ME634A
Mid-Sem Assignment Report
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Problem:

Consider the PDE:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=50000\cdot e^{[-50((1-x)^2+z^2)]}\cdot[100((1-x)^2+z^2)-2], \tag{1}$$

subjected to the following boundary conditions in x and z directions.

$$\phi(1,y,z)=100(1-z)+500e^{-50z^2} \tag{2}$$

$$\phi(0,y,z)=500e^{-50(1+z^2)} \tag{3}$$

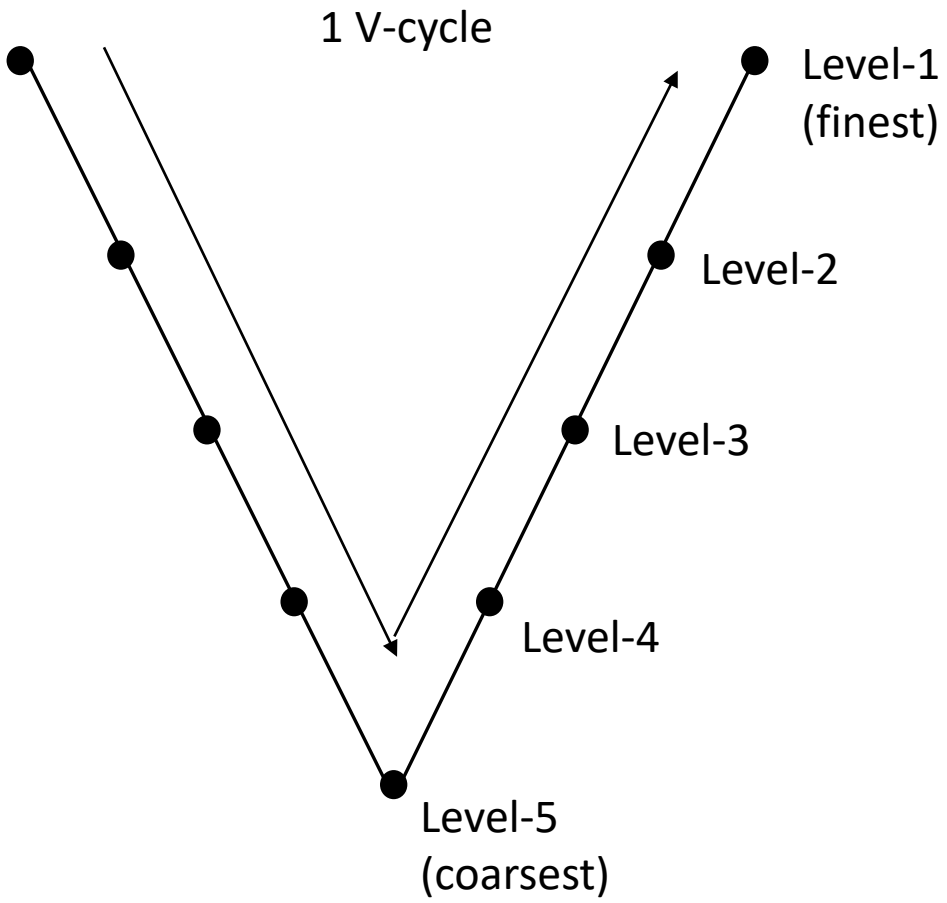
$$\phi(x,y,0)=100x+500e^{-50(1-x)^2} \tag{4}$$

$$\phi(x,y,1)=500e^{-50((1-x)^2+1)} \tag{5}$$

The y is treated periodically. The analytical solution of the PDE is given as:

$$\phi(x,y,z)=500e^{-50((1-x)^2+z^2)}+100x(1-z) \tag{6}$$

Domain size, Ω: [0,0,0] *to* [1,1,1]. Solve the above PDE numerically using Multigrid method in combination with Gauss-Seidal iterations. Use second order finite difference method to discretize equation 1.



2 sets of computations were performed:

Level	No. of internal nodes in each x, y, z direction		No. of GS iterations
	Set-1	Set-2	
1	32	64	10
2	16	32	8
3	8	16	6
4	4	8	4
5	2	4	4

Total no. of GS iterations in each V cycle = 60

Results

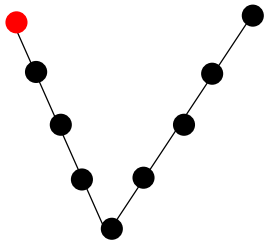
Comparison with original Gauss-Seidal (GS) results

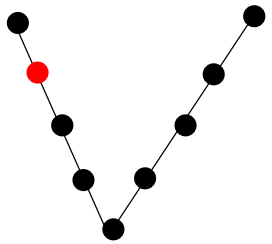
Number of internal nodes at the finest level	Total no. of GS iterations to reach the desired tolerance	
	Multi-Grid with GS	GS (without multigrid)
32 (Set-1)	1020 (17 V-cycles)	1501
64 (Set-2)	1860 (31 V-cycles)	5083

