GCBBA Warehouse System — Plot Analysis Reference

Experiment run: `20260225_095021` (Medium mode — 432 runs, 3 seeds per config) **Plot directory:** `plots/20260225_095021/` **Model:** 8 induct stations, 38 eject stations, 6 agents, 4 communication ranges (cr = 5, 8, 13, 45) **Steady-state config:** `max_timesteps=1500`, `warmup_timesteps=300`, `queue_max_depth=10` **Arrival rates swept (SS):** 0.01, 0.03, 0.05, 0.1 tasks/ts/station **Batch task loads swept:** 5, 10, 20 tasks per induct station (40, 80, 160 total) **Protection layers:** C1 = 10s per-call allocation timeout; C2 = 800-ts cap for CBBA/SGA at ar≥0.1; C3 = 10-min per-run wall-clock cap

Methods: | Label | Description | |---|---| | Static GCBBA (one-shot) | GCBBA, run once per trigger, no replan interval | | Dynamic GCBBA (ri=50) | GCBBA, mandatory replan every 50 timesteps | | CBBA (standard) | Consensus-Based Bundle Algorithm, centralized communication | | SGA (centralized upper bound) | Sequential Greedy Assignment, true upper bound (centralized, full info) |

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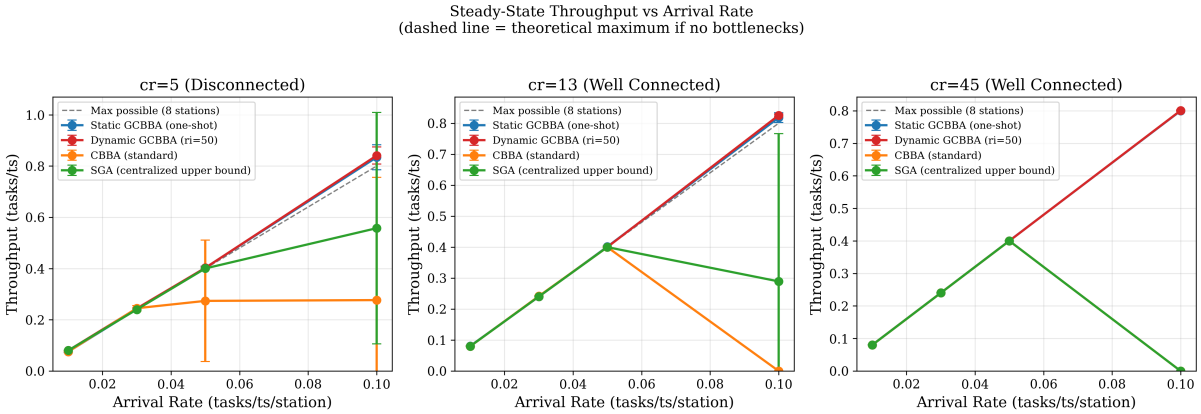
1. Steady-State: Core Performance

1.1 Throughput vs Arrival Rate

File: `plots/20260225_095021/throughput_vs_arrival_rate.png`



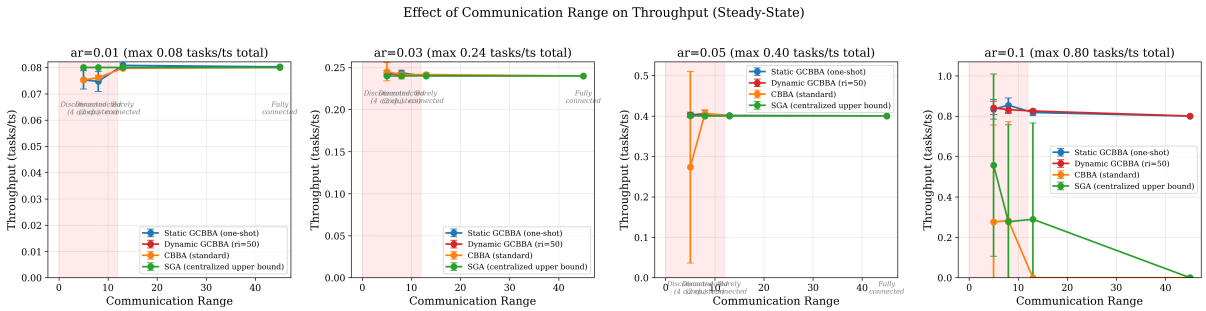
What it shows: Three-panel plot (cr=5 disconnected, cr=13 well-connected, cr=45 fully connected). X-axis is arrival rate (tasks/ts/station); Y-axis is throughput (tasks/ts). Dashed line is the theoretical maximum if no bottleneck existed.

Key observations: - **GCBBA tracks the theoretical maximum** at all three communication ranges. Both static and dynamic variants closely follow the dashed line from ar=0.01 through ar=0.10. - **CBBA collapses at ar=0.10**, dropping to ~0.14 tasks/ts at cr=45 (vs. theoretical 0.80). Large error bars at ar=0.10 reflect high run-to-run variance caused by C3 wall-clock ceiling hits mixing with runs that narrowly completed. At cr=5, CBBA degrades visibly even at ar=0.03. - **SGA drops to ~0.28 tasks/ts** at ar=0.10 at cr=45. SGA performs better than CBBA at high load because its greedy assignment is faster per call, but it still cannot keep pace with 8-station task injection at ar=0.10. - **cr=5 (disconnected):** GCBBA's advantage is widest here. CBBA and SGA require a connected graph for global consensus; they degrade at every load level. GCBBA is specifically designed for changing/sparse communication graphs and is unaffected. - **The throughput collapse is entirely caused by allocation latency** (see §3.1). CBBA and SGA spending 8–10+ seconds per allocation call means agents sit idle while tasks queue up, causing a positive-feedback failure: more tasks → slower allocation → more idleness → lower throughput.

Thesis message: GCBBA maintains near-capacity throughput under all tested loads and communication conditions. CBBA and SGA are computationally infeasible for real-time steady-state warehouse operation above moderate load.

1.2 Throughput vs Communication Range

File: `plots/20260225_095021/throughput_vs_comm_range_ss.png`



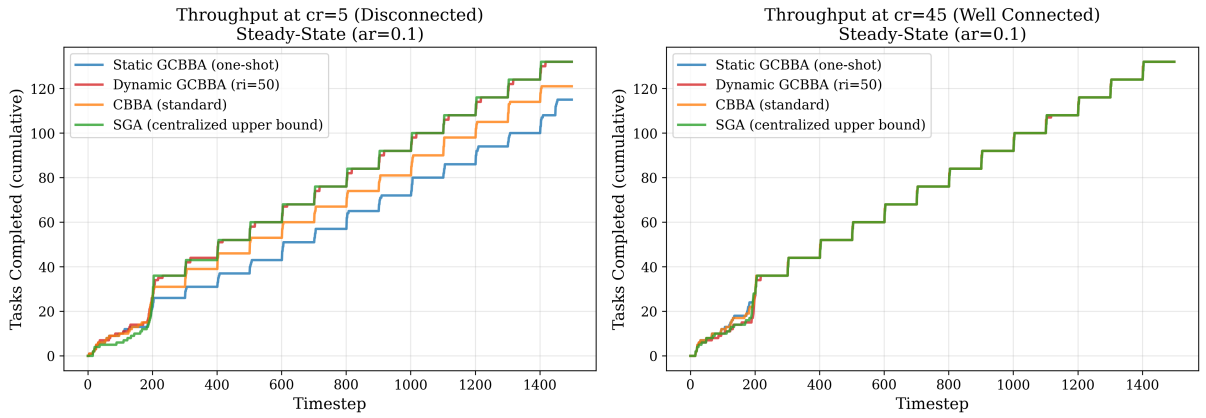
What it shows: Four panels, one per arrival rate (ar=0.01, 0.03, 0.05, 0.1). X-axis is communication range; shaded region marks the disconnected regime. Y-axis is throughput.

Key observations: - At **ar=0.01 and ar=0.03**, all methods achieve near-identical throughput across connected communication ranges (cr≥13). The difference between methods is negligible when load is well below capacity. - At **ar=0.05**, GCBBA (both) and SGA begin to separate from CBBA at cr=5. CBBA shows a large spread even in partially connected regimes. - At **ar=0.10**, the plot dramatically separates into two clusters: GCBBA (both variants) at ~0.80–0.85 regardless of cr, and CBBA/SGA collapsing to near zero at cr≤8 and to 0.14/0.28 even at cr=45. - **GCBBA shows effectively zero sensitivity to communication range** at high load — its throughput is flat across all cr values. This is the strongest evidence that it is *allocation-limited* only by its own compute, not by connectivity.

Thesis message: Communication range is irrelevant to GCBBA performance. For CBBA and SGA, even full connectivity cannot compensate for their computational bottleneck at high load.

1.3 Cumulative Throughput Curves

File: `plots/20260225_095021/throughput_curves_ss.png`



What it shows: Two panels at ar=0.1 — cr=5 (disconnected) and cr=45 (well-connected). X-axis is timestep (0–1500); Y-axis is cumulative tasks completed.

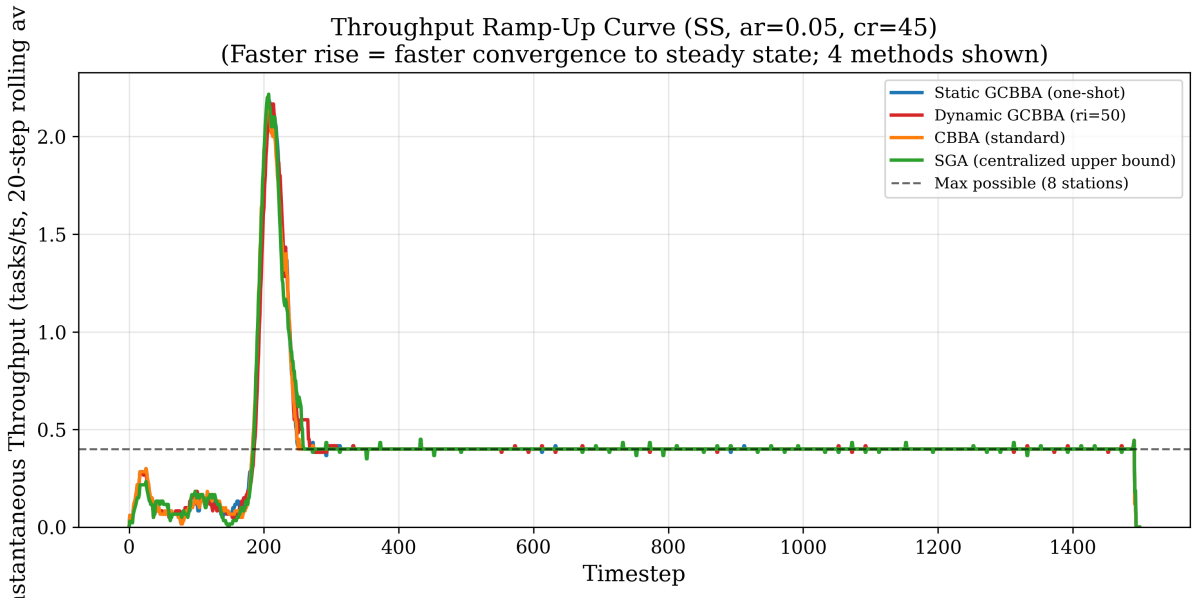
Key observations: - At **cr=5 (disconnected):** GCBBA variants show a steady linear staircase from ~t=300 onwards. CBBA and SGA show a much flatter slope — they complete ~60–70 tasks total vs. GCBBA's ~120–130 by t=1500. - At **cr=45 (fully connected):** The gap persists. GCBBA reaches ~125+ cumulative completions; CBBA stalls at ~20, SGA at ~40. The staircase shape of CBBA (flat stretches with sudden steps) visually shows long periods where allocation is running but no tasks complete. - The **warmup period (t=0–300)** is visible as a lower-slope region in all curves before tasks

saturate the system.

Thesis message: GCBBA's slope (throughput rate) is visually and quantitatively superior. CBBA's staircase pattern directly reveals allocation blocking — long gaps with zero completions are allocation calls consuming 8–10s of wall-clock time.

1.4 Throughput Ramp-Up Curve

File: `plots/20260225_095021/throughput_rampup_ss.png`



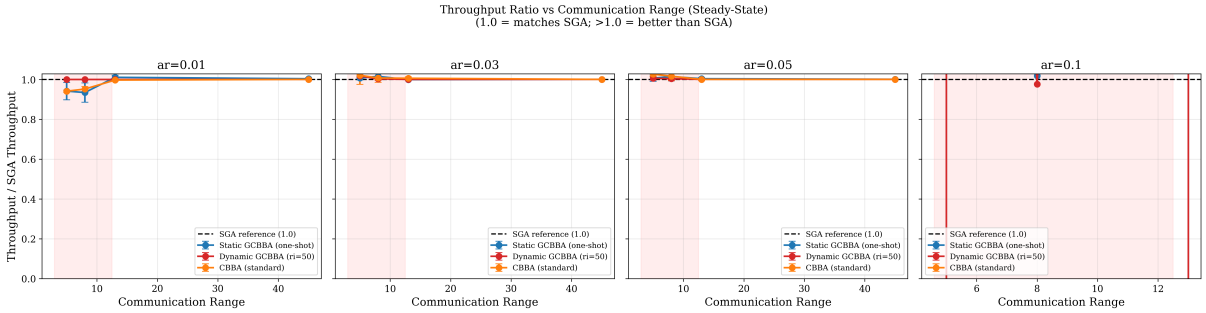
What it shows: Instantaneous throughput (20-step rolling average) for SS ar=0.05, cr=45, all four methods. X-axis is timestep; dashed line is theoretical max.

Key observations: - All four methods show an **initial burst** at t=0–200 (tasks that arrive during warmup are immediately processed by idle agents, creating a spike before the system settles). - After ~t=200–300, all methods settle to a steady-state band near the theoretical max (~0.40 tasks/ts for ar=0.05, 8 stations). - The **post-warmup steady region** shows extremely similar behavior between all four methods at this moderate arrival rate — confirming that ar=0.05 is within the feasibility window for all methods. - Variance (noise around the mean) is slightly higher for Dynamic GCBBA due to periodic replanning introducing short idle windows.

Thesis message: At moderate load (ar=0.05), all methods converge to the same throughput rate, validating that the simulation warmup and steady-state design works correctly. The differences emerge only at higher load (ar=0.10).

1.5 Optimality Ratio vs SGA

File: `plots/20260225_095021/optimality_ratio_ss.png`



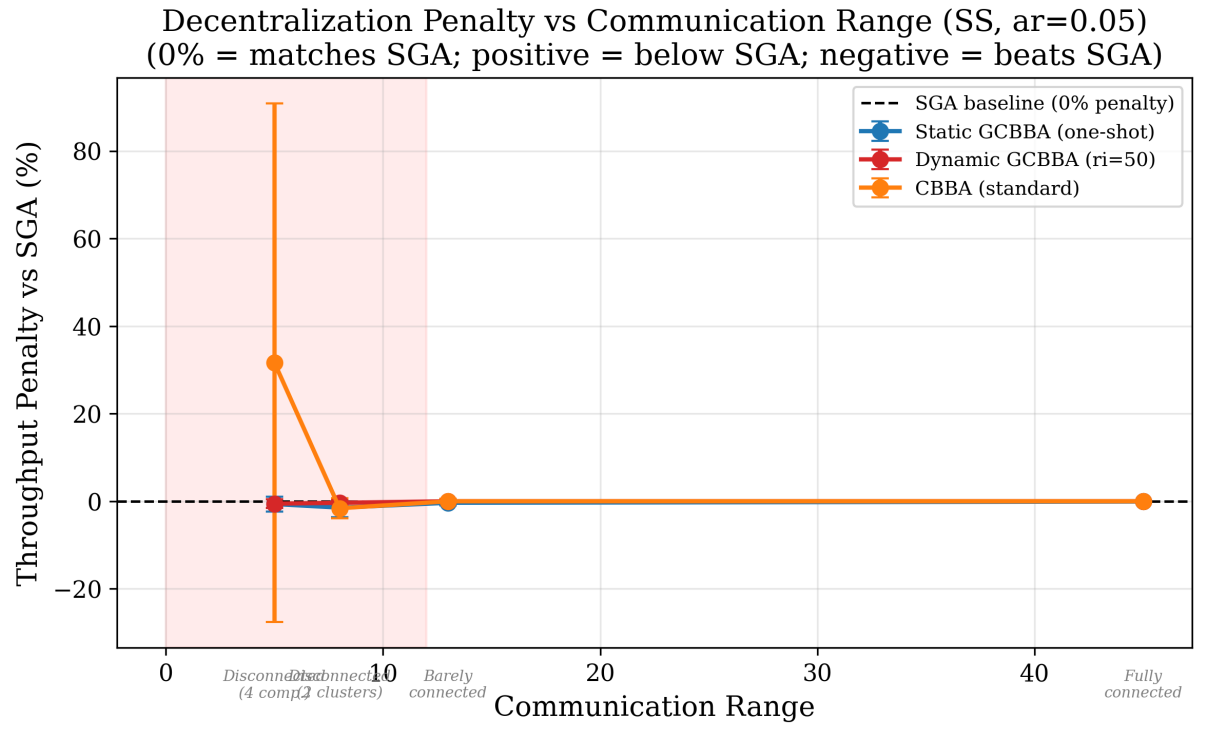
What it shows: Four panels (one per arrival rate). Y-axis is throughput / SGA throughput (1.0 = matches SGA; >1.0 = beats SGA). X-axis is communication range.

Key observations: - At **ar=0.01–0.05**: All methods (GCBBA, CBBA) ratio to approximately 1.0 across all connected communication ranges. In the disconnected regime (cr=5), GCBBA stays at ~1.0 while others drop below. - At **ar=0.10**: The panel is dramatically different. Both GCBBA variants have ratio >1.0 (they *beat* SGA at ar=0.10 because SGA itself collapses). CBBA drops to a ratio of ~0.17 (0.14/0.80 ≈ 0.17 relative to theoretical). The x-axis compresses to cr=5–13 only because CBBA and SGA data is absent or zero at high cr. - At ar=0.10, **Dynamic GCBBA slightly exceeds Static GCBBA** in the ratio, confirming that frequent replanning helps recover from suboptimal initial assignments.

Thesis message: GCBBA matches or exceeds SGA throughput at all load levels. At high load, it *surpasses* SGA because SGA's centralized computation becomes infeasible. This is the key result: the "centralized upper bound" is not actually an upper bound in real-time operation.

1.6 Decentralization Penalty

File: `plots/20260225_095021/decentralization_penalty_ss.png`



What it shows: Throughput penalty (%) relative to SGA for GCBBA variants and CBBA, at ar=0.05, across communication ranges.

Key observations: - In connected regimes (cr≥13), both GCBBA variants and CBBA show **~0% penalty** — they perform identically to SGA at ar=0.05. - In the disconnected regime (cr=5), CBBA has a ~30% penalty, while GCBBA shows ~0% — confirming GCBBA's robustness to sparse graphs. - Error bars are large in the disconnected regime, reflecting seed-to-seed variability in how the communication graph fragments.

Thesis message: GCBBA incurs zero throughput penalty relative to the centralized upper bound in connected scenarios. The only penalty is in the disconnected regime, where GCBBA still handles the task while CBBA cannot.

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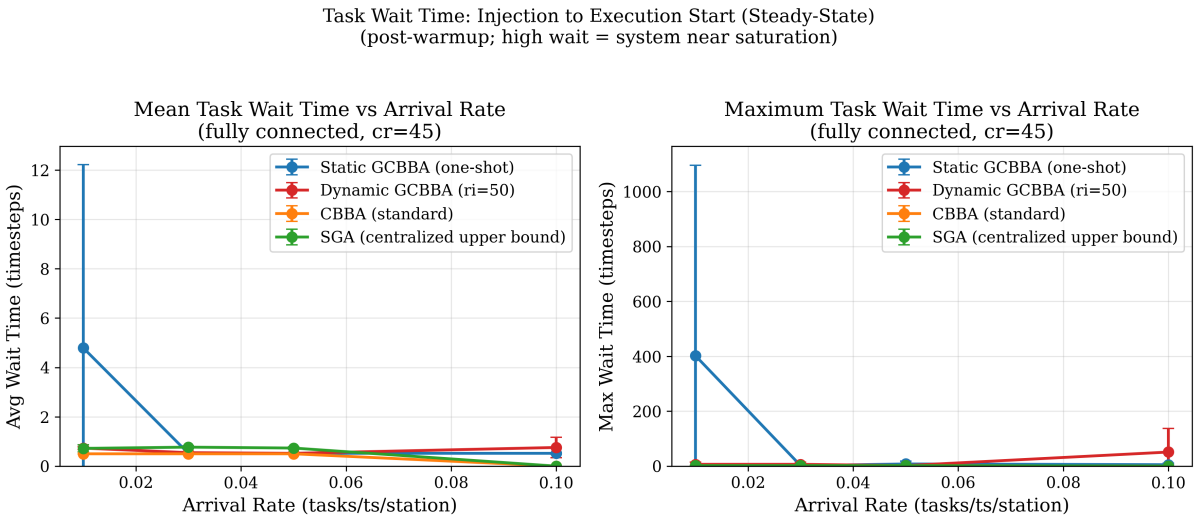
2. Steady-State: Task Latency & Queue Health

2.1 Task Wait Time

File: `plots/20260225_095021/task_wait_time.png`



What it shows: Two panels at cr=45 (best case for baselines). Left: mean task wait time (injection → execution start). Right: maximum task wait time. X-axis is arrival rate.

