

Module 2. Quantum Mechanics

EMW have both wave nature and particle nature.

↓
Interference
diffraction
Polarisation

↓
Photo electric effect
Compton effect
black body radiation
spectrum

De-Broglie Hypothesis :

for a particle of mass 'm' moving with velocity 'v' are associated with a waves and these waves are called "de-broglie matter waves"

$$\lambda = \frac{h}{p}$$

Characteristics of matter wave :-

- 1) $\lambda = \frac{h}{p} = \frac{h}{mv}$
- 2) de broglie waves are not Electro magnetic in nature.
- 3) The amplitude of de broglie matter waves gives the probability of finding the particle at a given point & at a given time.
- 4) particle velocity is equal to velocity of de broglie waves.
i.e., $v_{\text{particle}} = v_{\text{de-broglie}}$

W.K.T,

$$E = KE = \frac{1}{2} mv^2 = \frac{m^2 v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \Rightarrow \frac{p^2}{2m} = E$$

$$p^2 = 2mE$$

root on b.s

$$\sqrt{p^2} = \sqrt{2mE}$$

$$\boxed{p = \sqrt{2mE}}$$

$$1) \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$2) \Delta l \Delta \theta \geq \frac{\hbar}{2}$$

angular momentum

It is impossible to make measurements

* Applications of uncertainty principle :-

Prove that 1) electron is present inside the atom

2) proton is present inside the nucleus 10^{-14} meter

Using uncertainty principle

$$1) E = \frac{p^2}{2m}$$

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

$$\text{or } \Delta x \cdot \Delta p \cong \hbar$$

$$\Delta p \cong \frac{\hbar}{\Delta x} \rightarrow \Delta p = \frac{\hbar/2\pi}{\Delta x}$$

$$\therefore \Delta p = \frac{1.04 \times 10^{-24}}{10^{-10} \text{ m}}$$

$$\Delta p = \frac{6.626 \times 10^{-34}}{2 \times 3.14 \Delta x}$$

$$\Delta p = \frac{1.04 \times 10^{-24} \text{ m}}{10^{-10} \text{ m}}$$

$$\Delta p = \frac{1.04 \times 10^{-24} \text{ Js}}{\Delta x}$$

$$E = \frac{(1.04 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

dimension
of atom
is 10^{-10}

$$E = \frac{1.0816 \times 10^{-48}}{18.2 \times 10^{-31}} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

$$E = \frac{1.0816 \times 10^{-48}}{29.12 \times 10^{-50}}$$

$$E = 0.0375 \times 10^{-2}$$

$$E = 3.75 \text{ eV}$$

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \times 3600}{150 \times 10^{-2} \times 150 \times 10^{-3}}$$

$$\lambda = \frac{23853.6 \times 10^{-34}}{22500}$$

$$\lambda_{\text{ball}} = 1.06016 \times 10^{-34} \text{ m}$$

Nature of de-Broglie matter waves :-

Wave velocity = $V_{\text{phase}} \neq V_{\text{particle}}$

$$V_{\text{group}} = V_{\text{particle}}$$

$$V_{\text{phase}} = \frac{\omega}{k} \rightarrow \text{omega}$$

$$= \frac{2\pi f}{\frac{2\pi}{\lambda}} = f\lambda = \frac{c}{\lambda} \cdot \lambda = c$$

$$\therefore V_{\text{group}} = \frac{d\omega}{dk}$$

* wave group represents de-Broglie matter waves.

$$V_g = \frac{d\omega}{dk} = V_{\text{Particle}}$$



Group velocity

[indicates the probability of finding the position of a particle.]

It is impossible to make possible to a measurement for any 2 Conjugate.

Physical parameters like position and momentum

$$\Delta x \cdot \Delta p \geq \frac{h}{2}$$

The product of the uncertainty involved in the measurement should be greater than or equal to $\frac{h}{2}$

$$p = \sqrt{2mE}$$

$$E = eV$$

$$p = \sqrt{2meV}$$

$$\therefore \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

Calculate or express de broglie wavelength for electrons accelerated to a potential 'V'

$$\rightarrow \text{Mass of } e^- = 9.1 \times 10^{-31} \text{ kg}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$h = 6.626 \times 10^{-34}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{\sqrt{29.12 \times 10^{-50}}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{5.3 \times 10^{-25}} \times \frac{1}{\sqrt{V}}$$

$$\lambda = \frac{1.226}{\sqrt{V}} \text{ nm}^{***}$$

Calculate de broglie wavelength of a cricket ball 150 g moving with a velocity of 15 km/h

$$\rightarrow \text{Mass of ball} = 150 \text{ gram} = 150 \times 10^{-3} \text{ kg}$$

$$\text{Velocity} = 15 \text{ km/h}$$

$$= \frac{150 \times 10^3}{3600} \text{ m/s}$$

$$\lambda = \frac{6.626 \times 10^{-34} \times 3600}{\sqrt{2 \times 150 \times 10^{-3} \times 150 \times 10^3}}$$