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[Unit-IV] Graph Theory - I

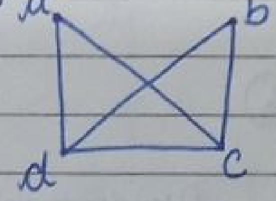
1. **Graph** → The Graph $G = (V, E)$ consists of V, non-empty set of vertices (nodes) and E, a set of edges. Each edge has either one or two end points associated with it.

2. **Finite Graph** → A Graph whose vertex is set is having finite elements is called finite graph.

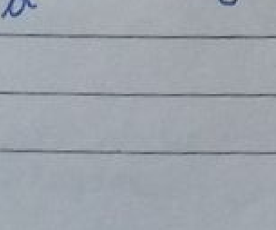
3. **Infinite Graph** → A Graph whose vertex is set is having infinite elements or vertices is called infinite graph.

4. **Simple Graph** → A Graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices, is called simple graph. [One edge or no edge between a pair of vertex]

example,



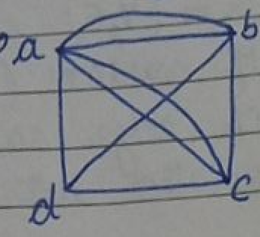
✓
simple graph



X
not simple graph

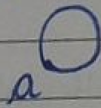
5. <u>multigraph</u> → Graphs that have multiple edges b/w the pair of vertices.

Example,



6. <u>Loop</u> → A single edge that connects a vertex with itself is called loop.

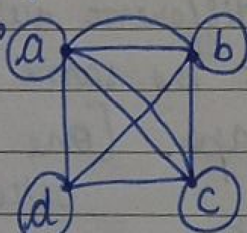
Example,

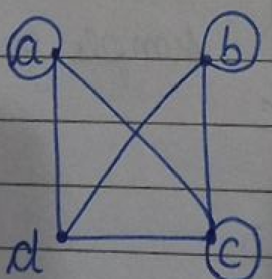


7. <u>Pseudo Graph</u> → A graph having loops and multiple edges is called a pseudo graph.

[Even if there are only loops then it is pseudograph]

Example,

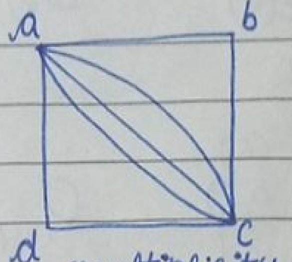




Pseudo graph

8. <u>multiplicity</u> → Let m be the no. of edges b/w the pair of vertices, then the multiplicity of the pair of vertices = m.

Example,



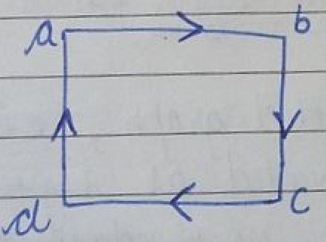
multiplicity

$\{a, b\} \rightarrow 1$

$\{a, c\} \rightarrow 3$

9. <u>Directed graph (digraph)</u> → A directed graph (V,E) consists of a non-empty set of vertices V and a set of directed edges E. For example, a directed edge associated with ordered pair (u, v) means an edge is initiating from u and terminating at v.

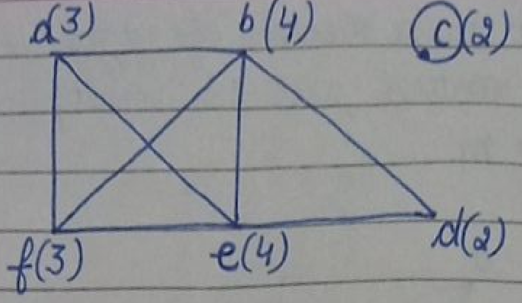
Example,

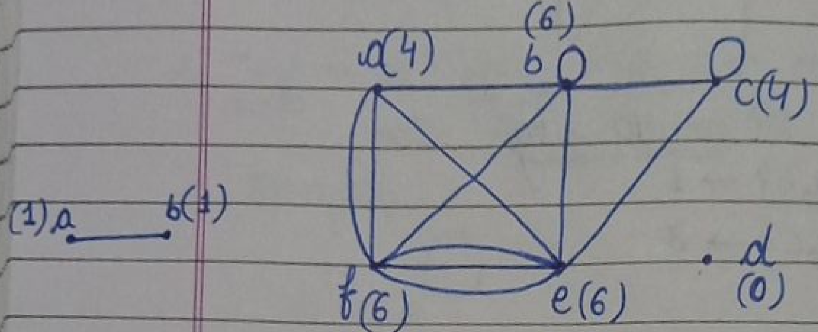


(a, b)
(d, a)
(c, d)
(b, c)

<u>Degree</u> → Degree of the vertex in an undirected graph is the number of edges that are incident with that vertex or the number of edges passing through the vertex.

In case of loop at the vertex it contributes twice to the degree of the vertex.





⇒ **Note** →

★ A vertex of degree = 0 is called $\boxed{\text{Isolated}}$.
★ A vertex of degree = 1 is called $\boxed{\text{pendant}}$.
★ multiple edges b/w same pair ~~pa~~ of vertices is called as $\boxed{\text{parallel edges}}$.

11. In-degree & Out-degree →

In case of directed graph, the in-degree of a vertex is represented as ~~Degree - deg (V)~~ $deg^-(V)$, is the no. of edges that terminate at the vertex V.

In case of out-degree it is represented as $deg^+(V)$, is the no. of edges that is initiating from the given vertex V.

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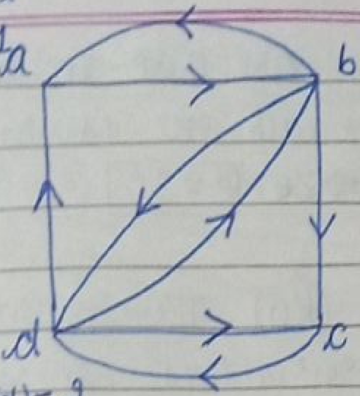
$deg^-(a)=2$
$deg^+(a)=1$



$deg^-(b)=2$
$deg^+(b)=3$

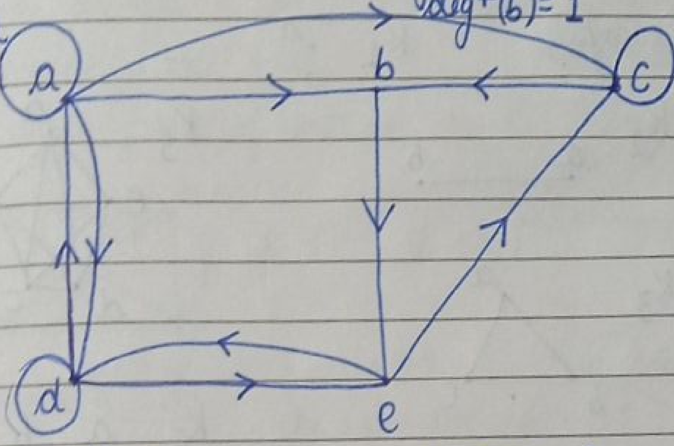
$deg^-(d)=2$
$deg^+(d)=3$

$deg^-(c)=2$
$deg^+(c)=1$

Q.

$deg^-(a)=2$
$deg^+(a)=4$

$deg^-(b)=2$
$deg^+(b)=1$



$deg^-(c)=3$
$deg^+(c)=2$

$deg^-(d)=3$
$deg^+(d)=3$

$deg^-(e)=2$
$deg^+(e)=2$

[★ Loop is counted one in each]

12. Handshaking Theorem → Let $G=(V,E)$ be an $\boxed{\text{undirected}}$ $\boxed{\text{graph}}$ with ~~n small~~ e edges then (i) $2e = \Sigma\, deg(V)$ and for $\boxed{\text{directed graph}}$

(ii) $e = \Sigma\, deg^-(V) = \Sigma\, deg^+(V)$

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Q. How many edges are there in a graph with 10 vertices & each one having degree 6?

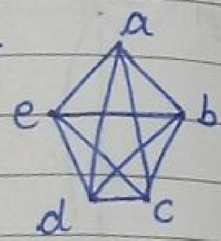
→ $2e = 10 \times 6 \Rightarrow \cancel{30}$ $\boxed{e = 30}$

13. <u>Complete graph</u> → (K_n) The complete graph on 'n' vertices is represented as K_n, is the $\boxed{\text{simple graph}}$ that contains exactly one edge b/w each pair of vertices.

