Optimization Assignment - 2

Ginna Shreyani

November 2022

1 Problem Statement

Show that if ABC is a triangle, and P any point then $(\mathbf{PA})^2 + (\mathbf{PB})^2 + (\mathbf{PC})^2$ will be minimum when $\mathbf{P}$ is at the centroid.

2 Solution:

Given, ABC is a triangle and $\mathbf{P}$ be any point, then

$$(\mathbf{P}-\mathbf{A})^2 + (\mathbf{P}-\mathbf{B})^2 + (\mathbf{P}-\mathbf{C})^2 \tag{1}$$

should be minimum when P is centroid.
For (1) to be minimum the differentiation of (1) should be equated to zero.

$$\frac{d}{d(x,y)}[(\mathbf{P}-\mathbf{A})^2 + (\mathbf{P}-\mathbf{B})^2 + (\mathbf{P}-\mathbf{C})^2] = 0 \tag{2}$$

By solving the (2), we get

$$2(\mathbf{P}-\mathbf{A})\frac{d}{d(P)}[(P-A)]+2(\mathbf{P}-\mathbf{B})\frac{d}{d(P)}[(P-B)]+2(\mathbf{P}-\mathbf{C})\frac{d}{d(P)}[(P-C)] = 0 \tag{3}$$

$$2((\mathbf{P}-\mathbf{A}) + (\mathbf{P}-\mathbf{B}) + (\mathbf{P}-\mathbf{C})) = 0$$
$$\mathbf{P} = \frac{\mathbf{A}+\mathbf{B}+\mathbf{C}}{3}$$

Therefore, minimum of (1) is obtained when P is the centroid of the triangle ABC.
Using cvxpy method,
Let us take the vertices of the triangle,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

The minimum value is given as,

$$\min_{\mathbf{P}}((\mathbf{P}-\mathbf{A})^{\mathbf{2}} + (\mathbf{P}-\mathbf{B})^{\mathbf{2}} + (\mathbf{P}-\mathbf{C})^{\mathbf{2}})$$

The minimum is obtained by equating the above equation to 0.

$$6\mathbf{P} = 2(\mathbf{A}+\mathbf{B}+\mathbf{C})$$
$$\mathbf{P} = \frac{1}{3}(\mathbf{A}+\mathbf{B}+\mathbf{C})$$
$$\mathbf{P} = \frac{1}{3}(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix})$$
$$\mathbf{P} = \begin{pmatrix} \frac{7}{3} \\ \frac{4}{3} \end{pmatrix}$$