UNIT-02

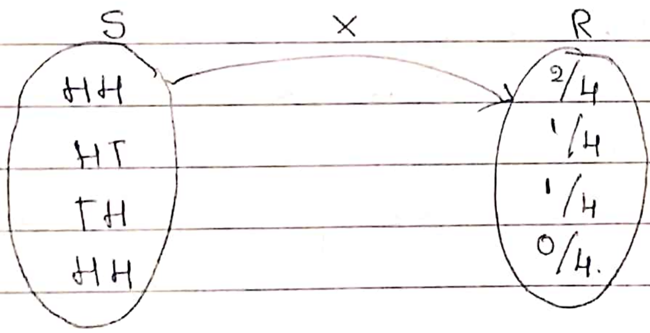
RADOM VARIABLE AND PROBABILTY

Probability distribution is a theoretical Counter part of frequency distribution

Random Variable: In an random Variable experiement if the real Variable is associated with every outcome then it is Called as a random Variable.

A random Variable 'x', On a Sample Space 'S' is a function $x : S \rightarrow R$ to a Set of real no's

eg: On tossing two Coins if random Variable.
X: getting a head then



X	P(x)	$\Sigma p(x)$
2	$2/4$	$2/4$
1	$1/4$	$3/4$
1	$1/4$	1
0	0	1

Random Variable can be classified into:
1) Discrete
2) Continuous

1) Discrete: A random variable x is said to be discrete if the set of all possible outcomes is countable (finite)
eg: 1) Tossing a coin
(ie getting a head)
2) No of children born in a year of city.

2) Continuous: A random variables is said to be " if the sample set 's' contains infinite no of values.
eg: 1) 'X' : no of values on earth.
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→ Probability distribution:
1) Discrete PD: The P.D of a discrete Random Variable 'X' is a list of each possible values of 'x'. to gether with the probability that the x, takes that value in one trial. of the expt. The probabilities that the x, takes that value in one trial of the exp. The probabilities in the distribution must satisfy 2 conditions.
(a) P(x) must be both 0 & 1

$$0 \le P(x) \le 1$$

b) The sum of all probabilities = 1
ie $\Sigma P(x) = 1$
c) Discrete probability function is also called as PMF

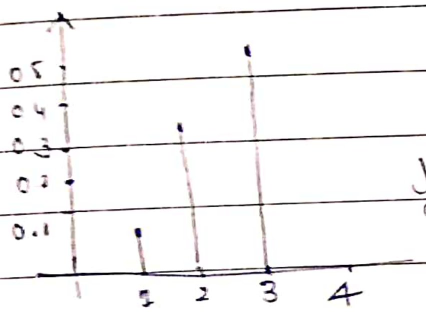
The table is represented as

X	x_1	x_2	...	x_n
$P(x)$	$P(x_1)$	$P(x_2)$	...	$P(x_n)$

they drawn find

eg $P(1) = \frac{1}{6} = 0.16$ $\quad P(3) = \frac{1}{2} = 0.5$

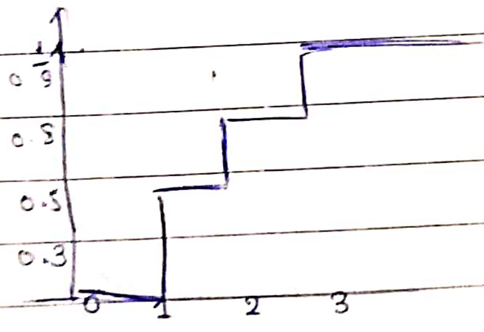
$P(2) = \frac{1}{3} = 0.33$



Cumulative distribution function (c.d.f) The distribution function $f(x)$ defined by.

$f(x) = P(X \leq x) = \sum_{x=1}^{p} P(x_i)$.

eg: $f(x) = \begin{cases} 0 & x < 1 \\ 1/2 & 1 \leq x \leq 2 \\ 5/6 & 2 \leq x \leq 3 \\ 1 & 3 \leq x. \end{cases}$



Mean Variance

1) Mean = $\Sigma x\, P(x)$

2) Variance = $\Sigma (x_i - \mu)^2 \cdot P(x_i)$

$V = \Sigma x_i^2\, P(x_i) - \mu^2$

* Continious P.D. (CPD)

if for every $x \in$ to the range of Continious random Variable (x) is assigned a real eg $f(x)$. that Satisfies the Conditions

$$f(x) \geq 0$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

then $f(x)$ is Called CPP or Pdf.

* If X is a Continious random Variable then the function, $f(x)$ defined by

$F(x) = P(X \leq x) = \int_{-\infty} f(x)\,dx$ is Called a

Cumulative distribution function

NOTE: mean : $\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$.

Variance : $(\sigma)^2 = \int_{-\infty}^{\infty} x^2\, f(x)\,dx - \mu^2$.

Q. Suppose a bag contains 3 white & 9 red balls, 3 balls are randomly chosen from the bag. Obtain a probability distribution for a white ball which is drawn and graph the results.

⇒ X. White balls is drawn   Total balls 12 → 3W, 9R.

$p(x=0)$ = Prob of getting no white balls

$$= \frac{{}^9C_3}{{}^{12}C_3}$$

$p(x=1)$ = ... 1 $= \frac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$ $= 0.3818$ ⇒ 0.4909

$p(x=2)$ = 2 $= \frac{{}^3C_2 \times {}^9C_1}{{}^{12}C_3}$ = 0.1227

$p(x=3)$ = 3 $= \frac{{}^3C_3}{{}^{12}C_3}$ = 0.0045.

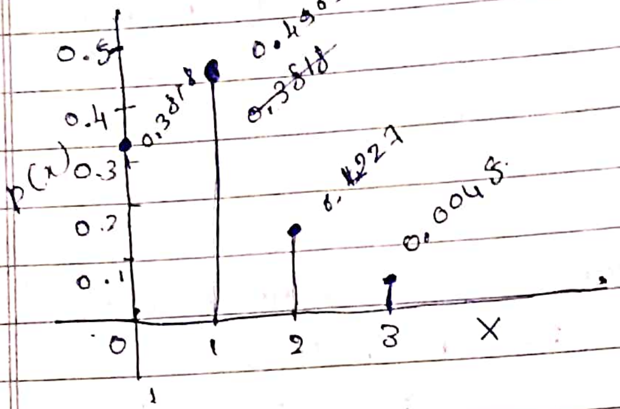
Probability distribution of X.

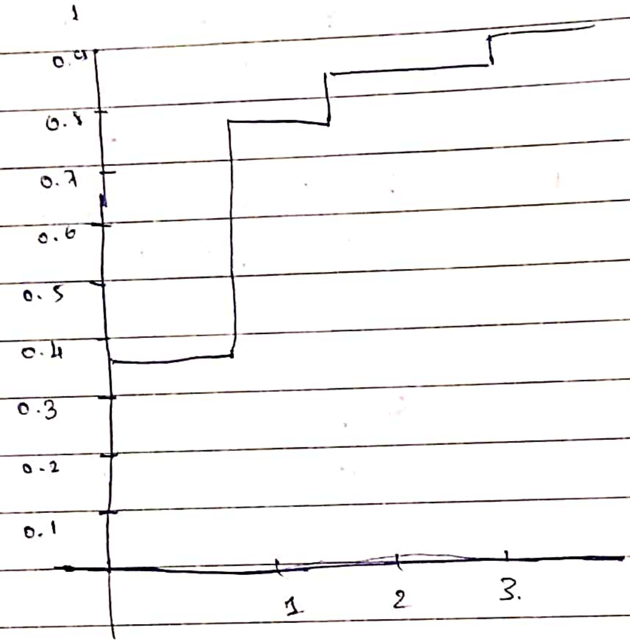
X	0	1	2	3
p(x)	${}^9C_3/{}^{12}C_3$ 0.3818	$\frac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$ = 0.4909	0.1227	0.0045
f(x) =	0.3818	0.8729	0.9954	≈ 1

+0.4904   +0.1227

Same as P.C.F

Prob dist'n Graph of X





Cummulative distn. function of X

Mean $= \Sigma x\, p(x)$.

$0 \times 3818 + 1 \times 0.4929 + 2 \times 0.1927 + 3 \times 0.0045$

from table.

$\boxed{\mu = 0.7498}$ ⟶ ⇒ Square 0.5622

Variance. $\Sigma x^2 (px) - \mu^2$

μ^2

$0^2 \times 3818 + 1^2 \times 0.4929 + 2^2 \times 0.1927 + 3^2 \times 0.0045 - 0.56$

$\boxed{= 0.46}$

LP:02 Pg No. 14

Q2.

Sol: Total no. of micro chips = 25

No. of defective micro chips = X

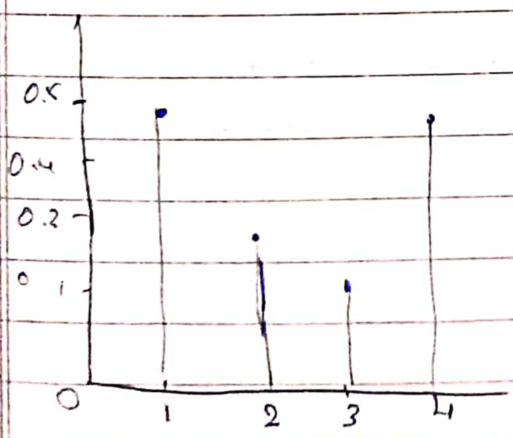
$P(x=0)$ P. of getting good micro chips = $\frac{{}^{20}C_4}{{}^{25}C_4} = 0.383$.

$P(x=1)$ P. of getting $\cancel{\frac{{}^{20}C_4}{{}^{25}C_4}} = \frac{{}^{20}C_3 \times {}^{5}C_1}{{}^{25}C_4} = 0.45$.

