

28th Feb '17

Date: ___/___/___
Page: 33

Lecture - 29.

* LR parsing algorithm:

Input:

An input string w and an LR parsing table with function ACTION & GOTO for a grammar A .

Output:

If w is in $L(G)$, the reduction steps for ~~parse~~

Method:

Initially s_0 on its stack & w in the input buffer.

Let, a' be the first symbol of w &
while(1)

{

let s be the state on top of the stack;
if (ACTION $[s, a'] = \text{shift } t$)

{

push t onto the stack;

let a be the next input symbol;

{

else if (ACTION $[s, a] = \text{reduce } A \rightarrow \beta$)

{

pop $\alpha | \beta$ symbols off the stack;

let state t now be the on top of stack;
 push $\text{GOTO}[t, A]$ onto the stack;
 output the production $A \rightarrow \beta$;
 }
 else if ($\text{ACTION}[s, a] = \text{accept}$)
 break;
 else error;
 }

Q. Consider an augmented grammar:

$$E' \rightarrow E$$

$$E$$

1st March '17

Lecture 30

(iii) SLR(1):

* Construction of $\text{pSLR}(1)$ or SLR parsing table:

Input : Augmented Grammar g'

Output : The $\text{SLR}(0)$ parsing table functions
 ACTION & GOTO for g'

Method :

Step 1 : Construct $C = \{ I_0, I_1, \dots, I_n \}$,
 the collection of sets of $\text{SLR}(0)$

items for g' .

Step-2:

Set i is constructed from I_i
The possible actions for state i are
determined as follows:

a.) If $A \rightarrow \alpha$ is in I_i^0 then set
production ACTION $[i, a]$ to reduce $A \rightarrow \alpha$
for all a in $\text{FOLLOW}(A)$.

b.)

c.)

Now, the SLR(1) parsing table for previous LR(0) parsing table.

I	ACTION						GOTO		
	id	+	*	()	\$	E	T	F
0	s ₅			s ₄		?			
1		s ₆				accept	1	2	3
2			s ₇						
3				s ₂	s ₂				
4	s ₅			s ₄					
5			s ₆	s ₆					
6	s ₅			s ₄				9	3
7	s ₅			s ₄					10
8		s ₆			s ₁₁				
9		s ₁	s ₇		s ₁	s ₁			
10		s ₃	s ₃		s ₃	s ₃			
11		s ₅	s ₅		s ₅	s ₅			

for I₂; s₂ \Rightarrow E \rightarrow T
 $\text{FOLLOW}(E) = \{ \$, +,) \}$

for I₃; s₄ \Rightarrow T \rightarrow F
 $\text{FOLLOW}(T) = \{ \$, +,) \}$

for I₅; s₆ \Rightarrow F \rightarrow id
 $\text{FOLLOW}(F) = \{ \text{FOLLOW}(T) \}$
 $= \{ \$, +,) \}$

for I₉; s₁ \Rightarrow E \rightarrow E + T
 $\text{FOLLOW}(E) = \{ \$, +,) \}$

for I₁₀; s₃ \Rightarrow T \rightarrow id
 $\text{FOLLOW}(T) = \{ \$, +,) \}$

Page _____
Date _____

for III; HS: FOLLOW (E) EQUIVALENT
= {0, 1, 2, 3, 4}

Parse the string:

from the LR(1) parsing table:

Stack	Top	Action
0	id * id + id \$	Shift
0 id \$	± id + id * \$	Reduce S → id
0 T 2	± id + id \$	Reduce T → id
0 T 2 * T	* id + id \$	Shift
0 T 2 * T id \$	id + id \$	Shift
0 T 2 * T P id \$	± id \$	Reduce P → id
0 T 2 * T P T id \$	± id \$	Reduce T → id
0 E 1	± id \$	Reduce E → T
0 E 1 + G	id \$	Shift
0 E 1 + G id \$	↓	Reduce E → T
0 E 1 + G F		Reduce E → T

DATE - / /
PAGE

Stack

0

0id\$

~~0~~ 0F\$

0T\$

0T2*E

0T2*Tid\$

0T2*Tid10

~~0~~ 0T*

0EL

0E1+6

0E1+6id\$

0E1+6F\$

0E1+6T\$

0E L

i/p

id * id + id + \$

* id + id + \$

* id + id + \$

id + id + \$

* id + id + \$

* id + id + \$

id + id + \$

* id + id + \$

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id + id + \$

* id + id + \$

Action

Shift ✓

Reduce $F \rightarrow id$ (Pop 2)

Reduce $T \rightarrow F$ (Pop 2)

Shift ✓

Shift ✓

Reduce $F \rightarrow id$ (Pop 2)

Reduce $T \rightarrow T * F$ (Pop 6)

Reduce $E \rightarrow T$ (Pop 2)

Shift ✓

Shift ✓

Reduce $F \rightarrow id$ (Pop 2)

Reduce $T \rightarrow F$ (Pop 2)

Reduce $E \rightarrow E + T$ (Pop 6)

accept ✓

2nd March '17
lecture 31

$S \rightarrow AA$

$A \rightarrow aA/b$

from LR(0) to SLR(1)

I	ACTION			GOTO	
1	a	b	\$	s	A
0	s ₃	s ₄		1	2
1			accept		
2	s ₃	s ₄			
3	s ₃	s ₄			
4	s ₃	s ₃	s ₃		
5			s ₁		
6	s ₂	s ₂	s ₂		

