Algorithm Analysis and Design

Advanced Data Structure
(Red Black Tree)

(Properties, Rotation and Insertion)

Lecture -26-29

Overview

- A variation of binary search trees.
- Balanced: height is O(lg n), where n is the number of nodes.
- Operations will take O(lg n) time in the worst case.

Red Black Tree

- A red-black tree is a binary search tree + 1 bit per node: an attribute color, which is either red or black.
- All leaves are empty (nil) and colored black.
  - A single sentinel, nil[T], is used for all the leaves of red-black tree T .
  - Color of [nil[T]] is black.
  - The root's parent is also nil[T].
- All other attributes of binary search trees are inherited by red-black trees (i.e. key, left, right, and p as parent).
- We don't care about the key in nil[T].

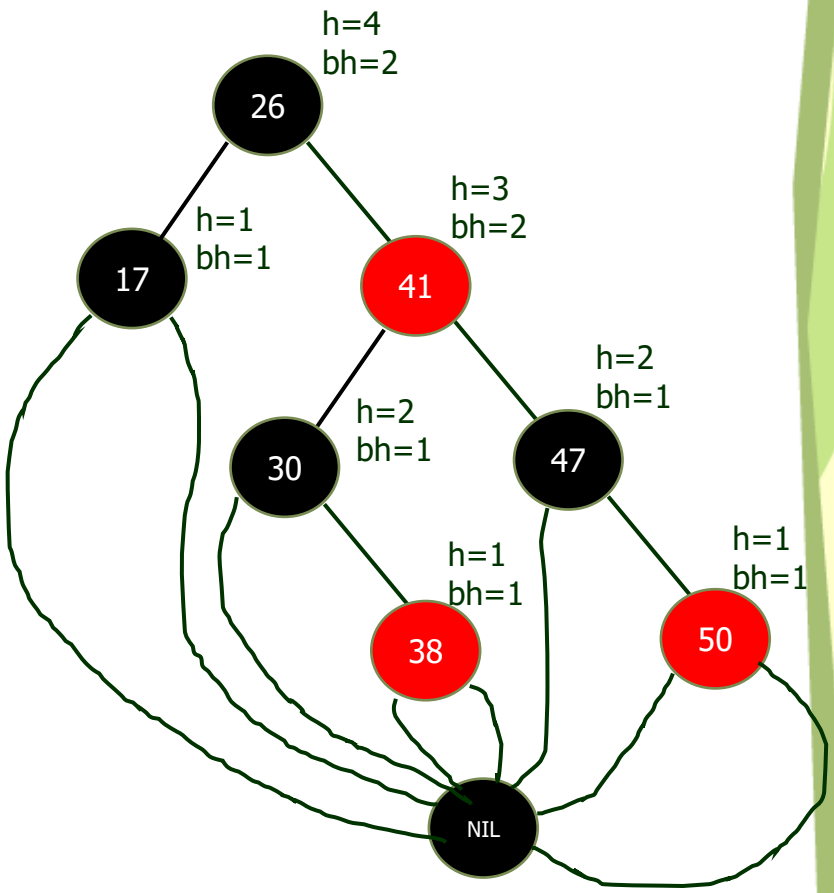
Red Black Tree

- **Properties:**
  1. Every node is either red or black.
  2. The root is always black.
  3. Every leaf (nil[T]) is black.
  4. If a node is red, then both its children are black. (Hence no two reds in a row on a simple path from the root to a leaf is allowed .)
  5. For each node, all paths from the node to descendant leaves contain the same number of black nodes.

Red Black Tree

Height of a red-black tree

- Height of a node is the number of edges in a longest path to a leaf.
- Black-height of a node x: bh(x) is the number of black nodes (including nil[T]) on the path from x to leaf, not counting x. By property 5, black-height is well defined.



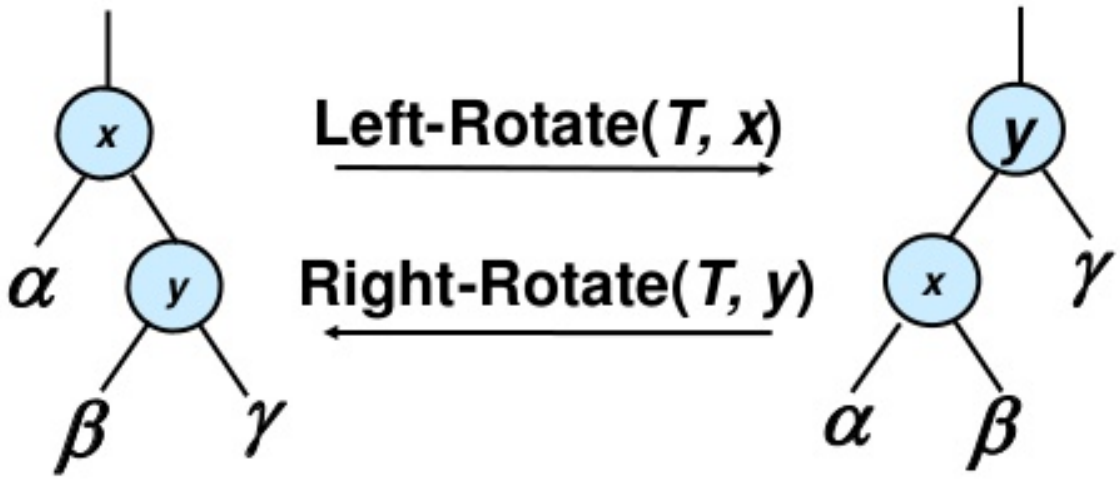
Red Black Tree

Rotations

- The basic tree-restructuring operation.
- Needed to maintain red-black trees as balanced binary search trees.
- Changes the local pointer structure. (Only pointers are changed.)
- Won't upset the binary-search-tree property.
- Have both left rotation and right rotation. They are inverses of each other.
- A rotation takes a red-black-tree and a node within the tree.

Red Black Tree

Rotations

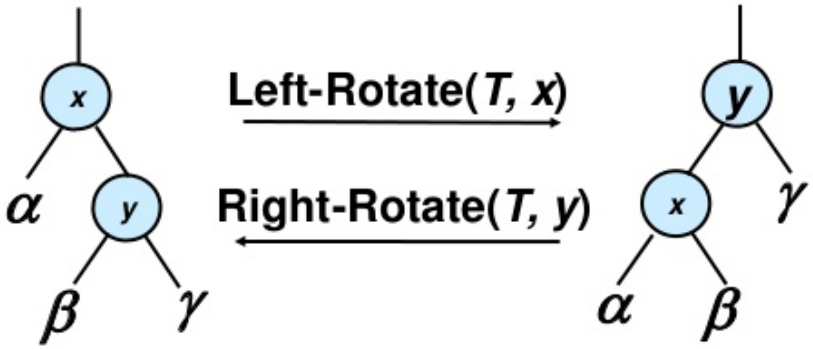


Red Black Tree

Rotations

LEFT-ROTATE*(T, x)*

```
y ← right[x]            //Set y.
right[x] ← left[y]      //Turn y.s left subtree into x.s right subtree.
if left[y] ≠ nil[T ]
    then p[left[y]] ← x
p[y] ← p[x]             //Link x.s parent
if p[x] = nil[T ]
    then root[T ] ← y
    else if x = left[p[x]]
            then left[p[x]] ← y
            else right[p[x]] ← y
left[y] ← x             //Put x on y.s left.
p[x] ← y
```



Red Black Tree

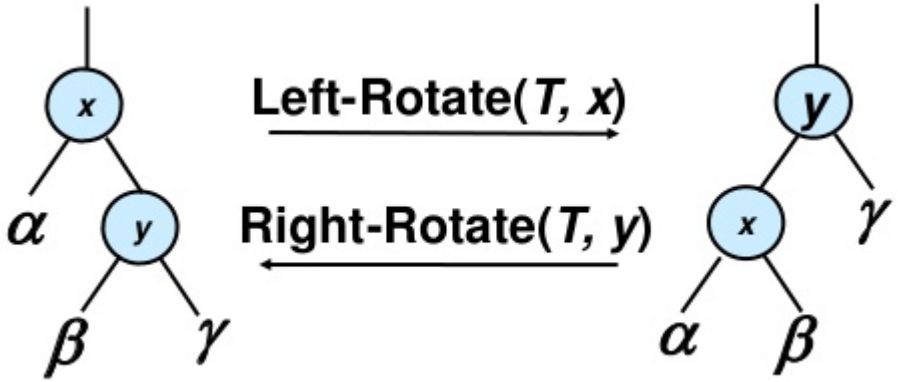
Rotations

The pseudocode for LEFT-ROTATE assumes that

right[*x*] = *nil*[*T*], and

root.s parent is *nil*[*T*].

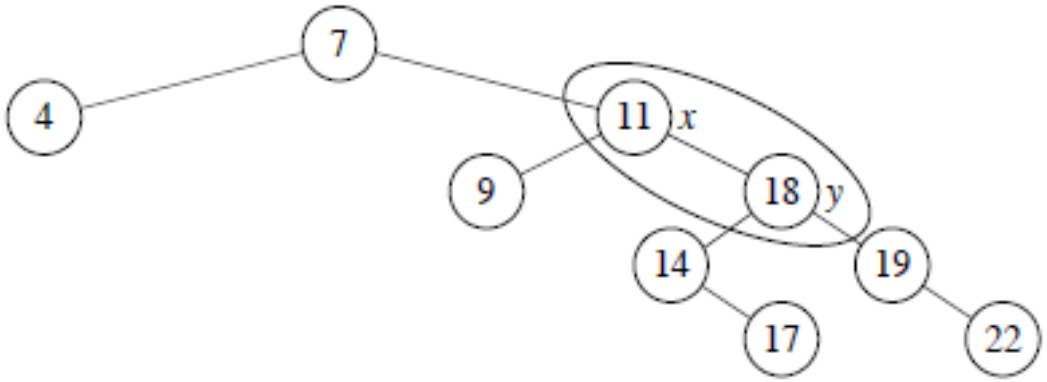
Pseudocode for RIGHT-ROTATE is symmetric: exchange *left* and *right* everywhere.



Red Black Tree

Rotations (Example)

Demonstrate of left rotation that maintains in-order ordering of keys.

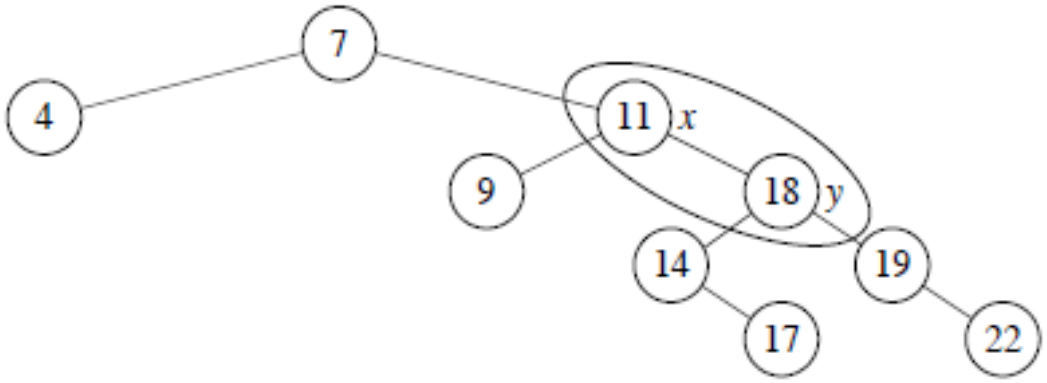


Red Black Tree

Rotations (Example)

Demonstrate of left rotation that maintains in-order ordering of keys.

LEFT-ROTATE(T, x)

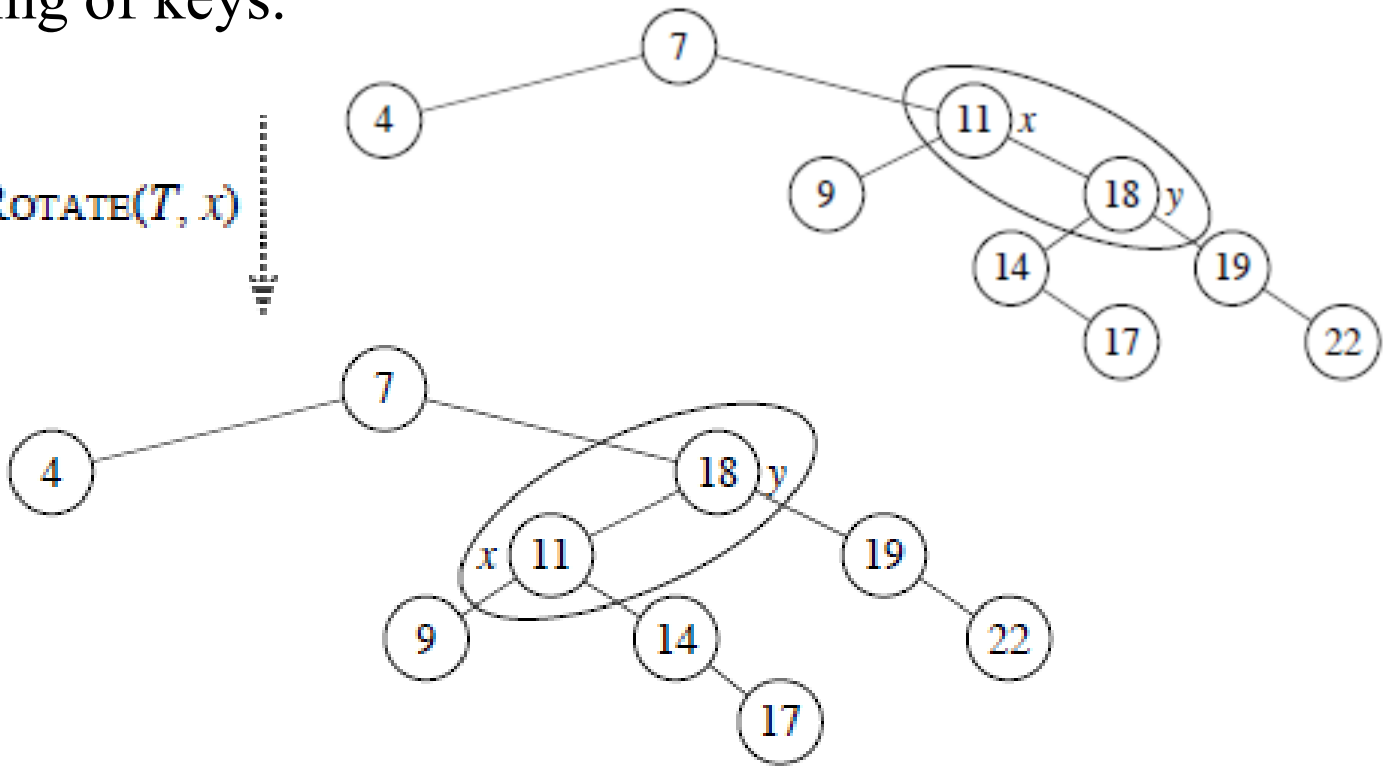


Red Black Tree

Rotations (Example)

Demonstrate of left rotation that maintains in-order ordering of keys.

LEFT-ROTATE(T, x)

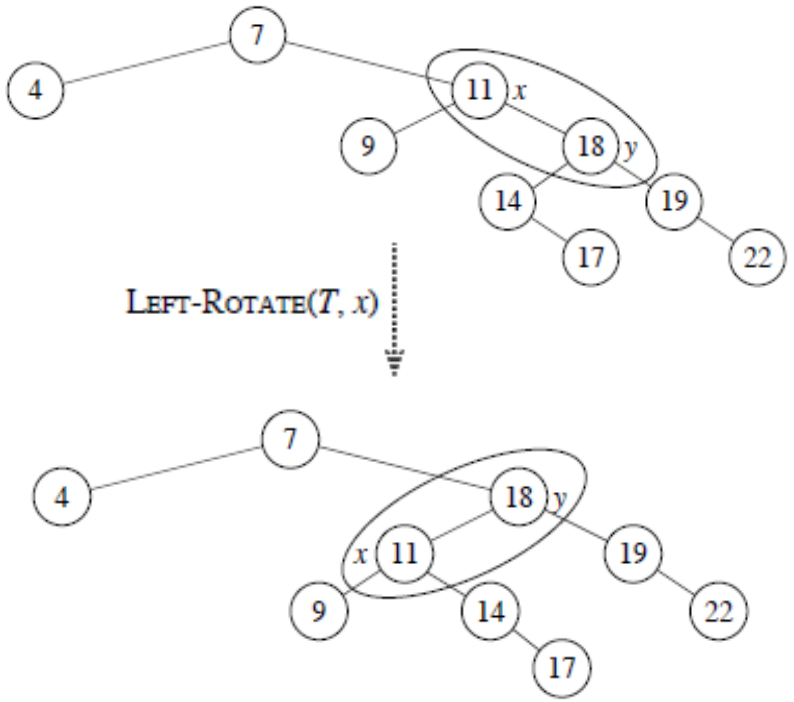


Red Black Tree

Rotations (Example)

Demonstrate of left rotation that maintains in-order ordering of keys.

- Before rotation: keys of $x's$ left subtree $\leq 11 \leq$ keys of $y's$ left subtree $\leq 18 \leq$ keys of $y's$ right subtree.
- Rotation makes $y's$ left subtree into $x's$ right subtree.
- After rotation: keys of $x's$ left subtree $\leq 11 \leq$ keys of $x's$ right subtree $\leq 18 \leq$ keys of $y's$ right subtree.
- Time complexity : O(1) for both LEFT-ROTATE and RIGHT-ROTATE, since a constant number of pointers are modified.



Red Black Tree

Insertion:

Start by doing regular binary-search-tree insertion:

Red Black Tree

Insertion:

RB-INSERT*(T, z)*
y ← *nil*[*T*]
x ← *root*[*T*]
while *x* = *nil*[*T*]
  do *y* ← *x*
    if *key*[*z*] < *key*[*x*]
    then *x* ← *left*[*x*]
    else *x* ← *right*[*x*]
p[*z*] ← *y*
if *y* = *nil*[*T*]
  then *root*[*T*] ← *z*
  else if *key*[*z*] < *key*[*y*]
      then *left*[*y*] ← *z*
      else *right*[*y*] ← *z*
left[*z*] ← *nil*[*T*]
right[*z*] ← *nil*[*T*]
color[*z*] ← RED
RB-INSERT-FIXUP*(T, z)*

Red Black Tree

Insertion:

- RB-INSERT ends by coloring the new node z red.
- Then it calls RB-INSERT-FIXUP to maintain the properties of a red-black Tree.

Which property might be violated?

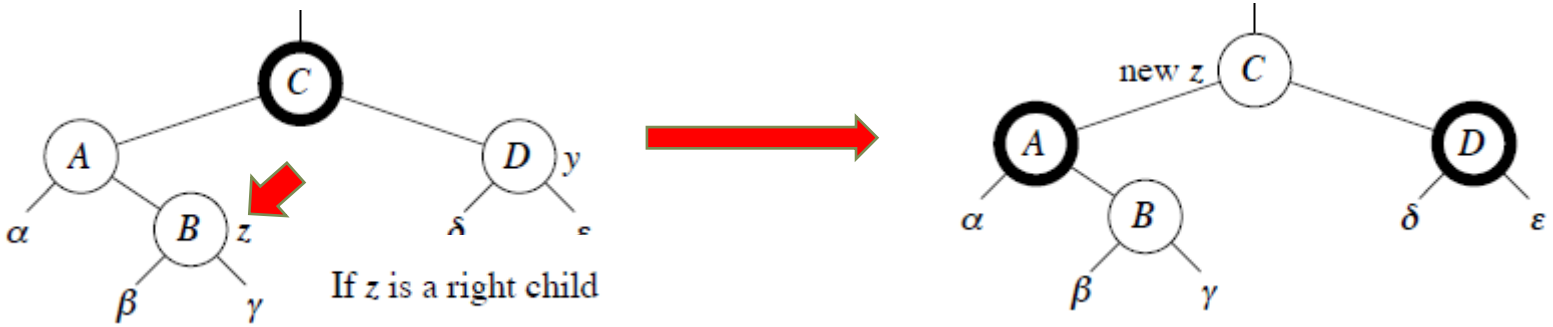
1. Every node is either red or black.	OK
2. The root is always black.	If z is the root, then there′s a violation. Otherwise, OK.
3. Every leaf (nil[T]) is black.	OK
4. If a node is red, then both its children are black.	If p[z] is red, there′s a violation: both z and p[z] are red.
5. For each node, all paths from the node to descendant leaves contain the same number of black nodes.	OK

Red Black Tree

Insertion (RB-INSERT-FIXUP CASE 1 (*y* is red))

[Nodes with bold outline indicate black nodes and light outline indicate red nodes]

if *color*[*y*] = RED
  then *color*[*p*[*z*]] ← BLACK //Case 1
    color[*y*] ← BLACK //Case 1
    color[*p*[*p*[*z*]]] ← RED //Case 1
    z ← *p*[*p*[*z*]] //Case 1

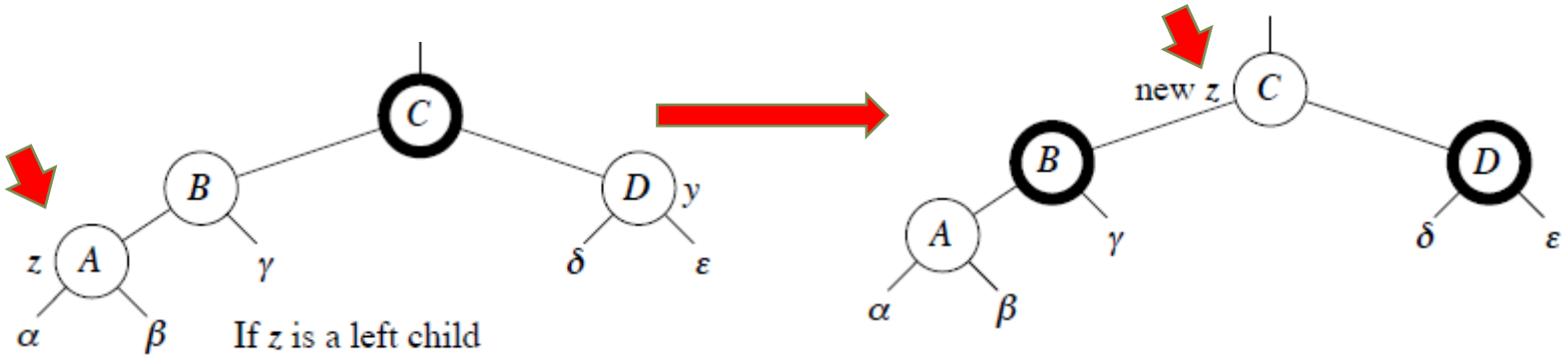


Red Black Tree

Insertion (RB-INSERT-FIXUP CASE 1)

