

Probability Assignment

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A coin is tossed twice, what is the probability that atleast one tail occurs?

Solution

By using binomial distribution we can find the probability of atleast one tail occurs when a coin is tossed twice.

Here we are assuming when ever a tail shows up it is a success(1)

The Binomial Distribution: Let

$$Y = \sum_{i=1}^n X_i \quad (1)$$

where n is the total number of times the coin is tossed.

Then X has a binomial distribution. Then, for

$$p_{X_i}(n) \stackrel{z}{=} P_{X_i}(z), \quad (2)$$

yielding

$$P_{X_i}(Z) = 1 - p + pz^{-1} \quad (3)$$

Since X_i are i.i.d.,

$$P_X(Z) = (1 - p + pz^{-1})^n \quad (4)$$

$$= \sum_{k=0}^n {}^nC_k p^k (1 - p)^{n-k} \quad (5)$$

$$p_X(k) = \begin{cases} {}^nC_k p^k (1 - p)^{n-k}, & \text{if } 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$$

As a result it is written as

$$p_X(k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad (6)$$

The cumulative distribution function of X is defined as

$$F_X(r) = \Pr(X \leq r) = \sum_{k=0}^r \binom{n}{k} p^k (1 - p)^{n-k} \quad (7)$$

Therefore, we get the number of trials i.e., the number of times the coin is tossed and the probability of getting atleast one tail is sum of the probability of getting one tail and probability of getting two tails. The probability of getting a tail when a coin is tossed is 0.5.

$$n = 2 \quad (8)$$

$$r = 1, 2 \quad (9)$$

$$p = 0.5 \quad (10)$$

Therefore,

$$\Pr(Y \geq 1) = \sum_{k=1}^2 \binom{n}{k} p^k (1 - p)^{n-k} \quad (11)$$

$$= \binom{2}{1} (0.5)^1 (1 - 0.5)^{2-1} + \binom{2}{2} (0.5)^2 (1 - 0.5)^{2-2} \quad (12)$$

$$\Pr(Y \geq 1) = 0.5 + 0.25 \quad (13)$$

$$\Pr(Y \geq 1) = 0.75 \quad (14)$$

Therefore, the probability of getting atleast one tail when the coin is tossed twice is 0.75.

x(tails)	0	1	2
P(X=x)	0.25	0.5	0.25
P(X >= x)	1	0.75	0.25

This is the code: Code