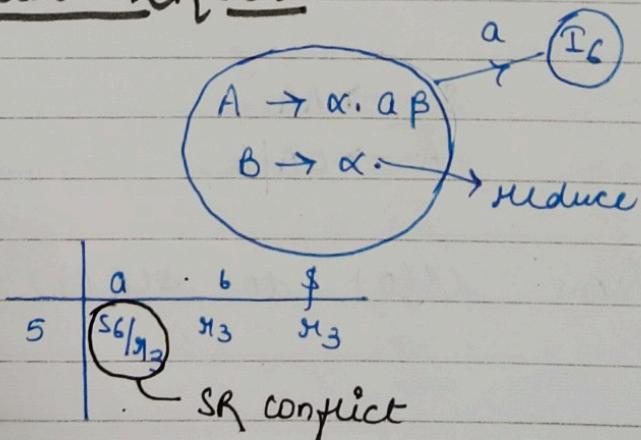
for I_4 ; $M_3 = A \rightarrow b$
 $\text{FOLLOW}(A) = \{ \$, a, b \}$

for I_5 ; $M_1 = S \rightarrow AA$
 $\text{FOLLOW}(S) = \{ \$ \}$

for I_6 ; $M_2 = A \rightarrow aA$
 $\text{FOLLOW}(A) = \{ \$, a, b \}$

* conflicts in LR(0) :

1.) SLR conflict :



2. RA conflict :

$$\begin{array}{c} A \rightarrow \alpha \\ B \rightarrow \alpha. \end{array}$$

	a	b	\$
5	x_1/x_3	x_4/x_3	x_4/x_3

& different actions on a terminal.

conflicts in SLR(1) :

1. SR conflict :

	a	b	\$
5	s_6/x_3	x_3	x_3

$$\begin{array}{c} A \rightarrow \alpha \cdot a\beta \\ B \rightarrow \alpha. \end{array}$$

$$\text{Follow}(B) = \{a\}$$

on finding $\text{Follow}(B)$ → solving by SLR(0) method we still get shift & reduce both operations on terminal 'a'. Hence, SR conflict still present.

2. RR conflict :

$$\begin{array}{c} A \rightarrow \alpha. \\ B \rightarrow \alpha. \end{array}$$

if $\text{Follow}(A) \cap \text{Follow}(B) \neq \emptyset$
then RR conflict.

q.

$$\begin{array}{l} S \rightarrow A/a \\ A \rightarrow a \end{array}$$

LL(1)

LR(0)

SLR(1)

$$\begin{array}{l} S \rightarrow A/a \\ A \rightarrow a \end{array} \quad \text{FIRST}(A) = \underline{\{a\}}$$

$$\text{FIRST}(a) = \underline{\{a\}}$$

So, not LL(1) grammar

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Date: ___/___/___
Page No. ___

3.

$$S \rightarrow dA | aB$$

$$A \rightarrow bA | c$$

$$B \rightarrow bB | c$$

$$S \rightarrow (L) | a$$

$$L \rightarrow L, S | S$$

4.

$$E \rightarrow E + T | T$$

$$T \rightarrow i$$

$$E \rightarrow T + E | T$$

$$T \rightarrow i$$

5.

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow TF$$

$$T \rightarrow F$$

$$F \rightarrow F * | a | b$$

6.

$L(1)_{\Sigma}$

$$S \rightarrow A/a$$

$$A \rightarrow a$$

FIRST

$$\{ \text{ } \}$$

$$\{ \text{ } \}$$

FOLLOW

$$\{ \text{ } \}$$

$$\{ \text{ } \}$$

$\Leftrightarrow L(1) \text{ not pass.}$

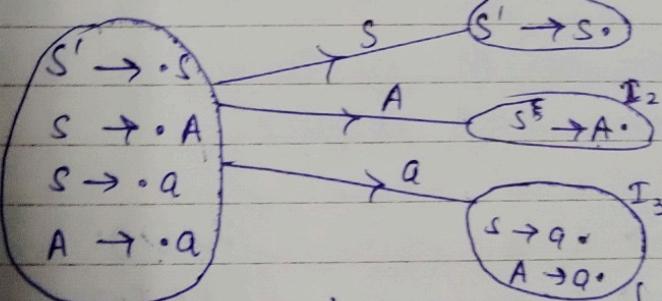
$LR(0)$:

$$S' \rightarrow S$$

$$S \rightarrow A/a$$

$$A \rightarrow a$$

I_0



NOT $LR(0)$

	ACTION	GO TO	
I	a. .	S	A
0	accpt	1	2
1			
2			
3	$\Sigma_0/\Sigma_3, \Sigma_1/\Sigma_3$		

Soln (1):

ACTION

GOTO

I a \$ 5 A
o S3 1 2
1 accept
2 H₄
3 (H₂ / H₃)

$$S \rightarrow dA \mid aB$$

$$A \rightarrow bA \mid c$$

$$B \rightarrow bB | c$$

		FIRST	FOLLOW
L2(1) :	$S \rightarrow dA \mid AB$	$\{d, a\}$	$\{ \$, b, c \}$
	$A \rightarrow bA \mid c$	$\{b, c\}$	$\{ \$, b, c \}$
	$B \rightarrow bB \mid c$	$\{b, c\}$	$\{ \$, b, c \}$

not LL(1)

