Signals

① Signals are variations, understood as patterns
② changes of something wrt to time/space
③ can be ideally just a shape.

Eg

- Electric Signal
  ↳ when we push a button or touchscreen, an electric signal is generated.
- Non electrical Signal
  ↳ temp, img, traffic light.

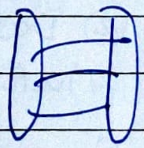
Mathematically, a signal is a function of one or more vars. $f(t) / f(x,y) / f(x,y,t)$

- 1D → voice
- 2D → img
- 3D → video

Signal as Mapping

↳ function: mapping b/w two sets
$\forall x \in D, \exists y \in R$

- Signal is a mapping from one set to another
  $f : X \to Y$
- collection of ordered pairs (x,y)



Representation

$x(t)$ → electric signals

$f(x,y)$ → digital image signals in 2 spatial axes → x & y.

Systems

- entity that processes a set of signals to yield another set of signals
- mapping across set of signals.
- for an i/p signal, system produces an o/p signal.

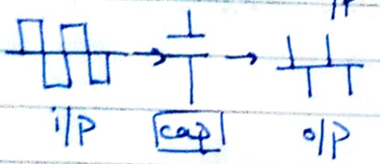
i/p signal		o/p signal
$x_1(t)$		$y_1(t)$
$x_2(t)$	$\rightarrow$ [System] $\rightarrow$	$y_2(t)$
⋮		⋮
$x_n(t)$		$y_n(t)$

- System can have physical components or an algo.
- Algo: a finite set of instructions that if followed accomplishes a certain task.

→ It is a physical abstraction of a physical process that relates i/p & o/p.

eg ① Computer Monitor: i/p → voltage signal from cpu
o/p – time varying signal in display

② Capacitor: i/p – terminal voltage
o/p – current



Goal is to develop general tools & techniques to analyze system, independent of their use.

Why study?

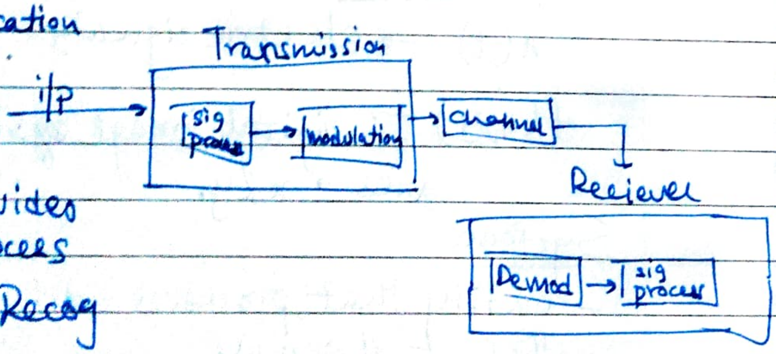
① Oxford Dictionary → Defn of IT

② Data organization, storage, processing, modelling, info extraction, interpretation, prediction.

Support

① Communication

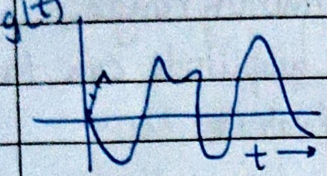
② System



③ Image/video process

④ Pattern Recog

Measure of Signals



~~area under curve~~

$\hookrightarrow \int_{-\infty}^{\infty} g(t)dt.$

Energy

Real valued $E_g = \int_{-\infty}^{\infty} g^2(t)dt$

Complex $E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$

Power

Real Valued $P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t)dt$

Complex $P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt.$

Is this E, P actual energy/power? NO!

Instantenous power $= g^2(t)/R$

Energy $= \int \frac{g^2(t)}{R} = \frac{E_g}{R}$

Energy of error signal = indicator of appoximation.

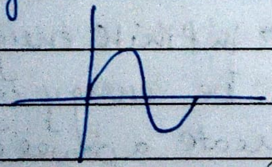
$SNR = \frac{\text{Power of signal}}{\text{Power of Noise}}$

Classification of signal

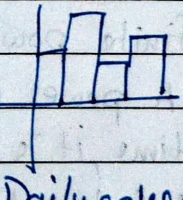
Continuous & Discrete Time Signals

$\hookrightarrow$ signal which is specified for all vals of t

$\hookrightarrow$ signal which are specified only at discrete values of t.



telephone, TV



Daily sales of stuff.

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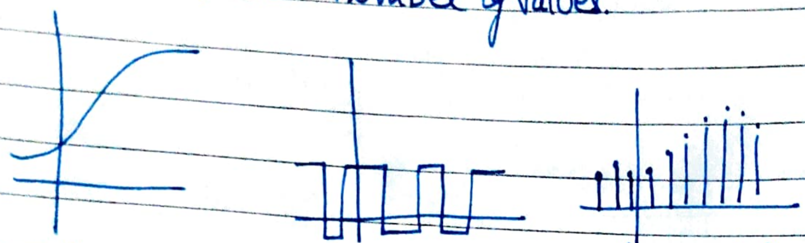
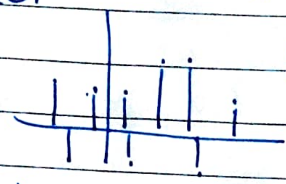
Analog Signal: Signal whose amplitude can take any value in the continuous range.

Digital Signal: ~~Signat~~ Signal whose amplitude can take only finite number of values.



Analog CT ~~Discrete~~ Digital CT Analog DT

Analog & Digital can both be CT|DT.



Digital DT

T	f(t)	Type
CT	cont in range	Analog CT
CT	discrete	digital CT
DT	Cont	Analog DT
DT	discrete	digital DT

Analog → Digital ⇒ Quantization

Power Signal: A signal whose P_x is finite.

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$0 < P_x < \infty.$$

finite power signal ⇒ infinite energy signal.

Imp A power signal can't be energy signal at same time, it is impossible to create a power signal with inf duration & inf energy.

Energy Signal: Signal with finite energy

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt < \infty$$

→ all real-life signals are energy signals.

→ all energy signals have 0 power.

