#### **Design and Analysis of Algorithm**

# Dynamic Programming (Longest Common Subsequence)



**Lecture – 57** 

### **Overview**

- Dynamic Programming is a general algorithm design technique for solving problems defined by recurrences with overlapping subproblems
- Invented by American mathematician "Richard Bellman) in the year 1950s to solve optimization problems and later assimilated by Computer Science.
- "Programming" here means "planning"

- "Method of solving complex problems by breaking them down into smaller sub-problems, solving each of those sub-problems just once, and storing their solutions."
- The problem solving approach looks like Divide and conquer approach.(which is not true)

Difference between Dynamic programming and Divide and Conquer approach.

Divide & Conquer	Dynamic Programming
Partitions a problem into independent smaller sub-problems	Partitions a problem into     overlapping sub-problems
Doesn't store solutions of sub- problems. (Identical sub-problems may arise - results in the same computations are performed repeatedly.)	Stores solutions of sub- problems: thus avoids calculations of same quantity twice
3. Top down algorithms: which logically progresses from the initial instance down to the smallest sub-instances via intermediate sub-instances.	3. Bottom up algorithms: in which the smallest sub-problems are explicitly solved first and the results of these used to construct solutions to progressively larger sub-instances

#### Is a Four-step methods

- 1. Characterize the structure of an optimal solution.
- 2. Recursively define the value of an optimal solution.
- 3. Compute the value of an optimal solution, typically in a bottom-up fashion.
- 4. Construct an optimal solution from computed information.

#### **Problems:**

- 1. 0/1 Knapsack Problem
- 2. Floyd-Warshall Algorithm
- 3. Longest Common Sub-sequence
- 4. Matrix Chain Multiplication

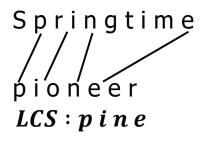
## Problem 3: Longest Common Subsequences (LCS)

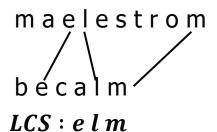
Problem:

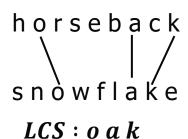
"Given two sequences  $X = \langle x_1, x_2, \dots, x_m \rangle$  and  $Y = \langle y_1, y_2, \dots, y_n \rangle$ . Find a subsequence common to both whose length is longest. A subsequence doesn't have to be consecutive, but it has to be in order."

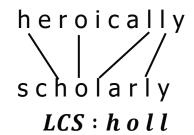
## Problem 3: Longest Common Subsequences (LCS)

Example:









#### **Problem 3: Longest Common Subsequence**

- It is used, when the solution can be recursively described in terms of solutions to subproblems (optimal substructure)
- Algorithm finds solutions to subproblems and stores them in memory for later use
- More efficient than "brute-force methods", which solve the same subproblems over and over again

#### **Problem 3: Longest Common Subsequence**

- Application: comparison of two DNA strings
- Example: X= {A B C B D A B }, Y= {B D C A B A}
   Longest Common Subsequence:

$$X = A B C B D A B$$

$$Y = BDCABA$$

 Brute force algorithm would compare each subsequence of X with the symbols in Y

#### **Problem 3: Longest Common Subsequence**

- if |X| = m, |Y| = n, then there are  $2^m$  subsequences of x; we must compare each with Y (n comparisons)
- So the running time of the brute-force algorithm is  $O(n \ 2^m)$
- Notice that the LCS problem has *optimal substructure*: solutions of subproblems are parts of the final solution.
- Subproblems: "find LCS of pairs of prefixes of X and Y"

#### **Problem 3: Longest Common Subsequence**

Step 1: Characterize the structure of an optimal solution

- Define  $X_i$ ,  $Y_j$  to be the prefixes of X and Y of length i and j respectively
- Define c[i,j] to be the length of LCS of  $X_i$  and  $Y_j$
- Then the length of LCS of X and Y will be c[m, n].
- We start with i = j = 0 (i.e empty substrings of x and y)
- Since  $X_0$  and  $Y_0$  are empty strings, their LCS is always empty (i.e. c[0,0] = 0)
- LCS of empty string and any other string is empty, so for every i and j: c[0, j] = c[i,0] = 0

