Mining Frequent Patterns without Candidate Generation

Introduction

- Terminology
- Apriori-like Algorithms
  - Generate-and-Test
  - Cost Bottleneck
- FP-Tree and FP-Growth Algorithm
  - FP-Tree: Frequent Pattern Tree
  - FP-Growth: Mining frequent patterns with FP-Tree

Terminology

- Item set
  - A set of items: $I = \{a_1, a_2, \ldots\ldots, a_m\}$
- Transaction database
  - $DB = \langle T_1, T_2, \ldots\ldots, T_n \rangle$
- Pattern
  - A set of items: A
- Support
  - The number of transactions containing A in DB
- Frequent pattern
  - A's support $\geq$ *minimum support threshold* ξ
- Frequent Pattern Mining Problem
  - The problem of finding the complete set of frequent patterns

Apriori-like Algorithms

- Algorithm
  - Anti-Monotone Heuristic
    - If any length *k* pattern is not in the database, its length *(k+1)* super-pattern can never be frequent
  - Generating candidate set
  - Testing candidate set
- Two non-trivial costs: (Bottleneck)
  - Candidate sets are huge. (They are pruned already but still increase exponentially with stage number k).
  - Repeated scan the database and test the candidate set by pattern matching.

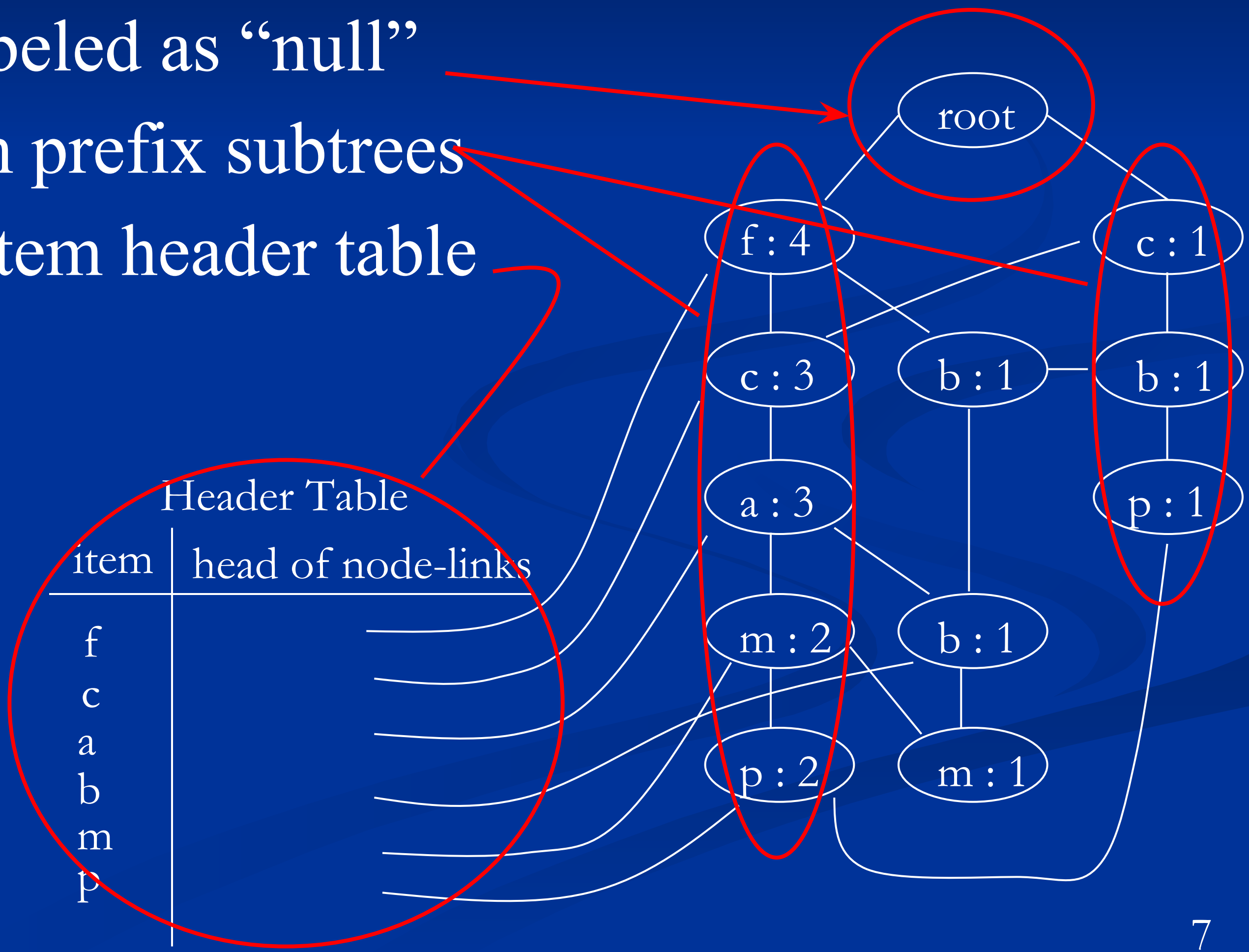
FP-Tree and FP-Growth Algorithm

- FP-Tree: Frequent Pattern Tree
  - Compact presentation of the DB without information loss.
  - Easy to traverse, can quickly find out patterns associated with a certain item.
  - Well-ordered by item frequency.
- FP-Growth Algorithm
  - Start mining from length-1 patterns
  - Recursively do the following
    - Constructs its conditional FP-tree
    - Concatenate patterns from conditional FP-tree with suffix
  - Divide-and-Conquer mining technique

- Constructing FP-Tree
  - Example 1

FP-Tree Definition

- Three components:
  - One root: labeled as "null"
  - A set of item prefix subtrees
  - A frequent-item header table



FP-Tree Definition (cont.)

- Each node in the item prefix subtree consists of three fields:
  - item-name
  - node-link
  - count
- Each entry in the frequent-item header table consists of two fields:
  - item-name
  - head of node-link

Example 1: FP-Tree Construction

- The transaction database used (fist two column only):

TID	Items Bought	(Ordered) Frequent Items
100	*f,a,c,d,g,i,m,p*	*f,c,a,m,p*
200	*a,b,c,f,l,m,o*	*f,c,a,b,m*
300	*b,f,h,j,o*	*f,b*
400	*b,c,k,s,p*	*c,b,p*
500	*a,f,c,e,l,p,m,n*	*f,c,a,m,p*

minimum support threshold $\xi = 3$

Example 1 (cont.)

- First Scan: //count and sort
  - count the frequencies of each item
  - collect length-1 frequent items, then sort them in support descending order into L, *frequent item list*.

  $L = \{(f{:}4), (c{:}4), (a{:}3), (b{:}3), (m{:}3), (p{:}3)\}$

Example 1 (cont.)