$P(x=2) = \frac{{}^{20}C_2 \times {}^{5}C_2}{{}^{25}C_4} = 0.15$.

$P(x=3) = \frac{{}^{20}C_1 \times {}^{5}C_3}{{}^{25}C_4} = 0.0158$.

$P(x=4) = \frac{{}^{5}C_3}{{}^{25}C_4} = 0.0004$.



Probability distribution of X.

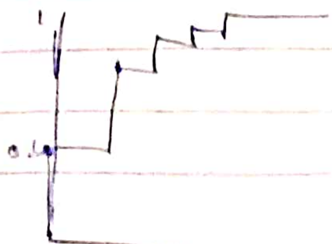
P(x)	0	1	2	3. 4
	0.383	0.45	0.15	0.0158 0.00
f(x)	0.3830	0.9838	0.9981	0.9991 ~ 1

$\Sigma x\, P(x)$

$\boxed{\text{Mean} = 0.8}$

Variance = $(1.2576) - (0.5140)^2$

$= 0.5844$



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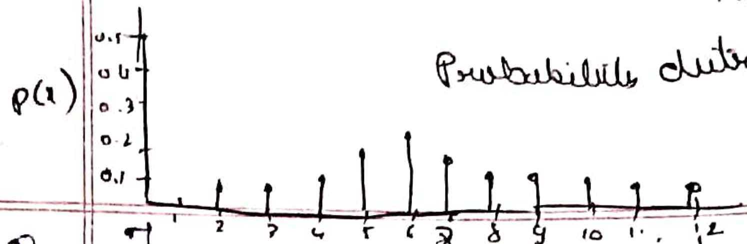
Probability distribution.



Q. In a random experiment a die was tossed twice. Suppose x is defined as sum of 2 no. turning up find the discrete probability Distribution and the Cummulative probability Distribution and graph the results. find mean & Variance

→ Total no. of observations = 36.

x = The number of sum of 2 no's. turning up.

possible values of x = {2, 3, 4, 5, 6, 7 . . . 12}

$P(x=2)$ = The sum of 2 no's is 2

$= \frac{1}{36}$

$P(x)=3$ = The sum of 2 no is 3

$\frac{3}{36}$

X	2	3	4	5	6	7	8	9	10
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36
f(x).	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36

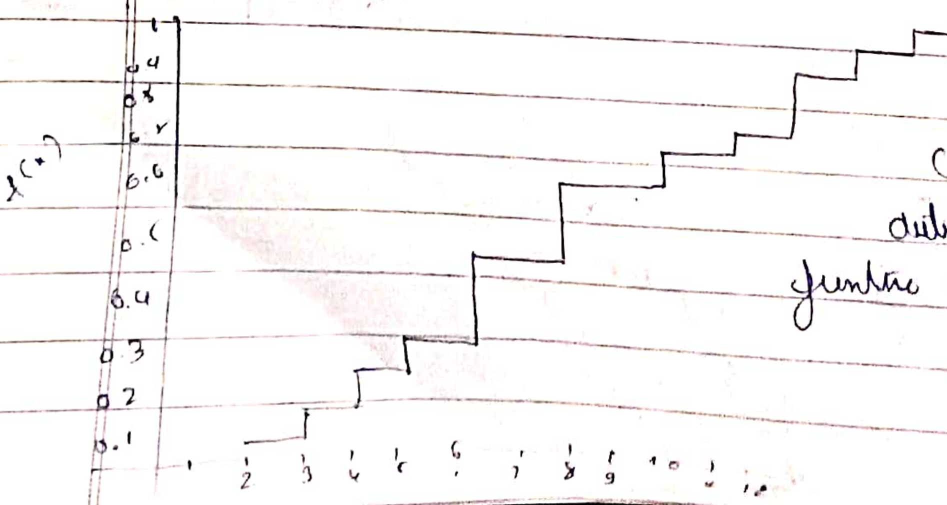
do L.M.R $\quad$ 1+2 $\quad$ 3+3 $\quad$ 6+4

Mean $\mu = 7$

X	~~10~~	11	12
P(x)		2/36	1/36
P(x)		35/36	36/36

σ^2 = Variance = (54.8333) - 49

= 5.8333



Cumulative distribution function of x.

In exam x write compulsory for all Problems,

Q The PMF of a Variable x is given by the following Table

x	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

for what value of K this represents the valid P.D. also find

$P(x \geq 5)$ $P(3 < x \leq 6)$ $P(x < 4)$

The given p.D is valid if $P(x)$ (1) $0 \leq P(x) \leq 1$

(2) $\sum P(x) = 1$

Hence $K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$$\boxed{K = \frac{1}{49}}$$

$P(x \geq 5) = P(5) + P(6) = 11K + 13K = \frac{24}{49}$

$P(3 < x \leq 6) = P(4) + P(5) + P(6) = 9K + 24K = 33/49$

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Q. A random Variable has the following distribution of x find the maximum Value of x.

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) find x = 0.1

ii) $P(x<6) = 0.81$

iii) $P(x \geq 6) = 0.19$

iv) $P(3 < x \leq 6) = P(4)+P(5)+P(6) = 0.33$

v) find cummulative distn of x

vi) If $P(x \leq a) > 1/2$ find the min Value of a

Sol: As $\Sigma P(x) = 1$.

$0+K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$

$10K^2 + 9K - 1 = 0$

$\boxed{K = \frac{1}{10}}$ Or $K = -1$

because $0 \leq x \leq 1$

ii) $P(x<6)$

v) Find C.D.F = of X

X	0	1	2	3	4	5	6	7
P(x)	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17
f(x)	0	0.1	0.3	0.5	0.8	0.81	0.83	1

vi) $P(x \le a) > \frac{1}{2}$ find the min Value of a.

Check for all, P(x).

So Now. $\frac{1}{2} = 0.5$.

$P(x \le 4) > \frac{1}{2}$ //

LP4] A random Variable X has the following probability

x	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K.

Find the (i) Value of K.

(ii) mean and Variance

(iii) $P(-1 < x \le 2)$

(iv) Express density function and C.d.F. graphically.

→ The given probability distribution is Value of.

(i) $0 \le P(x) \le 1$

(2) $\Sigma P(x) = 1$.

Hence, $0.1 + K + 0.2 + 2K + 0.3 + K = 1$

i) $0.6 + 4K = 1$

$4K = 0.4 \quad K = 0.4/4 = 0.1$ //

x:	-2	-1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1
$F(x)$	0.1	0.2	0.4	0.6	0.9	1

$(-2\times0.1)+(-1\times0.1)+(0\times0.2)+(1\times0.2)+(2\times0.3)+(3\times0.1)$

i) Mean $(\mu) = 0.8$

Variance $(\sigma^2) = (2.8) - (0.8)^2$

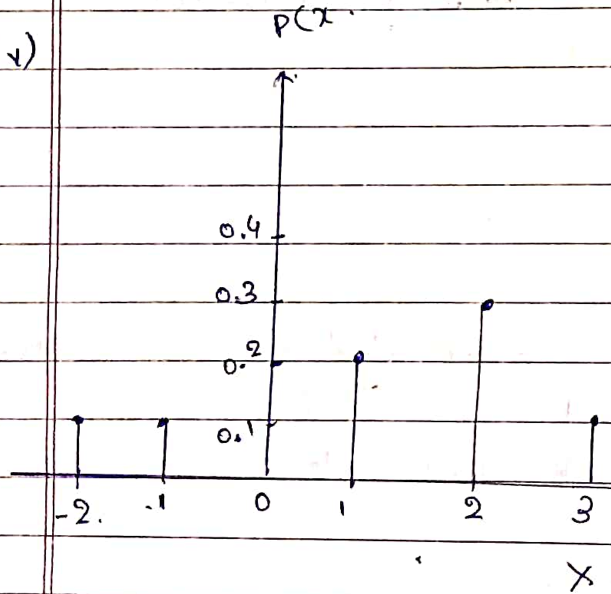
$= 2.16$

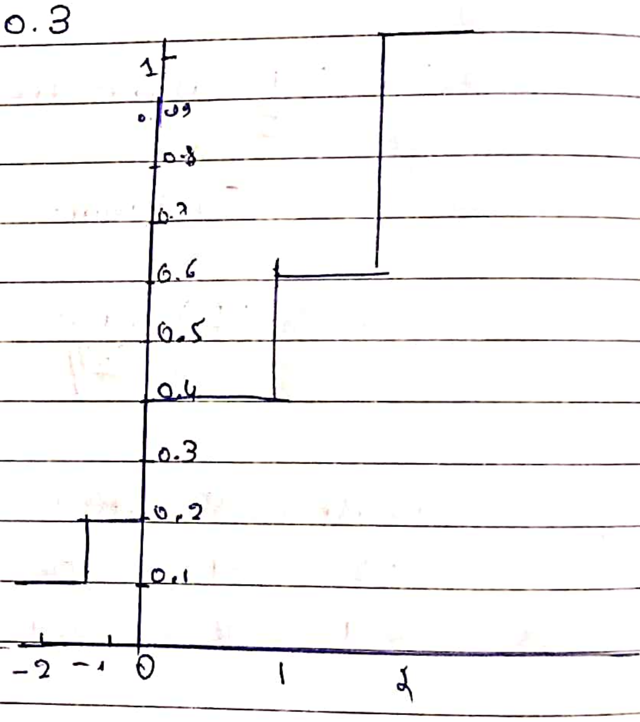
iii) $P(-1 < x \le 2) = P(0) + P(1) + P(2)$

$= 0.2 + 0.2 + 0.3$

$= 0.7$

iv)





Some of the Imp P D are

1) Binomial distribution
2) Poisson distribution (These are discrete probability distributions)
3) Normal " → (Continuous P. D)

① Binomial distribution

Defn: A random variate X is said to follow a Binomial distribution if it assumes only a +ve values. $P(X=x) = P(x) = \begin{cases} {}^nC_x p^x q^{n-x}, & x = 0, 1, 1, \ldots, n \text{ and } q = 1-p \\ 0 & \text{otherwise} \end{cases}$

and its P. Mass function given by

the ~~two~~ independent constants n & p known as the parameter

it can be represented in the table as

x	0	1	2		n
$P(x)$	q^n	${}^nC_1 p q^{n-1}$	${}^nC_2 p^2 q^{n-2}$		p^n

where $\Sigma P(x) = q^n + {}^nC_1 p q^{n-1} + \ldots\ p^n$

$= (p+q)^n$

$\Sigma P(x) = 1$

Note 1) $b(n, p, x) \geq 0$ for $x = 0, 1, \ldots n$

(ii) $\Sigma b(n, p, x) = 1$

(iii) If $p = 0.5$ we can expect the distribution is symmetrical

(iv) If $p \neq 0.5$ → skewed distribution

Necessary Conditions for B.D.

