and foul-smelling. This would appear to be an eminently civilized way of ordering

such encounters: the protester would have the satisfaction of lodging, or lobbing, his protest, and the protestee the person being protested against would have the option of choosing the ammunition of the protester. In this way, both parties could claim satisfaction.

In the al-Zaidi-Bush case, such mutual gratification cannot be ruled out. While in the Arab world, and indeed in the Indian subcontinent, footwear with its unclean, animal skin associations is considered a particularly offensive projectile to have hurled at oneself, in the rawhide Texan context of 10-gallon hats and cowboy boots that the exiting US president hails from, similar qualms of ritual pollution need not apply.

In Islamic and Indic traditions, footwear is removed before entering places of worship or even private houses. No such practice applies in the West, particularly in the fabled Wild West of which Dubya is in some ways an embodiment in which it was deemed an honour to have 'died with one's boots on'. Indeed, having ducked in time, the president didn't seem particularly fazed by being targeted by *jootis*, and later remarked that he must think up some good shoe jokes for future reference. Sound advice to all those who need to go public. Think of some good shoe jokes. Or good whatever-it-is-you-would-like-thrown-at-you jokes. And lay in stocks of your preferred tokens of protest to give to those who would protest against you. Eggs, tomatoes, chappals; the choice is yours.

1. Baghdad authorities have welcomed al-Zaidi's act. Yes/No
2. The shoe which was hurled was 9 in number. Yes/No
3. In Texas footwear is considered an offensive projectile. Yes/No
4. In Islamic tradition, footwear is removed before entering private houses. Yes/No
5. A Saudi tycoon bought the famous footwear for $10 million. Yes/No

Answers and Explanations

1. No, The passage authorities have taken a dim view of the episode.
2. No, it was 10 in number.
3. No, the passage says that it's a part of the Texan culture where cowboys don leather apparel.
4. Yes, given in the second last paragraph.
5. No, Saudi tycoon had only offered to buy the famous footwear, and the sale did not take place.

DIFFERENT CATEGORIES OF RC PASSAGES

The reading passages in IIM Indore-IPM and other B-School entrance tests are different subject areas like History, Philosophy, Literature, Economics etc. On the basis of the subject matter, viz., RCs can be broadly classified into different genre viz., Literature, Religion and Philosophy, Economics and Business, Psychology, Life Sciences, Physical Sciences etc.

A passage from sociology, arts, or philosophy can discombobulate a student from science background, and disturb the momentum of the entire paper.

Although the average length of passages has gone down significantly over the years to as low as 500–600 words, the subject matter can be quite unfamiliar and unfriendly to an average test taker. The primary reason behind this discomfiture is that students are seldom comfortable with the vocabulary and concepts used in these subject areas.

Therefore a serious aspirant must thoroughly read newspapers editorials, magazine articles and books from different areas which offer new ideas and challenging vocabulary. Expose yourself to new concepts and ideas through online glossaries and encyclopedia.

The idea is to develop a basic framework of knowledge so that the reader becomes comfortable

with different branches of knowledge. This can greatly reduce the shock factor one faces while confronting the abstract passages in the test paper.

Let's take a few sample passages from different disciplines. Be patient and keep your mind open to learning. Try to make sense of what the passage talks about, what the author is trying to convey.

PASSAGE 1

Physical Science

A large part of the North Polar Region the Arctic, Alaska and Greenland has lost over two trillion tonnes of ice over the last five years, according to scientists interpreting new data obtained from a NASA satellite. What is alarming is that the melt figure refers to depletion of landlocked ice, and more than half the loss is in Greenland, contributing to half a millimeter of sea level rise annually. The total ice melt from the entire region has led to sea levels rising by one-fifth of an inch in five years. Add to this the Arctic Amplification Effect. The increase in the volume of Arctic waters absorbs more heat in the absence of sunlight-reflecting white ice which warms the oceans even more in summer. In autumn, the heat is released into the air, leading to rise in air temperatures, up to 10 degrees warmer now than recorded in the 1980s.

The Intergovernmental Panel on Climate Change's fourth assessment report warned last year that if the current rate of emissions continues unabated, the world could reach a tipping point by 2050. Among other dire consequences, this would seriously impact the availability of drinking water, especially in Asia, Africa and Small Island Developing States.

James Hansen, director of NASA's Goddard Institute of Space Studies, says that the IPCC's estimates are conservative and the information outdated. New data reveal a situation that is far worse, calling for a 'Climate Code Red'. In other words, it's a planetary emergency. The December 2008 UN climate change conference in Poznan, Poland, did not reflect this urgency. Neither specifics nor timelines for curbing emissions were agreed upon that would help formulate a global plan of action at the Copenhagen December, 2009 summit to firefight climate change.

The year 2008 could be the tenth warmest year on record, according to the UN Meteorological Agency. Tackling the problem together ought to get top priority rather than assignment of blame, since emissions and their effects know no borders. The answer lies in sharing of clean technology and stepping up research and development efforts in alternative and renewable energy options despite cheaper oil. Industrialized countries should release the promised 2 per cent from carbon trade profits to the UN Adaptation Fund to help developing countries cope with the effects of climate change. India is among the hot spots identified by the UN for extreme weather events. Its National Action Plan on Climate Change shows India is taking the problem seriously. But this might be insufficient without a global plan.

Explanation

The passage begins with the alarming fact that the North Polar Region has lost over two trillion tonnes of ice over the last five years. The melting ice leads to rising sea levels. The increased water volumes absorb more heat and release it into the air, leading to rise in air temperature, which causes global warming. The recent IPCC report says that if the current rate of emissions goes unchecked the world could reach a tipping point by 2050.

James Hansen, director of NASA's Goddard Institute of Space Studies believes that the situation is even bleaker. The answer lies in sharing of clean technology and stepping up research and development efforts in alternative and renewable energy options. Industrialized countries should part with some of their carbon trade profits to help the developing countries cope with the effects of climate change.

PASSAGE 2

Politics/Government

The closing decades of the last century saw substantial offloading of responsibilities by national governments to those below both in the US and Canada. It would appear there is, what Kincaid (2002) calls a "federalist ferment" across the world. The ferment notwithstanding, federalism is facing challenges from several directions. First of all, the intellectual case for decentralization and federalism has come under some critical reappraisal. Even some of the key assertions regarding the virtues of decentralization and the assumptions underlying them have been challenged. Attention has been drawn to the possibility of decentralization failures and the merits of strong nationhood as a check against centrifugal forces gaining ascendancy and subverting the integrity of nations.

What is more, the very forces that led to the fall of oppressive statism and provided the impetus for decentralization, viz., globalization and the demise of statism, are now posing a threat to the sovereignty of nation states–their life blood–and along with them that of their constituent units with implications that are yet to unfold. Despite the moves towards decentralization and more room for junior governments in established federations, viz., the US, Canada and Australia, the signals are mixed. Federal government still accounts for 60 per cent of government expenditure in the US. Things have not changed much in Canada either.

In Australia, the trend, if any, is towards even more centralization. Some of the decentralized federal countries like Brazil are "recentralizing". Globalization has generated pressures for reform in the economic and political organisation and thereby intergovernmental relations of all developing countries. There are forces pulling in opposite directions, tending to centralize functions envisaged by second tier governments, like states in India, and decentralize some to tiers further down citing "subsidiarity".

The choice of the federal form for the US constitution that presaged the emergence of the federal idea across the world was motivated largely by the anxiety to have a central government that can act decisively when required unlike in a confederation, but with effective checks and balances by dividing powers between the federal government and the states.

What accounts for the current federalist ferment despite warnings about its risks and inefficiencies are basically two fold. One is the economic benefits of efficiency in the organisation and functioning of the public sector from decentralization1–now encapsulated by the principle of "subsidiarity" in the EU's Maastricht treaty–combined with the gains from the operation of a large common market. The other is commitment to diversity rather than homogeneity. And this is particularly relevant for a diverse country like India. There is also the strength that comes from unity, the ability to face calamities like the tsunami and threats to security like external aggression or terrorism.

Explanation

Federalism faces difficult challenges in the era of globalization, since the latter has generated pressures for reform in economic and political organizations and thereby in inter-governmental relations of all developing countries as well. There are forces in inter-governmental relations pulling in opposite directions, some tending to centralize functions of second tier governments, such as of the states in India, and others moving to decentralise to tiers further down, citing "subsidiarity".

PASSAGE 3

Philosophy/Religion

When I was a fairly precocious young man, I became thoroughly impressed with the futility of the hopes and strivings that chase most men restlessly through life. Moreover, I soon discovered the cruelty of that chase, which in

those years was much more carefully covered up by hypocrisy and glittering words than is the case today. By the mere existence of his stomach everyone was condemned to participate in that chase. The stomach might well be satisfied by such participation, but not man insofar as he is a thinking and feeling being.

As the first way out there was *religion*, which is implanted into every child by way of the traditional education-machine. Thus I came—though the child of entirely irreligious (Jewish) parents—to a deep religiousness, which, however, reached an abrupt end at the age of twelve. Through the reading of popular scientific books I soon reached the conviction that much in the stories of the Bible could not be true. The consequence was a positively fanatic orgy of freethinking coupled with the impression that youth is intentionally being deceived by the state through lies; it was a crushing impression. Mistrust of every kind of authority grew out of this experience, a skeptical attitude toward the convictions that were alive in any specific social environment-an attitude that has never again left me, even though, later on, it has been tempered by a better insight into the causal connections.

It is quite clear to me that the religious paradise of youth, which was thus lost, was a first attempt to free myself from the chains of the "merely personal," from an existence dominated by wishes, hopes, and primitive feelings. Out yonder there was this huge world, which exists independently of us human beings and which stands before us like a great, eternal riddle, at least partially accessible to our inspection and thinking. The contemplation of this world beckoned as a liberation, and I soon noticed that many a man whom I had learned to esteem and to admire had found inner freedom and security in its pursuit.

The mental grasp of this extra-personal world within the frame of our capabilities presented itself to my mind, half consciously, half unconsciously, as a supreme goal. Similarly motivated men of the present and of the past, as well as the insights they had achieved, were the friends who could not be lost. The road to this paradise was not as comfortable and alluring as the road to the religious paradise; but it has shown itself reliable, and I have never regretted having chosen it.

Explanation

This is a selection from an essay written by the great scientist Albert Einstein. The author says that most people chase material comforts throughout their lives, but it doesn't satisfy the thinking and feeling people like him. He believes that the traditional education 'machine' is rigid and mechanical. It dumbs down a person and limits one's freethinking.

The author wants to free himself from the chain of merely personal to something bigger and universal, something which can give inner freedom and security. He sums up his argument by saying that path of religion offers many shortcuts and comfortable solutions but the quest for truth is riddled with many challenges.

PASSAGE 4

Economics

China's lunar New Year sees the world's largest migration, as tens of millions of workers flock home. Deserting for a few days the factories that make the goods that fill the world's shops, they surge back to their native villages. This week, however, as they feasted to the deafening rattle of the firecrackers lit to greet the Year of the Ox, their celebrations had an anxious tinge. Many will not have jobs to go back to.

China's breakneck growth has stalled. The rest of East Asia, too, which had hoped that it was somehow "decoupled" from the economic trauma of the West, has found itself hit as hard as anywhere in the world—and in some cases harder. The temptation is to see this as a plague visited on the region from outside, which its governments are powerless to resist or cure. In truth, their policy errors have played their part in the downturn, so the remedies are partly in their hands.

The scale and speed of that downturn is breathtaking and broader in scope than in the financial crisis of 1997–98. China's GDP, which expanded by 13 per cent in 2007, scarcely grew at all in the last quarter of 2008 on a seasonally adjusted basis. In the same quarter, Japan's GDP is estimated to have fallen at an annualized rate of 10 per cent, Singapore's at 17 per cent and South Korea's at 21 per cent. Industrial-production numbers have fallen even more dramatically, plummeting in Taiwan, for example, by 32 per cent in the year to December.

The immediate causes are plain enough: destocking on a huge scale and a collapse in exports. Even in China, exports are spluttering, down by 2.8 per cent in December compared with the previous year. That month Japan's fell by 35 per cent and Singapore's by 20 per cent. Falls in imports are often even starker: China's were down by 21 per cent in December; Vietnam's by 45 per cent in January. Some had suggested that soaring intra-regional trade would protect Asia against a downturn in the West. But that's not happening, because trade within Asia is part of a globalized supply chain which is ultimately linked to demand in the rich world.

Some Asians are blaming the West. The Western consensus in favour of globalization lured them, they say, into opening their economies and pursuing export-led growth to satisfy the bottomless pit of Western consumer demand. They have been betrayed. Western financial incompetence has trashed the value of their investments and consumer demand has dried up. This explanation, which absolves Asian governments of responsibility for economic suffering, has an obvious appeal across the region.

Awkwardly, however, it tells only one part of the story. Most of the slowdown in regional economic growth so far stems not from a fall in net exports but from weaker domestic demand. Even in China, the region's top exporter, imports are falling faster than exports.

Domestic demand has been weak not just because of the gloomy global outlook, but also because of government policies. After the crisis a decade ago, many countries fixed their broken financial systems, but left their economies skewed towards exports. Savings remained high and domestic consumption was suppressed. Partly out of fright at the balance-of-payments pressures faced then, countries have run large trade surpluses and built up huge foreign-exchange reserves. Thus the savings of poor Asian farmers have financed the habits of spendthrift Westerners.

That's not at all bad. One consequence is that Asian governments have plenty of scope for boosting domestic demand and thus spurring economic recovery. China, in particular, has the wherewithal to make good on its promises of massive economic stimulus. A big public-works programme is the way to go, because it needs the investment anyway. When Japan spent heavily on infrastructure to boost its economy in the early 1990s, much of the money was wasted, because it was not short of the stuff. China, by contrast, could still do with more and better bridges, roads and railways.

Yet, infrastructure spending alone is not a long-term solution. This sort of stimulus will sooner or later become unaffordable, and growth based on it will run out of steam. To get onto a sustainable long-term growth path—and to help pull the rest of the world out of recession—Asia's economies need to become less dependent on exports in other ways.

Asian governments must introduce structural reforms that encourage people to spend and reduce the need for them to save. In China, farmers must be given reliable title to their land so that they can borrow money against it or sell it. In many countries, including China, governments need to establish safety-nets that ease worries about the cost of children's education and of health care. And across Asia, economies need to shift away from increasingly capital-intensive manufacturing towards labour-intensive services, so that a bigger share of national income goes to households.

For Asian governments trying to fix their countries' problems, the temptation is to reach for familiar tools—mercantilist currency policies to boost exports. But the region's leaders seem to realise that a round of competitive devaluation will help no one. China has responded to American accusations of currency "manipulation" by denying it has any intention of devaluing the Yuan to boost exports. Structural reforms to boost demand would not only help cushion the blow to Asia's poor and thus help avert an explosion of social unrest that governments such as China's fear; they would also help counter the relentless rise in protectionist pressure in the West.

If emerging Asia needs a warning of the dangers of relying on exports, it need look no further than Japan. Japan's decade-long stagnation ended in 2002, thanks to a boom in exports, especially to China. Now, largely because of its failure to tackle the root causes of weak domestic demand, it is taking more of an economic hiding than any other rich country. Japan used to see itself as the lead goose in a regional flight formation, showing the way to export-led prosperity. It is time for the other geese to break ranks.

Explanation

The article begins with a hint of irony that tens of millions of Chinese workers have been rendered jobless due to the economic recession amidst the backdrop of New Year celebrations. Many people believe that the economic crisis occurred because the Western countries lured the Asian countries into opening their economies and pursuing export-led growth. However, most of the slowdown in regional economic growth so far stems not from a fall in net exports but from weaker domestic demand. The domestic demand has been weak not just because of the gloomy global outlook, but also because of government policies, which left their economies skewed towards exports; The Savings remained high and domestic consumption was suppressed. But there is a silver lining too. There is a scope for boosting domestic demand and spend heavily on infrastructure. Asia's economies need to become less dependent on exports by introducing structural reforms that encourage people to spend and reduce the need for them to save. Moreover, the Asian economies need to shift away from increasingly capital-intensive manufacturing towards labour-intensive services.

PASSAGE 5

Literature

He that hath wife and children hath given hostages to fortune; for they are impediments to great enterprises, either of virtue or mischief. Certainly the best works, and of greatest merit for the public, have proceeded from the unmarried or childless men; which both in affection and means, have married and endowed the public. Yet it were great reason that those that have children, should have greatest care of future times; unto which they know they must transmit their dearest pledges. Some there are, who though they lead a single life, yet their thoughts do end with themselves, and account future times impertinences.

Nay, there are some other, that account wife and children, but as bills of charges. Nay more, there are some foolish rich covetous men that take a pride, in having no children, because they may be thought so much the richer. For perhaps they have heard some talk, Such an one is a great rich man, and another except to it, Yea, but he hath a great charge of children; as if it were an abatement to his riches. But the most ordinary cause of a single life, is liberty, especially in certain self-pleasing and humorous minds, which are so sensible of every restraint, as they will go near to think their girdles and garters, to be bonds and shackles.

Unmarried men are best friends, best masters, best servants; but not always best subjects; for they are light to run away; and almost all fugitives, are of that condition. A single life doth well with churchmen; for charity will hardly water the ground, where it must first fill a pool. It is indifferent for judges and magistrates;

for if they be facile and corrupt, you shall have a servant, five times worse than a wife. For soldiers, I find the generals commonly in their hortatives, put men in mind of their wives and children; and I think the despising of marriage amongst the Turks, maketh the vulgar soldier more base.

Certainly, wife and children are a kind of discipline of humanity; and single men, though they may be many times more charitable, because their means are less exhaust, yet, on the other side, they are more cruel and hardhearted (good to make severe inquisitors), because their tenderness is not so oft called upon. Grave natures, led by custom, and therefore constant, are commonly loving husbands, as was said of Ulysses, vetulam suam praetulit immortalitati.

Chaste women are often proud and forward, as presuming upon the merit of their chastity. It is one of the best bonds, both of chastity and obedience, in the wife, if she think her husband wise; which she will never do, if she find him jealous. Wives are young men's mistresses; companions for middle age; and old men's nurses. So as a man may have a quarrel to marry, when he will. But yet he was reputed one of the wise men, that made answer to the question, when a man should marry,—A young man not yet, an elder man not at all.

It is often seen that bad husbands, have very good wives; whether it be, that it raiseth the price of their husband's kindness, when it comes; or that the wives take a pride in their patience. But this never fails, if the bad husbands were of their own choosing, against their friends' consent; for then they will be sure to make good their own folly.

Explanation

This is one of the most famous essays written by Francis Bacon, noted English philosopher, statesman, scientist, lawyer, jurist, and author titled '*Marriage and Single Life*'. In this essay, he uses witty language to compare the merits and demerits of married life with that of single life. It begins on a cynical note that marrying is a hindrance to any great enterprise. The unmarried or childless people have done the greatest things. The essence of single life is liberty, especially for self-pleasing and humorous minds. Marriage makes a person more disciplined and humane. Although single men are quite charitable they are more cruel and hardhearted. It includes one of the most popular quotes of Bacon, 'Wives are young men's mistresses; companions for middle age; and old men's nurses.'

UNDERSTANDING THE STYLE AND TONE OF THE PASSAGE

Different writers adopt different ways to present their ideas, and even while using the similar writing techniques the author may reflect a different outlook, his way of looking at things. Therefore, to develop a better understanding of the text that is critical to answering questions in IIM Indore-IPM, it is important to understand the style and tone of the passage.

The style of the passage refers to how the ideas have been presented. It is the technique used by the author to convey his/her ideas. The style of writing depends on a variety of factors like the choice of words, clarity and accuracy of expression, sentence length, variety and structure, lucidity or complexity of language used.

Writing Styles

1. The author could use a *narrative style* in which things move like a story; with a definite beginning, middle and end. It is characterized by a personal touch to the description of events.
2. *Descriptive style* of writing reports the details of a person, place, thing or event. It is more like a news report you see in the front page of a daily newspaper. The writer begins in a general manner, and then offers a detailed description of the subject.
3. *Analytical style* of writing, as the name suggests, involves a detailed treatment of an issue or situation. The author dives

deep and tries to follow the chain of reasoning and draw inferences. The author weighs the different points of view in favour of or against his argument before arriving at any conclusion.

4. When the author uses the *argumentative style* of writing, he analyses the topic after taking a stand. He tries to persuade the reader using a chain of reasoning,evidence or suggestions. It is a variant of the analytical style of writing.

Tone of Writing

The word tone generally refers to the quality of sound. But, when we refer to the tone of the passage, it refers to the predominant emotion or absence of it displayed by the author. The tone also reflects the attitude of the author towards a subject or character. Any predominant emotion may, thus become the author's tone.

Broadly, the tones can be classified as subjective or objective in nature. An objective tone is used when the author does not choose any side, and remains a neutral and detached observer. On the contrary a subjective tone reflects the emotion displayed by the author. Some of the common tones used by the authors are listed as follows:

***Critical*:** Denotes negative or fault finding attitude of the author. Sometimes, the word critical is also used to denote deep analysis of the issue with a neutral outlook.

***Laudatory/Eulogistic*:** To shower high praise on somebody or something.

***Cynical*:** A higher degree of pessimism coupled with a sense of scepticism and helplessness.

***Satirical*:** To use humour as a tool for healthy criticism.

***Sarcastic*:** To use the words opposite to what you mean to taunt or make fun of somebody.

***Didactic*:** When the author tries to teach or instruct through his writing.

***Nostalgic*:** Conveys a sense of longing for the past.

Example 1

Identify the tone of the passages given below.

PASSAGE 1

Plants are not the only organisms that can be cloned naturally. The unfertilized eggs of some animals (small invertebrates, worms, and some species of fish, lizards and frogs) can develop into full-grown adults under certain environmental conditions—usually a chemical stimulus of some kind. This process is called parthenogenesis, and the offspring are clones of the females that laid the eggs.

Another example of natural cloning is identical twins. Although they are genetically different from their parents, identical twins are naturally occurring clones of each other.

Scientists have experimented with animal cloning, but have never been able to stimulate a specialized (differentiated) cell to produce a new organism directly. Instead, they rely on transplanting the genetic information from a specialized cell into an unfertilized egg cell whose genetic information has been destroyed or physically removed.

In the 1970s, a scientist named John Gurdon successfully cloned tadpoles. He transplanted the nucleus from a specialized cell of one frog (B) into an unfertilized egg of another frog (A) in which the nucleus had been destroyed by ultraviolet light. The egg with the transplanted nucleus developed into a tadpole that was genetically identical to frog B.

While Gurdon's tadpoles did not survive to grow into adult frogs, his experiment showed that the process of specialization in animal cells was reversible, and his technique of nuclear transfer paved the way for later cloning successes.

Explanation

The passage describes the process of cloning in animals. He describes the early experiments of animal cloning objectively; without taking

any stand or position. Therefore, the tone of the passage is *Objective* or *Scientific.*

PASSAGE 2

Everyone agrees that President George Bush's lobotomy has been a tremendous success.

Dick Cheney, the vice-president, declared that he was fully satisfied with it from his point of view. "Without the lobotomy," Mr Cheney told the American Academy of Neurology, "it might have proved difficult to persuade the president to start wars all around the world without any good pretext. But the removal of those parts of the brain associated with understanding the outcome of one's actions has enabled the president to function fully and without hesitation. Even when it is clear that disaster is around the corner, as it is currently in Iraq, the chief executive is able to go on TV and announce that everything is on course and that he has no intention of changing tactics that have already proved disastrous.

"I would like to commend the surgeons, nurses and all involved with the operation," said Mr Cheney. Similarly, Donald Rumsfeld regards the surgery as an unqualified success. He writes in this month's American Medical Association Journal: "The president's prefrontal leucotomy has successfully removed all neural reflexes resistant to war-profiteering. It is a tribute to the medical team who undertook this delicate operation that, no matter how close the connection between those instigating military action and the companies who benefit from it, the president is able to carry on as if he were morally in the right."

Paul Wolfowitz, the deputy secretary of defence, is also delighted at the beneficial effect that medical intervention has had on the president. "Just imagine how the president might have responded to Ariel Sharon's crazy schemes if we hadn't had the foresight to take out the neural pathways normally connected with perception and understanding," Mr Wolfowitz told a meeting of the Association of Muslim Neurosurgeons For An All-Jewish Israel. "The president is now capable of treating the man responsible for the massacres at Shatila and Sabra as a decent human being, whose advice on how to deal with the problems of Israel is worth not only listening to, but also taking."

With all this acclaim for the US president's lobotomy, it is scarcely surprising that Tony Blair, should have decided to follow suit and undergo similar psychosurgery.

Thanks to the inhibition of specific pre-synaptic terminals, Mr Blair now appears to feel totally comfortable giving his support to the US massacre in Falluja and to the activities of US snipers who have been so busy in that city shooting women, children and ambulance drivers in revenge for the murder of four mercenaries.

It is also believed that intervention in the motor speech area of his cortex now enables Mr Blair to describe Iraqis who respond negatively to having their houses blown up as "fanatics, extremists and terrorists". Similarly, ablation of the oculomotor nerve means that Mr Blair is now able to see Israeli plans to retain Jewish settlements in the West Bank as a big step forward in the Middle East peace process.

What has come as a complete surprise, however, is the recent revelation that Mr Blair's brain surgery may even predate President Bush's. For without the removal of large portions of his cerebellum, it is hard to understand how the British prime minister could have turned down Mr Bush's no-strings offer to keep British troops out of combat in Iraq.

Political commentators are thus finding it impossible to say whether it is Mr Bush or Mr Blair who has pioneered the use of executive lobotomies in the war against terrorism.

Explanation

This article is a satire on the former president of the US George W Bush. Satire uses humour as a tool of healthy criticism, especially to point out flaws of the society or system at large. Don't be bogged down by the medical jargon. Just amuse yourself and enjoy. The way the passage begins with 'The president's surgery has been a tremendous success' and then 'without the

lobotomy ... it might have proved difficult to persuade the president to start wars all around the world without any good pretext' has a touch of satire in it, as the author wants to criticize Bush's decision to start the wars around the world, hence the tone of the passage is *satirical.*

PASSAGE 3 (POEM)

I wandered lonely as a cloud
That floats on high o'er vales and hills,
When all at once I saw a crowd,
A host, of golden daffodils;
Beside the lake, beneath the trees,
Fluttering and dancing in the breeze.

Continuous as the stars that shine
And twinkle on the milky way,
They stretched in never-ending line
Along the margin of a bay:
Ten thousand saw I at a glance,
Tossing their heads in sprightly dance.

The waves beside them danced; but they
Out-did the sparkling waves in glee:
A poet could not but be gay,
In such a jocund company:
I gazed—and gazed—but little thought
What wealth the show to me had brought:

For oft, when on my couch I lie
In vacant or in pensive mood,
They flash upon that inward eye
Which is the bliss of solitude;
And then my heart with pleasure fills,
And dances with the daffodils.

Explanation

This is a classic poem written by one of the greatest romantic poets William Wordsworth. The poem has a great lyrical element. It is apparent that the poet was not just elated by the sight of the daffodils, but he reached a different plane of ecstasy. The poet has used vivid imagery. Expressions like 'sparkling wave of glee; A poet could not but be gay ... in such a jocund company' give us the impression that the tone of the passage is exalted or *laudatory.*

LOGICAL MAPPING OF THE PASSAGE

The author uses the reading passage as a tool to describe something, present his point of view on a particular subject, or elaborate a concept or idea. The author uses a web of words to convey his ideas and opinion.

The fact is only few words and key ideas are critical to understand the passage quickly and accurately. Rest of the words which form a mass of the passage are nothing but extensions,exam ples,explanations and facts used by the author to elaborate the theme of passage. Some of the words may also be used to display skillful use of language (Rhetoric).

Like an architect who makes a blueprint of the building, visualizing how the construction work will go in the future, an active reader makes a logical outline of ideas in his mind on the basis of how the author builds his ideas and what he wants to convey to the reader.

Underlining or making a mental note of the **key ideas** of each paragraph as they appear in the passage helps you to understand how the theme of the passage develops.

Since the passages are highly condensed, one is required to read between the lines too, to understand the important elements of the passage which are integral to answering the questions. Do not be misled by illustrations, examples or extensions given by the author but learn to seive the important details.

Such focused reading also helps you to read faster as you save your time by not getting into extraneous details. It also increases the comprehension as one does not miss out on the main points.

PASSAGE 4

Read the following passage carefully and underline the key points. Identify the logical flow of the passage. Time Allowed: 8 minutes

Education is a mess because politicians refuse to discipline teachers who sabotage primary education. Surveys show that government teacher absenteeism ranges from 20 per cent to 57 per cent

in different states, yet they earn thrice or more than private sector teachers.

Some teachers run businesses (shops, transport services). Others skip school in the morning but give paid tuitions to richer students in the afternoon.

No wonder half of all students drop out by Class 7. Barely 30–50 per cent can read the alphabet in Class1, and barely 40–50 per cent can read simply words in Class 2. Millions who complete school emerge functionally illiterate, unable to read simple paragraphs or do simple arithmetic. Yet no political party is willing to discipline teachers or demand performance.

An obvious reason is the power of teacher trade union. These often launch strikes just before school exams, impelling state governments to surrender rather than jeopardize the future of students. Hence teachers get ever-higher salaries while escaping accountability for performance. Teachers' salaries appropriate almost the whole educational budget, leaving hardly anything for other items such as teaching materials and textbooks. Between 1960 and 1980 in Uttar Pradesh, the share of non-salary pending in education fell from 12 per cent to 3 per cent in primary education, and from 28 per cent to 9 per cent in secondary education.

A seasoned politician gave me a big additional reason for teacher power. You see, he said, government teachers preside over polling booths at election time. So we must cosset them, not antagonize them. Otherwise teachers will help rival parties to rig elections, and we cannot afford that at any cost.

A recent book by Geeta Kingdon and Mohammed Muzammil (*The Political Economy of Education in India*) throws new light on teacher power in Uttar Pradesh. Teachers are politically strong because they themselves have become politicians in astonishingly large numbers. Masterji has become netaji.

The Constitution provides a quota for teachers in the Upper Houses of State Legislatures. Only large states have an Upper House, but the bulk of the population is in such states.

Second, while the law prohibits government servants from contesting elections, it makes an exception for teachers. Why should teachers be allowed to contest but not doctors, clerks, sanitary engineers or other officials? The only reason is teacher clout.

Third, teachers are often the best educated in rural areas, and so are natural leaders. Hence they are elected in large numbers to the lower houses of state legislatures too. Since they have so much spare time—they only teach in the mornings, if at all—many do political work. Some are really politicians pretending to be teachers in order to collect a regular salary and have an institutionalized position of power Fourth, politicized teachers help provide the troops needed for rallies and elections. Teachers help organize students in secondary schools to become political campaigners. This in turn produces a peculiar breed of "student leaders" who see a future in politics but none in education. They agitate for an automatic pass for all students, not high academic standards.

Kingdon and Muzammil give some stunning figures about the teacher-politician nexus in Uttar Pradesh. In the Upper House, 8.5 per cent of seats are reserved for teachers, yet the proportion actually elected to the Upper House varies from 13 per cent to 22 per cent. Clearly, the power of teachers far exceeds their Constitutional quota.

The Lower House has no teacher quota. Yet teachers accounted for 10.8 per cent of all elected MLAs in the 1993 election, and 8.7 per cent in the 1996 election, far above their 0.9 per cent share in the adult population.

Their share of Cabinet posts was even higher. This share has usually been in double digits since 1985, with a peak of 16.3 per cent in 1991–92. This high share persists regardless of which party is in power—Congress, BJP, Samajwadi, BSP. Mayawati, whose party is tipped to win the next election, is herself an ex-teacher.

This, then, explains why all state governments treat teachers with kid gloves, and in

the bargain ignore the mess in education. One obvious way to improve education and teacher accountability is to empower panchayats and parents' associations to discipline absentee teachers. But despite the Constitutional amendment seeking to devolve primary education to panchayats, all efforts at actual devolution have been sabotaged. The Kalyan Singh government in 1992 tried to give managers of aided private schools greater powers over teachers, but this led to a mass strike, and the government backed down. In the late 1990s the UP government tried to devolve some educational powers to panchayats, but once again teachers went on the rampage and the government caved in.

This is why many state governments prefer to let panchayats hire para-teachers—local people without proper teacher qualifications. These have helped improve basic literacy at a cost one-fifth that of regular teachers. That is a short-term gain, but para-teachers cannot provide quality education. Besides, in some states para-teachers are agitating to be recognized as regular teachers. What is the way out? Kingdon and Muzammil offer no panaceas. If villagers and panchayats get sufficiently angry with the mess in education, they could create a countervailing political force. That day still seems far off.

Explanation

The 1st paragraph highlights the main idea of the passage—the sorry state of education in India is primarily because of political unwillingness to reform primary education.

The second and the third paragraph bring facts to support that education is in a state of complete mess like: high drop out rates of students and high absenteeism among teachers because of their personal interests. The author then discusses the real reason behind the teacher power: powerful teacher unions and other political compulsions. The author presents facts from the book by Kingdon and Muzammil to support his main argument explaining the reasons behind teach power. The author presents some more facts and figures to explain the teacher-politician nexus. The last paragraph talks about one possible solution to this is to empower the local political bodies like village *panchayats* and parents' associations and the challenges in implementing it.

DIFFERENT TYPES OF RC QUESTIONS

RC is a tool which is used by the examiners to test a person's ability to understand and analyse text drawn from different areas. It also tests a person's ability to draw inferences, and if necessary to apply the inference in a new context or framework.

Reading passages do not directly test the general awareness or subject knowledge of any particular field, although a broad awareness of different areas is helpful in increasing the comfort level, and consequently the confidence level with which a person handles the passages. RC questions may test you on either what is given in the passage or what can be deduced from the passage. The idea is to get as close as possible to the mind of the author—what he wants to convey through the framework of ideas interwoven in the passage. From the examination point of view, the RC questions can be broadly subdivided into six categories. A student should try to master different RC question types to excoriate the fear of RC from his mind forever.

1. Main Idea Question
2. Explicit Detail or Direct Question
3. Inference Question
4. Logical Structure Question
5. Tone or Attitude based Question
6. Extended Application Question

Main Idea Question

Main idea of a passage can be defined as the most succinct summary that encompasses the passage.

These questions are very important and test one's ability to understand the overall theme of the passage, mainly 'what the passage is precisely about'. One needs to have a fair understanding of what the passage talks about 'as a whole',and not be confused by the facts, explanations and

examples given by the author to support the main idea. main idea is generally indicated in the first part of the passage. Rarely, it could also come in the concluding part of the article.

Sometimes, the main idea question may be based on supplying a suitable title for the passage. A suitable title is one which captures the major elements of the passage in the shortest logical manner.

Explicit Detail or Direct Question

These questions use the phrases like 'According to the passage ...', or 'The author/passage mentions the following except:' These questions are easier to tackle as they test one's ability to find specific information given in the passage. One has to locate the detail in the form of information, data or statistics as mentioned in the passage.

These questions provide direct clues like line references which make them easier to crack. Sometimes, these questions can also ask the contextual meaning of an underlined or italicized word as it has been used in the passage. This type of questions may not be frequently asked in IIM Indore-IPM now but appear in SNAP and FMS test.

Inference Questions

To infer is to draw or deduce something on the basis of what is given in or implied by the passage. These questions are favorite of IIM Indore-IPM. The students find these questions challenging, as answering these questions requires a careful reading of the passage, which includes the ability to read between the lines, ability to interconnect the different logical elements given in the passage.

These questions typically use words like infer, imply, arrive at, deduce, surmise etc. To handle these questions one must refer back to that part of the passage from where the inference has been drawn, correlate the concepts if necessary, and narrow down the answer choices. Both the aspects are important: to read the context given in the passage, and to gradually eliminate the answer choices to arrive at the best solution.

Generally, the students get stuck up between the last two options. One must settle for the answer choice which is more clearly and specific on the basis of the main idea of the passage.

Logical Structure Question

These questions ask about why the author introduces a specific point, gives a particular example, or quotes somebody in the passage. Sometimes, these questions also ask about overall development of the passage about 'WHY' the author uses a particular example, anecdote, refutation, or counter argument to develop the passage.

These questions mainly focus on 'WHY' of the subject matter. Therefore, it becomes important to read the mind of the author. These questions generally give you a line reference from where the example or quote has been taken. If you carefully read two to three lines above or below the line from where the text has been taken and work with the options, you can hit the bull's eye. Remember the answer can never be within the of line reference, as the question is 'WHY' that particular thing has been mentioned.

Tone/Attitude Based Question

These questions test your ability to find out the underlying emotion of a particular line, paragraph or passage as a whole. One has to judge the attitude of the author towards his subject. These questions are not common in IIM Indore-IPM and other B-School entrance tests, however, a fair understanding of tone helps one to understand the passage better and faster. One must develop the ability to understand the different shades of mood displayed by the author. Vocabulary plays an important role in handling these questions as the words like eulogistic or satirical are not used commonly used in colloquial or written English.

Attempt these questions only if you have a fairly clear idea of the emotional underpinning of the sentence or paragraph in question.

Extended Application Question

Application based questions take the logic a step further to that involved in inference questions. They test the ability to apply what you have learnt from the passage into a new context or framework, sometimes even unrelated to the passage. These questions are a bit challenging to the students, as they test one's ability to think creatively and see things in new light.

These questions generally look like: 'The author would most likely/least likely agree with...' 'Which of the following statements if true would most strongly strengthen/weaken the argument' 'the writer/target audience of the passage is most likely...'

To handle these questions one must meticulously narrow down the options keeping an eye on the main idea, scope and tone of the passage. These questions should be touched only when one has developed a clear understanding of the passage.

Another format of the application question is 'which of the following is likely to be the title of the forthcoming/next article written by the author.' These questions can be easily cracked if you carefully read the last paragraph and find out where the author leaves the passage, what are the unexplained issues or explanations which leave room for further discussion, analysis or elucidation.

2 Fill in the Blanks

INTRODUCTION

Fill in the blanks (FIB) questions are designed to measure your ability to understand the intended meaning of a sentence. Each question requires you to analyze the context of a sentence and determine which word or words best complete that sentence. It has been observed that both vocabulary-in-context and logic-based sentence completion questions are asked to test your grasp of the English language. You should be able to determine which answer choice best fills the blank(s) of the given sentence. Keep in mind that a complete sentence is clear and concise, conveys a logical meaning, and is uniform in grammar and style.

ANATOMY OF A FILL IN THE BLANKS QUESTIONS

Before getting to the strategies, let us understand what a *Sentence Completion question* looks like. Sentence completion questions consist of an incomplete sentence that includes one or two blanks, followed by five answer choices.

Consider the following example:

Example: Despite Tendulkar's ..... efforts, the team still suffered a ..... loss.

(a) complicated ... modest
(b) daring ... beneficial
(c) generous ... constructive
(d) heroic ... devastating **(Answer)**

GENERAL STRATEGIES FOR FILL IN THE BLANKS QUESTIONS

Every sentence contains hints that will help you select the correct answer. Each of the following strategies will help you decipher those hints, but remember that any given question might require you to use more than one approach:

1. Predict the best fit for the sentence
2. Use context, prefixes, suffixes, and cognates to define unfamiliar words
3. Identify "clue" words and phrases
4. Use connotation
5. Select an answer
6. Questions with two blanks

Predict the Best Fit for the Sentence

Before you look at the answer choices, think of a word that "Fits" the sentence. FIB questions usually test the standard meaning of a word. Pay attention to the logic and context of the sentence. Try to predict a word to insert in the blank or blanks as you read the sentence, and then look for your word or a *synonym* of your word among the answer choices. A synonym is a word with the same or a similar meaning. You should also look for *antonyms*, which are words that have the opposite meaning of your predicted word. If you locate any words among the answer choices that have a meaning opposite to the word that you would like to insert in the blank, eliminate those answer choices.

You should immediately begin to pick up on the idea the sentence is trying to convey, as well as any suggestions of tone or mood. Understanding the general meaning and nature of the sentence will help you to choose the most logical and stylistically appropriate answer.

Example: Crestfallen by having done poorly on the IIM IPM AT, Akansha began to question her abilities. Her self-confidence was .....

(a) appeased (b) destroyed
(c) placated (d) elevated

If somebody is crestfallen (despairing) and has begun to question herself, then her self-confidence would be destroyed. Hence, the answer is option (b).

Use Context, Prefixes, Suffixes and Cognates to Define Unfamiliar Words

Understanding the context of a sentence also helps to determine the meaning of any unfamiliar word you might encounter. Consider the following example:

Although the fossils were well preserved, paleontologists were unable to ..... the identity of the mammal species.

(a) display
(b) ascertain
(c) violate
(d) embellish

The best answer is option (b). You might not have heard the word "paleontologists" before, but you can deduce from the context of the sentence that they are most likely the scientists who study fossils. Another hint provided by the context is the word "although," which suggests a contrast between the condition of the fossils and the ability of the paleontologists to identify the species. Now, you can insert the words in the answer choices into the sentence to see which one best fits the context. It does not make sense that scientists would "display" or "violate" the identity of a mammal species, so eliminate answer choices (a) and (c). Likewise, scientists might "embellish" or "exploit" certain findings, but these words do not accurately describe what the scientists might do with the identity of a mammal species. If you did not know the meaning of "ascertain," you could arrive at it as the correct choice by using the context of the sentence to help you eliminate incorrect answer choices.

Also, if you have trouble establishing the meaning of an unfamiliar word from the context of the sentence, you can use your knowledge of *prefixes* and *suffixes* to help you. For example, the prefix "multi-" means "many," as in "multinational," and the suffix "-less" means "without," as in "careless."

Lastly, look for any recognizable *cognates* from French, Spanish, or Italian (the modern versions of Latin) in words that you are not familiar with. A *cognate* is a word that means the same or nearly the same thing in more than one language. For example, the word *amigo*, which means friend in Spanish, the word *ami*, which means friend in French, and the word *amicable*, which means friendly in English, all come from the Latin root word for friend, *amicus*.

Identify "Clue" Words and Phrases/Be Alert To Transitional Words

Transitional words tell you what is coming up. They indicate that the author is now going to draw a contrast with something stated previously, or support something stated previously.

Contrast Indicators

To contrast two things is to point out how they differ. In this type of sentence completion problem, we look for a word that has the opposite meaning (an antonym) of some key word or phrase in the sentence. Following are some of the most common contrast indicators:

But	**Yet**
Despite	**Although**
However	**Nevertheless**

Example: Although the warring parties had settled a number of disputes, past experience made them ..... to express optimism that the talks would be a success.

(a) rash
(b) ambivalent
(c) scornful
(d) overjoyed
(d) reticent

"Although" sets up a contrast between what has occurred—success on some issues—and what can be expected to occur—success for the whole talks. Hence, the parties are reluctant to express optimism. The common word "reluctant" is not offered as an answer-choice, but a synonym--reticent—is. The answer is option (e).

Support Indicators

Supporting words support or further explain what has already been said. These words often introduce synonyms for words elsewhere in the sentence. Following are some common supporting words:

And	**Also**
Furthermore	**Likewise**
In Addition	**For**

Example: Harshit is an opprobrious and ..... speaker, equally caustic toward friend or foe—a true curmudgeon.

(a) lofty
(b) vituperative
(c) unstinting
(d) retiring

"And" in the sentence indicates that the missing adjective is similar in meaning to "opprobrious," which is very negative. Now, vituperative—the only negative word—means "abusive." Hence, the answer is option (b).

Cause and Effect Indicators

These words indicate that one thing causes another to occur. Some of the most common cause and effect indicators are:

Because	**For**
Thus	**Hence**
Therefore	**If , Then .**

Example: Because the House has the votes to override a presidential veto, the President has no choice but to .....

(a) object
(b) abdicate
(c) abstain
(d) compromise

Since the House has the votes to pass the bill or motion, the President would be wise to compromise and make the best of the situation. The answer is option (d).

Apposition: This rather advanced grammatical structure is very common (Don't confuse "apposition" with "opposition": they have opposite meanings).

Words or phrases in apposition are placed next to each other, and the second word or phrase defines, clarifies, or gives evidence to the first word or phrase. The second word or phrase will be set off from the first by a comma, semicolon, hyphen, or parentheses.

Note: If a comma is not followed by a linking word—such as and, for, yet—then the following phrase is probably appositional.

Identifying an appositional structure, can greatly simplify a sentence completion problem since the appositional word, phrase, or clause will define the missing word.

Example

Man has no choice but to seek truth, he is made uncomfortable and frustrated without truth—thus, the quest for truth is part of what makes us ..... .

(a) noble
(b) different
(c) human
(d) intelligent

Answer: (c)

Solution

If man has no choice but to seek truth, then this is an essential characteristic of man. In other words, it is part of what makes us human.

Example

Though he claimed the business was ....., his irritability ..... that claim.

(a) sound ... belied
(b) expanding ... supported

(c) downsizing ... vindicated
(d) static ... contradicted
Answer: (a)

Solution

If the business was not sound, his irritability would belie (contradict) his claim that the business was sound.

Use Connotation

Each word expresses two things: A *definition* and a *connotation*. A *connotation* is a positive, negative, or neutral feeling that is implied by or associated with a word. Although context is the part of a sentence that surrounds a particular word or passage and determines its meaning, *connotation* refers to the emotion that is suggested by the word itself. For example, the adjective "thrifty" implies a positive connotation, whereas the adjective "cheap" implies a negative connotation. Both words have similar definitions, but very different connotations. Using connotations can help you determine the correct answer or at least eliminate some of the options.

Here is an example of how to use connotation to select the correct answer:

Example: Because of his ....., Max's guests felt very welcome and comfortable staying at his house for the weekend.

(a) animosity
(b) hospitality
(c) determination
(d) wittiness
(e) severity

The best answer is (b). The sentence has a positive connotation—Max's guests feel welcome and comfortable. In addition, the transition "because" indicates that something that belongs to Max has caused his guests to feel welcome and comfortable. "Animosity" and "severity" have a negative connotation and "determination" has a neutral connotation. "Hospitality" and "wittiness" both have positive connotations, but "hospitality" best fits the context of the sentence.

Select an Answer

Before you look at the answer choices, try to predict an answer. If your predicted word or words match one of the answer choices, it is most likely the correct choice. Remember that the test writers create incorrect answers in an attempt to distract you—if you predict an answer you are less likely to get caught up on these confusing incorrect answers.

Be careful to consider all of the choices before you confirm your answer, even if your predicted answer is among the choices. The difference between the best answer and the second best answer is sometimes very subtle. When you think that you have the correct answer, read the entire sentence to yourself, using your choice(s).

Questions with Two Blanks

If a sentence has two blanks, you can quickly eliminate incorrect answer choices if any word alone does not fit into the blank. When you select an answer choice for a two blank question, always ensure that *both* the words make sense in the sentence, both logically and stylistically. It helps to focus on one blank at a time. You can start with either the first or the second blank. Remember that if one word in the answer choice doesn't fit within the context of the sentence, you can eliminate the entire answer choice. Work on both blanks together only if you have not been able to eliminate all of the incorrect answers.

Note to the students: Each of the questions given ahead contains five options. This has been done deliberately to give you more opportunity to 'learn the art of eliminating the options'.

PRACTICE EXERCISES

EXERCISE 1

Direction for questions 1–10: ***The following sentences each contain one or two blanks, indicating that something has been left out of the sentence. Each answer choice contains one word or a set of words. Select the word or set of words, that, when inserted in the blank(s), best fits the context of the sentence.***

1. Her concern for the earthquake victims ..... her reputation as a callous person.
(a) restored (b) rescinded
(c) created (d) proved
(e) belied

2. The author presumably believes that all businessmen are ....., for her main characters, whatever qualities they may lack, are virtual paragons of ..... .
(a) clever ... ingenuity
(b) covetous ... greed
(c) virtuous ... deceit
(d) successful ... ambition
(e) cautious ... achievement

3. Lacking sacred scriptures or ....., Shinto is more properly regarded as a legacy of traditional practices and basic values than as a formal system of belief.
(a) followers (b) customs
(c) dogma (d) relics
(e) faith

4. Male sperm whales are normally ..... creatures; however, when they are jealously guarding their territory, they have been known to ships.
(a) docile ... attack
(b) aggressive ... strike
(c) large ... assault
(d) peaceful ... ignore
(e) powerful ... assail

5. Her charisma was a double-edged sword; in her friends, it aroused both admiration and ..... .
(a) envy (b) laudation
(c) equanimity (d) obstinacy
(e) affection

6. As the employee's motives were found to be ....., no disciplinary action will be taken against him for the mistake.
(a) absurd (b) gratuitous
(c) improvised (d) benign
(e) intentional

7. Devesh loves roses for the ..... appeal of their petals and leaves, while I am most ..... by their olfactory properties.
(a) aesthetic ... enthralled
(b) acrid ... interested
(c) nurturing ... persuaded
(d) visual ... displeased
(e) tacit ... disenchanted

8. His ..... for learning history should prove to be ..... during his studies to become a history teacher.
(a) disdain ... useful
(b) penchant ... practical
(c) dislike ... exceptional
(d) affinity ... futile
(e) appreciation ... gratuitous

9. We felt ..... once the committee issued its report that ..... our actions.
(a) angered ... supported
(b) abused ... endorsed
(c) vindicated ... authenticated
(d) helpless ... applauded
(e) ignorant ... dignified

10. The air in a room that contains several houseplants can be more ..... oxygen than a room that contains no plants.
(a) enjoyed by
(b) exhausted with
(c) obscured by

(d) saturated with
(e) complicated by

EXERCISE 2

Direction for questions 1–20: ***The following sentences each contain one or two blanks, indicating that something has been left out of the sentence. Each answer choice contains one word or a set of words. Select the word or set of words, that, when inserted in the blank(s), best fits the context of the sentence.***

1. Because of his success as a comedian, directors were loath to consider him for ..... roles.
(a) supporting (b) leading
(c) dramatic (d) comedic
(e) musical

2. The aspiring candidate's performance in the debate all but ..... any hope he may have had of winning the election.
(a) nullifies (b) encourages
(c) guarantees (d) accentuates
(e) contains

3. She is the most ..... person I have ever met, seemingly with an endless reserve of energy.
(a) jejune (b) vivacious
(c) solicitous (d) impudent
(e) indolent

4. Despite all its ....., a stint in the diplomatic core is invariably an uplifting experience.
(a) merits (b) compensation
(c) effectiveness (d) rigors
(e) mediocrity

5. Liharev talks about being both a nihilist and an atheist during his life, yet he never does ..... faith in God.
(a) affirm (b) lose
(c) scorn (d) aver
(e) supplicate

6. Existentialism can be used to rationalize evil: if one does not like the rules of society and has no conscience, he may use existentialism as a means of ..... a set of beliefs that are advantageous to him but injurious to others.
(a) thwarting (b) proving
(c) promoting (d) justifying
(e) impugning

7. These categories amply point out the fundamental desire that people have to express themselves and the cleverness they display in that expression; who would have believed that the drab, mundane DMV would become the ..... such creativity?
(a) catalyst for (b) inhibitor of
(c) disabler of (d) referee of
(e) censor of

8. This argues well that Chandresh exercised less free will than Ashutosh; for even though Chandresh was aware that he was misdirected, he was still unable to ..... free will.
(a) defer (b) facilitate
(c) proscribe (d) prevent
(e) exert

9. Man has no choice but to seek truth, he is made uncomfortable and frustrated without truth—thus, the quest for truth is part of what makes us ..... .
(a) noble (b) different
(c) human (d) intelligent
(e) aggressive

10. Though most explicitly sexist words have been replaced by gender-neutral terms, sexism thrives in the ..... of many words.
(a) indistinctness (b) similitude
(c) loquacity (d) implications
(e) obscurity

11. Though a small man, Abdul Kalam appeared to be much larger behind his desk; for, having skillfully designed his office, he was ..... by the perspective.
(a) augmented (b) comforted
(c) apprehended (d) lessened
(e) disconcerted

12. Man is violent and therefore any theory of conflict resolution between nations that ..... to account for this is ..... flawed.
(a) declines ... supposedly
(b) refuses ... pejoratively
(c) fails ... inherently
(d) consents ... manifestly
(e) flinches ... innately

13. Ironically, the foreign affairs policies of democracies are more likely to meet with protests than similar policies of totalitarian regimes because a democracy is ..... protest; whereas in a totalitarian regime, no one is listening.
(a) impassive to (b) indifferent to
(c) imperiled by (d) sensitive to
(e) inured to

14. Although the buildings and streets of this small beach town appear ....., the property values are quite ..... .
(a) expensive ... steep
(b) dilapidated ... high
(c) artistic ... pedestrian
(d) refurbished ... low
(e) quaint ... reasonable

15. Though he claimed the business was ....., his irritability ..... that claim.
(a) sound ... belied
(b) expanding ... supported
(c) downsizing ... vindicated
(d) static ... contradicted
(e) booming ... affirmed

16. The rules of engagement for United Nations troops stationed in Bosnia prohibit deadly force unless all ..... actions have been exhausted.
(a) comparable (b) menacing
(c) alternative (d) augmented
(e) extraordinary

17. Despite its lofty goal-truth-many scholars maintain that law as ..... is a highly regulated street fight.
(a) a dogma (b) a study
(c) a profession (d) a philosophy
(e) a lifestyle

18. In spite of the ..... vista of the country dismantled by war and its development clogged by illiteracy, locals like to ..... their nationalism.
(a) sickening ... unveil
(b) diverse ... exhibit
(c) unruly ... curb
(d) picturesque ... conceal
(e) chaotic ... flaunt

19. The citizenry had become so ..... by the government's ..... that the latest financial scandal did not even make the front page of the newspapers.
(a) fascinated ... impropriety
(b) disgusted ... peccadilloes
(c) distraught ... magnanimity
(d) regretful ... personification
(e) jaded ... indiscretions

20. In these politically correct times, it has become ..... to discuss certain subjects at all.
(a) safe (b) eccentric
(c) precarious (d) efficacious
(e) effortless

EXERCISE 3

Direction for questions 1–20: ***The following sentences each contain one or two blanks, indicating that something has been left out of the sentence. Each answer choice contains one word or a set of words. Select the word or set of words, that, when inserted in the blank(s), best fits the context of the sentence.***

1. By ..... celebrities from the sports, entertainment, or business arenas, the show narrates the stories of the ..... newsmakers from all walks of life.
(a) displaying ... pedestrian
(b) profiling ... influential
(c) parading ... effective

(d) narrating ... dominating
(e) setting forth ... ordinary

2. Behind their strange appearance and ..... for carrion, which has long singled them out for fear and loathing, hyenas present a ..... society in which females dominate.
(a) longing ... contrastive
(b) penchant ... realistic
(c) proclivity ... virtual
(d) appetite ... matriarchal
(e) yearning ... monarchal

3. At the cutting edge of research, scientists are developing new sunscreens of both ..... and internal varieties.
(a) polar (b) tropical
(c) territorial (d) atmospheric
(e) regional

4. Although the AIDS epidemic is in the limelight, there is a silent killer ..... through India, killing more people than AIDS itself. The ..... is that, unlike AIDS, this disease is easily cured.
(a) storming ... satire
(b) flaming ... ridicule
(c) raging ... parody
(d) rampaging ... irony
(e) traducing ... sarcasm

5. Knowing Hrithik was overshadowed by many other actors, she knew she was indulging in a bit of ..... when she wondered whether Hrithik was the greatest living actor ever.
(a) irony (b) overemphasis
(c) understatement (d) hyperbole
(e) injustice

6. Their courage is only ....., and a small show of strength is enough to call their bluff.
(a) ostentation (b) fortitude
(c) temperament (d) exhibition
(e) bravado

7. Life, as the film demonstrates, is too complex for ..... endings.
(a) facile (b) intricate
(c) straight (d) occult
(e) recognizable

8. Today, plastic has proved to be a ..... to the environment; the world over, steps are being taken to ban the ..... and non-recyclable material, which has silently taken over our lives.
(a) boon ... ominous
(b) threat ... jeopardizing
(c) menace ... non-eco-friendly
(d) inauspicious ... disastrous
(e) perquisite ... deleterious

9. While environmentalists and NGO's have welcomed the move, there has been strong ..... from many a quarter.
(a) opposition (b) rivalry
(c) approval (d) defiance
(e) acceptance

10. Suicide is the outcome of man's difficulty to ..... himself in society, so he does not feel isolated.
(a) materialize (b) isolate
(c) homogenize (d) secure
(e) integrate

11. Some are born with a ..... to commit suicide, whereas some commit suicide because they are unable to bear ..... changes in their lives.
(a) sentiment ... inimical
(b) resolution ... adverse
(c) predisposition ... cataclysmic
(d) prognosis ... miserable
(e) prodigy ... abrupt

12. It is a situation with a hard, practical edge which raises issues of life-threatening ..... and therefore demands to be addressed with cool, clear-headed ..... .
(a) existence ... involvement
(b) incidents ... erudition
(c) evidence ... cognizance
(d) illustrations ... expedience
(e) immediacy ... pragmatism

13. We landed at the airport with ..... notions of the country as ..... country where many parents are alleged to have sold their children in exchange for food.
(a) paradoxical ... an abounding
(b) incongruous ... an opulent
(c) preconceived ... an impoverished
(d) unwarranted ... an impotent
(e) germane ... a prolific

14. Located amidst the colossal green hills, what might have otherwise been ..... airfield in ..... part of the North assumed the overtones of the battlefield.
(a) an impressive ... an alien
(b) a blood-shattered ... an estranged
(c) a combatant ... a war torn
(d) a picturesque ... an exquisite
(e) a reposeful ... a hostile

15. A discerning publishing agent can ..... promising material from a mass of submissions, separating the good from the bad.
(a) supplant (b) dramatize
(c) finagle (d) winnow
(e) overhaul

16. Although some think the terms "bug" and "insect" are ....., the former term actually refers to ..... group of insects.
(a) parallel ... an identical
(b) precise ... an exact
(c) interchangeable ... a particular
(d) exclusive ... a separate
(e) useful ... a useless

17. The novel's protagonist, a pearl diver, naïvely expects that the buyers will compete among themselves to pay him the best price for his pearl, but instead they ..... to ..... him.
(a) venture ... reward
(b) pretend ... praise
(c) conspire ... reimburse
(d) refuse ... cheat
(e) collude ... swindle

18. His ..... sense of humor caused more ..... than he must have intended.
(a) debunk ... sobriety
(b) wry ... confusion
(c) prominent ... impudence
(d) dry ... jargon
(e) incorrigible ... paucity

19. Because Ranjan was generally ..... on the football field, he was genuinely touched by the amount of praise he received for his game-winning touchdown.
(a) indelible (b) methodical
(c) succinct (d) pious
(e) unheralded

20. Although the professor was quite prominent in her field, she wore an air of ..... while lecturing her students.
(a) unanimity (b) sanction
(c) modesty (d) parsimony
(e) morose

EXERCISE 4

Direction for questions 1–9: ***Each sentence below has three blanks, each blank indicating a missing set of words. Four numbered sets of words follow each sentence. Choose the option that fits each blank in the context of the whole sentence.***

1. The ..... difficulties produced by the marriage ..... prompted some intense ..... in Eliot's poetry.
(a) sentimental, clearly, mistakes
(b) adjustment, obviously, remarks
(c) emotional, evidently, passages
(d) familial, glaringly, smiles

2. Economists engage in ..... enquiry into the effects of those human ..... which are grouped under three broad headings: production, ..., and consumption.
(a) scientific, acts, redemption
(b) methodical, attitudes, supply
(c) systematic, activities, exchange
(d) pragmatic, function, exchange

3. Taipei is the capital, largest city, and ....., ..., and industrial centre of .....

(a) agricultural, educational, world
(b) educational, frugal, Taiwan
(c) educational, commerce, Taiwan
(d) educational, commercial, Taiwan

4. Eclecticism from the Greek 'eklektikos' ("selective"), is the practice of ..... elements ..... in style in a ..... work of art.
(a) compost, different, vague
(b) uniting, several, solid
(c) mixing, diverse, single
(d) converging, unlike, likely

5. It is a ..... held belief that ..... and productivity are a function of ..... or a set of new equipment.
(a) closely, inefficiency, technology
(b) commonly, quality, technology
(c) blindly, profit, management
(d) recently, durability, experts

6. Then in 1856, a similar skull, ... in the Neander Valley outside the German city of Düsseldorf, showed that at least one man's ..... ancestors, later named Neanderthal man, had a low sloping forehead, a receding chin, and thick ridges ..... his eye sockets.
(a) found, possible, in
(b) stalked, surmised, above
(c) unearthed, probable, on
(d) stumbled, closest, around

7. But five years ago, it could hardly have been ..... that a master's thesis ..... this recondite subject, published under the conservative imprint of the University of California Press, would become one of the _____ books of the early 70s.
(a) thought, upon, great
(b) guessed, on, bestselling
(c) considered, about, masterpiece
(d) surmised, over, bestseller

8. But it's there too—in the meetings the priests ..... to schedule their planting dates and combat the problem of crop pests; in the plans they ..... up to maintain aqueducts and police conduits; ..... the irrigation proposals they consider and approve, the dam proposals they reject or amend.
(a) call, set, of
(b) conduct, summon, for
(c) preside, step, with
(d) convene, draw, in

9. The ape-like part was above the waist, including the skull—more like a chimp in size than a human—and the arms, with ..... shoulders and long, curving fingers, well ..... to climbing, although is isn't clear whether they actually did climb ..... this point, or merely retained a more primitive feature.
(a) rounded, adapted, at
(b) moulded, built, around
(c) contoured, grounded, on
(d) hefty, groomed, within

Direction for questions 10–12: ***Each question is a sentence that contains blank spaces to be filled in from the words that are given below. The answer would be the number of blanks that get contextually and correctly filled in, with 1, 2, 3 and 4 standing respectively for the number of blanks that are correctly filled in.***

10. The poisoned chalice, as Shakespeare says, will return to ..... its inventor and propagating a culture of compassion and fellow feeling that ..... all barriers and religious labels is ..... and spiritual duty.

transcends, dominates, an enunciating, buttress, plague.