$df(0) :$

$$s' \rightarrow s$$

$$S \rightarrow dA$$

$S \rightarrow AB$

$$A \rightarrow b$$

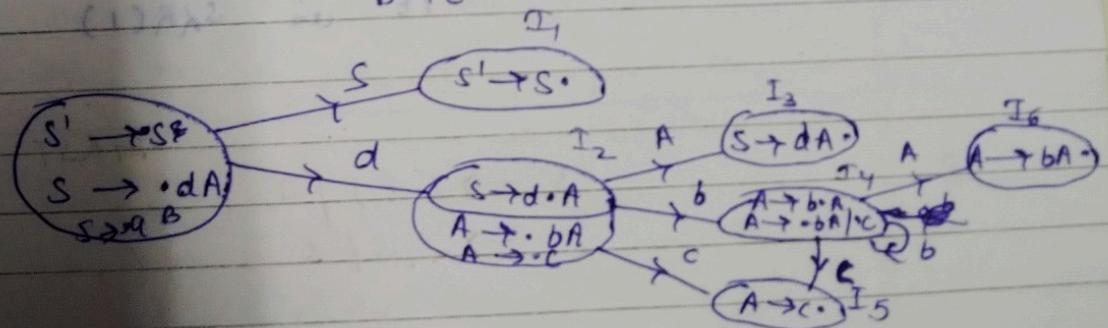
$$A \rightarrow C$$

$$B \rightarrow bB$$

B → C

1000

10



	ACTION					GOTO	
1	a	b	c	d	\$	S	A
0						S2	1
1						accept	
2		S4	S5				3
3	x1	x1	x1	x1	x1		
4	x4	x5					6
5	x4	x4	x4	x4	x4		
6	x3	x3	x3	x3	x3		

It is LR(0)

SLR(1) :

	ACTION					GOTO	
1	a	b	c	d	\$	S	A
0						S2	1
1						accept	
2	S4	S5					3
3					x1		8
4	S4	S5					6
5	x4	x4	x4	x4			
6	x3	x3	x3	x3			

It is SLR(1).

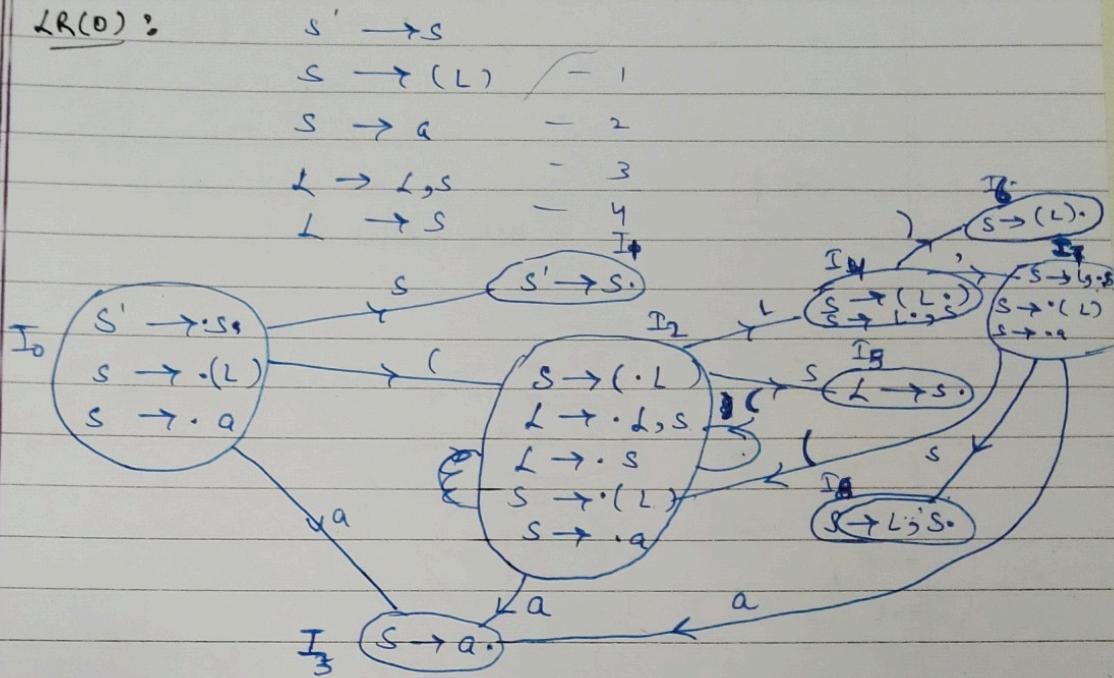
3

$$\begin{array}{l}
 \text{FIRST FOLLOW} \\
 S \rightarrow C(L) \mid a \quad \{ (, \alpha \} \quad \{ \$,), \beta \} \\
 L \rightarrow d, \beta \mid S \quad \{ \{, \alpha \} \quad \{ \}, \beta \}
 \end{array}$$

not Ld(1) : left recursion

Date: _____
Page: _____

LR(0):



ACTION

I () a, \$

0 S2 S3

1 ~~S2~~ S3 accept

2 S2 S3

3 H2 H2 H2 H2 H2

4 S6 S7

5 H4 H4 H4 H4 H4 H4

6 H4 H1 H4 H1 H4

7 S2 S3

8 H3 H3 H3 H3 H3

GOTO

S L

1

4

5

4

8

It is LR(0).

SLR(1):

∴ It is LR(0)

So, it is also SLR(1).

Date: ___/___/___
Page: ___
AP B

4 $E \rightarrow E + T \mid T$
 $T \rightarrow i^0$

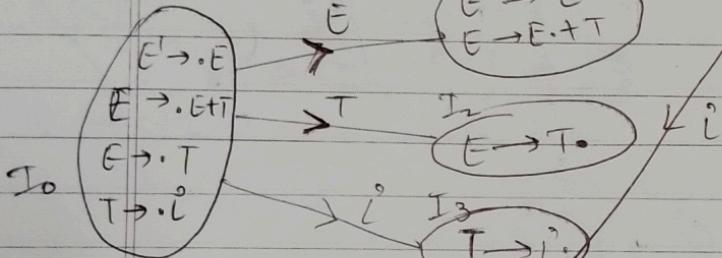
$\Rightarrow E' \rightarrow E$

$E \rightarrow E + T \quad \{ \begin{matrix} \{ \cdot \} \\ \{ + \} \end{matrix} \}$
 $E \rightarrow T \quad \{ \begin{matrix} \{ \cdot \} \\ \{ \cdot \} \end{matrix} \}$
 $T \rightarrow i^0 \quad \{ \begin{matrix} \{ \cdot \} \\ \{ + \} \end{matrix} \}$

