Coordinate Conversion Formulas

CYLINDRICAL TO RECTANGULAR	SPHERICAL TO RECTANGULAR	SPHERICAL TO CYLINDRICAL
$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$	$r = \rho\sin\phi$
$y = r\sin\theta$	$y = \rho\sin\phi\sin\theta$	$z = \rho\cos\phi$
$z = z$	$z = \rho\cos\phi$	$\theta = \theta$

Corresponding formulas for dV in triple integrals:

$$\begin{aligned} dV &= dx\,dy\,dz \\ &= dz\,r\,dr\,d\theta \\ &= \rho^2\sin\phi\,d\rho\,d\phi\,d\theta \end{aligned}$$

Equations Relating Spherical Coordinates to Cartesian and Cylindrical Coordinates

$$\begin{aligned} &r = \rho\sin\phi, \qquad x = r\cos\theta = \rho\sin\phi\cos\theta, \\ &z = \rho\cos\phi, \qquad y = r\sin\theta = \rho\sin\phi\sin\theta, \\ &\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}. \end{aligned} \tag{1}$$

TABLE 15.2 Moments of inertia (second moments) formulas

THREE-DIMENSIONAL SOLID

About the x-axis:	$I_x = \iiint (y^2 + z^2)\,\delta\,dV$	$\delta = \delta(x, y, z)$
About the y-axis:	$I_y = \iiint (x^2 + z^2)\,\delta\,dV$	
About the z-axis:	$I_z = \iiint (x^2 + y^2)\,\delta\,dV$	
About a line L:	$I_L = \iiint r^2(x, y, z)\,\delta\,dV$	$r(x, y, z) =$ distance from the point (x, y, z) to line L

TWO-DIMENSIONAL PLATE

About the x-axis:	$I_x = \iint y^2\,\delta\,dA$	$\delta = \delta(x, y)$
About the y-axis:	$I_y = \iint x^2\,\delta\,dA$	
About a line L:	$I_L = \iint r^2(x, y)\,\delta\,dA$	$r(x, y) =$ distance from (x, y) to L
About the origin (polar moment):	$I_0 = \iint (x^2 + y^2)\,\delta\,dA = I_x + I_y$	

TABLE 15.1 Mass and first moment formulas

THREE-DIMENSIONAL SOLID

Mass: $M = \iiint_D \delta \, dV$ $\quad \delta = \delta(x, y, z)$ is the density at (x, y, z).

First moments about the coordinate planes:

$$M_{yz} = \iiint_D x \, \delta \, dV, \qquad M_{xz} = \iiint_D y \, \delta \, dV, \qquad M_{xy} = \iiint_D z \, \delta \, dV$$

Center of mass: $\bar{x} = \dfrac{M_{yz}}{M}, \quad \bar{y} = \dfrac{M_{xz}}{M}, \quad \bar{z} = \dfrac{M_{xy}}{M}$

TWO-DIMENSIONAL PLATE

Mass: $M = \iint_R \delta \, dA$ $\quad \delta = \delta(x, y)$ is the density at (x, y).

First moments: $M_y = \iint_R x \, \delta \, dA, \quad M_x = \iint_R y \, \delta \, dA$

Center of mass: $\bar{x} = \dfrac{M_y}{M}, \quad \bar{y} = \dfrac{M_x}{M}$

Average Value of a Function in Space

The average value of a function F over a region D in space is defined by the formula

$$\textbf{Average value} \text{ of } F \text{ over } D = \frac{1}{\text{volume of } D} \iiint_D F \, dV. \tag{2}$$

If $f(x, y)$ and $g(x, y)$ are continuous on the bounded region R, then the following properties hold.

1. *Constant Multiple:* $\displaystyle\iint_R cf(x, y)\,dA = c\iint_R f(x, y)\,dA$ (any number c)

2. *Sum and Difference:*

$$\iint_R \big(f(x, y) \pm g(x, y)\big)\,dA = \iint_R f(x, y)\,dA \pm \iint_R g(x, y)\,dA$$

3. Domination:

(a) $\displaystyle\iint_R f(x, y)\,dA \geq 0$ if $f(x, y) \geq 0$ on R

(b) $\displaystyle\iint_R f(x, y)\,dA \geq \iint_R g(x, y)\,dA$ if $f(x, y) \geq g(x, y)$ on R

4. *Additivity:* If R is the union of two nonoverlapping regions R_1 and R_2, then

$$\iint_R f(x, y)\,dA = \iint_{R_1} f(x, y)\,dA + \iint_{R_2} f(x, y)\,dA$$