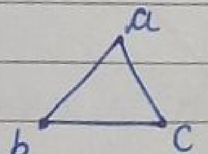
Example K_1 .a

K_2

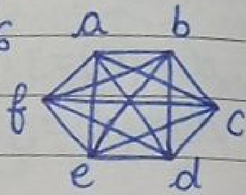
K_5



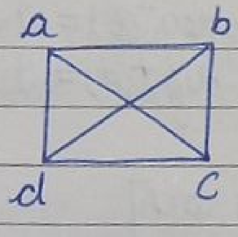
K_3



K_6

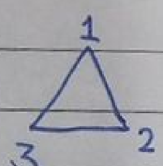


K_4

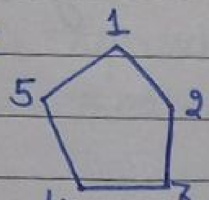


14. <u>Cycle (C_n)</u> → The cycle C_n ($n \geq 3$) consists of n vertices 1, 2, ... n and edges $\{1, 2\}, \{2, 3\}, \{3, 4\}, \ldots \{n-1, n\}, \{n$

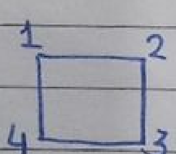
C_3



C_5

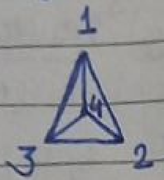


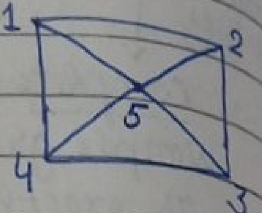
C_4



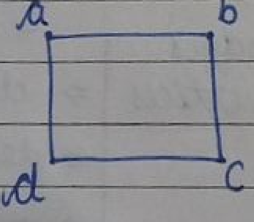
15. <u>Wheele (W_n)</u> → The wheele is obtained by adding a new vertex in the cycle (C_n) and connecting this vertex with all the previous 'n' vertices with new edges.
no. of vertices in $W_n = \boxed{n+1}$.

W_3 W_4

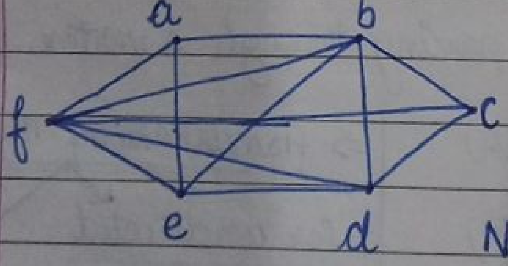




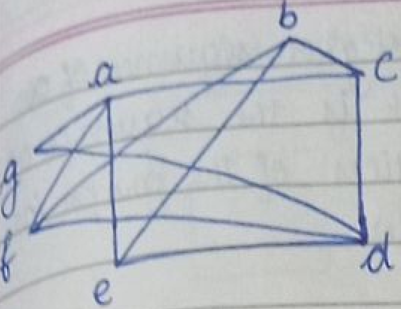
16. <u>Bipartite Graph</u> → A simple graph G is called bipartite if its vertex set V can be partitioned into 2 disjoined sets V_1 & V_2 such that every edge in the graph connects a vertex in V_1 & a vertex in V_2. so that no edge in G connects either 2 vertices of the vertex V_1 or V_2.



$V = \{a, b, c, d\}$
$V_1 = \{a, c\}$
$V_2 = \{b, d\}$

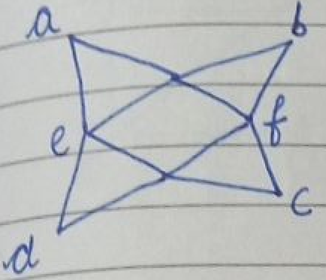


$V = \{a, b, c, d, e, f\}$
~~$V_1 = \{a,$~~
~~$V_2 = \{$~~
<u>Not bipartite</u>



$V = \{a, b, c, d, e, f, g\}$
$V_1 = \{a, b, d\}$
$V_2 = \{\cancel{b}\ g, c, f, e\}$

Bipartite

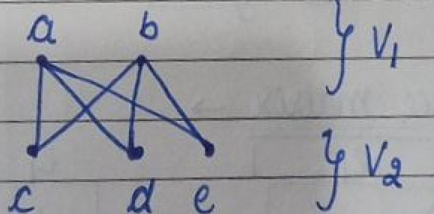


$V = \{a, b, c, d, e, f\}$
$V_1 = \{e, f\}$
$V_2 = \{a, b, c, d\}$

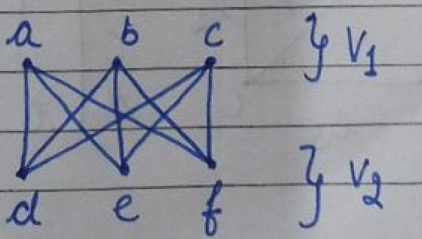
Bipartite

17. **Complete Bipartite Graph** → The complete bipartite graph ~~is~~ represented as $K_{m,n}$, is the graph that has its vertex set partitioned into two subsets of m & n vertices respectively. There is an edge ~~if~~ b/w 2 vertices iff one vertex is in the set 1 and second vertex is in the set 2.

$K_{2,3}$

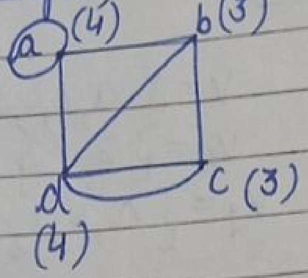


$K_{3,3}$



** 18. <u>Degree sequence</u> → The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in ~~as~~ non-increasing order.

Example,

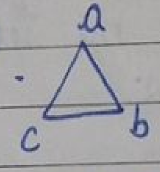


<u>Degree sequence:-</u>
4, 4, 3, 3

19. <u>Regular Graph</u> → A simple graph is called a regular graph if all the vertices of the graph have same degree and if the degree is n then it is called as n-regular graph.

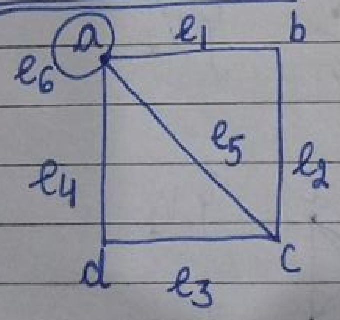
Example,

K_3



<u>2-regular</u>

20. <u>Incidence matrix</u> →



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

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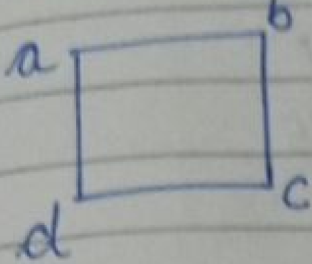
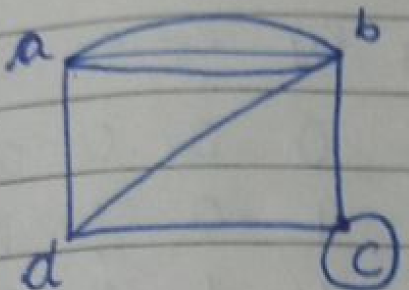
~~Adjan~~ Adjacency matrix →



$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

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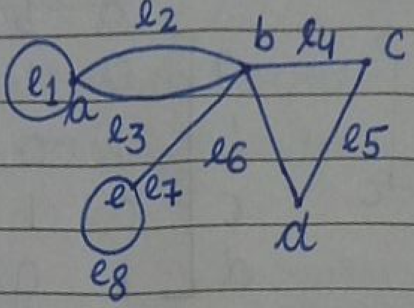
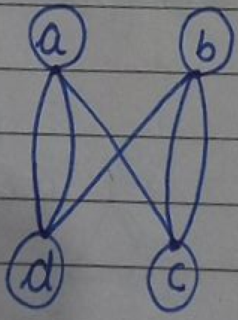
[Revision]

Q1 Draw Incidence matrix



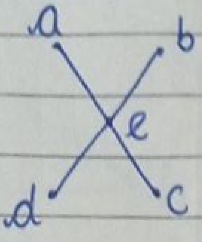
$$
\begin{array}{c}
\\ a \\ b \\ c \\ d \\ e
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
$$

Q2 Draw adjancy matrix



$$
\begin{array}{c}
\\ a \\ b \\ c \\ d
\end{array}
\begin{array}{c}
\begin{array}{cccc} a & b & c & d \end{array} \\
\begin{bmatrix}
1 & 0 & 1 & 2 \\
0 & 1 & 2 & 1 \\
1 & 2 & 1 & 0 \\
2 & 1 & 0 & 1
\end{bmatrix}
\end{array}
$$

Check whether graph is bipartite or not.



