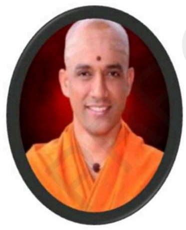

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Department of Mathematics
LECTURE NOTES
MATHEMATICS-3 FOR COMPUTER SCIENCE STREAM (BCS301)
MODULE - 1
RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS

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Random Experiment:

An activity that yield some results called the random experiment. The random variable means a real number, i.e. X associated with the outcomes of a random experiment.

Definition: Let S be a sample space associated with a random experiment with a real value function defined and taking its values is called a Random variable.

The random variables are two types. They are,

i) Discrete Random Variables (DRV)
ii) Continuous Random Variables (CRV)

Discrete Random Variables: A Discrete random variable is a variable which can only take a countable number of values.

For example, if a coin is tossed three times, the number of heads can be obtained is 0, 1, 2 or 3. The probabilities of each of these probabilities can be tabulated as shown.

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

X	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Continuous Random variables: A Continuous random variable is a random variable where the data can take infinitely many values. For example, a random variable measuring the time taken for something to be done is continuous since there are an infinite number of possible times that can be taken.

Ex: Temperature of the climate, Age of a person, etc.

Probability Mass Function:

Probability mass function is the probability distribution of a discrete random variable and provides the possible values and their associated probabilities.

1. $P(x_i) \geq 0$
2. $\sum_{i=1}^{n} P(X = x_i) = 1$
3. $0 \leq P(x) \leq 1$
4. $Mean\ \mu = \sum_{i=1}^{n} x_i P(x_i)$
5. $Variance\ \sigma^2 = \sum_{i=1}^{n} x_i^2 P(x_i) - \mu^2$

Probability Density Function:

Probability density function is the probability distribution of a continuous random variable and provides the possible values and their associated probabilities infinitely.

1. $P(x_i) \geq 0 \; or \; f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. Mean $\mu = \int_{-\infty}^{\infty} xf(x)dx$
4. Variance $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$
5. $P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b) = P(a < x < b) = \int_a^b f(x)dx$

PROBLEMS

1) Show that the following probabilities one satisfying the properties of discrete random variables, hence find it's mean and variance.

x	10	20	30	40
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Sol^n: Let X be the random variable for the random values,

$x_1 = 10$, $x_2 = 20$, $x_3 = 30$, $x_4 = 40$

and given

$$P(X = x_1) = P(x_1) = p_1 = \frac{1}{8}$$

$$P(X = x_2) = P(x_2) = p_2 = \frac{3}{8}$$

$$P(X = x_3) = P(x_3) = p_3 = \frac{3}{8}$$

$$P(X = x_4) = P(x_4) = p_4 = \frac{1}{8}$$

Let $\sum_{i=1}^{4} P(X = x_i) = P(X = x_1) + P(X = x_2) + P(X = x_3) + P(X = x_4)$

$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

$$= \frac{8}{8}$$

$$= 1$$

Hence the given probabilities can satisfy the DRV property.

Mean $\mu = \sum_{i=1}^{4} x_i P(x_i)$

$$= 10 \times \frac{1}{8} + 20 \times \frac{3}{8} + 30 \times \frac{3}{8} + 40 \times \frac{1}{8}$$

$$= \frac{200}{8}$$

$$= 25$$

Variance $\sigma^2 = \sum_{i=1}^{4} x_i^2 P(x_i) - \mu^2$

$$= 10^2 \times \frac{1}{8} + 20^2 \times \frac{3}{8} + 30^2 \times \frac{3}{8} + 40^2 \times \frac{1}{8} - 25^2$$

$$= 700 - 625$$

$$= 75$$

S.D=$\sqrt{Variance} = \sqrt{75} = 8.66$

2) Find the value of k, such that the following distribution represents discrete probability distribution. Hence find Mean, S.D, $P(x \le 1)$, $P(x > 1)$ and $P(-1 < x \le 2)$.

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

Sol^n: Let X be the random variable for the random values,
$x_1 = -3$, $x_2 = -2$, $x_3 = -1$, $x_4 = 0$, $x_5 = 1$, $x_6 = 2$, $x_7 = 3$
and the given probabilities are,

$P(X = x_1) = P(-3) = k$
$P(X = x_2) = P(-2) = 2k$
$P(X = x_3) = P(-1) = 3k$
$P(X = x_4) = P(0) = 4k$
$P(X = x_5) = P(1) = 3k$
$P(X = x_6) = P(2) = 2k$
$P(X = x_7) = P(3) = k$

We know that,

$\sum_{i=1}^{7} P(X = x_i) = 1$

$$\Rightarrow k + 2k + 3k + 4k + 3k + 2k + k = 1$$
$$\Rightarrow 16k = 1$$
$$\Rightarrow k = \frac{1}{16}$$

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
-3	K	-3k	9	9k
-2	2k	-4k	4	8k
-1	3k	-3k	1	3k
0	4k	0	0	0
1	3k	3k	1	3k
2	2k	4k	4	8k
3	K	3k	9	9k
$\sum$		0	-	40k

Mean $\mu = \sum_{i=1}^{4} x_i P(x_i) = 0$

Variance $\sigma^2 = \sum_{i=1}^{4} x_i^2 P(x_i) - \mu^2$

$= 40k - 0^2$

$= 40 \times \frac{1}{16}$

$= 2.5$

S.D $= \sqrt{2.5} = 1.5811$

$i)$ $P(x \le 1) = P(-3) + P(-2) + P(-1) + P(0) + P(1)$
$\Rightarrow P(x \le 1) = k + 2k + 3k + 4k + 3k$
$\Rightarrow P(x \le 1) = 13k = \frac{13}{16} = 0.8125$

$ii)$ $P(x > 1) = P(2) + P(3) = 2k + k = 3k = \frac{3}{16} = 0.1875$

$iii)$ $P(-1 < x \le 2) = P(0) + P(1) + P(2) = 4k + 3k + 2k = 9k = \frac{9}{16} = 0.5625$

3) Find the value of k, such that the following distribution represents discrete probability distribution. Hence find Mean, S.D, $P(x \geq 5)$ and $P(3 < x \leq 6)$.

x	0	1	2	3	4	5	6
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Sol^n: Let X be the random variable for the random values,

$x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4, x_6 = 5, x_7 = 6$

and the given probabilities are,

$P(X = x_1) = P(0) = k$

$P(X = x_2) = P(1) = 3k$

$P(X = x_3) = P(2) = 5k$

$P(X = x_4) = P(3) = 7k$

$P(X = x_5) = P(4) = 9k$

$P(X = x_6) = P(5) = 11k$

$P(X = x_7) = P(6) = 13k$

We know that,

$\sum_{i=1}^{7} P(X = x_i) = 1$

$$\Rightarrow k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$
$$\Rightarrow 49k = 1$$
$$\Rightarrow k = \frac{1}{49}$$

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	k	0	0	0
1	3k	3k	1	3k
2	5k	10k	4	20k
3	7k	21k	9	63k
4	9k	36k	16	144k
5	11k	55k	25	275k
6	13k	78k	36	468k
$\sum$		203k	-	973k

Mean $\mu = \sum_{i=1}^{4} x_i P(x_i) = 203k = \frac{203}{49} = 4.1428$

Variance $\sigma^2 = \sum_{i=1}^{4} x_i^2 P(x_i) - \mu^2$

$= 973k - 4.1428^2$

$= \frac{973}{49} - 17.1628$

$= 2.6943$

S.D $= \sqrt{2.6943} = 1.6414$

$$i)\ P(x \geq 5) = P(5) + P(6)$$
$$\Rightarrow P(x \geq 5) = 11k + 13k$$
$$\Rightarrow P(x \geq 5) = 24k = \frac{24}{49} = 0.4898$$

$$ii)\ P(3 < x \leq 6) = P(4) + P(5) + P(6) = 9k + 11k + 13k = 33k = \frac{33}{49} = 0.6734$$

4) A random variable X has a probability function for various values of x. Find i) k, ii) $P(x < 6)$, iii) $P(x \geq 6)$ and $P(3 < x \leq 6)$. Also find the probability distribution and distribution function of x.

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Sol^n: Let X be the random variable for the random values,

$x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$, $x_5 = 4$, $x_6 = 5$, $x_7 = 6$, $x_8 = 7$

and the given probabilities are,

$P(X = x_1) = P(0) = 0$

$P(X = x_2) = P(1) = k$

$P(X = x_3) = P(2) = 2k$

$P(X = x_4) = P(3) = 2k$

$P(X = x_5) = P(4) = 3k$

$P(X = x_6) = P(5) = k^2$

$P(X = x_7) = P(6) = 2k^2$

$P(X = x_7) = P(7) = 7k^2 + k$

We know that,

$\sum_{i=1}^{7} P(X = x_i) = 1$

$$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$\Rightarrow 10k^2 + 9k = 1$$

$$\Rightarrow 10k^2 + 9k - 1 = 0$$

$$\Rightarrow (10k - 1)(k + 1) = 0$$

$$\Rightarrow 10k - 1 = 0\,, k + 1 = 0$$

$$\Rightarrow k = \frac{1}{10}\,, k \neq -1$$

x	0	1	2	3	4	5	6	7
$P(x)$	O	0.1	0.2	0.2	0.3	0.01	0.02	0.17
CP	0	0.1	0.3	0.5	0.8	0.81	0.83	1

$$i)\ P(x < 6) = 1 - P(x \geq 6) = 1 - \{P(6) + P(7)\} = 1 - \{0.02 + 0.17\} = 0.81$$

$$ii)\ P(x \geq 6) = P(6) + P(7) = 0.02 + 0.17 = 0.19$$

$$iii)\ P(3 < x \leq 6) = P(4) + P(5) + P(6) = 0.3 + 0.01 + 0.02 = 0.33$$

5) A random variable has the following probability function for the various values of X=x. Find i) Value of k, ii) $P(x < 1)$, iii)$P(x \geq 1)$.

