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1 Part 1: Estimation of Robot's position

(a) Simulate the robot's motion in the grid from any starting position and record the sequence of sensor observations (given the the assumptions above) for T = 25 time steps.

Solution:

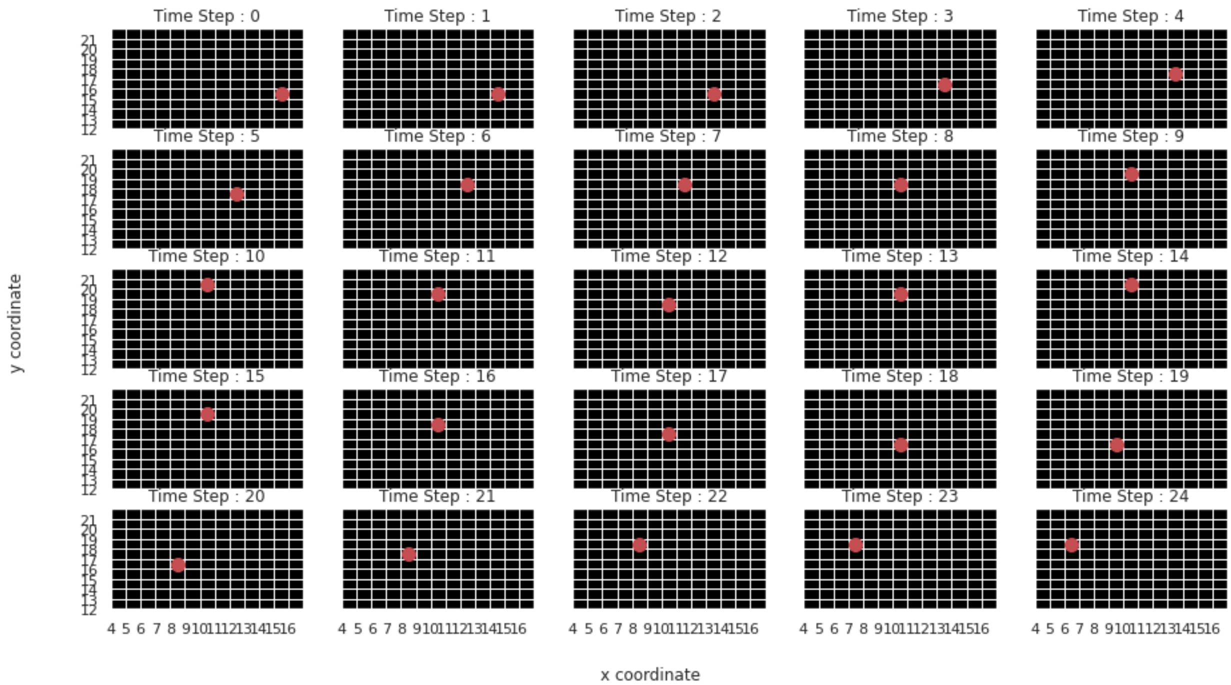

Figure 1.1: Simulated trajectory of the robot

The above plot shows the trajectory of the particle simulated by us in a 30X30 grid.

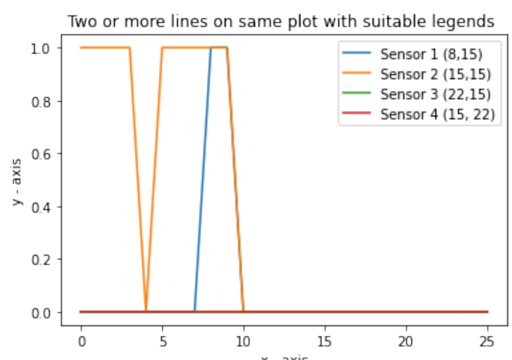

Figure 1.2: Sensor Data

This plot shows the data recorded by the 4 sensors. 1 indicates that the robot is detected and 0 means that it is undetected by the sensor at that time instance.

(b) Estimate the robot's current position at time t given the sequence of observations generated by the simulation above till time T. Visualize and plot the estimated log-likelihood over the grid locations at each time step. Plot the estimated and ground truth locations at each time step.

Solution:

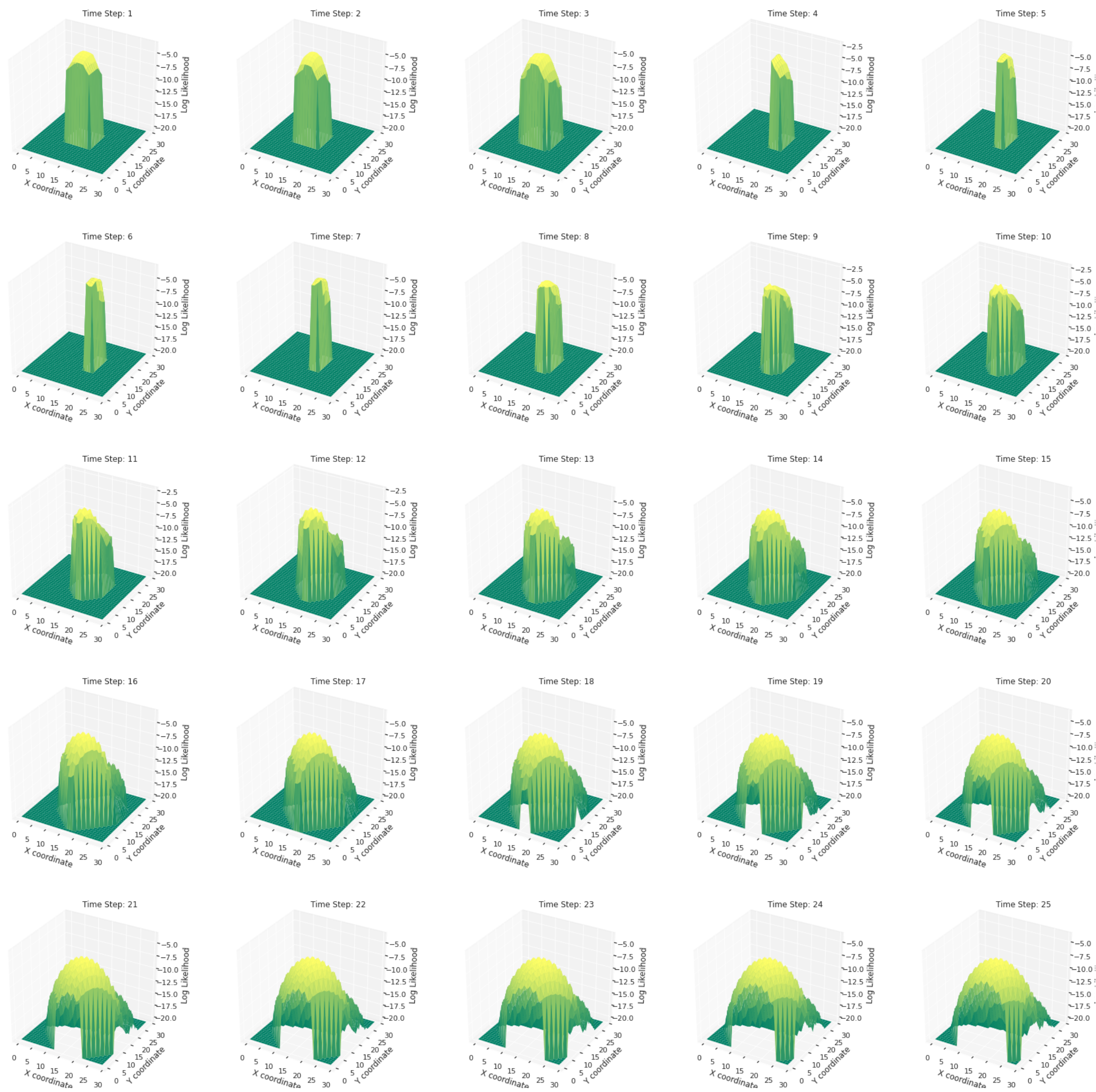

Figure 2.1: Log-likelihood over the grid locations at each time step

The above plot shows the estimated log-likelihood values at every grid location after performing filtering on the states using the sensor observations.

