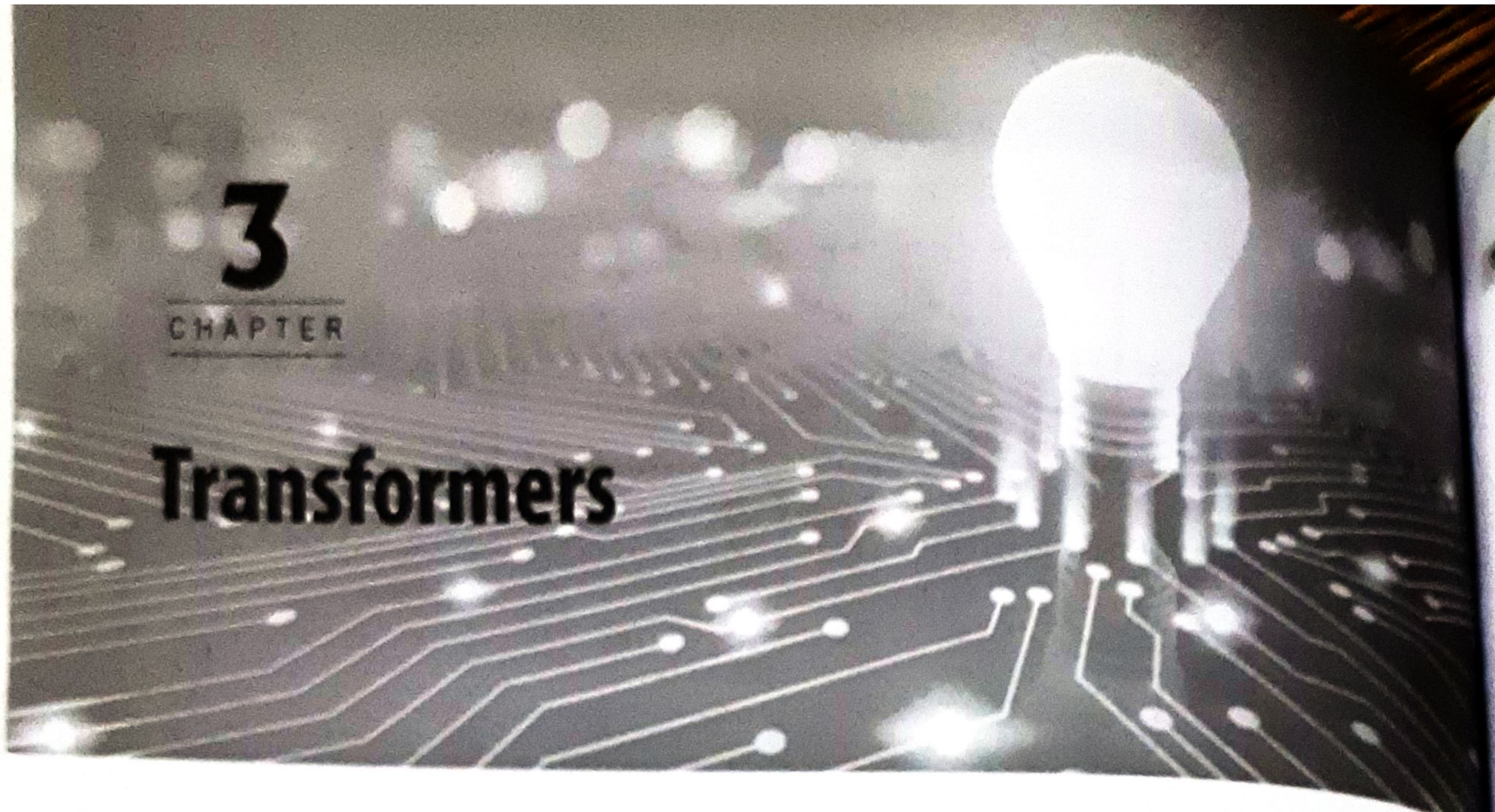


3 CHAPTER Transformers

Chapter Objectives

- To explain the importance of transformers
- To deal with various types of transformers
- To discuss how to develop an equivalent circuit of a transformer
- To discuss how to find the performance of a transformer
- To explain about special transformers

3.1 INTRODUCTION

The electric energy is seldom used at the point where it is generated. It has to be transmitted over long distances before it is used. Generally the generation of ac voltage at a generating station is 11–25 kV. But for economic reasons power will be transmitted at higher voltages such as 132, 220, 400, 765 kV, etc. At the same time, the voltage should be reduced to the safe level for the utilization of power at 400 V/230 V (commercial and residential loads). Hence, a device is needed to increase or decrease the voltage levels. Such a device is called *transformer*.

The transformer is a static device which transfers the electrical energy from one circuit to another circuit without changing the frequency. It does not involve any rotating parts; the efficiency of a transformer will be higher than that of a rotating equipment of the same rating. This is because there are no rotational losses in a transformer. For low capacity below 5 kVA, the efficiency is nearly 90–96% and for high capacity above 15 kVA to several hundred kVA, the efficiency varies from 96% to 99%. Transformers are extensively used in the day-to-day life. The ratings of transformers can be as small as a few volt-amperes or may be as high as several thousand volt-amperes.

3.2 BASIC PRINCIPLE OF TRANSFORMER

The principle of the operation of transformer is based on Faraday's laws of electromagnetic induction discovered by Michael Faraday (1791–1867). Figure 3.1 shows a simple transformer. It consists of two windings and is placed on the single magnetic core made up of the laminated and rewetted sheet of steel plate insulated and staggered together to form a magnetic core. The winding which is connected to ac supply is known as *primary* and the winding which is connected to the load is known as *secondary*. If the primary winding is excited with ac supply it draws the exciting current in primary and produces the required flux which flows through the magnetic core. The primary and secondary windings cut this flux and induce emf in the windings without changing frequency.

For example, if the primary winding is excited with an ac supply of $V_1 = V_m \sin \omega t$, the primary winding will draw the exciting current I_0 and produces the flux $\phi = \phi_m \sin \omega t$ and this flux circulates in the magnetic core as shown in Figure 3.1. The flux ϕ cuts both the windings, so there is self-induced emf e_1 in the primary winding and the mutual-induced emf e_2 in the secondary winding by the principle of magnetic induction.

$$\text{Induced emf, } e = -N\frac{d\phi}{dt} \tag{3.1}$$

$$\text{Induced emf on primary winding, } e_1 = -N_1\frac{d\phi}{dt} \tag{3.2}$$

$$\text{Induced emf on secondary winding, } e_2 = -N_2\frac{d\phi}{dt} \tag{3.3}$$

The magnitude of the induced emf depends on the number of turns of the windings.

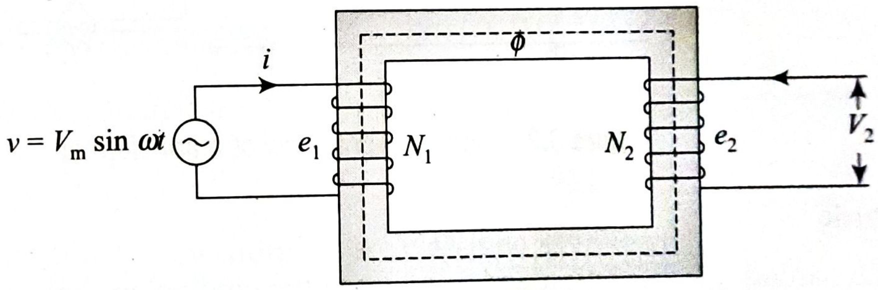


Figure 3.1 Simple transformer

Note: Why the transformers are not operated on dc supply?

The transformers cannot be connected to dc supply because the flux produced in the transformer core will not vary and remain constant in magnitude. Hence, the emf will not be induced in the windings. Thus, the transformer is not capable of raising or lowering the dc voltage. There is also possibility of getting the winding burnt because the winding resistance is quite low and high current starts flowing. That is why dc is never applied to a transformer.

3.3 CONSTRUCTION OF TRANSFORMERS

The constructional details of the transformer are shown in Figure 3.2. The construction has to ensure efficient removal of heat from the two sheets of heat generation core and winding. In all types of transformers, the core is constructed of transformer steel sheet laminations assembled to provide a continuous magnetic path with a minimum of airgap included. The steel used is of high silicon content. Sometimes heat is treated to produce a high permeability and a low hysteresis loss at the usual operating flux densities. The eddy current loss is minimized by laminating the core, the laminations being insulated from each other by a light coat of the core plate of varnish or by an oxide layer on the surface. The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz. It is seen that the joints in the alternate layer are staggered in order to avoid the presence of narrow gaps right through the cross section of the core. Such staggered joints are said to be imbricated.

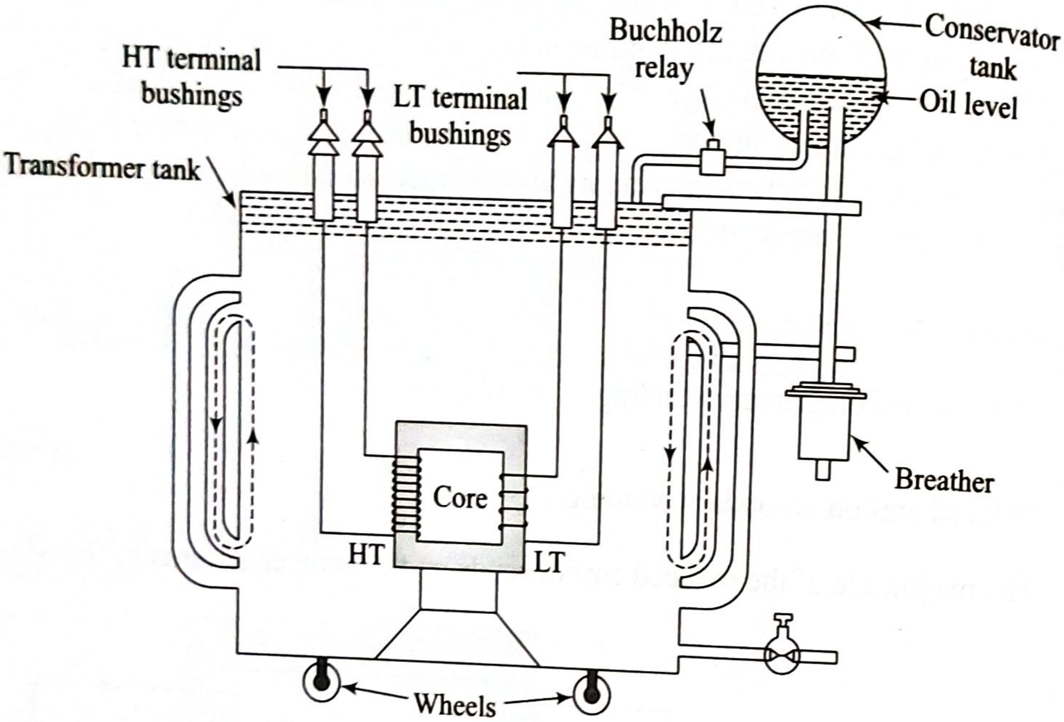


Figure 3.2 Constructional view of a transformer

3.3.1 Tank

To facilitate natural oil circulation and to increase the cooling surface exposed to the ambient, tubes or fans are provided on the outside of the tank walls. In large-size transformers, tubes may be cooled by air. For still large installations the best cooling system appears to be that in which the oil is circulated by a pump from the top of the transformer tank to a cooling plant returning when cold to the bottom of the tank.

In small sizes the transformers are directly placed in a protective housing or are enclosed in hard rubber molding and are air-cooled.

3.3.2 Conservator

The conservator is a smaller sized tank placed on the top of the main tank. This arrangement ensures that the surface area of transformer oil exposed to the atmosphere is limited so as to prevent the first oxidization and then the consequent deterioration of insulating properties of the oil.

3.3.3 Breather

Power transformers are provided with a conservator through which the transformer breathes into the atmosphere. Transformer oil exposed directly to the atmosphere may absorb moisture and dust from the environment and may lose its electrical properties in a very short time. To avoid this from happening, a breather is provided.

3.3.4 Bushings

The purpose of bushings is to provide proper insulation for the output leads to be taken out from the transformer tank. Bushings used are generally of two types.

(i) Porcelain type which are used for voltage ratings of up to 33 kV.

(ii) Condenser type and oil-filled type which are used for voltage ratings higher than 33 kV.

3.3.5 Transformer Oil

The temperature rise is limited to that allowed for the class of insulation employed. Further to prevent insulation deterioration, moisture ingress to it must not be allowed. These two objectives are simultaneously achieved in transformers. However, in very small sizes, these objectives are achieved by immersing the built up transformer in a closed tank filled with noninflammable insulated oil called transformer oil.

3.3.6 Buchholz Relay

The buchholz relay is designed to give alarm signals when there is internal fault in the transformers. The operation of this relay depends on the fact that most internal faults within the transformer generate gases.

3.4 TYPES OF TRANSFORMERS

The transformers are classified according to the type of core, voltage levels, phases, and applications.

3.4.1 Core

According to the core, the transformers are classified as follows:

Core-type transformer

In this case coils are wound around the limbs. Each limb carries one-half of the primary and secondary windings in order to reduce the leakage reactance of the windings. Here, the low-voltage (LV) winding is wound near to the core and the high-voltage (HV) winding is over to the LV winding reference to the core so as to reduce the insulation material.

Small transformers have rectangular or square cross-section with rectangular or circular coils. But it is wasteful in case of large capacity transformers. In case of large-sized transformers, a cruciform core with circular cylindrical coils is employed. From the economic point of view, this cruciform core is larger while in circular coils the winding is easy and provides more mechanical strength. The most important advantages of the cruciform structures are the high space factor and the turns of mean length which is reduced, finally reducing the copper losses. A core-type transformer is shown in Figure 3.3.

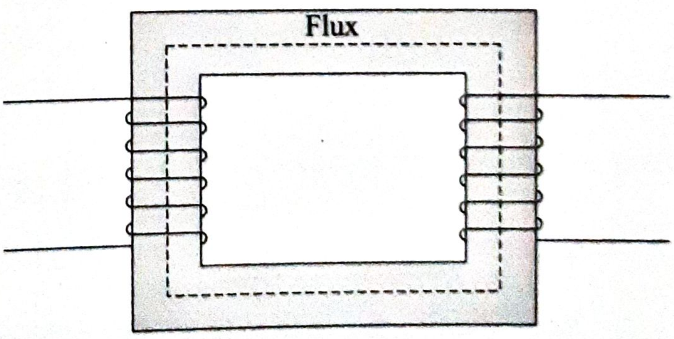


Figure 3.3 Core-type transformer

Shell-type transformer

In this case, a core is wound around the coils. Here, the total flux flows through the central limb and decides equally to the outer limbs. Ultimately, the cross section of the central limb is twice that of the outer limb. Generally, the sandwich-type winding is preferred for this type of transformer. Here, most of the core is supported by mechanical injuries to the windings. The shell-type construction is commonly used for small transformers. To have economic benefits it is advised to go for a square or rectangular core. Generally, for high-voltage transformers, a shell-type construction is preferred. Here, the disadvantage is that natural cooling does not exist. Because the windings are surrounded by the core and if one wants to remove any winding for maintenance, large number of laminations are required to be removed. Figure 3.4 shows the shell-type transformers.

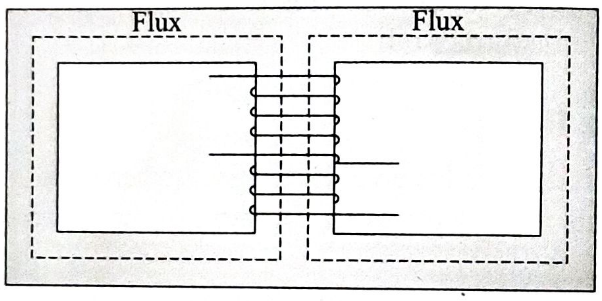


Figure 3.4 Shell-type transformer

Spiral-type transformer

In this case, magnetic circuit is distributed and this magnetic circuit is independent of having more than two circuits. In this core construction looks like spokes of the wheel. Here, the spokes are independent magnetic circuit. In this case, first the LV winding is wound and then the HV winding is wound. This is called as *Berry-type transformer.* Figure 3.5 shows Berry-type transformer.

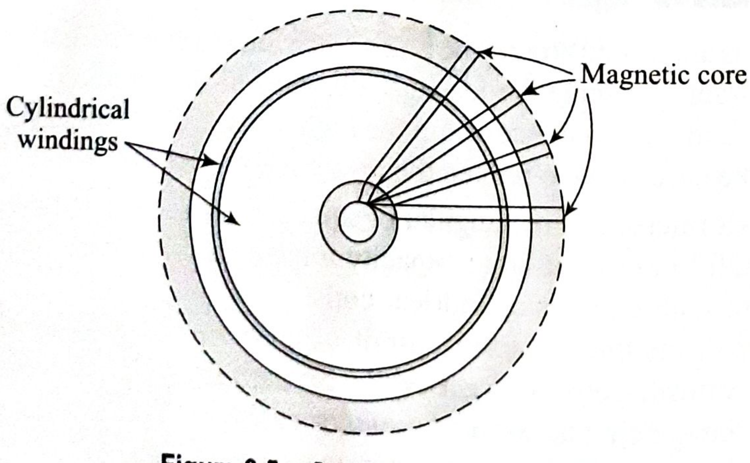


Figure 3.5 Spiral-type transformer

3.4.2 Voltage Level

According to the voltage level, the transformers are classified as follows:

Step-down transformer

A transformer in which the output voltage is less than its input (primary) voltage is called *step-down transformer*. It also depends on the number of turns. If $N_2 < N_1$, then the transformer is called step-down transformer. But in practice, there is small difference between the terminal voltage and the induced voltage. The terminal voltage is the voltage available at the load.

Step-up transformer

A transformer in which the output voltage is greater than its input (primary) voltage is called ***step-up transformer***. In terms of number of turns, if $N_2 > N_1$, then the transformer is called step-up transformer.

The same transformer can be used as step-up transformer or step-down transformer depending on the way it is connected in the circuit. When the transformer is used as a step-up transformer the LV winding is primary, while in the step-down transformer the HV winding is primary.

3.4.3 Phase

According to the phase level, the transformers are classified as follows:

Single-phase transformer

In this transformer, the input supply is single phase with specified frequency. So, it is called single-phase transformer.

Three-phase transformer

In this type of transformer, the input supply is three phase with specified frequency. Here, the phase difference between any two phases is 120° because 360/3 = 120°.

3.4.4 Application

According to the application, the transformers are classified as follows:

Power transformer

A transformer of rating above 200 kVA is used to step up the voltage for generating and transforming the voltage is called power transformer. They are put in operation during load hours and are disconnected during light load hours. Generally, these transformers are designed to have maximum efficiency at or near full load. Power transformers are usually made to use flux density of 1.5–1.7 Wb/m^2 and regulation 6–10%. There may be self-oil-cooled, forced oil-cooled, or forced water-cooled.

Distribution transformer

Transformers of rating less than 200 kVA and used to step down the voltage at distribution level are known as distribution transformers. They are always kept in operation in the entire day, i.e., 24 h whether they carry any load or not. Hence, in this type of transformer iron losses occur at all the time. But copper losses occur only during the load time. They are of the self-cooling type and are almost invariably oil-immersed. The regulation of this type of transformers is 4–8% only. They make use of cold rolled steel with flux density of 1.7 Wb/m^2.

Instrument transformer

Instrument transformers are divided into two types. These are current transformers and potential transformers. Current transformers are used to measure high current ratings of kilo-amperes and mega-amperes. In current transformers, the primary winding has few turns of heavy wire whereas the secondary winding has many turns of very fine wire.

Potential transformers are used to measure high voltages in the order of kilo volts. The primary winding has many turns and is connected across the high-voltage line. The secondary winding has few turns and is connected to a voltmeter.

3.5 EMF EQUATION OF TRANSFORMER

The transformer's primary winding is excited with ac supply V_1. So, the exciting current establishes the flux ϕ in the core. Consequently the primary winding has a flux linkage of λ_1 which induces the emf in primary winding with N_1 turns.

Primary winding flux linkage, $\lambda_1 = N_1\phi$ (3.4)

Instantaneous value of primary winding induced emf, $e_1 = \dfrac{d\lambda_1}{dt}$ (3.5)

$$\therefore \quad e_1 = N_1 \frac{d\phi}{dt} \tag{3.6}$$

Let the flux in the core be written as $\phi = \phi_m \sin \omega t$

where ϕ_m is the maximum value of the flux; ϕ is the instantaneous value of the flux, Wb; ω is the frequency of the supply, $2\pi f$ rad/s; and N_1 is the number of turns in primary winding.

$$\therefore \quad e_1 = N_1 \frac{d(\phi_m \sin \omega t)}{dt} \tag{3.7}$$

$$= \omega N_1 \phi_m \cos \omega t$$

$$= 2\pi f N_1 \phi_m \sin\left(\omega t + \frac{\pi}{2}\right) \tag{3.8}$$

$$= E_m \sin\left(\omega t + \frac{\pi}{2}\right) \tag{3.9}$$

where

E_m = Maximum value of induced emf in primary winding = $2\pi f N_1 \phi_m$ (3.10)

But the rms value of the induced emf = $(1/\sqrt{2}) \times$ maximum value of induced emf. The rms value of the induced emf in primary winding,

$$E_1 = \frac{1}{\sqrt{2}}[2\pi f N_1 \phi_m] \tag{3.11}$$

Therefore, the induced emf in primary winding,

$$E_1 = 4.44\phi N_1 \phi_m \text{ volts} \tag{3.12}$$

Similarly, the secondary winding induced emf,

$$E_2 = 4.44\phi N_2 \phi_m \text{ volts} \tag{3.13}$$

where N_2 is the number of turns in secondary winding.

3.5.1 Turns Ratio (Transformation Ratio)

The ratio of secondary to primary induced emf is the same as the ratio of secondary to primary turns, i.e., $N_2/N_1 = E_2/E_1$ is known as the *transformation ratio* and is denoted by the letter k. Based on the transformation ratio, the transformers are divided as

(i) If $k > 1$ or $N_2 > N_1$, the secondary induced emf or terminal voltage is greater than the primary induced emf or supply voltage. Such a transformer is known as the *step-up transformer.*

(ii) If $k < 1$ or $N_2 < N_1$, the secondary induced emf is less than the primary induced emf. Such a transformer is known as the *step-down transformer*.

(iii) If $k = 1$ or $N_1 = N_2$, the primary induced emf is equal to secondary induced emf. Such a transformer is known as the one-to-one transformer and is used generally where the isolation is required between the two circuits.

