Probability Assignment

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Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Solution

Using Baye's Rule:
Let the probability of students living in hostel be $\Pr(H = 1)$, therefore the students who are day scholars can be given as $\Pr(H = 0)$
The probability of the students getting grade A is given as $\Pr(A = 1)$.
By given information,

$$\Pr(H = 1) = \frac{60}{100} \tag{1}$$

$$\Pr(H = 0) = \frac{40}{100} \tag{2}$$

$$\Pr(A = 1 \mid H = 1) = \frac{30}{100} \tag{3}$$

$$\Pr(A = 1 \mid H = 0) = \frac{20}{100} \tag{4}$$

Input Variable	Value	Decription
H	0	Students who are day scholars
	1	Students who are hostlers
Y	0	Students who got Grade A
	1	Students who didnot get Grade A

Table 1: Declarations

Thus,

$$\Pr(A = 1) = \sum_{i=0}^{1} \Pr(A = 1 \mid H = i) \Pr(H = i) \tag{5}$$

S.No.	Expression	Value
1.	Pr(H=0)	40/100
2.	Pr(H=1)	60/100
3.	$\Pr(A = 1|H = 0)$	20/100
4.	$\Pr(A = 1|H = 1)$	30/100

Table 2: Given Data

$$\Pr(A = 1) = \Pr(A = 1 \mid H = 0) \Pr(H = 0) \\ + \Pr(A = 1 \mid H = 1) \Pr(H = 1) \tag{6}$$

$$\Pr(A = 1) = \left(\frac{20}{100} \times \frac{40}{100}\right) + \left(\frac{30}{100} \times \frac{60}{100}\right) \tag{7}$$

$$\Pr(A = 1) = \frac{26}{100} \tag{8}$$

$$\Pr(H = 1 \mid A = 1) = \frac{\Pr(A = 1 \mid H = 1) \Pr(H = 1)}{\Pr(A = 0)} \tag{9}$$

$$\Pr(H = 1 \mid A = 1) = \frac{9}{13} \tag{10}$$

The probability that the student is a hostlier who has A grade is $\frac{9}{13}$.