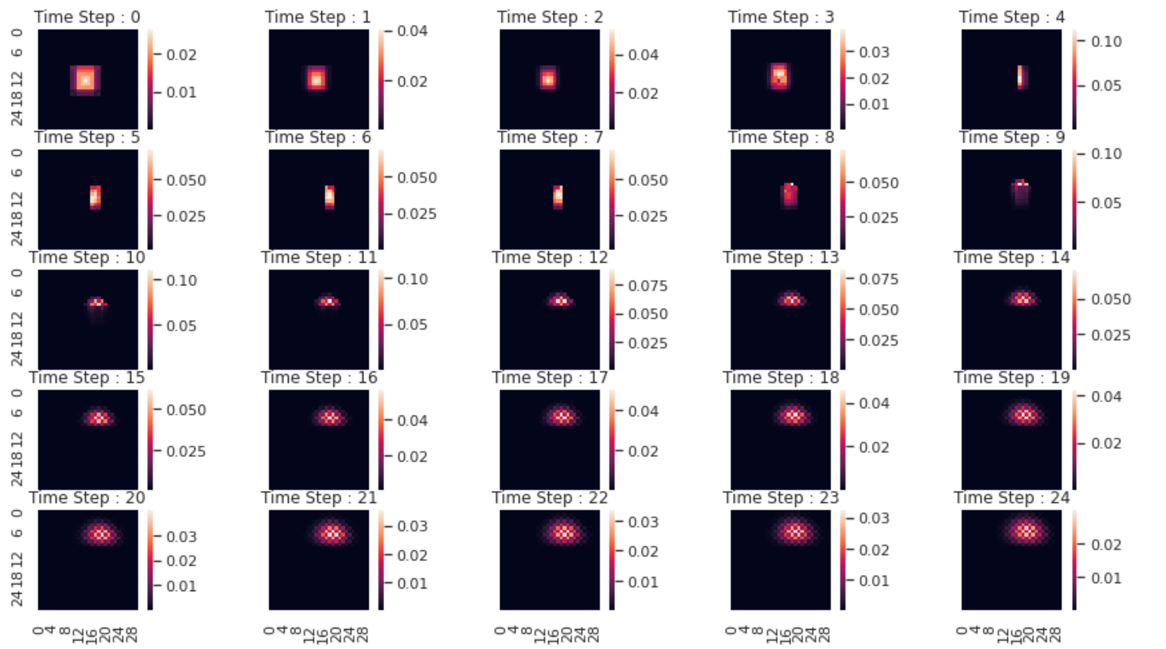

Figure 2.2: Heat-map of Likelihood probability over the grid locations at each time step

The above plot shows the likelihood probability values at every grid location after performing filtering on the states using the sensor observations.

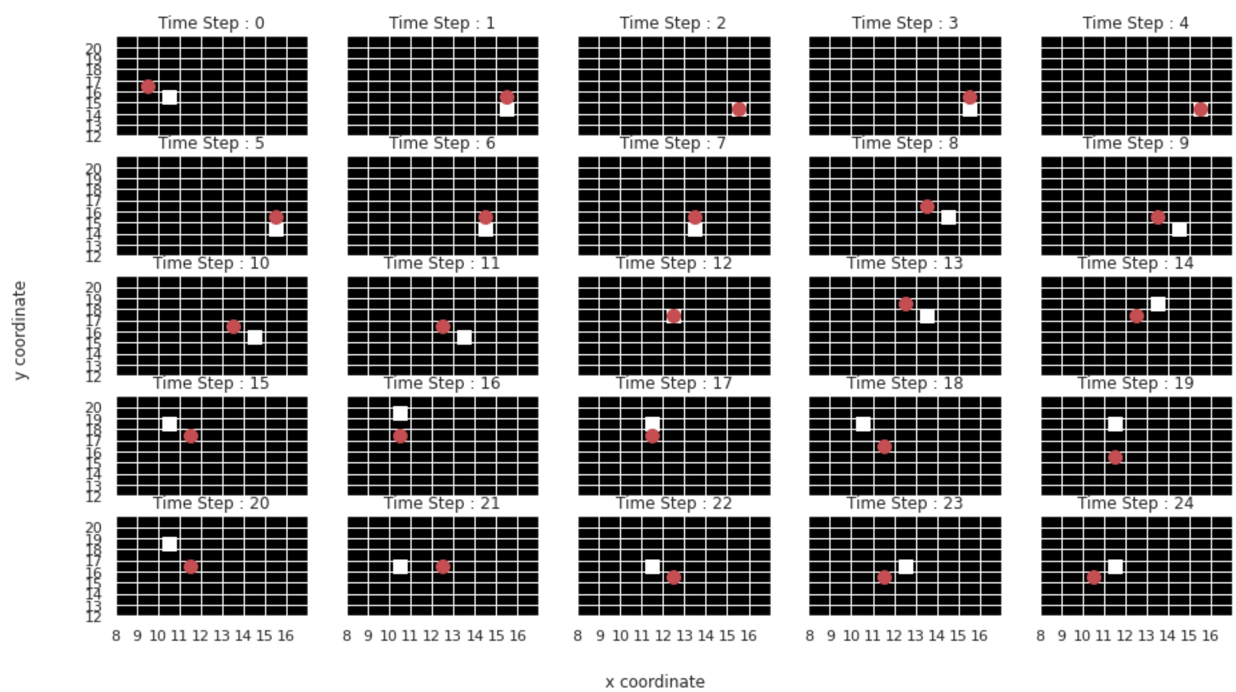

Figure 2.3: Estimated and Ground Truth locations

The white squares depict the estimated location and the red circles depict the ground truth location of the robot at every time instance.

From the above plots we see that initially the uncertainty in the state estimation was quite high but with passage of time more and more sensor data was collected to update our belief of the state and the estimation become more certain. But since the sensor data is also probabilistic, the errors in this data begin to accumulate and in the long term the prediction starts becoming uncertain again.

(c) Estimate agent's current and past positions given the sequence of observations received till time T. Plot the estimated and the ground truth locations at each time step.

Solution:

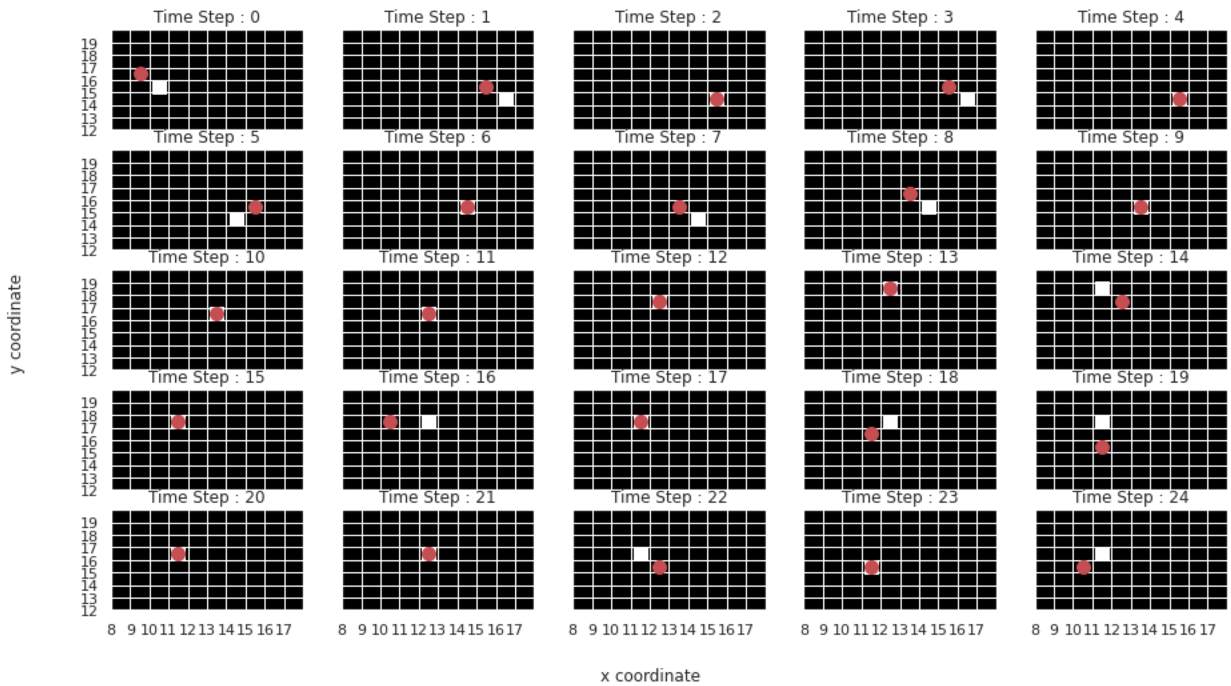

Figure 3.1: Estimated and Ground Truth locations after smoothing

The white squares depict the estimated location and the red circles depict the ground truth location of the robot at every time instance. We see that after performing the smoothing on the prior based on the posterior we improved our state estimation compared to part(b) where only filtering was performed.

