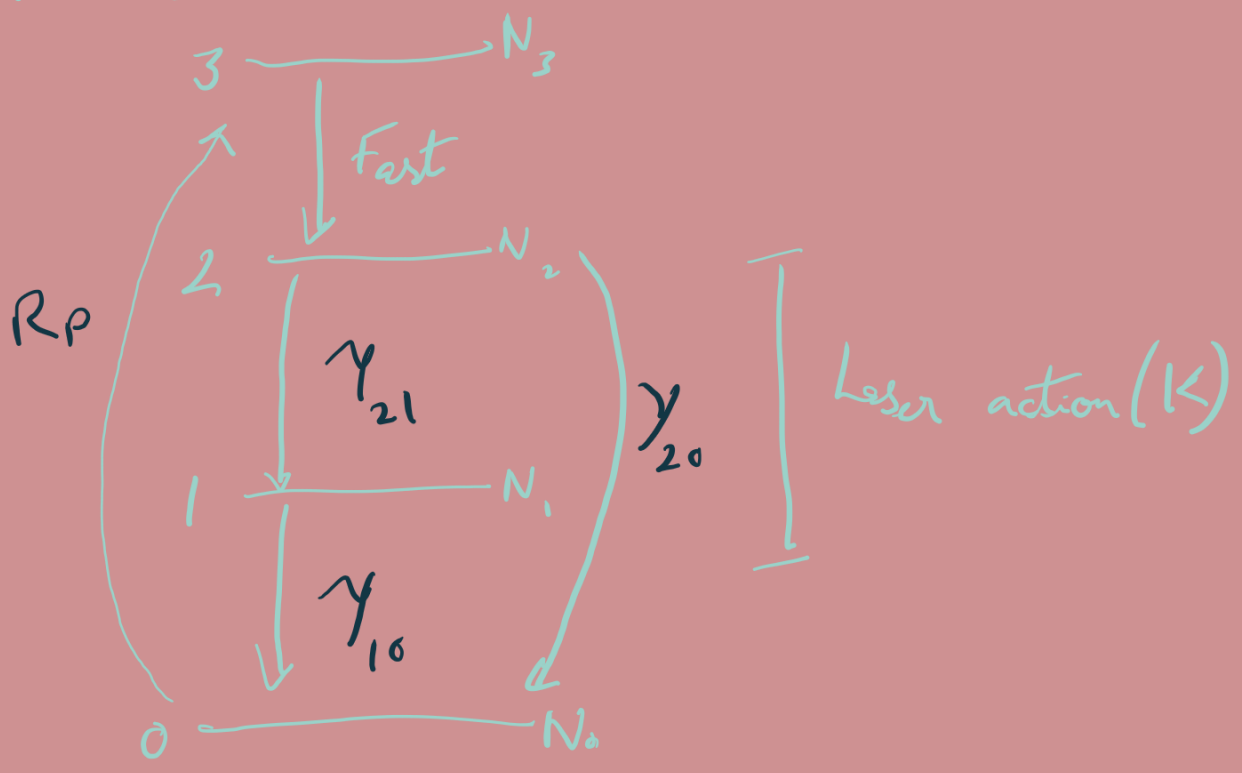
H.W 2

1.



Adding direct decay line of γ_{20}

$$\frac{dN_2}{dt} = R_p - \gamma_{21}N_2 - \gamma_{20}N_2 - K_n(N_2 - N_1) \quad \text{---(i)}$$

$$\frac{dN_1}{dt} = \gamma_{21}N_2 - \gamma_{10}N_1 + K_n(N_2 - N_1) \quad \text{---(ii)}$$

At steady state,

$$\frac{dN}{dt} = 0 \;\&\; K_n = 0$$

$$\therefore N_2 = \boxed{\frac{R_p}{\gamma_{21} + \gamma_{20}}}$$

Subst. in (ii),

$$\gamma_{10} N_1 = \gamma_{21} N_2 \text{ or, } N_1 = \boxed{\frac{\gamma_{21} R_p}{\gamma_{10}(\gamma_{21} + \gamma_{20})}}$$

For inversion,

$$N_2 > N_1 \text{ or, } 1 > \frac{\gamma_{21}}{\gamma_{10}}$$

$$\text{or, } \gamma_{10} > \gamma_{21}$$

Also,

$$\Delta N = N_2 - N_1 = \frac{R_p}{\gamma_{21} + \gamma_{20}} \left(1 - \frac{\gamma_{21}}{\gamma_{20}}\right)$$

2. Given,

Length $L = 7.5\,cm$

diameter $d = 1\,cm$

ion density $(n) = 1.58 \times 10^{17}$ ions $/cm^3$

$\lambda = 700\,nm$ (assumption) [round up of ruby]

