GARCH Models and VaR Backtesting Analysis

Comparing Component GARCH vs Standard GARCH for Risk Management

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Executive Summary

This analysis compares the performance of Component GARCH (CGARCH) and Standard GARCH models for Value-at-Risk (VaR) estimation using S&P 500 daily returns. The study demonstrates that both model structure and distributional assumptions are critical for reliable risk management.

Key Findings:

- Normal distribution assumptions lead to systematic underestimation of tail risk
- CGARCH with Student's t-distribution passes all statistical tests for VaR validity
- Standard GARCH fails to produce reliable risk estimates even with correct distributional assumptions

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1. Data and Methodology

Dataset

- **Source:** S&P 500 daily returns (2010-2025)
- **Total Observations:** 3,938
- **Training Set:** 2,756 observations (70%)
- **Testing Set:** 1,182 observations (30%)
- **VaR Level:** 99% ($\alpha = 0.01$)

Models Tested

1. **Component GARCH (CGARCH):** Allows for time-varying long-run volatility component
2. **Standard GARCH:** Traditional GARCH(1,1) specification
3. **Distributions:** Normal and Student's t-distribution

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2. The Failure of Normal Distribution Assumptions

Hypothesis

Both models will fail VaR backtests when assuming normal distributions due to thin tails that cannot capture extreme market events.

Results - Normal Distribution

Component GARCH (Normal)

Expected breaches: 11.8

Actual breaches: 27 (2.28%)

Kupiec Test (Unconditional Coverage):

- Test Statistic: 14.44

- p-value: 0.0001 ❌ FAIL

- Decision: Reject H0

Christoffersen Test (Conditional Coverage):

- Test Statistic: 14.65

- p-value: 0.0007 ❌ FAIL

- Decision: Reject H0

Standard GARCH (Normal)

Expected breaches: 11.8

Actual breaches: 28 (2.37%)

Kupiec Test (Unconditional Coverage):

- Test Statistic: 16.16

- p-value: 0.0001 ❌ FAIL

- Decision: Reject H0

Christoffersen Test (Conditional Coverage):

- Test Statistic: 16.32

- p-value: 0.0003 ❌ FAIL

- Decision: Reject H0

Interpretation

Both models catastrophically fail when assuming normal distributions, producing more than double the expected number of VaR breaches. This confirms that normal distributions have insufficient tail thickness for financial risk modeling.

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3. Definitive Analysis: Student's t-Distribution

Hypothesis

The superior theoretical structure of CGARCH, combined with appropriate fat-tailed distribution, will produce reliable VaR estimates while Standard GARCH may still exhibit flaws.

Results - Student's t-Distribution

Component GARCH (Student's t) ✅

Expected breaches: 11.8

Actual breaches: 17 (1.44%)

Kupiec Test (Unconditional Coverage):

- Test Statistic: 2.02

- p-value: 0.1553 ✅ PASS

- Decision: Fail to Reject H0

Christoffersen Test (Conditional Coverage):

- Test Statistic: 3.39

- p-value: 0.1833 ✅ PASS

- Decision: Fail to Reject H0

Standard GARCH (Student's t) ❌

Expected breaches: 11.8

Actual breaches: 20 (1.69%)

Kupiec Test (Unconditional Coverage):

- Test Statistic: 4.73

- p-value: 0.0296 ❌ FAIL

- Decision: Reject H0

Christoffersen Test (Conditional Coverage):

- Test Statistic: 5.62

- p-value: 0.0601 ⚠️ BORDERLINE PASS

- Decision: Fail to Reject H0

Statistical Interpretation

CGARCH Model Performance

- **Kupiec Test (p = 0.1553):** ✅ Produces statistically correct number of VaR breaches
- **Christoffersen Test (p = 0.1833):** ✅ Breaches are independent and not clustered
- **Conclusion:** Valid and reliable risk model suitable for practical use

Standard GARCH Performance

- **Kupiec Test (p = 0.0296):** ❌ Produces too many VaR breaches
- **Christoffersen Test (p = 0.0601):** ⚠️ Barely passes independence test
- **Conclusion:** Unreliable for risk management due to systematic bias

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4. Breach Clustering Analysis

CGARCH Model

Total breaches: 17

Average gap between breaches: 70.8 days

Minimum gap: 1 day

Consecutive breaches (gap = 1): 1

Breaches within 5 days: 1

Clustering percentage: 6.2%

Standard GARCH Model

Total breaches: 20

Average gap between breaches: 60.5 days

Minimum gap: 1 day

Consecutive breaches (gap = 1): 1

Breaches within 5 days: 1

Clustering percentage: 5.3%

Both models show minimal clustering, but CGARCH achieves this with the correct number of total breaches.

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5. Model Comparison Summary

Metric	CGARCH (t-dist)	Standard GARCH (t-dist)	CGARCH (Normal)	Standard GARCH (Normal)
Actual Breaches	17 (1.44%)	20 (1.69%)	27 (2.28%)	28 (2.37%)

Metric	CGARCH (t-dist)	Standard GARCH (t-dist)	CGARCH (Normal)	Standard GARCH (Normal)
Expected Breaches	11.8	11.8	11.8	11.8
Kupiec Test	✅ PASS (0.1553)	❌ FAIL (0.0296)	❌ FAIL (0.0001)	❌ FAIL (0.0001)
Christoffersen Test	✅ PASS (0.1833)	⚠️ PASS (0.0601)	❌ FAIL (0.0007)	❌ FAIL (0.0003)
Overall Assessment	**RELIABLE**	**UNRELIABLE**	**FAILED**	**FAILED**

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6. Long-Run Volatility Component Analysis

CGARCH q_t Component

The long-run variance component (q_t) from the CGARCH model provides economic intuition:

Average long-run variance (q_t): 1.26

Standard GARCH unconditional variance: 2.94

The q_t series correctly identifies:

- High-risk regimes following the 2020 crisis
- Gradual return to calmer market conditions
- Appropriate scale relative to unconditional variance

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7. Conclusions and Implications

Key Findings

1. **Distributional Assumptions Matter:** Normal distributions systematically underestimate tail risk, making them unsuitable for financial risk management.
2. **Model Structure is Critical:** Even with correct distributional assumptions, the Standard GARCH model fails to produce reliable VaR estimates due to its restrictive structure.
3. **CGARCH Superiority:** The Component GARCH model's ability to separate short-run volatility dynamics from long-run risk regimes enables superior risk forecasting.
4. **Practical Implications:** Only the CGARCH model with Student's t-distribution provides statistically valid and practically useful risk estimates.

Recommendations

For Risk Managers:

- Use Component GARCH models with fat-tailed distributions for VaR estimation
- Avoid normal distribution assumptions in risk models
- Implement proper statistical backtesting procedures

For Model Development:

- Focus on model structure improvements over distributional refinements alone
- Consider regime-switching capabilities in volatility modeling
- Validate models using both statistical tests and economic interpretation

Final Assessment

The analysis provides compelling evidence that successful risk modeling requires both appropriate model structure (CGARCH) and correct distributional assumptions (Student's t). The combination of these elements creates a model that is:

- **Statistically Valid:** Passes formal VaR backtests
- **Practically Useful:** Produces reliable risk estimates
- **Economically Interpretable:** Provides meaningful insights into market risk dynamics

This research demonstrates that theoretical sophistication in model design translates directly into practical improvements in risk management capability.