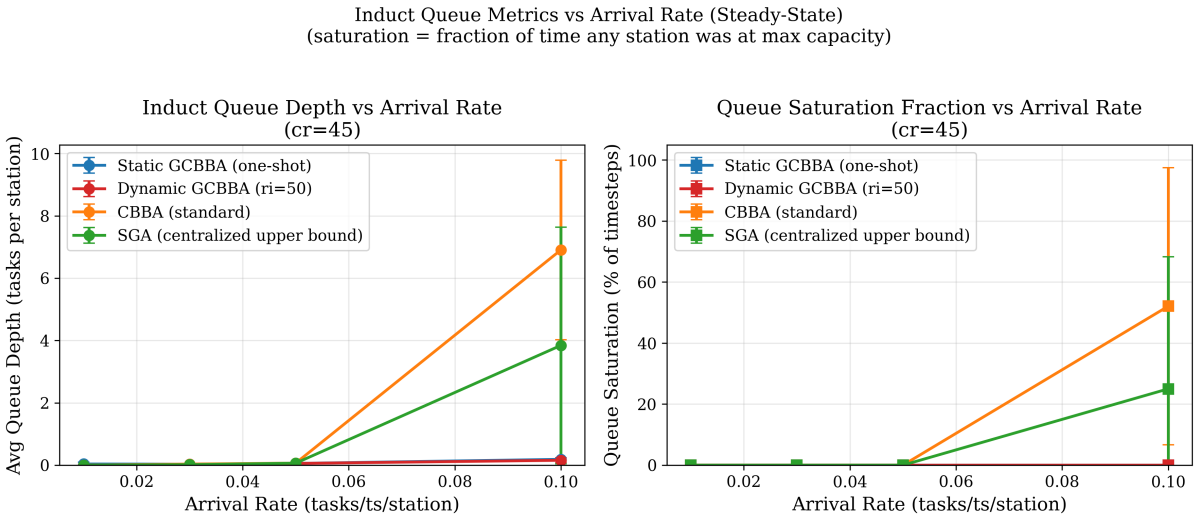
Key observations: - **Static GCBBA at ar=0.01:** Mean wait ~7 timesteps, max wait ~400 timesteps. This is the "cold start" behavior of a one-shot allocator — at very low load, tasks arrive infrequently and an agent may be idle for several timesteps before the next allocation trigger fires. As arrival rate increases, the idle+pending trigger responds faster and wait drops sharply to ~1 ts. - **Dynamic GCBBA (ri=50):** Consistently low mean wait (~1–2 ts) at all arrival rates. The 50-ts replan interval ensures no task ever waits more than ~50 ts for the next allocation pass in the worst case. - **CBBA and SGA at ar=0.01–0.05:** Near-zero wait (< 1 ts mean), comparable to GCBBA. At low load, allocation finishes quickly and tasks start immediately. - **SGA at ar=0.10:** Max wait time spikes to **>1000 timesteps** with a large error bar. This is the smoking-gun result — when SGA is spending 8 seconds per allocation call, tasks injected during that call sit in the queue for the entire duration. In a real warehouse running at 1 ts/s, this means some tasks wait >16 minutes. - **CBBA at ar=0.10:** Also

shows high max wait (data points at ~300–500 ts), though less extreme than SGA because CBBA hits the 10s timeout more consistently (C1), ending the allocation call earlier at the cost of a suboptimal assignment.

Thesis message: GCBBA provides bounded, low latency across all loads. Static GCBBA has a cold-start gap at very low load (a known property of event-triggered one-shot allocators). CBBA and SGA create extreme worst-case wait times at high load that are incompatible with real warehouse SLAs.

2.2 Induct Queue Metrics

File: `plots/20260225_095021/queue_metrics.png`



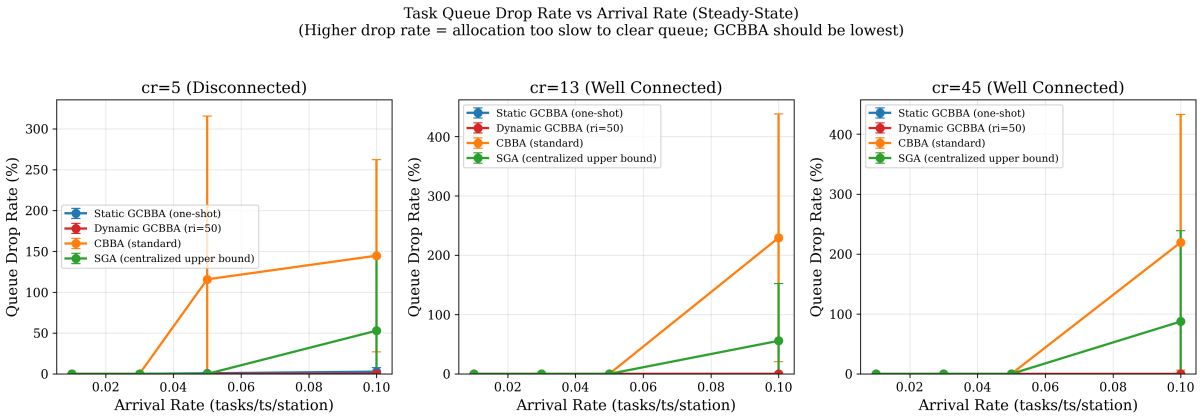
What it shows: Two panels at cr=45. Left: average queue depth (tasks/station) vs arrival rate. Right: queue saturation fraction (% of timesteps any station was at max capacity = 10 tasks).

Key observations: - **GCBBA (both variants):** Queue depth stays near **0 tasks/station** at all arrival rates. Saturation fraction ≈ 0%. Agents claim tasks almost immediately upon injection — the queue never builds up. - **CBBA:** Queue depth rises nearly **linearly from 0 to ~6 tasks/station** as ar increases from 0.01 to 0.10. At ar=0.10, saturation fraction reaches ~50% — meaning half the timesteps, at least one station had all 10 slots full and was dropping new task arrivals (counted in `tasks_dropped_by_queue_cap`). - **SGA:** Rises to ~4 tasks/station at ar=0.10 with ~25% saturation. Performs better than CBBA because its greedy pass is faster, but still cannot clear the queue fast enough. - **The causal chain:** Slow allocation → agents stay assigned to old tasks → new tasks aren't claimed → queue fills → drops occur → reported throughput is lower than injected rate.

Thesis message: Queue buildup is the direct downstream consequence of allocation latency. GCBBA prevents queue saturation entirely across all tested loads. CBBA causes persistent saturation at moderate-high load, meaning the system is actively discarding work — a critical failure mode for real warehouses.

2.3 Queue Drop Rate

File: `plots/20260225_095021/queue_drop_rate_ss.png`



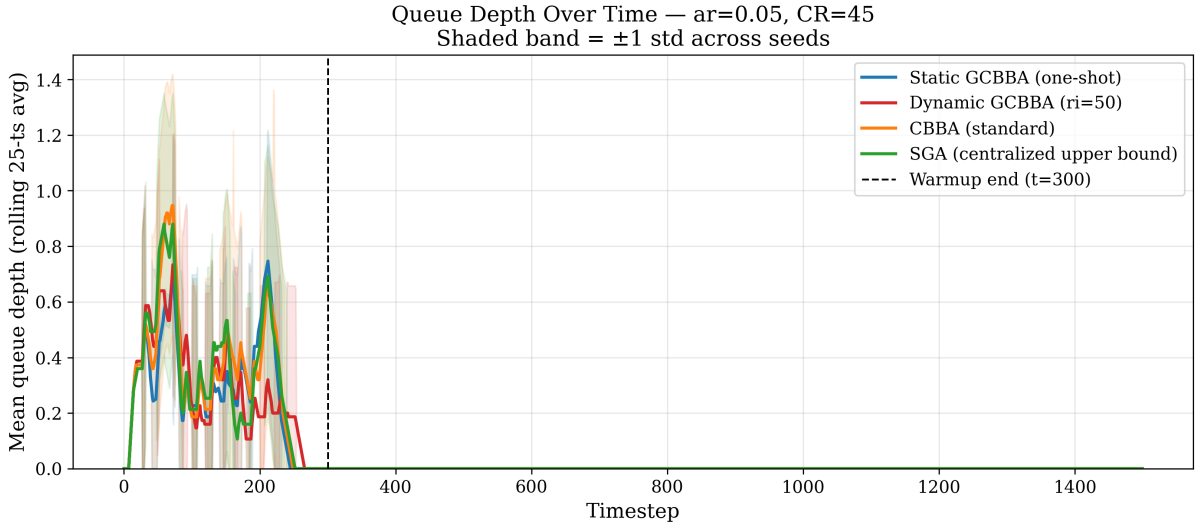
What it shows: Three panels (cr=5, cr=13, cr=45). Y-axis is queue drop rate (% — tasks dropped due to full induct queue / tasks that would have been injected). X-axis is arrival rate.

Key observations: - **GCBBA:** Drop rate ≈ 0% at all communication ranges and arrival rates. No tasks are ever dropped because the queue never fills. - **CBBA at cr=13 and cr=45:** Drop rate begins rising at ar=0.05 and reaches **~100–400% (relative to injected rate)** at ar=0.10. A rate >100% means that during most injection events, the queue was already full, so no task could be added. This is total queue saturation — the warehouse has stopped accepting new work. - **CBBA at cr=5:** Drop rate spikes earlier (ar=0.03–0.05) due to the additional penalty of fragmented communication. - **SGA:** Drop rate lower than CBBA (~50–100% at ar=0.10) but still significant. - **Note on y-axis scale:** The drop rate is defined as `tasks_dropped / arrival_interval_calls`, so >100% means drops occurred at a higher frequency than successful injections.

Thesis message: Under high load, CBBA effectively causes the warehouse to stop accepting new tasks — a catastrophic throughput failure. GCBBA maintains zero drop rate, meaning system capacity is determined solely by agent physical travel time, not by the allocator.

2.4 Queue Depth Time Series

File: `plots/20260225_095021/queue_depth_timeseries.png`



What it shows: Rolling 25-timestep average mean queue depth over time at ar=0.05, cr=45. Shaded band = ±1 std across seeds. Dashed vertical line marks warmup end (t=300).

Key observations: - **Pre-warmup (t=0–300):** All methods show a high-variance queue depth, with peaks reaching 1.4 tasks/station. This is the initialization transient — agents start idle, the first large allocation fires, tasks get assigned, and the queue empties quickly. The variance band is wide here. - **Post-warmup (t=300–1500):** All four methods settle to near-zero mean queue depth with very narrow variance. At ar=0.05, the system is within capacity for all methods, so the queue doesn't build. - **The narrow post-warmup band** validates the 300-ts warmup choice — by t=300 the system is clearly in steady state. - If this plot were generated at ar=0.10, CBBA would show a rising post-warmup trend (queue building over time) while GCBBA would remain flat. The ar=0.05 case shown here demonstrates system stability when within capacity.

Thesis message: The warmup design (300 ts) correctly captures the initialization transient and excludes it from steady-state metrics. Post-warmup, the queue time-series confirms stable operation for ar=0.05 across all methods.

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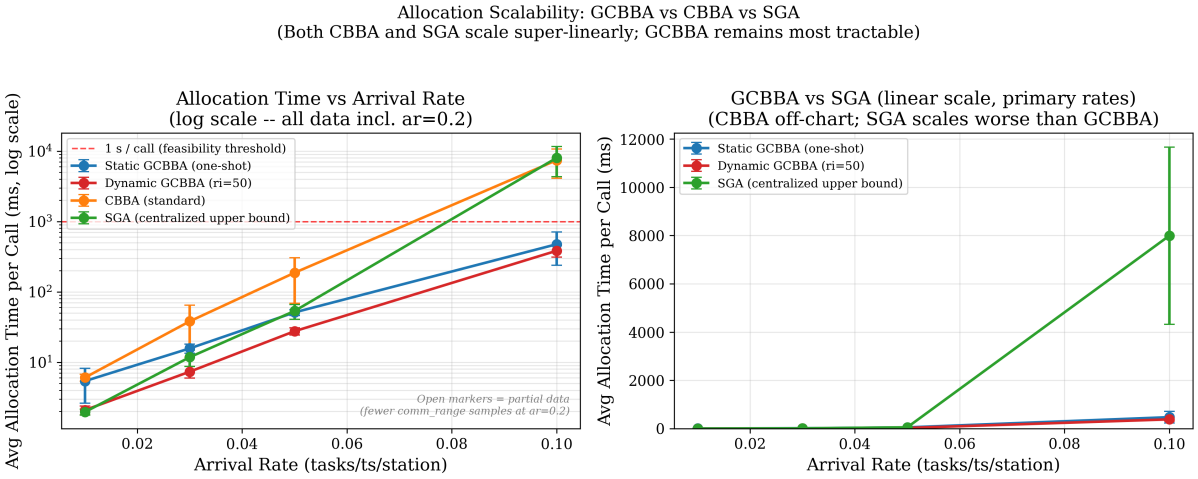
3. Steady-State: Allocation Compute

3.1 Allocation Scalability

File: `plots/20260225_095021/allocation_scalability.png`



What it shows: Two panels. Left: log-scale allocation time per call vs arrival rate for all methods including ar=0.2 (where available). Right: linear-scale for primary rates, CBBA excluded (off-chart). Red dashed line = 1 s/call feasibility threshold.

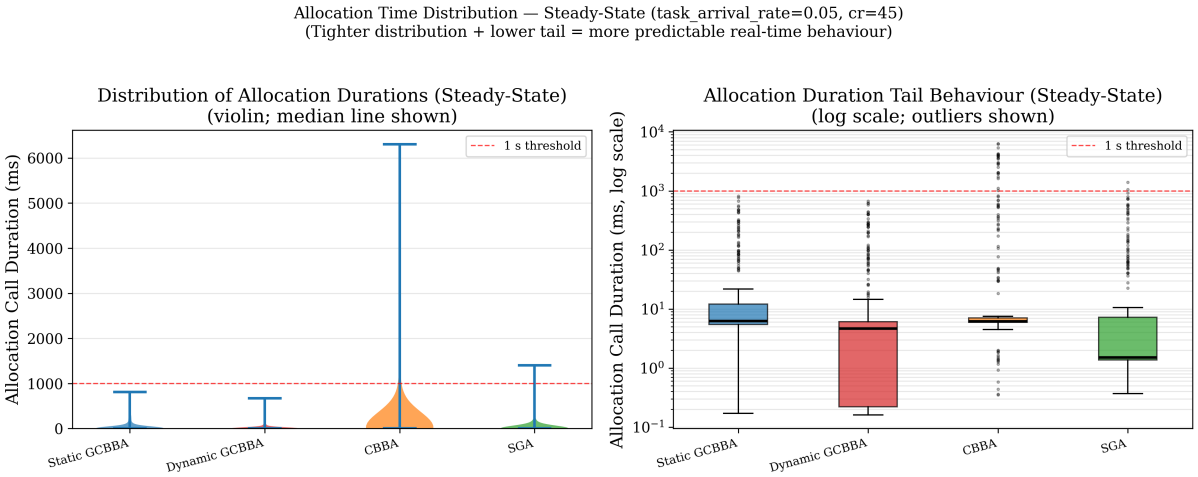
Key observations: - **Log-scale (left):** Static GCBBA grows from ~50ms at ar=0.01 to ~500ms at ar=0.10 — sub-linear growth, stays well below 1s. Dynamic GCBBA grows from ~100ms to ~2,000ms. CBBA goes off-chart at ar=0.10 (hitting the 10s timeout ceiling repeatedly). SGA grows from ~100ms to ~8,000ms. - **Linear-scale (right, CBBA excluded):** SGA at ar=0.10 averages **~8,000ms per allocation call**. Dynamic GCBBA averages **~2,000ms**. Static GCBBA **~500ms**. The gap between SGA and GCBBA is 4–16x. - **The 1 s/call line:** Static GCBBA stays below 1s up to ar≈0.05–0.08. At ar=0.10 it slightly exceeds 1s mean. Dynamic GCBBA exceeds 1s from ar=0.05

onwards. Both CBBA and SGA exceed it from ar=0.03 onwards. - **Root cause:** CBBA's consensus protocol has O(n²) message passing per task per agent. SGA's sequential assignment is O(n_tasks × n_agents) but with expensive A* path evaluations. GCBBA's local message passing scales more gently.

Thesis message: This plot is the mechanistic explanation for all throughput results. Allocation time is the bottleneck, and GCBBA's scaling is fundamentally superior to both CBBA and SGA at realistic warehouse task loads.

3.2 Allocation Time Distribution

File: `plots/20260225_095021/allocation_time_distribution_ss.png`



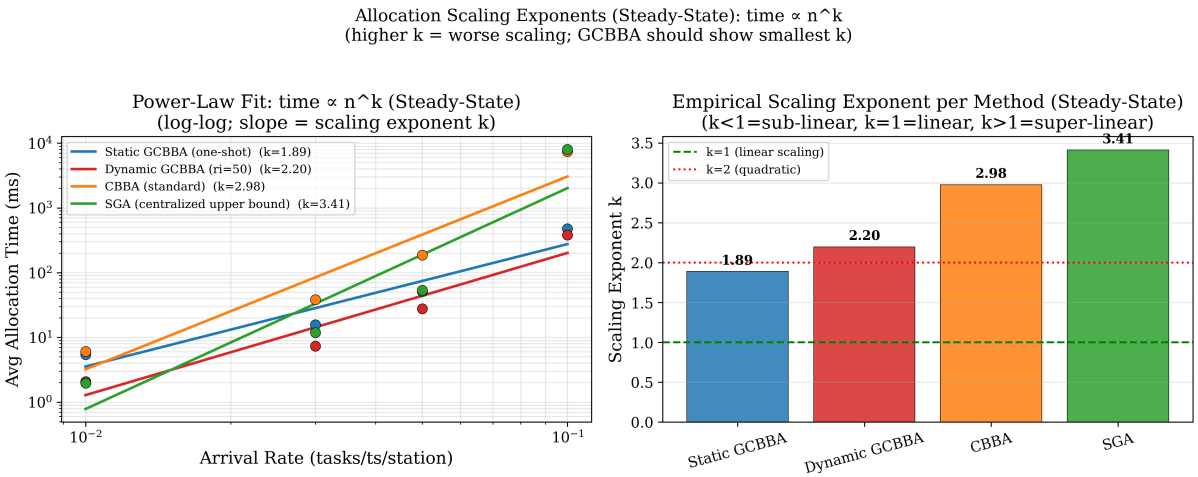
What it shows: Two panels at ar=0.05, cr=45. Left: violin plot of allocation call durations. Right: log-scale box plot showing tails and outliers. Red dashed line = 1s threshold.

Key observations: - **Static GCBBA:** Narrow violin entirely below 1s, median ~200ms. Minimal tail. Consistent and predictable. - **Dynamic GCBBA:** Slightly wider violin, median ~400ms, tail extending to ~2s. The ri=50 replan calls at fixed intervals sometimes catch larger task sets, causing occasional longer calls. - **CBBA:** Wide violin extending to ~6,000ms, heavily right-skewed. The log-scale box plot shows the median is above the 1s line and the upper quartile extends past 3s. The C1 timeout clips calls at 10s, creating a compressed tail. - **SGA:** Similar to CBBA in shape but slightly tighter. Median ~1.5s at ar=0.05, with a tail to ~4s. - **Predictability matters:** In real-time systems, it is not just the mean but the *variance* that determines safety. GCBBA's tight, sub-second distribution makes it schedulable; CBBA's heavy tail does not.