(d) Determine the error between actual path and estimated path obtained in parts (b) and (c) above using the Manhattan distance metric. Plot and describe your finding(s).

Solution:

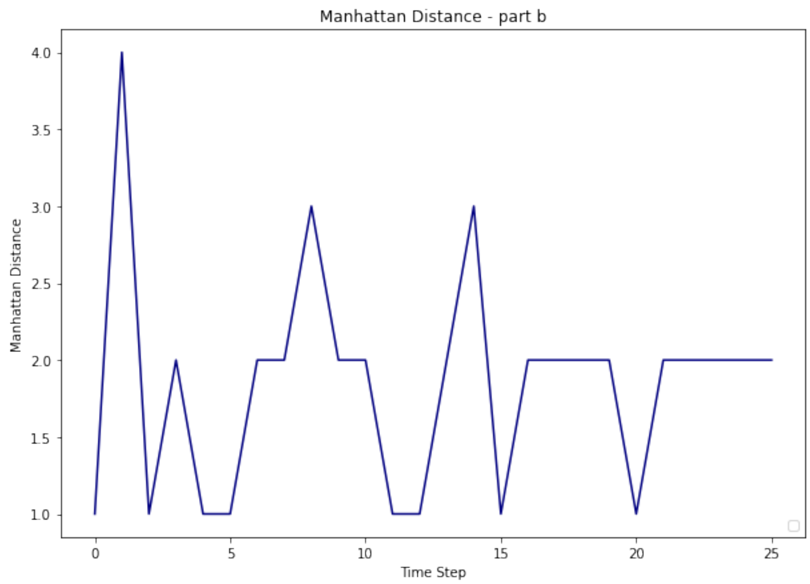

Figure 4.1: Manhattan distance - part b

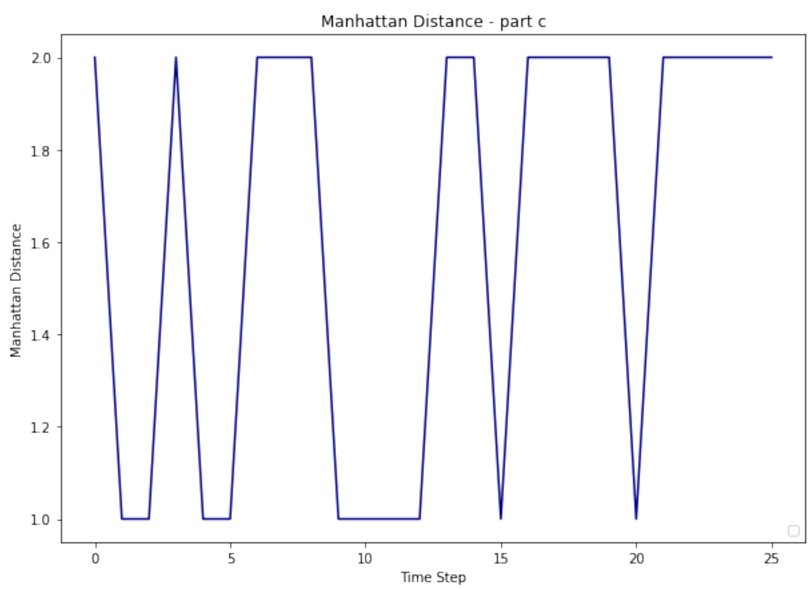

Figure 4.2: Manhattan distance - part c

The above two plots show the Manhattan distance between actual path and estimated path at each time instance for part b and part c respectively.
Average Manhattan distance in part(b) = 1.84
Average Manhattan distance in part(c) = 1.57
So, clearly if Manhattan distance is taken as a metric for accuracy in prediction then after smoothing (part c) the accuracy in state prediction improves compared to part(b).

(e) Compute the predictive likelihood over the robot's future location. Plot the likelihood over the next (i) 10 time steps and (ii) 25 time steps. Describe your finding(s).

Solution:

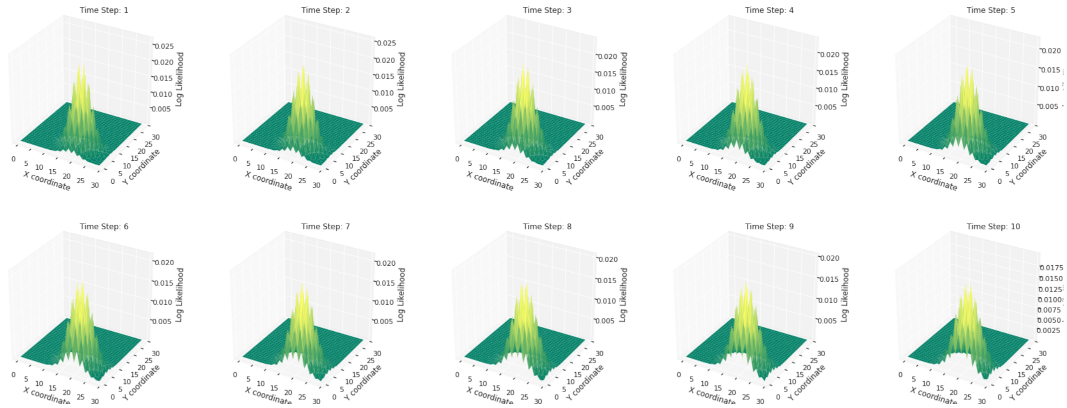

Figure 5.1: Predictive Likelihood over next 10 steps

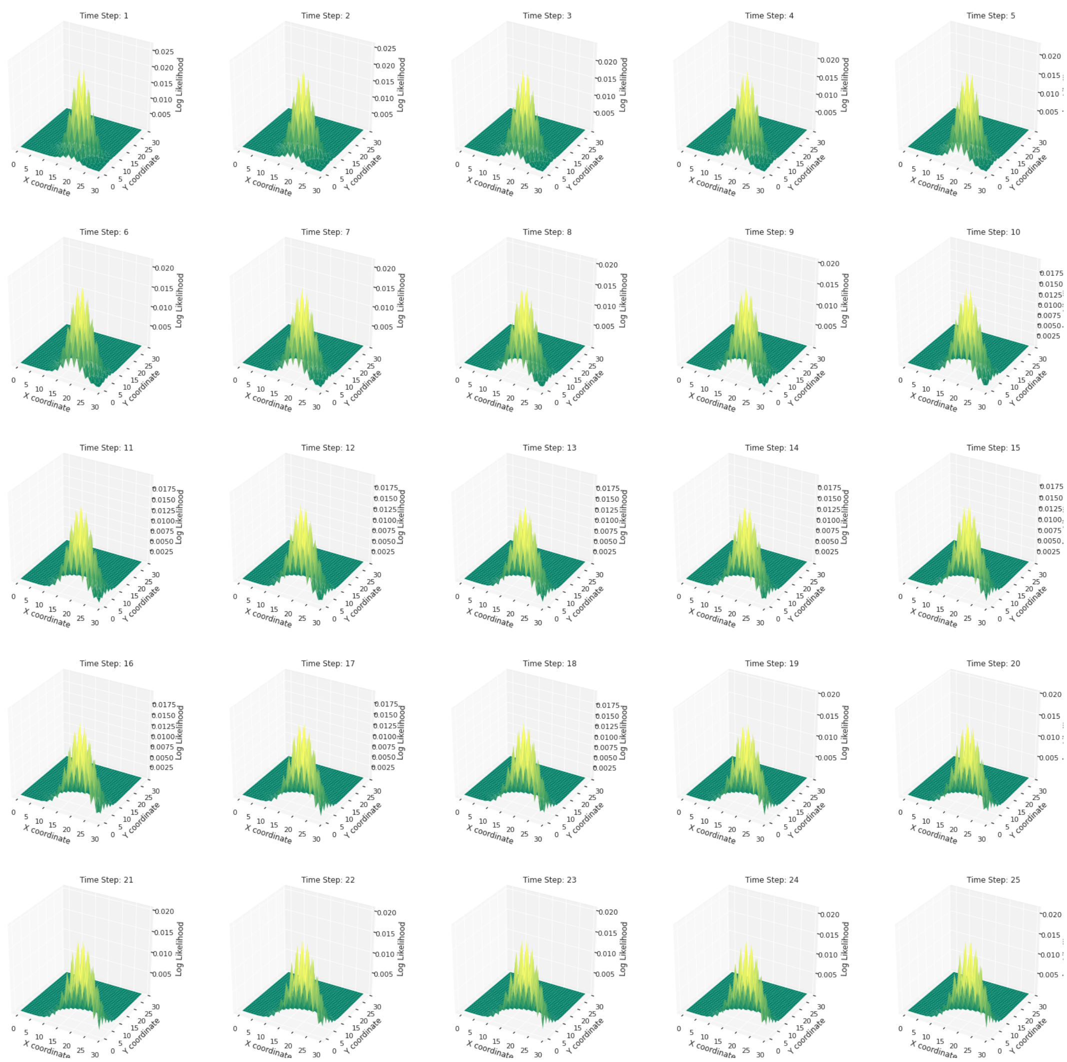

Figure 5.2: Predictive Likelihood over next 25 steps

Note that the above time steps are future time steps, i.e. after T=25. So time step 1 is T+1 = 26.
As you can see in the above graphs that the plots become less peaky and more broad as time increases. This is because state estimation further in future become more and more uncertain based on the data that we have collected till time T.

