Modeling and Simulation, MC312
Lab-7
Due Date: October 24, 2023

In this lab, we will look at the one-dimensional random walk problem, one of the most basic and important models of stochastic processes with widespread applicability in physical and natural processes. An important objective of this lab is to review your understanding of probability, random processes and the limit theorems.

1. Imagine a one-dimensional discrete lattice with unit spacing. At each time instant, the walker moves to the right with a probability p and to the left with a probability $q = 1 - p$. We first consider an **unbiased random walk** $p = q = 1/2$ problem.

   (a) Simulate this process for n steps and look at the trajectory of the random walker. Trajectory here implies the position of the random walker at steps $1,\ 2, \cdots,\ n$. Plot a few trajectories. Are they the same? If not, why?

   (b) Numerically obtain the expected position of the random walker at each time step. Comment based on your observation.

   (c) Numerically obtain the variance of the random walker at each time step. Comment based on your observation.

   (d) Plot the probability distribution $P_n(m)$, which is the probability that the random walker is at location m after n steps.

2. **Biased random walk** Assume that the random walker has a preferred direction ($p \neq q$). Repeat all the steps of the previous problem and record how the observations change.

Your report should discuss the simulation details also from a Monte Carlo Simulation perspective.