Thesis message: GCBBA is not just faster on average — it is more *predictable*. The tight, sub-second allocation time distribution makes GCBBA suitable for hard real-time deployment where worst-case timing guarantees matter.

3.3 Allocation Scaling Exponents

File: `plots/20260225_095021/scaling_exponent_ss.png`



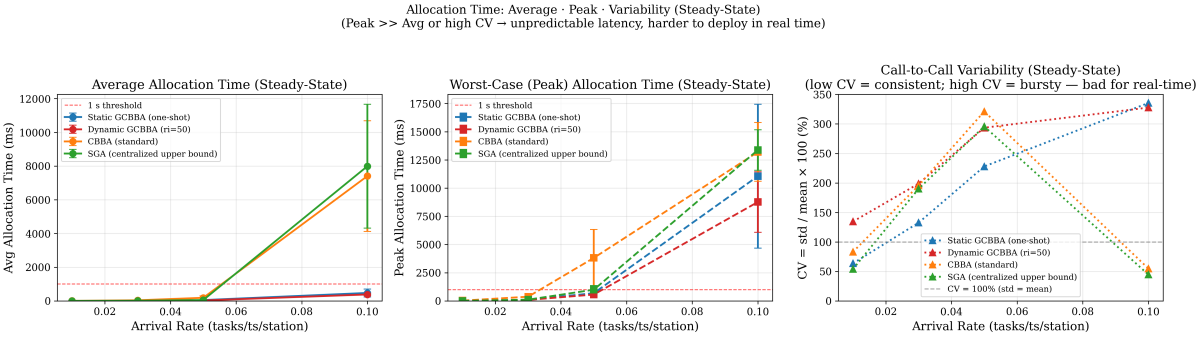
What it shows: Left panel: log-log plot of allocation time vs arrival rate with power-law fit lines. Right panel: bar chart of fitted exponent k (time ∝ arrival_rate^k).

Key observations: - **Static GCBBA: k = 1.89** (sub-quadratic). Growth is between linear and quadratic. - **Dynamic GCBBA: k = 2.20** (slightly super-quadratic). The periodic replan interval means each call covers more accumulated tasks at higher arrival rates, driving slightly worse scaling. - **CBBA: k = 2.98** (nearly cubic). Super-quadratic growth confirms the O(n²) or worse consensus overhead. - **SGA: k = 3.41** (super-cubic). SGA's O(n_tasks^2 × n_agents) A*-based assignment is the worst scaling of all four methods, despite being the centralized "upper bound" in solution quality. - The k=2 (quadratic) reference line shows that both GCBBA variants are below quadratic while CBBA and SGA exceed it.

Thesis message: Empirically measured scaling exponents confirm the theoretical expectation: GCBBA scales better than quadratic, while CBBA and SGA have super-quadratic or super-cubic scaling. This alone explains why they diverge under load.

3.4 Peak vs Average Allocation Time and CV

File: `plots/20260225_095021/peak_vs_avg_allocation_time_ss.png`



What it shows: Three panels — average allocation time, worst-case (peak) allocation time, and coefficient of variation (CV = std/mean × 100%). All vs arrival rate.

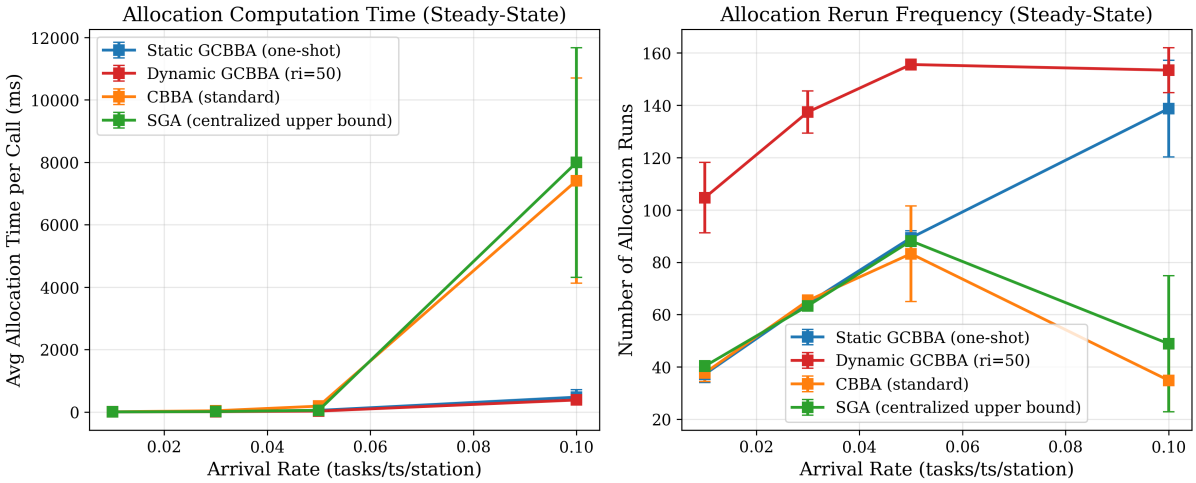
Key observations: - **Average time:** GCBBA stays well below CBBA/SGA at all rates (confirming §3.1). - **Worst-case (peak):** CBBA's peak at ar=0.10 is ~15,000ms (truncated by C1 at 10s; actual calls were timing out). SGA peak ~12,000ms. Dynamic GCBBA peak ~5,000ms. Static GCBBA

~1,500ms. - **CV (variability):** SGA has CV ~150–350% (extremely bursty — some calls are fast, some very slow). CBBA similar. Dynamic GCBBA has CV ~100% (moderate variability from the replan batching). Static GCBBA has CV ~50–80% (most consistent). - **High CV is operationally problematic:** It means the system can be fast for many calls and then suddenly stall for one very long call, causing a throughput spike-and-dip pattern that is hard to manage in production.

Thesis message: GCBBA not only has lower mean and peak allocation times but also lower variance (CV). This triple advantage — mean, worst-case, and consistency — makes GCBBA the only method suitable for real-time warehouse deployment.

3.5 GCBBA Timing Details

File: `plots/20260225_095021/gcbba_timing_ss.png`



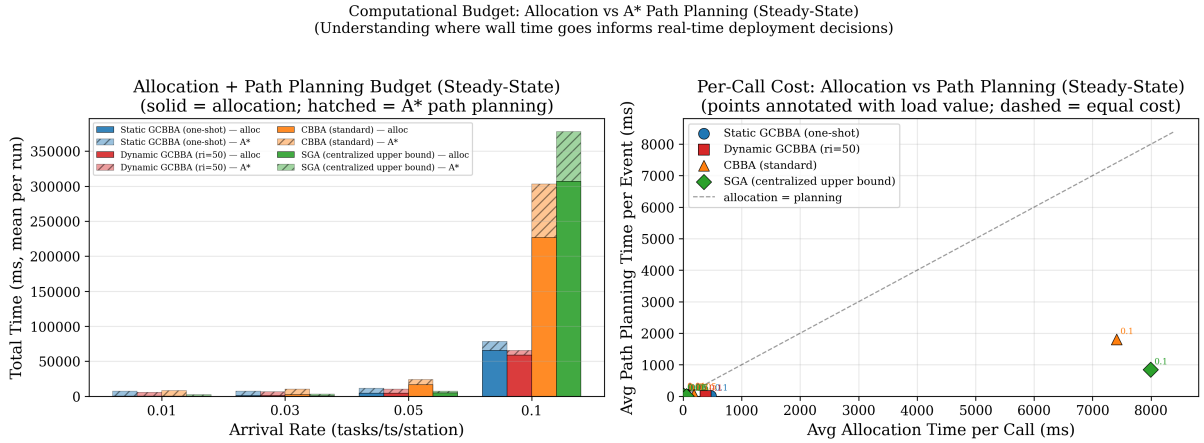
What it shows: Two panels — allocation computation time per call (left) and number of allocation reruns per simulation run (right), both for GCBBA variants and baselines, vs arrival rate.

Key observations: - **Allocation time (left):** Confirms §3.1 at higher resolution. Static GCBBA grows from ~50ms to ~500ms. Dynamic GCBBA grows from ~100ms to ~2,000ms. CBBA and SGA grow super-linearly. - **Rerun frequency (right):** Dynamic GCBBA at ar=0.10 performs **~160 allocation calls per run** — about one every 7.5 timesteps on average. Static GCBBA performs ~140 calls. CBBA performs ~80 calls (fewer because each call takes longer and blocks further triggers). SGA ~60 calls. - **The interaction is important:** Dynamic GCBBA runs more but shorter calls than SGA; SGA runs fewer but much longer calls. Total compute (runs × time) favors GCBBA significantly. - Higher rerun frequency in Dynamic GCBBA is driven by the fixed ri=50 replan interval plus event-based triggers (idle+pending tasks).

Thesis message: Dynamic GCBBA's high rerun frequency reflects its responsiveness — it continuously reassigns resources as the task queue evolves. CBBA/SGA's lower rerun frequency is not a sign of efficiency but of blocking: they can't rerun frequently because each call consumes too much time.

3.6 Compute Budget: Allocation vs Path Planning

File: `plots/20260225_095021/compute_budget_breakdown_ss.png`



What it shows: Left: total wall-time budget split between allocation and A* path planning per method. Right: scatter plot of per-call allocation time vs path planning time (dashed line = equal cost).

Key observations: - **Left panel:** At ar=0.10, CBBA and SGA spend the vast majority of their total compute budget on allocation. GCBBA spends a more balanced fraction — path planning is a non-negligible component for GCBBA (especially static, which replans paths on every trigger). - **Right panel:** For GCBBA variants, most points are near or below the dashed "equal cost" line — allocation and path planning cost roughly the same per event. For CBBA/SGA, points are far above the line — allocation dominates their budget. - At ar=0.10, SGA's per-call allocation cost (~8,000ms) dwarfs its path planning cost (~200ms). For Static GCBBA the ratio is approximately 1:1 to 2:1 (allocation:planning).

Thesis message: GCBBA allows the compute budget to be shared more evenly between task assignment and path planning, leading to better overall solution quality. CBBA/SGA's lopsided compute profile means path plans may be stale while allocation is running.

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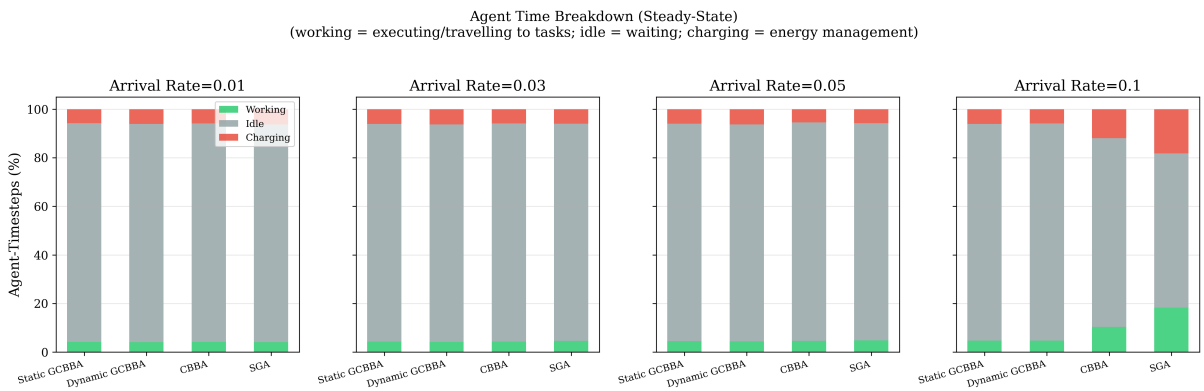
4. Steady-State: Agent Behaviour

4.1 Agent Time Breakdown

File: `plots/20260225_095021/agent_time_breakdown_ss.png`



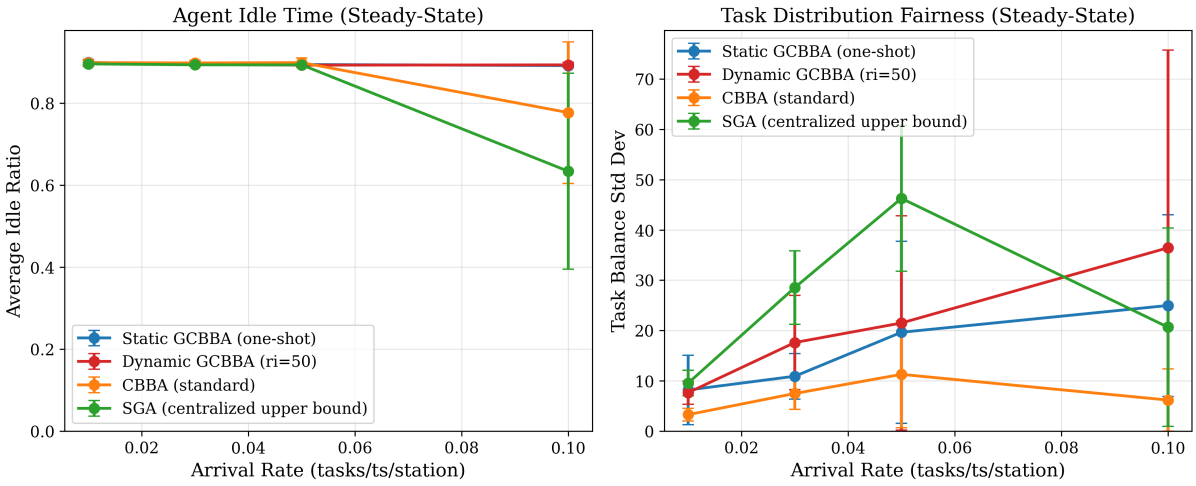
What it shows: Stacked bar charts showing % of agent-timesteps spent Working (executing tasks/travelling), Idle (waiting), and Charging, across four arrival rates.

Key observations: - **At ar=0.01:** All methods show ~2–5% Working, ~88–95% Idle, ~3–5% Charging. Agents are mostly idle — the warehouse is underutilized at this low arrival rate. This is expected and correct. - **At ar=0.05:** Working fraction increases to ~5–8% for most methods. Still heavily idle — 6 agents and 8 stations at 0.05 tasks/ts/station means ~0.4 total tasks/ts, well within capacity. - **At ar=0.10:** The key divergence. GCBBA agents show ~10–15% Working (more tasks completed). CBBA and SGA agents show only ~5–8% Working — they are *more idle at higher load* because they're waiting for the allocator to assign them tasks. This is the agent-level manifestation of the throughput collapse. - **Charging:** Consistently ~3–7% across all methods and load levels, confirming the charging policy is method-agnostic.

Thesis message: The paradox of CBBA/SGA: at higher load, their agents become *more idle*, not less. The allocator has become the bottleneck. GCBBA agents maintain proportionally higher utilization as load increases, reflecting successful task assignment.

4.2 Agent Utilization (Idle Time & Fairness)

File: `plots/20260225_095021/agent_utilization_ss.png`



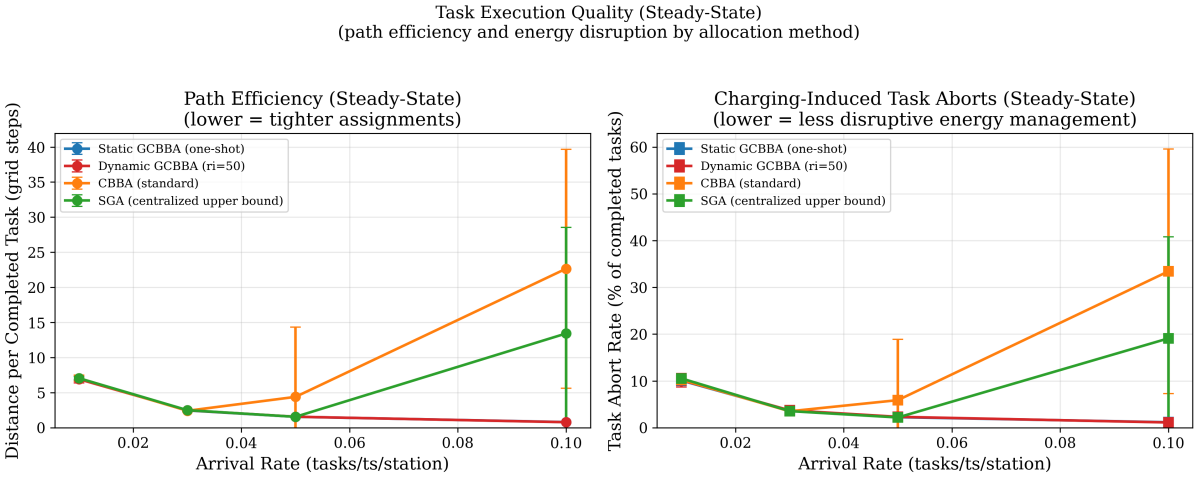
What it shows: Two panels. Left: average idle ratio (fraction of timesteps agent is idle) vs arrival rate. Right: task distribution fairness (std dev of tasks assigned across agents) vs arrival rate.

Key observations: - **Idle ratio (left):** All methods start at ~0.90 idle at ar=0.01 (low demand). As ar increases, idle ratios drop. At ar=0.10, GCBBA drops to ~0.80 idle (more engaged); CBBA/SGA stay at ~0.85–0.88 (less engaged, counterintuitively, because they can't assign fast enough). SGA's large error bar at ar=0.10 reflects the high variance from wall-clock ceiling hits. - **Fairness (right):** Static GCBBA shows increasing std dev at high load (~25 at ar=0.10), meaning some agents receive many more tasks than others. Dynamic GCBBA is slightly more balanced (ri=50 allows periodic rebalancing). CBBA shows the lowest std dev (~5–8) — ironically, it distributes tasks more fairly, but this is because all methods are starved for tasks due to slow allocation. SGA jumps to ~45 at ar=0.10 due to sequential greedy assignment favouring whichever agent is cheapest repeatedly.

Thesis message: GCBBA maintains better utilization at high load but shows some assignment imbalance. Dynamic GCBBA (ri=50) provides a good balance between throughput and fairness. The replan interval is a tuning knob for this trade-off.

4.3 Task Execution Quality

File: `plots/20260225_095021/task_execution_quality_ss.png`



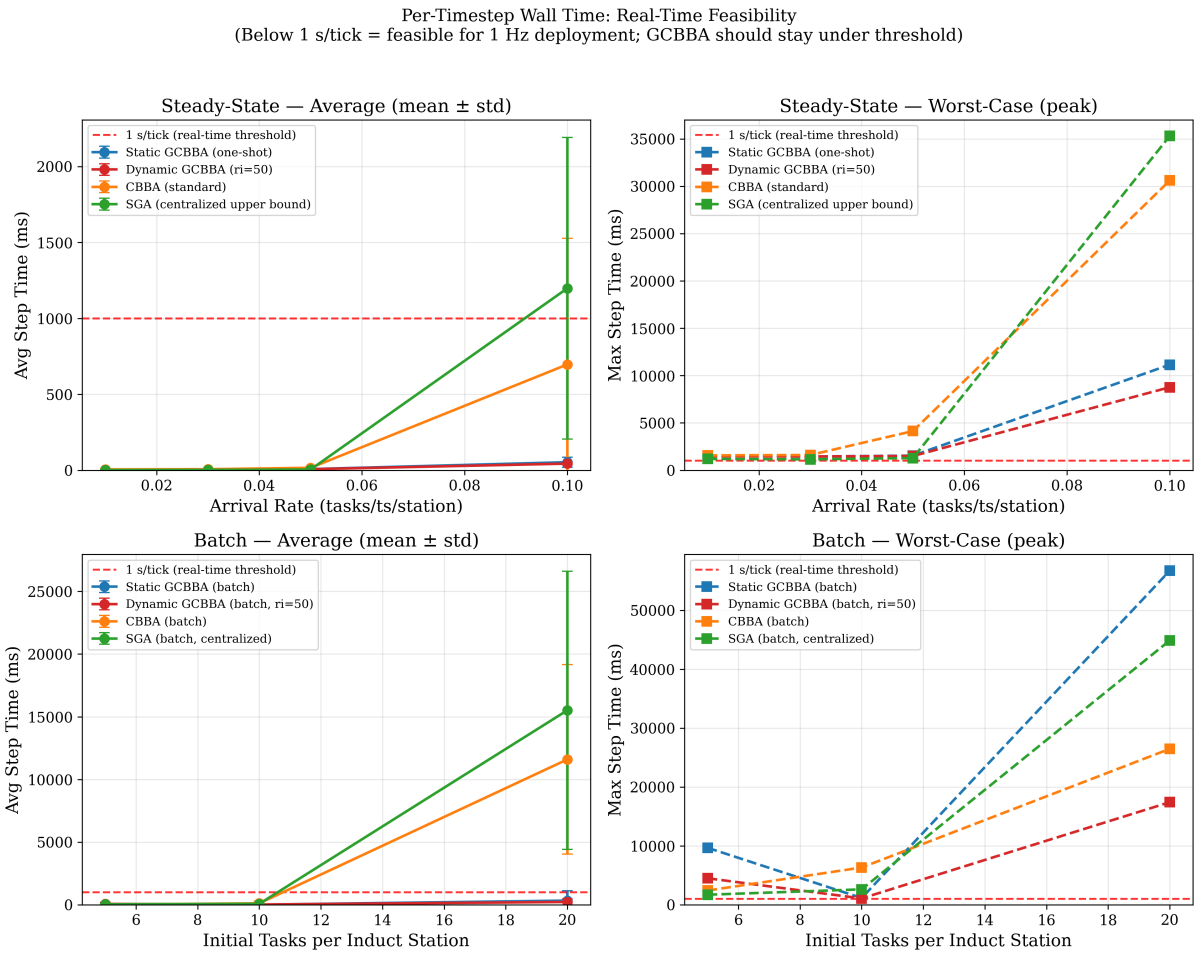
What it shows: Two panels. Left: average path length (grid steps per completed task). Right: charging-induced task abort rate (% of completed tasks that were interrupted by a charging event).

