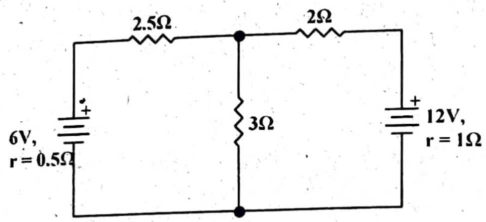
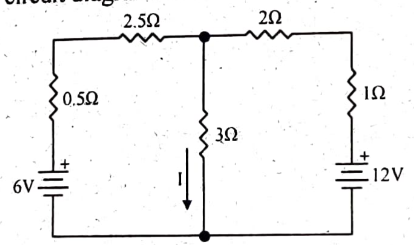
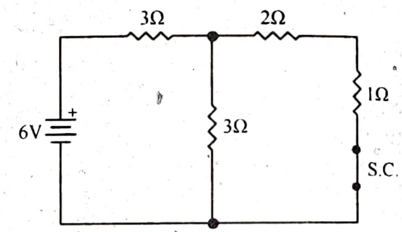
Short Answer Type Questions

2.1. Calculate the current through 3Ω resistor by Superposition Theorem. **[WBUT 2007]**



Answer:
To find I by using superposition Theorem.
First switch on 6V and switch off 12V.
The circuit diagram is shown below:



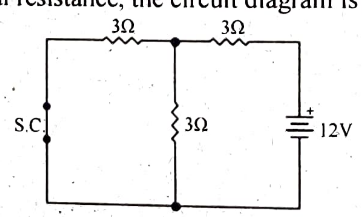


Resistance of the circuit as observed by the source $= \left(3+\frac{3\times 3}{3+3}\right)\Omega = 4.5\Omega$

$\therefore$ Source current $= \frac{6}{4.5}A = \frac{4}{3}A$.

Current through 3Ω resistor $= \left(\frac{3}{3+3}\times\frac{4}{3}\right)A = \frac{2}{3}A$

Next, find the current through the 3Ω resistor due to 12V source.
Replacing 6V by its internal resistance, the circuit diagram is as follows:



Resistance observed by source $= \left(3+\frac{3\times 3}{3+3}\right)\Omega = 4.5\Omega$.

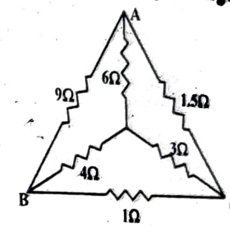
Source current $= \frac{12}{4.5}A = \frac{8}{3}A$.

Current through 3Ω resistor $= \left(\frac{8}{3}\times\frac{3}{3+3}\right)A = \frac{4}{3}A$.

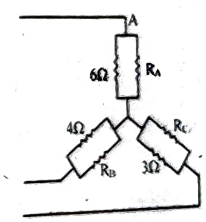
It is do be noted that current due to both voltage sources flows through the same direction for the given resistance.

$\therefore$ Total current thro 3Ω resistor $= \left(\frac{2}{3}+\frac{4}{3}\right)A = \frac{6}{3}A = 2A$.

2.2. A network of resistance is formed as given figure. Compute the resistance measured between A & B. **[WBUT 2008(EVEN)]**



Answer:
The network can be redrawn as



$$R_A = \frac{9\times 1.5}{9+1+1.5} = \frac{9\times 1.5}{11.5}\Omega = 1.1739\Omega$$

$$R_B = \frac{9\times 1}{11.5}\Omega = 0.7826\Omega$$

$$R_C = \frac{1.5}{11.5}\Omega = 0.1304\Omega$$

The equivalent resistance between A & B is

$$\frac{6\times 1.1735}{6+1.1739} + \frac{4\times 0.1304}{4+0.1304}\Omega = (0.9818+0.1263)\Omega = 1.1081\Omega$$

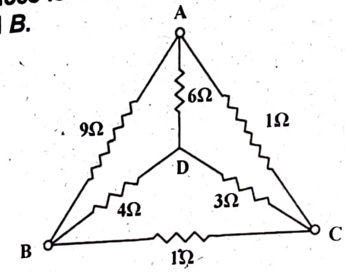
2.3. Define **[WBUT 2009]**
- a) **Linear circuit**
- b) **Non-linear circuit**
- c) **Bilateral circuit**
- d) **Unilateral circuit**
- e) **Network**

Answer:
a) ***Linear Circuit:*** A linear circuit is one which obeys the principle of superposition and homogeneity.
b) ***Non – linear circuit:*** It is that circuit which does not obey the principle of superposition.
c) ***Bilateral Circuit:*** A bilateral circuit is one whose properties or characteristics are same in either direction. It can work in both directions.

d) ***Unilateral Circuit:*** It is that circuit whose properties and characteristics change with the direction of operation i.e. it can work in one direction only.

e) ***Network:*** A network is a collection of interconnected components.

2.5. A network of resistances is formed as shown in figure. Compute the resistance between the points *A* and *B*. **[WBUT 2011]**



Answer:

Convert the star branch ADBC into equivalent delta, we get

$$R_{AC}=\left(6+3+\frac{6\times 3}{4}\right)\Omega=(9+4.5)\Omega=13.5\Omega$$

$$R_{BC}=\left(4+3+\frac{4\times 3}{6}\right)\Omega=(7+2)\Omega=9\Omega$$

$$R_{AB}=\left(6+4+\frac{6\times 4}{3}\right)\Omega=(10+8)\Omega=18\Omega$$

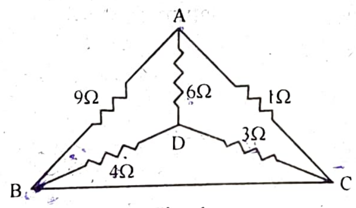
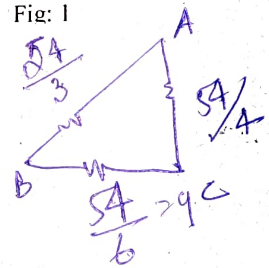


Fig: 1



Redrawing the circuit, we get

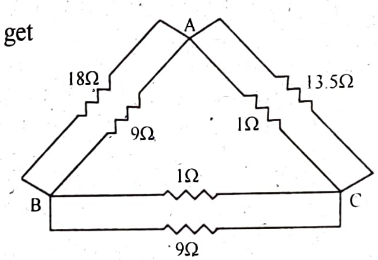


Fig: 2

Evaluating the equivalent resistances of the parallel branches, we get

$$R_{AC}=\frac{13.5\times 1}{14.5}\Omega=0.93\Omega$$

$$R_{BC}=\frac{1\times 9}{1+9}\Omega=0.9\Omega$$

$$R_{AB}=\frac{9\times 18}{27}\Omega=6\Omega$$

Resistance between A & B.

$$=\frac{(0.93+0.9)\times 6}{0.93+0.9+6}\Omega=\frac{10.98}{7.83}\Omega=1.4\Omega$$

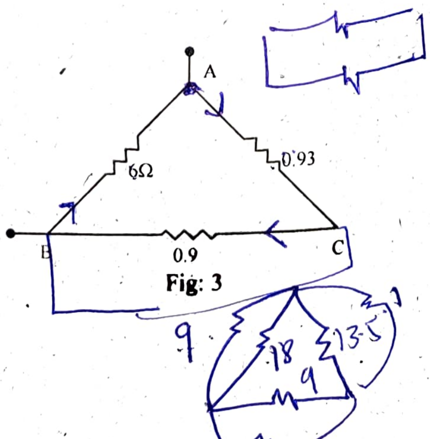


Fig: 3

2.6 Determine the value of *R* in Figure 1 such that 4Ω resistor consumes maximum power. **[WBUT 2011]**

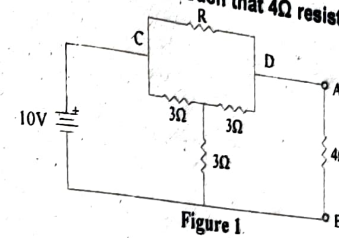
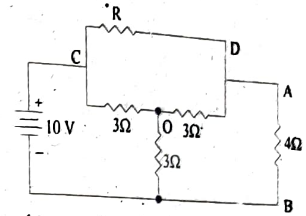


Figure 1

Answer:

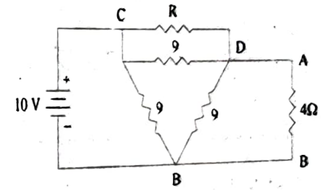


Taking B as the datum and converting the star connection of resistances into equivalent delta, we get

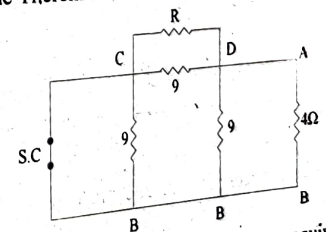
$$R_{CD}=\left(3+3+\frac{3\times 3}{3}\right)\Omega=9\Omega$$

$$R_{CB}=\left(3+3+\frac{3\times 3}{3}\right)\Omega=9\Omega$$

$$R_{DB}=\left(3+3+\frac{3\times 3}{3}\right)\Omega=9\Omega$$



In order to determine the Theremin resistance of the network as viewed from AB



The resistance $R_{CB}=9\Omega$ is series with a short circuit, hence equivalent resistance =0.

The equivalent resistance between C & D= $\frac{9R}{9+R}$

The equivalent resistance between D & B= $\frac{\frac{9R\times 9}{9+R}}{\frac{9R}{9+R}+9}=\frac{81R}{81+18R}$

For maximum power transfer $\frac{81R}{81+18R}=4$

$$\Rightarrow \frac{9R}{9+2R}=4$$
$$\Rightarrow 9R=36+8R$$
$$\text{or, } R=36\Omega$$

2.7. Establish the equivalence between Thevenin's and Norton's theorems. **[WBUT 2012]**

Answer:

Equivalence of Thevenin's & Norton's Theorems

Figure (i) shows the equivalency of Thevenin's and Norton's Theorems. It can be proved that the equivalent circuits given by Thevenin's and Norton's theorem yield exactly the same current and same voltage in the load impedance and are therefore effectively identical to one another. In any particular problem, either theorem can therefore be used. In most cases Thevenin's theorem is the easier to apply, although when the network impedance is high compared with the load impedance, the Norton's theorem concept may simplify calculations.

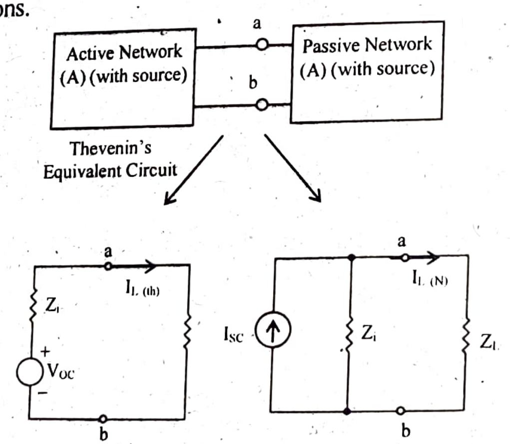


Fig: (i) Equivalency of Thevenin's and Norton's circuit

From Fig. (i), by applying Thevenin's theorem the load current is given by

$$I_{L(Th)}=\frac{V_{OC}}{Z_i+Z_L} \quad \text{...(i)}$$

where V_{OC} = Open circuit voltage (Thevenin's equivalent voltage source)

Z_i = Thevenin's equivalent impedance or (resistance for d.c. circuit)

Z_L = Load impedance of the total network.

On short – circuiting the terminals a and b of the Thevenin's equivalent circuit,

$$I_{SC}=\frac{V_{OC}}{Z_i} \quad \text{...(ii)}$$

From equation (ii),

$$V_{OC}=I_{SC}\times Z_i \quad \text{...(iii)}$$

However from Norton's equivalent circuit (i), the load current is given by

$$I_L(N)=\frac{I_{SC}\times Z_i}{Z_i+Z_L} \quad \text{...(iv)}$$

Substituting the equation (iii) in equation (iv),

$$I_L(N)=\frac{V_{OC}}{Z_i+Z_L} \quad \text{...(v)}$$

Compare equation (i) and equation (v),

$$I_{L(Th)}\equiv I_{L(N)} \quad \text{...(vi)}$$

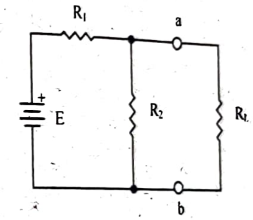
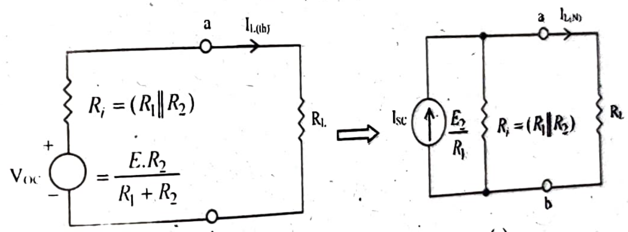


Fig: (ii) (a) Original Circuit



(b) (c)

Fig: (ii)(b) Thevenin's Equivalent Circuit,

Fig. (ii) (c) Norton's Equivalent Circuit

Thus for any passive network, being connected to an active network, one can have equivalent representation of Norton's equivalent or Thevenin's equivalent circuit (i.e., both the theorems are equivalent to each other). For easy understanding, a simple example is given below (Fig. (ii)).

From Fig. (ii) (b), the load current, $I_{L(Th)}=\dfrac{\dfrac{E.R_2}{(R_1+R_2)}}{\dfrac{R_1R_2}{R_1+R_2}+R_L}$

[$\because$ In Fig. (ii) (a), removing R_L the equivalent resistance 'R_i' looking back to the network from a-b, is $\{(R_1R_2)/(R_1+R_2)\}$ and V_{OC} is then

$$\left\{\left(\frac{E}{R_1+R_2}\right).R_2\right\}=\frac{ER_2}{R_1R_2+R_1R_L+R_2R_L}$$

On the other hand, from Fig. (ii) (c), the load current is given by

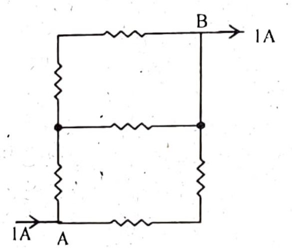
$$I_{L(N)}=\frac{\frac{E}{R_1}\times\frac{R_1R_2}{R_1+R_2}}{\frac{R_1R_2}{R_1+R_2}}=\frac{\frac{ER_2}{R_1+R_2}}{\frac{R_1R_2+R_1R_2+R_2R_2}{R_1+R_2}}$$

[$\because$ Removing R_L from a-b terminal and applying s/c at a-b, current through the terminals a-b is $I_{S/C}$ i.e., (E/R_1) while the internal resistance of the network is

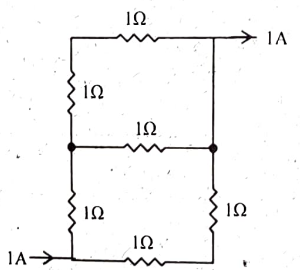
$$=\frac{ER_2}{R_1R_2+R_1R_L+R_2R_L}$$

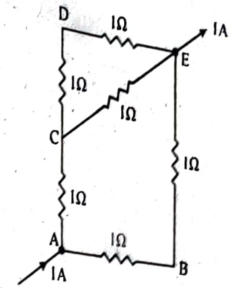
$$\therefore I_{L(Th)}\equiv I_{L(N)}=\frac{ER_2}{R_1R_2+R_1R_L+R_2R_L}$$

2.8. Find V_{AB} from the circuit if all the resistances are of same value of 1 ohm. **[WBUT 2012]**

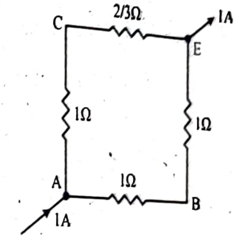


Answer:





The circuit can be redrawn as



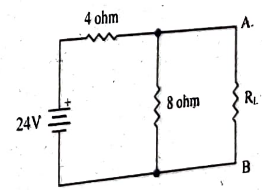
So equivalent resistance between C & E is

$$\frac{2\times1}{2+1}\Omega=\frac{2}{3}\Omega$$

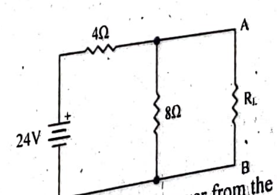
So equivalent resistance between A & E is

$$\left(\frac{\left(1+\frac{2}{3}\right)\times2}{1+\frac{2}{3}+2}\right)\Omega=\frac{10/3}{11/3}\Omega=\frac{10}{11}\Omega$$

2.9. Find the value of load resistance (R_L) for which the power source will supply maximum power. Also find the value of the maximum power for the network as shown below **[WBUT 2012]**

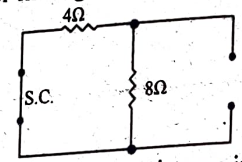


Answer:



The load resistance which will draw maximum power from the source = source resistance as viewed from the load.

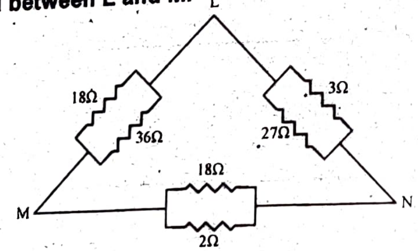
The circuit is redrawn for finding the source resistance.



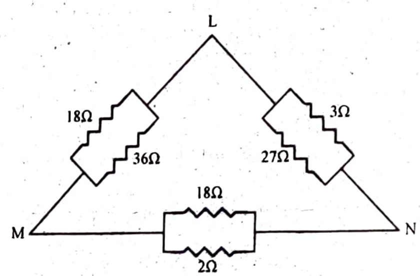
The equivalent resistance = Thevenin resistance if the source network = R_{th}

$$=\frac{8\times 4}{8+4}=\frac{32}{12}=\frac{8}{3}\Omega=2.67\Omega.$$

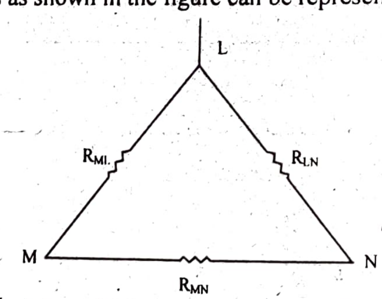
2.10 A network of resistance is formed as given in the figure. Compute the resistance measured between L and M. **[WBUT 2014]**



Answer:



The block of resistances as shown in the figure can be represented as



Where R_{LN} is the equivalent parallel resistance of 27Ω and 3Ω

$$R_{LN}=\frac{27\times 3}{27+3}\Omega=\frac{27\times 3}{30}\Omega=2.7\Omega$$

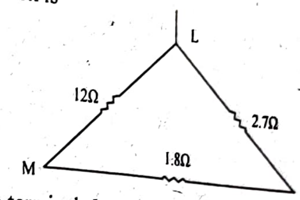
R_{MN} is the equivalent parallel resistance of 18Ω and 2Ω

$$R_{MN}=\frac{18\times 2}{18+2}\Omega=\frac{18\times 2}{20}\Omega=1.8\Omega$$

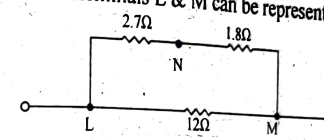
R_{ML} is the equipment parallel resistance of 18Ω and 36Ω

$$R_{ML}=\frac{18\times 36}{18+36}\Omega=\frac{18\times 36}{54}\Omega=12\Omega$$

Hence the equivalent block is

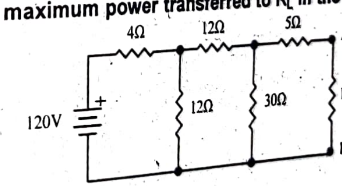


The resistance between terminals L & M can be represented as

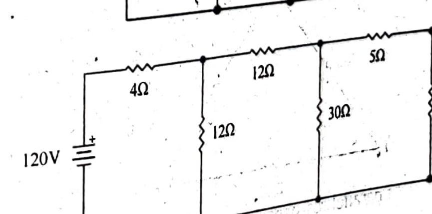


$$R_{LM}=\frac{4.5\times 12}{4.5+12}\Omega=\frac{12\times 4.5}{16.5}\Omega=3.27\Omega.$$

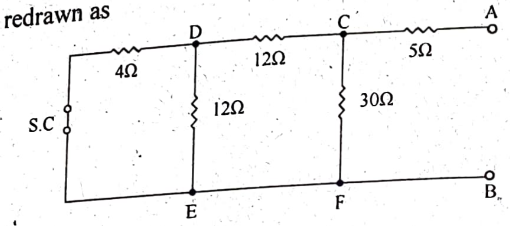
2.11. Obtain the maximum power transferred to R_L in the circuit and also the value of R_L. **[WBUT 2014]**



Answer:

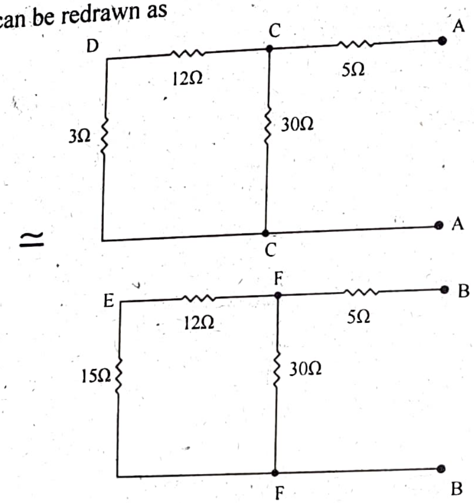


In order to find the resistance for which maximum power is to be transformed to R_2, the circuit needs to be redrawn as



The equivalent resistance between D & E is $\frac{4\times12}{4+12}\Omega=\frac{48}{16}\Omega=3\Omega$

The circuit can be redrawn as

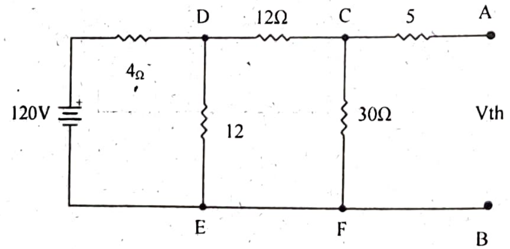


Equivalent resistance between C & F is $\frac{30\times15}{30+15}\Omega=\frac{450\Omega}{45}=10\Omega$

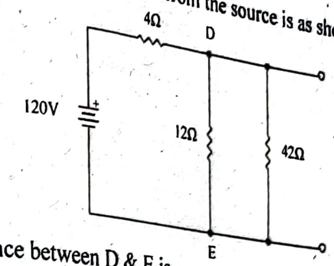
Hence equivalent resistance between A & B is (10+5)Ω = 15Ω and resistance is the Thevenin resistance R_{th} between A & B.

For maximum power transfer to R_L $R_{th}=R_L=10\Omega$

In order to find the maximum power transfer, the open-circuit voltage V_{th} is to be found.



The resistance between D & E as observed from the source is as shown below.



The equivalent resistance between D & E is

$\frac{12\times42}{12+42}\Omega=\frac{12\times42}{54}\Omega=\frac{28}{3}\Omega$

Resistance observed from source $=\left(\frac{28}{3}+4\right)\Omega=\frac{40}{3}\Omega$

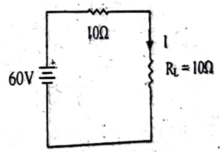
Hence, source current $=\frac{120}{40}\times3A=9A$

Current flowing through the series pair of 12Ω & 30Ω between D & E(F)

$=\frac{9\times12}{54}A=2A$

Hence voltage drop at 30Ω resistance between C & F = (30×2) = 60V

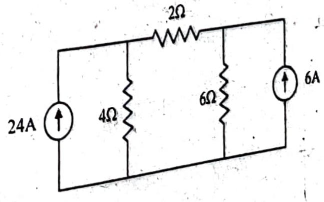
The circuit can be redrawn with V_{th} & R_{th} as



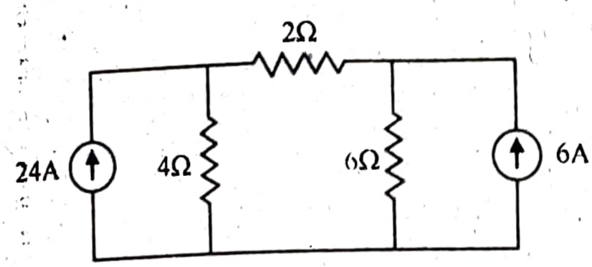
Current $I=\frac{60}{(10+10)}A=3A$

Power transferred to $R_L=I^2R_L=(9\times10)W=90W$

2.12. Applying Superposition theorem compute the current through 2Ω resistor. **[WBUT 2015]**



Answer:



For applying super position theorem, one source has to be alternatively turned off while keeping the other on.
At first the 24A is kept on and the 6A is switched off or replaced by an open circuit.

When 24A is on, by current division method, current through 2Ω resistor $= \frac{24 \times 4}{12} A = 8A$ from a to b.

When 6A is turned on and 24a is switched off, current through 2Ω resistor $= \frac{6 \times 6}{12} A = 3A$ from b to a.

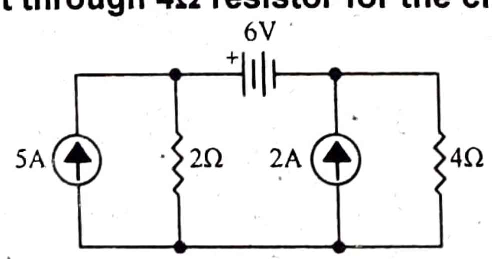
So the net current $= (8-3) = 5$ A from a to b.

2.13. Give an example of passive element. **[WBUT 2015]**

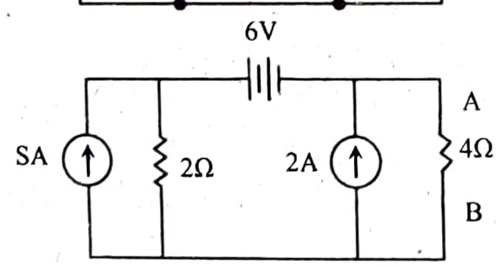
Answer:

Resistance R, Inductance (L) and capacitance (C)

2.14. Determine the current through 4Ω resistor for the circuit shown below: **[WBUT 2016]**



Answer:



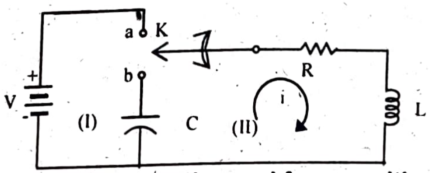
For $6V$, current through 4Ω resistor $= \frac{6}{6} A = 1A$ form $B\,to\,A$

For $5A$ current source, current through 4Ω resistor $= \frac{2 \times 5}{2+4} A = \frac{10}{6} A$ from $A\,to\,B$

For $2A$ current source, current through to Ω resistor $= \frac{2 \times 2}{6} A$ from $A\,to\,B$

So, the net current $= \left(\frac{10+4}{6} - 1\right) A$ from $A\,to\,B = 1.33A$ from $A\,to\,B$

2.16. **[MODEL QUESTION]**



In the network of the figure, K is changed from position a to b at $t=0$. Solve for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t=0$, if $R=1000\Omega$, $L=1H$, $C=0.1\mu F$, $V=100V$.

Answer:

Considering the position of the switch at b at $t=0^+$ the network equation applying KVL in (I) is as follows:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \qquad \ldots (1)$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 \qquad \ldots (2)$$

Evaluation of $\frac{d^2i}{dt^2}$ at $t=0^+$ requires knowledge of initial conditions.

Since this is an inductive circuit, $i(t=0^+) = i(t=0^-)$

Assuming the circuit (I) to be at steady – state condition at the juncture of changing the switch position

$i(t=0^-) = i(t=0^+) = V/R$ ($\because$ L behaves as short circuit under steady-state conditions)

Substituting the value of i in equation (2), we get,

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{V}{RC} = 0$$

Consider equation (1),

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

Since, $v_C(t=0^-) = v_C(t=0^+) = 0$

i.e. voltage across capacitance cannot change instantaneously equation (1) can be written as

$$Ri + L\frac{di}{dt} = 0\left[\frac{1}{C}\int i\,dt = v_C(t=0^+)\right] = 0$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = -\left.\frac{R}{L^2}\right|_{t=0^+} = \frac{-R.V}{L\ \ R} = -\frac{V}{L}$$

Substituting the value of $\left.\frac{di}{dt}\right|_{t=0^2}$ in equation (2),

We get

$$\left.\frac{d^2i}{dt^2}\right|_{t=0^+} = -\frac{R}{L}\left.\frac{di}{dt}\right|_{t=0^+} - \frac{V}{RC}$$

$$\left.\frac{d^2i}{dt^2}\right|_{t=0^+} = -\frac{R}{L}\left(-\frac{V}{L}\right) - \frac{V}{RC}$$

$$= \frac{VR}{L^2} - \frac{V}{RC}$$

$$= \frac{VR}{L}\left(\frac{1}{L} - \frac{L}{R^2C}\right)$$

$$= \frac{VR}{L}\left(\frac{1}{L} - \frac{L}{R}\cdot\frac{1}{RC}\right)$$

Putting the values of V, R, L & C we get

$$i(t=0^+) = \frac{100}{1000} = 0.1A$$

$$\frac{di}{dt}(t=0^-) = -100\,A\,sec^{-1}$$

$$\frac{d^2i}{dt^2} = \frac{100\times10^3}{1}\left(1 - \frac{1}{10^6\times0.1\times10^{-6}}\right) = -9\times10^5 = -0.9\times10^6\,A/sec^2$$

Long Answer Type Questions

3.1. Explain Thevenin's Theorem. **[WBUT 2005]**

OR,

State and explain Thevenin's Theorem. **[WBUT 2015]**

Answer:

Thevenin's theorem is used to simplify any two-terminal network comprising linear, passive bilateral parameters, being acted upon by many sources. A part of the complicated network can be substituted by a single voltage source in series with an equivalent resistance thereby helping easing the analysis of an electrical network.

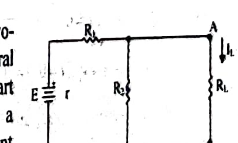


Fig: I(a)

To understand the theorem, the circuit as shown in fig. I(a) may be considered and it is required to find out current I_L through the resistance R_L.

In order to proceed in the matter, following steps are taken:

Step: 1

To disconnect and remove the resistance of the load resistors under consideration from the terminals A & B as shown in fig. I(b) to get open current voltage.

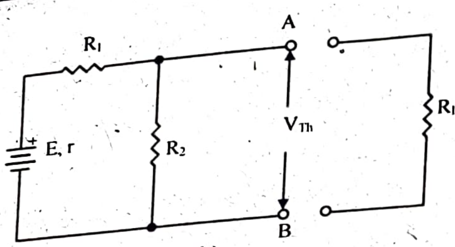


Fig: 1(b)

Step: 2
To calculate open circuit voltage V_{Th} across the free terminals A & B as shown in fig. 1(b). In this situation, the current when terminals A & B are opened is given by

$$I = \frac{E}{(r + R_1 + R_2)}$$

∴ open circuit voltage,

$$V_{Th} = \text{Potential drop across } R_2 = IR_2 = \frac{ER_2}{(r + R_1 + R_2)} \quad \cdots(i)$$

Step: 3
To remove the voltage source from the circuit leaving behind only its internal resistance (r) as shown in fig. 1(c). Next, to view the circuit inwards from the open terminals A & B and to find that the circuit consists of two parallel paths – one consisting resistance R_1 and r in series. Consequently the equivalent circuit resistance (R_{Th}) as viewed from the open – terminals A & B is given by:

$$R_{Th} = \frac{R_2(R_1 + r)}{R_2 + (R_1 + 2r)} \quad \cdots(ii)$$

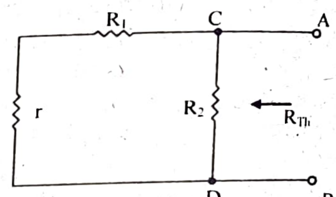


Fig: 1(c)

In other words, the entire open circuit network as viewed from A & B is reduced to a single source, called Thevenin's source, whose voltage and internal resistances are respectively given by V_{Th} and R_{Th} as depicted in fig. 1(d).

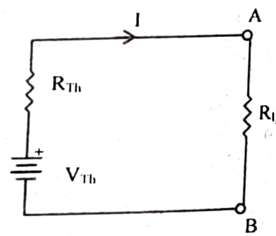


Fig: 1(d) Thevenin's Source

Step: 4
To connect back load resistance under consideration (R_L) across the terminals A & B from where it was removed earlier

Then the current flowing through the load resistance R_L is given by: $I = \dfrac{V_{Th}}{R_{Th} + R_L}$

Now, from the foregoing the Thevenin's in general can be stated as under:
The current I flowing through a resistance R_L connected across any two terminals (say A & B) of a passive, linear, bilateral network containing one or more sources of current/voltage is given by:

$I = \dfrac{V_{Th}}{R_L + R_{Th}}$ where V_{Th} = open circuit voltage between A & B i.e. with R_L disconnected and R_{Th} = equivalent resistance of the network with R_L and all voltage sources are removed, by their respective internal resistances.

3.2. State and explain Superposition theorem. Mention its limitations.

[WBUT 2006, 2009(EVEN)]

Answer:

Super Position Theorem
The theorem states that if a number of sources are acting simultaneously in any network comprising linear, bilateral and passive elements, then each source acts independently of the others.
Thus, the resultant value of current through (or voltage across) any circuit parameter can be obtained by "superposing" the current (or voltage) due to each source in the network, while all the other sources are replaced by their internal impedances.
To visualize the application of the super position theorem, the circuit as shown on fig.(a) below may be referred to in which there are two EMF sources in parallel supplying current to a load resistance R.
The EMF sources have open circuit EMF's E_1 & E_2 and internal resistances r_1 & r_2 respectively.

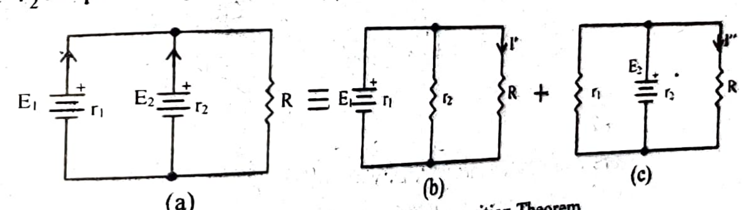


Fig: 1 Illustration of Superposition Theorem

Step: 1
The circuit is resolved into two circuits shown in fig. 1(b) and 1(c).

Step: 2
Fig. 1.(b) shows the circuit with EMF source E_2 removed and replaced by its internal resistance r_2. The partial load current in this case is

$$I' = \frac{E_1}{r_1 + \dfrac{Rr_2}{R + r_2}} \times \frac{r_2}{R + r_2} = \frac{E_1 r_2}{Rr_1 + r_1 r_2 + Rr_2}$$

Step: 3
In fig. 1.(c), E_1 is removed and replaced by its internal resistance r_1, and the partial load current is: $I'' = \frac{E_2 r_1}{Rr_1 + r_1 r_2 + Rr_2}$

Step: 4
Applying the super position theorem, the, load current in R is given by:

$$I = I' + I'' = \frac{E_1 r_2 + E_2 r_1}{Rr_1 + r_1 r_2 + Rr_2}$$

Limitations: Internal resistance of an ideal voltage source (i.e. one from which any amount of current can be drawn, but its voltage remains constant) is zero. Internal resistance of an ideal current source is infinity.

3.3. State & prove Maximum Power Transfer Theorem for D.C. networks.
[WBUT 2007, 2008, 2009(EVEN)]

OR,

State and prove maximum power transfer theorem. [WBUT 2010, 2015]

OR,

State and prove maximum power transfer theorem. Show that under maximum power transfer condition, efficiency is 50%. [WBUT 2013]

Answer:

Maximum Power Transfer Theorem

This theorem is applicable to both A.C and D.C circuits. When applied to D.C. circuits, the theorem can be stated as under:

A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all sources of energy removed leaving behind their internal resistances.

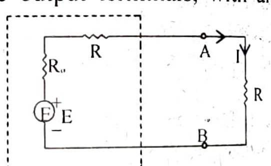


Fig: 1

A load resistance of R_L is connected across the terminals A & B of a network as shown in fig. and the network consists of a d.c. source of EMF 'E' and internal resistance R_g and a series resistance R. It is presumed that

$R_i = R + R_g$ = internal resistance of the network as viewed from A & B.

According to this theorem, R_L will abstract maximum power from the network when $R_L = R_i$.

Proof

Circuit current $I = \frac{E}{R_L + R_i}$

Power consumed by the load is: $P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2}$ ....(1)

For P_L to be maximum, $\frac{dP_L}{dR_L} = 0$

Differentiating equation (1) above

$$\frac{dP_L}{dR_L} = E^2\left[\frac{1}{(R_L + R_i)^2} + R_L\left(\frac{-2}{(R_L + R_i)^3}\right)\right] = E^2\left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3}\right]$$

$$0 = E^2\left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3}\right]$$

or, $2R_L = R_L + R_i$ or, $R_L = R_i$

under these conditions the voltage across load is half the open circuit voltage at the terminals A & B

$\therefore$ Maximum power is $P_{L_{max}} = \frac{E^2 R_L}{4R_L^2} = \frac{E^2}{4R_L}$

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{I^2 R_L}{I^2(R_L + R_i)}$$

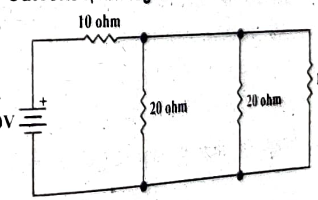
$$= \frac{I^2 R_L}{I^2 \times 2R_L} = \frac{1}{2} = 50\% \qquad (\because R_L = R_i)$$

Hence, the efficiency of power transfer

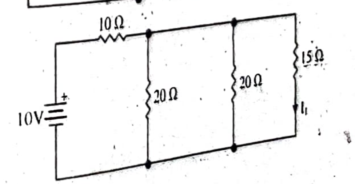
$$\eta = \frac{P_L}{P_S} = \frac{1}{2} \text{ or } 50\%$$

This is too low an efficiency for energy conversion devices; such devices must have load resistance far larger than that corresponding to the condition of maximum power transfer. However, in electronic devices the objective is to obtain maximum power output irrespective of device efficiency and hence the condition is always used.

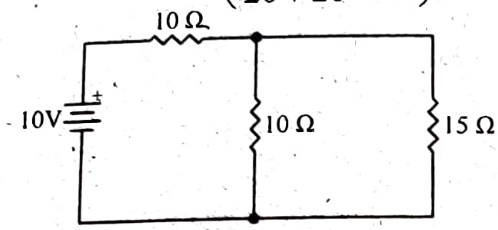
3.4. Determine the current I_1 through the 15 ohm resistor in the network given by Norton's Theorem. [WBUT 2007, 2008]



Answer:

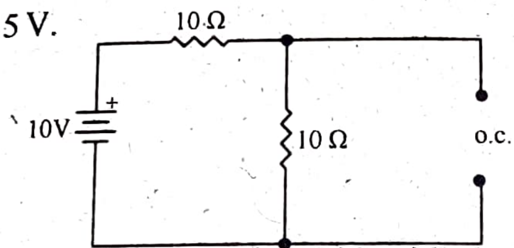


Before applying Norton's Theorem, circuit diagram can be simplified by replacing two 20Ω resistors in parallel with a single $10\Omega\left(\frac{20\times 20}{20+20}=10\right)$ resistor.



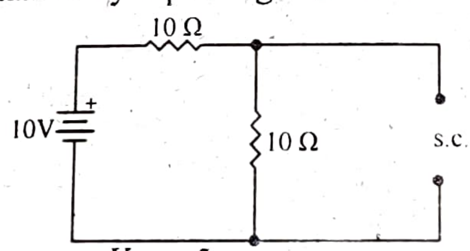
The open circuit voltage obtained by replacing 15Ω with o.c. is

$$V_{oc}=\left(\frac{10}{10+10}\right)\times 10=5\text{ V}.$$



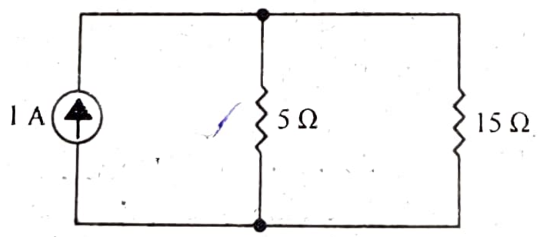
The short circuit current obtained by replacing 15Ω with short circuit

$$I_{sc}=\frac{10}{10}\text{A}=1\text{A}$$



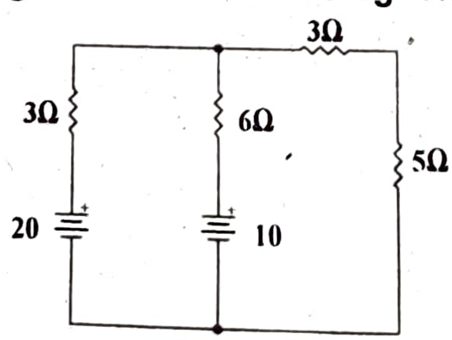
The Norton equivalent resistance $=\frac{V_{oc}}{I_{sc}}=\frac{5}{1}\Omega=5\Omega$.

The Norton equivalent circuit is drawn below:



Current through 15Ω resistor $=\frac{1\times 5}{15+5}\text{A}=\frac{5}{20}=0.25\text{A}$.

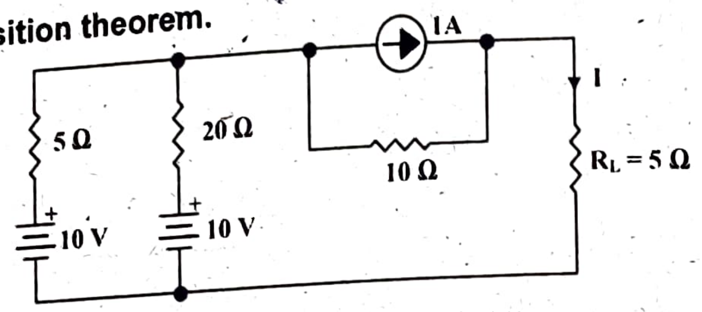
3.5. Find the current through 5Ω resistor using Thevenin's theorem in figure below: **[WBUT 2009(EVEN)]**



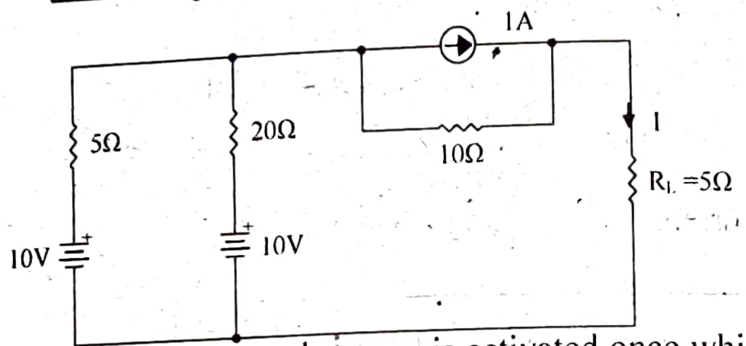
Answer:

It is required to determine the current through the 5Ω resistor using Thevenin's Theorem.

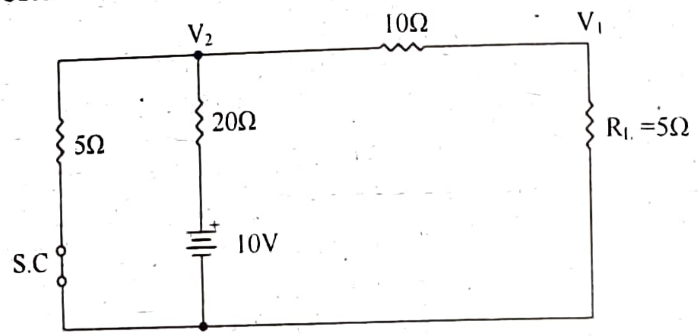
3.6. a) Find the current through resistance (R_2) for the network shown in the figure using the superposition theorem. **[WBUT 2009]**



Answer:



For applying superposition theorem, each source is activated once while all the others are switched off.



Applying KCL and writing node equation, we get,

$$\frac{V_2}{5}+\frac{V_2-10}{20}+\frac{V_2-V_1}{10}=0$$

or, $7V_2-2V_1=10$ ......(1)

$$\frac{V_1}{5}+\frac{V_1-V_2}{10}=0.$$

or, $3V_1-V_2=0$ ......(2)

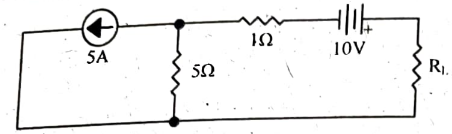
Substituting the value of V_2 in equation (1), we get,

$21V_1-2V_1=10$ or, $V_1=\frac{10}{19}V$

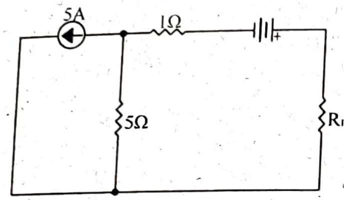
Hence current through the load resistance due to the 10V source in series with 20Ω resister $=\frac{10}{19\times 5}A=\frac{2}{19}A.$

Next, the 10V source in series with the 5Ω resister is activated.

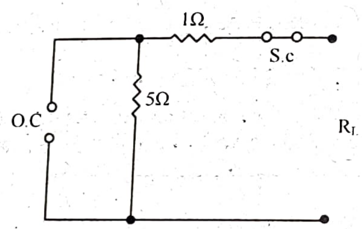
b) Find the value of R_2 for which the power transfer across R_2 is maximum. Find the value of this maximum power. **[WBUT 2009]**



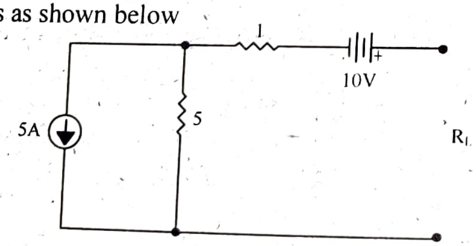
Answer:



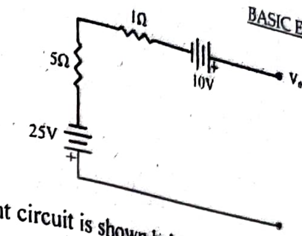
For evaluating the Thevenin resistance, for which power transfer will be maximum, the sources should be replaced by their interval resistances.
The circuit takes the form,



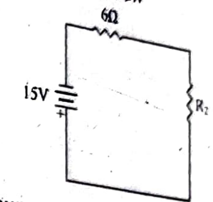
The resistance of the input circuit as viewed by the load terminals is $(5+1)\Omega = 6\Omega$. For determination of the maximum powers the open circuit voltage or V_{oh} needs to be evaluated.
The required circuit is as shown below



Since, the load terminals are open, the entire 5A current flows through the 5Ω resistor. Hence, a 25V voltage is developed across the 5Ω resistor.
By transforming the combination of 5A current source in parallel to 5Ω resistor as a series combination of 25V battery and a 5Ω resistor, the equivalent circuit can be drawn as shown.



The Thevenin equivalent circuit is shown below

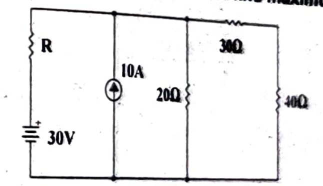


Hence, value of R_2 for maximum power transfer = 6Ω

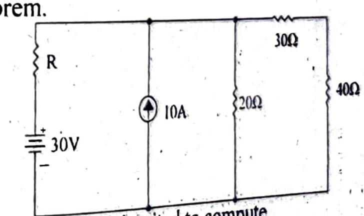
Current through the circuit for conditions of maximum power $= \frac{15}{12}A = 1.25A$

The maximum power is $= (1.25 \times 1.25) \times 6W = 9.375W$

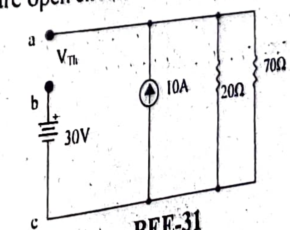
3.7. a) In the following circuit, find the value of the unknown resistance, R, so that maximum power will be transferred to load. Also find maximum power. **[WBUT 2010(EVEN)]**



Answer:
To compute the value of the unknown resistance, that consumes maximum power, we'll use Thevenin's Theorem.



The terminals across R are open circuited to compute



The equivalent resistance of the combination of resistors

$$R_{eq} = \frac{20\times 70}{90}\Omega = \frac{140}{9}\Omega = 15.56\Omega$$

The 10A current from the current source is flowing through this 15.56Ω

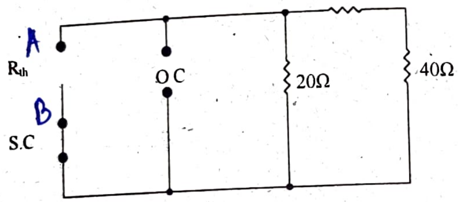
$\therefore$ The voltage across ac

$$V_{ac} = 15.56\times 10\text{V} = 155.6\text{V}$$

$$V_{ab} = V_{ac} - V_{bc} = (155.6-30)\text{V} = 125.6\text{V}$$

$$\therefore V_{th} = 125.6\text{V}.$$

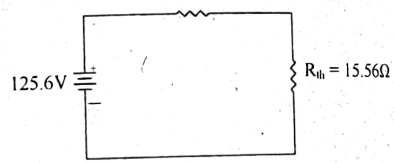
To compute R_{th} replace the active sources by their internal impedances. So the voltage source is replaced by a short circuit and the current source by an open circuit.



$$R_{th} = \frac{20\times 70}{20+70} = 15.56\Omega$$

Hence, the value of the unknown resistance required to ensure transfer of maximum power = 15.56Ω.

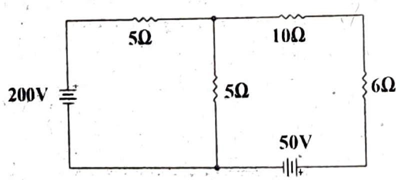
The circuit is redrawn with the computed values.



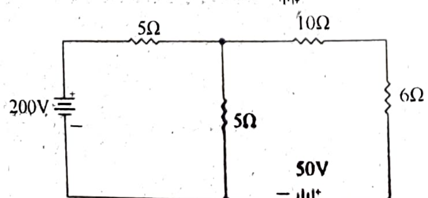
Current in the circuit = $= \frac{125.6}{15.56\times 2}\text{A} = 4\text{A}$

Value of maximum power $= I^2R = 4^2 \times 15.56\text{ W} = 248.96\text{W} \approx 249\text{ W}$

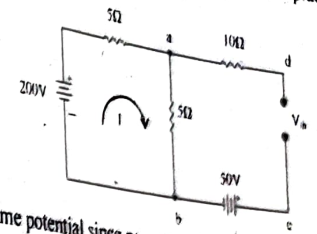
b) Apply the Thevenin's theorem to calculate the current in 6Ω resistor for the following circuit. **[WBUT 2010(EVEN)]**



Answer:



To apply Thevenin's Theorem to compute the current through 6Ω resistor, replace the 6Ω resistor first by an open circuit to compute V_{th}



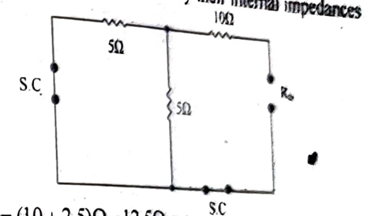
$$I = \frac{200}{5+5}\text{A} = \frac{200}{10}\text{A} = 20\text{A}$$

$$V_{ab} = 5\times 20\text{V} = 100\text{V}$$

The terminals a and d are at the same potential since no current flows between a & d.

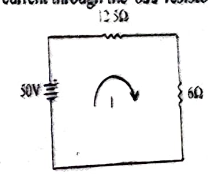
$$V_{dc} = V_{ac} = V_{ab} + V_{bc} = (100-50)\text{V} = 50\text{V}$$

To observe R_{th} replace all the active sources by their internal impedances



$$R_{th} = \left(10 + \frac{5\times 5}{5+5}\right)\Omega = (10+2.5)\Omega = 12.5\Omega$$

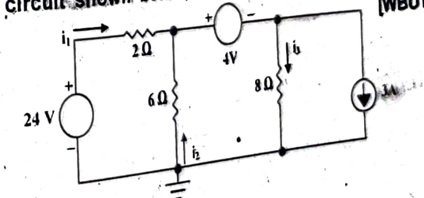
Redraw the circuit with V_{th} & R_{th} to compute the current through the 6Ω resistor.



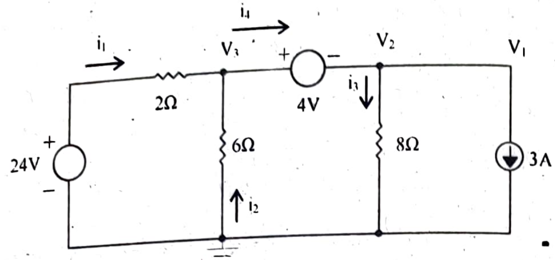
$$I = \frac{50}{(12.5+6)}\text{A} = \frac{50}{18.5}\text{A} = 2.7\text{A}.$$

Hence current through the 6Ω resistor = 2.7A.