$V = \{a, b, c, d, e\}$

~~$V_1 = \{a, b, e, c\}$~~

~~$V_2 = \{b, d, e, d, e\}$~~

~~Not bipartite~~

$V_1 = \{a, b, c, d\}$

$V_2 = \{e\}$

$V_1 \cap V_2 = \phi \Rightarrow$ **Bipartite**

(complete bipartite)

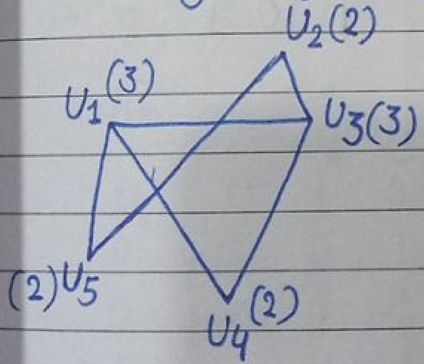
★ $\boxed{\text{Subjective Que ETE}}$

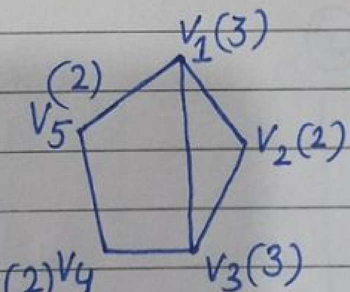
★ 10 marks
5 → mapping
5 → matrix

Isomorphism

Check
- no. of vertices
- no. of edges
- degree of vertices

- Mapping of degrees [Check for adjacent ~~matri~~ vertices]
- making Adjacency matrices for both graphs





— no. of vertices of $G_1 = G_2 = 5$

— no. of edges of $G_1 = G_2 = 6$

G_1: 3, 3, 2, 2, 2

G_2: 3, 3, 2, 2, 2

] Degree sequence is equal

- mapping

$f(u_1) = v_1$
$f(u_5) = v_5$
$f(u_2) = v_4$
$f(u_3) = v_3$
$f(u_4) = v_2$

- Adjacency matrix

(G1)

$$\begin{array}{c|c} & \begin{array}{ccccc} u_1 & u_5 & u_2 & u_3 & u_4 \end{array} \\ \begin{array}{c} u_1 \\ u_5 \\ u_2 \\ u_3 \\ u_4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(G2)

$$\begin{array}{c|c} & \begin{array}{ccccc} v_1 & v_5 & v_4 & v_3 & v_2 \end{array} \\ \begin{array}{c} v_1 \\ v_5 \\ v_4 \\ v_3 \\ v_2 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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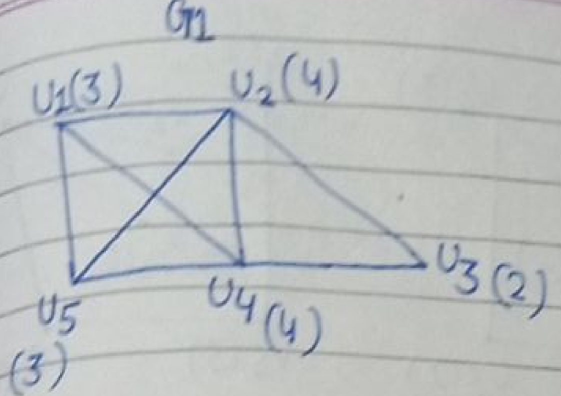
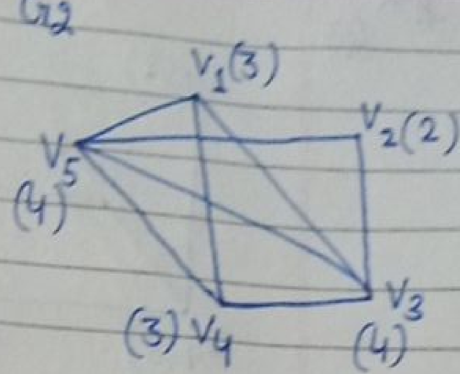




- no. of vertices of $G_1 = G_2 = 5$
- no. of ~~vertices~~ edges of $G_1 = G_2 = 8$

- degree sequence of $G_1 = G_2 = 4, 4, 3, 3, 2$

- mapping

$$f(u_1) = v_1$$
$$f(u_2) = v_5$$
$$f(u_3) = v_2$$
$$f(u_4) = v_3$$
$$f(u_5) = v_4$$

- matrix

G_1

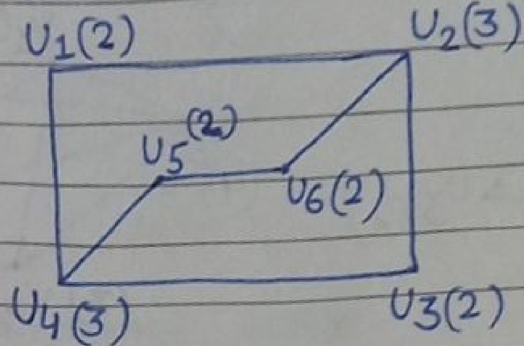
$$\begin{array}{c|c} & \begin{matrix} u_1 & u_2 & u_3 & u_4 & u_5 \end{matrix} \\ \begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

G_2

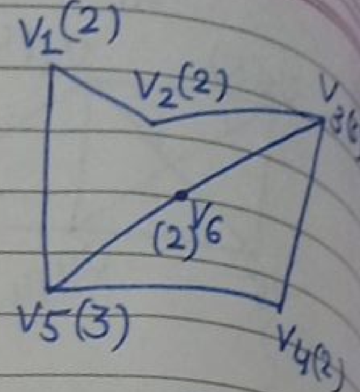
$$\begin{array}{c|c} & \begin{matrix} v_1 & v_5 & v_2 & v_3 & v_4 \end{matrix} \\ \begin{matrix} v_1 \\ v_5 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$\therefore$ G_1 & G_2 are isomorphic.

★★ Q3.



G_1



G_2

— no. of vertices of $G_1 = G_2 = 6$
— no. of edges of $G_1 = G_2 = 7$
— degree sequence of $G_1 = G_2 = 3,3,2,2,2,2$

— mapping

$f(U_1) = V_4$
$f(U_2) = V_3$
$f(U_3) = V_6$
$f(U_4) = V_5$
$f(U_5) = V_1$
$f(U_6) = V_2$

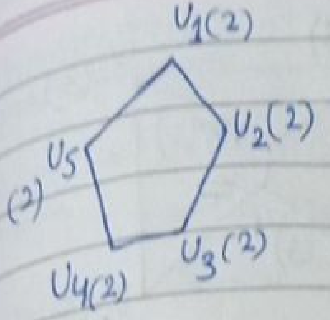
$$\begin{array}{c|c} & \begin{array}{ccccc} V_4 & V_3 & V_6 & V_5 & V_1 \end{array} \\ \begin{array}{c} V_4 \\ V_3 \\ V_6 \\ V_5 \\ V_1 \\ V_2 \end{array} & \left[\begin{array}{ccccc} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right. \end{array}$$

- matrix

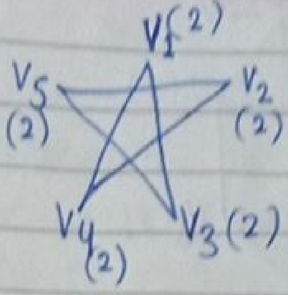
G_1

$$\begin{array}{c|c} & \begin{array}{cccccc} U_1 & U_2 & U_3 & U_4 & U_5 & U_6 \end{array} \\ \begin{array}{c} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$\therefore G_1$ & G_2 are isomorphic.



