

# AISI assignment-I

Roll no. MT2021130

① ⇒ closed form:

$$f(x, y, z) = x^2 + yz + 1$$

Here,  $\frac{\partial f}{\partial x} = 2x$ ,  $\frac{\partial f}{\partial y} = z$ ,  $\frac{\partial f}{\partial z} = y$

equating each equation to 0,

$$2x = 0 \Rightarrow x = 0$$

$$y = 0 \Rightarrow y = 0$$

$$z = 0 \Rightarrow z = 0$$

so, ans =  $(x, y, z) = (0, 0, 0)$

② ⇒ gradient descent:

$$f(x, y, z) = x^2 + yz + 1$$

learning rate  $\alpha = 0.1$

$$(x_{init}, y_{init}, z_{init}) = (2, 1, 1)$$

no. of iteration = 3



Here,

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2, \quad \frac{\partial f}{\partial z} = 4y$$

itn 1

Here  $(x_0, y_0, z_0) = (2, 1, 1)$

$$\left( \frac{\partial f}{\partial x} \right)_{(2,1,1)} = 4, \quad \left( \frac{\partial f}{\partial y} \right)_{(2,1,1)} = 2, \quad \left( \frac{\partial f}{\partial z} \right)_{(2,1,1)} = 4$$

$$\begin{aligned} x_1 &= x_0 - 0.1 \left( \frac{\partial f}{\partial x} \right)_{(x_0, y_0, z_0)} \\ &= 2 - (0.1)(4) \\ &= 1.6 \end{aligned}$$

Similarly,  $y_1 = 1 - (0.1)(2) = 0.8$

$$z_1 = 1 - (0.1)(4) = 0.6$$

itn 2

Here,  $(x_1, y_1, z_1) = (1.6, 0.8, 0.6)$

$$\text{So, } \left( \frac{\partial f}{\partial x} \right)_{(1.6, 0.8, 0.6)} = 3.2, \quad \left( \frac{\partial f}{\partial y} \right)_{(1.6, 0.8, 0.6)} = 1.6, \quad \left( \frac{\partial f}{\partial z} \right)_{(1.6, 0.8, 0.6)} = 3.2$$

$$\text{So, } x_2 = 1.6 - (0.1)(3.2) = 1.28$$

$$y_2 = 0.8 - (0.1)(1.6) = 0.64$$

$$z_2 = 0.6 - (0.1)(3.2) = 0.28$$



2+13

Here  $(x_2, y_2, z_2) = (1.28, 0.81, 0.81)$

$$\text{So, } \left( \frac{\partial f}{\partial x} \right)_{(x_2, y_2, z_2)} = 2.56 \quad \left( \frac{\partial f}{\partial y} \right)_{(x_2, y_2, z_2)} = 0.81 \quad \left( \frac{\partial f}{\partial z} \right)_{(x_2, y_2, z_2)} = 0.81$$

$$x_3 = 1.28 - (0.1)(2.56) = 1.024$$

$$y_3 = 0.81 - (0.1)(0.81) = 0.729$$

$$z_3 = 0.81 - (0.1)(0.81) = 0.729$$

So, after 3 it's

$$(x, y, z) = (1.024, 0.729, 0.729)$$

Q-3

(a) since, there are two input variables  $x_1, x_2$ . dataspace will be a 3 dimensional plot depending on data points.

(b) if we consider  $y = f(x_1, x_2)$   
Scenario ~~for~~ ~~modal~~ ~~space~~  
then hypothesis  $y$  for



Model space will be,

$$h_{\theta}(x_1, x_2) = y = \theta_1 + \theta_2 x_1 + \theta_3 x_2 \quad \text{--- (1)}$$

Cost function

$$J(\theta_1, \theta_2, \theta_3) = \sum_{i=1}^m (h_{\theta}(x_1, x_2) - y^i)^2 \quad \text{--- (2)}$$

So, from above cost function we can conclude that model space will be in  $\mathbb{R}^3$  space with discrete points.

