Lab -7 & 8

Shreyans Jain (202103009),* Aaditya Meher (202103024),† and Vraj Thakkar (202103052)‡
Dhirubhai Ambani Institute of Information & Communication Technology,
Gandhinagar, Gujarat 382007, India
MC312, Modeling and Simulation

In this lab, we numerically and analytically model one-dimensional and two-dimensional random walks. It is one of the most basic and important models of stochastic processes with widespread applicability in physical and natural processes.

I. INTRODUCTION

We will try to model four problems, concerned with random walk problem. First, we consider a One-dimensional lattice, where the random walker is equally probable to go to either side of the lattice. In the other case, we considered when random walker has a probability p to go to a particular direction in the lattice. Similarly, we extended the problem to 2-dimensional lattice also and drawn interesting conclusios. We also plotted trajectories, expected positions, variance and probability distribution of the observations gathered from the above problems.

II. MODEL

We have a one-dimensional discrete lattice with unit spacing. At each time instant, the walker moves to the right with a probability p and to the left with a probability $q = 1 - p$.

A. Unbiased One-dimensional Random Walk

For unbiased random walk case, we consider $p = q = \frac{1}{2}$. Therefore, the walker have an equal probability of moving to left and right at each time interval. Due to this, the walker's trajectory will be entirely stochastic. It can vary from one simulation to another.

Let us assume the initial/starting position of the walker to be at 0. At each step/interval, we generate a random number to decide whether the random walker moves to the left or right. Let's say, random number is generated by a fair coin flip. If the generated Random Number > 0.5, walker moves to the right, the position will be increased by 1 unit, however if Random Number < 0.5, walker moves to the left, the position will be decreased by 1 unit.
We simulate multiple trajectories and plot them to see how they yield different paths in the case of unbiased random walk. The trajectories differ from one another, reflecting the stochastic nature of the random walk.

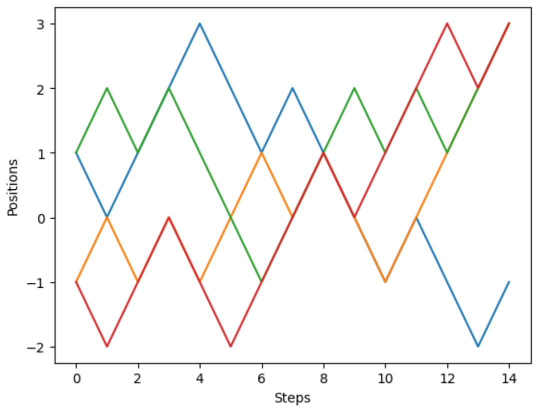

FIG. 1: We generate 4 different trajectory till $n = 15$ steps. We find different trajectories due to stochastic nature of the random walk.

To numerically obtain the expected position of the random walker, we simulate multiple random walks and then calculate the average position at each time step. After simulating all the trajectories, we calculate the average position at each time step by dividing the accumulated positions by the number of trajectories.

We let the initial point to be 0. After running simulations over larger number of trajectories, we observe that the expected position will remain around 0, indicating that, on average, the walker neither drifts to the left nor to the right.

*Electronic address: 202103009@daiict.ac.in
†Electronic address: 202103024@daiict.ac.in
‡Electronic address: 202103052@daiict.ac.in

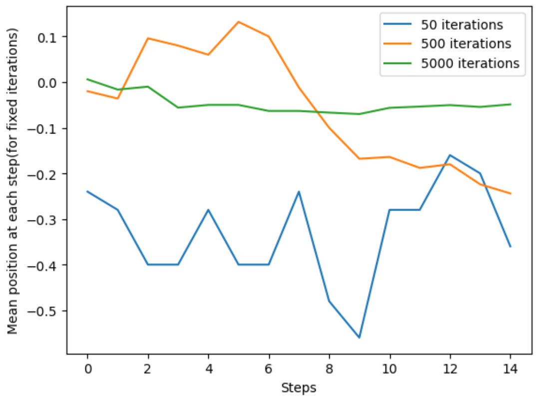

FIG. 2:

The variance provides a measure of how spread out or dispersed are the positions with respect to the average at each time interval. We observe that variance varies linearly with time, whoch shows that it is a difffusive process. The random walker is equally probable to move left or right at each time interval, resulting in a random walk with a variance that grows with time. The rate of growth of the variance will depend on the number of steps and the number of trials, but there is a linear increase in variance over time.

