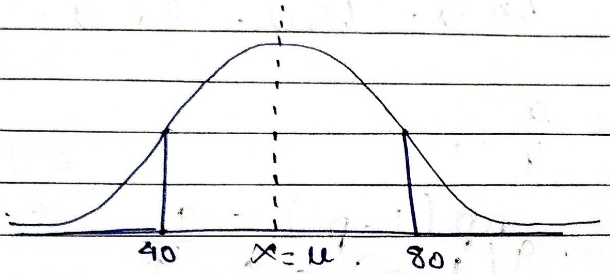
UNIT-4

CONTINUOUS PROBABILITY DISTRIBUTION.

NORMAL DISTRIBUTION.



A random variable x is said to follow normal distribution if it probability mass function is given by.

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$-\infty < x < \infty$

$-\infty < \mu < \infty$

$0 < \sigma < \infty$

$$X \sim N(\mu, \sigma^2)$$

This distribution has two parameter namely μ and σ^2.

STANDARD NORMAL VARIATE

$$z = \frac{x-\mu}{\sigma}$$

$x \rightarrow 10, 20, 30, 40, 50$

~~$\bar{x} = 30$~~ $\mu = 30$

$\sigma = 5$

$-4 \quad -2 \quad 0 \quad 2 \quad 4$

Find mean and variance of standard normal variate.

$$E(z) = E\left(\frac{x-\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma}\left[E(x-\mu)\right]$$

$$= \frac{1}{\sigma}\left[E(x) - E(\mu)\right] \qquad \boxed{\begin{matrix} E(C) = C \\ E(x) = \mu \end{matrix}}$$

$$= \frac{1}{\sigma}\left[E(x) - \mu\right]$$

$$= \frac{1}{\sigma}\left[\mu - \mu\right] = 0$$

$$var(x) = var\left(\frac{x-\mu}{\sigma}\right) \qquad var\, ax = a^2\, var(x)$$

$$var(x+b) = var(x)$$

$$= \frac{1}{\sigma^2}\, var(x-\mu) \qquad var(x) = \sigma^2$$

$$= \frac{var(x)}{\sigma^2} \qquad x \sim N(\mu, \sigma^2)$$

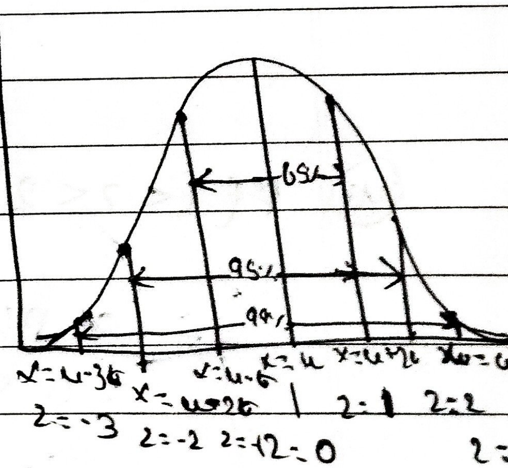
$$z \sim N(0,1)$$

$$= \frac{\sigma^2}{\sigma^2} = 1$$

$$\sigma^2 = 1$$

$$\sigma = 1$$

① Calculate the following ~~dist~~ probability for standard normal variate table.