x	-2	-1	0	1	2	3
$P(x)$	0.1	k	0.2	$2k$	0.3	k

Sol^n: Let X be the random variable for the random values,
$x_1 = -2$, $x_2 = -1$, $x_3 = 0$, $x_4 = 1$, $x_5 = 2$, $x_6 = 3$,
and the given probabilities are,
$P(X = x_1) = P(-2) = 0.1$
$P(X = x_2) = P(-1) = k$
$P(X = x_3) = P(0) = 0.2$
$P(X = x_4) = P(1) = 2k$
$P(X = x_5) = P(2) = 0.3$
$P(X = x_6) = P(3) = k$

i) We know that,
$\sum_{i=1}^{6} P(X = x_i) = 1$

$$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + k = 1$$

$$\Rightarrow 4k + 0.6 = 1$$

$$\Rightarrow 4k = 0.4$$

$$\Rightarrow k = 0.1$$

$$ii)\ P(x < 1) = P(-2) + P(-1) + P(0) = 0.1 + k + 0.2 = k + 0.3 = 0.1 + 0.3 = 0.4$$

$$iii)\ P(x \geq -1) = P(-1) + P(0) + P(1) + P(2) + P(3) = k + 0.2 + 2k + 0.3 + k = 4k + 0.5 = 0.9$$

6) A random variable has the following probability function for the various values of X=x. Find i)Value of k, ii)$P(x \leq 1)$, iii) $P(0 \leq x < 3)$.

x	0	1	2	3	4	5
$P(x)$	k	$5k$	$10k$	$10k$	$5k$	k

Sol^n: Let X be the random variable for the random values,
$x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $x_4 = 3$, $x_5 = 4$, $x_6 = 5$,
and the given probabilities are,
$P(X = x_1) = P(0) = k$
$P(X = x_2) = P(1) = 5k$
$P(X = x_3) = P(2) = 10k$
$P(X = x_4) = P(3) = 10k$
$P(X = x_5) = P(4) = 5k$
$P(X = x_6) = P(5) = k$

i) We know that,
$\sum_{i=1}^{6} P(X = x_i) = 1$

$$\Rightarrow k + +5k + 10k + 10k + 5k + k = 1$$

$$\Rightarrow 32k = 1$$

$$\Rightarrow k = \frac{1}{32}$$

$$ii)\ P(x \leq 1) = P(0) + P(1) = k + 5k = 6k = \frac{6}{32} = 0.1875$$

$$iii)\ P(0 \leq x < 3) = P(0) + P(1) + P(2) = k + 5k + 10k = 16k = \frac{16}{32} = 0.5$$

7) Show that the function $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$ is probability density function. Hence find $P(1.5 < x < 2.5)$.

Sol^n: Given probability function,

$$f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Let $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx$

$$= \int_{-\infty}^{0} 0\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

$$= 0 + \left[\frac{e^{-x}}{-1}\right]_0^\infty$$

$$= -[e^{-\infty} - e^0]$$

$$= -[0 - 1]$$

$$= 1$$

Hence the given probability function is p.d.f.

$$P(1.5 < x < 2.5) = \int_{1.5}^{2.5} f(x)dx$$

$$= \int_{1.5}^{2.5} e^{-x}dx$$

$$= -[e^{-x}]_{1.5}^{2.5}$$

$$= -[e^{-2.5} - e^{-1.5}] = \left[\frac{1}{e^{1.5}} - \frac{1}{e^{2.5}}\right]$$

8) A random variable X has probability density function $f(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0 & , otherwise \end{cases}$, Evaluate i) k ii) $P(x \leq 1)$, iii) $P(x > 1)$, iv) $P(1 \leq x \leq 2)$, v) $P(x \leq 2)$, vi) $P(x \geq 2)$.

Sol^n: Given probability function,

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 3 \\ 0 & , otherwise \end{cases}$$

$$i)\ \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx = 1$$

$$\Rightarrow 0 + \int_{0}^{3} kx^2dx + 0 = 1$$

$$\Rightarrow k\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$\Rightarrow 9k = 1$$

$$\Rightarrow k = \frac{1}{9}$$

$$ii)\ P(x \leq 1) = \int_{-\infty}^{1} f(x)dx$$

$$\Rightarrow P(x \leq 1) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx$$

$$\Rightarrow P(x \le 1) = 0 + \int_0^1 kx^2 dx$$

$$\Rightarrow P(x \le 1) = k\left[\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow P(x \le 1) = \frac{k}{3} = \frac{1}{27}$$

$$iii)\ P(x > 1) = \int_1^\infty f(x)dx$$

$$\Rightarrow P(x > 1) = \int_1^3 f(x)dx + \int_3^\infty f(x)dx$$

$$\Rightarrow P(x > 1) = 0 + \int_1^3 kx^2 dx$$

$$\Rightarrow P(x < 1) = k\left[\frac{x^3}{3}\right]_1^3$$

$$\Rightarrow P(x < 1) = \frac{26k}{3} = \frac{26}{27}$$

$$iv)\ P(1 \le x \le 2) = \int_1^2 f(x)dx$$

$$\Rightarrow P(1 \le x \le 2) = \int_1^2 kx^2 dx$$

$$\Rightarrow P(1 \le x \le 2) = k\left[\frac{x^3}{3}\right]_1^2$$

$$\Rightarrow P(1 \le x \le 2) = \frac{7k}{3} = \frac{7}{27}$$

$$v)\ P(x \le 2) = \int_{-\infty}^2 f(x)dx$$

$$\Rightarrow P(x \le 2) = \int_{-\infty}^0 f(x)dx + \int_0^2 f(x)dx$$

$$\Rightarrow P(x \le 2) = 0 + \int_0^1 kx^2 dx$$

$$\Rightarrow P(x \le 2) = k\left[\frac{x^3}{3}\right]_0^2$$

$$\Rightarrow P(x \le 2) = \frac{8k}{3} = \frac{8}{27}$$

$$vi)\ P(x \ge 2) = \int_2^\infty f(x)dx$$

$$\Rightarrow P(x \ge 2) = \int_2^3 f(x)dx + \int_3^\infty f(x)dx$$

$$\Rightarrow Px \ge 2) = 0 + \int_2^3 kx^2 dx$$

$$\Rightarrow P(x \ge 2) = k\left[\frac{x^3}{3}\right]_2^3$$

$$\Rightarrow P(x \ge 2) = \frac{19k}{3} = \frac{19}{27}$$

9) A random variable X has the pdf, $f(x)=\begin{cases} kx^2 & ,-3\le x\le 3 \\ 0 & ,\ otherwise \end{cases}$, find i) k, ii) $P(x\le 2)$, iii) $P(x\ge 2)$, iv) $P(x>1)$, v) $P(1\le x\le 2)$.

Sol^n^: Given probability function,

$$f(x)=\begin{cases} kx^2, -3\le x\le 3 \\ 0 \quad\quad\ , otherwise \end{cases}$$

$$i)\ \int_{-\infty}^{\infty} f(x)dx=1$$

$$\Rightarrow \int_{-\infty}^{\infty} f(x)dx=\int_{-\infty}^{-3} f(x)dx+\int_{-3}^{3} f(x)dx+\int_{3}^{\infty} f(x)dx=1$$

$$\Rightarrow 0+\int_{-3}^{3} kx^2dx+0=1$$

$$\Rightarrow k\left[\frac{x^3}{3}\right]_{-3}^{3}=1$$

$$\Rightarrow 18k=1$$

$$\Rightarrow k=\frac{1}{18}$$

$$ii)\ P(x\le 2)=\int_{-\infty}^{2} f(x)dx$$

$$\Rightarrow P(x\le 2)=\int_{-\infty}^{-3} f(x)dx+\int_{-3}^{2} f(x)dx$$

$$\Rightarrow P(x\le 2)=0+\int_{-3}^{2} kx^2dx$$

$$\Rightarrow P(x\le 2)=k\left[\frac{x^3}{3}\right]_{-3}^{3}$$

$$\Rightarrow P(x\le 2)=\frac{35k}{3}=\frac{35}{3}\times\frac{1}{18}=\frac{35}{54}$$

$$iii)\ P(x\ge 2)=\int_{2}^{\infty} f(x)dx$$

$$\Rightarrow P(x\ge 2)=\int_{2}^{3} f(x)dx+\int_{3}^{\infty} f(x)dx$$

$$\Rightarrow P(x\ge 2)=\int_{2}^{3} kx^2dx$$

$$\Rightarrow P(x\ge 2)=k\left[\frac{x^3}{3}\right]_{2}^{3}$$

$$\Rightarrow P(x\ge 2)=\frac{19k}{3}=\frac{19}{3}\times\frac{1}{18}=\frac{19}{54}$$

$$iv)\ P(x>1)=\int_{1}^{\infty} f(x)dx$$

$$\Rightarrow P(x>1)=\int_{1}^{3} f(x)dx+\int_{3}^{\infty} f(x)dx$$

$$\Rightarrow P(x>1)=\int_{1}^{3} kx^2dx$$

$$\Rightarrow P(x>1)=k\left[\frac{x^3}{3}\right]_{1}^{3}$$

$$\Rightarrow P(x>1)=\frac{26k}{3}=\frac{26}{3}\times\frac{1}{18}=\frac{26}{54}$$

$$v)\ P(1 \le x \le 2) = \int_1^2 f(x)dx$$

$$\Rightarrow P(1 \le x \le 2) = \int_1^2 kx^2 dx$$

$$\Rightarrow P(1 \le x \le 2) = k\left[\frac{x^3}{3}\right]_1^2$$

$$\Rightarrow P(1 \le x \le 2) = \frac{7k}{3} = \frac{7}{3} \times \frac{1}{18} = \frac{7}{54}$$

10) The diameter of an electric cable is assumed to be a CRV with pdf $f(x) = \begin{cases} kx(1-x) & ,0 \le x \le 1 \\ 0 & otherwise \end{cases}$, find i) value of k, ii) Mean & Variance.

