

Implementation of LMS FIR based Adaptive Filter for System Identification

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Abstract:

The method of detecting the coefficients of an unidentified system using an adaptive filter is known as system identification. These adaptive filters have been effectively used in Communications, radar, sonar, seismology, and biomedical engineering, etc. The focus of this report is to show the utilization of Adaptive Filters for System Identification. The results are generated using an adaptive filter based on FIR. Adaptive filters are used to improve SNR and provide a clear signal. Before employing the filter, it is necessary to identify the system; Least Mean Square is used for this (LMS). The Mean Squared Error gets less over time as the signal becomes more similar to the original signal. It is possible to execute our suggested technique for System Identification as it performed as expected.

Keywords: Adaptive Filters, Least Mean Square (LMS), Finite Impulsive Response (FIR), System Identification, Learning Curve

I. Introduction:

Digital Signal Processing systems are popular cause of the accuracy, flexibility, low cost, small physical size and reliability. Adaptive filter coefficients continuously and automatically adjust to a given signal in order to get the desired response and increase performance.[1]

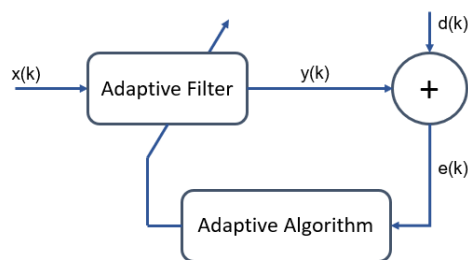


Figure 1: Block diagram of Adaptive filter configuration

Minimizing the error signal is the adaptive filter's prime objective. Algorithm, filter structure, and application are the three basic requirements for constructing an adaptive filter. There are many other types of structures, however the FIR filter structure is popular because of its stability. Numerous industries, including communications, radar, sonar,

seismology, and biomedical engineering, have successfully used adaptive filters. Despite the fact that these applications are very diverse from one another, they all have one fundamental characteristic: the computation of an estimation error, which is then utilized to regulate the values of a set of adjustable filter coefficients, using an input vector and a desired response. However, the method by which the required answer is retrieved differs significantly amongst the many uses of adaptive filtering. The algorithm is the process used to modify the adaptive filter coefficients in order to reduce an error signal, which is a preset criterion. Because the FIR filter and LMS algorithm are reasonably easy to design and implement, they are used in the majority of reported advancements and applications.[1]

The two types of adaptive digital filters are infinite impulse response (IIR) filter, which offers a potential improvement in performance and requires less processing power than the corresponding adaptive finite impulse response (FIR) filter, and other one is finite impulse response (FIR) filter, also known as an adaptive linear combiner and which is unconditionally stable.[2] Research on adaptive system identification has a rich history and has included a wide range of approaches, such as the use of neural networks, swarm optimization, and LMS adaptation algorithms on IIR and FIR adaptive filters in a variety of applications.[2-13] When IIR filter is used the limitation of the system is affected with multimodality of the error surface versus the filter coefficients, and the system might cling to local minima and differ from optimum solution. The LMS algorithm moves in the direction of the error gradient's inverse to find the global minimum of the error surface. The LMS algorithm, like the overwhelming majority of learning algorithms, may cause the filter to reach a local minimum in the event of a multi-modal error surface. This report focuses on the Implementation of LMS FIR based Adaptive Filter for System Identification on MATLAB. Section 2 contains Generalised Block Diagrams.

Followed by Section 3 with Simulation Results of the task. Section 4 consists of Conclusion and Section 5 is for references.