[Nodes with bold outline indicate black nodes and light outline indicate red nodes]

```
if color[y] = RED
        then color[p[z]] ← BLACK                    //Case 1
             color[y] ← BLACK                       //Case 1
             color[p[p[z]]] ← RED                   //Case 1
             z ← p[p[z]]                            //Case 1
```

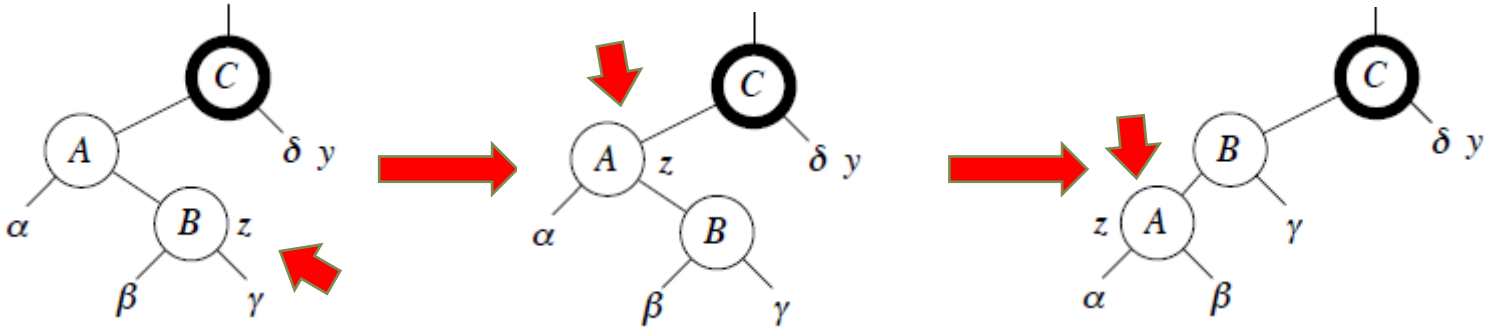


Red Black Tree

Insertion (RB-INSERT-FIXUP CASE 2 (*y* is black, *z* is a right child)

[Nodes with bold outline indicate black nodes and light outline indicate red nodes]

if $z = right[p[z]]$
  then $z \leftarrow p[z]$ //Case 2
    LEFT-ROTATE(T, z) //Case 2

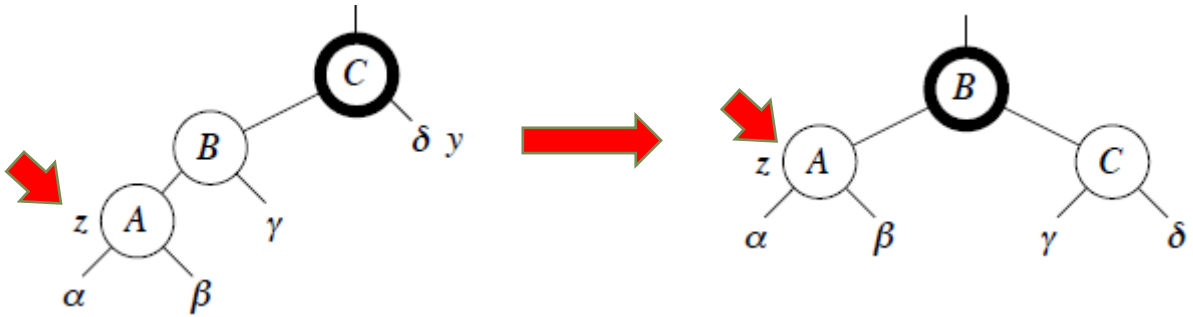


Red Black Tree

Insertion (RB-INSERT-FIXUP CASE 3 (*y* is black, *z* is a left child)

[Nodes with bold outline indicate black nodes and light outline indicate red nodes]

color[*p*[*z*]] ← BLACK // Case 3
color[*p*[*p*[*z*]]] ← RED // Case 3
RIGHT-ROTATE(*T*, *p*[*p*[*z*]]) // Case 3



Red Black Tree

Insertion:

```
RB-INSERT-FIXUP(T, z)
while color[p[z]] = RED
    do if p[z] = left[p[p[z]]]
        then y ← right[p[p[z]]]
             if color[y] = RED
               then color[p[z]] ← BLACK                    //Case 1
                    color[y] ← BLACK                      //Case 1
                    color[p[p[z]]] ← RED                  //Case 1
                    z ← p[p[z]]                           //Case 1
               else if z = right[p[z]]
                      then z ← p[z]                       //Case 2
                           LEFT-ROTATE(T, z)              //Case 2
                    color[p[z]] ← BLACK                   // Case 3
                    color[p[p[z]]] ← RED                  // Case 3
                    RIGHT-ROTATE(T, p[p[z]])              // Case 3
        else (same as then clause with .right. and .left. exchanged)
color[root[T ]] ← BLACK
```

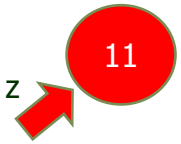
Red Black Tree

Insertion (Example):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -11



Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -11

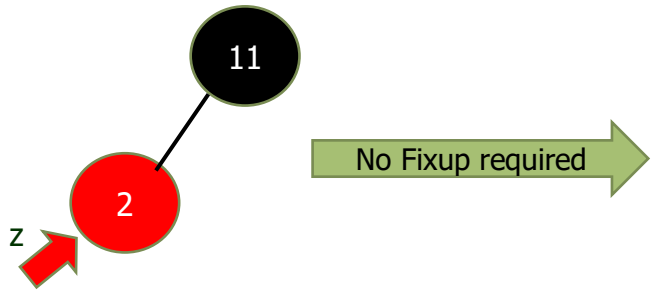
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -2



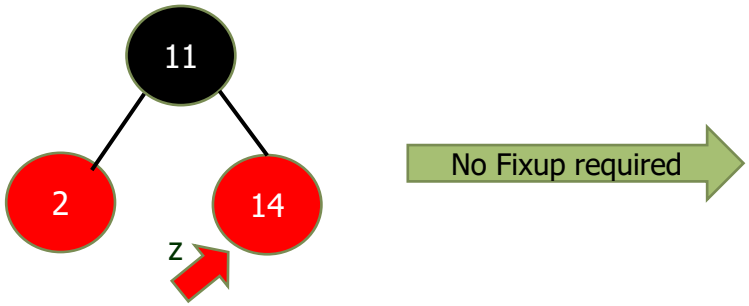
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -14



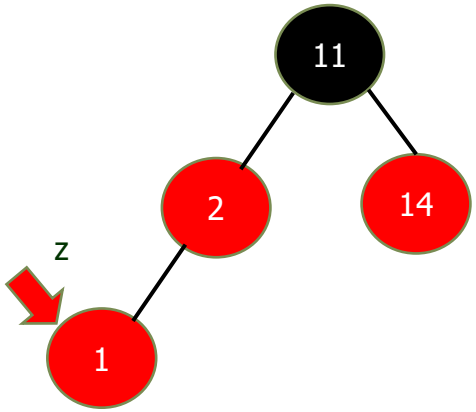
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -1



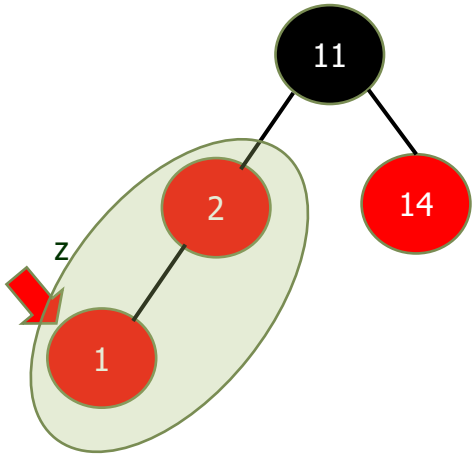
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -1



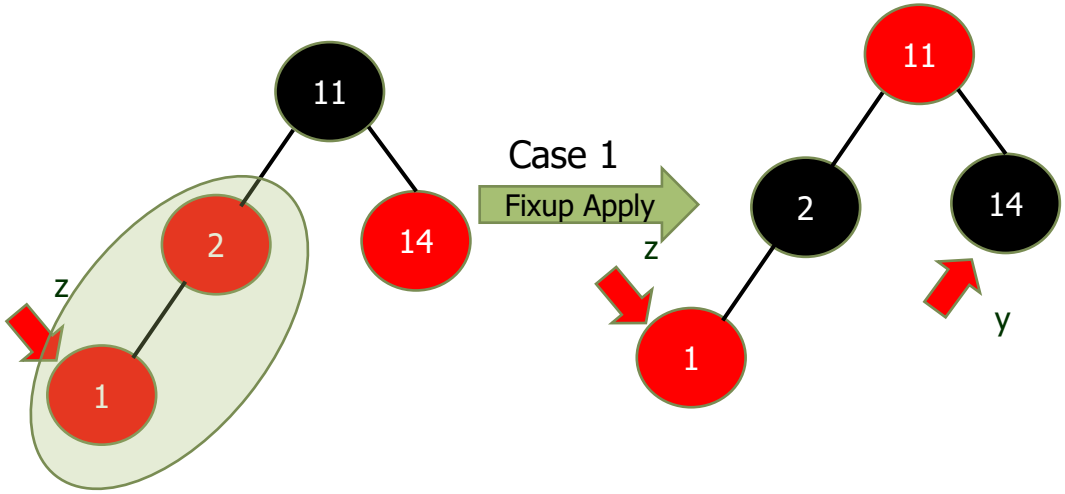
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -1



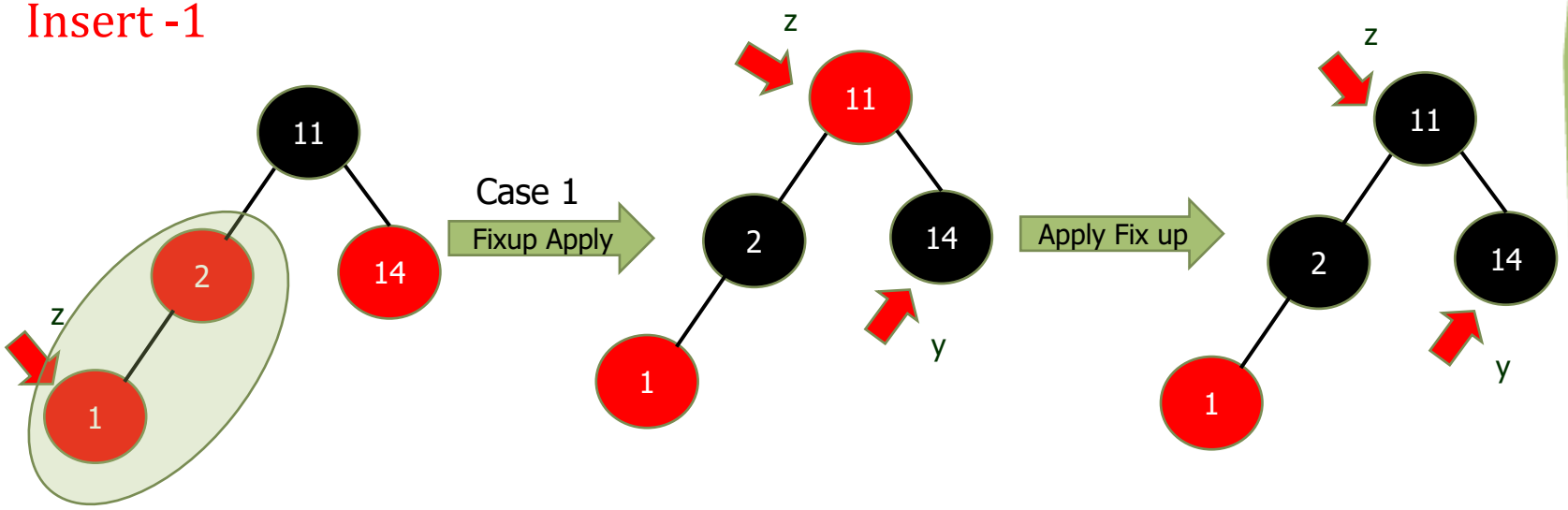
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -1



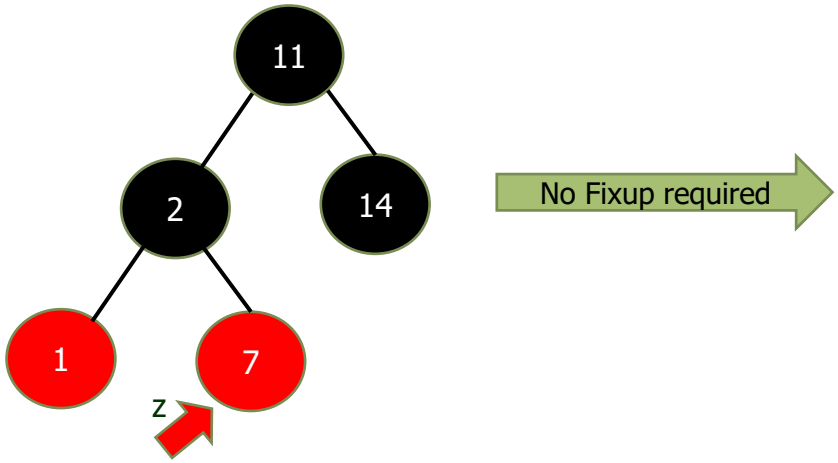
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -7



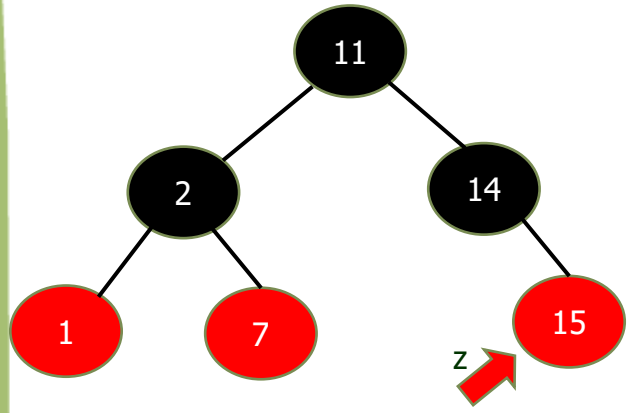
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -15



No Fixup required

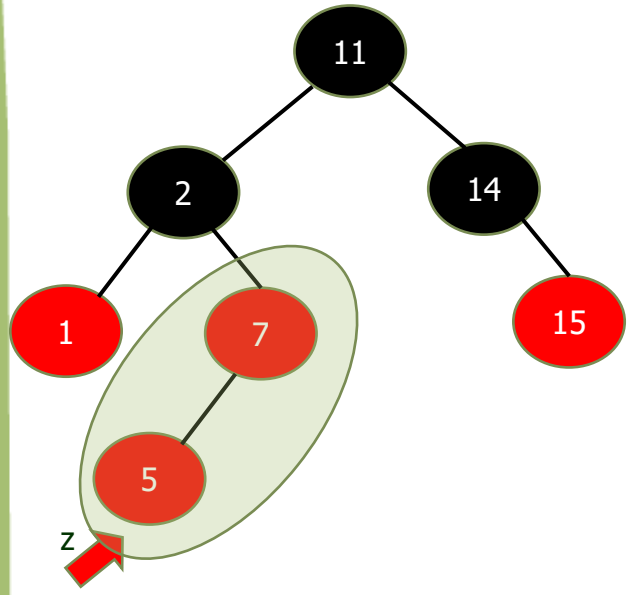
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -5



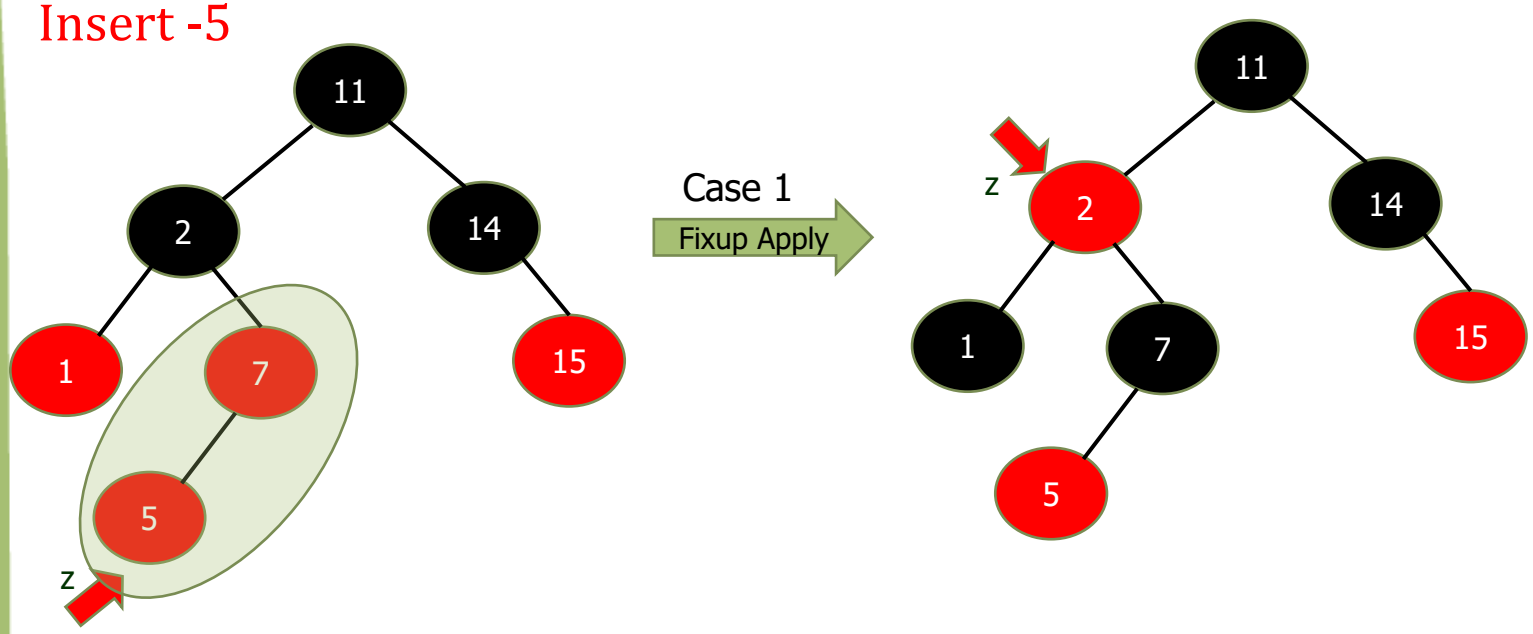
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -5



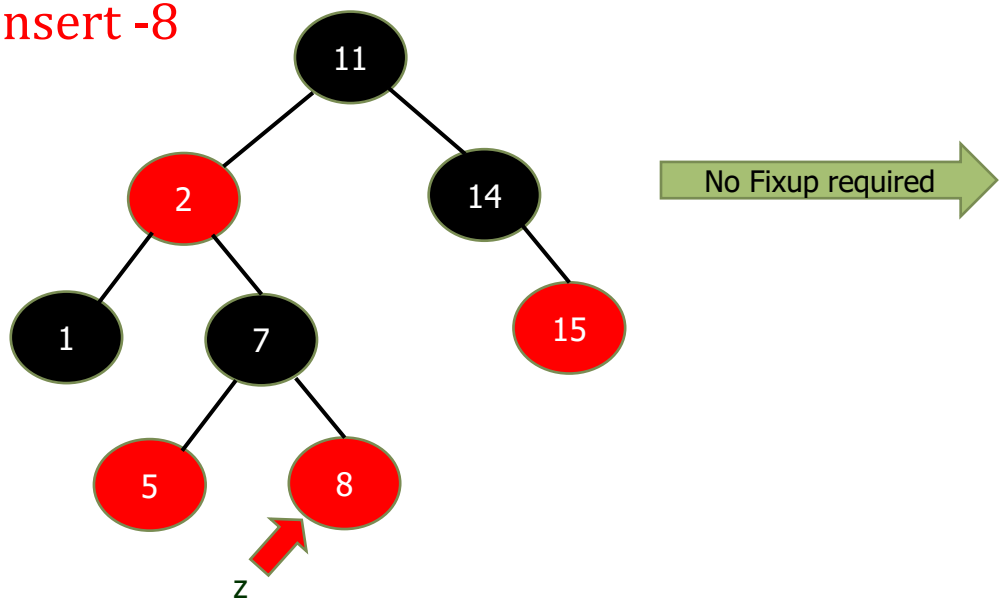
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -8



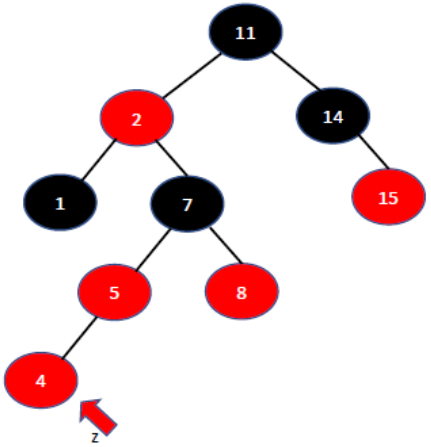
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