3.6 IDEAL TRANSFORMERS

3.6.1 Ideal Transformer on No Load

The ideal transformer has the following characteristics:

(i) It has no losses.

(ii) Its primary and secondary winding resistances are negligible.

(iii) Its leakage flux and leakage inductances are zero.

(iv) Its permeability of core is so high that the negligible current is required to establish the flux in it.

Consider an ideal transformer, as shown in Figure 3.6, when V_1 is applied to the transformer at no load, i.e., $I_2 = 0$. Here, I_1 chooses a closed path of primary winding. The magnetic circuit is magnetized due to I_1. Therefore, I_1 is called *magnetizing current* (I_m). The transformer is ideal, when the winding resistance is negligible and is purely inductive in nature. In a single inductive ac circuit, the current lags voltage by 90°. Therefore, I_m lags V_1 by 90° but I_m and ϕ are in phase because I_m is due to ϕ only.

The flux links both the windings producing induced emfs E_1 and E_2 in both primary and secondary windings, respectively. But according to Lenz's law the induced emf opposes the cause of producing it which is V_1; therefore, E_1 and V_1 are in antiphase with each other but equal in magnitude. The induced emf E_2 also opposes V_1. Therefore, E_2 is also in antiphase with V_1. Hence, E_1 and E_2 are in phase for ideal transformer on no load as shown in Figure 3.7.

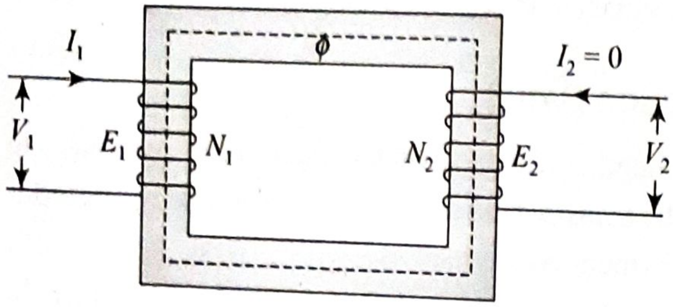


Figure 3.6 Ideal transformer

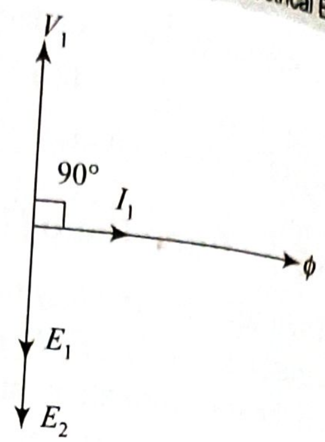


Figure 3.7 Phasor diagram of ideal transformer

From the phasor diagram as shown in Figure 3.7, it is observed that ϕ is taken as reference because it is common for both windings and I_m is due to ϕ only. Therefore, ϕ and I_m are in phase. Here, V_1 leads ϕ or I_m by 90°; in other words, I_m or ϕ lags V_1 by 90° because it is purely an inductive circuit.

E_1 and E_2 are in phase and both oppose supply voltage V_1. The power input to the transformer is $V_1 I_m \cos \phi$. Since $\phi = 90°$, the power input to transformer is zero. This is because on no-load output power is zero and for ideal transformer there are no losses. Hence, the input power is also zero.

3.6.2 Ideal Transformer on Load

If the secondary is loaded, the current I_2 flows through the load. The secondary mmf $N_2 I_2$ tends to oppose the core flux according to Lenz's law, but the excitation of the primary does not allow any change in the core flux. The two mmfs are equal (see Figure 3.8).

$$N_1 I_1 = N_2 I_2$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} = k \tag{3.14}$$

From the above equation, we can say that the current transforms in an ideal transformer is the inverse ratio of its turns. The phasor diagram for an ideal transformer under the load condition is shown in Figure 3.9 for power factor $\cos \theta$.

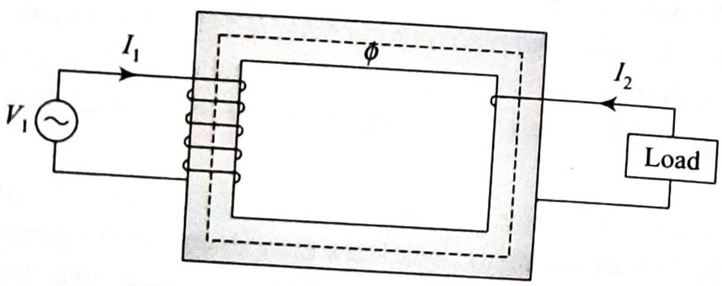


Figure 3.8 Ideal transformer on load

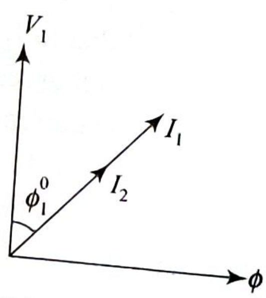


Figure 3.9 Phasor diagram on load

3.6.3 Equivalent Circuit of Ideal Transformer on No Load

When the secondary is open circuited the current flowing through the secondary is zero. There will be only very small amount of no-load primary current. The losses are now only iron losses.

Hence, the transformer can be represented on no load as shown in Figure 3.10.

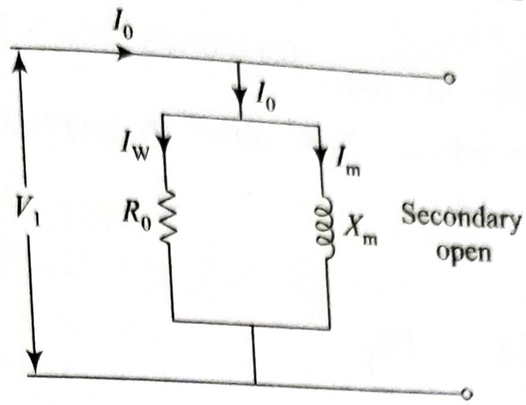


Figure 3.10 Equivalent circuit of a transformer on no load

$$R_0 = \frac{V_1}{I_w}$$

$$X_m = \frac{V_1}{I_m}$$

The primary no-load current and the transformer core loss are functions of the source voltage and hence the shunt branch consisting of R_0 and X_0 is connected across the supply source.

Note: Why rating of the transformer is always expressed in kVA?

Generally, the available output power depends on secondary power factor. As the secondary power factor can change depending on the load, the rating of the transformer is not specified in watts or kilowatts but indicated in as the product of voltage and current called kVA rating.

EXAMPLE 3.1 **The turns ratio of a transformer is 100/300, the primary winding is connected to a source of 3.3 kV, 50 Hz supply. The load impedance of (110 + j45) Ω is connected across the secondary terminals. Calculate the following:**

(i) Value of maximum flux in the core

(ii) Primary and secondary currents

(iii) The real and reactive power supplied by the source to the primary transformer

(iv) The value of impedance which is directly connected across the source would draw the same real and reactive power in (iii)

Solution Turns ratio = 100/300, i.e., $N_1 = 100$ and $N_2 = 300$

Primary voltage, $V_1 = 3.3$ kV

Supply frequency, $f = 50$ Hz

Load impedance, $(Z_2) = (110 + j45)\ \Omega$

(i) The transformer induced emf equation is

$$E_1 = \sqrt{2}\,\pi f \phi_m N_1$$

$$\phi_m = \frac{V_1}{\sqrt{2}\pi\ f\ N_1}$$

Therefore, the value of maximum flux in the core,

$$\phi_m = \frac{3.3 \times 10^3}{\sqrt{2}\pi \times 50 \times 100} = 0.1486 \text{ Wb}$$

(ii) Transformation ratio, $K = N_2/N_1$ (or) E_2/E_1 (or) I_1/I_2

$$K = \frac{N_2}{N_1} = \frac{300}{100} = 3$$

$$K = \frac{E_2}{E_1} \Rightarrow E_2 = KE_1$$

Secondary voltage, $E_2 = 3 \times 3.3 \times 10^3 = 9.9$ kV

$$I_2 = \text{Secondary current} = \frac{E_2}{Z_L} = \frac{9.9 \times 10^3}{(110 + j45)} = 83.3 \angle{-22.25} \text{ A}$$

$$K = \frac{I_1}{I_2} \Rightarrow I_1 = KI_2$$

Primary current, $I_1 = 3 \times 83.3 \angle{-22.25}$

$= 249.89 \angle{-22.25}$ A

(iii) The real and reactive power supplied by the source to the transformer is

$$S = V_1 I_1^* = V_2 I_2^*$$

$$= 3.3 \times 10^3 \times (249.89 \angle 22.25)$$

$$= 763.24 + j312.25$$

Real power, $P_1 = 763.24$ W

Reactive power, $Q_1 = 312.25$ VAr

(iv) The value of impedance which is connected directly across the source would draw the same real and reactive power, i.e., Z_L is transferred to primary.

$$Z_L' = Z_{L01} = \frac{Z_L}{(K)^2} = \frac{(110 + j45)}{(3)^2} = (12.22 + j5)\ \Omega$$

EXAMPLE 3.2 **A single-phase 50-Hz transformer has 100 turns on primary winding and the secondary winding turns are 500. The cross-sectional area of the core is 220 cm². A 240-V, 50-Hz voltage source is connected to the primary winding of the transformer. Calculate**

(i) The emf induced in the secondary winding.

(ii) The maximum value flux density in the core (B_m).

Solution Given data

Primary voltage, $V_1 = 240$ V, frequency of supply, $f = 50$ Hz

Primary turns, $N_1 = 100$, secondary turns, $N_2 = 500$

Area of the core, $A = 220$ cm^2

(i) The transformation ratio of the transformer is

$$K = \frac{N_2}{N_1} \text{ or } \frac{E_2}{E_1} \text{ or } \frac{I_1}{I_2}$$

$$K = \frac{500}{100} = 5$$

$$E_2 = KE_1 = 5 \times 240 = 1200 \text{ V}$$

(ii) The transformer-induced emf, $E_1 = \sqrt{2}\pi f \phi_m N_1$ volts

But maximum flux, $\phi_m = B_m \times A$,

where B_m = Maximum flux density

A = Area of the core

$$B_m = \frac{E_1}{\sqrt{2}\pi \times f \times A \times N_1} = \frac{240}{\sqrt{2}\pi \times 50 \times 220 \times 10^{-4} \times 100} = 0.4911 \text{ Wb/m}^2$$

EXAMPLE 3.3 **A 50-Hz single-phase transformer has 6600 V/400 V. Having emf per turn is 10 V and the maximum flux density in the core is 1.6 T. Find the**

(i) Suitable number of primary and secondary turns

(ii) Cross-sectional area of the core

Solution Given data

Primary voltage, V_1 = 6600 V, secondary voltage, V_2 = 400 V

The emf/turn in the transformer = 10 V

Maximum flux density in the core = 1.6 T

(i) Number of primary turns, $N_1 = \dfrac{\text{Primary voltage}}{\text{emf/turn}} = \dfrac{6600}{10} = 660$

Number of secondary turns, $N_2 = K \times N_1 = 40$

(ii) The emf equation of transformer is

$$E_1 = \sqrt{2}\pi f B_m A N_1 \text{ volts}$$

Area of the core, $A = \dfrac{6600}{\sqrt{2} \times \pi \times 50 \times 1.6 \times 660} = 281.3 \text{ cm}^2$

EXAMPLE 3.4 **A single-phase core-type 50-Hz transformer is in the square shape having 25 cm side, the maximum flux density in the core 1.2 Wb/m². Calculate the number of turns per limb on the high-voltage (HV) side and low-voltage (LV) side for a 3400 V/240 V ratio.**

Solution Given data

Maximum flux density in the core, B_m = 1.2 Wb/m²

Voltage ratio = 3400 V/240 V

Primary voltage, V_1 = 3400 V, secondary voltage, V_2 = 240 V

Supply frequency, f = 50 Hz

In this problem, HV is primary and LV is secondary.

Secondary voltage, $V_2 = \sqrt{2}\pi f B_m A N_1$ volts

$$240 = \sqrt{2}\pi \times 50 \times 1.2 \times 25 \times 10^{-2} \times 25 \times 10^{-2} N_2$$

$$= 16.66\, N_2$$

Secondary or LV turns, $N_2 = \dfrac{240}{16.66} = 14.41 \approx 15$

But in transformer the number of turns takes even numbers so the number of secondary turns or LV winding turns, $N_2 \approx 16$.

The transformation ratio, $K = V_2/V_1 = 240/3400 = 0.0706$. But $K = N_2/N_1$. Primary or HV turns, $N_1 = N_2/K = 16/0.0706 = 226.62 \approx 228$ turns.

Note: In a transformer, to reduce the leakage reactance, half of the turns should be on primary and half of the turns on secondary.

So the secondary turns on each limb = $N_2/2$ = 16/2 = 8 and the primary turns on each limb = $N_1/2$ = 228/2 = 114.

3.7 ACTUAL TRANSFORMER OR REAL TRANSFORMER

As in the ideal case of the transformer, the losses in core and winding are negligible. But when an actual transformer is loaded, there is always core loss or iron loss in the core and copper losses in the windings, which cannot be neglected. The operation of the transformer can be considered in two cases

(i) Under no load

(ii) With load

3.7.1 Transformer on No Load

Under no-load condition, the transformer draws the no-load current, I_0. It meets the no-load losses such as core losses. Figure 3.11 shows the transformer under no-load condition and assuming there is no leakage flux.

When the transformer is excited, it draws the no-load current of I_0 in the primary winding. This I_0 will set up the required flux in the core that will be used to induce the emf in secondary by the mutual induction principle. The induced emfs in primary and secondary are E_1 and E_2 and they are dependent on the primary and secondary turns N_1 and N_2, respectively.

$$E_1 = -N_1 \frac{d\phi}{dt}; \quad E_2 = -N_2 \frac{d\phi}{dt} \text{ volts}$$

The drawn current I_0 also meets the no-load losses in the transformer, i.e., iron losses in the core (hysteresis plus eddy current loss) and a small amount of copper loss in the primary winding, but the losses in the secondary are zero because of no load. The no-load current I_0 lags V_1 by an angle ϕ_0; it is always less than 90°.

The corresponding loss is given by $W_0 = V_1 I_0 \cos \phi_0$, where $\cos \phi_0$ = no-load power factor.

From the phasor diagram shown in Figure 3.12, the no-load current has two components, i.e., magnetizing component, $I_m = I_0 \sin \phi_0$, and working component, $I_w = \cos \phi_0$, and I_w is always very small as compared to I_m.

The no-load current is given by

$$I_0 = \sqrt{I_w^2 + I_m^2} \tag{3.15}$$

where I_m is the magnetizing component of the no-load current and I_w is the working component of the no-load current.

I_0 is very small as compared to the full-load current. Usually, it is about 5–10% of the full-load current. Therefore, the copper losses of the primary winding under no-load conditions are negligible. So the total power input under no-load conditions is equal to the iron loss in the transformer. These losses are constant throughout the operation of transformer at any load and power factor.

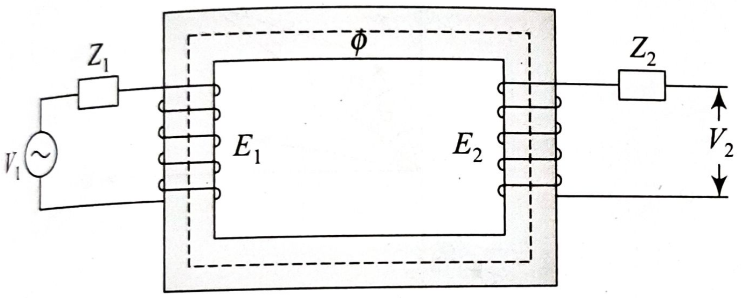


Figure 3.11 Transformer on no load

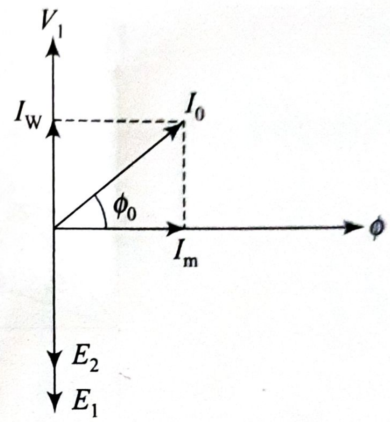


Figure 3.12 Phasor diagram

3.7.2 Transformer on Load

If the secondary of the transformer is connected to the load as shown in Figure 3.13, it supplies a load current or secondary current of I_2 amperes, at a terminal voltage of V_2 volts. The magnitude of the load current depends on the nature of the load connected. As per Faraday's law, the secondary current I_2 sets up its own flux ϕ_2 by its own mmf N_2I_2. This flux ϕ_2 is opposite to the main flux ϕ which is set up by the primary mmf N_1I_0. Hence, the net flux in the core is going to reduce momentarily which in turn reduces the primary induced emf E_1. Therefore, the primary winding draws the extra current of I_2', i.e., unequal in magnitude of I_2. This extra current I_2' produces the extra flux of ϕ_2' in the primary winding by the mmf of $I_2' N_1$ which is opposite to the secondary flux of ϕ_2. So, the magnetic effect produced by the secondary load current is neutralized by the additional primary current I_2', and the net flux available in the core always remains constant. So, the flux under no load and on load is always same.

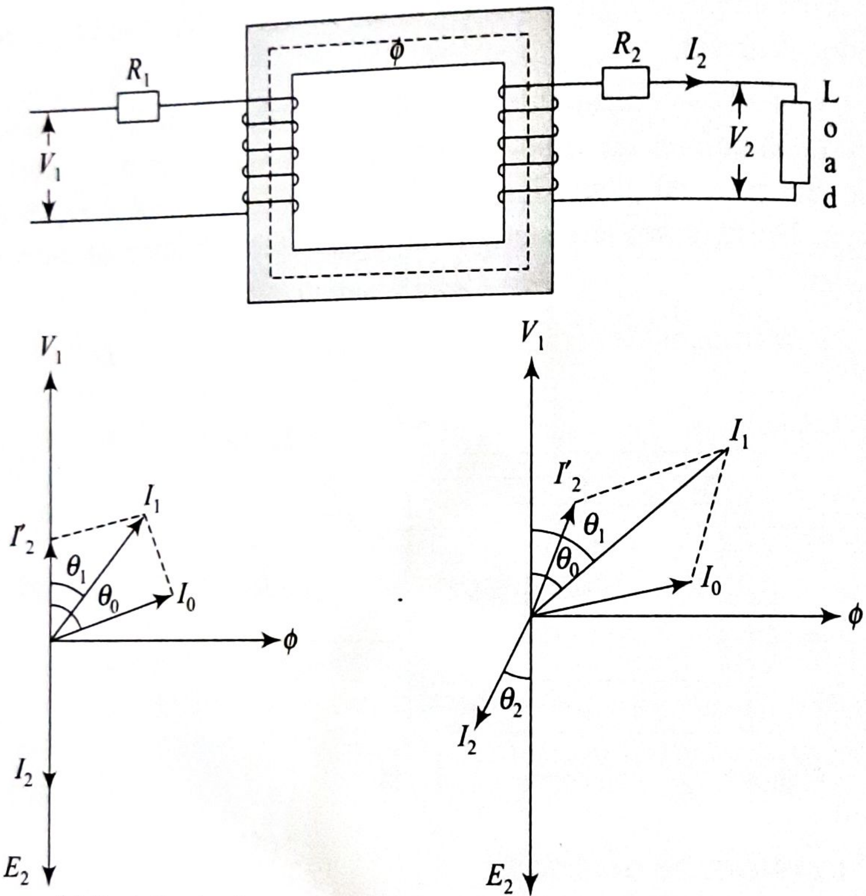
$$\phi_1 = \phi_2 \tag{3.16}$$

$$I_2' N_1 = N_2 I_2$$

$$I_2' = \left(\frac{N_2}{N_1}\right) I_2 \tag{3.17}$$

$$= KI_2$$

The transformer under no-load draws only the no-load current in the primary. When it is loaded, it draws both the no-load current (I_0) and load component in the primary, I_2'. Therefore, the net primary current is $\overline{I_1} = \overline{I_0} + \overline{I_2'}$ (phasor sum). The phasor diagram when the transformer is on load for resistive load and inductive load is shown in Figures 3.13(a) and (b).



(a) Resistive load (b) Inductive load

Figure 3.13 Transformer on load

EXAMPLE 3.5 **A transformer is having 160 turns on primary and 80 turns on secondary. The primary winding is excited by the 230-V, 50-Hz supply and the secondary side having a load impedance of 6 ∠30° is connected as shown in Figure 3.14. Neglect the losses of the transformer. Calculate (i) the primary and secondary currents, (ii) their power factor, and also (iii) the primary and secondary real powers.**

Solution Given data

Primary turns, $N_1 = 160$, secondary turns, $N_2 = 80$. Load impedance, $Z_L = 6\ \angle 30°$, primary voltage, $V_1 = 230$ V.

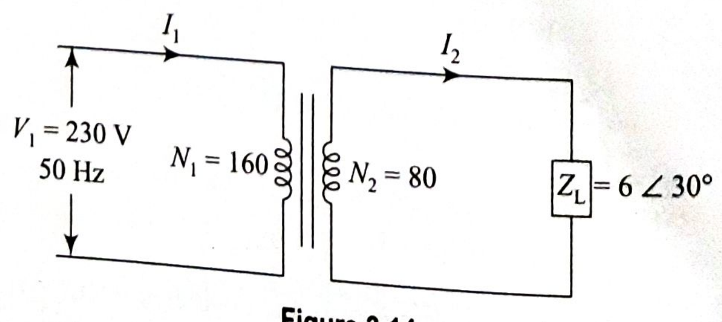


Figure 3.14

Assume the transformer to be ideal because of neglecting loss, take V_1 as the reference

Primary voltage, $\overline{V_1} = 230\ \angle 0°$ V

Secondary voltage, $\overline{V_2} = K\overline{V_1}$

Transformation ratio, $k = \dfrac{V_2}{V_1} = \dfrac{N_2}{N_1} = \dfrac{80}{160} = 0.5$

$$\therefore \qquad \overline{V_2} = 0.5 \times 230 = 115\ \angle 0°$$

Secondary current, $I_2 = \dfrac{V_2}{Z_L} = \dfrac{115\ \angle 0°}{6\ \angle 30°} = 19.2\ \angle{-30°}$ A

Primary current, $I_1 = k\, I_2 = 0.5 \times 19.2\ \angle{-30} = 9.6\ \angle{-30°}$ A

The phase angle is 30°. So the pf = cos 30° = 0.866 lagging pf (same as a secondary-side transformer being ideal).

