MA-202
PROJECT REPORT

Model of Aortic Blood Flow Using the Windkessel Effect

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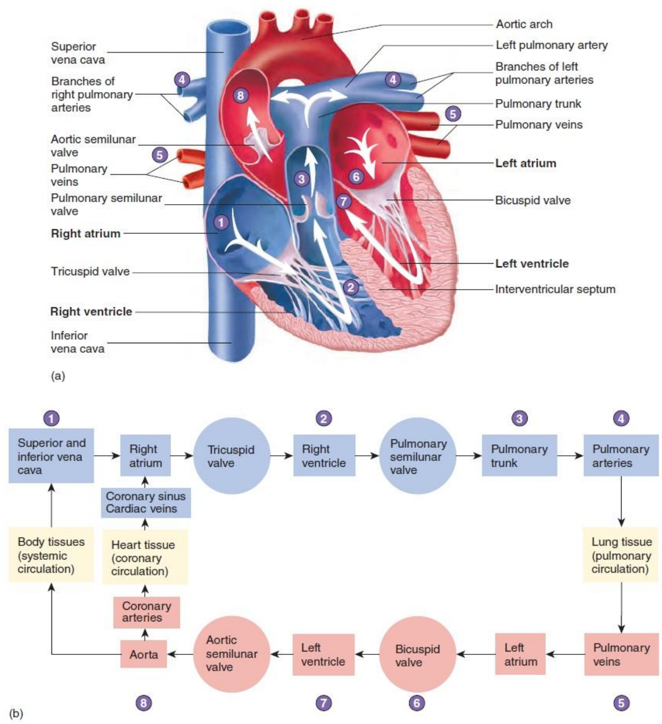
PROBLEM STATEMENT:

The study of blood flow through aorta is very crucial. The blood rate, blood pressure tells a lot about the wellness of the person in study. Blood flow through nerves and arteries can be taken as a dependent on blood pressure through different bio-vessels of the body. We aim to model the flow of blood in the human body via the well-known Windkessel Model . The flow of blood in the biological vessels can be modelled as an electrical model with the i(t) [current dependent on time] as the flow of flood, a capacitor derived from the physical constraints of aorta with a resistor in parallel to control the amount of current flow. With the help of the mathematical model we aim to compute the blood flow as a function of parameters defined on aorta itself using physical constraints and pressure limits.

Aim: Simulating a mathematical model for blood flow through Aorta.

INTRODUCTION:

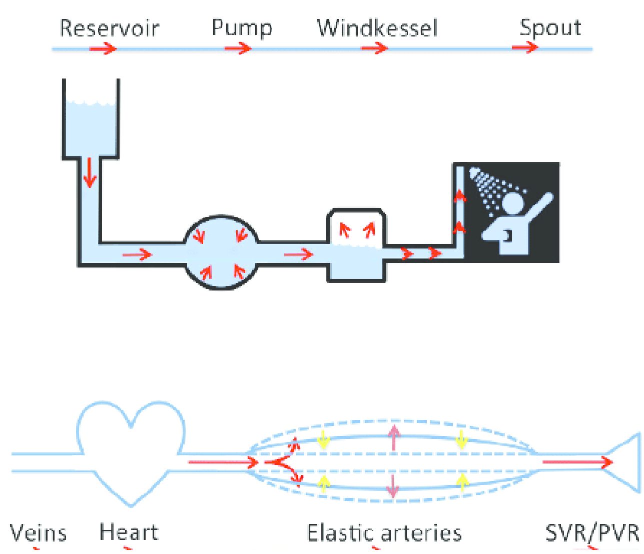
Heart is one of the most important internal organs of the human body. Since mammals have four chambered hearts, our hearts have four chambers: left atrium, right atrium, left ventricle, and right ventricle. The left part of the heart always contains pure blood that means oxygen contains blood, and the right part of the heart always contains impure blood that means carbon-di-oxide contains blood. Aorta is the largest artery of the human body that originates from the heart's left ventricle and extends down to the abdomen. When the impure blood comes to the right atrium through the vena cava, it comes down to the right atrium by tricuspid valve and then sent to the lungs where CO_2 is removed from the blood and oxygen is added. Then this oxygen containing blood comes back to the heart, in the left atrium. This oxygen-contained blood is sent to left ventricle and this blood is then sent to the entire body with the help of the aortic arch.



