4.(a) By Maxwell's,

$$\hat{k} \cdot E = 0$$

So, $\hat{k} \cdot (\hat{x} + \hat{y}) = 0$ —— (i)

Let $\hat{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$

For (i) to work,

$$-k_y = k_x$$

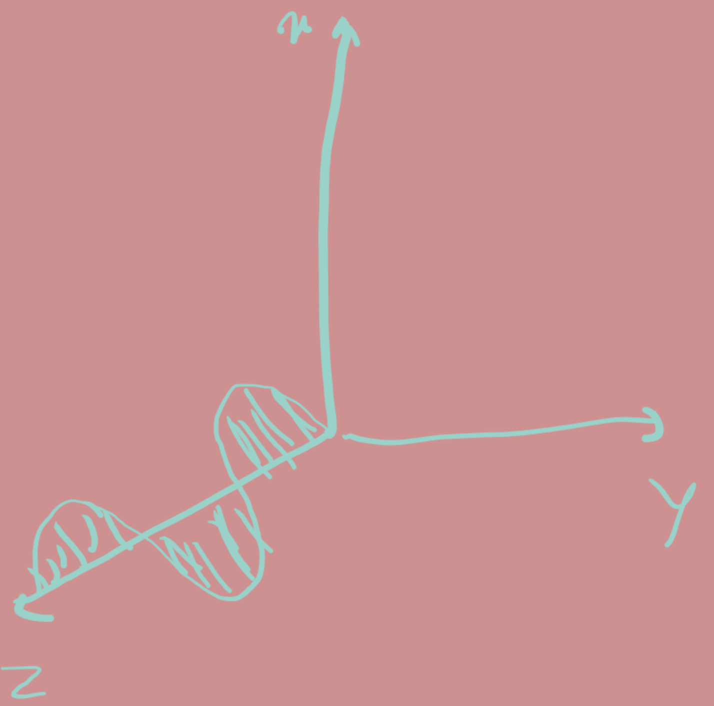
$$\therefore \hat{k} = \frac{1}{\sqrt{2k_x^2 + k_z^2}} \left(k_x \hat{x} - k_x \hat{y} + k_z \hat{z} \right)$$

B is perpendicular to E.

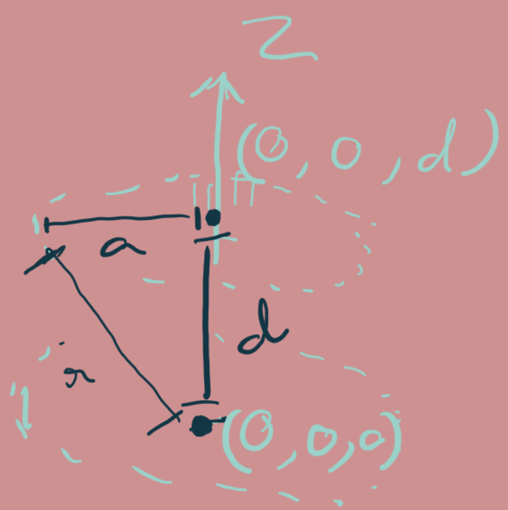
$$\therefore \hat{k} \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ k_x & k_y & k_z \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \boxed{k_z \hat{x} + (k_y - k_x)\hat{y} + (k_y - k_x)\hat{z}}$$

Simplest diagram: $[\hat{k} = \hat{z}]$



5.(a) $S = \frac{1}{\mu_0}(E \times B)$



[S is Poynting vector]

Given,

Distance from centre: r

Total power: P_{total}

We know,

Surface of radius r

$$P_{total} = \oint \langle S \rangle \cdot da = \langle S \rangle \langle 4\pi r^2 \rangle$$

or, $\langle S \rangle = P_{total} / 4\pi r^2$

Also, direction perpendicular to B & E.

$\therefore \hat{r}$.

$$\therefore S = \frac{P_{total}}{4\pi r^2}\hat{r}$$

(b) At mirror centre,

$r = d$.

$$\therefore S = \boxed{\frac{P_{total}}{4\pi d^2}\hat{z}}$$

At mirror edge,

$$r = \sqrt{a^2+d^2}$$

$$\hat{r}_{edge} = \frac{a\hat{x} + y\hat{y} + d\hat{z}}{\sqrt{a^2+d^2}}$$

$$\therefore S_{edge} = \frac{P_{total}}{4\pi(a^2+d^2)}(\hat{r}_{edge}) = \boxed{\frac{P_{total}}{4\pi(a^2+d^2)^{\frac{3}{2}}}\cdot(x\hat{x}+y\hat{y}+d\hat{z})}$$

At surface,

$$r = \sqrt{x^2+y^2+d^2}$$

$$\hat{r}_{surface} = \frac{x\hat{x}+y\hat{y}+d\hat{z}}{\sqrt{x^2+y^2+d^2}}$$

$$S_{surface} = \frac{P_{total}}{4\pi(x^2+y^2+d^2)^{\frac{3}{2}}} \cdot (x\hat{x}+y\hat{y}+d\hat{z})$$

(c). $I = |\langle S \rangle|$

$$= |S_{surface}|$$

$$= \boxed{\frac{P_{total}}{4\pi(x^2+y^2+d^2)}}$$

(d) Total power incident $= \int_{mirror} S_{surface} \cdot da$

$$= \int_0^a \int_0^{2\pi} \frac{P_{total}}{4\pi(r'^2+d^2)^{\frac{3}{2}}} (r' \cdot dr') \cdot d\phi$$

$$= \frac{\cancel{2\pi}\, P_{total}}{\cancel{4\pi}\, 2} \int_0^a \frac{r'\, dr'}{(r'^2+d^2)^{\frac{3}{2}}}$$ [$u = r'^2 + d^2$; $du = 2r' \cdot dr'$]

$$= \frac{P_{total}}{2}\left(1 - \frac{d}{\sqrt{a^2+d^2}}\right)$$

Comparing to P_{total},

$$\frac{P_{inc.}}{P_{total}} = \frac{1}{2}\left(1 - \frac{d}{\sqrt{a^2+d^2}}\right)$$

(l) $$P_{rad} = \frac{2I}{c}$$

$$= \frac{P_{total}}{2\pi c\,(a^2+d^2)}$$

$$F_{rad} = P_{rad} \times Area$$

$$= \frac{P_{total} \cdot a^2}{2c(a^2+d^2)}$$

$$a = \frac{F_{rad}}{m} = \frac{P_{total} \cdot a^2}{2mc(a^2+d^2)}$$