G_1



G_2

- no. of vertices of $G_1 = G_2 = 5$
- no. of edges of $G_1 = G_2 = 5$
- degree sequence of $G_1 = G_2 = 2, 2, 2, 2, 2$

- mapping

$f(U_1) = \cancel{v_1}\ V_1$
$f(U_2) = \cancel{v_2}\ V_3$
$f(U_3) = \cancel{v_3}\ V_5$
$f(U_4) = \cancel{v_4}\ V_2$
$f(U_5) = \cancel{v_5}\ V_4$

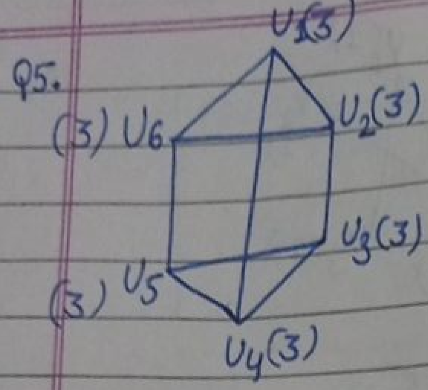
- matrix

G_1

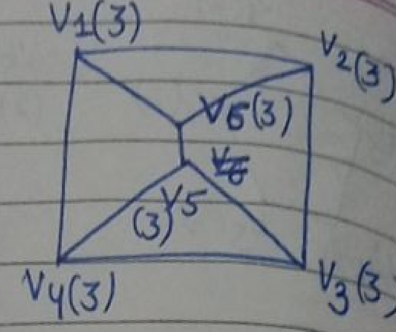
$$\begin{array}{c} \\ U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{array} \begin{array}{c} \begin{array}{ccccc} U_1 & U_2 & U_3 & U_4 & U_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ V_1 \\ V_3 \\ V_5 \\ V_2 \\ V_4 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_3 & V_5 & V_2 & V_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Yes, G_1 & G_2 are isomorphic.

Q5.



(G1)



(G2)

- no. of vertices of $G_1 = G_2 = 6$
- no. of edges of $G_1 = G_2 = 9$
- degree sequence of $G_1 = G_2 = 3,3,3,3,3,3$

- mapping

$f(U_1) = V_1$
$f(U_2) = V_2$
$f(U_3) = V_3$
$f(U_4) = V_4$
$f(U_5) = V_5$
$f(U_6) = V_6$

- matrix

$$\begin{array}{c|cccccc} & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 \\ \hline U_1 & 0 & 1 & 0 & 1 & 0 & 1 \\ U_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ U_3 & 0 & 1 & 0 & 1 & 1 & 0 \\ U_4 & 1 & 0 & 1 & 0 & 1 & 0 \\ U_5 & 0 & 0 & 1 & 1 & 0 & 1 \\ U_6 & 1 & 1 & 0 & 0 & 1 & 0 \end{array} \qquad \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0 & 1 & 0 & 1 & 0 \\ V_2 & 1 & 0 & 1 & 0 & 0 \\ V_3 & 0 & 1 & 0 & 1 & 1 \\ V_4 & 1 & 0 & 1 & 0 & 1 \\ V_5 & 0 & 0 & 1 & 1 & 0 \\ V_6 & 1 & 1 & 0 & 0 & 1 \end{array}$$

$\therefore G_1 \& G_2$ are isomorphic

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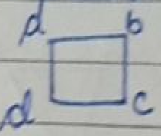
Connectivity :-

1. **Path** → A path is a sequence of edges that begins at a vertex and travels from vertex to another vertex (without repitition of vertex) along the edges of a graph.



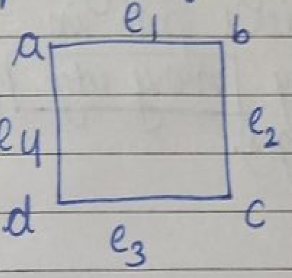
a → c ① a b c
② a d c

2. **simple path** → A path is called simple if it doesnot contain the same edge more than once (no repitition of edges).

3. **circuit** → A path is called a circuit if it begins and ends at the same vertex.

a → b → c → d → a

4. **walk** → A walk is defined to be an alternative sequence of vertices and edges.



a → c
① $a\,e_1\,b\,e_2\,c$
② $a\,e_4\,d\,e_3\,c$

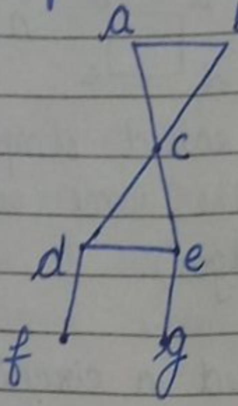
5. **closed walk** → a closed walk is used to represent a walk that begins and ends at the same vertex.

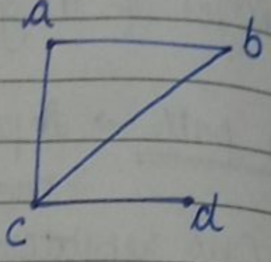
$$a\,e_1\,b\,e_2\,c\,e_3\,d\,e_4\,a$$

6. **Trail** → It is used to represent a walk with no repeated edges.

7. **Connected graph** → An undirected graph is called connected if there is a path b/w every pair of vertices of the graph.

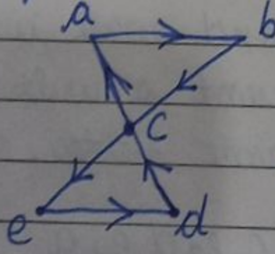
Example,





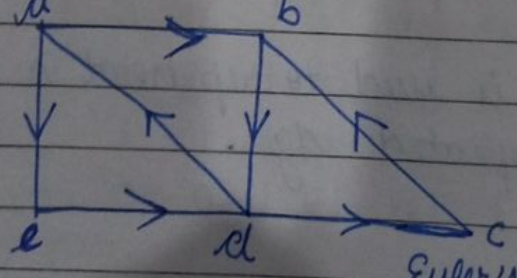
8. **Euler Path** → [~~Euler Circuit~~] An euler-circuit **Euler Circuit** in a graph G is a simple circuit containing every edge of G whereas in euler's path in G is a simple path containing every edge of G (without repetition of edges).

Example,



a → b → c → e → d → c → a

Euler's circuit



Euler's path

a → e → d → c → b → d → a → b

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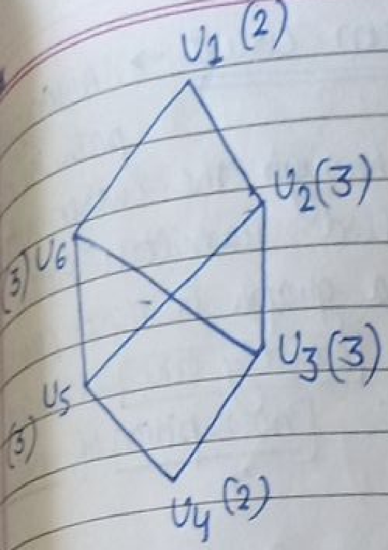
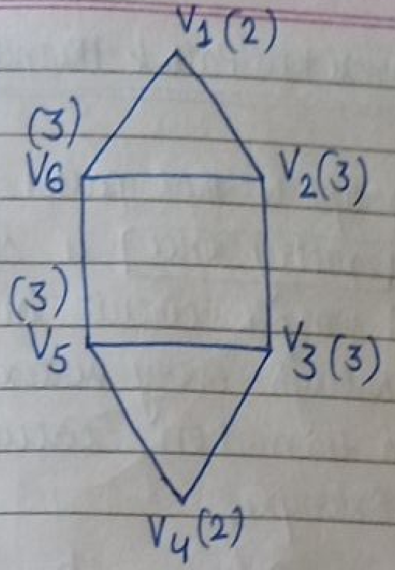




G_1

3 edges

$G_2 \rightarrow V_1 - V_6 - V_2 - V_1$

circuit of length 3

- no. of vertices $G_1 = G_2 = 6$
- no. of edges $G_1 = G_2 = 8$
- degree sequence of $G_1 = G_2 = 3,3,3,3,2,2$

- mapping

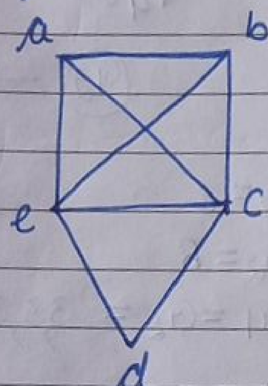
$f(U_1) = V_1$
$f(U_2) = V_2$
$f(U_3) = V_3$
$f(U_4) = V_4$
$f(U_5) = V_5$
$f(U_6) = V_6$