Periodic & Aperiodic signal

• ~~A signal g(t) is periodic, if for some const T,~~

$$g(t) = g(t + nT) \; ; \; n = 0, \pm 1, \ldots$$

· periodic signal is unchanged with one period shift

i.e g(t) starts @ $-\infty$.

· continues forever

Classification

a) Deterministic: Amp of $x(t)$ is completely specified at specified time const

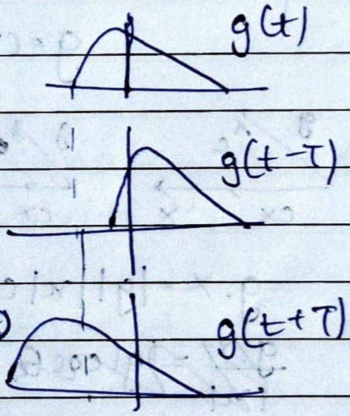
b) Non-Deterministic: Amp takes random value at time interval.

Properties of Signals

① Time Shifting

$$\phi(t+T) = g(t)$$

changes in g(t) reflects to $\phi(t)$

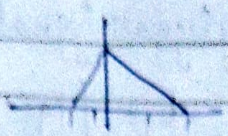


$T \to +ve \Rightarrow$ right shift

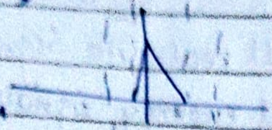
$T \to -ve \Rightarrow$ left shift

② Time Scaling

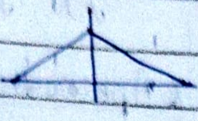
$\phi(t/2) = g(t) \Rightarrow \phi(t) = g(2t)$



$\phi(t) = g(at)$ $a > 1$
↳ compression



$\phi(t) = g(t/a)$ $a > 1$
↳ expansion

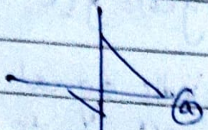


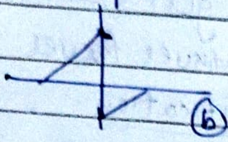
③ Time Inversion

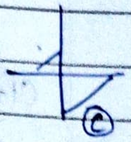
a) $\phi(-t) = g(t)$

b) $\phi(t) = g(-t)$

c) $\phi(t) = -g(t)$



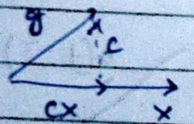


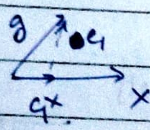


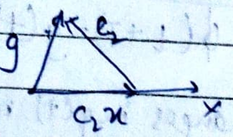
Signals & Vectors

- A vector can be represented by its components depending on the choice of the coordinate system.
- A signal can also be represented as a sum of components i.e basis signals.

$$g = cx + e$$







$g_1 = c_1x + e_1$

$g_2 = c_2x + e_2$

$g \cdot x = |g||x|\cos\theta$ $\qquad g \approx cx$

$\cancel{\frac{g \cdot x}{|x|} = |g|\cos\theta}$

- $c|x| = |g|\cos\theta$

$c|x|^2 = |g||x|\cos\theta$

$c = \frac{g \cdot x}{|x| \cdot |x|}$ $\qquad g, x$ are orthogonal.

Component of Signal

- concept of vector component and orthogonality can be extended to signal

Let a signal $g(t)$ be appoximated to a signal $x(t)$ over $[t_1, t_2]$; $g(t) = c\,x(t)$; $t_1 < t < t_2$

$$e(t) = \begin{cases} g - c\,x(t) & t_1 < t < t_2 \\ 0 & \text{else} \end{cases}$$

$$E_g = \int_{t_1}^{t_2} e^2(t)\,dt$$

$$= \int_{t_1}^{t_2} |g(t) - c\,x(t)|^2\,dt$$

$$\underbrace{\frac{d}{dc}\left[\int_{t_1}^{t_2} g^2(t)\,dt\right]}_{0} - \frac{d}{dc}\left[2c\int_{t_1}^{t_2} g(t)\,x(t)\,dt\right] + \frac{d}{dc}\left[c^2\int_{t_1}^{t} x^2(t)\,dt\right] = 0.$$

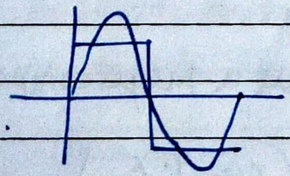
$$c = \frac{\int_{t_1}^{t_2} g(t)\,x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt} = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)\,x(t)\,dt.$$

So, g & x are orthogonal over $[t_1, t_2]$ iff

$$\int_{t_1}^{t_2} g(t)\,x(t)\,dt = 0.$$

Approximation of Signals

→ approximate a square signal $g(t)$ in terms of $x(t) = \sin t$ → $g(t) = c\,x(t)$



$$x(t) = \sin t$$

$$E_x = \int_0^{2\pi} \sin^2 t\,dt = \pi$$

$$c = \frac{1}{\pi}\int_0^{2\pi} g(t)\sin t\,dt = \frac{1}{\pi}\left[\int_0^{\pi} \sin t\,dt + \int_{\pi}^{2\pi} -\sin t\,dt\right]$$

$$= \frac{4}{\pi}$$

$$\boxed{g(t) = \frac{4}{\pi}\sin t}$$

Orthogonality in Complex Signals

Approximate a complex signal $g(t)$ as $cx(t)$ over $[t_1, t_2]$

$$g(t) = c\,x(t) \quad t_1 < t < t_2 \qquad \epsilon(t) = g - cx(t)$$

$$E_x = \int_{t_1}^{t_2} |x(t)|^2 dt \qquad E_\epsilon = \int_{t_1}^{t_2} |g(t) - cx(t)|^2 dt$$

$$E_\epsilon = \int_{t_1}^{t_2} |g(t)|^2 dt - \left|\frac{1}{\sqrt{E_x}} \int_{t_1}^{t_2} g(t)\,x(t)\,dt\right|^2 + \left|c\sqrt{E_x} - \frac{1}{\sqrt{E_x}} \int_{t_1}^{t_2} g(t)\,x(t)\,dt\right|^2$$

$$c = \frac{1}{E_x} \int_{t_1}^{t_2} g(t)\,x^*(t)\,dt \Rightarrow \int_{t_1}^{t_2} x_1(t)\,x_2(t) = 0$$

