Matching functions and approximations

Practice Quiz, 5 questions

4/5 points (80%)

Congratulations! You passed!

Next Item

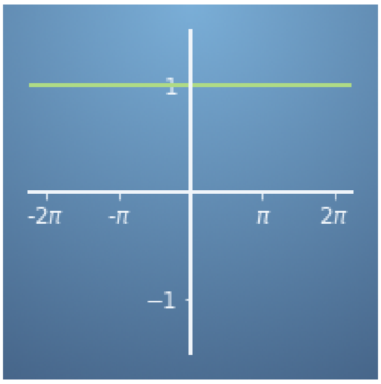
0 / 1 point

1.
As mentioned in the previous video, the Taylor series approximation can also be viewed as a power series, in which these approximations are used to build functions that are often simpler and easier to evaluate, particularly when using numerical methods. In the following questions, we are looking at developing our understanding of how the increasing order of a power series allows us to develop further information of a function.

Below are three graphs highlighting the zeroth, second and fourth order approximations of a common trigonometric function. Observe how increasing the number of approximations in the power series begins to build a better approximation, and determine which function these approximations represent.

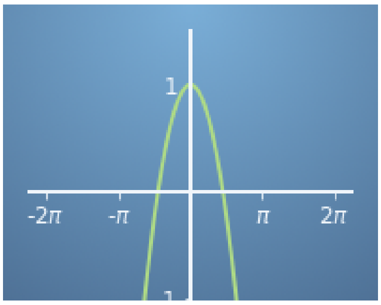
Zeroth order approximation:

$$f_0(x) = 1$$



Second order approximation:

$$f_2(x) = 1 - \frac{x^2}{2}$$



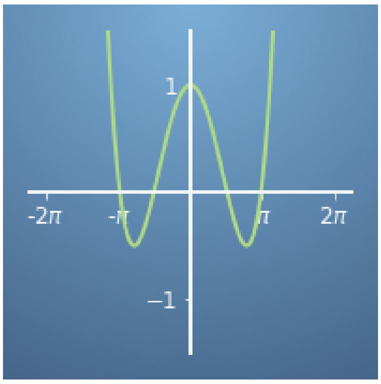
Matching functions and approximations

Practice Quiz, 5 questions

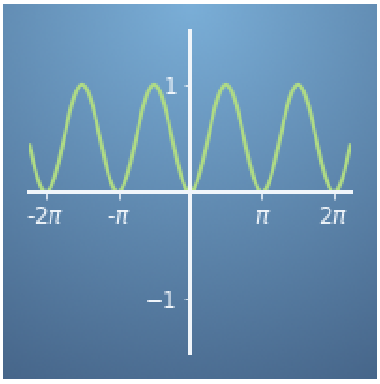
4/5 points (80%)

Fourth order approximation:

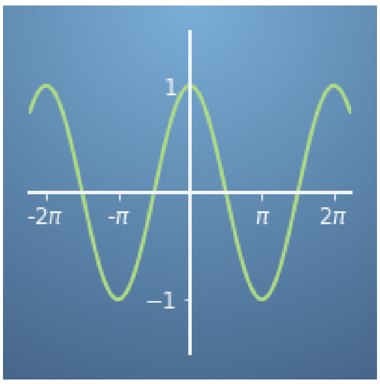
$$f_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$



- $f(x) = \sin^2(x)$



- $f(x) = \cos(x)$

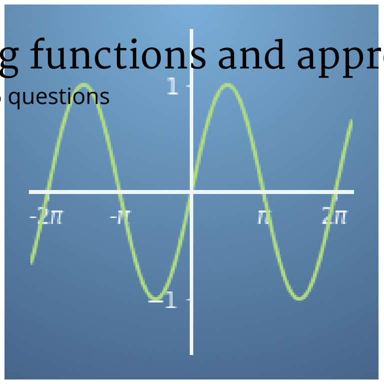


- $f(x) = \sin(x)$

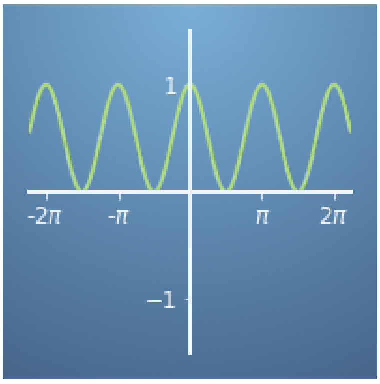
Matching functions and approximations

Practice Quiz, 5 questions

4/5 points (80%)