** since no circuit of length 3 exists in Graph 1 hence, they are not isomorphic

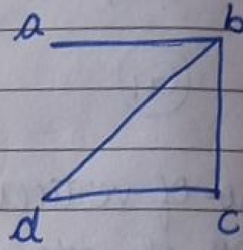
$$\begin{array}{c|cccccc} & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 \\ \hline U_1 & 0 & 1 & 0 & 0 & 0 & 1 \\ U_2 & 1 & 0 & 1 & 0 & 1 & 0 \\ U_3 & 0 & 1 & 0 & 1 & 0 & 1 \\ U_4 & 0 & 0 & 1 & 0 & 1 & 0 \\ U_5 & 0 & 1 & 0 & 1 & 0 & 1 \\ U_6 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{c|cccccc} & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\ \hline V_1 & 0 & 1 & 0 & 0 & 0 & 1 \\ V_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ V_3 & 0 & 1 & 0 & 1 & 1 & 0 \\ V_4 & 0 & 0 & 1 & 0 & 1 & 0 \\ V_5 & 0 & 0 & 1 & 1 & 0 & 1 \\ V_6 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}$$

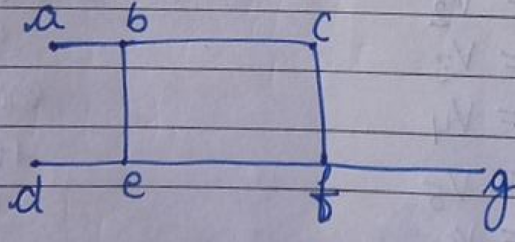
9. <u>Hamilton Path & Hamilton circuit</u> → A simple path in a graph G that passes through the every vertex $\boxed{\text{exactly once}}$ is called Hamilton Path.
A simple circuit in a graph G that passes through every vertex $\boxed{\text{exactly once}}$ is called a Hamilton circuit. [<u>no repition</u> of <u>vertices</u>]
Example,



G1
<u>Hamilton circuit</u>



G2
<u>Hamilton path</u>



G3
neither Hamilton's Path/circuit
nor Euler's Path/circuit

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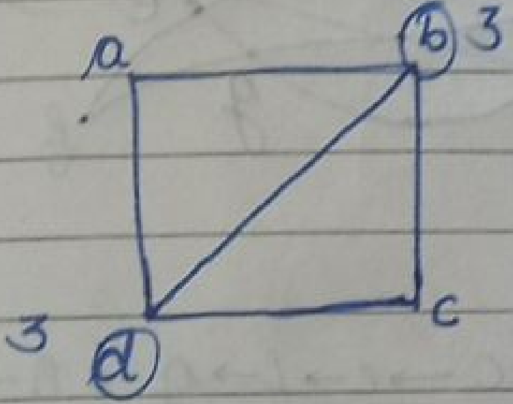
* Note →

1. A connected multigraph has an euler path but not an euler circuit iff it has exactly 2 vertices of odd degree.

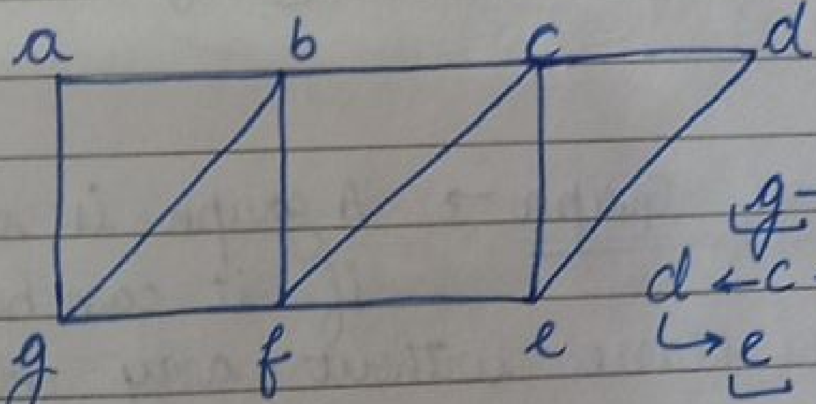
example,



G_1

b → d → c → b → a → d

Euler's path but not circuit



G_2

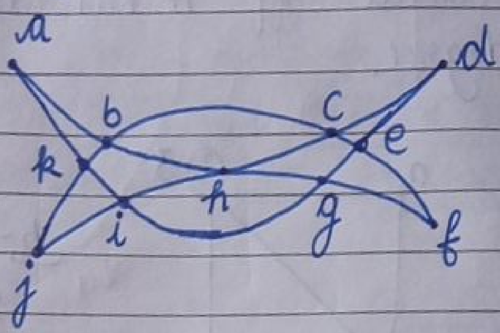
g → b → f → c → e → f

d ← c ← b ← a ← g

↳ e

Euler's Path

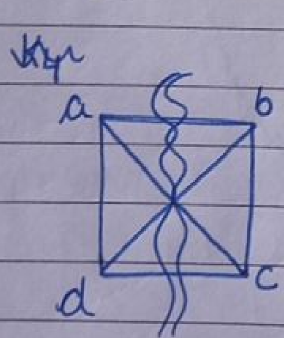
9. A connected multigraph with atleast 2 vertices has an Euler's circuit iff it's vertices are of even degree.

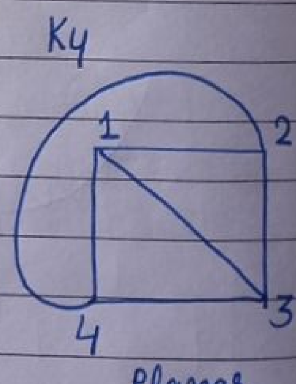


(i) → j → k → b → c → e → f → g → h → i → k → a → b

(i) ← g ← e ← d ← c ← h

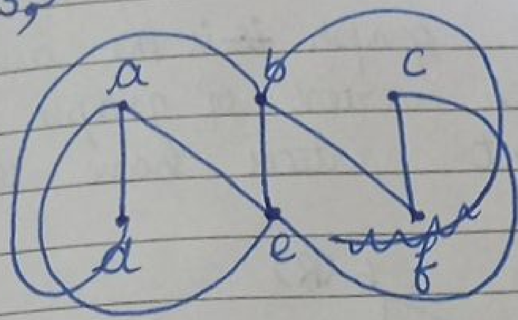
10. <u>Planar Graph</u> → A graph is called planar if it can be drawn in the plane without any edges crossing (where a crossing of edge is the intersection of lines at a point other than their common end points).





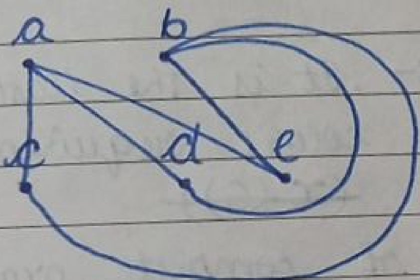
<u>Planar</u>

$K_{3,3}$



Not planar

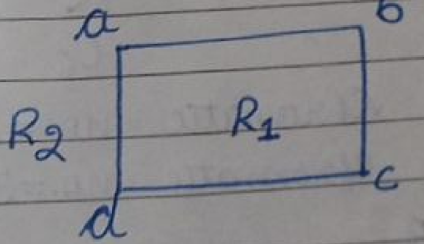
$K_{2,3}$



Planar

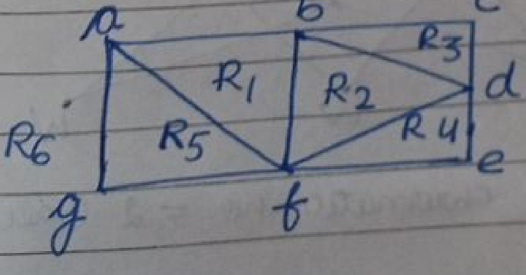
Euler's Formula → Let G be a connected planar simple graph with E edges & V vertices. Let R be the no. of regions in a planar representation of G. Then,

$$\boxed{R = E - V + 2}$$



$R = 4 - 4 + 2$

$\boxed{R = 2}$



$R = \cancel{10}\ 11 - 7 + 2$

$\boxed{R = 6}$

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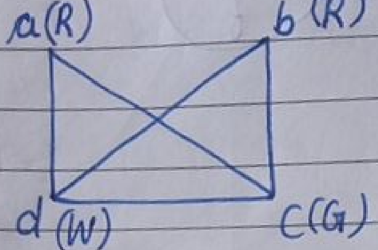
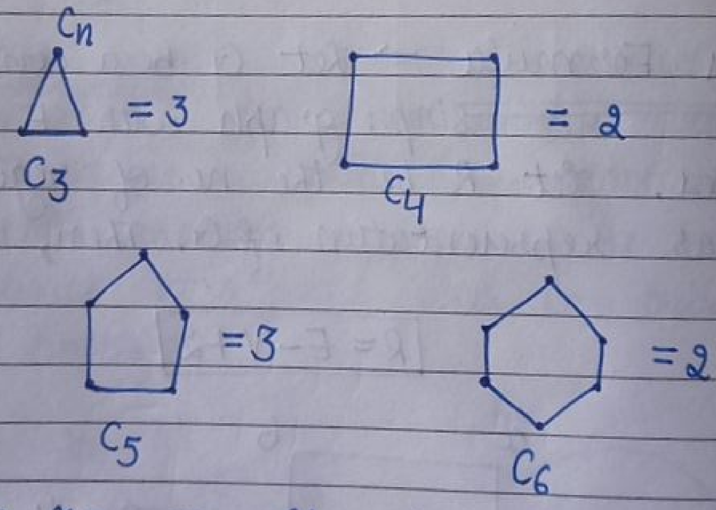
(1) **Graph colouring** :- ~~A~~ colouring of a simple graph ~~is~~ is the assignment of the colour to each vertex of graph so that no two adjacent vertices have the same colour.