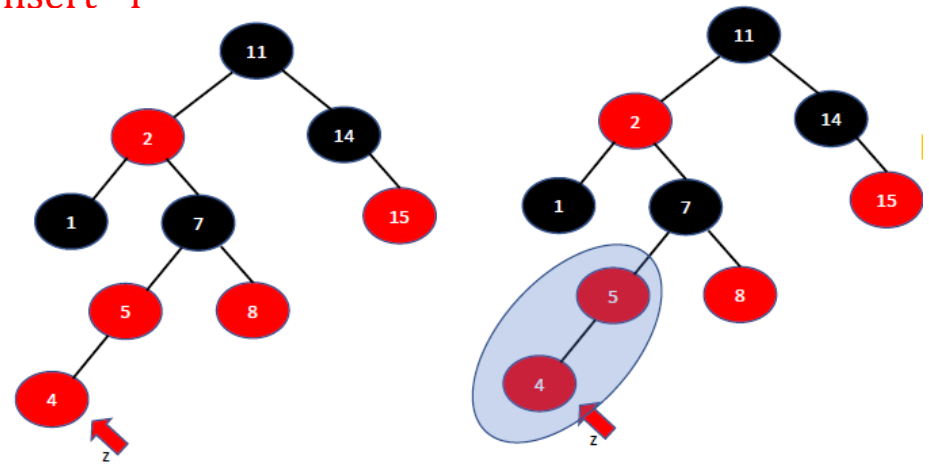
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



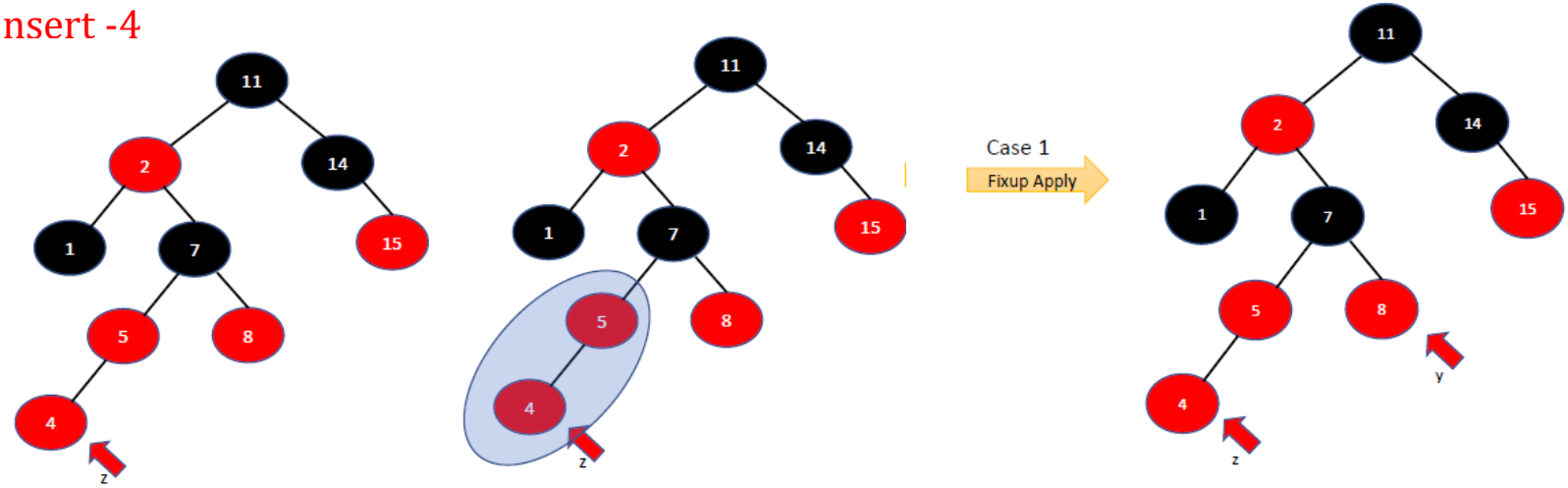
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



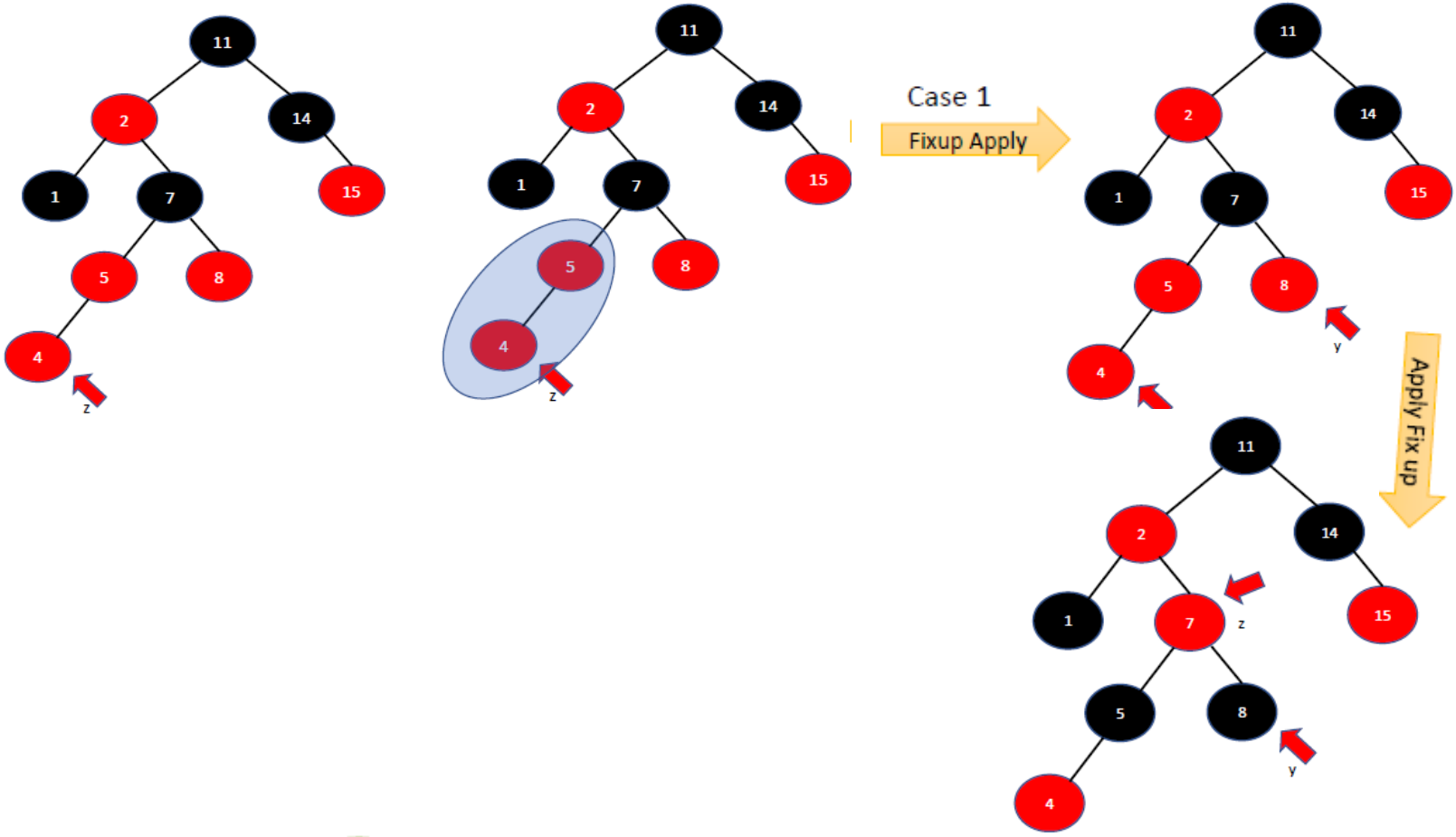
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



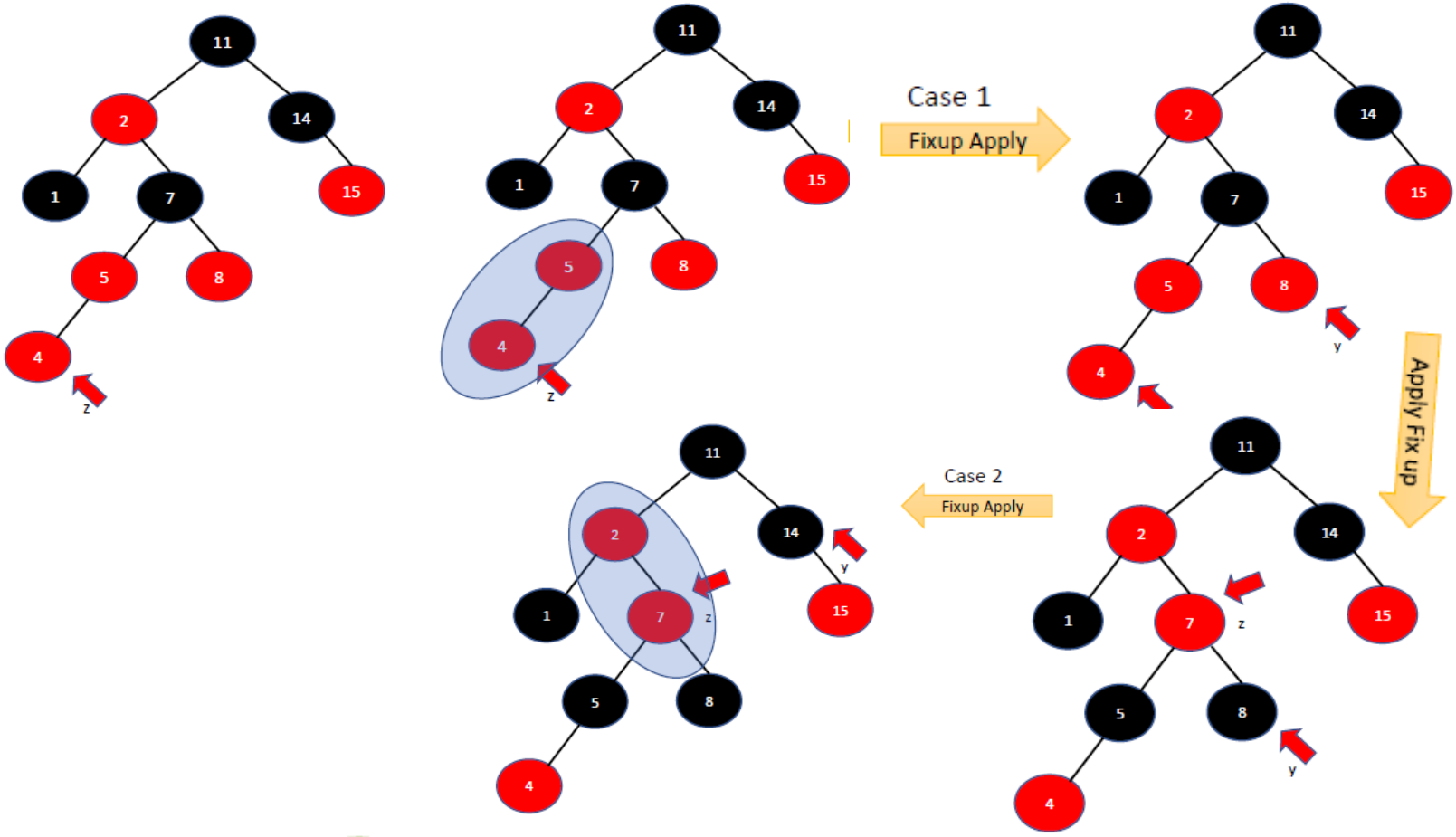
Red Black Tree

Insertion (Example):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



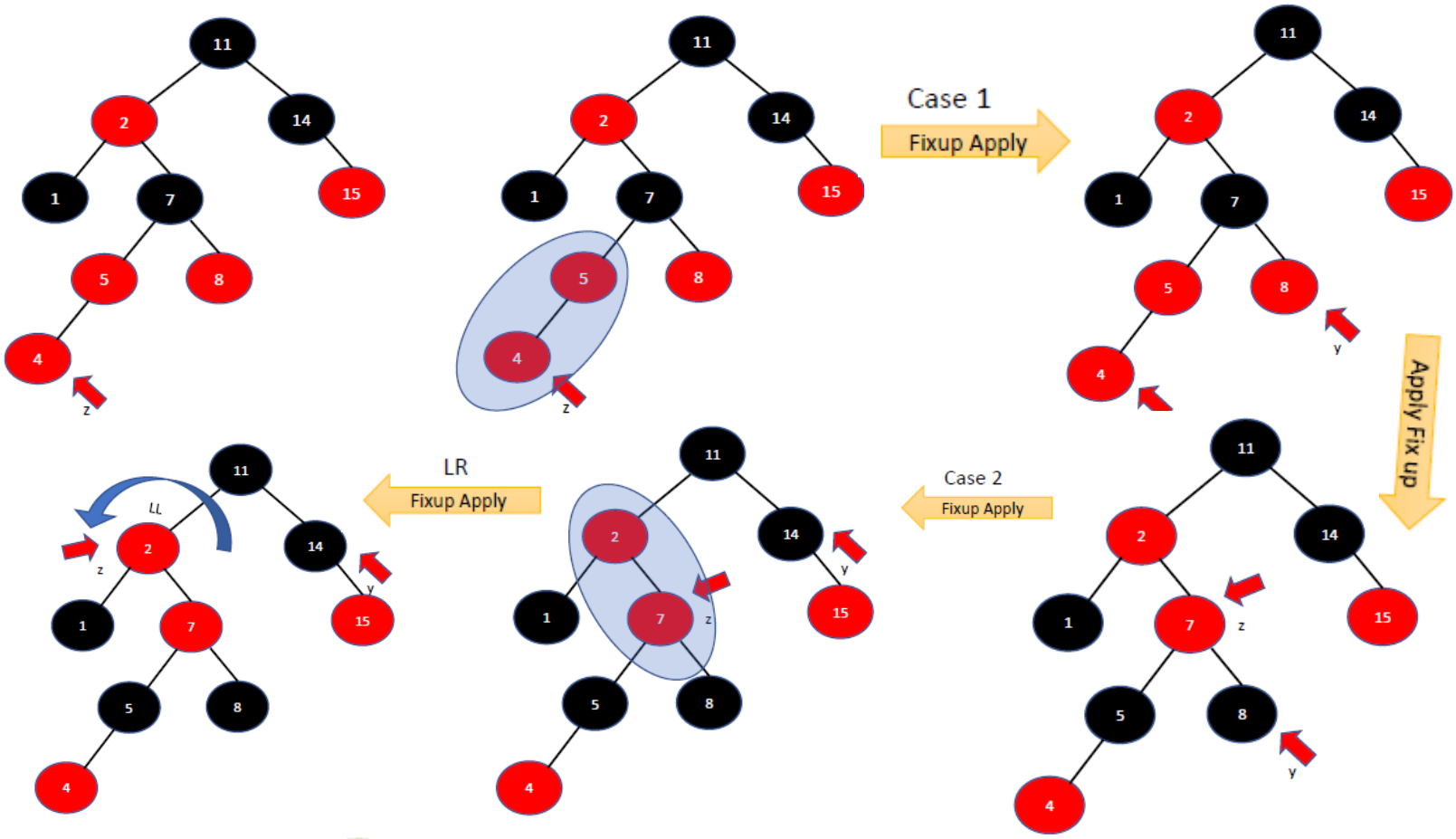
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



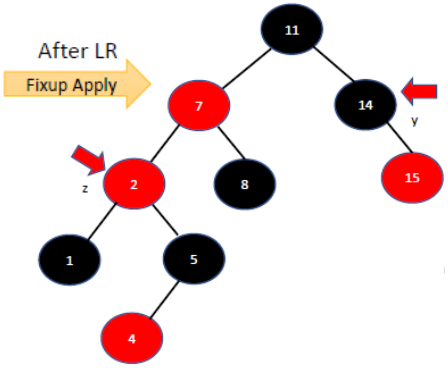
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



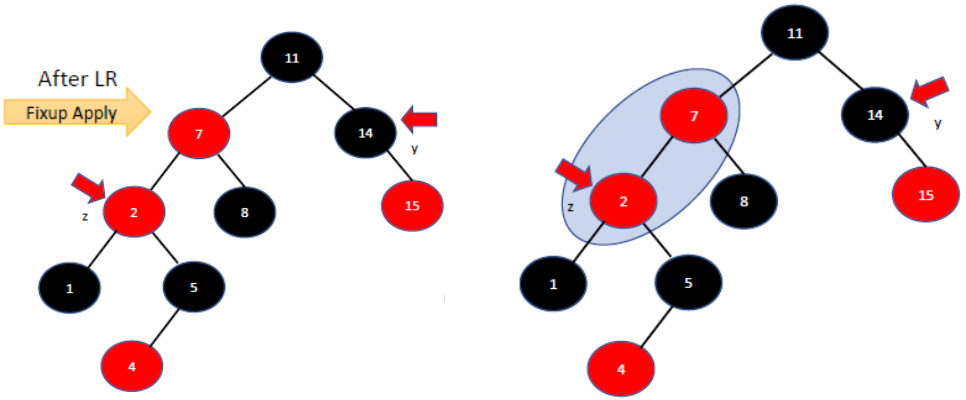
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



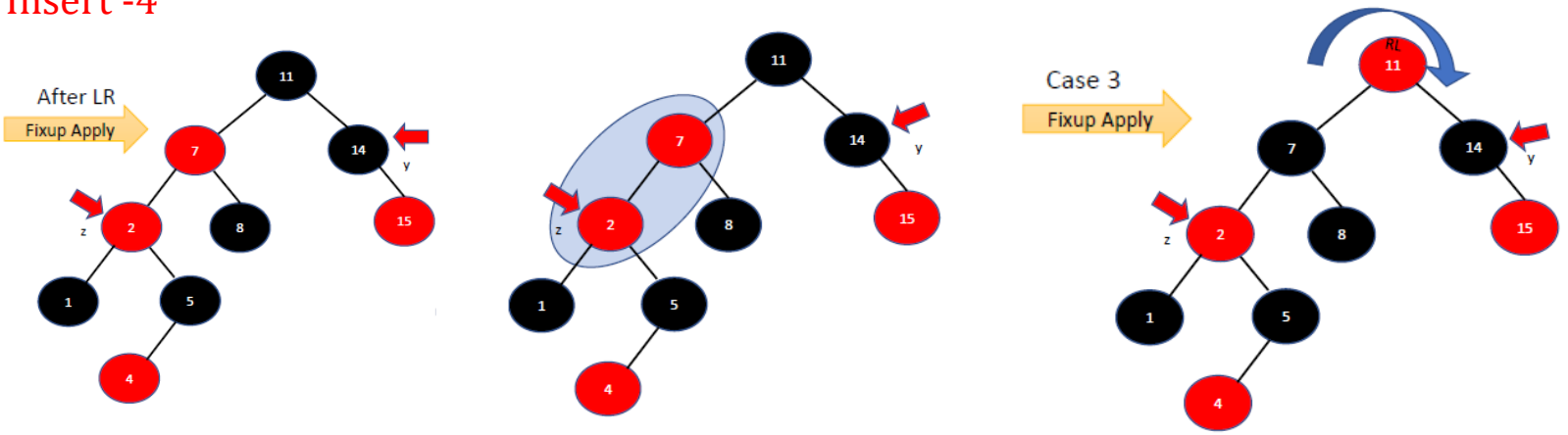
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



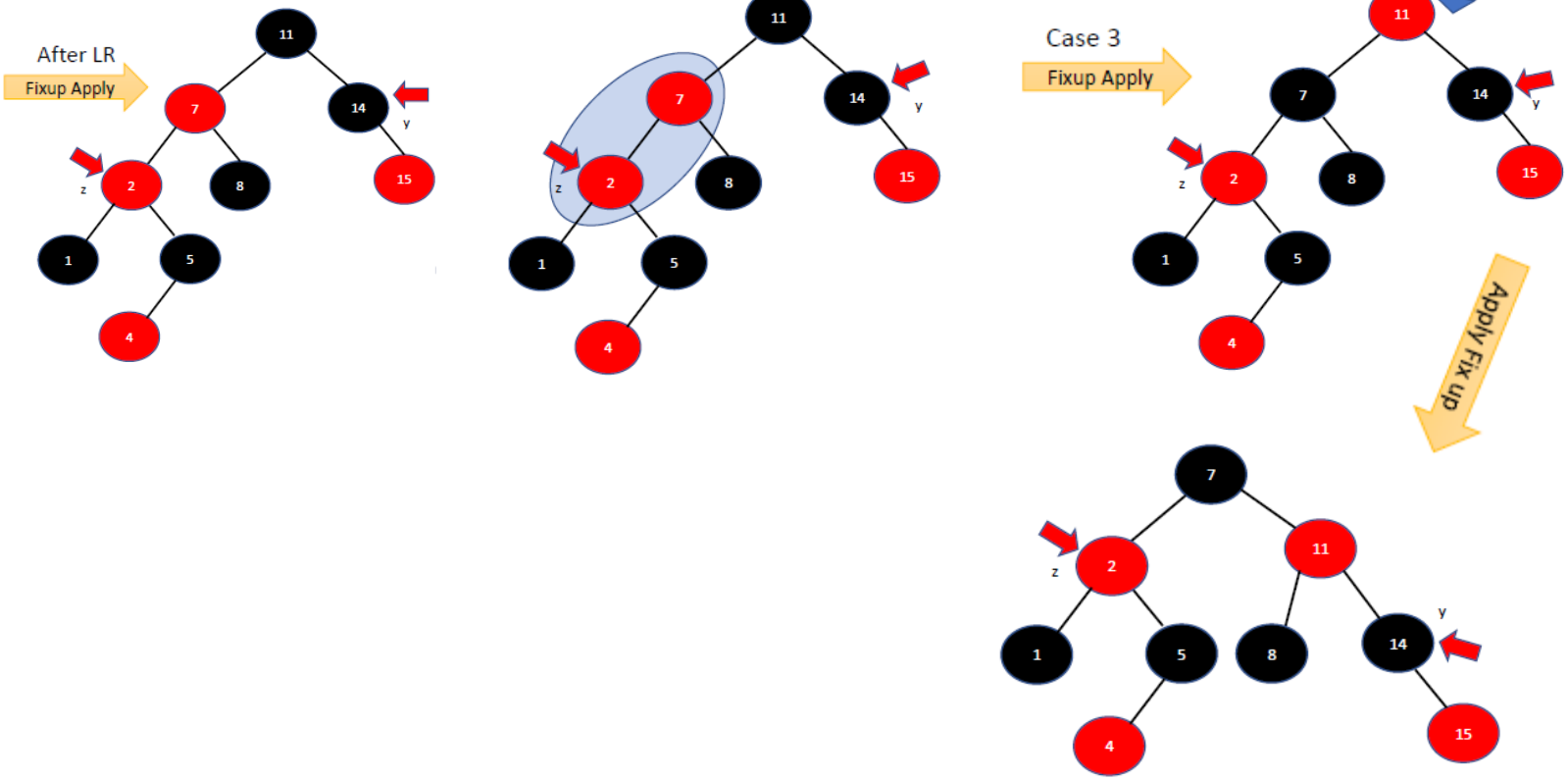
Red Black Tree

Insertion (Example 1):

Insert the following elements into an empty RB-Tree.

