Modeling and Simulation, MC312
Lab-3
Due: August 30, 2023

August 23, 2023

1. **Constrained Models of Population growth:** Fig. 1 shows the diffusion of innovations. Initially, the market share of a new product is zero. However, a group of people called early innovators initially adopted the product (technology) due to certain factors. As time progresses, the product increases its market share due to many factors such as advertising, distribution of prices, and contact between users and finally saturates to a maximum value. The behavior observed in Fig. 1 or some variant of it is commonly observed in many problems (e.g. number of Twitter users, people using smart phones, market share of apple phones, etc.). In this problem, we look at some models specifically for such problems.

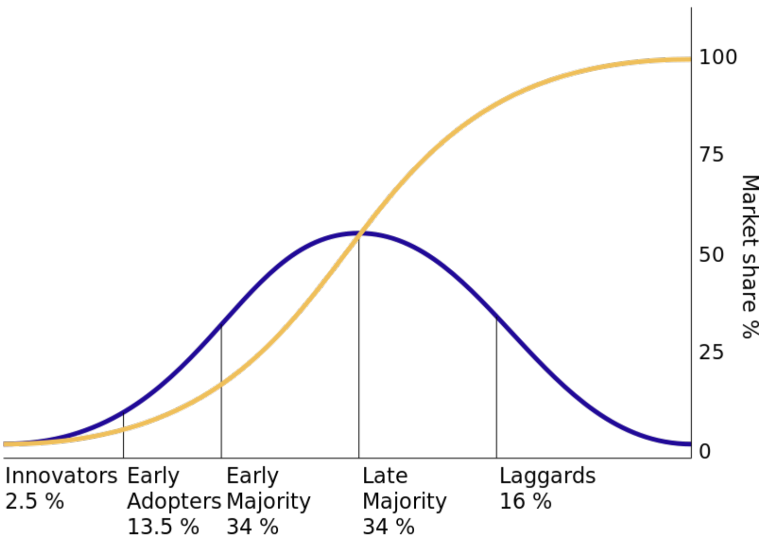

Figure 1: The diffusion of innovations according to Rogers. With successive groups of consumers adopting the new technology (shown in blue), its market share (yellow) will eventually reach the saturation level. The yellow curve is known as the logistic function. (figure and caption taken from wikipedia)

Typically the mathematical model for such problems has the form:

$$\dot{N} = \alpha(t)(C - N(t)) \tag{1}$$

where $\alpha(t)$ is the coefficient of diffusion, C is the maximum number of potential product users, and $N(t)$ is the total number who have adopted the product till time t. Three models are common:

- external influence model in which $\alpha(t) = p$, where p is a constant and captures the innovators or people who adopt the product on their own without being influenced by others.
- internal influence model $\alpha(t) = qN(t)/C$, in which the rate $\alpha(t)$ now captures the adoption due to the effect of the other users.
- mixed influence model (Bass): $\alpha(t) = p + qN(t)/C$, which captures both the effects.

Let us assume that the percentage of users can approximate the market share. The full (mixed influence) Bass Model has two parameters, p and q. The speed at which the product is being adopted or its longevity depends on the interplay between these parameters. We attempt to develop a qualitative understanding of these through numerical analysis. Provide a systematic analysis highlighting the significance and role of the parameters. For reasonable values of the parameters, you may refer to the original paper by Bass.

2. **(Not to be submitted)** In this problem, we try to see how well the different models we have looked at perform when modeling the human population. The data set for this exercise is provided as a .csv file. Consider two models of population growth linear and logistic. For both models, the growth rate r is around 0.0165. In the logistic model, determine the second parameter by trial and error. Starting with the base year as 1950, estimate the prediction of the human population by the two models. What is the steady state population, and when is it expected to be reached?