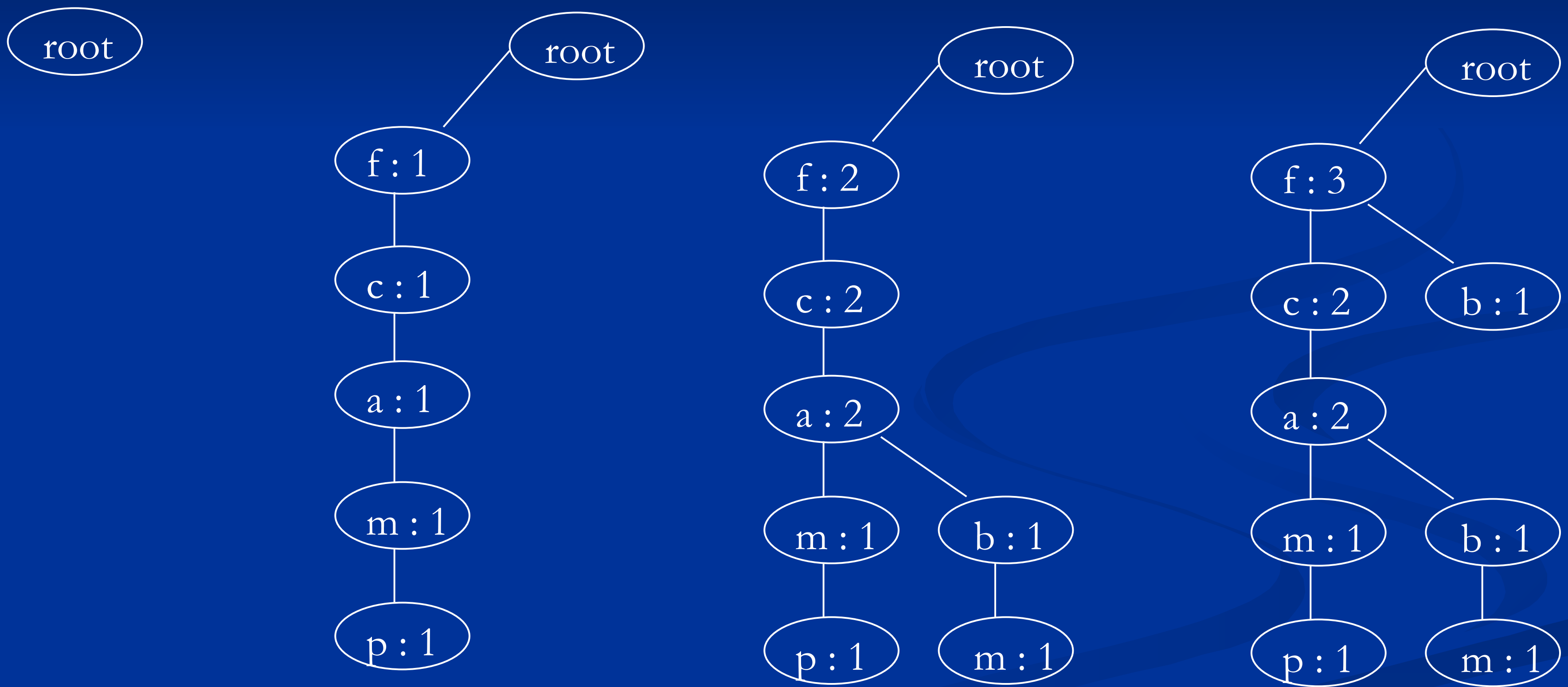
- Second Scan://create the tree and header table
  - create the root, label it as "*null*"
  - for each transaction *Trans*, do
    - select and sort the frequent items in *Trans*
    - increase nodes count or create new nodes

      If prefix nodes already exist, increase their counts by 1;

      If no prefix nodes, create it and set count to 1.
  - build the item header table
    - nodes with the same item-name are linked in sequence via node-links

Example 1 (cont.)

The building process of the tree



Create root

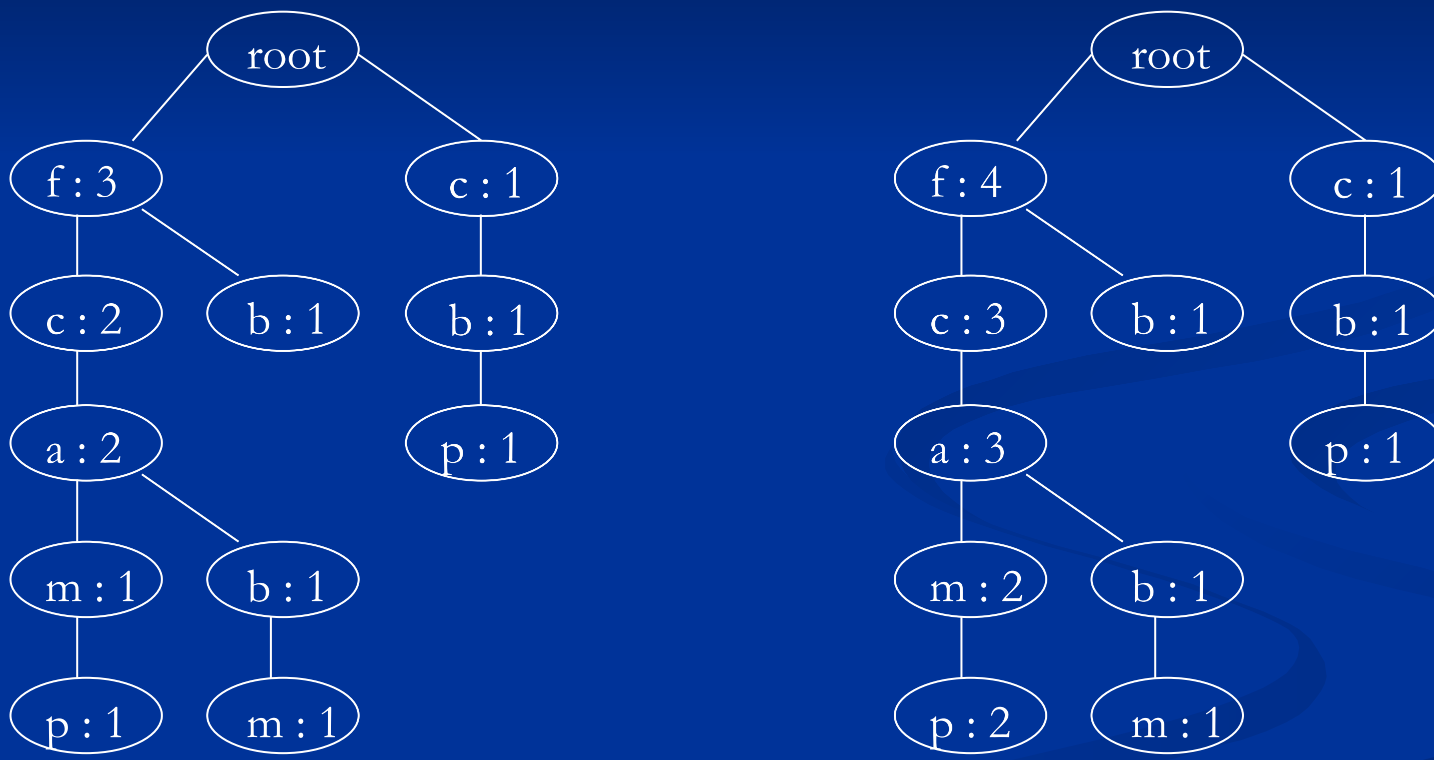
After trans 1
(f,c,a,m,p)

After trans 2
(f,c,a,b,m)

After trans 3
(f,b)

Example 1 (cont.)

