Assignment - 5

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Q-57 (June 2018) The covariance matrix of four dimensional random vector X is of the form $\begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho \\ \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & 1 \end{bmatrix}$, Where $\rho < 0$

If v is the variance of the first principle component then,

1. v cannot exceed 5/4
2. v can exceed 5/4 but cannot exceed 4/3
3. v can exceed 4/3 but cannot exceed 3/2
4. v can exceed 3/2

covariance matrix: In probability theory and statistics, a covariance matrix is a square matrix giving the covariance between each pair of elements of a given random vector

Solution Here, co-variance matrix is given and we've to find maximum value of variance of first principle component. For that, first we have to find principle components. And for that we have to find eigenvalues of covariance matrix.

By calculations we get 4 eigen values $1-\rho,\ 1-\rho,\ 1-\rho,\ 3\rho+1$ which will be the variance of their respective principle component,

here $\rho > 0$ means $(1-\rho) > (3\rho+1)$

So, order of principle components is $1-\rho,\ 1-\rho,\ 1-\rho,\ 3\rho+1$

here first 3 principle components are same so their variance will be same, let's assume $x = 1-\rho$ *and* $y = 3\rho+1$ and $x, y \geq 0$ because co-variance matrix is positive semi definite. Also note that variance of data remains same after applying linear transformation through eigan vectors. Hence,

$$\begin{aligned} 3x + y &= 4 \\ \therefore 3x &< 4 \ (because y > 0) \\ \therefore x &< 4/3 \end{aligned} \tag{1}$$

Means x cannot exceed 4/3 which is option 2