[11, 2, 14, 1, 7, 15, 5, 8, 4]

Insert -4



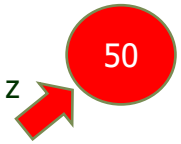
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -50



Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -50

Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -50

Red Black Tree

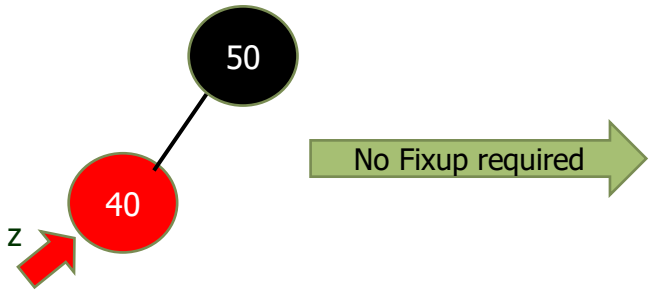
No Fixup required

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -40



Red Black Tree

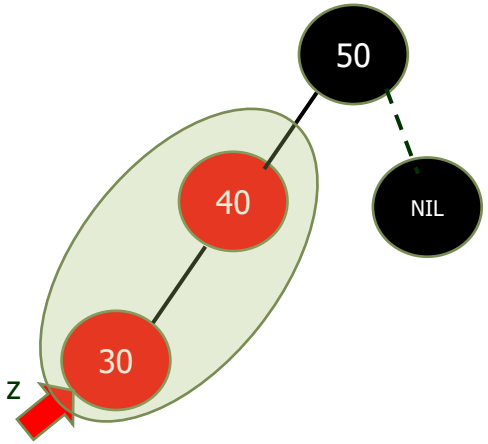
No Fixup required

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -30



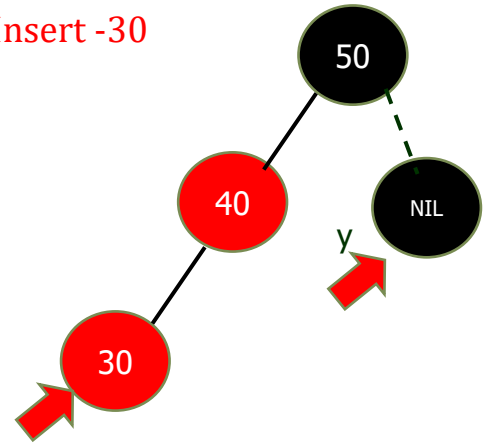
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

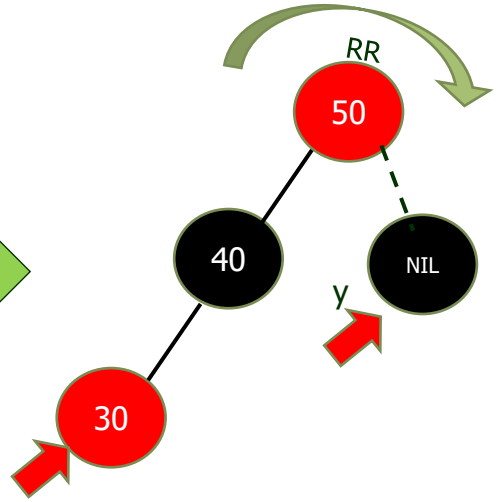
[50, 40, 30, 45, 20, 5]

Insert -30

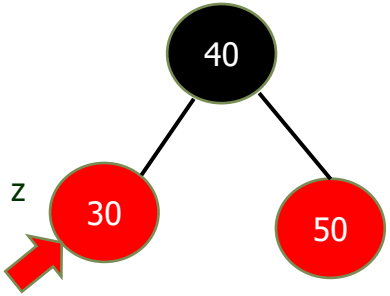


Case 3

Fixup Apply



Fixup Apply After RR



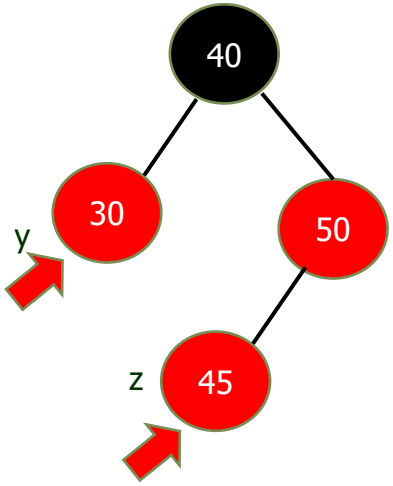
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -45



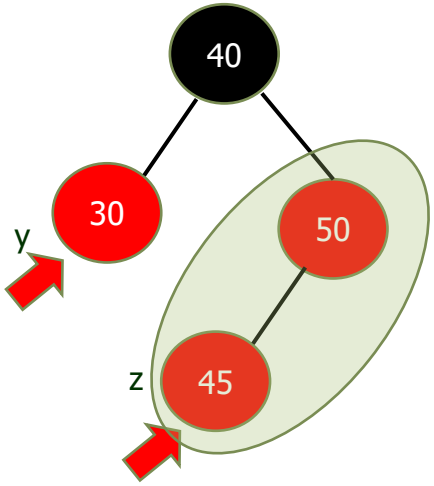
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -45



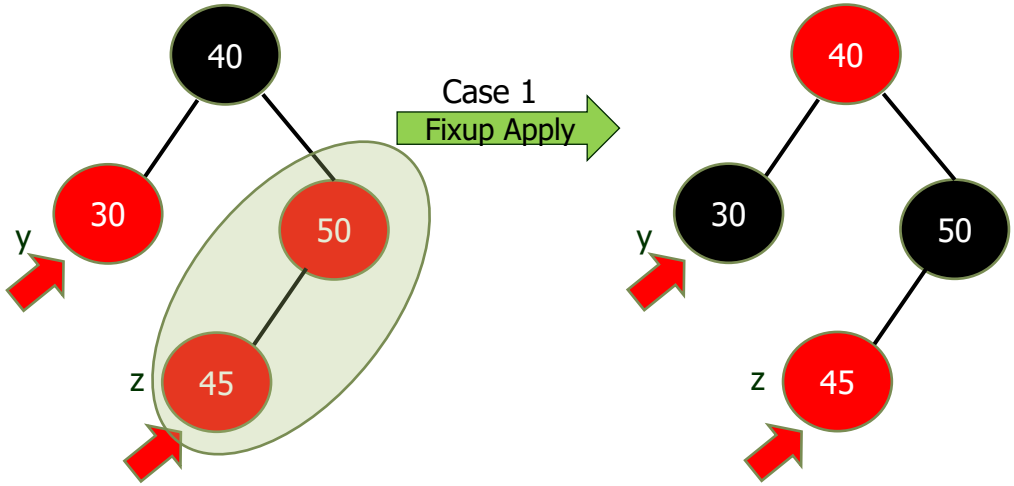
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -45



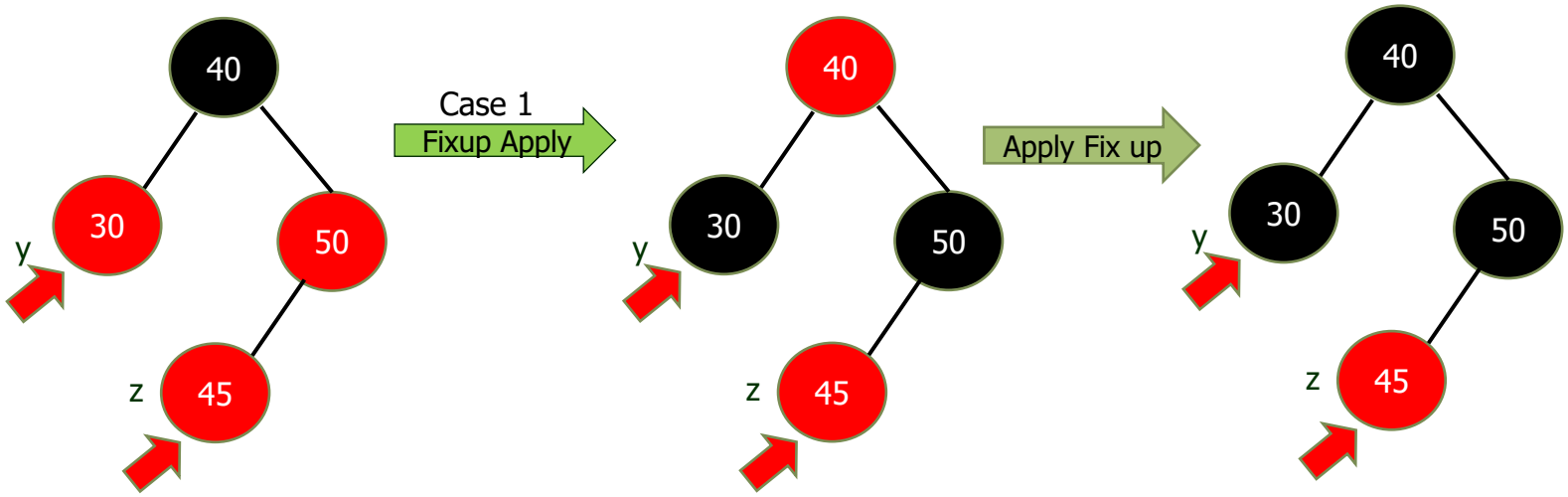
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -45



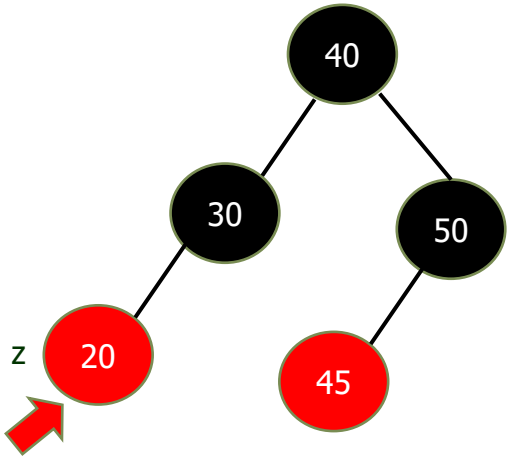
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -20



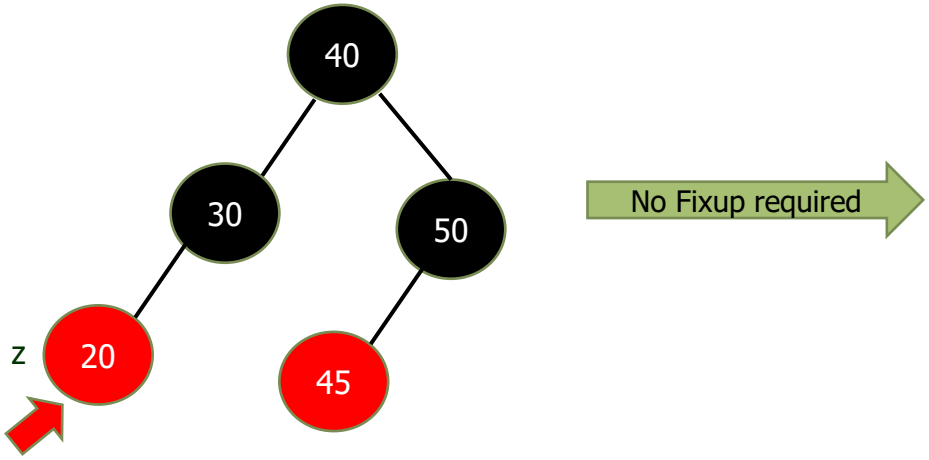
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -20



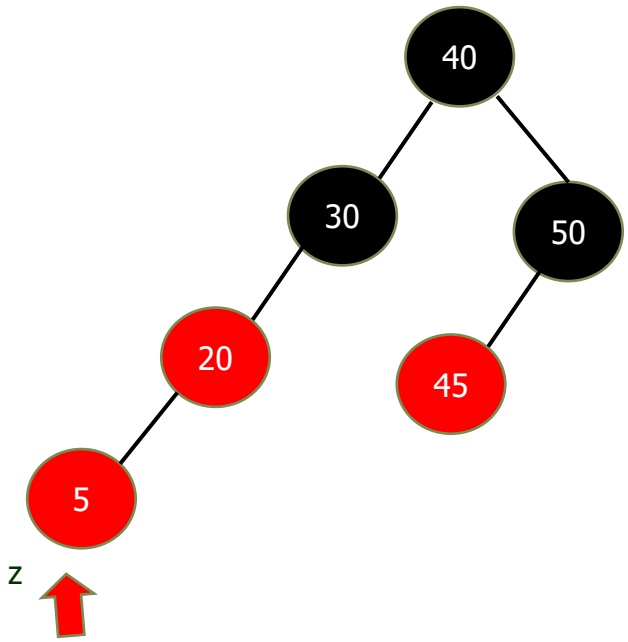
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -5



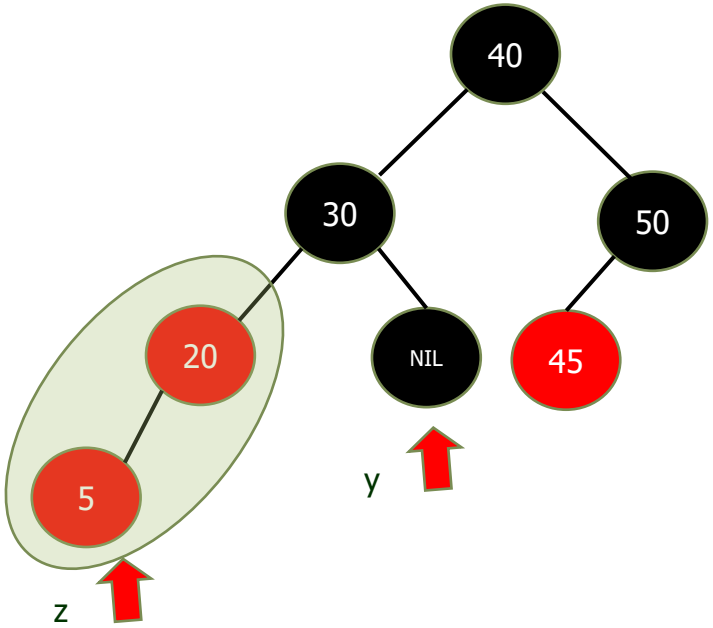
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -5



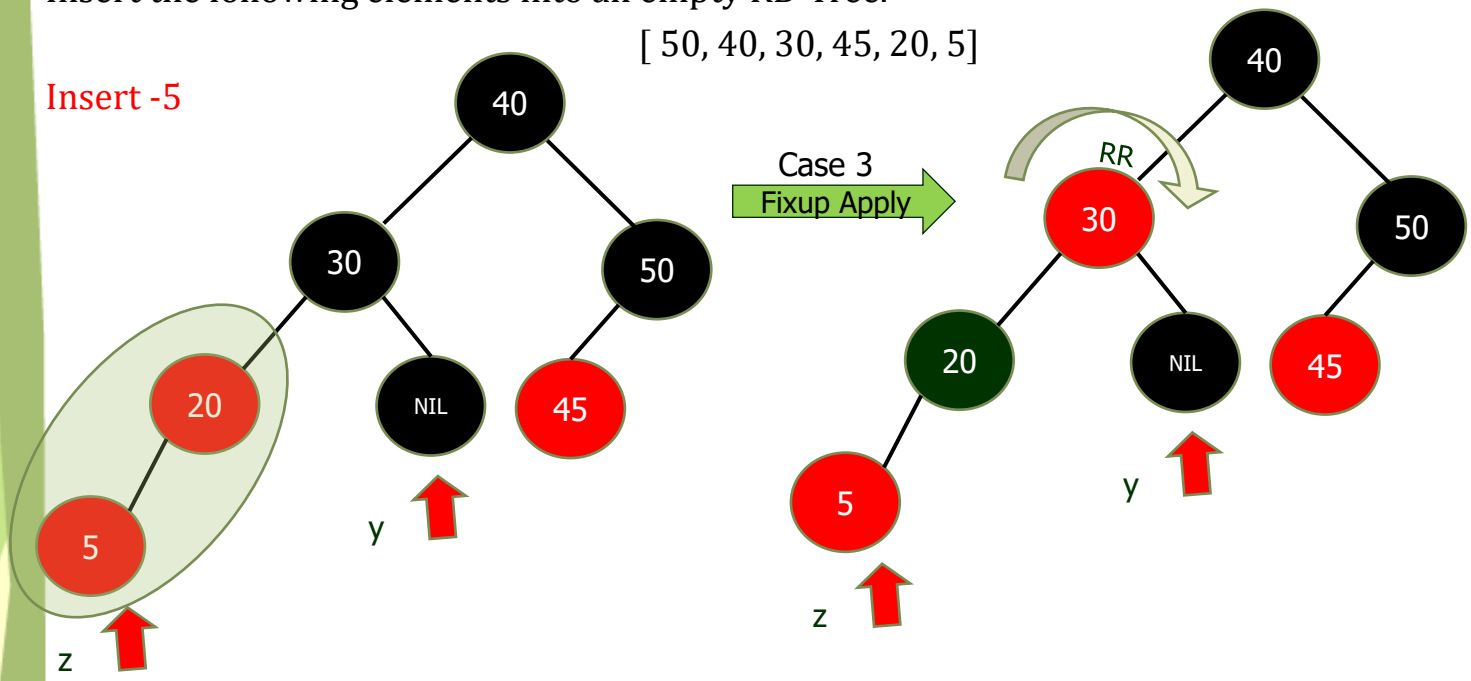
Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

Insert -5

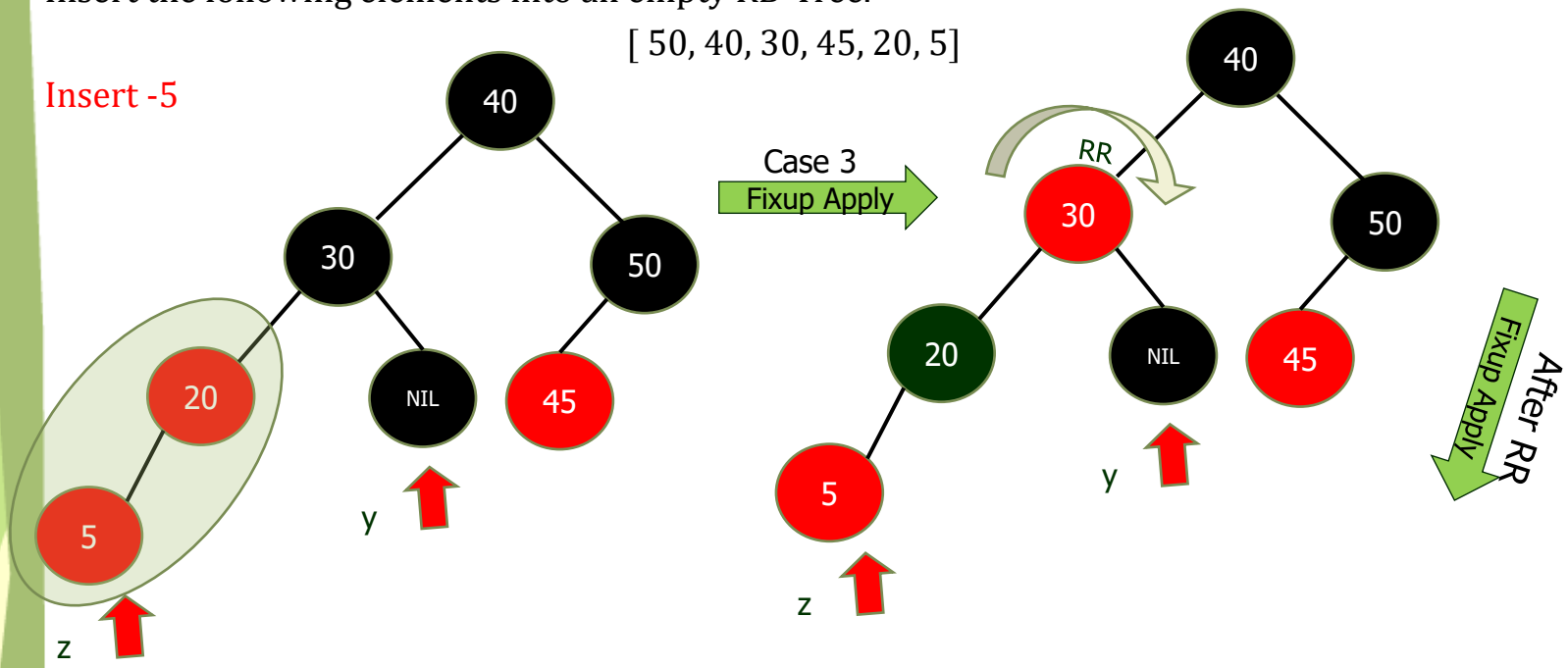


Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]