Energy of orthogonal signals

If X, Y are orthogonal & $Z = X + Y$,

$$|Z|^2 = |x|^2 + |Y|^2$$

$$z(t) = x(t) + y(t) \Rightarrow E(z) = E(x) + E(y)$$

$$\int_{t_1}^{t_2} |x(t) + y(t)|^2 dt = \int_{t_1}^{t_2} |x(t)|^2 dt + \int_{t_1}^{t_2} |y(t)|^2 dt + \int_{t_1}^{t_2} x^*(t)\,y(t)\,dt + \int_{t_1}^{t_2} x(t)\,y^*(t)\,dt$$

$$= \int_{t_1}^{t_2} |x(t)|^2 dt + \int_{t_1}^{t_2} |y(t)|^2 dt$$

Signal Comparison : Corellation

g & x are similar if g has a large component along x.

$\Rightarrow$ c has to be large $\left(c = \frac{g \cdot x}{|g|\,|x|}\right)$

$C_n \Rightarrow$ cosine coefficient $[-1 \le c_n \le 1]$

$$C_n = \frac{1}{\sqrt{E_g E_x}} \int_{-\infty}^{\infty} g(t)\,x(t)\,dt = \frac{g \cdot x}{|g|\,|x|}$$

Similarity Verification

Let $g(t) = k\,x(t)$, $C_n = 1$ if k is pos constant, $C_n = -1$ if k is neg const, $C_n = 0$ if g & x are orthogonal.

$C_n = 1 \Rightarrow$ Max similarity
$C_n = 0 \Rightarrow$ unrelatedness
$C_n = -1 \rightarrow$ Max dissimilarity

Corelation

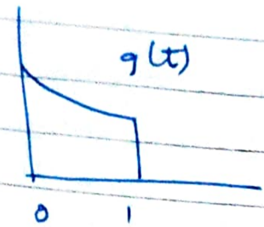
↳ Target detection by RADAR
↳ corelation b/w received pulse & transmitted pulse.
↳ binary data comms.
$1 \rightarrow$ +ve pulse / $0 \rightarrow$ neg pulse $\quad C_n = -1$
↳ antipodal scheme
if 0/1 are two states, its called ON/OFF or orthogonal.

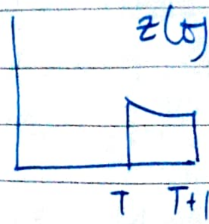
Why is antipodal better then ON/OFF?

In absense of noise, $C_n = \pm 1, 0$
→ signal transmission might have noise, dispersion
→ noise contaminates the signal, two adjacent waveforms might get overlapped
→ in such cases, we apply threshold detection → pulse is $p(t)$ or $-p(t)$.

CCF & ACF

CCF → Cross Corelation Function
ACF → Auto Corelation Function
In Radar, the corellation is the target.
Time delay → distance of target
Let $g(t)$ & $x(t)$ be two signals
if $C_n = 0$, it indicates no target.





g(t) needs corellation w z(t) shifted by T. Since they are diff funcs, they are called CCF.

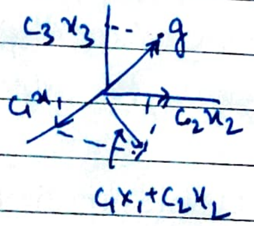
$$\psi_{gz}(T) = \int_{-\infty}^{\infty} g(t)\, z(t+T)\, dt \qquad \psi_{gz}(T) = \int_{-\infty}^{\infty} g(t)\, z(t+T)\, dt$$

Now if I relate g(t) with its own delayed versioned. its called ACF.

$$\psi_g(T) = \int_{-\infty}^{\infty} g(t)\, g(t+T)\, dt$$

Orthogonal Vector Sets

A vector can be represented as the sum of orthogonal vectors, a coordinate system of vector space.



$$g \simeq c_1x_1 + c_2x_2$$
$$e = g - (c_1x_1 + c_2x_2)$$
$$g = c_1x_1 + c_2x_2 + e$$
$$g = c_1x_1 + c_2x_2 + c_3x_3$$

→ unique choice of (c_1, c_2, c_3) makes the error vetor 0.

→ g is 3D vec and x_1, x_2, x_3 rep a complete set. Completeness as in its impossible to have a vector x_4 which is orthogonal to all 3 vectors (x_1, x_2, x_3). If $\{x_i\}$ is not complete, error func $\neq 0$.

→ if $\{x_i\}$ is mutually orthogonal, $x_m \cdot x_n = \begin{cases} 0 & m \neq n \\ |x_m|^2 & m = n \end{cases}$

→ if $\{x_i\}$ is complete, $c_i = \dfrac{g \cdot x_i}{x_i \cdot x_i} = \dfrac{g \cdot x_i}{|x_i|^2}$

Orthogonal Vector Space

Let the orthogonality of a signal set $x_1(t), x_2(t), \ldots, x_n(t)$ over $[t_1, t_2]$

$$\int_{t_1}^{t_2} x_m(t) \cdot x_n^*(t)\,dt = \begin{cases} 0 & m \neq n \\ E_n & m = n \end{cases}$$

→ if energy E_n is 1 $\forall n$, then the set is normalized and is called orthonormal.

→ divide by $\sqrt{E_n}$ to normalize a set.

Now, let's try to approximate a signal $g(t)$ over $[t_1, t_2]$ using set of orthogonal signals.

$$g(t) \simeq c_1 x_1(t) + c_2 x_2(t) + \cdots c_N x_N(t)$$

$$= \sum_{n=1}^{N} c_n x_n(t) \qquad t_1 \leq t \leq t_2$$

$$c_n = \frac{\int_{t_1}^{t_2} g(t)\, x_n^*(t)\,dt}{\int_{t_1}^{t_2} x_n^2(t)\,dt} = \frac{1}{E_n} \int_{t_1}^{t_2} g(t)\, x_n^*(t)\,dt$$

$n = 1, 2, \ldots, N$

if the orthogonal set is complete

error energy $= 0$

$$g(t) = \sum_{n=1}^{N} c_n x_n(t).$$

This is called the generalized fourier series of $g(t)$ wrt $\{x_i(t)\}$

→ when the set $\{x_i(t)\}$ is such that $E_e \to 0$ as $N \to \infty$ $\forall$ num of a class, we say that the set $\{x_n(t)\}$ is complete on $[t_1, t_2]$ for that class of signal $g(t)$ and the set $\{x_n(t)\}$ is called the set of basis functions or basis signals.