$\therefore$ Total ions $(N) = \pi r^2 L \cdot n$

$\approx 9.307 \times 10^{19}$ ions

$E_p = \frac{hc}{\lambda} \approx 2.8397 \times 10^{-19}\,J$

$E_{max} = N \cdot E_p$

$\approx 26.43\,J$

$P_{peak} = \frac{E_{max}}{\Delta T} = \frac{26.43}{5 \times 10^{-8}} \approx 5.29 \times 10^8\,W$

$= 0.529\,GW$

2.1. Power density $= \frac{5.29 \times 10^8}{0.75 (10^{-1})^2} \approx 70.5\,GW/cm^2$

$$|E|_{rot} = \sqrt{\frac{2I}{c\epsilon_0}} \qquad \epsilon_0 = 8.854 \times 10^{-12}$$

$$= \sqrt{3.98 \times 10^{17}}$$

$$\approx 6.31 \times 10^{8} \text{ V/m}$$

3.

$$E_{cav_1} = t_1 E_{in} e^{-ikL}$$

$$E_{cav_2} = t_1 E_{in} e^{-ikL} (r_1 r_2 e^{2ikL})$$

$$\vdots$$

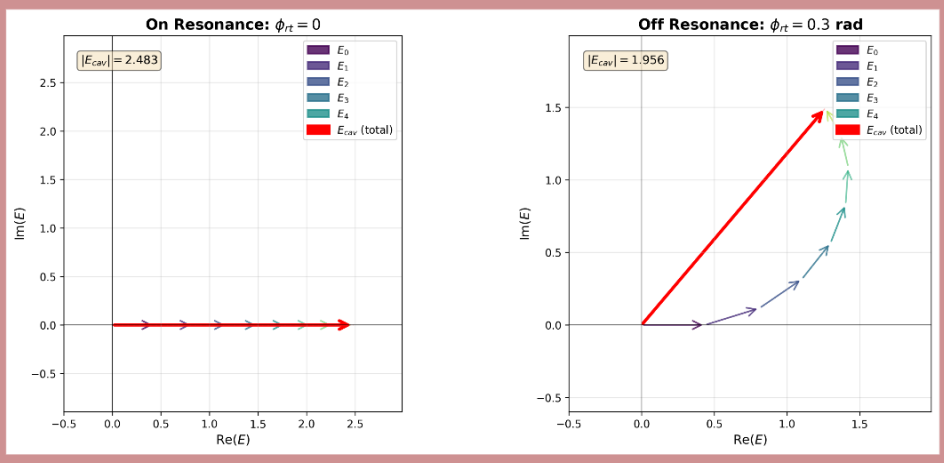
$$E_{cav_n} = t_1 E_{in} e^{-ikL} (r_1 r_2 e^{2ikL})^{n-1}$$

Adding all,

$$a(1 + r + r^2 + \dots) = \frac{ar^{n+1}}{1-r} \quad [\text{when } r < 1]$$

$$\therefore E_n = \frac{t_1 E_{in} e^{-ikL}}{1 - r_1 r_2} \quad \text{——— (a)}$$

3.2.



3.3.

1. round trip delay $(\tau_{rt}) = \frac{2L}{c}$

2. E_0 arrives at 0 second.

E_1 at τ_{rt}

E_2 at $2\tau_{rt}$.

⋮

E_n at $n \cdot \tau_{rt}$.

3. Insertity $e^{i2kL} = 1$ in (a),

$$E_{cav} = \frac{E_{in} t_1}{1 - r_1 r_2}$$

4. $E_{cav}(t) = \frac{t_1 E_{in}}{1 - r_1 r_2} \left(1 - r_1 r_2^{\,t/\tau_{rt}}\right)$ (not sure)

not sure how to proceed.

But,

$$\tau_{storage} = \frac{\tau_{rt}}{1-r}$$

$$\therefore \nu_p = \frac{F.S.R\ (1-r_1 r_2)}{2\pi\sqrt{r_1 r_2}}$$

$$F.S.R = \frac{c}{2L} = \frac{1}{2\tau_{rt}}$$

$$\therefore \nu_p = \frac{1-r_1 r_2}{4\pi\ \tau_{rt}\sqrt{r_1 r_2}}$$

If r close to 1,

$$\nu_p \approx \frac{1-r_1 r_2}{4\pi\,\tau_{rt}}$$

Somehow, we need it to give

$\frac{1}{2\pi\nu_p}$, as $\omega_p = 2\pi\nu_p$.

as the storage time and pole frequency should be inverse of each other.