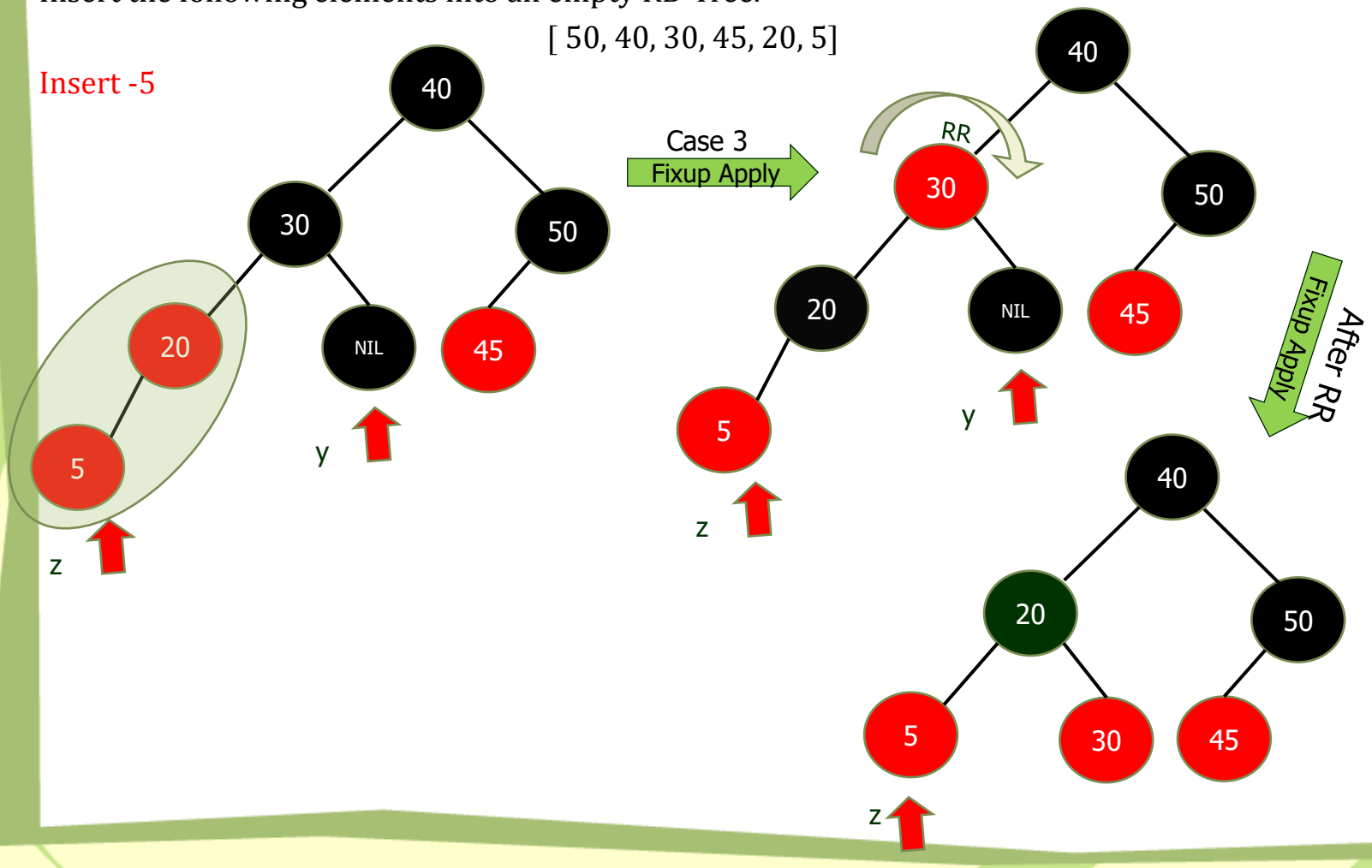


Red Black Tree

Insertion (Example 2):

Insert the following elements into an empty RB-Tree.

[50, 40, 30, 45, 20, 5]



Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -41

Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -50

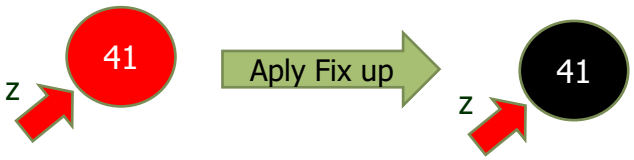
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -50



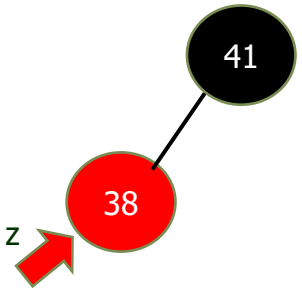
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -38



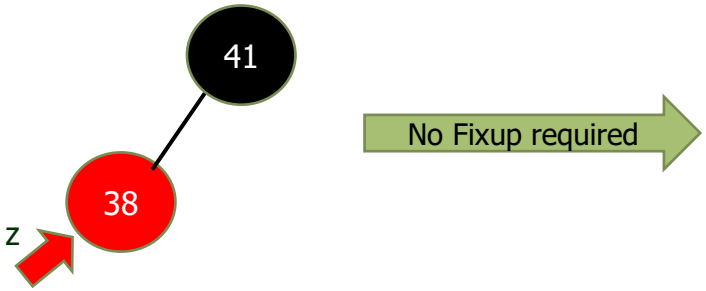
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -38



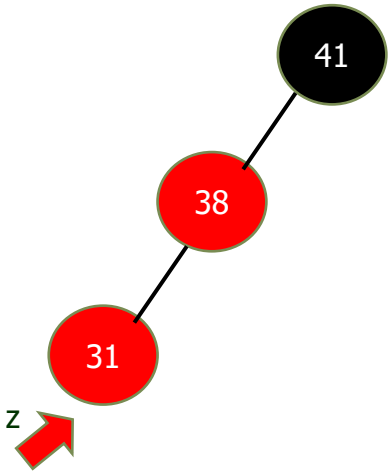
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -31



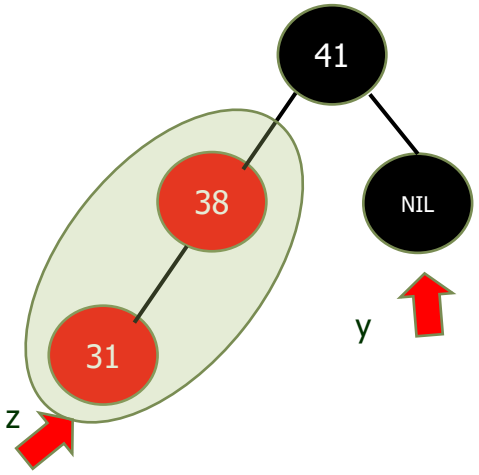
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -31



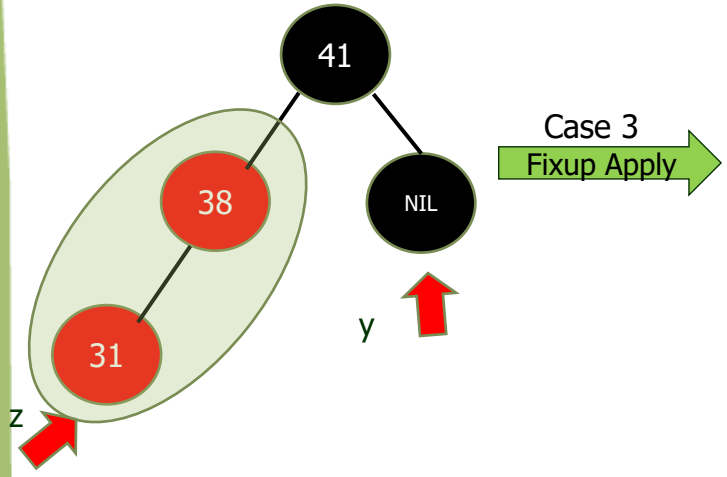
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -31



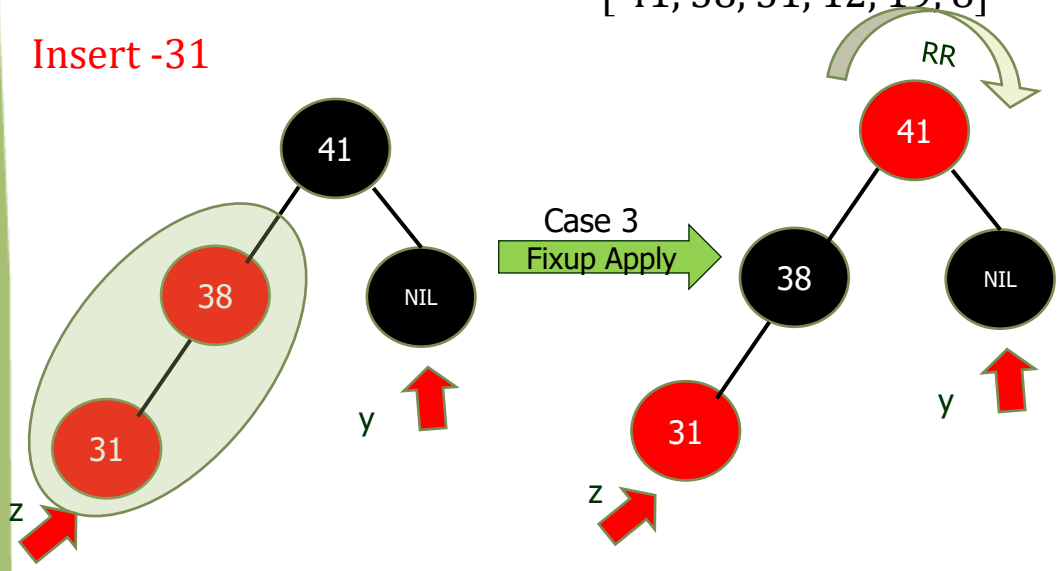
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -31



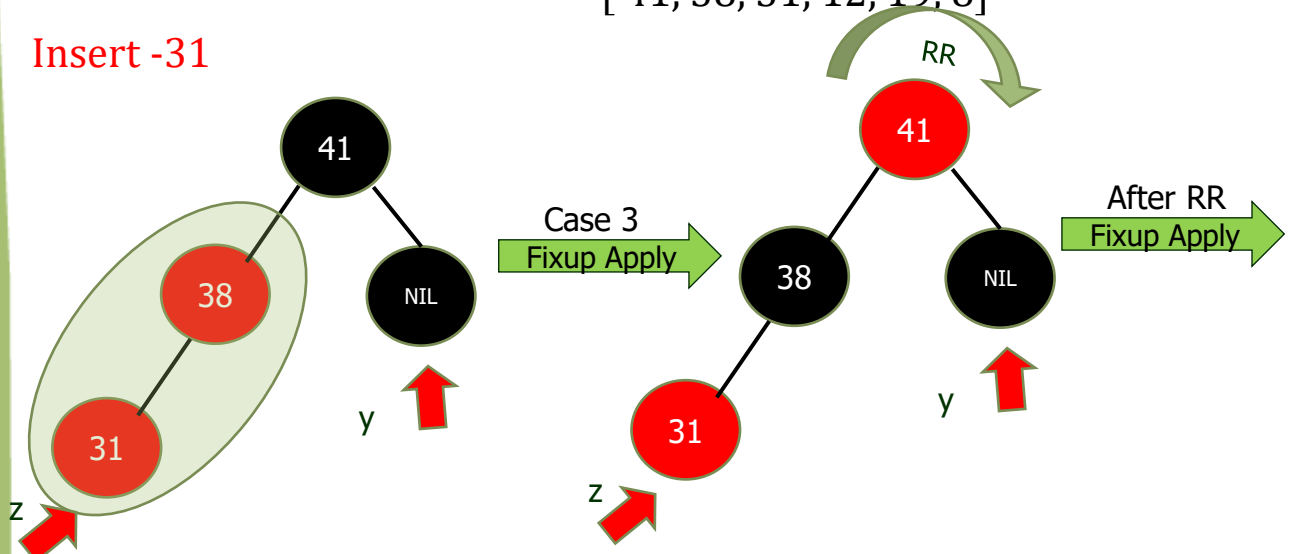
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -31



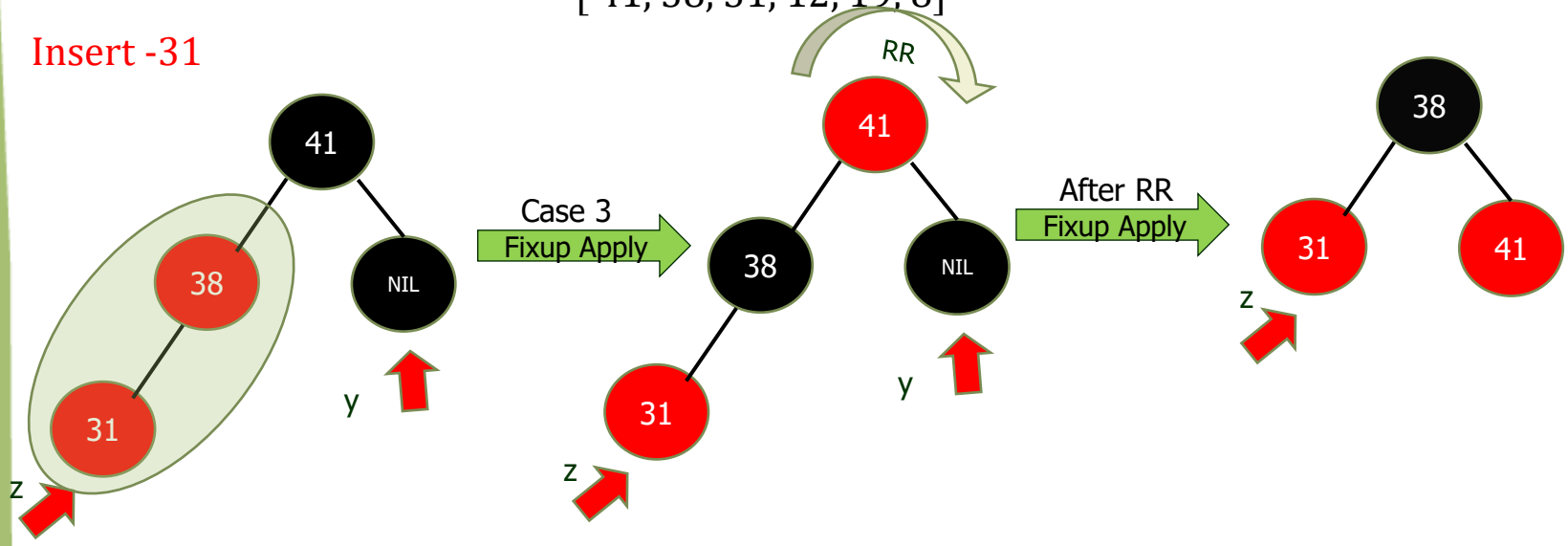
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -31



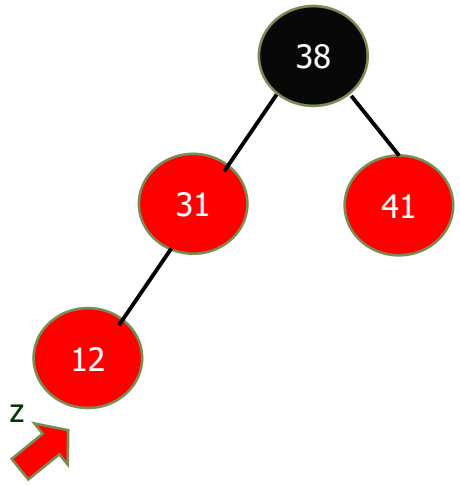
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -12



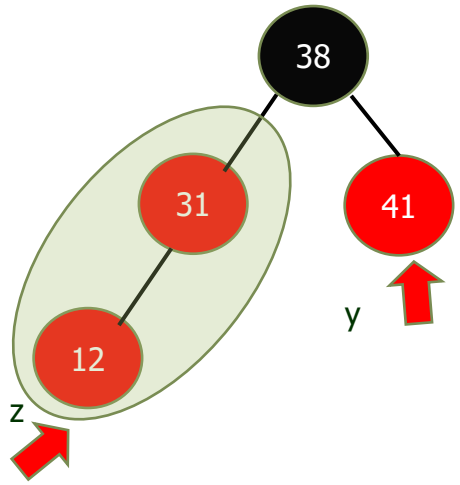
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -12



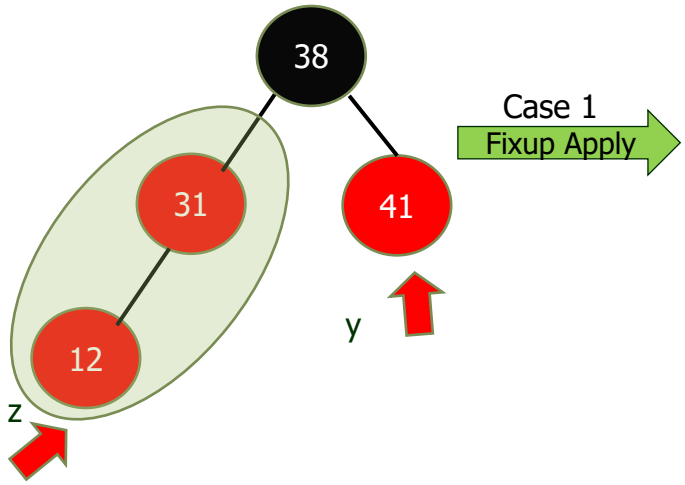
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -12



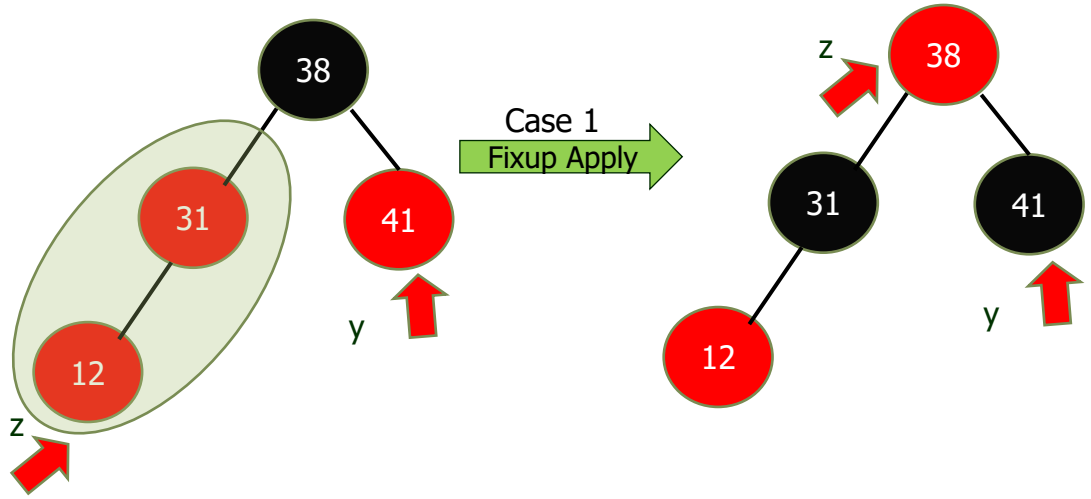
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -12



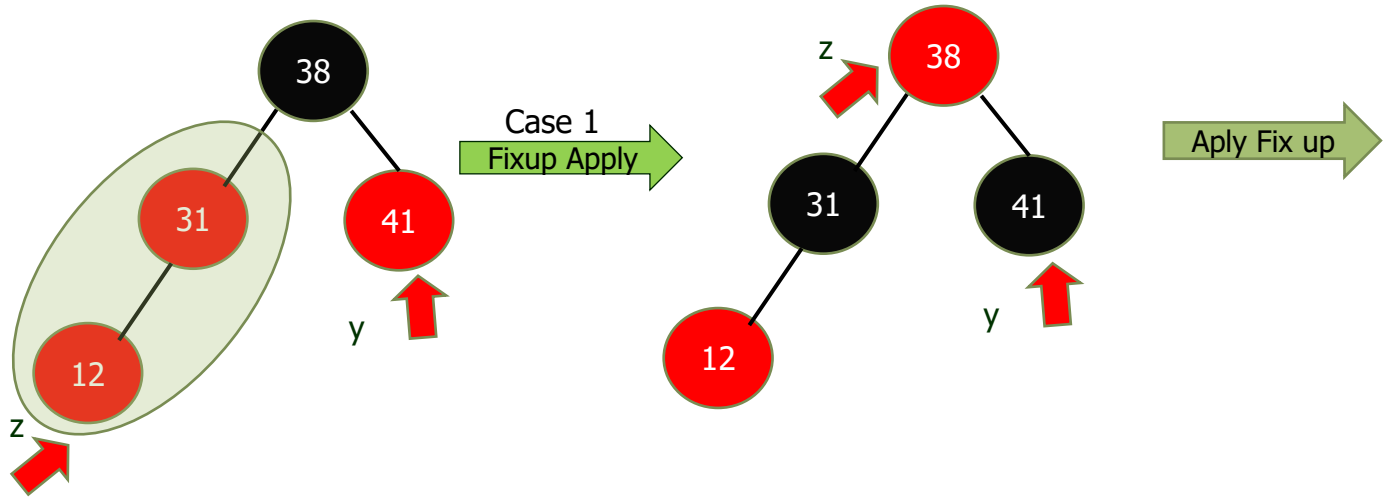
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -12