11. Earlier contributors to this discussion have adequately made the ..... point that in the Indian media, over recent years, ... commercial considerations have increasingly ..... journalistic ethics and editorial ..... .

spurious, crass, transcended, authority, corruption, maneuvered.

12. Is this metamorphosis of the gentleman's game into a ..... sport deplorable? Not at all, never mind the purists. There is more emotion that will ..... the stars. Despite the tag of being a team game there will be the clash of the ..... .

frenzy, midgets, glorified, gladiatorial, proscribe, contaminate.

EXERCISE 5

Direction for questions 1–20: ***Choose the best alternative that can fill in the spaces in the following passages.***

It is clean, smooth, fast and ..... (1) ..... . At every station stroll uniformed guards, truncheons in hand, deterring muggers and graffiti artists. Teams of moppers and sweepers keep platforms ..... (2) ..... . Air-conditioning chills the carriages and a soothing voice, in English and Thai, tells passengers where they are. Compared with the ..... (3) ....., overcrowded underground systems of London, Paris and New York, Bangkok's train on stilts is a ..... (4) ..... success.

Launched a year ago, after five years of building and many more before that of ..... (5) ..... dealings over contracts-the train is supposed to help cure the Thai capital's ..... (6) ..... traffic congestion. ..... (7) ..... and thick with exhaust fumes; Bangkok is one of the most difficult cities in which to get around. But with 600,000 people tempted on to the Skytrain each day, said the Bangkok Transit System Corporation (BTSC), the Skytrain's ..... (8) ....., Bangkok might "breathe again".

Not yet, though. Despite the distribution of 1 m leaflets and much ..... (9) ....., only 170,000 passengers are thought to take the train each day. Its 25 stations and 23 km (15 miles) of track, running along two lines, serve only part of the capital and stretch only some of the way to the city's airport. Most people seem to find it too ..... (10) .....: a short trip costs the same as a taxi and up to three times as much as a bus.

1. (a) frequent (b) vapid
(c) fashionable (d) sluggish

2. (a) unkempt (b) appealing
(c) pristine (d) befouled

3. (a) convenient (b) grubby
(c) sanitary (d) brand-new

4. (a) ludicrous (b) cloudy
(c) desolate (d) sparkling

5. (a) servile
(b) murky
(c) straightforward
(d) understandable

6. (a) fantastic (b) famous
(c) notorious (d) noted

7. (a) Deadlocked (b) Gridlocked
(c) Confined (d) Detained

8. (a) staff (b) advocates
(c) officials (d) authorities

9. (a) publicity (b) propaganda
(c) commercial (d) broadcasting

10. (a) unsafe (b) pricey
(c) onerous (d) confusing

One of the most famous, ... (11) ..... and influential African-American writers of this century, James Baldwin spent his life battling against a ..... (12) ..... enemy: racism. His sensitive and intelligent ..... (13) ..... helped to lay the groundwork for the black liberation movement in the 1950s and 1960s.

Growing up as a young black man in Harlem in the 1920s and 30s, James faced ..... (14) ..... and hatred every day. Finally, he snapped. When he was 18, he ..... (15) ..... a pitcher of water at a waitress who had refused to serve him. She ducked in time and the pitcher ..... (16) ..... a mirror on the wall behind her. When he later described this ..... (17) ....., he said, "My life, my real life, was in danger, and not from anything other people might do but from the hatred I carried in my own heart."

From then on he channeled his anger, frustration and ..... (18) ..... into his art. He

..... (19) ..... racism in classic novels and essay collections including *Notes of A Native Son, Go Tell It On The Mountain*, and *Nobody Knows My Name*. For his literary ..... (20) ....., Baldwin received a host of honors, including a Guggenheim Literary Fellowship, a Ford Foundation grant-in-aid, a Partisan Review Fellowship and a National Institute of Arts and Letters Award in 1956.

11. (a) profound (b) violent (c) bizarre (d) prolific

12. (a) docile (b) savage (c) dreadful (d) insipid

13. (a) prose (b) depiction (c) sketch (d) portrait

14. (a) decency (b) prejudice (c) propriety (d) integrity

15. (a) fractured (b) punctured (c) hurled (d) uplifted

16. (a) scattered (b) penetrated (c) scraped (d) shattered

17. (a) incident (b) memory (c) nightmare (d) episode

18. (a) anguish (b) anxiety (c) rage (d) felicity

19. (a) compliments (b) compromises (c) engages in (d) rails against

20. (a) revenges (b) triumphs (c) fiasco (d) collapse

ANSWER KEY

Exercise 1

1. (e) **2.** (a) **3.** (c) **4.** (a) **5.** (a) **6.** (d) **7.** (a) **8.** (b)
9. (c) **10.** (d)

Exercise 2

1. (c) **2.** (a) **3.** (b) **4.** (d) **5.** (b) **6.** (d) **7.** (a) **8.** (e)
9. (c) **10.** (d) **11.** (a) **12.** (c) **13.** (d) **14.** (b) **15.** (a) **16.** (c)
17. (c) **18.** (e) **19.** (e) **20.** (c)

Exercise 3

1. (b) **2.** (d) **3.** (b) **4.** (d) **5.** (d) **6.** (e) **7.** (a) **8.** (c)
9. (a) **10.** (e) **11.** (c) **12.** (e) **13.** (c) **14.** (d) **15.** (d) **16.** (c)
17. (e) **18.** (b) **19.** (e) **20.** (c)

Exercise 4

1. (c) **2.** (c) **3.** (d) **4.** (c) **5.** (b) **6.** (c) **7.** (b) **8.** (d)
9. (a) **10.** (c) **11.** (c) **12.** (a)

Exercise 5

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (b) **6.** (c) **7.** (b) **8.** (d)
9. (a) **10.** (b) **11.** (d) **12.** (b) **13.** (a) **14.** (b) **15.** (c) **16.** (d)
17. (a) **18.** (c) **19.** (d) **20.** (b)

HINTS AND EXPLANATIONS

Exercise 1

1. Callous means unfeeling, uncaring, but if this person has concern for the earthquake victims, her reputation must be an unfounded one, so the correct choice will mean contradicted or proved false. This is one of the meanings of belied, correct choice option (e). Option (b), rescinded is the second best answer. It means revoked or withdrawn, but you don't say that a reputation is rescinded. Option (a), (c), and (d) are the opposite of what we're looking for—they don't make sense in this context.

2. The author mentioned in this sentence believes that businessmen are models of some quality; whatever qualities they may lack implies that whatever bad points they possess, there's this one particular good thing about them. All of this should lead you to option (a)—if an author's main characters are businessmen, and if they're all paragons of "ingenuity" (meaning inventively talented), one could easily be led to the presumption that the author thinks all businessmen are "clever."

   Several of the wrong answers play off your possible biases about people in the business world, option (b) being the most blatant in that regard. That choice is tempting only because an author's use of many "greedy" businessman characters might suggest that the author thinks all businessmen are "covetous." But labeling businessmen as greedy contradicts the sense of "whatever qualities they may lack"—as we noted, we need a positive quality (also, paragons of greed is awkward). One who is morally upright or "virtuous" option (c) would hardly be a paragon of "deceit" (lying, falseness). Characters possessing great "ambition", option (d), wouldn't necessarily make one presume that the author believes all such people are "successful," since ambition and success in a field don't always go hand in hand; and there's even less connection between businessman characters who demonstrate great "achievement", option (e), and a conclusion that, in the author's opinion, all businessmen are "cautious."

3. We're looking for something that goes with sacred scriptures and implies a formal system of belief, but something whose absence doesn't rule out a legacy of traditional religious practices and basic values. We can eliminate choices (a), (b), and (e) because if Shinto lacked followers, customs, or faith it wouldn't be a legacy of traditional religious practices and basic values. Relics, option (d), are sacred objects but relics don't make something a formal system of beliefs. The best choice is option (c)—a dogma is a formal religious belief.

4. The first blank here will be filled with a word that describes what type of creatures male sperm whales normally are. The word "however" between these clauses indicates that a contrasting point will be made—these whales are normally something, but sometimes can be different. A clue in the second clause helps us figure this one out. In the second clause, we learn that something happens when the whales are jealously guarding their territory. They probably act aggressively or violently when guarding their territory, so the first blank will be a contrast to this. It must suggest that whales are not normally violent. The second blank will be filled with a word that tells us what the

whales do to ships when are guarding their territory.

A good prephrase for this answer would be "these whales are normally peaceful creatures; however when guarding their territory, they have been known to sink ships.

Starting with the first blank, two choices look good right away, option (a) and (d). Option (a) also looks good for the second blank, while the second word in option (d) is the opposite of what we need. Quickly checking the other choices, we see that option (b) the words aren't contradictory, and we need words that are opposite in meaning. Option (c) and (e) can be eliminated because each is a descriptive word that relates to the whale's physical characteristics rather than its temperament. So the answer is option (a).

5. A major sign here is the semicolon between the two clauses, signaling a continuation or consistency between the two parts. If her charisma is a double-edged sword, then it has contrasting points—good aspects and bad. The clause after the semicolon will elaborate on or give an example of these conflicting sides. In the second clause, we find that even in those who like her, her charisma arouses admiration, which would be the good side of the sword, and something else, which must be the bad side. This blank must be filled with a negative word.

A prephrase here might be: "Her charisma provoked both admiration and dislike in her friends."

Only two choices here have negative meanings, option (a) and (d). Option (a) makes the most sense since charisma could easily invoke envy in others. Option (d), obstinacy, is negative as well but means stubbornness; this answer does not make sense in the sentence. The remaining choices can be ruled out because they are positive in meaning, so the answer is option (a).

6. The context of the sentence indicates that the employee will not be disciplined as a result of his mistake, which suggests that the error was not intentional, and that the employee meant no harm. The word "benign" means "harmless," so it is the best choice based on the context of the sentence.

7. The best answer is option (a). Because the sentence indicates that Devesh "loves roses," the word that best fits in the first blank should have a positive connotation. The first word in answer choice (b) has a negative connotation, and the first words in both answer choices (d) and (e) have neutral connotations, so none of those choices will be best. "Aesthetic" refers to "the appreciation of beauty," which makes the most sense in this sentence. Likewise, "enthralled," which means "captivated," works well in the second blank.

8. To have a "penchant" for something means to have a fondness for it. A fondness for history would be practical or helpful for future studies in the field. Answer choices (a) and (c) are incorrect because anyone having "disdain" (hate) or "dislike" for history would certainly not study to become a history teacher.

9. "Vindicated" means "cleared of suspicion or doubt" and "authenticated" means "proved to be genuine." The rest of the answer choices are contradictory in nature and do not fit the context of the sentence.

10. The word "saturated" most accurately indicates the density of the oxygen in the room. The other answer choices do not fit the context of the sentence.

Exercise 2

1. If the public expects a comedian to always make them laugh, then they might not accept a comedian in a serious role. Hence, the directors would be loath (reluctant) to

cast a comedian in a dramatic role. The answer is option (c).

2. The phrase "all but" implies that the debate was a make-or-break event for the candidate. Suppose the candidate did well. Then his spirits would be high, and we would expect the missing word to be positive. However, a positive word in the phrase "*all but ..... any hope*" is awkward. Hence, the candidate must have done poorly in the debate and had his hopes for election nixed. So we turn to the answer-choices looking for "nixed." It's not there, but a synonym—nullifies—is. The answer is option (a).

3. Since no connecting word—such as *and, for, so*, etc.—follows the comma, the phrase *"seemingly with an endless reserve of energy*" defines the missing word. Now, a person with an endless reserve of energy would be lively, which is the meaning of "vivacious." The answer is option (b).

4. "Despite" sets up a contrast between the key phrase "uplifting experience" and the missing word. The implication is that in spite of the rewards, the job is harsh and trying; in other words, rigorous. The answer is option (d).

5. "Yet" draws a contrast between what one would expect an Atheist to do (renounce faith in God) and what Liharev did (maintained faith in God). In other words, he did not lose faith in God. The answer is option (b).

6. To rationalize evil is to make excuses for wrong doing. Now, the words following the colon explain how existentialism can be used to excuse or justify evil. The answer is option (d).

7. The phrase "*who would have believed*" implies that the reality is the opposite of what one would expect. Now, one would not expect the drab DMV to be a catalyst for creativity. The answer is option (a).

8. The sentence implies that even when Chandresh knows he is taking the wrong path in life, he still cannot stop. That is, he cannot exert free will. The answer is option (e).

9. If man has no choice but to seek truth, then this is an essential characteristic of man. In other words, it is part of what makes us human. The answer is option (c).

10. The sentence is saying that although a word may not be explicitly sexist it may contain sexist connotations or implications. The answer is option (d).

11. The passage states that when sitting behind his desk Abdul Kalam looked larger than he actually was. So the perspective must have increased the appearance of his size. The only word that means to increase is "augmented." The answer is option (a).

12. Since man is violent, any useful theory of conflict resolution must incorporate this fact. The answer is option (c).

13. The clause "whereas in a totalitarian regime, no one is listening" implies that a democracy does listen to protests. In other words, it is sensitive to protests. The answer is option (d).

14. "Although" sets up a contrast between what the property values are (high) and what one would expect them to be in a dilapidated (run down) community. The answer is option (b).

15. If the business was not sound, his irritability would belie (contradict) his claim that the business was sound. The answer is option (a).

16. (c) The word "exhausted" implies that all other actions (alternatives) have been tried. The answer is option (c).

17. The sentence is pointing out that as a practical matter the legal profession pursues the truth through a rough and tumble path. The answer is option (c).

18. The country under consideration has been dismantled by war and its development clogged by illiteracy. The author describes the overall situation of the country as "chaotic." One would expect the citizens to feel disgrace in such a situation. But the locals still "flaunt" their nationalism. The answer is option (e).

19. A financial scandal is an indiscretion; and it may not have made the front page because the public was jaded (worn out) by an excess of scandals. The answer is option (e).

20. The sentence is suggesting that it is risky to discuss certain subjects regardless of what you say. The answer is option (c).

Exercise 3

1. The show was organized to tell the stories of successful celebrities. By "profiling" these celebrities, the show narrates the stories of the "influential" newsmakers from all walks of life. The answer is option (b).

2. Since hyenas eat carrion (decaying flesh), they have an "appetite" for it. A society ruled by females is called "matriarchal." The answer is option (d).

3. The word "both" in the statement implies there are two different types of sunscreen. Since one is internal, the other one should be external or at least of a different type. The choices "territorial," "atmospheric," and "regional" do not imply the opposite of internal. "Polar" and "tropical" may indicate other varieties. Sunscreens are not used in polar regions; they are used in tropical regions. Hence, the answer is option (b).

4. The first sentence describes AIDS as a big threat; it also describes another disease that is unreported and is on a greater rampage. Yet, the author says there is a cure for this silent killer. It is ironic that the silent killer causes more harm than AIDS, yet it is curable. The answer is option (d).

5. The statement implies that Hrithik is certainly not the greatest Indian actor. To believe him to be the greatest actor would be to indulge in "hyperbole." The answer is option (d).

6. The word "only" in the first clause limits the meaning of courage: They have courage, but only to a point. The word "bluff" in the second clause indicates that their courage is merely bluster. This is the meaning of "bravado." The answer is option (e).

7. The phrase "too complex for" indicates that the missing word should have the opposite meaning of the word "complex." The opposite of complex is simple. Now, "facile" means simplistic, superficial. The answer is option (a).

8. The second clause states that worldwide steps are being taken to ban plastic. The word "and" joining the two clauses of the sentence indicates that the first clause should provide a reason for the worldwide ban of plastic. The reason is that plastic is proving to be a menace to the environment. The second blank needs to be filled by a word that supports the statement and shows the negative consequences of plastic usage. The suitable word is "non-eco-friendly." The answer is option (c).

9. The word "while" indicates a contrast between what is expressed in the first clause (welcoming the move) and what is expressed in the second clause (opposing the move). The answer is option (a).

10. Man is a social being; and to function properly, he needs to be an integral part of

society. When a person fails to integrate himself into society, he often feels alienated and incomplete, which, without a social support system, can lead to suicide. The author believes that those who commit suicide do so because they are unable to integrate themselves into the society. The answer is option (e).

11. The author states that some people are born with an inclination to commit suicide. Though the remaining people who have committed suicide do not have such a predisposition, they commit it because they are unable to bear sudden cataclysmic changes that occur in their lives. The answer is option (c).

12. According to the author, the core issue involves a hard, practical point that is to be dealt with a pragmatic approach. Pragmatism means "practical approach." The author stresses the immediacy of the situation since it is raising life threatening issues. The answer is option (e).

13. One would reasonably assume (preconceive) that a state where parents are alleged to have sold their children in exchange for food is an "impoverished" state. The answer is option (c). Note, option (d) is the second-best choice. Although an impoverished state is probably impotent, impoverished describes the situation better. Further, the sentence does not imply that the preconceived notions of the state were unwarranted (not justified).

14. The sentence implies that the location, though having splendid natural settings, is distressed by the ongoing battle. We can observe from the structure of the sentence that both blanks explain the natural beauty of the location. The author implies that if the location had no overtones of the battlefield, it would have been a "picturesque" airfield in an "exquisite" part of the North. The answer is option (d).

15. To "winnow" is to sort through and select the desirable part of something or to get rid of the unwanted or undesirable part. If the publishing agent goes through the submissions, "separating the good from the bad," then he or she can be said to be winnowing "promising material from a mass of submissions."

16. The word "although" indicates that the two parts of the sentence contrast with each other: although most people think about the terms "bug" and "insect" one way, something else is actually true about the terms. Choice (c) logically completes the sentence, indicating that while most people think the terms are "interchangeable," the term "bug" actually refers to a "particular" group of insects.

17. The sentence states that the pearl diver expected one kind of behavior from the buyers, but instead they behaved in a different way. Since the pearl diver expected the buyers to "compete among themselves to pay him the best price," the correct answer must be two words that make the last clause describe an opposite situation. Choice (e) is the best answer: Instead of competing to pay the pearl diver the best price, the buyers colluded (or plotted) to "swindle" him.

18. The clue word in this sentence is "caused". The speaker had some unintended effect on his audience. Wry means dry or twisted humor and confusion would certainly be a potential unintended effect from this speaking style. Hopefully, you were able to use process of elimination on many of these answer choices as the dual words doubles the chances you will know the meaning of at least one of the words and be able to eliminate the answer choice if it does not fit. The correct answer is option (b).

19. This sentence's clue word is "because". Unheralded means unappreciated or unnoticed. Clearly Ranjan was not the star

of this team and was not used to receiving accolades. Hence, correct answer is option (e).

20. Although is the clue word. Choice (a) may have confused you if you mistook unanimity with anonymity. This is an example of two words that are similar in appearance and enunciation, but vastly different in meaning. This is also another reason you should examine all 5 answer choices in the exam. The answer is option (c).

Exercise 4

1. The words 'mistakes', 'remarks', and 'smiles' in options (a), (b), (d) respectively, are wrong contextually for the third blank. the words 'sentimental', 'adjustment' and 'familial' of the same options are not proper for the first blank. The word 'evidently' in option (c) has to be preferred to 'clearly' 'obviously' and 'glaringly' in options (a), (b), (d) respectively. Thus the words 'emotional' 'evidently' and 'passages' in option (c) are correct for the three blanks and options (a), (b) and (d) are wrong.

2. The word 'redemption' in option (a) for the third blank is contextually wrong ('redemption' means 'returns or recovery'). The word 'attitudes' in option (b) for the second blank is wrong in the context of 'economists' 'supply' in option (b) will not be proper for the third blank contextually without its counterpart, 'demand'. The word 'pragmatic' in option (d) is not proper contextually for the first blank as an 'enquiry' need not be 'pragmatic'. 'Function' in the same option will be grammatically wrong for the second blank in the context of 'which are' succeeding it. Thus words 'systematic', 'activities' and 'exchange' in option (c) are correct and options (a), (b) and (d) are wrong.

3. The word 'World' in option (a) is wrong grammatically for the third blank ('the' should precede it). 'Frugal' of option (b) is inappropriate as it means 'sparing'. Finally, 'commerce' is incorrect in option (c)—it should have been 'commercial' to fit in. Thus 'educational', 'commercial' and 'Taiwan', in option (d) are correct for the three blanks and options (a), (b) and (c) are wrong.

4. The words 'compost' (mixed manure) and 'vague' in option (a) for the first and third blanks respectively are wrong contextually ('compost' is wrong grammatically also, following 'practice of'). The word 'likely' in option (d) is wrong contextually for the third blank as 'likely work of art' means 'nothing'. The word 'solid' in option (b) is wrong for the third blank in the context of 'work of art'. Thus, 'mixing ', 'diverse' and 'single' in option (c) are correct for the three blanks and options (a), (b) and (d) are wrong.

5. 'Closely and inefficiency' in option (a) for the first and second blanks respectively are wrong contextually. The words 'belief' not to be held 'closely' are in the context of production. 'Inefficiency', preceding 'productively' is wrong in the given context. 'Blindly' and 'profit' in option (c) for the first and second blanks respectively are wrong contextually as 'belief' need not be 'held' blindly and 'profit' comes in after successfully marketing, etc. It is the end-result of all the elements alongside 'productivity'. Also, in 'management' in option (c) for the third blank is not proper because it is a very broad term. 'Recently' in option (d) is also wrong for the first blank with 'held'. Also 'experts' doesn't fit in the third blank. Thus, 'commonly', 'quality' and 'technology' in option (b) are correct as 'commonly' is the most appropriate option preceding 'held belief' for the first blank and 'quality' and 'technology' are

the most correct option for the second and third blank respectively. Options (a), (c) and (d) are incorrect in their respective combinations.

6. The expression 'unearthed' (discovered by searching or rummaging) has to be preferred for the first blank to any of 'found' option (a), 'stalked' (pursue or approach stealthily), option (b) and 'stumbled' (found or encountered by chance) option (d), 'stumbled' also needing a preposition 'on' or 'upon'; 'probable' is also apt for the second; preposition 'on' has to be preferred to other prepositions, options for the third blank, being correct both grammatically and contextually, 'above' option (b) and 'around' option (d) being wrong contextually, 'over', option (b) and 'around' option (d) being wrong contextually, 'over' evoking a connotation of above, in or to a position higher than what is conceived.

7. In the context of or following 'could hardly have been' expression, 'guessed' has to be preferred for the first blank to any of 'thought' option (a), 'considered' option (c) and 'surmised' (conjectured about, or suspected the existence of truth of something); preposition 'on' is the most suitable, for the second, both grammatically and contextually, *i.e.*, preceding 'this recondite subject', compared to any of 'upon' (up + on) option (a), 'about' option (c) and 'over' option (d); adjective 'bestselling' is also appropriate grammatically and contextually, for the third, compared to 'great' option (a) 'masterpiece' (c) or 'bestseller'.

8. In the context of following 'in the plans they', verb 'draw' (compose or draft) is most suitable both grammatically and contextually and has to be preferred for the second blank to any of 'set' option (a), 'summon' (call upon to appear) option (b) and 'step' (increase, intensify) option (c); 'convene' (summon or arrange) option (d) is also appropriate for the first blank following 'in the meetings the priests'; in option (d) is also the most suitable preposition for the third blank in the context of or preceding the 'irrigation proposals', hence, option (d) is the answer.

9. Adjective 'rounded' (given a round shape) option (a) is most suitable for the first blank, in the context of preceding 'shoulders and long, curving fingers'; verb 'adapted' option (b) is also grammatically suitable for the second blank, preceding 'well' and following 'to climbing'; similarly, preposition 'at' is most suitable, for the third blank both grammatically and contextually preceding 'this point'.

10. All the blanks in the sentence can be correctly filled in. 'plague' fills in the first blank, 'transcends' the second, and 'an enunciating' fills the third. Hence option (c) is the answer.

11. 'Crass' fills in the second blank, 'transcends' the third and 'authority', the fourth, hence option (c) is the answer.

12. Only 'gladiatorial' fills in the first blank, hence, option (a) is the answer.

Exercise 5

Vocabulary for Questions 1–10

- **Vapid** = lacking spirit or liveliness
- **Sluggish** = slow
- **Unkempt** = lacking order or neatness
- **Pristine** = unspoiled
- **Befouled** = made dirty
- **Grubby** = dirty = slovenly
- **Sanitary** = free from filth or infective matter
- **Ludicrous** = laughable, ridiculous
- **Desolate** = deserted, abandoned
- **Sparkling** = brilliant
- **Servile** = befitting a slave or servant

- **Murky** = dark
- **Notorious** = widely and unfavorably known
- **Deadlock** = a stoppage of action because neither faction in a struggle will give in
- **Gridlock** = a traffic jam in which an intersection is so blocked that vehicles cannot move
- **Publicity** = public attention or acclaim
- **Pricey** = expensive
- **Onerous** = imposing or constituting a burden; troublesome

Vocabulary for Questions 11–20

- **Profound** = marked by intellectual depth or insight
- **Bizarre** = strange
- **Prolific** = productive
- **Docile** = easily taught, led, or managed; tractable
- **Savage** = cruel, fierce
- **Insipid** = lacking taste or savor
- **Prose** = the ordinary language people use in speaking or writing
- **Puncture** = to make a hole in : pierce
- **Hurl** = to throw down with violence
- **Uplift** = to lift or raise up; elevate
- **Scrape** = to damage or injure the surface of by contact with something rough
- **Shatter** = to dash or burst into fragments
- **Episode** = an incident in a course of events; occurrence
- **Anguish** = extreme pain or distress esp. of mind
- **Rage** = violent and uncontrolled anger
- **Felicity** = the quality or state of being happy
- **Rail against** = to complain angrily
- **Fiasco** = a complete failure

3 Parajumble

IMPORTANCE OF THE TOPIC

Parajumble questions are a regular feature at IIM IPM Aptitude Test. Sentence Arrangement or Jumbled Paragraphs are common test questions at competitive examinations. Examiner wants to assess an examinee's ***reading skills*** as well as ***reasoning prowess*** through these questions. In these questions, students are given a paragraph where the sentences are not in the proper order. You will have to rearrange the sentences around a theme so that they make sense. To solve these questions, what is more important is to look out for any piece of information that can help you connect the sentences together.

QUESTION PATTERNS

While creating a jumble, the examiners have four basic question patterns in mind. However, management entrance tests are dynamic in nature and the examiners can of course think of other innovative question patterns. But, be confident that if you practice on the four basic question types, given below, then you will be able to develop the general skills to handle other innovative question patterns.

Pattern One

The examiner conforms to the example above. There are no fixed sentences; the whole paragraph is jumbled up, and the examinee has to unjumble the lot.

Pattern Two

The examiner gives the examinee the first sentence in its proper position, and then jumbles the subsequent sentences. The examinee now has one anchor instead of two, but can still manage to find a link. An example of this type as follows:

1. To read the characters or the letters of the text does not mean reading in the true sense of the term.

A. This mere mechanism of reading becomes altogether automatic at an early period of life.

B. You will often find yourself reading words or characters automatically, while your mind is occupied with a totally different subject.

C. This can be performed irrespective of attention.

D. Neither can I call it reading when it is just to extract the narrative portion of a text from the rest simply for one's personal amusement.

(a) BACD (b) DCBA
(c) ADCB (d) CBDA

Pattern Three

Pattern Three is a variation on Pattern Two. In this pattern, the examiner gives the last sentence as an anchor, and jumbles the preceding sentences. An example of this type as follows:

A. The recovery of faith is a crucial problem of our age.

B. The deepest them of history is the conflict of faith and unbelief.

C. With it we are destined to enjoy triumphant and splendid advance, and without it we suffer failure and ignominy.

D. Those ages in which faith prevails, in whatever form, are noble and fruitful for the present and the future.

5. All ages in which unbelief, in whatever form, wins an unhappy victory vanish and are forgotten by posterity.

(a) ABCD (b) CDBA
(c) BCAD (d) ACBD

Pattern Four

The examiner gives the examinee the first and the last sentence as anchors, and jumbles the sentences in the middle. This type of jumble is considered to be the easiest because the examinee has two fixed points to guide him/her. An example of this type as follows:

1. An important tenet is that managements must not confuse awards with salaries.

A. An award is a one-time payment and could be anything up to 25% of the annual salary.

B. These can never be retrieved if performance is not stable throughout.

C. It would be better to introduce an increment-cum-award system based on the results.

D. Some companies give enormous salary hikes for excellent performance in a particular year.

E. In this system, the compensation becomes the hygiene factor, and the award, the motivator.

(a) DBCA (b) ABCD
(c) CDBA (d) BCDA

APPROACH FOR UNJUMBLING A JUMBLE

Most of the information given in parajumbles is unnecessary for the purpose at hand, *i.e.*, sorting the sentences. *In essence what we are looking for are things that can help us in connecting the sentences.* Some approaches are given below to help identify the sequence of sentences. Generally, in a given parajumble more than one approach will be applicable at the same time, therefore practices identifying which approach/approaches apply to the parajumbles you have to solve.

Noun-Pronoun Relationship Approach

In noun/pronoun relationships, we know that the noun will come first and will be referred to later using suitable pronouns.

Study the following example:

A. People can get infected by handling reptiles and then touching their mouths or an open cut.

B. At first they look the perfect pets: exotic, quiet and tidy.

C. A study estimates that in 1995, there were as many as 6,700 reptile-caused salmonella infections.

D. But lizards and other pets can harbor a salmonella bacterium that makes people sick.

(a) BCAD (b) BCDA
(c) ACDB (d) BDCA

Read sentences C and D carefully. Sentence D contains the noun phrase "a salmonella bacterium" and sentence C contains the noun phrase "salmonella infections". What is the relationship between the two? Since the phrase "a salmonella bacterium" introduces the bacterium, it should logically precede the phrase "salmonella infections". Therefore, the sentence that contains the phrase "a salmonella bacterium" should come before the sentence that contains the phrase "salmonella infections". So, sentence D should precede sentence C! Once you have a link between two sentences, look at the answer choices to see if you are on the right track. If you are, then you have the right answer [Answer (d)—BDCA] and it is time to move on to the next exercise.

Try this example:

1. These enormous "rivers"—quite inconstant, sometimes shifting, often branching and eddying in manners that defy explanation and prediction—occasionally cause disastrous results.

A. One example is El Nino, the periodic catastrophe that plagues the West Coast of America.
B. It is rich in life.
C. This coast is normally caressed by the cold, rich Humboldt Current.
D. Usually the Humboldt hugs the shore and extends 200 to 300 miles out to sea.
6. It fosters the largest commercial fishery in the world and is the home of one of the mightiest game fish on record, the black marlin.

(a) ABCD (b) DCAB
(c) ACDB (d) CBAD

Read sentences A and C carefully. Notice the noun/pronoun relationship between the two. Sentence A refers to "*the West Coast of America*" and sentence C talks about "*this coast*". Which coast? Obviously "*the West Coast of America*"! Therefore, sentences A and C are related and sentence A must come before sentence C. Now look again. Sentence C talks about "*the cold, rich Humboldt Current*" and sentence D refers to "*the Humboldt*" obviously these two sentences are also related. Which one should come first? Once you have decided, check the answer choices to see if you are correct [Answer (c)—ACDB].

Acronym Approach—Full Form vs. Short Form

When we introduce someone or something, we use the complete name or title. When we refer to the same someone or something later in the paragraph, we use just the surname or the first name if we are on familiar terms with the person being discussed. If we are discussing an object, we remove the modifiers and just use the noun or a pronoun to refer to it.

In Parajumbles we encounter full and short names or sometimes acronyms of some term or institution. Example: World Trade Organisation—WTO, Dr. Manmohan Singh—Dr. Singh, Karl Marx—Marx, President George W. Bush—President Bush or The President. The rule is that if both full form as well as short form is present in different sentences, then the sentence containing full form will come before the sentence containing short form. Let's see this technique at work through an example.

A. If you are used to having your stimulation come in from outside, your mind never develops its own habits of thinking and reflecting.
B. Marx thought that religion was the opiate, because it soothed people's pain and suffering and prevented them from rising in rebellion.
C. If Karl Marx was alive today, he would say that television is the opiate of the people.
D. Television and similar entertainments are even more of an opiate because of their addictive tendencies.

(a) BACD (b) ADBC
(c) BCDA (d) CBDA

Sentence B has Marx (short form) and sentence C has Karl Marx (full form). So sentence C will come before sentence B. Now look at the options. In options (a), (b) and (c), B is placed before C—hence rejected. Option (d) is the answer.

Time Sequence Approach (TSA)—Either Dates or Time Sequence Indicating Words

In a given parajumble, there may be a time indication given, either by giving years—or by using time indicating words. This provides a way for us to identify the correct sequence of the sentences by arranging the sentences using their proper time sequence. Some words through which a time sequence may be indicated are—before, after, later, when, etc.

Let's see an example illustrating this technique.

A. Then two astronomers—the German, Johannes Kepler, and the Italian, Galileo Galilei—started publicly sup-

porting the Copernican theory, despite the fact that the orbits they predicted did not quite match the ones observed.

B. His idea was that the sun was stationary at the centre and that the earth and the planets move in circular orbits around the sun.

C. A simple model was proposed in 1514 by a Polish priest, Nicholas Copernicus.

D. Nearly a century passed before this idea was taken seriously.

(a) CADB (b) BCAD
(c) CBDA (d) CDBA

In the above example you will observe that the flow of logic is in the form of a time sequence which flows from the oldest time period to a more contemporary time period. Therefore, sentence C will be the first sentence. Sentence B expands upon the "*simple model*" proposed, hence it will be the sentence following C. The next sentence in order of chronology is C—*nearly a century passed,* while the last sentence will be A which completes the sequence from older time to contemporary time thus, giving us the answer as CBDA. Hence, the answer is option (c).

Let us look at another example ...

A. By the time he got to Linjeflug four years later, he had learned many lessons. In fact, he began his second stint as top dog by calling the entire company together in a hanger and asking for help, a far cry from his barking out commands just 48 months back.

B. At SAS, he arrived at a time crisis.

C. This book is chock-a-block full of intrusive stories and practical advice, describing Carton's activities at Vingresor (where he assumed his first presidency at age 32), Linjeflug, and SAS in particular.

D. He began at Vingresor as an order giver, not a listener—neither to his people nor to his customers and made every mistake in the book.

(a) CDAB (b) CBAD
(c) BACD (d) BADC

Observe the sequence given. Again you will see a chronological order in the parajumble. Sentence C gives us a clear indication that the book is being talked about in current times. Sentence D then starts tracing Carton's career path from the beginning, thus, leading us to the correct sequence of CDAB. Hence, the answer is option (a).

Structure Approach

In order to unjumble a group of sentences quickly, it is essential for us to understand how language sticks together to form a cohesive unit. English provides certain sequencing words—firstly, secondly, then, however, consequently, on the other hand, etc.—which writers use to join sentences or ideas together and to provide a smooth flow from one idea to the next. It is essential to learn how to spot these words and learn how to use them correctly. Para-jumble sentences often contain several signal words, combining them in complex ways.

Cause and Effect Signals

Look for words or phrases explicitly indicating that one thing causes another or logically determines another. Some examples of such words are:

accordingly	in order to	because	so ... that
consequently	therefore	given	thus
hence	when ... then	if ... then	

Support Signal Words

Look for the words or phrases supporting a given sentence. Sentences containing these words will generally not be the opening sentence. These sentences will follow immediately the sentence supported. Some examples of such words are:

furthermore	additionally	also	and
indeed	besides	as well	too
likewise	moreover		

Contrast Signals

Look for function words or phrases (conjunctions, sentence adverbs, etc.) that indicate a contrast between one idea and another, setting up a reversal of a thought.

nevertheless	nonetheless	on the contrary	notwith-standing	and
even though	instead of	despite	in spite of	while
in contrast	although	however		

Let us put into practice what we have discussed so far. Here is a typical example, combining all the points discussed above.

A. When conclusions are carefully excluded, however, and observed facts are given instead, there is never any trouble about the length of the papers.
B. The reason for this is that those early paragraphs contain judgments that there is little left to be said.
C. A judgment ("He is a boy", "She is an awful bore") is a conclusion, summing up a large number of previously observed facts.
D. In fact, they tend to become too long, since inexperienced writers, when told to give facts, often give more than are necessary, because they lack discrimination between the important and the trivial.
E. It is a common observation among teachers that students almost always have difficulty in writing themes of the required length because their ideas give out after a paragraph or two.

(a) ECDAB (b) CEBAD
(c) EACBD (d) EBCAD

Sentence E states the situation in general and gives us information about why students have problems "*in writing themes of the required length*". Sentence B goes on to tell us "*the reason for this*", so the two sentences must be related. Similarly, sentence C is related to sentence B because both sentences contain the word "*judgment*", with sentence C explaining what the word means. Once a link of this nature is established, go to the answer choices to see if you are on the right track. Out of answer choices, option (d) offers this line of reasoning. Hence, option (d) is the answer.

In the above jumble, the word "*however*" in sentence A suggests a contrast to something mentioned previously. In situations of this kind, it is always a good idea to separate the argument clearly. Sentences A and D, therefore, should come together. Now you try.

1. To read the characters or the letters of the text does not mean reading in the true sense of the word.

A. This mere mechanism of reading becomes altogether automatic at an early period of life.
B. You will often find yourself reading words or characters automatically, while your mind is concerned with a totally different subject.
C. This can be performed irrespective of attention.
D. Neither can I call it reading when it is just to extract the narrative portion of a text from the rest simply for one's personal amusement.

(a) BACD (b) DCBA
(c) ADCB (d) CBDA

The word "*neither*" in sentence D will tell you that there is something additional that the writer wishes to discuss. Sentences 1, A, B and C all talk about the same idea. Therefore, sentence D should be the last sentence. Hence, option (a) is the answer.

Linking the Sentences

Example 1

Let us look at the following statements:

A. As a retention strategy, the company has issued many schemes including ESOPs.
B. Given the track record and success of our employees, other companies often look to us as hunting ground for talent.

C. The growth of the Indian economy has led to an increased requirement for talented managerial personnel and we believe that the talented manpower is our key strength.
D. Further in order to mitigate the risk we place considerable emphasis on development of leadership skills and on building employee motivation.

Start thinking like this:

Can A be the opening statement—Very unlikely. It does not introduce any idea or theme. Ideally the first statement would be an initiator of ideas or theme of the passage.

Can B be the opening statement—May be.

Can C be the opening statement—May be.

Can D be the opening statement—Very unlikely as it talks about an idea which is being "furthered" in this statement. You can also see that statement D talks about "Mitigating the risk". What is the risk? So now we would try to find out the "risk" in other statements. This "risk" is present in statement B in the words—"other companies often look to us as hunting ground for talent". So, statement B will come before statement D.

As discussed earlier, statement D furthers an idea, and that idea is present in statement A. Hence, statement A–D should come together.

Let us see all that we have established so far:

Link—Statement A–D and statement B will come before statement D and statement A cannot be the starting statement.

Now let us look at the options:

(a) Statements A, B, C, D—Ruled out and statements A–D link is not present.
(b) Statements B, A, D, C—This is the only option left out.
(c) Statements C, A, D, B—Ruled out as statement B comes after statement D.
(d) Statements D, A, C, B—Ruled out as statements A–D link is not present.

Hence, option (b) is the answer.

Example 2

Let us look at another example:

A. In America, highly educated women, who are in stronger position in the labour market than less qualified ones, have higher rates of marriage than other groups.

[A is the first statement. You are required to re-arrange the following four statements.]

B. Some work supports the Becker thesis, and some appears to contradict it.
C. And, as with crime, it is equally inconclusive.
D. But regardless of the conclusion of any particular piece of work, it is hard to establish convincing connections between family changes and economic factors using conventional approaches.
E. Indeed, just as with crime, an enormous academic literature exists on the validity of the pure economic approach to the evolution of family structures.

(Options Withheld *pro tem*)

I have deliberately not given the options here.

Read all the statements one by one, and try to find out the opening statement and any possible linkage between/among the statements.

Start thinking like this:

Can statement B be the opening statement—Very unlikely. There is no mention of "Becker thesis" in the opening statement.

Can statement C be the opening statement—Two words in the statement—"Crime" and "Inconclusive" make this as the statement after statement A very unlikely.

Can statement D be the opening statement—Though it furthers the idea presented in statement A, usage of word like "But" make it unlikely to be the statement coming just after statement A. Besides, statement A does not talk about any piece of work. In fact, statement A is just an opinion.

Can statement E be the opening statement—Yes. Statement E is the statement after statement A—both through elimination of other statements and selection.

Next statement should be C, as it again talks about 'Crime' and how inconclusive it is (despite "an enormous academics literature exists" as given in statement E).

Next statement D should be the last statement as it concludes the whole theme that "it is hard to establish". Now the whole point is—how do we place statement B and Becker thesis, which finds no mention in the whole passage. Only reasoning that can be given here is—Probably this passage has been taken from a book or project report which has something to do with Becker thesis, and this passage is just a small part of it.

Now let us look at the options and try to eliminate the options with the help of the conclusions that we have derived so far:

(a) BCDE—Ruled out as statement E is the first statement.
(b) DBEC—Ruled out as statement E is the first statement.
(c) BDCE—Ruled out as statement E is the first statement.
(d) ECBD—This is the only option left out.
(e) EBCD—Ruled out as statements EC_D is the link.

Hence, option (d) is the answer.

PRACTICE EXERCISES

EXERCISE 1

Direction for questions 1–15: ***The first and the last parts of the sentence are marked 1 and 6. The rest of the sentence is split into four parts and marked P, Q, R and S. These four parts are not given in their proper order. Read the sentences and find out which of the five combinations is correct.***

1. 1. It depends upon the
P. a new heaven on earth or to destroy
Q. user, whether science will be used to create
R. the world in a
S. outlook and mentality of the
6. common conflagration.
(a) PQRS (b) SPQR
(c) QPRS (d) SQRP
(e) SQPR

2. 1. The English
P. plundered the country
Q. and raw materials,
R. as a result, the Company became rich
S. of precious mineral wealth
6. and the Indian people became poor.
(a) PQSR (b) SQPR
(c) SRQP (d) PSQR
(e) PSRQ

3. 1. The power and pride
P. for in the courage
Q. of Sparta was above all
R. discipline and skill of these troops
S. in its army,
6. it found its security and its ideal.
(a) SPRQ (b) PQRS
(c) QSPR (d) PRQS
(e) QSRP

4. 1. It is the
P. the careful observer
Q. apparently trivial phenomena
R. which gives even the
S. intelligent eye of
6. their value.
(a) SPRQ (b) PQRS
(c) QSPR (d) PRQS
(e) SPQR

5. 1. Nuclear test explosions
P. food as well as
Q. present and future generations
R. directly injuring the
S. take place, contaminating air and water and
6. of mankind.
(a) RQSP (b) PSQR
(c) SPRQ (d) PQRS
(e) SQRS

6. 1. The club is an
P. nearby municipal school, for the children
Q. proper and healthy functioning of a
R. are constantly disturbed by its
S. intolerable nuisance to the
6. bizarre activities.
(a) QSRP (b) SQPR
(c) PQRS (d) RPSQ
(e) SQRP

7. 1. What the country needs
P. and change tactics
Q. who would encourage players
R. are coaches and officials
S. to read the game as it progresses
6. accordingly.
(a) RQPS (b) SPRQ
(c) QSPR (d) RQSP
(e) RSPQ

8. 1. The need of administration
P. which could bring together
Q. language of the rulers, should be the one link
R. apart from those of liberal education
S. render it necessary that English, as the
6. linguistic areas.
(a) RSQP (b) SRQP
(c) PQRS (d) QSPR
(e) RSPQ

9. 1. Economic domination
P. anger and
Q. a different language
R. of persons speaking
S. often causes
6. a sense of revolt.
(a) PSRQ (b) QRPS
(c) RQPS (d) SPQR
(e) RQSP

10. 1. It may be
P. of comparatively little consequence
Q. whilst everything depends upon
R. how a man is governed from outside
S. how he governs
6. himself from within.
(a) SPRQ (b) QSPR
(c) PRQS (d) RQPS
(e) PRSQ

EXERCISE 2

Direction for questions 1–10: ***Sentences given in each question, when properly sequenced, form a coherent paragraph. The first and last sentences are 1 and 6, and the four in between are labelled A, B, C and D. Choose the most logical order of these four sentences from among the five given choices to construct a coherent paragraph from sentences 1–6.***

1. 1. Ever since Pakistan signed on as an ally in the war on terror, human rights advocates have voiced concerns that innocent people would be caught up in the net.
A. Roughly half of the youths held are from families deemed uncooperative in the anti-terror fight.
B. Seven women and at least 54 children under the age of 18 are languishing in prisons in the tribal areas of the NWFP under the so-called collective responsibility clause, which permits imprisonment of a person for crimes committed by a relative, clan or tribe.
C. The decree has most recently been wielded to force tribal leaders to reveal the whereabouts of Qaeda and Taliban suspects.
D. Now they are pointing to evidence.

6. The rest were imprisoned for the alleged crimes of their fathers.

(a) BACD (b) DBCA
(c) BCDA (d) DBAC
(e) BDCA

2. 1. The smaller mobile phones get the more functions they seem to acquire.

A. Beginning early next year, TU Media, a consortium of SK Telecom and other Korean companies, will begin beaming television programmes via satellite to special Digital Multimedia Broadcasting phones.

B. The phones, made by Samsung, have 5.5-centimetre screens and cost $700.

C. Now a South Korean firm is rolling out mobiles that double as televisions—and high-definition ones, at that.

D. Based on early tests, executives at TU Media think that the new service will improve on SK's existing cellular-based television phone service, which has 3 million subscribers but is often interrupted due to poor transmission.

6. The biggest problem, though, is coming up with programming.

(a) BDAC (b) BADC
(c) CABD (d) CBAD
(e) CADB

3. 1. Although Maradona recounts his life with neither explanation for his actions (save his love of football) nor fluid penmanship, his story is gripping.

A. Maradona blames everyone but himself: luck, the media, managers, football executives—you name it, they did it.

B. And, ironically, the book's omissions are its most revealing element.

C. In passing the buck, El Diego reveals who he really is—a man accountable to no one.

D. It's a tale of cataclysmic ups and downs as he hopskotches from the junior leagues of Argentina to four World Cups with the national side, from solidarity with the people of Naples to seeking solace from that other fallen idol, Fidel Castro.

6. I now believe that Maradona truly is a god and that's why I am an atheist.

(a) CDAB (b) DABC
(c) DBCA (d) DBAC

4. 1. The journalists have all left Haiti.

A. The suffering caused by the floods is no longer news, and we now work far from the headlines.

B. Perhaps I have spent so much time in the Third World that I'm immune to it; one develops a thick skin, if only as a defense mechanism.

C. Some of the aid workers are going as well, myself among them.

D. How do I feel about the misery I've seen?

6. Otherwise you would be unable to do your job.

(a) CBAD (b) ACBD
(c) BCDA (d) ACDB
(e) ADCB

5. 1. Many space enthusiasts now warn that only private enterprise will truly drive human expansion into space, and yet America's government keeps ignoring them.

A. It may seem surprising, but there are large numbers of people who would spend hundreds of thousands of dollars on a trip into space.

B. Tourism and entertainment are both possibilities.

C. Two people, so far, have spent $20 million, and another two are on their way.

D. What might cause market forces to take up the mission?

6. Film and television companies would also spend tens of millions if they could.

(a) DCAB (b) BACD
(c) DBAC (d) ACBD
(e) DACB

6. 1. Zimbabwe provides a dramatic illustration of how stagy economic policies, corruptly enforced, swiftly impoverish.

A. The main reason the continent is so poor today is that Mugabe-style incompetent tyranny has been common since independence.

B. But Zimbabwe's curse is also Africa's.

C. The most important question for Africans now is whether Mr. Mugabe represents not only their past, but their future as well.

D. In the past five years, Mr. Mugabe's contempt for property rights has made half the population dependent on food aid, while his cronies help themselves to other people's land and savings, and build helipads for their own mansions.

6. There are encouraging signs that he does not.

(a) DBCA (b) CDBA
(c) DCAB (d) DBAC
(e) DABC

7. 1. If Africans are to have a chance of pulling themselves out of penury, they need governments that do not stand in their way.

A. Fiscal realism is more common now than a decade ago, as the continent's generally lower inflation rates attest.

B. They need leaders who uphold the law impartially, but otherwise let people do what they wish.

C. But graft is still widespread.

D. They need governments that pass sensible budgets and stick to them.

6. Angola's rulers were accused of having wasted or misappropriated $4 billion in five years—more than 9% of GDP each year.

(a) BADC (b) BDAC
(c) DACB (d) DBAC
(e) DBAC

8. 1. Africa's two most important countries—Nigeria and South Africa—are doing several things right.

A. If Africa as a whole is to prosper, the majority of its citizens will have to produce more, fashioning goods or providing services that the rest of the world wants to buy.

B. Given that most Africans are subsistence farmers, that will not be possible without a vast social upheaval, with unpredictable consequences.

C. Both have swapped tyranny for democracy, and both are using their diplomatic and military muscle to end some of their neighbours' wars.

D. But both governments are worryingly dependent on a single source of revenue: oil, in Nigeria's case, and white taxpayers, in South Africa's.

6. It is a daunting challenge, but the alternative is likely to be worse.

(a) CDAB (b) DCAB
(c) CBDA (d) DBAC
(e) CDBA

9. 1. The truth is that traditional economic labels are less and less useful.

A. Different organisations and banks use different groupings.

B. The old "third world" label has been replaced by "developing" or "emerging" economies to portray a more positive image.

C. "Industrial economies", the old name used for rich countries, is now meaningless: industry accounts for only 16% of jobs in America.

D. But the breakdown of old categories has bred confusion.

6. For instance, the United Nations and JP Morgan Chase count Hong Kong,

Singapore, South Korea and Taiwan as developing (or emerging) economies.

(a) ADBC (b) CBAD
(c) ABCD (d) CBDA
(e) DBCA

10. 1. Jane Austen died and came back as a fantasy writer.

A. The book itself has been called, by a media ever eager to summarise even 800-page hardcover tomes into a snappy catchphrase, "Harry Potter for adults".

B. Unlike her previous avatar, the 21st-century Ms Clarke (nee Austen) seems to be enjoying the attention showered upon her and far from publishing her first book under a pseudonym, has been a central performer at her own media circus.

C. The book, which she now calls her "debut", began attracting media attention long before publication and on release it's been universally lauded.

D. In her new avatar, she calls herself Susanna Clarke, lives in Cambridge, and has authored a fat historical fantasy novel set in the year 1860.

6. It's also been praised by perhaps the best living author of British fantasy novels, Neil Gaiman, as "unquestionably the finest English novel of the fantastic written in the past seventy years."

(a) BADC (b) DBCA
(c) DCAB (d) DCBA

EXERCISE 3

Direction for questions 1–6: ***In each of the following questions, four sentences are given which, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the given choices to construct a coherent paragraph.***

1. A. So Iraq and Afghanistan are now sun-drenched lands enjoying liberty and freedom, overflowing with joy and prosperity?

B. Yet in a new campaign ad, Republican Party strategists shamelessly take credit for "two more free nations" and "two less terrorist regimes," using footage of the Iraqi and Afghan Olympic teams.

C. The bylaws of the International Olympic Committee and the US Olympic Committee strictly prohibit the use of the word "Olympic" and related symbols for political ends.

D. The US Congress stipulates that the USOC "shall be nonpolitical and may not promote the candidacy of any individual seeking public office."

(a) CDBA (b) CDAB
(c) DABC (d) ACDB

2. A. He is the faceless man who is nowhere and everywhere.

B. He will live on bread and water, sleep on the streets, move from house to house.

C. It is impossible for the political leaders of a tame western world that wages its wars by remote control to understand the power of the marginal man, who has been deprived, stripped of his right and driven back to the wall.

D. This is a man who has lost everything, his home, his family, his country, even his personal history.

(a) CDAB (b) DABC
(c) CABD (d) CDBA

3. A. The first vision is cyclical and millennial.

B. It sees man originating from the greater anthropoid apes, climbing slowly through tribal groups, via the Renaissance and the early Industrial Age, into modern technological man.

C. The second is linear and utopian.

D. It sees man originating from a divine source, a fall, exile, a final battle, the regeneration of the world by a Divine Saviour and the foundation of the Kingdom of God on earth.

(a) ABCD (b) CBAD
(c) ACBD (d) ADCB

4. A. One may disagree with some of his contentions, but in the aftermath of September 11, one must give credence to his central point: that in the post-Cold War world, the critical distinctions between people are not ideological or economic—they are cultural.

B. Samuel P. Huntington's book *The Clash of Civilisations* and *The Remaking of World Order* is very topical these days.

C. Religious, ethnic and racial strands that constitute cultural identity are the central factors shaping associations and antagonisms.

D. Thus, cultural communities are replacing Cold War blocs and the fault-lines between civilisations are becoming the nerve-centres of conflicts in the world.

(a) BADC (b) CBAD
(c) BACD (d) DBAC

5. A. Even when it is self-willed, migration causes dislocation and alienation, creating the need for meaningful identities at the individual level.

B. The root cause is modernisation that has spurred migration in large numbers.

C. Simultaneously, education and development within non-western societies stimulates the revitalisation of indigenous identities and culture.

D. Huntington's analysis explains why we are witnessing this surging antagonism among Indian and Pakistani youth and expatriates.

(a) DBCA (b) BCDA
(c) DBAC (d) BADC

6. A. Not too far from the bright lights of Mumbai, a silent scourge is killing thousands of young children.

B. Their ill-fed bodies vulnerable to infection, most succumbed to ailments as minor as diarrhea.

C. Stalked by chronic hunger and disease, nearly 30,000 children below the age of six have died in the last year alone in the state's rural belt.

D. It may be considered the country's dollar magnet and envied for its robust industrial base but this image of Maharashtra cloaks a macabre reality.

(a) CBDA (b) DACB
(c) BDCA (d) CDBA

Direction for questions 7–10: ***Given below are five sentences S1, P, Q, R and S that make a paragraph. The sentences making the paragraph have been jumbled randomly, only the first sentence S1 is at its right place. Find out the correct sequence in which P, Q, R and S should follow after S1 so that a coherent and meaningful paragraph is made.***

7. S1. There was a time Egypt faced economic crisis.

P. Cotton is the chief export commodity of Egypt.

Q. Foreign trade depends on cultivation of cotton on large scale.

R. It became necessary for Egypt to boost cotton crops.

S. Only by means of increasing foreign trade Egypt could survive.

(a) PQRS (b) QRPS
(c) RPSQ (d) SRQP

8. S1. Exercising daily is a must for good health.

P. Luckily, there is no link between the amount of money spent and beneficent exercise, else the poor would have creaking bodies.

Q. While some cost you nothing, others may require the investment of some amount of money.
R. However, it is important to remember that exercises should not be overdone.
S. It can take any form from sedentary ones like walking to vigorous workouts like a game of squash.

(a) PQSR (b) QPRS
(c) RSQP (d) SQPR

9. S1. Urban problems differ from state to state and city to city.
P. Most of the cities have neither water nor the required pipelines.
Q. The population in these cities has grown beyond the planners' imagination.
R. However, certain basic problems are common to all cities.
S. Only broad macro-planning was done for such cities, without envisaging the future growth, and this has failed to meet the requirements.