(f) Estimate the robot's most-likely path given the full sequence of observations till time T.

Solution:

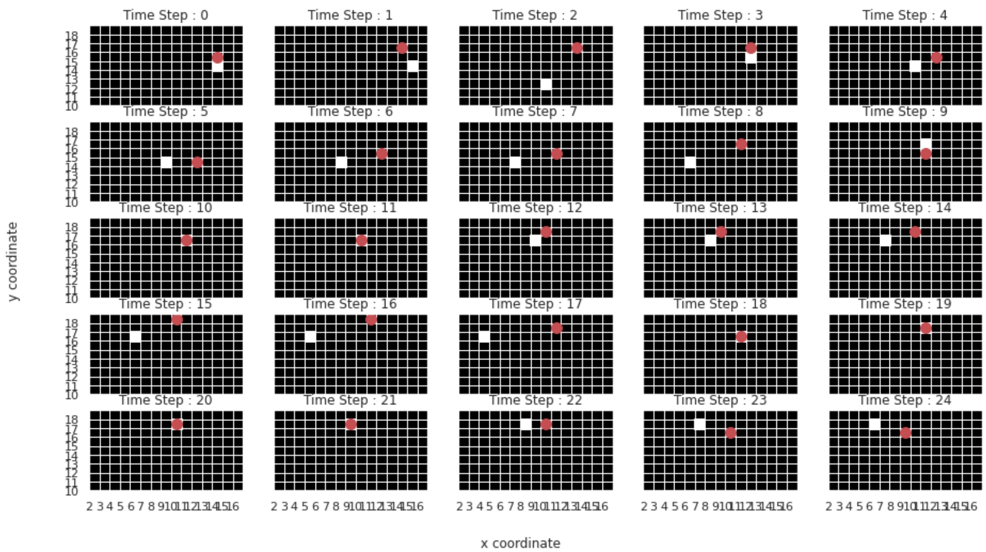

Figure 6.1: Most Likely path and ground truth path

The plot shows the position of robot according to the most likely path predicted from the algorithm and red circles show the ground truth location of the robot at each time instance.

2 Part 2: Estimation of Airplane's position

(a) Plot the actual trajectory and the observed trajectory of the vehicle.

Solution:

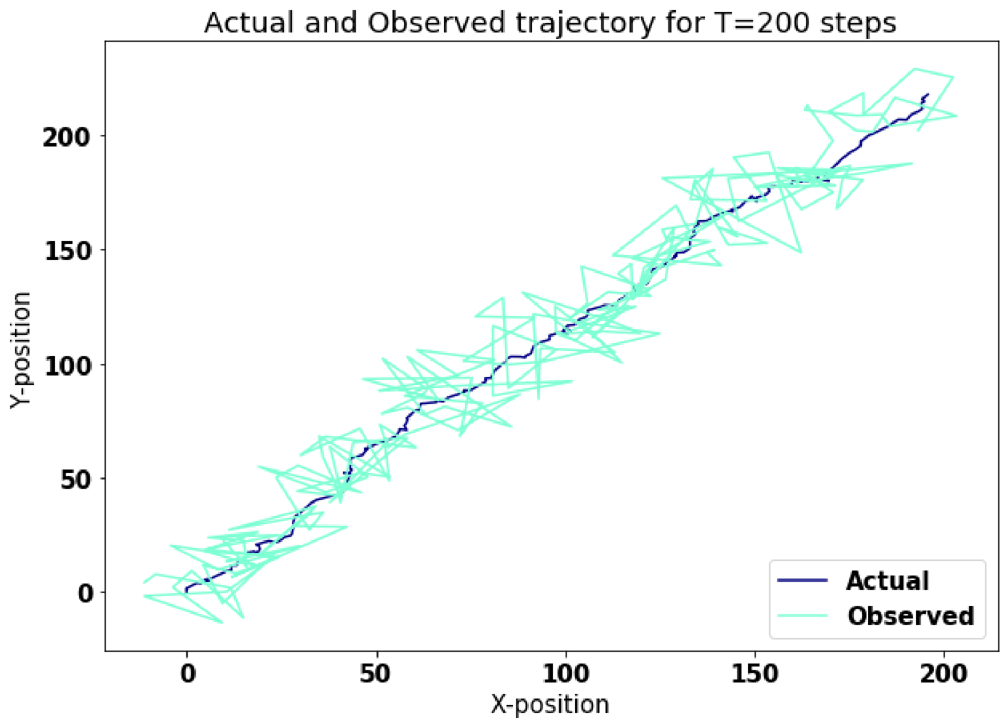

Figure 7: Actual trajectory and the Observed trajectory of the vehicle.

In the plot, the observed trajectory is very noisy as compared to the actual trajectory, and this is because the observation noise i.e. δ_t has a very high standard deviation and hence the observations are so noisy.

(b) Formally write down the the model for estimation.

Solution:

$X_{t+1} = A_t X_t + B_t u_t + \epsilon_t,\ \epsilon_t \sim N(0, R).$ $\qquad z_t = C_t X_t + \delta_t,\ \delta_t \sim N(0, Q)$

$$X_t = \begin{bmatrix} x_t \\ y_t \\ \dot{x}_t \\ \dot{y}_t \end{bmatrix} \qquad A_t = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B_t = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad C_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$u_t = \begin{bmatrix} \delta\dot{x}_t \\ \delta\dot{y}_t \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix} \qquad Q = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$$

The term of 0.5 in matrix B_t comes from the equations of motions $\because v = u + at,\ S = ut + \frac{1}{2}at^2$. Initial belief i.e. initial mean and covariance is already given in the question. The algorithm to compute the mean and covariance of next time step has been coded using the pseudocode given on slide 22 of this lecture.

(c) Plot the actual trajectory, the noisy observations, and the trajectory estimated by the filter. Additionally, plot the uncertainty ellipses for the estimated trajectory.

Solution: As observed in part (a) as well how the observed trajectory is noisy due to high standard deviation in δ_t. But the estimated trajectory is much more stabilised in comparison to observation. and it moves in tandem with the actual trajectory. The uncertainty ellipses are much smaller in comparison to their displacement since their standard deviation stabilises to an order of 3 units. Hence on the plot they look quite small and since they are overlapping the ellipse structure almost seems to disappear but we can still see the edges, especially towards the top right corner as the trajectory moves forward.

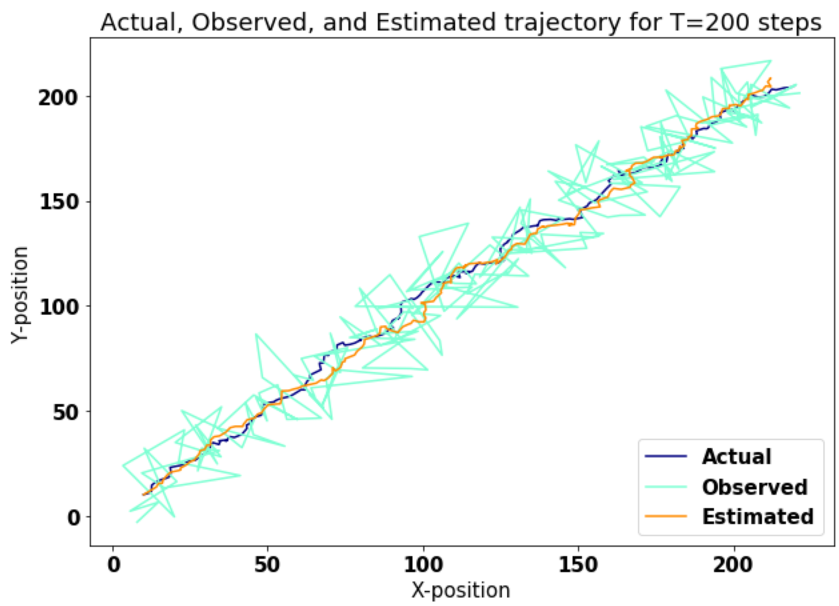

Figure 8.1: Actual, Observed, and Estimated trajectory of the vehicle.