$\lambda L(1)$:

not $\lambda L(1)$

$\lambda R(0)$:



$\lambda R(0)$ ✓

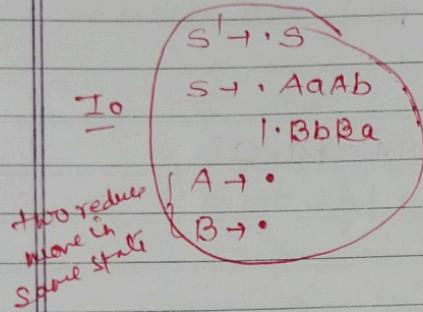
I	i^0	$+$	\cdot	E	T	no conflict
0	S_3			1	2	
1		S_4 accept				
2	H_2	H_2	π_2			
3	H_3	H_3	π_3			
4	S_3				5	
5	H_1	H_1	π_1			

$SLR(0)$:

✓

examples:

$S \rightarrow AaAb \rightarrow \textcircled{1}$	LL(1) ✓
$ BbBa \rightarrow \textcircled{2}$	LR(0) ✗
$A \rightarrow C \rightarrow \textcircled{3}$	SLR(1) ✗
$B \rightarrow C \rightarrow \textcircled{4}$	

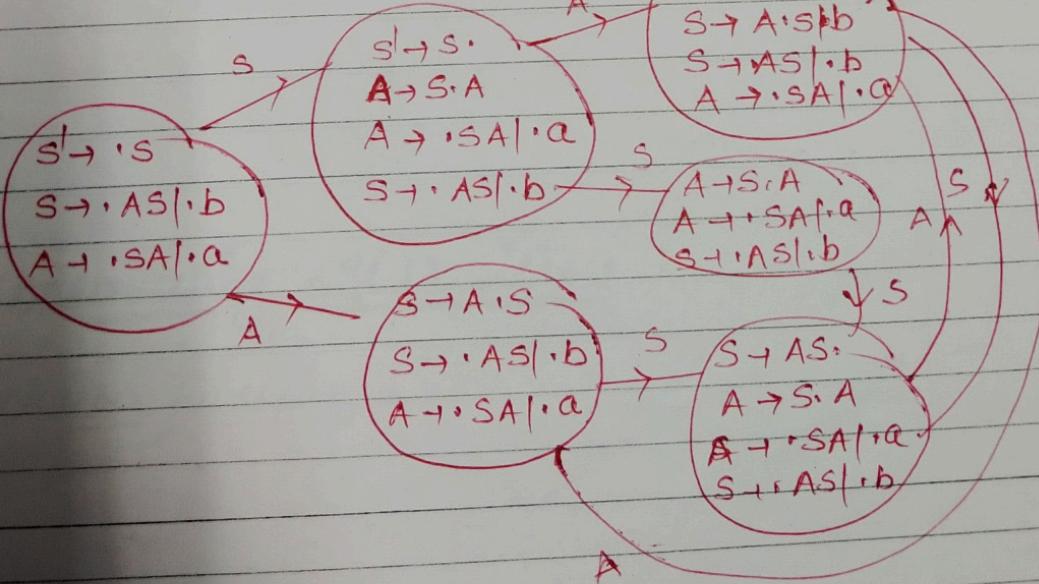


SLR Parsing table

	a	b	\$
0	r3/r4	r3/r4	

not SLR(1)

⑤ $S \rightarrow AS|b$ LL(1) ✗
 $A \rightarrow SA|a$ LR(0) ✗
 SLR(1) ✓



$S \rightarrow Aa \quad (a)$
 $| BA \quad (b)$
 $| dc \quad (d)$
 $| bda$

LU(1) X
LR(0) X
SLR(1) X

Date: ____ / ____ / ____
Page: ____

$$A \rightarrow a$$

S → S
S → Aa
↑ bAC
↑ dc
↑ bda
A → d

$\overset{d}{\rightarrow} \text{S} \xrightarrow{\text{d} \cdot C} \text{A} \xrightarrow{\text{d} \cdot i}$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline b & & & & 5/8 \end{array}$$

$$FOLLOW(A) = \{a, c\}$$

LR(0)
on RHS

LALR(1) CLR(1)

$$LR(1) \text{ items} = LR(0) \text{ items} + \text{lookahead}$$

$S \rightarrow \bullet aA, a \uparrow b$

$$S \rightarrow AA^*, C/d^*$$

3rd March '17

Lecture 32

Date: / /
Page: / /

First 10

Canonical LR (CLR):