The building process of the tree (cont.)

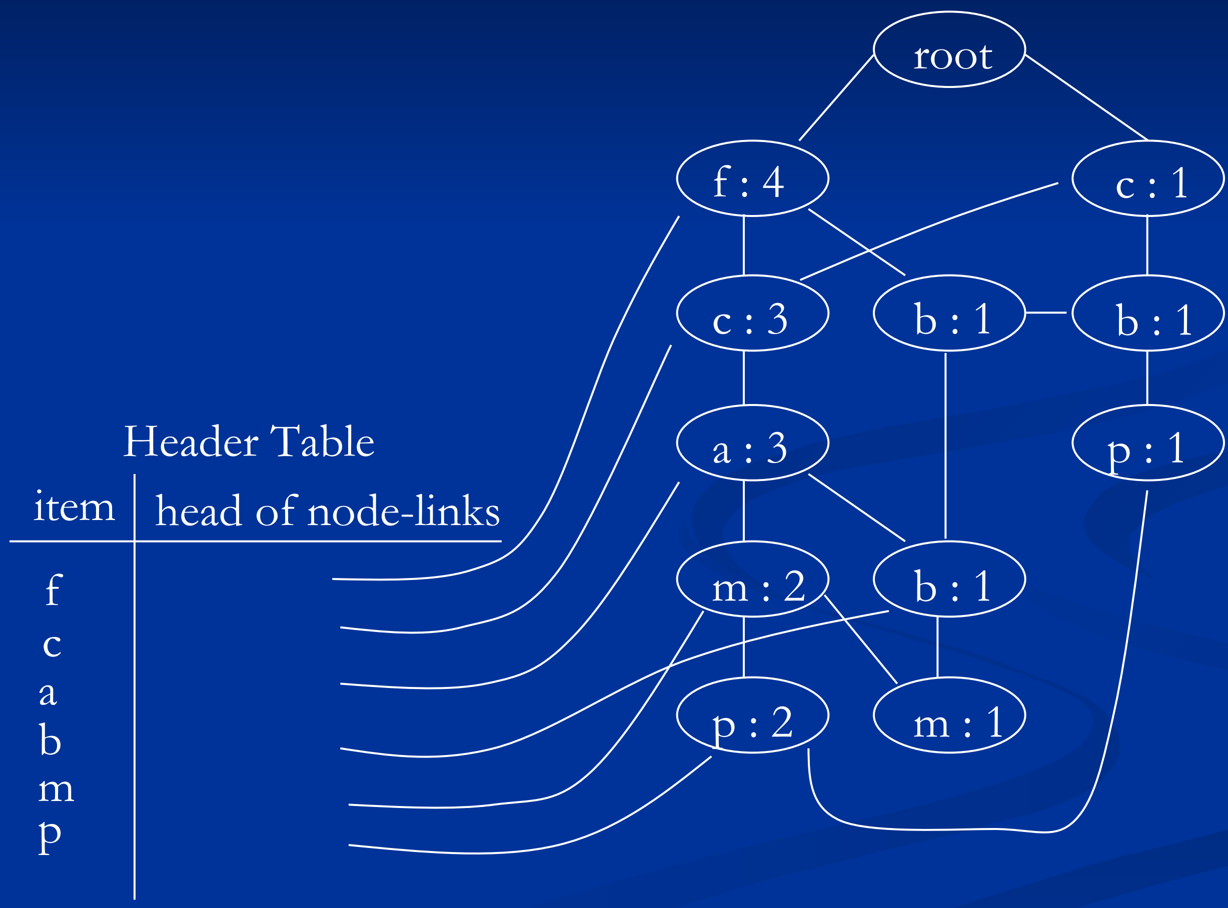


After trans 4
(c,b,p)

After trans 5
(f,c,a,m,p)

Example 1 (cont.)

Build the item header table



FP-Tree Properties

- Completeness
  - Each transaction that contains frequent pattern is mapped to a path.
  - Prefix sharing does not cause path ambiguity, as only path starts from root represents a transaction.
- Compactness
  - Number of nodes bounded by overall occurrence of frequent items.
  - Height of tree bounded by maximal number of frequent items in any transaction.

FP-Tree Properties (cont.)

- Traversal Friendly (for mining task)
  - For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links.
  - This property is important for divide-and-conquer. It assures the soundness and completeness of problem reduction.

FP-Growth Algorithm

- Functionality:
  - Mining frequent patterns using FP-Tree generated before
- Input:
  - FP-tree constructed earlier
  - minimum support threshold ξ
- Output:
  - The complete set of frequent patterns
- Main algorithm:
  - Call FP-growth(*FP-tree*, *null*)

FP-growth(*Tree*, α)

```
Procedure FP-growth(Tree, α)
{
  if (Tree contains only a single path P)
     {   for each combination β of the nodes in P
              {   generate pattern β Uα;
                  support = min(sup of all nodes in β)
              }
     }
  else // Tree contains more than one path
     {   for each a_i in the header of Tree
              {   generate pattern β= a_i Uα;
                  β.support = a_i.support;
                  construct β's conditional pattern base;
                  construct β's conditional FP-tree Tree_β;
                  if (Tree_β ≠ Φ)
                      FP-growth(Tree_β , β);
              }
     }

}
```