#### **Problem 3: Longest Common Subsequence**

Step 1: Characterize the structure of an optimal solution

- In the process of calculation of c[i,j], there are two cases:
- First case: x[i] = y[j]: one more symbol in strings X and Y matches, so the length of LCS  $X_i$  and  $Y_j$  equals to the length of LCS of smaller strings  $X_{i-1}$  and  $Y_{j-1}$ , plus 1.
- Second case: x[i]! = y[j]: As symbols don't match, our solution is not improved, and the length of  $LCS(X_i, Y_j)$  is the same as before (i.e. maximum of  $LCS(X_i, Y_{j-1})$  and  $LCS(X_{i-1}, Y_j)$

#### **Problem 3: Longest Common Subsequence**

Step 2: Recursively define the value of optimal solution.

• Define c[i,j] to be the length of LCS of  $X_i$  and  $Y_j$ . Then the length of LCS of X and Y will be calculated as c[m,n].

$$c[i,j] = \begin{cases} 0 & , & if \ i = 0 \ or \ j = 0 \\ c[i-1,j-1] + 1 & , & if \ i,j > 0 \ and \ X_i = Y_j \\ \max(c[i-1,j],c[i,j-1]), & if i,j > 0 \ and \ X_i \neq Y_j \end{cases}$$

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

```
LCS-Length(X, Y)
1.m = length(X) // get the # of symbols in X
2.n = length(Y) // get the # of symbols in Y
3. for i = 1 to m
         c[i,0] = 0 // special case: Y_0
4. for j = 1 to n
         c[0,j] = 0 // special case: X_0
5. for i = 1 to m // for all X_i
     for j = 1 to n // for all Y_i
7.
           if (X_i == Y_i)
               c[i,j] = c[i-1,j-1] + 1 and b[i,j] = " \land "
8.
           else c[i, j] = \max(c[i-1, j], c[i, j-1]) and
                            b[i,j] = " \uparrow " (if max is c[i-1,j])
                            b[i,j] = " \leftarrow " (if \max is c[i,j-1])
```

10. return c and b

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

Example 1: What do ABCB and BDCAB have in common?

$$X = ABCB$$

$$Y = BDCAB$$

What is the Longest Common Subsequence LCS(X,Y)?

$$X = A B C B$$
$$Y = B D C A B$$

Hence,

$$LCS(X,Y) = BCB$$

Note: The demonstration of this problem is given in the next page.

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

Example 1: What do ABCB and BDCAB have in common?

	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	Α	В
0	$X_{I}$						
1	Α						
2	В						
3	С						
4	В						

$$X = A B C B$$
$$Y = B D C A B$$

X = ABCB; m = |X| = 4 Y = BDCAB; n = |Y| = 5Allocate array c[5,6]

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	Α	В
0	$X_{I}$	0	0	0	0	0	0
1	Α	0					
2	В	0					
3	С	0					
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

for 
$$i = 0$$
 to m  $c[i,0] = 0$   
for  $j = 1$  to n  $c[0,j] = 0$ 

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_{j}$	B	D	С	Α	В
0	$X_{I}$	0	0	0	0	0	0
1	A	0	0				
2	В	0					
3	С	0					
4	В	0					

$$X = ABCB$$
$$Y = BDCAB$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
 $else c[i,j] = \max(c[i-1,j],c[i,j-1]) \text{ and}$ 
 $b[i,j] = " \downarrow " (if \max is c[i-1,j])$ 
 $b[i,j] = " \rightarrow " (if \max is c[i,j-1])$ 