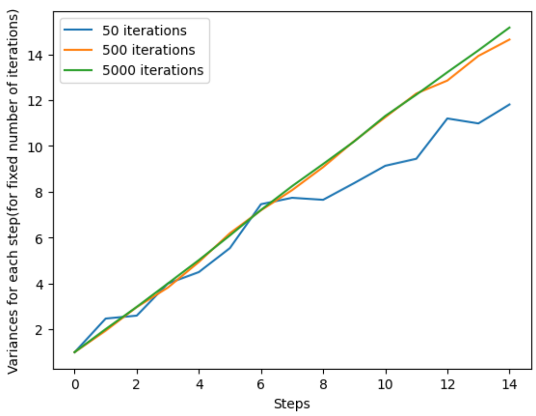

FIG. 3:

We'll try to find Probability distribution of the random walker is at location m after n steps by Monte-Carlo simulation. First, we will initialize a position variable to 0 (since walk starts from the origin). We simulate the random walk for a very large number of trial over n time intervals. Count the number of times the walker is at each position m after n steps. We can normalize the counts by dividing them by the total number of trials to obtain the estimated probability $P_n(m)$. Now, we can plot the following estimated probability distribution $P_n(m)$ as a function of m for 1000 iterations.

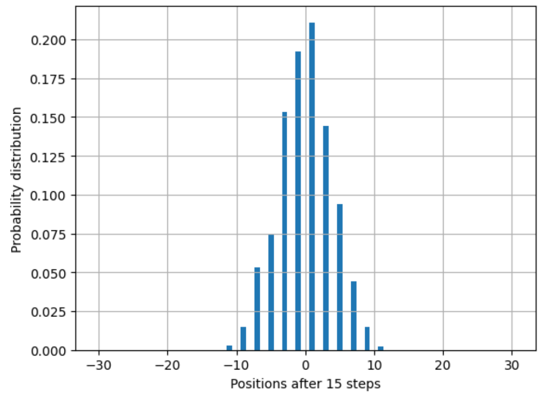

FIG. 4:

B. Biased One-Dimensional Random Walk

In this, $p \neq q$, so there will be complete randomness. The random walker will go to right of 0 and move in the right direction mostly, if p tends to 1. On the contrary, if $q = 1 - p$ tends to 1, the walker will be move to left of 0 and move in the left direction mostly.

We plotted different trajectories of walkers distinguished by their unique colours. The following graph clearly shows that biased random walker will create an illustration of complete randomness, which depends on the value of p used for plotting different trajectories.

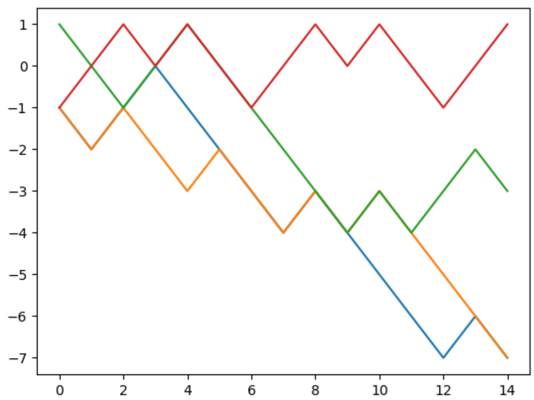

FIG. 5:

To obtain expected position of the random walker, we calculate average position at each time interval.

