Transport Phenomena

Course no: CHL4010

Pipe Poiseuille Flow

Report Submitted by :

Ayush Abad (B20CH011)

Arun Sahu (B20CH009)

Nitin Prasad (B20CH026)

Shreya Pagaria (B20CH042)

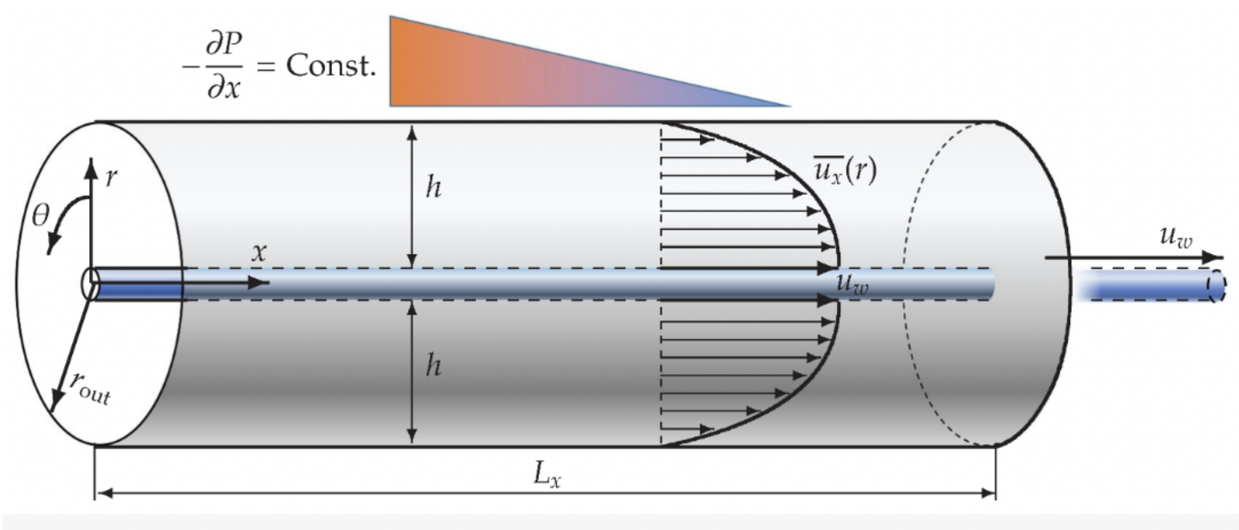
Suyash Surana (B20CH046)

Indian Institute of Technology Jodhpur

Department of Chemical Engineering

Introduction:-

Pipe Poiseuille flow is a fundamental fluid mechanics concept that describes the steady-state flow of an incompressible fluid through a long, straight pipe.In Pipe Poiseuille flow, the fluid is assumed to be Newtonian, meaning that it follows a linear relationship between shear stress and velocity gradient, and the flow is laminar, meaning that there is no turbulence or eddies present. The flow is driven by a pressure difference between the two ends of the pipe, and the velocity profile is parabolic, with the maximum velocity occurring at the center of the pipe and decreasing to zero at the pipe wall.



Pipe Poiseuille flow has a number of practical applications, including in the design of fluid transport systems, such as pipelines for oil and gas, and in medical devices, such as catheters. It is also a useful benchmark for computational fluid dynamics simulations and for experimental measurements of fluid properties. In this report, we will explore the theoretical and mathematical foundations of Pipe Poiseuille flow.

Question :-

Consider a steady fully developed fluid flow in a pipe for a Newtonian fluid with pressure difference $\Delta p = 10, 10^3, 10^5, 10^7$ Pa, viscosity $\mu = 0.492$ Pa s, pipe length L = 4.88 m, and radius R = 0.0025 m.

$$\frac{1}{r}\frac{d}{dr}\left(\mu r \frac{du}{dr}\right) = -\frac{\Delta p}{L}$$

Boundary conditions: $\frac{du}{dr}(r = 0) = 0$, u(r=R) = 0.

1. Derive the velocity profile and calculate the average velocity.
2. Compare analytical solutions with numerical solutions obtained using COMSOL/MATLAB. Plot the shear rate as a function of radial position. Plot the shear stress as a function of radial position.
3. When the fluid is non-Newtonian, it may not be possible to solve the problem analytically but can be solve numerically, for example, for the Bird–Carreau fluid (Bird et al., 1987, p. 171) the viscosity is

$$\mu = \frac{\mu_0}{\left[1 + \left(\lambda \frac{dv}{dr}\right)^2\right]^{(1-n)/2}}$$

   The viscosity depends on shear rate dv/dr . If $\mu_0 = 0.492$, $\lambda = 0.1$ and $n = 0.8$, obtain velocity profile.
4. Plot the velocity, shear rate and shear stress as a function of radial position. How do these curves change as the pressure drop is increased?