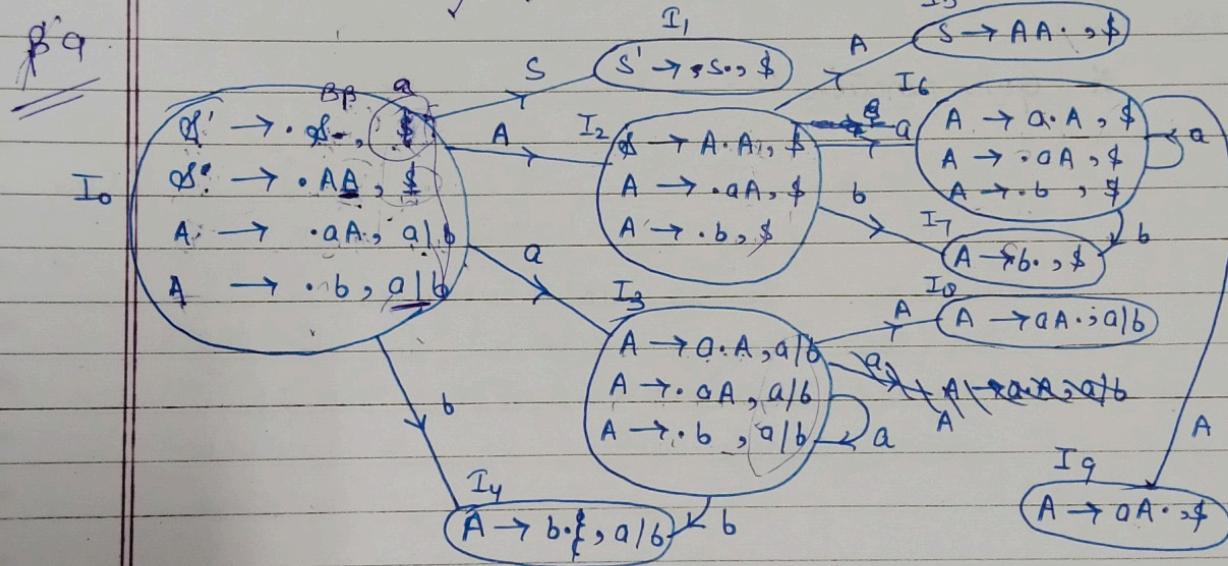
Consider an augmented grammar:

$$S' \rightarrow S$$

$$S \rightarrow AA$$

$$A \rightarrow aA \mid b$$

compute LR(1) items.



4th March '17

Lecture 33

Constructing LR(1) items:

* closure function:

closure (I)

{

repeat

for (each item $[A \rightarrow \alpha \cdot B\beta, a]$ in I)

for (each production $B \rightarrow \gamma$ in G')

for (each terminal b in FIRST(βa))

add $[B \rightarrow \gamma, b]$ to set I;

until no more items are added to I;

}

* GOTO function:

GOTO (I, x)

{

Initialize J to be the empty set;

for (each item $[A \rightarrow \alpha \cdot x\beta, a]$ in I)

add item $[A \rightarrow \alpha x \cdot \beta, a]$ to set J;

return (closure (J));

}

* Construction of canonical LR(1) parsing table:

Input : Augmented grammar G'

Output : The CLR(1) parsing table function
ACTION & GOTO for G'

Method :

Step-1 : Construct $C = \{I_0, I_1, \dots, I_n\}$ →
the collection of sets of
CLR(1) items for G' .

Step-2 :

State i is constructed from I_i^0 .
 The parsing actions for state i are determined as follows:

- If $[A \rightarrow \alpha \cdot a\beta, \bullet b]$ is in I_i^0 and $\text{GOTO}(I_i^0, a) = I_j^0$ then set $\text{ACTION}[i, a] = \text{"shift } j\text{"}$.
- If $[A \rightarrow \alpha \cdot, a]$ is in I_i^0 , $A \neq S'$ then set $\text{ACTION}[i, a]$ to "reduce $A \rightarrow \alpha$ ".
- If $[S' \rightarrow S \cdot, \$]$ is in I_i^0 , then set $\text{ACTION}[i, \$]$ to "accept".

Step-3 :

The GOTO transitions for state i are constructed for all non-terminals A using the rule:

if $\text{GOTO}(I_i^0, A) = I_j^0$ then
 $\text{GOTO}(I_i^0, A) = j^0$

Step-4 :

All entries not defined by rules 2 & 3 are made "error".

Step-5 :

The initial state of the parser is constructed from the set of items containing $[S' \rightarrow S \cdot, \$]$.

I	ACTIONS			GOTO	
	a	b	\$	S	A
0	s3	s4		1	2
1			accept		
2	s5s6	s7			5
3	s3	s4			0
4	s3	s3			
5				s1	
6	s6	s7			9
7				s3	
8	s2	s2			
9				s2	

iii. LALR (Look-ahead LR) :

* construction of LALR parsing table :

for the previous CLR(1) grammar
states 3 & 6, 4 & 7, 8 & 9 have
same LR(0) productions but different
look ahead symbols. So for LALR
combine them as follows:

(only for shift productions not for reduce)

I₃, I₆

I₄, I₇

I₈, I₉

I₃₆ : A → a·A , a/b/\$

A → ·aA , a/b/\$

A → ·b , a/b/\$

I₄₇ : A → b· , a/b/\$

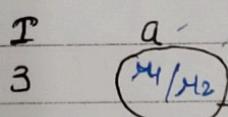
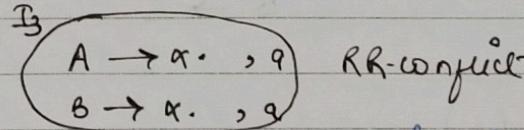
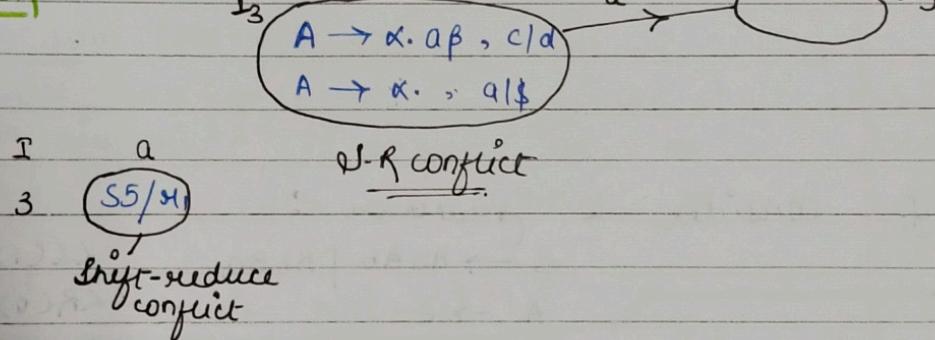
I₈₉ : A → aA· , a/b/\$