(a) PQSR (b) QPSR
(c) RQSP (d) RQPS

10. S1. Satyajit Ray made several films for children.
P. Later film-makers have followed his lead.
Q. Today other nations are making children's films in a big way.
R. This was at a time when no director considered children as potential audience.
S. Ray was, thus, a pioneer in the field.

(a) SQRP (b) RSQP
(c) PSQR (d) RSPQ

EXERCISE 4

Direction for questions 1–10: ***A number of sentences are given below which, when properly sequenced, form a COHERENT PARAGRAPH. Choose the most LOGICAL ORDER of sentence from the choices given to construct a COHERENT PARAGRAPH.***

1. A. He somehow knew he would find what he was looking for. So with missionary zeal, he started to climb.
B. So instead, for perhaps the first in this life he shed the shackles of reason and placed hi trust in his intuition.
C. At first he thought about hiring a Sherpa guide to aid him in his climb through the mountains, but, for some strange reason, his instincts told him this was one journey he would have to make alone.
D. The next morning, as the first rays of the Indian sun danced along the colorful horizon, Julian set out his trek to the lost land of Savanna.

(a) ABCD (b) ACBD
(c) CDAB (d) DCBA

2. A. It reverberates throughout the entire Universe. And you are transmitting that frequency with your thoughts!
B. The frequency you transmit reaches beyond cities, beyond cities, beyond countries beyond the world.
C. You are a human transmission tower, and you are more powerful than any television tower created on earth.
D. Your transmission creates your life and it creates the world.

(a) DACB (b) BDCA
(c) CDBA (d) ABCD

3. A. Asian economies will need alternative sources of growth to compensate for the rapid fall in demand from the western markets.
B. But the crisis has exposed the limits of region's dominant economic-growth model.
C. The export-led model that propelled many Asian economies so effectively for the past 30 year must be adapted to a different global economic context.

D. Asia is less exposed to the financial turmoil than the west is, because Asian countries responded to the previous decade's regional crisis by improving their current-account positions, accumulating reserves, and ensuring that their banking systems operated prudently.

(a) DBAC (b) ABCD
(c) CABD (d) BCDA

4. A. The dangers of conflicting irrational majoritarianism with enlightened consensus are, indeed, great in developing democracy.

B. Real democracy is about mediating the popular will through a network of institutional structure and the law of the land.

C. While law making and governance are meant to articulate the latter, the judiciary is supposed to protect the former from any kind of excess that might occur, unwittingly or otherwise, in the conduct of legislative and governmental functions.

D. The principle of separation of powers in meant to embody a desirable tension between individual rights and social consensus.

(a) ABCD (b) BACD
(c) DCAB (d) DABC

5. A. First may be necessary for immediate relief.

B. However, to cure the problem from the root the treatment at the elemental level is a must.

C. Therefore synergy of modern medical science and ancient Indian wisdom is in the interest of humanity.

D. Allopathic treatment is symptomatic while Ayurveda treats at an elemental level.

(a) DBAC (b) DABC
(c) DCBA (d) BDCA

6. A. Such inter-operability of a software service or product appears to be only one aspect, and the interoperable system is itself evolving.

B. Each software product introduces a variation and consequently a change in the system.

C. An operating system must work with applications and other elements in a hardware platform.

D. A software firm while introducing its product or service, therefore, does not strive for mute complementarities alone but tries to bring about a change in the existing structure.

E. In other words the components must be designed to be inter-operable.

(a) BCDAE (b) CEABD
(c) DAEBC (d) CBEAD

7. A. Moreover, as argued above, knowledge is entailed not by way of justification as such, but by the realization of good or fruit-ladenness of meaning and actions or iterated actions.

B. Knowledge is required in order to resolve doubts and thus, in order to act meaningfully.

C. Therefore the actions in a commonly led daily life are both meaningful and knowledge-driven.

D. Indian theorists argue for a common knowledge, which is obtained through iterated fruitful actions, through the authority of sentences (on words).

E. We argue for four sources of validation of knowledge, viz., sentence, inference, direct perception and analogy.

(a) AECBD (b) BDCEA
(c) BAECD (d) EADCB

8. A. But PST has also used satellite pictures to suggest that an ancient fortified town had existed 30 km from Junagadh.

B. Soil and vegetation patterns were used in the search.

C. The site matches the description of Krishna's town in an ancient scripture.
D. PST's primary job at Space Applications Centre has been tracking land use and forest cover with satellite images.
E. An archaeologist however cautioned that remote sensing and scriptures by themselves would not be enough to identify a town ship.
F. It was claimed that soil and vegetation patterns an ancient abandoned site reveal specific patterns that can be picked by satellite images.

(a) DACBEF (b) DACBFE
(c) FDCABE (d) FDACBE

9. A. But if, having done so, you did not spare constructive thought as to why your neighbourhood was becoming crime-prone, you would be leaving yourself vulnerable to similar future intrusions.
B. While the international community can only hope that Moscow's hostage drama is resolved without further loss of innocent lives, it would not be inappropriate once again to attempt to understand the nature and motivation of the archetypal terrorist who seeks martyrdom.
C. Today, in an era of globalisation, when the world is increasingly becoming an interpenetrative community, we need to ask whether misconceived state policies do not aid and abet terrorists in the guise of martyrs.
D. If you were to apprehend an armed nocturnal intruder in your house, your first—and correct—reaction would be to somehow render the criminal powerless.
E. Ill-regulated communities breed crime.

(a) BCDEA (b) DABEC
(c) CEDAB (d) BDAEC

10. A. Products of mutual funds and insurance companies all aimed specifically at women are an example of this fact.
B. For this purpose, we looked at the district wise figures for working women.
C. They influence a lot of consumption decisions and hence there is a distinctive attempt to target them.
D. The census provides interesting data on the presence of women in different areas in the country, but what is more important is the extent of employment.
E. Women are a very important target group for a whole host of corporates.

(a) DCEAB (b) DBECA
(c) ECABD (d) ECADB

ANSWER KEY

Exercise 1

1. (e) **2.** (d) **3.** (c) **4.** (a) **5.** (c) **6.** (b) **7.** (d) **8.** (a)
9. (e) **10.** (c)

Exercise 2

1. (b) **2.** (c) **3.** (d) **4.** (d) **5.** (c) **6.** (d) **7.** (b) **8.** (a)
9. (d) **10.** (d)

Exercise 3

1. (a) **2.** (b) **3.** (d) **4.** (c) **5.** (c) **6.** (b) **7.** (d) **8.** (d)
9. (d) **10.** (d)

Exercise 4

1. (d) **2.** (c) **3.** (a) **4.** (c) **5.** (b) **6.** (d) **7.** (b) **8.** (b)
9. (d) **10.** (d)

HINTS AND EXPLANATIONS

Exercise 1

1. SQPR, answer is option (e).

2. PSQR, answer is option (d).

3. QSPR, answer is option (c).

4. SPRQ, answer is option (a).

5. SPRQ, answer is option (c).

6. SQPR, answer is option (b).

7. RQSP, answer is option (d).

8. RSQP, answer is option (a).

9. RQSP, answer is option (e).

10. PRQS, answer is option (c).

Exercise 2

1. "They" in sentence D refers to "human rights advocates" in sentence 1. Hence, 1D is a sequence. Thus, option (a) and (c) are ruled out. Again, "the rest" in sentence 6 is obviously a sequel to "half of the youths" in sentence A. Which means sentence A6 is a sequence. Hence, option (d) is ruled out and answer is option (b).

2. Sentence C tells us about "the more functions" in sentence 1. Hence, sentence 1C is a sequence. Thus, option (a) and (b) are ruled out. And "the phones" in sentence B refers to the "special Digital Multimedia Broadcasting phones" in sentence A. Thus, sentence AB is a sequence. Hence, option (d) is ruled out and answer is option (c).

3. Look at the first word *It's* (focus on *it*) in sentence D. It refers to "his story" in sentence 1. Thus, sentence 1D is a sequence. Hence, option (a) is ruled out. And what is one of "the book's omissions"? That Maradona does not blame himself. In other words, sentence A is an explanation of sentence B. Thus, sentence BA is a sequence. Hence, option (b) is ruled out and option (d) is the answer.

4. The clue lies in "otherwise" in sentence 6, which here means "if you do not develop a thick skin". Obviously, sentence B6 is a sequence. Now, option (d) is the only such choice where sentence B can precede sentence 6. Hence, option (c) is the answer.

5. Sentence B is the answer to sentence D. Hence, sentence DB is a sequence. Besides, sentence C6 is another sequence. Hence, option (c) is the answer.

6. Sentence 6 is the answer to the question raised in sentence C. Thus, sentence C6 is an obvious sequence. Enough for our answer! Hence, option (d) is the answer.

7. Sentence 6 is an example of what is mentioned in sentence C. Again, we get sentence C6 as a sequence. Sentence A goes best with sentence D, thus, giving us

sentence DA as another sequence. Hence, option (b) is the answer.

8. Note the pronoun *both* in sentences C and D. It refers to "Nigeria and South Africa" mentioned in sentence 1. But D should come later because it is a sentence that begins with *But*. Hence, option (a) is the answer.

9. Here sentence 6 is an elaboration of sentence A. We thus, get sentence A6. Enough for our answer! Hence, option (d) is the answer.

10. Can sentence B come before sentence D? No. Because "Ms Clarke" in sentence B can come only after "Susanna Clarke" in sentence D. Hence, option (a) is ruled out. Again, note that "the book" in sentence C refers to "a fat historical fantasy novel" in sentence D. Hence, sentence DC is a sequence. Thus, option (b) is ruled out. Hence, option (d) is the answer.

Exercise 3

1. Short and perhaps therefore easy! C is the opening sentence. Sentence D elaborates it further. Sentence B gives a turn to the para. And sentence A comes as the conclusion. Hence, option (a) is the answer.

2. The only independent sentence is C and it should therefore come at the beginning. Don't be confused by "it" at the beginning: it is an indefinite pronoun here. The passage is about "the marginal man" talked about in sentence C. And then there is a gradual sequencing: *the marginal man, this is a man, he*. So either option (a) or (d). But option (a) is a better arrangement as far as tense is concerned. Hence, option (a) is the answer.

3. Easy one! The clue lies in the words "cyclical" and "linear". Hence, option (d) is the answer.

4. Clearly, B is the initial sentence and sentence D the conclusion. Hence, option (c) is the answer.

5. We begin with D, the only independent sentence. Also, sentence BA is a sequence. Hence, option (c) is the answer.

6. D is the first sentence as it outlines the paradoxical situation. The rest of the passage goes on to delineate the second part of the paradox. Hence, option (b) is the answer.

7. (d)
8. (d)
9. (d)
10. (d)

Exercise 4

6. The paragraph must start with sentence C as it introduces the subject, followed by sentence B which carries on the idea of software. Hence, option (d) is the answer.

7. The paragraph must start with sentence B as it introduces knowledge, and the idea is continued in sentence A. Hence, option (b) is the answer.

8. The paragraph must start with sentence D followed by sentence AC, then sentence F should precede sentence E. Hence, option (b) is the answer.

9. Sentence DA is obviously a sequence. Now, we have to find out which of the remaining sentences does DA elaborate. Or does it elaborate none and we can therefore start with sentence DA? On close scrutiny, we find sentence DA is an elaboration of sentence B. Hence, option (d) is the answer.

10. "This fact" at the end of sentence A refers to sentence C. Thus, sentence CA is a sequence and option (a) gets ruled out. Again, "this purpose" in sentence B is a reference to sentence D. Thus, sentence DB is a sequence and option (c) gets ruled out. "They" in sentence C is a reference to "women" in sentence E. So, clearly we have two sequences—ECA and DB. If you place sentence DB first, it doesn't make sense. Hence, option (b) is ruled out and option (d) the answer.

4 Para Completion

IIM Indore aptitude test paper has paragraph completion questions as a regular feature. It made its appearance in aptitude test 2013 as well.

In these questions, a short paragraph is given with a sentence from it removed. Generally, the last sentence of the paragraph is left as a blank, though it is not necessary. There may be some questions in which a statement from middle of the passage is removed. It is followed by four possible options and you will be required to choose that which one can fit into the paragraph best.

SOLVING THE PARA COMPLETION QUESTIONS

Quickly but carefully read the passage at least twice and figure out the scope, theme, tone and logical flow of the passage; and you are ready to eliminate the choices to figure out the best choice at hand!

Let's take a look at these individual elements in detail, and you shall find this section a cake walk for sure. Para completion questions can help you to boost your sectional and overall test score.

Scope of the Passage

Try to figure out what the paragraph is all about. What is the topic or subject area of the passage, and what are its limits. So, anything that strays out of the boundaries cannot be a part of the passage. Any answer choice that talks about something outside the scope of the passage is a outcast, and hence, should be eliminated in the first go. Read the following lines carefully, and decide what the scope of the passage is?

In my own constituency, in April, which is a hot month, we had hailstorms of the size of a tennis ball, which destroyed the entire wheat crop in the Tarai. We had snow in places where it had never snowed. We had snow late, we had flowering late and we lost large number of crops due to flooding. This is going to increase year after year.

Here the scope of the passage would be the changing weather conditions within a particular area, however it cannot be generalized.

Theme or Gist of the Passage

You have to think like the author of the passage. If you are able to put yourself into the shoes of the author, it should not be difficult to figure out what does not fit into the passage. Focus on the key words and try to figure out the essence of the passage. Try to rephrase the passage in your mind in simpler words, and ask yourself what the author is trying to convey. Read the following lines and try to guess the theme of the passage.

We must forever conduct our struggle on the high plane of dignity and discipline. We must not allow our creative protest to degenerate into physical violence. Again and again we must rise to the majestic heights of meeting physical force with soul force.

Here, the last line makes the theme amply clear; the author wants the people to uphold their sense of righteousness and integrity even in the midst of bitter struggle.

Tone of the Passage

Tone represents the predominant emotion or mood of the author towards the topic. Tone can be guessed by paying attention to the adjectives used

in the passage. Passages are normally the author's reaction to some issue. So, gauging the tone can help you to narrow down to the right choice. If the passage is eulogistic in tone, you do not expect the answer choice to be sarcastic or sardonic. Can you identify the tone of the passage?

American students are enrolling in college in record numbers, but they are also dropping out in droves. Barely half of those who start four-year colleges, and only a third of community college students, graduate. That's one of the worst records among developed nations, and it's a substantial drain on the economy. The American Institutes for Research estimates the cost of those dropouts, measured in lost earnings and taxes, at $4.5 billion. Incalculable are the lost opportunities for social mobility and the stillborn professional careers.

Clearly, the passage bears a negative tone. The author uses expressions like 'dropping in droves', 'drain on the economy' and finally 'incalculable are the lost opportunities', which clearly shows that the author displays a strong sense of disapproval for the flip side of the American education system.

Logical Flow of the Passage

You should read the passages like a detective, keeping an eye on the chain of event, and their logical sequence. If you mess up, you may make a blunder. Therefore, it is important to focus on key ideas and transition phrases used in the passage. One must be able to identify the thread of thought running through the paragraph. The flow of ideas in the passage should be maintained. Never pick an option which breaks or suddenly changes the flow to some other direction.

Remember, the last thought or idea in the paragraph has to be taken forward. Therefore, continuity is an important thing to be kept in mind in the process of arriving at the right answer.

Master the Art of Eliminating the Choices

It is important to master the art of eliminating the choices, especially in the verbal section. Any option that is out of scope of the argument or contradictory to the theme can be straightaway eliminated. Never pick an option which talks about things that are not mentioned in the paragraph. The correct option will be the one which relates itself to the core information mentioned in the paragraph. Beware of the choices that rephrase and repeat the topic. And avoid extreme choices.

Mma Ramotswe had a detective agency in Africa, at the foot of Kgale Hill. These were its assets: a tiny white van, two desks, two chairs, a telephone, and an old typewriter. Then there was a teapot, in which Mma Ramotswe—the only lady private detective in Botswana—brewed redbush tea. And three mugs—one for herself, one for her secretary, and one for the client. What else does a detective agency really need? Detective agencies rely on human intuition and intelligence, both of which Mma Ramotswe had in abundance.

(a) But there was also the view, which again would appear on no inventory.
(b) No inventory would ever include those, of course.
(c) She had an intelligent secretary too.
(d) She was a good detective and a good woman.
(e) What she lacked in possessions was more than made up by a natural shrewdness.

Solution

Needless to say, the passage revolves around Mma Ramotswe and the minimal assets of her little detective agency. Option (a) is off the mark as it takes the topic away from the main character, Mma Ramotswe or her possessions. Option (c) fails to sum up the theme of the passage, and digresses from it. The fourth one also fails to connect the possessions of her agency and her unique abilities. The second last sentence asks a question, and the last sentence tries to justify the case in a positive light, so option (e) does not fit. It's only the option (a), which suitably wraps up the discussion raised by the question in the second last line.

PRACTICE EXERCISES

EXERCISE 1

1. Perhaps the simplest and easiest to understand is the argument of the First Cause. I may say that when I was a young man and was debating these questions very seriously in my mind, I for a long time accepted the argument of the First Cause, until one day, at the age of eighteen, I read John Stuart Mill's Autobiography, and I there found this sentence: "My father taught me that the question 'Who made me'? cannot be answered, since it immediately suggests the further question 'Who made god'?" _____. If everything must have a cause, then God must have a cause. If there can be anything without a cause, it may just as well be the world as God, so that there cannot be any validity in that argument.

(a) There is no reason why the world could have come into being without a cause.
(b) That argument, I suppose, does not carry very much weight nowadays, because, in the first place, cause is not quite what it used to be.
(c) That very simple sentence showed me, as I still think, the fallacy in the argument of the First Cause.
(d) It brings us to the central truth that God is the ultimate source and essence of everything.

2. Then there is a very common argument from natural law. That was a favorite argument all through the eighteenth century, especially under the influence of Sir Isaac Newton and his cosmogony. People observed the planets going around the sun according to the law of gravitation, and they thought that God had given a behest to these planets to move in that particular fashion, and that was why they did so. _____.

(a) Nowadays we explain the law of gravitation in a somewhat complicated fashion that Einstein has introduced.
(b) Modern science has failed to explain this incongruity.
(c) You no longer have the sort of natural law that you had in the Newtonian system.
(d) That was, of course, a convenient and simple explanation that saved them the trouble of looking any further for explanations of the law of gravitation.

3. The fountains mingle with the river,
And the rivers with the ocean;
The winds of heaven mix forever,
With a sweet emotion;
_____;

(a) This is the power of love
(b) Nothing in the world is single
(c) This is the seed of creation
(d) What's life without love

4. All things by a law divine
In one another's being mingle: _____

(a) Why not I with thine?
(b) Let's make a jingle
(c) It takes two to tango
(d) God is not away from us

5. Conventional education makes independent thinking extremely difficult. _____ To be different from the group or to resist environment is not easy and is often risky as long as we worship success

(a) Creativity is crushed by orthodoxy
(b) Innovative thinking is the key
(c) This has ruined many careers
(d) Conformity leads to mediocrity

6. Though there is a higher and wider significance to life, of what value is our education if we never discover it? We may be highly

educated, but if we are without deep integration of thought and feeling, our lives are incomplete, contradictory and torn with many fears; _____.

(a) the 'well-educated' are ignoramus louts.
(b) and as long as education does not cultivate an integrated outlook on life, it has very little significance.
(c) and integrity is the key to spiritual evolution.
(d) it has been rightly remarked, "I never let school interfere in my education."

7. The function of education is to create human beings who are integrated and therefore intelligent. _____. We may take degrees and be mechanically efficient without being intelligent. Intelligence is not mere information; it is not derived from books, nor does it consist of clever self-defensive responses and aggressive assertions.

(a) Education should help us to discover lasting values so that we do not merely cling to formulas or repeat slogans.
(b) Education should not encourage the individual to conform to society or to be negatively harmonious with it.
(c) One who has not studied may be more intelligent than the learned.
(d) Unfortunately, the present system of education is making us subservient, mechanical and deeply thoughtless.

8. Politicians may be corrupt, but have to seek re-election, and to that extent are accountable to voters. But civil servants are virtually unsackable, unaccountable and widely corrupt. You cannot change this overnight. _____.

(a) However, you can create jobs for the unemployed.
(b) So, go easy on making temporary workers permanent.
(c) But you can halt the growth of unsackable, unaccountable staff.
(d) So, devise safeguards against false accusations.

9. The "grand sweep of history" has become a much overused cliché. It incorporated the belief that change stemmed from big ideas that motivated individuals, classes and nations. _____.

(a) The Bolshevik Revolution, whose impact dominated the 20th century, was prompted by disillusionment among the proletariat.
(b) This finds support in Namier's view that big ideas are less important than mundane and even base considerations.
(c) Mass movements, can't be judged by pronouncements of those who manage to filch them.
(d) Thus, the French Revolution happened because the idea of liberty, equality and fraternity motivated people to overturn the decrepit absolute monarchy.

10. A country that retains the death penalty needs constantly to fine-tune its clemency jurisprudence as the second best option. The Supreme Court's latest verdict on death row convicts is a thoughtful exposition of the law in this regard. _____.

(a) Commuting the death sentences of 15 convicts to life sentences has significantly expanded the scope for judicial intervention to save the lives of convicts after the rejection of their mercy petitions.
(b) The court has laid down fresh rules to humanise the treatment of those facing the gallows, right up to the moment of their execution and even after that.
(c) The breadth of this ruling is not as impressive: it fails to remove all lingering doubts about the rule against undue delay.
(d) The court has crafted a new rule that families of convicts ought to be informed in writing as soon as their mercy petitions are rejected.

11. After successfully eradicating smallpox in 1980, India has now gone three straight years without reporting any new case of poliomyelitis infection ("polio"). This qualifies it to receive the World Health Organisation's (WHO) certification for being polio-free. Undoubtedly, this is a victory that has been fought every inch of the way by myriad agencies on a number of fronts and against what seemed like insurmountable odds. _____.

(a) The lessons learnt are precious beyond words and the expectation is that these will be harnessed to fight other infectious diseases that plague the country.

(b) There is also the remaining challenge of treating and rehabilitating those who have already been crippled by the disease.

(c) Of course polio vaccination is not a cure-all solution for all infectious diseases.

(d) In the mid-1990s the vaccination programme that was undertaken involved the government, United Nations bodies, charitable organisations and private donors.

12. Up to this point, Jordan Belfort is no different from countless eager MBA graduates in India who work in the stock or bond markets for global financial firms. Belfort's lifestyle, while perhaps more (or less?) excessive than that of India's super-rich, is still something a lot of us covet. He acquires a harem, a hot blonde wife, a daily dose of recreational drugs, a yacht, a yellow Jaguar and a white Ferrari.

(a) As a job creator, he transforms hopeless, low-end drug dealers into corporate sharks, and even gives a desperate single mother benefits that the US' social welfare system overlooks.

(b) He lowers himself to unfathomed moral depths even as he soars to new heights of success.

(c) Surely many of us will laud Belfort when he says: "At least as a rich man, when I have to face my problems, I can show up in the back of a stretch limousine, wearing a two-thousand-dollar suit and a twenty-thousand-dollar gold watch!"

(d) These are charismatic brands that several of India's merchant princes flaunt and made more familiar to us through thousands of Bollywood fantasies.

13. The yearning for money as succour drives contemporary capitalism. If every revolution and alternative has failed, why not work to enable the one that actually exists, why not do what your stockbroker tells you, and keep investing to circulate money in the economy? The original *Forbes* magazine exposé that labelled Belfort "The Wolf of Wall Street" likened him to a "twisted Robin Hood" who takes from the rich and gives to himself and his squad of losers.

(a) It is this observation that makes Scorsese's *The Wolf of Wall Street* profound, locating and attacking the very appeal of money.

(b) Scorsese refuses to dish out false platitudes that "crime doesn't pay" nor does he echo the sentimentalism of Oliver Stone's Wall Street movies.

(c) As a job creator, he transforms hopeless, low-end drug dealers into corporate sharks.

(d) Is that not what we expect from the market, what keeps housewives glued to the television, watching CNBC for the latest stock information, and what drives many to start demat accounts?

14. Like *Company Limited*, Scorsese's *The Wolf of Wall Street* is an exploration of the contemporary world that few would have expected from these two artistes, given their refined sensibilities. *The Wolf*

of Wall Street attacks the lifestyle of the middle-class, the world of advertising and consumerism, the lust for the good life and the protection it offers.

(a) the lust for good life propels us to struggle and survive in this big bad world.
(b) behind the veneer of consumerism is an effort to assert one's individuality.
(c) the elite and the downtrodden are unaffected by this false glamour.
(d) The visible surface and texture of contemporary life corrupts us all, making us wolves thronging the pack of the alpha male rather than being benign, though gullible, sheep.

15. It is obvious today that America has defaulted on this promissory note insofar as her citizens of color are concerned. Instead of honoring this sacred obligation, America has given the Negro people a bad check, a check which has come back marked "insufficient funds."

(a) But we refuse to believe that the bank of justice is bankrupt.
(b) So we have come to cash this check—a check that will give us upon demand the riches of freedom and the security of justice.
(c) This note was a promise that all men, yes, black men as well as white men, would be guaranteed the unalienable rights of life, liberty, and the pursuit of happiness.
(d) In a sense we have come to our nation's capital to cash a check.

16. I am not unmindful that some of you have come here out of great trials and tribulations. Some of you have come fresh from narrow jail cells. Some of you have come from areas where your quest for freedom left you battered by the storms of persecution and staggered by the winds of police brutality.

(a) You have been the veterans of creative suffering.
(b) We cannot be satisfied as long as a Negro in Mississippi cannot vote and a Negro in New York believes he has nothing for which to vote.
(c) No, no, we are not satisfied, and we will not be satisfied until justice rolls down like waters and righteousness like a mighty stream.
(d) I say to you today, my friends, so even though we face the difficulties of today and tomorrow, I still have a dream.

17. Now the trumpet summons us again—not as a call to bear arms, though arms we need—not as a call to battle, though embattled we are—but a call to bear the burden of a long twilight struggle, year in and year out, "rejoicing in hope; patient in tribulation;"_____.

(a) A struggle against the common enemies of man—tyranny, poverty, disease, and war itself.
(b) Celebrating the pluralism of our cosmopolitan culture.
(c) A solemnization of the onerous fight against the injustice.
(d) For man holds in his mortal hands the power to abolish all forms of human poverty (does not explain the reason).

18. More fundamentally, the tiff has uncovered a deep rift in the two countries' perceptions of one another. From the Indian perspective, America remains unwilling to afford it the respect a true partner deserves. And from the American, the Indian response reveals both a brittle anxiety about its own status and a callous disregard for the well-being of the person the American justice system saw as the victim in this story—the maid. _____.

(a) Rather than partners, the two countries look like strangers.

(b) America is known for stringent labour laws and its ruthless enforcement.
(c) The Indo-US relations has therefore been damaged irreversibly.
(d) If the rift widens, it may impact the peace and stability in South Asia.

19. I believe that the school is primarily a social institution. Education being a social process, the school is simply that form of community life in which all those agencies are concentrated that will be most effective in bringing the child to share in the inherited resources of the race, and to use his own powers for social ends. I believe that education, therefore, is a process of living and not a preparation for future living.
(a) I believe that the school must represent present life—life as real and vital to the child as that which he carries on in the home, in the neighborhood, or on the play-ground.
(b) It must begin with a psychological insight into the child's capacities, interests, and habits.
(c) To prepare him for the future life means to give him command of himself; it means so to train him that he will have the full and ready use of all his capacities.
(d) Only true education comes through the stimulation of the child's powers by the demands of the social situations in which he finds himself.

20. We say Newton discovered gravitation. Was it sitting anywhere in a corner waiting for him? It was in his own mind; the time came and he found it out. All knowledge that the world has ever received comes from the mind; the infinite library of the universe is in your own mind. The external world is simply the suggestion, the occasion, which sets you to study your own mind, but the object of your study is always your own mind. The falling of an apple gave the suggestion to Newton, and he studied his own mind.
(a) Our mind is the root of all evil and good.
(b) The search for truth begins with the study of one's own mind.
(c) Mind is the element of a person that enables them to be aware of the world and their experiences, to think, and to feel; the faculty of consciousness and thought.
(d) He rearranged all the previous links of thought in his mind and discovered a new link among them, which we call the law of gravitation.

ANSWER KEY

Exercise 1

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (c)
9. (d)	**10.** (a)	**11.** (a)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (a)	**16.** (a)
17. (a)	**18.** (a)	**19.** (a)	**20.** (d)				

HINTS AND EXPLANATIONS

Exercise 1

1. (a) It is contrary to the main argument discussed in the passage, which tries to refute the argument of the First Cause.
(b) It does not go with the theme of the paragraph, as discussed above.
(d) Misleading choice. Out of scope of the passage, as it does not talk about the qualities of God.
(c) as the author believes that there is no validity in the argument of the First Cause, so the third choice is correct. Hence, option (c) is the answer.

2. (a) Gravitation is not the main focus of the argument; moreover the para talks about Newton, and not Einstein.
(b) Does not follow; why would Modern science explain a popular belief.
(c) Does not make sense as the passage nowhere says that natural law was a part of the Newtonian system.
(d) This option takes the argument forward, 'that was ...' is the link that takes the case further by questioning the validity of the conventional wisdom. Hence, option (d) is the answer.

3. The lines are simple and direct. You have to find something that concludes the idea. So (a), (d), and (c) are quite tempting, although quite general in inference.
(a) One can't deduce it from the argument.
(b) Option (b) concludes the idea mentioned in each line, hence, it is the answer.
(c) Again, this option is too farfetched.
(d) Once again, this option is too farfetched.

4. Now, this should be pretty logical for those who believe in love.
(a) hits the bulls eye, the only choice that logically completes the lines. Hence, option (a) is the answer.
(b) The choice is too creative although it rhymes well with the previous line.
(c) 'it takes two to two to tango' is generally used with a negative sense, so out of question.
(d) is out of scope, don't be misled by the word 'divine'.

5. The passage nowhere talks about creativity or innovative thinking, so options (a) and (b) are out. Option (c) take the negative tone too far, so we have to eliminate this choice. Option (d) is clearly the missing logical link, which is taken further by the last line of the passage. Mentioned in the last line 'to resist environment is not easy and often risky' supports this statement. Hence, option (d) is the answer.

6. (a) The first choice is downright offensive, therefore not possible.
(b) It includes both the elements 'education' and 'integration', hence, the right choice and is the answer.
(c) Out of scope as the passage is about the 'value of education' and not spiritual evolution.
(d) It is well said but out of place because the paragraph does not say that education is not necessary.

7. (a) It provides extra information which is out of scope of the passage.
(b) It does not go with the theme of the passage, which talks about the role of education in developing intelligence.
(c) It fills in the missing link. It has been illustrated in the last two lines.
(d) This does not go with the last line, which tries to define real intelligence.

8. (a) It strays from the theme of the topic and hence, eliminated.

(b) The previous line talks about the civil servants, so no connection between the two.
(c) It justifies the you-can't-change-it-overnight element in the previous line, hence, the answer.
(d) It is totally unrelated to the topic.

9. (a) The passage talks about big ideas motivating the individuals or nations, this option talks about negative motivation of the proletariat.
(b) It is counter to the theme of the paragraph which talks about the sweeping influence of the big ideas.
(c) This is contradictory to the main idea of the paragraph.
(d) Gives an illustration of how big ideas catch the popular imagination and create a wave of history.

10. (a) Option (a) provides a solid example to support the main idea about fine-tuning the clemency jurisprudence.
(b) It is an extension of the topic, but it deviate from the topic of mercy petitions or clemency jurisprudence.
(c) Goes against the tone of the paragraph which is positive, 'thoughtful exposition' has been used.
(d) It provides secondary information but fails to logically complement the last sentence.

11. (a) Goes with the tone of the paragraph which is laudatory.
(b) It does not follow the last sentence which is positive.
(c) It dilutes the tone of the paragraph which is positive.
(d) 'the vaccination programme' does not have its antecedent in the previous line.

12. (a) This could have been a possible choice, but it does not flow from the last sentence.
(b) This option is quite tempting, but misses on the 'India' element interspersed throughout the paragraph.
(c) Last sentence in option (c) does not refer to any 'problems' as mentioned in this statement.
(d) It gels both the elements mentioned in the given lines, global brands and the great Indian dream.

13. (a) It is not possible as the previous line talks about Forbes' review of the movie.
(b) The paragraph nowhere talks about Scorsese, so this option is eliminated.
(c) The last line does not have Belfort as the main subject, so 'he' can't be used for lack of clear antecedent.
(d) The author tries to argue that the greed of common man to make a fortune is not much different from a fraudster like Belfort who manipulates the stock market. Option (d) is in line with the theme of the paragraph. Hence, option (d) is the answer.

14. (a) It does not go with the negative tone of the last line, 'attacks the life-style ...'.
(b) Again, this option changes the tone of the passage which is negative.
(c) This is an extraneous piece of information which falls outside the scope of the discussion.
(d) As it explains how it 'attacks the middle class' and uses the analogy of wolves and sheep to support the case...

16. Let us eliminate the options:
Option (b) talks about how does 'voting' come into picture. Passage does not mention that or is not related to that. Hence, option (b) is not the answer.
Option (c) looks tempting but does not flow from the passage.

Option (d) sounds like a U turn and hence, cannot be the answer.
Option (a) is left out, Hence, option (a) is the answer.

18. (a) The first choice is correct, as it shows the effect of the two opposing views of the two countries engaged in tiff. Hence, this is the answer.

(b) It provides external information, so can't be the answer.

(c) This is an extreme choice to be eliminated.

(d) It is out of scope of the passage, as it talks about a hypothetical situation.

5 Idioms and Phrases

IDIOMS

An expression or phrase or collection of words, often informal, that has a meaning of its own which is not apparent from the meanings of its individual words is an IDIOM. In other words, meaning of IDIOMS is generated through the collection of individual words. For example "To Kick the bucket" is an idiom which means "to die." Knowledge of Idioms is important because in the absence of it, key meaning of passage or sentence may not be understood.

Following pages contain a list of the most commonly encountered idioms and their meanings.

A

Abide by—adhere to, agree to
Above board—honest, fair, frank
Account for—provide an explanation for
All agog—in a state of excitement
All and Sundry—everyone without exception
All intents and purposes—practically, in actual practice
An axe to grind—selfish motive
Answer a purpose—to serve a purpose
Answer for—take the responsibility for
Apple of discord—cause of quarrel, a bone of contention
Apple of one's eye—an object of love, the most valuable possession
Apple pie order—in perfect order
Ask after—inquire about one's condition or health
Ask for—asked to be given, to run the risk of something negative
Ask out—invite to a function (or an outing or a party)
At a low ebb—in a state of decline
At a pinch—in a difficulty
At a stone's throw—very near
At a stretch—continuously, without a break
At an arm's length—to keep aloof, to avoid, to keep at a distance
At any rate—in any event, under any circumstances
At home in—to be strong, to be comfortable with a subject
At one's beck and call—at one's disposal or command
At one's finger's tips or ends—to be an expert at something
At par—on level with, of a similar quality
At random—aimlessly, without goals
At sea—weak, perplexed
At sixes and sevens—in disorder, pell-mell
At stake—in danger
At the bottom—real cause
At the eleventh hour—at a late stage, at the last moment
At the mercy of—in the power of
At the outset—right in the beginning
A.B.C.—elementary facts
to Add fuel to the fire—to increase anger
to be At daggers drawn with—to be enemies, to be on bad terms
to be At one's wit's end—to be in a fix or confusion, to be perplexed
to be At large—to be free and not under any control

B

Back out—to withdraw, to break a promise
Bad blood—enmity, bad feelings
Bag and baggage—completely, leaving behind nothing
Be off one's head—to be mad, insane
Bear a grudge—to have bitter feelings towards someone
Bear in mind—remember
Bear up—keep one's spirits under hostile conditions
Bed of roses—full of joys and pleasures
Bed of thorns—full of sufferings
Behind one's back—in one's absence
Beside the mark—irrelevant, not to the point
Between the devil and the deep sea—to be in a fix or between two difficulties
Big gun—an important figure
Bird's eye view—a general view or study
Black sheep—a disgraceful person
Blessing in disguise—a certain thing which appears to be a curse in the beginning but proves to be a blessing in the end
Blow one's own trumpet—to speak proudly of one's achievements
Blow over—to end, to extinguish/terminate
Blue blood—a member of aristocracy
Bolt from the blue—a sudden shock or calamity
Bone of contention—a cause of quarrel
Bread and butter—livelihood
Break down—mental or nervous collapse of a person
Break in—enter with the aim of stealing
Bring up—to rear the children; to start a topic for discussion
Buckle under—to accept under pressure, to give in
Burning question—an important question or topic of the day
By far—beyond all comparison
By hook or by crook—by all means, fair or foul
By leaps and bounds—rapidly, at a rapid pace
By no means—in no way
By virtue of—on account of
to Bank upon—to depend, to rely
to Be born with a silver spoon in one's mouth—born in prosperous circumstances
to Be in the good books—to be in favour with
to Be in the bad books—to be in disfavour with
to Bear the brunt of—to bear the consequences
to Bear the palm—to win reward
to Beat about the bush—to talk irrelevant
to Beat a retreat—to retire, to move back
to Beat hollow—to defeat completely
to Bell the cat—to face a risk
to Bid fair—likely to be
to Break the ice—to break the silence, to get over initial shyness
to Bring credit to—to bring fame and honour
to Bring down the house—to win general praise
to Bring to book—to punish the guilty
to Burn one's fingers—to get into trouble
to Burn the candle at both ends—to waste one's energy, time and money uselessly
to Burn the mid-night oil—to work very hard till late at night
to Bury the hatchet—to forget a quarrel, to make peace

C

a Cat's paw—to make somebody a tool
a Chicken-hearted person—a cowardly person
a Child's play—something very easy
a Cock and bull story—an imaginary or false story
a Cool head—a calm judgment
a Cry in the wilderness—a useless cry
Call on—to pay a visit
Capital punishment—punishment of death
Care about—to feel for someone/something
Care for—have respect or regard for someone
Cast down—depressed, dejected
Casting vote—a vote which decides the issue when the voting is otherwise equal
Close shave—a narrow escape
Come of—come from, descended from

Come off—to be successful
Come round—become conscious; to visit; adopt an opinion
Come to hand—receive
Count on—to believe in, rely on
Count out—to disregard
Crocodile tears—false tears
Cut and dried—in a readymade form
to Call a spade a spade—to speak in plain terms, to speak out openly
to Call into question—to object to
to Carry fire and sword—to cause destruction
to Call for—to demand
to Carry off one's feet—to be wild with excitement
to Carry the day—to win, to succeed
to Cast a slur upon—to bring a blot, to bring discredit
to Cast a spell over—fascinate or attract
to Cast into the shade—to throw into obscure position
to Catch napping—to take unawares
to Catch tartar—to meet with a person who is more than a match for one
to Change colour—to turn pale with fear
to Chew the cud—to think deeply
to Clinch the issue—to decide the matter
to Come of age—to be major, to be twenty one
to Come off with flying colours—to succeed well
to Come to a head—to reach a crisis
to Come to a stand-still—to come to a stop
to Come to grief—to suffer, to be ruined, to fail
to Compare notes—to discuss impressions or opinions
to Cool one's heels—to wait for somebody patiently
to Cross one's mind—to come into one's mind
to Cross swords—to fight
to Cry for the moon—to wish for something impossible
to Cry over spilt milk—to waste time in vain, repent over lost opportunities
to Cut a sorry figure—to feel humbled, to make oneself ridiculous, to produce a poor result
to Cut no ice—to have no effect
to Cut the Gordian knot—to get over a difficulty by a bold strike

D

a Dead letter—no longer in use
a Deadlock—a position in which no progress can be made
a Dog in the manger—a selfish policy
a Drawn game or battle—in which no party wins
Dark horse—a competition of unknown capabilities
Dead of night—in the middle of
Dead shot—a marksman whose aim never goes wrong
Deal in—to trade in something, to be busy with something
Democle's sword—an impending danger
Die down—reduce to zero slowly, to diminish and finish
Die hard—customs that require a long period to die out
Dirt cheap—very cheap
Do away with—reject, get rid of
Do one proud—to give cause for feeling proud
Do up—put in better condition
Down and out—completely beaten
Draw in—to pull into something
Dress down—to scold, censure, rebuke
Dutch courage—fictitious courage induced somehow
to be Dashed to the ground—to fail
to Dance attendance upon—to wait on somebody always
to Dance to one's tune—to carry out orders
to Decline with thanks—to refuse or neglect
to Die in harness—to die while working till death
to Do full justice—to have one's fill, to do a thing thoroughly
to Drive home—to lay emphasis on, to make something very clear

E

at the Eleventh hour—at the last moment
Eat into—to destroy
Eat one out of house and home—to eat in a gluttonous manner
Eat one's heart out—to worry excessively
Eat one's words—to take back one's words
Eat out of one's hand—give no trouble
Enough and to spare—in plenty
Every inch—completely
Eye for eye—tit for tat
Eye wash—means of deceit
to Eat humble pie—to offer an humble apology
to Eat one's words—to go back on one's promise, to back out
to Egg on—to keep urging regarding something
to End in smoke—to come to nothing
to Extend the hand of friendship—to express a desire for friendship

F

a Fair hand—a beautiful and clear hand
a Fair weather friend—a selfish friend
a Fatal disease—a disease that ends in death
a Far cry—a long way or distance
a Feather in one's cap—another achievement to be proud of
a Fish out of water—to be in an uncomfortable position
a Flying visit—a hasty and brief visit
a Fool's paradise—state of joy based on false hopes
by Fits and starts—irregularly
Face to face—personally
Fair play—impartial treatment
Fall flat—collapse, flop, fail to generate interest
Fall foul of—to incur the disfavour of
Fall over one another—to try and do something before others
Fed up—to be tired of to be sick of
Few and far between—few and rare
First and foremost—important
Flesh and blood—human nature
Fly at—attack
Fly in the face of discretion—foolishly take a course that is not logical
Fly off at a tangent—to start discussing something totally irrelevant
For good—for ever
Foul play—bad intentions
From hand to mouth—a miserable existence
to Face the music—to face trouble, to face consequences of one's actions
to Fall to the ground—to come to nothing
to Fan the flames—to increase excitement
to Feather one's nest—to care for one's selfish interest
to Feel at home—to feel happy or at ease
to Fight shy of—to attempt to avoid a thing or person
to Fish in troubled waters—to take advantage of the troubles of others
to Fizzle out—failed out gradually
to Flog a dead horse—to waste one's energy and time
to Fly in the face of—to insult
to Follow in the foot-steps of—to follow somebody's example
to Follow suit—to follow example of
to Foot the bill—to pay the bill

G

Gala day—a day of rejoicings
Gift of the gab—fluency of speech
Give and take—the making of mutual concessions, quid pro quo
Give currency to—to give wide publicity
Go in for—to take up as an occupation
Go off the deep end—to lose temper
Go the rounds—be circulated
Grain of salt—to believe only a part of statement
to be Greek—unintelligible, not clear
to be a Good hand at—to be expert
to Gain ground—to progress
to Get at the bottom of—to find out the truth

to Get away with—to do something without the fear of any repercussion
to Get back at—to retaliate
to Get down to brass tacks—decide about the practical details
to Get into a mess—to get into muddle
to Get into a scrap—to be involved in a difficult situation
to Get into hot water—to get into scrap, to get into trouble
to Get on one's nerves—to be a source of worry
to Get the better of—to overpower, to defeat
to Get the sack—to be dismissed
to Get the upper hand—to become stronger, to get the better position
to Get wind of—to come to know of
to Gird up one's loins—to prepare oneself for a work
to Give a bit or a piece of one's mind—to rebuke; to scold
to Give a person the cold shoulder—to show a person apathy
to Give a slip—to escape
to Give a wide berth to—to avoid, to keep aloof from
to Give away—to reveal, betray, disclose
to Give chapter and verse—to give full proof
to Give in—to yield to pressure, to succumb
to Give oneself airs—to assume a superior attitude, to feel proud
to Give quarter to—to have sympathy with
to Give the cold shoulder—to receive in a cold and careless manner
to Give way—collapse, replace by
to Go against the grain—against one's likings
to Go all out for—to make the maximum possible effort
to Go by—to judge something by
to Go easy—not to use excessively
to Go hand in hand—to go together
to Go Scot free—to escape unpunished
to Go the whole hog—to go to the fullest extent; to agree
to Go through fire and water—to make every sacrifice
to Go to dogs—to be utterly ruined
to Go to the wall—to be ruined to, be hard pressed
to Go with the current—to follow the general trend
to Go without saying—to be clear
to Go home to—to appeal
to Grease the palm—to bribe

H

a Hair breadth escape—a narrow escape
a Hard nut to crack—a difficult problem, a stubborn person
Hair standing on end—a sign of fear and surprise
Hale and hearty—very healthy and sound
Hammer and tongs—with all might
Hand in gloves with—on very intimate terms
Hang by a thread—to be in a miserable condition
Hang heavy—difficult to pass
Happy go lucky—careless, depending on good luck
Hard and fast—strict
Hard of hearing—somewhat deaf
Hard up—in financial difficulty
Haunted house—in which ghosts or spirits are supposed to live
Heads and shoulders above others—very superior
Heart and soul—with full energy
Heels of Achilles—vulnerable point
Henpecked—a husband under the control or thumb of his wife
Herculean task—a work requiring great efforts
High spirits—to be very happy
High time—proper time
Hobson's choice—no choice at all
Hold good—to be valid
Hold on—to continue efforts
Hold out—not yield
Hole and corner—secret and underhand
Horns of dilemma—to be in a fix
Hue and cry—to raise a great cry and or stir, to raise alarm

Hush money—a bribe, price of silence
to be Hoisted with one's petard—to be killed with one's own sword
to Hammer out—to plan, to devise
to Hang in the balance—undecided
to Hang fire—delayed
to Harp on the same string or scheme—to talk continuously of the same matter
to Have a bee in one's bonnet—to take the remarks seriously
to Have a finger in every pie—to interfere unnecessarily
to Have a windfall—unexpected good fortune
to Have an old head on young shoulders—ripe in wisdom but young in years
to Have one's hands full—to be very busy
to Have one's way—to do one's own will
to Hit below the belt—to be mean, to fight unfairly
to Hit the ceiling—to give an outlet to anger
to Hit the nail on the head—to do what is proper at a proper time
to Hold a brief—to support be action or influence
to Hold a candle to—to be equal to, to be comparable to the other
to Hope against hope—to entertain hope when thee is no hope
to Hold the baby—be burdened with a task that should be shared by others too
to Hold one's own—to maintain one's position boldly
to Hold water—to be valid

I

an Iron hand—severe hand
an Iron will—strong determination
an Irony of fate—happening of events contrary to natural expectations
In a nut-shell—briefly, as a summary
In black and white—in writing
In cold blood—cruelly
In full swing—in great progress
In the air—wide-spread, prevalent
In the course of—during
In the face of—in spite of
In the guise of—in the dress of
In the light of—keeping in view
In the long run—in the end
In the nick of time—just in time
In the teeth of—in the face of
In the twinkling of an eye—in no time
In the wake of—behind
In tune with—in keeping with
In vogue—to be in fashion
Ins and outs—full details
Iron out—smoothen
to be In one's elements—in one's proper sphere

J

Jack of all trades—a person supposed to know everything partially
Jail bird—a person who has been to jail many times
Jump at—to accept immediately
Jump down one's throat—to reply in an angry manner
Jump on—to scold severely
Jump to the conclusion—to arrive at a conclusion soon

K

Keep to—stick to, adhere to
Keep up with—to keep pace with
Kick up the dust—to create disturbance
Kith and kin—relatives
to Keep a good table—to entertain
to Keep a straight face—to avoid smiling or laughing
to Keep abreast of—to be familiar with
to Keep body and soul together—to remain alive
to Keep one's head—to remain mentally calm in an emergency
to Keep one's head above water—to tide over difficulty, to escape debt
to Keep the ball rolling—to maintain interest of a conversation

to Keep the powder dry—to be ready for any work
to Keep the wolf from the door—to keep away, hunger and starvation
to Keep up appearances—to maintain outward show
to Kick up a row—to make a great noise
to Knock down—to defeat
to Knock off—to reduce; stop working

L

a Left-handed compliment—a false praise
at a Low ebb—on the decline
Labour of love—work undertaken not for profit but for service
Laughing stock—an object of ridicule
Lay off—to remove from work
Leap in the dark—a careless action
Lend oneself to—allow to be carried away
Let down—to fail to keep promise
Let off—released
Let up—to be sluggish
to the Letter—completely
Life and soul—main figure
Lion's share—a major share
Live up to—to maintain certain standards
Loaves and fishes—material comforts
Lock, stock and barrel—with all belongings
Look blue—to feel nervous r depressed
Look down one's nose—to regard others contemptuously
Look forward to—to expect with pleasure
Look on—watch carefully
Look out—take care
Look to—to request for help
Look up—to verify
Look up to—to admire something/someone
to Land an ear—to listen to
to Land on one's feet—to be lucky
to Laugh in one's sleeves—to laugh in secret but not openly
to Lead a cat and dog life—to lead a life of constant quarrelling
to Lead a dog's life—to lead a miserable life
to Lead astray—to misguide
to Leave in the lurch—to leave in time of difficulty and trouble
to Leave no stone unturned—to make all possible efforts
to Let bygones be bygones—to ignore the past
to Let off steam—to react aggressively to release the tensions
to Lie in wait for—to wait for in concealment
to Live in glass-houses—to be open to criticism
to Lose head—to lose balance of mind, to be proud

M

a Man of straw—a man with no voice or will of his own
a Moot point—a point or question still open to discussion
Maiden speech—a speech made for the first time
Man in the street—an ordinary person
Man of letters—a scholar with literary tastes
Man of moment—an important person
Man of word—a reliable person
Much ado about nothing—to make fuss
to Make a clean sweep of—to remove
to Make a dash—to go quickly
to Make a mark—to distinguish oneself
to Make a mess of—to bungle
to Make a mountain of mole hill—to exaggerate difficulties or trifles
to Make a pint of—to do something (certainly)
to Make amends for—to compensate for damage or injury
to Make an example of—to punish someone to make it a warning to others
to Make away with—to carry off
to Make both ends meet—to live within one's income
to Make do without—to manage without something
to Make free with—to take liberty with
to Make hay while the sun shines—to make the best use of the opportunity

to Make headway—to progress slowly and steadily
to Make light of—to treat lightly, to attach no importance
to Make much of—to make an issue of something trivial
to Make neither head nor tail of—not to understand
to Make one's way—to succeed, to prosper
to Make the flesh creep
to Make the heart bleed—to be filled with sorrow or pity
to Make up—to compensate
to Make up one's mind—to resolve
to Make up with—to compose one's differences
to Make way—to make room for others
wide of the Mark—irrelevant

N

a Narrow escape—to be saved with a great difficulty
a Necessary evil—something which cannot be avoided
Neck and neck—side by side; even in a race or contest
Nook and corner—everywhere
Not worth the salt—good for nothing
Now and again—occasionally
Null and void—invalid, of no effect
to Nip in the bud—to destroy a thing at the very beginning
to Nurse a grudge—to have jealousy or revenge

O

Odds and ends—different things, big and small
Off hand—without previous preparation
Of no avail—of no use
Of one's own accord—of one's own free will
On one's last legs—about to fall
On the sly—privately
On the spur of the moment—without any deliberation, at once
Order of the day—something common or general
Out of pocket—without money, short of
Out of the wood—out of danger or difficulty
Once for all—finally
Out of joint—in disorder and confusion
On the face of it—apparently
On the ground of—for
to be On the lookout for—to be in the search of
to be On the right scent—to be going in right direction
to be On the right side of—to be less than or below
to be On the wane—to decline
to be On the wrong side of—to be more than

P

Pillar to post—form one place of shelter to another
Point blank—frankly
Pros and cons—arguments for and against
to Pay in the same coin—to give tit for tat
to Pay lip service—sympathy, to pretend to be faithful
to Pay off old scores—to have revenge
to Play ducks and drakes—to spend lavishly
to Play fast and loose—to say one thing and to do another (be inconsistent)
to Play into the hands of—to be under the control of
to Play second fiddle—to be in a subordinate position
to Play to the gallery—to appeal to lower taste
to Play truant—to stay away form class
to Plead quality—to confess one's crime
to Pocket an insult—to bear insult quietly
to Poison one's ears—to prejudice
to Poke one's nose—to interfere with
to Pull a long face—to look sad and worried
to Pull one's legs—to make a fool of
to Put a spoke in one's wheel—to hinder one's progress
to Put heads together—to consult
to Put one's foot down—to show determination
to Put the best foot forward—to do one's best

to Put the cart before the horse—to do things in a wrong manner

R

a Rainy day—a time of difficulty or poverty
a Red letter day—auspicious day of rejoicing, lucky and important day
a Red rag to a bull—highly irritating, a cause for anger
a Rolling stone—one who is never constant to one work or the other
a Rough diamond—an illiterate but noble person
a Royal road—an easy way to achieve an end
Rain or shine—under all circumstances
Red tapism—official formalities causing excessive delays
Right hand man—a very useful person on whom one can depend
Rise from the ranks—to rise from a humble position
Root and branch—completely, entirely
Run over—go over
the Rank and file—the masses
to Rack one's brains—to think hard
to Read between the lines—to read carefully
to Rest on one's laurels—to rest satisfied with the honours already won
to Rest on one's oars—to rest after hard work
to Ride roughshod over—to be inconsiderable or cruel
to Rise to the occasion—to be found equal to the task
to Rub shoulders with—to come in close touch with
to Run amuck—to go mad
to Run down—weak in health
to Run riot—to wander without restraint
to Run the gauntlet—to undergo severe criticism

S

a Sharp tongue—a bitter tongue
a Sheet anchor—the chief support
a Square deal—a fair bargain, justice
a Square meal—full meal
a Square peg in a round hole—a misfit
a Stepping stone—source of success or help
a Storm in a tea cup—a quarrel for trifling reason
a Stumbling block—a great obstacle
by the Skin of one's teeth—very narrowly
Scapegoat—a person who is made to bear the blame of others
Scot free—to go without punishment
See through—understand
Shake in the shoes—to be in a state of fear
Shake off—to get rid of
Side issue—something not connected with the matter in hand
Sing low—to express one's views in an inconspicuous manner
Sink differences—to forget/overlook differences
Smooth sailing—no difficulty
Snake in the grass—a hidden enemy, a deceitful person
Sniff at—to show derision for
Spill one's sides—to laugh merrily
Stand by—to support
Steer clear of—to avoid, to keep aloof
Sum and substance—gist, purport
Sweet tooth—liking for sweetmeat
Swelled head—pride
Sword of Damocles—an impending danger
to do a Snow job—to fool someone
to Sail in the same boat—to be equally exposed to risk
to Sail under false colours—to pretend to be what one is not, to try to deceive
to Save one's skin—to accept without loss
to See a thing through coloured glasses—to see a thing with a prejudiced mind
to Set at naught—to disregard
to See eye to eye with—to agree
to Set Thames on fire—to try to achieve an impossible distinction
to Show a clean pair of heels—to run away
to Show the white feather—to show signs of cowardice

to Sing the blues—to exhibit a discouraged attitude
to Sit on the fence—to remain neutral
to Sit pretty—to be in a safe and comfortable position
to Sit up—to take notice of
to Smell a rat—to suspect something
to Sow the wild oats—to indulge in youthful follies
to Speak one's mind—to speak frankly
to Speak volumes—to bear sufficient evidence for or against
to Spill the beans—to give secret information
to Split hairs—to go into minute details
to Spread like a wild fire—to spread rapidly.
to Stand in good stead—to be useful and serviceable
to Stand on one's own legs—to be independent
to Stand on ceremony—to insist on formalities
to Stand up for—to support
to Stare in the face—to threaten
to Steal a march—to get the advantage secretly
to Stem the tide of—to check, to stop
to Step into another's shoes—t take another's place
to Stick to one's guns—to stand firm, to stick to one's principles
to Strain every nerve—to try one's best
to Stretch a point—to bend the rules
to Swallow the bait—to fall an easy victim to temptations

T

a Thorn in the flesh—to be a source of anger or displeasure
a Turncoat—a person who changes opinions
a Turning point—anything that brings change
on Tenterhooks—in a state of suspense and anxiety
Take ill—fall ill; consider unfavourably
Take the cake—to be the topmost
Tall talk—boastful and exaggerated talk
Thankless task—a selfless work for which we cannot expect anything
The thin edge of the wedge—small beginning with bright future
The three R's—reading, writing and arithmetic
Through thick and thin—under all circumstances
Tied to the apron string of—to be dependent upon somebody
to be Taken aback—to be extremely surprised
to Take a fancy to—to like something
to Take a leaf out of another's book—to follow somebody's example
to Take a thing lying down—to pocket an insult without a murmur
to Take after—to resemble in features
to Take an issue—to quarrel
to Take away one's breath—to surprise
to Take by storm—to conquer rapidly
to Take exception to—to object
to Take French leave—leave without permission
to Take heart—to pluck up courage
to Take off the hat to—show respect
to Take one's cue—to take a hint
to Take root—to become firmly established
to Take stock of—to observe and estimate
to Take the bull by the horns—to face a difficulty boldly
to Take the lead—to surge ahead in a competition
to Take the plunge—to take a bold decision
to Take time by the forelock—to act at once, to avoid delay
to Take to heart—to feel
to Take to heels—to run away
to Take to task—to call to account, to scold, to require explanation
to Take up the cudgels—to defend, to fight for somebody's claims
to Talk shop—to discuss exclusively of one's business on profession
to Talk (someone) into—persuade by talking
to Talk (someone) out of—to discourage
to Talk through a hat—to exaggerate or bluff or make wild statement
to the Tune of—to the amount of
to Throw cold water on—to discourage

to Throw down the gauntlet—to give an open challenge
to Throw mud at—to abuse, to vilify
to Tip off—to give a secret hint
to Turn a deaf ear to—to refuse to listen to
to Turn one's head—to be proud
to Turn over a new leaf—a change for the better
to Turn tail—to withdraw cowardly
to Turn the corner—to pass the critical stage
to Turn the tables on—to reverse the situation
to Turn turtle—to upset, to capsize
Tooth and nail—furiously, violently
True to one's salt—to be loyal to someone
Twinkling of an eye—very quickly

U

an Uphill task—a difficult work
Under a cloud—in disfavour or disgrace
Under lock and key—carefully
Under the thumb of—under the control of
Up and doing—active
Up one's sleeves—something hidden but ready for use in reserve
Up to the mark—up to certain standard
Ups and downs—good and bad times
Utopian scheme—a visionary scheme, not practicable

V

a Vexed question—a question regarding which there has been much controversy but no solution has been arrived at
to Vie with—to compete with

W

a Wet blanket—kill joy, a dull fellow who spoils our joy
a White elephant—an expensive burdensome but use less thing
a White lie—a harmless lie
a Wild goose chase—a foolish and useless search
a Wolf in a sheep's clothing—a hypocrite, a deceiver
to be Within an ace of—almost nearly
to Wash one's hands of—to have no connection
to Weather the storm—to come out safely through a difficulty
to While away—to pass in amusement
to Win laurels—to achieve success or win fame
to Wind up—to bring to an end
Wait upon—to serve
Walk over—very easy victory
Wash out—quite dull
Wear and tear—decrease in value due to constant use
Wide of the mark—beside the purpose
Willy-nilly—somehow or the other
Windfall—unexpected good fortune
With a grain of salt—with some reservation, not at the face value of
With a vengeance—excessively
With open arms—warmly and cordially
Word of mouth—a solemn promise

PHRASES

Phrasal Verbs

Many verbs (called 'phrasal verbs'), when followed by various prepositions or adverbs, acquire an idiomatic sense. Some examples are given below:

Example

- She backed up (supported) his boyfriend's claim.
- The current disturbances will soon blow over (pass off).
- The investigating officer produced evidence to bear out (substantiate) the charge of corruption.
- You must not build your hopes upon (rely upon) his promises.
- The matter has been cleared up (explained).
- I readily closed with (accepted) his offer.
- He is ready to dispose of (sell) his car for ₹ 1,500.
- Rust has eaten away (corroded) the plate.