① It is necessary that each observation is either a success or failure ② The probability of success remains the same for each observation in a trial.

③ The Individual observations are independent of one another.

Mean and Variance of B.D

$$\boxed{\begin{aligned} \text{Mean } (\mu) &= np \\ \text{Variance } (\sigma^2) &= npq \end{aligned}}$$

Pg 15
LP 3

It is a Binomial distribution $p(x)$

X = no. of heads - $\{0, 1, 2, 3, 4, 5\}$

$n = 5$

$p = q = \frac{1}{2}$

$p(x) = {}^nC_x \, p^x q^{n-x}$

$\Rightarrow {}^5C_3 \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$

$P(x) = {}^5C_x \left(\frac{1}{2}\right)^5$

x	0	1	2	3	4	5
$P(x)$	${}^5C_0(1/2)^5$	${}^5C_1(1/2)^5$	${}^5C_2(1/2)^5$	${}^5C_3(1/2)^5$	${}^5C_4(1/2)^5$	${}^5C_5(1/2)^5$
	$= 0.0313$	$= 0.1563$	$= 0.3125$	0.3125	0.1563	$= 0.313$
$F(x)$	0.0313	0.1876	0.5001	0.8126	0.9689	[illegible]

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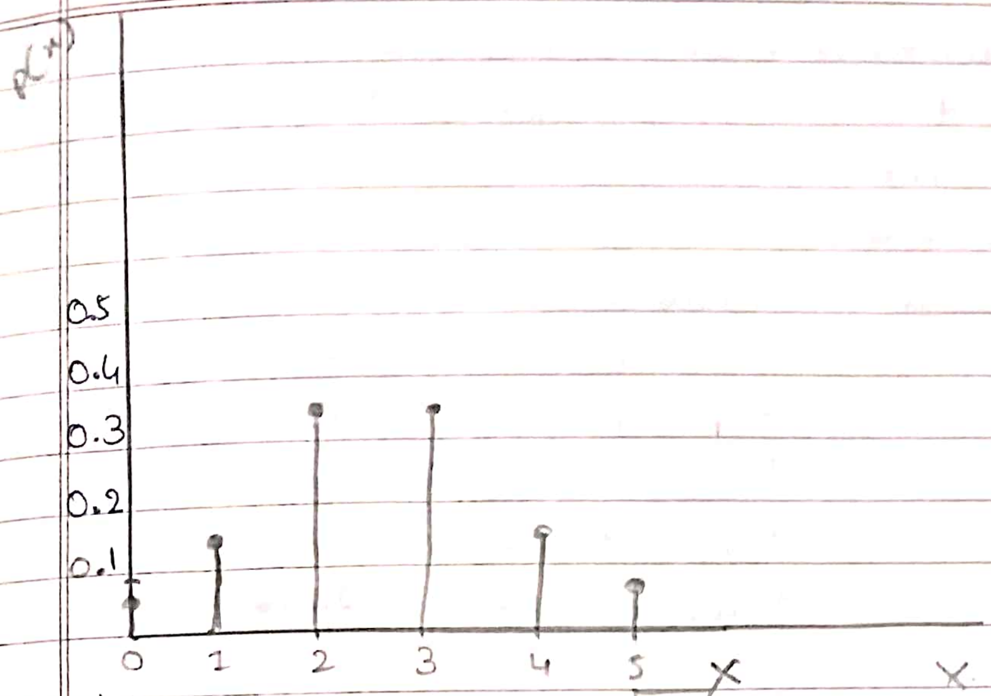

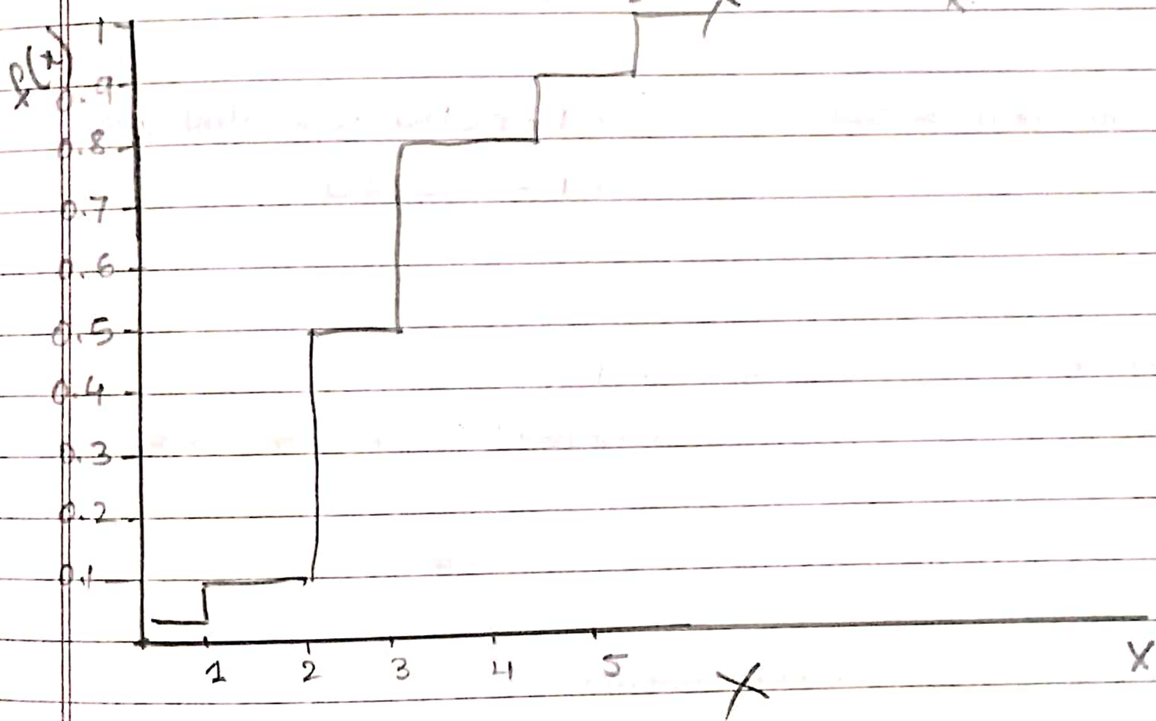

$4C_0 = 1 \quad 4C_1 = 4$

Pg No. 15.

LPQ5) Here. Binomial Distr'n is applied

$n = 4$ $\qquad P(x) = nC_x p^x q^{n-x}$

$p = 0.4$

$q = 0.6$

no. of Success	Probability.
0	$4C_0 (0.4)^0 (0.6)^4 = 0.1296$
1	$4C_1 (0.4)^1 (0.6)^3 = 0.3456$
2	$4C_2 (0.4)^2 (0.6)^2 = 0.3456.$
3	$4C_3 (0.4)^3 (0.6)^1 = 0.1536$
4	$4C_4 (0.4)^4 (0.6)^0 = 0.0256$

1 calculate then × $4C_1$

$1 - 0.1296$

i) P(One or more Successfull well) = 1 - P(No ~~successfull~~ well)

= 0.870

ii) expected no. of Success = np.
mean.

$\cancel{0 \times 0.1296} + 1 \times 0.3456 + 2 \times 0.3456 + 3 \times 0.1536 + 4 \times 0.0256$

$= 1.6$

iii) expected Gain = ~~total Success~~

= exp. Success - total Cost

= $1.6(600000) - 4(200000)$

= 160000.

iv) if Only One Will is Successfull

$\Rightarrow 1(600000) - 4(200000)$

= $-2,00,000$ (Loss)

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v) There will be a loss if 0 or 1 well will be successfull.

Probability of loss = P(0) + P(1)
= 0.4752

vi) $S.D = \sqrt{npq}$
$= 0.9798$

Q. The P. that the man ages 60 will be ~~lead~~ lived till 70 is 0.65, what is the P. that out of 10 men now aged 60
i) Exactly 9 will live till to be 70.
ii) at most 9 will " " " "
iii) " 7 will " " " "

~~100 − 65 = 35~~
~~= 0.35~~
$1-0.65 = 0.35$

do 7 4 one table.

Given $n = 10$, $P = 0.65$, $q = 0.35$

$b(10, 0.65, x) = {}^{10}C_x (0.65)^x (0.35)^{10-x}$

i) ${}^{10}C_9 (0.65)^9 (0.35)^1$
~~= 0.126~~
$= 0.0724$

ii) $P(x \leq 9) \Rightarrow P(0) + \dots P(9)$
$= 1 - P(10)$ $= \cancel{0.9276}$
0.9865

${}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 (0.5)^8 (0.3)^2 + \dots$

iii) $P(x \geq 7) \Rightarrow$
$\Rightarrow P(7) + P(8) + P(9) + P(10)$
$= 0.5138$

P(Error) < 5% then E is Unusual
P(Error) > 5% then E is Usual.