3.8. a) For the circuit shown below, determine the current i_1, i_2, i_3 using nodal analysis: **[WBUT 2010, 2016]**



Answer:



Assigning the $V_1, V_2 \& V_3$ as the mode voltages $V_3 - V_2 = V$

Let the current flowing from V_3 to V_2 be i_4

Writing the node equation at node V_2

$$i_3 + 3 - i_4 = 0 \quad \text{.... (1)}$$

Wring the node equation at node V_3

$$-i_2 - i_1 + i_4 = 0 \quad \text{.... (2)}$$

$$i_3 = \frac{V_2}{8}$$

$$i_2 = -\frac{V_3}{6}$$

$$i_1 = -\frac{(V_3 - 24)}{2}$$

Putting the values of V_2 & V_3 in equation (1) & (2)

$$\frac{V_2}{8} + 3 - i_4 = 0 \quad \text{.... (3)}$$

$$\frac{V_3}{6} + \frac{V_3 - 24}{2} + i_4 = 0 \quad \text{.... (4)}$$

Adding equation 3 & 4, we have

$$\frac{V_2}{8} + 3 + \frac{V_3}{6} + \frac{V_3 - 24}{2} = 0$$

$$3V_2 + 4V_3 + 12V_3 = 12 \times 24 - 3 \times 24$$

$$3V_2 + 16V_3 = 9 \times 24 \quad \text{.... (5)}$$

$$V_3 - V_2 = 4$$

$$3V_3 - 3V_2 = 12 \quad \text{.... (6)}$$

Adding (5) & (6)

$$19V_3 = 238$$

$$V_3 = 12.53\text{V}$$

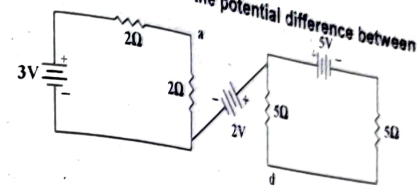
$$V_2 = 8.53\text{V}$$

$i_2 = \frac{12.53}{6}\text{A} = 2.07\text{A}$ flowing in the opposite direction

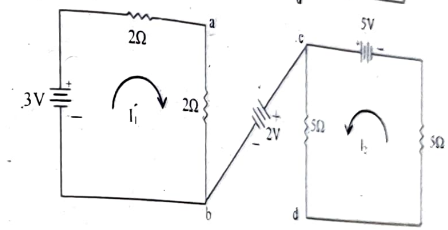
$$i_3 = \frac{8.53}{8}\text{A} = 1.07\text{A}$$

$$i_1 = \frac{24 - 12.53}{2}\text{A} = 5.735\text{A}$$

b) For the circuit shown below, find the potential difference between *a* and *d*: **[WBUT 2010]**



Answer:



Let I_1 be the current flowing through loop 1 and I_2 be the current flowing through loop 2

$$I_1 = \frac{3}{4}\text{A}$$

$$V_{ab} = I_1 \times 2 = \frac{3}{2}\text{V} = 1.5\text{V}$$

$$I_2 = \frac{5}{10}\text{A} = \frac{1}{2} = 0.5\text{A}$$

$$V_{cd} = 0.5 \times 5\text{V} = 2.5\text{V}$$

$$V_{ad} = V_{ab} + V_{bc} + V_{cd} = 1.5 - 2 + 2.5 = 2\text{V}$$

3.9. a) State and explain Thevenins theorem. **[WBUT 2011]**

b) Find the Thevenin equivalent of the circuit of Figure 1 as shown at terminal *XY*.

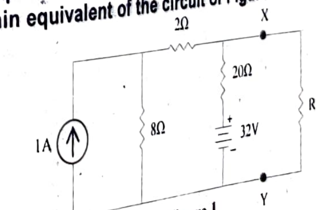
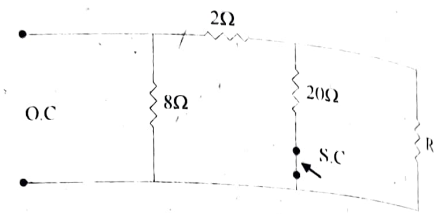


Figure 1

Answer:

a) ***Refer to Question No. 3.1.***

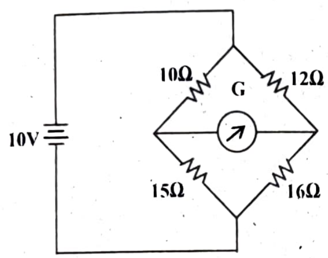


b) $R_{TH} = 8 + 2 \| 20$

$= 8 + 10 \| 20$

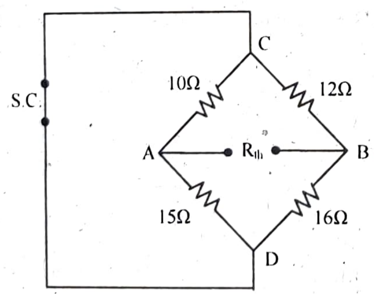
$= \frac{200}{30} = 6.67\ \Omega$

3.10 The galvanometer shown in the circuit has a resistance of 5 ohms. Find the current through the galvanometer using Thevenin's theorem. **[WBUT 2012]**

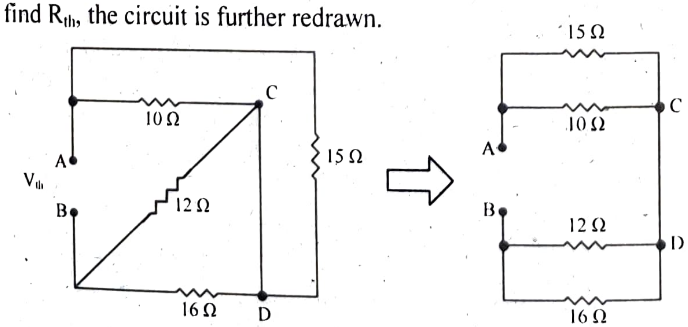


Answer:

For applying Thevenin's theorem, the circuit is redrawn as follows:



For find R_{th}, the circuit is further redrawn.

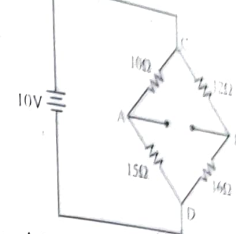


The equivalent resistance between A & B

$= \left(\frac{10 \times 15}{10 + 15}\right) + \frac{12 \times 16}{12 + 16} = \left(\frac{150}{25} + \frac{12 \times 16}{28}\right)\Omega$

$= \left(6 + \frac{48}{7}\right)\Omega = (6 + 6.857)\Omega = 12.857\ \Omega$

To find V_{th}, the circuit is redrawn.



Current flowing through CAD branch $= \frac{10}{25}\Omega = \frac{2}{5}\Omega = 0.4\ \Omega$

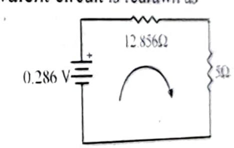
Current flowing through CDB branch $= \frac{10}{28}\Omega$

Voltage drop in CA $= \frac{2}{5} \times 10 = 4$ V

Voltage drop in CB $= \frac{10}{28} \times 12 = \frac{30}{7} = 4.286$ V

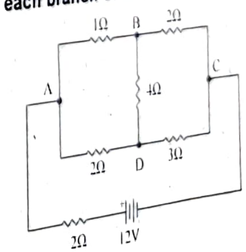
Hence voltage across AB $= (10 - 4) - (10 - 4.286) = 0.286$ V

The Thevenin equivalent circuit is redrawn as

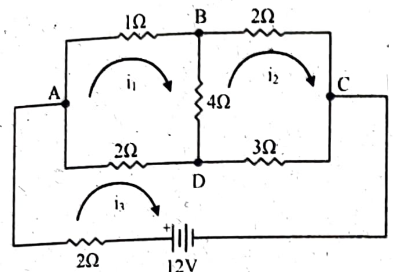


Current through galvanometer $= \frac{0.286}{12.856 + 5}$ A $= \frac{0.286}{17.856}$ A $= 0.016$ A.

3.11. Find the current in each branch of the network using Kirchoff's law. **[WBUT 2012]**



Answer:



Using KVL, the equations can be written as

$$2i_3 + 2(i_3 - i_1) + 3(i_3 - i_2) - 12 = 0$$

or,
$$-2i_1 - 3i_2 + 7i_3 = 12 \quad \ldots (1)$$
$$7i_1 - 4i_2 - 2i_3 = 0 \quad \ldots (2)$$
$$-4i_1 + 9i_2 - 3i_3 = 0 \quad \ldots (3)$$

Hence,
$$\begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$\det\begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} = 7(63-9) + (-1)(-4)(-28-6) - 2(12+18)$$

$$= 7\times 54 + 4\times(-34) - 2\times 30 = 378 - 136 - 60 = 378 - 196 = 182$$

Next, get, $\mathrm{Adj}\begin{bmatrix} 7 & -4 & -2 \\ -4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}$

Adj of the given matrix $= \begin{bmatrix} 54 & 34 & 30 \\ 34 & 45 & 29 \\ 30 & 29 & 47 \end{bmatrix}^{T} = \begin{bmatrix} 54 & 34 & 30 \\ 34 & 45 & 29 \\ 30 & 29 & 47 \end{bmatrix}$

Inverse of the given matrix $= \dfrac{1}{182}\begin{bmatrix} 54 & 34 & 30 \\ 34 & 45 & 29 \\ 30 & 29 & 47 \end{bmatrix}$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \frac{1}{182}\begin{bmatrix} 54 & 34 & 30 \\ 34 & 45 & 29 \\ 30 & 29 & 47 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Hence, $i_1 = \dfrac{30\times 12}{182}\text{A} = 1.978\text{ A}$; $i_2 = \dfrac{29\times 12}{182}\text{A} = 1.912\text{ A}$; $i_3 = \dfrac{47\times 12}{182}\text{A} = 3.098\text{ A}$

Hence current through source impedance = 3.098 A.
Flowing outwards to A.
Current through branch AB is 1.978 from A.
Current through branch BC is 1.912 from B to C.
Current through branch BD is $(1.978 - 1.912)\text{A} = (0.066)\text{A}$ from B to D.
Current through branch AD is $(3.098 - 1.978)\text{A} = 1.12\text{A}$ from A to D.
Current through branch DC is $(3.098 - 1.978)\text{A} = 1.186\text{ A}$ from D to C.

3.12. Explain (a) Star-delta conversion, (b) delta-star conversion with the help of a purely resistive circuit. **[WBUT 2012]**

Answer:

a) Star – delta transformation

Rule: The equivalent delta resistance between any two terminals is the sum of star resistances between the involved terminals plus the ratio of the product of these two star resistances to the remaining star resistance.

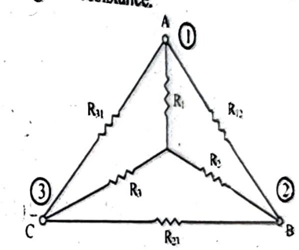


Fig. 1

Referring fig. (1) above, the total resistance between B & C must be same for both star and delta connections, so:

$$R_2 + R_3 = R_{23} \| (R_{31} + R_{12}) = \frac{R_{23}R_{31} + R_{23}R_{12}}{R_{23} + R_{31} + R_{12}} \quad \ldots(1)$$

Similarly it may be obtained:

$$R_3 + R_1 = R_{31} \| (R_{12} + R_{23}) = \frac{R_{31}.R_{12} + R_{31}.R_{23}}{R_{31} + R_{12} + R_{23}} \quad \ldots(2)$$

and
$$R_1 + R_2 = R_{12} \| (R_{31} + R_{23}) = \frac{R_{12}.R_{31} + R_{12}.R_{23}}{R_{12} + R_{31} + R_{23}} \quad \ldots(3)$$

Adding (1), (2) & (3): $2(R_1 + R_2 + R_3) = \dfrac{2(R_{23}R_{31} + R_{31}R_{12} + R_{12}R_{23})}{R_{12} + R_{23} + R_{31}}$

or, $$R_1 + R_2 + R_3 = \frac{R_{23}R_{31} + R_{31}R_{12} + R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} \quad \ldots(4)$$

Subtracting (1), (2), (3) from (4), turn by turn:

$$R_1 = \frac{R_{31}R_{12}}{R_{12}+R_{23}+R_{31}} \qquad \ldots(5)$$

$$R_2 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}} \qquad \ldots(6)$$

$$R_3 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}} \qquad \ldots(7)$$

dividing (5) by (6), $\frac{R_1}{R_2} = \frac{R_{12}}{R_{23}}$, or $R_{12} = \frac{R_1 R_{23}}{R_2}$ ...(8)

Similarly:

$$\frac{R_3}{R_1} = \frac{R_{23}}{R_{31}} \quad \text{or} \quad R_{31} = \frac{R_1 R_{23}}{R_3} \qquad \ldots(9)$$

Substituting the values of R_{31} & R_{12} from equation (3) and (9) respectively in equation (1), it may be obtained as under:

$$R_2 + R_3 = \frac{R_{23}\left(R_1.R_{23}/R_2\right) + R_{23}\left(R_1R_{23}/R_3\right)}{R_{23} + \left(R_1 R_{23}/R_2\right) + \left(R_1.R_{23}/R_3\right)}$$

$$\text{or, } R_2 + R_3 = \frac{\left(R_1.R_{23}/R_2\right) + R_1.R_{23}/R_3}{1+\left(\frac{R_1}{R_2}\right)+\left(R_1/R_2\right)} = \frac{R_1R_3R_{23} + R_1R_2R_{23}}{R_2R_3 + R_1R_3 + R_1R_2} = \frac{R_1R_{23}\left(R_3+R_2\right)}{R_2R_3 + R_1R_3 + R_1R_2}$$

$$\therefore R_1R_{23} = R_2R_3 + R_1R_3 + R_1R_2$$

$$\text{or} \quad R_{23} = \frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

Similarly:

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_3 + R_1 + \frac{R_3R_1}{R_2}$$

$$R_{12} = \frac{R_2R_3 + R_3R_1 + R_1R_2}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

b) Delta – star transformation

It is a method of obtaining equivalent star of a delta connection. A delta circuit ABC having three resistors $R_{12}R_{23}$ & R_{31} and its equivalent star circuit represented by $R_1, R_2, \& R_3$ as shown in fig (2) below may be referred to:

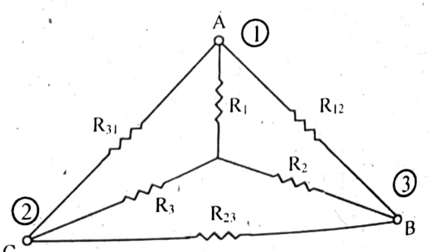


Fig: 2

Rule: 1

The resistance of any arm of the star is equal to the product of the resistances of two delta sides meeting it divided by the sum of the three delta resistances.

The resistances between terminals A & B in delta construction:

$$= \frac{R_{12}\times\left(R_{23}+R_{31}\right)}{R_{12}+\left(R_{23}+R_{31}\right)} \qquad \ldots(i)$$

and the resistances between the same terminals A & B in the star connection

$$= R_1 + R_2 \qquad \ldots(ii)$$

Since the terminals are same; so:-

$$R_1 + R_2 = \frac{R_{12}\times\left(R_{23}+R_{31}\right)}{R_{12}+\left(R_{23}+R_{31}\right)} \qquad \ldots(iii)$$

Similarly, by solving for terminals B & C and C & A, it can be obtained,

$$R_1 + R_3 = \frac{R_{23}\times\left(R_{31}+R_{12}\right)}{R_{23}+\left(R_{31}+R_{12}\right)} \qquad \ldots(iv)$$

and

$$R_3 + R_1 = \frac{R_{31}\times\left(R_{12}+R_{23}\right)}{R_{31}+\left(R_{12}+R_{23}\right)} \qquad \ldots(v)$$

On solving (iii), (iv) & (v) the delta – star transformation results as under:

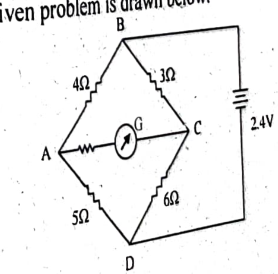
$$R_1 = \frac{R_{12}\times R_{31}}{R_{12}+R_{23}+R_{31}} \qquad R_2 = \frac{R_{23}\times R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$R_3 = \frac{R_{31}\times R_{23}}{R_{12}+R_{23}+R_{31}}$$

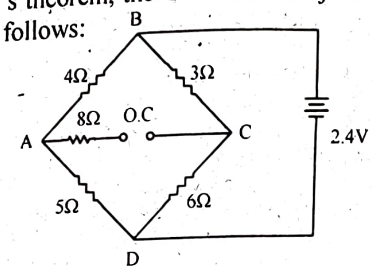
3.13. A Wheatstone bridge consists of $AB=4\Omega, BC=3\Omega, CD=6\Omega$ and $DA=5\Omega$. A 2.4 V battery is connected between points B and D. A galvanometer of 8Ω resistance is connected between A and C. Using Thevenin's theorem find the current through the galvanometer. **[WBUT 2013]**

Answer:

The diagram for the given problem is drawn below:



In order to use Thevenin's theorem, the Galvanometer junctions need to be opened. The new circuit diagram is as follows:



Total resistance of parallel pair $= \dfrac{9\times 9}{9+9} = 4.5\Omega$

Source current $= \dfrac{2.4}{4.5}$ A

Current through branch BAD $= \dfrac{2.4}{4.5\times 2}$ A $= \dfrac{1.2}{4.5}$ A

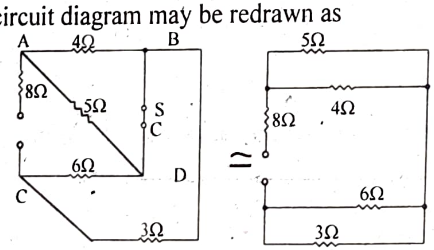
Current through branch BCD $= \dfrac{2.4}{4.5\times 2}$ A $= \dfrac{1.2}{4.5}$ A

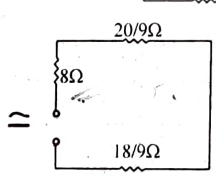
Voltage drop across BA $= \dfrac{4\times 1.2}{4.5}$ V

Voltage drop across BC $= \dfrac{3\times 1.2}{4.5}$ V

Hence voltage across AC = $V_{th} = \dfrac{4\times 1.2 - 3\times 1.2}{4.5} = \dfrac{1.2}{4.5}$ V $= 0.27$V

To find R_{th}, the circuit diagram may be redrawn as

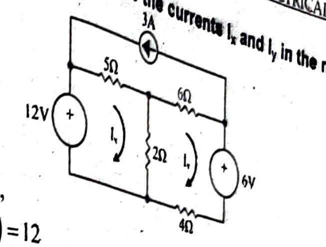




Hence $R_{th} = (8+2.2+2)\Omega = 12.2\Omega$

Current through the galvanometer $= \dfrac{0.27}{12.2}$ A $= 0.02$A

3.14. a) Using Mesh analysis, determine the currents I_x and I_y in the network shown below **[WBUT 2014]**



Answer:

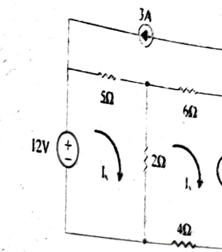
Writing mesh equations,

$$5I_x + 2(I_x - I_y) = 12$$

or,
$$7I_x - 2I_y = 12 \quad \text{... (1)}$$

$$6I_y + 4I_y + 2(I_y - I_x) + 6 = 0$$

$$12I_y - 2I_x = -6 \quad \text{... (2)}$$



Multiplying equation (1) by 2, we get

$$14I_x - 4I_y = 24 \quad \text{... (3)}$$

Multiplying equation (2) by 7, we get

$$84I_y - 14I_x = -42 \quad \text{... (4)}$$

Adding equation (3) & (4) we get

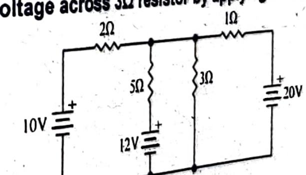
$$80I_y = -18$$

$$I_y = \frac{-9}{40}A$$

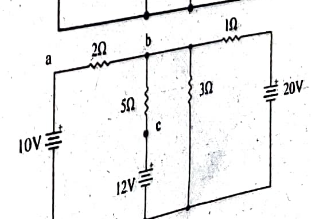
$$I_x = \frac{12I_y + 6A}{2} = \frac{\frac{-9\times 12}{40}+6}{2}A = \frac{\frac{27}{10}+6}{2}A = \frac{3.3}{2} = 1.65A$$

$I_x = 1.65A$, $I_y = 0.225A$ in the opposite direction.

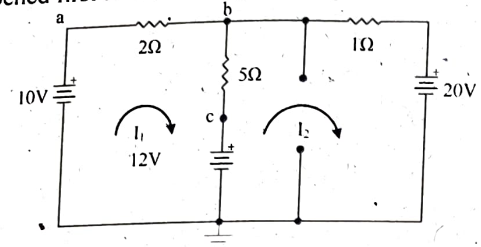
b) Determine the voltage across 3Ω resistor by applying Thevenin's Theorem in the following network. **[WBUT 2014]**



Answer:



The 3Ω resistor is opened first to find the V_{th} of the open circuit voltage.



Using Mesh Analysis, the currents I_1 & I_2 are found in both the meshes

$(2+5)I_1 - 5I_1 + 12 - 10 = 0$

or, $7I_1 - 5I_2 = -2$ ...(1)

$6I_2 - 5I_1 + 20 - 12 = 0$

or, $6I_2 - 5I_1 = -8$ ...(2)

Multiplying (1) by 6 and (2) by 5, we get

$42I_1 - 30I_2 = -12$

$-25I_1 + 30I_2 = -40$

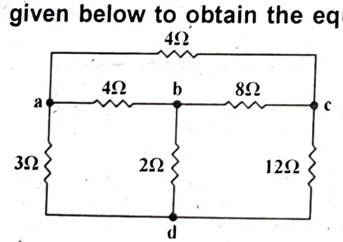
$17I_1 \quad = -52$

$I_1 = \dfrac{-52}{17}$

Voltage drop at 2Ω resistor $V_{ab} = -\dfrac{52}{17} \times 2 = -\dfrac{104}{17}$

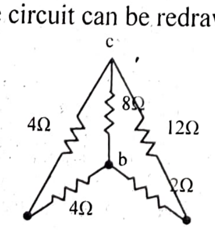
Voltage of the node 'b' with respect to **datum** $= \left(\dfrac{104}{17} + 10\right) V = \dfrac{274}{17} V = 16.11V$

3.15. Reduce the network given below to obtain the equivalent resistance as seen between nodes *ad*. [WBUT 2014]



Answer:

With the terminals a & d open, the circuit can be redrawn as,



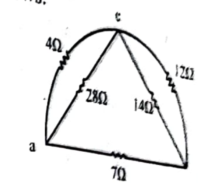
Star delta transformer can be used to redrawn the circuits follows:

$R_{ac} = \dfrac{8\times4+8\times2+4\times2}{2}\Omega = (16+8+4)\Omega = 28\Omega$

$R_{ad} = \dfrac{8\times4+8\times2+4\times2}{8}\Omega = (4+2+1)\Omega = 7\Omega$

$R_{cd} = \dfrac{8\times4+8\times2+4\times2}{8}\Omega = (8+4+2)\Omega = 14\Omega$

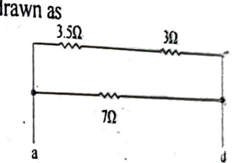
The modified circuit is as follows.



$R_{ac} = \dfrac{4\times28}{(4\times28)}\Omega = \dfrac{4\times7\times4}{8\times4}\Omega = 3.5\Omega$

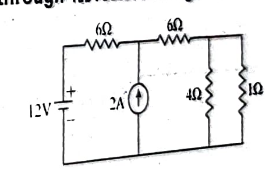
$R_{cd} = \dfrac{12\times4}{(12\times4)}\Omega = \dfrac{48}{16}\Omega = 3\Omega$

Equivalent circuit can be drawn as



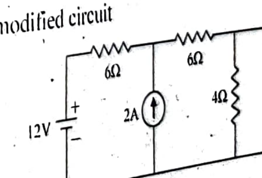
Equivalent Resistance $R_{ad} = \dfrac{6.5\times7}{6.5+7} = \dfrac{45.5}{13.5}\Omega = 3.4\Omega = (3.37)\Omega$

3.16. Find the current through 1Ω resistor using Thevenin's Theorem. [WBUT 2015]



Answer:

To find I_{th}, this is the modified circuit



Using superposition theorem, the 2A source is replaced by an open circuit.

Current through circuit $= \frac{12}{6+6+4}\text{A} = \frac{12}{16}\text{A}$

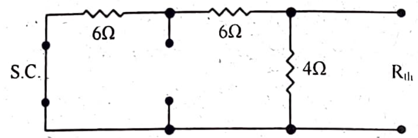
Now, the 12V source is replaced by a short circuit and the 2A current source by is on.

Current through 4Ω resistor $= \frac{2\times6}{16}\text{A}$

Total current through 4Ω resistor $= \left(\frac{12}{16}+\frac{12}{16}\right)\text{A} = \frac{24}{16}\text{A} = \frac{3}{2}\text{A}$

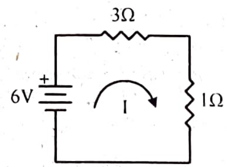
Voltage across 4Ω resistor $= \frac{3}{2}\times 4\,\text{V} = 6\,\text{V}$

To find R_{th}, following is the circuit



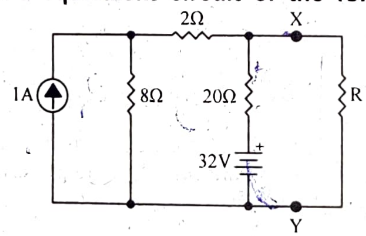
$$R_{th} = \frac{4\times12}{(4+12)}\Omega = \frac{48}{16}\Omega = 3\Omega$$

The equivalent circuit with V_{th} and R_{th}

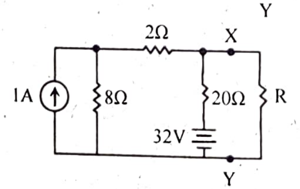


Current. $I = \frac{6}{4}\text{A} = 1.5\,\text{A}$

3.17. Find the Thevenin's equivalent circuit of the following figure between the terminals X-Y. **[WBUT 2016]**



Answer:



With the 32V source on and 1A current source switched of voltage at

$$XY = \left\{32 - 20\times\left(\frac{32}{20+2+8}\right)\right\}V = \left(32-\frac{20\times32}{30}\right)V = \left(32-\frac{64}{3}\right)V$$

Switching the, 32V source off and keeping the 1A source on, voltage drop across the 20Ω resistor

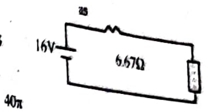
$$= \frac{1\times8}{30}\times 20 = \frac{16}{3}V$$

Thence $R_{th} = \left(32-\frac{64}{3}+\frac{16}{3}\right)V = \left(32-\frac{48}{3}\right)V = 16V$

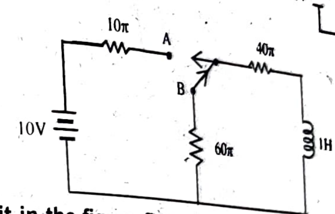
Looking from $X-Y$,

$$R_{th} = \frac{20\times10}{20+10}\Omega = \frac{20}{3}\Omega$$

The Thevenin equivalent circuit can be drawn as



3.18. a)

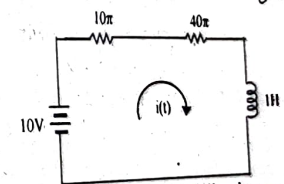


[MODEL QUESTION]

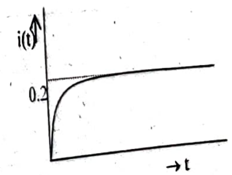
For the circuit in the figure, find how much time will be required for the current to reach 100mA after the switch is thrown to position A. If the switch is thrown to position B as soon as the current is 100mA, then how long after this the current will be 50mA.

Answer:

Considering the switch to be at position A, the circuit diagram for the problem can be drawn as follows:



Considering the inductor to be at zero–state initially, the governing differential equation is $50i + \frac{di}{dt} = 10$



Solving for, $i = 0.2\left(1-e^{-50t}\right)$

$i(t)$ can be graphically represented as

Put, $i = 100\times10^{-3}\,\text{A}$

$\therefore 0.2\left(1-e^{-50t}\right) = 100\times10^{-3}$

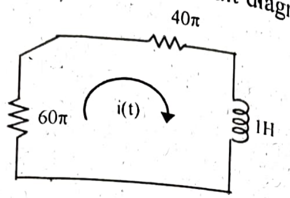
or, $1-e^{-50t} = 0.5$

or, $e = 0.5$

or, $-50t = \ln(0.5) = -0.693$

or, $t=\dfrac{0.693}{50}=0.01386\text{sec}$

After 0.01386 sec has lapsed, the switch is moved to position b. The new circuit diagram follows



The governing differential equation is

$100i_1+1\dfrac{di_1}{dt}=0$

$\dfrac{di_1}{i_1}=-100\,dt$

$\ln i_1=-100t+\ln K$ [$\ln K$ is the constant of integration]

$i_1=Ke^{-100t}$

Put $i_1=0.1$ A at $t=0$,

$K=0.1$

$\therefore i_1=0.1\,e^{-100t}$

Let t′ sec be the necessary time for the current to become 50m A,

$0.05=0.1\,e^{-100t'}$

or, $0.5=b\,e^{-100t'}$

or, $-100\,t'=\ln(0.5)=-0.693$

or, $t'=0.00693$ sec.

Ans. The current will take 0.01386 sec to become 100m A. After the switch is moved to position B, 0.00693 sec will be taken by the current to become 50mA.

b) Explain why current through an inductance and voltage across capacitance cannot change instantaneously. **[MODEL QUESTION]**

Answer:

The instantaneous voltage developed across an inductance is expressed as $v_l=L\dfrac{di}{dt}$

In order to bring about an instantaneous change in the current i.e., to obtain a finite di for $dt\to 0$, the voltage should assume an infinite value.

It is impossible to realize such a function physically. It can only be realized mathematically by a dirac delta function $\delta(t)$. So it can be concluded that voltage through inductance cannot change instantaneously.

Current flowing through an capacitance can be expressed as $i_c=C\dfrac{dv}{dt}$

An instantaneous change in the capacitor voltage would require a current source of infinite magnitude. It can only be realized mathematically by a direct delta function. As it is not possible to construct an impulse function in physical reality, voltage across capacitance cannot change instantaneously.

AC CIRCUITS

Chapter at a Glance

Series AC circuits R-L, R-C, R-L-C circuits. Impedance, Reactance, Phasor diagram, Impedance Triangle, Power Factor, Average power, Apparent power, Reactive power, Power triangle (Numerical)

R L C Series Circuit

1. The three elements R, L and C as shown in figure below are connected in series across an ac supply of R.M.S. voltage.

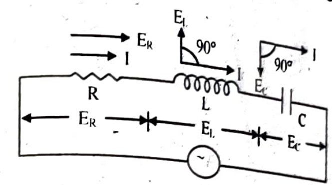


Fig. (a)

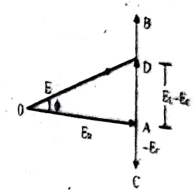


Fig. (b)

Assuming that: -

$E_R=IR=$ Voltage drop across 'R' (in phase with I)

$E_L=IX_L=$ Voltage drop across 'L' (leading I by $\pi/2$)

$E_C=IX_C=$ Voltage drop across 'C' (lagging I by $\pi/2$)

X_L And X_C are the inductive and capacitive reactance

$X_L=2\pi fL$ And $X_C=1/\omega C=1/2\pi fL$

Referring figure (b), $OD=\sqrt{(OA^2+AD^2)}$

i.e. $E=\sqrt{\left[(IR)^2+(IX_L-IX_C)^2\right]}=I\sqrt{\left[R^2+(X_L-X_C)^2\right]}$

$\therefore\quad I=\dfrac{E}{\sqrt{R^2+(X_L-X_C)^2}}=\dfrac{E}{\sqrt{R^2+X^2}}=\dfrac{E}{Z}$

The term $\sqrt{R^2+(X_L-X_C)^2}$ is known as the impedance of the circuit.

Phase angle ϕ is given by $\tan\phi=\dfrac{(X_L-X_C)}{R}=\dfrac{X}{R}=$ Net reactance / resistance.

Power factor, $\cos\phi=\dfrac{R}{Z}=\dfrac{R}{\sqrt{R^2+(X_L-X_C)^2}}$

Hence, it is seen that if the equation of the applied voltage is $e=E_m\sin\omega t$.

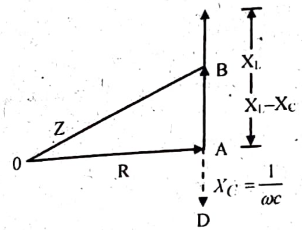


Fig. (c)

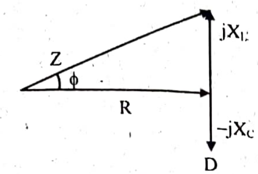


Fig. (d)

Then equation of the resulting current in an R-L-C circuit is given by $i = I_m \sin(\omega t \pm \phi)$.

The +ve sign is to be used when current leads i.e., when $X_C > X_L$ and the –ve sign is to be used when current lags i.e., when $X_L > X_C$.

In general, the current lags or leads the supply voltage, by an angle ϕ such that $\tan\phi = \frac{X}{R}$.

Using symbolic notation [figure (d)], $Z = R + j(X_L + X_C)$

Its phase angle is $\phi = \tan^{-1}\left[(X_L - X_C)/R\right]$

$$Z = Z\angle\tan^{-1}(X_L - X_C)/R$$

If $\quad V = V\angle 0$, then $I = \frac{V}{Z}$

$$\text{Power}(P) = VI\cos\phi$$

Apparent Power:
Apparent power *(S)* is the power delivered to an electrical circuit. The measurement of apparent power is in volt amperes (VA).

S = I²Z = IE

Where, S = apparent power (VA)
I = RMS current (A)
E = RMS voltage (V)
Z = impedance (Ω)

True Power:
True power *(P)* is the power consumed by the resistive loads in an electrical circuit. Equation below is a mathematical representation of true power. The measurement of true power is in watts.

$P = I^2R = EI\cos\theta$

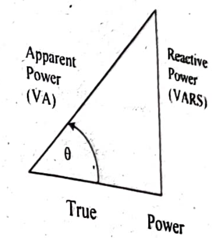
where, P = true power (watts)
I = RMS current (A)
E = RMS voltage (V)
R = resistance (Ω)
θ = angle between E and I sine waves

Reactive Power:
Reactive power (Q) is the power consumed in an AC circuit because of the expansion and collapse of magnetic (inductive) and electrostatic (capacitive) fields. Reactive power is expressed in volt-amperes-reactive (VAR). Equation below is a mathematical representation for reactive power.

$Q = I^2X = EI\sin\theta$

where, Q = reactive power (VAR)
I = RMS current (A)
X = net reactance (Ω)
E = RMS voltage (V)
θ = angle between E and I sine waves

Power Triangle:



Multiple Choice Type Questions

1.1. If the peak value of a sine wave is 100 volts, then its *rms* value will be [WBUT 2007]

a) 70.7 V b) 63.6 V c) 100 V d) 88 V

Answer: (a)

1.2. The power factor of a purely inductive circuit is [WBUT 2008, 2011, 2014]

a) zero b) one c) infinity d) 0.5

Answer: (a)

1.3. For an inductive circuit, current [WBUT 2008(EVEN)]

a) lags the voltage b) leads the voltage
c) is in phase with the voltage d) is independent of the voltage phase

Answer: (a)

1.4. A resistance of 8.0Ω and an inductive reactance of 6.0Ω will offer an impedance of [WBUT 2009(EVEN)]

a) 14 Ω b) 10 Ω c) 11 Ω d) none of these

Answer: (b)

1.5. If $e_1 = A\sin\omega t$ and $e_2 = B\sin(\omega t - \phi)$, then [WBUT 2009]

a) e_1 lags e_2 by ϕ b) e_2 lags e_1 by ϕ c) e_2 leads e_1 by ϕ d) e_1 is in phase with e_2

Answer: (b)

1.6. The form factor of a wave is 1. Its shape is [WBUT 2010, 2011, 2015]
a) sinusoidal b) triangular c) square d) sawtooth
Answer: (a)

1.7. Inductive reactance of a coil of inductance 0.2 H at 50 Hz is [WBUT 2011]
a) 62.8 Ω b) 628 Ω c) 0.2 Ω d) 20 Ω
Answer: (a)

1.8. In an electrical circuit, if the current lags the voltage by 60°, the circuit nature is [WBUT 2012]
a) $R-C$ b) $R-L$ c) LC d) none of these
Answer: (b)

1.9. If $E_1 = A\sin\omega t$ and $E_2 = A\sin(\omega t - \theta)$, then [WBUT 2012]
a) E_1 lags E_2
b) E_2 lags E_1
c) E_1 and E_2 are in phase
d) none of these
Answer: (b)

1.10. Time constant of LR circuit is given by [WBUT 2013]
a) L/R b) R/L c) 1/LR d) LR
Answer: (a)

1.11. A sinusoidal voltage is represented by $v = 141.4\sin(314.18t - 90°)$. The r.m.s. value of the voltage, its frequency and phase angle are respectively [WBUT 2013]
a) 141.42V, 314.16Hz, 90°
b) 100V, 50Hz, –90°
c) 87.92V, 60Hz, 90°
d) 200V, 56Hz, –90°
Answer: (b)

1.12. The admittance of a parallel circuit is $0.5\angle -30°$. The circuit is [WBUT 2016]
a) inductive b) capacitive c) resistive d) in resonance
Answer: (a)

1.13. In a three-phase star connected system, the relation between the phase and the line voltage is [WBUT 2006, 2008(EVEN)]
a) $V_p = V_L$ b) $V_p = \sqrt{3}\,V_L$ c) $V_p = \frac{V_L}{\sqrt{3}}$ d) $V_p = \frac{V_L}{3}$

Answer: (c)

1.14. In a 3 phase system, the *emfs* are [WBUT 2007, 2009(ODD)]
a) 30° apart b) 60° apart c) 90° apart d) 120° apart
Answer: (d)

Short Answer Type Questions

2.1. Derive an expression of
a) average
b) r.m.s value of a half-wave rectified voltage wave [WBUT 2007]

OR,

Derive the expression of (i) average (ii) R.M.S. value of a half-wave rectified voltage wave. [WBUT 2009]

OR,

Deduce an expression of average and RMS value of a half wave rectified voltage wave. [WBUT 2016]

Answer:
Mathematical derivation: Average Value

$$V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t\, d(\omega t) = \frac{1}{2\pi}\left(-V_m\cos\omega t\right)_0^{\pi} = \frac{-V_m}{2\pi}(0-1-1) = \frac{1}{\pi}V_m = 0.319V_m$$

Similarly, $I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t\, d(\omega t) = 0.319 I_m$

$$V_{r.m.s} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{4\pi}\int_0^{\pi}(1-\cos 2\omega t)\, d(\omega t)}$$

$$= V_m\sqrt{\frac{1}{4\pi}\int_0^{\pi} d(\omega t) - \frac{1}{4\pi}\int_0^{\pi}\cos 2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{4} - \frac{1}{8\pi}[\sin 2\omega t]_0^{\pi}}$$

$$= V_m/2 = 0.5V_m$$

Similarly, $I_{r.m.s} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\omega t\, d(\omega t)} = 0.5 I_m$

2.2. An alternating voltage is represented by $v = 62.35\sin 323t$. Determine (a) the maximum value (b) r.m.s. value (c) average value (d) frequency of the wave (e) form factor. [WBUT 2008]

Answer:
$v = 62.35\sin 323t$

(a) the maximum value = 62.35 V

(b) r.m.s. value $= \frac{\text{maximum value}}{\sqrt{2}} = \frac{62.35}{\sqrt{2}}V = 44.088V$

(c) average value $= \frac{1}{\pi}\int_0^{\pi} v_m \sin\omega t\, d(\omega t) = \frac{v_m}{\pi}[-\cos\omega t]_0^{\pi} = \frac{2v_m}{\pi} = 0.637 \times 62.35V = 39.7$

average = Area under curve / length of base

(d) $2\pi f = 323$

Frequency $f = \frac{323}{2.\pi}$ Hz = 51.40 Hz

(e) Form factor $= \frac{\text{r.m.s. value}}{\text{mean value}} = \frac{44.088}{39.7} = 1.11$

average

$$f(x)_{avg} = \frac{\int_a^b f(x)\,dt}{\int_a^b dt}$$

2.3. A full wave rectified sinusoidal voltage is clipped at $\frac{1}{\sqrt{2}}$ of its maximum value. Calculate the average and *r.m.s.* value of such a voltage waveform. Also calculate the form factor and peak factor. **[WBUT 2008(EVEN)]**

Answer:

The average value of the rectified sinusoidal voltage

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi}\frac{V_m}{V_2}\sin\omega t\, d(\omega t) = \frac{V_m}{\sqrt{2}\pi}\int_0^{\pi}\sin\omega t\, d(\omega t)$$

$$= \frac{V_m}{\sqrt{2}\pi}\left[-\cos\omega t\right]_0^{\pi} = \frac{-V_m}{\sqrt{2}\pi}\left[-1-1\right] = \frac{\sqrt{2}V_m}{\pi} = 0.45V_m$$

The r.m.s value of the rectified waveform

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi}\left(V_m/\sqrt{2}\right)^2\sin^2\omega t\, d(\omega t)}$$

$$= \frac{V_m}{\sqrt{2\pi}}\sqrt{\frac{1}{2}\int_0^{\pi}2\sin^2\omega t\, d(\omega)} = \frac{V_m}{2\sqrt{\pi}}\sqrt{\int_0^{\pi}\left(1-\cos^2\omega t\right)d(\omega t)}$$

$$= \frac{V_m}{2\sqrt{\pi}}\sqrt{\left(\pi-0\right)+\left.\frac{\sin^2\omega t}{2}\right|_0^{\pi}} = \frac{V_m}{2} = 0.5V_m$$

$$\text{Form factor} = \frac{\text{r.m.s value}}{\text{peak value}} = \frac{0.5V_m}{0.45V_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{peak value}}{\text{r.m.s value}} = \frac{V_m}{0.5V_m} = 2$$

$\sqrt{2} = 1.41$

2.4. Define R.M.S. value of alternating quantity & derive its expression for sinusoidal current. **[WBUT 2010(EVEN)]**

Answer:

Effective value of AC is the amount of AC that produces the same heating effect as an equal amount of DC. In simpler terms, one-ampere effective value of AC will produce the same amount of heat in a conductor, in a given time, as one ampere of DC. The heating effect of a given AC current is proportional to the square of the current. Effective value of AC can be calculated by squaring all the amplitudes of the sine wave over one period, taking the average of these values, and then taking the square root.

The effective value, being the root of the mean (average) square of the currents, is known as the root-mean-square, or RMS value.

Explanation (RMS value)

To understand the concept of effective value let us consider a sinusoidal current (I) wave (Figure 1).

The values of I are plotted on the upper curve, and the corresponding values of I^2 are plotted on the lower curve. The I^2 curve has twice the frequency of I and varies above and below a new axis. The new axis is the average of the I^2 values, and the square root of that value is the RMS, or effective value, of current.

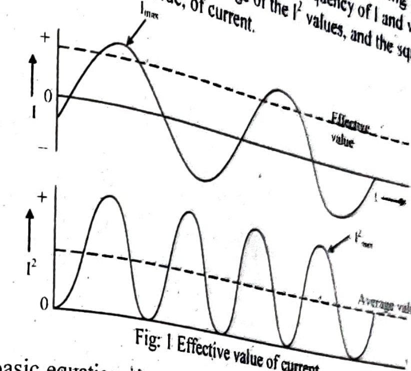


Fig: 1 Effective value of current

There are six basic equations that are used to convert a value of AC voltage or current to another value, as listed below.

The values of current (I) and voltage (E) that are normally encountered are assumed to be RMS values; therefore, no subscript is used.

Another useful value is the **average value** of the amplitude during the positive half of the cycle. The mathematical relationship between I_{avg}, I_{max}, and I_{rms} have been deduced below.

Mathematically,

$$V_{r.m.s} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}V_m^2\sin^2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{4\pi}\int_0^{2\pi}(1-\cos 2\omega t)\, d(\omega t)}$$

$$= V_m\sqrt{\frac{1}{4\pi}\int_0^{2\pi}d(\omega t) - \frac{1}{4\pi}\int_0^{2\pi}\cos 2\omega t\, d(\omega t)} = V_m\sqrt{\frac{1}{2} - \frac{1}{8\pi}\left[\sin 2\omega t\right]_0^{2\pi}}$$

$$= V_m/\sqrt{2} = 0.707V_m$$

Similarly, $I_{r.m.s} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}I_m^2\sin^2\omega t\, d(\omega t)} = 0.707I_m$

The **average value** of the amplitude of a sine wave is always calculated during the positive half of the cycle, since the average value evaluated over a complete cycle would be zero. The mathematical relationship between I_{avg} and I_{max} have been deduced below.

Mathematical derivation:

Average Value

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi}V_m\sin\omega t\, d(\omega t) = \frac{1}{\pi}\left(-V_m\cos\omega t\right)_0^{\pi} = \frac{-V_m}{\pi}(0-1-1) = \frac{2}{\pi}V_m = 0.637V_m$$

Similarly, $I_{avg} = \frac{1}{\pi}\int_0^{\pi}I_m\sin\omega t\, d(\omega t) = 0.637I_m$

Hence, $V_{av} = 0.637\ V_{max} = 0.90\ V_{rms}$
$I_{av} = 0.637\ I_{max} = 0.90\ I_{rms}$

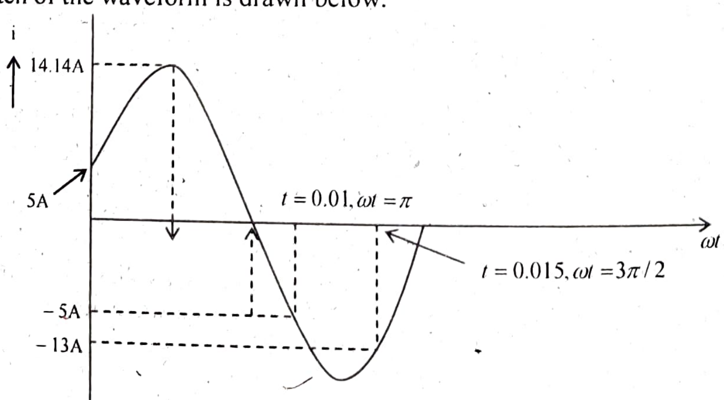
2.5. At $t = 0$, the instantaneous value of a 50 Hz, sinusoidal current is 5 Amp and increases in magnitude further. Its R.M.S. value is 10 Amp.
a) Write the expression for its instantaneous value
b) Find the current at $t = 0.01$ and $t = 0.015$ sec
c) Sketch the waveform indicating these values. **[WBUT 2010]**

Answer:
The R.M.S. value of the current is 10 Amp.
Hence, the peak value $= \sqrt{2} \times 10A = 14.14A$
The expression of instantaneous value of current $i = I_m \sin(\omega t + \phi)$
The phase angle is leading since the value of function (current) is positive at t = 0.
$I_m = 14.14\ A$
At $t = 0$
$i = I_m \sin\phi = \sqrt{2} \times 10 \sin\phi = 5$

or, $\sin\phi = \frac{1}{2\sqrt{2}} = 0.35$

or, $\phi = \sin^{-1} 0.35 = 20.7°$

(a) Hence, the expression of instantaneous value of current can be written as
$$i = 14.14\sin(\omega t + 20.7°)$$
(b) The value of current at $t = 0.01$sec can be computed as follows:
$$i = 14.14\sin\left(2\pi \times 50 \times 0.01 \times \frac{180°}{\pi} + 20.7°\right) = 14.14 \sin 200.7° = -5A$$
Value of current at $t = 0.015$sec
$$= i = 14.14 \sin(2\pi \times 50 \times 0.015 \times 180°/\pi + 20.7°)A = -13.2A$$
(c) The sketch of the waveform is drawn below.



$\omega = 2\pi f$ rad/sec $= 2 \times \pi \times 50$ rad/sec $= 100\pi$ rad/sec

2.6. Derive a mathematical expression for r.m.s. value of a sinusoidal voltage $v = V_m \sin\omega t$. **[WBUT 2011]**

Answer:
The elementary AC generator (Figure 1) consists of a conductor or loop of wire in a magnetic field that is produced by an electromagnet. The two ends of the loop are connected to slip rings, and they are in contact with two brushes. When the loop rotates it cuts magnetic lines of force, first in one direction and then the other.

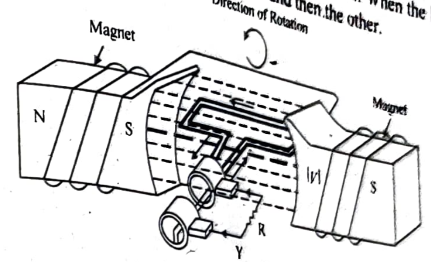


Fig: 1 Simple AC Generator

Development of a Sine-Wave Output

Step1:
At the instant the loop is in the vertical position (Figure 2, 0°), the coil sides are moving parallel to the field and do not cut magnetic lines of force. In this instant, there is no voltage induced in the loop.

Step2:
As the coil rotates in a counter-clockwise direction, the coil sides will cut the magnetic lines of force in opposite direction. The direction of the induced voltages depends on the direction of movement of the coil. The potential drop across resistor R will cause a current to flow through it.

Step3:
The voltage and hence current continues to increase until it reaches a maximum value when the coil is perpendicular to the magnetic lines of force (Figure 2, 90°) and is cutting the greatest number of magnetic lines of force.

Step4:
As the coil continues to turn, the voltage and current induced decrease until they reach zero, where the coil is again in the horizontal position (Figure 2, 180°).

Step5:
In the other half revolution, an equal voltage is produced except that the polarity is reversed (Figure 2, 270°, 360°). The current flow through R is in the opposite direction. The periodic reversal of polarity results in the generation of a voltage, as shown in Figure 2. The rotation of the coil through 360° results in an AC sine wave output.

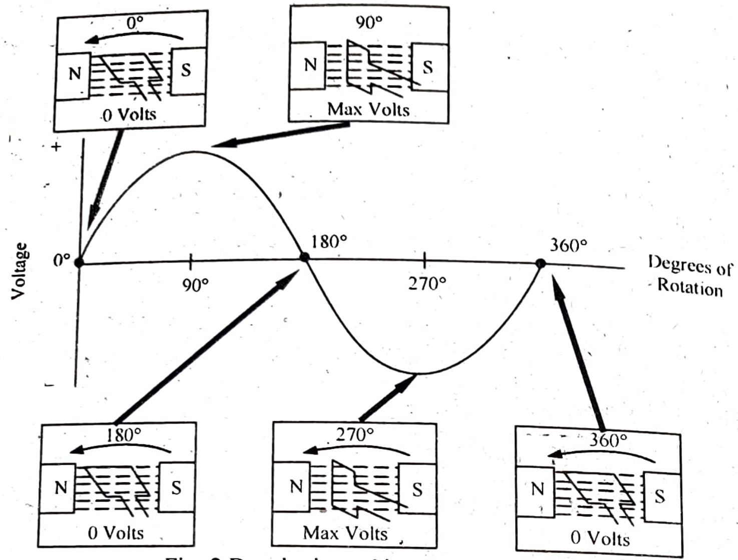


Fig: 2 Developing a Sine-Wave Voltage

AC generation is summarized below:

- A simple generator consists of a conductor loop turning in a magnetic field, cutting across the magnetic lines of force.
- The sine wave output is the result of one side of the generator loop cutting lines of force. In the first half turn of rotation this produces a positive current and in the second half of rotation produces a negative current. This completes one cycle of AC generation.
- When a voltage is produced by an AC generator, the resulting current varies in step with the voltage. As the generator coil rotates from 0° to 360°, the output voltage goes through one complete cycle.
- In one cycle, the voltage increases from zero to E_{max} in one direction, decreases to zero, increases to E_{max} in the opposite direction (negative E..), and then decreases to zero again.
- The value of E_{max} occurs at 90° and is referred to as peak voltage.
- The time taken by the waveform to complete one cycle is called the *period*, and the number of cycles per second is called the frequency (measured in hertz).
- The output voltage of an AC generator is sinusoidal in nature, as depicted in the figure 2. It can be expressed mathematically as

$E = E_{max} \sin \theta$

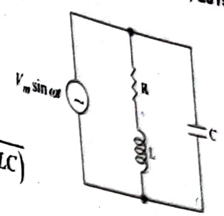
where, $\theta = \omega t$ and

$\omega = 2\pi f$, f being the frequency of the supply expressed in Hertz.

2.7. Derive an expression for the resonant frequency of a parallel circuit, one branch consisting of a coil of inductance L and a resistance R and the other branch of capacitance C. **[WBUT 2012, 2015]**

Answer:

The circuit diagram is drawn as given



The load impedance $= \dfrac{\frac{-j}{\omega C}\cdot(R+j\omega L)}{\frac{-j}{\omega C}+R+j\omega L}$

$$= \frac{\omega L - jR}{-j + R\omega C + j\omega^2 LC} = \frac{\omega L - jR}{R\omega C - j(1-\omega^2 LC)}$$

$$= \frac{(\omega L - jR)\left[R\omega C + j(1-\omega^2 LC)\right]}{\left[R\omega C - j(1-\omega LC)\right]}$$

$$= \frac{R\omega^2 LC - jR^2\omega C + j\omega L(1-\omega^2 LC) + R(1-\omega^2 LC)}{\left[R\omega C - j(1-\omega LC)\right]\left[R\omega C + j(1-\omega^2 LC)\right]}$$

$$= \frac{R\omega^2 LC - jR^2\omega C + j\omega L - j\omega^3 LC + R - R\omega^2 LC}{D}$$

For the impedance to be purely resistive,

$$-jR^2\omega C + j\omega L - j\omega^3 LC = 0$$

$$R^2 C - L + \omega^2 LC = 0$$

$$\omega^2 = \frac{L - R^2 C}{LC}$$

Resonant frequency, $\omega_r = \sqrt{\dfrac{L-R^2C}{LC}}$.

2.8. A two element series circuit consumes 700 V of power and has power factor of 0.707 leading when energized by a voltage source of waveform $v = 141\sin(314t + 30°)$. Find out the circuit elements. **[WBUT 2013]**

Answer:

Power consumed in a coil $= V_{rms} I_{rms} \cos\phi$

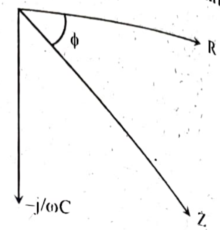
For this problem, $V_{rms} = \dfrac{V_{peak}}{V_2} V = \dfrac{141}{\sqrt{2}} V = 100V$

As given, $V_{rms} I_{rms} \cos\phi = 700$

$$I_{rms} = \frac{700}{100 \times 0.707} A = 7\sqrt{2} A$$

Impedance of the coil $= \dfrac{V_{rms}}{I_{rms}} = \dfrac{141}{\sqrt{2}} \times \dfrac{1}{7\sqrt{2}} \Omega = \dfrac{141}{14} \Omega = 10.04\Omega$

The power factor of the coil is 0.707 leading which means that the coil has a resistor and a capacitor. The phasor diagram for the impedance is drawn below.