Desired tolerance = 10^{-5}

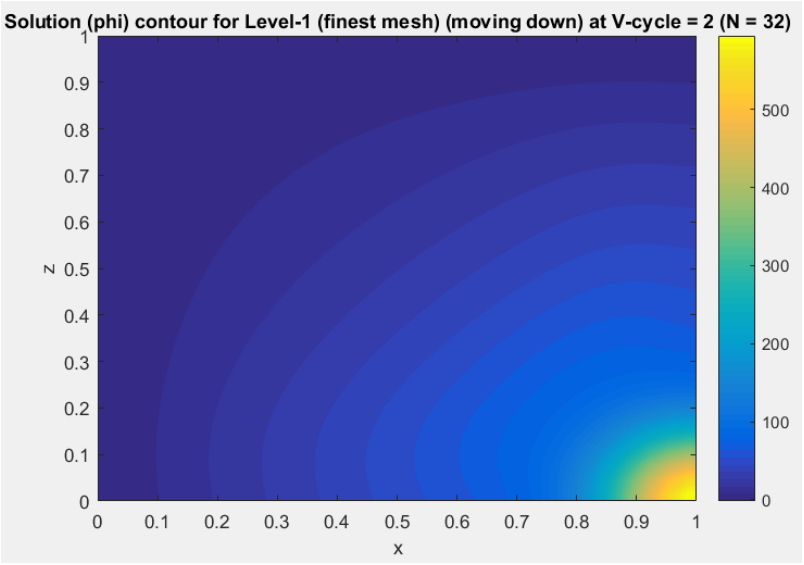
Results

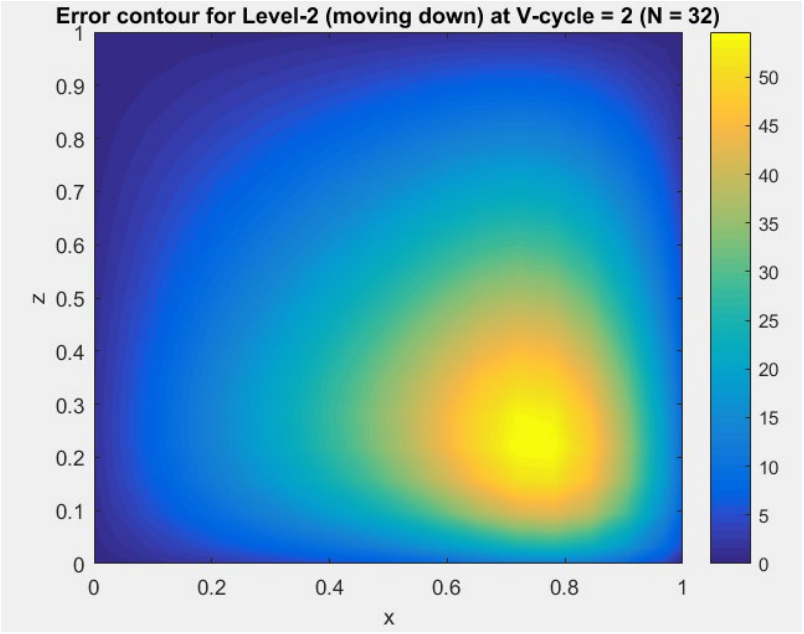
1. Nx, Ny, Nz = 32



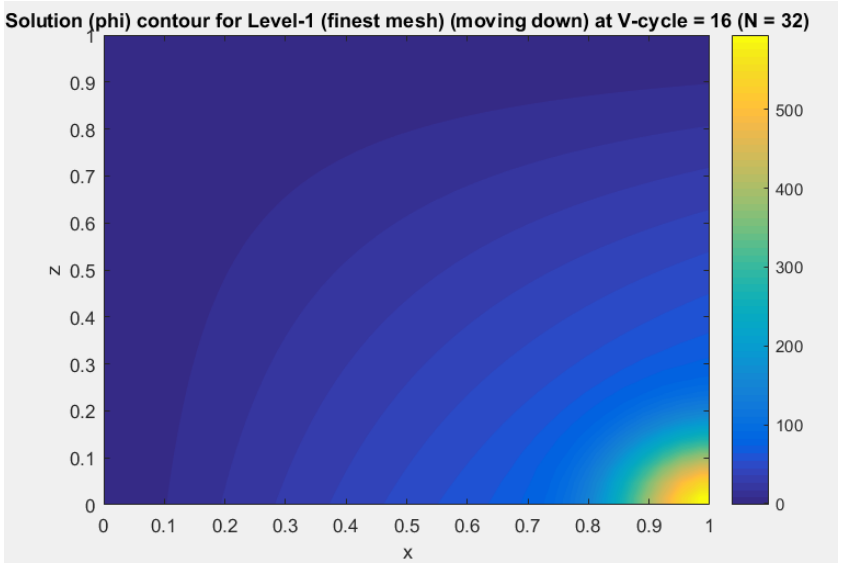


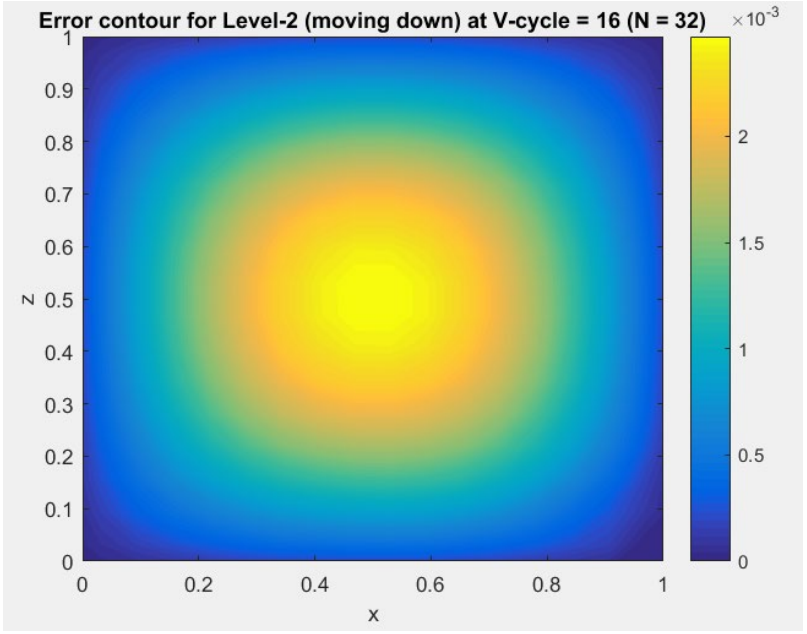
V-cycle = 2





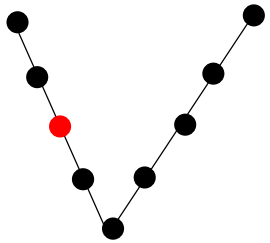
V-cycle = 16

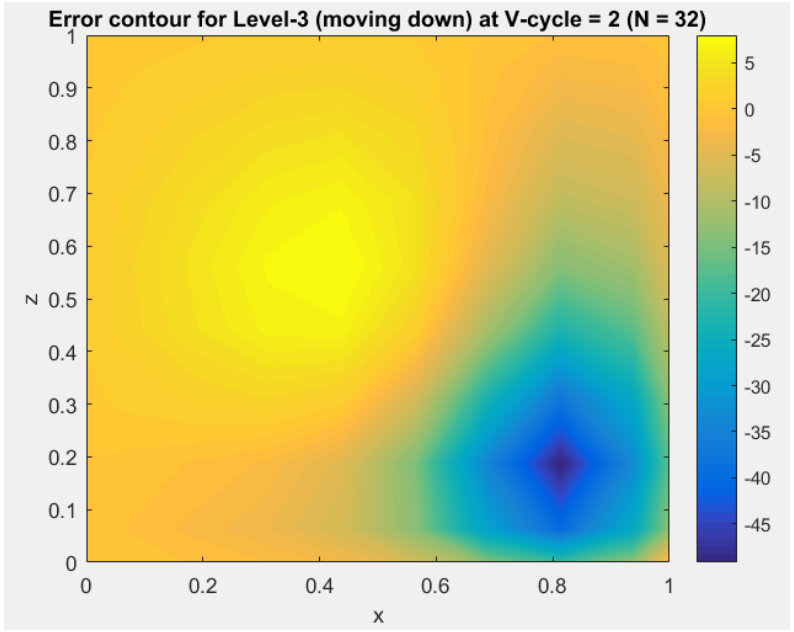


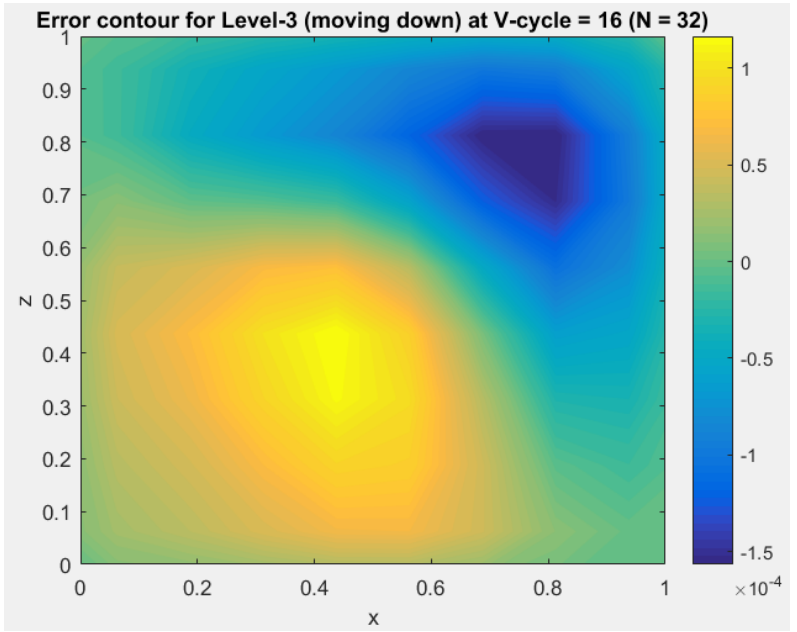