LP 8) $n = 20 \quad q = 0.9 \quad p = 0.1$

X = Silent paging error.

a) $P(x=0) \; {}^{20}C_0 (0.1)^0 (0.9)^{20}$

$= 0.1216$//

$P(x=0) \rightarrow$ The probability that no silent paging errors occur

b) $1 - P(0)$

$1 - 0.1216$

$= 0.8784$//

The probability that atleast one error occurs.

c) $1 - P(0) + P(1) + P(2) + P(3) + P(4)$

$= 1 - [0.1216 + {}^{20}C_1 (0.1)(0.9)^{19} +$
${}^{20}C_2 (0.1)^2 (0.9)^{18} + {}^{20}C_3 (0.1)^3 (0.9)^{17} +$
${}^{20}C_4 (0.1)^4 (0.9)^{16}]$

$= 1 - [0.216 + 0.2702 + 0.2852 + 0.1901 + 0.0898]$

$= 1 - 0.9569$

$= 0.0431 < 5\%$

$\therefore$ Error is unusual

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LPG: let p be the Probability bomb hits the target $\Rightarrow p = 50\% = 0.5$

N = No of bombs to be dropped that it 99% of destroying target $n = ?$

Given $P(x) = 99\% = 0.99$

$P(x) = {}^nC_x\, p^x q^{n-x}$

$0.99 = {}^nC_2 \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x}$

$0.99 = {}^nC_2 \left(\frac{1}{2}\right)^n$

$P(x \geq 2) = 0.99 = {}^nC_x \frac{1}{2^n}$

$[1-[P(0)+P(1)] \geq 0.99$

$1 - [{}^nC_0 \, 1/2^n + {}^nC_1 \frac{1}{2^n}] \geq 0.99$

$1 - 0.99 \geq \frac{1}{2^n} [{}^nC_0 + {}^nC_1]$

$0.01 \geq \frac{1}{2^n} [1+n]$

$2^n \geq 100 (1+n)$ Substi 11 in 2048 1200.

By Inspection

When $n = 11$ The above inequality is Satisfied hence the min. no. of bombs needed to destroy the target is 11

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Q The P of a man hitting a target is $\frac{1}{4}$. if he fires 7 times what is the P. of his i) hitting the target atleast twice.

ii) how many times must he fire so that the P of hitting target atleast one is $\frac{2}{3}$.

Soln: P = P. of hitting the target is $\frac{1}{4}$. $P = \frac{1}{4}$ $Q = \frac{3}{4}$

$Q = \frac{3}{4}$. $= \frac{1}{4}$ - "

$P(x)$ = P. of ~~hittin~~ getting x hits in 7 shots
$= {}^7C_x \left(\frac{1}{4}\right)^x \left(3/4\right)^{7-x}$

i]

Prob of atleast 2 hits

$P(x \geq 2) = 1 - (P(x=0) + P(x=1))$

$= 0.555$

ii] Given $P(x \geq 1) = \frac{2}{3}$

$1 - [P(0)] + \cancel{P(1)}] > \frac{2}{3}$

$\Rightarrow 1 - \left(\frac{3}{4}\right)^n > \frac{2}{3}$

$= 1 - \frac{2}{3} > \left(\frac{3}{4}\right)^n$.

$\frac{1}{3} > \left(\frac{3}{4}\right)^n$

taking ln both sides

$-\ln 3 > n(\ln 3 - \ln 4)$

$= -1.0986 > n(-0.2877)$

$1.0986 < n(0.2877)$

$n > \frac{1.0986}{0.2877}$

$n > 3.8$

$\boxed{n = 4}$

Q. The pro. of a man hitting the target is $\frac{1}{3}$

i) if he fires 5 times. What is the P. that he is getting the target atleast twice.
ii) How many times must he fire so that the P. of his getting target atleast once more than 90%.

$$P(x \geq 2) = 1 - [P(0) + P(1)]$$
$$= 1 - (0.1316 + 0.3292)$$
$$= 0.5392$$

ii) $P(x \geq 1) > 0.9$

$$1 - P(0) > 0.9$$
$$1 - 0.9 > (1/3)^n$$
$$0.1 > (2/3)^n$$
$$0.1 > 0.087$$
$$\left(\frac{2}{3}\right)^n - 0.087$$

$$\boxed{n = 6}$$

many other 3 Problems in Semp check (Home Contact)

Fit a Binomial distribution for the following data & Compare it with actual ones

x	0	1	2	3	4	5
$f(x)$	2	14	20	34	22	8

Sol: Here we have $n = 5$ $\Sigma N = \Sigma f(x) = 100$

$$\Rightarrow \frac{\Sigma x f(x)}{\Sigma f(x)} = 2.84$$

But for Binomial distribution distn mean $= np$.

$$np = 2.84$$
$$5p = 2.84$$
$$\therefore p = 0.568$$
$$q = 0.432$$

$$P(x) = {}^5C_x (0.568)^x (0.432)^{5-x}$$

$\therefore$ The prob distn is

x	$P(x)$	$f(x) = 100 \times P(x)$
0	${}^5C_0 (0.432)^5 = 0.015$	1.5
1	${}^5C_1 (0.568)(0.432)^4 = 0.989$	9.89
2	${}^5C_2 (0.568)^2 (0.432)^3 = 0.2601$	26.01
3	${}^5C_3 (0.568)^3 (0.432)^2 = 0.3420$	34.20
4	${}^5C_2 (0.568)^4 (0.432)^1 = 0.2248$	22.48
5	${}^5C_5 (0.568)^5 = 0.0591$	5.91

Comparing theoretical and actual frequencies

	x	0	1	2	3	4	5
actual freq	$f(x)$	2	14	20	34	22	8
theoretical freq	$F(x)$	1.5	9.89	26.01	34.20	22.48	5.9.

Yer this | 1st write what is x ?

Poison Distribution.

(question in 5 Samp.)

P. D is regarded as limiting case of B.D when n is very large ($n=\infty$). The P of success is very large ($P \to 0$) so that np tends to P. finite constant. Say $nP = \lambda$ then on substituting this in B.D we get

$$P(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}$$

Here P(x) is known as Poison Probability f'n
and x is " " ~~Gore~~ Poisson

$\boxed{\text{In this case mean is} \quad \text{mean} = \lambda \quad \text{Variance } (\sigma^2) = \sqrt{\lambda}}$

* when λ is less than 0.5 then apply ~~Binomial~~ Poisson Distribution

* when λ is > than 5 the Distribution is Symmetric

* Conditions for appling Poison Distribution.

1. The Probability of Occurring a S.I over Small Interval. is approximal proportional to the C.I

2. The probability of two events occuring in the same narrow interval is negligible ~~The P.O~~

3. The P of an event with a Certain Interval ~~Cannot~~ ~~cham~~ does not change over diff Intervals

4: The P. of an event in 1 Interval is Independent in any other Overlapping Intervals.

Q1: :

Q1. If the probability of a bad reaction from a certain injection is 0.001 determine the chance that out of 2000 individuals more than 2 will get a bad reaction.

→ X ⇒ no. of bad reactions

$P = 0.001$

$n = 2000.$

$\lambda = np \Rightarrow 2.$ $\boxed{\lambda = 2}$

So 2 is less than 25 then use poisson Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \Rightarrow \frac{2 e^{-2}}{x!}$$

Prob that more than 2 get bad reaction

$P(x) \Rightarrow 1 - [P(0) + P(1) + P(2)]$

$$1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2}{1!} + \frac{e^{-2} 2^1}{2!}\right]$$

$0.2707 + 0.2707 + 0.1353$

$= 1 - e^{-2}[1 + 2 + 2]$

$= 1 - 5e^{-2}$

$= 0.3233$

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Q2.

Q2. The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with

i. No accidents in a year

ii. more than 3 accidents in a year

→ $X \Rightarrow$ no of accidents in a year.

Given $\lambda = 3$.

$F(x) = 1000 P(x)$ no. of drivers

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^x e^{-3}}{x!}$$

i) No accidents

$$P(0) = 1000 e^{-3} = 49.7 \approx 50.$$

ii) more than 3 accidents in a year.

$$P(x>3) = \Rightarrow 1 - P(x \leq 3)$$

$$\Rightarrow 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$= 1 -$$

$P(x) = 0.3527$.

$F(x) = 1000 \times 0.3527 = 352.7$

No. of drivers with more than 3 accidents in a year $= 353$

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Q3.

Q3. In an automatic telephone exchange the probability that any one call is wrongly connected is 0.001. What is the minimum number of independent calls required to ensure a probability 0.9 that at least one call is wrongly connected?

→ $X \Rightarrow$ No. of Wrong Calls.

$p \Rightarrow 0.001$

$\cancel{P(x)} = P(X \geq 1) = 0.9$

p is Small So we Poisson.

$1 - P(0) = 0.9.$

$1 - e^{\lambda} = 0.9$

$1 - 0.9 = e^{\lambda}$

$0.1 = e^{\lambda}$

$-\lambda = \ln(0.1)$

$\lambda = 2.3026$

But $\lambda = np$

$\frac{2.3026}{0.001} = n.$

$\boxed{n \simeq 2303}$

x x 12

8

4

* for 8 2 × 8. for 12 → 3 × 12

Q4

Q4. Suppose in a bank customers arrive randomly on week day afternoons at average of 3.2 customers every 4 minute.

i. What is the probability of exactly 5 customers arriving in a 4 minute interval on a week day afternoon

j. What is the probability of having more than 3 customers in a 4 min interval on a week day afternoon?

k. What is the probability of having exactly 10 customers during an 8 min interval.