$|z| = 10.04\Omega$

$\phi = \cos^{-1} 0.707 = 45°$

$R = 10.04\cos 45° = 10.04 \times 0.707 = 7.1\Omega$

$X_C = \frac{1}{\omega C} = 10.04 \times \sin 45°\Omega = 10.04 \times 0.707 = 7.1\Omega$

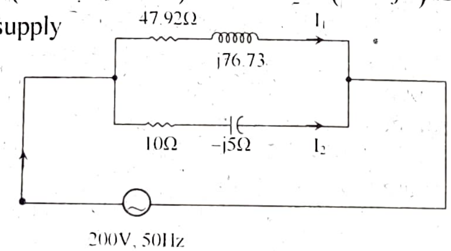
$C = \frac{1}{\omega . X_C} = \frac{1}{2\pi f \times 7.1} F = \frac{1}{314 \times 7.1} F = 0.45\,\text{mF}$

So, the circuit elements are a 7.1Ω resistor and 0.45 mF capacitor.

2.9. Two impedances $Z_1 = (47.92 + j76.73)\Omega$ and $Z_2 = (10 - j5)\Omega$ are connected in parallel across a 200 volt, 50 Hz supply. Find the current through each impedance and total current. What is the phase difference angle of each branch current with respect to the applied voltage? **[WBUT 2014]**

Answer:

Two impedances $Z_1 = (47.92 + j76.73)\Omega$ and $Z_2 = (10 - j5)\Omega$ are connected in parallel across a 200V, 50Hz supply



$Z_1 = (47.92 + j76.73)\Omega = 90.46\angle 58°\Omega$

$Z_2 = (10 - j5)\Omega = 11.18\angle -26.56°\Omega$

$\dot{I}_1 = \frac{200}{90.46\angle 58°} A = 2.2\angle -58° A = (1.166 - j1.866)\,A$

$\dot{I}_2 = \frac{200}{11.18\angle -26.56°} = 17.9\angle 26.56° A = (16 + j8)\,A$

So current through $Z_1 = 2.2A$

and current through $Z_2 = 17.9A$

Total current $= (1.166 - j1.866 + 16 + j8)\,A = (17.166 + j6.134)\,A = 18.23\angle 20° A$

Hence total current = 18.23A

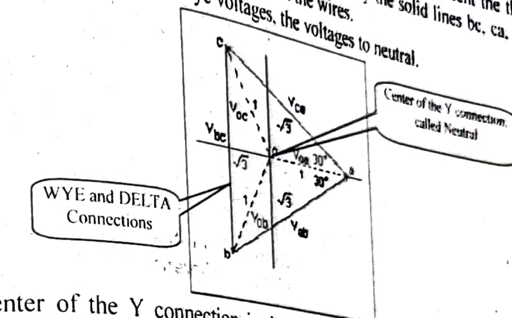
Phase different of current voltage source and Z_1 = 58° and that between Z_2 = 26.56°A

2.10. What is a three-phase balanced A.C. system? Show that, in a three-phase balanced a.c. circuit, the sum of current in the neutral is zero. **[WBUT 2012]**

Answer:

1st Part:

The key to understanding three-phase is to understand the phasor diagram for the voltages or currents. In the diagram, shown below, a, b and c represent the three lines, and o represents the neutral. The phasors are drawn by the solid lines bc, ca, ab are the line or delta voltages, the voltages between the wires. The dotted phasors are the wye voltages, the voltages to neutral.



The center of the Y connection is, in a way, equidistant from each of the three line voltages, and will remain at a constant potential. It is called the ***neutral***. They correspond to the two different ways a symmetrical load can be connected. We imagine the vectors to be rotating anticlockwise with time with angular velocity ω = 2πf, their projections on the horizontal axis representing the voltages as functions of time. e.g. V_{ab} is the voltage at point a relative to point b.

The same phasor diagram holds for the currents.

In this case, the line currents are the vectors, marked by the dotted lines and the vectors marked by the bold lines are the currents through a delta load.

The dotted and bold vectors

- Differ in phase by 30°, and
- In magnitude by a factor of $\sqrt{3}$, as is marked in the diagram.

2nd Part:

In the star connected system, $R_1, Y_1 \& B_1$ and joined together to form a neutral point, thus reducing the total number of conductors to three. If the neutral wire is also taken out the system will be a 3-phase 4 wire system. Such interconnection of the three phase is called star connection and is designated by a symbol Y. The junction of the three wires is normally called star or neutral point. If $i_R, i_Y \& i_B$ are the instantaneous values of the currents in the three phases, then their sum should be equal to zero to satisfy the property of interconnection. These currents may be represented by the following equations for a 3 phase symmetrical

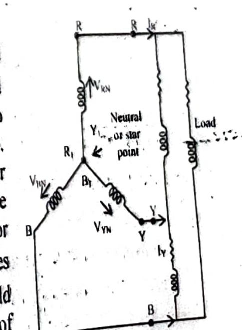


Fig: Star connected system

balanced system.

$$i_R = I_{max} \sin\theta$$
$$i_Y = I_{max} \sin(\theta - 120°)$$
$$i_B = I_{max} \sin(\theta - 240°)$$
$$i_R + i_Y + i_B = I_{max} [\sin\theta + \sin(\theta - 120°) + \sin(\theta - 240°)] = 0$$

Thus in a balanced 3 phase start connected circuit, the sum of the instantaneous currents in the three phases is always zero on the current in the neutral conductor is zero at every instant.

2.11. Proof that for a balanced start connected supply system connected to a balanced star connected load, the current through the neutral wire is zero. **[WBUT 2014]**

Answer:

In case of star-connected system V_{RN}, V_{YN} and V_{YN} are the rms values of the phase voltages.

The voltages across the lines, V_{RY}, V_{YB} and V_{BR} are obtained from the phase voltages as shown below.

Line voltage across terminals R and Y,

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} + (-V_{YN}) = V_{RN} - V_{YN} \text{ (Phasor difference)}$$

Similarly, line voltage across terminals Y and B, $V_{YB} = V_{YN} - V_{BN}$ and the line voltage across terminals B and R, $V_{BR} = V_{BN} - V_{RN}$. Hence the line voltages in a star-connected system are obtained by subtracting vectorially, the concerned phase voltages. Line voltages so obtained are equal and spaced 120°, apart, that is V_{YB} is lagging V_{RY} by 120° and V_{BR} is lagging V_{YB} by 120°, thus maintaining the phase sequence of the system as RYB. The phase voltages V_{RN}, V_{YN} and V_{BN} have been represented in Fig. 1, assuming the phase sequence of the system as RYB and taking V_{RN} as the reference phasor. The line voltage V_{RY} is obtained by adding V_{RN} and $V_{NY}(-V_{NY})$ vectorially. The other line voltages V_{YB} and V_{BR} are obtained in a similar manner. The complete phasor diagram showing phase and line voltages is shown in Fig. 1, in which the phase current is lagging its own phase voltage by an angle ϕ. Referring to Fig. 1, it is quite clear that the line voltages are 30° ahead of the phase voltages and are given by,

$$\text{Line voltage } V_{RY} = V_{RN} - V_{YN} = 2V_{RN} \cos 30° = 2V_{ph} \times \sqrt{3}/2 = \sqrt{3}V_{ph}$$

Hence, in a star-connected, 3-phase system,

Line voltage, $V_L = \sqrt{3}$ phase voltage (V_{ph}) ... (1)

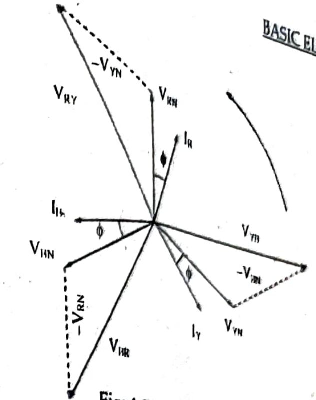


Fig: 1 Phasor diagram of star-connected circuit

It is quite obvious that the current in the line is equal to the phase current. Hence in a star connected 3-phase system,

Line current, I_L = phase current I_{ph} ... (2)

The angle between the line current and the corresponding the voltage is $(30° + \phi)$ in case of lagging loads and $(30° - \phi)$ in case of leading loads.

2.3. A star connected three-phase load draws a current of 15 A at a lagging power factor of 0.9 from a balanced 440 V, 50 Hz supply. Find the circuit elements in each phase of the elements connected in series. **[WBUT 2016]**

Answer:

The line to line voltage of the star-connected load is

$$V_{LL} = 440 \text{ V}$$

$$V_{ph} = \frac{440}{\sqrt{3}} = 254 \text{ Volts}$$

It draws a current of 15A.

So, the per phase impedance $= \frac{254}{15}\Omega = 16.9\Omega \approx 17\Omega$ The power factor is 0.9 lagging, so it can be inferred that there is a resistance and an inductance in series.

$$Z = 17\Omega.$$

Resistance $R = Z\cos\phi = 17 \times 0.9 = 15.3\Omega \approx 15\Omega$.

Inductive reactance $X_L = \omega L = 17\sin\phi = 17 \times 0.44 = 7.37\Omega$

Inductance $L = \frac{X_L}{\omega} = \frac{7.37}{2 \times \pi \times 50} H = \frac{2.346}{100} H = 23.46 \text{ mH}$

Long Answer Type Questions

3.1. Define phasor diagram, the phenomenon of resonance in a circuit containing an inductance, a capacitor and a resistor is series. **[WBUT 2005, 2009]**

OR,

What is resonance? Deduce the expression of frequency in a series RLC circuit at resonance. **[WBUT 2010, 2013, 2016]**

Answer:

Resonance may be described as the condition existing in any physical system when a fixed amplitude sinusoidal forcing function produces a response of maximum amplitude. The phenomenon of resonance occurs in electrical, mechanical, hydraulic, acoustic and many other physical systems. The following examples will serve to throw some light on the term.

The soldiers marching on a bridge are advised to break steps. This is done to prevent any undesirable vibrations which might occur due to the matching of the natural frequency of vibration of the bridge with that of the march of the soldiers.

or, for example, consider the case of an opera single who possesses the acumen of shattering crystal goblets by striking a particular note at a given frequency.

In each case, the phenomenon of resonance occurs through proper frequency matching. We shall however, restrict our focus to electrical systems only.

In view of study of resonance in electrical systems, the following points might be noted.

- Resonance occurs in an AC circuit when inductive reactance and capacitive reactance are equal to one another: $X_L = X_c$.
- When this occurs, the total reactance, $X = X_L - X_c$ becomes zero and the impendence is totally resistive.
- Because inductive reactance and capacitive reactance are both dependent on frequency, it is possible to bring a circuit to resonance by adjusting the frequency of the applied voltage.
- Resonant frequency (f_{Res}) is the frequency at which resonance occurs, or where $X_L = X_c$. Equation below is the mathematical representation for resonant frequency. $f_{Res} = \dfrac{1}{2\pi\sqrt{LC}}$

  where f_{Res} = resonant frequency (Hz)
  L = inductance (H)
  C = capacitance (f)

Series Resonance

In a series R-C-L circuit, as in the Figure below, at resonance the net reactance of the circuit is zero, and the impedance is equal to the circuit resistance; therefore, the current output of a series resonant circuit is at a maximum value for that circuit and is determined by the value of the resistance. (Z = R)

At resonance, the impedance of the circuit as observed by the constant amplitude, variable frequency sinusoidal forcing function is minimum since the reactive effect of the series inductance is nullified by the reactive effect of the capacitance. The input impedance of the source is assumed to be infinity.

The impedance of a network can be expressed as $Z = R + j\omega L - j/\omega C$

At resonance, $j\omega L = j/\omega C$

if the resonance frequency is designated by ω_0, then $\omega_0^2 = \dfrac{1}{LC} = \omega_n^2$

where ω_n is the natural frequency of oscillation of the circuit.

So, it can be concluded that resonance occurs in a series RLC circuit when the supply frequency is equal to the natural frequency of the network.

Since the nature of the impedance at resonance is purely resistive, it should be further noted that at resonance the supply voltage and source current are in phase.

Since $|Z| = R$; $\quad \phi = 0°$; $\quad |I| = \dfrac{|V|}{R}$

i.e., response or current assumes maximum value at resonance large voltage magnification occurs across inductance and capacitance, which however cancel each other.

The expressions of V_L and V_C are as follows:

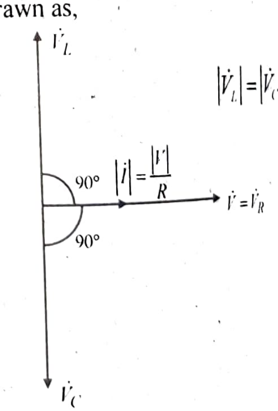
$$V_L = \frac{V_m}{R}\cdot\omega_0 L\sin(\omega t + 90°)$$

since voltage across inductance leads the current and thereby the supply voltage by 90°.

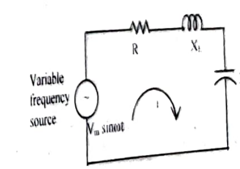
$$V_C = \frac{V_m}{R}\cdot\frac{1}{\omega_0 C}\sin(\omega t - 90°)$$

since voltage across capacitance lags the current and thereby the supply voltage by 90°.

The phasor diagram during resonant conditions with current as reference phasor can be drawn as,



$|\dot{V}_L| = |\dot{V}_C|$ at resonant frequency



Quality factor:

Quality Factor of a Series Resonant Circuit

The Q factor of an RLC series circuit can be defined in any of the following ways.

(i) It may be expressed as the voltage magnification that the circuit produces at resonance. During resonance current assumes maximum value, which is equal to V/R.
Voltage across inductance or capacitance equal to $I_{max} X_L = I_{max} X_c$.
Therefore, voltage magnification $= I_{max} X_L / V$

$$= (I_{max} X_L)/(I_{max} R)$$
$$= X_L/R = \omega_r L/R = 1/\omega_r RC$$

or, Q factor at resonance

$$Q_0 = \omega_r L / R = 1/\omega_r RC$$

(ii) Q Factor of a coil is the measure of its energy storage capability when it carries alternating current. It is a figure of merit of the coil. Mathematically, it is defined as $Q = 2\pi$ (max. energy stored / energy dissipated per cycle)

Since energy can be stored by either the inductor or the capacitor and dissipated by the resistor, Q might be expressed in terms of the instantaneous energy associated with each of the reactive elements and the average power dissipated in the resistor

$$Q = 2\pi \frac{\left[\omega_L(t) + \omega_C(t)\right]_{max}}{P_{avg} \cdot T}$$

where, P_{avg} is the average power lost in the resistor and T is the period of the sinusoidal frequency at which Q is being evaluated.

Consider the case of series resonance where the forcing function is $v = V_m \sin \omega t$ and the corresponding response at resonance is $i = \frac{V_m}{R} \sin \omega t$.

Energy stored by inductance $\omega_L(t) = \frac{1}{2} L_i^2 = \frac{1}{2} \cdot L \left(\frac{V_m}{R}\right)^2 \sin^2 \omega_0 t$

The instantaneous energy stored by the capacitor is $\omega_C(t) = \frac{1}{2} C_v^2$

$$= \frac{1}{2} C \left[\frac{1}{C} \int_0 i(t)\, dt\right]^2 = \frac{1}{2C} \frac{V_m^2}{R^2 \omega_0^2} \cos^2 \omega_0 t = \frac{1}{2R^2 C} \cdot V_m^2 \cdot LC \cos^2 \omega_0 t = \frac{V_m^2 \cdot L}{2R^2} \cos^2 \omega_0 t$$

The total instantaneous energy $= \frac{1}{2} \frac{L}{R^2} \cdot V_m^2 \left(\sin^2 \omega_0 t + \cos^2 \omega_0 t\right)$

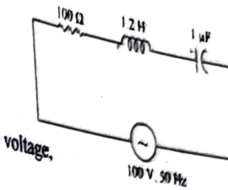
energy dissipated in resistor $= \frac{1}{2} I_m^2 R = \frac{1}{2} \frac{V_m^2}{R^2} \cdot R = \frac{1}{2} V_m^2 / R$

Multiplying with one period $P_{avg} T = \frac{1}{2 f_0} \frac{V_m^2}{R}$

3.2. A resistance of 100 ohms is connected with an inductance of 1.2 Henry and capacitance of microfarad in series. The combination is connected across 100 volts, 50 Hz supply.
Find.
a) Current in the resistance
b) Voltage across the capacitance
c) Power consumed.
d) Draw phasor diagram. **[WBUT 2007]**

Answer:

The circuit diagram is shown in the figure:



a) Current in the resistance $= \frac{V_m}{|z|} \sin(\omega t \pm \phi)$

Assuming that 100V is the r.m.s. value of supply voltage,

$$V_m = \sqrt{2} \times 100\text{V} = 141.4\text{V}$$

$$|z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\omega L = 2\pi \times 50 \times 1.2\Omega = 120\pi\,\Omega$$

$$\frac{1}{\omega C} = \frac{1}{2\pi \times 50} \times \frac{1}{10^{-6}} \Omega = \frac{10^6}{100\pi} \Omega = \frac{10^4}{\pi} \Omega$$

$$|z| = \sqrt{(100)^2 + \left(120\pi - \frac{10^4}{\pi}\right)^2}\,\Omega = \sqrt{(100)^2 + (376.99 - 3183.1)^2}\,\Omega$$

$$= \sqrt{(100)^2 + (2806.11)^2}\,\Omega = \sqrt{10^4 + 7874253.33}\,\Omega = 2807.89\,\Omega$$

The impedance diagram is drawn: $\phi = -\tan^{-1} \frac{2806.11}{100} = -87.958°$

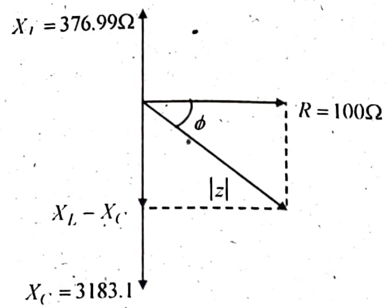
Current through resistance $= \frac{100 \times \sqrt{2}}{2807.89} \sin(\omega t + 87.958°) = 0.050 \sin(\omega t + 87.958°)$

b) Voltage across the capacitance $= 0.05 \cdot X_c \sin(\omega t + 87.958° - 90°)$

$$= 0.05 \times 3183.1 \sin(\omega t - 2.042°)$$
$$= 159.155 \sin(\omega t - 2.042°)$$

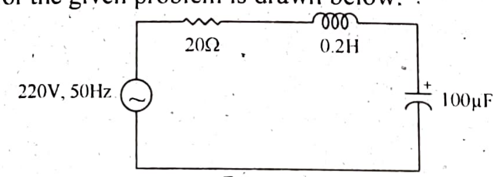
c) Power consumed $= I_{r.m.s}^2 R = \left(\frac{100}{2807.89}\right)^2 \cdot 100 = 0.1268\,\text{W}$

d)



3.3. A resistance of 20Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across 220 V, 50 Hz. Determine the following
a) Impedance **b) Current**
c) Voltage across R, L, C **d) Power factor and angle of lag**
e) Power in Watts and VA. **[WBUT 2008(EVEN)]**

Answer:
The circuit diagram for the given problem is drawn below:



(a) Impedance $= R + j\omega L - \frac{j}{\omega C} = 20 + j\left[2\pi\times 50\times 0.2 - \frac{1}{2\pi\times 50\times 100\times 10^{-6}}\right]$

$$= 20 + j\left[20\pi - \frac{100}{\pi}\right] = 20 + j\left[20\pi - \frac{100}{\pi}\right]$$

$$= 20 + j20\left[\pi - \frac{5}{\pi}\right] = 20 + j20[1.55]\Omega = 20 + j31\ \Omega$$

Impedance $= \sqrt{(20)^2 + (31)^2}\ \tan^{-1}\frac{31}{20} = 36.89\angle 57.17°\Omega$

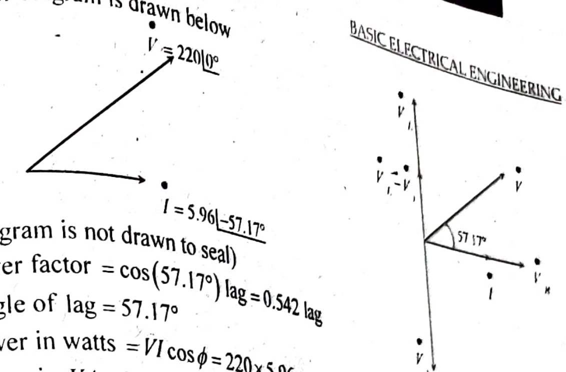
(b) Current $= \frac{\text{voltage}}{\text{Impedance}} = \frac{220}{36.89}\angle -57.17° = 5.96\angle -57.17°\text{A}$

(c) Voltage across resistance $= (5.96\times 20)$ V lagging the supply voltage by $57.17° = 119.2$ V lagging the supply voltage by 57.17°

Voltage across inductance $= 5.96\times 0.2\times 2\angle -57.17° + 90° \times \pi\times 50$
$= 374.48$ V by 32.83°

Voltage across capacitance $= 5.96\times\frac{1}{2\pi\times 50\times 100\times 10^{-6}}\text{V} = 5.96\times\frac{100}{\pi}\text{V}$
$= 189.7$ V lagging the supply voltage by $(57.17° + 90°)$

The phasor diagram is drawn below



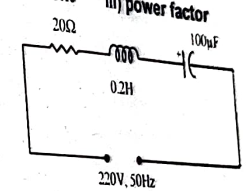
(The diagram is not drawn to seal)

(d) Power factor $= \cos(57.17°)$ lag $= 0.542$ lag
Angle of lag $= 57.17°$

(e) Power in watts $= VI\cos\phi = 220\times 5.96\times\cos 57.17°$ W $= 710.67$ W
Power in $VA = 220\times 5.96$ VA $= 1311.2$ VA

3.4. A resistance of 20 Ω, an inductance of 0.2 H and a capacitance of 100 μF are connected in series across a 220 V, 50 Hz supply.
a) Determine
i) impedance **ii) current** **iii) power factor** **iv) power consumed.** **[WBUT 2008]**

Answer:



(i) Impedance $= R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$$= 20 + j\left[2\times\pi\times 50\times 0.2 - \frac{1}{2\pi\times 50\times 100\times 10^{-6}}\right] = 20 + j\left[20\pi - \frac{100}{\pi}\right]$$

$$= 20 + j20\left[\frac{\pi^2 - 5}{\pi}\right] = 20 + j31 = 36.89\angle 57.17°$$

Impedance $= 36.89\Omega$

(ii) Current $= \frac{\text{Voltage}}{\text{impedance}} = 5.96$A

(iii) Power factor $= \cos 44.24° = 0.542$ lagging

(iv) Power consumed $= VI\cos\phi = 220\times 7.885\times 0.715 = 710.67$ watt

b) Draw the phasor diagram. **[WBUT 2008]**

Answer:

The phasor diagram is drawn with current as the reference phasor.

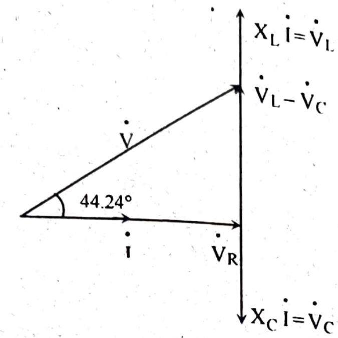
$\dot{I}=|I|\angle\phi=5.96\angle 0°$

$\dot{V}_R=|V_R|\angle\phi=|I|\,R\angle 0°$

$\dot{V}_L=|V_L|\angle\phi=|I|\,X_L\angle 90°$

$\dot{V}_C=|V_C|\angle\phi=|I|\,X_C\angle -90°$

$\dot{V}=|V|\angle\phi=220\angle 44.24°$



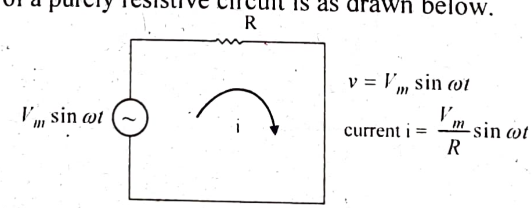
3.5. a) Draw the circuit diagram, waveform of voltage and current, phasor diagram of (i) purely resistive circuit (ii) purely inductive circuit (iii) purely capacitive circuit, supplied by sinusoidal voltage. **[WBUT 2009]**

OR,

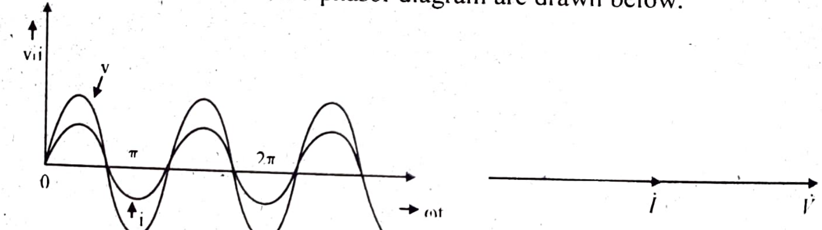
Prove that current in purely resistive circuit is in phase with applied A.C. voltage and current in purely capacitive circuit leads applied voltage by 90° and draw their waveforms. **[WBUT 2010]**

Answer:

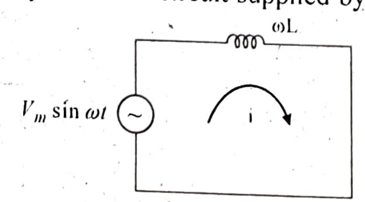
(i) Circuit diagram of a purely resistive circuit is as drawn below.



The voltage current waveforms and phasor diagram are drawn below.



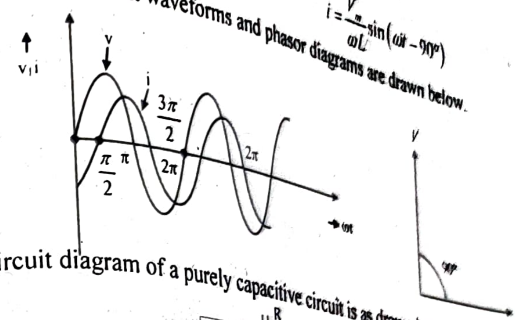
(ii) Circuit diagram of a purely inductive circuit supplied by sinusoidal sources is drawn below



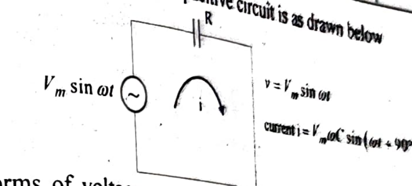
The mathematical expressions of and current are $v=V_m\sin\omega t$

$i=\frac{V_m}{\omega L}\sin(\omega t-90°)$

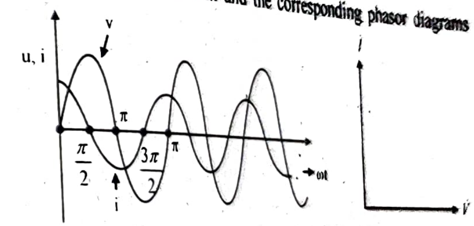
The voltage – current waveforms and phasor diagrams are drawn below.



(iii) Circuit diagram of a purely capacitive circuit is as drawn below



The waveforms of voltage, current and the corresponding phasor diagrams are drawn below.



b) A coil takes a current of 2A when connected to a 240 V, 50 Hz sinusoidal supply and consumes 200 W. Calculate the resistance, impedance and inductance of the coil. **[WBUT 2009]**

Answer:

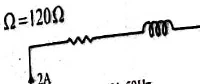
Supply voltage = 240V

Source current = 2A

Power consumed = 200W

Resistance of coil $R=\frac{\text{Power consumed}}{\text{Current Squared}}=\frac{200}{2\times 2}\Omega=50\Omega$

Impedance (magnitude) of the circuit $=\frac{\text{voltage}}{\text{Current}}=\frac{240}{2}\Omega=120\Omega$



Let L be the inductance of the coil.

Then ωL is the inductive reactance

From geometry, we know

$Z = \sqrt{R^2 + \omega^2 L^2}$

or, $(120)^2 = (50)^2 + \omega^2 L^2$

or, $(\omega L)^2 = 14400 - 2500 = 11{,}900$

or, $\omega L = 109.09\Omega$

or, $\omega = 2\pi f = 100\pi$ rad/sec

$L = \dfrac{109.09}{100\pi} = 0.347\text{ H} = 347\text{ mH}$

$\therefore$ Resistance $= 50\Omega$

Impedance $= 120\Omega$

Inductance $= 347$mH

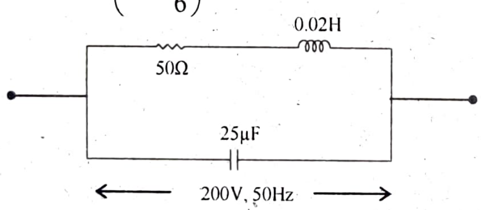
3.6. a) A coil having resistance of 50Ω and inductance of 0.02H is connected in parallel with a capacitor of 25 μF across a 200 V, 50 Hz supply. Find the current in the coil and the capacitor. Also find total current taken from the supply and overall power factor. Draw a neat phasor diagram. **[WBUT 2010(EVEN)]**

b) Find the resultant current in the following form:

$i = i_m \sin(\omega t \pm \phi)$, **if the current at a node are** $i_1 = 5\sin\omega t$, $i_2 = 10\sin\left(\omega t - \dfrac{\pi}{6}\right)$, $i_3 = 5\cos\left(\omega t + \dfrac{\pi}{6}\right)$ **and** $i_4 = 10\sin\left(\omega t + \dfrac{3}{6}\right)$.

Answer:

a)



The impedance of the coil $= (50 + j\omega \times 0.02)\Omega$

$\omega = 2\pi f = 2\pi \times 50$ rad/sec.

So, impedance of the coil $= (50 + j\,2\pi)\Omega = (50 + j\,6.28)\Omega = 50.39\angle 7.16°$

Hence current in the coil, $I_L = \left(\dfrac{200}{50.39}\right)A = 3.97A$

Lagging source voltage by 7.16°

Current in the capacitor, $\dot{I}_c = 200 \times 25 \times 10^{-6}\text{ A} = 5 \times 10^{-3}\text{ A}$

Leading the voltage by 90°

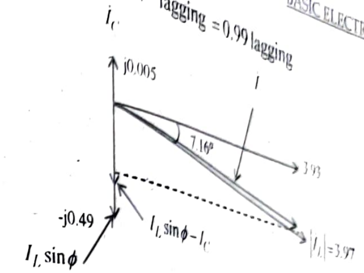
Total current taken form the supply $= \dot{I}_L + \dot{I}_c = 3.87\angle -7.16° + 5 \times 10^{-3}\angle 90°$

$= 3.93 - j0.49 + j0.005 = 3.93 - j0.485$

$\dot{I} = 3.96\angle -7.04°$ A

Total current taken from the supply is 3.96 A. Lagging the source voltage by 7.04°

Overall power factor $= \cos 7.04°$ lagging $= 0.99$ lagging



b) The four currents at a node are

$i_1 = 5\sin\omega t$; $\quad i_2 = 10\sin\left(\omega t - \dfrac{\pi}{6}\right)$;

$i_3 = 5\cos\left(\omega t + \dfrac{\pi}{6}\right)$; $\quad i_4 = 10\sin\left(\omega t + \dfrac{\pi}{6}\right)$

The currents can be expressed in phasor form as

$\dot{I}_1 = 5\angle 0°$; $\quad \dot{I}_2 = 10\angle -\dfrac{\pi}{6}$

$\dot{I}_3 = 5\angle \dfrac{2\pi}{3}$; $\quad \dot{I}_4 = 10\angle \dfrac{\pi}{6}$

Expressing the currents in the rectangular form from polar form we have

$\dot{I}_1 = 5 + j0 = 5$; $\quad \dot{I}_2 = 10\cos\dfrac{\pi}{6} - j10\sin\dfrac{\pi}{6} = 8.66 - j5$;

$\dot{I}_3 = 5\cos\dfrac{2\pi}{3} + j5\sin\dfrac{2\pi}{3} = -2.5 + j4.33$; $\quad \dot{I}_4 = 10\cos\dfrac{\pi}{6} + j10\sin\dfrac{\pi}{6} = 8.66 + j5$

To get the resultant current, we have to get the vector summation of the four currents

$\dot{I} = \dot{I}_1 + \dot{I}_2 + \dot{I}_3 + \dot{I}_4 = 5 + j0 + 8.66 - j5 - 2.5 + j4.33 + 8.66 + j5$

$= 19.82 + j4.33 = 20.29\angle 12.3°$

$i = 20.29\sin(\omega t + 12.3°)$

3.7. a) Explain what are meant by phase and phase difference of sinusoidal waves. **[WBUT 2010]**

Answer:

Phase:

Phase in sinusoidal waves is the fraction of a wave cycle which has elapsed. relative to an arbitrary point.

Phase Difference:

A term more commonly used is **phase difference**. The phase difference can be used to describe and compare two different voltages or currents or a current and a voltage that have the same frequency, which pass through zero values in the same direction at different times.

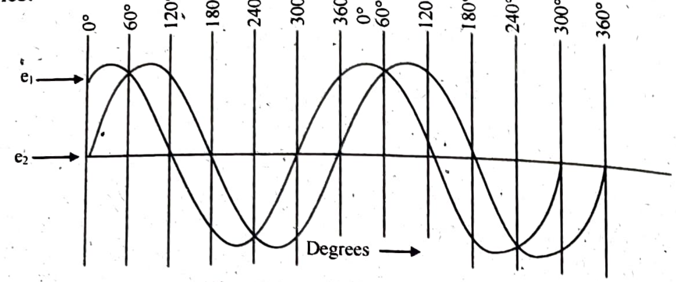


Fig: 1 Phase Relationship

If the phase difference between two currents, two voltages, or a voltage and a current is zero degrees, they are said to be "in-phase."
In the above diagram, e_2 is considered to be the reference voltage and the phase of e_1 is defined with respect to e_2. For example if
$e_2 = E_2 \sin\omega t$, then $e_1 = E_2 \sin(\omega t+\alpha)$ where α=60 electrical degrees since the zero crossing of e_2 occurs 60 electrical degrees before that of e_1 in the same direction.
The voltage e_1 is said to lead e_2 by 60 electrical degrees, or it can be said that e_2 lags e_1 by 60 electrical degrees.

b) A coil of resistance 30 Ω and inductance 320 mH is connected in parallel to a circuit consisting of 75 Ω in series with 150 μF capacitor. The circuit is connected to a 200 volt, 50 Hz supply. Determine supply current and circuit power factor.
[WBUT 2010]

Answer:
Resistance of the coil = 30Ω
Inductance of the coil = 320mH
Frequency of supply = 50 Hz

$$\omega = 2\pi f = 2\times\pi\times 50 \text{ rad/sec} = 100\pi \text{ rad/sec.}$$

Impedance of the coil

$$(30 + j\omega\times 320\times 10^{-3})\Omega = (30 + j\times 100\pi\times 320\times 10^{-3})\Omega$$
$$= (30 + j\,100.53)\Omega = 105\angle 73.38°\Omega$$

Impedance of the capacitive circuit $= (75 - \frac{j}{\omega C})\Omega = \left(75 - \frac{j}{100\pi\times 150\times 10^{-6}}\right)\Omega$

$$= \left(75 - j\frac{1000}{15\pi}\right)\Omega = (75 - j\,21.22)\Omega$$
$$= 78\angle -15.8°\ \Omega$$

For a 200V, 50 Hz supply,

Current through the coil $= \frac{200\angle 0°}{105\angle 73.38°}\text{A} = 1.9\angle -73.38°\text{A} = (0.53 - j\,1.82)\text{A}$

Current through the capacitive circuit $= \frac{200\angle 0°}{78\angle -15.8°} = 2.56\angle 15.8°\text{ A} = (2.46 + j0.7)\text{A}$

The source current $= (0.53 - j1.82 + 2.46 + j0.7)\text{A}$
$= (2.99 - j\,1.12)\text{A} = 4.24\angle -20.53°\text{A}$

So the supply current = 4.24A
Supply p.f. = cos(20.53°)lagging = 0.94 lagging

3.8. A circuit consists of series combination of elements as resistance of 6 Ω, inductance of 0.4 H and a variable capacitor across 100 V, 50 Hz supply. Calculate (i) value of capacitance at resonance, (ii) voltage drop across capacitor and (iii) Q factor of coil.
[WBUT 2010, 2013]

Answer:
The given circuit parameters are
Resistance $R = 6\Omega$
Inductance $L = 0.4\text{H}$
Capacitance is variable
Supply voltage = 100V
Supply frequency = 50 Hz

$$\omega = 2\pi f \text{ rad/sec} = 2\times\pi\times 50 \text{ rad/sec} = 100\pi \text{ rad/sec}$$

At resonance, $\omega L = \frac{1}{\omega C}$

$$C = \frac{1}{\omega^2 L}$$

(i) Capacitance at resonance

$$C = \frac{1}{(100\pi)^2\times 0.4}\text{F} = \frac{100\times 10^{-6}}{\pi^2\times 0.4}\text{F} = \frac{100}{\pi^2\times 0.4}\mu\text{F} = 25.33\mu\text{F}$$

(ii) Current during resonance $= \frac{100}{6}\text{A} = 16.67\text{A}$

Voltage drop across the capacitance during resonance $= X_c I = \frac{1}{\omega C} I$

$$= \frac{10^6}{100\pi\times 25.33}\times 16.67$$
$$= \frac{10^4\times 16.67}{25.33\times\pi}V = 2094.8V$$

iii) Q-factor of coil $= \frac{\omega L}{R} = \frac{100\pi\times 0.4}{6} = 20.9$

$$Q = \frac{1}{\omega_o RC} = \frac{10^6}{100\pi\times 6\times 25.33} = 20.9$$

$$Q = \text{voltage magnification} = \frac{\text{Voltage across capacitance}}{\text{Supply voltage}} = \frac{2094.8}{100} = 20.9$$

3.9. a) Derive a mathematical expression for the average real power delivered by a single phase a.c. source with an e.m.f. of $e=\sqrt{2}E_m \sin\omega t$ when the source current is $i=\sqrt{2}I_m \sin(\omega t-\theta)$.

b) Define power factor of an a.c. circuit. State the major disadvantages of poor power factor. **[WBUT 2011]**

OR,

Define power factor of an A.C. circuit. State the disadvantages associated with having a load power factor. **[WBUT 2012]**

Answer:

a) Consider the general case when the phase difference between voltage and current is ϕ. If the current lags the voltage by ϕ, the voltage and current may be written as

$$v=\sqrt{2}V_m \sin\phi,\ i=\sqrt{2}I_m \sin(\phi-\theta)\ \omega t=\phi$$

Instantaneous power, $p = vi = 2V_m I_m \sin\theta \sin(\theta-\phi)$

$$=\frac{2V_m I_m}{2}\left[\cos\phi-\cos(2\theta-\phi)\right]\left[\because \sin\alpha\sin\beta=\frac{1}{2}\cos(\alpha-\beta)-\frac{1}{2}\cos(\alpha+\beta)\right]$$

$$\text{Active Power } P=\frac{1}{2\pi}\int_0^{2\pi} p\,d\phi=\frac{2V_m I_m}{2\pi}\int_0^{2\pi}\frac{1}{2}\left[\cos\theta-\cos(2\phi-\theta)\right]d\phi$$

$$=\frac{2V_m I_m}{4\pi}\int_0^{2\pi}\cos\theta\,d\phi-\frac{2V_m I_m}{4\pi}\int_0^{2\pi}\cos(2\phi-\theta)\,d\phi$$

$$=\frac{2V_m I_m}{4\pi}\cos\theta\left[\phi\right]_0^{2\pi}-\frac{2V_m I_m}{4\pi}\left[\frac{1}{2}\sin(2\phi-\theta)\right]_0^{2\pi}$$

(since $\cos\theta$ is constant for a given circuit)

$$=\frac{2\left(\sqrt{2}V\right)\left(\sqrt{2}I\right)}{4\pi}(\cos\theta)(2\pi)-\frac{2V_m I_m}{8\pi}\left[\sin(4\pi-\theta)-\sin(-\theta)\right]$$

$$=2VI\cos\theta-\frac{2V_m I_m}{8\pi}(-\sin\theta+\sin\theta)$$

$$\therefore\quad P=2VI\cos\theta$$

b) 1[st] Part:

Power factor (pf) is defined as the ratio between true power and apparent power. True power is the power consumed by an AC circuit, and reactive power is the power that is stored in an AC circuit. $\cos\theta$ is called the power factor (pf) of an AC circuit. It is the ratio of true power to apparent power, where θ is the phase angle between the applied

voltage and current sine waves and also between P and S on a power triangle (Figure 1). Equation below is a mathematical representation of power factor.

$$\cos\theta=\frac{P}{S}$$

where $\cos\theta$ = power factor (pf)
P = true power (watts)
S = apparent power (VA)

Power factor also determines what part of the apparent power is real power. It can vary from 1, (when the phase angle is 0°), to 0, (when the phase angle is 90°). In an inductive circuit, the current lags the voltage and is said to have a lagging power factor, as shown in Figure 2.

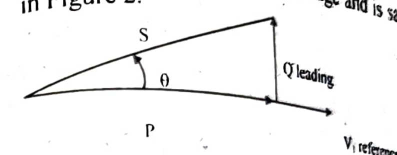


Fig: 1 Leading Power Factor

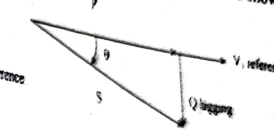


Fig 2 Lagging Power Factor

In a capacitive circuit, the current leads the voltage and is said to have a leading power factor.

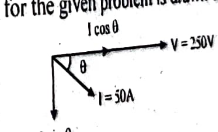
2[nd] Part:

1) If Power Factor is low the current rating of the electrical machinery increases which result in higher loss and overheating.
2) The Power Factor of a 3-phase system decreases, the current rises. The heat dissipation in the system rises proportionately by a factor equivalent to the square of the current rise.
3) Low power factor reduces an electrical system's distribution capacity by increasing current flow and causing voltage drops.
4) Low power factor shortens the lifespan of electrical appliances.

3.10. A circuit receives 50A current at a power factor of 0.8 lag from a 250V, 50 Hz, 1-ph A.C. supply. Calculate the capacitance of the capacitor which is required to be connected across the circuit to make the power factor unity. **[WBUT 2011]**

Answer:

The phasor diagram for the given problem is drawn below.



θ is power factor angle, $\theta=\cos^{-0.8}=36.86^\circ$

The component of current in phase with voltage $=I\cos\theta=50\times0.8=40A$

The component of current lagging the voltage by 90°

The component of current lagging the voltage by 90° $(\because \sin36.86^\circ=0.6)$

$I\sin\theta=50\times0.6=30\text{A}$

The role of the capacitor is to eliminate the lagging component of current.

Let the value of the attached capacitance $= C$ Farads

$$\frac{V}{X_c} = I$$

or, $V\omega C = I$

$$\omega C = \frac{I}{V} = \frac{30}{250} \text{ Siemens.}$$

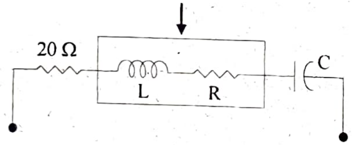
$$C = \frac{30}{250\times2\times\pi\times50}\text{ Farads} = \frac{30}{125\times2\times\pi\times100}\text{ Farads} = \frac{30}{25\times\pi\times10^3}\text{ Farads} = 382\mu F$$

So the required capacitance to make the p.f = unity is $382\mu F$.

3.11. A 20Ω resistor, a choke coil having some inductance and some resistance and a capacitor are connected in series across a 25V variable frequency source. When frequency is 400 Hz, the current is maximum and its value is 0.5A and the potential difference across the capacitor is 150V. Calculate the resistance and the inductance of the choke coil and the capacitance of the capacitor. [WBUT 2011]

Answer:

The circuit diagram for the given problem is shown below



The source impedance $= 20 + R + j\omega L - j/\omega C$

Maximum current flows through the coil at resonance.

During resonance, $\omega L = \dfrac{1}{\omega C}$

Current during resonance, $I = \dfrac{V}{20+R}$

or, $\dfrac{25}{20+R} = 0.5$

Hence, $20 + R = 50$

or, $R = 30\Omega$

Voltage drop across the capacitor, $V_c = IX_c = I/\omega C$

or, $\omega C = \dfrac{0.5}{150}$ Siemens.

$$\text{or, } C = \frac{0.5}{150\times2\times\pi\times400}\text{F} = \frac{5}{1500\times2\times\pi\times400}\text{F} = \frac{5}{120\times\pi\times10^4}\text{F} = \frac{5\times10^{-6}}{1.2\times\pi}\mu\text{F} = 1.33\mu\text{F}$$

During resonance, $\omega L = \dfrac{1}{\omega C}$

$$\omega^2 = \frac{1}{LC}$$

$$L = \frac{1}{\omega^2 C} = \frac{1\times1.33}{(2\times\pi\times400)^2\times10^{-6}}\text{H} = \frac{1.33}{64\times\pi^2\times10^4\times10^{-6}}\text{H} = \frac{133}{64\times\pi^2}\text{H} = 210\text{mH}$$

The resistance and inductance of choke coil are 30Ω and 210mH respectively. The capacitance of the capacitor is $1.33\mu F$.

3.12. Prove that the current in a purely resistive circuit is in phase with applied A.C. voltage and current in a purely capacitive circuit leads applied voltage by 90° and also draw their waveforms. [WBUT 2012]

Answer:

In a purely resistive circuit:

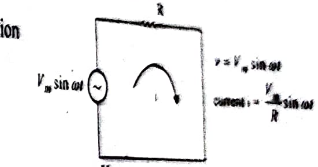
Consider a simple circuit consisting of a pure resistance R ohms connected across a voltage $v = V_m \sin\omega t$

$$i = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$

i.e. $i = \left(\dfrac{V_m}{R}\right)\sin(\omega t)$

This is the equation giving instantaneous value of the current.

Circuit diagram of a purely resistive circuit is as drawn below.



Comparing this with standard equation

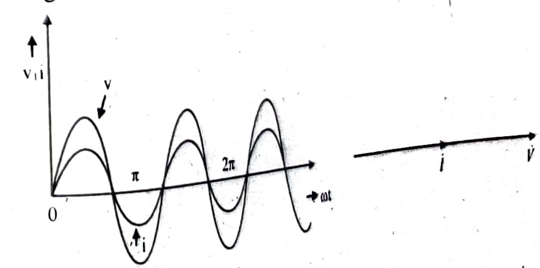
$i = I_m \sin(\omega t + \phi)$

$I_m = \dfrac{V_m}{R}$ & $\phi = 0$

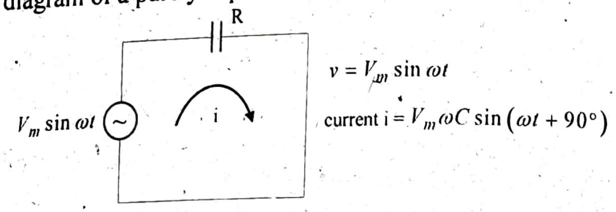
So, maximum value of alternating current i is $I_m = \dfrac{V_m}{R}$ while, as $\phi = 0$, it indicates that it is in phase with the voltage applied. There is no phase difference between the two.

The current is going to achieve its maximum (positive & negative) and zero whenever voltage is going to achieve its maximum (positive & negative) and zero values.

The voltage current waveforms and phasor diagram are drawn below.



In a purely capacitive circuit:
Consider a simple circuit consisting of a pure capacitor of C farads, connected across a voltage given by the equation, $v = V_m \sin \omega t$
Circuit diagram of a purely capacitive circuit is as drawn below



The current i charges the capacitor C. The instantaneous charge 'q' on the plates of the capacitor is given by

$q = c_v$

$q = CV_m \sin \omega t$

Now, current is rate of flow of charge.

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

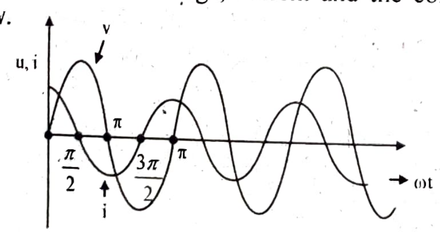
$$i = CV_m \frac{d}{dt}(\sin \omega t) = CV_m \omega \cos \omega t$$

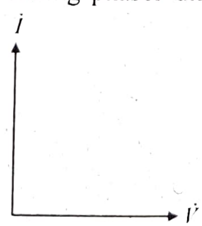
$$i = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where, $I_m = \frac{V_m}{X_C}$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

The above equation clearly shows that the current is purely sinusoidal and having phase angle of $\frac{\pi}{2}$ radians i.e. 90°.

This means current leads voltage applied by 90°. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor lags the current passing through the capacitor 90°.
The waveforms of voltage, current and the corresponding phasor diagrams are drawn below.





3.13. a) Derive the expression of quality factor of a series R-L-C circuit at resonance.
Answer: ***Refer to Question No. 3.1.***

b) A coil of resistance 10Ω and inductance 0.02 H is connected in series with another coil of resistance 6Ω and inductance 15mH across a 230 V, 50 Hz supply. Calculate: **[WBUT 2013]**
(i) impedance of the circuit
(ii) voltage drop across each coil
(iii) the total power consumed by the circuit. **[WBUT 2013]**

Answer:

i) Total resistance of the circuit $= (10+6)\Omega = 16\Omega$
Total inductance of the circuit $= 0.02\text{ H} + 15\text{ mH} = 20\text{ mH} + 15\text{ mH} = 35\text{ mH}$
Total inductive reactance of the circuit $= 0.035 \times 2 \times \pi \times 50 = 10.99\ \Omega \cong 11\ \Omega$
Total impedance of the circuit $= \sqrt{R^2 + \omega^2 L^2} = \sqrt{16^2 + 11^2} = 19.4\Omega$

Phase angle $= \tan^{-1}\frac{11}{16} = 34.5°$

ii) The current flowing through the circuit $= \frac{230}{19.4}\text{A} = 11.86\text{A}$
and it lags behind the supply voltage by 34.5°.
The impedance of the first coil

$$= (10 + j2\pi \times 50 \times 0.02)\Omega = \sqrt{10^2 + (2\pi)^2}\Omega = \sqrt{100 + (6.28)^2}\Omega = 11.81\Omega$$

Phase angle $= \tan^{-1}\frac{2\pi}{10} = \tan^{-1}\frac{\pi}{5} = 32.14°$

The voltage drop across first coil $= 11.86\angle -34.5° \times 11.81\angle 32.14°V = 140\angle -1.36°V$

Impedance of the second coil $= (6 + j2\pi \times 50 \times 0.015)\Omega$

$$= \sqrt{36 + (1.5\pi)^2}\Omega = \sqrt{58.2}\Omega = 7.629\Omega = 7.63\Omega$$

Phase angle $= \tan^{-1}\frac{1.5\pi}{6} = 38°$

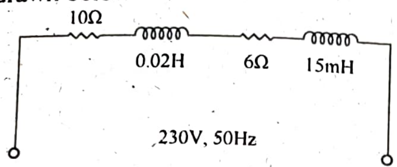
Voltage drop across the coil $= 11.86\angle -34.5° \times 7.63\angle 38°V = 90.5\angle 3.5°V$
Ans. Voltage drop across first coil is 140V and it lags the supply voltage by 1.36° whereas voltage drop across second coil is 90.5V and it leads the supply voltage by 3.5°.

iii) Total power consumed in the two coils $= I^2(R_1 + R_2)W = (11.86)^2 \times 16W = 2.25kW$

3.14. a) A coil of resistance 10Ω and inductance 0.02H is connected in series with another coil of resistance 6Ω and inductance 15mH across a 230V, 50Hz supply.
Calculate: **(i) impedance of the circuit**
(ii) the voltage drop across each coil and
(iii) the total power consumed by the circuit. **[WBUT 2014, 2016]**

Answer:

a) The circuit diagram is drawn below



Impedance of branch 1 $=(10+j2\pi\times50\times0.02)\Omega=(10+j2\pi)\Omega=(10+j6.282)\Omega$

$$=\sqrt{100+(6.28)^2}\,\angle\tan^{-1}\frac{6.282}{10}=11.81\angle 32.14^\circ$$

Impedance of branch 2 $=(6+j2\pi\times50\times0.015)\Omega=(6+j1.5\pi)\Omega=(6+j4.71)\Omega$

$$=7.63\Omega\,\angle\tan^{-1}\frac{4.71}{6}=7.63\angle 38.13^\circ$$

(i) Total impedance of the circuit $=(10+j2\pi+6+j1.5\pi)\Omega$

$$=(16+j3.5\pi)\Omega=(16+j11)\Omega=19.4\angle 34.5^\circ$$

Current through the circuit $\dfrac{230\angle 0^\circ}{19.4\angle 34.5^\circ}=11.86\angle -34.5^\circ$

Current through the circuit is 11.86A at 34.5° logging the supply voltage.

(ii) Voltage drop across coil 1 $=11.86\times11.81\angle -34.5^\circ+32.14^\circ=140^\circ\angle -2.36^\circ V$

Voltage drop across coil 1 is 140V lagging the supply voltage by 2.36.

Voltage drop across coil 2 $=11.86\times7.63\angle 38.13^\circ-34.15^\circ=90.5\angle 4^\circ$

(iii) Current flowing through the circuit = 19.4 A

Total resistance $=16\Omega$

Power consumption $=(19.4)^2\times16W=6021.76W=6kW$

b) Define power factor. Shown that the active power of a purely capacitive circuit over a complete cycle is zero. **[WBUT 2014, 2016]**

Answer:

1[st] Part: ***Refer to Question No. 3.9(b).***

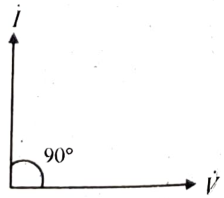
2[nd] Part:

For a capacitance, $z=\dfrac{1}{\omega C}$ $\qquad \phi=90^\circ$

The phasor diagram is shown below with current leading the voltage by 90°.

Instantaneous power

$$=vi=V_m\sin\omega t\cdot\frac{V_m}{z}\sin(\omega t+\phi)$$



$$=\frac{V_m^2}{2}\cdot\omega C\quad[2\sin\omega t\sin(\omega t+\phi)]$$

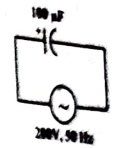
$$=\frac{V_m^2\omega C}{2}\left[\cos\{\omega t-(\omega t+\phi)\}-\cos\{\omega t+\omega t+\phi\}\right]$$

$$=\frac{V_m^2\omega C}{2}\left[\cos(-\phi)-\cos(2\omega t+\phi)\right]$$

$\phi=90^\circ$, hence $\cos(-\phi)=\cos\phi=0$ and average value of $\cos(2\omega t+\phi)=0$. Thus it can be shown that power dissipation in a pure capacitance in an a.c. circuit is equal to zero.