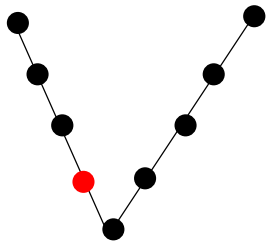
V-cycle = 2

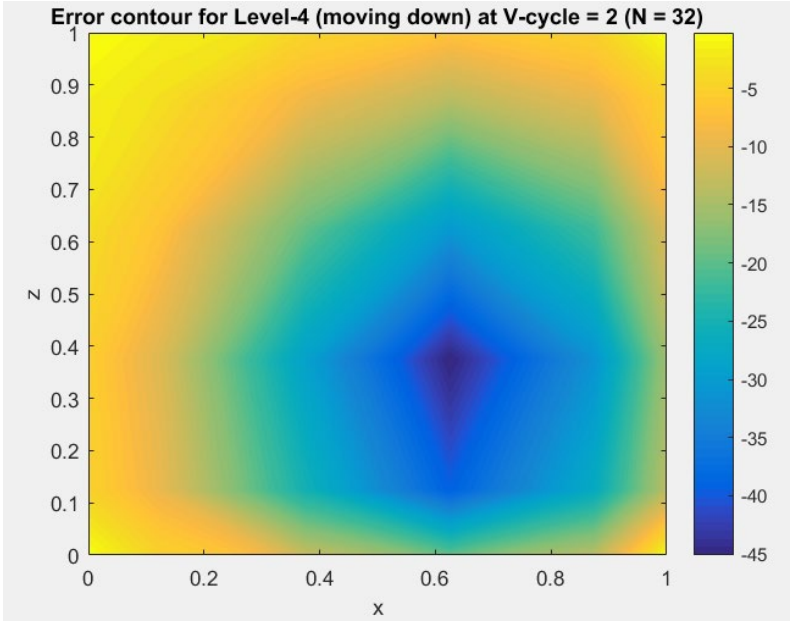
V-cycle = 16

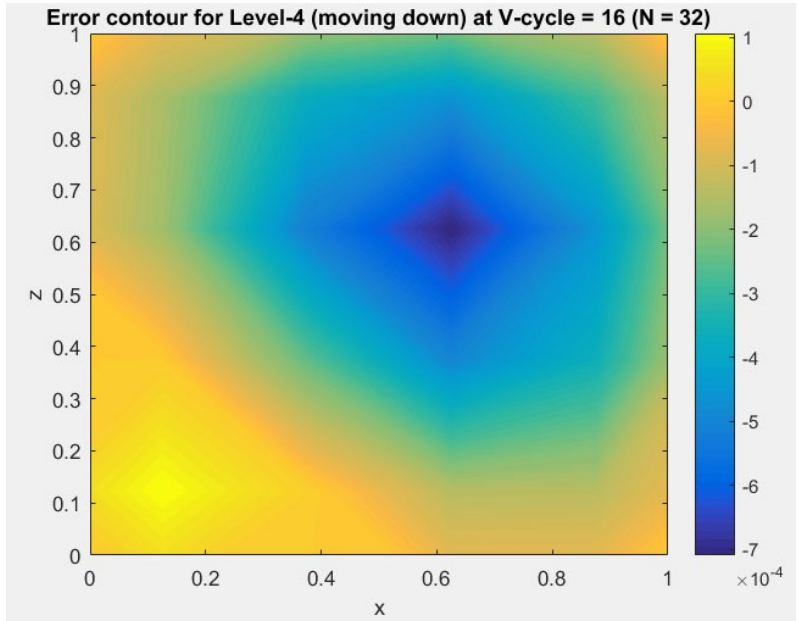






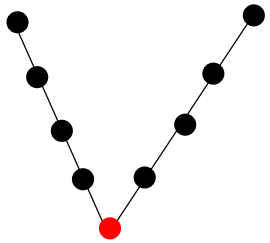


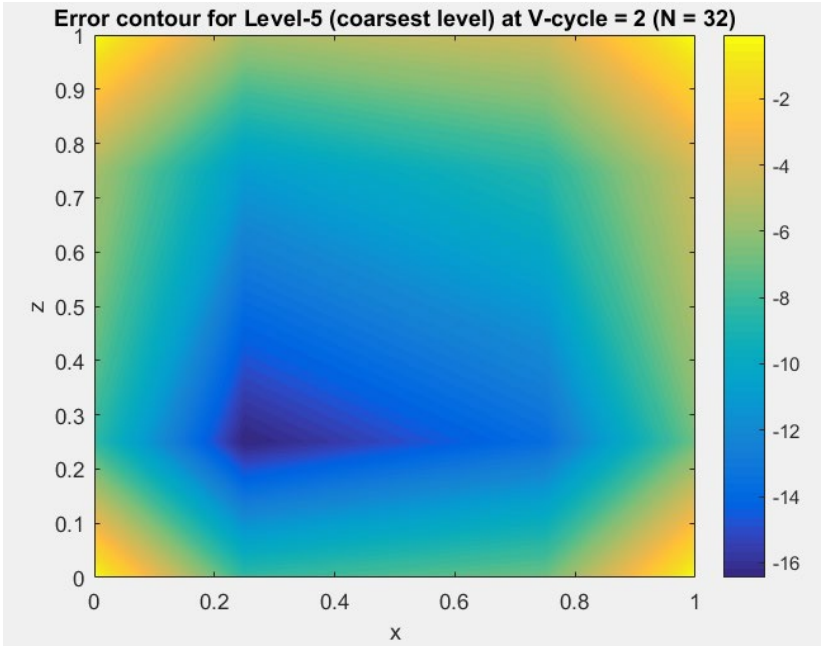


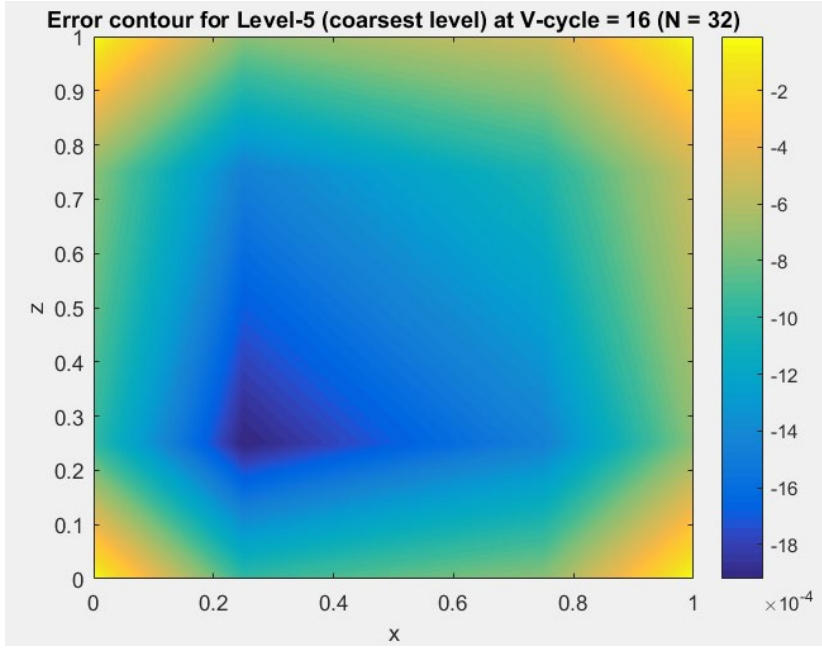


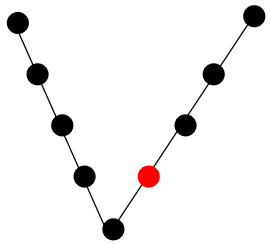
V-cycle = 2

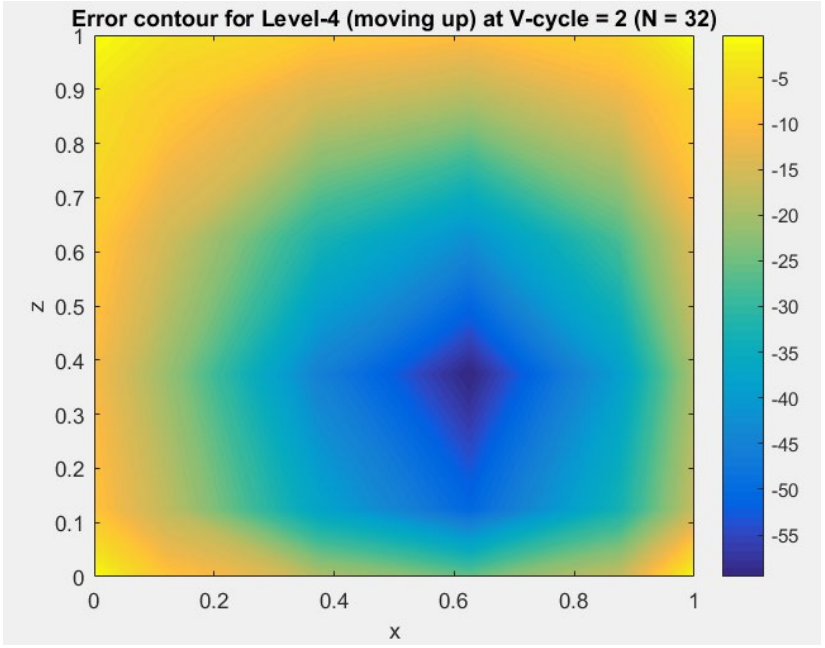
V-cycle = 16