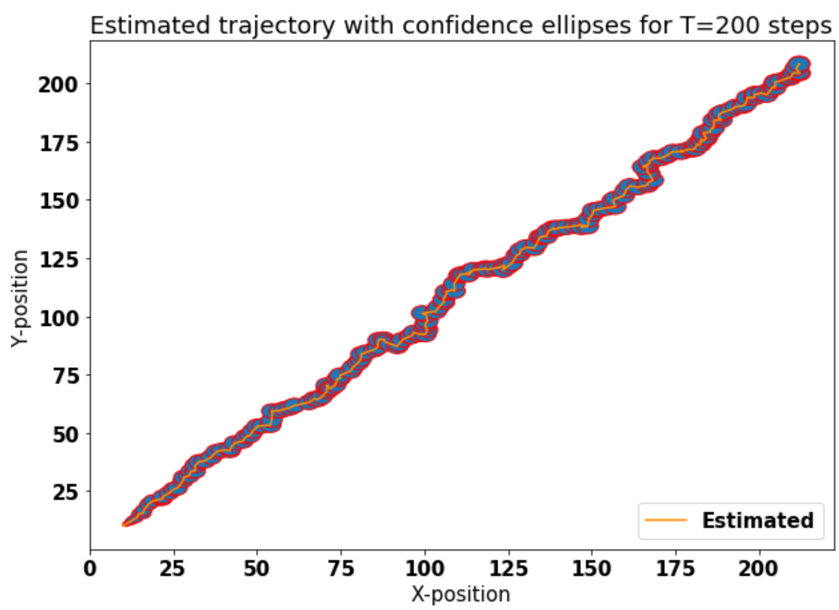

Figure 8.2: Uncertainty ellipses for the Estimated trajectory of the vehicle.

(d) Plot the true, observed and the estimated trajectories under the given control inputs. Compute and plot the error between the true and the estimated trajectory using the euclidean distance metric.

Solution:

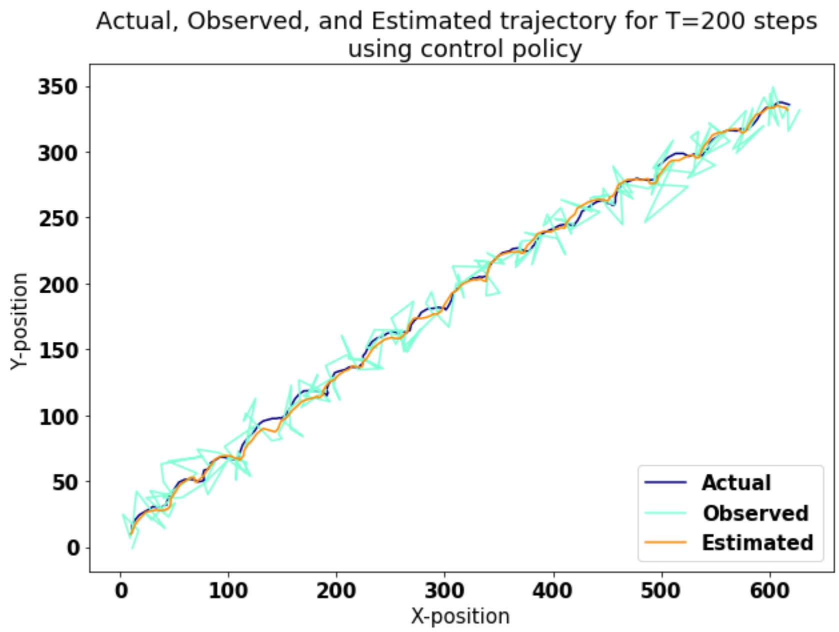

Figure 9.1: Actual, Observed, and Estimated trajectory of the vehicle under control policy.

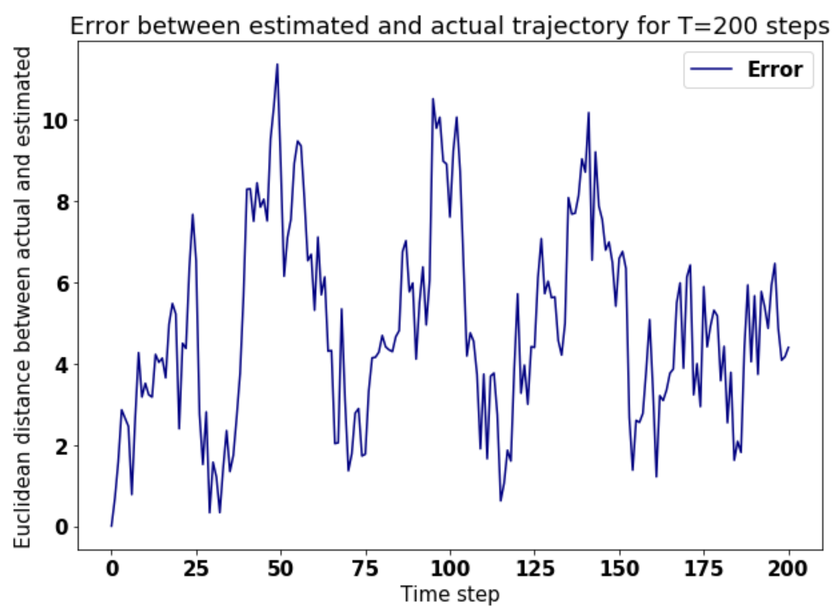

Figure 9.2: Error between the true and the estimated trajectory using the euclidean distance metric.

The velocity is varied as $sin(kt)$ and $cos(kt)$ where k is taken to be 0.5 for this part. As observed in the plot the actual trajectory moves in a wave fashion, as the velocity sometimes goes negative as well. Hence we observe the actual trajectory to be wavy and the estimated trajectory moves in tandem with the actual trajectory. The error between the estimated and actual trajectory also moves in a wave fashion as the error increases then decreases and then increases and so on. This is because the sine and cosine wave can't be exactly captured by the normal distribution hence at few points the difference between estimated and actual trajectory increases so does the error. This majorly happens after the peak of the small wavelets in the actual trajectory have been reached meaning the velocity is going from positive to negative crossing a zero at these points. And so the normal distribution fails to capture those exactly and computes an approximation.

(e) Plot the estimated trajectory and explain how the noise variation impacts the filter performance.

Solution: In the plots below, we can observe that as the standard deviation of δ_t is increased i.e. the uncertainty in sensor observation is increased, the error between the estimated and the actual trajectory increases. The filter performance degrades and it takes much more time to stabilise the error.

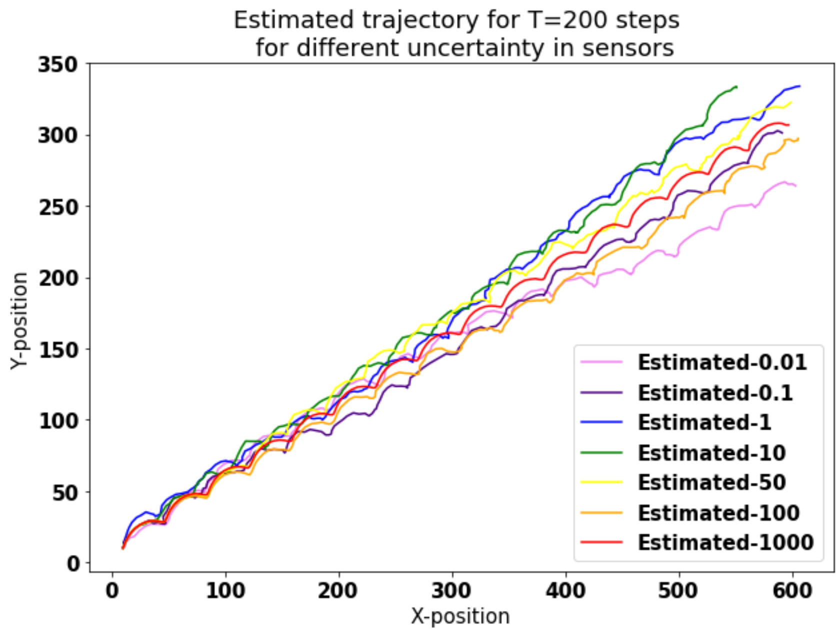

Figure 10.1: Estimated trajectory for different uncertainty in sensors.

