Assignment 1

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Q54) A random sample of size 7 is drawn from a distribution with p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1+x^2}{3\theta(1+\theta^2)}, & -2\theta \leq x \leq \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

and the observations are 12, -54, 26, -2, 24, 17, -39. What is the maximum likelihood estimation of θ.

1. 12	2. 24
3. 26	4. 27

Maximum likelihood estimation The goal of maximum likelihood estimation is to make inferences about the population that is most likely to have generated the sample, specifically the joint probability distribution of the random variables

Solution Here, p.d.f of distribution is

$$f(x) = \begin{cases} \frac{1+x^2}{3\theta(1+\theta^2)}, & -2\theta \leq x \leq \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

, so for data point $x_i = 12$, Probability $P_\theta(x_i = 12)$

$$= \begin{cases} \frac{1+12^2}{3\theta(1+\theta^2)}, & -2\theta \leq x \leq \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

$$= \begin{cases} \frac{145}{3\theta(1+\theta^2)}, & -2\theta \leq x \leq \theta, \theta > 0 \\ 0, & otherwise \end{cases}$$

for every x_i we can can calculate probability in the same way.
Now, for all random variables joint pdf function will be

$$f_\theta(x_1, x_2, ..., x_7|\theta) = \begin{cases} \frac{1+x_1}{3\theta(1+\theta^2)}, & -2\theta \leq x \leq \theta, \theta > 0 \\ 0, & otherwise \end{cases} * \begin{cases} \frac{1+x_2}{3\theta(1+\theta^2)}, & -2\theta \leq x \leq \theta, \theta > 0 \\ 0, & otherwise \end{cases} ....$$

Where, $x_1 = 12, X_2 = -54$ and so on.
Here from looking at the options given in question, 27(option 4) is the correct answer because it's the only option that satisfy condition $-2\theta \leq x \leq \theta, \theta > 0$. In all other cases we'll get joint p.d.f. 0.

Hence answer is 27(option 4)