The real power on primary side $= V_1 I_1 \cos\phi$

$$= 230 \times 9.6 \times 0.866 = 1.912 \text{ kW}$$

The real power on secondary side $= V_2 I_2 \cos\phi$

$$= 115 \times 19.2 \times 0.866 = 1.912 \text{ kW}$$

That it has no losses, i.e., $P_1 = P_2$

EXAMPLE 3.6 **A single-phase transformer having four windings, primary is 230 V. The primary is an 800-V winding with a 4-V center tapping and a 6-V winding for a net core area of 25 cm^2. Find the number of turns on each finding, the maximum value of flux not exceed 1.4 T.**

Solution Given data

Primary voltage, $V_1 = 230$ V; Area of the core, $A = 25$ cm^2

Maximum value of flux density, $B_m = 1.4$ T

Three secondary winding areas, 800 V, 4 V, 6 V with center tappings

The center tapping winding must have even number of turns and also first find the number of turns for low-voltage winding and from this find the induced voltage/turn, take the low-voltage winding as a 4 V winding.

$$V = 4 \text{ volts}$$

According to transformer emf equation

$$E_1 = \sqrt{2}\pi f \phi_m; \quad N_1 = \sqrt{2}\pi f B_m A N_1 \text{ V}$$

$$N_1 = \frac{E_1}{\sqrt{2}\pi B_m A f} = \frac{4}{\sqrt{2} \times \pi \times 50 \times 1.4 \times 25 \times 10^{-4}} = 5.15$$

Take the turns are even numbers, so 5.15 ≈ 6 turns and from these the emf induced/turn = voltage/number of turns = 4/6 = 0.67 V/turn. The induced emf/turn in a transformer is constant.

Number of turns on 230 V winding primary = voltage/emf/turn

$$= \frac{230}{0.67} = 343.28 \approx 344 \text{ turns}$$

Number of turns on the 4-V winding = voltage/emf/turn = $\frac{4}{0.67} = 6$ turns

Number of turns on the 6-V winding = $\frac{6}{0.67} = 8.95 \approx 10$ turns

Number of turns on the 800-V winding = $\frac{800}{0.67} = 1194$ turns

3.7.3 Resistance and Leakage Reactance of the Transformer

In the preceding section, an ideal transformer is discussed. According to our assumptions, it does not contain any resistance and leakage flux. But in practice, it is not possible to get such an ideal transformer.

In real transformer, both primary and secondary windings consist of finite resistances R_1 and R_2. Due to these resistances copper losses and voltage drop occur in the windings. The secondary terminal voltage V_2 is less than the secondary induced emf E_2. Mathematically, it can be written as $V_2 = E_2 - I_2 R_2$.

Similarly, in primary winding the induced emf, $E_1 = V_1 - I_1 R_1$. In both the cases, the effect of magnetic leakage is negligible.

The only flux that exists in an ideal transformer is ϕ_m. Previously it was assumed that all the flux links both the primary and secondary windings. But in actual practice, it is impossible to get such condition. A small amount of flux flows through the airgap and this is usually called the *leakage flux*. The primary leakage flux ϕ_1 linking with primary winding is produced by the primary ampere-turns only; therefore, it is proportional to primary current. On no-load since the primary current is negligible, the leakage flux produced by it is also negligible. But on load conditions, the primary current is high hence the leakage flux also increases. The primary leakage flux ϕ_1 produces self-induced emf E_1.

Its magnitude is given by $E_1 = 2\pi f L_1 I_1$, where L_1 is the self-inductance of the primary winding. Thus, the reactance of the primary winding,

$$X_1 = \frac{E_1}{I_1} = \frac{2\pi f L_1 I_1}{I_1} = 2\pi f L_1 \tag{3.18}$$

Similarly, the secondary leakage flux ϕ_2 is established by secondary ampere-turns and is proportional to the secondary current I_2. No leakage flux exists on no-load since no current flows. But on load, leakage flux ϕ_2 produces induced emf E_2 in the secondary winding. Mathematically, it can be written as $E_2 = 2\pi f L_2 I_2$, where L_2 is the self-inductance of the secondary winding. The secondary reactance is obtained by

$$X_2 = \frac{E_2}{I_2} = \frac{2\pi f L_2 I_2}{I_2} = 2\pi f L_2 \tag{3.19}$$

The effect of magnetic leakage is thus to produce emfs of self-inductances and, therefore, equivalent in effect to the addition of an inductive coil in series with each winding, the reactance is called the leakage reactance. A transformer with magnetic leakage and winding resistance is equivalent to an ideal transformer having inductive and resistive coils connected in series with each winding as shown in Figure 3.15.

3.8 EQUIVALENT RESISTANCE AND REACTANCE OF TRANSFORMER

In an actual transformer the windings have resistance and leakage reactances. Let the values be R_1, R_2, X_1, and X_2, respectively, where subscript 1 represents the primary winding and 2 represents the secondary winding. It is advantageous to transfer the resistances and reactances of the winding to any one winding. The main advantage is that the calculations become simpler. The copper loss in the secondary winding is $I_2^2 R_2$. This loss is supplied by the primary current I_1. Hence, if R_2' is the equivalent resistance in primary which would cause the same copper loss as R_2 in secondary, then

$$I_1^2 R_2' = I_2^2 R_2$$

$$\therefore \qquad R_2' = \left(\frac{I_2}{I_1}\right)^2; \quad R_2 = \frac{R_2}{K^2}$$

Similarly, the equivalent resistance of the primary winding referred to the secondary side is $R_1' = R_1 K^2$.

With these transformations we can calculate the total resistance of the transformer winding either referred to primary or secondary and is given by

Equivalent resistance referred to primary, $R_{01} = R_1 + R_2' = R_1 + \dfrac{R_2}{k^2}$

Equivalent resistance referred to secondary, $R_{02} = R_2 + R_1' = R_2 + k^2 R_1$

Similarly, leakage reactances can also be transferred from one side to another side and are given by

Equivalent reactance referred to primary, $X_{01} = X_1 + X_2' = X_1 + \dfrac{X_2}{k^2}$

Equivalent reactance referred to secondary, $X_{02} = X_2 + X_1' = X_2 + k^2 X_1$

The total impedance referred to either primary or secondary side is given by

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \quad \text{and} \quad Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

3.8.1 Equivalent Circuit

The analysis of the transformer can be better understood if the transformer equivalent circuit is known. The no-load primary current has two components I_m and I_w · I_m produces the flux and is assumed to flow through reactance X_0 called no-load reactance, while I_w is the working component representing core losses, hence is assumed to flow through the resistance R_0. This circuit consisting of R_0 and X_0 in parallel is called exciting circuit.

When the load is connected to the transformer then the secondary current I_2 flows. This causes voltage drop across R_2 and X_2. Due to I_2, the primary draws an additional current which is given by $I_2' = I_2/K$. Hence, primary current is the phasor addition of I_0 and I_2'. This causes the voltage drop across the primary resistance R_1 and reactance X_1. Hence, the equivalent circuit can be shown as in Figure 3.15(a).

The equivalent circuit is further simplified by transferring all the values to the primary or secondary. This makes calculations much simpler.

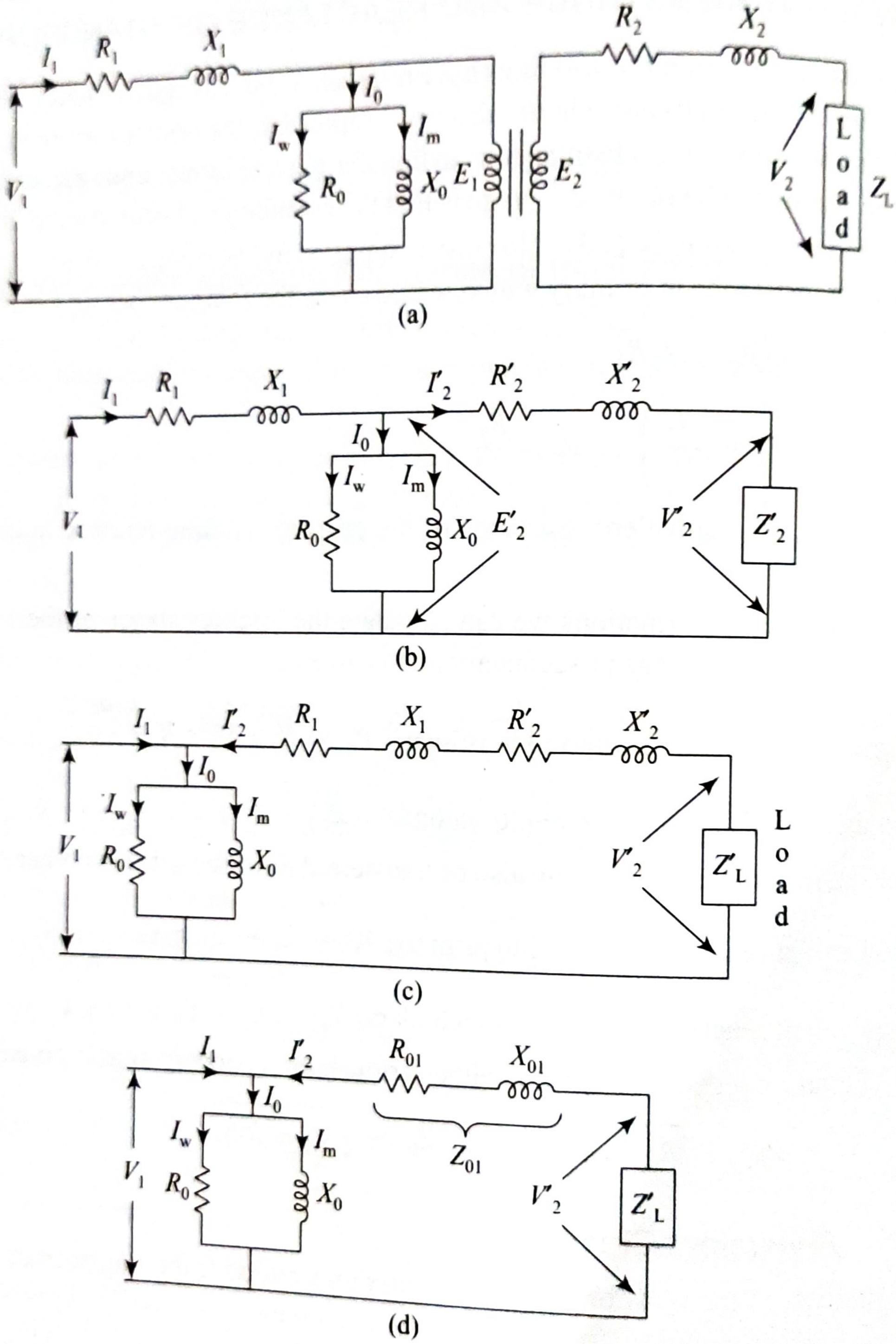


Figure 3.15 Equivalent circuit of transformer

Transferring secondary quantities to the primary side, we get

$$R_2' = \frac{R_2}{K^2}, \quad X_2' = \frac{X_2}{K^2}, \quad Z_2' = \frac{Z_2}{K^2}$$

while

$$E_2' = \frac{E_2}{K}, \; I_2' = KI_2$$

Thus the exact equivalent circuit referred to primary can be shown as in Figure 3.15(b). Similarly, we can also develop an exact equivalent circuit referred to secondary. Now as long as the no-load branch, i.e., the exciting branch is in between Z_1 and Z_2', the impedances cannot be combined. So, a further simplification of the circuit can be done. Such circuit is called approximate circuit.

To get approximate equivalent circuit, shift the no-load branch containing R_0 and X_0 to the left of R_1 and X_1. By doing this we are creating an error that the drop across R_1 and X_1 due to I_0 is neglected. Hence, such an equivalent circuit is called approximate equivalent circuit. So, the approximate equivalent circuit referred to primary can be as shown in Figure 3.15(c).

In this circuit, R_1 and R_2' can be combined to get equivalent resistance referred to primary R_{01} as discussed earlier. Similarly, X_1 and X_2' can be combined to get X_{01}, and equivalent circuit can be simplified as shown in the Figure 3.15(d).

In the similar fashion, the approximate equivalent circuit referred to secondary can also be obtained.

EXAMPLE 3.7 **A 2300/230-V transformer of 150 kVA is shown in Figure 3.16. The parameters of the circuit model are as follows:**

$V_1 = 0.25\ \Omega$	$V_2 = 3 \times 10^{-3}$
$X_1 = 0.5\ \Omega$	$X_2 = 5 \times 10^{-3}\ \Omega$
$R_1 = 10\ \text{k}\ \Omega$	$X_1 = 1.8\ \text{k}\ \Omega$

Draw the circuit model as shown on the HV side or equivalent resistance referred to the primary side.

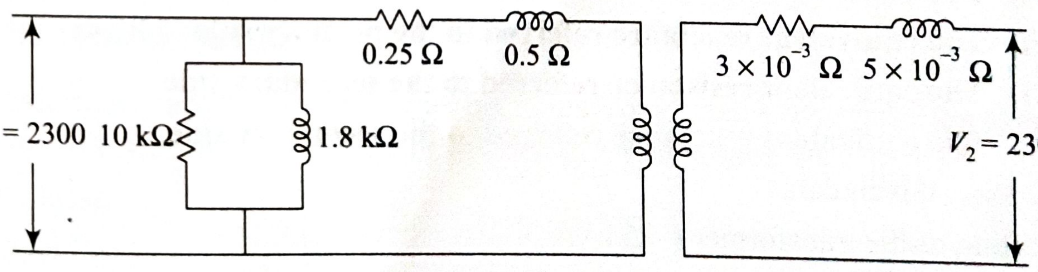


Figure 3.16

Solution Given data

Primary resistance, $R_1 = 0.25\ \Omega$, secondary resistance, $R_2 = 3 \times 10^{-3}\ \Omega$

Primary-side leakage reactance, $X_1 = 0.5\ \Omega$

Secondary-side leakage reactance, $X_2 = 5 \times 10^{-3}\ \Omega$

Magnetizing reactance or iron-cored reactance, $X_i = 1.8\ \text{k}\Omega$

Magnetizing resistance or iron-cored resistance, $R_i = 10\ \text{k}\Omega$

The transformation ratio of transformer, $k = \dfrac{V_2}{V_1} = \dfrac{230}{2300} = 0.1$

The total resistance referred to the primary side, $R_{01} = R_1 + \dfrac{R_2}{K^2} = 0.25 + \dfrac{3 \times 10^{-3}}{(0.1)^2} = 0.55\ \Omega$

The total reactance referred to the primary side, $X_{01} = X_1 + \dfrac{X_2}{(K)^2} = 0.5 + \dfrac{5 \times 10^{-3}}{(0.1)^2} = 1\ \Omega$

The total impedance referred to the primary side or HV side, $Z_{01} = R_{01} + j\,X_{01}$

$$= (0.55 + j1)\ \Omega$$

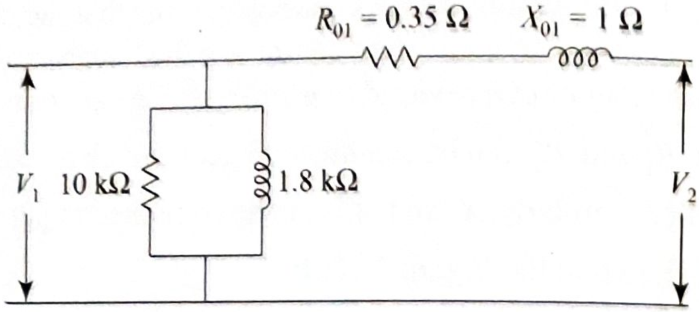


Figure 3.17

The equivalent circuit diagram of the total impedance referred to HV side is shown in Figure 3.17.

EXAMPLE 3.8 **A 2300/230-V, 50-Hz, single-phase transformer rating is 25 kVA. It has the following resistances and leakage reactances**

$$R_1 = 0.6\ \Omega \quad \text{and} \quad X_1 = 3.0\ \Omega$$
$$R_2 = 0.008\ \Omega \quad \text{and} \quad X_2 = 0.02\ \Omega$$

Calculate

(i) The equivalent resistance referred to the primary side

(ii) The equivalent reactance referred to the primary side

(iii) The equivalent resistance referred to the secondary side

(iv) The equivalent reactance referred to the secondary side

Solution Given data

Rating of the transformer = 25 kVA

Primary voltage, $V_1 = 2300$ V

Secondary voltage, $V_2 = 230$ V

Primary resistance, $R_1 = 0.6\ \Omega$

Primary leakage reactance, $X_1 = 3\ \Omega$

Secondary resistance, $R_2 = 0.008\ \Omega$

Secondary leakage reactance, $X_2 = 0.02\ \Omega$

The transformation ratio of transformer, $k = \dfrac{V_2}{V_1} = \dfrac{230}{2300} = 0.1$

(i) The equivalent resistance referred to the primary side. The secondary-side resistance referred to the primary side is denoted by R_2'.

$$\therefore \quad R_2' = R_2 \times \frac{1}{(K)^2} = 0.008 \times \frac{1}{(0.1)^2} = 0.8$$

The equivalent resistance referred to the primary side is denoted by R_{01}.

$$\therefore \quad R_{01} = R_1 + R_2' = 06 + 0.8 = 1.4\ \Omega$$

(ii) The equivalent reactance referred to the the primary side. The secondary reactance referred to the primary side is denoted by X_2'.

$$\therefore \quad X_2' = X_2 \times \frac{1}{(k)^2} = 0.02 \times \frac{1}{(0.1)^2} = 2\ \Omega$$

Total reactance referred to the primary side is denoted by X_{01}.

$$\therefore \quad X_{01} = X_1 + X_2' = 3 + 2 = 5\ \Omega$$

(iii) The equivalent resistance referred to the secondary side. The primary resistance referred to the secondary side is denoted by R_1'.

$$\therefore \quad R_1' = R_1 \times (k)^2 = 0.6 \times (0.1)^2 = 0.006\ \Omega$$

The total resistance referred to the secondary side is denoted by R_{02}.

$$\therefore \quad R_2' = R_1' + R_2 = 0.006 + 0.008 = 0.014\ \Omega$$

(iv) The total reactance referred to the secondary side. The primary reactance referred to the secondary side is denoted by X_1'.

$$\therefore \quad X_1' = X_1 \times (k)^2 = 3 \times (0.1)^2 = 0.03\ \Omega$$

The total reactance referred to the secondary side is denoted by X_{02}.

$$\therefore \quad X_2' = X_1' + X_2 = 0.03 + 0.02 = 0.05\ \Omega$$

3.9 LOSSES IN TRANSFORMER

Since a transformer is a static device, it does not contain any rotational losses. Hence, major losses that occur in a transformer on load can be divided into two groups:

(i) Core losses

(ii) Copper losses

3.9.1 Core Losses

The core flux in a transformer remains practically constant for all loads. Therefore, these losses are constant at any load. It is the sum of two losses, i.e., (i) hysteresis loss and (ii) eddy current loss.

(i) **Hysteresis loss.** These losses are directly proportional to the flux density of core, frequency, and volume of core.

Hysteresis losses, $W_h \propto B_{max}^{\eta} fv$

$$W_h = kB_{max}^{\eta} fv \text{ watts} \tag{3.20}$$

where η is the Steimntz constant, which is 1.6; B_m is the maximum flux density; f is the frequency; and v is the volume of the core. These losses can be reduced by selecting a suitable high silicon steel content material.

(ii) Eddy current loss. These losses are directly proportional to the square of flux density, frequency, and thickness.

Eddy current losses, $W_e \propto B_{max}^2 f^2 t^2$

$$W_e = kB_{max}^2 f^2 t^2 \text{W} \tag{3.21}$$

where k is the proportional constant; t is the thickness; f is the frequency; and B_{max} is the maximum flux density. These losses can be reduced by selecting thin sheets of laminations. If thin sheets are selected, automatically thickness is reduced.

EXAMPLE 3.9 **The transformer core losses are found to be 50 W at 40 Hz and 86 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current loss at 50 Hz.**

Solution Given data

Core losses at 40 Hz = 50 W

Core losses at 60 Hz = 86 W

In both cases, the flux density is same.

Total core losses, $W_i = Af + Bf^2$ (or) $W_i/f = A + Bf$

$$\frac{50}{40} = A + B \times 40$$

or

$$A + 40B = 1.25 \tag{1}$$

$$\frac{86}{60} = A + B \times 60$$

or

$$A + 60B = 1.43 \tag{2}$$

Solving Eqs. (1) and (2), we get

$$A = 0.89 \quad \text{and} \quad B = 0.009$$

The hysteresis and eddy current loss at 50 Hz are

$$\begin{aligned} W_i &= Af + Bf^2 \\ &= 0.89\,(50) + (0.009)\,(50)^2 \\ &= W_h + W_e \\ &= 44.5 + 22.5 = 67 \text{ W} \end{aligned}$$

Hysteresis loss at 50 Hz, $W_h = 44.5$ W

Eddy current loss at 50 Hz, $W_e = 22.5$ W

EXAMPLE 3.10 **A 10-kg specimen of steel sheet laminations of the maximum flux density and waveform factor are maintained constant. The test will be conducted on this transformer, and the following results are obtained:**

Frequency (Hz):	**20**	**40**	**50**	**60**	**80**
Total losses (W):	**16**	**35**	**48**	**65**	**102**

Calculate the eddy current loss per kg at a frequency of 50 Hz.

Solution Given data

Weight of the specimen = 10 kg

When the flux density and waveform factor remain constant, the expression for iron loss

$$W_{\text{i}} = Af + Bf^2 \text{ (or) } \frac{W_{\text{i}}}{f} = A + Bf$$

The values of W_{i}/f for different frequencies are as follows:

Frequency (Hz)	20	40	50	60	80
$\frac{W_{\text{i}}}{f}$	0.8	0.875	0.96	1.083	1.275

The graph between f and W_{i}/f has been plotted as shown in Figure 3.18.