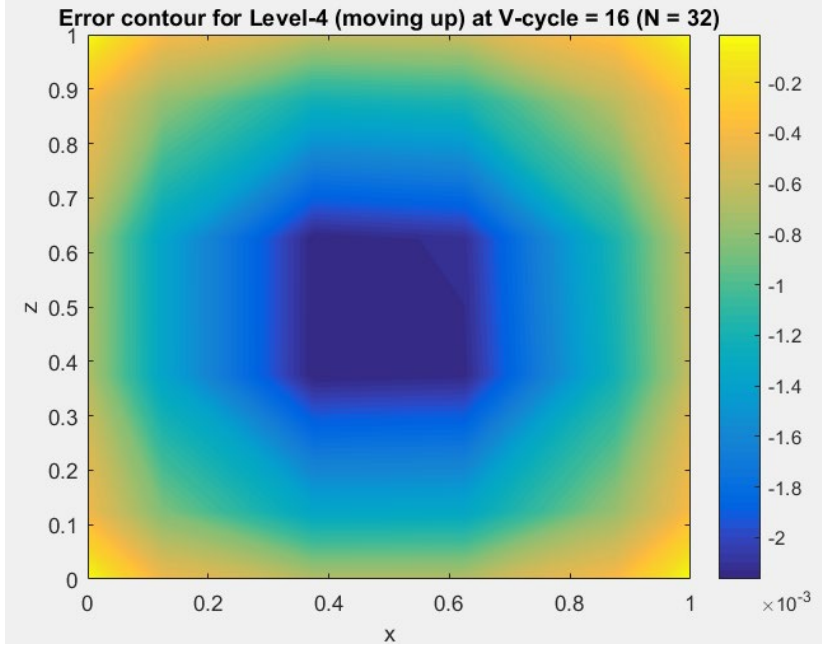






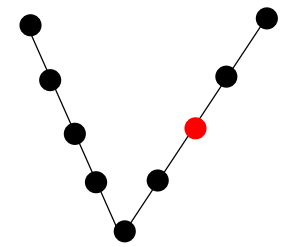


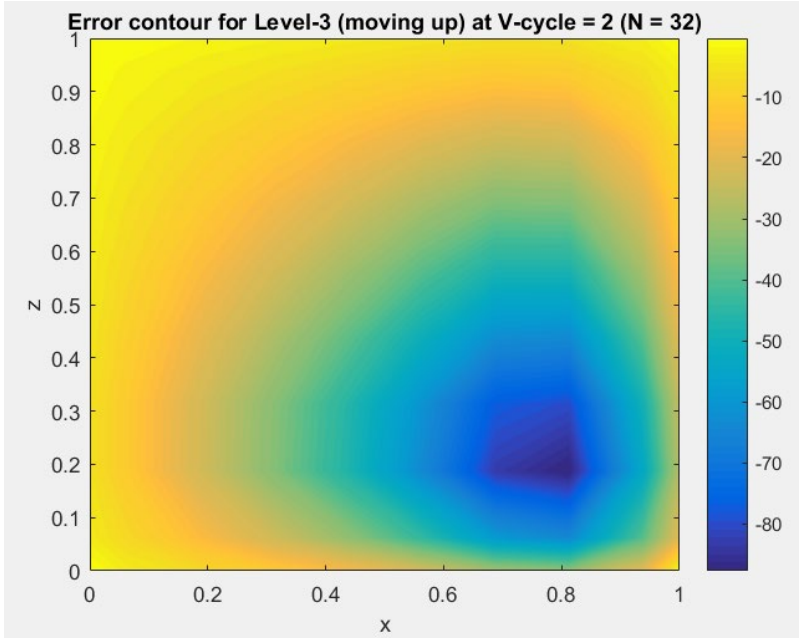


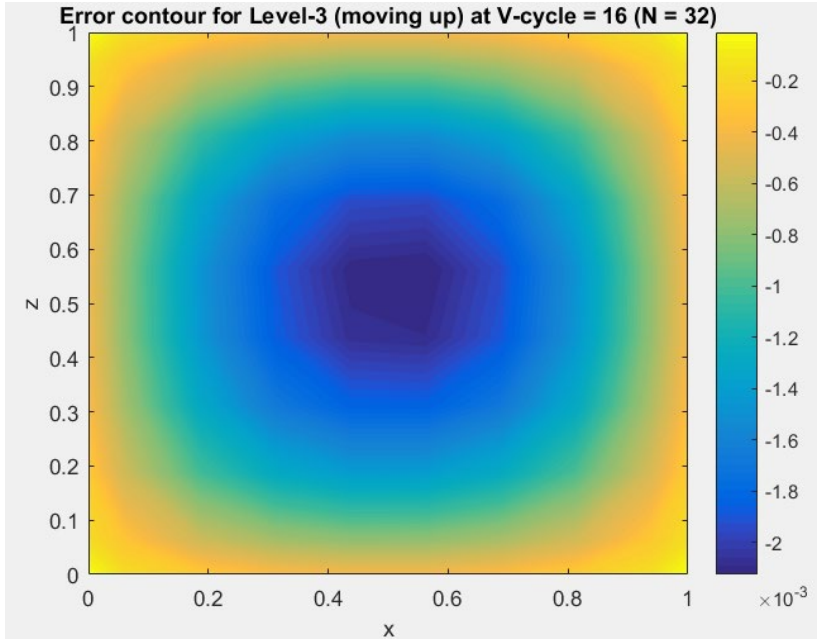


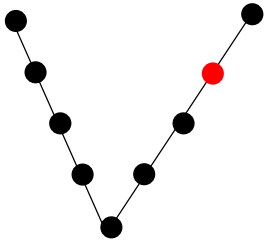
V-cycle = 2

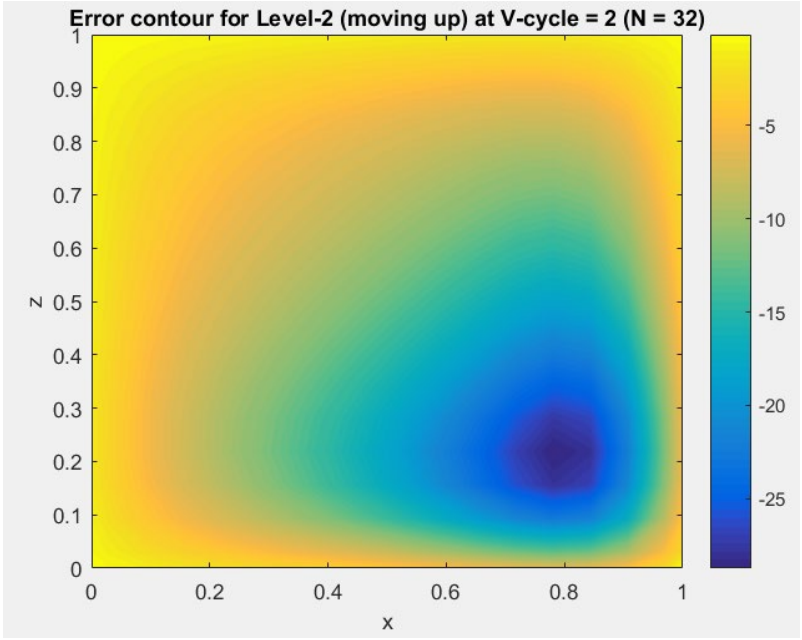
V-cycle = 16

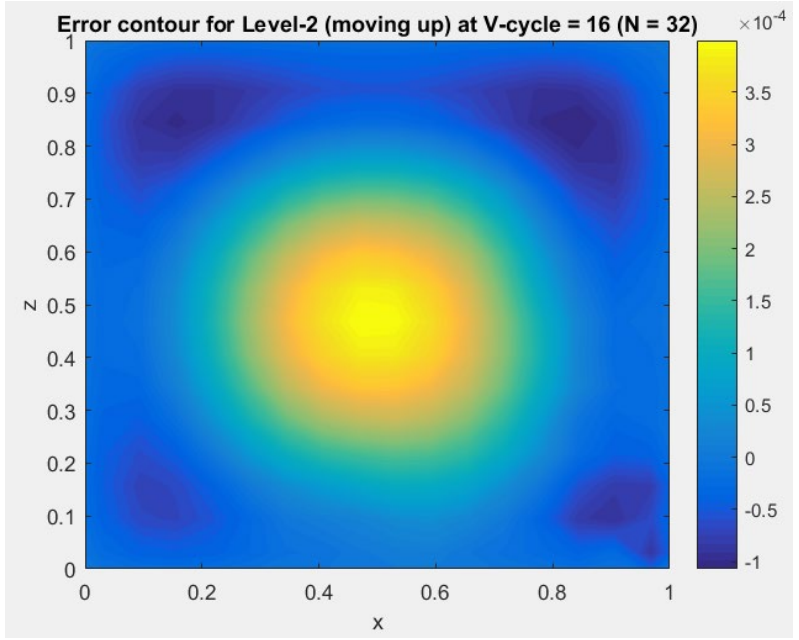


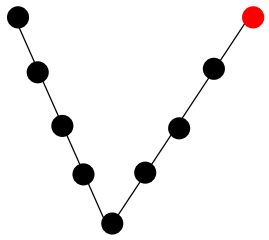




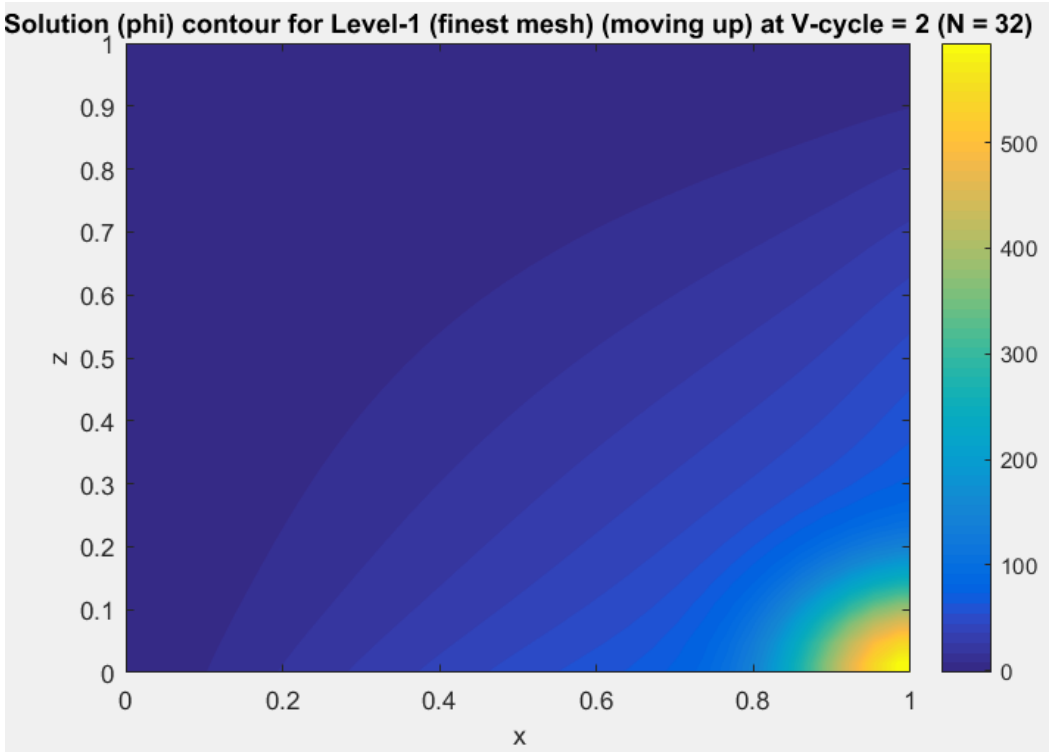