		ACTION			GOTO	
I		a	b	\$	S	A
0		s_{36}	s_{47}		1	2
1				accept		
2		s_{36}	s_{47}			5
36		s_{36}	s_{47}			89
47		h_3	h_3	h_3		
5				h_1		
89		h_2	h_2	h_2		

6th March '17 Lecture - 34

Conflicts in CLR(1) & LALR(1):

CLR:



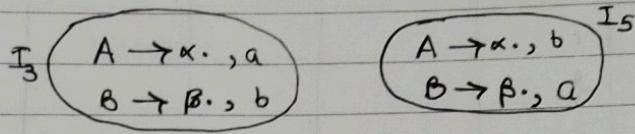
If it is a reduce operation & look-aheads of the productions in any item match then there will be RR conflict

* Note:

- 1.) If any grammar is not LR then it will definitely not be a LALR.
- 2.) If any grammar is CLR then it may or

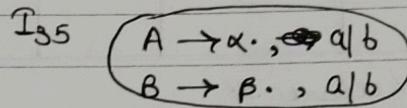
may not be LALR.

Example :



It is CLR(1) ~~not LALR~~

for LALR :



It is not LALR because
 there will be RR-conflict.
 Because $A \approx B$ will perform
 & they both will perform
 reduce on the same $a \& b$ so,
 RR-conflict not LALR

Q. Consider a grammar :

$$S \rightarrow AaAb \mid BbBa \quad LL(1)$$

$$A \rightarrow \epsilon \quad LR(0)$$

$$B \rightarrow \epsilon \quad SLR(1)$$

$$CLR(1)$$

$$LALR(1)$$

$$FOLLOW$$

$$\bullet \quad \underline{LL(1)}$$

$$S \rightarrow AaAb \quad \{a, b\} \quad \{\$\}$$

$$S \rightarrow BbBa$$

$$A \rightarrow \epsilon \quad \cancel{\{a\}} \quad \{a, b\}$$

$$B \rightarrow \epsilon \quad \cancel{\{b\}} \quad \{a, b\}$$

~~LL(1)~~ $LL(1)$

Date: ___/___/___
Page: ___

• LR(0):

Augmented grammar:

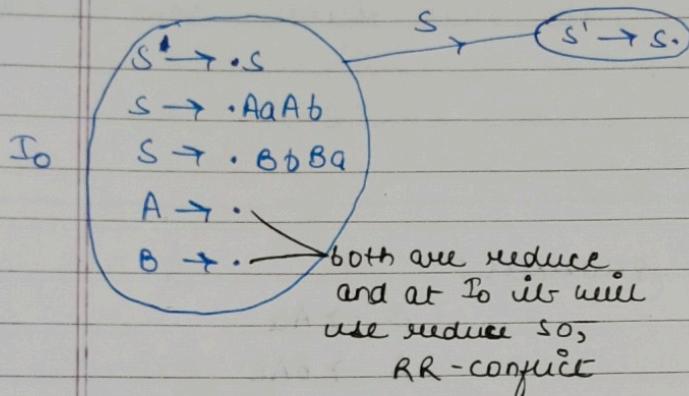
$$S' \rightarrow S$$

$$S \rightarrow AaAb \quad -1$$

$$S \rightarrow BbBa \quad -2$$

$$A \rightarrow \epsilon \quad -3$$

$$B \rightarrow \epsilon \quad -4$$



∴ NOT LR(0).

• SLR(1):

$$A \rightarrow \cdot \epsilon$$

$$\text{FOLLOW}(A) = \{a, b\}^{g_3 \text{ at } g_4, b}$$

$$B \rightarrow \cdot \epsilon$$

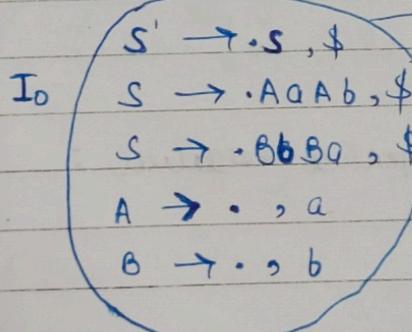
$$\text{FOLLOW}(B) = \{a, b\}^{g_4 \text{ at } g_3, b}$$

again R-R conflict as ~~at~~ at I_0 item
 a, b will contain g_3 & g_4 both

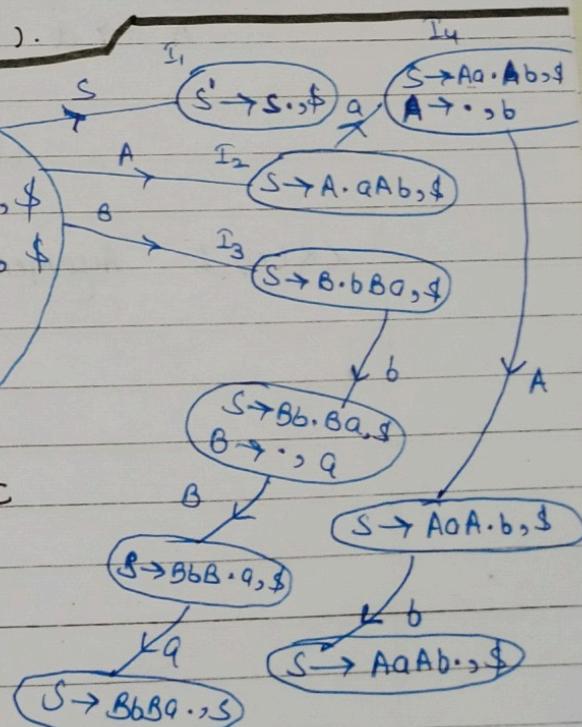
∴ NOT SLR(1).