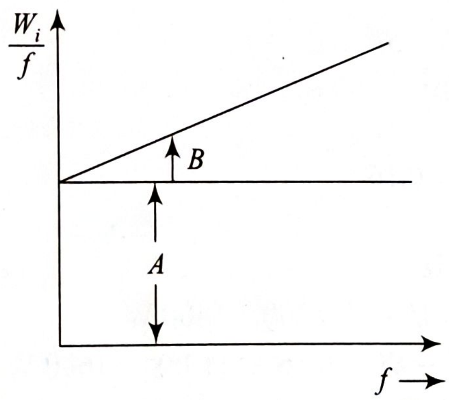


Figure 3.18

At

$$20 \text{ Hz} = \frac{W_{\text{i}}}{f} = A + Bf$$

$\Rightarrow$ $\quad 0.8 = A + 20B$

or

$$A + 20B = 0.8 \tag{1}$$

at

$$40 \text{ Hz} \Rightarrow 0.875 = A + 40B$$

$$A + 40B = 0.875 \tag{2}$$

Solving Eqs. (1) and (2), we get

$$A = 0.725 \quad \text{and} \quad B = 0.00375$$

Eddy current loss at 50 Hz = $Bf^2 = (0.00375)(50)^2 = 9.375$ W

Eddy current loss/kg = $\frac{9.375}{10} = 0.9375$ W

EXAMPLE 3.11 A transformer is connected to an 1100-V, 50-Hz supply, the core loss is 1100 W of which 700 W is the hysteresis loss and 400 W is the eddy current loss. If the applied voltage is raised to 2200 V and frequency to 100 Hz, find the core losses.

Solution Given data

Applied voltage, $V = 1100$ V

Frequency, $f = 50$ Hz

Core losses = 1100 W

Hysteresis losses, $W_h = 700$ W

Eddy current loss, $W_e = 400$ W

If the applied voltage and frequency are doubled, find the new core losses, i.e., $V = 2200$ V and $f = 100$ Hz. Here, voltage and frequency both are doubled, therefore, B_{max} is constant. With 1100 V at 50 Hz, the hysteresis loss, $W_h = Af$.

$$\Rightarrow \quad 700 = A \times 50$$

$$\therefore \quad A = \frac{700}{50} = 14$$

Eddy current loss, $W_e = Bf^2$

$$\Rightarrow \quad 400 = B(50)^2$$

$$\therefore \quad B = \frac{400}{(50)^2} = 0.16$$

With 2200 V at 100 Hz

Hysteresis loss, $W_h = Af = 14 \times 100 = 1400$ W

Eddy current loss, $W_e = Bf^2 = 0.16 \leftrightarrow (100)^2 = 1600$ W

New core losses, $W_i = W_h + W_e = 1400 + 1600 = 3000$ W

3.9.2 Copper Losses

Basically the copper losses are due to ohmic resistances of the transformer windings. If R_1 and R_2 are the resistances of primary and secondary windings, respectively, then the copper losses of primary and secondary are $I_1^2R_1$ and $I_2^2R_2$. Therefore, the total copper losses of transformer are $I_1^2R_1 + I_2^2R_2$. Generally, these losses vary as square of that particular current. These losses are also called as variable losses.

3.10 TESTING OF TRANSFORMER

The performance of a transformer can be analyzed by its equivalent circuit parameters shown in Figure 3.15. In order to determine the equivalent circuit parameters, the following two tests are performed on the transformer.

(i) Open-circuit (OC) test
(ii) Short-circuit (SC) test

3.10.1 Open-Circuit Test

This test is done to determine the core parameters such as R_0 and X_0. It is also known as core loss test. The circuit diagram for the OC test is shown in Figure 3.19.

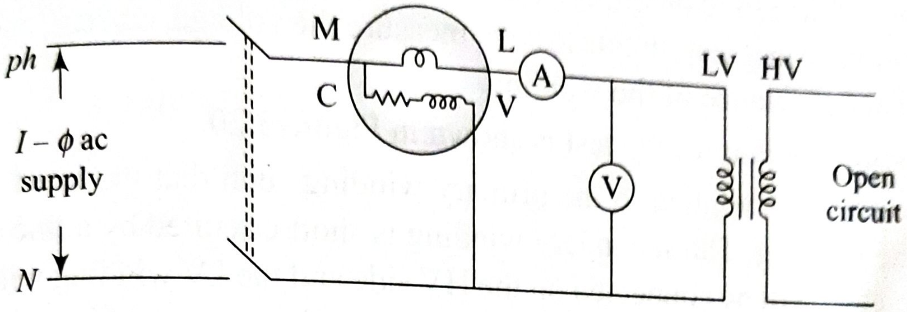


Figure 3.19 Circuit diagram for OC test

The primary is connected to the rated supply of the normal frequency through the wattmeter, ammeter, and voltmeter. The secondary of the transformer is left open. Normally for the OC test, the meters are connected on the LV side and the HV side will be open circuited to avoid the high current at the OC side. The wattmeter used is the low power factor (LPF) meter. Since the secondary is open circuited, there will not be any load current and hence no loss in the secondary side. In the primary, the no-load current is very less, i.e., 5–10% of the full-load current, so that the copper loss in the primary winding can also be neglected. Therefore, the power is utilized only to meet the core losses in this test.

Wattmeter reading, W_0 = Core losses

Voltmeter reading, V_0 = Applied voltage

Ammeter reading, I_0 = No-load current

$$\therefore \quad W_0 = P_i \text{ (core loss)}$$

$$= V_0 I_0 \cos \phi_0$$

No-load power factor, $\cos \phi_0 = \dfrac{W_0}{V_0 I_0}$

Working component of the no-load current, $I_w = I_0 \cos \phi_0$

Magnetizing component of the no-load current, $I_m = I_0 \sin \phi_0$

The shunt branch parameters can be calculated as

$$Y_0 = G_i - jB_m$$

$$= \frac{I_0}{V_0}$$

$$V_0^2 G_i = W_0$$

$$G_i = \frac{W_0}{V_0^2}; \quad P = \frac{V^2}{R}$$

$$B_m = \sqrt{Y_0^2 - G_i^2}$$

The values what we get are LV-side parameters. If we want to refer these values to the HV side, the values can be converted using the transformation ratio.

3.10.2 Short-Circuit Test

This test is conducted to determine the winding parameters such as R_{01}, X_{01} or R_{02}, X_{02}. It is also known as copper loss test. In this test we measure the voltage, current, and power to calculate the resistance and reactance of the winding.

The circuit diagram for SC test is shown in Figure 3.20.

A low voltage is applied to the primary winding such that the rated full-load current passes through the winding. The secondary winding is short circuited by a fine conductor. Normally, in this test, meters will be connected on the HV side and the LV winding will be short circuited. The wattmeter used in this test is unity power factor (upf) wattmeter. Since the applied voltage on the primary is very less, the flux setup is small so the core loss can be neglected.

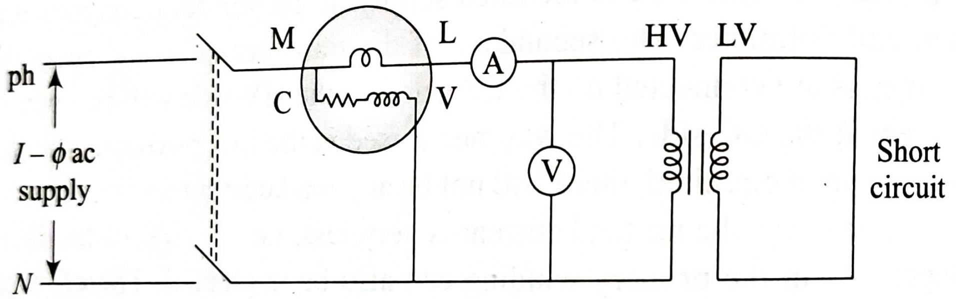


Figure 3.20 Circuit diagram for the SC test

Wattmeter reading, W_{SC} = Copper loss
Voltmeter reading, V_{SC} = Applied voltage
Ammeter reading, I_{SC} = Full-load current

$$\therefore \quad Z_{01} = \frac{V_{SC}}{I_{SC}}$$

and

$$R_{01} = \frac{W_{SC}}{I_{SC}^2}$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

The values are referred to the HV side. We can transfer these values to the LV side by using the transformation ratio.

EXAMPLE 3.12 **The no-load test is conducted on a single phase transformer. The following test data are obtained:**

Primary voltage, $V_1 = 230$ V, secondary voltage, $V_2 = 115$ V
Primary current, $I_0 = 0.6$ A, power input, $W_0 = 32$ W

Resistance of the primary winding, $R_1 = 0.5\ \Omega$

Find the following:

(i) Turns ratio

(ii) The magnetizing component of the no-load current

(iii) Its working (or) loss component

(iv) Iron loss

Draw no-load phasor diagram to scale.

Solution Given data

Primary voltage, $V_1 = 230$

Secondary voltage, $V_2 = 115$ V

Primary current, $I_0 = 0.6$ A

Power input, $W_0 = 32$ W

Primary winding resistance, $R_1 = 0.5\ \Omega$

(i) The turn's ratio of transformer, $k = \dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{115}{230} = 0.5$

(ii) The magnetizing component of the no-load current

The no-load power, $W_0 = V_1 I_0 \cos \phi_0$

$$\cos \phi_0 = \frac{W_0}{V_1 I_0} = \frac{32}{230 \times 0.6} = 0.232 \text{ lag}$$

$$\phi_0 = \cos^{-1}(0.232)\ 76.6$$

$$\sin \phi_0 = 0.973$$

The magnetizing current, $I_m = I_0 \sin \phi_0 = 0.6 \times 0.973 = 0.584$ A

(iii) Working component, $I_w = I_0 \cos \phi_0 = 0.6 \times 0.232 = 0.1392$ A

Primary copper losses $= I_0^2 R_1 = (0.6)^2 \times 0.5 = 0.18$ W

(iv) The core loss (or) iron loss = Input power (W_0) – Copper loss

$= 32 - 0.18 = 31.82$ W

The phasor diagram is shown in Figure 3.21.

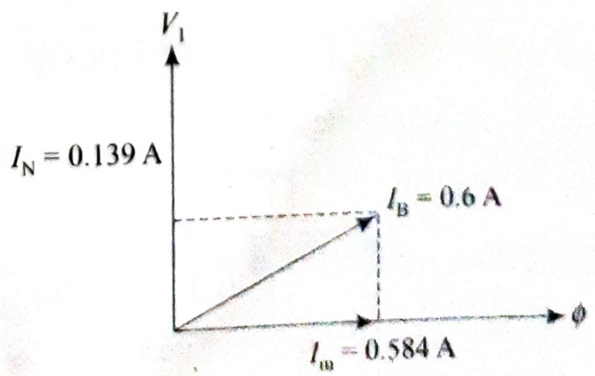


Figure 3.21

$$B_m = \sqrt{Y_0^2 - G_i^2}$$

The values what we get are LV-side parameters. If we want to refer these values to the HV side, the values can be converted using the transformation ratio.

3.10.2 Short-Circuit Test

This test is conducted to determine the winding parameters such as R_{01}, X_{01} or R_{02}, X_{02}. It is also known as copper loss test. In this test we measure the voltage, current, and power to calculate the resistance and reactance of the winding.

The circuit diagram for SC test is shown in Figure 3.20.

A low voltage is applied to the primary winding such that the rated full-load current passes through the winding. The secondary winding is short circuited by a fine conductor. Normally, in this test, meters will be connected on the HV side and the LV winding will be short circuited. The wattmeter used in this test is unity power factor (upf) wattmeter. Since the applied voltage on the primary is very less, the flux setup is small so the core loss can be neglected.

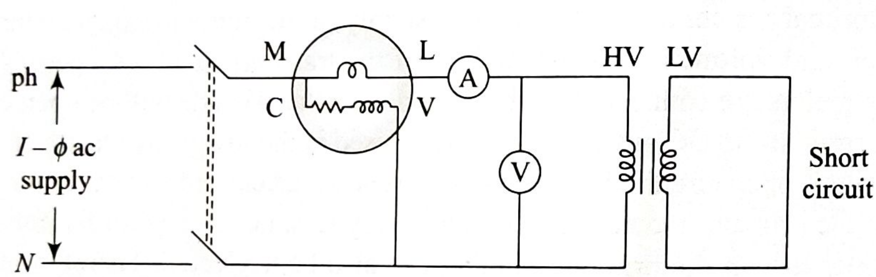


Figure 3.20 Circuit diagram for the SC test

Wattmeter reading, W_{SC} = Copper loss
Voltmeter reading, V_{SC} = Applied voltage
Ammeter reading, I_{SC} = Full-load current

$$\therefore \quad Z_{01} = \frac{V_{SC}}{I_{SC}}$$

and

$$R_{01} = \frac{W_{SC}}{I_{SC}^2}$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

The values are referred to the HV side. We can transfer these values to the LV side by using the transformation ratio.

EXAMPLE 3.12 The no-load test is conducted on a single phase transformer. The following test data are obtained:

Primary voltage, $V_1 = 230$ V, secondary voltage, $V_2 = 115$ V

Primary current, $I_0 = 0.6$ A, power input, $W_0 = 32$ W

Resistance of the primary winding, $R_1 = 0.5\ \Omega$

Find the following:

(i) Turns ratio

(ii) The magnetizing component of the no-load current

(iii) Its working (or) loss component

(iv) Iron loss

Draw no-load phasor diagram to scale.

Solution Given data

Primary voltage, $V_1 = 230$

Secondary voltage, $V_2 = 115$ V

Primary current, $I_0 = 0.6$ A

Power input, $W_0 = 32$ W

Primary winding resistance, $R_1 = 0.5\ \Omega$

(i) The turn's ratio of transformer, $k = \dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{115}{230} = 0.5$

(ii) The magnetizing component of the no-load current

The no-load power, $W_0 = V_1 I_0 \cos \phi_0$

$$\cos \phi_0 = \frac{W_0}{V_1 I_0} = \frac{32}{230 \times 0.6} = 0.232 \text{ lag}$$

$$\phi_0 = \cos^{-1} (0.232)\ 76.6$$

$$\sin \phi_0 = 0.973$$

The magnetizing current, $I_m = I_0 \sin \phi_0 = 0.6 \times 0.973 = 0.584$ A

(iii) Working component, $I_w = I_0 \cos \phi_0 = 0.6 \times 0.232 = 0.1392$ A

Primary copper losses = $I_0^2 R_1 = (0.6)^2 \times 0.5 = 0.18$ W

(iv) The core loss (or) iron loss = Input power (W_0) – Copper loss

$= 32 - 0.18 = 31.82$ W

The phasor diagram is shown in Figure 3.21.

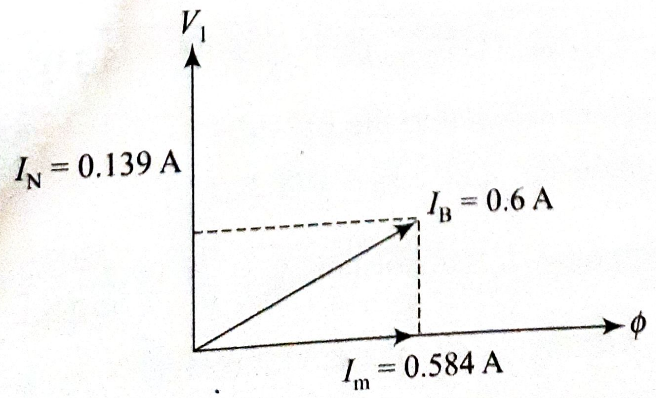


Figure 3.21

EXAMPLE 3.13 **A 25-kVA, 2200/220-V, 50-Hz, single-phase transformer obtained the following test results:**

OC test (LV side) = 220 V, 1.2 A, 100 W

SC test (HV side) = 100 V, 7 A, 310 W

Calculate the parameters of the equivalent circuit of the transformer referred to the LV side, and draw the equivalent circuit.

Solution Given data

Rating of the transformer = 25 kVA

Primary voltage, $V_1 = 2200$ V

Secondary voltage, $V_2 = 220$ V

From the OC test:

No-load power, $W_0 = V_0 I_0 \cos \phi_0$

No-load power factor, $\cos \phi_0 = \dfrac{W_0}{I_0 V_0} = \dfrac{100}{1.2 \times 220} = 0.38$ lag

$\sin \phi_0 = 0.925$

Working component current, $I_w = I_0 \cos \phi_0 = 0.38 \times 1.2 = 0.456$ A

Magnetizing component current, $I_m = I_0 \sin \phi_0 = 0.925 \times 1.2 = 1.11$ A

Magnetizing core resistance, $R_0 = \dfrac{V_0}{I_w} = \dfrac{220}{0.456} = 482.46\ \Omega$

Magnetizing core reactance, $X_0 = \dfrac{V_0}{I_m} = \dfrac{220}{1.11} = 198.2\ \Omega$

From the SC test:

$$Z_{SC} = \frac{V_{SC}}{I_{SC}} = \frac{100}{7} = 14.3\ \Omega$$

Copper loss $(P_{SC}) = I_{SC}^2 R_{SC}$

$$R_{SC\,HV} = \frac{P_{SC}}{I_{SC}^2} = \frac{310}{7^2} = 6.33\ \Omega$$

$$X_{SC\,HV} = \sqrt{Z_{SC}^2 - (R_{SC\,HV})^2} = \sqrt{(14.3)^2 - (6.33)^2} = 12.82\ \Omega$$

Equivalent parameters referred to the LV side

$R_0 = 482.46\ \Omega$ and $X_0 = 198.2\ \Omega$

The transformation ratio of transformer, $K = \dfrac{V_2}{V_1} = \dfrac{220}{2200} = 0.1$

$R_{SC\,LV} = K^2 R_{SC\,HV} (0.1)^2 \times 6.33 = 0.0633$

$X_{SC\,LV} = X_{SC\,HV} (K)^2 = 12.82\,(0.1)^2 = 0.1282\ \Omega$

The equivalent circuit diagram referred to the LV side is shown in Figure 3.22.

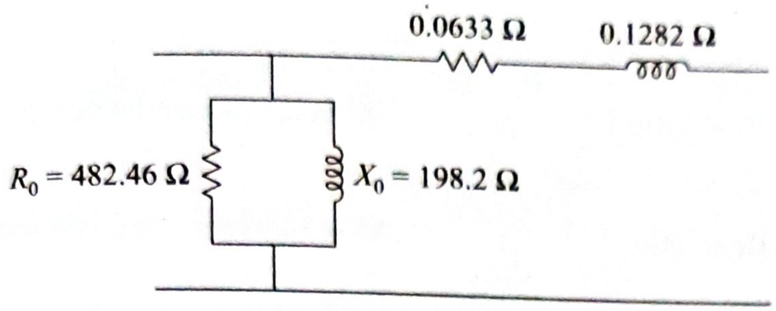


Figure 3.22

11 REGULATION OF TRANSFORMER

When the transformer on the secondary side is connected to load with constant excitation voltage on the primary side, the secondary voltage may decrease or increase with the lagging and leading power factors, respectively. This change in terminal voltage from no load to full load when the load is thrown off or removed at a specified power factor is known as the *regulation of a transformer.*

Figure 3.23 shows the appropriate equivalent circuit diagram by neglecting the shunt branch because the magnetizing current will not have any effect while calculating voltages.

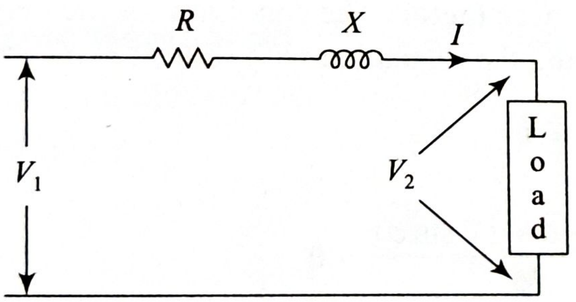


Figure 3.23 Approximate equivalent circuit

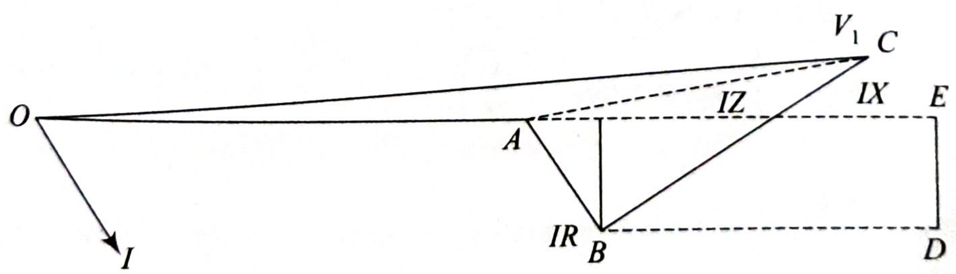


Figure 3.24 Phasor diagram

From Figure 3.24,

$$V_1 \cong OE$$

$$AE = V_1 - V_2 = IR \cos\phi + IX \sin\phi, \text{ for lagging pf}$$

$$AE = V_1 - V_2 = IR \cos\phi - IX \sin\phi, \text{ for leading pf}$$

When the load is removed, ${}_0V_2 = V_1$

$\therefore$

$$V_1 - V_2 = {}_0V_2 - V_2$$

Then

$$\% \text{ Regulation} = \frac{{}_0V_2 - V_2}{V_2} \times 100$$ is known as the percentage of voltage regulation up

$$\% \text{ Regulation} = \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$$ is known as the percentage of voltage regulation down

$$\% \text{ Regulation up} = \frac{IR\cos\phi + IX\sin\phi}{V} \tag{3.22}$$

$$= \frac{I_2(R_{02}\cos\phi + X_{02}\sin\phi)}{V_2} \text{ (referred to secondary)}$$

$$\therefore \quad \frac{I_2R_{02}}{V_2} = R_{pu} \quad \text{and} \quad \frac{I_2X_{02}}{V_2} = X_{pu} \tag{3.23}$$

The per unit regulation can be determined by $R_{pu}\cos\phi \pm X_{pu}\sin\phi$

3.11.1 Condition for Maximum Regulation

(i) For lagging power factor: The condition for the maximum regulation can be calculated by differentiating the regulation with respect to $d\phi$ and equating it to zero.

$$\frac{d(\text{reg})}{d\phi} = 0$$

$$\frac{d(IR\cos\phi + IX\sin\phi)}{d\phi} = 0 \tag{3.24}$$

$$-R\sin\phi + X\cos\phi = 0$$

$$R\sin\phi - X\cos\phi = 0$$

$$\tan\phi = \frac{X}{R} \Rightarrow \phi = \tan^{-1}\left(\frac{X}{R}\right) \tag{3.25}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + X^2}} \tag{3.26}$$

The voltage regulation will be maximum when the load power factor is equal to the impedance angle of the transformer.