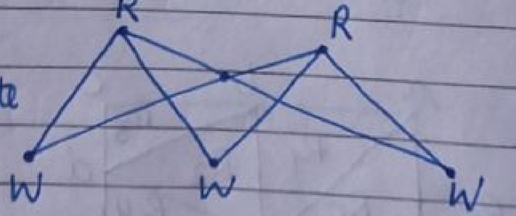
(2) **Chromatic no. (Cn)** :- It is the least number of colors required for the coloring of a graph. ~~$(\chi(G))$~~

* Chromatic no. of complete graph (K_n) = no. of vertices



For n = even, Chromatic no. = 2
For n = odd, chromatic no. = 3

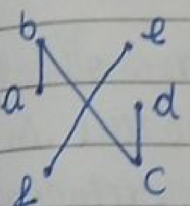
For Bipartite $K(m,n)$



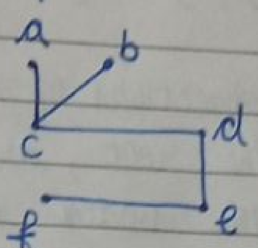
chromatic no. = 2 (always)

Four colouring theorem :- The chromatic no. of a planar graph is not greater than 4. K_4 is a planar graph.

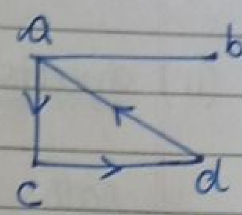
Tree :- a tree is a connected undirected graph with no simple circuit.



~~Tree~~ X

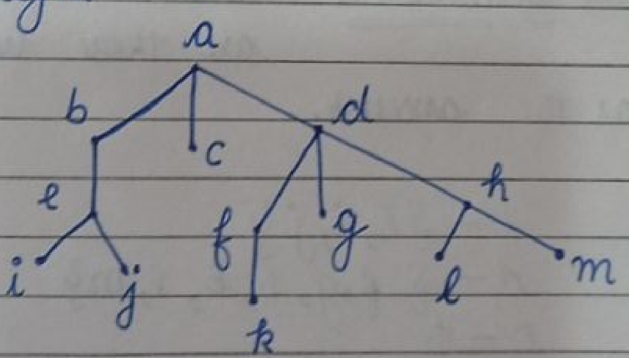


Tree ✓



~~Tree~~ X

5) Rooted Tree :- Its a tree in which one vertex is designated as root & rest of the vertices are directed away from root with the help of edges.



(1) Parent :- Parent of vertex V is the unique vertex u such that there is a direct edge between vertex V and u.

For example, (a, b, ~~c~~ d, e, f, h)

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(ii) **Child** :- When u is a parent of v then v is a child of u.

$(i, j, k, l, m, e, f, g, h)$

(iii) **Siblings** :- vertices with same parents.

$\{b, c, d\}$, $\{f, g, h\}$, $\{i, j\}$, $\{l, m\}$

(iv) **Ancestors** :- Ancestors of a vertex other than the root, are the vertices in the path from the vertex to root, excluding vertex itself & including root.

$i - \{a, b, e\}$
$k - \{f, d, a\}$
$h - \{d, a\}$

(v) ~~Decen~~ **Descendants** :- Descendants of vertex v are those vertices that have v as the ancestor.

$b - \{e, i, j\}$
$d - \{f, g, h, k, l, m\}$
$c - \phi$

(vi) **Leaf** :- A vertex of a tree is called leaf if it doesn't have any further extension.

(i, j, k, l, m, c, g)

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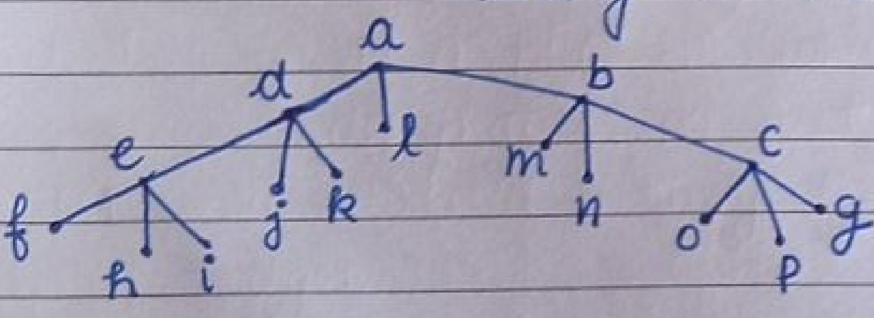
Internal vertices :- vertices that have children are called internal vertices.

$\{b, d, e, f, h\}$

m-ary tree :- A rooted tree is called m-ary tree if every internal vertex has not more than $\underbrace{m \text{ children.}}_{\text{maximum no. of children one can have.}}$

Full-m-ary :- If every internal vertices have exactly m-children.



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Formulae

1. A tree with n vertices has $n-1$ edges always.
2. A full-m-ary tree with i internal vertices contains $\boxed{n = mi+1}$ vertices & $\boxed{l = (m-1)i+1}$ leaves.
3. A full-m-ary tree with n vertices has $\boxed{i = \frac{n-1}{m}}$ internal vertices & $\boxed{l = \frac{(m-1)n+1}{m}}$ leaves.
4. A full-m-ary tree with l leaves has $\boxed{n = \frac{ml-1}{m-1}}$ vertices & $\boxed{i = \frac{l-1}{m-1}}$ internal vertices.

Q. How many edges does a tree with 10000 vertices have?

↳ $e = n-1 = 10000-1 = \underline{999}$ Ans.

Q. How many vertices does a full-5-ary tree with 100 internal vertices have?

↳ $n = mi+1$

$n = 5(100)+1 = \underline{501}$ Ans.

Q. How many edges does a full-binary-tree with 1000 internal vertices have?

↳ $e = n-1$

$= mi+1-1$

$= mi = \underline{2000}$ Ans.

Q. How many leaves does a full-3-ary tree with 100 vertices have?

A. $\cancel{l = (m-1)i+1}$

$\cancel{= (3-1)+}$

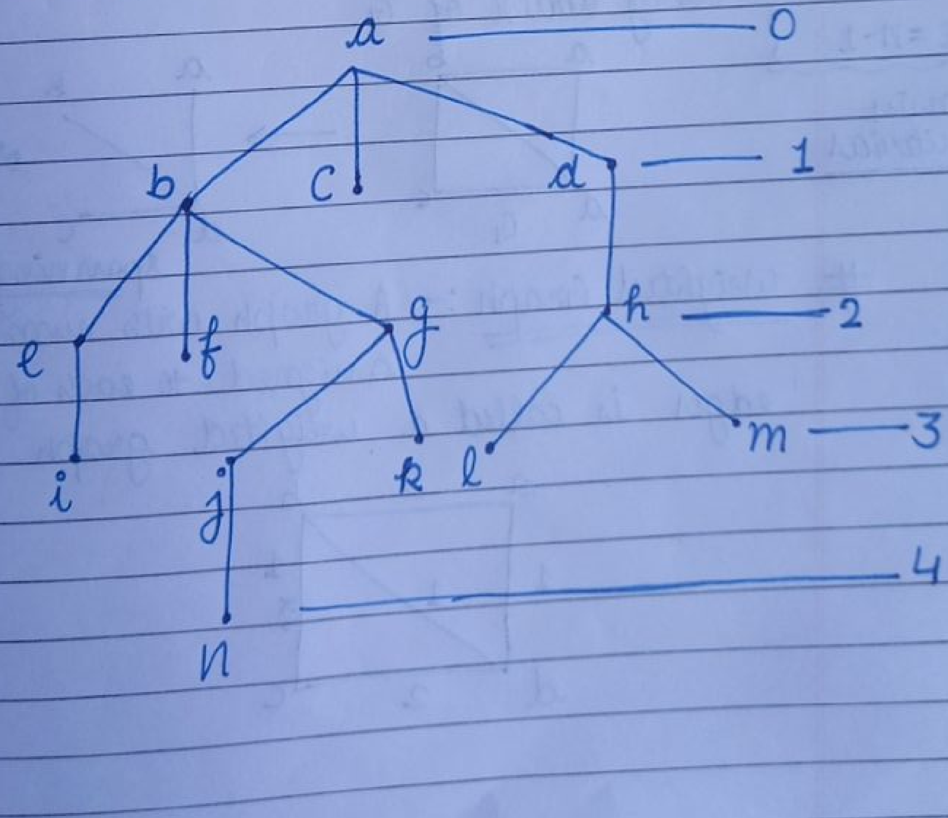
$$l = \frac{(m-1)n+1}{m}$$

$$= \frac{(3-1)100+1}{3} = \frac{201}{3}$$

$$l = 67$$ Ans.