4. $$\mathcal{F} = \frac{F.S.R}{\nu_{FWHM}}$$

$$\nu_{FWHM} = \frac{F.S.R\,(1 - r_1 r_2)}{2\pi\sqrt{r_1 r_2}} \quad [\text{Fabry perot}]$$

$$\therefore \mathcal{F} = \frac{2\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Given,

$$r_1 = 1 - T_1 - L_1$$

$$r_2 = 1 - T_2 - L_2$$

$$\therefore r_1 r_2 = 1 - T_1 - L_1 - T_2 - L_2 + (T_1 + L_1)(T_2 + L_2)$$

For high finesse, $T \ll L \ll 1$.

$$\therefore r_1 r_2 \approx 1 - (T_1 + T_2 + L_1 + L_2)$$

By binomial,

$$\sqrt{r_1 r_2} = \sqrt{1 - (T_1 + T_2 + L_1 + L_2)}$$

$$= 1 - \frac{T_1 + T_2 + L_1 + L_2}{2}$$

$$\therefore f = \frac{2\pi\left(1 - \frac{T_1 + T_2 + L_1 + L_2}{2}\right)}{1 - \left(1 - \frac{T_1 + T_2 + L_1 + L_2}{2}\right)}$$

$$= \frac{2\pi}{T_1 + T_2 + L_1 + L_2} - \pi$$

if $L \ll 1$,

$$= \frac{2\pi}{T_1 + T_2 + L_1 + L_2} \approx \frac{2\pi}{L_{total}}$$

5. For critically coupled,

$$T_1 = T_2 + L_1 + L_2$$

$$\therefore L_{total} = 2T_1$$

$$\therefore G_{ain} = G_{cav} = \frac{I_{circ}}{I_{in}} = \frac{T_1}{(1 - r_1 r_2)^2}$$

$$= \frac{T_1}{L_{total}^2}$$

$$f = \frac{2\pi}{L_{total}}$$

$$= \frac{1}{2L_{total}}$$

$$G_{cav} = \frac{F}{4\pi}$$

For over coupled,

$$T_1 > T_2 + L_1 + L_2 \;,\; T_2 \approx 0 \;\&\; L_2 \approx 0$$

$$L_{total} = T_1 + L_1$$

$$\approx T_1 \quad [L_1 << T_1]$$

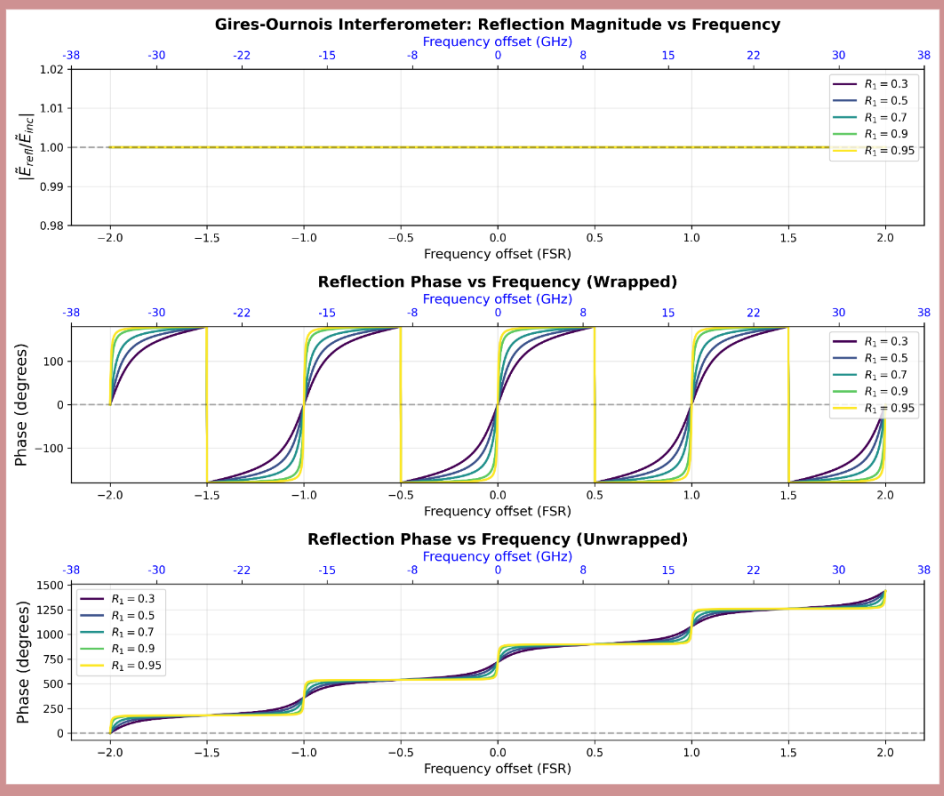
$$G_{cav} = \frac{T_1}{L^2_{total}} = \frac{1}{T_1} = \frac{1}{L_{total}}$$

