Q.3]

We have,

$$Pr(G = k \mid X = x) = \frac{\exp(\beta_{k0} + \beta_k^T x)}{1 + \sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x)}$$

& $$Pr(G = K \mid X = x) = \frac{1}{1 + \sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x)}$$

Now,

$$\sum_{k=1}^{K} Pr(G = k \mid X = x) = \sum_{k=1}^{K-1} Pr(G = k \mid X = x)$$

$$+ \sum Pr(G = K \mid X = x)$$

$$= \frac{\exp(\beta_{10} + \beta_1^T x)}{1 + \sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x)} + \frac{\exp(\beta_{20} + \beta_2^T x)}{1 + \sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x)}$$

$$+ \dots + \frac{\exp(\beta_{K-1\,0} + \beta_{K-1}^T x)}{1 + \sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x)}$$

$$+ \frac{1}{1 + \sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x)}$$

Let, $$\sum_{i=1}^{K-1} \exp(\beta_{i0} + \beta_i^T x) = Z.$$

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$$= \frac{\sum_{i=1}^{k-1} \exp(\beta_{0i0} + \beta_{i1}^T x)}{1+a} + \frac{1}{1+a}$$

$$= \frac{a+1}{1+a}$$

$$= 1.$$

$\therefore$ Sum of posterior probabilities of ~~a~~ classes is equal to one

b) $$p(x) = \frac{\exp(\beta_0 + \beta_1 x)}{1+\exp(\beta_0 + \beta_1 x)}.$$

$$\therefore \quad p(x)(1 - \exp(\beta_0 + \beta_1 x)) = \exp(\beta_0 + \beta_1 x)$$

$$\therefore \quad p(x) - p(x)(\exp(\beta_0 + \beta_1 x)) = \exp(\beta_0 + \beta_1 x)$$

$$\therefore \quad p(x) = \exp(\beta_0 + \beta_1 x)(1 - p(x))$$

$$\therefore \quad \frac{p(x)}{1-p(x)} = \exp(\beta_0 + \beta_1 x)$$