→ X : No. of customers arriving in a week day afternoon

3.2,

x 4 8 12

$\lambda = 3.2$ per 4 min $= 3.2/4$.

It is a poisson dist'n with $P(x) = \frac{e^{-3.2}(3.2)^x}{x!}$

i) P(5 customer arriving)

$$P(5) = \frac{e^{-3.2}(3.2)^5}{5!} = 0.1140$$

∴ Prob of 5 customers arriving in 4 min is 0.1140.

ii) P(more the 3 customer)

$P(x > 3) = 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - [e^{-3.2} + e^{-3.2}(3.2) + \frac{e^{-3.2}(3.2)^2}{2} + \frac{e^{-3.2}(3.2)^3}{6}]$

$= 0.3975$ //

iii) P(10 customer | 8 min)   Given $\lambda = 3.2/4$ min
∴ for 8 min

$\lambda = 6.4/8$ min
$$P(X=10) = \frac{e^{-6.4}(6.4)^{10}}{10!}$$ $\lambda (3.2)(2)$
= 6.4.

= 0.058 //.

Q5. Customers arrive at a checkout counter at an average rate of 1.5 per minute. What distribution will apply if reasonable assumptions are made? List those assumptions. Find the probabilities that a) exactly two will arrive in any given minute; b) at least three will arrive during an interval of two minutes; c) at most 8 will arrive during an interval of six minutes

→ X : no. of customers arriving at a counter
$$P(X=x) = \frac{\lambda^x e^{-x}}{x!}$$

Given $\lambda = 1.5$/min

a) P(exactly 2 arriving in given min) $P(x=2)$
$$P(2) = \frac{(1.5)^2 e^{-2}}{2!} = 0.251.$$

b) P(atleast three will arrive during 2 minutes.

$\lambda = 2(1.5) = 3.$
$P(X \geq 3) = 1 - P(0) + P(1) + P(2)]$ ~~P(3)]~~
$$= 1 - \left[e^{-3} + e^{-3}(3) + \frac{e^{-3}(3)^2}{2!}\right]$$

= 0.5768 //

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c) P (customers & will arrive in 6 min)

$\Rightarrow \lambda = (1.5 \times 6) = 9.$

$P(x \geq 8) = 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)]$

$e^{-9}\left[1 + 9 + \frac{9^2}{2!} + \frac{9^3}{3!} + \frac{9^4}{4!} + \frac{9^5}{5!} + \frac{9^6}{6!} + \frac{9^7}{7!} + \frac{9^8}{8!}\right]$

$= 0.4557$

Q6

Q6. At a certain auto service it is assumed that 7 customers arrive per hour
i. Compute the probability that more than 10 customers will arrive in a 2 hour period
ii. What is the mean number of arrivals during a 2 hour period?

→

X = No. of customers arrive

$\lambda = \frac{1}{2}$ hr.

it is a poisson distribution. b'cz λ is less than 9.

$7 \times 2 \Rightarrow$

$\lambda = 14/2$

a) $P(x > 10) = 1 - P(x \leq 10)$

$\Rightarrow 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)]$

$\Rightarrow 1 - [0 + 0 + 0.0001 + 0.0004 + 0.0013 + 0.0037 + 0.0087 + 0.0174 + 0.0304 + 0.0473 + 0.0663]$

$\Rightarrow 1 - 0.1756$

$\Rightarrow 0.8244$

ii) Mean $= \lambda$

$= 7 \times 2 = 14$

~~[illegible]~~

Mean$(\mu) = 14$

$P(x > 10) = 1 - [P(0) \cdots\cdots P(10)]$

$\Rightarrow 1 - e^{-14}\left[\frac{14^0}{0!} + \frac{14^1}{1!} + \frac{14^2}{2!} + \cdots \frac{14^{10}}{10!}\right]$

$\Rightarrow \underline{\underline{0.82432}}$

Q7)

Q7. Fit a Poisson Distribution to the following frequency distribution
Find the corresponding theoretical estimates for F

X	0	1	2	3	4
F	122	60	15	2	1

$\Rightarrow$ Mean $= \frac{\Sigma x f(x)}{f(x)}$

$\Rightarrow \frac{100}{200}$

$\lambda = 0.5$

$\lambda = 0.5$

$P(x) = \frac{(0.5)^x \times e^{-0.5}}{x!}$

X	0	1	2	3	4
F	122	60	15	2	1
~~F(x)~~	~~121.3~~	~~60.6~~	~~15.6~~		
P(x) =	0.6065	0.303	0.0758	0.0126	0.0016
F(x) =	121.3	60.6	15.16	2.52	0.32

$$P(x=0) = \frac{e^{-0.5}(0.5)^0}{0!} = 0.605$$

$$P(x=2) \Rightarrow \frac{e^{-0.5} \times (0.5)^2}{1!} = 0.3033$$

$$P(x=2) \Rightarrow \frac{e^{-0.5} \times (0.5)^2}{2!} = 0.0758$$

$$P(x=3) = \frac{e^{-0.5} \times (0.5)^3}{3!} = 0.0126$$

$$P(x=4) = \frac{e^{-0.5} \times (0.5)^4}{4!} = 0.0016$$

X	0	1	2	3	4
F	122	60	15	2	1
P(x)	0.6065	0.3033	0.0758	0.0126	0.0016
f(x) 200 X P(x)	121.30	60.66	15.16	2.52	0.32

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CONTINOUS RANDOOM VARAIBALE

Proab Denth'n fuct'n

PDF → $P(a \leq x \leq b) = \int_a^b f(x)\,dx$.

$P(x<3) = \int_{-a}^{3} f(x)\,dx$.

$P(x>3) = \int_{-a}^{3} f(x)\,dx$.

$P(x>3) = \int_{3}^{\infty} f(x)\,dx$.

(i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ ii) $0 \leq f(x) \leq 1$.

Cummulative Distribution functin (CDF).
if t is any real no. then
$F(t) = P(x \leq t) = \int_0^t f(x)\,dx$.

Note ① $F(-\infty) = 0$
$F(\infty) = 1$

$P(a \leq x \leq b) = P(a<x<b)$
$= P(a \leq x < b)$
$= P(a < x \leq b)$

$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$.

$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$.

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Find which of the following function is PDF

Q1) $F(x) = \begin{cases} 2x \ldots\ldots 0<x<1 \\ 0 \ldots\ldots \text{otherwise.} \end{cases}$

Sol: (i) $f(x)$ = is +ve.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^1 2x\,dx = 1$

Q2) $f(x) \begin{cases} 2x \ldots\ldots 0<x<1 \\ 4-4x \ldots\ldots 1<x<2 \\ 0 \quad \text{otherwise.} \end{cases}$

→ i) $2x \ldots 0<x<1 \ldots f(x)$ is +ve

ii) $4-4x \ldots\ldots 1<x<2$ $f(x)$ is -ve,

Hence 1[st] Condit'n fails So its not a Pdf

Q3) find the constant K Such that $f(x) = \begin{cases} Kx^2 \ldots 0<x< \\ 0 \ldots \text{other} \end{cases}$ is a Pdf.

i) $P(1<x<2)$

ii) $P(x \leq 1)$

iii) $P(x>1)$

iv) Mean

v) Variance

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Q. The P.D.f of a random Variable is given by $f(x) = \begin{cases} 0, \dots & x<1. \\ b/x \dots & 1<x<5 \\ 0 \dots & x>5. \end{cases}$

(i) find b

(ii) Value at $x = 2$.

(iii) Find $P(2 < x < 4)$

iv) find cummulative Distb'n. of x.

v) $E(x)$, $Var(X)$, $S.D(x)$

Sol: For the given fun'n to be a probability it must satisfy conditions. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

iv)

→. (1) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{1} \cancel{0\,dx} + \int_{1}^{5} \frac{b}{x^2}\,dx + \int_{0}^{\infty} \cancel{0\,dx} = 1$$

$$b\left(\frac{-1}{x}\right)^{5} = 1$$

$$b\left(\frac{-1}{5}+1\right) = 1$$

$$\boxed{b = \frac{5}{4}}$$

~~as~~ Since 2 lies in the interval 1 to 5 ~~Int~~

ii)

$$f(2) = \frac{b}{x^2} = \frac{5}{4(4)} = \frac{5}{16}$$

iii) $P(2<x<4)$

$$= \int_{2}^{4} \frac{b}{x^2}\,dx = \cancel{\frac{5}{4}\left(\frac{x^3}{3}\right)^{4}} = \frac{5}{4}\left(\frac{-1}{x}\right)_{2}^{4} = \frac{5}{16}$$

iv) Cummulative Distribution of CDF we have.

$$F(t) = P(x \le t) = \int_{-\infty}^{t} f(x)\,dx.$$

For $(-\infty < t)$

$$F(t) = \int_{-\infty}^{1} 0\,dx = 0.$$

for $(1 < t < 5)$

$$F(t) = \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{\cancel{5}t} f(x)\,dx$$

$$F(x) = 0 + \int_{1}^{5} \frac{b}{x^2}\,dx = \frac{5}{4}\left(1 - \frac{1}{t}\right)$$

for $(5 < x < \infty)$

$$F(t) = \int_{0}^{1} 0\,dx + \int_{1}^{5} \frac{5}{4x^2}\,dx + \int_{5}^{t} 0\,dx.$$

$$= \frac{5}{4}\left(1 - \frac{1}{5}\right) = \underline{1}$$

$\therefore$ The CDF is defined as $\therefore$

$$F(x) = \begin{cases} 0 & -\infty < t < 1 \\ \frac{5}{4}\left(1 - \frac{1}{t}\right) & 1 < t < 5 \\ 1 & t > 5 \end{cases}$$

$$\text{Mean} = \int_{-\infty}^{\infty} x\, f(x)\, dx.$$

$$= \frac{5}{4}\int_{1}^{5} \frac{x}{x^2}\, dx$$

$$= \frac{5}{4}\left(\ln x\right)_1^5$$

$$= \frac{5}{4}\left(\ln 5 - \ln 1\right) = \frac{5}{4}\ln 5.$$

$$= 2.0118.$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 . f(x)\, dx - \mu^2$$

$$\Rightarrow \frac{5}{4}\int_{1}^{5} \frac{x^2}{x^2}\, dx - (2.0118)^2$$

$$\Rightarrow \frac{5}{4}(4) - (2.0018)^2.$$

$$\sigma^2 = 0.9527$$

$$S.D = \sqrt{0.9527}$$

$$= 0.9760.$$

Q. Random Variable is given by

$$f(x) = \begin{cases} x & \dots \quad 0 \le x < 1 \\ 2-x & \dots \quad 1 \le x \le 2 \\ 0 & \dots \quad \text{elsewhere.} \end{cases}$$

i) find CDF.
ii) $P(x \geq 1.5)$.