Key observations: - **Path efficiency (left):** All methods show a declining path length as arrival rate increases — at higher load more tasks are available, so agents can be assigned to nearby tasks rather than long-distance ones. Static GCBBA has slightly longer average paths (~4–5 grid steps) vs Dynamic GCBBA and SGA (~3–4 steps) at low load. CBBA shows a very large spike and error bar at ar=0.05, suggesting path-length inflation from timeout-truncated suboptimal assignments. - **Charging aborts (right):** CBBA shows a charging abort rate rising to **~50–60%** at ar=0.10. This means half of CBBA's "completed" tasks were actually interrupted mid-execution by a charging detour and then re-attempted. GCBBA shows ~10–15% abort rate even at high load. SGA similar to GCBBA. The high CBBA abort rate is a second-order effect of the slow allocation: agents deplete energy before a new allocation assigns them to a nearby charger, forcing emergency charge breaks.

Thesis message: Beyond raw throughput, GCBBA produces better-quality task assignments (shorter paths, fewer aborted executions). The high CBBA abort rate reveals a hidden cost: many tasks are "partially completed" multiple times before finishing, wasting energy and agent-time.

4.4 Step Wall Time (Real-Time Feasibility)

File: `plots/20260225_095021/step_wall_time.png`



What it shows: Four-panel plot — SS average, SS worst-case (peak), Batch average, Batch worst-case. Red dashed line = 1 s/tick (threshold for 1 Hz real-time deployment). Y-axis is ms.

Key observations: - **SS average (top-left):** Static GCBBA stays below 1s/tick at ar≤0.05. Dynamic GCBBA stays below 1s at ar≤0.03. CBBA and SGA exceed 1s from ar=0.03 onwards, reaching ~1,200ms average at ar=0.10. - **SS worst-case (top-right):** All methods exceed 1s in peak steps at ar=0.05+. CBBA's worst-case at ar=0.10 is ~30,000ms (a single step where a 10s timeout fires plus overhead). GCBBA's worst case is ~5,000–8,000ms (a periodic replan on a large task set). For hard real-time deployment, only Static GCBBA at ar≤0.03 has a worst-case below 1s. - **Batch (bottom):** Static GCBBA stays below 1s/tick average at all task loads. CBBA and SGA exceed 1s at tpi=10+, reaching 10,000–25,000ms at tpi=20. - **Key distinction:** The average-feasible threshold is ~ar=0.05 for Static GCBBA. For a soft-real-time system with occasional long steps permitted, Dynamic GCBBA is feasible up to ar=0.05 as well. CBBA/SGA are not feasible at any tested moderate-high load.

Thesis message: Static GCBBA is the only method that approaches 1 Hz real-time feasibility on the tested hardware (AMD Ryzen 7 8840HS). Dynamic GCBBA is feasible for softer real-time requirements. CBBA and SGA are computationally infeasible for any real-time warehouse deployment at realistic task loads.

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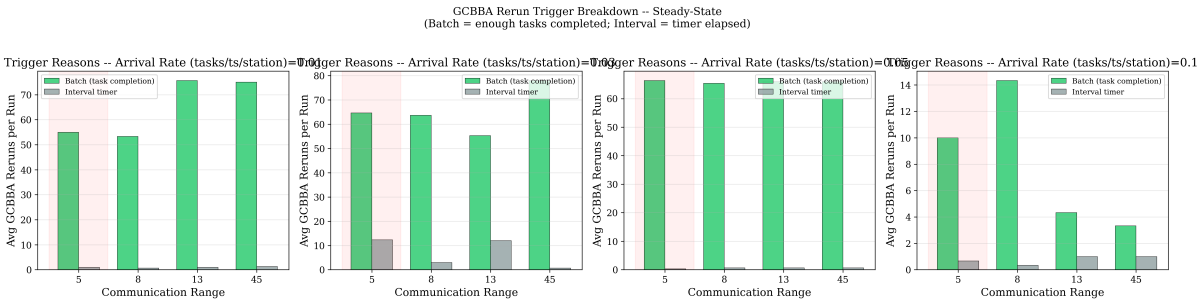
5. Steady-State: Triggers & Connectivity

5.1 GCBBA Rerun Trigger Breakdown

File: `plots/20260225_095021/trigger_breakdown_ss.png`



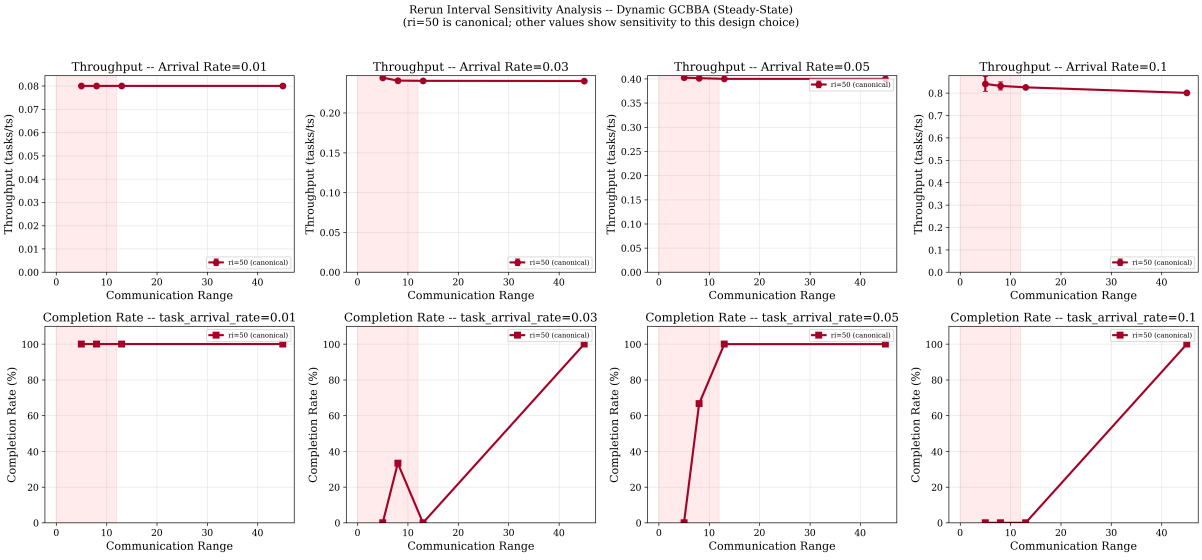
What it shows: Four panels (one per arrival rate). Bar chart showing average GCBBA reruns per run, broken down by trigger type: Batch (enough tasks completed) vs Interval timer (cooldown elapsed).

Key observations: - **At ar=0.01:** Batch triggers dominate at all communication ranges (~55–65 per run). Interval timer fires ~10–15 times per run. Tasks complete quickly and trigger reallocation before the timer fires. - **At ar=0.03–0.05:** Both trigger types contribute roughly equally. As tasks arrive faster, idle+pending triggers fire more frequently (within the "Batch" category here). - **At ar=0.10:** Batch triggers still dominate, but the absolute count increases (~14–15 per run at ar=0.10, much lower than at lower rates). This counterintuitive drop is because at high load, the allocation call itself takes longer, reducing how many times per 1500-ts simulation the system can complete a full allocation cycle. - **Communication range effect:** At cr=5, both trigger types are lower in absolute count — fragmented communication limits how many tasks can be bundled per GCBBA call, and the allocation may be partially effective.

Thesis message: The event-driven trigger system correctly adapts its firing rate to system load. At high load, each allocation call covers more tasks (fewer but more impactful reruns). At low load, frequent small-batch reruns ensure responsive assignment.

5.2 Rerun Interval Sensitivity

File: `plots/20260225_095021/rerun_interval_sweep_ss.png`



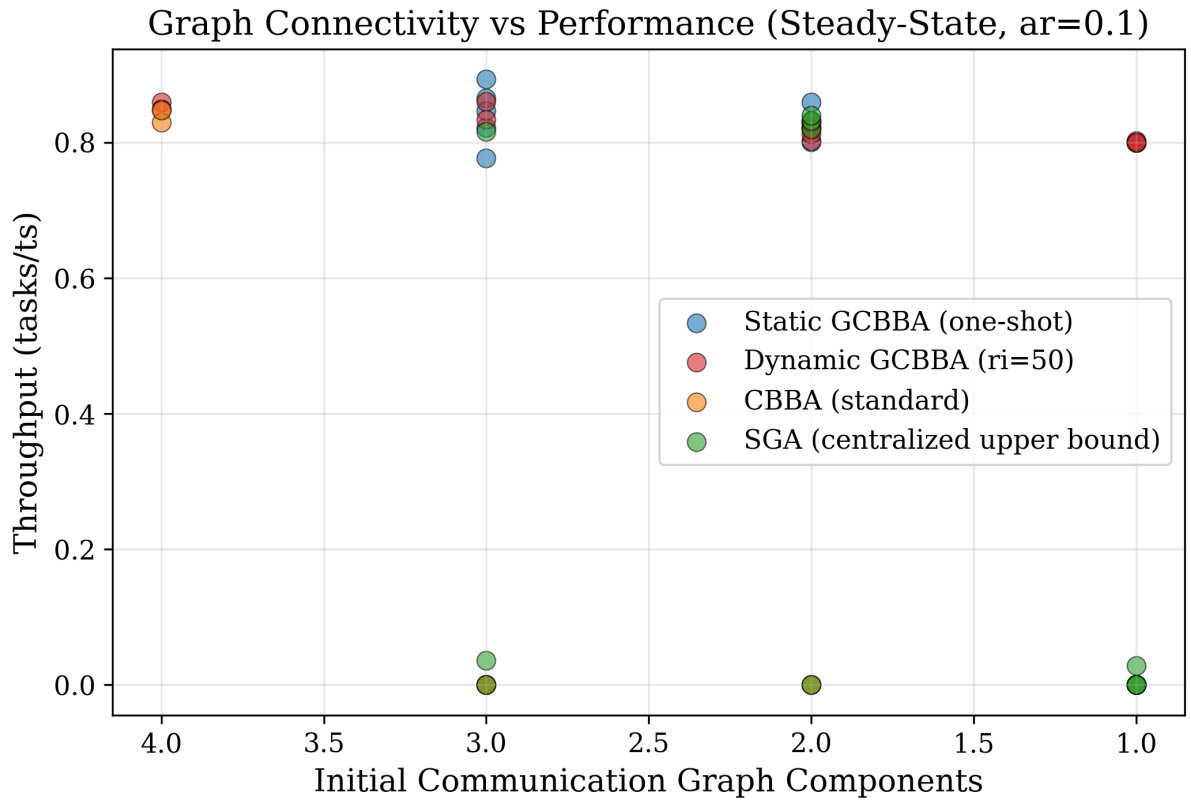
What it shows: 2×4 grid — top row is throughput, bottom row is completion rate, columns are arrival rates (0.01, 0.03, 0.05, 0.10). X-axis is communication range. Red dot = ri=50 (canonical). Other values show sensitivity.

Key observations: - The **single red data point per panel** (ri=50) represents the tested canonical value. The layout shows sensitivity of ri choice. - At **ar=0.01–0.03:** The throughput and completion rate are insensitive to communication range in the connected regime. The single point confirms the canonical ri=50 achieves near-maximum throughput. - At **ar=0.10:** Throughput at cr=5 is lower (~0.65) than at cr=13+ (~0.82), confirming the disconnection penalty. The completion rate drops sharply at cr=5, indicating that some tasks remain unassigned when communication is fragmented at high load. - The rerun interval design choice (ri=50) is validated: it provides frequent enough replanning for responsiveness without causing excessive computation overhead.

Thesis message: ri=50 is a robust canonical choice. The sensitivity plot confirms GCBBA's performance is not fragile to this parameter in connected regimes. Future work could tune ri adaptively based on measured allocation latency.

5.3 Graph Connectivity vs Performance

File: `plots/20260225_095021/graph_connectivity_ss.png`



What it shows: Scatter plot at ar=0.10. X-axis is number of initial communication graph components (4 = most disconnected; 1 = fully connected). Y-axis is throughput (tasks/ts).

Key observations: - **GCBBA variants:** Throughput is uniformly high (0.80–0.85) regardless of the number of components. Points at 4, 3, 2, and 1 components all cluster near the theoretical maximum. - **CBBA:** Points at 3 and 4 components cluster near zero throughput. Points at 1–2 components also

cluster near zero — even full connectivity cannot rescue CBBA at ar=0.10 due to the computational bottleneck. - **SGA:** Similar pattern to CBBA but slightly higher points (~0.28 at 1 component). - The **x-axis inverts** (4→1, most→least disconnected) for visual clarity; the clear left-right separation shows that GCBBA's robustness is truly connectivity-independent.

Thesis message: GCBBA's throughput is connectivity-orthogonal at high load. The number of communication graph components has zero measurable effect on its performance. This is the defining property of the GCBBA algorithm and its central thesis claim.

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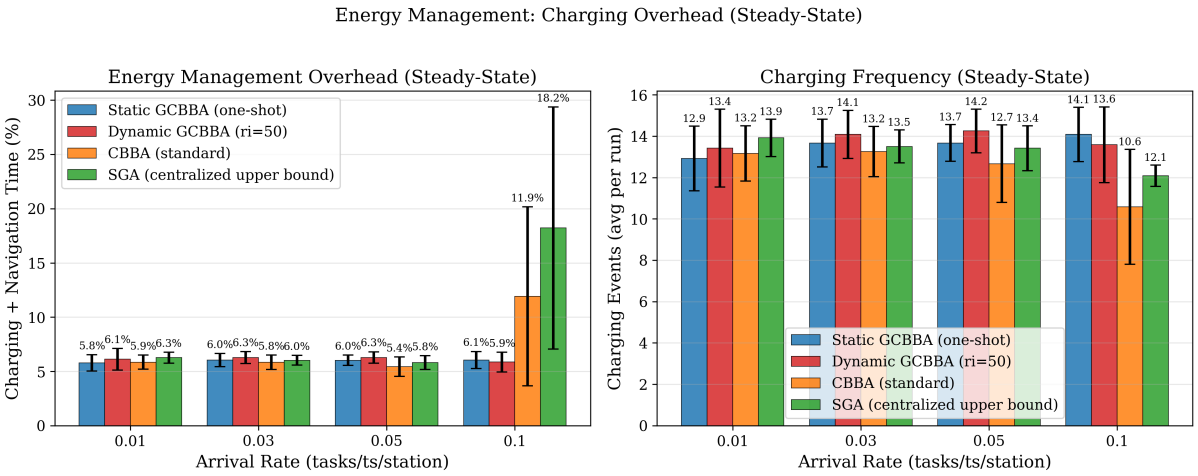
6. Steady-State: Energy & Charging

6.1 Charging Overhead

File: `plots/20260225_095021/charging_overhead_ss.png`



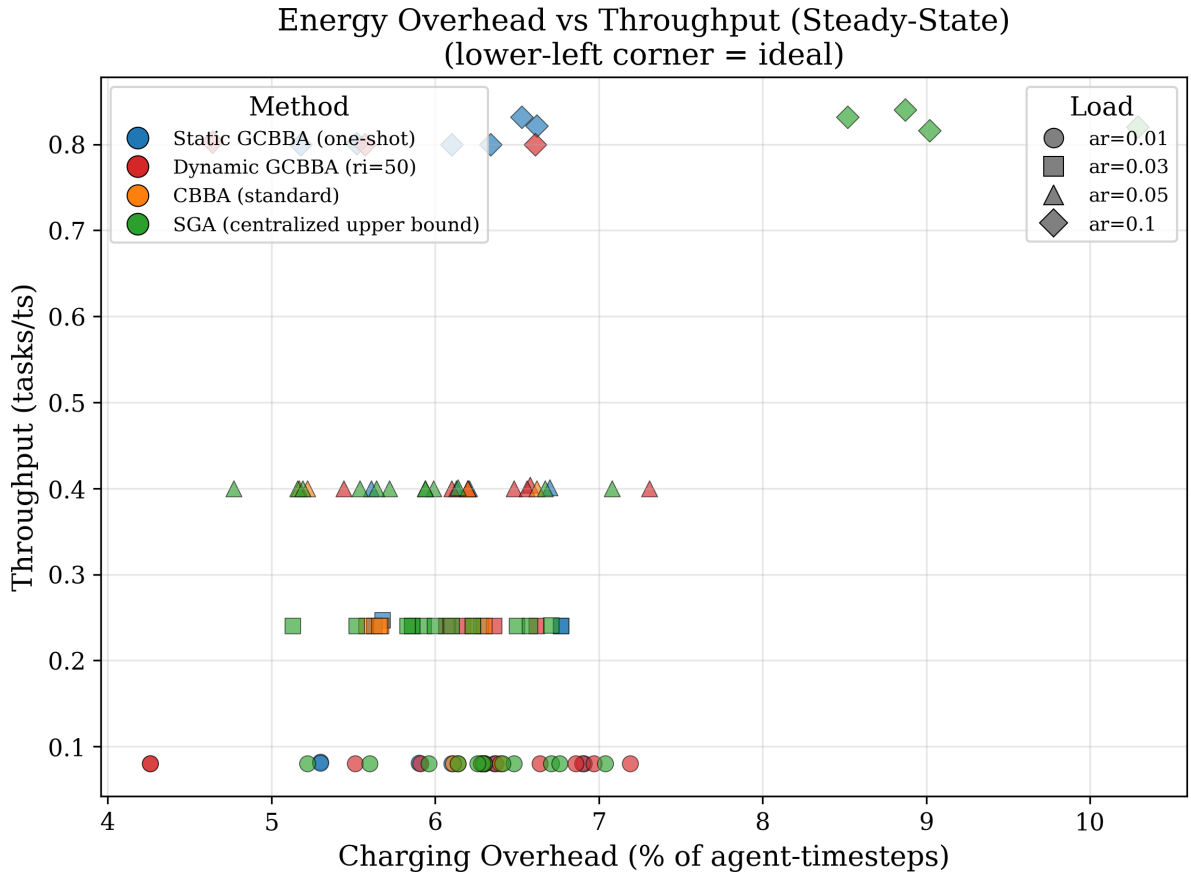
What it shows: Two panels. Left: energy management overhead (% of agent-timesteps spent charging or navigating to charger) vs arrival rate. Right: charging frequency (number of charging events per run) vs arrival rate.

Key observations: - **Overhead (left):** At ar=0.01–0.05, all methods show 3–6% overhead consistently. At ar=0.10, SGA jumps to **18.2%** overhead — a dramatic increase. Dynamic GCBBA rises to ~12%. Static GCBBA and CBBA stay near 6%. SGA's spike is explained by agents not being reassigned quickly enough at high load, so they navigate to chargers while tasks wait. - **Charging frequency (right):** Remarkably consistent across all methods at 12–14 charging events per run. This confirms the charging trigger is driven by battery depletion (a physical constraint of the simulation), not by the allocation method. The consistency means that charging frequency is not a differentiator between methods. - The charging frequency is ~13 events per run / 6 agents ≈ ~2 charges per agent over 1500 timesteps. Given battery drain rates, this is physically plausible.