- Average position when the tossing is positive biased is a line with positive slope implying that the position of of the random walker is in the positive direction as the steps taken increases.
- For unbiased random walks, considering 1000 walks we can almost cancel out all the forward and backward movement possibilities, as we dicussed previously. Hence, the graph is a straight line on 0.

- Average position when the tossing is negative biased is a line with negative slope implying that the position of of the random walker is in the negative direction as the steps taken increases.

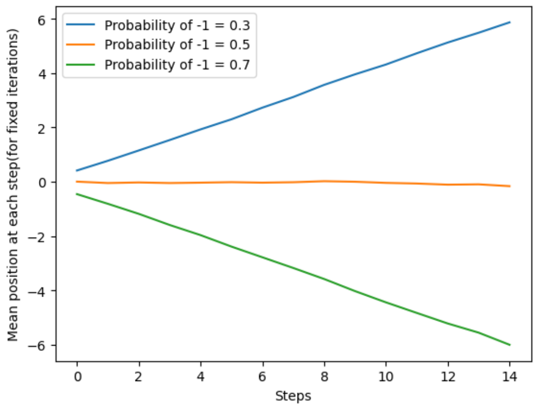

FIG. 6:

To obtain variance of the random walker, we run simulation on a large number of trajectories and calculate variance on each time interval.

- When $p > 0.5$, walker has a preferred direction to the right. Therefore, variance increases more rapidly than in an unbiased random walk.
- When $p = 0.5$, walker has no preferred direction, and the variance will increase linearly with time, similar to an unbiased random walk discussed earlier.
- When $p < 0.5$, walker has a preferred direction to the right. Therefore, variance decreases more rapidly than in an unbiased random walk.

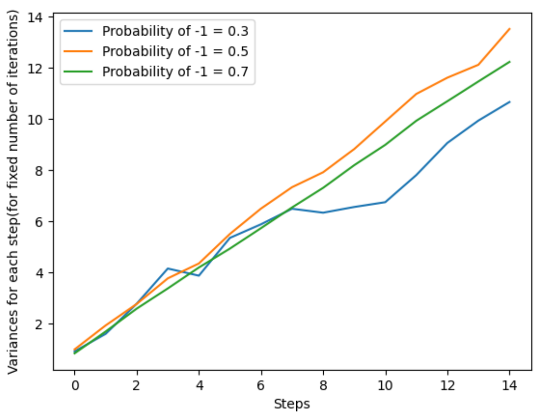

FIG. 7:

We can perform Mote-Carlo simulation to estimate the probabilities of being at different positions after n steps. The following graph illustrates how the preferred direction influences the probability distribution, with more probability mass shifting in the direction corresponding to the preferred probability.

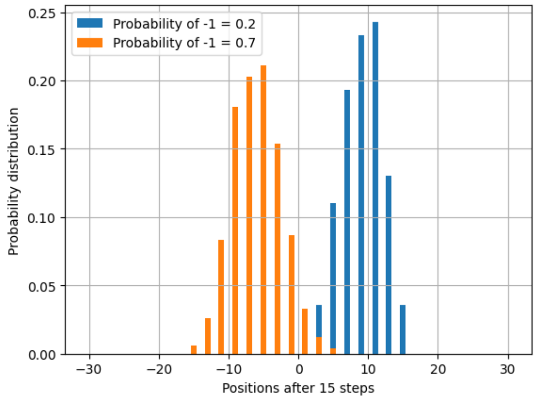

FIG. 8:

C. 2D Random Walk on a lattice

Fig. 9 is an example in which we generate a 2D random walk with 100 steps. In the plot we observe that each point represents the walker's position after a certain number of steps. The pseudo function is intended to generate a random walk, where at each time step, the walker moves diagonally in one of four directions: Northeast NE, Northwest NW, Southeast SE, or Southwest SW.