- They fixed upon (chose) him to do the work.
- The habit of chewing tobacco has been growing upon (is having stronger and stronger hold over) him.
- About a day ago I saw a beggar hanging about (loitering about) our bungalow.
- These events led up to (culminated in) the establishment of a republic.
- During excavations one of the workmen lighted upon (chanced to find, discovered) a gold idol.
- During her long illness she often longed for (desired) death.
- I could not prevail on (persuade, induce) him to attend the meeting.
- For years I could not shake off (get rid of) my malaria.
- I threatened to show him up (expose him).
- All eyes turned to him because he was the only person who could stave off (prevent, avert) the impending war.
- He is sticking out for (persists in demanding) better terms.
- I must think the matter over (*i.e.*, consider it).
- Train up (educate) a child in the way he should go; and when he is old he will not depart from it.
- That fellow trumped up (concocted, fabricated) a story.
- He seems to be well off (in comfortable circumstances).

BEAR

- Rajeev bore away (won) many prizes at the school sports.
- The new sultan has been able to bear down (overthrow, crush) all opposition.
- His evidence bears out (confirms, corroborates), the evidence of the first witness.
- In his misfortune god gave him strength to bear up (to keep up spirits, not to despair).
- A religious hope bears up (supports) a man in his trials.
- His evidence did not bear upon (was not relevant to) the inquiry.
- I trust you will bear with (have patience with, show forbearance to) me a few minutes more.

BRING

- Idleness and luxury bring forth (produce, cause) poverty and want.
- Our teacher often tells us a story to bring out (show) the meaning of a lesson.
- The publishers have recently brought out (published) a cheap edition of their new dictionary.
- He found great difficulty in bringing her round (converting her) to his views.
- She brought up (reared) the orphan as her own child.

CALL

- His master called for (demanded) an explanation of his conduct.
- New responsibilities often call out (draw forth) virtues and abilities unsuspected before.
- Call in (summon, send for) a doctor immediately.
- He called on me (paid me a brief visit) yesterday.
- The old man could not call up (recollect) past events.

CARRY

- He agreed to carry out (execute) my orders.
- His passion carried him away (i.e., deprived him of self control).
- His son carried on (managed) his business in his absence.
- Many persons were carried off (killed) by plague.

CAST

- The ship was cast away (wrecked) on the coast of Africa.
- He was much cast down (depressed) by his loss.
- Some snakes cast off (throw away) their outer skins seasonally.

COME

- At last the truth has come out (transpired).
- The taxes come to (amount to) a large sum.
- The question came up (was mooted or raised for discussion) before the Municipal Corporation last week.
- I expect he will come round (recover) within a week.
- I hope he will come round (agree) to our views.

CRY

- Men of dissolute lives cry down (depreciate) religion, because they would not be under the restraints of it.
- He cried out against (protested against) such injustice.
- That young author is cried up (extolled) by his friends.

CUT

- He was cut off (died) in the prime of life.
- You must cut down (reduce) your expenditure.
- He is cut out for (specially fitted to be) a sailor.
- His wife's death cut him up (affected him, distressed him) terribly.

DO

- I am done for (ruined).
- Having walked twenty miles, he is quite done up (fatigued, exhausted).
- She has done up (decorated, furnished) her apartment beautifully.

FALL

- At last the rioters fell back (retreated, yielded).
- At my friend's wedding reception, I fell in with (met accidentally) my old time friend.
- The measure falls in with (happens to meet) the popular demand.
- The scheme has fallen through (failed) for want of support.
- I am told the two brothers have fallen out (quarreled).
- It is said that the standard of efficiency in public service has recently fallen off (deteriorated).
- In the second school term the attendance fell off (diminished).

GET

- His friends expected that he would get off (escape) with a fine.
- The thief got away (escaped) with my cash box.
- I can't get out (remove) this stain.
- The revolt of the tribal chiefs has been got under (subdued).
- The dog tried to get at (attack) me.
- He has got through (passed) his examination.
- They soon got the fire under (under control) by pouring buckets of water over it.
- You were lucky to get out of (escape from) his clutches.

GIVE

- We are credibly informed that the murderer has given himself up (surrendered himself) to the police.
- The doctors have given him up (*i.e.*, have no hope of his recovery).
- Soon after it was given forth (published) and believed by many, that the king was dead.
- The fire gave off (emitted) a dense smoke.
- The strikers seem determined and are not likely to give in (submit, yield).

- It was given out (published, proclaimed) that he was a bankrupt.
- The horses gave out (were exhausted) at the next milestone.
- Give over (abandon) this foolish attempt.
- In his cross examination he ultimately gave himself away (betrayed himself).

GO

- You cannot always go by (judge from) appearances.
- It is a good rule to go by (to be guided by) the exact procedure.
- He promised to go into (examine, investigate) the matte.
- Have you anything to go upon (*i.e.*, any foundation for your statement)?
- We have no data to go upon (on which to base our conclusions).
- The story won't go down (be believed).
- The concept went off well (was a success).
- The auditor went over (examined) the balance sheet.
- The poor woman has gone through (suffered) much.
- I must first go through (examine) the accounts.

HOLD

- The rebels held out (offered resistance) for about a month.
- He holds out (gives) no promise of future prospects.
- They were held up (stopped) on the highway and robbed by bandits.
- The subject is held over (deferred, postponed) till the next meeting.

KEEP

- A few boys were kept in (confined) after school hours.
- I was kept in (confined to the house) by a bad cold.
- They kept up (carried on) a long conversation.
- Little disputes and quarrels are chiefly kept up (maintained) by those who have nothing else to do.
- He is trying his best to keep up (maintain) the reputation of his family.
- The rubber syndicate keeps up (maintain) the price.
- She kept on (continued) talking.
- I shall keep back (conceal) nothing from you.

KNOCK

- He has knocked about (wandered about) the world a great deal.
- The dressing table was knocked down (sold at an auction) for fifty rupees.
- We were greatly knocked up (exhausted) after our steep climb.

LAY

- The rebels laid down (surrendered) their arms.
- He had laid out (invested) a large sum in railway shares.
- Foolish people, who do not lay out (spend) their money carefully, soon come to grief.
- He is laid up (confined to his bed) with fever.
- He resolved to lay be (save for future needs) a part of his income.

LET

- I was let into (made acquainted with) her secret.
- This being his first offence he was let off (punished leniently) with a fine.

LOOK

- His uncle looks after (takes care of) him.
- He looks down upon (despises) his poor cousins.
- Look up (search for) the word in the dictionary.
- The old man is looking forward to (expecting with pleasure) the visit of his grandchildren.

- I will look into (investigate) the matter.
- I look on (regard) him as my son.
- Some look to (rely on) legislation to hasten the progress of social reforms.
- Look to (be careful about) your manners.
- Prices of piece goods are looking up (rising).
- Things are looking up (improving).
- His friends look up to (respect) him.
- He will not look at (*i.e.*, will reject) your offer.

MAKE

- Contentment makes for (conduces to happiness.
- He made over (presented, gave in charity) his bungalow to the Islam orphanage.
- I cannot make out (discover) the meaning of this verse.
- I cannot make out (read, decipher) his handwriting.
- You have failed to make out (prove) your case.
- Some time ago the two brothers quarreled but they have now made it up (become reconciled).

PULL

- Unless we pull together (co-operate, work together in harmony) we cannot succeed.
- My cousin pulled through (passed with difficulty) the examination.
- The doctor says the patient will pull through (recover from his illness).
- It is far easier to pull down (demolish) than to build up.
- He was pulled up (scolded, rebuked) by the president.

PUT

- Please put out (extinguish) the light.
- He was put out (vexed, annoyed) when I refused his request for a loan.
- The plaintiff was put out (disconcerted) when the suit was dismissed.
- He tried to put me off (evade me, satisfy me) with promises.
- He has put in (made, sent in) a claim for compensation.
- He put off (postponed) his departure for a week.
- The measure was put through (passed) without opposition.

RUN

- On account of overwork he is run down (enfeebled).
- He always runs down (disparages) his rivals.
- The lease of our premises has run out (expired, come to an end).
- He has run through (squandered away) his fortune.
- The tailor's bill has run up to (amounted to) a large amount.
- He has run into (incurred) debt.
- While turning the corner I ran against (chanced to meet) an old fried.
- Recently my expenses have run up (increased) considerably.
- The cistern is running over (overflowing).

SEE

- I saw through (detected) the trick.
- It is hard to see into (discern) his motive.
- His friends were present at the station to see him off (witness his departure).

SET

- The high court set aside (annulled) the decree of the lower court.
- He immediately set about (took steps towards) organizing the department.
- He set off (started) for Peshawar early this morning.
- The frame sets off the picture (*i.e.*, enhances its beauty by contrast).
- He has set up started business) as a banker.
- I have enough capital to set me up (establish myself) in trade.
- He hired a palatial bungalow and set up for (pretended to be) a millionaire.

- I was obliged to set him down (snub him).
- You may set down (charge) this loss to me.
- Who set you on (instigated you) to do it.
- These seats are set apart (reserved) for ladies.
- In his speech on prohibition, he set forth (explained, made known his views) at length.
- The robbers set upon (attacked) the defenseless travelers.
- Winter in England sets in (begins) about December.

SPEAK

- In Bombay there is no free library to speak of (worth mentioning).
- I was determined to speak out (express my opinion freely).

STAND

- They are determined to stand up for (vindicate, maintain) their rights.
- Let this matter stand over (be deferred or postponed) for the present.
- It is hard but I think I can stand it out (endure it to the end without yielding).
- He is always standing up for (championing the cause of) the weak and oppressed.
- We shall be formidable if we stand by (support) one another.

STRIKE

- He is struck down with (attacked by) paralysis.
- The medical council struck off (removed) his name from the register of medical practitioners.

TAKE

- The piano takes up (occupies) too much room.
- He takes after (resembles) his father.
- At present I am reading the essays of Bacon but it is sometimes.
- Difficult to take in (comprehend, understand) his meaning.
- Recently he has taken to (become addicted to) opium eating.
- Finally, he was talked into saying (convinced) yes to the proposal.

TALK

- We talked over (discussed) the matter for an hour.
- I hope to talk him over (convince him by talking) to our view.

THROW

- My advice was thrown away (wasted) upon him, because he ignored it.
- The bill was thrown out (rejected) by the Assembly.
- In disgust he threw up (resigned) his appointment.
- When he became rich became rich he threw over (abandoned or deserted) all his old friends.

TURN

- The factory turns out (produces, manufactures) 20,000 lbs of cloth a day.
- If he is lazy, why don't you turn him off (dismiss him)?
- He turned out (proved) to be a scholar.
- His very friends turned against (became hostile to) him.
- Who can say what will turn up (happen) next?
- He promised to come but he never turned up (appeared).

WORK

- We tempted him with many promises but nothing would work on (influence) him.
- He worked out (solved) the problem in a few minutes.
- He is sure to work up (excite) the mob.
- He worked upon (influenced) the ignorant villagers.

6 Antonym and Synonym

ANTONYMS

Vocabulary is an essential part of IIM Indore IPM examination. Questions from this chapters are asked in two ways—Antonyms and Synonyms. This chapter covers Antonyms, Synonyms and Confusing Words.

Typically, vocabulary questions can be a great addition to the score as they are big time savers.

Our advice to you will be to go through the words given in this chapter. Though, needless to say, it is always advisable to go through an English newspapers on a daily basis to have a good "Vocab bank".

In these questions, student will be expected to choose a word or phrase that is most nearly opposite in meaning to the word. As some of the questions require you to distinguish fine shades of meaning, one has to be sure that all the choices have been considered before marking the answer.

Ideally, the best way to improve antonyms is to improve the vocabulary. But there are certain techniques to ace these words.

You will find three types of words in the question paper. The words you know, the words you sort of know, and the words you have never seen. The words that are totally unfamiliar should not be touched to avoid negative marking.

For the words, you are comfortable with; do not look at the options in the first go.

Try to think of an opposite meaning word in simple English. Look at the options and eliminate those, which do not match your shadow word.

If you are not very clear about the meaning of the word:

- Try to find the context in which the word is generally used like mitigating circumstances and abject poverty. You may also make a short sentence using the word.
- Try to find some prefix, suffix or word root. Pre-means before so precognition means awareness of events before they take place, and post-means after, so posterity means future generation.
- Try to guess whether the word is positive or negative in shade. If the word is positive, all the positive and neutral and related options can be eliminated and vice versa.

Example

1. Censure:

(a) Extol (b) Impartial
(c) Slander (d) Castigate
(e) Admonish

The options (c), (d) and (e) can be eliminated straight away as they are also negative like censure which means to criticize. Impartial means unbiased is a neutral word, so it can also be eliminated. Therefore, we are left with extol (to praise), which is the right answer.

- Consider the secondary meanings of the words like pedestrian, august, cataract, etc.

2. Pedestrian:

(a) Ordinary (b) Static
(c) Jejune (d) Motor Car
(e) Imaginative

The answer is (e) Imaginative, as the secondary meaning of pedestrian is unimaginative.

- Look at the answer choices to determine the part of speech of the question word.

3. Polish:
   (a) Ruthless (b) Honesty
   (c) Indolent (d) Gaucheness
   (e) Complexity

   Here, polish has nothing to do with rubbing and shining. The noun Polish means refinement and culture, so the opposite is gaucheness or awkwardness.

- You can also eliminate the choices that don't have opposites like chair, flower, philosophy, priority, birthright, etc. You can also eliminate the answer choices that are synonyms of each other as no question can have two answers.
- Looks can be deceptive, so beware! Some of the simple looking words can be really tricky and insidious. They are the land mines laid by the examiners. The word simplistic does not mean simple, and the word redoubtable has least to do with doubting again.

4. Redoubtable:
   (a) Decisive
   (b) Resolved
   (c) Awesome
   (d) Formidable
   (e) Minnow

   Redoubtable means generating respect and fear or formidable, so its opposite is minnow, which means a small and unimportant person or company, so (e) is the answer.

PRACTICE EXERCISES (ANTONYM)

EXERCISE 1

Direction for questions 1–30: ***In the following questions, a word is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys opposite meaning as the word given in bold letters.***

1. **Fickle:**
   (a) Aggressive (b) Persistent
   (c) Miraculous (d) Hard working
   (e) Timid
2. **Tranquil:**
   (a) Serene (b) Disturbed
   (c) Cowardly (d) Beautiful
   (e) Sumptuous
3. **Gloomy:**
   (a) Disgusting (b) Comical
   (c) Versatile (d) Spirited
   (e) Dark
4. **Cacophonous:**
   (a) Loud (b) Melodious
   (c) Raucous (d) Harsh
   (e) Loud
5. **Zenith:**
   (a) Crest (b) Pinnacle
   (c) Interior (d) Nadir
   (e) Acme
6. **Advance:**
   (a) Retreat (b) Goad
   (c) Plod (d) Defeat
   (e) Cash
7. **Coy:**
   (a) Shy (b) Reserved
   (c) Optimistic (d) Brazen
   (e) Comely
8. **Dearth:**
   (a) Terror (b) Abundance
   (c) Paucity (d) Levity
   (e) Life
9. **Alleviate:**
   (a) Motivate (b) Pep up
   (c) Aggravate (d) Malign
   (e) Godly

10. Callous:
(a) Frugal (b) Nonchalant
(c) Sensitive (d) Stingy
(e) Cellular

11. Oriental:
(a) Fatal (b) Occidental
(c) Fatalistic (d) Actuarial
(e) Bank

12. Nascent:
(a) Primal (b) Senescent
(c) Primordial (d) Modish
(e) Secondary

13. Brood:
(a) Support (b) Exult
(c) Slander (d) Fragmented
(e) Vermin

14. Blasphemy:
(a) Irreverence (b) Scandalous
(c) Cursing (d) Respect
(e) Assail

15. Shelter:
(a) Pillar (b) Imperil
(c) Security (d) Refuge
(e) Protégé

16. Condescend:
(a) Surrender (b) Resist
(c) Laud (d) Disdain
(e) Come down

17. Profound:
(a) Minnow (b) Deep
(c) Shallow (d) Petty
(e) Excessive

18. Archetype:
(a) Unique (b) Quixotic
(c) Duplicate (d) Modern
(e) Ancient

19. Frugal:
(a) Wasteful (b) Ugly
(c) Provident (d) Stylish
(e) Miserly

20. Repugnant:
(a) Odious (b) Coercive
(c) Sensitive (d) Agreeable
(e) Repulsive

21. Scramble:
(a) Simplify (b) Decipher
(c) Cook eggs (d) Denigrate
(e) Jumble

22. Elite:
(a) Plebeian (b) Ignoble
(c) Boring (d) Gentry
(e) Top class

23. Ostensible:
(a) Crooked (b) Apparent
(c) Hidden (d) Avian
(e) Equine

24. Modest:
(a) Complacent (b) Haughty
(c) Jovial (d) Barbaric
(e) Decent

25. Irrevocable:
(a) Alterable (b) Ultimate
(c) Fixed (d) Moving
(e) Permanent

26. Ludicrous:
(a) Awesome (b) Awful
(c) Lively (d) Grave
(e) Somber

27. Congenital:
(a) Inborn (b) Societal
(c) Acquired (d) Hereditary
(e) Genetic

28. Hilarious:
(a) Eulogistic (b) Morose
(c) Paltry (d) Sportive
(e) Comical

29. Impecunious:
(a) Affluent (b) Comatose
(c) Ruthless (d) Superficial
(e) Ravenous

30. Absolve:
(a) Confront (b) Charge
(c) Accuse (d) Confuse
(e) Vindicate

EXERCISE 2

Direction for questions 1–30: ***In the following questions, a word is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys opposite meaning as the word given in bold letters.***

1. **Ratify:**
   (a) Abrogate (b) Pass
   (c) Rat race (d) Competition
   (e) Pass a law
2. **Fruitful:**
   (a) Productive (b) Abortive
   (c) Messy (d) Dingy
   (e) Prolific
3. **Amplify:**
   (a) Rake (b) Abbreviate
   (c) Assail (d) Mark
   (e) Magnify
4. **Abstract:**
   (a) Confused (b) Perplexed
   (c) Concrete (d) Open
   (e) Derive
5. **Clumsy:**
   (a) Adroit (b) Messy
   (c) Convoluted (d) Boorish
   (e) Gauche
6. **Ally:**
   (a) Adversary (b) Partner
   (c) Fence-sitter (d) Almighty
   (e) Partner
7. **Oblivious:**
   (a) Apparent (b) Unperturbed
   (c) Nonchalant (d) Alert
   (e) Absent minded
8. **Affirmative:**
   (a) Obliging (b) Uncivilized
   (c) Platonic (d) Negative
   (e) Approving
9. **Ambiguous:**
   (a) Unequivocal (b) Perplexing
   (c) Befuddled (d) Murky
   (e) Uncertain
10. **Abhorrence:**
   (a) Disgust (b) Admiration
   (c) Animus (d) Pathos
   (e) Loathsome
11. **Meagre:**
   (a) Insufficient (b) Ample
   (c) Marginal (d) Extraneous
   (e) Scanty
12. **Analysis:**
   (a) Dissection (b) Criticize
   (c) Projection (d) Synthesis
   (e) Dialysis
13. **Condemn:**
   (a) Penalize (b) Censure
   (c) Punish (d) Approve
   (e) Castigate
14. **Apposite:**
   (a) Opposite (b) Ruddy
   (c) Truthful (d) Inappropriate
   (e) Apt
15. **Inflexible:**
   (a) Rigid (b) Caustic
   (c) Amenable (d) Acrid
   (e) Infirm
16. **Disparage:**
   (a) Indict (b) Slander
   (c) Appreciate (d) Honour
   (e) Downsize
17. **Impede:**
   (a) Tolerate (b) Recede
   (c) Assist (d) Bother
   (e) Cripple
18. **Dissent:**
   (a) Fall (b) Debacle
   (c) Approval (d) Antagonism
   (e) Ascent
19. **Reasoned:**
   (a) Logical (b) Arbitrary
   (c) Nurtured (d) Confused
   (e) Objective

20. Fetid:
(a) Rotten (b) Aromatic
(c) Gibberish (d) Bucolic
(e) Fowl smelling

21. Ardent:
(a) Zealous (b) Fanatical
(c) Apathetic (d) Bullish
(e) Spirited

22. Contentious:
(a) Irascible (b) Placatory
(c) Jaundiced (d) Myopic
(e) Irritable

23. Buoyant:
(a) Lively (b) Disinterested
(c) Affable (d) Morose
(e) Relaxed

24. Coalesce:
(a) Converge (b) Assimilate
(c) Disperse (d) Moderate
(e) Assemble

25. Antediluvian:
(a) Modern (b) Antiquated
(c) Stylish (d) Artistic
(e) Lucky

26. Overbearing:
(a) Servile (b) Dictatorial
(c) Haughty (d) Irate
(e) Arrogant

27. Lucid:
(a) Diurnal (b) Pellucid
(c) Indifferent (d) Obscure
(e) Transparent

28. Puerile:
(a) Obliging (b) Uncivilized
(c) Platonic (d) Mature
(e) Credulous

29. Pulchritude:
(a) Ugliness (b) Fitness
(c) Daintiness (d) Robustness
(e) Charm

30. Eschew:
(a) Taunt (b) Embrace
(c) Possess (d) Castigate
(e) Abstain

EXERCISE 3

Direction for questions 1–25: ***In the following questions, a word is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys opposite meaning as the word given in bold letters.***

1. Fledgling:
(a) Lobbyist (b) Sophomoric
(c) Veteran (d) Sapling
(e) Green horn

2. Apathy:
(a) Nonchalance (b) Rebuttal
(c) Antipathy (d) Empathy
(e) Fortune

3. Bucolic:
(a) Penalize (b) Censure
(c) Punish (d) Civic
(e) Rustic

4. Veneration:
(a) Dissuasion (b) Ignorance
(c) Passivity (d) Contempt
(e) Honour

5. Vitriolic:
(a) Humble (b) Retiring
(c) Pleasant (d) Pure
(e) Bitter

6. Soporific:
(a) Mature (b) Impervious
(c) Exciting (d) Steep
(e) Dull

7. Ameliorate:
(a) Conquer (b) Worsen
(c) Circumvent (d) Exaggerated
(e) Better

8. Voluble:
(a) Tender (b) Capacious
(c) Fragile (d) Taciturn
(e) Garrulous

9. **Fission:**
(a) Splitting (b) Joining
(c) Spinning (d) Discursive
(e) Nuclear

10. **Paucity:**
(a) Intricacy (b) Glut
(c) Suffering (d) Fastidious
(e) Scarcity

11. **Platitude:**
(a) Genuine (b) Unoriginal
(c) Boring (d) Jejune
(e) Abundance

12. **Sober:**
(a) Egalitarian (b) Gory
(c) Morose (d) Inebriated
(e) Smart

13. **Invigorate:**
(a) Debilitate (b) Enliven
(c) Revel (d) Carouse
(e) Animate

14. **Insalubrious:**
(a) Pale (b) Sickly
(c) Morbid (d) Wholesome
(e) Diseased

15. **Spiritual:**
(a) Stellar (b) Apocryphal
(c) Carnal (d) Vernal
(e) Humane

16. **Diaphanous:**
(a) Flimsy (b) Thin
(c) Opaque (d) Angry
(e) Lamenting

17. **Facetious:**
(a) Comical (b) Grave
(c) Satirical (d) Ironical
(e) Ardent

18. **Perfidy:**
(a) Felony (b) Loyalty
(c) Mockery (d) Antagonism
(e) Agnosticism

19. **Fatuous:**
(a) Silly (b) Inane
(c) Sensible (d) Perplexed
(e) Impressive

20. **Heretical:**
(a) Heroic (b) Orthodox
(c) Villainous (d) Climactic
(e) Rebellious

21. **Gregarious:**
(a) Unsociable (b) Sociable
(c) Motivating (d) Ecstatic
(e) Enlightened

22. **Furtive:**
(a) Clandestine (b) Fugitive
(c) Expatriate (d) Open
(e) Hidden

23. **Fleeting:**
(a) Vanishing (b) Passing
(c) Affable (d) Permanent
(e) Mesmerizing

24. **Flowery:**
(a) Decorated (b) Magical
(c) Heavenly (d) Fragrant
(e) Unadorned

25. **Fecund:**
(a) Sterile (b) Poor
(c) Amateur (d) Debacle
(e) Damsel

EXERCISE 4

Direction for questions 1–30: ***In the following questions, a word is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys opposite meaning as the word given in bold letters.***

1. **Imperious:**
(a) Docile (b) Unimportant
(c) Pacific (d) Sloppy
(e) Puerile

2. **Illustrious:**
(a) Nasty (b) Hazy
(c) Undistinguished (d) Rich
(e) Respectful

3. Immaculate:
(a) Contaminated (b) Charge
(c) Incarcerate (d) Virgin
(e) Germinal

4. Inculpate:
(a) Exonerate (b) Arraign
(c) Incarcerate (d) Indict
(e) Debase

5. Impudent:
(a) Saucy (b) Respectful
(c) Irreverent (d) Onerous
(e) Crazy

6. Invective:
(a) Humour (b) Praise
(c) Honour (d) Grace
(e) Divine

7. Indigent:
(a) Crooked (b) Reserved
(c) Wealthy (d) Saintly
(e) Occidental

8. Imperturbable:
(a) Stoic (b) Calm
(c) Peaceful (d) Tranquil
(e) Excitable

9. Impeach:
(a) Allege (b) Extradite
(c) Invoke (d) Slander
(e) Vindicate

10. Imminent:
(a) Likely (b) Probable
(c) Possible (d) Remote
(e) Inevitable

11. Guileful:
(a) Ingenuous (b) Spiteful
(c) Malicious (d) Insidious
(e) Vague

12. Intelligible:
(a) Perceptive
(b) Insightful
(c) Incomprehensible
(d) Sagacious
(e) Insipid

13. Languid:
(a) Energetic (b) Jaded
(c) Jocular (d) Delighted
(e) Lachrymose

14. Innocuous:
(a) Genetic (b) Genuine
(c) Cursing (d) Malicious
(e) Harmful

15. Indigenous:
(a) Crafty (b) Foreign
(c) Crafty (d) Wicked
(e) Awkward

16. Jocular:
(a) Comical (b) Insular
(c) Solemn (d) Dull
(e) Prosaic

17. Jaunty:
(a) Lively (b) Sporty
(c) Comatose (d) Sedate
(e) Moribund

18. Intransigent:
(a) Rigid (b) Squalid
(c) Flexible (d) Filthy
(e) Prim

19. Jaundiced:
(a) Optimistic (b) Diseased
(c) Vulnerable (d) Strong
(e) Meek

20. Malignant:
(a) Pernicious (b) Benign
(c) Rigid (d) Roomy
(e) Impervious

21. Nullify:
(a) Eradicate (b) Ratify
(c) Expedite (d) Activate
(e) Motivate

22. Lax:
(a) Lenient (b) Easy
(c) Rigorous (d) Intractable
(e) Refractory

23. Macrocosm:
(a) Equable (b) Equitable
(c) Balanced (d) Microcosm
(e) Behemoth

24. Munificent:
(a) Generous (b) Lavish
(c) Stingy (d) Provident
(e) Prudent

25. Benevolent:
(a) Cheerful (b) Optimistic
(c) Malevolent (d) Edgy
(e) Bad tempered

26. Piquant:
(a) Tangy (b) Racy
(c) Risqué (d) Bland
(e) Vulgar

27. Perseverance:
(a) Patience (b) Joy
(c) Harmony (d) Inconstancy
(e) Love

28. Unobstrusive:
(a) Arcane (b) Morose
(c) Noticeable (d) Amoral
(e) Amorphous

29. Aggravate:
(a) Worsen (b) Mitigate
(c) Imperil (d) Redeem
(e) Coach

30. Naive:
(a) Tyro (b) Sophisticated
(c) Beginner (d) Veteran
(e) Inveterate

ANSWER KEY

Exercise 1

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (d)	**6.** (a)	**7.** (d)	**8.** (b)
9. (c)	**10.** (c)	**11.** (b)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (c)
17. (c)	**18.** (c)	**19.** (a)	**20.** (d)	**21.** (b)	**22.** (a)	**23.** (c)	**24.** (b)
25. (a)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (a)	**30.** (c)		

Exercise 2

1. (a)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (d)
9. (a)	**10.** (b)	**11.** (b)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)
17. (c)	**18.** (c)	**19.** (b)	**20.** (b)	**21.** (c)	**22.** (b)	**23.** (d)	**24.** (c)
25. (a)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (b)		

Exercise 3

1. (c)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (d)
9. (b)	**10.** (b)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (c)
17. (b)	**18.** (b)	**19.** (c)	**20.** (b)	**21.** (a)	**22.** (d)	**23.** (d)	**24.** (e)
25. (a)							

Exercise 4

1. (b)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (e)
9. (e)	**10.** (d)	**11.** (a)	**12.** (c)	**13.** (e)	**14.** (e)	**15.** (b)	**16.** (c)
17. (d)	**18.** (c)	**19.** (a)	**20.** (b)	**21.** (b)	**22.** (c)	**23.** (d)	**24.** (c)
25. (c)	**26.** (d)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (b)		

HINTS AND EXPLANATIONS

Exercise 1

1. Fickle means changing unpredictably and persistent means consistent, which does not change.

2. Tranquil means peaceful, so its antonym will be disturbed.

3. Gloomy means sad and depressed, spirited means lively.

4. Cacophonous means harsh and unpleasant.

5. Zenith is the topmost point, and nadir is the lowest point.

6. Advance means to move ahead and retreat means to withdraw.

7. Coy means shy, brazen means rude and shameless.

8. Dearth means scarcity, so abundance is the opposite.

9. Alleviate means to make the things more bearable, aggravate means to worsen.

10. Callous means insensitive, so sensitive is the antonym.

11. Oriental means pertaining to East so occidental, which means pertaining to West.

12. Nascent means new so senescent means old is the opposite.

13. Brood means to sulk, opposite can be exult, to praise highly.

14. Blasphemy means insulting God or religion, so respect is the opposite.

15. Shelter means to protect, so imperil, *i.e.*, to risk or endanger oneself is the answer.

16. Condescend means to belittle, so laud which means to praise is the opposite.

17. Shallow profound means deep, so shallow.

18. Archetype means original, so duplicate.

19. Frugal means economical, so wasteful.

20. Repugnant means deserving hate, so agreeable.

21. Scramble means to jumble or mix randomly, so decipher, which means to crack a code.

22. Elite, belonging to the upper class, so plebeian, pertaining to the masses.

23. Ostensible means apparent, so hidden.

24. Modest means unassuming, so haughty means arrogant.

25. Irrevocable means something which can't be undone, so alterable.

26. Ludicrous means humorous, so grave.

27. Congenital means acquired by birth, so acquired.

28. Hilarious means comical, so morose means sorrowful or depressive.

29. Impecunious means poor, so affluent.

30. Absolve, means to free from charges or blame, so accuse is the opposite.

Exercise 2

1. Ratify means to pass a law, so abrogate, *i.e.*, to cancel a law.
2. Fruitful means successful, so abortive which means unsuccessful.
3. Amplify means increase, so abbreviate.
4. Abstract means hazy and intangible, so concrete.
5. Clumsy means awkward, so adroit which means skilled.
6. Ally means a friend and supporter, so adversary.
7. Oblivious means unaware, so alert.
8. Affirmative means positive.
9. Ambiguous means unclear, so unequivocal which means definite is the antonym.
10. Abhorrence means hatred, so admiration.
11. Meager means less.
12. Synthesis is the opposite of analysis.
13. Condemn means to criticize, so approve.
14. Apposite means appropriate, so inappropriate.
15. Inflexible is the opposite of amenable which means flexible.
16. Disparage means to belittle, so opposite is appreciate.
17. Impede means to hinder, so opposite is assist.
18. Dissent means disagreement, so opposite is approval.
19. Arbitrary means not clear.
20. Fetid means rotten, so opposite is aromatic.
21. Ardent means very interested, so opposite means apathetic or insensitive.
22. Contentious means argumentative or quarrelsome, so opposite is placatory.
23. Buoyant means lively or spited, so opposite is morose.
24. Coalesce means to join, so disperse.
25. Antediluvian means very old, so modern.
26. Overbearing means haughty, so servile.
27. Lucid means easy to understand, so obscure.
28. Puerile means childish, so mature.
29. Pulchritude means beauty, so ugliness.
30. Eschew means to avoid, so embrace.

Exercise 3

1. Fledgling means young and inexperienced, so veteran.
2. Empathy is the opposite of apathy.
3. Bucolic means village like, so civic.
4. Veneration means respect, so opposite is contempt.
5. Vitriolic means bitter and biting, so pleasant.
6. Soporific means causing sleep, so exciting.
7. Ameliorate means to improve, so worsen.
8. Voluble means expressing fluently, so taciturn.
9. Fission means splitting, so joining.
10. Paucity means lack of, so glut which means abundance.
11. Platitude means clichéd, so unoriginal.
12. Sober, means not drunk, so inebriated means drunken.
13. Invigorate means to energize, so debilitate means to cripple.
14. Insalubrious means unhealthy, so wholesome.

15. Carnal means showing physical desire, so carnal.

16. Diaphanous means transparent, so opaque.

17. Facetious means humorous, so grave.

18. Perfidy means disloyalty, so loyalty.

19. Fatuous means silly and pointless, so sensible.

20. Heretical means against established religious views, so orthodox.

21. Gregarious means group loving, so unsociable.

22. Furtive means secretive, so open.

23. Fleeting means passing and disappearing, so permanent.

24. Flowery means decorated so unadorned, so unadorned.

25. Fecund means fertile, so sterile.

Exercise 4

1. Imperious means authoritative, so docile.

2. Undistinguished.

3. Immaculate means pure, so contaminated.

4. Inculpate means to blame, so indict.

5. Impudent means showing casual disrespect, so respectful.

6. Invective means abuse or verbal insult, so praise.

7. Indigent means poor, so wealthy.

8. Imperturbable means one who can't be disturbed.

9. Impeach means to charge a public official, so vindicate.

10. Imminent means likely to happen, so remote.

11. Guileful means deceptive, so ingenuous.

12. Intelligible means perceptible.

13. Languid means tired, so energetic.

14. Innocuous means harmless.

15. Indigenous means native of, so foreign.

16. Jocular means comical, so solemn, serious.

17. Jaunty means lively and cheerful, so sedate.

18. Intransigent means stubborn, so flexible.

19. Jaundiced means prejudiced, so optimistic.

20. Benign is the opposite of malignant.

21. Nullify means to cancel a law, so ratify.

22. Lax means not strict.

23. Microcosm.

24. Munificent means generous, so stingy.

25. Benevolent means kind and generous, so malevolent.

26. Piquant means sharp and biting, so bland.

27. Perseverance means consistent.

28. Unobtrusive means something which is not very clear or distinct.

29. Aggravate means to worse, so mitigate, means to make bearable.

30. Naïve means unworldly, so sophisticated.

PRACTICE EXERCISES (SYNONYM)

EXERCISE 1

Direction for questions 1–20: ***In the following questions, a word is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys the same meaning as the word given in bold letters.***

1. **Amply:**
   (a) Sufficiently
   (b) To dress
   (c) Prior
   (d) A brief account of some interesting event or incident
2. **Comport:**
   (a) Agree with
   (b) Fitting in
   (c) Too dirty or discoloured
   (d) With no shape; unorganized
3. **Abjure:**
   (a) Hard coal
   (b) To recant, renounce, repudiate under oath
   (c) A book whose leaves are so made to form paper frames for holding photographs
   (d) One who or that which accompanies
4. **Hostility (opposition):**
   (a) Cacophonous
   (b) Comeliness
   (c) Bombastic
   (d) Antagonism
5. **Alley:**
   (a) Quantity or extent of land, especially of cultivated land
   (b) A narrow street, garden path, walk, or the like
   (c) Largeness
   (d) To represent beforehand in outline or by emblem
6. **Wrong; awry:**
   (a) Contentious
   (b) Collusion
   (c) Amiss
   (d) Conglomeration
7. **Anglophobia:**
   (a) Hatred or dread of England or of what is English
   (b) Having the right or privilege of entry
   (c) To make shorter in words, keeping the essential features, leaning out minor particles
   (d) Sourness, with bitterness and astringency
8. **Touching; or adjoining and close, but not touching:**
   (a) Contiguous (b) Abase
   (c) Antagonism (d) Apposite
9. **Advent:**
   (a) Cheerful willingness
   (b) The coming or arrival, as of any important change, event, state, or personage
   (c) A portable free-reed musical instrument
   (d) The act or state of lying concealed for the purpose of surprising or attacking the enemy
10. **Trite (without freshness or originality):**
    (a) Baroque (b) Boor
    (c) Chimera (d) Banal
11. **Alter:**
    (a) Flood
    (b) One chosen to act in place of another, in case of the absence or incapacity of that other
    (c) To make change in
    (d) The act of cutting off, as in a surgical operation

12. Analogy:
(a) Unnecessary activity or ceremony
(b) Induction or elevation, as to dignity, office, or government
(c) Reasoning in which from certain and known relations or resemblance, others are formed
(d) That which is near or bordering upon

13. Corroborate:
(a) To confirm the validity
(b) Tasting sour; harsh in language or temper
(c) Unselfish devotion to the welfare of others
(d) Controversy; dispute

14. Augment:
(a) One who is skeptical of the existence of know ability of a god or any ultimate reality
(b) Urge; plead for
(c) Increase
(d) Pertaining to soil deposits left by running water

15. To become a semisolid, soft mass; to clot:
(a) Coagulate (b) Advocate
(c) Copious (d) Abysmal

16. Amazement or terror that causes confusion:
(a) Conjure (b) Appease
(c) Analogy (d) Consternation

17. To determine the quality of a substance:
(a) Consequential (b) Cacophonous
(c) Assay (d) Agrarian

18. A fortified place or strong defense:
(a) Adamant (b) Astringent
(c) Bungler (d) Bastion

19. To go away hastily or secretly; to hide:
(a) Abaft (b) Abscond
(c) Clemency (d) Aseptic

20. Airy:
(a) Slight sickness
(b) Delicate, ethereal
(c) To hate violently
(d) Anything gained, or made one's own, usually by effort or labour.

EXERCISE 2

Direction for questions 1–20: ***In the following questions, a word/phrase is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys the same meaning as the word/phrase given in bold letters.***

1. Eager readiness or speed:
(a) Cloying (b) Bode
(c) Alacrity (d) Arbiter

2. Acknowledgment:
(a) Recognition
(b) Willing and ready to submit
(c) Very hateful
(d) Answering yes; to a question at issue

3. Of the land:
(a) Complacent (b) Approbatory
(c) Beholden (d) Agrarian

4. Anachronism:
(a) Pertaining to the act or sense of hearing
(b) Anything occurring or existing out of its proper time
(c) Self denial
(d) The entire number, sum, mass, or quantity of something

5. Being too long, as in a description or expression; a roundabout, indirect, or ungainly way of expressing something:
(a) Abysmal
(b) Circumlocutory
(c) Complacent
(d) Awry

6. Acquit:
(a) Able to move or act quickly, physically, or mentally
(b) To free or clear, as from accusation

(c) The art or practice of flying aircraft
(d) Portion

7. **To come down from one's position or dignity:**
(a) Adage (b) Condescend
(c) Cajole (d) Abandon

8. **Aide-de-camp:**
(a) A white or delicately tinted fine-grained gypsum
(b) Profound devotion
(c) An officer who receives and transmits the orders of the general
(d) Good-by; farewell

9. **To move towards one point (opposite: diverge):**
(a) Askance (b) Converge
(c) Analogy (d) Asperity

10. **Annuity:**
(a) An annual allowance, payment or income
(b) The superior of a community of monks
(c) To warn of a fault
(d) To pile or heap together

11. **Accede:**
(a) Without determinate shape
(b) Misfortune
(c) Of unknown authorship
(d) To agree

12. **To overlook; to forgive:**
(a) Charisma (b) Condone
(c) Contempt (d) Consecrate

13. **Arrogant:**
(a) Allure (b) Cacophony
(c) Bumptious (d) Conjoin

14. **To forgive; to acquit:**
(a) Chary (b) Aghast
(c) Absolve (d) Accretion

15. **Afoot:**
(a) In progress
(b) To warn of a fault
(c) To refer incidentally
(d) An adulterating substance

16. **Something that is abnormal:**
(a) Cohesion (b) Connotative
(c) Atypical (d) Alacrity

17. **Aggregate:**
(a) Aa series of tables giving the days of the week together with certain astronomical information
(b) The entire number, sum, mass, or quantity of something
(c) The practical unit of electric-current strength
(d) The state of being attached or joined

18. **To acknowledge; admit:**
(a) Concede (b) Communal
(c) Attenuate (d) Arrogate

19. **Anode:**
(a) The point where or path by which a voltaic current enters an electrolyte
(b) Friendship
(c) Of or pertaining to the times, things, events before the great flood in the days of Noah
(d) Sharpness or bitterness of speech or temper

20. **With no shape; unorganized:**
(a) Asperity (b) Amorphous
(c) Conviviality (d) Constrain

EXERCISE 3

Direction for questions 1–20: ***In the following questions a word/phrase is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys the same meaning as the word/phrase given in bold letters.***

1. **Lack of emotion or interest:**
(a) Apathy (b) Complacent
(c) Caliber (d) Anecdote

2. **Ailment:**
(a) Colourless
(b) A discharge from accusation by judicial action
(c) Slight sickness

(d) Having fine and penetrating discernment

3. **Changeable; fickle**:
(a) Condescend (b) Complaisance
(c) Cascade (d) Capricious

4. **Abhorrent:**
(a) Very repugnant; hateful
(b) A member of an academy of literature, art, or science
(c) Easy to approach
(d) Profound devotion

5. **Acrimonious:**
(a) Full of bitterness
(b) Not conformed to the ordinary rule or standard
(c) A book whose leaves are so made to form paper frames for holding photographs or the like
(d) To cause to appear greatly

6. **One who believes that a formal government is unnecessary:**
(a) Confluence (b) Anarchist
(c) Compromise (d) Aghast

7. **Abdominal:**
(a) One who manages affairs of any kind
(b) Not mandatory
(c) A condensed form as of a book or play
(d) Of, pertaining to, or situated on the abdomen

8. **A symbolic description:**
(a) Conjure (b) Amiss
(c) Allegory (d) Chaffing

9. **To lump together, causing confusion; to damn:**
(a) Confound (b) Abbreviate
(c) Arcane (d) Candid

10. **Americanism:**
(a) Beginning, ending, or changing suddenly or with a break
(b) Opposing or opposed
(c) A charge of crime, misdemeanor, or error
(d) A peculiar sense in which an English word or phrase is used in the United States

11. **Suitable (as land) for plowing:**
(a) Amity (b) Arable
(c) Adjure (d) Blatant

12. **Alabaster:**
(a) Before noon
(b) Of, pertaining to, or involving an accusation
(c) A white or delicately tinted fine-grained gypsum
(d) A condensed form as of a book or play

13. **Not yielding, firm:**
(a) Awry (b) Contrite
(c) Abase (d) Adamant

14. **Antipathy:**
(a) Urge; plead for (b) Calm; pacify
(c) Aversion; dislike (d) Increase

15. **Advert:**
(a) To speak to
(b) To make explanatory or critical notes on or upon
(c) To fight
(d) To refer incidentally

16. **Fitting in:**
(a) Comport (b) Attenuate
(c) Allure (d) Beholden

17. **Abdicate:**
(a) Aan officer who receives and transmits the orders of the general
(b) A person or thing that aids the principal agent
(c) Primitive; unsophisticated
(d) To give up (royal power or the like)

18. **Conferring benefits; kindly:**
(a) Beneficent (b) Amortize
(c) Carte blanche (d) Audacious

19. **Abrupt:**
(a) Designed to excite love

(b) Beginning, ending, or changing suddenly or with a break
(c) The branch of pneumatics that treats of the equilibrium, pressure, and mechanical properties
(d) To move faster

20. Ablution:
(a) A manually skilled worker
(b) Dry; barren
(c) Ash-coloured; deadly pale
(d) Washing

EXERCISE 4

Direction for questions 1–20: ***In the following questions, a word/phrase is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys the same meaning as the word/phrase given in bold letters.***

1. Abbot:
(a) A discharge from accusation by judicial action
(b) A white or delicately tinted fine-grained gypsum
(c) The superior of a community of monks
(d) Sufficiently

2. Friendly; amiable:
(a) Affiliate
(b) Abstemious
(c) Affable
(d) Abeyance

3. Academy:
(a) Any institution where the higher branches of learning are taught
(b) Induction or elevation, as to dignity, office, or government
(c) The act of detesting extremely
(d) To refer incidentally

4. Erratic:
(a) Irresponsible, eccentric; lacking a fixed purpose erratic behaviour
(b) Display or wave boastfully
(c) Relating to the countryside
(d) Dry; barren

5. Affront:
(a) The setting forth of a subject under the guise of another subject of aptly suggestive likeness
(b) A record of events in their chronological order, year by year
(c) The character '&'; and
(d) An open insult or indignity

6. Benevolent:
(a) Art authoritative statement; a saying
(b) Kindly; charitable
(c) To pierce through with a pointed instrument
(d) A distortion of the face to express an attitude or feeling

7. Affix:
(a) To contend angrily or zealously in words
(b) Practicing an art or occupation for the love of it, but not as a profession
(c) To stick fast or together
(d) To fasten

8. Expound:
(a) To express sorrow or grief over
(b) To set forth in detail; to explain
(c) To make gestures, or indicate feelings by motions
(d) A god

9. Alienable:
(a) Occurring or existing before birth
(b) To make inefficient or worthless; muddle
(c) Anything forbidden, as by social usage
(d) Capable of being aliened or alienated, as lands

10. Acute:
(a) Historian
(b) Of, pertaining to, or involving an accusation
(c) Estrangement
(d) Having fine and penetrating discernment

11. Devout:
(a) Devoted to religious observances
(b) To beg earnestly
(c) Pertaining to public discussion or law courts
(d) A picture or other description of a person which exaggerates ludicrously one or more of his distinctive features

12. Abet:
(a) To use for one's selfish purpose
(b) To encourage or support
(c) Origin
(d) Rudely abrupt

13. Condole:
(a) Sociable, courteous, and agreeable in manner
(b) Despotic
(c) To express sympathy with another in sorrow, pain, or misfortune
(d) Inclined to believe anything; easily imposed upon

14. Affiliate:
(a) Some auxiliary person or thing
(b) Of or pertaining to an academy, college, or university
(c) To recognize (v.); to admit the genuineness or validity of
(d) Plentiful (adj)

15. Heresy:
(a) Despotic
(b) Historical records
(c) An opinion held in opposition to the traditional view
(d) Coward

16. Devoid:
(a) Implied but not clearly expressed; unquestioning
(b) Lacking in; not possessing
(c) Sharp or harsh in language or temper
(d) Positive in expressing an opinion; asserting an opinion as though it were an undisputed fact

17. Aggrieve:
(a) Reasoning in which from certain and known relations or resemblance others are formed
(b) A vehicle fitted for conveying the sick and wounded
(c) To give grief or sorrow to
(d) A volatile, inflammable, colourless liquid of a penetrating odour and burning taste

18. Cringe:
(a) Self-satisfied
(b) To shrink in fear
(c) Prejudiced
(d) Habitually fond of associating in a company or herd

19. Acrimonious:
(a) Sharp or harsh in language or temper
(b) Of low morals; corrupt
(c) To make a mistake or to do something wrong
(d) One who denies that God exists

20. Craven:
(a) Implied but not clearly expressed; unquestioning
(b) Of low morals; corrupt
(c) Coward
(d) Rudely abrupt

EXERCISE 5

Direction for questions 1–20: ***In the following questions a word/phrase is given in bold letters followed by four alternatives marked (a)–(d). Select the alternative that conveys the same meaning as the word/phrase given in bold letters.***

1. Antarctic:
(a) Pertaining to the south pole or the regions near it
(b) Corresponding (to some other) in certain respects, as in form, proportion, relations

(c) Some auxiliary person or thing
(d) Eagerly desirous and aspiring

2. Egregious:
(a) A distortion of the face to express an attitude or feeling
(b) Conversation which is amusing and not serious
(c) A record of a person's or a family's ancestors or relatives
(d) Often of mistakes, extremely and noticeably bad

3. Adjuration:
(a) To pile or heap together
(b) To wear away the surface or some part of by friction
(c) A vehement appeal
(d) Anything gained, or made one's own, usually by effort or labour

4. Annuity:
(a) Solemn curse; someone or something that is despised
(b) Unreasonable or capricious; tyrannical
(c) Preventing infection; having a cleansing effect
(d) Yearly allowance

5. Aboriginal:
(a) Primitive; unsophisticated
(b) Passive consent
(c) To assign or affix a date to earlier than the actual one
(d) To represent beforehand in outline or by emblem

6. Fetish:
(a) A swamp
(b) Something that is believed to have magical powers an object of unreasoning devotion and worship
(c) To soil or dirty
(d) To quicken, speed tip

7. Anterior:
(a) A member of a municipal legislative body, who usually exercises also certain judicial functions
(b) Prior
(c) Having the right or privilege of entry
(d) The point where or path by which a voltaic current enters an electrolyte or the like

8. Apathy:
(a) To free from blame
(b) To give a false idea of
(c) Lack of feeling, emotion, or interest
(d) To express sympathy with another in sorrow, pain, or misfortune

9. Exploit:
(a) Spirited; ardent
(b) A swamp
(c) To use for one's selfish purpose
(d) Quick to find fault about trifles

10. Abrogate:
(a) Lacking in freshness, originality, or vigor
(b) Liable to make mistakes or be deceived
(c) A public command or proclamation issued by an authority
(d) To abolish or render void

11. Antithesis:
(a) Pertaining to the clergy or the church
(b) To declare positively; to confirm
(c) Departure, emigration
(d) Contrast; the direct opposite

12. Deluge:
(a) Thoughtless; taking little care
(b) To abolish or render void
(c) A brief summary of the main ideas of a larger work
(d) A great flood; downpour

13. Advocate:
(a) Diversion
(b) One who pleads the cause of another, as in a legal or ecclesiastical court
(c) Any raised place or structure on which sacrifices may be offered or incense burned
(d) Change or modification

14. Crass:
(a) Personal peculiarity
(b) Coarse and stupid
(c) Well-deserved (applied chiefly to punishment)
(d) Wicked; hateful

15. Aldermanship:
(a) Invulnerable
(b) The art or practice of flying aircraft
(c) The dignity, condition, office, or term of office of an alderman
(d) To warn of a fault

16. Disparity:
(a) Inequality; difference in image, quantity, character, or rank
(b) To punish or criticize severely
(c) To declare positively; to confirm
(d) Coarse and stupid

17. Aggrandize:
(a) Having fine and penetrating discernment
(b) To cause to appear greatly
(c) To utter with a shout
(d) The setting forth of a subject under the guise of another subject of aptly suggestive likeness

18. Cumbrous:
(a) Burdensome and clumsy
(b) Mentally distressed; distracted
(c) To express sympathy with another in sorrow, pain, or misfortune
(d) A tombstone inscription

19. Annex:
(a) To add or affix at the end
(b) To recant, renounce, repudiate under oath
(c) An abiding
(d) To move faster

20. Cabal:
(a) A feeling of hatred
(b) Obnoxiously conceited or self-assertive
(c) Pertaining to public discussion or law courts
(d) A small group of persons engaged in plotting

ANSWER KEY

Exercise 1

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (a)
9. (b)	**10.** (d)	**11.** (c)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (a)	**16.** (d)
17. (c)	**18.** (d)	**19.** (b)	**20.** (b)				

Exercise 2

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (b)	**8.** (c)
9. (b)	**10.** (a)	**11.** (d)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (c)
17. (b)	**18.** (a)	**19.** (a)	**20.** (b)				

Exercise 3

1. (a)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (d)
9. (a)	**10.** (a)	**11.** (b)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (d)	**16.** (a)
17. (d)	**18.** (a)	**19.** (b)	**20.** (d)				

Exercise 4

1. (c)	**2.** (c)	**3.** (a)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (d)	**8.** (b)
9. (d)	**10.** (d)	**11.** (a)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (b)
17. (c)	**18.** (b)	**19.** (a)	**20.** (c)				

Exercise 5

1. (a)	**2.** (d)	**3.** (d)	**4.** (d)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (c)
9. (c)	**10.** (d)	**11.** (d)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (a)
17. (b)	**18.** (a)	**19.** (a)	**20.** (d)				

CONFUSING WORDS

Mark Twain said, "The difference between the right word and the almost right word is the difference between lightning and the lightning bug".

To excel in this chapter, having a good vocabulary only may not be sufficient. A good vocabulary can be useful when used to trigger a word already in your vocabulary bank. But simply plucking out a near-synonym can result in some disastrous situation because the word may not fit the context of the question. And there comes the need of understanding the usage of words and having mental alertness to identify the minor differences between/among them.

Specially, in today's time, when students have got access to computer/MS Word/T9 in mobiles, which prompt predictive typing or suggested right word for every wrong word, at a very early stage, mental alertness is of much more importance.

We start this chapter by providing a list of Commonly Mistaken Word followed by exercises. Though, needless to say, any such list cannot be exhaustive and students are advised to keep looking for such words in their day to day reading too.

LIST OF COMMONLY MISTAKEN WORD

Sl. No.	Mistaken Word	Meaning	Example
1.	Emigrant	A person who leaves one country to move to another	The Bangladeshi *emigrants* got on to the boat at Chittagong.
	Immigrant	A person who enters one country from another	The *immigrants* to India got off the boat in Howrah.
2.	Eminent	Well-known	Newton is an *eminent* scientist.
	Imminent	About to happen	The documentary movie predicted that a flood was *imminent.*
3.	Feign	To give a false appearance of	To *feign* ignorance.
	Faint	To lose consciousness briefly	Some people often *faint* at the sight of a mouse.
4.	Farther	At a greater distance	I can see him going *farther.*
	Further	More	I have nothing *further* to tell you.

Sl. No.	Mistaken Word	Meaning	Example
5.	Flaunt	To make a gaudy display	Paris likes to *flaunt* her wealth.
	Flout	To defy	*Flouting* traffic rules may put you in difficulty.
6.	Ideal	Perfect	There is no *ideal* solution to the poverty problem in India.
	Idol	Object of worship	*Idol* worship has made him blind to his coach's faults.
7.	Ingenious	Clever, Imaginative	*Ingenious* minds dream ambitious plans.
	Ingenuous	Frank and open	Deenanath Chauhan has an *ingenuous* way of believing everything she hears.
8.	Moral	Having high moral values as prescribed in religious books	*Immortality* has permeated in the lives of politicians to a great extent.
	Morale	State of mind in terms of confidence and courage	The *morale* of our troops is high.
9.	Ordinance	Rule of order	A new *ordinance* has been issued by the Government regarding the economic policy.
	Ordnance	Arms, Arsenal	The *ordnance* factory at army HQ is quite old.
10.	Precede	To go in advance of	A *precedes* B in the alphabet.
	Proceed	To go on or go forward	If there are no doubts, I will *proceed* with the next concept.
11.	Prescribe	To recommend or set down a rule to be followed	The doctor *prescribed* complete rest for the patient.
	Proscribe	To ban something	Doctor *proscribed* curd for the patient.
12.	Principal	(i) Major, capital as opposed to interest	Carelessness is a *principal* cause of highway accidents.
		(ii) The principal of the school attended our concert	The *principal* was ₹ 10,000 and the interest was 5% a year.
	Principle	A general rule or truth	A *principled* life is required to lead a happy life.
13.	Quiet	Calm, Still, Silent	Keep *quiet*, you might altert the birds.
	Quit	To stop doing something	People who *quit* smoking leads a healthy life.
	Quite	Entirely, Really, Noticeably	Preeti felt *quite* ill during dinner.
14.	Rain	Liquid precipitation	There will be *rain* today—as per MET forecast.
	Rein	A device used to guide a horse	Adjust the saddle and *reins*.
	Reign	The rule of a sovereign	It happened in the *reign* of Lord Mountbatten.

Sl. No.	Mistaken Word	Meaning	Example
15.	Raise	To lift something	Police ordered the thief to *raise* both the hands and surrender.
	Raze	To demolish	The authorities began to *raze* the illegal complex.
16.	Septic	A bacterial infection	The wound developed into a *septic* because of negligence.
	Skeptic	A person who always doubts	Fake placement records of B-Schools make students *skeptical.*
17.	Simulate	To imitate or Pretend	Now a days, drivers can get themselves trained on *simulators.*
	Stimulate	Excite, Provoke, Inspire	Anna's speech could *stimulate* the whole Nation.
18.	Sore	Unhappy, Bitter	He was quite *sore* over his defeat.
	Sour	Not sweet, Pungent	The grapes were quite *sour.*
	Soar	To rise high	His spirits *soared* when he got the call letter.
19.	Stationary	Static, Motionless	The car rammed into the *stationary* truck.
	Stationery	Writing material	Mr. Randhir owns the best *stationery* shop in town.
20.	Straight	Not curved or Crooked	A *straight* line is the shortest distance between two points.
	Strait	A narrow passage of water connecting two large bodies of water, a distressing situation	(i) We passed through the *strait* of Gibraltar. (ii) Our next door neighbor is in dire financial *straits*.
21.	Suit	A coat with matching trousers or skirt, a proceeding in a law court	(i) Grandfather still wears a blue *suit* every Sunday. (ii) Ram Bikemalani argued his *suit* eloquently.
	Suite	A set of rooms, of matching furniture	Ramesh reserved the bridal *suite* at The Taj for his wedding.
22.	Tamper	To meddle with	Ask children not to *tamper* with the equipments.
	Temper	Disposition, Nature	He has a bad *temper* and that is why he is not very popular.
23.	Urban	Belonging to the city or town	The villagers are fast migrating to *urban* areas.