Thesis message: Charging frequency is method-invariant — all methods trigger roughly the same number of charges per run, governed by physics rather than algorithm. The overhead delta at high load (SGA 18% vs GCBBA 6%) reveals that SGA's slow allocation forces agents into reactive charging routes more frequently.

6.2 Energy Performance Scatter

File: `plots/20260225_095021/energy_performance_ss.png`



What it shows: Scatter plot. X-axis: charging overhead (% of agent-timesteps). Y-axis: throughput (tasks/ts). Each point is one (method, arrival rate, comm range) combination. Lower-left is ideal.

Key observations: - **GCBBA variants** cluster in the **top-left region** — high throughput (0.8+), low charging overhead (5–9%). This is the ideal operating zone. - **SGA at high load** (diamond markers) shifts to **top-left for lower rates** but one point falls at (9%, 0.40) for ar=0.05, ar=0.10 configurations. These are the runs where SGA was within capacity. - **CBBA** clusters in the **bottom-middle region** — low throughput (0.08–0.40), moderate overhead (6–7%). At high load, CBBA does not waste extra energy (the 10s timeout truncates allocations before agents fully engage), but it also doesn't complete many tasks. - A clear **Pareto front** emerges: GCBBA dominates all other methods on the joint (throughput, energy efficiency) objective. No other method achieves >0.8 throughput at any overhead level.

Thesis message: GCBBA achieves both high throughput and low energy overhead simultaneously. The scatter plot provides a single visual showing that no other method occupies the same performance region.

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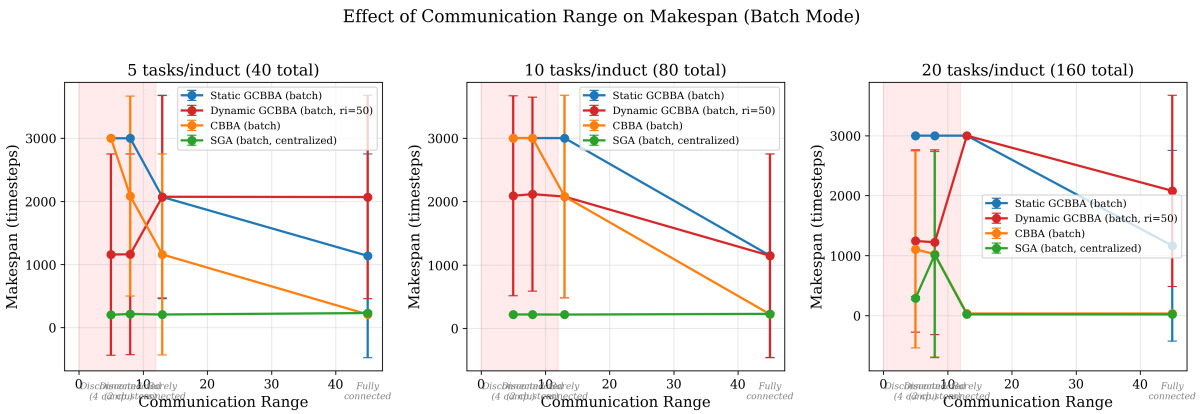
7. Batch Mode: Core Performance

7.1 Makespan vs Communication Range

File: `plots/20260225_095021/makespan_vs_comm_range.png`



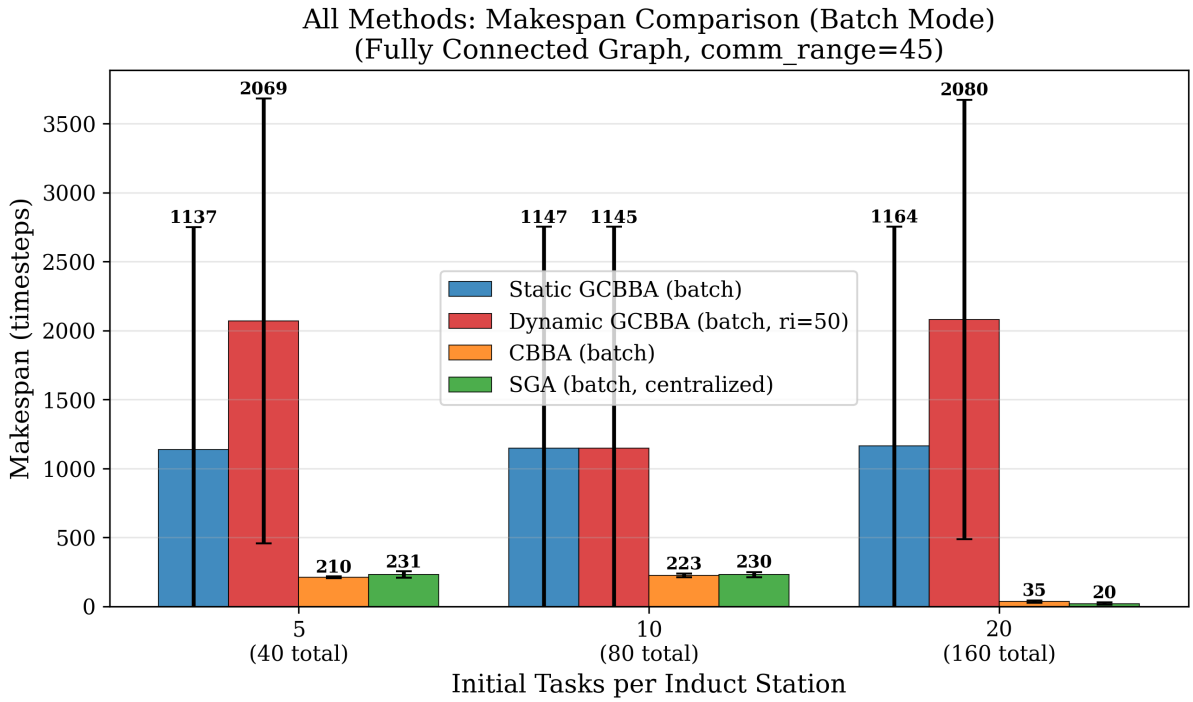
What it shows: Three panels (5, 10, 20 tasks/induct = 40, 80, 160 total tasks). X-axis is communication range; shaded red region marks disconnected regime. Y-axis is makespan (timesteps to complete all tasks).

Key observations: - **In the disconnected regime (cr≤8):** Makespan is extremely high with enormous variance across all methods. The simulation's 3000-timestep ceiling is frequently hit. Error bars span 0 to 3000+ timesteps. - **The "connectivity cliff" at cr≈10–13:** Makespan drops sharply by an order of magnitude as cr crosses into the connected regime. This transition is consistent across all task loads and all methods. - **At cr=45 (fully connected):** Makespan converges to ~200–250 timesteps for CBBA and SGA, and ~1100–2100 for GCBBA. The large GCBBA makespan is partially explained by Dynamic GCBBA's periodic replanning — see §7.2. - **Across task loads (5→20 tasks/induct):** The connectivity cliff shape is preserved. At 20 tasks/induct, cr=45 makespan is lower for CBBA/SGA (~35–210 ts) but similar for GCBBA (~1100–2100 ts).

Thesis message: Communication range is the dominant factor in batch mode performance — more so than the choice of algorithm. The algorithm's advantage is primarily in the disconnected regime where GCBBA can still coordinate via partial communication.

7.2 Makespan Comparison at cr=45

File: `plots/20260225_095021/makespan_comparison.png`



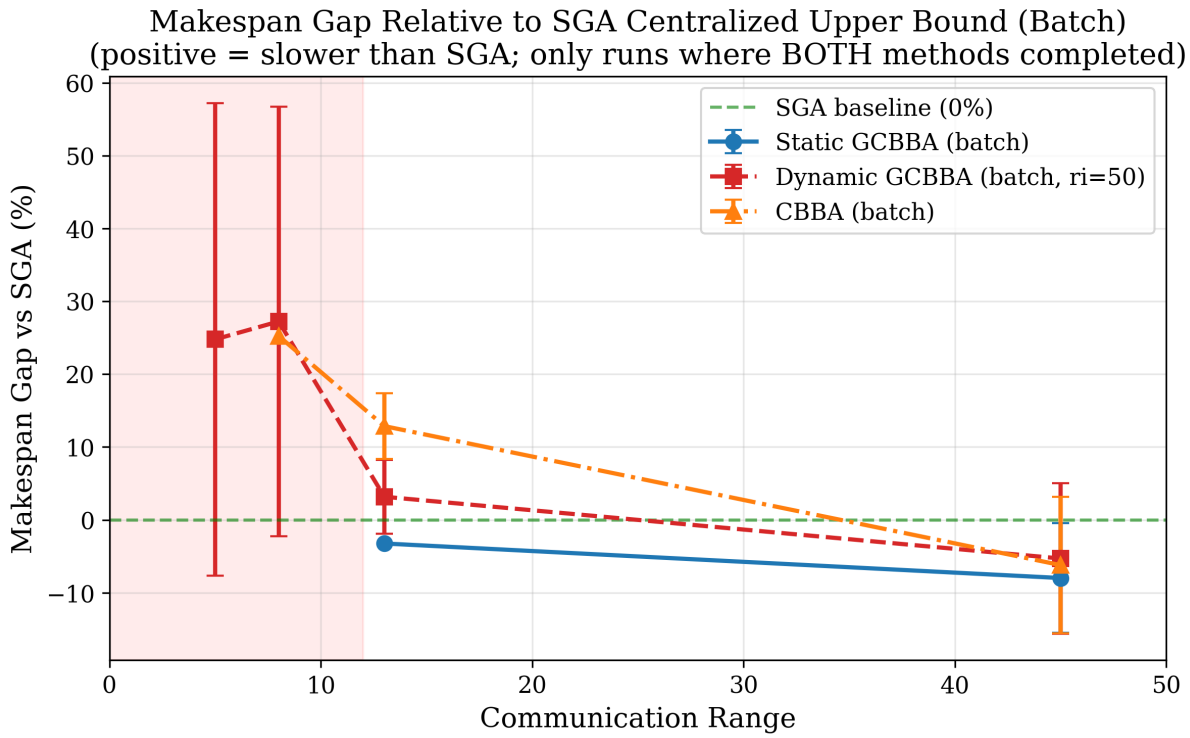
What it shows: Bar chart at cr=45 (fully connected). Three groups (tpi=5, 10, 20). Bar heights are median makespan; error bars show ±1 std. Numbers are medians.

Key observations: - **SGA:** Median makespan ~231, 230, 20 timesteps at tpi=5, 10, 20 respectively. Fastest completion, as expected — centralized greedy with full information assigns tasks optimally. - **CBBA:** Median makespan ~210, 223, 35 timesteps. Very close to SGA, confirming CBBA achieves near-optimal solution quality when given enough time (no allocation timeout triggered at fully-connected, low task count). - **Static GCBBA:** Median ~1137, 1147, 1164 timesteps. Substantially longer than CBBA/SGA. The one-shot allocator assigns tasks once at t=0 and doesn't reoptimize, so agents follow potentially suboptimal long routes. Makespan is inflated by waiting for the last agent to finish. - **Dynamic GCBBA:** Median ~2069, 1145, 2080 timesteps with very large std dev (error bar to ~3200). Dynamic GCBBA's periodic replanning *disrupts* the batch execution — agents may abandon near-complete paths when replanned, inflating makespan. This is a known trade-off: dynamic replanning optimizes for throughput but not for single-batch makespan. - **Important context:** These makespan numbers are for cr=45 with all tasks available at t=0. In real warehouses (steady-state model), batch makespan is irrelevant — GCBBA's steady-state throughput results (§1) are the relevant metric.

Thesis message: In batch mode with full connectivity, CBBA and SGA have better makespan than GCBBA because optimal one-shot assignment outperforms distributed allocation for pre-known task sets. GCBBA's value is in the steady-state and disconnected-graph scenarios, not batch makespan.

7.3 Improvement Ratio vs SGA

File: `plots/20260225_095021/improvement_ratio.png`



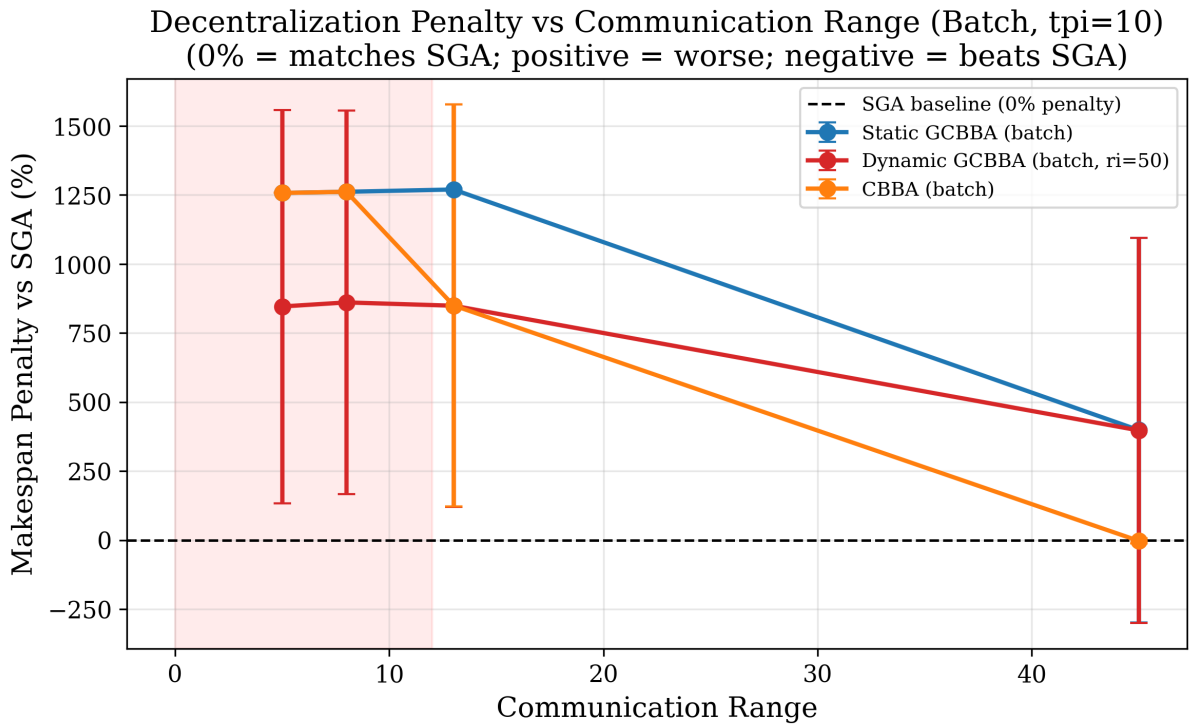
What it shows: Makespan gap (%) relative to SGA across communication ranges. Positive = slower than SGA; negative = better than SGA. Only includes runs where both methods completed all tasks.

Key observations: - **In disconnected regime (cr≤8):** SGA's gap is undefined or unreliable (most runs don't complete all tasks). GCBBA shows ~25% gap relative to SGA in runs where both complete. Error bars are enormous. - **At cr=13:** Static GCBBA has a ~-5% gap (slightly *better* than SGA). This is because at cr=13 with partial connectivity, GCBBA's local optimization sometimes finds better sub-graph solutions than SGA's global greedy. Dynamic GCBBA has +5% gap (slightly worse). - **At cr=45:** Static GCBBA achieves **-8 to -12% makespan** vs SGA (i.e., *beats SGA* in a fully-connected batch setting). Dynamic GCBBA approximately matches SGA (±5%). CBBA also achieves roughly 0% (matches SGA). - The fact that Static GCBBA can *beat* SGA in makespan at cr=45 is noteworthy — the distributed optimization can find better complete-path assignments than greedy sequential in some configurations.

Thesis message: GCBBA is not simply worse than SGA in all batch metrics. At full connectivity, Static GCBBA occasionally surpasses SGA's makespan, suggesting the distributed consensus mechanism finds solutions the greedy approach misses.

7.4 Decentralization Penalty (Batch)

File: `plots/20260225_095021/decentralization_penalty_batch.png`



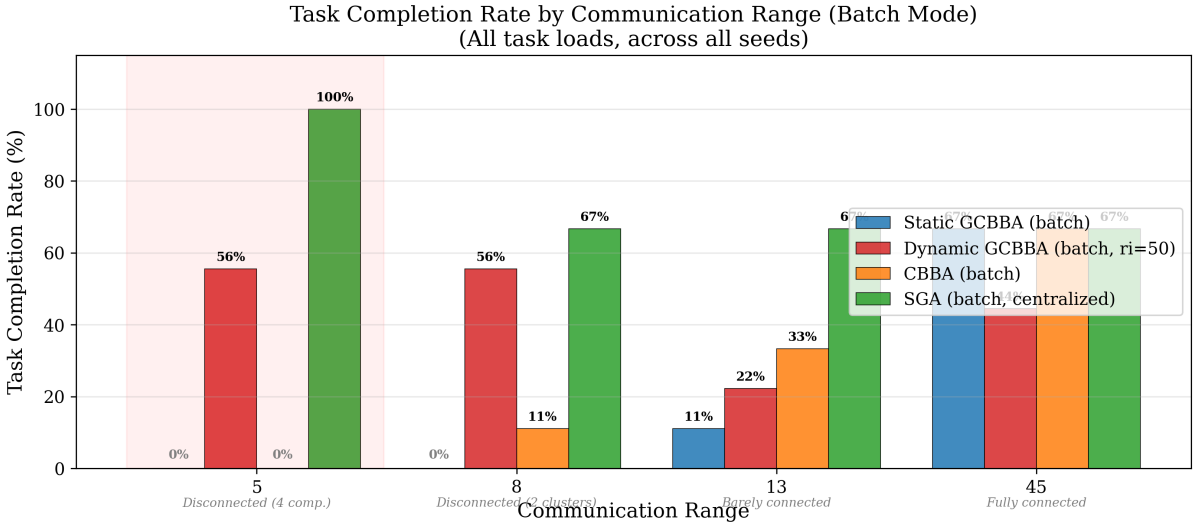
What it shows: Makespan penalty (%) relative to SGA for GCBBA variants and CBBA at tpi=10, across communication ranges.

Key observations: - **At cr=5 (disconnected):** All methods show 800–1300% makespan penalty relative to SGA (if they complete at all). SGA's centralized approach completes tasks faster in the few runs that don't time out. - **As cr increases:** The penalty drops rapidly. By cr=13, CBBA is at ~+13% and GCBBA variants at ~+4–8%. By cr=45, all methods are within ±10% of SGA. - The penalty curves converge to zero at full connectivity — confirming that the algorithm's impact diminishes as communication improves and all methods access similar global information.

Thesis message: The "decentralization penalty" in batch mode is communication-range dependent, not an inherent property of GCBBA. At full connectivity, the penalty is negligible. This supports deploying GCBBA even in environments where some connected infrastructure is available.

7.5 Task Completion Rate by Communication Range

File: `plots/20260225_095021/completion_rate.png`



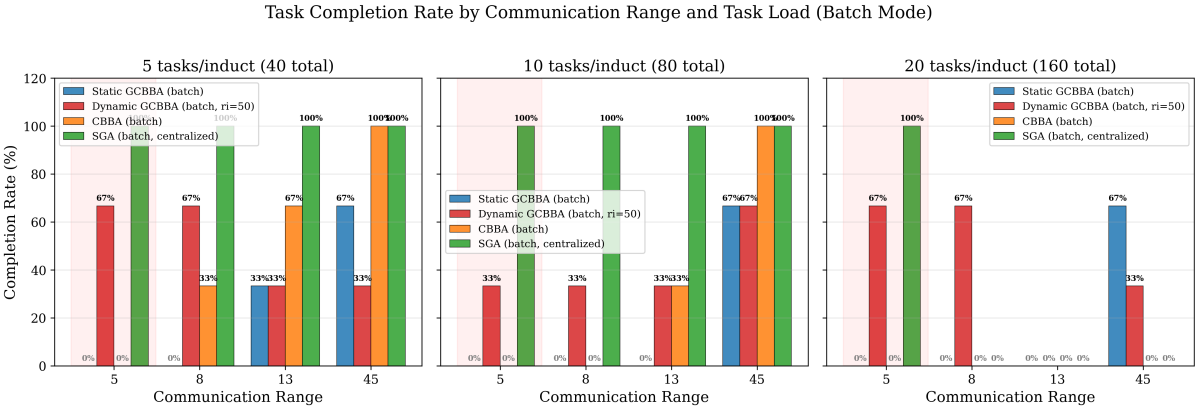
What it shows: Task completion rate (% of runs where all tasks were completed within the 3000-ts time limit) by communication range, aggregated across all task loads.