① $P(0<z<2)$ ⑧ $P(|z| \le 2)$

② $P(z>2)$

③ $P(z<2)$

④ $P(-1<z<2)$

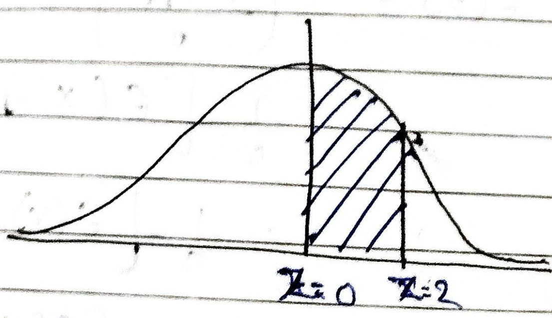
⑤ $P(z<-1)$

⑥ $P(1<z<2)$

⑦ $P(-1.9<z<-1)$

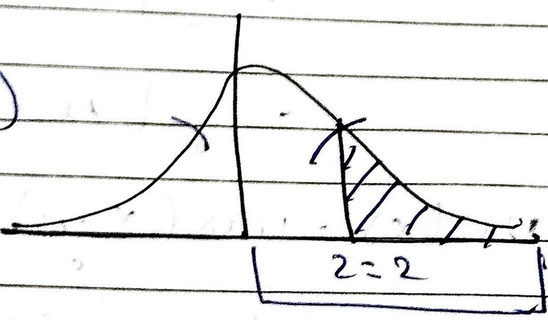
① $P(0<z<2)$

$= 0.4772$

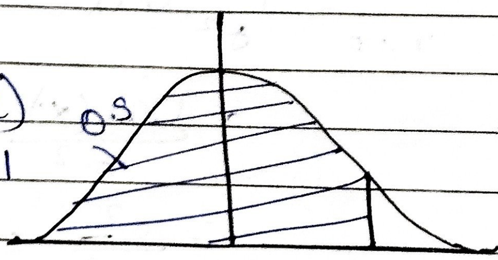


② $P(z>2)$

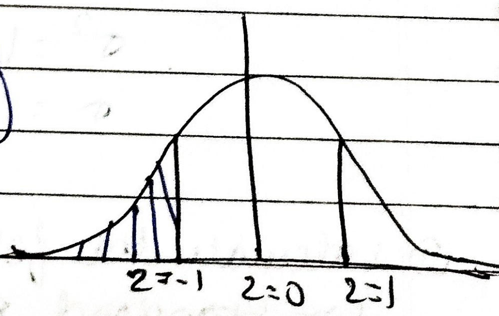
$= 0.5 - P(0<z<2)$
$= 0.5 - 0.4772$
$= 0.0228$



Q3. $P(z<2)$
$= 0.5 + P(0<z<2)$
$= 0.5 + 0.4771$
$= 0.9771$

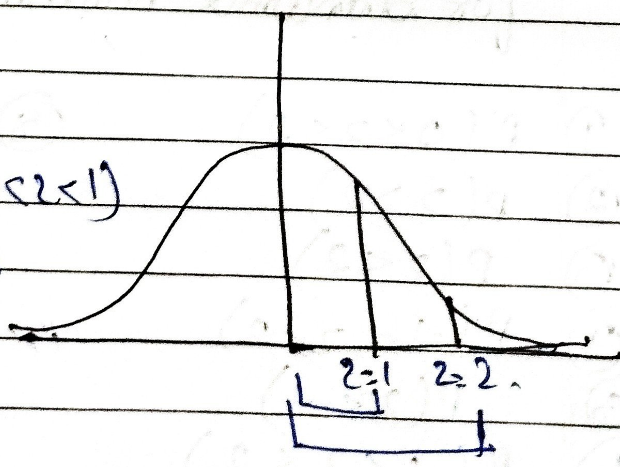


Q4. $P(z<-1) = 0.5 - P(-1<z<0)$
$= 0.5 - P(0<z<1)$
$= 0.5 - 0.3413$
$= 0.1587$



(4i) $P(1<z<2)$

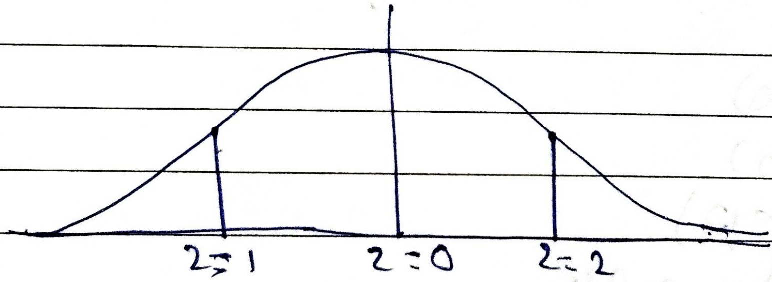
$= P(0<z<2) - P(0<z<1)$
$= 0.4772 - 0.3413$
$= 0.1359.$



(7) $P(-1.5 < z < 2)$

$= P(0 < z < 1.5) + P(0 < z \leq 2)$
$= 0.4332 + 0.4772$
$= 0.9104.$

(8) $P(|z| \leq 2)$
$= P(-2 \leq z \leq 2)$
$= P(-2 \leq z \leq 0) + P(0 \leq z \leq 2)$
$= P(0 \leq z \leq 2) + P(0 \leq z \leq 2)$
$= 2 \times P(0 \leq z \leq 2)$
$= 2 \times 0.4772$
$= 0.9544.$



(1) $P(0 < z < 2.5)$
(2) $P(2.5 < z < 3.5)$
(3) $P(z > 2.5)$
(4) $P(|z| < 1.75)$

(1) $P(0 < z < 2.5)$
$= 0.4938.$

(ii) $P(2.5 < z < 3.5)$
$= P(0 < z < 3.5) - P(0 < z < 2.5)$
$= 0.5004 - 0.4938$
$= 0.0069.$