$f(x) = \cos^2(x)$



This should not be selected

Although $f(0) = 1$ and the function is symmetric about the point $x = 0$, the second order approximation tells us the function we are approximating should become negative for particular values of x.

1 / 1 point

2.

Matching functions and approximations

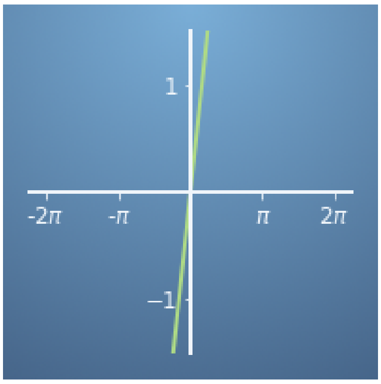
Practice Quiz, 5 questions

4/5 points (80%)

Below are three graphs highlighting the first, third and fifth order approximations of a common trigonometric function. Observe how the power series begins to build the function, and determine which function these approximations represent.

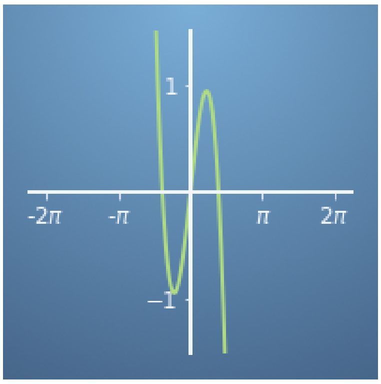
First Order:

$$f_1(x) = 2x$$



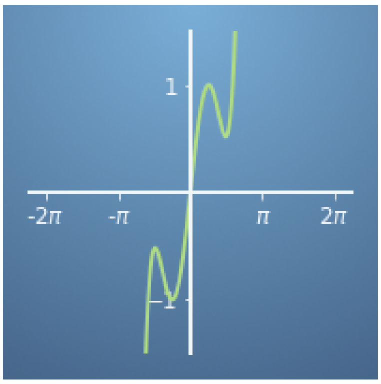
Third Order:

$$f_3(x) = 2x - \frac{4x^3}{3}$$



Fifth Order:

$$f_5(x) = 2x - \frac{4x^3}{3} + \frac{4x^5}{15}$$

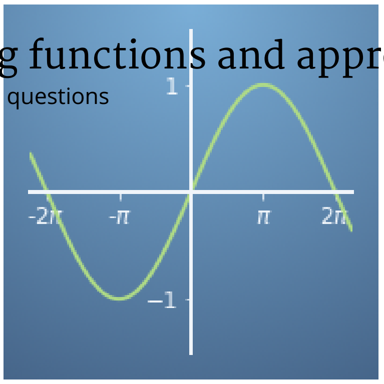


- $f(x) = \sin(\frac{x}{2})$

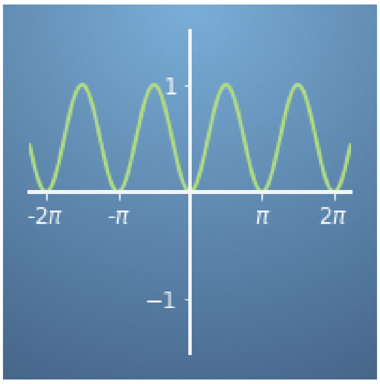
Matching functions and approximations

Practice Quiz, 5 questions

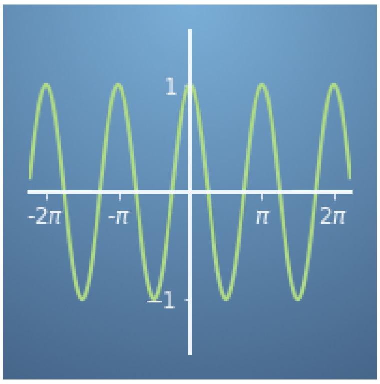
4/5 points (80%)