Solution 2(Numerical solution using MATLAB):-

/MATLAB Drive/TPQ2.m

```
syms u(r) du dr mu_r delta_p L R
du_dr = diff(u,r);
d_mu_r = diff(mu_r*r*du_dr,r);
eqn = (1/r)*d_mu_r == -(delta_p/L);
cond1 = u(R) == 0;
cond2 = du_dr(0) == 0;
conds = [cond1 cond2];
sol = dsolve(eqn,conds);
```

Command Window

```
>> tpq2
>> TPQ2
>> sol

sol =

(delta_p*R^2 - delta_p*r^2)/(4*L*mu_r)

>>
```

Figure 1: Code Snippet of velocity profile in the MATLAB

Both the velocity profiles obtained from the MATLAB and analytically have come out to be equal which suggests that the analytical solution is correct.

```
syms u(r) du dr
delta_p = [10 10^3 10^5 10^7];
L = 4.88;
mu_r = 0.492;
m = delta_p/2*L*mu_r;
Du = -(m*r);
fplot(Du, r)
title('Shear rate v/s r')
xlabel('r')
ylabel('du/dr')
```

Figure 2: Code Snippet of the shear rate in the MATLAB

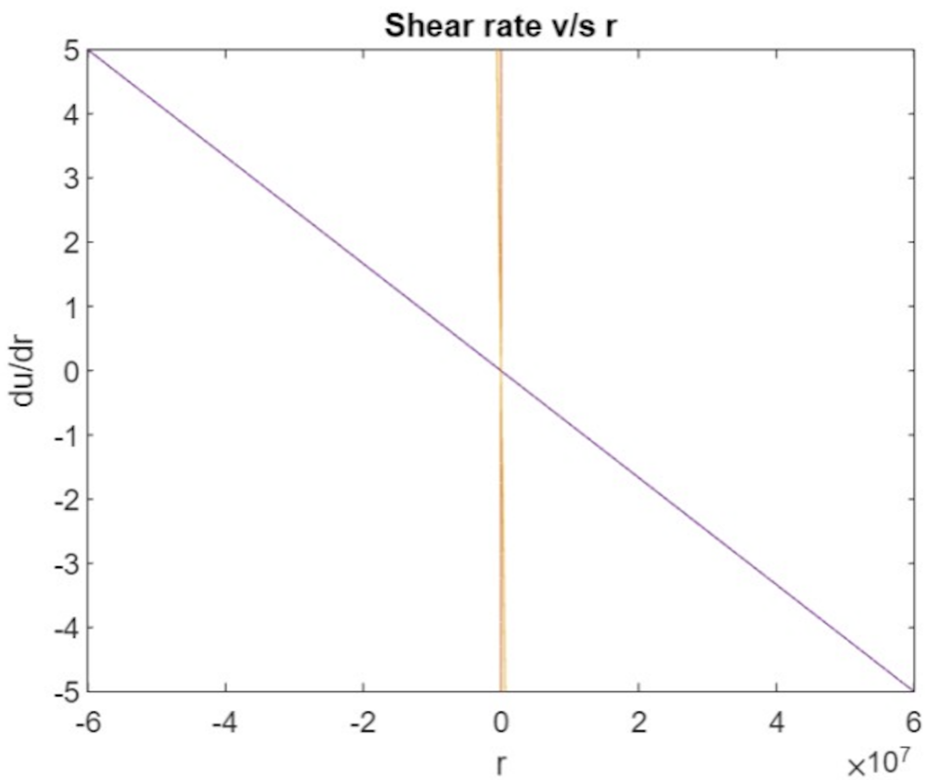


Figure 3: Plot of shear rate as a function of radial position

```
syms u(r) du dr
delta_p = [10 10^3 10^5 10^7];
L = 4.88;
mu_r = 0.492;
m = delta_p/2*L;
Du = -(m*r);
fplot(Du, r)
title('Shear stress v/s r')
xlabel('r')
ylabel('\mudu/dr')
```

