Assignment 1

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Q54) A random sample of size 7 is drawn from a distribution with p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1+x^2}{3\theta(1+\theta^2)}, & -2\theta \le x \le \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

and the observations are 12, -54, 26, -2, 24, 17, -39. What is the maximum likelihood estimation of θ.

1. 12	2. 24
3. 26	4. 27

Here, Given x the maximum likelihood estimate (MLE) for the parameter θ is the value of θ that maximizes the likelihood $P(x|\theta)$. That is, the MLE is the value of θ for which the data is most likely.
Here, p.d.f of distribution is

$$f(x) = \begin{cases} \frac{1+x^2}{3\theta(1+\theta^2)}, & -2\theta \le x \le \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

, so for data point $x_i = 12$, Probability $P_\theta(x_i = 12)$

$$= \begin{cases} \frac{1+12^2}{3\theta(1+\theta^2)}, & -2\theta \le x \le \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

$$= \begin{cases} \frac{145}{3\theta(1+\theta^2)}, & -2\theta \le x \le \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

for every x_i we can can calculate probability in the same way.
Now, for all random variables joint pdf function will be

$$f_\theta(x_1, x_2, ..., x_7|\theta) = \begin{cases} \frac{1+x_1}{3\theta(1+\theta^2)}, & -2\theta \le x \le \theta, \theta > 0 \\ 0, & otherwise \end{cases} * \begin{cases} \frac{1+x_2}{3\theta(1+\theta^2)}, & -2\theta \le x \le \theta, \theta > 0 \\ 0, & otherwise \end{cases} ....$$

Where, $x_1 = 12, X_2 = -54$ and so on.

Here from looking at the options given in question, 27(option 4) is the correct answer because it's the only option that satisfy condition $-2\theta \leq x \leq \theta, \theta > 0$. In all other cases we'll get joint p.d.f. 0.

Hence answer is 27(option 4)