$$= \frac{F}{2\pi}$$

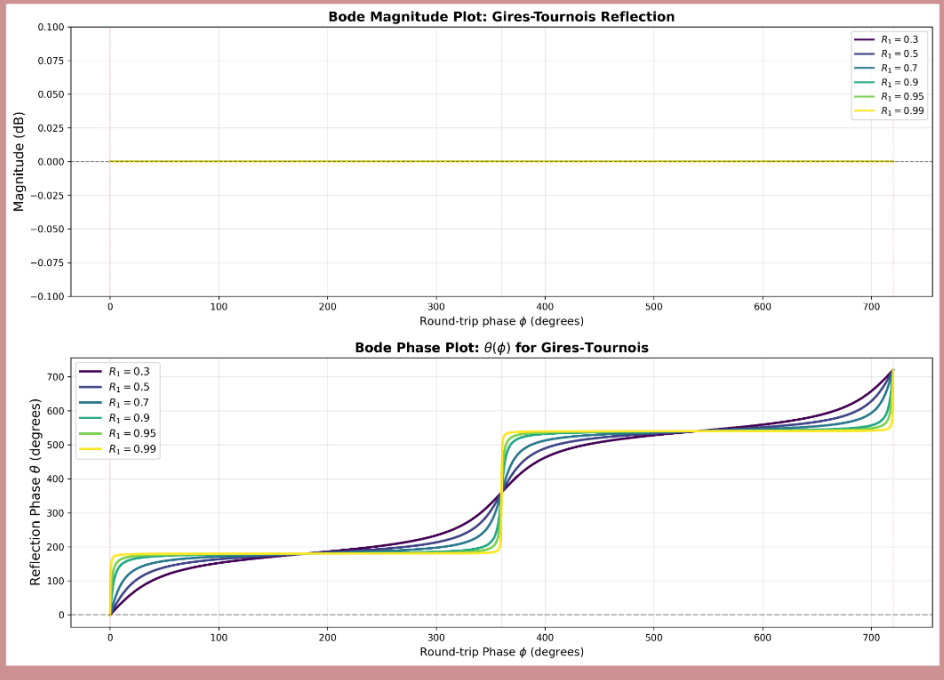
Clearly, we have more power for over coupled.

6. $R_1 =$ finite, $L_2 = 100\%$, $\delta_0 = 0$ [no absorption]

$$\frac{E_{refl}}{E_{inc}} = \frac{-r_1 + e^{i\phi}}{1 - r_1 e^{i\phi}}$$


Gires-Ournois Interferometer: Reflection Magnitude vs Frequency
Frequency offset (GHz)
-38 -30 -22 -15 -8 0 8 15 22 30 38
$|\tilde{E}_{refl}/\tilde{E}_{inc}|$
1.02 1.01 1.00 0.99 0.98
$R_1 = 0.3$
$R_1 = 0.5$
$R_1 = 0.7$
$R_1 = 0.9$
$R_1 = 0.95$
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
Frequency offset (FSR)
Reflection Phase vs Frequency (Wrapped)
Frequency offset (GHz)
-38 -30 -22 -15 -8 0 8 15 22 30 38
Phase (degrees)
100 0 −100
$R_1 = 0.3$
$R_1 = 0.5$
$R_1 = 0.7$
$R_1 = 0.9$
$R_1 = 0.95$
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
Frequency offset (FSR)
Reflection Phase vs Frequency (Unwrapped)
Frequency offset (GHz)
-38 -30 -22 -15 -8 0 8 15 22 30 38
Phase (degrees)
1500 1250 1000 750 500 250 0
$R_1 = 0.3$
$R_1 = 0.5$
$R_1 = 0.7$
$R_1 = 0.9$
$R_1 = 0.95$
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5 2.0
Frequency offset (FSR)

6.1.


Bode Magnitude Plot: Gires-Tournois Reflection
Magnitude (dB)
0.100 0.075 0.050 0.025 0.000 −0.025 −0.050 −0.075 −0.100
$R_1 = 0.3$
$R_1 = 0.5$
$R_1 = 0.7$
$R_1 = 0.9$
$R_1 = 0.95$
$R_1 = 0.99$
0 100 200 300 400 500 600 700
Round-trip phase ϕ (degrees)
Bode Phase Plot: $\theta(\phi)$ for Gires-Tournois
Reflection Phase θ (degrees)
700 600 500 400 300 200 100 0
$R_1 = 0.3$
$R_1 = 0.5$
$R_1 = 0.7$
$R_1 = 0.9$
$R_1 = 0.95$
$R_1 = 0.99$
0 100 200 300 400 500 600 700
Round-trip Phase ϕ (degrees)