Figure 4: Code Snippet of the shear stress in the MATLAB

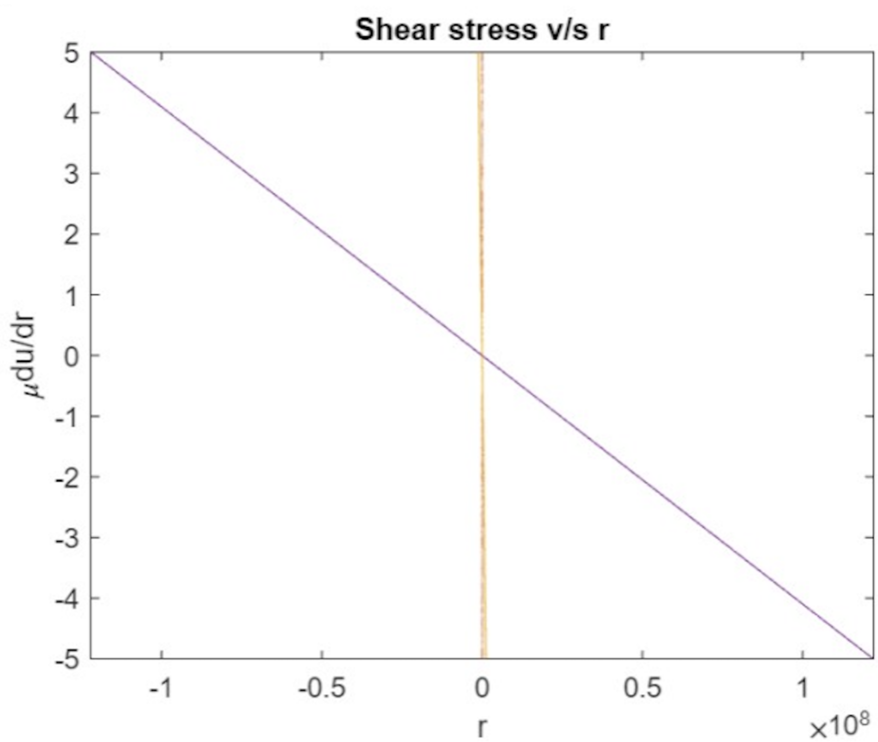


Figure 5: Plot of shear stress as a function of radial position

Solution 4(Numerical solution using MATLAB):-

```matlab
%for the bird carreau model
syms u(r) du dr
Du = diff(u,r);
R = 0.0025;
delta_p = 10;
b = 24.6*(log((1+20.3252*sqrt(0.0024-0.001*delta_p^2*R^2))*(1-20.3252*sqrt(0.0024-0.001*delta_p^2*r^
u = 49.2*log(R/r)+ 1000*(sqrt(0.0024-0.001*delta_p^2*r^2) - sqrt(0.0024-0.001*delta_p^2*R^2))+ b;
u_final1 = u/delta_p;

fplot(u_final1, r,'green')
title('velocity v/s r')
xlabel('r')
ylabel('u')
```