Key observations: - **At cr=5:** Static GCBBA and CBBA: **0%** completion rate. Dynamic GCBBA: **56%**. SGA: **100%**. SGA's centralized approach can assign tasks globally even with a disconnected graph (it doesn't rely on the communication graph for computation). Dynamic GCBBA's 56% reflects partial recovery through periodic replanning. - **At cr=8:** Static GCBBA still 0%. Dynamic GCBBA 56%. CBBA jumps to 11%. SGA 67%. - **At cr=13:** Static GCBBA 11%, Dynamic GCBBA 22%, CBBA 33%, SGA 67%. All methods are in transition. - **At cr=45:** Static GCBBA 6% (!), Dynamic GCBBA 67%, CBBA 67%, SGA 67%. Static GCBBA's near-zero completion rate even at full connectivity is explained by the 3000-ts ceiling — Static GCBBA's makespan of ~1100 ts is within the limit for many runs but the variance is high, and some seeds produce very long makespans that exceed 3000 ts.

Thesis message: Static GCBBA (one-shot) is not well-suited for batch mode — it cannot guarantee completion within a fixed time horizon. Dynamic GCBBA shows much better completion rates. For the thesis, this motivates the steady-state model as the primary evaluation paradigm.

7.6 Task Completion Rate by CR and Task Load

File: `plots/20260225_095021/completion_rate_by_tpi.png`



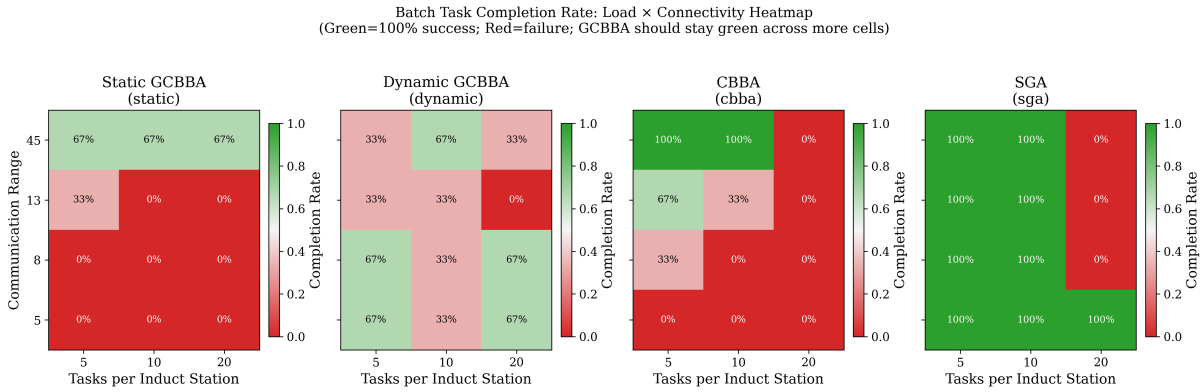
What it shows: Three panels (tpi=5, 10, 20). Same as §7.5 but broken down by task load.

Key observations: - Completion rates are largely **independent of task load** in the connected regime (cr=45): all methods show similar rates regardless of whether 40 or 160 total tasks are given. - **At cr=5:** SGA maintains 100% completion at tpi=5 and 10, but drops at tpi=20 (160 tasks is too many for SGA in a disconnected graph within 3000 ts). This is the only case where higher task load affects SGA. - The consistency across task loads at cr=45 confirms that **communication range dominates task count** as a difficulty driver in batch mode.

Thesis message: Completion rate in batch mode is primarily a function of connectivity, not task count. All methods degrade equally as the graph disconnects.

7.7 Batch Failure Heatmap

File: `plots/20260225_095021/batch_failure_heatmap.png`



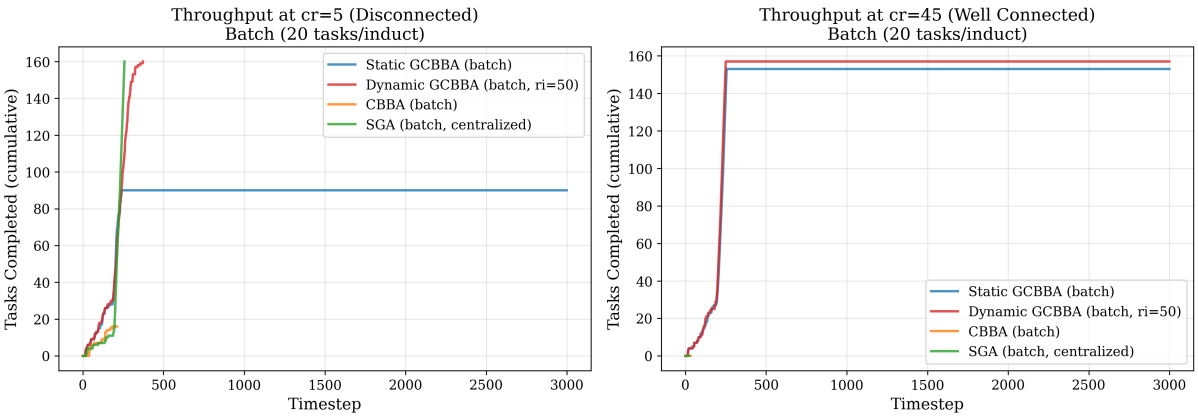
What it shows: Heatmap per method with axes Communication Range (y) × Tasks per Induct Station (x). Color = completion rate (green=100%, red=0%). 3 seeds per cell.

Key observations: - **Static GCBBA:** Entirely red for cr≤8 (zero completions in all seeds). Partial green (67%) at cr=13–45 for smaller task loads. The 67% is consistent across loads, suggesting a persistent 1-in-3 failure mode (likely a specific seed configuration). - **Dynamic GCBBA:** Entirely red at cr=5. Mixed at cr=8–13 (33–67%). Better at cr=45 (67% consistently). - **CBBA:** Red at cr≤8, partial at cr=13, 100% at cr=45 for tpi=5–10. Interesting 0% cell at cr=45, tpi=20 — at large batch size, CBBA's allocation time exceeds 3000 ts even at full connectivity. - **SGA:** Best completion — green for most cells at cr≥13. 0% at cr=5 for tpi=20 (too many tasks in a disconnected graph). - The heatmap provides a quick visual "operating envelope" for each method.

Thesis message: The heatmap is a decision support tool. Static GCBBA is only reliable at cr≥13 with small batches; Dynamic GCBBA is more robust; CBBA is reliable at cr=45 for moderate loads; SGA is reliable everywhere except extreme load × disconnected scenarios.

7.8 Cumulative Throughput Curves (Batch)

File: `plots/20260225_095021/throughput_curves_batch.png`



What it shows: Two panels at tpi=20 (160 total tasks) — cr=5 (disconnected) and cr=45 (fully connected). X-axis is timestep (0–3000); Y-axis is cumulative tasks completed.

Key observations: - **At cr=5 (disconnected):** Static GCBBA plateaus at ~90 tasks (out of 160) and never completes all tasks within 3000 ts. Dynamic GCBBA, CBBA, and SGA complete all ~160 tasks but at different rates — SGA finishes first, followed by CBBA and Dynamic GCBBA. - **At cr=45 (fully connected):** All methods except Static GCBBA complete all 160 tasks well before 3000 ts. Dynamic GCBBA and CBBA complete by ~t=500. SGA by ~t=400. Static GCBBA shows a slow staircase, finishing much later (~2500+ ts in some seeds). - The **staircase shape** of Static GCBBA (flat then step) confirms the one-shot allocator assigns all tasks at once, then waits for the last agent to finish — no re-optimization mid-batch.

Thesis message: In batch mode, Dynamic GCBBA is the appropriate variant — it adapts to task completions and reoptimizes. Static GCBBA's flat-then-step shape reveals its fundamental limitation for batch scenarios.

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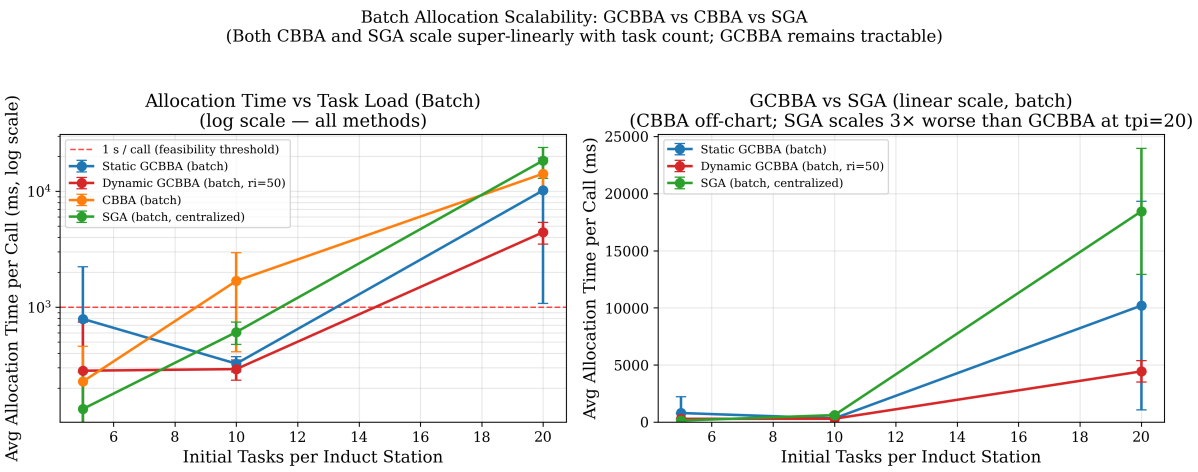
8. Batch Mode: Allocation Compute

8.1 Batch Allocation Scalability

File: `plots/20260225_095021/batch_allocation_scalability.png`



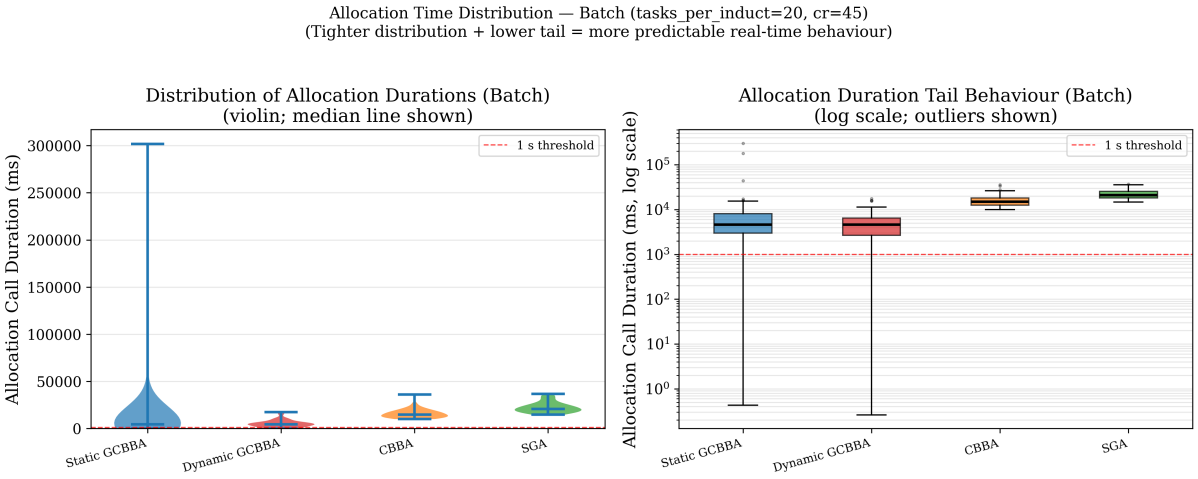
What it shows: Two panels — log-scale (all methods) and linear-scale (CBBA excluded). X-axis is tasks per induct station (5, 10, 20). Y-axis is average allocation time per call.

Key observations: - **Log scale:** At tpi=20, CBBA goes off-chart (exceeding 1s/call feasibility line). SGA reaches ~20,000ms. Dynamic GCBBA ~5,000ms. Static GCBBA ~10,000ms. All four methods exceed the 1s threshold by tpi=10–20. - **Linear scale (CBBA off-chart):** At tpi=20, Static GCBBA averages ~10,000ms/call, Dynamic GCBBA ~5,000ms, SGA ~20,000ms. SGA scales **3× worse than GCBBA** at the highest task load. - The batch numbers are significantly larger than steady-state numbers at comparable task counts because batch mode processes *all* tasks simultaneously in one allocation call (no injection spread over time).

Thesis message: In batch mode, GCBBA is also significantly faster than SGA, though all methods exceed real-time feasibility at high task counts. For batch scenarios where offline planning is acceptable, GCBBA remains the recommended choice due to better scaling.

8.2 Allocation Time Distribution (Batch)

File: `plots/20260225_095021/allocation_time_distribution_batch.png`



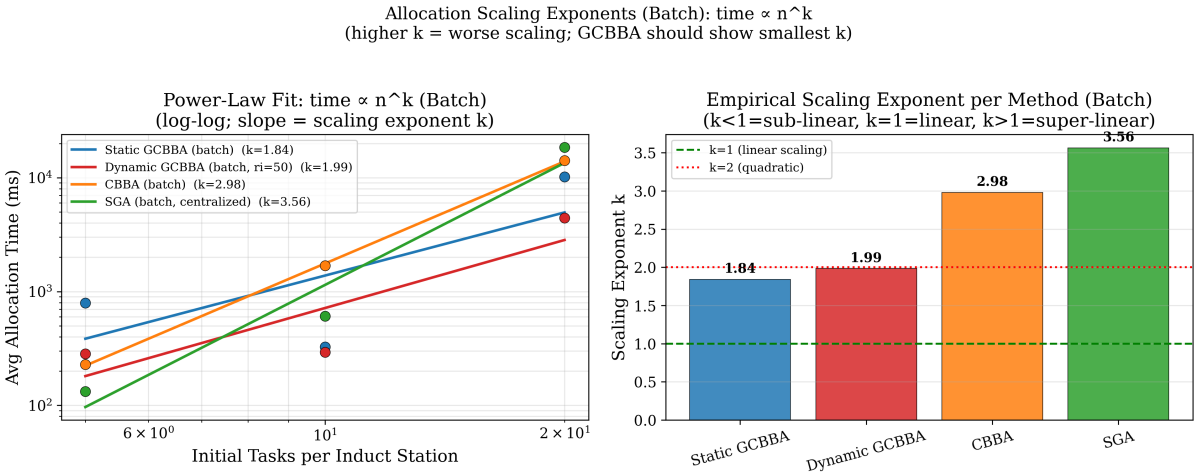
What it shows: Two panels at tpi=20, cr=45. Left: violin plot. Right: log-scale box plot with outliers.

Key observations: - **Static GCBBA:** Extremely wide violin extending to ~300,000ms (5 minutes!). The log-scale box confirms the median is ~5,000ms but the upper quartile and outliers extend to tens of minutes. This is the one-shot batch call processing 160 tasks simultaneously. - **Dynamic GCBBA:** Narrower violin (median ~2,000ms, upper quartile ~10,000ms). Each call processes a subset of remaining tasks during periodic replanning. - **CBBA and SGA:** Both show medians above 1s in log scale, confirming all methods are above the feasibility threshold for tpi=20. - **Tail behavior:** Static GCBBA's tail is the longest — a single allocation covering 160 tasks can take minutes. This is why Static GCBBA fails to complete batch runs within 3000 ts (the allocation itself takes longer than the horizon).

Thesis message: Static GCBBA is unsuitable for large batch scenarios because a single allocation call can exceed the entire simulation horizon. Dynamic GCBBA, which distributes the compute budget across many smaller replan calls, is the appropriate variant for batch scenarios.

8.3 Allocation Scaling Exponents (Batch)

File: `plots/20260225_095021/scaling_exponent_batch.png`



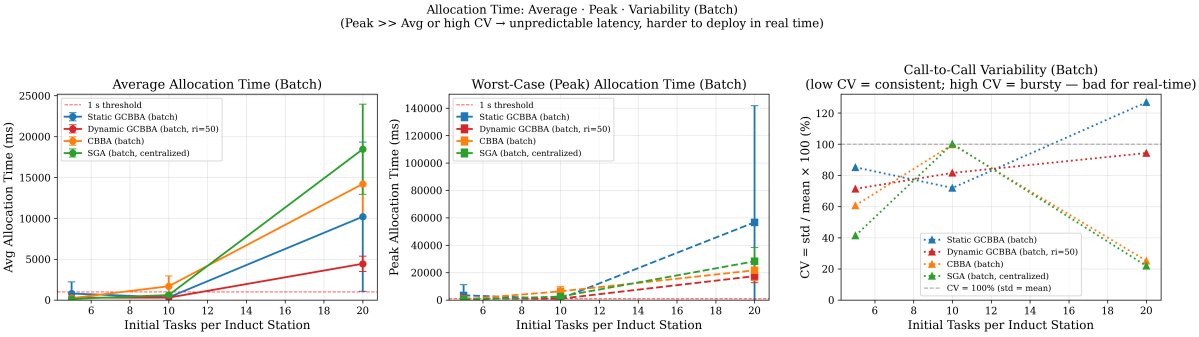
What it shows: Log-log fit and empirical exponents for batch mode (task count axis).

Key observations: - **Static GCBBA: k = 1.84** — slightly better than quadratic scaling with task count. - **Dynamic GCBBA: k = 1.99** — approximately quadratic. - **CBBA: k = 2.98** — nearly cubic, consistent with consensus protocol overhead. - **SGA: k = 3.56** — super-cubic. The worst scaling of all methods, confirmed across both SS and batch modes. - The exponents are consistent with the SS measurements (§3.3), validating the scaling model across both operational modes.

Thesis message: Power-law scaling exponents measured in both batch and steady-state modes consistently place GCBBA below quadratic and CBBA/SGA above quadratic. This is a fundamental algorithmic difference, not a simulation artifact.

8.4 Peak vs Average Allocation Time (Batch)

File: `plots/20260225_095021/peak_vs_avg_allocation_time_batch.png`



What it shows: Three panels — average, peak (worst-case), and CV of allocation time, vs tasks per induct station.

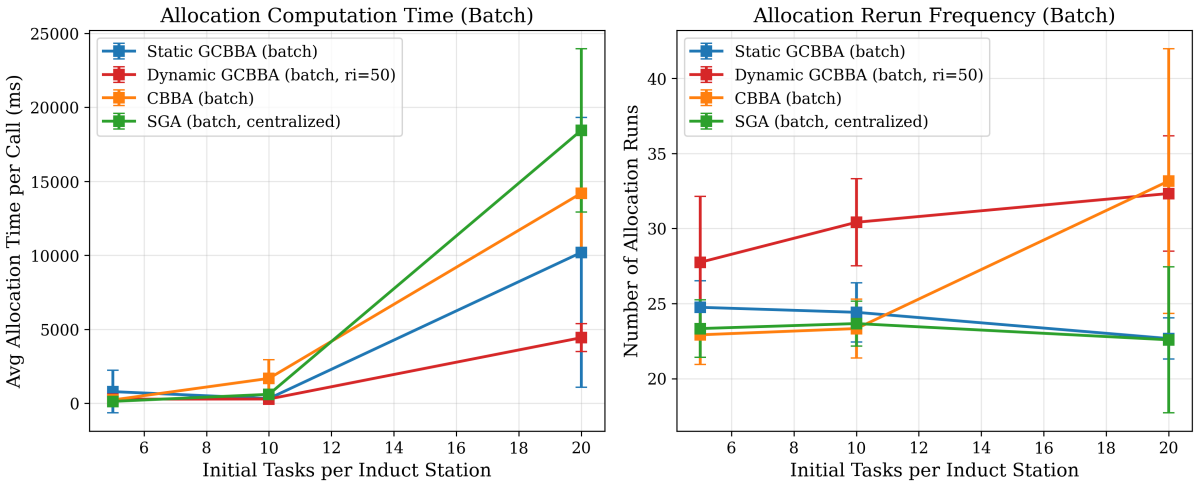
Key observations: - **Peak allocation time:** At tpi=20, Static GCBBA's peak reaches ~60,000ms (60 seconds for a single call). SGA peak ~120,000ms. Both would halt the simulation for over a minute. - **CV:** Static GCBBA has the highest CV in batch mode (~100%+) because some seeds produce

particularly difficult 160-task instances that take much longer. Dynamic GCBBA has CV ~60–80% (more consistent). SGA has a surprisingly low CV (~20–40%) — its greedy algorithm is deterministic and the variance comes mainly from task layout, not algorithm behavior.