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→ If set is complete, it doesn't mean set is equal but it actually means $E_e \to 0$.

→ converse is not true.

Parseval's Thm

→ Energy of the sum of orthogonal signals is equal to the sum of these energies.

→ Energy of LHS of GFS is sum of individual orthogonal components.

$$E_g = c_1^2 E_1 + c_2^2 E_2 + \dots + c_n^2 E_n = \sum_n c_n^2 E_n.$$

This is Parseval's Thm.

Trigonometric Fourier Series

- Consider a signal set $\{1, \cos\omega_0 t, \cos 2\omega_0 t, \dots, \cos n\omega_0 t, \sin\omega_0 t, \dots, \sin n\omega_0 t\}$ $\left(T_0 = \frac{2\pi}{\omega_0}\right)$ forms an orthogonal complete set.

$$\int_{T_0} \cos n\omega_0 t \cos m\omega_0 t\,dt = \begin{cases} 0 & m \neq n \\ T_0/2 & m = n \neq 0 \end{cases}$$

To prove, $\cos mx \cos nx = \frac{1}{2}(\cos(m+n)x + \cos(m-n)x)$

$$\int_{T_0} \sin n\omega_0 t \sin m\omega_0 t\,dt = \begin{cases} 0 & n \neq m \\ T_0/2 & n = m \neq 0 \end{cases}$$

$$\int_{T_0} \sin n\omega_0 t \cos m\omega_0 t\,dt = 0 \quad (\text{always } \forall n, m)$$

A signal $g(t)$ can be expressed as

$$g(t) = a_0 + a_1 \cos\omega t + \dots + b_1 \sin\omega_0 t + \dots$$

$$= a_0 \int_{T_0} dt + \sum_{n=1}^{\infty} a_n \int_{T_0} \cos n\omega t + \sum_{n=1}^{\infty} b_n \int_{T_0} \sin n\omega_0 t. \quad \left(\omega_0 = \frac{2\pi}{T_0}\right)$$

a_0

$$\int_{T_0} x(t)\,dt = a_0 \int_{T_0} dt = a_0 T_0 \Rightarrow a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt$$

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a_n

$$\int_{T_0} x(t)\cos m\omega_0 t\,dt = a_0 \underbrace{\int_{T_0} \cos m\omega_0 t\,dt}_{0} + \sum_{n=1}^{\infty}\left[a_n\int_{T_0} \cos m\omega_0 t \cos n\omega_0 t\,dt + b_n \underbrace{\int_{T_0} \cos m\omega_0 t \sin n\omega_0 t\,dt}_{0}\right]$$

$$\int_{T_0} x(t)\cos m\omega_0\,dt = \frac{a_m T_0}{2}$$

$$a_m = \frac{2}{T_0}\int_{T_0} \cos m\omega_0\,dt.$$

b_n

multiply by $\sin m\omega_0 t$

$$\Rightarrow b_m = \frac{2}{T_0}\int_{T_0} \sin m\omega_0\,dt$$

Compact TFS

TFS contains sine & cosine terms of same freq. They can be combined in a single term of same freq by that identity.

$$a_n\cos n\omega_0 t + b_n \sin n\omega_0 t$$
$$= C_n \cos(n\omega_0 t + \theta_n)$$

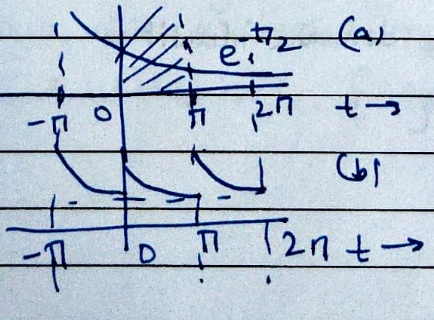
✓ $C_n = \sqrt{a_n^2 + b_n^2}$ ✓ $\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

Sums

Q Find CFS for $e^{-t_0/2}$ for $t\in[0,\pi]$

Ans



in $t \in [0,\pi]$ $T_0 = \pi$ $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi} = 2$ rad/s

$$g(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos 2nt + b_n \sin 2nt\right)$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} e^{-t/2}\,dt = 0.504$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\cos 2nt\,dt = 0.504\left(\frac{2}{1+16n^2}\right)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\cos 2nt\,dt = 0.504\left(\frac{8n}{1+16n^2}\right)$$

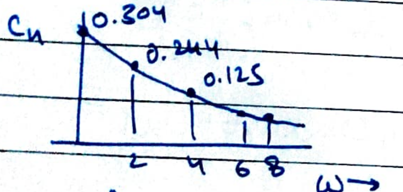
$$g(t) = 0.504\left[1 + \sum_{n=1}^{\infty}\left(\frac{2}{1+16n^2}\right)\left(\cos 2nt + 4n\sin 2nt\right)\right]$$

$$C_0 = a_0 = 0.504$$

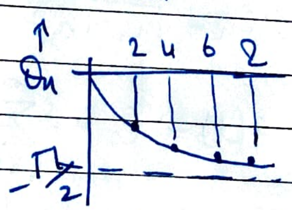
$$C_n = \sqrt{a_n^2 + b_n^2} = 0.504\left(\frac{2}{\sqrt{1+16n^2}}\right)$$

$$\theta_n = \tan^{-1}(-4n) = -\tan^{-1}(4n)$$

$$g(t) = 0.504 + 0.504\sum_{n=1}^{\infty}\frac{2}{\sqrt{1+16n^2}}\cos\left(2nt - \tan^{-1}4n\right)$$



Amp spectrum



phase spectrom

$x(t)$ → time domain identity

FS → freq " "

C_n/ω & θ_n/ω ↗

_/_/_

Periodicity of TFS

A arbitary signal $g(t)$ may be expressed as a TFS over any interval of T_0.