PROCESS Figure 12.10 AP|R **Blood Flow Through the Heart**

(*a*) Frontal section of the heart, revealing the four chambers and the direction of blood flow (*purple numbers*). (*b*) Diagram listing in order the structures through which blood flows in the systemic and pulmonary circulations. The heart valves are indicated by *circles*, deoxygenated blood by *blue*, and oxygenated blood by *red*.

WINDKESSEL EFFECT : Stephen Hales first gave the idea of capacitance in a hemodynamic model ,also known as ‘Windkessel effect’. The windkessel effect was first founded by Otto Frank in 1887, Windkessel is a German word that means ‘air chamber’. The windkessel model shows the heart anatomy into a hydraulic system which contains fluid as blood. Therefore in this analogy blood pressure acts as a pressure which helps to maintain the flow of the blood in the body. Analogy can be seen by the following diagram:

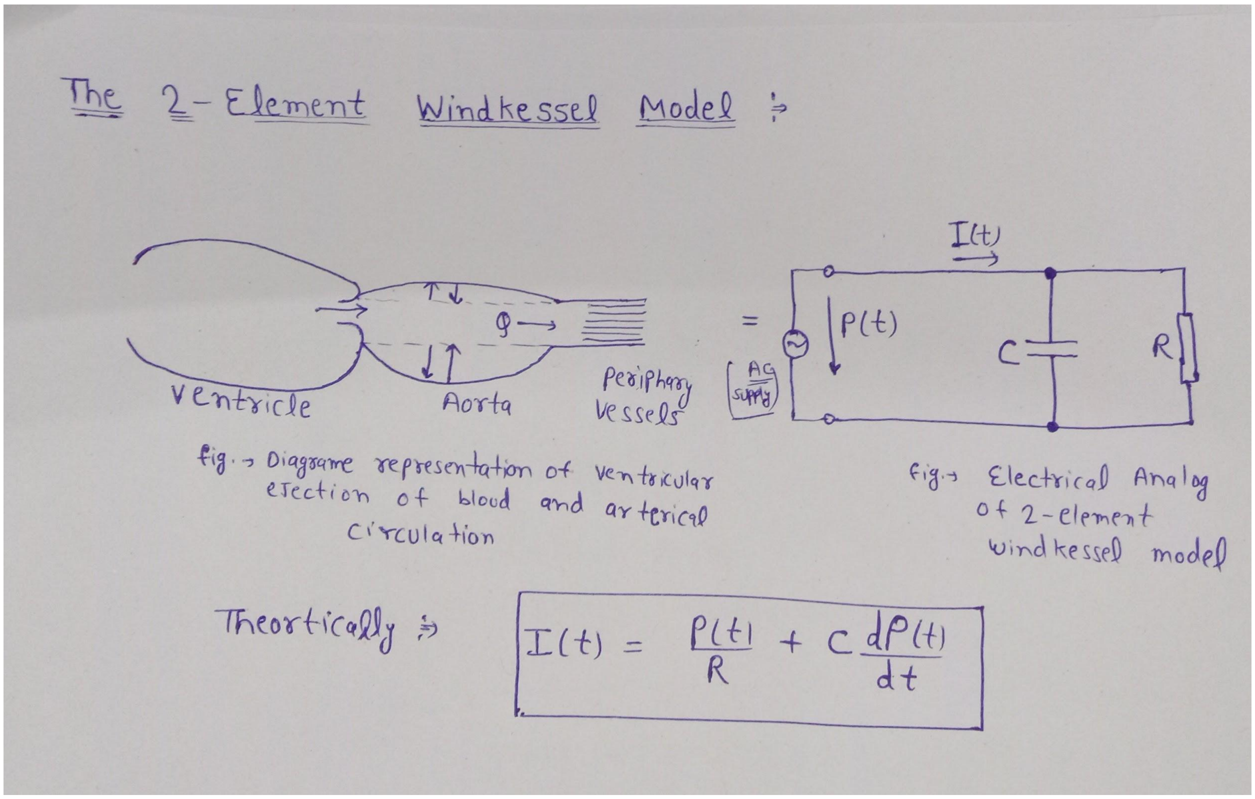


ASSUMPTIONS:

- The flow resistance encountered by the blood as it flows through the systemic arterial system (peripheral resistance) has been shown by a resistor.
- Elasticity and extensibility of the major artery during the cardiac cycle (arterial compliance) has been shown by a capacitor.
- The inertia of the blood as it is cycled through the heart (inertia) has been shown by an inductor.
- Systole (the phase in which the ventricle contracts and releases blood from the heart to the body) is the starting point of the cardiac system.

- The time taken by the systole phase is two-fifth of the total time taken in a complete cardiac cycle.

THE WINDKESSEL ELEMENT WITH TWO ELEMENTS:



This is the simplest form of the Windkessel model in which only two elements are presented in the form of a hydraulic system. One of the elements are total peripheral resistance which acts as a resistor in the circuit, shown by ‘R’ and its dimension is mmHg.s/cm^3. The second element is arterial compliance, which acts as a capacitor in the circuit, shown by ‘C’, and its dimension is cm3/mmHg.

In the circuit, the pressure difference (p2-p1) acts as the potential difference in the circuit and the flow rate of the blood in arteries acts as the electric current in the circuit. Thus the diagram of blood flow through the aorta, its equivalent electrical circuit diagram and the equation for the current in the circuit will be:

Analytic solution of Mathematical Models ⇒

1. for Two-element Windkessel ⇒

⇒ flow can be written → $\boxed{Q(t) = C\frac{dP(t)}{dt} + \frac{P(t)}{R}}$

⇒ Laplace domain ⇒

$$Q = C.s.P + \frac{P}{R}$$

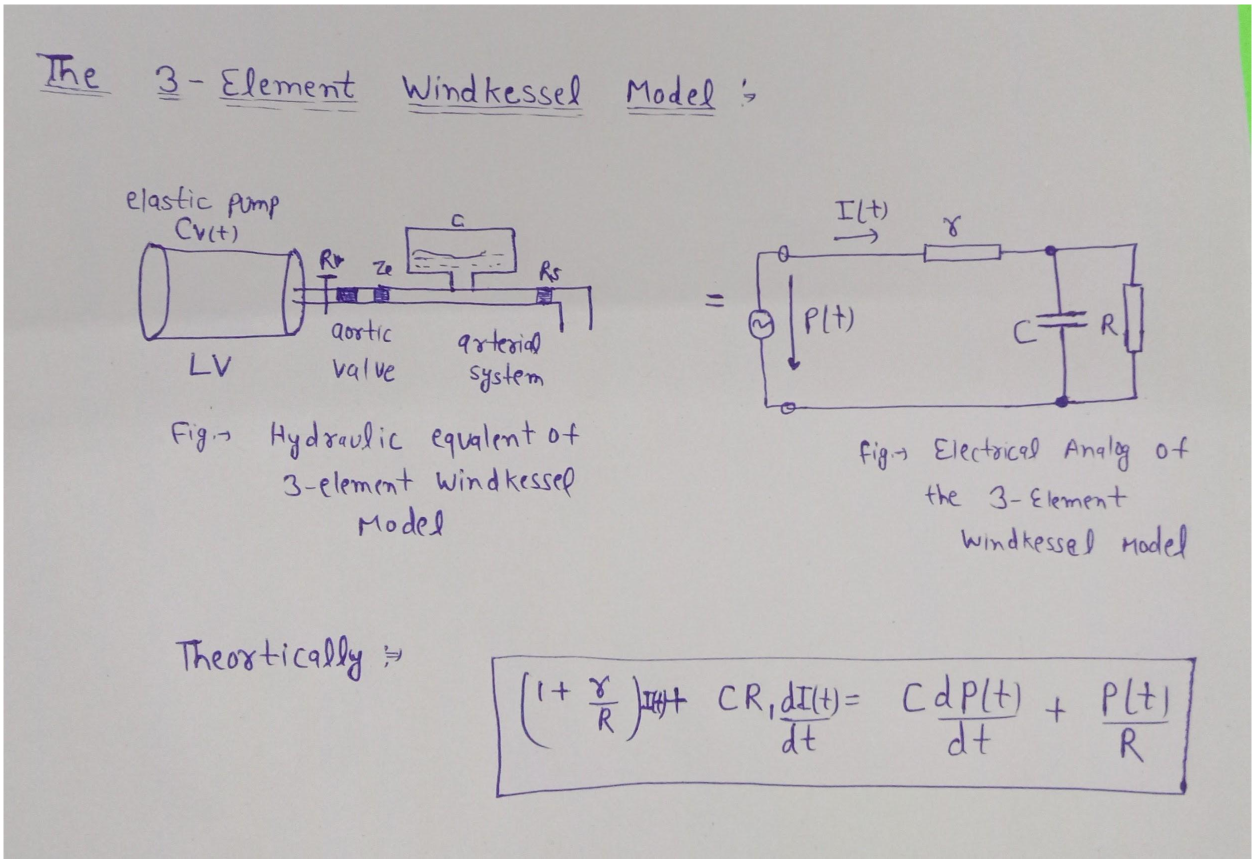
$$Q = P\left(C.s + \frac{1}{R}\right)$$

⇒ solving for the pressure ⇒

$$\boxed{P(i) = \left[Q(i) . \frac{R.\Delta t}{C} + P(i-1)R\right] \cdot \frac{1}{\left(\frac{\Delta t}{C} + R\right)}}$$

* we need Q and P signal.

THE WINDKESSEL MODEL WITH THREE ELEMENTS:



In this type of model, three elements are presented in the form of hydraulic system; in which 2 elements are peripheral resistance (one is added in the main branch and one is added in the side branch) and 1 arterial compliance. peripheral resistances shown by ‘R’ and ‘r’ and the arterial compliance shown by ‘C’. All three elements have the same dimensions as mentioned in the 2-element model. Therefore the diagram of blood flow through the aorta, its equivalent electrical circuit diagram and the equation for the current in the circuit will be:

Three element Windkessel :

⇒ flow can be written :

$$\boxed{Q(t)\left(1+\frac{r}{R}\right) + C\cdot r\,\frac{dQ(t)}{dt} = \frac{P(t)}{R} + C\,\frac{dP(t)}{dt}}$$