Sol^n: Given probability function,

$$WKT, \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow i) \int_{-\infty}^{0} f(x)dx + \int_0^1 f(x)dx + \int_1^{\infty} f(x)dx = 1$$

$$\Rightarrow 0 + \int_0^1 kx(1-x)dx + 0 = 1$$

$$\Rightarrow k\int_0^1 (x - x^2)dx = 1$$

$$\Rightarrow k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$$

$$\Rightarrow \frac{k}{6} = 1$$

$$\Rightarrow k = 6$$

$$ii)\ Mean\ \mu = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_0^1 xkx(1-x)dx$$

$$= k\int_0^1 x^2(1-x)dx$$

$$= k\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= k\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{k}{12} = \frac{6}{12} = \frac{1}{2}$$

$$iii)\ Variance\ \sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

$$= \int_0^1 kx^3(1-x)dx - \mu^2$$

$$= k\int_0^1 (x^3 - x^4)dx - \left[\frac{1}{2}\right]^2$$

$$= k\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 - \frac{1}{4}$$

$$= \frac{k}{20} - \frac{1}{4} = \frac{6}{20} \times \frac{1}{4} = \frac{1}{20}$$

11) Find the constant k such that $f(x) = \begin{cases} kxe^{-x} & ,0 < x < 1 \\ 0 & otherwise \end{cases}$, is pdf. Find the mean.

Sol^n: Given probability function,

$$f(x) = \begin{cases} kxe^{-x} & ,0 < x < 1 \\ 0 & otherwise \end{cases}$$

also given f(x) represents pdf for the CRV 'X'.

$$WKT, \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$$

$$\Rightarrow \int_{0}^{1} kxe^{-x}dx = 1$$

$$\Rightarrow k\left\{\int_{0}^{1} e^{-x}dx - \int_{0}^{1}\left(1 \times \int e^{-x}dx\right)dx\right\} = 1$$

$$\Rightarrow k(-e^{-1} - e^{-1} + 1 = 1$$

$$\Rightarrow k\left(1 - \frac{2}{e}\right) = 1$$

$$\Rightarrow k = \frac{e}{e-2}$$

$$Mean\ \mu = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_{0}^{1} xkxe^{-x}dx$$

$$= k\int_{0}^{1} x^2e^{-x}dx$$

$$= k\left\{x^2\int_{0}^{1} e^{-x}dx - \int_{0}^{1}\left(2x \times \int e^{-x}dx\right)dx\right\}$$

$$= k\left\{-(x^2e^{-x})_0^1 + 2\int_{0}^{1} xe^{-x}dx\right\}$$

$$= k(2 - 5e^{-1})$$

$$= k\left(2 - \frac{5}{e}\right)$$

$$= \frac{e}{e-2} \times \frac{2e-5}{e}$$

$$\mu = \frac{2e-5}{e-2}$$

Binomial Distribution:

Let X be a discrete random variable, 'p' be the probability of success and let 'q' be the probability of failure, then the probability mass function of the binomial distribution can be defined as,

$$P(X=x)=b(n,p,x)=\begin{cases} n_{c_x} p^x q^{n-x} & ,x\geq 0 \\ 0 & ,Otherwise \end{cases}$$

where, n is the number of trials and n & p are the parameters which follows,

i) $P(X=x)=b(n,p,x)\geq 0$

ii) p + q = 1

iii) $\sum_{x=0}^{n} n_{c_x} p^x q^{n-x}=1$

iv) The mean of B.D $\mu=np$, Vairance $\sigma^2=npq$ and S.D is $\sigma=\sqrt{npq}$.

MEAN & VARIANCE OF A BINOMIAL DISTRIBUTION:

WKT, the probability mass function of the binomial distribution is,

$$P(X=x)=f(x)=\begin{cases} n_{c_x} p^x q^{n-x}, & x\geq 0 \\ 0 \quad , & otherwise \end{cases}$$

i) Mean:

$$\mu=E(x)=\sum_{x=0}^{n} xP(X=x)$$

$$=\sum_{x=0}^{n} x\, n_{c_x} p^x q^{n-x}$$

$$=\sum_{x=0}^{n} x\frac{n!}{x!\,(n-x)!}p^{x-1}\,p^1 q^{n-x}$$

$$=\sum_{x=0}^{n} x\frac{n\,(n-1)!}{x\,(x-1)!\,(n-x)!}p^{x-1}\,p^1 q^{n-x}$$

$$=np\sum_{x=1}^{n}\frac{(n-1)!}{(x-1)!\,(n-x)!}p^{x-1}\,q^{n-x}$$

$$=np\sum_{x=1}^{n}\frac{(n-1)!}{(x-1)!\,((n-1)-(x-1))!}p^{x-1}\,q^{(n-1)-(x-1)}$$

$$=np\sum_{x=1}^{n}(n-1)_{c_{(x-1)}}p^{x-1}q^{(n-1)-(x-1)}$$

$$=np(1)$$

$$\mu=E(x)=np$$

ii) Variance:

$$\sigma^2=E(x^2)-[E(x)]^2\ ----(1)$$

$$\Rightarrow E(x^2)=E(x(x-1)+x)$$

$$\Rightarrow E(x^2)=E\big(x(x-1)\big)+E(x)----(2)$$

$$\therefore E(x(x-1))=\sum_{x=0}^{n} x(x-1)p(x)$$

$$=\sum_{x=0}^{n} x(x-1)\,n_{c_x}p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1)\frac{n!}{x!\,(n-x)!}p^{x}\,q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1)\frac{n\,(n-1)(n-2)!}{x\,(x-1)(x-2)!\,(n-x)!}p^{x-2}\,p^{2}q^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,(n-x)!}p^{x-2}\,p^{2}q^{n-x}$$

$$= n(n-1)p^{2}\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!}p^{x-2}\,q^{n-x}$$

$$= n(n-1)p^{2}\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,((n-2)-(x-2))!}p^{x-2}\,q^{(n-2)-(x-2)}$$

$$= n(n-1)p^{2}\sum_{x=2}^{n} (n-2)_{c_{(x-2)}}p^{x-2}q^{(n-2)-(x-2)}$$

$$= n(n-1)p^{2}(1)$$

$$\therefore E(x(x-1)) = n(n-1)p^{2}$$

$$(2) \Rightarrow E(x^{2}) = E\big(x(x-1)\big) + E(x)$$

$$\Rightarrow E(x^{2}) = n(n-1)p^{2} + np$$

$$(1) \Rightarrow \sigma^{2} = E(x^{2}) - [E(x)]^{2}$$

$$\Rightarrow \sigma^{2} = n(n-1)p^{2} + np - [np]^{2}$$

$$\Rightarrow \sigma^{2} = n^{2}p^{2} - np^{2} + np - n^{2}p^{2}$$

$$\Rightarrow \sigma^{2} = np - np^{2}$$

$$\Rightarrow \sigma^{2} = np(1-p)$$

$$but\ \ 1-p = q$$

$$\therefore \sigma^{2} = npq$$

PROBLEMS

1) Let X be a binomially distributed random variable based on 6 repetitions of an experiment. If p=0.3, evaluate the following probabilities i) $P(x \le 3)$, ii) $P(X > 4)$.

Sol^n: Given p=0.3 and n=6, hence q = 1-p = 1-0.3 = 0.7

and $P(X = x) = b(6, 0.3, x) = 6_{C_x}(0.3)^{x}(0.7)^{6-x}$

i) $P(x \le 3) = P(0) + P(1) + P(2) + P(3)$

$$= 6_{C_0}(0.3)^{0}(0.7)^{6} + 6_{C_1}(0.3)^{1}(0.7)^{5} + 6_{C_2}(0.3)^{2}(0.7)^{4} + 6_{C_3}(0.3)^{3}(0.7)^{3}$$

$$= 0.1176 + 0.3025 + 0.3241 + 0.1852$$

$$= 0.9294$$

ii) $P(x \le 3) = P(5) + P(6)$

$$= 6_{C_5}(0.3)^{5}(0.7)^{1} + 6_{C_6}(0.3)^{6}(0.7)^{0}$$

$$= 0.0102 + 0.0007$$

$$= 0.0109$$

2) The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that
i)Exactly 2 pens will be defective
ii)Atmost 2 pens will be defective
iii)None will be defective

Sol^n: Let the probability that a pen manufactured is defective, p=0.1
then, q = 1-p = 1-0.1 = 0.9 and given n=12
Hence $P(X=x)=b(6,0.3,x)=12_{C_x}(0.1)^x(0.9)^{12-x}$

i) The probability that exactly 2 pens will be defective, $P(2)=12_{C_2}(0.1)^2(0.9)^{12-2}$
$=(66)(0.01)(0.3487)$
$=0.2301$

ii) The probability that atmost 2 pens will be defective, $P(x\leq 2)=P(0)+P(1)+P(2)$
$=12_{C_0}(0.1)^0(0.9)^{12}+12_{C_1}(0.1)^1(0.9)^{11}+12_{C_2}(0.1)^2(0.9)^{10}$
$=0.2824+0.3766+0.2301$
$=0.8891$

iii) The probability that none will be defective, $P(0)=12_{C_0}(0.1)^0(0.9)^{12}$
$=(1)(1)(0.2824)$
$=0.2824$

3) The number of telephonic lines busy at an instant is a binomial variant with a probability 0.1. If 10 lines are chosen at random, what is the probability that,
i)No line is busy
ii)All lines are busy
iii)Atleast one line is busy
iv)Atmost two lines are busy

Sol^n: Let the probability that a telephonic line is busy p=0.1
then q = 1-p = 1-0.1 = 0.9 and number of lines chosen is n = 10
Hence $P(X=x)=b(10,0.1,x)=10_{C_x}(0.1)^x(0.9)^{10-x}$

i) The probability that no line is busy, $P(0)=10_{C_0}(0.1)^0(0.9)^{10}$
$=(1)(1)(0.3487)$
$=0.3487$

ii) The probability that all lines are busy, $P(10)=10_{C_{10}}(0.1)^{10}(0.9)^0$
$=(1)(10^{-10})(1)$
$=10^{-1}$

iii) The probability that atleast one line is busy, $P(x\geq 1)=1-P(0)$
$=1-10_{C_0}(0.1)^0(0.9)^{10}$
$=1-0.3487$
$=0.6513$

iv) The probability that atmost two lines are busy, $P(x\leq 2)=P(0)+P(1)+P(2)$
$=10_{C_0}(0.1)^0(0.9)^{10}+10_{C_1}(0.1)^1(0.9)^9+10_{C_2}(0.1)^2(0.9)^8$
$=0.3487+0.3874+0.1937$
$=0.92968$

4) When a coin is tossed 4 times, find the probability of getting i)Exactly one head, ii)Atmost three heads, iii)Atleast two heads.