3.15. a) A capacitor of 100 µF is connected across a 200V, 50 Hz single phase supply. Calculate (i) the reactance of the capacitor, (ii) r.m.s. value of current, (iii) the maximum current. **[WBUT 2015]**

Answer:



a) (i) Reactance of the capacitor $=X_C=\dfrac{1}{j\omega C}=\dfrac{1}{2\pi\times f\times C}$

$$=\frac{1}{2\pi\times50\times100\times10^{-6}}\Omega$$

$$=\frac{10^6}{\pi\times10^4}\Omega=\frac{100}{\pi}\Omega=31.84\Omega$$

(ii) Taking the r.m.s voltage to be voltage to be 200V,

R.M.S. current $=\dfrac{200}{31.84}A=6.28A$

(iii) The maximum current $=(6.28\times\sqrt{2})A=8.88A$.

b) What is meant by bandwidth? With a neat sketch of waveform find out the expression for the bandwidth of a resonant circuit. **[WBUT 2015]**

Answer:

Bandwidth is defined as the difference between the upper cut off frequency and the lower cut off frequency. These two frequencies are also termed as the half power frequencies of the network.

The current varies inversely with the variation in the impedance and therefore it is maximum at resonance frequency when impedance is minimum and decreases with the variation in frequency on both sides of the resonant frequency (as impedance is large). The curve drawn below depicting the relation between circuit current and the frequency of the source voltage is called the resonance curve.

Dependence of the Resonance Curve

The shape of the Resonance Curve depends largely on the value of the resistance R. For small values of R it is sharply peaked and flat for larger values of R.

Depending on the value of R, the circuit is termed as ***sharply resonant*** or ***highly selective.*** On the other hand, high resistance circuits have flat resonance curves and poor selectivity.

However, it should be emphasized that the height of the response curve depends only upon the value of R for constant amplitude excitation, the width of the curve or the steepness of the sides depends upon the other two element values also. This width of the response curve which determines the selectively of a network is related to a quantity called bandwidth.

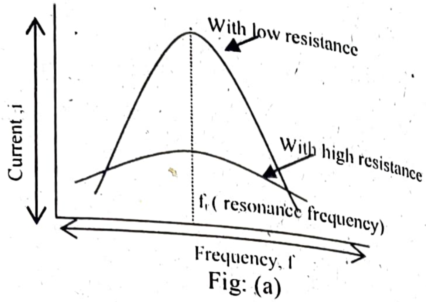


Fig: (a)

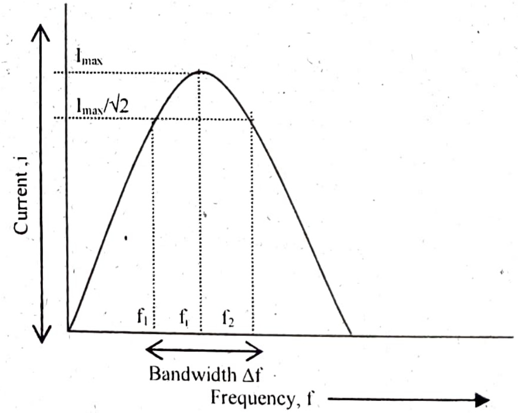
Selectivity of different resonant circuits is compared in terms of their bandwidths. Bandwidth is defined as the difference between the upper cut off frequency and the lower cut off frequency. These two frequencies are also termed as the half power frequencies of the network. When the source frequency assumes these two critical values, the power transferred to the circuit is equal to half the power transmitted during resonance. Hence the name half power frequencies. During this condition the current in the circuit is $1/\sqrt{2}$ times the current at resonant condition.

Bandwidth $\Delta f = f_2 - f_1$

The actual power input at frequencies at f_1 and f_2, $P = I^2 R = (I_{max} / \sqrt{2})^2 R$

$= (I^2_{max} R) / 2$

$= P_{max} / 2$ ..... where P_{max} is the power transferred at resonance.



Resonance

It can be established mathematically that the sharpness of the response curve of any resonant circuit is determined by the maximum amount of energy that can be stored in a circuit, compared to the energy that is lost during one complete period.

3.16. a) Prove that the current in purely capacitive circuit leads the applied voltage by an angle 90° and draw their waveforms. Also calculate the average power of capacitive circuit. **[WBUT 2015]**

Answer:

a) The capacitive reactance, $X_c = \frac{1}{j\omega C}$

In phasor form, $X_C = \frac{1}{\omega C \angle 90°}$

Current through capacitance $= \frac{\text{Voltage across capacitance}}{\text{Capacitive reactance}}$

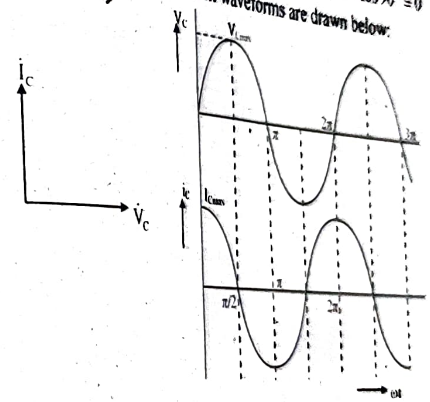
$$I_C = \frac{V_C}{X_C} = V_C \cdot \omega C \angle 90°$$

This clearly proves that the current phasor leads the voltage phasor by 90° for a capacitance.

Average power of a capacitive circuit $= V_C I_C \cos\phi_C$

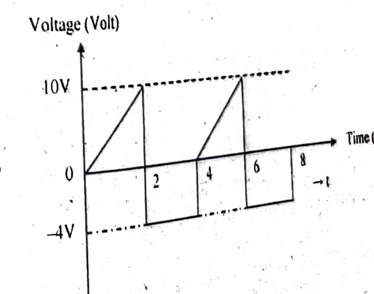
$= V_C V_C \omega C \cos 90° = V_C^2 \omega C \cos 90° = 0$

The phasor diagrams and voltage-current waveforms are drawn below:



b) Find the Form Factor of the given waveform. **[WBUT 2015]**

$\frac{\frac{1}{2}\times 10 \times 2}{2}$ 5



Answer:

Form factor $= \frac{\text{RMS value}}{\text{Average value}}$

Average value $= \frac{1}{T}\left[\int_0^T f(t)\,dt\right]$

$f(t) = 5t$ for $0 \le t \le 2$

$= -4$ for $2 \le t \le 4$

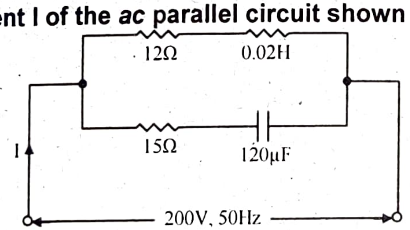
$\therefore$ Average value $= \frac{1}{4}\int_0^2 5t\,dt - \frac{1}{4}\int_2^4 4\,dt = \frac{1}{4}\times 5 \times \frac{t^2}{2}\Big|_0^2 - \frac{1}{4}4t\Big|_2^4 = \frac{10-8}{4} = \frac{1}{2} = 0.5$

RMS value $= \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-4)^2\,dt\right]} = \sqrt{\frac{1}{4}\left[\frac{25t^3}{3}\Big|_0^2 + 16t\Big|_2^4\right]}$

$= \sqrt{\frac{1}{4}\left[\frac{200}{3} + 32\right]} = \sqrt{\frac{296}{12}} = \sqrt{\frac{74}{3}} = 4.97$

Form factor $= \frac{4.97}{0.5} = 9.9$

3.17. Find the net current I of the *ac* parallel circuit shown in figure below. **[WBUT 2016]**



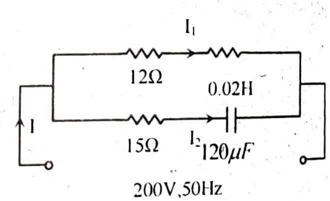
Answer:

Impedance Z_1

$= 12 + j0.02 \times 2 \times \pi \times 50 = 12 + j2\pi = (12 + j6.28)\,\Omega$

$= 13.54\angle 27.6°\,\Omega$

current



$I_1 = \frac{200}{13.54\angle 27.6°} A = 14.8\angle -27.6 A = 13.1 - j6.84 A$

$Z_2 = 15 - \frac{j}{2\times\pi\times 50\times 120\times 10^{-6}} = 15 - j\frac{1000}{12} = 15 - j\frac{250}{3} = 15 - j83.33$

$= 84.67\angle -80°\ \Omega$

current $I_2 = \frac{200}{84.67} = 2.36\angle 80° A = (0.41 + j2.32)\,A$

Hence, total current $= 13.1 - j6.84 + 0.41 + j2.32 = 13.51 - j4.52$

$= 14.25$ lagging the voltage by $\tan^{-1}\frac{4.52}{13.51}$ degrees $= 18.5°$

3.18. Explain the method of measurement of balanced three phase power by two wattmeter method under different power factor conditions. **[WBUT 2006, 2009]**

OR,

Explain the 2-Wattmeter method of power measurement for a 3-ϕ balanced load. Draw the necessary phasor diagrams. Also show how the power factor can be measured from this method. **[WBUT 2014]**

OR,

Show that the power in a three phase circuit can be measured using 2 wattmeters. **[WBUT 2015]**

OR,

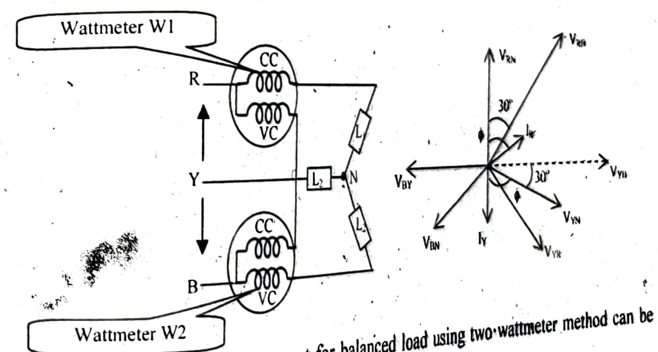
Draw the necessary phasor diagram of power measurement for 3 phase star connected balanced inductive load and show how power factor can be calculated from this method. **[WBUT 2017]**

Answer:

Power Measurement in three-phase Circuit: The two-wattmeter Method

Suppose

- The loads L_1, L_2 and L_3 are connected in star configuration, as shown in the figure below.
- The current coils of the two wattmeters are connected in any two lines, say the 'red' and 'blue' lines, and the voltage circuits are connected between these lines and the third line.
- V_{RN}, V_{YN} and V_{BN} to be the potential, differences (p.ds) across the loads.
- I_R, I_Y and I_B to be the corresponding values of the line (and phase) currents.
- Load is inductive in nature.



Three-phase power measurement for balanced load using two-wattmeter method can be understood from the phasor diagram.

Presume, coil of wattmeter-I measures V_{RB} and current coil measures I_R.

Power measured by wattmeter-I $= V_{RB}\, I_R \cos(\phi - 30°)$

Similarly,

Presume coil of wattmeter-II measures V_{YB} and current coil measures I_Y.

Power measured by wattmeter-II $= V_{YB}\, I_Y \cos(30° + \phi)$

Total power measured

$= V_{RB}\, I_R \cos(\phi - 30°) + V_{YB}\, I_Y \cos(30° + \phi)$

$= \sqrt{3}V_{ph}I_{ph}[\cos 30° \cos\phi + \sin 30° \sin\phi + \cos 30° \cos\phi - \sin 30° \sin\phi]$

$= \sqrt{3}V_{ph}I_{ph}[\sqrt{3}\cos\phi] = 3V_{ph}I_{ph}\cos\phi = \sqrt{3}V_L I_L \cos\phi$

Measurement of Power Factor using two–Watt-Meter Method

Let W_1 & W_2 be the readings of two Wattmeters.

$W_1 = V_{LL}I_{LL}\cos(30° - \phi)$

$W_2 = V_{LL}I_{LL}\cos(30° + \phi)$

$$\frac{W_1+W_2}{W_1-W_2} = \frac{V_{LL}I_{LL}\left[\cos(30°-\phi)+\cos(30°+\phi)\right]}{V_{LL}I_{LL}\left[\cos(30°-\phi)-\cos(30°+\phi)\right]} = \frac{2\cos 30°\cos\phi}{2\sin 30°\sin\phi} = \frac{2\times\frac{\sqrt{3}}{2}\cos\phi}{2\times\frac{1}{2}\sin\phi} = \frac{\sqrt{3}}{\tan\phi}$$

$$\tan\phi = \frac{\sqrt{3}(W_1-W_2)}{(W_1+W_2)} \Rightarrow \phi = \tan^{-1}\frac{\sqrt{3}(W_1-W_2)}{(W_1+W_2)}$$

$$\text{p.f.} = \cos\phi \qquad \Rightarrow \text{p.f.} = \cos\left\{\tan^{-1}\frac{\sqrt{3}(W_1-W_2)}{(W_1+W_2)}\right\}$$

3.19. A three-phase 230 V load has a power factor 0.7. Two wattmeters are used to measure power which shows the input to be 10 kW. Find the reading of each wattmeter. **[WBUT 2009(EVEN)]**

Answer:

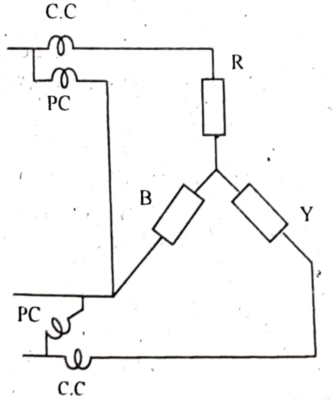
From the given data, $V_{LL} = 230\,\text{V}$ $\quad \cos\phi = 0.7 \quad \phi = \cos^{-1}0.7 = 45.58°$

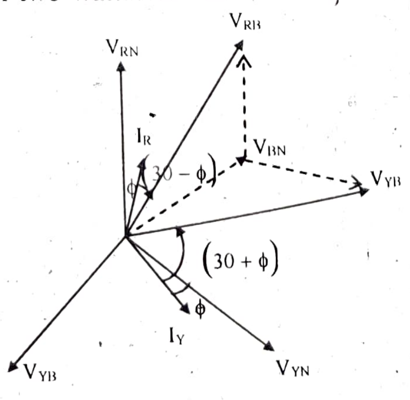
There phase power is given by $\sqrt{3}V_{LL}I_{LL}\cos\phi$

Let us assume that the load is connected in star.

For star connection, $I_{LL} = I_\phi = \dfrac{10\times10^3}{\sqrt{3}\times230\times0.7}A = 35.86A$

The schematic diagram and phasor diagram for two-wattmeter scheme are drawn below.





The readings of two wattmeters are V_{BN}

$\sqrt{3}V_\phi I_\phi \cos(30° - \phi)$ & $\sqrt{3}V_\phi I_\phi \cos(30° + \phi)$

$$\left[\begin{aligned} &\because \sqrt{3}V_\phi I_\phi\cos(30°-\phi)+\sqrt{3}V_\phi I_\phi\cos(30°+\phi) \\ &= \sqrt{3}V_\phi I_\phi\left[\frac{\sqrt{3}\cos\phi}{2}+\frac{\sin\phi}{2}+\frac{\sqrt{3}}{2}\cos\phi-\frac{\sin\phi}{2}\right] \\ &= \sqrt{3}V_\phi I_\phi\cos\phi = \sqrt{3}V_{LL}I_{LL}\cos\phi \end{aligned}\right]$$

Hence the readings of two wattmeters are

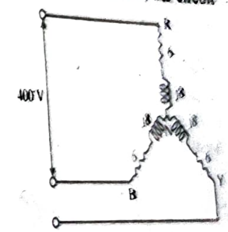
$230\times35.86\cos 15.58°W$ & $230\times35.86\cos 75.58°W$

i.e., $7.945kW$ & $2.05kW$ respectively.

3.20. Three equal impedances $(6+j8)\,\Omega$ are connected a cross a 400 V, 3-phase, 50 Hz supply. Calculate-
i) the line current & the phase current
ii) power factor
iii) active & reactive drawn by load per phase. **[WBUT 2012]**

Answer:

(i) Assuming the three equal impedances $(6+j8)\Omega$ to be connected in star, the circuit diagram is drawn.



Let 400V be the line voltage.

The per phase voltage is $\dfrac{400}{\sqrt{3}}V = 231V$

Per phase impedance $\sqrt{8^2+6^2}\,\Omega = 10\Omega$

Hence, per phase current $= \dfrac{231}{10}A = 23.1A$

Line current = phase current for star connection = 23A.

(ii) Power factor angle, $\phi = \tan^{-1}\dfrac{8}{6} = 53.1°$, Power factor $= \cos\phi = \cos 53.1° = 0.6$

(iii) Active power drawn per phase $= V_{ph}I_{ph}\cos\phi = 231\times23.1\times0.6\,\text{Watt} = 3.202\,\text{kW}$

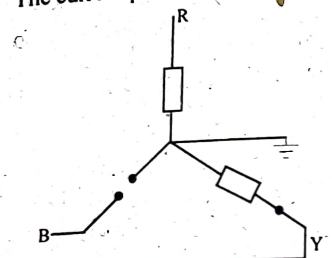
Reactive power per phase

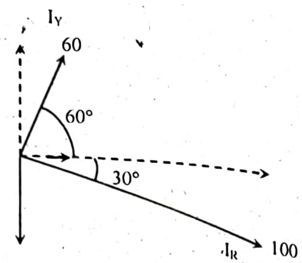
$= V_{ph}I_{ph}\sin\phi = 231\times23.1\times0.8 \quad \left[\sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-(0.6)^2} = 0.8\right]$

$= 4.269\,\text{kVAR}.$

3.21. In a three phase four wire power distribution system, phase B is open while current through R & Y are $100\angle-30°$ & $60\angle60°$. Find the current through the neutral connection. **[WBUT 2015]**

Answer:
The current phasors along R & Y are drawn





The component of I_R along the reference x-direction

$$=100\cos 30° A=\frac{\sqrt{3}\times 100}{2}A=86.6A$$

Component of I_Y along the x-direction $=60\cos 60° A=\frac{60}{2}A=30A$

Total current along x-direction $=(86.6+30A)=116.6A$

Component of I_Y along the y-direction $=60\sin 60° A=\frac{60\times\sqrt{3}}{2}=51.96A$

Component of I_R along the y-direction $=-100\sin 30° V=-\frac{100}{2}A=50A$

Resultant current along the y-direction $=(51.96-50)A=1.96V$

Resultant current $=\sqrt{(116.6)^2+(1.96)^2}=116.6A$

Leading the reference x-direction by $\tan^{-1}\frac{1.96}{116.6}=0.54$ degrees

The current through the neutral to balance this current $=116.6\angle 180° A$

TRANSFORMERS

Chapter at a Glance

Equivalent circuit of the transformer

It is already seen that in a transformer there are two circuits i.e. primary circuit and secondary circuit and as such one equivalent circuit can be obtained by either transforming all the parameters of secondary to primary side or transferring all the parameters of primary to secondary side. In order to proceed in the matter it is presumed that:

N_1 = No. of turns in primary winding
N_2 = No. of turns in secondary winding
V_1 = Applied voltage in primary
V_2 = Secondary terminal voltage
E_1 = Induced emf in primary winding
E_2 = Induced-emf in secondary winding
R_0 = Resistance of the core
R_1 = Resistance of primary winding
R_2 = Resistance of secondary winding
X_1 = Leakage reactance of primary winding
X_2 = Leakage reactance of secondary winding
X_m = Reactance of the core
I_1 = Primary current
I_2 = Secondary current
I_2' = Load component of current I_1
I_0 = No load component of current I_1
I_ω = Core loss component of no load current
I_m = Magnetizing component of no load current

The circuit model of the transformer considering resistance and leakage reactance in the winding and core parameters (R_0, X_m) is shown below.

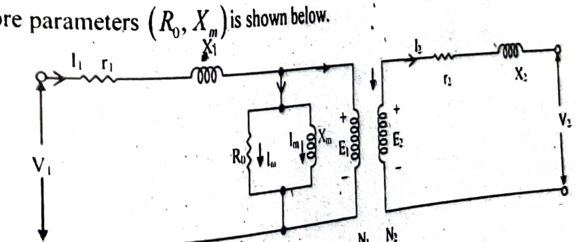


Fig: Equivalent Circuit

Equivalent resistance and reactance:

$R_1 = r_1 + K^2 r_2$ = Total resistance of the winding w.r.t primary side

and $X_1 = x_1 + K^2 x_2$ = Total leakage resistance w.r.t primary side.

$R_2 = r_2 + \frac{r_1}{K^2} = r_2 + r_1'$ = Total resistance of the windings w.r.t secondary

$X_2 = x_2 + \frac{x_1}{K^2} = x_2 + x_1'$ = Total leakage reactance w.r.t secondary

where $K = \frac{N_1}{N_2}$ = transformation ratio

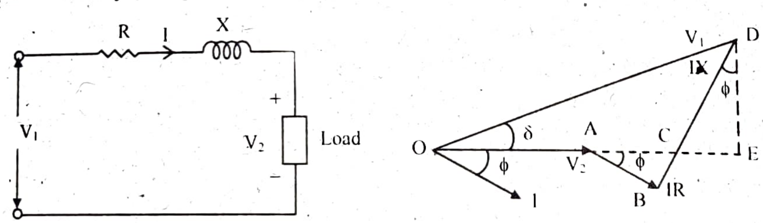
Regulation of a Transformer

The voltage drop in a transformer take place due to the resistances of the windings and for leakage reactance. Mathematically voltage regulation can be expressed as:-

$$\% \text{ Voltage regulation } = \frac{{}_0V_2 - V_{2f}}{{}_0V_2} \times 100$$

where V_{2f} = Rated secondary voltage at full load,

${}_0V_2$ = Secondary voltage at no load



Phasor Diagram

Since the volt drops IR & IX of the transformer are very small and hence the angle between V_1 & V_2 i.e. δ is negligible. So $V_1 = OE$

and $V_1 - V_2 = AE = IR\cos\Phi + I \times \sin\phi$ for ϕ lagging.

$= IR\cos\phi - I \times \sin\phi$ for ϕ leading

Again at no load ${}_0V_2 = V_1$

where, ${}_0V_2 - V_2 = I(R\cos\phi \pm X\sin\phi)$

I = Full load secondary current.

V_2 = Full load secondary voltage (rated)

$$\% \text{ voltage regulation } = \frac{({}_0V_2 - V_2)}{V_2} \times 100 = \frac{I(R\cos\phi \pm \sin\phi)}{V_2} \times 100$$

From the above equation it may be seen that voltage regulation varies with power factor.

The following points may be remembered:

1. % Regulation 'down' $= \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$
2. % Regulation 'up' $= \frac{{}_0V_2 - V_2}{V_2} \times 100$
3. Unless stated otherwise regulation is to be taken as regulation down.
4. The less the value of regulation, the better the transformer

Multiple Choice Type Questions

1.1. For a coil with N-turns the self-inductance will be proportional to [WBUT 2006, 2009, 2011]

a) N b) $\frac{1}{N}$ c) N^2 d) $\frac{1}{N^2}$

Answer: (c)

1.2. Hysteresis loss in a transformer can be reduced by using [WBUT 2007, 2008]

a) laminated Core b) silicon Steel Core
c) oil d) none of these

Answer: (a)

1.3. The reluctance of a magnetic circuit is given by [WBUT 2008(EVEN), 2013]

a) $l/\mu_r\mu_o A$ b) $\frac{\phi}{Nl}$ c) $1/\mu_o A$ d) $\frac{l}{\mu_r A}$

Answer: (a)

1.4. The unit of magnetic flux density is [WBUT 2009(EVEN)]

a) weber b) tesla c) coulomb d) none of these

Answer: (b)

1.5. Which of the following is not true of leakage flux? [WBUT 2010(EVEN)]

a) It links both the winding through air
b) It links the primary winding through air
c) It links the secondary winding through air
d) It does not link both the windings

Answer: (d)

1.6. In a magnetic circuit, once a flux is set up [WBUT 2010(EVEN)]

a) no further energy is required
b) energy is continuously required to maintain the flux
c) energy is released in the form of heat
d) none of these

Answer: (b)

1.7. The force experienced by a small conductor of length L, carrying a current I, placed in a magnetic field $\vec{B}$ at an angle θ with respect to $\vec{B}$ is given by [WBUT 2010, 2014]

a) BIL b) $BIL\sin\theta$ c) $BIL\cos\theta$ d) zero

Answer: (b)

1.8 Area hysteresis loop is a measure of [WBUT 2011]

a) retentivity b) coercivity c) saturated flux density d) energy loss

Answer: (d)

1.9. Conductance is analogous to [WBUT 2012]

a) permeance b) flux c) reluctance d) inductance

Answer: (a)

1.10. Energy stored by a capacitor is given by [WBUT 2012]

a) $\frac{1}{2}CV^2$ b) $\frac{1}{2}QV$ c) $\frac{Q^2}{2C}$ (d)

Answer: (a)

1.11. The unit of m.m.f is [WBUT 2014]

a) AT/m b) N/Wb c) both (a) & (b) d) Wb/m^2

Answer: (a)

1.12. The transformer core is laminated to reduce [WBUT 2009, 2016]

a) copper loss b) eddy current loss c) hysteresis loss d) none of these

Answer: (b)

1.13. In a transformer, electric power is transformed from the primary to the secondary without the change in [WBUT 2009]

a) voltage b) current c) frequency d) none of these

Answer: (d)

1.14. In a transformer connected to constant voltage source, increase in secondary load current will ultimately [WBUT 2011]

a) reduce the mutual flux in the core
b) increase the mutual flux in the core
c) not cause any change in the magnitude of mutual flux
d) decrease the mutual flux when the secondary current is below 0.8pf lag

Answer: (c)

1.15. The regulation of a transformer is negative, if the load at the secondary side is [WBUT 2012]

a) resistive b) inductive c) capacitive
d) combination of resistive, inductive & capacitive

Answer: (c)

1.16. In a transformer zero voltage regulation at full load is [WBUT 2014]

a) not possible
b) possible at unity power factor load
c) possible at leading power factor load
d) possible at lagging power factor load

Answer: (c)

1.17. In a transformer, the flux phasor [WBUT 2015, 2017]

a) leads the induced emf by 90°
b) lags the induced emf by 90°
c) leads the induced emf by slightly less than 90°
d) lags the induced emf by by slightly less than 90°

Answer: (b)

1.18. When the phase sequence of a three phase sinusoidal system is stated as $a-b-c$, it implies that [WBUT 2017]

a) phase voltage *a* leads the phase voltage *b* by 120°
b) phase voltage *a* lags the phase voltage *c* by 120°
c) phase voltage *b* leads the phase voltage *c* by 120°
d) all of these

Answer: (d)

Short Answer Type Questions

2.1. Derive an expression for the hysteresis loss in a magnetic material. [WBUT 2008]

Answer:

Hysteresis Loss

The Hysteresis Loop

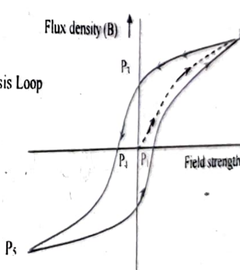


Fig.1 shows the effect of hysteresis in ferromagnetic materials.

We start with an unmagnetized sample at the origin (P_1) where both field strength and flux density are zero. The field strength is increased in the positive direction and the flux begins to grow along the dotted path until we reach P_2. This is called the initial magnetization curve.

If the field strength is now relaxed then some curious behavior occurs. Instead of retracing the initial magnetization curve the flux falls more slowly. In fact, even when the applied field is returned to zero there will still be a remaining (***remnant or remanent***) flux density at P_3. It is this phenomenon which makes permanent magnets possible. To force the flux to go back to zero we have to reverse the applied field (P_4). The field strength here is called the *coercivity*. We can then continue reversing the field to get to P5, and so on round this type of magnetization curve called (by J. A. Ewing) a hysteresis loop.

The expression of Hysteresis loop can be written as $W_h = K_h v f B_m^{1.6}$ W

Where K_h is a constant which depends on material and range of flux density, v is volume of the core material, f is the frequency of the alternation of the current passing through the magnetizing coil, B_m is the maximum value of flux density in the core in Wb/m^2

2.2. An iron ring of mean length 50 cm has an air-gap of 1 mm and a winding of 200 turns. The relative permeability of iron is 300 when 1 amp current flows through the coil. Determine the flux density. [WBUT 2009, 2013]

Answer:

Given data

Mean Length of iron ring

$l = 50\,\text{cm} = 0.5\,\text{m}$

$l_{ag} = 1\,\text{mm} = 1\times10^{-3}\,\text{m}$

$I = 1\,\text{Amp.}$

Number of Turns $N = 200$

Relative permeability of iron $\mu_r = 300$

For finding the flux density, the ampere Turns for iron and air gap are to be determined. Leakage and fringing effects are neglected.

At iron = $H \times l$

Where H is the magnetic field intensity

$$H = \frac{B}{\mu_0\mu_r} = \frac{B}{4\pi\times10^{-7}\times300}\ \text{AT/m} = 2652.58B\ \text{AT/m}$$

where B is the flux density.

Hence AT for iron $= Hl$ AT $= 2652.58B\times0.5$ AT $= B26.29$ AT 1326.29 AT

$$\text{AT for Air gap} = H_{\text{air gap}}\times l_{ag}\ \text{AT} = \frac{B}{4\pi\times10^{-7}}\times10^{-3}\ \text{AT} = 795.77B\ \text{AT}$$

Total ampere turns $= B(1326.29+795.77)$ AT. $= 2122.06B$ AT.

Total ampere turns can be expressed as $N\times I = 200\times1$ AT $200\times1 = 2122.06B$

$$\text{Hence flux density } B = \frac{200}{2122.06}\ \text{Wb/m}^2 = 0.094\ \text{Wb/m}^2$$

2.3. Two coils A of 1000 turns and B of 500 turns are mutually coupled with 80% coupling. If a current of 5A in coil A produces a flux of 0.25 mWb, find the mutual inductance and co-efficient of coupling between the coils. [WBUT 2016]

Answer:

$$M = \frac{N_2\phi_1}{I_1}\ H$$

Flux produced in $A = 0.25\,\text{mWb} = 0.25\times10^{-3}$ Wb

Flux linked with $B = 0.25\times10^{-3}\times0.8 = 0.2\times10^{-3}$ Wb

$$M = \frac{500\times0.2\times10^{-3}}{5} = 20\times10^{-3} = 0.02\,\text{H}$$

$$L_1 = \frac{1000\times0.25\times10^{-3}}{5} = \frac{0.25}{5} = 0.05\,\text{H}$$

$$L_2 = \frac{500\times0.25\times10^{-3}}{5} = 0.025\,\text{H}$$

$$k = \frac{M}{\sqrt{L_1L_2}} = \frac{0.02}{\sqrt{0.05\times0.025}} = \frac{0.02}{0.0355} = 0.05633$$

2.4. A 20 kVA transformer has 400 turns on the primary and 40 turns on the secondary winding. The primary is connected to 2 kV, 50 Hz supply. Find the full load primary and secondary currents, secondary emf and the maximum flux in the core. Neglect leakage drop and no-load primary current. [WBUT 2006, 2017]

Answer:

The kVA rating of the transformer is 20 kVA.

No. of turns in the primary, $N_1 = 400$

No. of turns in the secondary, $N_2 = 40$

Primary voltage, $V_1 = 2\times10^3$ V

Full load primary current, $I_1 = \dfrac{20\times10^3}{2\times10^3}\,\text{A} = 10\,\text{A}$

For an ideal transformer,

$$I_1N_1 = I_2N_2$$

$$I_2 = I_1\times\frac{N_1}{N_2} = 10\times\frac{400}{40}\,\text{A} = 100\,\text{A}$$

Full load secondary current = 100 A

Secondary emf

$$V_2 = V_1\times\frac{N_2}{N_1}\quad\left(\because \frac{V_1}{V_2} = \frac{N_1}{N_2}\right)$$

$$= 2\times10^3\times\frac{40}{400}\,\text{V} = 200\,\text{V}.$$

The emf equation of a transformer is expressed as, $E = 4.44\phi_m fT$ where E is the induced emf, ϕ_m, the maximum flux in the core, f, the frequency and, T, the number of turns,

$$E_1 = 4.44\phi_m fT$$

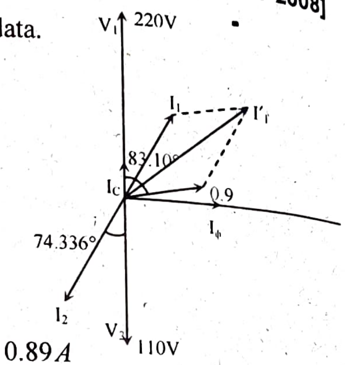
$$\phi_m = \frac{E_1}{4.44 f T_1} = \frac{2\times10^3}{4.44\times50\times400} = 22.5 \text{ m weber.}$$

[For an ideal transformer, $V_1 = E_1$]

2.5. A 220/110 V transformer is having no load current of 0.9A at 0.12 p.f (lag) and a secondary current of 95 A at 0.27 p.f (lag). Find its primary current. **[WBUT 2008]**

Answer:

The transformer phasor diagram is drawn with the given data.



$$\frac{N_1}{N_2} = \frac{220}{110} = 2$$

$I_1 N_1 = I_2 N_2$

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{1}{2}\times 95A = 47.5A$$

$I_0 = 0.9$ lagging

V_1 by $83.10°$

The magnetizing component of $I_0 = 0.9\times\sin 83.10°\,;\, I_\phi = 0.89A$

The core-loss component of $I_0, I_c = 0.9\times0.12\text{A} = 0.108\text{A}$

The component of I_1 in phase with $I_c = 47.5\times0.27\text{A} = 12.825\text{A}$

The component I_1 in phase with $I_\phi = 47.5\times\sin 74.336° = 45.736$ A

Total current in phase with $I_\phi = (45.736+0.89)\text{A} = 46.626A$

Total current in phase with $I_c = (12.825+0.108)\text{A} = 12.933\text{A}$

$$I_1' = \sqrt{(46.626)^2 + (12.933)^2}\,A = 48.386\,A \text{ at } \tan^{-1}\frac{46.62}{12.933} \text{ with } V_1 = 48.386\,A \text{ at } 0.27\text{lag}$$

2.6. Explain what will happen to transformer if we give DC supply to it.[WBUT 2010]

Answer:

If we give D.C. supply to transformer, no alternating current will flow through the primary and hence no alternating flux would be generated. In the absence of alternating flux, no induced e.m.f. would be generated, or in other words there will be no transformer action. The result would be that the transformer coil would get burn due to its low resistance.

2.7. Draw and explain the phasor diagram of a single phase transformer under lagging p.f. **[WBUT 2013, 2017]**

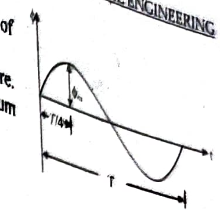
Answer: ***Refer to Question No. 3.13(a).***

2.8. Show that for a single phase transformer, $E_p = 4.44 f \phi_m N_p$ where the symbols have their usual meanings. **[WBUT 2014]**

Answer:

The transformer works based on the Faraday's law of electromagnetic induction.

The sinusoidal waveform of flux is as shown in figure.



The instantaneous value of flux, reaches the maximum value ϕ_m in time T/4 seconds.

Change in flux $d\phi = \phi_m - 0 = \phi_m$

Change in time $dt = T/4$ sec.

$\therefore$ Average value of emf induced $= \frac{d\phi}{dt} = \frac{\phi_m 4}{T}$

and $\frac{1}{T} = f$ frequency in Hz. .... (1)

$\therefore$ Average emf induced per conductor $= 4f\phi_m$ volts.

for the N number of conductor, the average emf. $= 4f\phi_m N$ volts.

RMS value $= 1.11\times$ Average value (for Sinusoidal signal). .... (2)

RMS value of emf induced $= 4.44 f\phi_m N$ volts

For primary $N = N_1$ .... (3)

and induced emf $\Rightarrow e_1 = 4.44 f\phi_m N_1$ .... (4)

For secondary $N = N_2$ and the induced emf $e_2 = 4.44 f\phi_m N_2$ volts. .... (5)

In an ideal transformer i.e. the transformer in which there are no losses and no impedance drops, $V_1 = e_1$ & $e_2 = V_2$ where V_2 is the terminal voltage. .... (6)

Voltage transformation Ratio (K)

From equation (5) and (6), it may be obtained as:

$$\frac{e_2}{e_1} = \frac{N_2}{N_1} = K$$

This constant is known as voltage transformation ratio

(i) If $N_2 > N_1$ i.e. $K > 1$, then transformer is called STEP UP transformer.

(ii) If $N_2 < N_1$ i.e. $K < 1$ then transformer is known as STEP DOWN transformer.

Again for an ideal transformer

Input VA = Output VA

$$\therefore \quad V_1 I_1 = V_2 I_2 \quad \text{or,} \quad \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (Voltage) transformation ratio.

2.9. Explain the working of a transformer on load. **[WBUT 2016]**

Answer:

Transformer on Load

When a load is connected, the actual load current flows through the windings, and therefore actual losses will occur in the core and windings. And there will be voltage drops in the windings also.
The drop in voltage in the winding is due to two components. One is due to the winding resistance and the other is due to the leakage reactance of the winding.
When V_1 is the voltage supplied to the primary winding, it circulates a current I_1. This currents produces a flux ϕ which links both primary and secondary windings. Since this flux is of alternating in nature, it induces emfs E_1 & E_2 on primary and secondary respectively.

$$\therefore V_1 = E_1 + I_1(R_1 + jX_1) \quad \text{.... (i)}$$

$$V_2 = E_2 - I_2(R_2 + jX_2) \quad \text{.... (ii)}$$

where, V_1 = primary input voltage V

Note: V_1 supplies the drops in primary and it produces the induced emf.

I_1 = Primary current in A

R_1, X_1 = Resistance and leakage reactance of primary in Ω

E_1 = Primary induced emf in V

E_2 = Secondary induced emf in V

R_2, X_2 = Resistance and leakage reactance of secondary in Ω.

Note: It is emf E_2, which supplies the terminal voltage V_2 and also the drops in secondary.

Note: Further wherever the transformer is loaded the flux passing through the core remain the same and therefore the core loss almost remains constant.

Let I_2' be the primary equivalent of the secondary current I_2 (Refer equivalent circuit).

$$\text{Where } I_2' = \left(\frac{N_2}{N_1}\right) I_2 \quad \text{.... (iii)}$$

Therefore the primary current I_1 is the vector summation of I_0 and I_2'

$$I_1 = \sqrt{I_0^2 + I_2'^2} \quad \text{.... (iv)}$$

Phasor diagrams

Not only the magnitude but also the nature of the load plays an important role on the loading characteristics of the transformer.
To draw a phasor diagram and to understand a given phasor diagram the following points to be remembered:

1. Flux is taken as the reference vector.
2. Primary voltage V_1 is drawn 90° ahead of ϕ_1.
3. $E_1 = E_2$ and E_1 & E_2 are drawn 180° behind V_1
4. I_0 – No load current is drawn such that I_0 is lagging V_1 by an angle ϕ_0

- I_2 is drawn with the necessary angle of leading or lagging (ϕ_2) with respect to E_2
- I_2' is drawn which is the projection I_2 vector on primary side i.e. I_2' is lagging V_1 by ϕ_2
- I_2' & I_0 are added to get the resultant I_1 (vector addition)
- With E_1 resistive and reactive drops are added to obtain vector V_1
- similarly using equation 12, V_2 is obtained

Long Answer Type Questions

3.1. Compare magnetic circuit with electric circuit. [WBUT 2006, 2009(EVEN), 2009]

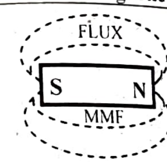
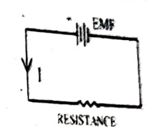
OR,

Compare electric and magnetic circuits with respect to their similarities and dissimilarities. [WBUT 2010]

OR,

Draw comparison between electrical and magnetic circuit. [WBUT 2013]

Answer:

Magnetic circuits	Electrical circuits
 	
1. Flux flows from N-pole to S-pole	Current flows from +ve to –ve
2. Flux flow is due to mmf.	Current flows due to e.m.f.
3. Resistance to flow of flux is offered by reluctance.	Resistance to flow of current is offered by resistance R.
Flux $= \frac{\text{mmf}}{\text{reluctance}}$ (i) mmf is expressed in ampere turns. (ii) Flux is expressed in webers. (iii) Flux density is expressed in Wb/m$_2$	Current $= \frac{\text{emf}}{\text{resistance}}$ (i) emf is expressed in volts (ii) Current is expressed in amperes. (iii) Current density is expressed in (A/m^2).
4. Reluctance $R = \frac{1}{\mu A}$ where μ is permeability.	4. Resistance $R = \frac{1}{\sigma A}$ where σ is the conductivity.
5. Permeance $= \frac{1}{\text{reluctance}}$	5. Conductance $= \frac{1}{\text{resistance}}$
6. Permeability $= \frac{1}{\text{reluctivity}}$	6. Conductivity $= \frac{1}{\text{relutivity}}$

3.2. Define self and mutual inductance. What do you mean by co-efficient of coupling? **[WBUT 2007, 2008(EVEN), 2009]**

OR,

Define self-inductance and mutual inductance. **[WBUT 2009]**

OR,

Two coils have self inductances L_1 and L_2 and mutual inductance between them is M. Derive a mathematical expression for co-efficient of coupling k for these coils. **[WBUT 2011, 2015]**

Answer:

Definition:

Self Inductance: It is the property of an electric circuit or component that causes an e.m.f. to be generated in it as a result of a change in the current flowing through the circuit.

The property of self-inductance is a particular form of electromagnetic induction. Self-inductance is defined as the induction of a voltage in a current-carrying wire when the current in the wire itself is changing. In the case of self-inductance, the magnetic field created by a changing current in the circuit itself induces a voltage in the same circuit. Therefore, the voltage is self-induced.

The term ***inductor*** is used to describe a circuit element possessing the property of inductance and a coil of wire is a very common inductor. In circuit diagrams, a coil or wire is usually used to indicate an inductive component.

The number of turns in the coil has an effect on the amount of voltage that is induced into the circuit. Increasing the number of turns or the rate of change of magnetic flux increases the amount of induced voltage. Therefore, **Faraday's Law** must be modified for a coil or wire and becomes the following. $V_L = N\dfrac{d\phi}{dt}$

Where:

V_L = the induced voltage in volts

N = the number of turns in the coil

$d\phi/dt$ = the rate of change in magnetic flux in webers per second

The equation simply states that the amount of induced voltage (V_L) is proportional to the number of turns in the coil and the rate of change of the magnetic flux ($d\phi/dt$).In other words, when the frequency of the flux is increased or the number of turns in the coil is increased, the amount of induced voltage will also increase.

In a circuit, it is much easier to measure current than it is to measure magnetic flux so the following equation can be used to determine the induced voltage if the inductance and frequency of the current are known. This equation can also be reorganized to allow the inductance to be calculated when the amount of induced voltage could be determined and the current frequency is known. $V_L = L\dfrac{di}{dt}$

Where

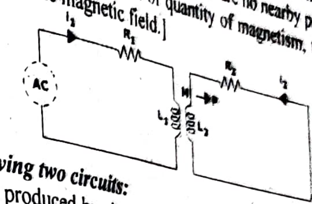
V_L = the induced voltage in volts

L = the value of inductance in henries

di/dt = the rate of change in current in amperes per second

Mutual Inductance & Co-efficient of Coupling:

The magnetic flux through a circuit can be related to the current in that circuit and the currents in other nearby circuits, assuming that there are no nearby permanent magnets. [Note: Magnetic flux is a measurement of quantity of magnetism, taking account of the strength & the extent of the magnetic field.]



Consider the following two circuits:

The magnetic field produced by circuit 1 will link the wire in circuit 2 and induce emf in circuit 2 causing a current to flow. The induced current flow in circuit 2 will have its own magnetic field, which will interact with the magnetic field of circuit 1. At some point P on the magnetic field consists of a part due to i_1 and a part due to i_2. These fields are proportional to the currents producing them.

The coils in the circuits are labeled L_1 and L_2 and this term represents the self-inductance of each of the coils. The values of L_1 and L_2 depend on the geometrical arrangement of the circuit (i.e. number of turns in the coil) and the permeability of the material. The constant M is called the mutual inductance of the two circuits and it is dependent on the geometrical arrangement of both circuits. In particular, if the circuits are far apart, the magnetic flux through circuit 2 due to the current i_1 will be small and the mutual inductance will be small. L_2 and M are constants.

We can write the flux, ϕ_B through circuit 2 as the sum of two parts.

$$\phi_{B2} = L_2 i_2 + i_1 M$$

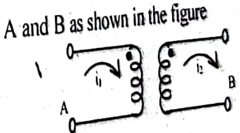
An equation similar to the one above can be written for the flux through circuit 1.

$$\phi_{B1} = L_1 i_1 + i_2 M$$

Though it is certainly not obvious, it can be shown that the mutual inductance is the same for both circuits. Therefore, it can be written as follows: $M_{1,2} = M_{2,1}$

Mutual Inductance:

Let there be two coils A and B as shown in the figure



Let, L_A and L_B be the self – inductance of coil A and B respectively.

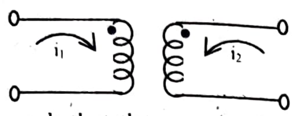
In the given case resultant flux is subtractive when current flows in both coils.

The voltage induced in coil A $|e_A| = L_A\, di_1/dt - M\, di_2/dt$

Where M is the mutual inductance voltage induced in coil B.

The voltage induced in coil B $|e_B| = L_B\, di_2/dt - M\, di_1/dt$

If the dots are placed in such a way that the resultant flux is additive when current flows, the induced voltages will be as follows:



$$|e_A| = L_A\, di_1/dt + M\, di_2/dt$$

$$|e_B| = L_B\, di_2/dt + M\, di_1/dt$$

If both the coils are wound on the same core such that they magnetise in the same way when current flows through shown (i.e. the resultant flux is additive), the induced emf

$$e = L_1\, di/dt + M\, di/dt + L_2\, di/dt + M\, di/dt$$

Where L_1 & L_2 are the self – inductances and M is the mutual inductance.
Equivalent inductance = $L_1 + L_2 + 2M$.
When the orientation of the coils is such that the resultant flux is subtractive,

$$e = L_1\, di/dt + L_2\, di/dt - 2M\, di/dt$$

Equivalent inductance = $L_1 + L_2 - 2M$

Co-efficient of coupling:
The mutual inductance gives an indication of the amount of closeness or proximity of the coupling between the two coils.
Co-efficient of coupling provides a quantitative measure of the magnetic coupling of two coils whose self-inductances are L_1 and L_2 respectively.

Mathematically, $K = \dfrac{M}{\sqrt{L_1 L_2}}$

where M is the mutual inductance. Maximum value of K can be unity when the entire magnetic flux set up by coil L_1 is linked with all the turns of the coil L_2.

3.3. Derive an expression for energy stored in a magnetic field.
[WBUT 2008(EVEN), 2013]

OR,

Deduce an expression of energy stored of in a magnetic circuit.
[WBUT 2009(DEDCMBER)]

OR,

Derive the expression for energy stored in electric field. **[WBUT 2010(EVEN)]**

Answer:

Energy Stored in the Magnetic Field of an Inductor

The above inductor having inductance L is connected to the battery through Key K. In this case inducted e m f is given by: $c = -L\dfrac{dI}{dt}$

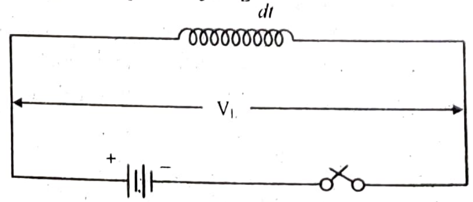


Fig: (i)

$$e = -L.\frac{dt}{dt} \qquad \text{....(1)}$$

To drive the current through the inductor against the induced emf 'e', the external voltage is applied from the battery which has emf E and in this case E = - e.

Hence equation (1) becomes, $E = L\dfrac{di}{dt}$

Let an infinitesimal charge dq be driven through the inductor. So the work done by the external voltage to do this work is given by:

$$dw = E\, dq$$

$$\text{or, } dw = L\frac{di}{dt}.dq = L.di\left(\frac{dq}{dt}\right)$$

$$\text{But } \frac{dq}{dt} = i$$

$$\therefore dw = L\ i\ di$$

Total work done to maintain the maximum value of current (I_o) through the inductor is given by

$$\int dw = \int_0^{I_o} Li\, di \qquad W = L\left[\frac{i^2}{2}\right]_0^{I_o} = L\left[\frac{I_o^2}{2}\right]$$

The work done in increasing the current flowing through the inductor is stored as the potential energy (U) in its magnetic field. $U = \frac{1}{2}LI_o^2$

3.4. What is meant by hysteresis in a magnetic circuit? What is the significance of B-H curve? **[WBUT 2012, 2015]**

Answer:

Hysteresis Loss

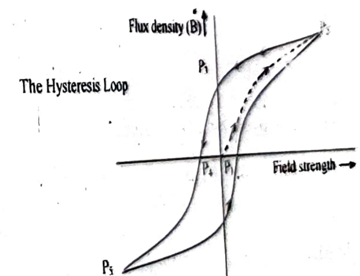


Fig.1 shows the effect of hysterisis in ferromagnetic materials.

We start with an unmagnetized sample at the origin (P_1) where both field strength and flux density are zero. The field strength is increased in the positive direction and the flux begins to grow along the dotted path until we reach P_2. This is called the initial magnetization curve.

If the field strength is now relaxed then some curious behavior occurs. Instead of retracing the initial magnetization curve the flux falls more slowly. In fact, even when the applied field is returned to zero there will still be a remaining (***remnant or remanent***) flux density at P_3. It is this phenomenon which makes permanent magnets possible. To force the flux to go back to zero we have to reverse the applied field (P_4). The field strength here is called the *coercivity*. We can then continue reversing the field to get to P5, and so on round this type of magnetization curve called (by J. A. Ewing) a hysteresis loop.

It is that we have had to spend energy in order to set up the remnant flux. To show this more clearly we'll look at separate figures. The area (shown shaded) between the B-H curve and the B axis represents the work done (per unit volume of material).

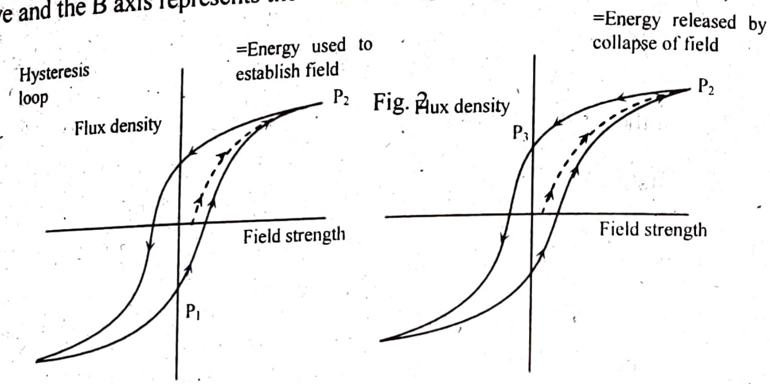


Fig. 2

3.5. a) State Faraday's Laws of Electromagnetic Induction. Show that $M = K\sqrt{L_1 L_2}$ where M is the mutual inductance between the coils L_1 and L_2 and K is the co-efficient of coupling. **[WBUT 2014]**

Answer:

Faraday, in 1831, showed that, whenever the number of lines of magnetic flux linking with an electric circuit is changed, an electromotive force is induced in the circuit, the magnitude of which is proportional to the rate of change of flux. Thus, if e is the e.m.f. induced by a rate of change of flux of $\frac{d\Phi}{dt}$, $e \propto \frac{d\Phi}{dt}$

The e.m.f. is also proportional to the number of turns of wire, N, in the circuit in which the e.m.f. is induced.

Thus $e \propto N\frac{d\Phi}{dt}$

If e is expressed in volts and Φ in webers, then $e = -N\frac{d\Phi}{dt}$.

The negative sign is used to establish the fact that the e.m.f. induced always opposes the cause, as stated in Lenz's law.

The unit of magnetic flux is defined by this relationship. The weber is the magnetic flux which, linking a circuit of 1 turn, produces in it an e.m.f. of 1 volt as the flux is reduced to zero at a uniform rate, in one second.

The phenomenon of electromagnetic induction plays a crucial role in the three most useful of all electrical devices: the electric generator, the electric motor, and the transformer.

Refer to Question No. 3.2.

b) A coil of 250 turns carrying a current of 2A produces a flux of 0.3mWb. When the current is reduced to zero in 2ms, the voltage induced in a nearby coil is 60V. Calculate the self-inductance of each coil and the manual inductance between the two coils. Assume co-efficient of coupling to be 0.7. **[WBUT 2014]**

Answer:

The self inductance of the first coil, $L_1 = N_1\frac{\phi}{I_1} = 250 \times \frac{3\times10^{-3}}{2} = 37.5mH$

When the current in first coil is changed, the induced emf in the second coil is given a.

$$e_2 = M\frac{di_1}{dt}$$

Hence we have, $60 = M\frac{2-0}{2\times10^{-3}}$ $\Rightarrow M\times10^3 = 60$ $\Rightarrow M = 60mH$

The self inductance of the second coil can now be calculated from the relation

$$M = k\sqrt{L_1 L_2} \quad \Rightarrow L_2 = \left(\frac{M}{k}\right)^2\frac{1}{L_1} = \left(\frac{60}{.7}\right)^2 \times \frac{1}{37.5} mH = 195.91mH$$

3.6. What do you mean by the terms 'Hysteresis' and 'Eddy current losses'? **[WBUT 2016]**

Answer:

Hysteresis: ***Refer to Question No. 2.1.***

Eddy Current Losses:

Eddy current losses occur whenever the core material is electrically conductive. Most ferromagnetic materials contain iron: a metal that has fairly low resistivity. The solid iron core is subjected to varying flux fields. So 'eddy emfs' are induced in the core and the solid core serves as a conducting path.

In any resistive circuit the power is proportional to the applied voltage. The induced voltage is itself proportional to f×B and so the eddy losses are proportional to fB also. The flux is also related to the size of the loop. The same principle applies to motors and generators too. Usage of a solid iron core results in large circulating currents. So, instead, the core is made up of a stack of thin (~0.5 millimetre) sheets (cross section C). The lines of magnetic flux can still run around the core within the plane of the laminations. The

situation for the eddy currents is different. The surface of each sheet carries an insulating oxide layer formed during heat treatment. This prevents current from circulating from one lamination across to its neighbours.
Power loss (the reduction of which is our aim) is proportional to the square of induced voltage. Induced voltage is proportional to the rate of change of flux, and each of our laminations carries one quarter of the flux. So, if the voltage in each of the four laminations is one quarter of what it was in the solid core then the power dissipated in each lamination is one sixteenth the previous value.
The eddy current losses of the core are expressed as $P_e = k_e f^2 B_m^2$ where k_e is the proportionality constant whose value depends on the volume and resistivity of the core material, thickness of the laminations and the units employed.
B_m is the maximum flux density of the core and f is the frequency of the alternating supply.