for Laplace domain ⇐

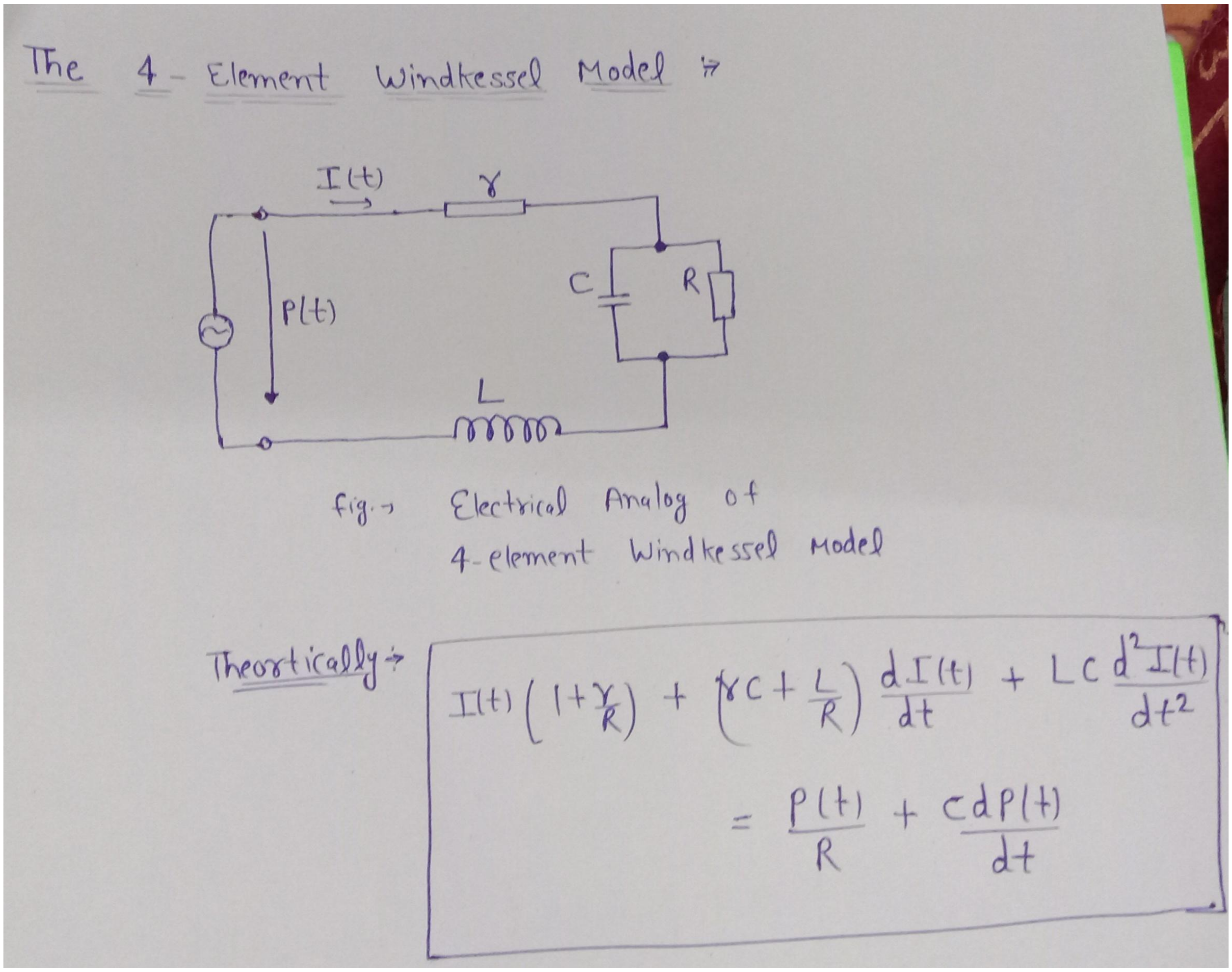
$$Q\left(1+\frac{r}{R} + C\cdot r\cdot s\right) = P\cdot\left(\frac{1}{R} + C\cdot s\right)$$

⇒ Laplace definition can be used in frequency behaviour analysis.

⇒ solving for the pressure :

$$P(i) = \frac{1}{\left(1+\frac{C\cdot R}{\Delta t}\right)}\left\{P(i-1)\cdot\left(\frac{CR}{\Delta t}\right) + R\left[Q(i)\cdot\left(\left\{1+\frac{r}{R}\right\} + \frac{C\cdot r}{\Delta t}\right) - Q(i-1)\left(\frac{C\cdot r}{\Delta t}\right)\right]\right\}$$

THE WINDKESSEL MODEL WITH FOUR ELEMENTS:



In this type of model, four elements are presented in the form of hydraulic system; in which two elements are peripheral resistance (one is added in the main branch and one is added in the side branch), one arterial compliance in the main branch and one more element which is represent the inertia in the flow of blood in the hydrodynamic model. The symbol ‘L’ is used for the inertia in the blood flow, and other symbols are the same as before. Then the diagram of blood flow through the aorta, its equivalent electrical circuit diagram and the equation for the current in the circuit will be:

Four - Element - series windkessel ⇒

flow ⇒

$$q(t)\left[\left(1+\frac{\gamma}{R}\right)+\frac{dq(t)}{dt}\left(\frac{L}{R}-C\cdot\gamma\right)+C\cdot L\cdot\frac{d^2q(t)}{dt^2}\right]$$
$$=P(t)\cdot\left(\frac{1}{R}\right)+C\cdot\frac{dP(t)}{dt}$$

Laplace domain ⇒

$$Q\cdot\left[\left(1+\frac{\gamma}{R}\right)+s\cdot\left(\frac{L}{R}-C\cdot\gamma\right)+C\cdot L\cdot s^2\right]$$
$$=P\cdot\left(\frac{1}{R}+C\cdot s\right)$$