II. Generalised Block Diagrams

a. Adaptive Filters:

Figure 1 depicts the generalised block diagram of Adaptive filters. Where $x(k)$ is the input signal which is passed through adaptive filter and the output we get is $y(k)$. This output is then compared with the reference signal $d(k)$ and an error signal $e(k)$ is generated. This error signal is then passed through the adaptive algorithm which generates feedback which is connected to adaptive filter and input signal is modified according to the received feedback. Adaptive filters are systems that can change or vary their structure such that their behaviour or performance gets better when they interact with their environment. Other properties may be found in [14]. These systems often have the ability to automatically adapt to changing surroundings, can be trained to achieve certain filters, and do not require the complex synthesis techniques required for non-adaptive systems.[2]

b. LMS Algorithm:

The LMS adaptive algorithm is primarily used to iteratively update the weight vector at each sample instance in order to reduce approximately the mean-square error. The assumption while implementing LMS algorithm is that the process is ergodic, i.e. the

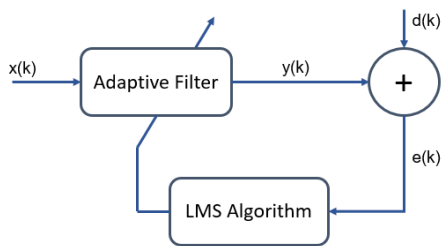


Figure 2: Block diagram of LMS adaptive filter
ensemble mean can also be determined by time averaging a sample function of the process.

$$J(\mathbf{w}) = E\{e^2[k]\} \equiv \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{k=0}^{M-1} e^2[k], \quad \dots(1)$$

And the error signal should be

$$e[k] = d[k] - \mathbf{w}^T \mathbf{x}[k] \quad \dots(2)$$

To keep it in short, At the initialization of LMS algorithm

$$\mathbf{w}[0] = 0, \mu > 0 \quad \dots(3)$$

In each iteration $k=0,1,2,\dots$

a) Update input signal vector,

$$\mathbf{x}[k] = [x[k], x[k-1], \dots, x[k-N+1]]^T \dots(4)$$

and output signal of adaptive filter

$$y[k] = \mathbf{w}^T[k] \mathbf{x}[k] \quad \dots(5)$$

b) Error signal

$$e[k] = d[k] - y[k] \quad \dots(6)$$

c) Update of filter coefficient vector

$$\mathbf{w}[k+1] = \mathbf{w}[k] + \mu e[k] \mathbf{x}[k] \quad \dots(7)$$

It is still by far the most often used algorithms for the adaptation of a FIR-based adaptive filter given how simple it is.

The correction vector for the LMS algorithm consists of an gradient estimate that causes gradient noise. Due to that, the gradient estimate is not necessarily pointing into the direction of steepest decent.

- For $k \rightarrow \infty$ (and an appropriate step-size μ), the gradient method converges, but the Wiener solution is not necessarily obtained.

- Only the expected value of the coefficient vector $E\{\mathbf{w}[k]\}$ converges to the Wiener solution for $k \rightarrow \infty$

c. System Identification:

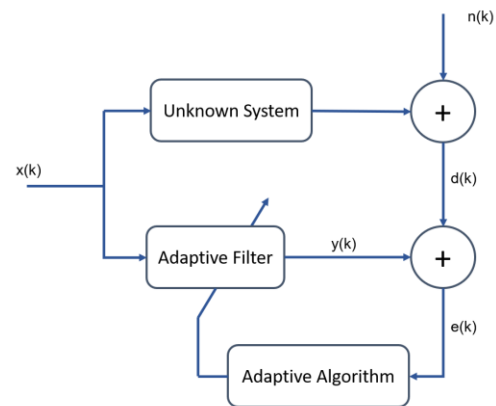


Figure 3: Block Diagram of System Identification

Analytical and design methods may be successfully applied to real-world issues using mathematical models of physical events. Using basic physical principles and knowledge of the system's components and their interconnections, a mathematical model can often be created. This strategy, however, is frequently less successful since the physical system or phenomena is too complicated and poorly understood. In these situations, it is necessary to develop this mathematical model based on measurements of the input and output. Usual practise is to presume that the unknown system may be modeled mathematically as a linear temporal system. The system identification problem is defined as the issue of obtaining a model of a system from measurements of its input and output.[1]

III. Task & Simulation Results:

a. LMS Algorithm

In the given task, we simulated the LMS adaptive algorithm in FIR based adaptive filter. Figure 3 represents the block diagram of the LMS algorithm. In this, the output of the adaptive filter is denoted by $y[k]$, the desired signal is denoted as $d[k]$, the input signal is denoted as $x[k]$ and the error signal is denoted as $e[k]$. The input signal is passed through the adaptive filter and the output signal is then compared with the desired signal, which generates an error signal. This error signal is then fed into the LMS adaptive algorithm and the output is changed at every iteration. The LMS adaptation algorithm is designed to reduce the Mean Square Error (MSE).