3.7. Sketch the equivalent circuit of a single-phase transformer referred to the primary. Also draw the phasor diagram for a lagging p.f. load. [WBUT 2006]
OR,
Draw the exact equivalent circuit of a transformer & describe briefly the various parameters involved in it. [WBUT 2015]

Answer:

Equivalent circuit of the transformer

It is already seen that in a transformer there are two circuits i.e. primary circuit and secondary circuit and as such one equivalent circuit can be obtained by either transforming all the parameters of secondary to primary side or transferring all the parameters of primary to secondary side.
In order to proceed in the matter it is presumed that:

N_1 = No. of turns in primary winding; N_2 = No. of turns in secondary winding
V_1 = Applied voltage in primary; V_2 = Secondary terminal voltage
E_1 = Induced emf in primary winding; E_2 = Induced emf in secondary winding
R_0 = Resistance of the core; R_1 = Resistance of primary winding
R_2 = Resistance of secondary winding;
X_1 = Leakage reactance of primary winding
X_2 = Leakage reactance of secondary winding; X_m = Reactance of the core
I_1 = Primary current; I_2 = Secondary current
I_2' = Load component of current I_1; I_0 = No load component of current I_1
I_ω = Core loss component of no load current
I_m = Magnetizing component of no load current

The circuit model of the transformer considering resistance and leakage reactance in the winding and core parameters (R_0, X_m) is shown below: (fig. 1)

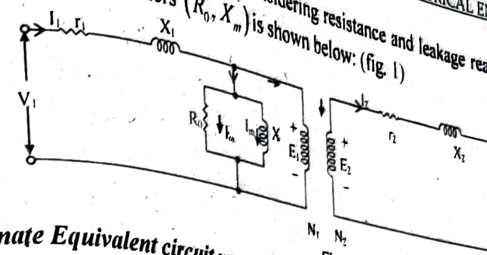


Fig (1)

Approximate Equivalent circuit w.r.t primary

Since r_1 & x_1 are very small, then $V_1 \approx E_1$. Hence the shunt branch can be shifted to the input of the primary side as shown in fig. (2) below:

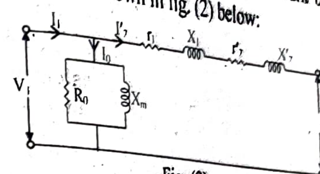


Fig: (2)

The equivalent circuit as shown in fig. 2 can be reduced as shown in fig. 3

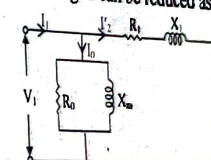


Fig: 3

Here, $R_1 = r_1 + K^2 r_2$ ...(1)
= Total resistance of the winding w.r.t primary side and $X_1 = x_1 + K^2 x_2$ ...(2)
= Total leakage resistance w.r.t primary side.

Equivalent circuit at no load:
At no load $I_2' = 0$, Hence $I_1 = I_0$
The equivalent circuit of the transformer w.r.t primary side is shown in fig. 4.

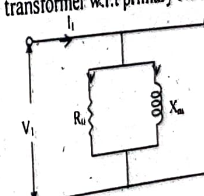


Fig: 4

3.8. a) Prove that the efficiency of a transformer is maximum when iron loss = copper loss. [WBUT 2008, 2009, 2012]
b) Following test data were obtained on a 20 KVA, 50 Hz, 1 ph, 2000/200 V for
No-load test : 200 V, 1 A, 120 W

Short = circuit test : 60 V, 10 A, 300 W

Find (i) the efficiency of the transformer at ½ of the full load and 0.8 p.f. lagging.

(ii) Maximum efficiency and the load at which it occurs. **[WBUT 2008(EVEN)]**

Answer:

a) The efficiency (η) of the transformer is given by the ratio of output power to the input power. Mathematically it can be expressed as

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}} \quad \text{.... (1)}$$

(since $P_{in} = P_{out} + P_{losses}$)

Also but, $P_{out} = P_{in} - P_{losses}$, so the equation (1) can also be expressed as

$$\eta = \frac{P_{in} - P_{losses}}{P_{in}} = 1 - \frac{P_{losses}}{P_{in}} \quad \text{.... (2)}$$

It is presumed that the transformer is supplying power at a terminal voltage V_2 to a load having power factor $\cos\phi_2$ from its secondary as shown in fig. 1.

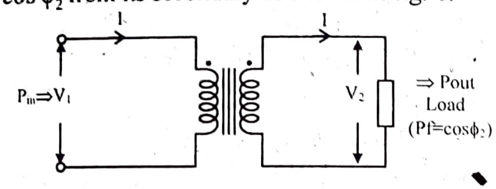


Fig: 1

In this case $P_{out} = V_2 I_2 \cos\phi_2$ .... (3)

The equation 3 shows that the output power of the transformer depends on the power factor of the load. Power losses in the transformer can be expressed as

$$P_{losses} = P_i + P_c \quad \text{.... (4)}$$

The iron loss, P_i is constant and it is independent of load. The copper loss, P_c is variable and it depends on the load. It can be expressed as

$$P_c = I_1^2 r_1 + I_2^2 r_2 = I_2^2 R_2 \quad \text{.... (5)}$$

where, r_1 = Resistance of primary winding; r_2 = Resistance of secondary winding
I_1 = Primary current; I_2 = Secondary current
R_2 = Total resistance of the transformer with respect to secondary side

Now, the efficiency of the transformer can be expressed as

$$\eta = \frac{P_0}{P_0 + P_i + P_c} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_2} \quad \text{.... (6)}$$

The equation (6) shows that for a given power factor, the efficiency of the transformer varies with load current.

Condition for Maximum Efficiency

Using the equation (6) the efficiency of the transformer can be expressed as

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + \frac{P_i}{I_2} + I_2 R_2} \quad \text{.... (7)}$$

for the maximum value of η, the denominator of the equation (7) should be minimum. The condition for maximum η is obtained by differentiating the denominator and equating to zero, i.e. $-\frac{P_i}{I_2^2} + R_2 = 0$

or, $P_i = I^2 R_2 = P_C$

or, copper loss = Iron loss. .... (8)

Now, the expression for maximum efficiency will be

$$\eta_m = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + 2P_i} \quad \text{.... (9)}$$

b) (i) From the available data: Iron loss = 120 W

Copper loss = $300 \times \frac{1}{2} \times \frac{1}{2}$ W [at 50% full load] = 75 W

Total loss at $\frac{1}{2}$ of the full load conditions = (120 + 75)W = 195W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{20 \times 0.5 \times 0.8}{20 \times 0.5 \times 0.8 + 0.195} = \frac{8}{8.195}\% = 97.62\%$$

(ii) Maximum efficiency will be attained when iron loss = copper loss
i.e. copper loss = fixed iron loss = 120W

Percentage of load at maximum efficiency $\sqrt{\frac{120}{300}} = \sqrt{0.4} = 63.24\%$

$$\text{Maximum efficiency} = \frac{\text{Output (at 63.24\% load)}}{\text{Output} + \text{Losses}} \times 100 = \frac{20 \times 0.6329}{20 \times 0.6324 + 0.24} \times 100$$

12.658

$$= \frac{12.649}{12.889} \times 100 = 98.14\%$$

12.898

3.9. Following test data were obtained on a 20 kVA, 50Hz, 1ph, 2000/200 V Transformer.
No load Test: 200V, 1A, 120W Secondary
Short circuit test: 60V, 10A, 300W
Find
i) the efficiency of the transformer at $\frac{1}{2}$ of the full load and 0.8 p.f lagging. **[WBUT 2008]**
ii) maximum efficiency and the load at which it occurs.

Answer:

Available data

No. load test: 200V, 1A, 120W

Short circuit test: 60V, 10A, 300W

(i) The half full load output of the transformer at $p.f = 0.8$ lagging

$$= \frac{1}{2} \times 20 \times 10^3 \times 0.8\ W = 8000\ W$$

Efficiency of the transformer at half-full load condition

$$= \frac{\text{output at half - full load condition}}{\text{output + losses}}$$

$$= \frac{8000}{8000 + \text{Fixed Iron loss} + (\text{fraction of load})^2 \times \text{Full-load copper loss}}$$

$$= \frac{8000}{8000 + 120 + \frac{1}{4} \times 300} = \frac{8000}{8195} \times 100 = 97.6\%$$

(ii) Maximum efficiency occurs when

Iron loss = copper loss

Let 'η' be the percentage of load at which man efficiency occurs

$$\eta^2 \times \text{full load copper loss} = \text{iron loss}$$

$$\eta^2 \times 300 = 120; \qquad \eta^2 = \frac{12}{30} = \frac{4}{10}; \qquad \eta = \frac{2}{\sqrt{10}} = 63.2\%$$

Maximum efficiency of the transformer assuming $0.8\,p.f$ lag.

$$= \frac{0.632 \times 20 \times 10^3 \times 0.8}{0.632 \times 20 \times 0.8 \times 10^3 + 120 + 120} = \frac{10112}{10112 + 240} = 97.68\%$$

10|27

3.10. Explain voltage regulation of a single-phase transformer with the help of phasor diagram. **[WBUT 2008]**

Answer:

Regulation of a Transformer

The voltage drop in a transformer take place due to the resistances of the windings and for leakage reactance. Accordingly the output voltage under no load conditions is different from the output voltage under load conditions. Voltage regulation of the transformer in fact indicates the figure of merit that determines the voltage drop characteristics. It can be defined as the change in secondary voltage when rated load at a specified power is removed. It is expressed as a percentage of rated terminal voltage. Mathematically voltage regulation can be expressed as:

$$\%\ \text{Voltage regulation} = \frac{{}_0V_2 - V_{2f}}{{}_0V_2} \times 100 \qquad \frac{E_2 - V}{E_2} \times 100$$

where V_{2f} = Rated secondary voltage at full load

${}_0V_2$ = Secondary voltage at no load

To find the expression for voltage regulation in terms of total resistance and leakage reactance of the transformer it is considered that the equivalent circuit of the transformer

with respect to secondary as shown in fig. 1 and the corresponding phasor diagram as shown in fig.2.

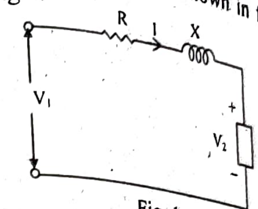


Fig: 1

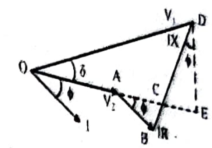


Fig 2

Since the volt drops IR & IX of the transformer are very small and hence the angle between V_1 & V_2 i.e. δ is negligible. So $V_1 = OE$

and $\quad V_1 - V_2 = AE = IR\cos\Phi + IX\sin\phi$ for ϕ lagging.

$\quad = IR\cos\phi - IX\sin\phi$ for ϕ leading

Again at no load ${}_0V_2 = V_1$

where, ${}_0V_2 - V_2 = I(R\cos\phi \pm X\sin\phi)$; I = Full load secondary current.

V_2 = Full load secondary voltage (rated).

$$\%\ \text{voltage regulation} = \frac{({}_0V_2 - V_2)}{V_2} \times 100 = \frac{I(R\cos\phi + X\sin\phi)}{V_2} \quad \ldots (1)$$

From the equation (1) it may be seen that voltage regulation varies with power factor, The following points may be remembered:

1. % Regulation 'down' $= \frac{{}_0V_2 - V_2}{{}_0V_2} \times 100$
2. % Regulation 'up' $= \frac{{}_0V_2 - V_2}{V_2} \times 100$
3. Unless stated otherwise regulation is to be taken as regulation down.

3.11. a) A 2200/250 volt transformer has primary resistance and reactance of 5 Ω and 6.2 Ω respectively. The secondary resistance and reactance values are 0.03 Ω and 0.06 Ω.

Calculate,

i) equivalent resistance referred to primary side
ii) equivalent resistance referred to secondary side
iii) equivalent reactance referred to primary side
iv) equivalent reactance referred to secondary side.

b) The open circuit & short circuit tests on a 4 kVA, 200/400V, 50 Hz, single phase transformer gave the following results:
OC test on the LV side: 200V, 1A, 100W
SC test with the LV side opened: 15 V, 10 A, 85 W;
i) Determine the parameters of the equivalent circuit.
ii) Draw the equivalent circuit referred to the LV side. **[WBUT 2010]**

Answer:

a) Given data:

Primary resistance $r_1 = 5\Omega$; Primary reactance $x_1 = 6.2\Omega$

Secondary resistance $r_2 = 0.03\Omega$; Secondary reactance $x_2 = 0.06\Omega$

i) Equivalent resistance referred to primary side

$$r_{e1} = r_1 + r_2 \cdot \left(\frac{N_1}{N_2}\right)^2 = 5 + 0.03 \times (8.8)^2\ \Omega = 5 + 2.32\Omega = 7.32\Omega$$

ii) Equivalent resistance referred to secondary side $= \left\{0.03 + 5 \times \left(\frac{250}{2200}\right)^2\right\}\Omega = 0.095\Omega$

iii) Equivalent reactance referred to primary

$$= x_1 + x_2 \cdot \left(\frac{N_1}{N_2}\right)^2 = 6.2 + 0.06 \times (8.8)^2 = (6.2 + 4.65)\Omega = 10.85\Omega$$

iv) Equivalent reactance referred to secondary side $= \left\{0.06 + 6.2 \times \left(\frac{250}{2200}\right)^2\right\}\Omega = 0.14\Omega$

b) Given data:

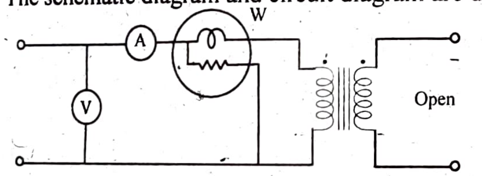
KVA → 4 KVA; 200/400 V, 50 Hz.

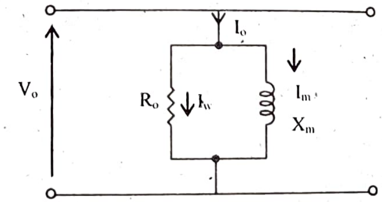
OC test on the LV side: 200 V, 1 A 100 W

SC test on the LV side: 15 V, 10 A, 85 W

i) $V_0 = 200\text{V}$; $I_0 = 1\text{A}$; $W_0 = 100\text{ W}$

The schematic diagram and circuit diagram are drawn below:





Magnetising current $I_m = I_0 \sin\phi_0$

Core-loss component $I_w = I_0 \cos\phi_0$

No-load p.f., $\cos\phi_0 = \frac{W_0}{V_0 I_0} = \frac{100}{200 \times 1} = \frac{1}{2} = 0.5$

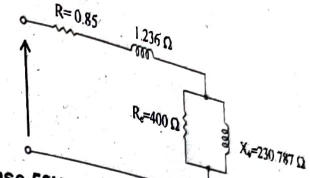
$$R_0 = \frac{V_0}{I_w} = \frac{V_0}{I_0 \cos\phi_0} = \frac{200}{1 \times 0.5}\Omega = 400\Omega;\ X_m = \frac{V_0}{I_m} = \frac{V_0}{I_0 \sin\phi_0} = \frac{200}{1 \times 0.866} = 230.787$$

The impedance as observed from the short-circuit side $Z = \frac{V_{sc}}{I_{sc}} = \frac{15}{10}\Omega = 1.5\Omega$

The resistance $R = \frac{P_{cf}}{I_{sc}^2} = \frac{85}{10^2}\Omega = 0.85\Omega$

$$X = \sqrt{X^2 - R^2} = \sqrt{(1.5)^2 - (0.85)^2}\ \Omega = 1.236\Omega$$

ii) The equivalent circuit diagram



3.12. a) A single phase 50Hz core type transformer has core of cross-section area 400 sq.cms. The permissible maximum flux density is 1Wb/m². Calculate the number of turns on the high & low voltage sides for a 3000/220V ratio. [WBUT 2011]

Answer:

Frequency, $f = 50\text{ Hz}$

Cross-sectional area, $A = 400\text{ sq.cm} = 400 \times 10^{-4}\text{ m}^2$

Maximum permissible flux density, $B_{max} = 1\text{ Wb/m}^2$

Maximum flux $= B_{max} \times A = 1 \times 400 \times 10^{-4}\text{ Wb}$.

Let N_1 be the number of primary turns

Induced e.m.f.

$$E_1 = \sqrt{2}\pi f N_1 \phi_{max} = \sqrt{2} \times \pi \times 50 \times N_1 \times 0.04 = 3000$$

$$N_1 = \frac{3000}{\sqrt{2} \times 2 \times \pi} = 337.6 = 338\text{ turns}$$

Let N_2 be the number of secondary turns

$$\frac{N_1}{N_2} = \frac{3000}{220}$$

$$N_2 = N_1 \times \frac{220}{3000} = 25\text{ turns.}$$

b) Determine the full – load efficiency at unity power factor for the 4kVA 200/400V, 50 Hz single phase transformer of which the following are test figures. [WBUT 2011]

O.C test: 200 V, 0.8 A, 70 W

S.C test: 17.5V, 9A, 50W

Answer:

The core-loss = 70 W

Full-load copper loss is directly proportional to full-load current.

Short-circuit test is conducted on the low-current high voltage side.

Rated current on the high-voltage side $= \frac{4 \times 10^3\text{ VA}}{400\text{ V}} = 10\text{ A}$

Hence, full load copper loss $=\frac{50}{(9)^2}\times(10)^2=61.73$ W

Total losses = core-loss + copper loss $=(70+61.73)$ W $=131.73$ W

Efficiency of the transformer

$$=\frac{\text{Output}}{\text{Output}+\text{Losses}}=\frac{4\times10^3\times1}{4000+131.73}\times100=96.8\%=97\%$$

3.13. a) Draw & explain the phasor diagram of a single phase transformer under loaded condition. **[WBUT 2011]**

OR,

Explain the principle of operation of a transformer under loaded condition. **[WBUT 2012]**

Answer:

When a load is connected, the actual load current flows through the windings, and therefore actual losses will occur in the core and windings. And there will be voltage drops in the windings also.

The drop in voltage in the winding is due to two components. One is due to the winding resistance and the other is due to the leakage reactance of the winding.

When V_1 is the voltage supplied to the primary winding, it circulates a current I_1. This currents produces a flux ϕ which links both primary and secondary windings. Since this flux is of alternating in nature, it induces emfs E_1 & E_2 on primary and secondary respectively.

$$\therefore V_1=E_1+I_1(R_1+jX_1) \quad ...(1)$$

$$V_2=E_2-I_2(R_2+jX_2) \quad ...(2)$$

where V_1 = primary input voltage V

Note: V_1 supplies the drops in primary and it produces the induced emf.

I_1 = Primary current in A

R_1, X_1 = Resistance and leakage reactance of primary in Ω

E_1 = Primary induced emf in V

E_2 = Secondary induced emf in V

R_2, X_2 = Resistance and leakage reactance of secondary in Ω.

Note: It is emf E_2, which supplies the terminal voltage V_2 and also the drops in secondary.

Note: Further wherever the transformer is loaded the flux passing through the core remain the same and therefore the core loss almost remains constant.

Let I_2' be the primary equivalent of the secondary current I_2 (Refer equivalent circuit).

Where $I_2'=\left(\frac{N_2}{N_1}\right)I_2$ ...(3)

Therefore the primary current I_1 is the vector summation of I_0 and I_2'

$$I_1=\sqrt{I_0^2+I_2'^2} \quad ...(4)$$

Phasor diagrams

Not only the magnitude but also the nature of the load plays an important role on the loading characteristics of the transformer.

To draw a phasor diagram and to understand a given phasor diagram the following points to be remembered: -

1. Flux is taken as the reference vector.
2. Primary voltage V_1 is drawn 90° ahead of ϕ_1.
3. $E_1=E_2$ and E_1 & E_2 are drawn 180° behind V_1
4. I_0 – No load current is drawn such that I_0 is lagging V_1 by an angle ϕ_0
   - I_2 is drawn with the necessary angle of leading or lagging (ϕ_2) with respect to E_2
   - I_2' is drawn which is the projection I_2 vector on primary side i.e. I_2' is lagging V_1 by ϕ_2
   - I_2' & I_0 are added to get the resultant I_1 (vector addition)
   - With E_1 resistive and reactive drops are added to obtain vector V_1
   - similarly using equation 2, V_2 is obtained

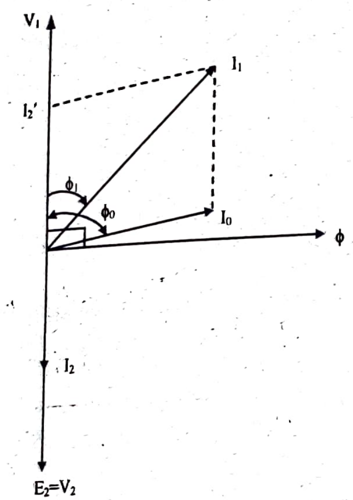
Note: Usually resistive drop is drawn parallel to the current vector, inductive drop perpendicular to the current vector. Both vectors are added to obtain the resultant impedance drop.

i.e. $R_1+jX_1=Z_1$ ...(5)

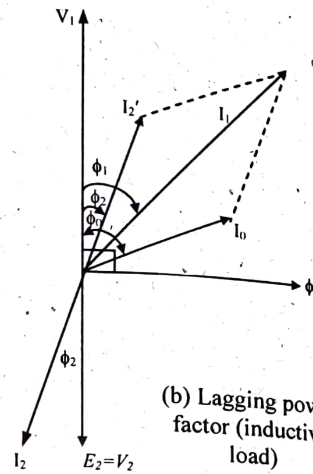
$R_2+jX_2=Z_2$ ...(6)

Note: since actual power output in watts or kilowatts $(V_2I_2\cos\phi_2)$ is depending on the power factor and hence the nature of the load which can't be predicted in advance, the rating of transformer is given VA or KVA instead of W or KW.

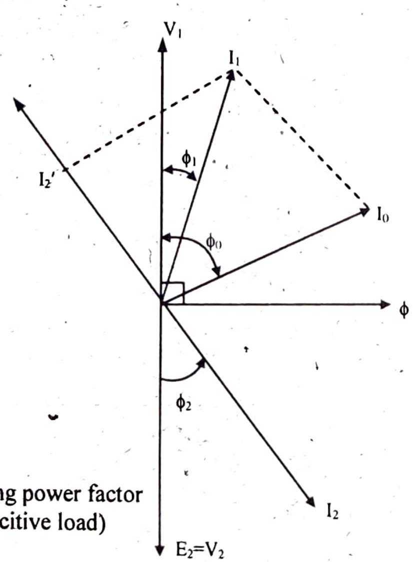
And further the losses also independent of power factor i.e., iron loss is dependent on supply voltage and copper loss is on load current and not on power factor.



(a) Unity power factor (Resistive load)



(b) Lagging power factor (inductive load)



(c) Leading power factor (Capacitive load)

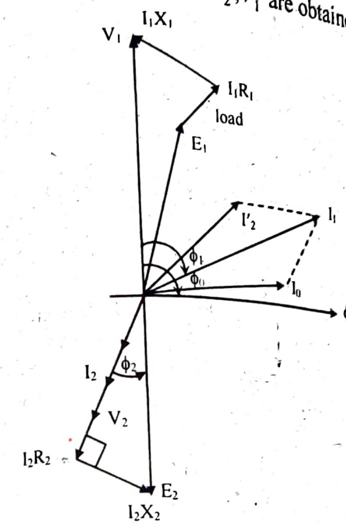
Fig: 1

The phasor diagrams drawn above fig.(1) are not the actual ones, because the resistive and reactive drops are not considered.

The actual phasor diagrams for different types of loads can be drawn with slight changes as shown in fig. (2).

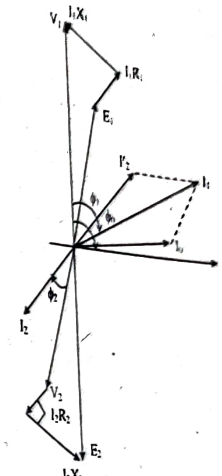
- E_1 is drawn with 90° lead of ϕ(flux).
- E_2 is 180° lagging E_1.

- V_2 is always drawn with some lagging angles of E_2.
- I_2 is drawn with respect to V_2 i.e. in phase, or lagging or leading.
- From E_1 & E_2, V_1 are obtained.



(a)

Fig: 2



(b) Lagging pf load

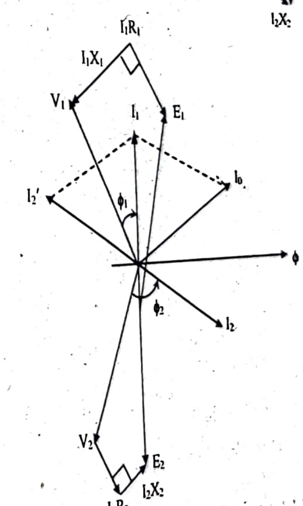


Fig: 2 (c) Leading pf load

b) A 200 kVA transformer has 400 turns on the primary & 40 turns on the secondary winding. The primary is connected to 2 kV, 50 Hz supply. Find the full load primary & secondary current, secondary emf & the maximum flux in the core. Neglect leakage drop & no-load primary current. **[WBUT 2012]**

Answer:

The rating of the transformer is 200 kVA.

$$\text{Full load primacy current} = \frac{\text{Rated kVA}}{\text{Applied voltage}} = \frac{200}{2}\text{ A} = 100\text{ A}.$$

Assuming the transformer to be an ideal one, $I_1N_1 = I_2N_2$

where, I_1 & I_2 are the primary and secondary currents respectively and N_1 & N_2 are the primary and secondary turns.

$$N_1 = 400,\ N_2 = 40$$

$$I_2 = I_1 \cdot \frac{N_1}{N_2} = 100 \times \frac{400}{40} = 1000\text{ A}.$$

Also, $\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2}$

So, $V_2 = V_1 \cdot \dfrac{N_2}{N_1} = 2 \times 10^3 \times \dfrac{40}{400} = 200\text{ V}$

Secondary emf is 200 V.

The induced emf is expressed as $e_1 = \sqrt{2}\pi f \phi N_1$; $e_{1\max} = \sqrt{2}\pi f \phi_{\max} N_1$

The maximum flux in the core

$$\phi_{\max} = \frac{e_{1\max}}{\sqrt{2}\pi f N_1} = \frac{2000}{\sqrt{2} \times \pi \times 50 \times 400}\text{ Weber} = 0.023\text{ Weber}.$$

3.14. a) Draw the phasor diagram of a single phase transformer under no load condition. **[WBUT 2014]**

Answer:

Under no load the primary winding will be connected to the input supply and on secondary, there will not be any load connected or secondary is just open circuited. If I_0 be current taken by the primary winding from the supply under no load, then current will lag the impressed voltage V_1 by an angle which is about 90°. This I_0 can be resolved into two components as shown i.e. I_ω & I_μ.

Note: since no load is connected to the secondary, the power drawn by the primary is fully taken to overcome the losses and some amount of power is utilized in creating and maintaining the flux.

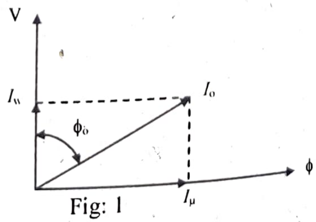


Fig: 1

No load power $P_0 = V_1 I_0 \cos\phi_0$ .... (1)

where V_1 = primary voltage

I_0 = No load current

$\cos\phi_0$ = No load power factor

From the no load phasor diagram

I_ω = Active or working or iron loss component.

This component is in quadrature with V_1 and it supplies the iron loss and small amount of copper loss in primary ($\because$ there is no load on the secondary, there will be no copper loss on secondary).

I_μ = Magnetizing component or watt loss component.

This is in phase with V_1 and in phase with the flux. Its function is to maintain the flux in the core.

and also $I_\omega = I_0 \cos\phi_0$ .... (2)

$I_\mu = I_0 \sin\phi_0$ .... (3)

$I_0 = \sqrt{I_\omega^2 + I_\mu^2}$ .... (4)

Under no load the power input supplies the iron loss and a small amount of primary copper loss. Since no load current value is of very small around 10 percent of the full load current, the copper loss produced by this small value of the primary winding will also be very less.

And therefore compared to the iron loss the copper loss value will be negligible. In general the no load power is assumed to be equal to the core loss alone.

b) The efficiency at unity power factor of a 6600/384 V, 200 KVA single phase transformer is 98% both at full load and at half load. Calculate the full load Cu Loss and Core Loss. **[WBUT 2014]**

Answer:

The efficiency at unity p.f. of a 6600/384 V, 200 kVA single-phase transformer is 98% at full load and at half load.

At full load, the output = 200 × 1 kW = 200 kW

$$\text{The efficiency of a transformer} = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{200}{200 + \text{Losses}} = 0.98$$

$$\text{Losses} = \frac{200 \times 0.02}{0.98}\text{ kW} = \frac{400}{98}\text{ kW} = \frac{100}{24.5}\text{ kW} = 4.0816\text{ kW} = 4082\text{ W}.$$

Losses = Fixed loss + Variable loss.

= Iron loss + Copper loss

$= k_l + I_a^2 k$ {Copperless $\propto$ Square of current} .... (1)

= 4082

At half load, the output = 100 kW

$$\text{Efficiency of transformer} = \frac{\text{Output}}{\text{Output} + \text{Losses}} = 0.98$$

$$\frac{100}{100+\text{Losses}}=0.98$$

$$\text{Losses}=\frac{100\times 0.02}{0.98}\text{kW}=\frac{200}{98}\text{kW}=\frac{100}{49}\text{kW}=2041\text{ W}$$

Losses = Iron loss + Copper loss

$$=k_I+\left(\frac{I_a}{2}\right)^2 k \quad \{\text{Copper loss} \propto \text{square of current}\}$$

$$=k_I+\frac{I_a^2 k}{4}=2041 \qquad \text{.... (2)}$$

$$(1)-(2)=\frac{3}{4}I_a^2 k=2041.$$

$$I_a^2 k=\frac{2041\times 4}{3}=2721.33 \qquad \text{.... (3)}$$

$$(1)-(3)=k_I=(4082-2721.33)\text{ W}=1360.67\text{ W}$$

Ans. Full load copper loss = 1360.67 W
Core loss = 2721.33 W.

3.15. a) Why is the open circuit test on a transformer conducted at a rated voltage? Explain.
b) A 20 KVA, 2000/200V single phase transformer has a primary resistance of 2.1Ω and a secondary resistance of 0.02Ω. If the total iron loss equals 200W, find the efficiency on (i) full load & a p.f. of 0.5 laging (ii) half load & p.f. of 0.8 leading.
[WBUT 2015]

Answer:

a) As the values of shunt branch elements of the equivalent circuit R_i and X_m depend on the applied voltage, the rated voltage is applied to one winding while the other winding is open circuited.

b) Given data:
Full load output power = 4kW
Full load efficiency = 90%
p.f = 0.8
Line voltage = 400 V,
Frequency = 50 Hz

$$\text{Input power during full-load condition}=\frac{\text{output}}{\text{afficiency}}=\frac{4\times 10^3}{0.9}W=4444W$$

Input three phase power $=\sqrt{3}V_{LL}I_{LL}\cos\phi$ where I_{LL} is the line current.

$$=\sqrt{3}V_{LL}I_{LL}\cos\phi=4444W$$

$$I_L=\frac{4444}{\sqrt{3}\times 400\times 0.8}A=8.02A$$

ELECTRICAL MACHINES

Chapter at a Glance

Principle of D.C. Machines

D.C. Machines are based on Faraday's Laws of electromagnetic induction. According to this law 'the voltage induced in a coil is proportional to rate of change of flux linking with the coil' i.e.

$$e=-N.\frac{d\phi}{dt} \qquad \text{.... (1)}$$

where N = number of turns in a coil, and $\frac{d\phi}{dt}$ = rate of change of flux linking with the coil.

The negative sign indicates that the current produced by it opposes the change in flux linkages.

Case–I: In case of a generator, when a conducting coils is rotated in a magnetic field, the flux linked with the coil changes due to motion of the coil, and due to this change in flux linkage, emf is induced in the coil (as per Faraday's laws). This induced emf is given by:-

$$e=Blv \text{ volts} \qquad \text{.... (2)}$$

where B = flux density of the magnetic field (in wb/m²),
ℓ = length of conductor in m,
and v = velocity of the conductor in m/s.

The induced emf is used (i) to overcome the voltage drop in the resistance of the windings and (ii) to supply the load. The electromagnetic torque produced in the moving current carrying conductor opposes the mechanical torque rotating the coil.

Case-II: In case of a motor, when a current carrying conductor is placed in a magnetic field, a mechanical force is exerted on the conductor (called Lorentz's force), which is given by:-

$$F=BIl\sin\theta \qquad \text{.... (3)}$$

where θ = angle between length (ℓ) and magnetic field (B).

In case of a generator the direction of induced emf or current is determined by Fleming's right hand rule. The thumb fore finger and middle finger of the right hand are stretched mutually perpendicular to each other, then if the thumb represents direction of motion of the conductor and fore-finger represents the direction of field, then middle finger will indicate the direction of induced emf.

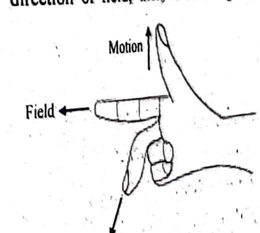


Fig: Right-hand rule

8-F-

In case of a motor, the rectangular coil of length 'ℓ', width 'r', carrying a current 'i', the two arms of the conductor, according to Fleming's left hand rule are acted upon by two forces, equal in magnitude, opposite in direction and their lines of action are different.

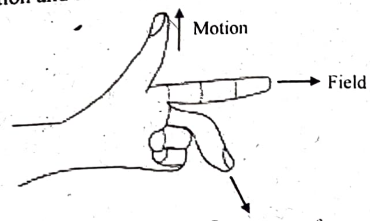


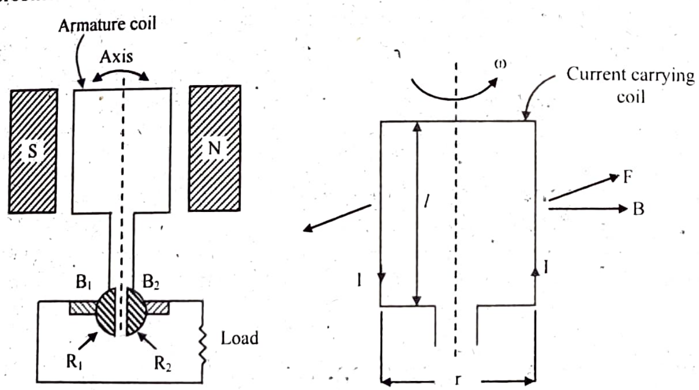
Fig: Left-hand rule

and their lines of actions are different. Thus the two forces form a couple and the torque (twisting effect) is given by:-

$$T = F.r = B\ell I \sin\theta.r = BAI\sin\theta \qquad \text{.... (4)}$$

where A = area of coil $= \ell \times r$.

In a motor, once its current carrying coil starts rotating in the magnetic field, a rotational emf, called back emf is produced. This back emf opposes the applied emf. Mechanical torque produced is used to overcome the frictional/damping torque, and to supply the load torque.



(a) Generator (b) Motor

Fig: Working Principle of a d.c. (a) Generator (b) Motor

It may be noted that:

i) In both generator and motor, there is a relative motion between the conducting coil and the magnetic field.

ii) The generator and motor actions go hand in hand. Only the direction of current flow can ascertain whether the machine is working as a generator or motor.

Multiple Choice Type Questions

1.1. The d.c. motor needs a starter during starting to control [WBUT 2007, 2008]

a) speed b) current c) voltage d) flux

Answer: (b)

1.2. For a wave connected d.c. machine, for no. of poles = 4, the no. of parallel path is [WBUT 2008(EVEN)]

a) 4 b) 2 c) 8 d) None of these

Answer: (b)

1.3. For traction d.c. motor used is [WBUT 2008(EVEN), 2014]

a) shunt b) compound c) series d) None of these

Answer: (c)

1.4. The number of parallel paths in an 8-pole D.C. generator is [WBUT 2009(EVEN)]

a) 8 b) 4 c) 2 d) 16

Answer: (a)

1.5. The direction of rotation in D.C. motor can be determined by [WBUT 2009(EVEN)]

a) Lenz's law b) Fleming's right hand rule
c) Fleming's left hand rule d) none of these

Answer: (c)

1.6. In order to a dc generator be able to excite & generate voltage, the value of the field winding resistance should be [WBUT 2010(EVEN)]

a) of any value b) less than the critical value
c) equal to the critical resistance d) greater than the critical resistance

Answer: (d)

1.7. If the field of a d.c. shunt motor is opened [WBUT 2011]

a) it will continue to run at its rated speed
b) the speed of the motor will become very high
c) the motor will stop
d) the speed of the motor will decrease

Answer: (b)

1.8. If a d.c series motor is started at no load, the speed will be [WBUT 2012]

a) rated speed b) zero
c) very high d) half of the rated speed

Answer: (c)

1.9. A series motor drawing armature current I_a is operated under saturation condition. The torque will be proportional to [WBUT 2014]

a) $1/I_a$ b) $1/I_a^2$ c) I_a^2 d) I_a

Answer: (c)

1.10. The output voltage of a dc generator is [WBUT 2015]

a) ac square wave b) ac sinusoidal wave
c) pulsating dc d) pure dc

Answer: (c)

1.11. Power developed by dc motor is maximum when the ratio of back emf & applied voltage is [WBUT 2015]

a) double b) zero c) unity d) half

Answer: (d)

1.12. The critical resistance of a dc generator refers to the resistance of [WBUT 2015]

a) load b) brushes c) field d) armature

Answer: (c)

1.13. Iron losses in a *dc* machine are independent of variation in [WBUT 2016]

a) frequency b) load c) voltage d) flux density

Answer: (b)

1.14. A series motor will run at very high speed when [WBUT 2016]

a) the load is increased b) the field is opened
c) the armature is opened d) the load is removed

Answer: (d)

1.15. In order that a dc generator be able to excite and generate voltage, the value of the field winding resistance should be [WBUT 2017]

a) of any value b) less than the critical value
c) equal to the critical resistance d) greater than the critical resistance

Answer: (b)

1.16. If the direction of current following in a conductor is in the plane of the paper, the magnetic flux lines by it are [WBUT 2017]

a) concentric circles in the clockwise direction
b) concentric circles in the anti-clockwise direction
c) straight lines parallel to the conductor and in the opposite direction of current flow
d) straight lines parallel to the conductor and in the direction of current flow

Answer: (a)

1.17. The slip of 400 V, 3-phase, 50 Hz, 4-pole induction motor when rotating at 1440 r.p.m. is [WBUT 2009]

a) 2% b) 3% c) 4% d) 5%

Answer: (c)

1.18. What happens when the phase sequence of the voltage applied to the stator of a three-phase induction motor is changed? [WBUT 2010]

a) Motor does not run b) Slip changes
c) Direction of rotation is reversed d) Motor gets heated

Answer: (c)

1.19. The power drawn by 3-phase induction motor when first run is star connection & then delta is [WBUT 2011]

a) same b) √3 times that of star
c) 3 times that of star d) 1/√3 times that of star

Answer: (a)

1.20. Frequency of the induced emf in the rotor circuit is [WBUT 2011]

a) maximum at stand still b) maximum at synchronous speed
c) zero at standstill d) none of these

Answer: (a)

1.21. The speed in which stator magnetic field rotates is called [WBUT 2012]

a) actual speed b) synchronous speed
c) slip speed d) super-synchronous speed

Answer: (b)

1.22. If the induction motor is supplied with voltage having frequency f and s is the slip of the motor, then the frequency of rotor current is given by [WBUT 2016]

a) sf b) f c) $sf+f$ d) $sf-f$

Answer: (a)

1.23. Capacitor start and run induction motor is basically a [MODEL QUESTION]

a) single phase induction motor b) two-phase induction motor
c) three-phase induction motor d) single phase reluctance motor

Answer: (a)

1.24. The backward rotor slip in a single-phase induction motor is equal to [MODEL QUESTION]

a) $1-s$ b) s c) $2-s$ d) $s/2$

Answer: (c)

1.25. The direction of rotation of a single-phase induction motor can be reversed by [MODEL QUESTION]

a) reversing the leads of main winding
b) reversing the leads of auxiliary winding
c) reversing the supply leads
d) either (a) or (b)

Answer: (d)

1.26. In a synchronous generator operating at zero power factor lagging, the effect of armature reaction is **[MODEL QUESTION]**

a) Magnetizing
b) Demagnetizing
c) Cross-magnetizing
d) Both magnetizing and cross-magnetizing

Answer: (b)

1.27. In a salient pole synchronous machine, where $X_d = d$ - axis synchronous reactance, X_q = quadrature axis synchronous reactance. **[MODEL QUESTION]**

a) $X_q = X_d$
b) $X_q > X_d$
c) $X_q < X_d$
d) $X_q = 0$

Answer: (c)

1.28. A 400V-60 Hz synchronous machine has 6 poles. The synchronous speed in rpm is **[MODEL QUESTION]**

a) 1200
b) 3000
c) 1000
d) 1500

Answer: (a)

1.29. In a split phase motor, the running winding should have **[MODEL QUESTION]**

a) High resistance and low inductance
b) High resistance and high inductance
c) low resistance and high inductance
d) Low resistance and low inductance

Answer: (d)

1.30. If the capacitor of a single-phase motor is short-circuited **[MODEL QUESTION]**

a) The motor will not start
b) The motor will run in the same direction at reduced speed
c) The motor will run in reverse direction
d) None of these

Answer: (a)

1.31. Which of the following motor will have relatively higher power factor? **[MODEL QUESTION]**

a) Capacitor start motor
b) Shaded pole motor
c) Capacitor run motor
d) Split phase motor

Answer: (c)

1.32. A two phase servomotor is commonly used in feedback control system to drive the loads and as sensors to measure **[MODEL QUESTION]**

a) Speed of controlled element
b) Position of controlled element
c) Both (a) & (b)
d) None of these

Answer: (c)

Short Answer Type Questions

2.1. A 220 volt separately excited DC machine has an armature resistance of 0.4 Ω. If the load current is 20 A, find the induced emf when the machine operates
i) as a motor and
ii) as a generator. **[WBUT 2006, 2017]**

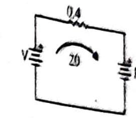
Answer:

The circuit diagram of the d.c. machine operating as a motor is shown below:

(i) Terminal voltage $V = 220$ V

Back e.m.f. $= V - I_L \cdot r_a$

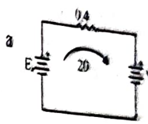
(I_L = load current, r_a = armature resistance)



$\therefore E_b = V - I_L r_a = (220 - 20 \times 0.4)\text{V} = (220 - 8)V = 212\text{V}$

Hence induced emf = 212V.

(ii) The circuit diagram of the d.c. machine operating as a generator is shown below:



For a generator. Induced emf, $E_a = V + I_L r_a$

Hence, $E_a = (220 + 20 \times 0.4)\text{V} = 228\text{V}$

2.2. Derive speed-torque characteristics of a d.c series motor. Discuss its applications. **[WBUT 2011]**

Answer:

1st Part:

Speed-Torque characteristics

This can be obtained from the two plots as made in fig (1) i.e. with increase in armature current (I_a), the torque T increases and with increase in I_a, the speed (N) decreases slightly. And therefore with increase of torque T there will be slightly decrease in speed. In fact speed-torque characteristics is similar to speed armature current characteristic is similar to speed armature current characteristic as shown in fig (1).

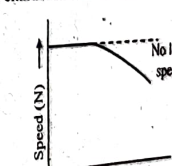


Fig: 1

2nd Part:

i) Traction work
ii) Hoists and Cranes.
iii) Electric locomotives
iv) Trolley Cars
v) Conveyor.

2.3. Derive the expression of torque of a d.c. series motor. **[WBUT 2012]**

Answer:

Torque of a D.C. Machine

The voltage equation of a d.c. motor is:

$$V = E_b + I_a R_a \quad \text{.... (1)}$$

Multiplying both the sides of equation (1) by I_a:

$$V I_a = E_b I_a + I_a^2 R_a \quad \text{.... (2)}$$

But $V I_a$ = electrical power input to the armature

$I_a^2 R_a$ = Copper loss in the armature.

It is known that: Input = Output + losses .... (3)

Comparing equation (2) & (3), it may be seen that $E_b I_a$ = electrical equivalent of gross mechanical power developed by the armature (electro magnetic power).

It is presumed that T = average electromagnetic torque developed by the armature in Nm. At this value of torque the electro mechanical power conversion takes place.

So mechanical power developed by the armature:

$$P_m = \omega T = 2\pi NT$$

Therefore,

$$P_m = E_b I_a = \omega T = 2\pi NT \quad \text{.... (4)}$$

But $E_b = \dfrac{\phi Z N}{60} \times \left(\dfrac{P}{A}\right)$

Therefore, $\dfrac{\phi Z N}{60} \times \left(\dfrac{P}{A}\right) I_a = 2\pi NT$

and $T = \dfrac{PZ}{2\pi A} . \phi \, I_a$ .... (5)

Equation (5) is called the torque equation of d.c. motor. For a given d.c. machine, P, Z, A are constant and as such $\left(\dfrac{PZ}{2\pi A}\right)$ is also a constant.

If it is presumed that $\dfrac{PZ}{2\pi A} = K$, then

$$T = K \phi I_a \quad \text{.... (6)}$$

or, $T \, \alpha \, \phi . I_a$ .... (7)

Hence the torque developed by a d.c. motor is directly proportional to the flux per pole and armature current.

The schematic diagram of a series motor is shown in fig (1) and its equivalent circuit in fig (2).

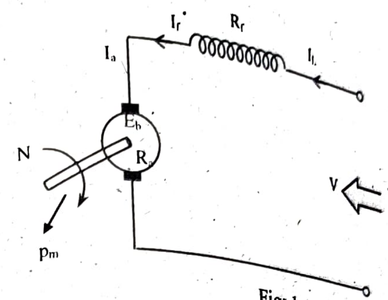


Fig: 1

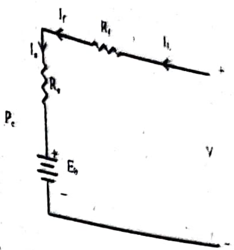


Fig 2

From the circuit (Fig 2), $I_L = I_f = I_a$ .... (8)

By applying KVL in the meshes:-

$$V = I_a R_a + I_f R_f + E_b$$

$$= I_a (R_a + R_f) + E_b \quad \text{.... (9)}$$

or, $E_b = V - I_a (R_a + R_f)$ .... (10)

Torque developed, $T = \dfrac{E_b I_a}{2\pi N / 60} = \dfrac{60 E_b I_a}{2\pi N}$ .... (11)

Electrical power input, $P_e = V I_L$

$= VI_f = VI_a$ .... (12)

Mechanical power output,

$$P_m = T \times \frac{2\pi N}{60} = E_b I_a = E_b I_f = E_b I_L \quad \text{.... (13)}$$

2.4. Draw and explain the speed-torque characteristics of a (i) dc shunt motor (ii) dc series motor. **[WBUT 2013]**

Answer:

For d.c shunt motors:

Speed-Torque characteristics

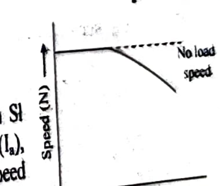


Fig: 1

This can be obtained from the two plots as made in Sl No.(1) & (2), i.e. with increase in armature current (I_a), the torque T increases and with increase in I_a, the speed (N) decreases slightly. And therefore with increase of torque T there will be slightly decrease in speed.

For d.c series motors:

Speed vs Torque Characteristics (N vs. T)

Sine a series motor develops a high initial torque at low Speeds, and a low torque at his speed, so speed torque Characteristic of a series motor is a hyperbola. High initial Torque at low speeds enables, even a small series motor, to start a heavy load. However, when the starting friction is overcome the motor begins to accelerate, the counter EMF increases, and current and torque decreases correspondingly as the motor speeds up.

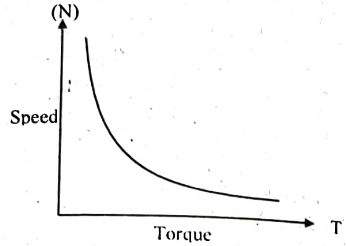
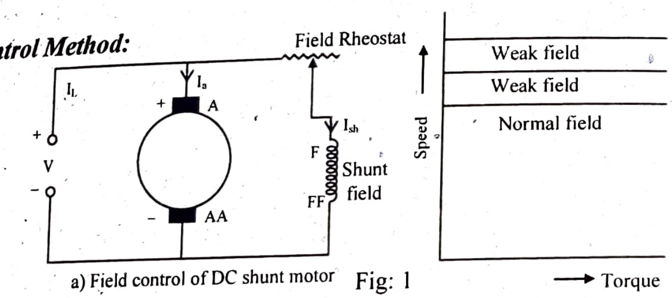


Fig: 2 Speed torque characteristics

2.5. Explain how the speed of a D.C. shunt motor can be controlled by flux control method. **[WBUT 2014]**

Answer:

Field Control Method:



a) Field control of DC shunt motor Fig: 1

As the resistance connected to the field circuit is increased, the field current will decrease, the flux will decrease and finally the speed can be increased. Therefore using this methods speed above the rated speed can be obtained.

$$R_{rhe}\ \alpha\ \frac{1}{I_f} \qquad I_f\ \alpha\ \phi \qquad \phi\ \alpha\ \frac{1}{N}$$

$\therefore$ As R_{rhe} increases N also increases.

2.6. Explain why the rotor of an induction motor rotates in the same direction as the stator magnetic field. Why the speed of the motor is less than the synchronous speed? **[WBUT 2011]**

Answer:

An induction motor operates on the principle of Faraday's Laws of electromagnetic induction. When a three-phase supply is connected to the stator terminals, a rotating magnetic field is produced. This rotating magnetic field cuts the rotor conductors by virtue of which an emf is induced in the rotor. Current flows through the short-circuited rotor which in turn produces a force acting on the rotor conductors. The rotor conductors experience a motion under the action of this force. By Lenz' Law the rotor conductors move in a direction so as to oppose the cause (i.e., the rotating magnetic field). So they move in the direction of the stator flux.

If there is no relative motion between the stator flux and the rotor conductors, slip will be equal to zero. Hence frequency of the rotor circuit will be equal to zero. No emf would be induced in the rotor circuit since there is no transformer action. Therefore no rotor current would flow and there will be no motoring action. So, relative motion is necessary. In the motoring mode the speed of the rotor will be less than the synchronous speed. If the speed of the rotor is more than the synchronous speed, the machine will act as an induction generator.

2.7. What is slip? Deduce a relationship between rotor current frequency & supply frequency in terms of slip of an induction motor. **[WBUT 2015]**

Answer:

1st Part:

It is the difference between the flux speed (N_s) and the rotor speed (N_r).

$$s = N_s - N_r$$

Slip is always expressed as a percentage of synchronous speed (N_s) and so

$$\%s = \frac{N_s - N_r}{N_s} \times 100$$

or, Fractional slip or per unit slip $= \dfrac{N_s - N_r}{N_s} = s$

$$sN_s = N_s - N_r$$

$$N_r = N_s - sN_s = N_s(1-s)$$

i.e., speed of the rotor $N_r = N_s(1-s)$ where N_s the synchronous speed in r.p.m. and s is the fractional slip.

2nd Part:

It is known that frequency of the stator circuit (stator voltage and stator current) is the supply frequency and it can be obtained from the equation.

$$N_s = \frac{120f}{P}; \qquad f = \frac{N_s P}{120}$$

The frequency of the rotor circuit f_r is mainly depending on the relative velocity $N_s - N_r$ and can be obtained from

$$N_s - N_r = \frac{120 f_r}{P}$$

$$f_r = \frac{(N_s - N_r)P}{120}$$

From equation 10 and 11

$$\frac{N_s - N_r}{N_s} = \frac{f_r}{f}$$

i.e., $s = \frac{f_r}{f}$

so $f_r = sf$ where f_r or f is the frequency of rotor circuit.

At stand still, $s = 1$ (i.e.) at no load $N_r = 0$, $s = 1$ and therefore $f_r = f$

When $N_r = N_s, s = 0$, and $f_r = 0$

2.8. What is the function of centrifugal starting switch in a single-phase induction motor? **[MODEL QUESTION]**

Answer:

The centrifugal switch is connected in series with the starting winding. The primary function of the centrifugal switch is to produce rotating flux in conjunction with main winding at the time of starting. When the motor has started and reaches nearly 75% of synchronous speed, it produces its own rotating field from the cross field effect. The starting winding now has no function to perform and is removed from the circuit by a centrifugally operated switch.

2.9. What is the principle of operation of shaded-pole induction motor? **[MODEL QUESTION]**

Answer:

A shaded-pole motor is basically a small single-phase squirrel cage motor in which the starting winding is composed of short-circuited copper ring (called shading coil) surrounding one-third of each pole. The effect of the shading coil is to cause a flux to sweep across the pole faces, from unshaded to shaded portion of the pole, producing a weak rotating magnetic field. As a result, the rotor is set in motion due to induction principle.

2.10. Why is the starting torque of a resistance split-phase induction motor not high? **[MODEL QUESTION]**

Answer:

The starting torque is given as, Ts = K Im Is Sin Φ

Where

K = constant whose magnitude depends upon the design of the motor

(i) The angle between Is and Im is small (approximately 25 degree) in a resistance split-phase induction motor, so the starting torque is small.

(ii) Since currents Is and Im are not equal in magnitude, the rotating magnetic field is not uniform and the starting torque produced is small.

