Taylor series - Special cases

Practice Quiz, 5 questions

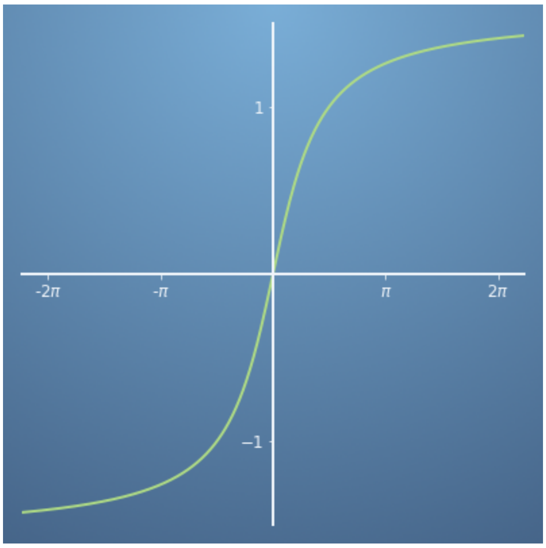
4.2/5 points (84.00%)

Congratulations! You passed!

Next Item

1 / 1 point

1.
The graph below shows the function $f(x) = \tan^{-1}(x)$



By using the Maclaurin series or otherwise, determine whether the function shown above is even, odd or neither.

- Odd

Correct
For an odd function, $-f(x) = f(-x)$. We can also determine if a function is odd by looking at its symmetry. If it has rotational symmetry with respect to the origin, it is an odd function.

- Even
- Neither odd nor even

1 / 1 point

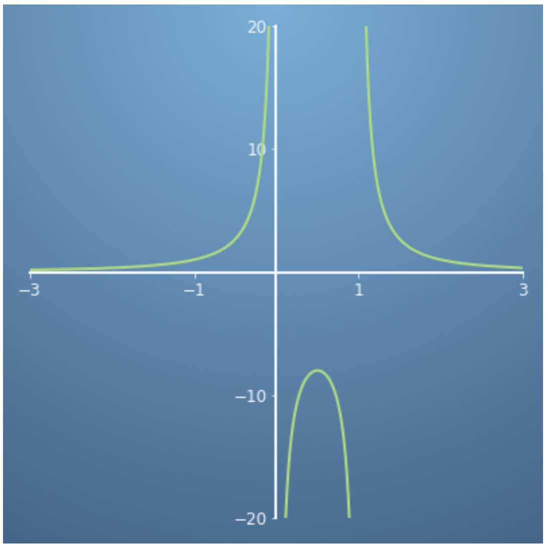
2.

The graph below shows the discontinuous function $f(x) = \frac{2}{(x^2-x)}$. ~~For this function, select the starting points that will allow a Taylor approximation to~~ be made.

Taylor series - Special cases

Practice Quiz, 5 questions

4.2/5 points (84.00%)



- [ ] $x = 2$

Correct
A Taylor approximation centered at $x = 2$ will allow us to approximate $f(x)$ for $x > 1$only.

- [ ] $x = -3$

Correct
A Taylor approximation centered at $x = -3$ will allow us to approximate $f(x)$ for $x < 0$ only.

- [ ] $x = 0.5$

Correct
A Taylor approximation centered at $x = 0.5$ will allow us to approximate $f(x)$ for $0 < x < 1$ only.

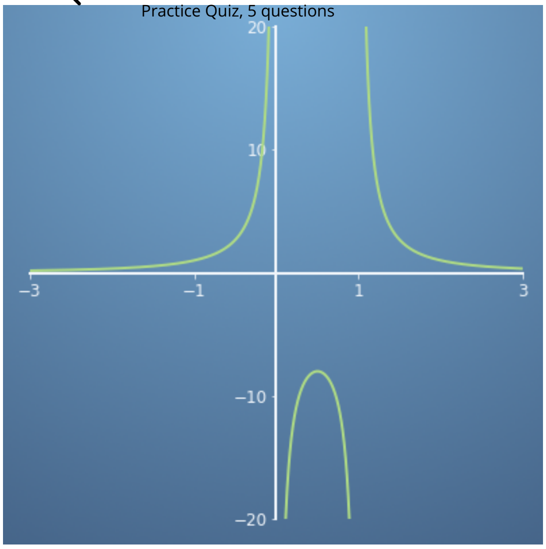
- [ ] $x = 1$

Un-selected is correct

0.80 / 1
point

3.