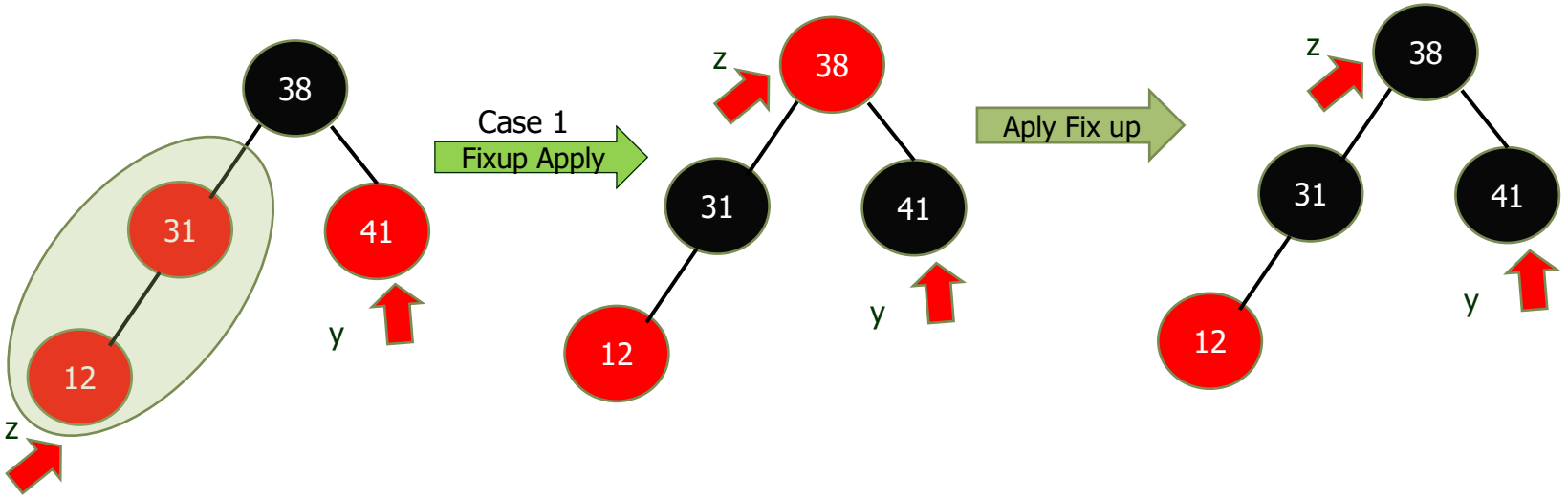
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -12



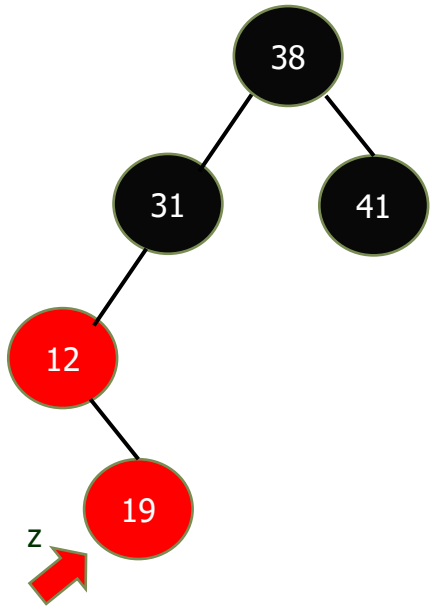
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -19



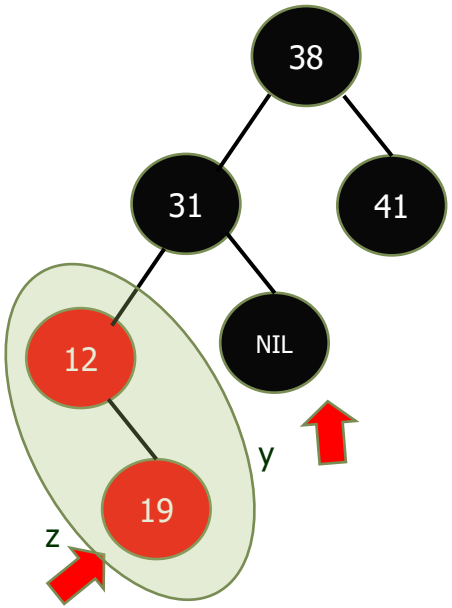
Red Black Tree

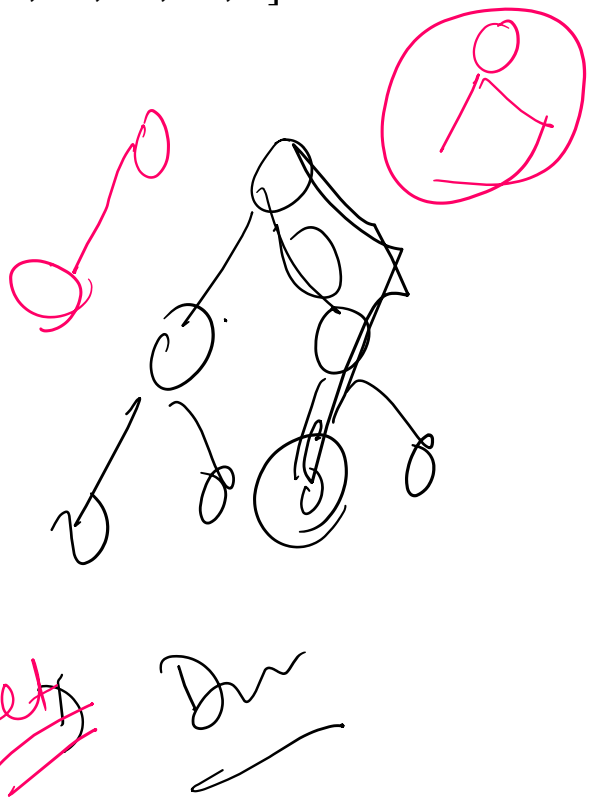
Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -19





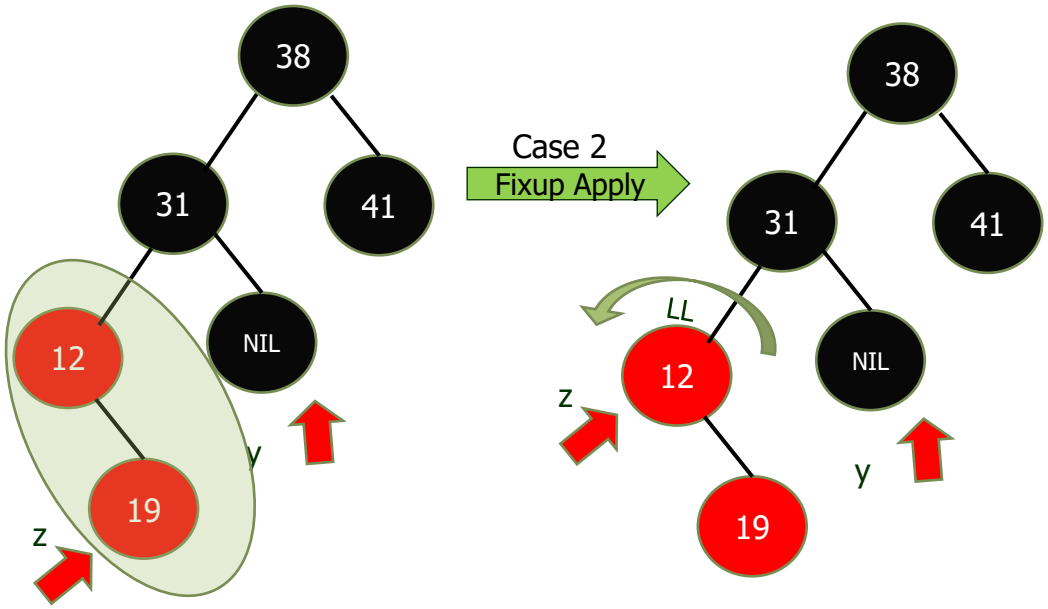
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -19



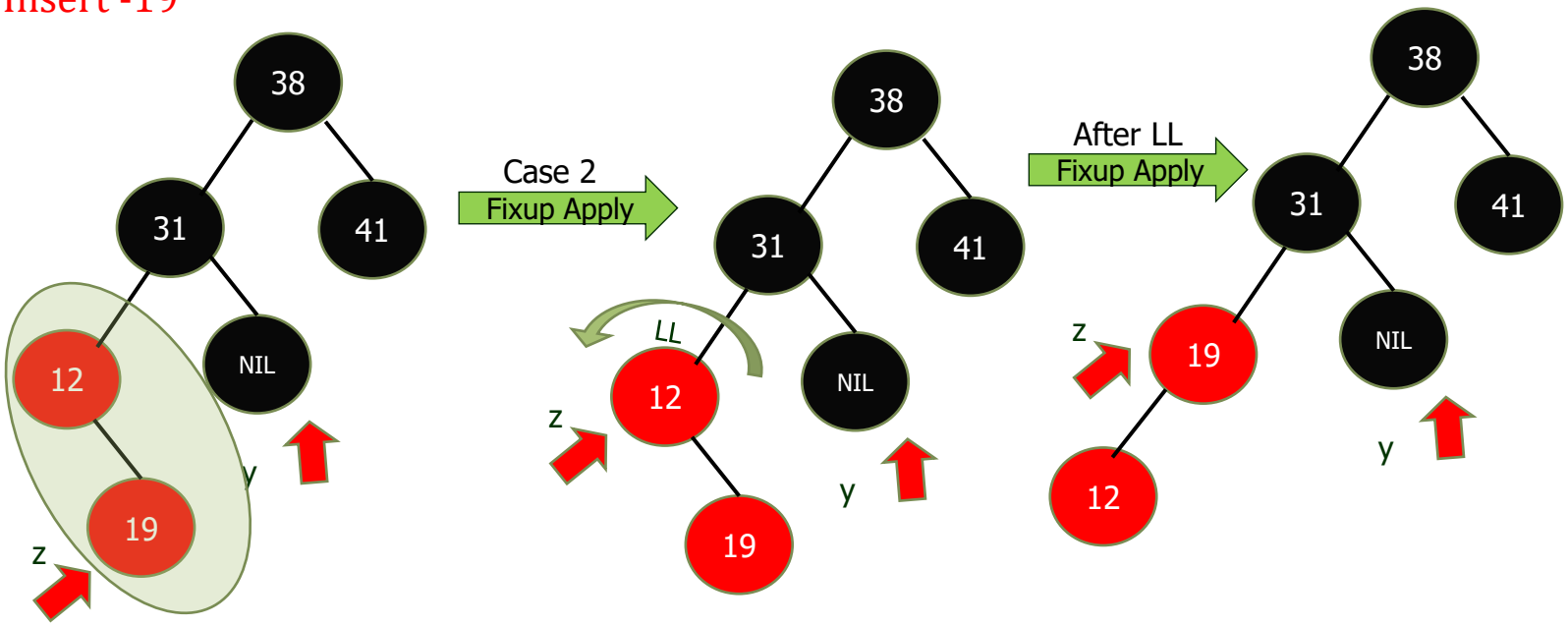
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -19



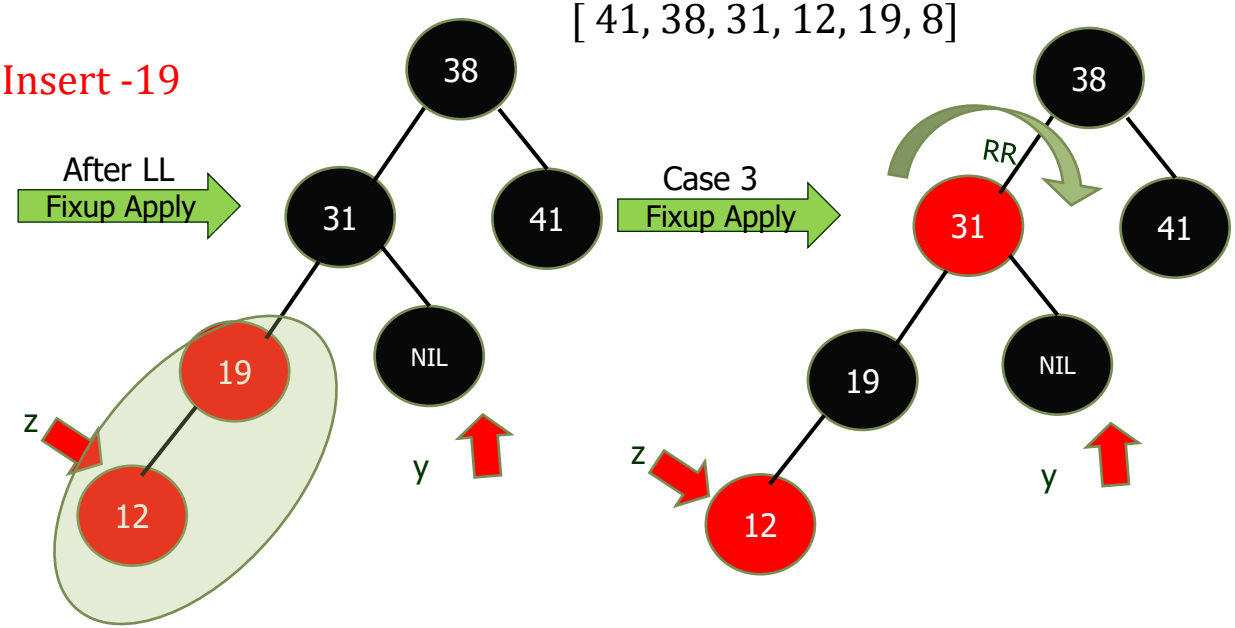
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -19

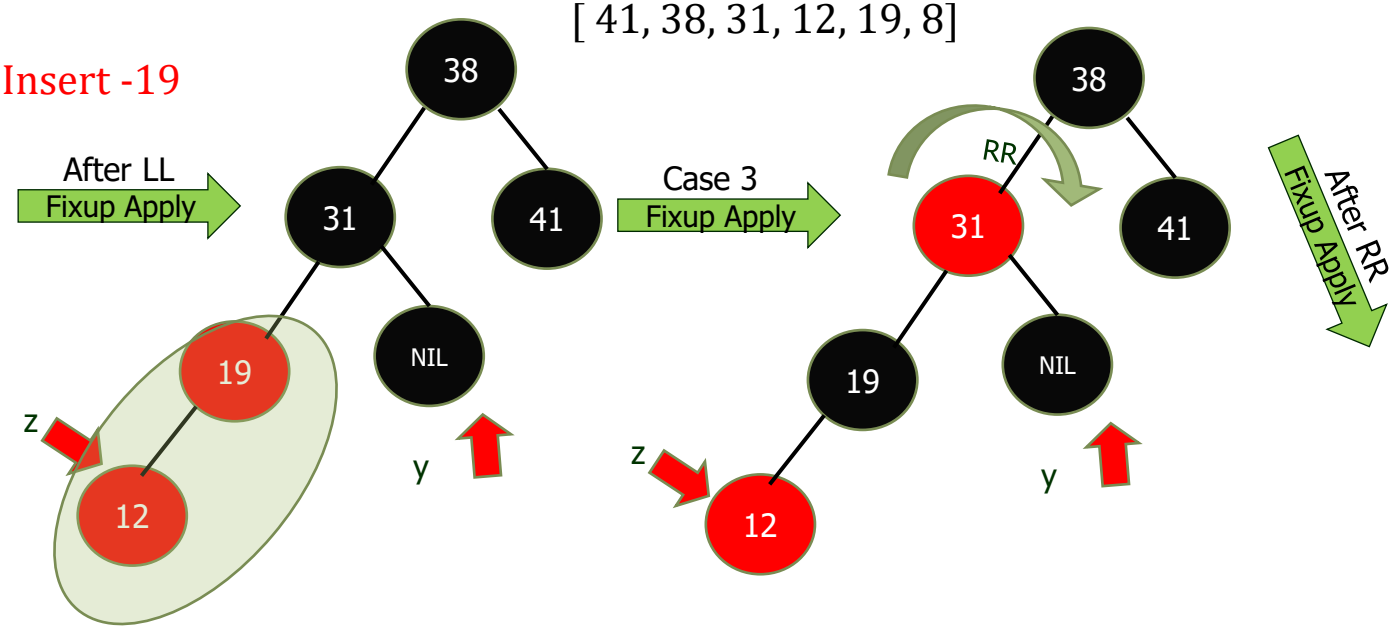


Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

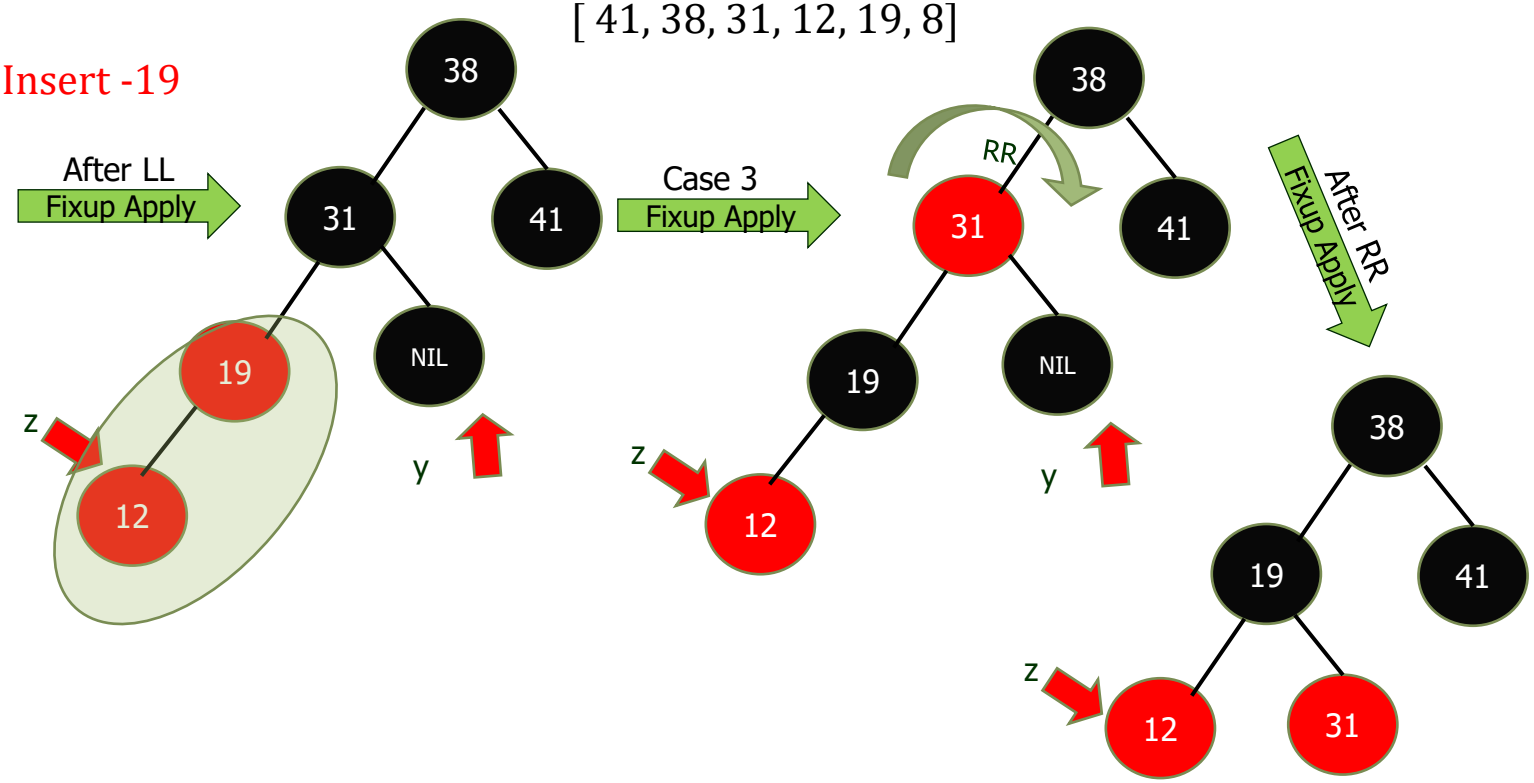


Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]



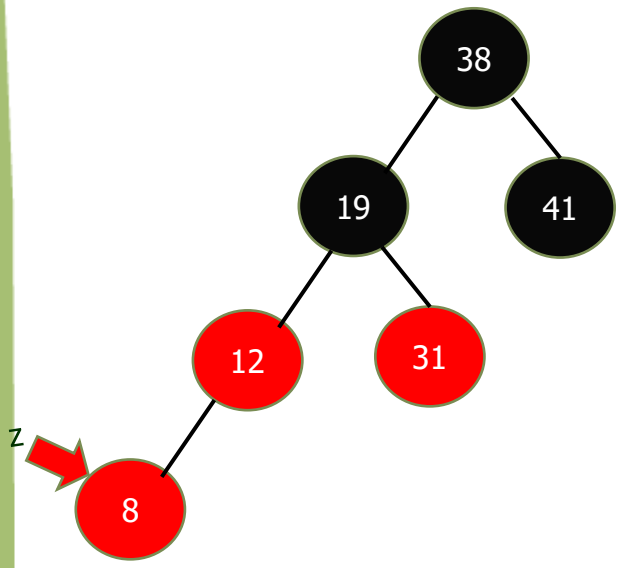
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -8



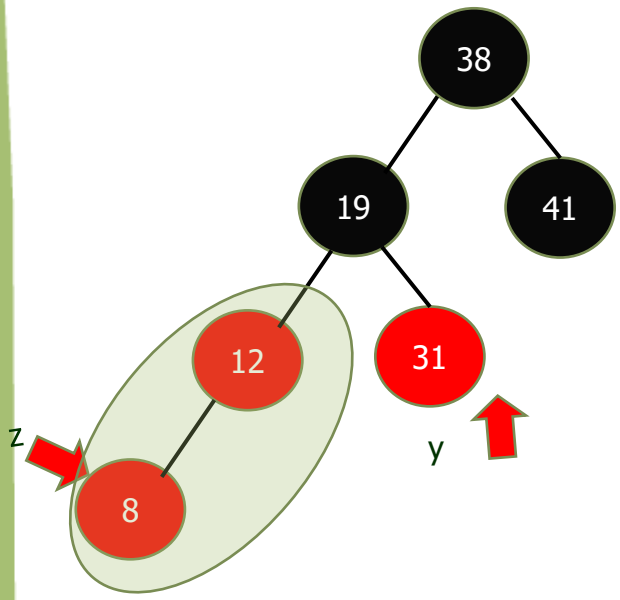
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -8