Figure 6: Code Snippet of the shear stress in MATLAB

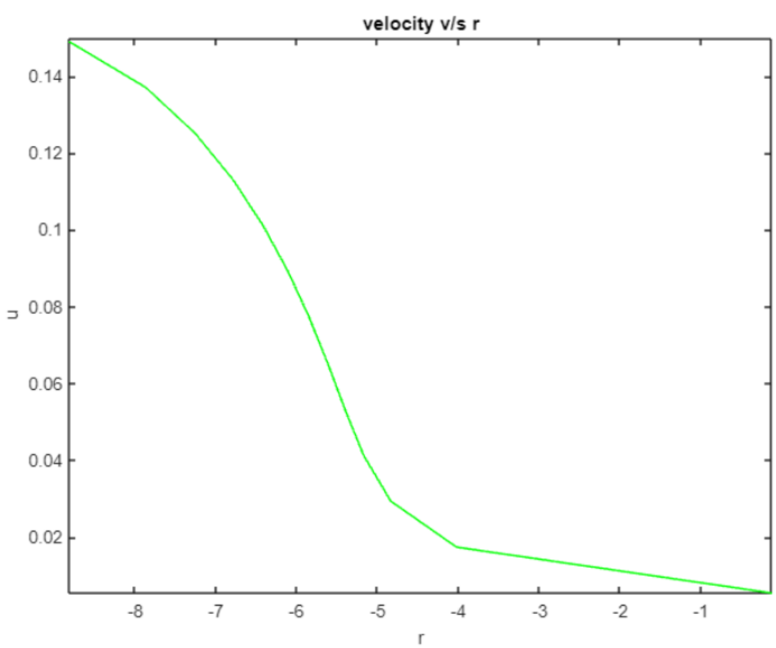


Figure 7: Plot of velocity vs radial position

ATLAB Drive/tpq4_shearrate.m

```
%for the bird carreau model
syms u(r) du dr
R = 0.0025;
delta_p = 10;
Du = -0.0492 + sqrt(0.0024 - (0.001*delta_p^.2*r.^2))/0.001*delta_p*r;
fplot(Du, r)

delta_p2 = 1000;
Du2 = -0.0492 + sqrt(0.0024 - (0.001*delta_p2^.2*r.^2))/0.001*delta_p2*r;

delta_p3 = 100000;
Du3 = -0.0492 + sqrt(0.0024 - (0.001*delta_p3^.2*r.^2))/0.001*delta_p3*r;

delta_p4 = 10000000;
Du4 = -0.0492 + sqrt(0.0024 - (0.001*delta_p4^.2*r.^2))/0.001*delta_p4*r;

fplot(Du, r,'green')
hold on
fplot(Du2, r, 'blue');
hold on
fplot(Du3, r, 'yellow');
hold on
fplot(Du4, r, 'red');
title('shear rate v/s r')
xlabel('r')
ylabel('du/dr')
```

Figure 8: Code Snippet of the shear rate in MATLAB

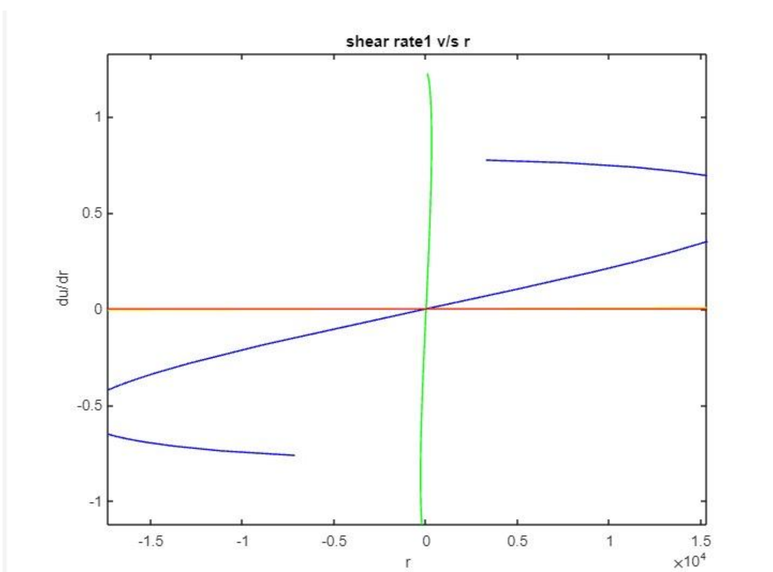


Figure 9: Plot of shear rate vs radial position

MATLAB Drive/tpq4_shearstress2.m

```
%for the bird carreau model
syms u(r) du dr
R = 0.0025;
delta_p = 10;
mu_r = 0.492;
Du = mu_r*(-0.0492 + sqrt(0.0024 - (0.001*delta_p^.2*r.^2))/0.001*delta_p*r);
fplot(Du, r)

delta_p2 = 1000;
Du2 = mu_r*(-0.0492 + sqrt(0.0024 - (0.001*delta_p2^.2*r.^2))/0.001*delta_p2*r);

delta_p3 = 100000;
Du3 = mu_r*(-0.0492 + sqrt(0.0024 - (0.001*delta_p3^.2*r.^2))/0.001*delta_p3*r);

delta_p4 = 10000000;
Du4 = mu_r*(-0.0492 + sqrt(0.0024 - (0.001*delta_p4^.2*r.^2))/0.001*delta_p4*r);

fplot(Du, r,'green')
hold on
fplot(Du2, r, 'blue');
hold on
fplot(Du3, r, 'yellow');
hold on
fplot(Du4, r, 'red');
title('shear stress v/s r')
xlabel('r')
ylabel('\mudu/dr')
```