$\Rightarrow$ for $(0 < x < 1)$

$$F(t) = \int_0^t x\,dx = \frac{t^2}{2}$$

for $1 < x < 2$,

$$F(t) = \int_0^1 x\,dx + \int_1^t (2-x)\,dx = \frac{1}{2} +$$

for $(2 < x < \infty)$

$$F(t) = \int_0^1 x\,dx + \int_1^2 (2-x)\,dx + \int_2^t 0.$$

$$= 1$$

H/W

34 $\because x^2$ is always +ve

$\therefore Kx^2$ is +ve.

(i) $\int_0^3 f(x)\,dx = 1$

$$\int_0^3 Kx^2\,dx = 1$$

$$K\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$K\left(\frac{27}{3}\right) = 1$$

$$9K = 1$$

$$\boxed{K = \frac{1}{9}}$$

(i) $P(1 < x < 2)$

$$\int_1^2 Kx^2\,dx = K\left(\frac{x^3}{3}\right)\Big|_1^2$$

$$= \frac{1}{9 \times 3}(8 - 1)$$

$$= \frac{7}{27}$$

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ii) $P(x<1) = \int_0^1 kx^2 dx = k\left(\frac{x^3}{3}\right)_0^1$

$= \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}$

iii) $P(x>1) = \int_1^3 kx^2 dx = \frac{1}{9}\left[\frac{x^3}{3}\right]_1^3$

$= \frac{1}{9 \times 3}[27-1]$

$= \frac{26}{27}$

iv) Mean $= \int_0^3 x f(x)\, dx.$

$= \int_0^3 x . kx^2 dx.$

$= k\left[\frac{x^4}{4}\right]_0^3 = \frac{1}{9 \times 4}(81) = \frac{9}{4}.$

v) Variance $(\sigma^2) = \int_0^3 x^2 f(x) dx - \mu^2.$

$= \int_0^3 (x^2 . kx^2) dx - (9/4)^2$

$= \frac{1}{9}\left[\frac{x^5}{5}\right]_0^3 - \frac{81}{16}$

$= \frac{27}{80} = 0.3375$

$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty}$

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LP 11 E

Pg No: 16.

Given X = Time in millions of hours

$\therefore$ it is continuous.

(i) Average time = (mean) $\boxed{\int_{-\infty}^{\infty} x f(x)\, dx}$

$$K \Rightarrow \int_0^{\infty} x e^{-x} dx$$

$$\Rightarrow K \int_0^{\infty} \cancel{K} x e^{-x} dx$$

$$\Rightarrow K \int \left[x(-e^{-x}) - 1(e^{-x}) \right]_0^{\infty}$$

$$= K\,(0 - \cancel{1}(0-1)]$$

upper lim. lower lim.

(b) Variance = $\boxed{\int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu^2}$

$$\Rightarrow K \int_0^{\infty} x^2 e^{-x} dx - K^2$$

Integrate

$$\Rightarrow K.\left[x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^{\infty} - K^2$$

$$\Rightarrow K\,[0 - (-2)] - K^2$$

$$\Rightarrow 2K - K^2$$

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LP12] Given X : life in hours of a certain part of radio tube. $\therefore f(x) = \frac{100}{x^2}$.

i) P. that it is replaced in 150 hours.
P(replaced in first 150 hours) = $P(x<150)$

$\Rightarrow P(x<50) \Rightarrow \int_{100}^{150} f(x)\,dx$

Inquith ils ≥ 100 & ~~other~~ 150 so take $\int_{100}^{150}$ $\Rightarrow \int_{100}^{150} \frac{100}{x^2}\,dx$.

$\Rightarrow 100\int_{100}^{150} \frac{1}{x^2}\,dx \Rightarrow 100\left[\frac{-1}{x}\right]_{100}^{150}$

$\Rightarrow 100\left[\frac{-1}{150} + \frac{100}{100}\right]$

$\Rightarrow 100\,[3.33]$

$\Rightarrow 0.333 = \frac{1}{3}$

$$\boxed{\Rightarrow 100\left[\frac{-1}{x}\right]_{100}^{\infty} = 100\left[0 + \frac{1}{50}\right] = \frac{2}{3}}$$

Ans is $\frac{1}{3}$

P(all 3 tubes to be replaced) = $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$.

ii) P_2 = ~~Pouel~~ P(not replaced before 150 hrs)
$P(x \geq 150) = \int_{150}^{\infty} \frac{100}{x^2}\,dx$.

$1 - \frac{1}{3}$

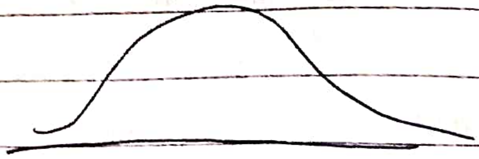
$= \frac{2}{3}$.

P(None of three tubes are replaced) $= \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$.

NORMAL DISTRIBUTING

→ Unimodel.
→ Symmetric.
→ bell Shape.
→ it has 2 parameter μ & σ
$N(\mu, \sigma^2)$

$\mu \downarrow$ mean, $\sigma \downarrow$ SD



Conditions for N.D.

* N.D is a limiting Case of B.D. if
(1) * if → n → no. of trials is very large.
(2) p and q are very closer to each other.
(3) X is Continuous.
(4) when np is > 10
(5) BD → ND.

n > 30 → N.D & it should be Continuous.

C. Rand. variable X is Set to have N.D if its Pdf (Probabil Dist'n functn) is given by

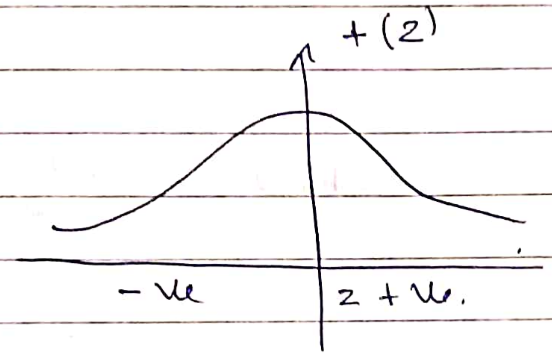
$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

where $-\infty < x < \infty$
$-\infty < \mu < \infty$
$\sigma > 0$

To make it simpler we substitute $\frac{x-\mu}{\sigma} = z$

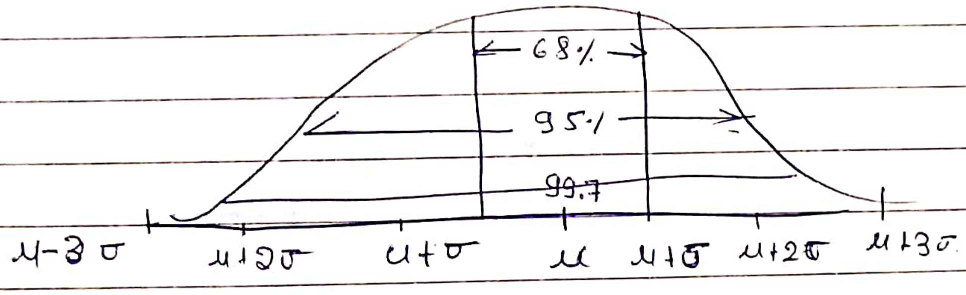
Thus the standard form of N.D if
X is a R.V with mean μ & S.D $= \sigma$
then for the standardized R.V. z is

$$f(x)\; f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \quad \text{where } \mu = 0,\ \sigma = 1$$



Strict rule that governs N.D. Is

$\boxed{\text{68 - 95 - 99.7 rule}}$

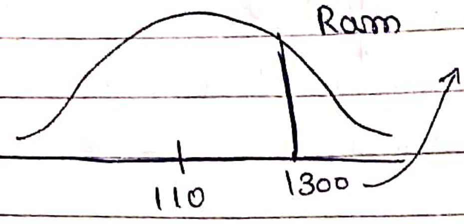


The mean and SD on total scores for gate and CAT are

	Gate	Cat
mean	110	21
SD	200	6

Suppose both are nearly normal. And ram scored 1300 in Gate, and ravi scored 24 in cat. who performed better.

Sol: Gate



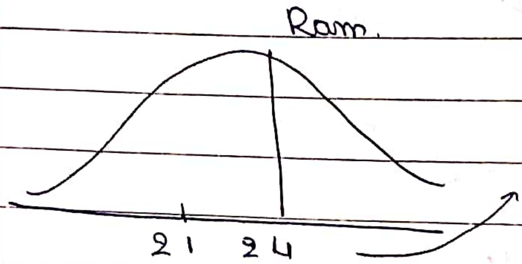
$= 1350$

$110 + 200$

$\mu + \sigma$

$\sigma = 1$

Cat



$\cancel{21+1}\ 21+3 = \mu + \frac{1}{2}\sigma$

$\sigma = \frac{1}{2}$

Ram took more space.
So ram performs better.