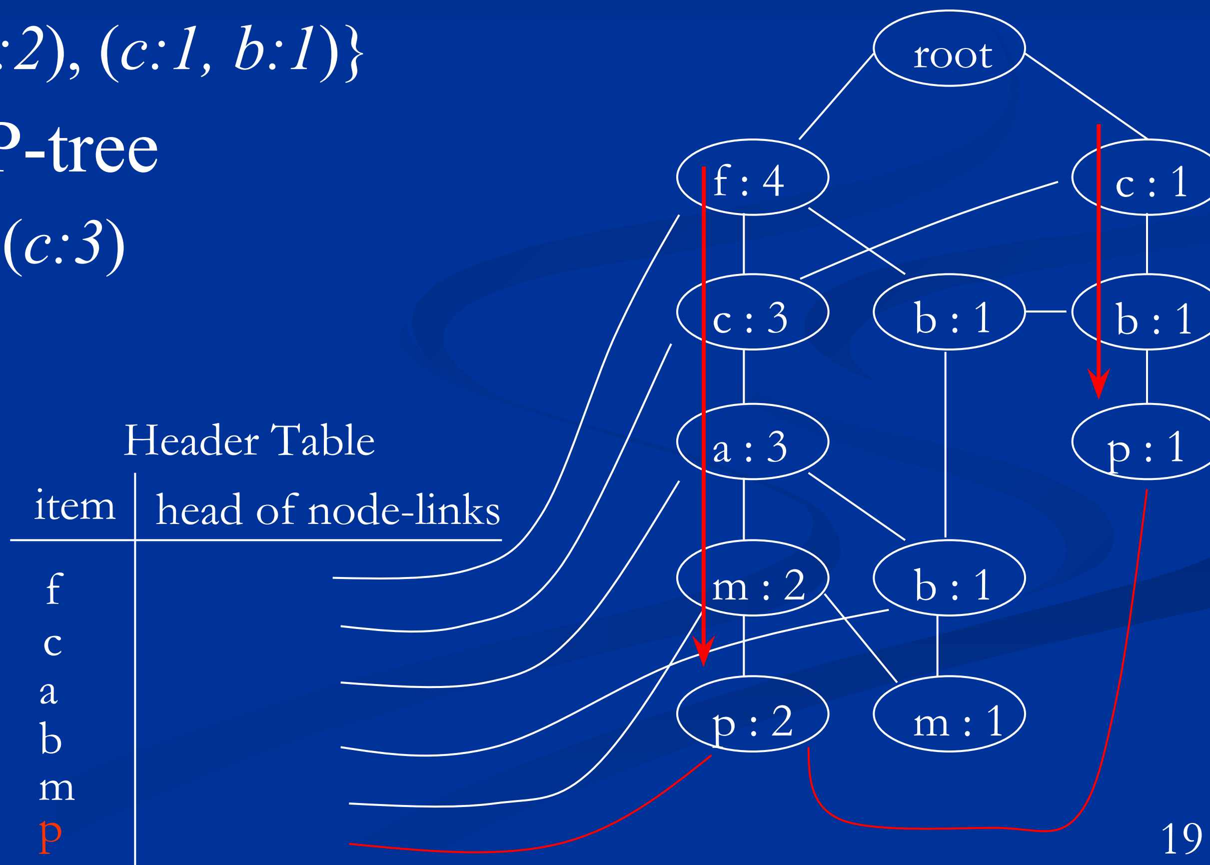
Example 2

- Start from the bottom of the header table: node *p*
- Two paths
- *p*'s conditional pattern base
  - {(*f:2, c:2, a:2, m:2*), (*c:1, b:1*)}

transformed prefix path

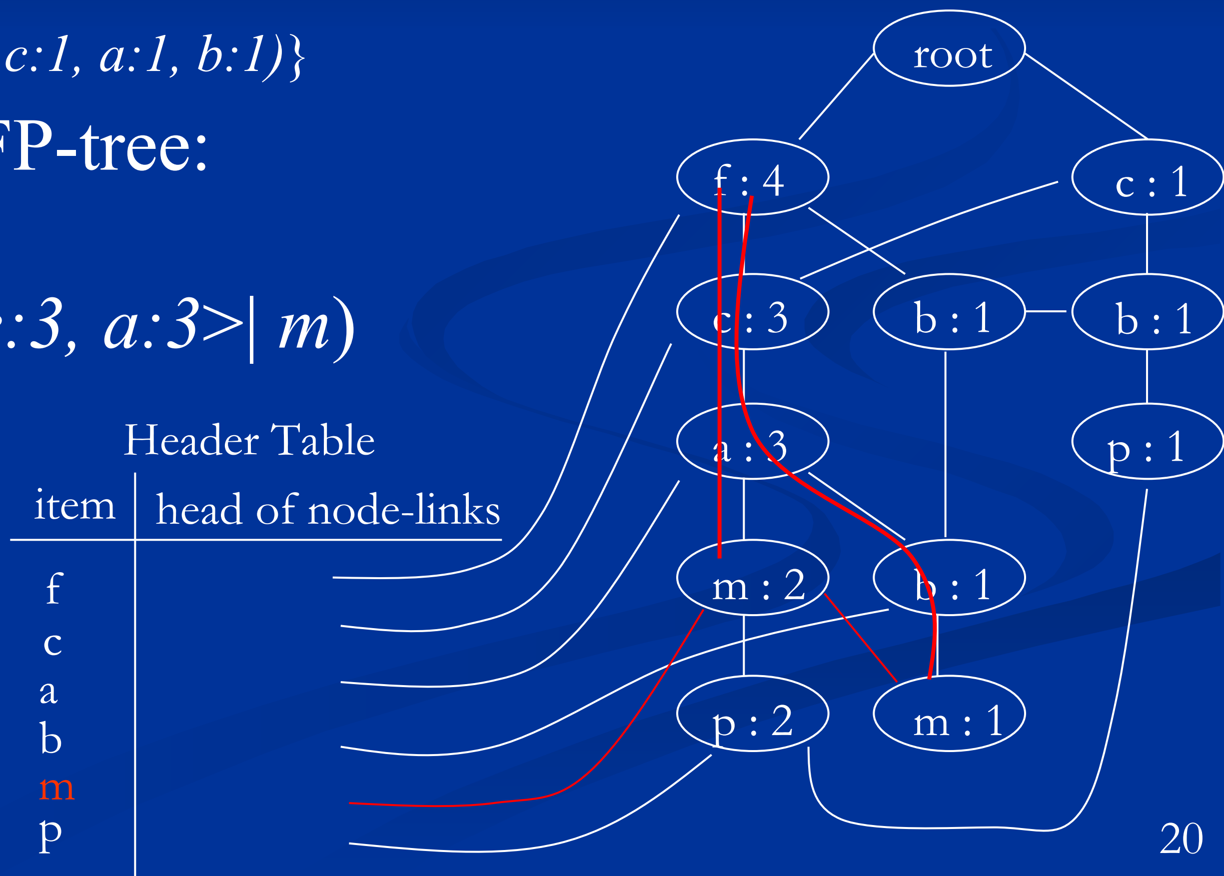
- *p*'s conditional FP-tree
  - Only one branch (*c:3*)

  □pattern *(cp:3)*
- Patterns:
  - *(p:3)*
  - *(cp:3)*



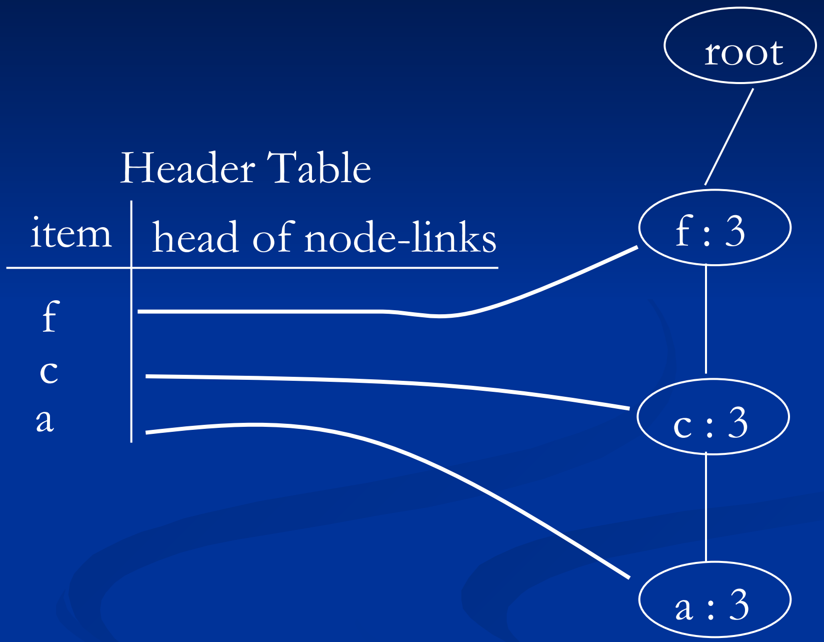
Example 2 (cont.)