To execute the simulation, we used MATLAB's function " $y = \text{lms1}(x, d, N, \mu)$ ". The specification of the function are as follows:

```
function y = lms1(x, d, N, mu)
% y = lms1(x, d, N, mu)
% Adaptive transversal filter using LMS
% INPUT
% x : column vector containing the samples of the input
signal x[k]
% size(x) = [xlen,1]
% d : column vector containing the samples of the desired
output
% signal d[k]
% size(d) = [xlen,1]
% N : number of coefficients
% mu : step-size parameter
% OUTPUT
% y : column vector containing the samples of the output
signal y[k]
% size(y) = [xlen,1]
```

By varying the step size between 0.01, 0.1, and 0.5, we evaluate our function using the signals $x[k]=2$ and $d[k]=1$ for $N=1$. To retain the stability property, the step-size of the adaptive algorithm should be smaller than $2/\lambda_{(\max)}$ Where $\lambda_{(\max)}$ is the covariance matrix R 's highest possible eigenvalue. However, in order to ensure a convergence curve that is monotonically declining, must actually be selected to be two orders of magnitude smaller. The following photos illustrate the outcomes for plotting $x[n]$, $y[k]$, and $d[k]$ into the same figure for $\mu=0.01$, 0.1, and 0.5 using the plot ($[x, y, d]$) function, accordingly.

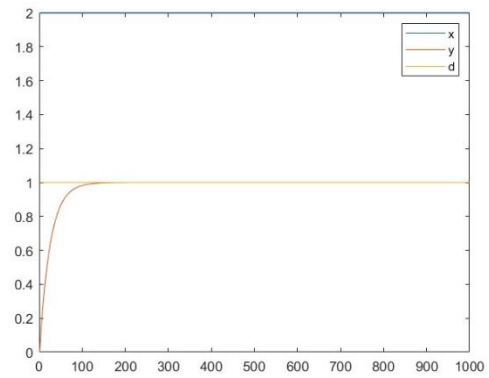


Figure 4: The graphs of $x[n]$, $y[k]$, and $d[k]$
 $\mu=0.01$

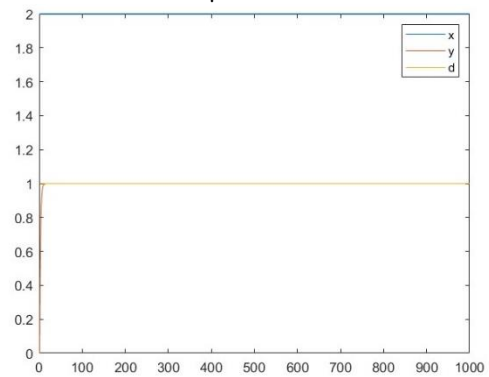


Figure 5: The graphs of $x[n]$, $y[k]$, and $d[k]$
 $\mu=0.1$

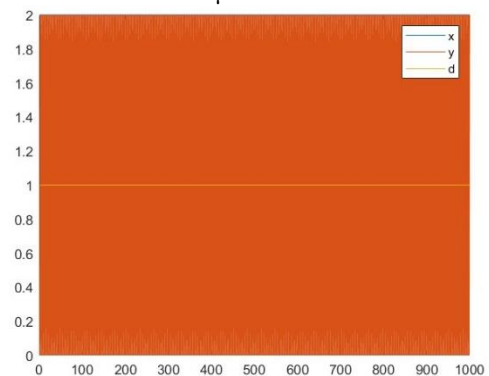


Figure 6: The graphs of $x[n]$, $y[k]$, and $d[k]$
 $\mu=0.5$

The Figure 4,5,6 allow us to establish that there is a relationship between μ , time, and error. The pace of the convergence and error is growing as the μ increases.

