Group#2 **(Pipe Poiseuille Flow)**

Consider a steady fully developed fluid flow in a pipe for a Newtonian fluid with pressure difference $\Delta p = 10, 10^3, 10^5, 10^7$ Pa, viscosity $\mu = 0.492$ Pa s, pipe length L = 4.88 m, and radius R = 0.0025 m.

$$\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}\right) = -\frac{\Delta p}{L}$$

Boundary conditions: $\frac{du}{dr}(r = 0) = 0$, u(r=R) = 0.

1. Derive the velocity profile and calculate the average velocity.
2. Compare analytical solutions with numerical solutions obtained using COMSOL/MATLAB. Plot the shear rate as a function of radial position. Plot the shear stress as a function of radial position.
3. When the fluid is non-Newtonian, it may not be possible to solve the problem analytically but can be solve numerically, for example, for the Bird–Carreau fluid (Bird et al., 1987, p. 171) the viscosity is

$$\mu = \frac{\mu_0}{\left[1 + \left(\lambda\frac{dv}{dr}\right)^2\right]^{(1-n)/2}}$$

   The viscosity depends on shear rate dv/dr . If $\mu_0 = 0.492$, $\lambda = 0.1$ and $n = 0.8$, obtain velocity profile.
4. Plot the velocity, shear rate and shear stress as a function of radial position. How do these curves change as the pressure drop is increased?