③ as described in question 3⑥'s equation 1,

We need to calculate 3 parameters  $\theta_1, \theta_2, \theta_3$  for model space to create

a model.



+ Code + Text

✓ RAM   
Disk 

Editing



```
[1] import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```



### Gradient descent with 3 variables

$$f(x, y, z) = x^2 + yz + 1$$

$$\alpha = 0.1$$

Initially,  $(x, y, z) = (2, 1, 1)$

```
[2] def three_dimensional_loss(x, y, z):
    return x**2 + y*z + 1
```

```
[3] def three_dimensional_loss_gradient(x, y, z):
    ...
    partially derivating f(x,y,z) w.r.t x,y and z
    ...
    return (2*x, z, y)
```

RAM  
Disk

Editing



+ Code + Text

```
[3] three_dimensional_loss(x, y, z)

[4] x_init, y_init, z_init = 2, 1, 1

[5] learning_rate = 0.1  
num_iters = 2000

[6] x, y, z = x_init, y_init, z_init  
costs = []  
for _ in range(3):  
    cost = three_dimensional_loss(x, y, z)  
    costs.append(cost)  
    dx, dy, dz = three_dimensional_loss_gradient(x, y, z)  
    x = x - learning_rate*dx  
    y = y - learning_rate*dy  
    z = z - learning_rate*dz

[7] plt.plot(costs)  
plt.xlabel("Iteration")  
plt.ylabel("Cost")  
  
Text(0, 0.5, 'Cost')
```

6.0



0s

completed at 10:17 PM







+ Code + Text

✓ RAM   
Disk 

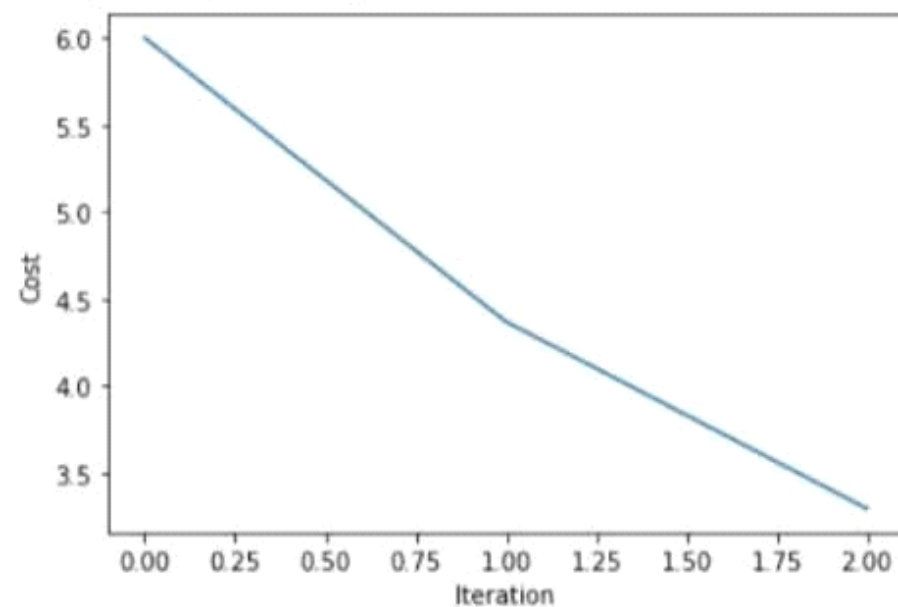
Editing



```
✓ [6] y = y - learning_rate*dy  
0s z = z - learning_rate*dz
```

```
✓ [7] plt.plot(costs)  
0s plt.xlabel("Iteration")  
plt.ylabel("Cost")
```

Text(0, 0.5, 'Cost')



```
✓ [8] (x, y, z)  
0s  
(1.024, 0.7290000000000001, 0.7290000000000001)
```

