CS31005 Algorithms – II, Autumn 2021–2022

Second Test

Oct 05, 2021, 2:15pm — Maximum marks: 50

Duration: 75 minutes (60 minutes for answering questions + 15 minutes for download/submission)

All your answers MUST BE HANDWRITTEN on paper. Scan all papers with your answers in a SINGLE pdf, and upload it as the answer to the Quiz in Moodle. The size of the final pdf must be less than 10 MB. You must upload the pdf strictly by 3:30 pm Moodle server time, the quiz submission will close after that.

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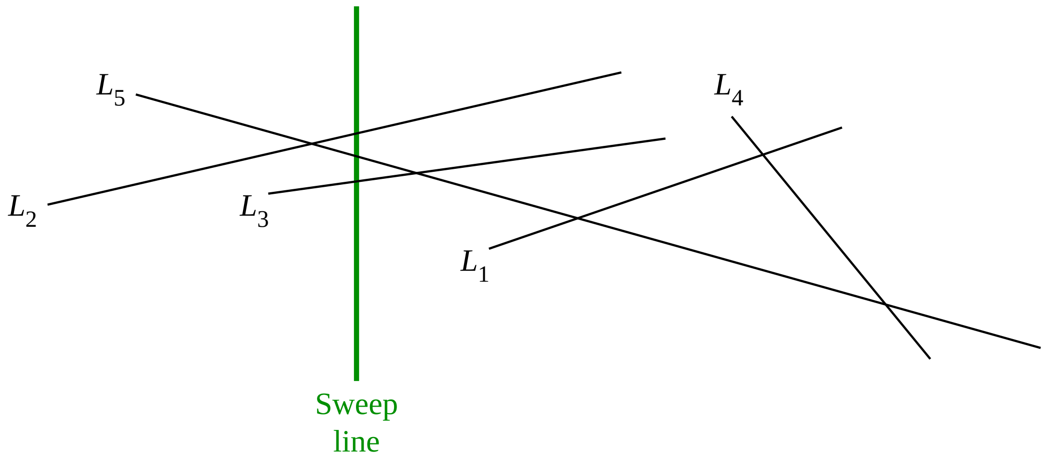
1. You are given the four corners of a convex quadrilateral along with n other points in the interior of the quadrilateral. Your task is to identify the four corners of the quadrilateral from the given set S of $n+4$ points, and output them in clockwise order. Assume that the points of S are in general position. To solve the problem, you compute the leftmost point L, the rightmost point R, the topmost point T, and the bottommost point B in S, and output the sequence L,T,R,B. Prove/Disprove the correctness of the algorithm. **(6)**

2. Consider the line-sweep algorithm covered in the class for computing the intersections of line segments. The algorithm is applied to the five lines in the following figure. Suppose that the sweep line is vertical, moves from left to right, and is currently at the position as shown in the figure. Denote the enter-segment events as $ES(L_i)$, the leave-segment events as $LS(L_i)$, and the intersection events as $\cap(L_i, L_j)$.



(a) List, in the order of occurrence, all the events that the sweep line encounters as it completes its sweep from the current position. **(4)**

(b) Which of these events are stored in the event queue at the current position of the sweep line? Assume that the event queue is initialized by all enter-segment and leave-segment events. **(3)**

3. We run Fortune's line-sweep algorithm to compute the Voronoi diagram of the three points P,Q,R as given in the following figure. Assume that the sweep line is horizontal, and moves from top to bottom.

(a) In what order do the events (site and circle) occur in the execution of the algorithm? **(2)**

(b) Describe the beach line just before and just after the circle event. Do not draw the beach line. Specify only the sequence of sites contributing parabolic arcs to the beach line from left to right. If some site(s) contribute(s) multiple arcs on the beach line, list the site(s) the required number(s) of times. **(4)**

(c) Is the circle event a fork-in event or a fork-out event? **(1)**

4. Let S be a set of n points (in general position) in the two-dimensional plane. We want to reorder ***all*** the points of S to a list $P_1, P_2, \ldots, P_n$ in such a way that

(1) no segment P_iP_{i+1} intersects with any other segment P_jP_{j+1} except perhaps at the endpoints, and
(2) $P_iP_{i+1}P_{i+2}$ is a right turn for all $i = 1, 2, \ldots, n-2$.

Propose an efficient algorithm to solve this problem. What is the time complexity of your algorithm? **(8 + 2)**

5. A forest is inhabited by n prides of lions. The i-th pride lives in a den at location $P_i = (x_i, y_i)$, and requires a circular area of a fixed radius r around P_i for hunting. A pride is called safe if its hunting area does not intersect with the hunting area of any other pride. A survey gives the forest ranger the den locations x_i, y_i, and the hunting radius r (same for all prides). The ranger wants to figure out which prides are safe.

Let S be the set of the n den locations (assumed to be in general position). Propose an algorithm that, given $\text{Vor}(S)$, solves the ranger's problem in $O(n)$ time. Assume that each segment (finite or semi-infinite) of $\text{Vor}(S)$ stores the indices i, j such that the segment belongs to the perpendicular bisector of P_iP_j. Justify the correctness of your algorithm, and that its running time is $O(n)$. **(10)**

6. Let $\Phi(x_1, x_2, \ldots, x_n)$ be a Boolean formula in the conjunctive normal form (CNF). We say that Φ is all-but-one satisfiable if there is a truth assignment of the variables for which all except exactly one of the clauses of Φ evaluate to true. By AB1SAT, we denote the problem of deciding whether the given CNF formula Φ is all-but-one satisfiable.

(a) Prove that AB1SAT is in NP. **(4)**

(b) Prove that AB1SAT is NP-complete. (**Hint:** Use reduction from CNFSAT.) **(6)**