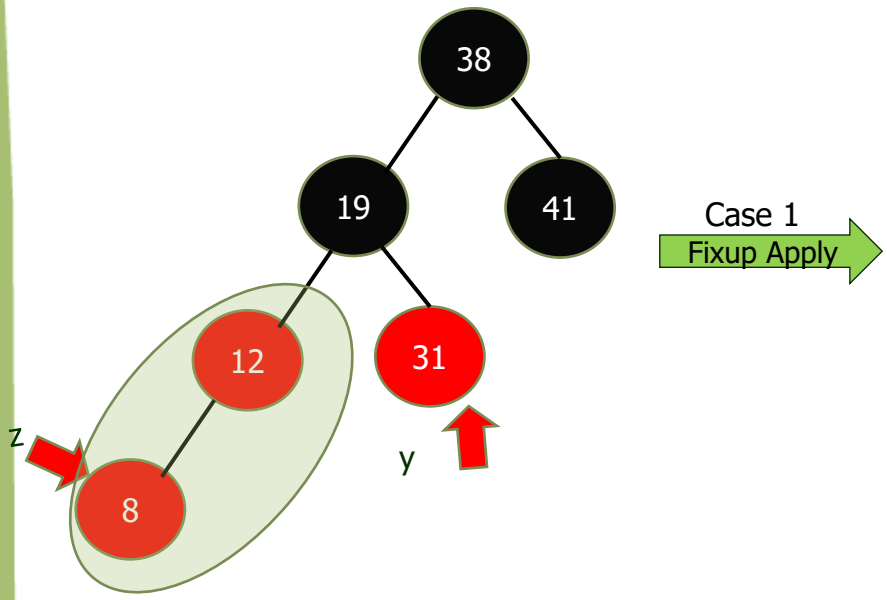
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -8



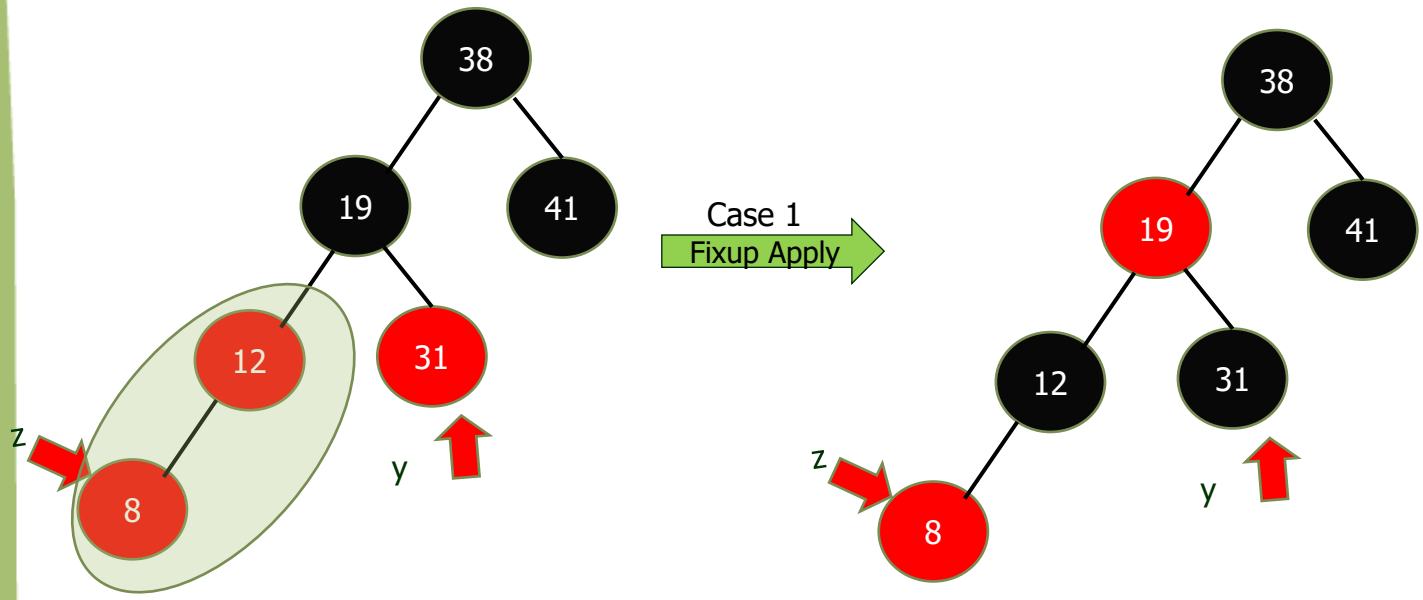
Red Black Tree

Insertion (Example 3):

Insert the following elements into an empty RB-Tree.

[41, 38, 31, 12, 19, 8]

Insert -8



Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -5

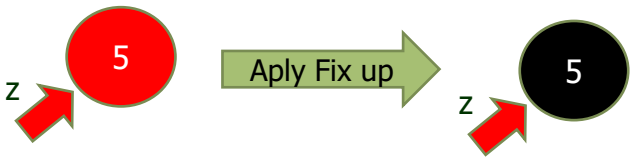
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -5



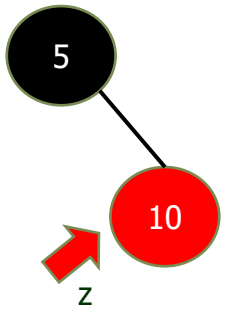
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -10



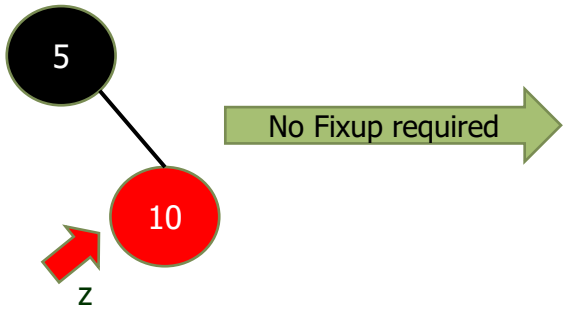
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -10



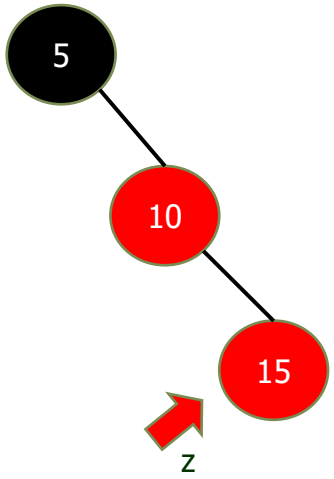
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -15



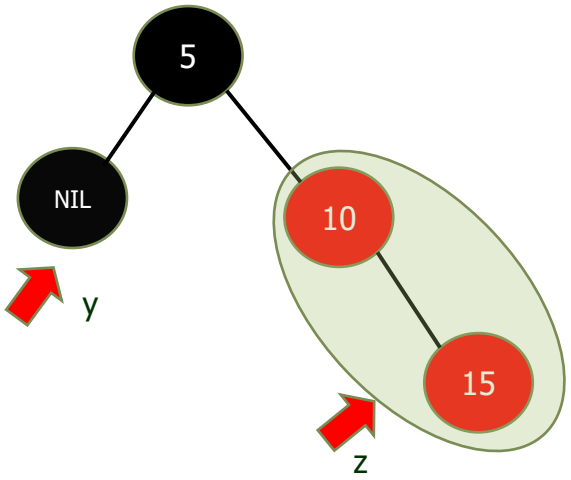
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -15



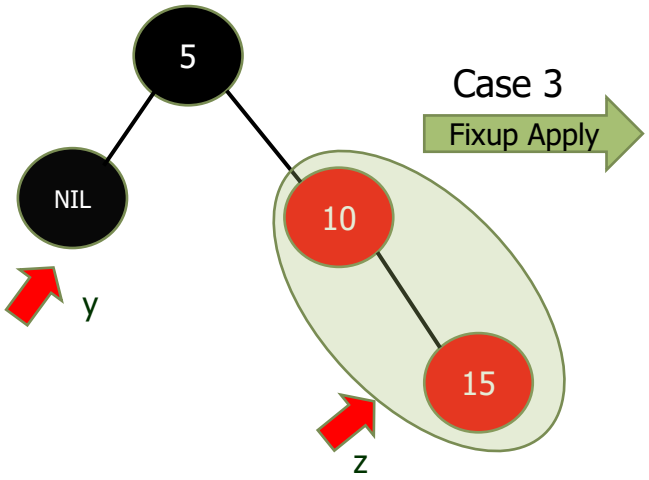
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -15



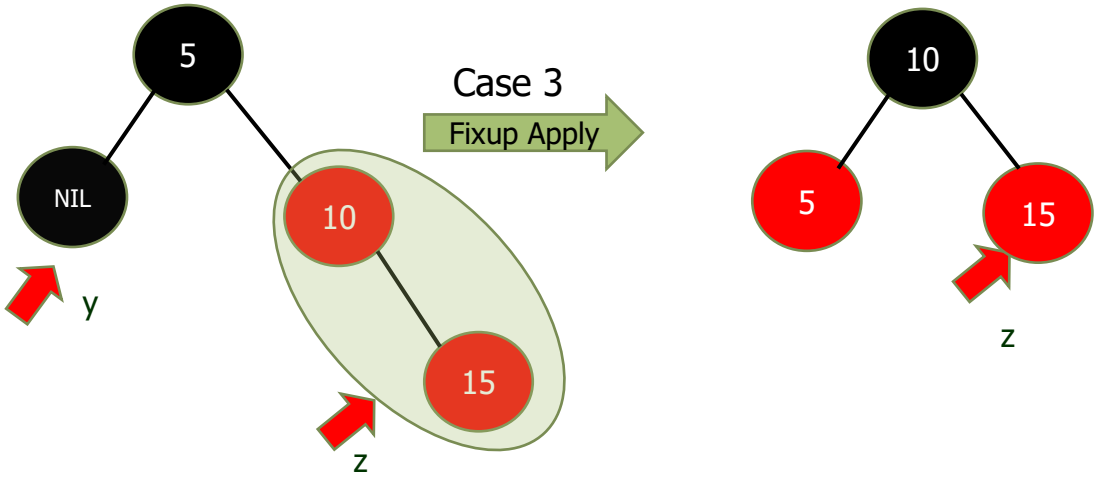
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -15



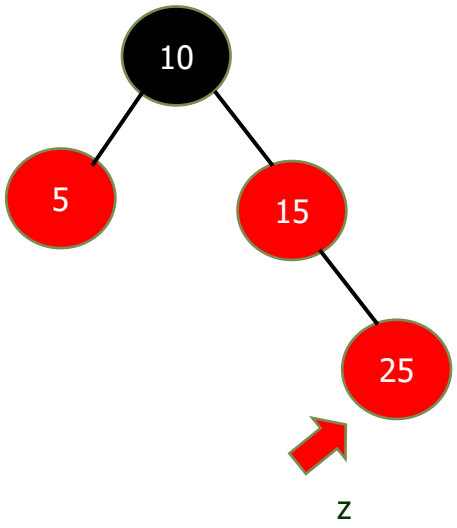
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -25



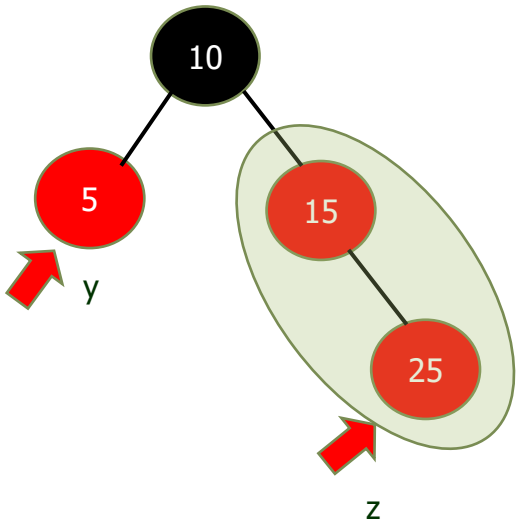
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -25



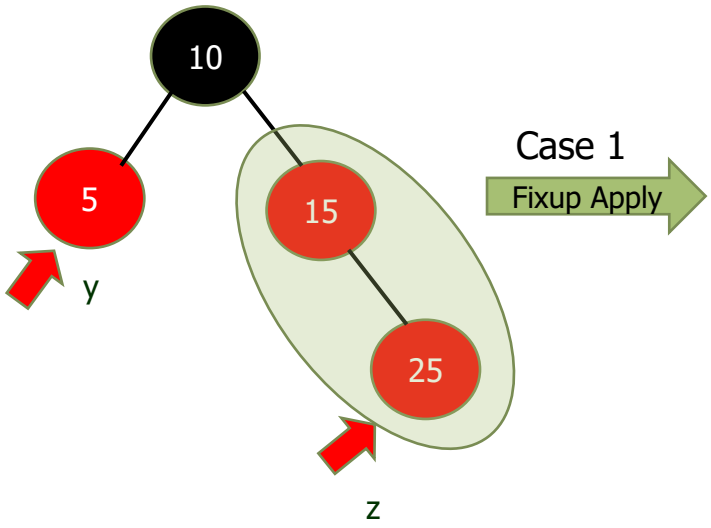
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -25



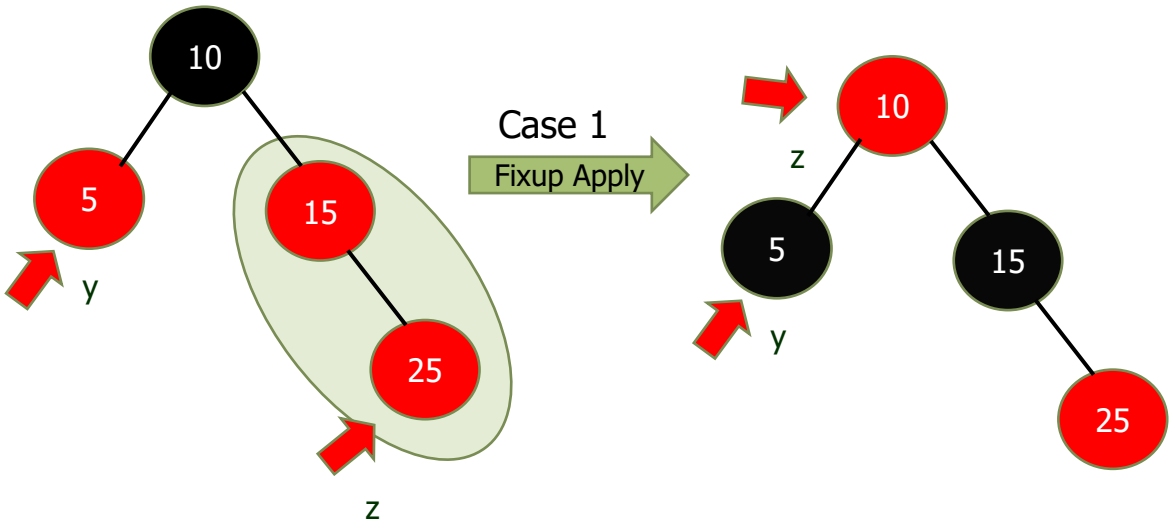
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -25



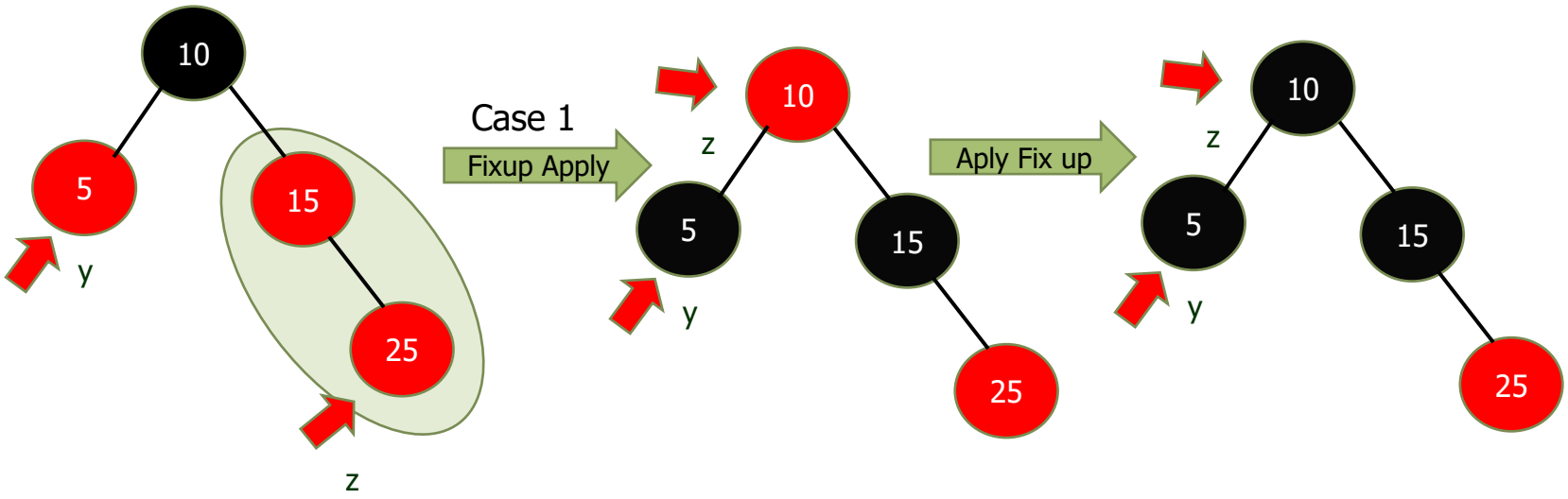
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -25



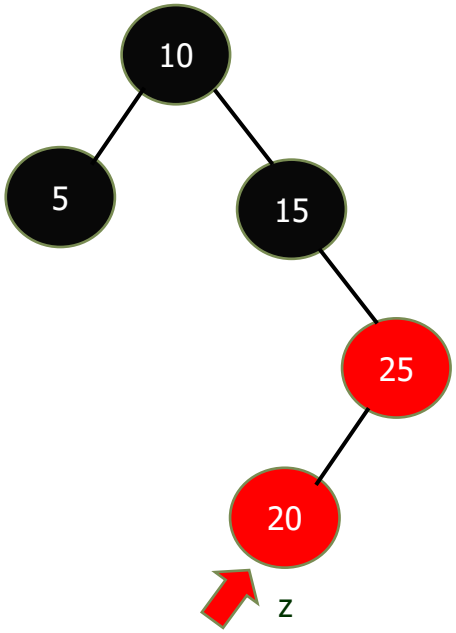
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



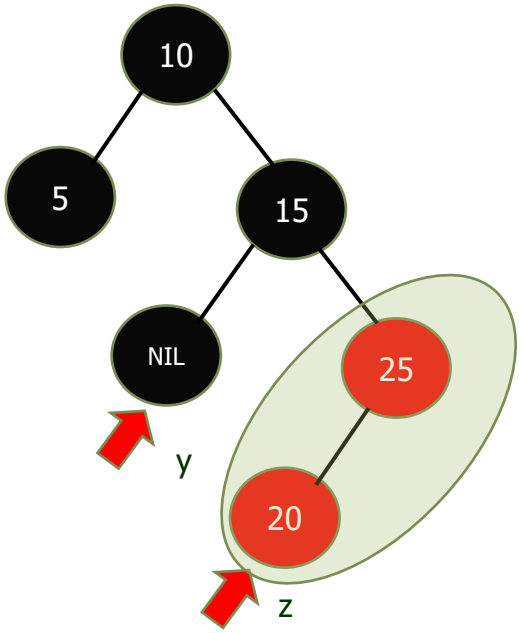
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



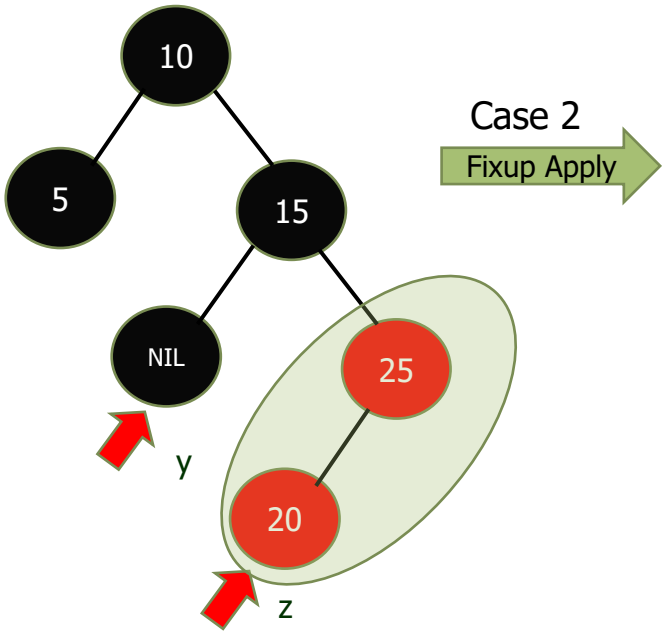
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



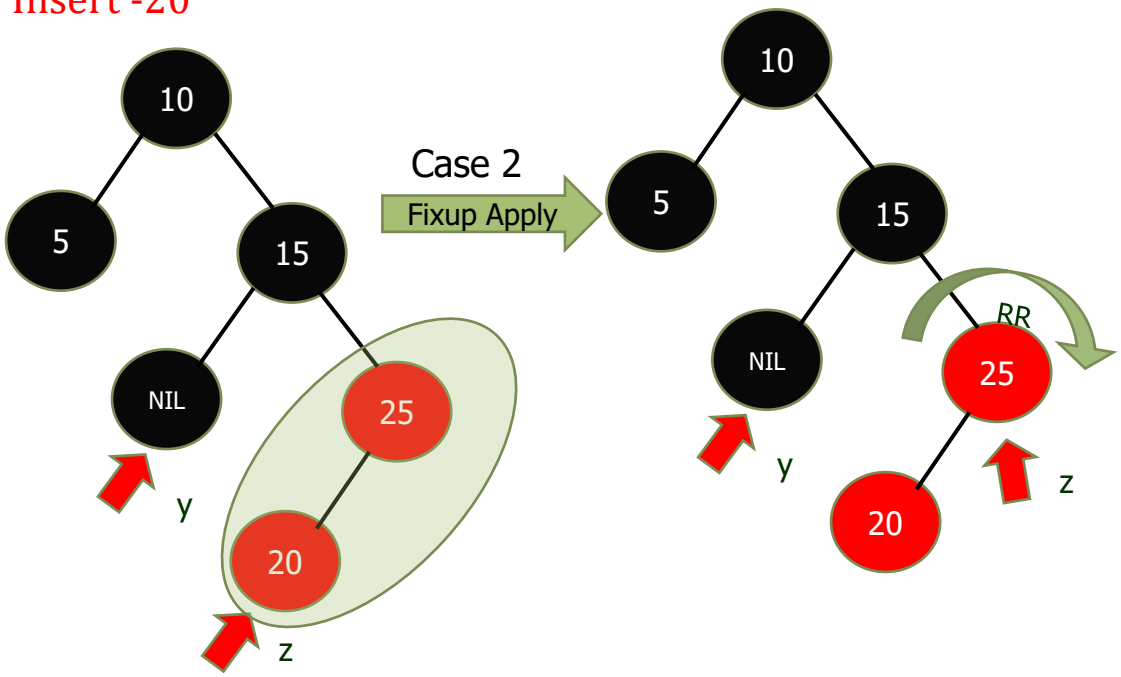
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