Z - SCORE

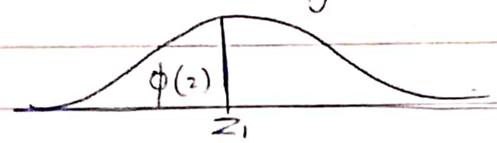
The Z-Score of an observation given by.

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

Note : when $|z|>2$ then it is Unusual observation.

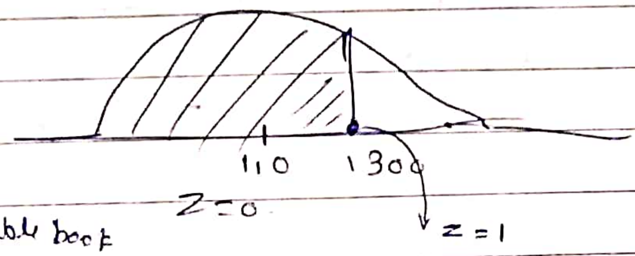
PERSENTILE

P. is the % of observation that falls below given data pt. Graphically it is the area below the curve to the left.



$\boxed{\text{Pnorm}(x, \mu, SD)}$ in R to get z value.

What fraction of people in gate Scores are below rams Score of 1300



(in Table book)

$\phi(z) = 0.8413$

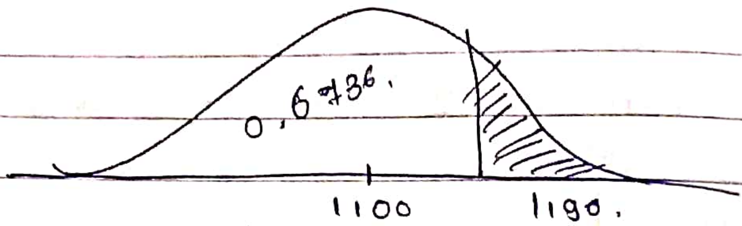
84% of people have perform low than ram.

⇒ more means $1 - 0.8413$.

	Gate	cad
mean	1100	21
S.D.	200	6

Q. Sham is another person who's GATE Score are about 1190, what will be his P. to that Score.

Sol:



$\mu = 1100$

$\sigma = 200$

$X = 1190$

$$z = \frac{1190 - 1100}{200} = 0.45.$$

$z = 0.45 \leftarrow$ z table

$\Rightarrow 0.6736.$

$\Rightarrow$ 67% of the P he scored.

Q Karthik earned 1030 on his gate what is his percentile $\mu = 1100$ $\sigma = 200$, $X = 1030$.
what is his %

$\Rightarrow$

$$z = \frac{x - \mu}{\sigma} = -0.35 \Rightarrow \text{check in table.}$$

$\phi(z) \Rightarrow 0.3631$

$\Rightarrow 36\%$

NOTE WHEN S.D & V. is giving direct check where it is not given remaining which is left check for that NO. what is Z Ex.

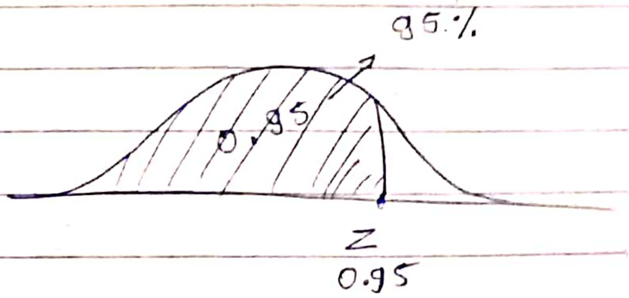
30% $\Rightarrow$ 0.3 cumulative in table check for 0.7.

where it will come and give(Z)

LP 13 14 15 ⇒ Normal D. Problems

Q In a Gate Score what is the 95th % and 97.5%

⇒ for 95%



$Z_{0.95} = 1.65$ from table.

$z = \frac{x-\mu}{\sigma}$

$1.65 = \frac{x-1100}{200}$ $X = 1430$

97.5% U do.

Given

LP13] let X : time in bus

is a $N(\mu, \sigma^2)$

Given $\mu = 10$ (mean)

$\sigma^2 = 9$ (variance)

$\sigma = 3.$

(i) $P(x \leq 15) = ?$

when $x = 15$ ⇒ $Z = \frac{x-\mu}{\sigma}$

$= \frac{15-10}{3}$

$= 1.67$

$P(x \leq 15) = P(Z \leq 1.67)$

↓

from table.

	0.07
1.6	0.9525

⇒ 0.9525

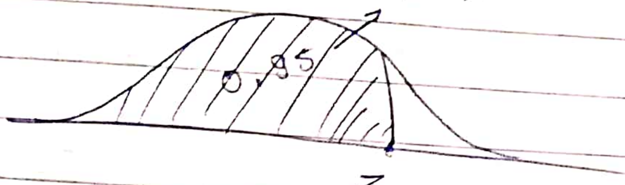
⇒ 95%.

LP 13 14 15 $\Rightarrow$ Normal D. Problems.

Q In a Gate Score what is the 95th % and 97.5 %.

$\Rightarrow$ for 95 %

95 %



Z
0.95

$Z_{0.95} = 1.65$ from table.

$z = \frac{x - \mu}{\sigma}$

$1.65 = \frac{x - 1100}{200}$ $\qquad x = 1430$

97.5 % u do.

given

LP 13] ~~let~~ X : time in hrs

w a $N(\mu, \sigma^2)$

given $\mu = 10$ (mean)

$\sigma^2 = 9$ (variance)

$\sigma = 3$.

(i) $P(x \leq 15) = ?$

when $x = 15 \Rightarrow Z = \frac{x - \mu}{\sigma}$

$= \frac{15 - 10}{3}$

$= 1.67$//

$P(x \leq 15) = P(z \leq 1.67)$

from table.

	0.07
1.6	0.9525

$\Rightarrow 0.9525$

$\Rightarrow 95\%$

means it is area so find z
~~mean~~

ii) Percentage they gave so 5% of are

Given : $\phi(z) = 0.05$
from the table $z = -1.64$

$$z = \frac{x - \mu}{\sigma}$$

$$x = \frac{z - \mu}{\sigma} = -1.64 = \frac{x - 10}{3}$$

Nearly 5 hours to complete $x = 5.08$ hours

$-1.64 \times 3 \Rightarrow -4.92$

$-4.92 = x - 10$

$x = 4.92 - 10$

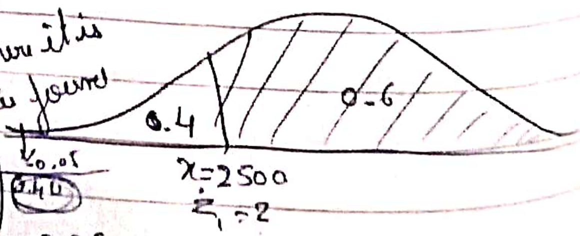
$= 5$

LP14] Here also they gave % we using that find z

Given : x = lifetime of bulbs
mean $= \mu$
S.D $= \sigma$

60% longer than 2500 so
$P(x > 2500) = 0.6$

from table check 0.4 where it is found so it is



$z = -0.25$

	0.05
-0.2	0.40

so ans is -0.25

$$z_1 = \frac{x_1 - \mu}{\sigma} \Rightarrow -0.25 = \frac{2500 - \mu}{\sigma}$$

$2500 - \mu = -0.25\sigma \longrightarrow (1)$

30% greater than 3000

ii) $P(x_2 > 3000) = 0.3$

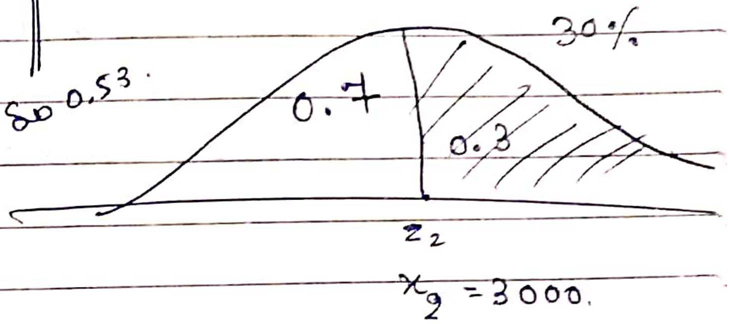
$\Rightarrow 1 - P(z < z_2)$

$\Rightarrow 0.7 \quad \phi(z)$

	0.03
0.5	0.7
0.5	0.53

from table

$z_2 = 0.53$



$$z_2 = \frac{x_2 - \mu}{\sigma}$$

$$0.53 = \frac{3000 - \mu}{\sigma}$$

$3000 - \mu = 0.53\sigma \rightarrow (2)$

Solving (1) and (2).

$3000 - \mu = 0.53\sigma$

$2500 - \mu = -0.25\sigma$

160.25

$-0.25\sigma = 2500 - \mu$

$0.~~25~~53\sigma = 3000 - \mu$

$(-) \quad (-) \quad +$

$-0.78\sigma = -500$

$\boxed{\sigma = 641}$

Substitute

Checking

$-0.25 \times 641 = 2500 - \mu$

$\boxed{\mu \approx 2660}$ $\Rightarrow 160.25 + 2500$

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Q3. The average grade for an exam is 75 and the S.D is 7. if 12% of the class is given grade 'A' and grades are curved to follow a N.D. what is the lowest possible A and the highest possible B?

⇒ In question it is average so it is known that it is mean.