(iii) $P(z > 2.5)$

$= 0.5 - P(0 < z < 2.5)$
$= 0.5 - 0.4938$
$= 0.0062$

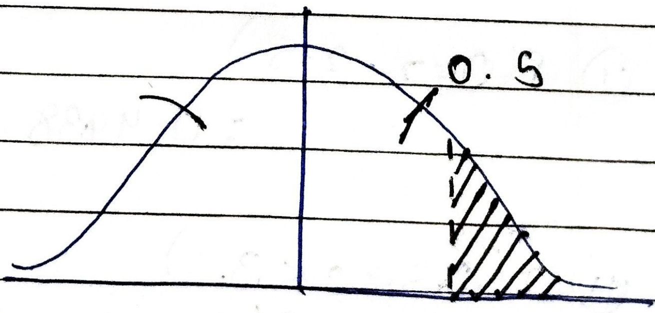
(iv). $P(1 z < 1.75)$
$= P(0 < z < 1.75) + P(0 < z < 1.75)$
$= 2(P < z < 1.75)$
$= 2 \times 0.4599$
$= 0.9198$

1. If x is normally distributive and mean of x is 12 and standard deviation is 4 find out probability

(i). $P(x \geq 20)$
(ii) $P(x < 20)$
(iii) $P(0 \leq x \leq 12)$
(iv) $P(x > x) = 0.24$.

$z = \frac{x - \mu}{\sigma}$ $\mu = 12, \sigma = 4$

(i) $P(x \geq 20)$
$P\left(\frac{x-\mu}{\sigma} \geq \frac{20-\mu}{\sigma}\right)$
$= P\left(z \geq \frac{20-12}{4}\right)$
$= P(z \geq 2)$
$= 0.5 - P(0 < z < 2)$
$= 0.5 - 0.4772$
$= 0.0228$.

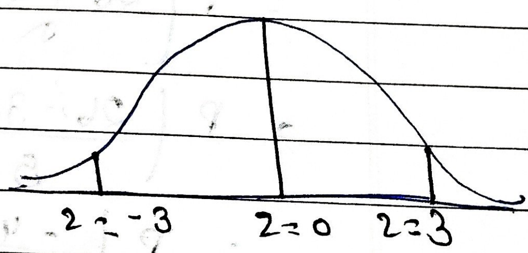


(ii) $P(x<20)$

$$= 1 - P(x>20)$$
$$= 1 - 0.0228$$
$$= 0.9772.$$

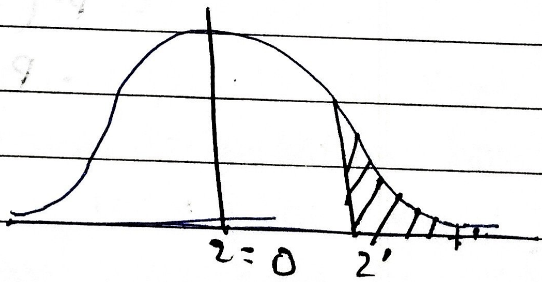
(iii) $P(0 \le x \le 12)$

$$= P\left(\frac{0-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{12-\mu}{\sigma}\right)$$
$$= P\left(\frac{-12}{4} \le z \le \frac{12-12}{4}\right)$$
$$= P(-3 \le z \le 0)$$
$$= P(0 \le z \le 3)$$
$$= 0.4987.$$



(iv). $P(x>x')$

$$P\left(\frac{x-\mu}{\sigma} > \frac{x'-\mu}{\sigma}\right) = 0.24$$
$$P(z>z') = 0.24.$$
$$0.5 - P(0<z<z') = 0.24.$$
$$P(0<z<z') = 0.5 - 0.24.$$
$$P(0<z<z') = 0.26.$$



$$z' = 0.71.$$
$$\frac{x'-\mu}{\sigma} = 0.71$$

$$x'-\mu = 0.71\sigma$$
$$x' = 0.71\sigma+\mu$$
$$= 12+0.71\sigma$$
$$= 12+0.71\times 4.$$
$$= 12+2.84$$
$$= 14.84.$$

2. If x is a normal variate with mean 30 and S.D is 5. find the probability of

(i) $P(26 \leq x \leq 40)$
(ii) $P(x \geq 45)$
(iii) $P(|x-30| > 5)$

$$z = \frac{x-\mu}{\sigma}$$

(i) $P(26 \leq x \leq 40)$

$$= P\left(\frac{26-\mu}{\sigma} \leq \frac{x-\mu}{\sigma} \leq \frac{40-\mu}{\sigma}\right)$$

$$= P\left(\frac{26-30}{5} \leq \frac{x-\mu}{\sigma} \leq \frac{40-30}{5}\right)$$