Sl. No.	Mistaken Word	Meaning	Example
	Urbane	Refined, Polished, Cultured	Having an *urbane* personality is desirable for all the sales people.
24.	Vain	Conceited, Useless	(i) That model is an unusually *vain* girl. (ii) He made several *vain* attempts to get a job.
	Vane	A direction pointer	The weekend farmers bought a new weather *vane* for the barn.
	Vein	A blood vessel	In today's medical class, we were told about *veins* and arteries.
25.	Venal	Evil, Corrupt, Immoral	The *venal* acts of bureaucrats have damaged the nation.
	Venial	Something that can be pardoned	The *venial* mischief of the child are often overlooked.
26.	Waist	The narrow part of the body above the hips	He has a 34 inches *waist*.
	Waste	Needless consumption or destruction	Facebook is a total *waste* of time.
27.	Weather	Day to day climate	What is the *weather* forecast for today?
	Whether	If it be the case that	I don't know *whether* she would marry me or not?
28.	Whose	The possessive of the pronoun who	*Whose* book is this?
	Who's	'Who is'	*Who's* going to dinner with me?
29.	Your	The possessive of the pronoun You	Is this *your* book?
	You're	'You are'	*You're* late for work.
30.	Can	Capable	Despite the warning, hooligans *can* create problems.
	May	Permission	*May* I go out Sir?
31.	Able	Power or strength in general	Despite going through Chemotherapy, Yuvraj is *able* to play Cricket.
	Capable	Power or strength in particular	He is *capable* of running for 2 hours continuously.
32.	Advice	Suggestion (Noun)	People who give unsolicited *advice* are not respected in the society.
	Advise	Counsel (Verb)	I *advise* you to try to stay put in the hostel.
33.	Beside	Adjacent, Nearby	My house is *beside* church.
	Besides	Also, As well as	*Besides* getting a technical knowhow, doing MBA will also give me practical exposure.

Sl. No.	Mistaken Word	Meaning	Example
34.	Decent	Suitable; Respectable	
	Descent	The act of coming down	
	Dissent	To differ	Despite knowing the opinion of the minister, the officer *dissented* on the whole issue.
35.	Desert	Waste land, To abandon	(i) Sahara is one of the biggest *deserts* in the world.
			(ii) Mr Bhatt *deserted* Ms Parveen after using her talent.
	Dessert	The course at the end of the dinner	Delicious *dessert* was served after dinner.
36.	Dual	Double	All long range aircrafts have *dual* engine.
	Duel	Contest between two people	*Duel* between MSD and VS for captaincy is detrimental for Team performance.

PRACTICE EXERCISES

EXERCISE 1

Direction for questions 1–10: ***Each question contains one sentence. Each sentence has a pair of words/phrases that are italicized/highlighted. From the italicized/highlighted word(s)/phrase(s), select the most appropriate word(s)/phrase(s) to form correct sentences. Then choose the best option.***

1. Lawyer's ***spacious [A]/specious [B]*** argument was lengthy but it could not convince the judge.
   Answer = A/B
2. Agencies have informed us that he was one of those ***whom [A]/who [B]*** they arrested last month.
   Answer = A/B
3. Usually it is junior managers who are not able to ***conform [A]/confirm [B]*** to the prevailing pattern of thinking and norms.
   Answer = A/B
4. Trends in consumer behavior and the individual's preference for one brand name ***affect [A]/effect [B]*** the situation.
   Answer = A/B
5. Persistent questioning by the lawyer failed to **elicit [A]/illicit [B]** a clear answer from the respondent.
   Answer = A/B
6. Frustration is ***implicit [A]/explicit [B]*** in any attempt to express the deepest self.
   Answer = A/B
7. In this invoice ***is [A]/are [B]*** included the charges for the past billing cycle.
   Answer = A/B
8. With our granting of this credit ***go [A]/goes [B]*** our best wishes for the success in your business venture.
   Answer = A/B
9. The judge warned the prisoner that he will ***countermand [A]/countervail [B]*** the bail of the offender if he misbehaved in the court.
   Answer = A/B

10. It is important to ***cite [A]/site [B]*** as many examples as possible in your answer to fetch good marks.
Answer = A/B

EXERCISE 2

Direction for questions 1–25: ***There are few sentences in each question. Each sentence has pairs of words/phrases that are italicized/highlighted. From the italicized/highlighted word(s)/phrase(s), select the most appropriate word(s)/phrase(s) to form correct sentences. Then choose the best option.***

1. (i) Several pages of the book have come ***loose [A]/lose [B].***
(ii) I am totally ***uninterested [A]/disinterested [B]*** in grammar.
(a) AA (b) AB
(c) BA (d) BB

2. (i) Almost every decade someone or the other comes out with the ***prophecy [A]/prophesy [B]*** that the world is going to end.
(ii) I ***loath [A]/loathe [B]*** to admit that I was totally jealous.
(a) AB (b) BA
(c) BB (d) AA

3. (i) The storm ***wreaked [A]/wrecked [B]*** havoc along the coast.
(ii) ***Accessory [A]/Excess [B]*** minerals are disregarded by petrologists while classifying rocks.
(iii) I hope we have good ***whether [A]/weather [B]*** during our trip.
(a) AAB (b) ABB
(c) BAB (d) BAA

4. (i) Though the story was focused on ***historic [A]/historical [B]*** figures, it was mainly fiction.
(ii) The accused made a passionate effort before the judge to ***ensure [A]/assert [B]*** his innocence.
(a) AB (b) BB
(c) BA (d) AA

5. (i) The authorities ***complemented [A]/complimented [B]*** her for completing the project ahead of schedule.
(ii) The invasion was ***preceded [A]/proceeded [B]*** by a massive mobilization of armed forces at the site.
(a) AB (b) BA
(c) AA (d) BB

6. (i) We had to offer our house as a ***guarantee [A]/warranty [B]*** for getting loan.
(ii) The golfer ***waddled [A]/waggled [B]*** the club before hitting the ball.
(iii) I was so ***ticked [A]/pickled [B]*** at the party that I sat down.
(iv) Everything in the room was covered ***by [A]/with [B]*** dust.
(v) I cannot ***tolerate [A]/endure [B]*** your laziness any more.
(a) BBBBA (b) ABABA
(c) ABBBA (d) BBABB

7. (i) I don't think I can proceed with the matter without your expert ***advise [A]/advice [B]***.
(ii) Election dates in Uttarakhand were decided after taking into account the ***climatic [A]/climactic [B]*** conditions.
(iii) Did he ever ***prophesy [A]/prophecy [B]*** anything but bad news for me?
(iv) The poor nations are finding it difficult to reconcile concerns about the environment with ***economic [A]/economical [B]*** growth.
(a) ABBA (b) BBAA
(c) BABA (d) BAAA

8. (i) Malaria is ***pandemic [A]/endemic [B]*** to the areas where mosquitoes breed.
(ii) The cleaner ***scunged [A]/scrunched [B]*** the trash cans for reuse.
(iii) His work consists ***in [A]/of [B]*** dealing with people.

(iv) The child ***tumbled [A]/stumbled [B]*** over the stone.
(v) He and I intend to leave ***to [A]/for [B]*** USA after two days.
(a) BBABB (b) ABABB
(c) BBABA (d) BAABB

9. (i) Daily wagers pay ***continuous [A]/constant [B]*** attention to the weather.
(ii) This year's rains that persisted ***continuously [A]/continually [B]*** in winter are attributed to climate change.
(iii) She has a habit of talking to her ***imaginative [A]/imaginary [B]*** friend.
(iv) The General declared that the victory came with the ***minimum [A]/minimal [B]*** loss of lives.
(a) ABAB (b) ABBB
(c) BBBB (d) BBBA

10. (i) He hired several ***aids [A]/aides [B]*** to help him in his work.
(ii) Her ***masterful [A]/masterly [B]*** personality soon dominated the movement.
(iii) The chartered accountant went through the ***maize [A]/maze [B]*** of papers to file tax return.
(iv) During the high ***tide [A]/tied [B]*** the fishermen **tied [A]/tide [B]** their boats to the pier.
(a) ABABA (b) ABBAB
(c) BABAA (d) BAABA

11. (i) The college ***affected [A]/effected [B]*** a new policy on leave encashment by teachers.
(ii) The ***rationale [A]/rational [B]*** behind current curricula is that students need to learn language and thinking skills, not specific information.
(a) AA (b) BB
(c) BA (d) AB

12. (i) All the bickering and noise we witness in the legislature over petty politics is pure ***bathos [A]/pathos [B].***
(ii) After the accident the victim went into ***comma [A]/coma [B]*** for several days.
(a) BB (b) AB
(c) BA (d) AA

13. (i) Our confidence in cricket has been seriously ***undermined [A]/underrated [B]*** by the recent match fixing scandal.
(ii) His ***valid [A]/vapid [B]*** talk failed to impress the investors.
(iii) It was so hot that he walked in the ***shade [A]/shadow [B]*** of the tree.
(a) BBA (b) ABA
(c) ABB (d) BAB

14. (i) The messenger gave him a letter written in a hurried ***scroll [A]/scrawl [B].***
(ii) You are ***deflecting [A]/detracting [B]*** from the agenda of discussion.
(iii) The receptionist ***scowled [A]/screamed [B]*** at me when I made a vacuous inquiry.
(iv) The items I liked most were the rosewood carvings and the teak wood ***furnitures [A]/furniture [B]*** of Gothic style.
(a) BABB (b) BAAB
(c) BBAB (d) BAAB

15. (i) The ***warp [A]/woof [B]*** of India's economic structure is still agriculture.
(ii) My friend always does the opposite ***of [A]/to [B]*** what I advise him.
(iii) Compact disks with ***prurient [A]/prudish [B]*** content are secretly in circulation in the assembly.
(iv) He believed that the existence of god cannot be proved; he was not an ***atheist [A]/agnostic [B]***.
(a) AAAA
(b) ABAA
(c) ABBB
(d) BAAB

16. (i) I have kept the book ***besides [A]/beside [B]*** the computer on the table.
(ii) You can attempt 20 questions from ***among [A]/between [B]*** 30 questions.
(iii) It was so hot in the sun that they moved into the ***shadow [A]/shade [B].***

(iv) ***Although [A]/Despite [B]*** his having known her for years, he behaved like a stranger.

(a) ABAB (b) ABBA
(c) BAAA (d) BABB

17. (i) I don't know who ***discovered [A]/invented [B]*** the computer.

(ii) You should get your car's ***breaks [A]/brakes [B]*** repaired before the long drive to Ladakh.

(iii) ***As long as [A]/As far as [B]*** my opinion is concerned, I will not say anything against the case.

(iv) Lack of awareness of environmental policies of the government has led the company ***tangle [A]/dangle [B]*** into the legal problems.

(a) ABAB (b) BBBA
(c) BABB (d) BAAB

18. (i) Within a short time span, she became his trusted ***confident [A]/confidant [B]***.

(ii) We were required to make a ***custom [A]/costume [B]*** design for the online shop.

(iii) He was declared ***illegible [A]/eligible [B]*** for the post despite not having required work experience.

(iv) Several ***eminent [A]/imminent [B]*** academicians are expected to be present for the seminar.

(a) ABAB (b) ABBA
(c) BABA (d) BAAB

19. (i) On television, we have been witnessing the ***baneful [A]/baleful [B]*** results of war as Syrian army attacks civilians.

(ii) The wrestler appeared to be tall and ***burly [A]/burley [B]*** as he stood by the referee.

(iii) Clouds smoke ***billowed [A]/bellowed [B]*** into the air as the fire engulfed the building.

(iv) The old patriarch uttered a ***malediction [A]/benediction [B]*** against the rival clan.

(a) BAAA (b) AAAA
(c) ABAB (d) AAAB

20. (i) The English poem has been ***translated [A]/transliterated [B]*** in Devanagri script.

(ii) The ***valuation [A]/evaluation [B]*** of a building is made by a team of engineers and MBAs.

(iii) No sooner did he start talking about his personal achievements, ***then [A]/than [B]*** his students began to yawn.

(iv) He has a great ***zeal [A]/zest [B]*** to go to America for a pleasure trip.

(a) BABB (b) BBBB
(c) BAAB (d) BABA

ANSWER KEY

Exercise 1

1. B **2.** A **3.** A **4.** A **5.** A **6.** B **7.** A **8.** B
9. A **10.** B

Exercise 2

1. (a) **2.** (a) **3.** (a) **4.** (b) **5.** (b) **6.** (c) **7.** (d) **8.** (a)
9. (c) **10.** (c) **11.** (a) **12.** (a) **13.** (b) **14.** (b) **15.** (d) **16.** (d)
17. (b) **18.** (a) **19.** (a) **20.** (a)

Section Test 2

Verbal Ability

- There are 40 questions in this test.
- Each question carries 4 marks.
- There is one-fourth negative marking.
- Total time allotted is 50 minutes.
- Qualifying marks for this test is 98 marks.

Direction for questions 1 and 2: ***Each of the following questions has a paragraph from which a sentence has been deleted. From the given options, choose the sentence that completes the paragraph in the most appropriate way.***

1. Osho Rajneesh, a prominent 20th century cult leader in India articulated his thoughts the way many other rational people did. Like them, Osho too questioned the dogmas, myths and contradictions of religion and life, but he did so with a revolutionary clarity. As is the case with such people normally, the governments and masses did not like him too much for his piercing, thought-provoking and rebellious ideas and constantly criticized him. So much so that he had to leave Oregon in the US when the locals protested against his utterances. By the time he died in 1989, however, his criticism of organized, ritualistic religion and his celebration of religion as something private, bringing joy and happiness were widely shared by the middle class. _____.

(a) With the passage of time, Osho's path breaking vision became a conventional outlook.
(b) Slowly but surely, complacency crept into Osho's cult and it became as bad its competitors.
(c) Thus finally, Osho was proven to be incorrect.
(d) However, Osho's cult spread to far-off locations and became the torch-bearer of a new religious reality.

2. _____. Give people positions of power and discretion, and whether they are church officials or border guards, some will use their position to enrich themselves. The problem can be big enough to drag a country's development. A study shows that bribes account for 8% of the total cost of running a business in Somalia, while another one finds that corruption boosted the price of hospital supplies in Ougugdu by 15%.

(a) Corruption is universal.
(b) Corruption is inevitable.
(c) Corruption cannot be tamed at all.
(d) The only people who are not corrupt are without power and discretion.

Direction for questions 3–7: ***Each of the following questions has a sentence with two blanks. Given below each question are four pairs of words. Choose the pair that best completes the sentence.***

3. _____, the scope and depth of the exhibition is such that a visitor doesn't feel _____ by the inevitable gift shop and with a populist bent, it wisely highlights not only the beauty and intelligence of the New Kingdom-era Egyptians, but also its considerable weirdness.

(a) Happily, oppressed
(b) Surprisingly, overwhelmed
(c) Naturally, confined
(d) Mercifully, attracted

4. This is a rich and exuberant biography of a man who _____ better known, as well as a rare window on gay life in an era known mostly for its _____ and repression.
   (a) was, ebullience
   (b) deserves to be, prudishness
   (c) should be, permissiveness
   (d) ought to be, complicity
5. Doctors and lawyers _____ that advertising will certainly _____ their profession negatively.
   (a) trust, damage (b) believe, affect
   (c) think, boost (d) argue, effect
6. Ravi was frustrated with Mohit, who would not _____ himself to a deadline. Mohit claimed he was working well without a deadline, but Ravi _____ and finally influence Mohit.
   (a) encourage, enforced
   (b) cooperate, compromised
   (c) commit, insisted
   (d) declared, pressurized
7. Lack of _____ is basic to good teamwork but our ability to work with others depends on our _____.
   (a) rigidity, compatibility
   (b) professionalism, friendly
   (c) positivism, flexibility
   (d) consideration, acumen

Direction for questions 8–12: ***Read the passage below and solve the questions based on it.***

The modern world requires us to repose trust in many anonymous institutions. We strap ourselves in a flying tin can with two hundred other people not because we know the pilot but because we believe that airline travel is safe. Our trust in these institutions depends on primarily two factors: skill and ethics. And we expect that the people who run these institutions know well what they are doing. We expect that they build and operate machines that work as they are supposed to and that they are looking out for our welfare even though we are completely strangers.

When one of these factors is weak or absent, trust breaks down and we either pay a high price in safety—as in the Bhopal tragedy—or a large welfare premium such as the elaborate security measures at airports. Trust-deficient environments work in the favour of the rich and powerful, who can command premium treatment and afford welfare premium. Poor people can command neither; which is why air travel is safer than train travel, which in turn is safer than train travel, which in turn is safer than walking by the road side.

Every modern society depends on the trust in the skills and ethics of a variety of institutions such as schools and colleges, hospital and markets. If we stopped believing in the expertise of our teachers, doctors and engineers, we will stop being a modern society.

As the Institution among institutions, it is the duty of the state to ensure that all other institutions meet their ethical obligations. The Indian state has failed in its regulatory role. We produce good graduates and well trained engineers but we cannot guarantee that our graduates and engineers will turn out good products.

Last year, I was invited to speak at an undergraduate research conference. Most of the participants in this conference were students at the best engineering colleges in the State. One student who was driving me back and forth recounted a story about the previous year's final exam. One of his papers had a question from a leading text book to which the textbook's answer was wrong. The student was in a dilemma: should he write the wrong answer as given in the textbook or should he write the right answer using his own analytical skills. He decided to do the latter and received a zero on that question. Clearly, as the student had suspected, the examiners were looking at the textbook answer while correcting the examination papers instead of verifying its correctness.

The behaviour of these examiners is a breakdown of institutional morals, with consequences for the skills acquired by students. I say institutional morals, for the failure of these examiners is not a personal failure. At the same

conference I met a whole range of college teachers, all of whom were drafted as examiners at some time or the other. Without exception, they were dedicated individuals who cared about the education and welfare of their students. However, when put in the institutional role of evaluating an anonymous individual, they fail in fulfilling their responsibilities. When some of our best colleges are run in this fashion, is it any wonder that we turn out unskilled engineers and scientists? It, as we are led to expect, there is a vast increase in education at all levels and the regulatory regime is as weak as it is currently, isn't it likely that the trust deficit is only going to increase?

We are all aware of the consequences of ignoring corruption at all jewels of society. While institutional failures in governance are obvious, I think the real problem lies deeper, in the failure of every day institutions that are quite apart from institutions that impinge on our lives only on rare occupying on our lives only on rare occasions. It is true that our lives are made more miserable by government officials demanding bribes for all sorts of things, but what about the everyday lying and cheating and breaking of rules with people who are strangers?

Let me give you an example that many of us have experienced. I prefer buying my fruits and vegetables from roadside vendors rather than chain stores. To the vendor, I am probably an ideal customer, since I do not bargain and I do not take hours choosing the best pieces, instead, letting the vendor do the selecting. The market near my house is quite busy: as a result, most vendors are selling their wares to strangers. It takes a while before a particular vendor realizes that I am a repeat customer, in such a situation trust is crucial. I have a simple rule: if a vendor palms off a bad piece whose defects are obvious, I never go back to that person again it is amazing how often that happens.

In my opinion, the failure of institutional ethics is as much about these little abuses of trust as anything else. Everyday thievery is like roadside trash: if you let it accumulate the whole neighbourhood stinks.

8. What is the meaning of the phrase palms off as used in the passage?
(a) Steals from
(b) Hides away from
(c) Buys quickly
(d) Passes on by deception

9. Why according to the author do people repose trust in Institutions they do not know?
(a) The marketing strategies adopted by these institutions make them trustworthy in the eyes of the public.
(b) Many other people before them have done the same thing thus they feel safe.
(c) People learn from the experiences of their richer counterparts who have availed of the services of these institutions.
(d) They believe that these institutions have the requisite knowledge and will act only in favour of the general public.

10. Which of the following is possibly the most appropriate title for the passage?
(a) Modifications in Institutional Ethics
(b) Little Deceptions Add to Larger Trust-Deficit
(c) India—A Country Without Ethics
(d) Failure of the Government

11. What according to the author happens when there is a breakdown of trust?
A. Less affluent people have to compromise on quality.
B. Our wellbeing is compromised.
C. We pay a higher prices for services.
(a) Only A (b) Only A and C
(c) Only B and C (d) All A, B, and C

12. Why according to the author, do institutional failures in governance not matter on a larger scale?
(a) The general public does not care about the failures of these institutions as they do not feature in their lives at all.

(b) Such institutional failures are almost non-existent and do not occur often to matter to the public.
(c) These intrude in our lives only under exceptional circumstances whereas he everyday lies contribute to much more.
(d) These are a part of every country's problems and are taken with a pinch of salt by the public.

Direction for questions 13–17: ***The sentences given in each question, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the given choices to construct a coherent paragraph.***

13. A. The celebrations of economic recovery in Washington may be as premature as that "Mission Accomplished" banner hung on the USS Abraham Lincoln to hail the end of the Iraq war.
B. Meanwhile, in the real world, the struggles of families and communities continue unabated.
C. Washington responded to the favourable turn in economic news with enthusiasm.
D. The celebrations and high-fives up and down Pennsylvania Avenue are not to be found beyond the Beltway.
E. When the third quarter GDP showed growth of 7.2% and the monthly unemployment rate dipped to 6%, euphoria gripped the US capital.
(a) ACEDB (b) CEDAB
(c) ECABD (d) ECBDA

14. A. The logic was that stripping the plane's paint would lighten it by 360 pounds, thereby reducing air resistance and cutting expenses by $24,000 per plane a year in terms of fuel saving.
B. In a market of soaring expenses and plunging profits, there are two ways to survive—constantly streamline operations and increase revenue even as you strive to keep customers happy.
C. While the trial was limited to only one Boeing, the airline planned to extend the experiment to the rest of the fleet, if it worked.
D. Believe it or not, in November last year, Air Canada decided to call in strippers.
E. While airlines in India haven't yet felt the need to go so far, each one is finding ways to get a bigger bang for the buck.
F. Whether you are a full service airline or low cost carrier, the passenger is looking for a deal.
(a) BDCAEF (b) DCAFBE
(c) BCDFAE (d) DAEFBC

15. A. A whole range of information-based industries and applications has come up creating new sources of employment and earnings.
B. Vision 2020 conceives of India as evolving into an information society and knowledge economy where IT and telecommunications will be the springboard of growth.
C. Already telecom is improving opportunities for people across different social strata.
D. It has improved access in the fields of education, healthcare, governance and business services.
E. An efficient and rapid flow of information is a catalyst for economic and social development.
F. Information and communication technology has caused rapid innovations in other areas of material sciences.
(a) ABCEFD (b) ECABDF
(c) BAEFDC (d) EBCAFD

16. A. Call it the third wave sweeping the Indian media.
B. Now, they are starring in a new role, as suave dealmakers who are in a hurry to strike alliances and agreements.

C. Look around and you will find a host of deals that have been inked or are ready to be finalized.

D. Then the media barons wrested back control from their editors, and turned marketing warriors with the brand as their missile.

E. The first came with those magnificent men in their mahogany chambers who took on the world with their mighty fountain pens.

(a) ACBED (b) CEBDA
(c) CAEBD (d) AEDBC

17. A. The wall does not simply divide Israel from a putative Palestinian state on the basis of the 1967 borders.

B. A chilling omission from the road map is the gigantic 'separation wall' now being built in the West Bank by Israel.

C. It is surrounded by trenches, electric wire and moats; there are watchtowers at regular intervals.

D. It actually takes in new tracts of Palestinian land, sometimes 5 or 6 kms at a stretch.

E. Almost a decade after the end of South African apartheid, this ghastly racist wall is going up with scarcely a peep from Israel's American allies who are going to pay for most of it.

(a) EBCAD (b) BADCE
(c) AEDCB (d) ECADB

18. Board member: As a longtime member of the college's board of trustees, I believe that the board has functioned well in the past because each of its members has a broad range of experience and interest. Thus, if in the future any members are elected primarily to press for a particular policy, such as reducing tuition, the board will function less well.

In drawing the conclusion above, the board member must have been making which of the following assumptions?

(a) The college will suffer financially if the board reduces tuition.

(b) The college will not be able to operate if and when the board functions less well than it does currently.

(c) The board functions well because its members are primarily interested in particular academic policies rather than in financial policies such as the level of tuition.

(d) Each of the people who would be elected to the board primarily to press for a particular policy lacks a broad range of experience or interests.

19. When school administrators translate educational research into a standardized teaching program and mandate its use by teacher, students learn less well than they did before, even though the teachers are the same. The translation by the administrators of theory into prescribed practice must therefore be flawed.

The argument above is based on which of the following assumptions?

(a) Teachers differ in their ability to teach in accordance with standardized programs.

(b) The educational research on which the standardized teaching programs are based is sound.

(c) Researchers should be the ones to translate their own research into teaching programs.

(d) The ways in which teachers choose to implement the programs are ineffective.

20. According to a report issued by the government of a certain country, in 1990 the public sector and the private sector each employed the same number of people. Between 1990 and 1994, according to the government report, total employment decreased in the public sector more than it increased in the private sector.

If, according to governmental figures, the unemployment rate in this country was the same in both 1990 and 1994, which of the following statements must be true about this country?

(a) Fewer people were in the labor force, as counted by the government, in 1994 than in 1990.
(b) The competition for the available work increased between 1990 and 1994.
(c) The government's figures for total employment increased between 1990 and 1994.
(d) The number of people counted by the government as unemployed was the same in 1990 and 1994.

Direction for questions 21–25: ***In each of the questions, a part of the question is underlined. Choose the option that best represents the idea given in the underlined part.***

21. A new proposal has been made to trim the horns from rhinoceroses to discourage animal poachers; though the larger question is <u>whether tourists will continue to visit sanctuaries and see rhinoceroses after their horns are</u> trimmed.

(a) whether tourists will continue to visit sanctuaries and see rhinoceroses after their horns are
(b) whether tourists will continue to visit sanctuaries to see one once their horns are
(c) whether tourists will continue to visit sanctuaries to see rhinoceroses once the animals' horns have been
(d) if tourists will continue to visit sanctuaries and see rhinoceroses once the animals' horns are

22. A recent study of the govt. schools shows that <u>there are now one computer for every 28 students, four times as many than there were</u> four years ago.

(a) there are now one computer for every 28 students, four times as many than there were
(b) there is now one computer for every 28 students, four times as many than there were
(c) there is now one computer for every 28 students, four times as many as there were
(d) every 28 students now have one computer, four times as many than there were

23. A recent editorial condemned the election committee for, first of all, failing to replace faulty voting machines prone to technical snags, and <u>secondarily, for their failure to</u> address allegations of corruption.

(a) secondarily, for their failure to
(b) secondly, for their failure to
(c) secondly, that they failed and did not
(d) second, that they failed to

24. A recent report has found that within the past six years, many medical professionals <u>had elected early retirement rather than face</u> the constant threats of lawsuits.

(a) had elected early retirement rather than face
(b) had elected early retirement instead of facing
(c) have elected retiring early instead of facing
(d) have elected to retire early rather than facing

25. A new documentary on ancient clay deposits has given new evidence <u>supporting the hypothesis of forest fires ignited by a meteorite impact that</u> contributed to the extinction of the dinosaurs and many other creatures some 65 million years ago.

(a) supporting the hypothesis of forest fires ignited by a meteorite impact that
(b) supporting the hypothesis that forest fires ignited by a meteorite impact

(c) that supports the hypothesis of forest fires that were ignited by a meteorite impact and that
(d) in support of the hypothesis that forest fires were ignited by a meteorite impact and that

Direction for questions 26–28: ***For the following questions, substitute the given sentence of phrase with the most suitable word from the given options.***

26. A person who is all-knowing would be termed as being .....
(a) Omnipotent (b) Omnivorous
(c) Omnifarious (d) Omniscient

27. An audacious and an extremely brash person is often described as being .....
(a) Insouciant (b) Avaricious
(c) Temerarious (d) None of these

28. A person who is unbeatable is most definitely .....
(a) Incorrigible (b) Insuperable
(c) Intransigent (d) Inept

Direction for questions 29–30: ***For each of the words below, a contextual usage is provided. Pick the word from the alternatives that is most inappropriate in the given context.***

29. PAROCHIAL: Although it is national newspaper, its reporting is quite parochial in its outlook.
(a) Provincial (b) Restricted
(c) Catastrophic (d) Insular

30. REFURBISH: The interior decorators refurbished the house so well that it had a new look altogether.
(a) Renovate (b) Reiterate
(c) Revamp (d) Redo

Direction for questions 31–35: ***In the following questions four alternatives are given for the idiom/phrase italicized in the sentence. Choose the one that best expresses the meaning of the given idiom/phrase:***

31. I am sure they will *fight tooth and nail* for their rights.
(a) with all their rights
(b) with all their mights
(c) without any weapon
(d) very cowardly

32. His most trusted friend proved to be *a snake in the grass.*
(a) an unreliable person
(b) a hidden enemy
(c) low and mean
(d) cowardly and brutal

33. The captors of the kidnapped boy *kept his family on tenter-hooks*.
(a) an anxious suspense
(b) on constant move
(c) in seething anger
(d) in excited wait

34. I have come to know of your *hole-and-corner methods* of dealing with people.
(a) suspicious (b) secret
(c) servile (d) strict

35. The detective *left no stone unturned* to trace the culprit.
(a) took no pains
(b) did very irrelevant things
(c) restored to illegitimate practices
(d) used all available means

Direction for questions 36–40: ***Read the passage below and solve the questions based on it.***

Giving loans to impoverished women to make ceramics or to farmers to buy milk cows were not seen as great business. Microfinance was an industry championed by antipoverty activists. Today it is on the verge of a revolution, with billions of dollars from big banks, private-equity shops and pension funds pouring in, driving growth of 30–40% this year alone. In 1998, a non-profit microfinance organization in Peru, converted into bank (called Mibanco). This demonstrated that the poor are good risks who repay loans on time and getting them together, not

only chips away at poverty but also turns a profit. The success of Mibanco has piqued the interest of commercial banks, which has previously shunned the country's poor. Now big banks are going after Mibanco's clients with low-rate loans and realizing it takes special know-how to work with the unbanked—are hiring away Mibanco's staff.

But with the emergence of players who are only out for profit, microfinance schemes cold end up milking the poor. This could happen in countries where lenders don't have to disclose interest rates. When a Mexican micro financier went public, revealing its loans had rates of about 86% annually; the Consultative Group to Assist the Poor (CGAP) criticized it for putting shareholders ahead of clients. The pressure of turn a profit also forces microfinance's to change their business models in ways that depart from the industry's core mission: to help poor people lead better lives. Such shifts have caused the average loan size to triple. Moreover smaller loans being costlier to service, a lower percentage of loans go to women because, according to CGAP, with the flood of new large entities there is the risk that a large percentage of cross-border funds go to Latin America and Eastern Europe, the world's most developed microfinance markets. "The poorest of the world's poor who are predominantly in Asia and Africa get left out," says the CEO of the nonprofit Grameen Foundation, which helps develop microfinance institutions.

Segmenting the industry, might be worthwhile if it allows more of the poor to get access to credit. Multinational corporations could take the top microfinance institutions to the next level and the remainder could be the responsibility of development groups and regional banks. Yet making loans to poor people is hardly a poverty cure. Property rights and the rule of law matter too. One cannot over idealize what microfinance alone can do. Most nonprofits started with lending simply because local laws prohibited nonbank from offering deposit accounts. With an increase in competition and marketing efforts, poverty alleviation experts are concerned that people will be talked into loans they wouldn't otherwise want, for example organizations like Mibanco are providing consumer loans. There is nothing wrong with buying TVs and microwaves on credit, but certain markets, like Mexico, have been flooded with loans that have nothing to do with providing capital to aspiring entrepreneurs—just increasing household debt.

36. What does the transformation of Peru's non-profit organization into bank illustrate?

A. To compete with commercial banks, microfinance institutions should convert into banks and offer a wide variety of services.

B. Microfinance institutions turn higher profits as banks since interest rates on loans are at their discretion.

C. The poor prefer to go to large banks rather than NGOs to obtain loans.

(a) None
(b) Both A and B
(c) All A, B and C
(d) Only A

37. Why did most microfinance institutions initially provide only credit services?

(a) They were unable to compete with the interest rates offered on deposits by commercial banks.
(b) They have to operate purely on a non-profit basis.
(c) Government restrictions prevented them from offering additional services.
(d) To ensure the poor have access to modern necessities like microwaves.

38. What is CGAP's fear with respect to new entities providing microfinance?

(a) NGO will be unable to survive in an environment of cut throat competition.
(b) The poor will hesitate to take advantage of credit facilities because of the formalities involved.
(c) The poor in the developed world will be overlooked.
(d) The interests of the most deserving among the poor will be neglected.

39. What is the author's opinion about the competition for customers among micro-financiers?
(a) It benefits the poor by providing them with loans they would have otherwise not had access to.
(b) It is futile since the poor have to pay high rates of interest in case of property loans.
(c) It is not beneficial since firms waste their profits on marketing rather than helping the poor.
(d) None of these

40. Which of the following is/are consequence/s of microfinanciers altering their business models?
A. Larger loan amounts get sanctioned.
B. Debt among the poor has fallen in some countries.
C. Drop in the loans awarded to women.
(a) Both A and B
(b) Both A and C
(c) Only C
(d) All A, B and C

MY PERFORMANCE SHEET IN SECTION TEST 2

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWER KEY

1. (a)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (a)	**8.** (d)
9. (d)	**10.** (b)	**11.** (d)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (d)	**16.** (d)
17. (b)	**18.** (b)	**19.** (b)	**20.** (a)	**21.** (c)	**22.** (c)	**23.** (d)	**24.** (d)
25. (b)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (b)	**31.** (b)	**32.** (a)
33. (b)	**34.** (a)	**35.** (d)	**36.** (d)	**37.** (b)	**38.** (d)	**39.** (d)	**40.** (c)

HINTS AND EXPLANATIONS

1. Note the words carefully ..... *were widely shared by the middle class*. Something shared by the middle class on a large scale ought to be popular and with time, it should become a commonly held position. On the other hand, there is nothing to justify option (d) which talks of the cult spreading to far-off locations as there is nothing to suggest it. Besides, the option is wrong *ab initio* as *however* is not justified here since

there is no contrast between the essence of the last line and the meaning of this option. Hence, option (a) is the answer.

2. Option (a) is a deceptively good answer but upon closer examination, you will discover that it is a naughty one. The passage nowhere hints at this idea of universality of corruption. All it says is that wherever we have humans in positions of power and discretion, corruption will always happen. This is an idea best conveyed by option (b). The rest are either irrelevant or go beyond the scope of the passage.
Hence, option (b) is the answer.

3. The completed sentence should convey a sense of relief over what happens there. The only two such words are happily and mercifully, but the word attract renders the meaning irrelevant. The second blank needs to have a word with a slightly negative connotation.
Hence, option (a) is the answer.

4. We require a word for the second blank which gels with the idea of *repression.* The only such choice is *prudishness.*
Hence, option (b) is the answer.

5. Question ends with a negative word. So, option (c) can be ruled out because of a positive word "boost". If we look at option (a), "trust" doesn't fit in the first blank. Now, between options (b) and (d), option (b) is better because there is no need to "argue" with anyone. So, option (b) is the answer and makes the sentence meaningful.

6. The word in the blank should explain the reason for Ravi's frustration with Mohit. Now, if we look at the answer choices, only option (c) looks appropriate. Words in option (a), (b), and (d) don't make sense with "himself to a deadline". Therefore, option (d) is the answer.

7. Answer would be a word which goes against team work. If we look at the answer choices, only option (a) has such a word 'rigidity' that hinders teamwork and the second word flexibility also fits well in the second blank. Remaining options (b), (c), (d) can be eliminated.

13. If we take option (a), statements AC poses a problem because of the repetition of "Washington". Again, statement B should come before statement D because "meanwhile" changes the tone of the paragraph. Hence, option (b) is ruled out. Option (c) is ruled out because statement B should be in contrast with its preceding sentence. But statements AB does not bring out this contrast. Hence, option (d) is the answer.

14. The opening statement is clearly B, which introduces the idea of the challenge before the industry. Keeping the customer happy in statement B is clearly linked to 'passenger looking for a deal' in statement F. Then, we move to Air Canada's experiment statement D, the logic behind at statement A and its manner of executing its plan statement C. So, statements D AC go together. The para concludes with the comparison with the airlines in India. Hence, option (d) is the answer.

15. Statement E is a general statement and begins the para. Statement B cites the example of how India has identified IT and telecom as springboards of growth. The benefits which telecom and information industries have brought is outlined in statements C and A 'other areas of material sciences' in statement F is listed in statement D. So, statements FD link can't altered. The correct sequence is EBCAFD.
Hence, option (d) is the answer.

16. EDB is a chronological sequence. Hence, option (d) is the answer.

17. An easy one! Only statement B qualifies as an independent sentence. So the paragraph should begin with statement B. Hence, option (b) is the answer.

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SECTION 3

Full Length Test Papers and Analysis

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Mock Aptitude Test (Mock AT) 1

This test paper contains 100 questions.

1. There are 2 sections in this test paper—Section 1 containing 60 questions and Section 2 containing 40 questions.
2. You have 2 hours to complete this test.
3. All questions carry 4 marks each. Each wrong answer will attract a penalty of 1 mark.

Direction for answering the questions are given before each group of questions. Read these directions carefully and answer the questions. Each question has only one correct answer.

SECTION 1

1. A man generated his email password a 4-digit number 'abcd'. Since he didn't log into his account for quite some time, he forgot the password. To recollect the original password again, he writes 9 separate probable passwords:
2186, 5127, 6924, 4351, 5916, 8253, 4521, 6384, 8517
In each of the nine passwords, at least one of the digits a, b, c, and d occurs in the correct position.
What is the value of d?
(a) 1 (b) 3
(c) 4 (d) 6

2. Points A, B, C and D lie on a circle as shown in Fig. 1 with $AB = 4$ units and $BC = 2$ units. AC is the diameter of the circle and $\angle ABD = \angle CBD$. What is the length of BD?

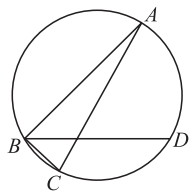


Fig. 1

(a) $\frac{9}{\sqrt{5}}$ (b) $3\sqrt{2}$
(c) 4 (d) $2\sqrt{3}$

3. Three persons start walking together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?
(a) 25 m 20 cm (b) 50 m 40 cm
(c) 75 m 60 cm (d) 100 m 80 cm

4. Set A contains all the even numbers between 2 and 50 inclusive. Set B contains all the even numbers between 102 and 150 inclusive. What is the difference between the sum of elements of set B and that of set A?
(a) 2,500 (b) 1,250
(c) 1,500 (d) 5,000

5. A milk and water mixture contains 5% water. What quantity of pure milk should be added to 10 litres of this mixture to reduce the water concentration to 2%?
(a) 5 litres
(b) 7 litres
(c) 15 litres
(d) Cannot be determined

6. 405 chocolates were distributed equally among students in such a way that the number of chocolates received by each student is 20% of the total number of student. How many chocolates did each student receive?
(a) 9 (b) 15
(c) 18 (d) 45

Direction for questions 7–8: ***Go through the information given below and solve the questions based on it.***

Table 1 presents the data regarding 8th Lok Sabha elections held in 1984 in India. In this table, 10 states are mentioned with the total registered voters and total votes polled in these states.

Table 1

Sl. no.	State	Total registered voters	Total votes polled
1.	Andhra Pradesh	3,35,46,487	2,31,36,116
2.	Arunachal Pradesh	3,19,049	2,40,762
3.	Bihar	4,33,41,090	2,54,84,520
4.	Goa	5,86,656	4,21,257
5.	Gujarat	1,88,43,760	1,09,16,331
6.	Haryana	77,25,946	51,63,799
7.	Himachal Pradesh	23,14,024	14,22,000
8.	J&K	34,00,010	22,58,113
9.	Karnataka	2,11,02,484	1,38,57,272
10.	Kerala	1,42,76,072	1,10,11,029

7. Polling percentage is defined as total votes polled as a percentage of total registered voters. For how many states, polling percentage is more than 70%?
(a) 1 (b) 2
(c) 3 (d) 4

8. Polling percentage is defined as total votes polled as a percentage of total registered voters. Which of the states mentioned have the second best polling percentage?
(a) Arunachal Pradesh
(b) Kerala
(c) Goa
(d) Haryana

9. Consider regular convex polygons of sides n, where $2 < n < 21$. For how many values of n in the given range, ratio of one internal angle to one external angle will be an integer?
(a) 5 (b) 7
(c) 9 (d) None of these

10. The numbers a, b, c are the digits of a three digit number which satisfy $49a + 7b + c = 284$. What is the sum of the digits of the three digit number abc?
(a) 11 (b) 14
(c) 17 (d) 15

11. Three numbers form an arithmetic sequence, the common difference being 11. If the first number is decreased by 6, the second is decreased by 1 and the third is doubled, the resulting numbers are in geometric sequence. How many such arithmetic progressions are possible?
(a) 2 (b) 3
(c) 4 (d) 0

Direction for questions 12 and 13: ***Go through the information given below and solve the questions based on it.***

An Airline Company has divided the potential market into three categories through a market survey for the purpose of implementing a Customer Relationship Management Programme.

The rectangle given in the diagram as shown in Fig. 2 represents Economy Class passengers. The circle represents working Executives and the triangle represents the Indians.

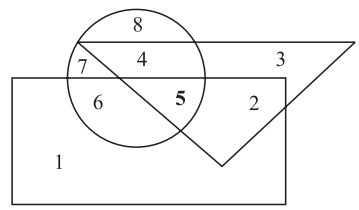


Fig. 2

12. A signature hotel is launching a service for Executives who travel Economy Class but are not Indians. Which segments should it concentrate in Fig. 2?
(a) 7 (b) 6
(c) 5 and 6 (d) None of these

13. A travel agency wishes to target passengers who are Executives and who travel by Economy Class. Which segments in the diagram should it aim?
(a) 6 and 5 (b) 6 and 7
(c) 2, 5 and 8 (d) None of these

14. Given is the graph of quadratic equation $y = ax^2 + bx + c = 0$ (Fig. 3)

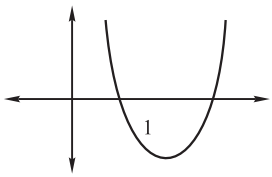


Fig. 3

How many of the following expression/s is/are positive?
ab, ac, b, $a + b + c$, $a - b + c$
(a) 1 (b) 2
(c) 3 (d) 4

15. Rupesh is driving on a straight road with a constant speed. He is XYZ kms away from destination at 2 p.m. X, Y and Z are whole numbers with $X \geq 1$ and $Y = 0$. At 2:18 p.m., he is ZX kms away from his destination and at 3 p.m., he is XZ kms away from his destination. At what time did he arrive at destination?
(a) 3:12 p.m. (b) 3:24 p.m.
(c) 3:30 p.m. (d) 3:36 p.m.

16. Given that $\frac{1025}{1024} = 1.0009765625$. What is the sum of digits of 510?
(a) 36 (b) 40
(c) 44 (d) 52

Direction for questions 17–19: ***Go through the data set given below and solve the questions based on it.***

Table 2 gives the marks obtained by 6 students A, B, C, D, E and F in 6 subjects—History, Science, Geography, English, Hindi and Math. Marks in bracket show total marks in that subject. For example—History (150) shows that total marks in History = 150.

17. How many students have got more than 50% marks in each of the subjects and more than 65% marks in total?
(a) 0 (b) 1
(c) 2 (d) 3

18. What is the difference between the percentage marks obtained by the student B in History and percentage marks obtained by C in Hindi?
(a) 25 (b) 20
(c) 35 (d) 30

Table 2

Marks obtained by 6 students in 6 subjects

Students	History (150)	Science (200)	Geography (150)	English (200)	Hindi (100)	Math (200)
A	75	110	90	140	75	170
B	105	130	75	130	80	140
C	95	105	80	150	90	160
D	85	115	95	125	65	135
E	115	135	110	145	70	125
F	120	160	96	110	55	145

19. For C, what is the difference between the percentage of marks in English and the percentage of marks obtained in total?

(a) 82 (b) $38\frac{1}{3}$

(c) 7 (d) 14

20. In June, the ratio of boys to girls in a school was 3:2. In September, there were 80 fewer boys and 20 fewer girls in the school and the ratio of boys to girls was 7:5. What was the total number of students at the school in June?

(a) 1,200 (b) 1,300

(c) 900 (d) 750

Direction for questions 21 and 22: ***Read the information given below and solve the questions based on it.***

There is a carpenter who sells only tables (with two legs only) or chairs (with four legs only). The top part of table and top part of chair (the wooden plank) are same and one wooden plank is required to configure one chair or one table. Carpenter gets the legs and top wooden plank separately and then assembles it to make either chair or table.

Its also given that profit per table = ₹ 1,000 unit and profit per chair = ₹ 1,500 unit.

21. If there is limited supply of 1,200 legs and top wooden plank, which is more profitable to produce—table or chair to maximize the overall profit?

(a) Table

(b) Chair

(c) (a) or (b)

(d) Cannot be determined

22. There is limited supply of 1,200 legs and top wooden plank. Carpenter takes 4 hours to assemble a table and 3 hours to assemble a chair. If there is limited supply of 1,000 man-hour labor, what is the total maximum profit that can be generated?

(a) 2,40,000 (b) 2,80,000

(c) 4,60,000 (d) None of these

23. What is the value of the following expression:

$$6\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}}\right)$$

(a) –1 (b) 4

(c) 7 (d) 10

24. What is the highest power of 3 in $3^3 \times 4^4 \times 5^5 \times 24^{24}$?

(a) 108 (b) 135

(c) 72 (d) 144

25. In a class of 80 students, 60% of those who like tea also like coffee, and 60% of those who don't like tea also don't like coffee. How many students like only one of the two drinks?

(a) 32 (b) 40

(c) 48 (d) None of these

26. In how many ways 12 persons can be seated on a rectangular table with 4 seats and 2 seats on the longer and shorter side respectively?

(a) 11! (b) 2 × 11!

(c) 6 × 11! (d) 4 × 11!

27. $f(x)$ is a quadratic polynomial such that $f(1) = f(2)$ and $f(4) = 0$. If $f(y) = 0$ and y is not equal to 4, what is the value of y?

(a) 1 (b) –1

(c) 4 (d) –4

Direction for questions 28 and 29: ***Read the information given below and solve the questions based on it.***

A started a business in January, 1985 by investing some amount 'a'. After some months, B joined him investing an amount 'b'. At the end of 1985, they shared the profit equally. Sometimes during 1986, C joined the two of them investing an amount 'c', and at the end of 1986, A, B and C split the year's profit in the ratio 4:6:5. B then left the company. During 1987, D joined the group investing an amount 'd', and at the end of 1987, A, C and D split the year's profit in the ratio 2:3:4. A then left the company, and a year

later, at the end of 1988, C and D split the year's profit in the ratio 1:2.

28. For how many total months was B a part of the business?
(a) 8 (b) 20
(c) 12 (d) 32

29. At the starting of which month in 1986 did C join the business?
(a) March (b) August
(c) June (d) September

30. Four circles of radius 1 cm are arranged in such a way that their centres form a square. A 5th circle, largest in radius, is placed in the space created among four circles. What is the radius of the 5th circle?
(a) $\sqrt{2}$ (b) $\sqrt{2}-1$
(c) $\sqrt{2}+1$ (d) None of these

Direction for question 31–33: ***Read the information given below and solve the questions based on it.***

31. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $(\alpha/2, 2\beta)$ be the roots of the equation $x^2 - qx + r = 0$. What is the value of r?
(a) $2/9\ (p-q)\ (2q-p)$
(b) $2/9\ (q-p)\ (2p-q)$
(c) $2/9\ (q-2p)\ (2q-p)$
(d) $2/9\ (2p-q)\ (2q-p)$

32. Let S be the set of integers x such that:
I. $100 \le x \le 200$
II. x is odd and
III. x is divisible by 3 but not by 7
How many elements does S contain?
(a) 16 (b) 12
(c) 11 (d) 13

33. $ABCD$ is a rectangle. The points P and Q lie on AD and AB respectively. If the triangles PAQ, QBC and PCD all have the same areas and $BQ = 2$, then $AQ = ?$
(a) $1+\sqrt{5}$ (b) $1-\sqrt{5}$
(c) $\sqrt{7}$ (d) $2\sqrt{7}$

Direction for questions 34–40: ***Each question is followed by two statements.***

Mark (a) as the answer—If the question can be answered by one of the statements alone and not by the other.

Mark (b) as the answer—If the question can be answered by using either statement alone.

Mark (c) as the answer—If the question can be answered by using both the statements together, but cannot be using either statement alone.

Mark (d) as the answer—If the question cannot be answered even by using both the statements together.

34. Is $X + Y + Z$ is positive?
Statement 1: $X + Y$ is positive.
Statement 2: $Y + Z$ is positive.

35. Is x an even number?
Statement 1: $4x + 3y$ is an even number.
Statement 2: $3x + 4y$ is an even number.

36. Is the number completely divisible by 99?
Statement 1: The number is divisible by 9 and 11 simultaneously.
Statement 2: If the digits of the number are reversed, the number is divisible by 9 and 11 both.

37. How much did Amit earn in 2006?
Statement 1: Amit earned ₹ 12,000 in 2007 which is 12% more than what he earned in 2006.
Statement 2: Amit and his wife earned ₹ 24,000 in 2007. Amit's wife earned 20% more in 2007 than what she earned in 2006.

38. What is the average marks of the whole class?
Statement 1: Boys are more than twice in number than girls and their average marks is 78%.
Statement 2: Average marks of the girls, who are 20 less in number than boys, is 84%.

39. F and M are father and mother of S, respectively. S has four uncles and three aunts. F has two siblings. The siblings of F

and *M* are unmarried. How many brothers does *M* have?
(a) *F* has two brothers
(b) *M* has five siblings

40. If *A* and *B* run a race, then *A* wins by 60 seconds. If *B* and *C* run the same race, then *B* wins by 30 seconds. Assuming that *C* maintains a uniform speed, what is the time taken by *C* to finish the race?
(a) *A* and *C* run the same race and *A* wins by 375 metres.
(b) The length of the race is 1 km.

Direction for questions 41–44: ***Go through the data set given below and solve the questions based on it.***

Pie charts as shown in Fig. 4(a) and 4(b) present the expenses of Goyal family for two consecutive years 2013 and 2014. In 2012, data given is in percentage and in 2013, data given is in degrees.

Total expenses in 2013 = ₹ 64,000 and total expenses in 2014 = ₹ 96,000.

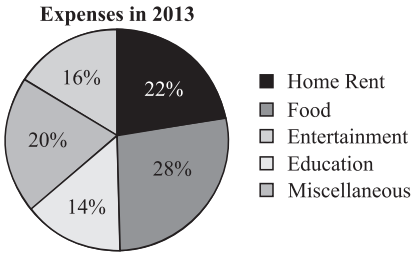


Fig. 4(a)

22% = Home Rent and so on in clock wise direction.

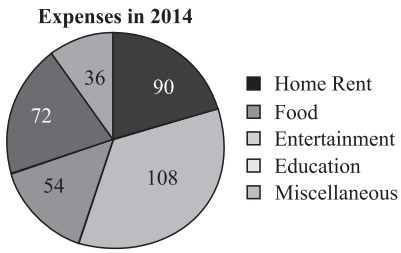


Fig. 4(b)

90 degrees = Home Rent and so on in clock wise direction.

41. For how many sectors, spending in 2014 is less than the spending in 2013?
(a) 1 (b) 2
(c) 3 (d) 0

42. Which sector has seen the largest percentage increase in its spending in 2014 over 2013?
(a) Home rent (b) Food
(c) Education (d) Entertainment

43. What is the percentage growth in the spending on Education in 2014 over 2013?
(a) 54% (b) 60%
(c) 64% (d) 68%

44. Which sector has seen the largest absolute increase in spending in 2014 over 2013?
(a) Home rent (b) Food
(c) Education (d) Entertainment

Direction for questions 45 and 46: ***Fill the missing number after analyzing the logic given to produce the other numbers in the series.***

45. 3, 7, 13, 21, 31, _____
(a) 43 (b) 34
(c) 56 (d) 45

46. 6, 15, 33, 69, _____, 285
(a) 137 (b) 141
(c) 143 (d) 147

47. A band passes around all the wheels so that they can be turned by the driving wheel. When the driving wheel turns in the direction as shown in Fig. 5. Which way will the wheel *B* turn?

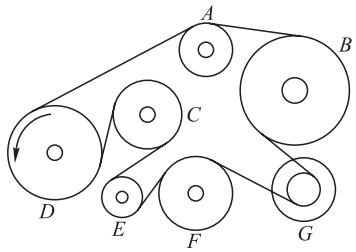


Fig. 5

(a) Clockwise (b) Cannot move
(c) Anti-clockwise (d) Either way

48. Each child in a family has at least 4 brothers and 3 sisters. What is the smallest number of children the family might have?
(a) 7 (b) 8
(c) 9 (d) 10

Direction for questions 49 and 50: ***In Fig. 6, circle stands for "educated", square stands for 'hard working', triangle for 'urban people' and rectangle for 'honest'. The different regions of the diagram are numbered from 1 to 12. Study the diagram carefully and answer the questions*:**

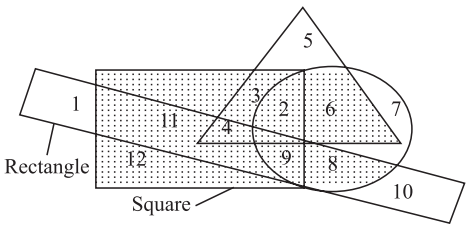


Fig. 6

49. Uneducated urban hard-working and honest people are indicated by:
(a) 3 (b) 11
(c) 9 (d) 4

50. Non-urban educated people who are neither hard-working nor honest are indicated by:
(a) 5 (b) 7
(c) 10 (d) 11

51. In this question insert the missing number at (?).

8	4	9	5
5	7	3	4
3	4	5	8
39	44	60	?

(a) 62 (b) 72
(c) 60 (d) 70

52. In the Sunday bazzar, Jamuna sells her lemons at ₹ 0.50 for two. Her neighbor Seema has a little smaller lemons; she sells hers at ₹ 0.50 for three. After a while, when both ladies have the same number of lemons left, Seema is called away. She asks her neighbor to take care of her goods. To make things simple, Jamuna puts all lemons in one big pile, and starts selling five lemons per one rupee. When Seema returns, at the end of the day, all lemons have been sold. But when they start dividing the money, there appears to be a shortage of ₹ 3.50. Supposing they divide the money equally, how much does Jamuna lose with deal?
(a) ₹ 10.50 (b) ₹ 11.50
(c) ₹ 42.00 (d) ₹ 52.50

53. There are two cups, one containing orange juice and one containing an equal amount of lemonade. One teaspoon of the orange juice is taken and mixed with the lemonade. Then a teaspoon of this mixture is mixed back into the orange juice. Is there more lemonade in the orange juice or more orange juice in the lemonade?
(a) More orange juice in the lemonade
(b) More lemonade in the orange juice
(c) Equal amount of each juice between the two cups
(d) None of the above

Direction for questions 54 and 55: ***Read the information given below and answer the questions that follow*:**

There are five friends—Babu, Om, Qadir, Munu and Ani—each having some marbles with them. Following is given the sum of the marbles with them taken three at a time, making sure no group of three friends is repeated—212, 214, 219, 216, 215, 217, 222, 217, 221, and 219.

It is also known that:

Ani has more marbles than Om but less than Munu.

Babu has the lowest number of marbles.

Average number of marbles with Qadir and Munu is equal to the number of marbles with Ani.

Maximum number of marbles with any of the friends can be 100, however it is not known if anyone of them actually possessed 100 marbles or not?

54. How many marbles are there with Qadir?
(a) 70 (b) 72
(c) 74 (d) None of these

55. What is the average of number of marbles with all of them?
(a) 73.2 (b) 72.4
(c) 71.3 (d) 70.5

56. A certain quantity of 40% solution is replaced with 25% solution such that the new concentration is 35%. What is the fraction of the solution that was replaced?
(a) 1/4 (b) 1/3
(c) 1/2 (d) 2/3

57. My name is AMEET. But my son accidentally types the name by interchanging a pair of letters in my name. What is the probability that despite this interchange, the name remains unchanged?
(a) 5% (b) 10%
(c) 15% (d) 20%

58. Out of 20 surveyed students 8 study math and 7 study both Math and Chemistry. If 10 students do not study either of these subjects, how many students study Chemistry but not Math?
(a) 1 (b) 2
(c) 4 (d) 5

59. If $-2 \le x \le 2$ and $3 \le y \le 8$, which of the following represents the range of all possible values of $y - x$?
(a) $5 \le y - x \le 6$ (b) $1 \le y - x \le 5$
(c) $1 \le y - x \le 6$ (d) $1 \le y - x \le 10$

60. Neha crosses a toll bridge twice everyday—one for coming to her office and then in the evening going back to her home. A person is required to pay the toll tax everytime one crosses the bridge. There are two schemes for the toll tax: Scheme A : ₹ 13/month + ₹ 0.2/time, Scheme B: ₹ 0.75/time. How many days at least she passes the bridge in a month so that *A* becomes more economical than *B*?
(a) 11 (b) 12
(c) 13 (d) 14

SECTION 2

Direction for questions 61–63: *Go through the passage below and answer the questions that follow*:

This confusion concerns nothing less than the concept of socialism itself. It may mean, and is often used to describe, merely the ideals of social justice, greater equality and security which are the ultimate aims of socialism. But it means also the particular method by which most socialists hope to attain these ends and which many competent people regard as the only methods by which they can be fully and quickly attained. In this sense socialism means the abolition of private enterprise, of private ownership of the means of production, and the creation of a system of planned economy in which the entrepreneur working for profit is replaced by a central planning body.