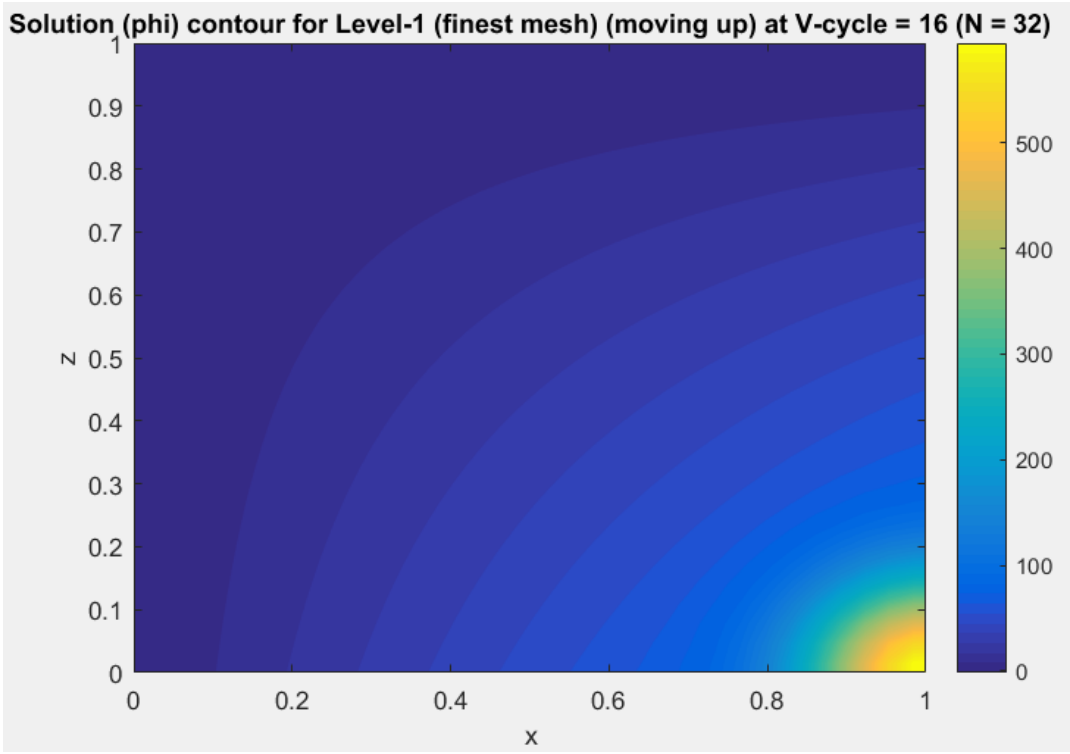


V-cycle = 2



Residual (L2 norm) = 6.25689924E-02

V-cycle = 16

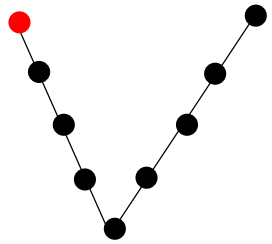


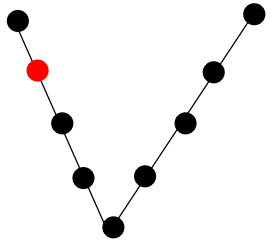
Residual (L2 norm) = 1.31175593E-05

NOTE that the contours are plotted at { j = $(Ny_{level} + 2)/2$ } y-node where Ny_{level} = Total no. of internal y-nodes at a given level.