$$= P\left(-\frac{4}{5} \leq z \leq 2\right)$$

$$= P(-0.8 \leq z \leq 2)$$

$$= P(-0.8 \leq z \leq 0) + P(0 \leq z \leq 2)$$

$$= P(0 \leq z \leq 0.8) + P(0 \leq z \leq 2)$$

$$= 0.2881 + 0.4772$$

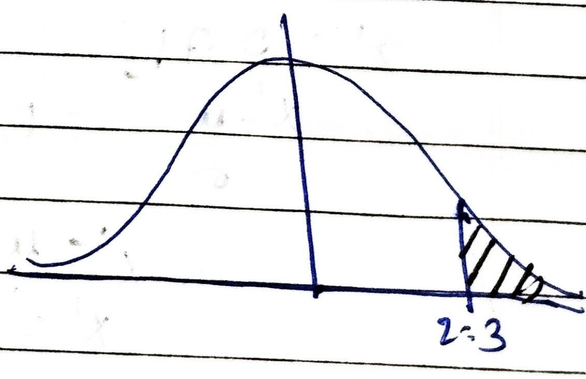
$$= 0.7653$$

(ii) $P(x \geq 45)$

$$= P\left(\frac{x-\mu}{\sigma} \geq \frac{45-\mu}{\sigma}\right)$$

$$= P\left(z \geq \frac{45-30}{5}\right)$$

$$= P\left(z \geq \frac{15}{5}\right)$$



$$= P(z \geq 3)$$

$$= 0.5 - P(0 \leq z \leq 3)$$

$= 0.5 - 0.4987$

$= 0.0013.$

$|x| \le \lambda$

$-\lambda \le x \le \lambda.$

(iii) $P(|x-30| > 5)$

$= 1 - P(|x-30| \le 5)$

$= 1 - P(-5 \le x - 30 \le 5)$

$= 1 - P\left(-\frac{5}{5} \le \frac{x-30}{5} \le \frac{5}{5}\right)$

$= 1 - P(-1 \le z \le 1)$

$= 1 - [P(-1 \le z < 0) + P(0 \le z \le 1)]$

$= 1 - [P(0 \le z \le 1) + P(0 \le z \le 1)]$

$= 1 - 2 \times P(0 \le z \le 1)$

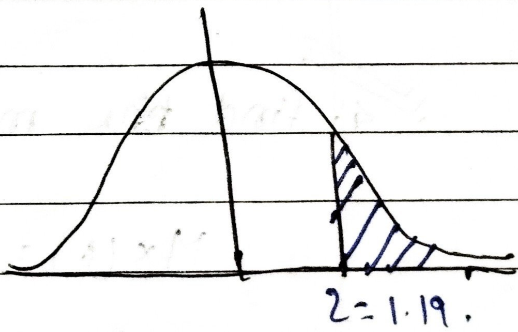
$= 1 - 2 \times 0.3413$

$= 1 - 0.6826$

$= 0.3174.$

8. The mean production from one acre of plot is 662 Kilos. with S.D of 32 Kilos. Assuming normal distribution how many one acre plot in a batch of 1000 plot would you expect to give production. ~~over~~ first part over 700 kilos and second part below 650 kilos.

$\mu = 662, \quad \sigma = 32$



(i) $P(x > 700)$

$= P\left(\frac{x-\mu}{\sigma} > \frac{700-662}{32}\right)$

$= P\left(z > \frac{38}{32}\right) = P(z > 1.1875)$

$= P(z > 1.19)$

$= ~~0.3830~~$

$= 0.5 - P(0 \le z \le 1.19)$

$= 0.5 - 0.3830$

$= 0.117.$

no. of plots $= N \times P$

$= 1000 \times 0.117.$

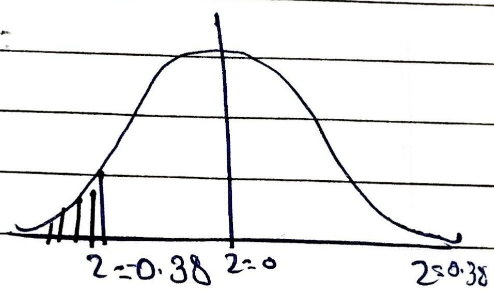
$= 117.$

(ii) $P(x < 650)$

$= P\left(\frac{x-\mu}{\sigma} < \frac{650-662}{32}\right)$

$= P\left(z < \frac{-12}{32}\right)$

$= P(z < -0.375)$



$= P(0.375 < z < 0)$

$= P(-0.38 < z < 0) = 0.5 - P(0 < z < 0.375)$

$= 0.5 - P(0 < z < 0.38)$

$= 0.5 - 0.1480$

$= 0.352.$

no. of plots $= np = 0.352 \times 1000$

$= 352$

4. find the mean and variance of the normal distribution

$M_x(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Diff w.r.t t.