• CLR(1):

I_0



∴ It is CLR(1) grammar.



• LALR(1) :

" There are no common productions with different look ahead values.

so, it will remain same as CLR(1).

∴ It is also LALR(1).

Q. consider the grammar:

LL(1)
LR(0)
SLR(1)
CLR(1)
LALR(1)

$S \rightarrow Aa$
 $S \rightarrow bAC$
 $S \rightarrow dc$
 $S \rightarrow bda$
 $A \rightarrow d$

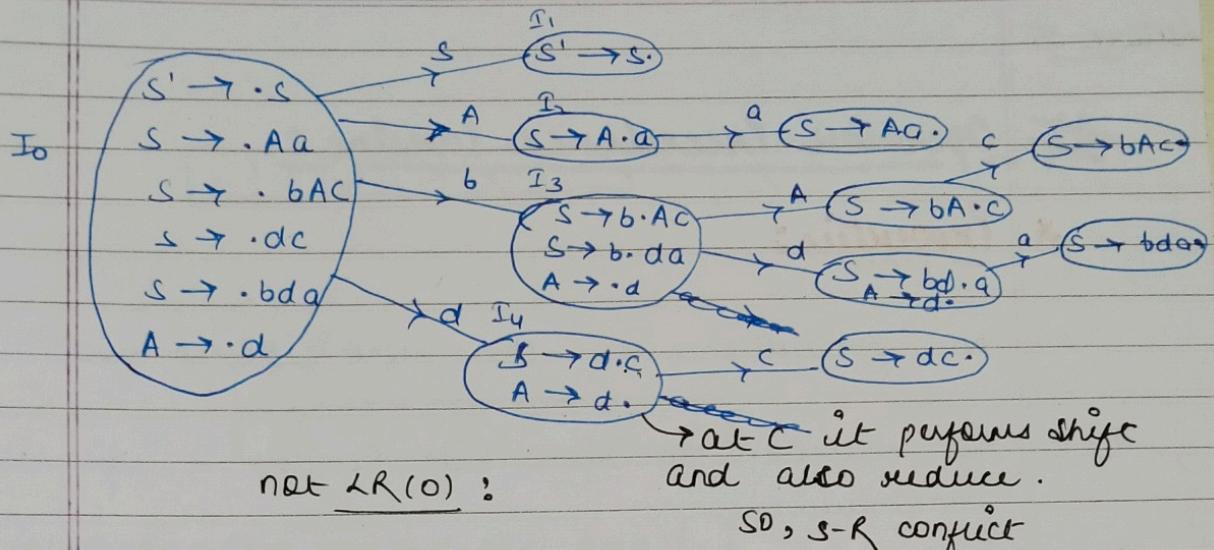
LL(1) :

$S \rightarrow Aa$	$ $	bAC	$ $	dc	$ $	bda	$\{d, b\}$	FIRST	FOLLOW
$A \rightarrow d$	\swarrow	<u>same entries</u>			$\{d\}$	$\{a, c\}$	$\{\$\}$	$\{a, c\}$	

It is ^{not} LL(1).

LR(0) : Augmented grammar :

$S' \rightarrow S$	
$S \rightarrow Aa$	-1
$S \rightarrow bAC$	-2
$S \rightarrow dc$	-3
$S \rightarrow bda$	-4
$A \rightarrow d$	-5



CLR(1) :

$S' \rightarrow \cdot S, \$$
 $S \rightarrow \cdot Aa, \$$
 $S \rightarrow \cdot bAC, \$$
 $S \rightarrow \cdot dc, \$$
 $S \rightarrow \cdot bda, \$$
 $A \rightarrow$

7th March '17

Lecture-35.

Operator Precedence Parsing:

* Properties:

- NO production on the right side contains ϵ
- NO production on right side contains 2 adjacent non-terminals.

Example:

$$E \rightarrow EAE \mid id$$

$$A \rightarrow + \mid *$$

It is not operator grammar bcz
(ii)nd condition is not satisfied
"E" & "A" non-terminals are adjacent.

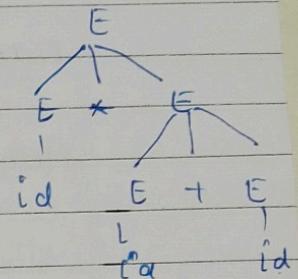
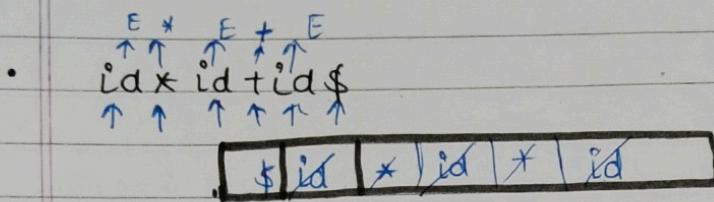
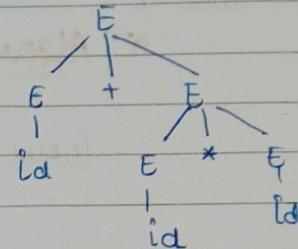
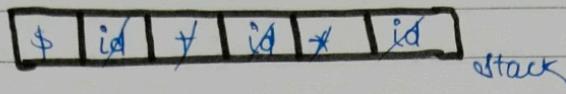
$$E \rightarrow E+E \mid E*E \mid id$$

now, it is operator grammar

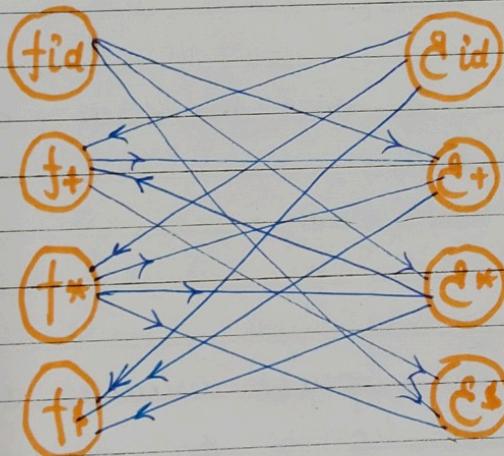
→ Operator Precedence Relation Table:

		id	+	*	\$ (g)
		-	⇒	⇒	⇒
		+	≤	⇒	⇒
		*	≤	⇒	⇒
		\$	≤	≤	accept

• $id + id * id \$$



* Оператор precedence граф:



* Operator precedence table ^{function}

	+	*	id	\$
+	2	4	4	0
t				
c	1	3	5	0

$$\begin{array}{l}
 \text{if } f+ = 2 \\
 g* = 3 \\
 \text{C* has high value} \\
 \text{than } f+ \text{ so,} \\
 \text{* high precision than +}
 \end{array}$$

fid $\rightarrow c^* \rightarrow f^+ \rightarrow c^+ \rightarrow f\$$ qid $\rightarrow f^* \rightarrow c^* \rightarrow f^+ \rightarrow c^+ \rightarrow f\$$ } longest paths

8th March '17

Lecture-36

Date: ___/___/___
Page: ___

* Algorithm:

Input: The precedence relation from some operator precedence grammar & input string terminals from that grammar.

Output:

Method:

while(1)

{

if only ϵ is in onto the stack & only ϵ is in the input then
accept and break;

else

{

let 'a' be the topmost terminal symbol on the stack and let 'b' the current input symbol;

if $a < b$ or $a = b$ then shift 'b' onto the stack;

else if $a > b$ then (*reduce*)

repeat pop the stack;

until the top stack terminal is related by $<$ to the terminal most recently popped;

else

call the error correcting routine;

{

* Algorithm for finding-precedence function for a table:

Step-1:

Create symbols $f_a \leq g_a$ for each a ; i.e., terminal or $\$$.

Step-2:

Partition the created symbol into as many group as possible in such a way that $a \leq b$ then $f_a \leq g_b$ in the same group.

Step-3:

Create a direct graph whose nodes are the groups found in step 2. For any $a \leq b$ if $a \leq b$, place an edge from the group of g_b to group f_a .

Step-4:

If the graph constructed in step 3 has a cycle then no precedence function exist. If there is no cycle, then let $f(a)$ be the length of the longest path beginning from the group of f_a and let $g(a)$ be the longest path from the group of g_a .

Example :

$$\begin{aligned}
 P &\rightarrow SR | S \\
 R &\rightarrow bSR | bS \\
 S &\rightarrow wbS | w \\
 w &\rightarrow L * w | L \\
 L &\rightarrow id
 \end{aligned}$$

It is not operator grammar bcz
1st & 2nd production have 2 adjacent
non-terminals.

Now, making it an operator grammar.

$$\begin{aligned}
 P &\rightarrow SbSR | Sbs | S \\
 &\quad \xrightarrow{P \rightarrow SR}
 \end{aligned}$$

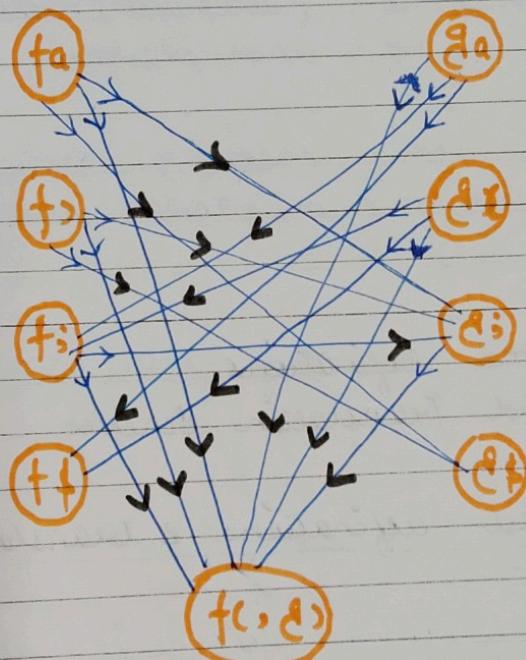
$$\begin{aligned}
 P &\rightarrow SbP | Sbs | S \\
 S &\rightarrow wbS | w \\
 w &\rightarrow L * w | L \\
 L &\rightarrow id
 \end{aligned}$$

	id	*	b	\$
id	-	>	>	>
*	<	<	>	>
b	<	<	<	>
\$	<	<	<	accept

Q.

convert the given relation table into function table :

	a	()	;	\$
a			↗	↗	↗
(↖	↖		↖	
)				↗	↗
;	↖	↖	↗	↗	
\$	↖	↖			



$$f(a) \Rightarrow f_a \rightarrow g_a \rightarrow f_{(,g)}$$

$$f() \Rightarrow f() \rightarrow g() \rightarrow f_{(,g)}$$

$$f(); \Rightarrow f(); \rightarrow g(); \rightarrow f_{(,g)}$$

*(\$)

$$g(a) \Rightarrow g_a \rightarrow f(); \rightarrow g(); \rightarrow f_{(,g)}$$

$$g() \Rightarrow g() \rightarrow f(); \rightarrow g(); \rightarrow f_{(,g)}$$

$$g(); \Rightarrow g(); \rightarrow f(); \rightarrow f_{(,g)}$$

	a	()	;	\$
f	2	0	2	2	0
g	3	3	0	1	0