$f(x) = \sin^2(x)$



$f(x) = \cos(2x)$

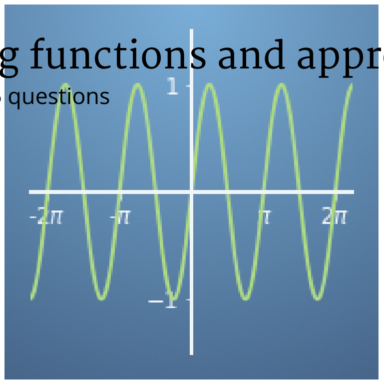


$f(x) = \sin(2x)$

Matching functions and approximations

Practice Quiz, 5 questions

4/5 points (80%)



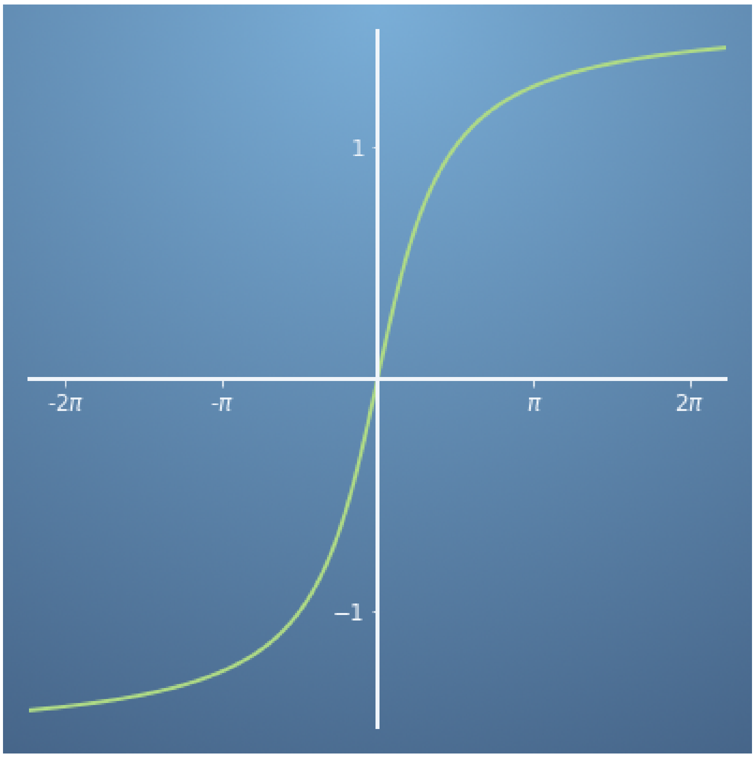
Correct

The function $f(x) = sin(2x)$ has rotational symmetry about the origin. Furthermore, we can see that the period is much shorter, also evident from the three approximations shown.

1 / 1 point

3.

The graph below shows the function $f(x) = \tan^{-1}(x)$, select all the power series approximations that can be used to obtain an approximation for this function.