$\frac{d[M_x(t)]}{dt} = e^{\mu t + \frac{1}{2}\sigma^2 t^2} \cdot \frac{d}{dt}\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$

$$= e^{\mu t+\frac{1}{2}\sigma^2 t^2}\left(\mu+\frac{1}{2}\sigma^2(2t)\right)$$

$$= e^{\mu t+\frac{1}{2}\sigma^2 t^2}\left(\mu+\frac{1}{2}\sigma^2 . 2t\right)$$

$$= e^{\mu t+\frac{1}{2}\sigma^2 t^2}\left(\mu+\sigma^2 t\right)$$

Set $t=0$

$E(x) = \mu$.

$$\frac{d^2 M_x(t)}{dt} = \frac{d}{dt}\left[e^{\mu t+\frac{1}{2}\sigma^2 t^2}(\mu+\sigma^2 t)\right]$$

$$= e^{\mu t+\frac{1}{2}\sigma^2 t^2} . \frac{d}{dt}(\mu+\sigma^2 t) + \mu+\sigma^2 t . \frac{d}{dt} e^{\mu t+\frac{1}{2}\sigma^2 t^2})$$

$$= e^{\mu t+\frac{1}{2}\sigma^2 t^2} . (\sigma^2) + (\mu+\sigma^2 t) . e^{(\mu t+\frac{1}{2}\sigma^2 t^2)} . (\mu+\sigma^2 t)$$

Set $t=0$

$$= e^0 . \sigma^2 + \mu^2 . e^0$$

$$E(x^2) = \sigma^2 + \mu^2$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$= \sigma^2+\mu^2-\mu^2$$

$$= \sigma^2.$$

H.W.

5. In a school of 1000 student with mean height of the students 150cm and S.D is 10. find the following probability

(i) $P(150 \le x \le 170)$

(ii) $P(x \ge 179)$

(iii) $P(x \le 140)$

M.G.F of Normal distribution

$$M_x(t) = E[e^{tx}]$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx.$$

$$= \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} e^{tx}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

put $\frac{x-\mu}{\sigma} = z$

$$x - \mu = z\sigma$$
$$x = \mu + z\sigma$$
$$dx = \sigma\, dz$$

When $x \to -\infty$ $\quad z \to -\infty$
When $x \to \infty$ $\quad z \to \infty$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} e^{t(\mu + \sigma z)}\, e^{-\frac{1}{2}z^2}\, \sigma\, dz.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\mu t + t\sigma z - \frac{1}{2}z^2}\, dz.$$

$$\boxed{= \cancel{\frac{1}{\sqrt{2\pi}} e^{\mu t} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2t\sigma z - \mu t)}}\, dz}$$

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$$= \frac{1}{\sqrt{2\pi}} e^{\mu t} \int_{-\infty}^{\infty} e^{\left(t\sigma z - \frac{z^2}{2}\right)} dz.$$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu t} \int_{-\infty}^{\infty} e^{\frac{(2t\sigma z - z^2)}{2}} dz.$$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu t} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2t\sigma z)} dz.$$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu t} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(z^2 - 2t\sigma z + (\sigma t)^2 - (\sigma t)^2\right)} dz.$$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu t + \frac{1}{2}\sigma^2 t^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z - \sigma t)^2} dz.$$

Put $z - \sigma t = y$.

$dz = dy$.

$z \to -\infty$. $\quad y \to -\infty$

$z \to \infty$ $\quad y \to \infty$.

$$= \frac{1}{\sqrt{2\pi}} e^{\mu t + \frac{1}{2}\sigma^2 t^2} \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dz.$$

$$= \frac{1}{\sqrt{2\pi}} e^{\mu t + \frac{1}{2}\sigma^2 t^2} \sqrt{2\pi}$$

$$= e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

~~$M_z(t) = e^{\mu t} + \frac{1}{2}$~~ $\quad \therefore M_x(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$M_z(t) = e^{\frac{1}{2}t^2}$

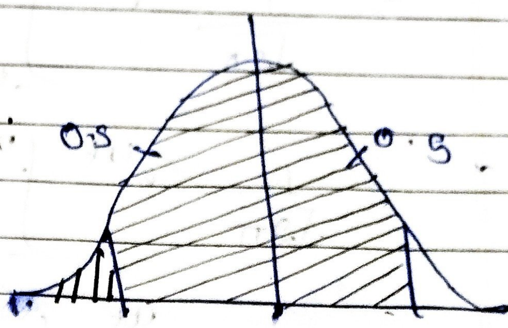
$E(z) = 0.14.$

$Var(z) = 110$

Q1. In a distribution exactly normally 7% of the items are under 35 and 89% of the items are under 63. Find the mean and the standard deviation.