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

TFS ⇒ time period T_0

Let RHS be $\phi(t)$

$$\phi(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$\phi(t+T_0) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 (t+T_0) + \theta_n)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + 2\pi n + \theta_n)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$= \phi(t) \quad \forall t.$$

TFS is a periodic func of T_0.

Existence of FS

① For FS to exist, a_0, a_n, b_n must be finite.
$g(t)$ must be integrable over T_0. [Weak Dirchelet]

$$\int_{T_0} |g(t)|\, dt < \infty$$

② The function $g(t)$ have only a finite number of maxima and minima in T_0 and a " " " discontinuities in T_0. [Strong Dirchelet]

_/_/_

Exponential FS

TFS can be expressed as exponential → $e^{jn\omega t}$

over $T_0 = \frac{2\pi}{\omega_0}$

$$\int_{T_0} \left(e^{jm\omega t}\right)\left(e^{jn\omega t}\right)^* dt = \int_{T_0} e^{j(m-n)\omega t} = \begin{cases} 0 & m \neq n \\ T_0 & m = n \end{cases}$$

↳ $$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$\Rightarrow \int_{T_0} x(t) e^{-jm\omega_0 t} = \sum_{-\infty}^{\infty} D_n \int e^{j(n-m)\omega_0 t} dt$$

$$\Rightarrow \int_{T_0} x(t) e^{-jm\omega_0 t} dt = D_m T_0$$

$$\Rightarrow D_m = \frac{1}{T_0} \int_{T_0} x(t) e^{-jm\omega_0 t} dt.$$

Comparison with TFS

$$D_0 = \frac{1}{T_0} \int x(t)\, dt = a_0$$

$$D_n = \underbrace{\frac{1}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\, dt}_{a_n/2} - \underbrace{\frac{j}{T_0} \int_{T_0} x(t) \sin n\omega_0 t}_{b_n/2}$$

$$= \frac{1}{2}(a_n - jb_n) \quad \text{for } n \neq 0$$

$$D_{-n} = \frac{1}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\, dt + \frac{j}{T_0} \int_{T_0} x(t) \sin n\omega_0 t\, dt$$

$$= \frac{1}{2}(a_n + jb_n)$$

→ $D_n = D_{-n}^*$

Proof $\quad a_n - jb_n = \sqrt{a_n^2 + b_n^2}\, e^{j\tan^{-1}\left(\frac{-b_n}{a_n}\right)} = C_n e^{j\theta_n}$

Hence, $D_0 = a_0 = C_0$

and $D_n = \frac{1}{2} C_n e^{j\theta_n}$ & $D_{-n} = \frac{1}{2} C_n e^{-j\theta_n}$

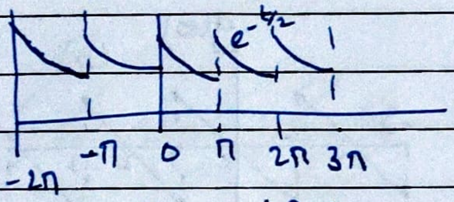
$|D_n| = |D_{-n}| = \frac{1}{2} C_n$ and $\angle D_n = \theta_n$ / $\angle D_{-n} = -\theta_n$

→ EFS has two sided spectrum, TFS has one.

→ EFS → even symmetry amp spectra; EFS → odd symmetry phase spectra

EFS of Impulse Train

Q



$T_0 = \pi \quad \omega_0 = 2$

Ans

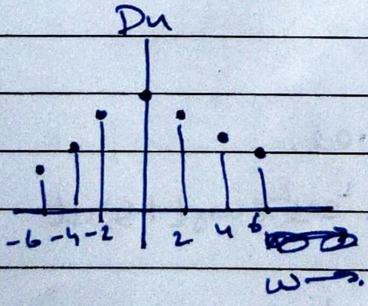
$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt}$$

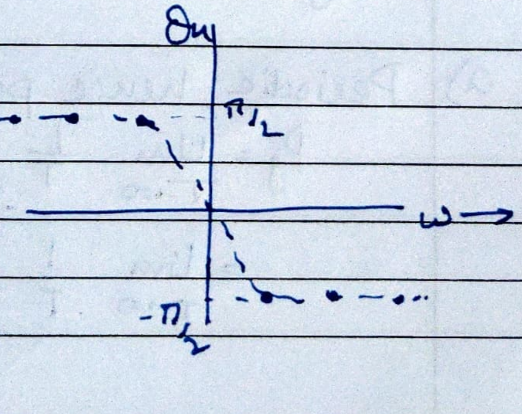
$$D_n = \frac{1}{T_0} \int_{T_0} \phi(t) e^{-j2nt} \, dt$$

$$= \frac{1}{\pi} \int_0^{\pi} e^{-t/2} e^{-j2nt} \, dt = \frac{1}{\pi} \int_0^{\pi} e^{-(\frac{1}{2} + 2nj)t} \, dt$$

$$= \frac{-1}{\pi(\frac{1}{2} + j2n)} e^{(-\frac{1}{2} + j2n)t} \Big|_0^{\pi} = \frac{0.504}{1 + 4nj}$$

$$g(t) = 0.504 \sum_{-\infty}^{\infty} \frac{1}{1 + j4n} e^{j2nt}.$$





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Parseval's Thm

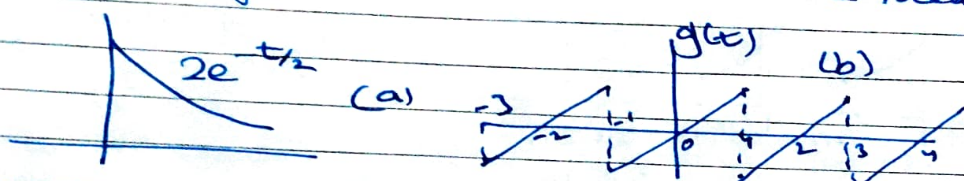
↳ A periodic signal is a power signal and every term on FS is also a power signal. power of the signal is the power of its FS components

$$P_g = C_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 \;\Big/\; P_g = \sum_{n=-\infty}^{\infty} |D_n|^2$$

$$= D_0 + 2\sum_{n=1}^{\infty} |D_n|^2$$

Signal Analysis (SUMS)