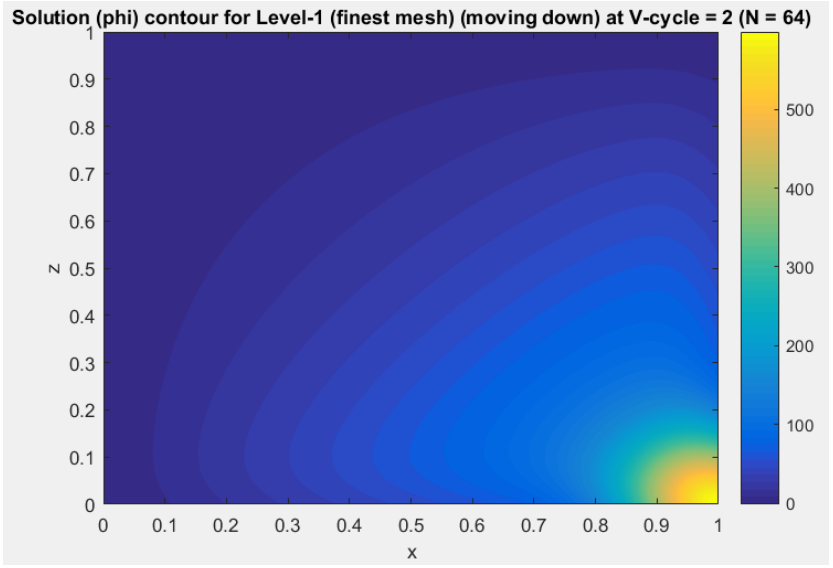
Results

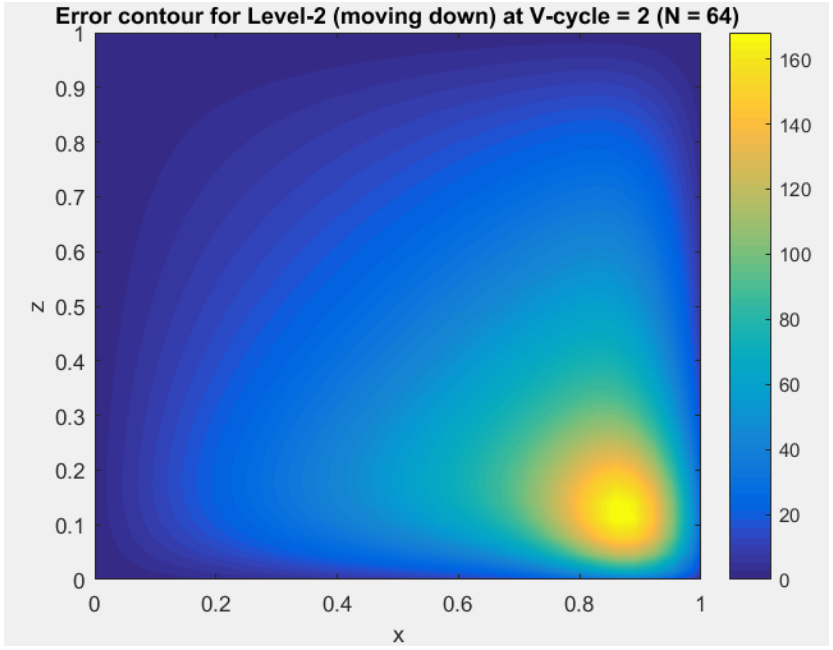
2. <u>Nx, Ny, Nz = 64</u>



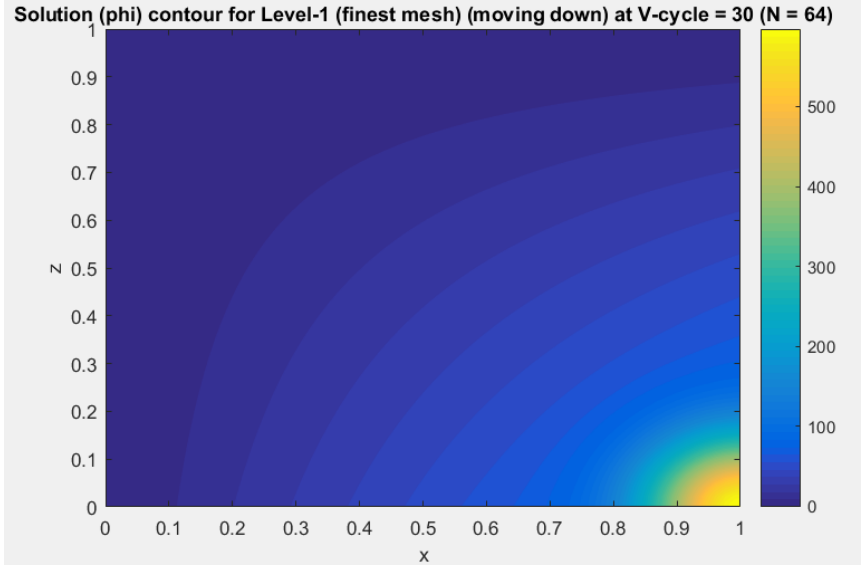


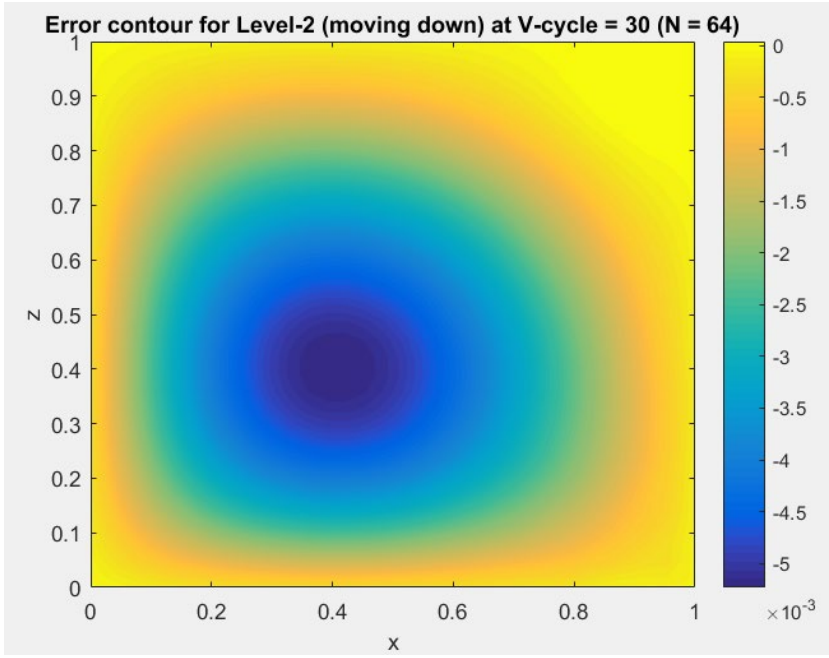
V-cycle = 2





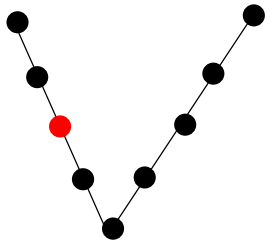
V-cycle = 30

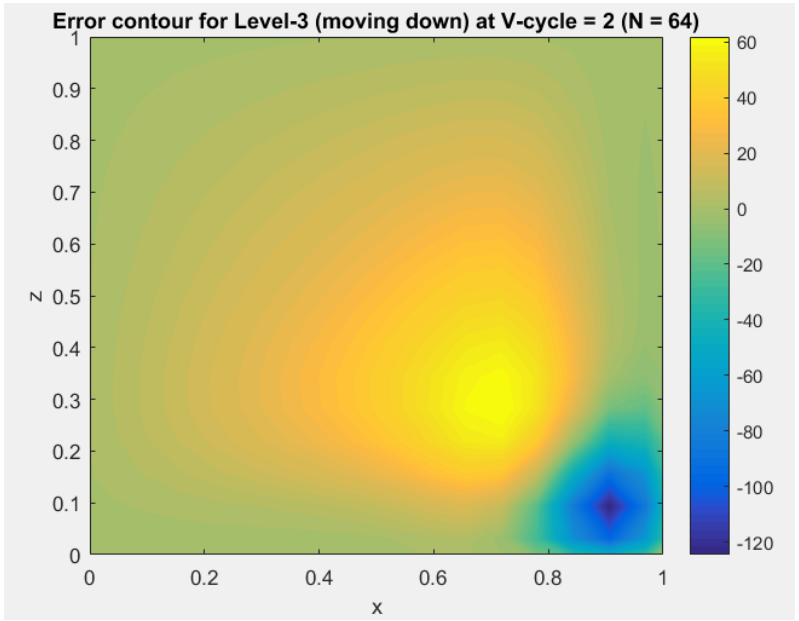


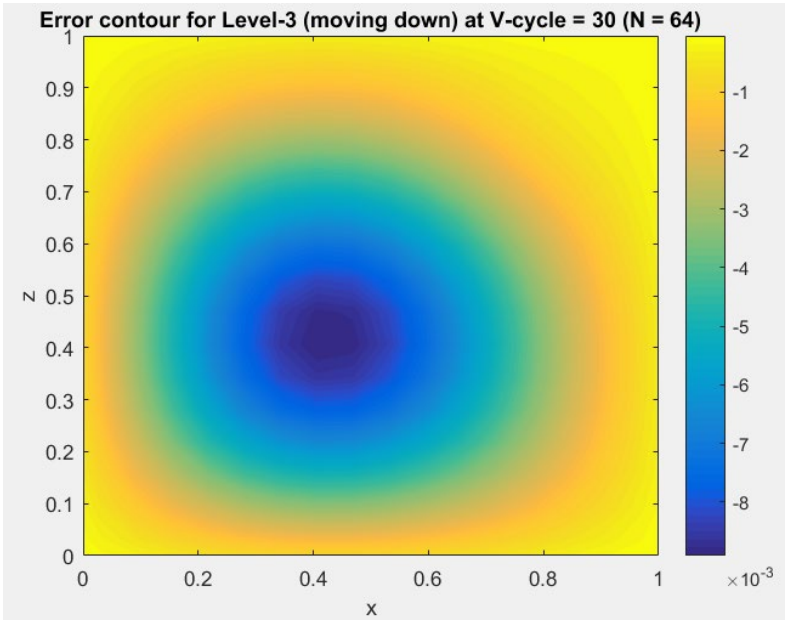


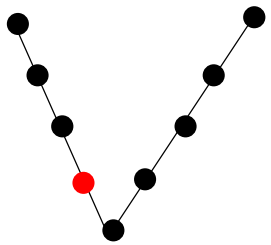
V-cycle = 2

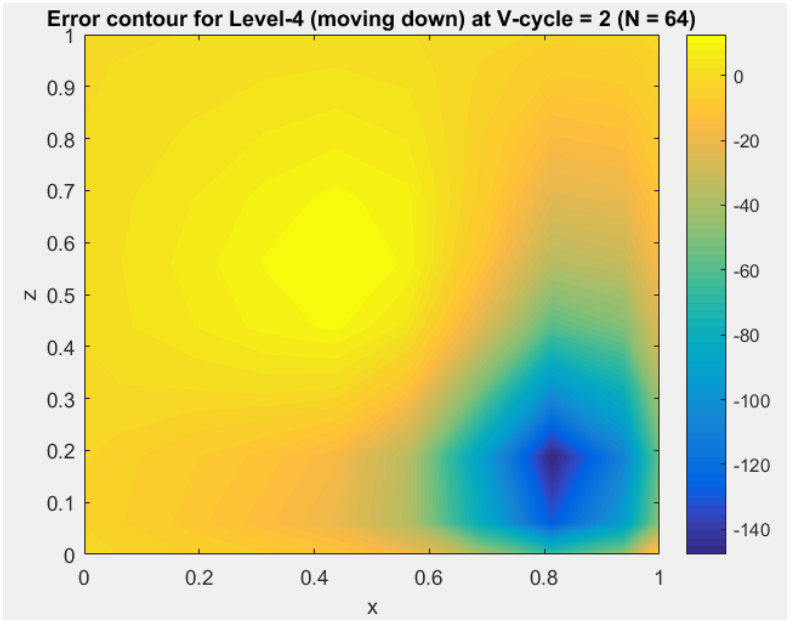