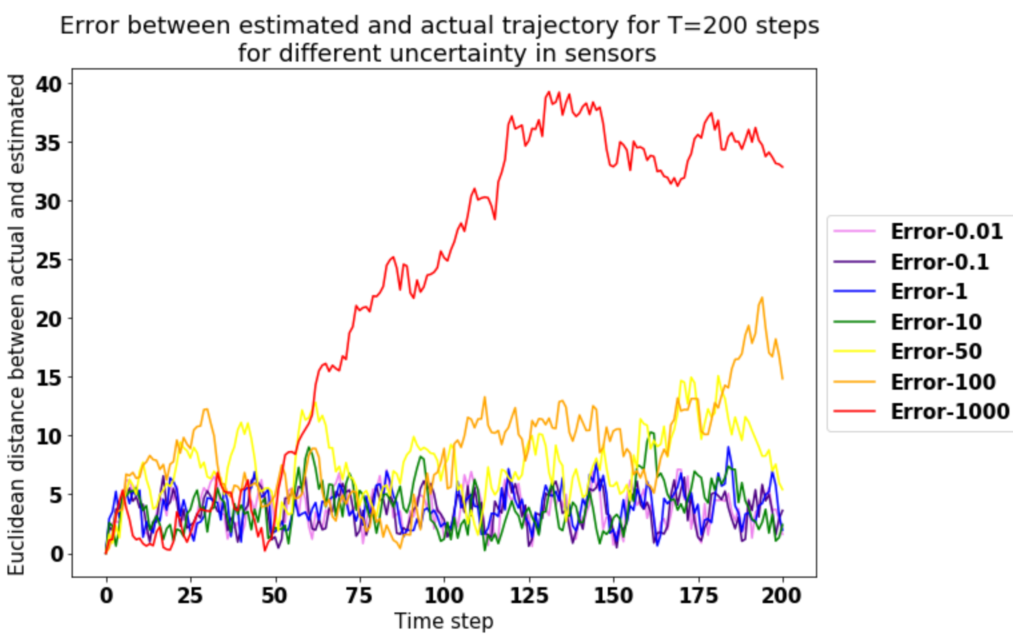

Figure 10.2: Error between the true and the Estimated trajectory for different uncertainty in sensors.

(f) Plot the true, observed and the estimated trajectories under the sine-cosine control policy for higher uncertainty in the initial belief.

Solution: When the uncertainty in initial belief is increased, the difference between the estimated and the actual trajectory increases abruptly for the first few time steps and so does the error. After that the error stabilises and follows the same patter as in part (d).

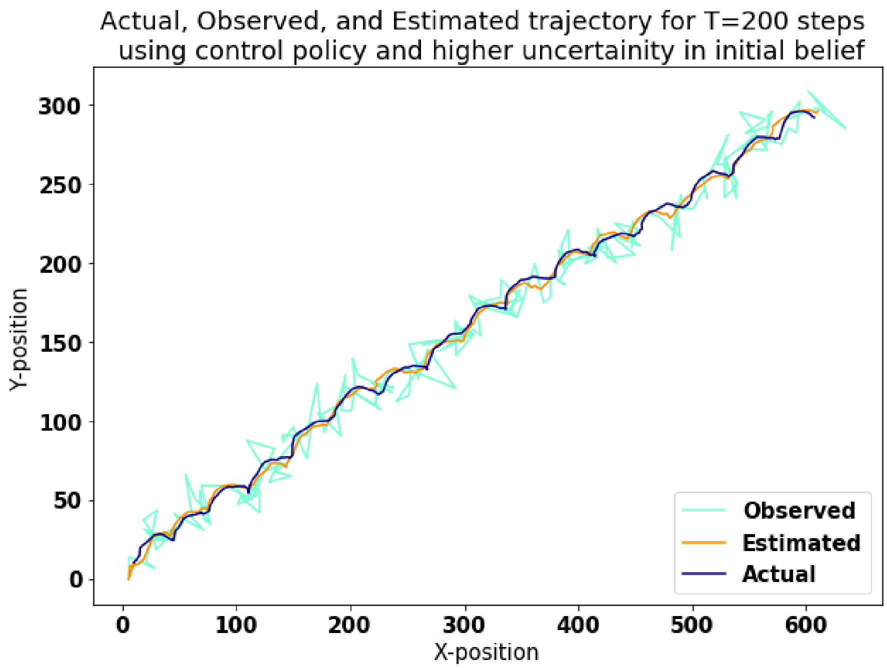

Figure 11.1: Actual, Observed, and Estimated trajectory for higher uncertainty in initial belief.

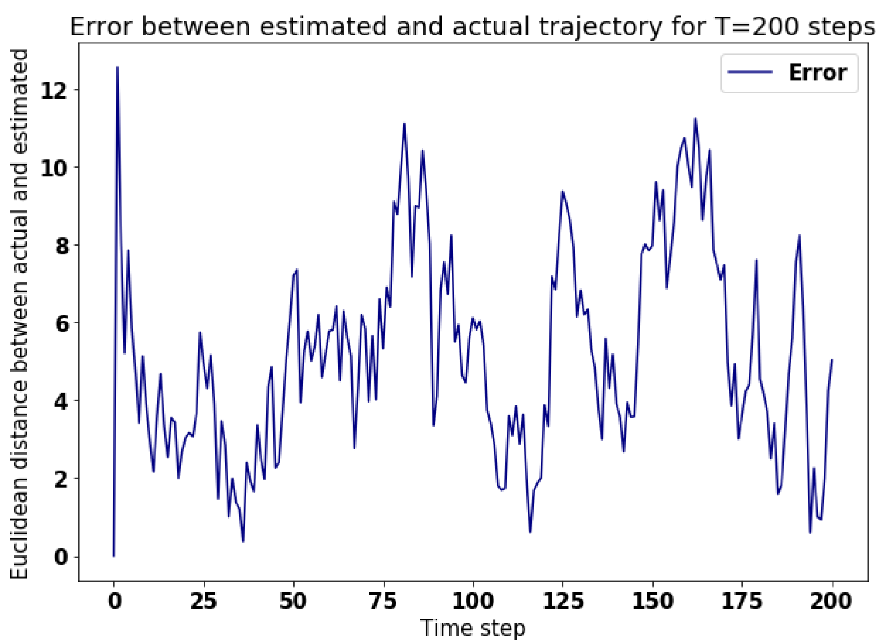

Figure 11.2: Error between true and Estimated trajectory for higher uncertainty in initial belief.

(g) Simulate and show the evolution of uncertainty in the vehicle's position by plotting the uncertainty ellipses.

Solution:

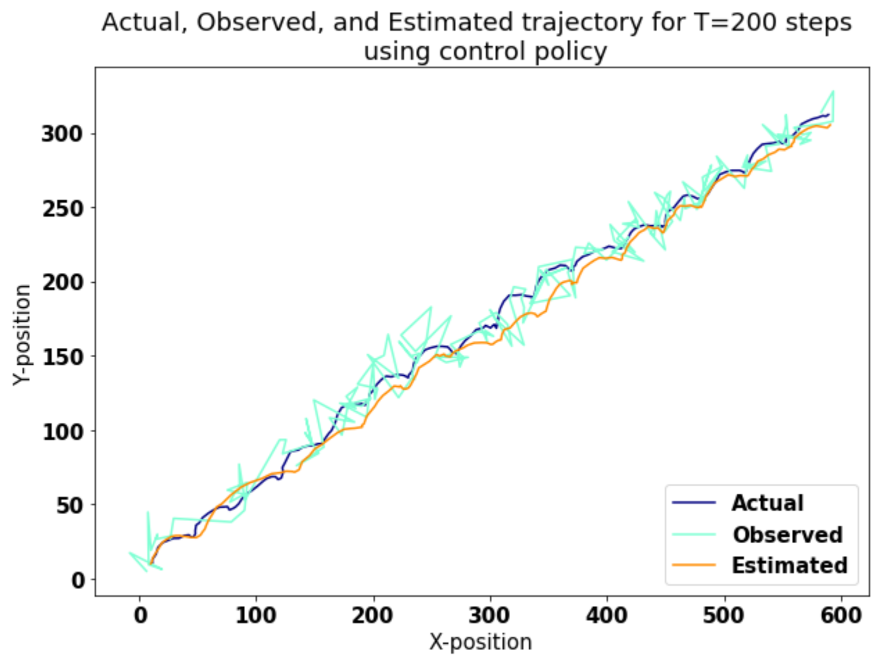

Figure 12.1: Actual, Observed, and Estimated trajectory with sensor observations drop out.