FIG. 9: 2D Random Walk Implementation

The animateWalk is a function that generates a series of steps of a random walk based on a list of points. We will depict the same by considering random points and plotting its graph. We first start at Step 0, our initial position. We then carry out the function for n number of steps, in this case we take $n = 8$:

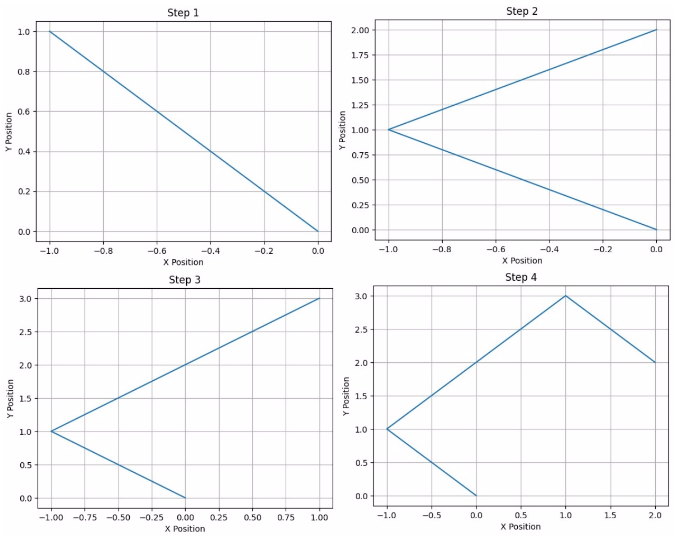

FIG. 10: Step 1 to Step 4

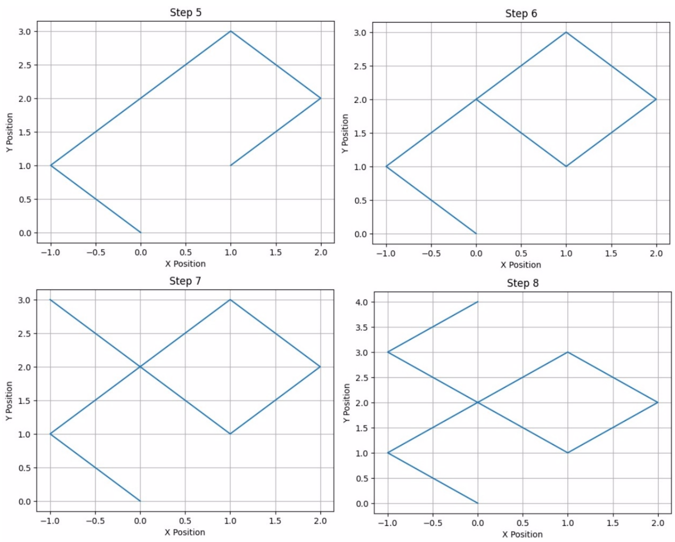

FIG. 11: Step 5 to Step 8

Coming to the next part of the abstract that involves plotting the average distance traveled versus the number of steps. We do so by just combining the distances traveled at each step and then dividing by the total number of steps. Initially we observe some significant fluctuations since the walker hasn't yet taken enough steps to establish a trend. As the number of steps increases, we notice that the average distance tends to increase. This is a predictable observation because, in a random walk, the walker moves in a random direction at each step. Over time, they tend to cover a larger area, leading to a greater average distance from the origin. Even as the average distance increases, we notice fluctuations in the data. These fluctuations are a result of the randomness in the walker's movements. Sometimes, the walker takes steps that bring them closer to the origin, and at other times, they move further away. These observations can be verified through the following graph:

