1) $S = 0, 2, 6, 12, 20, 30, 42, \cdots$

The differences between consecutive terms grows by 2, thus the recurrence relation here is $a_{n+1} = a_n + 2n$ with initial condition $a_1 = 0$. In other words, each element in this series is twice the value of the simple sums series. Thus, the explicit formula is $a_n = n(n+1)$.

2)

a) Let $a_n$ denote the number of strings of length n of letters of the alphabet. For $n = 1$, we have $A_n = 26$. For every increment of $n$, we have 26 ways to append to each of the previous strings, thus the recurrence relation is $a_{n+1} = 26a_n$ with initial condition $a_1 = 26$. Thus the explicit formula for the series is $a_n = 26^n$.

b) If no adjacent letters can be the same, the recurrence relation is now $a_{n+1} = 25a_n$ with the same inital condition $a_1 = 26$, since one letter now cannot be appended to each string, namely the last letter of the string. Thus, the explicit formula is $a_n = 26 \cdot 25^{n-1}$.

c) Assuming we know $a_n$, then we have $26 - n$ valid letters that can be appended to each string of length $n$ with no repeats. Thus, the recurrence relation is $a_{n+1} = a_n(26 - n)$. with initial condition $a_1 = 26$. The explicit formula is then $a_{n+1} = (26 - n)!$

3) $b_n = 2b_{n-1} + n - 2^n \quad \text{and} \quad b_0 = 5$

a) $b_{n-1} = 2b_{n-2} + n - 1 - 2^{n-1}$

b) $b_n = 2b_{n-1} + n - 2^n$, substituting $b_{n-1}$ from (a) gives us

$b_n = 2(2b_{n-2} + n - 1 - 2^{n-1}) + n - 2^n$

$b_n = 4b_{n-2} + 2n - 2 - 2^n + n - 2^n$

$\boxed{b_n = 4b_{n-2} + 3n - 2 - 2(2^n)}$

c) $b_n = 4b_{n-2} + 3n - 2 - 2(2^n)$ from (b). Expanding $b_{n-2}$ gives us

$b_n = 4(2b_{n-3} + n - 2 - 2^{n-2}) + 3n - 2 - 2^{n+1}$

$b_n = 8b_{n-3} + 4n - 8 - 2^n + 3n - 2 - 2^{n+1}$

$\boxed{b_n = 8b_{n-3} + 7n - 10 - 3(2^n)}$

d)

$$\boxed{b_n = -2 + 7(2^n) - n(2^n + 1)}$$

A formal proof for this closed form would likely be derived by using generating functions.

4)

$$\sum_{i=1}^{50} 3i$$

5) $a = 4$, $r = 1/2$, $n = 9$. The sum is thus

$$S_n = a\frac{1 - r^n}{1 - r} = 4 \cdot \frac{1 - (1/2)^9}{1 - 1/2}$$

$$S_n = 4 \cdot \frac{511/512}{1/2} = \boxed{\frac{511}{256}}$$

6) Here, $a = 1$ and $r = x^2$. The sum is

$$S = \frac{a}{1-r} = \frac{1}{1-x^2}$$

7) The number of prints is just $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

8) $x^2$ grows faster than $x \log x$ so the upper bound for $x \log x$ is $x^2$ but the upper bound for $x^2$ is not $x \log x$. Thus $x \log x = O(x^2)$ but $x^2 \neq O(x \log x)$.

9) For all $n > 1$, $\log (n+1) < \log 2n = O(\log n)$. Thus $\log (n+1)$ is $O(\log n)$

Similarly, for all $n > 1$, $\log (n^2+1) < \log (4n^2) = 2 \log 2n = O(\log n)$. Thus $\log (n^2+1)$ is $O(\log n)$