P1: Given a signal below find its suitable measure



a) Since amp → 0, t → ∞. suitable measure is energy

b) periodic → power.

a) $E_g = \int_{-\infty}^{\infty} g^2(t)\,dt = \int_{-1}^{0} 2^2\,dt + \int_{0}^{\infty} 4e^{-t}\,dt = 8$

b) $P_g = \frac{1}{2}\int_{-1}^{1} g^2(t)\,dt = \frac{1}{3}$

P2: Determine powe and rms values

a) $g(t) = C\cos(\omega_0 t + \theta)$

b) $g(t) = C_1\cos(\omega_1 t + \theta_1) + C_2\cos(\omega_2 t + \theta_2)$ $\quad \omega_1 \neq \omega_2$

c) $g(t) = De^{j\omega_0 t}$

a) Periodic, hence power.

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} C^2\cos^2(\omega_0 t + \theta)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{C^2}{2}\left[1 + \cos(2\omega_0 t + 2\theta)\right]dt.$$

$$= \lim_{T\to\infty} \frac{c^2}{2T}\left[\int_{-T/2}^{T/2} dt + \int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta)\,dt\right]$$

$$= \frac{c^2}{2}$$

b) $$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2}\left[c_1^2\cos^2(\omega_1 t+\theta_1) + c_2^2\cos^2(\omega_2 t+\theta_2) + 2c_1c_2\underbrace{\cos(\omega_1 t+\theta_1)\cos(\omega_2 t+\theta_2)}_{0}\right]dt$$

$$= \frac{c_1^2}{2} + \frac{c_2^2}{2}$$

$$\text{rms} = \sqrt{\frac{c_1^2+c_2^2}{2}}$$

~~if $\omega_1 = \omega_2$~~

$$\cancel{P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} c_1^2}$$

if $\omega_1 = \omega_2 = \omega$,

$$P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g^2(t)\,dt$$

$$g^2(t) = c_1^2\cos^2(\omega t+\theta_1) + c_2^2\cos^2(\omega t+\theta_2) + 2c_1c_2\cos(\omega t+\theta_1)\cos(\omega t+\theta_2)$$

$$\cos(\omega t+\theta_1)\cos(\omega t+\theta_2) \to \cos(\theta_1-\theta_2)$$

$$= \frac{1}{2}\left(\frac{c_1^2}{2} + \frac{c_2^2}{2} + \frac{2c_1c_2\cos(\theta_1-\theta_2)}{2}\right)$$

Ans

c) Periodic, so Power.

$$P_g = \frac{1}{T_0}\int_{T_0} |De^{j\omega_0 t}|^2\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0} |D|^2\,dt$$

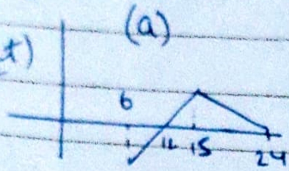
$$= |D|^2$$

rms $\to |D|$.

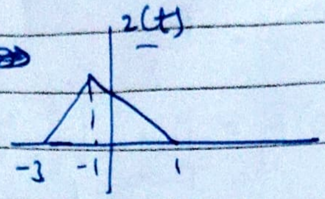
P3: for $g(t)$, $z(t)$, find $g(3t)$ and $z(t/2)$

A: (a) (b)

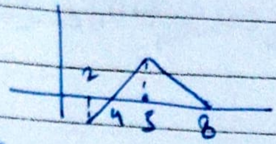
$g(t)$



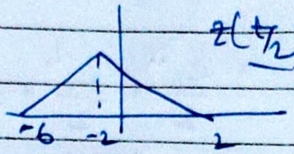
$z(t)$



$g(3t)$

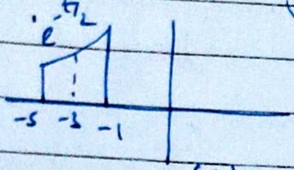


$z(t/2)$

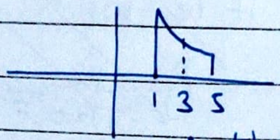


P4: for $g(t)$, sketch $g(-t)$

A:



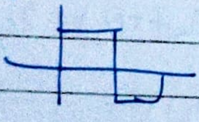
$g(t)$ →



$g(-t)$

P5: Apporimate a signal in terms of $\sin t$.

A:



$$E_x = \int_0^{2\pi} \sin^2 t = \pi$$

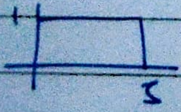
$$g(t) = c \sin t$$

$$c = \frac{1}{\pi} \int_0^{2\pi} g(t) \sin t = \frac{1}{\pi}[2+2] = \frac{4}{\pi}$$

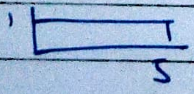
Ans $g(t) = \frac{4}{\pi} \sin t$.

P6. Find corelattion.

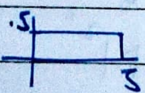
$x(t)$



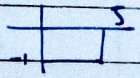
$g_1(t)$



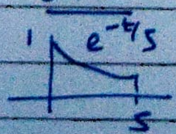
$g_2(t)$



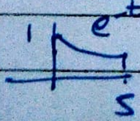
$g_3(t)$



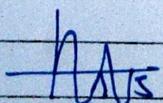
$g_4(t)$



$g_5(t)$



$g_6(t)$



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A: $E_x = 5$ $\quad E_{g_3} = 5$

$E_{g_1} = 5$ $\quad E_{g_4} = \int_0^5 e^{-\cancel{2t}/5}\, \cancel{dt}$

$E_{g_2} = 1.25$

$= \frac{1}{2.5}\left(1 - e^{-\frac{2t}{5}t}\right)$

$= 2.1617$

$E_{g_5} = 0.5$

$E_{g_6} = \int_0^5 \sin^2 2\pi t\, dt = 2.5$.