Sol^n: The number o times a coin is tossed, n=4

Let x be the binomial variant getting head, p=0.5

then q = 1-p = 1-0.5 = 0.5

Hence $P(X=x) = b(4,0.5,x) = 4_{C_x}(0.5)^x(0.5)^{4-x}$

$= 4_{C_x}(0.5)^4 = 4_{C_x}(0.0625)$

i) The probability of getting exactly one head, $P(1) = 4_{C_1}(0.0625) = 4 \times 0.0625 = 0.25$

ii) The probability of getting atmost three heads, $P(x \le 3) = 1 - P(4)$

$= 1 - 4_{C_4}(0.0625)$

$= 1 - 0.0625$

$= 0.9375$

iii) The probability of getting atleast two heads, , $P(x \ge 2) = P(2) + P(3) + P(4)$

$= 4_{C_2}(0.0625) + 4_{C_3}(0.0625) + 4_{C_4}(0.0625)$

$= 0.375 + 0.25 + 0.0625$

$= 0.6875$

5) The probability of germination of a seed in a packet of seeds is found to be 0.7. If 10 seeds are taken for experimenting on germination in a laboratory, find the probability that
i)8 seeds germinate
ii)Atleast 8 seeds germinate
iii)Atmost 8 seeds germinate

Sol^n: Let X be the binomial variant of seed germination.

Given the number of seeds taken for experimenting in laboratory, n=10

The probability of germination of a seed in a packet of seeds is, p=0.7

then q = 1-p = 1-0.7 = 0.3

Hence, $P(X=x) = 10_{C_x}(0.7)^x(0.3)^{10-x}$

i) The probability that exactly 8 seeds germinate, $P(8) = 10_{C_8}(0.7)^8(0.3)^2$

$= (45)(0.0576)(0.09)$

$= 0.2334$

ii) The probability that atleast 8 seeds germinate, $P(x \ge 8) = P(8) + P(9) + P(10)$

$= 10_{C_8}(0.7)^8(0.3)^2 + 10_{C_9}(0.7)^9(0.3)^1 + 10_{C_{10}}(0.7)^{10}(0.3)^0$

$= 0.2334 + 0.1210 + 0.0282$

$= 0.3826$

iii) The probability that atmost 8 seeds germinate, $P(x \le 8) = 1 - \{P(9) + P(10)\}$

$= 1 - \{10_{C_9}(0.7)^9(0.3)^1 + 10_{C_{10}}(0.7)^{10}(0.3)^0\}$

$= 1 - \{0.1210 + 0.0282\}$

$= 0.8508$

6) A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probability of,
i)No error during a micro second
ii)1 error
iii)Atleast 1 error
iv)2 error
v)Atmost 2 error

Sol^n: Let X be the binomial variant of Transmission error.

Given the number of pulses per micro second, n=12

Let p be the probability of transmission error, p=0.001

then q = 1-p = 1-0.001 = 0.999

The pmf of binomial distribution is, $P(X=x)=P(x)=n_{C_x}p^x q^{n-x}$
$= 12_{C_x}(0.001)^x(0.999)^{12-x}$

i) The probability of no error during a micro second, $P(0)=12_{C_0}(0.001)^0(0.999)^{12}$
$=(1)(1)(0.9880)$
$=0.9880$

ii) The probability of only one error during a micro second, $P(1)=12_{C_1}(0.001)^1(0.999)^{11}$
$=(12)(0.001)(0.9890)$
$=0.01186$

iii) The probability of atleast one error during a micro second, $P(x\geq 1)=1-P(0)$
$=1-12_{C_0}(0.001)^0(0.999)^{12}$
$=1-0.9880$
$=0.0120$

iv) The probability of two error during a micro second, $P(2)=12_{C_2}(0.001)^2(0.999)^{10}$
$=(66)(0.000001)(0.9900)$
$=0.00006534$

v) The probability of atmost two error during a micro second, $P(x\leq 2)=P(0)+P(1)+P(2)$
$=12_{C_0}(0.001)^0(0.999)^{12}+12_{C_1}(0.001)^1(0.999)^{11}+12_{C_2}(0.001)^2(0.999)^{10}$
$=0.9880+0.01186+0.00006534$
$=0.999925$

7) In 800 families with 5 children each, how many family would be expected to have,
i)3 boys
ii)5 girls
iii)Atmost 2 girls
iv)Either 2 or 3 boys
by assuming probability for boys and girls to be equal.

Sol^n: The total number of families given is 800 and number of children per family is, n=5
Given the probability of boy or girl to born, p=0.5
then q = 1-p = 1-0.5 = 0.5

The pmf of binomial distribution is, $P(X=x)=P(x)=n_{C_x}p^x q^{n-x}=5_{C_x}(0.5)^x(0.5)^{5-x}$
$=5_{C_x}(0.5)^5=5_{C_x}(0.03125)$

i) The probability to have exactly 3 boys, $P(3)=5_{C_3}(0.03125)$
$=(10)(0.03125)$
$=0.3125$
$\therefore$The total number of families may have exactly 3 boys $=800\times 0.3125=250.$

ii) The probability to have exactly 5 girls, $P(5)=5_{C_5}(0.03125)$
$=(1)(0.03125)$
$=0.03125$
$\therefore$The total number of families may have exactly 5 girls, $=800\times 0.03125=25.$

iii) The probability to have atmost two girls, $P(x\leq 2)=P(0)+P(1)+P(2)$
$=5_{C_0}(0.03125)+5_{C_1}(0.03125)+5_{C_2}(0.03125)$
$=0.03125+0.15625+0.3125$
$=0.5$
$\therefore$The total number of families may have atmost two girls, $=800\times 0.5=400.$

iv) The probability to have either 2 or 3 boys, $P(2 \le x \le 3) = P(2) + P(3)$

$$= 5_{C_2}(0.03125) + 5_{C_3}(0.03125)$$
$$= 0.03125 + 0.03125$$
$$= 0.0625$$

$\therefore$ The total number of families may have either 2 or 3 boys, $= 800 \times 0.0625 = 500$.

Poisson Distribution:

Let X be the discrete random variable for any real value λ , such that the probability mass function of poisson distribution can be defined as,

$$P(X = x) = P(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & , x \ge 0 \\ 0, & otherwise \end{cases}$$

where, λ is called the parameter and,

i) $P(X = x) = P(x) \ge 0$

ii) $\sum_{x=0}^{n} P(x) = \sum_{x-0}^{n} \frac{e^{-\lambda}\lambda^x}{x!} = 1$

iii) Mean $\mu = np = \lambda$

iv) Vairance $\sigma^2 = \lambda$, S.D = $\sqrt{\lambda}$

The poisson distribution can be used to find the probabaility that an event might happen a definite number of times based on how often it usually occurs and the companies can utilize the poisson distribution to examine how they may be able to take steps to improve their operational effieciency.

MEAN & VARIANCE OF A POISSON DISTRIBUTION:

WKT, the probability mass function of the poisson distribution is,

$$P(X = x) = P(x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^x}{x!} & , \quad x \ge 0 \\ 0 & , \quad otherwise \end{cases}$$

i) Mean:

$$\mu = E(x) = \sum_{x=0}^{\infty} xP(x)$$
$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$$
$$= \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x-1}\lambda}{x(x-1)!}$$
$$= \lambda \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}$$
$$= \lambda(1)$$
$$\mu = \lambda$$

ii) Variance:

$$\sigma^2 = E(x^2) - \mu^2 \quad ----(1)$$

$$= E(x(x-1)+x) - \mu^2$$

$$= E\big(x(x-1)\big) + E(x) - \mu^2 \quad ----(2)$$

$$\therefore E(x(x-1)) = \sum_{x=0}^{\infty} x(x-1)P(x)$$

$$= \sum_{x=0}^{\infty} x(x-1)\frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1)\ \frac{e^{-\lambda}\lambda^{x-2}\lambda^2}{x(x-1)(x-2)!}$$

$$= \lambda^2 \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^{x-2}}{(x-2)!}$$

$$= \lambda^2(1)$$

$$E(x(x-1)) = \lambda^2(1)$$

$$(2) \Rightarrow \sigma^2 = \lambda^2 + \lambda - \lambda^2$$

$$\Rightarrow \sigma^2 = \lambda$$

$$S.D = \sigma = \sqrt{\lambda}$$

$$Mean = \lambda = np$$

PROBLEMS

1) The number of accidents in a year to taxi drivers in a city follows a poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with,
i)No accident in a year
ii)More than 3 accidents in a year.

Sol^n: Let X be the poisson variant follows accident in the year of the poisson distribution.

The probability mass function of the poisson distribution is $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given the mean of poisson distribution is $\mu = \lambda = 3$

$P(X=x) = \frac{e^{-3}3^x}{x!}$

i) No accident in a year out of 1000 taxi drivers $= 1000 \times P(0)$

$$= 1000 \times \frac{e^{-3}3^0}{0!}$$
$$= 1000 \times 0.05$$
$$= 50$$

Hence 50 drivers out of 1000 having no accidents in a year.

ii) More than 3 accidents in a year out of 1000 taxi drivers $= 1000 \times P(x > 3)$

$$= 1000 \times [1 - P(x \le 3)]$$
$$= 1000 \times [1 - P(0) - P(1) - P(2]) - P(3)]$$
$$= 1000 \times \left[1 - \frac{e^{-3}3^0}{0!} - \frac{e^{-3}3^1}{1!} - \frac{e^{-3}3^2}{2!} - \frac{e^{-3}3^3}{3!}\right]$$
$$= 1000 \times \left(1 - \left(e^{-3} + e^{-3}(3) + e^{-3}\left(\frac{9}{2}\right) + e^{-3}\left(\frac{27}{6}\right)\right)\right)$$

$$= 1000 \times (1 - 0.06472) = 352.8$$
$$= 353$$

Therefore 353 drivers out of 1000 have done more than 3 accidents in the year.