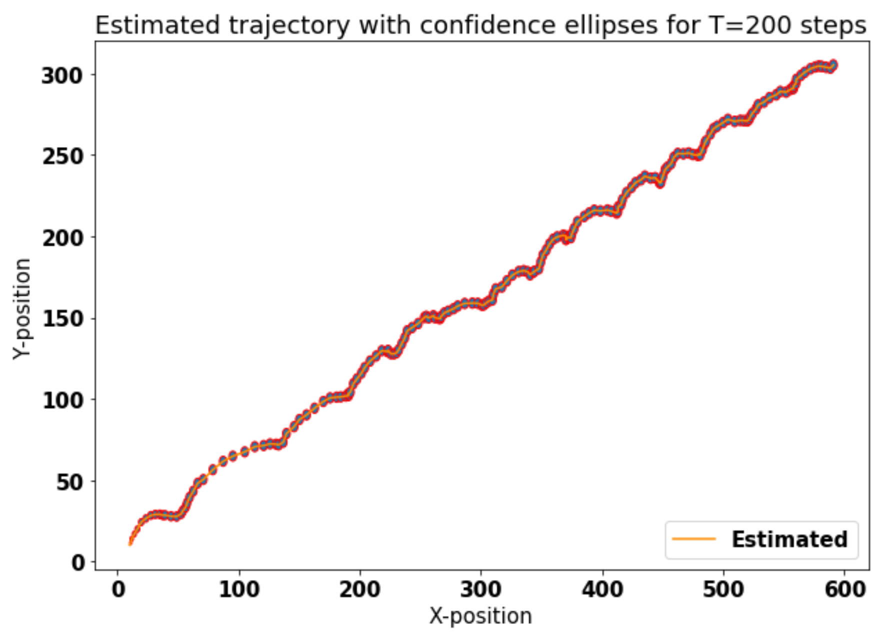

Figure 12.2: Uncertainty ellipses for the Estimated trajectory with sensor observations drop out.

The estimated and actual trajectory diverge and the error increases after the observations drop out and it starts to converge once the observations start to come in. And then the estimator stabilises after a while when the observations after T=40 become quite old and the the actual and estimated trajectory move in tandem as in part (d) and the error term also follows the same pattern as in part (d).

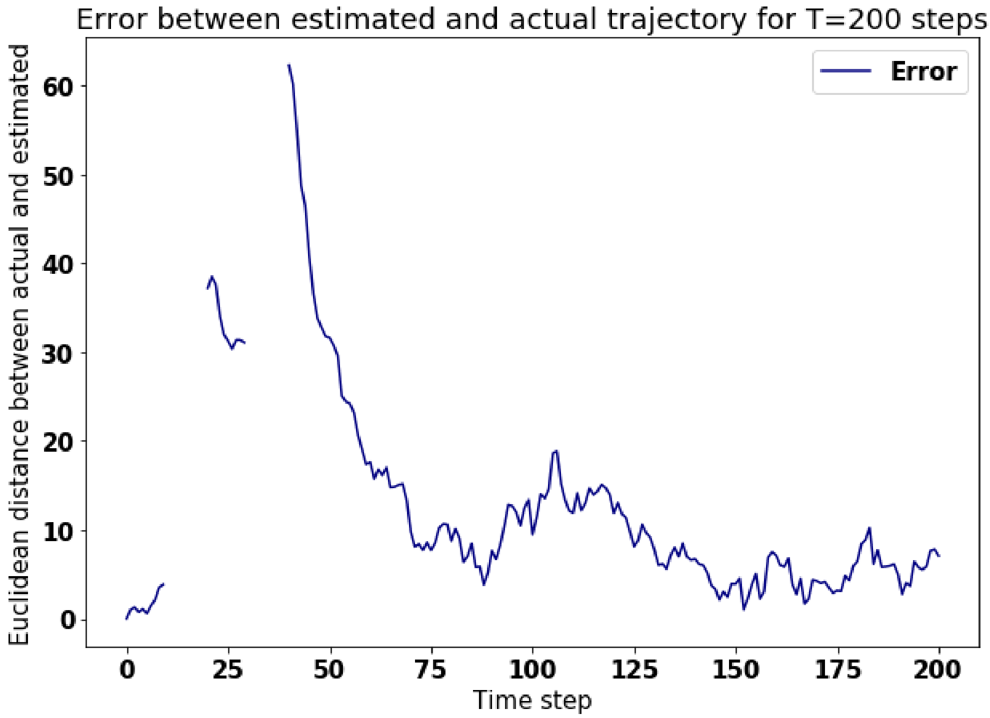

Figure 12.3: Error between true and Estimated trajectory with sensor observations drop out.

(h) Plot the estimated velocities and the true velocities of the vehicle.

Solution:

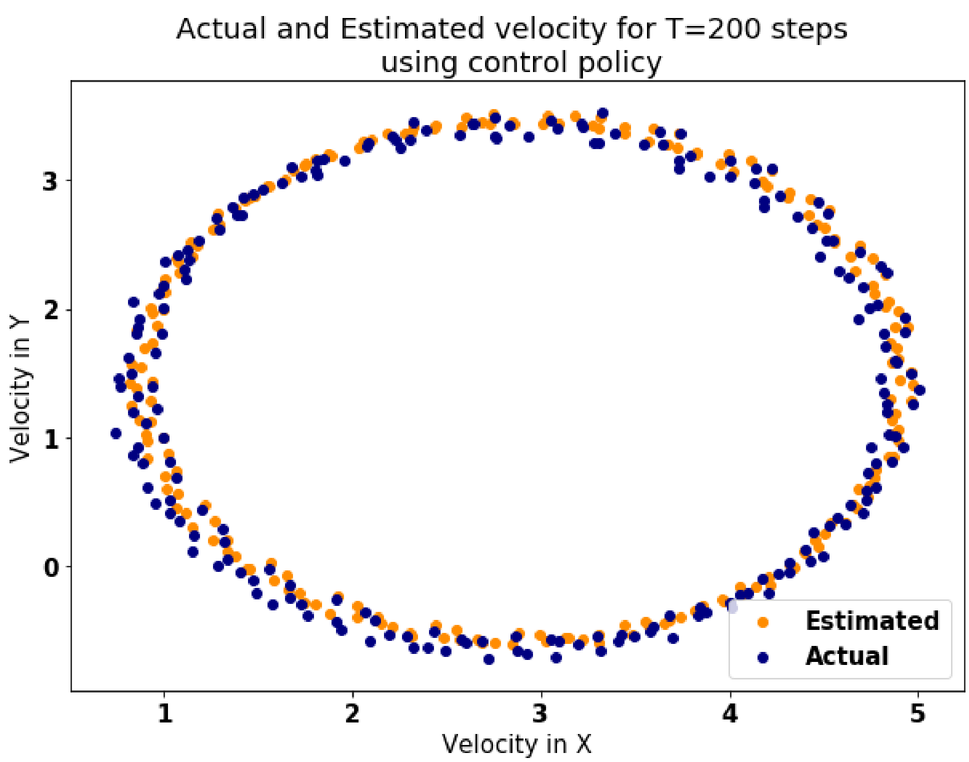

Figure 13.1: Actual and Estimated velocity of the vehicle under control policy.

The estimator is able to track the velocities and does so very finely. The error in between the actual and estimated velocity is quite low. This happens because even though the observed parameters are the positions, the velocity is basically displacement over time and displacement is basically change in position. So the estimator is able to track the velocity by estimating the change in velocity over between two time steps. And this is also observed in the plots.