- Continue with node *m*
- Two paths
- *m*'s conditional pattern base
  - *{(f:2, c:2, a:2), (f:1,c:1, a:1, b:1)}*
- *m*'s conditional FP-tree:
  - *(f:3, c:3, a:3)*
- Call mine(<*f:3, c:3, a:3*>| *m*)
- Patterns:
  - *(m:3)*
  - *see next slide*



mine(<(*f*:3, *c*:3, *a*:3>| *m*)

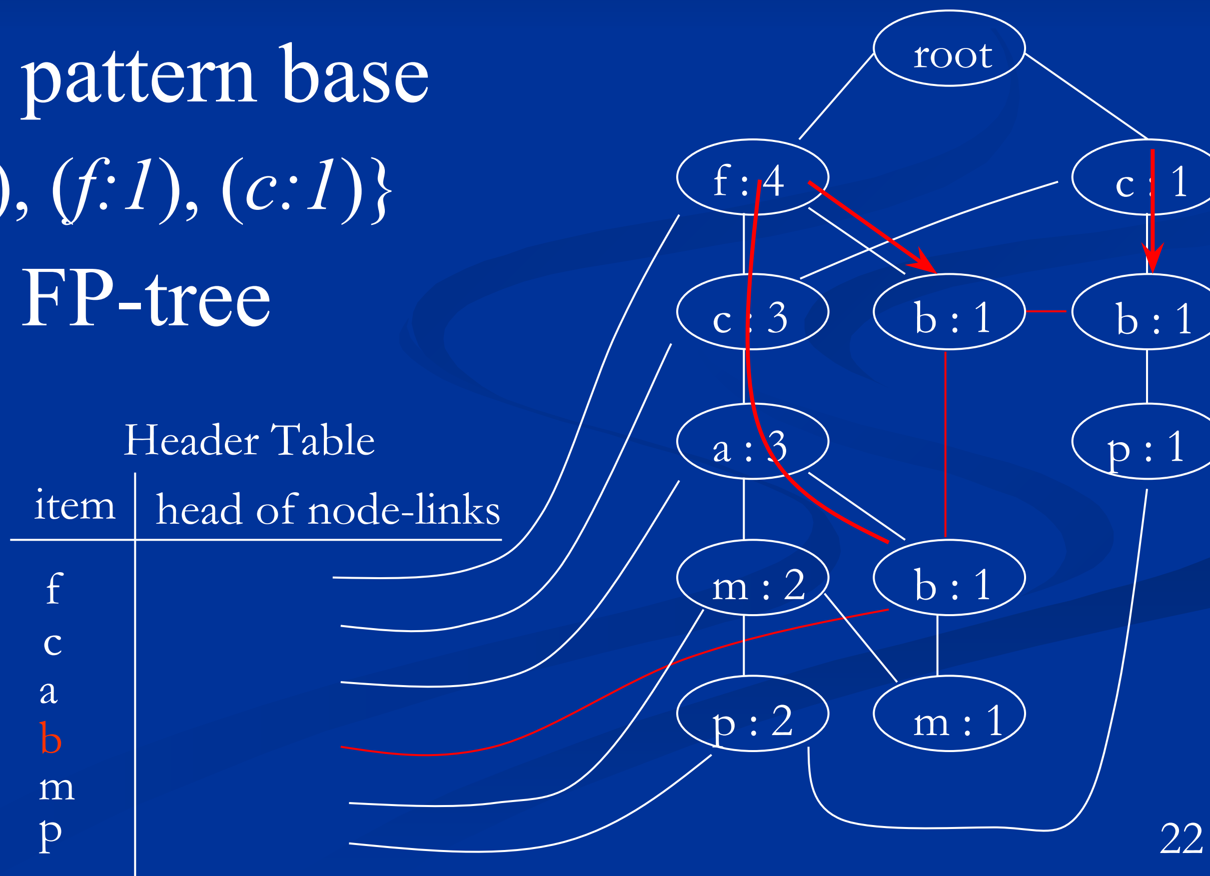
- node *a*:
  - *(am:3)*
  - call mine(<*f*:3, *c*:3>|*am*)
    - *(cam:3)*
    - call(<*f*:3)|*cam*)
      - (*fcam*:3)
    - *(fam:3)*
- node *c*:
  - *(cm:3)*
  - call mine(<*f*:3>|*cm*)
    - *(fcm:3)*
- node *f*:
  - *(fm:3)*
- All the patterns: *(m:3, am:3, cm:3, fm:3, cam:3, fam:3, fcm:3, fcam:3)*
- Conclusion: *A single path FP-Tree can be mined by outputting all the combination of the items in the path.*



conditional FP-tree of “m”

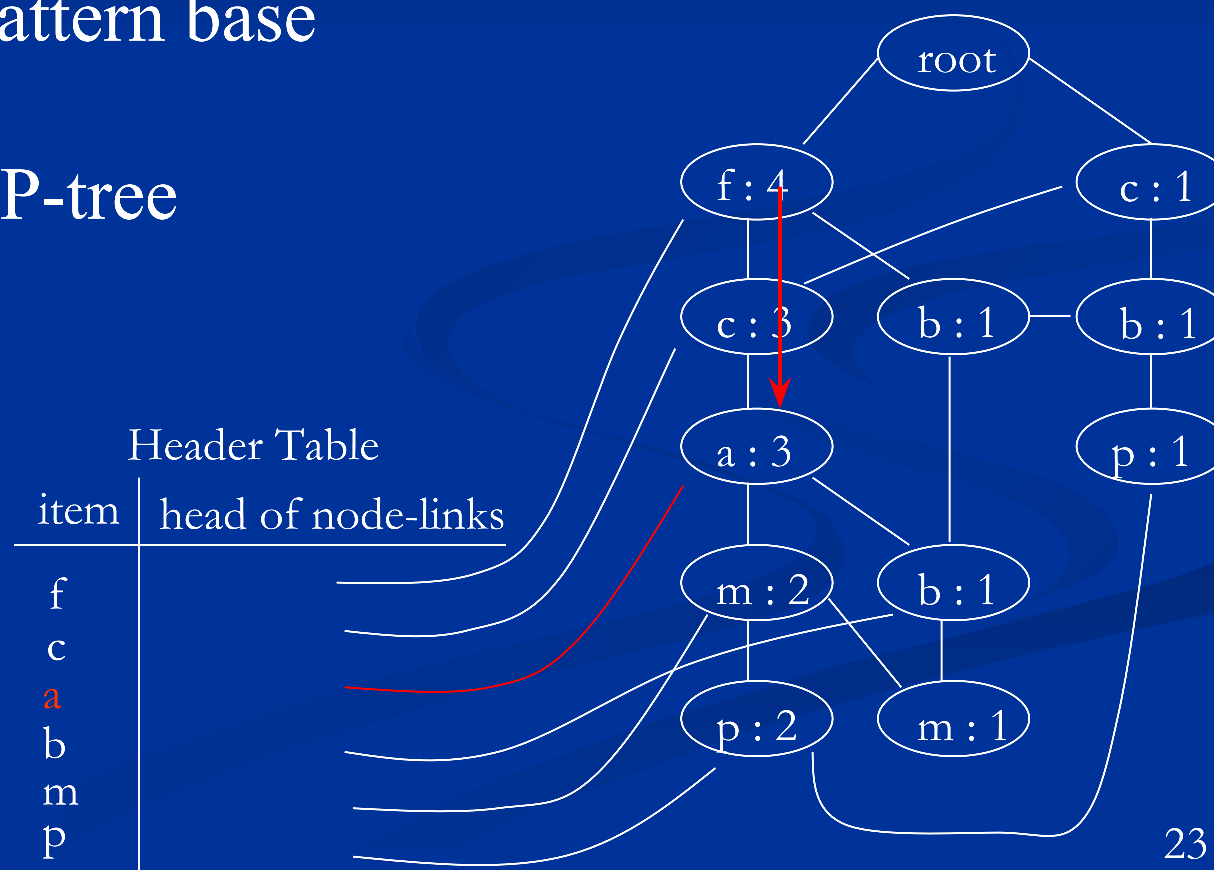
Example 2 (cont.)