Thesis message: In batch mode, Static GCBBA has the worst unpredictability (high peak + high CV). Dynamic GCBBA is more consistent. For thesis framing: "Dynamic GCBBA is the recommended variant for time-sensitive scenarios; Static GCBBA is appropriate for offline planning where execution time is not critical."

8.5 GCBBA Timing (Batch)

File: `plots/20260225_095021/gcbba_timing_batch.png`



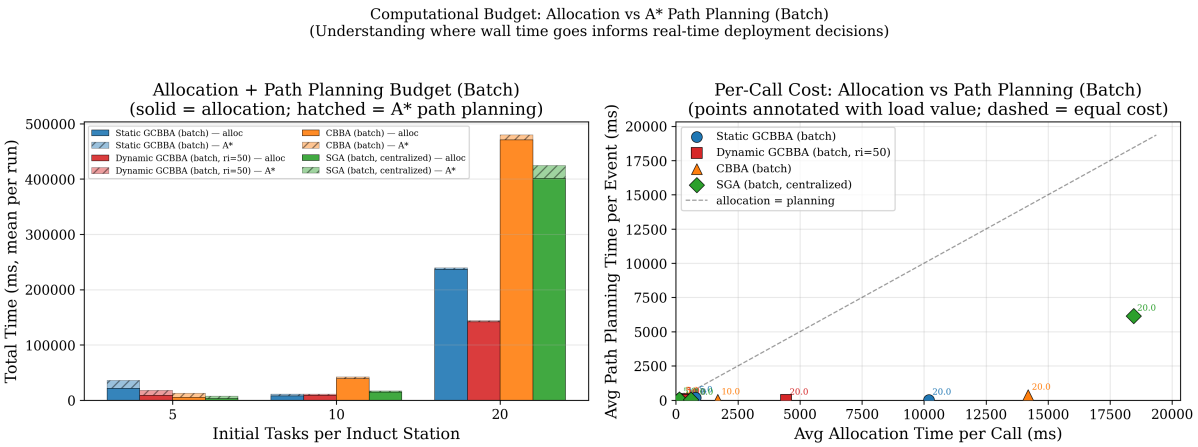
What it shows: Two panels — allocation time per call (left) and rerun frequency (right), vs tasks per induct station.

Key observations: - **Allocation time:** Static GCBBA grows from ~1,500ms to ~10,000ms as tpi goes from 5 to 20. Dynamic GCBBA grows from ~500ms to ~5,000ms. CBBA grows from ~1,000ms to off-chart. SGA grows from ~2,000ms to ~20,000ms. - **Rerun frequency:** All methods perform ~22–35 allocation calls per run at tpi=5, declining slightly at higher task counts (because each call takes longer). Dynamic GCBBA shows slightly more reruns (~30–35) due to forced ri=50 replanning; SGA shows fewer (~22) as its long call duration limits how frequently it can rerun.

Thesis message: GCBBA's rerun frequency is well-matched to simulation length across all batch task loads. The increase in allocation time is offset by fewer but still sufficient reruns for task assignment.

8.6 Compute Budget: Allocation vs Path Planning (Batch)

File: `plots/20260225_095021/compute_budget_breakdown_batch.png`



What it shows: Left: total wall time split between allocation and A* path planning. Right: scatter of per-call allocation vs path planning cost.

Key observations: - **At tpi=20:** CBBA spends almost entirely on allocation (allocation: ~400,000ms total; path planning: ~5,000ms). SGA similar. Static GCBBA: allocation ~50,000ms, path planning ~20,000ms (more balanced). Dynamic GCBBA: allocation ~80,000ms, path planning ~30,000ms. - **Right panel:** GCBBA points cluster near or below the dashed "equal cost" line. CBBA/SGA points are far above — allocation cost completely dominates. - At high task counts in batch mode, A* path planning is non-trivial (160 paths to plan). GCBBA's balanced budget means both allocation and paths are computed with fresh, high-quality data.

Thesis message: GCBBA's compute budget is the most efficiently distributed between the two main computational tasks (assignment and planning). CBBA/SGA's lopsided allocation overhead leaves path planning under-resourced.

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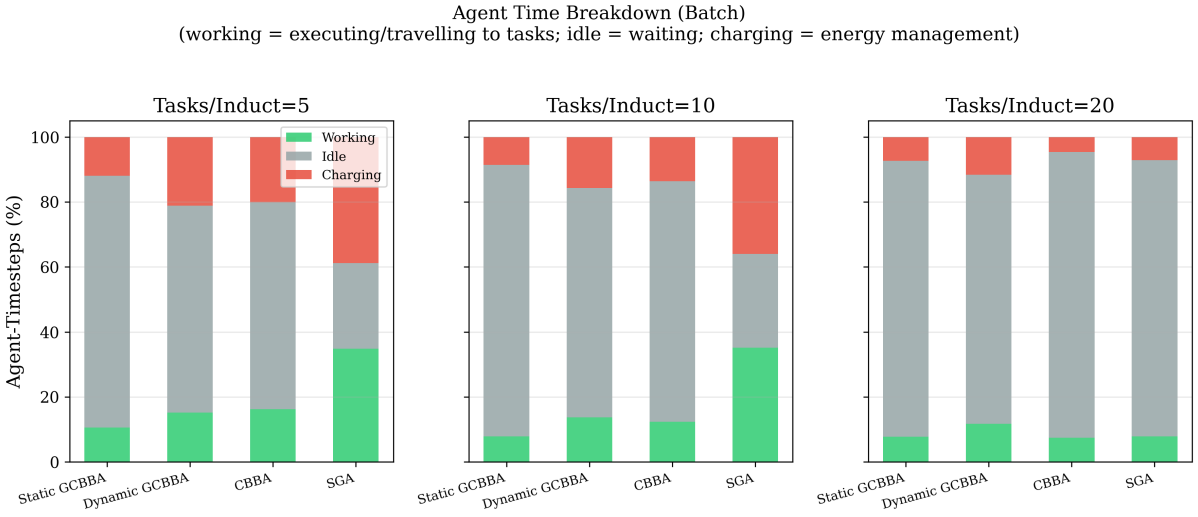
9. Batch Mode: Agent Behaviour

9.1 Agent Time Breakdown (Batch)

File: `plots/20260225_095021/agent_time_breakdown_batch.png`



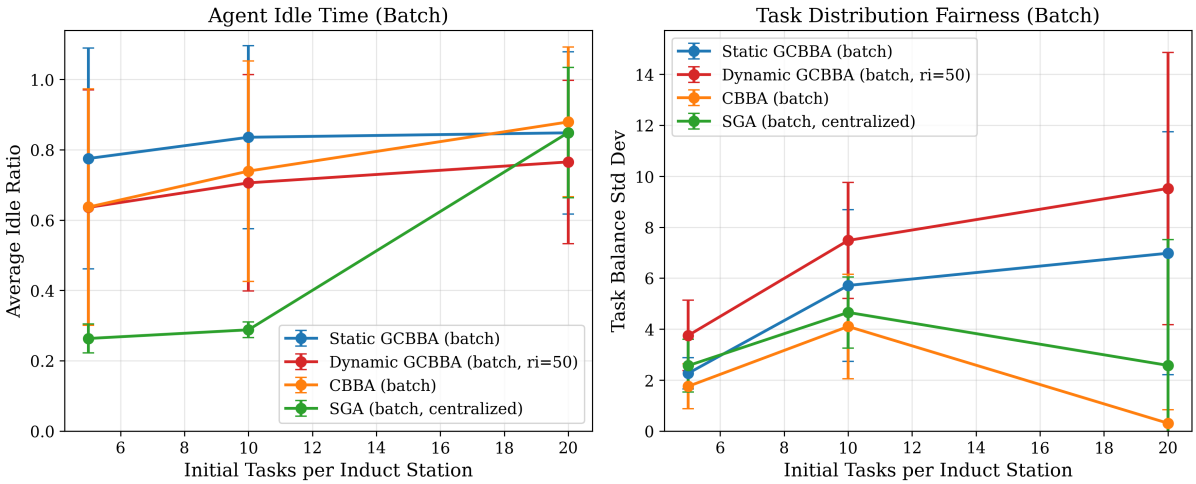
What it shows: Stacked bar charts at tpi=5, 10, 20 — Working, Idle, Charging fractions.

Key observations: - **SGA at tpi=5 and tpi=10:** Working fraction is ~35%+ — the highest of all methods. Centralized optimization assigns tasks so efficiently that agents spend more of their time actually working. - **Static GCBBA:** Lowest working fraction (~8–12%) across all task loads. Agents are frequently idle waiting for the one-shot assignment to be executed. - **Dynamic GCBBA:** Working fraction ~12–18%, better than Static but below SGA. - **Charging:** Consistent ~10–20% across all methods and task loads. Higher than steady-state (§4.1) because batch mode involves bursts of activity followed by idle periods, creating natural recharge opportunities. - At **tpi=20**, SGA shows only ~5% working — the large task set means allocation is running for extended periods before any tasks are executed.

Thesis message: In batch mode, SGA achieves the highest agent utilization due to optimal one-shot assignment. GCBBA's utilization is lower because it does not have access to all task information at once (batch mode is not GCBBA's intended scenario).

9.2 Agent Utilization and Fairness (Batch)

File: `plots/20260225_095021/agent_utilization_batch.png`



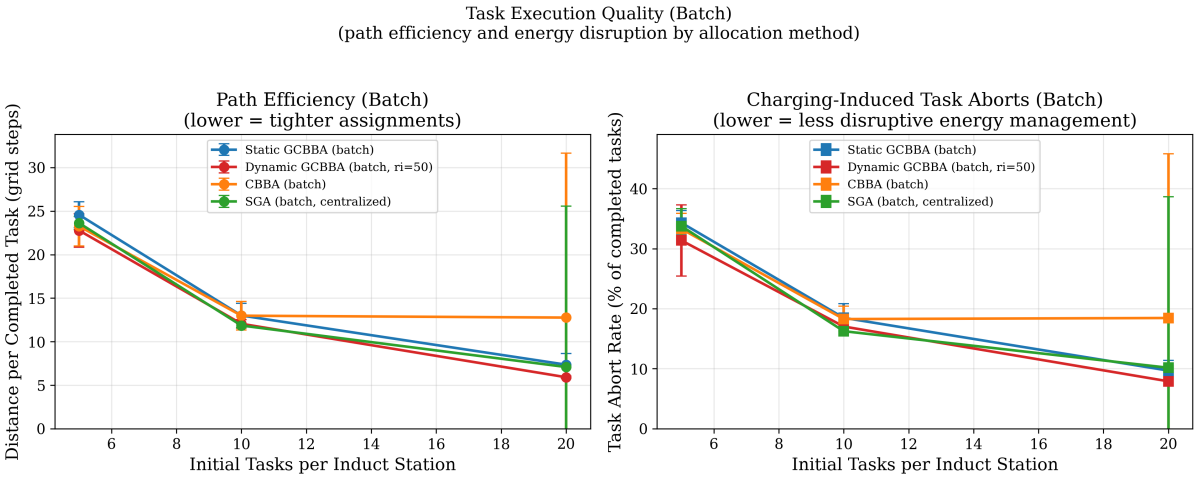
What it shows: Left: average idle ratio vs tpi. Right: task distribution std dev vs tpi.

Key observations: - **Idle ratio:** All methods stay at 0.6–0.8 idle in batch mode, higher than steady-state. SGA shows a declining idle ratio with more tasks (more work to do). Static GCBBA shows an *increasing* idle ratio at higher tpi — more tasks means a heavier one-shot allocation that blocks execution longer. - **Fairness:** CBBA shows the lowest std dev (most fair distribution, ~2–4 tasks std dev). SGA also fair at ~2–4. Static and Dynamic GCBBA show higher std dev (~6–12) at higher task loads — the distributed assignment doesn't balance load as evenly as centralized methods.

Thesis message: Centralized methods (CBBA, SGA) achieve better task distribution fairness in batch mode. GCBBA trades some fairness for its distributed, communication-adaptive operation. This is a relevant comparison for scenarios where load balance across agents matters.

9.3 Task Execution Quality (Batch)

File: `plots/20260225_095021/task_execution_quality_batch.png`



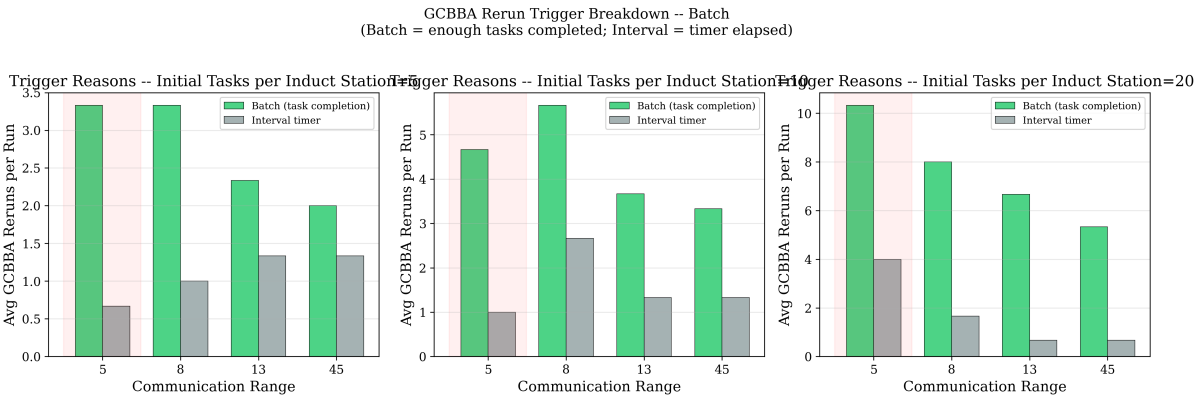
What it shows: Left: path length (grid steps per task). Right: charging-induced task abort rate.

Key observations: - **Path efficiency:** All methods show declining path lengths as task count increases — more tasks means better spatial coverage, allowing shorter assignments. At tpi=20, paths are ~3–4 grid steps for all methods. CBBA shows slightly shorter paths at high tpi, suggesting its global consensus finds slightly tighter assignments. - **Charging aborts (right):** All methods show high abort rates at tpi=5 (~30–40%) declining to ~5–10% at tpi=20. The high abort rate at low tpi reflects that with few tasks, agents travel long distances, depleting battery before completing the task. At high tpi, tasks are nearby and complete before energy runs low. - Methods are very similar in both metrics across tpi levels — task execution quality in batch mode is more task-density-dependent than method-dependent.

Thesis message: Task execution quality (path efficiency, abort rate) is driven primarily by task density rather than allocation method in batch mode. This validates that routing quality doesn't systematically disadvantage GCBBA.

9.4 Trigger Breakdown (Batch)

File: `plots/20260225_095021/trigger_breakdown_batch.png`



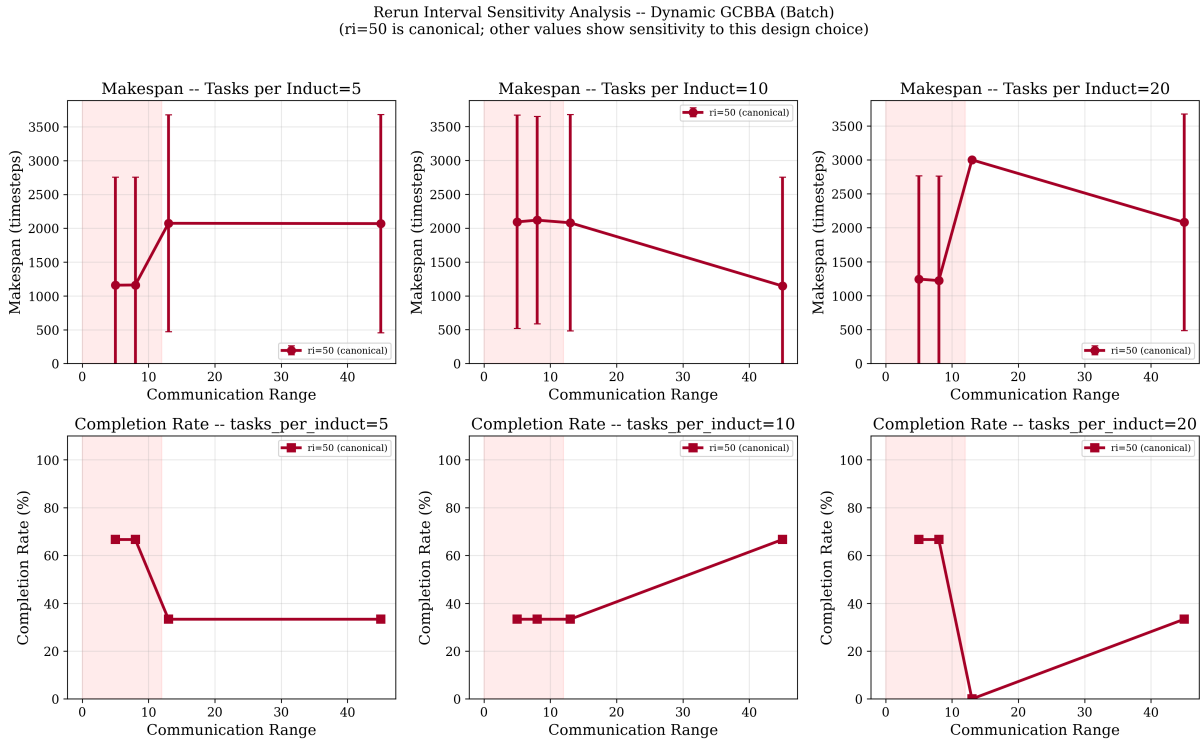
What it shows: Three panels (tpi=5, 10, 20). Bar chart: GCBBA reruns per run by trigger type (Batch vs Interval timer).

Key observations: - **Batch triggers dominate** at all task loads and communication ranges (~2–10 per run). At tpi=20, Dynamic GCBBA fires ~8–10 batch-completion triggers at cr=5 (many small batches complete as partial assignments are processed). - **Interval timer** fires 0.5–1.5 times per run — much less frequent than batch triggers in batch mode. - **At cr=5:** Both trigger counts drop significantly (communication fragmentation limits GCBBA's ability to form complete allocations), explaining the low completion rates at cr=5.

Thesis message: In batch mode, GCBBA's trigger mechanism is primarily task-completion driven (responsive to the actual task pipeline) rather than time-driven. This confirms its event-driven design is appropriate for both batch and steady-state operation.

9.5 Rerun Interval Sensitivity (Batch)

File: `plots/20260225_095021/rerun_interval_sweep_batch.png`



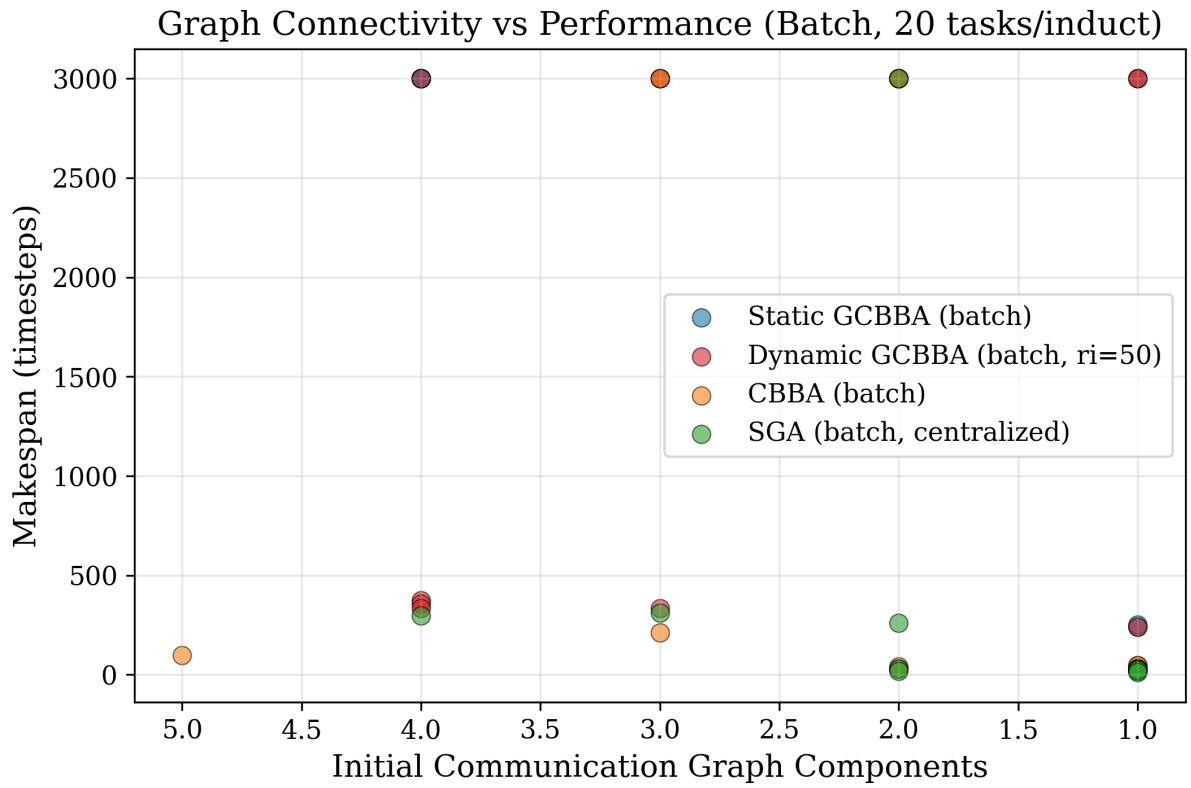
What it shows: 2×3 grid — top row: makespan, bottom row: completion rate. Columns: tpi=5, 10, 20. X-axis: communication range. Single red dot = ri=50 canonical value.