2) In a certain factory turning out razor blades there is a small probability of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a packets of 10. Use poisson distribution to calculate approximate number of packets containing,
i)No defective
ii)2 defective
iii)3 defective
in the consignment of 10000 packets.

Sol^n: Let X be the poisson variant follows the blades to be defective of the poisson distribution.

The probability mass function of the poisson distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given, p=$\frac{1}{500} = 0.002$, n=10, $\mu = np = 0.002 \times 10 = 0.02 = \lambda$

$$\therefore P(X = x) = \frac{e^{-0.02}(0.02)^x}{x!}$$

i) No blades are defective out 10000 packets $= 10000 \times P(x = 0)$

$$= 10000 \times \frac{e^{-0.02}(0.02)^0}{0!}$$
$$= 10000 \times 0.9802$$
$$= 9802$$

$\therefore$ 9802 packet blades are not defective out of 10000 packets.

ii) 2 defective blades out of 10000 packets $= 10000 \times P(x = 2)$

$$= 10000 \times \frac{e^{-0.02}(0.02)^2}{2!}$$
$$= 10000 \times 0.0002$$
$$= 2$$

$\therefore$ 2 packets blades are 2 defective out of 10000 packets.

iii) 3 defective blades out of 10000 packets $= 10000 \times P(x = 3)$

$$= 10000 \times \frac{e^{-0.02}(0.02)^3}{3!}$$
$$= 10000 \times 0.0000$$
$$= 0$$

$\therefore$ No packets blades are 3 defective out of 10000 packets.

3) If the probability of bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals more than 2 will get a bad reaction.

Sol^n: Let X be the poisson variant follows the bad reaction of the injection.

WKT, The probability mass function of the poisson distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given, n=2000, p=0.001 and $\mu = np = 2000 \times 0.001 = 2 = \lambda$

$\therefore P(X = x) = \frac{e^{-2}(2)^x}{x!}$

The probability that of more than two individuals get bad reaction $= P(x > 2)$

$$= 1 - P(x \le 2)$$
$$= 1 - [P(0) + P(1) + P(2)]$$
$$= 1 - \frac{e^{-2}(2)^0}{0!} - \frac{e^{-2}(2)^1}{1!} - \frac{e^{-2}(2)^2}{2!}$$
$$= 1 - \frac{5}{e^2} = 0.3233$$

4) The probability that a news reader commits no mistakes in reading the news is $\frac{1}{e^3}$. Find a probability on a particular news broadcast he commits,
i)Only 2 mistakes

ii)More than 3 mistakes
iii)Atmost 3 mistakes

Sol^n: Let X be the poisson variant follows the news reader do mistakes of the poisson distribution.

The probability mass function of the poisson distribution is $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$

Given, $P(X=0)=\frac{1}{e^3}$

$$\frac{e^{-\lambda}\lambda^0}{0!}=\frac{1}{e^3}\Rightarrow\frac{1}{e^\lambda}=\frac{1}{e^3}\Rightarrow\lambda=3$$

$$\therefore P(X=x)=\frac{e^{-3}(3)^x}{x!}$$

i) The probability that news reader can do 2 mistakes $=P(2)$

$$=\frac{e^{-3}(3)^2}{2!}$$
$$=0.2240$$

ii) The probability that the news reader can do more than 3 mistakes $P(x>3)=1-P(x\le 3)$

$$\Rightarrow P(x>3)=1-[P(0)+P(1)+P(2)+P(3)]$$
$$\Rightarrow P(x>3)=1-e^{-3}\left[\frac{3^0}{0!}+\frac{3^1}{1!}+\frac{3^2}{2!}+\frac{3^3}{3!}\right]$$
$$\Rightarrow P(x>3)=1-0.05(1+3+4.5+4.5)$$
$$\Rightarrow P(x>3)=1-0.65$$
$$\Rightarrow P(x>3)=0.3500$$

iii) The probability that the news reader can do atmost 3 mistakes $=P(x\le 3)$

$$\Rightarrow P(x\le 3)=P(x=0)+P(x=1)+P(x=2)+P(x=3)$$
$$\Rightarrow P(x\le 3)=e^{-3}\left[\frac{3^0}{0!}+\frac{3^1}{1!}+\frac{3^2}{2!}+\frac{3^3}{3!}\right]$$
$$\Rightarrow P(x\le 3)=(0.05)(1+3+4.5+4.5)$$
$$\Rightarrow P(x\le 3)=0.6500$$

5) Suppose 300 misprints are randomly distributed throughout a book of 500 pages, find the probability that a given page contains,
i)Exactly 3 misprints
ii)Less than 3 misprints
iii)4 or more misprints

Sol^n: Let X be the poisson variant of misprints throughout a book of 500 pages.

The probability mass function of the poisson distribution is $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$

Given, suppose 300 misprints are randomly distributed throughout a book of 500 pages.

$\therefore$Mean $\lambda=\frac{300}{500}=0.6$

WKT, the pmf of poisson distribution is $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$

i) The probability that exactly 3 misprints $=P(3)$

$$=\frac{e^{-0.6}(0.6)^3}{3!}$$
$$=0.01975$$

ii) The probability that there are less than three misprints $P(x<3)=P(0)+P(1)+P(2)$

$$\Rightarrow P(x<3)=\frac{e^{-0.6}(0.6)^0}{0!}+\frac{e^{-0.6}(0.6)^1}{1!}+\frac{e^{-0.6}(0.6)^2}{2!}$$
$$\Rightarrow P(x<3)=e^{-0.6}[1+0.6+0.18]$$
$$\Rightarrow P(x<3)=0.5488\times 1.78$$

$$\Rightarrow P(x < 3) = 0.9768$$

iii) The probability that there are 4 or more misprints, $P(x \geq 4) = 1 - P(x < 4)$

$$\Rightarrow P(x \geq 4) = 1 - P(0) - P(1) - P(2) - P(3)$$

$$\Rightarrow P(x \geq 4) = 1 - \frac{e^{-0.6}(0.6)^0}{0!} - \frac{e^{-0.6}(0.6)^1}{1!} - \frac{e^{-0.6}(0.6)^2}{2!} - \frac{e^{-0.6}(0.6)^3}{3!}$$

$$\Rightarrow P(x \geq 4) = 1 - e^{-0.6}(1 + 0.6 + 0.18 + 0.036)$$

$$= 1 - 0.5488 \times 1.816 = 0.00338$$

6) A certain screw making machine produces an average 2 defective out of 100 and packs of them in boxes of 500. Find the probability that the box contains,
i)3 defective
ii)Atleast 1 defective
iii)Between 2 & 4 defective

Sol^n: Given the machine producing an average defective screw is $p = \frac{2}{100} = 0.02$
also given, n=500, $\mu = np = 500 \times 0.02 = 10 = \lambda$

The probability mass function of the poisson distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$\therefore P(X = x) = \frac{e^{-10}(10)^x}{x!}$$

i) The probability that exactly 3 defective $= P(3)$

$$= \frac{e^{-10}(10)^3}{3!}$$
$$= 0.007566$$

ii) The probability that atleast 1 screw is defective $= 1 - P(0)$

$$= 1 - \frac{e^{-10}(10)^0}{0!}$$
$$= 1 - 0.0000454$$
$$= 0.9999546$$

iii) The probability that between 2 & 4 screw will be defective $= P(2 \leq x \leq 4)$

$$= P(2) + P(3) + P(4)$$

$$= \frac{e^{-10}(10)^2}{2!} + \frac{e^{-10}(10)^3}{3!} + \frac{e^{-1}\ (10)^4}{4!}$$

$$= e^{-10}\left(\frac{100}{2} + \frac{1000}{6} + \frac{10000}{24}\right)$$

$$= e^{-10} \times 633.32$$

$$= 0.02875$$

Exponential Distribution:

Let X be a continuous random variable for any real value $\alpha > 0$, then the probability density function of an exponential distribution can be defined as, $P(X = x) = f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0 & , otherwise \end{cases}$, it follows:

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

iii) $Mean\ \mu = \frac{1}{\alpha}$

iv) $Variance\ \sigma^2 = \frac{1}{\alpha^2}$

v) $S.D\ of\ Exponential\ Distribution, \sigma = \frac{1}{\alpha}$

PROBLEMS

1) If X is an Exponential variant with mean 3, then find $P(x > 1)$ & $P(x < 3)$.

Sol^n: Given X be a continuous random variable of an Exponential distribution is,

$$P(X = x) = f(x) = \begin{cases} \alpha e^{-\alpha x} & , for x \geq 0 \\ 0 & , Otherwise \end{cases}$$

and given the mean of exponential distribution is 3.

$\Rightarrow \mu = 3 \Rightarrow \frac{1}{\alpha} = 3 \Rightarrow \alpha = \frac{1}{3}$

$$\therefore f(x) = \begin{cases} \frac{1}{3} e^{-\frac{x}{3}} & for x \geq 0 \\ 0 & Otherwise \end{cases}$$

i) $P(x > 1) = \int_1^{\infty} f(x)dx = \frac{1}{3}\int_1^{\infty} e^{\frac{-x}{3}}\,dx = -\left[e^{\frac{-x}{3}}\right]_1^{\infty} = -\left[e^{-\infty} - e^{\frac{-1}{3}}\right] = -\left[0 - e^{\frac{-1}{3}}\right] = e^{\frac{-1}{3}}$

ii) $P(x < 3) = \int_{-\infty}^{3} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_0^3 f(x)dx = 0 + \frac{1}{3}\int_0^3 e^{\frac{-x}{3}}\,dx = -\left[e^{\frac{-x}{3}}\right]_0^3 = -[e^{-1} - e^0] = 1 - \frac{1}{e}$

2) If X is an exponential variant with mean 4, then find $P(0 < x < 1), P(x > 2)$ & $P(-\infty < x < 10)$.