Level of a vertex :- The level of a vertex in a rooted tree is the length of the unique path from the root to this vertex.

The level of the root is defined to be zero.



Height of a tree :- The height of a rooted tree is the maximum of the levels of vertices or it is the length of the longest path from root to any vertex.

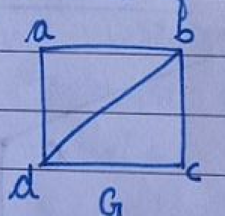
example, in previous fig.,
$H =$ height of tree is 4.

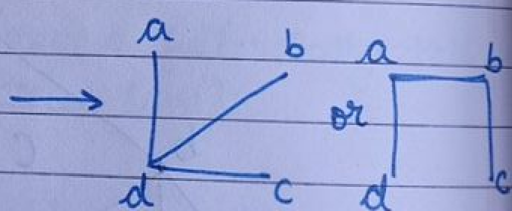
Balanced tree :- A rooted m-ary tree of height H is called balanced tree if all the leaves are at the level H or H−1.

Spanning tree :- Let G be a simple graph. A spanning tree of G is a sub-graph of G i.e. a tree containing every vertex of G.

→ connected
→ no circuit
→ $e = n-1$

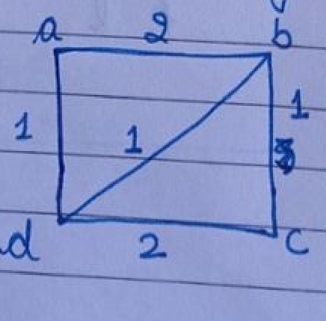
3 main criterias





spanning tree

Weighted Graph :- A graph with numbers assigned to each of its edges is called a weighted graph.



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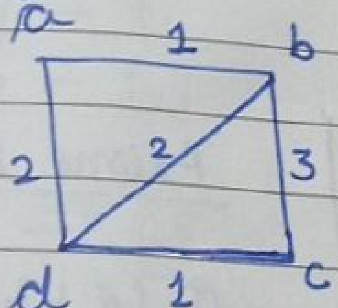
* Impt.

Minimum Spanning Tree :- A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible ~~sub~~ sum of weights of its edges.



G

changing to minimum spanning tree

Kruskal's Algorithm

$n = 4$

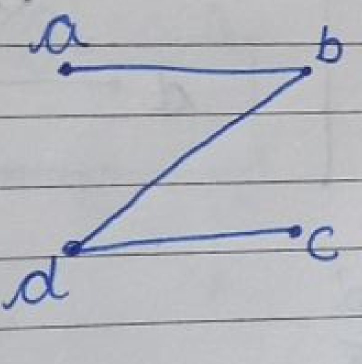
$e = n - 1$

$= 4 - 1$

$= 3$

1. $\{a, b\} - 1$
2. $\{c, d\} - 1$
3. $\{b, d\} - \underline{2}$

$\underline{4}$



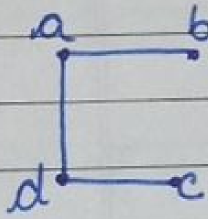
Prims' Algorithm

$n = 4$

$e = 3$

1. $\{a, b\} - 1$
2. $\{a, d\} - 2$
3. $\{d, c\} - \underline{1}$

$\underline{4}$



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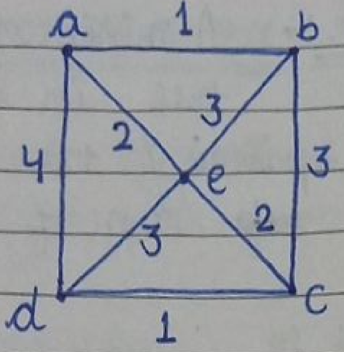
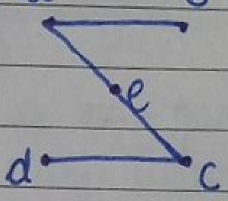
Q1

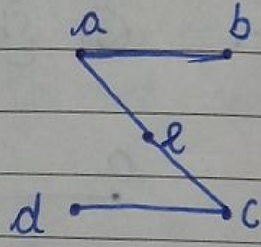


Kruskal's Algorithm	Prims Algorithm
① {a, b} − 1	① {a, b} − 1
② {c, d} − 1	② {a, e} − 2
③ {a, e} − 2	③ {e, c} − ~~3~~ 2
④ {e, c} − 2	④ {d, c} − 1
6	6

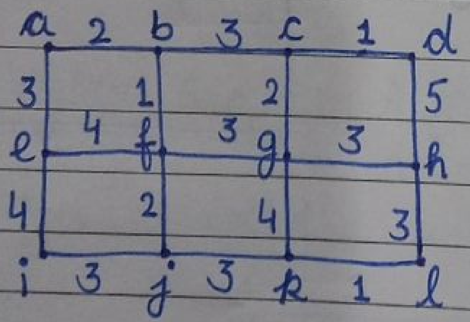
$n = \cancel{4}\ 5$

$e = \cancel{3}\ 4$





Q2



$n = 12$

$e = 12 - 1$

$= 11$

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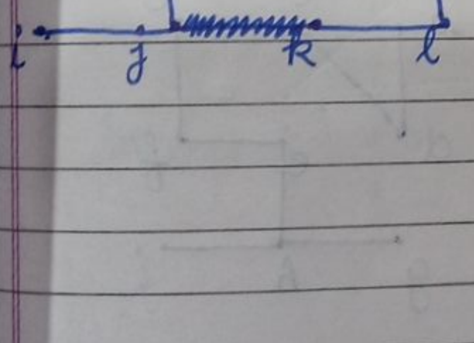
Kruskal's Algorithm

1. $\{c, d\} - 1$
2. $\{b, f\} - 1$
3. $\{k, l\} - 1$
4. $\{a, b\} - 2$
5. $\{c, g\} - 2$
6. $\{f, j\} - 2$
7. $\{b, c\} - 3$
8. $\{a, e\} - 3$
9. $\{g, h\} - 3$
10. $\{h, l\} - 3$
11. $\{i, j\} - 3$

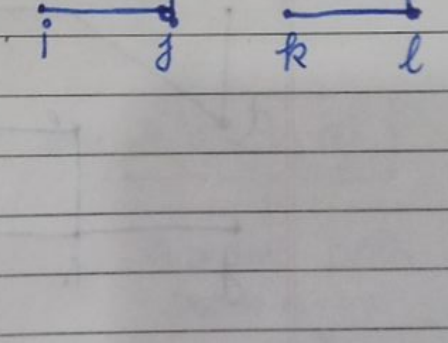
$$\overline{24}$$



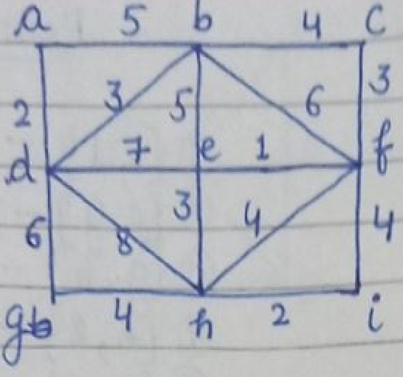
Prims Algorithm

1. $\{c, d\} - 1$
2. $\{c, g\} - 2$
3. $\{g, h\} - 3$
4. $\{h, l\} - 3$
5. $\{k, l\} - 1$
6. $\{b, c\} - 3$
7. $\{b, f\} - 1$
8. $\{f, j\} - 2$
9. $\{j, i\} - 3$
10. $\{a, b\} - 2$
11. $\{a, e\} - 3$

$$\overline{24}$$



Q3



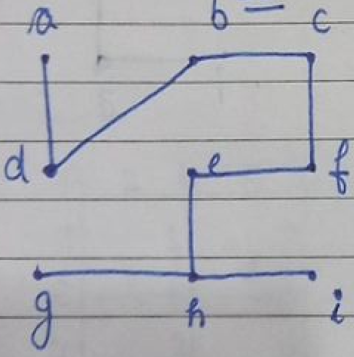
$n = 9$

$e = 8$

Krushal's Algorithm

1. $\{e, f\} - 1$
2. $\{h, i\} - 2$
3. $\{a, d\} - 2$
4. $\{c, f\} - 3$
5. $\{b, d\} - 3$
6. $\{e, h\} - 3$
7. $\{g, h\} - 4$
8. $\{b, c\} - \underline{4}$