- Continue with node *b*
- Three paths
- *b*'s conditional pattern base
  - $\{(f{:}1,\ c{:}1,\ a{:}1),\ (f{:}1),\ (c{:}1)\}$
- *b*'s conditional FP-tree
  - Φ
- Patterns:
  - *(b:3)*



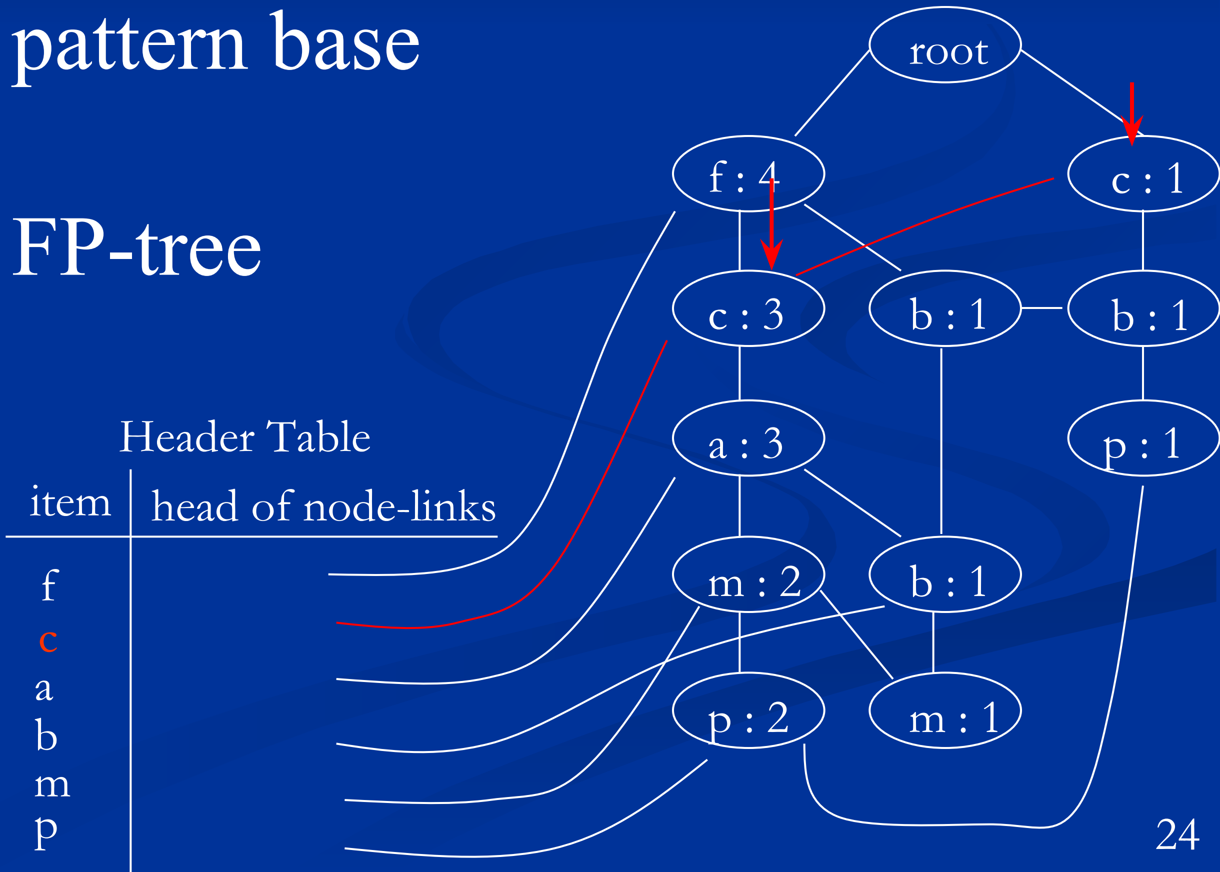
Example 2 (cont.)

- Continue with node *a*
- One path
- *a*'s conditional pattern base
  - *{(f:3, c:3)}*
- *a*'s conditional FP-tree
  - *{(f:3, c:3)}*
- Patterns:
  - *(a:3)*
  - *(ca:3)*
  - *(fa:3)*
  - *(fca:3)*