2.11. A pure single-phase induction motor does not have starting torque but has running torque. **[MODEL QUESTION]**

Answer:

When the stator winding of a single-phase induction motor is connected to single-phase A.C. supply, a magnetic field is developed, whose axis is always along the axis of stator coils. The magnetic field produced by the stator coils is pulsating, varying sinusoidally

with time. Currents are induced in the rotor conductors by transformer action, these currents being in such a direction as to oppose the stator mmf. Then the axis of the rotor mmf wave coincides with that of the stator field, the torque angle is, therefore, zero and no torque is developed on starting. As such the single-phase does not have starting torque. However, if the rotor is given a push by hand or by other means in either direction, it will pick up the speed and continue to rotate in the same direction developing running or operating torque. This behaviour can be explained either by double revolving field theory or cross-field theory

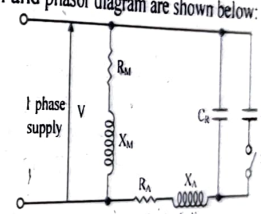
2.12. What are the methods adopted to start a single phase induction motor? Explain with the help of connection diagram and phasor diagram, the principle of operation of capacitor start and capacitor run motor. **[MODEL QUESTION]**

Answer:

For the purpose of starting a single phase induction motor, the following methods are broadly classified:

i) split phase starting
ii) shaded pole starting
iii) repulsion motor starting
iv) reluctance starting.

In order to explain the principle of operation of capacitor start and capacitor run motor, the connection diagram and phasor diagram are shown below:



Connection diagram

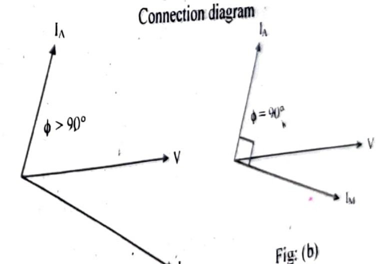


Fig: (a) Fig: (b)

Phasor diagram

The motor uses two capacitors C_S (Starting Capacitor) and C_R (Running Capacitor) which are connected in parallel at starting. In order to obtain a high starting torque, a large current is required. For this purpose, the capacitive reactance X in the starting winding should be low. Since $X_R = \frac{1}{2\pi f C_R}$, the value of C_R should be small. As the motor approaches synchronous speed, the capacitor C_S is disconnected by a centrifugal switch S_C. The capacitor C_R is permanently connected in the circuit. It is usually oil filled paper construction and long time rated for continuous running. Since one capacitor C_S is used only at starting and other C_R for continuous running, this motor is called capacitor start capacitor run motor. If the phasor diagrams are referred to it may be seen that at starting both the capacitors are in circuit and $\phi > 90°$ [Fig. (a)]. When the capacitor C_S in disconnected ϕ be comes 90° (electrical) as shown in Fig. (b).

2.13. Explain what causes hunting in a synchronous machine. Explain the methods adopted to minimize hunting. **[MODEL QUESTION]**

Answer:
The phenomenon of oscillation of the rotor of a synchronous machine about its final equilibrium position is called hunting. The main causes of hunting are
1. Sudden changes of load.
2. Fault occurring in the system, which the generator supplies.
3. Sudden changes in the field current.
4. Cyclic variations of the load torque.

Due to hunting there may be (i) loss of synchronism (ii) undesirable lamp flickering for variation of supply voltage (iii) possibility of resonance (iv) development of mechanical stress in the rotor shaft (v) increase of machine losses and rise of temperature of the machine. Some of the methods that are adopted for reduction of hunting are:

a) Use of flywheels: The prime-mover is provided with a large and heavy flywheel. This increases the inertia of the prime-mover and hence in maintaining the rotor-speed.
b) Damper winding: Damper winding is nothing but copper bars placed inside the holes present in the rotor face, which are short-circuited. EMF is induced in the bar whenever there is oscillation, which will circuit current in the bars there by producing torque that opposes the oscillation.
c) By designing synchronous machines with suitable synchronizing power co-efficients.

2.14. What is distribution factor? What are the advantages of distributing a winding in slots? **[MODEL QUESTION]**

Answer:
1st Part:
The ratio of the vector sum of the emfs induced in all the coils distributed in a number of slots under one pole to the arithmetic sum of the emfs induced (or to be resultant of the emfs induced in all the coils concentrated in one slot under one pole) is known as distribution factor

$$K_d = \frac{\text{emf induced in a distributed winding}}{\text{emf induced of the winding would have been concentrated}} = \frac{\text{Vector sum}}{\text{Arithmetic sum}}$$

The distribution factor is less than unity.

2nd Part:
Advantages of distributing a winding in slots are:
i) Mechanical strength of the coils is increased.
ii) Improved waveform of the induced emf.
iii) Less copper loss.
iv) Less winding space resulting in a shorter unit.
v) Distinct reduction in machine harmonics.
vi) Reduction of tooth ripples.

Long Answer Type Questions

3.1. a) Deduce the emf equation of a DC generator? **[WBUT 2006, 2007, 2008, 2017]**
OR,
Deduce the emf equation of a d.c. machine.
b) What do you mean by back e.m.f.? **[WBUT 2009]**
c) A 4-pole 220 V, DC shunt motor has armature and shunt field resistances at 0.2 Ω and 220 Ω respectively. It takes 20 A at 220 V from a source while running at a speed of 1000 r.p.m. find. **[WBUT 2008, 2009, 2015]**
(i) field current
(ii) armature current
(iii) back e.m.f.
(iv) torque developed **[WBUT 2009]**

Answer:
a) E. M. F Equation of A D.C. machine

Let P = No. of poles.
ϕ = Flux production per pole in webers.
Z = No. of armature conductors.
N = Rotational spaced of armature in rpm.
A = No. of parallel paths in armature.
And E_g = Generated emf per parallel paths in volts.

According to Faraday's law, the induced emf is given by:

$$e = -\frac{d\phi}{dt} \text{ volts} \quad \text{.... (1)}$$

[Negative sign indicates that the induced current will flow in such a direction as to oppose the very cause that produced it - Lenz's law]

As ϕ is the flux per pole, the total flux produced for P number of poles = $P\phi$
This is the amount of flux cut by a conductor in one revolution i.e.

$$\therefore d\phi = P\phi$$

It is known that N = revolutions per minute i.e. time taken for N revolutions = 60 seconds
∴ Time taken for one revolution .... (2)

$$dt = \frac{60}{N}$$

$$\therefore\ e=\frac{d\phi}{dt}=\frac{P\phi N}{60} \qquad \text{.... (3)}$$

is the emf in one conductor.

$$\text{For Z number of conductors} = \frac{P\phi N}{60}Z \qquad \text{.... (4)}$$

$\therefore E_g$ = emf induced / parallel path

$$=\frac{P\phi ZN}{60}\times\frac{1}{A}=\frac{\phi ZN}{60}\times\left(\frac{P}{A}\right)\text{volts} \qquad \text{.... (5)}$$

Where A = 2 – Wave wound armature = P – Lap wound armature.
Further, the polarity of the induced emf depends upon the direction of the magnetic field and the direction of rotation. It either of the two is reversed, the polarity of the induced emf i.e. brushes is reversed, but both are reversed, the polarity remains unchanged.
The induced emf is fundamental phenomenon to all D.C. machines whether they are operating as generators or motors. However, when the machine is operating as a generator, this induced emf is called the generated emf E_g where as in case of a machine operating as a motor it is called the counter or back emf, E_b.

b) When the motor armature rotates, its conductors cut the magnetic flux resulting production of induced emf in them. This emf of rotation in a motor is known as back emf or counter emf, denoted by E_b and it opposes the applied voltage. Since the back emf is induced due to generator action its magnitude is therefore, given by the same expression as that for the generated emf in a d.c. Generator, i.e. $E_b=\frac{\phi ZN}{60}\times\left(\frac{P}{A}\right)$ where the symbols have their usual meaning. For the sake of convenience, the armature of d.c. motor is represented by an equivalent circuit having back emf with a series armature resistance as shown in fig (1) & fig (2).

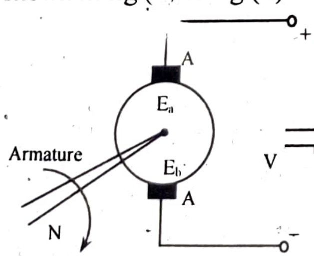


Fig: 1

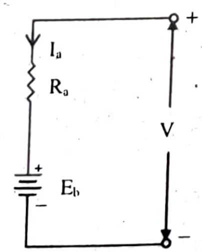
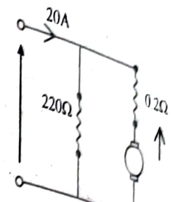


Fig: 2

c) (i) Field current $I_f=V_t/R_f$ [where V_t is the applied terminal voltage and R_f is the field circuit resistance].

$$=\frac{220}{220}\text{ A}=1\text{ A}.$$



(ii) Armature current = input current – field current = (20 – 1) A = 19 A.

(iii) Back e.m.f. $=V_t-I_ar_a=(220-19\times0.2)\text{ V}=(220-3.8)\text{ V}=216.2\text{ V}$

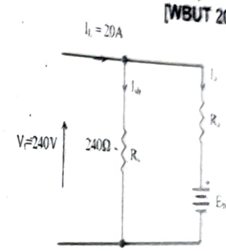
(iv) Torque developed $=\frac{\text{Power transmitted}}{\text{Speed}}=\frac{E_bI_a}{\omega}=\frac{216.2\times19\times60}{2\times\pi\times1000}=39.227$ W-sec.

3.2. a) A 4-pole, 240 V D.C. shunt motor has armature and shunt field resistances of 0.24 Ω and 240 Ω respectively. It takes 20 A at 240 V while running at a speed of 1000 r.p.m. Find (i) field current (ii) armature current (iii) back e.m.f. (iv) torque developed. **[WBUT 2009]**

Answer:

$$P=4,\ R_a=0.24\Omega,\ R_{sh}=240\Omega$$

The circuit diagram is drawn below



(i) Field current $I_{sh}=\frac{V_t}{R_{sh}}=\frac{240}{240}\text{A}=1\text{A}$

(ii) Armature current $I_a=(20-1)\text{A}=19\text{A}$

(iii) Back emf $E_b=V_t-I_aR_a=(240-0.24\times19)\text{V}=235.44\text{ V}$

(iv) Electrical power converted to mechanical output

$$=E_b\times I_a=235.44\times19\text{ Watt}=4473.36\text{ Watt}=4.47\text{ kW}$$

$$\text{Torque developed}=\frac{\text{Power}}{\text{Rotational speed}}=\frac{4473.36}{\frac{1000\times2\pi}{60}}\text{N-m}=42.72\text{ N-m.}$$

b) A shunt generator delivers 200A at 250V. The armature resistance & shunt field resistance are 0.02Ω and 50Ω respectively. The iron & friction losses are equal to 950W, Determine- **[WBUT 2009, 2011]**
(i) the *emf* generated **(ii) copper losses**
(iii) output of the prime mover **(iv) efficiency of the generator.**

Answer:

$$r_a=0.02\Omega;\ r_{sh}=50\Omega;\ I_L=200\text{ A};\ V_t=250\text{ V}$$

$$I_{sh}=\frac{V_t}{r_{sh}}=\frac{250}{50}\text{A}=5\text{A}$$

$$I_a=I_{sh}+I_L=(5+200)\text{A}=205\text{A}$$

The armature resistance voltage drop $= r_a \times I_a\ \text{V} = (0.02 \times 205)\ \text{V} = 4.1\ \text{V}$

(i) Generated e.m.f. $= (V_t + I_a r_a)\ \text{V} = (250 + 4.1)\ \text{V} = 254.1\ \text{V}$

(ii) Copper losses

$$= \left(I_a^2 r_a + I_{sh}^2 r_{sh}\right)\ \text{W} = (205)^2 \times 0.02 + (5)^2 \times 50 = (840.5 + 1250)\ \text{W} = 2090.5\ \text{W}$$

(iii) Output of the generator = power delivered to the load

$$= (200 \times 250)\ \text{W} = 50 \times 10^3\ \text{W} = 50\ \text{kW}$$

Total losses = Copper loss + Iron & friction losses $= (2090.5 + 950)\ \text{W} = 3040.5\ \text{W}$

Input to the generator = output + losses $= (50000 + 3040.5)\ \text{W} = 53040.5\ \text{W}$

Output of the prime mover = Input to the generator $= 53040.5\ \text{W}$

(iv) Efficiency of the generator $= \dfrac{\text{Output}}{\text{Output} + \text{Losses}} = \dfrac{50000}{53040.5} \times 100 = 94.3\%$

3.3. a) Explain the open circuit characteristics (OCC) of a DC generator. **[WBUT 2012]**

Answer:

This is also called open circuit characteristics (O.C.C). This characteristics show the plot between the no load induced emf and the field current.

Two different kinds of excitations may be considered such as:

a) Separately excited:

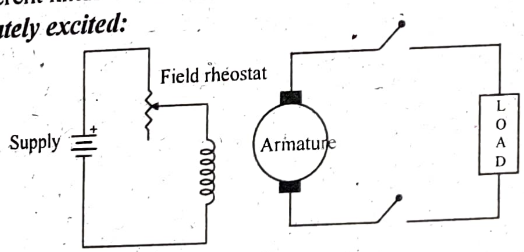


Fig: 1

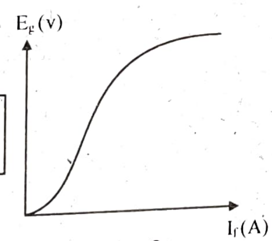


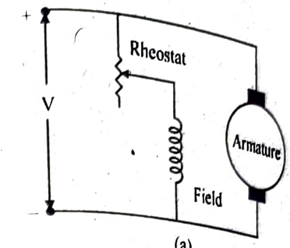
Fig: 2

The generator with the above connections is made to run at a constant speed. The field current is varied by the rheostat variation. An ammeter will be used to measure the field current and a voltage across the open circuited generator is measured by a voltmeter. With the variation of filed current, the induced emf also varies by the relation

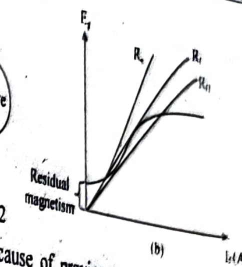
$$\phi \,\alpha\, I_f \ldots(1) \text{ and } E_g \,\alpha\, \phi \left[\therefore E_g = \frac{\phi Z N}{60}\left(\frac{P}{A}\right)\right] \ldots(2).$$

With constant speed e.g. is directly proportional to the I_f. Therefore the initial portion of OCC is a straight line. But after a certain range the relationship, $\phi \,\alpha\, I_f$ is not satisfied due to the saturation of magnetic poles. So the flux will be constant with increase in field current and hence the emf also becomes constant which makes the graph a flat one as shown above.

b) Self excited:



(a)



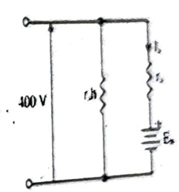
(b)

Fig: 2

In case of self-excited generator, because of previous usage there will be some flux existing in the field poles, without any field current. Therefore with the value of zero field current itself there will be some emf induced and this is known as residual emf or this flux is known as **residual Magnetism.** Therefore the graph is not starting from the origin. In the other aspects it is similar to the case of separately excited itself.

b) An 8-pole, 400 V shunt motor has 960 wave connected armature conductors. The full load armature current is 40 A & flux per pole is 0.02 Wb. The armature resistance is 0.1Ω and the contact drop is 1V per brush. Calculate the full load speed of the motor. **[WBUT 2012]**

Answer:



The back emf developed in the motor is given by $E_b = \dfrac{P\phi Z N}{60a}$

where, P is the no. of poles, ϕ, the flux per poles Z, the number of parallel conductors, a the no. of parallel paths and N, the full load speed of the motor.

In the given problem,

The armature circuit drop $= 0.1 \times 40\ \text{V} = 4\ \text{V}$.

The total brush drop $= 1 \times 2 = 2\ \text{V}$.

Back emf $= (400 - 4 - 2)\ \text{V} = 394\ \text{V}$.

$$394 = \frac{8 \times 0.02 \times 960 \times N}{60 \times 2} \quad [\because \text{For wave connection } a = 2]$$

or, $$N = \frac{394 \times 60 \times 2}{8 \times 0.02 \times 960}\ \text{rpm} = 308\ \text{rpm}$$

c) Why starter is needed to start a d.c. motor? **[WBUT 2012]**

Answer:

A starter is necessary in a d.c. motor to limit the starting current. The relation between applied voltage and current in the armature is given by $V_t = E_b + I_a r_a$ where V_t is the applied voltage, E_b the back emf developed, I_a, the armature current and r_a the armature resistance neglecting the brush contact drop.

Armature current, $I_a = \dfrac{V_t - E_b}{r_a}$.

At starting, the speed of the motor is zero, hence back emf developed is zero

($\because E_b = \dfrac{P\phi Z_n}{60a}$, where, n is the speed of the motor).

In the absence of E_b, the armature current is very high, which is potentially detrimental to the armature circuit. A starter is an additional resistance which limits the starting current and ensures smooth operation of the motor, hence it is necessary.

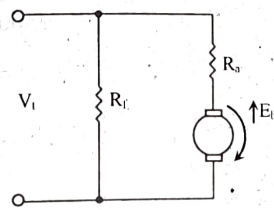
3.4. A shunt motor has a rated armature current of 40A when connected to 200V. The rated speed of the motor is 1000 rpm. The armature resistance is 0.2 ohm. Find the speed of the motor if total torque is reduced to 70% of that at rated load and a 3 ohm resistance is inserted in series with the armature. [WBUT 2013]

Answer:

At rated load

$V_t = 200V$

$E_b = V_t - I_a r_a = (200 - 0.2\times 10)\,\text{V} = (200-2)\,\text{V} = 198\,\text{V}$



Torque is proportional to current.

Let I_1 be the rated current at full load,

Let I_2 be the current when torque is 70% of that at rated load.

$\therefore \quad \dfrac{T_1}{T_2} = \dfrac{I_1}{I_2}$

or, $\quad I_2 = I_1 \cdot \dfrac{T_2}{T_1} = 10\times 0.7 = 7\text{A}$

Hence back e.m.f $= \{200 - (3+0.2)\times 7\}\,\text{V} = 177.6\,\text{V}$

$\dfrac{E_{b1}}{E_{b2}} = \dfrac{N_1}{N_2}$ [Since back emf is proportional to speed at constant flux]

$N_2 = N_1 \times \dfrac{E_{b2}}{E_{b1}} = \dfrac{1000\times 177.6}{198}\,\text{rpm} = 897\,\text{rpm}$

3.5. a) A 120 V D.C. Shunt Motor having an armature resistance of 0.2 Ω and field resistance of 60 Ω, draws a line current of 40 A at full load. The brush voltage drop is 3 V and the rated full load speed is 1800 rpm. Calculate the speed at half load and 125% of full load. [WBUT 2014]

Answer:

The relevant data for the D.C. shunt motor are listed below:

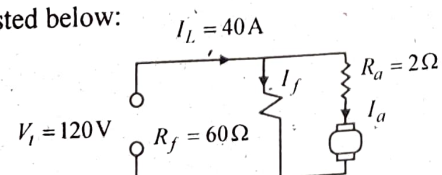
Terminal voltage $\quad V_t = 120\,\text{V}$

Armature resistance $\quad R_a = 0.2\Omega$

Field resistance $\quad R_f = 60\Omega$

Line current $\quad I_L = 40\,\text{A}$

Brush voltage drop $= 3\,\text{V}$



Rated full load speed = 1800 rpm

The circuit diagram is drawn:

The field current, $I_f = \dfrac{V_t}{R_f} = \dfrac{120}{60}\text{A} = 2\,\text{A}$

Line current = Field current + Armature current

$I_L = I_f + I_a$

$I_a = 40 - 2 = 38\,\text{A}$

Back emf $E_b = V_t - I_a r_a$ – Brush contact drop

$= (120 - 0.2\times 38 - 3)\,\text{V} = (120 - 7.6 - 3)\,\text{V} = 109.4\,\text{V}$

$E_b = \dfrac{P\phi ZN}{60A}$

or, $\quad \dfrac{PQ}{60A} = \dfrac{E_b}{N} = \dfrac{109.4}{1800}\,\text{V/rpm}$

At half load, line current $= \dfrac{40}{2}\text{A} = 20\text{A}$

The field current remains constant for constant terminal voltage i.e., 2A,

Hence, $\quad I'_a = (20-2)\,\text{A} = 18\,\text{A}$

The new back emf

$E'_b = V_t - I'_a r_a$ – Brush contact drop

$= (120 - 0.2\times 18 - 3)\,\text{V} = (120 - 3.6 - 3)\,\text{V} = 113.4\,\text{V}$

$\dfrac{E'_b}{N'} = \dfrac{PQ}{60A} = \dfrac{109.4}{1800}$

{The flux remains constant for constant field current}

$N' = E'_b \times \dfrac{1800}{109.4} = \dfrac{113.4\times 1800}{109.4}\,\text{rpm} = 1865.8\,\text{rpm} = 1865\,\text{rpm}$

At 125% of full load, load current $= 40\times\dfrac{125}{100}\text{A} = 50\,\text{A}$

The armature current, $I'_a = 50 - 2 = 48\,\text{A}$

The back emf $= (120 - 48\times 0.2 - 3)\,\text{V} = (120 - 9.6 - 3)\,\text{V}$

$E'_b = 107.4\,\text{V}$

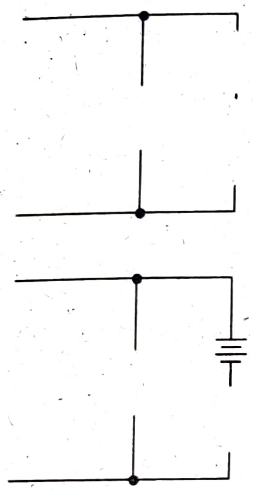
$\dfrac{E'_b}{N'} = \dfrac{P\phi}{60A} = \dfrac{109.4}{1800}$

{The flux remains constant for constant field current and terminal voltage}

$N' = \dfrac{107.4\times 1800}{109.4}\,\text{rpm} = 1767\,\text{rpm}.$

b) A 4-pole 240 V dc shunt motor has armature and shunt field resistances of 0.24 Ω and 240 Ω respectively. It takes 20 A from a 240 V *dc* supply while running at a speed of 1000 rpm. Find —
(i) field current, (ii) armature current, (iii) back *emf*, (iv) torque developed in Nm.
[WBUT 2014, 2016]

Answer:
The schematic and circuit diagram of d.c. shunt motor are drawn below:



(i) Field current $I_f = \dfrac{240}{240}\text{A} = 1\text{ A}$

(ii) Armature current $I_a = I_L - I_f = (20-1)\text{ A} = 19\text{ A}$

The voltage drop at the armature resistance $= 0.24 \times 19\text{ V} = 4.56\text{ V}$

(iii) The back emf $= V_t - I_r r_a$

$$E_b = (240 - 4.56)\text{ V} = (235.44)\text{ V} \simeq 235\text{ V}$$

(iv) Torque × Speed = Power

Electromagnetic power $= E_b I_a = 235 \times 19\text{ W} = 4465\text{ W}$

$$\text{Torque} = \frac{\text{Power}}{\text{Speed}}$$

$$\text{Torque (in N-m)} = \frac{\text{Power in watts}}{\text{Speed in rad/sec}}$$

$$T(\text{N-m}) = \frac{4465}{\frac{1000}{60} \times 2\pi} = 42.64\text{ N-m.}$$

3.6. Derive an expression for the torque-slip characteristic of a three phase induction motor. **[WBUT 2008]**

Answer:
Torque slip curve

The torque equations of the motor is, $T = k_1 \dfrac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$ ....(1)

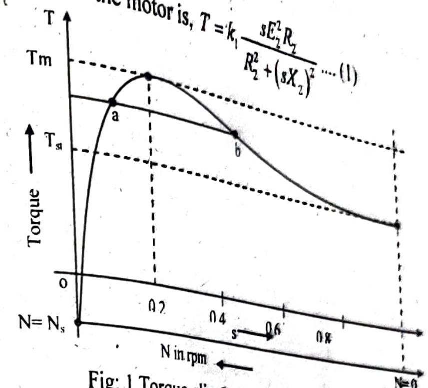


Fig: 1 Torque-slip & Torque-speed characteristics

1. When the motor is running at synchronous speed N_s, slip = 0, hence $T = 0$.
2. When motor speed is nearer to synchronous speed, slip is very small and hence sX_2 can be neglected so.

$$T = \frac{k_1 s E_2^2 R_2}{R_2^2} = \frac{k_1 s E_2^2}{R_2}$$

$\therefore \quad T \propto s$

Since all other terms are constants $T \propto s$ i.e., all low values of slip the torque characteristic is just a straight line.

3. Torque increases as the slip increases and reaches the maximum value when

$s = s_m = \dfrac{R_2}{X_2}$ and value $T_{max} = \dfrac{k_1 E_2^2}{X_2}$ is also called as break down (or) pull out torque.

4. When the speed is very low slip is high (above the maximum slip) $(sX_2)^2 >> R_2^2 \therefore R_2$ can be neglected and so $T = \dfrac{k_1 s E_2^2 R_2}{s^2 X_2^2}$

$$T = \frac{k_1 E_2^2 R_2}{sX_2}; \quad T \propto \frac{1}{s} \text{ (since } E_2, R_2, X_2 \text{ are constant)}$$

Hence after s_m if s increases T decreases. Since s increases when load increases, at higher loads torque decreases and machine slows down and stops.

Torque slips characteristics for various rotor resistances R_{21}, R_{22} and R_{23} are shown in fig.2.

It is found that the maximum torque remains constant even if rotor resistance is varied but the position at which it occurs is varied. By increasing the rotor resistance to R_{24} one can obtain the maximum torque even at the starting. This variation of rotor resistance is possible only for slip ring induction motor.

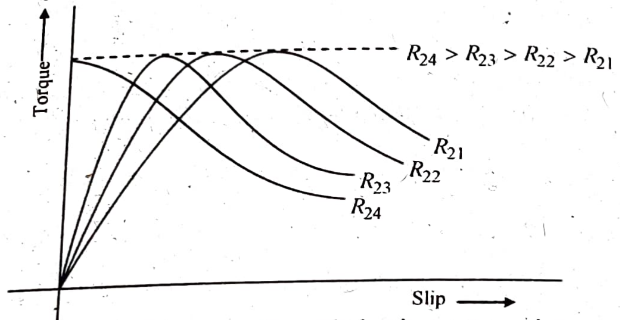


Fig: 2 T vs. speed for variation in rotor r-resistance

b) A 4 pole, 3ph, 275kW, 440V, 50Hz induction motor is running with the slip of 4%
Find
i) Synchronous speed
ii) Rotor speed
iii) Frequency of the rotor induced emf. **[WBUT 2008]**

Answer:

(i) Synchronous speed $N_S = \frac{120 \times f}{P}$ rpm $= \frac{120 \times 50}{4}$ rpm $= 1500$ rpm

(ii) Let the rotor speed be N_r

$$\frac{N_S - N_r}{N_s} \times 100 = 4 \text{ or, } \frac{1500 - N_r}{1500} \times 0.04$$

$$N_r = 1500 - 60 = 1440 \text{rpm}$$

(iii) Frequency of the rotor induced emf $= sf = 0.04 \times 50 = 2$Hz

3.7. The no-load speed of an induction motor is 1500 rpm. When it is connected across a voltage source of frequency 50 cycles/sec, the motor speed is 1200 rpm at full load. Determine,
i) the number of poles ii) slip iii) rotor frequency iv) speed of the rotor field with respect to the rotor v) rotor speed with respect to the stator vi) the speed of the rotor field in the air gap with respect to the stator field. **[WBUT 2010]**

Answer:

a) $N_s = 1500$ rpm ; $N_r = 1200$ rpm

i) $N_s = \frac{120f}{P} = 1500$; $P = \frac{120 \times 50}{1500} = 100 = 20\%$

ii) Slip $= \frac{N_s - N_r}{N_s} \times 100 = \frac{1500 - 1200}{1500} \times 100 = 20\%$

iii) Rotor frequency $= sf = 0.2 \times 50$ Hz $= 10$ Hz

iv) Speed of rotor field = speed of stator field = 1500 rpm.

Speed of rotor field with respect to the rotor $= (1500 - 1200)$ rpm $= 300$ rpm

v) Rotor speed with respect to stator $= (1200 - 0)$ rpm $= 1200$ rpm

vi) The speed of the rotor field in the air-gap w.r.t stator field $= 0$

3.8. a) A 6 pole, 3 ϕ, 50 Hz induction motor develops a full – load torque of 150nm when the rotor current makes 120 complete cycles per minute. Determine the shaft power output. **[WBUT 2011]**

Answer:

Frequency of rotor current $= sf = \frac{120}{60} = 2$; $f = 50$ Hz, $s = \frac{2}{50} = \frac{1}{25} = 0.04$

Rotor speed $= (1 - s) N_s$

$$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm.}$$

$$N_r = (1 - s) N_s = (1 - 0.04) \times 1000 \text{ rpm} = 960 \text{ rpm.}$$

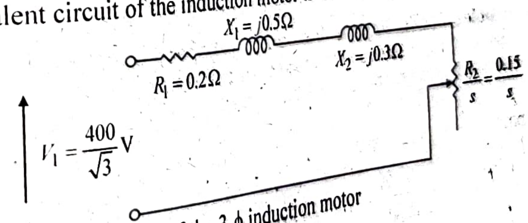
Rotor speed in rad/sec. $= \frac{960 \times 2\pi}{60} = 100.5$ rad/sec.

Shaft power output = Torque × speed = 150 N-m × 100.5 rad/sec.
$= 15{,}080$ W $= 15.08$ kW.

b) A 3 – phase 50 Hz, 4 – pole, 400 volt induction motor has the following data referred to stator side: $R_1 = 0.2\Omega, R_2 = 0.15\Omega, X_1 = j0.5\Omega, X_2 = j0.3\Omega$
The motor operates at 3% slip & the total losses including copper & constant losses are 2000 watt. The core effect may be neglected.
Determine
i) the stator current
ii) p.f
iii) efficiency of the motor. **[WBUT 2011]**

Answer:

The equivalent circuit of the induction motor is drawn



Assuming star connection of the 3-ϕ induction motor

$$V_\phi = \frac{V_{LL}}{\sqrt{3}} = \frac{400}{\sqrt{3}} \text{ V}$$

The stator impedance $= R_1 + \frac{R_2}{s} + j(X_1 + X_2) = 0.2 + \frac{0.15}{0.03} + j(0.5 + 0.3)$

$$=0.2+5+j0.8=5.2+j0.8=\sqrt{(5.2)^2+(0.8)^2}\angle\tan^{-1}\frac{0.8}{5.2}=5.26\angle 8.746^\circ.$$

(i) The stator current $=\dfrac{400\angle 0^\circ}{\sqrt{3}\times 5.26\angle 8.75^\circ}$ A

$=43.9\text{ A}=44\text{ A}$ lagging the voltage by 8.75°

(ii) p.f $=\cos\phi=\cos 8.75^\circ=0.99$ lag

(iii) Stator input $=\sqrt{3}\times 400\times 44\times 0.99=30{,}179$ W

Shaft power output = power input – total losses $=(30179-2000)\text{ W}=28{,}179$ W

Efficiency of the motor $=\dfrac{\text{Shaft output}}{\text{Power input}}\times 100=\dfrac{28{,}179}{30{,}179}\times 100=93.4\%$.

3.9. a) A three-phase induction motor is self starting. Explain. **[WBUT 2012]**

Answer:

In a 3-phase induction motor, a rotating magnetic field is created in the air gap by virtue of three phase balanced voltages being applied to the stator. This rotating magnetic field is capable of generating electromagnetic torque which in turn rotates the shaft in the direction of the torque. So, an induction motor is self-starting.

b) Obtain the relation between the slip and frequency of the rotor induced emf. **[WBUT 2012]**

Answer:

It is known that frequency of the stator circuit (stator voltage and stator current) is the supply frequency and it can be obtained from the equation.

$$N_s=\frac{120f}{P} \quad \text{.... (1)}$$

$$f=\frac{N_sP}{120}$$

The frequency of the rotor circuit f_r is mainly depending on the relative velocity N_s-N_r, and can be obtained from

$$N_s-N_r=\frac{120f_r}{P} \quad \text{.... (2)}$$

$$f_r=\frac{(N_s-N_r)P}{120}$$

From equation 10 and 11

$$\frac{N_s-N_r}{N_s}=\frac{f_r}{f} \quad \text{i.e.,} \quad s=\frac{f_r}{f}$$

so $f_r=sf$ .... (3) where f_r or f is the frequency of rotor circuit.

Note:

At stand still, $s=1$ (i.e.,) at no load $N_r=0, s=1$ and therefore $f_r=f$

When $N_r=N_s, s=0$, and $f_r=0$

c) A 4-pole, 3-phase, 275 kW, 440 V 50 Hz induction motor in running with a slip of 4%. Find-
i) Synchronous speed
ii) rotor speed
iii) frequency of the rotor induced emf. **[WBUT 2012]**

Answer:

(i) Synchronous speed, $N_s=\dfrac{120f}{P}=\dfrac{120\times 50}{4}\text{ rpm}=1500$ rpm.

(ii) Slip$_s$ $=\dfrac{N_s-N_r}{N_s}\times 100$ where N_s is the synchronous speed. N_r the rotor speed.

$$\frac{1500-N_r}{1500}\times 100=4$$

$$N_r=1500-60=1440\text{ rpm.}$$

(iii) Frequency of the rotor induced emf $=sf=0.04\times 50\text{ Hz}=2$ Hz.

3.10. a) Discuss briefly the principle of speed control of a 3-phase induction motor by variation of input voltage frequency.

b) A three phase, 415 V, 50 Hz star connected 4-pole induction motor has stator impedance $Z_1=(0.2+10.5)\Omega$ and rotor impedance referred to stator side is $Z_2=(0.1+10.5)\Omega$ per phase. The magnetizing reactance is 10 Ω and resistance representing core loss is 40Ω on per phase basis. Determine the rotor current at slip of 0.04. **[WBUT 2013]**

Answer:

a) This method provides a wide variation in speed with gradual variation of supply frequency.

If an induction motor is to be operated at different frequencies with practically constant values of efficiency, power factor, over load capacity and a constant absolute slip and with the iron unsaturated, it is necessary to vary the supply voltage V accordingly with the frequency. $\dfrac{V'}{V}=\dfrac{f'}{f}\cdot\sqrt{\dfrac{T'}{T}}$ where V' and T' are the voltage and torque corresponding to the frequency f' and V and T to the frequency f.

For constant torque $\dfrac{V'}{V}=\dfrac{f'}{f}$ $T'=T$, we have $\dfrac{V'}{V}=\dfrac{f'}{f}$ i.e., the voltage applied to the stator must vary in proportion to the frequency.

However, a change of frequency will result in a change of flux unless the induced emf is changed in the same ratio. An imbalance might cause either an excessive flux and saturation or a reduced torque per ampere of current.

The applied voltage $V = E_1$, the induced emf in the state.

The synchronous speed $\left(N_s = \frac{120f}{P}\right)$ is proportion to the frequency for a particular motor. Hence, $\frac{V}{f}$ constant implies that $\frac{E_1}{N_s}$ would also remain constant.

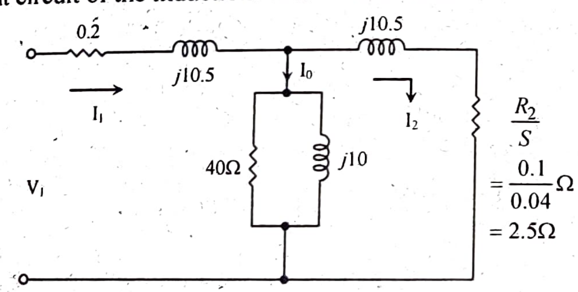
Now, for a given flux level, torque developed is proportional to

$$I_2' \cos\phi_2 \propto \frac{sE_2}{z_2'} \cdot \frac{R_2'}{z_2'} \propto \frac{N_s - N}{N_s} \cdot \frac{E_1 \cdot R_2'}{z_2'^2}$$

So if, $\frac{E_1}{N_s}$ is constant for a particular torque, $(N_s - N)$, the speed difference is also constant.

Hence, if N_s, the synchronous frequency is varied with f, N, the rotor speed varies accordingly.

b) The equivalent circuit of the induction motor is drawn below.



The equivalent impedance of the magnetizing and core loss pair

$$= \frac{j10 \times 40}{j10 + 40} = \frac{400\angle 90^\circ}{41.23\angle 14.03^\circ}\Omega = 9.7\angle 76^\circ\Omega = 2.35 + j9.4\Omega$$

The equivalent impedance of magnetizing pair and rotor impedance referred to stator side.

$$= \frac{(2.35 + j9.4)(2.5 + j10.5)}{2.35 + j9.4 + 2.5 + j10.5}\Omega = \frac{9.7\angle 76^\circ \times 10.79\angle 77^\circ}{2.85 + j19.9}\Omega$$

$$= \frac{104.7\angle 153^\circ}{20.1\angle 81.85^\circ}\Omega = 5.21\angle 71{,}15^\circ\Omega = 1.68 + j4.93\Omega$$

The total per phase impedance $= (1.68 + 0.2) + j(10.5 + 4.93)\Omega$

$$= (1.88 + j15.43)\Omega = 15.54\angle 83.05^\circ$$

The per phase current or line current I_1

$$= \frac{415}{\sqrt{3} \times 15.54\angle 83.05^\circ} = 15.42\angle -83.05^\circ \text{ A} = (1.87 - j15.3)\text{ A}$$

From the equivalent circuit diagram, the rotor current per phase (I_2)
= The stator current per phase (I_1) – the magnetizing current (I_0)
I_2 can be found using current division method.

$$I_2 = I_1 \times \frac{\text{Impedance of magnetising branch}}{\text{total impedance of parallel branches}}$$

$$= \frac{15.42\angle -83.05^\circ \times 9.7\angle 76^\circ}{5.21\angle 71.15^\circ}\text{ A} = 28.7\angle -78.2^\circ \text{ A}$$

3.11. a) The power input to a 400 V, 6 poles, 50 Hz, 3-ϕ induction motor running at 975 rpm is 40 KW. The Stator losses are 1 KW and Friction and windage losses are 2 KW. Find the efficiency of the motor. **[WBUT 2014]**

Answer:

The synchronous speed of an induction motor

$$= \frac{120f}{P}\text{ rpm} = \frac{120 \times 50}{6}\text{ rpm} = 1000\text{ rpm}.$$

$$\text{Slip } s = \frac{N_s - N_r}{N_s} \times 100 = \frac{100 - 975}{1000} \times 100 = \frac{2500}{1000} = 2.5\% = 0.025$$

The power input to the induction motor $= 40$ kW

Air-gap power = Power Input – Stator Losses

$$P_g = (40 - 1)\text{ kW} = 39\text{ kW}$$

Rotor copper losses $= sP_g = 39 \times 0.025 = 0.975\text{ kW} = 975\text{ W}$

Efficiency of the motor

$$\eta = \frac{\text{Input} - \text{losses}}{\text{Input}} \times 100 = \frac{40 - 1 - 0.975 - 2}{40} \times 100 = \frac{40 - 3.975}{40} \times 100 = \frac{36.025}{40} \times 100$$

$$= 90.06\% = 90.1\%.$$

b) A three phase 4kW, 400V, 50Hz, induction motor is working at full load with an efficiency of 90% at a power factor of 0.8 lagging. Calculate (i) the input power (ii) the line current. **[WBUT 2014, 2015]**

Answer:

Given data: Full load output power = 4kW

Full load efficiency = 90%

p.f = 0.8

Line voltage = 400 V, Frequency = 50 Hz

Input power during full-load condition $= \frac{\text{output}}{\text{afficiency}} = \frac{4 \times 10^3}{0.9} W = 4444W$

Input three phase power $= \sqrt{3} V_{LL} I_{LL} \cos\phi$

Input three phase power $= \sqrt{3} V_{LL} I_{LL} \cos\phi = 4444W$

Where I_{LL} is the line current $= \sqrt{3} V_{LL} I_{LL} \cos\phi = 4444W$

$$I_L = \frac{4444}{\sqrt{3}\times 400\times 0.8}A = 8.02A$$

3.12. Explain the principle of operation of a 3 phase induction motor. Draw the equivalent circuit and phasor diagram of the motor. **[WBUT 2017]**

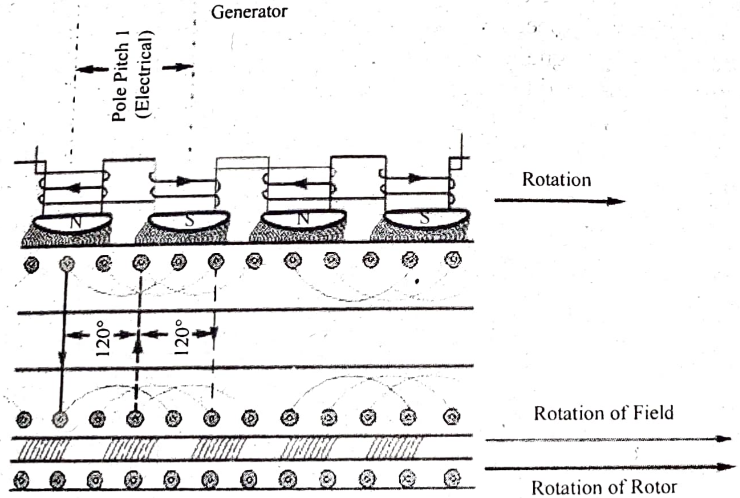
Answer:

1st Part:

An **electric motor** converts electrical energy into a mechanical energy which is then supplied to different types of loads. 3 phase induction motors are most widely used for industrial applications mainly because they do not require a starting device.

A 3 phase induction motor derives its name from the fact that the rotor current is induced by the magnetic field, instead of electrical connections.

The operating principle of a 3 phase induction motor is based on the production of r.m.f.



Production of a rotating magnetic field

The **stator** of an induction motor consists of a number of overlapping windings offset by an electrical angle of 120°. When the primary winding or stator is connected to a three phase alternating current supply, it establishes a rotating magnetic field which rotates at a synchronous speed.

The direction of rotation of the motor depends on the phase sequence of supply lines, and the order in which these lines are connected to the stator. Thus interchanging the connection of any two primary terminals to the supply will reverse the direction of rotation.

The number of poles and the frequency of the applied voltage determine the synchronous speed of rotation in the motor's stator. Motors are commonly configured to have 2, 4, 6 or 8 poles. The synchronous speed, a term given to the speed at which the field produced by primary currents will rotate, is determined by the following expression.

Synchronous speed of rotation = (120* supply frequency) / Number of poles on the stator

Production of magnetic flux

A rotating magnetic field in the stator is the first part of operation. To produce a torque and thus rotate, the rotors must be carrying some current. In induction motors, this current comes from the rotor conductors. The revolving magnetic field produced in the stator cuts across the conductive bars of the rotor and induces an e.m.f.

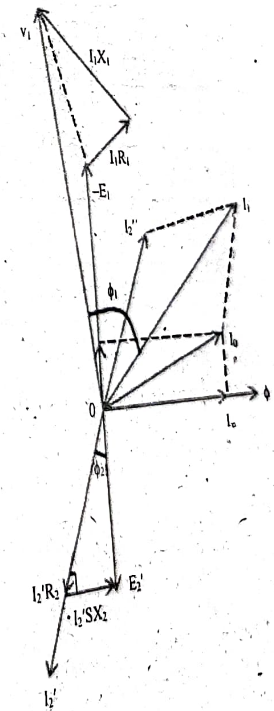
The rotor windings in an induction motor are either closed through an external resistance or directly shorted. Therefore, the e.m.f induced in the rotor causes current to flow in a direction opposite to that of the revolving magnetic field in the stator, and leads to a twisting motion or torque in the rotor.

As a consequence, the rotor speed will not reach the synchronous speed of the r.m.f in the stator. If the speeds match, there would be no e.m.f. induced in the rotor, no current would be flowing, and therefore no torque would be generated. The difference between the stator (synchronous speed) and rotor speeds is called the slip.

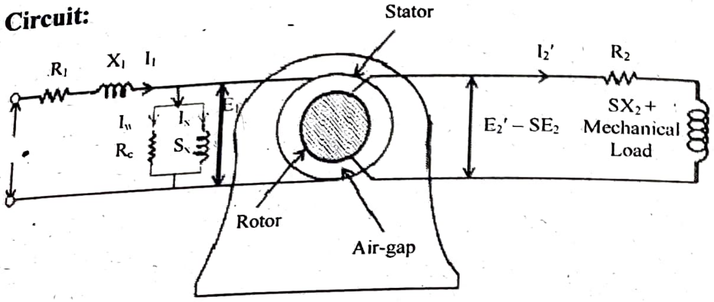
The rotation of the magnetic field in an induction motor has the advantage that no electrical connections need to be made to the rotor.

2nd Part:

Phasor Diagram



Equivalent Circuit:



3.13. Write short notes on the following:

a) Speed control of dc motor by armature voltage control **[WBUT 2016]**

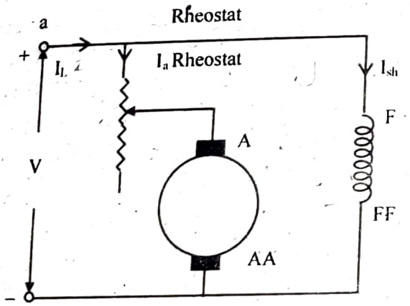
b) 3-point starter **[WBUT 2016]**

Answer:

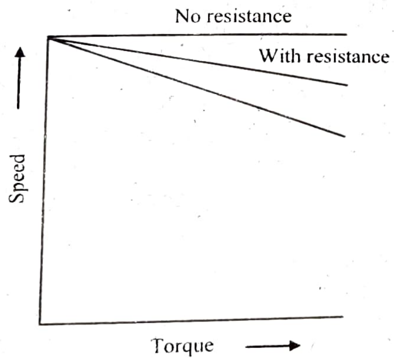
a) Speed control of dc motor by armature voltage control:

Armature Control method:

In this method the field current is kept constant and the armature current is varied by using



Armature resistance control method for DC shunt motor



Speed- torque characteristics of DC shunt motor with armature control

It is know, $N \alpha \dfrac{V - I_a R_a}{\phi}$

Here, $N \ \alpha \ V - I_a R_a$

As the armature circuit resistance i.e. $(R_a + R_{rhe})$ is increased from minimum value, the armature drop will increase, which will decrease the term $V - I_a R_a$ and finally the speed will be decreased. Hence using this method speed below the rated speed can be obtained. Since it increases the armature circuit resistance, the armature copper loss increases which will reduce the efficiency of the machine.

b) Three Point D.C. Motor Starter:

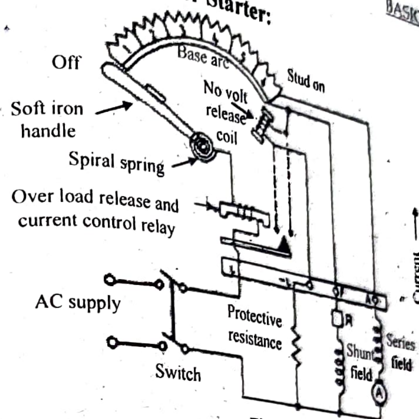


Fig: 1

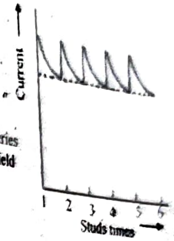


Fig 2

It essentially consists of a rheostatic resistance, divided into several sections through Studs (fig. 1). To start the motor, the D.C. supply is switched on and the starter handle is swiftly moved from the 'OFF' position to "ON" position at stud No 1, so that the total resistance of the rheostat comes in series with the armature circuit and the current in armature is low. As the motor picks up speed, the armature develops back emf, and consequently, the current in it falls. Hence the handle is moved over in steps to reduce the series resistance. At each step, the current in armature falls, and then rises as shown in fig (2) above. Finally, when the handle is in the 'ON' position, there is no resistance in series, and the armature is connected directly to the supply line. In addition to rheostatic resistances, following to protective devices are provided:

i) No Volt Coil:

a) When the handle is in position of 'ON' (i.e. last contact), the position is maintained by the electromagnet of the no-volt coil against the action of the coiled spring, which tries to pull the handle towards the 'OFF' position.

b) If by chance the supply fails, when the motor is running, the electromagnet of the no volt coil loses energy (to attract the handle towards itself) and consequently, the handle is pulled by the coiled spring of the starter and goes back to the 'OFF position.

ii) Overload release coil:

It provides protection to the motor against any excessive flow of current, when the motor is over loaded. When the motor is over loaded, it draws more current than the rated value. However, this current flows through the overload release coil, and magnetic it to such an extent that it pulls the lever of the current adjuster relay upwards, thereby no volt release coil is short-circuited. Under such circumstances, the no volt release coil gets demagnetized, there by releasing the handle from 'ON' position, which then returns to 'OFF' position by the action of the coiled spring. Thus the motor is automatically disconnected from the supply lines when over loaded.

3.14. a) Develop the equivalent circuit of a 1-phase induction motor with two revolving field theory. **[MODEL QUESTION]**

Answer:

The equivalent circuit of a single-phase induction motor can be developed on the basis of two revolving field theory. For developing the equivalent circuit it is first imperative to consider stand still or blocked rotor conditions. Under blocked rotor condition, it will act like a transformer with its secondary short-circuited and its equivalent circuit will be as shown in Fig. (a) below, E_m being emf induced in the stator.

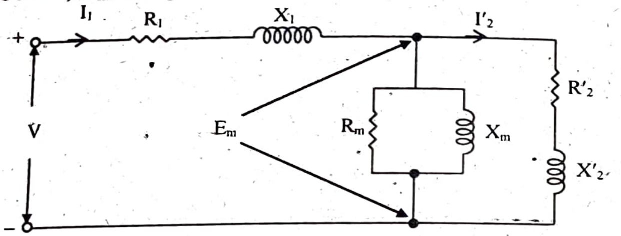


Fig: (a) Equivalent circuit of a 1ϕ induction motor at stand still

- Fig. (b) as shown below is the equivalent circuit of a single-phase induction motor at stand-still on the basis of two revolving field theory. The phasor sum of E_{mf} and E_{mb} equals the applied voltage V (less the voltage drops is stator resistance R_1 and leakage reactance X_1).

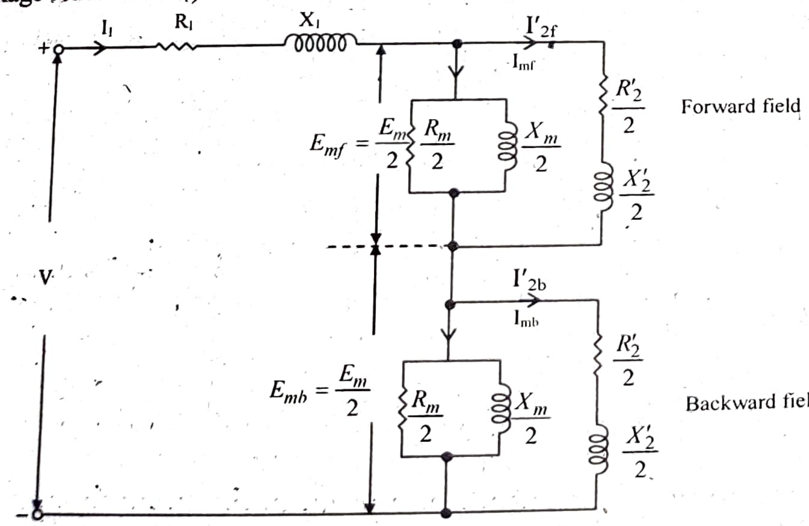


Fig: (b) Equivalent circuit of 1ϕ induction motor at standstill on the basis of two revolving field theory

When the rotor runs at speed N with respect to forward field, the slip is 's' with respect to forward field and (2 – s) w.r.t backward field and the equivalent circuit becomes as shown in Fig. (c) below:

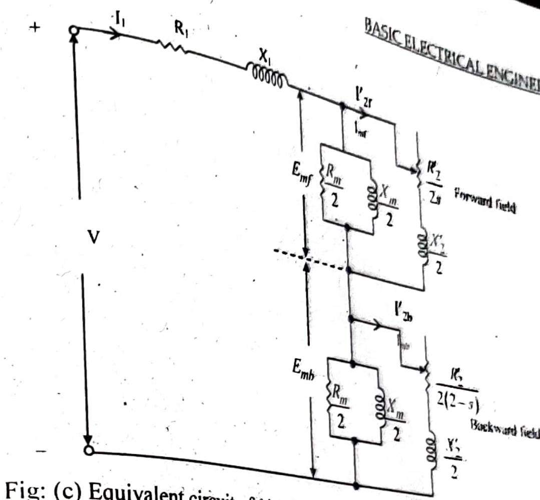


Fig: (c) Equivalent circuit of 1ϕ induction motor under normal operating condition

- If the core losses are neglected the equivalent circuit is modified as shown in Fig. (d) below. The core losses, here, are handled as rotational losses and subtracted from the power converted into mechanical power, the amount of error thus introduced is relatively small.