V-cycle = 30

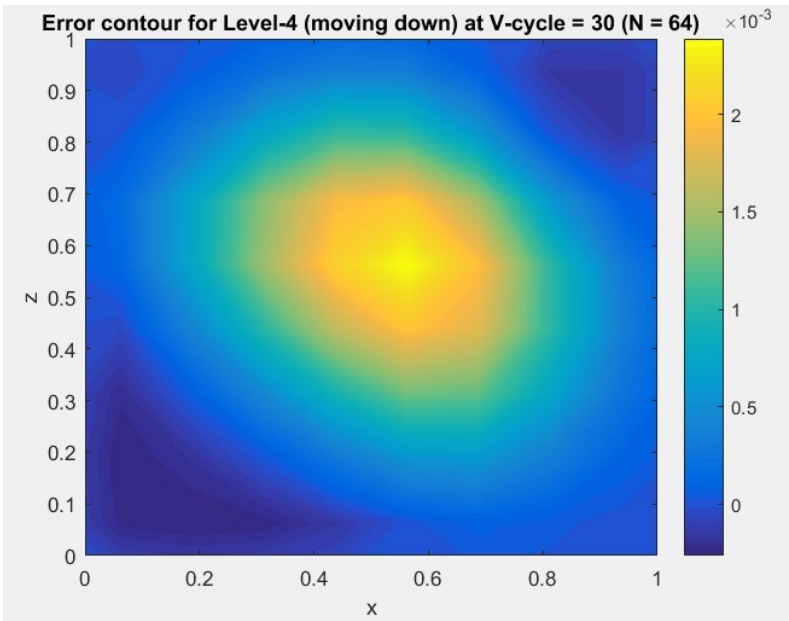






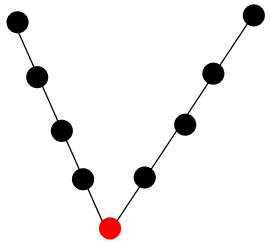


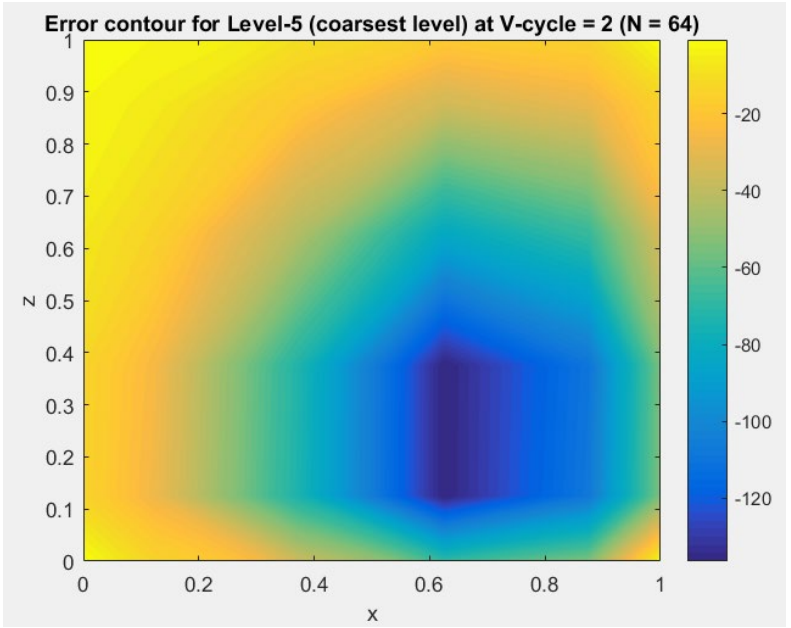


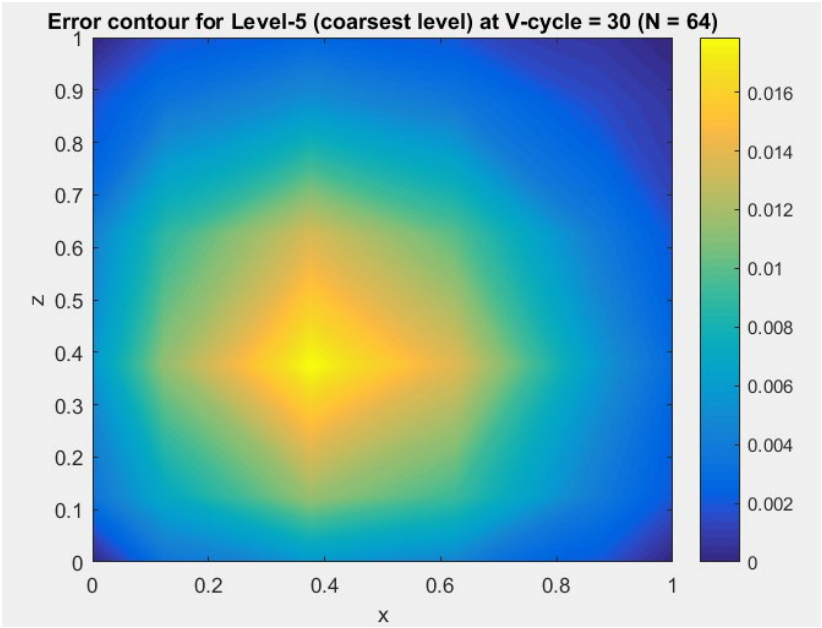


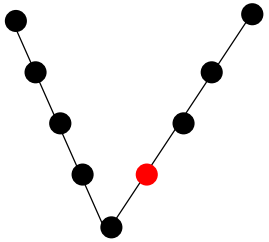
V-cycle = 2

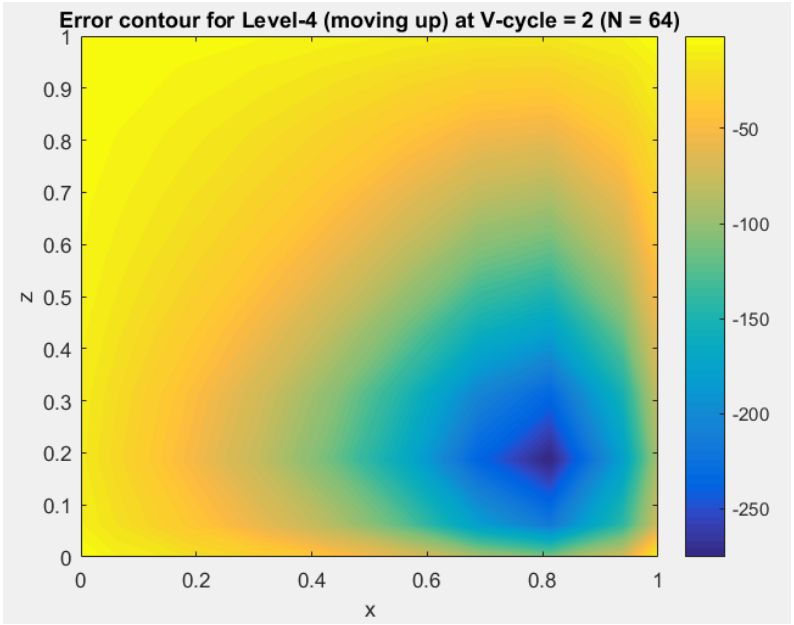
V-cycle = 30

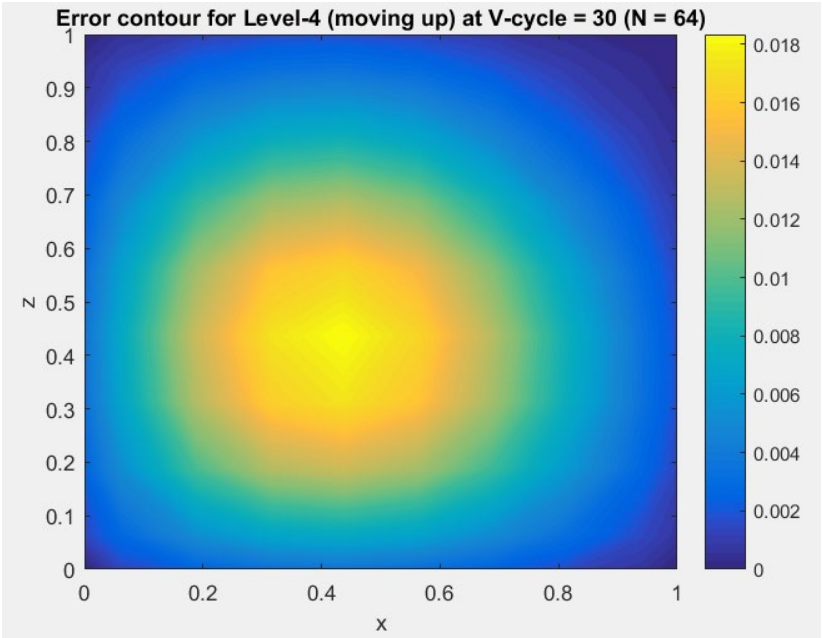












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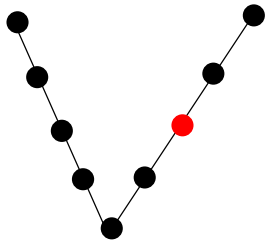