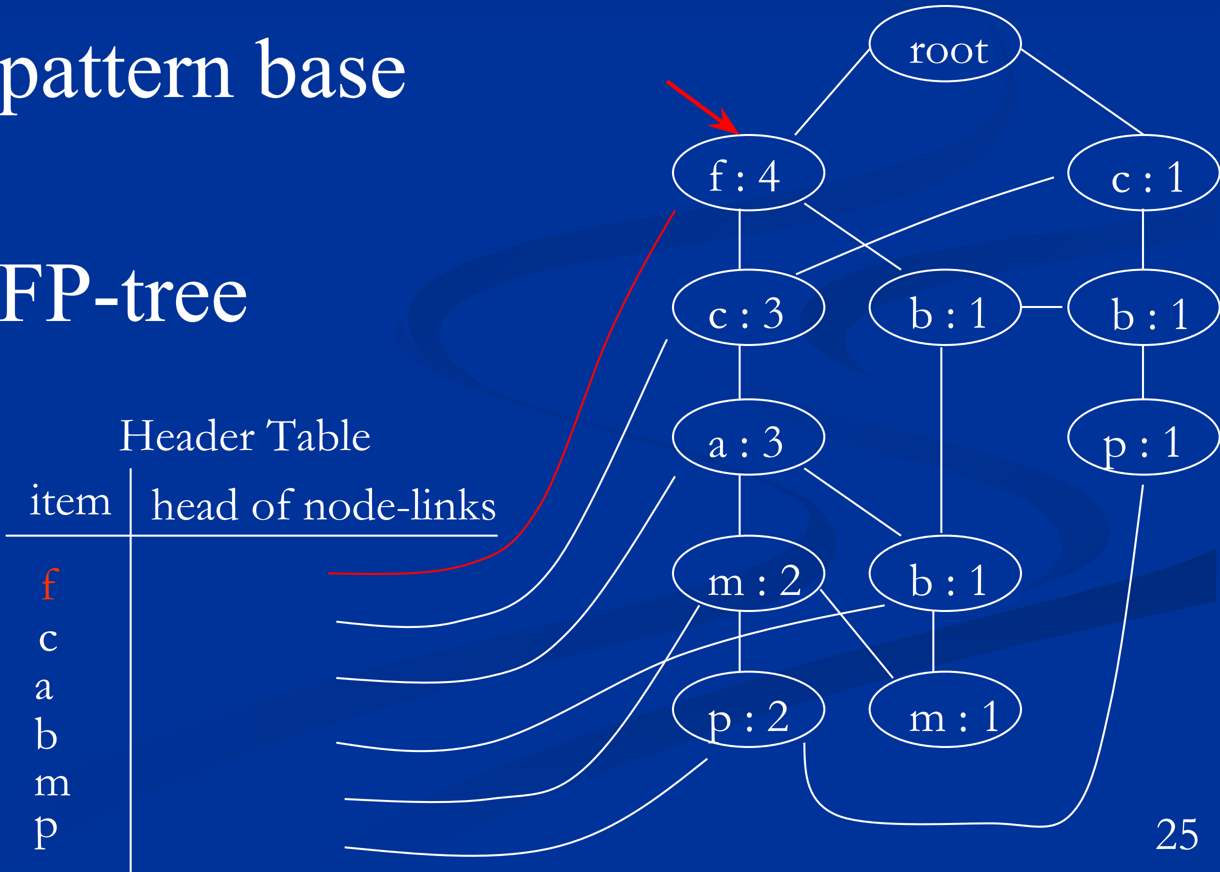
Example 2 (cont.)

- Continue with node *c*
- Two paths
- *c*'s conditional pattern base
  - {(*f:3*)}
- *c*'s conditional FP-tree
  - {(*f:3*)}
- Patterns:
  - *(c:4)*
  - *(fc:3)*



Example 2 (cont.)

- Continue with node *f*
- One path
- *f*'s conditional pattern base
  - *Φ*
- *f*'s conditional FP-tree
  - *Φ*
- Patterns:
  - *(f:4)*



Example 2 (cont.)

- Final results:

item	conditional pattern base	conditional FP-tree
p	{(f:2, c:2, a:2, m:2), (c:1, b:1)}	{(c:3)}| p
m	{(f:4, c:3, a:3, m:2), (f:4, c:3, a:3, b:1, m:1)}	{(f:3, c:3, a:3)}| m
b	{(f:4, c:3, a:3, b:1), (f:4, b:1), (c:1, b:1)}	Φ
a	{(f;3, c:3)}	{(f:3, c:3}| a
c	{(f:3)}	{(f:3)}| c
f	Φ	Φ

FP-Growth Properties

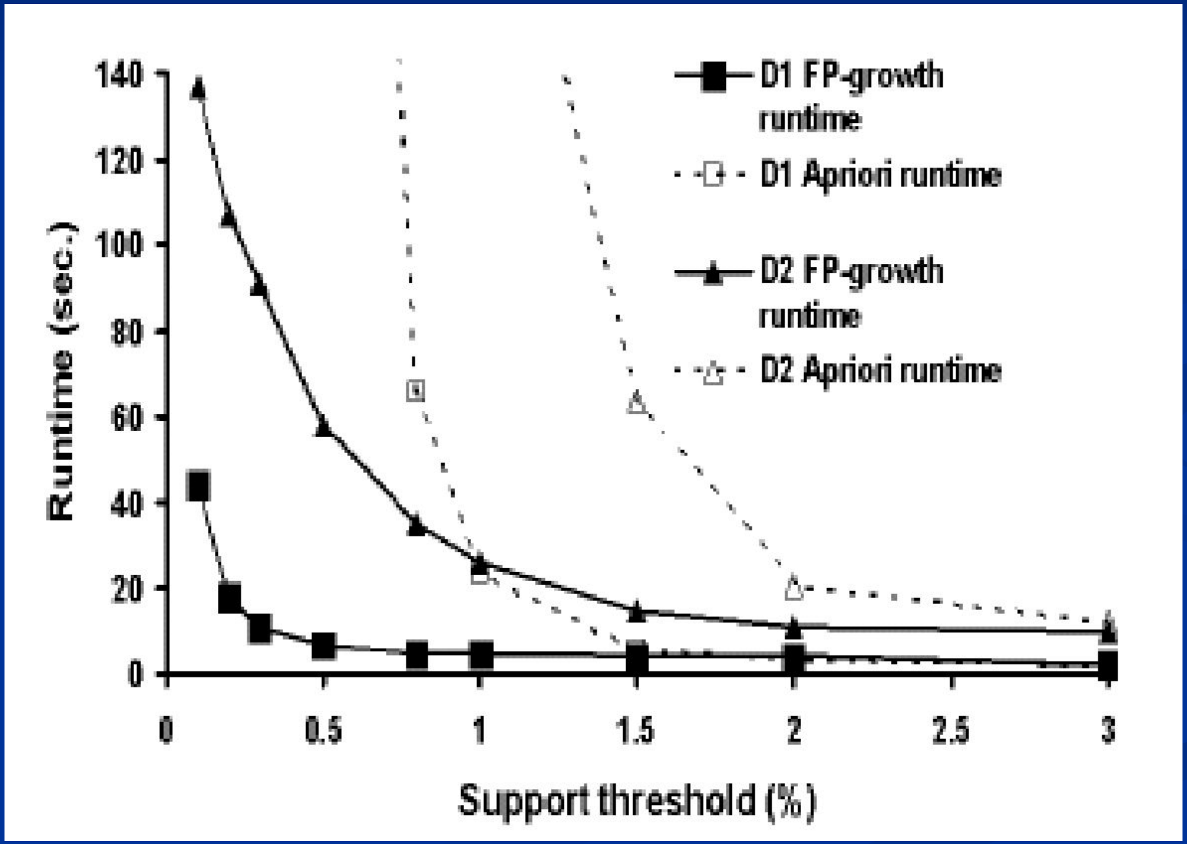
- Property 3.2 : Prefix path property
  - To calculate the frequent patterns for a node a_i in a path P, only the prefix subpath of node a_i in P need to be accumulated, and the frequency count of every node in the prefix path should carry the same count as node a_i.
- Lemma 3.1 : Fragment growth
  - Let α be an itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then the support of $\alpha U \beta$ in DB is equivalent to the support of β in B.

FP-Growth Properties (cont.)