(ii) For leading power factor:

$$\frac{d(\text{reg})}{d\phi} = 0$$

$$\frac{d(IR\cos\phi - IX\sin\phi)}{d\phi} = 0 \tag{3.27}$$

$$-R\sin\phi - X\cos\phi = 0$$

$$\tan\phi = \frac{-X}{R} \Rightarrow \phi = \tan^{-1}\left(\frac{-X}{R}\right) \tag{3.28}$$

The voltage regulation is negative for the leading power factor. Figure 3.25 shows the variation of regulation for both lagging and leading power factors.

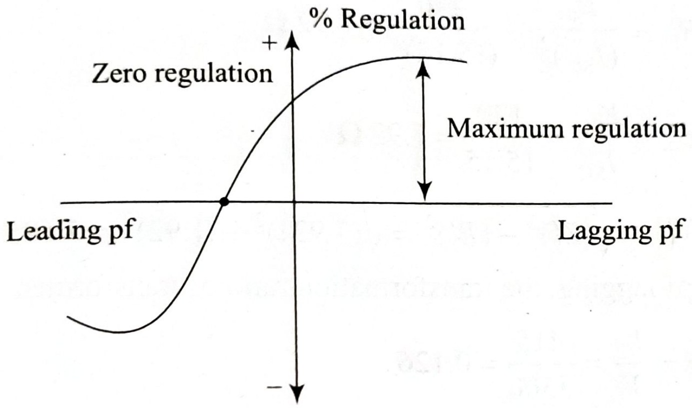


Figure 3.25 Variation of regulation with power factor

3.11.2 Condition for Zero Regulation

The condition for zero regulation is obtained as

$$I(R\cos\phi + X\sin\phi) = 0 \tag{3.29}$$

$$\tan\phi = \frac{-R}{X} \tag{3.30}$$

$$\phi = \tan^{-1}\left(\frac{-R}{X}\right) \tag{3.31}$$

$$\cos\phi = \cos\left[\tan^{-1}\left(\frac{-R}{X}\right)\right]$$

At this $\cos\phi$ the voltage regulation of the transformer becomes zero.

EXAMPLE 3.14 **The OC and SC tests were conducted on a 50-kVA transformer and the following test results were obtained:**

OC test: primary voltage, 3300 V; secondary voltage, 415 V; and primary power (W_0), 440 W.

SC test: primary voltage, 120 V; primary current, 15.15 A; primary power, 440 W; and secondary currents equal to full-load current. Find the voltage regulation for 0.7 power factor (i) lagging and (ii) leading.

Solution Given data

Rating of transformer = 50 kVA

V_{OC} = 3300 V, W_0 = 440 W, secondary voltage = 415 V

V_{SC} = 120 V, I_{SC} = 15.15 A, P_{SC} = 440 W

Secondary current = Full-load current = Short-circuit current

Full-load current, $I_2 = I_{SC} = \dfrac{\text{Transformer rating}}{\text{Voltage}}$

$$I_{SC} = I_{fl} = \frac{50 \times 1000}{415} = 120.5 \text{ A}$$

$$R_1' = \frac{P_{SC}}{(I_{SC})^2} = \frac{440}{(15.15)^2} = 1.92\ \Omega$$

$$Z_1' = \frac{V_{SC}}{I_{SC}} = \frac{120}{15.15} = 7.92\ \Omega$$

$$X_1' = \sqrt{(Z_1')^2 - (R_1')^2} = \sqrt{(7.921)^2 - (1.92)^2} = 7.685\ \Omega$$

(i) At 0.7 pf lagging, the transformation ratio of transformer,

$$k = \frac{V_2}{V_1} = \frac{415}{3300} = 0.126$$

$$R_2' = R_1'(k)^2$$

$$= 1.92 \times (0.126)^2 = 0.0304\ \Omega$$

$$X_2' = X_1'(k)^2 = 7.685 \times (0.126)^2 = 0.122\ \Omega$$

$$\cos\phi_0 = 0.7 \text{ lag, } \phi_0 = \cos^{-1} 0.7 = 45.6° \Rightarrow \sin(45.6°) = 0.714$$

$$\%\text{ Regulation} = \frac{I_2 R_2' \cos\phi + I_2 X_2' \sin\phi}{V_2} \times 100$$

$$\%\text{ Regulation} = \frac{120.5 \times 0.0304 \times 0.7 + 120.5 \times 0.122 \times 0.714}{415}$$

$$= 0.0315 \times 100 = 3.15\ \%$$

(ii) $\%$ Regulation at 0.7 pf leading $= \dfrac{I_2 R_2' \cos\phi - I_2 \times X_2' \sin\phi_2}{V_2} \times 100$

$$= \frac{120.5 \times 0.0304 \times 0.7 - 120.5 \times 0.122 \times 0.714}{415} \times 11$$

$$= -0.0191 \times 100 = -1.91\%$$

EXAMPLE 3.15 **A single-phase, 50-Hz, 2.5-kVA, 415/240-V transformer has the following parameters referred to the HV side:**

$$\mathbf{R_{01} = 3.5\ \Omega \text{ and } X_{01} = 4.5\ \Omega}$$

Determine the regulation of transformer when operating at (i) full load with 0.8 pf lagging, (ii) full load with 0.8 pf leading, and (iii) half load at 0.8 pf leading.

Solution Given data

Rating of transformer = 2.5 kVA

Primary voltage, $V_1 = 415$ V

Secondary voltage, $V_2 = 240$ V

Total resistance referred to the primary side, $R_{01} = 3.5\ \Omega$
Total reactance referred to the primary side, $X_{01} = 4.5\ \Omega$

(i) At 0.8 pf lagging

Primary current at full load, $I_1 = I_p = \dfrac{2.5 \times 10^3}{415} = 6.02$ A

$$\cos\phi = 0.8 \Rightarrow \phi_0 = \cos^{-1}(0.8) = 36.86$$
$$\sin\phi = \sin(36.86) = 0.6$$

$$\% \text{ Regulation} = \frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi}{V_1} \times 100$$
$$= \frac{6.02 \times 3.5 \times 0.8 + 6.02 \times 4.5 \times 0.6}{415} \times 100$$
$$= 0.0798 \times 100$$
$$= 7.98\%$$

(ii) For 0.8 leading pf, % regulation $= \dfrac{I_1 R_{01} \cos\phi - I_1 X_{01} \sin\phi}{V_1} \times 100$

$$= \frac{6.02 \times 3.5 \times 0.8 - 6.02 \times 4.5 \times 0.6}{415} \times 100$$
$$= 0.00145 \times 100$$
$$= 0.145\%$$

(iii) Half load at 0.8 pf leading

Half full-load current $= 6.02 \times \dfrac{1}{2} = 3.01$ A

$$\% \text{ Regulation} = \frac{I_1 R_{01} \cos\phi - I_1 X_{01} \sin\phi}{V_1} \times 100$$
$$= \frac{3.01 \times 3.5 \times 0.8 - 3.01 \times 4.5 \times 0.6}{415} \times 100$$
$$= 7.25 \times 10^{-4} \times 100$$
$$= 0.0725\%$$

EXAMPLE 3.16 **A transformer is having the copper loss of 3% of the output and the reactance drop of 6%. Calculate the voltage regulation when the power factor is (i) 0.8 pf lagging and (ii) 0.8 pf leading.**

Solution Given data

Copper loss = 3% of the output

Reactance drop = 6%

(i) % Regulation at 0.8 pf lagging

Copper loss $= (I_S)^2 R_{02} = 0.03\ (V_S I_S)$

$$I_S R_S = 0.03 V_S$$

Reactance drop = $I_S X_{02} = 0.06V_S$

$$\% \text{ Regulation for lagging pf} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{V_2} \times 100$$

Here,

$R_S = R_2, I_2 = I_S, V_2 = V_S$ and $X_S = X_2$

$\cos\phi = 0.8 \Rightarrow \phi = \cos^{-1} 0.8 = 36.86°$

$\sin\phi = \sin(36.86) = 0.6$

$$\% \text{ Regulation} = \frac{(0.03V_S)\times 0.8 + (0.06V_S)\,0.6}{V_S} \times 100 = \frac{0.06V_S}{V_S} \times 100 = 6\%$$

(ii) Regulation at 0.8 pf leading

$$\% \text{ Regulation for leading pf} = \frac{I_S R_{02} \cos\phi - I_s X_{02} \sin\phi}{V_S} \times 100$$

$$= \frac{(0.03V_S)\times 0.8 - (0.06V_S)0.6}{V_S} \times 100$$

$$= \frac{-0.012V_S}{V_S} \times 100 = -1.2\%$$

EXAMPLE 3.17 A single-phase 6600/415-V transformer has an equivalent resistance of 0.015 pu and an equivalent reactance of 0.045 pu. Calculate the full-load voltage regulation at 0.8 pf lag if the primary voltage is 6600 V. Find also the secondary terminal voltage at full load.

Solution Given data

Primary-side voltage, $V_1 = 6600$ V

Secondary-side voltage, $V_2 = 415$ V

Resistance drop = 0.015 pu

Reactance drop = 0.045 pu

(i) % Voltage regulation at full load with 0.8 pf lagging

Per unit voltage regulation = $E_r \cos\phi + E_x \sin\phi$

E_r = Per unit resistance drop = 0.015 pu

E_x = Per unit reactance drop = 0.045 pu

$\cos\phi = 0.8$ and $\phi = \cos^{-1}(0.8) = 36.86$

$\sin\phi = \sin(36.86) = 0.6$

$$\frac{E_2 - V_2}{V_2} = E_r \cos\phi + E_x \sin\phi$$

$$= 0.015 \times 0.8 + 0.045 \times 0.6 = 0.039$$

(ii) For a primary voltage of 6600 V, the secondary no-load voltage (E_2) is 415 V. The change in secondary terminal voltage

$$\frac{E_2 - V_2}{V_2} = 0.039$$

$$\frac{415-V_2}{V_2}=0.039$$

The secondary terminal voltage, $V_2 = 399.42$ V

EXAMPLE 3.18 **A single-phase transformer has 1000 turns on the primary and 250 turns on the secondary side. The no-load current is 2.4 A and the no-load power factor is 0.21 lagging. Calculate the current and power factor of the primary circuit when the secondary draws a current 310 A at 0.8 pf lagging.**

Solution Given data

Primary turns, $N_1 = 1000$

Secondary turns, $N_2 = 250$

No-load current, $I_0 = 2.4$ A

No-load pf $= 0.217$ lagging

Secondary current, $I_2 = 310$ A

The transformation ratio, $k=\dfrac{N_2}{N_1}=\dfrac{250}{1000}=0.25$

The relation from the turns and current

$$k=\frac{N_2}{N_1}=\frac{I_1}{I_2}\Rightarrow I_1=\frac{N_2}{N_1}\times I_2$$

$$I_1=\frac{250}{1000}\times 310=77.5\ \text{A}$$

$$I_0=2.4\ \text{A}$$

$$\cos\phi_0=0.217\Rightarrow \phi_0=\cos^{-1}(0.217)=77.47^\circ$$

$$\sin\phi_0=\sin(77.47^\circ)=0.976$$

$$I_1\cos\phi_1=I_2^1\cos\phi_2+I_0\cos\phi_0$$

$$=77.5\times 0.8+2.4\times 0.217=62.52\ \text{A}$$

$$I_1\sin\phi_1=I_2\sin\phi_2+I_0\sin\phi_0$$

$$=77.5\times 0.6+2.4\times 0.976=48.84\ \text{A}$$

$$I_1=\sqrt{(I_1\cos\phi_1)^2+(I_1\sin\phi_1)^2}$$

$$=\sqrt{(62.52)^2+(48.84)^2}=79.34\ A$$

$$\tan\phi_1=\frac{I_1\sin\phi_1}{I_1\cos\phi_1}=\frac{48.84}{62.52}=0.7812$$

$$\phi_1=\tan^{-1}(0.7812)=38.63^\circ$$

Primary power factor, $\cos\phi_1=\cos 38.63=0.781$ lagging.

3.12 EFFICIENCY OF TRANSFORMER

The efficiency of any device can be defined as the ratio of the output power to the input power. So, the transformer efficiency can be defined as

$$\text{Efficiency, } \eta = \frac{\text{Output power}}{\text{Input power}}$$

But due to losses present in transformer the output power will not be equal to the input power. Input power = output power + total losses

$$\eta = \frac{\text{Output power}}{\text{Output power} + \text{Total losses}}$$

$$= \frac{\text{Output power}}{\text{Output power} + I_2^2 R_{02} + P_i}$$

Here, the output power = $V_2 I_2 \cos \phi_2$

R_{02} = Total resistance of the transformer referred to secondary side

Efficiency at any particular load "x" of transformer is given as

$$\eta_{\text{at particular load}} = \frac{V_2 I_2 \cos \phi_2 \cdot x}{x \cdot V_2 I_2 \cos \phi_2 + x^2 P_{cu} + P_i} \quad (3.32)$$

From previous discussion, we come to know that the net flux is constant irrespective of loading conditions. Hence, the iron losses are constant. But the copper losses vary with the load current. Hence efficiency also varies with variation in load. At light loads the efficiency is very poor and as the load increases the increased efficiency becomes constant after 20–30% of the load and there becomes a slight fall at around 120% of the load. The efficiency versus load curves for different power factors is shown in Figure 3.26.

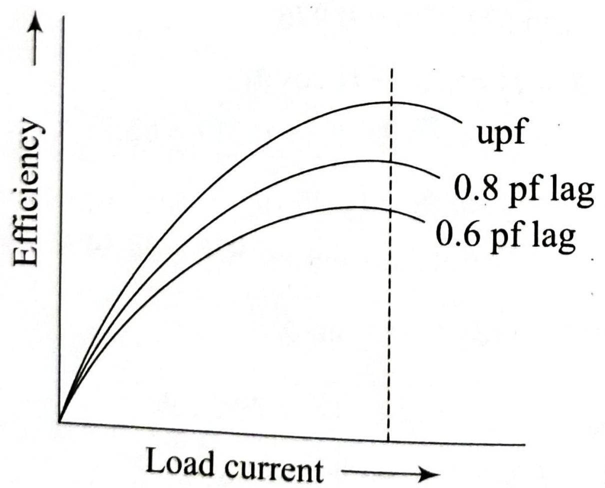


Figure 3.26 Variation of efficiency with load current for different power factors

3.12.1 Condition for Maximum Efficiency

According to the efficiency equation of transformer, the efficiency is function load current I_2. Let us assume that $\cos \phi_2$ is constant. Therefore, the secondary terminal voltage V_2 is also constant.

3.12 EFFICIENCY OF TRANSFORMER

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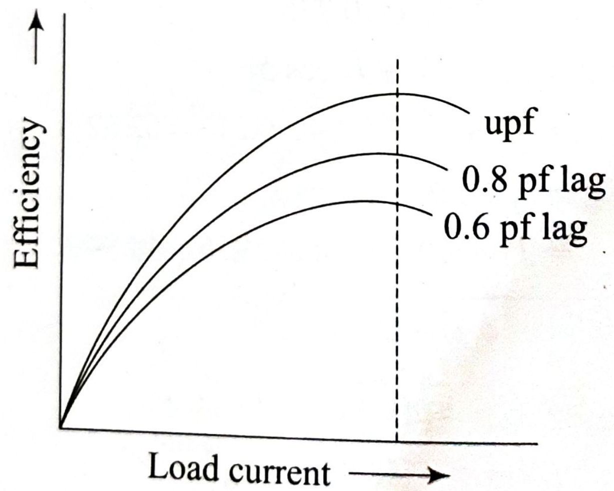

Figure 3.26 Variation of efficiency with load current for different power factors

3.12.1 Condition for Maximum Efficiency

According to the efficiency equation of transformer, the efficiency is function load current I_2. Let us assume that $\cos \phi_2$ is constant. Therefore, the secondary terminal voltage V_2 is also constant.

The condition for the maximum efficiency of transformer is

$$\frac{d\eta}{dI_2} = 0$$

$$\frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2\, I_2 \cos\phi_2}{V_2\, I_2 \cos\phi_2 + I_2^2\, R_{02} + P_i}\right] = 0 \tag{3.33}$$

(3.34)

Simplifying further

$$P_i + I_2^2\, R_{02} - 2 I_2^2\, R_{02} = 0$$

$$P_i - I_2^2 R_{02} = 0$$

$$P_i = I_2^2 R_{02}$$

$$P_i = P_{cu} \tag{3.35}$$

(3.36)

The condition to achieve maximum efficiency of a transformer is that copper losses must be equal to iron losses.

kVA supplied at maximum efficiency

At constant V_2, the kVA supplied is function of load current

$$\text{kVA at } \eta_{max} = I_2\, V_2$$

$$= V_2\, I_2 \times \sqrt{\frac{P_i}{P_{cu} \text{ at fl}}} \tag{3.37}$$

$$\text{kVA supplied at } \eta_{max} = \text{kVA rating at fl}\sqrt{\frac{P_i}{P_{cu} \text{ at fl}}} \tag{3.38}$$

EXAMPLE 3.19 **A single phase, 50-Hz transformer has the following data: turns ratio 40:2, $R_1 = 30\ \Omega, X_1 = 90\ \Omega, R_2 = 0.05\ \Omega, X_2 = 0.3\ \Omega$. No-load current is 1.4 A, leading flux by 35°. The secondary delivers 190 A at a terminal voltage of 415 V and at a pf of 0.8 lagging. Determine the following by the aid of a vector diagram:**

The primary applied voltage

The primary power factor

The efficiency

Solution Given data

Turns ratio = 40:2

Primary winding resistance, $R_1 = 30\ \Omega$

Secondary winding resistance, $R_2 = 0.05\ \Omega$

Primary winding leakage reactance, $X_1 = 90\ \Omega$

Secondary winding leakage reactance, $X_2 = 0.3\ \Omega$

No-load current, $I_0 = 1.4$ A, leading flux by an angle = 35°

The secondary current, $I_2 = 190$ A and $V_2 = 415$ V at a pf of 0.8 lagging

(i) Take V_2 as reference vector

$$V_2 = 415\ \angle 0° = (415 + j0)\ \text{V}$$

$I_2 = 190(0.8 - j0.6) = (152 - j114)$ A

$Z_2 = (0.05 + j0.3)\ \Omega$

$E_2 = V_2 + I_2 Z_2$

$= (415 + j0) + (152 - j114) \times (0.05 + j0.3)$

$E_2 = (456.8 + j39.9) = 458.54 \angle 5°$ V

Obviously $\beta = 5°$

$$E_1 = \frac{E_2}{k}$$

The transformation ratio of transformer is

$$k = \frac{E_2}{E_1} \Rightarrow E_1 = \frac{E_2}{k} = 20\,(456.8 + j39.9) = (9136 + j798) - E_1 = (-9136 - j798)$$
$$= 9170.79 \angle{-175°}$$

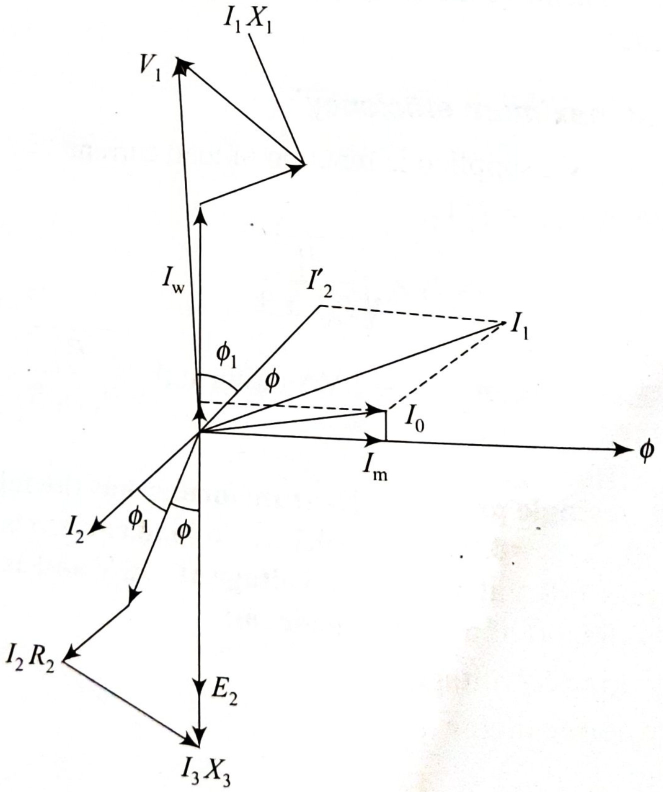

Figure 3.27

Secondary current referred to primary

$I_2' = -kI_2 = -0.05\,(152 - j114) = (-7.6 + j5.7) = 9.5 \angle{-143.13°}$A

$I_2' = 95 \angle{-143.13°}$ A

As seen from the vector diagram shown in Figure 3.27, I_0 leads by an angle

$5° + 90° + 35° = 130°$

$I_0 = 1.4 \angle 130° = 1.4\,(\cos 130 + j \sin 130°)$

$= 1.4\,(-0.643 + j\,0.766) = (-0.9 + j\,1.073)$

Primary current, $I_1 = -I_2' + I_0$

$= (-7.6 + j\,5.7) + (-0.9 + j1.073) = (-8.5 + j\,6.773)$

$I_1 = 10.87\ \angle 141.45°$ A

$V_1 = -E_1 + I_1\,Z_1$

$= (-9136 - j798) + (-8.5 + j6.733)\,(30 + j90)$

$= (-9996.97 - j1361.01) = 10089.19\ \angle{-172.25°}$ V

(ii) Primary power factor

Phase angle between I_1 and V_1, $\phi_1 = 172.3° - 141.45° = 30.85°$

$\therefore$ Primary power factor = cos (30.85°) = 0.858 lagging

(iii) Efficiency of the transformer

No-load input primary power, $W_0 = V_1 I_0 \cos\phi_0$

$W_0 = 10089.19 \times 1.4 \times \cos 55° = 8101.69$ W

$R_{02} = R_2 + K^2 R_1 = 0.05 + (0.05)^2 \times 30 = 0.125\ \Omega$

Total copper loss as referred to secondary

$I_2^2\,R_{02} = (190)^2 \times 0.125 = 4512.5$ W

Secondary output $= V_2 I_2 \cos\phi$

$= 415 \times 190 \times 0.8 = 63080$ W

Total losses = Copper loss + No-load loss

$= 4512.5 + 8104.43 = 12614.19$ W

Input = Output + Total losses

$= 63080 + 12614.93 = 75694.19$ W

$$\text{Efficiency of transformer} = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{63080}{75694.93} = 0.8333 \times 100 = 83.33\%$$

Example 3.20 **The OC test and SC test will be conducted on a 230/460-V transformer**

OC test (LV side) = 230 V, 1.2 A, 85 W

SC test (HV side) = 30 V, 14 A, 105 W (LV winding short circuited)

Determine

(i) The circuit constant

(ii) The applied voltage and efficiency when the output is 12 A, 415 V

Solution Given data

Primary voltage, $V_1 = 230$ V

Secondary voltage, $V_2 = 460$ V

From OC test:

Open-circuit voltage, $V_{OC} = 230$ V, no-load current, $I_0 = 1.2$ A

No-load power, $W_0 = 85$ W

From SC test:

Short-circuit voltage, $V_{SC} = 30$ V, short-circuit current, $I_{SC} = 14$ A

Short-circuit power, $W_{SC} = 105$ W

(i) The circuit constants

The transformation ratio of transformer, $k = \dfrac{V_2}{V_1} = \dfrac{460}{230} = 2$

No-load power, $W_0 = V_0 I_0 \cos \phi_0$

$$\cos \phi_0 = \frac{W_0}{I_0 V_0} = \frac{85}{1.2 \times 230} = 0.308 \text{ lag}$$

Watt full component (or) working component of current, $I_w = I_0 \cos \phi_0$

$$= 1.2 \times 0.308 = 0.37 \text{ A}$$

Magnetizing component of current, $I_m = I_0 \sin \phi_0$

$\cos \phi_0 = 0.308 \Rightarrow \phi_0 = \cos^{-1}(0.308) = 72.1$

$\sin(72.1) = 0.9514$

$I_m = I_0 \sin \phi_0 = 1.2 \times 0.9514 = 1.142$ A

Now resistance representing the core losses

$$R_0 = \frac{V_{OC}}{I_w} = \frac{230}{0.37} = 621.62\ \Omega$$

Magnetizing reactance, $X_0 = \dfrac{V_{OC}}{I_m} = \dfrac{230}{1.142} = 201.4\ \Omega$

As the primary is short circuited all values are referred to secondary winding

$$R_{02} = \frac{P_{SC} \text{ (or) } W_{SC}}{(I_{SC})^2} = \frac{105}{(14)^2} = 0.536\ \Omega$$

$$Z_{02} = \frac{V_{SC}}{I_{SC}} = \frac{30}{14} = 2.143\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(2.143)^2 - (0.536)^2} = 2.075\ \Omega$$

As R_{02} and X_{02} are referred to primary, let us transfer these values to primary as follows:

$$R_{01} = \frac{R_{02}}{(K)^2} = \frac{0.536}{(2)^2} = 0.134\ \Omega$$

$$X_{01} = \frac{X_{02}}{(K)^2} = \frac{2.075}{(2)^2} = 0.519\ \Omega$$

$$Z_{01} = \frac{Z_{02}}{(K)^2} = \frac{2.143}{(2)^2} = 0.536\ \Omega$$

The equivalent circuit is shown in Figure 3.28.