Sol^n: Given X be a continuous random variable of an exponential distribution is,

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , for x \geq 0 \\ 0 & , Otherwise \end{cases}$$

and given the mean of exponential distribution is 4.

$\Rightarrow \mu = 4 \Rightarrow \frac{1}{\alpha} = 4 \Rightarrow \alpha = \frac{1}{4}$

$$\therefore f(x) = \begin{cases} \frac{1}{4} e^{-\frac{x}{4}} & , x \geq 0 \\ 0 & , otherwise \end{cases}$$

$$P(0 < x < 1) = \int_0^1 fr(x)dx = \frac{1}{4}\int_0^1 e^{\frac{-x}{4}}\,dx = -\left[e^{\frac{-x}{4}}\right]_0^1 = -\left[e^{\frac{-1}{4}} - e^0\right] = 1 - \frac{1}{e^{\frac{1}{4}}}$$

$$P(x > 2) = \int_2^{\infty} fr(x)dx = \frac{1}{4}\int_2^{\infty} e^{\frac{-x}{4}}\,dx = -\left[e^{\frac{-x}{4}}\right]_2^{\infty} = -\left[e^{-\infty} - e^{\frac{-2}{4}}\right] = e^{\frac{-1}{2}}$$

$$P(-\infty < x < 10) = \int_{-\infty}^{10} fr(x)dx = \int_{-\infty}^{0} fr(x)dx + \int_0^{10} fr(x)dx = 0 + \frac{1}{4}\int_0^{10} e^{\frac{-x}{4}}\,dx$$

$$\Rightarrow P(-\infty < x < 10) = -\left[e^{\frac{-x}{4}}\right]_0^{10} = -\left[e^{\frac{-10}{4}} - e^0\right] = 1 - \frac{1}{e^{\frac{5}{2}}}$$

3) In a certain town the duration of shower has mean 5 minutes, what is the probability that shower will last for,
i)10 minutes and more
ii)Less than 10 minutes

iii)Between 10 & 12 minutes.

Sol^n: Given X be a continuous random variable of an exponential distribution is,

$$f(x)=\begin{cases}\alpha e^{-\alpha} & ,x\geq 0\\ 0 & ,otherwise\end{cases}$$

and given the mean of exponential distribution is 5.

$\Rightarrow \mu=5\Rightarrow \frac{1}{\alpha}=5\Rightarrow \alpha=\frac{1}{5}$

$$\therefore f(x)=\begin{cases}\frac{1}{5}e^{-\frac{x}{5}} & ,x\geq 0\\ 0 & ,otherwise\end{cases}$$

i) The probability that the shower will last 10 minutes and more is,

$$P(x\geq 10)=\int_{10}^{\infty}f(x)dx=\int_{10}^{\infty}\frac{1}{5}e^{-\frac{x}{5}}dx=\frac{1}{5}\int_{10}^{\infty}e^{-\frac{x}{5}}dx=-\left[e^{-\frac{x}{5}}\right]_{10}^{\infty}=-[0-e^{-2}]=\frac{1}{e^2}$$

ii) The probability that the shower will last less than 10 minutes is,

$$P(x<10)=\int_{-\infty}^{10}f(x)dx=\int_{-\infty}^{0}f(x)dx+\int_{0}^{10}f(x)dx$$

$$\Rightarrow P(x<10)=0+\int_{0}^{10}\frac{1}{5}e^{-\frac{x}{5}}dx=\frac{1}{5}\int_{0}^{10}e^{-\frac{x}{5}}dx=-\left[e^{-\frac{x}{5}}\right]_{0}^{10}=-[e^{-2}-1]=1-\frac{1}{e^2}$$

iii) The probability that the shower will last between 10 & 12 minutes is,

$$P(10<x<12)=\int_{10}^{12}f(x)dx=\int_{10}^{12}\frac{1}{5}e^{-\frac{x}{5}}dx=\frac{1}{5}\int_{10}^{12}e^{-\frac{x}{5}}dx$$

$$\Rightarrow P(10<x<12)=-\left[e^{-\frac{x}{5}}\right]_{10}^{12}=-\left[e^{\frac{-12}{5}}-e^{-2}\right]=\frac{1}{e^{\frac{12}{5}}}-\frac{1}{e^2}$$

4) The life of a TV tube manufactured by a company is known to have mean 200 months. Assuming that the life of tube has an exponential distribution, find the probability that the life of a tube manufactured by a company is,
i)Less than 200 months
ii)Between 100 & 300 months
iii)More than 200 months

Sol^n: Given X be a continuous random variable of an exponential distribution is,

$$f(x)=\begin{cases}\alpha e^{-\alpha x} & ,x\geq 0\\ 0 & ,otherwise\end{cases}$$

and given the mean of exponential distribution is 200 months.

$\Rightarrow \mu=200\Rightarrow \frac{1}{\alpha}=200\Rightarrow \alpha=\frac{1}{200}$

$$\therefore f(x)=\begin{cases}\frac{1}{200}e^{-\frac{x}{200}} & ,x\geq 0\\ 0 & ,otherwise\end{cases}$$

i) The probability that the life of a tube is less than 200 months is,

$$P(x<200)=\int_{-\infty}^{200}f(x)dx=\int_{0}^{200}\frac{1}{200}e^{\frac{-x}{200}}dx=\frac{1}{200}\int_{0}^{200}e^{\frac{-x}{200}}dx=-\left[e^{\frac{-x}{200}}\right]_{0}^{200}=-[e^{-1}-e^{0}]=1-\frac{1}{e}$$

ii) The probability that the life of a tube is between 100 & 300 months is,

$$P(100\leq x\leq 300)=\int_{100}^{300}f(x)dx=\int_{100}^{300}\frac{1}{200}e^{-\frac{x}{200}}dx=\frac{1}{200}\int_{100}^{300}e^{-\frac{x}{200}}dx$$

$$\Rightarrow P(100 \le x \le 300) = -\left[e^{-\frac{x}{200}}\right]_{100}^{300} = -\left[e^{\frac{-3}{2}} - e^{\frac{-1}{2}}\right] = \frac{1}{e^{\frac{1}{2}}} - \frac{1}{e^{\frac{3}{2}}}$$

iii) The probability that the life of a tube is more than 200 months is,

$$P(x > 200) = \int_{200}^{\infty} f(x)dx = \int_{200}^{\infty} \frac{1}{200} e^{\frac{-x}{200}} dx$$

$$\Rightarrow P(x > 200) = \frac{1}{200}\int_{200}^{\infty} e^{\frac{-x}{200}} dx = -\left[e^{\frac{-x}{200}}\right]_{200}^{\infty} = -\left[e^{\frac{-\infty}{200}} - e^{-1}\right] = e^{-1} = \frac{1}{e}$$

5) The length of a telephone conversation is an exponential variant with mean 3 minutes. Find the probability that a call,
i)ends in less than 3 minutes
ii)ends between 3 & 5 minutes
iii)ends in more than 4 minutes

Sol^n: Given X be a continuous random variable of an exponential distribution is,

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , x \ge 0 \\ 0 & , otherwise \end{cases}$$

and given the mean of exponential distribution is 3 minutes.

$$\Rightarrow \mu = 3 \Rightarrow \frac{1}{\alpha} = 3 \Rightarrow \alpha = \frac{1}{3}$$

$$\therefore f(x) = \begin{cases} \frac{1}{3} e^{-\frac{x}{3}} & , x \ge 0 \\ 0 & , otherwise \end{cases}$$

i) The probability that the conversation ends in less than 3 minutes is,

$$P(x < 3) = \int_{-\infty}^{3} f(x)dx = \int_{0}^{3} \frac{1}{3} e^{\frac{-x}{3}} dx = \frac{1}{3}\int_{0}^{3} e^{\frac{-x}{3}} dx = -\left[e^{\frac{-x}{3}}\right]_{0}^{3} = -[e^{-1} - e^{0}] = 1 - \frac{1}{e}$$

ii) The probability that the conversation ends in between 3 & 5 minutes is,

$$P(3 \le x \le 5) = \int_{3}^{5} f(x)dx = \int_{3}^{5} \frac{1}{3} e^{\frac{-x}{3}} dx = \frac{1}{3}\int_{3}^{5} e^{\frac{-x}{3}} dx$$

$$\Rightarrow P(100 \le x \le 300) = -\left[e^{-\frac{x}{3}}\right]_{3}^{5} = -\left[e^{\frac{-5}{3}} - e^{-1}\right] = \frac{1}{e} - \frac{1}{e^{\frac{5}{3}}}$$

iii) The probability that the conversation ends in more than 4 minutes is,

$$P(x > 4) = \int_{4}^{\infty} f(x)dx = \int_{4}^{\infty} \frac{1}{3} e^{\frac{-x}{3}} dx$$

$$\Rightarrow P(x > 4) = \frac{1}{3}\int_{4}^{\infty} e^{\frac{-x}{3}} dx = -\left[e^{\frac{-x}{3}}\right]_{4}^{\infty} = -\left[e^{\frac{-\infty}{3}} - e^{\frac{-4}{3}}\right] = e^{\frac{-4}{3}} = \frac{1}{e^{\frac{4}{3}}}$$

Normal Distribution:

Let X be a continuous random variable for any real μ and σ^2 , the normal distribution can be defined as,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} ,$$

where $-\infty \le x \le \infty$, $-\infty \le \mu \le \infty$ and here $\mu, \sigma^2 (>0)$ are called the mean and variance of the normal distribution i.e., widely used in statistical inference, hypothesis testing, data analysis, i.e., to analysis the data when there is an equal chance for the data to be above and below the average value of the continuous data. The normal is also known as Gaussian distribution (or) Probability Bell Curve. The normal distribution is a probability distribution i.e., symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.