- Corollary 3.1 (Pattern growth)
  - Let α be a frequent itemset in *DB*, *B* be α's conditional pattern base, and β be an itemset in *B*. Then $\alpha U \beta$ is frequent in *DB* if and only if is β frequent in *B*.
- Lemma 3.2 (Single FP-tree path pattern generation)
  - Suppose an FP-tree *T* has a single path *P*. The complete set of the frequent patterns of *T* can be generated by the enumeration of all the combinations of the subpaths of *P* with the support being the minimum support of the items contained in the subpath.

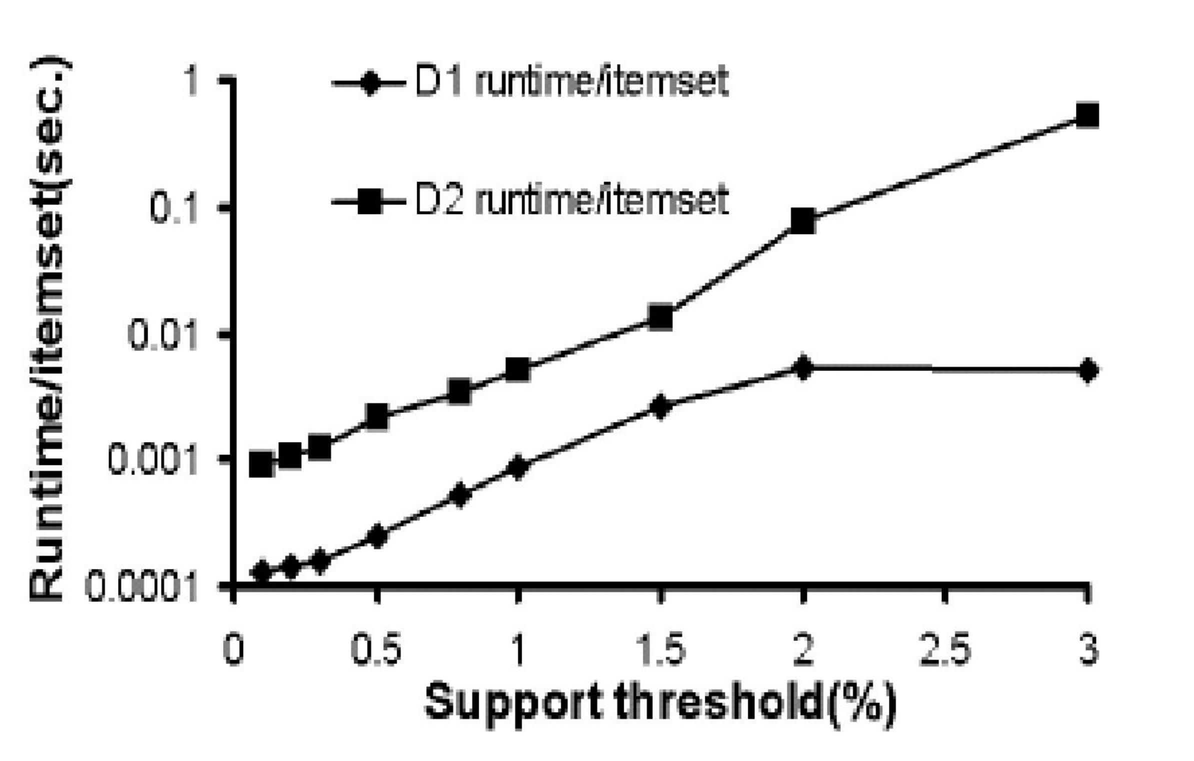
Performance Evaluation: FP-Tree vs. Apriori

- Scalability with Support Threshold



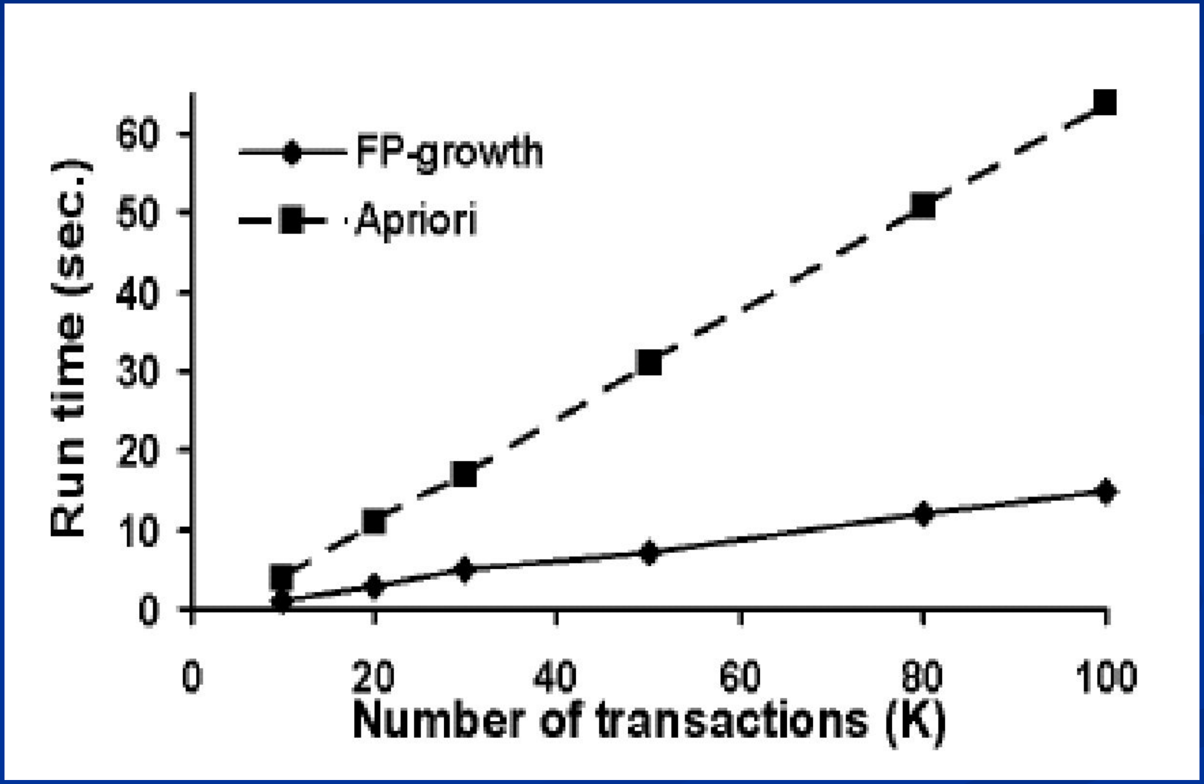
Performance Evaluation: FP-Tree vs. Apriori (Cont.)

- Per-item runtime actually decreases with support threshold decrease.



Performance Evaluation: FP-Tree vs. Apriori (Cont.)

- Scalability with DB size.



Discussions

- When database is extremely large.
  - Use FP-Tree on projected databases.
  - Or, make FP-Tree disk-resident.
- Materialization of an FP-Tree
  - Construct it independently of queries, with an reasonably fit-majority minimum support-threshold.
- Incremental updates of an FP-Tree.
  - Record frequency count for every item.
  - Control by watermark.