b. Learning Curve

The error signal is not steady throughout the learning (transient) phase, and the MSE relies on the time k . This task looks at how the filter coefficients w adjust and how the error behaves over time. To achieve this, $[y,e,w]=\text{lms2}(x,d,N,\mu)$ must be added to the function from the previous exercise, $\text{lms1}()$, to offer extra return values.

Using the same input as in the prior exercise, we test the function and use the $\text{semilogy}()$ function to plot the squared error $e^2[k]$ (learning curve). The

intended signal $d[k]$ with variance should be multiplied by a random Gaussian noise component, $n[k]$.

$$\sigma_2^2 = 10^{-4}$$

This might be measurement noise or an erroneous signal from another signal source. Figure 8 illustrates the convergence behaviour of the filter coefficients for $N=2$ and the learning curve for the LMS method using the noisy desired signal $d[k]$.

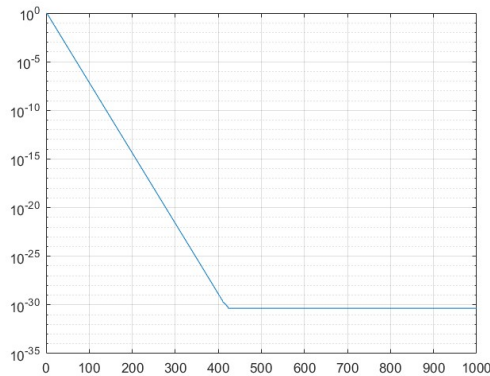


Figure 7: Learning Curve without noise

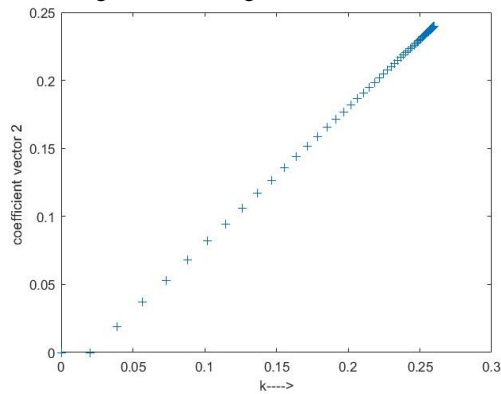


Figure 8: Convergence of filter coefficient

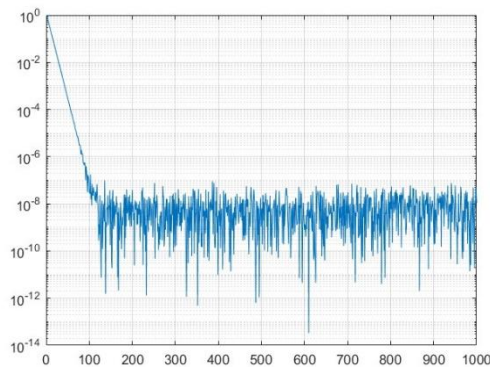


Figure 7: Learning Curve without noise

c. System Identification

The capacity of a FIR-based adaptive filter to approximate an unknown system defined by a FIR filter coefficient vector of length L using the formula $h=[h_1, h_2, \dots, h_L]$. h is a filter impulse response vector that contains the coefficients from the unknown system. An FIR-based adaptive filter with a filter coefficient vector w that roughly

approximates the unknown system h is used to estimate this coefficient vector.

This system identification is accomplished using the `lms2()` function from the previous task. The objective of this task is System Identification for different N , which is 7 for this task, in order to establish the best length N of the FIR-based adaptive filter. We may obtain the vector h 's closest integer integers in this situation. We do this experiment while taking into account various step-size values in order to demonstrate the filter coefficients w_i where $i = 1..N$, rounded to the next decimal place.