There are many people who call themselves socialists although they care only about the first, who fervently believe in those ultimate aims of socialism but neither care nor understand how they can be achieved, and who are merely certain they must be achieved, whatever the cost. But to nearly all those to whom socialism is not merely a hope but an object of practical politics, the characteristic methods of modern socialism are as essential as the ends themselves. Most people, on the other hand, who value the ultimate ends of socialism no less than the socialists, refuse to support socialism because of the dangers of other values they see in the methods proposed by the socialists. The dispute about socialism has thus become largely a dispute about means and not about ends-although the question whether the different ends of socialism can be simultaneously achieved is also involved.

This would be enough to create confusion. And the confusion has been further increased by the common practice of denying that those who repudiate the means value the ends. But this is not all. The situation is still more complicated by the fact that the same means, the economic planning which is the prime instrument for socialist reform, can be used for many other purposes. We must centrally direct economic activity if we want to make the distribution of income conform to current ideas of social justice. Planning, therefore, is wanted by all those who demand that production for use be substituted for production for profit. But such planning is no less indispensable if the distribution of incomes is to be, regulated in a way which to us appears to be the opposite of just. Whether we should wish that more of the good things of this world should go to some racial elite, the Nordic men, or the members of a party or an aristocracy, the methods which we shall have to employ are the same as those which could ensure an equalitarian distribution.

It may, perhaps, seem unfair to use the term socialism to describe its methods rather than its aims, to use for a particular method a term which for many people stand for an ultimate ideal. It is probably preferable to describe the methods which can be used for a great variety of ends as collectivism and to regard socialism as a species of that genus. Yet, although to most socialists only one species of collectivism will represent true socialism, it must always be remembered that socialism is a species of collectivism and that therefore everything which is true for collectivism as such must apply also to socialism. Nearly all the points which are disputed between socialists and liberals concerns the methods common to all forms of collectivism and not the particular ends for which the socialists want to use them; and all the consequences with which we shall be concerned in this book follow from the methods of collectivism irrespective of the ends for which they are used. It must also not be forgotten that socialism is not only by far the most important species of collectivism or planning; but that it is socialism which has persuaded liberal-minded people to submit once more to that regimentation of economic life which they had overthrown because, in the words of Adam Smith, it puts governments in a position where to support themselves they are obliged to be oppressive and tyrannical.

61. According to the author, those who agree to the aims of socialism would:
(a) refuse to support socialism because of possible consequences of the methods advocated to achieve those ends.
(b) agree on the collectivization and centralization of economic activities since both require strong and powerful governments.
(c) have disagreed if only they had known that the only way to achieve the equalitarian ends of socialism required centralization and collectivization of economic activities.
(d) explicitly advocate strict control of economic and social life always since it is a matter of practical politics for them.

62. From the passage, it may be inferred that the author is a supporter of:
(a) the ideals of socialism and does not care much about the means required to achieve them.
(b) ends as portrayed by supporters of socialism but is skeptical of the efficacy of the means advocated to reach those ends.
(c) the ends of socialism, since the means required to achieve those ends always involve collectivization and centralization.
(d) all the possible means of collectivization as they are the tools which can be used to achieve equalitarian distribution in society.

63. The statement that the author of the passage would support the most is:
(a) The development of society, if left to individuals, would be impossible

as each individual would exploit opportunities for plundering and exploiting one's fellow citizens.

(b) The development of society is best done by individuals working for selfish interests within free economic environment that collectively ends up improving the condition of society since the outcomes of each action of individuals are shared by the members of the society.

(c) In the absence of a strong government, society would degenerate into anarchy as each individual tries to achieve success at the cost of one's fellow citizens.

(d) Individuals would create mechanisms to put a system in place that would have rules and regulations so that the society does not break down into chaos.

64. Which of the following choices most accurately captures the meaning of the word 'Crucible' as used in the sentence below?
General Charles Krulak, the Commandant, introduced "The Crucible" to Marine training—an incredibly grueling fifty-four straight hours of live fire exercises, long marches, and sleep deprivation at the end of basic training. After climbing the final hill in this test, recruits are presented with the eagle, globe, and anchor emblem.

(a) Cauldron (b) Nightmare
(c) Albatross (d) Victory

65. Which of the following choices will be most opposite to the word 'Juvenile' as used in the sentence below?
Juvenile delinquents and depressive or suicidal psychiatric patients show orphanhood rates similar to those of the eminent public figures.

(a) Puerile (b) Naïve
(c) Jejune (d) Tyke

66. "The correlation between technological intensity and diversification may be spurious, as both may be related to selling to the government or to other dominant customers". In the above sentence, spurious may refer to all of the following but:

(a) counterfeit (b) questionable
(c) contrived (d) forged

Direction for questions 67–69: ***Analyze the statements given and answer the questions that follow*:**

67. Prediction, the hallmark of natural sciences, appears to have been possible by reducing phenomena to mathematical expressions. Some social scientists also want the power to predict accurately and assume they ought to perform the same reduction. But this would be a mistake, it would neglect data that are not easily mathematised and thereby would only distort the social phenomena. Which one of the following most accurately expresses the main conclusion of the argument?

(a) The social sciences do not have as much predictive power as the natural sciences.

(b) Mathematics plays a more important role in the natural sciences than it does in the social sciences.

(c) Phenomena in the social sciences should not be reduced to mathematical formulae.

(d) There is a need in the social sciences to improve the ability to predict.

68. Chandra: Hailey's Comet, now in a part of its orbit relatively far from the Sun, recently flared brightly enough to be seen by telescope on earth. This has never happened before, so such a flare must be highly unusual.
Suzya: Nonsense. Usually no one bothers to observe comets when they are so far from the Sun. This flare was observed only because an observatory was tracking Hailey's Comet very carefully. Surya challenges Chandra's reasoning by

(a) offering an alternative explanation for the evidence Chandra seeks to explain.
(b) drawing attention to an inconsistency between two of Chandra's claims.
(c) presenting evidence that directly contradicts Chandra's evidence.
(d) pointing out that Chandra's use of the term "observed is excessively vague.

69. Most antidepressant drugs cause weight gain. While dieting can help reduce the amount of weight gained as one takes such antidepressants, some weight gain is unlikely to be preventable. The information above most strongly supports which one of the following?
(a) A physician should not prescribe any antidepressant drug for an overweight patient.
(b) People who are trying to lose weight should not ask their doctors for an antidepressant drug.
(c) At least some, patients gain weight as a result of taking antidepressant drugs.
(d) All patients taking antidepressant drugs should diet to maintain their weight.

Direction for questions 70 and 71: ***In each question, there are five sentences. Each sentence has a pair of words that are Italicized and highlighted. From the Italicized and highlighted words, select the most appropriate words (A or B) to form correct sentences. The sentences are followed by options that indicate the words, which may be selected to correctly complete the set of sentences. From the options given, choose the most appropriate one.***

70. I. The delectable delicacies at the fund-raising dinner were a ***gourmand's (A)/gourmet's (B)*** delight.
II. The ***discriminatory (A)/discriminating (B)*** attitude of the white rulers alienated the populace and led to their huge unpopularity.
III. The area was ***depopulated (A)/unpopulated (B)***, so as to make way for holding the massive rally.
IV. Being always caught up with the sad realties of real life is an ***impractical (A)/impracticable (B)*** thing to do.
V. The ***respectable (A)/respectful (B)*** junior endeared herself to all with her kind demeanor.
(a) BAAAB (b) BBAAA
(c) BAABB (d) BBAAB

71. I. There was a global ***building up (A)/build up (B)*** of arms and ammunition in the Cold War days.
II. The ***global (A)/globalizing (B)*** economy of India cannot afford to be left behind in the current race for economic integration worldwide.
III. It was a ***calculative (A)/calculated (B)*** move to undermine his authority by not so-obvious means.
IV. The marital counselor was in no doubt that the wife was quite ***cool (A)/cold (B)*** to any kind of conciliatory gesture.
V. You should include some rest breaks in your time-table that you feel ***rested (A)/restive (B)*** in the afternoon shift.
(a) BABBA (b) BBBBB
(c) BBBBA (d) BBABA

Direction for questions 72 and 73: ***Each of the following questions has a sentence with two blanks. Given below each question are four pairs of words. Choose the pair that best completes the sentence.***

72. _____, the scope and depth of the exhibition is such that a visitor doesn't feel _____ by the inevitable gift shop and with a populist bent, it wisely highlights not only the beauty and intelligence of the New Kingdom-era Egyptians, but also its considerable weirdness.
(a) Happily, oppressed
(b) Surprisingly, overwhelmed
(c) Naturally, confined
(d) Mercifully, attracted

73. Since the encryption keys rely on familiar groupings—passport number, driving license number, social-security number as well as place and date of birth—they tend to be _____ sequences with a fair degree of _____.
(a) familiar, familiarity
(b) common, uniqueness
(c) structured, predictability
(d) complex, irascibility

Direction for questions 74–78: ***For each question below, determine the relationship between the pair of words in the question, and then select, from the answer options, the pair of words having a similar relationship to the first pair.***

74. Aviary : Birds ::
(a) Zoo : Tigers
(b) Bank : Money
(c) Post-office : Letters
(d) Arboretum : Plants

75. Convict : Imprisonment ::
(a) Student : School
(b) Exile : Banishment
(c) Delinquent : Orphanage
(d) Prisoner : Court

76. Water : Flood ::
(a) Rain : River (b) Wind : Sleet
(c) Snow : Blizzar (d) Ice : Floe

77. Penitent : Obdurate ::
(a) Pensive : Thoughtless
(b) Vacuous : Empty
(c) Cumulative : Aggressive
(d) Happy : Ecstatic

78. Fortitude : Resolution ::
(a) Timidity : Cowardice
(b) Heroics : Stoicism
(c) Medal : Bravery
(d) Poem : Poet

79. I recently returned from a trip. Today is Tuesday. I returned three days before the day after the day before tomorrow. On which day did I return?
(a) Monday (b) Tuesday
(c) Sunday (d) Friday

Direction for questions 80–83: ***Go through the passage below and answer the questions that follow*:**

Of all our many thousands of genes, the ones involved in our behaviour are among the most controversial. The search for these genes has already prompted panic. Many fear that genetic arguments might be used to excuse criminal acts or justify divorce. Despite these worries, researchers are pressing ahead and finding some thought-provoking links between our genetic code and our personality.

If you apply for a job you'll be asked to list your personal qualities. Employers are assuming that your personality is to an extent fixed and won't change much from one year to the next. Most of us can relate to that idea, but where does our personality come from? Is it in our genes, or is it created more by the circumstances of our childhood?

Of course the answer is both. There are bound to be genes which influence our behaviour simply because our brain and the chemicals that operate within it are made by genes. But trying to find any one of the hundreds of genes involved is notoriously difficult. The genetics of behaviour is complex because personalities are complex. Scientists are only just beginning to have any success understanding how genes influence behaviour.

Some people like their kicks to be fast and frequent. They are thrill-seekers who are easily bored. Whether they get their thrills from sleeping with lots of partners, taking drugs or indulging in extreme sports the basic need is the same—for adventure. Others like to stay at home and put the kettle on. They are fully satisfied just keeping life simple and sticking to a familiar routine. There is now known to be a "thrill-seeking" gene which has an influence on this kind of variation in personality. The protein it produces receives signals from a chemical in

the brain called Dopamine. It turns out that there are two possible forms of the gene, one which leads to high levels of dopamine in certain parts of the brain and another which results in much lower levels.

So what does Dopamine do? It has been described as a motivation chemical....too little and a person lacks the initiative to try new things, too much and they become excessive thrill-seekers. When researchers looked at this gene in different personality types they did find a link between the gene which causes higher levels of Dopamine and a thrill-seeking behaviour.

But this gene is only the beginning of the story. It is believed to account for no more than 4% of thrill-seeking behaviour. Up to ten other genes are expected to be involved in the same process. Every behaviour is expected to be controlled by a large number of genes working together.

Are violent aggressive personality types simply acting according to their genes? In Holland a family has been found where three generations of men have been criminals. They were discovered to have an extremely rare gene which affected their brain chemistry. It does seem to explain why men in this family were more likely to fall into a life of crime but the gene is so rare it could not be extended to most other criminals.

A genetic argument has already been put forward in one American trial of a death row convict accused of murder. The potentially explosive social implications of this kind of research has led to some scientists to call for it to be stopped.

Scientists are finding genetic influences are at work even in very complex social behaviours. There may be a genetic influence on whether or not our marriages last. It has been discovered that identical twins were more likely to follow the same patterns of divorce than non-identical twins. But genes don't seem to influence whether we choose to get married in the first place. It's been suggested that the genetic influence on divorce is related to other factors which have a genetic component such as drug abuse, depression and alcoholism.

Studying any kind of gene which might influence the way we behave is proving to be highly controversial. From the gay gene to those linked with criminal behaviour there's a fear that suggests that people are "born that way" could be used to incite intolerance... or excuse anti-social acts.

80. If we come across 100 cases of persons seeking thrill-based experience, it can be concluded that:
(a) All of them are due to a hyperactive dopamine level.
(b) A significant number among them are biochemically dependent on dopamine.
(c) Some of them may be so because of the dopamine levels in their brains.
(d) High dopamine levels, coupled with other genetic influences are responsible for such thrill-seeking behaviour in humans.

81. A suitable title for the above passage can be:
(a) Behaviour: Is It All In Genes?
(b) Games: The Real Culprits
(c) It's In My Genes
(d) Behaviour: The Ever-Present Dilemma

82. The study of gene-based behaviour is rooted in controversy because:
(a) There is no agreement on the research methods.
(b) Of multiplicity of explanations available to answer relevant questions.
(c) The findings have the potential of being misused to justify anti-social or intolerant behaviour.
(d) It is a very costly business and the results may not justify the money spent on it.

83. Primarily, the author wants to:
(a) Present a case for putting a stop to all research work trying to link behaviour with genes.

(b) Present some findings and the potential problem areas in an upcoming field of research.
(c) Decry the findings of same new research by presenting counter evidence.
(d) Convince us of the desirability of conducting such research.

Direction for questions 84–86: ***In each of the following questions, there is one underlined statement. Choose the best way of presenting that statement from the given options.***

84. My dad longs for a Landcruiser, but this is not true for me, because I've always wanted a Pazero.
(a) But this is not true for me, because I've always
(b) But I have always
(c) As opposed to me, who has always
(d) I have always

85. The circle of oak trees in my grandmother's back yard remind me of fairy tales and thoughts of olden times.
(a) remind me of fairy tales and thoughts of olden times.
(b) remind me of fairy tales, and olden times are thought of.
(c) reminds me of fairy tales, which caused a thought about olden times.
(d) reminds me of fairy tales and olden times.

86. Trying not to alienate either side in the debate, the commission focused on the condemned buildings less than they did on the proposed new parkland.
(a) commission focused on the condemned buildings less than they did on
(b) commission's focus was less on the condemned buildings than it was on
(c) commissioners focused less on the condemned buildings than on
(d) commission focused on certain things, and this focus being placed less on the condemned buildings than about

87. From the given choices, choose the best antonym for the word *Restive*.
(a) Patient (b) Rigorous
(c) Active (d) Cautious

88. From the given choices, choose the best synonym for the word *Mollify*.
(a) Mitigate (b) Engender
(c) Mollify (d) Satiate

Direction for questions 89–93: ***A number of sentences are given below which, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the five given choices to construct a coherent paragraph.***

89. A. Gilad was to pocket $ 3 m if the project got off the ground.
B. More controversially, Mr Appel hired Mr Sharon's farmer son, Gilad, who had no experience in tourism, at a huge salary, ostensibly to help with the project.
C. The main charge concerns a tourism project that Mr Appel tried to launch in 1999–2000 on a Greek island.
D. But he also allegedly provided each of them with logistical help in primary-election campaigns in this period.
E. He sought the help of the then foreign minister, Mr Sharon, and the then Mayor of Jerusalem, Mr Olmert, to persuade Greek officials to give the go-ahead.
(a) CEDBA (b) ECBDA
(c) CADBE (d) BDEAC

90. A. This would let the Kurds keep a wide measure of autonomy in the three provinces north of Kirkuk, but not Kirkuk itself, pending a new Iraqi constitution due to come into force next year.
B. He favours a return to the status quo that prevailed before the American invasion.

C. The Kurds suspect Mr Bremer of betraying their war-time alliance by seeking to re-impose rule from Baghdad.
D. Down in Baghdad, Paul Bremer, Iraq's American proconsul, is in a bind.
E. But that, at present, satisfies no one—and all sides blame the Americans.
(a) DAEBC (b) DBAEC
(c) CBADE (d) CBDAE

91. A. This may seem unlikely as southern Sudan is the scene of Africa's longest-burning civil war.
B. If Jerome K Jerome were alive today, he would be proud.
C. Over a century after he wrote it, There Men in a Boat, his quintessentially English comic novel about accident-prone Victorian gentlemen paddling down the river Thames, is a bestseller in southern Sudan.
D. How could they relate to a comedy about chaps in red-and-orange blazers sculling to Hampton Court and getting lost in the hedge maze there?
E. Its people have for decades lived in fear of death or enslavement at the hands of mounted militiamen.
(a) DCAEB (b) CBDAE
(c) BCEAD (d) BCAED

92. A. "Never been to school," he explained.
B. There are few jobs in Liberia, and the ex-fighters are among the least employable people anywhere.
C. This muscular 15-year-old can wield an AK-47 with considerable expertise, but he cannot read, write, or even spell his own name.
D. Even if the gunmen are disarmed, rehabilitating them will be hard.
E. Take Boima Dagosary.
(a) EBDAC (b) DAEBC
(c) DBECA (d) BECAD

93. A. Cases can be heard up to three times before final sentencing.
B. Perhaps oddly, the bill does little to tackle the biggest defect in Italian justice: its slowness.
C. Might the hope be to make them more susceptible to government control?
D. Likewise, the bill proposes a more hierarchical structure for prosecutors.
E. Might this be because the prime minister has been a prime beneficiary of delay, several charges against him having run beyond a statute of limitations?
(a) EACDB (b) BAEDC
(c) ACDBE (d) BDCAE

Direction for questions 94–96: ***Each question is a logical sequence of statements with a missing link, the location of which is shown parenthetically [(.....)]. From the four options available, choose the one that best fits into the sequence.***

94. After buying a few thousand books and a building, Booth spread the word that Hay-on-Wye would be the used-book capital of Britain. Over the years, he has acquired eight properties to turn into bookstores, which he filled with purchases made on expeditions to Britain and to the continent and North America. By the time Anselot made his first visit to Hay in 1981, Booth had been in successful operation for 20 years. (.....) There were now 15 bookstores in Hay, with over half a million volumes in stock. All of these books were primarily meant to be treasure of the bygone days and hence were priced at astonishingly high value.
(a) Other merchants had followed Booth's lead.
(b) A considerable number of used books were generated out of the scheme.
(c) Particular references to past authors and their times generated even more interest.
(d) The business grew at a small pace initially.

95. The Indian legal system is weighted down with the demands made on it. Not only do the courts have to deal with the daily law-and-order problems, but they also have to resolve disputes related to matters of religious faith, probity of politician and bureaucrats and political questions like the dismissals of governments. The result of all this is that the system is so tardy and the citizens don't get justice readily. The absence of speedy justice has had a two-fold effect on society (.....).

(a) It has made the law abiders, the public-spirited and the good Samaritans despondent and encouraged the vested interests of wrongdoers and lawbreakers.
(b) It has made the police, the lawyers and the judiciary vexed.
(c) It has made people lose faith in the bar and the bench.
(d) It has established the necessity for the training of more lawyers and the appointment of more judges.

96. How do you do it? Your colleagues, neighbours, family and friends, how do they all do it? "I follow my nose," says Dan Rhodes, author of Gold, "I am always on the hunt for the next book that's going to rock my world … my favorite thing is still going into a shop and coming out with something I'd never heard of." But if you stand in any bookstore, you're unlikely to see many people using their noses, they just head straight for the "new" Salman Rushdie or the "latest" Chetan Bhagat or the "most recent" Shobhaa De or the "new bestseller" from Paulo Coelho: (.....).

(a) Because they believe that books can change life.
(b) May be that is the Catcher effect-most of us had read Catcher in the Rye and Salinger's other books in our late teens.
(c) It doesn't seem to work that way now.
(d) It's a matter of judging every book by its author.

Direction for questions 96–98: ***In each of the questions, there is a word given which is used in four different ways, numbered from option (a) to option (d). Choose the option in which the usage of the word is incorrect or inappropriate.***

96. Cite:

(a) The most commonly cited reason for retention problems was a perceived lack of career progression.
(b) In our terms, some of the old testament passages cited are clearly taken out of context.
(c) Cited as a major factor that contributes to global warming
(d) All are correct

97. Help:

(a) This syrup will help your cold.
(b) I can't help the colour of my skin.
(c) Ranjit may help himself with the beer in the fridge.
(d) Do you really expect me to help you out with cash?

98. Reason:

(a) Your stand is beyond all reason.
(b) Has she given you any reason for her resignation?
(c) There is little reason in your pompous advice.
(d) How do you deal with a friend who doesn't listen to a reason?

Direction for questions 99 and 100: ***For the word given at the top of each table should match the dictionary definitions on the left (A, B, C, D) with their corresponding usage on the right (E, F, G, H). Out of the four possibilities given in the boxes below the table, select the one that has all the definitions and their usages correctly matched.***

99. Measure:

	Dictionary definition		Usage
A	Size or quantity found by measuring	E	A measure was instituted to prevent outsiders from entering the campus.
B	Vessel of standard capacity	F	Sheila was asked to measure each item that was delivered.
C	Suitable action	G	The measure of the cricket pitch was 22 yards.
D	Ascertain extent or quantity	H	Ramesh used a measure to take out one litre of oil.

	A	B	C	D
(a)	H	F	E	G
(b)	G	E	F	H
(c)	G	H	E	F
(d)	F	H	E	G

100. Bound:

	Dictionary definition		Usage
A	Obliged, constrained	E	Dinesh felt bound to walk out when the discussion turned to kickbacks.
B	Limiting value	F	Buffeted by contradictory forces he was bound to lose his mind.
C	Move in a specified direction	G	Vidya's story strains the bounds of credulity.
D	Destined or certain to be	H	Bound for a career in law, Jyoti was reluctant to study Milton.

	A	B	C	D
(a)	F	H	G	E
(b)	E	G	H	F
(c)	E	H	F	G
(d)	F	G	E	H

ANSWER KEY

Section 1

1. (d)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (a)
9. (c)	**10.** (b)	**11.** (a)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)
17. (d)	**18.** (b)	**19.** (c)	**20.** (b)	**21.** (a)	**22.** (c)	**23.** (b)	**24.** (b)
25. (a)	**26.** (c)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (b)	**31.** (d)	**32.** (d)
33. (a)	**34.** (d)	**35.** (d)	**36.** (b)	**37.** (a)	**38.** (d)	**39.** (a)	**40.** (d)
41. (a)	**42.** (b)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (c)
49. (d)	**50.** (b)	**51.** (b)	**52.** (a)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (b)
57. (b)	**58.** (b)	**59.** (d)	**60.** (b)				

Section 2

61. (a)	**62.** (d)	**63.** (b)	**64.** (a)	**65.** (d)	**66.** (d)	**67.** (a)	**68.** (a)
69. (d)	**70.** (d)	**71.** (c)	**72.** (a)	**73.** (c)	**74.** (d)	**75.** (b)	**76.** (c)
77. (a)	**78.** (a)	**79.** (c)	**80.** (d)	**81.** (a)	**82.** (c)	**83.** (b)	**84.** (a)
85. (d)	**86.** (c)	**87.** (a)	**88.** (a)	**89.** (a)	**90.** (b)	**91.** (d)	**92.** (c)
93. (b)	**94.** (a)	**95.** (a)	**96.** (d)	**97.** (c)	**98.** (d)	**99.** (c)	**100.** (b)

MY PERFORMANCE SHEET (MOCK AT 1)

	Total Qs	Attempts	Right	Wrong	Marks	% Accuracy
Section 1 (Q1 to Q60)	60					
Section 2 (Q61 to Q100)	40					
Overall	100					

(Every correct response: +4 marks, every wrong response: –1 mark (minus 1).

Key Learnings from this test:

1. .....
.....

2. .....
.....

HINTS AND EXPLANATIONS

Section 1

1. In each of the 9 passwords, one of the digits a, b , c and d appears in the correct position. As there are exactly 4 possibilities for a correct digit in a correct position, there must be at least 3 passwords that contain one of a, b, c and d in a correct position. As 2 is the only digit that occurs 3 times in the same position and this happens in the third position, we deduce that c = 2. Of the 6 passwords that do not have 2 in the third position, there are no more than two occurrences of the same digits in the same position. It follows that each of a, b and d must occur in its correct position exactly twice among these 6 passwords. This gives d = 6.

Hence, option (d) is the answer.

2. Since AC is the diameter,

$$AC^2 = BC^2 + AB^2 = 2\sqrt{5}$$

Following is the Fig. 7:

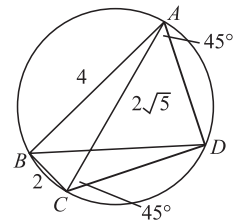


Fig. 7

Now, chord CD subtends

$$\angle CBD = \angle CAD = 450$$

In right angled Isosceles triangle ACD,

$$\text{Sin } 45° = \frac{AD}{AC} = \frac{AD}{2\sqrt{5}}$$

Hence, $AD = \sqrt{10} = CD$

Using Ptolemy's theorem of cyclic quadrilateral,

$AB \times CD + AD \times BC = AC \times BD$

Or, $4 \cdot \sqrt{10} + \sqrt{10} \cdot 2$

$= 2\sqrt{5} \cdot BD$

So, $BD = 3\sqrt{2}$

Hence, option (b) is the answer.

3. *LCM* of 40, 42, 45 = 23 × 32 × 51 × 71 = 2,520 cm = 25.2 m
Hence, option (a) is the answer.

4. Option (a) is the answer.

5. Quantity of water in 10 litres = 5% of 10 litres = 0.5 litres.
Let x litres of pure milk be added.

Then, $\frac{0.5}{10} + x = \frac{2}{100}$

$\Rightarrow$ $2x = 30$

So $x = 15$

Hence, option (c) is the answer.

6. Let the total number of student be x.
Then, $x \times (20\% \text{ of } x) = 405$

or, $\frac{x^2}{5} = 405$

or, $x^2 = 2{,}025$

Hence, $x = 45$

Hence, number of chocolates received by each student = 20% of 45 = 9.
Hence, option (a) is the answer.

7. Polling percentage is more than 70% for the following states = Arunachal Pradesh, Goa and Kerala.
Hence, option (c) is the answer.

8. Arunachal Pradesh 0.75
Kerala 0.77
Kerala has best polling percentage. 2nd best is Arunachal Pradesh.
Hence, option (a) is the answer.

9. $(2n - 4)$ should be divisible by 4. This is possible for n = All the even values = All the even values from 4 to 20 = 9 values.
Hence, option (c) is the answer.

10. $49a + 7b + c = 284$

$\Rightarrow$ $7(7a + b) + c = 7\,(40) + 4$

or, $c = 4$ and $7a + b = 40$

$\Rightarrow$ $a = b = 5$

$100a + 10b + c = 100 \times 5 + 10 \times 5 + 4$
$= 554$

Hence, option (b) is the answer.

11. Let the three terms of an *AP* be $(a - 11)$, a, $(a + 11)$. Then, the terms of *GP* will be $(a - 17)$, $(a - 1)$, $2(a + 11)$. According to the question and properties of *GP*, $(a - 1)^2 = (a - 17)\,2(a + 11)$
This gives us a quadratic equation and consequently we get two values of a. Hence, two different *GP* and *AP*. The two *AP* are 14, 25, 36 and –26, –15, –4.
Hence, option (a) is the answer.

12. Option (b) is the answer.

13. Option (a) is the answer.

14. Curve is upward sloping, hence $a > 0$. Since both the roots are positive, hence, a and c are positive and b is negative. Also $f(1)$ is negative and $f(-1)$ is positive.
Hence, ac and $a - b + c$ are positive.
Hence, option (b) is the answer.

15. At 2:00, he is $100X + Z$ kms from destination. In 18 minutes he travels $100X + Z = (10Z + X) = 99X - 9Z$ kms. So he has travelled at a rate of $11X - Z$ kms every 2 minutes. In 60 minutes he travels $100X + Z - (10X + Z) = 90X$ kms. So he has travelled at a rate of $3X$ kms every 2 minutes. Thus, $11X - Z = 3X$, so that $Z = 8X$. Since X and Z are digits with $X \geq 1$, we get $X = 1$ and $Z = 8$. Hence, at 2:00 he was 108 kms from destination and he travelled at a constant speed of 3 kms each 2 minutes. It follows that it took her 36 × 2 = 72 minutes to reach destination.
Hence, option (a) is the answer.

16. $\frac{1025}{1024} = \frac{2^{10} + 1}{2^{10}}$

$= 1 + \frac{1}{2^{10}}$

$= 1.0009765625$ (Given)

So, $\frac{1}{2^{10}} = 0.0009765625$

$= 9765625\ (10^{-10})$

Hence, $5^{10} = 9765625$

So, sum of digits = 40

Alternatively, this question could have been done by finding the value of 5^{10} through actual calculation.
Hence, option (b) is the asnwer.

17. C, E and F are the students. [We have to consider more than 50%].
Hence, option (d) is the answer.

18. $B \rightarrow \frac{105}{150} \times 100 = 70\%$

$C \rightarrow = 90\%$

Reqd. difference $= 90 - 70 = 20$

Hence, option (b) is the answer.

19. Percentage of marks of English

$$= \frac{105}{200} \cdot 100\% = 75\%$$

Average percentage of marks

$$= \frac{680}{1000} \cdot 100\% = 68\%$$

Therefore required percentage

$$= 75\% - 68\% = 7\%$$

Hence, option (c) is the answer.

20. Suppose that there were $3k$ boys and $2k$ girls in the school in June, for some positive integer k.
In September, there were thus $3k - 80$ boys and $2k - 20$ girls in the school. Since, the new ratio is 7:5, then $\left(\frac{3k-80}{2k-20}\right) = \frac{7}{5}$

Therefore, the total number of the students in the school in June was $3k + 2k = 5k = 5$ (260) = 1,300
Hence, option (b) is the answer.

21. In this situation, for any limited supply of legs, it is more profitable to make table (if at all production to be made). We can simply see it through the concepts of average that in case of table, profit per leg = ₹ 500, and for chair profit per leg = ₹ 375/leg. We are not considering the top wooden plank because no. of unit required for each table and chair is same = 1 top wooden plank.
Hence, option (a) is the right answer.

22. Assume that 'x' table and 'y' chairs are to be made.
Equation for legs
$\Rightarrow \quad 2x + 4y \leq 1{,}200$
(we are taking '$\leq$' and not '=' sign because we may not be using all the units supplied)
Equation for man-hour labor
$\Rightarrow \quad 4x + 3y \leq 1{,}000$
(we are taking '$\leq$' and not '=' sign because we may not be using all the units supplied)
Total profit $= 1000x + 1500y$
Objective is to maximize total profit
$\Rightarrow$ maximize $(1000x + 1500y)$
In this question, we have only one pair of equation formed from the constraints *viz.*, of legs and man-hour labor.

$2x + 4y = 1200$ (i)

$4x + 3y = 1000$ (ii)

Solving these two equations,
we get $y = 280, x = 40$
Hence, carpenter should make 40 tables and 280 chairs to maximize his profit.
Hence, total profit

$= 1000x + 1500y$

$= 1000 \times 40 + 1500 \times 280$

$=$ ₹ 4,60,000

Hence, option (c) is the answer.

23. Solution: Let,

$$x = \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}}}}\ldots$$

$$\Rightarrow \quad x = \frac{1}{3\sqrt{2}}\sqrt{4 - x}$$

$$\Rightarrow \quad 18x^2 + x - 4 = 0$$

$$\Rightarrow \quad x = \frac{4}{9} \text{ or } -\frac{1}{2}$$

$\therefore$ Reqd. expression = 6 + log3/2 x

$\because \quad x > 0$

$$\Rightarrow \quad x = \frac{4}{9}$$

Hence, net value of the expression = 6 – 2 = 4.
Hence, option (b) is the answer.

24. Answer would be = 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 9 + 18 = 135. Hence, option (b) is the answer.

25. Assume that x students like tea and y students don't like tea. Hence, $x + y = 80$. Answer would be 60% of x + 60% of $y = 32$. Hence, option (a) is the answer.

26. 1st person can choose his seat either on length side or breadth side in (4 + 2) = 6 ways.
Once he is seated, seating arrangement of remaining 11 persons is similar to linear arrangement = 11!. Hence, total ways of arrangement = 6 × 11!. Hence, option (c) is the answer.

27. Question is asking for other root of the equation.
Assume $f(x) = ax^2 + bx + c$
Since $f(1) = f(2)$
So, $a + b + c = 4a + 2b + c$
Or, $3a + b = 0$
Hence $-b/a = 3$ = Sum of roots. Since one root of the equation is 4, so other root must be –1. Hence, option (b) is the answer.

28. B was in business for 8 months in 1985 and 12 months of 1986, hence, a total of 20 months. Hence, option (b) is the answer.

29. C was there for 10 months, hence, he must have joined in March. Hence, option (a) is the answer.

30. Let A and B be the centers of two bigger circles and C be the centre of the smaller circle. AB = 2 units, $AC = \sqrt{2}$ units. Hence radius of the smaller circle = $\sqrt{2} - 1$. Hence, option (b) is the answer.

31.
$$\alpha + \beta = p$$
$$\alpha\beta = r$$
$$\frac{\alpha}{2} + 2\beta = q$$
$$\beta = \frac{2q - p}{3}$$
$$\alpha = \frac{4p - 2q}{3}$$
$$r = 2/9\,(2p - q)\,(2q - p)$$
Hence, option (d) is the answer.

32. Numbers which are divisible by 3 (between 100 and 200) are 33. Numbers which are divisible by 21, *i.e.*, *LCM* of 7 and 3 (between 100 and 200) are 5. Out of the 33 numbers divisible by 3, 17 are even and 16 are odd. Out of the 5 numbers divisible by 7, 3 are odd. Hence, the numbers of odd numbers divisible by 3 but not by 7 is (16 – 3) = 13.
Hence, option (d) is the answer.

33.
$$\frac{1}{2} \times a \times 2 = a$$
$$= \frac{1}{2} \times (b - 2) \times AP$$
$$= \frac{1}{2} \cdot b \cdot PD$$

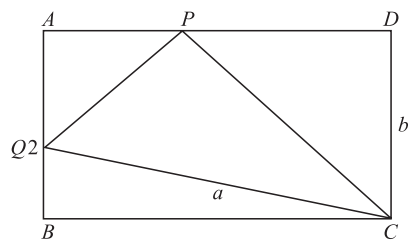


Fig. 8

$$AP = \frac{2a}{b-2},\ PD = \frac{2a}{b}$$
$$1 = \frac{2b + 2(b-2)}{b(b-2)}$$
$$b^2 - 2b = 4b - 4$$
$$b^2 - 6b + 4 = 0$$
$$b = \frac{6 \pm \sqrt{36+16}}{2} = 3 \pm \sqrt{5}$$
if $b = 3 \pm \sqrt{5}$
then $AQ = 1 + \sqrt{5}$
if $b = 3 - \sqrt{5}$
then $AQ = 1 - \sqrt{5}$

Which is a negative number

So, $AQ = 1 + \sqrt{5}$

Hence, option (a) is the answer.

Solution to questions 41–44

Story Line: There are two set of data—One for 2013 and other for 2014. Pie chart given for 2013 has the data in percentage and pie chart given for 2014 has the data in degrees. To compare the data from one pie chart to other, we need to convert percentage into degrees or degrees into percentage. In my opinion, it is always convenient to convert degrees into percentage data wherever required.

Let us first convert data given in the second pie chart into percentage (Table 3):

360 degrees = 100%

⇒ 3.6 degrees = 1%

⇒ 18 degrees = 5 %

Table 3

Particulars	Expenses in 2014	In Percentage
Home Rent	90	25%
Food	108	30%
Education	54	15%
Entertainment	72	20%
Miscellaneous	36	10%

I would suggest you to write down the percentage value in the second pie chart as shown in Fig. 9:

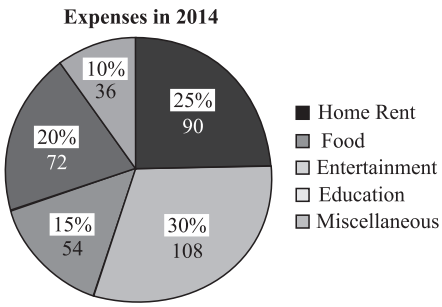


Fig. 9

So, finally the tables look like Fig. 10:

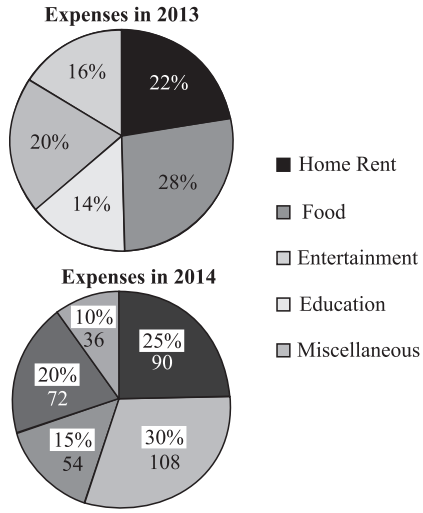


Fig. 10

41. Except in case of Miscellaneous, everywhere else percentage value has gone up. Since total value has gone up to ₹ 96,000 in 2014 from ₹ 64,000 in 2013, with an increase in percentage share, value of sectors like Home Rent, Food, Education, Entertainment are bound to rise.

Now we will verify for Miscellaneous sector:

Value in 2013 = 16% of 64,000 = 16% of 60,000 + 16% of 4,000 = 9,600

Value in 2014 = 10% of 96,000 = 9,600

Hence, value in 2013 > value in 2014

Hence, option (a) is the answer.

42. To calculate the sectoral percentage change in 2014 over 2013, we can see that there are two factors responsible for it: (i) percentage increase in the share, (ii) total value has increased from ₹ 64,000 to ₹ 96,000.

Now, this second factor is constant for each of the sectors, hence it has equal impact over all the sectors. So we can neglect this and calculate only factor (i) to answer this question.

$$\text{Percentage increase} = \frac{\text{Increase}}{\text{Initial Value}} \cdot 100$$

Entertainment has remained constant at 20% and Miscellaneous has decreased. So we are not required to check for these two. It can be seen that 'Home rent' has seen the maximum change in the percentage value = 25% – 22% = 3%. To calculate percentage change, we are supposed to divide this by Initial value (22). However, it can be further visualized that 3/22 is greater than 2/28 (Food sector) and 1/14 (education). Hence, Home Rent is the answer. Hence, option (a) is the answer.

43. See Table 4:

Table 4

Percentage Share in 2013	Percentage Share in 2014
14%	15%

There are two methods of solving this question:

Method 1: Doing the actual calculation

Spending in 2013 = 14% of ₹ 64,000
= ₹ 8,960

Spending in 2014 = 15% of ₹ 96,000
= ₹ 14,400

Percentage increase $= \frac{\text{Increase}}{\text{Initial Value}} \cdot 100$

$= \frac{14400 - 8960}{8960} \times 100$

= 60.7%

Method 2: Successive percentage change

There are two factors responsible for the percentage increase—(a) rise in percentage share and (b) total spending has increased. We will calculate these one by one and use successive percentage increase to find out the net percentage increase.

(a) Rise in percentage share:

$$= \frac{15-14}{14} \times 100 = 7\% \text{ (approx.)}$$

(b) Rise in total spending:

$$\frac{96000 - 64000}{64000} \times 100 = 50\%$$

Net percentage increase

$$7 + 50 + \frac{7 \cdot 50}{100} = 60.5\%$$

Hence, option (b) is the answer.

44. Answer will be one of Home Rent or Food.

Note: Just because, increase in the percentage share for Home Rent is 3%, and that of Food is 2%, we cannot say that absolute increase will be more in case of Home Rent. It may be or may not be.

To be accurate, it is advisable that you go through the actual calculations (Table 5).

It can be seen that maximum absolute increase occurs in case of Food sector. Hence, option (b) is the answer.

45. Series is $2^2 - 1, 3^2 - 2, 4^2 - 3, 5^2 - 4, 6^2 - 5$. So next term = $7^2 - 6 = 43$. Hence, option (a) is the answer.

46. Difference between two consecutive terms are like = multiple by two and then add 3. Hence, required term = 69 × 2 + 3 = 141. Hence, option (b) is the answer.

47. How cogged wheels move : If one wheel is moving clockwise, then the wheels directly attached to it will move anticlockwise and vice versa. In this case, given that D moves anti clockwise $\Rightarrow$ A will move clockwise $\Rightarrow$ B will move anti clockwise $\Rightarrow$ G will move clockwise $\Rightarrow$ F will move anti clockwise $\Rightarrow$ E will move clockwise $\Rightarrow$ C will move anti clockwise $\Rightarrow$ D will move clockwise $\Rightarrow$ which is a contradiction. Hence, option (b) is the answer.

Table 5

Expenses	2014			2013			Difference
	%	Total spend	Sector spend	%	Total spend	Sector spend	
Home Rent	0.25	96000	24000	0.22	64000	14080	9920
Food	0.3	96000	28800	0.28	64000	17920	10880

48. Number of brothers and sisters in the family respectively = 5 and 4. Hence, number of children = 9. Hence, option (c) is the answer.

Solution to questions 49 and 50

49. Uneducated urban = 3, 4, 5. Rectangle is for honest.
Out of 3, 4, and 5, only 4 is inside the rectangle. Hence, option (d) is the answer.

50. Non-urban educated people = 7, 8 and 9. Out of this, neither hardworking nor honest = 7. Hence, option (b) is the answer.

51. Sequence goes like:
$(8 + 5) \times 3 = 39$
$(4 + 7) \times 4 = 44$
$(9 + 3) \times 5 = 60$
$(5 + 4) \times 8 = ? = 72$
Hence, option (b) is the answer.

52. Let us assume that each one of them had x lemons before Jamuna decided to mix it.
Total revenue earned if each of them would have sold it individually

$$= \frac{x}{4} + \frac{x}{6} = ₹\frac{5x}{12}$$

Total revenue earned if sold @ five lemons per rupee

$$= ₹\frac{2x}{5}$$

Loss in revenue

$$= \frac{5x}{12} - \frac{2x}{5} = ₹\frac{x}{60}$$

According to the question,

$$= ₹\frac{x}{60} = ₹3.5$$

$$\Rightarrow \quad x = 210$$

So, total revenue earned if sold @ five lemons per rupee

$$= ₹\frac{2x}{5} = ₹84$$

Out of this, share of Jamuna = ₹ 42 [they divide ₹ 84 equally]

Total revenue earned by Jamuna if she would have sold her lemons individually

$$= \frac{210}{4} = ₹52.5$$

Net loss to Jamuna

$$= ₹\ 52.5 - ₹\ 42 = ₹\ 10.5$$

Hence, option (a) is the answer.

53. It can be seen by assuming some values that in such cases, percentage of one liquid in other remains same. Hence, option (c) is the answer.

57. There are actually 20 ways to interchange the letters, namely, the first letter could be one of 5, and the other letter could be one of 4 left. So total pairs by product rule = 20. Now, there are two cases when it wouldn't change the name. First, keeping them all the same. Second, interchanging the two EEs together. Thus, 2 options would leave the name intact. Probability $= \frac{2}{20} = 0.1$, or 10%. Hence, option (b) is the answer.

58. Assume Math = M and Chemistry = C
Total $= M + C -$ both + neither
$20 = 8 + C - 7 + 10$
$C = 9$, 9 students study both C and M. Now given that 7 study both the subjects, hence $9 - 7 = 2$ students study only C.

59. One can solve this by subtracting the two inequalities. To do this, they need to be in the opposite direction; when you subtract them preserve the sign of the inequality from which you are subtracting.

$$3 < y < 8$$

multiply the second one by (–1) to reverse the sign

$$2 > x > -2$$

Subtract them to get

$$3 - 2 < y - x < 8 - (-2)$$
$$1 < y - x < 10$$

Hence, option (d) is the answer.

Section 2

61. Most people, on the other hand, who value the ultimate ends of socialism no less than

socialists, refuse to support socialism because of the dangers of other values they see in the methods proposed by the socialist.

62. Yet, although to most socialists only one species of collectivism will represent true socialism, it must always be remembered that socialism is a species of collectivism and that therefore everything which is true for collectivism as such must apply also to socialism.

70. See Table 6:
Hence, option (d) is the answer.

71. Option (c) is the answer.

72. The completed sentence should convey a sense of relief over what happens there. The only two such words are happily and mercifully, but the word attract renders the meaning irrelevant. The second blank needs to have a word with a slightly negative connotation.
Hence, option (a) is the answer.

73. The parameters for encryption mentioned here are fairly commonplace. Such commonplace things lead to a good amount of guessability. Esoteric and complex are ruled out as they go against the spirit of the idea here. Besides, something esoteric does not lead to comprehensibility in the first place. Hence, option (c) is the answer.

87. The word, restive, is tricky, because it sounds like it has something to do with rest. Therefore, (c) active would be the best antonym. However, this is a trick. Restive actually means restless. So, the correct answer is option (a) patient, a good antonym for restless.

88. Mollify means "to reduce or soften the pain". Hence, option (a) is the answer.

89. Only two sentences—A and C—appear to be independent. Fortunately, no choice begins with A. So, we come to the conclusion that C is the first sentence. This rules out choices 2 and 4. Now, look at D, which mentions "each of them". This refers to "Mr Sharon" and "Mr Olmert" in E. Thus ED is a sequence. This rules out option (c).

90. Both C and D are independent sentences. And so it seems the paragraph may begin with either of them. But there is a subtle clue: D gives the full name "Paul Bremer" but C mentions him as only "Mr Bremer". On this basis alone, D seems to be better poised for the first sentence. Thus 3 and 4 are ruled out. Again, the "he" in B refers to "Paul Bremer" in D. Also, the "this" in A refers to the action in B. Thus DBA emerges as a sequence. Hence, rule out (a) and go for (b).

91. Being the only independent sentence, B qualifies as the first in the paragraph. Thus (a) and (b) are ruled out. Next, look at E. Clearly, it elaborates A. Thus AE is a sequence, which rules out (c).

92. A is an explanation for C: why "he cannot read, write, or even spell his own name." And "this muscular 15-year-old" in C refers to Boima Dagosary" in E. Thus ECA is a sequence. Hence, (a) and (b) are easily ruled out. Again, ECA tries to substantiate D. Therefore, D can't come after ECA.

93. Only A and B qualify as independent sentences. Thus, (a) is ruled out. Again, if you look carefully, A is an elaboration of the "slowness" mentioned in B. Thus BA is a sequence, which leads us to (b).

Table 6

discriminatory	partial	*discriminating*	sharp, astute
depopulated	emptied of population by choice	*unpopulated*	without population
impractical	inadvisable, undesirable	*impracticable*	not implementable
respectable	worthy of respect	*respectful*	one who is full of respect

Mock Aptitude Test (Mock AT) 2

This test paper contains 100 questions.

1. There are 2 sections in this test paper—Section 1 containing 60 questions and Section 2 containing 40 questions.
2. You have 2 hours to complete this test.
3. All questions carry 4 marks each. Each wrong answer will attract a penalty of 1 mark.

Direction for answering the questions are given before each group of questions. Read these directions carefully and answer the questions. Each question has only one correct answer.

SECTION 1

1. If p is a prime number greater than 11, and p is the sum of the two prime numbers x and y, then x could be which of the following?

(a) 2 (b) 5
(c) 7 (d) 9

2. In a certain shipment 2 percent of the boxes shipped were damaged. If the loss per damaged box was ₹ 35 and the total loss due to damage was ₹ 700, how many boxes were shipped?

(a) 2,000 (b) 1,000
(c) 200 (d) 100

3. A, B and C purchase a gift worth ₹ 120. A pays half of what B and C together has paid. B pays 1/3rd of what A and C together has paid. What is the sum paid by C?

(a) ₹ 30 (b) ₹ 40
(c) ₹ 60 (d) ₹ 50

Direction for questions 4–6: ***Answer the following questions based on the following information.***

Table 1 gives the education level of 570 married couples surveyed. The surveyed members are distributed into four categories namely A (Std. X passes), B (Std. XII passed), C (Graduates) and D (Post Graduates). Answer the questions based on the information given in the Table 1 below. Row is for wives and column is for husbands:

Table 1

Husband ↓/ Wife→	*A*	*B*	*C*	*D*
A	25	30	35	42
B	32	36	45	23
C	46	39	39	33
D	37	45	41	22

4. In how many married couples have exactly one of the pair finished his/her education at least till graduation?

(a) 72 (b) 110
(c) 312 (d) 156

5. How many of the above married couples is one of them more educated than the other?

(a) 400 (b) 427
(c) 463 (d) 448

6. What is the difference between the number of couples with one PG and another 10th and one grad and another 12th?

(a) 6 (b) 5
(c) 7 (d) None of these

Direction for questions 7–9: ***Go through the data set given below and solve the questions based on it.***

Following bar charts as shown in Figs. 1 and 2 provide the sales value (in ₹ lac) and profit (in ₹ lac) of a company over the years.

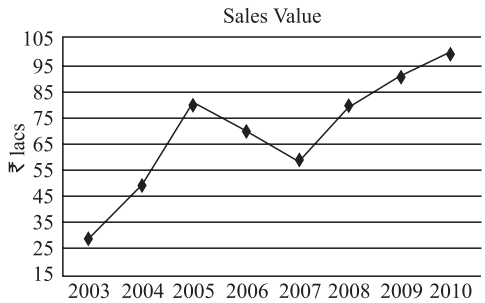


Fig. 1

All the sales values are in the multiples of 100.

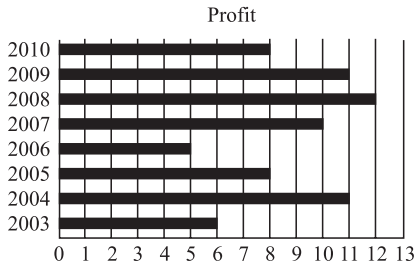


Fig. 2

7. Which year's profit percentage is maximum?
(a) 2003 (b) 2004
(c) 2005 (d) None of these

8. Company wants to calculate its average profit percentage for the given years. For this to calculate, company 1st calculates average sales value for the given years, and then average profit value for the given years. Finally average percentage profit is calculated using this data. What is the value of average profit percentage (approximately)?
(a) 12% (b) 15%
(c) 18% (d) 22%

9. In which of the year from 2004 to 2009 inclusive, did profits change by the greatest percentage over the previous year?
(a) 2004 (b) 2005
(c) 2007 (d) 2008

10. If a rectangular picture that measures 4 feet from side to side is hung exactly in the middle of a rectangular wall that measures 13 feet from side to side, then the left edge of the picture is how many feet from the left edge of the wall?
(a) 2.0 (b) 3.0
(c) 4.5 (d) 6.5

11. In a solid cylinder, sum of the height and radius of the base is equal to 37 cm. Total surface area of cylinder is 1,628 cm^2. What is the volume of the cylinder (in cubic cm)?
(a) 4,640 (b) 4,620
(c) 6,420 (d) 6,460

12. A company paid ₹ 5,00,000 in merit raises to employees whose performances were rated *A*, *B* or *C*. Each employee rated *A* received twice the amount of the raise that was paid to each employee rated *C*; each employee rated *B* received 1.5 times the amount of the raise that was paid to each employee rated *C*. If 50 workers were rated *A*, 100 were rated *B*, and 150 were rated *C*, how much was the raise paid to each employee rated *A*?
(a) ₹ 370 (b) ₹ 625
(c) ₹ 1,250 (d) ₹ 2,500

13. For each of *n* people, Margie bought a hamburger and a soda at a restaurant. For each of *n* people, Paul bought 3 hamburgers and a soda at the same restaurant. If Margie spent a total of \$5.40 and Paul spent a total of \$12.60, how much did Paul spend just for hamburger? (Assume that all hamburgers cost the same and all sodas cost the same)
(a) \$10.80 (b) \$9.60
(c) \$7.20 (d) \$3.60

14. What is the number of squares of perimeter 36 into which a rectangle with width 36 and length 72 can be partitioned?
(a) 2 (b) 4
(c) 8 (d) 32

15. Find the area of the enclosed area made by the lines/curves:
$|x + y| = 2$, $|x| = 2$ and $|y| = 2$
(a) 6 (b) 9
(c) 12 (d) 24

16. x, y and z are consecutive positive integers such that $x < y < z$; which of the following must be true?
I. xyz is divisible by 6
II. $(z - x)(y - x + 1) = 4$
III. xyz is divisible by 12
(a) I and II only (b) I only
(c) II only (d) All three

17. The ratio of a compound, by weight, consisting only of substances x, y, and z is 4:6:10, respectively. Due to a dramatic increase in the surrounding temperature, the composition of the compound is changed such that the ratio of x to y is halved and the ratio of x to z is tripled. In the changed compound, if the total weight is 58 gms, how much does substance x weigh?
(a) 6 (b) 12
(c) 24 (d) 36

18. $x^2 - bx + b = 0$ has two roots r_1 and r_2. What is the value of b such that the roots differ by less than 5?
(a) $b < \sqrt{29}$ (b) $b > \sqrt{29}$
(c) $b < 2 + \sqrt{29}$ (d) $b = 0$

19. If consecutive natural numbers are written and each perfect square is deleted, then what will be the 2009th natural number?
(a) 2,052 (b) 2,053
(c) 2,054 (d) 2,055

20. There are two swimmers who start towards each other from opposite banks of lake of still water. First time they meet at a point 900 m from one shore. They cross each other, touch the opposite bank and return. They meet each other again, this time, 300 m from the other shore. What is the width of the lake?
(a) 1,500 m (b) 600 m
(c) 2,400 m (d) 1,200 m

21. Six machines working at the same rate constant rate complete a job in 12 days. How many additional machines working at same rate will be needed to complete the job in 8 days?
(a) 2 (b) 3
(c) 4 (d) 9

22. Real x and y satisfy the following equations: $\log_8 x + \log_4 y^2 = 5$; $\log_8 y + \log_4 x^2 = 7$. Find xy.
(a) 256 (b) 512
(c) 1,024 (d) 128

23. Circular gears A & B start rotating at the same time at constant speeds. Gear A rotates at 10 revolutions/min and gear B at 40 revolutions/min. How many seconds after gears start rolling will B have made exactly 6 more revs than A?
(a) 8 (b) 10
(c) 12 (d) 15

24. Vertices of a regular hexagon $ABCDEF$ are denoted on the Cartesian coordinate system. Vertex $A = (1, 0)$ and vertex $B = (2, 0)$. Which of the following will be the equation of the straight line CE?
(a) $x\sqrt{3} + y = 4$
(b) $y\sqrt{3} + x = 4$
(c) $x\sqrt{3} + y = 4\sqrt{3}$
(d) $x + \sqrt{3}y = 4\sqrt{3}$

25. Let a, b, c be distinct digits. Consider a two digit number 'ab' and a three digit number 'ccb', both defined under the usual decimal number system. If $(ab)^2 = ccb$ and $ccb > 300$ then the value of b is:
(a) 1 (b) 0
(c) 5 (d) 6

26. Let x and y be positive integers such that x is prime and y is composite. Then,
(a) $y - x$ cannot be an even integer
(b) xy cannot be an even integer
(c) cannot be an even integer
(d) None of these

27. Let x, y and z be the natural number less than 10 such that:
$(100x + 10y + x)^2 = (x + y + z)^5$
What is the value of $x^2 + y^2 + z^2$?
(a) 21 (b) 23
(c) 29 (d) 33

28. For real number $x \geq 3$, let $f(x) = \log_2 (\log_3 x) - \log_3 (\log_2 x)$. Which one of the following is true?
(a) $f(x) > 0$ for all $x \geq 3$
(b) $f(x) < 0$ for all $x \geq 3$
(c) $f(x) = 0$ for all $x \geq 3$
(d) $f(x) = 0$ for exactly one vlaue of $x \geq 3$

29. An old lady is driving on a straight road at a constant speed. She is XYZ miles from her home in Patna at 2 p.m., where x, y and z are digits with x is necessarily a digit from 1 to 9 and $y = 0$. At 2:18 p.m., she is ZX miles from her home at and 3 p.m. she is XZ miles from her home. At what time does she arrive at her home?
(a) 3:10 p.m. (b) 3:12 p.m.
(c) 3:24 p.m. (d) 3:30 p.m.

30. Given that the equations below all hold for x, y, z, t and w, determine which of the following options is true:

$$x + y + z + t + w = 1$$
$$x + y + 2z + 2t + 2w = 1$$
$$x + 2y + 3z + 4t + 5w = 2$$
$$x + y + 3z + 2t + w = 3$$

(a) x must be equal to –2 (minus 2).
(b) z must be either 0 or negative.
(c) y must be negative.
(d) None of x, y, z, t or w can be uniquely determined.

31. The smallest perfect square divisible by 7:
(a) 44,100 (b) 176,400
(c) 705,600 (d) 19,600

32. In green view apartment, the house of a row is numbered consecutively from 1 to 49. Assuming that there is a value of 'x' such that the sum of the numbers of the houses preceding the house numbered 'x' is equal to the sum of the numbers of the houses following it. Then what will be the value of 'x'?
(a) 21 (b) 30
(c) 35 (d) 42

33. For all the values of x and y, which of the following will not divide the natural number $N = (x^{36} - y^{36})$?
(a) $x - y$ (b) $x^3 - y^3$
(c) $x^{12} - y^{12}$ (d) $x^{24} - y^{24}$

34. P is a prime number > 71. $N = (P + 2)(P - 1)(P + 1)(P - 2)$. Which of the following will definitely divide N for all the values of P?
I. 120 II. 240
III. 360 IV. 480
(a) I and II only
(b) I and III only
(c) II and IV only
(d) I, II and III only

35. If $x = 17 + 12\sqrt{2}$, then find the value of $x^3 + \frac{1}{x^3}$?
(a) 38,202 (b) 39,202
(c) 39,302 (d) 38,302

36. P, Q and 1 are the roots of the equation $x^3 - ax^2 - bx - c = 0$, where a, b and c are in Arithmetic Progression. Which of the following is true?
(a) $PQ + P + Q = \frac{1}{3}$
(b) $PQ + P + Q = -\frac{1}{3}$
(c) $PQ - (P + Q) = -\frac{1}{3}$
(d) $PQ - (P + Q) = \frac{1}{3}$

37. Consider the expression $y = x^2 - 2kx + k^2 + 1$. For which of the following values of k, y will be always greater than zero for any value of x?
(a) $k > 0$
(b) k is any real number
(c) $k > 2$
(d) $-2 < k < 7$

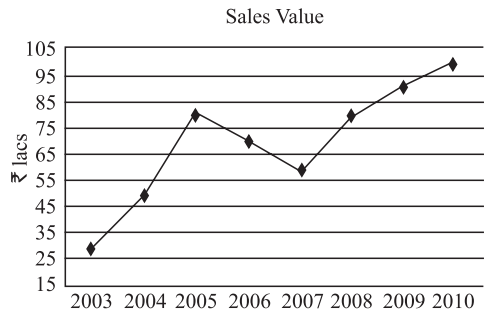
38. *HCF* of $[(N-1)!$ and $N] > 1$. Consider the statements given below:
(i) N is a natural number
(ii) N is a composite number
(iii) N is a prime number
(iv) N is a rational number
Choose the best option:
(a) Exactly one of the statements is definitely true.
(b) At least one of the statements is definitely true.
(c) Exactly two of the statements are definitely true.
(d) At max two of the statements are definitely true.

