Parameter Estimation

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1) Let $(X_1, X_2, \ldots X_n)$ be a random sample of size 'n' from normal population having parameters:

mean $= \theta_1$ & var $= \theta_2$.

Find max. likelihood estimates of these:

Ans) pdf :→ $f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-1/2 \left(\frac{x-\theta_1}{\sqrt{\theta_2}}\right)^2}$

$\therefore \; L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{-1/2 \frac{(x_i-\theta_1)^2}{\theta_2}}$

$\Rightarrow \log L = -\frac{n}{2} \log(2\pi\theta_2) + \left(\frac{-1}{2\theta_2}\right) \sum_{i=1}^{n} (x_i-\theta_1)^2$

Differentiate:—

$\frac{1}{L}\frac{\partial L}{\partial \theta_1} = -\frac{1}{2\theta_2} \sum_{i=1}^{n} 2(x_i-\theta_1)(-1)$

$\frac{1}{L} \cdot \frac{\partial L}{\partial \theta_1} = \frac{1}{2\theta_2} \sum 2(x_i-\theta_1)$

$\therefore \frac{\partial L}{\partial \theta_1} = 0 \Rightarrow \frac{L}{2\theta_2} \sum 2(x_i-\theta_1) = 0$

$\Rightarrow L = 0$ or $\frac{1}{2\theta_2} \sum 2(x_i-\theta_1) = 0$

$\boxed{L=0}$ not possible.

$\frac{1}{2\theta_2} \sum 2(x_i-\theta_1) = 0$

$\Rightarrow \boxed{\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}}$ where θ_1 = sample mean

Differentiate w.r.t θ_2

$$\frac{\partial L}{\partial \theta_2}\left(\frac{1}{L}\right) = -\frac{n}{2} \cdot \frac{2\pi}{2\pi\theta_2} + \sum_{i=1}^{n} (x_i - \theta_1)^2 \left(\frac{1}{2\theta_2^2}\right)$$

equate to 0

$$\therefore \boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

$\theta_2 \rightarrow$ sample variance

— x ————

2) Let $X_1, X_2 \ldots X_n$ be a random sample from $B(m, \theta)$ distr. where $\theta \in (0,1)$ is unknown & m is a known positive integer. Compute value of θ using MLE.

Ans) Pmf :→ $P(X=k) = {}^{m}C_k \cdot \theta^k (1-\theta)^{m-k}$

$$L(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i} \, \theta^{x_i} (1-\theta)^{m-x_i}$$

where for x_i, it repr. the no. of success in ith trial.

$$\log L = \sum_{i=1}^{n} \left(\log {}^{m}C_{x_i} + x_i \log \theta + (m - x_i) \log (1-\theta)\right)$$

Differentiate w.r.t θ & equate to 0.

$$L\left(\frac{1}{\theta} \sum x_i - \frac{1}{(1-\theta)} \sum m - x_i\right) = 0$$

L can't be 0

$$\therefore \cancel{\theta}(1-\theta) \sum_{i=1}^{n} x_i = \theta_n m - \theta \sum_{i=1}^{n} x_i$$

$$\therefore \boxed{\theta = \frac{1}{m}\left(\frac{1}{n} \sum x_i\right)}$$

$\theta = \frac{\text{sample mean}}{m}$

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