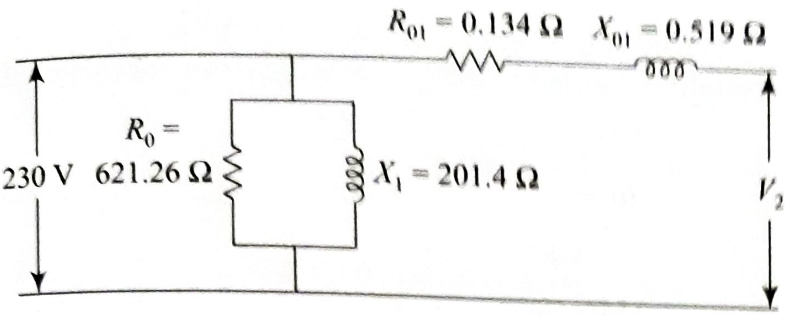

Figure 3.28

(ii) From Figure 3.29, the applied voltage V_1 is the vector sum of V_1 and $I_1 Z_{01}$ output current, $I_2 = 12$ A

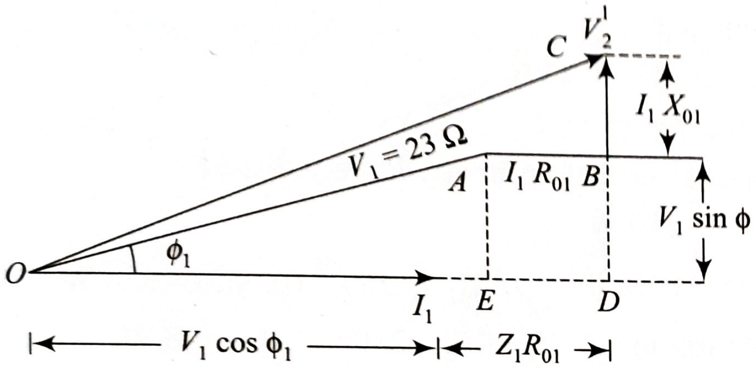

Figure 3.29

Input current, $I_1 = I_2 K = 12 \times 2 = 24$ A

Primary current, $I_1 = 24$ A

Now,

$$I_1 R_{01} = 24 \times 0.134 = 3.216 \text{ V}$$
$$I_1 X_{01} = 24 \times 0.519 = 12.46 \text{ V}$$
$$\cos\phi_1 = 0.8 \Rightarrow \phi_1 = \cos^{-1} 0.8 = 36.86°$$
$$\sin 36.86 = 0.6$$

Refer from the phasor diagram neglecting the angle between V_1 and V_1^1, we have

$$OC^2 = OD^2 + DC^2$$
$$OC = V_1^1 = \sqrt{(OE + ED^2) + (DB + BC)^2}$$
$$= \sqrt{(V_1 \cos\phi_1 + I_1 R_{01})^2 + (V_1 \sin\phi_1 + I_1 X_{01})^2}$$
$$= \sqrt{(230 \times 0.8 + 3.216)^2 + (V_1 \sin\phi_1 + I_1 X_{01})^2} = 240.18 \text{ V}$$

Hence, applied voltage, $V_1 = 240.18$ V

Iron loss (or) no-load losses = 85 W

Copper loss = $W_{SC} = I_2^2 R_{02} = (12)^2 \times 0.536 = 77.184$ W

$$\text{Output} = V_2 I_2 \cos\phi_2 = 415 \times 12 \times 0.8 = 3984 \text{ W}$$

$$\text{Input} = \text{Output} + \text{Losses} = 3984 + 85 + 77.184 = 4146.184 \text{ W}$$

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}} \times 100$$

$$= \frac{3984}{4146.184} \times 100 = 96.08\%$$

EXAMPLE 3.21 **A single-phase, 50-Hz, 50-kVA transformer has a full-load primary current of 260 A and total resistance referred to primary is 0.005 Ω if the iron loss amounts to 210 W. Find the efficiency at full load, and half load at (i) unity power factor and (ii) 0.8 power factor.**

Solution Given data

Transformer rating = 50 kVA

Full-load primary current, $I_1 = 260$ A

Total resistance referred to primary, $R_{01} = 0.005\ \Omega$

Iron losses, $W_i = 210$ W

Full-load copper loss, $W_c = I_1^2 R_{01} = (260)^2\ (0.005) = 338$ W

Total losses at full load $= W_i + W_c = 210 + 338 = 548$ W

(i) Unity power factor

Full load $= 50 \times 1 = 50$ kW

$$\text{Efficiency at full load with upf} = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{50 \times 10^3}{50 \times 10^3 + 548} \times 100$$

$$= 98.92\%$$

(ii) η at 0.8 pf

Full load $= 50 \times 0.8 \times 10^3 = 40$ kW

$$\text{Efficiency at full load with 0.8 pf} = \frac{40}{40 + 0.548} \times 100 = 98.64\%$$

Efficiency at half load:

$$\text{Half full-load output} = \frac{1}{2} \times 50 = 25 \text{ kVA}$$

Efficiency at 0.8 power factor:

Half full-load output $= 25 \times 0.8 = 20$ kW

$$\text{Copper loss at half load} = \left(\frac{I_1}{2}\right)^2 \times R_{01}$$

$$= \left(\frac{260}{2}\right)^2 \times 0.005 = 84.5 \text{ W}$$

$$\eta \text{ at half load at 0.8 pf} = \frac{20 \times 10^3}{20 \times 10^3 + \text{Losses}}$$

Total losses = $W_c + W_i = 210 + 84.5 = 294.5$ W

$$\eta = \frac{20\times10^3}{20\times10^3 + 294.5}\times100 = 98.55\%$$

Efficiency at half-load upf:

Half load at upf = $25 \times 1 = 25$ kW

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Total losses}} = \frac{25\times10^3}{25\times10^3 + 294.5}\times100 = 98.84\%$$

EXAMPLE 3.22 **A transformer 1100/230 V has primary and secondary resistances of 0.4 Ω and 0.03 Ω, respectively, if the iron loss amounts to be 250 W. Determine (i) the secondary current at which maximum efficiency occurs and (ii) the maximum efficiency at 0.8 pf lagging.**

Solution Given data

Primary voltage, $V_1 = 1100$ V

Secondary voltage, $V_2 = 230$ V

Primary resistance, $R_1 = 0.4\ \Omega$

Secondary resistance, $R_2 = 0.03\ \Omega$

Iron losses = 250 W

(i) The transformation ratio of transformer, $K = \dfrac{V_2}{V_1} = \dfrac{230}{1100} = 0.2091$

Total resistance referred to secondary, $R_{02} = R_2 + R_1 (K)^2 = 0.03 + 0.4(0.2091)^2 = 0.0475\ \Omega$

Let I_2 be the secondary current at maximum efficiency.

At maximum efficiency:

Copper loss = Iron loss

$$I_2^2 R_{02} = 250$$

$$I_2^2 = \frac{250}{R_{02}} = \frac{250}{0.0475} = 5263.16$$

$$I_2 = 72.55 \text{ A}$$

Hence, the secondary current at maximum efficiency = 72.55 A

(ii) Output at maximum efficiency at 0.8 power factor is

$$V_2 I_2 \cos\phi_2 = 230 \times 72.55 \times 0.8 = 13349.2 \text{ W}$$

At maximum efficiency:

Copper loss = Iron loss

$$P_c = P_i$$

The total losses at maximum efficiency = $P_i + P_i = 2P_i$

$$P_{total} = 250 \times 2 = 500 \text{ W}$$

Maximum efficiency = $\dfrac{\text{Output}}{\text{Output} + \text{Total losses}} \times 100$

$$= \frac{13349.2}{13349.2 + 500} \times 100 = 96.39\%$$

EXAMPLE 3.23 **The efficiency of a 250-kVA, single-phase, 50-Hz transformer is 95% when delivering full load at 0.8 power factor lagging and 96.2% at delivering half load at upf. Calculate the efficiency at 75% of the full load at 0.8 power factor lagging.**

Solution Given data

Rating of the transformer = 250 kVA

Efficiency at full load with 0.8 pf = 95%

Efficiency at half load with upf = 96.2%

At full load:

Output at 0.8 pf lagging = 250 × 0.8 = 200 kW

$$\text{Input} = \frac{\text{Output}}{\text{Efficiency}} = \frac{200 \times 10^3}{0.95} = 210.53 \text{ kW}$$

Total losses = Input – Output

$$= 210.53 - 200 = 10.53 \text{ kW} \qquad (1)$$

At half load:

$$\text{Output at upf} = \frac{250}{2} \times 1 = 125 \text{ kW}$$

$$\text{Input} = \frac{\text{Output}}{\text{Efficiency}} = \frac{125 \times 10^3}{0.962} = 129.94 \text{ kW}$$

Total losses = 129.94 – 125 = 4.94 kW

$$\text{Copper loss at half load} = \left(\frac{1}{2}\right)^2 P_c = \frac{1}{4} P_c$$

$$\text{Total losses} = P_i + \frac{1}{4} \times P_c = 4.94 \text{ kW} \qquad (2)$$

Subtracting Eq. (2) from Eq. (1), we get

$$\frac{3}{4} P_c = 10.53 - 4.94 = 5.59 \text{ kW}$$

$$P_c = 5.59 \times \frac{4}{3} = 7.45 \text{ kW}$$

$$P_i = 10.53 - 7.45 = 3.08 \text{ kW}$$

Copper loss at 75% of full load = $(0.75)^2 \times P_c = (0.75)^2 \times 7.45 = 4.191$ kW

Total loss at 75% at full load = $P_i + P_c = 3.08 + 4.191 = 7.27$ kW

Output at 75% of full load at 0.8 pf = 250 × 0.75 × 0.8 = 150 kW

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{150 \times 10^3}{150 \times 10^3 + 7.27 \times 10^3} \times 100 = 95.38\%$$

Example 3.24 **A 10-kVA, 415/230-V, single-phase, 50-Hz transformer has maximum efficiency of 93% at 81% of full load and upf. Determine the efficiency at 0.8 pf lagging.**

Solution Given data

Rating of the transformer = 10 kVA

Primary voltage, $V_1 = 415$ V

Secondary voltage, $V_2 = 230$ V

The maximum efficiency occurs at 81% of full load = 93%, i.e.,

$$\eta_{max} = 93\% \text{ at } 81\% \text{ of full load.}$$

Output at 81% of full load at upf = $10 \times 0.81 \times 1 = 8.1$ kW

$$\text{Input at 81\% of full load} = \frac{\text{Output}}{\text{Efficiency}} = \frac{8.1 \times 10^3}{0.93} = 8.709 \text{ kW}$$

Total losses = Input – Output = 8.709 – 8.1 = 0.609 kW

At maximum efficiency

Copper loss = Core loss

$$P_i = (0.81)^2 P_c = 0.6561 P_c$$

Total losses at maximum efficiency = $2P_i = 0.609$ kW

$$P_i = \frac{0.609}{2} = 0.3045 \text{ kW}$$

$$P_c = \frac{P_i}{0.6561} = \frac{0.3045}{0.6561} = 0.4641 \text{ kW}$$

Total full-load losses = $P_i + P_c = 0.3045 + 0.4641 = 0.7686$ kW

Full-load output at 0.8 pf lagging = $10 \times 0.8 = 8$ kW

Efficiency of full load at 0.8 pf lagging is

$$\frac{\text{Output}}{\text{Output + Total losses}} = \frac{8}{8 + 0.7686} \times 100 = 91.235\%$$

Example 3.25 **A 10-kVA, 440/230-V, single-phase transformer gave the following results: short-circuit test (HV side), 50 V, 22.73 A, 160 W, the maximum efficiency occurs at upf and 1.3 times of full-load current. Determine the full-load efficiency and also the maximum efficiency.**

Solution Given data

Rating of the transformer = 10 kVA

Primary voltage, $V_1 = 440$ V

Secondary voltage, $V_2 = 230$ V

From the SC test (HV side)

$$V_{SC} = 50 \text{ V}, I_{SC} = 22.73 \text{ A and } W_{SC} = 160 \text{ W}$$

The maximum efficiency occurs at upf and 1.3 times of full-load current.

$$\text{Full-load current on HV side} = \frac{10 \times 10^3}{440} = 22.73 \text{ A}$$

The short-circuit test is conducted on the HV side only, therefore, the full-load copper loss at upf will be 160 W.

The maximum efficiency occurs at 1.3 times of full-load current.

Hence at upf, corresponding copper loss = $(1.3)^2 \times 160 = 270.4$ W

At maximum efficiency

Copper loss = Core loss at upf

Input power = $1.3 \times 10000 \times 1 = 13000$ W

Hence, the maximum efficiency at upf is

$$\frac{13}{13+0.27+0.27} \times 100 = 0.9601 \times 100 = 96.01\%$$

Full-load efficiency at 0.8 pf

Output power at full load and 0.8 pf = $10000 \times 0.8 = 8$ kW

Core losses = 270.4 W

Copper loss at full load = 160 W

Total losses = $P_i + P_c = 270.4 + 160 = 430.4$ W

$$\eta = \frac{\text{Output}}{\text{Output + Losses}} \times 100 = \frac{8 \times 10^3}{8 \times 10^3 + 430.4} \times 100 = 94.89\%$$

EXAMPLE 3.26 **A single-phase, 50-Hz, 40-kVA transformer has an iron loss of 450 W and full-load copper loss of 900 W. Find the load at which the maximum efficiency is achieved at upf.**

Solution Given data

Rating of the transformer = 40 kVA

Iron losses = 450 W

Copper loss = 900 W (full-load copper loss)

If

X = fraction of rated load at which maximum efficiency

P_i = Iron losses = 450 W

P_c = full-load copper loss = 900 W

$$X^2 P_c = P_i$$

Substituting the values of P_i and P_c in the above equation, we get

$$X^2 (900) = 450 \Rightarrow X^2 = \frac{1}{2}$$

$\therefore$ $X = 0.707$

Hence, the efficiency is maximum at 70.7% of the rated load.

At these loads, $P_i = P_c$

Corresponding output = $40 \times 0.707 = 28.28$ kW

Corresponding efficiency = $\dfrac{28.28}{28.28 + 0.45 + 0.45} \times 100 = 96.92\%$

EXAMPLE 3.27 **A transformer has a resistance of 2.0% and a reactance of 6%. At full load, what is the power factor at which the regulation will be**

(i) **Zero**

(ii) **Positive maximum**

(iii) **If the maximum efficiency of the transformer occurs at full load (at upf) what will be the efficiency under these condition?**

Solution Given data

Resistance = 2.0%

Reactance = 6.0%

Approximate percentage regulation = 2.0 cos $\phi \pm$ 6.0 sin ϕ

(i) If regulation is zero, negative sign must be applicable. This happens at leading pf

$$\text{Corresponding pf} = \tan\phi = \frac{2}{6} = 0.333$$

$$\phi = \tan^{-1}(0.333) = 18.44° \text{ leading}$$

(ii) For a maximum positive regulation lagging pf the result can be obtained.

Corresponding $\tan\phi = 6/2 = 3$

$$\phi = \tan^{-1} 3 = 71.57°$$

% voltage regulation = $2 \times \cos 71.57 + 6 \times \sin 71.57 = 6.325\%$

(iii) Maximum efficiency occurs at such load when

Iron losses = Copper losses

The copper loss depends on the resistance of transform

Take iron losses = 2 copper losses

$$\text{Efficiency} = \frac{100}{100 + 2 \times 2} = \frac{100}{104} \times 100 = 96.15\%$$

EXAMPLE 3.28 **If P_1 and P_2 are the iron loss and copper loss of a transformer on full load, find the ratio of P_1 and P_2 such that maximum efficiency occurs at 75% of full load.**

Solution Given data

If P_1 = Iron losses

P_2 = Copper losses at full load

Copper loss at 75% of full load = $P_2(0.75)^2 = \frac{9}{16} P_2$

At maximum efficiency, $P_{\text{Iron}} = P_{\text{Cu}}$

$$P_1 = \frac{9}{16} P_2 = \frac{P_1}{P_2} = 9/16$$

EXAMPLE 3.29 **The rating of a 2300/230-V, 50-Hz single-phase transformer is 25 kVA. The efficiency at upf is 96% at rated load and also at half rated load. Determine the transformer core losses and ohmic loss.**

Solution Given data

Rating of the transformer = 25 kVA

Primary voltage, $V_1 = 2300$ V

Secondary voltage, $V_2 = 230$ V

The efficiency at upf full load = 96%, and also at half load the efficiency is same.

The transformer core losses and ohmic loss

$$\text{At full load, } \eta = 1 - \frac{\text{Losses}}{\text{Out put} + \text{Losses}}$$

$$0.96 = 1 - \frac{P_C + P_{SC}}{25000 \times 1 + P_C + P_{SC}}$$

$$0.04 = \frac{P_C + P_{SC}}{25000 + (P_C + P_{SC})}$$

$$P_C + P_{SC} = 1000 + 0.04\, P_C + 0.04\, P_{SC}$$

$$P_C + P_{SC} = \frac{1000}{0.96} = 1041.67 \text{ W}$$

$$\text{At half load, } \eta = 1 - \frac{P_C + \left(\frac{1}{4}\right) P_{SC}}{\text{Output} + P_C + \left(\frac{1}{4}\right) P_{SC}}$$

$$= 1 - \frac{P_C + \left(\frac{1}{4}\right) P_{SC}}{(25000) \times \frac{1}{2} + P_C + \left(\frac{1}{4}\right) P_{SC}}$$

$$0.96 = 1 - \frac{P_C + 0.25\, P_{SC}}{12500 + P_C + 0.25\, P_{SC}}$$

$$\Rightarrow \quad 0.04 = \frac{P_C + 0.25\, P_{SC}}{12500 + P_C + 0.25\, P_{SC}}$$

$$P_C + 0.25\, P_{SC} = 500 + 0.04\, P_C + 0.01\, P_{SC}$$

$$P_C + \left(\frac{1}{4}\right) P_{SC} = \frac{500}{0.96} = 520.83 \text{ W}$$

$$P_C + P_{SC} = 1041.67 \text{ W}$$

$$P_C + \frac{1}{4} P_{SC} = 520.83 \text{ W}$$

Solving the above equations, we get

$$P_{SC} = P_C = 694.44 \text{ W}$$

EXAMPLE 3.30 **The maximum efficiency of a single phase, 50 Hz, 100 kVA transformer is 96% and it occurs at 82% of the full load at 0.8 power factor. If the leakage impedance is 6%, find the voltage regulation at rated load of 0.8 pf lagging.**

Solution Given data

Rating of the transformer = 100 kVA

Maximum efficiency = 96%

The load at which maximum efficiency occurs = 82% of full load

Leakage impedance = 6%

$$\text{Efficiency } (\eta) = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

$$\frac{1}{\eta} = \frac{\text{Output} + \text{Losses}}{\text{Output}} = 1 + \frac{\text{Losses}}{\text{Output}}$$

$$\therefore \text{ Losses} = \left(\frac{1}{\eta} - 1\right) \text{Output}$$

Output at maximum efficiency = (100000) (0.82) (0.8) = 65600 W

$$\text{Total transformer losses} = \left[\frac{1}{0.96} - 1\right] \times [65600] = 2733.33 \text{ W}$$

At maximum efficiency, ohmic loss = Core loss

$$P_C = P_{SC} = \frac{2733.33}{2} = 1366.67 \text{ W}$$

This ohmic losses of 1366.67 W occur at 82% of full-load current.

$$\therefore \text{ Ohmic loss at full load} = 1366.67 \left[\frac{100}{82}\right] = 1666.671 \text{ W}$$

$$R_{epu} = \frac{1666.67}{100000} = 0.0167 = E_{rpu}$$

It is given that $Z_{epu} = 0.06$

$$X_{epu} = \sqrt{(0.06)^2 - (0.0167)^2} = 0.05763 = E_x \text{ pu}$$

Voltage regulation = $E_r \cos \phi_2 + E_x \sin \phi_2$

$$\cos \phi_2 = 0.8 \Rightarrow \phi_2 = \cos^{-1} (0.8) = 36.86°$$

$$\sin \phi_2 = \sin (36.86°) = 0.6$$

Voltage regulation = $0.0167 \times 0.8 + 0.05763 \times 0.6 = 0.0479$

% Voltage regulation = $0.0479 \times 100 = 4.794\%$

EXAMPLE 3.31 **A 15-kVA, 1100/110-V, 50-Hz, single-phase transformer has the following test results:**

OC test (LV side): 110 V, 0.8 A, 90 W

SC test (HV side): 70 V, 12 A, 100 W

Determine the following:

(i) Core losses of the transformer

(ii) Equivalent resistance and leakage reactance referred to HV side

(iii) Equivalent resistance and leakage reactance referred to LV side

(iv) Regulation of transformer at full load and half load at 0.8 pf lagging

(v) Transformer terminal voltage at full load at 0.8 pf lagging

(vi) Efficiency of the transformer at full load and half load at 0.8 pf lagging

Solution Given data

Rating of the transformer = 15 kVA

Primary voltage, $V_1 = 1100$ V

Secondary voltage, $V_2 = 110$ V and frequency, $f = 50$ Hz

No-load voltage, $V_{0c} = 110$ V

No-load current, $I_0 = 0.8$

No-load power, $W_0 = 90$ W

Short-circuit voltage, $V_{SC} = 70$ V

Short-circuit current, $I_{SC} = 12$ A

Short-circuit power, $W_{SC} = 100$ W

Transformation ratio of transformer, $K = \dfrac{V_2}{V_1} = \dfrac{110}{1100} = 0.1$

(i) From SC test $V_{SC} = 70$ V, $I_{SC} = 12$ A, $P_{SC} = 100$ W

$$Z_{01} = \frac{V_{SC}}{I_{SC}} = \frac{70}{12} = 5.833\ \Omega$$

$$R_{01} = \frac{P_{SC}}{(I_{SC})^2} = \frac{100}{(12)^2} = 0.694\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(5.833)^2 - (0.694)^2} = 5.792\ \Omega$$

(ii) Equivalent resistance and leakage reactance referred to the HV side (or) primary side are 0.694 Ω and 5.792 Ω.