39. Inside a quadrilateral *ABCD* (right angled at *B*), a circle is drawn in such a way that all the four sides of the quadrilateral are tangent to the circle. Circle touches side *CD* at *P* and side *BC* at *R*. Given that *PD* = 9 cm and *RC* = 11 cm. If radius of the circle = 5 cm, find the length of *BC*.
(a) 16 (b) 12
(c) 18 (d) 14

40. Using the digits 1, 2, 3 and 4, how many numbers of at most 4 digits can be formed (repetition allowed)?
(a) 340 (b) 256
(c) 64 (d) None of these

Direction for questions 41–44: ***Go through the data set given below and solve the questions based on it.***

Following bar charts as shown in Fig. 3 provides the sales value (₹ in lac) and profit (₹ in lac) of a company over the years.



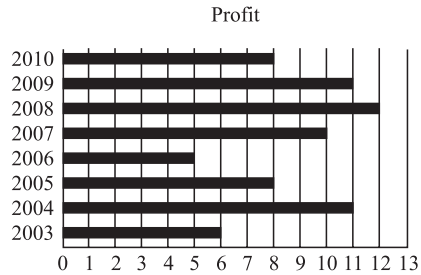


Fig. 3

41. For the years 2004 to 2009 inclusive, what was the amount of the greatest increase (₹ in thousands) in sales from one year to the next?
(a) 50 (b) 500
(c) 30 (d) 300

42. For the period from 2003 to 2007 inclusive, what were the average sales per year (₹ in thousands)?
(a) 570 (b) 580
(c) 600 (d) 610

43. In which of the years from 2004 to 2009 inclusive, did profits change by the greatest percentage over the previous year?
(a) 2004 (b) 2005
(c) 2007 (d) 2008

44. Company considers a 'Performance year' to be any year in which profits were at least 20 percentage of sales. How many of the years shown are 'Performance years'?
(a) Zero (b) One
(c) Two (d) Three

45. P is a root of the equation $x^2 + ax + b = 0$ and $0 \le b \le a \le 1$. Which of the following gives the sharpest bound for P?

(a) $|P| \le 1$ (b) $|P| \le \frac{3}{2}$

(c) $|P| \le 2$ (d) $|P| \le \frac{4}{2}$

46. $x^2 + x + 1$ is a factor of $(x + 1)\, n - xn - 1$, whenever:

(a) n is odd
(b) n is odd and a multiple of 3
(c) n is an even multiple of 3
(d) n is odd and not a multiple of 3

Direction for questions 47–49: ***Go through the directions given below and solve the questions based on it.***

(a) If the question can be answered by one of the statements alone and not by the other.
(b) If the question can be answered by using either statement alone.
(c) If the question can be answered by using both the statements together, but cannot be using either statement alone.
(d) If the question cannot be answered even by using both the statements together.

47. If p is an integer, what is the value of $p^2 - 9p + 20$?
Statement 1: $p = 4$
Statement 2: $p = 4$ or 5

48. How many integers, x, satisfy the inequality $n < x < m$?
Statement 1: $m - n = 78$
Statement 2: $m > 100$ and $n < 50$

49. A, B and C are three points in the same plane. Is Y the midpoint of XZ?
Statement 1: ZY and YX have the same length.
Statement 2: XZ is the diameter of a circle with centre Y.

50. Find the remainder when $(3)^{53!}$ is divided by 80.

(a) 0
(b) 1
(c) 2
(d) Cannot be determined

51. When 96 is added to a perfect square, another perfect square is obtained. How many such pairs of perfect squares exist?

(a) 0 (b) 1
(c) 2 (d) 4

52. If $a * b = \frac{ab}{a+b}$, find $3*(3*-1)$

(a) –3 (b) –1.5
(c) 2/3 (d) 3

53. How many numbers between 3,000 and 7,000 (both inclusive) can be formed with the digits 0, 2, 3, 5, 6, 7 and 8 with repeated digits?

(a) 1,029 (b) 1,028
(c) 1,030 (d) 1,031

54. Shyam travels the distance from A to B. He covers 1/3rd of the distance at 60 kmph, 3/5th of the remaining at 40 kmph and the remaining distance at 70 kmph and 30 kmph for equal time. Find the average speed (in kmph) during the course of journey?

(a) 72 (b) 74
(c) 60 (d) 48

55. Which term in the series 3 + 10 + 17 + ... and 63 + 65 + 67 + ... is common?

(a) 9 (b) 13
(c) 14 (d) 15

56. $\sqrt{999999998000000001} = ?$

(a) 999999999 (b) 99999999
(c) 9999999999 (d) 999999991

57. Given that m and n are natural numbers. What is the unit digit of $215^{n^m} + 1$?

(a) 6
(b) 4
(c) 5
(d) More than one value possible

58. If $[x]$ denotes the integral part of x, then

$$\left[\frac{1}{5}\right]+\left[\frac{1}{5}+\frac{1}{300}\right]+\left[\frac{1}{5}+\frac{2}{150}\right]+\left[\frac{1}{5}+\frac{3}{100}\right]+\ldots+\left[\frac{1}{5}+\frac{299}{300}\right]$$

is equal to which of the following?

(a) 61 (b) 60
(c) 300 (d) 240

59. The surface area of cuboids is 24 cm^2. Find the maximum volume of the cuboids.

(a) 4 cm^3 (b) 6 cm^3
(c) 10 cm^3 (d) 8 cm^3

60. If for real x and y satisfy the following equations; $\log_8 x + \log_4 y^2 = 5$; $\log_8 y + \log_4 x^2 = 7$. Find xy.

(a) 256 (b) 512
(c) 1,024 (d) 128

SECTION 2

Direction for questions 61–63: ***In each question, there are five sentences. Each sentence has a pair of words that are italicized and highlighted. From the italicized and highlighted words, select the most appropriate words (A or B) to form correct sentences. The sentences are followed by options that indicate the words, which may be selected to correctly complete the set of sentences. From the options given, choose the most appropriate one.***

61. I. The ***enquiry (A)/inquiry (B)*** report of the panel is yet to be submitted because there has been inordinate delay in collecting the relevant evidence.

II. Though she was an attendee at the seminar, we could get only a ***taciturn (A)/tacit (B)*** approval to the proposal from her.

III. Since the act of downing the drink leads to a ***pleasurable (A)/pleasing (B)*** sensation, it tends to become a habit.

IV. He ***refused (A)/refuted (B)*** to believe the charges as he thought that they were motivated and biased.

V. They were convinced that the mistake was ***purposely (A)/purposefully (B)*** done as to mislead the examiners.

(a) ABAAA (b) BBBAA
(c) AAAAA (d) BBBAA

62. I. It was a spiritually ***satisfactory (A)/satisfying (B)*** experience to come across such a saintly person after such a long time.

II. You need to ***extract (A)/extricate (B)*** the essential points from this book in order to make readable notes.

III. The danseuse' ***gracious (A)/graceful (B)*** acceptance of our suggestion, despite the seeming impracticability of it all, won our hearts.

IV. In a pretty much ***exceptionable (A)/exceptional (B)*** move, she made a suggestion which was taken quite uncharitably by all those present there.

V. Owing to constant ***misuse (A)/disuse (B)*** the snakes' ears have disappeared over several millions of years, says the theory of evolution.

(a) BBAAA (b) BBBAA
(c) AABAB (d) BAAAB

63. I. Slowly and slowly, the TV channel ***uncovered (A)/discovered (B)*** the murky goings-on beyond the public gaze in the corridors of power.

II. If one is really keen to know the reality, one needs to go ***beyond (A)/behind (B)*** the apparent simplicity of behaviour shown by her at the audition.

III. Several instances of this kind prove his ***metal (A)/mettle (B)*** beyond any reasonable doubt.

IV. The ***hospitable (A)/hostile (B)*** terrain proved a major dampener in our way.

V. It is an immeasurable delight to watch a ***baby's growth (A)/baby grow (B)***.

(a) BABAA (b) ABBBA
(c) ABBBB (d) AABBB

Direction for questions 64–66: ***Each of the following questions has a sentence with two blanks. Given below each question are five pairs of words. Choose the pair that best completes the sentence.***

64. As international broadcasters turn to popular music and soap operas to retain their audiences, such gripes will grow _____. Having long trumpeted the cause of freedom, the big Western broadcasters must now learn to _____ its consequences.

(a) louder, live with
(b) often, face
(c) loud, tackle
(d) shriller, dance to

65. China is the most powerful exhibit for those who would like to argue that authoritarian regimes are no _____ to growth but most disastrous growth performances have also been under authoritarian regime and somewhat _____, the postwar era democracies have not done much better than non-democratic regimes.

(a) strangers, rationally
(b) enemies, predictably
(c) way linked to, expectedly
(d) impediments, puzzlingly

66. Mr Acer is no apologist for prejudice, and at times _____ what he sees as a decline in Arab intellectualism and good sense and also rebukes those Arabs who stoke anti-Semitism by _____ the distinction between Jews and Zionists.

(a) likes, analyses
(b) harshly criticizes, blurring
(c) is biased, abolishing
(d) looks at, confusing

Direction for questions 67 and 68: ***Read each of the following passages carefully and select the best option that forms the best concluding line in each case.***

67. While many people would consider philosophy as irrelevant in today's highly scientific world, the truth is that it has played a major role in the development of almost every aspect and code of conduct of every institution on the planet. This is why philosophy remains the best tool in arriving at knowledge about God and his nature. While many would consider religion as the only option in understanding God, it is actually philosophy that is best suited for the task. This is due mainly to the inherent limitations of each religion such as parochial mentality–the thinking that only the God of my denomination or religion exists and nothing else – that restricts a bias-free idea about what God is and its nature.

(a) Philosophy is not hindered by biases derived from religious belief and it is free to study God from all angles.
(b) On the other hand, God belongs to all, whatever be one's creed, race, color or nationality.
(c) Philosophy is totally unencumbered by any belief systems.
(d) The idea of God is basic to any kind of philosophy and all philosophical discussions are necessarily premised on God and His relations with nature.

68. To better understand moral relativism, we need to know about the nature of objective truths. Truths that are deemed independent of our beliefs or perceptions are called objective truths. A typical example is the activity of the moon around the Earth. The moon orbits the Earth regardless of whether people believe it or not. Another example is the orbit of the Earth around the sun.

Subjective truths, on the other hand, hold at least in part due to our beliefs and individual perceptions. The taste of pork, for instance, varies from person to person. Some people find pork delicious while others find it repulsive, either due

to sensual perception or belief. Whether pork is delicious or not is considered subjective.

(a) I am OK, so are you.
(b) I am OK, but you aren't.
(c) The truth lies somewhere between these two extremes.
(d) We both are OK, but they aren't.

Direction for questions 69–71: ***Each question given below has on opening statement and a closing statement. In between, there are four other sentences labeled A to D. You are required to choose the most coherent and logical sequencing of the four sentences.***

69. I. The prospects for democracy in the Third World are again a subject of practical concern.
A. They have generally failed to produce material prosperity or political stability.
B. At worst their oppressiveness involved incarceration and brutality.
C. Various forms of dictatorship have been tried.
D. Their repressiveness has at best curtailed freedom of expression.
IV. But is awareness of the consequences of dictatorship enough to ensure the triumph of democracy?

(a) ABCD (b) CADB
(c) DBAC (d) CBDA

70. I. The basic economic resource is knowledge.
A. The wealth creating activities will be the allocation to productive uses of neither capital nor labour.
B. The representative social groups of the knowledge society will neither be the capitalist nor the worker.
C. They will centre around productivity and innovation, both applications of knowledge to work.
D. The ruling group will be the knowledge workers, knowledge executives, knowledge professionals and knowledge entrepreneurs.
VI. Practically all of them will be employed, either originally, or eventually in knowledge organizations.

(a) BACD (b) BDCA
(c) ACBD (d) ACDB

71. I. Science has sought to escape from the doctrine of perpetual flux by finding some permanent substratum amid changing phenomena.
A. Accordingly it was supposed that atoms are indestructible, and that all change in the physical world consists merely in re-arrangement of persistent elements.
B. It was found that fire, which appears to destroy, only transmutes.
C. Elements are recombined, but each atom that existed before combustion still exists when the process is completed.
D. Chemistry seemed to satisfy this desire.
VI. The view prevailed until the discovery of radioactivity, when it was found that atom could disintegrate.

(a) ACBD (b) CADB
(c) DABC (d) DBCA

Direction for questions 72–80: ***Read the passages given and solve the questions based on it.***

Passage 1: Evidently, in the process of animal evolution, the foot appeared suddenly when the backboned creatures started living on the dry land—that is, with the frogs. How it happened is a question which still haunts the phylogenists, who have been unable to find a sure pedigree for the frog. But there it is, anyhow, and the noteworthy thing about it is that a frog's foot is not a rudimentary thing, but an authentic standard foot, just like the yard measure kept in the Tower of London, of which all other feet are replicas or adaptations. This tool, as part of the original outfit

given to the pioneers of the brainy, backboned, and four-limbed species, when they were sent out to proliferate and replenish the earth, is certainly worth considering well. It comprises essentially a sole, or palm, having small bones and five separate digits, each having several joints.

In the hind foot of a frog, the toes are extremely long and webbed from point to point. In this, it differs significantly from the toad. The "heavy-gaited toad", content with sour ants, hard beetles, and other such fare it can lay its hands on, and grown nasty in consequence, so that no other predator feasts on it, has hobbled through life, like a plethoric old gentleman, to the present day, on its original feet. The more versatile and nimble-witted frog, seeking better diet and greater security of life, went back to its original environment in which it was bred, and, swimming much, became better adapted to swimming. The soft, elastic skin between the fingers or toes is just the sort of tissue which responds most readily to inward impulses, and we find that the same change has come about in those birds and beasts which live much in water. I know that this is not the kind of evolutionary theory much accepted today, but I am waiting till it becomes so. We all tend to develop in the direction of our tendencies, and shall, I am sure, be wise enough someday to give animals leave to do the same.

As we progress from reptiles to birds, lo! an astonishing thing has happened. We already know about the flying reptiles in the fossil ages and the flying beasts in our own. But their wings are simple mechanical contraptions, which the imagination of a child could explain. The hands of a bat are hands still, and, though the fingers are hampered by their awkward gloves, the thumbs are free. The giant fruit bats of the tropics prance about the trees quite gymnastically with their thumbs and feet. Here is a whole sub-kingdom of the animal world which has unreservedly and irrevocably bartered one pair of its limbs for a flying-machine. The apparatus is made of feathers—a new invention, unknown to amphibian or saurian, whence, obtained nobody can say—and these are grafted into the transformed frame of the old limbs. The bargain was worth it, for the winged creature at once zoomed away, in all senses, from the creeping things of earth, and became a more ethereal being. But the price it paid was quite heavy: it must get sustain its entire on one pair of feet and its mouth.

Let us note that the art of standing began with the birds. Frogs are in the habit of sitting, and, as far as I know, every reptile, be it lizard, crocodile, alligator, or tortoise, lays its body on the ground while not actually carrying it. And all these have four fat legs each. Contrast with the flamingo, which, has only two, and like willow wands, tucks up one of them and sleeps poised high on the other, like a tulip on its stem. Also note that one toe has been altogether discarded by birds as superfluous. The germ, or bud, must be there, for the Dorking fowl has produced a fifth toe under some influence of the poultry-yard, but no natural bird has more than four. Except in swifts, which never perch, but cling to rocks and walls, one is turned backwards, and, by a cunning contrivance, the act of bending the leg draws them all automatically together. So a hen closes its toes at every step it takes, as if it grasped something, and, of course, when it settles down on its roost, it clasps it tight till morning. But to non-perching birds, this mechanism is nothing more than an encumbrance. So many birds, like the plovers, abolish the hind toe entirely, and the king of all two-legged runners, the ostrich, gets rid of one of the front toes also, retaining just two.

72. The author primarily intends to talk of:
- (a) the evolution of reptiles and birds in the context of animal evolution in general.
- (b) how frogs and toads have overcome various barriers to their successful evolution.
- (c) the importance of the development of feet for frogs and birds.
- (d) how the organs of movement developed within the animal kingdom as a whole, with special reference to the frog, toad and birds.

73. The passage can be faulted on the grounds of:
A. not exemplifying the ideas presented
B. being full of too many details
C. having a lot of technical jargon
(a) A and C (b) B and C
(c) B only (d) C only

74. Which inference/s can be drawn from the above selection?
A. The author's notion of how the soft tissue on the feet of birds have developed is debatable.
B. The predators rarely, if at all, attack the toad.
C. The water bodies was the original home of the frogs.
(a) A only (b) B and C
(c) C only (d) A, B and C

75. The author's tone in the above passage is:
(a) narrative
(b) analytical
(c) scientific
(d) non-committal

Passage 2: In all languages, the most frequently used words evolve at the slowest pace, say researchers in a paper published in *Nature*. In one of the papers, Harvard University researchers researched the evolution of English verb conjugations spanning 1,200 years while the Reading University researchers reviewed cognates (words sounding similar in different languages and carrying the same connotation, such as "*water*" and the German "*wasser*") to discover how all Indo-European tongues have evolved from a single, common ancestor.

Pagel and his co-workers examined some 210 words in 87 Indo-European languages, including those for "water," "two," "to die" and "where." The number of cognate classes for each word ranged from one for frequent concepts such as *numbers* to 46 different basic sounds to portray a single entity like a *bird*. The word used to describe the idea of *three* in all Indo-European languages and English is quite similar: from *tres* in Spanish to *drei* in German to the Hindi *teen*. Contrarily, *bird* has several sounds like *pajaro* in Spanish and *oiseau* in French.

Thereafter, they narrowed their attention to their usage frequency in four Indo-European languages—English, Spanish, Greek and Russian. It was found that they were used at similar rates even if the synonymous words were not cognates. "The high frequency words in Spanish are the same as in those the high frequency English", say he. "That points to the possibility of our coming up with an Indo-European frequency of use".

The researchers have found that it would take just 750 years to replace less-used words and up to 10,000 years for new words to come into existence. The Harvard researchers studied the roots of the English language, tracing verb conjugations from 1,200 years ago to its current form. Over time, many past tense forms of verbs have died out in the English language and now only one persists as a rule: adding "-ed" to the verb-ending.

Some research on grammatical texts from Old English catalogued all the irregular verbs. Among them: the still irregular "sing"/"sang", "go"/"went" as well as the now-regularized "smite" which once was "smote" in Old English but since has become "smited" and "slink", which is now "slinked" but 1,200 years ago was "slunk". The researchers identified 177 irregular verbs in Old English and 145 that were still irregular in Middle English; however today, only 98 of the 177 verbs remain not "regularized".

After computing their usage frequency, the researchers concluded that the words that evolved most quickly into regular forms were used less than others. In reality, given two verbs, if one was used 100 times less frequently than others, it would evolve 10 times faster than them. They also predict that the past tense of *wed* will regularize from *wed* to *wedded* in near future.

Bela Sen, in her *The Computational Nature of Language Learning and Evolution* says—these findings are in line with lexical evolution models. "Languages are constantly changing", she notes. "In biological evolution, that fact has

received great attention, but linguistically, this is happening constantly".

76. The principal purpose of the passage is to:
(a) talk about the usage frequency of common words in four different Indo-European languages.
(b) make the point that even now, there is uncertainty about the origins of some words in Indo-European languages.
(c) inform the reader of the controversy surrounding a particular issue in linguistic research.
(d) emphasize the idea that in linguistic research, certain issues are still not resolved.

77. If you were to complete the last paragraph above, which of the following would be the best bet?
(a) Languages, all the more, are evolving over time just like the animal species do.
(b) This constant linguistic evolution has been the reason for all the confusion about the usage of specific words.
(c) In a nutshell, the survival of the fittest applies here, too.
(d) In the last analysis, nothing succeeds like success in language, too.

78. Which of the following cannot be inferred from the passage?
A. The past tense verb of drink should become drinked instead of drank earlier than cost becomes costed in future.
B. Many irregular verbs have been "regularized" with time.
C. English has many features in common with Spanish.
(a) A and C (b) A and B
(c) B only (d) A only

Passage 3: We go to school through lanes and back streets so that we won't meet the respectable boys who go to the Christian Brothers' School or the rich ones who go to the Jesuit school, Crescent College. The Christian Brothers' boys wear tweed jackets, warm woolen sweaters, shirts, ties, and shiny new boots. We know they're the ones who will get jobs in the civil service and help the people who run the world. The Crescent College boys wear blazers and school scarves tossed around their necks and over their shoulders to show they're cock o' the walk. They have long hair which falls across their foreheads and over their eyes so that they can toss their quaffs like Englishmen. We know they're the ones who will go to university, take over the family business, run the government, run the world. We'll be the messenger boys on bicycles who deliver their groceries or we'll go to England to work on the building sites. Our sisters will mind their children and scrub their floors unless they go off to England, too. We know that. We're ashamed of the way we look and if boys from the rich schools pass remarks we'll get into a fight and wind up with bloody noses or torn clothes. Our masters will have no patience with us and our fights because their sons go to the rich schools and, Ye have no right to raise your hands to a better class of people so ye don't.

79. The "we" the author uses throughout the passage refers to:
(a) his family.
(b) the poor children in his neighborhood.
(c) the children who attend rich schools.
(d) the reader and writer.

80. The passage suggests that the author goes to school:
(a) in shabby clothing.
(b) in a taxi cab.
(c) in warm sweaters and shorts.
(d) on a bicycle.

Direction for questions 81–85: ***A number of sentences are given below which, when properly sequenced, form a coherent paragraph. Each sentence is labelled with a letter. Choose the most logical order of sentences from among the five given choices to construct a coherent paragraph.***

81. A. But that conceals some wide differences.
B. Slovenia's GDP per head rises to more than 70% of the EU average.
C. The eight newcomers to the EU have an average GDP per head of about one-quarter of today's EU average.
D. However, measured by purchasing-power parity, which takes prices into account and so gives a better indication of local living standards, the whole region looks better off.
E. Incomes in the richest country, Slovenia, are three times those in the poorest, Latvia—a far greater gap than exists inside the EU now.
(a) CDAEB (b) ECDAB
(c) CAEDB (d) ECABD

82. A. Today's 15 EU countries make do with 11 national languages among them, or one for every 34 m people.
B. Half of central Europe's national languages are Slavic in origin.
C. Each has a different national language (if one accepts Czech and Slovak as separate languages), or roughly one for every 6 m people, not counting Russian and the Romany dialects spoken by the region's largest minorities.
D. Linguistically, the central Europeans are more diverse than the Union they join.
E. Latvian and Lithuanian stem from the Baltic branch of the Indo-European tree.
(a) ABCED (b) DCAEB
(c) BECDA (d) DCABE

83. A. A second view was that Dr Kelly may have been at fault for speaking too freely to an untrustworthy journalist, but that the government had exposed him when they should have shielded him.
B. When the news came out, many thought so.
C. Did government ill-treatment drive David Kelly to suicide?
D. The third and mildest critique was that although it might have been necessary to release his name, the government did it in a shabby, underhand way, thus contributing to his suicide.
E. Some saw Dr Kelly as a whistle-blower, a public-spirited civil servant who had exposed official deceit.
(a) CBEAD (b) CBEDA
(c) EACBD (d) EACDB

84. A. These activities are still crimes, but the authorities now turn a blind eye.
B. Seven years ago, anyone taking a drive with a member of the opposite sex, or wearing make-up, was punished by jail or a lashing.
C. Since 1997, Tehran has become a more humane, even permissive, place.
D. The movement should not be seen to be worthless or to have achieved nothing.
E. No one who remembers Iran before Mr Khatami was elected would dispute that it has changed.
(a) BAECD (b) CBEAD
(c) EDCBA (d) DECBA

85. A. Cynics should perhaps beware.
B. Many people scoffed when the city announced plans in 1990 to develop what was then just an expanse of marsh land, villages and old factories into its new financial district.
C. The "build it and they will come" mentality of Shanghai has, after all, paid off handsomely before.
D. Last year, it is reckoned, it sucked in just under $ 6 billion, more than a tenth of the total for the entire country.
E. Today Pudong, as the area is called, is a stunning conglomeration of soaring office towers and hi-tech factories that has attracted tens of billions of dollars in foreign investment.
(a) CBEAD (b) ECDBA
(c) ACBED (d) ACBDE

Direction for questions 86–91: ***In these questions, choose the word from the four alternatives (a), (b), (c) and (d) that is most nearly similar in meaning to the word given in CAPITAL letters.***

86. REVULSION:
(a) Apathy (b) Violence
(c) Disgust (d) Avenge

87. GRIT:
(a) Bold (b) Courage
(c) Grease (d) Level

88. SALACIOUS:
(a) Obscene (b) Wise
(c) Wholesome (d) Confident

89. CLOUT:
(a) Fear (b) Claw
(c) Joke (d) Power

90. FOMENT:
(a) Instigate (b) Shield
(c) Frustrate (d) Waver

91. REPERCUSSION:
(a) Reaction (b) Acceptance
(c) Resistance (d) Magnificence

Direction for questions 92–97: ***In these questions, choose the word from the four alternatives (a), (b), (c) and (d) that is most nearly opposite in meaning to the word given in CAPITAL letters.***

92. COMPASSIONATE:
(a) Indecisive (b) Unsympathetic
(c) Unlawful (d) Untrustworthy

93. RESTIVE:
(a) Buoyant (b) Placid
(c) Resistant (d) Insolent

94. GRADUAL:
(a) Energetic (b) Dynamic
(c) Rapid (d) Enthusiastic

95. RUDIMENTARY:
(a) Developed (b) Polite
(c) Pale (d) Weak

96. SALVAGE:
(a) Burn (b) Remove
(c) Confuse (d) Lose

97. SEDENTARY:
(a) Vivid (b) Afraid
(c) Indolent (d) Active

Direction for questions 98–100: ***Each question is a logical sequence of statements with a missing link, the location of which is shown parenthetically [(.....)]. From the four options available, choose the one that best fits into the sequence.***

98. This is the part of the world where amassing the money seems to be the only standard of success. As a professor of environmental science says, "People won't sacrifice their living standards for the sake of the environment". The lesson that Asia is yet to learn is that an improved environment is as much a part of higher living standard as lower fertility rates and better education. The remedies are known to all but too many ignore them—and will continue to do so until their own lives are touched by death, disease or pollution. Asia's blinkered attitude to the world it actually lives in, is perfectly summed up in: (.....).
(a) Let the next man clear up the mess!
(b) Your house is someone else's backyard!
(c) "You cry only when you see the coffin".
(d) There is a higher reality than dirt, pollution and human lives.

99. The DN Academy was born on a train. Last January, when Academy's accounts director began scouting for trainees, he would end up exhausted and exasperated. After days of backbreaking interviews, he discovered that most of the candidates didn't know the difference between an accounts manger and the chief accountant. A few days later, as he and Academy's

chief operating officer were travelling to Kharagpur, they had a brainwave – they could start a training institute of their own. In just this year, the agency set up the DN Academy in Dehradun offering a six-month comprehensive course in advertisement. (.....) And this is how their efforts have started to bear fruits.

(a) They were entirely unsuccessful in their initial efforts.
(b) Some of their difficulties seemed to last forever.
(c) Its first batch of 35 students passed earlier this month.
(d) Though they always wondered if he would ever be able to train the students successfully.

100. State officials warned raw-milk drinkers in the country that their risk of contracting Salmonella Dublin, a potentially lethal bacterium, was more than 150 times greater than the people who drink pasteurised milk. The official, however, had not stated the actual risk of contracting the bacterium through raw milk. (.....) If the chances of something happening to you are minuscule-even if your risk is ten times greater than someone else's don't lose sleep over it. Every small hazard posed by nature cannot be avoided at will and of course, cannot be eradicated forever. What will happen, will happen.

(a) Some dangers in life should be avoided at all cost, more so, when they can be fatal.
(b) The risk is considerably high and should compel you to think alternative ways out.
(c) The risk is considerably lower and worth panicking over.
(d) A particular bacteria can make you lose your sleep over itself.

ANSWER KEY

Section 1

1. (a)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (b)
9. (b)	**10.** (c)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (a)
17. (b)	**18.** (c)	**19.** (c)	**20.** (c)	**21.** (b)	**22.** (b)	**23.** (c)	**24.** (b)
25. (a)	**26.** (d)	**27.** (c)	**28.** (d)	**29.** (b)	**30.** (a)	**31.** (b)	**32.** (c)
33. (d)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (a)	**39.** (a)	**40.** (a)
41. (d)	**42.** (b)	**43.** (c)	**44.** (c)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (d)
49. (a)	**50.** (b)	**51.** (d)	**52.** (a)	**53.** (c)	**54.** (d)	**55.** (b)	**56.** (a)
57. (a)	**58.** (b)	**59.** (d)	**60.** (b)				

Section 2

61. (a)	**62.** (d)	**63.** (c)	**64.** (a)	**65.** (d)	**66.** (b)	**67.** (a)	**68.** (a)
69. (b)	**70.** (c)	**71.** (d)	**72.** (d)	**73.** (c)	**74.** (d)	**75.** (b)	**76.** (d)
77. (a)	**78.** (a)	**79.** (b)	**80.** (a)	**81.** (c)	**82.** (d)	**83.** (a)	**84.** (d)
85. (c)	**86.** (c)	**87.** (b)	**88.** (a)	**89.** (d)	**90.** (a)	**91.** (a)	**92.** (b)
93. (a)	**94.** (c)	**95.** (a)	**96.** (a)	**97.** (d)	**98.** (c)	**99.** (c)	**100.** (d)

MY PERFORMANCE SHEET (MOCK AT 2)

	Total Qs	Attempts	Right	Wrong	Marks	% Accuracy
Section 1 (Q1 to Q60)	60					
Section 2 (Q61 to Q100)	40					
Overall	100					

(Every correct response: +4 marks, every wrong response: –1 mark (minus 1).

Key Learnings from this test:

1. .....
.....

2. .....
.....

HINTS AND EXPLANATIONS

Section 1

1. Since p is a prime number greater than 11, hence, it is odd. Now, $p = x + y$. Obviously one of x and y has to be 2, and other one will be odd. Since question is asking for "could be", x could be 2. Hence, option (a) is the answer.

2. Number of boxes damaged = 700/35 = 20 = 2% of total boxes. Hence, total number of boxes = 1,000. Hence, option (b) is the answer.

3. Assume that (B + C) together have paid ₹ $2x$. Hence, A has paid ₹ x.
So, sum paid by all three = $x + 2x$ = ₹ 120.
$\Rightarrow$ sum paid by $A = x$ = ₹ 40
Similarly, sum paid by B = ₹ 30
Hence, sum paid by C = ₹ 120 – ₹ 40 – ₹ 30 = ₹ 50. Hence, option (d) is the answer.

Solution to questions 4–6

4. Following is the data (Table 2):

Table 2

Husband/Wife	*A*	*B*	*C*	*D*
A			35	42
B			45	23
C	46	39		
D	37	45		
Total				**312**

Hence, option (c) is the answer.

5. Following is the data (Table 3):

Table 3

Husband/ Wife	*A*	*B*	*C*	*D*
A	25			
B		36		
C			39	
D				22
Total				**122**

Table 3 gives the data for the couples with same education level. Hence, answer would be 570 – 122 = 448. Hence, option (d) is the answer.

6. Clearly the required difference is (45 + 39) – (42 + 37) = 5. Hence, option (b) is the answer.

Solution to questions 7–9

Table 4 is the data set:

Table 4

Years	Sales	Profit	Cost
2003	30	6	24
2004	50	11	39
2005	80	8	72
2006	70	5	65
2007	60	10	50
2008	80	12	68
2009	90	11	79
2010	100	8	92

7. Maximum profit percentage is in 2004 = 28% (approx.). Hence, option (b) is the answer.

8. Average sales value per year = ₹ 70 lacs and average profit value per year = ₹ 8.87 lacs ⇒ Average Cost per year = ₹ 61.12 lacs. Hence, average profit percentage = 14.5% (approximately). Hence, option (b) is the answer.

10. Answer = 6.5 feet (half of wall) – 2 feet (half of picture). = 4.5 feet

11. Total surface area = $2\pi r\,(r + h) = 1628$. Or, $2\pi r \times 37 = 1628$, or, $r = 7$ cm. Hence, $h = 30$ cm. Hence, volume = $\pi r^2 h = 4620$ cubic cm. Hence, option (b) is the answer.

12. If ₹ X is given to each C rated employee, then ₹ $1.5x$ and $2x$ are given to each B and A rated employee respectively. So total amount payment = $400x$ = ₹ 5,00,000. Hence, x = ₹ 1,250.
So each A type rated employee gets = $2x$ = ₹ 2,500.

14. Square of perimeter 36 cm = Square of side 9 cm
So, number of squares formed = $(36 \times 72)/9 = 32$. Hence, option (d) is the answer.

16. Since these numbers are consecutive integers, one of these has to be divisible by 3, and atleast one of these three will be divisible by 2. Hence, product xyz is divisible by 6. Using the same logic, we can verify statement III, that xyz will be divisible by 12 only if x is an even number. If x is an odd number, then xyz cannot be certainly divisible by 12. Hence, I is true but III is false. Now let us verify statement II:
Since x, y and z are consecutive integers such that $x < y < z$, we can rewrite y and z in terms of x:$y = x + 1$, and $z = x + 2$. Substitute these values in the equation:

$$(z - x)\,(y - x + 1) = 4$$

or, $[(x + 2) - x]\,[(x + 1) - x + 1] = 4$
Now, simplify the equation:

$$(x - x + 2)\,[(x - x + 1 + 1] = 4$$
$$(2)\,(1 + 1) = 2\,(2) = 4$$

Hence, statement II is true.
Hence, option (a) is the answer.

17. The old ratio of x to y was 4:6. If this ratio is cut in half, then the new ratio of x to y is 2:6.
The old ratio of x to z was 4:10. If this ratio is tripled, then the new ratio of x to z is 12:10.
In order to combine these two ratios into a new ratio of x:y:z, we must rewrite them so that the element in common, x, has the same coefficient. With the same x-coefficient, we can compare the ratios of x:y and x:z. Using a multiplier of 6 on the first ratio (x:y = 2:6) yields x:y = 12:36.
Since the new ratio of x:z is 12:10, we can combine the new x:y ratio that we multiplied by 6/6 with the new x:z ratio in order to arrive at an x:y:z ratio of 12:36:10. Hence, weight of x = 12 gms.
Hence, option (b) is the answer.

19. Going through the options:
(a) 2052—There are 45 perfect squares from 1 to 2052 (last one being 2025)—Hence, number of numbers written = 2052 – 45 = 2007. Same way of checking for option (b) too.

(c) 2054—There are 45 perfect squares from 1 to 2052 (last one being 2025)—Hence, number of numbers written = 2054 – 45 = 2009.

Hence, option (c) is the answer.

21. Total work = 6 × 12 = 72 days, hence, machines required to finish the work in 8 days = 72/8 = 9 machines. So, additional machines required = 3. Hence, option (b) is the answer.

22. $\log_8 x + \log_4 y^2 = 5$

$\Rightarrow \log_2 x^{1/3} + \log_2 y = 5$

or $yx^{1/3} = 25$ (i)

Similarly,

$\log_8 y + \log_4 x^2 = 7$

$\Rightarrow xy^{1/3} = 27$ (ii)

Multiplying (i) and (ii), we have

$(xy)^{4/3} = 212$

or $xy = 29$

or $xy = 512$

Hence, option (b) is the answer.

23. In 60 seconds, gear *B* takes 30 revolutions more than *A*. So, to take 6 more revolutions, it will take 12 seconds. Hence, option (c) is the answer.

Alternatively, this question can be done by options too.

25. $(ab)^2 = ccb$, the greatest possible value of 'ab' to be 31, since 312 = 961 so 18 < ab < 31. So the possible value of ab which satisfies $(ab)^2 = ccb$ is 21. So 212 = 441, a = 2, b = 1, c = 4. Hence, option (a) is the answer.

26. Let us solve this question through elimination 9 – 7 = 2 is even, therefore, option (a) not possible. 2 × 9 = 18 is even, therefore, option (c) is not possible. $\frac{3+9}{3} = \frac{12}{3} = 4$ is even, therefore, option (c) is not possible.

The correct option is (d).

28. Observe that

$f(3) = \log_2(\log_3 3) - \log_3(\log_2 3)$

$= \log_2 1 - \log_3(\log_2 3)$

$= -\log_3(\log_2 3) < 0$

We know that $\log_b a = \frac{\log_c a}{\log_c b}$

Using this and other rules of logarithm, we can deduce

$(x) = \log_2(\log_3 x) - \log_3(\log_2 x)$

$$= \log_2\left(\frac{\log_2 x}{\log_2 3}\right) - \frac{\log_2 \log_2 x}{\log_2 3}$$

$$= \log_2 \log_2 x - \log_2 \log_2 3 - \frac{\log_2 \log_2 x}{\log_2 3}$$

$$= \left(1 - \frac{1}{\log_2 3}\right)\log_2 \log_2 x - \log_2 \log_2 3$$

The definition of the logarithm implies $\log_2 x$ is an increasing function for $x > 0$ tends to infinity with x, we obtain that $\log_2 \log_2 x$ is increasing for $x > 1$ and tends to infinity with x. Thus $f(3) < 0$, $f(x)$ is increasing for $x > 3$, and tends to infinity with x. Hence, option (d) is true and the answer.

29. At 2 p.m., the lady is $100x + z$ miles from her home. In 18 minutes, she travels $100x + z - (10z + x) = 99x - 9z$ miles. So she has travelled at a rate of $11x - z$ miles every 2 minutes. In 60 minutes, she travels $100x + z - (10x + z) = 90x$ miles. So she has travelled at a rate of $3x$ miles every 2 minutes. Hence, $11x - x = 3x \Rightarrow z = 8x$. Since x and z are digits with x being a natural number, we get $x = 1$ and $z = 8$. Hence, at 2 p.m. she was 108 miles away from her home and she has travelled at a constant speed of 3 miles each 2 minutes. It follows that it took her 36 × 2 = 72 minutes to get home. Hence, she reached home at 3:12 p.m. Hence, option (b) is the answer.

31. $7! = 2^4 \times 3^2 \times 5 \times 7$

The required perfect square should be divisible by 16, 9, 5 and 7.

16 and 9 are already squares. Since 5 and 7 are not perfect squares, we multiply 7! by 5 and 7 to make it a perfect square.
So required perfect square
$= 24 \times 32 \times 5 \times 7 = 176400$
Hence, option (b) is the answer.

32. Sum of the number before x = sum of numbers after x
$\Rightarrow 1 + 2 + 3 + ... + (x - 1)$
$= (x + 1) + (x + 2) + ... + 49$
Adding $[1 + 2 + 3 + 4 + ... + (x - 1) + x]$ on both the sides
$\Rightarrow 2(1 + 2 + 3 + ... + (x - 1) + x$
$= (1 + 2 + 3 + ... + 49)$
$$2x\frac{(x-1)x}{2} + x = \frac{49 \cdot 50}{2}$$
$\Rightarrow \quad x^2 = 49 \times 25$
$\Rightarrow \quad x = 7 \times 5 = 35$
Hence, option (3) is the answer.

33. It will be divisible for all the factors of 36. Since 24 is not a factor or 36, hence, $x^{24} - y^{24}$ will not divide $x^{36} - y^{36}$.

34. P is a prime number > 71, hence, P is an odd number.
$(P - 1)$ and $(P + 1)$ are two consecutive even natural numbers, hence, one of these two numbers will be divisible by 4 and other by 2.
Also P will be of the format $6K \pm 1$, hence, one of $(P - 1)$ or $(P + 1) = 6K$ [if $6K + 1$, then $P - 1 = 6K$ and if $6K - 1$, then $P + 1 = 6K$]. Since we have already considered these two number to be even numbers, hence, $(P - 1)(P + 1)$ will be divisible by 24 $(= 2 \times 4 \times 3)$. (i)
Next P will be of the format $6K \pm 1$, so one of $(P + 2)$ or $(P - 2)$ will be a multiple of 3. (ii)
Also, since P is an odd prime number, so unit digit of $P = 1$ or 3 or 7 or 9.
If unit digit = 3 or 7, then one of $(P - 2)$ or $(P + 2)$ will be a multiple of 5. Or if unit digit = 1 or 9, then one of $(P - 2)$ or $(P + 2)$ will be a multiple of 5. (iii)
Combining (i), (ii) and (iii), N is a multiple of $24 \times 3 \times 5 = 360$.
Hence, N is definitely divisible by 120 and 360. It may or may not be divisible by 240.
So, option (b) is the answer.

35. Rationalize $\dfrac{1}{[17 + 12\sqrt{2}]}$
$$\Rightarrow \quad \frac{1}{x} = 17 - 12\sqrt{2}$$
Hence, $\quad x + \dfrac{1}{x} = 34$
Using the formula:
$$a^3 + b^3 = (a+b)^3 - 3ab\,(a+b)$$
Hence, answer $= 34^3 - 34 \times 3$
$= 39{,}202$
Hence, option (b) is the answer.

37. $Y = (x - k)^2 + 1 > 0$. Hence, K can take any real value. Hence, option (b) is the answer.

39. $BC = BR + RC = 5 + 11 = 16$ cm. Hence, option (a) is the answer.

40. Total number of single digit = 4
Total number of two digits = 16
Total number of three digits = 64
Total number of four digits = 256
Total numbers = 4 + 16 + 64 + 256 = 340
Hence, option (a) is the answer.

45. Since both a and b are positive, sum of roots > 0, and using Descartes' rule, both the roots will be positive.
Sum of roots $= \dfrac{-b}{a} = -a$. Now sum of roots > 0, $\Rightarrow -a > 0$. Now $0 \le a \le 1$, so sum of roots lies in between $-1 \le a \le 0$. Since sum of roots lies in between –1 and 0, hence, both the roots should lie in between –1 and 0. Hence, option (a) is the answer.

46. Option (a) can be eliminated by assuming $n = 1$ and showing that it is not a factor.
Option (b) can be eliminated by assuming $n = 3$ and showing that it is not a factor.

Option (c) can be eliminated by assuming $n = 6$ and expanding it through binomial theorem to show that it is not a factor.

Option (d) should be the answer through elimination. Further, assume $n = 5$, and it can be seen that it is a factor. Hence, option (d) is the answer.

47. 1st statement alone definitely gives the value of expression. Even second alone statement also gives the same value of expression for both the values of p. Hence, answer is option (b).

48. Question cannot be solved by: 1st statement—if m and n are fractions, then the answer would be different that had m and n been the integers. 2nd statement—This does not provide any limit. Even both together cannot solve the question. Hence, answer is option (d).

49. Statement 2 alone is sufficient, but statement 1 alone is not sufficient. Hence, answer is option (a).

50. As 3^4 gives remainder 1 when divided by 80.

We can write $(3)^{53!} = (3^4)^{\text{some even}}$, will give remainder 1 when divided by 80.

Hence, option (b) is the answer.

51.
$$A^2 = B^2 + 96$$
$$\Rightarrow \quad A^2 - B^2 = 96$$
$$\Rightarrow \quad (A - B)(A + B) = 96$$

Both of A and B should be even numbers for this calculation to hold good.

$(A - B)(A + B) = 48 \times 2$

or $(A - B)(A + B) = 24 \times 4$

or $(A - B)(A + B) = 16 \times 6$

or $(A - B)(A + B) = 12 \times 8$

These four combinations will give four different sets.

$A + B = 48$

and $A - B = 2$

$\Rightarrow$ $A = 25$

$B = 23$ and so on.

Hence, option (d) is the answer.

52. Given expression

$$= 3 * \left(\frac{3 \times (-1)}{3 + (-1)}\right) = 3 * \left(\frac{-3}{2}\right)$$

$$= \frac{3 \times \left(\frac{-3}{2}\right)}{3 + \left(\frac{-3}{2}\right)} = \frac{\frac{-9}{2}}{\frac{3}{2}} = -3$$

Hence, option (a) is the answer.

53. Required number of numbers = 3 × 7 × 7 × 7 + 1= 1029 + 1 = 1,030

Hence, option (c) is the answer.

54. Shyam travels 1/3rd of the distance at 60 kmph.

3/5 of (1 – 1/3) = 2/5th of the remaining at 40 kmph

Remaining (1 – 1/3 – 2/5) = 4/15 at (70 + 30)/2 = 50 kmph

Therefore, Average speed

$$= \frac{1}{\left(\frac{1}{3}\frac{1}{60}\right) + \left(\frac{2}{5}\frac{1}{40}\right) + \left(\frac{4}{15}\frac{1}{50}\right)}$$

= 40 kmph

Hence, option (d) is the answer.

55.
$$3 + (n - 1)7 = 63 + (n - 1)2$$
$$\Rightarrow \quad n = 13$$

Hence, option (b) is the answer.

56.
$$9^2 = 81$$
$$99^2 = 9801$$
$$999^2 = 998001$$
$$9999^2 = 99980001$$

...

...

$999999999^2 = 999999998000000001$

Hence, option (a) is the answer.

57. Unit digit of $215^{n^m} = 5$ for all the values of n and m. Hence, final unit digit = 6.

Hence, option (a) is the answer.

58. $[n] = 0$, if $n < 1$ and n is positive

From $\left[\frac{1}{5}\right]$ to $\left[\frac{1}{5} + \frac{239}{300}\right]$ all terms is zero.

From $\left[\frac{1}{5}+\frac{240}{300}\right]$ to $\left[\frac{1}{5}+\frac{299}{300}\right]$ all terms equal to 1.
Total number of terms is equal to 60.
Therefore, required expression is equal to 60.
Hence, option (b) is the answer.

59. Given that $2(ab+bc+ca)=24$; if a, b and c are the sides of the cuboids.
Applying $AM-GM$ inequality to ab, bc, ca

$$\frac{ab+bc+ca}{3} \geq \sqrt[2]{ab.bc.ca}$$

$$\Rightarrow \quad abc \leq 8$$

Therefore, the maximum volume of the cuboid is 8 cm^3.
Hence, option (d) is the answer.

60. $\log_8 x + \log_4 y^2 = 5$
$\Rightarrow \quad \log_2 x^{1/3} + \log_2 y = 5$
or $\quad yx^{1/3} = 2^5 \quad$ (i)
Similarly,
$\log_8 y + \log_4 x^2 = 7$
$\Rightarrow \quad xy^{1/3} = 27 \quad$ (ii)
Multiplying (i) and (ii), we have
$(xy)^{4/3} = 212$
or $\quad xy = 29$
or $\quad xy = 512$
Hence, option (b) is the answer.

Section 2

61. See Table 5:
Hence, option (a) is the answer.

62. See Table 6:
Hence, option (d) is the answer.

63. See Table 7:
Hence, option (c) is the answer.

64. The word *grow* in the first line indicates that the said tendency was there before too. So a second-degree adjective is warranted here. Between options (a) and (c), the former is certainly better. One does *not dance* to *consequences*, thus ruling out option d. We are left with options (a) and (b), of which the former is better due to the second-degree of the adjective.
Hence, option (a) is the answer.

65. The words *also been* are a clear indicator of the contrast which is intended by the author in this sentence. The second idea talks of the contrast between democratic set-ups and non-democratic set-ups. *Puzzlingly* is very much apt here as it captures the sense of surprise over something expected not happening, *i.e.*, democracies doing better than authoritarian regimes.
Hence, option (d) is the answer.

Table 5

enquire	investigate, probe	*inquiry*	try to know (informally)
tacit	unspoken	*taciturn*	speaking little
pleasing	is most often used to describe manners		
refuted	does not require any preposition		
purposely	deliberately	*purposefully*	meaningfully

Table 6

satisfactory	just alright, passable	*satisfying*	serving to satisfy, pleasing
extract	take out	*extricate*	rescue, save
gracious	kind	*graceful*	elegant
exceptionable	objectionable	*exceptional*	unique
misuse	wrong use	*disuse*	lack of use

Table 7

uncover	reveal deliberately	*discover*	become clear
hospitable	gracious	*hostile*	opposed

66. Note the words *rebuke* and *prejudice* carefully, which point to the *suitability* of *harshly criticizes* for the first blank. Now *rebuke* does not gel with *dislikes* as much as it does with *harshly criticizes*.
Hence, option (b) is the answer.

67. The passage mainly speaks about the idea of philosophy being the best placed to study God. Why it is in an ideal position to do so is duly explained by option (a). Options (b) and (c) are actually secondary remarks, totally unrelated to the idea in the passage.
Hence, option (a) is the answer.

68. "Some find it good, some do not"—the idea is best captured by option (a). The rest cannot be justified in the context of the passage.
Hence, option (a) is the answer.

69. Option (b) is the answer.

70. Option (c) is the answer.

71. Option (d) is the answer.

72. Specifically, the focus of the passage is on the evolution of feet, as is very obvious from the introductory lines as well as the subsequent discussion. Thus, choice (a) is very much generic while choice (b) is too narrow, as the passage also talks of birds. Option (c) is misguiding as the author nowhere discusses why evolution happened.
Hence, option (d) is the answer.

73. No technical term has been used anywhere, thus helping us rule out choice (c). A lot of examples exist in the passage to corroborate the ideas, especially from the bird kingdom.
Hence, choice (a) cannot be supported.
True, it can be faulted on the grounds having too many details, which can put off a reader. Hence, option (c) is the answer.

74. Choice (a) is amply supported by the last line of the second paragraph. Choices (b) and (c) find enough justification in the opening 3–4 lines of the second paragraph.
Hence, option (d) is the answer.

75. Since the author delves deep into the process and gives reasons for whatever he discusses, the tone is predominantly analytical. The term *scientific* applies to the topic, not the tone. Had he been non-committal (indifferent), why would have he written this piece in the first place?
Hence, option (b) is the answer.

76. Option (a) represents one of the ideas explored by Pagel (paragraph 3) in the course of his research to prove his point. So it is part of the whole story, certainly not the whole itself.
There is no controversy at all being discussed here, thereby ruling out option (c). In fact, the entire passage talks of complete agreement on the issue between the two sets of researchers.
Option (b) is not only factually wrong, but also is only a part of the focus of this entire passage.
Thus option (d) happens to be the best one in the sense that it is all-encompassing, beyond any doubt and is very clearly mentioned in the first two paragraphs.
Hence, option (d) is the answer.

77. The last few lines emphasize the idea of evolution of words in language like that of specie in biological evolution. A suitable closing line should be one that either negates or corroborates the idea most logically.
Option (b) does not make any logical sense as there is no confusion at all either hinted at or suggested here.
Option (c) is a misleading one as in the passage, the author does not even hint at the fact there is some competition among words.
Hence, option (a), which carries forward the idea appropriately (note the word *constantly* in the last line, which supports *all the more* in the option), is the most suitable answer.
Hence, option (a) is the answer.

78. Since *eat* is a higher-frequency word in contrast with *cost* in general, it should evolve into a regular form much more slowly than *cost*, in terms of the passage. So *A* is wrong. *B* is correct because the last line of paragraph 5 clearly mentions this idea. The idea contained in *C* is not supported by the contents of the passage. The only thing mentioned in paragraph 3 is the fact both Spanish and English have the same high-frequency words. Hence, option (c) is an incorrect choice here.
Hence, option (a) is the answer.

79. Option (b) is the answer.

80. Option (a) is the answer.

Solution to questions 81–85

81. If you go through the sentences, it becomes clear that the passage is about "the eight newcomers to the EU". Hence, *C* is the first sentence and this rules out (b) and (d). Note the word "rises" in *B*. This is a consequence of "measuring by purchasing-power parity", mentioned in *D*. Thus DB is a sequence. Hence, rule out (a).

82. "Each" in *C* refers to the central European countries mentioned in *D*. Hence, *DC* is a sequence which eliminates (a) and (c). *B* continues the theme of language referred to in *A*, hence, *DCAB* is better than *DCAE*, hence, option (d) *DCABE* is the answer.

83. This one is clearly connected. *C* raises the issue and *B* refers to *C*. This is followed by the three views: the first (though the word "first" has not been mentioned) in *E*, the "second view" in *A* and the "third ... critique" in *D*.

84. Note the words "these activities" in *A*. Obviously, they refer to the activities in *B*. Hence, *BA* is a sequence. Thus (b) is ruled out. Now, note the sentence *D*. What do the words "the movement" refer to? Certainly, nothing in the paragraph. Which implies *D* should be the first sentence referring to some movement in an earlier paragraph.

85. What does "the city" in *B* refer to? "Shanghai" in sentence *C*. Hence, *CB* is a sequence. Again, in *D* we are told in "it sucked in just under $6 billion ..." Which is a continuation of *E*, which tells us about the foreign investment Pudong attracts. Hence, *ED* is a sequence.

86. 'Revulsion' means—a strong feeling of distaste, or dislike. Out of the options given, option (c) 'Disgust' matches the meaning. Hence, option (a) 'Apathy' can be eliminated on the ground that 'Apathy' means 'not concerned' or 'lack of interest'.

87. 'Grit' means—firmness of character, or indomitable spirit. For example, 'Grit' can be used in a sentence like this:
Through sheer grit and determination, his company has weathered the recession and they're now ready to go places.
Out of the given options, option (b) 'Courage' matches the meaning of 'Grit' most.

88. 'Salacious' means—lustful, obscene. Hence, option (a) is clearly the answer.

89. 'Clout' means—a blow, or strong influence. Hence, option (d) matches the meaning of 'Clout' most.

90. 'Foment' means—to instigate somebody for doing something. Hence, option (a) is clearly the answer.

91. 'Repercussion' means—reaction or impact of an action. Hence, option (a) is clearly the answer.

92. 'Compassionate' means—a person who shows sympathy or is kind towards others. Hence, option (b) is clearly the antonym for this.

93. 'Restive' means—restless or impatient. Hence, option (b) 'Placid' means calm (remember movie—Lake Placid). Hence, it is the right answer.

94. 'Gradual' means—slowly, step by step. Hence, option (c) 'Rapid' means suddenly and thus, it is the answer.

95. 'Rudimentary' means—elementary. Hence, option (a) 'Developed' is opposite in meaning.

96. 'Salvage' means—saving (from fire, or any other calamity, etc). Hence, option (a) 'Burn' is opposite in meaning.

97. 'Sedentary' means—accustomed to sit or rest. Option (d) 'Active' is the opposite of this.

Mock Aptitude Test (Mock AT) 3

This test paper contains 100 questions.

1. There are 2 sections in this test paper—Section 1 containing 60 questions and Section 2 containing 40 questions.
2. You have 2 hours to complete this test.
3. All questions carry 4 marks each. Each wrong answer will attract a penalty of 1 mark.

Direction for answering the questions are given before each group of questions. Read these directions carefully and answer the questions. Each question has only one correct answer.

SECTION 1

Direction for questions 1–5: ***For each of the following questions, select the option which is CLOSEST in meaning to the capitalized word.***

1. Abandon:
(a) Recklessness (b) Ignore
(c) Caution (d) Refuse

2. Abjure:
(a) Discussion (b) Elope
(c) Perjure (d) Renounce

3. Baleful:
(a) Impious (b) Serious
(c) Restrain (d) Menace

4. Cajole:
(a) Coerce (b) Convince
(c) Persuade (d) Compel

5. Debase:
(a) Degrade
(b) Inception
(c) Strong Foundation
(d) Strong Character

Direction for questions 6–10: ***Questions ahead consist of sentences each of which contains one or two idioms. Four possible meanings labeled (a) through (d) are provided below each sentence. Choose the one which best expresses the meaning of the idiom.***

6. I would like to see Ramu do his own work for a change instead of always back-seat driving.
(a) driving a car form the back
(b) being critical of work being done by others
(c) offering advice
(d) annoying

7. He proved to be quite a wet blanket at the party for he spoke to no one and morosely sat by himself.
(a) he was crying
(b) discouraged the rest from having fun
(c) he wet the blanket
(d) he had covered himself with a wet blanket

8. The ship was about to weigh anchor when the storm came.
(a) weight the anchor
(b) leave the docking berth
(c) raise the anchor
(d) drop the anchor

9. Debu had a chequered carres since I first knew him as a clerk in the local bank.
(a) had a variety of jobs and experiences
(b) a career which helped him make lots of money
(c) a career where he signed a lot of cheques
(d) did odd jobs

10. Women should be paid the same as men when they do the same job, for surely, what is sauce for gander is sauce for the goose.
(a) both goose and gander should be equally treated
(b) what is thought suitable for a man should also be for a woman
(c) goose and gander eat the same sauce
(d) the principle of equal treatment should be implemented

Direction for questions 11 and 12: ***In each of the sentences, part/s of the sentences are left blank. Beneath each sentence, four different ways of completing the sentence are indicated. Choose the word or set of words for each blank that best fits the meaning of the sentence as a whole.***

11. Even as we are becoming more fully aware of their enormous _____ and _____ value, coral reefs are being lost in the United States, just as they are being destroyed in other parts of the world.
(a) marketing–sociological
(b) ecological–economic
(c) physical–biological
(d) geographical–tourist

12. As members of a weird religious cult with _____ rituals and bizarre edicts, rock band Evanescence were always going to struggle crossing over into the musical _____.
(a) esoteric–mainframe
(b) abstruse–norm
(c) strange–edict
(d) arcane–mainstream

Direction for questions 13–17: ***Each of the following questions has a paragraph from which the last sentence has been deleted. From the given options, choose the one that completes the paragraph in the most appropriate way.***

13. Yes, nobody wants to be Japan, the fallen angel that went from one of the fastest growing economies in the world for more than three decades to one that has slowed to a crawl for the past 18 years. No one wants to live with the trauma of the deflation (falling prices) that Japan has repeatedly experienced. No one wants to navigate the precarious government-debt dynamic that Japan faces, with debt levels far above 100% of GDP-even if one factors in the Japanese government's vast holdings of foreign-exchange reserves. _____.
(a) No one wants to go from being a world-beater to a poster child for economic stagnation.
(b) And yet, visitors to Tokyo today see prosperity everywhere.
(c) Although hardly in crisis yet, Japan's fiscal situation grows more alarming by the day.
(d) Until now, the government has been able to finance its vast debts locally, despite paying paltry interest rates even on longer-term borrowings.

