IIIT-Bangalore
Course: BS 109 - Probability and Statistics
Problem Set 1

1. What is the probability of an odd sum when two dice are thrown? (Ans. $\frac{1}{2}$)

2. Two cards are drawn from a well-shuffled pack. Find the probability that at least one of them is spade. (Ans. $\frac{15}{34}$)

3. Two urns contain respectively 3 white, 7 red, 15 black balls and 10 white, 6 red and 9 black balls. One ball is drawn from each urn. Find the probability that both the balls are of same colour. (Ans. $\frac{207}{625}$)

4. The numbers $1, 2, \ldots, n$ ($n \geq 2$, $n \leq 9$) are arranged in random order. What is the probability that the numbers 1 and 2 are always together? (Ans. $\frac{2}{n}$)

5. From an urn containing n balls any number of balls are drawn. Show that the probability of drawing an even number of balls is $\frac{2^{n-1}-1}{2^n-1}$. (Ans. $\frac{2^{n-1}-1}{2^n-1}$)

6. If an even number of cards are drawn from a full pack find the probability these consist of half red and half black cards. (Ans. $\frac{\frac{52!}{(26!)^2}-1}{2^{51}-1}$)

7. Find the minimum number of times a die has to be thrown such that the probability of no six is less than half. (Ans. $\left\lceil \frac{\log 2}{\log 6 - \log 5} \right\rceil$)

8. A coin is tossed $(m+n)$ times $(m > n)$. Show that (i) the probability of exactly m consecutive heads is $(n+3)/2^{m+2}$ and (ii) the probability of at least m consecutive heads is $(n+2)/2^{m+1}$.

9. The integers x and y are chosen at random with replacement from nine natural numbers $1, 2, \ldots, 9$. Find the probability that $|x^2 - y^2|$ is divisible by 2. (Ans. $\frac{41}{81}$)

10. What is the probability that a bridge hand will contain (i) all the aces (ii) at least one ace. (Ans. (i) $\frac{{}^4C_4 \times {}^{48}C_9}{{}^{52}C_{13}}$, (ii) $1 - \frac{{}^{48}C_{13}}{{}^{52}C_{13}}$)