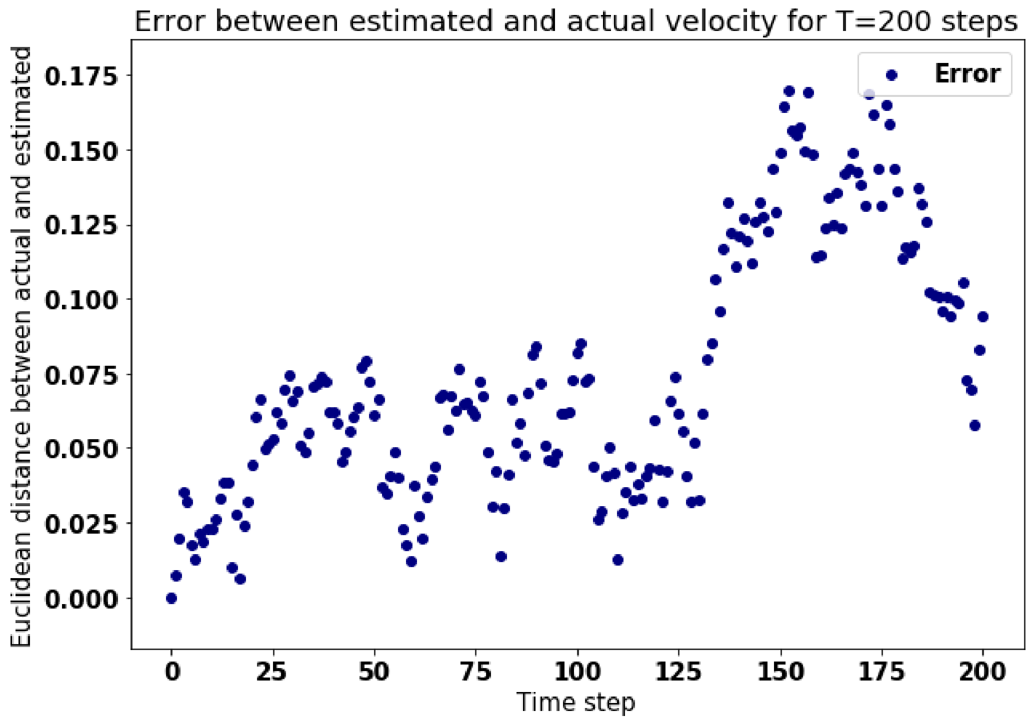

Figure 13.2: Error between the true and the estimated velocity using the euclidean distance metric.

(i) Implement a data association strategy and study its behavior in your simulation. Can your solution scale to more (4-5) agents?
Solution: Say there are n vehicles and n observations. In the data association step calculate the estimation of each of the n vehicles with respect to each of the observations. Hence you will have n^2 estimations. This can be stored in the form of a matrix E where the rows denote the vehicles and columns denote the observations. Entry E_{ij} denote the entry corresponding to the estimation of agent i with respect to observation j. The entry stores the frobenius norm of the covariance matrix obtained for the observation of that entry. Now comes the task of mapping each estimation to each of the observations exclusively.

Pick the matrix element with least value, say (i, j). This means vehicle i is estimated by the observation j. Now delete the i^{th} row and j^{th} column from the matrix E and then recursively repeat this step. This will be done atmost n times and then you will have an association of the vehicles with the observations. And yes this strategy is scalable for more(4-5) agents.

As evident in the plots below, the strategy works really well and is able to associate the observations to the vehicles. In both the plots the actual and estimated trajectories move in tandem. The difference between actual and observed trajectory in plot b is not much visible because the range on x-axis and y-axis is bigger in (b) as compared to (a). This is because velocity of vehicle (b) has a smaller time period as compared to vehicle (a) and hence the vehicle (b) moves faster.

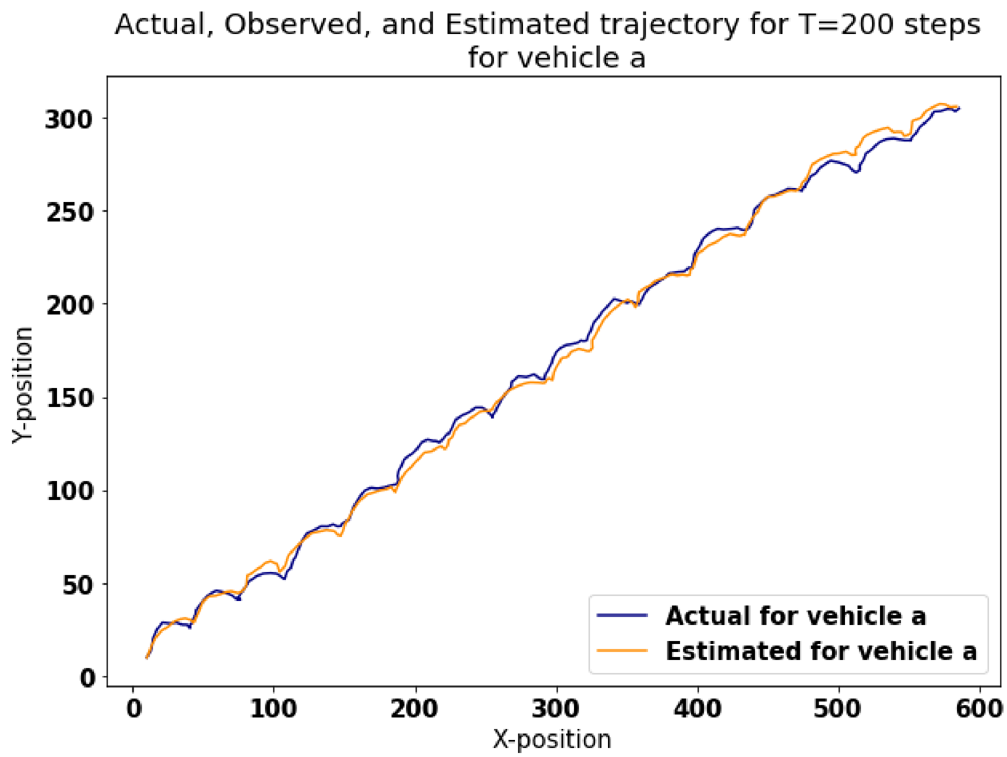

Figure 14.1: Actual and estimated trajectory for vehicle a using the data association strategy explained above.

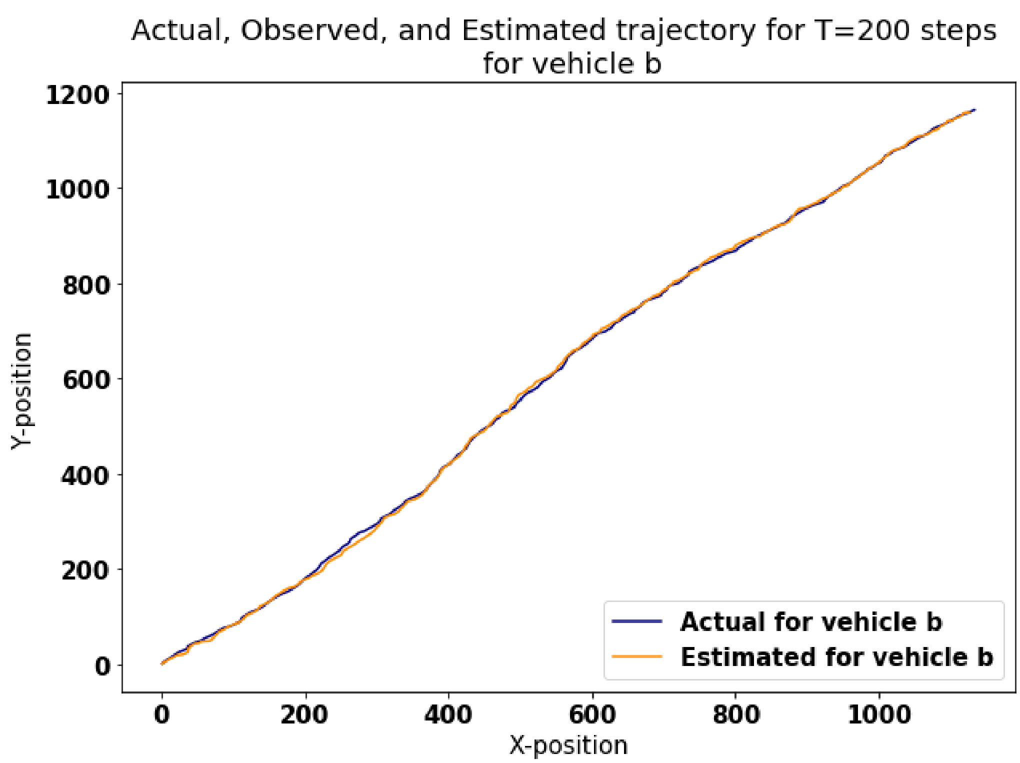

Figure 14.2: Actual and estimated trajectory for vehicle b using the data association strategy explained above.