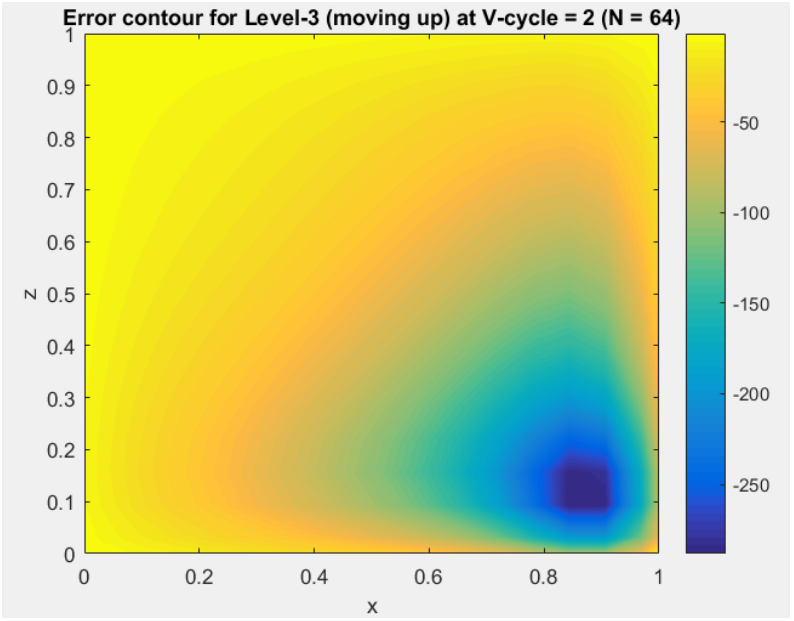
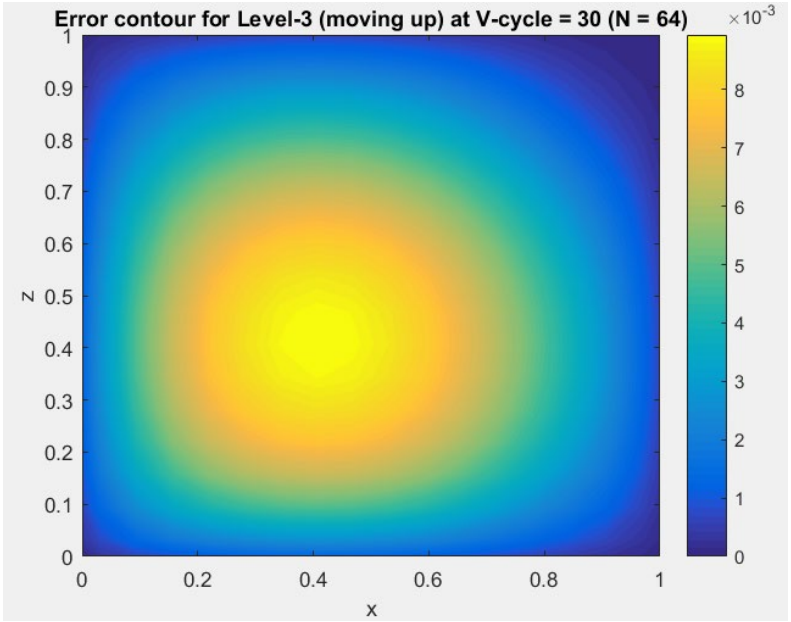
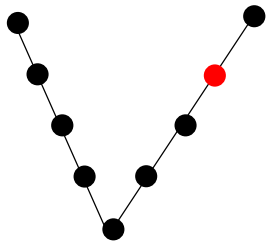
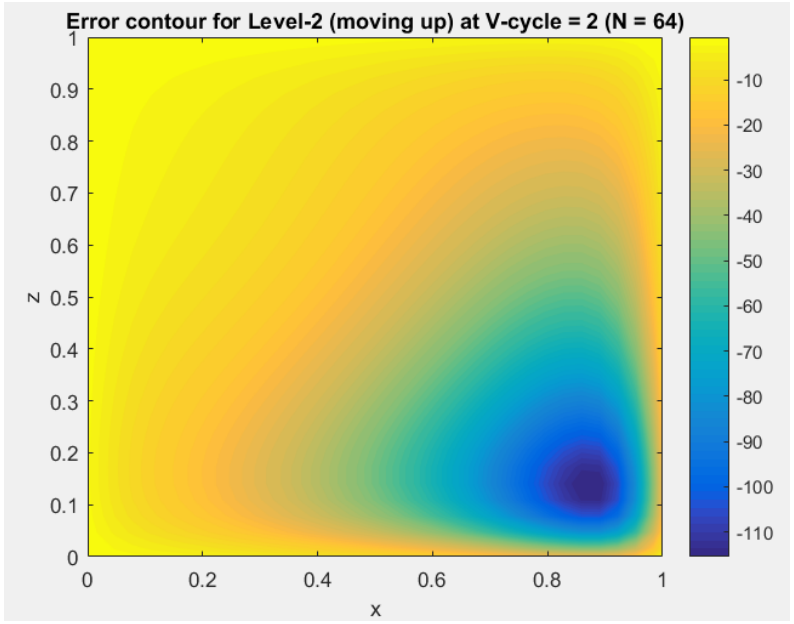
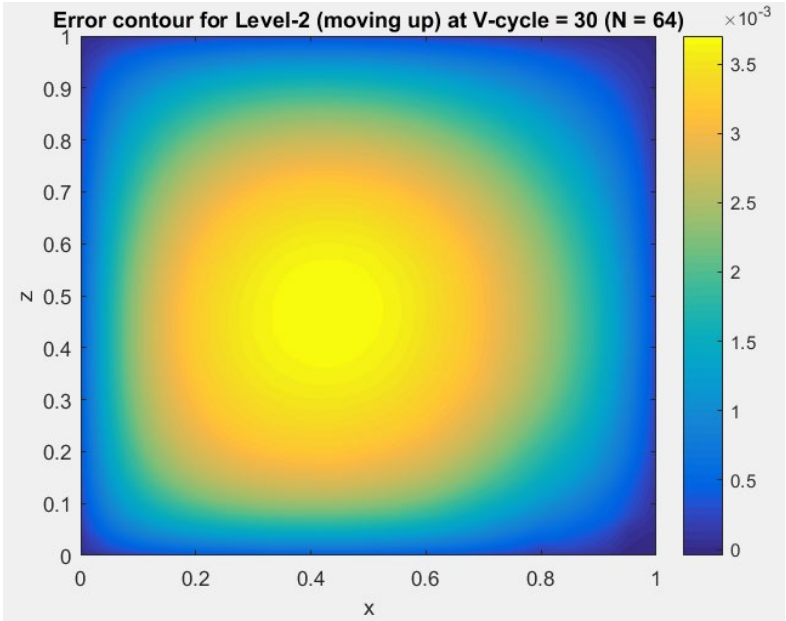


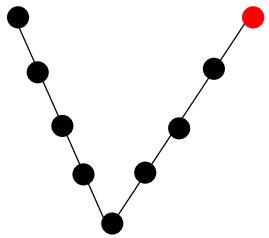




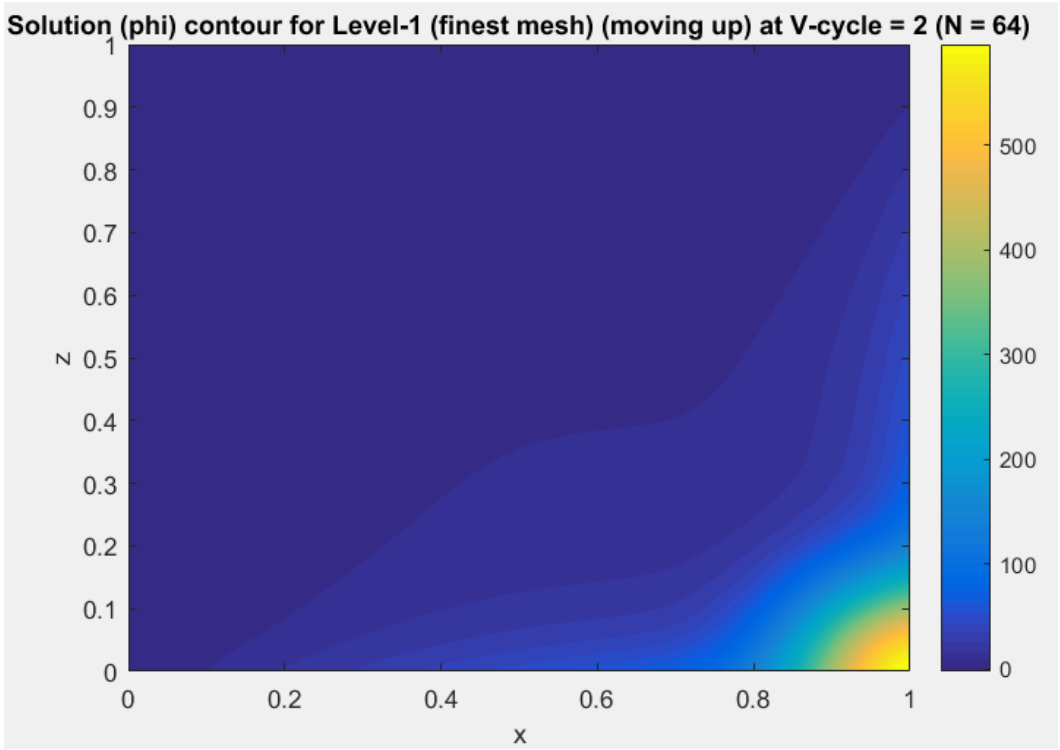






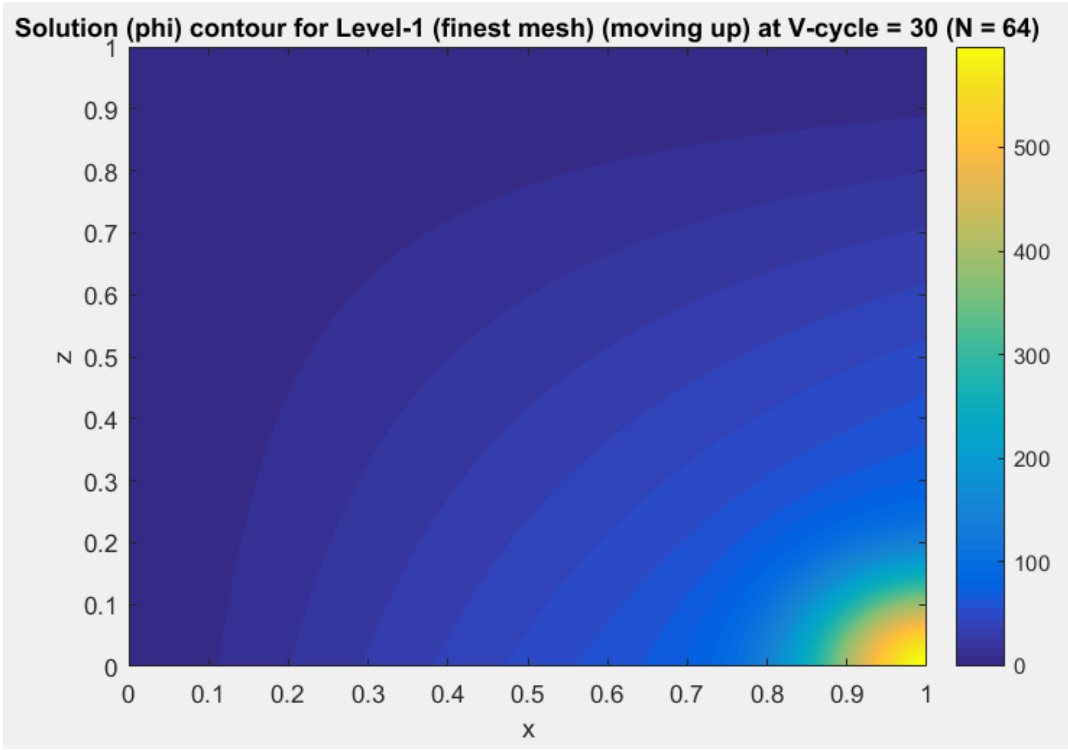


V-cycle = 2



Residual (L2 norm) = 0.124821179

V-cycle = 30



Residual (L2 norm) = 1.44640144E-05