(1) $\frac{1}{\sqrt{5.5}} \int_0^5 dt = 1$ $\quad$ (2) $\frac{1}{\sqrt{5 \times 1.25}} \int_0^5 0.5\, dt = 1$

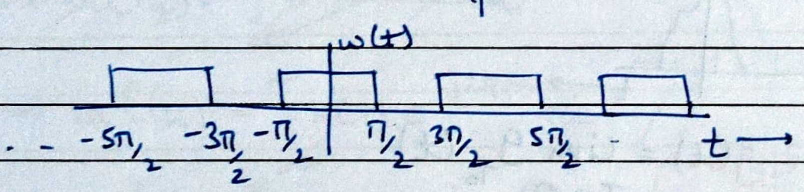
(3) $\frac{1}{\sqrt{5.5}} \int_0^5 (-1)\, dt = -1$ $\quad$ (4) $\frac{1}{\sqrt{2.16 \times 5}} \int_0^5 e^{-t/5}\, dt = 0.961$

(5) $\frac{1}{\sqrt{0.5 \times 5}} \int_0^5 e^{-t}\, dt = 0.6028$ $\quad$ (6) $\frac{1}{\sqrt{2.5 \times 5}} \int_0^5 \sin^2 2\pi t\, dt$

$= 0.$

P8: Find TFS and sketch amp.

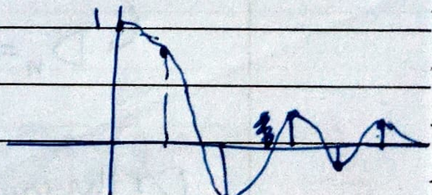
A:



$\omega(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right)$

$T_0 = \pi \quad \omega_0 = 2.$

$a_0 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} dt = 1/2$



$a_n = \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} \cos n\omega_0 t\, dt = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right)$

amp graph

$b_n = \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} \sin nt\, dt = 0$

$\omega(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_0 t - \frac{1}{2} \cos 3\omega_0 t + \dots\right)$

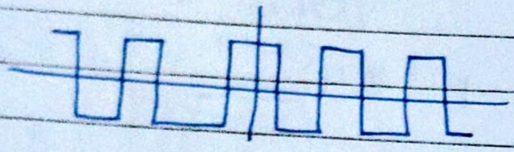
CTFS

$c_0 = \frac{1}{2}$

$c_n = \begin{cases} 0 & \text{even} \\ \frac{2}{n\pi} & \text{odd} \end{cases}$

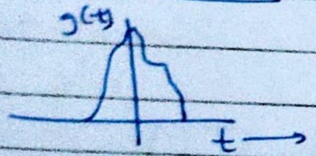
$\theta_n = \begin{cases} 0 & n \neq 3, 7, 11 \ldots \\ -\pi & n = 3, 7, 11, \ldots \end{cases}$

Bipolar Square Wave



$\hookrightarrow w(t) = \frac{4}{\pi}\left(\cos\omega_0 t - \cos(3\omega_0 t) + \ldots\right) + \frac{1}{2}$

Fourier Transform



$$g(t) = \lim_{T \to \infty} g_{T_0}(t)$$

$$g_{T_0}(t) = \sum_{-\infty}^{\infty} D_n e^{jn\omega_0 t} \quad -①$$

$$\hookrightarrow D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-jn\omega_0 t}\, dt \quad -②$$

Orthogonal Signal Space

Let $\Delta f = \frac{1}{T_0}$ $\qquad \omega_0 = \frac{2\pi}{T_0}$

$f_n = \frac{n}{T_0}$

$D_0 T_0 = G(f_n) = G(f_n)$

$$G(f_n) = \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-jn\omega_0 t}\, dt \quad -②$$

$$\lim_{T_0\to\infty} G(f_n) = \lim_{T_0\to\infty} \int_{-T_0/2}^{T_0/2} g_{T_0}(t) e^{-jn\omega_0 t} dt.$$

$$\lim_{T_0\to\infty} G\left(\frac{n}{T_0}\right) = \lim_{T_0\to\infty} \int_{-T_0/2}^{T_0/2} g_{T_0}(t) e^{-j2\pi f_n t} dt.$$

$$\Rightarrow \lim_{T_0\to\infty} \frac{n}{T_0} = \lim f_n = f$$

$$\Rightarrow G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$$

(i) $g_{T_0}(t) = \sum_{-\infty}^{\infty} D_n e^{-j\omega_0 n t}$

$$\lim_{T_0\to\infty} g_{T_0}(t) = \lim_{T_0\to\infty} \sum_{-\infty}^{\infty} \frac{D_n}{T} T_0 e^{-jn\frac{2\pi}{\omega_0}t}$$

$$g(t) = \lim_{T_0\to\infty} \sum_{n=-\infty}^{\infty} G(f_n) e^{j2\pi f_n t} dt.$$

$$= \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} dt.$$

~~$\Rightarrow G(f_n) = \int_{-\infty}^{\infty} e^{j2\pi f t} dt$~~

$$\Rightarrow G(f_n) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$$

$$\Rightarrow g(t) = \int_{-\infty}^{\infty} G(f) e^{-j2\pi f t} dt$$

imp

Conjugate Symmetry Property

↳ if $g(t)$ is a real func of t, then $G(\omega)$ and $G(-\omega)$ are complex conjugates i.e $G(\omega) = G^*(-\omega)$

i) $|G(-\omega)| = |G(\omega)|$

ii) $\theta g(-\omega) = -\theta g(\omega)$

$|G(\omega)| \rightarrow$ even function.

