Basic Trigonometric Formulas

In Right Triangle ABC

$$\sin\theta = \frac{\text{side opposite to angle }\theta}{\text{hypotenuse}}$$

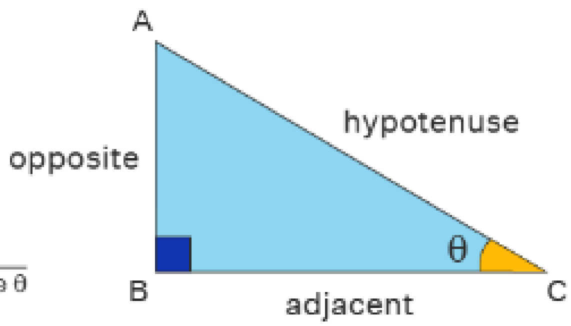
$$\cos\theta = \frac{\text{side adjacent to angle }\theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{side opposite to angle }\theta}{\text{side adjacent to angle }\theta}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{side opposite to angle }\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{side adjacent to angle }\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{side adjacent to angle }\theta}{\text{side opposite to angle }\theta}$$



Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Trigonometry Table

Angles (In Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
Angles (In Radians)	0	π/6	π/4	π/3	π/2	π	3π/2	2π
sin	0	1/2	1/√2	√3/2	1	0	-1	0
cos	1	√3/2	1/√2	1/2	0	-1	0	1
tan	0	1/√3	1	√3	∞	0	∞	0
cot	∞	√3	1	1/√3	0	∞	0	∞
cosec	∞	2	√2	2/√3	1	∞	-1	∞
sec	1	2/√3	√2	2	∞	-1	∞	1

Periodicity Identities (in Radians)

$$\sin(\theta + 2\pi n) = \sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta$$

$$\tan(\theta + 2\pi n) = \tan\theta$$

$$\csc(\theta + 2\pi n) = \csc\theta$$

$$\sec(\theta + 2\pi n) = \sec\theta$$

$$\cot(\theta + 2\pi n) = \cot\theta$$

Cofunction Identities (in Degrees)

$\sin(90^\circ - x) = \cos x$

$\cos(90^\circ - x) = \sin x$

$\tan(90^\circ - x) = \cot x$

$\cot(90^\circ - x) = \tan x$

$\sec(90^\circ - x) = \text{cosec } x$

$\text{cosec}(90^\circ - x) = \sec x$

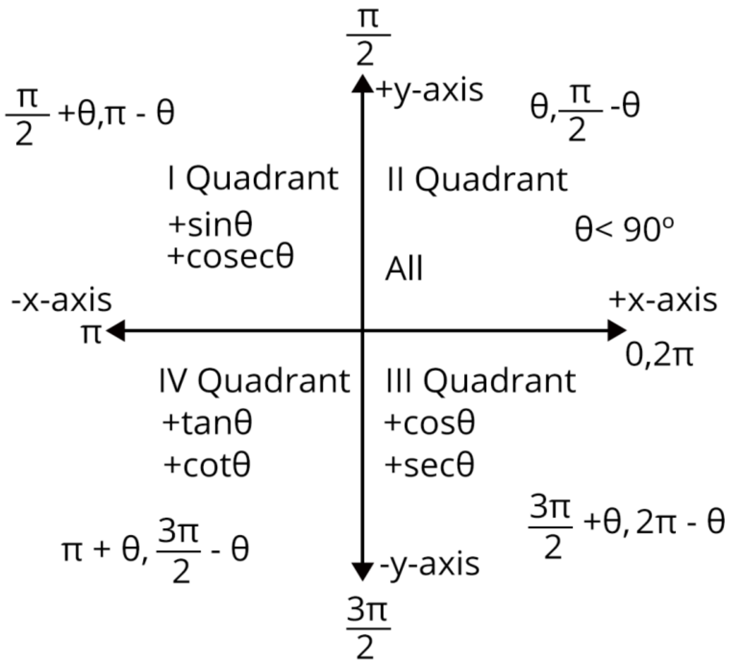
Pythagorean Identities List

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

All Silver Tea Cups



First Quadrant:

- $\sin(\pi/2 - \theta) = \cos\theta$
- $\cos(\pi/2 - \theta) = \sin\theta$
- $\sin(2\pi + \theta) = \sin\theta$
- $\cos(2\pi + \theta) = \cos\theta$

Third Quadrant:

- $\sin(\pi + \theta) = -\sin\theta$
- $\cos(\pi + \theta) = -\cos\theta$
- $\sin(3\pi/2 - \theta) = -\cos\theta$
- $\cos(3\pi/2 - \theta) = -\sin\theta$

Second Quadrant:

- $\sin(\pi/2 + \theta) = \cos\theta$
- $\cos(\pi/2 + \theta) = -\sin\theta$
- $\sin(\pi - \theta) = \sin\theta$
- $\cos(\pi - \theta) = -\cos\theta$

Fourth Quadrant:

- $\sin(3\pi/2 + \theta) = -\cos\theta$
- $\cos(3\pi/2 + \theta) = \sin\theta$
- $\sin(2\pi - \theta) = -\sin\theta$
- $\cos(2\pi - \theta) = \cos\theta$

Sum & Difference Identities

sin(x + y) = sin(x)cos(y) + cos(x)sin(y)
cos(x + y) = cos(x)cos(y) - sin(x)sin(y)
tan(x + y) = (tan x + tan y)/(1 - tan x • tan y)

sin(x − y) = sin(x)cos(y) - cos(x)sin(y)
cos(x − y) = cos(x)cos(y) + sin(x)sin(y)
tan(x - y) = (tan x - tan y)/(1 + tan x • tan y)

Half Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta} = \csc\theta - \cot\theta$$

$$\cot\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$$

Half angle formulas using
double angle formulas

- $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$
- $\cos A = \cos^2\frac{A}{2} - \sin^2\frac{A}{2}$

  $= 1 - 2\sin^2\frac{A}{2}$

  $= 2\cos^2\frac{A}{2} - 1$
- $\tan A = \dfrac{2\tan\left(\frac{A}{2}\right)}{1-\tan^2\left(\frac{A}{2}\right)}$

Double Angle Identities

(i) $\sin 2A = 2 \sin A \cos A = \dfrac{2 \tan A}{1 + \tan^2 A}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2 \sin^2 A$

$= \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(iii) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Triple Angle Identities

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3 \cos x$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

Product identities

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x-y) + \cos(x+y)\right]$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\tan x \tan y = \frac{\tan x + \tan y}{\cot x + \cot y}$$

$$\tan x \cot y = \frac{\tan x + \cot y}{\cot x + \tan y}$$

Sum to Product Identities

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\tan x+\tan y=\frac{\sin(x+y)}{\cos x\cos y}$$

$$\tan x-\tan y=\frac{\sin(x-y)}{\cos x\cos y}$$

Inverse Trigonometry Formulas

$\sin^{-1}(-x) = -\sin^{-1} x$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\tan^{-1}(-x) = -\tan^{-1} x$

$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\cot^{-1}(-x) = \pi - \cot^{-1} x$

Double of Inverse Trigonometric Function Formulas

$2\sin^{-1}x = \sin^{-1}(2x.\sqrt{(1 - x^2)})$

$2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$

$2\tan^{-1}x = \tan^{-1}(2x/1 - x^2)$

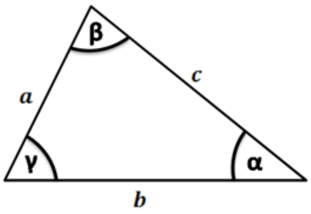
Triple of Inverse Trigonometric Function Formulas

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

$3\tan^{-1}x = \tan^{-1}(3x - x^3/1 - 3x^2)$

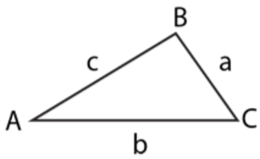
Law of Sines



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Law of Cosines

$$a^2=b^2+c^2 - 2bc\cos A$$
$$b^2=a^2+c^2 - 2ac\cos B$$
$$c^2=a^2+b^2 - 2ab\cos C$$



Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan tan\left(\frac{A-B}{2}\right)}{\tan tan\left(\frac{A+B}{2}\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan tan\left(\frac{B-C}{2}\right)}{\tan tan\left(\frac{B+C}{2}\right)}$$

$$\frac{c-a}{c+a} = \frac{\tan tan\left(\frac{C-A}{2}\right)}{\tan tan\left(\frac{C+A}{2}\right)}$$

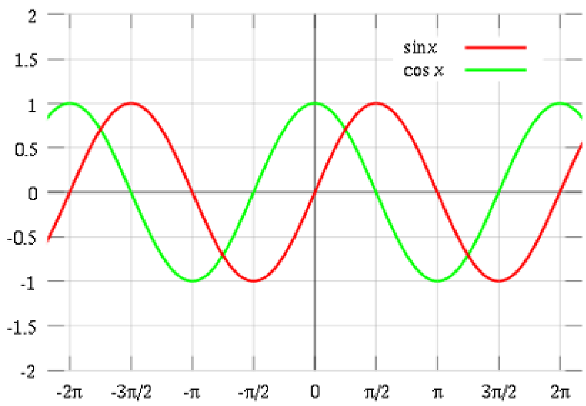
We know tan (-θ) = -tan θ for any angle θ, we can rewrite the laws of tangent as-

$$\frac{b-a}{b+a} = \frac{\tan tan\left(\frac{B-A}{2}\right)}{\tan tan\left(\frac{B+A}{2}\right)}$$

$$\frac{c-b}{c+b} = \frac{\tan tan\left(\frac{C-B}{2}\right)}{\tan tan\left(\frac{C+B}{2}\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan tan\left(\frac{A-C}{2}\right)}{\tan tan\left(\frac{A+C}{2}\right)}$$

Sin vs Cos Graph



Domain and Range of Trigonometric Functions Using Graph