For the same function as previously discussed, $f(x) = \frac{2}{(x^2-x)}$, select all of the statements that are true about the resulting Taylor approximation.



- [ ] Approximation ignores segments of the function

Correct
Due to the discontinues function and the range of x values in which it remains well behaved, the starting point of the Taylor series dictates the domain of the function we are trying to approximate.

- [ ] Approximation ignores the asymptotes

Correct
Taylor series approximations often find it difficult to capture asymptotes correctly. For example, the zeroth and first order terms cut directly through an asymptote in most cases.

- [ ] Approximation accurately captures the asymptotes

Un-selected is correct

- [ ] The approximation converges quickly

This should not be selected
Often the case with similar functions and as mentioned in the previous chapter video, there is often a sign flip in the approximation, showing very slow convergence however, this is not always the case.

- [ ] This is a well behaved function

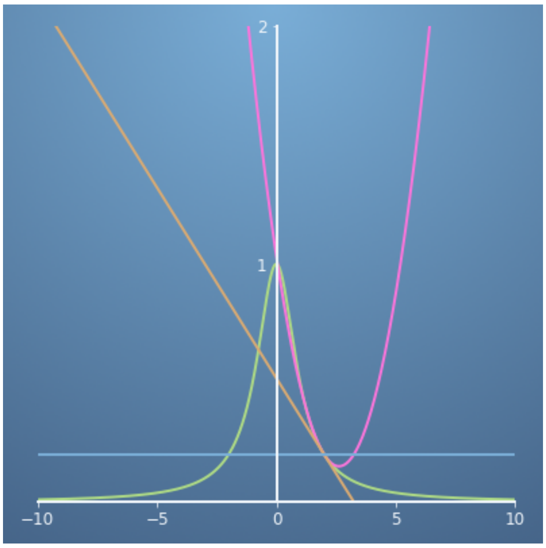
Un-selected is correct

0.40 / 1
point

4.

The graph below highlights the function $f(x) = \frac{1}{(1+x^2)}$ (green line), with the Taylor expansions for the first 3 terms also shown about the point $x = 2$.

The Taylor expansion is $f(x) = \frac{1}{5} - \frac{4(x-2)}{25} + \frac{11(x-2)^2}{125} + \ldots$. Although the function looks rather normal, we find th[illegible]oes a bad approximation further from its starting point, not capturing the turning point. What could be the reason why this approximation is poor for the function described.



None of these options

This should not be selected
Consider the approximations again and what may be the reason for the poor approximation of the Taylor expansion.

Function does not differentiate well

Un-selected is correct

Asymptotes are in the complex plane

This should be selected

The function has no real roots

Un-selected is correct

It is a discontinuous function in the complex plane

This should be selected

1 / 1
point

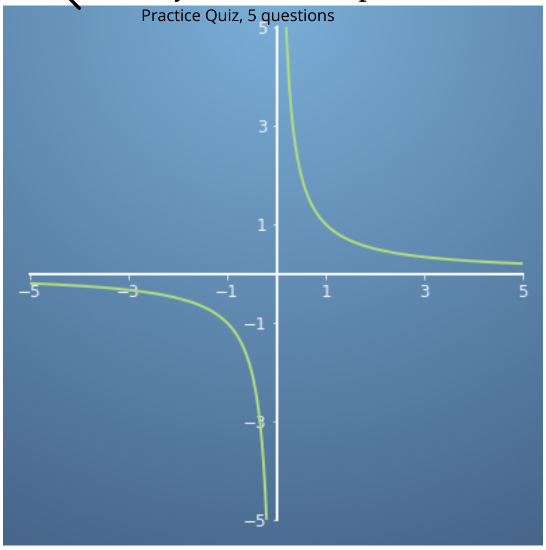
5.

For the function $f(x) = \frac{1}{x}$, provide the linear approximation about the point $x = 4$, ensuring it is second order accurate.

Taylor series - Special cases

Practice Quiz, 5 questions

4.2/5 points (84.00%)



- [x] $f(x) = 1/4 - (x-4)/16 + O(\Delta x^2)$

Correct

Second order accurate means we have a first order Taylor series. All the terms above are sufficiently small, assuming Δx is small.

- [ ] $f(x) = 1/4 - x/16 + O(\Delta x^2)$
- [ ] $f(x) = 1/4 - (x-4)/16 + O(\Delta x)$
- [ ] $f(x) = 1/4 + x/16 - O(\Delta x^2)$