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	(D)	С	Α	В
0	$X_{I}$	0	0	0	0	0	0
1	A	0	0	0			
2	В	0					
3	С	0					
4	В	0					

$$X = ABCB$$
$$Y = BDCAB$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	<u>(c)</u>	Α	В
0	$X_{I}$	0	0	0	) 0	0	0
1	A	0	0	0	0		
2	В	0					
3	С	0					
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
 $else c[i,j] = \max(c[i-1,j],c[i,j-1]) \text{ and}$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	A	В
0	$X_{I}$	0	0	0	0	0	0
1	A	0	0	0	0	1	
2	В	0					
3	С	0					
4	В	0					

$$X = ABCB$$
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	Α	(B)
0	$X_{I}$	0	0	0	0	0	0
1	A	0	0	0	0	1 -	1
2	В	0					
3	С	0					
4	В	0					

$$X = ABCB$$
$$Y = BDCAB$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_{j}$	B	D	С	Α	В
0	$X_{I}$	0	) 0	0	0	0	0
1	Α	0	0	0	0	1	<b>→</b> 1
2	B	0	1				
3	C	0					
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
 $else c[i,j] = \max(c[i-1,j],c[i,j-1]) \text{ and}$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	Α	В
0	$X_{I}$	0	0	) 🗖	0	0	0
1	Α	0	0	0	0	1	<b>→</b> 1
2	B	0	1	<b>1</b>			
3	C	0					
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	(c)	Α	B
0	$X_{I}$	0	0	0	0	0	0
1	Α	0	0	0	0	1 -	<b>1</b>
2	$\bigcirc$ B	0	1	<b>→</b> 1	<b>→</b> 1		
3	C	0					
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

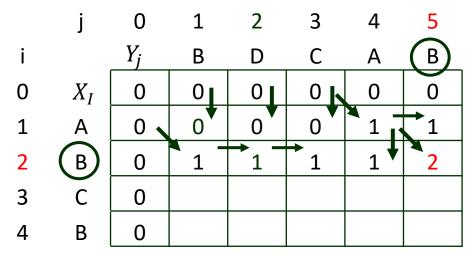
	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	A	В
0	$X_{I}$	0	0	0	0	0	0
1	Α	0	0	0	0	1 1	<b>→</b> 1
2	$\bigcirc$ B	0	1	<b>→</b> 1	<b>1</b>	1 ♥	
3	C	0					
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.



$$X = A B C B$$
$$Y = B D C A B$$

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 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution for cost.

	j	0	1	2	3	4	5
i		$Y_j$	B	D	С	Α	В
0	$X_{I}$	0	) 0	0	0	0	0
1	Α	0	0	0	0	1 1	<b>1</b>
2	В	0	1	<b>→</b> 1 -	<b>→</b> 1	1 ♦	2
3	(c)	0	1 ♥				
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

	j	0	1	2	3	4	5
i		$Y_j$	В	D	С	Α	В
0	$X_{I}$	0	0	)	0	0	0
1	Α	0 🔪	0	0	0	1 1	<b>1</b>
2	В	0	1	<b>1</b> 1	<b>†</b> 1	1 ♥	2
3	(c)	0	1 ♥	1			
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

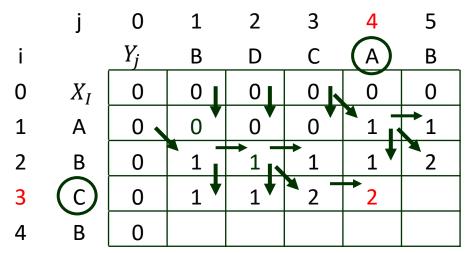
	j	0	1	2	3	4	5
i		$Y_j$	В	D	(c)	Α	В
0	$X_{I}$	0	0	0	0	0	0
1	Α	0	0	0	0	1 1	1
2	В	0	1	→ 1 •	<b>1</b>	1 ♥	2
3	(c)	0	1 ♥	1	2		
4	В	0					