The normal distribution follows as,

$\therefore P(a \le x \le b) = \int_a^b f(x)dx$

$$\Rightarrow P(a \le x \le b) = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

$$\Rightarrow P(a \le x \le b) = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\left(\frac{x-\mu}{\sigma}\right)^2}{2}} dx$$

$$Let\ Z = \frac{x-\mu}{\sigma}$$

Where $z = \frac{x-\mu}{\sigma}$, $z_1 = \frac{a-\mu}{\sigma}$, $z_2 = \frac{b-\mu}{\sigma}$

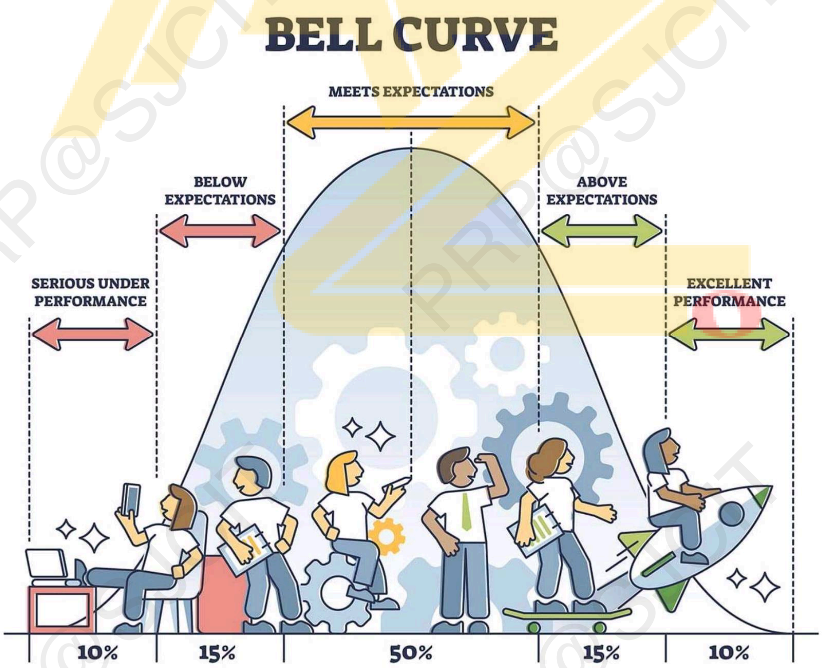
and $F(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ is called the standard normal function

and $z = \frac{x-\mu}{\sigma}$ is called the standard normal variate.

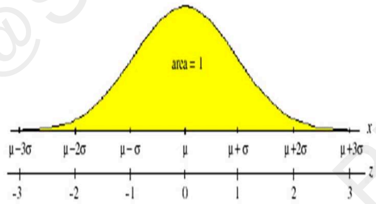
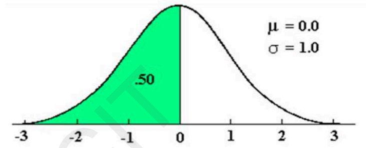
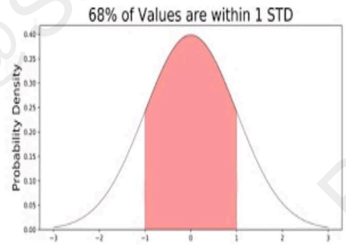
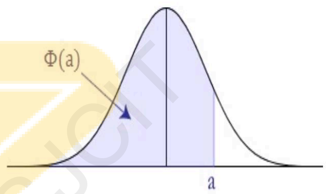
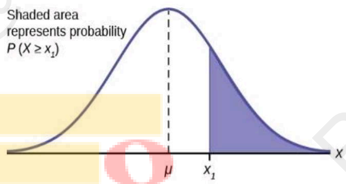
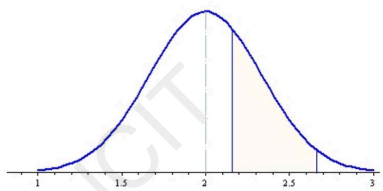
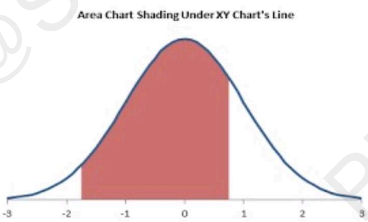
when $z_1 = 0$, $z_2 = z$, then the normal curve over 0 to z is defined as

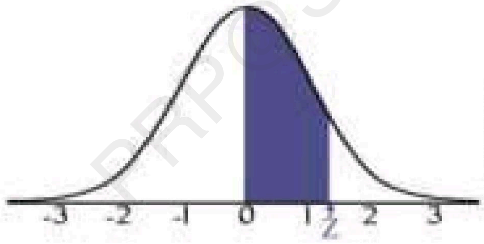
$$A(z) = \varphi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz$$

where these values will be taken from Area table of normal distribution.



Sl.No.	Probability Range	Result	Graph

1	$P(-\infty < z < \infty)$	1	
2	$P(-\infty < z < 0) = P(0 < z < \infty)$	0.5	
3	$P(-z_1 < z < z_1) = 2P(0 < z < z_1)$	$2A(z_1)$	
4	$P(-\infty < z < z_1) = 0.5 + P(0 < z < z_1)$	0.5+$A(z_1)$	
5	$P(z_1 < z < \infty) = P(0 < z < \infty) - P(0 < z < z_1)$	0.5-$A(z_1)$	
6	$P(z_1 < z < z_2) = P(0 < z < z_2) - P(0 < z < z_1)$	$A(z_2)$ - $A(z_1)$	
7	$P(-z_1 < z < z_2) = P(0 < z < z_2) + P(0 < z < z_1)$	$A(z_2)$ +$A(z_1)$	



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

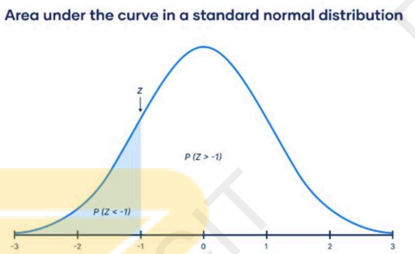
PROBLEMS

1) The marks of 1000 students in an examination follows normal distribution with mean 70 and standard deviation 5. Find the number students whose marks will be
i) Less than 65
i) More than 75
ii) Between 65 and 75. [A(1)=0.3413]

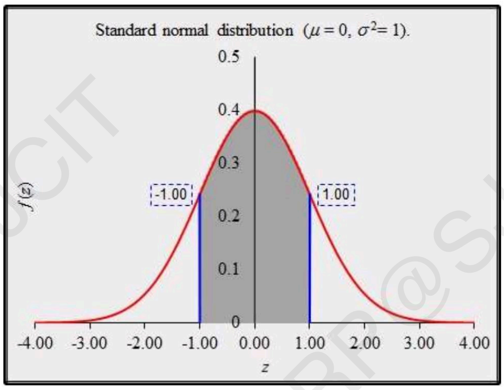
Sol.

Let X be the continuous random variable
Given
Mean of the Normal distribution $\mu = 70$
Standard deviation of the Normal distribution $\sigma = 5$
∴The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-70}{5}$

When x =65 then $z = \frac{65-70}{5} = -1$
When x =75 then $z = \frac{5-70}{5} = 1$
i) No.0f students scored less than 65 marks=$P(x < 65) = P(z < -1) =$
$P(z > 1) = 0.5 - A(1) = 0.5 - 0.3413 = 0.1587$
No.0f students scored less than 65 marks out of 1000 students=1000x0.1587=158.7=159



ii) No.0f students scored more than 75 marks=$P(x > 75) = P(z > 1) =$
$P(z > 1) = 0.5 - A(1) = 0.5 - 0.3413 = 0.1587$
No.0f students scored more than 75 marks out of 1000 students=1000x0.1587=158.7=159
iii) No.0f students scored marks between 65 and 75 =$P(65 < x < 75) = P(-1 < z < 1)$
$= 2P(0 < z < 1)$
$= 2A(1)$
$= 2 \times 0.3413$
$= 0.6826$
No.0f students scored between65 and 75 marks out of 1000 students=1000x0.6826
=682.6=683



2) 200 students appeared in an examination, distribution of marks is assumed to be normal with mean 30 and standard deviation 6.25, how many students are expected to get marks .
i) Between 20 and 40
ii) Less than 35 [A(1.6)=0.4452 , A(0.8)=0.2881]

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 30$

Standard deviation of the Normal distribution $\sigma = 6.25$

$\therefore$The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-30}{6.25}$

When x =20 then $z = \frac{20-}{6.25} = -1.6$

When x =40 then $z = \frac{40-}{6.25} = 1.6$

When x =35 then $z = \frac{35-30}{6.25} = 0.8$

The probability that number of students expected to score between 20 and 40 marks:

$$P(20 < x < 40) = P(-1.6 < z < 1.6)$$
$$\Rightarrow P(20 < x < 40) = 2 \times P(0 < z < 1.6)$$
$$\Rightarrow P(20 < x < 40) = 2 \times A(1.6)$$
$$\Rightarrow P(20 < x < 40) = 2 \times 0.4452 = 0.8904 = 0.9$$

The number of students expected to score between 20 and 40 marks out of 200:

The probability that number of students expected to score less than 35:

$$= P(x < 35) = P(z < 0.8)$$
$$= A(0.8) = 0.2881 = 0.3$$

The number of students expected to score less than 35 marks out of 200:

=200x0.3=60

3) The weekly wages of workers in a company are normally distributed with mean of Rs.700 and S.D. of Rs.50.Find the probability that the weekly wage of randomly chosen workers is i) Between Rs.650 and Rs.750 ii) More than Rs.750.