(iii) Equivalent resistance and leakage reactance referred to the LV side (or) secondary side

$$R_{02} = [R_{01}] \times k^2 = 0.694 \times [0.1]^2 = 0.00694\ \Omega$$

$$X_{02} = [X_{01}] \times k^2 = 5.792 \times [01]^2 = 0.05792\ \Omega$$

(iv) % Regulation of transformer at full load and half load at 0.8 pf lagging

(a) Secondary current at full load, $I_2 = \dfrac{15 \times 10^3}{110} = 136.36$ A

$$\cos\phi = 0.8 \Rightarrow \phi = \cos^{-1} 0.8 = 36.86$$

$$\sin\phi = \sin(36.86) = 0.6$$

Approximate voltage drop = $I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$

$$= 136.36 (0.00694 \times 0.8 + 0.05792 \times 0.6) = 5.496 \text{ V}$$

Voltage regulation of the transformer = $\dfrac{5.496}{110} = 0.04996$ pu

% Regulation = $0.04996 \times 100 = 4.996\%$

(b) Secondary current at half load, $I_2 = \dfrac{7.5 \times 10^3}{110} = 68.2$ A

At half load, the approximate voltage drop of transformer = $I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$

Regulation in pu = $\dfrac{2.75}{110} = 0.025$ pu

% Voltage regulation = $0.025 \times 100 = 2.5\%$

At half-load terminal voltage, $V_2 = 110 - 2.75 = 107.5$ V

(v) At full load the transformer terminal voltage at 0.8 pf lagging = $110 - 5.496 \text{ V} = 104.504$ V

(vi) Efficiency of the transformer

(a) At full load at 0.8 pf lagging

Copper loss at full load, $P_{SC} = I_2^2 R_{02} = (136.36)^2 \times 0.00694 = 129.04$ W

Total losses of the transformer = $P_i + P_{SC} = 90 + 129.04 = 219.04$ W

At full-load output power = $V_2 I_2 \cos \phi_2$

$$= 104.504 \times 136.36 \times 0.8 = 11400.132 \text{ W}$$

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} \times 100$$

$$= \frac{11400.132}{11400.132 + 219.04} \times 100 = 98.12\%$$

(b) η at half load at 0.8 pf

$$\text{Copper loss at half load} = \left(\frac{1}{2}\right)^2 \times \text{Full-load copper loss}$$

$$= \frac{1}{4} \times 129.04 = 32.26 \text{ W}$$

Total losses at half load = $P_i + P_c = 90 + 32.26 = 122.26$ W

Output at half load = $V_2 I_2 \cos \phi_2$

$$= 107.5 \times \left(\frac{136.36}{2}\right) \times 0.8 = 5863.48 \text{ W}$$

$$\text{Efficiency, } \eta = \frac{5863.48}{5863.48 + 122.26} \times 100 = 97.96\%$$

3.13 ALL-DAY EFFICIENCY OF TRANSFORMER

The all-day efficiency of a transformer can be defined as the ratio of output energy over 24 h to input energy for the same time

$$\eta_{AD} = \frac{\text{Output in kWh}}{\text{Input in kWh}}$$

Generally, the all-day efficiency of a transformer can be calculated for distribution transformer. Since the distribution transformer does not supply the rated load for the entire day. Therefore, all-day efficiency of transformer will always be less than the commercial or ordinary efficiency of transformer.

EXAMPLE 3.32 **A 400-kVA distributed transformer whose copper and iron losses at full load are 5 kW and 4 kW, respectively. During a day of 24 h it is loaded as under**

Number of hours	Loading in kW	Power factor
6	320	0.8
10	240	0.75
4	80	0.8
4	0	—

Find the all-day efficiency.

Solution Given data

Rating of the transformer = 400 kVA

Full-load copper loss = 5 kW

Iron losses = 4 kW

Load of 320 kW at 0.8 pf $= \dfrac{320}{0.8} = 400$ kVA

Similarly for 240 kW at 0.75 pf $= \dfrac{240}{0.75} = 320$ kVA

For 100 kW at 0.8 pf $= \dfrac{80}{0.8} = 100$ kVA, i.e., one-fourth of full load

Copper loss at full load of 400 kVA = 5 kW

Copper loss at 320 kVA $= 5 \times \left[\dfrac{320}{400}\right]^2 = 3.2$ kW

Copper loss at 100 kVA $= 5 \times \left[\dfrac{100}{400}\right]^2 = 0.3125$ kW

Total copper loss in 24 h $= (6 \times 5) + (10 \times 32) + 0.3125 + 4 \times 0$

$\Rightarrow \quad (6 \times 5) + (10 \times 3.2) + 0.3125 \times 4 + 4 \times 0 = 63.25$ kWh

Iron loss for 24 h $= 24 \times 4 = 96$ kWh

Total transformer losses for 24 h $= 63.25 + 96 = 159.25$ kWh

Transformer output for 24 h = $(6 \times 320) + (10 \times 240) + (4 \times 80) + 4 \times 0$
$= 4640$ kW

$$\text{All-day efficiency} = \frac{\text{Output for 24 h}}{\text{Output for 24 h + Total losses for 24 h}}$$

$$\eta_{AD} = \frac{4640}{4640 + 159.25} \times 100 = 96.68\%$$

EXAMPLE 3.33 **A 125-kVA transformer is loaded as follows: load is increased from 0 to 83.25 kVA in 3 h from 7:00 a.m. to 10:00 a.m., stays at 83.25 kVA from 10:00 a.m. to 6:00 p.m., and then the transformer is disconnected till next day. Assuming the load to be resistive and the core loss is equal to the copper loss of 1.2 kW, determine the all-day efficiency and ordinary efficiency of transformer.**

Solution Given data

Rating of transformer = 125 kVA

From 7:00 a.m. to 10:00 a.m., the load increased from 0 to 83.25 kVA in 3 h.

$$\text{The average load in 3 h} = \left(\frac{0 + 83.25}{2}\right) = 41.67 \text{ kVA}$$

$$41.63 \text{ kVA} \left(\frac{1}{3} \text{ of full load kVA}\right)$$

$$\text{Load from 10:00 a.m. to 6:00 p.m.} = 83.25 \text{ kVA} \left(\frac{2}{3} \text{ of full load kVA}\right)$$

Copper loss = Iron loss = 1.2 kW

Ordinary efficiency:

In this case, load variations are not relevant

Output = $125 \times 1 = 125$ kW, Iron loss = Copper loss = 1.2 kW

In this case the load is resistive so take pf as upf.

Total losses = 2.4 kW = $P_i + P_c$

$$\text{Ordinary efficiency} = \frac{\text{Output}}{\text{Output + Losses}} = \frac{125}{125 + 2.4} \times 100 = 98.12\%$$

All-day efficiency:

$$\text{Copper loss from 7 a.m. to 10 a.m.} = 3 \times \left(\frac{1}{3}\right)^2 \times 1.2 = 0.4 \text{ kWh}$$

$$\text{Copper loss from 10 a.m. to 6 p.m.} = 8 \times \left(\frac{2}{3}\right)^2 \times 1.2 = 4.3 \text{ kWh}$$

Total copper loss for 24 h = 0.4 + 4.3 = 4.67 kWh

Total iron loss for 24 h = $24 \times 1.2 = 28.8$ kWh

Losses for a day of 24 h = 28.8 + 4.67 = 33.47 kWh

Output for 24 h = 3 × (41.67) + 8 × (83.25) = 791.01 kWh

$$\text{All-day efficiency, } \eta = \frac{\text{Output for 24 h}}{\text{Output for 24 h + Total losses for 24 h}} \times 100$$

$$= \frac{791.01}{791.01 + 33.47} \times 100 = 95.94\%$$

EXAMPLE 3.34 **A 10-kVA, 1–ϕ, transformer has a core loss of 50 W and full-load ohmic loss of 110 W. The daily variation of load on the transformer is as follows:**

6:00 a.m. to 1:00 p.m. 3 kW at 0.60 pf

1:00 p.m. to 5:00 p.m. 8 kW at 0.8 pf

5:00 p.m. to 1:00 a.m. full load at upf

1:00 a.m. to 1:00 p.m. no load

Solution Given data

Rating of the transformer = 10 kVA

Core losses = 50 W

Copper losses (full load) = 110 W

Fractional loading (x) and output kWh corresponding to load variations can be worked out in the tabulation form as below.

S. No.	Number of hours	$x = \dfrac{\text{Load kVA}}{\text{Transformer rating}}$	$x^2 P_c$ in kW	Output in kWh	Copper loss in kWh
1.	7	$\dfrac{3/0.6}{10} = 0.5$	$(0.5)^2 \times 0.11 = 0.0275$	$3 \times 7 = 21$	$0.0275 \times 7 = 0.1925$
2.	4	$\dfrac{8/0.8}{10} = 1$	$(1)^2 \times 0.11 = 0.11$	$8 \times 4 = 32$	$0.11 \times 4 = 0.44$
3.	8	$\dfrac{10/1}{10} = 1$	0.11	$10 \times 8 = 80$	$0.11 \times 8 = 0.88$
4.	5	Zero	Zero	Zero	Zero

Output in kWh = 21 + 32 + 80 = 133

Total ohmic loss in kWh = 0.1925 + 0.44 + 0.88 = 1.513 kWh

$$\text{Core losses during 24 h} = \frac{50}{1000} \times 24 = 1.2 \text{ kWh}$$

$$\text{Energy efficiency} = \text{All-day efficiency} = \frac{133}{133 + 1.513 + 1.2} \times 100 = 98\%$$

EXAMPLE 3.35 **The all-day efficiency of a 250-kVA transformer is 97% when it is loaded as follows.**

Losses for a day of 24 h = 28.8 + 4.67 = 33.47 kWh

Output for 24 h = 3 × (41.67) + 8 × (83.25) = 791.01 kWh

$$\text{All-day efficiency, } \eta = \frac{\text{Output for 24 h}}{\text{Output for 24 h + Total losses for 24 h}} \times 100$$

$$= \frac{791.01}{791.01 + 33.47} \times 100 = 95.94\%$$

EXAMPLE 3.34 **A 10-kVA, 1–ϕ, transformer has a core loss of 50 W and full-load ohmic loss of 110 W. The daily variation of load on the transformer is as follows:**

6:00 a.m. to 1:00 p.m. 3 kW at 0.60 pf

1:00 p.m. to 5:00 p.m. 8 kW at 0.8 pf

5:00 p.m. to 1:00 a.m. full load at upf

1:00 a.m. to 1:00 p.m. no load

Solution Given data

Rating of the transformer = 10 kVA

Core losses = 50 W

Copper losses (full load) = 110 W

Fractional loading (x) and output kWh corresponding to load variations can be worked out in the tabulation form as below.

S. No.	Number of hours	$x = \frac{\text{Load kVA}}{\text{Transformer rating}}$	$x^2 P_c$ in kW	Output in kWh	Copper loss in kWh
1.	7	$\frac{3/0.6}{10} = 0.5$	$(0.5)^2 \times 0.11 = 0.0275$	$3 \times 7 = 21$	$0.0275 \times 7 = 0.1925$
2.	4	$\frac{8/0.8}{10} = 1$	$(1)^2 \times 0.11 = 0.11$	$8 \times 4 = 32$	$0.11 \times 4 = 0.44$
3.	8	$\frac{10/1}{10} = 1$	0.11	$10 \times 8 = 80$	$0.11 \times 8 = 0.88$
4.	5	Zero	Zero	Zero	Zero

Output in kWh = 21 + 32 + 80 = 133

Total ohmic loss in kWh = 0.1925 + 0.44 + 0.88 = 1.513 kWh

$$\text{Core losses during 24 h} = \frac{50}{1000} \times 24 = 1.2 \text{ kWh}$$

$$\text{Energy efficiency} = \text{All-day efficiency} = \frac{133}{133 + 1.513 + 1.2} \times 100 = 98\%$$

EXAMPLE 3.35 **The all-day efficiency of a 250-kVA transformer is 97% when it is loaded as follows.**

No. of hours	Load (kW)	Power factor
10	150	0.8
9	180	0.9
5	No load	—

If maximum efficiency of a transformer occurs at 82% of full load, find the iron loss and full-load copper loss.

Solution Given data

Rating of the transformer = 250 kVA

All-day efficiency of transformer = 97%

Maximum efficiency occurs at 82% of full load.

For 10 h, the load is 150 kW at 0.8 pf.

For 9 h, the load is 180 kW at 0.9 pf.

For 5 h, the load is zero.

$$\text{Output/day} = 150 \times 10 + 180 \times 9 + 5 \times 0 = 3120 \text{ kWh}$$

$$\text{All-day efficiency} = \frac{\text{Output in kWh}}{(\text{Output} + \text{Losses}) \text{ in kWh}}$$

$$0.97 = \frac{3120}{3120 + \text{Losses}}$$

$$0.97\,(3120 + \text{Losses}) = 3120$$

$$\text{Losses} = \frac{3120 - 3026.4}{0.97} = 96.5 \text{ kWh}$$

Let P_i be the iron loss in kW and P_c be the full-load copper loss in kW.

$$\text{Copper loss at 150 kW} = \frac{\left(\frac{150}{0.8}\right)^2}{(250)^2} \times P_c = 0.5625\, P_c$$

$$\text{Copper loss for 10 h at 150 kW} = 0.5625 \times 10 \times P_c$$
$$= 5.625\, P_c \text{ kWh}$$

$$\text{Copper loss at 180 kW} = \frac{(180/0.9)^2}{(250)^2} \times P_c = 0.64\, P_c$$

$$\text{Copper loss at 180 kW for 9 h} = 0.64 \times 9 \times P_c = 5.76\, P_c$$
$$= 5.76\, P_c \text{ kWh}$$

$$\text{Iron loss for 24 h} = P_i \times 24 \text{ kWh}$$

$$\text{Total losses/day} = (5.625\, P_c + 5.76\, P_c) + 24\, P_i = 11.385\, P_c + 24\, P_i \text{ kWh}$$

$$\text{But total losses/day} = 96.5 \text{ kWh}$$

$$11.385\ P_c + 24\ P_i = 96.5 \tag{1}$$

Copper loss at 82% of full load $= (0.82)^2\ P_c = 0.6724\ P_c$

At maximum efficiency, $P_i = 0.6724\ P_c$

Substituting this P_i value in Eq. (1), we get

$$11.385\ P_c + 24\ (0.6724\ P_c) = 96.5$$

$$11.385\ P_c + 16.14\ P_c = 96.5 \Rightarrow P_c = \frac{96.5}{27.53} = 3.51\ \text{kW}$$

Full-load copper loss = 3.51 kW

Iron loss, $P_i = 0.6724\ P_c$

$$= 0.6724 \times 3.51 = 2.36\ \text{kW}$$

$$P_c = 3.51\ \text{kW}$$

$$P_i = 2.36\ \text{kW}$$

3.14 AUTOTRANSFORMER

Figure 3.30 shows an autotransformer. The winding AC with N_1 number of turns is connected across V_1 supply, whose induced emf is E_1. Let the winding BC consists of N_2 turns and terminal voltage V_2, whose induced emf is E_2. But $E_2/E_1 = N_2/N_1$ if the circuit is closed through the load, then E_2 drives a current I_2 through the load. Here the two mmfs $N_1\ I_1$ and $N_2\ I_2$ will be equal and opposite. If B is a variable point then the output voltage V_2 is varied.

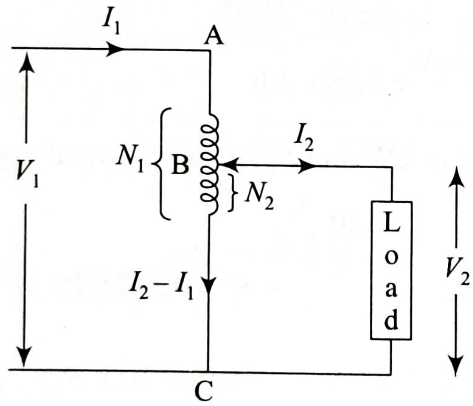

Figure 3.30 Autotransformer

(i) Step-down autotransformer. In Figure 3.30, if V_1 is greater than V_2 it is called as step-down autotransformer. According to ideal condition

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = k \tag{3.39}$$

The currents are distributed through the windings as shown in Figure 3.30. The VA delivered to the load is $V_2\ I_2$

$$V_2\ I_2 = V_2\ I_1 + V_2\ (I_2 - I_1) \tag{3.40}$$

where $V_2 I_1$ is the volt-amperes transferred conductively to the load through the AB portion of $(N_2 - N_1)$ turns and $V_2\,(I_2 - I_1)$ is the volt-amperes transferred inductively to the load through the BC portion having N_2 turns. Therefore,

$$\frac{\text{Output VA of autotransformers}}{\text{Transferred VA}} = \frac{\text{Output VA of a transformer}}{\text{Output VA of equivalent ordinary transformer}}$$

$$= \frac{V_2\, I_2}{V_2(I_2 - I_1)} = \frac{1}{1-k} \tag{3.41}$$

An ordinary transformer connected as a step-down transformer will have a volt-ampere rating $1/(1 - K)$ times of rating of ordinary transformer. Therefore, this result shows that the ordinary transformer connected as an autotransformer will have greater VA rating than those connected as ordinary transformers.

(ii) **Step-up autotransformer.** Figure 3.31 shows a step-up autotransformer. If V_1 is less than V_2 or the number of turns in portion AB, i.e., $(N_1 - N_2)$, is less than the number of turns in portion BC, i.e., N_2, it is called step-up autotransformer. The current directions are distributed as shown in Figure 3.31. According to ideal conditions

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} = k \tag{3.42}$$

The volt-ampere input $V_1 I_1$ can be written as

$$V_1 I_1 = V_2 I_2 + (I_1 - I_2) \tag{3.43}$$

where $V_2 I_1$ is the volt-amperes transferred conductively from input to the load through the $(N_1 - N_2)$ turns and $V_1(I_1 - I_2)$ is the volt-amperes transferred inductively from input to the load through N_2 turns. Therefore,

$$\frac{\text{Output VA of autotransformers}}{\text{Transferred VA}} = \frac{V_1 I_1}{V_1(I_1 - I_2)} = \frac{K}{K-1} \tag{3.44}$$

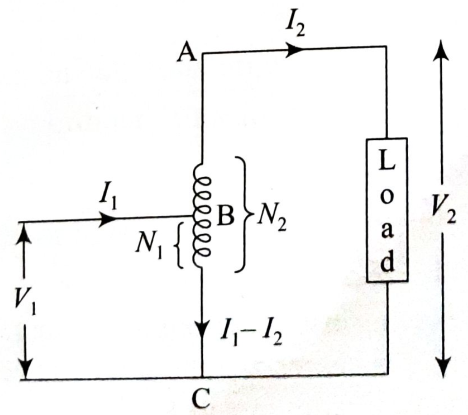

Figure 3.31 Step-up autotransformer

This result shows that a step-up transformer can be obtained from a step-down transformer by interchanging the currents $(I_1 - I_2)$ and I_2 provided that "K" is defined in the same manner in both cases.

3.14.1 Copper Saving in an Autotransformer

For the same capacity and voltage ratio, an autotransformer requires less winding material than a two winding transformer. The cross section of the conductor is proportional to the current carried by the conductor and length of the conductor is proportional to the number of turns. Hence, the weight of the copper is proportional to the product of the number of turns and current. For ordinary transformers,

Weight of copper in primary $\propto N_1 I_1$

Weight of copper in secondary $\propto N_2 I_2$

Total weight of copper $\propto N_1 I_1 + N_2 I_2$

But in an autotransformer (Figure 3.30) the turns $(N_1 - N_2)$ across AB carry a current of I_1. The weight of copper in section AB is proportional to $(N_1 - N_2)I_1$. In the same way, the turns N_2 across BC carry a current of $(I_2 - I_1)$. The weight of copper in section BC is proportional to $N_2(I_2 - I_1)$.

$$\text{Total weight of copper} \propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1)$$

$$\propto N_2 I_2 + (N_1 + 2N_2) I_1 \tag{3.45}$$

$$\frac{\text{Weight of copper in autotransformers}}{\text{Weight of copper in ordinary transformer}} = \frac{N_2 I_2 + (N_1 - 2N_2) I_1}{N_1 I_1 + N_2 I_2} \tag{3.46}$$

Divide entire equation by $N_2 I_1$

$$\frac{\dfrac{N_2 I_2}{N_2 I_1} + \dfrac{I_1 N_1 - 2N_2 I_1}{N_2 I_1}}{\dfrac{N_1 I_1}{N_2 I_1} + \dfrac{N_2 I_2}{N_2 I_1}}$$

$$\frac{\dfrac{I_2}{I_1} + \dfrac{N_1}{N_2} - 2}{\dfrac{N_1}{N_2} + \dfrac{I_2}{I_1}}$$

But

$$\frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

$$= 1 - K$$

These results show that the saving in the copper of autotransformer is $(1 - K)$ times the weight of copper in ordinary transformer. If K is nearer to 1 the saving is very large. Hence, the autotransformer is more useful for transformation ratio near to 1.