$\theta g(\omega) \rightarrow$ odd function

Q: find fourier transform of $e^{-at}u(t)$

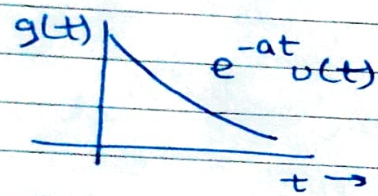
Ans

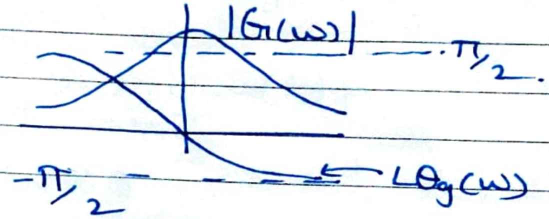
$$G(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt.$$

$$= \int_{0}^{\infty} e^{-(a+j\omega t)}\, dt = \frac{-1}{a+j\omega}\, e^{-(a+j\omega)t}\Big|_{0}^{\infty}$$

$$= \frac{1}{a+j\omega}$$

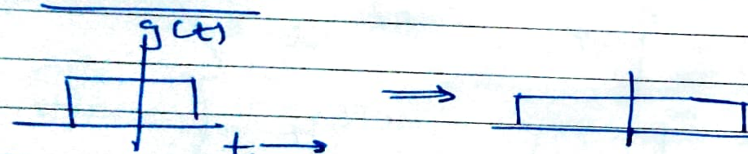
$$|G(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \theta_g(\omega) = \tan^{-1}\left(\frac{\omega}{a}\right)$$





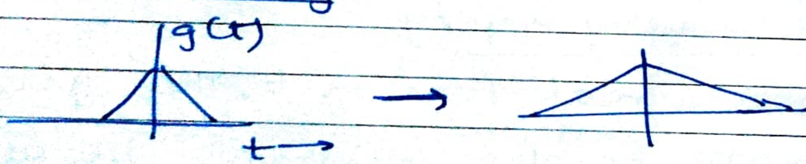
FT of some funcs

① Unit Gate



$$g(t) = \begin{cases} 0 & |x| > \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 1 & |x| < \frac{1}{2} \end{cases}$$

② Unit triangle

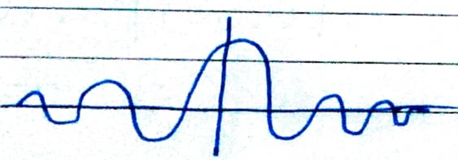


$$g(t) = \begin{cases} 0 & |x| \geq \frac{1}{2} \\ 1-2|x| & |x| < \frac{1}{2} \end{cases}$$

③ sinc

$$\text{sinc}(x) = \frac{\sin x}{x} \quad [Sa(x)]$$

interpolation func

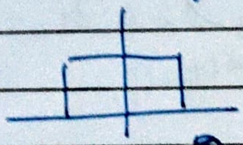


Properties of sinc

- ~~$sinc(x) = \frac{\sin x}{x}$ is an even func~~
- is 0 when $\sin x = 0, x \neq 0$ / or at $\pm n\pi$
- $sinc(0) = 1$
- ~~its the product of $\sin x$~~ & $\frac{1}{x}$ ← monotic decreasing
  ($\sin x$ ↑ oscillatory)
- sinosodial oscillation with decaying amp.

Q find FT of $rect(t/T)$

Ans

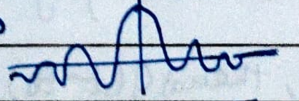
 $\Rightarrow$?

$$G(\omega) = \int_{-T_0}^{} e^{-j\omega t}\,dt = -\frac{1}{j\omega}\left(e^{-j\omega T/2} - e^{j\omega T/2}\right)$$

$$= \frac{2\sin(\omega T/2)}{\omega}$$

$$= \frac{\sin(\omega T/2)}{\omega T/2}\,T = T\,sinc(\omega T/2)$$

FT $\Rightarrow$



Bandwidth
↳ diff b/w highest and lowest freq.

$g(t) \rightarrow \boxed{\int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt} \rightarrow G(\omega)$

$G(\omega) \rightarrow \boxed{\frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}\,dt} \rightarrow g(t)$

Properties of FT

① Linearity property

$g_1(t) \Leftrightarrow G_1(\omega)$ and $g_2(t) \Leftrightarrow G_2(\omega)$

$$a_1 g_1(t) + a_2 g_2(t) \Leftrightarrow a_1 G_1(\omega) + a_2 G_2(\omega)$$

② Time duality property

$$g(t) \rightarrow \boxed{\int_{-\infty}^{\infty} g(t) e^{-j\omega t}\, dt} \rightarrow G(\omega)$$

$$G(\omega) \rightarrow \boxed{\frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega) e^{j\omega t}\, dt} \rightarrow g(t)$$

③ Time frequency duality

For any result b/w $g(t)$ and $G(\omega)$, $\exists$ a dual result on relationship that can be obtained by interchanging $g(t)$ & $G(\omega)$ in original result along w some modification

→ Time shifting has duality of freq shifting

→ if $g(t) \Leftrightarrow G(\omega)$, then a) $g(t-t_0) \Leftrightarrow G(\omega)e^{-j\omega t_0}$

b) $g(t)e^{j\omega_0 t} \Leftrightarrow G(\omega-\omega_0)$

④ Symmetry property

if $g(t) \Leftrightarrow G(\omega)$, $G(t) \Leftrightarrow 2\pi g(-\omega)$

eg $\underbrace{\tau \operatorname{sinc}\left(\frac{\tau t}{2}\right)}_{G(t)} \Leftrightarrow \underbrace{2\pi \operatorname{rect}\left(\frac{-\omega}{\tau}\right)}_{2\pi g(-\omega)} = 2\pi \operatorname{rect}\left(\frac{\omega}{\tau}\right)$

Proof $g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(x) e^{-jxt}\, dx$

$$2\pi g(t) = \int_{-\infty}^{\infty} G(x) e^{-jxt}\, dt$$

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⑤ Scaling property

if $g(t) \Leftrightarrow G(\omega)$ then

for any const a, $g(at) \Leftrightarrow \frac{1}{|a|} G\left(\frac{\omega}{a}\right)$

Proof:

for +ve const a,

$$f(g(at)) = \int_{-\infty}^{\infty} g(at) e^{-j\omega t} dt$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} g(x) e^{-j\omega/a\, t} dx \qquad x = at$$

$$= \frac{1}{a} G\left(\frac{\omega}{a}\right)$$

if $a < 0$,

$$g(at) \Leftrightarrow -\frac{1}{a} G\left(\frac{\omega}{a}\right)$$