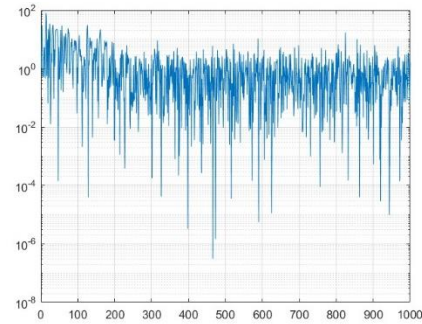


Figure 8: System Identification for $N=4, \mu=0.01$

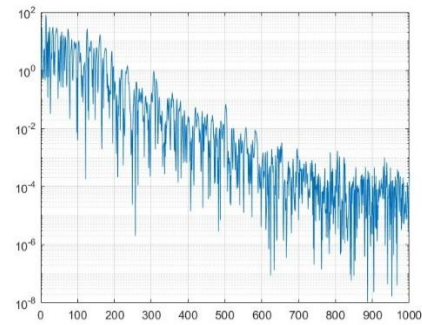


Figure 9: System Identification for $N=7, \mu=0.01$

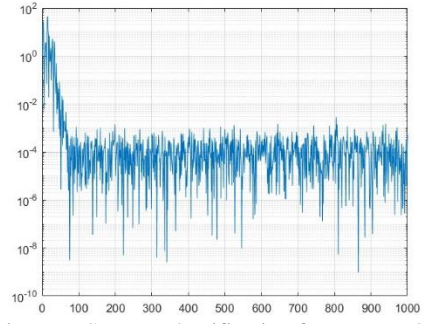


Figure 9: System Identification for $N=7, \mu=0.1$

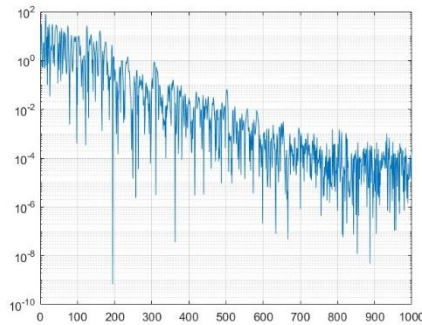


Figure 10: System Identification for $N=8, \mu=0.01$

Here we have taken input by loading the '06_task3_x_d.mat' file which had values of x (input signal) and d(Desired Signal), We chose $N=2$ and $\mu = 0.01$. For achieving the system Identification, it is necessary to choose the value of the μ and N precisely.

IV. Conclusion

The project's findings demonstrate that the adaptive filter accurately predicts and converges on the unidentified system coefficients. Numerous studies have looked at how step size and iteration count affect filter performance, including mean square error and estimate accuracy. While increasing the convergence time, the step size reduction decreases steady state error. The Least Mean Square (LMS) approach, which determined the system based on its input and output responses, was initially used to carry out the prediction process. The forecast and the initial signal diverged in a reasonable amount. The analysis's conclusion is as follows: The Least Mean Square (LMS) approach, which identified the system based on its input and output responses, was first used to carry out the prediction operation. The forecast was rather close to the first indication. The outcomes of the analysis were as follows: In the early stages of the prediction phase, the system was identified using the Least Mean Square (LMS) approach based on its input and output responses. Within a reasonable range, the forecast and the original signal matched. The LMS algorithm's performance is significantly impacted by the step size fluctuation. The intended output is achieved by applying the appropriate amount of correction to the input signal at each iteration, which is determined by the step size. It is discovered that if the step size is short, the algorithm requires more iterations to produce the correct signal when a sine wave is the system's target signal. We may thus conclude that a larger number of iterations will be required for the system to converge if the step size is very tiny. The step size is selected such that it is neither too low nor too large since if the step size is maintained excessively large, the system's convergence rate is sped up but the system also becomes unstable, which is unanticipated.

V. References

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