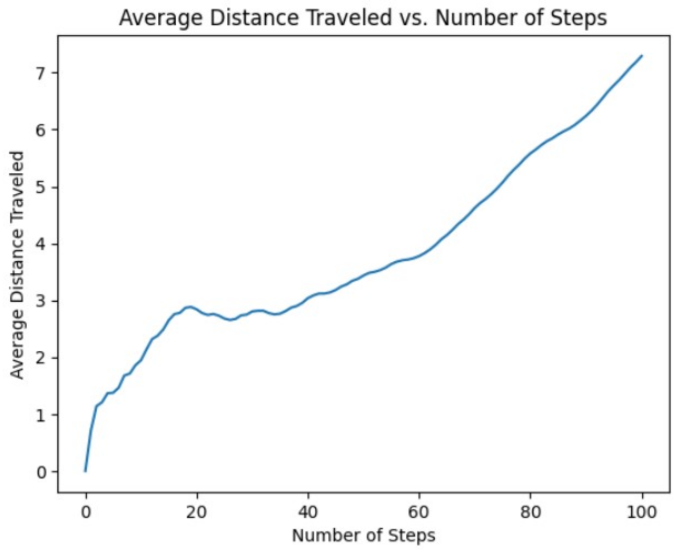

FIG. 12: Average Distance Traveled

D. Hiker Problem

We notice from the probabilities given for each direction, that the hiker's preference for rightward movement is relatively high. This in turn leads to an increase in chances of the hiker taking NE or SE steps. Therefore we notice that the hiker tends to move diagonally to the right more frequently. Since the nature of the process of taking a path is random we deduce and observe the non-uniform paths in the graph. We also see that the hiker often clusters around diagonal directions (NE and SE), resulting in an overall trend of moving to the upper right or lower right quadrants of the plot, the reason being the higher probabilities associated with NE and SE. The boundaries of the plot may appear irregular, this happens due to the combination of the hiker's random movements and the impact of the tendency to drift towards the right.

These observations can be seen in Fig13, which basically highlight the intricate and changing nature of the hiker's path in the above simulation.

III. CONCLUSIONS

In conclusion, this lab report demonstrates the modeling and simulation of one-dimensional and two-

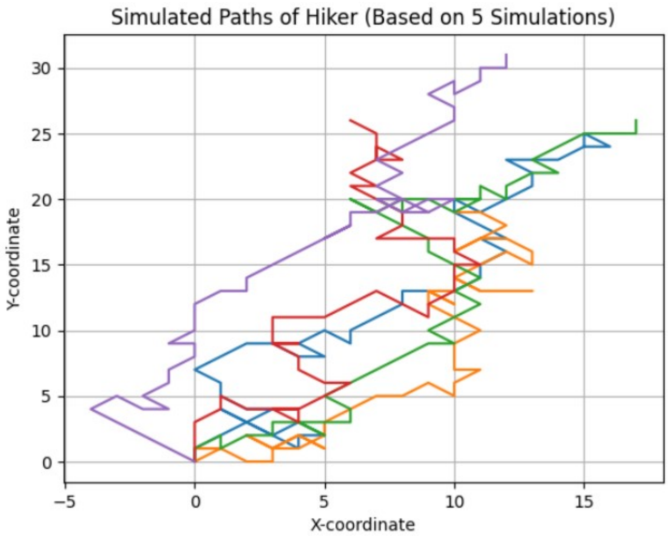

FIG. 13: Simulated Paths

dimensional random walks, which are fundamental stochastic processes with widespread applicability in physical and natural processes. The report explores unbiased and biased one-dimensional random walks, as well as a 2D random walk on a lattice. The results show that the walker's trajectory is entirely stochastic and varies from one simulation to another. The average position, variance, and probability distribution of the walker's position were analyzed, and the observations highlight the intricate and changing nature of the walker's path in the simulations. Overall, this lab report provides valuable insights into the behavior of random walks and their potential applications in various fields.

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[1] A. Shiflet and G. Shiflet, *Introduction to Computational Science: Modeling an Simulation for the Sciences*, Princeton University Press.3, 276 (2006).

[2] A. Einstein and N. Rosen, Phys. Rev.**48**, 73 (1935).