14. On 1st March, Philip Morris, a tobacco giant, sued eight American retailers for selling counterfeit versions of its Marlboro cigarettes. Governments are also boosting their efforts to crack down on counterfeiting which deprives them of tax revenue in addition to harming legitimate businesses. Thanks to the rise of the internet and of extended international supply chains, and more recently, to the global economic downturn, counterfeit goods are everywhere. Fake Porsches and Ferraris zoom along the streets of Bangkok. A German bank has discovered an ersatz gold ingot made of tungsten in its reserves,

according to a German television channel investigating persistent reports that many of the world's financial institutions have been similarly hoodwinked.

(a) Counterfeiting used to be a luxury goods problem, but now people are trying to traffic counterfeit items that have a wider effect on the economy.
(b) NASA, America's space agency, has even bought suspect materials.
(c) Several factors have contributed to the growth of counterfeiting in recent years.
(d) Fake goods are proliferating, to the dismay of companies and governments.

15. The rate of conviction in SC and ST atrocity cases in the State has reached 22 per cent from 10 per cent last year due to the proper investigations done by the departments concerned and the increased awareness that enabled the aggrieved persons to get justice. The government has been appointing special public prosecutors in most sensational and long-pending cases and has issued guidelines to increase people's understanding of the SC/ST (Prevention of Atrocities) Act. _____.

(a) The people's representatives should now acquaint people with the legal remedies available for them.
(b) The Act has been implemented swiftly on complaints of atrocities.
(c) The Police Department act swiftly on complaints of atrocities.
(d) These measures facilitated speedy disposal of SC and ST atrocity cases.

16. Make no mistake: the setting matters. There are many ways to listen to classical Indian music-in the private, somewhat sterile perfection of the CDs and DVDs we play at home; in the concert sabhas of Mylapore and T. Nagar; on the music channels on TV or on YouTube, which now carries a little or a lot of almost everything, often in choppy, byte-size pieces. But I happen to think that this music sounds best outside, on a hot spring or summer night, with the taste of pollen and dust on your tongue and mosquitoes circling around your feet.

(a) We tend to forget that much of the classical music of India, both Hindustani and Carnatic, was meant for intimate settings like the royal court.
(b) Taking classical music out of 'sabhas' into vibrant cultural spaces brings out its ethereal dimensions.
(c) Music is a tangible thing, to be felt in the pores of your skin no less than in the ear; you just can't do that very well in the concert halls.
(d) One of the important things about the settings is the recreation of a context for listening in which subtlety and intimacy can assume their natural role.

17. In 2009, China overtook Germany to become the world's largest exporter. Exactly half the trade disputes that were filed at the World Trade Organization (WTO) last year involved China. These facts are not unrelated. As Pascal Lamy, the WTO's chief, pointed out in January, the scope for trade friction increases as countries trade more. _____.

(a) Hence, China will test the WTO's dispute-settlement system.
(b) Hence, disputes between China and other countries are only to be expected.
(c) Hence, China's increasing propensity to bring disputes to the WTO is part of a broader shift.
(d) Hence, China has moved from learning-by-watching, to being an active participant in formal dispute settlement.

Direction for questions 18–20: ***Choose the best way of summarizing the text given in the question.***

18. When the next full-scale global financial crisis hits, let it not be said that the International Monetary Fund never took a stab at forestalling it. Recently, the IMF proposed a new global tax on financial institutions loosely in proportion to their size, as well as a tax on banks' profits and bonuses.

(a) The IMF has proposed a new global tax on financial institutions and banks in an attempt to forestall future financial crises.

(b) The IMF new global tax on financial institutions and banks will prevent future financial crises.

(c) The IMF has proposed a new global tax on financial institutions and banks which will forestall financial crises.

(d) The IMF has proposed a tax on financial institutions/and banks' profits and bonuses that will prevent financial crises.

19. Russia and the United states have signed a new strategic nuclear-arms reduction treaty. Officially, the treaty cuts their weapons by one-third; in fact, each party will decommission only several dozen. Nevertheless, the treaty is a considerable achievement. It normalizes political relations between the two countries, thereby facilitating their further cooperation and rapprochement.

(a) The new strategic arms reduction treaty signed between Russia and the United States may facilitate further cooperation between them.

(b) The new strategic arms reduction treaty signed between Russia and the United States may lead to cordial relations between them.

(c) Russia and the United States have signed a strategic nuclear-arms reduction treaty to reduce weapons by one-third.

(d) Russia and the United States have signed a new strategic nuclear-arms reduction treaty which normalizes political relations between the two countries.

20. A severe food crisis currently threatens southern Sudan. In East Africa, where millions of people already are dependent on food aid, a sharp rise in the cost of staple crops looms. These are just the latest sources of concern in a turbulent period that began two years ago when food shortages hit many countries in Africa and Asia due to a worldwide spike in prices. Higher food prices meant that poor people, already struggling to meet basic human needs, were pushed deeper into poverty.

(a) An impending food crisis looms over southern Sudan where higher food prices have pushed people deeper into poverty.

(b) The food crisis in Africa and Asia, especially in southern Sudan already struggling under food shortages, higher prices, and poverty may worsen owing to further a rise in the cost of staple crops.

(c) As many countries in Africa and Asia are experiencing prolonged food shortages, an impending food crisis threatens Sudan due to a rise in the cost of staple crops.

(d) The food crisis in Africa and Asia already struggling under food shortages, higher prices, and poverty may worsen owing to further a rise in the cost of staple crops.

Direction for questions 21–25: ***In each of the following sentences, a part of the sentence is underlined. Beneath each sentence, four different ways of phrasing the underlined part are indicated. Choose the best alternative from among the four.***

21. The MP <u>rose up to say that, in her opinion, she thought the Women's Reservation Bill should be passed on</u> unanimously.

(a) rose to say that she thought the Women's Reservation Bill should be passed
(b) rose up to say that, the Women's Reservation Bill should be passed on
(c) rose to say that, in her opinion, she thought that the Women's Reservation Bill should be passed
(d) rose to say that, in her opinion, the Women's Reservation Bill should be passed on

22. Mr. Pillai, the president of the union and <u>who is also a member of the community group</u>, will be in charge of the negotiations.
(a) since he is a member of the community group
(b) also being a member of the community group
(c) a member of the community group
(d) in addition, who is a member of the community group

23. Since the advent of cable television, at the beginning of <u>this decade, the entertainment industry took</u> a giant stride forward in our country.
(a) this decade saw the entertainment industry taking
(b) this decade, the entertainment industry has taken
(c) this decade, the entertainment industry had taken
(d) this decade, the entertainment industry took

24. His mother made great sacrifices to educate him, moving house on three occasions, and <u>severing the thread on her loom's shuttle whenever Mencius neglected his lessons to make him understand the need to persevere.</u>
(a) severing the thread on her loom's shuttle whenever Mencius neglected his lessons to make him understand the need to persevere.
(b) severed the thread on her loom's shuttle whenever Mencius neglected his lessons to make him understand the need to persevere.
(c) severed the thread on her loom's shuttle whenever Mencius neglected his lessons to make him understand the need for persevering.
(d) severing the thread on her loom's shuttle whenever Mencius neglected his lessons, to make them understand the need to persevere.

25. It was <u>us who had left before he arrived.</u>
(a) we who had left before time he had arrived.
(b) us who had went before he arrived.
(c) us who had went before had arrived.
(d) we who had left before he arrived.

Direction for questions 26–30: ***Read the passage given below and solve the questions based on it.***

Most discussions of colonial rule distinguish between "direct" and "indirect" rule. Although the defining principles of the tow forms of rule differed considerably, in practice direct rule administrations-most of which were in French colonies-incorporated elements of indirect rule, especially during the later colonial era.

Under direct rule, colonies were divided into districts administered by European appointees. In French West Africa, the chief district administrator, the commandant de circle, was responsible for regional tax collection, labor and military recruitment, public works, education, local court cases, and the execution of dictates handed down from the colony's governor. Although an Africans staffed the lower levels of the commandant's bureaucracy, most of the top officials were European. The commandant was also expected to maintain public order and discourage offensive or "backward" local customs. Thus direct rule was seen as a means of "civilizing" as well as controlling African populations.

Frequently, regional administrative borders cut through preexisting African polities and ethnic communities. In parts of French West Africa, these borders marked a deliberate policy to divide and weaken militarily powerful groups such as the Baule and the Fon, who for years had resisted foreign occupation. Another part of this policy, at least in the years during and after conquest, was a campaign to dethrone the "great chiefs" of these and other African kingdoms and replace them with more malleable appointees. In general, these traditional authorities were viewed not only as political threats but as obstacles to the "civilizing mission" of direct rule.

Direct rule encountered at least three practical problems. First, European governments lacked the personnel needed to administer their African colonies effectively. Although schools of colonial administration were well established in Europe by the early 20th century, debilitating diseases and harsh climates discouraged officers from seeking posts in Africa. European colonies often had fewer than one colonial officer for every 22,000 Africans, as was the case in Kenya in 1921. the Belgian Congo had a mere 2,384 officials for an African population of 9.4 million; French West Africa, with an African population of 15 million, employed only 3,660 French personnel.

Second, these administrators were responsible not only for large subject populations, but for vast territories. Travel over long distances was slow and arduous, and communication difficult. As a result, colonial governors had little contact with remote areas, where district administrators often proved incapable or uninterested in carrying out the dictates of their superiors.

Finally, colonial officials' limited knowledge of local languages and customs, combined with their often total lack of legitimacy, undermined their ability to recruit labor, collect taxes, or carry out other administrative duties effectively. In the end, both the "great chiefs" and local village chiefs-both of which had greater legitimacy and more local-level contacts-proved indispensable to the project of colonial administration. For all these reasons, most European powers began to modify their approach to colonial rule in the early 20th century.

26. Which of the following statements is not probably right?

(a) There is clear-cut demarcation between "direct" and "indirect rule" under colonization making them water tight compartments.

(b) Definition-wise the tenets of operation of the two form of rule, *viz.*, direct and indirect are at significant variance from each other.

(c) Division of French colonies into districts placed under the charge of European appointees was a tenet of direct rule.

(d) Contrary to theoretical explanation, in practice, the methodology of direct rule has embedded in it elements of indirect rule as well.

27. Which of the following is not an expected duty of European appointees?

(a) Regional tax collection, labour and military recruitment, public works education, local litigation.

(b) Ensuring that the lower level of African staff abided by the dictates of the Colonial Governor.

(c) Act as an instrument propagating civilization through withdrawal of support to offensive local customs.

(d) None of these

28. According to the author, the Regional administrative borders through already de-marked African racial groups sought to achieve other than which of the following:

(a) To thwart the pace of the civilization mission upheld by direct rule.

(b) To pose a threat to Baule and the Fon which comprised militarily powerful groups who had been waging a war against foreign occupation.

(c) To substitute the acclaimed chiefs of African kingdoms with appointees who are less resistant.
(d) To dispense with and combat political threats created by traditional authorities.

29. The statement that correctly sums up the author's views on the factors debilitating implementation of direct rule is:
(a) Schools of colonial administration were well established in Europe by the early 20th century.
(b) Colonies in Kenya, Belgian Congo and French West Africa had identical disadvantage of disproportionate distribution of Colonial officer's vis-à-vis local residents.
(c) Lack of effective administrative personnel in European Government, dissuading African climatic conditions, insufficient supervisory staff posted over a vast territory devoid of transport and communication facilities and the ignorance of local language and custom.
(d) The jurisdiction of the administration was unwieldy.

30. Though relevant to the issue under discussion, one of the following statements is not the most important of the practical problems that the European rulers faced that led to the modification of approach to colonial rule of European powers in the early 20th century in Africa and elsewhere.
(a) The ratio between Colonial officers and local Africans in European colonies was in the range of 1:2200.
(b) There was absence of constant interaction between colonial governors and local district administrators who were either incapable or far from interested in adhering to instructions of the former.
(c) Lack of an effective tax collection mechanism that suffered a further setback on account of lack of legitimacy of colonial officers.
(d) Lack of greater legitimacy and local level contacts, a privilege confined to the great chiefs and level village chiefs, was becoming increasingly inevitable for colonial administration.

Direction for questions 31–33: ***Sentences given in each question containing either four statements or five statements, when properly sequenced, form a coherent paragraph. Each sentence is labeled with a letter. Choose the most logical order of sentences from among the given choices to construct a coherent paragraph.***

31. A. Inching along on six claw-like legs, the insect carts its shelter everywhere, lashing it to a branch when it stops to rest or molt.
B. To be able to mate the male needs to stretch his abdomen several times the normal length of his body.
C. To lessen the perils of living outdoors, the evergreen bagworm constructs a mobile home of silk which it camouflages with bits of foliage from its host plants.
D. Both male and female larvae construct bags, but as adults only males are winged and able to fly free.
(a) ACDB (b) ACDB
(c) CADB (d) CABD

32. A. With Talaash she incorporated a more South Indian element.
B. The Indian diva is known for her distinctive blend of Carnatic with Western, Traditional with Pop, East-meets-West kind of music.
C. Ask her how she's evolved musically and she'll point to her forthcoming album.
D. Although she began with plain Western music and Hindi lyrics, by Dhuan she

was using Indian instruments, albeit north Indian ones.

(a) BADC (b) ACBD
(c) BDAC (d) ABCD

33. A. Similarly, turning to caste, even though being lower caste is undoubtedly a separate cause of disparity, its impact is all the greater when the lower-caste families also happen to be poor.

B. Belonging to a privileged class can help a woman to overcome many barriers that obstruct women from less thriving classes.

C. It is the interactive presence of these two kinds of deprivation—being low class and being female—that massively impoverishes women from the less privileged classes.

D. A congruence of class deprivation and gender discrimination can blight the lives of poorer women very severely.

E. Gender is certainly a contributor to societal inequality, but it does not act independently of class.

(a) EABDC (b) EBDCA
(c) DAEBC (d) BECDA

Direction for questions 34–37: ***Read the passage below and solve the questions based on it.***

Indians are known for their obsessive and compulsive fascination for gold. India is the largest importer and largest consumer of the yellow metal as Indians buy about 25 percent of the world's gold. In 2008, India imported around 400 tons of it. About 80 percent of the world's extracted gold is **fashioned** as jewellery. However, most of us don't know or don't think about the environmental cost of the metal. For instance, extracting enough gold to **forge** a solitary, no-frills wedding band ultimately translates into roughly 20–30 tons of waste. At some mines in Nevada (USA), 100 tonnes or more of earth have been excavated for a single ounce of gold.

The waste is of two forms: redundant rock, which is typically piled as flat heaps in locations near the mining site and the effluent or tailings which are a result of chemical processing of the mined ore. Sulphides in the redundant rock react with oxygen heavy metals like cyanide, cadmium lead and mercury harmful to people even at miniscule concentrations. The tailings component is typically a thick slurry laced with cyanide, aluminum, copper, lead, and mercury: enough to **decimate** fish populations of water environments it is disposed of into. Disposal of wet tailings into water bodies has been effectively banned in developed countries but it continues to be practiced in most developing nations. There is also a very real danger of surface water and groundwater table contamination on account of these heavy metals.

In fact, gold mining generates more waste per ounce than any other metal and the effects are starting. Mining for gold has left huge gouges on the face of the earth so **massive** that they can be seen from space.

According to a study, respiratory ailments, soil and water contamination, thick blankets of dust. Withering of coconut trees and changes in land pattern use are some of the common features of the urban area around a particular gold mine in Karnataka. Many areas are reported to have become in fertile because of soil contamination. They contain a percentage of heavy metals enough to retard plant growth.

Similarly according to another report in 2008, nearly seven years after the closure of these mines, the people of this region continue to face serious environment and health problems, particularly in July and August, due to winds in these months that carry with them cyanide particles from the dust piles in the abandoned mines. When the mines were operational, a layer of red soil used to be put over these dust piles before these crucial months to prevent the cyanide particles from being carried away be the heavy winds. Now that the mines have been closed, the imitative measures have **ceased** as well.

People from socially and economically marginalized communities turn to mining to escape acute poverty, unemployment, and landlessness. In some cases, their homes and farms may be acquired for large scale gold mining. While compensation is promised to them, it may take a year or two to kick in. Till then forced to eke out a bare livelihood mostly in a kind of lottery system, they resort to crude methods to separate any flecks of gold that may be there in the discarded waste rock using mercury. In the process, destroy themselves slowly as well as their environment. The shanty towns which **inevitably** come up around the large-scale mining sites only serve to add to the problem. Given their illegal and therefore unrecognized nature, they lack basic amenities like garbage disposal and water supply and sanitation, becoming another unsightly blot on the landscape.

According to the World Gold Council, while estimates of numbers engaged in artisanal mining vary widely. They range between 13 and 20 million men, women and children from over 50 developing countries. Indeed, it is believed that as much as a quarter of the world's gold is supplied by artisanal manners. Their efforts to earn themselves a daily wage have resulted in huge habitual loss and destruction. For example, huge patches of land, once home to lush trees in the island of Borneo in Indonesia, are being swiftly rendered treeless and lifeless pits of waste. Incidentally, the island is highly famed for its rich biodiversity. Combined with heavy pressures from the logging lobby and need for cheap power through hydroelectricity and relentless mining activity. It is hard to imagine if Borneo will manage to retain its crown.

Why should these facts about gold mining bother us? After all, we just import the metal: we do not mine it here to the extent other countries do. That is about to change though. New Delhi has big plans to **fuel** growth in the mining sector and is looking to open investment in gold mining in the country—and in a big way.

However, India's environmental track record in mining has been anything but stellar. And this is something that requires close attention in light of the planned increased forays into gold mining. Even with the comparatively minuscule amounts of gold mining done so far, we have tripped up on environmental considerations. Geologically, India's terrain is very similar to those in other parts of the world where there have been huge gold finds. What we need to do is to learn from the mistakes committed by certain developed countries in their own backyard. We have a whole series of examples of where things have gone wrong from other developing countries. We need to use these insights to our advantage and quickly.

34. According to the author, how are gold mines detrimental to the environment as well as public health even after their closure?

(a) The layer of red soil used to cover dust piles in these mines seeps into the ground water thereby making it unfit for consumption.

(b) The mines weaken land mass and increase the chances of occurrence of earthquakes especially after there is no one looking after them.

(c) The imitative measures adopted after the closure of these miens are not supervised adequately hence, are highly damaging.

(d) Winds in specific months carry harmful heavy metal particles from the dust heaps accumulated in these mines.

35. Which of the following is/are ill effect of gold mining as mentioned in the passage?

A. Waste generated while mining for gold is harmful even in small quantities.

B. Groundwater gets polluted due to the release of heavy metals generated from the mining of gold.

C. Gold mining activities cause respiratory illnesses in people.

(a) Only A

(b) All A, B, and C

(c) Only B and C
(d) Only B

36. What is the author's intention behind writing this passage?
(a) To discourage the Indian Government from investing in gold mining.
(b) To bring forward the problems associated with gold mining for India's benefit
(c) To discourage Indians from buying gold based on its repercussions
(d) To advocate the import of gold instead of mining of it.

37. Why according to the author, is the environmental cost of gold very high?
(a) As gold is the only metal which generates harmful waste on its excavation.
(b) As excavation of gold releases the highest amount of pollutants into the air as compared to any other metal.
(c) As gold excavation is a very tedious process and requires usage of a number of environmentally destructive resources.
(d) As the amount of gold recovered in proportion to the land excavated is negligible.

Direction for questions 38–40: ***In each question, the word at the top of the table is used in four different ways, numbered 1 to 4. Choose the option in which the usage of the word is incorrect or inappropriate.***

38. Near:

1.	I got there just after you left—near miss!
2.	She and her near friend left early
3.	The war led to an near doubling of oil prices
4.	They came near to tears seeing the plight of the victims

39. Hand:

1.	I have my hand full, I cannot do it today
2.	The minister visited the jail to see the breach at first hand
3.	The situation is getting out of hand there
4.	When the roof of my house was blown away, he was willing to lend me a hand

40. For:

1.	He has a great eye for detail
2.	We are waiting for the day
3.	I can't bear for her to be angry
4.	It couldn't be done for ever

SECTION 2

41. Sum of the first n terms ($n > 1$) of an AP is 153 and the common difference is 2. If the first term is an integer, then the number of possible values of n is:
(a) 2 (b) 3
(c) 4 (d) 5

42. What is the minimum number of squares that can be cut out of a cloth of 16 m × 6 m such that no cloth is wasted?
(a) 96 (b) 24
(c) 5 (d) 6

43. For what value of 'n' will the remainder of 451^n and 448^n be the same when each of the numbers is divided by 9?
(a) 2 (b) 3
(c) 6 (d) 4

44. Which of the following is closest to $\frac{10^8 - 10^2}{10^7 - 10^3}$?
(a) 1 (b) 10
(c) 10^2 (d) 10^3

45. A right circular cone, a right circular cylinder and a hemisphere, all have same radius. Heights of cone and cylinder equal their diameters. Then their volumes are proportional, respectively, to:

(a) 1:3:1 (b) 2:1:3
(c) 3:2:1 (d) 1:2:3

46. Average age of A, B and C is 82 years, average age of A, B and D is 79 years, average age of A, C and D is 83 years and the average age of D, B and C is 81 years. Who among A, B, C and D is the oldest?

(a) A (b) B
(c) C (d) D

47. Find the value of

$$\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc}\ ?$$

(a) 0 (b) 1
(c) 2 (d) Infinity

48. In solving a problem that reduces to a quadratic equation one student makes a mistake only in the constant term of the equation and obtains 8 and 2 for the roots. Another student makes a mistake only in the coefficient of the first degree term and finds –9 and –1 for the roots. The correct equation is:

(a) $x^2 - 10x + 9 = 0$
(b) $x^2 + 10x + 9 = 0$
(c) $x^2 - 10x + 16 = 0$
(d) $x^2 - 8x - 9 = 0$

49. Of the following sets, the one that includes all values of x which will satisfy $2x - 3 > 7 - x$ is:

(a) $x > 4$ (b) $x < 10/3$
(c) $x = 10/3$ (d) $x > 10/3$

Direction for questions 50–54: ***Study the Table 1 carefully and answer the questions based on it.***

Table 1 shows the percentage population of six states below poverty line and the proportion of male and female.

Table 1

State	Percentage of Total Population Below Poverty Line	Proportion of Male and Female	
		Below Poverty Line	Above Poverty Line
		Male: Female	Male: Female
A	12	3:2	4:3
B	15	5:7	3:4
C	25	4:5	2:3
D	26	1:2	5:6
E	10	6:5	3:2
F	32	2:3	4:5

Everybody is either above the poverty line or below the poverty line.

Answer to some of the questions may be fractional. For the sake of calculation convenience, take integral values in such cases.

50. The total population of State A is 3,200 thousands, then what is the approximate number of females above poverty line in state A (in thousands)?

(a) 1,206 (b) 2,112
(c) 1,800 (d) 1,950

51. If the total population of State C and D together is 18,000, then what is the total number of females below poverty line in these two states?

(a) 5,000
(b) 5,500
(c) 4,800
(d) Cannot be determined

52. If the population of males below poverty line in State B is 500 then what is the total population of State B?

(a) 14,400 (b) 6,000
(c) 8,000 (d) 7,600

53. If the population of males below poverty line in State A is 3,000 and that in State E is 6,000, then what is the ratio of the total population of State A and E?

(a) 3:4 (b) 4:5
(c) 21:25 (d) None of these

54. If in State E population of females above poverty line is 19,800 then what is the population of males below poverty line in that state?
(a) 5,500 (b) 3,000
(c) 2,970 (d) None of these

55. In a group of cows and chickens, the number of legs was 14 more than twice the number of heads. The number of cows was:
(a) 5 (b) 7
(c) 10 (d) 12

Direction for questions 56–59: ***Go through the data set given in Table 2 and solve the questions based on it.***

Table 2

Company	2011	2010
Maruti	77,086	92,874
Hyundai	26,677	28,601
Tata	16,829	25,312
Mahindra	15,664	13,796
Toyota	11,693	6,361
Ford	7,392	7,925
Honda	6,987	3,531
Skoda	1,812	1,512
Mercedes	614	573

Table 2 presents the sales volume of the car manufacturing companies in the month of August for the given two years.

56. Which car manufacturing company listed above has shown maximum percentage increase in the sales volume in August, 2011 over August, 2010?
(a) Toyota (b) Skoda
(c) Honda (d) None of these

57. How many companies have shown a growth in the sales volume in 2011 as compared to 2010?
(a) 4
(b) 5
(c) 6
(d) Cannot be determined

58. Which of the following statements is true for the month of August?
(a) Number of companies seen the growth in sales volume > Number of companies seen a decrease in sales volume
(b) Number of companies seen the growth in sales volume < Number of companies seen a decrease in sales volume
(c) Number of companies seen the growth in sales volume = Number of companies seen a decrease in sales volume
(d) Cannot be determined

59. For the month of August, 2011, sales volume of which company is closest to the average sales value of August, 2011?
(a) Hyundai
(b) Tata
(c) Mahindra
(d) Cannot be determined

60. Instead of walking along two adjacent sides of a rectangular field, a boy took a short-cut along the diagonal of the field and saved a distance equal to 1/2 of the longer side. The ratio of the shorter side of the rectangle to the longer side is:
(a) 1/2 (b) 2/3
(c) 1/4 (d) 3/4

61. In a house, there are six 40-watt lamps which are on for 5 hours a day and three 80-watt fans which are on for 10 hours a day. If electricity costs ₹ 2 per kilowatt hour, what is the monthly electricity bill?
(a) ₹ 216 (b) ₹ 280
(c) ₹ 315 (d) ₹ 400

62. A driver's income consists of his salary and tips. During one week his tips were 5/4 of his salary. What fraction of his income came from tips?
(a) 4/9 (b) 5/9
(c) 5/8 (d) 5/4

63. What is the value of the following expression?
$(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 - x)$
(a) $1 + x^{16}$ (b) $1 - x^{16}$
(c) $x^{16} - 1$ (d) $x^8 + 1$

64. Find the unit digit of the expression $199^{2n} + 144^{3n}$, where n is a natural number.
(a) 5 (b) 7
(c) Either 5 or 7 (d) 3

65. A square hole of cross-sectional area 4 cm^2 is drilled across a cube with its length parallel to a side of the cube. If an edge of the cube measures 5 cm, what is the total surface area of the body so formed?
(a) 158 cm^2 (b) 190 cm^2
(c) 166 cm^2 (d) 182 cm^2

66. Product of 1st 15 natural numbers will not be a multiple of:
(a) 98 (b) 57
(c) 224 (d) 459

67. A stick of length 15 cm leans of a sphere of radius 5 cm lying on the floor. A certain length of the stick extends beyond its point of contact with the sphere. What is the extended length if the point of contact is exactly 8 cm above the floor?
(a) 4 cm
(b) 5 cm
(c) 6 cm
(d) Cannot be determined

68. Maulik and Gyan work in the same office and stay in the same home. When they walk to their office with normal speed, Maulik reaches 15 minutes before Gyan. But if Gyan leaves 10 minutes before his usual time, Maulik has to walk at twice his original speed to reach 15 minutes before Gyan reaches office. What is the ratio of speed of Gyan to Maulik?
(a) 3:4
(b) 4:7
(c) 5:7
(d) Cannot be determined

69. The sum of three numbers is 98, the ratio of the first to the second is 2/3, and the ratio of the second to the third is 5/8. The second number is:
(a) 15 (b) 20
(c) 30 (d) 32

70. Two candles of the same height are lighted at the same time. The first is consumed in 4 hours and the second in 3 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the height of the second?
(a) 3/4 hrs (b) 1½ hrs
(c) 2 hrs (d) None of these

71. The perimeter of a rectangular field is 84 m. If the length of the field is increased by 3 m and its breadth decreased by 1 m, then the area of the field is increased by 15 m^2. What is the difference between the length and breadth of the original rectangle?
(a) 12 m (b) 15 m
(c) 41 m (d) 8 m

72. How many times in a 24 hour day will the minute and the hour hand be at 90º or 270º to each other?
(a) 44 times (b) 66 times
(c) 96 times (d) 48 times

73. A and B working together can finish a job in T days. If A works alone and completes the job, he will take $T + 5$ days. If B works alone and completes the same job, he will take $T + 45$ days. What is T?
(a) 25 days (b) 60 days
(c) 15 days (d) 225 days

74. A discount of 18% on the marked price of book enables a man to buy a pen worth ₹ 72. How much did he pay for the book?
(a) ₹ 328 (b) ₹ 288
(c) ₹ 400 (d) ₹ 648

75. In a community of 175 persons, 40 read the Times, 50 read the Samachar and 100 do not read any. How many persons read both the papers?

(a) 10 (b) 15
(c) 20 (d) 25

76. Two rockets approach each other, one at 42,000 mph and the other at 18,000 mph. They start 3,256 miles apart. How far are they apart (in miles) 1 minute before impact?
(a) 1,628 (b) 1,000
(c) 826 (d) 1,200

77. *A* bought 4 bottles of *B* and *B* bought one bottle of *L*, *L* per bottle costing twice that of the *B*. *C* bought nothing but paid ₹ 50 for his share of the drink which they mixed together and shared equally. If *C*'s ₹ 50 covered his share, then what is the cost of the *L* per bottle?
(a) 50 (b) 75
(c) 30 (d) 46

78. If $|x^2 - 7x + 12| > x^2 - 7x + 12$, then:
(a) $x \le 3$ or $x \ge 4$
(b) $3 \le x \le 4$
(c) $3 < x < 4$
(d) x can take any value except $x = 3$ and $x = 4$.

79. 300 persons are participating in a meeting in India, out of which 120 are foreigners, and the rest are Indians. Out of the Indians, there are 110 men who are not judges; 160 are men or judges, and 35 are women judges. There are no foreign judges. How many Indian women attended the meeting?
(a) 60 (b) 45
(c) 3.55 (d) 40

80. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in AP then n is:
(a) 8 (b) 9
(c) 14 (d) 9

Direction for questions 81–83: ***Read the passage below and solve the questions based on it.***

Each of the questions is followed by two statements (a) and (b). Mark the following as your answer as per the directions given:

Mark 1: If one of the statements is sufficient to answer the question but other statement is not sufficient to answer the question.

Mark 2: If either statement (a) or statement (b) alone is sufficient to answer the question.

Mark 3: Question can be answered only by using both the statements together, and not by any statement alone.

Mark 4: Question cannot be answered by using even both the statements together.

81. What is the maximum value of $\frac{a}{b}$?
(a) a, $a + b$ and $a + 2b$ are three sides of a triangle.
(b) a and b both are positive.

82. Five integers *A*, *B*, *C*, *D* and *E* are arranged in such a way that there are two integers between *B* and *C* and *B* is not the greatest. There exists one integer between *D* and *E* and *D* is smaller than *E*. *A* is not the smallest integer. Which one is the smallest?
(a) *E* is the greatest.
(b) There exists no integer between *B* and *E*.

83. What is the inradius of the right angled triangle *ABC*?
(a) Perimeter of the triangle *ABC* = 90 units
(b) Length of hypotenuse = 41 units

84. Product of two natural numbers *A* and *B* = 10,000. How many distinct HCF of *A* and *B* are possible?
(a) 1 (b) 9
(c) 16 (d) Infinite

85. How many odd integers from 1,000 to 8,000 have none of its digits repeated?
(a) 1,628 (b) 1,736
(c) 392 (d) 1,344

86. For how many integers n, $\frac{n}{20 - n}$ is the square of an integer?
(a) 0 (b) 1
(c) 2 (d) 3

87. Two varieties of rice are mixed in the ratio 2:3. Price of mixture is ₹ 12 per kg and price of the variety having lower weight is ₹ 10 per kg. Find the price (₹/kg) of other variety.
(a) 13.33 (b) 12.66
(c) 13 (d) 12

88. In a maternity centre, 5% of all the child-birth cases result in twins and all other deliveries result in single child. What is the approx. percentage of twins out of total children born?
(a) 5% (b) 7.6%
(c) 9.5% (d) 10.4%

89. There is a natural number which when divided by 4 and 5 gives 3 as the remainder in each case. What is the lowest such number?
(a) 23 (b) 43
(c) 103 (d) 13

Direction for questions 90–94: ***Go through the chart as shown in Fig. 1 and solve the questions based on it.***

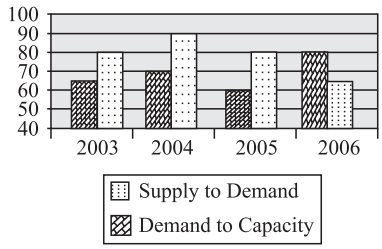


Fig. 1

Following bar chart as shown in Fig. 1 presents supply as a percentage of demand of a product and demand as a percentage of capacity of the industry to produce the same product.

All the values are multiples of 5.

90. If product supply for the year 2005 is 6,000 kgs, then the total capacity of the product is (in kg):
(a) 12,500 (b) 16,500
(c) 10,000 (d) None of these

91. If capacity of the industry to produce the product remains constant at 10,000 kgs, what is the approx. average supply (in kgs) for the given period?
(a) 5,100 (b) 5,375
(c) 4,400 (d) 6,200

92. If the product supply remains unchanged for the given period then what does the trend of industry capacity signify?
(a) Industry capacity steadily increases over the given period.
(b) Industry capacity steadily decreases over the given period.
(c) Industry capacity fluctuates over the given period.
(d) None of these

93. Which year shows the highest increase in the difference between the industry capacity and supply over the previous year, if demand for the product is 10,000 kgs for each of the given years?
(a) 2003 (b) 2004
(c) 2005 (d) 2006

94. If supply is constant for each of the years given above, then for which year capacity is highest?
(a) 2003 (b) 2004
(c) 2005 (d) 2006

95. Five students tried to get the sum of the first 21 positive primes, and only one student was correct. *A* got 709, *B* got 711, *C* got 712, *D* got 717 and *E* got 713. Who was correct?
(a) *A* (b) *B*
(c) *C* (d) *D*

96. Which term in the series 3 + 10 + 17 + ... and 63 + 65 + 67 + ... is common?
(a) 9 (b) 13
(c) 14 (d) 15

97. If x and y are positive integers such that $3x + 7y$ is a multiple of 11, then which of the following will also be divisible by 11?
(a) $4x + 6y$ (b) $x + y + 4$
(c) $4x - 9y$ (d) $9x + 4y$

98. How many numbers between 3,000 and 7,000 (both inclusive) can be formed with the digits 0, 2, 3, 5, 6, 7 and 8 with repeated digits?
(a) 1,029 (b) 1,028
(c) 1,030 (d) 1,031

99. In a party, where everybody shakes hand with everybody else, total number of handshakes = 28. What is the number of persons present in the party?
(a) 14 (b) 28
(c) 56 (d) 8

100. A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, by 8 leaves a remainder of 7, etc., down to where, when divided by 2, it leaves a remainder of 1, is:
(a) 59 (b) 419
(c) 1,259 (d) 2,519

ANSWER KEY

Section 1

1. (b)	**2.** (d)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (b)	**8.** (c)
9. (a)	**10.** (b)	**11.** (b)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (d)
17. (b)	**18.** (a)	**19.** (d)	**20.** (c)	**21.** (a)	**22.** (c)	**23.** (b)	**24.** (a)
25. (d)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (c)	**30.** (c)	**31.** (c)	**32.** (c)
33. (b)	**34.** (d)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (b)	**39.** (a)	**40.** (c)

Section 2

41. (d)	**42.** (c)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (a)
49. (d)	**50.** (a)	**51.** (d)	**52.** (c)	**53.** (d)	**54.** (b)	**55.** (b)	**56.** (c)
57. (d)	**58.** (b)	**59.** (d)	**60.** (d)	**61.** (a)	**62.** (b)	**63.** (b)	**64.** (c)
65. (d)	**66.** (d)	**67.** (b)	**68.** (b)	**69.** (c)	**70.** (d)	**71.** (a)	**72.** (a)
73. (c)	**74.** (a)	**75.** (b)	**76.** (b)	**77.** (a)	**78.** (c)	**79.** (c)	**80.** (c)
81. (a)	**82.** (d)	**83.** (c)	**84.** (b)	**85.** (b)	**86.** (b)	**87.** (a)	**88.** (c)
89. (d)	**90.** (a)	**91.** (b)	**92.** (d)	**93.** (c)	**94.** (c)	**95.** (c)	**96.** (b)
97. (c)	**98.** (c)	**99.** (d)	**100.** (d)				

MY PERFORMANCE SHEET (MOCK AT 3)

	Total Qs	Attempts	Right	Wrong	Marks	% Accuracy
Section 1 (Q1 to Q40)	40					
Section 2 (Q41 to Q100)	60					
Overall	100					

(Every correct response: +4 marks, every wrong response: –1 mark (minus 1).

Key Learnings from this test:

1. .....
.....

2. .....
.....

HINTS AND EXPLANATIONS

Section 1

11. Coral reefs cannot be of 'physical', 'sociological' or 'geographical' value. So only option (b) is appropriate. Hence, option (b) is the answer.

12. All the first words of the options may do. But the second option relates to the musical 'mainstream', so none of the other words apply. Hence, option (d) is the answer.

18. The key elements are: *IMF's proposal of new global tax on the profits and bonuses of financial institutions and banks-intend to prevent future global financial crises*. Options (b), (c) and (d) say 'will prevent'. Hence, option (a) is the best without distortion though some elements are left out; there is no distortion. Hence, option (a) is the answer.

19. Options (a) and (b) mention 'arms treaty' instead of 'nuclear arms treaty'. Option (c) misses out on the normalizing of relationship and rapprochement. Option (d) is the best concise description. Hence, option (d) is the answer.

20. The gist of the passage is: *Asia and Africa are already experiencing food shortages for two years, the cost of crops is expected to rise in Sudan*—this may trigger a severe to Asia and Africa, especially the rise in the cost of crops. Option (c) gives the gist without distortion of idea. Hence, option (c) is the answer.

21. This is an example of 'error of verbosity'. We don't need 'in her opinion' and 'she though' together. One of these would be sufficient. Also rose and up, are redundant rose is sufficient. If should be 'passed' not 'passed on'. Hence, option (a) is the answer.

22. We don't need the pronoun 'who' in this sentence.
The part between the commas qualifies Mr Pillai. Hence, option (c) is the answer.

23. With 'since'—a point of time present perfect time (has taken) should be used. Hence, option (b) is the answer.

24. The given sentence is right—'severing' is parallel to 'moving'. 'Them' in option (d) is incorrect. Hence, option (a) is the answer.

25. The subjective pronoun 'we' should be used here. 'Had left' precedes 'arrived'. Hence, option (a) is the answer.

26. The opening paragraph of the passage clearly states that a water tight compartment does not exist between direct and indirect rule. Hence, option (a) is the answer.

27. All of the activities mentioned in options (a) to (c) are mentioned as duties expected of European appointees. Hence, option (d) is the answer.

28. The first few sentences of the third paragraph have the answer. The idea of demarcating already established ethnic communities is to weaken any political threats by traditional authorities. Hence, option (d) is correct.

29. Option (c) summarizes the author's views on why direct rule by European colonies faced trouble in places like Africa. Hence, option (c) is the answer.

30. The reason for the modification of approach to colonial rule in the early 20th century was surely not on account of lack of effective tax collection mechanism. Hence, option (c) is the answer.

31. The passage needs to begin with an introduction to the bagworm, hence, it needs to

begin with sentence C. The sentence that follows need to elaborate on the bags that they form, hence, sentence A. There is a link between sentences A and D. Hence, option (c) is the answer.

32. Since the sentence B introduces the person and the subject, sentence B is the opening line. Sentence A obviously follows sentence D since it qualifies the information given in sentence D about her musical diversity, from North Indian to South Indian elements. Since sentence C ends in the present tense, it's the concluding line that is a pointer to further information. Hence, option (c) is the answer.

33. Sentence E is an introductory statement about gender and class. Statements B, D and C elaborate this idea. Statement A talks of caste, which is an entirely new topic (caste and class); it cannot follow sentence E. Hence, option (b) is the answer.

38. 'Near friend' is incorrect usage. It should be replaced with 'a close friend.' Option (b) is the answer.

39. 'Hand full' should be replaced with 'Hands full,' which is the correct idiomatic format. Hence, option (a) is the answer.

40. The word 'for' is redundant. The statement could be rephrased as 'I can't bear to see her angry.' Hence, option (c) is the answer.

Section 2

41. Using the formula of summation of n terms of an AP, we can say that

$$n\,[2a + (n - 1)\,2] = 153 \times 2 = 306$$

Now factorizing 306 gives

$= 306 \times 1,\ 153 \times 2,\ 102 \times 3,\ 51 \times 6,$
$34 \times 9,\ 18 \times 17.$

Out of these sets, following sets will give us the integral value of n as well as a:

n	a
1	153
153	–151
3	49
51	–47
9	9
17	–7

Out of these $n = 1$ is not possible. Hence, option (d) is the answer.

42. Generally this question is misinterpreted and common answer given is $(16 \times 6)/(2 \times 2) = 24$, which is wrong.

Question nowhere mentions that all the squares should be equal squares hence, approach to the question will be like this. The first 2 squares of 6×6 units will be cut. Then out of remaining 4×6 cloth third square of 4×4 will be cut. Then from remaining cloth of 4×2, two squares of 2×2 will be cut. Hence, 5 square in total. Hence, option (c) is the answer.

43. When 451 is divided by 9, the remainder is 1. When 448 is divided by 9, the remainder is (–2). Let us look at answer choice. When $n = 3$, When $451n$ is divided by 9, the remainder will be $13 = 1$. When and $448n$ is divided by 9, the remainder will be $(-2)^3 = (-8) = 1$. Hence, option (b) is the answer.

44. $$\frac{[10^2\,(10^6 - 1)]}{[10^3\,(10^4 - 1)]}$$

$$= \frac{[10^2\,(10^2 - 1)\,(10^4 + 10^2 + 1)]}{[10^3\,(10^2 - 1)\,(10^2 + 1)]}$$

$$= \frac{[(10^4 + 10^2 + 1)]}{[10\,(10^2 + 1)]}$$

Since question is asking for approximate value, we can neglect (+1) in the numerator.

$$= \frac{[(10^4 + 10^2)]}{[10(10^2 + 1)]} = 10$$

Hence, option (b) is the answer.

45. Take radius = 10 cm, then height = 20 cm. Use the formula for volume. Hence, option (a) is the answer.

46. Average age of A, B and C, *i.e.*,

Av (A, B and C) = 82 (i)

Average age of A, B and D, *i.e.*,

Av (A, B and D) = 79 (ii)

Average age of A, C and D, *i.e.*,

Av (A, C and D) = 83 (iii)

Average age of B, C and D, *i.e.*,

Av (B, C and D) = 81 (iv)

From equations (i) and (ii), C left and D joined the group, average age decreases,

Age of C is more than D.

Similarly, from equations (i) and (iii), age of D is more than B

From equations (ii) and (iv), age of C is more than A.

Therefore, we can deduce that C is the eldest among A, B, C and D.

Hence, option (c) is the answer.

47. $$\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc}$$

$$= \frac{\log_{ab}}{\log abc} + \frac{\log_{bc}}{\log abc} + \frac{\log_{ca}}{\log abc}$$

$$= \frac{\log_{ab} + \log_{bc} + \log_{ca}}{\log abc} = \frac{\log abc^2}{\log abc}$$

$$= 2\frac{\log abc}{\log abc} = 2$$

Therefore, given expression is equal to 2. Hence, option (c) is the answer.

50. Total population above poverty line

= 100% – 12% = 88%

So, total population above poverty line

= 88% of 3200 = 2816

Ratio of Male: Female in above poverty line = 4:3

Number of females above poverty line in State A

$$= \frac{3}{7} \cdot 2816 = 1206$$

Hence, option (a) is the answer.

51. From the given information, we cannot find the population of States C and D individually; so we can't find the required value. Hence, option (d) is the answer.

52. Population of State B below poverty line = 15%

Ratio of males to females below poverty line = 5:7 ⇒ So males are $\frac{5}{12}$ of the total population below poverty line.

So, $\frac{5}{12}$ of 15% population of State A = 500

So, total population of State A

$$= 500 \cdot \frac{12}{5} \cdot \frac{100}{15} = 8000$$

Hence, option (c) is the answer.

53. Population of State A below poverty line = 12%

Ratio of males to females below poverty line = 3:2 ⇒ So males are $\frac{3}{5}$ of the total population below poverty line.

So, $\frac{3}{5}$ of 12% population of State A = 3000

So, total population of State A

$$= 3000 \cdot \frac{5}{3} \cdot \frac{100}{12}$$

[At this stage, we are not going to calculate the exact value of population of State A. Because, we have to find out the ratio of population of States A and E.]

Similarly, total population of State E

$$= 6000 \cdot \frac{11}{6} \cdot \frac{100}{10}$$

Therefore, Required ratio

$$= \frac{3000 \cdot 5 \cdot 100 \cdot 6}{6000 \cdot 3 \cdot 12 \cdot 11 \cdot 10} = \frac{25}{66}$$

Hence, option (d) is the answer.

54. Population of State E below poverty line = 10%. Hence, population of State E above poverty line = 90% of the total population. Ratio of males to females above poverty line = 3:2 ⇒ So females are $\frac{2}{5}$ of the total population above poverty line.

So, $\frac{2}{5}$ of 90% population of State E

= 19800

So, total population of State E

$$= 19800 \cdot \frac{5}{2} \cdot \frac{100}{90} = 55000$$

So, number of males below poverty line

= 10% of total = 10% of 55000

= 5500

Ratio of males : females below poverty line

= 6:5

Hence, males are $\frac{6}{11}$ of the total population below poverty line

$$= \frac{6}{11} \cdot 5500 = 3000$$

Hence, option (b) is the answer.

55. Go through the options. Option (b) is the answer.

56. In this question, with a bit of guestimation, answer can be calculated easily.

Following companies have shown growth: Mahindra, Toyota, Honda, Skoda and Mercedes.

Out of these companies, Toyota and Ford have become almost twice, and clearly one of these two is going to be the answer.

Observing the data for Toyota and Ford, we can observe that in case of Honda, August, 2011 value is almost equal to twice the value of August, 2010, whereas in case of Toyota, it is slightly less than the twice the value of August, 2010.

Hence, option (c) is the answer.

57. Data given in Table 2 is for August month only, whereas question is asking for the whole year. Hence, question cannot be answered.

Hence, option (d) is the answer.

58. Number of companies seen the growth in sales volume = 4

Number of companies seen a decrease in sales volume = 5

Hence, option (b) is the answer.

59. Table 2 given in the question set gives the data for only these companies. There may be more companies manufacturing cars. It is also not given these companies are the top companies.

Hence, option (d) cannot be determined is the answer.

61. Consumption per day

= 6 × 40 × 5 + 3 × 80 × 10

= 1200 + 2400 = 3600 watt

= 3.6 kilowatt

Consumption in a month

= 3.6 × 30 = 108 kilowatt.

So monthly bill = 108 × 2 = ₹ 216

Hence, option (a) is the answer.

63. Given expression:

$= (1 - x)(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$

$= (1 - x^2)(1 + x^2)(1 + x^4)(1 + x^8)$

$= (1 - x^4)(1 + x^4)(1 + x^8)$

$= (1 - x^8)(1 + x^8) = 1 - x^{16}$

Hence, option (b) is the answer.

64. For any n, 199^{2n} has last digit as 1.

But the last digit of 144^{3n} is 4 for odd values of n and 6 for even values of n.

Therefore last digit of the given expression is either 5 or 7.

Hence, option (c) is the answer.

65. The resultant figure will be like a square tunnel inside a cube.

Total surface area of the cube

$= 6\,(\text{side})^2 = 150$

New surface area added

$= 4 \times (2 \times 5)$ [surfaces × side of square cross section × depth]

Total old surface area to be subtracted

$= 4 + 4 = 8$ (Area of top + Area of bottom)

Hence, net surface area

$= 150 + 40 - 8 = 182\ \text{cm}^2$

Hence, option (d) is the answer.

66. If P represents the product of the first 15 integers, P would consist of the prime numbers that are below 15.

2, 3, 5, 7, 11, 13

Any value that has a prime higher than 13 would not be a value of P.

$459 = 27 \times 17$

19 is a prime greater than 13.

So the answer is option (d).

68. If Gyan starts 10 minutes early, Maulik has to cover the same distance in 10 minutes less than his usual time so that he reaches 10 minutes before Gyan. So, assume speed of Maulik = y m/min and time taken = t mins.

So, $yt = 2y\,(t - 10)$.

Hence, $t = 20$ minutes

So Gyan takes 35 minutes and Maulik takes 20 minutes.

Hence, option (b) is the answer.

71. Let the length be 'L' and breadth be 'B'

The perimeter $= 2\,(L + B) = 84$

$\Rightarrow \quad L + B = 42 \Rightarrow L = 42 - B \qquad$ (i)

Area = LB. When the changes are made area

$= (L + 3)\,(B - 1)$

$= LB + 3B - L - 3$

It is given that $LB = 3B - L - 3$ is equivalent to $LB + 15$

$\therefore \quad LB + 15 = 1B + 3B - L - 3$

Hence, $LB + 15 = LB + 3B - L - 3$

or, $\quad 3B - L = 18$

Putting the value of L from equation (i), we obtain $4B = 60$, hence $B = 15$

Substituting in (i) we get, $L = 27$. So, the difference is 12.

Hence, option (a) is the answer.

72. The hands of a clock are at right angles twice in every hour, but in 12 hours, they are at right angles 22 times and thus 44 times in a day.

Between 2'O clock and 4'O clock, there are only 3 positions in which the angles of the hands of a clock are at right angles. Similarly, between 8'O clock and 10'O clock there are only 3 positions.

Hence, option (a) is the answer.

73. The time it will take when A and B work together is given by the formula 5 * 45 = 225 = 15 days. Where 5 and 45 are the extra time that A and B take to complete the job if they work alone as against working together.

Hence, option (c) is the answer.

74. 18% of marked price = ₹ 72

Marked price $= \dfrac{72 \cdot 100}{18} =$ ₹ 400.

So, the man would have paid ₹ 72 lesser, *i.e.*, ₹ 328. Hence, option (a) is the answer.

75. See Fig. 2.

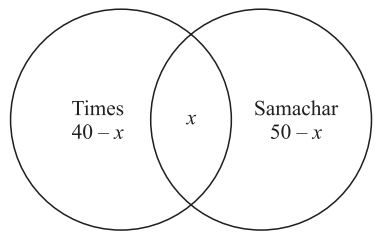


Fig. 2

$(40 - x) + x + (50 - x) = 75$. Hence, $x = 15$.

Option (b) is the answer.

76. Because they cover 60,000 miles in 60 minutes, *i.e.*, 1,000 miles in one minute, therefore they should be 1,000 miles apart, 1 minute before impact.
Hence, option (b) is the answer.

77. Let cost of B = ₹ x/bottle
and cost of L = ₹ $2x$/bottle
A spent ₹ $4x$ and B spent ₹ $2x$. C paid ₹ 50. So, cost of L = ₹ 50 per bottle and cost of B = ₹ 12.50 per bottle. Hence, option (a) is the answer.

78. If $|A| > A$, then $A < 0$
So, $x^2 - 7x + 12 < 0$ and
$\Rightarrow (x-3)(x-4) < 0$
$\Rightarrow 3 < x < 4$
Hence, option (c) is the answer.

81. Sum of two sides in a triangle should be greater than third side, *i.e.*,

$$a + 2b > a + a + b$$

$$b > a \Rightarrow \frac{a}{b} < 1$$

Maximum value of $\frac{a}{b}$ can be 1.
Only statement 1 is sufficient to get answer.

82. Neither of the statements is sufficient as it is not clear whether integers have been arranged in increasing or decreasing order.

83. Obviously none of the statements alone can give the answer.
Using both the statements together, Inradius of a right angled triangle
= Semi-perimeter – length of hypotenuse
Using this we can solve the question.

84. $A \times B = 10{,}000$
Highest HCF possible of A and B = 100 (Highest HCF occurs if both the numbers are same).
Now, each of the factors of highest HCF (= 100) will give rise to a HCF of 100.
So total number of distinct HCF
= Factors of 100 = 9
[To find out the factors of 100:

$$100 = 2^2 \times 5^2$$

Number of factors,

$$= (2 + 1) \times (2 + 1) = 9]$$

Hence, option (b) is the answer.

85. There are two restrictions operating in this question:
(i) For a number to be odd, unit digit should be either 1 or 3 or 5 or 7 or 9.
(ii) Thousand's place cannot be filled with 8 or 9.

For unit's digit—When it is filled with 9, thousand's place can be filled in 7 ways namely any digit from 1 to 7, and remaining two places can be filled in

$$8 \times 7 = 56 \text{ ways}$$

So, total number of numbers formed in this way

$$= 56 \times 7 = 392$$

Now, if unit's place is filled with any of the four digits 1, 3, 5 or 7, the thousand's place can be filled in 6 ways (0 will be excluded), and remaining two places can be filled in

$$8 \times 7 = 56 \text{ ways}$$

So, total number of numbers formed in this way

$$= 56 \times 6 \times 4 = 1344$$

So, total number of numbers

$$= 392 + 1344 = 1736$$

Hence, option (b) is the answer.

86. For $n = 16$

$$\frac{n}{20-n} = \frac{16}{20-16} = \frac{16}{4} = 4 = (2)^2$$

$\frac{n}{20-n}$ can be square only for $0 < n < 20$

Checking different values of integer n, it is clear that only $n = 16$ satisfies the criteria.
Hence, option (b) is the answer.

87. Using methods of alligation (Fig. 3),

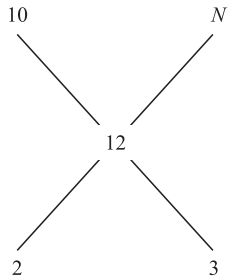


Fig. 3

$[N - 12]\ [12 - 10]\%$

Now, $2:3 = \dfrac{[N-12]}{[12-10]} = \dfrac{N-12}{2}$

So, N = ₹ 13.33 per kg. Hence, option (a) is the answer.

88. 5% child birth cases results in twins ⇒ out of 100 child birth cases, 5 cases will be of twins.

Total children born in 100 birth cases

= 105 (since there are 5 twins cases)

Total twins children = 10

Hence, percentage of twins in the total children

$$= \frac{10}{105} \cdot 100 = 9.52\%$$

Hence, option (c) is the answer.

89. Lowest such number = 3

Hence, option (d) is the answer.

Solution to questions 90–94

The Story Line:

1st bar mentions the ratio Supply to Demand, *i.e.*, Supply:Demand. For example, as mentioned in the bar of 2003, 65% is the value for supply to demand. It means that supply is 65% of demand. In other words, it means that if demand is of 100 units, then supply is 65 units. Similarly, the second bar gives the value for demand to capacity. For example, as mentioned in the bar of 2003, 80% is the value for demand to capacity. It means that if capacity is of 100 units, then demand is 80 units.

It can also be observed that if we join the values in both the bars, then a relation between Supply and capacity can be established using ratio or percentage calculation.

90. Method 1:

In year 2005, percentage of supply to demand = 60%. So supply is 60% of the demand.

So, 60% of demand = 6,000 kg ⇒ 100% of demand = total demand = 10,000 kg.

In year 2005, percentage of demand to capacity = 80%. So demand is 80% of the capacity.

So, 80% of the capacity = 10,000 kg ⇒ 100% of the capacity = total capacity

$= 10{,}000 \times \dfrac{100}{80} = 12{,}500$ kg.

Method 2:

In year 2005, 80% of capacity = demand, and 60% of demand = supply.

Assume that capacity = p, then demand = 80% of p = $0.8p$. Further, 60% of $0.8p$ = $0.48p$ = supply

Using the information given in the question, $0.48p = 6{,}000 \Rightarrow p = 6{,}000 \times \dfrac{100}{48}$ = 12,500.

Hence, option (a) is the answer.

91. In year 2003, supply = 65% of demand.

Further, demand = 80% of capacity

⇒ supply = 65% of 80% of capacity

= (65/100 × 80)% of capacity

= 52% of capacity

Similarly, in year 2004, supply = 70% of 90% of capacity = 63% of capacity

In year 2005, supply

= 60% of 80% of capacity

= 48% of capacity

In year 2006, supply

= 80% of 65% of capacity

= 52% of capacity

Since capacity of the industry is constant every year, to find out the average supply per year, we can simply find out the average percentage supply and multiply it with 10,000.

Average percentage supply per annum

$$= \frac{52\% + 63\% + 48\% + 52\%}{4}$$

$$= \frac{215\%}{4} = 53.75\%$$

Hence, average supply per annum

= 53.75% of 10,000 = 5375.

Hence, option (b) is the answer.

92. Using data from previous question: Supply as a percentage of demand as given in Table 3.

Table 3

2003	2004	2005	2006
52%	63%	48%	52%

It is given that supply remains constant. Since there is no fixed pattern of supply as a percentage of demand (as given in above table), hence, there is no fixed trend of capacity.

93. Following data can be obtained (Table 4):

Table 4

	2003	**2004**	**2005**	**2006**
Supply	6500	7000	6000	8000
Capacity	12500	11111.11	12500	15384.62
Difference	6000	4111.11	6500	7384.6

It can be easily seen that maximum increase occurs in 2005. Hence, option (c) is the answer.

94. Given that supply is same for each of the years. Using the data for question 3, supply as a percentage of demand is minimum in 2005 (= 48%). Hence, total supply for the year 2005 will be maximum. Hence, option (c) is the answer.

95. The key is to note that out of first 21 primes, 20 will be odd and one will be even. And when we add these 21 numbers, the sum should be even. One of the five has to be correct. So, option (c) is the answer.

96. $3 + (n - 1)\,7 = 63 + (n - 1)\,2$

$n = 13$.

Hence, option (b) is the answer.

97. $3x + 7y$ is a multiple of 11, we have to find one particular solution of $3x + 7y = 11k$ (where k is any positive integer)

It is possible when $x = 9$ and $y = 4$ or multiple of that values.

$3x + 7y = 55$, *i.e.*, it is multiple of 11.

Now, substitute these values of x and y in the answer option and check the divisibility by 11.

Hence, option (c) is the answer.

98. Required number of numbers

$$= 3 \times 7 \times 7 \times 7 + 1$$

$$= 1029 + 1 = 1030$$

Hence, option (c) is the answer.

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