$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

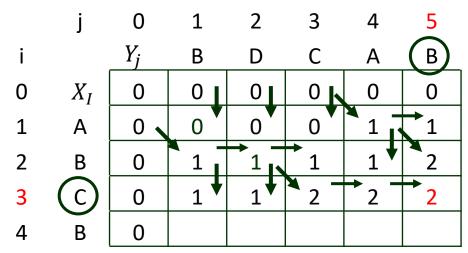


$$X = A B C B$$
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

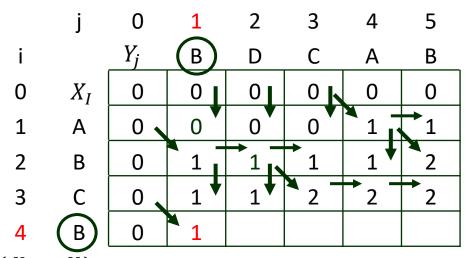


$$X = A B C B$$
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$$if (X_i == Y_j)$$
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.



$$X = A B C B$$
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 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

$$X = A B C B$$
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$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

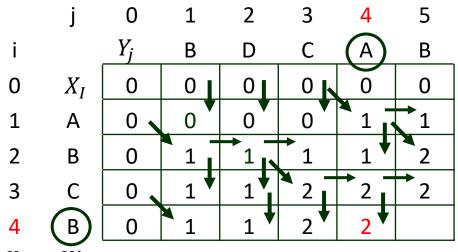
$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

Example 1: What do ABCB and BDCAB have in common?



$$X = A B C B$$
$$Y = B D C A B$$

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
 $else c[i,j] = \max(c[i-1,j],c[i,j-1]) \text{ and}$ 
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#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

Example 1: What do ABCB and BDCAB have in common?

j 0 1 2 3 4 5

i 
$$Y_j$$
 B D C A B

0  $X_I$  0 0 0 0 0 0 0

1 A 0 0 0 0 1 1

2 B 0 1 1 1 2 2 2

3 C 0 1 1 2 2 2

4 B 0 1 1 2 2 3

if  $(X_i == Y_j)$ 
 $c[i,j] = c[i-1,j-1] + 1$  and  $b[i,j] = "\cdot"$ 

 $else\ c[i,j] = \max(\ c[i-1,j],\ c[i,j-1]\ )$  and

 $b[i,j] = " \downarrow " (if \max is c[i-1,j])$  $b[i,j] = " \rightarrow " (if \max is c[i,j-1])$ 

$$X = A B C B$$
$$Y = B D C A B$$

#### **Problem 3: Longest Common Subsequence**

Step 3: Compute optimal solution cost.

Example 1: What do ABCB and BDCAB have in common?

$$if (X_i == Y_j)$$
 $c[i,j] = c[i-1,j-1] + 1 \text{ and } b[i,j] = " \setminus "$ 
 $else c[i,j] = \max(c[i-1,j],c[i,j-1]) \text{ and}$ 
 $b[i,j] = " \downarrow " (if \max is c[i-1,j])$ 
 $b[i,j] = " \rightarrow " (if \max is c[i,j-1])$ 

$$X = A B C B$$
$$Y = B D C A B$$

The running time= O(m \* n) since each c[i,j] is calculated in constant time, and there are m\*n elements in the array

### **Problem 3: Longest Common Subsequence**

Step 4: Construct / print the optimal solution.

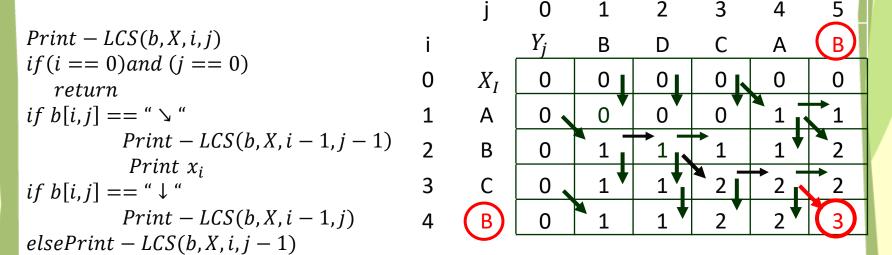
Example 1: What do ABCB and BDCAB have in common?

```
\begin{aligned} & Print - LCS(b, X, i, j) \\ & if (i == 0) and \ (j == 0) \\ & return \\ & if \ b[i, j] == " \searrow " \\ & Print - LCS(b, X, i - 1, j - 1) \\ & Print \ x_i \\ & if \ b[i, j] == " \searrow " \\ & Print - LCS(b, X, i - 1, j) \\ & elsePrint - LCS(b, X, i, j - 1) \end{aligned}
```