EXAMPLE 3.36 **A load of 8 kW is supplied by an autotransformer at 140 V and at upf if the primary voltage is 230 V. Determine the (i) transformation ratio, (ii) secondary current, (iii) primary current, (iv) number of turns across the secondary if the total number of turns is 300, (v) power transformed, and (vi) power conducted directly from supply mains to load.**

Solution Given data

Load = 8 kW

Primary voltage, $V_1 = 230$ V

Secondary voltage, $V_2 = 140$ V

Total number of turns on primary, $N_1 = 300$

(i) Transformation ratio, $K = \frac{V_2}{V_1} = \frac{140}{230} = 0.61$

(ii) Secondary current, $I_2 = \frac{8 \times 1000}{V_2 \cos \phi} = \frac{8000}{140 \times 1} = 57.14$

(iii) Primary current, $I_1 = KI_2 = 0.61 \times 57.14 = 34.86$ A

(iv) Number of turns across secondary, $N_2 = KN_1 = 0.61 \times 300 = 183$

(v) Power transformed = Load $\times (1 - K) = 8\,(1 - 0.61) = 3.12$ kW

(vi) Power conducted directly from the supply mains to load

= Total load supplied by the autotransformer – Power transformed

$= 8 - 3.12 = 4.88$ kW

EXAMPLE 3.37 The primary and secondary voltage of an autotransformer are 500 V and 400 V, respectively. Show with the aid of a diagram the current distribution in the windings when the secondary current is 200 A. Calculate the economy in copper.

Solution Given data

Primary voltage, $V_1 = 500$ V

Secondary voltage, $V_2 = 400$ V

Secondary current, $I_2 = 200$ A

Transformation ratio, $K = \frac{V_2}{V_1} = \frac{400}{500} = 0.8$

Primary current, $I_1 = KI_2 = 0.8 \times 200 = 160$ A

We know that

$$\frac{\text{Weight of copper on autotransformer}}{\text{Weight of copper on ordinary transformer}} = 1 - K$$

Economy in copper

$$= \frac{\text{Weight of copper on ordinary transformer} - \text{Weight of copper on autotransformer}}{\text{Weight of copper on ordinary transformer}}$$

$$= 1 - \frac{\text{Weight of copper on autotransformer}}{\text{Weight of copper on ordinary transformer}}$$

$$= 1 - (1 - K)$$

$$= 1 - (1 - 0.8) = 80\%$$

EXAMPLE 3.38 A 230/330-V autotransformer draws power from a 230-V source and supplies a 6-kW load with a power factor of 0.8 lagging. A second load of 2 kW is supplied at upf.

Neglecting losses, calculate the current drawn by the transformer from the 230-V line and its power factor.

Solution Given data

$V_2 = 230$ V

$V_1 = 330$ V

6 kW at 0.8 pf lagging

2 kW at upf

The connection diagram is shown in Figure 3.32.

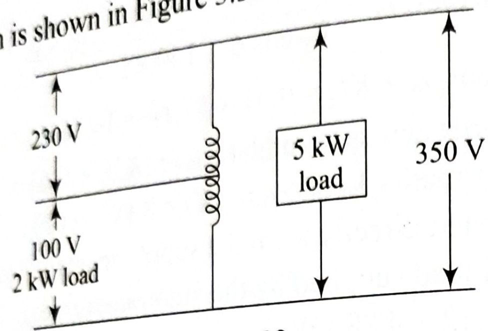

Figure 3.32

$$\text{Transformation ratio for primary and 6-kW load} = \frac{330}{230} = 1.435$$

$$\text{Current drawn by the first load, } I_1 = \frac{6 \times 1000}{330 \times 0.8} = 22.73 \text{ A}$$

$\therefore$ Current drawn by the primary to this supply load $= 22.73 \times 1.435$

$= 32.62$ A

$= I_2 \times K = I_1$ (at 0.8 pf lagging)

$$\text{Current drawn by the secondary load} = \frac{2 \times 1000}{100 \times 1} = 20 \text{ A}$$

$$\text{Transformation ratio for primary and 2-kW load} = \frac{100}{230} = 0.435$$

$\therefore$ Current drawn by primary to supply this load $= 0.435 \times 20 = 8.7$ A at upf

Hence, the total primary current drawn from the 230-V supply is the vector sum of

(i) 32.62 A at 0.8 pf lagging

(ii) 8.7 A at upf

Resolving these currents into their X- and Y-components, we get

X-component $= 32.62 \times 0.8 + 8.7 \times 1 = 34.8$ A

Y-component $= 32.62 \times 0.6 = 19.57$ A

$\therefore$ Total primary current, $I_3 = 34.8 + j\,19.57 = 39.93$ A

$$\text{Power factor} = \frac{34.8}{39.93} = 0.872 \text{ lagging}$$

EXAMPLE 3.39 **A 5-kVA, 1 − ϕ, 50-Hz transformer has a full-load efficiency of 96% and iron of 50 W. The transformer is connected now as an autotransformer to 230-V supply if it delivers 5 kW load at upf to a 115-V circuit, calculate the efficiency of the operation and the current drawn by the high voltage side.**

Solution Given data

Rating of the two-winding transformers = 5 kVA

Full-load efficiency, $\eta = 96\%$

Iron loss = 50 W

The transformer is connected as an autotransformer with 230-V supply.

The load is 5 kW at upf 115-V circuit.

Two-winding transformer is shown in Figure 3.33.

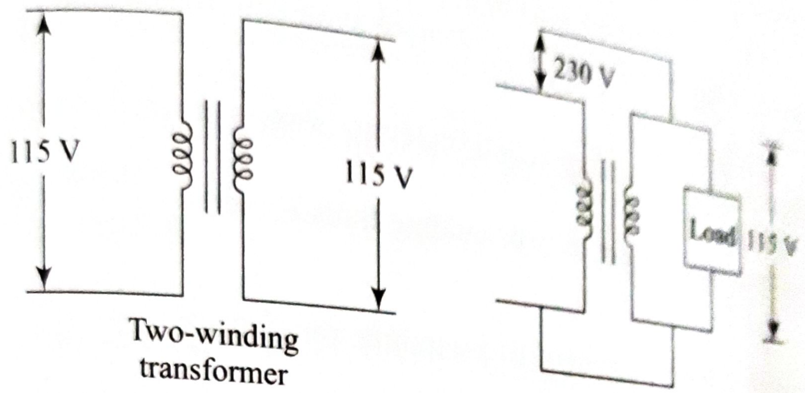

Figure 3.33

The same transformer unit is connected as an autotransformer to a 230 V supply. Since two windings are connected in series hence the voltage across each is 115 V.

In both the connections the iron loss would remain the same since each of the autotransformer windings will carry the currents but half of the currents as the conventional two-winding transformer, the copper loss will be one-fourth of the previous value.

Two-winding transformer:

Efficiency, $\eta = 96\%$

$$0.96 = \frac{\text{Output}}{\text{Output} + \text{Losses}} = 0.96 = \frac{5000}{5000 + \text{Losses}}$$

Losses = (Iron loss + Copper loss)

$$0.96 = \frac{5000}{5000 + (\text{Iron loss} + \text{Copper loss})} = 0.96 = \frac{5000}{5000 + 50 + \text{Copper loss}}$$

$$5000 = 4800 + 48 + \text{Copper loss}$$

Copper loss = 5000 – 4848 = 152 W

Copper loss = 152 W

Autotransformer:

$$\text{Copper loss} = \frac{152}{4} = 38\text{ W}$$

Iron loss = 50 W

$$\therefore \text{ Efficiency} = \frac{5000}{5000 + 38 + 50} \times 100 = 98.27\%$$

EXAMPLE 3.40 **A two-winding transformer is rated at 2300/230 V, 50 kVA. It is reconnected as a step-up autotransformer with 2300 V input. Calculate the rating of the autotransformer and the inductively and conductively transferred powers while delivering the rated output at upf.**

Solution Given data

Two-winding transformer rating = 50 kVA

$V_1 = 2300$ V and $V_2 = 230$ V

The two-winding transformer is connected to an autotransformer

$V_{1auto} = 2300$ V

The inductively and conductively transferred powers at rated output at upf:

$$\text{The rated primary current of two-winding transformer} = \frac{50000}{2300} = 21.74 \text{ A}$$

$$\text{The rated secondary current of two-winding transformer} = \frac{50000}{2300} = 21.74 \text{ A}$$

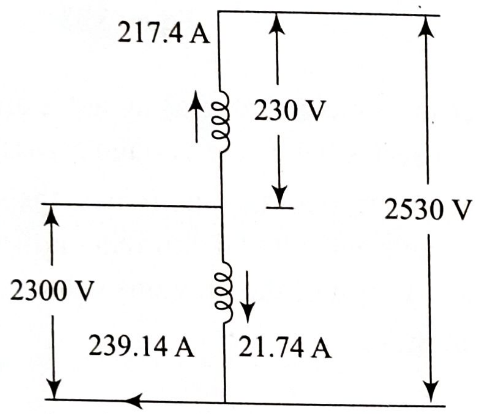

Figure 3.34

With required reconnection the 2300-V winding works as a common winding to input and the output current can be 217.4 A with voltage of 2530 V which means that the output of these autotransformer = 2530 × 217.4 = 550.022 kV

$$\text{The corresponding input current} = 217.4 \times \frac{2530}{2300} = 239.14 \text{ A}$$

$$\text{The ratio of turns in these cases is given by } K = \frac{2530}{2300} = 1.1$$

With step-up job and $k = 1.1$,

$$\frac{\text{Rating of autotransformer}}{\text{Rating of two-winding transformer}} = \frac{K}{K-1} = \frac{1.1}{0.1} = 11$$

This gives the rating of autotransformer of 550 kVA.

At upf, the rated load = 550 W

Out of this, inductively transferred power = Power handled by the common winding

$= (2300 \times 21.74) \times 10^{-3} = 50.002$ kW

Rated output of two-winding transformer = 50.002 kW

The remaining power of 500 kW is conductively transferred as is clear from the division of current at the input mode out of the total current of 239.14 A from the source of 217.43 A goes straight to the output of the remaining current of 21.74 A is through the common and inductive path as marked in Figure 3.34.

3.15 PARALLEL OPERATION OF TRANSFORMERS

Sometimes the load on substation is more than the rating of the existing transformer. Here, one more transformer is needed to share the load which is connected in parallel so the operation of parallel transformers should have the following conditions:

(i) The same polarity
(ii) The same voltage ratio
(iii) The same phase sequence
(iv) Equal per unit impedances in magnitude and phase angle

(a) **Polarity test.** If two transformers are to be connected in parallel they should not have the same instantaneous value. If they have the same instantaneous value then a dead short circuit is formed. So, before operating the transformers in parallel check the polarity. Their polarity may be either positive or negative.

(b) **Voltage ratio.** If the two transformers are connected in parallel, their voltage ratio must be the same, otherwise circulating currents will exist. Once circulation currents exist the copper losses will increase finally and the efficiency of the transformer will decrease.

(c) **Phase sequence.** The phase sequence will be applicable for three-phase transformers.

(d) **Per unit impedances.** Transformers connected in parallel may not have the same kVA rating. But they will share the load on the ratio of their rating. If the primary and secondary sides of transformers are parallel with their per unit impedances then their voltage regulation will be same, otherwise it will differ.

If X/R ratios of two transformers are not equal, they would not operate at the same power factor.

3.15.1 Load Sharing in Equal kVA

Figure 3.35 shows the equivalent circuit for two transformers A and B which are connected in parallel. Transformer A supplies load current of I_A and transformer B supplies a current of I_B to load. Total load current is $I = I_A + I_B$ of impedance Z. Let the voltage across load is V. The secondary emfs are E_A and E_B with impedances Z_A and Z_B, respectively. If E_A and E_B are equal in magnitude and in phase, then there will not be any circulating current between the transformers.

Then

$$E_A = E_B$$
$$I_A Z_A = I_B Z_B$$

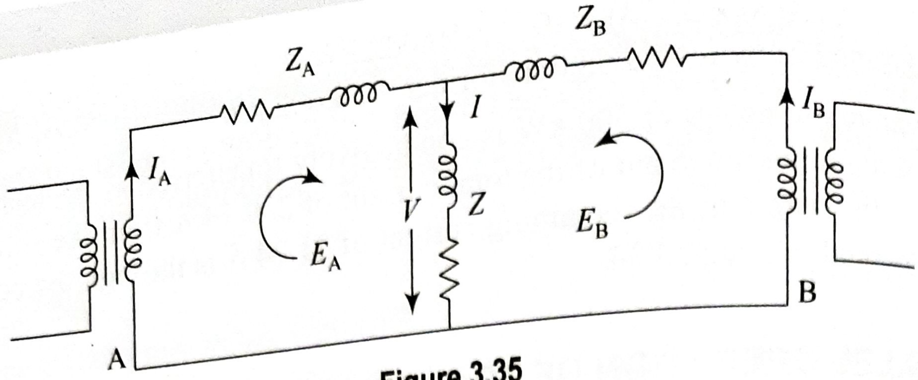

Figure 3.35

But

$$I = I_A + I_B$$

From the current division principle,

$$I_A = I \cdot \frac{Z_B}{Z_A + Z_B} \quad (3.4$$

Similarly,

$$I_B = I \cdot \frac{Z_A}{Z_A + Z_B} \quad (3.4$$

If Eqs. (3.47) and (3.48) are multiplied by V, then we get VA delivered by each transform VI_A is W_A, i.e., VA delivered by transformer A and VI_B is W_B, i.e., VA delivered by transformer Then VI is the total volt-ampere.

$$W_A = W \cdot \frac{Z_B}{Z_A + Z_B}$$

$$W_B = W \cdot \frac{Z_A}{Z_A + Z_B}$$

In the above equations, Z_A and Z_B can be expressed in ohms or in pu. From the above equation it can be concluded that the transformers will share the load in proportion to their kVA rating. B leakage impedances are inversely proportional to their kVA rating.

3.15.2 Load Sharing in Unequal kVA

Applying the KVL to the left loop of Figure 3.35,

$$E_A = I_A Z_A + IZ$$
$$= I_A Z_A + (I_A + I_B)Z$$

Similarly,

$$E_B = I_B Z_B + IZ$$
$$= I_B Z_B + (I_A + I_B)Z \quad (3.49$$

The solution for the above equation

$$I_A = \frac{E_A Z_B + (E_A - E_B)Z}{Z_A Z_B + (Z_A + Z_B)Z}$$

$$I_B = \frac{E_A Z_A - (E_A - E_B)Z}{Z_A Z_B + (Z_A + Z_B)Z} \tag{3.50}$$

$$I = \frac{E_A Z_B + E_B Z_A}{Z_A Z_B + (Z_A + Z_B)Z} \tag{3.51}$$

But

$$V = I.Z$$

$$= \frac{E_A Z_B + E_B Z_A}{\frac{Z_A Z_B}{Z} + (Z_A + Z_B)}$$

From the above equation, it can be observed that if E_A and E_B are not equal then the circulating current will exist between, and the current is given by

$$I = \frac{(E_A - E_B) Z}{Z_A Z_B + (Z_A + Z_B)Z} \tag{3.52}$$

3.16 SPECIAL TRANSFORMERS

These are classified as two types based on the application in electronics and telecommunications:

(i) Audio frequency transformers

(ii) Current transformers

3.16.1 Audio Frequency Transformers

These transformers are used in audio frequency ranges from 20 to 20,000 Hz. They are used in electronic and telecommunication (control systems and measurements are also used for boosting the voltage or impedance) circuits. Basically, they are small in size and have core losses.

The most important point in electronic circuit is distortion and it should be as low as possible. The input voltage should be amplified equally and the phase should be zero. For all frequencies, here two characteristics are important. One is amplitude frequency characteristics (ratio of output voltage to input versus frequency) and other is phase characteristics (i.e., phase angle between output voltages and input voltage versus frequency) and these are used for frequency response. Generally, a flat response is desirable over audio frequency response and this frequency response can be determined by exact equivalent circuit diagram.

At low frequencies the leakage reactances are not important but the shifting of magnetizing branch is important. The ratio of V_2/V_1 decreases with decrease in frequency, intermediate frequency range; none of the inductances is effective and the equivalent circuit reduces to purely resistive circuit. The ratio of V_2/V_1 is constant in this range. In high-frequency ranges the leakage reactance becomes more. Due to inductances and capacitances the ratio of V_2/V_1 decreases with increase in frequency, the response is shown in Figure 3.36.

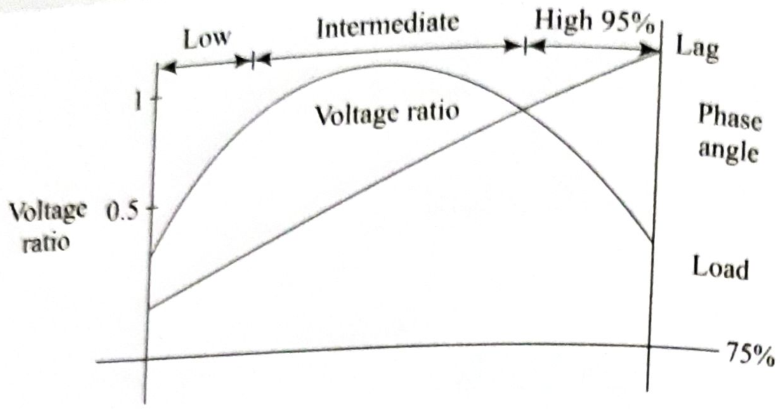

Figure 3.36

3.16.2 Current Transformers

The function of a current transformer is to step-down a high-magnitude alternating current. It can be easily measured by a low-range ammeter. The primary winding of a current transformer will consist of a few turns of thick wire and is connected in series with the load. The secondary winding has a large number of turns of thick wire and is usually in the range of 1–5 A. Therefore, a current transformer is a step-up transformer. Its secondary is always short circuited by a small resistance or relay coil.

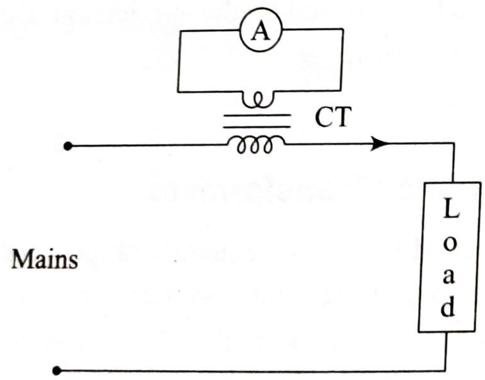

Figure 3.37 Current transformer

CHAPTER AT A GLANCE

- Transformer is a static device used for power transmission from one voltage level to another without changing the supply frequency.
- Transformer basically works on the principle of electromagnetic induction.
- It mainly consists of two windings and one core.
- These can be used at different applications such as power transformers, distribution transformers and instrument transformers.
- The induced emf in primary winding is given by $E_1 = 4.44 f N_1 \phi_m$ volts.
- The induced emf in secondary winding is given by $E_2 = 4.44 f N_2 \phi_m$ volts.
- The transformation ratio is given by $N_2/N_1 = E_2/E_1 = I_1/I_2 = k$.
- An ideal transformer is one which does not have any losses.

- A real transformer is one which has losses.
- The magnitude of the no-load current of a transformer is given by $I_0 = \sqrt{I_w^2 + I_m^2}$.
- There are two types of losses in transformer: one is iron loss and the other is copper loss.
- Iron losses are the sum of hysteresis and eddy current loss, $W_h = kB_{max}^{\eta} fv + W_e = kB_{max}^2 f^2 t^2$.
- The total copper losses in a transformer are given by $I_1^2 R_1 + I_2^2 R_2$.
- The net flux flowing through a transformer is constant, i.e., whether it is loaded or unloaded.
- Copper losses are dependent on loading condition where as iron losses are independent and constant.
- Mathematically, an analysis can be made if we know the electrical equivalent circuit of the transformer.
- The total resistance referred to a primary is given by $R_{01} = R_1 + R_2' = R_1 + (R_2/K^2)$.
- The total resistance referred to a secondary is given by $R_{02} = R_2 + R_1' = R_2 + R_1K^2$.
- The total reactance referred to a primary is given by $X_{01} = X_1 + X_2' = X_1 + (X_2/K^2)$.
- The rotal reactance referred to a secondary is given by $X_{02} = X_2 + X_1' = X_2 + X_1 K^2$.
- Without loading a transformer, the efficiency can be predetermined with open-circuit test and short-circuit test.
- The efficiency of a transformer is defined as the ratio of output to input in percent.

$$\eta = \frac{\text{Output power}}{\text{Output power + Total losses}}$$

- The condition for maximum efficiency in copper loss is equal to iron loss.
- The kVA supplied at maximum efficiency = kVA rating at fl $\sqrt{\frac{P_i}{P_{cu} \text{ at fl}}}$.
- Regulation is another performance criterion of the transformer. It is defined as the ratio of rise in the voltage at the load end when the load is thrown off to the full-load voltage.

$$\frac{{}_0V_2 - V_2}{V_2} \times 100 = \frac{IR\cos\phi + IX\sin\phi}{V_2} \times 100$$

- Regulation will be high for lagging power factor and low and sometimes even negative for leading power factors.
- All-day efficiency of a transformer is defined as the ratio of output energy to the input energy for a period of 24 h, η_{AD} = output in kWh/input in kWh.
- In autotransformer, only one winding is used in primary winding and secondary winding.
- The saving in the copper of autotransformer is $(1 - K)$ times the weight of copper in an ordinary transformer.
- When the load is heavy, then two or more transformers can be operated in parallel.
- While transformers are operating in parallel they must satisfy certain conditions: the polarity, voltage ratio, and phase sequence must be same.
- These can also be used as instrument transformers such as current transformers (CT) and potential transformers (PT).