Table 1: Key Linear Algebra Concepts with Intuitive Explanations

Term	**Symbol/Notation**	**Intuitive Explanation**
Vector	$\vec{v}$	An element of a vector space; a list of numbers that represent direction and magnitude in space.
Matrix	A	A rectangular grid of numbers representing a linear transformation or operator acting on vectors.
Transpose	A^T	Flips a matrix over its diagonal, swapping rows and columns; geometrically, it corresponds to reflecting vectors in certain ways.
Conjugate Transpose (Hermitian adjoint)	$A^\dagger$	Transpose combined with complex conjugation; preserves inner product structure in complex vector spaces.
Determinant	$\det(A)$	Scalar value showing how much a matrix scales volume or area; zero means the matrix squashes space into a lower dimension (non-invertible).
Rank	$\text{rank}(A)$	The dimension of the space spanned by the matrix columns; how many directions the matrix can "reach."
Inverse	A^{-1}	A matrix which reverses the effect of A, so $AA^{-1} = I$; exists only if $\det(A) \neq 0$.
Eigenvalue	λ	Scalars such that multiplying a vector by the matrix just scales it by λ, without changing direction.
Eigenvector	$\vec{x}$	A nonzero vector that changes only in scale (by λ) under the transformation A.
Orthogonal Vectors	$\vec{u} \perp \vec{v}$	Two vectors at right angles, with zero dot product, meaning they are independent directions.
Orthonormal Basis		A set of vectors that are mutually orthogonal and each of unit length; provides a "nice" coordinate system.
Unitary Matrix	U	A complex square matrix that preserves lengths and angles, satisfies $U^\dagger U = I$; quantum operations are unitary.
Hermitian Matrix	H	A matrix equal to its own conjugate transpose, $H = H^\dagger$; corresponds to measurable observables in quantum mechanics.
Positive Definite Matrix	M	Matrix for which $\vec{x}^\dagger M \vec{x} > 0$ for all nonzero $\vec{x}$; represents "positive energy" or "distance" in many contexts.
Singular Value Decomposition (SVD)	$A = U\Sigma V^\dagger$	Factorization expressing any matrix as rotation (or unitary) U, scaling Σ, and another rotation $V^\dagger$.
Norm	$\\|\vec{v}\\|$	A measure of vector length or size; commonly Euclidean norm $\sqrt{\sum_i v_i^2}$.
Inner Product	$\langle \vec{u}, \vec{v} \rangle$	Generalizes dot product; measures "angle" and length relationships between vectors.
Projection	$\text{proj}_{\vec{u}}\vec{v}$	The component of vector $\vec{v}$ along vector $\vec{u}$; "shadow" cast on $\vec{u}$.
Linear Independence		Vectors are linearly independent if no vector can be written as a combination of others; crucial for basis formation.
Span		The set of all possible linear combinations of a set of vectors; the "space" they cover.
Kernel (Null Space)	$\text{Ker}(A)$	Set of vectors sent to zero by A; shows directions lost in transformation.
Image (Range)	$\text{Im}(A)$	Set of all outputs $A\vec{x}$; the space reachable by transformation A.
Diagonalization	$A = PDP^{-1}$	Expressing a matrix as a product of eigenvector matrix P, diagonal eigenvalue matrix D, and inverse P^{-1}.
Trace	$\text{tr}(A)$	Sum of diagonal elements; equals sum of eigenvalues; related to total "scaling" effect.
Condition Number	$\kappa(A)$	Ratio of largest to smallest singular value; measures sensitivity of matrix inversion to errors.