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 700$

Standard deviation of the Normal distribution $\sigma = 50$

$\therefore$The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-700}{50}$

When x =650 then $z = \frac{650-700}{50} = -1$

When x =750 then $z = \frac{750-700}{50} = 1$

The probability of the weekly wages between Rs.659 and Rs.750 is:

$$P(650 < x < 750) = P(-1 < z < 1)$$
$$\Rightarrow P(650 < x < 750) = 2 \times P(0 < z < 1)$$
$$\Rightarrow P(650 < x < 750) = 2 \times A(1)$$
$$\Rightarrow P(650 < x < 750) = 2 \times 0.3413 = 0.6826$$

The probability of the weekly wages of more than Rs.750 is:

$$= P(x > 750) = P(z > 1)$$
$$= 0.5 - P(z < 1)$$
$$= 0.5 - 0.3413$$
$$= 0.17065$$

4) In a test on 2000 electric bulbs , it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours . Estimate the number of bulbs likely to burn for...

i) More than 2150 hours

ii) Less than 1950 hours

iii) Between 1920 and 2160 hours.

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 2040$

Standard deviation of the Normal distribution $\sigma = 60$

$\therefore$The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-204}{60}$

When x =2150 then $z = \frac{2150-204}{60} = 1.83$

When x =1950 then $z = \frac{1950-2040}{60} = -1.5$

When x =1920 then $z = \frac{1920-2040}{60} = -2$

When x =2160 then $z = \frac{2160-2040}{60} = 2$

i) The probability that the number of bulbs likely to burn of more than 2150 hours:

$$P(x > 2150) = P(z > 1.83) =$$

$$P(z > 1.83) = 0.5 - A(1.83) = 0.5 - 0.4664 = 0.0336$$

The number of bulbs likely to burn of more than 2150 hours out of 2000 bulbs :

=2000x0.0336

=67.2=67

ii) The probability that the number of bulbs likely to burn of less than 1950 hours:

$$=P(x < 1950) = P(z < -1.5) =$$

$$P(z > 1.5) = 0.5 - A(1.5) = 0.5 - 0.4332 = 0.0668$$

The number of bulbs likely to burn of less than 1950 hours out of 2000 bulbs :

=2000x0.0668

=133.6=137

iii) The probability that the number of bulbs likely to burn between 1920 and 2160 hours

$$=P(1920 < x < 2160) = P(-2 < z < 2)$$

$$= 2P(0 < z < 2)$$

$$= 2A(2)$$

$$= 2 \times 0.4772$$

$$= 0.9544$$

The number of bulbs likely to burn between 1920 and 2160 hours out of 2000 bulbs :

=2000x0.9544

=1908.8=1909

5) If the life time of a certain types electric bulbs of a particular brand was distributed normally with an average life of 2000 hours and S.D.60 hours. If a firm purchases 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2100 hours

(ii) less than 1950 hours

(iii) between 1900 and 2100 hours.

Sol.

Let X be the continuous random variable

Given

Mean of the Normal distribution $\mu = 2000$

Standard deviation of the Normal distribution $\sigma = 60$

$\therefore$ The standard normal variate $z = \frac{x-\mu}{\sigma} \Rightarrow z = \frac{x-2000}{60}$

When x =1950 then $z = \frac{1950-2000}{60} = -0.83$

When x =1900 then $z = \dfrac{1900-2000}{60} = -1.66$

When x =2100 then $z = \dfrac{2100-2000}{60} = 1.66$

i) The probability that the number of bulbs likely to burn of more than 2100 hours:

$$P(x > 2100) = P(z > 1.66) = 0.5 - A(1.66) = 0.5 - 0.4515 = 0.0485$$

The number of bulbs likely to burn of more than 2100 hours out of 2500 bulbs:

=2500x0.0485

=121.25=121

ii) The probability that the number of bulbs likely to burn of less than 1950 hours:

$$=P(x < 1950) = P(z < -0.83) = 0.5 - A(0.83) = 0.5 - 0.2967 = 0.2033$$

The number of bulbs likely to burn of less than 1950 hours out of 2500 bulbs :

=2500x0.2033=508.25=508

iii) The probability that the number of bulbs likely to burn between 1900 and 2100 hours

$$= P(1900 < x < 2100) = P(-1.66 < z < 1.66)$$
$$= 2P(0 < z < 1.66)$$
$$= 2A(1.66)$$
$$= 2 \times 0.4515$$
$$= 0.9030$$

The number of bulbs likely to burn between 1900 and 2100 hours out of 2500 bulbs :

=2500x0.9030

=2257.5=2258

6) In a normal distribution , 7% of items are under 35 and 89% of the items are under 63.Find the mean and standard deviation of the distribution.

Sol.

Let X be the continuous random variable

Given

Let μ and σ be the Mean and Standard deviation of the distribution

$\therefore$The standard normal variate $z = \frac{x-\mu}{\sigma}$ -------------(1)

When x=35 the standard normal variate $z = \frac{35-\mu}{\sigma} = z_1(Say)$

When x=63 the standard normal variate $z = \frac{63-\mu}{\sigma} = z_2(Say)$

Given

$$P(x < 35) = P(z < z_1) = 0.07$$
$$\Rightarrow P(z < z_1) = P(-\infty < z < 0) - P(0 < z < z_1) = 0.07$$
$$\Rightarrow 0.5 - A(z_1) = 0.07$$
$$\Rightarrow A(z_1) = 0.5 - 0.07$$

$$\Rightarrow A(z_1) = A(-1.47)$$
$$\Rightarrow z_1 = -1.47$$
$$\Rightarrow \frac{35-\mu}{\sigma} = -1.47$$
$$\Rightarrow \mu - 1.47\sigma = 35 ------(2)$$

And $P(x < 63) = P(z < z_2) = 0.89$

$$\Rightarrow P(z < z_2) = P(-\infty < z < 0) + P(0 < z < z_2) = 0.89$$
$$\Rightarrow 0.5 + A(z_2) = 0.89$$

$$\Rightarrow A(z_2) = 0.89 - 0.5 = 0.39$$
$$\Rightarrow A(z_2) = A(1.23)$$
$$\Rightarrow z_2 = 1.23$$
$$\Rightarrow \frac{63 - \mu}{\sigma} = 1.23$$
$$\Rightarrow \mu + 1.23\sigma = 63 --------(3)$$

Solving eq(2) and (3)
we get

$$\mu = 50.2915$$
$$\sigma = 10.332$$

7) In a normal distribution , 31% of items are under 45 and 8% of the items are 0ver 64.Find the mean and standard deviation of the distribution.

Sol.

Let X be the continuous random variable
Given
Let μ andσ be the Mean and Standard deviation of the distribution
∴The standard normal variate $z = \frac{x-\mu}{\sigma}$ ------------(1)

When x=35 the standard normal variate $z = \frac{45-\mu}{\sigma} = z_1(Say)$
When x=63 the standard normal variate $z = \frac{64-\mu}{\sigma} = z_2(Say)$

Given

$$P(x < 35) = P(z < z_1) = 0.31$$
$$\Rightarrow P(z < z_1) = P(-\infty < z < 0) - P(0 < z < z_1) = 0.31$$
$$\Rightarrow 0.5 - A(z_1) = 0.31$$
$$\Rightarrow A(z_1) = 0.5 - 0.31 = 0.19$$

$$\Rightarrow A(z_1) = A(0.5)$$
$$\Rightarrow z_1 = 0.5$$
$$\Rightarrow \frac{45 - \mu}{\sigma} = 0.5$$
$$\Rightarrow \mu + 0.5\sigma = 45 ------(2)$$

And $P(x > 64) = P(z > z_2) = 0.08$

$$\Rightarrow P(z > z_2) = 0.5 - P(0 < z < z_2) = 0.08$$
$$\Rightarrow 0.5 - A(z_2) = 0.08$$
$$\Rightarrow A(z_2) = 0.08 - 0.5 = -0.42$$
$$\Rightarrow A(z_2) = A(1.4)$$
$$\Rightarrow z_2 = 1.4$$
$$\Rightarrow \frac{64 - \mu}{\sigma} = 1.4$$
$$\Rightarrow \mu + 1..4\sigma = 64 --------(3)$$

Solving eq(2) and (3)
we get

$$\mu = 50$$
$$\sigma = 10$$

8) In an examination 7% of the students scored less than 35% of the marks and 89% of the students scored less than 60% of the marks. Find the mean and standard deviation if marks are normally distributed.

Sol.

Let X be the continuous random variable
Given

Let μ and σ be the Mean and Standard deviation of the distribution

$\therefore$ The standard normal variate $z = \frac{x-\mu}{\sigma}$ -------------(1)

When x=35 the standard normal variate $z = \frac{35-\mu}{\sigma} = z_1(Say)$

When x=63 the standard normal variate $z = \frac{60-\mu}{\sigma} = z_2(Say)$

Given

$$P(x < 35) = P(z < z_1) = 0.07$$
$$\Rightarrow P(z < z_1) = P(-\infty < z < 0) - P(0 < z < z_1) = 0.07$$
$$\Rightarrow 0.5 - A(z_1) = 0.07$$
$$\Rightarrow A(z_1) = 0.5 - 0.07$$

$$\Rightarrow A(z_1) = A(-1.47)$$
$$\Rightarrow z_1 = -1.47$$
$$\Rightarrow \frac{35-\mu}{\sigma} = -1.47$$
$$\Rightarrow \mu - 1.47\sigma = 35 ------(2)$$

And $P(x < 60) = P(z < z_2) = 0.89$

$$\Rightarrow P(z < z_2) = P(-\infty < z < 0) + P(0 < z < z_2) = 0.89$$
$$\Rightarrow 0.5 + A(z_2) = 0.89$$
$$\Rightarrow A(z_2) = 0.89 - 0.5 = 0.39$$
$$\Rightarrow A(z_2) = A(1.23)$$
$$\Rightarrow z_2 = 1.23$$
$$\Rightarrow \frac{60-\mu}{\sigma} = 1.23$$
$$\Rightarrow \mu + 1.23\sigma = 60 --------(3)$$

Solving eq(2) and (3)

we get

$\mu = 48.65$

$\sigma = 9.25$

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