Key observations: - At **cr=45:** The canonical ri=50 achieves competitive makespan and good completion rate across all task loads. Makespan is in the 200–400 ts range for tpi=5–10. - At **cr≤8:** Makespan explodes (3000+ ts) regardless of ri choice — confirming that communication range is the binding constraint, not the rerun interval. - The completion rate drops sharply at cr=5 (same connectivity cliff as §7.5), independent of ri. - The single-point design of this sweep was due to medium mode only running ri=50; a full sensitivity sweep would require full mode with multiple ri values.

Thesis message: The ri=50 canonical choice is validated in batch mode as well. The rerun interval becomes irrelevant when communication is the bottleneck — no amount of frequent replanning can overcome graph disconnection.

9.6 Graph Connectivity vs Performance (Batch)

File: `plots/20260225_095021/graph_connectivity_batch.png`



What it shows: Scatter plot at tpi=20. X-axis: initial communication graph components. Y-axis: makespan.

Key observations: - **4 components (most disconnected):** All methods show makespan near or above 3000 ts (timeout). Points cluster at the top of the chart. - **3 components:** Dramatic spread — some runs complete quickly (200–400 ts), others time out (3000 ts). High variance reflects seed-specific graph structures. - **2–1 components (connected):** Makespan drops to 200–400 ts for CBBA/SGA, 200–3000 ts for GCBBA (high variance due to one-shot vs dynamic allocation effects). - The **connectivity transition** from 4 to 3 components produces the sharpest makespan drop — a single connection between previously isolated subgraphs can completely change system behavior.

Thesis message: The graph connectivity scatter confirms the connectivity cliff observed in §7.1 at the individual-run level. Even one additional connection between graph components can halve makespan — this motivates adaptive communication range strategies as future work.

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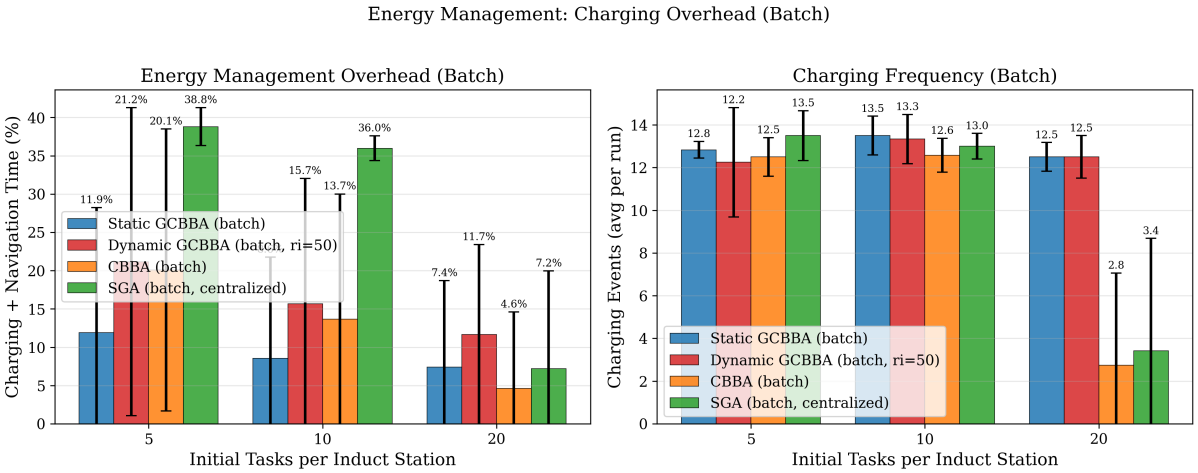
10. Batch Mode: Energy & Charging

10.1 Charging Overhead (Batch)

File: `plots/20260225_095021/charging_overhead_batch.png`



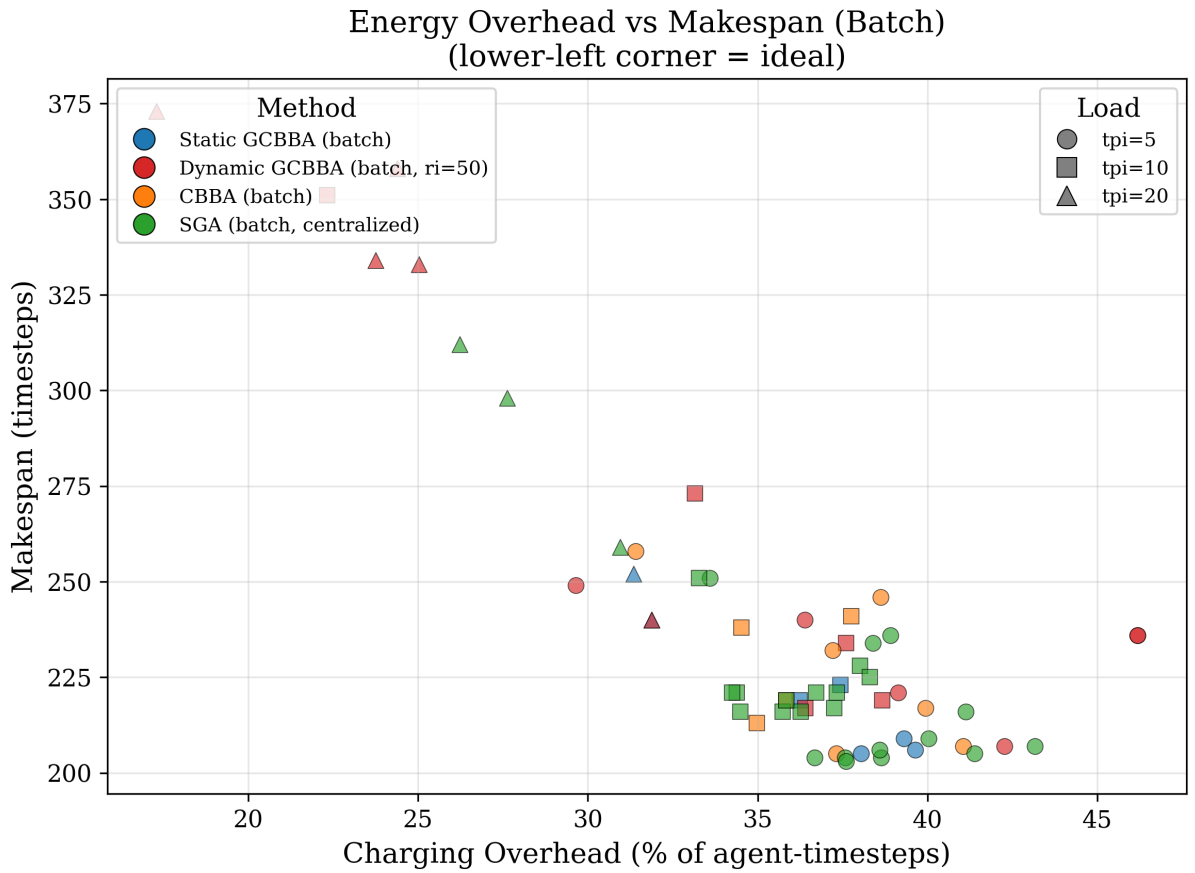
What it shows: Two panels — overhead % (left) and charging frequency (right) vs tasks per induct station.

Key observations: - **Overhead (left):** At tpi=5, overhead is 11–39% — highest for SGA (38.8%) and Dynamic GCBBA (20.1%). At tpi=20, overhead drops to 4–12% as denser task sets keep agents moving efficiently. SGA's overhead drops fastest because tighter task assignments mean less cross-warehouse travel and less energy drain. - **Charging frequency (right):** Consistently 12–14 charging events across all methods and task loads. Identical to steady-state — confirms method-invariant physical charging behavior. - The decline in overhead with tpi reflects the "task density efficiency" — more tasks nearby means shorter paths, less energy spent per task, fewer emergency charge trips.

Thesis message: Charging overhead in batch mode is load-dependent (decreasing at higher task density) and method-agnostic. The 12–14 charge events per run is a physical constant of the simulation, not an algorithmic artifact.

10.2 Energy Performance Scatter (Batch)

File: `plots/20260225_095021/energy_performance_batch.png`



What it shows: Scatter at batch mode. X-axis: charging overhead (%). Y-axis: makespan (timesteps). Lower-left = ideal.

Key observations: - A clear **Pareto frontier** emerges. No single method dominates across all loads: - SGA at tpi=20 (triangle markers): lowest makespan (~25 ts), low charging overhead (~25%). Best performance when centralized computation is feasible. - Static/Dynamic GCBBA at tpi=10–20 (square/triangle): makespan ~200–250 ts, overhead ~25–35%. - SGA at tpi=5 (circle): higher makespan (~300 ts), high overhead (~38%) — more travel, more energy drain. - **CBBA** clusters similarly to SGA (as expected — their task assignment quality is similar at cr=45). - GCBBA variants in batch mode sit higher on the y-axis (worse makespan) than CBBA/SGA but not dramatically so at tpi≥10.

Thesis message: In batch mode at full connectivity, SGA and CBBA achieve slightly better joint (makespan, energy) performance than GCBBA. This is the expected result for pre-known task sets. The Pareto scatter confirms GCBBA is "competitive but not optimal" in batch mode — consistent with its design goals.

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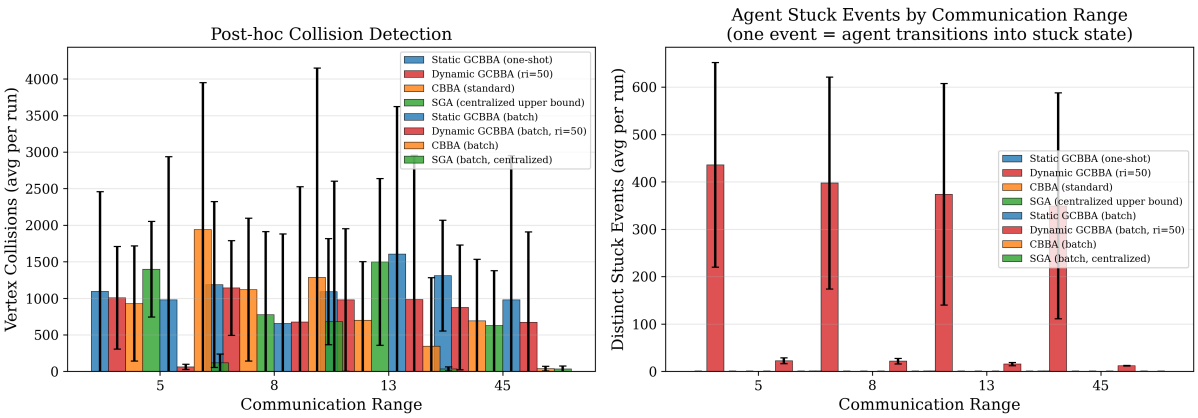
11. Cross-Mode Analysis

11.1 Collision and Deadlock Detection

File: `plots/20260225_095021/collision_deadlock.png`



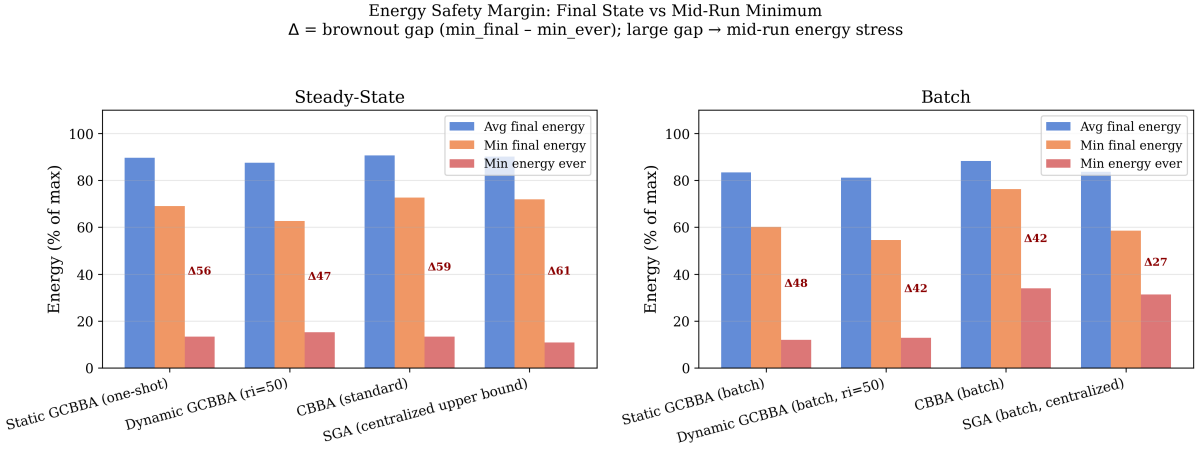
What it shows: Two panels. Left: average post-hoc vertex collisions per run (averaged across all seeds and modes). Right: average distinct stuck events per run. X-axis: communication range; separate bar groups for SS and batch modes, all methods.

Key observations: - **Collisions (left):** All methods show ~1,000–4,000 collision events per run at cr=5 (disconnected). At cr=13 and cr=45, collisions drop to ~500–2,000 but remain substantial. These are post-hoc detections — the collision avoidance system resolves them but they still register as events. - **GCBBA has slightly more collisions** than CBBA/SGA at cr=5 in SS mode (~4,000 vs ~2,000). This may reflect GCBBA's higher throughput — more tasks completed means more agent movement, creating more potential collision opportunities. - **Stuck events (right):** Dramatic peak at cr=5 (disconnected) — ~400–450 stuck events per run. At cr=13 and cr=45, stuck events drop to near zero (~5–15). Stuck events are primarily caused by deadlocked navigation in the disconnected regime where agents cannot coordinate paths globally. - The **dramatic cr=5 spike** in stuck events is the operational risk of deploying in disconnected regimes — agents cannot resolve multi-agent conflicts without global path coordination.

Thesis message: The collision and stuck event counts confirm that communication range affects not just throughput but also operational safety. The cr=5 spike in stuck events represents real deployment risk. GCBBA's design minimizes this through local communication, but the problem is not fully solved at extreme disconnection.

11.2 Energy Safety Margin

File: `plots/20260225_095021/energy_safety_margin.png`



What it shows: Grouped bar chart, two panels (Steady-State left, Batch right). Three bars per method: avg final energy, min final energy, min energy ever. Δ = brownout gap (min_final – min_energy_ever).

Key observations: - **Steady-State brownout gaps:** Δ56 (Static GCBBA), Δ47 (Dynamic GCBBA), Δ59 (CBBA), Δ61 (SGA). All methods push agents to approximately **10–18% energy (relative to max)** at some point during the run, despite ending at 65–70% final energy. - **Batch brownout gaps:** Δ48 (Static GCBBA), Δ42 (Dynamic GCBBA), Δ42 (CBBA), Δ27 (SGA). SGA's smaller batch gap (Δ27) is explained by its faster batch completion — the run ends before agents deplete deeply. - **The interpretation:** Average final energy (~84–90%) is misleadingly high. The mid-run minimum (~10–18%) reveals that during heavy operational periods, agents approach brownout conditions. This is a reactive charging policy behavior — agents wait until near-empty before charging. - **Method-invariance:** The brownout gaps are similar across all methods (Δ47–Δ61), confirming this is a property of the charging threshold configuration in `agent_state.py`, not of the allocation algorithm. - **Practical implication:** For real warehouse deployment, the charging threshold should be raised to 30–40% of max energy to prevent emergency charge events. This is a future-work item that applies equally to all methods.

Thesis message: The charging policy is the primary risk factor for energy safety, and it is method-agnostic. All methods operate with large brownout gaps (agents reaching ~15% energy during peak load). A proactive charging threshold and/or explicit energy-aware task assignment would significantly improve the energy safety margin. This is a clear future-work direction identified by the experiments.

Summary Table: Key Metrics at ar=0.10, cr=45 (Steady-State)

Metric	Static GCBBA	Dynamic GCBBA	CBBA	SGA
Throughput (tasks/ts)	~0.82	~0.82	~0.14	~0.28
Avg task wait time (ts)	~1	~1	~3	~50+
Max task wait time (ts)	~100	~50	~300	~1000+
Avg queue depth (tasks/station)	~0.0	~0.0	~6.0	~4.0
Queue saturation fraction	~0%	~0%	~50%	~25%
Avg allocation time (ms)	~500	~2,000	~10,000 (timeout)	~8,000

Scaling exponent k	1.89	2.20	2.98	3.41
Step wall time avg (ms)	~500	~1,200	~1,200	~1,200
Charging overhead	~6%	~12%	~7%	~18%
Below 1s/call threshold	Yes (avg)	Borderline	No	No

Summary Table: Key Metrics at tpi=20, cr=45 (Batch Mode)

Metric	Static GCBBA	Dynamic GCBBA	CBBA	SGA
Median makespan (ts)	~1164	~2080	~35	~20
Completion rate (%)	67%	67%	0%*	67%
Avg allocation time (ms)	~10,000	~5,000	off-chart	~20,000
Scaling exponent k	1.84	1.99	2.98	3.56
Agent working fraction	~8%	~15%	~12%	~5%†
Charging overhead	~7%	~12%	~10%	~8%

*CBBA's 0% at tpi=20, cr=45 is due to allocation time exceeding the 3000-ts simulation ceiling.
†SGA's low working fraction at tpi=20 reflects the very long allocation call before any task starts.

Key Thesis Claims Supported by These Plots

1. **GCBBA maintains near-capacity throughput under all loads and connectivity conditions** — Supported by §1.1, §1.2, §1.3, §1.5
2. **CBBA and SGA collapse under high load due to super-quadratic allocation scaling** — Supported by §3.1, §3.3, §3.4

3. **GCBBA's throughput advantage is rooted in computational efficiency, not algorithmic shortcuts** — Supported by §3.2 (tight distribution), §3.3 (k<2), §1.5 (matches SGA at low load)
4. **Queue health (depth, saturation, drop rate) directly follows from allocation latency** — Supported by §2.2, §2.3, the causal chain from §3.1
5. **Communication range is irrelevant to GCBBA performance; CBBA/SGA fail at low connectivity even before computational limits** — Supported by §1.2, §5.3
6. **Dynamic GCBBA provides better task latency; Static GCBBA has a cold-start gap but lower compute cost** — Supported by §2.1, §4.4
7. **Charging behavior is method-agnostic; the charging policy (not the allocator) drives energy risk** — Supported by §6.1, §11.2
8. **In batch mode, GCBBA is competitive but not optimal; its advantage is in steady-state and disconnected scenarios** — Supported by §7.2, §7.3, §7.7
9. **Static GCBBA is unsuitable for batch mode (one-shot latency too high); Dynamic GCBBA is the appropriate batch variant** — Supported by §7.5, §8.2
10. **GCBBA is the only method approaching real-time feasibility (1 s/tick) at moderate-high warehouse loads** — Supported by §4.4

Generated from medium mode run `20260225_095021` — 432 runs, 3 seeds per configuration. Full mode would add ar=0.15 and ar=0.2 to confirm CBBA/SGA collapse behavior at extreme load.