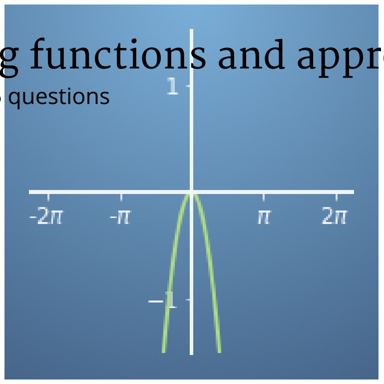


- [ ] $f(x) = -x^2 \ldots$

Matching functions and approximations

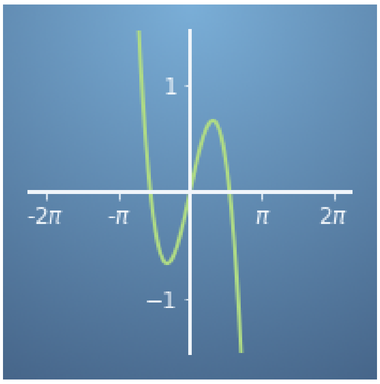
Practice Quiz, 5 questions

4/5 points (80%)



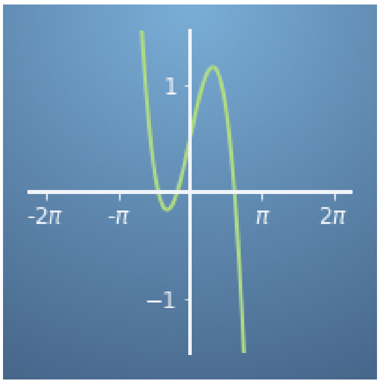
Un-selected is correct

- [ ] $f(x) = x - \frac{x^3}{3} \dots$



Correct
We can see this approximation goes through the origin and also looks as if it fits the function well between $-0.5 < x < 0.5$.

- [ ] $f(x) = \frac{1}{2} + x - \frac{x^3}{3} \dots$



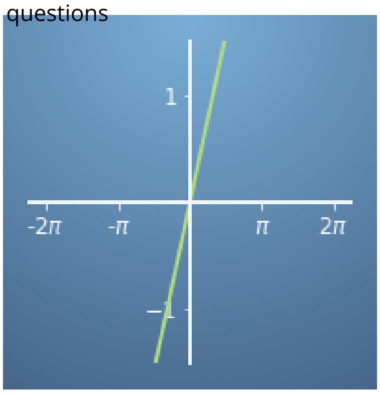
Un-selected is correct

Matching functions and approximations

Practice Quiz, 5 questions

4/5 points (80%)

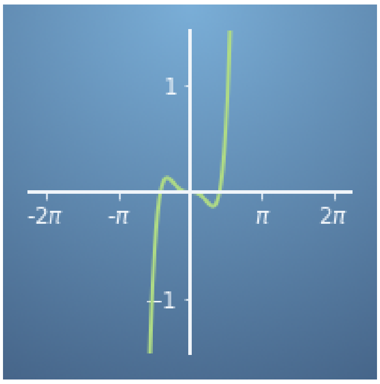
$f(x) = x$ ...



Correct

We can see this approximation goes through the origin and also looks as if it fits the function well between $-0.5 < x < 0.5$. As this is a linear function, this is a first-order approximation.

$f(x) = -\frac{x^3}{3} + \frac{x^5}{5} \ldots$



Un-selected is correct

1 / 1 point

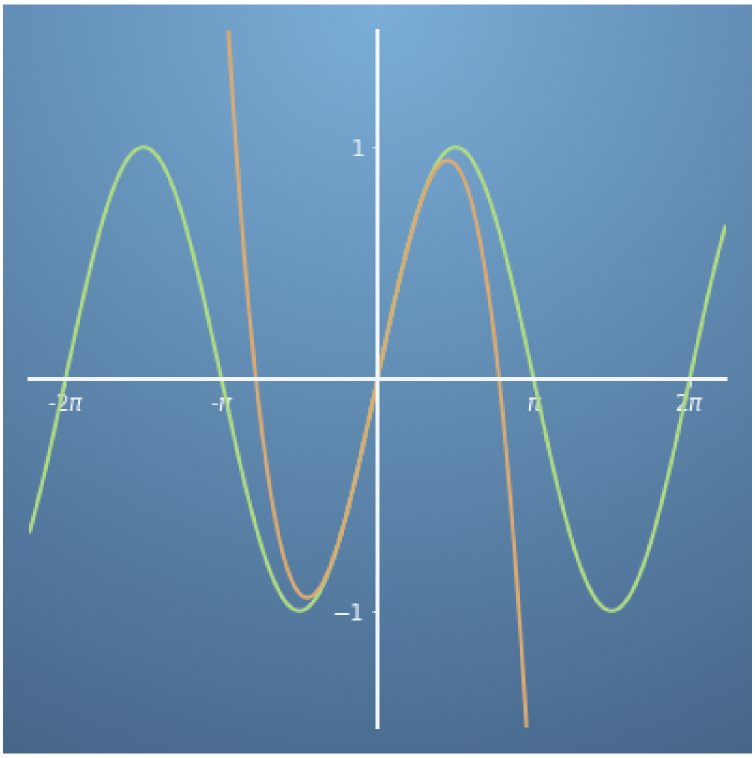
4.