$P(x<33) = 0.07$

$P\left(\frac{x-\mu}{\sigma} < \frac{33-\mu}{\sigma}\right) = 0.07$



$P(z<z_1) = 0.07$

$0.5 - P(z_1<z<0) = 0.07$

$0.5 - 0.07 = P(z_1<z<0)$

$P(z_1<z<0) = 0.43$

$z_1 = -1.48$

$\frac{33-\mu}{\sigma} = -1.48$

$33-\mu = -1.48\sigma$ —— (1)

$P(x<63) = 0.89$

$P\left(\frac{x-\mu}{\sigma} < \frac{63-\mu}{\sigma}\right) = 0.89$

$P(z<z_2) = 0.89$

$0.5 + P(0<z<z_2) = 0.89$

$P(0<z<z_2) = 0.89 - 0.5$

$P(0 < z < z_2) = 0.39$.

$z_2 = 1.23$.

$z_2 = 1.23$

$\frac{63 - \mu}{\sigma} = 1.23$.

$63 - \mu = 1.23\sigma$ — (2)

(2) - (1) ⇒

$(63 - \mu) - (35 - \mu) = 1.23\sigma + 1.48\sigma$

$28 = 2.71\sigma$

$\sigma = \frac{28}{2.71} = 10.33$.

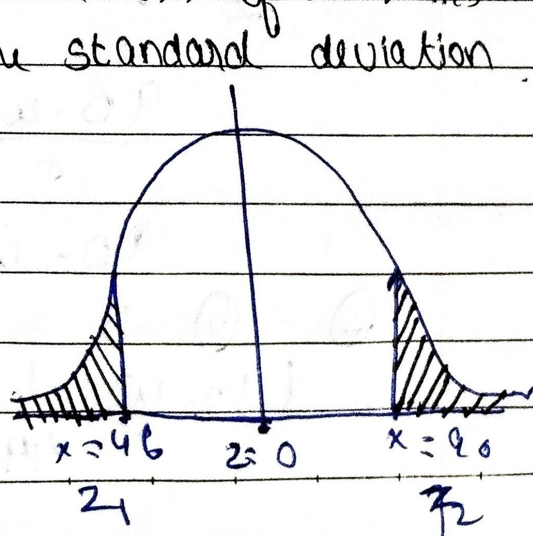
from (1)

$35 - \mu = -1.48(6.64)$

$35 + 1.48(10.33) = \mu$.

$\mu = 50.28$.

(2) If 10% of the items are below 46 and 16% of the items are above 90. find the mean and the standard deviation.



$P(x < 46) = \frac{10}{100}$.

$P(x < 46) = 0.1$

$$P\left(\frac{X-\mu}{\sigma} < \frac{46-\mu}{\sigma}\right) = 0.1$$

$$P(z < z_1) = 0.1$$

$$0.5 - P(z_1 < z < 0) = 0.1$$

$$P(z_1 < z < 0) = 0.5 - 0.1$$

$$P(z_1 < z < 0) = 0.4.$$

$$z_1 = -1.29.$$

$$\frac{46-\mu}{\sigma} = -1.29.$$

$$46 - \mu = -1.29\sigma \quad \text{—— (i)}$$

$$P(x > 90) = \frac{16}{100}$$

$$P(x > 90) = 0.16.$$

$$P\left(\frac{x-\mu}{\sigma} > \frac{90-\mu}{\sigma}\right) = 0.16$$

$$P(z > z_2) = 0.16$$

$$0.5 - P(0 < z < z_2) = 0.16$$

$$P(0 < z < z_2) = 0.5 - 0.16$$

$$P(0 < z < z_2) = 0.34$$

$$z_2 = 1$$

$$\frac{90-\mu}{\sigma} = 1$$

$$90 - \mu = \sigma \quad \text{—— (ii).}$$

(ii) - (i) $\Rightarrow$

$$(90-\mu) - (46-\mu) = \sigma + 1.29\sigma$$

$$44 = 2.29\sigma$$

$$\sigma = \frac{44}{2.29}$$

$$\sigma = 19.21$$

from (11) and (1)

$$90 - \mu = 19.21$$

$$\mu = 90 - 19.21$$

$$\mu = 70.79.$$

Exponential distribution

A random variable x is said to follow exponential distribution if it is following.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0 \quad 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

λ is called rate of waiting time / rate of failure of a component.

(1) $$f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0 \quad 0 < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Avg life = 5 year.

failure rate $= \frac{1}{5} = 0.2$ per year.