#### **Problem 3: Longest Common Subsequence**

Step 4: Construct / print the optimal solution.

Example 1: What do ABCB and BDCAB have in common?

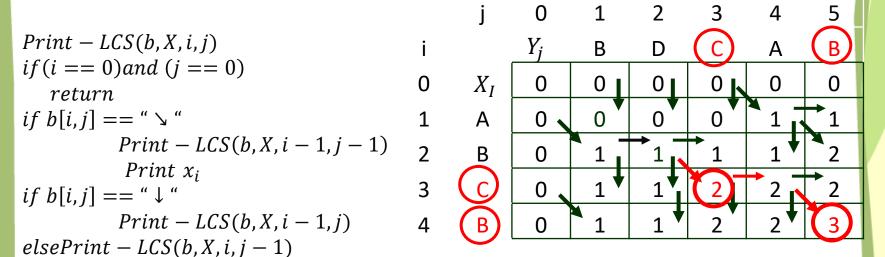


B

### **Problem 3: Longest Common Subsequence**

Step 4: Construct / print the optimal solution.

Example 1: What do ABCB and BDCAB have in common?

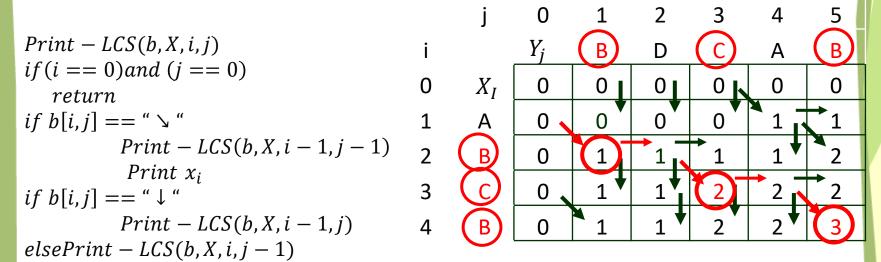


CB

#### **Problem 3: Longest Common Subsequence**

Step 4: Construct / print the optimal solution.

Example 1: What do ABCB and BDCAB have in common?



B C B

#### **Problem 3: Longest Common Subsequence**

Step 4: Construct / print the optimal solution.

Example 1: What do ABCB and BDCAB have in common?

print reverse(B C B) = B C B

### **Problem 3: Longest Common Subsequence**

Step 4: Construct / print the optimal solution.

Example 1: What do ABCB and BDCAB have in common?

The initial call is Print - LCS(b, X, X. length, Y. length)

Print 
$$-LCS(b, X, i, j)$$
  
 $if(i == 0)and (j == 0)$   
 $return$   
 $if b[i,j] == " \supset "$   
 $Print - LCS(b, X, i - 1, j - 1)$   
 $Print x_i$   
 $if b[i,j] == " \downarrow "$   
 $Print - LCS(b, X, i - 1, j)$   
 $elsePrint - LCS(b, X, i, j - 1)$ 

This algorithm required  $\Theta(m+n)$  time for execution

#### **Problem 3: Longest Common Subsequence**

Example 2: What do ABCBDAB and BDCABA have in common?

X = ABCBDAB

Y = BDCABA

What is the Longest Common Subsequence LCS(X,Y)?

Example 3: What do AGGTA and GXTYAY have in common?

X = A G G T A

Y = G X T Y A Y

What is the Longest Common Subsequence LCS(X,Y)?

Self practice