Matching functions and approximations

Practice Quiz, 5 questions

4/5 points (80%)

The sinusoidal function $f(x) = \sin(x)$ (green line) centered at $x = 0$ is shown in the graph below. The approximation for this function is shown through the series $f(x) = x - \frac{x^3}{6} \ldots$ (orange line). Determine what polynomial order is represented by the orange line.



- Zeroth Order
- First Order
- Third Order

Correct
The highest power of x in the approximation is 3, therefore this approximation is a third order approximation.

- Fifth Order
- None of the above

---

1 / 1
point

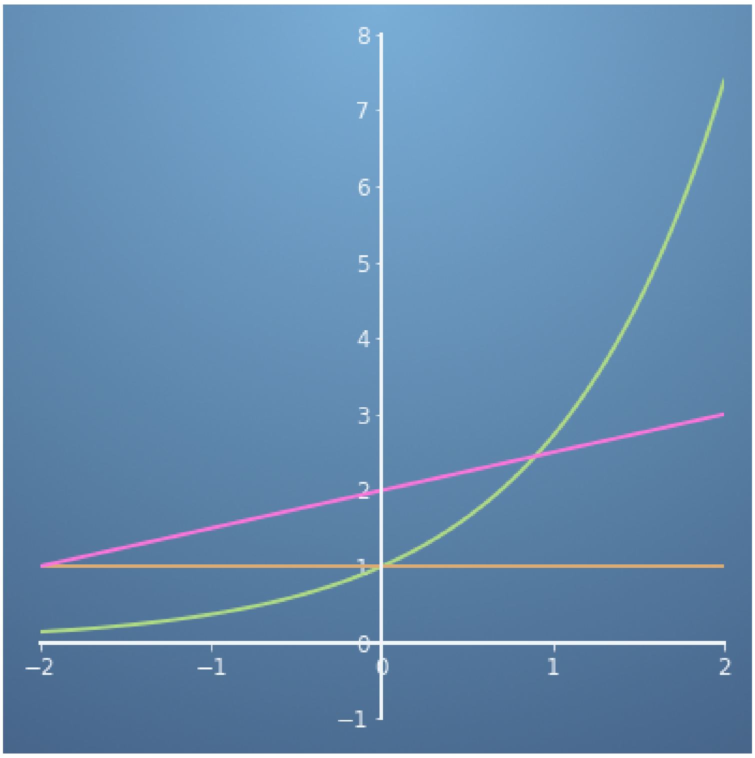
5.

Matching functions and approximations

Practice Quiz, 5 questions

4/5 points (80%)

The graph below shows the function $f(x) = e^x$ (green line), the exponential function so widely used in science and mathematics today. The orange line represents the zeroth order approximation for the exponential function, centred at $x = 0$. Determine if the pink line shown on the graph is, in fact, an approximation and if so, what order is this approximation.



- First Order
- Second Order
- Third Order
- Not a correct approximation

Correct

The approximation shown is not tangent to this point and is, therefore, a poor approximation of the function.