Q: Find the M.g.f of the exponential distribution.

$$M_x(t) = E[e^{tx}]$$

$$= \int_0^\infty e^{tx} f(x)\, dx.$$

$$= \int_0^\infty e^{tx} \lambda e^{-\lambda x}\, dx.$$

$$= \lambda \int_0^\infty e^{tx - \lambda x}\, dx. \qquad e^{\infty} = \infty, \quad e^{-\infty} = 0.$$

$$= \lambda \int_0^\infty e^{(t-\lambda)x}\, dx$$

$$= \lambda \int_0^\infty e^{-(\lambda - t)x}\, dx \qquad \lambda - t > 0, \quad \lambda > t.$$

$$= \frac{-\lambda}{(\lambda - t)} \left[e^{-(\lambda - t)x} \right]_0^\infty = \frac{-\lambda}{\lambda - t} [0 - 1]$$

$$= \frac{\lambda}{\lambda - t} = \frac{1}{\left(1 - \frac{t}{\lambda}\right)}$$

$$M_x(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}$$

$$\frac{d}{dx}[M_x(t)] = -\left(1 - \frac{t}{\lambda}\right)^{-2} \frac{d}{dt}\left(1 - \frac{t}{\lambda}\right)$$

$$= -\left(1 - \frac{t}{\lambda}\right)^{-2}\left(-\frac{1}{\lambda}\right)$$

$$= \left(1 - \frac{t}{\lambda}\right)^{-2} \cdot \frac{1}{\lambda}$$

Set $t = 0$

$$E(x) = \frac{1}{\lambda}$$

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$$\frac{d^2}{dt^2}[M_x(t)] = -2\left(1-\frac{t}{\lambda}\right)^{-3}\frac{d}{dt}\left(1-\frac{t}{\lambda}\right)\left(\frac{1}{\lambda}\right)$$

$$= -2\left(1-\frac{t}{\lambda}\right)^{-3}\left(-\frac{1}{\lambda}\right)\left(\frac{1}{\lambda}\right)$$

$$= \frac{2}{\lambda^2}\left(1-\frac{t}{\lambda}\right)^{-3}$$

Set $t = 0$.

$$E(x^2) = \frac{2}{\lambda^2}$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

Q. A component avg life time is 5 year. further suppose that life of this component can be modelled using exponential distribution. find the probability that component will survive more than 8 years.

average life of component = 5 years

$$\lambda = \frac{1}{5} = 0.5 \text{ per/year}$$

$$P(x>8) = \int_8^\infty \lambda e^{-\lambda x}\,dx$$

$$= \frac{\cancel{\lambda}}{-\cancel{\lambda}}\left(e^{-\lambda x}\right)_8^\infty$$

$$= -\left(0 - e^{-8\lambda}\right)$$

$$= e^{-8\lambda} = e^{-8\times 0.2} = e^{-1.6}$$

1. Suppose that length of phone call in min is exponential random variable. If ~~B~~ λ value is $\frac{1}{12}$. If another person arrives at the public telephone just before you. Find the probability that you have to wait more the 8 min.

$$\lambda = \frac{1}{12}$$

$$P(x>5) = \int_5^\infty f(x)\,dx$$

$$= \int_5^\infty \lambda e^{-\lambda x}\,dx$$

$$= \frac{\lambda}{-\lambda}\left(e^{-\lambda x}\right)_5^\infty$$

$$= -1\left(0 - e^{-5\lambda}\right)$$

$$= e^{-5\lambda}$$

$$= e^{-5/12}.$$

2. Suppose that a system contains a ~~ba~~ certain types of component whose times t is failure is given. The random variable T is modelled nicely by exponential distribution with mean time to failure 5 years. If 5 of these components are installed in different system what is the probability that atleast two ~~are~~ are still functional at the end of 8 years.

M.T.T.F = 5 years.

$e^{-1.6} = 0.2$.

$\lambda = \frac{1}{5} = 0.2$

$P(T>8) = \int_8^\infty f(x)\,dx$.

$= \int_8^\infty \lambda e^{-\lambda x}\,dx$.

$= \frac{\lambda}{-\lambda}\left(e^{-\lambda x}\right)_8^\infty$

$= -\left[e^{-\lambda x}\right]_8^\infty$

$= -\left[0 - e^{-8\lambda}\right]$

$= e^{-8\lambda} = e^{-8(0.2)}$

$= e^{-1.6}$.

$= 0.2$.

$p = 0.2, \quad q = 1-p = 1-0.2$
$= 0.8$

let r.v x count no. of components which services for more than 8 years.

$P(x \geq 2) = 1 - P(x<2)$

$= 1 - \left[P(x=0) + P(x=1)\right]$

$= 1 - \left[{}^5C_0 p^0 q^5 + {}^5C_1 p^1 q^4\right]$

$= 1 - \left[(0.8)^5 + 5(0.2)(0.8)^4\right]$

Date

GAMMA DISTRIBUTION.