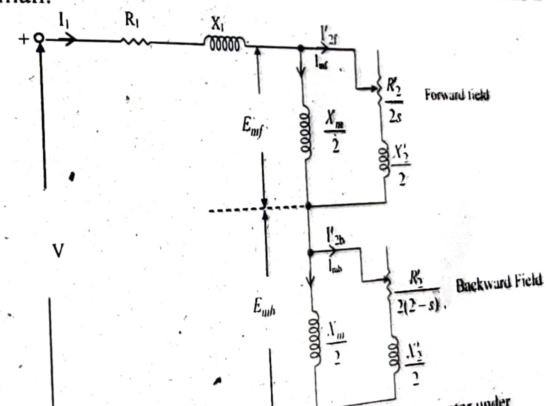


Fig: (d) Approximate equivalent circuit of 1ϕ induction motor under normal operating condition

b) The resistance and total inductance of a single-phase fractional horse power series motor are 30 Ω and 0.5 H respectively. It draws 0.8 A current and runs at 2000 r.p.m. when connected to a 250 V d.c. supply. Calculate the speed and power factor when connected to a 250 V, 50 Hz supply and takes the same load current. How much voltage is required for getting 2000 r.p.m. with a.c. supply? Assume resistance and reactance remains constant. **[MODEL QUESTION]**

Answer:

$R_a = 30\Omega \quad L = 0.5\text{H} \quad I_a = 0.8\text{A}$

$E_{b\text{-}ac} = ?\ E_{b\text{-}dc} = ?\text{V} \quad V = 250\text{V}$

For A.C. operation

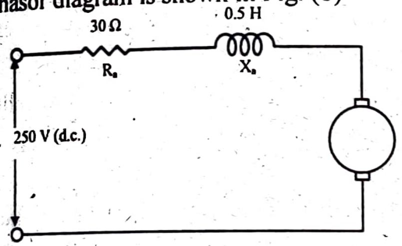
$X_a = 2\pi \times 50 \times 0.5 = 157\Omega$

$R_a = 30\Omega$

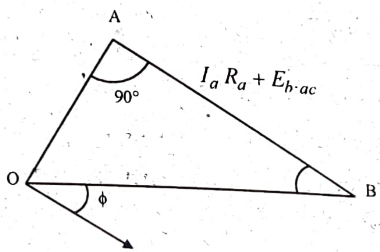
$I_a R_a = 0.8 \times 30 = 24\text{V}$

$I_a X_a = 0.8 \times 157 = 125.6\text{V}$

The phasor diagram is shown in Fig. (b)



(a)



(b)

From phasor diagram, we get

$$V^2 = \left(E_{b\text{-}ac} + I_a R_a\right)^2 + \left(I_a X_a\right)^2$$

or, $$250^2 = \left(E_{b\text{-}ac} + 24\right)^2 + (125.6)^2$$

or, $$\left(E_{b\text{-}ac} + 24\right)^2 = 250^2 - 125.6^2$$

or, $$E_{b\text{-}ac} + 24 = 216.1588$$

or, $$E_{bac} = 216.1588 - 24 = 192.1588\text{ V}$$

In case of D.C. operation

$$E_{b\text{-}dc} = V - I_a R_a$$

$$E_{b\text{-}dc} = 250 - (0.8 \times 30)$$

$$= 250 - 24 = 226\text{V}.$$

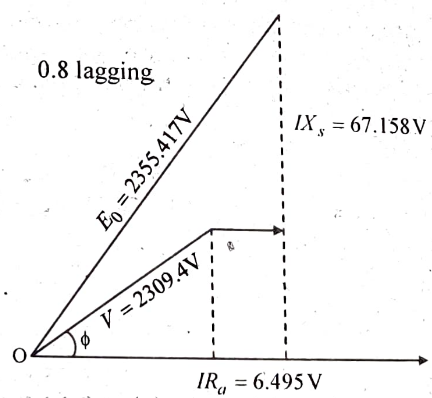
Now, $$\frac{E_{b\text{-}ac}}{E_{b\text{-}dc}} = \frac{N_{ac}}{N_{dc}}$$

or, $$\frac{192.1588}{226} = \frac{N_{ac}}{2000}$$

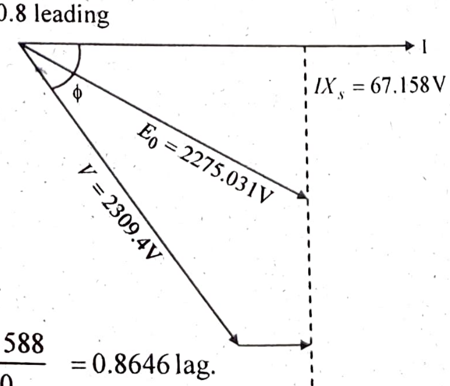
or, $$N_{ac} = \frac{2000 \times 192.1588}{226} = 1700.52\text{ r.p.m.}$$

$$\cos\phi = \frac{E_{b\text{-}ac} + I_a R_a}{V} = \frac{192.1588 + 24}{250} = \frac{216.1588}{250} = 0.8646\text{ lag.}$$

0.8 lagging



0.8 leading



3.15. a) Define voltage regulation of an alternator. Is it possible to have the full-load terminal voltage greater than the no-load terminal voltage? Explain. **[MODEL QUESTION]**

Answer:

When an alternator is subjected to a varying load, the voltage at the armature terminals varies to a certain extent, the amount of this variation determines the regulation of the machines. The numerical value of regulation is defined as the percentage rise in voltage when full load at the specified power factor is switched off, the excitation being adjusted initially to give normal voltage. Thus, % Regulation $= \frac{E_0 - V}{V} \times 100$ where E_0 = no load emf. This being the voltage induced in armature in the absence of IR_a, IX_L and IX_a. Hence it represents the maximum value of induced emf. V = terminal voltage. It is vectorically less then E, the induced emf after allowing armature reaction, by IZ where $Z = \sqrt{R^2 + X_L^2}$ and I, the armature current per phase. The voltage regulation depends upon the power factor of the load. When lagging p.f. load is thrown off the terminal voltage rise will be greater than the no load terminal voltage.

b) A 3-ϕ star connected alternator is rated 1600 KVA, 13500 V. The armature effective resistance and synchronous reactance are 1.5Ω and 30Ω respectively per phase. Calculate the percentage regulation for a load of 1280kW at a p.f. of (i) 0.8 leading, (ii) unity and (iii) 0.8 lagging. **[MODEL QUESTION]**

Answer:

$$P_{3\phi} = \sqrt{3} V_L I_L \cos\phi$$

$$1280 \times 10^3 = \sqrt{3} \times 13{,}500 I_L \times 0.8$$

$$I_L = \frac{1280 \times 10^3}{\sqrt{3} \times 13{,}500 \times 0.8} = 68.43A = I_a$$

(i) $\cos\phi = 0.8,\ \sin\phi = 0.6$

$$R_a = 1.5\Omega\ X_s = 30\Omega\ V_p = \frac{13500}{\sqrt{3}} = 7794.5\text{V}$$

For leading power factor,

$$E_p^2 = \left(V_p \cos\phi + I_a R_a\right)^2 + \left(-V_p \sin\phi + I_a X_s\right)^2$$

$$= (7794.5 \times 0.8 + 68.43 \times 1.5)^2 + (-7794.5 \times 0.6 + 68.43 \times 30)^2$$

$$= (6338)^2 + (-2623.8)^2$$

$$E_p = 6859.6\text{ V}$$

Voltage regulation $= \frac{E_p - V_p}{V_p} \times 100 = \frac{6859.6 - 7794.5}{7794.5} \times 100 = -11.99\%.$

(ii) Unity power factor:

$\cos\phi = 1$ $\quad\quad$ $\sin\phi = 0$

$$P_{3\phi} = \sqrt{3}V_L I_L \cos\phi$$

$$1280\times10^3 = \sqrt{3}\times13{,}500\, I_L \times 1$$

$$I_L = \frac{1280\times10^3}{\sqrt{3}\times13{,}500} = 54.74A = I_a$$

$$E_p^2 = (V_p + I_a R_a)^2 + (I_a X_s)^2 = (7794.5 + 54.74\times1.5)^2 + (54.74\times30)^2$$

$$= (7876.6)^2 + (1642.2)^2$$

$$E_p = 8046\text{ V.}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p}\times100 = \frac{8046-7794.5}{7794.5}\times100 = 3.227\%.$$

(iii) p.f. at 0.8 lagging:

$$E_p^{\,2} = (V_p\cos\phi + I_a R_a)^2 + (V_p\sin\phi + I_a X_s)^2$$

$$= (7794.5\times0.8 + 68.43\times1.5)^2 + (7794.5\times0.6 + 68.43\times30)^2$$

$$= (6338)^2 + (6729.6)^2$$

$$E_p = 9244.4\text{ A.}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p}\times100 = \frac{9244.4-7794.5}{7794.5}\times100 = 18.6\%.$$

3.16. a) Draw & explain the phasor diagram of salient pole alternator supplying full-load lagging power current. Show that the power output per phase is given by

$$P = \frac{EV}{X_d}\sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\sin 2\delta.$$

[MODEL QUESTION]

Answer:

1st Part: Two reaction theory proposes to resolve the given armature mmfs into two mutually perpendicular components, one located along the axis of the rotor salient pole, called direct-axis (or d-axis) component and the other located perpendicular to the axis of rotor salient pole which is known as the quadrature axis (or q axis) component.

The d-axis component of the armature mmf F_a is denoted by F_d which is either magnetizing or demagnetizing and q axis component F_q which results in a cross-magnetizing effect.

If ψ is the angle between the armature current I_a and the excitation voltage E_f and F_a is the amplitude of the armature mmf then

$$F_d = F_a\sin\psi \quad ; \quad F_q = F_a\cos\psi$$

If ϕ_d is the direct axis flux and ϕ_q is the quadrature axis flux, then assuming reluctance of direct axis flux path, it may be written as:-

$$\phi_d = \frac{F_d}{R_d} \quad \text{and} \quad \phi_q = \frac{F_q}{R_q}$$

It may be noted that:

i) As $R_d < R_q$, the direct axis component of mmf F_d produces more flux than the Quadrature axis component of mmf F_q.

ii) The direct and quadrature axis stator fluxes produce voltages in the stator winding by armature reaction.

The complete phasor diagram of a salient-pole synchronous generator based on two axis theory is shown in Fig. (1).

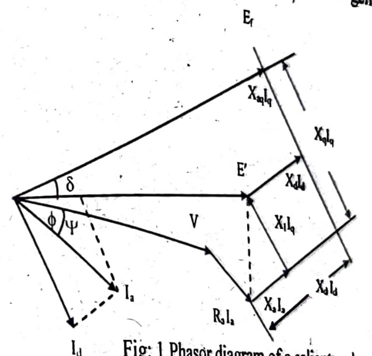


Fig: 1 Phasor diagram of a salient-pole synchronous generator at lagging power factor

The simplified phasor diagram is shown in Fig. 2.

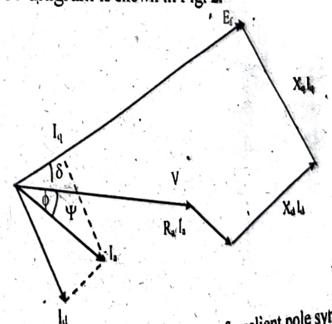


Fig: 2 Simplified phasor diagram of a salient pole synchronous generator at lagging power factor

In the phasor diagram of Fig. 2 the angle $\Psi = \phi + \delta$, is not known for given values of V, I_a and ϕ. The components I_d and I_q of the armature current are usually not given. These component current depend upon δ, the torque angle and from the phasor diagram,

$|E_f| = V\cos\delta + R_a I_q + X_d I_d$ since $\psi = \delta + \phi$ and $\delta = \psi - \phi$. ϕ is negative for leading power factor since ϕ is taken positive for lagging p.f.

2nd Part:
The resistance R_a of the armature has negligible effect on the relationship between the power output of a synchronous machine and its torque-angle δ. It may, therefore, be neglected. The phasor diagram at lagging *pf* for a salient-pole synchronous generator, neglecting R_a, is shown in the figure (A).

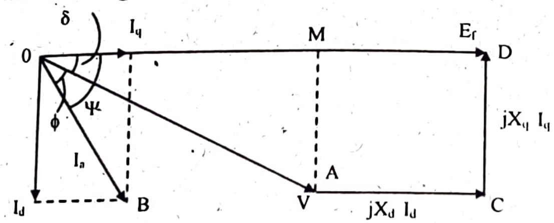


Fig: (A) Phasor diagram at lagging pf of a salient-pole generator, neglecting R_a

From the phasor diagram of the figure (A).
The complex power output per phase

$$S_{1\phi} = V I_a \qquad \text{.... (1)}$$

Taking E_f as the reference phasor.

$$\therefore \quad V = V\angle -\delta = V\cos\delta - jV\sin\delta$$

$$I_a = I_q - jI_d$$

$$I_a^* = I_q + jI_d$$

$$\therefore \quad S_{1\phi} = VI_a^* = (V\cos\delta - jV\sin\delta)(I_q + jI_d) \qquad \text{.... (2)}$$

From the phasor diagram of figure (A)

$$X_q I_q = CD = AM = V\sin\delta$$

$$\therefore \quad I_q = \frac{V\sin\delta}{X_q} \qquad \text{.... (3)}$$

$$X_d I_d = AC = MD = OD - OM = E_f - V\cos\delta$$

$$\therefore \quad I_d = \frac{E_f - V\cos\delta}{X_d} \qquad \text{.... (4)}$$

Substituting the values of I_q and I_d in Eqn. (2) we get

$$S_{1\phi} = (V\cos\delta - jV\sin\delta)\left(\frac{V\sin\delta}{X_q} + j\frac{E_f - V\cos\delta}{X_d}\right)$$

$$= \left(\frac{V^2}{X_q}\sin\delta\cos\delta + \frac{VE_f}{X_d}\sin\delta - \frac{V^2}{X_d}\sin\delta\cos\delta\right) + j\left(\frac{VE_f}{X_d}\cos\delta - \frac{V^2}{X_d}\cos^2\delta - \frac{V^2}{X_q}\sin^2\delta\right)$$

$$= \left[\frac{VE_f}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right] + j\left[\frac{VE_f}{X_d}\cos\delta - \frac{V^2}{2X_d}(1+\cos 2\delta) - \frac{V^2}{2X_q}(1-\cos 2\delta)\right]$$

$$= \left[\frac{VE_f}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right] + j\left[\frac{VE_f}{X_d}\cos\delta - \frac{V^2}{2X_dX_q}\left\{(X_d+X_q) - (X_d-X_q)\cos 2\delta\right\}\right]$$

Also, $S_{1\phi} = P_{1\phi} + jQ_{1\phi}$ .... (5)

Therefore the real power per phase in watts is

$$P_{1\phi} = \frac{VE_f}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta \qquad \text{.... (6)}$$

POWER CONVERTERS

Chapter at a Glance

In recent years power electronic devices have great importance due to increase in production, efficiency and control in Industrial application. Power electronics devices applied in electrical circuits are also called power converter which signifies to convert electrical power at high frequency through a matrix of power semiconductor switch. The converter system consists of switch (mainly diode or thyristor or transistor etc) reactive component L, C and transformer. This chapter describes basically various types of chopper circuit, inverter circuit which are useful in our daily appliances.

Multiple Choice Type Questions

1.1 In the dc chopper, for chopping period of the output voltage can be controlled by frequency modulation method by varying [MODEL QUESTION]

a) T and keeping T_{on} constant
b) T and keeping T_{off} constant
c) T_{on} and keeping T constant
d) both (a) and (b)

Answer: (d)

1.2. A chopper has source voltage V_s and load resistance R. If α is the duty cycle, then rms value of load voltage [MODEL QUESTION]

a) $\alpha . V_s$
b) $\alpha^{1/2} . V_s$
c) $V_s / \alpha^{1/2}$
d) none of these

Answer: (b)

1.3. In a dc chopper per unit ripple is maximum, when the duty cycle is [MODEL QUESTION]

a) 0
b) 0.5
c) 1
d) none of these

Answer: (b)

1.4. A step up chopper has V_s as source voltage and α as duty cycle, the output voltage for this chopper is given by [MODEL QUESTION]

a) $V_s.(1+\alpha)$
b) $V_s.(1-\alpha)$
c) $V_s/(1+\alpha)$
d) $V_s/(1-\alpha)$

Answer: (d)

1.5. Chopper control of a dc motor provides variation on [MODEL QUESTION]

a) frequency
b) current
c) Input voltage
d) All of these

Answer: (d)

1.6. In a buck converter as the duty cycle increases, the output voltage [MODEL QUESTION]

a) increases
b) decrease
c) increases at no load and decreases at full load
d) decreases at no load and increases at full load

Answer: (a)

1.7. In a voltage source inverters, [MODEL QUESTION]

a) Load voltage waveform Vo depends on load impedance Z but load current Io does not depends on Z
b) Both Vo and Io depends on Z
c) Vo does not depend on Z but Io depends on Z
d) None of these

Answer: (c)

1.8. A single phase full bridge VSI has inductor L as the load. For a constant source voltage, the current through the inductor is [MODEL QUESTION]

a) Square wave
b) triangular wave
c) sine wave
d) pulsated wave

Answer: (b)

1.9. The advantage of 180° conduction mode of three phase inverter circuit over 120° conduction mode is [MODEL QUESTION]

a) It needs less number of switches
b) there is no paralleling of switches
c) devices in series are not simultaneously switches
d) load terminals are not left open during switching

Answer: (d)

1.10. In a single phase voltage source inverter, the shape of the load current depends on the [MODEL QUESTION]

a) Source voltage
b) duration of conduction of SCRs
c) load impedance
d) duration of conduction of feedback diodes

Answer: (c)

Short Answer Type Questions

2.1. a) What do you mean by chopper? [MODEL QUESTION]
b) Classify different power converter circuit and their application?
c) What is duty ratio?

Answer:

a) A chopper converts fixed dc input voltage to a variable dc output voltage. The dc output voltage may be different in amplitude than the source input voltage. The circuits are designed using semiconductor devices such as power transistors, IGBT, power MOSFET, thyristor and GTOs.

b) Power converter circuit basically converts the electrical power at high frequency through a matrix of power semiconductor switch. It can be classified into five groups.
(i) AC-DC converter. (Phase controlled rectifier): This converter converts fixed ac voltage/current to a variable low ripple dc voltage / current. It is applied as battery charger circuit, DC motor drive, regulated DC power supplies etc.
(ii) Inverter (DC to AC converter): An inverter converts a fixed dc voltage to an ac voltage of variable frequency and of fixed or variable magnitude. It is widely used in photography camera, Uninterrupted power supply (UPS), Induction and synchronous motor drives, High voltage DC transmission system etc.

(iii) **Cycloconverter (AC to AC converter):** This circuit converts input power at one frequency to output power at a different frequency. It is applied ac motor drives etc.
(iv) **AC Voltage Controller (AC Regulator):** This converter converts fixed ac voltage directly to a variable ac voltage at the same frequency. It is widely used in lighting control, speed control of large fans and pumps etc.
(v) **Choppers (DC to DC converter):** A chopper converts fixed dc input voltage to a variable dc output voltage. It is used mainly electric traction, switch mode power supply (SMPS), battery driven vehicles etc.

c) The total time period of one cycle of output waveform of a chopper circuit is constant. The average output voltage is directly proportional to the on time of the chopper. The ratio of ON time to total time is defined as duty cycle.

The duty cycle $=\left(\frac{T_{ON}}{T}\right)=\left(\frac{T_{ON}}{T_{ON}+T_{OFF}}\right)$ where T_{ON} is the ON time and T is the total time.

2.2. a) What is an inverter? **[MODEL QUESTION]**
b) Classify the different inverter circuit?

Answer:
a) An inverter converts a fixed dc voltage to an ac voltage of variable frequency and of fixed or variable magnitude.

b) Depending upon:
(i) The number of phases inverter are two types:
(a) Single phase inverter
(b) Three phase inverter
(ii) The circuit configuration
(a) Half bridge
(b) Full bridge
(iii) Waveform of output voltage
(a) square wave inverter
(b) quasi square wave inverter
(c) sine wave inverter
(iv) Types of source
(a) Voltage source inverter (VSI)
(b) Current source inverter (CSI)
(v) Method of switching and control
(a) switched mode PWM
(b) resonant inverter
(vi) Method of commutation of conducting devices
(a) Line commutated thyristor Inverter
(b) Forced commutated thyristor Inverter

2.3. A chopper circuit is operating on time ratio control (TRC) principal at a frequency of 2KHz on a 220V d.c supply. If the load voltage is 170 V, compute the conduction and blocking period of thyristor in each cycle. **[MODEL QUESTION]**

Answer:

We know that, $V_0=V_S.T_{ON}.\frac{1}{T}=V_S.T_{ON}.f$

$f=2\text{ kHz},\ V_S=220\text{ V},\ V_0=170\text{ V}$

Conduction period, $T_{ON}=\frac{V_0}{V_S f}=\frac{170}{220\times2\times10^3}=0.386\text{ ms}$

Chopping period, $T=\frac{1}{f}=\frac{1}{2\times10^3}=0.5\text{ms}$

Blocking period, $T_{OFF}=T-T_{ON}=0.5-0.386=0.114\text{ ms}$

2.4. In a 110V dc chopper drive using current limit control scheme, the maximum possible value of the accelerating current is 300A, the lower limit current pulsation is 140 A. The ON and OFF periods are 15ms and 12ms. Calculate the limit of current pulsation, chopping frequency, duty cycle and output voltage. **[MODEL QUESTION]**

Answer:
Given, $T_{ON}=15\text{ ms},\ T_{OFF}=12\text{ms},\ I_{max}=300\text{ A},\ I_{min}=140\text{ A}$
Maximum limit of current pulsation $=300-140=160\text{ A}$

Chopping frequency $=\frac{1}{T}=\frac{1}{15+12}=37\text{ Hz}$, duty ratio$=\frac{T_{ON}}{T}=\frac{15}{27}=0.56$

Output voltage, $V_0=V_S.T_{ON}.\frac{1}{T}=110\times0.56=61.60\text{ V}$

2.5. A boost converter is used to deliver a load voltage of 500V from a 220V dc supply. If the blocking period of the thyristor is 80μs. Compute the required pulse width. **[MODEL QUESTION]**

Answer:

We know that for boost converter, $V_0=V_S\left(\frac{T}{T_{OFF}}\right)$

$500=220\frac{T_{ON}+80\times10^{-6}}{80\times10^{-6}}$

Equating we get, $T_{ON}=101.6\times10^{-6}=101.6\,\mu\text{S}$

2.6. A buck converter has a resistive load of 15ohm and input voltage 200V dc supply. When the chopper remains ON its voltage drop is 2.5V. The chopper frequency is 1 kHz. If the duty cycle is 50% find (i) average output voltage (ii) RMS output voltage (iii) converter efficiency (iv) effective input resistance of the converter. **[MODEL QUESTION]**

Answer:
(i) Average output voltage, $V_0=K(V_S-V_d)=0.5(200-2.5)=98.75\text{ V}$

(ii) RMS output voltage $=(V_S-V_d)\sqrt{K}=(200-2.5)\sqrt{0.5}=139.653$

(iii) Converter efficiency:

Output Power, P_o

$$(V_0.I_0)_{rms}=\sqrt{K}V_S.\frac{\sqrt{K}V_S}{R}=\frac{KV_S^2}{R}=\frac{K(V_S-V_d)^2}{R}=\frac{0.5(200-2.5)^2}{15}=1300.21 \text{ watt}$$

Input Power, P_i

$$\frac{1}{T}\int_0^T V_S.i_S.dt=\frac{1}{T}\int_0^{T_{ON}} V_S.\frac{(V_S-V_d)}{R}dt=\frac{V_S.(V_S-V_d)}{TR}\int_0^{T_{ON}} dt$$

$$\frac{V_S.(V_S-V_d)}{TR}|t|_0^{T_{ON}}=\frac{V_S.(V_S-V_d)}{TR}(T_{ON}-0)=\frac{V_S.(V_S-V_d)}{TR}KT$$

$$\frac{V_S.(V_S-V_d)K}{R}=\frac{200(200-2.5)0.5}{15}=1316.67 \text{ watt}$$

$$\text{Efficiency}=\eta=\frac{P_o}{P_i}=\frac{1300.21}{1316.67}=0.9874=98.74\%$$

2.7. A single phase half bridge inverter has a resistive load of 10 ohm and the center tap dc input voltage is 96V. Compute
(i) RMS value of the output voltage.
(ii) RMS value of fundamental component of the output voltage waveform.
(iii) First five harmonics
(iv) Fundamental power consumed by the load.
(v) RMS power consumed by the load. **[MODEL QUESTION]**

Answer:

(i) RMS value of the output voltage, $V_o(rms)=\frac{E_{dc}}{2}=96$ V

(ii) RMS value of fundamental frequency component of output voltage

$$V_{o,1}(rms)=\frac{\sqrt{2}E_{dc}}{\pi}=0.9\times96=86.40 \text{ V}$$

(iii) First five harmonics are given by,

Third harmonics $=\frac{V_{o,1}(rms)}{3}=\frac{56.4}{3}=28.8$ V

Fifth harmonics $=\frac{V_{o,1}(rms)}{5}=\frac{86.4}{5}=17.28$ V

Seventh harmonics $=\frac{V_{o,1}(rms)}{7}=\frac{86.4}{7}=12.34$ V

Ninth harmonics $=\frac{V_{o,1}(rms)}{9}=\frac{86.4}{9}=9.6$ V

Eleventh harmonics $=\frac{V_{o,1}(rms)}{11}=\frac{86.4}{11}=7.85$ V

(iv) Fundamental Power, $P_0(fund)=\frac{V_{o,1}(rms)}{R}=\frac{(86.0)^2}{10}=746.5$ watt

(v) RMS power, $P_0(rms)=\frac{V_o(rms)^2}{R}=\frac{(96)^2}{10}=921.6$ watt

2.8. A three phase bridge inverter is fed from 500V DC source. The inverter is operated in 180° conduction mode and it is supplying a purely resistive, star connected load with R=15 ohm per phase. Determine the
a) RMS value of phase voltage
b) RMS value of line voltage
c) RMS value of load current
d) RMS and average value of switch current
e) Power delivered to the load and
f) The average source current

Answer: **[MODEL QUESTION]**

a) RMS value of phase voltage, $V_{rms/phase}=\frac{\sqrt{2}}{3}V_S=\frac{\sqrt{2}}{3}\times500=235.7$ V

b) RMS value of line voltage, $V_{rms/line}=\sqrt{\frac{2}{3}}V_S=\sqrt{\frac{2}{3}}\times500=408.25$ V

c) RMS value of load current, $I_{rms/phase}=\frac{V_{ph}}{R}=\frac{235.7}{15}=15.71$ A

d) RMS value of current through switch, $I_{rms/phase}=\frac{I_{rms/phase}}{\sqrt{2}}=\frac{15.71}{\sqrt{2}}=11.11$ A

Average value of switch current, $I_{average/switch}=\frac{1}{2\pi}\int_0^{\pi} i_0 d\omega t=0.22\frac{V_S}{R}=0.22\left(\frac{500}{15}\right)=7.33$ A

e) Power delivered to the load, $P_L=3I^2_{rms/ph}R=3\times(15.71)^2\times15=11.106$ KW

f) Average Source current, $I_{average}=\frac{P_L}{V_S}=\frac{11.106\times1000}{500}=22.21$ A

(Assuming lossless inverter)

2.9. Repeat the problem 6 for 120° mode of conduction. **[MODEL QUESTION]**

Answer:

a) RMS value of phase voltage, $V_{rms/phase}=\frac{V_S}{\sqrt{6}}=\frac{500}{\sqrt{6}}=204.12$ V

b) RMS value of line voltage, $V_{rms/line}=\frac{V_S}{\sqrt{2}}=\frac{500}{\sqrt{2}}=353.55$ V

c) RMS value of load current, $I_{rms/phase} = \frac{V_{ph}}{R} = \frac{204.12}{15} = 13.61\,\text{A}$

d) RMS value of current through switch, $I_{rms/phase} = \frac{I_{rms/phase}}{\sqrt{2}} = \frac{13.61}{\sqrt{2}} = 9.62\,\text{A}$

Average value of switch current, $I_{average/switch} = \frac{1}{2\pi}\int_0^{\pi} i_0 d\omega t = \frac{V_S}{6R} = 5.56\,\text{A}$

e) Power delivered to the load, $P_L = 3I_{rms/ph}^2 R = 3\times(13.61)^2 \times 15 = 8.335\,\text{kW}$

f) Average Source current, $I_{average} = \frac{P_L}{V_S} = \frac{8.335\times 1000}{500} = 16.67\,\text{A}$

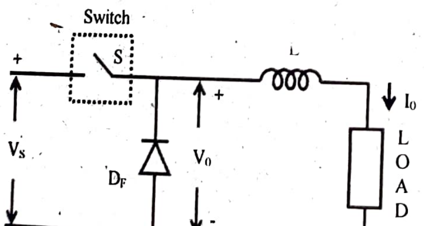
(Assuming lossless inverter)

Long Answer Type Questions

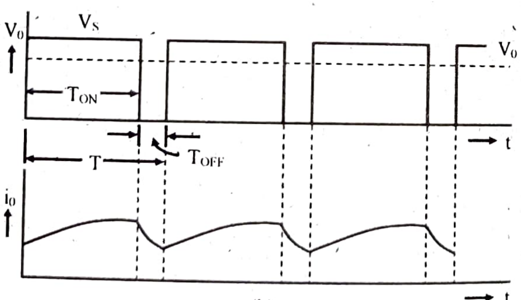
3.1. Explain the working principle of a buck converter with necessary circuit diagram and waveforms. **[WBUT 2012]**

Answer:

A buck converter (dc-dc) is shown in Fig. (a) consists of a switch, a diode (termed as freewheeling is used to allow the load current to flow through it, when the switch is turned off) and inductive (R-L) load. In some cases, a battery (or back emf) is connected in series with the load (inductive). Due to the load inductance, the load current must be allowed a path, which is provided by the diode; otherwise, in the absence of the above diode, the high induced emf of the inductance, as the load current tends to decrease, may cause damage to the switching device. If the switching device used is a thyristor, this circuit is called as a step-down chopper, as the output voltage is normally lower than the input voltage.



(a)



(b)

The output voltage and current waveforms of the circuit are shown Fig. (b). The output voltage V_0 is same as the input voltage V_S. When the switch is ON, during the period, $T_{ON} \geq t \geq 0$. The switch is turned on at $t = 0$, and then turned off at $t = T_{ON}$, this is called **ON period**. During the next time interval $T \geq t \geq T_{ON}$. The output voltage V_0 is zero. as the diode, D_F start to conduct. The OFF period is $T_{OFF} = T - T_{ON}$ with the time period being $T = T_{ON} + T_{OFF}$

The average value of the output voltage is $V_0 = \frac{1}{T}\int_0^T v_0\,dt = \frac{1}{T}\int_0^{T_{ON}} V_S\,dt = V_S\left(\frac{T_{ON}}{T}\right) = KV_S$,

where, $K = \left(\frac{T_{ON}}{T}\right) = \left(\frac{T_{ON}}{T_{ON}+T_{OFF}}\right)$

K is known as the duty ratio and its range $1.0 \geq K \geq 0$

The average value of the output current is $i_0 = \frac{V_0}{R} = \frac{KV_S}{R}$

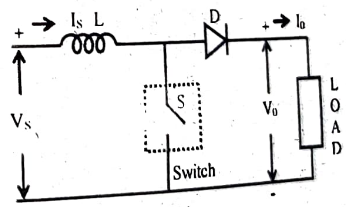
The RMS value of the output voltage is $V_{RMS} = \sqrt{V_0^2} = V_S\sqrt{K}$

Due to turn-on delay of the device used, the duty ratio (k) is not zero, but has some positive value and due to requirement of turn-off time of the device, the duty ratio (k) is less than 1.0. So the output voltage is lower than the input voltage. If the duty ratio (k) is increased, the average output voltage increases. So, a variable dc output voltage is obtained from a constant dc input voltage. The load current is assumed to be continuous as shown in Fig. (b). The load current increases in the ON period, as the input voltage appears across the load, and it (load current) decreases in the OFF period, as it flows in the diode, but is positive at the end of the time period, T.

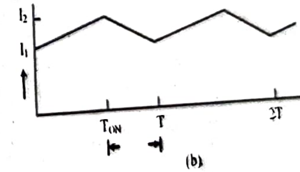
3.2. Explain with a neat circuit diagram, the principle of a Boost converter. **[WBUT 2008, 2011]**

Answer:

A Boost converter (dc-dc) is shown in Fig. (a) where a diode is used in series with the load. The R-L load is considered where the inductance is small. An inductance, L is assumed in series with the input supply.



(a)



(b)

When the switch is ON during the period $T_{ON} \geq t \geq 0$, the output voltage V_0 is zero. If no battery (back emf) is connected in series with the load, the load inductance is small. The current from the source i_S flows in the inductance L. The value of current increases linearly with time in this interval, with $\frac{di}{dt}$ being positive. As the current through L increases, the polarity of the induced emf is considered as positive. During ON period

$$V_S = L\frac{di_S}{dt} \qquad \text{or} \qquad \frac{di_S}{dt} = \frac{V_S}{L}$$

$E_1 = I_{ON} \cdot I_{ON}$ $= V \cdot I \cdot T_{ON}$ $I = \left(\frac{I_1+I_2}{2}\right)$

The switch, S is put OFF during the period, $T \geq t \geq T_{ON}$. The current through L decreases, with the induced emf reverses and added with the supply voltage. The current $I_S = I_0$ decreases linearly in the time interval, T_{OFF}, as the output voltage assumed nearly constant.

$E_1 = E_2$

During OFF period,

$$V_S = V_0 + L\frac{di_S}{dt} \quad \text{or} \quad \frac{di_S}{dt} = \left(\frac{V_S - V_0}{L}\right)$$

The source current waveform is shown in Fig. (b), the current varies linearly from $I_1(I_{min})$ to $I_2(I_{max})$ during the time interval, T_{ON} and similarly from $I_2(I_{max})$ to $I_1(I_{min})$ during the time interval, T_{OFF}

Here o/p voltage is greater than i/p voltage $\left(\frac{1}{1-\alpha}\right)$ $\alpha < 1$.

During ON period, $I_2 - I_1 = I_{max} - I_{min} = \left(\frac{V_S}{L}\right)T_{ON}$

During OFF period, $I_2 - I_1 = I_{max} - I_{min} = \left(\frac{V_0 - V_S}{L}\right)T_{OFF}$

Equating above two equations we get, $\left(\frac{V_S}{L}\right)T_{ON} = \left(\frac{V_0 - V_S}{L}\right)T_{OFF}$

The average value of the output voltage is,

$$V_0 = V_S\left(\frac{T}{T_{OFF}}\right) = V_S\left(\frac{T}{T - T_{ON}}\right) = V_S\left(\frac{1}{1 - \frac{T_{ON}}{T}}\right) = V_S\left(\frac{1}{1-k}\right)$$

The duty ratio $= K = \left(\frac{T_{ON}}{T}\right) = \left(\frac{T_{ON}}{T_{ON} + T_{OFF}}\right)$ with the range $1.0 \geq K \geq 0$

In this case, the output voltage is higher than the input voltage, as contrasted with buck converter (dc-dc). So, this is called boost converter (dc-dc).

3.3. Explain the time ratio control strategies of chopper circuit. [MODEL QUESTION]

Answer:

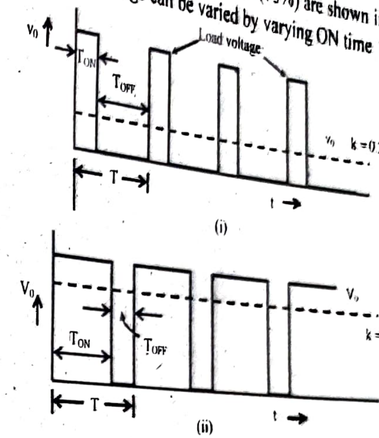
The average value of the output voltage of chopper circuit can be controlled by periodic opening and closing of the switches. In time ratio control the duty ratio $K = \left(\frac{T_{ON}}{T}\right)$ can be controlled in two ways.

(i) Constant frequency operation, (ii) Variable frequency operation.

(i) Constant Frequency Operation

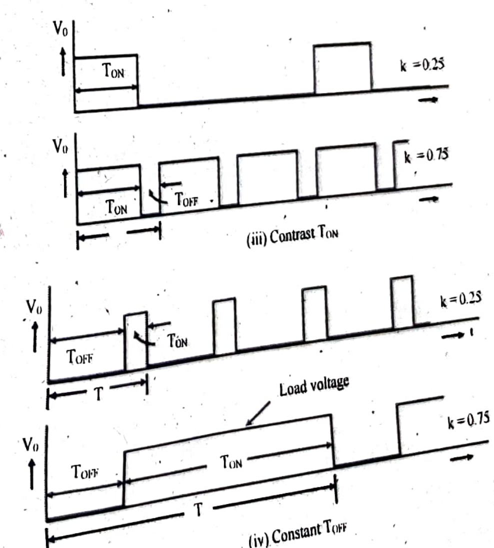
In this control strategy, the ON time T_{ON} is varied, keeping the frequency $\left(f = \frac{1}{T}\right)$ or time period T constant. This is also called as ***pulse width modulation control*** **(PWM).**

Two cases with duty ratios, as (a) 0.25 (25%), (b) 0.75 (75%) are shown in Fig. (i) & (ii) respectively. Hence, the output voltage can be varied by varying ON time T_{ON}.



(ii) Variable Frequency Operation

In this control strategy, the frequency $\left(f = \frac{1}{T}\right)$ or time period T is varied, keeping either the ON time T_{ON} or the OFF time T_{OFF} constant. This is also called as ***frequency modulation control.***



Two cases with the ON time T_{ON} constant and the OFF time T_{OFF} constant, with variable frequency or time period T are shown in Fig. (iii) and (iv) respectively. The output voltage can be varied in both cases with the change in duty ratio.

3.4. a) What is Voltage source inverter?
b) How a single phase full bridge inverter differs from a half bridge inverter according to the principle. **[WBUT 2008, 2012]**

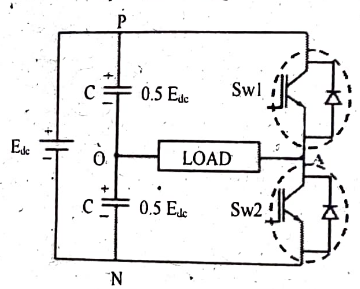
Answer:

a) An inverter converts a fixed dc voltage to an ac voltage of variable frequency and of fixed or variable magnitude. If the input of a converter is a dc voltage source, the inverter is called a voltage source inverter (VSI). If the input of a converter is a current source, the inverter is called a current source inverter (CSI),

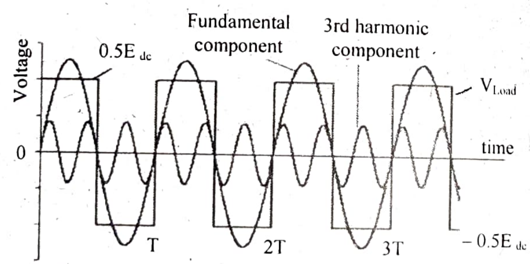
b) A single-phase square wave type voltage source inverter produces square shaped output voltage for a single-phase load. Depending upon the thyristor connection and commutating component single phase VSI is classified as:
(i) Single phase half bridge VSI
(ii) Single phase full bridge VSI

(i) Single-phase VSI

The circuit for single phase half bridge VSI is shown in Fig. Assuming ideal circuit condition, input dc voltage is constant and the switches are lossless.



A 1-phase half bridge VSI
(a)



Square wave load voltage output by half-bridge inverter
(b)

In half bridge circuit the input dc voltage is split in two equal parts through an ideal and loss-less capacitive potential divider. The half bridge circuit consists of one leg (one pole) of switches whereas the full bridge circuit has two such legs. Each leg of the inverter consists of two series connected electronic switches shown within dotted lines in the Figures. Each of these switches consists of an IGBT type controlled switch across which an uncontrolled diode is put in anti-parallel manner. These switches are capable of conducting bi-directional current but they need to block only one polarity of voltage. The junction point of the switches in each leg of the inverter serves as one output point for the load. In half bridge circuit the single-phase load is connected between the mid-point of the input dc voltage and the junction point (marked as 'O' and 'A') of the two switches. The switches Sw1 and Sw2 may be assumed to be controlled mechanical switches that

open and close in response to the switch control signal. If the switches Sw1 and Sw2 are turned on alternately with duty ratio of each switch kept equal to 0.5, the load voltage V_{Load} will be square wave with a peak-to-peak magnitude equal to input dc voltage E_{dc} shown in Fig. (b). V_{Load} acquires a magnitude of $+0.5E_{dc}$ when Sw1 is on and the magnitude reverses to $-0.5E_{dc}$ when Sw2 is turned on. The fundamental frequency component of the square wave voltage is also shown in this Figure, its peak-to-peak magnitude being equal to $\frac{4}{\pi}E_{dc}$. The two switches of the inverter leg are turned on in a complementary manner. For a general load, the switches should neither be on or off simultaneously. Simultaneous turn-on of both the switches will amount to short circuit across the dc bus and will cause the switch currents to rise rapidly. For an inductive load, containing an inductance in series, one of the switches must always conduct to maintain continuity of load current. In case of inductive load, the load current may not change abruptly even though the switching frequency is very high, for which switches must have bidirectional current carrying capability.

Circuit Analysis:

(A) With purely resistive (R) load

(i) RMS Output Voltage: The average value of the output voltage is given by

$$V_o(avg) = \frac{1}{2\pi}\int_0^{2\pi} v_0(\omega t)\,d\omega t \qquad \frac{\int_0^{2\pi} V_o(\omega t)\,d\omega t}{\int_0^{2\pi} d(\omega t)}$$

Rms value of the output voltage is given by

$$V_o(rms) = \sqrt{\frac{1}{2\pi}\int_0^{\frac{\pi}{2}} v_0^2(\omega t)\,d\omega t}$$

$$= \sqrt{\frac{4}{2\pi}\int_0^{\pi/2} v_0^2(\omega t)\,d\omega t} \qquad \text{(due to square wave symmetry)}$$

$$= \sqrt{\frac{2}{\pi}\int_0^{\pi/2}\left(\frac{E_{dc}}{2}\right)^2 d\omega t}$$

$$= \frac{E_{dc}}{2}$$

(ii) Instantaneous output (load) voltage: Using Fourier series analysis the instantaneous output voltage wave form may be expressed as

$$V_{Load} = v_0(\omega t) = \sum_{n=1,3,5,7,\ldots\alpha} \frac{2E_{dc}}{n\pi}\sin(n\omega t)$$

$= 0$ for n

$= 2, 4, 6$ for even values of n

where n is harmonic order and f is switching frequency of the inverter switches. The square wave load voltage consists of all the odd harmonics and their magnitudes are inversely proportional to their harmonic order.

The fundamental frequency component of output voltage $= \dfrac{2E_{dc}}{\pi}$

RMS value of fundamental frequency component of output voltage $= \dfrac{2E_{dc}}{\sqrt{2}\pi} = \dfrac{\sqrt{2}E_{dc}}{\pi}$

The nth harmonic voltage (n being odd integer) has a peak magnitude of $= \dfrac{2E_{dc}}{n\pi}$

RMS value of nth harmonic component of output voltage $= \dfrac{\sqrt{2}E_{dc}}{n\pi}$

The magnitudes of very high order harmonic voltages become negligibly small. In most applications, only the fundamental component in load voltage is of practical use and the other higher order harmonics are undesirable distortions.

(iii) Output current: The average current of a square wave with a peak value of $\dfrac{E_{dc}}{2R}$

$$I_o(avg) = \frac{1}{T}\int_0^{T/2} \frac{E_{dc}}{2R}\,dt = \frac{E_{dc}}{4R}$$

$$I_o(rms) = \sqrt{\frac{1}{T}\int_0^{T/2}\left(\frac{E_{dc}}{2R}\right)^2 dt} = \frac{E_{dc}}{2\sqrt{2}R}$$

(B) With R-L Load

With an inductive load, the output voltage waveform is similar to that with a resistive load. However the load current cannot change immediately with the output voltage. Under steady state the load current waveform in a particular output cycle will repeat in successive cycles and hence only one square wave period has been considered. Let, $t = 0$ be the instant when the positive half cycle of the square wave starts and let I_{Load} be the load current at this instant. The negative half cycle of square wave starts at $t = 0.5T$ and extends up to T.

(i) Instantaneous Output Current:

With R-L load the load impedance Z may be represented as

$$Z = R + j\omega L = \sqrt{R^2 + (\omega L)^2} \text{ and } \phi = \tan^{-1}\frac{\omega L}{R}$$

Putting, $\omega = \dfrac{2\pi}{T}$, we get, $Z = \sqrt{R^2 + \left(\dfrac{2\pi L}{T}\right)^2}$

For the fundamental harmonic frequency the load impedance (Z_1) and load power factor angle (ϕ_1) can be calculated as

$$Z_1 = \sqrt{R^2 + \left(\frac{2\pi L}{T}\right)^2} \text{ and phase angle, } \phi_1 = \tan^{-1}\frac{2\pi L}{TR}$$

The load impedance and load power factor angle for the n[th] harmonic component (Z_n and ϕ_n) respectively will similarly be given by,

$$Z_n = \sqrt{R^2 + \left(\frac{2\pi nL}{T}\right)^2} \text{ and phase angle, } \phi_n = \tan^{-1}\frac{2\pi nL}{TR}$$

As the square shape load voltage may be taken as superposition of different harmonic voltages. The load current may similarly be taken as superposition of harmonic currents produced by the different harmonic voltages. The load current may be expressed in terms of these harmonic currents.

Instantaneous load current

$$i_0(t) = \sum_{n=1,3,5,7,\ldots\alpha} \frac{2E_{dc}}{n\pi Z_n}\sin(n\omega t - \phi_n)$$

The fundamental component of load current is expressed by,

$$(I_{load})_1 = i_{01}(t) = \frac{2E_{dc}}{\pi Z_1}\sin(\omega t - \phi_1)$$

The n^{th} harmonic component of load current is expressed by,

$$(I_{load})_n = i_{0n}(t) = \frac{2E_{dc}}{n\pi Z_n}\sin(n\omega t - \phi_n)$$

From above equations it may be seen that the contribution to load current from very higher order harmonics become negligible and hence the infinite series based expression for load current may be terminated beyond certain values of harmonic order 'n'. For L/R ratio = $2T$, the individual harmonic components of load current normalized against a base current of $\dfrac{0.5E_{dc}}{R}$ have been calculated as:

$$(I_{load})_{1,normalized} = \frac{4}{\pi\sqrt{1+16\pi^2}}\sin(\omega t - \tan^{-1}4\pi) = 0.1\sin(\omega t - 1.491)$$

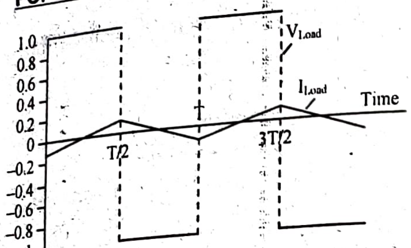
$$(I_{load})_{3,normalized} = \frac{4}{3\pi\sqrt{1+144\pi^2}}\sin(3\omega t - \tan^{-1}12\pi) = 0.011\sin(3\omega t - 1.544)$$

$$(I_{load})_{5,normalized} = \frac{4}{5\pi\sqrt{1+400\pi^2}}\sin(5\omega t - \tan^{-1}20\pi) = 0.004\sin(5\omega t - 1.555)$$

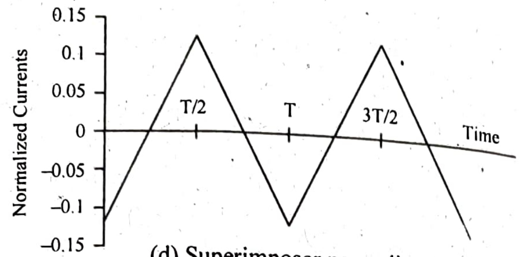
$$(I_{load})_{7,normalized} = \frac{4}{7\pi\sqrt{1+784\pi^2}}\sin(7\omega t - \tan^{-1}28\pi) = 0.002\sin(7\omega t - 1.559)$$

$$(I_{load})_{11,normalized} = \frac{4}{11\pi\sqrt{1+193\pi^2}}\sin(11\omega t - \tan^{-1}44\pi) = 0.0008\sin(11\omega t - 1.564)$$

It may be concluded that for L/R = 2T, the contribution to load current from 13[th] and higher order harmonics are less than 1% of the fundamental component and hence they may be neglected without any significant loss of accuracy. Fig.(c) shows the load voltage and normalized the load current waveform, whereas the superimposed load current waveform is shown In Fig.(d)



(c) Normalized load voltage and load current for L/R = 2T



(d) Superimposer normalized load current waveforms for L/R = 2T

(ii) Fundamental Output Power:

The output power at fundamental frequency (n=1) is given by $P_{1rms} = E_{1rms}.I_{1rms} = I_{1rms}^2 R$

where E_{1rms}, I_{1rms} are the rms value of fundamental output voltage and current respectively.

$$I_{1rms} = \frac{2E_{dc}}{\pi\sqrt{2}}.\frac{1}{\sqrt{R^2+(\omega L)^2}}$$

$$P_{1rms} = \left[\frac{2E_{dc}}{\pi\sqrt{2}}.\frac{1}{\sqrt{R^2+(\omega L)^2}}\right]^2.R = \frac{2E_{dc}{}^2 R}{\pi^2\left(R^2+\omega^2 L^2\right)}$$

(iii) Single phase Full bridge VSI

The single-phase full bridge circuit Fig.(e) can be thought of as two half bridge circuits sharing the same dc bus. The full bridge circuit will have two pole-voltages V_{AO} and V_{BO}. Both V_{AO} and V_{BO} of the full bridge circuit are square waves but they will, in general, have some phase difference. Fig. (f) shows these pole and line voltages output.

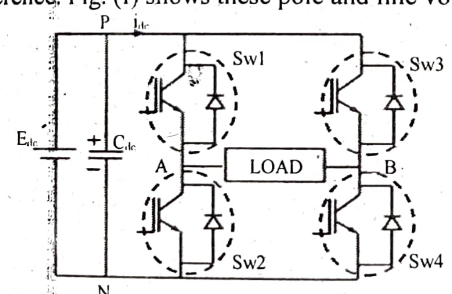


Fig: (e) A 1-phase full-bridge VSI

The pole voltage VAO of the full bridge inverter may be written as

$$V_{AO} = \sum_{n=1,3,5,7....\alpha} \frac{2E_{dc}}{n\pi}\sin(n\omega t)$$

Similarly the pole voltage VBO of the full bridge inverter may be written as

$$V_{BO} = \sum_{n=1,3,5,7....\alpha} \frac{2E_{dc}}{n\pi}\sin n(\omega t-\phi) \text{ where phase shift angle } \phi = \frac{2\pi t}{T}$$

Here 't' is the time by which the two pole voltages are staggered and 'T' is the time period of the square wave pole voltages.

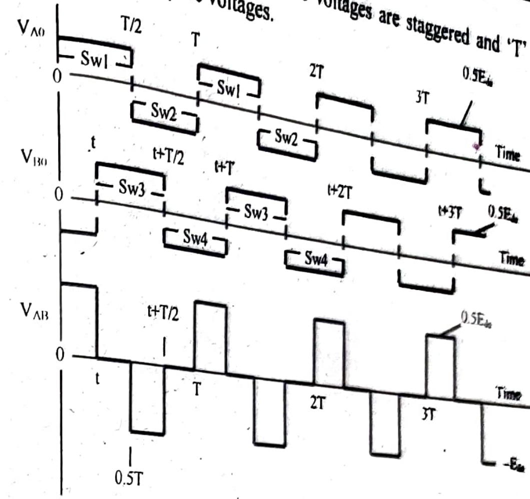


Fig. (f): Pole and line voltages output by 1-ph full bridge inverter

In full bridge inverter the single phase load is connected between points 'A' and 'B' and the line or the load voltage V_{AB} can be obtained by Taking difference of the voltage V_{AO} and V_{BO},

We can write

$$V_{AB} = V_{AO} - V_{BO} = \sum_{n=1,3,5,7,....\alpha} \frac{2E_{dc}}{n\pi}\left[\sin(n\omega t) - \sin n(\omega t-\phi)\right]$$

The fundamental component of V_{AB} may be written as

$$V_{AB,1} = \frac{2E_{dc}}{\pi}\left[\sin\omega t - \sin(\omega t-\phi)\right] = \frac{4E_{dc}}{\pi}\cos\left(\omega t-\frac{\phi}{2}\right)\sin\frac{\phi}{2}$$

The n[th] harmonic component of VAB may be similarly written as

$$V_{AB,n} = \frac{2E_{dc}}{n\pi}\left[\sin n\omega t - \sin n(\omega t-\phi)\right] = \frac{4E_{dc}}{n\pi}\cos n\left(\omega t-\frac{\phi}{2}\right)\sin\frac{n\phi}{2}$$

The rms magnitude of the fundamental component of load voltage may be written as

$$\left(V_{AB,1}\right)_{rms} = 0.9E_{dc}\sin\frac{\phi}{2}$$

The rms magnitude of load voltage can be changed from zero to a peak magnitude of $0.9E_{dc}$. The peak load voltage magnitude corresponds to $\phi = 180°$ and the load voltage will be zero for $\phi = 0°$. For $\phi = 180°$, the load voltage waveform is once again square wave of time period T and instantaneous magnitude E. As the phase shift angle changes from 0° to 180°, the width of voltage pulse in the load voltage waveform increases. Thus the fundamental voltage magnitude is controlled by pulse-width modulation.

3.5. Explain briefly the three phase voltage source inverter with necessary circuit diagram & output waveforms with purely resistive load. **[WBUT 2009]**

Answer:

With 3-phase balanced load. Fig. (a) shows the power circuit of the three-phase inverter. This circuit may be identified as three single-phase half-bridge inverter circuits put across the same dc bus. The individual pole voltages of the 3-phase bridge circuit are identical to the square pole voltages output by single phase half bridge or full bridge circuits. The three pole voltages of the 3-phase square wave inverter are shifted in time by one third of the output time period. These pole voltages along with some other relevant waveforms have been plotted in Fig. (b). The horizontal axis of the waveforms in Fig. (b) has been represented in terms of 'ωt', where 'ω' is the angular frequency (in radians per second) of the fundamental component of square pole voltage and 't' stands for time in second. In Fig. (b) the phase sequence of the pole voltages is taken as V_{AO}, V_{BO} and V_{CO}. The numbering of the switches in Fig. (b) has some special significance for the output phase sequence.

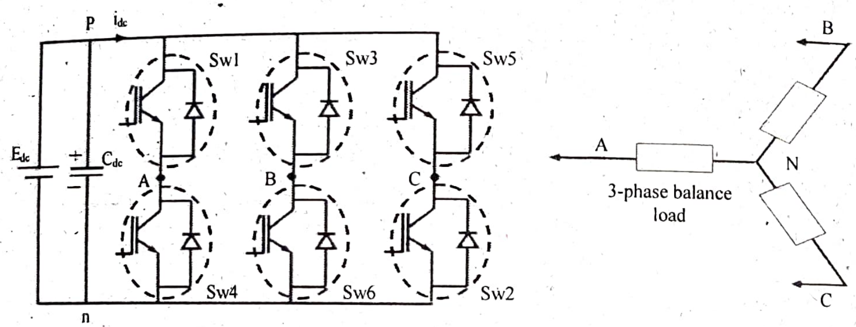


Fig: (a) A 3-phase Voltage Source Inverter (VSI) feeding a balanced load

From the conduction pattern of the switches marked in Fig. (b) shown that with the chosen numbering the switches turn on in the sequence: Sw1, Sw2, Sw3, Sw4, Sw5, Sw6, Sw1, Sw2, ...and so on. Identifying the switching cycle time as 360° (2π radians), it can be seen that each switch conducts for 180° and the turning on of the adjacent switch is staggered by 60°. The upper and lower switches of each pole (leg) of the inverter conduct in a complementary manner. To reverse the output phase sequence, the switching sequence may simply be reversed.