Figure 10: Code snippet of Shear stress in MATLAB

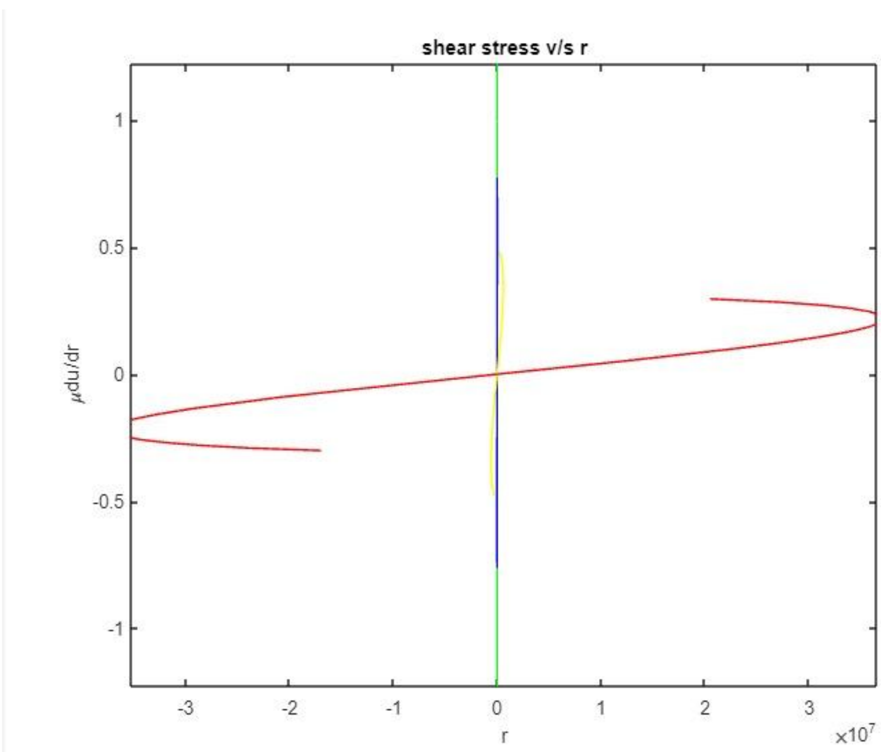


Figure 11: Plot of shear stress vs radial position

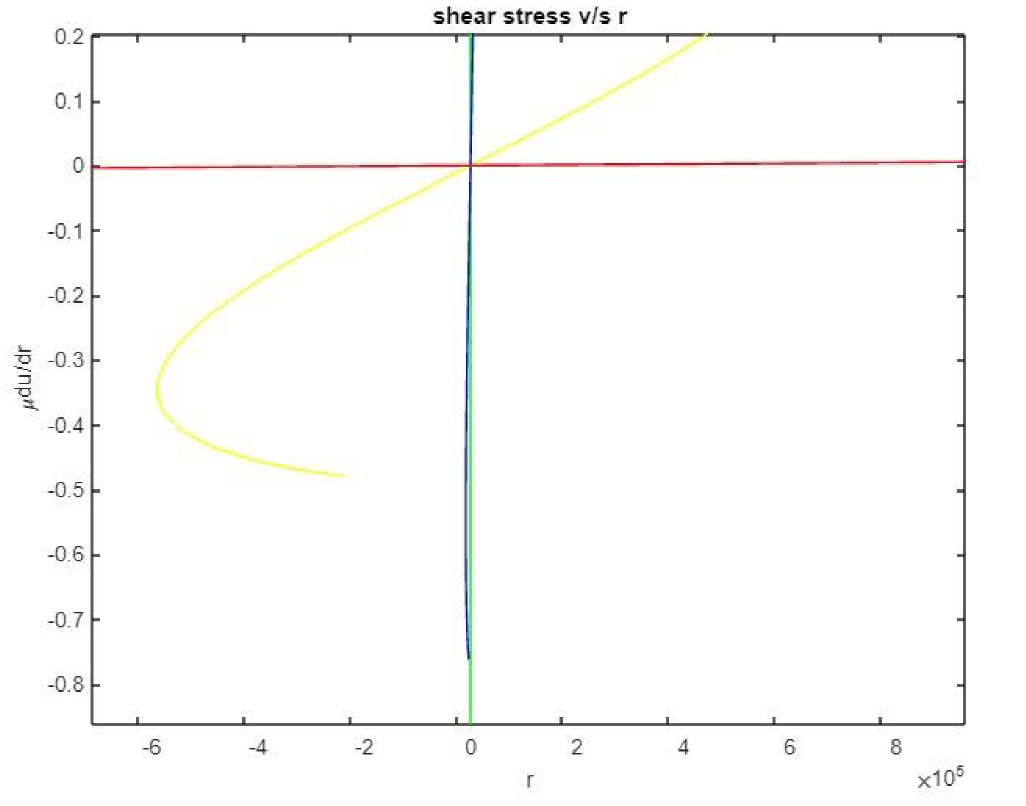


Figure 12: Plot of shear stress vs radial position(enlarged)

THANK YOU!