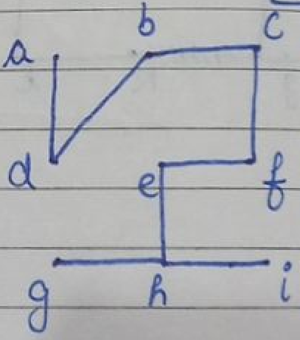
22



Prims Algorithm

1. $\{e, f\} - 1$
2. $\{e, h\} - 3$
3. $\{h, i\} - 2$
4. $\{g, h\} - 4$
5. $\{c, f\} - 3$
6. $\{b, c\} - 4$
7. $\{b, d\} - 3$
8. $\{a, d\} - \underline{2}$

22



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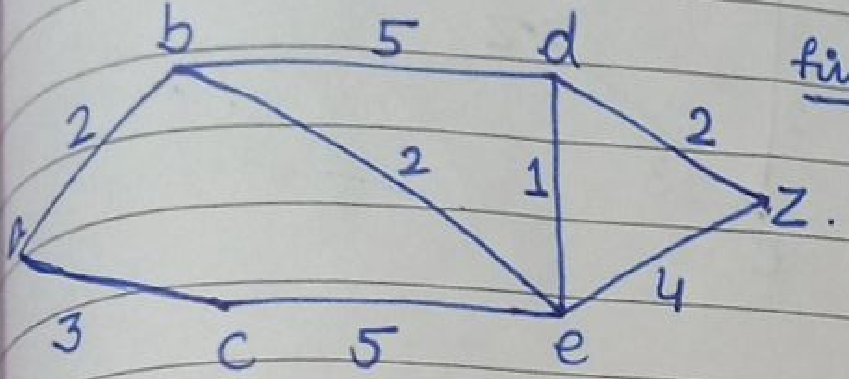
Dijkstra's Algorithm [Shortest Path]



find Shortest path b/w $\boxed{a \to z}$ – ?

[For no edge → ∞]

Source	b	c	d	e	z
{a}	$\boxed{2(a)}$	3(a)	∞	∞	∞
	smallest path				
~~{b}~~					
{a, b}	—	$\boxed{3(a)}$	7(a,b)	$\boxed{4(a,b)}$	∞
{a, b, c}	—	—	7(a,b)	$\boxed{4(a,b)}$	∞
{a, b, e}	—	—	$\boxed{5(a,b,e)}$	—	8(a,b,e)
{a, b, e, d}	—	—	—	—	$\boxed{7(a,b,e,d)}$

[back tracing]

skip the back tracing step

Shortest path from a → z

[a → b → e → d → z = 7]

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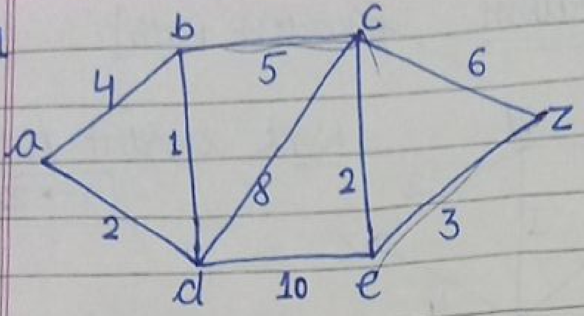
Q1



(a→z)

source	b	c	d	e	z
{a}	4(a)	~~9(a)~~ ∞	$\boxed{2(a)}$	~~12(e)~~ ∞	∞
{a, d} ~~a, b~~	$\boxed{3(a,d)}$	10(a,d)	—	12(a,d)	∞
{a, d, b}	—	$\boxed{8(a,d,b)}$	—	∞	∞
{a, d, b, c}	—	—	—	$\boxed{10(a,d,b,c)}$	13 (a,d,b,c)
{a, d, b, c, e}	—	—	—	—	$\boxed{13(a,d,b,c,}$

$\boxed{a \to z = 13}$

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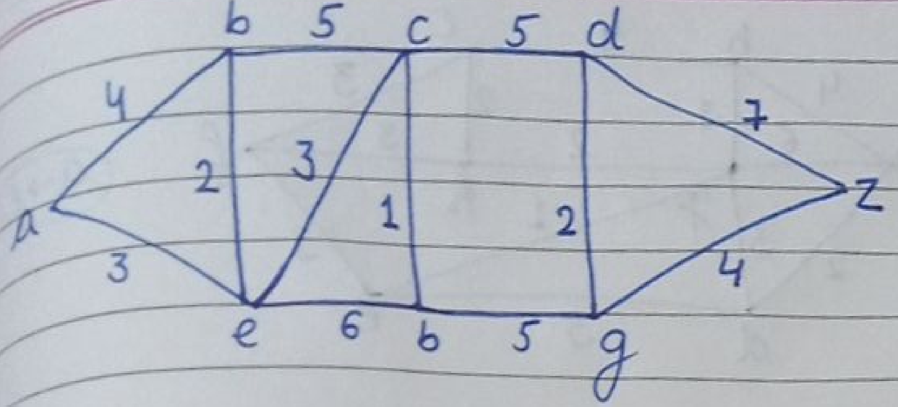

a → z

source	b	c	d	e	f	g	z
{a}	4(a)	∞	∞	$\boxed{3(a)}$	∞	∞	∞
{a,e}	4(a)	$\boxed{6(a,e)}$	∞	—	9(a,e)	∞	∞
{a,e,c}	—	—	11(a,e,c)	—	$\boxed{7(a,e,c)}$	∞ ~~a,e,c~~	∞
{a,e,c,f}	—	—	11(a,e,c)	—	—	12(a,e,c,f)	∞
{a,e,c,f,g}	—	—	—	—	—	—	16(a,e,c,f)

$a \to e \to c \to f \to g \to z = \boxed{16}$ Ans.

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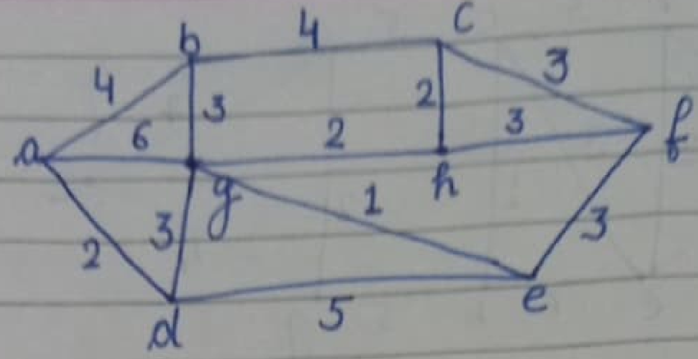
Q3.



$a \to f$

Source	b	c	d	e	g	h	f
{a}	4(a)	∞	$\boxed{2a}$	∞	6(a)	∞	∞
{a, d}	$\boxed{4(a)}$	∞	—	7(a,d)	5(a,d)	∞	∞
{a, d, b}	—	8(a,b)	—	7(a,d)	$\boxed{5(a,d)}$	∞	∞
{a, d, b, g}	—	8(a,b)	—	$\boxed{6(a,d,g)}$	—	7(a,d,g)	∞
{a, d, b, g, e}	—	8(a,b)	—	—	—	$\boxed{7(a,d,g)}$	9(a,d,g,e)
{a, d, b, g, e, h}	—	—	—	—	—	—	10(a,d,g,h,f)
*{adbgehf}	—	—	—	—	—	—	$\boxed{11(a,b,c,f)}$

$a \to d \to g \to e \to f = \boxed{9 \text{ units}}$