⇒ Solving for the pressure ⇒

$$P(i)=\frac{1}{\left(\frac{1}{R}+\frac{C}{\Delta t}\right)}\Bigg\{P(i-1)\cdot\frac{C}{\Delta t}+Q(i)\left[\left(1+\frac{\gamma}{R}\right)+\frac{\left(\frac{L}{R}-C\cdot\gamma\right)}{\Delta t}+\frac{C\cdot L}{\Delta t^2}\right]$$
$$-Q(i-1)\cdot\left[\frac{\left(\frac{L}{R}-C\cdot\gamma\right)}{\Delta t}+\frac{2\cdot C\cdot L}{\Delta t^2}\right]$$
$$+Q(i-2)\cdot\frac{C\cdot L}{\Delta t^2}\Bigg\}$$

THE MODEL OF THE BLOOD FLOW TO THE AORTA:

Now we need to calculate the rate of blood flow to the aorta i.e. I(t). for this we will consider two cases: first one is in the case of diastole and the second is the case of systole.

$$I(t) = I_0 \sin\left[\pi * \frac{\mathrm{mod}(t, T_c)}{T_s}\right]$$

$$g_0 = \int_0^{T_c} I_0 \sin\left[\pi * \frac{\mathrm{mod}(t, T_c)}{T_s}\right] dt$$

$$I_0 = \int_0^{T_c} \left(\frac{1}{g_0}\right) \sin\left(\pi * \frac{\mathrm{mod}(t, T_c)}{T_s}\right] dt$$

For systole: In the process of systole, I(t) will be a sinusoidal curve with amplitude I_0 .

Then the value of I_0 will be:

For diastole: when diastole happens, all ventricles are in the relaxed form. So the blood

Flow through the aorta will be zero.

Thus,

$I_0 = 0$

CALCULATION OF SYSTOLIC PHASE:

Systolic Phase : Inhomogeneous solution

$$I(t) = \frac{P(t)}{CR} + \frac{dP(t)}{dt}$$

Integrating factor $u(t) = \int \frac{dt}{CR} = e^{\frac{t}{RC}}$

$$\frac{dP(t)}{dt} e^{\frac{t}{RC}} + e^{\frac{t}{RC}} \frac{P(t)}{CR} = \frac{I_0}{C} \sin\left(\frac{\pi t}{T_s}\right) e^{\frac{t}{RC}}$$

also,

$$\frac{dP(t)}{dt} e^{\frac{t}{RC}} + e^{\frac{t}{RC}} \frac{P(t)}{CR} = \frac{d}{dt}\left[e^{\frac{t}{RC}} P(t)\right]$$

$$\int d\left[e^{\frac{t}{RC}} P(t)\right] = \int \frac{I_0}{C} \sin(\pi t/T_s) \cdot e^{\frac{t}{RC}} \cdot dt$$

Solution is ⇒

$$\boxed{y(t) = \frac{C_1 e^{-\frac{t}{RC}} + -e^{\frac{t}{CR}} T_s I_0 R\left(C\pi R \cos\left(\frac{\pi t}{T_s}\right) - T_s \sin\left(\frac{\pi t}{T_s}\right)\right)}{T_s^2 + C^2 \pi^2 R^2}}$$

<u>CONCLUSION</u>:

We know that the blood pressure for a healthy heart is 120/80mmHg. That means the blood pressure will work against time according to the sinusoidal curve with the higher peak at 120mmHg and lowest peak at 80mmHg. So with the help of this model we can easily calculate, whether a person's heart is healthy or not. We can also find how much healthy someone's heart is. We can also find fluctuation in the blood flow in a single cardiac cycle. In this report, we have calculated the value of flow rate of the 2-element Windkessel model, 3-element Windkessel model, 4-element Windkessel model by calculating numerical formula and then insert it in the MATLAB code. One thing to notice that, we got a perfect sine curve for the blood pressure; but in real life scenario this curve would not be a perfect sine curve, but a distorted sine curve. Thus the Windkessel model works better in the real life scenario.

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