Considering the symmetry in the switch conduction pattern, it may be found that at any time three switches conduct. It could be two from the upper group of switches, which are connected to positive dc bus, and one from lower group or vice-versa (i.e., one from upper group and two from lower group). According to the conduction pattern indicated in Fig. (b). There are six combinations of conducting switches during an output cycle: (Sw5, Sw6, Sw1), (Sw6, Sw1, Sw2), (Sw1, Sw2, Sw3), (Sw2, Sw3, Sw4), (Sw3, Sw4, Sw5), (Sw4, Sw5, Sw6). Each of these combinations of switches conducts for 60° in the sequence mentioned above to produce output phase sequence of A, B, C. and the fundamental component of the three output line-voltages will be balanced.

Circuit Analysis

Fig. (a) shows a star connected balanced 3-phase load. The three load terminals are connected to the three output points (A, B, C) of the inverter. The neutral point 'N' of the load is left open. The load side phase voltages V_{AN}, V_{BN}, V_{CN} can be determined from the conduction pattern of the inverter switches.

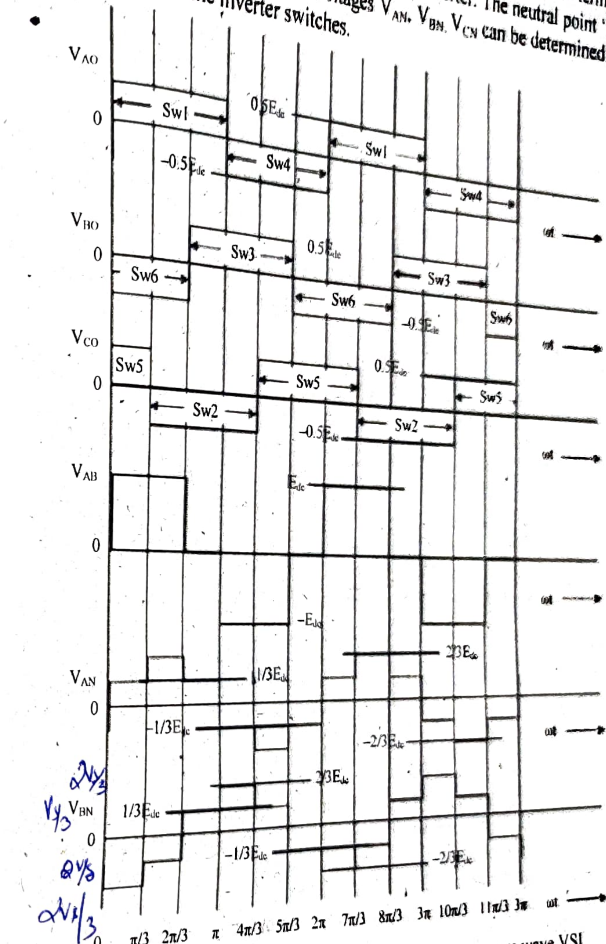


Fig: (b) Some voltage waveforms output by a 3-phase square wave VSI

With reference to Fig. (b), it may be seen that for $0 \le \omega t \le \pi/3$, switches Sw5, Sw6 and Sw1 conduct. Under the assumption of ideal switches Fig. (c) will represent the equivalent inverter and load circuit during the time interval $0 \le \omega t \le \pi/3$. In the equivalent circuit representation the non-conducting switches have been omitted and a cross (X) sign is used to represent a conducting switch. For a balanced 3-phase load the instantaneous phase voltage for the following two cases are considered.

For a balanced three-phase load, the instantaneous magnitude of any phase current can be determined by superposition of different harmonic currents of the phase. For a simple three-phase R-L load, the phase-A current i_A through R-L load is

$$i_A = \sum_{n=1,5,7,11,13....\alpha} \frac{2E_{dc}}{n\pi\sqrt{R^2+\omega^2 L^2}} \sin\left(n\omega t - \tan^{-1}\frac{n\omega L}{R}\right)$$

$$i_B = \sum_{n=1,5,7,11,13....\alpha} \frac{2E_{dc}}{n\pi\sqrt{R^2+\omega^2 L^2}} \sin\left(n\left(\omega t - \frac{2\pi}{3}\right) - \tan^{-1}\frac{n\omega L}{R}\right)$$

$$i_C = \sum_{n=1,5,7,11,13....\alpha} \frac{2E_{dc}}{n\pi\sqrt{R^2+\omega^2 L^2}} \sin\left(n\left(\omega t + \frac{2\pi}{3}\right) - \tan^{-1}\frac{n\omega L}{R}\right)$$

Due to the reduction in harmonic for a purely inductive load, load current wave form is very close to sinusoidal shape.

3.6. What do you mean by sinusoidal pulse width modulation?[MODEL QUESTION]

Answer:

In Sine-PWM inverter the widths of the pole-voltage pulses, over the output cycle, vary in a sinusoidal manner. The scheme involves comparison of a high frequency triangular carrier voltage with a sinusoidal modulating signal that represents the desired fundamental component of the pole voltage waveform. The peak magnitude of the modulating signal should remain limited to the peak magnitude of the carrier signal. Fig. (i) shows an op-amp based comparator output along with sinusoidal and triangular signals as inputs. In the comparator the triangular and sinusoidal signals are fed to the inverting and the non-inverting input terminals respectively and the comparator output magnitudes for high and low levels are assumed to be $+V_{CC}$ and $-V_{CC}$.

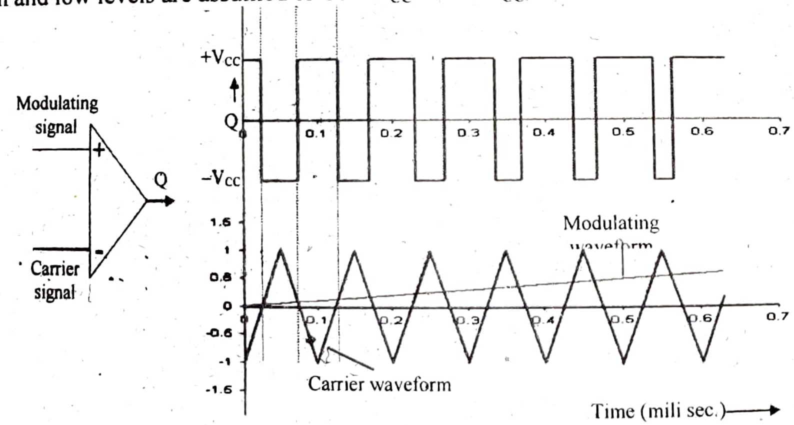


Fig: (i) A schematic circuit for comparison of Modulating and Carrier signals

The comparator output signal 'Q' is used to turn-on the high side and low side switches of the inverter pole. When 'Q' is high, upper (high side) switch of the particular pole is turned on and when 'Q' is low the lower switch is turned on.

When 'Q'= $+V_{CC}$, the pole voltage is $+0.5E_{dc}$ and

When 'Q'= $-V_{CC}$, the pole voltage is $-0.5E_{dc}$

The harmonic contents in the comparator output voltage and the pole voltage waveforms are identical as E_{dc} is constant. A slowly varying sinusoidal voltage, with the following constraints considered as the modulating signal:

The peak magnitude of the sinusoidal signal is less than or equal to the peak magnitude of the carrier signal. This ensures that the instantaneous magnitude of the modulating signal never exceeds the peak magnitude of the carrier signal.

2. The frequency of the modulating signal is several orders lower than the frequency of the carrier signal. A typical value of 50 Hz for the modulating signal and 20 Kilohertz for the carrier signal. Under such high frequency ratios, the magnitude of modulating signal will be virtually constant over any particular carrier-signal time period.

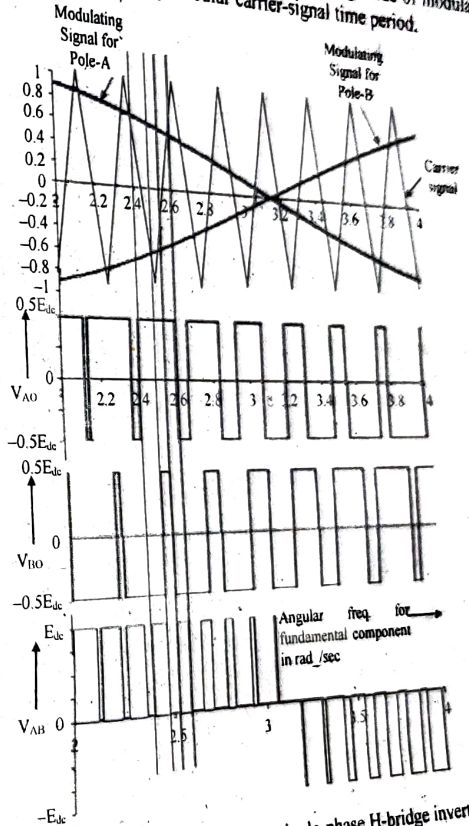


Fig: (ii) Sine-PWM waveform for single-phase H-bridge inverter

ELECTRICAL INSTALLATION

Chapter at a Glance

An electrical installation deals with proper safety of personnel and the protection of electrical equipment from electrical fault such as short circuit, over voltage / current, overheating etc. For example, a worn out bearing may cause overloading of a motor. A tree falling or touching an overhead line may cause a fault. A lightning strike can cause insulation failure. Pollution may result in degradation in performance of insulators which may lead to breakdown. Under frequency or over frequency of a generator may result in mechanical damage of turbine. To avoid these abnormal operating regions for safety of the equipment, it is necessary to isolate and de-energize the equipment from fault region. So the apparatus and system must be protected.

The most popular methods of protection are

- Earthing or grounding of equipment
- Use of fuses or circuit breakers (such as the Miniature Circuit Breaker – MCB)
- Use of earth leakage and residual current circuit breakers.

Wiring is necessary for distribution of electrical energy from supply meter board to home appliances such as lamp, fan and other purposes. It can be done using two ways-

- Joint Box/Tee system
- Loop-in system.

All electrical wiring systems and all electrical apparatus associated with wiring must be protected to:

- Prevent damage by fire or shock
- Maintain continuity of the supply
- Disconnect faulty apparatus from the remainder of the system
- Prevent damage to wiring and equipment
- Minimize the system interruptions under fault conditions.

Protection must be provided against excess currents and earth leakage. Protective equipment must possess the following features:

- Certainty and reliability of operation under fault conditions and non-operation under normal conditions.
- Discrimination
- Rapidity of operation
- Simplicity, low initial and maintenance cost (e) Easy adjustment and testing.

Daily electricity consumption can be calculated in the following way-

$$\text{Electricity consumption day (kWh/day)} = \frac{\text{Meter reading end-Meter reading start}}{\text{Days in period}}$$

Electricity consumption year = Electricity consumption day (kWh/day) × 365

Electricity savings (kWh/yr)

$$= 365 \times \left(\frac{\text{Electricity consumption previous year} - \text{Electricity consumption current year}}{\text{Days in period}} \right)$$

Multiple Choice Type Questions

1.1. To keep RRRV within the rating of a circuit breaker for resistance switching, the critical value of resistance R is [MODEL QUESTION]

a) $\frac{1}{3}\sqrt{L/C}$
b) $\frac{1}{2}\sqrt{L/C}$
c) $\sqrt{L/C}$
d) $\frac{1}{2}\sqrt{C/L}$

Answer: (b)

1.2. Which of the following circuit breakers is not suitable for auto reclosing purpose? [MODEL QUESTION]

a) Airblast circuit breaker
b) Oil circuit breaker
c) SF circuit breaker
d) Vaccum circuit breaker

Answer: (b)

1.3. The stability of arc in vacuum depends on [MODEL QUESTION]

a) the contact material only
b) the contact material and the vapour pressure
c) the circuit parameters only
d) the combination of (a) and (c)

Answer: (d)

1.4. The arc voltage in a circuit breaker is [MODEL QUESTION]

a) in phase with the arc current
b) lagging the arc current by 90°
c) leading the arc current by 90°
d) in phase opposition of the arc current

Answer: (a)

1.5. Use of high speed circuit breakers [MODEL QUESTION]

a) improves transient stability
b) deteriorates transient stability
c) has no effect on the system
d) none of these

Answer: (a)

1.6. Circuit breakers usually operate under [MODEL QUESTION]

a) steady short circuit current
b) sub-transient state of short circuit current
c) transient state of short circuit current
d) none of these

Answer: (c)

1.7. Which of the following circuit breakers is not suitable for autoreclosing purpose? [MODEL QUESTION]

a) Airblast circuit breaker
b) Oil circuit breaker
c) SF6 circuit breaker
d) Vacuum circuit breaker

Answer: (b)

1.8. Compared to rewirable fuse a HRC fuse has **[MODEL QUESTION]**
a) high rupturing capacity b) high speed of operation
c) no aging effect d) all of these
Answer: (a)

1.9. In SF_6 circuit breakers, the time of interruption is approximately **[MODEL QUESTION]**
a) 3 microseconds b) 3 milliseconds c) 3 seconds d) 3 cycles
Answer: (d)

1.10. RRRV of a circuit breaker means **[MODEL QUESTION]**
a) rate of rise in restricting voltage b) rate of rise in recovering voltage
c) rate of rise in restricting current d) rate of rise in recombining charges
Answer: (a)

1.11. Which of the following circuit breakers has the lowest operating voltage? **[MODEL QUESTION]**
a) SF_6 gas b) Air break c) Air blast d) Minimum oil
Answer: (b)

1.12. Compared to the breaking capacity of circuit breaker its making capacity should normally be **[MODEL QUESTION]**
a) more b) equal
c) less d) the two are unrelated to each other
Answer: (a)

1.13. Re-wirable fuse has **[MODEL QUESTION]**
a) inverse time current characteristic b) linear time current characteristic
c) square-law time current characteristic d) none of these
Answer: (a)

1.14. HRC fuses provide best protection against **[MODEL QUESTION]**
a) overload b) short circuits c) open circuits d) reverse current
Answer: (b)

1.15. Making capacity of a circuit breaker is **[MODEL QUESTION]**
a) 1.55 times the symmetrical breaking current
b) 2.55 times the symmetrical breaking current
c) 2.55 times the asymmetrical breaking current
d) 2.55 times the peak symmetrical current
Answer: (b)

Short Answer Type Questions

2.1. Explain the two method of wiring system. **[MODEL QUESTION]**

Answer:
The wiring system for domestic purpose can be done by two methods:
i) Joint box system or Tee system
ii) Loop-in system

Joint Box or Tee or Jointing System:
In this method of wiring, connections to appliances are made through joints. These joints are made in joint boxes by means of suitable connectors or joints cutouts. This method of wiring doesn't consume too much cables size. This method is suitable for temporary installations and it is cheap.

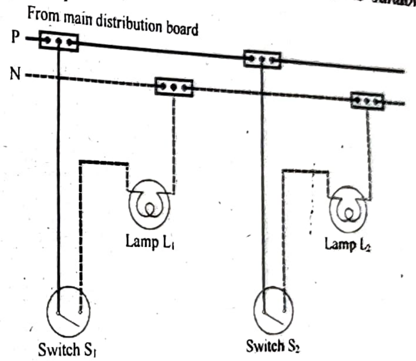


Fig: 1 Joint box system

Loop-in or Looping System:
This method of wiring is universally used in wiring. Lamps and other appliances are connected in parallel so that each of the appliances can be controlled individually. When a connection is required at a light or switch, the feed conductor is looped in by bringing it directly to the terminal and then carrying it forward again to the next point to be fed. The switch and light feeds are carried round the circuit in a series of loops from one point to another until the last on the circuit is reached. The phase or line conductors are looped either in switchboard or box and neutrals are looped either in switchboard or from light or fan. Line or phase should never be looped from light or fan.

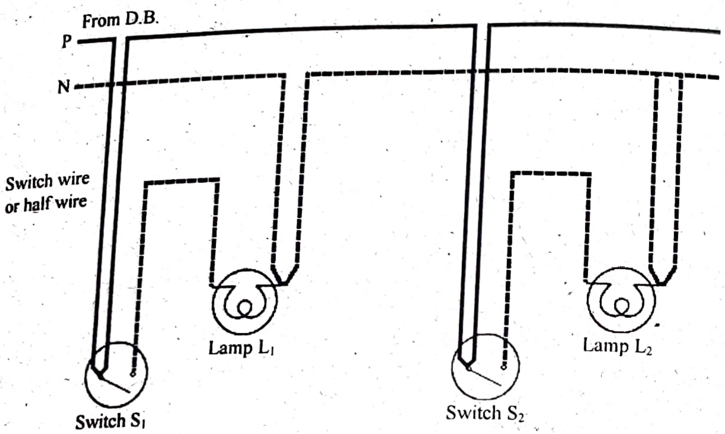


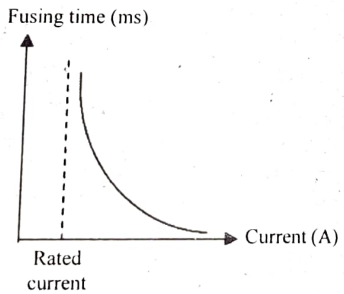
Fig: 2 Loop-in-system

2.2. a) What is fuse? **[MODEL QUESTION]**
b) Explain the working principle of fuse.

Answer:

a) A 'fuse' is a device that opens a circuit with fusible part, which is heated and severed by current flowing through it. The fusible part is also called the "Element".

b) The fuse operation involves two phases
1) Melting and
2) Current interruption.



When current flows in a fuse, heat is generated and the element temperature rises. If the current is within (less or equal to) its continuous rated value, then the steady state temperature is such that the fuse does not melt. However, if the current has large enough magnitude, it will lead to the fuse element to melt before the steady state temperature conditions are achieved. After melting, an arc may be struck. Fuse provides economy in protection as well as flexibility in rating and time current characteristic. It is used for overcurrent protection of transformers, capacitors and lateral taps in distribution systems. The inverse time characteristics of fuse are shown in Fig.

2.3. a) What do you mean by circuit breaker? **[MODEL QUESTION]**
b) Write the advantages of miniature circuit breaker (mcb) over fuse?

Answer:

a) The circuit breaker is a device for making and breaking a circuit (under normal and abnormal conditions). A circuit breaker is selected for a particular duty taking the following into consideration (a) the normal current it will have to carry and (b) the amount of current which the supply system will feed into the circuit under a fault (which

current the circuit breaker will have to interrupt without damage to itself). It has a mechanism which, when it is in the closed position, holds the contacts together and separated during fault condition.

Miniature Circuit Breakers (mcb) are commonly used in domestic installations, it incorporate most of the features of the circuit breaker in a compact form and are being fitted in place of fuses in consumer units in the home or office.

b) The advantages of ***Miniature Circuit Breakers* (mcb)** over fuses are –
- Non destructive determination of tripping characteristics
- Shorter tripping times under moderate over currents than with fuses
- Immediate indication of faulty circuit
- Reclosing can be effected at once after the fault has been cleared - No stock of fuses are required - Can be easily used as a circuit control switch when needed

2.4. Define a) Fusing current b) Fusing factor c) Current rating d) Electric Shock. **[MODEL QUESTION]**

Answer:

a) Fusing current is the minimum current that will cause the fuse element to heat up melt or blow.

b) Fusing factor is the ratio of the fusing current to current rating.

c) Current rating of the fuse is the maximum current, which it will carry for an indefinite period without undue deterioration of its element.

d) Electric Shock

If an electric current passes through the human body, the effects could be an electric shock or even death. The degree of danger of electric shock depends on the value of the body current and the time for which the current flows.

2.5. Describe the operating characteristics of fuse. **[MODEL QUESTION]**

Answer:

Fuses are characterized by 'thermal' and 'interrupting' characteristics.

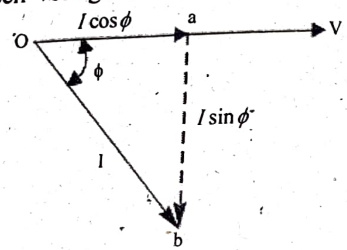
1) **Thermal Characteristics:** As the magnitude of the current increases, melting time reduces. The larger magnitude currents will lead to higher power dissipation I^2R in the fuse and hence faster rise in temperature of the element. This would imply that melting time of the fuse should be inversely proportional to magnitude of square of current. The relationship between the magnitude of the current that causes melting current and the time needed for it to melt is given by the fuse's melting time current characteristics (TCC).

2) **Interrupting Characteristics:** Due to the presence of inductive elements in power apparatus and systems, melting of a fusing element is not sufficient to interrupt the current. Consequently, there is always some period of arcing before the current is interrupted. During this period, fuse must withstand any immediate transient voltage condition and subsequent steady state recovery voltage. Addition of melting time and this arcing overhead gives the total clearing time. For lower currents, melting time can be large and arcing time small because of lower stored energy in induction

circuit. In contrast, for large currents, melting time is small but the arcing time is large. Hence, time current characteristic (TCC) for melting time and total clearing time diverges as I increases.

2.6. a) What is power factor?
b) Write the disadvantages of low power factor. **[MODEL QUESTION]**

Answer:
a) The cosine angle between voltage and current of an ac circuit is known as power factor.



For an inductive circuit as shown in Fig., current I is lagging behind a supply voltage V by an angle ϕ which is known as power factor angle.

Power in a Three Phase AC Circuit $= P = \sqrt{3}V \times I\cos\phi$

And Current in a Three Phase AC Circuits $= I = \dfrac{P}{(3V \times \cos\phi)}$

$$I \propto \frac{1}{\cos\phi} \qquad \text{.... (1)}$$

Also, power in a Single Phase AC Circuits $= P = V \times I\cos\phi$

And Current in a Three phase AC Circuits $= I = \dfrac{P}{(V \times \cos\phi)}$

$$I \propto \frac{1}{\cos\phi} \qquad \text{.... (2)}$$

From both Eqns. (1) and (2) it is seen that Current " I " is inversely proportional to $\cos\phi$ i.e. Power Factor. This implies that when Power Factor increases, Current Decreases, and when Power Factor decreases, Current Increases.

So, In case of Low Power Factor, Current will be increased, and this high current will cause the following disadvantages:

1) Large Line Losses (Copper Losses):
We know that Line Losses is directly proportional to the squire of Current I.
Power Loss $= I^2 \times R$ i.e., the larger the current, the greater the line losses.

2) Large kVA rating and Size of Electrical Equipments:
For all Electrical Machinery (Transformer, Alternator, Switchgears etc) rated in kVA, Power factor is inversely proportional to the kVA i.e.

$$\cos\phi = \frac{kW}{kVA}$$

Therefore, the Lower the Power factor means the larger the kVA rating of machines and the larger size of machines cause the larger the Cost of machines.

3) Greater Conductor Size and Cost:
In case of low power factor, current will be increased, thus, to transmit this high current, for which the larger size of conductor. Also, the cost of large size of conductor will be increased.

4) Poor Voltage Regulation and Large Voltage Drop:
Voltage Drop $= V = IZ$
For low Power factor, Current will be increased. So larger the current, larger the Voltage Drop.

Also Voltage Regulation $= V \cdot R = \dfrac{(V_{\text{No Load}} - V_{\text{Full Load}})}{V_{\text{Full Load}}}$

In case of Low Power Factor (lagging Power factor) there would be large voltage drop which cause low voltage regulation. Therefore, keeping Voltage drop in the particular limit, we need to install Extra regulation equipments i.e. Voltage regulators.

5) Low Efficiency:
In case of low Power Factor, there would be large voltage drop and large line losses and this will cause the system or equipments efficiency too low. For instant, due to low power factor, there would be large line losses; therefore, alternator needs high excitation, thus, generation efficiency would be low.

2.7. Describe the different methods of improvement of power factor. **[MODEL QUESTION]**

Answer:
The following devices and equipment are used for Power Factor Improvement.
i) Static Capacitor
ii) Synchronous Condenser
iii) Phase Advancer

i) Static Capacitor: For Power factor improvement purpose, Static capacitors are connected in parallel with those devices which work on low power factor. These static capacitors provide leading current which neutralize (totally or approximately) the lagging inductive component of load current (i.e. leading component neutralize or eliminate the lagging component of load current) thus power factor of the load circuit is improved. These capacitors are also installed in Induction motors and transformers etc, and improve the load circuit power factor to improve the system or devises efficiency.

Advantages:
- Capacitor bank offers several advantages over other methods of power factor improvement.
- Losses are low in static capacitors.

- There is no moving part, therefore need low maintenance.
- It can work in normal conditions (i.e. ordinary atmospheric conditions).
- Do not require a foundation for installation.
- They are lightweight so it is can be easy to installed.

Disadvantages:
- The age of static capacitor bank is less (8-10 years).
- With changing load. we have to ON or OFF the capacitor bank; which causes switching surges on the system.
- If the rated voltage increases, then it causes damage it.
- Once the capacitors spoiled, then repairing is costly.

ii) Synchronous Condenser: When a Synchronous motor operates at No-Load and over-exited then it's called a synchronous Condenser. Whenever a Synchronous motor is over-exited then it provides leading current and works like a capacitor. When a synchronous condenser is connected across supply voltage (in parallel) then it draws leading current and partially eliminates the re-active component and this way, power factor is improved. Generally, synchronous condenser is used to improve the power factor in large industries.

Advantages:
- Long life (almost 25 years).
- High Reliability.
- Step-less adjustment of power factor.
- No generation of harmonics of maintenance.
- The faults can be removed easily.
- It's not affected by harmonics.
- Require Low maintenance (only periodic bearing greasing is necessary).

Disadvantages:
- It is expensive (maintenance cost is also high) and therefore mostly used by large power users.
- An auxiliary device has to be used for this operation because synchronous motor has no self starting torque.
- It produces noise.

iii) Phase Advancer: Phase advancer is a simple AC exciter which is connected on the main shaft of the motor and operates with the motor's rotor circuit for power factor improvement. Phase advancer is used to improve the power factor of induction motor in industries. As the stator windings of induction motor takes lagging current 90° out of phase with Voltage, therefore the power factor of induction motor is low. If the exciting ampere-turns are excited by external AC source, then there would be no effect of exciting current on stator windings. Therefore the power factor of induction motor will be improved. This process is done by Phase advancer.

Advantages:
- Lagging kVAR (Reactive component of Power or reactive power) drawn by the motor is sufficiently reduced because the exciting ampere turns are supplied at slip frequency (f_s).
- The phase advancer can be easily used where the use of synchronous motors is Unacceptable

Disadvantage:
- Using Phase advancer is not economical for motors below 200 H.P. (about 150kW)

Power factor improvement in three phase system by connecting a capacitor bank in a) Delta connection and b) Star Connection

2.8. The current reading of a Residential Consumption meter is 20803 while the previous reading was 14803. What is the amount of electricity bill? [MODEL QUESTION]

Answer:

Meter reading = electricity consumption $= (20803 - 14803) = 6000\,\text{kWh}$

For unit:

0 - 3000 kWh	0.01 Rs./kWh	Amount $3000 \times 0.01 = \text{Rs.}30/-$
3001 - 5000 kWh	0.015 Rs./kWh	Amount $2000 \times 0.015 = \text{Rs.}30/-$
5001 - 6000 kWh	0.02 Rs./kWh	Amount $1000 \times 0.02 = \text{Rs.}20/-$

Total amount of electricity bill $= (\text{Rs.}30 + \text{Rs.}30 + \text{Rs.}20) = \text{Rs.}80/-$

Long Answer Type Questions

3.1. Explain the operating principle of different types of fuse. [MODEL QUESTION]

Answer:

Fuse is a devise for opening a circuit by means of a conductor designed to melt when an excessive current flows along it. Fuse can be classified into two types:

(A) Non-Current Limiting type fuse
(B) Current Limiting type fuse

Non-Current Limiting type fuse may be two types:
1. Expulsion
2. Vacuum

(A) Non-Current Limiting Fuses

1. Expulsion type: The expulsion type fuse is used where expulsion gases cause no problem such as in overhead circuits and equipment. These fuses can be termed as current awaiting types; and the function of interrupting medium is similar to that of an ac circuit breaker. The temperature of arc is of the order of 4000-5000K. At this temperature special materials located in close proximity to fuse element rapidly create gases. Preferred gas generating materials are fiber, melamine, boric acid and liquids such as oil or carbon tetrachloride. These gases help to create a high pressure turbulent medium surrounding the arc, thus when the current does reach to zero and the arc channel reduces to a minimum; the ablated gases rapidly mix with remaining ionized gas and thereby deionize them as well as remove them from 'arc area'. In turn, this leads to rapid build-up of dielectric strength that can withstand the transient recovery voltage (TRV) and steady state power system voltage. TRV for expulsion fuse is shown in Fig. (i). Note that in an inductive circuit, current zero occurs at lag to voltage i.e. when voltage is at maximum value. The action of interrupting medium causes TRV to be seen in this region.

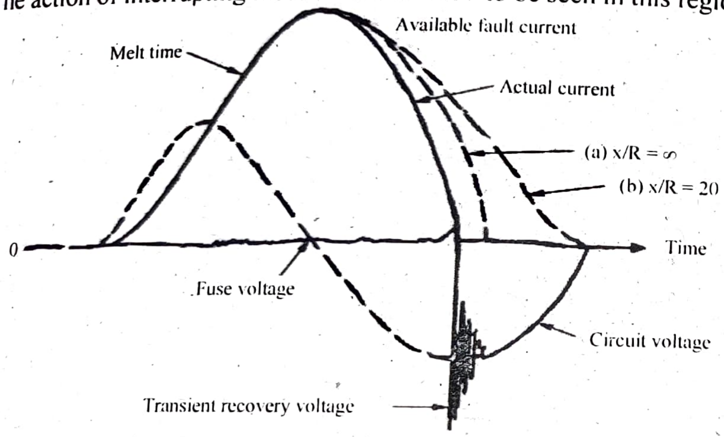


Fig: (i) Current-Voltage-Time relationship of expulsion type fuse

2. Vacuum Fuse: Vacuum fuse is a non expulsive fuse but still a current zero awaiting type. The design, operation and current-voltage-time relationship of this fuse closely matches with that of an expulsion fuse. The main difference is that it is a completely sealed unit and no expulsion action. Interruption occurs because of rapid dielectric build up that occur in a vacuum after current zero is reached.

B) Current Limiting Fuse: A typical current limiting fuse is shown in Fig. (ii). In this case, the fusible element is very long. The element is completely surrounded with filler material, typically silica sand, to contain the arc as well as maintain a very high pressure in the long restricted arc area caused by the practically simultaneous melting of the full length of element. This then allows the fuse to produce a very high resistance in the circuit in a very short period of time (typically hundreds of μsec).

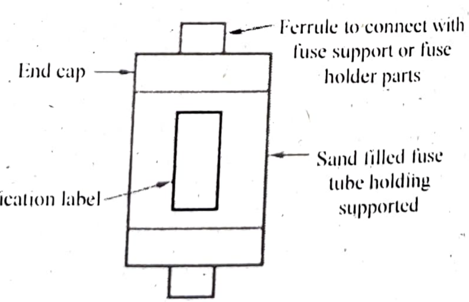


Fig: (ii) Typical current limiting fuse

This would then improve the power factor in the fault circuit as fault current and voltage are in same phase. So when the arc is extinguished temporarily at current zero, the applied voltage across it will also be zero. This zero voltage quickly de-ionize the dielectric medium. This leads speeding in fuse action. Thus current limiting fuses attempt to constrict the arc and it is cooled by sand.

3.2. Describe the operation of different circuit breaker. **[MODEL QUESTION]**

Answer:

A Circuit Breaker (CB) is basically a switch used to interrupt the flow of current. It ...ens on relay command. The relay command initiates mechanical separation of the contacts. It is a complex element because it has to handle large voltages (few to hundreds of kV's) and currents (in kA's). Interrupting capacity of the circuit breaker is therefore expressed in MVA. It is able to provide a more accurate degree of over current protection than that normally provided by different fuses.

Miniature Circuit Breakers (MCB): It is commonly used in domestic installations. It incorporate most of the features of the circuit breaker in a compact form and are being fitted in place of fuses in consumer units in the home or office. An MCB eliminates the cost of fuse replacement and may be used as a switch for isolating circuits. In the MCB, the automatic operation is by **magnetic** or **thermal** means. The reason for the two characteristics is to have proper operation during both short circuit and overload conditions.

Magnetic Mechanism: The magnetic mechanism in Fig. (a) uses a solenoid with an iron piece. It is used for short circuit (fault) protection, as high fault currents have to be isolated almost instantly. When the circuit current is above a certain level, the magnetic field strength increases to cause the iron piece to move in the direction of solenoid. This operates the tripping linkage and open the contacts. Even if the MCB is closed again, the contacts will not hold while the fault is still present.

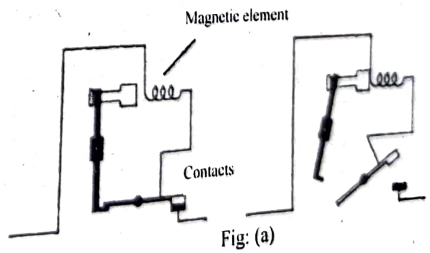


Fig: (a)

Thermal mechanism: The thermal mechanism as shown in Fig. (b) uses a heat sensitive bimetal element. When the element is heated to a pre-determined temperature, the resultant deflection trips the circuit breaker. The time taken to heat the element to this temperature depends on the magnitude of the current and provides the necessary time delay characteristics (tripping by this means is not as rapid as with magnetic tripping).

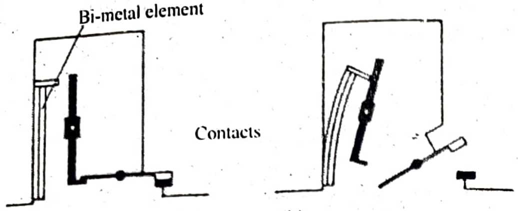


Fig: (b)

When a small sustained overload occurs, the thermal trip will come into operation after a few seconds or even minutes. However, when a heavier over load occurs, the magnetic trip coil operates quickly to disconnect the faulty circuit. This time delay characteristic is useful to avoid unwanted interruptions during the starting of motors and similar instances where the initial current may be high, but not an overload condition.

Earth Leakage Circuit Breaker (ELCB): For proper operation of ELCB (shown in Fig. (c)) two earth terminals are required. These are the frame earth to which all non-metallic parts of equipment are connected, and the ELCB reference earth. The ELCBs normally operate when the voltage across the coil, which corresponds to the voltage of the frame earth with respect to the reference earth, exceeds about 40 V.

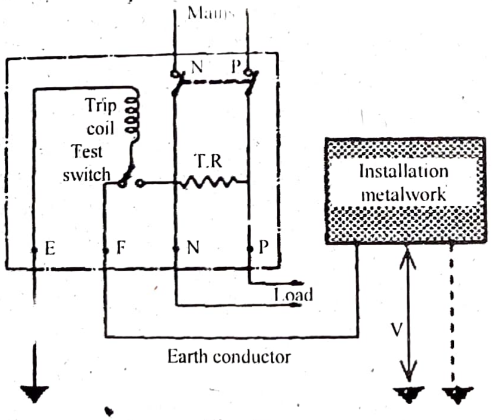


Fig: (c)

Residual Current Circuit Breakers (RCCB) or Residual Current Device (RCD): The operation is based on a fault current, causing a difference between the line current and the neutral current (the difference need not actually flow to earth but back to the circuit through an unplanned path). This difference is used to energize the solenoid, which causes the switch to open. Under normal operating conditions, two identical windings, m_1 and m_2 will carry the main current. Since the currents are equal and opposite through the two windings, there is mmf balance and there will be no induced emf on the detector winding. Thus the operating coil will not be energized. However, in case of a fault the line and neutral currents will not equal and the trip coil will be energized due to

the induced currents in the detector winding. In both the ELCB and the RCCB (Fig. (d)), a test switch 'T' is provided to create an artificial fault for test purposes.

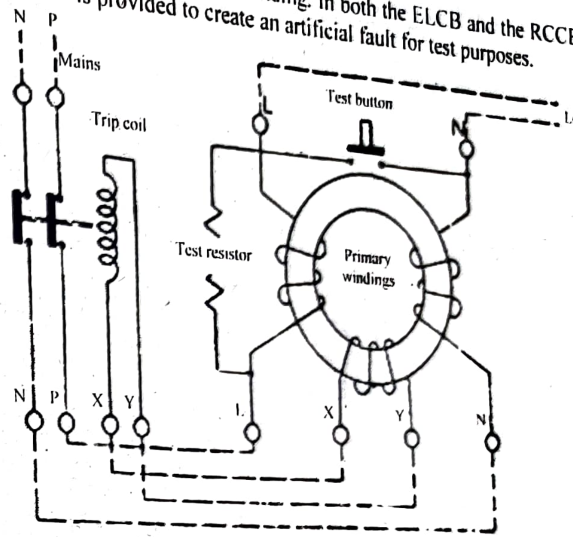


Fig: (d)

Advantages of RCCB:

If the live wire is exposed (the insulation is open) and then somebody touches it, he may get a shock if a current passes through him. In the case of voltage operated ELCB this earth current is not going through the tripping coil and will cause danger. But in the case of RCCB, the return path is going to loose part of the current, which passed through the human body, which in turn would cause a resultant flux within the ring energizing the tripping circuit.

3.3. a) What is earthing or grounding?
b) Describe different earthing. **[MODEL QUESTION]**

Answer:

a) Earthing is carried out in an electrical installation for the purpose of
(a) limiting the potential (voltage) of current carrying conductors forming a part of the system called "neutral earthing"
(b) limiting the potential of non-current carrying metal work associated with equipment, apparatus and appliances in the system called as "equipment earthing". The potential of an installation is measured with respect to the general mass of the earth or commonly called earth. Thus the potential is limited with respect to earth.

Neutral earthing: This is important because the performance of the system in terms of short circuits, stability, protection, etc., is greatly affected by the state of the neutral conductor. When the neutral is properly grounded, voltages of the phases are limited to near phase to ground voltage.

Equipment earthing: This refers to grounding of all metal work of equipment other than the parts which are normally current carrying. This is governed by various regulations such as the IEE regulations. The objective of this grounding is to ensure effective and rapid operation of the protective gear in the event of earth fault currents which might otherwise be undetected and cause fire and also protect against danger to life through shock due to installation metal work being maintained at a dangerous potential relative to earth.

b) *Types of earthing arrangement:*
In the regulations for electrical installations, the types of earthing systems are identified as follows, depending on the relationship of the source (supply authority network) and of the exposed conductive parts of the installation, to earth.
These are
(1) TN - earthing of the installation is done to that supplied by the supply authority.
(2) TT - supply authority earth and the installation earth is independent.
(3) IT - supply authority has effectively an isolated neutral and the installation has an independent earth.

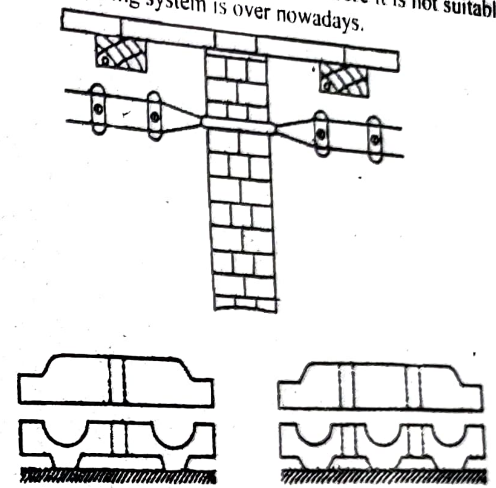
In these, the first letter denotes the earthing arrangement at the supply authority side and the second letter denotes the relationship of the exposed conductive parts of the installation to earth.
With the First letter, **T** (short for terra or earth) refers to a direct connection of one or more points of the source to earth, and **I** (short for isolated) indicated that all live parts are isolated from earth or one point connected to earth through a high impedance.
With the Second letter, **T** denotes a direct electrical connection of the exposed conductive parts of the consumers installation to earth, independently of the earthing of any point of the supply authority side, while **N** denotes a direct electrical connection of the exposed conductive parts to the earthed point of the supply authority side, which for ac is usually the neutral point.

3.4. Describe different types of Electrical Wiring system and their advantages and disadvantages. [MODEL QUESTION]

Answer:
The types of internal wiring usually used are
a) Cleat wiring
b) Wooden casing and capping wiring
c) CTS or TRS or PVC sheath wiring
d) Lead sheathed or metal sheathed wiring
e) Conduit wiring

There are additional types of conduit wiring according to Pipes installation (Where steel and PVC pipes are used for wiring connection and installation).
i) Surface or open Conduit type
ii) Recessed or concealed or underground type Conduit

a) Cleat Wiring: This system of wiring (Fig. 1) comprises of ordinary VIR or PVC insulated wires (occasionally, sheathed and weather proof cable) braided and compounded held on walls or ceilings by means of porcelain cleats, Plastic or wood. Cleat wiring system is a temporary wiring system therefore it is not suitable for domestic premises. The use of cleat wiring system is over nowadays.



i. Cleat with two grooves   ii. Cleat with three grooves

Fig: 1

Advantages of Cleat Wiring:
- It is simple and cheap wiring system
- Most suitable for temporary use i.e. under construction building or army camping
- As the cables and wires of cleat wiring system is in open air, therefore fault in cable scan be seen and repair easily.
- Cleat wiring system installation is easy and simple.
- Customization can be easily done in this wiring system e.g. alteration and addition.
- Inspection is easy and simple.

Disadvantages of Cleat Wiring:
- Appearance is not so good.
- Cleat wiring can't be use for permanent use because, Sag may be occur after sometime of the usage.
- In this wiring system, the cables and wiring is in open air, therefore, oil, steam, humidity, smoke, rain, chemical and acidic effect may damage the cables and wires.
- It is not lasting wire system because of the weather effect, risk of fire and wear & tear.

- It can be only used on 250/440 Volts on low temperature.
- There is always a risk of fire and electric shock.
- It can't be used in important and sensitive location and places.
- It is not lasting, reliable and sustainable wiring system.

b) Casing and Capping wiring: Casing and Capping wiring system was famous wiring system in the past but, it is considered obsolete this days because of Conduit and sheathed wiring system. The cables used in this kind of wiring were either VIR or PVC or any other approved insulated cables.
The cables were carried through the wooden casing enclosures. The casing is made up of a strip of wood with parallel grooves cut length wise so as to accommodate VIR cables. The grooves were made to separate opposite polarity. The capping (also made of wood) used to cover the wires and cables installed and fitted in the casing.

Advantages of Casing Capping Wiring:
- It is cheap wiring system as compared to sheathed and conduit wiring systems.
- It is strong and long-lasting wiring system.
- Customization can be easily done in this wiring system.
- If Phase and Neutral wire is installed in separate slots, then repairing is easy.
- Stay for long time in the field due to strong insulation of capping and casing..
- It stays safe from oil, Steam, smoke and rain.
- No risk of electric shock due to covered wires and cables in casing & capping.

Disadvantages Casing Capping Wiring:
- There is a high risk of fire in casing & capping wiring system.
- Not suitable in the acidic, alkalies and humidity conditions
- Costly repairing and need more material.
- Material can't be found easily in the contemporary
- White ants may damage the casing & capping of wood.

c) Batten Wiring (CTS or TRS): Single core or double core or three core TRS cables with a circular oval shape cables are used in this kind of wiring. Mostly, single core cables are preferred. TRS cables are chemical proof, water proof, steam proof, but are slightly affected by lubricating oil. The TRS cables are run on well seasoned and straight teak wood batten with at least a thickness of 10mm.
The cables are held on the wooden batten by means of tinned brass link clips (buckle clip) already fixed on the batten with brass pins and spaced at an interval of 10cm for horizontal runs and 15cm for vertical runs.

Advantages of Batten Wiring
- Wiring installation is simple and easy
- cheap as compared to other electrical wiring systems

- Paraphrase is good and beautiful
- Repairing is easy
- strong and long-lasting
- Customization can be easily done in this wiring system.
- less chance of leakage current in batten wiring system

Disadvantages of Batten Wiring
- Can't be install in the humidity. Chemical effects, open and outdoor areas.
- High risk of fire
- Not safe from external wear & tear and weather effects (because, the wires are openly visible to heat, dust, steam and smoke.
- Heavy wires can't be used in batten wiring system.
- Only suitable below then 250V.
- Need more cables and wires.

d) Lead Sheathed Wiring: The type of wiring employs conductors that are insulated with VIR and covered with an outer sheath of lead aluminum alloy containing about 95% of lead. The metal sheath given protection to cables from mechanical damage, moisture and atmospheric corrosion.
The whole lead covering is made electrically continuous and is connected to earth at the point of entry to protect against electrolytic action due to leaking current and to provide safety in case the sheath becomes alive. The cables are run on wooden batten and fixed by means of link clips just as in TRS wiring.

e) Conduit Wiring: There are two additional types of conduit wiring according to pipe installation.
i) Surface Conduit Wiring
ii) Concealed Conduit Wiring

i) Surface Conduit Wiring: If conduits installed on roof or wall, it is known as surface conduit wiring. In this wiring method, they make holes on the surface of wall on equal distances and conduit is installed then with the help of rawal plugs.
ii) Concealed Conduit wiring: If the conduits are hidden inside the wall slots with the help of plastering, it is called concealed conduit wiring. In other words, the electrical wiring system inside wall, roof or floor with the help of plastic or metallic piping is called concealed conduit wiring. It is the **most popular, beautiful, stronger and common electrical wiring system** nowadays. In conduit wiring, steel tubes known as conduits are installed on the surface of walls by means of pipe hooks (surface conduit wiring) or buried in walls under plaster and VIR or PVC cables are afterwards drawn by means of a GI wire of size if about 18SWG.

Types of Conduit
Following conduits are used in the conduit wiring systems (both concealed and surface conduit wiring) which are shown in the above image.

- Metallic Conduit
- Non-metallic conduit

Metallic Conduit:
Metallic conduits are made of steel which are very strong but costly as well.
There are two types of metallic conduits.

- Class A Conduit: Low gauge conduit (Thin layer steel sheet conduit)
- Class B Conduit: High gauge conduit (Thick sheet of steel conduit)

Non-metallic Conduit:
A solid PVC conduit is used as non-metallic conduit now a days, which is flexible and easy to bend.

Size of Conduit:
he common conduit pipes are available in different sizes genially, 13, 16.2, 18.75, 20, 5, 37, 50, and 63 mm (diameter) or 1/2, 5/8, 3/4, 1, 1.25, 1.5, and 2 inch in diameter.

dvantage of Conduit Wiring Systems:

- It is the safest wiring system (Concealed conduit wring)
- Appearance is very beautiful (in case of concealed conduit wiring)
- No risk of mechanical wear & tear and fire in case of metallic pipes.
- Customization can be easily done according to the future needs.
- Repairing and maintenance is easy.
- There is no risk of damage the cables insulation.
- It is safe from corrosion (in case of PVC conduit) and risk of fire.
- It can be used even in humidity, chemical effect and smoky areas.
- No risk of electric shock (In case of proper earthing and grounding of metallic pipes).
- It is reliable and popular wiring system.
- Sustainable and long-lasting wiring system.

vantages of Conduit Wiring Systems:

- It is expensive wiring system (Due to PVC and Metallic pipes, Additional earthing for metallic pipes Tee(s) and elbows etc.
- Very hard to find the defects in the wiring.
- Installation is not easy and simple.
- Risk of Electric shock (In case of metallic pipes without proper earthing system)
- Very complicated to manage additional connection in the future.

3.5. Describe the different types of batteries. **[MODEL QUESTION]**

Answer:

A battery is a combination of one or more electrochemical cells that are capable of converting stored chemical energy into electrical energy. Depending upon the working principle batteries may be divided into two groups:

i) Primary batteries: Primary batteries are disposable because their electrochemical reaction cannot be reversed. Once these batteries are used they cannot be recharged and active materials may not return to their original forms. Battery manufacturers recommend against recharge of primary cells. Some of the examples for the disposable/Primary batteries are the normal AA, AAA batteries which are used in wall clocks, television remote etc.

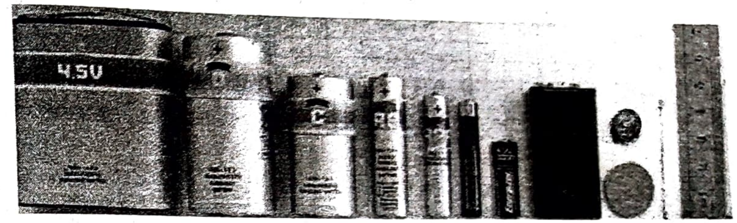


Fig: (i) Primary batteries

ii) Secondary batteries: Secondary batteries are rechargeable, because their electrochemical reaction can be reversed by applying a certain voltage to the battery in the opposite direction of the discharge. Chargers are devices which supply the required voltage. Some examples for these rechargeable batteries are the batteries used in mobile phones, MP3 players etc. Devices such as hearing aids and wristwatches use miniature cells and in places such as telephone exchanges or computer data centre's, larger batteries are used.

Fig: (ii) Secondary batteries

Secondary (rechargeable) Batteries are classified as

a) SMF
b) Lead Acid
c) Li and
d) Nicd

a) SMF: It is a sealed maintenance free battery, designed to offer reliable, consistent and low maintenance power for UPS applications. These batteries can be subject to deep cycle applications and minimum maintenance in rural and power deficit areas. These batteries are available from 12V.

Fig: (iii)

b) Lead Acid Battery: Lead Acid batteries are widely used in automobiles, inverters, backup power systems etc. Unlike tubular and maintenance free batteries, Lead Acid batteries require proper care and maintenance to prolong its life. The Lead Acid battery consists of a series of plates kept immersed in sulphuric acid solution. The plates have grids on which the active material is attached. The plates are divided into positive and negative plates. The positive plates hold pure lead as the active material while lead oxide is attached on the negative plates.

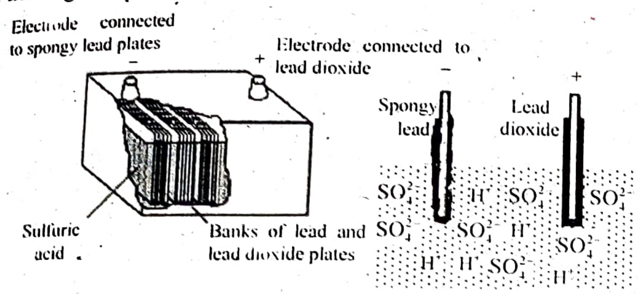


Fig: Lead acid battery

A completely charged battery can discharge its current when connected to a load. During the process of discharge, the sulphuric acid combines with the active materials on the positive and negative plates resulting in the formation of Lead sulphate. Water is the single most important step in maintaining a Lead Acid battery. The frequency of water depends on usage, charge method and operating temperature. During process, the hydrogen atoms from the sulphuric acid react with oxygen to form water. This results in the release of electrons from the positive plates which will be accepted by the negative plates. This leads to the formation of an electric potential across the battery. The electrolyte in the Lead Acid battery is a mixture of Sulphuric acid and water which has a specific gravity. Specific gravity is the weight of the acid-water mixture compared to equal volume of water. The specific gravity of pure ions free water is 1. The lead-acid batteries provide the best value for power and energy per kilowatt-hour; have the longest life cycle and a large environmental advantage in that they are recycled at an extraordinarily high rate.

c) Lithium (Li) Battery: We all use it in portable devices such as cell phone, a laptop computer or a power tool. The lithium battery has been one of the greatest achievements in portable power in the last decade; with use of lithium batteries we have been able to shift from black and white mobile to colour mobiles with additional features like GPS, mail alerts etc. These are the high energy density potential devices for higher capacities. And relatively low self-discharge batteries. Also Special cells can provide very high current to applications such as power tools.

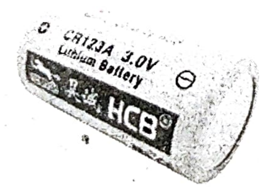


Fig: Li Battery

Lithium-Ion batteries are now popular in majority of electronic portable devices like Mobile phone, Laptop, Digital Camera, etc due to their long lasting power efficiency. These are the most popular rechargeable batteries with advantages like best energy density, negligible charge loss and no memory effect. Li-Ion battery uses Lithium ions as the charge carriers which move from the negative electrode to the positive electrode during discharge and back when charging. During charging, the external current from the charger applies an over voltage than that in the battery. This forces the current to pass in the reverse direction from the positive to the negative electrode where the lithium ions get embedded in the porous electrode material through a process called Intercalation. The Li-ions pass through the non aqueous electrolyte and a separator diaphragm. The electrode material is intercalated lithium compound.

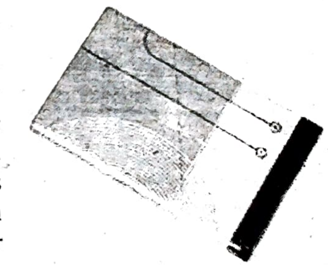


Fig: Li-ion battery

The negative electrode of the Li-Ion battery is made up of carbon and the positive electrode is a metal oxide. The most commonly used material in the negative electrode is Graphite while that in the positive electrode may be Lithium cobalt oxide, Lithium ion phosphate or Lithium manganese oxide. Lithium salt in an organic solvent is used as the electrolyte. The electrolyte is typically a mixture of organic carbonates like Ethylene carbonate or Diethyl carbonate containing lithium ions. The electrolyte uses anion salts like Lithium hexa fluoro phosphate, Lithium hexa fluoro arsenate monohydrate, Lithium per chlorate, Lithium hexa fluoro borate etc. Depending upon the salt used, the voltage, capacity and life of the battery varies. Pure lithium reacts with water vigorously to form lithium hydroxide and hydrogen ions. So the electrolyte used is non aqueous organic solvent. The electrochemical role of the electrodes charge between anode and cathode depends on the direction of current flow. In the Li-Ion battery, both the electrodes can accept and release lithium ions. During the Intercalation process, the lithium ions move into the electrode. During the reverse process called de intercalation, the lithium ions move back. During discharging, the positive lithium ions will be extracted from the negative electrodes and inserted into the positive electrode. During the charging process, the reverse movement of lithium ions takes place.

Advantages of Lithium-Ion Battery:

Lithium Ion batteries outperform NiCd batteries and other secondary batteries. Some of the advantages are

- Light weight compared to other batteries of similar size
- Available in different shape including Flat shape
- High open circuit voltage that increases the power transfer at low current
- Lack of memory effect.
- Very low self discharge rate of 5-10% per month. Self discharge is around 30% in NiCd and NiMh batteries.
- Eco-friendly battery without any free lithium metal