∴

mean = 75 S.D is 7 12% => 0.12.
z = 1.17.

check z val

0.12

is 1.17

$\Rightarrow z = \frac{x - \mu}{\sigma}$

$\Rightarrow 1.17 = \frac{x - 75}{7}$

$\Rightarrow 1.17 * 7 = x - 75$

$= x - 75 = 8.2$

$\Rightarrow x = 75 + 8.2$

$\Rightarrow 83$ //

∴ The lowest possible A and highest possible B is 83 //.

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Q4. A In a N.D. 77% of items are under 35 and ~~99%~~ 89% are under 63. What are the mean and SD of the ND.

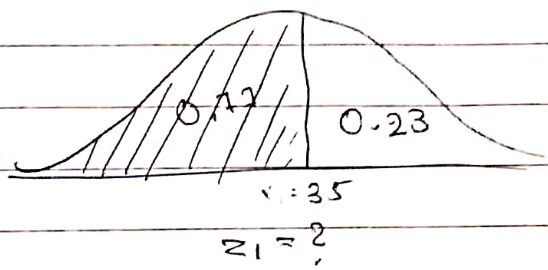
$\Rightarrow$

(i) 77% of items are under 35

So 77% = 0.77

$X = 35$

$Z = -0.73$.



$\cancel{X \leftarrow Z}$

$$Z = \frac{X-\mu}{\sigma}$$

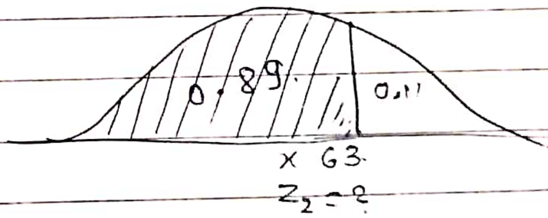
$$-0.73 = \frac{35-\mu}{\sigma}$$

$$\boxed{35-\mu = -0.73\sigma} \rightarrow (1)$$

ii) 89% are under 63.

$X = 63$

$Z = -1.23$.



$$Z = \frac{X-\mu}{\sigma} \Rightarrow -1.23 = \frac{63-\mu}{\sigma}$$

$$\Rightarrow \boxed{63-\mu = -1.23\sigma} \rightarrow (2)$$

Now equate = (1) and (2)

$63-\mu = -1.23\sigma \Rightarrow -0.73\sigma = 35-\mu$.

$35-\mu = -0.73\sigma \qquad -1.23\sigma = 63-\mu$.

$28 \cdot \mu = 1.96\sigma \qquad 1.96\sigma = -28$.

$\boxed{\sigma = -14.28}$

Substitute

~~$35-\mu = -0.73(-14.28)$~~ $\quad -1.23\sigma = 63-\mu$

~~$35-\mu = 10.42$~~ $\Rightarrow$ $\quad \Rightarrow 17.6 = 63-\mu$

$\boxed{\mu = -45.4}$

JOINT PROBABILITY DISTRIBUTION.

LP (16, 17, 18, 19, 20)

if X and Y are 2 random Variables that define the P.D that defines the Simultaneous behaviour is Called as J.P.D.

Denoted by $P(X=x, Y=y) = f_{xy}(x, y)$

where $f(x,y)$ Satisfy the Cond'n.

(i) $f(x,y) \geq 0$

(ii) $\sum_{x} \sum_{y} f(x,y) = 1$

The Value of Joint Probability are represented in the form of Joint P. Table

X \ Y	y_1	y_2	....	y_n	Sum
x_1	J_{11}	J_{12}	...	J_{1n}	$f(x_1)$
x_2	J_{21}	J_{22}	— —	J_{2n}	$f(x_2)$
x_m	J_{m2}	J_{m2}	......	J_{mn}	$f(x_m)$
Sum	$g(y_1)$	$g(y_2)$	-	$g(y_n)$	1

→ Marginal Probability Distribution:

The P. $f(x_1), f(x_2) \ldots f(x_n)$ and $g(y_1), g(y_2) \ldots g(y_n)$ are Called Marginal P

NOTE:

INDEPENDENT RANDOM VARIABLES.

The discrete R.V. x and Y are said to be independent R.V. if

$P(x = x, y = y) = P(X = x) \cdot P(Y = y)$

$J_{ij} = f(x_i) g(y_j)$

→ Expectation (Mean) $E(x)$ or μ_x.

⇒ $E(x) = \Sigma x f(x)$

$E(Y) = \Sigma y g(y)$

$E(XY) = \sum_i \sum_j x_i y_j J_{ij}$

* Variance

$V(x) = E(X^2) - \mu_x^2$; $E(X^2) = \Sigma x^2 f(x)$

$V(Y) = E(Y^2) - \mu_y^2$; $E(Y^2) = \Sigma y^2 g(y)$

* SD $= \sqrt{\text{variance}}$

* Co Variance of X & Y

$Cov(X, Y) = E(XY) - \mu_x \mu_y$.

* Co-relation X & Y $\rho(x, y)$

$$\rho(x, y) = \frac{Cov(x, y)}{\sigma_x \sigma_y}$$

NOTE: If x and Y are independent variable. then

i) $E(XY) = E(X)E(Y)$

ii) $Cov(XY) = 0$

hence $\rho(x, y) = 0$

Q. i) Find the Marginal Distb'n

1) $E(x)$ & $E(V)$

2) $Cov(x, y)$

3) $P(x, y)$

4) ~~Rel~~ are x and Y independent.

$x \backslash y$	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/6

$\Rightarrow$

$x \backslash y$	-			Sum.
				1/2
				1/2
Sum	3/8	3/8	1/4	1

a) Marginal Distb'n of ~~x~~ $f(x)$

x_i	1	5
$f(x_i)$	1/2	1/2.

M.D of $f(y)$.

y_i	-4	2	7
$g(y_i)$	3/8	3/8	1/4

$\rightarrow$ 2m

c table $\Rightarrow 1 \times 0.5 + 5 \times 0.5 = 3.$

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b) $E(x) = \sum x f(x) = 3$

$E(y) = \sum y g(y) = 1.$

$\downarrow$ same

(3) $Cov(xy) = E(xy) - E(x)E(y)$

$E(xy) = \frac{1}{8}(-4)(1) + \frac{1}{4}(2)(1) + \frac{1}{8}(7)(1) + \frac{1}{4}(-4)(5) + \frac{1}{8}(2)(5) + \frac{1}{8}(7)(5)$

$E(xy) = \frac{3}{2}.$

-0.5

$Cov(xy) = E(xy) - E(x) \cdot E(y)$

$= \frac{3}{2} - 3(1) = -\frac{3}{2}$ //.

(4) $\rho(xy) = \frac{Cov(xy)}{\sigma_x \sigma_y}.$

$V.(x) = \sum x^2 f(x) - \mu_x^2$

c table $\Rightarrow (1)^2 \times (0.5) + (5^2) \times (0.5)$

$\Rightarrow 13 - \mu_x^2$

$\Rightarrow 13 - 9$

$= 4$

$\boxed{\sigma_x = \sqrt{4} = 2}$

3+4

1.5

$V(y) = \sum y^2 g(y) - \mu_x^2.$

$-4 \times -4 = +16$, $4^2 = 16$

$\Rightarrow (-4^2)\left(\frac{3}{8}\right) + 2^2\left(\frac{3}{8}\right) + 7^2\left(\frac{1}{4}\right) - 1$

$= 18.75$

$\sigma_y = \sqrt{18.75} = 4.33$

$$\rho(xy) = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{-1.5}{(2)(4.33)}$$

$$= 0.1732$$

Since Co-variance of (x,y) is not equal to 0

$\Rightarrow \therefore x$ & y are not independent

LP17]

X : number of 4's
Y : " of 5's

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2), (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2)
(6,6)

no of 5's

no 4 but 5

$X \backslash Y$	0	1	2
0	16/36	8/36	1/36
1	6/36	2/36	0
2	1/36	0	0

2 ⟹ 2 5's

no. of 4's

2 ⟹ 2 4's

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(P18) LD $x = 0 \le x \le 2 \; \{0, 1, 2\}$

$y = 0 \le y \le 3 = \{0, 2, 3\}$

y

x \ y	y = 0	y = 1	y = 2	y = 3	
x = 0	0	k	2k	3k	6k
x = 1	k	2k	3k	4k	10k
3	2k	3k	4k	5k	14k
Sum	3k	6k	9k	12k	30k

$30k \Rightarrow \frac{1}{30} = 1$ Assume

$f(x,y) = k(x+y) \Leftarrow$ given.

[P16]

Red = 3
White = 5
Total = 8

$P(\text{red ball}) = \frac{3}{8}$

$X = \{1, 2, 3\}$
$Y = \{1, 2, 3\}$

$P(\text{white ball}) = \frac{5}{8}$

In question. Drawing 3 balls so only do

white balls.

no. of red balls.

X\Y	0	1	2	3	Sum
0	0	0	0	$P(0,3)$ $(\frac{5}{8})^3$	
1	0	0	$P(1,2)$ $3(\frac{3}{8})(\frac{5}{8})^2$	0	
2	0	$2+1=3$ $(\frac{3}{8})^2(\frac{5}{8})$	0	0	
3	$(\frac{3}{8})^3$	0	0	0	
Sum					

$\Rightarrow P(0,3) = \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} = \frac{125}{512}$

$\rightarrow$

$P(1,2) = RWW + WRW + WWR$

$= \cancel{\frac{3}{8}} +$

$= 3\left(\frac{3}{8}\right)\left(\frac{5}{8}\right)^2$

$P(2,1) \Rightarrow 2+1 = 3.$

because X is 2.

2 is red balls So $\left(\frac{3}{8}\right)^2 \left(\frac{5}{8}\right)^{1 \rightarrow Y=1}$

$\boxed{\text{Final ans do all (+)}}$