Shrey Joshi Net ID: SXJ210081

P1)

$$\log_2 64 = \boxed{\frac{\log_{10} 64}{\log_{10} 2}}$$

P2)

$$\sum_{i=1}^{k} 2i + 1 = 2\sum_{i=1}^{k} i + \sum_{i=1}^{k} 1 = 2\left(\frac{k(k+1)}{2}\right) + k$$

$$= k(k+1) + k = \boxed{k^2 + 2k}$$

P3) Take $n = 4$. $2^4 \not> 4!$

P4) Assume there exists path P from A to B such that $\text{len}(P) < \text{len}(A \to B \to C)$. But then the shortest path from A to D must go through P instead of $A \to B \to C$. Since this is not a case, there is a contradiction, implying that such a path P cannot exist.

P5) Base case $n = 1$:

$$\sum_{i=1}^{1} 2i - 1 = 1^2$$

Inductive Step: assume the claim holds for some n. Then,

$$\sum_{i=1}^{n} 2i - 1 = n^2$$

$$\sum_{i=1}^{n} 2i - 1 + 2(n+1) - 1 = n^2 + 2(n+1) - 1$$

$$\sum_{i=1}^{n+1} 2i - 1 = n^2 + 2n + 1$$

$$\sum_{i=1}^{n+1} 2i - 1 = (n+1)^2$$

P6) $T(10) = T(9) + 3 = T(8) + 6 = \cdots = T(1) + 27 = 30$ $\boxed{T(10) = 30}$

P7) $T(N) = 3N$

P8) Base case: $T(1) = 3$ Inductive step: assume the claim holds for some n. Then, $T(n) = 3n$ so $T(n+1) = T(n) + 3 = 3n + 3$ So $T(n+1) = 3(n+1)$