A r.v x is said to follow gamma distribution if its probability density function is given by $\frac{e^{-x}x^{\lambda-1}}{\Gamma(\lambda)}$

$$f(x) = \begin{cases} \frac{e^{-x}x^{\lambda-1}}{\Gamma(\lambda)} & \lambda > 0,\ 0 < x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

R (1) $\Gamma(\lambda) = \int_0^\infty e^{-x} x^{\lambda-1} dx.$

R (2) $\Gamma(\lambda) = \underline{|\lambda-1}$

Q. Prove that this function is probability density ~~distribution~~ function

$$\int_0^\infty f(x)dx = \int_0^\infty \frac{e^{-x} . x^{\lambda-1}}{\Gamma(\lambda)} dx.$$

$$= \frac{1}{\Gamma(\lambda)} \int_0^\infty e^{-x} x^{\lambda-1} dx.$$

$$= \frac{1}{\Gamma(\lambda)} \times \Gamma(\lambda)$$

$$= 1$$

$f(x)$ is a probability density function.

Q. Find the ~~mean~~ m.g.f of the gamma distribution and after finding the m.g.f find mean and variance of gamma distribution

$$M_x(t) = E[e^{tx}]$$

$$= \int_0^\infty e^{tx} f(x)\, dx.$$

$$= \int_0^\infty e^{tx} \frac{e^{-x} x^{\lambda-1}}{\Gamma(\lambda)} dx.$$

$$= \frac{1}{\Gamma(\lambda)} \int_0^\infty e^{(tx-x)} \cdot x^{\lambda-1} dx.$$

$$= \frac{1}{\Gamma(\lambda)} \int_0^\infty e^{-(1-t)x} x^{\lambda-1} dx.$$

Put $(1-t)x = z$.

$$x = \frac{z}{1-t}$$

$$dx = \frac{dz}{1-t}$$

when $x \to 0$, $z \to 0$

when $x \to \infty$, $t \to \infty$

$$M_x(t) = \frac{1}{\Gamma(\lambda)} \int_0^\infty e^{-z} \left(\frac{z}{1-t}\right)^{\lambda-1} \frac{dz}{(1-t)}$$

$$= \frac{1}{\Gamma(\lambda)} \int_0^\infty e^{-z} z^{(\lambda-1)} \frac{1}{(1-t)^\lambda} dz$$

$$= \frac{1}{\Gamma(\lambda)} \cdot \frac{1}{(1-t)^\lambda} \int_0^\infty e^{-z} z^{\lambda-1} dz.$$

$$= \frac{1}{\Gamma(\lambda)} \cdot \frac{1}{(1-t)^{\lambda}} \Gamma(\lambda)$$

$$= \frac{1}{(1-t)^{\lambda}}$$

$$M_x(t) = (1-t)^{-\lambda}$$

$$\frac{d}{dx} M_x(t) = -\lambda(1-t)^{-\lambda-1} \frac{d}{dt}(1-t)$$

$$= -\lambda(1-t)^{-\lambda-1}(-1)$$

$$= +\lambda(1-t)^{-\lambda-1}$$

$$\frac{d}{dt}\boxed{M_x(t)} = \lambda(1-t)^{-\lambda-1}$$

Sub $t=0$

$$\boxed{E(x) = \lambda}$$

$$\frac{d^2}{dt^2}[M_x(t)] = \lambda(-\lambda-1)(1-t)^{-\lambda-2} \frac{d}{dt}(1-t)$$

$$= -\lambda(\lambda-1)(1-t)^{-\lambda-2}(-1)$$

$$= \lambda(\lambda+1)(1-t)^{-\lambda-2}$$

Sub $t=0$

$$E(x^2) = \lambda(\lambda+1) = \lambda^2+\lambda$$

$$\text{var}(x) = E(x^2) - [E(x)]^2$$

$$= \lambda^2+\lambda-(\lambda)^2$$

$$= \lambda$$

Mean and variance is equal in continuous ~~rando~~ gamma distribution ~~of gamn~~

Two parameter gamma distribution.

A r.v X is said to follow ~~a~~ gamma distribution with two parameter, if it probability density function is given by.

$$f(x) = \begin{cases} \dfrac{a^{\lambda} e^{-ax} x^{\lambda-1}}{\Gamma(\lambda)} & a>0,\ \lambda>0,\ 0<x<\infty \\ 0 & \text{otherwise.} \end{cases}$$

$X \sim (a, \lambda)$

Set $a = 1$

$$f(x) = \begin{cases} \dfrac{e^{-x} x^{\lambda-1}}{\Gamma(\lambda)} & \lambda>0,\ 0<x<\infty \\ 0 & \text{otherwise.} \end{cases}$$