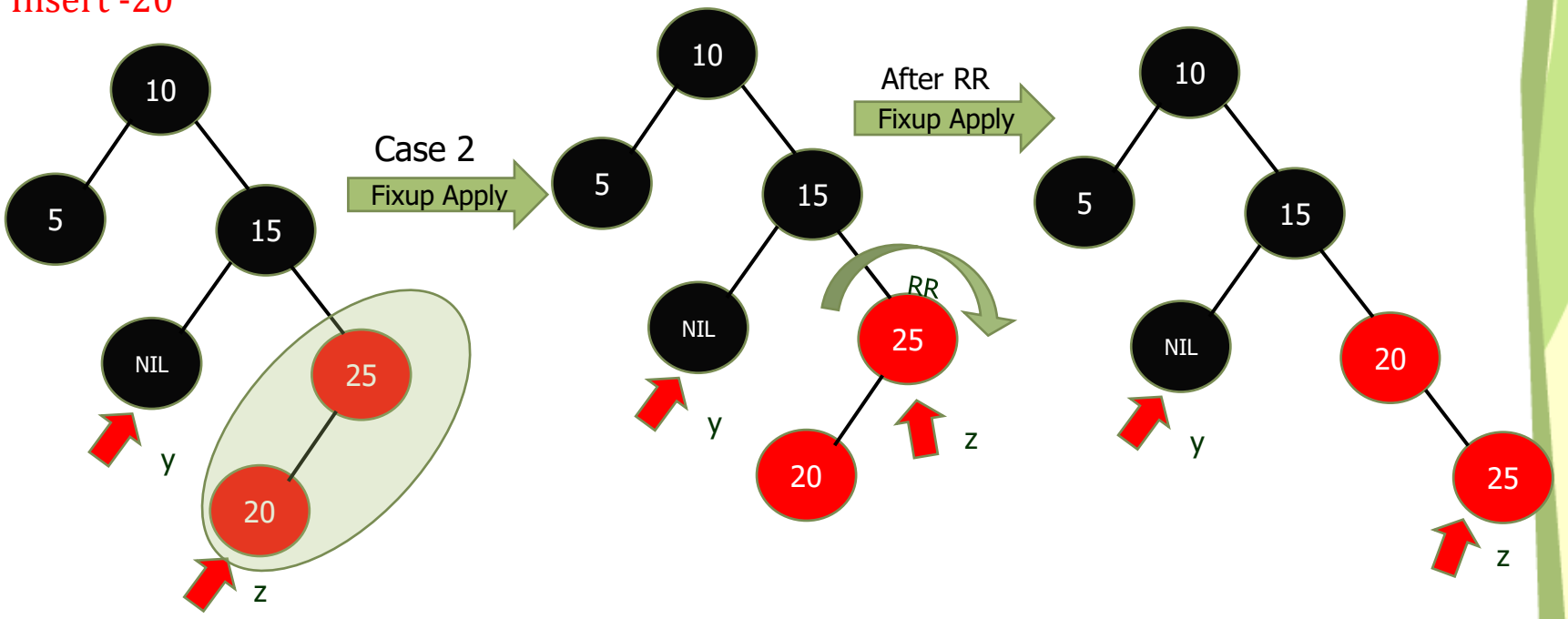
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



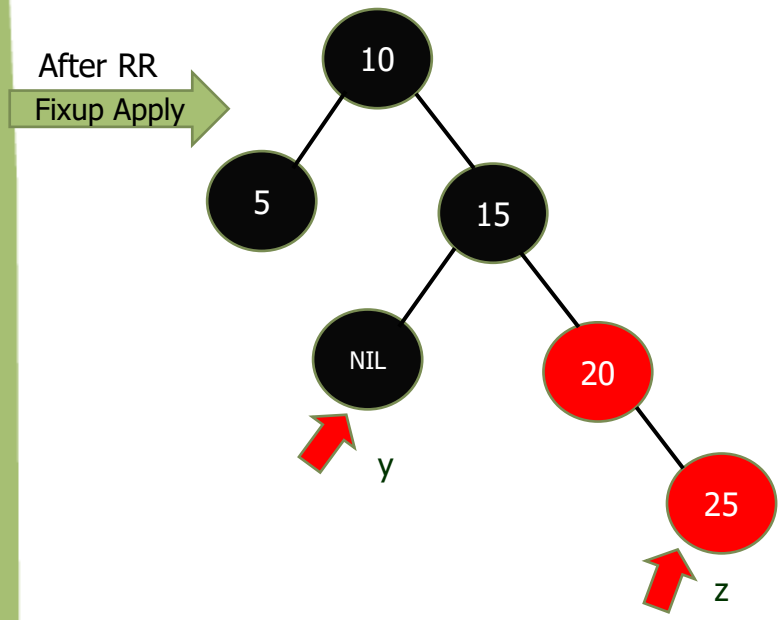
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



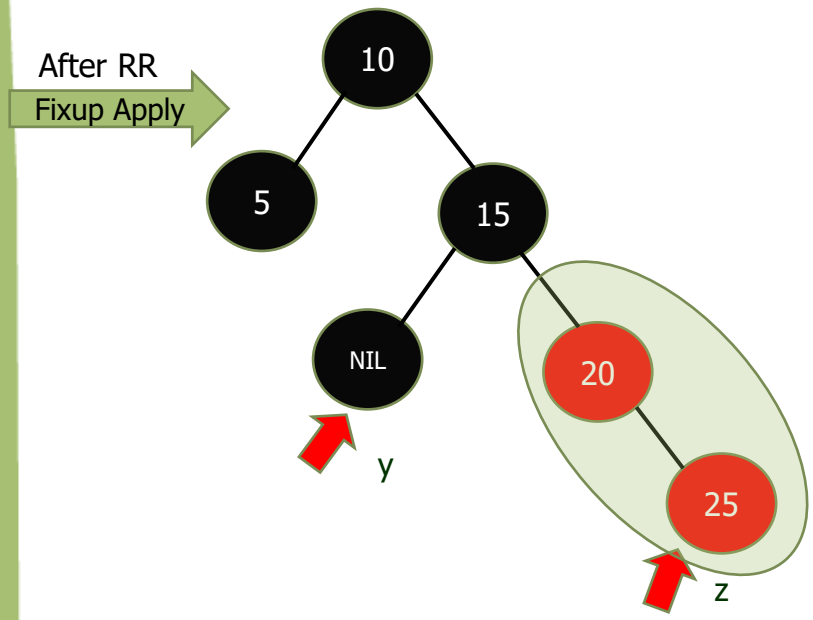
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



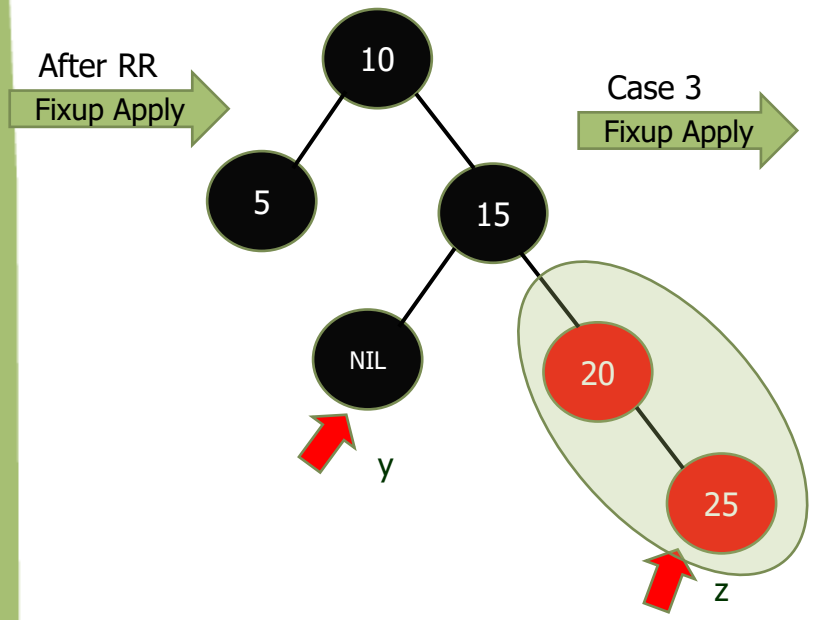
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



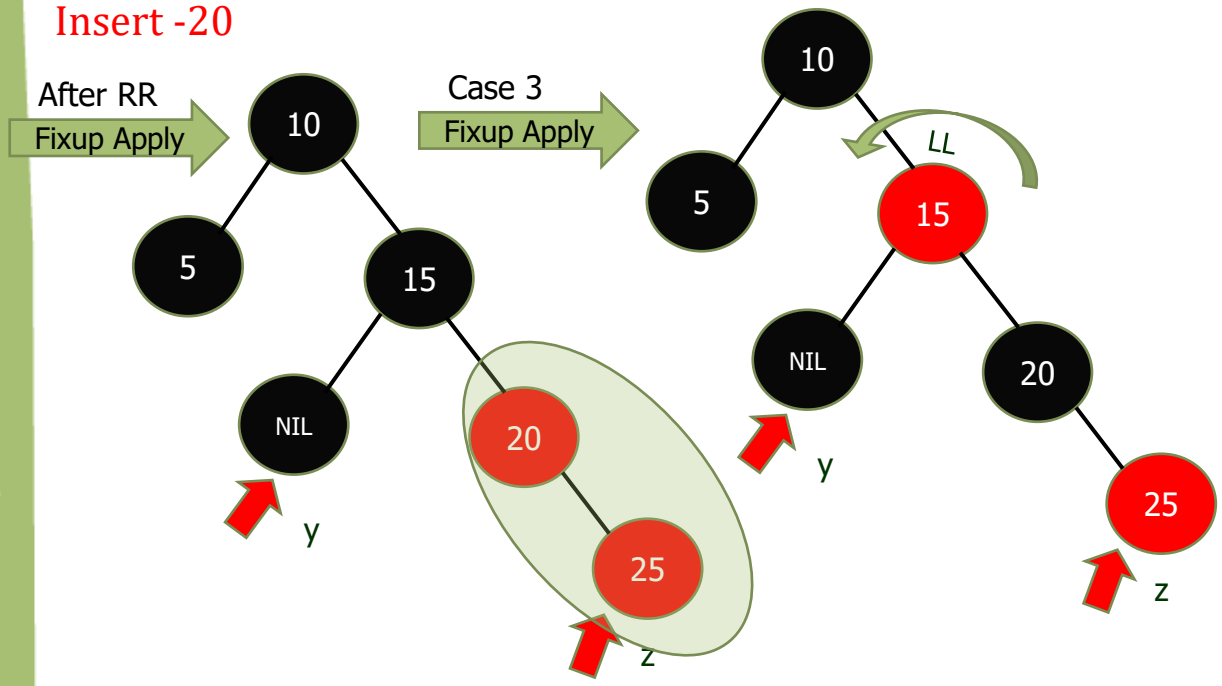
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



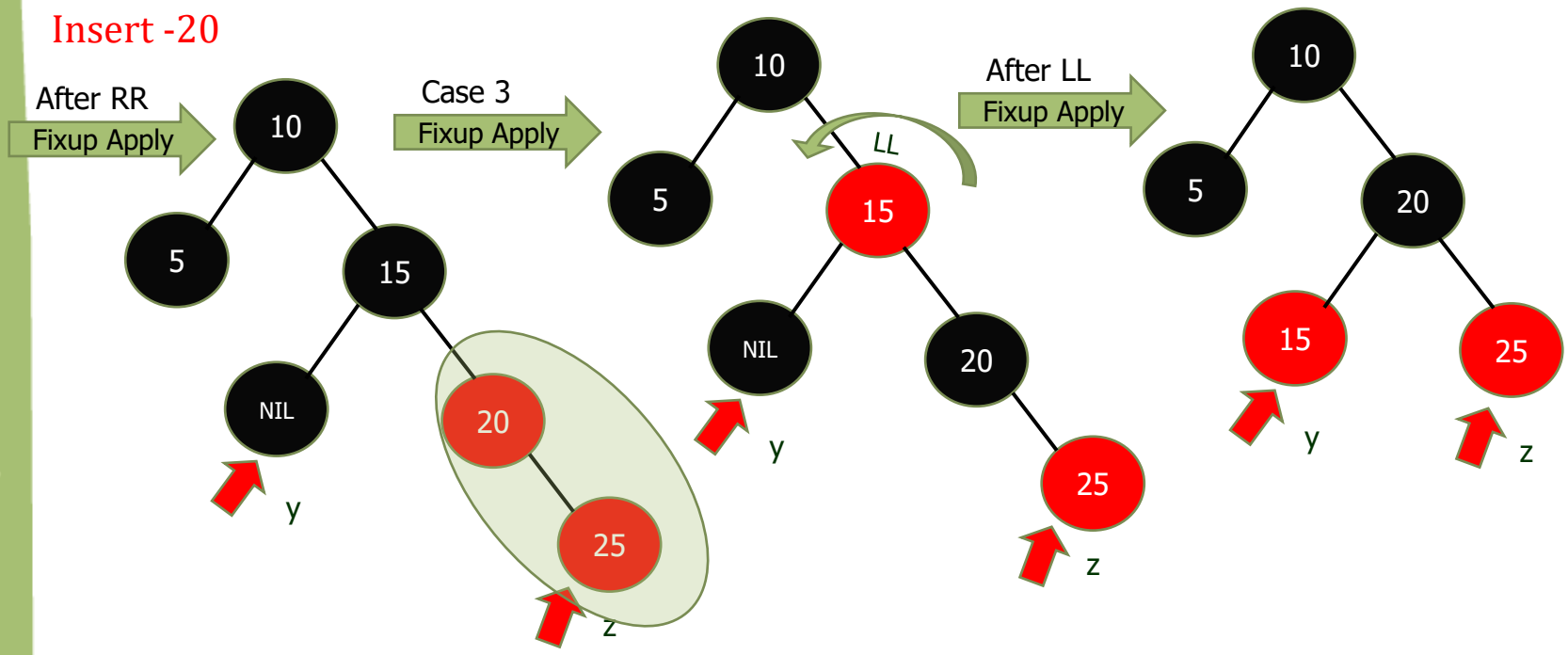
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -20



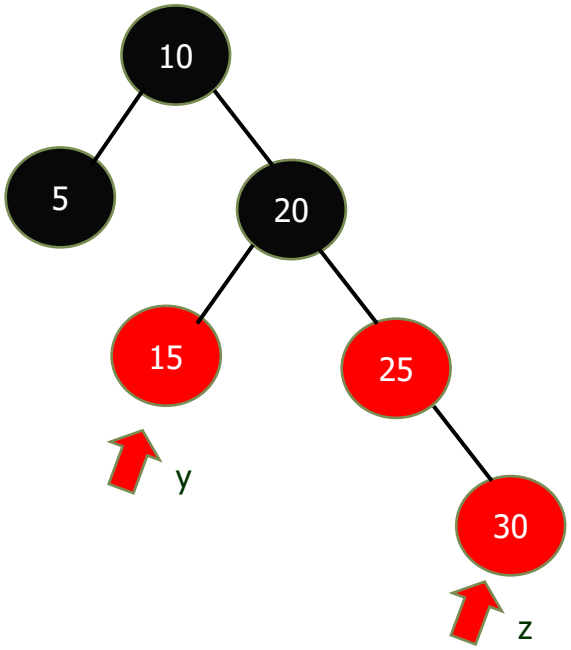
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -30



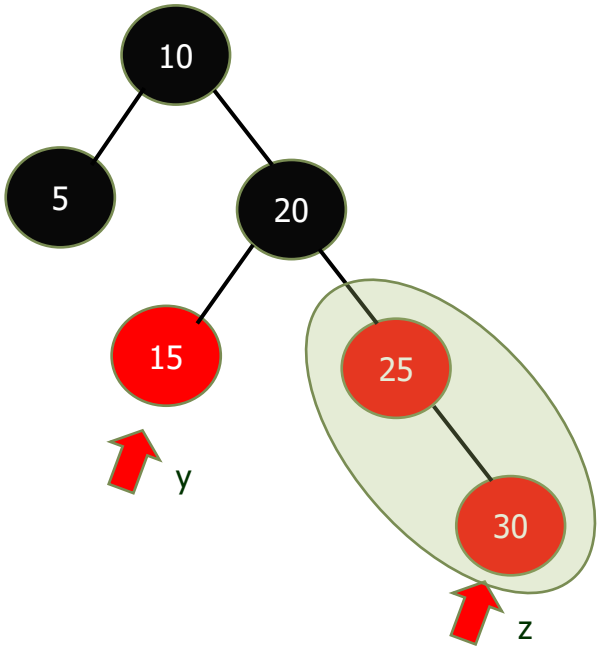
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -30



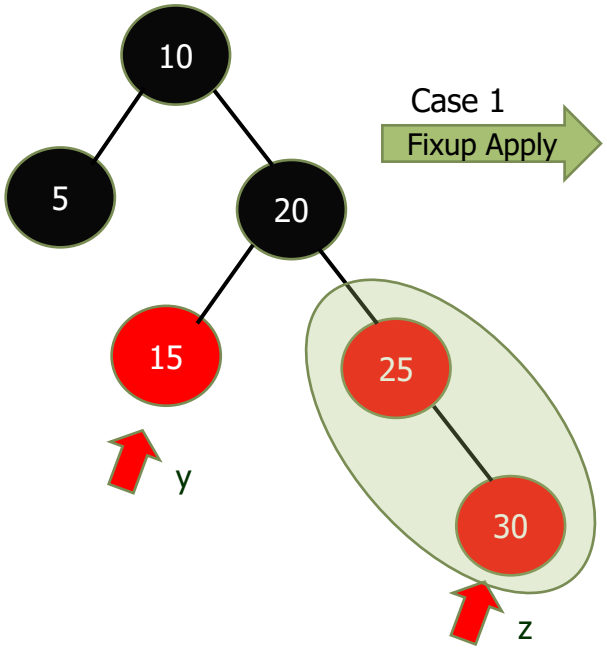
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -30



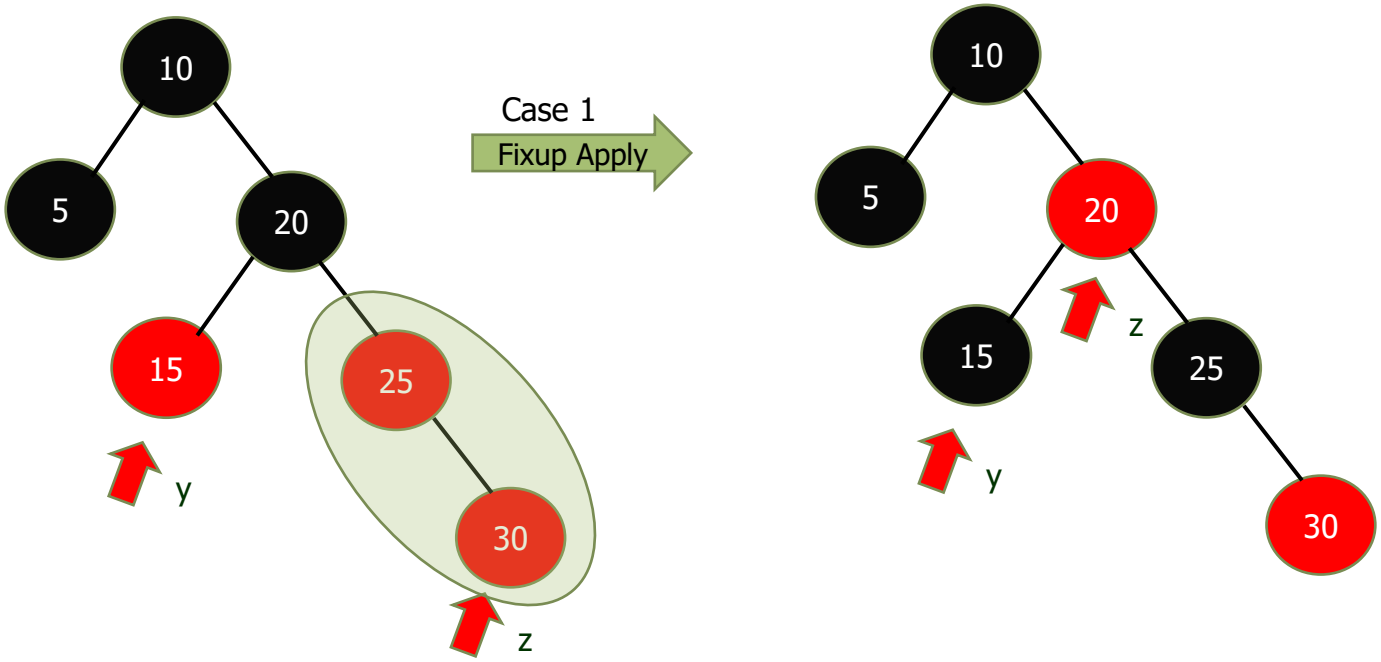
Red Black Tree

Insertion (Example 4):

Insert the following elements into an empty RB-Tree.

[5, 10 ,15 , 25, 20 ,30]

Insert -30




Thank U